Power corrections in $e^+e^− \to \pi^+\pi^−$, $K^+K^−$ and $B \to K\pi, \pi\pi$

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CLEO-c measurements of the timelike form factors $F_\pi, F_K$ at $\sqrt{s} = 3.671$ GeV provide a direct probe of power corrections (PC’s) at energies near $m_B$. PC’s in $F_{\pi,K}$ and $B \to K\pi, \pi\pi$ are separated into perturbative parts, and soft parts obtained from fits to data. In $F_{\pi,K}$ the latter are $O(10)$ larger. Soft dominance of same order is expected in the QCD penguin PC’s, and is required by the $B \to K\pi, \pi\pi$ fits. The CP asymmetries $S_{K^+,\pi^0}$, $C_{K^+,\pi^0}$ are well determined, agree with experiment, and provide a clean test for new physics. Greater PC soft-overlaps for $\pi\pi$, and much larger ones for $PP$ than $VP, VV$ shed light on the special nature of the pseudo-Nambu-Goldstone bosons.

Much effort has gone into the theoretical description of $B$ decays into light meson pairs. Apart from being of interest in QCD, the issue has important implications for new physics search strategies which rely on comparing decay rates and CP asymmetries in different final states. The decay amplitudes can be organized into expansions in powers of $1/m_b$. The leading power (LP) contributions are calculable in QCD factorization (QCDF) \cite{1} in terms of universal non-perturbative quantities. Numerous leading power predictions for $B \to M_1M_2$ decays are in gross conflict with the data. In $B \to K\pi, \pi\pi$ the direct CP asymmetry $A_{\pi^+\pi^-}$ is too small, $A_{K^+\pi^-}$ is too small and of wrong sign, $A_{K^+\pi^+}$ is not accounted for, contrary to observation, and the branching ratios $B_{\pi^0\pi^0}$, $B_{\pi^0\pi^0}$ are too small. A possible explanation is that certain PC’s are of same order as or larger than their LP counterparts and have large strong phases, due to non-perturbative effects.

Continuum $e^+e^− \to M_1M_2$ light meson cross sections at $\sqrt{s} \approx 3.7$ and 10.58 GeV at the charm and $B$ factories provide a direct probe of PC’s in the timelike vector-current matrix elements $\langle M_1M_2|\bar{q}\gamma\mu|0\rangle$. Perturbative calculations of PC’s on the light-cone contain IR logarithmic terms of the form $\alpha_s(m_b)/(\sqrt{s})^n \ln^{m_n}(\sqrt{s}/\Lambda)$, signaling the breakdown of short/long-distance factorization. A represents a physical IR cutoff on the longitudinal momentum of, e.g., a valence quark in the convolution integrals of light meson light-cone distribution amplitudes (LCDA’s) with hard scattering amplitudes. We therefore divide the PC’s into perturbative and non-perturbative (soft overlaps), where the former are defined by imposing $\Lambda, m_b \gtrsim 1$ GeV, and fits to the data give ranges for the latter. For example, the vector-current form factor PC’s are written as $\delta F = \delta F^{\text{pert}}, \delta F^{\text{non-pert}}$. In this letter we focus on PC’s in $e^+e^− \to \pi^+\pi^−$, $K^+K^−$ and $B \to K\pi, \pi\pi$. Details, and results for $VP, VV$ final states will be given elsewhere \cite{2}.

The continuum timelike form factors $F_{K,\pi} at \sqrt{s} = 3.671$ GeV are \cite{3}

$$|F_{\pi}| = 0.075 \pm 0.009, \quad |F_K| = 0.063 \pm 0.004. \quad (1)$$

The calculable LP contributions arise at twist-2 in the LCDA’s, and fall like $1/s$ \cite{4}. We obtain

$$F_{\pi}^{\text{LP}} = -0.01^{+0.002}_{-0.004}, \quad F_K^{\text{LP}} = -0.014^{+0.002}_{-0.006}. \quad (2)$$

FIG. 1: $\delta F_{K,\pi}$ vs. $\Lambda$. Solid curves are for $\alpha_s$ and central values of LCDA parameters evaluated at $\mu_0 = \Lambda$. Inner bands (blue) are for variations of LCDA parameters. Outer bands (yellow) include variation of $\mu_0 \in [\max[1, \Lambda/2], \sqrt{s}]$. Errors are added in quadrature. Dashed lines outline the ‘outer bands’ for asymptotic LCDA’s.

where the errors due to variation of the first two LCDA Gegenbauer coefficients \cite{5}, and the scale at which they and $\alpha_s$ are evaluated, $\sqrt{s}$ to $\sqrt{s}/2$, are added in quadrature. Thus, the form factors must be dominated by PC’s. A dual-resonance model calculation of $F_{\pi}$ \cite{6}, which includes the first three $\rho$ resonances explicitly, is consistent with the twist-2 result for $\sqrt{s} > 3$ GeV \cite{7}. Evidently, duality has set in, implying sensitivity to PC’s.

