IntelligentPooling:
Practical Thompson Sampling for mHealth

Sabina Tomkins
Stanford University
stomkins@stanford.edu

Peng Liao
Harvard University
pengliao@g.harvard.edu

Predrag Klasnja
University of Michigan
klasnja@umich.edu

Susan Murphy
Harvard University
samurphy@fas.harvard.edu

August 5, 2020

Abstract

In mobile health (mHealth) smart devices deliver behavioral treatments repeatedly over time to a user with the goal of helping the user adopt and maintain healthy behaviors. Reinforcement learning appears ideal for learning how to optimally make these sequential treatment decisions. However, significant challenges must be overcome before reinforcement learning can be effectively deployed in a mobile healthcare setting. In this work we are concerned with the following challenges: 1) individuals who are in the same context can exhibit differential response to treatments 2) only a limited amount of data is available for learning on any one individual, and 3) non-stationary responses to treatment. To address these challenges we generalize Thompson-Sampling bandit algorithms to develop IntelligentPooling. IntelligentPooling learns personalized treatment policies thus addressing challenge one. To address the second challenge, IntelligentPooling updates each user’s degree of personalization while making use of available data on other users to speed up learning. Lastly, IntelligentPooling allows responsivity to vary as a function of a user’s time since beginning treatment, thus addressing challenge three. We show that IntelligentPooling achieves an average of 26% lower regret than state-of-the-art. We demonstrate the promise of this approach and its ability to learn from even a small group of users in a live clinical trial.

1 Introduction

Mobile health (mHealth) applications deliver treatments in users’ everyday life to support healthy behaviors. mHealth offers an opportunity to impact health across a diverse range of domains from substance use [42], to disease self-management [23] to physical inactivity [13]. For example, to help users increase their physical activity, an mHealth application might send a walking suggestions at times and in contexts (e.g. weather, current physical activity, location) when a user is likely to be able to pursue the suggestions. The effectiveness of an mHealth app requires providing treatments in contexts in which users need the support, while avoiding over-treatment as it leads to user disengagement (e.g., treatments are ignored, app is deleted). Consequently, the goal is to be able to learn an optimal policy for when and how to intervene for each user and context. Contextual bandit algorithms appear ideal for this task.
1.1 Challenges

There are significant challenges to learning optimal policies in mHealth. This work primarily addresses the challenge of learning personalized user policies from limited data. Contextual bandit algorithms can be viewed as algorithms that use the user’s context to adapt treatment. While this approach can have advantages compared to ignoring the user’s context, it fails to address that users can respond differentially to treatments even when they appear to be in the same context. This occurs since sensors on the smart devices are unlikely to record all aspects of a user’s context that affect their health behaviors. For example, the context may not include social constraints on the user (e.g., care-giving responsibilities), which may influence the user’s ability to be active. Thus, algorithms that can learn from the differential responsiveness to treatment are desirable. This motivates the need for an algorithm that not only incorporates contextual information, but that can also learn personalized policies. A natural first approach would be to use the algorithm separately for each user, but the algorithm is likely to learn very slowly if data on a user is sparse and/or noisy. However, typically in mHealth studies multiple users are using the application at any given time. Thus an algorithm that pools data over users intelligently so as to speed up learning of personalized policies is desirable.

An additional challenge is non-stationary responses to treatment (i.e. non-stationary reward function). For example, in the beginning of a study, a user might be excited to receive a treatment, however after a few weeks this excitement can wane. This motivates the need for algorithms that can learn time-varying treatment policies.

1.2 Contributions

We develop IntelligentPooling, a type of Thompson sampling contextual bandit algorithm specifically designed to overcome the above challenges. Our main contributions are:

- **IntelligentPooling**: A Thompson sampling contextual bandit algorithm for rapid personalization in limited data settings. This algorithm employs classical random effects in the reward function and empirical Bayes to adaptively adjust the degree to which policies are personalized to each user. We present an analysis of this adaptivity in Section 3.3 showing that IntelligentPooling can learn to personalize to a user as a function of the observed variance in the treatment effect both between and within users.

- A high probability regret bound for IntelligentPooling.

- An empirical evaluation of IntelligentPooling in a simulation environment constructed from mHealth data. IntelligentPooling not only achieves 26% lower regret than state-of-the-art approaches, it also is better able to adapt to the degree of heterogeneity present in a population than this approach.

- Feasibility of IntelligentPooling from a pilot study in a live clinical trial. We demonstrate that IntelligentPooling can be executed in a real-time online environment and show preliminary evidence of this method’s effectiveness.

- We show how to modify IntelligentPooling to learn in non-stationary environments.

Next, in Section 2 we discuss relevant related work. In Section 3 we present IntelligentPooling and provide a high-probability regret bound for this algorithm. We then describe how
we use historical data to construct a simulation environment and evaluate our approach against state-of-the-art in Section 4. Next, in Section 5 we introduce the feasibility study and provide preliminary evidence into the benefits of this approach. We then discuss how to extend this work to include time-varying effects in Section 6. Finally, we discuss the limitations with our approach in Section 7 before concluding.

2 Related Work

To put the proposed work in a broader healthcare perspective, an overview of similar work in mHealth is provided by Section 2.1. Next, we discuss the extent to which reinforcement learning/bandit algorithms have been deployed in mHealth settings (Section 2.1). INTELLIGENTPOOLING has similarities with several modeling approaches, here we discuss the most relevant: multi-task learning, meta-learning, Gaussian processes for Thompson Sampling contextual bandits, and time-delayed bandits. These topics are discussed in Section 2.2 - Section 2.4.

2.1 Connections to Bandit algorithms in mHealth

Bandit algorithms in mHealth have typically used one of two approaches. The first approach is person-specific, that is, an algorithm is deployed separately on each user, such as in [41], [24], [19], and [34]. This approach makes sense when users are highly heterogeneous, that is, their optimal policies differ greatly one from another. However, this approach can present challenges for learning the policy when data is scarce and/or noisy, as in our motivating example of encouraging activity in an mHealth study where only a few decision time-points occur each day. The second approach completely pools users’ data, that is one algorithm is used on all users so as to learn a common treatment policy both in bandit algorithms [38, 54], and in full reinforcement learning algorithms [12, 56]. This second approach can potentially learn quickly but may result in poor performance if there is large heterogeneity between users.

In INTELLIGENTPOOLING we follow strike a balance between these two extremes, adjusting the degree of pooling to the degree that users are similarly responsive. When users are heterogeneous, INTELLIGENTPOOLING achieves lower regret than the second approach while learning more quickly than the first approach. When users are homogeneous our method performs as well as the second approach.

2.2 Connections to multi-task learning and meta-learning

Researchers have proposed pooling data in a variety of ways. For example, Deshmukh et al. [15] proposed pooling data from different arms of a single bandit problem. Li and Kar [32] used context-sensitive clustering to produce aggregate reward estimates for the bandit algorithm. More relevant to this work is multi-task Gaussian Process (GP), e.g., [30, 4, 52], however these have been proposed in the prediction as opposed to the reinforcement learning setting. The Gang of Bandits approach [9], which is a generalization from the original LinUCB algorithm for a single task [31], has been shown to be successful when there is prior knowledge on the similarities between users. For example, a known social network graph might provide a mechanism for pooling. It was later extended to the Horde of Bandits in [51] which used Thompson Sampling, allowing the algorithm to deal with large number of tasks.
Each of the multi-task approaches introduces some concept of similarity between users. The extent to which a given user’s data contributes to another user’s policy is some function of this similarity measure. This is fundamentally different from the approach taken in IntelligentPooling. Rather than determining the extent to which any two users are similar, IntelligentPooling determines the extent to which a given user’s reward function parameters differ from parameters in a population (average over all users) reward function. This approach has the advantage of requiring less hyper-parameters, as we do not need to learn a similarity function between users. Instead, of a pairwise similarity function it is as if we are learning a similarity between each user and the population average. In the limited data setting we expect this simpler model to be advantageous.

In meta-learning, one exploits shared structure across tasks to improve performance on new tasks. IntelligentPooling thus shares similarities with meta-learning for reinforcement learning \cite{37, 18, 17, 57, 22, 47}. At a high level, one can view our method as a form of meta-learning where the population-level parameters are learned from all available data and each user’s parameters represent deviations from the shared parameters. However, while meta-learning might require a large collection of source tasks, we demonstrate the efficacy of our approach on data on the small scale found in clinical mHealth studies.

2.3 Connections to Gaussian process models for Thompson sampling contextual bandits

IntelligentPooling is based on Bayesian mixed-effects model of the reward, which is similar to using a Gaussian Process (GP) model with a simple form of the kernel. GP models have been used for multi-armed bandits \cite{11, 6, 49, 13, 53, 16, 3}, and for contextual bandits \cite{31, 28}. However the above approaches are not structuring the way in which the pooling of data across users occurs. IntelligentPooling uses a mixed-effects GP model to pool across users in structured manner. Although mixed effects GP model have been previously used for off-line data analysis \cite{48, 35}, to our best knowledge it has not been previously used in online decision making problems considered in this work.

2.4 Connection to non-stationary linear bandits

There is a growing literature investigating how to adapt linear bandit algorithms to changing environments. A common approach is for the learning algorithm to differentially weight data across time. Differential weighting is used by both Russac et al. \cite{44} (using a LinUCB algorithm) and Kim and Tewari \cite{25} (using perturbation-based algorithms). Cheung et al. \cite{10} use a linear moving window to estimate the parameters in the reward function and Zhao et al. \cite{55} restart the algorithm at regular intervals discarding the prior data. Similarly Bogunovic et al. \cite{3}, using GP-based UCB algorithms, accommodate non-stationarity by both restarting and using an autoregressive model for the rewards function.

IntelligentPooling allows for non-stationary reward functions by the use of time-varying random effects. The correlation between the time-varying random effects induces a weighted estimator whereby more weight is put on the recently collected samples, similar to the discounted estimators in \cite{44} and \cite{25}. In contrast to existing approaches, IntelligentPooling considers both individual and time-specific variation.
3 Intelligent Pooling

IntelligentPooling is a generalization of a Thompson sampling contextual bandit for learning personalized treatment policies. We first outline the components of IntelligentPooling and then introduce the problem definition in Section 3.2. As our approach offers a natural alternative to two commonly used approaches, we begin by describing these simpler methods in Section 3.3. We introduce our method in Section 3.4.

3.1 Overview

The central component of IntelligentPooling is a Bayesian model for the reward function. In particular, IntelligentPooling uses a Gaussian mixed effects linear model for the reward function. Mixed effects models are widely used across the health and behavioral sciences to model the variation in the linear model parameters across individuals [43, 29] and within an individual across time. Use of these models enhances the ability of domain scientists to inform and critique the model used in IntelligentPooling. The properties and pitfalls of these models are well understood; see [40] for an application of a mixed effects model in mHealth. IntelligentPooling uses Bayesian inference for the mixed effects model. As discussed in Section 2.3, a Bayesian mixed effects linear model is a GP model with a simple kernel. This opens the door to increasing the flexibility of the model for the reward function, given sufficient data.

Furthermore, IntelligentPooling uses Thompson sampling [50], also known as posterior sampling [45], to select actions. At each decision point, the parameters in the model for the reward function are sampled from their posterior distribution, thus inducing exploration over the action space [46]. These sampled parameters are then used to form an estimated reward function and the action with the highest estimated reward is selected.

The hyper-parameters (e.g., the variances of the random effects) control the extent of pooling across individuals and across decision times. The right amount of pooling depends on the heterogeneity among individuals and the non-stationarity, which is often difficult to pre-specify. Unlike other bandit algorithms in which the hyper-parameters are set at the beginning [15, 9, 51], IntelligentPooling includes a procedure for updating the hyper-parameters online. In particular, Empirical Bayes [7] is used to update the hyper-parameters in the online setting, as more data becomes available.

3.2 Problem formulation

Consider an mHealth study with $N$ users. Let $i \in [N] = \{1, \ldots, N\}$ be a user index. For each user, we use $k \in \{1, 2, \ldots\}$ to index decision times, i.e., times at which a treatment could be provided. Denote by $S_{i,k}$ the contextual features at the $k$-th decision time of user $i$. We denote by $t_{i,k}$ the calendar time of user $i$’s $k$-th decision time. Since users enroll in the study in a staggered fashion or download a wellness application at different times, the calendar time is indexed by the user. Throughout for simplicity we focus on the case where the action is binary, i.e., $A_{i,k} \in \{0, 1\}$. The algorithm can be easily generalized to case with more than two actions. In the feasibility study in Section 5, each participant specifies five walking suggestion times during the day and the action corresponds to sending an activity-suggestion message.

Below for simplicity we consider a simpler setting where the parameters in the reward is assumed time-stationary and we discuss how to generalize the algorithm to the non-stationary setting in
Section 6. The goal is to learn personalized treatment policies for each of the \( N \) users. We treat this as \( N \) contextual bandit problems due as the reward function may differ between users. Recall that in mHealth settings this might occur due to the inability of sensors to record the user’s entire context. Section 3.3 reviews two approaches for using Thompson Sampling \([2]\) and Section 3.4 presents IntelligentPooling, our approach for learning the treatment policy for any specific user.

### 3.3 Two Thompson Sampling instantiations

First consider learning the treatment policy separately per person. We refer to this approach as **Person-Specific**. At each decision time \( k \), we would like to select a treatment \( A_{i,k} \in \{0, 1\} \) based on the context \( S_{i,k} \). We model the reward \( R_{i,k} \) by a Bayesian linear regression model: for user \( i \) and time \( k \)

\[
R_{i,k} = \phi(S_{i,k}, A_{i,k})^\top w_i + \epsilon_{i,k},
\]

where \( \phi(s, a) \) is the feature vector of context and treatment (e.g., those described in Section 4.2), \( w_i \) is a parameter vector which we will learn, and \( \epsilon_{i,k} \sim N(0, \sigma^2) \) is the error term. The parameters \( \{w_i\} \) are assumed independent across users and to follow a common prior distribution \( w_i \sim N(\mu_w, \Sigma_w) \).

