Propagation of vacuum polarized photons in topological black hole spacetimes

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Abstract

The one-loop effective action for QED in curved spacetime contains equivalence principle violating interactions between the electromagnetic field and the spacetime curvature. These interactions lead to the dependence of photon velocity on the motion and polarization directions. In this paper we investigate the gravitational analogue to the electromagnetic birefringence phenomenon in the static and radiating topological black hole backgrounds, respectively. For the static topological black hole spacetimes, the velocity shift of photons is the same as the one in the Reissner-Nordström black holes. This reflects that the propagation of vacuum polarized photons is not sensitive to the asymptotic behavior and topological structure of spacetimes. For the massless topological black hole and BTZ black hole, the light cone condition keeps unchanged. In the radiating topological black hole backgrounds, the light cone condition is changed even for the radially directed photons. The velocity shifts depend on the topological structures. Due to the null fluid, the velocity shift of photons does no longer vanish at the apparent horizons as well as the event horizons. But the "polarization sum rule" is still valid.

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I. INTRODUCTION

The Hawking radiation [1] of black holes is a remarkable prediction of quantum field theory in curved spacetime. This has been leading to a quite active field now: black hole physics. Compared to the Hawking radiation, the superluminal (“faster than light”) photon propagation in gravitational backgrounds, one of the predictions of quantum field theory in curved spacetime, is little to know. This phenomenon was discovered first by Drummond and Hathrell [2] in 1980. They calculated in QED the contribution to the photon effective action from one-loop vacuum polarization on a general gravitational background and used it to investigate the correction to the local propagation of photons in the optics approximation. They found that the quantum corrections introduce tidal gravitational forces on the photons which in general alter the characteristics of propagation. They investigated this phenomenon in the Schwarzschild spacetime, de Sitter space, gravitational wave background and the Robertson-Walker spacetimes. The photons may travel at speed greater than unity in certain motion and polarization directions, except for the case of de Sitter space. Shortly after this discovery, Ohkuwa [3] obtained the similar result for the massless neutrino propagating in the Robertson-Walker spacetime.

It was not further studied until the work of Daniels and Shore [4], in which they generalized the analysis of Drummond and Hathrell to the Reissner-Nordström black hole spacetimes, and further confirmed this phenomenon: for the radial motion photons the light cone is unchanged and the photon velocity is still the unity; but for the orbital photons the velocity may depend on the polarization directions. This phenomenon is in fact the gravitational analogue to the electromagnetic birefringence [5,4]. Due to the vacuum polarization in QED, the photons exist for part of the time as a virtual $e^+e^-$ pair, thereby acquiring an effective size of the order of the Compton wavelength of the electron ($\lambda = 1/m_e$, $m_e$ is the mass of the electron). Thus the motion of photons could be changed by the tidal effects of the spacetime curvature. But, “faster than light” photons do not violate the causality [2,4], because the effective action contains the equivalence principle violating interactions. More recently, this phenomenon has been studied further in the Kerr black hole background [6] and dilaton black hole spacetime [7]. Due to the rotating feature of Kerr metric, Daniels and Shore [6] found that the light cone is also changed even for the radial motion photons. But the velocity shifts are always equal and opposite for two physical polarization directions, and vanish at the black hole event horizons. According to these observations, Shore [6] proposed two theorems about the propagation of vacuum polarized photons: “horizon theorem” and “polarization sum rule”. The “horizon theorem” states that the velocity of radial motion photons remains equal to $c$ at the event horizons. This is satisfied by now for the de Sitter space, Schwarzschild black hole, Reissner-Nordström black hole, Kerr black hole and the dilaton black hole. This theorem seems to ensure that the geometric event horizon for black holes remains a true horizon for real photon propagation in QED. The “polarization sum rule” states that the sum of the averaged velocity shifts for two physical polarization directions is proportional to the energy-momentum tensor of matter. Hence for Ricci flat spacetime, the sum over the two physical polarizations of the velocity shift is zero. In [9] Lafrance and Myers investigated the propagation of photons with the emphasis on the dispersive property of photon propagating in gravitational backgrounds and the validity of the equivalence principle in QED.
The velocity shift of photons also takes place in non-gravitational backgrounds, for instance, in electromagnetic field [5,4], Casimir-type regions with boundaries [10,11], and in the finite temperature and/or density backgrounds [12]. Except the case in the Casimir-type regions, the velocity of photons is always less than $c$. In a flat spacetime, indeed the propagating velocity larger than $c$ may threat the causality. For the case of electromagnetic wave traveling in vacuum between two parallel conducting plates, Ben-Menahem [13] showed that the wavefront still travels at exactly $c$. The two-loop effect poses thus no threat to causality in QED. Contrary to the situation in flat spacetime, in curved spacetime the propagation of the so-called “faster than light” photons is possible and does not necessarily imply that the causality must be violated. Usually, the establishment of a causal paradox needs at least two conditions: spacelike motion and Poincaré invariance, in a flat spacetime. In curved spacetime the Poincaré invariance is lost and replacing it is the principle of equivalence. Just as mentioned above, due to the interacting terms between the electromagnetic field and the spacetime curvature, the principle of equivalence is lost in QED in curved spacetime. The superluminal photons are therefore allowed in curved spacetime. This may be an important feature of quantum field theory in curved spacetime.

Although the phenomenon of the superluminal photon propagation is unmeasurable almost [2], it is still a quite interesting phenomenon at least in principle. In order to better understand and to find its general feature, it is necessary to investigate this phenomenon thoroughly. In the present paper we would like to extend previous investigations in three directions. First, according to [2], the velocity of vacuum polarized photons remains unchanged in de Sitter space due to the isotropy of the spacetime. It is therefore interesting to see whether the cosmological constant affects the velocity shift in other spacetimes with a cosmological constant. Secondly, we hope to know whether the topological structure of backgrounds has effects on the propagation of vacuum polarized photons. Thirdly, the “polarization sum rule” and “horizon theorem” have been checked for the static and stationary black hole backgrounds only. It is of some interest to consider the phenomenon in the dynamical black hole spacetimes. For the sake of generality, as the gravitational background we choose the solutions to Einstein-Maxwell equations with a cosmological constant. This theory contains the static, spherically symmetric Reissner-Nordström-(anti-)de Sitter black hole solutions. The spacetime is asymptotically anti-de Sitter or de Sitter one, depending on the sign of the cosmological constant. The topologies of their event horizons are both a two-sphere $S^2$. When the cosmological constant is negative, the theory has also the so-called topological black hole solutions [14–21]. That is, the two-dimensional hypersurface of event horizon may have zero curvature or negative constant curvature. Thus, the topology of event horizon is no longer the two-sphere. These topological black hole solutions also exist in the dilaton gravity [22].

