Bounds on the Capacity of MIMO Broadband Power Line Communications Channels

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Abstract—Communications over power lines in the frequency range above 2 MHz, commonly referred to as broadband (BB) power line communications (PLC), is a central communications scenario for smart power grids. BB-PLC channels are characterized by a dominant colored non-Gaussian additive noise, as well as by periodic variations of the channel impulse response and the noise statistics, induced by the mains voltage. In this work we study the fundamental rate limits for multiple input-multiple output (MIMO) BB-PLC channels, modeled as periodic channels with additive non-Gaussian noise and finite memory. We present bounds on the capacity of these channels by exploiting a bijection with time-invariant MIMO channels of extended dimensions. We illustrate the resulting fundamental limits in a numerical analysis corresponding to practical MIMO BB-PLC channels.

I. INTRODUCTION

Power line communications (PLC) is an emerging technology which utilizes the existing power grid infrastructure for data transmission. PLC systems operating in the frequency range of 2 – 100 MHz are commonly referred to as broadband (BB) PLC. In order to improve performance, BB-PLC systems may utilize all three wires of the indoor power transmission network to realize multiple transmit and receive ports, giving rise to multiple input-multiple output (MIMO) BB-PLC scenarios [1]. The resulting MIMO channel exhibits periodicity in both the channel impulse response (CIR) as well as in the noise statistics. In particular, the CIR in MIMO BB-PLC channels is typically modeled as a multipath channel [2] with periodic variations [3], [4], where the channel outputs contain crosstalk from other wires [1], [5], while MIMO BB-PLC noise is generally modeled as a temporally correlated [2], [6], [7], spatially correlated [1], [8], cyclostationary [9] multivariate process. Furthermore, MIMO BB-PLC noise is typically non-Gaussian [2], [6], [10], where common models for the marginal probability density function (PDF) of BB-PLC noise include the Nakagami-m distribution [10] and the Gaussian mixture (GM) distribution [6]. Consequently, MIMO BB-PLC channels fall into the class of MIMO periodic channels with additive non-Gaussian noise and finite memory.

The unique model of MIMO BB-PLC channels introduces several major challenges when attempting to characterize their capacity. To avoid handling the technical difficulties, previous works which attempted to characterize the fundamental rate limits for BB-PLC channels used very simplified models which do not capture many of the special characteristics of these channels: The work [7] evaluated the capacity of BB-PLC channels by modeling them as having a linear time-invariant (LTI) CIR with additive colored stationary Gaussian noise; the work [4] modeled BB-PLC channels as linear, periodically time-varying (LPTV) channels with additive white Gaussian noise (AWGN), and evaluated the achievable rate by using a transmission scheme which utilizes orthogonal frequency division multiplexing (OFDM) signalling. Other related works are [11], which characterized the capacity of PLC channels in the narrowband frequency range (0 – 500 kHz), modeled as periodic channels with finite memory in which the noise is additive and Gaussian, and [12], which studied the capacity of periodic MIMO channels where again the noise was additive and Gaussian. We emphasize that [4], [7], [11], [12] derived expressions assuming Gaussian noise. Previous works on the capacity of channels with additive non-Gaussian noise, e.g., [13], considered channels with memoryless and fixed CIR with i.i.d. non-Gaussian noise, and are thus not applicable to the characterization of the fundamental rate limits of MIMO BB-PLC channels. To the best of our knowledge, the fundamental limits for MIMO BB-PLC channels, accounting for the periodic variations of the CIR and of the noise statistics, as well as for the non-Gaussianity and the temporal correlation of the noise, have not been characterized to date.

