Non-Abelian braiding has attracted substantial attention because of its pivotal role in describing the exchange behaviour of anyons—candidates for realizing quantum logics. The input and outcome of non-Abelian braiding are connected by a unitary matrix that can also physically emerge as a geometric-phase matrix in classical systems. Hence it is predicted that non-Abelian braiding should have analogues in photonics, although a feasible platform and the experimental realization remain out of reach. Here we propose and experimentally realize an on-chip photonic system that achieves the non-Abelian braiding of up to five photonic modes. The braiding is realized by controlling the multi-mode geometric-phase matrix in judiciously designed photonic waveguide arrays. The quintessential effect of braiding—sequence-dependent swapping of photon dwell sites—is observed in both classical-light and single-photon experiments. Our photonic chips are a versatile and expandable platform for studying non-Abelian physics, and we expect the results to motivate next-generation non-Abelian photonic devices.

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follow a Schrödinger-like equation $H(z)\psi(z) = -i\partial_z |\psi(z)\rangle$, where the Hamiltonian reads

$$H(z) = \begin{bmatrix} \beta_X & \kappa_{AX}(z) & \kappa_{AX}(z) & \kappa_{AS}(z) \\ \kappa_{AX}(z) & \beta_A & 0 & 0 \\ \kappa_{BS}(z) & 0 & \beta_B & 0 \\ \kappa_{SS}(z) & 0 & 0 & \beta_S \end{bmatrix}.$$  \hspace{1cm} \text{(1)}$$

In our waveguide system, $\beta_{AX,BS} = \beta_\mu$, which is the waveguide’s propagation constant, and $|\psi(z)\rangle = [\varphi_X(z), \varphi_A(z), \varphi_B(z), \varphi_S(z)]^T$ is the state vector, where $\varphi_i$ ($i = X, A, B$ and $S$) denotes the wavefunction in waveguide $i$.

The two-mode braiding is carried out in three steps as indicated in Fig. 1a,b. In step I, $\kappa_{SS}(\kappa_{AX})$ smoothly decreases (increases) from its maximum (zero) to zero (its maximum), while $\kappa_{BS}$ is kept at zero. Figure 1c plots the eigenvalues of equation (1) as functions of $\kappa_{AX}$ and $\kappa_{AS}$, with $\kappa_{SS} = 0$. Two out of the four eigenvalues are independent of the changes in $\kappa_{SS}$ and are always degenerate at a constant eigenvalue $\beta_\mu$ (see the blue sheet). The two-fold degeneracy is protected by the chiral symmetry $\Gamma^*HF\Gamma = -H$ with $\Gamma = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I_3 \end{bmatrix}$, where $I_3$ is a 3 x 3 identity matrix, when we set $\beta_\mu = 0$. We prepare a single-site injection at waveguide A, that is, $|\psi(0)\rangle = [0, 0, 0, 1]^T$ (upper-left inset of Fig. 1c). The adiabatic evolution of $|\psi(z)\rangle$ follows the trajectory on the blue sheet and becomes $|\psi(\frac{L}{3})\rangle = [0, 0, 0, -1]^T$, which occupies waveguide S (lower-left inset of Fig. 1c) and picks up a geometric phase of $\pi$. On the other hand, a different injection at waveguide B $|\psi(0)\rangle = [0, 0, 1, 0]^T \rightarrow |\psi(\frac{L}{3})\rangle = [0, 0, 1, 0]^T$ remains unchanged (right insets in Fig. 1c). The dynamical phases accumulated for both injections are $\beta_\mu L/3$. Therefore, the total phases accumulated in the above two processes differ by $\pi$, which is the geometric phase.

Steps II and III can be understood similarly. In step II, the state dwelling in waveguide B is relocated to A and acquires a geometric phase $\pi$,
that is, \( |\psi_3(L)\rangle = [0, 0, 1, 0]^T \) \( \rightarrow \) \( |\psi_2(L)\rangle = [0, -1, 0, 0]^T \). In step III, the state occupying waveguide S with \( |\psi(L)\rangle = [0, 0, 0, 0]^T \) transfers to the waveguide B state \( |\psi(L)\rangle = [0, 0, 0, 0]^T \) and also obtains the \( \pi \) phase (Supplementary Fig. 2). The mode-switching behaviour can be verified via the calculated state vectors in the braiding process, as plotted in Fig. 1d. To summarize, the net behaviour can be verified via the calculated state vectors in the obtained \( \psi \rangle \) transfers to the waveguide B state \( |\psi\rangle \) (see Supplementary Note 2 for a rigorous proof of the geometric phase), which is a unitary matrix \( Y = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \) (see Supplementary Note 2 for a rigorous proof of the geometric phase), which is a \( U(2) \) operation also known as the \( Y \) gate in quantum logic.

