LEPTOGENESIS, NEUTRINO MIXING DATA AND THE ABSOLUTE NEUTRINO MASS SCALE\(^a\)

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Recent developments in thermal leptogenesis are reviewed. Neutrino mixing data favor a simple picture where the matter-anti matter asymmetry is generated by the decays of the heavy RH neutrinos *mildly* close to thermal equilibrium and, remarkably, in the full non relativistic regime. This results into predictions of the final baryon asymmetry not depending on the initial conditions and with minimized theoretical uncertainties. After a short outline of a geometrical derivation of the $CP$ asymmetry bound, we derive analytic bounds on the lightest RH neutrino mass and on the absolute neutrino mass scale. Neutrino masses larger than 0.1 eV are not compatible with the minimal leptogenesis scenario. We discuss how the results get just slightly modified within the minimal supersymmetric standard model. In particular a conservative lower bound on the reheating temperature, $T_R \gtrsim 10^9$ GeV, is obtained in the relevant effective neutrino mass range $\tilde{m}_1 \gtrsim 3 \times 10^{-3}$ eV. We also comment on the existence of a ‘too-short-blanket problem’ in connection with the possibility of evading the neutrino mass upper bound.

\(^a\)Compendium of \(^1\) and \(^2\) mostly based on \(^3\) with some new results in 3.4, 3.8 and 3.9.
1 Introduction

Cosmic rays and CMBR observations indicate that our observable Universe is baryon asymmetric. Moreover the observation of the acoustic peaks in the power spectrum of CMBR, combined with large scale structures observations, provide a precise and robust measurement of such an asymmetry that can be expressed in terms of the baryon to photon number ratio at the recombination time,

$$\eta^{CMB}_B = (6.3 \pm 0.3) \times 10^{-10},$$

in very good agreement with the latest determination from (NACRE updated) Standard BBN and primordial Deuterium measurements that give

$$\eta^{SBBN}_B = (6.1 \pm 0.5) \times 10^{-10}.$$

At the same time there is a growing evidence that an inflationary stage occurred during the early Universe. In this case this would have diluted any pre-existing initial asymmetry to a level many orders of magnitude below the measured value, thus requiring an explanation of the observed baryon asymmetry in terms of a dynamical generation, the aim of a model of baryogenesis that necessitates the accomplishment of the three famous Sakharov’s conditions: C and CP violation, B violation and departure from thermal equilibrium. Within the Standard Model all three conditions are fulfilled, yet the observed value is too large to be explained and therefore a successful model of baryogenesis requires some new physics ingredient. A host of models have been proposed since the first Sakharov idea. Some examples of typologies of baryogenesis models are: Planck scale baryogenesis, baryogenesis from phase transitions, Affleck-Dine models, baryogenesis from black holes evaporation, models of spontaneous baryogenesis.

Even though leptogenesis exhibit, from a particle physics point of view, substantial differences, they can be jointly regarded as two different examples belonging to the oldest class of models of baryogenesis from heavy particle decays. Such a classification privileges the thermodynamical aspect enlightening general properties that do not depend on the specific particle physics framework. We will thus discuss the kinetic theory of heavy particle decays in the first part, while in the second part we will see how leptogenesis is a specific remarkable example in which the new physics ingredient is provided by the seesaw mechanism and such that the observed baryon asymmetry is nicely related to neutrino mixing data.

2 Baryogenesis from heavy particle decays

2.1 Out-of-equilibrium decays

Let us consider a self-conjugate heavy ($M_X \gg M_{EW}$) particle $X$ whose decays are CP asymmetric, in such a way that the decaying rate into particles, $\Gamma$, is in general different from the decaying rate into anti-particles, $\bar{\Gamma}$, and such that the single decay process into
particles (anti-particles) violate $B - L$ by a quantity $\Delta_{B-L} (-\Delta_{B-L})$. The CP asymmetry parameter is then conveniently defined as

$$\varepsilon = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}.$$ \hfill (3)

For a joint discussion of baryogenesis ($\Delta_{B-L} > 0$) and leptogenesis ($\Delta_{B-L} < 0$) models, it is useful to introduce the quantity $\bar{\varepsilon} = \Delta_{B-L} \varepsilon$. The total decay rate $\Gamma_D = \Gamma + \bar{\Gamma}$ is the product of the total decay width, $\Gamma_D^{\text{rest}}$, times the averaged dilation factor $\langle 1/\gamma \rangle$

$$\Gamma_D = \Gamma_D^{\text{rest}} \langle \frac{1}{\gamma} \rangle.$$ \hfill (4)

Sphaleron processes, while inter-converting $B$ and $L$ separately, leave $B - L$ unchanged\textsuperscript{13} and for this reason the kinetic equations get much simpler if the $B - L$ evolution is tracked instead of the separate $B$ or $L$ evolution. Moreover it is convenient to use, as an independent variable, the quantity $z = M_X/T$ and to introduce the decay factor $D = \Gamma_D/(Hz)$. Another useful choice is to track the number of $X$ particles, $N_X$, and the amount of the asymmetry, $N_{B-L}$, in a portion of comoving volume $R^3$ normalized in such a way to contain, averagely in ultra-relativistic thermal equilibrium, just one $X$ particle (i.e. $N_X^\text{eq}(z \ll 1) = 1$).

The simplest case is when the $X$ life-time, $\tau = 1/\Gamma_D^{\text{rest}}$, is much longer than the age of the Universe, $t_U = (2H)^{-1}$, at $z = 1$, when the $X$ particles become non relativistic. In this way decays will occur when the temperature is much below the $X$ mass and the $X$-production from inverse decays, or other possible processes, is Boltzmann suppressed. In this situation decays are the only relevant processes and the kinetic equations for the $X$-abundance and the $B - L$ asymmetry are particularly simple to be written,

$$\frac{dN_X}{dz} = -D(z) N_X(z)$$ \hfill (5)

$$\frac{dN_{B-L}}{dz} = -\bar{\varepsilon} \frac{dN_X}{dz},$$ \hfill (6)

and solved,

$$N_{B-L}(z) = N_{B-L}^i + \bar{\varepsilon} \left[ N_X^i - N_X(z) \right]$$ \hfill (7)

$$N_X(z) = N_X e^{-\int_{z_i}^{z} dz' D(z')}.$$ \hfill (8)

The solutions can be fully described just in terms of the decay parameter

$$K = \frac{\Gamma_D^{\text{rest}}}{H|_{z=1}},$$ \hfill (9)

in terms of which $D = K z \langle 1/\gamma \rangle$. The dilation factor, averaged on the Boltzmann statistics, is simply approximated by\textsuperscript{3}

$$\langle \frac{1}{\gamma} \rangle \simeq \frac{z}{z + 15/8},$$ \hfill (10)
a useful simple expression that makes possible to solve analytically the integral in the Eq. (8), yielding the result

\[ N_X(z) \simeq N^i_X e^{-K \left[ \frac{z^2}{2} - \frac{15z}{8} + \left( \frac{15}{8} \right)^2 \ln \left( 1 + \frac{8}{15} z \right) \right]} \] .

(11)

In particular the final $B - L$ asymmetry is given by

\[ N^f_{B-L} = N^i_{B-L} + \bar{\varepsilon} N^i_X. \]

(12)

The baryon to photon ratio at recombination can then be obtained dividing by the number of photons at recombination $N^\gamma_{\text{rec}}$ (about thirty times the number of photons at the onset of $X$ decays) and taking into account that sphalerons will convert only a fraction $a_{\text{sph}} \simeq 1/3$ of the $B - L$ asymmetry into a baryon asymmetry. In this way one can write:

\[ \eta_B \simeq \frac{1}{3} \frac{N^f_{B-L}}{N^\gamma_{\text{rec}}}. \]

(13)

It is useful to introduce the efficiency factor defined as the ratio of the asymmetry produced from the $X$ decays, excluding the contribution from a possible initial quantity, to the $CP$
asymmetry, i.e.

\[ \kappa(z) \equiv \frac{N_{B-L}(z)|_{N_{B-L}^i=0}}{\bar{\varepsilon}}. \]  

(14)

In the case of out of equilibrium decays one has \( \kappa(z) = N_X^i - N_X(z) \) and the Eq. (11) can be re-casted as

\[ N_{B-L}(z) = N_{B-L}^i + \varepsilon \kappa(z). \]  

(15)

This definition is such that the final efficiency factor, \( \kappa_f \equiv \kappa(\infty) = N_X^i \), is equal to unity in the case of an initial thermal abundance with \( z_i \ll 1 \). In Fig. 1 we show two examples of out of equilibrium decays, for \( K = 10^{-2} \) and \( K = 10^{-4} \), assuming an initial thermal X abundance (\( N_X^i = 1 \)) and zero initial asymmetry (\( N_{B-L}^i = 0 \)). The numerical results are compared with the analytic expression (cf. (11)).

The out-of-equilibrium picture is an efficient way to produce an asymmetry from decays. However it relies on the possibility that an initial X abundance was thermalized by some unspecified mechanism at \( T \gtrsim M_X \) and that one can neglect a possible \( N_{B-L}^i \) generated during or after inflation and before the onset of X decays. Therefore, it is evident that this picture is plagued by a strong sensitivity to the initial conditions and hence it requires to be complemented with a model able to specify them, for example a detailed description of the inflationary stage.

