Effect of spatial variability of soil properties on permanent seismic displacements of slopes with uniform load

Nikolaos Alamanis¹ and Panos Dakoulas²

ABSTRACT: The purpose of this study is to investigate the development of permanent displacements of earth slopes, with spatially varying properties and a uniform surface load, subjected to a strong seismic excitation. The uncertainties associated with the determination of the mechanical properties of the slopes and the intensity of the seismic excitation, make the use of stochastic methods very attractive. As opposed to deterministic methods, stochastic methods allow an acceptable risk level based on the specifications of each project. Recent research described the spatial variability of soil parameters using the Random Field Theory. Apart from the prescribed statistical characteristics and cross-correlation of the various soil properties, the generated random field variables exhibit autocorrelation, a trend in which the soil properties of a point appear to be correlated with the soil properties of neighbouring points. Among the various random field algorithms, a particularly effective one is the Local Average Subdivision (LAS) method by Fenton and Vanmarcke [1]. To this end, a large number of random fields of soil properties is generated for a natural slope, having prescribed mean value \( \mu \), standard deviation \( \sigma \), cross-correlation coefficients \( \rho_{ij} \) of properties \( i \) and \( j \), and spatial autocorrelation lengths \( l_x \) and \( l_y \) for the vertical and horizontal directions, respectively. The numerical simulations are achieved using an automated procedure, based on collaboration of Mathematica, the incorporated LAS algorithm, and the finite difference program FLAC. The studied slope is loaded with a uniform load and subjected to a seismic shaking of various intensities using a series of acceleration records obtained from the five different earthquakes. The paper focuses mainly on the development of permanent seismic displacements of the slope at the end of ground shaking. It is demonstrated that the spatial variability of soil properties has a significant effect on the values of permanent displacements, resulting to a wide range of displacement variation.

Keywords: slope stability, spatial variability, random fields, nonlinear behaviour, seismic analysis, permanent seismic displacement.

¹ PhD, MSc, Assistant Professor, General Department, University of Thessaly
² Professor, Department of Civil Engineering, University of Thessaly
1. Introduction
Numerous studies have been undertaken in recent years to develop probabilistic approaches that account for the uncertainties of soil properties [1,2,3,4,5,6]. The present study examines the influence of the spatial variability of soil properties on the stability and seismic performance of highway slopes (with uniform load). The methodology is based on the development of random fields of soil properties characterized by specific mean, variance, cross-correlation coefficients and autocorrelation lengths, using the Local Average Subdivision method (LAS) proposed by Fenton and Vanmarcke [1].

2. Numerical methodology

2.1 Methodology
A fully automated procedure is created for the development of appropriate random fields of material properties, followed in a seamless manner by numerical analysis of an earth slope using the finite difference program FLAC [7]. In each analysis, an evaluation of the slope stability is conducted initially, using the stress-strain relationship of the soil leading to a direct determination of the true failure mechanism, without any need for trial failure surfaces. Subsequently, a simulation of the seismic behavior of the slope is performed aiming at the evaluation of the permanent horizontal and vertical displacements at the end of the shaking. The numerical analysis is used in a Monte Carlo simulation procedure to examine parametrically the effect of the spatial variability of soil properties, the slope inclination and the earthquake excitation characteristics.

2.2 Model
Figure 1 illustrates the geometry and the numerical discretization of a slope with inclination equal to 2:1. The constitutive model used for the soil is the elastoplastic Mohr-Coulomb model, which is defined by the cohesion c, friction angle φ, density ρ, modulus of elasticity Young E, Poisson's ratio ν, and dilatancy angle ψ. All material properties are considered as randomly varied in space, except of the angle of dilatancy ψ and Poisson's ratio ν which are taken as constant.

The mean values, \( \mu \), and standard deviations, \( \sigma \), of soil properties used in this study are given in Table 1. The correlation coefficients \( \rho_{ij} \) of the above soil properties, obtained from available published data, are given in Table 2 [8,9,10].

Each of the above properties is considered as a Gaussian random field defined by its mean value \( \mu \), variance \( \sigma^2 \) and autocorrelation function \( \rho(x,y) \). A widely used exponential autocorrelation function is employed, expressed as

\[
\rho(x,y) = \exp\left(-\frac{|x-x'|}{l_x} - \frac{|y-y'|}{l_y}\right)
\]  

(1)
where $l_x$ and $l_y$ are the characteristic autocorrelation lengths in the horizontal and vertical directions, respectively. The values of $l_x$ and $l_y$ considered in this study are equal to 20 and 2 m, respectively.

For the seismic analysis, the Mohr-Coulomb model is enhanced with a hysteretic sig3 model with parameters $a = 1$, $b = -0.55$ and $x_o = -1.22$ [7].