We show experimental results to verify the design. Injection at waveguide A and B is performed using a laser at 808 nm (CNI, MDL-III-808L). The light-diffraction patterns at the output facet were recorded using a CCD (XG5000, XV[GW]). The photographs are shown in Fig. 1c, where the swapping of light-dwelling sites is clearly seen. The braiding was also verified in the quantum-mechanical limit using single-photon injections, where indistinguishable pairs of photons at 810 nm were generated using a quantum setup (Supplementary Fig. 4). One photon was injected into the braiding structure via waveguide A or B, while the other one propagated in a single-mode reference optical fibre. We used two avalanche photodetectors to respectively collect the single photons at the output (that is, waveguide A or B) and the reference fibre. Coincidence measurements were then performed using the collected data. In the results displayed in Fig. 1f, the output waveguide exhibiting the dominating coincidence is always different from the input waveguide—clear evidence of the two-mode braiding described by the \( Y \) gate.

### Measurement of the geometric phase

To measure the geometric phase that is a key characteristic of two-mode braiding, we have designed an interference experiment, as illustrated in Fig. 2a. The structure consists of three stages. In the injection stage, injected photons are equally split into two identical waveguides. Two separate braiding processes are then carried out in the second stage, which contains two copies of braiding structures that are identical to the one in Fig. 1a. Two experiments are performed with different configurations. In experiment I (II), injection of the lower braiding structure is via waveguide B’ (A’). The upper braiding structure is injected at waveguide A for both experiments. After the braiding, in stage III, the two output waveguides are equally split into four arms, two of which are merged again so that photons from the upper and lower clusters can interfere. The three terminal ports are labelled Y1, Y2 and Y3.

The results of the two experiments with light are shown in Fig. 2b,c. Strong light intensities are seen at ports Y1 and Y3 for both cases, which indicates the successful braiding-induced mode switching. However, discrepancies are seen at Y2. In experiment I (with injections at A and B’), the image at Y2 is dark (Fig. 2b), which suggests destructive interference of the light after braiding. Because the light propagating through the upper and lower braiding structures accumulates the same dynamical phase, this result indicates a phase difference of \( \pi \), which can only be the consequence of a geometric phase. For comparison, in experiment II (with injections at A and A’), port Y2 lights up (Fig. 2c), which suggests constructive interference due to their same phase accumulation. These experimental results are strong evidence of the \( \pi \) geometric-phase difference induced by the two-mode braiding.

### Three-mode non-Abelian braiding

The two-mode braiding demonstrates the effectiveness of our photonic platform, which we now expand for three modes. The three-mode braid group has two generating operations \( G_i \): \( |\psi\rangle \rightarrow \{ |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle \} \) \( \rightarrow \{ |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle \} \) \( \rightarrow \{ |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle \} \). In step II, we will next demonstrate. Figure 3a illustrates the schematic diagram for the three-mode braiding structure. Here, the system has seven waveguides. Waveguides A, B and C are sufficienty far apart that they can only couple via waveguides X1 and X2. The system’s Hamiltonian thus has a similar structure to equation (1) but sustains three degenerate zero modes that form the braiding subspace (see Supplementary Note 3 for more details). The waveguide array has a length of 80 mm and is divided into two sections, with the fitted coupling coefficients shown in Fig. 3b. The first section swaps the modes in waveguides A and B, which executes \( G_1 \). The second section exchanges modes in B and C, so that the net result is \( G_2 G_1 \) \( \rightarrow \) \( \{ |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle \} \) \( \rightarrow \) \( \{ |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle \} \) (see the braiding diagram in the left panel of Fig. 3c). The experimental results of \( G_2 G_1 \) are summarized in Fig. 3c. We employ a double-exposure-assisted scattering technique (Methods) in which point-like scatterers were fabricated inside all the waveguides so that the light passages can be captured using a camera (Andor, Zyla 5.5 sCMOS) as shown in the middle panels. We find that when the injection is at waveguide A, the light successively propagates to waveguide S1, B, S2, and finally outputs at C. By contrast, when the injection is at waveguide B (C), the output is at A (B). The output patterns are shown in Fig. 3c (right panels), which are clearly the intended three-mode permutation described by \( G_2 G_1 \).

We fabricated another system with two sections arranged in the opposite order so that \( G_2 G_1 \) \( \rightarrow \) \( \{ |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle \} \) is executed (see the braiding diagram in the left panel of Fig. 3d). The experimental results shown in Fig. 3d demonstrate the intended outcome. Comparing Fig. 3c,d, it is evident that \( G_2 G_1 \neq G_1 G_2 \), which unambiguously demonstrates the non-Abelian nature of the three-mode braiding.
Non-Abelian braiding is also successfully realized in single-photon experiments using the same photonic chips. The measured coincidences confirm that single photons also follow non-Abelian braiding, as shown in Fig. 3e,f for $G_2G_1$ and $G_2G_1$, respectively. Another property of three-mode braiding is the equivalence of $G_2G_2$, and $G_2G_1$ (ref. 7), both of which induce $[\varphi_1, \varphi_2, \varphi_3, \varphi_4]^T \rightarrow [\varphi_1, -\varphi_2, -\varphi_3, -\varphi_4]^T$. These braiding operations can be used to design a quantum X gate (by discarding one state after the operation), which is confirmed in separate experiments (Supplementary Figs. 7–10).

