New physics effects in radiative leptonic $B_s$ decay

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(Dated: May 8, 2018)

There are several anomalous measurements in the decays induced by the quark level transition $b \to s \mu^+ \mu^-$, which do not agree with the predictions of the Standard Model. These measurements could be considered as signatures of physics beyond the Standard Model. Working within the framework of effective field theory, several groups have performed global fits to all $b \to s \mu^+ \mu^-$ data to identify the Lorentz structure of possible new physics. Many new physics scenarios have been suggested to explain the anomalies in the $b \to s \mu^+ \mu^-$ sector. In this work, we investigate the impact of these new physics scenarios on the radiative leptonic decay of $B_s$ meson. We consider the branching ratio of this decay along with the ratio $R_\gamma$ of the differential distribution $B_s \to \mu^+ \mu^- \gamma$ & $B_s \to e^+ e^- \gamma$ and the muon forward-backward asymmetry $A_{FB}$. We find that the predicted values of these observables are close to the SM results for all allowed new physics solutions. Hence we find that the present $b \to s \mu^+ \mu^-$ data does not allow large deviation in any of these $B_s \to \mu^+ \mu^- \gamma$ observables from their SM predictions.
I. INTRODUCTION

Over the last few years there have been several measurements in the $B$-meson sector, more specifically in decays induced by the flavor changing neutral current (FCNC) quark level transition $b \rightarrow s l^+ l^-$, which are incompatible with the predictions of the Standard Model (SM). The following measurements have been of rigorous attention:

- In 2012, the LHCb collaboration reported the measurement of the ratio $R_K \equiv \Gamma(B^+ \rightarrow K^+ \mu^+ \mu^-)/\Gamma(B^+ \rightarrow K^+ e^+ e^-)$ performed in the low dilepton invariant mass-squared $q^2$ range $(1.0 \leq q^2 \leq 6.0 \text{ GeV}^2)$ \cite{1}, which deviates from the SM prediction of order $\approx 1$ \cite{2, 3} by $2.6 \sigma$. This could be an indication of lepton flavor universality violation in the $b \rightarrow s l^+ l^-$ sector.

- The measurement of $R_K$ was further corroborated recently by the measurement of the $R_{K^*} \equiv \Gamma(B^0 \rightarrow K^{*0} \mu^+ \mu^-)/\Gamma(B^0 \rightarrow K^{*0} e^+ e^-)$ in the low $(0.45 \leq q^2 \leq 1.1 \text{ GeV}^2)$ and central $(1.1 \leq q^2 \leq 6.0 \text{ GeV}^2)$ $q^2$ bins \cite{1}. These measurements differ from the SM prediction of order $\approx 1$ \cite{2, 3} by 2.2-2.4$\sigma$ and 2.4-2.5$\sigma$, in the low and central $q^2$ regions, respectively.

- The experimentally measured values of some of the angular observables in $B \rightarrow K^* \mu^+ \mu^-$ \cite{5-7} disagree with their SM predictions \cite{8}. In particular, the angular observable $P_5'$ in the $q^2$-bin 4.3-8.68 disagrees with the SM at the level of 4$\sigma$. The recent ATLAS \cite{9} and CMS \cite{10} measurements confirm this disagreement.

- The measured value of the branching ratio of $B_s \rightarrow \phi \mu^+ \mu^-$ \cite{11, 12} does not agree with its SM value. This disagreement is at the level of 3$\sigma$.

The measurements of $R_K$ and $R_{K^*}$ could be an indication of presence of new physics in $b \rightarrow s \mu^+ \mu^-$ and/or $b \rightarrow s e^+ e^-$ sector where as the discrepancies in $P_5'$ and the branching ratio of $B_s \rightarrow \phi \mu^+ \mu^-$ are related to $b \rightarrow s \mu^+ \mu^-$ sector only. Hence it is quite natural to account for all of these anomalies by assuming new physics only in the $b \rightarrow s \mu^+ \mu^-$ sector. A recent global fit \cite{13} also favours this point of view.

In order to probe the Lorentz structure of new physics in $b \rightarrow s \mu^+ \mu^-$, several model independent analyses have been performed. It is observed that the present $b \rightarrow s \mu^+ \mu^-$ data can be accommodated by new physics in the form of vector ($V$) and axial-vector operators ($A$) \cite{13, 21}. However there is no unique solution. For e.g., new physics operator $O_9 = (\bar{s} \gamma^\mu P_L b)(\bar{l} \gamma^\mu l)$ alone as well as a combination of operators $O_9$ and $O_{10} = (\bar{s} \gamma^\mu P_L b)(\bar{l} \gamma^\mu \gamma^5 l)$ can both account for all
\[ b \rightarrow s \mu^+ \mu^- \] anomalous data. Therefore one needs additional observables to discriminate between various possible solutions, and hence identify the Lorentz structure of new physics in \( b \rightarrow s \mu^+ \mu^- \) sector. In this work we consider radiative leptonic decay of \( B_s \) meson to explore such a possibility.