### 2.2 Inverse decays

The out-of-equilibrium picture is strictly valid only in the limit \( K \to 0 \). If one defines \( z_d \) as the value \( M_X/T_D \) such that the X life time coincides with the age of the Universe \( (\tau = t_U(z_D)) \) then, for \( K \ll 1 \), one has \( z_d \approx \sqrt{2/K} \). Thus for \( K \gtrsim 1 \) the X’s will decay for \( T_d = M_X/z_d \gtrsim M_X/\sqrt{2} \) and the inverse decays have to be taken into account. The kinetic equations (5) and (6) are then generalized in the following way:

\[
\frac{dN_X}{dz} = -D(z)N_X(z) + D(z)N_X^{eq}(z) \tag{16}
\]

\[
\frac{dN_{B-L}}{dz} = -\varepsilon \frac{dN_X}{dz} - W_{ID}(z)N_{B-L}(z). \tag{17}
\]

In the equation for \( N_X \) the second term accounts for the inverse decays that, remarkably, can now produce the X’s. On the other hand one can see that a new term appears in the second equation for the asymmetry too, a wash-out term that tends to destroy what is generated from the decays. This term is controlled by the (inverse decays) wash-out factor given by

\[
W_{ID} = \frac{m}{2} D \frac{N_X^{eq}}{N_{b,l}^{eq}} \propto K, \tag{18}
\]

\[ ^{b}\text{The equations (16) and (17) are actually not only accounting for decays and inverse decays but also for the real intermediate state contribution from } 2 \leftrightarrow 2 \text{ scattering processes. This term exactly cancels a } CP \text{ non conserving term from inverse decays that would otherwise lead to an un-physical asymmetry generation in thermal equilibrium}^{13} \]
where \( m \) is the number of baryons or leptons in the \( X \) decay final state (\( m = 1 \) in the case of leptogenesis). Note that the decay parameter \( K \) is still the only parameter in the equations and thus the solutions will still depend only on \( K \). They can be again worked out in an integral form\(^{12}\). In the case of the \( B - L \) asymmetry one can write the final asymmetry as

\[
N_{B-L}^f = N_{B-L}^i e^{- \int_{z_i}^{\infty} dz' W_{ID}(z')} + \tilde{\varepsilon} \kappa_f, \tag{19}
\]

where now the efficiency factor is given by the integral

\[
\kappa_f(K, z_i) = - \int_{z_i}^{\infty} dz' \left[ \frac{dN_X}{dz'} \right] e^{- \int_{z_i}^{\infty} dz'' W_{ID}(z'')} . \tag{20}
\]

In the limit \( K \to 0 \) the out-of-equilibrium case is recovered. In general one can see that the wash-out has the positive effect to damp a pre-existing asymmetry but also the negative one to damp the same asymmetry generated from decays, thus reducing the efficiency of the mechanism. A quantitative analysis is crucial and it is very useful to discuss separately the regime of strong wash out for \( K \gtrsim 1 \) and the regime of weak wash-out for \( K \lesssim 1 \).

### 2.3 Strong wash-out regime

The strong wash-out regime is characterized by the existence, for \( K \gtrsim 3 \), of an interval \([z_{in}, z_{out}]\) such that \( W_{ID} \gtrsim 1 \) and thus such that inverse decays are in equilibrium. Practically all the asymmetry produced at \( z > z_{out} \) is washed-out including, remarkably, an initial one. Moreover the calculation of the residual asymmetry is made very simple by the possibility to use the *close equilibrium approximation* given by

\[
\frac{dN_X}{dz'} \approx \frac{dN_X^{eq}}{dz} = - \frac{2}{K z} W_{ID}(z) . \tag{21}
\]

In this way the integral in the Eq. (20) can be easily evaluated\(^{12,23}\). Indeed, this can be regarded as a Laplace integral, that means an integral of the form

\[
\int_0^{\infty} dz' e^{-\psi(z', z)}, \tag{22}
\]

that receives a dominant contribution only from a small interval centered around a special value \( z_B \) such that \( d\psi/dz = 0 \). In this way one can use the approximation of replacing \( W_{ID}(z'') \) with \( W_{ID}(z_B)/z'' \) in the Eq. (20). With this approximation and assuming \( z_i \ll 1 \) the integral can be easily solved obtaining

\[
k_f \approx \frac{2}{m K z_B} \left( 1 - e^{-\frac{m K z_B}{2}} \right) . \tag{23}
\]

For large \( K \gg 1 \) and for \( m = 2 \) this expression coincides with that one found in\(^{12,23}\). The calculation of the important quantity \( z_B \) proceeds from its definition, \((d\psi/dz)_{z_B} = 0\), approximately equivalent to the equation

\[
W_{ID}(z_B) = \left( \frac{1}{\gamma} \right)^{-1} (z_B) - \frac{3}{z_B}. \tag{24}
\]

\(^c\)Note however that the definition \(^9\) for \( K \) has to be used instead of \( K = (1/2)(\Gamma_D/H)_{z=1} \).
This is a transcendental algebraic equation and thus one cannot find an exact analytic solution (see [3] for an approximate procedure). However the expression

$$z_B(K) \simeq 1 + 4.5 (m K)^{0.13} e^{-\frac{5}{2} m K},$$

provides quite a good fit that can be plugged into the Eq. (23) thus getting an analytic expression for the efficiency factor. At vary large $K$ this behaves as a power law $\kappa_f \propto K^{-1.13}$.

In Fig. 2 we compare the analytic solution for $\kappa_f$ (cf. (23)) with the numerical solution (for $m = 1$). One can see how for $K \gtrsim 4$ the agreement is quite good. Note that the Eq. (24) implies that for large values of $K$ one has $z_B \simeq z_{\text{out}}$, that particular value of $z$ corresponding to the last moment when inverse decays are in equilibrium ($W_{1D} \geq 1$). In this way almost all the asymmetry produced for $z \lesssim z_B$ is washed-out and most of the surviving asymmetry is produced in the period just around the inverse decays freeze out, simply because the $X$ abundance gets rapidly Boltzmann suppressed. An example of this picture is illustrated in Fig. 3 for $K = 100$ (from [3]). Instead of the abundance we plotted the deviation from the equilibrium value, the quantity $\Delta = N_X - N_X^{\text{eq}}$. The deviation grows until the $X$'s decay.
at $z \simeq z_d$, when it reaches a maximum, and decrease afterwards when the abundance stays close to thermal equilibrium. Correspondingly the asymmetry grows for $z \lesssim z_d$, reaching a maximum around $z \simeq 1$, and then it is washed-out until it freezes at $z_B \simeq z_{\text{out}}$. The evolution of the asymmetry $N_{B-L}(z)$ can induce the wrong impression that the residual asymmetry is some fraction of what was generated at $z \simeq 1$ and that one cannot relax the assumption $z_i \ll 1$ without reducing considerably the final value of the asymmetry. Actually what is produced is also very quickly destroyed. A plot of the quantity $\psi(z, \infty)$, as defined in the Eq. (22) and shown in Fig. 4 (from 3), enlightens some interesting aspects. This is the final asymmetry that was produced in an infinitesimal interval around $z$. It is evident how just the asymmetry that was produced around $z_B$ survives and, for this reason, the temperature $T_B = M_X / z_B$ can be rightly identified as the \textit{temperature of baryogenesis} for these models. It also means that in the strong wash out regime the final asymmetry was produced when the $X$ particles were fully non relativistic implying that the simple kinetic equations (16) and (17), employing the Boltzmann approximation, give actually accurate results and corrections from use of the exact quantum statistics can be safely neglected.

This is not the only nice feature of the strong wash-out regime. Since any asymmetry generated for $z \lesssim z_B$ gets washed-out, one can also rightly neglect any pre-existing initial asymmetry $N_{B-L}^i$. At the same time the final asymmetry does not depend on the initial $X$ abundance. In Fig. 5 we show how even starting from a zero abundance, the $X$’s are
rapidly produced by inverse decays in a way that well before $z_B$ the number of decaying neutrinos is always equal to the thermal number. The final asymmetry does not even depend on the initial temperature as far as this is higher than $\sim T_B$ and thus if one relaxes the assumption $z_i \ll 1$ to $z_i \lesssim z_B - \Delta z_B$, the final efficiency factor gets just slightly reduced (for example for $\Delta z_B \simeq 2$ this is reduced approximately by 10%).

Summarizing we can say that in the strong wash out regime the reduced efficiency is compensated by the remarkable fact that, for $T_i \gtrsim T_B$, the final asymmetry does not depend on the initial conditions and all non relativistic approximations work very well. These conclusions change quite drastically in the weak wash-out regime.

2.4 Weak wash-out regime

For $K \lesssim 1$ one can see that $z_B$ rapidly tends to unity (cf. (25)). In Fig. 2 the analytic solution for the efficiency factor, Eq. (23), is compared with the numerical solution. It can appear surprising that, in the case of an initial thermal abundance, the agreement is excellent not only at large $K \gtrsim 4$, but also at small $K \lesssim 0.4$, with some appreciable deviation just around $0.4 \lesssim K \lesssim 4$. The reason is that when the wash-out processes get frozen, the efficiency factor depends only on the initial number of neutrinos and not on its derivative and thus the approximation Eq. (21) introduces a sensible error only in the transition regime $K \sim 1$.

The Eq. (23) can be easily generalized to any value of the initial abundance until one can neglect the $X$’s produced by inverse decays. More generally, one has to calculate such a contribution and it is convenient to consider the limit case of a zero initial $X$ abundance.
Figure 5: Fast thermalization of the X abundance in the strong wash-out regime. The final \( N_{B-L} \) abundance (for \( \varepsilon_1 = 0.75 \times 10^{-6} \)) is the same as in the case of an initial thermal abundance (cf. Fig. 3) and it is independent on the evolution at \( z \ll z_B \).