The plan of this paper is as follows. In next section we introduce the one-loop effective action in QED, and obtain the equation of vacuum polarized photons using the geometric optics approximation. In section 3 we investigate the propagation of photons in the static topological black hole backgrounds. In section 4 we discuss the case in the radiating topological black holes. We present our conclusion in section 5.
II. EFFECTIVE ACTION AND EQUATIONS OF MOTION

The one-loop effective action of QED in a general gravitational back ground has been calculated in [4], and has been used already to discuss the propagation of photons in some gravitational fields, such as the Schwarzschild black holes [2], Reissner-Nordström black holes [4], Kerr black holes [6] and dilaton black holes [7]. The effective action has been also used to explain the production of primordial magnetic field in the early universe [23]. In order to compare with these known results, we use the following one-loop effective action up to the same approximation,

\[ S = -\frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} \]

\[ - \frac{1}{m_e^2} \int d^4x \sqrt{-g} \left( a R F_{\mu\nu} F^{\mu\nu} + b R_{\mu\nu} F^{\mu\sigma} F^{\nu}_{\sigma} + c R_{\mu\nu\sigma\tau} F^{\mu\sigma} F^{\nu\tau} + d \nabla_{\mu} F^{\mu\nu} \nabla_{\sigma} F^{\sigma}_{\nu} \right) \]

\[ + \frac{1}{m_e^4} \int d^4x \sqrt{-g} \left( z (F_{\mu\nu} F^{\mu\nu})^2 + y F_{\mu\nu} F_{\sigma\tau} F^{\mu\sigma} F^{\nu\tau} \right), \]

where \( a, b, c, d, z \) and \( y \) are constants. For the QED correction, these coefficients are [2,23]

\[
\begin{align*}
    a &= -\frac{5}{720} \frac{\alpha}{\pi}, & b &= \frac{26}{720} \frac{\alpha}{\pi}, & c &= -\frac{2}{720} \frac{\alpha}{\pi}, \\
    d &= -\frac{1}{30} \frac{\alpha}{\pi}, & z &= -\frac{5}{180} \alpha^2, & y &= \frac{14}{180} \alpha^2.
\end{align*}
\]

(2.2)

Here \( \alpha \) is the fine structure constant. It is clear that the first line in (2.1) is the usual action of Maxwell field. The first three terms in the second line reveal the influences of the curvature, and the fourth survives even in flat spacetime and represents off-mass-shell effects in the vacuum polarization. The third line is the Euler-Heisenberg terms. This action (2.1) is valid in the approximation of weak curvature and low frequency photons. Therefore the higher powers of the curvature tensor and extra covariant derivatives in the interacting terms can be neglected. Varying the action (2.1) yields the equation of motion for the electromagnetic field,

\[
\nabla_{\mu} F^{\mu\nu} + \frac{1}{m_e^2} \left[ 4a \nabla_{\mu} (R F^{\mu\nu}) + 2b \nabla_{\mu} (R^{\mu}_{\nu\sigma} F^{\sigma\mu} - R^{\nu}_{\mu\sigma} F^{\sigma\mu}) + 4c \nabla_{\mu} (R_{\mu\nu\sigma\tau} F^{\sigma\tau} + 2d (\nabla^2 \nabla_{\sigma} F^{\sigma\nu} - \nabla_{\mu} \nabla_{\nu}\nabla_{\sigma} F^{\mu\sigma})) \right]
\]

\[
- \frac{1}{m_e^4} \left[ 8z (F^{\sigma\tau} F_{\sigma\tau} \nabla_{\mu} F^{\mu\nu} + 2F^{\mu\nu} F_{\sigma\tau} \nabla_{\mu} F^{\sigma\tau}) + 8y (F^{\nu\tau} F_{\sigma\tau} \nabla_{\mu} F^{\mu\sigma} + F^{\mu\sigma} F_{\sigma\tau} \nabla_{\mu} F^{\nu\tau} + F^{\mu\sigma} F^{\nu\tau} \nabla_{\mu} F^{\sigma\tau}) \right] = 0.
\]

(2.3)

In order to study the propagation of photons, we expand the field strength \( F^{\mu\nu} \) using background field method,

\[ F^{\mu\nu} = \bar{F}_{\mu\nu} + \hat{f}_{\mu\nu}, \]

(2.4)

where \( \bar{F}_{\mu\nu} \) is the background electromagnetic field. Substituting (2.4) into (2.3) and linearizing the equation in \( \hat{f}_{\mu\nu} \), one can get the equation describing the propagation of photons,
\[
\n\nabla_{\mu} \hat{f}^{\mu\nu} + \frac{1}{m^2_e} [2b R^{\mu}_{\sigma} \nabla_{\mu} \hat{f}^{\sigma\nu} + 4c R^{\mu\nu}_{\sigma\tau} \nabla_{\mu} \hat{f}^{\sigma\tau}] - \frac{1}{m^4_e} \left[ 16z \hat{F}^{\mu\nu} \hat{F}^{\sigma\tau} \nabla_{\mu} \hat{f}^{\sigma\tau} + 8y \left( \hat{F}^{\mu\sigma} \hat{F}^{\sigma\tau} \nabla_{\mu} \hat{f}^{\nu\tau} + \hat{F}^{\nu\sigma} \hat{F}^{\nu\tau} \nabla_{\mu} \hat{f}^{\sigma\tau} \right) \right] \right] = 0.
\]

(2.5)

Here some approximations have been used to derive Eq. (2.5). The quantum corrections are retained up to the first order of \( \alpha \) from the background curvature and electromagnetic field. The typical variations of the background electromagnetic field and gravitational field, characterized by the scale \( L \), are assumed to be much small than the one of the photons. That is, \( L >> \lambda \), here \( \lambda \) is the wavelength of photons. Thus some derivative terms, such as \( \nabla_{\sigma} \hat{F}^{\mu\nu}, \nabla_{\sigma} R^{\mu\nu}, \) etc. can be omitted. Without the one-loop correction, we have \( \nabla_{\mu} F^{\mu\nu} = 0 \). Hence \( \nabla_{\mu} \hat{f}^{\mu\nu} \) gives at least the first order correction of \( \alpha \). Such terms in interacting terms can also be neglected. Other assumption in Eq. (2.5) is that the wavelength of photons is much larger than that of the Compton wavelength: \( \lambda >> \lambda_c \). The details of these approximations can be found in [4] and [7].

In order to investigate the behavior of propagation of vacuum polarized photons, a simple method is to employ the geometric optics approximation [2]. In this method, one can write

\[
\hat{f}^{\mu\nu} = f^{\mu\nu} e^{i\theta},
\]

(2.6)

where \( f^{\mu\nu} \) is a slowly varying amplitude and \( \theta \) the rapidly varying phase. The wave vector is \( k_{\mu} = \nabla_{\mu} \theta \). In the quantum mechanics, it can be regarded as the momentum of photons. Thus from the electromagnetic Bianchi identity

\[
\nabla_{\lambda} F^{\mu\nu} + \nabla_{\mu} F_{\lambda\nu} + \nabla_{\nu} F_{\lambda\mu} = 0,
\]

(2.7)

one has

\[
k_{\lambda} f^{\mu\nu} + k_{\mu} f^{\nu\lambda} + k_{\nu} f^{\lambda\mu} = 0.
\]

(2.8)

Further we can write down

\[
f^{\mu\nu} = k_{\mu} a_{\nu} - k_{\nu} a_{\mu},
\]

