On model-independent measurement of the angle $\phi_3$ using Dalitz plot analysis.

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This report shows the latest results on the study of the method to determine the angle $\phi_3$ of the unitarity triangle using Dalitz plot analysis of $D^0$ decay from $B^\pm \to DK^\pm$ process in a model-independent way. We concentrate on the case with a limited charm data sample, which will be available from the CLEO-c collaboration in the nearest future, with the main goal to find the optimal strategy for $\phi_3$ extraction.

I. INTRODUCTION

The measurement of the angle $\phi_3$ ($\gamma$) of the unitarity triangle using Dalitz plot analysis of the $D^0 \to K^0_S \pi^+ \pi^-$ decay from $B^\pm \to DK^\pm$ process, introduced by Giri et al. [1] and Belle collaboration [2] and successfully implemented by BaBar [3] and Belle [4], presently offers the best constraints on this quantity. However, this technique is sensitive to the choice of the model used to describe the three-body $D^0$ decay. Currently, this uncertainty is estimated to be $\sim 10^\circ$ and due to large statistical error does not affect the precision of $\phi_3$ measurement. As the amount of $B$ factory data increases, though, this uncertainty will become a major limitation. Fortunately, a model-independent approach exists (see [5]), which uses the data of the $\tau$-charm factory to obtain the missing information about the $D^0$ decay amplitude.

In our previous study of the model-independent Dalitz analysis technique [3] we have implemented a procedure proposed by Giri et al. involving the division of the Dalitz plots into bins, and shown that this procedure allows to measure the phase $\phi_3$ with the statistical precision only 30–40% worse than in the unbinned model-dependent case. We did not attempt to optimize the binning and mainly considered a high-statistics limit with an aim to estimate the sensitivity of the future super-B factory.

The data useful for model-independent measurement are presently available from the CLEO-c experiment [5]. CLEO-c collected an integrated luminosity of 280 pb$^{-1}$ at the $\psi(3770)$ resonance decaying to $D\bar{D}$. By the end of CLEO-c operation this statistics will grow up to 750 fb$^{-1}$ [4]. This corresponds to $\sim 1000$ events where $D$ meson in a $CP$ eigenstate decays to $K^0_S \pi^+ \pi^-$, and twice as much events of $\psi(3770) \to D^0\bar{D}^0$ with both $D$ mesons decaying to $K^0_S \pi^+ \pi^-$. Both of these processes include the information useful for a model-independent $\phi_3$ measurement. In this paper, we report on studies of the model-independent approach with a limited statistics of both $\psi(3770)$ and $B$ data, using both $D_{CP} \to K^0_S \pi^+ \pi^-$ and $(K^0_S \pi^+ \pi^-)_D \to (K^0_S \pi^+ \pi^-)_D$ final states.

II. MODEL-INDEPENDENT APPROACH

The density of $\bar{T}^0 \to K^0_S \pi^+ \pi^-$ Dalitz plot is given by the absolute value of the amplitude $f_D$ squared:

$$p_D = p_D(m^2_+, m^2_-) = |f_D(m^2_+, m^2_-)|^2$$

(1)

In the case of no $CP$-violation in $D$ decay the density of the $D^0$ decay $\bar{p}_D$ equals to

$$\bar{p}_D = |\bar{f}_D|^2 = p_D(m^2_+, m^2_-).$$

(2)

Then the density of the $D$ decay Dalitz plot from $B^\pm \to DK^\pm$ process is expressed as

$$p_{B\pm} = |f_D + r_B e^{i(\delta_B \pm \phi_3)} \bar{f}_D|^2 = p_D + r_B^2 \bar{p}_D + 2 \sqrt{p_D \bar{p}_D} (x \pm y \pm s),$$

(3)

where $x$, $y$, $s$ include the value of $\phi_3$ and other related quantities, the strong phase $\delta_B$ of the $B^\pm \to DK^\pm$ decay, and amplitude ratio $r_B$:

$$x = r_B \cos(\delta_B - \phi_3); \quad y = r_B \sin(\delta_B - \phi_3).$$

(4)

