Mathematical simulation of heat transfer for atmospheric air cooling of heat exchange tube

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Abstract. The laboratory facility with horizontal, atmospheric air cooled heat exchange tube has been engineered, manufactured and put to trial. The test has proven the reliability of the employed analysis methods and hardware. Mathematical simulation of horizontal tube heat transfer process has been fulfilled with the use of ideal displacement and mixing, cellular and longitudinal diffusion models. The in-tube hot water temperature pattern has been evaluated according to the aforementioned models. The results comparison has been fulfilled for experimental determination of in-tube water temperature differential and calculation data in accordance with given models. The longitudinal diffusion model has proven itself as the most suitable for describing the heat transfer process in a horizontal, atmospheric air cooled tube. The longitudinal diffusion coefficient value has been deducted from experiments. The research data can be utilized during the development of tubular heat exchangers.

1. Introduction.
A liquid flow in a heat exchange tube has a complex nature: chaotic curls appear in the turbulent stream; the speed of a liquid is lower at the tube walls compared to the speed at the stream core; reverse flows are tending to appear. Thereby, the thorough mathematical formulation of the real stream appears to be very difficult to make [1]. Hence the researchers often replace the truthful process with a simplified model omitting the effects which complicate the process description, but do not have a severe influence on calculation data [2-5]. The following models are used to describe the in-tube heat exchange process: the models of ideal displacement, ideal diffusion, cellular and longitudinal diffusion models [6-9]. The purpose of this study is to evaluate which of the models presented is the most valid for describing the heat transfer process in the horizontal, gravity-type air cooled tube. For this purpose, the experiments using the laboratory facility were conducted, the water temperature differential inside the heat-exchanging tube has been evaluated and the analysis of this temperature differential was made in accordance with various mathematical models. The comparison of analyzed and experimental temperature differentials will indicate which mathematical model is the most valid in this particular case.

2. The laboratory facility tests.
The layout of the laboratory facility utilized in the experiment is shown in figure 1. The facility consists of the horizontal heat exchange tube (1), 40 liters water holding container (2), receiving container (3), valve (4), electric heater (5), supporting piers (6), joint hoses, flow meter (7),
temperature gauges (8-10). Water level difference in the containers (2) and (3) is 1.8m. The horizontal tube elevation above floor level is 1m. The hot water flow is fed under gravity through the heat exchange tube from container (2) to container (3). The heat exchange tube is cooled externally by an atmospheric air under gravity convection.

![Diagram of the laboratory facility](image)

**Figure 1.** The laboratory facility

The laboratory facility was tested on a first priority. For this purpose, the coefficient of heat transfer from hot water to atmospheric air through the tube wall has been evaluated and compared with heat transfer coefficient which was calculated using common acquainted equations. The experimental evaluation of heat transfer coefficient of the horizontal, gravity-type air cooled tube is made under the following conditions: steady state mode; water flow quantity is 26.3 kg/h; initial water temperature in tube $t_1=92.2 \, ^\circ\text{C}$; final water temperature in tube $t_2=90.4 \, ^\circ\text{C}$; tube length $l=1.5$ m; tube diameter $d=0.033$ m; ambient temperature $t_a=26 \, ^\circ\text{C}$.

The experimental average heat transfer coefficient (W/(m²-deg)) over the tube length is calculated using the equation:

$$K = \frac{Q}{F \cdot \Delta t_{avg}},$$

where $Q$ is the heat current (load) (J/sec); $F$ is the heat transfer area (m²); $\Delta t_{avg}$ is the average driving force for the heat transfer process ($^\circ$C). The experimental heat transfer coefficient equaled to $K=5.27$ W/(m²-deg).

