Axions and a Gauged Peccei-Quinn Symmetry

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Abstract

The axion solution to the strong CP problem requires an anomalous global $U(1)$ symmetry. We show that the existence of such a symmetry is a natural consequence of an extra dimension in which a gauged $U(1)$ is spontaneously broken on one of two branes, leaving an accidental global symmetry on the other brane. Depending on where the standard model matter lives, the resulting axion can be either the DFSZ or hadronic type. Gaugino-mediated supersymmetry breaking fits comfortably in our framework. In addition, we present a model in which the supersymmetry-breaking and Peccei-Quinn breaking scales are naturally of the same size.

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1 Introduction

The Peccei-Quinn mechanism \([1]\) is probably the most elegant solution to the strong CP problem. It introduces a new global \(U(1)_{\text{PQ}}\) symmetry, spontaneously broken at a scale \(f_{\text{PQ}}\). The CP violating parameter \(\theta\) in the QCD Lagrangian,

\[
\bar{\theta} \frac{g_3^2}{32\pi^2} G_{\alpha\mu} \tilde{G}_{\alpha\mu},
\]

then becomes a dynamical degree of freedom, corresponding to the Nambu-Goldstone boson of the broken \(U(1)_{\text{PQ}}\) symmetry, called the axion, \(\bar{\theta} \to a(x)/f_{\text{PQ}}\) \([2]\). The \(U(1)_{\text{PQ}}\) is anomalous with respect to \(SU(3)_C\), and the QCD instanton effects generate a periodic potential for the axion,

\[
V(\bar{\theta}) \sim m^2_\pi f_{\text{PQ}}^2 (1 - \cos \bar{\theta}).
\]

The minimum is at the CP conserving value \(\bar{\theta} = 0\), therefore it solves the strong CP problem.

Astrophysics and cosmology put strong bounds on the \(U(1)_{\text{PQ}}\) breaking scale,

\[
10^9 \text{ GeV} \lesssim f_{\text{PQ}} \lesssim 10^{12} \text{ GeV}.
\]

The lower bound comes from the energy losses in globular-cluster stars and the supernova SN 1987A \([3]\), and the upper bound comes from the requirement that the axion density does not overclose the universe \([4, 5, 6]\). If there is some late entropy production which dilutes the axion density, the upper bound may be raised to \(\sim 10^{15} \text{ GeV}\) \([7]\). The bounds also depend on the nature of the axion, which can be divided into two classes, the DFSZ axion \([8]\) and the hadronic (or KSVZ) axion \([9]\). In the DFSZ-type model, it requires two Higgs doublets and the standard model quarks and leptons carry \(U(1)_{\text{PQ}}\) charges, while in the hadronic axion model, one introduces new heavy quarks which carry Peccei-Quinn charges and the ordinary quarks and leptons are neutral under \(U(1)_{\text{PQ}}\). For the hadronic axion, if the axion-photon coupling is small, \((C_{a\gamma\gamma} < 0.1)\), there is a small window at \(f_{\text{PQ}} \sim 10^6 \text{ GeV}\) which may be allowed \([10, 11]\).

The problem of the Peccei-Quinn mechanism is to understand why there exists such a global symmetry. Global symmetries are argued to be broken by quantum gravity effects \([12]\). In fact, even without considering quantum gravity, \(U(1)_{\text{PQ}}\) is not an exact symmetry, explicitly broken by the anomaly from QCD. If it is an accidental symmetry at the renormalizable level, one expects that it will be violated by the Planck scale physics, which may be represented by the Planck scale suppressed operators. However, such explicit \(U(1)_{\text{PQ}}\) violating effects contribute to the axion potential and can change the minimum, therefore render the strong CP solution unnatural. It has been shown that the higher dimensional operators have
to be suppressed by more than $M_{\text{pl}}^8$ in order to satisfy the neutron electric dipole moment constraint $[13, 14, 15]$. There have been attempts which use additional discrete or gauge symmetries to forbid these Peccei-Quinn violating interactions to such high orders $[13, 16]$.