The functions $c$ and $s$ are the cosine and sine of the strong phase difference $\Delta \delta_D$ between the symmetric Dalitz plot points:

$$c = \cos(\delta_D(m^2_+, m^2_-) - \delta_D(m^2_-, m^2_+)) = \cos \Delta \delta_D;$$

$$s = \sin(\delta_D(m^2_+, m^2_-) - \delta_D(m^2_-, m^2_-)) = \sin \Delta \delta_D.$$  

(5)

The phase difference $\Delta \delta_D$ can be obtained from the sample of $D$ mesons in a $CP$-eigenstate, decaying to $K^0_S \pi^+ \pi^-$. The Dalitz plot density of such decay is

$$p_{CP} = |f_D \pm \bar{f}_D|^2 = p_D \pm \bar{p}_D \pm 2 \sqrt{p_D \bar{p}_D} c$$

(6)

(normalization is arbitrary). Decays of $D$ mesons in $CP$ eigenstate to $K^0_S \pi^+ \pi^-$ can be obtained in the process, e.g. $e^+e^- \to \psi(3770) \to D\bar{D}$, where the other (tag-side) $D$ meson is reconstructed in the $CP$ eigenstate, such as $K^+K^- \text{ or } K^0_S\omega$.

Another possibility is to use a sample, where both $D$ mesons (we denote them as $D$ and $D'$) from the $\psi(3770)$ meson decay into the $K^0_S \pi^+ \pi^-$ state [5]. Since $\psi(3770)$ is a vector, two $D$ mesons are produced in a $P$-wave, and the wave function of the two mesons is antisymmetric. Then the four-dimensional density of two correlated

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Dalitz plots is
\[ p_{\text{corr}}(m_i^2, m_j^2, m_k^2, m_l^2) = |f_D f_D^* - f_D^* f_D|^2 = \frac{p_D p_D^* + \bar{p}_D \bar{p}_D^* - 2 \sqrt{p_D p_D^* \bar{p}_D \bar{p}_D^*}}{|p_D^*|^2}, \]
(7)
This decay is sensitive to both \(c\) and \(s\) for the price of having to deal with the four-dimensional phase space.

In a real experiment, one measures scattered data rather than a probability density. Two options of dealing with real data are possible: a binned approach, or a scatter plot smoothing using nonparametric density estimation. The latter could be useful to reach the statistical sensitivity equivalent to the model-dependent case. However, in this paper we show that using the appropriate binning this is also possible.

### III. Binned Analysis with \(D_{CP}\) Data

The binned approach was proposed by Giri et al. [1]. Assume that the Dalitz plot is divided into \(2N\) bins symmetrically to the exchange \(m_i^2 \leftrightarrow m_j^2\). The bins are denoted by the index \(i\) ranging from \(-N\) to \(N\) (excluding 0); the exchange \(m_i^2 \leftrightarrow m_j^2\) corresponds to the exchange \(i \leftrightarrow -i\). Then the expected number of events in the bins of the Dalitz plot of \(D_{CP}\) decay from \(B_{CP}^\pm \rightarrow DK_{CP}^\pm\) is
\[ \langle N_i \rangle = h_B [K_i + r B_K_{\rightarrow i} + 2 \sqrt{K_{\rightarrow i}}(xc_i + ys_i)], \]
(8)
where \(K_i\) is the number of events in the bins in the Dalitz plot of the \(D^0\) in a flavor eigenstate, \(h_B\) is the normalization constant. Coefficients \(c_i\) and \(s_i\), which include the information about the cosine and sine of the phase difference, are given by
\[ c_i = \frac{\int_{D_i} \sqrt{p_D p_D^*} \cos(\delta_D(m_i^2, m_j^2)) dD}{\sqrt{\int_{D_i} p_D dD \int_{D_i} \bar{p}_D dD}}, \]
(9)
\(s_i\) is defined similarly with cosine substituted by sine. Here \(D_i\) is the bin region, over which the integration is performed. Note that \(c_0 = 1, c_i = -s_i\) and \(c_i^2 + s_i^2 \leq 1\) (the equality \(c_i^2 + s_i^2 = 1\) being satisfied if the amplitude is constant across the bin).