2.3 Earthquake Excitation

The numerical excitation consists of five acceleration records obtained from the Kalamata (1986), Lefkada (2003), Kobe (1995), Friuli (1976) and Northridge (1994) earthquakes.

| Table 1. Mean value and standard deviation of soil properties for the geometry A. |
|-------------------|-----|-----|
| **Parameter**     | $\mu$ | $\sigma/\mu$ |
| $c$, kPa          | 30   | 0.3   |
| $\varphi^\circ$, deg | 20$^\circ$ | 0.2 |
| $E$, MPa          | 60   | 0.2   |
| $\rho$, t/m$^3$   | 2    | 0.1   |
| $\psi^\circ$, deg | 0$^\circ$ | 0   |
| $v$               | 0.3  | 0     |

| Table 2. Correlation coefficient of soil properties. |
|-------------------|-----|-----|-----|-----|-----|
| **Correlation factor $\rho_{ij}$** |
| Property | $c$ | $\varphi^\circ$ | $\rho$ | $E$ | $v$ |
| $c$      | 1   | -0.5           | 0.5    | 0.2 | 0   |
| $\varphi^\circ$ | -0.5 | 1               | 0.5    | 0.2 | 0   |
| $\rho$   | 0.5 | 0.5            | 1      | 0   | 0   |
| $E$      | 0.2 | 0.2            | 0      | 1   | 0   |
| $v$      | 0   | 0              | 0      | 0   | 1   |

Table 3 provides the characteristics of the historical records used in this study. These records have been adjusted to match the Eurocode 8 acceleration spectra for hard soil and rock sites, calibrated for a peak ground acceleration of 0.3g. Fig. 2 plots the acceleration spectra of the five excitations and the Eurocode 8 design spectra.
Figure 2. Acceleration spectra of modified seismic excitations and Eurocode 8 spectra for rock sites.

Table 3. Characteristics of Historical Seismic Excitation Records.

| Earthquake          | Mw  | R (km) | Recording | Component | PGA (g) |
|---------------------|-----|--------|-----------|-----------|---------|
| Kalamata (1986)     | 6.0 | 12     | Prefecture| Hor.      | 0.25g   |
| Lefkada (2003)      | 6.4 | 10     | Lefkada   | Trans.    | 0.60g   |
| Kobe (1995)         | 7.2 | 20     | Port Island| horizontal| 0.57g   |
| Northridge (1994)   | 6.7 | 30     | Rinaldi   | Hor. 318 | 0.47g   |
| Friuli (1976)       | 6.5 | 19     | Friuli    | Hor.      | 0.35g   |

3. Illustrative example
As an illustrative example, the earth slope in Fig. 1 is considered, having the soil properties given in Table 1 and correlation coefficients equal to those in Table 2. The autocorrelation lengths are $l_x = 20$ m in the horizontal direction and $l_y = 2$ m in the vertical direction. Figures 3a, 3b and 3c illustrate one realization of the spatial distribution of the random fields of soil cohesion $c$, friction angle $\phi$ and Young's elastic modulus $E$. 

(a) (b)
3.1 Static Stability
The slope stability is analyzed using the strength reduction method to determine the factor of safety FS. This is achieved using a trial-and-error technique whereby numerical simulations are performed for a range of parameter values until the critical strength parameters are found.

The failure surface is given in Fig. 4a, which shows the distribution of the maximum equivalent shear strain increment within the slope. For comparison purposes, Fig. 4b, shows the corresponding failure surface of a similar slope with a homogeneous soil and properties equal to the mean values of those used in the above example. It is noted that the spatial variability of properties affects the factor of safety. Results from additional analyses indicate that it may also affect the location of the failure surface.

3.2 Seismic behavior
Figure 5 shows the horizontal permanent displacement $u_x$ at the end of the seismic shaking, having a maximum value of about 2.10 m in the area of the lower half of the
slopes. The settlement $u_y$, not shown here, has a maximum value about 1.30 m of the top horizontal surface. The results of the analysis show that a significant mass of soil moves to the left due to the inertial forces that develop during seismic vibration.

Figure 5. Permanent horizontal displacement after the end of seismic vibration.

The evolution of the horizontal and vertical displacement at point A (see Fig. 1) is given in Fig. 6, for Kalamata earthquake with biggest residual values at the end of shaking equal to 1.10m and 1.15 m, respectively. It is evident from the results of Fig. 6a and 6b that there are significant dispersions in the response and permanent displacement of the slope due to the spatial variability of the soil properties, despite that the average values of soil properties are the same for all random fields. Consequently, spatial variability of properties has a significant effect on the permanent displacements of the slope.

Figure 6. (a) Horizontal displacement at point A for heterogeneous soil (b) Vertical displacement at point A for heterogeneous soil (Kalamata earthquake).

