Optical computation of a spin glass dynamics with tunable complexity

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Spin glasses (SGs) are paradigmatic models for physical, computer science, biological, and social systems. The problem of studying the dynamics for SG models is nondeterministic polynomial-time (NP) hard; that is, no algorithm solves it in polynomial time. Here we implement the optical simulation of an SG, exploiting the \( N \) segments of a wavefront-shaping device to play the role of the spin variables, combining the interference downstream of a scattering material to implement the random couplings between the spins (the \( J_{ij} \) matrix) and measuring the light intensity on a number \( P \) of targets to retrieve the energy of the system. By implementing a plain Metropolis algorithm, we are able to simulate the spin model dynamics, while the degree of complexity of the potential energy landscape and the region of phase diagram explored are user defined, acting on the ratio \( P/N = \alpha \). We study experimentally, numerically, and analytically this Hopfield-like system displaying a paramagnetic, ferromagnetic, and SG phase, and we demonstrate that the transition temperature \( T_g \) to the glassy phase from the paramagnetic phase grows with \( \alpha \). We demonstrate the computational advantage of the optical SG where interaction terms are realized simultaneously when the independent light rays interfere on the detector’s surface. This inherently parallel measurement of the energy provides a speedup with respect to purely in silico simulations scaling with \( N \).

The Optical Simulator

As a starting point for our experiment, it is easy to observe that, in the simplified case in which the \( r \)th light ray field has brain functions (14), random lasers (15, 16), and quantum chromodynamics (17). Indeed, novel methods for the calculation of the equilibrium states and of the dynamics of a spin glass system are highly desiderate. Here we propose an optical system able to compute the energy of a given spin glass state. We integrated such an optical layer onto a standard digital computation layer to realize an optical spin glass (OSG) dynamics simulation. Our idea stems from the observation that the overall intensity

\[ I = \sum_{\nu=1}^{P} f^{(\nu)} \]

at \( P \) given points \( \nu \) on a screen placed at the downstream of a strongly scattering medium shone with \( N \) coherent light rays from a single laser can be formally written as a spin glass Hamiltonian. Thus scattering, coupled with an adaptive optical element, has been proposed as an instrument to access the spin glass dynamics and employed for low-complexity (small \( P \) values) simulations (18). This approach is revealed to be very promising and has been successively employed to find ground-state transmission matrices (19).

Significance

Optics is a unique platform to speed up computation, providing advantages especially for inherently parallel operations. Here we demonstrate an optical experiment enabling the simulation of a paradigmatic prototype model for statistical mechanics: the spin glass (SG). SGs are systems presenting all the aspects of complexity, such as many equilibrium states producing multiple relaxation times and nontrivial phenomenology. With the optical approach, the spin state is mapped on the mirrors of a fast adaptive optics device illuminated by a laser, and the energy calculation is performed by collecting the intensity after all the controlled light rays interfered in strongly scattering medium. The optical simulation enables beating the digital calculation in terms of speed for large-sized SG.

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a complex amplitude $a_i = A_i e^{i\phi_i}$, the single target contribution $f^{(\nu)}$ reads

$$f^{(\nu)} = E^{(\nu)} E^{(\nu)\dagger} = \sum_{i=1}^{N} |\xi_i^{(\nu)} a_i|^2 = \sum_{i,j}^{N} \xi_i^{(\nu)} \xi_j^{(\nu)*} a_i a_j,$$

where $\xi_i^{(\nu)} = |\xi_i^{(\nu)}| e^{i \arg \xi_i^{(\nu)}}$ are the complex transmission matrix elements (20) from the $i$th incoming beam to the target $\nu$. Input illumination is controlled by a digital micromirror device (DMD, Vialux V-7000) with the superpixel method (ref. (21); see also SI Appendix). This approach enables separation of the input laser wavefront into many segments (up to $2 \cdot 10^5$), each one composed of four DMD mirrors. Each segment can be then programmed into a state characterized by a phase factor $\phi_i = \pm \pi$, equivalent to an amplitude factor of $S_i \in \{-1, 1\}$, so that the single target intensity (2) can be rewritten as

