Population Synthesis Handling Three Geographical Resolutions

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Abstract: In this paper, we develop a synthetic population as the first step in implementing an integrated land use/transport model. The model is agent-based, where every household, person, dwelling, and job is treated as an individual object. Therefore, detailed socioeconomic and demographic attributes are required to support the model. The Iterative Proportional Updating (IPU) procedure is selected for the optimization phase. The original IPU algorithm has been improved to handle three geographical resolutions simultaneously with very little computational time. For the allocation phase, we use Monte Carlo sampling. We applied our approach to the greater Munich metropolitan area. Based on the available data in the control totals and microdata, we selected 47 attributes at the municipality level, 13 attributes at the county level, and 14 additional attributes at the borough level for the city of Munich. Attributes are aggregated at the household, dwelling, and person level. The algorithm is able to synthesize 4.5 million persons in 2.1 million households in less than 1.5 h. Directions regarding how to handle multiple geographical resolutions and how to balance the amount and order of attributes to avoid overfitting are presented.

Keywords: population synthesis; microscopic land use model; travel demand; agent based

1. Introduction

Synthetic populations are used in transportation modeling when individual records of households and persons are not available due to privacy reasons, insufficient resolution, or missing attributes. Within the context of transportation modelling, population synthesis is the process of creating a representation of a complete, disaggregate population by combining a sample of disaggregate members of a population in a way as to match key distributions for the entire population [1]. Key distributions—also known as control attributes—can be at the household level, such as household size, at the person level, such as gender, age or employment status, or at the dwelling level, such as construction year or living space. Moreover, control attributes can be aggregated at different geographical resolutions, such as boroughs, municipalities, or counties.

Synthesizing a population has two main phases: optimization (fitting) and allocation. The first phase fits a disaggregate sample of agents (microdata) to aggregated constraints (control totals), while the second phase replicates actual agents for the synthetic population using a probabilistic selection [2]. While the procedure on the second stage is usually the same across population synthesizers and relies on Monte Carlo sampling, there is a broad range of procedures for the first stage.

There is an ongoing debate about which procedures and enhancements are best suited at the optimization phase. As seen in Table A1, the Iterative Proportional Fitting (IPF) procedure is a well-established algorithm for fitting [1,3–17]. This method, first proposed by Deming and Stephan [18], identifies weights for the microdata iteratively by adjusting an n-dimensional array until every dimension matches to the control totals. The same method is often called matrix balancing in
computer science or RAS method in input-output analysis. The main disadvantage of the procedure is that it can only handle one level of aggregation (person or household) and geographical resolution (municipality or county) at each time. Some authors enhanced the procedure by: substituting the n-dimensional array with sparse lists to accommodate a large number of control attributes without exponentially increasing computational requirements [7]; using two-step IPF to accommodate person level and household level attributes in sequence [11]; incorporating more heterogeneity into the initial seed [17,19]; and combining IPF with spatial microsimulation [15] or reweighting IPF results using Iterative Proportional Updating (IPU) [17]. Recent work has evolved IPF into IPU, which calculates a set of weights for each one of the microdata records in an iterative approach. IPU is capable of closely matching household-level, dwelling-level, and person-level control totals at the same time [20] and it can accommodate control attributes defined at municipality-level and county-level simultaneously [21]. As with IPF, IPU is from the static family of models. Other procedures that can handle person and household-level attributes are entropy maximization [16,22,23], hierarchical IPF [24,25], combinatorial optimization [26,27], Monte Carlo Markov Chain [27,28], Hidden Markov Models [29], or multinomial regression models [30,31]. Most of the procedures are compared to IPF and usually tend to better match observed distributions in multiple dimensions, although the convergence time can be very high [26].

In terms of control attributes, all studies for transportation engineering include at least household size, age, and gender, as summarized in Table A2. Employment status has been included at the household [4,5,11,21,27] or person level [8,13,26,31–33]. While household income is available for most of the studies in the United States and Canada [4,7,9–11,20–22,26], it is commonly not included in European countries and Australia [8,13,23,27,32]. Other variables are the number of cars, number of children, type of dwelling, or ethnicity. Dwelling attributes are less common, with only a few studies including dwelling tenure [4,9,34] or dwelling type [4,11,32].

The aim of this work is to synthesize the population of the greater Munich metropolitan area. This paper does not intervene in the methodological debate by comparing performance of alternative procedures but rather gathers alternative procedures available and selects one suitable to the case study needs. The available data is limited in several respects, which triggered our need to create a new multiresolution solution. Firstly, person and household attributes are aggregated at the municipality level, but most dwelling attributes are aggregated at the county level. Secondly, the German administrative division classifies the city of Munich as a single municipality-county of 1.3 million inhabitants in 0.7 million households. A higher resolution is required to synthesize demographic and dwelling differences across boroughs. Thirdly, the data do not cover all attribute dimensions of households, individuals, and dwellings that the model requires. Specifically, data on individual income, car availability, land price, or number of bedrooms are missing. The first and second constraints lead to implementing one optimization procedure that can enable control at household, dwelling, and person levels simultaneously and can deal with different geographical resolutions in a reasonable amount of time. The third constraint is not fundamental and results in having a few uncontrolled attributes that are directly copied from the microdata.

2. Materials and Methods

The algorithm builds on several of the methods and techniques that have been introduced thus far in the field of population synthesis and expands it to three geographical levels. The method includes three stages: (1) selecting geographical resolution and scales of analysis; (2) optimization; (3) allocation. The following subsections will describe each stage and the application.

