Joint Power Control and Fronthaul Rate Allocation for Throughput Maximization in Broadband Cloud Radio Access Network

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Abstract

The performance of cloud radio access network (C-RAN) is constrained by the limited fronthaul link capacity under future heavy data traffic. To tackle this problem, extensive efforts have been devoted to design efficient signal quantization/compression techniques in the fronthaul to maximize the network throughput. However, most of the previous results are based on information-theoretical quantization methods, which are hard to implement due to the extremely high complexity. In this paper, we consider using practical uniform scalar quantization in the uplink communication of a broadband C-RAN system, where the mobile users are assigned with orthogonal sub-carriers for multiple access. In particular, we consider joint wireless power control and fronthaul quantization design over the sub-carriers to maximize the system end-to-end throughput. Efficient algorithms are proposed to solve the joint optimization problem when either information-theoretical or practical fronthaul quantization method is applied. Interestingly, we find that the fronthaul capacity constraints have significant impact to the optimal wireless power control policy. As a result, the joint optimization shows significant performance gain compared with either optimizing wireless power control or fronthaul quantization alone. Besides, we also show that the proposed simple uniform quantization scheme performs very close to the throughput performance upper bound, and in fact overlaps with the upper bound when the fronthaul capacity is sufficiently large. Overall, our results would help reveal practically achievable throughput performance of C-RAN, and lead to more efficient deployment of C-RAN in the next-generation wireless communication systems.

Index Terms

Cloud radio access network (C-RAN), fronthaul constraint, quantize-and-forward, multi-carrier system, power control, throughput maximization.

I. INTRODUCTION

A. Motivations and Contributions

The dramatic increase of mobile data traffic in the recent years has posed imminent challenges to the current 4G cellular systems, requiring higher throughput, larger coverage, and smaller communication delay. The 5G cellular system on the roadmap is expected to achieve up to 1000 times of throughput improvement over today’s 4G standard. As a promising candidate for the future 5G standard, cloud radio access network (C-RAN) enables a centralized processing architecture, using multiple relay-like base stations, named remote radio heads (RRHs), to serve mobile users cooperatively under the coordination of a central unit (CU) \[\text{CU}\]. For the practical deployment of C-RAN, a cluster-based C-RAN system is
shown in Fig. 1 where in adjacent clusters orthogonal frequency bands are used to avoid interference. Within each C-RAN cluster, the RRHs are connected to a CU that is further connected to the core network via high-speed fiber fronthaul and backhaul links, respectively. In a C-RAN, a mobile user could be associated with multiple RRHs. However, unlike the base stations in conventional cellular systems which encode/decode user messages locally, the RRHs merely forward the signals to/from the mobile users, while leaving the joint encoding/decoding complexity to a baseband unit (BBU) in the CU. The use of inexpensive and densely deployed RRHs, along with the advanced joint processing mechanism, could significantly improve upon the current 4G system with enhanced scalability, increased throughput and extended coverage.

The distributed antenna system formed by the RRHs enables spectrum efficient spatial division multiple access (SDMA) in C-RAN, which has gained extensive research attentions [2]-[10]. In the uplink communication of a SDMA based C-RAN, all mobile users in the same cluster transmit on the same spectrum and at the same time, while the BBU performs multi-user detection (MUD) to separate the user messages. In practice, however, the implementation of MUD is hurdled by the high computational complexity and the difficulty in signal synchronization. Similarly, the downlink communication using SDMA is also of high complexity in the encoding design to mitigate the co-channel interference. In practical broadband cellular systems, multi-carrier modulations are mostly used, where users are allocated with orthogonal sub-carriers (SCs) free of co-channel interference, such as the orthogonal frequency-division multiple access (OFDMA) scheme used in the 4G LTE system. Instead of employing complex MUD in the uplink, in multicarrier-based C-RAN simple maximal-ratio combining (MRC) technique is performed over the forwarded signals from the RRHs to decode a user message transmitted on different SCs allocated to
it. Considering its potential implementations in future wireless systems, we adopt multi-carrier based multiple access scheme for the cluster-based C-RAN (see Fig. [1]) in this paper.

The performance of a C-RAN system is constrained by the fronthaul link capacity. In the uplink communication, for instance, the RRHs need to sample, quantize, and then forward the received radio frequency (RF) signals to the CU. With densely deployed RRHs, the fronthaul traffic generated from a single user signal of MHz bandwidth could be easily scaled up to multiple Gbps [1]. In practice, a commercial fiber link with tens of GHz capacity could thus be easily overwhelmed even under moderate mobile traffic. To tackle this problem, many signal compression/quantization methods have been proposed to optimize the system performance under fronthaul capacity constraints [3]-[9]. Despite of their respective contributions to the understanding of the theoretical limits of C-RAN, most of the proposed quantization methods are based on information-theoretical distributed source coding techniques, which are practically hard to implement due to the extremely high complexity in obtaining and exchanging the quantization codebooks in real-time. Therefore, it still remains as a question about the practically achievable throughput of C-RAN using practical quantization methods, such as uniform scalar or vector quantization used in common A/D modules [11].

For a multi-carrier C-RAN, fronthaul signal quantization design is equivalent to allocating the fronthaul data rate (capacity) to the orthogonal SCs. Besides the fronthaul rate allocation, the end-to-end throughput performance of a multi-carrier C-RAN is also affected by the wireless resource allocation. With a given SC assignment, the problem becomes to optimally allocate the user transmit power among the assigned SCs to maximize the network throughput. In a system without fronthaul constraint, the optimal power allocation problem is extensively studied, e.g., it follows the celebrated water-filling policy for a single user case [12]. However, the behavior of optimal SC power allocation in a fronthaul constrained system like C-RAN is still unknown to the authors’ best knowledge.

In this paper, we address to the above two important problems in a multi-carrier C-RAN. In particular, we consider using simple uniform scalar quantization to replace the information-theoretical quantization method, and propose joint wireless power control and fronthaul rate allocation design to maximize the system throughput performance. Our main contributions are detailed as follows:

- In the uplink communication of a multi-carrier C-RAN with a given SC assignment, we formulate the optimization problem of joint wireless power control and fronthaul rate allocation to maximize the sum-rate performance. Efficient algorithms are proposed to solve the joint optimization problem when either information-theoretical or practical fronthaul quantization method is applied. Significant performance gain is observed for the joint optimization compared with either optimizing wireless power control or fronthaul rate allocation alone.

\[^1\]In the uplink communication, the achievable end-to-end data rate is always strictly smaller than the wireless link rate, as it requires infinite fronthaul capacity to perfectly represent an analog RF signal.
• By investigating the single-user and single-RRH special case, we highlight the insights obtained from joint wireless power control and fronthaul rate allocation over SCs. An efficient alternating optimization algorithm is proposed to solve the special case, and exploit the solution structures when either wireless power allocation or fronthaul rate allocation is fixed.

• We demonstrate the impact of fronthaul constraint to optimal wireless resource allocation. With a fixed fronthaul rate allocation, we find that the optimal power allocation is a threshold based policy with respect to the channel power of a SC, i.e., no power is allocated to a SC if the channel power is below the threshold. In particular, with an equal fronthaul rate allocation, the common threshold increases with the allocated fronthaul rate, and the power allocation policy reduces to the celebrated water-filling method when the fronthaul rate approaches infinity. Interestingly, we find that the power allocation in general does not follow a water-filling policy that always allocates more power to SC with higher channel power. The inconsistency is especially evident in low-fronthaul-rate region, where the SC with the highest channel power may receive the least transmit power, and vice versa.

• We also show that, with a fixed transmit power allocation, the optimal fronthaul rate allocation is also a threshold based policy with respect to the signal-to-noise ratio (SNR) of a SC, where a larger fronthaul rate is allocated to a SC with a larger SNR. Besides, our observation shows that a joint wireless power and fronthaul rate allocation is in favor of SCs with high channel power, such that some SCs with relatively low channel power receive no power and fronthaul rate at all.

• We quantify the performance gap between the proposed simple uniform quantization scheme from the throughput upper (cut-set) bound. In particular, experiments show that the throughput performance of the simple uniform quantization scheme is very close to the performance upper bound, and in fact overlaps with the upper bound when the fronthaul capacity is sufficiently large.

In summary, our proposed design paradigm employs simple uniform scalar quantization methods, and exploits the performance gain from joint wireless and fronthaul resource allocation. The obtained insights on wireless power and fronthaul rate allocation are also useful to develop reduced-complexity heuristics in practical implementations. Overall, our results would help reveal practically achievable throughput performance of C-RAN, and lead to more efficient deployment of C-RAN in the future 5G systems.

B. Related Work

As the key component of C-RAN fronthaul signal processing, the quantize-and-forward scheme was initially studied in relay channel as an efficient way for the relay to deliver the received signal from the source to the destination [13], [14], [15]. From an information theoretical perspective, the effect of data quantization could be modeled as a test channel (often Gaussian for simplicity of analysis) with the uncompressed signals as the input and compressed signals as the output. In a Gaussian test channel model, the output signal is generated by the input signal corrupted by an additive Gaussian compression noise.
The design of codebook is equivalent to setting the variance of the compression noise. An interesting result shows that by simply setting the quantization noise power proportional to the background noise level at each relay, the quantize-and-forward scheme can achieve a capacity within a constant gap to the throughput performance upper (cut-set) bound [14], [15].

C-RAN is a special case of relay channel model with a wireless first-hop link and wired (fiber) second-hop link. In the uplink communication, when multiple RRHs compress and forward their received signals to the CU, the compression design becomes setting the covariance matrix of the compression noises across different RRHs [4], [5], [6]. In this setting, distributed Wyner-Ziv lossy compression is used at the RRHs, exploiting the signal correlation across the multiple BSs. However, the implementation of distributed Wyner-Ziv compression is difficult mainly due to the high complexity in determining the optimal joint compression codebook and the joint decompressing/decoding at the CU. Alternatively, independent compression method are proposed where the quantization codebook of a RRH is only determined by its local channel state information (CSI), and the decompression operation at the CU is also on a per-relay basis [7], [9]. Although independent quantization reduces the fronthaul complexity compared to joint compression design, the compression codebook generation is still based on information-theoretical source coding techniques, which is highly complex and impractical in a fast varying wireless environment. In other words, the throughput performance of C-RAN derived by previous studies is in fact not yet achievable in practical wireless systems. In this paper, we consider using practical quantization methods, i.e., uniform scalar quantization, to replace the information-theoretical methods, and show that its throughput performance is in fact very close to the performance upper bound.

