The temporal rich club phenomenon

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Identifying the hidden organizational principles and relevant structures of complex networks is fundamental to understand their properties. To this end, uncovering the structures involving the prominent nodes in a network is an effective approach. In temporal networks, the simultaneity of connections is crucial for temporally stable structures to arise. Here, we propose a measure to quantitatively investigate the tendency of well-connected nodes to form simultaneous and stable structures in a temporal network. We refer to this tendency as the temporal rich club phenomenon, characterized by a coefficient defined as the maximal value of the density of links between nodes with a minimal required degree, which remain stable for a certain duration. We illustrate the use of this concept by analysing diverse data sets and their temporal properties, from the role of cohesive structures in relation to processes unfolding on top of the network to the study of specific moments of interest in the evolution of the network.

Many natural, technological and social systems can be represented as networks of agents (nodes) and their interactions (edges), including communication systems, transportation infrastructures, biological and ecological systems and social interactions. Such representation offers a common framework to analyse these systems, link their structure and dynamics and investigate processes on top of them. In particular, one challenge consists in identifying relevant network structures, and several approaches have been put forward to characterize networked data sets and their central elements, such as hubs (nodes with very large numbers of connections) or a central core of well-connected nodes (core–periphery structure). The k-core decomposition moreover decomposes the network into subgraphs of increasing connectedness, and increasing influence in spreading processes, while the rich club coefficient quantifies whether the hubs tend to form more tightly interconnected groups.

While these approaches concern static networks, many data sets include temporal information about edges. Static networks are often aggregated representations of the resulting temporal networks. Thus, any structure found in such static networks could in fact be formed by edges that were active at unrelated times. For instance, a static hub might have different numbers of neighbours and different neighbours at different times. Temporal motifs can be defined as the repetition over time of simultaneous subgraphs or of the connections in a temporal subgraph in a given order. Well-connected structures such as cores need to be defined on specific time intervals.

Structures and hierarchies in temporal networks thus need to be defined taking into account (1) the temporality and the set of edges simultaneously active at a given time (the network at each timestamp), represented as a series of instantaneous snapshots of the network at each timestamp (Fig. 1a and 1b); and (2) the time span on which the structure exists. Here, we propose a new way to investigate the cohesion of increasingly central nodes in a temporal network: the temporal rich club (TRC) coefficient. Given a temporal network, our aim is to quantify whether nodes that interact with increasing numbers of other nodes (that is, with increasing degree in the aggregate network) tend to interact with each other simultaneously and in a stable way. We first define the Δ-cohesion of a group of nodes at time t as the density of links persistently connecting the group’s nodes during [t, t + Δt]. We then consider groups of nodes of increasing degree in the aggregated network, and measure the maximum value of their Δ-cohesion over time. This quantifies whether these groups are tightly and simultaneously interconnected for a duration Δ.

Moreover, a natural question is whether the simultaneous connections between high-degree nodes could exist just by chance, so we compare the result with an adequate null model for temporal networks. To show the interest of this new analysis tool, we consider empirical temporal networks representing very different systems: an air transportation infrastructure, social interaction networks and a network of neurons exchanging information. By computing the TRC coefficient for the data and their respective null model, we highlight how it unveils interesting properties of the data. The TRC coefficient provides a new item in the toolbox for the analysis of temporal networks, complementary to other structures such as stable or unstable hubs, dynamic motifs or span-cores.

Results

The TRC. We consider a temporal network in discrete time on a time interval [1, T], represented as a series of instantaneous snapshots of the network at each timestamp (Fig. 1a and 1b). We denote by temporal edges the interactions between pairs of nodes in each snapshot. The temporal aggregation over [1, T] yields a static network G = (V, E) with a set of nodes V and a set of edges E (Fig. 1b), in which an edge is drawn between two nodes i and j if they have at least one shared temporal edge, with a weight wij given by the number of temporal edges between i and j. The degree k of a node in G is the number of distinct other nodes with which it has interacted at least once, and its strength s is the total number of temporal edges it has participated in.

