COEFFICIENT ESTIMATES FOR BAZILEVIČ FUNCTIONS OF BI-PRESTARLIKE FUNCTIONS

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Abstract. In the present article, we introduce and study two certain classes of holomorphic prestarlike and bi-univalent functions associated with Bazilevič function. We determinate upper bounds for the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions belonging to these classes. Further we point out certain special cases for our results.

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1. Introduction

Let $A$ indicate the collection of all holomorphic functions in the unit disc $U = \{z \in \mathbb{C} : |z| < 1\}$ that have the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k. \quad (1.1)$$

We also denote by $S$ the sub-collection of the set $A$ containing of functions in $U$ satisfying (1.1) which are univalent in $U$.

A function $f \in A$ is called starlike of order $\delta$ ($0 \leq \delta < 1$), if

$$\text{Re} \left\{ z f'(z) \over f(z) \right\} > \delta, \quad (z \in U).$$

For $f \in A$ given by (1.1) and $g \in A$ defined by

$$g(z) = z + \sum_{k=2}^{\infty} b_k z^k,$$

the "Hadamard product" of $f$ and $g$ is defined (as usual) by

$$(f \ast g)(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k, \quad (z \in U).$$
Ruscheweyh [8] defined and considered the family of "prestarlike functions" of order $\delta$, which are the function $f$ such that $f \ast I_\delta$ is a starlike function of order $\delta$, where

$$I_\delta(z) = \frac{z}{(1-z)^{2(1-\delta)}}, \quad (0 \leq \delta < 1, z \in U).$$

The function $I_\delta$ can be written in the form:

$$I_\delta(z) = z + \sum_{k=2}^{\infty} \varphi_k(\delta)z^k,$$

where

$$\varphi_k(\delta) = \prod_{i=2}^{k} \frac{(i-2\delta)}{(k-1)!}, \quad k \geq 2.$$

We note that $\varphi_k(\delta)$ is a decreasing function in $\delta$ and satisfies

$$\lim_{k \to \infty} \varphi_k(\delta) = \begin{cases} \infty, & \text{if } \delta < \frac{1}{2} \\ 1, & \text{if } \delta = \frac{1}{2} \\ 0, & \text{if } \delta > \frac{1}{2} \end{cases}.$$

Singh [9] (also see Kim and Srivastava [4]) introduced and studied the family of Bazilevič functions $f \in A$ satisfying the condition:

$$\Re \left\{ \frac{z^{1-\gamma}f'(z)}{(f(z))^{1-\gamma}} \right\} > 0, \quad (z \in U, \gamma \geq 0).$$

According to the Koebe one-quarter theorem (see [3]) "every function $f \in S$ has an inverse $f^{-1}$ which satisfies $f^{-1}(f(z)) = z$, $(z \in U)$ and $f(f^{-1}(w)) = w$, $(|w| < r_0(f), r_0(f) \geq \frac{1}{4})", where

$$g(w) = f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_3^3 - 5a_2a_3 + a_4)w^4 + \cdots. \quad (1.2)$$

For $f \in A$, if both $f$ and $f^{-1}$ are univalent in $U$, we say that $f$ bi-univalent function in $U$. We indicate by $\Sigma$ the family of bi-univalent functions in $U$ given by (1.1). In fact, Srivastava et al. [18] have actually revived the study of holomorphic and bi-univalent functions in recent years. Some examples of functions in the family $\Sigma$ are

$$z, \quad \frac{1}{1-z}, \quad \frac{1}{2} \log \left( \frac{1+z}{1-z} \right) \quad \text{and} \quad -\log(1-z)$$

with the corresponding inverse functions

$$\frac{w}{1+w}, \quad \frac{e^{2w} - 1}{e^{2w} + 1} \quad \text{and} \quad \frac{e^w - 1}{e^w},$$

respectively. Other common examples of functions is not a member of $\Sigma$ are

$$z - \frac{z^2}{2} \quad \text{and} \quad \frac{z}{1-z^2}.$$
Recently, many authors introduced various subfamilies of the bi-univalent functions family Σ and investigated upper bounds for the first two coefficients |a_2| and |a_3| in the Taylor-Maclaurin series expansion (1.1) (see, for example [1, 5, 10–17, 19–24]).

We require the following lemma that will be used to prove our main results.