4. Parametric analysis
This section summarizes some key results of the parametric investigation using the slope geometry of Fig. 1. This section summarizes some key results of the parametric investigation using the slope geometry of Fig. 1. The soil properties used in the parametric analysis for the geometry A are already defined in Table 1 and in Table 2 while the seismic analyses were per-formed using the excitation records in Table 3.
4.1 Slope stability under static conditions
Initially, the effect of spatial variability of soil properties on the factor of safety $FS$ under static conditions is considered. To this end, the distribution of the ratio $FS / FS_0$ is examined, where $FS_0$ is the factor of safety of a homogeneous slope having similar geometry and material properties equal to the average value $\mu$ of properties of the heterogenous slopes. Fig. 7 plots the distribution of $FS / FS_0$ from 150 similar slope stability analyses. It can be seen that the results can be described with a normal distribution with average value slightly less than one ($\mu = 0.996$) and a relatively small standard deviation ($\sigma = 0.066$), corresponding to a range of variation from 0.8 to 1.2.

Fig. 8 plots the probability of density of the factor of safety. It is evident that a slope with average values of strength parameters that result into an average factor of safety value $FS = 1.55$, may have a real $FS$ factor which ranges between 1.25 and 1.85 due to the spatial variability of soil properties. The reduction of the $FS$ from 1.55 obtained for the homogeneous slope to a probable 1.25 due to the variability of soil properties, shows that there is a 50% chance that the $FS$ value is less that the value based on the assumption of a homogeneous slope.

4.2 Seismic behavior-accumulation of permanent deformations
To investigate the effect of the spatial variability of soil properties, it is desirable to separate the effect of the characteristics of the seismic excitation used in the analysis from the influence of spatial variability of material properties. To this end, the ratio $f_x = \frac{u_x}{\bar{u}_x}$ is considered, where $u_x$ is the permanent horizontal displacement at the end of the vibration and $\bar{u}_x$ is the permanent hor. displacement of a homogeneous slope with property values equal to the average values of those of the heterogeneous slope.

![Figure 7. Data distribution and probability density of the ratio FS / FS0.](image)

![Figure 8. Probability density of the factor of safety.](image)
Figure 9a shows the probability of density of the results of permanent horizontal displacement ratio \( f_x = \frac{u_x}{\bar{u}_x} \) at the top of the slope (point A in Fig. 1) from all seismic analyses using geometry A for a peak ground acceleration of 0.3g. Similarly, Fig. 9b plots the probability density of the permanent vertical displacement ratio \( f_y = \frac{u_y}{\bar{u}_y} \) at the top of the slope. As shown in Fig. 9a, the average permanent hor. displacement of slope A with uniform load with spatial variability of properties is about 11% greater than the horizontal displacement of a homogeneous slope with the same geometry subjected to the same excitation.

![Figure 9](image)

**Figure 9.** Distribution of (a) hor. displacement ratio \( f_x = \frac{u_x}{\bar{u}_x} \) and (b) vert. displacement ratio \( f_y = \frac{u_y}{\bar{u}_y} \) and probability density function of the normal distribution. (Slope 2:1).

The maximum theoretical deviation of the displacement corresponding to \( \mu + 3\sigma \) is equal to 1.95, i.e. 95% greater than the horizontal displacement value of the homogeneous slope. This means that about 66% of cases of heterogeneous slopes will undergo permanent displacements that will be larger than those predicted for homogeneous slopes.

![Figure 10](image)

**Figure 10.** Distribution of (a) hor. displacement \( u_x \) and (b) vert. displacement \( u_y \) and probability density of the Weibull distribution. (Slope 2:1).
Moreover, the results of all analyses show that, in addition to the spatial variability of soil properties, the distribution of the horizontal and vertical displacements \( u_x \) and \( u_y \) depends substantially on the frequency characteristics of the seismic excitation. Figs. 10a and 10b plot the distribution of the permanent hor. displacement \( u_x \) and (b) vert. displacement \( u_y \), from all numerical simulations using geometry A subjected to peak ground acceleration of 0.3g. The range of variation for \( u_x \) is from 0.20 m to 2.20 m and for \( u_y \) from 0.20 m to 2.0 m.

5. Conclusions
For the spatial variability of soil properties considered in this study, the main conclusions derived from the results of the parametric investigation are as follows:

1. The spatial variability of soil properties may affect the factor of safety by approximately ±20%.
2. Slopes (with uniform load) with spatially variable soil properties have a 66% probability to experience larger permanent horizontal displacements compared to those predicted for homogeneous slopes (with uniform load).
3. The permanent seismic displacements of the soil slopes, are significantly affected by the spatial variability of the soil properties as it is evident from the results of the parametric analysis.
4. The probabilistic approach used in this study allows a realistic evaluation of the effects of the spatial variability and earthquake excitation characteristics on the slope permanent displacement. Differences in the response due to spatial variability of soil properties are quite significant.
5. The automated Local Average Subdivision methodology for the development of cross-correlated and autocorrelated random fields of material properties allowed an efficient realization of random fields that satisfy successfully the target statistical parameters. Such random fields can be used for numerical analysis in Monte Carlo simulations to investigate the effects of spatial variability of soil properties on the seismic performance of various geotechnical systems.

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