Novel insights into the $\gamma\gamma^* \rightarrow \pi^0$ transition form factor

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BaBar’s observation of significant deviations of the pion transition form factor (TFF) from the asymptotic expectation with $Q^2 > 9$ GeV$^2$ has brought a serious crisis to a fundamental picture established for such a simplest $q\bar{q}$ system by perturbative QCD, i.e. the dominance of collinear factorization at high momentum transfers for the pion TFF. We show that non-factorizable contributions due to open flavors in $\gamma\gamma^* \rightarrow \pi^0$ could be an important source that contaminates the pQCD asymptotic limit and causes such deviations with $Q^2 > 9$ GeV$^2$. Within an effective Lagrangian approach, the non-factorizable amplitudes can be related to intermediate hadron loops, i.e. $K^{(*)}$ and $D^{(*)}$ etc, and their corrections to the $\pi^0$ and $\eta$ TFFs can be estimated.

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I. INTRODUCTION

Since the foundation in late 1970s [1], perturbative QCD (pQCD) has been a powerful tool to explore the strong interaction phenomena at large momentum transfers. This is such a kinematic region where the exclusive transition matrix element can be expressed as the convolution of the perturbative calculable coefficient and non-perturbative light-cone distribution amplitude (DA) that describes the longitudinal momentum fraction of quarks, namely the so-called collinear factorization. In particular, with the availability of clean electromagnetic probes, the photon-pion transition form factor (TFF) will be the most ideal subject to test the validity of pQCD and partonic structure of pion.

Experimentally, the pseudoscalar mesons’ TFFs have been measured by CELLO and CLEO collaborations with the virtuality of one photon $Q^2$ up to 9 GeV$^2$ and the other nearly on-shell [2, 3]. The data fit very well the pQCD leading order interpolation formula [4], which stitches together the chiral anomaly and pQCD asymptotic limit,

$$F_{BL}^{\gamma\pi^0}(Q^2) = \frac{1}{4\pi^2 f_\pi} \frac{1}{1 + (Q^2/8\pi^2 f_\pi)^2},$$  \hspace{1cm} (1)$$

where $f_\pi = 92.3$ MeV, $f_\eta = 97.5$ MeV [8] are the corresponding decay constants of the pseudoscalar mesons. In fact, it has been shown that nearly all the analyses up to next-to-leading order (NLO) radiative and twist-four corrections favor the endpoint-suppressed or asymptotic-like pion DA, which means that the first two nontrivial Gegenbauer moments of the DA are small and have opposite signs in the region of $Q^2 = 9$ GeV$^2$ [5, 6].

However, the celebrated confirmation of pQCD dominance at large momentum transfers was shortly overwhelmed by the surprising BaBar measurement [7, 8]. It shows that the rescaled pion TFF exhibits a continuous growth at $9 < Q^2 < 40$ GeV$^2$, even beyond the asymptotic limit of pQCD. In contrast, the $\eta$ and $\eta'$ TFFs seem to be consistent with the pQCD expectations. Thus, a coherent understanding of the $\pi^0$ and $\eta$ (and $\eta'$) TFFs would be a necessity for resolving this puzzle.

The BaBar observation immediately arouses tremendous interests in the pion TFF. In the pQCD scheme, the scaling violation of the BaBar data can be better explained by either a wider DA with deep midpoint or even a flat one [9, 13]. However, they both conflict with some other constraints such as the data at low $Q^2$, midpoint value of the pion DA from light-cone sum rules [14], or operator product expansion (OPE) analysis for the leading handbag contribution [9] etc. Moreover, a flat DA means pion is an elementary field and can interact with quarks locally. Thus, a quark loop can also give the logarithmic enhancement but with rather low constituent quark masses [15, 16]. In this case, the QCD evolution is switched off, and the whole process is totally non-factorizable [5]. Therefore, a rather flat DA with vanishing endpoints may still be a temporary solution [14] taking into account that it is not supported by the $u, d$ quark behavior in the $\eta$ and $\eta'$ TFFs [13, 18]. It is even claimed that a peculiar mechanism beyond the standard model is needed to explain the BaBar puzzle [14, 21].