The coefficients \(K_i\) are obtained precisely from a very large sample of \(D^0\) decays in the flavor eigenstate, which is accessible at \(B\)-factories. The expected number of events in the Dalitz plot of \(D_{CP}\) decay equals to
\[ \langle M_i \rangle = h_{CP} [K_i + K_{\rightarrow i} + 2 \sqrt{K_{\rightarrow i}}(c_i + s_i)], \]
(10)
and thus can be used to obtain the coefficient \(c_i\). As soon as the \(c_i\) and \(s_i\) coefficients are known, one can obtain \(x\) and \(y\) values (hence, \(\phi_3\) and other related quantities) by a maximum likelihood fit using equation [5].

Note that now the quantities of interest \(x\) and \(y\) (and consequently \(\phi_3\)) have two statistical errors: one due to a finite sample of \(B_{CP}^\pm \rightarrow DK_{CP}^\pm\) data, and due to \(D_{CP} \rightarrow K_{CP}^0 \pi^\mp \pi^-\) statistics. We will refer to these errors as \(B\)-statistical and \(D_{CP}\)-statistical, respectively.

Obtaining \(s_i\) is a major problem in this analysis. If the binning is fine enough, so that both the phase difference and the amplitude remain constant across the area of each bin, expressions [6] reduce to \(c_i = \cos(\delta D)\) and \(s_i = \sin(\delta D)\), so \(s_i\) can be obtained as \(s_i = \pm \sqrt{1 - c_i^2}\). Using this equality if the amplitude varies will lead to the bias in the \(x, y\) fit result. Since \(c_i\) is obtained directly, and \(s_i\) is overestimated by the absolute value, the bias will mainly affect \(y\) determination, resulting in lower absolute values of \(y\).

Our studies [5] show that the use of equality \(c_i^2 + s_i^2 = 1\) is satisfactory for the number of bins around 200 or more, which cannot be used with presently available \(D_{CP}\) data. It is therefore essential to find a relatively coarse binning (the number of bins being 10–20) which a) allows to extract \(s_i\) from \(c_i\) with low bias, and b) has the sensitivity to the \(\phi_3\) phase comparable to the unbinned model-dependent case.

Fortunately, both the a) and b) requirements appear to be equivalent. To determine the \(B\)-statistical sensitivity of a certain binning, let’s define a quantity \(Q\) — a ratio of a statistical sensitivity to that in the unbinned case. Specifically, \(Q\) relates the number of standard deviations by which the number of events in bins is changed by varying parameters \(x\) and \(y\), to the number of standard deviations if the Dalitz plot is divided into infinitely small regions (the unbinned case):
\[ Q^2 = \frac{\sum_i \left( \frac{1}{\sqrt{N_i}} \frac{dN_i}{dx} \right)^2 + \left( \frac{1}{\sqrt{N_i}} \frac{dN_i}{dy} \right)^2 \int_D \left( \frac{|f_D|^2}{\sqrt{|f_B|^2}} \right)^2 \left( \frac{|f_B|^2}{\sqrt{|f_B|^2}} \right)^2 dD}{\int_D \left( \frac{|f_D|^2}{\sqrt{|f_B|^2}} \right)^2 \left( \frac{|f_B|^2}{\sqrt{|f_B|^2}} \right)^2 dD}, \]
(11)
where \(f_D = f_D + (x + iy)f_D\), \(N_i = \int_{D_i} |f_B|^2 dD\).

Since the precision of \(x\) and \(y\) weakly depends on the values of \(x\) and \(y\) [5], we can take for simplicity \(x = y = 0\). In this case one can show that
\[ Q^2|_{x=y=0} = \sum_i (c_i^2 + s_i^2)N_i / \sum_i N_i \]
(12)
Therefore, the binning which satisfies \(c_i^2 + s_i^2 = 1\) \((i.e.\ the absence of bias if \(s_i\) is calculated as \(\sqrt{1 - c_i^2}\)) also has the same sensitivity as the unbinned approach. The factor \(Q\) defined this way is not necessarily the best measure of the binning quality (the binning with higher \(Q\) can be insensitive to either \(x\) or \(y\), which is impractical from the point of measuring \(\phi_3\)), but it allows an easy calculation and correctly reproduces the relative quality for a number of binnings we tried in our simulation.