To ensure the experiments actuality the heat transfer coefficient has been calculated under the same conditions using the acquainted equations from studies [10-11]:

$$Nu_L = 1.55 \cdot \left(Pe_L \frac{d}{T}\right)^{1/3},$$

where $Nu_L$ is the Nusselt number for water; $Pe_L$ is the Peclet number for water;

$$Nu_a = 0.5 \cdot (Gr_a \cdot Pr_a)^{0.25},$$

where $Nu_a$ is the Nusselt number for air under gravity convection; $Pr_a$ is the Prandtl number for air; $Gr_a$ is the Grashof number for air;
\[ \alpha_{L,a} = \frac{Nu_{L,a} \cdot \lambda_{L,a}}{d}, \]

where \( \alpha_{L,a} \) is the heat transmittance coefficients of water and air, respectively, (W/(m\(^2\)-deg));

\[ K_p = \frac{1}{\frac{1}{\alpha_L} + \frac{1}{\alpha_a} + \frac{\delta_c}{\lambda_c}}, \]

where \( \delta_c \) is the wall thickness, m; \( \lambda_c \) is the heat conductivity coefficient of the wall, W/(m\(^2\)-deg); \( K_p \) is the calculated heat transfer coefficient (W/(m\(^2\)-deg)). The calculated heat transfer coefficient equaled to \( K_p = 4.82 \) W/(m\(^2\)-deg).

The deviation between the experimental and calculated coefficients is:

\[ \Delta\% = \frac{K - K_p \cdot 100\%}{K_p} = \frac{5.27 - 4.82}{5.27} = 8.54\% \]

The calculated heat transfer coefficient is 8.54% lower than the experimental one. Thus, the experimental heat transfer coefficient value has noncritical deviation from the theoretical coefficient value which makes it possible to understand the reliability of employed analysis methods and hardware, as well as the veracity of the results of experiments.

3. Mathematical simulation and the results of experiments.

The next stage of the study is to evaluate which type of the mathematical model is the most valid for describing the heat transfer process in the horizontal tube with the external gravity-type air cooling. For that purpose, let’s calculate the water temperature at the tube downstream using four types of mathematical models: the models of the ideal displacement, ideal diffusion, cellular and longitudinal diffusion models. The most valid model will be the one which provides the closest calculated temperature value to that observed by utilizing the laboratory facility.

The following assumptions are at the core of the ideal displacement model: a liquid flow in a tube resembles a forcer, i.e. the liquid temperature and the flow speed is constant in the cross-section of a tube, the liquid temperature changes only longitudinally, the longitudinal diffusion is absent.

The input values for the calculation in compliance with the ideal displacement are: initial water temperature \( t_1, ^\circ\text{C} \); atmospheric air temperature \( t_a, ^\circ\text{C} \); tube length \( l, \text{m} \); tube diameter \( d, \text{m} \); volumetric water discharge \( V_L, \text{m}^3/\text{sec} \); water density \( \rho, \text{kg/m}^3 \); heat capacity of water \( c_p, \text{J/(kg-deg)} \); heat transfer coefficient \( K, \text{W/(m}^2\text{-deg)} \); heat transfer area \( F, \text{m}^2 \).

The differential equation of the ideal displacement model is written as:

\[ \frac{dT_L}{dx} = \frac{2 \cdot K \cdot (t_L - t_a)}{w \cdot r \cdot \rho \cdot c_p}, \]

where \( w \) is the liquid speed in tube, m/sec; \( r \) is the internal tube radius, m; \( dt_L \) is the water temperature differential in an elementary segment of a tube, \(^\circ\text{C} \); \( dx \) is the length of an infinitely small tube segment, m;

The solution to this equation is the expression:

\[ t_L = t_a + (t_1 - t_a) \cdot e^{-\frac{x}{d}}, \]

where \( x \) is the distance between the given tube section area and the tube nose, m; \( t_L \) is the water temperature at the given tube section area, \(^\circ\text{C} \); and
The equation (8) allows to evaluate the water temperature at any section area of the tube including the exit section area in compliance with the ideal displacement model. Using the equation (8) the final temperature of water in the tube is calculated under conditions of the aforementioned experiment: initial water temperature $t_1=92.2^\circ$C; atmospheric air temperature $t_a=26$ °C; tube length $l=1.5$m; tube diameter $d=0.033$ m; volumetric water discharge $V_L=7.5315\times10^{-6}$ m$^3$/sec; water density $\rho=970$ kg/m$^3$; heat capacity of water $c_p=4190$ J/(kg-deg); heat transfer coefficient $K=4.82$W/(m$^2$-deg). In accordance with the ideal displacement model the water temperature at the tube downstream should be $t_2=90.58$ °C, i.e. in this terms the water temperature differential along the tube length should be 1.62 °C. The observed temperature equaled 90.4 °C and the temperature differential equaled 1.80 °C.

