FORMATION OF LOW-MASS STARS AND BROWN DWARFS

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Abstract. These lectures attempt to expose the most important ideas, which have been proposed to explain the formation of stars with particular emphasis on the formation of brown dwarfs and low-mass stars. We first describe the important physical processes which trigger the collapse of a self-gravitating piece of fluid and regulate the star formation rate in molecular clouds. Then we review the various theories which have been proposed along the years to explain the origin of the stellar initial mass function paying particular attention to four models, namely the competitive accretion and the theories based respectively on stopped accretion, MHD shocks and turbulent dispersion. As it is yet unsettled whether the brown dwarfs form as low-mass stars, we present the theory of brown dwarfs based on disk fragmentation stressing all the uncertainties due to the radiative feedback and magnetic field. Finally, we describe the results of large scale simulations performed to explain the collapse and fragmentation of molecular clouds.

1 Introduction

Stars are the building blocks of our universe and understanding their formation and evolution is one of the most important problems of astrophysics. It is well established that stars form by gravitational collapse of molecular dense cores. These cores are themselves embedded in molecular clouds and often, though not always, inside filaments. However, the conditions through which the dense cores form, collapse and fragment remain a matter of debate. Three questions are of particular importance: the star formation rate (hereafter SFR), the problem of the initial mass function (hereafter IMF), and the problem of disk formation and evolution that may give birth to brown dwarfs, low-mass stars and planets.

As pointed out by Zuckerman & Evans (1974), if all the molecular gas observed in the Galaxy was collapsing in a freefall time, then the star formation rate would...
be 10 to 100 times higher than the star formation rate of \( \simeq 3 \, M_\odot \, \text{yr}^{-1} \) observed in the Galaxy. A lot of efforts have been devoted to explain this low efficiency of star formation. Two main ideas have been explored so far. The first is that magnetic field provides an efficient support against gravity and delays the star formation (e.g. Shu et al. 1987). The second postulates that turbulent motions observed in molecular clouds, prevent the clouds to collapse in a freefall time (Mac Low & Klessen 2004, McKee & Ostriker 2007).

Of particular importance is the problem of the initial mass function of stars (Salpeter 1955, Scalo 1986, Kroupa 2002, Chabrier 2003) largely because the star properties, evolution and influence on the surrounding interstellar medium strongly depend on their masses. It is generally found that the number of stars per logarithmic bin of masses, \( dN/d\log M \), can be described by a lognormal distribution below 1 \( M_\odot \), peaking at about \( \simeq 0.3 \, M_\odot \), and a power-law of slope \(-1.3\) for masses between 1 and 10 \( M_\odot \) (e.g. Chabrier 2003). It should be stressed that the IMF of more massive stars is extremely poorly known.

The conservation of angular momentum during collapse leading to the so-called centrifugal barrier is another long-standing problem in astrophysics. Angular momentum is responsible for the formation of ubiquitous circumstellar disks (e.g. Haisch et al. 2001, Watson et al. 2007) and probably, at least in part for the formation of binaries (Duquennoy & Mayor 1991). Indeed, the fragmentation of massive early, class-0, disks is possibly a mode for the formation of low-mass stars and brown dwarfs while the formation of planets occurs within disks probably at different stages of their evolution.

In the following, we first present the basic theory of self-gravitating isothermal gas, including Jeans length and mass, self-gravitating equilibrium and collapse. We also infer the mass of the smallest star that could possibly form in present day universe. We then discuss the question of the star formation rate, presenting the two main schools of thought which have been proposed so far, namely the magnetic and the turbulent regulation of star formation. Note that as this aspect does not constitute the main goal of this lecture but is presented for completeness, only the most important aspects are discussed. The third section is devoted to the question of the initial mass function and the various theories, which have been proposed along the years. Four different theories will be presented, namely the competitive accretion scenario, the theories based on stopped accretion, the MHD shock based theory, and the turbulent dispersion theory. In the fourth section, we first discuss self-gravitating disk formation, evolution and stability. Then we present the brown dwarf formation by disk fragmentation theory stressing the uncertainties due to the radiative feedback and magnetic field. Finally, in the fifth section, we describe the results of large scale numerical simulations which have been performed so far to study the formation of stars in massive collapsing clumps. In particular, we present two other scenarii for the formation of brown dwarfs which have been

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1 It should be kept in mind that the *initial* mass function is measured only up to about 8 \( M_\odot \), in young stellar clusters, and is inferred only indirectly for larger masses (see e.g. Kroupa 2002)
studied in these simulations, namely the formation by ejection and the formation in collapsing filaments. Then we discuss the IMF which have been obtained in these simulations as well as the physical explanations that have been proposed.

2 Physical processes

2.1 Gravity and thermal support

Before going into the more complex situations of magnetized and turbulent clouds, we start by establishing the basic principles of self-gravitating isothermal gas dynamics.

2.1.1 Ratio of thermal and gravitational energies

It is instructive to start by computing the ratio between the thermal energy, \( E_{\text{therm}} = \frac{M}{(\gamma - 1)m_p} k_b T \) and the gravitational energy \( E_{\text{grav}} = (3/5)M^2 G/R \), where \( M \) is the cloud mass, \( R \) its radius, \( m_p \) the mean mass per particle, \( T \) the temperature, \( k_b \) and \( G \) are respectively the Boltzmann and the gravitational constants, and \( \gamma \) is the adiabatic index, which depends on the internal degrees of freedom of the constituents. For a polytropic cloud, the thermal pressure is given by \( P = K \rho \Gamma \), where \( \Gamma \) is an effective adiabatic exponent that depends on the cooling processes. With these expressions, we get

\[
\frac{E_{\text{therm}}}{E_{\text{grav}}} \propto R^{4-3\Gamma}.
\]

This expression clearly shows that \( \Gamma = 4/3 \) is a critical case below which thermal pressure is unable to support the cloud against gravitational collapse because the ratio between support and gravitational energy drops with the radius. While this is in particular true for the isothermal case, \( \Gamma = 1 \), gravitational collapse will be stopped by thermal pressure if the gas is unable to cool efficiently both for a monoatomic gas (\( \Gamma \simeq \gamma = 5/3 \)) and for a diatomic one (\( \Gamma \simeq \gamma = 7/5 \)).

2.1.2 Jeans length, Jeans mass, and freefall time

The Jeans length (Jeans 1905, Lequeux 2005) is easily derived by performing a linear analysis of the self-gravitating fluid equations. Let us consider a cloud of density \( \rho_0 \), radius \( R \), and sound speed \( C_s \) (note that strictly speaking a self-gravitating isothermal cloud cannot have a uniform density because the pressure forces should compensate the gravitational forces). A linear analysis leads to the dispersion relation

\[
\omega^2 = C_s^2 k^2 - 4\pi G \rho_0,
\]

which reveals that when the wave number, \( k \), is smaller than \( \sqrt{4\pi G \rho_0}/C_s \), the waves cannot propagate and perturbations are amplified. From this we obtain the
Jeans length, $\lambda_J$,

$$\lambda_J = \sqrt{\frac{\pi C_s^2}{G \rho_0}}, \quad (2.3)$$

where $G$ is the gravitational constant. The Jeans length can be physically understood in the following way. Self-gravity tends to induce contraction in a time scale of the order of $1/\sqrt{G \rho_0}$. On the other hand, thermal pressure tends to reestablish uniform density in a sound crossing time, $R/C_s$. If $1/\sqrt{G \rho_0} < R/C_s$, then the waves cannot erase the pressure fluctuations induced by the gravitational contraction before the whole cloud collapses.

The Jeans mass is naturally defined as the mass contained in a volume of typical size $\lambda_J$. The Jeans mass is generally determined as

$$M_J = \frac{4\pi/3}{\rho_0} (\lambda_J/2)^3 = \frac{\pi^{5/2}}{6} \frac{C_s^3}{(G^3 \rho_0)^{1/2}} \quad (2.4)$$

though there is no fundamental justification for this choice within a factor of a few.

Equation (2.4) shows that, for an isothermal gas, the Jeans mass decreases with density. Therefore, in a collapsing cloud the Jeans mass value increases as the collapse proceeds. Based on this argument, Hoyle (1953) proposed the concept of recursive fragmentation by which a cloud is fragmenting more and more as it becomes denser. However, as shown by eq. (2.2), the growth rate of the gravitational instability, decreases with $k$, meaning that the large scale perturbations evolve more rapidly than the smaller scale perturbations. Thus the recursive fragmentation scenario suffers a timescale problem. Note that there is an inconsistency in inferring eq. (2.2) as a uniform medium is not an exact solution of the self-gravitating fluid equations (a force should balance gravity). When exact solutions of the equations (see next section) are perturbed (e.g. Nagai et al. 1998 for layers or Fiege & Pudritz 2000 for filaments), it is generally obtained that the growth rate tends to zero as $k \rightarrow 0$ while the fastest growing mode corresponds to a few times the Jeans length.

In general, it is not possible to analytically compute the time for a cloud to collapse. However, in the ideal case of a cold spherical cloud with uniform density, one can calculate it exactly (see e.g. Lequeux 2005). The result, known as the freefall time, is

$$\tau_{ff} = \sqrt{\frac{3\pi}{32G \rho_0}} \quad (2.5)$$

2.1.3 The smallest Jeans mass in contemporary molecular clouds

The expression stated by eq. (2.4) assumes that the gas follows a barotropic equation of state and neglects cooling and heating. This is only valid above a certain
limit that eventually sets the value of the smallest Jeans mass. To estimate its value we closely follow the approach of Rees (1976) and Whitworth et al. (2007). The first condition that must be fulfilled is that the size of the piece of fluid, $R$, must be of the order of the Jeans length as explained above. Thus $R \simeq \lambda_J$, leading to

$$R \simeq \frac{6}{\pi^2} \frac{G}{c_s^2} M_J. \quad (2.6)$$

The second condition is that the thermal energy due to the gravitational contraction, should be efficiently radiated. If this condition is not fulfilled, as explained above, the effective adiabatic index, $\Gamma$, will be larger than $4/3$ and thermal pressure will halt the collapse. The heating rate is given by the work of the thermal pressure $P_{\text{dV}}$. Assuming that the collapse is near freefall, we get $v = d_t R \simeq \sqrt{GM/R}$ and thus

$$P_{\text{dV}} = \rho c_s^2 \frac{d}{dt} \left( \frac{4\pi}{3} R^3 \right) \simeq \frac{3c_s^2 M}{R} \sqrt{\frac{GM}{R}}. \quad (2.7)$$

The cooling is due to the radiative loss which in the optically thick regime is given by (e.g. Mihalas & Mihalas 1984)

$$\mathcal{L} = \frac{16\pi}{3\chi_R} R^2 \sigma T^4 \partial_R T \quad (2.8)$$

where $\chi_R = \rho \kappa$ and $\kappa$ is the opacity (e.g. Semenov et al. 2003) while $\sigma = 2\pi^5 k_b^4/15h^3 c^2$ the Stefan-Boltzmann constant. Thus the second condition that the piece of gas must fulfill to be gravitationally unstable is given by

$$P_{\text{dV}} \simeq \frac{3c_s^2 M}{R} \sqrt{\frac{GM}{R}} \simeq \mathcal{L} \simeq \frac{16\pi}{3\tau} R^2 \sigma T^4, \quad (2.9)$$

where the optical depth $\tau \simeq \chi_R R$. This leads to

$$R \simeq \left( \frac{3^6 \pi^2}{2^{10} \pi^{12}} \right)^{1/7} \left( \frac{G^2 h \sigma T^4}{\kappa_{\text{s}}^2 m_p^8} \right)^{1/7} \tau^{2/7} M^{3/7}. \quad (2.10)$$

