Spectroscopic Temperature Determination of Degenerate Fermi Gases

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We suggest a simple method for measuring the temperature of ultra-cold gases made of fermions. We show that by using a two-photon Raman probe, it is possible to obtain lineshapes which reveal properties of the degenerate sample, notably its temperature $T$. The proposed method could be used with identical fermions in different hyperfine states interacting via s-wave scattering or identical fermions in the same hyperfine state via p-wave scattering. We illustrate the applicability of the method in realistic conditions for $^4$Li prepared in two different hyperfine states. We find that temperatures down to 0.05 $T_F$ can be determined by this in-situ method.

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I. INTRODUCTION

The experimental realization of Bose-Einstein condensation (BEC) in atomic samples \footnote{Rb} have stimulated a new wave of theoretical and experimental studies of degenerate systems in the dilute and weakly interacting regime. Several recent experiments have successfully reached the degenerate regime for ultra-cold atomic samples of fermions. For example, temperatures of 0.5 $T_F$ or less, where $T_F$ is the Fermi temperature of the cold fermions, have been reported for $^{40}$K \footnote{K}, or $^6$Li \footnote{Li}. One of the goals of reaching even lower temperatures is the formation of Cooper pairs of interacting fermions and the detection of the Bardeen-Cooper-Schrieffer (BCS) phase transition in a trapped gas of fermionic atoms. Such transitions in atomic gases will allow the study of phenomena usually associated with condensed matter physics, such as superconductivity. Recently, “resonance superfluidity” \footnote{Rb} has been predicted in the strong-coupling regime occurring near a Feshbach resonance. This regime is being explored experimentally \footnote{Li}.

One of the key parameters in studying these systems is the accurate value of the temperature $T$. Various methods are being employed to determine $T$, such as fitting the tail of the fermion energy distribution \footnote{K}, using mixtures with bosons \footnote{K}, or using impurities as probes \footnote{K}. Each of these approaches have limitations. For example, considering that the thermal contribution in a Fermi gas vanishes with temperature, fitting the tail of the energy distribution becomes very challenging. Even in a fermion-boson mixture with large attractive interactions, such as $^{40}$K-$^{87}$Rb \footnote{K}, or $^6$Li-$^7$Li \footnote{Li}, it is questionable whether bosons and fermions stay in thermal equilibrium when the temperature is lowered to degenerate regime. Even if so, the determination of the bosons temperature is not easy below $T_{c}/2$ \footnote{K} $(T_{c}$: BEC critical temperature) \footnote{K}. Typically, temperature is read by imaging the velocity distribution of a freely expanded sample.

In this paper, we describe an in situ method to measure the temperature of a sample based on spectroscopic measurements. It offers the potential for very accurate determination of extremely low temperatures in degenerate Fermi systems as well as other properties (e.g., their Fermi temperature $T_F$). Furthermore, if used in systems made of fermions only, it avoids the problem arising from the superfluidity of Bose-Einstein condensates when $T$ is below the BEC critical temperature $T_c$, which suppresses scattering \footnote{Rb}. The approach is based on Doppler-free two-color spectroscopy. Such techniques have been successfully employed to create ultra-cold dimers in ultracold bosonic gases and in BEC \footnote{K}.

II. METHOD

The schematic of the two-photon transition is illustrated in Fig. 1 together with the relevant quantities. While two atoms approach each other along the molecular ground state with a given relative kinetic energy $\epsilon$, two co-propagating lasers can induce a two-photon transition into a bound level $E_{b2}$ of the molecular ground state. The first laser of intensity $I_1$ is detuned from the excited molecular bound level $E_{b1}$ by $\delta_1 = E_{b1} - \nu_1$. Similarly, the second laser of intensity $I_2$ is detuned by an amount $\delta_2 = E_{b2} - (\nu_1 - \nu_2)$ from the bound level energy $E_{b2}$ of the ground molecular state. The rate coefficient for the photoassociation (PA) process is given by \footnote{K}

