Testing Causal Quantum Theory

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Causal quantum theory assumes that measurements or collapses are well-defined physical processes, localised in space-time, and never give perfectly reliable outcomes and that the outcome of one measurement only influences the outcomes of others within its future light cone. Although the theory has unusual properties, it is not immediately evident that it is inconsistent with experiment to date. I discuss its implications and experimental tests.

INTRODUCTION

There are fairly compelling theoretical and experimental reasons [1–17] to believe that nature violates local causality and that local hidden variable theories are incorrect. However, while loopholes remain, the case is not quite conclusive. In particular, the collapse locality loophole [18] remains largely untested. Moreover, causal quantum theory [18] is a theoretically interesting example of a local hidden variable theory that remains consistent with Bell experiments to date by exploiting this loophole.

Causal quantum theory assumes that measurements or collapses are well-defined physical processes, localised in space-time, and never give perfectly reliable outcomes. Unlike standard quantum theory, it also assumes that measurement outcomes respect a strong form of Minkowski or Einstein causality, in that the outcome of one measurement only affects the probability distributions for the outcomes of other measurements within its future light cone. This gives it very peculiar properties, which evoke the suspicion that it must already be excluded by experiments other than Bell experiments and/or cosmological observations. Perhaps it is, but this has not yet been shown. It is an interesting challenge to our understanding of physics to seek conclusive evidence for quantum theory against this peculiar and even more counter-intuitive alternative.

I first review the definition of causal quantum theory, its theoretical motivation, and its very radical implications. I then review the evidence from Bell experiments. As is well known, a long sequence of successively more sophisticated Bell experiments have confirmed the predictions of standard quantum theory. However (with one only partial exception discussed below) all Bell experiments to date are also consistent with causal quantum theory if the collapse process only takes place when macroscopic displacements of masses or significantly different gravitational fields are superposed. They are also consistent with causal quantum theory if collapse requires a measurement outcome enters the consciousness of an observer. Since these are the most popular collapse hypotheses, causal quantum theory is not definitively excluded by Bell experiments to date.

Finally, I consider other experiments that could exclude causal quantum theory.

CAUSAL QUANTUM THEORY

Quantum theory and special relativity are related rather subtly and beautifully, in a way that allows quantum theory to violate local causality without allowing superluminal signalling. There is nothing evidently problematic in this relationship, except perhaps that it is not so obvious how to extend it to include a theory of gravity. Still, it is interesting to consider alternatives, even if ultimately only to be sure we understand just how compelling the evidence is for standard quantum theory.

Causal quantum theory assumes that measurements or collapses are well-defined physical processes, localised in space-time, and never give perfectly reliable or definitive outcomes. It then ensures consistency with special relativity by postulating that measurement or collapse outcomes respect a strong form of Minkowski causality, in that a measurement’s outcome only influences the probability distribution for the outcomes of other measurements within its future light cone. There may be no strong reason to prefer these assumptions over those of standard quantum theory, even in the absence of experimental evidence. However, they are all reasonably well motivated: the following paragraphs sketch some reasons.

Collapse hypotheses can be motivated as solutions to the quantum reality (or measurement) problem, as alternative routes to unifying quantum theory and gravity without necessarily quantising gravity in any standard sense, or even...
as speculative ways of connecting consciousness and physics. All of these motivations are questionable, but all have thoughtful proponents.

If collapses are objective, it is quite plausible that they are typically well localized events, and indeed this is a feature of some explicit collapse models [19, 20]. As far as we know, all real world measurements are imperfect, and it seems very plausible that this is true of all possible real world measurements or observations, including those made by our organs of perception and brains. It is also a feature of most well known explicit collapse models that collapse outcomes are never definitive – “tails” in the wave function persist. Also, as far as we know, quantum states in the real world never precisely lie in the kernel of real world measurement operators.

The strong form of Minkowski causality that defines causal quantum theory holds in classical physical models respecting the relativity principle, and the similarly strong version of Einstein causality holds in general relativity. All else being equal, imposing this version of causality on a physical theory is the simplest way to ensure that it respects at least the signalling constraints implied by special and general relativity.