On the other hand, most of the current studies in C-RAN focus on using SDMA scheme for multiple access in the wireless link to enhance the spectral efficiency [2]-[10]. This is, however, costly to implement due to the high decoding complexity and the difficulty of signal synchronization, especially in the uplink. In practice, multi-carrier modulation methods are commonly used in a broadband cellular system, such as 4G LTE, where each user is assigned with a set of orthogonal SCs. In this case, wireless resource allocation, especially SC transmit power control, plays an important role in determining the spectral efficiency. In the context of C-RAN, the end-to-end performance is determined jointly by both the wireless and fronthaul links. With the above practical considerations, we adopt the multi-carrier C-RAN architecture in this paper, and propose to jointly optimize the wireless power control and fronthaul quantization to maximize the system throughput. In particular, we show that significant performance gain could be obtained than either optimizing wireless resource allocation or fronthaul quantization alone.

C. Organization of the Paper

The rest of this paper is organized as follows. We first introduce in Sections II and III the system model of C-RAN and the quantization techniques used in the fronthaul signal processing, respectively.
In Section [IV] we formulate the end-to-end sum-rate maximization problems for both the Gaussian test channel and uniform scalar quantization models. Sections [V] and [VI] solve the formulated problems for the special case of single-user and single-RRH and general case of multi-user and multi-RRH, respectively. Finally, we conclude the paper and point out future research directions in Section [VII].

II. SYSTEM MODEL

We consider the uplink of a clustered C-RAN. As shown in Fig. [I] each cluster consists of one BBU, $M$ single-antenna RRHs, denoted by the set $\mathcal{M} = \{1, \cdots, M\}$, and $K$ single-antenna users, denoted by the set $\mathcal{K} = \{1, \cdots, K\}$. It is assumed that each RRH $m$, $\forall m \in \mathcal{M}$, is connected to the BBU through a noiseless wired fronthaul link of capacity $\bar{T}_m$ bps. In the uplink, each RRH receives user signals over the wireless link and forwards to the BBU via its fronthaul link. Then, the BBU jointly decodes the users’ messages based on the signals from all the RRHs within the cluster and forwards the decoded information to the core network through a backhaul link. The detailed signal models in the wireless and the fronthaul links are introduced in the following.

A. Multi-Carrier Wireless Transmission

In this paper, we consider multi-carrier uplink information transmission between the $K$ users and the $M$ RRHs over a wireless link of a $B$ Hz total bandwidth equally divided into $N$ SCs. The SC set is denoted by $\mathcal{N} = \{1, \cdots, N\}$. It is assumed that each SC $n \in \mathcal{N}$ is only allocated to one user. Denote $\Omega_k$ as the set of SCs allocated to user $k$, $\forall k \in \mathcal{K}$. For convenience, it is assumed in this paper that the SC allocation among users, i.e., $\Omega_k$’s, is pre-determined.

Specifically, in the uplink each user $k$, $\forall k \in \mathcal{K}$, first generates a multi-carrier modulated signal over its assigned SCs and then transmits to the RRHs in the same cluster. As shown in Fig. [II] each RRH $m$, $\forall m \in \mathcal{M}$, first downconverts the received RF signal to the baseband, and then demodulates the baseband signal into $N$ parallel streams. Suppose that $n \in \Omega_k$, then the equivalent baseband complex symbol

Note that our studied multi-carrier communication is a general model that covers many practical systems. For example, if the multi-carrier modulation and demodulation is performed by inverse fast Fourier transform (IFFT) and FFT, then it reduces to the OFDMA system.
received by RRH $m$ at SC $n$ can be expressed as

$$y_{m,n} = h_{m,k,n} \sqrt{p_{k,n}} s_{k,n} + z_{m,n},$$

where $s_{k,n} \sim \mathcal{CN}(0, 1)$ denotes the transmit symbol of user $k$ at SC $n$ (which is modelled as a circularly symmetric complex Gaussian random variable with zero-mean and unit-variance), $p_{k,n}$ denotes the transmit power of user $k$ at SC $n$, $h_{m,k,n}$ denotes the channel from user $k$ to RRH $m$ at SC $n$, and $z_{m,n} \sim \mathcal{CN}(0, \sigma_{m,n}^2)$ denotes the aggregation of additive white Gaussian noise (AWGN) and (possible) out-of-cluster interference at RRH $m$ at SC $n$. It is assumed that $z_{m,n}$’s are independent over $m$ and $n$.

### B. Quantize-and-Forward Processing at RRH

To forward the baseband symbols $y_{m,n}$’s to the BBU via the fronthaul links, the so-called “quantize-and-forward” scheme is applied, where each RRH first quantizes its baseband received signal and then sends the corresponding digital codewords to the BBU. Specifically, since at each RRH the received symbols at all the SCs are independent with each other and we assume independent signal quantization at different RRHs, a simple scalar quantization on $y_{m,n}$’s is optimal as shown in Fig. 2. The baseband quantized symbol of $y_{m,n}$ is then given by

$$\tilde{y}_{m,n} = y_{m,n} + e_{m,n} = h_{m,k,n} \sqrt{p_{k,n}} s_{k,n} + z_{m,n} + e_{m,n},$$

where $e_{m,n}$ denotes the quantization error for the received symbol $y_{m,n}$ with zero mean and variance $q_{m,n}$. Note that $e_{m,n}$’s are independent over $n$ due to scalar quantization at each SC, and over $m$ due to independent compression among RRHs. Then, each RRH transforms the parallel encoded bits $\tilde{y}_{m,n}$’s into the serial ones and sends them to the BBU via its fronthaul link for joint information decoding.

After collecting the digital codewords, the BBU first recovers the baseband quantized symbols $\tilde{y}_{m,n}$ based on the quantization codebooks used by each RRH. Then, to decode $s_{k,n}$, the BBU applies a linear combining on the quantized symbols at SC $n$ collected from all RRHs:

$$\hat{s}_{k,n} = w_n^H \tilde{y}_n = w_n^H h_{k,n} \sqrt{p_{k,n}} s_{k,n} + w_n^H z_n + w_n^H e_n, \quad n \in \Omega_k, \ k = 1, \cdots, K,$$

where $\tilde{y}_n = [\tilde{y}_{1,n}, \cdots, \tilde{y}_{M,n}]^T$, $h_{k,n} = [h_{1,k,n}, \cdots, h_{M,k,n}]^T$, $z_n = [z_{1,n}, \cdots, z_{M,n}]^T$, and $e_n = [e_{1,n}, \cdots, e_{M,n}]^T$. According to (3), the SNR for decoding $s_{k,n}$ is expressed as

$$\gamma_{k,n} = \frac{p_{k,n} |w_n^H h_{k,n}|^2}{w_n^H \left( \text{diag}(\sigma_{1,n}^2, \cdots, \sigma_{M,n}^2) + \text{diag}(q_1, \cdots, q_M) \right) w_n}, \quad n \in \Omega_k, \ k = 1, \cdots, K,$$

where $\text{diag}(a)$ denotes a diagonal matrix with the main diagonal given by vector $a$. It can be shown that the optimal combining weights that maximize $\gamma_{k,n}$’s are obtained from the well-known MRC [12]:

$$w_n^* = \left( \text{diag}(\sigma_{1,n}^2, \cdots, \sigma_{M,n}^2) + \text{diag}(q_1, \cdots, q_M) \right)^{-1} h_{k,n}, \quad n = 1, \cdots, N.$$
With the above MRC receiver, $\gamma_{k,n}$ given in (4) reduces to
\[
\gamma_{k,n} = \sum_{m=1}^{M} \frac{|h_{m,k,n}|^2 p_{k,n}}{\sigma_{m,n}^2 + q_{m,n}}, \quad n \in \Omega_k, \ k = 1, \ldots, K.
\] (6)

III. QUANTIZATION SCHEMES

The key issue to implement the quantize-and-forward scheme introduced in Section II is how each RRH should quantize its received signal at each SC in practice. In this section, we first study a theoretical quantization model by viewing (2) as a test channel and derive its achievable sum-rate based on the rate-distortion theory, which can serve as a performance upper bound. Then, we investigate the practical uniform scalar quantization scheme in details, which can be easily applied at each RRH, and derive the corresponding achievable end-to-end sum-rate.

A. Gaussian Test Channel

In this subsection, we assume that the quantization errors given in (2) are Gaussian distributed, i.e., $e_{m,n} \sim \mathcal{CN}(0, q_{m,n})$, $\forall m, n$. With Gaussian quantization errors, (2) can be viewed as a Gaussian test channel [16]. As a result, to forward the received data at SC $n$, the transmission rate in RRH $m$’s fronthaul link is expressed as [16]
\[
T^{(G)}_{m,n} = \frac{B}{N} \log_2 \left( 1 + \frac{|h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2}{q_{m,n}} \right).
\] (7)
Since quantization is performed at each RRH independently, \{\{y_{m,1}, \ldots, y_{m,N}\}\} can be reliably transmitted to the BBU if and only if \[\sum_{n=1}^{N} T^{(G)}_{m,n} = \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2 \left( 1 + \frac{|h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2}{q_{m,n}} \right) \leq \bar{T}_m, \quad m = 1, \ldots, M. \] (8)

Next, consider the end-to-end performance of the users. With Gaussian noise in (2), the achievable rate of user $k$ at SC $n$ is expressed as
\[
R^{(G)}_{k,n} = \frac{B}{N} \log_2 (1 + \gamma_{k,n}) = \frac{B}{N} \log_2 \left( 1 + \sum_{m=1}^{M} \frac{|h_{m,k,n}|^2 p_{k,n}}{\sigma_{m,n}^2 + q_{m,n}} \right).
\] (9)
where (a) is obtained by substituting $q_{m,n}$ by $T^{(G)}_{m,n}$ according to (7). Notice that as the allocated fronthaul rate $T^{(G)}_{m,n} \to 0$ (versus $\infty$), the achievable end-to-end rate in (9) converges to zero (or that of the wireless link capacity). Then, the achievable throughput of all users is expressed as
\[
R^{(G)}_{\text{sum}} = \sum_{k=1}^{K} \sum_{n \in \Omega_k} R^{(G)}_{k,n} = \frac{B}{N} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2 \left( 1 + \sum_{m=1}^{M} \frac{|h_{m,k,n}|^2 p_{k,n}}{\sigma_{m,n}^2 + \frac{|h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2}{2^{N T^{(G)}_{m,n} / B} - 1}} \right).
\] (10)
From (10), it is clearly seen that the sum-rate performance depends on both the users’ power allocations, \{\{p_{k,n}\}\}, and the RRHs’ fronthaul rate allocations, \{\{T^{(G)}_{m,n}\}\}, over the SCs.
B. Uniform Scalar Quantization

In practice, it is very difficult to find the quantization codebooks to achieve the throughput given in (10) subject to the fronthaul capacity constraints given in (8). In this subsection, we consider using practical uniform scalar quantization technique at each RRH and derive the achievable sum-rate.