Our goal is to quantify a TRC effect, that is, whether nodes of increasing degree in G tend to be more connected than by chance simultaneously and for a certain duration. We recall that the rich club coefficient was defined for a static network as the density of edges in the subset SΔ of the NΔ nodes with degree larger than k (refs. 16,17,21), that is, \( \phi(k) = \frac{2E_{\Delta k}}{N_{\Delta k}(N_{\Delta k} - 1)} \), where \( E_{\Delta k} \) is the number of edges among \( S_{\Delta k} \). An increasing \( \phi(k) \) indicates that nodes of...
larger degree tend to form increasingly connected groups of nodes (the ‘rich club’ effect)\(^1\),\(^2\),\(^3\), while \(\phi(k)\) can also collapse to 0 at large \(k\), for instance in very disassortative networks\(^4\). To take into account temporality, we define at each time \(t\) the \(\Delta\)-cohesion \(\epsilon_{\Delta}(t,\Delta)\). It is the number \(|E_{\Delta}(t,\Delta)|\) of ties \(E_{\Delta}(t,\Delta)\) (between the nodes of \(S_{\Delta}\)) that remain stable over \([t, t + \Delta]\) (Fig. 1c), normalized by its maximal possible value \(N_{\Delta}(N_{\Delta} - 1)/2\). Note that \(\epsilon_{\Delta}(t,\Delta = 1)\) is the instantaneous density between the nodes of \(S_{\Delta}\) that is, a kind of instantaneous static rich club coefficient calculated with the same ranking of nodes in all snapshots (given by the aggregated degree), which is different from the rich club coefficient of the instantaneous snapshot at time \(t\) computed using the instantaneous values of the degree, as the latter can fluctuate\(^5\). We then define the TRC coefficient as the maximal density of temporal edges observed in a stable way for a duration \(\Delta\) among nodes of aggregated degree larger than \(k\):

\[
M(k, \Delta) = \max_{\Delta} \epsilon_{\Delta}(t, \Delta).
\]

\(M(k,\Delta)\) quantifies (1) whether the static rich club patterns correspond to a structure that actually existed at some instant, (2) how dense and stable such structure was or (3) whether the static rich club is formed by links that appeared at unrelated times and not in a simultaneous way. A \(M(k,\Delta)\) increasing with \(k\) denotes that the most connected nodes tend to be increasingly connected with each other in a simultaneous way for a duration of at least \(\Delta\). This is a different requirement from distinguishing stable and unstable hubs\(^6\),\(^7\), even if a stable hub would statistically tend to contribute to such connectedness. Indeed, \(M(k,\Delta)\) focuses on the links between hubs. A hub might have a consistently large instantaneous degree, but towards small-degree nodes or towards changing neighbours, while another might have a fluctuating degree but maintain some links towards other high-degree nodes, hence contributing to a TRC. We compare \(M(k,\Delta)\) with the value \(M_{\text{ran}}(k,\Delta)\) obtained in a suitable null model of the temporal network. Thus, \(\mu(k,\Delta) \equiv M(k,\Delta)/M_{\text{ran}}(k,\Delta) > 1\) indicates that the nodes of degree larger than \(k\) are more connected simultaneously on at least one time interval of duration \(\Delta\) than expected by chance, denoting a TRC ordering. Although there is a large variety of null models for temporal networks\(^8\),\(^9\), we focus here on the simultaneity of connections and consider a randomization procedure that preserves the number of temporal edges at each time and the degree of each node and weight of each link (the number of snapshots in which the link is active) in the aggregated graph. We thus consider the list of all temporal edges, in the form \((i_{pq}, j_{pq}, t_{pq})\), denoting an interaction between nodes \(i_{pq}\) and \(j_{pq}\) at time \(t_{pq}\) and we randomly permute the timestamps \(t_{pq}\) of all temporal edges. The resulting aggregated structure and activity timeline are the same as in the original data. Differences in the cohesion between the data and the null model thus differentiate between the simultaneity
of links due purely to, for example, bursts of activity or to more meaningful structures.

Furthermore, the time evolution of the Δ-cohesion \( e_{\Delta}(t, \Delta) \) indicates the moments of highest simultaneous connectivity of \( S_\Delta \), and whether this cohesion is stable or fluctuates (just as single nodes can have stable or fluctuating, high degree\(^3\)). This quantity can be shown, as in Fig. 1d,e, as a colour map of \( e_{\Delta}(t, \Delta) \) versus \( t \) and \( \Delta \) at fixed \( k \) (or \( t \) and \( \Delta \) at fixed \( k \)), or as curves of \( e_{\Delta}(t, \Delta) \) versus \( t \) at fixed \( \Delta \) and \( k \). This allows one to distinguish between stable or recurrent and transient rich club effects. In the former case, \( e_{\Delta}(t, \Delta) \) reaches its maximum \( M(k, \Delta) \) repeatedly, or remains close to it, while in the latter, \( M(k, \Delta) \) is reached only once or at specific moments. Comparing \( e_{\Delta}(t, \Delta) \) for the data with the null model reveals also which temporal patterns cannot be simply explained by, for example, periods of higher activity.