In this Letter we show that this problem may be easily solved with a gauged $U(1)_{\text{PQ}}$ if the gauge fields propagate in an extra dimension. Extra dimensions provide a natural framework for the accidental global symmetries $[17, 18, 19]$. If there are fields which are charged under some gauge symmetry localized at different branes in the extra dimensions, then effectively there will be a separate symmetry on each brane if there are no light bulk fields connecting them. If the gauge symmetry is broken by the fields on two different branes, there will be two (sets of) Nambu-Goldstone bosons. One linear combination is eaten by the gauge field and becomes heavy. The other corresponds to the Nambu-Goldstone boson of the broken accidental global symmetry. It can obtain some small mass if the accidental symmetry is anomalous or broken by some bulk-brane interactions. However, there will be no Planck scale physics violating the symmetry on the branes since the symmetry is gauged. The contributions from the bulk fields will be exponentially suppressed if the bulk fields are heavier than the inverse of the distance of the branes.

2 Models

We consider an extension of the supersymmetric (SUSY) standard model (SM) with an additional $U(1)_{\text{PQ}}$ gauge symmetry. All gauge fields propagate in an extra dimension. We assume that the size of the extra dimension is larger than the 4-dimensional Planck length, but smaller than $f_{\text{PQ}}^{-1}$. The SM matter fields, including right-handed neutrinos, $Q_i, U^c_i, D^c_i, L_i, E^c_i, \nu_R^i, i = 1, 2, 3$, and the two Higgs doublets, $H_U, H_D$, are localized on a 3-brane (SM brane). All the ordinary matter fields have charge +1 under $U(1)_{\text{PQ}}$ and the Higgs fields have charge $-2$ so that the Yukawa couplings are allowed. In addition, on the same brane there are $U(1)_{\text{PQ}}$ charged SM singlets which are responsible for breaking the $U(1)_{\text{PQ}}$ symmetry. We choose them to be $P(+2)$ and $N(-2)$ for simplicity. When they get nonzero vacuum expectation values (VEVs) at the intermediate scale, they can generate the Majorana masses for the right-handed neutrinos through the $N\nu_{Ri}\nu_{Rj}$ interactions, and the $\mu$-term through the non-renormalizable interactions

$$\frac{1}{M} P^2 H_U H_D,$$  \hspace{1cm} (4)

where $M$ is the fundamental Planck scale.

To cancel the anomalies of the $U(1)_{\text{PQ}}$, we need to add additional fields charged under the SM gauge group and $U(1)_{\text{PQ}}$. We assume that these fields are localized on a different brane.
(hidden brane) from where the ordinary matter resides. A simple choice consists of 3 pairs of $\bar{d}, d(-2)$, 2 pairs of $\bar{\ell}, \ell(-2)$ fields, which are vector-like under the SM gauge group and transform like the $SU(2)_W$-singlet down-type quarks and the $SU(2)$-doublet leptons, and 3 SM gauge singlets $X_i(+4), i = 1, 2, 3$.\footnote{This choice is not unique. Discussion of finding anomaly-free combinations can be found in Refs.\cite{20}.} We require that one of the $X_i$ fields (denoted by $X$) gets a nonzero VEV and gives large masses to the $\bar{d}, d$ and $\bar{\ell}, \ell$ fields through the interactions, $X\bar{d}d, X\bar{\ell}\ell$. The total particle content is anomaly free which can be seen easily by noticing that $U(1)_{PQ}$ can be embedded into the $E_6$ gauge group. To cancel the anomalies everywhere there should be Chern-Simons terms in the bulk which provide the anomaly inflow from one brane to another \cite{21}.