**Measurement model and assumptions**

We can define *causal quantum theory* [18] by an abstract black box model of measurements, rather than specifying a particular localized collapse theory or measurement model. We will focus here on measurements that are approximations to an ideal detection of a single particle: it is easy to generalize our discussion to other types of local measurements. In our model, a measurement $M$ is defined by a set of Kraus operators $\{A_i\}_{i \in I}$ and is a physical operation that takes place inside a finite black box, which produces an output (the measurement outcome $i$). To simplify, we suppose the box is of negligible size and that the measurement takes negligible time, so that in our model we can approximate the measurement output as being created at a definite point $x_M$ in space-time. We assume a fixed background space-time with no closed time-like curves. Our discussion applies in any such space-time, but for definiteness we consider Minkowski space unless otherwise specified.

The measurement operators include a distinguished operator, $A_0$, whose outcome is supposed to correspond approximately to the event that no particle was detected in the box. The other outcomes $i \neq 0$ are supposed to correspond approximately to a detection of a particle in the box, perhaps together with other information – for example, the type of particle detected and/or information about internal degrees of freedom. So, suppressing internal degrees of freedom to simplify the illustration, if $\psi(x_1, \ldots, x_n)$ is an $n$-particle state with zero particle probability density inside the box, we have $A_i \psi \approx 0$ for $i \neq 0$.\[31\]

The measurement operators $A_i$ are supposed to roughly approximate projections, $A_i \approx P_i$, where $P_i$ is a projection operator corresponding to the relevant outcome. The sense in which this approximation holds depends on the details of the collapse or measurement model in question. A good illustration is a GRW [19] localization operator $A_{x,0}^n$, which acts on single particle wave functions $\psi(x)$ by

$$\psi(x) \rightarrow C \exp((x-x_0)^2/a^2)\psi(x),$$

where $a$ is a constant of the localisation model and $C$ is a normalisation constant. This approximates a projective measurement of position onto the interval $[x_0 - a, x_0 + a]$.

While the Kraus operators may correspond to the specified outcomes to very good approximation, the simplest version of our model requires that none of them has a zero eigenstate. For example, for any $\psi$ as above and any $i$, $A_i \psi \neq 0$, although $|A_i \psi|$ may be very small. GRWP [19, 20] spontaneous localisation model collapses have this feature.

One way to interpret this in models of measurement is to think of the measurements as carried out by imperfect detectors that always have some nonzero probability of giving false positive or negative detections, of misidentifying the particle type, and so on. As far as we know, every real world detector, including the perception organs and brains of conscious observers, indeed has these properties.

A more general version of our model requires only the weaker condition that, given any physically realisable initial state $\psi_S$ defined on a hypersurface $S$ prior to space-time points $x_1, \ldots, x_n$, the prescribed unitary evolution law, and any sequence of measurement operators $A_{x_1}^1, \ldots, A_{x_n}^n$ localised near the space-time points $x_1, \ldots, x_n$, the final state $\psi_S'$ arising on any hypersurface $S'$ to the future of $x_1, \ldots, x_n$ is well-defined (i.e. non-zero). This could be justified by noting that in practice we can never produce pure states of a precisely specified form, and every physically realised state contains small uncontrolled components that are not annihilated by any possible sequence of measurements. This may also be true of the cosmological initial state.

The effect of obtaining outcome $i$ is, as usual, to replace an initial state $\rho$ by the post-measurement state

$$A_i \rho A_i^\dagger / \text{Tr}(A_i \rho A_i^\dagger).$$

\[2\]
We allow here the possibility that $\rho$ may be mixed. Since the Kraus operators have no zero eigenstates, the denominator is always nonzero, and so this effect is always well defined.

So far, this picture is consistent with standard formulations of quantum theory, modulo some vagueness about whether we can think of measurement outcomes as arising at a definite space-time point (and if so exactly which point). In a version of Copenhagen quantum theory with prescribed Heisenberg cuts defining measurement apparatus, we can think of our black boxes as defined spatially by measurement apparatus and temporally by the measurement process, or by regions within these space-time regions. On Wigner’s hypothesis, we can define the black boxes spatially by the brains of conscious observers and temporally by their conscious perception times.

The Dirac-von Neumann treatment of measurements corresponding to projection operators can be understood in this model – as generally in modern formulations of quantum theory – as a mathematically useful but physically unrealistic limiting case.