$\delta F_{K,\pi}$ enter at $1/E^2$, or twist-4 perturbatively, and to first approximation fall like $1/s^2$. We obtain $\delta F_{K,\pi}^{\text{pert}}$ from convolutions of two twist-3 valence quark LCDA’s with the tree-level hard-scattering amplitudes (twist-4 valence quark LCDA’s contribute negligibly). The model parameters of \cite{5} are employed for the LCDA’s. Perturbative higher Fock state effects are of same twist and order of magnitude, and therefore would not alter our result for the soft PC’s. Fig. 1 shows the ranges obtained as the cutoff $\Lambda$ in the divergent terms is varied from $\sqrt{s}$ to 1 GeV. $\Lambda$ is roughly the lowest gluon virtuality allowed. Ranges for asymptotic LCDA’s are also shown. There are large accidental cancelations between asymptotic and
non-asymptotic effects at lower $\Lambda$. The magnitudes of each separately therefore give a better indication of the size of perturbative effects for $\Lambda$ near 1 GeV. Comparing the asymptotic plots to Eq. [4] it is clear that the dominance of the PC’s in $F_{K,\pi}$ is due to their soft parts,

$$\delta F_{\pi}^{\text{np}} / \delta F_{\pi}^{\text{pert}} = O(10), \quad \delta F_{K}^{\text{np}} / \delta F_{K}^{\text{pert}} = O(10). \quad (3)$$

Similar soft enhancement would account for $F_{\pi}(m_{J}/\psi) \approx 0.10$, as extracted from $J/\psi$ decays [5].

At LP, the form factors obey canonical $SU(3)_F$ flavor symmetry breaking, i.e., $(F_{\pi}/F_{K})_{\text{LP}} \approx f_2^2/f_K^2 \approx 0.7$. However, $|F_{\pi}/F_{K}|_{\text{exp.}} = 1.20 \pm 0.17$, implying that $|\delta F_{\pi}/\delta F_{K}| > 1$, e.g., $\langle 1.9 \pm 0.36 \rangle f_2^2/f_K^2$ for constructive interference between LP and PC effects. Apparently, the soft-overlap is greater for $\pi\pi$ than $KK$, providing a source for large $SU(3)_F$ breaking.

The $SU(3)_F$ diagrammatic representation gives a convenient general classification of the $B \to K\pi\pi$ amplitudes [1]. The $B \to K\pi$ amplitudes are

$$A_{K\pi,\pi^-} = \lambda_p (A_{3} p_{\pi u} + p_{\pi p} - 1/3 p_{\pi EW} - 1/3 p_{\pi EW})$$

$$- A_{K-\pi^+} = \lambda_p (T p_{\pi u} + p_{\pi p} + 1/3 p_{\pi EW} - 1/3 p_{\pi EW})$$

$$- \sqrt{2} A_{K-\pi^0} = \lambda_p (T + C + A) p_{\pi u} + p_{\pi p} + p_{\pi EW}$$

$$+ 2/3 p_{\pi EW} + 3/3 p_{\pi EW})$$

$$\sqrt{2} A_{K\pi,\pi^0} = A_{K\pi,\pi^-} + \sqrt{2} A_{K-\pi^0} - A_{K-\pi^+}. \quad (4)$$

The CKM factor $\lambda_p = V_{pb}^* V_{ps}^*$, and there is a sum over $p = u, c$. The $B^- (B^0)$ Br’s are given by $|A|^2 (T_{\pi 0}/B_{\pi 0} |A|^2)$ in our normalization. $T(a_1)$ and $C(a_2)$ are the color-allowed and color-suppressed ‘tree’ amplitudes. $P_{\pi E}(a_{4,5})$, $P_{\pi EW} (a_{7,9})$, and $P_{\pi EW} (a_{10,10})$ are the QCD, electronweak penguin (EWP), and color-suppressed EWP amplitudes, respectively. In $\pi\pi$ the corresponding quantities are primed (’). These amplitudes consist of LP parts (the corresponding QCDF coefficients $a_i$) are given in parenthesis) $T_{\text{LP}}, C_{\text{LP}}, \text{etc.}$, and PC’s $\delta T, \delta C, \text{etc.}$. $P_{\pi EW}$ and $A$ are EWP and ‘tree’ weak-annihilation (WA) PC’s.