Now at the \( k^{th} \) decision time with the context \( S_{i,k} \), **Person-Specific** selects the treatment \( A_{i,k} = 1 \) with probability

\[
\pi_{i,k} = \Pr\{\phi(S_{i,k}, 1)^\top \tilde{w}_{i,k} > \phi(S_{i,k}, 0)^\top \tilde{w}_{i,k}\}
\]

where \( \tilde{w}_{i,k} \) follows the posterior distribution of \( w_i \) given the user’s history \( D_{i,k} = \{(S_{i,o}, A_{i,o}, R_{i,o}) : o \leq k - 1\} \). Note that in this formulation the posterior distribution of \( w_i \) is formed based on the user’s own data.

The opposite approach is to learn a common bandit model for all users. In this approach, the reward model is a single Bayesian regression model with no individual-level parameters:

\[
R_{i,k} = \phi(S_{i,k}, A_{i,k})^\top w + \epsilon_{i,k},
\]

where the common parameters, \( w \), follows the prior distribution \( w \sim N(\mu_w, \Sigma_w) \). We then use the posterior distribution of the parameter \( w \) to sample treatments for each user. This approach, which we refer to as **Complete**, may suffer from high bias when there is significant heterogeneity among users.

### 3.4 Intelligent pooling across bandit problems

**IntelligentPooling** is an alternative to the two methods mentioned above. Specifically, in **IntelligentPooling** data is pooled across users in an adaptive way, i.e., when there is strong homogeneity observed in the current data, the algorithm will pool more from others than when there is strong heterogeneity. Consider the Bayesian linear regression model \([1]\). The Bayesian Gaussian mixed effects linear model imposes structure on the \( w_i \)'s, in particular, a random-effects structure \([43, 29]\) on \( w_i \):

\[
w_i = w_{\text{pop}} + u_i,
\]
where \( w_{\text{pop}} \) is a population-level parameter and \( u_i \) is a “random effect” that represents the person-specific deviation from \( w_{\text{pop}} \) for user \( i \). See section 4.5 for a discussion of how the posterior means for \( w_{\text{pop}} \) and \( u_i \) are based on user \( i \)'s data as well as other users depending on the variances of the random effects. The prior on \( w_{\text{pop}} \) is Gaussian with prior mean \( \mu_w \) and variance \( \Sigma_w \): \( u_i \) has mean 0 and covariance \( \Sigma_u \); and \( u_i \perp u_j \) for \( i \neq j \) and \( w_{\text{pop}} \perp \{u_i\} \).

The prior parameters \( \mu_w, \Sigma_w \) as well as the variance of the random effect \( \Sigma_u \), and the residual variance \( \sigma^2 \) are hyper-parameters. In \([1]\), there is a the random effect, \( u_i \) on each element of \( w \). In practice, one can use domain knowledge to specify which of the parameters should include random effects; this will be the case in the feasibility study described in Section 6 below.

We denote by \( T \), the set of update times. These are the calendar times that the posterior distribution is updated; see the choice of update times in the feasibility study in Section 5. We now discuss how the posterior distribution at a given update time, \( T \in T \) can be calculated given the hyper-parameters. Let \( \mathcal{U}_T \subseteq \{N\} \) be the set of users that are currently in or have finished the trial. The history available at time \( T \) is \( D_T = \{(S_{i,k}, A_{i,k}, R_{i,k}, i) : i \in \mathcal{U}_T, t_{i,k} \leq T\} \). Suppose the number of tuples in \( D_T \) is \( n_T \).

The posterior distribution of each \( w_i \) is Gaussian with mean and variance determined by a kernel function \( K \) induced by the mixed effects model (Eqns. \([1],[4]\)): for any two tuples in \( D_T \), e.g., \( x_l = (S^{(l)}, A^{(l)}, R^{(l)}, i_l) \), \( l = 1, 2 \)

\[
K(x_1, x_2) = \phi_1^\top(\Sigma_w + \mathbb{1}_{(i_1=i_2)} \Sigma_u)\phi_2
\]

(5)

where \( \phi_l = \phi(S^{(l)}, A^{(l)}) \). Note that the above kernel depends on \( \Sigma_w \) and \( \Sigma_u \). The kernel matrix \( K_{n_T} \) is of size \( n_T \times n_T \) and each element is the kernel value between two tuples in \( D_T \). The posterior mean and variance of \( w_i \) given \( D_T \) can be calculated by

\[
\hat{w}_{i,T} = \mu_w + M_i^\top(K_{n_T} + \sigma^2 \mathbb{1}_{n_T})^{-1}\hat{R}_{n_T}
\]

\[
\Sigma_i,T = \Sigma_w + \Sigma_u - M_i^\top(K_{n_T} + \sigma^2 \mathbb{1}_{n_T})^{-1}M_i
\]

(6)

where \( \hat{R}_{n_T} \) is the vector of the rewards centered by the prior means, i.e., each element corresponds to a tuple \((S, A, R, j, h)\) in \( D_T \) given by \( R - \phi(S, A)^\top\mu_w \), and \( M_i \) is a matrix of size \( n_T \) by \( p \), with each row corresponding to a tuple \((S, A, R, j)\) in \( D_T \) given by \( \phi(S, A)^\top(\Sigma_w + \mathbb{1}_{(j=i)} \Sigma_u) \).

**Treatment selection**

To select a treatment for user \( i \) at the \( k \)-th decision time, we use the posterior distribution of \( w_i \) formed at the most recent update time \( T \). That is, for the context \( S_{i,k} \) of user \( i \) at the \( k \)-th decision time, INTELLIGENTPOOLING selects the treatment \( A_{i,k} = 1 \) with the probability calculated in the same formula as in \([2]\) but with the different posterior distribution \( \hat{w}_{i,k} \sim N(\hat{w}_{i,T}, \Sigma_{i,T}) \).

**Selecting hyper-parameters**

Thus far the degree of pooling across users has been determined by the choice of the hyper-parameters. The prior mean \( \mu_w \) and variance \( \Sigma_w \) of the population parameter \( w_{\text{pop}} \) can be set according to previous data or domain knowledge. In Section 5, we discuss how we set the prior distribution \( \mu_w \) and \( \Sigma_w \) in the feasibility study. Also the influence of the prior mean and variance on the Thompson Sampling algorithm decreases as data accrues and is used by the algorithm. However the influence of the variance components for the random effects on the degree of pooling...
persists even with increasing user data. Thus IntelligentPooling uses, at the update times, an empirical Bayes approach to update $\lambda = (\Sigma_u, \sigma_u^2)$. The updated values maximize the marginal log-likelihood of the observed reward, marginalized over the population parameters $w_{pop}$ and the random effects. At every update time, $T$, we set the hyper-parameters as $\hat{\lambda} = \text{argmax } l(\lambda|\mathcal{D}_T)$, the maximizer of the marginal likelihood $l(\lambda|\mathcal{D}_T)$:

$$l(\lambda|\mathcal{D}_T) = -\frac{1}{2} \left[ \tilde{R}_n^\top (K_n(\lambda) + \sigma^2 I_n)^{-1} \tilde{R}_n ight.$$  
$$+ \log \det(K_n(\lambda) + \sigma^2 I_n) + n_T \log(2\pi) \left.] \right)$$  

where $K_n(\lambda)$ is the kernel matrix as a function of parameters $\lambda = (\Sigma_u, \sigma_u^2)$. See Algorithm 1.

**Algorithm 1 IntelligentPooling**

1: Set $\hat{w}_{i,0} = \mu_w, \Sigma_{i,0} = \Sigma_w + \Sigma_u$
2: for $t \in [0, T]$ do
3: Receive user index $i$ and decision time index $k$
4: Collect state variable $S_{i,k}$
5: Calculate randomization probability $\pi_{i,k} = \Pr\{\phi(S_{i,t}, 1)^\top \tilde{w} > \phi(S_{i,t}, 0)^\top \tilde{w}\}$ where $\tilde{w} \sim \mathcal{N}(\hat{w}_i, \Sigma_i)$
6: Sample treatment $A_{i,t} \sim \text{Bern}(\pi_{i,k})$
7: Collect reward $R_{i,t}$
8: $\mathcal{D} \leftarrow \mathcal{D} \cup \{S_{i,t}, A_{i,t}, R_{i,t}, i\}$
9: if $t \in T$ then
10: Update the hyper-parameters: $\hat{\lambda} = \text{argmax } l(\lambda|\mathcal{D})$ in Eqn 7
11: Update the posterior mean and covariance $\hat{w}_i, \Sigma_i$ by Eqns 6 with $\hat{\lambda}$
12: end if
13: end for

### 3.5 Intuition for the use of random effects

IntelligentPooling uses random effects to adaptively pool users’ data based on the degree to which users exhibit heterogeneous rewards. That is, the person-specific random effect should outweigh the population term if users are highly heterogeneous. If users are highly homogeneous, the person-specific random effect should be outweighed by the population term. The amount of pooling is controlled by the hyper-parameters, e.g., the variance components of the random effects.

To gain intuition, we consider a simple setting where the feature vector $\phi$ in the reward model (Eqn. 1) is one-dimensional (i.e., $p = 1$) and there are only two users (i.e., $i = 1, 2$). Denote the prior distributions of population parameter $w_{pop}$ by $\mathcal{N}(0, \sigma_w^2)$ and the random effect $u_i$ by $\mathcal{N}(0, \sigma_u^2)$. Below we investigate how the hyper-parameters (e.g., $\sigma_u^2$ in this simple case), impact the posterior distribution.

Let $k_i$ be the index of decision time of user $i$ at the updating time $T$. In this simple setting, the posterior mean of $\hat{w}_1$ can be calculated explicitly by

$$\hat{w}_1 = \frac{[\delta \gamma + (1 - \gamma^2) S_2] Y_1 + \delta \gamma^2 Y_2}{(1 - \gamma^2) S_1 S_2 + \delta \gamma (S_1 + S_2) + (\delta \gamma)^2}$$
where for $i = 1, 2$, $S_i = \sum_{k=1}^{k_i} \phi(A_{i,k}, S_{i,k})^2$, $Y_i = \sum_{k=1}^{k_i} \phi(A_{i,k}, S_{i,k})R_{i,k}$, $\gamma = \sigma_w^2/(\sigma_w^2 + \sigma_u^2)$ and $\delta = \sigma^2/\sigma_w^2$. Similarly, the posterior mean of $w_2$ is given by

$$\hat{w}_2 = \frac{[\delta \gamma + (1 - \gamma^2)S_1]Y_2 + \delta \gamma^2 Y_1}{(1 - \gamma^2)S_1 S_2 + \delta \gamma (S_1 + S_2) + (\delta \gamma)^2}$$

When $\sigma_u^2 \to 0$ (i.e., the variance of random effect goes to 0), we have $\gamma \to 1$ and both posterior means

$$\hat{w}_1, \hat{w}_2 \to \frac{Y_1 + Y_2}{S_1 + S_2 + \delta},$$

which is the posterior mean under the model COMPLETE (Eqn 3) using prior $N(0, \sigma_w^2)$. On the other hand, when $\sigma_u^2 \to \infty$, we have $\gamma \to 0$ and

$$\hat{w}_1 \to \frac{Y_1}{S_1}, \quad \hat{w}_2 \to \frac{Y_2}{S_2},$$

where correspond to the person-specific estimation of $\hat{w}_1$ and $\hat{w}_2$ under the model PERSON-SPECIFIC (Eqn 1) using a non-informative prior. Fig. 1 illustrates that when $\gamma$ goes from 0 to 1, the posterior mean $\hat{w}_i$ smoothly transits from the population estimates to the person-specific estimates.

### 3.6 Regret

We prove a regret bound for a modification of INTELLIGENTPOOLING similar to that in [2, 51] in a simplified setting. Further details are provided in Appendix A. Let $d$ be the length of the parameter vector $w_i$ in the Bayesian mixed-effect model of the reward in Eqn. 1. Recall that $\Sigma_w$ is the prior covariance of the parameter vector $w_{pop}$, $\Sigma_u$ is the covariance of the random effect $u_i$.
and \( \sigma^2 \) is the variance of the error term. Let \( K_i \) be the number of decision times for user \( i \) up to a given calendar time and \( T = \sum_{i=1}^{N} K_i \) be the total number of decision times encountered by all \( N \) users in the study up to the calendar time. We define the regret of the algorithm after \( T \) decision times by

\[
\mathcal{R}(T) = \sum_{i=1}^{N} \sum_{k=1}^{K_i} \max_a \phi(S_{i,k}, a)^T w_i - \phi(S_{i,k}, A_{i,k})^T w_i
\]

**Theorem 1.** With probability \( 1 - \delta \), where \( \delta \in (0, 1) \) the total regret of the modified Thompson Sampling with IntelligentPooling after \( T \) total number of decision times is:

\[
\mathcal{R}(T) = \tilde{O}\left( dN \sqrt{T} \sqrt{\log \left( \frac{(\text{Tr}(\Sigma_w) + \text{Tr}(\Sigma_u) + \text{Tr}(\Sigma_u^{-1}))}{d} \right) + \frac{T}{\sigma^2 dN} \log \frac{1}{\delta}} \right)
\]

### 4 Experiments

This work was conducted to prepare for deployment of IntelligentPooling in a live trial. Thus, to evaluate IntelligentPooling we construct a simulation environment from a precursor trial, HeartStepsV1 [26]. This simulation allows us to evaluate the proposed algorithm under various settings that may arise in implementation. For example, heterogeneity in the observed rewards may be due to unknown subgroups across which users’ reward functions differ. Alternatively, this heterogeneity may vary across users in a more continuous manner. We consider both scenarios in simulated trials. In Sections 4.1-4.3 we evaluate the performance of IntelligentPooling against baselines and state-of-the-art algorithms. In Section 5 we assess feasibility of IntelligentPooling in a pilot deployment in a clinical trial.