$$f^{(\nu)} = \sum_{i,j}^{1,N} \nu_{ij} S_i S_j,$$  

where all transmission matrix elements and input field amplitudes have been included in the coefficient

$$\nu_{ij} \equiv A_i A_j \xi_i^{(\nu)} \xi_j^{(\nu)*}.$$

Let us stress that, although $\nu_{ij}$ are complex valued, the intensity $f^{(\nu)}$ is always a real number because $\nu_{ij} = \nu_{ij}$ and the sum in Eq. 3 runs on all $i, j = 1, \ldots, N$. Amplitudes $A_i$ are defined by the laser intensity. By using a laser with a Gaussian beam and expanding it to retrieve a homogeneous distribution of the intensity on the active DMD area, it is possible to approximate all of the $A_i$, for any $i$, to a constant over the DMD segments. Maximizing the overall intensity (Eq. 1) with respect to DMD spin configurations finally corresponds to minimize the following Hamiltonian:

$$\mathcal{H}[S] = -\frac{1}{2} \sum_{i,j}^{1,N} J_{ij} S_i S_j,$$

where we have introduced the interaction matrix

$$J_{ij} \equiv \frac{1}{N} \sum_{\nu=1}^{P} \nu_{ij}^{(\nu)}.$$

Fig. 1. A sketch of the experimental setup. Laser light (Azure Light 532 NM, reflected by the DMD, is scattered by an opaque medium after passing through a relay system including an iris. The far facet of the scattering medium is then imaged on a camera after passing through a linear polarizer. Inset shows the measured $P(\delta_{\text{off-diag}})$ and $P(\delta_{\text{diag}})$. They have been fitted with the function $P(\delta) = A_0 \exp(-\delta_0^2 / \delta_0^2)$. We retrieve $\delta_{\text{off-diag}} = 0.08$ and $\delta_{\text{off-diag}} = 0.04$. The values of $\delta_0$ are consistent with the predicted behavior of $P(\delta_{\text{diag}}) = \|\zeta_1\|^2$ and $P(\delta_{\text{off-diag}}) = \|\zeta_2\|^2 + 2 \|\xi_1\|^2 - 2 \|\zeta_1\| \|\zeta_2\|$ (SI Appendix); that is, $P(\delta_0) = 1/(2\sigma^2) \exp(-\delta_0^2 / \delta_0^2)$ and $P(\delta_0) = 1/(2\sigma^2) \exp(-\delta_0^2 / \delta_0^2)$.
intensity $I^{(c)}$ changes on the targets. The targets, in our experiment, identified with a set of binned camera pixels, are chosen at a distance larger than the typical speckle size, in order to avoid correlated values in $\xi^{(c)}$ for neighboring targets. The simplest configuration is the one with targets organized into a square lattice, with lattice sides larger than the size of the speckles (intensity correlation size); see Fig. 2A, Inset.

If $I^{(c)}$ increases, then the change is accepted; otherwise, the change is accepted only with a probability $p = e^{\Delta I/T}$, where $\Delta I$ is the measured variation in intensity following the spin/micromirror flip, and $T$ is a user-defined system temperature. The spin flipping is performed acting on the relative segment on the DMD (flipping time is $18 \mu$s), while the spin coupling is realized optically thanks to the nearly instantaneous light propagation into the disordered medium; the intensity reading is performed through the camera, while the move acceptance is performed digitally by the computer.