Combining the conditions stated by eqs. (2.6)-(2.10), one gets a characteristic mass given by

$$M \simeq \left( \frac{5^2 \pi^2}{2^{17} 3} \right)^{1/4} \left( \frac{h c}{G} \right)^{3/2} m_p^{-2} \left( \frac{c_s}{c} \right)^{1/2} \tau^{1/2} \simeq \frac{m_{\text{Planck}}}{m_p} \left( \frac{c_s}{c} \right)^{1/2} \tau^{1/2}, \quad (2.11)$$

where $m_{\text{Planck}}$ is the Planck mass. The exact value of the numerical factor depends on the assumptions that have been made and may vary from one author to another. One finds, for an optical depth $\tau \simeq 1$, that it is of the order of a few Jupiter masses. Note that as discussed by Whitworth & Stamatellos (2006) and Masunaga & Inutsuka (2000), $\tau \geq 1$ is not a necessary condition although it appears reasonable in this context.
2.1.4 Equilibrium configurations

Equilibrium configurations are obtained when pressure forces compensate gravitational forces. Such static solutions of the fluid equations are useful and convenient guides. They allow to test numerical codes, perform more rigorous stability analysis than the Jeans analysis and can sometimes be compared directly to the observations. The equations of equilibrium, namely the hydrostatic equation and the Poisson equation, are respectively:

\[- C_s^2 \partial_X \rho + \rho \partial_X \phi = 0, \tag{2.12} \]
\[ \frac{1}{X^{D-1}} \partial_X (X^{D-1} \partial_X \phi) = -4\pi G \rho. \tag{2.13} \]

Combining these 2 equations leads to the so-called Lane-Emden equation:

\[ \frac{1}{X^{D-1}} \partial_X \left( X^{D-1} \frac{\partial_X \rho}{\rho} \right) = -\frac{4\pi G}{C_s^2} \rho, \tag{2.14} \]

where \( D \) is the dimension and \( X \) the spatial coordinate.

In plane-parallel geometry (\( D = 1 \)), \( X \) represents the usual Cartesian coordinate, \( z \), whereas in cylindrical geometry (\( D = 2 \)), \( X \) represents the cylindrical radius, \( r \). In the first case, a self-gravitating layer solution has been inferred by Spitzer (1942) whereas in the second, a self-gravitating filament has been obtained by Ostriker (1964). These two solutions are fully analytical. They are characterized by a flat density profile near \( X = 0 \). The former presents an exponential decrease for large \( z \) whereas the latter decreases as \( r^{-4} \).

In spherical geometry (\( D = 3 \)) (\( X \) represents the spherical radius, \( r \)), the solutions of eq. (2.14) are the so-called Bonnor-Ebert spheres (Bonnor 1956). In general, these solutions are not analytical and must be obtained by solving numerically eq. (2.14). There is however a noticeable exception which is the singular isothermal sphere whose density is given by \( \rho_{\text{SIS}} = \frac{C_s^2}{(2\pi G r^2)} \). The density profile of the Bonnor-Ebert sphere is flat in the inner part and tends toward the density of the singular isothermal sphere at large radii. Since it is physically required that the cloud has a finite radius, the solutions are obtained by truncating this profile at any arbitrary radius, assuming pressure equilibrium with the medium outside the cloud which is supposed to be diffuse and warm. Therefore a whole family of equilibrium solutions is obtained. They can be characterized by the density contrast between the center and the edge. Stability analysis reveals that the solutions which have a density contrast smaller than about 14 are stable and unstable otherwise.

As mentioned in the previous section, stability analysis of the self-gravitating layer and filament have been performed in various studies (e.g. Larson 1985, Fiege & Pudritz 2000). Both are unstable to perturbations of wavelengths comparable to the Jeans length. In particular, this suggests that cores or filaments distributed periodically, could form through gravitational instability within self-gravitating filaments and layers respectively. As the interstellar medium is out of equilibrium
and subject to supersonic motions, it is always difficult to assess this scenario quantitatively but qualitatively at least, spatially roughly periodically distributed cores and filaments are often observed (Dutrey et al. 1991).

2.1.5 Gravitational collapse

The gravitational collapse of a spherical cloud has been investigated in some details both analytically and numerically. Since even in spherical geometry, the cloud is described by two non linear equations, the analytical models have mainly focused on self-similar solutions (e.g. Larson 1969; Penston 1969; Shu 1977; Whitworth & Summers 1985) which allow to reduce the equations of the problem to simpler ordinary equations. These solutions have been useful to understand the physics of the collapse and sometimes used in various contexts to provide easily time-dependent density and velocity fields. Two main types of solutions have been inferred. Larson (1969) and Penston (1969) derive a solution which presents supersonic infall velocity at large radii ($\simeq 3.3C_s$) whereas Shu (1977) obtains a solution in which the gas is initially at rest and undergoes inside-out collapse. A rarefaction wave which propagates outwards at the sound speed, is launched from the cloud center when the protostar forms. All self-similar solutions have constant accretion rate equal to few up to several times $C_s^3/G$. Note that in all solutions the density field is proportional to $r^{-2}$ in the outer part and to $r^{-3/2}$ in the inner region which has been reached by the rarefaction wave. Finally, note that the density of the Larson-Penston solution is about 8 times larger than the density of the Shu’s solution at infinity.

The collapse has also been investigated numerically. Larson (1969) starts with a uniform density cloud and calculates the gas contraction up to the formation of the protostar by using some simplified radiative transfer (see also Masunaga & Inutsuka 2000). He shows that a first accretion shock develops at the edge of the thermally supported core which forms when the dust becomes opaque to its own radiation, i.e. at a density of about $10^{-13}$ g cm$^{-3}$. This core is sometimes called the first Larson core. A second accretion shock forms at the edge of the protostar at much higher density ($\simeq 10^{-2}$ g cm$^{-3}$). Foster & Chevalier (1993) start with a slightly unstable Bonnor-Ebert sphere. Interestingly, they find that the collapse occurs very slowly in the outer part where only subsonic infall velocities develop. In the inner part, however, supersonic motions appear. Indeed, they show that convergence towards the Larson-Penston solution is achieved deep inside the cloud. On the other hand, in the outer part of the envelope, the density profile turns out to stay close to $\rho_{SIS}$. Triggered collapse has also been investigated by various authors (e.g. Hennebelle et al. 2003). Faster infall velocities are then obtained as well as densities few times denser than $\rho_{SIS}$. A common feature shared by the numerical solutions is that the accretion rate varies significantly along time, unlike what is inferred from the self-similar solutions.
2.2 Influence of magnetic field

As recalled previously, magnetic field has early been proposed to provide an important mechanical support to the gas (see e.g. Shu et al. 1987) which could possibly explain the low star formation efficiency within the Milky Way. In this section, we expose the basic principles of the magnetically controlled theory of star formation.

2.2.1 Magnetic support

The effect of the magnetic field is not easy to visualize because unlike the thermal pressure, it is highly non isotropic. In particular, the magnetic forces, $\mathbf{j} \times \mathbf{B}$, where $\mathbf{j}$ is the electric current, vanishes along the field lines. An easy way to estimate the magnetic support, is to compute the ratio of the magnetic over gravitational energies. For simplicity let us consider again a spherical and uniform cloud of mass $M$, volume $V$, radius $R$, threaded by an uniform magnetic field of strength $B$. The magnetic flux within the cloud, $\psi$ is equal to $\psi = \pi R^2 B$. As long as the magnetic field remains well coupled to the gas (see next section), the magnetic flux threading the cloud will remain constant along time. The ratio of magnetic over gravitational energies for uniform density cloud threaded by a uniform magnetic field, is:

$$\frac{E_{\text{mag}}}{E_{\text{grav}}} = \frac{B^2 V}{8\pi} \times \frac{2R}{5GM^2} \propto \frac{B^2 R^4}{M^2} \propto \left(\frac{\psi}{M}\right)^2.$$  \hfill (2.15)

Remarkably, the ratio of magnetic over gravitational energies is independent of the cloud radius. This implies that if the cloud contracts or expands, the relative importance of these two energies remains the same. This is unlike the thermal energy of an isothermal gas, which becomes smaller and smaller compared to the gravitational energy as the cloud collapses (e.g. eq. 2.1). It is clear from eq. (2.15), that there is a critical value of the magnetic intensity for which the gravitational collapse is impeded even if the cloud was strongly compressed. Mouschovias & Spitzer (1976) have calculated accurately the critical value of the mass-to-flux ratio using the virial theorem and numerical calculations of the cloud bidimensional equilibrium. A cloud which has a mass-to-flux ratio smaller (larger) than this critical value cannot collapse and is called subcritical (supercritical). It is usual to define $\mu = (M/\psi)/(M/\psi)_{\text{crit}}$. Large values of $\mu$ correspond to small magnetic fields and thus supercritical clouds.

Another important effect of the magnetic field is its ability to brake a rotating cloud. This is discussed rapidly later in this manuscript.

2.2.2 Ambipolar diffusion

At microscopic scales, the neutrals are not experiencing the Lorentz force which applies only on charged particles. Strictly speaking, this implies that at least two fluids should be considered, the neutrals and the ions (in different contexts
more than one fluid of charged particles must be considered), to treat the problem properly. Since the two fluids are coupled to each other by collisions, the neutrals are nevertheless influenced by the magnetic field if the gas is sufficiently ionized. Since the ionization in molecular clouds is usually of the order of $10^{-7}$ or less, the density of the ions is much smaller than the density of the neutrals. It is thus possible to neglect the inertia of the ions and assume mechanical equilibrium between the Lorentz force and the drag force. This leads to:

$$\left(\nabla \times B\right) \times B = \gamma \rho_i (V_i - V), \tag{2.16}$$

where $\rho_i$ and $V_i$ are the ion density and velocity respectively, $\gamma \simeq 3.5 \times 10^{13}$ cm$^{-3}$ g$^{-1}$ s$^{-1}$ is the drag coefficient (Mouschovias & Paleologou 1981). From eq. (2.16), the ion velocity can easily be expressed as a function of the neutral velocity and the Lorentz force. Considering now the induction equation, which entails the velocity of the ions, and using eq. (2.16), we obtain

$$\partial_t B + \nabla \times (B \times V) = \nabla \times \left(\frac{1}{4\pi \gamma \rho_i} \left((\nabla \times B) \times B\right)\right). \tag{2.17}$$

The left part of this equation is identical to the induction equation except that the velocity of the neutrals appears instead of the velocity of the ions. The right term is directly responsible for the slip between the neutrals and the magnetic field. Although it is of the second order, it is not rigorously speaking a diffusion term. From this equation, it can easily be inferred a typical timescale for the ambipolar diffusion

$$\tau_{ad} \simeq \frac{4\pi \gamma \rho_i L^2}{B^2}, \tag{2.18}$$

where $L$ is the typical spatial scale relevant for the problem. In the context of star formation, $L$ could be the size of the cores, $R$. Ionization equilibrium allows to estimate that the ions density is about $\rho_i = C\sqrt{\rho}$, where $C = 3 \times 10^{-16}$ cm$^{-3/2}$ g$^{1/2}$.