The X production lasts until \( z = z_{\text{eq}} \), when the abundance is equal to the equilibrium value, such that

\[
N_X(z_{\text{eq}}) = N_X^{\text{eq}}(z_{\text{eq}}).
\]  

(26)

At this time the number of decays equals the number of inverse decays. For \( z \leq z_{\text{eq}} \) decays can be neglected and the Eq. (16) becomes

\[
\frac{dN_X}{dz} = D(z) N_X^{\text{eq}}(z).
\]  

(27)

For \( z \ll 1 \) one then simply finds

\[
N_X(z) = \frac{K}{6} z^3.
\]  

(28)

In the weak wash out regime the equilibrium is reached very late, when neutrinos are already non relativistic and \( z_{\text{eq}} \gg 1 \). In this way one can see that the number of \( N_X \) reaches, at \( z \approx z_{\text{eq}} \), a maximum value given by

\[
N_X(z_{\text{eq}}) \approx N(K) \equiv \frac{3\pi}{4} K.
\]  

(29)

It is possible to interpolate between the two asymptotical regimes getting a global solution for any \( z \leq z_{\text{eq}} \). For \( z > z_{\text{eq}} \) inverse decays can be neglected and the X’s decay out of equilibrium in a way that

\[
N_X(z > z_{\text{eq}}) \approx N_X(z_{\text{eq}}) e^{-\int_{z_{\text{eq}}}^z dz' D(z')}.
\]  

(30)
Let us now consider the evolution of the asymmetry calculating the efficiency factor. Its value can be conveniently decomposed as the sum of two contributions, a negative one, $\kappa^{-}$, generated at $z < z_{eq}$, and a positive one, $\kappa^{+}$, generated at $z > z_{eq}$. In the limit of zero wash-out we know that the final efficiency factor must vanish, since we are assuming an initial zero abundance. This implies that the negative and the positive contributions have to cancel each other. The effect of wash-out is to suppress the negative contribution more than the positive one, in a way that the cancellation is only partial. In the weak wash-out regime it is possible in first approximation to neglect completely the wash-out at $z \geq z_{eq}$. In this way it is easy to derive from the Eq. (20) the following expression for the final efficiency factor:

$$\kappa_f \simeq N(K) - 2 \left(1 - e^{-\frac{1}{2} N(K)}\right).$$  \hfill (31)

One can see how it vanishes at the first order in $N(K) \propto K$ and only at the second order one gets $k_f \simeq \left(9 \pi^2 / 64\right) K^2$.

### 2.5 Final efficiency factor: summary

Generalizing the procedure seen for the strong wash-out it is possible to find a global solution for $\kappa_f(K)$ valid for any $K$. The calculation proceeds separately for $\kappa^{-}$ and $\kappa^{+}$ and the final results are given by

$$\kappa^{-}_f(K) = -2 e^{-\frac{1}{2} N(K)} \left(e^{\frac{1}{2} N(K)} - 1\right)$$  \hfill (32)

and

$$\kappa^{+}_f(K) = \frac{2}{z_B(K) m K} \left(1 - e^{-\frac{1}{2} z_B(K) m K N(K)}\right).$$  \hfill (33)

The function $\overline{N}(K)$ extends, approximately, the definition of $N(K)$ to any value of $K$

$$\overline{N}(K) = \frac{N(K)}{\left(1 + \sqrt{\frac{N(K)}{N_{eq}}}\right)^2}.\hfill (34)$$

The sum of the Eq.’s (33) and (32) is plotted, for $m = 1$, in Fig. 2 (short-dashed line) and compared with the numerical solution (solid line).

We can now outline some conclusions about a comparison between the weak and the strong wash-out regimes. A large efficiency in the weak wash-out regime relies on some unspecified mechanism that should have produced a large (thermal or non thermal) $X$ abundance before their decays. On the other hand the decrease of the efficiency at large $K$ in the strong wash-out regime is only (approximately) linear and not exponential. This means that for moderately large values of $K$ a small loss in the efficiency would be compensated by a full thermal description such that the predicted asymmetry does not depend on the initial conditions, a nice situation that resembles closely the Standard Big Bang Nucleosynthesis scenario for the calculation of the primordial nuclear abundances.
3 Leptogenesis

Let us see now how the results that hold for generic baryogenesis models from heavy particle decays get specialized in the case of leptogenesis. This is the cosmological consequence of the seesaw mechanism, explaining the lightness of the ordinary neutrinos through the existence of three new heavy RH neutrinos \( N_1, N_2, N_3 \) with masses respectively \( M_1 \leq M_2 \leq M_3 \), much larger than the electroweak scale. The simple seesaw formula,

\[
m_\nu = -m_D \frac{1}{M} m_D^T ,
\]

relates the neutrino mixing matrix \( m_\nu \) to the RH neutrino mass matrix \( M \) and to the Dirac neutrino mass matrix \( m_D = h v \) generated by the Yukawa coupling matrix \( h \), where \( v \) is the Higgs vacuum expectation value. Both light and heavy neutrinos are predicted to be Majorana neutrinos. All mass matrices are in general complex and this provides a natural source for the \( CP \) asymmetry while the new RH neutrinos are the natural candidates to play the role of the \( X \) particles. In this case things are apparently more complicated since there are three of them. We will assume that the decays and inverse decays of the two heavier neutrino decays do not influence the value of the final asymmetry. This assumption holds for example either if the asymmetry produced by the two heavier RH neutrinos is negligible or if this is produced and then washed out by the inverse decays of the lightest (heavy RH). In this way we can straightforwardly apply the general picture of baryogenesis from \( X \) decays to leptogenesis, with the \( N_1 \)'s playing the role of the \( X \) particles.

3.1 Decay parameter and neutrino masses

The total \( N_1 \) decay width is given by

\[
\Gamma_D = \frac{\tilde{m}_1 M_1^2}{8 \pi v^2} ,
\]

where the effective neutrino mass is defined as

\[
\tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} .
\]

It is then easy to see that the decay parameter is related to \( \tilde{m}_1 \) by the following relation

\[
K = \frac{\tilde{m}_1}{m_\star} ,
\]

where the equilibrium neutrino mass \( m_\star \) can be written as

\[
m_\star = \frac{v^2}{M_\star} \simeq 10^{-3} \text{ eV} ,
\]

with the quantity \( M_\star \) given by

\[
M_\star = \frac{3 \sqrt{5}}{16 \pi^{5/2} \sqrt{g_*}} \simeq 3 \times 10^{16} \text{ GeV} .
\]
It is quite non trivial that the value of $m_\star$ is close to the neutrino mixing mass scales and we will show soon the relevance of this result. For the moment note that the value of $m_\star$ is independent on the well known success of the seesaw mechanism in explaining the atmospheric and solar neutrino mass scales and this is why we wrote $m_\star$ in a sort of seesaw-like form, introducing the scale $M_\star$. Apart from the very general consideration that the logarithm of $M_\star$ is expected to be close to the Planck scale, this is not related to the grand unified scale, rather to the expansion rate at the baryogenesis time$^d$.

Let us now assume that the simple decays plus inverse decays picture studied in the previous section is a good approximation of leptogenesis. It is then crucial to determine the value of the the effective neutrino mass $\tilde{m}_1$, and thus, from the Eq. (38), the value of the decay parameter $K$, in order to answer the important question whether leptogenesis lies in the strong or in the weak wash-out regime.

It is always possible to work in a basis in which the heavy neutrino mass matrix is diagonal, such that $M = \text{diag}(M_1, M_2, M_3) \equiv D_M$. Moreover one can also simultaneously diagonalize the light neutrino mass matrix $m_\nu$ by mean of the unitary MNS matrix $U$, such that

$$U^\dagger m_\nu U^* = -D_m.$$  

In this way the seesaw formula gets specialized in the following way:

$$D_m = U^\dagger m_D D^{-1}_m m_D^T U^*.$$  

This expression can be also re-casted as an orthogonality condition,

$$\Omega \Omega^T = \Omega^T \Omega = I,$$  

for the $\Omega$ matrix defined as

$$\Omega = D_m^{-1/2} U^\dagger m_D D^{-1/2}_M$$  

and whose matrix elements are then simply given by

$$\Omega_{ij} = \frac{v \tilde{h}_{ij}}{\sqrt{m_i M_j}},$$

where $\tilde{h} = U^\dagger h$. The $\Omega$ matrix is fully determined by three complex parameters. Four of them are needed to fix the three first column entries $\Omega_{11}$, $\Omega_{21}$ and $\Omega_{31}$, particularly important for leptogenesis. This because if one inverts the relation, in a way to get an expression of $m_D$ in terms of $\Omega$, and plugs it into the effective neutrino mass definition (cf. (37)), then one easily gets

$$\tilde{m}_1 = m_1 |\Omega^2_{11}| + m_2 |\Omega^2_{21}| + m_3 |\Omega^2_{31}|.$$  

$^d$It is then quite curious that the value of $M_\star$ is just the value of the supersymmetric unification scale.
From the orthogonality of $\Omega$ it follows that $\tilde{m}_1 \geq m_1$. This is the only fully model independent restriction on $\tilde{m}_1$. For configurations such that

$$\sum_j |\Omega_{j1}^2| \sim \sum_j |\Omega_{j1}^2| = 1$$

one has $\tilde{m}_1 \lesssim m_3$. Models with $\tilde{m}_1 \gg m_3$ rely on the possibility of strong phase cancellations.

Neutrino mixing data provide two important pieces of information on the neutrino mass spectrum. In the case of normal hierarchy one has $m_3^2 - m_2^2 = \Delta m_{atm}^2$ and $m_2^2 - m_1^2 = \Delta m_{sol}^2$. In the case of inverted hierarchy $m_3^2 - m_2^2 = \Delta m_{sol}^2$ and $m_2^2 - m_1^2 = \Delta m_{atm}^2$. The third, still undetermined, independent information, the absolute neutrino mass scale, can be conveniently expressed in terms of the lightest neutrino mass $m_1$. The two heavier neutrino masses are then given, for normal (inverted) hierarchy, by

$$m_3^2 = m_1^2 + m_{atm}^2,$$

$$m_2^2 = m_1^2 + \Delta m_{sol}^2 (\Delta m_{atm}),$$

where we defined $m_{atm} = \sqrt{\Delta m_{atm}^2 + \Delta m_{sol}^2}$. The latest measurements give

$$\Delta m_{atm}^2 = (2.6 \pm 0.4) \times 10^{-3} \text{eV}^2,$$

and for solar neutrinos

$$\Delta m_{sol}^2 \simeq (7.1^{+1.2}_{-0.6} \times 10^{-5}) \text{eV}^2,$$

from which it follows that

$$m_{atm} = (0.051 \pm 0.004) \text{eV}. $$

These relations imply that for $m_1^2 \gg m_{atm}^2$ neutrinos are quasi-degenerate ($m_3 \simeq m_2 \simeq m_1$), whereas for $m_1^2 \ll m_{atm}^2$ they are hierarchical ($m_1 \ll m_2, m_3$).