$$K(T, L_1, L_2) = \left\langle \frac{\pi v}{\kappa^2} \sum_l (2l + 1)|S_l(\epsilon, L_1, L_2)|^2 \right\rangle , \quad (1)$$

where $L_i = \{I_i, \delta_i\}$ represent the parameters of laser $i$, $\epsilon = \hbar^2 k^2 / 2\mu = \mu v^2 / 2$, $\mu$ is the reduced mass, and $v$ is the relative velocity of the colliding pair. In Eq. (1) the sum goes over contributing partial waves $l$, and $S_l$ represents the scattering matrix element for producing the final state $E_{b2}$ from the initial continuum state. Averaging over relative velocities is implied by $\langle \ldots \rangle$. The scattering matrix is well approximated by \footnote{K}

$$|S_l|^2 = \frac{(\epsilon - \delta_2)^2 \gamma_1 \gamma_s}{(\epsilon - \Delta_+)^2 (\epsilon - \Delta_-)^2 + (\gamma^2/4)(\epsilon - \delta_2)^2} , \quad (2)$$

where $\gamma_1$ is the width of the intermediate bound level $E_{b1}$ and $\gamma_s \simeq 4\pi^2 (I_1/\epsilon)|D(R)|^2$ is the simulated width from the continuum initial state $|\epsilon, l\rangle$ to the
intermediate state \( |E_{0i}\rangle \). Here, \( D(R) \) is the molecular dipole transition moment, and in the zero-energy limit, \( \gamma_s \propto e^{1/2+1} \). Also, \( \gamma = \gamma_1 + \gamma_s \simeq \gamma_1 \) if the laser intensities are not too large (this is the regime we are interested in). Finally, \( \Delta_\pm \) represent a split in the single resonance due to the second laser, and is given by

\[
\Delta_\pm = \frac{1}{2}(\delta_1 + \delta_2) \pm \frac{1}{2}\sqrt{(\delta_1 - \delta_2)^2 + 4\hbar^2\Omega_2^2},
\]

where \( \hbar^2\Omega_2^2 = \frac{(2\pi I_2/c)|\langle E_{0i}|D(R)|E_{0i}\rangle|^2}{c^2} \) defines the Rabi frequency \( \Omega_2 \). From the analytical form of \( |S_i|^2 \), we expect two peaks located at \( \epsilon \simeq \Delta_\pm \), and a minimum \( (|S_i|^2 = 0) \) located between the peaks at \( \epsilon = \delta_2 \) (due to destructive interference between the two scattering paths). Because of the \( e^{1/2+1} \) Wigner’s threshold behavior of \( \gamma_s \), the peak at \( \Delta_- \) will be weaker than the one at \( \Delta_+ \).

We consider samples of identical fermions of mass \( m \) in the ultra-cold regime (so that \( 2\mu = m \)). While there is no s-wave \( (l = 0) \) scattering between two identical fermionic atoms in the same hyperfine state (the lowest contribution is then a p-wave \( (l = 1) \) which vanishes as \( \epsilon \to 0 \)), s-wave scattering occurs between different hyperfine states. We will concentrate our treatment to the latter case, and assume two identical fermions in hyperfine states labeled 1 and 2, respectively.

To compute the rate coefficient \( K \), we need to average over the distribution of relative velocities in the system. The velocity distribution of each ensemble follows a Fermi distribution characterized by its temperature \( T \) and Fermi energy \( E_F = k_BT \). Because of the setups in most experiments, we consider the atoms trapped in an harmonic potential. For large traps with frequencies \( \omega_1, \omega_2, \) and \( \omega_3 \), the density of state for the hyperfine state \( i \) is given by \( \rho(\vec{k}_i) = \hbar^3 k_i^3 / m^3 \omega_1 \omega_2 \omega_3 \).

