Analysis of Bending Moment-Curvature and the Damage Limits of Reinforced Concrete Circular Columns

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Abstract

In this study; the effect of axial load levels, longitudinal reinforcement ratio, transverse reinforcement diameter and transverse reinforcement spacing were investigated on the moment curvature relationships of reinforced concrete columns. For this purpose, circular reinforced concrete columns having different parameters were designed considering the regulations of the Turkish Building Earthquake Code (2018). The behavior of the columns were investigated from the moment-curvature relation, by considering the nonlinear behavior of the materials taken into account. The moment-curvature relationships of the reinforced concrete column cross-sections having different axial load levels have been obtained by considering Mander model, which considers the lateral, confined concrete strength. Moment-curvature relationships were obtained by SAP2000 Software, which takes the nonlinear behavior of materials into consideration. The designed reinforced concrete cross section models are considered to be composed of three components; cover concrete, confined concrete and reinforcement steel. The examined behavioral effects of the parameters were evaluated by the curvature and moment carrying capacity of the cross-sections. From the obtained moment-curvature relationship, cracking and destruction in cover and core concrete, yield and hardening conditions in reinforcement steel were calculated and the results were presented in charts and graphs. The confining effect in the core concrete is taken into account in the calculations. The behavior of the circular column sections and the types of refraction were interpreted according to the results obtained from the moment-curvature relationship of the sections. It is observed that the variation of the axial load, longitudinal reinforcement ratio, transverse reinforcement diameter and transverse reinforcement spacing have an important effect on the moment-curvature behavior of the reinforced concrete columns. The load bearing capacity of reinforced concrete column sections ends by destruction of the core concrete. Reinforced concrete column sections damaged by reinforcement yield before crushing of cover concrete exhibit more ductile behavior.

Keywords: Transverse reinforcement, nonlinear behavior, confined concrete strength, axial load, moment-curvature,

Betonarme Dairesel Kolonların Eğilme Momenti-Eğrilik ve Hasar Sınırlarının Analizi

Öz

Bu çalışmada; eksenel yük seviyesi, boyuna donatı oranı, sargı donatı çapı ve sargı donatı aralığının değişiminin betonarme kolonların moment-eğrilik ilişkisine olan etkisi incelenmiştir. Bu amaçla, farklı parametrelerle sahip betonarme dairesel kolon modelleri Türkiye Bina Deprem Yönetmeliği (2018) hükümlerine uyularak tasarlanmıştır. Betonarme kolonların davranış, malzemelerin doğrusal olmayan davranışlarını dikkate alan SAP2000 programı ile elde edilmiştir. Betonarme kolon kesitlerinin moment-eğrilik ilişkileri farklı eksenel yük seviyeleri için yanal sargı basıncını göz önüne alan Mander modeli ile elde edilmiştir. Moment-eğrilik ilişkileri, malzemelerin doğrusal olmayan davranışlarını dikkate alan SAP2000 programı ile elde edilmiştir. Sunulan betonarme kesit modellinin kabuk betonu, sargılı beton ve donatı çeligi olarak üç farklı ussurdan oluşturulmuş ve SAP2000 programı ile elde edilmiştir. Parametrelerin incelenen davranışlarsal etkileri, kesitlerin eğrilik ve moment şamba kapasitesi kullanılanlarak değerlendirilmştir. Elde edilen Moment- eğrilik ilişkilerinden, kabuk ve çekirdek betonunda çatlama ve kırsal, donatı çeligi ve peklemeye durumları hesaplanarak sonuçlar çizelgeler ve grafikler halinde sunulmuştur. Çekirdek betonundaki sargı etkisi hesaplamada gözönune alınmıştır. Dairesel kesitli

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kolonların davranış ve kıırılma tipleri, kesitlerin moment- eğrilik ilişkisinden elde edilen sonuçlara göre yorumlanmıştır. Eksenel yükün, boyuna donatı oranının, sargı donatı çapının ve sargı donatı aralığının değişiminin betonarme kolonların moment- eğrilik davranışının üzerinde önemli bir etkiye sahip olduğu gözlemlemiştir. Betonarme kolon kesitlerinin yük taşıma kapasitesi, çekirdik betonun ezilerek kıırılması ile sona ermektedir. Kabuk betonun ezilmesinden önce donatı akması ile hasar gören betonarme kolon kesitleri daha fazla güneş davranış göstermektedir.