A typical method to implement the uniform quantization is via separate in-phase/quadrature (I/Q) quantization, where the architecture is shown in Fig. 3. Specifically, the received complex symbol \( y_{m,n} \) given in (1) could be presented by its I and Q parts:

\[
y_{m,n} = y_{I,m,n} + jy_{Q,m,n}, \quad \forall m,n,
\]

(11)

where \( j^2 = -1 \), and the I-branch symbol \( y_{I,m,n} \) and Q-branch symbol \( y_{Q,m,n} \) are both real Gaussian random variables with zero mean and variance \( (|h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2)/2 \). Then, the uniform quantization is conducted on the I-branch and Q-branch symbols separately, the details of which are shown as follows.

First, each RRH normalizes the I-branch and Q-branch symbols at each SC into the interval \([-1, 1]\) for quantization by the following scaling process:

\[
\bar{y}_{m,n}^\chi = \frac{y_{\chi,m,n}}{\eta_{m,n}}, \quad \chi \in \{I, Q\}, \quad \forall m,n.
\]

(12)

Since \( y_{I,m,n} \)'s and \( y_{Q,m,n} \)'s are both real Gaussian random variables the instantaneous power of which can go to infinity in some instances, the probability of overflow should be controlled by a proper selection of the scaling factors \( \eta_{I,m,n} \)'s and \( \eta_{Q,m,n} \)'s. In this paper, we apply the so-called “three-sigma rule” \([11]\) to select the scaling factors. Specifically, since the average power of \( y_{I,m,n} \) and \( y_{Q,m,n} \) are both \( (|h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2)/2 \), we set

\[
\eta_{I,m,n} = \eta_{Q,m,n} \equiv \eta_{m,n} = 3\sqrt{\frac{|h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2}{2}}, \quad \forall m,n.
\]

(13)

As a result, the probability of overflow for both the I-branch and Q-branch symbols is expressed as

\[
P(|\bar{y}_{I,m,n}^\chi | > 1) = P(|\bar{y}_{Q,m,n}^\chi | > 1) = 2Q(3) = 0.0027, \quad \forall m,n.
\]

(14)
Note that in the case of overflow, the quantized value can be set to be 1 if the scaled symbol is larger than 1 or -1 if it is smaller than -1.

Next, RRH $m$ implements uniform quantization on the normalized symbols $\tilde{y}_{m,n}^I$’s and $\tilde{y}_{m,n}^Q$’s at each SC in the interval $[-1, 1]$. We assume that RRH $m$ uses $D_{m,n} \geq 1$ bits to quantize the symbol received on SC $n$, resulting $2^{D_{m,n}}$ quantization levels, for which the quantization step size is given by

$$\Delta_{m,n} = \frac{2}{2^{D_{m,n}}} = 2^{1-D_{m,n}}, \quad \forall m,n. \quad (15)$$

Furthermore, for each normalized symbol $\tilde{y}_{m,n}^I$ or $\tilde{y}_{m,n}^Q$, its quantized value is given by

$$\hat{y}_{m,n}^\chi = \left[\frac{2^{D_{m,n}-1}\tilde{y}_{m,n}^\chi}{2^{D_{m,n}-1}}\right] - \frac{1}{2^{D_{m,n}}}, \quad \chi \in \{I, Q\}, \quad \forall m,n, \quad (16)$$

where $\lfloor x \rfloor$ denotes the minimum integer that is no smaller than $x$. Then, $\hat{y}_{m,n}^I$’s and $\hat{y}_{m,n}^Q$’s are encoded into digital codewords $\hat{y}_{m,n}^I$’s and $\hat{y}_{m,n}^Q$’s and transmitted to the BBU.

In the following, we derive the end-to-end achievable throughput of all users subject to the fronthaul capacity constraints under the uniform scalar quantization technique described above. First, we derive the transmission rate required from each RRH to the BBU over the fronthaul link. Note that the I, Q symbols, i.e., $y_{m,n}^I$’s and $y_{m,n}^Q$’s, are obtained by sampling of the I, Q waveforms, the bandwidth of which is $B/2N$, $\forall m,n$. As a result, at each RRH, the Nyquist sampling rate for the I, Q waveforms at each SC is $B/N$ samples per second. Furthermore, since at RRH $m$, each sample at SC $n$ is represented by $D_{m,n}$ bits, the corresponding transmission rate in the fronthaul link is expressed as

$$T_{m,n}^{(U)} = \frac{BD_{m,n}}{N} + \frac{BD_{m,n}}{N} = \frac{2BD_{m,n}}{N}. \quad (17)$$

Then, the overall transmission rate from RRH $m$ to the BBU in the fronthaul link is given as

$$T_m^{(U)} = \sum_{n=1}^{N} T_{m,n}^{(U)} = \frac{2B}{N} \sum_{n=1}^{N} D_{m,n}, \quad m = 1, \ldots, M, \quad (18)$$

which should not exceed the fronthaul link capacity $T_m$.

To derive the end-to-end sum-rate, we need to calculate the power of the quantization error given in (2), i.e., $q_{m,n}, \forall m,n$. Note that in (2) we have $y_{m,n} = \eta_{m,n}(\hat{y}_{m,n}^I + j\hat{y}_{m,n}^Q)$, $\forall m,n$. According to Widrow Theorem [17], if the number of quantization levels (i.e., $2^{D_{m,n}}$) is large, and the signal varies by at least some quantization levels from sample to sample, the quantization noise can be assumed to be uniformly distributed. As a result, we assume that the quantization errors for both I, Q signals, which are denoted by $e_{m,n}^I$ and $e_{m,n}^Q$, are uniformly distributed in $[-\eta_{m,n}\Delta_{m,n}/2, \eta_{m,n}\Delta_{m,n}/2]$, $e_{m,n} = e_{m,n}^I + j e_{m,n}^Q$, $\forall m,n$.

$^3$For the case when $D_{m,n}$ is small, the quantization noise is practically non-uniformly distributed. In this paper, we assume uniformly distributed quantization noise for obtaining an approximate closed-form expression of the quantization noise power $q_{m,n}$ as shown in (19), which has been verified by simulation to be quite accurate even for the case of small $D_{m,n}$. For example, by setting $|h_{m,k,n}|^2 p_{k,n} + \sigma^2_{m,n} = 1$, when $D_{m,n} = 1, 2, 3$, $q_{m,n}$ obtained by (19) is 0.16667, 0.04167, 0.01042, while that obtained by simulation is 0.18948, 0.04177, 0.01047, indicating the approximation becomes more accurate as $D_{m,n}$ increases.
∀m, n. Then we have
\begin{align*}
q_{m,n} &= \int_{-\eta_{m,n}\Delta_{m,n}}^{\eta_{m,n}\Delta_{m,n}} \frac{(e_{m,n})^2}{\eta_{m,n}\Delta_{m,n}} de_{m,n} + \int_{-\eta_{m,n}\Delta_{m,n}}^{\eta_{m,n}\Delta_{m,n}} \frac{(e_{Q_{m,n}})^2}{\eta_{m,n}\Delta_{m,n}} de_{m,n} \\
&= \frac{\eta_{m,n}^2 \Delta_{m,n}^2}{6} = 3(|h_{m,n}|^2 p_{k,n} + \sigma_{m,n}^2) 2^{-2D_{m,n}} \\
&\overset{(a)}{=} 3(|h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2) 2^{-2\frac{NT(U)}{B_m}},
\end{align*}

where (a) is obtained by substituting \(D_{m,n}\) by \(T(U)_{m,n}\) according to (17). Then according to (6), a lower bound for the achievable rate of user \(k\) at SC \(n\), by viewing \(e_{m,n}\)'s as the worst-case Gaussian noise in (2) (although they are uniformly distributed), can be expressed as
\begin{align*}
R_{k,n}^{(U)} &= \frac{B}{N} \log_2 \left( 1 + \frac{\sum_{m=1}^{M} |h_{m,k,n}|^2 p_{k,n}}{\sigma_{m,n}^2 + q_{m,n}} \right) \\
&= \frac{B}{N} \log_2 \left( 1 + \frac{\sum_{m=1}^{M} |h_{m,k,n}|^2 p_{k,n}}{\sigma_{m,n}^2 + 3(|h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2) 2^{-2\frac{NT(U)}{B_m}}} \right). 
\end{align*}

Notice that (20) holds when \(T(U)_{m,n} \geq (2B)/N\) (i.e., \(D_{m,n} \geq 1\)) according to (17). The end-to-end throughput of all users is thus expressed as
\begin{align*}
R_{\text{sum}}^{(U)} &= \frac{B}{N} \sum_{k=1}^{K} \sum_{n \in \Omega_k} R_{k,n}^{(U)} \\
&= \frac{B}{N} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2 \left( 1 + \frac{\sum_{m=1}^{M} |h_{m,k,n}|^2 p_{k,n}}{\sigma_{m,n}^2 + 3(|h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2) 2^{-2\frac{NT(U)}{B_m}}} \right). 
\end{align*}

Similar to (10) for the ideal case of Gaussian compression, the sum-rate in (21) with the uniform scalar quantization also jointly depends on both the users’ power allocations, \(\{p_{k,n}\}\), and the RRHs’ fronthaul rate allocations, \(\{T(U)_{m,n}\}\), over the SCs. Furthermore, given the same set of power and fronthaul rate allocations, the achievable rate in (21) is always strictly less than that in (10) provided that \(T(G)_{m,n} = T(U)_{m,n} \geq (2B)/N, \forall m, n.\)

IV. Problem Formulation

In this paper, given the wireless bandwidth \(B\), each user \(k\)'s SC allocation \(\Omega_k\)'s as well as transmit power constraint \(\bar{P}_k\)'s, and each RRH \(m\)'s fronthaul link capacity \(\bar{T}_m\)’s, we aim to maximize the end-to-end throughput of all the users subject to each RRH’s fronthaul link capacity constraint by jointly optimizing the wireless power control and fronthaul rate allocation. Specifically, for the benchmark scheme, i.e., the theoretical Gaussian test channel based scheme in Section [III-A] we are interested in solving the
following problem.

(P1): Maximize \( \{p_{k,n},T_{m,n}\} \) \( R^{(G)}_{\text{sum}} \)

Subject to

\[
\sum_{n=1}^{N} T^{(G)}_{m,n} \leq \bar{T}_m, \quad \forall m \in \mathcal{M},
\]

\[
\sum_{n \in \Omega_k} p_{k,n} \leq \bar{P}_k, \quad \forall k \in \mathcal{K},
\]

where \( R^{(G)}_{\text{sum}} \) is given in (10) and \( T^{(G)}_{m,n} \) is given in (7). Furthermore, for the proposed uniform scalar quantization based scheme in Section III-B, we are interested in solving the following problem.