In the chiral limit approximation, there is only one large physical scale in the hard part of pQCD calculation. So the power law is explicit and concrete. In turn, the abnormal growth of the rescaled pion TFF implies that an additional mass scale would be present. A natural mechanism for generating a mass scale should satisfy the following constraints: i) It should keep the general OPE analysis. In another word, the solution should not differ from non-OPE or non-power corrections if only light quarks are involved; ii) The new mechanism should not leave the chiral anomaly intact at $Q^2 = 0$ since the room for the theoretical and experimental improvement is very small [22]; iii) The pQCD asymptotic prediction should be valid, though the current experimental region in the exclusive processes seems to be still far away from the...
asymptotic region determined in inclusive processes [23]; iv) The flavor symmetry breaking of the DA should be explained in the same scheme, and it is better to keep SU(3) flavor symmetry taking into account its big success in the effective theory; v) Pion should be treated as a bound state of current quarks. It is necessary to make the new mechanism compatible with the numerous analyses for both the asymptotic-like DA and collinear factorization before the BaBar data since they have been cross checked based on different powerful tools.

To find a solution that satisfies the above constraints simultaneously seems not easy. In the literature there are also proposals to suggest that the BaBar puzzle may need a full understanding of the evolution of non-perturbative mechanism from low to high $Q^2$. In principle, any solution satisfying those constraints based on QCD should be treated seriously.

In this work, we propose a novel prescription to address above questions as a compensation of pQCD. Since non-perturbative flavor changing processes (FCP) may play a role as “mass transmutation” [10] above certain momentum transfers, we assume that such non-perturbative mechanisms can be expressed in terms of hadronic degrees of freedom. With an effective Lagrangian approach (ELA), we shall show that intermediate meson loops have a peculiar evolution in terms of $Q^2$, which could be an important mechanism causing the TFF deviations from the asymptotic behavior in the region of $9 < Q^2 < 40$ GeV$^2$.

BaBar Collaboration recently also measured the $\eta_c$ TFF [24]. A continuous growth of the rescaled TFF (or slow decrease of the TFF) is observed clearly with $Q^2$, which is caused by the large charm quark mass [23]. It is also shown that a small charm component in $\eta'$, which is needed to explain the anomalously large branch ratio of $B \to K\eta'$ and $B \to X_s\eta'$ [26, 27], may cause large deviations from the asymptotic prediction in the medium momentum transfer region while affects little the low momentum transfer region [23]. So it is natural to speculate that the charm quark may play a role in the deviations from the asymptotic predictions in the BaBar kinematic region. The first possible QCD diagram is the photons fuse into a charm quark loop which then couples to two gluons. These two gluons then evolve to the pion through triangle anomaly. This factorizable charm loop is also a possible correction for the chiral anomaly [22], and cannot be recognized by the OPE based only on the massless quarks. Although the correction at $Q^2 = 0$, i.e. the chiral anomaly, is tiny, its evolution with the photon virtuality could make it relatively important in larger photon virtuality. Unfortunately, for the case of pion transition it is an isospin suppressed process proportional to the $u, d$ quark mass difference. Therefore, its influence on $\eta$ is much stronger than that on the pion. To avoid the isospin suppression, the $u, d$ quarks would interact with photons more directly, and the gluon exchanges between charm quark and light quark must be non-perturbative to evade the strong coupling suppressions. Such a scenario can be equivalent to the introduction of the intermediate meson loops based on the quark-hadron duality argument. It will be our focus in this work to investigate such a non-perturbative effect on the pion TFF in the BaBar kinematic region.

The rest of this paper is organized as follows: The effective Lagrangian approach for the intermediate meson loops is introduced in Sec. II. Numerical results and discussions are presented in Sec. III, and a brief summary is given in Sec. IV.