**Standard relativistic quantum theory**

Now suppose that we have a relativistically covariant quantum evolution law, with quantum states $\psi_S$ defined on space-like hypersurfaces $S$ via the Tomonaga-Schwinger formalism. We need to include the effects of measurements within this framework.

The standard way to do this is to define $\psi_S$ by applying the unitary evolution from the initial hypersurface $S_0$ to $S$, together with the measurement transformations (2) for all measurements taking place between $S_0$ and $S$. The outcome probabilities of measurements on (or in the near future of) $S$ are then obtained from $\psi_S$ in the usual way. In particular, the outcome probabilities for a measurement localized at the point $x \in S$ depend only on the local density matrix of $\psi_S$ at $x$. Since the quantum measurement postulate is consistent with Minkowski causality, this prescription gives a well-defined answer and is Lorentz covariant. In particular, it does not matter in which order (2) is applied for space-like separated measurement events. This prescription defines standard relativistic quantum theory within our model of measurements.

**Causal quantum theory**

By contrast, in causal quantum theory, the outcome probabilities for a measurement localized at the point $x$ depend only on the local density matrix of $\psi_{\Lambda(x)}$ at $x$, where, loosely speaking, $\psi_{\Lambda(x)}$ is the wave function defined on the past light cone $\Lambda(x)$ of $x$. More precisely, the local density matrix is the limit of the local density matrices for the wave functions of spacelike hypersurfaces $S$ tending to $\Lambda(x)$. Equivalently, we calculate the local density matrix at $x$ from $\psi_S$, defined for any spacelike hypersurface $S$ through $x$, but for this calculation we define $\psi_S$ allowing only for the outcomes of measurements inside $\Lambda(x)$. According to this prescription, if $i$ and $j$ are possible outcomes of measurements at spacelike separated points $x$ and $y$, then $\text{Prob}(i) = \text{Prob}(i|j)$. In other words, conditioning on space-like separated measurement events makes no difference.

This prescription, clearly, is not consistent with standard quantum theory. For example, it predicts that if we can arrange spacelike separated measurement events that correspond approximately to spin measurements of two separated particles in a spin singlet, the outcomes will be random (as in standard quantum theory) but also uncorrelated, whatever measurements are chosen. Standard quantum theory predicts approximate anti-correlation when the measurements are the same. We now consider this in more detail.

**Internal consistency of causal quantum theory**

Is this prescription self consistent? At first sight it may seem that spacelike separated measurements at $x$ and $y$ can give inconsistent outcomes. For example, spacelike separated $s_z$ measurements on a singlet state

$$\Psi_- = \frac{1}{\sqrt{2}}(|\uparrow\rangle_L |\downarrow\rangle_R - |\downarrow\rangle_L |\uparrow\rangle_R)$$

(3)

could give outcomes corresponding to $|\uparrow\rangle_L$ and $|\uparrow\rangle_R$. If this were precisely true, the quantum state and local density matrix would be undefined at points in the joint future of $x$ and $y$, and we would have no prescription for the probabilities of measurement outcomes there.
Recall, though, that our model requires that the Kraus operators defining measurement outcomes have no zero eigenvalues. However closely a product of such operators approximates a projection operator, it cannot annihilate the quantum state. The resulting state may have very small norm, but the denominator in (2) ensures a normalised post-measurement state after any sequence of measurement outcomes.

We could also invoke the fact that, as far as we know, it is impossible to prepare or find in nature a system represented precisely by the state (3). A full discussion would take this into account, noting that at best we can prepare a mixed state dominated by states of the form

$$\Psi = a_{↑↑} |↑⟩_L |↓⟩_R + a_{↑↓} |↑⟩_L |↓⟩_R + a_{↓↑} |↓⟩_L |↑⟩_R + a_{↓↓} |↓⟩_L |↓⟩_R, \quad (4)$$

where the four coefficients are all non-zero, with $a_{↑↑} = 1/\sqrt{2}$, $a_{↑↓} = -1/\sqrt{2}$, $a_{↓↑} \approx 0$, $a_{↓↓} \approx 0$. For simplicity of illustration, we neglect this here (although it is an important defence of causal quantum theory and the details may significantly affect its implications in any given example) and focus on the first point. Suppose then that the state of the relevant system is precisely (3). The operator corresponding to the outcome we label “$s_z = |↑⟩$” might for example take the form

$$A^\dagger = (1 - \epsilon)^{1/2} |↑⟩⟨↑| + \epsilon^{1/2} |↓⟩⟨↓|, \quad (5)$$

and similarly the outcome we label “$s_z = |↓⟩$” is

$$A^\dagger = (\epsilon)^{1/2} |↑⟩⟨↑| + (1 - \epsilon)^{1/2} |↓⟩⟨↓|. \quad (6)$$

Our model allows $\epsilon$ to be arbitrarily small, but requires $\epsilon > 0$; we take $\epsilon \ll 1$.