If a dense core is initially subcritical (therefore magnetically supported), the diffusion of the field will progressively reduce the magnetic flux within the cloud. So after a few ambipolar diffusion times, the cloud is becoming supercritical and the magnetic field is not able to prevent the collapse any more. The important and interesting question at this stage is, how much is the collapse delayed by this process? In order to estimate this time, it is usually assumed that the cloud is in virial equilibrium that is to say: $B^2/4\pi \simeq M \rho G/R$ (within a factor of a few) and to compute the ratio of $\tau_{ad}$ and $\tau_{ff}$, the freefall time (Shu et al. 1987). This leads to:

$$\frac{\tau_{ad}}{\tau_{ff}} \simeq 8. \tag{2.19}$$

Remarkably enough, $\tau_{ad}/\tau_{ff}$ is independent of $R$ and $M$, the size and mass of the cloud (as long as the virial assumption is verified). The important point is of course
that $\tau_{\text{ad}}$ is roughly 10 times higher than $\tau_{\text{ff}}$. This implies therefore that ambipolar diffusion could possibly reduce the star formation rate significantly making it closer to the observed value.

2.2.3 Predictions of ambipolar diffusion theory

In order to make quantitative predictions, numerical simulations of magnetized collapse controlled by ambipolar diffusion, have been performed (e.g. Basu & Mouschovias 1995). These simulations are generally one dimensional and assume a thin disk geometry. They explore a wide range of magnetic intensities, from nearly critical cores, $\mu \simeq 1$, to very subcritical cores, $\mu \ll 1$. They also investigate the effect of changing the ionization which results in a weaker coupling between the magnetic field and the neutral gas. Velocity and density profiles potentially useful for comparison with observations, are therefore available in the literature. Here we simply draw some of the most important features.

When the dense core is very subcritical, with values of $\mu$ as low as 0.1, Basu & Mouschovias (1995) find that the infall velocity in the outer part of the envelope is only a small fraction of the sound speed with values as low as $0.2 \times C_s$ whereas in the inner part, it gradually increases and reaches values of about $0.5 - 0.8 \times C_s$. The collapse is significantly delayed and occurs in about 15 freefall times. For nearly critical cores, $\mu \simeq 1$, the infall velocity is roughly twice higher than in the previous case whereas the collapse occurs after about $\simeq 3$ freefall times. Another interesting prediction, is the evolution of the central mass-to-flux ratio. The value of $\mu$ in the centre, smaller than 1 initially, grows with time as the cloud loses its flux and eventually becomes larger than 1. By the time of protostar formation, typical values of $\mu$ are about $\simeq 2$. Interestingly, this does not depend too much on the initial value of $\mu$. An important prediction of the magnetically controlled models, is therefore that typically, values of $\mu$ around 2 should be measured. Values significantly higher than 2, would certainly indicate that the collapse is not magnetically controlled.

Recently, Kunz & Mouschovias (2009) proposed a theory of the initial mass function based on the development of gravitational instability within magnetically supported molecular clouds.

2.3 Role of turbulence

During the last two decades, the theory of magnetically controlled star formation, has been challenged by a new theory based on supersonic turbulence (e.g. Mac Low & Klessen 2004, McKee & Ostriker 2007). The general idea of this theory is that turbulence prevents most of the gas to collapse in a freefall time and regulates the star formation, though different aspects have been emphasized by various authors.

2.3.1 Turbulent support and decay of turbulence

Unlike magnetic field, it is not straightforward to anticipate the influence of turbulence on the star formation rate. This is because, on one hand, the turbulent
motions tend to spread out the gas, reducing its ability to collapse, but on the other hand, the turbulent motions may also increase the gas density locally when the flow is globally convergent. Moreover, the difficulty with theories involving turbulence is that it appears hopeless to search for exact analytical descriptions, even for highly idealized situations. Therefore most of our theoretical knowledge of the interstellar turbulence is provided by numerical simulations (e.g. Kritsuk et al. 2007, Schmidt et al. 2009). It appears nevertheless highly wishable to draw simple trends. To this purpose let us assume that the turbulence is sufficiently isotropic and is an additional support that can be described by a sound speed. Let $V_{\text{rms}}$ be the root mean square of the velocity. The effective sound speed of the flow is $C_{s,\text{eff}} \simeq \sqrt{C_s^2 + V_{\text{rms}}^2/3}$. Since the turbulence observed in molecular clouds is highly supersonic, $V_{\text{rms}}^2$ is larger than $C_s^2$ by typically a factor of 25 to 100. Therefore, $C_{s,\text{eff}} \simeq V_{\text{rms}}/\sqrt{3}$ and the turbulent Jeans mass, $M_J \propto V_{\text{rms}}^2/\sqrt{\rho}$. On the other hand, turbulence creates density enhancements that can be estimated by the Rankine-Hugoniot jump conditions for an isothermal gas, $\rho/\rho_0 = (V/C_s)^2$, where $\rho_0$ is the mean cloud density. Combining these two relations, we get the turbulent Jeans mass $M_J \propto V_{\text{rms}}$, which indicates that turbulence is globally supporting the cloud. This is certainly the case when turbulence is large and dominates over gravity. However the dual role of turbulence, which is also compressing the gas through converging motions, requires a more complex treatment as presented in section 3.2.

To go further than these very simple analytic estimates, it is necessary to perform numerical calculations. Before describing some results of these simulations, it is important to emphasize the fast decay of turbulence which constitutes a severe issue of the turbulent theory. Consider a turbulent piece of fluid of size $L$, a robust conclusion seems to be that a significant fraction (say more than half) of the initial turbulent energy is dissipated in about one crossing time, $L/V_{\text{rms}}$. This result, well established in the case of nearly incompressible fluids, has also been inferred for numerical simulations of supersonic turbulence with and without magnetic field (Mac Low & Klessen 2004). Therefore, if not driven (that is to say when no external forcing is applied continuously to the flow), the turbulence decays quickly and thus cannot delay very significantly the collapse of a self-gravitating cloud. In order to explain the low star formation rate in the Galaxy, the turbulence theory must invoke a driving source which continuously replenishes the turbulent energy. Various sources of energies able to compensate for the dissipation have been proposed. A first category of models is based on feedback from stars due to their wind, jet or HII regions (e.g. Matzner 2002, 2007) while a second category invokes the role of continuous accretion of diffuse atomic gas onto the molecular clouds (Klessen & Hennebelle 2010, Goldbaum et al. 2011).

Various numerical simulations have been performed to study directly the influence of turbulence on star formation. The influence of turbulence on the IMF is discussed in the following sections (3 and 5), while below we restrict the discussion to its influence on the star formation rate. As it is not the main subject of these lectures, only a short and qualitative description is given.
2.3.2 Hydrodynamical turbulence

As anticipated above, decaying turbulence cannot delay star formation significantly. Calculations done by Klessen et al. (2000) and Bate et al. (2003), indeed, show that within a few freefall times, most of the gas has been accreted.

Driven turbulence can reduce the star formation rate if the driving is sufficient. With a large scale driving (that is to say $k = 1 - 2$ where $k$ counts the number of driving wavelengths in the box), providing an effective Jeans mass of 0.6 in a box which contains a total mass equal to 1 (that is to say that the box contains more than one turbulent Jeans mass), Klessen et al. (2000) find that more than half of the mass is accreted within one freefall time. The problem is less severe if the driving is on smaller scales ($k = 3 - 4$ or $k = 7 - 8$) since in that case 3 to 6 (depending on the scales at which driving is applied) freefall times are needed to accrete half of the mass. If the driving is stronger, providing an effective Jeans mass of 3.2, these numbers are typically multiplied by a factor of 3 to 4. Star formation can be entirely suppressed if sufficiently strong driving is applied at scales smaller than the thermal Jeans length inside the box. Note that analytical estimates of the star formation rate have been derived by Krumholz & McKee (2005), Padoan & Nordlund (2011) and Hennebelle & Chabrier (2011). While the first conclude that purely hydrodynamical turbulence can explain the very low efficiency of star formation within galaxies, the two others found much higher values, which are nevertheless in good agreement with star formation rate observed in molecular clouds (e.g. Heiderman et al. 2010) although possibly higher by a factor of a few.

2.3.3 Turbulence in magnetized clouds

Supercritical magnetic fields are probably reducing the star formation rate by a factor of a few, that is to say the star formation rate is smaller in a cloud that has a supercritical magnetic field than in the otherwise identical non magnetized cloud (Price & Bate 2009, Padoan & Nordlund 2011) although the exact physical mechanism by which this happens is not well understood.

Turbulence in subcritical clouds in the presence of ambipolar diffusion, has been investigated by Basu & Ciolek (2004) and Li & Nakamura (2005). These simulations combine the magnetic support and the turbulent motions. Since the clouds are initially subcritical, ambipolar diffusion plays an important role since it allows to reduce locally the magnetic flux. Interestingly, it has been found that in this context, turbulence tends to accelerate the star formation. This is because, turbulence creates stiff gradients, due to the formation of shocks, in which the ambipolar diffusion takes place quickly. Indeed, eq. 2.18 shows that the ambipolar diffusion time decreases with the spatial scale, $L$. A very interesting result is that these simulations are able to reproduce the low star formation efficiency observed in the Milky Way provided the initial value of $\mu$ is small enough (typically $\mu \simeq 1$) and the turbulence is sufficiently strong (typically $V_{\text{rms}} \simeq 10 \times C_s$).
3 Origin of the Initial mass function

Several theories have been proposed to explain the origin of the IMF, invoking various physical processes that we tentatively classify in four categories: theories based on recursive fragmentation or pure gravity (e.g. Larson 1973), theories based on pure statistical argument, invoking the central limit theorem (e.g. Zinnecker 1984, Elmegreen 1997, Adams & Fatuzzo 1996), theories based on accretion and, finally, theories invoking the initial Jeans mass in a fluctuating environment. We will focus on the two latter ones, which appear to be favored in the modern context of star formation.

3.1 Theories based on accretion

3.1.1 Competitive accretion

The theory of competitive accretion has been originally proposed by Zinnecker (1982) and Bonnell et al. (2001). It has then been used to interpret the series of numerical simulations similar to the ones performed by Bate et al. (2003).

The underlying main idea is that the accretion onto the stars is directly linked to its mass in such a way that massive stars tend to accrete more efficiently and thus become disproportionally more massive than the low-mass stars. The accretion rate is written as:

$$\dot{M}_* = \pi \rho V_{rel} R_{acc}^2,$$

where $\rho$ is the gas density, $V_{rel}$ is the relative velocity between the star and the gas while $R_{acc}$ is the accretion radius. Bonnell et al. (2001) consider two situations, namely the cases where the gravitational potential is dominated by the gas or by the stars.