(P2): Maximize \( \{p_{k,n},T^{(U)}_{m,n}\} \) \( R^{(U)}_{\text{sum}} \)

Subject to

\[
\sum_{n=1}^{N} T^{(U)}_{m,n} \leq T_m, \quad \forall m \in \mathcal{M},
\]

\[
\sum_{n \in \Omega_k} p_{k,n} \leq P_k, \quad \forall k \in \mathcal{K},
\]

\[
T^{(U)}_{m,n} = \frac{2BD_{m,n}}{N}, \quad D_{m,n} \in \{1,2,\cdots\} \text{ is an integer, } \forall m \in \mathcal{M}, \forall n \in \mathcal{N},
\]

where \( R^{(U)}_{\text{sum}} \) is given in (21) and \( T^{(U)}_{m,n} \) is given in (17).

Recall that with the same rate allocations in the fronthaul links for the two schemes, i.e., \( T^{(G)}_{m,n} = T^{(U)}_{m,n} \geq (2B)/N, \forall m,n, \) \( R^{(G)}_{\text{sum}} \) in (10) is always larger than \( R^{(U)}_{\text{sum}} \) given in (21). Furthermore, uniform scalar quantization requires that the fronthaul rate allocated at each SC must be an integer multiplication of \((2B)/N\). Due to the above two reasons, in general the optimal value of problem (P2) is smaller than that of problem (P1), i.e., \( R^{(U)}_{\text{sum}} < R^{(G)}_{\text{sum}} \).

It can be also observed that both problems (P1) and (P2) are non-convex since their objective functions are not concave over \( p_{k,n} \)'s and \( T_{m,n} \)'s; thus, it is difficult to obtain their optimal solutions in general. In the following two sections, we first study the special case of problems (P1) and (P2) with one user and one RRH to shed some light on the mutual influence between the wireless power allocation and fronthaul rate allocation, and then propose efficient algorithms to solve problems (P1) and (P2) for the general case of multiple users and multiple RRHs.

V. SPECIAL CASE: SINGLE USER AND SINGLE RRH

In this section, we study problems (P1) and (P2) for the special case of \( K = 1 \) and \( M = 1 \). For convenience, in the rest of this section we omit the subscripts of \( k \) and \( m \) in all the notations in problems (P1) and (P2).
A. Gaussian Test Channel

It can be shown that problem (P1) is still a non-convex problem for the case of $K = 1$ and $M = 1$. In this subsection, we propose to apply the alternating optimization technique to solve this problem. Specifically, first we fix the fronthaul rate allocation $T_n^{(G)} = \hat{T}_n^{(G)}$'s in problem (P1) and optimize the wireless power allocation by solving the following problem.

Maximize $\{p_n\} \quad \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 p_n}{\sigma_n^2 + \frac{|h_n|^2 p_n + \sigma_n^2}{2^{N T_n^{(G)}/B - 1}}} \right)$

Subject to $\sum_{n=1}^{N} p_n \leq \bar{P}$. \hspace{1cm} (22)

Let $\{\hat{p}_n\}$ denote the optimal solution to problem (22). Next, we fix the wireless power allocation $p_n = \hat{p}_n$'s in problem (P1) and optimize the fronthaul rate allocation by solving the following problem.

Maximize $\{T_n^{(G)}\} \quad \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 \hat{p}_n}{\sigma_n^2 + \frac{|h_n|^2 \hat{p}_n + \sigma_n^2}{2^{N T_n^{(G)}/B - 1}}} \right)$

Subject to $\sum_{n=1}^{N} T_n^{(G)} \leq \bar{T}$. \hspace{1cm} (23)

Let $\{\hat{T}_n^{(G)}\}$ denote the optimal solution to problem (23). The above update of $\{p_n\}$ and $\{T_n^{(G)}\}$ is iterated until convergence. In the following, we show how to solve problems (22) and (23), respectively.

First, it can be shown that the objective function of problem (22) is concave over $p_n$'s. As a result, problem (22) is a convex problem, and thus can be efficiently solved by the Lagrangian duality method [18]. We then have the following proposition.

**Proposition 5.1:** The optimal solution to problem (22) is expressed as

$$\hat{p}_n = \begin{cases} \frac{-\alpha_n + \sqrt{\alpha_n^2 - 4\eta_n}}{2}, & \text{if } \frac{|h_n|^2}{\sigma_n^2} > f_n(T_n^{(G)}), \quad n = 1, \cdots, N, \\ 0, & \text{otherwise.} \end{cases} \hspace{1cm} (24)$$

where

$$\alpha_n = \frac{\sigma_n^2(2^{N T_n^{(G)}/B} + 1)}{|h_n|^2}, \hspace{1cm} (25)$$

$$\eta_n = \frac{\sigma_n^2(2^{N T_n^{(G)}/B} - 1)}{|h_n|^4} - \frac{\sigma_n^2(2^{N T_n^{(G)}/B} - 1)}{\lambda N|h_n|^2 \ln 2}, \hspace{1cm} (26)$$

$$f_n(T_n^{(G)}) = \frac{2^{N T_n^{(G)}/B} - 1}{2^{N T_n^{(G)}/B} - 1} \lambda N \ln 2, \hspace{1cm} (27)$$

and $\lambda$ is a constant under which $\sum_{n=1}^{N} \hat{p}_n = \bar{P}_n$.

**Proof:** Please refer to Appendix A.
It can be shown that as $\hat{T}_n^{(G)}$'s go to infinity, i.e., the case without fronthaul link constraint in problem (P1), the optimal power allocation given in (24) reduces to

$$\hat{p}_n = \begin{cases} \frac{1}{\lambda N \ln 2} - \frac{\sigma_n^2}{|h_n|^2}, & \text{if } \frac{|h_n|^2}{\sigma_n^2} > \lambda N \ln 2, \\ 0, & \text{otherwise}, \end{cases} \quad n = 1, \cdots, N,$$

(28)

which is consistent with the conventional water-filling based power allocation. In the following, we discuss about the impact of fronthaul rate allocation on the optimal power allocation given in (24) with finite values of $\hat{T}_n^{(G)}$'s.

It can be observed from (24) that the optimal wireless power allocation with given $\hat{T}_n^{(G)}$'s is threshold-based. In the following, we give a numerical example to investigate the monotonicity of the threshold $f_n(\hat{T}_n^{(G)})$ over $\hat{T}_n^{(G)}$, $\forall n$ (note that in (27) $\lambda$ is also a function of $\hat{T}_n^{(G)}$'s). In this example, the bandwidth of the wireless link is assumed to be $B = 100$MHz, which is equally divided into 4 SCs. The channel powers are given as $|h_1|^2 = 1.276 \times 10^{-9}$, $|h_2|^2 = 6.12 \times 10^{-10}$, $|h_3|^2 = 2.9 \times 10^{-11}$, $|h_4|^2 = 1.8 \times 10^{-11}$. Moreover, the power spectral density of the background noise is assumed to be $-169$dBm/Hz, and the noise figure due to receiver processing is 7dB. The transmit power of the user is 23dBm. It is further assumed that the fronthaul rates are equally allocated among SCs, i.e., $\hat{T}_n^{(G)} = \bar{T}/4$, $\forall n$, and thus $f_n(\hat{T}_n^{(G)})$'s are of the same value. Fig. 4 shows the plot of $f_n(\hat{T}_n^{(G)})$ versus $\hat{T}_n^{(G)}$ by increasing the value of $\bar{T}$ in problem (22). It is observed in this particular setup (and many others used in our simulations for which the results are not shown here due to the space limitation) that in general $f_n(\hat{T}_n^{(G)})$ is increasing with $\hat{T}_n^{(G)}$. This implies that as $\hat{T}_n^{(G)}$ increases, more SCs with weaker channel powers tend to be shut down. The reason is as follows. The dynamic range of the received signal at the SC with stronger channel power is larger, and thus with equal $\hat{T}_n^{(G)}$'s, the corresponding quantization noise level is also larger. When $\hat{T}_n^{(G)}$'s are small, quantization noise dominates the end-to-end rate performance and thus the relatively small quantization
noise level at the SC with weaker channel power may offset the loss due to the poor channel condition. However, as $\hat{T}_n^{(G)}$ increases, the quantization noise becomes smaller, until the wireless link dominates the end-to-end performance. In this case, we should shut down some SCs with poor channel conditions just as water-filling based power allocation given in (28).

To verify the above analysis, Fig. 5 shows the optimal power allocation among the 4 SCs versus different values of $\hat{T}_n^{(G)} = \bar{T}/4$ in the above numerical example. It is observed that when $\hat{T}_n^{(G)}$ is small, in general the SCs with poorer channel conditions are allocated higher transmit power since the quantization noise levels are small at these SCs. As $\hat{T}_n^{(G)}$ increases, the SCs with poorer channels are allocated less and less transmit power. Specially, when $\hat{T}_n^{(G)} \geq 252.5$Mbps or $\bar{T} \geq 1.1$Gbps, SC 4 with the poorest channel condition is shut down for transmission. It is also observed that when $\hat{T}_n^{(G)}$ is sufficiently large such that the quantization noise is negligible, the power allocation converges to the water-filling based solution given in (28).

Next, similar to problem (22), it can be shown that problem (23) is a convex problem and thus can be efficiently solved by the Lagrangian duality method. We then have the following proposition.

**Proposition 5.2:** The optimal solution to problem (23) can be expressed as

$$\hat{T}_n^{(G)} = \begin{cases} \frac{B}{N} \log_2 \frac{1-B\beta}{\beta B} + \frac{B}{N} \log_2 \nu(n), & \text{if } \nu(n) > \frac{\beta B}{1-\beta B}, \\ 0, & \text{otherwise}, \end{cases} \quad n = 1, \ldots, N, \quad (29)$$

where

$$\nu_n = \frac{|h_n|^2 \hat{p}_n}{\sigma_n^2}, \quad (30)$$

and $\beta < \frac{1}{B}$ is a constant under which $\sum_{n=1}^N \hat{T}_n^{(G)} = \bar{T}$.

**Proof:** Please refer to Appendix B.
Similar to the optimal power allocation given in (24), it can be inferred from Proposition 5.2 that the optimal fronthaul rate allocation with given $\hat{p}_n$’s is also threshold-based. If the received signal SNR, $\nu_n$, at SC $n$ is below the threshold $\beta B/(1 − \beta B)$, the RRH should not quantize and forward the signal at this SC to the BBU for decoding. On the other hand, if $\nu_n > \beta B/(1 − \beta B)$, more quantization bits should be allocated to the SCs with higher values of $\nu_n$’s.