For fully hierarchical neutrinos ($m_1 = 0$) there is practically no restriction on $\tilde{m}_1$. However the case $\tilde{m}_1 \ll m_2, m_3$ requires $|\Omega_{21}^2| << 1$ and $|\Omega_{31}^2| << m_2/m_3$. This situation cannot be excluded but, because of the observed large mixing angles in the mixing matrix $U$, it relies on a fine tuning between the $U$ and $m_D$ matrix elements (cf. (44)), such that the off-diagonal terms are very small. This qualitative and general argument is supported by different investigations on specific models or classes of models for which typically one finds $m_{sol} \simeq m_2 \lesssim \tilde{m}_1 \lesssim m_3 \simeq m_{atm}$. Therefore, in the case of normal hierarchy one has that the favored range for the $\tilde{m}_1$ value is given by $O(m_{sol}) \leq \tilde{m}_1 \leq O(m_{atm})$, that in terms of the decay parameter (cf. (38)) gets translated into the range

$$O(K_{sol} \simeq 7) < K < O(K_{atm} \simeq 50),$$

while for inverted hierarchy the situation is even simpler since $\tilde{m}_1 = O(m_{atm})$ and $K = O(K_{atm} \simeq 50)$. One thus arrives to the interesting conclusion that neutrino mixing data
favor leptogenesis to lie in a *mildly strong wash out regime*, strong enough to benefit from the advantages we discussed, independence on the initial conditions plus minimal theoretical uncertainties, but not too much to result in an untenable efficiency loss. This conclusion derives because both the two independent experimental quantities, $m_{\text{sol}}$ and $m_{\text{atm}}$, are about ten times $m_\ast$ and so now one can better appreciate the nice matching of the theoretical quantity $m_\ast$ with the experimental data\(^{\text{c}}\). In the range $K_{\text{sol}} \lesssim K \lesssim K_{\text{atm}}$ a good fit of the final efficiency factor (cf. Eq. (23)) is given by the power law

$$\kappa_f = \frac{0.5}{K^{1.2}} \simeq 3 \times 10^{-2} \left( \frac{10^{-2} \text{eV}}{\bar{m}_1} \right)^{1.2},$$

shown in Fig. 2 (dot-dashed line). These conclusions hold under the assumption that leptogenesis is well approximated by the simple decays plus inverse decays picture and we have now to verify whether they are drastically modified or just corrected by the account of $N_1$ scatterings and $\Delta L = 2$ processes.

### 3.2 Scatterings

The $N_1$’s can also be destroyed or produced in $\Delta L = 1$ scatterings involving the top quark. These are mediated by the Higgs and can occur in the s channel, like $N_1 + l \leftrightarrow t + q$, or in the t channel, like $N_1 + t \leftrightarrow l + q$. The account of these processes modify the kinetic equations (16) and (17) in the following way:

$$\frac{dN_{N_1}}{dz} = -(D + S) \left( N_{N_1} - N_{N_1}^{\text{eq}} \right), \quad (55)$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon_1 D \left( N_{N_1} - N_{N_1}^{\text{eq}} \right) - (W_{ID} + W_{\Delta L = 1} + 1) N_{B-L}. \quad (56)$$

Note that scatterings have two effects: they contribute both to the neutrino production (the $S$ function) and to the wash-out (the $W_{\Delta L = 1}$ function).

The first one is important in the weak wash-out regime. As one can see from the Eq.’s (55) and (56), the production of the $N_1$’s from the $S$ function is not associated to a production of the asymmetry, simply because these processes do not violate CP. In Fig. 5 (from\(^{\text{b}}\)) we show an example of $N_1$ production for $\bar{m}_1 = 10^{-5}$ eV ($K \simeq 0.01$), comparing the case when scatterings are included with the case when they are neglected. It can be seen how at $z = z_{\text{eq}}$ the number of neutrinos is approximately doubled while the final asymmetry is two orders of magnitude larger. The reason is that the neutrino production from the scatterings is not associated to a production of a negative asymmetry. On the other hand all produced neutrinos yield a positive contribution when they decay. The expression (31) for the final efficiency factor in the weak wash-out regime gets thus modified in the following way at the first order in $K$

$$\kappa_f \simeq \left. \frac{N_{N_1} D}{D + S} \right|_{z = z_{\text{eq}}} - N(K) \propto K. \quad (57)$$

\(^{\text{c}}\)Note that this is also a consequence of the recent exclusion of the low solution in the solar neutrino data, that would have implied $K_{\text{sol}} \ll 1$. 

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[^c]: Note that this is also a consequence of the recent exclusion of the low solution in the solar neutrino data, that would have implied $K_{\text{sol}} \ll 1$. 

If scatterings are switched off the negative and the positive contribution cancel at the first order like we saw already. If $S \neq 0$ the positive term is enhanced while the negative one remains unchanged and in this way the sum does not vanish any more. Hence this effect makes more efficient the asymmetry production at small $K$, without having to assume an initial thermal abundance. There is however a drawback. The final result is quite sensitive to the theoretical assumptions. The scattering cross section depends on the ratio, $M_h/M_1$, of the Higgs mass to the RH neutrino mass. The case depicted in Fig. 5 is for $M_h/M_1 = 10^{-5}$. For smaller values of this ratio the result does not change much. However it has been recently pointed out\textsuperscript{27} that the Higgs mass is better described by its thermal mass such that $M_h/M_1 \simeq 0.4/z$. The relevant values of $z$ for neutrino production are $z \simeq 1$ and so the ratio $M_h/M_1 \simeq 0.4$. Such an high value has the effect to suppress heavily the $S$ term and the suppression is made even stronger by the account of the running of the top Yukawa coupling at high temperature. In this way the simple decays plus inverse decays picture is practically recovered. On the other hand in\textsuperscript{28,27} it has been noticed how scatterings involving gauge bosons should also be included. These scatterings yield an additional contribution to the $S$ function such that the final result is between a situation where scatterings are neglected and one where scatterings involving top quark and small $M_h/M_1$ are taken into account. The conclusion is that in the weak-wash out regime the theoretical uncertainties are such that

Figure 6: comparison between the case when scatterings are included (thick lines) with the decays plus inverse decays picture (solid thin lines) for $\tilde{m}_1 = 10^{-5}$ eV.
it seems that any result between the simple decays plus inverse decays picture, for which \( \kappa_f \propto K^2 \) or a behavior \( \kappa_f \propto K \) (cf. Eq. (57) ) cannot be firmly excluded at the moment. These large theoretical uncertainties, represented in Fig. 6 with the short-dashed region, are in addition to the model dependence in the description of the initial conditions.

In the strong wash-out regime all difficulties get considerably reduced. The theoretical uncertainties in the description od scatterings can change the final efficiency factor no more than \((20 \div 30)\%\) and this is clearly shown in Fig. 6, where at large \( K \) the (thin solid line) range shrinks considerably compared to the (short-dashed line) range at small \( K \). This because the thermal abundance limit is saturated at \( z_{eq} \ll 1 \) anyway and therefore the number of decaying neutrinos does not depend on the \( S \) function. A residual source of uncertainty is still present because of the scattering contribution, \( W_{\Delta L=1} \), to the wash-out. The effect of this term is however small, for the simple reason that in the strong wash out regime the surviving asymmetry is produced sharply around \( z_B \gg 1 \) and at such low temperatures inverse decays are dominant compared to scatterings. The conclusion is that in the strong wash-out regime the simple decays plus inverse decays picture does not get modified by scatterings within the theoretical uncertainties. It has been also pointed out that an accurate description of the dynamics of sphalerons in converting the lepton number into a baryon number is expected to lead to a suppression of the final asymmetry of a \( \mathcal{O}(1) \) factor and since this is currently neglected it gives an additional contribution to the theoretical uncertainties. Taking into account all these effects, an expression for the final efficiency factor in the strong wash out regime that accounts for the theoretical uncertainties

---

**Figure 7:** efficiency factor when scatterings are included.
is given by the power law

\[ \kappa_t = (2 \pm 1) \times 10^{-2} \left( \frac{10^{-2} \text{eV}}{m_1} \right)^{1.1 \pm 0.1}. \]  \hspace{1cm} (58)

The central value corresponds to the curve represented in Fig. 5 with circles (more precisely this is obtained for a power law \( \tilde{m}_1^{-1.13} \)), while the range that is spanned by the error, corresponds approximately to the thin solid line area. The upper values of this range is the power law \( \tilde{m}_1 \), that well describes the simple decays plus inverse decays picture where the wash-out from scatterings is neglected.

### 3.3 \( \Delta L = 2 \) processes

There is another important contribution to the wash-out term arising from the \( \Delta L = 2 \) processes like \( lH \leftrightarrow \bar{l}H \) and mediated by the RH neutrinos. In the non relativistic regime this contribution tends simply to

\[ \Delta W(z \gg 1) \simeq \frac{\omega}{z^2} \left( \frac{M_1}{10^{10} \text{GeV}} \right) \left( \frac{\tilde{m}}{\text{eV}} \right)^2, \]  \hspace{1cm} (59)

with \( \tilde{m}^2 = m_1^2 + m_2^2 + m_3^2 \), and dominates on the other Boltzmann suppressed wash-out terms arising from inverse decays and scatterings. A well known problem is that at temperatures \( T \sim M_1 \) one has to be sure that the cross section of \( \Delta L = 2 \) processes does not double count the on-shell contribution already accounted by inverse decays followed by decays to a final state with opposite lepton number (i.e. \( l + \bar{H} \rightarrow N_i \rightarrow \bar{l} + H \)). In \[^{17}\] it has been found that the subtraction procedure usually employed in the previous literature gives arise to a washout \( \Delta L = 2 \) term that is very well approximated by the asymptotical non-relativistic limit Eq. (59) plus a term that is just half the washout from inverse decays. In \[^{27}\] this second term has been shown to be spurious and to disappear when a proper subtraction procedure is employed. This result has been confirmed in \[^{18}\] Therefore the effect of the \( \Delta L = 2 \) processes is entirely well approximated by its non-relativistic limit. It is easy to see that for \( M_1 < 10^{14} \text{GeV} \left( 0.05 \text{eV/}\tilde{m} \right)^2 \), this term can be neglected. Thus, for sufficiently small neutrino masses and in the strong wash-out regime, we can conclude that leptogenesis is well approximated by a simple decays plus inverse decays picture.