• Gas dominated potential

Let $R$ be the spherical radius, $\rho$ the gas density and $n_*$ the number density of stars. In the gas dominated potential case, Bonnell et al. (2001) assume, following Shu (1977), that the gas density profile is proportional to $R^{-2}$. They further assume that $n_* \propto R^{-2}$. The accretion radius, $R_{acc}$ is assumed to be equal to the tidal radius given by

$$R_{tidal} \simeq 0.5 \left( \frac{M_*}{M_{enc}} \right)^{1/3} R,$$

where $M_{enc}$ is the mass enclosed within radius $R$. This choice is motivated by the fact that a fluid particle located at a distance from a star smaller than $R_{tidal}$ is more sensitive to the star than to the cluster potential and will thus be accreted onto this star.

The mass of gas within a radius $R$, $M(R)$, is proportional to $R$ (since $\rho \propto R^{-2}$). The infall speed is about $V_{in} \simeq \sqrt{GM(R)/R}$, and, assuming that the stars
are virialized, one gets $V_{\text{rel}} \approx V_{\text{in}}$. The number of stars, $dN_*$, located between $R$ and $R + dR$ is given by the relation $dN_* = n_*(R) \times 4\pi R^2 dR \propto dR$. Thus eq. (3.1) combined with the expression of $V_{\text{rel}}$ and $R_{\text{acc}}$ leads to the relation $\dot{M}_* \propto (M_* / R)^{2/3}$ and, after integration, $M_* \propto R^{-2}, R \propto M_*^{-1/2}$. Consequently, we obtain:

$$dN \propto M_*^{-3/2} dM_*.$$  \hspace{1cm} (3.3)

Even though the mass spectrum is too shallow compared to the fiducial IMF, $dN/dM \propto M^{-2.3}$, it is interesting to see that this power law behaviour can be obtained from such a simple model. Note, however, that the model implies that stars of a given mass are all located in the same radius, which points to a difficulty of the model.

- **Star dominated potential**

When the potential is dominated by the stars located at the centre of the cloud, the density is given by $\rho \propto R^{-3/2}$, which corresponds to the expected density distribution after the rarefaction wave has propagated away (Shu 1977). The velocity is still assumed to be $V_{\text{rel}} \propto R^{-1/2}$. The accretion radius is now supposed to correspond to the Bondi-Hoyle radius as the gas and the star velocities are no longer correlated. This leads to $\dot{M}_* = \pi \rho V_{\text{rel}} R_{BH}^2$, where $R_{BH} \propto M_* / V_{\text{rel}}^2$. It follows

$$\dot{M}_* \propto M_*^{-2}.$$  \hspace{1cm} (3.4)

One can then show (Zinnecker 1982, Bonnell et al. 2001) that under reasonable assumptions, $dN \propto M_*^{-2} dM_*$. This estimate is in better agreement with the Salpeter exponent, although still slightly too shallow. These trends seem to be confirmed by the simulations performed by Bonnell et al. (2001) which consists in distributing 100 sink particles in a cloud of total mass about 10 times the total mass of the sinks initially (their fig. 3).

- **Difficulties of the competitive accretion scenario**

As obvious from the previous analytical derivations, finding an explanation for the Salpeter exponent with the competitive accretion scenario appears to be difficult, even though numerical simulations (as the ones presented in Bonnell et al. 2001) seem to successfully achieve this task. However, although the IMF exponent is close to the Salpeter one in the star dominated potential case, this scenario entails by construction the Bondi-Hoyle accretion, which is at least a factor 3 lower than the mass infall rate resulting from gravitational collapse at the class-0 and I stages and leads to too long accretion times compared with observations (Andrè et al. 2007, 2009). Another difficulty of this scenario is that it does not explain the peak of the IMF which might be related to the Jeans mass (see § 3.2). Finally, it is not clear that competitive accretion can work in the case of non-clustered
star formation, for which the gas density is much too small. As no evidence for substantial IMF variation among different regions has yet been reported (e.g. Bastian et al. 2010), this constitutes a difficulty for this model as a general model for star formation. Perhaps this scenario applies well to the formation of massive stars.

3.1.2 Stopped accretion

The principle of this type of models is to assume that the accretion of gas onto the stars or the dense cores is a non-steady process, which stops because of either the finite reservoir of mass or the influence of an outflow which sweeps up the remaining gas within the vicinity of the accreting protostar.

The first studies were performed by Silk (1995) and Adams & Fatuzzo (1996). They first relate the mass of the stars to the physical parameters of the cloud such as sound speed and rotation and then assume that an outflow whose properties are related to the accretion luminosity stops the cloud collapse. Using the Larson (1981) relations, they can link all these parameters to the clump masses. Since the mass spectrum of these latter is known (e.g. Heithausen et al. 1998), they infer the IMF.

A statistical approach has been carried out by Basu & Jones (2004). These authors assume that the dense core distribution is initially lognormal, justifying it by the large number of processes that control their formation (and invoking the central limit theorem). Then, they argue that the cores grow by accretion and postulate that the accretion rate is simply proportional to their mass, $\dot{M} = \gamma M \rightarrow M(t) = M_0 \exp(\gamma t)$, leading to $\log M = \mu = \mu_0 + \gamma t$. Finally, they assume that accretion is lasting over a finite period of time given by $f(t) = \delta \exp(-\delta t)$. The star mass distribution is thus obtained by summing over the accretion time distribution.

$$f(M) = \int_0^\infty \frac{\delta \exp(-\delta t)}{\sqrt{2\pi}\sigma_0 M} \exp \left( -\frac{(\ln M - \mu_0 - \gamma t)^2}{2\sigma_0^2} \right) dt$$  \hspace{1cm} (3.5)

$$f(M) = \frac{\alpha}{2} \exp(\alpha \mu_0 + \alpha^2 \sigma_0^2 / 2) M^{-1-\alpha} \text{erf} \left( \frac{1}{\sqrt{2}} \left( \alpha \sigma_0 - \ln M - \frac{\mu_0}{\sigma_0} \right) \right),$$

where $\alpha = \delta / \gamma$ and $\sigma_0$ characterizes the width of the initial dense core distribution. As $\delta$ and $\alpha$ are controlled by the same types of processes, their ratio is expected to be of the order of unity and thus $f(M)$ exhibits a powerlaw behaviour close to the fiducial IMF.

In a recent study, Myers (2009) develops similar ideas in more details, taking into account the accretion coming from the surrounding background. Adjusting two parameters, he reproduces quite nicely the observed IMF (his figure 5).

A related model has also been developed by Bate & Bonnell (2005) based on an idea proposed by Price & Podsiadlowski (1995). They consider objects that form by fragmentation within a small cluster and are ejected by gravitational interaction with the other fragments, which stops the accretion process. Assuming a lognormal accretion rate and an exponential probability of being ejected, these
authors construct a mass distribution that can fit the IMF for some choices of parameters.

In summary, the stopped accretion scenario presents interesting ideas and, providing (typically 2 or more) adequate adjustable parameters, can reproduce reasonably well the IMF. However, the very presence of such parameters, which characterizes our inability to precisely determine the processes that halt accretion, illustrates the obvious difficulties of this class of models.

3.2 Gravo-turbulent theories

While turbulence is not determinant in the accretion models, it is one of the essential physical processes for the two theories presented in this section, although the role it plays differs in both models, as shown below. The theories proposed along this line seemingly identify cores or pre-cores and are motivated by the strong similarity between the observed core mass function (hereafter) CMF and the IMF (e.g. Andrés et al. 2010).

The first theory that combined turbulence and gravity was proposed by Padoan et al. (1997). In this paper, the authors consider a lognormal density distribution - density PDF computed from numerical simulations (e.g. Vázquez-Semadeni 1994, Kritsuk et al. 2007, Schmidt et al. 2009, Federrath et al. 2010a) are indeed nearly lognormal - and select the regions of the flow which are Jeans unstable. By doing so, they get too stiff an IMF (typically $dN/dM \propto M^{-3}$) but nevertheless find a lognormal behaviour at small masses, a direct consequence of the lognormal density distribution, and a powerlaw one at large masses.

3.2.1 Formation of cores by MHD shocks

The idea developed by Padoan & Nordlund (2002) is slightly different. These authors consider a compressed layer formed by ram pressure in a weakly magnetized medium. They assume that the magnetic field is parallel to the layer and thus perpendicular to the incoming velocity field. The postshock density, $\rho_1$, the thickness of the layer, $\lambda$, and the postshock magnetic field, $B_1$, can be related to the Alfvinic Mach numbers, $M_a = v/v_a$ ($v$ is the velocity and $v_a$ the Alfvén speed), and preshocked quantities, $\rho_0$ and $B_0$ according to the shock conditions:

$$\frac{\rho_1}{\rho_0} \simeq M_a, \quad \frac{\lambda}{L} \simeq M_a^{-1}, \quad \frac{B_1}{B_0} \simeq M_a,$$

(3.6)

where $L$ is the scale of the turbulent fluctuation. Note that for classical hydrodynamical isothermal shocks, the jump condition is typically $\propto M^2$. The dependence on $M_a$ instead of $M^2$ stems from the magnetic pressure which is quadratic in $B$. As we will see, this is a central assumption of this model.

The typical mass of this perturbation is expected to be

$$M \simeq \rho_1 \lambda^3 \simeq \rho_0 M_a \left( \frac{L}{M_a} \right)^3 \simeq \rho_0 L^3 M_a^{-2}.$$  

(3.7)
As the flow is turbulent, the velocity distribution depends on the scale and $v \simeq L^\alpha$, with $\alpha = (n - 3)/2$, $E(k) \propto k^{-n}$ being the velocity powerspectrum. Combining these expressions with eq. (3.7), they infer

$$M \simeq \frac{\rho_0 L_0^3}{\mathcal{M}_{a,0}} \left( \frac{L}{L_0} \right)^{6-n},$$

where $L_0$ is the largest or integral scale of the system and $\mathcal{M}_{a,0}$ the corresponding Mach number. To get a mass spectrum, it is further assumed that the number of cores, $N(L)$, formed by a velocity fluctuation of scale $L$, is proportional to $L^{-3}$. Combining this last relation with eq. (3.8) leads to

$$\frac{dN}{d \log M} \simeq M^{-3/(6-n)}.$$  

(3.9)

For a value of $n = 3.74$ (close to what is inferred from 3D numerical simulations), one gets $dN/d \log M \simeq M^{-1.33}$, very close to the Salpeter exponent.

So far, gravity has not been playing any role in this derivation and the mass spectrum that is inferred is valid for arbitrarily small masses. In a second step, these authors consider a distribution of Jeans masses within the clumps induced by turbulence. As the density in turbulent flows presents a lognormal distribution, they assume that this implies a lognormal distribution of Jeans lengths and they multiply the mass spectrum (3.9) by a distribution of Jeans masses, which leads to

$$\frac{dN}{d \log M} \simeq M^{-3/(6-n)} \left( \int_0^M p(m_J) dm_J \right).$$  

(3.10)

The shape of the mass spectrum stated by eq. (3.10) is very similar to the observed IMF (see for example the figure 1 of Padoan & Nordlund 2002).