Main Contributions: In this work we study the fundamental rate limits for discrete-time (DT) MIMO periodic channels with additive non-Gaussian noise and finite memory, which is the common model for MIMO BB-PLC channels. We note that when the noise is not a Gaussian process, obtaining a closed-form expression for the capacity is generally not a simple task, even for memoryless channels, and often times the approach is to characterize upper and lower bounds on the capacity, see, e.g., [14, Ch. 7.4]. To facilitate the derivation of such bounds, we first derive bounds on the capacity of a general LTI MIMO channel with additive stationary non-Gaussian noise. Then, we prove that the capacity of finite-memory periodic MIMO channels with additive non-Gaussian noise can be obtained from the capacity of LTI MIMO channels with additive stationary non-Gaussian noise having extended dimensions compared to those of the original periodic MIMO channel, via a proper selection of the parameters of the extended LTI channel. Lastly, we apply the bounds on the capacity of the extended LTI model to obtain the corresponding bounds for the MIMO BB-PLC channel. This approach yields capacity bounds which depend on the PDF of the noise process only through its entropy rate and autocorrelation function. We use the derived bounds to numerically evaluate the capacity of practical MIMO BB-PLC models, and demonstrate that, in the high signal-to-noise ratio (SNR) regime, the achievable rate of cyclostationary Gaussian signaling is within a small gap of capacity. We also show that assuming the noise is Gaussian may result in significantly underestimating the capacity.

The rest of this paper is organized as follows: Section II formulates the problem; Section III derives the capacity.
bounds, and Section IV presents numerical examples; Lastly, Section V provides concluding remarks.

II. PROBLEM DEFINITION

Notations: We use upper-case letters, e.g., $X$, to denote random variables (RVs), and lower-case letters, e.g., $x$, to denote deterministic values. Column vectors are denoted with boldface letters, e.g., $X$ for a deterministic vector and $X$ for a random vector. We use Sans-Serif fonts to denote matrices, e.g., $A$, the all-zero $k \times l$ matrix is denoted with $0_k \times l$, and the $n \times n$ identity matrix is denoted with $I_n$. Complex conjugate, Hermitian transpose, Euclidean norm, determinant, stochastic expectation, differential entropy, and mutual information are denoted by $(\cdot)^*$, $(\cdot)^H$, $||\cdot||$, $|\cdot|$, $\text{E} \{ \cdot \}$, $h(\cdot)$, and $I(\cdot; \cdot)$, respectively, and $a^n$ denotes max $\{0,a\}$. The sets of non-negative integers, integers, and of real numbers are denoted by $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{R}$, respectively. All logarithms are taken to base-2. Lastly, for any sequence $y[i]$, $i \in \mathbb{Z}$, and integers $b_1 < b_2$, $y_{b_2}^b_1 = \{y[b_1], \ldots, y[b_2]\}$ and $y_{b_1}^b_2 \equiv y_{b_1}^{b_2}$. Definitions: We shall use the following definitions:

Definition 1 (MIMO channel with finite-memory). A DT $n_t \times n_t$ MIMO channel with finite memory consists of an input sequence $X[i] \in \mathbb{R}^{n_t}$, $i \in \mathbb{N}$, an output sequence $Y[i] \in \mathbb{R}^{n_t}$, $i \in \mathbb{N}$, an initial state $S_0 \in \mathbb{S}_0$, and a sequence of PDFs $\{p(Y[i]|X[i], S_0)\}_i^{\infty}$. The set $(x_{-1}^i, l_{-1})_{i=1}^{2n}$ is referred to as the codebook of the $[R, l]$ code. Assuming $U$ is uniformly selected from $U$, the average probability of error, when the initial state is $S_0$, is:

$$P_e^{\infty}(s_0) = \frac{1}{2^{2l}} \sum_{l=1}^{2^l} \sum_{y(l-1) \neq u} P_r(\hat{d}_l(l-1) = u | U = u, S_0 = s_0).$$

Definition 3 (Achievable rate). A rate $R_c$ is called achievable if, for every $\epsilon_1, \epsilon_2 > 0$, there exists a positive integer $l_0 > 0$ such that for all integer $l > l_0$, there exists an $[R, l]$ code which satisfies $\sup_{s_0 \in S_0} P_e^{\infty}(s_0) < \epsilon_1$ and $R \geq R_c - \epsilon_2$.

Definition 4 (Capacity). Capacity is defined as the supremum of all achievable rates.