### 3.4 CP asymmetry and seesaw geometry

So far we concentrated on the kinetic theory of leptogenesis and we have seen how neutrino mixing data favor a very simple regime in which predictions are model independent and theoretical uncertainties are minimized. We have now to answer the crucial question whether the resulting final asymmetry can explain the measured CMB value (cf. \[^{11}\]). The thermodynamical point of view, i.e. the efficiency factor, is not enough to answer this question,

\[^{f}\] The result obtained in \[^{24}\] corresponds to this situation because at \( z \approx z_B \gg 1 \) the Higgs thermal mass suppresses the wash-out from scatterings involving the top quark while the contribution from scatterings involving gauge bosons is negligible.
since one needs to know the value of the $CP$ asymmetry too. This is a specific 
leptogenesis issue that concerns what can be called the \emph{seesaw geometry}.

A perturbative calculation from the interference between tree level and vertex plus self energy one-loop diagrams yields

$$
\varepsilon_1 \simeq \frac{1}{8\pi} \sum_{i=2,3} \frac{\text{Im} [ (hh^\dagger)^2 ]}{(hh^\dagger)_{11}} \times \left[ f_V (\frac{M_2^2}{M_1^2}) + f_S (\frac{M_2^2}{M_1^2}) \right].
$$

(60)

The function $f_V$, describing the vertex contribution, is given by

$$
f_V(x) = \sqrt{x} \left[ 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right],
$$

(61)

while the function $f_S$, describing the self-energy contribution, is given by

$$
f_S(x) = \frac{\sqrt{x}}{1 - x}.
$$

(62)

In the limit $x \gg 1$, corresponding to have a mild RH neutrinos mass hierarchy with $M_{2,3} \gg M_1^2$, one has

$$
f_V(x) + f_S(x) \simeq -\frac{3}{2\sqrt{x}}.
$$

(63)

In this limit and barring strong phase cancellations, the expression (60) simplifies into

$$
\varepsilon_1 \simeq \frac{3}{16\pi} \frac{\text{Im} [ (h^\dagger h) M^{-1} h^T h^* ]_{11}}{(hh^\dagger)_{11}}.
$$

(64)

Replacing $h$ with $\Omega$ (cf. (45)) one then gets

$$
\varepsilon_1 \simeq \frac{3}{16\pi} \frac{M_1 m_{\text{atm}}}{v^2} \beta(m_1, \bar{m}_1, \Omega_{j1}^2),
$$

(65)

where we introduced the convenient quantity

$$
\beta(m_1, \bar{m}_1, \Omega_{j1}^2) = \sum_j \frac{m_j^2 \text{Im}(\Omega_{j1}^2)}{m_{\text{atm}} \sum_j m_j |\Omega_{j1}^2|}.
$$

(66)

The final asymmetry is proportional to the product of the $CP$ asymmetry times the final efficiency factor that, in the simplified decays plus inverse decays picture, depends only on the effective neutrino mass $\bar{m}_1$. The expression (65) shows that the $CP$ asymmetry depends on the three complex numbers $\Omega_{j1}^2$ and thus it introduces a model dependence in the prediction of the final asymmetry that one was hoping to have removed in the calculation of the final efficiency factor. It is however possible to maximize the absolute value of the $CP$ asymmetry respect to the ‘geometrical’ parameters $\Omega_{j1}^2$ and thus finding a non trivial maximum $\varepsilon_1^{\text{max}}(M_1, \bar{m}_1, m_1)$ depending only on $M_1$, $\bar{m}_1$ and $m_1$. One can then define an \emph{effective leptogenesis phase} $\delta_L$ such that the expression (66) can be re-casted in the following way

$$
\beta(m_1, \bar{m}_1, \Omega_{j1}^2) = \beta^{\text{max}}(m_1, \bar{m}_1) \sin \delta_L(\bar{m}_1, m_1, \Omega_{j1}^2).
$$

(67)
Figure 8: Seesaw geometry. Different configurations of $\Omega_{j1}^2$ (see text for explanation) with the same value of $\tilde{m}_1$.

The maximum of the absolute value of the CP asymmetry and of the function $\beta$ are thus realized for those particular geometrical configurations, corresponding to some $\Omega_{j1}^2$’s values, such that $\sin \delta_L = 1$. A general procedure for the calculation of $\varepsilon_{1}^{\text{max}}$ and $\sin \delta_L$ is presented in [36]. Here we just sketch some general features and describe two particularly interesting limit cases.

If one represents the three $\Omega_{j1}^2$ in the complex plane, the orthogonality condition fixes the sum of the three to start from the origin and to end up onto the real axis at the point $\text{Re}(\sum_j \Omega_{j1}^2) = 1$, as shown in Fig. 8 for a generic configuration (solid line arrows). Using the orthogonality condition, defining $\Omega_{j1}^2 = X_j + iY_j$ and using the definition of $\tilde{m}_1$ (cf. [46]), this can be re-casted as

$$\beta(m_1, \tilde{m}_1, \Omega_{j1}^2) = \frac{\Delta m^2_{32} Y_3 + \Delta m^2_{21} Y}{m_{\text{atm}} m_1},$$

with $Y = Y_2 + Y_3$. The absolute value of $\beta$ has to be maximized for $\tilde{m}_1$ constant. In general one always finds that

$$\beta_{\max}(m_1, \tilde{m}_1) = \beta_{\max}(m_1) f(m_1, \tilde{m}_1) \leq 1,$$

with $\beta_{\max}(m_1) = (m_3 - m_1)/m_{\text{atm}}$, $f(m_1, \tilde{m}_1) \leq 1$ and $f(m_1, \infty) = 1$.

An interesting limit case is that of fully hierarchical neutrinos for $m_1 = 0$. In this case $\beta_{\max} = 1$ and there is no global suppression. Moreover one has $\tilde{m}_1 = m_2 |\Omega_{21}^2| + m_3 |\Omega_{31}^2|$ and thus, for any change configuration such that $|\Omega_{21}^2|$ and $|\Omega_{31}^2|$ are constant, the quantity $\tilde{m}_1$ is also constant while $|\Omega_{11}^2|$ can be arbitrarily modified. Hence $|\beta(\tilde{m}_1, m_1, \Omega_{j1}^2)|$ is maximized
for configurations such that $X_2 = X_3 = 0$. It is then easy to see that it is further maximized for $Y_2 = 0$ and $Y_3 = \tilde{m}_1/m_3$, corresponding to the configuration shown in Fig. 8 with dotted line arrows. In this case one has very simply $f(0, \tilde{m}_1) = 1$. Therefore, the case $m_1 = 0$ corresponds, for a fixed $M_1$, to an absolute maximum of the $CP$ asymmetry given by $^{37,35}$ (cf. (65) and (66))

$$
\varepsilon_{1}^{\text{max}}(M_1) = \frac{3}{16\pi} \frac{M_1 m_{\text{atm}}^2}{v^2} \approx 10^{-6} \left( \frac{M_1}{10^{10} \text{GeV}} \right) \left( \frac{m_{\text{atm}}}{0.05 \text{eV}} \right). \quad (70)
$$

Note that with this last definition of $\varepsilon_{1}^{\text{max}}(M_1)$, together with the expressions (67) and (69), the Eq. (65) for the $CP$ asymmetry can be re-casted like

$$
\varepsilon_1 = -\varepsilon_{1}^{\text{max}}(M_1) \beta_{\text{max}}(m_1) f(m_1, \tilde{m}_1) \sin \delta_L(m_1, \tilde{m}_1, \Omega_{j1}^2), \quad (71)
$$

showing the sequence of different maximization steps.

In the quasi-degenerate limit the expression (46) for $\tilde{m}_1$ becomes simply $\tilde{m}_1 \approx m_1 \sum_j |\Omega_{j1}^2|$. Thus the condition $\tilde{m}_1 = \text{const}$ is equivalent to select all those configurations for which $\sum_j |\Omega_{j1}^2|$ is constant. Hence it is straightforward to conclude that $|\beta|$ is maximum for a configuration such that $Y_2 = X_2 = 0$ and $X_3 = 1/2$, shown in Fig. 8 with dashed line arrows. Using that in the quasi degenerate limit $m_{\text{atm}}^2 \approx 2 m_1 (m_3 - m_1)$, one obtains $^9$

$$
f(m_1, \tilde{m}_1) = \sqrt{1 - \frac{m_1^2}{m_2^2}}. \quad (72)
$$

Note that in both the two limit cases the maximum $CP$ asymmetry is obtained for configurations such that $\Omega_{21} = 0$. It is possible to show that this result holds in general, for any value of $m_{1}$. $^{46}$

Therefore, for maximal $CP$ asymmetry ($\sin \delta_L = 1$), one can still express all predictions in terms just of $M_1, \tilde{m}_1$ and $m_1$. In particular it is possible to express the CMB constraints, for all neutrino models, just in terms of these three parameters. For specific models it can happen of course that $\sin \delta_L < 1$ and the constraints get, in general, more restrictive.

### 3.5 CMB bound

From the Eq.’s (13) and (19) one obtains for the predicted baryon to photon number ratio

$$
\eta_B = d \varepsilon_1 \kappa_f, \quad (73)
$$

where the quantity $d$ is defined as

$$
d = \frac{a_{\text{sph}}}{N_{\gamma}^\text{rec}}. \quad (74)
$$

In the Standard Model case one has $a_{\text{sph}} = 28/79$, while the number of photons at recombination, assuming a standard thermal history, is given by

$$
N_{\gamma}^\text{rec} = \frac{4 g_{\text{SM}}}{3 g_{\text{rec}}} = \frac{4697}{129} \approx 36 \quad (75)
$$

$^9$This limit expression has been first shown in $^{32}$ using the approximation $m_{\text{sol}} = 0$. Here we derived it in a more general way.
and thus $d \simeq 0.97 \times 10^{-2}$.