The $K\pi$ PC fit is dominated by $|\delta P^{\pi}| e^{i\delta T}, |\delta C| e^{i\delta C}, |\delta T| e^{i\delta T}$. The strong phases are defined relative to the corresponding naive-factorization amplitudes. Allowing for a soft contribution that is an order of magnitude greater than its perturbative part leaves $A$ negligible. The electromagnetic $u, c$-loop penguin contributions are negligible in the EWP PC’s, so we drop their ‘p’ superscripts. The approximate relation (up to an unknown $SU(3)_F$ breaking factor) $\delta P_{\pi EW} \approx 3/2 C_9/C_1 \delta C$, and the order of magnitude relations (equalities in the one-gluon exchange approximation) $\delta P_{\pi EW} \approx 3/2 C_9/C_1 \delta C$ and $\delta P_{\pi EW} \approx 3/2 C_9/C_2 A$ relegate the EWP PC’s to a minor role. The $C_i$ are Wilson coefficients for the $\Delta B = 1$ effective Hamiltonian [1]. In terms of the measured uncertainties, the impact of $\delta P_{\pi EW}$ on $B_{K-\pi^0}, B_{K-\pi^+}$, $A_{K^0-\pi^-}$ is found to be $\lesssim 2.5\sigma, 1.5\sigma, 1.5\sigma$, respectively. The impact of the remaining EWP PC’s and $A$ on all observables is $< 1\sigma$. The $\pi\pi$ PC fit is dominated by $|\delta P^\pi| e^{i\delta T}, |\delta C| e^{i\delta C}, |\delta T| e^{i\delta T}$. The ‘tree’ WA amplitude $E$ plays a significant role in $\gamma\pi^-, \pi\pi^-$, and the $\pi\pi$ EWP PC’s are negligible.

All Br’s and $A_{K^0-\pi^-}, A_{K^0+}, A_{K^0\pi^-}, A_{K^0\pi^-}, S_{\pi^-}$ are required to lie within their $1\sigma$ HAG experimental errors [10]. The LP amplitudes are evaluated in QCDF to NLO [1]. The NNLO corrections [11] would not have a substantial impact on our fit results. The LP inputs are varied uniformly within their errors. The Wilson coefficients, $\alpha_s$, and the LCDA parameters [5] are evaluated at the scale $\mu_h \in [m_\psi/2, m_B]$. The fit allows $|\delta P^{\pi}| u, c$ and $|\delta P^{\pi}| u, c$ to differ substantially, as suggested by their perturbative contributions, see Fig. 4c. In fact, $|\delta P^{\pi}/\delta P^\pi| \leq 2, 3$ for $69\%, 87\%$ ($|\delta P^{\pi}/\delta P^\pi| \leq 2, 3, 4$ for $55\%, 81\%, 92\%$ of the $K\pi$ (\pi\pi) scatter points in Fig. 3c. According to Figs. 3a, c $|\delta P^{\pi}| \sim |P_{\pi L}|$, and $\delta C$ dominates in $C$ with a significant strong phase $\delta T \approx 30^\circ$. The fit results for $C/T$ in Fig. 3d allow for a low $\pi^\pi$ magnitude than obtained in $SU(3)_F$ fits, see e.g. [12]. The need for a large strong phase difference $\delta C - \delta T$ is well known. The SM predictions for the time-dependent CP asymmetries $S_{C,\pi^-} \in [0.62, 0.86]$ and $C_{K^0\pi^-} = -A_{K^0\pi^-} \in [0.07, 0.18]$ are consistent with the measured ranges [10], see Fig. 3c. Nevertheless, the measured uncertainties exceed those for the fits, making these observables a good place to look for new physics, see e.g., [13]. Note that the $K\pi$ results show little dependence on $\delta P^{\pi}/\delta P^\pi$.