We then have that

$$A^\dagger_L A^\dagger_R \Psi_- = \frac{1}{\sqrt{2}} (1 - \epsilon)^{1/2} \epsilon^{1/2} (|↑⟩_L |↓⟩_R - |↓⟩_L |↑⟩_R), \quad (7)$$

which has norm $\epsilon(1 - \epsilon)$ and normalises to $\Psi_-$. The key point here is that, while the norm is small, it is non-zero. Hence, we still have a well-defined state and a consistent prescription for obtaining measurement probabilities in the joint future of $x$ and $y$.

Now consider an extension of this experiment, in which the L and R wings are widely separated enough that a sequence of $N$ measurements can be carried out within a region $R_L$ on the L wing, and a sequence of $N$ measurements within a region $R_R$ on the R wing, with the regions $R_L$ and $R_R$ space-like separated.

Suppose that the first measurement on the L wing produces outcome $A^\dagger_L$. According to causal quantum outcome $A^\dagger_L$. According to causal quantum theory, the relevant unnormalised state for calculating outcomes of the second measurement on the L wing is

$$A^\dagger_L \Psi_- = \frac{1}{\sqrt{2}} (1 - \epsilon)^{1/2} \epsilon^{1/2} |↑⟩_L |↓⟩_R - |↓⟩_L |↑⟩_R. \quad (8)$$

This normalises to

$$(1 - \epsilon)^{1/2} |↑⟩_L |↓⟩_R - \epsilon^{1/2} |↓⟩_L |↑⟩_R. \quad (9)$$

The second measurement thus has outcome $A^\dagger_L$ with probability

$$(1 - \epsilon)^2 + \epsilon^2 = 1 - 2\epsilon + 2\epsilon^2 \quad (10)$$

Assuming this outcome is realised, the third measurement has the same outcome with probability

$$1 - \epsilon - \epsilon^2 + O(\epsilon^3), \quad (11)$$

If all previous measurements produced outcome $A^\dagger_L$, each successive measurement produces the same outcome with conditional probability closer to $(1 - \epsilon)$.

Given that the first outcome is $A^\dagger_L$, with high probability, the first several outcomes will be $A^\dagger_L$, making the local state closer and closer to $|↑⟩_L |↑⟩_L$. Outcomes $A^\dagger_L$ will continue to have probability $\epsilon + O(\epsilon^2)$, and will be realised with frequency $\approx \epsilon$. Nonetheless, with probability close to 1, the local state will tend asymptotically to $|↑⟩_L |↑⟩_L$. A sequence of $N$ measurements with proportion $\approx (1 - \epsilon)$ having outcome $A^\dagger_L$ will give local observers on this wing
increasingly strong evidence that, for practical purposes, they may take their local state to be very close to $|\uparrow\rangle_L \langle \uparrow|_L$, so long as measurement outcomes elsewhere may be neglected.

Now suppose the first measurement on the R wing also produces outcome $A_R^\uparrow$. This will be the case with probability $\frac{1}{2}$, independent of the outcomes of the measurements on the L wing. The same discussion applies. So, with probability close to 1, the local state on the R wing will tend asymptotically to $|\uparrow\rangle_R \langle \uparrow|_R$, and local observers will obtain increasing evidence that, for practical purposes, they may take their local state to be very close to $|\uparrow\rangle_R \langle \uparrow|_R$, again so long as measurement outcomes elsewhere may be neglected.

Now suppose both observers pause after $N$ measurements, and both wait until all the measurements from the other wing are in their past light cone. At this point their local states revert to $\frac{1}{2}I_L$ and $\frac{1}{2}I_R$ respectively. Whether or not they are aware of the measurement outcomes in the other wing, their outcome probabilities for their next measurement change: according to causal quantum theory, $A_L^\uparrow$ and $A_R^\uparrow$ now both have probability $\frac{1}{2}$ (see Fig. 1).