The choice of the optimal binning naturally depends on the \(D^0\) model. In our studies we use the two-body amplitude obtained in the latest Belle \(\phi_3\) Dalitz analysis [4].
FIG. 1: Divisions of the $D^0 \to K^0_S \pi^+ \pi^-$ Dalitz plot. Uniform binning of $\Delta \delta_D$ strong phase difference with $N = 8$ (left), and the binning obtained by variation of the latter to maximize the sensitivity factor $Q$ (right).

From the consideration above it is clear that a good approximation to the optimal binning is the one obtained from the uniform division of the strong phase difference $\Delta \delta_D$. In the half of the Dalitz plot $m_+^2 < m_-^2$ (i.e., the bin index $i > 0$) the bin $D_i$ is defined by condition

$$2\pi(i-1/2)/N < \Delta \delta_D(m_+^2, m_-^2) < 2\pi(i+1/2)/N, \quad (13)$$

in the remaining part ($i < 0$) the bins are defined symmetrically. We will refer to this binning as $\Delta \delta_D$-binning. As an example, such a binning with $N = 8$ is shown in Fig. 1 (a). Although the phase difference variation across the bin is small by definition, the absolute value of the amplitude can vary significantly, so the condition $c_i^2 + s_i^2 = 1$ is not satisfied exactly. The values of $c_i$ and $s_i$ in this binning are shown in Fig. 2 with crosses.

Figure 1 (b) shows the division with $N = 8$ obtained by continuous variation of the $\Delta \delta_D$-binning to maximize the factor $Q$. The sensitivity factor $Q$ increases to 0.89 compared to 0.79 for $\Delta \delta_D$-binning.

We perform a toy MC simulation to study the statistical sensitivity of the different binning options. We use the amplitude from the Belle analysis $3$ to generate decays of flavor $D^0$, $D_{CP}$, and $D$ from $B^\pm \to DK^\pm$ decay to the $K^0_S \pi^+ \pi^-$ final state according to the probability density given by $1$, $2$, and $3$, respectively. To obtain the $B$-statistical error we use a large number of $D^0$ and $D_{CP}$ decays, while the generated number of $D$ decays from the $B^\pm \to DK^\pm$ process ranges from 100 to 100000. For each number of $B$ decay events, 100 samples are generated, and the statistical errors of $x$ and $y$ are obtained from the spread of the fitted values. A study of the error due to $D_{CP}$ statistics is performed similarly, with a large number of $B$ decays, and the statistics of $D_{CP}$ decays varied. Both errors are checked to satisfy the square root scaling.

The binning options used are $\Delta \delta_D$-binning with $N = 8$ and $N = 20$, as well as “optimal” binning with maximized $Q$ obtained from these two with a smooth variation of the bin shape. Note that the “optimal” binning with $N = 20$ offers the $B$-statistical sensitivity only 4% worse than an unbinned technique. For comparison, we use the binnings with the uniform division into rectangular bins (with $N = 8$ and $N = 19$ in the allowed phase space, the ones which are denoted as 3x3 and 5x5 in $3$).

The $B$- and $D_{CP}$-statistical precision of different binning options, recalculated to 1000 events of both $B$ and $D_{CP}$ samples, as well as their calculated values of the factor $Q$, are shown in Table I. In the present study we use the errors of parameters $x$ and $y$ rather than $\phi_3$ as a measure of the statistical power since they are nearly independent of the actual values of $\phi_3$, strong phase $\delta$ and amplitude ratio $r_B$. The error of $\phi_3$ can be obtained from these numbers given the value of $r_B$. The factor $Q$ reproduces the ratio of the values $\sqrt{1/\sigma_x^2 + 1/\sigma_y^2}$ for the binned and unbinned approaches with the precision of $\sim 1\%$. While the binning with maximized $Q$ offers better $B$-statistical sensitivity, the best $D_{CP}$-statistical precision of the options we have studied is reached for the $\Delta \delta_D$-binning. However, for the expected amount of experimental data of $B$ and $D_{CP}$ decays the $B$-statistical error dominates, therefore, slightly worse precision due to $D_{CP}$ statistics does not affect significantly the total precision.