**Lemma 1** ([3]). If h ∈ P, then |c_k| ≤ 2 for each k ∈ N, where P is the family of all functions h holomorphic in U for which

\[ \Re(h(z)) > 0, \quad (z \in U), \]

where

\[ h(z) = 1 + c_1 z + c_2 z^2 + \cdots, \quad (z \in U). \]

2. COEFFICIENT ESTIMATES FOR THE FUNCTIONS FAMILY Ω^∗(Ω)

**Definition 1.** A function f ∈ Σ given by (1.1) is called in the family Ω^∗(λ, γ, δ; α) if it fulfils the conditions:

\[ \left| \arg \left[ \frac{1}{2} \left( \frac{z^{1-\gamma} (f * I_\delta)'(z)}{((f * I_\delta)(z))^{1-\gamma}} + \left( \frac{z^{1-\gamma} (f * I_\delta)'(z)}{((f * I_\delta)(z))^{1-\gamma}} \right)^{\frac{1}{2}} \right) \right] \right| < \frac{\alpha \pi}{2}, \quad (z \in U) \quad (2.1) \]

and

\[ \left| \arg \left[ \frac{1}{2} \left( \frac{w^{1-\gamma} (g * I_\delta)'(w)}{((g * I_\delta)(w))^{1-\gamma}} + \left( \frac{w^{1-\gamma} (g * I_\delta)'(w)}{((g * I_\delta)(w))^{1-\gamma}} \right)^{\frac{1}{2}} \right) \right] \right| < \frac{\alpha \pi}{2}, \quad (w \in U), \quad (2.2) \]

(0 < α ≤ 1, 0 < λ ≤ 1, γ ≥ 0, 0 ≤ δ < 1),

where the function g = f^{-1} is given by (1.2).

**Remark 1.** It should be remarked that the family Ω^∗(Ω) is a generalization of well-known families considered earlier. These families are:

1. For λ = 1 and δ = 1/2, the family Ω^∗(Ω) reduce to the family P^∗(α, γ) which was introduced by Prema and Keerthi [7];
2. For λ = 1, γ = 0 and δ = 1/2, the family Ω^∗(Ω) reduce to the family S^∗_Ω(α) which was given by Brannan and Taha [2];
3. For λ = γ = 1 and δ = 1/2, the family Ω^∗(Ω) reduce to the family H^\alpha_Ω which was investigated by Srivastava et al. [18].

**Theorem 1.** Let f ∈ Ω^∗(Ω, γ, δ; α) (0 < α ≤ 1, 0 < λ ≤ 1, γ ≥ 0, 0 ≤ δ < 1) be given by (1.1). Then

\[ |a_2| \leq \frac{2\alpha \lambda}{\sqrt{\alpha \lambda (\gamma + 2)(\lambda + 1)(1 - \delta)(2\gamma(1 - \delta) + 1) + \Gamma(\alpha, \lambda)(\gamma + 1)^2(1 - \delta)^2}} \]
and

$$|a_3| \leq \frac{4\alpha\lambda}{(\lambda + 1)(1 - \delta)} \left[ \frac{\alpha\lambda}{(\gamma + 1)^2(\lambda + 1)(1 - \delta)} + \frac{1}{(\gamma + 2)(3 - 2\delta)} \right],$$

where

$$\Upsilon(\alpha, \lambda) = 2\alpha(1 - \lambda) + (1 - \lambda)(\lambda + 1)^2. \quad (2.3)$$

**Proof.** It follows from conditions (2.1) and (2.2) that

$$\frac{1}{2} \left( \frac{z^{1-\gamma}(f * I_\delta)'(z)}{((f * I_\delta)(z))^{1-\gamma}} + \left( \frac{z^{1-\gamma}(f * I_\delta)'(z)}{((f * I_\delta)(z))^{1-\gamma}} \right)^{1/2} \right) = |p(z)|^\alpha \quad (2.4)$$

and

$$\frac{1}{2} \left( \frac{w^{1-\gamma}(g * I_\delta)'(w)}{((g * I_\delta)(w))^{1-\gamma}} + \left( \frac{w^{1-\gamma}(g * I_\delta)'(w)}{((g * I_\delta)(w))^{1-\gamma}} \right)^{1/2} \right) = |q(w)|^\alpha, \quad (2.5)$$

where $g = f^{-1}$ and $p, q$ in $P$ have the following series representations:

