Eccentric comets approach $v_{esc}$ at perihelion

\[ v_{peri}^2 = \mu \left( \frac{2}{q} - \frac{1}{a} \right) \]

\[ v_{esc}^2 = \mu \frac{2}{q} \]
Small particles are subject to radiation pressure

- Radiation pressure follows inverse square law
- Reduces central potential by $\beta$:

$$\beta = \frac{F_r}{F_g}$$

- Effect is inversely proportional to size (and density):

$$\beta \propto \frac{1}{s}$$
Meteoroids can be ejected directly onto escape trajectories. For \( \beta \geq 1 \), there are no bound orbits. For \( \beta < 1 \), \( v_{\text{esc}} \) is reduced. Comet's velocity alone exceeds \( v_{\text{esc}} \) for:

\[
v_{\text{esc}}^2 = \mu (1 - \beta)^{\frac{2}{q}}
\]

\[
\beta > \frac{1 - e}{2}
\]

Burns, Lamy, & Soter (1979)
Ejection speed can give meteoroids a boost

- Meteoroids ejected in the direction of the comet’s motion get a boost; trailing particles the opposite.
- For large particles:
  \[ \Delta v = v_0 \sqrt{\beta} \]
  (Whipple, 1951; Jones, 1995; etc.)
- The value of \( \beta \) above which particles are unbound has an analytical solution. For leading particles:
  \[ y = \sin^{-1} \left( \frac{v_{peri}}{\sqrt{v_0^2 + v_{esc}^2}} \right) - \text{atan2}(v_{esc}, v_0) \]
  \[ \beta_L = \sin^2 y \]
- A similar equation exists for trailing particles
The only thing left to do is calculate $\beta$:

$$\beta = 5.7 \times 10^{-4} \text{ kg m}^{-2} \times \left(\frac{Q_{pr}}{\rho s}\right)$$

Geometric optics: $Q_{pr} = 1$

But there are some complications ...
What about small particles?

- For small particles:
  \[ \Delta v \propto \sqrt{\beta} \]

- Instead, we must numerically integrate (see Jones, 1995):
  \[
  \frac{d^2x}{dt^2} = \frac{\Gamma}{2} m^{-1/3} \rho_d^{-2/3} \rho_{\text{gas}}(x) \left[ v_{\text{gas}}(x) - \frac{dx}{dt} \right]^2
  \]

- Then:
  \[
  \Delta v = \left. \frac{dx}{dt} \right|_{t \to \infty}
  \]
What about small particles?

- $\Delta v$ has no analytic form, but is very close to:

$$\Delta v \simeq v_{\text{gas},0} \left( 0.38532 + 0.50341 \cdot \xi^{-1.054} \right)^{-0.949}$$

$$\xi = \frac{A \Gamma}{2} m^{-1/3} \rho_d^{-2/3} \rho_{\text{gas},0} \times c$$

Ugly, but easy to code up.

- Calculating $\beta$ is another matter.
Calculating $\beta$ for small particles and real materials

$$\beta = 5.7 \times 10^{-4} \text{ kg m}^{-2} \times (Q_{pr}/\rho s)$$

- **Geometric optics**: $Q_{pr} = 1$
- **“Ideal material”**: $Q_{pr} = 1$ for $\lambda < 2\pi s$, 0 otherwise
- **Real materials**: Calculate $Q_{pr}$ using Mie theory
  (Python code available from Navarro & Werts, 2012)
Calculating $\beta$ for real materials

- I’ll compare the “ideal material” case with one real material
- Tholins are a reddish brown polymer found on icy bodies
Perseids

Ideal material

Tholin

\[ v \text{ (km s}^{-1}\text{)} \]

\[ m \text{ (g)} \]

- Leading ejecta
- Trailing ejecta
- Escape velocity
- Ejecta lost
Forbidden mass ranges for 10 major showers

- Ideal material
- Tholin

| Shower | Ideal material | Tholin |
|--------|----------------|--------|
| CAP    |                |        |
| DRA    |                |        |
| GEM    |                |        |
| LEO    |                |        |
| LYR    |                |        |
| NTA    |                |        |
| ORI    |                |        |
| PER    |                |        |
| QUA    |                |        |
| URS    |                |        |

- $m$ (g)

- Leading ejecta lost
- Accompanying ejecta lost
- All ejecta lost

- Ideal material: $10^{-14}$ to $10^{-6}$
- Tholin: $10^{-10}$ to $10^{-6}$
Small meteoroids originating from eccentric comets may be on unbound orbits. We’ve extended this to handle the ejection velocity imparted by the sublimation process:

- Analytic solution for $\beta$ limit for large particles
- Semi-numerical solution for $\Delta v$ (and thus $\beta$ limit) for all particles
- New $\Delta v$ equation also useful for stream modeling
- We’ve calculated $\beta$ for small particles/real materials.
  - Ideal material: very small particles may remain in stream
  - Tholins: small particles do not remain in stream

- Large comets: some small particles can still be ejected
- Eccentric comets: excluded range can be large: no Lyrids smaller than $4 \times 10^{-7}$ g