Results and Discussion

By storing spin configurations $S(t)$ at each time step $t$, we are able to extract the temporal behavior of the connected autocorrelation function,

$$F_{\text{self}}(\tau) = \frac{1}{N} \sum_{i} \langle S_i(t) S_i(t+\tau) \rangle_c,$$

where $\langle \ldots \rangle$ indicates the averaging over Monte Carlo steps $t$. As an instance, we show the case of an optical spin model with $N = 225$ spins whose dynamics we simulated for $t_{\text{max}} = 2 \times 10^5$ Monte Carlo steps (each step consists of $N$ micromirror flips, with micromirrors selected uniformly and in random fashion) from which we extract the behavior of the correlation $F_{\text{self}}(\tau)$ on a more limited temporal window for $\tau$ in order to perform a proper temporal averaging.

![Diagram](https://doi.org/10.1073/pnas.2015207118)
The results for a single target (number of targets $P = 1$) are reported in Fig. 2A, displaying $F_{\text{self}}(\tau)$. In this case, the correlation function decays rapidly to zero at high temperature, and, lowering the temperature, its behavior crosses over to a power-law decay. This slowing down of the dynamics is called critical relaxation (28–32). Eventually, at low enough $T$, $F_{\text{self}}(\tau)$ tends toward a nonzero plateau. To check further the nature of the critical relaxation, results a reasonable interpolation, while, in Eq. (4), we have a transition to a ferromagnetic phase.

Indeed, the occurrence of only a pair of opposite states may be explained if we go back to the formal description of the $J_{ij}$ in Eqs. 4-6, with a $P = \nu = 1$ target,

$$H[S] = -\frac{1}{2N} \sum_{i=1}^{N} \xi^{(1)}(i) S_i^2 = -\frac{\langle S(1) \cdot S \rangle^2}{2N}, \tag{9}$$

in which we exploited the approximation of constant amplitudes for all of the incoming light rays, and the Hamiltonian is just proportional to the scalar product of the spin configuration array and the array of the transmission matrix elements $\xi(i)$ from the micromirror-created spins $(i)$ to the target $(1)$. This is the maximum for two configurations: the one maximizing the field along the transmission vector ($\uparrow$; Fig. 1) and the other maximizing the field along the opposite direction. It is, in fact, well known that, given a certain transmission matrix, it is possible to find the best input configuration producing maximum intensity (33) and that this configuration is “unique.”

The interaction matrix $J_{ij} = \xi_i \xi_j / N$ is a diadic matrix constructed on a single vector $\xi^{(1)}$ with the Hebb rule (34). Resulting from a diadic matrix, our scattering system is, thus, behaving as a sort of trained optical memory in which the pattern $\xi^{(1)}$ is the memory learned by the network. In this neural network context, “learning” means varying the couplings. The network is designed in such a way that the learned patterns $\xi$ are retrieved as stable

![Fig. 3](image-url)

**Fig. 3.** Results for $P = 36, N = 225$. (A) The $F_{\text{self}}$ as a function of $\tau$ indicating the sweeps. The inset reports the image on the camera, with white circles highlighting the target positions. In $\bar{A}$, we report the degree of similarity obtained for clustered states resulting many optimizations at zero temperature. The phase space results clustered in many states. In $C$–$F$, we report, respectively, $A$, $\tau_i$, and $a$ as a function of $1/T$, comparing the results from the digital simulation (red full dots) with those of the optical simulation (open blue symbols) (see SI Appendix). In $D$, $\xi_i$ is reported only in the region where the exponential contribution results a reasonable interpolation, while, in $E$, $a$ is reported only in the critical region.
configurations, that is, in this example, the spin configurations $\uparrow S$ and $\downarrow S$. In optical terms, instead, when $P = 1$, the experiment is identical to a standard wavefront-shaping experiment (35) in which there exists a single spin configuration maximizing intensity (36). The case where the $\zeta$ are Boolean, rather than complex and Gaussian distributed, is also called the Mattis model (37).