Note, however, that difficulties with this theory have been pointed out by McKee & Ostriker (2007) and Hennebelle & Chabrier (2008). Eqn (3.6), in particular, implies that in the densest regions where dense cores form, the magnetic field is proportional to the density, in strong contrast with what is observed both in simulations (Padoan & Nordlund 1999, Hennebelle et al. 2008) and in observations (e.g. Troland & Heiles 1986). This is a consequence of the assumption that the magnetic field and the velocity field are perpendicular, which again is not the trend observed in numerical simulations. In both cases, it is found that at densities lower than about $10^3$ cm$^{-3}$, $B$ depends only weakly on $\rho$ while at higher densities, $B \propto \sqrt{\rho}$. This constitutes a problem for this theory, as the index of the power law slope is a direct consequence of eq. (3.6). Assuming a different relation between $B$ and $\rho$, as the aforementioned observed one, would lead to a slope stiffer than the Salpeter value. Furthermore, the Salpeter IMF is recovered in various purely hydrodynamical simulations (e.g. Bate et al. 2003), while the Padoan & Nordlund theory predicts a stiffer distribution ($dN/dM \propto M^{-3}$) in the hydrodynamical case.

---

$n$ is denoted $\beta$ in Padoan & Nordlund (2002), more precisely $n - 2 = \beta$
3.2.2 Turbulent dispersion

Recently, Hennebelle & Chabrier (2008, 2009) proposed a different theory which consists in counting the mass of the fluid regions within which gravity dominates over the sum of all supports, thermal, turbulent, and magnetic, according to the virial condition (see also Hopkins 2012). In this approach, the role of turbulence is dual: on one hand it promotes star formation by locally compressing the gas but on the other hand, it also quenches star formation because of the turbulent dispersion of the flow, which is taken into account in the selection of the pieces of fluid that collapse.

Here instead of presenting the full formalism that can be found in Hennebelle & Chabrier (2008), we qualitatively describe the procedure and important physical features and we then focus on simpler scaling relations that show how the Salpeter slope can be simply recovered using the turbulent support (Chabrier & Hennebelle 2011). The theory is formulated by deriving an extension of the Press & Schechter (1974) statistical formalism, developed in cosmology. The principle of the method is the following. First, the density field is smoothed at a scale $R$, using a window function. Then, the total mass contained in areas which, at scale $R$, have a density contrast larger than the specified density criterion $\delta^c_R$, is obtained by integrating accordingly the density PDF. This mass, on the other hand, is also equal to the total mass located in structures of mass larger than a scale dependent critical mass $M^c_R$, which will end up forming structures of mass smaller than or equal to this critical mass for collapse. The two major differences are (i) the underlying density field, characterized by small and Gaussian fluctuations in the cosmological case, while lognormal in the star formation case, and (ii) the selection criterion, a simple scale-free density threshold in cosmology while scale-dependent, based on the virial theorem in the second case. That is, fluid particles which satisfy the criterion

$$\langle V_{\text{rms}}^2 \rangle + 3(C_s^{eff})^2 < -E_{\text{pot}}/M$$

are assumed to collapse and form a prestellar bound core. The turbulent rms velocity obeys a power-law correlation with the size of the region, the observed so-called Larson relation, with $V_0 \simeq 1 \text{ km s}^{-1}$ and $\eta \approx 0.4-0.5$ (Larson 1981).

The theory is controlled by two Mach numbers, namely a Mach number (called $M_*$) defined as the non-thermal velocity to sound speed ratio at the mean Jeans scale $\lambda^*_J$ (and not at the local Jeans length), and the usual Mach number, $M$, which represents the same quantity at the scale of the turbulence injection scale, $L_i$, assumed to be the characteristic size of the system. The global Mach number, $M_*$, broadens the density PDF, as $\sigma^2 = \ln(1 + b^2M^2)$, illustrating the trend of supersonic turbulence to promote star formation by creating new overdense collapsing seeds. The effect described by $M_*$ is the additional non thermal support induced by the turbulent dispersion. In particular, at large scales the net effect of turbulence is to stabilize pieces of fluid that would be gravitationally unstable if only the thermal support was considered.

The condition to select a marginally unstable piece of fluid of mass $M$ and
size $R$, with internal velocity dispersion $\sigma_R \equiv \sigma$, simply stems from the virial condition: $2E_{\text{kin}} + E_{\text{pot}} = 0$, where $E_{\text{kin}} \sim (1/2)\rho \sigma^2$ denotes the kinetic energy and $E_{\text{pot}} \sim \rho G M^2 / R$ the gravitational energy. Using dimensional analysis, this yields the relation

$$M \sigma^2 \sim G \frac{M^2}{R}, \quad \text{i.e.} \quad M \sim \frac{R \sigma^2}{G}. \quad (3.12)$$

In the limit where the velocity dispersion is dominated by thermal motions, and assuming nearly isothermal conditions, $\sigma$ does not depend on $R$ and condition (3.12) yields

$$M_{\text{th}} \propto R \quad \text{(THERMAL SUPPORT).} \quad (3.13)$$

Conversely, in the limit where the velocity dispersion is dominated by turbulent motions, $\sigma \propto R^{(n-3)/2}$, yielding for the marginal instability condition

$$M_{\text{turb}} \propto R^{(n-2)} \propto R^{2\eta+1} \quad \text{(TURBULENT SUPPORT).} \quad (3.14)$$

with $\eta = (n-3)/2$. The thermal case is recovered for $n = 3$, i.e. $\eta = 0$. For $n > 3$, i.e. $\eta > 0$, we note the stronger dependence of the mass $M$ upon the size $R$ in the presence of turbulent support, compared with the pure thermal case (eq. 3.13). The complete relation is obtained in Hennebelle & Chabrier (2008; their eq.(26)) and illustrates that a bound stable region can achieve larger masses for a given size thanks to the non-thermal support at the characteristic Jeans length, naturally leading to a larger number of high mass reservoirs (then of massive prestellar cores), which would otherwise have collapsed long before. This impact of nonthermal support on the mass-size relation of massive bound cores has been confirmed by recent numerical simulations (Schmidt et al. 2010, their fig.7).

Assuming a large enough multi-scale structured space of size $L$ and dimension $D$, thus volume $L^D$, the probability of having $N$ gravitationally bound structures on a scale larger than $R$ is given by the probability law $P(N_R) \equiv N(> R) \propto R^{-D}$. In Fourier space, this simply means that the number of fluctuations of wavenumber $k \sim 1/R$ is proportional to the volume of a fluctuation, i.e. $dN(k) \propto d\vec{k} = k^{D-1}dk$. The density probability, i.e. the probability to have a structure of size $\in [R, R+dR]$, thus reads as

$$dN(R) \propto R^{-(1+D)}. \quad (3.15)$$

A homogeneous volume in 3 dimensions obviously corresponds to $D = 3$, i.e. a uniform density probability $dN(k)/d\vec{k} = \text{constant}$, as assumed e.g. in Padoan & Nordlund (2002).

From Eqs. (3.14) and (3.15), the probability of having a bound structure of mass $\in [M(R), M + dM(R + dR)]$, which defines the CMF/IMF, reads

$$\frac{dN}{dM} \propto M^{-\alpha}$$

with $\alpha = \frac{n+1+(D-3)}{n-2} = \frac{2\eta+4+(D-3)}{2\eta+1} \quad (3.16)$
As seen from Eq. (3.16), for \( D = 3 \), pure thermal support \((n = 3, \eta = 0)\) yields a power-law IMF \( \frac{dN}{dM} \propto M^{-4} \), substantially steeper than the Salpeter value, \( \alpha = 2.36 \). In the two limiting cases of incompressible (Kolmogorov) \((n = 11/3, \eta = 1/3)\) and pressureless (Burgers) \((n = 4, \eta = 1/2)\) turbulence, the turbulent support yields \( \alpha = 2.8 \) and \( \alpha = 2.5 \), respectively. In molecular clouds, turbulence is assumed to scale according to the observed Larson (1981) velocity dispersion - size relation with a typical value \( \eta \sim 0.4-0.5 \). This value is well recovered for the aforementioned power spectrum index inferred from numerical simulations of supersonic turbulence, \( n = 3.8 \). According to eq. (3.16), this yields a slope for the CMF/IMF \( \alpha = 2.66 \), between the Kolmogorov and Burgers values. Note that the index \( D \) observed in molecular clouds is typically smaller than \( D = 3 \) and closer than \( D \approx 2.5 \) which leads to values for \( \alpha \approx 2.2 - 2.5 \) very close to the Salpeter exponent (see Chabrier & Hennebelle 2011) for details.

Comparisons with the Chabrier (2003) IMF have been performed for a series of cloud parameters (density, size, velocity dispersion) and good agreement has been found (Hennebelle & Chabrier 2009) for clouds typically 3 to 5 times denser than the mean density inferred from Larson (1981) density-size relation. Comparisons with numerical simulations have also been performed. In particular, Schmidt et al. (2010), performing supersonic isothermal simulations with various forcing, have computed the mass spectrum of cores supported either by pure thermal support or by turbulent plus thermal support. Their converged simulations show very good quantitative agreement with the present theory, confirming that turbulent support is needed to yield the Salpeter index. Note that Schmidt et al. (2010) use for the density PDF the one they measure in their simulations which is nearly, but not exactly lognormal. Comparisons with the results of SPH simulations (Jappsen et al. 2005) including self-gravity and thermal properties of the gas have also been found to be quite successful (Hennebelle & Chabrier 2009).

3.2.3 Difficulties of the gravo-turbulent theories

One natural question about any IMF theory is to which extent it varies with physical conditions. Indeed, there is strong observational support for a nearly invariant form and peak location of the IMF in various environments under Milky Way like conditions (see e.g. Bastian et al. 2010). Jeans length based theories could have difficulty with the universality of the peak position, since it is linked to the Jeans mass which varies with the gas density. Various propositions have been made to alleviate this problem. Elmegreen et al. (2008) and Bate (2009) propose that the gas temperature may indeed increase with density, resulting in a Jeans mass which weakly depends on the density, while Hennebelle & Chabrier (2008) propose that for clumps following Larson relations, there is a compensation between the density dependence of the Jeans mass and the Mach number dependence of the density PDF, resulting in a peak position that is insensitive to the clump size.

\footnote{Assuming a Gaussian (lognormal) density probability instead of a uniform one yields in that case \( \frac{dN}{dM} \propto M^{-3} \) (eq. (37) of HC08).}
A related problem is the fact that massive stars are often observed to be located in the densest regions, where the Jeans mass is smaller. Indeed, $M_J \propto \rho^{-1/2}$ when a purely thermal support is considered, whereas $M_J \propto \rho^{-2}$ when turbulence is taken into account (assuming that $V \propto L^{0.5}$). This constitutes a difficulty for theories based on Jeans mass although, as seen above, the issue is much less severe when turbulent support is considered as massive stars can be formed at densities only few times smaller than the densities at which low-mass stars form. Another possibility is that dynamical interactions between young protostars may lead to the migration of massive stars in the center of the gravitational well.

Furthermore, the dependence of the freefall time on the Jeans mass should also modify the link between the CMF and the IMF, as pointed out by Clark et al. (2007). This is particularly true for theories which invoke only thermal support. When turbulent support is included, the free-fall time is found to depend only weakly on the mass, with $t_{ff} \propto M^{1/4}$ (see McKee & Tan 2003 and Hennebelle & Chabrier 2009 App. C), resolving this collapsing time problem. We stress that this time represents the time needed for the whole turbulent Jeans mass to be accreted. It is certainly true that, within this turbulent Jeans mass, small structures induced by turbulent compression will form rapidly.