**Static rich clubs versus TRCs.** We first apply our measure to a data set describing the US air transportation infrastructure from 2012 to 2020, with temporal resolution of 1 month, for 105 snapshots (Methods and Table 1). In this temporal network, the \( N=1,920 \) nodes represent airports, and a temporal edge in one snapshot represents the existence of a direct connection in the corresponding month. We show in Supplementary Note 1 that the degree distributions of the aggregated network and of several snapshots are broad and stable across months, despite fluctuations in the nodes’ instantaneous degrees\(^3\).

Figure 2a shows the \( k-\Delta \) diagram of the TRC coefficient \( M(k, \Delta) \) as a colour plot. \( M(k, \Delta) \) is small for small and intermediate \( k \) but decreases rapidly as \( \Delta \) increases. Many small airports have fluctuating activity, sometimes seasonal, so that many temporal edges involving these airports are not very stable\(^3\), leading to a small cohesion at the global level. The maximal cohesion, however, increases with \( k \). Airports with more connections tend to be more interconnected and with increasingly stable connections\(^3\). \( M(k, \Delta) \) reaches very large values around \( k \approx 315 \), even at large \( \Delta \), indicating a stable and very cohesive structure. In fact, most of the \( 51 \) airports in \( S_{310} \), are hubs of the US air transportation system, which are largely interconnected with very stable connections. We note that: (1) the cohesion reaches 1 for \( \Delta = 1 \) (there exists at least one month in which these nodes are all simultaneously interconnected) but (2) the cohesion remains lower than 1 for \( \Delta > 1 \) (not all these connections are stable). For higher values of \( k \), \( M(k, \Delta) \) decreases again, especially at large \( \Delta \), with a final increase close to the maximum possible value of \( k \) (such that \( |S_{310}| \geq 2 \)). This pattern indicates that, when restricting to \( k > 380-390 \), the interconnections of the nodes of \( S_{310} \) are less simultaneous and stable than in \( S_{310} \). Some airports with degree larger than \( 380-390 \) have less stable connections than others with degree \( 315 < k < 380 \), even if their instantaneous degrees remain stable (Fig. 2d). Note that this counter-intuitive behaviour of nodes with very high degree (showing less stable connections than nodes with slightly less high degree, as seen also in Fig. 1d,e) cannot be deduced from the simply increasing behaviour of the static rich club coefficient (Fig. 2c).

We further investigate this behaviour in Fig. 2b,d. Figure 2b shows the 20 airports with largest aggregated degree (from 350 to 498). We highlight in red the airports that are also among the 20 nodes with largest aggregated strength. While the red nodes are well-known hubs, the other nodes include airports such as Burbank-Hollywood (BUR) or Westchester County Airport (HPN). These airports serve as reliever airports for hubs such as Los Angeles (LAX) and New York (JFK), respectively. They are extremely well connected in the aggregated network but have fluctuating connections, depending on the needs of the neighbouring hubs. Figure 2d highlights the differences between the two types of nodes, that is, the ‘real’ hubs and the reliever airports. On the one hand, both hubs and relievers have rather stable values of their instantaneous degree \( k(t) \). They are not unstable hubs\(^3\), that is, with a fluctuating instantaneous degree. On the other hand, the bottom panel displays the Jaccard index between the connections of O’Hare International Airport (ORD) and HPN in successive months. ORD has a very stable neighbourhood, while HPN, despite having the largest aggregated degree value, undergoes changes of up to 80% of its neighbourhood from a month to the next. It is thus the dynamics of their neighbourhoods that identifies reliever airports, rather than the evolution of their instantaneous degree.