Using $\Delta \delta_D$-binning, the following combination of the binned and unbinned approaches is possible, which allows to reach $B$-statistical precision equivalent to the unbinned case. Assume the number of $\Delta \delta_D$-bins is large enough, so $\cos \Delta \delta_D$ and $\sin \Delta \delta_D$ remain almost constant across the bin area. At the level of current precision this is reached already for a number of bins as small as 10–20. Then

$$c_i = \cos \Delta \delta_D \frac{\int_{D_i} \sqrt{p_D dD} \int_{D} d\bar{D}}{\int_{D_1} \sqrt{p_D dD} \int_{D_1} p_D dD}, \quad (14)$$

where the integrals can be calculated from the flavor $D$ data sample. Therefore, it is possible to obtain $\cos \Delta \delta_D$ from $c_i$ and consequently $\sin \Delta \delta_D$, and use them in the expression for the probability density $3$ to perform the unbinned $B$-data fit thus obtaining the best possible $B$-statistical precision.

We have considered the choice of the optimal binning only from the point of statistical power. However, the conditions to satisfy low model dependence are quite different. Since the bins in the binning options we have considered are sufficiently large, the requirement that the phase does not change over the bin area is a strong model assumption. We have performed toy MC simulation to study the model dependence. While the binning was kept the same as in the statistical power study (based on the phase difference from the default $D^0$ amplitude), the amplitude used to generate $D^0$, $D_{CP}$ and $B^\pm \to DK^\pm$ decays was altered in the same way as in the Belle study of the model-dependence in the unbinned analysis $3$. As a result, the same bias of $\Delta \phi_3 \sim 10^\circ$ is observed as in unbinned analysis. We remind that the cause of this bias is a fixed relation between the $c_i$ and $s_i$. Therefore, proposed binning options, although providing good statistical precision, are not flexible enough to provide also
a low model dependence. To minimize the model dependence, the bin size should be kept as small as possible, therefore, uniform binning is more preferred.

In a real analysis, one can control the model error by testing if the amplitude used to define binning is compatible with the observed $D_{CP}$ data. This can be done, e.g., by dividing each bin and comparing calculated values of $c_i$ in its parts, or by comparing the expected and observed numbers of events in each bin.

We conclude that the method of $\phi_3$ determination using only $D_{CP}$ data is only asymptotically model-independent, since for any finite bin size the calculation of $s_i$ is done using model assumptions of the $\Delta\delta_D$ variations across the bin. Increasing the $D_{CP}$ data set, however, allows to apply a finer binning and therefore reduce the model error due to the variation of the phase difference.

IV. BINNED ANALYSIS WITH CORRELATED $D^0 \rightarrow K^0\pi\pi$ DATA

The use of the $\psi(3770)$ decays where both neutral $D$ mesons decay to the $K_S^0\pi^+\pi^-$ state allows to significantly increase the amount of data useful to extract phase information in $D^0$ decay. It is also possible to detect events of $\psi(3770) \rightarrow (K_S^0\pi^+\pi^-)_D(K_L^0\pi^+\pi^-)_D$, where $K_L^0$ is not reconstructed, and its momentum is obtained from kinematic constraints. The number of these events is approximately twice that of $(K_S^0\pi\pi)^2$ due to combinatorics. We will refer to both of these processes as $(K^0\pi\pi)^2$.