The maximum baryon asymmetry $\eta_B^{\max}(M_1, \tilde{m}_1, m_1)$ is defined like the asymmetry corresponding to the maximum CP asymmetry

$$
\epsilon_1^{\max}(M_1, \tilde{m}_1, m_1) = \epsilon_1^{\max}(M_1) \beta_{\max}(m_1) f(m_1, \tilde{m}_1).
$$

The CMB bound is then simply equivalent to require

$$
\eta_B^{\max}(M_1, \tilde{m}_1, m_1) \geq \eta_B^{CMB}
$$

and therefore will yield constraints on the space of the three parameters $M_1, m_1$ and $\tilde{m}_1$.

### 3.6 Lower bounds on the lightest RH neutrino mass and on the reheating temperature

We have seen that the absolute maximum of the CP asymmetry is obtained for $m_1 = 0$. For $m_1 > 0$ the function $\beta_{\max}(m_1)$ suppresses the CP asymmetry. Furthermore the $\Delta L = 2$ wash-out term gets enhanced when the absolute neutrino mass scale increases (cf. (59)). Therefore, the maximum baryon asymmetry $\eta_B^{\max}$ is maximal when $m_1 = 0$. In this case the allowed region in the space of the parameters $M_1$ and $\tilde{m}_1$ and compatible with the CMB constraint is maximum and one finds an interesting lower bound on the $M_1$ value just plugging the expression (70) into the CMB constraint (cf. (77))

$$
M_1 \geq M_1^{\min} = \frac{1}{d} \left( \frac{16 \pi}{3} \frac{v^2}{m_{\text{atm}}} \frac{\eta_B^{CMB}}{\kappa_f} \right) \simeq 6.4 \times 10^8 \text{ GeV} \left( \frac{\eta_B^{CMB}}{6 \times 10^{-10}} \right) \left( \frac{0.05 \text{ eV}}{m_{\text{atm}}} \right) \kappa_f^{-1}.
$$

For an initial thermal abundance and in the limit $\tilde{m}_1/m_* \to 0$, one has, by definition, $\kappa_f = 1$ and so one finds

$$
M_1 \geq (6.6 \pm 0.8) \times 10^8 \text{ GeV} \gtrsim 4 \times 10^8 \text{ GeV},
$$

where the last inequality is the $3\sigma$ bound and we have used the experimental values Eq. (1) and Eq. (52). The case of a dynamically generated $N_1$ abundance is more significative and in this case the peak value $\kappa_f \simeq 0.18$, implies

$$
M_1 \geq (3.6 \pm 0.4) \times 10^9 \text{ GeV} \gtrsim 2 \times 10^9 \text{ GeV}.
$$

The most interesting situation corresponds to the range $(m_{\text{sol}}/m_{\text{atm}})$, for which the power law Eq. (58) can be used for $\kappa_f$, thus giving

$$
M_1 \gtrsim [3.3(2.2) \pm 0.4(0.3)] \times 10^{10} \text{ GeV} \left( \frac{\tilde{m}_1}{10^{-2} \text{ eV}} \right)^{1.1(1.2)} \gtrsim [1.5(1) \pm 10(9)] \times 10^{10} \text{ GeV},
$$

where we have used the central value (upper value) in the Eq. (58). The $M_1$ lower bound can be translated into a lower bound on the initial temperature $T_i$ that, within inflationary models, can be identified with the reheating temperature, corresponding to that temperature...
below which a radiation dominated regime holds. So far we assumed that this is much larger than $M_1$. If one relaxes this assumption then the final efficiency factor gets reduced. For small $\tilde{m}_1$ the threshold value is given approximately by $M_1$ itself since below this temperature either, assuming an initial thermal abundance, the $N_1$ abundance is thermally suppressed or the production gets considerably suppressed for an initial zero abundance. Therefore, for small values $\tilde{m}_1 \lesssim 10^{-3}$ eV, the same bounds (79) and (80) apply also to the reheating temperature.

In the more interesting case of strong wash-out, since the 90% of the surviving abundance is produced in an interval $z \simeq z_B \pm 2$, then the reheating temperature can be $\simeq z_B - 2$ times lower than $M_1$, without any appreciable change in the final predicted asymmetry. In the interesting range $(m_{\text{sol}}, m_{\text{atm}})$ one has that $z_B$ spans between 6 and 8 and thus the bound Eq. (81) gets relaxed from 4 to 6 times giving

$$T_i \gtrsim [4 (2.5) \times 10^9 - 2 (1.5) \times 10^{10}] \text{ GeV}.$$  (82)

This is another interesting result showing how the support of neutrino mixing data to the range $\tilde{m}_1 \sim O(m_{\text{sol}}, m_{\text{atm}})$ not only makes leptogenesis working in a simple and predictive way but also how the loss in the efficiency is compensated by a non relativistic production of the final asymmetry such that the lower bound on the reheating temperature gets just slightly more restrictive compared to the small $\tilde{m}_1$ range. Note that in case of modifications of the theoretical assumptions such that the maximum baryon asymmetry $\eta_B^{\text{max}} \rightarrow \xi \eta_B^{\text{max}}$, one has correspondingly $M_1^{\text{min}}, T_i^{\text{min}} \rightarrow M_1^{\text{min}}/\xi, T_i^{\text{min}}/\xi$.

### 3.7 Upper bound on the absolute neutrino mass scale

For large values of the absolute neutrino mass scale the $\Delta W$ wash-out term cannot be neglected. The final efficiency factor can be calculated in the approximation that $\Delta W$ starts to be effective for $z > z_B$, when the asymmetry generation from decays already stopped. This is a very good approximation in the strong wash-out regime and since $\tilde{m}_1 \geq m_1$ it does not introduce any restriction for $m_1 \gtrsim m_* \simeq 10^{-3}$ eV. Within this approximation one has simply

$$\kappa_f(\tilde{m}_1, M_1 m_2) = \kappa_f(\tilde{m}_1) e^{-\frac{\tilde{m}_1}{z_B^{10^{10} \text{ eV}}}} (\frac{m_1}{10^{10} \text{ eV}})^2,$$  (83)

where $\kappa_f(\tilde{m}_1)$ is the efficiency factor calculated in the regime of small neutrino masses neglecting the $\Delta W$ term. We use the simple limit $k_i = 2/(z_B K)$ that corresponds to neglect, conservatively, the contribution of scatterings to the wash-out and approximately to the upper values in the Eq. (58). A search of the peak values $(M_1, \tilde{m}_1)$ for which the maximum baryon asymmetry $\eta_B^{\text{max}}$ has an absolute maximum yields

$$\frac{\eta_B^{\text{peak}}(m_1)}{\eta_B^{\text{CMB}}} \sim \frac{2}{3^{7/2}} \chi m_* \frac{\xi}{m_i^4},$$  (84)

with the constant $\chi \simeq 1.6 \text{ eV}^3$. Hence the CMB bound implies an interesting constraint on the neutrino mass, given by $m_i < 0.1 \text{ eV}$. A precise calculation has to take into account
the running of neutrino masses\(^{(10)}\). The atmospheric neutrino mass scale at temperatures \(T \sim 10^{13}\) GeV is higher than at zero temperature. On the other hand the bound on neutrino masses that is obtained at large temperatures gets lower when calculated at low temperatures. This second effect is dominant and thus the account of neutrino mass running will make the neutrino mass bound more restrictive. The smallest effect is obtained for an Higgs mass \(M_h \approx 150\) GeV and makes the bound 20\% more stringent. Taking into account this effect one then obtains the 3\(\sigma\) bound\(^{(8)}\)

\[
m_i < 0.12 \text{ eV } \xi^{1/4}. \tag{85}\]

Note however that a (1 figure) bound \(m_i < 0.1\) eV conservatively accounts for the theoretical uncertainties\(^{\text{k}}\). As defined in the previous subsection, a value of \(\xi \neq 1\) describes a possible variation of the maximum baryon asymmetry in the case of modified theoretical assumptions, like for example in the supersymmetric case that will be studied in the next subsection, or in the presence of possible different effects, like an enhancement of the \(CP\) asymmetry due to a degenerate heavy neutrino spectrum\(^{(31)}\) that would relax the bounds\(^{(30, 32, 12)}\) or simply for a variation of the input values of the experimental quantities (note that \(\xi \propto n^2_{\text{atm}}/\eta_B CMB\)). However the strong suppression of the baryon asymmetry for an increasing absolute neutrino mass scale \((\eta_B \propto 1/m_i^4)\) makes the bound quite stable\(^{(79)}\). It is important to realize that the bound can be evaded but not trivially. This means that a measurement of a value of the absolute neutrino mass scale above the leptogenesis bound will necessarily imply some drastic modifications of the minimal leptogenesis scenario. These include particular neutrino models within the simple seesaw formula\(^{(39, 32)}\), non thermal leptogenesis scenarios\(^{(43)}\) or a non minimal seesaw formula, like that one arising in theories with a triplet Higgs\(^{(44)}\).

### 3.8 The supersymmetric case

Leptogenesis can be also studied within the minimal supersymmetric standard model (MSSM)\(^{(15, 16, 27)}\). In this case the asymmetry is generated not only from the \(N_1\) decays but also from the decays of their scalar partners, the \(\tilde{N}_1^c\)’s and their antiparticles \(\tilde{N}_1^c\), with the same mass \(M_1\). Since the decay width and thus also the inverse decay wash-out term are the same, these yield an additional equal contribution\(^{\text{i}}\). Therefore, from a thermodynamical point of view, the \(N_1\)’s and the \(\tilde{N}_1\)’s will play the same role and it is simply like if the ‘X-abundance’

\(^{\text{k}}\)If instead of the upper values we were using the central (lower) values in the Eq. \(^{(58)}\), then the bound would have been \(\sim 0.005\) (0.01) eV more stringent. As we said already, these values could arise from a possible contribution to the wash-out from scatterings or for the account of spectator processes. Note also that we used the latest published results on \(\Delta m^2_{\text{atm}}\)\(^{(24)}\). Preliminary results from the SuperKamiokande collaboration\(^{(11)}\) find \(\Delta m^2_{\text{atm}} = (1.3 - 3.0) \times 10^{-3}\) eV\(^2\) at 90\% c.l. (best fit \(\Delta m^2_{\text{atm}} = 2.0 \times 10^{-3}\) eV\(^2\)), from which, at 1\(\sigma\), \(m_{\text{atm}} = (0.045 \pm 0.006)\) eV, implying a \(\sim 0.01\) eV more stringent bound \(m_i < 0.11\) eV. In\(^{(27)}\) a bound \(m_i < 0.15\) eV, when this second value of \(m_{\text{atm}}\) is employed, has been obtained. A difference \(\sim 0.02\) eV can be ascribed to a different estimation of the running of neutrino masses. The remaining 0.02 eV difference can be safely included in the account of the theoretical uncertainty.