The Fermi energy \( E_F \) of the atoms in a particular hyperfine state is the energy of the highest occupied state in the harmonic potential at absolute zero and so is determined by the total number of atoms sharing the same hyperfine state. To have the same number of atoms in each hyperfine state provides two benefits: both group of atoms have the same Fermi energy, and the s-wave scattering rate is maximal. This requirement in not too stringent experimentally: if the sample is to be prepared from the atoms in a single hyperfine state by transferring \( \sim 50\% \) of the atoms to another hyperfine state, then the maximal number difference between the two is determined by the degeneracy of the trap potential at the Fermi energy. E.g., for isotropic harmonic trap with the degeneracy of the \( m \)-th energy level \( \sim m^2 \) it follows \( \Delta N/N \approx N^{-1/3} \). In what follows we assume that both hyperfine states have identical number of atoms so that their Fermi energies are identical as well.

The distribution of the relative momentum \( \vec{k} \) (normalized to unity) is

\[
g_{12}(\kappa) = \int d^3k_1 d^3k_2 \, g(\vec{k}_1) \, g(\vec{k}_2) \, \delta(\vec{k}_1 - \vec{k}_2 - 2\vec{k}),
\]

where \( g(\vec{k}_i) \equiv \rho(\vec{k}_i)f_{FD}(\vec{k}_i,E_F,T)/N_i \). Here, the function \( f_{FD}(\vec{k}_i,E_F,T) = \{1 + \exp[\epsilon(\vec{k}_i) - E_F]/k_BT\}^{-1} \) is the Fermi-Dirac distribution with \( E_{\vec{k}_i} = \hbar^2k_i^2/2m \), and \( N_i = \int d^3k_i\rho(\vec{k}_i)f_{FD}(\vec{k}_i,E_F,T) \) is the number of fermions in the hyperfine state \( i \). Note that \( g_{12}(\kappa) \) depends on \( E_F \) and \( T \), but not on the direction of \( \vec{k} \); the integration removes that dependence. In Fig. 2 we illustrate the distribution \( g_{12} \) in term of the relative energy \( \epsilon \) for various temperatures and compare it with the corresponding Maxwell-Boltzmann (MB) distributions. As the temperature drops under \( T_F \), the difference between the two types of distribution becomes more apparent; as the system becomes more degenerate, the \( g_{12} \) distribution spreads over a larger range of energies (when compared to MB distributions). As \( T \) approaches zero, \( g_{12} \) is entirely contained between \( \epsilon = 0 \) and \( 2E_F \). As opposed to the single particle distribution \( g(E) \) (see inset), which is contained between \( E = 0 \) and \( E_F \) at zero-\( T \) and decreases sharply to zero at \( E_F \), \( g_{12} \) goes to zero more gradually. In fact, for the case of free particles, one can show that \( g_{12}(\epsilon) \) is a triangle with values 1 at \( \epsilon = 0 \) and \( \epsilon = 2E_F \), while \( g(E) \) is the standard step function between \( E = 0 \) and \( E_F \).

We want to use 2-photon scattering to probe the relative velocity (or energy) distribution and infer the temperature of the system. At ultra-cold temperatures, Eq. (4) becomes

\[
K(T, L_1, L_2) = \frac{4\pi^2}{\hbar} \int_0^\infty d\epsilon \, \epsilon \, g_{12}(\epsilon) |S_0(\epsilon, L_1, L_2)|^2 / \kappa, \tag{5}
\]

Besides \( T \) and \( E_F \), \( K \) depends on the detunings \( \delta_1 \) and \( \delta_2 \) and the Rabi frequencies \( \Omega_1 \) and \( \Omega_2 \). Because of the important variation of \( |S_0|^2 \) with respect to \( \delta_2 \), lineshapes obtained as a function of \( \delta_2 \) contain precious information about the degenerate Fermi gas. As \( T \) is reduced, the distribution \( g_{12} \) becomes more sharply defined for \( \epsilon \) between 0 and \( 2E_F \), and by scanning \( \delta_2 \) for nearby values, the large variations in the integrand of Eq. (5) will probe \( g_{12} \).