Assuming that there is no light $U(1)_{PQ}$ charged field in the bulk which interact with fields on both branes, there is effectively one (anomalous) $U(1)_{PQ}$ symmetry on each brane. After they are broken by the VEVs of the $P, N$ and $X$ fields on these two branes, there are two corresponding Nambu-Goldstone bosons. One linear combination is eaten by the $U(1)_{PQ}$ gauge field and becomes heavy. The other remains light and gets a small mass from the anomaly, due to the Chern-Simons terms in the bulk. This becomes the axion which relaxes $\theta$ to zero. There is no Planck scale physics violating $U(1)_{PQ}$ because it is a gauge symmetry. If there are heavy bulk fields charged under $U(1)_{PQ}$ which couple to both branes, they can contribute to the axion potential because they connect the two $U(1)_{PQ}$ symmetries on the two branes. However, their effects are suppressed exponentially and can be made safe easily if their masses are much larger than the inverse of the distance, $L$, of the two branes.

The axion in this model can be either the DFSZ type or the hadronic type depending on the $U(1)_{PQ}$ breaking scales on the two branes. If the $X$ VEV is larger than the VEVs of $P$ and $N$, the axion lies mostly on the SM brane, and it is the DFSZ axion. On the other hand, if the $X$ VEV is smaller, the axion lies in the hidden brane, it becomes the hadronic axion. It interpolates between the two type of axions if the VEVs on the two branes are comparable.

This two-brane setup also fits well with the gaugino-mediated SUSY breaking scenario \cite{22}. If supersymmetry is broken on the hidden brane, SUSY breaking can be transmitted to the SM sector through the gauge fields in the bulk. It solves the supersymmetric flavor problem by giving flavor-universal contributions to the superpartners of the SM quarks and leptons. The coincidence of the SUSY breaking scale and the Peccei-Quinn breaking scale also hints at the tantalizing possibility that these two breaking scales are correlated. In the following we describe two models which give rise to a a hadronic axion and the DFSZ axion respectively.

\footnote{This choice is not unique. Discussion of finding anomaly-free combinations can be found in Refs.\cite{20}.}
2.1 A hadronic axion model

We assume that SUSY breaking occurs on the hidden brane which contains the $\overline{d}$, $d$, $\overline{\ell}$, $\ell$, and the $X$ fields, so these fields and the bulk gauge fields can couple directly to the SUSY-breaking field and obtain SUSY-breaking masses of the order of the weak scale. The ordinary squarks and sleptons receive SUSY-breaking masses from the running contributions of the gaugino masses below the scale $L^{-1}$. These contributions are positive and comparable to the gaugino masses due to the large logarithm enhancement. The $U(1)_{\text{PQ}}$ gauge symmetry can be broken on the SM brane if $P$ and $N$ fields have negative squared masses. This happens if the coupling between the $U(1)_{\text{PQ}}$ gauge field and the SUSY-breaking field is suppressed so that the $U(1)_{\text{PQ}}$ SUSY-breaking gaugino mass is vanishingly small. Then, the dominant contributions to the SUSY-breaking masses of the SM singlets on the SM brane ($P$, $N$, $\nu_R$) are the two-loop running contributions of the soft masses of the hidden brane fields, and the anomaly-mediated contribution [23]. These contributions to the squared masses of the scalars are negative. The fields $P$, $N$ can then get large VEVs to break $U(1)_{\text{PQ}}$. The right-handed sneutrinos are prevented from getting VEVs due to the interactions $N\nu_{Ri}\nu_{Rj}$.

The VEVs of the $P$, $N$ fields can be stabilized by the non-renormalizable interactions $^2$

$$\frac{\lambda}{M} P^2 N^2,$$

and will be of the order

$$v \sim \sqrt{\frac{m_{\tilde{P}} M}{\lambda}} \sim \frac{10^9 \text{GeV}}{\sqrt{\lambda}},$$

where $m_{\tilde{P}}$ is the size of the soft scalar SUSY-breaking mass of the $P$ field, and is expected to be $O(1 \text{ GeV})$, and $M$ is the fundamental Planck scale which is close to but somewhat smaller than the effective four-dimensional Planck scale $2.4 \times 10^{18} \text{ GeV}$ because of the existence of extra dimensions larger than the fundamental length scale. We assume that the coupling $\lambda$ is small ($\lambda < 10^{-2}$, $v > 10^{10} \text{ GeV}$) so that the $\mu$-term of the Higgs superpotential can be generated by the operator