The decay \( B_s \rightarrow \mu^+ \mu^- \gamma \) has several advantages over its non radiative counterpart \( B_s \rightarrow \mu^+ \mu^- \). In the SM, the decay \( B_s \rightarrow \mu^+ \mu^- \) is chirally suppressed, and hence has a smaller branching ratio. On the other hand, the decay \( B_s \rightarrow \mu^+ \mu^- \gamma \) is free from chirality suppression owing to the emission of a photon in addition to the muon pair. The photon emission, however, suppresses the branching ratio of \( B_s \rightarrow \mu^+ \mu^- \gamma \) by a factor of \( \alpha_{em} \). The SM prediction of the branching ratio of \( B_s \rightarrow \mu^+ \mu^- \gamma \) is of the order \( \sim 10^{-8} \) and hence would be helpful in the experimental observation.

Further, \( B_s \rightarrow \mu^+ \mu^- \) is sensitive to a wider range of new physics operators as compared to that of \( B_s \rightarrow \mu^+ \mu^- \). It is sensitive to the new physics operators \( O_9, O_{10} \), and also their chirality flipped counterparts \( O'_9 = (\bar{s}\gamma\mu P_R b) (\bar{l}\gamma\mu l) \) and \( O'_{10} = (\bar{s}\gamma\mu P_R b) (\bar{l}\gamma\mu\gamma^5 l) \). Hence it is sensitive to all new physics operators which can provide a possible explanation for present \( b \rightarrow s \mu^+ \mu^- \) anomalies.

From the experimental point of view, the observation of \( B_s \rightarrow \mu^+ \mu^- \gamma \) decay is a challenging task. The photon in the final state is difficult to detect, in general, the detection efficiency of photon is smaller as compared to that of the charged leptons. Further, the photon makes the other daughter particles softer which results in smaller reconstruction efficiencies. Hence the observation of the \( B_s \rightarrow \mu^+ \mu^- \gamma \) decay seems to be a rather challenging task. However due to the fact that its branching ratio is \( \sim 10^{-8} \), this decay mode might not be beyond the reach of the higher runs of the LHC. In ref. [22], a method was suggested for the detection of \( B_s \rightarrow \mu^+ \mu^- \gamma \) by making use of the event sample selected for the measurements of the branching ratio of \( B_s \rightarrow \mu^+ \mu^- \). This would be applicable at Run 2 of the LHC.

The decay \( B_s \rightarrow \mu^+ \mu^- \gamma \) has been studied in [23–34]. In this work we perform a model independent analysis of \( B_s \rightarrow \mu^+ \mu^- \gamma \) decay by considering all new physics \( V \) and \( A \) operators. Apart from the branching ratio, \( B(B_s \rightarrow \mu^+ \mu^- \gamma) \), we consider the ratio \( R_\gamma \equiv \Gamma(B_s \rightarrow \mu^+ \mu^- \gamma)/\Gamma(B_s \rightarrow e^+ e^- \gamma) \) and the forward backward asymmetry \( (A_{FB}) \) of muons. We obtain predictions for these observables for the various new physics solutions which provide a good fit to the present \( b \rightarrow s \mu^+ \mu^- \) data. We intend to identify the new physics interactions which can provide large deviation in these observables. We find that the present \( b \rightarrow s \mu^+ \mu^- \) data do now allow large deviations in any of these observables.

The paper is organized as follows. In Section II, we provide theoretical expressions for various observables in \( b \rightarrow s \mu^+ \mu^- \gamma \) decay in the presence of new physics in the form of \( V \) and \( A \) operators. The predictions for \( B(B_s \rightarrow \mu^+ \mu^- \gamma), R_\gamma \) and \( A_{FB} \) of muons for the existing new physics solutions
II. $B_s \to \mu^+ \mu^- \gamma$ DECAY

The effect of new physics in the $b \to s l^+ l^- \gamma$ decays can be most conveniently probed by making use of the effective field theory approach where the new physics contributions are encoded in the Wilson coefficients of the operators of the $b \to s l^+ l^-$ effective Hamiltonian. The decay $B_s \to l^+ l^- \gamma$ is induced by the effective Hamiltonian for the quark level transition $b \to s l^+ l^-(l = e, \mu)$, and is given by,

$$H_{\text{eff}}^{SM}(b \to s l^+ l^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{2\pi} V_{ts} V_{tb}^* \left\{ -2i m_b \frac{C_7(\mu)}{q^2} \cdot \vec{s} \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b \cdot \vec{l} \gamma^\mu l \\
+ C_9(\mu) \cdot \vec{s} \gamma_\mu (1 - \gamma_5) b \cdot \vec{\gamma}^\mu l + C_{10}(\mu) \cdot \vec{s} \gamma_\mu (1 - \gamma_5) b \cdot \vec{l} \gamma^\mu \gamma_5 l \right\}, \tag{1}$$

where $G_F$ is the Fermi constant, and $V_{ts}, V_{tb}$ are the elements of the Cabbibo-Kobayashi-Maskawa (CKM) matrix. The Wilson Coefficients $C_9$ and $C_{10}$ above are associated with the standard short-distance semi-leptonic operators $O_9$ and $O_{10}$ respectively,

$$O_9 = (\bar{s} \gamma^\mu P_L b) (\bar{l} \gamma^\mu l), \quad O_{10} = (\bar{s} \gamma^\mu P_L b) (\bar{l} \gamma^\mu \gamma_5 l) \tag{2}$$

where $P_{L,R} = (1 \mp \gamma^5)/2$.