We computed lineshapes for realistic systems. The Fermi energies \( E_F \) reached in todays experiments are in the range of few \( \mu K \). In addition to this experimental constraint, one needs to be detuned far enough from resonance to not populate the level \( v_1 \), with typical detunings \( \delta_1 \sim 150 \text{ MHz} \) (7.2 nK) [14]. The lifetime of most levels \( v_1 \) is about 10-20 nsec, giving \( \gamma_1 \sim 16 - 32 \text{ MHz} \) (0.76-0.38 nK). Finally, one needs intensities large enough to drive the transition, but not too large to heat the system (through broadening). Typical values for high lying
states \(v_1\) at 500 GHz below the asymptote are \(\Omega_1 = 100\) kHz (4.8 \(\mu\)K) and \(\Omega_2 \approx 2-3\) MHz (96-144 \(\mu\)K). Under these conditions, we have \(\delta_1 \gg h\Omega_2\). With large \(\delta_1\), \(\Delta_+\) will also be large, and the only way to get a peak of \(|S|^2\) with \(\epsilon \sim E_F\) will come from \(\Delta_-\): hence, we must have \(\delta_2 \sim E_F\) as well. Then, \(\delta_1 \gg h\Omega_2 \gg \delta_2\) and we will have peaks at \(\Delta_- \sim \delta_2 - h^2\Omega_2^2/\delta_1\) and \(\Delta_+ \sim \delta_1 + h^2\Omega_2^2/\delta_1\), as well as a minimum at \(\epsilon = \delta_2\).

III. RESULTS AND DISCUSSION

We obtained lineshapes for various temperatures by varying \(\delta_2\) in \(\delta\). We selected parameters consistent with the case of \(^6\)Li (although similar results can be obtained for other species): \(E_F = 1\) \(\mu\)K, \(\delta_1 = 7\) mK, \(\gamma_1 = 0.76\) mK and \(h\Omega_2 = 100\) \(\mu\)K. The remaining parameters lead to a multiplicative constant in \(K\) that can be factored out for all lineshapes. We show representative lineshapes normalized to unity (in arbitrary units) in Fig. 3 as \(T\) is lowered below \(T_F\), the peak becomes narrower and sharper. Many other features can be identified on those lineshapes. As \(\delta_2\) grows, \(K\) reaches a minimum value \(K_{\text{min}}\), increases rapidly to its maximum value \(K_{\text{max}}\), and then decreases more slowly to a “background” value \(K_{\text{bg}}\). In addition, inflection points on both sides of the peak can be identified. All these features can be extracted from the lineshapes, and could be used to extract temperature. For example, one could take the ratio \(K_{\text{max}} - K_{\text{bg}}/K_{\text{min}} - K_{\text{bg}}\) of a given lineshape, in principle removing the multiplicative constant in \(K\). However, this ratio varies very little for \(T/T_F \sim 0.5\) and below. Besides the effect of a background signal/noise would also affect/reduce the sensitivity of the temperature determination. One could also map the location (i.e. the value of \(\delta_2\)) of these features as a function of \(T\), but again, their variations are small below say \(T/T_F \sim 0.1\).

Alternative features, such as ratios of the derivatives at the inflection points (on each side of the main peak) could be used. In this case, the background would not play any role, nor the multiplicative constant in \(K\). A simple measure incorporating both criteria is the ratio of the curvature of the signal at the peak value and its height as defined from the minimum value of \(K\) (see inset in Fig. 3), i.e.

\[
C \equiv -\frac{K''(\delta_{\text{max}})}{K_{\text{max}} - K_{\text{min}}} E_F^2. \tag{6}
\]

While finding the second derivative of a lineshape with respect to \(\delta_2\) may be challenging in general, at the local peak maximum this should not be a problem as the first derivative there is zero.