$$\frac{\lambda'}{M} P^2 H_U H_D.$$

On the hidden brane the Peccei-Quinn symmetry can be broken radiatively. The interactions

$$\kappa_d X \overline{d} d, \quad \kappa_\ell X \overline{\ell} \ell,$$

$^2$ The mass term $PN$ may be forbidden by a parity under which $P$ changes sign. Alternatively, we can assign $P$ with a different charge and cancel the anomaly with fields on the hidden brane. For $P$ field of charge $+2p$, $(p > 2)$, the VEVs can be stabilized by the superpotential $PN^p/M^{p-2}$ and the $\mu$-term can be generated by $PN^{p-2} H_U H_D / M^{p-2}$. The SM brane sector then resembles the model of Ref. [24].
can drive the SUSY-breaking squared mass of the $X$ scalar, $m_X^2$, to negative in running down to low energies if $\kappa_d$, $\kappa_\ell$ are big enough. Including radiative corrections, the size of the $X$ VEV will be stabilized at the scale where $m_X^2$ changes sign \[25\], which depends on the couplings $\kappa_d$, $\kappa_\ell$ and the soft masses of the $X$, $\bar{d}$, $d$, $\bar{\ell}$, $\ell$ fields. If the VEV of the $X$ field, $v_X$, is smaller than the VEVs of the $P$, $N$ fields, $U(1)_{PQ}$ gauge symmetry is mostly broken by the $P$, $N$ VEVs. The $X$ VEV then breaks the left over accidental global Peccei-Quinn symmetry on the hidden brane and hence $f_{PQ} \approx v_X$. The resulting axion is of the hadronic type and lies mostly in the $X$ field. The axion-photon coupling \[26\]

$$C_{a\gamma\gamma} = \frac{E_{PQ}}{N_{PQ}} - 1.92 \pm 0.08,$$

is small in this model because the ratio of the electromagnetic and the color anomalies of Peccei-Quinn symmetry in the hidden sector, $E_{PQ}/N_{PQ}$, is 2. Therefore, this model is viable if $v_X$ lies in the hadronic axion window $\sim 10^6$ GeV or in the conventional range $10^9 - 10^{12}$ GeV (with $v_X < v$).

If $v_X$ is larger than $\sqrt{m_P M/\lambda}$, then the $N$ field will get a VEV of the order of $v_X$ while the $P$ field will be prevented from getting a VEV due to the $U(1)_{PQ}$ $D$-term interactions. In this case, the axion will be a comparable mixture of the $X$ and $N$ fields with $f_{PQ} \sim v_X$. However, we will need some other way to generate the $\mu$-term because $\langle P \rangle = 0$.

### 2.2 A DFSZ axion model with correlated SUSY and Peccei-Quinn breaking scales

Now we include an explicit model of supersymmetry breaking using the shining method \[27\]. This can be accomplished by adding a pair of uncharged chiral superfields $\Phi, \Phi^c$ with mass $m$ to the bulk. In the language of four-dimensional $\mathcal{N} = 1$ superspace, the superpotential now contains

$$\Phi^c(x_5) \left( \partial_5 + m \right) \Phi(x_5).$$

Adding the source $-J\Phi^c \delta(x_5)$ on the hidden brane (at $x_5 = 0$) and a coupling $S\Phi \delta(x_5 - L)$ to a field $S$ on the SM brane (at $x_5 = L$) gives the following $F$-term equations:

$$-F_{\Phi^c}^* = \left( \partial_5 + m \right) \phi - J \delta(x_5),$$

$$-F_{\Phi}^* = \left( -\partial_5 + m \right) \phi^c + S \delta(x_5 - L),$$

$$-F_S^* = \phi(x_5 = L).$$

\[11\]
The first and third lines cannot be made to vanish simultaneously. The first line vanishes if
\[ \phi = \frac{Je^{-mx_5}}{1 - e^{-2mL}}, \tag{12} \]
where we have assumed a compactification length of \(2L\). The above gives \(F_S \sim Je^{-mL}\) and thus supersymmetry is broken [27].