The remaining dominant contribution to this decay emerges from the Wilson Coefficient $C_7$, associated with the magnetic penguin operator, $O_7 = (\bar{s} \sigma_{\mu\nu} q^\nu P_R b) (\bar{l} \gamma^\mu l)$. The operators $O_{7,9,10}$ present in the effective Hamiltonian contributing to the $B_s^0 \to \mu^+ \mu^- \gamma$ amplitude can be parameterised in terms of the $B_s \to \gamma^*$ form factors as follows [34],

$$\langle \gamma^*(k, \epsilon) | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}_s(p) \rangle = i e \epsilon_\alpha^* \left( g_{\mu\alpha} k' k - k'_\alpha k_\mu \right) \frac{F_A(k'^2, k^2)}{M_{B_s}},$$

$$\langle \gamma^*(k, \epsilon) | \bar{s} \gamma_\mu b | \bar{B}_s(p) \rangle = e \epsilon_\alpha^* \epsilon_{\mu\alpha\xi\eta} k'_\xi k_\eta \frac{F_V(k'^2, k^2)}{M_{B_s}},$$

$$\langle \gamma^*(k, \epsilon) | \bar{s} \sigma_{\mu\nu} \gamma_5 b | \bar{B}_s(p) \rangle = e \epsilon_\alpha^* \left( g_{\mu\alpha} k' k - k'_\alpha k_\mu \right) \frac{F_{TA}(k'^2, k^2)}{M_{B_s}},$$

$$\langle \gamma^*(k, \epsilon) | \bar{s} \sigma_{\mu\nu} b | \bar{B}_s(p) \rangle = i e \epsilon_\alpha^* \epsilon_{\mu\alpha\xi\eta} k'_\xi k_\eta \frac{F_{TV}(k'^2, k^2)}{M_{B_s}},$$

(3)

where $k$ is the momentum of the photon emitted from the valence quark of the $B_s$ meson and $k'$ is the momentum emitted from the $b \to s$ penguin vertex. The form factors relevant for this process are $F_i(k'^2 = q^2, k^2 = 0) = F_i(q^2)$ and $F_i(k'^2 = 0, k^2 = q^2) = F_i(0, q^2)$ with $q^2$ being the momentum of the lepton pair and $i = \{V, A, TV, TA\}$. These form factors can be parameterised as,

$$F_i(q^2) = \frac{F_i(0)}{(1 - q^2/M_{R_i}^2)(1 - \sigma_1(q^2/M_{R_i}^2) + \sigma_2(q^2/M_{R_i}^2)^2)} \tag{4}.$$
The above form factor definition uses a modified pole parameterization as opposed to single pole one \[26\], to explicitly take into account the poles at \( q^2 = M^2_R \) where \( M_R \) is the mass of the meson and provide better precision in the entire region of \( q^2 \). The details of the calculation of these form factors as well as the numerical values of the parameters can be found in Ref. \[34\]. Further,

\[
F_{TV,TA}(0, q^2) = F_{TV,TA}(0, 0) - \sum_V 2 f^e.m. g^{B \rightarrow V}(0) \frac{q^2/M_V}{q^2 - M^2_V + iM_V \Gamma_V}, \tag{5}
\]

where the values of the mass and width of the vector meson resonances, \( M_V \) and \( \Gamma_V \), respectively and the \( B \rightarrow V \) transition form factors \( g^{B \rightarrow V}(0) \) can be found in Ref. \[34\]. In our analyses of the different observables for this decay, we consider systematic uncertainty arising from the form factors to be about 10% in the low \( q^2 \) region and 20% in the high \( q^2 \) region.

The global analyses of the \( b \rightarrow s l^+ l^- \) anomalies have shown that if new physics is present, it will contribute mainly via the operators \( O_9 \) and \( O_{10} \) and their chirality flipped counterparts. Therefore to study new physics effects in the \( B_s \rightarrow l^+ l^- \gamma \) transition, we consider new physics in the form of \( O^{(\prime)}_9 \) and \( O^{(\prime)}_{10} \) operators. The effective Hamiltonian in the presence of these additional new physics operators is,

\[
H_{\text{eff}}(b \rightarrow s l^+ l^-) = H^{\text{SM}}_{\text{eff}}(b \rightarrow s l^+ l^-) + H^{\text{VA}}_{\text{eff}}(b \rightarrow s l^+ l^-), \tag{6}
\]

where \( H^{\text{VA}}_{\text{eff}}(b \rightarrow s l^+ l^-) \) is given by,

\[
H^{\text{VA}}_{\text{eff}}(b \rightarrow s l^+ l^-) = \frac{\alpha G_F}{\sqrt{2}\pi} V^*_{ts} V_{tb} \left[ C^{NP}_9 (\bar{s} \gamma^\mu P_L b)(\bar{l} \gamma_\mu l) + C^{NP}_{10} (\bar{s} \gamma^\mu P_L b)(\bar{l} \gamma_\mu \gamma_5 l) 
+ C^{tNP}_9 (\bar{s} \gamma^\mu P_R b)(\bar{l} \gamma_\mu l) + C^{tNP}_{10} (\bar{s} \gamma^\mu P_R b)(\bar{l} \gamma_\mu \gamma_5 l) \right], \tag{7}
\]

where \( C^{NP}_9 \) and \( C^{NP}_{10} \) are the new physics couplings associated with the operators \( O_9 \) and \( O_{10} \), respectively while \( C^{tNP}_9 \) and \( C^{tNP}_{10} \) are the coefficients of the the primed operators \( O'_9 \) and \( O'_{10} \), respectively which are obtained by replacing \( P_L \) by \( P_R \) in Eq. \[2\].