Model and Problem Formulation: We consider a DT MIMO BB-PLC channel modeled as a linear, non-Gaussian, MIMO periodic channel (LNGMC) with $n_t$ receive ports and $n_t$ transmit ports. Let $\tilde{m}$ be a non-negative integer which represents the length of the memory of the channel, $\tilde{m}$ be a positive integer which represents the period of the CIR, and $\tilde{p}$ be a positive integer which represents the period of the noise statistics. Let $W[i] \in \mathbb{R}^{n_t}$ be a real-valued, $n_t$-dimensional, zero-mean, strict-sense cyclostationary, non-Gaussian additive noise. Thus, for any of $k$ integer indexes $\{i_j\}_{j=1}^k$, $k > 0$, the joint PDF of $W[i_1], W[i_2], \ldots, W[i_k]$ is equal to the joint PDF of $W[i_1 + \tilde{p}], W[i_2 + \tilde{p}], \ldots, W[i_k + \tilde{p}]$. Since the channel memory is $\tilde{m}$, then noise vectors which are more than $\tilde{m}$ instances apart are mutually independent, i.e., $\forall i_1, i_2, l_1, l_2 \in \mathbb{N}$ such that $l_2 > i_1 + l_1 + \tilde{m}$, the random vectors $W[i_1 + l_1] \oplus W[i_2 + l_2]$ are mutually independent. We further assume that there is no deterministic dependence between instances of $W[i]$, i.e., $\tilde{p}$ for which $W[i_0]$ can be expressed as a linear combination of $\{W[i]\}_{i \neq i_0}$. Let $\{G[i, \tau]\}_{i=0}^{\tilde{m}}$ denote the LPTV CIR of the MIMO BB-PLC channel, $G[i, \tau] \in \mathbb{R}^{n_t \times n_t}$. The periodicity of the CIR implies that $G[i, \tau] = G[i + \tilde{p}_G, \tau]$. Lastly, in Subsection III-C, we prove that the input-output relationship for the LNGMC with input codeword length $l$ is given by

$$Y[i] = \sum_{\tau=0}^{\tilde{m}} G[i, \tau]X[i - \tau] + W[i], \quad i \in \{0, 1, \ldots, l-1\}, \quad (1)$$

where the initial state of the channel (i.e., prior to the beginning of reception) is given by $S_0 = \left[\left(\tilde{x}_{-1}^{-1} \right)^T, \left(\tilde{w}_{-1}^{-1} \right)^T\right]^T$. The channel input is subject to a time-averaged power constraint $P$, as in [11, Eq. (7)]:

$$\frac{1}{l} \sum_{i=0}^{l-1} \mathbb{E}\left\{\|\tilde{X}[i]\|_2^2\right\} \leq \tilde{P}. \quad (2)$$

Letting $\tilde{p}$ be the least common multiple of $\tilde{p}_G$ and $\tilde{p}_W$ which satisfies $\tilde{p} > \tilde{m}$, we obtain that the CIR and the statistics of the noise of the LNGMC (1) are periodic with a period $\tilde{p}$, hence we refer to $\tilde{p}$ as the period of the channel. We also note that while the above model was stated for real signals, complex (baseband) channels can be accommodated by representing all complex vectors and matrices using real vectors and matrices.

In the following section we study the capacity of LNGMCs, defined in (1)–(2), denoted $C_p$.

III. THE CAPACITY OF LNGMPCS

Our main result is the characterization of upper and lower bounds on the capacity of LNGMPCs, defined in (1)–(2). This result is obtained via three steps: First, in Subsection III-A, we define a general LTI $n_t \times n_t$ MIMO channel with stationary non-Gaussian noise, to which we refer as the linear non-Gaussian MIMO channel (LNGMC). We express the capacity of the LNGMC as a limit of the mutual information between its input and its output when the blocklength increases to infinity. Next, in Subsection III-B, we derive computable upper and lower bounds on the capacity of the LNGMC, which are stated in terms of the CIR, and of the entropy rate and autocorrelation function of the noise. Lastly, in Subsection III-C, we prove that the capacity of the LNGMC can be obtained as the capacity of an equivalent $\tilde{p} \times \tilde{p}$ LNGMC, and use the bounds derived in Subsection III-B to state the corresponding capacity bounds for the LNGMC. We give only a sketch of the proofs of the results. Full details of the proofs can be found in [15].