Figure 2. Water temperature distribution along the tube length in accordance with mathematical models: 1 - ideal displacement model; 2 - ideal diffusion model; 3 - cellular model; 4 - longitudinal diffusion model

Let’s evaluate the deviation between calculated using the ideal displacement model and experimentally observed temperature differential values.

$$\Delta\% = \frac{1.80-1.62}{1.80} \times 100\% = 10.0\%$$

The deviation is 10.0%. The calculation data for hot water temperature differential along the length of the gravity-type air cooled tube in terms of the ideal displacement model are shown in figure 2 (line 1). Figure 2 shows that under the existing conditions and in accordance with the ideal displacement model the water temperature distribution along the tube length is linear.

Let’s calculate the water temperature differential in the tube using the ideal diffusion model. The ideal diffusion model assumes that there is such an effective diffusion in the tube, that the temperature of liquid is constant along the length of the tube and equals to the final temperature. That means that when liquid enters the tube its temperature rises spasmodically to the final temperature value. That is possible if material particles entering the tube are distributed momentarily and evenly throughout the tube. Temperature transfer and heat balance equations are accounted for setting up the mathematical description of the model. The equation of the ideal diffusion model is written as:
The calculation for the final temperature of water in the tube is made in accordance with the ideal diffusion model for the same conditions. The water temperature at the tube downstream equaled 90.60 °C. Thus, in accordance with the ideal diffusion model the temperature differential of water in the tube should be 1.60 °C. Let’s evaluate the deviation between calculated using the ideal diffusion model and experimentally observed temperature differential values:

$$\Delta \% = \frac{1.8 - 1.6}{1.8} \cdot 100\% = 11.1\%$$  \(12\)

The deviation is 11.1%. The water temperature distribution along the tube length is shown in figure 2 (line 2). The diagram shows a horizontal line because the water temperature at each point of the tube is identical and equals to the final water temperature.

According to the cellular model, the whole tube is being conceptually split into some number of cells considering that each cell has the ideal diffusion in it. The water temperature is constant along the cell, rising spasmodically between cells. When the number of cells \(n\) goes to infinity the ideal displacement model is valid, when the number of cells \(n=1\) the valid model is ideal diffusion.

The temperature in each cell can be calculated using the equation similar to the ideal diffusion equation. The equation is composed for each cell; in generic form it is written as:

$$t_L = \frac{l_i \cdot K \cdot F \cdot t_a + w \cdot V \cdot \rho \cdot c_p \cdot t_l}{w \cdot V \cdot \rho \cdot c_p + l_i \cdot K \cdot F}.$$  \(11\)

The calculation data for hot water temperature differential along the length of the gravity-type air cooled tube in terms of the cellular model of ideal displacement with the number of cells \(n=4\) are shown in figure 2 (line 3). The single cell length is 0.0375m. The line 3 in figure 2 joins centers of cells.

The calculation shows that in accordance with the cellular model the final water temperature should equal 90.59 °C. Thus, according to the cellular model the temperature differential of water in the tube where \(n=4\) should be 1.61 °C. Let’s evaluate the deviation between calculated using the cellular model and experimentally observed temperature differential values:

$$\Delta \% = \frac{1.80 - 1.61}{1.80} \cdot 100\% = 10.56\%$$  \(14\)

The deviation is 10.56%. In terms of the longitudinal diffusion model it is assumed that a liquid flows across a tube under conditions of the ideal displacement where both downstream and upstream longitudinal diffusion occur. It is assumed that the longitudinal diffusion can be described by equations similar to those of molecular diffusion. The longitudinal heat transfer intensity is characterized by the longitudinal diffusion coefficient \(D\) which is similar to the molecular diffusion coefficient by physical meaning and dimension. If \(D\) goes to infinity then the flow approaches the ideal diffusion flow, if \(D\) tends to zero then the flow the ideal displacement flow.