This is a general feature of causal quantum theory: evidence about the state of a local system determines the future behaviour of that state only insofar as measurement outcomes elsewhere remain causally separated. This is true no matter how compelling that evidence would be in standard quantum theory. And it is true for systems of any size or complexity. In principle, a dynamical collapse model version of causal quantum theory could allow an apparently stable galaxy obeying quasiclassical laws to persist for a long period, only to be “overwritten” in the future when its past light cone includes currently space-like separated measurement outcomes that are inconsistent with its state. Inhabitants of such a galaxy who know they live in a universe described by causal quantum theory might (depending on the details of the relevant collapse or measurement hypothesis) be motivated to frantically carry out more and more measurements confirming its and their existence, and to broadcast the data in the hope that it might be measured again elsewhere, with the aspiration of out-weighting any currently space-like separated and inconsistent measurement data.
Is causal quantum theory too strange to contemplate?

Causal quantum theory might be criticized as maintaining internal consistency only by invoking the possibility of highly improbable measurement errors and/or improbable outcomes arising from wave function tails. This is circular reasoning, though. According to standard quantum theory, the measurement errors and outcomes that routinely arise in causal quantum theory are indeed highly improbable. However, causal quantum theory is a new theory with different rules for the probability of a sequence of measurement outcomes, and according to causal quantum theory its measurement outcome predictions are not improbable. An unbiased scientific comparison of standard quantum theory and causal quantum theory cannot invoke statements that apply only within one theory as reasons for disbelieving the other.

Again, it is worth emphasizing that, however strange causal quantum theory may be, it does have a respectable theoretical motivation. To recap, this runs as follows. First, we accept the empirical success of non-relativistic quantum theory when dealing with individual systems, and take this as evidence that the basic mathematical formalism of quantum theory is an essential part of the description of nature. Second, we note that measurement plays a key role in standard quantum theory. We note too that an adequately general and realistic account of quantum measurements is given by considering Kraus operators with no zero eigenvalues, and that it is not absurdly unreasonable to postulate that all measurements are defined by such operators. Third, noting the EPR argument, and the a priori puzzling relationship between quantum theory and special and general relativity, we make the assumption that measurement outcomes influence one another in a way that respects strict Minkowski and Einstein causality. That is, any given measurement outcome only affects events in its causal future. Although this last assumption is non-standard, it seems a priori quite natural within the framework of special and general relativity. Causal quantum theory is the result.

One could even imagine a counter-factual history in which the theory of approximate measurement was developed soon after 1926, and the idea that measurement might be a definitely localized physical process was taken seriously from the start. Given the then unresolved tension between quantum theory and special relativity, causal quantum theory might conceivably have been proposed as a logically consistent possibility in, say, 1930. After the EPR argument was presented, one could just about imagine causal quantum theory and standard quantum theory initially being seen as genuine competitors, which needed to be distinguished empirically.

Of course, history followed another path. The EPR argument ultimately led to Bell’s theorem and Bell experiments, which are generally taken to be compelling confirmations of standard quantum theory and Bell non-locality. The theory of approximate measurements was developed much later than the EPR argument. And, although the idea that measurement is a localized physical process is quite well aligned with ideas explored by some of the founders of quantum theory, it was not generally seen as an idea that might have testable implications worth exploring until the work of Ghiradi-Rimini-Weber-Pearle [19, 20], Diosi [21], Penrose [22] and others.

What is the fundamental status of causal quantum theory?

The extent to which ordinary quantum theory is well-defined is already a delicate question. Some standard textbook versions of quantum theory offer versions of a Copenhagen interpretation, in which measuring devices or macroscopic amplifications play a fundamental role in defining the theory. No precise definition of measuring device or macroscopic is available, and so there is an ineliminable vagueness in principle in these versions of quantum theory. Wigner’s speculative idea [23] consciousness plays a fundamental role, and is essential to the definition of measurement, can be thought of as another Copenhagen-like interpretation. Again, though, there is an ineliminable vagueness, since we lack any precise model of consciousness.