(2.9)

where \( a_{\nu} \) can be interpreted as the polarization vector of photons and satisfies \( k_{\mu} a^{\mu} = 0 \). Substituting (2.6) into (2.3) and using the relation (2.9), we arrive at

\[
k^{2} a^{\nu} + \frac{1}{m^2_e} \left[ 2b R^{\mu}_{\sigma} k_{\mu} (k^{\sigma} a^{\nu} - k_{\nu} a^{\sigma}) + 8c R^{\mu\nu}_{\sigma\tau} k_{\mu} k^{\sigma} a^{\tau} \right]
\]

\[
- \frac{1}{m^4_e} \left[ 32z \hat{F}^{\mu\nu} \hat{F}^{\sigma\tau} k_{\mu} k^{\sigma} a^{\tau} + 8y \left( \hat{F}^{\mu\sigma} \hat{F}^{\sigma\tau} k_{\mu} (k^{\nu} a^{\tau} - k^{\tau} a^{\nu}) - \hat{F}^{\nu\sigma} \hat{F}^{\nu\tau} k_{\nu} k_{\tau} a_{\sigma} \right) \right]
\]

\[
= 0.
\]

(2.10)

To study the propagation of photons in curved spacetimes, it is convenient to introduce the orthonormal frame by using the vierbeins defined as \( q_{\mu\nu} = \eta_{ab} e^{a}_{\mu} e^{b}_{\nu} \), where \( \eta_{ab} \) is the metric of Minkowski space. In the orthonormal frames the equation (2.10) becomes
\[ k^2 a^b + \frac{1}{m_e^2} [2bR^a c k_a (k^c a^b - k^b a^c) + 8cR^{ab} c d k_a k^c a^d] \]
\[ - \frac{1}{m_e^2} [32z F^{ab} c d k_a k^c a^d + 8y \left( F^{ac} c d k_a (k^b a^d - k^d a^b) - F^{ad} c d k_a k^b a^c \right)] \]
\[ = 0. \]  
(2.11)

In the next section, we will use this equation to investigate the propagation of photons in the topological black hole backgrounds.

### III. PHOTONS IN STATIC TOPOLOGICAL BLACK HOLE SPACETIMES

Consider the Einstein-Maxwell equations with a cosmological constant, \( \Lambda \),

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}^{\text{em}}, \]

(3.1)

\[ \nabla_\mu F^{\mu\nu} = 0, \]

(3.2)

where \( T_{\mu\nu}^{\text{em}} \) is the energy-momentum tensor of electromagnetic field,

\[ T_{\mu\nu}^{\text{em}} = F_{\mu\sigma} F^{\sigma\nu} - \frac{1}{4} g_{\mu\nu} F^2. \]

(3.3)

In Eqs. (3.1) and (3.2), there are well-known static, spherically symmetric Reissner-Nordström-(anti-)de Sitter solutions,

\[ ds^2 = -(f(r)dt)^2 + f^{-1}(r)dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \]

(3.4)

\[ F_{tr} = \frac{Q}{4\pi r^2}, \]

(3.5)

where

\[ f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{4\pi r^2} - \frac{1}{3}\Lambda r^2. \]

(3.6)

Here in order to compare with the case of Reissner-Nordström black holes, we have used the same units as those in [4]. In equation (3.6) the constants \( M \) and \( Q \) are the mass and electric charge of black holes, respectively. As \( \Lambda < 0 \), the solution (3.4) describes an asymptotically anti-de Sitter spacetime. Usually, the equation, \( f(r) = 0 \), has two positive roots. The large one \( r_+ \) is the location of outer event horizon of black holes and the small \( r_- \) the inner horizon of black holes. As \( \Lambda > 0 \), the solution (3.4) is the Reissner-Nordström-de Sitter spacetime. In this case, the equation \( f(r) = 0 \) may have three positive roots. The largest one \( r_c \) is the cosmological horizon, the intermediate is the outer event horizon of the black holes and the smallest is the inner horizon.

Although the asymptotic behaviors are quite different for the Reissner-Nordström-anti-de Sitter black holes and Reissner-Nordström-de Sitter black holes, the topological structures of their event horizons are same. They are both the two-sphere \( S^2 \), as that of Reissner-Nordström black holes. In recent years, many authors have found that, when the cosmological constant is negative, the two-dimensional hypersurface of event horizon may have zero
or negative constant curvature [14–22]. The topology of event horizon of black holes is no longer the two-sphere \(S^2\). For instance, in the Einstein-Maxwell equations (3.1) and (3.2), we have exact static solutions,

\[
\begin{align*}
\text{(3.7)} & \quad ds^2 = - \left\{ -\left( -\frac{2M}{r} + \frac{Q^2}{4\pi r^2} - \frac{1}{3}\Lambda r^2 \right) dt^2 + \left( -\frac{2M}{r} + \frac{Q^2}{4\pi r^2} - \frac{1}{3}\Lambda r^2 \right)^{-1} dr^2 + r^2(d\theta^2 + \theta^2d\phi^2) \right\}, \\
\text{(3.8)} & \quad ds^2 = - \left\{ -1 - \frac{2M}{r} + \frac{Q^2}{4\pi r^2} - \frac{1}{3}\Lambda r^2 \right\} dt^2 + \left( -1 - \frac{2M}{r} + \frac{Q^2}{4\pi r^2} - \frac{1}{3}\Lambda r^2 \right)^{-1} dr^2 + r^2(d\theta^2 + \sinh^2 \theta d\phi^2).
\end{align*}
\]

Obviously, when \(\Lambda < 0\), both the solutions (3.7) and (3.8) are of the black hole structure in certain parameter regime. But, the scalar curvature of event horizon surface is zero for the solution (3.7), and minus one for the solution (3.8). By appropriately identifying the coordinates \(\theta\) and \(\phi\), one may obtain the topological black hole solutions whose event horizons possess different topological structures. Combining the Reissner-Nordström-(anti-)de Sitter solution (3.4) with the solutions (3.7) and (3.8), we have the unified form

\[
\begin{align*}
\text{(3.9)} & \quad ds^2 = - f(r)dt^2 + f^{-1}(r)dr^2 + r^2(d\theta^2 + h_k(\theta)d\phi^2),
\end{align*}
\]

where

\[
\begin{align*}
\text{(3.10)} & \quad f(r) = k - \frac{2M}{r} + \frac{Q^2}{4\pi r^2} - \frac{1}{3}\Lambda r^2, \\
\text{(3.11)} & \quad h_k(\theta) = \begin{cases} 
\sin \theta, & \text{for } k = 1 \\
\theta, & \text{for } k = 0 \\
\sinh \theta, & \text{for } k = -1.
\end{cases}
\end{align*}
\]

For the solution (3.9) the appropriate basis 1-forms are

\[
\begin{align*}
\text{(3.12)} & \quad e^0 = \sqrt{f}dt, \quad e^1 = (\sqrt{f})^{-1}dr, \quad e^2 = r d\theta, \quad e^3 = rh_k d\phi.
\end{align*}
\]