2.1. Selecting Geographical Resolution and Scales of Analysis

Two main data items required to synthesize populations are individual household structures (or household microdata) and aggregate distributions at a certain geographical resolution (or control totals).
Household microdata is provided by many statistics bureaus in the form of the microcensus. It includes basic socioeconomic information, current and previous employment, and location. Control totals are usually available at statistics bureaus. The data can be aggregated to several geographical resolutions: borough, municipality, county, state, or nationwide.

After data were collected, we selected attributes as control attributes. Control attributes must be meaningful for the model, included in both databases, and have equal or comparable stratifications in both databases.

2.2. Optimization: IPU with Three Geographical Resolutions

The optimization uses Iterative Proportional Updating (IPU). It was proposed by Konduri et al. [21] for two geographical resolutions, and it was expanded for this research to three geographical resolutions. The IPU procedure consists of adjusting the set of weights for each household of the microdata to minimize the error between control totals and calculated distributions of each attribute for each geographic resolution.

Before starting the IPU procedure, it is required to summarize each microdata record according to the categories of the control attributes. The result is stored in the frequency matrix. The frequency matrix shows the household and dwelling type and the frequency of different person types within each household for the sample. The dimension of the matrix is \( N \times M \), where \( N \) is the number of households in the microdata and \( M \) is the number of control attributes (household, person and dwelling type).

The set of weights is provided at the lowest geographical resolution. An initial set of weights is set to one. In the next iterations, weights are updated after considering each control attribute. All attributes, regardless of whether they are household, dwelling, or person type, are treated equally. Weights are only updated in the households where the frequency of the control attribute is different than zero. Attributes at the lowest geographical resolution (i.e., municipality) update only the weight of one record, while attributes at the higher geographical resolution (i.e., county) update a set of weights of all nested areas (i.e., municipalities or boroughs).

After all control attributes are considered, we calculate the relative difference in absolute difference between control total and calculated distribution for each attribute. The average error is compared to the previous iteration. If the absolute difference of average deviation values between two full iterations satisfies a set of tolerance criteria, the algorithm stops updating household weights. The default threshold is set equal to 0.01% and can be modified by the user. Average absolute relative difference across all constraints has been used previously by Ye et al. [20] and Konduri et al. [21] in the original IPU procedure and by others [3,5,23,31,35]. Other indicators for goodness of fit include standardized root mean square error [7,11,24,27,36], difference on counts [1,8,10], or error percentages [9]. Additional stopping criteria include the maximum number of iterations (default value set to 1500) and average error threshold (default value of \( 1 \times 10^{-7} \)). The process converges after several iterations depending on the number of control attributes and number of municipalities within one county.

Table A3 shows the pseudocode for the IPU with three geographical resolutions and two aggregation levels.

2.3. Allocation: Monte Carlo Sampling

To generate the synthetic population, households of the microdata are randomly drawn based on their weights. Once a household is selected for the municipality, it is allocated to a traffic analysis zone (TAZ) within the municipality or borough. The zone system nests TAZs within the municipal regions respecting municipal boundaries. The probability for each TAZ is the ratio between the population in the TAZ and the total population on the municipality.

The value of control attributes is directly copied from the microdata into the synthetic population.

Finally, we computed the absolute difference between the generated population and control totals. The goodness of fit of the allocation is evaluated as the average relative error of the control attributes at the highest-ranking geography (i.e., county).
2.4. Application

We applied our approach to the greater Munich metropolitan area (Figure 1). The region includes the cities of Munich, Augsburg, Ingolstadt, Landshut, and Rosenheim and their respective suburbs to cover the large commuting shed of the Munich region. The delineation of the study area was defined by the share of commuter flow into these five central cities. Municipalities were included in the area if at least 25% of workers commute to one of the central cities. The area includes a population of 4.5 million living in 2.1 million households. It has 444 municipalities distributed in 28 counties. The number of municipalities by county varied from 1 to 46.

Figure 1. Greater Munich metropolitan area: counties (in color) and municipalities.
After delineating the study area, municipalities are divided in Traffic Analysis Zones (TAZs) using a gradual raster-based zone system [37]. The total number of TAZs is 4950 (Figure 2). Twenty-nine percent of the population lives in the city of Munich itself, while the average population per municipality is below 10,000. Therefore, a higher spatial resolution was created for the city of Munich than for other less populated municipalities.

Figure 2. Greater Munich metropolitan area: counties (in color) and traffic analysis zones.
The German household microdata was purchased from the German State Statistical Office. Due to privacy considerations, individual records are anonymized and not geolocated. In fact, the only geographical resolution of the reference is the state of residence (e.g., Bavaria). Control totals are available online at the German 2011 Household Census [38], the GENESIS Online database [39], the Census Hub [40], and the website of the city of Munich [41].