Figure 2a (bottom) displays the maximal cohesion \( M_{\text{ran}}(k, \Delta) \) for the randomized version of the data. \( M_{\text{ran}}(k, \Delta) \) shows similar patterns but smaller values than \( M(k, \Delta) \) for all \( (k, \Delta) \), showing that a TRC ordering is present. For any \( S_{310} \), the interactions tend to be more simultaneously cohesive than expected by chance. In more detail, the ratio \( \mu(k, \Delta) \) (Fig. 2c) is above \( 1 \) and almost constant over a large range of \( k \) values, and decreases for \( 320 \leq k \leq 380 \). In this range of \( k \) values, \( S_{310} \) is a mix of hubs and reliever airports, with both very stable and other much less stable connections. The randomization does not perturb the most stable connections, so that \( M \) and \( M_{\text{ran}} \) are closer. For the largest aggregated degree values, \( \mu(k, \Delta) \) reaches again very large values. Many of the remaining connections are to reliever airports (more than 50% of the edges between the nodes of \( S_{310} \), for \( k > 350 \), and even if these connections are not necessarily very stable nor simultaneous, they are more so than by chance.

We also show in Supplementary Note 1 the analysis of the data set if we merge each reliever we have identified with its corresponding hub. The TRC coefficient is then simply an increasing function of \( k \) at fixed \( \Delta \), with \( M(k, \Delta) \) close to 1 for all \( k > 310 \) and all \( \Delta \), and the large values of \( \mu(k, \Delta) \) at large \( k \) are suppressed. The patterns of Fig. 2 are thus due to the co-existence of hubs and relievers, whose different nature could not simply be inferred from the static rich club coefficient nor by the fluctuation of their instantaneous degrees. We note indeed that the static rich clubs of the two cases (with or without merging of hubs and relievers) are very similar. However, the temporal patterns are very different, especially for the very high degrees. When hubs and relievers are merged, \( e_{\Delta}(t, \Delta) \) remains close to 1 for all \( k > 310 \) and all \( \Delta \), indicating an extremely stable, densely connected structure present at all times, differently from the original data where, for \( k=410 \), it is possible to identify the co-existence between (1) a very stable structure with density \( \sim 0.3 \) and (2) links that allow a larger cohesion to be reached but only at specific moments in time and for limited durations. Overall, the analysis of the US air transportation network under the lens of the TRC has thus shed light on the different roles of well-connected nodes, highlights how temporal and static rich club can co-exist albeit with different patterns and how a given static rich club pattern can correspond to very different TRC dynamics.

**The TRC and spreading processes.** The second data set we consider is a temporal network of face-to-face interactions between

### Table 1 | Some properties of the data sets

| Data                        | \( N \) | \( k_{\text{min}} \) | \( k_{\text{max}} \) | \( k \) | No. of temporal edges | Time resolution, \( T \) |
|-----------------------------|--------|---------------------|---------------------|-------|-----------------------|--------------------------|
| US airways                  | 1,920  | 1                   | 498                 | 44    | 1,286,616             | \( t = 1 \) month, \( T = 105 \) |
| Primary school              | 242    | 1                   | 98                  | 49    | 53,056                | \( t = 5 \) min, \( T = 103 \) |
| Information-sharing network | 67     | 14                  | 66                  | 53    | 511,174               | \( t = 1 \) s, \( T = 2,284 \) |

Data | \( N \) | \( k_{\text{min}} \) | \( k_{\text{max}} \) | \( k \) | No. of temporal edges | Time resolution, \( T \) |
232 students and 10 teachers of a primary school in France\textsuperscript{12}. Each temporal edge between two nodes correspond to a face-to-face interaction between the two corresponding individuals, detected by wearable sensors\textsuperscript{11,12,25,31} (Methods). The time resolution of the data is 20 s, and to smoothen the short-time noisy dynamics, we perform a temporal coarse graining on successive time windows of 5 min. We consider in the main text the first school day only (duration \(T = 103\) timestamps, Table 1). Results for the whole data set and for a finer temporal resolution are shown in Supplementary Note 2. We discuss in Supplementary Note 3 other data sets describing face-to-face interactions in other contexts.

Figure 3a displays the \(k-\Delta\) diagrams of \(M(k,\Delta)\) for the original temporal network and its randomized version. At fixed \(\Delta\), \(M(k,\Delta)\) tends to increase with \(k\). Moreover, \(M(k,\Delta)\) decreases more slowly with \(\Delta\) when \(k\) increases. Nodes with higher degree in the aggregated network tend to be more tightly interconnected, and in a more stable way. While the static rich club coefficient \(\phi(k)\) increases with \(k\) (Fig. 3c), as with other social networks\textsuperscript{11,12,25,31}, this gives an additional insight into the social dynamics of the school. The fact that children with a larger diversity of contacts tend to be more interconnected is not only due to contacts occurring at unrelated times, but a cohesive structure between the high-degree nodes actually took place in a simultaneous way.