#### 4.1 Simulation environment

HeartStepsV1 was a 6-week micro-randomized trial of an Android-based physical activity intervention with 41 sedentary adults. The intervention consisted of two “push” interventions: planning and contextually-tailored activity suggestions. Activity suggestions acted as action cues and were designed to provide users with actionable options for engaging in short bouts of activity in their current situation. The content of the suggestions was tailored based on the users’ location, weather, time of day, and day of the week. For each individual, on each day of the study, the HeartSteps system randomized whether or not to send an activity suggestion five times a day. The intended outcome of the suggestions—the proximal outcome used to evaluate their efficacy—was the step count in the 30 minutes following suggestion randomization.

HeartStepsV1 data was used to construct all features within the environment, and to guide choices such as how often to update the feature values. Recall that \( S_{i,k} \) and \( R_{i,k} \) denote the context features and reward of user \( i \) at the \( k^{th} \) decision time. The reward is the log step counts in the thirty minutes immediately following a decision time. In HeartStepsV1 the treatment action was binary; \( A_{i,k} = 1 \) corresponded to a smartphone notification containing an activity suggestion that should take around 3 minutes to perform and \( A_{i,k} = 0 \) corresponded to not sending a message. However, in the simulation, \( A_{i,k} = 0 \) corresponds to a less burdensome suggestion of a very brief (30 second) activity. Fig. 2 describes the simulation while Table 1 describes context features and rewards. Each context feature in Table 1 was constructed from HeartStepsV1 data. For example, we found that in HeartStepsV1 data splitting participants’ prior 30 minute step count into the two categories of high or low best explained the reward. Additional details about this process are included in Section 3.
Figure 2: Contextual features for a simulated User are composed of both general environmental features (such as time of day) and individual features (such as location). At decision times a simulated user receives a message determined by the current treatment policy. Periodically this policy is updated according to a learning algorithm which outputs a new posterior distribution for each User.

The temperature and location are updated throughout a simulated day according to probabilistic transition functions constructed from HeartStepsV1. The step counts for a simulated user are generated from participants in HeartStepsV1 as follows. We construct a one-hot encoding containing the group-ID of a participant, the time of day, the day of the week, the temperature, the preceding activity level, and the location. Then for each possible realization of the one-hot encoding we calculate the empirical mean and empirical standard deviation of all step counts observed in HeartStepsV1.

A simulated user’s context is encoded via the same one-hot encoding to produce \( f(S_{i,k}) \). The corresponding empirical mean and empirical standard deviation from HeartStepsV1 form \( \mu_{f(S_{i,k})} \) and \( \sigma_{f(S_{i,k})}^2 \) respectively. At each 30 minute window, if a treatment is not delivered step counts are generated according to

\[
R_{i,k} = \mathcal{N}(\mu_{f(S_{i,k})}, \sigma_{f(S_{i,k})}^2) \tag{8}
\]

Heterogeneity This model, which we denote HETEROGENEITY, allows us to compare the performance of the approaches under different levels of population heterogeneity. The step count after a decision time is a modification of Eqn. 8 to reflect the interaction between context and treatment on the reward and heterogeneity in treatment effect. Let \( \beta \) be a vector of coefficients of \( f(S_{i,k}) \) which weigh the relative contributions of the entries of \( f(S_{i,k}) \) that interact with treatment on the reward. The magnitude of the entries of \( \beta \) are set using HeartStepsV1. Step counts \( (R_{i,k}) \) are generated as

\[
R_{i,k} = \mathcal{N}(\mu_{f(S_{i,k})}, \sigma_{f(S_{i,k})}^2) + A_{i,k}(f(S_{i,k})^T \beta_i + Z_i) \tag{9}
\]

The inclusion of \( Z_i \) will allow us to evaluate the relative performance of each approach under different levels of population heterogeneity. Let \( \beta_i^l \) be the coefficient of the location term for the \( i^{th} \) user. We consider three scenarios (shown in Table 2) to generate \( Z_i \), the person-specific effect,
### State Features

| Name                  | Value                                      | User Specific |
|-----------------------|--------------------------------------------|---------------|
| Time of day           | Morning(0) 9:00 and 15:00; Afternoon(1)    |               |
|                       | 15:00 and 21:00; Night(2) 21:00 and 9:00  |               |
| Day of the week       | Weekday(0) or Week-end(1)                  |               |
| Temperature           | Cold(0) or Hot(1)                          |               |
| Preceding activity    | Low(0) or High(1)                          |               |
| Location              | Home/work(1)                               |               |
| Interception          | 1                                          |               |
| Reward                | Continuous on log scale                    |               |

Table 1: The value used in encoding each feature is shown in parentheses. For example cold(0) indicates that cold is coded as a 0 wherever this feature is used.

| Homogeneous | Bi-modal | Smooth |
|-------------|----------|--------|
| \( Z_i = 0 \), \( \beta_i = 0 \) | \( Z_i, \beta_i = \begin{cases} z_1, \beta_1^1 & \text{if } i \in \text{group one} \\ z_2, \beta_1^2 & \text{if } i \in \text{group two} \end{cases} \) | \( Z_i \sim N(0, \sigma^2_i) \), \( \beta_i \sim N(0, \sigma^2_i) \) |

Table 2: Settings for \( Z \) in three cases of homogeneous, bimodal and smoothly varying populations.

and \( \beta_i \) the location-dependent effect. The performance of each algorithm under each scenario will be analyzed in Section 4.3. In the smooth scenario, \( \sigma \) is equal to the standard deviation of the observed treatment effects \( f(S_{i,k})^T \beta : S_{i,k} \in \text{HeartStepsV1} \) and \( \beta_i \) is set to 0.1.

In the bi-modal scenario each simulated user is assigned a base-activity level: low-activity users (group 1) or high-activity users (group 2). When a simulated user joins the trial they are placed into either group one or two with equal probability. Whether or not it is optimal to send a treatment for user \( i \) at their \( k^{th} \) decision time depends both on their context, and on the values of \( z_1, \beta_1^1 \) and \( z_2, \beta_2^1 \). The values of \( z_1, \beta_1^1 \) and \( z_2, \beta_2^1 \) are set so that for all users in group 1, it is optimal to send a treatment under 75% of the contexts they will experience. Yet for all users in group 2, it is only optimal to send a treatment under 25% of the contexts they will experience. Group membership is not known to any of the algorithms.

### 4.2 Model for the reward function in IntelligentPooling

In Section 3 we introduced the feature vector \( \phi \), recall that \( \phi \) is the vector \( \phi(S_{i,k}, A_{i,k}) \in \mathbb{R}^p \) used in the model for the reward. The features in the reward model for all algorithms considered here are,

\[
\phi(S_{i,k}, A_{i,k})^T = (g(S_{i,k}, A_{i,k})^T, \pi_{i,k} f(S_{i,k})^T, (A_{i,k} - \pi_{i,k}) f(S_{i,k})^T)
\]  

(10)

where both \( f(S_{i,k}) \) and \( g(S_{i,k}) \) contain: an intercept term (equal to 1), time of day, day of the week, preceding activity level, and location. Recall that the bandit algorithms produce \( \pi_{i,k} \) which is the probability that \( A_{i,k} = 1 \).

The inclusion of the term \((A_{i,k} - \pi_{i,k}) f(S_{i,k})\) is motivated by [33, 5, 21], who demonstrated that action-centering can protect against mis-specification in the baseline effect (e.g., the expected
reward under the action 0. In HeartStepsV1 we observed that users varied in their overall responsiveness and that a user’s location was related to their responsiveness. In the simulation, we assume the person-specific random effect on four parameters in the reward model (i.e., the coefficients of terms in g and f involving the intercept and location).

Finally, we constrain the randomization probability to be within [0.1, 0.8] to ensure continual learning. The update time for the hyper-parameters is set to be every 7 days. All approaches are implemented in Python and we implement GP regression with the software package GPytorch [20].

4.3 Simulation results

In this section we compare the use of mixed effects model for the reward function in IntelligentPooling to two standard methods used in mHealth, Complete and Person-Specific from Section 3.3. Recall that IntelligentPooling includes person-specific random effects, as described in Eqn. 13. In Person-Specific, all users are assumed to be different and there is no pooling of data and in Complete, we treat all users the same and learn one set of parameters across the entire population.

Additionally, to assess IntelligentPooling’s ability to pool across users we compare our approach to Gang of Bandits [9], which we refer to as GangOB. As this model requires a relational graph between users, we construct a graph using the generative model [9] and Table 2 connecting users according to each of the three settings: homogeneous, bi-modal and smooth. For example, with knowledge of the generative model users can be connected to other users as a function of their $Z_i$ terms. As we will not have true access to the underlying generative model in a real-life setting we distort the true graph to reflect this incomplete knowledge. That is we add ties to dissimilar users at 50% of the strength of the ties between similar users.

From the generative model [9], the optimal action for user $i$ at the $k^{th}$ decision time is $a^*_{i,k} = \mathbb{1}_{\{f(S_{i,k})^T \beta_i^* + Z_i \geq 0\}}$. The regret is

$$regret_{i,k} = |f(S_{i,k})^T \beta_i^* + Z_i| \mathbb{1}_{\{a^*_{i,k} \neq A_{i,k}\}} \quad (11)$$

where $\beta_i^*$ is the optimal $\beta$ for the $i^{th}$ user.
Group one optimal policy = send activity suggestion
Group two optimal policy = don’t send suggestion

|                  | Group one | Group two |
|------------------|-----------|-----------|
| **COMPLETE**     | 0.49      | 0.46      |
| **PERSON-SPECIFIC** | 0.65      | 0.49      |
| **GANGOB**       | 0.57      | 0.35      |
| **INTELLIGENT-POOLING** | 0.59      | 0.36      |

Table 3: Average fraction of times activity suggestion treatment was sent (action=1), over 50 simulations (bi-modal generative model $Z^b$).

In these simulations each trial has 32 users. Each user remains in the trial for 10 weeks and the entire length of the trial is 15 weeks, where the last cohort joins in week six. The number of users who join each week is a function of the recruitment rate observed in HeartStepsV1. In all settings we run 50 simulated trials.

First, Fig. 3 provides the regret averaged across all users across 50 simulated trials where the reward distribution follows (9) for each of the Table 2 categories. The horizontal axis in Fig. 3 is the average regret over all users in their nth week in the trial, e.g. in their first week, their second week, etc. In the bi-modal setting there are two groups, where all users in group one have a positive response to treatment when experiencing their typical context, while the users in group two have a negative response to treatment under their typical context. An optimal policy would learn to not send treatments to users in the first group, and to send them to users in the second.

To evaluate each algorithm’s ability to learn this distinction we show the percentage of time each group received a message in Table 3. The relative performance of the approaches depends on the heterogeneity of the population. When the population is very homogenous COMPLETE excels, while its performance suffers as heterogeneity increases. PERSON-SPECIFIC is able to personalize; as shown by Table 3 it can differentiate between individuals. However, it learns slowly and can only approach the performance of COMPLETE in the smooth setting of Table 2 where users differ the most in their response to treatment. Both INTELLIGENT-POOLING and GANGOB are more adaptive than either COMPLETE or PERSON-SPECIFIC. GANGOB consistently outperforms PERSON-SPECIFIC and achieves lower regret than COMPLETE in some settings. In the homogeneous setting we see that GANGOB can utilize social information more effectively than PERSON-SPECIFIC does while in the smooth setting it can adapt to individual differences more effectively than COMPLETE. Yet, INTELLIGENT-POOLING demonstrates stronger and swifter adaptability than does GANGOB, consistently achieving lower regret at quicker rates. Finally, the algorithms differ in their suitability for real-world applications, especially when data is limited. GANGOB requires reliable values for hyper-parameters and can depend on fixed knowledge about relationships between users. INTELLIGENT-POOLING can learn how to pool between individuals over time and without prior knowledge.
5 IntelligentPooling Feasibility Study

The simulated experiments provide insights into the potential of this approach for a live deployment. As we see reasonable performance in the simulated setting, we now discuss an initial pilot deployment of IntelligentPooling in a real-life physical activity clinical trial.

5.1 Feasibility Study Design

The feasibility study of IntelligentPooling involves 10 participants added to a larger 90-day clinical trial of HeartSteps v2, an mHealth physical activity intervention. The purpose of the larger clinical trial is to optimize the intervention for individuals with Stage 1 hypertension. Study participants with Stage 1 hypertension were recruited from Kaiser Permanente Washington in Seattle, Washington. The study was approved by the institutional review board of the Kaiser Permanente Washington Health Research Institute.

HeartSteps v2 is a cross-platform mHealth application that incorporates several intervention components, including weekly activity goals, feedback on goal progress, planning, motivational messages, prompts to interrupt sedentary behavior, and—most relevant to this paper—actionable, contextually-tailored suggestions for individuals take a short physical activity (suggesting, roughly, a 3 to 5 minute walk). In this study physical activity is tracked with a commercial wristband tracker, the Fitbit Versa smart watch.

As in the first version of the intervention, activity suggestions are randomized five times per day for each participant on each day of the 90-day trial. These decision times are specified by each user at the start of the study, and they roughly correspond to the participant’s typical morning commute, lunch time, mid-afternoon, evening commute, and after dinner periods. The treatment options for activity suggestions are binary: at a decision time, the system can either send or not send a notification with a activity suggestion. When provided, the content of the suggestion is tailored to current sensor data (location, weather, time of day, and day of the week). Examples of these suggestions are provided in [27]. At a decision time, activity suggestions are randomized only if the system considers that the user is available for the intervention—i.e., that it is appropriate to intervene at that time (see Figure 5 for criteria used to determine if it is appropriate to send an activity suggestion at a decision time). Subject to these availability criteria, IntelligentPooling determines whether to send a suggestion at each decision time. The posterior distribution was updated once per day, prior to the beginning of each day. Fig. 4 provides a schematic of the feasibility study.

The feasibility study included the second set of 10 participants in the trial of HeartSteps v2, following the initial 10 enrolled participants. IntelligentPooling (Algorithm 1) is deployed for each of the second set of 10 participants. At each decision time for these 10 participants, IntelligentPooling uses all data up to that decision time (i.e. from the initial ten participants as well as from the subsequent ten participants). Thus the feasibility study allows us to assess performance of IntelligentPooling after the beginning of a study instead of the performance at the beginning of the study (when there is little data) or the performance at the end of the study (when there is a large amount of data and the algorithm can be expected to perform well).