Based on the available data in the control totals and microdata, we selected 60 control attributes: 47 attributes at the municipality level and 13 attributes at the county level. For the city of Munich, we selected 14 additional attributes at the borough level. Table 1 summarizes the control attributes by type and geographical resolution.

| Attribute     | Categories                                                                 | Geographical Resolution(s) |
|---------------|-----------------------------------------------------------------------------|----------------------------|
| Household     | Sum of households                                                           | Municipality and borough   |
| Household size| 1, 2, 3, 4, 5+                                                              | Municipality               |
| Household size| 1                                                                          | Borough                    |
| Household with children | Household with person(s) younger than 18 years old | Borough                    |
| Person        | Population                                                                  | Municipality and borough   |
| Age by gender | Male/Female + age (under 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, over 80) | Municipality               |
| Age           | Male (reference), female                                                    | Borough                    |
| Nationality   | German (reference), foreigner                                               | Municipality and borough   |
| Employment status | Male/Female + employed                                                      | Municipality and borough   |
| Dwelling      | Owned, rent                                                                 | Municipality               |
| Dwelling living space (m²) | Less than 60, 61–80, 81–100, 101–120, more than 120 | County                    |
| Building size by construction year | Smaller/Larger (2 or less dwellings in the building, 3 or more dwellings in the building) + construction year (before 1948, 1949–1990, 1991–2000, after 2001) | County                    |

The land use/transport model for which this synthetic population was generated requires other attributes that are not in the microdata, such as person income, person workplace, dwelling monthly cost, dwelling quality, or number of cars on the household. They are obtained from other submodels, such as car ownership model or land price model, and assigned by Monte Carlo sampling.

3. Results

This section presents the results of the synthetic population for the greater Munich metropolitan area. We analyze the differences between the aggregate distributions and the resulting synthetic population by: (1) geographical resolution and (2) attribute.

The optimization phase is deterministic, but the allocation phase produces slightly different model runs every simulation due to the random seed used while sampling. To provide some impression of the range of the sampling error, we ran the allocation five times. The average result of the five runs and the standard deviation between the five model runs are indicated.

3.1. By Geographical Resolution

Firstly, we analyzed the average error of control attributes at the municipality level (Figure 3). We took the absolute value of each control attribute’s error to calculate the average error by municipality. The errors after the optimization are shown in green, and the errors of the allocation are shown in orange. The errors after the optimization phase are generally below 6%. In fact, only 7 out of 444 municipalities had an average error higher than 6%. Those municipalities are located in large...
counties containing 46 and 23 municipalities, respectively. Additionally, some of the municipalities had a higher concentration of students or retired persons living in large households, which were scarcely observed on the microdata. The microdata seemed to underrepresent selected combinations of attributes, making it difficult to match all control totals very well.

![Figure 3. Average error (%) per municipality at the optimization and allocation phase.](image)

The errors of the allocation phase are, as expected, highly dependent on the size of the municipality. In small municipalities, it is likely that many weights are near zero. Microrecords are sampled proportionally to their weights an integer number of times. The “law of large numbers” generally compensated the error on the attributes for a large number of draws, but it led to lower accuracy for small municipalities with a small number of draws, given the high number of possible combinations of attributes. In this sense, the size of the municipality, the number of microrecords, and the number of combinations of attributes should be balanced. In our case study, the balance was produced for municipalities with 5000 inhabitants or more (asymptote in Figure 3).

The standard deviation between the five runs is also highly dependent on the size of the municipality (Figure 4). The standard deviation is generally lower than 1% for municipalities higher than 10,000. The optimization phase is deterministic, and therefore the deviation in five runs is equal to zero. As a consequence, only the allocation phase is plotted in Figure 4.

Secondly, we analyzed the errors at the county level for all attributes (Figure 5). Given the size dependency of the allocation error, the average error at the county level was calculated weighting the municipality error by its population. Weighted average error was used at the optimization phase as well for consistency.

After the optimization phase, the majority of the counties had a weighted average error lower than 1.5%, and only two counties exceeded 3%. Not surprisingly, the weighted average error increased during the allocation phase. Rural counties with smaller municipalities presented the largest weighted average errors, although they were below 6%. Compared to the municipality level, the weighted average error at the county level is lower. The lower error of measurement for the larger zones is only a natural effect of the scale-dependent accuracy of geoinformation, perhaps due to spatial autocorrelation of the data and that at the county level, the number of microrecords is multiplied by the number of municipalities within the county, having a bigger sample to distribute the control totals.
Figure 4. Standard deviation between five model runs (%) per municipality at the optimization and allocation phase.

Figure 5. Weighted average error (%) per county after: (a) optimization phase; (b) allocation phase.
Finally, we analyzed the results for the city of Munich at three geographical resolutions: county, municipality, and borough. The error for the city of Munich was 2%, slightly higher than other large cities. The attribute with the highest error is single-person households, which provided an average error of 10%, similar across all boroughs. The error is produced due to inconsistencies between borough and municipality control totals, as the total number of single-person households in Munich differs from 368,447 at the municipality level to 410,993 as the sum of the boroughs. The difference is around 10%, which is the same result obtained from the algorithm.

Figure 6 shows the share of single-person households across Munich boroughs and the difference between control totals and the results after the optimization. The differences in absolute value are below 0.2%, providing practically the same distribution of the attributes across all boroughs. Therefore, the sole analysis of the error could be misleading when control totals at different resolutions are not consistent.