Because of the rapid increase of \(K\) from its minimum value to its peak value, only a small range of \(\delta_2\) needs to be scanned (as compared to the ratio of the derivatives at the inflection points): for \(E_F \sim 1\) \(\mu\)K, scans with kHz precision are required. In Fig. 4 we show \(C\) as a function of \(T/T_F\): it decreases monotonically with \(T\), and even at \(T/T_F \sim 0.05\), its variation is noticeable. So, by measuring three features on the lineshape, namely its minimum, maximum, and the curvature at the maximum, one can evaluate \(C\) and determine the temperature of the degenerate Fermi sample by reading it from Fig. 3. Although the exact values of \(C\) depend on other parameters (\(\gamma_1\), \(\delta_1\), and \(\Omega_2\)), its general shape follows the example shown in Fig. 4.

For illustration purposes, we also include two other curves (corresponding to other sets of \(\delta_1\) and \(\Omega_2\)). The curve with the largest variations at low \(T/T_F\) is found when we have \(\delta_1 \gg h\Omega_2 \gg \delta_2\) with a factor of 100 between each parameter.

The lineshapes shown in Fig. 4 are not very sensitive to small variations in \(\gamma_1\), \(\delta_1\) or \(\Omega_2\); variations of few \% give very similar curves. Of these parameters, \(\gamma_1\) cannot be varied experimentally, except by changing the intermediate molecular level \(E_{\text{bg}}\). However, for most atoms with Fermi species considered in todays experiments, i.e., alkali metals, \(\gamma_1\) varies little with \(E_{\text{bg}}\) (e.g., for \(^6\)Li in the triplet excited state, \(\gamma_1\) varies from \(\sim 11.7\) MHz down to \(3.2\) MHz (or 0.56 mK to 0.16 mK). However, systems with even smaller \(\gamma_1\) (about 1-10 \(\mu\)K) would offer the possibility of measuring the velocity distribution \(g_{12}\) directly by scanning \(\delta_2\). In fact, \(|S|^2\) then becomes proportional to the delta-function \(\delta(\epsilon - \delta_2)\). The extremely small variations in \(g_{12}\) as \(T \to 0\) could possibly be detected. Another possibility to enhance the sensitivity of this type of measurement could be realized using Feshbach resonances: a sharp increase in the integrand of Eq. (5) at \(\epsilon \sim \epsilon_{\text{res}}\), where \(\epsilon_{\text{res}}\) is the resonant relative energy, would emphasize the corresponding region of the velocity distribution. By positioning the resonance appropriately (e.g., via external magnetic fields), strong contribution of regions of \(g_{12}\) where small variations occur as \(T \to 0\) could be obtained, hence more precise information could be extracted from the lineshapes. Notice that p-wave Feshbach resonances \(12\) could be used to enhance p-wave scattering in a single-fermion population.

IV. CONCLUSION

We have shown that the lineshapes obtained using a two-photon Raman probe can yield very precise determination of the temperature in degenerate Fermi gases. One of the key advantages of this approach, beside its non-destructive nature, is the extremely low temperatures measurable: in the example considered here, as low as \(\sim 0.05T_F\). Although we have illustrated this two-color scheme for identical fermions in two different internal states with equal populations, this method can be made much more flexible. For example, if the populations are very different, the \(g_{12}\) distribution may exhibit sharper structures that could be used advantageously. The enhancement due to Feshbach resonances could also help probing chosen regions of \(g_{12}\), and extend the method to samples of identical fermion in a single internal state, by using p-wave Feshbach resonances. Finally, when the
interaction between the fermions is attractive such as for $^6$Li, Cooper pairs may form. Photoassociation of a Cooper pair should demand less energy than the photoassociation of the two fermions produced by breaking either one or two Cooper pairs \cite{16}. These processes should contribute additional features on top of the simple lineshapes, their locations and magnitudes probing the energy gap and the number of Cooper pairs.