A. The Capacity of the LNGMC

We begin with the definition of the LNGMC: Let $m$ be a non-negative integer which represents the length of the memory of the channel, and let $G[i, \tau]_{\tau=0}^{\tilde{m}}$ denote a set of $m + 1$ real-valued $n_t \times n_t$ CIR matrices. Additionally, let $W[i] \in \mathbb{R}^{n_t}$.
be a multivariate, real-valued, strict-sense stationary non-Gaussian additive noise process, whose mean is zero and whose temporal dependence spans a finite interval of length \( m \), i.e., \( \forall i_1, i_2, l_1, l_2 \in \mathcal{N} \) such that \( i_2 > i_1 + l_1 + m \), the random vectors \( \mathbf{W}_{i_1+i_1} \) and \( \mathbf{W}_{i_2+i_2} \) are mutually independent. For the transmission of a block of \( l \) symbols, \( \{ \mathbf{X}[i] \}_{i=0}^{l-1} \), the input-output relationship is defined as

\[
\mathbf{Y}[i] = \sum_{\tau=0}^{m} \mathcal{G}[\tau] \mathbf{X}[i-\tau] + \mathbf{W}[i], \quad i \in \{0, 1, \ldots, l-1\},
\]

where the initial state of the channel is given by \( \mathbf{S}_0 = \left( (\mathbf{X}_{-m}^{-1})^T, (\mathbf{W}_{-m}^{-1})^T \right)^T \). The channel input is subject to a time-averaged power constraint \( P \), i.e.,

\[
\frac{1}{T} \sum_{i=0}^{T-1} \mathbb{E}\left[ ||\mathbf{X}[i]||^2 \right] \leq P.
\]

The capacity of the LNGMC defined above is stated in the following proposition:

**Proposition 1.** The capacity of the LNGMC defined in (3), subject to (4), is given by

\[
C_{L} = \lim_{n \to \infty} \frac{1}{n} \sup_{p(\mathbf{X}^{n-1})} \mathcal{I}(\mathbf{X}^{n-1}; \mathbf{Y}^{n-1}|\mathbf{X}_{-m}^{0} = \mathbf{0}_{n m}) .
\]

**Proof outline:** Note that Prop. 1 corresponds to the capacity of an information stable channel [17]. Since stationary channels with finite memory are known to be information stable, see, e.g., [17, Sec. 1.5]1, the proposition follows.

**Comment 1.** Previous works on the capacity of finite-memory channels with Gaussian noise, e.g., [19], obtained a capacity result in the frequency domain, by transforming the channel into a set of parallel independent channels, for which capacity is expressed as an explicit integral. When the noise is non-Gaussian, switching to the frequency domain results in the noise components at different frequency bins having statistical dependence (even if the noise samples are independent in the time domain). For this reason, our analysis is carried out in the time domain, and the capacity is stated in terms of an asymptotic limit. Nonetheless, the bounds on the capacity of LNGMCs, derived in Prop. 2, are stated in closed-form (not as limiting expressions) in the frequency domain.

Prop. 1 implies that the capacity of the LNGMC can be computed by setting \( \mathbf{X}_{-m}^0 = \mathbf{0}_{n m} \). We note that setting the signal component in the initial state to zero was stated as a model assumption in [19] and [20], which studied the capacity of point-to-point channels with memory and Gaussian noise.

**B. Bounds on the Capacity of the LNGMC**

Next, based on the capacity expression in Prop. 1, we derive upper and lower bounds on \( C_L \), which depend on the PDF of the non-Gaussian noise \( \mathbf{W}[i] \) only through its autocorrelation function, \( \mathcal{C}_W[\tau] = \mathbb{E}\left[ \mathbf{W}[i+\tau] \mathbf{W}^H[i] \right] \), and its entropy rate, \( \bar{H}_W = \lim_{T \to \infty} \frac{1}{T} h(\mathbf{W}^{T-1}) \). Note that the strict-sense stationarity and finite memory of \( \mathbf{W}[i] \) imply that \( \bar{H}_W = h(\mathbf{W}[m] | \mathbf{W}^{m-1}) \) [16, Ch. 12.5].