The decay \( B^0_s \rightarrow \mu^+ \mu^- \gamma \) receives contributions from many channels \[33, 34\]. We present the amplitudes for these channels in the presence of additional new physics VA operators, defined in Eq. \[7\] as follows:

- The amplitude for the emission of a real photon from the valence quarks of \( B_s \) meson and a
lepton pair from the FCNC vertex is given by,

\[
A^{(1)} = \langle \gamma(k', \epsilon), l^+(p_1), l^-(p_2) | H_{eff}^{b\rightarrow s l\rightarrow l} | B_s(p) \rangle = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{2\pi} e \epsilon_\alpha^* \\
\times \left[ (C_{9}^{NP} + C_{9}^{NP}) P_{\mu\alpha}^{\perp} F_{V}(q^2) - (C_{9}^{NP} + C_{9}^{NP}) P_{\mu\alpha}^{\parallel} F_{A}(q^2) M_{B_s} \right] \bar{l}(p_2) \gamma_\mu l(-p_1) \\
+ \left( C_{10}^{NP} + C_{10}^{NP} \right) P_{\mu\alpha}^{\perp} F_{V}(q^2) M_{B_s} - \left( C_{10}^{NP} + C_{10}^{NP} \right) P_{\mu\alpha}^{\parallel} F_{A}(q^2) M_{B_s} \right] \bar{l}(p_2) \gamma_\mu \gamma_5 l(-p_1) \\
+ \frac{2C_{7\gamma}(\mu)}{q^2} m_b \left( P_{\mu\alpha}^{\perp} F_{TV}(q^2, 0) - P_{\mu\alpha}^{\parallel} F_{TA}(q^2, 0) \right) \bar{l}(p_2) \gamma_\mu l(-p_1),
\]

where

\[
P_{\mu\alpha}^{\perp} = \epsilon_{\mu\alpha\xi\eta} k_\xi^l k_\eta^l, \quad P_{\mu\alpha}^{\parallel} = i (g_{\mu\alpha} k^l - k_\alpha^l k_\mu^l).
\]

In this process, the momentum from the FCNC vertex, \( k' = q \) and the momentum emitted from the valence quark, \( k = p - q \). So \( k'^2 = q^2 \) and \( k^2 = 0 \) and the form factors \( F_i(q^2, 0) \) given in Eq. 4 contribute.

- The amplitude for the emission of a virtual photon from the valence quark of \( B_s \) meson and a real photon from the FCNC vertex is,

\[
A^{(2)} = \langle \gamma(k', \epsilon), l^+(p_1), l^-(p_2) | H_{eff}^{b\rightarrow s \gamma} | B_s(p) \rangle = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{2\pi} e \epsilon_\alpha^* \bar{l}(p_2) \gamma_\alpha l(-p_1) \\
\times \left[ \frac{2m_bC_{7\gamma}(\mu)}{q^2} \right] \left( P_{\mu\alpha}^{\perp} F_{TV}(0, q^2) - P_{\mu\alpha}^{\parallel} F_{TA}(0, q^2) \right),
\]

where \( k^2 = q^2 \) and \( k'^2 = 0 \) and the form factors appearing in the amplitude are \( F_{TV}(0, q^2) \) and \( F_{TA}(0, q^2) \) defined in Eq. 5.

- The amplitude for bremsstrahlung emission from the leptons in the final state is given by,

\[
A^{Brems} = -i e \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{2\pi} V_{td}^* V_{tb} \frac{f_{B_s}}{M_{B_s}} \frac{2m_i}{M_{B_s}^2} \bar{l}(p_2) \left( \frac{(\gamma\epsilon^*) (\gamma p)}{t - \hat{m}_i^2} - \frac{(\gamma p) (\gamma\epsilon^*)}{\hat{u} - \hat{m}_i^2} \right) \gamma_5 l(-p_1) \\
\times \left( C_{10}(\mu) + C_{10}^{NP} - C_{10}^{NP} \right)
\]

This contribution is suppressed compared to \( A^{(1)} \) by the lepton mass but is important in the high \( q^2 \) region, when \( q^2 \) approaches \( M_{B_s}^2 \).

In an effective theory approach, the heavy degrees of freedom like the top quark and W/Z boson masses are integrated out, however the effects of lighter degrees of freedom like charm and up quarks need to be taken into account in the loops. The contribution of the charm quarks to the
\( B \rightarrow \gamma^* \gamma^* \) amplitude, at the lowest order arise from the charming penguin topology and the weak annihilation topology.