Generally speaking, the fragmentation that occurs during the collapse could constitute a problem for theories invoking Jeans masses. Although this problem is far from being settled, it should be stressed that while the presence of small self-gravitating condensations induced by turbulence at the early stages of star formation and embedded into larger ones is self-consistently taken into account in the turbulent dispersion theory, the fragmentation induced by gravity, i.e. by the dense regions produced during collapse (see section 5.2.2) is not included in this theory. It certainly constitutes another problem if this mode turns out to be dominant.

4 Formation of brown dwarfs through disk fragmentation

In the previous section, the mechanism by which stars and brown dwarfs formed, namely the selection by gravity of dense fluctuations induced by turbulence, was identical. Other scenarios have been proposed in which the formation of brown dwarfs is different. This is in particular the case for the fragmentation of massive disks as recently emphasized by Whitworth & Stamatellos (2006) and Matzner & Levin (2005). It is the purpose of this section to present this model.

4.1 Basics

Here we start by presenting the basics of disk physics in particular the processes which have been used by these authors to infer their theories.
4.1.1 Rotational support

Let us consider a cloud of mass, \( M \), rotating at a rate \( \Omega \). If angular momentum is conserved, we have the relation \( R^2 \Omega = j \), where \( j \) is the specific angular momentum. The rotation energy is proportional to \( MR^2 \Omega^2 \) and thus

\[
\frac{E_{\text{rot}}}{E_{\text{grav}}} \propto \frac{MR^2 \Omega^2}{M^2 G/R} = \frac{j^2}{GM} \frac{1}{R^2}.
\] (4.1)

Thus, as the cloud collapses, the radius \( R \to 0 \) and the energy ratio increases. This shows that the centrifugal support becomes dominant even if rotation is small initially compared to gravity. This well known and very important behaviour is called the centrifugal barrier. It is responsible for disk formation in astrophysics and makes the transport of angular momentum a crucial issue. Indeed, if angular momentum was conserved, eq. (4.1) shows that stars would probably never form.

4.1.2 Centrifugal radius and disk growth

Let us consider a star of mass \( M \) and a fluid particle whose specific angular momentum is \( j \). The centrifugal radius, \( r_d \), is simply obtained when centrifugal and gravitational forces on the fluid particle are equal, which leads to

\[
r_d = \frac{j^2}{GM}.
\] (4.2)

This expression shows that the disk formation in a collapsing cloud, crucially depends on the specific angular momentum distribution. To illustrate this more quantitatively let us consider a cloud in solid body rotation, whose density profile is initially proportional to \( 1/R^2 \) (e.g. Shu 1977). In this case, the mass enclosed in a sphere of radius \( R_0 \) is \( M(R_0) \propto R_0 \) while the specific angular momentum \( j(R_0) = R_0^2 \Omega \). Therefore \( j(R_0) \propto M(R_0)^2 \). As accretion proceeds, the shells of increasing \( R_0 \) are reaching the central part, say the star-disk system. The centrifugal radius for a fluid particle initially located in \( R_0 \) is given by \( r_d = j(R_0)^2/(GM(R_0)) \propto M(R_0)^3 \) (Terebey et al. 1984).

On the other hand, for a cloud in solid body rotation for which the density profile is initially flat, i.e. the density is uniform, we have \( M(R_0) \propto R_0^3 \) and \( j \propto M(R_0)^{2/3} \) leading to \( r_d \propto M(R_0)^{1/3} \). As the mass delivered within the cloud is typically of the order of a few \( C_s^2/(G)t \), we see that while in the first case, the disk grows like \( t^3 \), i.e. extremely slowly (since \( t \approx 0 \) initially), it grows much faster in the second case.

4.1.3 Equations of thin disk and steady accretion

After averaging along the disk axis, the continuity and angular momentum conservation equations, one obtains (see e.g. Pringle 1981)

\[
\partial_t \Sigma + \partial_r (r \Sigma V_r) = 0,
\] (4.3)
\[ \partial_t (\Sigma r^2 \Omega) + \frac{1}{r} \partial_r (r^3 \Omega \Sigma V_r) = \frac{1}{r} \partial_r (\nu r^3 \frac{d\Omega}{dr} \Sigma), \quad (4.4) \]

where \( \Sigma \) is the column density through the disk, \( V_r \) is the radial velocity, and \( \nu \) is the viscosity. While the left hand terms in eq. (4.4) are the standard conserved angular momentum and angular momentum flux, the right hand side is the torque exerted through viscosity by the neighboring annulus.

In steady state, eq. (4.4) leads to \( \Omega V_r = \nu \frac{d\Omega}{dr} \). As the mass flux \( \dot{M} = 2\pi r \Sigma V_r \), we get \( \dot{M} = 2\pi \nu r \frac{d\Omega}{dr} \). For a Keplerian disk, mechanical equilibria in radial and axial directions lead to \( GM/r^2 \simeq r \Omega^2 \) and \( GMh/r^3 \simeq c_s^2/h \) and thus \( h \simeq c_s/\Omega \). It is usual to introduce the adimensional quantity, \( \alpha = \nu/(c_s h) \), which leads to the relation

\[ \dot{M} = 3\pi \alpha \frac{\Sigma c_s^2}{\Omega}, \quad (4.5) \]

4.1.4 Thermal balance

In the absence of other source, the heating is due to the viscous dissipation and is given by (e.g. Balbus & Hawley 1998, Hartmann 2009)

\[ Q_+ = (\nu \Sigma r^2 \frac{d\Omega}{dr}) \times \frac{d\Omega}{dr} = \nu \Sigma \left( r \frac{d\Omega}{dr} \right)^2 \simeq \alpha \Sigma c_s^2 \Omega \left( \frac{d\log \Omega}{d\log r} \right)^2, \quad (4.6) \]

which is simply the rate of work per units of length of the viscous torque. For a Keplerian disk this becomes

\[ Q_+ = \frac{3}{4\pi} \Omega^2 \dot{M}. \quad (4.7) \]

If \( \tau_{\text{cool}} \) is the cooling time, the cooling rate is about

\[ Q_- = \frac{U_{\text{th}}}{t_{\text{cool}}} = \frac{\Sigma c_s^2}{\gamma (\gamma - 1) t_{\text{cool}}}, \quad (4.8) \]

At thermal equilibrium heating and cooling compensate each other and thus \( Q_+ = Q_- \) leading to

\[ \alpha = \left( \frac{d\log \Omega}{d\log r} \right)^{-2} \frac{1}{\gamma (\gamma - 1) t_{\text{cool}}}, \quad (4.9) \]

Thus, disks that cool fast must have a large viscosity to provide enough heat to compensate the efficient radiative loss.
4.1.5 The Toomre criterion

The stability of a centrifugally supported disk is governed by the Toomre criterion \( Q = C_s \kappa / (\pi G \Sigma) \) where \( \kappa^2 = 4 \Omega^2 + Rd\Omega^2/dR \) is the epicyclic frequency and \( \Sigma \) is the column density through the disk (Toomre 1964). The complete derivation of this relation can be found in Binney & Tremaine (1987) and is beyond the scope of the present manuscript where we follow a more phenomenological but simpler approach.

Let us consider a piece of fluid of size \( \delta R \) and mass \( \delta M = \pi \delta R^2 \Sigma \). Its thermal, gravitational and rotational energies are about \( \delta E_{th} \approx c_s^2 \delta M \propto \delta R^2 \), \( \delta E_{gr} \approx G \delta M^2 / \delta R \propto \delta R^3 \) and \( \delta E_{rot} \approx \delta M \Omega (\Omega \delta R)^2 \propto \delta R^4 \). Thus it appears that while at small scales, when \( R \leq R_{th} = c_s^2 / (\pi G \Sigma) \), the disk is stabilized by thermal support, it is stabilized at large scales when \( R \geq R_{rot} = \pi G \Sigma / \Omega^2 \) by the rotational one. Thus only the intermediate scales between \( R_{th} \) and \( R_{rot} \) are possibly unstable.

The condition for disk stability is that \( R_{th} > R_{rot} \) in which case all scales are stabilized either by thermal or by rotational support. This leads to \( c_s^2 / (\pi G \Sigma) \geq \pi G \Sigma / \Omega^2 \) and finally \( Q = c_s \Omega / (\pi G \Sigma) \geq 1 \), which is almost, although not exactly, the famous Toomre’s criterion as the epicyclic frequency replaces the rotation, except for a Keplerian disk for which \( \kappa = \Omega \).

4.1.6 Self-gravitating disks and self-regulation

So far, it has been assumed that an effective viscosity was responsible of the transport of angular momentum without attempting to relate it to a physical process. When the disk is self-gravitating the angular momentum conservation equation can be written as (e.g. Lodato & Rice 2004)

\[
\partial_t (\Sigma r^2 \Omega) + \frac{1}{r} \partial_r (r^3 \Sigma V_r \Omega + T_{r\phi}) = 0 \tag{4.10}
\]

where \( T_{r\phi} = \int \frac{r^2 \partial \rho \phi}{4 \pi G} dz + \Sigma \delta V_r \delta V_\phi \) and \( \delta V_r = V_r - <V_r>, \delta V_\phi = V_\phi - <V_\phi> \) also known as a Reynolds decomposition. The first is obtained by using the Poisson equation to write the gravitational term as \( (r^2 \partial \rho \phi/r) \partial \theta \phi = (r^{-1} \partial_r (r \partial_r \phi) + \partial_\theta^2 \phi/r^2 + \partial_z^2 \phi) \partial \theta \phi / (4 \pi G r) \) and integrating along the \( z \)-axis. The second one is the Reynolds stress and comes from the Reynolds decomposition of the velocity field.

Equation (4.10) suggests that the gravitational stress provides an effective viscosity which is due to the gravitational field itself but also to the non-axisymmetric velocity fluctuations that it generates. One important difference however with a simple viscosity arises from the non-local nature of gravity that could in principle deeply modify its behaviour. It seems however that gravitational stress behaves in a way which closely resembles a local viscous stress (Lodato & Rice 2004). This has important implications because according to the Toomre criterion, when the disk is cold, it is unstable. Thus, non-axisymmetric spiral modes develop making the gravitational stress higher and thus according to eq. (4.10) the heating increases and thus the disk becomes more stable. This suggests that self-gravitating disks...
self-regulate (Paczynski 1978), that is they tend to maintain their Toomre parameter, $Q$ around 1. This has been extensively observed in numerical simulations as for example by Lodato & Rice (2004). Varying the initial parameters of their disks, they find that except close to the center or the edge, $Q$ stays typically between 1 and 1.5.

4.1.7 Non-linear stability

The analysis of the previous section shows that the disk stability is ultimately linked to its ability to cool. This has been studied in details by Gammie (2001) who run a series of disk simulations (using the shearing box approximation) in which he varies the cooling time. Indeed, eq. (4.9) shows that a short cooling time implies a large effective viscosity and thus a large gravitational stress. As the latter is the consequence of non-axisymmetric perturbations, it is intuitive that large values of the $\alpha$ parameter will tend to be associated with very perturbed and therefore very unstable disks. Gammie (2001) shows that the critical value for the cooling time is about $t_{\text{cool}} \approx 3\Omega^{-1}$. Below this value the disk fragments while it remains stable above. Note that the exact value of the coefficient $t_{\text{cool}}\Omega$ remains controversial (e.g. Meru & Bate 2011).