In the feasibility study, the feature used in the reward model were selected to be predictive of the baseline reward and/or the treatment effect, based on the data analysis of HeartStepsV1; see section 6.2 in [34] for details. The features used in the reward model are shown in Table 4. The feature engagement represents the extent to which a user engages with the mHealth application.
A user is available to receive an activity suggestion under the following conditions:

- She is not currently active and has not had a large amount of activity in the last two hours.
- She has not recently received a notification with a HeartSteps intervention.
- Her phone has an internet connection and can communicate with the HeartSteps server.
- Her smart watch has been able to communicate with the HeartSteps server in the last ten minutes to provide the current location and step count data.

Figure 5: Availability criteria

measured as a function of how many screen views are made within the application within a day. The feature dosage represents the extent to which a user has received treatments (activity suggestions). This feature increases and decreases depending on the number of activity suggestions recently received. The feature location refers to whether a user is at home or work (encoded as a 1) or somewhere else (encoded as a 0). The temperature feature value is set according to the temperature at a user’s current location (based off of phone GPS). The variation feature value is set according to the variation in step count in the hour around that decision point over the prior seven-day period. We provide a full description of these features in Section E. The prior distribution was also constructed based on HeartStepsV1; see Section 6.3 in [34] for more details. As this feasibility study only includes a small number of users, a simple model with only two person-specific random effects, each on the intercept term in $g$ and $f$ (Eqn. 10) was deployed.

Here we discuss how much data we have to personalize the policy to each user. Recall the 10 users only receive interventions when they meet the availability criteria outlined in Fig. 5; thus we find that in practice we have a limited number of decision points to learn a personalized policy.
| Name                          | Value          | User Specific | Included in f |
|-------------------------------|----------------|---------------|---------------|
| Temperature                   | Continuous     | Yes           | No            |
| Yesterday’s step count        | Continuous     | Yes           | No            |
| Prior 30-minute step count    | Continuous     | Yes           | No            |
| Step variation level          | Discrete       | Yes           | Yes           |
| Engagement with mobile app    | Discrete       | Yes           | Yes           |
| Dosage                        | Continuous     | Yes           | Yes           |
| Location                      | Discrete       | Yes           | Yes           |
| Intercept                     | 1              | Yes           | Yes           |
| Reward                        | Step count     | Yes           | NA            |

Table 4: State feature descriptions for FeasibilityStudy.

Figure 6: We see that IntelligentPooling covers the full range of treatment selection probabilities. The tendency seems to be to send with a lower rather than higher probability.

From. In the case of perfect availability, we would have at most 450 decision points per person. However due to the criteria in Fig. 5, the algorithm is used with only approximately 23% of each user’s decision points. Pooling users’ data allows us to learn more rapidly. On the day that the first pooled user joined the feasibility study there were 107 data points from the first set of 10 users.

The 10 users received an average number of .20 (±0.015) messages a day. The average log step count in the 30-minute window after a suggestion was sent was 4.47, while it was 3.65 in the 30-minute windows after suggestions were not sent. Fig. 6 shows the entire history of treatment selection probabilities for all of the users who received treatment according to IntelligentPooling. We see that the treatment probabilities tended to be low, though they covered the whole range of possible values.

**Personalization** By comparing how the decisions to treat under IntelligentPooling differ from those under Complete, we gather preliminary evidence concerning whether IntelligentPooling personalizes to users. Fig. 7 shows the posterior mean of the coefficient of the $A_{i,k}$ term in $f$, for all users in the feasibility study on the 90th day after the last user joined the study. We
Figure 7: Posterior mean and standard deviation of the coefficient of $A_{i,k}$ in Eqn. 10 for all users in the feasibility study.

Figure 8: Posterior mean of the coefficient of $A_{i,k}$ in Eqn. 10 for users A and B in the feasibility study.

Figure 9: Mean squared distance between posterior and prior mean of the coefficients of $A_{i,k}$
show this term not only for IntelligentPooling but for Complete and Person-Specific. We see that for some users this coefficient is below zero while for others it is above. While the terms under IntelligentPooling differ from Complete they do not vary as much as those learned by Person-Specific. Yet, crucially, the variance is much lower for these terms.

Fig. 8 displays the posterior mean of the coefficient of the $A_{i,k}$ term in $f$. This coefficient represents the overall effect of treatment on one of the users, “User A.” During the prior 7 days User A had not experienced much variation in activity at this time and the user’s engagement is low. Note that the treatment appears to have a positive effect on a different user, User B, in this context whereas on User A there is little evidence of a positive effect. If Complete had been used to determine treatment, User A might have been over-treated.

**Speed of policy learning** We consider the speed at which IntelligentPooling diverges from the prior, relative to the speed of divergence for Person-Specific. Fig. 9 provides the Euclidean distance between the learned posterior and prior parameter vectors (averaged across the data from the 10 users at each time). From Fig. 9 we see that Person-Specific hardly varies over time in contrast to IntelligentPooling and Complete, which suggests that Person-Specific learns more slowly.

In conclusion IntelligentPooling was found to be feasible in this study. In particular the algorithm was operationally stable within the computational environment of the study, produced decision probabilities in a timely manner, and did not adversely impact the functioning of the overall mHealth intervention application. Overall, IntelligentPooling produced treatment selection probabilities which covered the full range of available probabilities, though treatments tended to be sent with a low probability.

## 6 Non-stationary environments

An additional challenge in mHealth settings is that users’ response to treatment can vary over time. To address this challenge we show that our underlying model can be extended to include time-varying random effects. This allows each policy to be aware of how a user’s response to treatment might vary over time. We propose a new simulation to evaluate this approach and show that IntelligentPooling achieves state-of-the-art regret, adjusting to non-stationarity even as user populations vary from heterogenous to homogenous.

### 6.1 Time-varying random effect

In addition to user-specific random effects we extend our model to include time-specific random effects. Consider the Bayesian mixed effect model with person-specific and time-varying effects: for user $i$ at the $k$-th decision time,

$$ R_{i,k} = \phi(S_{i,k}, A_{i,k})^\top w_{i,k} + \epsilon_{i,k}. $$

(12)

In addition, we impose the following additive structure on the parameters $w_{i,k}$:

$$ w_{i,k} = w_{\text{pop}} + u_i + v_k, $$

(13)

where $w_{\text{pop}}$ is the population-level parameter and $u_i$ represents the person-specific deviation from $w_{\text{pop}}$ for user $i$ and $v_k$ is the time-varying random effects allowing $w_{i,k}$ varying with time in the study.
Figure 10: **Disengagement generative model** Regret averaged across all users for each week in the trial, i.e. average regret of all users in their first week of the trial.

The prior terms for this model are as introduced in Section 3.4. Additionally, \( v_k \) has mean 0 and covariance \( D_v \). The covariance between two relative decision times in the trial is
\[
\text{Cov}(v_k, v_{k'}) = \rho(k, k')D_v,
\]
where \( \rho(k, k') = \exp(-\text{dist}(k, k')^2/\sigma_\rho) \) for a distance function, \( \text{dist} \) and \( \theta_{\text{pop}} \perp \{u_i\}\{v_k\} \).

There is no change to Algorithm 1 except that now the algorithm would select the action based on the posterior distribution of \( w_{i,k} \), which depends on both the user and time in the study.

### 6.2 Experiments

We now modify our original simulation environment so that users’ responses will vary over time. To do so we introduce the generative model **Disengagement**. This generative model captures the phenomenon of disengagement. That is as users are increasingly exposed to treatment over time they can become less responsive. This model adds a further term to (9),
\[
A_{i,k}X_w^T\beta_w
\]
where \( X_w \) is defined as follows. At time \( k \) during the trial, let \( w_{i,k} \) be the highest number of weeks user \( i \) has completed at time \( k \); \( X_w \) encodes a user’s current week in a trial, \( X_w = [1\{w_{i,k}=0\}, \ldots, 1\{w_{i,k}=11\}] \).

We set \( \beta_w \) such that the longer a user is in the trial the less they respond to treatment. When a simulated user is at a decision time the user will receive a treatment according to whichever RL policy is being run through the simulation.

In order to evaluate the effectiveness of our time-varying model we compare to Time-Varying Gaussian Process Thompson Sampling (TV-GP) \[3\]. This approach incorporates temporal information for non-stationary environments and was shown to be competitive to stationary models. To compare this method to **INTELLIGENTPOOLING** we use a linear kernel for the spatial component. We then modify Eqn. \[6\] to compute the posterior distribution by removing the random-effects and modifying the kernel (Eqn. \[5\]) to include the temporal terms introduced in \[3\].
Table 5: Average fraction of times treatment was sent (action=1), over 50 simulations (generative model HETEROGENEITY with homogenous $Z^h$ setting).

|                          | Cohort One Week 10 | Cohort Six Week 10 |
|--------------------------|--------------------|--------------------|
| COMPLETE                 | 0.62               | 0.44               |
| PERSON-SPECIFIC          | 0.76               | 0.59               |
| HORDEOB-                 | 0.50               | 0.57               |
| TV-GP                    | 0.64               | 0.31               |
| INTELLIGENT-POOLING      | 0.30               | 0.06               |

Fig. 10 provides the regret averaged across all users across 50 simulated trials where the reward distribution follows generative model DISENGAGEMENT. As before the horizontal axis in Fig. 10 is the average regret over all users in their nth week in the trial, e.g. in their first week, their second week, etc. In DISENGAGEMENT, the time-specific response to treatment is set so that a negative response to treatment is introduced in the seventh week of the trial.

In the DISENGAGEMENT condition as users become increasingly less responsive to treatment good policies should learn to treat less. Thus, Table 5 provides the average number of times a treatment is sent in the last week of the trial for both the first and last cohort. We expect that a policy which learns not to treat will treat less often in the last week of the last cohort than in the last week of the first cohort.

7 Limitations

A significant limitation with this work is that our pilot study involved a small number of participants. Our results from this work must be considered with caution as preliminary evidence towards the feasibility of deploying INTELLIGENTPOOLING, and bandit algorithms in general, in mHealth settings. Moreover, we cannot claim to provide generalizable evidence that this algorithm can improve health outcomes; for this larger studies with more participants must be run. We offer our findings as motivation for such future work.

Our proposed model is designed to overcome the challenges faced when learning personalized policies in limited data settings. As such, if data was abundant our model would likely have limited effectiveness compared to more complex models. For example, a more complex model could allow us to pool between users as a function of their similarity. Our current model instead determines the extent to which a given user deviates from the population and does not consider between-user similarities. A limitation with our current understanding of mHealth is that it is unclear what a good similarity measure would be. We leave the question of designing a data-efficient algorithm for learning such a measure as future work.

A component of INTELLIGENTPOOLING is the use of empirical Bayes to update the model hyperparameters. Here, we used an approximate procedure. However, with our model it is possible to produce exact updates in a streaming fashion and we are currently developing such an approach.
Finally, IntelligentPooling can incorporate a time-specific random effect to capture the phenomenon of responsivity changing over the course of a study. There is much to be improved with this model. For example, the first cohort in a study will not have prior cohorts to learn from, and the final cohort will have the greatest amount of data to benefit from. Other models might treat different cohorts with greater equality. Furthermore, this representation does not incorporate alternative temporal information, such as continually shifting weather patterns, where temperatures might change slowly and gradually alter one’s desire to exercise outside.

8 Conclusion

When data on individuals is limited a natural tension exists between personalizing (a choice which can introduce variance) and pooling (a choice which can introduce bias). In this work we have introduced a novel algorithm for personalized reinforcement learning, IntelligentPooling that presents a principled mechanism for balancing this tension. We demonstrate the practicality of our approach in the setting of mHealth. In simulation we achieve improvements of 26% over a state-of-the-art-method, while in a live clinical trial we show that our approach shows promise of personalization on even a limited number of users. We view adaptive pooling as a first step in addressing the trade-offs between personalization and pooling. The question of how to quantify the benefits and risks for individual users is an open direction for future work.

Acknowledgements

This material is based upon work supported by: NIH/NIAAA R01AA23187, NIH/NIDA P50DA039838, NIH/NIBIB U54EB020404 and NIH/NCI U01CA229437. The views expressed in this article are those of the authors and do not necessarily reflect the official position of the National Institutes of Health, or any other part of the U.S. Department of Health and Human Services

References

[1] Marc Abeille, Alessandro Lazaric, et al. Linear thompson sampling revisited. *Electronic Journal of Statistics*, 11(2):5165–5197, 2017.

[2] Shipra Agrawal and Navin Goyal. Analysis of thompson sampling for the multi-armed bandit problem. In *Conference on Learning Theory*, 2012.

[3] Ilija Bogunovic, Jonathan Scarlett, and Volkan Cevher. Time-varying gaussian process bandit optimization. In *Artificial Intelligence and Statistics*, 2016.

[4] Edwin V Bonilla, Kian M Chai, and Christopher Williams. Multi-task gaussian process prediction. In *Advances in neural information processing systems*, 2008.

[5] Audrey Boruvka, Daniel Almirall, Katie Witkiewitz, and Susan A Murphy. Assessing time-varying causal effect moderation in mobile health. *Journal of the American Statistical Association*, 113(523), 2018.

[6] Eric Brochu, Matthew W Hoffman, and Nando de Freitas. Portfolio allocation for bayesian optimization. *arXiv preprint arXiv:1009.5419*, 2010.
[7] Bradley P Carlin and Thomas A Louis. Bayes and empirical Bayes methods for data analysis. Chapman and Hall/CRC, 2010.

[8] George Casella. An introduction to empirical bayes data analysis. The American Statistician, 39(2):83–87, 1985.

[9] Nicolo Cesa-Bianchi, Claudio Gentile, and Giovanni Zappella. A gang of bandits. In Advances in Neural Information Processing Systems, pages 737–745, 2013.

[10] Wang Chi Cheung, David Simchi-Levi, and Ruihao Zhu. Learning to optimize under non-stationarity. arXiv preprint arXiv:1810.03024, 2018.

