AN EMPIRICAL EXPLANATION OF THE ANOMALOUS INCREASES IN THE ASTRONOMICAL UNIT AND THE LUNAR ECCENTRICITY

L. Iorio

Viale Unità di Italia 68 70125 Bari (BA), Italy; lorenzo.iorio@libero.it

Received 2011 May 5; accepted 2011 June 17; published 2011 July 25

ABSTRACT

The subject of this paper is the empirically determined anomalous secular increases of the astronomical unit, of the order of some cm yr\(^{-1}\), and of the eccentricity of the lunar orbit, of the order of 10\(^{-12}\) yr\(^{-1}\). The aim is to find an empirical explanation of both anomalies as far as their orders of magnitude are concerned. The methods employed are working out perturbatively with the Gauss equations the secular effects on the semi-major axis \(a\) and the eccentricity \(e\) of a test particle orbiting a central body acted upon by a small anomalous radial acceleration \(A\) proportional to the radial velocity \(v_r\) of the particle-body relative motion. The results show that non-vanishing secular variations \(\dot{a}\) and \(\dot{e}\) occur. If the magnitude of the coefficient of proportionality of the extra-acceleration is of the same order of magnitude as the Hubble parameter \(H_0 = 7.47 \times 10^{-11}\) yr\(^{-1}\) at the present epoch, they are able to explain both astrometric anomalies without contradicting other existing observational determinations for the Moon and the other planets of the solar system. Finally, it is concluded that the extra-acceleration might be of cosmological origin, provided that the relative radial particle-body motion is accounted for in addition to that due to the cosmological expansion only. Further data analyses should confirm or disprove the existence of both astrometric anomalies as genuine physical phenomena.

Key words: celestial mechanics – ephemerides – gravitation – Moon – planets and satellites: general

1. INTRODUCTION

Recently, the main features of the anomalous secular increases of both the astronomical unit and the eccentricity \(e\) of the lunar orbit have been reviewed (Anderson & Nieto 2010). While the first effect, obtained by several independent researchers (Krasinsky & Brumberg 2004; Standish 2005; Pitjeva & Standish 2005; E. V. Pitjeva 2008, private communication to P. Noerdlinger; Anderson & Nieto 2010), should be of the order of a few cm yr\(^{-1}\), the second one (Williams et al. 2001; Williams & Dickey 2003) amounts to \(\dot{e} = (9 \pm 3) \times 10^{-12}\) yr\(^{-1}\), according to the latest data analysis (Williams & Boggs 2009).

Such phenomena attracted the attention of various scientists dealing with them in different contexts (Ni 2005; Kopeikin 2007; Lämmel & Dittus 2007; Carrera & Giulini 2010; Kopeikin 2010; Li et al. 2010a, 2010b; Goldhaber & Nieto 2010; Sharma 2010; Speliotopoulos 2010; Zhang et al. 2010; Lämmel 2011; Zhang & Kelley 2011). Thus, several more or less sound attempts to find, or to rule out, possible explanations (Krasinsky & Brumberg 2004; Iorio 2005; Mashhoon & Singh 2006; Östvang 2007; Mashhoon et al. 2007; Khokhlov 2007; Noerdlinger et al. 2008; Verbiest et al. 2008; Lämmel 2008; Amin 2009; Arakida 2009; Miura et al. 2009; Ito 2009; Li & Chang 2009; Lämmel 2009; Brumberg 2010; Nyambuya 2010; Bel 2010; Arakida 2010; Anderson & Nieto 2010; Rasko 2010; Iorio 2011; Arakida 2011) for both anomalies have been proposed so far, both in terms of standard known gravitational physical phenomena and of long-range modified models of gravity.

Here, we propose an empirical formula that is able to accommodate both anomalies, at least as far as their orders of magnitude are concerned.

2. ANOMALOUS ACCELERATION PROPORTIONAL TO THE RADIAL VELOCITY OF THE TEST PARTICLE

Let us assume that, in addition to the usual Newtonian inverse-square law for the gravitational acceleration imparted to a test particle by a central body orbited by it, there is also a small radial extra-acceleration of the form

\[
A_{\text{pert}} = k H_0 v_r.
\]

In it \(k\) is a positive numerical parameter of the order of unity to be determined from the observations, \(H_0 = (73.8 \pm 2.4)\) km s\(^{-1}\) Mpc\(^{-1}\) \((7.47 \pm 0.24) \times 10^{-11}\) yr\(^{-1}\) (Riess et al. 2011) is the Hubble parameter at the present epoch, defined in terms of the time-varying cosmological scaling factor \(S(t)\) as \(H_0 = \dot{S}/S_0\), and \(v_r\) is the component of the velocity vector \(v\) of the test particle’s proper motion about the central body along the common radial direction. The radial velocity for a Keplerian ellipse is (Brouwer & Clemence 1961)

\[
v_r = \frac{nae \sin f}{\sqrt{1 - e^2}}.
\]

where \(n\) is the Keplerian mean motion, \(a\) is the semi-major axis, and \(f\) is the true anomaly reckoning the instantaneous position of the test particle along its orbit: \(v_r\) vanishes for circular orbits.