II. EFFECTIVE LAGRANGIAN APPROACH FOR THE INTERMEDIATE MESON LOOPS

The effective Lagrangian for the $D^{(*)}$ mesons ($D^{(*)0}, D^{(*)+}, D_s^{(*)+}$) couplings to light pseudoscalar mesons [24] has the following expression:

$$\mathcal{L} = -ig_{D^+D^0}(D^i \partial^\mu P_{ij} D'^{j\mu} - D'^{i\mu} \partial^\mu P_{ij} D^j) + \frac{1}{2} g_{D^+D^0} \epsilon_{\mu\nu\alpha\beta} D^i_{\mu} \partial^\nu P_{ij} \epsilon^{\alpha\beta\gamma\delta} D'^j_{\gamma} - \frac{1}{2} D^i_{\mu} \partial^\nu P_{ij} \epsilon^{\alpha\beta\gamma\delta} D'^j_{\nu} D'^j_{\gamma} - \frac{1}{2} D'_{\mu} \partial^\nu P_{ij} D_{\nu} D'^j_{\gamma} - \frac{1}{2} D'_{\mu} \partial^\nu P_{ij} D^j_{\nu} D^j_{\gamma}$$

where $\epsilon^{\alpha\beta\gamma\delta} \equiv \epsilon^{\alpha\beta\gamma\delta} - \epsilon^{\alpha\beta\delta\gamma}$, and $P$ denotes the pseudoscalar octet mesons

$$P = \begin{pmatrix} \pi^0 + \eta_8 & \pi^+ & K^+ \\ \pi^- & -\pi^0 + \eta_8 & K^0 \\ K^- & K^0 & -\sqrt{2} \eta_8 \end{pmatrix}$$

The corresponding Lagrangians for the photon and $D^{(*)}$ couplings are

$$\mathcal{L}_{DD^\gamma} = i e A_\mu D^- \partial^\mu D^+ + i e A_\mu D_- \partial^\mu D'^+ - \frac{1}{4} e F_{\mu\nu} D^0_{\mu} D^0_{\nu} + h.c.$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, D^0_{\mu} = \partial^\mu A^0, D^{\mu+} = D^{\mu-} = \partial^\mu A^0, D^{\mu+} = \partial^\mu A^0$,
where the signs and relative size can also be deduced from constituent quark model, see also [31] for the lattice calculation. Similarly, the coupling constant \( g_{D^*D^0} \equiv g_{D^*D\pi} = 17.9 \) [32], and \( g_{D^*D^0} \) can be obtained from the heavy quark effective theory

\[
g_{D^*D^0} = \frac{g_{D^*D\pi}}{\sqrt{M_{D^*}M_D}}. \tag{8}
\]

Four types of loops would contribute to the pion TFF, i.e. \( D^*D(D), D^*D(D^*), D^*D^*(D^*), \) and \( D^*D^*(D) \), where the \( D^{(*)} \) mesons in the parentheses are the exchanged mesons between two photons. The kinematic conventions are illustrated by Fig. 1. The transition amplitude can be expressed as follows:

\[
M_{fi} = \sum_{\text{Polarization}} \int \frac{d^4p_3}{(2\pi)^2}T_3T_4T_5 F(p_3^2), \tag{9}
\]

where \( T_{3,4,5} \) are the vertex functions given by the effective Lagrangians, and \( a_{i=3,4,5} \) are the denominators of propagators of the intermediate mesons, respectively.

At hadronic level, the calculation of intermediate meson loops only includes \( S \)-wave ground state mesons. In our case, the intermediate charged mesons include \( D \) \((D_s)\) and \( D^* \) \((D_s^*)\), and in the strange sector \( K \) and \( K^* \) are considered. One essential question which is very often raised is the contributions from higher excited meson loops apart from those \( S \)-wave states. Intuitively, one would expect that a complete set of intermediate meson loops should be included based on the quark-hadron duality argument. In our approach, the following reasons would allow us to only consider contributions from the \( S \)-wave states. Firstly, for a given kinematic condition, we would expect that the local EM dipole couplings in Eq. 7 would be much larger than the excited states due to quantum mechanical selection rules. For electric dipole couplings, e.g. \( \gamma D^+D^- \), the ground state would be less suppressed by the wavefunction overlaps than excited states. Second, in many cases the quark-hadron duality is locally broken such that a subset of hadron loops would play a dominant role in observables. Moreover, we find in numerical simulations that apart from the vertex couplings the loop integrals general decrease when the masses of the intermediate mesons increase. In this sense the contributions from the ground state \( S \)-wave intermediate meson loops can be regarded as a reasonable approximation. Meanwhile, although we still lack a systematic evaluation of how good such an approximation would be in different circumstances, it interests us to explore such a possible mechanism that accounts for the non-factorizable contributions to the pion TFF.

