Vacuum decay into Anti de Sitter space

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We propose an interpretation of decays of a false vacuum into an $AdS$ region. The $AdS$ region is interpreted in terms of a dual field theory living on an end of the world brane which expands into the false vacuum.
1. Introduction

In a classic paper \cite{1}, Coleman and de Luccia studied vacuum decay in theories of gravity. They pointed out that decays into $AdS$ generically produce a singularity. This has been interpreted by some authors as an obstacle for applying AdS/CFT \cite{2} to this problem. Here we point out that AdS/CFT can be applied to this problem in a way that gives a non-singular description of the decay process.

The idea is very simple. The bubble decay geometry contains only a portion of $AdS$. This portion can be interpreted as a field theory with a cutoff. The field theory lives on the domain wall. The domain wall has a de-Sitter geometry. Thus we have a field theory on de Sitter space. A field theory on de-Sitter space is well defined, provided we choose the standard Euclidean vacuum. Thus, this field theory gives a dual description of the whole interior of the domain wall. When we are away from the thin wall approximation, the conformal field theory is perturbed by an irrelevant (or relevant) operator. This picture is most clear in cases where there is a parametric separation between the radius of $AdS$ and the radius of curvature of the de-Sitter expanding geometry.

The conclusion is that decays into $AdS$ are conceptually similar to decays via end of the world branes, or bubbles of nothing \cite{3}.

2. A static domain wall

Here we consider a static (non-expanding) domain wall separating an $AdS$ region from flat space. Let us start in the thin wall approximation. We adjust the tension to a critical value, $T_{cr}$, in order to have a static domain wall\footnote{The critical tension is $T_{cr} = 12 \frac{M_{pl}^2}{R_{AdS}} \sim M_{pl}\sqrt{\Lambda_{AdS}}$.}. With a domain wall at $\rho = 0$, we have Minkowski space for $\rho > 0$ and $AdS$ space for $\rho < 0$. The $AdS$ metric is

$$ds^2 = d\rho^2 + e^{2\rho/R_{AdS}} dx_{2+1}^2 , \quad \text{for } \rho < 0$$ (2.1)

Via the AdS/CFT duality, we can now replace the $AdS$ region by a conformal field theory. See fig. 1. Since we only get a portion of $AdS$ space, we get a field theory with a UV cutoff. This field theory becomes conformal at low energies. Thus the whole set up can be viewed as flat space with an “end of the world” brane. On this end of the world brane we have a set of degrees of freedom which becomes conformal at low energies. In the coordinates

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in (2.1) the UV cutoff on the CFT is of the order of the radius of $AdS$, $R_{AdS}$. The rough number of degrees of freedom of the CFT is $c \propto R_{AdS}^2 M_{pl}^2$, where $M_{pl}$ is the Planck mass.

Let us now move away from the thin wall approximation and consider a potential with two minima, one with zero potential energy and one with a negative potential energy. The potential has to be tuned so that we get a static domain wall. There is a transition region joining the flat space and the $AdS$ space region. The scalar field is non-zero throughout $AdS$, but it approaches zero as $\rho \to -\infty$, which is the IR region. This can be interpreted as a field theory with a UV cutoff, which is perturbed by an irrelevant operator (dual to the scalar field) at the cutoff scale. The detailed shape of the domain wall is telling us how the field theory is coupled to the flat space region at the cutoff scale. (Supersymmetric domain walls of this kind were studied in [4].)