In the statement of the bounds we make use of the following additional definitions: For any \( \omega \in [-\pi, \pi] \), we define the \( n_t \times n_t \) matrix \( \mathcal{G}(\omega) \triangleq \sum_{\tau=0}^{\tau=m} \mathcal{G}[\tau] e^{-j\omega\tau} \), and the \( n_t \times n_t \) matrix \( \mathcal{C}_W(\omega) \triangleq \sum_{\tau=0}^{\tau=m} \mathcal{C}_W[\tau] e^{-j\omega\tau} \), and we let \( \lambda_k(\omega) \) and \( \lambda_k^*(\omega) \) denote the eigenvalues of \( \mathcal{G}(\omega) \mathcal{G}^H(\omega) \) and of \( \mathcal{G}(\omega) \mathcal{G}^H(\omega) \), respectively. Next, let \( \bar{H}_C, \bar{H}_N \) denote the entropy rate of a zero-mean \( n_t \times 1 \) multivariate Gaussian process whose autocorrelation function is equal to \( \mathcal{C}_W[\tau] \). From [21, Sec. III] the entropy rate \( \bar{H}_C, \bar{H}_N \) can be expressed as

\[
\bar{H}_C, \bar{H}_N = \frac{1}{4 \pi} \int_{-\pi}^{\pi} \log |2\pi e \mathcal{C}_W(\omega)| \, d\omega .
\]

Let \( \bar{C}_G \) denote the capacity of the channel defined in (3) when the noise \( \mathbf{W}[i] \) is Gaussian, subject to the constraint (4) and to setting \( \mathbf{X}_{-m} = \mathbf{0}_{n m} \). In [20, Eqn. (9)] the capacity of LTI MIMO channels with additive stationary Gaussian noise was characterized2, assuming \( \mathbf{X}_{-m} = \mathbf{0}_{n m} \). Using [20, Eqn. (9)] we can write

\[
\bar{C}_G = \frac{1}{4 \pi} \sum_{k=0}^{n_t-1} \int_{-\pi}^{\pi} \log \left( 1 - \lambda_k(\omega) - \lambda_k^*(\omega) \right) \, d\omega . \tag{5b}
\]

where \( \Delta' \) is set s.t. \( \frac{1}{4 \pi} \sum_{k=0}^{n_t-1} \int_{-\pi}^{\pi} \left( \Delta' - \lambda_k(\omega) - \lambda_k^*(\omega) \right) \, d\omega = P. \)

We next state an upper bound and two lower bounds on the capacity of the LNGMC using \( \bar{H}_W, \bar{H}_G, \) and \( C_G \). These bounds are stated in the following proposition:

**Proposition 2.** The capacity of the LNGMC defined in (3), subject to the constraint (4), satisfies

\[
C_G \leq C_L \leq C_G + \bar{H}_G, \bar{H}_W . \tag{6b}
\]

Moreover, if \( n_t = n_t \) and \( \mathcal{G}[0] \) is invertible, then \( C_L \) satisfies

\[
C_L \geq \frac{n_t}{2} \log \left( \frac{2\pi e P}{n_t \sum_{k=0}^{n_t-1} \int_{-\pi}^{\pi} \log(\alpha_k(\omega)) \, d\omega + 2\pi B_W} \right) . \tag{6a}
\]

**Proof outline:** Eq. (6a) is obtained by first defining

\[
S_n \triangleq \frac{1}{n} \sup_{p(\mathbf{X}^{n-1})} \mathcal{I}(\mathbf{X}^{n-1}; \mathbf{Y}^{n-1}|\mathbf{X}_{-m}^{0} = \mathbf{0}_{n m}) .
\]

By Prop. 1, \( C_L = \lim_{n \to \infty} S_n \). Then, \( S_n \) is upper and lower bounded for any finite \( n \) using results from [14, Ch. 7], and lastly, letting \( n \to \infty \) in the bounds we arrive at (6a). To obtain (6b), we lower bound \( S_n \) for any finite \( n \) using the entropy power inequality [16, Thm. 17.7.3]. Then, letting \( n \to \infty \), and using the extension of Szegö’s theorem to block-Toeplitz matrices [20, Appendix A.2], we arrive at (6b).

**C. Capacity Analysis for LNGMPCs**

In order to obtain bounds on the capacity of LNGMPCs, we first prove that any LNGMPC can be equivalently represented

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1 Although [17, Sec. 1.5] proves the information stability of finite memory stationary channels with discrete and finite alphabets, this results also holds for arbitrary alphabets, see [18, Thm. 6].