We also note that in the process of \( \gamma \gamma^* \rightarrow \pi^0 \), there exists a significant difference between \( g_{D^+D^+} \), and \( g_{D^0D^0}\gamma \gamma \) as given in Eq. 7. In some other diagrams, the neutral meson loops even vanishes, e.g. due to the \( \gamma D^0D^0 \) vertex. It means that the isospin violation effects are actually enhanced by the electromagnetic interaction. Also, a nonvanishing charmed meson loop contribution must be present in the low \( Q^2 \) region which comes from the destructive sum between the charged and neutral \( D^{(*)} \) meson loops. At sufficiently high \( Q^2 \) the meson loop contributions should become negligible as expected by pQCD. The interesting question is at which range of \( Q^2 \) such an effect becomes negligible. Meanwhile, an associated difficulty is to model-independently quantify the loop contributions in a broad range of \( Q^2 > 0 \). In this sense our study is motivated to provide a qualitative estimate of the intermediate meson loop effects as part of the non-factorizable contributions in terms of \( Q^2 \).

Generally speaking, the loops suffer from ultraviolet divergence which means that we should replace the local coupling constants by the non-local form factors to suppress the divergence. As usually done, we introduce the form factor \( F(p_3^2) \) to take into account the off-shell effect of each vertex, which also reflects the substructure of external hadrons. With the large space-like momentum transfers, each vertex would be far off-shell. Therefore, we adopt the following tri-monopole form in the calculation:

\[
F(p_3^2) = \prod_{i=3}^{5} \frac{(\Lambda_i^2 - m_i^2)}{(\Lambda_i^2 - p_i^2)}. \tag{10}
\]

This turns out to be a reasonable consideration by our numerical simulation. The cutoff \( \Lambda_i \) can be parameterized as \( \Lambda_i = m_i + \alpha QCD \) with \( QCD = 220 \text{ MeV} \), and \( m_i \) is the exchanged meson mass. Parameter \( \alpha \) is usually taken at \( O(1) \) and not necessarily the same for different open flavors.

Apart from the \( D \) loops, the Kaon and \( B \) loops can also contribute. The Kaon loops are similar to Fig. 1 where the coupling constants can be determined by experimental data [33], i.e. \( g_{K^+\pi^+} = 0.84 \text{ GeV}^{-1} \), \( g_{K^0\pi^0} = -1.27 \text{ GeV}^{-1} \), \( g_{K^+\pi^0} = -
The introduction of an empirical form factor will bring about model-dependent features. In order to give a meaningful interpretation of the results, we outline the conditions for constraining the parameter $\alpha$ as follows:

i) For an adopted value of $\alpha$, the meson loop corrections to the TFF at $Q^2 = 0$ should be negligibly small in order not to conflict with the well established theoretical and experimental results for the chiral anomaly [22]. In practise, the $D$ or Kaon meson loop contributions to the anomaly are restricted to be less than 0.005 GeV$^{-1}$ (absolute value) in comparison with Eq. (1), $F_{\pi}(0) = 0.27$ GeV$^{-1}$. This serves as a strong constraint for the upper limit of $\alpha$.

ii) The form factor of Eq. (10) introduces additional singularities into the integrals empirically [34], which means that in order to reduce the model-dependence, parameter $\alpha$ should have a sufficiently large value, e.g. $\alpha > 1$. This condition can be satisfied in $\gamma\gamma \to \pi^0$ since all the internal exchanged mesons are highly off-shell.

iii) We neglect $B$ meson loops because the numerical calculation shows that apart from couplings, their contributions are usually one order magnitude smaller than $D$ and $K$ loops. In addition, we do not include the non-strange light meson loops to avoid double counting.