The consequences of Equation (1) on the trajectory of the particle can be straightforwardly worked out with the standard Gauss equations for the variation of the Keplerian orbital elements (Brouwer & Clemence 1961) which are valid for any kind of perturbing acceleration, whatever its physical origin may be. For the semi-major axis and the eccentricity they are

\[
\begin{align*}
\frac{da}{dt} &= \frac{2}{n\sqrt{1 - e^2}} \left[ e A_R \sin f + A_T \left( \frac{p}{r} \right) \right], \\
\frac{de}{dt} &= \frac{\sqrt{1 - e^2}}{na} \left[ A_R \sin f + A_T \left[ \cos f + \frac{1}{e} \left( 1 - \frac{r}{a} \right) \right] \right].
\end{align*}
\]
In Equation (3) $p = a(1 - e^2)$ is the semi-latus rectum, and $A_0$ and $A_T$ are the radial and transverse components of the disturbing acceleration, respectively; in our case, Equation (1) is entirely radial. In a typical first-order perturbative calculation such as the present case, the right-hand sides of Equation (3) have to be computed onto the unperturbed Keplerian ellipse, characterized by

$$r = \frac{p}{1 + e \cos \phi},$$

and integrated over one orbital period by means of

$$dt = \frac{1}{n} \left( \frac{r}{a} \right)^2 \frac{1}{\sqrt{1 - e^2}} df.$$

It turns out that both the semi-major axis $a$ and the eccentricity $e$ of the test particle’s orbit secularly increase according to

$$\langle \dot{a} \rangle = 2kaH_0(1 - \sqrt{1 - e^2}),$$

$$\langle \dot{e} \rangle = kH_0(1 - e^2 e^2 / \sqrt{1 - e^2}).$$

The formulae in Equation (6), which were obtained by taking an average over a full orbital revolution, are exact to all orders in $e$.

Since $\varepsilon_{\text{Moon}} = 0.0647$, it turns out that Equation (6) is able to reproduce the measured anomalous increase of the lunar orbit for $2.5 \lesssim k \lesssim 5$. Moreover, for such values of $k$ Equation (6) yields an increase of the lunar semi-major axis of just $0.3$--$0.6$ mm yr$^{-1}$. It is, at present, undetectable, in agreement with the fact that, actually, no anomalous secular variations pertaining to such an orbital element of the lunar orbit have been detected so far. If we assume the terrestrial semi-major axis $a_0 = 1.5 \times 10^{13}$ cm as an approximate measure of the astronomical unit and consider that $e_{\oplus} = 0.0167$, Equation (6) and the previous values of $k$ yield a secular increase of just a few cm yr$^{-1}$. Also in this case, it can be concluded that Equation (6), if applied to other situations for which accurate data exist, does not yield results in contrast with empirical determinations for $a$ and $e$. Indeed, for the eccentricity of the Earth, Equation (6), with $2.5 \lesssim k \lesssim 5$, yields $\langle \dot{e} \rangle = (1.7$--$3.4) \times 10^{-12}$ yr$^{-1}$. Actually, such an anomalous effect cannot be detectable since, according to Table 3 of Pitjeva (2008), the present-day formal, statistical accuracy in determining $e$ from the observations amounts to just $3.6 \times 10^{-12}$; it is well known that the realistic uncertainty can be up to one order of magnitude larger. Similar considerations hold for the other planets.

3. CONCLUSIONS

Here we do not intend to speculate too much about possible viable physical mechanisms yielding the extra-acceleration of Equation (1).\footnote{\textsuperscript{3}Indeed, it can be easily inferred that Equation (1) is of the order of $10^{-15}$ m s$^{-2}$ for Earth’s motion around the Sun, while its Newtonian solar monopole term is as large as $10^{-3}$ m s$^{-2}$. The same holds for the Earth–Moon system as well. Indeed, Equation (1) yields about $10^{-16}$ m s$^{-2}$ for the lunar geocentric orbit, while the Newtonian monopole acceleration due to the Earth is of the order of $10^{-3}$ m s$^{-2}$.}

It might be argued that, reasoning within a cosmological framework, the Hubble law may give Equation (1) for $k = 1$ if the proper motion of the particle about the central mass is taken into account in addition to its purely cosmological recession which, instead, yields the well-known local\textsuperscript{4} extra-acceleration of tidal type (Cooperstock et al. 1998; Mashhoon et al. 2007; Klioner & Soffel 2005)

$$A_{\text{cosmol}} = -q_0 H_0^2 r,$$

where $q_0 = -(\dot{\bar{H}}/\bar{H}) H_0^{-2}$ is the deceleration parameter at the present epoch.