2 We note that [20, Thm. 1] is stated for a per-codeword power constraint. However, it follows from [20, Sec. 3.1] and from [14, Ch. 7.3] that the proof of [20, Thm. 1] also holds subject to the time-averaged power constraint (4).
as an LNGMC, and then apply the capacity bounds derived for LNGMCs in Prop. 2 to bound the capacity of the equivalent representation. To that aim, define the \( \tilde{p} \cdot \tilde{n}_r \times 1 \) random vector 
\[
W_{\text{DCD}}[\tilde{i} ] \triangleq W_{\tilde{i},\tilde{p}}^{(\tilde{m}+1)\tilde{p}^{-1}} ,
\]
and additionally, define two \( \tilde{p} \cdot \tilde{n}_r \times \tilde{p} \cdot \tilde{n}_r \) matrices, \( G_{\text{DCD}}[0] \) and \( G_{\text{DCD}}[1] \), as follows:
\[
G_{\text{DCD}}[0] \triangleq \begin{bmatrix}
G[0,0] & \cdots & 0 & \cdots & 0 \\
G[\tilde{m},\tilde{m}] & \cdots & \tilde{G}[\tilde{m},0] & \cdots & 0 \\
0 & \cdots & \tilde{G}[\tilde{p}-1,\tilde{m}] & \cdots & \tilde{G}[\tilde{p}-1,0]
\end{bmatrix} ,
\]
\[
G_{\text{DCD}}[1] \triangleq \begin{bmatrix}
0 & \cdots & 0 & \tilde{G}[0,\tilde{m}] & \cdots & \tilde{G}[0,1] \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & \tilde{G}[\tilde{m}-1,\tilde{m}] & \cdots & \tilde{G}[\tilde{m}-1,1] \\
\vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0
\end{bmatrix} .
\]

As \( W_{\text{DCD}}[\tilde{i}] \) is given by the decimated components decomposition (DCD) \([22]\) of \( W[\tilde{i}] \), the strict-sense cyclostationarity of \( W[\tilde{i}] \) induces a strict-sense stationarity for \( W_{\text{DCD}}[\tilde{i}] \). Using these definitions, we construct an LNGMC with a \( \tilde{p} \cdot \tilde{n}_r \times 1 \) input \( X_{\text{DCD}}[\tilde{i}] \) and a \( \tilde{p} \cdot \tilde{n}_r \times 1 \) output \( Y_{\text{DCD}}[\tilde{i}] \), which satisfies the following input-output relationship for a sequence of \( l \) channel inputs:
\[
Y_{\text{DCD}}[\tilde{i}] = \sum_{\tau=0}^{l-1} G_{\text{DCD}}[\tilde{\tau}] X_{\text{DCD}}[\tilde{\tau} - \tilde{\tau}] + W_{\text{DCD}}[\tilde{i}] , \tag{7}
\]
\( \tilde{i} \in \{0,1,\ldots,l-1\} \), where the channel input to the LNGMC (7) has to satisfy an average power constraint
\[
\frac{1}{l} \sum_{\tau=0}^{l-1} E \left\{ \left\| X_{\text{DCD}}[\tilde{i}] \right\|^2 \right\} \leq P_{\text{DCD}} = \tilde{p} \cdot \tilde{P} . \tag{8}
\]

Since \( \tilde{p} > \tilde{n}_r \), the initial state of the LNGMC is \( S_{0,\text{DCD}} = [X_{\text{DCD}}[\tilde{l}-1],W_{\text{DCD}}^{[\tilde{l}-1]}]^T \). The relationship between the capacity of the LNGMC in (1)–(2), denoted \( C_P \), and the capacity of the LNGMC in (7)–(8), denoted \( C_{\text{DCD}} \), is stated in the following theorem:

**Theorem 1.** The capacity of the LNGMC defined in (1), subject to (2), satisfies
\[
C_P = \frac{1}{\tilde{p}} C_{\text{DCD}} . \tag{9}
\]

**Proof outline:** We first show that the capacity of the LNGMC (1) with the constraint (2) can be characterized by considering only codes whose blocklength is an integer multiple of \( \tilde{p} \). Then, we show that the capacity of LNGMCs constrained to using only codes whose blocklength is an integer multiple of \( \tilde{p} \) satisfies (9).