In the case of a binned analysis, the number of events in the region of the $(K_S^0\pi\pi)^2$ phase space is

$$\langle M \rangle_{ij} = h_{corr}[K_i K_{-j} + K_{-i} K_j - 2\sqrt{K_i K_{-i} K_{-j} K_j (c_i c_j + s_i s_j)}].$$

In the case of the $(K_S^0\pi^+\pi^-)(K_L^0\pi^+\pi^-)$ final state, the interference term changes its sign. Here two indices correspond to two $D$ mesons from $\psi(3770)$ decay. It is logical to use the same binning as in the case of $D_{CP}$ statistics to improve the precision of the determination of $c_i$ coefficients, and to obtain $s_i$ from data without model assumptions, contrary to $D_{CP}$ case. The obvious advantage of this approach is its being unbiased for any finite $(K^0\pi\pi)^2$ statistics (not asymptotically as in the case of $D_{CP}$ data).

Note that in contrast to $D_{CP}$ analysis, where the sign of $s_i$ in each bin was undetermined and has to be fixed using model assumptions, $(K_S^0\pi^+\pi^-)$ analysis has only a four-fold ambiguity: change of the sign of all $c_i$ or all $s_i$. In combination with $D_{CP}$ analysis, where the sign of $c_i$ is fixed, this ambiguity reduces to only two-fold. One of the two solutions can be chosen based on a weak model assumption (incorrect sign corresponds to complex-conjugated $D$ decay amplitude, which violates causality requirement when parameterized with the Breit-Wigner amplitudes).

Coefficients $c_i$, $s_i$ can be obtained by minimizing the negative logarithmic likelihood function

$$-2 \log \mathcal{L} = -2 \sum_{i,j} \log P(M_{ij}, \langle M \rangle_{ij}),$$

where $P(M, \langle M \rangle)$ is the Poisson probability to get $M$ events with the expected number of $\langle M \rangle$ events.

The number of bins in the 4-dimensional phase space is $4N^2$ rather than $2N$ in the $D_{CP}$ case. Since the expected number of events in correlated $K_S^0\pi\pi$ data is of the same order as for $D_{CP}$, the bins will be much less populated. This, however, does not affect the precision of $c_i$, $s_i$ determination since each of the free parameters is constrained by many bins.

The toy MC simulation was performed to study the procedure described above. Using the amplitude from the Belle analysis, we generate a large number of $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays and several sets of $(K_S^0\pi\pi)^2$ decays (according to the probability density given by (1)). We use the same binning options as in $D_{CP}$ study with $N = 8$. The negative logarithmic likelihood (15) is then minimized with $c_i$, $s_i$ and $h_{corr}$ as free parameters. We constrain $|c_i|, |s_i| < 1$ in the fit to improve the convergence. The coefficients $c_i$, $s_i$ are then used in the fit to $B$ decay data to obtain the $x, y$ error due to $(K^0\pi\pi)^2$ decay statistics. The number of $(K_S^0\pi\pi)^2$ decays ranges from 1000 to 10000. The obtained errors show a square root scaling. The best $(K^0\pi\pi)^2$-statistical error is obtained for $\Delta\delta_D$-binning and recalculated to 1000 events yields $\sigma_x = 0.0050$,$\sigma_y = 0.0095$, which is only slightly worse than the error obtained with the same amount of $D_{CP}$ data (see Table I for comparison). We also check that

| Binning | $Q$ | $B$-stat. err. $\sigma_x$ | $\sigma_y$ | $D_{CP}$-stat. err. $\sigma_x$ | $\sigma_y$ | $(K^0\pi\pi)^2$-stat. err. $\sigma_x$ | $\sigma_y$ |
|---------|-----|-----------------------------|-------------|-----------------------------|-------------|---------------------------------|-------------|
| $N = 8$ (uniform) | 0.57 | 0.0331 | 0.0600 | 0.0053 | 0.0097 | 0.0145 | 0.0322 |
| $N = 8$ ($\Delta\delta_D$) | 0.79 | 0.0273 | 0.0370 | 0.0042 | 0.0072 | 0.0050 | 0.0095 |
| $N = 8$ (optimal) | 0.89 | 0.0232 | 0.0324 | 0.0058 | 0.0114 | 0.0082 | 0.0114 |
| $N = 19$ (uniform) | 0.69 | 0.0274 | 0.0549 | 0.0042 | 0.0112 | - | - |
| $N = 20$ ($\Delta\delta_D$) | 0.62 | 0.0266 | 0.0350 | 0.0048 | 0.0074 | - | - |
| $N = 20$ (optimal) | 0.96 | 0.0223 | 0.0290 | 0.0078 | 0.0110 | - | - |