After problems (22) and (23) are solved by Propositions 5.1 and 5.2 we are ready to propose the overall algorithm to solve problem (P1), which is summarized in Table I. It can be shown that a monotonic convergence can be guaranteed for Algorithm I since the objective value of problem (P1) is increased after each iteration and it is practically bounded.

| TABLE I |
| A LGORITHM I | ALGORITHM FOR PROBLEM (P1) WHEN $K = 1$ AND $M = 1$ |

1. Initialize: Set $T^{(G,0)}_n = \frac{N}{T}, \forall n, R^{(0)} = 0$, and $i = 0$;
2. Repeat
   a. $i = i + 1$;
   b. Update $\{p_n^{(i)}\}$ by solving problem (22) with $T^{(G)}_n = T^{(G,i-1)}_n, \forall n$, according to Proposition 5.1
   c. Update $\{T^{(G,i)}_n\}$ by solving problem (23) with $\hat{p}_n = p_n^{(i)} , \forall n$, according to Proposition 5.2
3. Until $R^{(i)}_n - R^{(i-1)}_n \leq \varepsilon$, where $R^{(i)}_n$ denotes the objective value of problem (P1) achieved by $\{p_n^{(i)}\}$ and $\{T^{(G,i)}_n\}$, and $\varepsilon$ is a small value to control the accuracy of the algorithm.

With the proposed Algorithm I to solve (P1), we provide a numerical example to analyze the properties of the resulting wireless power and fronthaul rate allocation among SCs. The setup of this example is the same as that for Figs. 4 and 5, while the fronthaul link capacity is assumed to be $\bar{T} = 400$Mbps. Fig. 6 (a) and Fig. 6 (b) show the wireless power allocation and the fronthaul rate allocation at each SC, respectively, obtained via Algorithm I. For comparison, in Fig. 6 (a) we also provide the power allocation at each SC obtained by solving problem (22) with equal fronthaul rate allocation, as well as the water-
filling based power allocation at each SC (obtained without considering fronthaul link constraint), and in Fig. 6 (b) the equal fronthaul rate allocation as well as the fronthaul rate allocation obtained by solving problem (23) with water-filling based power allocation. It is observed in Fig. 6 (a) that Algorithm I results in a more greedy power allocation solution among SCs than the water-filling based method: besides SC 4, SC 3 with the second poorest channel condition is also forced to shut down, and the saved power and quantization bits are allocated to SCs 1 and 2 with better channel conditions. This is in sharp contrast to the case of equal fronthaul rate allocation for which SC 3 is allocated the highest transmit power and even SC 4 with the poorest channel condition is still used for transmission. Moreover, in Fig. 6 (b), the fronthaul rate allocations at SCs 1−4 obtained by Algorithm I are 213.54Mbps, 186.46, 0Mbps, and 0Mbps, respectively. As a result, different from equal fronthaul rate allocation, Algorithm I tends to allocate more quantization bits to the SCs with strong channel power to explore their good channel conditions, while allocating less (or even no) quantization bits to the SCs with weaker power. A similar fronthaul rate allocation is observed for the water-filling power allocation case.

B. Uniform Scalar Quantization

In this subsection, we study problem (P2) in the case of $K = 1$ and $M = 1$ to evaluate the efficiency of the uniform quantization based scheme. We first solve problem (P2) in this case by extending the results in Section V-A. It can be observed that without the last set of constraints involving integer $D_n$’s, problem (P2) is very similar to problem (P1). As a result, in the following we propose a two-stage algorithm to solve problem (P2). First, we ignore the integer constraints in problem (P2), which is denoted by problem (P2-NoInt), and apply an alternating optimization based algorithm similar to Algorithm I to solve it (the details of which are omitted here for brevity). Let $\{\hat{p}_n, \hat{T}_n(U)\}$ denote the converged wireless power and fronthaul rate allocation solution to problem (P2-NoInt). Next, we fix $p_n = \hat{p}_n$’s and find a feasible solution of $T_n(U)$’s based on $\{\hat{p}_n, \hat{T}_n(U)\}$ such that $D_n = NT_n(U)/2B$’s are integers, $\forall n$, in problem (P2).

This is achieved by rounding each $NT_n(U)/2B$ to its nearby integer as follows:

$$
\frac{NT_n(U)}{2B} = \begin{cases} 
\left\lfloor \frac{NT_n(U)}{2B} \right\rfloor, & \text{if } \frac{NT_n(U)}{2B} - \left\lfloor \frac{NT_n(U)}{2B} \right\rfloor \leq \alpha, \\
\left\lceil \frac{NT_n(U)}{2B} \right\rceil, & \text{otherwise,}
\end{cases} \quad n = 1, \cdots, N, \tag{31}
$$

where $0 \leq \alpha \leq 1$, and $\left\lfloor x \right\rfloor$ denotes the maximum integer that is no larger than $x$. Note that we can always find a feasible solution of $T_n$’s by simply setting $\alpha = 1$ in (31) since in this case we have $\sum_{n=1}^{N} T_n(U) \leq \sum_{n=1}^{N} \hat{T}_n(U) \leq \bar{T}$. In the following, we show how to find a better feasible solution by optimizing $\alpha$. It can be observed from (31) that with decreasing $\alpha$, the values of $T_n(U)$’s will be non-decreasing, $\forall n$. As a result, the objective value of problem (P2) will be non-decreasing, but the fronthaul link constraint in problem (P2) will be more difficult to satisfy. Thereby, we propose to apply a simple bisection method to find the optimal value of $\alpha$, denoted by $\alpha^*$, which is summarized in Table II. After
α∗ is obtained, the feasible solution of Tn(U)s can be efficiently obtained by taking α∗ into (31). Notice that by (31) the number of quantization bits per SC, Dn, is now allowed to be zero, instead of being a strictly positive integer as assumed in Sections III and IV.

### Table II

| Algorithm | Algorithm to Find Feasible Solution of Tn(U)s to Problem (P2) |
|-----------|-------------------------------------------------------------|
| 1.        | Initialize αmin = 0, αmax = 1;                              |
| 2.        | Repeat                                                      |
|           | a. Set α = (αmin + αmax) / 2;                               |
|           | b. Take α into (31). If Tn(U)s, ∀n, satisfy the fronthaul link capacity constraint in problem (P2), set αmax = α; otherwise, set αmin = α; |
| 3.        | Until αmax − αmin < ε, where ε is a small value to control the accuracy of the algorithm; |
| 4.        | Take α into (31) to obtain the feasible solution of Tn(U)s, ∀n. |

Next, we evaluate the end-to-end rate performance of the uniform scalar quantization based scheme in the case of K = 1 and M = 1. Note that a cut-set based capacity upper bound of our studied C-RAN is

\[ C = \min \left( \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 p_{n_{w}}}{\sigma_n^2} \right), \frac{T}{B} \right) \text{ bps/Hz,} \]  

(32)

where \( \{p_{n_{w}}\} \) is the water-filling based optimal power solution given in (28).

**Proposition 5.3:** In the case of K = 1 and M = 1, let \( \bar{R}_{\text{sum}}^{(G)} \) denote the optimal value of problem (P1) with an additional set of constraints of

\[ q_n = \frac{|h_n|^2 p_n + \sigma_n^2}{2N (\bar{R}_{\text{sum}}^{(G)})^2 B} = \sigma_n^2, \quad n = 1, \ldots, N. \]  

(33)

Then we have \( \bar{R}_{\text{sum}}^{(G)}/B \geq C - 1. \)

**Proof:** Please refer to Appendix C.

Proposition 5.3 implies that with the simple solution \( \{p_n = \tilde{p}_n, T_n^{(G)} = (B/N) \log_2(2 + |h_n|^2 \tilde{p}_n/\sigma_n^2)\} \) with \( \tilde{p}_n \)'s denoting the optimal solution to problem (54) given in Appendix C, the Gaussian test channel based scheme can achieve a capacity to within 1bps/Hz. Next, for the uniform scalar quantization, by setting the quantization noise level given in (19) as \( q_n = 3\sigma_n^2, \forall n, \) in problem (P2-NoInt), we have the following proposition.

**Proposition 5.4:** In the case of K = 1 and M = 1, \( \{p_n = \tilde{p}_n, T_n^{(U)} = (B/N) \log_2(1 + |h_n|^2 \tilde{p}_n/\sigma_n^2)\} \) is a feasible solution to problem (P2-NoInt). Let \( \bar{R}_{\text{sum}}^{(U)} \) denote the objective value of problem (P2-NoInt) achieved by the above solution, we then have \( \bar{R}_{\text{sum}}^{(U)}/B > \bar{R}_{\text{sum}}^{(G)}/B - 1. \)

**Proof:** Please refer to Appendix D.

\(^4\)In the case of Dn = 0 and hence Tn(U) = 0, for any SC n, the achievable end-to-end rate for the uniform scalar quantization given in (20) no longer holds, which instead should be set to zero intuitively.
TABLE III  
SETUP OF NUMERICAL RESULTS

|                        | Value            |
|------------------------|------------------|
| Channel Bandwidth      | 100 MHz          |
| Number of SCs          | 32               |
| User Transmit Power    | 23 dBm           |
| AWGN                   | -169 dBm/Hz      |
| Noise Figure           | 7 dB             |
| User Distance (\(d\)) | 50 m             |
| Pass Loss Model        | 30.6 + 36.7 \(\log_{10}(d)\) dB |

It can be inferred from Propositions 5.3 and 5.4 that \(\bar{R}_{\text{sum}}/(U)/B > \bar{R}_{\text{sum}}/(G)/B - 1 \geq C - 2\). As a result, we have the following corollary.

**Corollary 5.1:** Without the constraints that the number of quantization bits per SC is an integer, with the simple solution \(\{p_n = \bar{p}_n, T_n^{(G)} = (B/N) \log_2(1 + |h_n|^2 \bar{p}_n/\sigma_n^2)\}\), the uniform scalar quantization based scheme at least achieves a capacity to within 2 bps/Hz in the case of \(K = 1\) and \(M = 1\).

Corollary 5.1 gives a worst-case performance gap of the proposed uniform quantization based scheme to the cut-set upper bound \(C\) in (32) if we ignore the constraints that each quantization level is represented by an integer number of bits. However, it is difficult to analyze the performance loss due to these integer constraints. In the following subsection, we will provide a numerical example to show the impact of the integer constraints on the end-to-end rate performance.