![Figure 6](image-url)
Table 2. Weighted average error and standard deviation for household attributes.

| Attribute            | Weighted Average Error (%) and Standard Deviation (%) |
|----------------------|-------------------------------------------------------|
|                      | Optimization Phase | Allocation Phase |
| Total                | 0.0 [-]            | 0.0 [0.00]       |
| Size: 1              | 0.1 [-]            | 1.5 [0.08]       |
| Size: 2              | 0.2 [-]            | 1.5 [0.08]       |
| Size: 3              | 0.3 [-]            | 2.2 [0.12]       |
| Size: 4              | 0.7 [-]            | 2.8 [0.10]       |
| Size: 5+             | 0.6 [-]            | 3.8 [0.11]       |

Table 3. Weighted average error and standard deviation for dwelling attributes.

| Attribute                                      | Weighted Average Error (%) and Standard Deviation (%) |
|------------------------------------------------|-------------------------------------------------------|
|                                                | Optimization Phase | Allocation Phase |
| Tenure status: owned                          | 0.7 [-]            | 1.4 [0.09]       |
| Tenure status: rented                         | 0.2 [-]            | 1.3 [0.05]       |
| Living space: <60 sqm                         | 0.1 [-]            | 1.4 [0.08]       |
| Living space: 60–80 sqm                       | 0.1 [-]            | 0.6 [0.08]       |
| Living space: 80–100 sqm                      | 0.1 [-]            | 1.2 [0.20]       |
| Living space: 100–120 sqm                     | 0.1 [-]            | 1.5 [0.12]       |
| Living space: >120 sqm                        | 0.2 [-]            | 1.4 [0.09]       |
| Smaller building constructed before 1948      | 0.2 [-]            | 2.1 [0.22]       |
| Smaller building constructed 1949–1990        | 0.1 [-]            | 1.3 [0.16]       |
| Smaller building constructed 1991–2000        | 0.2 [-]            | 1.8 [0.38]       |
| Smaller building constructed after 2001       | 0.3 [-]            | 1.6 [0.11]       |
| Larger building constructed before 1948       | 0.2 [-]            | 1.4 [0.33]       |
| Larger building constructed 1949–1990         | 0.1 [-]            | 0.6 [0.07]       |
| Larger building constructed 1991–2000         | 0.1 [-]            | 1.2 [0.29]       |
| Larger building constructed after 2001         | 0.0 [-]            | 0.8 [0.09]       |

Table 4. Weighted average error and standard deviation for person attributes.

| Attribute          | Male (Phase O) | Male (Phase A) | Female (Phase O) | Female (Phase A) | All (Phase O) | All (Phase A) |
|--------------------|----------------|----------------|------------------|------------------|---------------|---------------|
| Total persons      | 1.9 *          | 1.9 *          | 2.1 *            | 2.2 *            | 2.0 [-]       | 2.0 [-]       |
| Workers            | 0.6 [-]        | 1.8 [0.06]     | 0.9 [-]          | 2.3 [0.05]       | 0.7 *         | 1.8 *         |
| Foreigners         |                |                |                  |                  | 0.7 [-]       | 4.6 [0.12]    |
| Age: <4 years old  | 2.3 [-]        | 6.5 [0.18]     | 2.8 [-]          | 6.9 [0.45]       | 2.4 *         | 4.9 *         |
| Age: 5–9 years old | 2.7 [-]        | 6.9 [0.27]     | 2.6 [-]          | 6.8 [0.32]       | 2.5 *         | 5.0 *         |
| Age: 10–14 years old | 3.0 [-] | 6.9 [0.32] | 2.8 [-] | 6.5 [0.26] | 2.7 * | 4.7 * |
| Age: 15–19 years old | 2.7 [-] | 6.2 [0.25] | 3.1 [-] | 7.0 [0.36] | 2.8 * | 5.1 * |
| Age: 20–24 years old | 1.9 [-] | 5.1 [0.26] | 1.4 [-] | 5.5 [0.53] | 1.6 * | 4.2 * |
| Age: 25–29 years old | 1.3 [-] | 4.5 [0.13] | 1.3 [-] | 4.5 [0.24] | 1.3 * | 3.2 * |
| Age: 30–34 years old | 1.7 [-] | 4.2 [0.24] | 2.1 [-] | 4.7 [0.12] | 1.8 * | 3.3 * |
| Age: 35–39 years old | 2.2 [-] | 4.4 [0.11] | 2.8 [-] | 4.9 [0.15] | 2.4 * | 3.8 * |
| Age: 40–44 years old | 2.0 [-] | 4.1 [0.19] | 2.9 [-] | 4.6 [0.15] | 2.3 * | 3.3 * |
| Age: 45–49 years old | 2.0 [-] | 3.7 [0.17] | 2.4 [-] | 3.9 [0.15] | 2.1 * | 3.3 * |
| Age: 50–54 years old | 1.8 [-] | 3.7 [0.13] | 1.9 [-] | 3.5 [0.14] | 1.8 * | 2.9 * |
| Age: 55–59 years old | 1.3 [-] | 3.6 [0.10] | 1.7 [-] | 3.8 [0.28] | 1.5 * | 3.1 * |
| Age: 60–64 years old | 1.2 [-] | 3.8 [0.16] | 1.5 [-] | 4.1 [0.23] | 1.3 * | 3.1 * |
| Age: 65–69 years old | 2.0 [-] | 4.6 [0.30] | 2.0 [-] | 5.0 [0.20] | 1.9 * | 3.8 * |
| Age: 70–74 years old | 1.7 [-] | 4.3 [0.33] | 2.3 [-] | 4.9 [0.29] | 1.9 * | 4.1 * |
| Age: 75–79 years old | 2.4 [-] | 6.0 [0.32] | 2.0 [-] | 5.4 [0.35] | 2.1 * | 4.8 * |
| Age: >80 years old | 1.5 [-] | 5.5 [0.27] | 2.0 [-] | 5.4 [0.18] | 1.7 * | 4.2 * |

Notes: O stands for optimization and A stands for allocation; * Not used to calculate the average error, included for comparison purposes.
The error in the total number of households is equal to zero because the method adjusted weights to match that number. During optimization, the total number of households was the last attribute to compute. As a consequence, the final weight provides a perfect match for the total number of households. During allocation, households are drawn N-times, with N being the total number of households at the control total.