The higher is the intensity of upstream diffusion, the bigger is coefficient \(D\). It indicates the quantity of heat being transmitted in a tube in a direction opposite to the liquid flow with temperature differential of 1 degree per 1 meter in 1 second through the flow cross-section of 1m². The coefficient \(D\) is evaluated experimentally.
Let’s calculate the hot water temperature distribution along the air cooled tube in terms of the longitudinal diffusion model and evaluate the longitudinal diffusion coefficient $D$. We will inspect the tube element section with the length of $dx$ from the perspective of the longitudinal diffusion model. The differential equation of the longitudinal diffusion model for a horizontal, gravity-type air cooled tube is written as:

$$- \frac{D}{c_p} \frac{d^2 t_L}{dx^2} + w \frac{dt_L}{dx} - \frac{K \cdot 2 \cdot \pi \cdot r}{S \cdot c_p} (t_L - t_a) = 0.$$  \hspace{1cm} (15)

The equation (15) is second-order differential equation. By solving the equation we shall find the temperature distribution of the liquid $t_L$ along the tube $x$, i.e. the function formula: $t_L = f(x)$. The solution to the equation (15) is the dependence:

$$t_L = C_1 \cdot e^{k_1 x} + C_2 \cdot e^{k_2 x} + t_a,$$  \hspace{1cm} (16)

where $t_a$ is the air temperature, $C_1, C_2$ are the constants and

$$k_1 = \frac{wc_p}{D} + \frac{\left(\frac{wc_p}{D}\right)^2 + 4 \frac{Hc_p}{D}}{2},$$  \hspace{1cm} (17)

$$k_2 = \frac{wc_p}{D} - \frac{\left(\frac{wc_p}{D}\right)^2 + 4 \frac{Hc_p}{D}}{2},$$  \hspace{1cm} (18)

$$H = \frac{2 \cdot K \cdot \pi \cdot r}{S \cdot c_p},$$  \hspace{1cm} (19)

where $S$ is the sectional area of the tube, $m^2$.

Let’s determine values of constants $C_1$ and $C_2$ from boundary conditions. Boundary conditions are the conditions at the tube nose and at the tube end. It is known that at the tube nose, at $x=0$ $t_L = t_1$, at $x=l$ ($l$ is the tube length) the temperature stops changing, ergo the temperature derivative with respect to length is zero. Boundary conditions are written as:

$$x = 0 \quad t_L = t_1$$
$$x = l \quad \frac{dt_L}{dx} = 0$$

Let’s find the derivative of the right part of the equation (16):

$$\frac{dt_L}{dx} = C_1 \cdot k_1 \cdot e^{k_1 x} + C_2 \cdot k_2 \cdot e^{k_2 x} = 0$$  \hspace{1cm} (20)

If $x=0$ then in accordance with the equation (16):

$$t_1 = C_1 + C_2 + t_a$$  \hspace{1cm} (21)

If $x=l$ then in accordance with the equation (20):

$$C_1 \cdot k_1 \cdot e^{k_1 l} + C_2 \cdot k_2 \cdot e^{k_2 l} = 0$$  \hspace{1cm} (22)

The set of equations (21) and (22) contain two indeterminates.
\[ C_1 = t_1 - C_2 - t_a \]  
\[ C_2 = \frac{t_a \cdot k_1 \cdot e^{k_1l} - t_1 \cdot k_1 \cdot e^{k_1l}}{k_t e^{k_2l} - k_t e^{k_1l}} \]  
(24)

The calculation data for hot water temperature differential along the length of the gravity-type air cooled tube in terms of the longitudinal diffusion model by the equation (16) are shown in figure 2 (line 4). Figure 2 shows that under the existing conditions and in accordance with the longitudinal diffusion model the temperature distribution along the tube length is curvilinear.

The longitudinal diffusion coefficient \( D \) is determined by trial-and-error method and equals: \( D = 22440 \) m²/sec. The water temperature at the tube end calculated in accordance with the longitudinal diffusion model corresponds strictly to the observed temperature of 90.40 °C.

4. Conclusion.
As can be seen from above, comparing the experimental results to calculation data has indicated that the longitudinal diffusion model describes the temperature distribution in the heat exchanger tube most closely, compared to the model of ideal displacement, the cellular model or the ideal diffusion model. The results of the study (the longitudinal diffusion coefficient \( D \) value, conclusions on mathematical models sufficiency) can be utilized during the research and development of tubular heat exchangers.

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