**C. Numerical Example**

In this subsection, we provide a numerical example to verify our results for the case of \(K = 1\) and \(M = 1\). The setup of this example is summarized in Table III. First, we evaluate the performance of the proposed uniform scalar quantization based scheme against that of the Gaussian test channel based scheme as well as the capacity upper bound given in (32). Fig. 7 shows the end-to-end rate achieved by various schemes versus the fronthaul link capacity. Note that with the algorithm proposed for problem (P2-NoInt) in Section V-B, we use \(\{p_n = \bar{p}_n, T_n^{(G)} = (B/N) \log_2(1 + |h_n|^2 \bar{p}_n/\sigma_n^2)\}\) as the initial point such that the worst-case performance gap shown in Corollary 5.1 can be guaranteed. It is observed from Fig. 7 that for various values of \(\bar{T}\), uniform scalar quantization based scheme without the integer constraints in problem (P2) does achieve a capacity within 2 bps/Hz to \(C\). Moreover, it is observed that with Algorithm II, the performance loss due to the integer constraints is negligible. However, if we simply set \(\alpha = 1\) in (31) to find feasible \(T_n^{(U)}\)'s, there will be a considerable rate loss. As a result, our proposed Algorithm II is practically useful for setting \(\alpha\) such that uniform scalar quantization based scheme can perform very close to the capacity upper bound. Last, it is observed that the performance gap of all the schemes to the upper bound \(C\) vanishes as the fronthaul link capacity increases. This is because if \(\bar{T}\) is sufficiently large at the RRH, each symbol can be quantized by a large number of bits such that the specific quantization
method does not affect the quantization noise significantly.

To further illustrate the gain from joint optimization of wireless power and fronthaul rate allocation, in the following we introduce some benchmark schemes where either wireless power or fronthaul rate allocation is optimized, but not both.

- **Heuristic Solution 1: Equal Power Allocation.** In this scheme, the user allocates its transmit power equally to each SC, i.e., $p_n = \bar{P}/N$, $\forall n$. Then, with the given equal power allocation, we optimize the fronthaul rate allocation at the RRH to maximize the end-to-end rate.

- **Heuristic Solution 2: Water-Filling Power Allocation.** In this scheme, the user ignores the fronthaul link constraints and allocates its transmit power based on water-filling solution as shown in (28). Then, with the given water-filling based power allocation, we optimize the fronthaul rate allocation at the RRH to maximize the end-to-end rate.

- **Heuristic Solution 3: Equal Fronthaul Rate Allocation.** In this scheme, the RRH equally allocates its fronthaul link capacity among SCs, $T_n^{(U)} = \bar{T}/N$. Then, with the given equal fronthaul rate allocation, we optimize the transmit power of the user to maximize the end-to-end rate.

- **Heuristic Solution 4: Equal Power and Fronthaul Rate Allocation.** In this scheme, the user allocates its transmit power equally to each SC, and the RRH equally allocates its fronthaul link bandwidth among SCs.

Fig. 8 shows the performance comparison among various proposed solutions for the uniform scalar quantization based scheme. It is observed that compared with Heuristic Solutions 1-4 where only either wireless power or fronthaul rate allocation is optimized, our joint optimization solution proposed in Section V-B achieves a much higher end-to-end rate, especially when the fronthaul link capacity is small, e.g., $\bar{T} \leq 0.5$ Gbps. Furthermore, it is observed from Heuristic Solutions 1 and 3 that when $\bar{T}$ is small,
fronthaul rate optimization plays the dominant role in improving the end-to-end rate performance, while when $\bar{T}$ is large, most of the optimization gain comes from the wireless power allocation. Furthermore, when $\bar{T}$ is sufficiently large, the performance of Heuristic Solutions 2 and 3, for which wireless power allocation is optimized, even converges to the joint optimization solution proposed in Section \[V-B\].

VI. General Case: Multiple Users and Multiple RRHs

In this section, we consider the joint wireless power allocation and fronthaul rate allocation in the general C-RAN with multiple users and multiple RRHs, i.e., $K \geq 1$ and $M \geq 1$.

A. Gaussian Test Channel

In this subsection, we solve problem (P1). It is worth noting that different from Section \[V-A\] in the case of multiple RRHs, the throughput $R_{\text{sum}}^{(G)}$ given in (10) is not concave over $T_{m,n}$'s with given $p_{k,n}$'s due to the summation over $m$ in (6). As a result, the alternating optimization based solution proposed in Section \[V-A\] cannot be directly extended to the general case of $K \geq 1$ and $M \geq 1$.

To deal with the above difficulty, we change the design variables in problem (P1). Define

$$\psi_{m,n} = 2 \frac{N T_{m,n}^{(G)}}{B} - 1, \quad \forall m, n.$$  \hspace{1cm} (34)

Then, by changing the design variables of problem (P1) from $\{p_{k,n}, T_{m,n}^{(G)}\}$ to $\{p_{k,n}, \psi_{m,n}\}$, problem (P1)
is transformed into the following problem.

\[
\text{Maximize} \quad \frac{B}{N} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2 \left( 1 + \sum_{m=1}^{M} \frac{|h_{m,k,n}|^2 p_{k,n} \hat{\psi}_{m,n}}{\sigma_{m,n}^2 \psi_{m,n} + |h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2} \right)
\]

Subject to \( \frac{B}{N} \sum_{n=1}^{N} \log_2 (1 + \psi_{m,n}) \leq \bar{T}_m, \quad \forall m, \) \( \sum_{n=1}^{N} p_{k,n} \leq \bar{P}_k, \quad \forall k. \) (35)

Problem (35) is still a non-convex problem. In the following, we propose to apply the techniques of alternating optimization as well as convex approximation to solve it.

First, by fixing \( \psi_{m,n} = \hat{\psi}_{m,n} \)'s, we optimize the transmit power allocation \( p_{k,n} \)'s by solving the following problem.

\[
\text{Maximize} \quad \frac{B}{N} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2 \left( 1 + \sum_{m=1}^{M} \frac{|h_{m,k,n}|^2 p_{k,n} \hat{\psi}_{m,n}}{\sigma_{m,n}^2 \hat{\psi}_{m,n} + |h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2} \right)
\]

Subject to \( \sum_{n=1}^{N} p_{k,n} \leq \bar{P}_k, \quad \forall k. \) (36)

Let \( \hat{p}_{k,n} \)'s denote the optimal solution to problem (36). Then, by fixing \( p_{k,n} = \hat{p}_{k,n} \)'s, we optimize the fronthaul rate allocation by solving the following problem.

\[
\text{Maximize} \quad \frac{B}{N} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2 \left( 1 + \sum_{m=1}^{M} \frac{|h_{m,k,n}|^2 \hat{p}_{k,n} \hat{\psi}_{m,n}}{\sigma_{m,n}^2 \hat{\psi}_{m,n} + |h_{m,k,n}|^2 \hat{p}_{k,n} + \sigma_{m,n}^2} \right)
\]

Subject to \( \frac{B}{N} \sum_{n=1}^{N} \log_2 (1 + \psi_{m,n}) \leq \bar{T}_m, \quad \forall m. \) (37)

Let \( \hat{\psi}_{m,n} \)'s denote the optimal solution to problem (37). Then, the above update of \( p_{k,n} \)'s and \( \psi_{m,n} \)'s is iterated until convergence. In the following, we provide how to solve problems (36) and (37), respectively.

First, we consider problem (36). We have the following lemma.

**Lemma 6.1:** The objective function of problem (36) is a concave function over \( \{p_{k,n}\} \).

**Proof:** Please refer to Appendix E.

According to Lemma 6.1, problem (36) is a convex optimization problem. As a result, its optimal solution can be efficiently obtained via the interior-point method [18].

Next, we consider problem (37). Similar to Lemma 6.1, it can be shown that the objective function of problem (37) is a concave function over \( \hat{\psi}_{m,n} \)'s. However, the fronthaul link capacity constraints in problem (37) are not convex. In the following, we apply the convex approximation technique to convexify the fronthaul link capacity constraints. Specifically, since according to (34) \( T_m^{(G)} \) is concave over \( \psi_{m,n} \)'s, its first-order approximation serves as an upper bound to it, i.e.,

\[
T_m^{(G)} = \frac{B}{N} \sum_{n=1}^{N} \log_2 (1 + \psi_{m,n}) \leq \frac{B}{N} \sum_{n=1}^{N} \left( \log_2 (1 + \hat{\psi}_{m,n}) + \frac{\psi_{m,n} - \hat{\psi}_{m,n}}{(1 + \hat{\psi}_{m,n}) \ln 2} \right), \quad m = 1, \ldots, M. \quad (38)
\]
Note that the above inequality holds given any \( \tilde{\psi}_{k,n} \)'s. As a result, we solve the following problem via a relaxation of problem (37).

\[
\begin{align*}
\text{Maximize} & \quad \frac{B}{N} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2 \left( 1 + \sum_{m=1}^{M} \frac{|h_{m,k,n}|^2 \hat{p}_{k,n} \psi_{m,n}}{\sigma_{m,n}^2 \psi_{m,n} + |h_{m,k,n}|^2 \hat{p}_{k,n} + \sigma_{m,n}^2} \right) \\
\text{Subject to} & \quad \frac{B}{N} \sum_{n=1}^{N} \left( \log_2(1 + \tilde{\psi}_{m,n}) + \frac{\psi_{m,n} - \tilde{\psi}_{m,n}}{(1 + \psi_{m,n}) \ln 2} \right) \leq \bar{T}_m, \quad \forall m.
\end{align*}
\] (39)

Problem (39) is a convex problem, and thus its optimal solution, denoted by \( \tilde{\psi}_{m,n} \)'s, can be efficiently obtained via the interior-point method [18]. Then we have the following lemma.

**Lemma 6.2:** Suppose that \( \tilde{\psi}_{m,n} \)'s is a feasible solution to problem (37), i.e., \( \frac{B}{N} \sum_{n=1}^{N} \log_2(1 + \tilde{\psi}_{m,n}) \leq \bar{T}_m, \forall m \). Then, \( \tilde{\psi}_{m,n} \)'s is a feasible solution to problem (37) and achieves an objective value no smaller than that achieved by the solution \( \tilde{\psi}_{m,n} \)'s.

**Proof:** Please refer to Appendix F.

Since the optimal solution to problem (37), i.e., \( \tilde{\psi}_{m,n} \)'s, is difficult to obtain, in the following we use \( \tilde{\psi}_{m,n} \) as the solution to (37) according to Lemma 6.2, i.e., \( \hat{\psi}_{m,n} = \tilde{\psi}_{m,n}, \forall m,n \).