In fact, we find that the above conditions can indeed provide a stringent constraint on $\alpha$, and $1 < \alpha_K < 2$, $1 < \alpha_D < 4$ can be determined in Fig. 3(b).

In Fig. 2 we plot pion TFF given by the exclusive $D$ and Kaon loops with $\alpha_D = 3$ and $\alpha_K = 1.5$ in terms of $Q^2$. It should be stressed that, the meson loop contributions to the chiral anomaly are nearly zero because of the destructive interferences from the $D$ and $K$ loops. It is indicated by the thick solid line at zero momentum transfer point in Fig. 2. The dominance of $D$ loops over the Kaon loops can be recognized. In particular, the $D^*D^*(D)$ loop exhibits an enhancement in the BaBar kinematics due to the relatively large coupling differences between the charged and neutral meson loops as shown in Eq. (7), i.e. the vertex coupling product of the neutral $D$ meson loops is 16 times larger than that of the charged $D$ mesons. The Kaon loops are much smaller because of the destructive interferences among different loop amplitudes and smaller $\alpha_K$.

A sensible feature of the $D$ loops is that, the total meson loop contribution decreases faster than pQCD with the increasing $Q^2$. Thus, it guarantees the asymptotic prediction of pQCD at high $Q^2$. This is different from the logarithmic [16] or double logarithmic [15, 17] growth predicted by some analyses with flat DA or equivalent.

III. RESULTS AND DISCUSSIONS

The introduction of an empirical form factor will bring about model-dependent features. In order to give a meaningful interpretation of the results, we outline the conditions for constraining the parameter $\alpha$ as follows:

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In Fig. 2(a), the shadowed band is the exclusive contributions from the intermediate meson loops plus the pQCD prediction, which in this paper means the interpolation result involving only the $u, d$ quarks. The range of the shadowed area is given by $\alpha_D = 2$ (lower) and 3 (upper bound) with $\alpha_K = 1.5$ fixed. This is a rather conservative estimate of the meson loop contributions based on the above constraints on $\alpha$. It shows that the inclusion of the meson loops significantly improves

$-g_{K^{\ast}}g_{\pi^{0}} = -9.17/\sqrt{2}$, $g_{\rho\pi \gamma} = 0.69$ GeV$^{-1}$ and $g_{\rho\pi \gamma} = -0.73$ GeV$^{-1}$. The SU(3)$_F$ symmetry then gives $g_{K^{\ast}}g_{\pi^{0}} = -g_{K^{\ast}K^{\ast} \pi^{0}} = -g_{\rho\omega\pi^{0}}/2 = -11.58/2$ GeV$^{-1}$.

\[ F_{\pi}^\ast(Q^2) [\text{GeV}^4] \]

\[ F_{\rho}^\ast(Q^2) [\text{GeV}^4] \]

FIG. 2: (a) $D$ meson loop contributions to the pion TFF. The thin solid, dashed, dot-dashed, and dotted lines denote $D^\ast D(D^\ast)$, $D^\ast D(D^\ast)$, $D^\ast D^\ast(D^\ast)$, $D^\ast D^\ast(D)$ loops, and the thick solid line for the total loops, dot-dot-dashed for Eq. (1). (b) Kaon loop contributions with the same notations as (a), but $D^\ast$ replaced by Kaons.