Through a straightforward calculation, in the orthonormal frame the nonvanishing components of curvature tensor are

\[
\begin{align*}
\text{(3.13)} & \quad R_{0101} = \frac{f''}{2} \equiv A(r), \quad R_{0202} = R_{0303} = \frac{f'}{2r} \equiv B(r), \\
R_{1212} = R_{1313} = -\frac{f'}{2r} = -B(r), \quad R_{2323} = \frac{k - f}{r^2} \equiv D(r).
\end{align*}
\]

where a prime stands for the derivative with respect to \(r\). Introducing the notation \(U_{a0}^{01} \equiv \delta_a^0\delta_0^1 - \delta_0^1\delta_a^0\), etc., the Riemann tensor can be expressed as

\[
\begin{align*}
\text{(3.14)} & \quad R_{abcd} = AU_{ab}^{01}U_{cd}^{01} + B(U_{ab}^{02}U_{cd}^{02} + U_{ab}^{03}U_{cd}^{03}) - B(U_{ab}^{12}U_{cd}^{12} + U_{ab}^{13}U_{cd}^{13}) + DU_{a0}^{23}U_{cd}^{23},
\end{align*}
\]

and the background electric field
In order to solve the equation of motion of photons (2.11), following [6], we introduce some linearly independent combinations of momentum components

\[ l_b = k^a U_{ab}^{01}, \quad m_b = k^a U_{ab}^{02}, \quad n_b = k^a U_{ab}^{03}, \]  

(3.16)

and some dependent combinations

\[ p_b = k^a U_{ab}^{12}, \quad q_b = k^a U_{ab}^{13}, \quad r_b = k^a U_{ab}^{23}. \]  

(3.17)

Using \( l_a, m_a, \) and \( n_a \) to contract (2.11), respectively, with the help of Eqs. (3.14) and (3.15), we have

\[
k^2 (a \cdot v) + \frac{2b}{m_e^2} (a \cdot v) [A l^2 + B (m^2 + n^2 - p^2 - q^2) + D r^2]
\]

\[
+ \frac{8c}{m_e^2} [A(a \cdot l)(l \cdot v) + B(m \cdot v)(a \cdot m) + B(n \cdot v)(a \cdot n)
\]

\[
- B(a \cdot p)(v \cdot p) - B(a \cdot q)(v \cdot q) + D(r \cdot a)(r \cdot v)]
\]

\[
- \frac{1}{m_e^4} \left( \frac{Q}{4 \pi r^2} \right)^2 \left\{ 32 z (a \cdot l)(l \cdot v) + 8y [l^2 (a \cdot v) + (l \cdot a)(l \cdot v)] \right\}
\] = 0

(3.18)

where \( v = l, m, n, \) respectively. Now we discuss the radial and orbital photon motions, respectively.

(i). **Radial photon motion.** In this case, we have \( k^2 = k^3 = 0. \) Hence,

\[ l^a = (k^1, k^0, 0, 0), \quad m^a = (0, 0, k^0, 0), \quad n^a = (0, 0, 0, k^0), \]  

(3.19)

\[ p_b = \frac{k^1}{k^0} m_b, \quad q_b = \frac{k^1}{k^0} n_b, \quad r_b = 0. \]  

(3.20)

For the longitudinal polarization \( a \cdot l, \) it follows from Eq. (3.18)

\[
\left[ k^2 + \frac{2b}{m_e^2} (A + 2B) l^2 + \frac{8c}{m_e^2} A l^2 - \frac{1}{m_e^4} \left( \frac{Q}{4 \pi r^2} \right)^2 (32z + 16y) l^2 \right] (a \cdot l) = 0.
\]  

(3.21)

Note that \( l^2 = -k^2 \) from (3.19). It is easy to see that the light cone condition \( (k^2 = 0) \) is unchanged and the velocity \( |k^0/k^1|_r = 1 \) for the longitudinal polarization of radial photons. For the physical transverse polarizations \( a \cdot m \) and \( a \cdot n, \) simplifying (3.18) one can get

\[
k^2 + \frac{2b}{m_e^2} (A + 2B) l^2 - \frac{8c}{m_e^2} B k^2 - \frac{8y}{m_e^4} \left( \frac{Q}{4 \pi r^2} \right)^2 l^2 = 0,
\]  

(3.22)

from which again we have \( k^2 = -l^2 = 0. \) That is,

\[
\left| \frac{k^0}{k^1} \right|_\theta, \phi = 1.
\]  

(3.23)
Therefore, the velocity of photons is still $c$ for the radially directed photons in the static topological black hole background. This is independent of the topological structure of spacetimes.

(ii). Orbital photon motion. In this case we set $k^1 = k^2 = 0$ and consider photon propagation in the orbital ($\phi$)-direction. Thus we have

$$l^a = (0, k^0, 0, 0), \quad m^a = (0, 0, k^0, 0), \quad n^a = (k^3, 0, 0, k^0),$$

$$p_b = 0, \quad q_b = -\frac{k^3}{k^0} l_b, \quad r_b = -\frac{k^3}{k^0} m_b.$$  \hspace{1cm} (3.24)

For the unphysical longitudinal polarization ($a \cdot n$), from (3.18) one has

$$\left\{ k^2 + \frac{2b}{m_e^2} [A(k^0)^2 + 2B(k^0)^2 - 2B(k^3)^2 + D(k^3)^2] + \frac{8c}{m_e^2} B[(k^0)^2 - (k^3)^2] - \frac{8y}{m_e^4} \left( \frac{Q}{4\pi r^2} \right)^2 (k^0)^2 \right\} (a \cdot n) = 0. \hspace{1cm} (3.26)$$

The photon velocity is, up to the first order correction,

$$\left| \frac{k^0}{k^3} \right|_\phi = 1 + \frac{b}{m_e^2} (A + D) - \frac{4y}{m_e^4} \left( \frac{Q}{4\pi r^2} \right)^2. \hspace{1cm} (3.27)$$

For the physical radial polarization ($a \cdot l$),

$$\left\{ k^2 + \frac{2b}{m_e^2} [A(k^0)^2 + 2B(k^0)^2 - 2B(k^3)^2 + D(k^3)^2] + \frac{8c}{m_e^2} [A(k^0)^2 - B(k^3)^2] - \frac{1}{m_e^4} \left( \frac{Q}{4\pi r^2} \right)^2 [32z + 16y](k^0)^2 \right\} (a \cdot l) = 0, \hspace{1cm} (3.28)$$

from which we obtain the velocity of photons

$$\left| \frac{k^0}{k^3} \right|_r = 1 + \frac{b}{m_e^2} (A + D) + \frac{4c}{m_e^2} (A - B) - \frac{16z + 8y}{m_e^4} \left( \frac{Q}{4\pi r^2} \right)^2. \hspace{1cm} (3.29)$$

For the other physical polarization ($a \cdot m$), we have

$$\left\{ k^2 + \frac{2b}{m_e^2} [A(k^0)^2 + 2B(k^0)^2 - 2B(k^3)^2 + D(k^3)^2] + \frac{8c}{m_e^2} [B(k^0)^2 + D(k^3)^2] - \frac{8y}{m_e^4} \left( \frac{Q}{4\pi r^2} \right)^2 (k^0)^2 \right\} (a \cdot m) = 0. \hspace{1cm} (3.30)$$