For instance, the seven nodes of \(S_{st}\) reach a maximal cohesion \(M(k,\Delta)\approx0.28\) at \(\Delta = 1\) and have some long-lasting stable contacts \((M(k,\Delta) \geq 0.09\) up to \(\Delta = 25)\). The students of \(S_{st}\) actually belong to different classes. The static rich club could thus a priori be due to random contacts occurring at unrelated times between them, but the value of \(M(k,\Delta)\) shows that a part of the structure found statically is indeed found as simultaneous links at least once. However, the instantaneous cohesion remains lower than the static rich club coefficient. Only a fraction of the links of the aggregated network are present simultaneously at any instant.

The temporal structures disappear in the randomized version of the temporal network, with much lower cohesion values on the whole \(k-\Delta\) domain, indicating a TRC ordering. This is confirmed in Fig. 3b through the temporal evolution of the cohesion \(c_{st}(t,\Delta)\). For the original data, the simultaneous cohesion of these nodes fluctuates strongly at small \(\Delta\), is 0 in many snapshots and reaches its maximum in the periods of high overall activity (namely breaktime and lunch break\textsuperscript{11}), forming a transient but repeated TRC (just as single nodes can repeatedly have large numbers of neighbours\textsuperscript{12}). Note that, while the analysis of static rich clubs on each snapshot could reveal that cohesive structures between these nodes appear repeatedly, it would not allow to investigate their stability. We show for instance in Supplementary Note 4 the cohesion diagrams as a function of time and degree, for various values of \(\Delta\). \(c_{st}(t,\Delta = 1)\) reaches \(\approx 0.25\) in several periods, but a same density of 0.25 in successive times could correspond to completely different links. This is actually the case in the reshuffled data. \(c_{st}(t,\Delta > 1)\) remains larger than zero for \(\Delta \geq 5\) only for the original data and during the lunch break. In other words, patterns of apparent stability seen in instantaneous
Fig. 3 | Primary school temporal network. a, Top: the size $|S_\Delta|$ of the sub-network of nodes of aggregate degree larger than $k$ as a function of $k$ for the primary school temporal network. Middle: the maximal cohesion $M(k,\Delta)$ as a function of $k$ and $\Delta$. Bottom: the $M_{\text{avg}}(k,\Delta)$ diagram of the randomization-preserving aggregate node statistics and overall activity timeline. b, Top: the activity timeline of the network, that is, the number of temporal edges at each time step. Middle: the colour map of the cohesion $c_{\text{coh}}(t,\Delta)$ versus $t$ and $\Delta$. Bottom: the colour map of the same cohesion for the randomized data. c, Top: the static rich club coefficient $\phi(k)$ computed for the aggregated graph as a function of the aggregate degree $k$. Bottom: the ratio $\phi(k,\Delta)$ in Fig. 3c. It is higher for larger $\Delta$, remains stable for a broad range of $k$ values and tends to decrease at larger $k$. This indicates that the nodes of the temporal network are connected in a much more simultaneous way than expected by chance, especially when considering stable interactions (that is, temporal edges lasting over many successive snapshots).

Interactions among individuals can be the support of many processes, and in particular of the spread of information or infectious diseases. We thus investigate whether the TRC ordering plays a role in the unfolding of such processes. We consider the paradigmatic susceptible—infected—susceptible (SIS) and susceptible—infected—recovered (SIR) models of spreading processes. In the SIS case, nodes can be either susceptible (S) or infectious (I). A susceptible can become infectious upon contact with an infectious, with probability $\lambda$ per time step. Infectious individuals recover with probability $\nu$ at each time step and become susceptible again. In the SIR case, nodes enter the R compartment upon recovering and cannot be infected again. We quantify the interplay between the temporal network and the spread by the epidemic threshold $\lambda_c$ at given $\nu$ (the thresholds of the SIS and SIR models coincide\ref{40}). This threshold separates a phase at $\lambda < \lambda_c$ in which the epidemic dies out from a phase at $\lambda > \lambda_c$ where it reaches a non-zero fraction of the population. We compute the epidemic threshold, using the method of ref.\ref{40}, in (1) the original data set ($\lambda_c^{\text{data}}$) and (2) versions of the data set in which the timestamps of the temporal edges connecting the nodes in $S_{\text{res}}$ are randomized ($\lambda_c^{\text{rand}}$), disrupting their simultaneity (Methods). Figure 3d displays the relative difference between the two obtained values as a function of $k$, $\lambda_c^{\text{data}}$ is systematically lower than $\lambda_c^{\text{rand}}$, indicating that the spreading process is favoured by the TRC of the data, that is, by the stronger simultaneity of connections than in the randomized versions\ref{29}. The TRC analysis thus reveals cohesive simultaneous structures of prominent nodes that affect the spreading dynamics, similarly to the fluctuation or stability of hubs or other structures\ref{26,29,33}.