FIG. 3: (a) The rescaled pion TFF in comparison with the experimental data. The dashed line is given by the pQCD interpolation formula. The meson loops combining the pQCD results give the shadowed band with $\alpha_D = 2$ and $\alpha_D = 3$ for the lower and upper bound. (b) The $\alpha$ dependence of the meson loop contributions to the chiral anomaly. The dashed, dotted, solid lines are for $D$ loops, Kaon loops, and their sum with the same $\alpha$, respectively.
the region of $Q^2 > 9$ GeV$^2$ while affects little in $Q^2 < 5$ GeV$^2$. Therefore, the CELLO, CLEO and BaBar experiments can be explained simultaneously without modifying conclusions obtained before. We also note that the discrepancies with the data at low $Q^2$ are mainly caused by the pQCD interpolation formula Eq.\( (1) \) which would break down due to the growing importance of the QCD corrections with negative sign \[6\]. We also emphasize that we do not try to fit the data and judge the possible contributions of the meson loops to the chiral anomaly. The dashed, dotted and solid lines are from the $D$ loops, Kaon plus $\phi\eta(1^+)$ loops, and their sum with the same $\alpha$, respectively.

The meson loops can also contribute to the $\eta$ and $\eta'$ TFFs. It is thus natural to expect that the same formulation in the SU(3)$_F$ symmetry limit should keep consistent with the theoretical and experimental results for $\eta$ and $\eta'$, or at least for $\eta$. Interestingly, the data \[8\] show that the rise of the rescaled TFFs is much weaker than the rise of $\pi^0$ case. It implies that the $u, d$ components in $\eta$ and $\eta'$ would be significantly different from that in $\pi^0$ if the analysis is only based on pQCD \[13\]. Therefore, it is interesting to examine the meson loop contributions here as a direct test of the proposed mechanism. It should be pointed out that a strict anomaly sum rule shows that the same non-perturbative non-OPE correction to the continuum as in the pion TFF should also be present in the $\eta$ and $\eta'$ TFF \[33, 34\], although the $\eta$ TFF favors the result from the pQCD interpolation formula. Here we do not discuss the $\eta'$ TFF because of its possible mixing with the gluonic and/or charm component \[37\].

In Fig.\( (4) \) the rescaled $\eta$ TFF is plotted. Considering the $\eta$-$\eta'$ mixing, an extended interpolation formula for $\eta$

$$F_{\eta BL}^\eta(Q^2) = \frac{6C_qf_q^\eta}{Q^2 + 4\pi^2f_q^2} + \frac{6C_sf_s^\eta}{Q^2 + 4\pi^2f_s^2}, \quad (11)$$

where $C_q = 5/9\sqrt{2}$ and $C_s = 1/9$, and $f_q$ and $f_s$ are decay constants of the $q\bar{q}$, $d\bar{d}$ for the additional $\phi\eta(0^1)$ loops.

The datum at $Q^2 = 112$ GeV$^2$ is actually measured at the time-like point $Q^2 = -112$ GeV$^2$ \[42\]. Due to the analyticity of QCD, the time and space-like form factors can be related to each other at $Q^2 \rightarrow \infty$. We mark the time-like datum point in Fig.\( (4) \) assuming that the $|Q^2|$ value for the time-like virtual photon is high enough for the analytical continuation of the TFF. In this sense, it may provide a guidance for the magnitude of the corresponding space-like form factor at $Q^2 \approx 112$ GeV$^2$. It is interesting to see that our result converges at high $Q^2$ as expected. Namely, the meson loop contributions should vanish at high $Q^2$. Although the pQCD interpolation formula of $\eta$ and $\eta'$ are not as well-established as that of pion, e.g. radiative and higher-twist corrections need to be systematically included, the role played by the meson loops turns out to fit well the observed pattern.
IV. SUMMARY

In summary, we have proposed that the meson loops as a non-perturbative component of the pion TFF may still play an important role up to $Q^2 \approx 40 \text{ GeV}^2$, hence cause deviations from the collinear factorization results. This mechanism is non-factorizable and different from the factorizable charm quark loop at quark-gluon level. The factorizable charm quark loop in the pion TFF must be suppressed by the $u$-$d$ quark mass difference as compared with the $q$. The meson loops in terms of quark-hadron duality may correspond to some continuum corrections in the spirits of the strict anomaly sum rule \cite{35}. It should be stressed that this solution keeps consistent with the chiral anomaly without violating the SU(3)$_F$ symmetry. Nevertheless, this mechanism does not bring conflicts to against the pQCD analyses before the BaBar result, and is an economic explanation for the BaBar puzzle.

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