If we take these interpretations as requiring some qualitative distinction between macroscopic and microscopic, or conscious and unconscious, then they imply that quantum theory is not universal. Alternatively, if we try to set out an Everettian version [24] in which quantum theory is universal, we run into problems of structure, probability and the problem of theory confirmation for many-worlds theories [25], and we find a lack of consensus amongst many-worlders on the answers.

Similarly, we can consider Copenhagen-like versions of causal quantum theory, in which measuring devices (or conscious minds) are taken to be qualitatively distinct from quantum systems. In these versions, the output of a measuring device is treated as a stable classical record. However, because we assume that measurements are necessarily imperfect, successive measurements on a quantum system do not necessarily produce the same output.

Alternatively, we can think of causal quantum theory as a causal version of some form of objective collapse theory [19, 20], with localised collapses, that aspires to give a unified treatment of microscopic and macroscopic physics. Dynamical collapse models are designed to resolve the problems of Everettian quantum theory by postulating objective
probabilities that effectively single out one quasi-classical world as realised from among infinitely many possible such worlds. Whether they completely succeed is debated \[26\]. Fully satisfactory relativistic collapse models have proven elusive. Also, of course, collapse models make testably distinct predictions from standard quantum theory, and may be refuted \[27\].

Causal quantum theory is thus an umbrella term for a class of theories. While this is true of standard quantum theory in a sense, it is true of causal quantum theory in a stronger sense: causal quantum theory needs some localized collapse hypothesis, and different localized collapse hypotheses can make very different predictions in causal quantum theory, even when they are effectively indistinguishable when applied to standard quantum theory.

**EMPIRICAL TESTS OF CAUSAL QUANTUM THEORY**

**Stronger Bell experiments**

Causal quantum theory can be thought of as a local hidden variable theory, in which the local hidden variable is the local quantum state, and is influenced by measurement outcomes in its past light cone. Any given version of causal quantum theory can thus be excluded by Bell experiments that close the relevant version of the collapse locality loophole, by ensuring that, according to the relevant collapse or measurement model, space-like separated measurement actually take place in the two wings.

Indeed, causal quantum theory is somewhat easier to exclude than a generic local hidden variable theory exploiting the same version of the collapse locality loophole, since it makes specific predictions that differ from those of standard quantum theory even for experiments with a single measurement choice on each wing. For example, as discussed above, causal quantum theory predicts that, if we prepare an approximate singlet state

$$|\psi\rangle \approx \frac{1}{\sqrt{2}}(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B) ,$$

and ensure space-like separated measurements of $s_Z$ on each wing, the outcomes in the two wings will be uniformly random and uncorrelated:

$$P_{\text{causal qt}}(|\uparrow\rangle_A |\uparrow\rangle_B) \approx P_{\text{causal qt}}(|\uparrow\rangle_A |\downarrow\rangle_B) \approx P_{\text{causal qt}}(|\downarrow\rangle_A |\uparrow\rangle_B) \approx P_{\text{causal qt}}(|\downarrow\rangle_A |\downarrow\rangle_B) \approx 1/4 ,$$

whereas

$$P_{\text{standard qt}}(|\uparrow\rangle_A |\downarrow\rangle_B) \approx P_{\text{standard qt}}(|\downarrow\rangle_A |\uparrow\rangle_B) \approx 1/2 , \quad P_{\text{standard qt}}(|\uparrow\rangle_A |\uparrow\rangle_B) \approx P_{\text{standard qt}}(|\downarrow\rangle_A |\downarrow\rangle_B) \approx 0 .$$

There has been some progress in this area since causal quantum theory was first described \[18\]. In particular, motivated by these ideas, Salart et al. \[28\] carried out a Bell experiment closing the collapse locality loophole assuming that collapses arise to prevent superpositions of distinguishable gravitational fields and can be characterised by quantitative guesstimates due to Diosi and Penrose. This experiment refuted a version of causal quantum theory based on the same assumptions. It left open the question of whether causal quantum theory could still hold if Diosi and Penrose’s estimates of collapse time were increased by a factor of $\approx 10^2$, or other versions of gravitational collapse model were implied, or other collapse or measurement models were assumed. Techniques for implementing stronger Bell experiments that should be able to test the collapse locality loophole for most interesting collapse models were described in Ref. \[?\]. Such experiments could also refute causal quantum theory.