We now discuss the contribution of the penguin diagrams containing the charm quarks in the loop. The amplitudes of these penguin diagrams can be found in Ref. [34]. These amplitudes have the same Lorentz structure as that of the amplitudes \( A^{(1)} \) and \( A^{(2)} \) defined in Eq. (8) and Eq. (10). Hence, the form factor in these penguin diagram amplitudes can be expressed as corrections to the Wilson coefficient \( C_9 \) appearing in the amplitudes \( A^{(1)} \) and \( A^{(2)} \). These corrections arising due to the charm loop effects consist of factorizable contribution and non-factorizable contribution arising due to soft gluon exchanges. The non-factorizable corrections due to charm-loop for the \( B \rightarrow \gamma l^+ l^- \) amplitude have not been calculated yet. These non-factorizable corrections have been computed for \( B \rightarrow K^* l^+ l^- \) amplitude [35], and are a good approximation for \( B \rightarrow \mu^+ \mu^- \gamma \) at low \( q^2 \). We implement these factorizable and non-factorizable corrections in the low \( q^2 \) region by adding to \( C_9 \), a simplified \( q^2 \)-dependent parameterization of these corrections, taken from Ref. [36]. For larger \( q^2 \) region, above the \( D \bar{D} \) threshold, the invariant amplitude is expressed in the form of a dispersion relation with a spectral density function. The spectral density function is modeled by an effective pole, the details of which can be found in Ref. [36].

The amplitude of the weak annihilation diagrams including the QCD corrections is [34],

\[
A^{WA} = -\frac{G_F}{\sqrt{2}} \alpha_{em} \epsilon \cdot a_1 \{ V_{ub} V^*_{ud} + V_{cb} V^*_{cd} \} \frac{16}{3} \epsilon_{\mu\nu\rho\sigma} q^\nu q^\rho \bar{l}_\gamma l^\mu.
\] (12)

The amplitude \( A^{(2)} \) has a structure similar to the \( C_7 \gamma \) part of \( A^{(1)} \) except for the form factors. Similarly, the Lorentz structure of the Weak annihilation amplitude as given in Eq. (12) is the same as that of \( C_7 \gamma P_{\mu\alpha}^\perp \) in the amplitude \( A^{(1)} \). These two amplitudes can therefore be combined with \( A^{(1)} \) by redefining the form factors as follows,

\[
\bar{F}_{TV}(q^2) = F_{TV}(q^2, 0) + F_{TV}(0, q^2) - \frac{16}{3} \frac{V_{ub} V^*_{us} + V_{cb} V^*_{cs}}{V_{ub} V^*_{ts}} \frac{a_1 f_B}{C_7 \gamma m_b},
\] (13)

\[
\bar{F}_{TA}(q^2) = F_{TA}(q^2, 0) + F_{TA}(0, q^2).
\] (14)

The double differential decay rate for \( B_s \rightarrow \mu^+ \mu^- \gamma \) process in the presence of new physics \( V \) and \( A \) operators can now be calculated using the amplitudes and form factors defined above. It is expressed as a sum of three contributions:

- \( d^2 \Gamma^{(1)}/d\hat{s}d\hat{t} \), which receives contributions from the combined amplitudes \( (A^{(1)} + A^{(2)} + A^{(WA)}) \),

- \( d^2 \Gamma^{(2)}/d\hat{s}d\hat{t} \), which is the contribution from the bremsstrahlung amplitude \( A^{\text{Brems}} \), and
These decay rates are as given below,
\[
\frac{d^2\Gamma^{(1)}}{d\hat{s} \, d\hat{t}} = \frac{G_F^2 \alpha_{em}^3 M_{B_s}^5}{210 \pi^4} |V_{tb} V_{ts}^*|^2 \left[ x^2 B_0 (\hat{s}, \hat{t}) + x \xi (\hat{s}, \hat{t}) \hat{B}_1 (\hat{s}, \hat{t}) + \xi^2 (\hat{s}, \hat{t}) \hat{B}_2 (\hat{s}, \hat{t}) \right],
\]
where
\[
B_0 (\hat{s}, \hat{t}) = (\hat{s} + 4\hat{m}_t^2) (F_1 (\hat{s}) + F_2 (\hat{s})) - 8\hat{m}_t^2 \left| C_{10}(\mu) + C_{10}^{NP} + C_{10}^{tNP} \right|^2 F_V(q^2) + \left| C_{10}(\mu) + C_{10}^{NP} - C_{10}^{tNP} \right|^2 F_A(q^2),
\]
\[
B_1 (\hat{s}, \hat{t}) = 8 [\hat{s} F_V(q^2) F_A(q^2) Re \left( (C_{9}^{eff} \mu, q^2) + C_{9}^{NP} + C_{9}^{tNP} \right) (C_{10}(\mu) + C_{10}^{NP} - C_{10}^{tNP})] + \hat{m}_b F_V(q^2) Re \left[ C_{7}(\mu) F_{TV}(q^2) (C_{10}(\mu) + C_{10}^{NP} - C_{10}^{tNP}) \right] + \hat{m}_b F_A(q^2) Re \left[ C_{7}(\mu) F_{TA}(q^2) (C_{10}(\mu) + C_{10}^{NP} - C_{10}^{tNP}) \right],
\]
\[
B_2 (\hat{s}, \hat{t}) = \hat{s} (F_1 (\hat{s}) + F_2 (\hat{s})),
\]
and
\[
F_1 (\hat{s}) = \left( \left| C_{9}^{eff} \mu, q^2 \right| + C_{9}^{NP} + C_{9}^{tNP} \right)^2 + \left| C_{10}(\mu) + C_{10}^{NP} + C_{10}^{tNP} \right|^2 F_V(q^2) + \left( \frac{2\hat{m}_b}{\hat{s}} \right)^2,
\]
\[
F_2 (\hat{s}) = \left( \left| C_{9}^{eff} \mu, q^2 \right| + C_{9}^{NP} - C_{9}^{tNP} \right)^2 + \left| C_{10}(\mu) + C_{10}^{NP} - C_{10}^{tNP} \right|^2 F_A(q^2) + \left( \frac{2\hat{m}_b}{\hat{s}} \right)^2.
\]