If $P$ is larger and, in particular, if it is so large as to scale with the number of variables $N$, the situation is similar to the Hopfield model of Amit et al. (24). In that seminal paper, the authors demonstrate that it is possible to store states into a neural network memory by exploiting a sum of diadic matrices generated from that many different vectors. However, there is a limit to how many different vectors. However, there is a limit to how many different vectors can be stored in such a kind of memory (38). When this limit is surpassed, the multiple states start to combine, thus generating additional states to those related to the transmission patterns. All these states are minima of a complex energy landscape displaying a spin glass phase in a given region of the phase diagram. To experimentally engineer the complexity of the spin glass phase in our optical system, then, the only thing that we need to do is to increment the number of targets. We, therefore, perform optical Monte Carlo simulation with the Hamiltonian (5) and increasing $P$.

We perform OSG simulations with $N = 225$ and $P = 4, 9, 16, 25, 36, 225$ targets, corresponding to $\alpha \approx 0.018, 0.04, 0.071, 0.11, 0.16, 1$ (where $\alpha$ is the ratio between the number of targets $P$ and the number of degrees of freedom $N$). We report the results for $P = 36$ in Fig. 3. In Fig. 3A, we show the auto-correlation functions of $\sigma$ as a function of $\tau$. It is readily noticed that the $F_{\text{self}}$ moves from an exponential to a nonexponential relaxation in time as temperature decreases, which is a typical aspect of many complex systems (29, 30, 39). In particular, the $F_{\text{self}}$ of the OSG with many targets can be typically described by the generic interpolating function approaching the critical point from high temperature,

$$F_{\text{self}}(\tau) = (1 - A)e^{-\frac{\tau}{\tau_0}} + A\frac{C}{\tau^n},$$

in which the first term represents the exponential relaxation time, while the second term represents the power law decay appearing.

Fig. 4. Probability distribution of the Parisi parameter $q$ as a function of $\beta$, for $n = 225$. Optical simulations have been performed for a few values of $P$. (A) $P = 4; \alpha = 0.02$. (B) $P = 16; \alpha = 0.07$. (C) $P = 25; \alpha = 0.11$. $P(q)$ in a given $\beta, \alpha$ configuration has been obtained by performing 50 fast (25 sweeps) simulations (see SI Appendix). In D, we report simulations retrieved by performing two very long (8,000 sweeps) simulations (see SI Appendix) for $P = 225, \alpha = 1$. In E, we report the estimated $T_G$ by optical simulations, together with the curve $T_G = A(1 + \sqrt{2g})$ predicted from Replica theory for $N \to \infty$ (SI Appendix, Eq. S30). Optical simulation provides a value of $A$ of $1.4 \pm 0.6$ compatible with theory (in which $A = 1$). Inset in E shows the Kurtosis of the $P(q)$ as a function of $\beta$. Continuous line is to guide the eye.
approaching the glassy phase, that is, crossing over for large $\tau$. The $\Lambda$ parameter represents the relative weight of the two terms; the larger it is, the more the power-law-like term is relevant. The behavior of the fitting parameters is reported in Fig. 3, estimated both by the data obtained in standard “digital” numerical simulations and with the optical Monte Carlo methods introduced in the present work. As expected, as temperature is decreased from the paramagnetic phase, we observe 1) an increase of the correlation time $\tau^*$ (Fig. 3D), 2) a very sharp increase of the weight $\Lambda$ of the power-law decay contribution in the critical region (Fig. 3C), and 3) a lowering of the power-law decay exponent $\alpha$ in the critical region (Fig. 3E). The critical slowing down of the relaxation is a signature of a multistate (free-)energy landscape. Further evidence of complexity is also shown in an experiment in which multiple short relaxation dynamics at $T = 0$ have been retrieved. In Fig. 3B, we report the parameter $q(p_1, p_2)$ for a thousand different simulations. Here the pattern shows many different clusters, indicating that many equilibrium states are populating the potential energy landscape. We have been using $k$ means to organize data into 36 clusters. In SI Appendix, we add up finite size effects and the role of the $\alpha$.