[11] Sayak Ray Chowdhury and Aditya Gopalan. On kernelized multi-armed bandits. In Proceedings of the 34th International Conference on Machine Learning-Volume 70, 2017.

[12] Shanice Clarke, Luis G Jaimes, and Miguel A Labrador. mstress: A mobile recommender system for just-in-time interventions for stress. In 2017 14th IEEE Annual Consumer Communications & Networking Conference (CCNC), 2017.

[13] Sunny Consolvo, David W McDonald, Tammy Toscos, Mike Y Chen, Jon Froehlich, Beverly Harrison, Predrag Klasnja, Anthony LaMarca, Louis LeGrand, Ryan Libby, et al. Activity sensing in the wild: a field trial of ubifit garden. In Proceedings of the SIGCHI conference on human factors in computing systems, pages 1797–1806, 2008.

[14] Thomas Desautels, Andreas Krause, and Joel W Burdick. Parallelizing exploration-exploitation tradeoffs in gaussian process bandit optimization. The Journal of Machine Learning Research, 15(1), 2014.

[15] Aniket Anand Deshmukh, Urun Dogan, and Clay Scott. Multi-task learning for contextual bandits. In Advances in Neural Information Processing Systems, 2017.

[16] Josip Djolonga, Andreas Krause, and Volkan Cevher. High-dimensional gaussian process bandits. In Advances in Neural Information Processing Systems, 2013.

[17] Chelsea Finn, Kelvin Xu, and Sergey Levine. Probabilistic model-agnostic meta-learning. In Advances in Neural Information Processing Systems, 2018.

[18] Chelsea Finn, Aravind Rajeswaran, Sham Kakade, and Sergey Levine. Online meta-learning. arXiv preprint arXiv:1902.08438, 2019.

[19] Evan M Forman, Stephanie G Kerrigan, Meghan L Butryn, Adrienne S Juarascio, Stephanie M Manasse, Santiago Ontañón, Diane H Dallal, Rebecca J Crochiere, and Danielle Moskow. Can the artificial intelligence technique of reinforcement learning use continuously-monitored digital data to optimize treatment for weight loss? Journal of behavioral medicine, 2018.

[20] Jacob Gardner, Geoff Pleiss, Kilian Q Weinberger, David Bindel, and Andrew G Wilson. Gpytorch: Blackbox matrix-matrix gaussian process inference with gpu acceleration. In Advances in Neural Information Processing Systems, 2018.

[21] Kristjan Greenewald, Ambuj Tewari, Susan Murphy, and Predag Klasnja. Action centered contextual bandits. In Advances in neural information processing systems, 2017.
[22] Abhishek Gupta, Russell Mendonca, YuXuan Liu, Pieter Abbeel, and Sergey Levine. Meta-reinforcement learning of structured exploration strategies. In Advances in Neural Information Processing Systems, 2018.

[23] Saee Hamine, Emily Gerth-Guyette, Dunia Faulx, Beverly B Green, and Amy Sarah Ginsburg. Impact of mhealth chronic disease management on treatment adherence and patient outcomes: a systematic review. Journal of medical Internet research, 17(2):e52, 2015.

[24] Luis G Jaime, Martin Llofriu, and Andrew Raij. Preventer, a selection mechanism for just-in-time preventive interventions. IEEE Transactions on Affective Computing, 7(3), 2016.

[25] Baekjin Kim and Ambuj Tewari. Near-optimal oracle-efficient algorithms for stationary and non-stationary stochastic linear bandits. arXiv preprint arXiv:1912.05695, 2019.

[26] Predrag Klasnja, Eric B Hekler, Saul Shiffman, Audrey Boruvka, Daniel Almirall, Ambuj Tewari, and Susan A Murphy. Microrandomized trials: An experimental design for developing just-in-time adaptive interventions. Health Psychology, 34(S):1220, 2015.

[27] Predrag Klasnja, Shawna Smith, Nicholas J Seewald, Andy Lee, Kelly Hall, Brook Luers, Eric B Hekler, and Susan A Murphy. Efficacy of Contextually Tailored Suggestions for Physical Activity: A Micro-randomized Optimization Trial of HeartSteps. Annals of Behavioral Medicine, 53(6):573–582, 09 2018. ISSN 0883-6612. doi: 10.1093/abm/kay067. URL https://doi.org/10.1093/abm/kay067.

[28] Andreas Krause and Cheng S Ong. Contextual gaussian process bandit optimization. In Advances in Neural Information Processing Systems, 2011.

[29] Nan M Laird, James H Ware, et al. Random-effects models for longitudinal data. Biometrics, 38(4), 1982.

[30] Neil D Lawrence and John C Platt. Learning to learn with the informative vector machine. In Proceedings of the twenty-first international conference on Machine learning, 2004.

[31] Lihong Li, Wei Chu, John Langford, and Robert E Schapire. A contextual-bandit approach to personalized news article recommendation. In Proceedings of the 19th international conference on World wide web, pages 661–670, 2010.

[32] Shuai Li and Purushottam Kar. Context-aware bandits. arXiv preprint arXiv:1510.03164, 2015.

[33] Peng Liao, Predrag Klasnja, Ambuj Tewari, and Susan A Murphy. Sample size calculations for micro-randomized trials in mhealth. Statistics in medicine, 35(12):1944–1971, 2016.

[34] Peng Liao, Kristjan Greenewald, Predrag Klasnja, and Susan Murphy. Personalized heartsteps: A reinforcement learning algorithm for optimizing physical activity. Proceedings of the ACM on Interactive, Mobile, Wearable and Ubiquitous Technologies, 4(1):1–22, 2020.

[35] Linkai Luo, Yuan Yao, Furong Gao, and Chunhui Zhao. Mixed-effects gaussian process modeling approach with application in injection molding processes. Journal of Process Control, 62, 2018.
[36] Carl N Morris. Parametric empirical bayes inference: theory and applications. *Journal of the American statistical Association*, 78(381):47–55, 1983.

[37] Anusha Nagabandi, Chelsea Finn, and Sergey Levine. Deep online learning via meta-learning: Continual adaptation for model-based rl. *arXiv preprint arXiv:1812.07671*, 2018.

[38] Pablo Paredes, Ran Gilad-Bachrach, Mary Czerwinski, Asta Roseway, Kael Rowan, and Javier Hernandez. Poptherapy: Coping with stress through pop-culture. In *Proceedings of the 8th International Conference on Pervasive Computing Technologies for Healthcare*, 2014.

[39] Fabian Pedregosa et al. Scikit-learn: Machine learning in Python. *Journal of Machine Learning Research*, 12, 2011.

[40] Tianchen Qian, Predrag Klasnja, and Susan A Murphy. Linear mixed models under endogeneity: modeling sequential treatment effects with application to a mobile health study. *arXiv preprint arXiv:1902.10861*, 2019.

[41] Mashfiqui Rabbi, Min Hane Aung, Mi Zhang, and Tanzeem Choudhury. Mybehavior: automatic personalized health feedback from user behaviors and preferences using smartphones. In *Proceedings of the 2015 ACM International Joint Conference on Pervasive and Ubiquitous Computing*, 2015.

[42] Mashfiqui Rabbi, Meredith Philyaw-Kotov, Jinsok Lee, Anthony Mansour, Laura Dent, Xiaolei Wang, Rebecca Cunningham, Erin Bonar, Inbal Nahum-Shani, Predrag Klasnja, et al. Sara: a mobile app to engage users in health data collection. In *Proceedings of the 2017 ACM International Joint Conference on Pervasive and Ubiquitous Computing and Proceedings of the 2017 ACM International Symposium on Wearable Computers*, pages 781–789, 2017.

[43] Stephen W Raudenbush and Anthony S Bryk. *Hierarchical linear models: Applications and data analysis methods*, volume 1. 2002.

[44] Yoan Russac, Claire Vernade, and Olivier Cappé. Weighted linear bandits for non-stationary environments. In *Advances in Neural Information Processing Systems*, pages 12017–12026, 2019.

[45] Daniel Russo and Benjamin Van Roy. Learning to optimize via posterior sampling. *Mathematics of Operations Research*, 39(4):1221–1243, 2014.

[46] Daniel J. Russo, Benjamin Van Roy, Abbas Kazerouni, Ian Osband, and Zheng Wen. A tutorial on thompson sampling. *Foundations and Trends in Machine Learning*, 11(1):1–96, 2018. ISSN 1935-8237. doi: 10.1561/2200000070. URL [http://dx.doi.org/10.1561/2200000070](http://dx.doi.org/10.1561/2200000070).

[47] Steindór Sæmundsson, Katja Hofmann, and Marc Peter Deisenroth. Meta reinforcement learning with latent variable gaussian processes. *arXiv preprint arXiv:1803.07551*, 2018.

[48] JQ Shi, B Wang, EJ Will, and RM West. Mixed-effects gaussian process functional regression models with application to dose–response curve prediction. *Statistics in medicine*, 31(26), 2012.
[49] Niranjan Srinivas, Andreas Krause, Sham M Kakade, and Matthias Seeger. Gaussian process optimization in the bandit setting: No regret and experimental design. *arXiv preprint arXiv:0912.3995*, 2009.

[50] William R Thompson. On the likelihood that one unknown probability exceeds another in view of the evidence of two samples. *Biometrika*, 25(3/4):285–294, 1933.

[51] Sharan Vaswani, Mark Schmidt, and Laks Lakshmanan. Horde of bandits using gaussian markov random fields. In *Artificial Intelligence and Statistics*, 2017.

[52] Yuyang Wang and Roni Khardon. Nonparametric bayesian mixed-effect model: A sparse gaussian process approach. *arXiv preprint arXiv:1211.6653*, 2012.

[53] Zi Wang, Bolei Zhou, and Stefanie Jegelka. Optimization as estimation with gaussian processes in bandit settings. In *Artificial Intelligence and Statistics*, 2016.

[54] Elad Yom-Tov, Guy Feraru, Mark Kozdoba, Shie Mannor, Moshe Tennenholtz, and Irit Hochberg. Encouraging physical activity in patients with diabetes: intervention using a reinforcement learning system. *Journal of medical Internet research*, 19(10), 2017.

[55] Peng Zhao, Lijun Zhang, Yuan Jiang, and Zhi-Hua Zhou. A simple approach for non-stationary linear bandits. In *Proceedings of the 23rd International Conference on Artificial Intelligence and Statistics (AISTATS)*, page to appear, 2020.

[56] Mo Zhou, Yonatan Mintz, Yoshimi Fukuoka, Ken Goldberg, Elena Flowers, Philip Kaminsky, Alejandro Castillejo, and Anil Aswani. Personalizing mobile fitness apps using reinforcement learning. In *IUI Workshops*, 2018.

[57] Luisa M Zintgraf, Kyriacos Shiarlis, Vitaly Kurin, Katja Hofmann, and Shimon Whiteson. CAML: Fast context adaptation via meta-learning, 2019.
A Regret Bound

In this section we prove a high probability regret bound for a modification of IntelligentPooling in a simplified setting. We modify the Thompson sampling algorithm in IntelligentPooling by multiplying the posterior covariance by a tuning parameter, following [2]. This is mainly due to the technical reasons; see [1] for a discussion. We also simplify the setting in this regret analysis. Specifically, we assume that the posterior distribution of all users is updated after every decision time and the hyper-parameters are fixed throughout the study.

Vaswani et al. [51] also provided a regret bound for the Thompson Sampling Horde of Bandits algorithm where the data is pooled using a known, prespecified, social graph. Our proof follows along similar lines with the primary difference being how the prior covariance of all parameters is formulated. Specifically, the prior variance in [51] is constructed by the Laplacian matrix of the social graph, whereas ours is constructed based on the Bayesian mixed effect model (4). As a result, while in Vaswani et al. [51] the regret bound is stated in terms of properties of the social graph, our bound depends on properties of our mixed effect model (e.g., the covariance matrix of the random effects).

Recall that $\Sigma_w$ is the prior covariance of the parameter vector $w_{\text{pop}}$, $\Sigma_u$ is the covariance of the random effect $u_i$ and $\sigma^2$ is the variance of the error term. We assume that both $w_{\text{pop}}$ and $u_i$ have the same dimensions and that $\Sigma_u$ is invertible. Additionally, for simplicity of presentation we assume that the largest eigenvalue in $\Sigma_w$ is at most $d$ and the largest eigenvalue of $\Sigma_u$ is at most $dN$.

Recall that Theorem [1] bounds the regret of IntelligentPooling at time $T$ by:

$$R(T) = \bar{O}\left(dN\sqrt{T}\sqrt{\log\left(\frac{(\text{Tr}(\Sigma_w) + \text{Tr}(\Sigma_u) + \text{Tr}(\Sigma_u^{-1}))}{d}\right) + \frac{T}{\sigma^2 d N} \log \frac{1}{\delta}}\right)$$

with probability $1 - \delta$.

**Proof Sketch of Theorem [1]** We align the decision times from all users by the calendar time. Specifically, for a given time $t$, we retrieve the user index encountered at time $t$ by $i(t)$ and retrieve this user’s decision time index by $k(t)$. IntelligentPooling selects an action $A_{i(t),k(t)} \in \mathcal{A}$ for time $t \in [1, \ldots, T]$. We denote the selected action at time $t$ by $A_t$.

In this setting, we combine each user specific variable into a global shared variable. Recall that a feature vector $\phi(A_{i,k}, S_{i,k})$ encodes contextual variables for the action and state of user $i$ at their $k$-th decision time. For simplicity, we denote by $A_t$ the action $A_{i(t),k(t)}$ at time $t$ and denote the vector $\phi(A_{i(t),k(t)}, S_{i(t),k(t)})$ at time $t$ by $\phi_{A_t}$. Additionally, we let $\phi_{a,t}$ refer to $\phi(a, S_{i(t),k(t)})$ for any $a \in \mathcal{A}$. We introduce a sparse vector $\varphi_{A_{i,t}} \in \mathbb{R}^{dN}$, which contains $\phi_{A_{i,t}}$ vector among $N$ $d$-dimensional vectors, the rest of which are zeros.