4.2 Brown dwarf formation through disk fragmentation

4.2.1 Basic scenario

The formation of brown dwarfs through disk fragmentation has been recently investigated by Matzner & Levin (2005) and Whitworth & Stamatellos (2006), using similar although not identical arguments. Below we present the arguments of Matzner & Levin (2005) and mention the difference with the ones used by Whitworth & Stamatellos (2006).

Matzner & Levin consider a disk, which due to the self-regulation has a Toomre parameter $Q \approx 1$. It is accreting at a constant rate equal to the accretion rate within the parent collapsing core equal to $\epsilon C_s^3/G$ (Shu 1977) where $\epsilon$ is a factor of a few. Assuming stationarity, this accretion rate must be equal to the viscous rate given by eq. (4.5) leading to $\dot{M} = 3\pi\alpha \Sigma c_s^2 / \Omega = \epsilon C_s^3/G$. Assuming that the disk is marginally critical, that is the criteria of Gammie (2001) is just satisfied, they estimate the $\alpha$ viscosity parameter to be about 0.23.

With eq. (4.7), the heating rate, $F_v$, due to the viscous dissipation in the disk is thus known. The cooling rate per face is assumed to be the classical black body radiation $F_r = (16/3)\sigma T^4 / (\Sigma \kappa)$, where $\kappa \simeq \kappa_R T^2$ is the Rosseland opacity taken from Semenov et al. (2003) and estimated to be $\kappa_R = 3 \times 10^{-4} \text{cm}^2\text{g}^{-1}$. The thermal balance leads to the relation $F_r = F_v$.

Together with the conditions $Q > 1$, the expression of the flux $\dot{M} = \epsilon C_s^3/G$ and the thermal balance lead to an estimate for the critical value of the rotation
frequency, $\Omega_{\text{crit}}$ above which the disk is stable

$$\Omega > \Omega_{\text{crit}} \simeq 3.6 \left( \frac{G^2 \mu^2 \sigma}{\alpha \kappa R^2 k_B T} \right)^{1/3} \simeq 4.3 \times 10^{-10} \text{s}^{-1},$$

(4.11)

where the relation $c_s^2 = k_0 T/\mu$ has been used.

For a Keplerian disk, we have $\Omega^2 = GM/R^3$, which leads to

$$\Omega \simeq 2 \times 10^{-10} \left( \frac{M}{1M_\odot} \right)^{1/2} \left( \frac{R}{100 \text{AU}} \right)^{-3/2} \text{s}^{-1}.$$

(4.12)

The combination with eq. (4.11) shows that typically fragmentation can occur in mechanically heated disks, for radius larger than $\simeq 60$ AU. This offers an appealing explanation for the brown dwarf desert (McCarthy & Zuckerman 2004), i.e. the marked dearth for close brown dwarfs companions around stars. In these conditions, the typical temperature within the disk is about 15 K while the typical fragment mass is a few Jupiter mass.

Whitworth & Stamatellos (2006) arrive to similar conclusions using related although slightly different arguments. First, they do not assume that the disk is mechanically heated but rather heated from the star radiation. Second, instead of using the Gammie criterion, they follow more closely the derivation presented in section 2.1.3 writing that the mechanical $PdV$ heating during contraction must be compensated by the cooling. They estimate a slightly larger radii for the fragmentation to occur and also propose that the brown dwarf desert can be naturally explained in this context as follows (e.g. Stamatellos & Whitworth 2009, figure 6). Fragments that form in the disk (around 70-100 AU) migrate into the inner disk region and grow in mass fast to become low-mass stars (they start from 1-3 Jupiter mass and grow to 0.2-0.3 solar mass) while fragments that form further out, tend to migrate in the inner disc region as well but they endure 3-body interactions with the most massive fragments (and the central star) which are there. They are thus thrown out back to outer disk region and the region close to the central star is kept clear of brown dwarfs.

4.2.2 The role of irradiation

The heating due to the central star is likely important although still controversial. An estimate for the temperature is simply given by the radiative equilibrium

$$\sigma T^4 = \frac{L_*}{4\pi R_d^2},$$

(4.13)

which leads to

$$T = \frac{L_*^{1/4}}{(4\pi)^{1/4} R_d^{3/2}} \simeq 300 K \left( \frac{L_*}{L_\odot} \right)^{1/4} \left( \frac{R_d}{1 \text{AU}} \right)^{-1/2}.$$  

(4.14)

This shows that the temperature is typically larger that when the disk is heated by mechanical dissipation only for radius of the order of 100 AU. Thus, the radius of
the brown dwarf desert estimated by Whitworth & Stamatellos (2006) is slightly larger and about $\approx 150$ AU.

However, the luminosity is not only due to the stellar luminosity but also to the accretion luminosity. For a fiducial one solar mass star accreting at a rate of about $10^{-5} M_\odot \, \text{s}^{-1}$, the accretion luminosity is about

$$L_{\text{acc}} = \frac{GM_*}{R_*} \dot{M} \simeq 150 L_\odot \left( \frac{R_*}{R_\odot} \right)^{-1} \frac{M_*}{1M_\odot} \frac{\dot{M}}{10^{-5} M_\odot \text{yr}^{-1}},$$

which is much larger than the stellar luminosity. Consequently, as noted by Matzner & Levin (2005) and confirmed by the numerical simulations performed by Bate (2009) and Offner et al. (2009), radiation substantially reduces the fragmentation of self-gravitating disks. Indeed Matzner & Levin (2005) even conclude that it may entirely suppress the formation of brown dwarfs by disk fragmentation.

At this stage, another complication must be taken into account. The accretion may not be continuous but instead could occur in a burst mode as advocated by Vorobyov & Basu (2006). In their case, this is due to disk non-linear dynamics which leads to the formation of small dense clumps within the disks that increase drastically the accretion when they fall into the central object. If this is the case, the implication is that the heating due to the accretion luminosity may also be intermittent leaving the possibilities that fragmentation could occur in between two bursts of accretion. Recent simulations have been performed by Stamatellos et al. (2011b) along this line. They indeed conclude that while disk fragmentation does not happen when the accretion is assumed to be constant, it does happen between the burst of accretion when it is irregular.

To conclude, it seems that the possibilities of brown dwarfs being formed by disk fragmentation is eventually linked to how accretion proceeds within the disk and to a large extent remains a matter of debate.

### 4.2.3 The role of the magnetic field

So far, the impact of the magnetic field has not been considered while as emphasized in several studies, it may have a crucial impact on disk formation and evolution. Here we simply summarize the most important ideas without going into the details.

- **Magnetic braking**

Due to the generation of torsional Alfvén waves which propagate and transfer angular momentum from the cloud to the intercloud medium (Mouschovias & Paleologou 1979, Shu et al. 1987), the magnetic field is able to efficiently brake interstellar clouds. To estimate the time scale over which this process is occurring, let us consider an intercloud medium of density $\rho_{\text{icm}}$ and let us assume that the magnetic field is parallel to the rotation axis. The waves propagate at the Alfvén speed, $V_a = B / \sqrt{4\pi \rho_{\text{icm}}}$ along a cylinder parallel to the magnetic field. Significant braking will arise when the waves have transmitted to the intercloud medium a substantial fraction of the cloud angular momentum. This is the case, when the waves have reached a distance from the cloud, $l$, such that $l \times \rho_{\text{icm}} \simeq R \times \rho_0$. That
is to say the waves have been able to transfer angular momentum to a mass of intercloud medium comparable to the mass of the cloud. This gives an estimate for the magnetic braking time, in case where the magnetic field and the rotation axis are aligned:

$$\tau_{br} \simeq \frac{R}{V_a} \frac{\rho_0}{\rho_{icm}}. \quad (4.16)$$

The braking time increases when $\rho_{icm}$ decreases because if the intercloud medium has a low inertia, its angular momentum is small.

- **The magnetic braking catastrophe**

Thanks to the progress of numerical schemes and computers, numerical calculations of the collapse of magnetized cores have been recently performed. One of the conclusions shared by several authors (e.g. Allen et al. 2003, Price & Bate 2007, Mellon & Li 2008, Hennebelle & Fromang 2008) is that the magnetic field has a very severe impact on disk formation. Indeed, these authors conclude that even for relatively weak fields, the formation of big massive disks that form in the hydrodynamical case, can be suppressed at least in the early phase of the collapse though Hennebelle & Ciardi (2009) found that the problem is less severe when the magnetic field is inclined with respect to the rotation axis. This is because the magnetic field is strongly amplified during the collapse and transports very efficiently the angular momentum (e.g. Galli et al. 2006). This behaviour has been found to persist when non-ideal MHD is taken into account (e.g. Mellon & Li, 2009 for ambipolar diffusion) though small scale disks can form at a scale of a few tens of AU due to ohmic dissipation at high densities (Dapp & Basu 2011, Machida et al. 2011). These models have received some support from the comparisons between high resolution observations and synthetic ones performed using various models (Maury et al. 2010). Observational data are in better agreement with magnetized models than with purely hydrodynamical ones. The main difference comes from the presence of a big massive disk in the latter case. An alternative view has been proposed by Stamatellos et al. (2011a) who argue that the massive disks do not live long because they quickly fragment, thus the chance to see them is low.

- **Disk stabilization by magnetic field line twisting**

When the magnetic field is sufficiently low, the magnetic braking is not strong enough to remove efficiently the angular momentum and disks form. However, it has been found that fragmentation is much reduced if not suppressed (Hennebelle & Teyssier 2008, Machida et al. 2008, Duffin & Pudritz 2009). This is due to the twisting of the magnetic field lines by the Keplerian velocity profile that quickly triggers the growth of a toroidal component. The latter tends to stabilize the disk since it operates, in the loosely sense, as an effective pressure that adds up to the thermal pressure. Note that at this stage, the impact of non-ideal MHD has not been investigated. This conclusion could change, if the field turns out to be not sufficiently coupled to the gas in the disk.
The general trend is therefore that the magnetic field tends to quench the formation of brown dwarfs by disk fragmentation because it tends to suppress the formation of big massive disks at the class 0 phase (leaving probably small tens of AU disk) and whenever they form, magnetic field tends to stabilize them. The final answer will depend on the distribution of magnetic intensities in dense cores and also on the non-ideal MHD processes.

5 Statistics from large scale numerical simulations

The approaches developed in previous sections attempted to identify a particular situation responsible of the formation of low-mass stars and brown dwarfs. More specifically, the gravo-turbulent theory emphasizes the role of turbulent density fluctuations while the fragmentation disk theory emphasizes the role of disk instabilities. In reality, both processes should be considered simultaneously and treated as a continuum. In particular, the impact of rotation and gravitational density enhancement which are at the very heart of the disk fragmentation theory should be taken into account in the gravo-turbulent theory. Large scale numerical simulations have been performed along this line using either smoothed particle hydrodynamics or adaptive mesh refinement techniques. This approach, which avoids many of the assumptions made in the previous sections, has the advantage of providing direct statistics which can be compared with the observational data. Given their complexity, it is however not always easy i) to check the reliability of these predictions (numerical convergence and sensibility to the physical processes) and ii) to interpret the physical mechanisms at play.