As we already mentioned, in Eqs. 5 and 6, the number of targets $P$ plays the same role as the number of memories to be stored into a Hebbian matrix in the Amit Gutfreund Sompolinsky theory (24, 26) predicting the impossibility of memory retrieval, that is, the inability to retrieve the encoded $\xi$ states, for $\alpha$ larger than a given critical value $\alpha_c$. Considering our optical simulator as a Hopfield memory, the retrieval process corresponds to the thermalization in the small-$\alpha$, low-T regime, in which, starting from a random configuration, a low-energy (high intensity) state is retrieved, minimizing the scalar product between $S_{\text{final}}$ and the $\xi^\alpha$. The retrieved state corresponds then to the $\nu$th memory element. This qualitative change in the degree of complexity of our system can be demonstrated in Fig. 4, where we report the temperature behavior of the probability density function $P(q)$ of the overlap (see Eq. 8),

$$q_{\text{eb}} \equiv \frac{1}{N} \sum_{k=1}^{N} S_k^{(a)} S_k^{(b)},$$

between couples of replicas $S^{(a)}$ and $S^{(b)}$. At high temperature, each equilibrium configuration will be uncorrelated from the others, because of strong thermal fluctuations. For low $T$, at $\alpha \simeq 0$, that is $P/N \to 0$ as $N \to \infty$, the model dynamics will tend to align, or counteralign, to the $\xi$ patterns, as mentioned in Eq. 9 for the generalized Mattis model. The values of the overlap between such states will, then, converge to only two possible values as $N$ increases, one the inverse of the other, and the distribution will display two distinct symmetric peaks. As the number of patterns to be satisfied increases with the number of micromirrors, however, frustration arises, as it becomes more and more unfeasible to satisfy any of them with a configuration of $S$ minimizing the Hamiltonian $H(S)$; compare Eq. 5. The change of the $P(q)$ from a low-temperature two-peak distribution into a multipeak distribution is clear evidence of the onset of such frustration and the relative complexity in the equilibrium state organization. We stress that, in Fig. 4, the overlap distributions displayed are for a single realization of the random couplings. Moreover, from these curves, we extracted the values of $T_c$ (see SI Appendix), and these are reported in Fig. 4E as a function of $\alpha$ for $N = 225$ (green dots fitted with green curve).

To compare data with theory, we calculated the phase transition equation with the replica symmetry-breaking approach (reported in SI Appendix). The curves in Fig. 4E represent the fit with the model (SI Appendix, Eq. S30), leaving $\Lambda$ as a free parameter. By fitting the data for $T_c$ with the model, we retrieve a constant $\lambda = 1.44 \pm 0.6$, thus resulting, within the fit error, with the theoretically predicted value ($\lambda = 1$).

In order to test our optical procedure, we performed numerical experiments with a few spins, from $N = 16$ to $N = 225$; however, the real advantage of the proposed method comes about when the sample size and the number of targets are large. In Fig. 5, we report a measurement with a large fully connected spin glass ($N = 12,100$ micromirror-combined spins organized into a $110 \times 110$ 2D square lattice) with $\alpha = 0.113$. We retrieve a minimum time per step (TPS) of $\simeq 10^{-4}$ s per step for this large optical experiment (average TPS is 4.5 s per step, due to the interrupts of our operating system; see SI Appendix). This TPS turns out to be only barely dependent on $N$, as we show in Fig. 5, Insert, in which we report the TPS for the optical simulation as red circles. The TPS for the digital simulation is, instead, growing with $N$, as expected (Fig. 5, Insert, blue circles). The optical simulation can thus outperform the digital one at $N \approx 12,000$. We note that this configuration is exploiting only a small portion of the full DMD size (48,400 mirrors out of 786,432, i.e., 1/16), thus potentially still providing a further advantage with respect to the fully digital approach for simulations with larger $N$.