State-specific TRCs: similar static but distinct temporal patterns. We finally investigate the TRC patterns of the time-resolved functional connectivity of $N=67$ neurons in the entorhinal cortex and hippocampus of an anesthetized rat. The nodes represent neurons, and the temporal edges correspond to significant mutual information between the firing patterns of pairs of neurons in a sliding window of 10 s (refs.\ref{28,44}) (Fig. 4a). Successive time windows are shifted by 1 s. This is the temporal resolution of the network, which lasts 2,284 s (Table 1). We also consider in Supplementary Note 5 a similar data set describing the temporal functional connectivity of neurons in an epileptic, anesthetized rat\ref{45}.
Fig. 4 | Temporal network of information-sharing neurons. a. Sketch of the brain areas (entorhinal cortex, mEC; hippocampus, HPC) and electrodes (left) recording the firing signal of single neurons (right). b. Top: the size $|S_{\Delta}|$ of the sub-network of nodes of aggregate degree larger than $k$ as a function of $k$. Bottom: the maximal cohesion $M(k, \Delta)$ as a function of $k$ and $\Delta$. c. The temporal network similarity matrix, where entry $(t, t')$ is given by the similarity between the instantaneous snapshots of the network at times $t$ and $t'$. The red blocks around the diagonal indicate periods in which the network remains similar to itself, that is, ‘states’ of the network\cite{28,29}. The timeline of these states is shown below the matrix as a coloured barcode (where each colour represents a different state), as extracted by clustering of the similarity matrix in ref. 28. In addition, the colour map of the cohesion $\varepsilon_{\Delta}(t, \Delta)$ of the $N_{\text{tot}} = 8$ nodes of aggregate degree larger than $k = 65$ as a function of $t$ and $\Delta$ is presented below this timeline, showing patterns at different times and that the TRC is transient and mostly concentrated at the times corresponding to a specific state. d. For three different states (states 1, 3 and 5), the size $|S_{\Delta}|$ of the sub-network of nodes of aggregate degree larger than $k$ and the maximal cohesion $M(k, \Delta)$ (left column) and the cohesion $\varepsilon_{\Delta}(t, \Delta)$ of the nodes with highest degree in the aggregate graphs of the corresponding states (with respective largest degree of 45, 56 and 41) (right column). The sets have sizes $|S^1_{k=44}| = 5$, $|S^5_{k=55}| = 7$ and $|S^3_{k=40}| = 7$. $S^1_{k=44}$ has one node in common with $S^5_{k=55}$, $S^3_{k=40}$ has no nodes in common with $S^1_{k=44}$ or $S^5_{k=55}$.

The aggregated network is so dense (average degree $\langle k \rangle = 54$) that the static rich club ordering cannot be assessed, because randomization of the links is impossible to achieve in practice. Taking into account temporality reveals a more interesting picture. Figure 4b shows that the TRC coefficient $M(k, \Delta)$ increases with $k$ for each $\Delta$ and that groups of nodes with increasing $k$ are increasingly interconnected for increasing durations. The group of eight neurons with largest degree are very strongly interconnected in a simultaneous way, with $M(k, \Delta) \geq 0.5$ up to $\Delta = 150$ (reaching $M(k, \Delta) = 1$), that is, the fully connected link structure of these eight neurons seen in the static aggregated graph does occur at some time in a simultaneous way). $M_{\text{max}}(k, \Delta)$ takes much smaller values for most values of $k$ and $\Delta$ (Supplementary Note 4), indicating the existence of a TRC ordering in this data set. In the null model, the corresponding structure is never simultaneously fully connected, nor stable.