On the other hand, our empirical results, which are not in contrast with other observational determinations for the Moon and the other planets of the solar system, may be simply interpreted, in a purely phenomenological way, in terms of a radial extra-acceleration proportional to the radial component $v_r$ of the proper velocity of the test particle about its primary through a coefficient having the dimensions of $T^{-1}$ and a magnitude close to that of $H_0$: its physical origin should not necessarily be of cosmological origin.

Finally, we want to say some words about the nature of the anomalies considered. As shown, the anomalous increase of the astronomical unit has attracted the attention of several researchers so far. It was determined as a solve-for parameter by using different ephemerides which neither use the same dynamical force models nor the same observational records. Further processing of more extended data sets, with more accurate dynamical modeling, will be useful in shedding further light on such an anomaly. Anderson & Nieto (2010, p. 194) conclude their review of the anomalous variation of the astronomical unit by writing: “If the reported increase holds up under further scrutiny and additional data analysis, it is indeed anomalous. Meanwhile it is prudent to remain skeptical of any real increase. In our opinion the anomalistic increase lies somewhere in the interval 0--20 cm yr$^{-1}$, with a low probability that the reported increase is a statistical false alarm.” On the other hand, it must be noted that a clear definition for the change of the astronomical unit is still lacking since some researchers believe that the astronomical unit is a redundant unit, like to the gravitational parameter $GM$ of the Sun, which should, instead, be empirically determined from data processing as a solve-for parameter (Klioner 2008; Fienga et al. 2010; Capitaine et al. 2010). It is likely that at the IAU meeting in 2012 a fixed numerical value for it will be adopted.\textsuperscript{7} Anyway, this would have the effect of just shifting the detected anomaly to another physical quantity. Concerning the Moon’s orbit, the first report of the lunar eccentricity anomaly dates back to 2001 (Williams et al. 2001); such a phenomenon is still here (Williams & Dickey 2003; Williams & Boggs 2009), despite the increasing accuracy in lunar laser ranging (LLR) observations and modeling occurred in the last decade due to the steady efforts of a wide community of researchers engaged in LLR science and technology. On the other hand, it is certainly not unreasonable to expect that further modeling of classical effects occurring in the lunar interior may finally be able to explain the observed anomaly. Until it actually happens, looking for alternative explanations remains a task worth being pursued.

\textsuperscript{6} For a recent review of the influence of global cosmological expansion on the local dynamics and kinematics, see Carrera & Giulini (2010).

\textsuperscript{7} W. M. Folkner 2011, private communication.
REFERENCES

Amin, M. Y. 2009, arXiv:0912.2443

Anderson, J. D., & Nieto, M. M. 2010, in Proc. IAU Symp. 261, Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis, ed. S. A. Klioner, P. K. Seidelmann, & M. H. Soffel (Cambridge: Cambridge Univ. Press), 189

Arakida, H. 2009, New Astron., 14, 264

Arakida, H. 2010, Adv. Space Res., 45, 1007

Arakida, H. 2011, Gen. Rel. Grav.

Bel, Ll. 2010, arXiv:1005.5442

Brouwer, D., & Clemence, G. M. 1961, Methods of Celestial Mechanics (New York: Academic)

Brumberg, V. A. 2010, Celest. Mech. Dyn. Astron., 106, 209

Capitaine, N., Guinot, B., & Klioner, S. 2010, Proposal for the Re-definition of the Astronomical Unit of Length Through a Fixed Relation to the SI Metre, Rencontres de l’Observatoire Journées 2010 “Syst`emes de r´ef´erence Spatio-Temporels”, New Challenges for Reference Systems and Numerical Standards in Astronomy, Observatoire de Paris Ecole Normale Sup´erieure, 2010 September, 20–22, 6

Carrera, M., & Giulini, D. 2010, Rev. Mod. Phys., 82, 169

Cooperstock, F. I., Faraoni, V., & Vollick, D. N. 1998, ApJ, 503, 61

Fienga, A., Manche, H., Kuchynka, P., Laskar, J., & Gastineau, M. 2010, arXiv:1011.4419

Goldhaber, A. S., & Nieto, M. M. 2010, Rev. Mod. Phys., 82, 939

Iorio, L. 2005, J. Cosmol. Astropart. Phys., JCAP09(2005)006

Iorio, L. 2011, MNRAS

Ito, Y. 2009, PASJ, 61, 1373

Kholhlov, D. L. 2007, arXiv:0710.5862

Klioner, S. A. 2008, A&A, 478, 951

Klioner, S. A., & Soffel, M. M. 2005, in Proc. Symp. The Three-Dimensional Universe with Gaia, ed. C. Turon, K. S. O’Flaherty, & M. A. C. Perryman (ESA SP-576), 305