The distribution of household sizes provides individual errors below 1% after the optimization phase and below 4% after the allocation phase. Errors increase slightly with household size. Large households have more possible combinations of attributes and they are less frequent in the microdata, which increases the error. The initial categories of household size were reduced from six to five to reduce the allocation errors in small municipalities for very large household sizes.

Similar conclusions are obtained from dwelling attributes. There are very few microdata records with dwellings in small buildings constructed after 2005. After a series of trials and errors, the initial categories of dwelling construction period were reduced from six to four categories.

The errors for person attributes are higher than for household and dwelling attributes. It is caused by the high number of possible cross-classifications by age and gender (equal to 34). Age-by-gender distributions are a key demographic characteristic for the land use model. Therefore, we opted to maintain the relatively large number of stratifications. The errors were below 3.1% after the optimization phase and varied between 3.5% and 7.0% after the allocation phase. If only age is taken into account, the error decreases considerably and is below 5%. Given that census data have a small error term as well (between 5% and 10% for the number of microrecords used in this case study [42]), the results are assumed to represent the population of the study area reasonably well.

4. Discussion

In this paper, we developed the synthetic population of the greater Munich metropolitan area. The algorithm is written in Java 8, taking advantage of memory efficiency benefits and parallelization of some methods for the optimization procedure. As with some other authors, we used IPU for optimization [17,20,21,25] and, like almost all synthesizers, Monte Carlo sampling for allocation. The runtime of the optimization phase is 17 min, while the allocation phase takes 1 h. Household, person, and dwelling objects are created during the allocation phase sequentially, increasing the total runtime of the program. The population synthesizer outperforms other synthesizers, which usually take between a couple of hours to an overnight run [17] for a similar scale.

We developed, for the first time, one application for three geographical areas. This feature could be beneficial in large municipalities with different characteristics across boroughs and case studies with control totals clustered at different geographical resolutions. We observed that the attributes controlled at the higher geographical resolution (i.e., county) presented lower errors than those controlled at the lower geographical resolution (i.e., municipality). At the county level, the number of microrecords is multiplied by the number of municipalities within the county. Having a bigger sample of microrecords to calculate the weights leads to better distributions of control totals, and therefore, to more accurate allocation results at the county level. Nevertheless, the local distribution at the municipality level is not controlled and could lead to higher errors if there are strong differences across the municipalities. In some cases, the algorithm can produce better results if the attributes are controlled at the higher geographical resolution, and the lower resolution is only used for validation. In the case of inconsistencies between control totals at two geographical levels, it is misleading to solely use the average error. The results should be analyzed with caution to determine whether the share across the lower level resembles the control totals distribution. Another approach could be rescaling the control totals distributions of the less reliable area to have the same total in both resolutions.

Another important discussion is the order of attributes at the optimization phase. The algorithm updates sequentially the set of weights to match the control totals. Therefore, the weights better represent those attributes that are matched at the end. As a consequence, the most important attributes...
shall come last, while the least reliable (or least relevant) attributes shall come first. During this exercise, all county attributes were considered before starting with municipality attributes due to reliability.

The application presents one of the highest numbers of attributes found in the literature (see Table A2). As far as the authors know, there is no open discussion regarding the number of attributes and their categorization to avoid overfitting. Some categories were merged to reduce the number of possible combinations of household, person, and dwelling types. Having a fast algorithm allowed for testing multiple combinations of attributes and categories that reduced the resulting error without significantly decreasing the fidelity of the results. In this case study, the initial categories for household size were reduced from six to five to control the errors at small municipalities. Similarly, the number of dwelling ages by building size was reduced from 12 to 8.

5. Conclusions

Our algorithm to create a synthetic population maintained the advantages of the original IPU algorithm and added two important improvements. Foremost, it expanded the geographical scope from two to three geographic levels. This allowed for a more accurate synthesis of the city of Munich, which contains 29% of the population and has differentiated demographic profiles by boroughs. The method can include different attributes at each geographical level.

The algorithm is also able to synthesize 4.5 million persons in 2.1 million households in less than 1.5 h. Having a faster algorithm can improve the accuracy of the synthesized population. A series of trials with different algorithm stratifications and a different order in which attributes are controlled are usually required to better replicate the population. The order of control attributes during the optimization alters the results. Control attributes with lower reliability or less relevance for the task at hand should be tested first to reduce their influence on the final results. The most important control attributes, such as total population or total number of households, should be tested last to assure that the final weights accommodate their control totals.

Equally important is to avoid overfitting. An excessive number of attributes could drastically increase the number of possible combinations of household, person, and dwelling types and lead to very scarce combinations of those attributes on the microdata. This issue becomes more relevant for larger counties with a high number of municipalities. In such cases, the user needs to balance the relative importance and reliability of having more categories from the attributes and the error that could be produced after allocation. This consideration is especially important for small municipalities.