While this analysis corresponds to the TRC computed over the whole temporal network, the functional connectivity goes through several ‘states’\cite{28}. These states are identified by the hierarchical clustering of the network similarity matrix, in which each element $(t, t')$ gives the similarity between the snapshots of the network at times $t$ and $t'$ (refs. 27,28,38,44) (Fig. 4c). The ‘states’ are identified as periods of large similarity values (red blocks along the diagonal in Fig. 4c, with
the timeline of successive states underneath). The figure also shows the cohesion $\epsilon_{s\Delta}$ of the nodes with highest degree in the network aggregated on the whole recording. It reaches very large values only during one specific state. Even if the TRC analysis by itself does not allow to uncover and define precisely the network states, $\epsilon_{s\Delta}$ is thus an indication that (1) there exists at least one period of stability in the network, in which the static rich club nodes are very cohesive and (2) there are specific periods of time with distinct dynamics, and thus it is relevant to look for network states. Such insights could not be obtained by investigating only the activity timeline and the static rich club, nor the instantaneous rich club on each timestamp. This is made clear in Supplementary Note 4 by comparing the data and the null model. Indeed, (1) the states are scrambled in the null model and (2) $\epsilon_{s\Delta}$ reaches high values during several periods for both the data and null model, indicating the existence of simultaneous connections among the high-degree nodes in both cases. As soon as $\Delta$ increases, however, this cohesion disappears for the null model, showing that no stable structure is in place, while it remains high for large values of $\Delta$ at specific periods in the real data.

We thus construct an aggregated network $G'$ for each state $s$ by aggregating the temporal edges in the snapshots belonging to $s$, and defining $S_{s,k}$ as the set of nodes with degree larger than $k$ in $G'$. Nodes are not similarly active in each state and have thus different degrees in the different $G'$s. This leads us to measure the state-specific TRC coefficients $M(k,\Delta)$. Figure 4d shows that the corresponding $k-\Delta$ diagrams for states 1, 3 and 5 have in each case more stable simultaneously interconnected sets of nodes as $k$ increases, and a TRC ordering (Supplementary Note 4). The sizes of the sets of nodes with largest degree are comparable in the three states ($S_{3,3,4}$ = 5, $S_{3,3,5}$ = 7 and $S_{3,3,0}$ = 6). However, the nodes belonging to these three sets are mostly different. Of the nodes in $S_{3,3,4}$, only one is also in $S_{3,3,5}$, and $S_{3,3,0}$ has an empty intersection with the other sets. Moreover, the cohesion of these sets of nodes (Fig. 4d and Supplementary Note 4) shows that (1) their instantaneous cohesion is maximal (reaching even 1) precisely in the timestamps of the corresponding state and (2) different dynamics are observed despite the similarity in their state-wise static rich club ordering concept is observed in Supplementary Note 3, we also exhibit a case where a static rich club does not correspond to any temporally simultaneous structure. In the third data set explored, the static rich club ordering concept is irrelevant in practice, but there is a TRC ordering (Fig. 4). The evolution of the instantaneous cohesion reveals an additional diversity of patterns, as the TRC can be present in a rather stable way (Fig. 1d), or concentrated around specific periods (Figs. 3b and 4c). The temporal patterns of the cohesion can even hint at the presence of different temporal network states (Fig. 4c).

A limit of our analysis comes from the fact that we have considered the degree of nodes in the aggregate network as the reference for centrality. A natural extension would be to consider instead the strength of the nodes, that is, the number of temporal edges in which they have participated. The focus would be on the simultaneity of the connections within the set $S_{s\Delta}$ of nodes with more than $s$ temporal edges. As strength and degree are generally correlated, the results are expected to be similar, but differences might emerge, as in the example of the air transportation temporal network where the reliever airports have a very high degree but relatively low strength.

The TRC perspective provides a new tool to study temporal networks and to unveil the relevance of simultaneous interactions of increasingly connected nodes in processes unfolding on top of the temporal network, suggesting the addition of this new measure to the repertoire of methods to study processes in networks and build and validate models of temporal networks. Moreover, TRC patterns provide an additional way to characterize network states and their dynamics and, possibly, investigate their function. For instance, key processes in neural information processing, such as synaptic plasticity, are critically affected by the timing of neuronal interactions and different TRCs in different states may enable flexible computations within a same circuit. TRCs might even provide a connection to so-called cell assemblies, with different cell assemblies being recruited in different states. In a cell assembly, many neurons must remain strongly functionally connected for a certain time in order for the assembly to be detected. Such firing coordination (both distributed in space and lasting in time) is captured only partially by common methods, which seek either for instantaneous synchrony or for sequential firing. The TRC notion captures both criteria needed to qualify a set of co-firing neurons as an assembly. We thus expect the TRC to join the toolbox of network neuroscience for the investigation of dynamic functional connectivity patterns. In conclusion, our work provides a new procedure to detect relevant temporal and structural patterns in a temporal network, enabling a new quantitative perspective on the temporal patterns of data sets coming from very different fields.