Of course, one can also consider an end of the world brane which does not have many degrees of freedom in the IR, such as an orbifold plane (which has none), or a brane with a gauge theory with a small number of colors, or with a field theory which is gapped and not conformal in the IR. A string theory example of an end of the world brane with a CFT in the IR is the Horava-Witten end of the world brane [5] in a $T^6$ compactification to five dimensions. At low energies we have an $E_8$, $\mathcal{N} = 4$ super Yang Mills on the brane. Here the end of the world brane is extended along the $T^6$ and four of the five dimensions.
3. An expanding domain wall

We now consider the case where the tension is very slightly below the critical value $T < T_{cr}$\textsuperscript{2}. In this case the domain wall expands. The geometry of the domain wall is a three dimensional de-Sitter space of radius of curvature $R_{dS}$. It is convenient to slice the space with $dS_3$ slices to obtain a full geometry of the form \[ ds^2 = R_{AdS}^2 [d\rho^2 + \sinh^2 \rho dS_3^2] , \quad \rho < \rho_0 \]
\[ ds^2 = dr^2 + r^2 dS_3^2 , \quad r_0 < r \]
\[ R_{dS} \equiv r_0 = R_{AdS} \sinh \rho_0 , \quad e^{-\rho_0} = \frac{T_{cr} - T}{T_{cr} + T} \]

where the value of $\rho_0$ is obtained from the junction conditions. Here $dS_3^2$ is just the metric of three dimensional de-Sitter with unit radius. And $T_{cr}$ is the critical tension for a given $R_{AdS}$.

When the tension of the domain wall is very close to the critical tension the expansion of the wall is very slow. In this regime, it is clear that we can think of the domain wall again as an end of the world brane plus a conformal field theory. The end of the world brane is expanding, because it has a non-zero effective tension, $\tilde{T} < 0$. The effective cutoff on the end of the world brane is again of the order of $1/R_{AdS}$. If $\rho_0$ is very large, we see that the acceleration of the wall is very small compared to this UV cutoff in the field theory, $\frac{H_{UV}}{R_{UV}} \sim \frac{R_{AdS}}{R_{dS}} \sim e^{-\rho_0}$.

The situation is very similar if we consider a thick brane. Let us first consider a thick brane whose tension is close to the critical value so that again we have a large range of $\rho$ where the geometry is well approximated by $AdS$. The scalar field tries to approach the minimum of the potential but it is still non-zero at $\rho = 0$. In the dual field theory description, the fact that we have a scalar field which is approaching the minimum corresponds to the fact that we have a CFT deformed by an irrelevant operator. The effects of this irrelevant operator become of order one where the thick brane is localized. This is the UV cutoff region for the dual field theory. The effects of this irrelevant deformation become smaller as we go to the IR, but when we reach the $dS_3$ Hubble scale, this irrelevant deformation has not yet died completely. This scale corresponds to $\rho \sim 0$ in the above geometry.

\textsuperscript{2} If the tension of the domain wall is $T > T_{cr}$ the decay cannot proceed.\[ ]
Fig. 2: In (a) we have the Euclidean solution, with $\rho = 0$ marking the center of the $AdS$ region. In (b) we see the Lorentzian solution in the thin wall regime. We can continue past $\rho = 0$ into an FRW cosmology with $H_3$ slices. This FRW cosmology expands and then contracts again. In the thin wall approximation the space is $AdS$ and the collapse is a coordinate singularity. In (c) we see the Lorentzian solution in the thick wall regime. The nonzero energy density at $\rho = 0$ or $\tau = 0$, which is due to the displacement of the field from the minimum of the potential, generically produces a singularity at $\tau = \pi$. (d) Same decay described by an Euclidean solution with an end of the world brane. The false vacuum is outside and there is nothing inside. (e) End of world brane picture for a Lorentzian solution. In this case we have “nothing” inside the wall, but we have a field theory on the wall. Here we drew the picture for a flat space false vacuum, but similar pictures can be drawn for $AdS$ or $dS$ false vacua.

As shown in [1], we can continue behind the region $\rho = 0$ into an FRW geometry which, as a first approximation, is

$$ds^2 = R_{AdS}^2 [-d\tau^2 + \sin^2 \tau ds_{H_3}^2]$$

where $\tau = 0$ is the same as $\rho = 0$. This corresponds to $AdS_4$ in hyperbolic slices. The universe expands and collapses again at $\tau = \pi$. If we were in $AdS$ this would be just a coordinate singularity. However, as pointed out in [1,6], the fact that the scalar field is slightly displaced from the minimum at $\tau = 0$ generically leads to a singularity as we approach $\tau = \pi$. See fig. 2.