The $\pi\pi P^\pi$ fit in Fig. 3b is multiplied by $f_K/f_\pi$ for comparison with Fig. 3a. This time, $|\delta P^{\pi}| \gtrsim |P_{\pi L}|$. Canonical $SU(3)_F$ breaking at LP gives $f_K/f_\pi P^{\pi}_{\text{LP}} \approx P_{\pi L}$. However, $f_K/f_\pi \delta P^{\pi}$ appears to be larger than $\delta P^{\pi}$, in accord with $\delta P^{\pi} > \delta F_K$. This conclusion is reinforced if $|\delta P^{\pi}/\delta P^{\pi}|$ is bounded from above, e.g., $\lesssim 3$. 

![FIG. 2: Diagrams for perturbative $B \to M_1M_2$ PC’s. Crosses indicate all places where the gluon can attach. The $\Delta B = 1$ effective Hamiltonian operators [1].](image-url)
see Fig. 3b. $C'/T'$ is plotted in Fig. 3d. The lower bound on its magnitude increases from roughly 0.15 to 0.25 for $|\delta P'/\delta P|^2 \lesssim 3$. The predicted ranges for $A_{g^+\pi^0}$, $A_{g^\pi^0}$ are $[-0.06, +0.06]$, $[-0.95, 0.55]$, respectively, show little dependence on $|\delta P'/\delta P|^2$, and are consistent with the HFAG averages $0.06 \pm 0.05$, $0.43 \pm 0.25$.

The perturbative $B \to K\pi\pi$ PC's are obtained from the diagrams of Fig. 2. They depend on two renormalization scales: (i) $\mu_b$, linked to the energy release of the decay, as in the LP amplitudes, and (ii) $\mu_h$, linked to the IR cutoff $\Lambda$. The Wilson coefficients and one $\alpha_s$ factor in Figs. 2b,d (associated with the $u$, $c$-loops) are evaluated at $\mu_b$. The other $\alpha_s$ factors and the LCDA parameters are evaluated at $\mu_h$. The Wilson coefficients in Figs. 2a,c,e are NLO, and $C_1$ is LO in Figs. 2b,d. We have checked that the loop diagrams in Fig. 2 eliminate the dominant leading $\log \mu_b$ scale dependence ($\propto C_1\alpha_s/\pi$ in $\delta P_{p,p}$). Products of twist-2,3 $\times$ twist-3 $K,\pi$ valence quark LCDA’s are included in the convolutions with the hard-scattering amplitudes. Again, higher Fock-state effects would not alter our conclusions.

Our results are summarized in Fig. 4. The largest real and imaginary contributions to $\delta P^{(i)p}_{p,p}$ come from Fig. 2b. The QCD dipole operator $Q_{g\pi}$ (Fig. 2c) and QCD penguin WA ($Q_{3\ldots6}$ in Fig. 2a) contributions are a factor of 2-3 smaller in magnitude, and real. All three are dominated by diagrams in which a gluon is not attached to the $B$. They contribute at twist-3 ($1/m_b$). (Other contributions to $\delta P^{(i)p}_{p,p}$, e.g. $Q_{3\ldots6}$ in Fig. 2c and Fig. 2d, are negligible.) In the limit $\Lambda \to 0$, they would give rise to two energetic outgoing quarks, and soft-overlaps for both light mesons, as indicated by a quadratic dependence on $\log m_B/\Lambda$. This is also the case for $\delta F_{\pi,K}$, which depend quadratically on $\log \sqrt{s}/\Lambda$. Therefore, the relative importance of non-perturbative PC’s in $\delta P^{(i)p}$ should be similar to Eq. 3 given the proximity of energy scales. Indeed, comparison of Figs. 4a,b,c with the fit results in Figs. 3a,b implies $\delta P^{(i)c,n.p.}/\delta P^{(i)p}_{p,p} = O(10)$.

$\delta C^{(i)}_{p,p}$ is due to $Q_1$ in Fig. 2c. The diagrams again enter at twist-3 ($1/m_b$). In the $\Lambda \to 0$ limit only the spectator quark light meson would be produced via a soft-overlap, as indicated by a linear dependence on $\log m_B/\Lambda$. Therefore, we expect $\delta C^{(i), n.p.}/\delta C^{(i)}_{p,p}$ to be smaller than the QCD penguin ratio. According to Figs. 3c, 4d (for low $\Lambda$) this may well be the case, but the errors are too large to reach a definitive conclusion.