\[
\frac{d^2\Gamma^{(2)}}{d\hat{s} \, d\hat{t}} = \frac{G_F^2 \alpha_{em}^3 M_{B_s}^5}{210 \pi^4} |V_{tb} V_{ts}^*|^2 \left( \frac{8 f_{B_s}}{M_B} \right)^2 \hat{m}_t^2 \left| C_{10}A(\mu) + C_{10}^{NP} - C_{10}^{tNP} \right|^2 \times
\]
\[
\left[ \frac{\hat{s} + x^2 / 2}{(\hat{u} - \hat{m}_t^2)(\hat{t} - \hat{m}_t^2)} - \left( \frac{x \hat{m}_t^2}{(\hat{u} - \hat{m}_t^2)(\hat{t} - \hat{m}_t^2)} \right)^2 \right],
\]
\[
\frac{d^2\Gamma^{(12)}}{d\hat{s} \, d\hat{t}} = -\frac{G_F^2 \alpha_{em}^3 M_{B_s}^5}{210 \pi^4} |V_{tb} V_{ts}^*|^2 \frac{16 f_{B_s}}{M_{B_s}} \hat{m}_t^2 \frac{x^2}{(\hat{u} - \hat{m}_t^2)(\hat{t} - \hat{m}_t^2)} \left[ \frac{2 \hat{m}_b}{\hat{s}} Re \left( (C_{10}A(\mu) + C_{10}^{NP} - C_{10}^{tNP}) \right) \right]
\]
\[
+ x F_V(q^2) Re \left[ (C_{10}A(\mu) + C_{10}^{NP} - C_{10}^{tNP})(C_{9}^{eff} \mu, q^2) + C_{9}^{NP} + C_{9}^{tNP} \right] \]
\[
+ \xi(\hat{s}, \hat{t}) F_A(q^2) \left| C_{10}A(\mu) + C_{10}^{NP} - C_{10}^{tNP} \right|^2 \right].
\]

Here
\[
\hat{s} = \frac{(p - k)^2}{M_{B_s}^2}, \quad \hat{t} = \frac{(p - p_1)^2}{M_{B_s}^2}, \quad \hat{u} = \frac{(p - p_2)^2}{M_{B_s}^2},
\]
with \( \hat{s} + \hat{t} + \hat{u} = 1 + 2 \hat{m}_l^2, \hat{m}_l^2 = m_l^2/M_{B_s}^2, \hat{m}_b = m_b/M_{B_s}, \) and \[26\]

\[
x = 1 - \hat{s}, \quad \cos \theta = \frac{\xi(\hat{s}, \hat{t})}{x \sqrt{1 - 4 \hat{m}_l^2/\hat{s}}}, \quad \xi(\hat{s}, \hat{t}) = \hat{u} - \hat{t}.
\]

The total differential branching ratio is then given by,

\[
\frac{dB(B_s \to \mu^+ \mu^- \gamma)}{dq^2} = \frac{\tau_{B_s}}{m_{B_s}^2} \int dt \left( \frac{d^2 \Gamma^{(1)}}{d\hat{s} dt} + \frac{d^2 \Gamma^{(2)}}{d\hat{s} dt} + \frac{d^2 \Gamma^{(12)}}{d\hat{s} dt} \right).
\]

(23)

We also consider the ratio,

\[
R_\gamma(q^2) \equiv \frac{d\Gamma(B_s \to \mu^+ \mu^- \gamma)/dq^2}{d\Gamma(B_s \to e^+ e^- \gamma)/dq^2},
\]

(24)

along with the forward backward asymmetry of muons,

\[
A_{FB}(q^2) = \frac{\int_{-1}^{1} d\cos \theta \frac{d^2 \Gamma(B_s \to l^+ l^- \gamma)}{dq^2 d\cos \theta} - \int_{-1}^{0} d\cos \theta \frac{d^2 \Gamma(B_s \to l^+ l^- \gamma)}{dq^2 d\cos \theta}}{\int_{-1}^{1} d\Gamma(B_s \to l^+ l^- \gamma)/dq^2},
\]

(25)

where \( \theta \), the angle between the momentum of \( B_s \) meson \( \vec{p} \) and \( \vec{p}_2 \), the momentum of the lepton, can be expressed in terms of \( \hat{t} \) as given in Eq. 22.

In the next section, we obtain predictions for these observables for various new physics solutions which provide a good fit to the present \( b \to s \mu^+ \mu^- \) data.