Next, using Thm. 1 and Prop. 2, we derive lower and upper bounds on the capacity of the LNGMC. To that aim, define the \( \tilde{p} \cdot \tilde{n}_r \times \tilde{p} \cdot \tilde{n}_r \) autocorrelation function
\[
C_{\text{DCD}}[\tilde{\tau}] \triangleq E \left\{ W_{\text{DCD}}[\tilde{i} + \tilde{\tau}] W_{\text{DCD}}[\tilde{i}]^T \right\} ,
\]
the entropy rate \( H_{W_{\text{DCD}}} \triangleq \lim_{n \to \infty} \frac{1}{n} \log \left( WD_{\text{DCD}}^{-1} \right) \), the \( \tilde{p} \cdot \tilde{n}_r \times \tilde{p} \cdot \tilde{n}_r \) matrix \( G'_{\text{DCD}}(\omega) \triangleq \sum_{\tau=0}^{l-1} G_{\text{DCD}}[\tilde{\tau}] e^{-j\omega \tilde{\tau}} \), and the \( \tilde{p} \cdot \tilde{n}_r \times \tilde{p} \cdot \tilde{n}_r \) matrix \( C_{W_{\text{DCD}}}(\omega) \triangleq \sum_{\tau=0}^{l-1} C_{W_{\text{DCD}}}[\tilde{\tau}] e^{-j\omega \tilde{\tau}} \). Additionally, let \( \{ \alpha_{\text{DCD},k}(\omega) \}_{k=0}^{\tilde{n}_r-1} \) and \( \{ \lambda_{\text{DCD},k}(\omega) \}_{k=0}^{\tilde{n}_r-1} \) be the eigenvalues of \( G'_{\text{DCD}}(\omega) \) and \( G_{\text{DCD}}(\omega) \), respectively.

**Theorem 2.** The capacity of the LNGMC defined in (1), subject to (2), satisfies
\[
\frac{1}{\tilde{p}} C_{\text{DCD}} \leq C_P \leq \frac{1}{\tilde{p}} \left( C_{\text{DCD}} + H_{G,W_{\text{DCD}}^{-1}} - H_{W_{\text{DCD}}} \right) . \tag{10a}
\]

Moreover, \( \tilde{n}_r = \tilde{n}_r \) and \( \tilde{G}[\tilde{i},\tilde{j}] \) is non-singular for every \( \tilde{i} \in \tilde{P} \), then \( C_P \) also satisfies
\[
C_P \geq \tilde{n}_r \log \left( \frac{2\pi \tilde{P} \tilde{P}}{\tilde{n}_r} \right) + \tilde{P} \sum_{\omega = -\infty}^{\infty} \log (\alpha_{\text{DCD},k}(\omega)) d\omega \tag{10b}
\]

**Comment 2.** Note that (6b) also lower bounds the achievable rate of the LNGMC with stationary Gaussian input. This implies that (10b) constitutes a lower bound on the achievable rate of LNGMCs with cyclostationary Gaussian input. Consequently, when (10b) coincides with the upper bound in (10a), then cyclostationary Gaussian inputs are optimal.

**IV. NUMERICAL EXAMPLES**

In this section we numerically evaluate the capacity bounds derived in Section III for MIMO BB-PLC channels. As BB-PLC channels exhibit a broad range of frequency responses and noise power values, depending on the topology of the power line network and on the appliances connected to the network, [2], [7], [9], we consider a wide range of SNRs.

We consider a passband 2 × 2 MIMO BB-PLC channel. The multivariate LPTV CIR \( G[\tilde{i},\tau] \) is generated using the method proposed in [5] for generating MIMO BB-PLC channels. Specifically, we first generate four real LPTV CIRs with period \( \tilde{p}_G = 240 \) and memory length \( \tilde{n} = 4 \) using the channel generator proposed in [3], and denote the generated channels as \( \{ g_k[\tilde{i},\tau] \}_{\tilde{i}=1}^{\tilde{p}} \). Then, setting \( \rho = 0.9 \), the multivariate LPTV CIR is obtained via
\[
G[\tilde{i},\tau] = \left[ \begin{array}{c|c}
\tilde{p} & 1 \\
\hline
\tilde{p} & 1 \\
\end{array} \right]^{1/2} \left[ \begin{array}{c}
g_1[\tilde{i},\tau] \\
g_2[\tilde{i},\tau] \\
\end{array} \right] \left[ \begin{array}{c}
g_3[\tilde{i},\tau] \\
g_4[\tilde{i},\tau] \\
\end{array} \right] \left[ \begin{array}{c}
g_5[\tilde{i},\tau] \\
g_6[\tilde{i},\tau] \\
\end{array} \right]^{1/2} .
\]