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots \quad (2.6)$$

and

$$q(w) = 1 + q_1 w + q_2 w^2 + q_3 w^3 + \cdots. \quad (2.7)$$

Comparing the corresponding coefficients of (2.4) and (2.5) yields

$$\frac{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}{\lambda} a_2 = \alpha p_1, \quad (2.8)$$

$$\frac{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}{2\lambda} a_3$$

$$+ \frac{\lambda(\gamma + 2)(\gamma - 1)(\lambda + 1) + (\gamma + 1)^2(1 - \lambda)}{\lambda^2} (1 - \delta)^2 a_2^2 = \alpha p_2 + \frac{\alpha(\alpha - 1)}{2} p_1^2, \quad (2.9)$$

and

$$\frac{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}{\lambda} a_2$$

$$+ \frac{\lambda(\gamma + 2)(\gamma - 1)(\lambda + 1) + (\gamma + 1)^2(1 - \lambda)}{\lambda^2} (1 - \delta)^2 a_2^2 = \alpha q_2 + \frac{\alpha(\alpha - 1)}{2} q_1^2, \quad (2.10)$$

In view of (2.8) and (2.10), we conclude that

$$p_1 = -q_1 \quad (2.12)$$
and
\[ \frac{2(\gamma + 1)^2(\lambda + 1)^2(1 - \delta)^2}{\lambda^2} a_2^2 = \alpha^2 (p_1^2 + q_1^2). \] (2.13)

Also, by using (2.9) and (2.11), together with (2.13), we find that
\[
\left( \frac{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}{\lambda} + 2 \left[ \frac{\lambda(\gamma + 2)(\gamma - 1)(\lambda + 1) + (\gamma + 1)^2(1 - \lambda)}{\lambda^2} (1 - \delta)^2 \right] \right) a_2^2
\]
\[= \alpha(p_2 + q_2) + \frac{\alpha(\alpha - 1)}{2} (p_1^2 + q_1^2) = \alpha(p_2 + q_2) + \frac{\alpha(\gamma + 1)^2(\lambda + 1)^2(1 - \delta)^2}{\alpha^2} a_2^2. \]

Further computations show that
\[ a_2^2 = \frac{\alpha^2 \lambda^2 (p_2 + q_2)}{\alpha \lambda (\gamma + 2)(\lambda + 1)(1 - \delta) (2\gamma(1 - \delta) + 1) + \Upsilon(\alpha, \lambda)(\gamma + 1)^2(1 - \delta)^2}, \] (2.14)

where \( \Upsilon(\alpha, \lambda) \) is given by (2.3).

By taking the absolute value of (2.14) and applying Lemma 1 for the coefficients \( p_2 \) and \( q_2 \), we have
\[ |a_2| \leq \frac{2\alpha \lambda}{\sqrt{\alpha \lambda (\gamma + 2)(\lambda + 1)(1 - \delta) (2\gamma(1 - \delta) + 1) + \Upsilon(\alpha, \lambda)(\gamma + 1)^2(1 - \delta)^2}}. \]

To determine the bound on \( |a_3| \), by subtracting (2.11) from (2.9), we get
\[ \left( \frac{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}{\lambda} \right) (a_3 - a_2^2) = \alpha(p_2 - q_2) + \frac{\alpha(\alpha - 1)}{2} (p_1^2 - q_1^2). \] (2.15)

Now, substituting the value of \( a_2^2 \) from (2.13) into (2.15) and using (2.12), we deduce that
\[ a_3 = \frac{\alpha^2 \lambda^2 (p_1^2 + q_1^2)}{2(\gamma + 1)^2(\lambda + 1)^2(1 - \delta)^2} + \frac{\alpha \lambda(p_2 - q_2)}{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}. \] (2.16)

Taking the absolute value of (2.16) and applying Lemma 1 once again for the coefficients \( p_1, p_2, q_1 \) and \( q_2 \), it follows that
\[ |a_3| \leq \frac{4\alpha \lambda}{(\lambda + 1)(1 - \delta)} \left[ \frac{\alpha \lambda}{(\gamma + 1)^2(\lambda + 1)(1 - \delta)} + \frac{1}{(\gamma + 2)(3 - 2\delta)} \right]. \]

\[ \square \]

Remark 2. In Theorem 1, if we choose
1) \( \lambda = 1 \) and \( \delta = \frac{1}{2} \), then we have the results which was given by Prema and Keerthi [7, Theorem 2.2];
(2) \( \lambda = 1, \gamma = 0 \) and \( \delta = \frac{1}{4} \), then we have the results obtained by Murugusundaramoorthy et al. [6, Corollary 6];
(3) \( \lambda = \gamma = 1 \) and \( \delta = \frac{1}{2} \), then we obtain the results obtained by Srivastava et al. [18, Theorem 1].