### III. RESULTS AND DISCUSSIONS

After the measurement of \( R_{K^*} \), several groups have performed global fits to all the \( b \to s \mu^+ \mu^- \) data to identify one or combinations of Wilson coefficients which provide a good fit to the data \[13\] \[20\]. In most of the analyses, three scenarios: (I) \( C_9^{\mu\mu} \) (NP) \(< 0 \), (II) \( C_9^{\mu\mu} \) (NP) = \(- C_{10}^{\mu\mu} \) (NP) \(< 0 \) and (III) \( C_9^{\mu\mu} \) (NP) = \(- C_{10}^{\mu\mu} \) (NP) \(< 0 \) were suggested as an explanation of anomalies in the \( b \to s \mu^+ \mu^- \) sector \[13\]. The numerical values of the Wilson coefficients corresponding to these scenarios are listed in Table II.

Eventually these scenarios must arise in some NP models. The simplest NP models that can give rise to these scenarios involve the tree-level exchange of a leptoquark or a \( Z' \) boson. There are three leptoquark models that can explain the data in the \( b \to s \mu^+ \mu^- \) sector. They are scalar triplet with \( Y = 1/3 \) (\( S_3 \)), vector isosinglet with \( Y = -2/3 \) (\( S_3 \)) and vector isotriplet with \( Y = -2/3 \) (\( U_3 \)). All of these leptoquark models give rise to scenario (II). The first and third scenarios can only be achieved in \( Z' \) models. \( Z' \) couples vectorially to \( sb \) in scenarios (I) and (II), and hence one can easily construct NP models. Scenario (III) requires an axial-vector coupling of the \( Z' \) to
TABLE I: New physics scenarios suggested as an explanation for all $b \to s \mu^+ \mu^-$ data. The numerical values of Wilson coefficients are taken from [20].

| Scenario         | WC                  | Operator                                      |
|------------------|---------------------|-----------------------------------------------|
| (I) $C_9^{\mu\mu}$ (NP) | $-1.25 \pm 0.19$   | $[\bar{s}\gamma_{\mu} P_L b] [\bar{\mu}\gamma_{\mu} \mu]$ |
| (II) $C_9^{\mu\mu}$ (NP) = $- C_{10}^{\mu\mu}$ (NP) | $-0.68 \pm 0.12$   | $[\bar{s}\gamma_{\mu} P_L b] [\bar{\mu}\gamma_{\mu} P_L \mu]$ |
| (III) $C_9^{\mu\mu}$ (NP) = $- C_9'^{\mu\mu}$ (NP) | $-1.11 \pm 0.17$   | $[\bar{s}\gamma_{\mu} \gamma_{5} b] [\bar{\mu}\gamma_{\mu} \mu]$ |

FIG. 1: Left and right panel correspond to the differential branching ratio, $dB/dq^2$, in low and high-$q^2$ regions, respectively. The green curve corresponds to the central value of the SM prediction. The red, blue and orange curves correspond to $dB/dq^2$ for scenarios (I), (II) and (III), respectively at the best fit values of the WCs.

The differential branching ratio $dB/dq^2$ in the low and high $q^2$ regions for various NP scenarios listed in Table I are depicted in Fig. 1. It can be seen from the figure that none of the NP scenarios can provide large deviation from the SM prediction. Hence the decay $B_s \to \mu^+ \mu^-$ is expected to be observed with a branching ratio close to its SM prediction.
the integrated values of the branching ratio of $B_s \rightarrow \mu^+ \mu^- \gamma$. The SM predictions are,

\[ B(B_s \rightarrow \mu^+ \mu^- \gamma)_{q^2 \in (1,6) \text{GeV}^2} = (6.05 \pm 0.49) \times 10^{-9}, \]

\[ B(B_s \rightarrow \mu^+ \mu^- \gamma)_{q^2 \in (16,24) \text{GeV}^2} = (2.69 \pm 0.81) \times 10^{-10} \]  \hfill (26)

Here we have added uncertainties in the form-factor and the uncertainty in the contribution of the light vector meson $\phi$ (in the low $q^2$ region) in quadrature. In the low-$q^2$ region, the prediction for $B(B_s \rightarrow \mu^+ \mu^- \gamma)$ for the NP scenarios (I), (II) and (III) are $(6.09 \pm 0.50) \times 10^{-9}$, $(6.05 \pm 0.49) \times 10^{-9}$ and $(6.08 \pm 0.49) \times 10^{-9}$ respectively. The $B(B_s \rightarrow \mu^+ \mu^- \gamma)$ for scenarios (I), (II) and (III) in the high-$q^2$ region are $(2.30 \pm 0.65) \times 10^{-10}$, $(1.85 \pm 0.57) \times 10^{-10}$ and $(2.60 \pm 0.81) \times 10^{-10}$, respectively. Thus we see that the predictions for all new physics scenarios are consistent with the SM.