We emphasize that the similarity of Eq. 5 to Eq. 3 supports the importance of the charm-loop [14], dipole, and WA PC’s in the full QCD penguin amplitudes, and increases our confidence that the $B \to K\pi\pi$ ‘puzzles’ can be accounted for as in the fits. Given the penguin PC soft dominance, the origin of $f_K/f_\pi$ $\delta P'_{\pi c} > \delta P'_{\pi c}$ is the same as $\delta F_{\pi c} > \delta F_K$, a larger $\pi\pi$ soft-overlap. That certain soft PC’s are of same order as their LP counterparts should be viewed as accidental, rather than a breakdown of the $1/E$ expansion. The latter would be signaled by a violation of power counting among the PC’s themselves.

The asymptotic LCDA plots in Figs. 4a,b imply that the charm quark loops can yield large strong phases in the full penguin PC’s. Their presence may be indicated by the fits for $\delta C^{(i)}$ in Figs. 3a,b. However, a sizable strong phase could also originate from soft-overlaps in light meson production, independently of such loops, explaining the large strong phase in $C$. The existence of such a mechanism would be confirmed by a non-vanishing strong phase difference between the LL and LT
FIG. 4: $B \rightarrow K\pi,\pi\pi$ perturbative PC's vs. $\Lambda$. Solid curves are for $m_b = m_b$, $m_b = 0$, central values for other inputs. Inner bands (blue) are for $m_b = \Lambda$, $m_b \in [m_b/2, m_b]$, other inputs varied within uncertainties. Outen bands (yellow) include variation of $m_b \in [\max[1, \Lambda/2], m_b]$. Errors added in quadrature. Dashed curves outline 'outer bands' for asymptotic LCDAs. Inner brown, outer orange bands in (a) are for $Q_{8g}$. Black (brown) curves in (c) are for $f_K/f_\pi$ for $\delta P^{\mu}_{\perp}(u)$.

(L=longitudinal, T=transverse) polarization amplitudes in $e^+e^- \rightarrow \rho^+\rho^-$ at the $\Upsilon(4S)$. This can be searched for with the current $B$ factory data sets.

The asymptotic LCD plots for $\delta P^{\mu}_{\perp}(u)$ and $\delta P^{o,\perp}_{\perp}(u)$ in Fig. 4c support the possibility, allowed by the PC fits, that the total up and charm penguin PC's can have a large hierarchy, e.g., $\delta P^{\mu}_{\perp}(u)/\delta P^{o,\perp}(u) \lesssim 3$. The perturbative analysis also implies that they have large strong phase differences. Similar considerations in $B \rightarrow \phi K$ imply that a large negative shift in the time-dependent $CP$-asymmetry relative to $\sin 2\beta$ is possible e.g., $\delta \phi_{BCP} \approx -(0.10 - 0.15)$.

The QCD dipole operator contribution to $\delta P_{\perp}^{\text{pert}}$ is also plotted in Fig. 4a. Eq. 5 then implies that the total $Q_{8g}$ PC could be of order $P_{\text{LP}}^{\mu}$, see Fig. 3a. It is noteworthy that this is an order of magnitude larger than its LP matrix element. Therefore, the size of new CP violating dipole operator contributions currently probed could be $O(10)$ smaller than expected based on LP analyses.

Continuum $e^+e^- \rightarrow VP,VV$ cross sections at $\sqrt{s} \approx 3.7$ and 10.58 GeV and penguin-dominated $B \rightarrow VP, VV$ data can be accounted for if soft-to-perturbative PC ratios vary from $O(1)$ to $O(\text{few})$ $\delta P^{\text{pert}}$, rather than $O(10)$ as for $PP$. This appears to shed light on the "$\rho - \pi$ puzzle" (and the smaller "$K^* - K$ puzzle"), i.e., how to simultaneously understand the $\pi$ as a $q\bar{q}$ state like the $\rho$, and a nearly massless Nambu-Goldstone boson. In a physical picture of the $\pi$ (and $K$) that addresses this puzzle, the valence $q$ and $\bar{q}$ are always a distance $< 1/\Lambda_QCD$ apart. However, $r_\pi \approx r_\rho < 1/\Lambda_QCD$ is accounted for by a much larger soft cloud of higher Fock states, attributed to a zitterbewegung-like motion of the tightly bound valence quarks. This may account for the large $PP$ soft-overlaps, as well as the larger $\pi\pi$ soft-overlaps because the $\pi$ approximates a Nambu-Goldstone boson more closely than the $K$.

Note added: After completion of the work reported here we were informed by L. Silvestrini that his group has also carried out a $K\pi$ PC fit, with similar predictions for $\delta S_{K^*\pi^0}, C_{K^*\pi^0}$, which has since appeared [17]. For an earlier discussion by them, see M. Pierini in [14].

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