After problems (36) and (37) are solved, we are ready to propose the overall iterative algorithm to solve problem (35), which is summarized in Table IV. Note that in Step 2.c., we set \( \tilde{\psi}_{m,n} = \psi^{(i-1)}_{m,n} \) in problem (39). According to Lemma 6.2 \( \psi^{(i)}_{m,n} \)'s will achieve a sum-rate that is no smaller than that achieved by \( \psi^{(i-1)}_{m,n} \)'s. To summarize, a monotonic convergence can be guaranteed for Algorithm III since the objective value of problem (35) is increased after each iteration and it is upper-bounded by a finite value.

| TABLE IV | ALGORITHM III: ALGORITHM FOR PROBLEM (35) |
|----------|------------------------------------------|
| 1. Initialize: Set \( \psi^{(0)}_{m,n} = \frac{2^{R^{(0)}} - 1}{B} \), \( R^{(0)} = 0 \), and \( i = 0 \); |
| 2. Repeat |
| a. \( i = i + 1 \); |
| b. Update \( \{\hat{p}_{k,n}^{(i)}\} \) by solving problem (36) with \( \tilde{\psi}_{m,n} = \psi^{(i-1)}_{m,n}, \forall m,n \), via interior-point method; |
| c. Update \( \{\psi^{(i)}_{m,n}\} \) by solving problem (39) with \( \hat{p}_{k,n}^{(i)} = \hat{p}_{k,n}^{(i-1)} \) and \( \tilde{\psi}_{m,n} = \psi^{(i-1)}_{m,n}, \forall m,n \), via interior-point method; |
| 3. Until \( R^{(i)} - R^{(i-1)} \leq \varepsilon \), where \( R^{(i)} \) denotes the objective value of problem (35) achieved by the solution \( \{\hat{p}_{k,n}^{(i)}, \psi^{(i)}_{m,n}\} \), and \( \varepsilon \) is a small value to control the accuracy of the algorithm. |

**B. Uniform Scalar Quantization**

In this subsection, we propose an efficient algorithm to solve problem (P2) by jointly optimizing the wireless power allocation as well as the fronthaul rate allocation. To be consistent with the solution to problem (P1) proposed in Section VI-A, we define

\[
\psi_{m,n} = \frac{2^{R^{(i)}_{m,n}}}{B} = 2^{2D_{m,n}}, \quad \forall m,n.
\] (40)
Then, by changing the design variables from \( \{ p_{k,n}, T_{m,n}^{(U)} \} \) into \( \{ p_{k,n}, \psi_{m,n} \} \), problem (P2) is transformed into the following problem.

\[
\text{Maximize} \quad \frac{B}{N} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2 \left( 1 + \frac{1}{\sigma_{m,n}^2 \psi_{m,n}} \left| \frac{h_{m,k,n}}{p_{k,n} \psi_{m,n}} \right|^2 + \frac{3}{\psi_{m,n}^2} \right)
\]

\[
\text{Subject to} \quad \frac{B}{N} \sum_{n=1}^{N} \log_2 \psi_{m,n} \leq T_m, \quad \forall m,
\]

\[
\sum_{n \in \Omega_k} p_{k,n} \leq P_k, \quad \forall k,
\]

\[
\psi_{m,n} = 2^{2D_{m,n}}, \quad D_{m,n} \in \{1, 2, \cdots \} \text{ is an integer}, \quad \forall m, n. \tag{41}
\]

It can be observed that if we ignore the last set of constraints involving integers \( D_{m,n} \)'s, then problem (41) is very similar to problem (35). As a result, we propose a two-stage algorithm to solve problem (41). First, we ignore the last constraints in problem (41) and apply an alternating optimization based algorithm similar to Algorithm III to solve it (the details of which are omitted here for brevity). Let \( \{ \hat{p}_{k,n}, \hat{\psi}_{m,n} \} \) denote the obtained solution. Then we fix \( p_{k,n} = \hat{p}_{k,n} \)'s and find a feasible solution \( \psi_{m,n} \)'s based on \( \hat{\psi}_{m,n} \)'s such that \( D_{m,n} = \frac{1}{2} \log_2 \psi_{m,n} \)'s are integers. For any given \( m = \bar{m} \), this is done by rounding \( \frac{1}{2} \log_2 \hat{\psi}_{\bar{m},n} \)'s, \( \forall n \), to their nearby integers as follows:

\[
\frac{1}{2} \log_2 \psi_{m,n} = \begin{cases} \lfloor \frac{1}{2} \log_2 \hat{\psi}_{m,n} \rfloor, & \text{if } \frac{1}{2} \log_2 \hat{\psi}_{\bar{m},n} - \lfloor \frac{1}{2} \log_2 \hat{\psi}_{m,n} \rfloor \leq \alpha_{\bar{m}}, \\ \lceil \frac{1}{2} \log_2 \hat{\psi}_{m,n} \rceil, & \text{otherwise}, \end{cases} \quad n = 1, \cdots, N, \tag{42}
\]

where \( 0 \leq \alpha_{\bar{m}} \leq 1, \forall \bar{m} \). Similar to Algorithm III for the special case of \( K = 1 \) and \( M = 1 \), the optimal value of \( \alpha_{\bar{m}} \) can be efficiently obtained via a simple bisection method, and thus a feasible solution of \( \psi_{m,n} \)'s, \( \forall n \), is obtained according to (42). Last, by searching \( \bar{m} \) from 1 to \( M \), the overall feasible solution \( \{ \psi_{m,n} \} \) is obtained.

C. Numerical Example

In this subsection, we provide a numerical example to evaluate the sum-rate performance of the proposed uniform scalar quantization based scheme in a single C-RAN cluster with \( M = 7 \) RRHs and \( K = 16 \) users randomly distributed in a circular area of radius 100m. It is assumed that the \( B = 300 \)MHz bandwidth of the wireless link is equally divided into \( N = 64 \) SCs, and each user is pre-allocated \( \frac{N}{K} = 4 \) SCs. It is further assumed that the capacities of all the fronthaul links are identical, i.e., \( T_m = T, \forall m \). The other setup parameters are the same as those shown in Table III. Similar to the single-user single-RRH case in Section V-C, we provide various heuristic solutions as benchmark schemes. Note that Heuristic Solutions 1-4 introduced in Section V-C can be simply extended to the general case of \( K \geq 1 \) and \( M \geq 1 \). Furthermore, we also consider Heuristic Solution 5 where each user is only served by its nearest RRH, while the other RRHs do not quantize and forward their received signals at the
SCs possessed by this user (in order to remove any correlation among the quantized signals by different RRHs to minimize the fronthaul overhead). In this case, the BBU does not need to perform a MRC processing to decode the message at each SC as shown in (3). As a result, each cluster of the studied C-RAN reduces to $M$ independent sub-systems each consisting of only one RRH and its served users. In each sub-system, the joint optimization of wireless power allocation and fronthaul rate allocation can be solved similarly as by Algorithm I proposed in Section V.

Fig. 9 shows the end-to-end sum-rate performance versus the common fronthaul link capacity, $T$, achieved by uniform quantization, Gaussian test channel, as well as Heuristic Solutions 1-5. It is observed that with our proposed algorithm in Section VI-B the sum-rate achieved by the uniform scalar quantization based scheme is very close to that achieved by the Gaussian test channel based scheme for various fronthaul capacities. Furthermore, this performance gap vanishes as the fronthaul link capacities increase at all RRHs. It is also observed that compared with Heuristic Solutions 1-4 where only either wireless power or fronthaul rate allocation is optimized, our joint optimization solution proposed in Section VI-B achieves a much higher sum-rate, especially when the fronthaul link capacities are not sufficiently high. By comparing with Fig. 8 it is observed that the joint optimization gain is more significant over the case of single user and single RRH. Last, it is observed that when $T$ is small, Heuristic Solution 5 performs better than Heuristic Solutions 1-4. However, as $T$ increases, there is a considerable performance gap of Heuristic Solution 5 to other solutions. The reason is as follows. When $T$ is small, each RRH can only forward a very small number of users’ data to the BBU due to its limited fronthaul capacity. In this case, the performance loss of Heuristic Solution 5 to MRC-based joint decoding at BBU is small, but the performance gain due to joint wireless power and fronthaul rate allocation is also significant due
to the strict fronthaul link constraints. On the other hand, when $T$ is large, more and more users’ data can be forwarded by the RRHs to the BBU, and thus the joint decoding gain will dominate the sum-rate performance over the joint resource allocation gain.

VII. CONCLUSIONS AND FUTURE WORK

In this paper, we have proposed joint wireless power control and fronthaul rate allocation optimization to maximize the throughput performance of a broadband C-RAN system. In particular, we have considered using practical uniform scalar quantization to replace the information-theoretical quantization method in the system design. Efficient algorithms have been proposed to solve the joint optimization problems. Our results showed that the joint design achieves significant performance gain compared to either optimizing wireless power control or fronthaul rate allocation alone. Besides, we showed that the throughput performance of the proposed simple uniform scalar quantization is very close to the performance upper bound. This has verified that high throughput performance could be practically achieved with C-RAN using simple fronthaul signal quantization methods.

There are also many interesting topics to be studied in the area of fronthaul-constrained broadband C-RAN system. For instance, the impact of imperfect fronthaul link with packet loss of quantized data; dynamic SC allocation among mobile users; multiple users coexist on one SC to further improve the spectral efficiency; distributed quantization among RRHs to exploit the signal correlations; and the corresponding downlink joint wireless resource and fronthaul rate allocations, etc.

APPENDIX

A. Proof of Proposition 5.1

The Lagrangian of problem (22) is expressed as

$$\mathcal{L}(\{p_n\}, \lambda) = \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 p_n}{\sigma_n^2 \left( \frac{|h_n|^2 p_n + \sigma_n^2}{2N^2\bar{P}} \right) + \sigma_n^2} \right) - \lambda \left( \sum_{n=1}^{N} p_n - \bar{P} \right),$$

(43)

where $\lambda$ is the dual variable associated with the transmit power constraint in problem (22). Then, the Lagrangian dual function of problem (22) is expressed as

$$g(\lambda) = \max_{p_n \geq 0, \forall n} \mathcal{L}(\{p_n\}, \lambda).$$

(44)

The maximization problem (44) can be decoupled into parallel subproblems all having the same structure and each for one SC. For one particular SC, the associated subproblem is expressed as

$$\max_{p_n \geq 0} \mathcal{L}_n(p_n),$$

(45)
where
\[
\mathcal{L}_n(p_n) = \frac{1}{N} \log_2 \left( 1 + \frac{|h_n|^2 p_n}{\sigma_n^2 + \frac{|h_n|^2 p_n + \sigma_n^2}{2N^2T_n^{(G)}}} \right) - \lambda p_n, \quad n = 1, \ldots, N. \tag{46}
\]

It can be shown that \( \mathcal{L}_n(p_n) \) is concave over \( p_n \), \( \forall n \). The derivative of \( \mathcal{L}_n(p_n) \) over \( p_n \) is expressed as
\[
\frac{\partial \mathcal{L}_n(p_n)}{\partial p_n} = \frac{|h_n|^2 \sigma_n^2 \left( 2 \frac{N^2 T_n^{(G)}}{B} - 1 \right)}{\left( |h_n|^2 p_n + \sigma_n^2 \right)} - \lambda, \quad \forall n. \tag{47}
\]

By setting \( \partial \mathcal{L}_n(p_n)/\partial p_n = 0 \), we have
\[
p_n^2 + \alpha_n p_n + \eta_n = 0, \quad n = 1, \ldots, N, \tag{48}
\]
where \( \alpha_n \)'s and \( \eta_n \)'s are given in (25) and (26), respectively. If \( \eta_n < 0 \), then there exists a unique positive solution to the quadratic equation (48), denoted by \( \tilde{p}_n = (-\alpha_n + \sqrt{\alpha_n^2 - 4\eta_n})/2 \). In this case, \( \mathcal{L}_n(p_n) \) is an increasing function over \( p_n \) in the interval \((0, \tilde{p}_n)\), and decreasing function in the interval \( [\tilde{p}_n, \infty) \). As a result, \( \mathcal{L}_n(p_n) \) is maximized when \( p_n = \tilde{p}_n \). Otherwise, if \( \eta \geq 0 \), there is no positive solution to the quadratic equation (48), and thus \( \mathcal{L}_n(p_n) \) is a decreasing function over \( p_n \) in the interval \((0, \infty)\). In this case, \( \mathcal{L}_n(p_n) \) is maximized when \( p_n = 0 \).