The population synthesizer is incorporated with the open-source land use model SILO [43] and is available at the GitHub repository: https://github.com/msmobility/silo. The pseudocode for the optimization phase is in Table A3. Implementations of this work in Cape Town and Kagawa (Japan) demonstrated the adaptability of the algorithm to other study areas. Further plans to implement this work in Sydney and Teheran, and to compare the results in Cape Town with an alternative approach will appraise the adaptability of the algorithm to other study areas and modeling scales. More empirical work still needs to be done to evaluate the model performance of the algorithm in land use models or other applications, such as travel demand models [44,45] or public health models [46], and to improve allocation based on dasymetric mapping [47]. Including agents’ microlocation (or explicit coordinates) is also under investigation.

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Appendix A

Table A1. Summary of previous population synthesis based on optimization technique.

| Model            | Ref.            | Optimization Procedure ¹ | Allocation Procedure          |
|------------------|-----------------|--------------------------|-------------------------------|
| TRANSIMS         | [1]             | IPF for hh and pp        | Monte Carlo                   |
| CEMDAP           | [3]             | IPF for hh and pp        | Monte Carlo with replacement  |
| ILUMASS          | [4]             | Microsimulation for hh and pp, IPF for dd and jj | Monte Carlo                   |
| PopSynWin        | [5,13]          | IPF for hh and pp        | Monte Carlo                   |
| ILUTE            | [6,7,36,48]     | IPF for hh and families with sparse list | Conditioned Monte Carlo         |
| ALBATROSS        | [8]             | IPF with relation matrix at pp level and IPF for hh | Monte Carlo                   |
| Zhu and Ferreira | [11]            | Two step IPF for hh and pp | Monte Carlo                   |
| FSUTMS           | [9]             | IPF                      | Monte Carlo                   |
| Lovelace et al.  | [14]            | IPF for pp               | Monte Carlo                   |
| Whitworth et al. | [15]            | IPF with spatial microsimulation | Monte Carlo                   |
| Bar-Gera et al.  | [22]            | IPF, entropy maximization | Monte Carlo                   |
| Barthelemy and Toint | [23]       | IPF, entropy maximization | Monte Carlo for household head |
| PopSyn           | [10]            | IPF, entropy maximization | Monte Carlo                   |
| Rose et al.      | [16]            | IPF, entropy maximization | Monte Carlo                   |
| PopGen           | [12,13,20,21,32]| IPU                      | Monte Carlo                   |
| Fournier et al.  | [17]            | IPF, integerization, IPU | Monte Carlo                   |
| Mueller and Axhausen | [49]     | Hierarchical IPF          | Monte Carlo                   |
| Ryan et al.      | [50]            | Combinatorial optimization | With fitting                  |
| Synthesizer      | [26]            | Combinatorial optimization | With fitting                  |
| Farooq et al.    | [27]            | Full or Partial conditionals using discrete choice models | Monte Carlo Markov chains |
| Saadi et al.     | [19,28]         | Hidden Markov Models      | Monte Carlo                   |
| Saadi et al.     | [28]            | Partial conditionals using Hidden Markov Models | Monte Carlo                   |
| SILC             | [31]            | Multinomial regression model | Monte Carlo                   |
| Agenter          | [30,33]         | Multinomial regression model | Choice modeling               |

¹ hh stands for households; pp stands for persons.
Table A2. Summary of previous population synthesis based on geographical area, location, and variables.

| Model       | Ref. | Application                  | Geographical Resolutions | Household Attributes                                                                 | Person Attributes                                      | Dwelling Attributes          |
|-------------|------|------------------------------|--------------------------|------------------------------------------------------------------------------------|--------------------------------------------------------|-----------------------------|
| TRANSIMS    | [1]  | Bay Area                     | Local                    | Size                                                                                | -                                                      | -                           |
| CEMDAP      | [3]  | Dallas/Forth Worth MA        | Target area              | Family, type, children, size, zone                                                | Gender, age, race                                      | -                           |
| TRI-LAT     | [4]  | Netanya, 159,000 persons 50,000 households | Zones                   | Size, workers, income, cars                                                        | Age, gender, religion, education, workplace            | -                           |
| ILUMASS     | [4]  | Dortmund, 2.6 M persons in 1.1 M households | Zones                   | Size, workers, income, cars                                                        | Age, gender, religion, education, workplace            | -                           |
| PopSynWin   | [5]  | Chicago, 1.5 M persons in 1.5 M households | Block groups            | Size, income, workers, zone                                                         | Gender, age, ethnicity                                 | -                           |
| PopSynWin   | [13] | Melbourne, 4 M persons in 1.4 M households | Census zones            | Type, size, cars                                                                   | Gender, age, employment                                | -                           |
| ILUTE       | [6,7,36,48] | Toronto, 3.4 M persons 1.1 M households | Census tracts           | Size                                                                                | Gender, Income, age by family, age by labor, age by education, education by labor, occupation | -                           |
| ALBATROSS   | [8]  | The Netherlands, 6.4M households | Zones and regions       | HH type, region and density                                                         | Gender, age, employment                                | -                           |
| FSUTMS      | [9]  | Florida state 87,800 persons in 23,000 households | Census tracts           | Workers, income, cars, size, structure                                            | Age, gender, ethnicity, working hours, citizenship    | Size, tenure                |
| Zhu and Ferreira | [11] | Singapore                     | TAZ                     | Size, income, workers                                                              | Age, gender, ethnicity                                 | Type                        |
| Whitworth et al. | [15] | Wales                         | MSOAs                   | -                                                                                  | Age by sex, employment, quals, health, region          | Tenure                      |
| Lovelace et al. | [14] | South Yorkshire               | Area code               | -                                                                                  | Age by gender, travel mode, distance to work, income   | -                           |
| Rose et al.  | [16] | Bangladesh, 150 M persons     | Fit to division and estimate district | -                                                                | Age by gender by school attendance, average size of household, electricity, tenure status, rural/urban division | -                           |
| Bar-Gera et al. | [22] | Maricopa county               | -                       | Type, size, income                                                                 | Gender, age, ethnicity                                 | -                           |
| Barthelemy and Toint | [23] | Belgium, 10 M persons in 4.3 M households | Districts               | Type, children, other adults                                                        | Age, gender, activity, education, driving license      | -                           |
| PopSyn      | [10] | Maricopa, 4 M persons         | County, TAZ, MAZ        | Size, type, income, workers                                                         | Age                                                    | -                           |
| PopGen      | [20] | Maricopa county, 5.4 M persons, 2.0 M households, 0.1 M quarters | Block groups            | Type, size, income                                                                 | Gender, age, ethnicity                                 | -                           |
| PopGen      | [21] | Baltimore metro council       | County, PUMA, TAZ       | Size, income, workers                                                              | Age, employment                                        | -                           |
| PopGen      | [32] | Sydney, 4.9 M persons 1.8 M households | TAZ                     | Type, size, cars                                                                   | Age, gender, employment                                | Type                        |
| PopGen      | [12] | Southern California 17 M persons 5.5 M households | TAZ                     | Family, head age, size, type, children, income                                      | Age, gender, ethnicity                                 | -                           |
### Table A2. Cont.