In proving the regret we consider the equivalent way of selecting the action. Instead of randomizing the action by the probability, here to select an action we assume the algorithm draws a sample $\tilde{w}_t = \tilde{w}_{i(t),k(t)}$ and then selects the action $A_t = A_{i(t),k(t)} = \arg\max_{a \in \mathcal{A}} \phi_{a,t}^T \tilde{w}_t$ that maximizes the sampled reward. Analogously to $\phi_{a,t}$, we define $\tilde{w}_t$ and $\bar{w}_t$ as the sparse vectors which contain $\tilde{w}_{i(t),k(t)}$ and $\bar{w}_{i(t),k(t)}$ respectively as the $i(t)$-th vector among $Nd$-dimensional vector, the rest of which are zeros.
We concatenate the person-specific parameters \( w_i \) into \( w \in \mathbb{R}^{dN} \). Let the prior covariance of \( w \) be \( \Sigma_0 = 1_{N \times N} \otimes \Sigma_w + I_N \otimes \Sigma_u \). At time \( t \), all contexts observed thus far, for all users, can be combined into one matrix \( \Phi_t \in \mathbb{R}^{t \times dN} \) where a single row \( s \) corresponds to \( \varphi_{a,s} \), the sparse context vector associated with the action \( A \) taken for user \( i(s) \) at their \( k(s) \)-th decision time. Let, \( \Omega_t = \frac{1}{\sigma_t^2} \Phi_t^\top \Phi_t + \Sigma_0 \). At each decision time \( t \) we draw a feature vector \( \tilde{w}_t \sim N(\tilde{w}_t, \sigma_t^2 \Omega_t^{-1}) \).

Now, within this framework, we rewrite the instantaneous regret as \( \Delta_t = \varphi_{a,t}^\top \Delta w_t - \varphi_{A_t}^\top \Delta w_t \). We prove that with high probability both \( \varphi_{a,t}^\top \tilde{w}_t \) and \( \varphi_{A_t}^\top \tilde{w}_t \) are concentrated around their respective means. The standard deviation around the reward at decision time \( t \) for action \( a \) is thus \( s_{a,t} = \sqrt{\varphi_{a,t}^\top \Omega_t^{-1} \varphi_{a,t}} \). We proceed as in [2, 51] by bounding three terms, the event \( \mathcal{E}^\theta_t \), the event \( \mathcal{E}^w_t \) and \( \sum_{t=1}^T s_{A_t,t}^2 \).

**Definition 1.** Let \( \sigma_{\text{umin}}^{-1} \) be the inverse of the smallest eigenvalue of \( \Sigma_w \), \( \sigma_{\text{umax}} \) be the largest eigenvalue of \( \Sigma_w \), \( \sigma_{\text{pmax}} \) be the largest eigenvalue of \( \Sigma_u \) and \( \sigma_{\text{max}} = \sigma_{\text{umax}} + \sigma_{\text{pmax}} \). We assume that \( \sigma_{\text{umax}} \leq dN \) and \( \sigma_{\text{pmax}} \leq d \).

**Definition 2.** For all \( a \), define \( \theta_{a,t} = \varphi_{a,t}^\top \tilde{w}_t \).

**Definition 3.**

\[
\begin{align*}
l_t &= \sqrt{dN \log \left( 1 + \frac{\sigma_{\text{max}} \sigma_{\text{umin}}^{-1}}{\delta} + \frac{t \sigma_{\text{umin}}^{-1}}{dN \delta} \right) + \sqrt{N} \sigma_{\text{pmax}} + \sigma_{\text{umax}}} \\
v_t &= 2\sqrt{dN \log \left( 1 + \frac{\sigma_{\text{max}} \sigma_{\text{umin}}^{-1}}{\delta} + \frac{t \sigma_{\text{umin}}^{-1}}{dN \delta} \right)} \\
g_t &= \min \{ \sqrt{4dN \ln(t)}, \sqrt{4 \ln(|A_t|)} \} v_t + l_T.
\end{align*}
\]

**Definition 4.** Define \( \mathcal{E}^w_t \) and \( \mathcal{E}^\theta_t \) as the events that \( \varphi_{a,t}^\top \tilde{w}_t \) and \( \theta_{A_t,t} \) are concentrated around their respective means. Recall that \( |A| \) is the total number of actions. Formally, define \( \mathcal{E}^w_t \) as the event that

\[
\forall a : \left| \varphi_{a,t}^\top \tilde{w}_t - \varphi_{a,t}^\top \Delta w_t \right| \leq l_t s_{a,t}.
\]

Define \( \mathcal{E}^\theta_t \) as the event that

\[
\forall a : \left| \theta_{A_t,t} - \varphi_{A_t,t}^\top \tilde{w}_t \right| \leq \min \{ 4dN \log(t), 4 \log(|A_t|) \} v_t s_{a,t}.
\]

Let \( \zeta = \frac{1}{4 \sqrt{d} \pi} \). Given that the events \( \mathcal{E}^w_t \) and \( \mathcal{E}^\theta_t \) hold with high probability, we follow an argument similar to Lemma 4 of [2] and obtain the following bound:

\[
\mathcal{R}(T) \leq \frac{3 \sqrt{T}}{\zeta} \sum_{t=1}^T s_{A,t} + \frac{2 \sqrt{T}}{\zeta} \sum_{t=1}^T \frac{1}{\sqrt{2}} + 6\sqrt{T} \sqrt{|A| T \log(2/\delta)}.
\]

(14)

To bound the variance of the selected actions, \( \sum_{t=1}^T s_{A,t} \), we follow an argument similar to [51], and include the prior covariance terms of our model. We prove the following inequality:

\[
\sum_{t=1}^T s_{A,t} \leq \sqrt{dNT} \sqrt{C \left( \log \left( \frac{\text{Tr}(\Sigma_w) + \text{Tr}(\Sigma_u) + \text{Tr}(\Sigma_u^{-1})}{d} \right) + \frac{T}{\sigma_{\text{umin}}^2 dN} \right)},
\]

(15)

28
where $C$ is a constant equal to $\frac{\sigma_{\text{min}}^{-1}}{\log(1+\frac{\sigma_{\text{min}}}{\sigma^2})}$. By combining Eqn. 14 and Eqn. 4 we obtain the bound given in Theorem 1. □

B Supporting Lemmas

Definition 5. Recall that at time $t$ we define as $D_t$ as the history of all observed states, actions, and rewards up to time $t$. Define filtration $F_{t-1}$ as the union of history until time $t - 1$, and the contexts at time $t$, i.e., $F_{t-1} = \{D_{t-1}, \varphi_{a,t}, a \in A\}$. By definition, $F_1 \subseteq F_2 \cdots \subseteq F_{t-1}$. The following quantities are also determined by the history $D_{t-1}$ and the contexts, $\varphi_{a,t}$ and are included in $F_{t-1}$.

- $\hat{w}_t, \Omega_{t-1}$
- $s_{a,t}$ ∀ $a$
- the identity of the optimal action $a^*_t$
- whether $\mathcal{E}_t^w$ is true or not
- the distribution of $\mathcal{N}(\hat{w}_t,\sigma^2_t\Omega_{t-1}^{-1})$

Note that the actual action $A_t$ which is selected at decision point $t$ is not included in $F_{t-1}$.

We now address the lemmas used in the proof which differ from [2, 51].

Lemma 1. For $\delta \in (0,1)$:

$$Pr(\mathcal{E}_t^w) \geq 1 - \frac{\delta}{2}$$

Proof The true reward at time $t$, $R_t = \varphi_{A_t,t}^\top w + \epsilon_t$. Let, $\Omega_t \hat{w}_t = \frac{b_t}{\sigma^2_t}$. Define $S_{t-1} = \sum_{t-1}^{t-1} \epsilon_t \varphi_{a,t}$. Then,

$$S_{t-1} = \sum_{l=1}^{t-1} (R_l - \varphi_{a_l,t}^\top w)\varphi_{a_l,t} = \sum_{l=1}^{t-1} (R_l \varphi_{a_l,t} - \varphi_{a_l,t} \varphi_{a_l,t}^\top w)$$

$$S_{t-1} = b_{t-1} - \sum_{l=1}^{t-1} (\varphi_{a_l,t} \varphi_{a_l,t}^\top w) = b_{t-1} - \sigma^2_t(\Omega_{t-1} \hat{w}_t - \Omega_{t-1} w + \Sigma_0 w)$$

$$\hat{w}_t - w = \Omega_t^{-1} \left( \frac{S_{t-1}}{\sigma^2_t} - \Sigma_0 w \right).$$

The following holds for all $a$:

$$|\varphi_{a,t}^\top \hat{w}_t - \varphi_{a,t}^\top w| = |\varphi_{a,t}^\top (\hat{w}_t - w)|$$

$$\leq |\varphi_{a,t}^\top \Omega_t^{-1} \left( \frac{S_{t-1}}{\sigma^2_t} - \Sigma_0 w \right)|$$

$$\leq \|\varphi_{a,t}\|_{\Omega_t^{-1}} \left( \left\| \frac{S_{t-1}}{\sigma^2_t} - \Sigma_0 w \right\|_{\Omega_t^{-1}} \right).$$
By the triangle inequality,
\[ |\varphi_{a,t}^\top \hat{w}_t - \varphi_{a,t}^\top w| \leq \left( \frac{\|S_{t-1}\|}{\sigma^2} \right) + \|\Sigma_0 w\|_{\Omega_{t-1}^{-1}} \]
(16)

We now bound the term \(\|\Sigma_0 w\|_{\Omega_{t-1}^{-1}}\). Recall that the prior covariance of \(w\), \(\Sigma_0 = \mathbf{1}_{N\times N} \otimes \Sigma_w + \mathbf{I}_N \otimes \Sigma_u\).

\[ \nu_{\text{max}}(\Sigma_0) = \nu_{\text{max}}(1_{N\times N} \otimes \Sigma_w + \mathbf{I}_N \otimes \Sigma_u) \]
\[ = \nu_{\text{max}}(1_{N\times N}) \cdot \nu_{\text{max}}(\Sigma_w) + \nu_{\text{max}}(\mathbf{I}_N) \cdot \nu_{\text{max}}(\Sigma_u) \]
\[ = N \nu_{\text{max}}(\Sigma_w) + \nu_{\text{max}}(\Sigma_u) \]
\[ = N \sigma_{\text{pmax}} + \sigma_{\text{umax}} \]

\[ \|\Sigma_0 w\|_{\Omega_{t-1}^{-1}} \leq \|\Sigma_0 w\|_{\Omega_{t-1}^{-1}} = \sqrt{w^\top \Sigma_0^{-1} \Sigma_0 w} \]
\[ \leq \sqrt{\nu_{\text{max}}(\Sigma_0)} \|w\|_2 \]
\[ \leq \sqrt{\nu_{\text{max}}(\Sigma_0)} \]
\[ \leq \sqrt{N \sigma_{\text{pmax}} + \sigma_{\text{umax}}} \]

For bounding \(\|\varphi_{a,t}\|_{\Omega_{t-1}^{-1}}\), note that
\[ \|\varphi_{a,t}\|_{\Omega_{t-1}^{-1}} = \sqrt{\varphi_{a,t}^\top \Omega_{t-1}^{-1} \varphi_{a,t}} = s_{a,t} \]

We can thus write Eqn. 16
\[ |\varphi_{a,t}^\top \hat{w}_t - \varphi_{a,t}^\top w| \leq s_{a,t} \left( \frac{1}{\sigma^2} \|S_{t-1}\|_{\Omega_{t-1}^{-1}} + \sqrt{n \sigma_{\text{pmax}} + \sigma_{\text{umax}}} \right) \]
(17)

We now bound \(\|S_{t-1}\|_{\Omega_{t-1}^{-1}}\).

**Theorem 2.** For any \(d > 0\), \(t \geq 1\), with probability at least \(1 - \delta\),
\[ \|S_{t-1}\|_{\Omega_{t-1}^{-1}}^2 \leq 2\sigma_e^2 \log \left( \frac{\det \Omega_{t-1}^{\frac{1}{2}} \det \Sigma_0^{-1}}{\delta} \right) \]
\[ \leq 2\sigma_e^2 \left( \log(\det \Omega_t) + \log(\det \Sigma_0^{-1}) - \log(\delta) \right) \]
\[ \leq \sigma_e^2 \left( \log(\det \Omega_t) + \log(\det \Sigma_0^{-1}) - 2 \log(\delta) \right). \]

For any \(n \times n\) matrix \(A\), \(\det(A) \leq \left( \frac{\text{Tr}(A)}{n} \right)^n\). This implies, \(\log(\det(A)) \leq n \log \left( \frac{\text{Tr}(A)}{n} \right)\). Applying this inequality for both \(\Omega_t\) and \(\Sigma_0^{-1}\), we obtain:
\[ \|S_{t-1}\|_{\Omega_{t-1}^{-1}} \leq dN \sigma_e^2 \left( \log \left( \frac{\text{Tr}(\Omega_t)}{dN} \right) + \log \left( \frac{\text{Tr}(\Sigma_0^{-1})}{dN} \right) - \frac{2}{dN} \log(\delta) \right) \]
(18)
Next, we use the fact that

\[ \Omega_t = \Sigma_0 + \sum_{l=1}^{t} \phi_{a,l} \phi_{a,l}^\top \Rightarrow \text{Tr}(\Omega_t) \leq \text{Tr}(\Sigma_0) + t \]

\[ \text{Tr}(\Sigma_0) = \text{Tr}(I_{N \times N} \otimes \Sigma_w + I_{N} \otimes \Sigma_u) \]

\[ = \text{Tr}(I_{N \times N}) \cdot \text{Tr}(\Sigma_w) + \text{Tr}(I_{N}) \cdot \text{Tr}(\Sigma_u) \]

\[ = N \text{Tr}(\Sigma_w) + N \text{Tr}(\Sigma_u) = N(\text{Tr}(\Sigma_w) + \text{Tr}(\Sigma_u)) \]