TABLE I: Statistical precision of $(x, y)$ determination using different binnings and with an unbinned approach. The errors correspond to 1000 events in both the $B$ and $D_{CP}$ ($(K^0\pi\pi)^2$) samples.
changing the model used to define the binning does not lead to the systematic bias (although it does decrease the statistical precision). Figure 2 demonstrates the precision of the determination of $c_i$, $s_i$ coefficients in our toy MC study and the absence of the systematic bias for both $c_i$ and $s_i$.

Since the number of $(K^0\pi\pi)^2$ decays in $\psi(3770)$ data is approximately twice larger than the number of $D_{CP}$ decays, the statistical errors due to $\psi(3770)$ data for the two approaches are nearly equal. The same binning can be used in both approaches, therefore improving the accuracy of $c_i$ determination. The approach based on $(K^0\pi\pi)^2$ data allows to extract both $c_i$ and $s_i$ without additional model uncertainties, so it can be used to check the validity of the constraint $c_i^2 + s_i^2 = 1$ and therefore to test the sensitivity of the particular binning.

V. CONCLUSION

We have studied the model-independent approach to $\phi_3$ measurement using $B^{\pm} \to DK^{\pm}$ decays with neutral $D$ decaying to $K_S^0\pi^+\pi^-$. The analysis of $\psi(3770) \to D\bar{D}$ data allows to extract the information about the strong phase in $D^0 \to K_S^0\pi^+\pi^-$ decay that is fixed by model assumptions in a model-dependent technique. We specially consider the case with a limited $\psi(3770) \to D\bar{D}$ data sample which will be available from CLEO-c in the nearest future.

In the binned analysis, we propose a way to obtain the binning that offers an optimal statistical precision (close to the precision of an uninned approach). Two different strategies of the binned analysis are considered: using $D_{CP} \to K_S^0\pi^+\pi^-$ data sample, and using decays of $\psi(3770)$ to $(K^0\pi^+\pi^-)_D(K^0\pi^+\pi^-)_D$. The strategy using $D_{CP}$ decays alone cannot offer a completely model-independent measurement: it provides only the information about $c_i$ coefficients, while $s_i$ for low $D_{CP}$ statistics has to be fixed using model assumptions. However, as the $D_{CP}$ data sample increases, model-independence can be reached by reducing the bin size. The strategy using the $\psi(3770) \to (K^0\pi^+\pi^-)_D(K^0\pi^+\pi^-)_D$ sample, in contrast, allows to obtain both $c_i$ and $s_i$ with an accuracy comparable to $D_{CP}$ approach. Both strategies can use the same binning of the $D^0 \to K_S^0\pi^+\pi^-$ Dalitz plot and therefore can be used in combination to improve the accuracy due to $\psi(3770)$ statistics.

The expected sensitivity is obtained based on the $D^0$ decay model from Belle analysis. For the CLEO-c statistics of 750 pb$^{-1}$ (1000 $D_{CP}$ events and 2000 $(K^0\pi\pi)^2$ events) the expected errors of parameters $x$ and $y$ due to $\psi(3770)$ statistics are $\sigma_x = 0.003$ and $\sigma_y = 0.007$. For $r_B = 0.1$ it gives the $\phi_3$ precision $\sigma_{\phi_3} = \max(\sigma_x, \sigma_y)/(\sqrt{2}r_B) \simeq 3^\circ$, which is far below the expected error due to present-day $B$ data sample.

In our study, we did not consider the experimental systematic uncertainties e.g. due to imperfect knowledge of the detection efficiency or background composition. We believe these issues can be addressed in a similar manner as in already completed model-dependent analyses.

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