In this polarization direction, the velocity of photons is

$$\left| \frac{k^0}{k^3} \right|_{m_e} = 1 + \frac{b}{m_e^2} (A + D) + \frac{4c}{m_e^2} (B + D) - \frac{4y}{m_e^4} \left( \frac{Q}{4\pi r^2} \right)^2. \hspace{1cm} (3.31)$$

From Eq. (3.13), a straightforward calculation gives
Finally, it is worth noticing that, when $\Lambda$ is set to zero, the expressions for the velocities of light do not change. This reflects that the propagation of photons is not sensitive to the topological structures of spacetimes also does not enter the expressions of velocity. In fact, this should be expected. When $M = Q = 0$, the solution (3.32) reduces to the (anti-)de Sitter space. Therefore these results (3.36)-(3.38) should go back to those in (anti-) de Sitter space. In addition, when $Q = 0$, these expressions are the same as those in the Schwarzschild black hole background [2]. It is clear from (3.36) and (3.38) that the velocity shifts of the two physical polarizations are equal and opposite. The averaged velocity shift is thus zero for neutral topological black holes. That is, the cosmological constant does not change the behavior of spacetimes dose not affect the propagation of photons. In fact, this should be expected. When $M = Q = 0$, the solution (3.32) reduces to the (anti-) de Sitter space. Therefore these results (3.36)-(3.38) should go back to those in (anti-) de Sitter space. In addition, when $Q = 0$, these expressions are the same as those in the Schwarzschild black hole background [2]. It is clear from (3.36) and (3.38) that the velocity shifts of the two physical polarizations are equal and opposite. The averaged velocity shift is thus zero for neutral topological black holes. That is, the cosmological constant does not change the result in the Ricci flat spacetimes. The second point is that the parameter $k$ representing different topological structures of spacetimes also does not enter the expressions of velocity. This reflects that the propagation of photons is not sensitive to the topological structures of spacetimes. Finally, it is worth noticing that, when $\Lambda < 0$, there is still black hole structure in Eq. (3.33) even as $M = Q = 0$.

$$A = -\frac{2M}{r^3} + \frac{3Q^2}{4\pi r^4} - \frac{1}{3}\Lambda,$$

$$B = \frac{M}{r^3} - \frac{Q^2}{4\pi r^4} - \frac{1}{3}\Lambda,$$

$$D = \frac{2M}{r^3} - \frac{Q^2}{4\pi r^4} + \frac{1}{3}\Lambda.$$  \hspace{1cm} (3.32)

Substituting the above into Eqs. (3.29), (3.31) and (3.27), we obtain

$$\left| \frac{k^0}{k^3} \right|_{\phi} = 1 + \frac{b}{m^2_e 4\pi r^4} - \frac{4y}{m^4_e} \left( \frac{Q}{4\pi r^2} \right)^2,$$

$$\left| \frac{k^0}{k^3} \right|_{r} = 1 + \frac{b}{m^2_e 4\pi r^4} + \frac{4c}{m^2_e} \left( -\frac{3M}{r^3} + \frac{4Q^2}{4\pi r^4} \right) - \frac{16z + 8y}{m^4_e} \left( \frac{Q}{4\pi r^2} \right)^2,$$

$$\left| \frac{k^0}{k^3} \right|_{\theta} = 1 + \frac{b}{m^2_e 4\pi r^4} + \frac{4c}{m^2_e} \left( \frac{3M}{r^3} - \frac{2Q^2}{r^4} \right) - \frac{4y}{m^4_e} \left( \frac{Q}{4\pi r^2} \right)^2.$$  \hspace{1cm} (3.33)

(3.34)

(3.35)

For QED correction (2.2), they reduce to

$$\left| \frac{k^0}{k^3} \right|_{\phi} = 1 + \frac{13}{45} \frac{\alpha}{m^2_e} \left( \frac{Q}{4\pi r^2} \right)^2 - \frac{14}{45} \frac{\alpha^2}{m^4_e} \left( \frac{Q}{4\pi r^2} \right)^2,$$

$$\left| \frac{k^0}{k^3} \right|_{r} = 1 + \frac{1}{30} \frac{\alpha M}{m^2_e r^3} + \frac{1}{9} \frac{\alpha}{m^2_e} \left( \frac{Q}{4\pi r^2} \right)^2 - \frac{8}{45} \frac{\alpha^2}{m^4_e} \left( \frac{Q}{4\pi r^2} \right)^2,$$

$$\left| \frac{k^0}{k^3} \right|_{\theta} = 1 - \frac{1}{30} \frac{\alpha M}{m^2_e r^3} + \frac{17}{45} \frac{\alpha}{m^2_e} \left( \frac{Q}{4\pi r^2} \right)^2 - \frac{14}{45} \frac{\alpha^2}{m^4_e} \left( \frac{Q}{4\pi r^2} \right)^2.$$  \hspace{1cm} (3.36)

(3.37)

(3.38)

Comparing with the results in [4,7], we find that the velocities of photon propagating in the topological black hole spacetimes are totally the same as those in the Reissner-Nordström black hole backgrounds. This reveals at least three noticeable features. The first is that the physical polarizations are equal and opposite. The averaged velocity shift is thus zero for neutral topological black holes. That is, the cosmological constant does not change the behavior of spacetimes dose not affect the propagation of photons. In fact, this should be expected. When $M = Q = 0$, the solution (3.32) reduces to the (anti-)de Sitter space. Therefore these results (3.36)-(3.38) should go back to those in (anti-) de Sitter space. In addition, when $Q = 0$, these expressions are the same as those in the Schwarzschild black hole background [2]. It is clear from (3.36) and (3.38) that the velocity shifts of the two physical polarizations are equal and opposite. The averaged velocity shift is thus zero for neutral topological black holes. That is, the cosmological constant does not change the result in the Ricci flat spacetimes. The second point is that the parameter $k$ representing different topological structures of spacetimes also does not enter the expressions of velocity. This reflects that the propagation of photons is not sensitive to the topological structures of spacetimes. Finally, it is worth noticing that, when $\Lambda < 0$, there is still black hole structure in Eq. (3.33) even as $M = Q = 0$.

$$ds^2 = -(1 - \Lambda r^2/3)dt^2 + (1 - \Lambda r^2/3)^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2).$$  \hspace{1cm} (3.39)

This so-called massless topological black hole [17] has an event horizon at $r_+ = \sqrt{3/|\Lambda|}$. In this case, the light cone keeps unchanged and the velocity of photons is $c$ for any motion.
and polarization directions. This massless topological black hole can be constructed by identifying certain points in a four dimensional anti-de Sitter space. This is reminiscent of the Bañados-Teitelboim-Zanelli (BTZ) black hole in three dimensions [24]. The metric of BTZ black holes can be written as

\[ ds^2 = -N^2(r)dt^2 + N^{-2}(r)dr^2 + r^2[N^\phi(r)dt + d\phi]^2, \]

where

\[ N^2(r) = -M - \Lambda r^2 + \frac{J^2}{4r^2}, \quad N^\phi(r) = -\frac{J}{2r^2}, \]