The gauged \(U(1)_{\text{PQ}}\) symmetry is assumed to be broken on the hidden brane at a high scale (\(> \sqrt{F_S}\)). We need to introduce an additional pair of \(X_4(\pm 4), X(-4)\) fields on the hidden brane so that there is a \(D\)-flat direction where \(X, \overline{X}\) can get large VEVs. The gauge \(U(1)_{\text{PQ}}\) breaking can occur radiatively as described in the previous scenario, or by the superpotential interaction,
\[ Z(X\overline{X} - v_X^2). \tag{13} \]
This breaking will add \(D\)-term contributions to soft masses if the soft masses \(m_X^2\) and \(m_{\overline{X}}^2\) are not equal. The matter contributions will be universal, and are positive if \(m_X^2 < m_{\overline{X}}^2\) (with negative contributions to Higgs soft masses).

To break the remaining anomalous global \(U(1)_{\text{PQ}}\) symmetry on the SM brane, we add a singlet \(T\) to the SM brane with superpotential couplings
\[ T(PN - k\Phi^c). \tag{14} \]
We see that \(F_T = 0\) requires (assuming approximately equal soft masses)
\[ P \sim N \sim \sqrt{k}e^{-mL/2}. \tag{15} \]
If all couplings are of order unity in units of the fundamental Planck scale, a compactification length of \(mL \sim 32\) gives a supersymmetry-breaking and the global \(U(1)_{\text{PQ}}\)-breaking scale of a few times \(10^{10}\) GeV. A right-handed neutrino mass will be generated at the same scale, and a \(\mu\) term of the correct size will be produced by the operator in equation (14).

In this simple model supersymmetry is broken on the SM brane. The scalar superpartners of the SM fermions can receive soft masses via the contact terms which can be flavor violating. Models with SUSY breaking on the hidden brane so that gaugino-mediation is responsible for scalar masses can also be constructed by including shining in both directions with sources on both the SM brane and the hidden brane [28].

\[^{3}\text{Bounds on operators which explicitly violate the global } U(1)_{\text{PQ}} \text{ require any charged bulk field to have a mass } m_c \text{ such that } m_cL \lesssim 130, \text{ or } m_c \gtrsim 4m \text{ [13].}\]
3 Discussion and Conclusions

A nice feature of the models discussed in the previous sections is that $R$ parity is automatically conserved. The $U(1)_{PQ}$ gauge symmetry is broken only by fields with even charges, while all the ordinary matter superfields $Q_i, U^c_i, D^c_i, L_i, E^c_i, \nu_R$ have charge +1. A $Z_2$ matter parity, equivalent to the $R$ parity, is left unbroken, so the $R$ parity conservation is an automatic consequence of the $U(1)_{PQ}$ gauge symmetry.

There are several dark matter candidates in these type of the theories. Axions with $f_{PQ} \sim 10^{12}$ GeV have been known as a popular cold dark matter candidate [4]. The hadronic axion in the hadronic axion window, $f_{PQ} \sim 10^6$ GeV, can serve as a hot dark matter component [11]. The superpartner of the axion, the axino, is also a good cold dark matter candidate if it is the lightest supersymmetric particle (LSP) [29], and it can relax the bounds on the ordinary superpartner masses from the constraint $\Omega_\chi h^2 < 1$ in the neutralino ($\chi^0$) LSP scenario. Finally, the superheavy fields $\tilde{d}, \tilde{d}, \tilde{\ell}, \ell$ may also be the dark matter [30].