Kopeikin, S. M. 2007, in AIP Conf. Proc. 886, New Trends in Astrodynamics and Applications III, ed. E. Belbruno (Melville, NY: AIP), 268

Kopeikin, S. M. 2010, in IAU Symp. 261, Relativity in Fundamental Astronomy: Dynamics, Reference Frames, and Data Analysis, Proceedings of the International Astronomical Union, ed. S. A. Klioner, P. K. Seidelmann, & M. H. Soffel (Cambridge: Cambridge Univ. Press), 7

Krasinsky, G. A., & Brumberg, V. A. 2004, Celest. Mech. Dyn. Astron., 90, 267

Lämmerzahl, C. 2008, Eur. Phys. J. Spec. Top., 163, 255

Lämmerzahl, C. 2009, Space Sci. Rev., 148, 501

Lämmerzahl, C. 2011, in Mass and Motion in General Relativity Fundamental Theories of Physics, Vol. 162, ed. L. Blanchet, A. Spallicci, & B. Whiting (Berlin: Springer), 25

Lämmerzahl, C., & Dittus, H. 2007, Int. J. Mod. Phys. D, 16, 2455

Lämmerzahl, C., Preuss, O., & Dittus, H. 2008, in Lasers, Clocks and Drag-Free Control Exploration of Relativistic Gravity in Space, ed. H. Dittus, C. Lämmerzahl, & S. G. Turyshchev (Astrophysics and Space Science Library, Vol. 349; Berlin: Springer), 75

Li, X., & Chang, Z. 2009, arXiv:0911.1890

Li, X., Chang, Z., & Li, M. 2010a, arXiv:1001.0066

Li, X., Chang, Z., & Mo, X. 2010b, arXiv:1001.2667

Mashhoon, B., Mobed, N., & Singh, D. 2007, Class. Quantum Grav., 24, 5031

Mashhoon, B., & Singh, D. 2006, Phys. Rev. D., 74, 124006

Miura, T., Arakida, H., Kasai, M., & Kuramata, S. 2009, PASJ, 61, 1247

Ni, W.-T. 2005, Int. J. Mod. Phys. D, 14, 901

Noerdlinger, P. 2008, arXiv:0801.3807

Nyambuya, G. G. 2010, MNRAS, 403, 1381

Ostvang, D. 2007, Grav. Cosmol., 13, 1

Pritjeva, E. V. 2008, in Proc. IAU Symp. 248, A Giant Step: from Milli- to Micro-arcsecond Astrometry, ed. W. J. Jin, I. Platais, & M. A. C. Perryman (Cambridge: Cambridge Univ. Press), 20

Pritjeva, E. V., & Standish, E. M. 2005, in Proc. IAU Colloq. 196, Transits of Venus: New Views of the Solar System and Galaxy, ed. D. W. Kurtz (Cambridge: Cambridge Univ. Press), 177

Rasor, N. S. 2010, Phys. Essays, 23, 383

Riess, A. G., Macri, L., Casertano, S., et al. 2011, ApJ, 730, 119

Sharma, B. K. 2010, Earth Moon Planets, 108, 15

Speliotopoulos, A. D. 2010, Gen. Rel. Grav., 42, 1537

Standish, E. M. 2005, in Proc. IAU Colloq. 196, Transits of Venus: New Views of the Solar System and Galaxy, ed. D. W. Kurtz (Cambridge: Cambridge Univ. Press), 163

Verbiest, J. P. W., Bailes, M., van Straten, W., et al. 2008, ApJ, 679, 675

Williams, J. G., & Boggs, D. H. 2009, in Proc. 16th International Workshop on Laser Ranging, Lunar Core and Mantle, SLR—The Next Generation, ed. S. Schilliak, http://cddis.gsfc.nasa.gov/lw16/docs/papers/sci_1_Williams_p.pdf

Williams, J. G., Boggs, D. H., Yoder, C. F., Ratcliff, J. T., & Dickey, J. O. 2001, J. Geophys. Res., 106, 27933

Williams, J. G., & Dickey, J. O. 2003, in Proc. 13th International Workshop on Laser Ranging, Toward Millimeter Accuracy, ed. R. Noomen, S. Klosko, C. Noll, & M. Pearlman, (NASA/CP-2003-212248), 75, http://cddis.nasa.gov/lw13/docs/papers/sci_williams_1m.pdf

Zhang, W. J., & Kelley, N. 2011, Adv. Sci. Lett., 4, 574

Zhang, W. J., Li, Z. B., & Lei, Y. 2010, Chin. Sci. Bull., 55, 4010