(3.40)

(3.41)

\( M \) and \( J \) are two integration constants and represent the mass and angular momentum of the holes, respectively. When \( J < M/\sqrt{|\Lambda|} \), the BTZ black holes have two horizons determined by the equation \( N^2(r) = 0 \). The most remarkable feature of the black hole is that this is a negative constant curvature solution and can be constructed by identifying certain points in a three-dimensional anti-de Sitter space. Therefore the Riemann tensor of the solution can be expressed as [24]

\[ R_{abcd} = \Lambda(\eta_{ac}\eta_{bd} - \eta_{bc}\eta_{ad}). \]

(3.42)

We now consider the propagation of vacuum polarized photons in the BTZ black hole background (3.40). In this case, the one-loop effective action in three dimensions is different from the one in four dimensions (2.1). But we assume that this action (2.1) is also valid in three dimensions, of course, the coefficients must not be those in Eq. (2.2). Note that we can set \( d = z = y = 0 \) in this case to the first order correction. Using the equations (2.3) and (3.42), we can obtain

\[ \beta \nabla_\mu F^{\mu\nu} = 0, \]

(3.43)

where \( \beta \) is a constant. Thus, this situation is the same as that of the (anti-) de Sitter space in four dimensions [4]. Therefore the light cone is also unchanged in the BTZ black hole background. Note that the BTZ black hole is the analogue of Kerr black hole in three dimensions. That is the BTZ black hole solution is also a stationary spacetime as that of Kerr black hole. This implies that the velocity of photons keeps probably unchanged even for stationary black hole spacetimes.

Before the end of this section, let us consider the propagation of photons in the magnetically charged black hole backgrounds. In this case, the background magnetic field is

\[ \bar{F}_{ab} = \frac{Q_m}{4\pi r^2} U_{ab}^{23}, \]

(3.44)

where \( Q_m \) is the magnetic charge of black holes. The equation (3.18) describing the propagation of photons becomes

\[ k^2(a \cdot v) + \frac{2b(a \cdot v)}{m^2_e}[A l^2 + B(m^2 + n^2 - p^2 - q^2) + Dr^2] + \frac{8c}{m^2_e}[A(a \cdot l)(l \cdot v) + B(m \cdot v)(a \cdot m) + B(n \cdot v)(a \cdot n) - B(a \cdot p)(v \cdot p) - B(a \cdot q)(v \cdot q) + D(r \cdot a)(r \cdot v)] - \frac{1}{m^4_e}\left(\frac{Q_m}{4\pi r^2}\right)^2 \left\{ 32z(a \cdot r)(r \cdot v) + 8y[r^2(a \cdot v) + (r \cdot a)(r \cdot v)] \right\} = 0. \]

(3.45)
It is easy to check that the velocity of radial photons is still $c$, but the velocities of orbital photons become

\[
\begin{align*}
\frac{k^0}{k^3}_\phi &= 1 + \frac{b}{m_e^2}(A + D) - \frac{4y}{m_e^2} \left( \frac{Q_m}{4\pi r^2} \right)^2, & (3.46) \\
\frac{k^0}{k^3}_r &= 1 + \frac{b}{m_e^2}(A + D) + \frac{4c}{m_e^2}(A - B) - \frac{4y}{m_e^4} \left( \frac{Q_m}{4\pi r^2} \right)^2 & (3.47) \\
\frac{k^0}{k^3}_\theta &= 1 + \frac{b}{m_e^2}(A + D) + \frac{4c}{m_e^2}(B + D) - 16z + 8y \left( \frac{Q_m}{4\pi r^2} \right)^2. & (3.48)
\end{align*}
\]

Compared with Eqs. (3.27), (3.29) and (3.31), the contribution from the gravitational background keeps unchanged reasonably, the contributions from the magnetic field to two physical polarizations interchanges their places.

**IV. PHOTONS IN RADIATING TOPOLOGICAL BLACK HOLE SPACETIMES**

In this section we would like to extend the above discussions to dynamical black hole spacetimes, in which apparent horizons and event horizons do not coincide. It is an interesting problem to see whether or not the “horizon theorem” of Shore \[8\] holds for the apparent horizons and event horizons in the dynamical black hole spacetimes. For simplicity, we consider the topological black hole solutions with a null fluid radiation,

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}^{\text{em}} + 8\pi T_{\mu\nu}^{\text{rad}}.
\]

Here $T_{\mu\nu}^{\text{em}}$ is the energy-momentum tensor of electromagnetic field (3.3) and $T_{\mu\nu}^{\text{rad}} = \rho l_\mu l_\nu$, is the energy-momentum tensor of null fluid with the energy density $\rho$ and the four-velocity $l_\mu$. In the advanced time coordinates, we have \[16\]

\[
d s^2 = -f(v, r) dv^2 + 2dvdr + r^2[d\theta^2 + h^2(\theta)d\phi^2],
\]

where

\[
f(v, r) = k - \frac{2M(v)}{r} + \frac{Q^2(v)}{4\pi r^2} - \frac{1}{3}\Lambda r^2.
\]

This is a generalization of the Bonnor-Vaidya metric. The solution (4.2) implies that the energy density of null fluid radiation is

\[
\rho = \frac{\dot{M}(v)}{4\pi r^2} - \frac{Q(v)\dot{Q}(v)}{(4\pi)^2 r^3},
\]

where an overdot represents differentiation with respect to $v$. In the solution (4.2) the appropriate basis one-forms are

\[
e^0 = \sqrt{f}(dv - f^{-1}dr), \ e^1 = (\sqrt{f})^{-1}dr, \ e^2 = rd\theta, \ e^3 = rhkd\phi.
\]

The nonvanishing spin connection one-forms are
In this case, the equation of motion (3.18) for photons becomes

\[
w^0_1 = \left( \frac{f'}{2\sqrt{f}} - \frac{\dot{f}}{2f \sqrt{f}} \right) e^0 - \frac{\dot{f}}{2f \sqrt{f}} e^1, \]

\[
w^1_2 = -\frac{\sqrt{f}}{r} e^2, \quad w^1_3 = -\frac{\sqrt{f}}{r} e^3, \quad w^2_3 = -\frac{h_{k\theta}}{r} e^3. \tag{4.6}\]

Using (4.3) and (4.6), we obtain the nonvanishing components of Riemann tensor

\[
R_{0101} = \frac{f''}{2} \equiv A, \quad R_{0202} = R_{0303} = \frac{f'}{2r} - \frac{\dot{f}}{2rf} \equiv B, \quad R_{0212} = R_{0313} = -\frac{\dot{f}}{2rf} \equiv E
\]

\[
R_{1212} = R_{1313} = -\frac{f'}{2r} - \frac{\dot{f}}{2rf} \equiv C, \quad R_{2323} = \frac{1}{r^2} (k - f) \equiv D. \tag{4.7}\]