**Testing for multiple detections of a single particle**

Recall that our black box model of measurement assumes some characterisation of measurement or collapse-inducing devices, which may for example involve reconfiguring mass distributions or perceptions in human brains. Suppose that these devices detect whether or not a particle is in the black box, and have efficiency $(1 - \epsilon)$, in the following sense. The measurement for box $i$ is defined by two operators $A_{i0}, A_{i1}$. Let $\psi_i^0$ be any single particle state with support outside the box, and $\psi_i^1$ be any single particle state with support inside the box. Let $\rho_i^0$ and $\rho_i^1$ be the corresponding density matrices. Then we assume

$$\text{Tr}(A_{i0}^\dagger \rho_i^0(A_{i0})^\dagger) = 1 - \epsilon ,$$

$$\text{Tr}(A_{i1}^\dagger \rho_i^1(A_{i1})^\dagger) = 1 - \epsilon ,$$

\[15\] \[16\]
We also assume that \( A_i^0 |\psi_i^0\rangle \) and \( A_i^1 |\psi_i^1\rangle \) have support inside the box, and that \( A_i^0 |\psi_0\rangle \) and \( A_i^1 |\psi_0\rangle \) have support inside the same region as \( |\psi_0\rangle \) (and so, in particular, have support outside the box).

Thus, in a single experiment with a single measurement box \( i \) on a single particle state with support inside the box, the detector will click with probability \( 1 - \epsilon \); if the state has support outside the box, the detector will click with probability \( \epsilon \).

Now consider an experiment with \( N \) spacelike separated (hence disjoint) boxes, labelled by \( 1 \leq i \leq N \), on a quantum state

\[
|\psi\rangle = \frac{1}{\sqrt{N}} \left( \sum_i |\psi_i^1\rangle \right),
\]

where the \( |\psi_i^1\rangle \) are normalised states with support in box \( i \), as above. According to standard quantum theory, if we obtain a click from box \( i \), the post-detection state is

\[
|\psi_i\rangle = \frac{A_i^1 |\psi\rangle}{|A_i^1 \psi\rangle} = (1 + (N-2)\epsilon)^{-1/2}(A_i^1 |\psi_i^1\rangle + \sum_{j \neq i} A_i^1 |\psi_j^1\rangle).
\]

Here the normalisation factor is obtained using that the states \( A_i^1 |\psi_i^1\rangle \) for \( 1 \leq j \leq N \) are all orthogonal, with \( |A_i^1 |\psi_i^1\rangle|^2 = 1 - \epsilon \) and \( |A_i^1 |\psi_i^1\rangle|^2 = \epsilon \). The probability \( P_{QM}(i) \) of getting a click in any given box \( i \) is thus roughly \( N^{-1} \); the conditional probability \( P_{QM}(j|i) \) of getting a click in box \( j \) given that we get one in a specified box \( i \) is \( O(\epsilon) \). More generally, the conditional probability \( P_{QM}(k|i_1 \ldots i_m) \), where \( k \) is distinct from \( i_1, \ldots, i_m \), is also \( O(\epsilon) \). To \( O(\epsilon) \) the probabilities \( P_{QM}(n) \) of getting \( n \) clicks are given by \( P_{QM}(1) = 1, P_{QM}(n) = 0 \) for \( n \neq 1 \).

In causal quantum theory, on the other hand, so long as the measurement events are spacelike separated, the probability of each outcome is independent of the others. We have \( P_{CQM}(i) = P_{CQM}(j|i) = P_{CQM}(k|ji) = \ldots \approx N^{-1} \).

The probabilities \( P_{CQM}(n) \) of getting \( n \) clicks obey

\[
P_{CQM}(n) \approx \frac{1}{n!} e^{-1},
\]

for \( n \ll N \).

The expectation value of the number of clicks is 1 in both theories. However, the variances are different. Causal quantum theory predicts multiple and zero click outcomes that are very unlikely according to standard quantum theory. Note that causal quantum theory does not predict that, when multiple detections are made, the system goes on indefinitely to behave as though it now contains multiple particles. Events in the joint future of all the detectors are predicted by allowing for all the detector measurements, in a way consistent with the possibility that all but one of the detectors produced a false positive result – a possibility which always has nonzero probability. Trying to bring the “detected particles” together in order to verify violation of a conservation law (which has zero probability in standard quantum theory) will thus always fail.