The CFT gives a non-singular description of the decay process. In other words, if we had a CFT with a known $AdS$ dual and we now put this CFT on de-Sitter space and we add an irrelevant deformation, then we will also find a similar singular space. However, in this
case the CFT description is perfectly non-singular. We do not expect singularities for field theories in de-Sitter space. The gauge gravity duality on de-Sitter space was studied in [8,9,10,11,12,13,14,15,16,17,18,19,20,21,22], and references therein. Thus, the singularity is resolved in the same way that the black hole singularity is resolved by AdS/CFT. Namely, there is an “in principle description” via the field theory, but the concrete field theory computation that describes a local observer near the singularity remains mysterious.

The question of whether we can continue time past the crunch is related to the question of whether we can continue time past the time that the expanding brane hits the boundary of the false vacuum. That is an interesting question, but it seems that to answer this question we need to go beyond the degrees of freedom of the CFT, i.e. it involves energies above the UV cutoff of the field theory.

Here we have discussed the case where the potential has an $AdS$ minimum. However, it can also have an $AdS$ maximum, as long as the curvature of the potential (or mass of the tachyon) obeys the Breitenlohner-Freedman bound [23]. In this case, the scalar field corresponds to a relevant deformation of the field theory and the scalar field becomes larger as we approach $\rho = 0$. In some cases, these relevant deformations can remove the singularity completely by producing a mass gap which is above the $dS_3$ Hubble scale. In theories with gravity duals, this often appears as a kind of end of the world brane that is cutting off the geometry at a finite warp factor of the $dS_3$ slices, see [21,22] for recent discussions.

4. Conclusions

This paper pointed out that vacuum decay processes where we produce $AdS$ regions can conceptually be viewed as bubble of nothing decays, by replacing the whole $AdS$ region by an effective field theory that lives on the domain wall. The particular CFT that we obtain depends on the theory we are considering. Note that this field theory is defined with a UV cutoff. The physics at the UV cutoff is encoded by the precise bulk brane solution which joins the $AdS$ region into the false vacuum. In cases where there is a large hierarchy between $R_{AdS}$ and $R_{dS}$ the field theory modes can be sharply separated from the dynamics of the thick brane. In cases where there is no sharp separation, the acceleration

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3 A field theory with an irrelevant deformation is not well defined at all energy scales. However, here we are only interested in the low energy behavior of such field theory.

4 For a dissident view see [7].
of the brane is comparable to the UV cutoff of the field theory. It is also likely that we can still view the setup as dual to an end of the world brane, but we do not have a clear argument.

The false vacuum itself does not appear to be well defined, since it can decay! Thus we do not expect to have the field theory defined with a precision better than that of the vacuum where it lives. The false vacuum can be flat, AdS, or dS, but we are discussing decays into $AdS$ spaces. Unstable $AdS$ vacua were studied from the point of view of AdS/CFT in [24] and references therein. Note that the false vacuum will produce multiple bubbles that will collide. The description of these collisions takes us away from the field theory regime, and one would have to treat it using the full bulk solution. Thus, in our discussion we have ignored such collisions. Finally, we should point out that field theories in de-Sitter with a small relevant deformations are an interesting situation for understanding big crunch singularities. Very closely related solutions were studied in [25,26].