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\begin{thebibliography}{99}
\bibitem{1} F. Dalfovo and S. Stringari, Phys. Rev. A \textbf{53}, 2477 (1996).
\bibitem{2} B. DeMarco and D.S. Jin, Science \textbf{285}, 1703 (1999).
\bibitem{3} A. Truscott, K. Strecke, W. McAlexander, G. Partridge, and R. Hulet, Science \textbf{291}, 2570 (2001).
\bibitem{4} M. Holland et al., Phys. Rev. Lett. \textbf{87}, 120406 (2001); J.N. Milstein, S.J.J.M.F. Kokkelmans, and M.J. Holland, Phys. Rev. A \textbf{66}, 043604 (2002).
\bibitem{5} K.M. O’Hara et al., Science \textbf{298}, 2179 (2002).
\bibitem{6} M.E. Gehm et al. \texttt{arXiv:cond-mat/0212499} (2002).
\bibitem{7} G. Ferrari, Phys. Rev. A \textbf{59}, R4125 (1999).
\bibitem{8} G. Ferrari, M. Inguscio, W. Jastrzebski, G. Modugno, G. Roati, and A. Simoni, Phys. Rev. Lett. \textbf{89}, 053202 (2002).
\bibitem{9} H. Hu, X.-J. Liu, and M. Modugno, E-Print No. \texttt{cond-mat/0301182} (2003); G. Modugno et al., Science \textbf{297}, 2240 (2002).
\bibitem{10} G. Modugno (private communication).
\bibitem{11} E. Timmermans and R. Côté, Phys. Rev. Lett. \textbf{80}, 3419 (1998); A.P. Chikkatur et al., Phys. Rev. Lett. \textbf{85}, 483 (2000).
\bibitem{12} R. Wynar et al., Science \textbf{287}, 1016 (2000); J.M. Gerton, D. Strekalov, I. Prodan, and R.G. Hulet, Nature \textbf{408}, 692 (2000); C. McKenzie et al., Phys. Rev. Lett. \textbf{88}, 120403 (2002).
\bibitem{13} J. Bohn and P. Julienne, Phys. Rev. A \textbf{54}, R4637 (1996).
\bibitem{14} R.G. Hulet (private communication).
\bibitem{15} C.A. Regal et al. \texttt{arXiv:cond-mat/0209071} (2002).
\bibitem{16} P. Törmä and P. Zoller, Phys. Rev. Lett. \textbf{85}, 487 (2000).
\end{thebibliography}
FIG. 1: Two-photon scheme. In this setup the detunings are $\delta_1 = E_{b1} - \nu_1 > 0$ and $\delta_2 = E_{b2} - \nu_1 + \nu_2 < 0$. 
Relative energy $\varepsilon / E_F$

Normalized Distribution $g_{12}(\varepsilon)$

Energy $E / E_F$

Distribution $g(E) E^{1/2}$

$T / T_F = 0.01$

$T / T_F = 0.5$

$T / T_F = 1.0$

FIG. 2: Relative velocity distribution for various temperatures: $g_{12}(\varepsilon)d^3\kappa = 2\pi(2\mu/\hbar)^{3/2}g_{12}(\varepsilon)\sqrt{\varepsilon}d\varepsilon$ (here, $g_{12}(\varepsilon)\sqrt{\varepsilon}$ is normalized to unity). The effect of degeneracy becomes apparent when $g_{12}$ (solid lines) are compared to the corresponding Maxwell-Boltzmann distributions (dashed lines). The inset shows the corresponding single-particle distribution $g(E)\sqrt{E}$.

Normalized Signal (arb.units)

Signal (arb.units)

$T / T_F = 0.1$

$T / T_F = 0.5$

$T / T_F = 0.9$

FIG. 3: Lineshapes as the functions of $\delta_2$ for temperatures $T / T_F = 0.1, 0.5, \text{and} 0.9$, with $\gamma_1 = 3 \text{ mK}$, $\delta_1 = 0.56 \text{ mK}$, and $\Omega_2 = 100 \mu\text{K}$. The definitions of $K''$, $K_{\max}$, $K_{\min}$, and $\delta_2^{\max}$ are illustrated in the inset.
FIG. 4: Ratio $C$ from Eq. (6) as a function of $T/T_F$. The sensitivity of $C$ on other parameters is also illustrated by two other cases (dashed lines).