Nonetheless, spacelike separated sets of detectors should often register anomalous multiple detections from a single particle, and in our measurement model evidence of such multiple detections persists even in the intersection of their future light cones. It is interesting to ask whether we should expect to have seen evidence of something like this in existing experiments, observations such as cosmic ray tracks in mica, or elsewhere, if causal quantum theory were correct. It is not obvious that we should, given that the expectation value of the number of detections is the same in causal quantum theory as in standard quantum theory. One reason is that our measurement model does not always apply: the probability of a track appearing in mica in the absence of a cosmic ray is likely astronomically small. Another more generally relevant reason is that, as in Bell experiments, the prediction requires specifically designed space-like separated measurement devices appropriate for the relevant collapse postulate.

The case of \( N = 2 \) suffices for experimental tests distinguishing causal and standard quantum theory, since the rate of anomalous double clicks would be high (\( \frac{1}{4} + O(\epsilon) \)) according to causal quantum theory and low (\( O(\epsilon) \)) according to standard quantum theory. Causal quantum theory can thus be tested with a single photon source and beamsplitter together with appropriate space-like detectors. This makes it somewhat easier to refute causal quantum theory than to close the collapse locality loophole in general: the latter requires a source of entangled particles as a component of the experiment, although not necessarily long range entanglement distribution [?].
A very nice experiment of this type was carried out and carefully analysed by Guerreiro et al.\[30\] In their experiment, single heralded photons were observed at one (and always only one) of two photodetectors, arranged so that the possible detection events were necessarily spacelike separated. Unlike the experiment of Salart et al. \[28\], discussed above, these detection events were not macroscopically amplified in spacelike regions. The Guerreiro et al. experiment thus did not test a version of causal quantum theory associated with any particularly plausible of well known localized collapse model. However, the techniques of Refs. \[28, 30\] could certainly be combined, and extended to other collapse hypotheses. Indeed, we hope this discussion will stimulate further experimental work in this area.

Causal quantum theory thus can and very likely will be refuted by experiment. Nonetheless it is a reasonably well motivated radical alternative to standard relativistic quantum theory that has essentially gone unnoticed for many decades and quite plausibly still awaits conclusive refutation. It raises the question: what else might we be missing?

Conclusions

Causal quantum theory relies on some form of objective localized collapse or measurement model. These models may of course be incorrect, and their motivations, though reasonable, are not universally appreciated. Causal quantum theory is also a strange theory that makes predictions that run counter to the intuition of anyone familiar with standard quantum theory. It may perhaps already be contradicted by experiment or observation, though it does not seem obvious that it is.

Different people may reasonably assign different weights to each of these arguments, but all of them give some reason to be sceptical about causal quantum theory, and in combination they probably give very strong reasons to be sceptical. Even so, a conclusive experimental refutation would be more compelling, and I hope the present discussion may stimulate experimental work.

One might, perhaps, also entertain the idea that causal quantum theory could apply to some part of nature but not all. The most obvious candidate here is the gravitational field, given that Einstein causality is a fundamental feature of general relativity. A theory in which the gravitational field is determined by causal quantum theory rules, while matter correlations follow standard quantum predictions, might seem even stranger than fully-fledged causal quantum theory. Again, though, a conclusive experimental refutation would be a more compelling argument.

Competing Interests

There are no competing interests.

Data accessibility

This work does not have any experimental data.

Ethics statement

This work did not involve any active collection of human data

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There are analogies in present-day politics, alas. As Marx might have put it, all that is solid melts into air. For simplicity, we assume here the particles are distinguishable, but that the detector in this illustration does not distinguish. Thiago Guerreiro, Bruno Sanguinetti, Hugo Zbinden, Nicolas Gisin, and Antoine Suarez. Single-photon space-like anti-collapse theories. In Edward N. Zalta, editor, The Scientist Speculates, pages 307–354. Oxford University Press, 2010.

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[31] For simplicity, we assume here the particles are distinguishable, but that the detector in this illustration does not distinguish them. In general, we allow any type of detector that locally detects particles, including detectors that detect some types but not others. It is also straightforward to deal with bosons or fermions, at the price of slightly complicating the notation. In principle, our definitions are also meant to extend to relativistic quantum fields, modulo the standard problems in rigorizing their measurement theory.

[32] As Marx might have put it, all that is solid melts into air.