3. Coefficient estimates for the functions family \( \Omega_{\Sigma}^\ast(\lambda, \gamma; \delta; \beta) \)

**Definition 2.** A function \( f \in \Sigma \) given by (1.1) is called in the family \( \Omega_{\Sigma}^\ast(\lambda, \gamma; \delta; \beta) \) if it fulfills the conditions:

\[
\text{Re} \left\{ \frac{1}{2} \left( \frac{z^{1-\gamma}(f * I_\delta)'(z)}{(f * I_\delta)(z)^{1-\gamma}} + \left( \frac{z^{1-\gamma}(f * I_\delta)'(z)}{(f * I_\delta)(z)^{1-\gamma}} \right)^\frac{1}{\lambda} \right) \right\} > \beta, \quad (z \in U) \tag{3.1}
\]

and

\[
\text{Re} \left\{ \frac{1}{2} \left( \frac{w^{1-\gamma}(g * I_\delta)'(w)}{(g * I_\delta)(w)^{1-\gamma}} + \left( \frac{w^{1-\gamma}(g * I_\delta)'(w)}{(g * I_\delta)(w)^{1-\gamma}} \right)^\frac{1}{\lambda} \right) \right\} > \beta, \quad (w \in U), \tag{3.2}
\]

\[0 \leq \beta < 1, \quad 0 < \lambda \leq 1, \quad \gamma \geq 0, \quad 0 \leq \delta < 1,
\]

where the function \( g = f^{-1} \) is given by (1.2).

**Remark 3.** It should be remarked that the family \( \Omega_{\Sigma}^\ast(\lambda, \gamma; \delta; \beta) \) is a generalization of well-known families consider earlier. These families are:

1. For \( \lambda = 1 \) and \( \delta = \frac{1}{2} \), the family \( \Omega_{\Sigma}^\ast(\lambda, \gamma; \delta; \beta) \) reduce to the family \( P_{\Sigma}(\beta, \gamma) \) which was introduced by Prema and Keerthi [7];
2. For \( \lambda = 1, \gamma = 0 \) and \( \delta = \frac{1}{2} \), the family \( \Omega_{\Sigma}^\ast(\lambda, \gamma; \delta; \beta) \) reduce to the family \( S_{\Sigma}(\beta) \) which was given by Brannan and Taha [2];
3. For \( \lambda = \gamma = 1 \) and \( \delta = \frac{1}{2} \), the family \( \Omega_{\Sigma}^\ast(\lambda, \gamma; \delta; \beta) \) reduce to the family \( H_{\Sigma}(\beta) \) which was investigated by Srivastava et al. [18].

**Theorem 2.** Let \( f \in \Omega_{\Sigma}^\ast(\lambda, \gamma; \delta; \beta) \) \((0 \leq \beta < 1, \quad 0 < \lambda \leq 1, \quad \gamma \geq 0, \quad 0 \leq \delta < 1)\) be given by (1.1). Then

\[
|a_2| \leq \frac{2\lambda \sqrt{1-\beta}}{\sqrt{\lambda(\gamma+2)(\lambda+1)(1-\delta)(2\gamma(1-\delta)+1)+2(1-\lambda)(\gamma+1)^2(1-\delta)^2}}
\]

and

\[
|a_3| \leq \frac{4\lambda(1-\beta)}{(\lambda+1)(1-\delta)^2} \left[ \frac{\lambda(1-\beta)}{(\gamma+1)^2(\lambda+1)(1-\delta)} + \frac{1}{(\gamma+2)(3-2\delta)} \right].
\]

**Proof.** In the light of the conditions (3.1) and (3.2), there are \( p, q \in \mathcal{P} \) such that

\[
\frac{1}{2} \left( \frac{z^{1-\gamma}(f * I_\delta)'(z)}{(f * I_\delta)(z)^{1-\gamma}} + \left( \frac{z^{1-\gamma}(f * I_\delta)'(z)}{(f * I_\delta)(z)^{1-\gamma}} \right)^\frac{1}{\lambda} \right) \right\} = \beta + (1-\beta)p(z) \tag{3.3}
\]
and

\[
\frac{1}{2} \left( w^{1-\gamma} (g * I_\delta)'(w) \left( \frac{w^{1-\gamma} (g * I_\delta)'(w)}{((g * I_\delta)(w))^{1-\gamma}} \right)^{1/2} \right) = \beta + (1 - \beta) q(w), \tag{3.4}
\]