Online content
Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41567-022-01634-8.

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Discussion
The analysis of temporal networks necessitates specific tools. We have proposed here a novel concept to quantify the patterns of simultaneous interconnectedness of nodes, namely the TRC coefficient, defined as the maximal value of the density of links stable during at least a duration $\Delta$ between nodes having aggregated degree at least $k$. We compare the TRC coefficient obtained on each empirical data set with those reached in a randomized version of the data, to measure whether the simultaneity and stability of the connections of groups of nodes are higher than expected by chance.

Different patterns can be observed in terms of TRCs, at a given static rich club: the links of the static network can indeed correspond to interactions occurring at different times. In the US air transportation case (Fig. 2), the static rich club coefficient increases with $k$, but the existence of reliever hubs leads to more fluctuating links around high-degree nodes, with a non-monotonic behaviour of $M(k,\Delta)$ with $k$. In the school contact network instead, both static and temporal coefficients show an increasing trend with $k$ (Fig. 3).
Methods

Data. We consider publicly available data sets. We have moreover gathered them at https://github.com/nicolaPedre/Temporal-Rich-Club/.

Air transportation network. This data set represents the connections between US airports, with temporal resolution of 1 month, from January 2012 to September 2020, for a total of 105 timestamps. The $N=1,920$ nodes of the temporal network represent the airports, and in each monthly snapshot a temporal edge is drawn between two nodes if there was at least one direct flight between the corresponding airports during that month. The degree of a node in the aggregated network is thus the number of other airports to which it has been connected directly once, and its strength is its total number of temporal edges. We show in Supplementary Note 1 the degree distribution of the aggregated network and of some monthly snapshots, as well as some additional temporal properties of the data.

The data are publicly available on the website of the Bureau of Transportation Statistics (https://www.transtats.bts.gov/), Air Carrier Statistics (From 41 Traffic) - U.S. Carriers database).

Face-to-face interactions. This data set describes the face-to-face close-proximity contacts between 232 children and 10 teachers in a primary school in Lyon, France, during two days in 2009, as collected by the SocioPatterns collaboration using wearable devices. The original data are publicly available from the SocioPatterns website (http://www.sociopatterns.org/datasets/primary-school-temporal-network-data/). The original data are a temporal network with temporal resolution of 20 s, where the nodes represent the individuals and each temporal edge corresponds to the detection of a face-to-face contact between them. Here, we perform a temporal coarse graining on successive time windows of 5 min to remove short-time noise. We show in Supplementary Note 2 the degree distribution of the aggregated network and some timelines of instantaneous degree.

The results in the main text correspond to the first day of data, while the analysis performed on the whole data set can be found in Supplementary Note 1, as well as the results obtained with a coarse graining on time windows of 1 min.

Information-sharing neurons. This data set describes the functional connectivity between neurons in the hippocampus and medial entorhinal cortex of an anesthetized rat. The data were first presented in ref. 42 and further analyzed between neurons in the hippocampus and medial entorhinal cortex of an information-sharing neurons. This data set describes the functional connectivity well as the results obtained with a coarse graining on time windows of 1 min. Analysis performed on the whole data set can be found in Supplementary Note 2, as the instantaneous degree.

Note 2 the degree distribution of the aggregated network and some timelines of instantaneous degree.

For each data set, we compute 100 realizations of the randomized data set and compute the average $M_{\text{temp}}$ of the TRC coefficient over these realizations.

Data availability

All the data used in this work are available at https://github.com/nicolaPedre/Temporal-Rich-Club/.

Code availability

The analysis presented here were performed in Python. Example notebooks are available at https://github.com/nicolaPedre/Temporal-Rich-Club/. The randomized reference models were obtained thanks to the publicly available notebooks by the authors of ref. 34, namely at https://github.com/mygenois/RandTempNet.

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Author contributions

N.P., D.B. and A.B. designed the study and the experiment. N.P. developed the numerical tools, and produced the figures. N.P., D.B. and A.B. analysed the data and results and wrote and reviewed the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

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