The additive multivariate noise is a temporally and spatially correlated cyclostationary GM process generated as follows: First, we generate a real i.i.d. 2 × 1 GM process \( \tilde{U}[^1] \)

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whose PDF is a weighted sum of three Gaussian PDFs \( \mathcal{N}([5, 4]^T, 5 \cdot I_2), \mathcal{N}([ -8, -16]^T, 2 \cdot I_2), \) and \( \mathcal{N}([-10, 4]^T, I_2) \), with weights \( \{0.7, 0.2, 0.1\} \), respectively. Then, we generate a multivariate LPTV spectral shaping filter, \( \tilde{F}[i, \tau] \), with period \( \tilde{R}_W = 120 \) (i.e., \( \tilde{p} = 240 \)) and memory length \( \tilde{m} = 4 \), based on the method described in [8] for constructing a spectral profile for MIMO BB-PLC channels: Let \( \rho_W(\omega) \) be a \( 2\pi \)-periodic function representing the spectral variations in the spatial correlation. Following [8, Fig. 5], we set \( \rho_W(\omega) = 0.7 - \frac{0.3}{\pi} |\omega| \) for \( |\omega| < \pi \). Let \( s[i, \omega] \) be the instantaneous power spectral density (PSD), corresponding to the ‘heavily disturbed’ profile based on [9]. Finally, we set

\[
\tilde{F}[i, \omega] = \left[ \begin{array}{c} 1 \\ \rho_W(\omega) \\ 1 \end{array} \right]^{1/2} \begin{bmatrix} s[i, \omega] & 0 \\ 0 & s[i, \omega] \end{bmatrix}^{1/2},
\]

and \( \tilde{F}[i, \tau] = \frac{1}{\pi} \int_{\omega = -\pi}^{\omega = \pi} \tilde{F}[i, \omega]e^{j\omega\tau} d\omega \). The noise signal \( \tilde{W}[i] \) is obtained as the output of the LPTV filter \( \tilde{F}[i, \tau] \) whose input is \( \tilde{U}[i] \).

Defining \( \text{SNR} = \frac{\tilde{P}}{\frac{1}{2} \sum_{l \in \mathcal{O}} E[|\tilde{W}[l]|^2]} \), we depict in Fig. 1 the capacity bounds for the MIMO BB-PLC channel vs. SNR. Note that the lower bound in (10b) is much tighter than the lower bound in (10a) for the entire SNR region, indicating the significant mismatch induced by assuming that the noise is Gaussian. We also note that the gap between the maximal lower bound and the upper bound in Fig. 1 varies from 3.05 bps/Hz at SNR of 0 dB to 0.45 bps/Hz at high SNRs. We conclude that by using cyclostationary Gaussian inputs, it is possible to obtain an achievable rate which is very close to capacity at high SNRs. Finally, we note that for the considered channel, a \( 2 \times 2 \) MIMO BB-PLC system utilizing a frequency band of 100 MHz, as in the ITU-T G.9963 standard [23], can achieve data rates approaching one Gbps at high SNRs.

V. CONCLUSIONS

In this paper we derived upper and lower bounds on the capacity of MIMO BB-PLC channels, modeled as finite-memory periodic MIMO channels with additive non-Gaussian noise. The capacity bounds derived depend on the noise distribution only through its entropy rate and autocorrelation function. Our numerical evaluations demonstrate the tightness of the proposed bounds at high SNRs, and illustrate the significant loss resulting from assuming that the noise is Gaussian in the computation of the capacity. We conclude that the Gaussian noise assumption may lead to inherently suboptimal schemes.

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