Now the Riemann tensor can be rewritten as

\[
R_{abcd} = A U^{01}_{ab} U^{01}_{cd} + B (U^{02}_{ab} U^{02}_{cd} + U^{03}_{ab} U^{03}_{cd}) + E (U^{02}_{ab} U^{12}_{cd} + U^{02}_{ab} U^{12}_{cd})
\]

\[
+ U^{03}_{ab} U^{13}_{cd} + U^{03}_{ab} U^{13}_{cd} + C (U^{12}_{ab} U^{12}_{cd} + U^{13}_{ab} U^{13}_{cd}) + DU_{ab}^{23} U_{cd}^{23}, \tag{4.8}\]

and the background electric field is

\[
F_{ab} = \frac{Q(v)}{4\pi r^2} U_{ab}^{01}. \tag{4.9}\]

In this case, the equation of motion (3.18) for photons becomes

\[
k^2 (a \cdot v) + \frac{2b}{m_e^2} (a \cdot v) [A l^2 + B (m^2 + n^2) + C (p^2 + q^2) + Dr^2 + 2E (m \cdot p + n \cdot q)]
\]

\[
+ \frac{8c}{m_e^2} [A (l \cdot v) (a \cdot l) + B (m \cdot v) (m \cdot a) + B (n \cdot v) (n \cdot a)
\]

\[
+ C (p \cdot v) (p \cdot a) + C (q \cdot v) (q \cdot a) + D (r \cdot v) (r \cdot a)
\]

\[
+ E (m \cdot v) (p \cdot a) + E (p \cdot v) (m \cdot a) + E (q \cdot v) (n \cdot a) + E (n \cdot v) (q \cdot a)]
\]

\[
- \frac{1}{m_e^4} \left( \frac{Q(v)}{4\pi r^2} \right)^2 \left\{ 32z (l \cdot v) (l \cdot a) + 8y [l^2 (a \cdot v) + (l \cdot v) (l \cdot a)] \right\} = 0. \tag{4.10}\]

Now it is a position to discuss the propagation of photons in the radiating topological black hole spacetimes. Let us consider first the case of the radially directed photons.

(i). **Radial photon motion.** In this case we have \( k^2 = k^3 = 0 \). From Eq. (4.10), for the longitudinal polarization \( (a \cdot l) \) we have

\[
\left\{ k^2 + \frac{2b}{m_e^2} [A l^2 + 2B (k^0)^2 + 2C (k^1)^2 + 4E (k^0 k^1)]
\right.
\]

\[
+ \frac{8c}{m_e^2} A l^2 - \frac{1}{m_e^4} \left( \frac{Q(v)}{4\pi r^2} \right)^2 (32z + 16y) l^2 \right\} (a \cdot l) = 0, \tag{4.11}\]

from which it follows

\[
\left| \frac{k^0}{k^1} \right| = 1 + \frac{2b}{m_e^2} (B + C \pm 2E). \tag{4.12}\]
Here the plus sign corresponds to \( k^1 > 0 \) (outgoing photons) and the minus sign to \( k^1 < 0 \) (ingoing photons). For two physical polarizations \((a \cdot m)\) and \((a \cdot n)\), the corresponding equations are

\[
\begin{align*}
\left\{ k^2 + \frac{2b}{m_e^2} [A k^2 + 2B(k^0)^2 + 2C(k^1)^2 + 4E(k^0 k^1)] \\
+ \frac{8c}{m_e^2} [B(k^0)^2 + C(k^1)^2 + 2E(k^0 k^1)] - \frac{8y}{m_e^2} \left( \frac{Q(v)}{4\pi r^2} \right)^2 \right\} (a \cdot m) &= 0, \quad (4.13)
\end{align*}
\]

and

\[
\begin{align*}
\left\{ k^2 + \frac{2b}{m_e^2} [A k^2 + 2B(k^0)^2 + 2C(k^1)^2 + 4E(k^0 k^1)] \\
+ \frac{8c}{m_e^2} [B(k^0)^2 + C(k^1)^2 + 2E(k^0 k^1)] - \frac{8y}{m_e^2} \left( \frac{Q(v)}{4\pi r^2} \right)^2 \right\} (a \cdot n) &= 0, \quad (4.14)
\end{align*}
\]

respectively. From Eqs. (4.13) and (4.14), we obtain the velocity of photons

\[
\left| \frac{k^0}{k^1_{\theta,\phi}} \right| = 1 + \frac{(2b + 4c)}{m_e^2} (B + C \pm 2E), \quad (4.15)
\]

where the meaning of “±” signs is the same as that in (4.12). Here it should be noted that the velocity shift of photons is changed even for the radial photons due to the different motion directions (ingoing and outgoing photons).

(ii). Orbital photon motion. In this case we have \( k^1 = k^2 = 0 \) for the \((\phi)\)-direction photons. For the longitudinal polarization \((a \cdot n)\) it follows from (4.14)

\[
\begin{align*}
\left\{ k^2 + \frac{2b}{m_e^2} [A(k^0)^2 + 2B(k^0)^2 + C(k^1)^2 + D(k^3)^2] \\
+ \frac{8c}{m_e^2} Bn^2 - \frac{8y}{m_e^2} \left( \frac{Q(v)}{4\pi r^2} \right)^2 (k^0)^2 \right\} (a \cdot n) - \frac{8cEn^2}{m_e^2} \left( \frac{k^3}{k^0} \right) (a \cdot l) &= 0.
\end{align*}
\]

Note that \( n^2 = -k^2 \). The last term in the above equation can be neglected because it gives the second correction to the light-cone. Thus we have

\[
\left| \frac{k^0}{k^3} \right| _{\phi} = 1 + \frac{b}{m_e^2} (A + B + C + D) - \frac{4y}{m_e^2} \left( \frac{Q(v)}{4\pi r^2} \right)^2.
\]

For the transverse polarization \((a \cdot l)\),

\[
\begin{align*}
\left\{ k^2 + \frac{2b}{m_e^2} [A(k^0)^2 + 2B(k^0)^2 + C(k^3)^2 + D(k^3)^2] \\
+ \frac{8c}{m_e^2} [A(k^0)^2 + C(k^3)^2] - \frac{(32z + 16y)}{m_e^4} \left( \frac{Q(v)}{4\pi r^2} \right)^2 (k^0)^2 \right\} (a \cdot l) - \frac{8c}{m_e^2} E k^0 k^3 (a \cdot n)
\end{align*}
\]

\[
= 0,
\]

\[ (4.18) \]
from which we obtain the velocity of photons

$$\left| \frac{k^0}{k^3} \right|_r = 1 + \frac{b}{m_e^2}(A + B + C + D) + \frac{4c}{m_e^2}(A + C) - \frac{16z + 8y}{m_e^4} \left( \frac{Q(v)}{4\pi r^2} \right)^2. \tag{4.19}$$

And for the other physical polarization \((a \cdot m)\)

$$\{k^2 + \frac{2b}{m_e^2}[A(k^0)^2 + 2B(k^0)^2 - B(k^3)^2 + C(k^3)^2 + D(k^3)^2]$$

$$+ \frac{8c}{m_e^2}[B(k^0)^2 + D(k^3)^2] - \frac{8y}{m_e^4} \left( \frac{Q(v)}{4\pi r^2} \right)^2 (k^0)^2 \} (a \cdot m) = 0. \tag{4.20}$$