Anahtar Kelimeler: Sargı donatısı, doğrusal olmayan davranış, sarglı beton dayanımı, eksenel yük, moment- eğrilik.

1. Introduction

In reinforced concrete structures, reinforced concrete columns are one of the most crucial elements under earthquake loads. Column mechanisms are very critical to prevent total collapse in earthquakes. The objective performance levels of reinforced concrete structures could not be ensured due to the failure of some critical reinforced concrete columns. Because of this, determining the behavior of the structures should be known well to design earthquake-resisting structures (Dok et al., 2017). In seismic zones, it is important to design structures, with power ranging deformation beyond the elastic deformations without losing its ability to stay in service, in other words designing structures with ductile behavior. The current philosophy used in the seismic design of reinforced concrete frames auto-stable is based on the hypothesis of the formation of plastic hinges at critical sections, the ability of the latter to resist several cycles of inelastic deformations without significant loss in bearing capacity is evaluated in terms of available ductility (Youcef and Chemrouk, 2012).

The behavior of reinforced concrete elements are determined by the cross-sectional behavior of elements. Cross-sectional behavior depends on the materials designed of the cross-section and the loading on that particular cross-section. The behavior of a reinforced concrete cross-section under bending moment or bending moment plus axial force can be monitored from moment-curvature relationship (Xie et al., 1994).

The bending moment-curvature curve can be widely applied in cross-section seismic analysis of reinforced concrete as the seismic performance that evaluates the cross-section. The bending moment-curvature curve is obtained by section size and reinforcement. The method of using this curve to evaluate the cross-section seismic performance is simple and able to save the analysis time (Jun and Hui, 2015).

Bedirhanoglu and Ilki (2004) obtained the analytical moment-curvature relationships for reinforced concrete cross-sections by using three different models for confined concrete. The theoretical moment-curvature relationships were then compared with experimental data reported in the literature. The results showed that the theoretical moment-curvature relationships obtained by all of these three models were in quite good agreement with experimental data. In the second part, a parametric investigation was carried out for examining the effects of various variables on the moment-curvature relationships, such as quality of concrete, level of axial load, amount and arrangement of transverse reinforcement.

Foroughi and Yuksel (2020) investigated the effect of the material model, axial load, longitudinal reinforcement ratio, transverse reinforcement ratio and transverse reinforcement spacing on the behavior of square reinforced concrete cross-sections. The effect of axial load, transverse reinforcement diameter and transverse reinforcement spacing on the behavior of reinforced concrete column models have been analytically investigated. The moment-curvature relationships for different axial load levels, transverse reinforcement diameter and transverse reinforcement spacing of the reinforced concrete column cross-sections were obtained considering the Mander confined model (Mander et. al, 1988). It was examined behavioral effects of the parameters were evaluated by comparing the curvature ductility and the cross-section strength. It has been found that transverse reinforcement diameters and transverse reinforcement spacing are effective parameters on the ductility capacities of the column sections. Axial load is a very important parameter affecting the ductility of the section. It has been observed that the cross-sectional ductility of the column sections increases with the decrease in axial load.