**Conclusions**

In conclusion, we demonstrate the possibility to simulate the dynamics of a system with $N$ spins and $N^2$ couplings with random couplings and a low-temperature spin glass phase. Our approach has the advantage of 1) being scalable: the possibility to simulate very large systems without affecting the calculation speed, because the instantaneous interference enables to access the energy difference without requiring to individually calculate all of the coupling terms; 2) varying the constraints density so as to survey both a ferromagnetic-like ordering, retrieving the transmission matrix patterns, and a spin glass freezing, that is, loss of memory, by playing with the number of optical target in which the intensity is monitored; and 3) varying the dynamic variables of the system, for Ising spins, each one needing four micromirrors to be constructed, both to multiple discrete state variables and to complex continuous phases. 4) The same approach may be employed with multistate spins, employing higher-dimensional superpixel capable of generating many different phase and amplitude values. In this configuration, as far as the number of targets per spin is below the critical $\alpha_c(T)$ value of memory retrieval, the analysis of the ferromagnetic Mattis-like states allows reconstruction of the transmission matrix elements of the random opaque medium through the states $S$ visited in the optical simulation, as memories learned in a neural network.

**Data Availability.** All study data are included in the article and SI Appendix.
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1. N. C. Harris et al., Quantum transport simulations in a programmable nanophotonic processor. Nat. Photonics 11, 447 (2017).

2. C. Sparrow et al., Simulating the vibrational quantum dynamics of molecules using photonics. Nature 557, 660–667 (2018).

3. X. Lin et al., All-optical machine learning using diffractive deep neural networks. Science 361, 1004–1008 (2018).

4. A. Peruzzo, J. McClean, S. Alan, J. L. O. Brien, A variational eigenvalue solver on a photonic quantum processor. Nat. Commun. 5, 4213 (2014).

5. C. Roques-Carmes et al., “Photonic recurrent ising sampler” in Conference on Lasers and Electro-Optics (Optical Society of America, 2019), p. Flu4C.2.

6. N. H. Farhat, D. Paik, A. Prata, P. Paik, Optical implementation of the Hopfield model. Appl. Opt. 42, 1469–1475 (1985).

7. S. F. Edwards, P. W. Anderson, Theory of spin glasses. J. Phys., Lett. 5, 965 (1975).

8. M. Mézard, G. Parisi, M. Virasoro, Spin Glass Theory and Beyond: An Introduction to the Replica Method and Its Applications (Lecture Notes in Physics, World Scientific, 1987), vol. 9.

9. A. P. Young, Spin Glasses and Random Fields (Series on Directions in Condensed Matter Physics, World Scientific, 1998), vol. 12.

10. D. S. Fisher, D. A. Huse, Equilibrium behavior of the spin-glass ordered phase. Phys. Rev. B 38, 386–411 (1988).

11. L. Leuzzi, G. Parisi, F. Ricci-Tersenghi, J. J. Ruiz-Lorenzo, Ising spin-glass transition in a magnetic field outside the limit of validity of mean-field theory. Phys. Rev. Lett. 103, 267201 (2009).

12. T. Temesvari, The Ising spin glass in finite dimensions: A perturbative study of the free energy. Nucl. Phys. B 829, 534–554 (2010).

13. M. Batty-Jesi et al., Aging rate of spin glasses from simulations matches experiments. Phys. Rev. Lett. 120, 267203 (2018).

14. D. J. Amit, D. J. Amit, Modeling Brain Function: The World of Attractor Neural Networks (Cambridge University Press, 1992).

15. N. Ghofraniha et al., Experimental evidence of replica symmetry breaking in random lasers. Nat. Commun. 6, 6058 (2015).

16. F. Antenucci, Statistical Physics of Wave Interactions (Springer, 2016).

17. M. A. Halasz, A. D. Jackson, R. E. Shrock, Misha. A. Stephanov, The phase diagram of QCD. Phys. Rev. D 58, 096007 (1998).