The low energy theory is simply the supersymmetric standard model (SSM) with one or more SM singlet fields which contain the axion. Their couplings to the SSM fields are highly suppressed by the intermediate scale $f_{PQ}$, so they are difficult to produce at the colliders. However, if the axino is the LSP, the next-to-lightest supersymmetric particle will decay to the axino. Although the average proper decay length is likely much larger than a typical collider detector, with a large sample of SUSY events the decay will occasionally happen in the detector, giving rise to a spectacular signal [31]. In addition, if the lightest ordinary superpartner is a charged slepton, as can happen in the gaugino mediation models [22], the slowly moving long-lived charged sleptons will produce highly ionizing tracks which will easily be discovered [32].

In conclusion, we have shown that an accidental Peccei-Quinn global symmetry can arise naturally in a theory with gauge fields propagating in an extra dimension. The resulting axion from the broken accidental Peccei-Quinn symmetry only receives its mass from the anomaly, but not from any Planck scale physics providing there are no other $U(1)_{PQ}$ charged bulk fields communicating between the two branes. Therefore it provides a viable solution to the strong CP problem. The similar setup can also be used to generate other possible global symmetries and pseudo-Nambu-Goldstone bosons. For example, the quintessence field which explains the dark energy in the universe could be the ultra-light pseudo-Nambu-Goldstone boson from a broken accidental global symmetry in theories with extra dimensions [33, 34].

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References

[1] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. 38, 1440 (1977), Phys. Rev. D 16, 1791 (1977).

[2] F. Wilczek, Phys. Rev. Lett. 40, 279 (1978); S. Weinberg, Phys. Rev. Lett. 40, 223 (1978).

[3] See G. G. Raffelt, Ann. Rev. Nucl. Part. Sci. 49, 163 (1999) [hep-ph/9903472], for a review and references therein.

[4] J. Preskill, M. B. Wise and F. Wilczek, Phys. Lett. B 120, 127 (1983); L. F. Abbott and P. Sikivie, Phys. Lett. B 120, 133 (1983); M. Dine and W. Fischler, Phys. Lett. B 120, 137 (1983).

[5] R. L. Davis, Phys. Lett. B 180, 225 (1986); R. L. Davis and E. P. Shellard, Nucl. Phys. B 324, 167 (1989); R. A. Battye and E. P. Shellard, Phys. Rev. Lett. 73, 2954 (1994), Erratum-ibid.76:2203-2204,1996 [astro-ph/9403018].

[6] D. Harari and P. Sikivie, Phys. Lett. B 195, 361 (1987); C. Hagmann and P. Sikivie, Nucl. Phys. B 363, 247 (1991); C. Hagmann, S. Chang and P. Sikivie, [hep-ph/0012361].

[7] P. J. Steinhardt and M. S. Turner, Phys. Lett. B 129, 51 (1983); G. Lazarides, C. Pangiotakopoulos and Q. Shafi, Phys. Lett. B 192, 323 (1987); M. Kawasaki, T. Moroi and T. Yanagida, Phys. Lett. B 383, 313 (1996) [hep-ph/9510461].

[8] A. R. Zhitnitsky, Sov. J. Nucl. Phys. 31, 529 (1980); M. Dine, W. Fischler and M. Srednicki, Phys. Lett. B 104, 199 (1981).

[9] J. E. Kim, Phys. Rev. Lett. 43, 103 (1979); M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 166, 493 (1980).

[10] S. Chang and K. Choi, Phys. Lett. B 316, 51 (1993) [hep-ph/9306216].

[11] T. Moroi and H. Murayama, Phys. Lett. B 440, 69 (1998) [hep-ph/9804291].
12. S. B. Giddings and A. Strominger, Nucl. Phys. B 307, 854 (1988); S. Coleman, Nucl. Phys. B 310, 643 (1988); G. Gilbert, Nucl. Phys. B 328, 159 (1989); S. W. Hawking, Commun. Math. Phys. 43, 19 (1975).

13. M. Kamionkowski and J. March-Russell, Phys. Lett. B 282, 137 (1992) [hep-th/9202003].

14. S. M. Barr and D. Seckel, Phys. Rev. D 46, 539 (1992).

15. R. Holman, S. D. Hsu, T. W. Kephart, E. W. Kolb, R. Watkins and L. M. Widrow, Phys. Lett. B 282, 132 (1992) [hep-ph/9203200].