**Expandability of the photonic chips and multi-mode braiding**

Comparing the structures of two-mode and three-mode braiding, it becomes clear that the photonic platform can be straightforwardly expanded to realize the braiding of an arbitrary number of modes, as shown in Fig. 4a. As a proof of concept, we present a five-mode braiding design. The targeted braid diagrams are shown in Fig. 4b–c (upper), which depict two operation sequences: $M_1M_1M_1M_1M_1: [\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5]^T \rightarrow [\varphi_1, -\varphi_2, -\varphi_3, -\varphi_4, -\varphi_5]^T$ and $M_2M_2M_2M_2: [\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5]^T \rightarrow [\varphi_1, -\varphi_2, -\varphi_3, -\varphi_4, -\varphi_5]^T$ (see Supplementary Table 1 for definitions). The measured diffraction patterns using lasers are given in Fig. 4b,c, and the measured coincidences using single photons are summarized in Fig. 4d,e, respectively. The observed permutation outcomes align well with theoretical predictions (Supplementary Figs. 11 and 12).

We further remark that within each building block (the red dashed box in Fig. 4a), only one waveguide needs to be bent to achieve the modulation of three coupling coefficients. As a result, the top and bottom rows are all identical straight waveguides that can be prefabricated, which makes the design of subsequent braiding quite flexible. With these characteristics and the expandability as demonstrated, our scheme becomes a versatile and convenient on-chip platform for realizing more complex non-Abelian operations.

**Conclusion**

To conclude, we have realized non-Abelian braiding for both classical light and single photons in a photonic on-chip platform. The key characteristics of non-Abelian braiding—permutations of degenerate states and the order-dependent braiding outcomes—are both definitively observed. We remark that our scheme for photonic non-Abelian braiding is a purely geometric-phase effect on multiple degenerate modes protected by chiral symmetry. As a result, the braiding is robust to perturbations such as those on the evolution path and coupling coefficients. How to incorporate topological protection to make the braiding operations even more robust is a valuable future goal. The proposed versatile photonic platform is expected to reveal more non-Abelian physics related to the multi-mode Berry phase, and its capability in generating a variety of unitary matrices may lead to a new generation of non-Abelian photonic devices for the manipulation of light and photons.

**Online content**

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary
Fig. 4 | Multi-mode braiding. a, The expandability of the braiding design. b,c, Two examples of five-mode braiding (upper: braid diagram) and their experimental realization with light (lower) for $M_1 M_2 M_3 M_4$ (b) and $M_2 M_1 M_4 M_3$ (c) braiding. d,e, Results of single-photon five-mode braiding experiments for $M_4 M_1 M_2 M_3$ (d) and $M_3 M_2 M_1 M_4$ (e) braiding.

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Methods

Sample fabrication. The waveguides inside boroaluminosilicate glass (Corning, EAGLE®) were fabricated using femtosecond-laser direct-writing techniques. In the experiment, a Ti:sapphire laser (Light Conversion, Carbide 5W) was focused below the surface of the glass using a ×40 microscope (numerical aperture = 0.75). The motion of the glass chip was controlled using an Aerotech system to fabricate the waveguide structures inside the glass. The moving speed of the laser is 40 mm s⁻¹, under which the fabricated cross-section size of the waveguide is ~6.9 × 5.3 μm. The refractive index contrast between the waveguide and the background is ~2.5 × 10⁻³.

Double exposure-assisted scattering technique. This technique is employed to obtain images of light propagation in the braiding structures shown in Fig. 3c. The fabricated waveguides can confine the photons well since they exhibit a transmission loss of ~0.03 dB mm⁻¹. Therefore, it is not easy to directly observe the light/photon passages. To overcome this issue, we first fabricated the waveguides by controlling the trajectory of the lasers. In the second exposure procedure, the laser was again focused on a series of points inside the straight waveguides. This procedure further alters the local refractive index and produces point scatterers inside the waveguides. The exposure time is 10 ms and the spacing between adjacent scatterers is kept at 140 μm. These scatterers can out-couple the light/photons, thus enabling us to visualize the light passage within the braiding structure. This technique was only used in the measurements for Fig. 3c.

Data availability

The data that support the findings of this work are available from the corresponding authors upon reasonable request.

Code availability

The codes used for performing the theoretical analysis and numerical simulations are available from X.-L.Z. upon reasonable request.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (11922416, 11974140, 61825502, 61827826 and 61960206003), China Postdoctoral Science Foundation (2019T120234) and the Hong Kong Research Grants Council (12302420, 12300419, 22302718 and C6013–18G). X.-L.Z. and G.M. thank C. T. Chan and R.-Y. Zhang for fruitful discussions.

Author contributions

X.-L.Z. and G.M. conceived of the idea. X.-L.Z., Z.-G.C. and G.M. performed the theoretical analysis. X.-L.Z. performed numerical simulations and designed the experiment. F.Y. carried out the experimental measurements under the supervision of Z.-N.T. and Q.-D.C. The manuscript was written by X.-L.Z. and G.M. with inputs from all the authors. The project was supervised by G.M. and H.-B.S.

Competing interests

The authors declare no competing interests.

Additional information

Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s41566-022-00976-2.

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Peer review information Nature Photonics thanks the anonymous reviewers for their contribution to the peer review of this work.

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