In this study, reinforced concrete circular columns were designed and the effects of the longitudinal reinforcement ratio, axial load levels, transverse reinforcement diameter and transverse reinforcement spacing on the behavior of these models were investigated. The behavior of the reinforced concrete column models was investigated through the relation of moment-curvature. Forty-eight circular reinforced concrete columns having different longitudinal and transverse reinforcements were analyzed. Moment-curvature relations were obtained and presented in graphical form using SAP2000 Software (CSI, V.20.1.0) which takes nonlinear behavior of materials into consideration. The designed reinforced concrete cross section models are considered to be composed of three components; core concrete, confined concrete and reinforcement steel. The SAP2000 Software material models are defined considering the Mander unconfined concrete model for cover concrete, and the Mander confined concrete model for core concrete. A concrete model proposed by Mander et al. (1988) which is widely used, universally accepted and mandated in Turkish Building Earthquake Code (TBEC, 2018) has been used to determine the moment-curvature relationships of reinforced concrete members. For reinforcement modeling stress-strain relationship given in TBEC (2018) was used. The examined behavioral effects of the parameters were evaluated by the curvature and moment carrying capacity of the cross-section. From the moment-curvature relationships obtained, the limits of damage zones were calculated in circular column sections. From the moment-curvature relationships, the limits of the damage zone were calculated based on limit states of strain in concrete and reinforcement bars in the section. From the obtained moment-curvature relationship, cracking and destruction in cover and core concrete, yield and hardening conditions in reinforcement steel were calculated and the results were presented in charts and graphs. The confining effect in the core concrete is taken into account in the calculations. The behavior of the circular section columns and the types of refraction were interpreted according to the results obtained from the moment-curvature relationship of the section.
2. Material and Method

The aim of this paper is to examine the influence of four parameters on the moment-curvature and the limits of the damage zone of reinforced concrete columns. SAP2000 software was used to predict the moment-curvature of reinforced concrete columns having different axial load levels \( (N/N_{\text{max}}) \). In order to investigate the effect of longitudinal reinforcement ratio, transverse reinforcement diameter, transverse reinforcement spacing and axial load levels, forty-eight reinforced concrete circular column models having dimensions 450mm diameter circular cross-sections were designed (Table 1). The parameters investigated in the moment-curvature relations of the reinforced concrete circular column models are the longitudinal reinforcement ratio, transverse reinforcement diameter, transverse reinforcement spacing and axial load levels. By using the Mander model (Mander et al., 1988), the moment-curvature relationships of the reinforced concrete circular columns are obtained by using the SAP2000 software, which performs non-linear analysis for different models designed. For all RC column models, C30 was chosen as concrete grade and B420C was selected as reinforcement for the reinforcement behavior model. The stress-strain relationship for materials given in TBEC (2018) were used (Table 2 and Figure 1).

Different transverse reinforcement diameters; \( \Phi8 \)mm and \( \Phi10 \)mm and the transverse reinforcement spacing; 50mm were selected in order to investigate the effect of the transverse reinforcement on the cross-section behavior. In the column models the longitudinal column reinforcement was \( \Phi20, \Phi22, \Phi24, \Phi26, \Phi28 \) and \( \Phi30 \) selected. Six different longitudinal reinforcement diameters and two different transverse reinforcement diameters are used for each reinforced concrete circular column models. In order to examine the effect of longitudinal reinforcement diameter on cross-sectional behavior, six different longitudinal reinforcement diameters (\( \Phi20 \), \( \Phi22 \) mm, \( \Phi24 \) mm, \( \Phi26 \) mm, \( \Phi28 \) mm ve \( \Phi30 \) mm) were selected.

The combined effect of vertical and seismic loads (\( N_{\text{dn}} \)), gross section area of column shall satisfy the condition \( A_c \geq N_{\text{dmax}}/0.40f_{\text{ck}} \) (TBEC, 2018). In this section, the moment-curvature relationships of the column sections were investigated for the values of \( N/N_{\text{max}} \) ratios of 0.10, 0.20, 0.30 and 0.40. To investigate the effect of axial force on the cross-section behavior the circular columns models were investigated under four different axial loads (480 kN, 960 kN, 1440 kN and 1920 kN). The aim of this paper is to examine the influence of different axial load levels, transverse reinforcement diameter and transverse reinforcement spacing on the moment-curvature and the limits of the damage zone for the designed column cross-sections are presented. The results obtained from the analyzes for reinforced concrete columns with different parameters were compared and interpreted.