After problem (44) is solved given any \( \lambda \), in the following we explain how to find the optimal dual solution for \( \lambda \). It can be shown that the objective function in problem (22) is an increasing function over \( \{p_n\} \), and thus the transmit power constraint must be tight in problem (22). As a result, the optimal \( \lambda \) can be efficiently obtained by a simple bisection method such that the transmit power constraint is tight in problem (22). Proposition 5.1 is thus proved.

### B. Proof of Proposition 5.2

Let \( \beta \) denote the dual variable associated with the fronthaul link capacity constraint in problem (23). Similar to Appendix A, it can be shown that problem (23) can be decoupled into the \( N \) subproblems with each one formulated as
\[
\max_{T_n^{(G)} \geq 0} \mathcal{L}_n(T_n^{(G)}), \tag{49}
\]
where
\[
\mathcal{L}_n(T_n^{(G)}) = \frac{1}{N} \log_2 \left( 1 + \frac{|h_n|^2 \hat{p}_n}{\sigma_n^2 + \frac{|h_n|^2 \hat{p}_n + \sigma_n^2}{2N^2 T_n^{(G)} / \beta - 1}} \right) - \beta T_n, \quad n = 1, \ldots, N. \tag{50}
\]

The derivative of \( \mathcal{L}_n(T_n^{(G)}) \) over \( T_n^{(G)} \) is expressed as
\[
\frac{\partial \mathcal{L}_n(T_n^{(G)})}{\partial T_n^{(G)}} = \frac{1}{B} - \beta - \frac{\sigma_n^2 \frac{N^2 T_n^{(G)}}{B}}{B \left( |h_n|^2 \hat{p}_n + \sigma_n^2 \frac{N^2 T_n^{(G)}}{B} \right)}, \quad n = 1, \ldots, N. \tag{51}
\]
If $\beta \geq \frac{1}{B}$, then $\frac{\partial \mathcal{L}_n(T(G))}{\partial T_n(G)} \leq 0$, i.e., $\mathcal{L}_n(T_n(G))$ is a decreasing function over $T_n(G)$, $\forall n$. In this case, we have $\bar{T}_n(G) = 0$, $\forall n$, which cannot be the optimal solution to problem (23). As a result, the optimal dual solution must satisfy $\beta < \frac{1}{B}$. In this case, it can be shown that $\mathcal{L}_n(T_n(G))$ is an increasing function over $T_n(G)$ when $T_n(G) < \frac{B}{N} \log_2 \left( \frac{(1-\beta B)|h_n|^2p_n}{\beta B\sigma_n^2} \right)$, and decreasing function otherwise. As a result, $\mathcal{L}_n(T_n(G))$ is maximized at $T_n(G) = \max(\frac{B}{N} \log_2 \left( \frac{(1-\beta B)|h_n|^2p_n}{\beta B\sigma_n^2} \right), 0)$. After problem (49) is solved given any $\beta < \frac{1}{B}$, the optimal $\beta$ that is the dual solution to problem (23) can be efficiently obtained by a simple bisection method over $(0, \frac{1}{B})$ such that the fronthaul link capacity constraint is tight in problem (23). Proposition 5.2 is thus proved.

C. Proof of Proposition 5.3

With constraints given in (33), $R_{\text{sum}}(G)$ given in (10) reduces to

$$
\frac{R_{\text{sum}}(G)}{B} = \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2p_n}{2\sigma_n^2} \right). \quad \text{(52)}
$$

Moreover, it can be shown from (33) that

$$
\frac{T(G)}{B} = \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 2 + \frac{|h_n|^2p_n}{\sigma_n^2} \right) = \frac{R_{\text{sum}}(G)}{B} + 1. \quad \text{(53)}
$$

Thereby, with the additional constraints given in (33), problem (P1) can be simplified as the following power control problem.

Maximize $\begin{cases} p_n \end{cases}$ \quad \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2p_n}{2\sigma_n^2} \right)

Subject to $\begin{cases} \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2p_n}{\sigma_n^2} \right) + 1 \leq \bar{T} \\sum_{n=1}^{N} p_n \leq \bar{P} \end{cases} \quad \text{(54)}$

Let $\{\tilde{p}_n\}$ and $\{\hat{p}_n\}$ denote the optimal power solution to problem (54) and the relaxed version of problem (54) without the first fronthaul link constraint, respectively. If $(1/N) \sum_{n=1}^{N} \log_2(1 + |h_n|^2\tilde{p}_n/2\sigma_n^2) + 1 \leq \bar{T} / B$, we have $\tilde{p}_n = \hat{p}_n$, $\forall n$. Otherwise, it can be shown that any feasible solution to the following problem is optimal to problem (54):

Find $\{p_n\}$ \quad \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2p_n}{2\sigma_n^2} \right) + 1 = \bar{T} \\sum_{n=1}^{N} p_n \leq \bar{P} \quad \text{(55)}$

To summarize, the cut-set bound based optimal value of problem (54) is expressed as

$$
\bar{R}_{\text{sum}}(G) = \min \left\{ \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2\tilde{p}_n}{2\sigma_n^2} \right), \frac{\bar{T}}{B} - 1 \right\}. \quad \text{(56)}
$$
In the following, we compare this optimal value with the capacity upper bound \( C \) given in (32). First, we have
\[
\frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 p_{n}^{\text{wf}}}{\sigma_n^2} \right) - 1 < \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 p_{n}^{\text{wf}}}{2\sigma_n^2} \right)
\leq (a) \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 \hat{p}_n}{2\sigma_n^2} \right),
\]
where (a) is because \( \{\hat{p}_n\} \) is the optimal power solution to problem (54) without the fronthaul link constraint. It then follows that
\[
\bar{R}_{\text{sum}}^{(G)}(\sum) = \min \left\{ \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 \tilde{p}_n}{\sigma_n^2} \right), \frac{\bar{T}}{B} - 1 \right\}
\geq \min \left\{ \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 p_{n}^{\text{wf}}}{\sigma_n^2} \right) - 1, \frac{\bar{T}}{B} - 1 \right\}
= C - 1.
\]
Proposition 5.3 is thus proved.

D. Proof of Proposition 5.4

First, it follows that
\[
\bar{R}_{\text{sum}}^{(G)}(\sum) = \min \left\{ \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 \tilde{p}_n}{\sigma_n^2} \right), \frac{\bar{T}}{B} - 1 \right\}
\geq \min \left\{ \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 p_{n}^{\text{wf}}}{\sigma_n^2} \right) - 1, \frac{\bar{T}}{B} - 1 \right\}
= C - 1.
\]
As a result, \( \{p_n = \tilde{p}_n, T_n^{(G)} = (B/N) \log_2 (1 + |h_n|^2 \tilde{p}_n/\sigma_n^2)\} \) is a feasible solution to problem (P2-NoInt). Furthermore, with \( \{p_n = \tilde{p}_n, T_n^{(G)} = (B/N) \log_2 (1 + |h_n|^2 \tilde{p}_n/\sigma_n^2)\} \), \( R_{\text{sum}}^{(U)} \) given in (21) reduces to
\[
R_{\text{sum}}^{(U)} = \frac{B}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 \tilde{p}_n}{4\sigma_n^2} \right).
\]
It then follows that
\[
\frac{\bar{R}_{\text{sum}}^{(U)}}{B} > \frac{1}{N} \sum_{n=1}^{N} \log_2 \left( 1 + \frac{|h_n|^2 \tilde{p}_n}{2\sigma_n^2} \right) - 1
= \frac{\bar{R}_{\text{sum}}^{(G)}}{B} - 1.
\]
Proposition 5.4 is thus proved.

E. Proof of Lemma 6.1

Define
\[
\varphi_{m,k,n}(p_{k,n}) = \frac{|h_{m,k,n}|^2 p_{k,n} \hat{\psi}_{m,n}}{\sigma_{m,n}^2 \hat{\psi}_{m,n} + |h_{m,k,n}|^2 p_{k,n} + \sigma_{m,n}^2}, \quad \text{if } n \in \Omega_k, \forall m, n.
\]
Then, it can be shown that $\varphi_{m,k,n}(p_{k,n})$ is concave over $p_{k,n}, \forall m, n$. As a result, $\sum_{m=1}^{M} \varphi_{m,k,n}(p_{k,n})$ is concave over $p_{k,n}, \forall k, n$. According to the composition rule \[18\], $\log_2(1 + \sum_{m=1}^{M} \varphi_{m,k,n}(p_{k,n}))$ is concave over $p_{k,n}, \forall k, n$. It then follows that the objective function of problem (36), i.e., $\sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2(1 + \sum_{m=1}^{M} \varphi_{m,k,n}(p_{k,n}))$, is concave over $\{p_{k,n}\}$. Lemma 6.1 is thus proved.

F. Proof of Lemma 6.2

First, due to the inequality given in (38), any feasible solution to problem (39) must be a feasible solution to problem (37). Thereby, $\tilde{\psi}_{m,n}$'s must be feasible to problem (37). Next, it can be observed that if $\tilde{\psi}_{m,n}$'s is feasible to problem (37), it must be feasible to problem (39). Since $\tilde{\psi}_{m,n}$'s is the optimal solution to problem (39), the sum-rate achieved by it must be no smaller than that achieved by $\tilde{\psi}_{m,n}$'s. Lemma 6.2 is thus proved.

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