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Appendix A. Field theories in de-Sitter and cosmological singularities

In the main part of the paper we discussed an interpretation of Coleman de Luccia AdS bubbles that arise through vacuum decay. Here we give a short discussion field theories in de-Sitter space and their gravity duals. See [8-22] for further discussion. This is a completely well defined setup, which contains similar geometries in the interior. We start with a conformal field theory in de-Sitter. We then add a relevant deformation to the theory. For example, we can consider the theory describing M2 branes and add a mass deformation, corresponding to a dimension one operator. If the mass term is small enough the resulting field theory is stable. By a small mass, we mean a mass small compared to the Hubble scale of the three dimensional de-Sitter space. The gravity solution is the

\footnote{We can reinterpret the O(1,3) symmetric solutions in [25,26] as being dual to three dimensional field theories on $dS_3$ with a mass deformation. The boundary conditions are different from the ones in [25,26], but the actual solutions are the same. This is explained better in Appendix A}

\footnote{This appendix was added in January 2011, upon the suggestion of T. Banks.}
Fig. 3: In (a) we see a Euclidean solution with $SO(4)$ symmetry corresponding to a field theory on $S^3$ with a relevant deformation. In (b) we can see the scalar field profile through a cross section of the solution indicated by the red line in (a). In (c) we schematically depict the lorentzian continuation of the solution. $\rho = 0$ becomes a lightcone. Outside the lightcone we can foliate the spacetime with $dS_3$ slices on which the $SO(1, 3)$ symmetry acts. Inside the light cone $SO(1, 3)$ acts on $H_3$ slices. These slices expand and contract into a singularity.

$O(1, 3)$ symmetric solution in [25,26]. The geometry of these solutions is summarized in fig. 3. The Euclidean solution corresponds to the field theory on $S^3$. For a small mass the geometry is a small deformation of Euclidean $AdS_4$ (or hyperbolic space) with a scalar field that approaches zero at the boundary and it is non-zero, but relatively small, everywhere else, see fig. 3(b). The solutions in [25,26] contain a bulk scalar field, which, for large $\rho$ behaves as

$$\phi(\rho) \sim \alpha e^{-\rho} + \beta e^{-2\rho}$$

(A.1)

with both $\alpha$ and $\beta$ nonzero. The quantization that makes this field dual to an operator of dimension one on the boundary corresponds to the one where we view $\beta$ as the parameter we fix at the $AdS$ boundary. Thus $\alpha$ corresponds to a vev and $\beta$ to the boundary condition [27]. This boundary condition is adding a term of the form $\beta O$ to the field theory lagrangian, where $O$ is an operator of dimension one. We can embed these solutions into eleven dimensions so that they are asymptotic to $AdS_4 \times S^7/Z_k$ [28,29]. The the dual field theory is known [29], and the operator has a form given by $O = \alpha_f^I Tr[C^I(C^J)^I]$, where $\alpha_f^I$ are some constants, chosen so that the operator is real, $I = 1, \cdots 4$ and $\alpha_f^I = 0$. For example, we can choose $\alpha_1^1 = -\alpha_2^1 = 1$ and the rest zero. This operator gives mass squared terms which are positive for some fields and negative for others. However, all fields have a positive contribution due to the conformal coupling to the positive curvature of $S^3$ or $dS_3$. Thus, as long as the coefficient of $O$ is small enough, the theory is stable. The classical solutions are the same as those considered in [25,26] but the interpretation is different.
The quantum corrections around the classical solutions would also be different since were choosing different boundary conditions for the fields. With the boundary conditions in the setup was unstable. With the boundary conditions chosen here the theory is stable for small enough masses.

When we continue this solution to Lorentzian signature the origin of the Euclidean solution maps to a light cone. In the interior of the lightcone the $SO(1,3)$ symmetry acts on the spatial slices which have an $H_3$ geometry. In the interior the solution is time dependent and gives rise to a crunch singularity. The scalar field has a negative mass squared and a potential which is unbounded below.

In the main part of the paper we discussed potentials which had minima (rather than maxima) and for that reason we considered irrelevant perturbations. In a field theory context we cannot consider irrelevant perturbations at the UV boundary, since the field theory has to be well defined in the UV. This is, of course, not a problem for the situation in the bulk of the paper where we had a UV cutoff. Of course, we can have an irrelevant perturbation if we UV complete the theory in a suitable way (via another UV CFT, for example).
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