18. H. Erik, Optical spin glasses: A new model for glassy systems. Master thesis, 23 October 2018 Univ. Sapienza. arXiv [Preprint] (2020), https://arxiv.org/abs/2010.13400 (Accessed 7 May 2021).

19. D. Pierangeli, M. Rafayelyan, C. Conti, S. Gigan, Scalable spin-glass optical simulator. arXiv [Preprint] (2020), https://arxiv.org/abs/2006.00828 (Accessed 7 May 2021).

20. C. W. J. Beenakker, Random-matrix theory of quantum transport. Rev. Mod. Phys. 69, 731 (1997).

21. S. A. Goorden, J. Bertolotti, A. P. Mosk, Superpixel-based spatial amplitude and phase modulation using a digital micromirror device. Opt. Expr 22, 17999–18009 (2014).

22. J. J. Hopfield, Neural networks and physical systems with emergent collective computational abilities. Proc. Natl. Acad. Sci. U.S.A. 79, 2554–2558 (1982).

23. J. J. Hopfield, Neurons with graded response have collective computational properties like those of two-state neurons. Proc. Natl. Acad. Sci. U.S.A. 81, 3088–3092 (1984).

24. D. J. Amit, H. Gutfreund, H. Sompolinsky, Storing infinite numbers of patterns in a spin-glass model of neural networks. Phys. Rev. Lett. 55, 1530 (1985).

25. D. J. Amit, H. Gutfreund, H. Sompolinsky, Spin-glass models of neural network. Phys. Rev. A 32, 1007 (1985).

26. D. J. Amit, H. Gutfreund, H. Sompolinsky, Statistical mechanics of neural networks near saturation. Ann. Phys. 173, 30–67 (1987).

27. N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, E. Teller, Equation of state calculations by fast computing machines. J. Chem. Phys. 21, 1087–1092 (1953).

28. A. Crisanti, L. Leuzzi, Equilibrium dynamics of spin-glass systems. Phys. Rev. B 75, 144301 (2007).

29. W. Götze, Complex Dynamics of Glass-Forming Liquids, A Mode-Coupling Theory (Oxford University Press, 2009).

30. F. Caltagirone et al., Critical slowing down exponents of mode coupling theory. Phys. Rev. Lett. 108, 085702 (2012).

31. F. Caltagirone, U. Ferrari, L. Leuzzi, G. Parisi, T. Rizzo, Critical slowing down exponents in structural glasses: Random orthogonal and related models. Phys. Rev. B 86, 064204 (2012).

32. U. Ferrari, L. Leuzzi, G. Parisi, T. Rizzo, Two-step relaxation next to dynamic arrest in mean-field glasses: Spherical and ising p-spin model. Phys. Rev. B 86, 014204 (2012).

33. S. M. Popoff et al., Measuring the transmission matrix in optics: An approach to the study and control of light propagation in disordered media. Phys. Rev. Lett. 104, 100601 (2010).

34. D. D. Hebb, The Organization of Behavior: A Neuropsychological Theory (Science Editions, 1962).

35. I. M. Vellekoop, A. P. Mosk, Focusing coherent light through opaque strongly scattering media. Opt. Lett. 32, 2309–2311 (2007).

36. S. M. Popoff et al., Measuring the transmission matrix in optics: An approach to the study and control of light propagation in disordered media. Phys. Rev. Lett. 104, 100601 (2010).

37. D. C. Mattis, Solvable spin systems with random interactions. Phys. Lett. 56A, 421 (1975).

38. J. Buhmann, K. Schulten, Influence of noise on the function of a physiological neural network. Proc. Natl. Acad. Sci. U.S.A. 105, 10857–10862 (2008).

39. W. Kob, C. Donati, S. J. Plimpton, P. H. Poole, S. C. Götzter, Dynamical heterogeneities in a supercooled Lennard-Jones liquid. Phys. Rev. Lett. 79, 2827 (1997).