The ratio $R_\gamma(q^2)$ in the low and high $q^2$ regions for various NP scenarios are presented in Fig. 2. It can be seen from the figure that the SM prediction of $R_\gamma(q^2)$ is close to 1 in the entire low-$q^2$ region. In the high-$q^2$ region, $R_\gamma(q^2) \sim 1$ for $q^2 < 18 \text{ GeV}^2$. Above $q^2 = 18 \text{ GeV}^2$, the value of $R_\gamma(q^2)$ starts to increase from unity and at the extreme end of the $q^2$ spectrum, $R_\gamma(q^2)$ increases up to 3.5. The value of $R_\gamma$ deviates from unity mainly due to lepton mass effects in the bremsstrahlung contribution to the $B_s \rightarrow l^+ l^- \gamma$ decay rate. At low $q^2$, the bremsstrahlung amplitude is suppressed by $O(m_l/m_{B_s})$ compared the amplitude $A^{(1)}$. At higher $q^2$ values when $q^2 \rightarrow M^2_{B_s}$, the contribution from bremsstrahlung being proportional to $m_t^2/M^3_{B_s} \times (M^4_{B_s} + q^4)/(M^2_{B_s} - q^2)$, starts to dominate the total branching ratio and hence increases $R_\gamma$ above 1. The contribution from the interference
amplitude, being proportional to \( m_\ell^2 (M_{Bs}^2 - q^2)^2 \), is small compared to the bremsstrahlung contribution at large \( q^2 \). The terms containing the Wilson coefficients \( C_9 \) and \( C_{10} \) are mainly dominant at large \( q^2 \) while the contribution from terms containing \( C_{7\gamma} \) are small and can be neglected.

It can be seen from the left panel of Fig. 2 that the prediction for \( R_\gamma(q^2) \), in low-\( q^2 \) region, for scenario (III) is almost the same as that of the SM whereas there is marginal suppression for scenarios (I) and (II). In the high-\( q^2 \) region, \( R_\gamma(q^2) \) for scenarios (II) and (III) are similar to that of the SM whereas there is only a marginal enhancement for scenarios (I). Thus we see that none of the new physics scenarios can provide large deviations in \( R_\gamma(q^2) \) from its SM prediction.

![Graph](image)

**FIG. 3:** Left and right panels correspond to \( A_{FB}(q^2) \) in low and high-\( q^2 \) regions, respectively. The green curve corresponds to the central value of the SM prediction. The red, blue and orange curves correspond to \( A_{FB}(q^2) \) for scenarios (I), (II) and (III), respectively at the best fit values of the WCs.

The predictions for \( A_{FB}(q^2) \) in the low and high \( q^2 \) regions for various NP scenarios are depicted in Fig. 3. It can be observed that \( A_{FB}(q^2) \) is marginally suppressed in the entire low-\( q^2 \) region for scenario (III). For scenarios (I) and (II), there is marginal suppression for \( q^2 < 4 \text{ GeV}^2 \) whereas an enhancement is possible above \( q^2 > 4 \text{ GeV}^2 \). In the high-\( q^2 \) region too, large deviation in \( A_{FB}(q^2) \) is not possible for any of the new physics scenarios. This can also be seen from the integrated value of \( A_{FB}(q^2) \), \( \langle A_{FB}(q^2) \rangle \), in the high \( q^2 \) ([16, 24] GeV\(^2 \)) region. The SM prediction for \( \langle A_{FB}(q^2) \rangle \) in the high-\( q^2 \) region is \((-0.38 \pm 0.09)\). For scenarios (I), (II) and (III), the prediction for \( \langle A_{FB}(q^2) \rangle \) are \((-0.24 \pm 0.08), (-0.35 \pm 0.10) \) and \((-0.39 \pm 0.10) \), respectively. Thus we see that these predictions are consistent with the SM value.
IV. CONCLUSIONS

The measurements of several observables in the decays induced by the quark level transition $b \to s \mu^+ \mu^-$ do not agree with the predictions of SM. These measurements could be considered as hints of physics beyond the SM. Several new physics scenarios, all in the form of vector and axial-vector operators, were suggested as an explanation of anomalies in the $b \to s \mu^+ \mu^-$ sector. Therefore it is worth to consider the impact of these solutions on other related decay modes.

In this work we study new physics effects on the radiative leptonic decay of $B_s$ meson in the light of the present $b \to s \mu^+ \mu^-$ data. We consider contributions to the $b \to s \mu^+ \mu^- \gamma$ decay from: (i) direct emission of real or virtual photons from the valence quarks of the $B_s$ meson, (ii) emission of real photon from the $b \to s$ loop, (iii) weak annihilation, and (iv) bremsstrahlung from leptons in the final state and compute the differential branching ratio in the presence of the additional NP vector and axial-vector operators. The form factors relevant for this decay are defined in the modified pole parameterization. We also include in our analyses the charm loop effects in the form of corrections to the Wilson coefficients. We then obtain predictions for the branching ratio of $B_s \to \mu^+ \mu^- \gamma$, the ratio $R_\gamma$ of the differential distribution $B_s \to \mu^+ \mu^- \gamma$ and $B_s \to e^+ e^- \gamma$, and the muon forward-backward asymmetry $A_{FB}$ for the allowed new physics solutions which fit the present $b \to s \mu^+ \mu^-$ data best. We find that none of the solutions permit large enhancement in $B(B_s \to \mu^+ \mu^- \gamma)$, $R_\gamma$ and $A_{FB}$ as compared to their SM values.

V. ACKNOWLEDGMENT

We thank S. Uma Sankar, Dinesk Kumar and Jacky Kumar for useful discussions and suggestions.

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