We now return to Eqn. 18

\[ \left\| S_{t-1} \right\|_{\Omega_{t-1}}^2 \leq dN \sigma_t^2 \left( \log \left( \frac{\text{Tr}(\Sigma_0) + t}{dN} \right) + \log \left( \frac{\text{Tr}(\Sigma_0^{-1})}{dN} \right) - \frac{2}{dN} \log(\delta) \right) \]

\[ \leq dN \sigma_t^2 \left( \log \left( \frac{\text{Tr}(\Sigma_0) \text{Tr}(\Sigma_0^{-1}) + t \text{Tr}(\Sigma_0^{-1})}{d^2 N^2} \right) - \log(\delta^2) \right) \]

\[ = dN \sigma_t^2 \left( \log \left( \frac{\text{Tr}(\Sigma_0) \text{Tr}(\Sigma_0^{-1}) + t \text{Tr}(\Sigma_0^{-1})}{d^2 N^2 \delta} \right) \right) \]

\[ \leq dN \sigma_t^2 \left( \log \left( \frac{\sigma_{\text{max}} \sigma_{\text{min}}^{-1}}{\delta} + \frac{t \sigma_{\text{min}}^{-1}}{dN \delta} \right) \right) \]

\[ \left\| S_{t-1} \right\|_{\Omega_{t-1}} \leq \sigma_t \sqrt{dN \log \left( \frac{\sigma_{\text{max}} \sigma_{\text{min}}^{-1}}{\delta} + \frac{t \sigma_{\text{min}}^{-1}}{dN \delta} \right)} \]

\[ \left\| S_{t-1} \right\|_{\Omega_{t-1}} \leq \sigma_t \sqrt{dN \log \left( 1 + \frac{\sigma_{\text{max}} \sigma_{\text{min}}^{-1}}{\delta} + \frac{t \sigma_{\text{min}}^{-1}}{dN \delta} \right)} \]

\[ \left| \phi_{a,t}^\top w_t - \phi_{a,t}^\top w \right| \leq s_{a,t} \sqrt{dN \log \left( 1 + \frac{\sigma_{\text{max}} \sigma_{\text{min}}^{-1}}{\delta} + \frac{t \sigma_{\text{min}}^{-1}}{dN \delta} \right) + \sqrt{N \sigma_{\text{max}} + \sigma_{\text{min}}} \right) \]

\[ \leq s_{a,t} \]

\[ \square \]

**Lemma 2.** With probability \( 1 - \frac{\delta}{2} \),

\[ \sum_{t=1}^{T} \text{regret}(t) \leq \sum_{t=1}^{T} \frac{3g_t}{\zeta} s_t + \sum_{t=1}^{T} \frac{2g_t}{\zeta t^2} s_t + \sqrt{2 \sum_{t=1}^{T} \frac{36g_t^2}{\zeta^2} \ln\left( \frac{2}{\delta} \right)} \]  

(19)

**Proof** Let \( Z_t \) and \( Y_t \) be defined as follows:

\[ Z_t = \text{regret}(t) - \frac{3g_t}{\zeta} s_t - \frac{2g_t}{\zeta t^2} s_t \]

\[ Y_t = \sum_{l=1}^{t} Z_l \]

31
Hence, $Y_t$ is a super-martingale process:

$$
E[\Delta_t | F_{t-1}] \leq E[\Delta_t] \leq \frac{3g_t}{\zeta} s_t - \frac{2g_t}{\zeta l^2} s_t
$$

We now apply Azuma-Hoeffding inequality. We define $Y_0 = 0$. Note that $|Y_t - Y_{t-1}| = |Z_t|$ is bounded by $1 + 3g_t - 2g_t$. Hence, $c = 6g_t$. Setting $a = \sqrt{2 \ln(\frac{2}{\delta}) \sum_{t=1}^{T} c_t^2}$ in the above inequality, we obtain that with probability $1 - \frac{\delta}{2}$,

$$
Y_t \leq \sqrt{2 \ln(\frac{2}{\delta}) \sum_{t=1}^{T} 36g_t^2}
$$

$$
\sum_{t=1}^{T} \left(\text{regret}(t) - \frac{3g_t}{\zeta} s_t - \frac{2g_t}{\zeta l^2} s_t\right) \leq \sqrt{2 \ln(\frac{2}{\delta}) \sum_{t=1}^{T} 36g_t^2}
$$

$$
\sum_{t=1}^{T} \left(\text{regret}(t)\right) \leq \sum_{t=1}^{T} \frac{3g_t}{\zeta} s_t + \sum_{t=1}^{T} \frac{2g_t}{\zeta l^2} s_t + \sqrt{2 \ln(\frac{2}{\delta}) \sum_{t=1}^{T} 36g_t^2}
$$

Lemma 3. (Azuma-Hoeffding). If a super-martingale $Y_t$ (with $t \geq 0$) and its the corresponding filtration $F_{t-1}$, satisfies $|Y_t - Y_{t-1}| \leq ct$ for some constant $c$ for all $t = 1, \ldots, T$ then for any $x \geq 0$:

$$
Pr(Y_t - Y_0 \geq x) \leq \exp\left(\frac{-x^2}{2 \sum_{t=1}^{T} c_t^2}\right)
$$

Lemma 4. $\sum_{t=1}^{T} s_{A_t,t} \leq \sqrt{dNT} \left(\sqrt{\log\left(\frac{\log(\frac{T}{\sigma^2}) + \log(\frac{T}{\sigma^2}) + \log(\frac{T}{\sigma^2})}{d} + \frac{T}{\sigma^2 dN}\right)}\right)$

For simplicity, we let $s_{A_t,t} = s_t$ below.

$$
\log(\det|\Omega_t|) \geq \log(\det|\Sigma_0|) + \sum_{t=1}^{T} \log(1 + \frac{s_t^2}{\sigma^2})
$$

$$
\geq \log(\det|\Omega_t|) + \log(\det|\Sigma_{u,t} \otimes \Sigma_u|) + \sum_{t=1}^{T} \log(1 + \frac{s_t^2}{\sigma^2})
$$

$$
= n \log(\det|\Sigma_u|) + \sum_{t=1}^{T} \log(1 + \frac{s_t^2}{\sigma^2})
$$
\begin{align*}
\text{Tr}(\Omega_t) & \leq \text{Tr}(\Sigma_0) + \frac{T}{\sigma^2} \\
& = \text{Tr}(\mathbf{1}_{N \times N} \otimes \Sigma_w) + \text{Tr}(\mathbf{1}_N \otimes \Sigma_u) + \frac{T}{\sigma^2} \\
& = \text{Tr}(\mathbf{1}_{N \times N})\text{Tr}(\Sigma_w) + \text{Tr}(\mathbf{1}_N)\text{Tr}(\Sigma_u) + \frac{T}{\sigma^2} \\
& = N\text{Tr}(\Sigma_w) + N\text{Tr}(\Sigma_u) + \frac{T}{\sigma^2}
\end{align*}

Using the determinant-trace inequality, we have the following relation:

\[
\left( \frac{1}{dN} \text{Tr}(\Omega_t) \right)^{dN} \geq \det|\Omega_t| \\
dN \log\left( \frac{1}{dN} \text{Tr}(\Omega_t) \right) \geq \log(\det|\Omega_t|)
\]

\[
dN \log\left( \frac{1}{dN} \text{Tr}(\Omega_t) \right) \geq \log(\det|\Omega_t|) \\
dN \log\left( \frac{1}{dN} \left( \text{Tr}(\Sigma_0) + \frac{T}{\sigma^2} \right) \right) \geq \log(\det|\Omega_t|) \geq N \log(\det|\Sigma_u|) + \sum_{t=1}^{T} \log\left( 1 + \frac{s_t^2}{\sigma^2} \right)
\]

\[
dN \log\left( \frac{1}{dN} \left( \text{Tr}(\Sigma_0) + \frac{T}{\sigma^2} \right) \right) \geq N \log(\det|\Sigma_u|) + \sum_{t=1}^{T} \log\left( 1 + \frac{s_t^2}{\sigma^2} \right) \\
dN \log\left( \frac{1}{dN} \left( \text{Tr}(\Sigma_0) + \frac{T}{\sigma^2} \right) \right) - N \log(\det|\Sigma_u|) \geq \sum_{t=1}^{T} \log\left( 1 + \frac{s_t^2}{\sigma^2} \right)
\]

\[
dN \log\left( \frac{1}{dN} \left( \text{Tr}(\Sigma_0) + \frac{T}{\sigma^2} \right) \right) + N \log(\det|\Sigma_u^{-1}|) \geq \sum_{t=1}^{T} \log\left( 1 + \frac{s_t^2}{\sigma^2} \right)
\]

\[
dN \log\left( \frac{1}{dN} \left( \text{Tr}(\Sigma_0) + \frac{T}{\sigma^2} \right) \right) + dN \log\left( \frac{1}{d} \text{Tr}(\Sigma_u^{-1}) \right) \geq \sum_{t=1}^{T} \log\left( 1 + \frac{s_t^2}{\sigma^2} \right)
\]

33
\[ dN \left( \log \left( \frac{1}{dN} \text{Tr}(\Sigma_0) \right) + \log \left( \frac{1}{dN} \text{Tr}(\Sigma_{u}^{-1}) \right) \right) \geq \sum_{t=1}^{T} \log \left( 1 + \frac{s_t^2}{\sigma^2} \right) \]

\[ dN \left( \log \left( \frac{\text{Tr}(\Sigma_0) \sigma^2_T + T}{\sigma^2_T dN} \right) + \log \left( \frac{1}{dN} \text{Tr}(\Sigma_{u}^{-1}) \right) \right) \geq \sum_{t=1}^{T} \log \left( 1 + \frac{s_t^2}{\sigma^2} \right) \]

\[ dN \left( \log \left( \frac{\text{Tr}(\Sigma_0) \sigma^2_T + T + N \text{Tr}(\Sigma_{u}^{-1}) \sigma^2_T}{\sigma^2_T dN} \right) \right) \geq \sum_{t=1}^{T} \log \left( 1 + \frac{s_t^2}{\sigma^2} \right) \]

\[ dN \left( \log \left( \frac{n \text{Tr}(\Sigma_w) + N \text{Tr}(\Sigma_u) + T + N \text{Tr}(\Sigma_{u}^{-1}) \sigma^2_T}{d \sigma^2_T dN} \right) \right) \geq \sum_{t=1}^{T} \log \left( 1 + \frac{s_t^2}{\sigma^2} \right) \]

Let, \( s_t^2 \leq \sigma_{\text{umin}}^{-1} \). For all \( y \in [0, \sigma_{\text{umin}}^{-1}] \) \( \log(1 + \frac{y}{\sigma^2}) \geq \frac{1}{\sigma_{\text{umin}}} \log(1 + \frac{\sigma_{\text{umin}}^{-1}}{\sigma^2} y) \)

(See argument in [31]).

\[ \log \left( 1 + \frac{s_t^2}{\sigma^2} \right) \geq \frac{1}{\sigma_{\text{umin}}} \log \left( 1 + \frac{\sigma_{\text{umin}}^{-1}}{\sigma^2} s_t^2 \right) \]

\[ \frac{1}{\sigma_{\text{umin}} \log \left( 1 + \frac{1}{\sigma_{\text{umin}} \sigma^2} \right)} \log \left( 1 + \frac{s_t^2}{\sigma^2} \right) \leq s_t^2 \]

\[ \sum_{t=1}^{T} s_t^2 \leq C \sum_{t=1}^{T} \log \left( 1 + \frac{s_t^2}{\sigma^2} \right) \]

Where, \( C = \sigma_{\text{umin}} \log \left( 1 + \frac{1}{\sigma_{\text{umin}} \sigma^2} \right) \)

By Cauchy Schwartz

\[ \sum_{t=1}^{T} s_t \leq \sqrt{T} \sqrt{\sum_{t=1}^{T} s_t^2} \]

\[ \sum_{t=1}^{T} s_t \leq \sqrt{T} \sqrt{C \sum_{t=1}^{T} \log \left( 1 + \frac{s_t^2}{\sigma^2} \right)} \]

\[ \sum_{t=1}^{T} s_t \leq \sqrt{T} \sqrt{CdN \left( \log \left( \frac{\text{Tr}(\Sigma_w) + \text{Tr}(\Sigma_u) + \text{Tr}(\Sigma_{u}^{-1})}{d} + \frac{T}{\sigma^2_T dN} \right) \right)} \]

\[ \sum_{t=1}^{T} s_t \leq \sqrt{dNT} \sqrt{C \left( \log \left( \frac{\text{Tr}(\Sigma_w) + \text{Tr}(\Sigma_u) + \text{Tr}(\Sigma_{u}^{-1})}{d} + \frac{T}{\sigma^2_T dN} \right) \right)} \]

□
C Simulation

We include additional information about the simulation environment. We first explain general information about the simulation environment. We then provide the procedures for generating state variables (features) in the simulation. Finally, we discuss how we used HeartStepsV1 to arrive at the feature representations used in the simulation.

Simulation dynamics Within the simulation states are updated every thirty minutes. Each thirty minutes is associated with a date-time, thus we can acquire the month from the current time which is useful in updating the temperature. The decision times are set roughly two hours apart from 9:00 to 19:00.

Availability In the real-study users are not always available to receive treatment for a suite of reasons. For example, they may be driving a vehicle or they might have recently received treatment. Thus, at each decision time we update the context feature $Available_i \sim Bernoulli(.8)$. for the $i$th user where $Available_i$ is drawn from a Bernoulli. This condition reduces the distance between the settings in the environment and those in a real-world study. At each decision time interventions are only sent to users who are available; i.e. user $i$ cannot receive an intervention when $Available_i = 0$.

Recruitment We follow the recruitment rate observed in HeartStepsV1. For example, if 20% of the total number of participants were recruited in the third week of HeartStepsV1 we recruit 20% of the total number of participants who will be recruited in the third week of the simulation. To explore the effect of running the study for varying lengths we scale the recruitment rates. For example, if the true study ran for 8 weeks, and we want to run a simulation for three weeks, we proportionally scale the recruitment in each of the three weeks so that the relative recruitment in each week remains the same. In these experiments we would like to recruit the entire population within 6 weeks. Thus about 10% of participants are recruited each week, except for the second week of the study where about 30% of all participants are recruited. This reflects the recruitment rates seen in the study, which were more of less consistent throughout besides one increase in the second week.