\(^{\text{i}}\)The discussion in this subsection is from\(^{(2)}\).
gets doubled. We will still track the $B - L$ asymmetry in the co-moving volume containing, on average in ultra-relativistic thermal equilibrium, one RH neutrino $N_1$ and thus now also one $\tilde{N}_1$. In this way we will still write the final baryon asymmetry as in the Eq. (13) but now $d = 2a_{sph}/N_{\gamma}^{rec}$, with the additional factor 2 taking into account the contribution from the $\tilde{N}_i$s. Let us analyze how the different quantities involved in the calculation of the final asymmetry get modified from the SM to the MSSM case. To this aim it will prove convenient to introduce the variations

$$\xi_X \equiv \frac{X^{MSSM}}{X^{SM}}$$

for any quantity $X$. The sphaleron conversion coefficient is given by $a_{sph} = 8/23$ and thus it is almost unchanged. The number of degrees of freedom is given by $g_{MSSM} = 915/4$ and this approximately doubles the number of photons at recombination given by

$$\left( N_{\gamma}^{rec} \right)^{MSSM} = 4 \frac{g_{MSSM}}{3 g_{rec}} = 3355 \frac{43}{43} \simeq 78.$$ 

In this way one gets $d^{MSSM} \simeq 0.89 \times 10^{-2}$, almost unchanged compared to the SM case ($\xi_d \simeq 0.92$). The $CP$ asymmetry in the MSSM case is given by

$$\varepsilon_1 \simeq -\frac{1}{8\pi} \sum_{i=2,3} \text{Im} \left[ (hh^\dagger)_{ii} \right] \times \left[ f_V \left( \frac{M_i^2}{M_1^2} \right) + f_S \left( \frac{M_i^2}{M_1^2} \right) \right],$$

where

$$f_V(x) = \sqrt{x} \ln \left( \frac{1 + x}{x} \right)$$

and

$$f_S(x) = \sqrt{x} \frac{1}{1 - x}.$$ 

In the limit $x \gg 1$, corresponding to have a mild RH neutrinos mass hierarchy with $M_i^2 \gg M_1^2$ and barring strong phase cancellations, one has

$$f_V(x) + f_S(x) \longrightarrow \frac{3}{\sqrt{x}}$$

and thus, compared to the SM case (cf. (60) and (63)), the absolute value of the $CP$ asymmetry and hence also of its maximum $\varepsilon_1^{max}$ get doubled ($\xi_\varepsilon = 2$).

Let us now calculate how the efficiency factor at small values of $M_1 \tilde{m}^2$ gets modified in the MSSM case. We have seen that SM leptogenesis is well approximated, in the strong wash-out regime, by the simple decays plus inverse decays picture. It is then interesting to study how the results change within such a simple picture. Since the $N_1$'s can now decay in two new channels ($N_1 \rightarrow \tilde{l}_i \tilde{l}_i$, $\tilde{l}_i \tilde{l}_i$) that give exactly the same contribution to the decay width as the other two standard ones, this gets doubled compared to the standard

\[\text{The other possibility would have been to choose a halved co-moving volume in a way that the factor 2 was absorbed in } N_{\gamma}^{rec}.\]
case (cf. \(36\)). This makes lifetime shorter and the inverse decays wash out rate stronger. However the increase of the degrees of freedom makes the expansion faster and this partially compensates. Recalling the definition of the equilibrium neutrino mass, Eq. \(39\), one has simply

\[
\xi_{m_{\nu}} = \sqrt{\frac{\xi_{g_{\nu}}}{\xi_{\Gamma_{\nu}}}} = \frac{1}{2} \sqrt{\frac{915}{427}} \simeq 0.73,
\]

implying \(m_{\nu}^{\text{MSSM}} \simeq 0.8 \times 10^{-3} \text{eV}\). Therefore now one has that the transition to the strong wash-out regime occurs for slightly smaller values of \(m_{1}(\approx 1/\sqrt{2} \text{smaller})\) and thus for the decay parameter one has \(\xi_K \simeq \sqrt{2}\). The range of \(K\) values favored by neutrino mixing data will thus be given by

\[
\mathcal{O}(K_{\text{sol}}^{\text{MSSM}} \simeq 10) \lesssim K^{\text{MSSM}} \lesssim \mathcal{O}(K_{\text{atm}}^{\text{MSSM}} \simeq 65),
\]

to be compared with the Eq. \(53\) in the SM case. In the strong wash out regime the efficiency factor, calculated in the decays plus inverse decays picture, will be still given by the expression Eq. \(23\). For \(K\) in the range \(93\) one has \(\kappa_{f} \propto K^{-1.15}\) (cf. \(54\)) and thus

\[
\xi_{k_{f}} \simeq \xi_{m_{\nu}}^{1.15} \simeq \frac{1}{\sqrt{2}}.
\]

Assuming that the effect of scatterings in the MSSM goes in the same direction as in the SM and thus that the result \(\xi_{k_{f}} \simeq 1/\sqrt{2}\) holds approximately also when scatterings are included, one can write the analogous of the Eq. \(20\) in the MSSM case

\[
\kappa_{f}^{\text{MSSM}} = (1.5 \pm 0.7) \times 10^{-2} \left(\frac{10^{-2} \text{eV}}{\tilde{m}_{1}}\right)^{1.1 \pm 0.1}.
\]

A more detailed analysis is needed to verify the role of scatterings in the MSSM. Note that again the upper values of this range correspond to the power law \(34\) translated in the MSSM case (i.e. by replacing \(m_{\nu}^{\text{SM}} \rightarrow m_{\nu}^{\text{MSSM}}\)).

Let us now investigate the consequences for the lower bound on \(M_{1}\). The Eq. \(78\) will now become

\[
M_{1} \geq M_{1}^{\text{min}} = \frac{1}{d} \frac{8 \pi}{3} \frac{v^{2}}{m_{\text{atm}}} \frac{\eta_{CMB}^{\text{CMB}}}{\kappa_{f}}
\]

\[
\simeq 3.5 \times 10^{8} \text{ GeV} \left(\frac{\eta_{B}^{\text{CMB}}}{6 \times 10^{-10}}\right) \left(\frac{0.05 \text{ eV}}{m_{\text{atm}}}\right) \kappa_{f}^{-1}.
\]

From this expression one obtains, for the case of initial thermal abundance, zero initial abundance and in the strong wash-out regime respectively, the constraints

\[
M_{1} \geq (3.7 \pm 0.4) \times 10^{8} \text{ GeV} \gtrsim 2.5 \times 10^{8} \text{ GeV},
\]

\[
M_{1} \geq (1.7 \pm 0.2) \times 10^{9} \text{ GeV} \gtrsim 1.1 \times 10^{9} \text{ GeV}
\]
and
\[
M_1 \gtrsim [2.4 (1.6) \pm 0.2 (0.15)] \times 10^{10} \, \text{GeV} \left( \frac{\tilde{m}_1}{10^{-2} \, \text{eV}} \right)^{1.1 (1.2)} \gtrsim [1.5 (0.9) - 10 (8)] \times 10^{10} \, \text{GeV},
\]
(99)

where we have used the central (upper) value in the Eq. (95). These have to be compared with the constraints obtained in the SM case (cf. Eq.'s (79), (80) and (81)). In the first two cases the constraints are approximately twice looser, because of the CP asymmetry enhancement, while in the case of strong wash out one has \( \xi_{M_1} \min \simeq (\xi_\varepsilon \xi_{\kappa_f})^{-1} \simeq 0.7 \) for the central value, while the 3\( \sigma \) bound remains practically unchanged because the experimental error gets reduced. For the lower bound on the initial temperature, corresponding to the reheating temperature within inflation, the same considerations as in the SM case hold. For \( \tilde{m}_1 \lesssim 10^{-3} \, \text{eV} \) the same lower bounds valid for \( M_1 \) apply approximately also to \( T_i \).

In the relevant strong wash-out regime for \( K \gtrsim 4 \), corresponding to \( \tilde{m}_1 \gtrsim 3.2 \times 10^{-3} \, \text{eV} \), the relaxation compared to the \( M_1 \) lower bound is larger than twice. In this way one obtains conservatively, using the upper values for \( \kappa_f \) (cf. (95)),
\[
T_i \gtrsim 1.5 \times 10^9 \, \text{GeV}.
\]
(100)

This is an appropriate conservative value for a general comparison with the upper bounds on the reheating temperature that arise by imposing that a gravitino thermal production is not in conflict with cosmological observations (see [77] for a recent discussion and references). For a more precise comparison one should calculate the lower bound on \( T_i \) for a specific value of \( \tilde{m}_1 \). For example, in the range \( m_{\text{sol}} \lesssim \tilde{m}_1 \lesssim m_{\text{atm}} \), the relaxation compared to the \( M_1 \) lower bound is practically the same as in the SM case (between 4 and 6 times) and thus one gets
\[
T_i \gtrsim [3.5 (2) \times 10^9 - 2 (1.5) \times 10^{10}] \, \text{GeV}.
\]
(101)

Note that these analytic results are in good agreement with the numerical ones in [77].