| Model           | Ref.   | Application                        | Geographical Resolutions | Household Attributes                  | Person Attributes                                      | Dwelling Attributes |
|-----------------|--------|------------------------------------|---------------------------|---------------------------------------|-------------------------------------------------------|---------------------|
| PopGen          | [13]   | Melbourne, 4 M persons 1.4 M households | Census tracts             | Type, size, cars                      | Gender, age, employment                                | -                   |
| Mueller and     | [49]   | Switzerland, 7 M persons 3.1 M households | Municipality              | Size, type, children, age of head, age oldest child, age youngest child | Age, gender, foreigner, marital status, education, workplace location, commute mode | -                   |
| Axhausen        |        | Boston, 4.6 M persons in 1.7 M households | Census tracts             | Size, cars, income, race              | Gender, age, work hours, school enrollment, relationship, travel time, industry, occupation | Type                |
| Fournier et al. | [17]   | California STDM, 33.9 M persons, 11.5 M households 0.8 M group quarters | TAZ                       | Size, income, cars, resident          | Age, occupation, grade level                           | Type                |
| Synthesizer     | [26]   | Brussels, 1.2 M households          | Regions                   | Size, workers, children, cars, education, income | -                                                      | -                   |
| Farooq et al.   | [27]   | Switzerland                         | Sectors                   | Household size                        | Age, gender, education                                 | -                   |
| Saadi et al.    | [19,29]| Belgium                             | Municipality              | -                                     | Age, gender, education, travel distance, profession    | -                   |
| Saadi et al.    | [28]   | Belgium                             | Municipality              | -                                     | Age, gender, socio-professional status, working time expenditure, public transport subscription, driver license | -                   |
| SILC            | [31]   | Austria, 10 M persons 3.8 M households | Region                    | Size, region, urbanization            | Age, gender, employment                                | -                   |
| Agenter         | [30,33]| Beijing                             | TAZ                       | Income, size, parcel ID, distance to center | Age, gender, marriage, education, occupation           | -                   |
Table A3. Pseudo code of IPU with three geographical resolutions and two levels.

(a) Main routine, to be repeated for each county

**Require:** Reference sample $H$ in frequency matrix  
**Require:** County $C$ control totals $T_{c,a}, T_{c,a'}$,...  
**Require:** Municipalities $m$ control totals $T_{m_1,\beta}, T_{m_1,\beta'},...; T_{m_2,\beta}, T_{m_2,\beta'}$,...  
**Require:** Boroughs $b$ control totals $T_{b_1,y}, T_{b_1,y'},...; T_{b_2,y}, T_{b_2,y'}$,...  
**Require:** Initial set of weights $w_{bh}^{(0)}$  
**Ensure:** Set of weights $w_{bh}$ for each borough $b$ of the county and each household $h$ from the reference sample $H$ obeying all control totals  
\[ w_{bh} \leftarrow w_{bh}^{(0)} \text{ for all } h \in H \]
while convergence not reached do  
\[ w_{bh} \leftarrow \text{County IPU} (H, w_{bh}, T_{c,a}, T_{c,a'}) ... \]
\[ w_{bh} \leftarrow \text{Municipality IPU} (H, w_{bh}, T_{m_1,\beta}, T_{m_1,\beta'}) ...; T_{m_2,\beta}, T_{m_2,\beta'} ...) \]
\[ w_{bh} \leftarrow \text{Borough IPU} (H, w_{bh}, T_{b_1,y}, T_{b_1,y'}) ...; T_{b_2,y}, T_{b_2,y'} ...) \]
Check convergence  
return $w_{bh}$

(b) Subroutine County IPU

**Require:** Households $h$ from the reference sample $H$ in frequency matrix  
**Require:** County $C$ control totals $T_{c,a}, T_{c,a'}$,...  
**Ensure:** Improved set of weights $w_{bh}$ that fits control totals at the county level  
For all attributes $a$ at the county level do  
\[ H_a \leftarrow \{h_p : p \in P_a \} \]
\[ f \leftarrow T_{c,a} / \sum_{b} \sum_{p} (w_{bh} \cdot h_p) \]
For all boroughs $b$ of the county $C$  
\[ w_{bh} \leftarrow w_{bh} \cdot f \quad \forall h \in H_a \]
return $w_{bh}$