We generate states from historical data. Given relevant context we search historical data for states which match this given context. This subset of matching states can be used to generate new states. We discuss this in more detail in Section C.1 Then, we describe in more detail how we generate temperature, location and step counts.

C.1 Querying history

Algorithm 2 is used to obtain relevant historical data in order to form a probability distribution over some target feature value. For example, if we would like a probability distribution over discretized temperature IDs under a given context, we would search over the historical data for all temperature IDs present under this context. This set of context-specific temperature IDs can then be used to form a distribution to simulate a new ID. This process of querying historical data is used throughout the simulation and is outlined in Algorithm 2. For example, it is used in generating new step counts, new locations and new temperatures.
Algorithm 2 QueryHistory

1: INPUT = historical data \([x_i; i = [1, N]]\), conditioning state \(x^*\), target data variable \(y = f(x)\),
2: \(S = \{\}\)
3: for \(i = 1\) to \(N\) do
4: if \(x_i == x^*\) then
5: \(\text{Add } f(x_i) \text{ to } S\)
6: end if
7: end for
8: OUTPUT = \(S\)

As the simulation environment simulates draws stochastically from a variety of probability distributions, it is possible it draws a state which was not present in the historical dataset. In this case there is a process for finding a matching state. Similarly we might have a state in the historical dataset with insufficient samples to form an informative (not overly-noisy) distribution. In this case we also find a surrogate state with which to generate future step counts. The idea of the process is to find the closest state to the current state, such that this close state has sufficient data to generate a good distribution. Again, given a state, we want to be able to generate a step count from a distribution with sufficient data to inform its parameters. The pseudocode for how we do so is shown in Algorithm 3.

This algorithm takes as input a target state, \(s^*\). We also have a dictionary (hasmap) formed from the historical dataset. The keys to this dictionary are the states which existed in the dataset. A value is an array of step counts for this state.

Algorithm 3 FindMatch

1: INPUT = current state \(s^* \in \mathbb{R}^d\), dictionary of existing states to step counts \(\mathcal{D} = \{s : \left[c_1, \ldots, c_N\right]\}\)
2: match\(\leftarrow\)None
3: if \(s^* \in \mathcal{D}\) and \(\text{len(\mathcal{D}[s^*]) > 30}\) then
4: match\(\leftarrow s^*\)
5: else
6: \(\text{new}_\text{size} = d-1\)
7: while match is None do
8: #find state of size new size with most data points in historical dataset
9: rank states \(s\) by \(\text{len(\mathcal{D}[s])}\)
10: choose state with greatest len
11: \(\text{temp} \leftarrow \text{max}_s(\text{len(\mathcal{D}[s])})\)
12: if \(\mathcal{D}[\text{temp}] > 30\) then
13: match\(\leftarrow \text{temp}\)
14: end if
15: \(\text{new}_\text{size} = \text{new}_\text{size} - 1\)
16: end while
17: end if

This procedure gives the closest state with the most data points to our current state.
To be more explicit about lines 8-11. A state is a vector of some length, for example \([1, 0, 1]\). When we consider all subsets of size 2, we are considering the subsets \([1, 0],[1, 1]\), and \([0, 1]\). For each of these we can look in the historical data set and find all points where this state was true. Thus for each subset we’ll get a new list of points, \([1, 0] = [c_1, \ldots, c_{N1}]\) \([1, 1] = [c_1, \ldots, c_{N2}]\), \([0, 1] = [c_1, \ldots, c_{N3}]\). We now look at \(N1, N2, N3\) and choose the state with the highest value. For example, if the lists were: \([1, 0] = [c_1, \ldots, c_{100}]\) \([1, 1] = [c_1, \ldots, c_2]\), \([0, 1] = [c_1, \ldots, c_{300}]\), we would choose \(s = [0, 1]\). Now if we encounter the state \([1, 0, 1]\) and there is insufficient data to form a distribution from this state, we will instead form it from the values found under the state \([0, 1]\), \([c_1, \ldots, c_{300}]\).

C.2 Generating temperature

We mimic a trial where everyone resides in the same general area, such as a city. In this setting everyone experiences the same global temperature. We describe how to obtain temperature at any point in time in Algorithm 4. The temperature is updated exactly five times a day.

In the following algorithms \(t\), refers to a timestamp, \(D\) refers to a historical dataset, \(K_t\) refers to a set of temperature IDs, and \(w_{t-1}\) refers to the temperature at the previous timestamp. Here, \(D = \text{HeartStepsV1}\) and \(K_t = \{\text{hot, cold}\}\). The contextual features which influence temperature are time of day, day of the week and the month \(\text{tod}, \text{dow}\) and \(\text{month}\) respectively. Furthermore, at all times besides the first moment in the trial, the next temperature depends on the current temperature \(w_{t-1}\).

Algorithm 4 GetTemperature

1: INPUT \(= t, D, K_t, w_{t-1}\),
2: \(\text{tod} \leftarrow \text{tod}(t)\)
3: \(\text{dow} \leftarrow \text{dow}(t)\)
4: \(\text{month} \leftarrow \text{month}(t)\)
5: if \(w_{t-1}\) is Null then
6: \(q \leftarrow [\text{tod}, \text{dow}, \text{month}]\)
7: else
8: \(q \leftarrow [\text{tod}, \text{dow}, \text{month}, w_{t-1}]\)
9: end if
10: \(p \leftarrow [0]_{K_t}\)
11: \(T \leftarrow \text{QUERYHistory}(D, q, w)\)
12: for \(k \in K_t\) do
13: \(p_k = \frac{1}{|T|} \sum_{i=1}^{|T|} \mathbb{1}_{l_i == k}\)
14: end for
15: \(w_t \sim \text{Categorical}([p_{\text{cold}}, p_{\text{hot}}])\)
16: OUTPUT \(w_t\)

C.3 Generating location

In the following algorithms \(t\), refers to a timestamp, \(g_u\) refers to the group id of user \(i\), \(D\) refers to a historical dataset, \(K_t\) refers to a set of location IDs, and \(l_{t-1}\) refers to the location at the previous timestamp. Here, \(D = \text{HeartStepsV1}\) and \(K_t = \{\text{other, home or work}\}\).
1. User is at a decision time
   (a) User is available
   (b) User is not available

2. User is not at a decision time

As in generating temperature, the contextual features which influence location are time of day, day of the week and the month \(\text{tod, dow and month}\) respectively. Generating location is different from generating temperature in that each user moves from location to location independently. Whereas we model users to share one common temperature, they move from one location to another independently of other users. Thus we also include group id in determining the next location for a given user.

**Algorithm 5 GetLocation**

1: INPUT = \(t, g_u, D, K_l\)
2: \(\text{tod} \gets \text{tod}(t)\)
3: \(\text{dow} \gets \text{dow}(t)\)
4: Find \(t_0\) in \(D\)
5: if \(l_{t-1}\) is Null then
6:   \(q \gets [\text{tod}, \text{dow}, g_u]\)
7: else
8:   \(q \gets [\text{tod}, \text{dow}, g_u, l_{t-1}]\)
9: end if
10: \(L \leftarrow \text{QUERYHISTORY}(D, q, l)\)
11: \(p \leftarrow [0]_{K_l}\)
12: for \(k \in K_l\) do
13:   \(p_k = \frac{1}{|L|} \sum_{l=0}^{|L|} 1_{l_1=k}\)
14: end for
15: \(l_t \sim \text{Categorical}([p_{\text{other}}, p_{\text{home or work}}])\)
16: OUTPUT \(l_t\)

C.4 Generating step-counts

A new step-count is generated for each User active in the study, every thirty-minutes according to one of the following scenarios:

Scenarios 1b and 2 are equivalent with respect to how step-counts are generated; a User’s step count either depends on whether or not they received an intervention (when they are at a decision time and available) or it does not (because they were either not at a decision time or not available). Recall, that if a user is available the final step count is generated according to Eqn. \ref{eqn:stepcount}. This equation requires sufficient statistics from \textsc{HeartStepsV1}. The procedure for obtaining these statistics is shown explicitly in Algorithm 6.

\[
R_{i,k} = N(\mu_{h(S_{i,k})}, \sigma_{h(S_{i,k})}^2) + A_{i,k}(f(S_{i,k})^T \beta_i + Z_i).
\] (24)
Algorithm 6 StepStatistics

1: INPUT = t, g_i, w_i, l_i, D
2: #Compute variables included in conditioning context
3: tod ← tod(t)
4: dow ← dow(t)
5: y ← yst(t, u)
6: q ← [g_i, w_t, tod, dow, y, l_i, u, a]
7: #Obtain step counts from D conditioned on q
8: S ← QueryHistory(D, q, c)
9: \( \hat{\mu}_S \leftarrow \frac{1}{|S|} \sum_{i=0}^{S} s_i \)
10: \( \hat{\sigma}^2_S \leftarrow \frac{1}{|S|} \sum_{i=0}^{S} (s_i - \hat{\mu}_S)^2 \)
11: OUTPUT \( \hat{\mu}_S, \hat{\sigma}^2_S \)

Here, t, g_i, w_i, l_i, D refer to the current time in the trial, the group id of the i^{th} user, the temperature at time t, the location of the i^{th} user, and a historical dataset, respectively. To find sufficient statistics of step counts, we also employ the time of day and day of the week, tod and dow respectively. Finally, yst(t, u) describes the previous step count as high or low.

D Feature construction

We provide more details on the processes used for feature construction. As stated in the paper we rely heavily on the dataset HeartStepsV1 to make all feature construction decisions. The one exception is in the design of the location feature, for which we had domain knowledge to rely on (more detail below)

D.1 Baseline activity

Each user is assigned to one of two groups: a low-activity group or a high-activity group. These groups are found from the historical data. We perform hierarchical clustering using the method hcluster in scikit-learn [39]. We used a euclidean distance metric to cluster the data and found that two groups naturally arose. These groups were consistent with the population of HeartStepsV1, which consisted of participants who were generally either office administrators or students.

D.2 State features

We now briefly outline the decisions for the remaining features: time of day, day of the week, and temperature. For each feature we explored various categorical representations. For each, the question was how many categories to use to represent the data. For each feature we followed the same procedure.

1. We chose a number of categories (k) to threshold the data into
2. We partitioned the data into k categories
3. We clustered the step counts according to these k categories
4. We computed the Calinski-Harabasz score of this clustering.

5. We chose the final $k$ to be that which provided the highest score.

For example, consider the task of representing temperature. Let $l$ be a temperature, $x$ be a step count and $x_{lb}$ be a thirty-minute step count occurring when the temperature $l$ was assigned to bucket $b$. Given a historical dataset, we have a vector $x$ where each entry $x_{i,t}$ refers to the thirty-minute step count of user $i$ at time $t$.

- Let $p$ be a number of buckets. We create $p$ buckets by finding quantiles of $l$. For example, if $p=2$, we find the 50th quantile of $l$. A bucket is defined by a tuple of thresholds $(th_1, th_2)$, such that for a data point $d$ to belong to bucket $i$, $d$ must be in the range of the tuple $(th_1 \leq d < th_2)$.

- For each temperature, we determine the bucket label which best describes this temperature. That is the label $y$ of $l$, is the bucket for which $th_y^1 \leq \bar{s}_l < th_y^2$.

- We now create a vector of labels $y$, of the same length as $x$. Each $y_{i,t}^l$ is the bucket assigned to $l_{i,t}$. For example, if the temperature for user $i$ at time $t$ falls into the lowest bucket, 0 would be the label assigned to $l_{i,t}$. This induces a clustering of step-counts where the label is a temperature bucket.

- We determine the Calinski-Harabasz score of this clustering.

We test this procedure from $p$ equal to 1, through 4.

For example, consider determining a representation for time of day. We choose a partition to be morning, afternoon, evening. For each thirty-minute step count, if it occurred in the morning we assign it to the morning cluster, if it occurred in the afternoon we assign it to the afternoon cluster, etc. Now we have three clusters of step counts and we can compute the C score of this clustering. We repeat the process for different partitions of the day.

**Time of day** To discover the representation for time of day which best explained the observed step counts, we considered all sequential partitions from length 2-8. We found that early-day, late-day, and night best explained the data.

**Day of the week** To discover the representation for day of the week which best explained the observed step counts, we considered two partitions: every day, or weekday/weekend. We found weekday/weekend to be a better fit to the data.

**Temperature** Here we choose different percentiles to partition the data. We consider between 2 and 5 partitions (percentiles at 50, to 20, 40, 60, 80). Here we found two partitions to best fit the step counts. We also tried more complicated representations of weather combined with temperature, however for the purpose of this paper we found a simple representation to best allow us to explore the relevant questions in this problem setting.

**Location** In representing location we relied on domain knowledge. We found that participants tend to be more responsive when they are either at home or work, than in other places. Thus, we decided to represent location as belonging to one of two categories: home/work or other.
E Feasibility Study

In the clinical trial we describe users’ states with the features described in Table 4. The two features which differ from the simulation environment are engagement and exposure to treatment. We clarify these features below.

**Engagement** The engagement variable measures the extent to which a user engages with the mHealth application deployed in the trial. There are several screens within the application that a user can view. Across all users we measure the 40th percentile of number of screens viewed on day $d$. If user $i$ views more than this percentile, we set their engagement level to 1, otherwise it is 0.

**Exposure to treatment** This variable captures the extent to which a user is treated, or the treatment dosage experienced by this user. Let $D_i$ denote the exposure to treatment for user $i$. Whenever a message is delivered to a user’s phone $D_i$ is updated. That is, if a message is delivered between time $t$ and $t + 1$, $D_{t+1} = \lambda D_t + 1$. If a message is not delivered, $D_{t+1} = \lambda D_t$. Here, we set $\lambda$ according to data from HEARTSTEPSV1 and initialize $D$ to 0.