It is straightforward to get the velocity of photons for \((\theta)\)-direction polarization,

$$\left| \frac{k^0}{k^3} \right|_\theta = 1 + \frac{b}{m_e^2}(A + B + C + D) + \frac{4c}{m_e^2}(B + D) - \frac{4y}{m_e^4} \left( \frac{Q(v)}{4\pi r^2} \right)^2. \tag{4.21}$$

From the components of Riemann tensor (4.7), we get

$$B + C \pm 2E = -\frac{\dot{f}}{rf} \mp \frac{f'}{rf}, \tag{4.22}$$

$$A + B + C + D = \frac{f''}{2} - \frac{\dot{f}}{rf} + \frac{k - f}{r^2}, \tag{4.23}$$

$$A + C = \frac{f''}{2} - \frac{f'}{2r} - \frac{\dot{f}}{2rf}, \tag{4.24}$$

$$B + D = \frac{f'}{2r} - \frac{\dot{f}}{2rf} + \frac{k - f}{r^2}. \tag{4.25}$$

According to the definition of apparent horizon and event horizon \([25]\), for static and stationary black holes, the apparent horizon and event horizon coincide with each other. But for the radiating black holes they do no longer coincide. Up to the \(O(L_M, L_Q)\), where \(L_M = -\dot{M}(v)\) and \(L_Q = -\dot{Q}(v)\), the apparent horizon is defined as surface whose expansion \(\Theta = 0\) and the event horizon are null surface such that \(d\Theta/dv \approx 0 \tag{26}\). Generalizing the analysis of Koberlein and Mallett \([27]\), for the radiating topological black holes (4.2) the apparent horizons can be obtained by roots of the equation \(f(v, r) = 0\). Inspecting Eqs. (4.22)-(4.25), these quantities all diverge at the apparent horizons because \(\dot{f} \neq 0\) at apparent horizons. This results in the divergence of velocity shift of photons at the apparent horizons. It is worth noting that this does not mean that the velocity of photons becomes infinity at the apparent horizon. In fact, it reflects that the approximation employed above breaks down in this case. But an important fact is that the velocity of photons is no longer \(c\) at the apparent horizon. Another quite interesting result is that, from (4.12) and (4.15), we find that the velocity is still \(c\) for the ingoing photons \((k^1 < 0)\) of any polarization directions; but does not for the outgoing photons \((k^1 > 0)\) and diverges at the apparent horizons.

For the radiating topological black holes (4.2), the event horizons, up to \(O(L_M, L_Q)\), can also be obtained by the equation \(f(v, r) = 0\), but \(M\) and \(Q\) in (4.3) should be replaced by \(M^*\) and \(Q^* \tag{27}\). Here
\[ M^* = M - \frac{L_M}{\kappa}, \quad Q^* = Q - \frac{L_Q}{\kappa}, \quad (4.26) \]

with

\[ \kappa = \frac{M(v)}{r_{oah}^2} - \frac{Q^2(v)}{4\pi r_{oah}^4} - \frac{1}{3}\Lambda r_{oah}, \quad (4.27) \]

is the surface gravity of outer apparent horizon \((r_{oah})\) of black holes. In this case, these quantities in (4.22)-(4.25) have not any strange behavior and finite on the event horizon. Therefore the velocity of outgoing radial photons is also no longer \(c\) at the event horizon. That is, the “horizon theorem” of Shore [8] does not hold for the radiating black holes. Inspecting [8] reveals that the “horizon theorem” is derived under the condition that the spacetime is stationary. Therefore our result is not inconsistent with the “horizon theorem” because our spacetime is time-dependent. Furthermore, due to the presence of function \(f\) in Eqs. (4.24)-(4.26), the parameter \(k\) reflecting the topological structure of spacetimes enters the expressions of velocity of photons. The velocity shift of vacuum polarization photons becomes dependent on the topological structure of spacetimes.

Now we check the “polarization sum rule” in the radiating topological black holes. With help of (4.8) and (4.15), for the radial motion photons, we have

\[ \sum_{\theta,\phi} k^2 = -\frac{(8b + 16c)}{m_e^2} (B + C \pm 2E)(k^1)^2 \]

\[ = -\frac{(4b + 8c)}{m_e^2} R_{ab} k^a k^b. \quad (4.28) \]

For the orbital photons, we can obtain

\[ \sum_{r,\theta} k^2 = -\frac{(4b + 8c)}{m_e^2} (A + B + C + D)(k^3)^2 \]

\[ = -\frac{(4b + 8c)}{m_e^2} R_{ab} k^a k^b, \quad (4.29) \]

where we have assumed \(Q(v) = 0\). From the above, we can see clearly that the “polarization sum rule” still holds for the radiating topological black holes.

**V. CONCLUSION**

The equivalence principle is violated in the quantum field theory in curved spacetimes, because the effective action of quantum fields contains some interacting terms between quantum fields and spacetime curvature, which violate the equivalence principle. Therefore the propagation of superluminal photons does not necessarily imply that the causality must be violated in curved spacetime. In this work, using the one-loop effective action for QED we have investigated, respectively, the propagation of photons in the static and radiating topological black hole backgrounds. We have found that in the static topological black hole spacetimes, the light cone condition \((k^2 = 0)\) keeps unchanged for the radial motion.
photons. That is the velocity of vacuum polarized photons is still \( c \) for the radial photons. The velocity shifts of orbital photons are same as those in the Reissner-Nordström black hole background. It means that the cosmological constant \( \Lambda \) and the topological parameter \( k \) do not enter the expressions of velocity. This indicates that the propagation of vacuum polarized photons is not sensitive to the asymptotic behavior and topological structure of backgrounds in this case. The light cone condition is also unchanged in the massless topological black hole and BTZ black holes, as in the de Sitter space. Note that the velocity is changed even for the radially directed photons due to the stationary feature in the Kerr background \([3]\), and the BTZ black hole spacetime is also a stationary one. We conclude that the velocity of photons is not changed necessarily even for the stationary black hole spacetimes. It is important to notice that all the Riemann tensors have the form \((3.42)\) for the massless topological black holes, BTZ black holes, and (anti-)de Sitter space, in which the light cone condition retains unchanged.

For the propagation of photons in the radiating topological black holes, we have obtained the expressions of velocity shifts for radial and orbital photons. The velocity of photons becomes dependent on the cosmological constant and topological structure of backgrounds. We noted that the light cone condition of radial photons is changed due to the null fluid matter. The velocity of radial photons is no longer \( c \). The velocity will become infinite at the apparent horizons, which implies the breakdown of the approximation. The velocity is also not \( c \) at the event horizons. Here it should be pointed out that this does not violate the “horizon theorem” of Shore, because the latter is proved for stationary black hole spacetimes only. The “polarization sum rule” is still valid for the radiating topological black holes. This is clear because the “polarization sum rule” depends on only some properties of curvature tensors \([3]\), but the null geodesic plays a crucial role in the definition of apparent horizon and event horizon. Therefore it is unsurprising that the velocity of vacuum polarized photon is no longer \( c \) at the apparent and event horizons. Finally we would like to mention that the velocity is still \( c \) for the ingoing radial photons. We have not yet understood it.

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