where \( p(z) \) and \( q(w) \) have the forms (2.6) and (2.7), respectively. Comparing the corresponding coefficients in (3.3) and (3.4) yields

\[
(\gamma + 1)(\lambda + 1)(1 - \delta) a_2 = (1 - \beta) p_1, \tag{3.5}
\]

\[
\frac{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}{2\lambda} a_3 + \left[ \frac{\lambda(\gamma + 2)(\gamma - 1)(\lambda + 1) + (\gamma + 1)^2 (1 - \lambda)}{\lambda^2} \right] (1 - \delta)^2 a_2 = (1 - \beta) p_2, \tag{3.6}
\]

\[
- \frac{\gamma + 1)(\lambda + 1)(1 - \delta)}{\lambda} a_2 = (1 - \beta) q_1 \tag{3.7}
\]

and

\[
\frac{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}{2\lambda} (2a^2 - a_3) + \left[ \frac{\lambda(\gamma + 2)(\gamma - 1)(\lambda + 1) + (\gamma + 1)^2 (1 - \lambda)}{\lambda^2} \right] (1 - \delta)^2 a_2 = (1 - \beta) q_2. \tag{3.8}
\]

From (3.5) and (3.7), we get

\[
p_1 = -q_1 \tag{3.9}
\]

and

\[
\frac{2(\gamma + 1)^2 (\lambda + 1)^2 (1 - \delta)^2}{\lambda^2} a_2 = (1 - \beta)^2 (p_1^2 + q_1^2). \tag{3.10}
\]

Adding (3.6) and (3.8), we obtain

\[
\left( \frac{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}{\lambda} + 2 \left[ \frac{\lambda(\gamma + 2)(\gamma - 1)(\lambda + 1) + (\gamma + 1)^2 (1 - \lambda)}{\lambda^2} \right] (1 - \delta)^2 \right) a_2^2 = (1 - \beta)(p_2 + q_2). \tag{3.11}
\]

Hence, we find that

\[
a_2^2 = \frac{\lambda^2(1 - \beta)(p_2 + q_2)}{\lambda(\gamma + 2)(\lambda + 1)(1 - \delta)(2\gamma(1 - \delta) + 1) + 2(1 - \lambda) (\gamma + 1)^2 (1 - \delta)^2}.
\]
By applying Lemma 1 for the coefficients $p_2$ and $q_2$, we deduce that
\[ |a_2| \leq \frac{2\lambda \sqrt{1 - \beta}}{\sqrt{\lambda (\gamma + 2)(\lambda + 1)(1 - \delta)(2\gamma(1 - \delta) + 1) + 2(1 - \lambda)(\gamma + 1)^2(1 - \delta)^2}}. \]

To determinate the bound on $|a_3|$, by subtracting (3.8) from (3.6), we get
\[ \frac{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}{\lambda} (a_3 - a_2^2) = (1 - \beta)(p_2 - q_2), \]
or equivalently
\[ a_3 = a_2^2 + \frac{\lambda(1 - \beta)(p_2 - q_2)}{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}. \quad (3.12) \]
Substituting the value of $a_2^2$ from (3.10) into (3.12), it follows that
\[ a_3 = \lambda^2(1 - \beta)^2(p_1^2 + q_1^2) + \frac{\lambda(1 - \beta)(p_2 - q_2)}{(\gamma + 2)(\lambda + 1)(1 - \delta)(3 - 2\delta)}. \]

By applying Lemma 1 once again for the coefficients $p_1$, $p_2$, $q_1$ and $q_2$, we deduce that
\[ |a_3| \leq \frac{4\lambda(1 - \beta)}{(\lambda + 1)(1 - \delta)} \left[ \frac{\lambda(1 - \beta)}{(\gamma + 2)(\lambda + 1)(1 - \delta)} + \frac{1}{(\gamma + 1)^2(1 - \delta)} \right]. \]

□

Remark 4. In Theorem 2, if we choose
1. $\lambda = 1$ and $\delta = \frac{1}{2}$, then we have the results which was given by Prema and Keerthi [7, Theorem 3.2];
2. $\lambda = 1$, $\gamma = 0$ and $\delta = \frac{1}{2}$, then we have the results obtained by Murugusundaramoorthy et al. [6, Corollary 7];
3. $\lambda = \gamma = 1$ and $\delta = \frac{1}{2}$, then we obtain the results obtained by Srivastava et al. [18, Theorem 2].

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