(c) Subroutine Municipality IPU

**Require:** Households $h$ from the reference sample $H$ in frequency matrix  
**Require:** Municipalities $m$ control totals $T_{m_1,\beta}, T_{m_1,\beta'},...; T_{m_2,\beta}, T_{m_2,\beta'}$,...  
**Ensure:** Improved set of weights $w_{bh}$ that fits control totals at the municipality level  
For all attributes $\beta$ at the municipality level do  
For all municipalities $m$ within the county do  
\[ H_\beta \leftarrow \{h_p : p \in P_\beta \} \]
\[ f \leftarrow T_{m,\beta} / \sum_{b} \sum_{p} (w_{bh} \cdot h_p) \]
For all boroughs $b$ of the municipality do  
\[ w_{bh} \leftarrow w_{bh} \cdot f \quad \forall h \in H_\beta \]
return $w_{bh}$

(d) Subroutine Borough IPU

**Require:** Households $h$ from the reference sample $H$ in frequency matrix  
**Require:** Boroughs $b$ control totals $T_{b_1,y}, T_{b_1,y'},...; T_{b_2,y}, T_{b_2,y'}$,...  
**Require:** Attributes at the borough level $\gamma, \gamma'$...  
**Require:** Boroughs $b$ of the county $c$  
**Require:** Current set of weights $w_{bh}$  

Ensure: Improved set of weights $w_{bh}$ that fits control totals at the borough level
For all attributes $\gamma$ at the borough level do
  \begin{align*}
  &H_{\gamma} \leftarrow \{ h_p : p \in P_{\gamma} \} \\
  &f \leftarrow T_{b,\gamma} / \sum_p (w_{bh} \cdot h_p) \\
  &w_{bh} \leftarrow w_{bh} \cdot f \quad \forall h \in H_{\beta} \\
  &\text{return } w_{bh}
  \end{align*}

(e) Subroutine Check convergence

Require: Households $h$ from the reference sample $H$ in frequency matrix
Require: County $c$ control totals $T_{c,a_1}, T_{c,a_2}, \ldots$
Require: Attributes at the county level $a_1, a_2, \ldots$
Require: Municipalities $m$ control totals $T_{m_1,\beta_1}, T_{m_1,\beta_2}, \ldots, T_{m_2,\beta_1}, T_{m_2,\beta_2}, \ldots$
Require: Attributes at the municipality level $\beta_1, \beta_2, \ldots$
Require: Boroughs $b$ control totals $T_{b_1,\gamma_1}, T_{b_1,\gamma_2}, \ldots, T_{b_2,\gamma_1}, T_{b_2,\gamma_2}, \ldots$
Require: Attributes at the borough level $\gamma_1, \gamma_2, \ldots$
Require: Boroughs $b$ of the county $c$
Require: Current set of weights $w_{bh}$
Require: Previous average error in absolute value $\bar{\varepsilon}(0)$
Require: Threshold for the average error in absolute value $\varepsilon_{\text{min}}$
Require: Threshold for difference between two iterations $\Delta \varepsilon_{\text{min}}$
Require: Maximum number of iterations $i_{\text{max}}$

Ensure: Check for stopping criteria based on the average error in absolute value $\bar{\varepsilon}$ and iteration $i$

For all attributes $\alpha$ at the county level do
  \begin{align*}
  &H_{\alpha} \leftarrow \{ h_p : p \in P_{\alpha} \} \\
  &\text{For all boroughs } b \text{ within the county do} \\
  &\quad \varepsilon \leftarrow \varepsilon + \left| \left( T_{c,\alpha} - \sum_b (w_{bh} \cdot h_p) \right) / T_{c,\alpha} \right| \\
  &\quad n \leftarrow n + 1
  \end{align*}
For all attributes $\beta$ at the municipality level do
  \begin{align*}
  &H_{\beta} \leftarrow \{ h_p : p \in P_{\beta} \} \\
  &\text{For all municipalities } m \text{ within the county do} \\
  &\quad \varepsilon \leftarrow \varepsilon + \left| \left( T_{m,\beta} - \sum_b (w_{bh} \cdot h_p) \right) / T_{m,\beta} \right| \\
  &\quad n \leftarrow n + 1
  \end{align*}
For all attributes $\gamma$ at the borough level do
  \begin{align*}
  &H_{\gamma} \leftarrow \{ h_p : p \in P_{\gamma} \} \\
  &\text{For all boroughs } b \text{ within the county do} \\
  &\quad \varepsilon \leftarrow \varepsilon + \left| \left( T_{b,\gamma} - \sum_p (w_{bh} \cdot h_p) \right) / T_{b,\gamma} \right| \\
  &\quad n \leftarrow n + 1
  \end{align*}
\[ \bar{\varepsilon} \leftarrow \varepsilon / n \]
If $\bar{\varepsilon} < \varepsilon_{\text{min}}$ stop
If \( (\bar{\varepsilon} - \bar{\varepsilon}(0)) / \bar{\varepsilon}(0) < \Delta \varepsilon_{\text{min}} \) stop
If $i = i_{\text{max}}$ stop
\text{return } stop \text{ or continue}

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