5.1 Numerical and physical setup

The typical setup used in this kind of calculations consists in a cloud of hundreds of solar masses (e.g. Bate et al. 2003). The shape is most of the time spherical and the density profile goes from a uniform density to an \( r^{-2} \) dependence (Girichidis et al. 2011). The problem is largely determined by the values of the thermal and turbulent energies which are typically a few percent or less for the former and a factor of a few for the latter. A turbulent velocity field, which has a powerspectrum close to the Kolmogorov one and random phases, is initially setup. Finally, sink particles that mimic the formation of stars, are usually used. Such Lagrangian entities (e.g. Federrath et al. 2010b) can accrete the surrounding gas and interact with it through gravity. To mimic the dust opacity, a barotropic equation of state is usually employed leading for example to \( c_s^2 = c_{s,0}^2 (1 + (n/n_c)^{2/3}) \) with \( n_c \approx 10^{10} \) cm\(^{-3}\) (note that the exponent \( 2/3 = 5/3 - 1 \) is valid until the internal rotation level of H\(_2\) are not excited after which it should typically be equal to \( 2/5 = 7/5 - 1 \)). As described below, calculations including radiative transfer and heating by the stars have been performed as well though due to very large computing overhead, the statistics obtained so far are only preliminary. The same is true for the magnetic
field. Simulations of magnetized massive clumps (Price & Bate 2009, Peters et al. 2010, Hennebelle et al. 2011, Seifried et al. 2011, Commerçon et al. 2011) have attempted to demonstrate trends rather than to obtain statistics. The general conclusion matches the trends inferred from the core collapse calculation presented above, magnetic field tends to reduce the fragmentation of these massive clouds.

5.2 Two new mechanisms for the formation of brown dwarfs

From the results of large scale numerical simulations, two new mechanisms for the formation of brown dwarfs have been inferred. The first one is the formation of brown dwarfs by ejection and the second one is the formation within dense filaments induced by gravity during collapse. Both mechanisms rely on strong density enhancements triggered by gravity.

5.2.1 Formation of brown dwarfs by ejection

In collapsing massive, turbulent clumps, massive centrifugally supported disks form as turbulence provides angular momentum. These disks quickly fragment as described above (at least when radiative transfer and magnetic field are ignored). As usually a small cluster of objects formed, they undergo N-body interactions and the less massive objects can easily be ejected from the parent cloud when the velocity they acquire during a close encounter is large enough. When they leave the cloud, there is no more gas around, that they could accrete and therefore their masses do not increase along time as it is the case for an object embedded in the parent clump. This mechanism has originally been proposed by Watkins et al. (1998) and Reipurth & Clarke (2001) and later observed and quantified in large scale simulations (Bate et al. 2002, Bate 2012).

5.2.2 Formation of brown dwarfs in dense filaments

The physics of this mechanism is simply that as the clump collapses, dense filaments form by a combination of shocks and local gravitational collapse (which in particular amplifies the anisotropies and tend to form more filaments). Within these dense structures, the density is larger and the Jeans mass is smaller, thus smaller objects including brown dwarfs can form. Note that while the preceding mechanisms was essentially a formation by disk fragmentation, the present one is very similar to the gravo-turbulent mechanism presented previously. The main difference is that the PDF instead of being due to turbulent compression, is due to gravitational contraction. This mechanism has been studied in detail by Bonnell et al. (2008). They show for example that there is a correlation between the initial Jeans mass and the final mass of the stars, suggesting that indeed this mechanism is at play.

Bate et al. (2002) estimate that in their barotropic calculations, the first mechanism is dominating over the second by a factor of the order of $\lesssim 3$. However, as the fragmentation of disk is much affected by the heating from the central star and the magnetic field, it is unclear whether this conclusion is really robust.
5.3 Impact of the radiative transfer

Before discussing some of the quantitative results obtained in large scale numerical simulations, it is useful to discuss qualitatively the influence of the radiative feedback further. Indeed, it has been found to play an important role in reducing the fragmentation by Krumholz et al. (2007), Bate (2009) and Commerçon et al. (2011). Generally speaking the reason is similar to what is discussed in section 4.2.2, that is to say around the accreting protostars, the gas is heated due to the intense accretion luminosity and thus the temperature is higher than what is typically obtained by using a barotropic equation of state. Although no simple estimate of the efficiency in reducing the fragmentation has been provided, this effect is found to be substantial in the above mentioned studies. For example, Bate (2009) concludes that the number of brown dwarfs may be lower by a factor of about $\simeq 3$ in his radiative calculations.

Another possible consequence, though not firmly established yet, is that the universality of the peak of the IMF could be due to the heating by the stars themselves. This is based on a scaling argument proposed by Bate (2009). The argument is as follows.

In the optically thick regime, the temperature around a star is given by $L/4\pi\sigma T^4$ while a Jeans length in this region is given by the relation $c_s^2 = kT/m_p \simeq GM/\lambda_J$ that is to say the thermal support is roughly equal to the gravitational energy for a mass $M \simeq (4\pi/3)\rho\lambda_J^3$. This leads to the relation

$$\lambda_J^{5/2} \simeq \frac{3kL_s^{1/4}}{(4\pi)^{5/4}Gm_p\sigma^{1/4}\rho^{-1}}. \quad (5.1)$$

The Jeans mass which is simply estimated as $M_J \simeq (4\pi/3)\rho\lambda_J^3$ is then found to be

$$M_J = (4\pi/3)^{-1/5}(k/Gm_p)^{6/5}L_s^{1/10}(4\pi\sigma)^{-1/10}\rho^{-1/5}. \quad (5.2)$$

Thus instead of getting a density dependence $\rho^{-1/2}$ as in the isothermal case, one gets $M_J \propto \rho^{-1/5}$ which is much shallower dependence seemingly leading to an IMF which is less dependent on the cloud parameters. Although it constitutes an interesting idea, various aspects of the scheme need further explanations. For example, the first generation of stars that formed in a molecular cloud, is not affected by this heating and therefore would have different characteristics unless this effect starts operating while the stars are still forming (for example by modifying the Jeans mass that would otherwise be accreted by the central star). Another question is the fraction of the gas in the molecular cloud which is affected by this heating. As this effect strongly depends on the distance to the stars, the pieces of gas which are too far from stars are not heated significantly.
5.4 The IMF and its sensibility to the initial conditions

Numerical simulations performed with SPH (e.g., Jappsen et al. 2005, Smith et al. 2008, Bate 2009, 2012) or using AMR and sink particles (e.g., Girichidis et al. 2011) tend to produce distributions which present similarities with the observed IMF, that is to say they present a powerlaw at high masses, which has an index compatible with the Salpeter value and a peak at some smaller mass. While the index of the high mass part appears to be seemingly robust (although the distributions are not always clearly powerlaws), the position of the peak vary as expected with the initial conditions (for example Bate & Bonnell 2005 show that it depends on the thermal energy) and the equation of state (e.g. Jappsen et al. 2005). The dependence on the initial density profiles found by Girichidis et al. (2011) is very drastic. For example, while when they start with a uniform density profile, they obtain the formation of many objects that mimics the IMF (though peaking at too small mass), they find in the case of an $r^{-2}$ profile, that a single object can be obtained. In general, these simulations tend to find that the peak of the mass distribution is too small compared to the peak of the IMF (which is typically equal to 0.3 $M_{\odot}$). For example Bate et al. (2003) conclude that too many brown dwarfs (typically a factor 3) form in his simulations.

Interestingly, Bate (2012) includes radiative transfer as described in § 5.3 and finds that the fragmentation is significantly reduced with respect to his former simulations (Bate et al. 2003) and obtains a mass distribution very similar to the IMF. Note that at this stage, this result is obtained for one particular clump having a specific initial density and turbulence strength. One of the remaining question is therefore what determines the initial cloud parameters (mean density, mass and Mach number) which all influence the mass distribution. As argued by Bate (2009) when radiation is included, the mass distribution may depend only weakly on, at least, the initial density though this should clearly be demonstrated by running simulations with different initial conditions and high resolution. Another word of caution is the influence of the magnetic field, not treated in these work. As advocated in Commerçon et al. (2010, 2011), the magnetic braking has a drastic influence on the radiative feedback by transporting angular momentum away and leading to enhanced local accretion rates. It is also difficult to properly simulate as it requires a high numerical resolution.

5.5 Physical origin of the IMF in numerical simulations

The mechanism responsible for setting the IMF in the above mentioned numerical simulations, is not easy to identify and remains a matter of debate.

Bate (2012) argues that it is due to competitive accretion. His main argument is that the more massive stars at the end of the simulations are the ones, which have accreted longer. Moreover, the accretion often ends after the star undergoes a dynamical interaction and is therefore ejected from the densest part of the parent cloud. Let us stress that the first part of this argument is also compatible with the massive stars having a larger accretion reservoir available as suggested by
Hennebelle & Chabrier (2008). This is because bigger cores require more time to collapse. An important test to distinguish between the various scenarii would be to quantify the spatial distribution of the gas accreted by the sink.

On the other hand, the SPH simulations performed by Smith et al. (2008) show a clear correlation between the initial masses (see also Bonnell et al. 2008) within the gravitational well and the final sink masses up to a few local freefall times (see Chabrier & Hennebelle 2010 for a quantitative analysis), suggesting that the initial prestellar cores do not fragment into many objects. As time goes on, the correlation becomes weaker but seems to persist up to the end of their run. Massive stars, on the other hand, are weakly correlated with the mass of the potential well in which they form. Whether their mass was contained into a larger more massive well with which the final sink mass would be well correlated remains an open issue, which needs to be further investigated.

Finally, we reiterate that the magnetic field (e.g. Machida et al. 2005, Hennebelle & Teyssier 2008, Price & Bate 2009, Peters et al. 2010, Hennebelle et al. 2011) and even more the combination between magnetic field and radiative feedback (Commerçon et al. 2011) seem to reduce the fragmentation, suggesting that the core-sink correlation found in Smith et al. (2008) should improve if such processes were included. Clearly, these questions require careful investigations.

6 Conclusion

We have reviewed the most recent theories which have been proposed to explain the origin of the formation of low-mass stars and brown dwarfs. First, we have described the physical processes at play in star formation as well as the most fundamental results. Second, we have presented the theories which attempt to explain the origin of the IMF. Two main categories received particular attention: the theories based on accretion and the ones based explicitly on turbulence. It should be stressed that these theories are not all exclusive of each other and may apply in different ranges of mass. For instance, the turbulent dispersion theory calculates the distribution of the initial mass accretion reservoirs; it is not incompatible with the stopped accretion theories and with the competitive accretion as long as mass redistribution/competition occurs within one parent core reservoir. The question as to whether one of these mechanisms is dominant is yet unsettled. Detailed comparisons between systematic sets of simulations, as done in Schmidt et al. (2010), or observations, and the various analytical predictions is clearly mandatory to make further progress. Third, we have described the theory for brown dwarf formation based on massive disk fragmentation. While it could in principle lead to the formation of at least a fraction of the brown dwarfs, large uncertainties remain regarding the role of magnetic field and radiation that could drastically reduce the importance of this mechanism. Finally, we described the large numerical simulations, which have been performed to study massive clump collapse and fragmentation. The introduction of magnetic field and radiative feedback has been a significant improvement of the last years, which unsurprisingly has revealed that both have a major impact on the outcome of star formation.
One important challenge for the next decades is certainly to perform large scale simulations which include both effects, including non-ideal MHD, with a sufficient resolution.

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