Let us now study how the upper bound on the neutrino masses gets modified. This can be easily done calculating the value of the total variation of the final asymmetry given by
\[
\xi = \frac{\xi_\varepsilon \xi_{\kappa_f}}{\xi_\omega},
\]
(102)

where \( \xi_\omega \) is the variation of the wash-out term from \( \Delta L = 2 \) processes that is crucial for the determination of the neutrino mass bound and that can be expressed through the parameter \( \omega \) in the Eq. (59). We have already seen that \( \xi_\varepsilon = 2 \). Moreover since the peak of the asymmetry lies in the strong wash-out regime we can also use the result Eq. (94) for \( \xi_{\kappa_f} \). Therefore, we miss only to determine \( \xi_\omega \). There are two effects to be considered. The first is the increase of the number of degrees of freedom that speeds up the expansion reducing the efficiency of wash-out processes. The second is the presence of new different additional \( \Delta L = 2 \) processes and this clearly strengthens the rate \( \Gamma_{\Delta L=2} \). From the expressions given in [16] one can find \( \xi_\Gamma_{\Delta L=2} = 5/3 \) and thus \( \xi_\omega = \xi_\Gamma_{\Delta L=2}/\sqrt{\xi_\varepsilon} \simeq 5/(3 \sqrt{2}) \). Putting all together one finds \( \xi \simeq 6/5 \) and, when the running of neutrino masses is neglected, the
bound is about 5% more relaxed compared to the SM case, namely $m_1^{\text{MSSM}} \lesssim 0.16 \text{eV}$. The effect of running of neutrino masses, as in the SM, goes into the direction to make the bound more stringent. However the effect can be as small as $\sim 7\%$ (for $\tan\beta \sim 10$), roughly half than in the SM case, and thus one obtains in the end (at $3\sigma$)

$$m_1^{\text{MSSM}} < 0.15 \text{eV}. \quad (103)$$

### 3.9 A ‘too-short-blanket problem’

If one requires that $M_1$ is lower than a certain cut-off value $M_1^*$ then the upper bound on the neutrino masses becomes more stringent.\textsuperscript{49} Such a cut-off can either arise directly from neutrino models\textsuperscript{48} or indirectly from an upper bound on the reheating temperature $T_R^\ast$. In this second case one has $M_1^* \simeq z_* T_R^\ast$ where $z_* \simeq 1$ in the weak wash-out regime and $z_* \simeq z_B - 2$ in the strong wash-out regime.\textsuperscript{k} It is then quite interesting to study the dependence $m_1^{\text{bound}}(M_1^*)$, or equivalently $m_1^{\text{bound}}(T_R^\ast)$. This is done in detail in\textsuperscript{36} here we just sketch some general features and results. For definiteness we will refer to the supersymmetric case, since in this case the avoidance of the gravitino problem implies an upper bound on the reheating temperature.

First, note that if $T_R^\ast < T_R^{\text{min}} \simeq 10^9 \text{GeV}$ (cf.(100)), then simply there is no allowed value for $m_1$. Moreover until one has $m_1^{\text{bound}} \ll m_{\text{atm}}$, there is a strong dependence on $T_R^\ast$, since the maximum baryon asymmetry grows linearly with $T_R^{\text{min}}$ while is very slightly dependent on $m_1$. This means that the function $m_1^{\text{bound}}(T_R^\ast)$ has a vertical asymptote in $T_R^{\text{min}} \simeq 10^9 \text{GeV}$. For $m_1^{\text{bound}} \sim m_{\text{atm}}$ the suppression factor $\beta\text{max}(m_1) = m_{\text{atm}}/(m_1 + m_3)$ in the CP asymmetry and the loss in the efficiency for $\tilde{m}_1 > m_1 \gtrsim 10^{-3} \text{eV}$ compensate the increase of $T_R^\ast$ and the growth of $m_1^{\text{bound}}$, for increasing $T_R^\ast$, slows down and eventually, for $T_R^\ast \geq T_R^{\text{peak}} \simeq 3 \times 10^{12} \text{GeV}$, saturates to its maximum value (cf.(103)) and stays constant.\textsuperscript{l} Values $T_R^\ast \sim 10^{10} \text{GeV}$ are particularly interesting since they correspond to maximum allowed values from gravitino problem arguments. In this case one expects $m_1 \sim m_{\text{atm}} \gtrsim 10^{-3} \text{eV}$ and, since $\tilde{m}_1 \geq m_1$, one can use the strong wash-out limit for the final efficiency factor $\kappa_R \simeq 2/(K z_B)$. Moreover the wash-out factor $\Delta W$ from $\Delta L = 2$ processes can be neglected. In this case it is possible to show the following approximate bound

$$\frac{m_1}{m_{\text{atm}}} \lesssim \frac{A}{\sqrt{1+2A}} \quad \text{with} \quad A \simeq \frac{0.2}{\xi_{\text{atm}}} \frac{T_R^\ast}{10^{10} \text{GeV}}, \quad (104)$$

where $\xi_\eta = n_B^{\text{CMB}}/6 \times 10^{-10}$ and $\xi_{\text{atm}} = m_{\text{atm}}/0.051 \text{eV}$.

Let us consider two examples using $\xi_\eta = \xi_{\text{atm}} = 1$. For an upper bound $T_R^\ast = 3 \times 10^{10} \text{GeV}$ one finds $m_1 \lesssim 0.4 m_{\text{atm}} \simeq 0.02 \text{eV}$, implying $m_3 \lesssim 0.055 \text{eV}$. For $T_R^\ast = 10^{11} \text{GeV}$ one finds $m_1 \lesssim 0.9 m_{\text{atm}} \simeq 0.045 \text{eV}$, implying $m_3 \lesssim 0.07 \text{eV}$.

\textsuperscript{k}The value of $z_*$ has to be evaluated for that particular value of $\tilde{m}_1$ that maximizes the asymmetry.

\textsuperscript{l}Note that at the peak one has $z_B \simeq 10$ and $M_1^{\text{peak}} \simeq 2 \times 10^{13} \text{GeV}$ and thus $T_R^{\text{peak}} \simeq M_1^{\text{peak}}/8 \simeq 3 \times 10^{12} \text{GeV}$.
This exercise shows that it is difficult to conciliate reheating temperatures close to the minimum allowed one \( T_R \lesssim 10^{10} \text{GeV} \) and at the same time to make thermal leptogenesis compatible with quasi-degenerate neutrino masses by evading the upper bound: the two things go into opposite directions, a typical too-short-blanket problem. There are two interesting consequences. The first is that if a stringent upper bound on the reheating temperature is placed, like \( T_R \lesssim 3 \times 10^{10} \text{GeV} \), then it becomes difficult to evade the bound invoking a quasi-degenerate heavy neutrino spectrum since the bound still falls in a transition region where light neutrinos exhibit a partial hierarchy. Vice versa if one requires quasi-degenerate light neutrinos to be compatible with the minimal thermal leptogenesis scenario then the problem of a large minimum reheating temperature gets exacerbated. Indeed, it is difficult in this case to avoid high values \( T_R \gtrsim 10^{11} \text{GeV} \), unless one invokes a strong degenerate heavy neutrino spectrum such to have a resonant enhancement of the asymmetry.\(^{42}\) Anyway further investigations are needed to understand the exact conditions on the degeneracy of the heavy neutrino spectrum and how they depend on a cut-off on \( T_R \) or directly on \( M_1 \).

4 Final discussion

Leptogenesis is a specific realization of the simplest and oldest baryogenesis class of models where the asymmetry is generated from heavy particle decays. Its minimal version, thermal leptogenesis, is based crucially on neutrino properties and because of the great experimental neutrino physics achievements it became in the last year a testable model. The decay parameter, the key quantity in models of baryogenesis from heavy particle decays, is a quantity closely related to neutrino masses. This cannot be exactly determined from data but there is an emerging favored range of values, \( K_{\text{lep}} \sim 5 - 50 \), that implies just a small departure from thermal equilibrium, however large enough to explain the observed value of the asymmetry and moreover with some nice consequences. The predicted baryon asymmetry is independent on the initial conditions, both on the initial value of the asymmetry and on the initial number of decaying RH neutrinos. Moreover, the theoretical uncertainties are minimized and the final asymmetry is predicted with a precision that is within half order of magnitude. These features can be synthesized saying that thermal leptogenesis predictions are quite stable and model independent, a picture that resembles very closely the Standard Big Bang Nucleosynthesis in predicting the primordial nuclear abundances. The drawback is that values of \( K \sim 10 \) determine a loss in the efficiency between one and two orders of magnitude. This has to be compensated by an increase of the \( CP \) asymmetry of the same amount, implying a more stringent lower bound on \( M_1 \). On the other hand we have seen how in the strong wash out regime the temperature of baryogenesis gets much smaller than \( M_1 \) and this relaxes the lower bound on the reheating temperature compared to the lower bound on \( M_1 \) of a factor \( \sim 5 \). Therefore, there seems to be an intriguing conspiracy between neutrino mixing data and the explanation of the observed baryon asymmetry. Actually considerations on the maximum allowed value of the effective neutrino mass show
that the conspiracy is even deeper\cite{47} and future experimental information on the absolute neutrino mass scale could give a further support.

If the leptogenesis upper bound on the absolute neutrino mass scale, \( m_1 < 0.1 \text{ eV} \), will be fully tested with cosmology, neutrinoless double beta decay and Tritium beta decay experiments, then thermal leptogenesis can work in its minimal way. On the other hand, if neutrino masses higher than 0.1 eV will be found, then this can be either regarded as the effect of the existence of some level of degeneracy in the heavy neutrino spectrum, to be understood whether easily realized or not within the simple seesaw mechanism, or, more likely, as a drastic departure from the minimal thermal leptogenesis picture, at the expense of predictivity. In any case it should be clear, from the discussion on the ‘too-short-blanket problem’, that the two statements for which thermal leptogenesis requires dangerously large reheating temperatures within the supersymmetric framework and that the neutrino mass bound can be evaded within minimal thermal leptogenesis, can be very difficultly made compatible with each other.

Another interesting aspect is that if the lightest neutrino mass \( m_1 \) will be found to be higher than \( m_\star \simeq 10^{-3} \text{ eV} \), then it will be possible to conclude model independently that thermal leptogenesis lies in the strong wash-out regime and in this way all pieces of the experimental information have fitted within the theoretical best expectations. Therefore, if the absolute neutrino mass scale will be found to lie within the window \( (10^{-3} - 10^{-1}) \text{ eV} \), the picture will receive further strong support from the data.

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