16. E. J. Chun and A. Lukas, Phys. Lett. B 297, 298 (1992) [hep-ph/9209208].

17. N. Arkani-Hamed and S. Dimopoulos, hep-ph/9811353.

18. H.-C. Cheng, Phys. Rev. D 60, 075015 (1999) [hep-ph/9904252].

19. N. Arkani-Hamed, D. E. Kaplan, H. Murayama and Y. Nomura, JHEP 0102, 041 (2001) [hep-ph/0012103].

20. H.-C. Cheng, B. A. Dobrescu and K. T. Matchev, Nucl. Phys. B 543, 47 (1999) [hep-ph/9811310]; J. Erler, Nucl. Phys. B 586, 73 (2000) [hep-ph/0006051].

21. C. G. Callan and J. A. Harvey, Nucl. Phys. B 250, 427 (1985).

22. D. E. Kaplan, G. D. Kribs and M. Schmaltz, Phys. Rev. D 62, 035010 (2000) [hep-ph/9911293]; Z. Chacko, M. A. Luty, A. E. Nelson and E. Ponton, JHEP 0001, 003 (2000) [hep-ph/9911323].

23. L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999) [hep-th/9810155].

24. H. Murayama, H. Suzuki and T. Yanagida, Phys. Lett. B 291, 418 (1992).

25. S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

26. D. B. Kaplan, Nucl. Phys. B 260, 215 (1985); M. Srednicki, Nucl. Phys. B 260, 689 (1985); P. Sikivie, UFTP-86-28 Based upon lectures given at Les Houches Summer School on the Architecture of Fundamental Interactions at Short Distances, Les Houches, France, Jul 1-Aug 8, 1985 and at the 27th Int. GIFT Seminar on Cosmology and Particle Physics, Peniscola, Spain, Jun 2-7, 1986.
[27] N. Arkani-Hamed, L. Hall, D. Smith and N. Weiner, Phys. Rev. D 63, 056003 (2001) [hep-ph/9911421].

[28] M. Schmaltz and W. Skiba, Phys. Rev. D 62, 095005 (2000) [hep-ph/0001172]; and for models of this type which truly avoid flavor-changing neutral currents, see D. E. Kaplan and N. Weiner, to appear.

[29] L. Covi, J. E. Kim and L. Roszkowski, Phys. Rev. Lett. 82, 4180 (1999) [hep-ph/9905212]; E. J. Chun, H. B. Kim and D. H. Lyth, Phys. Rev. D 62, 125001 (2000) [hep-ph/0008133]; L. Covi, H. Kim, J. E. Kim and L. Roszkowski, hep-ph/0101003.

[30] D. J. Chung, E. W. Kolb and A. Riotto, Phys. Rev. D 59, 023501 (1999) [hep-ph/9802238]; Phys. Rev. Lett. 81, 4048 (1998) [hep-ph/9805473]; Phys. Rev. D 60, 063504 (1999) [hep-ph/9809453]; L. Hui and E. D. Stewart, Phys. Rev. D 60, 023518 (1999) [hep-ph/9812345]; T. Asaka, M. Kawasaki and T. Yanagida, Phys. Rev. D 60, 103518 (1999) [hep-ph/9904438].

[31] S. P. Martin, Phys. Rev. D 62, 095008 (2000) [hep-ph/0005110].

[32] J. L. Feng and T. Moroi, Phys. Rev. D 58, 035001 (1998) [hep-ph/9712499].

[33] C. T. Hill, D. N. Schramm and J. N. Fry, Comments Nucl. Part. Phys. 19, 25 (1989). J. A. Frieman, C. T. Hill and R. Watkins, Phys. Rev. D 46, 1226 (1992). J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, Phys. Rev. Lett. 75, 2077 (1995) [astro-ph/9505060].

[34] S. M. Carroll, Phys. Rev. Lett. 81, 3067 (1998) [astro-ph/9806099].