Table 1. Details for the designed column model cross-sections

| No | Cross-sectional dimensions | Longitudinal reinforcement | Transverse reinforcement | Axial Load (N/N_{\text{max}}) |
|----|---------------------------|---------------------------|--------------------------|-----------------------------|
| A  |                           | \( \Phi8 \)20 mm           | \( \Phi8/50 \) mm         | 0.10                        |
| B  |                           | \( \Phi8 \)22 mm           |                          | 0.20                        |
| C  |                           | \( \Phi8 \)24 mm           |                          | 0.30                        |
| D  |                           | \( \Phi8 \)26 mm           |                          | 0.40                        |
| E  |                           | \( \Phi8 \)28 mm           |                          |                             |
| F  |                           | \( \Phi8 \)30 mm           |                          |                             |
| J  |                           | \( \Phi8 \)20 mm           | \( \Phi10/50 \) mm        | 0.10                        |
| H  |                           | \( \Phi8 \)22 mm           |                          | 0.20                        |
| I  |                           | \( \Phi8 \)24 mm           |                          | 0.30                        |
| G  |                           | \( \Phi8 \)26 mm           |                          | 0.40                        |
| K  |                           | \( \Phi8 \)28 mm           |                          |                             |
| L  |                           | \( \Phi8 \)30 mm           |                          |                             |

Table 2. Material parameters for concrete and reinforcement (TBEC, 2018)

| Standard Strength | Parameters                                      | Values |
|-------------------|------------------------------------------------|--------|
| Concrete: C30     | Strain at maximum stress of unconfined concrete (\( \varepsilon_{\text{cr}} \)) | 0.002  |
|                   | Ultimate compression strain of concrete (\( \varepsilon_{\text{cu}} \)) | 0.0035 |
|                   | Characteristic standard value of concrete compressive strength (\( f_{\text{ck}} \)) | 30 MPa |
| Reinforcement: B420C | Yield strain of reinforcement (\( \varepsilon_{\text{yr}} \)) | 0.0021 |
|                   | Spalling strain in reinforcing steel (\( \varepsilon_{\text{sp}} \)) | 0.008  |
|                   | Strain in reinforcing steel at ultimate strength (\( \varepsilon_{\text{su}} \)) | 0.080  |
|                   | Characteristic yield strength of reinforcement (\( f_{\text{ys}} \)) | 420 MPa |
|                   | Ultimate strength of reinforcement (\( f_{\text{fu}} \)) | 550 MPa |
3. Numerical Study

In this study, the design parameters of reinforced concrete members are investigated to determine the behavior of reinforced concrete circular columns. Theoretical moment-curvature analysis for reinforced concrete circular columns indicating the available bending moment and curvature can be constructed providing that the stress-strain relations for both concrete and steel are known. The objective of this study is to analyze the moment-curvature and the limits of the damage zone of forty-eight reinforced concrete circular columns with different parameters. Moment-curvature relationships were obtained by SAP2000 Software which takes the nonlinear behavior of materials into consideration. In this part of the study, the moment-curvature relations are obtained by changing the longitudinal reinforcement ratio, transverse reinforcement diameter, transverse reinforcement spacing and axial load levels. The numerical model was employed to calculate the moment and curvature values at the limit of the damage zone of reinforced concrete circular columns with different parameters. The moment-curvature relationships of reinforced concrete circular columns were determined and the results were prepared are given in Figure 2. In Figure 2, moment-curvature relationships are presented comparatively for different axial load levels. For different axial load levels, critical points in moment-curvature relations of circular cross-section column models are determined and presented in tables.
From the moment-curvature relationships of reinforced concrete circular column sections, the limits of the damage zone were calculated. Three material models are defined as cover concrete, reinforcing steel and core concrete for each section. From the obtained moment-curvature relationship, cracking and destruction in cover and core concrete, yield and hardening conditions in reinforcement steel were calculated and the results were presented in charts and graphs. The behavior of the circular section columns and the types of destruction were interpreted according to the results obtained from the moment-curvature relationship of the section. The values obtained according to different parameters for each material model in circular column sections are given in Tables 3 to 8 and 10 to 15, respectively. The units for the moment \( M \) is kN.m and the units for the curvature \( C \) is rad/m in all Tables. The circular reinforced concrete column sections given in the tables are prepared for four different axial loads, six different longitudinal rebar diameters and two different transverse reinforcement diameters and spacings. Using the values obtained from the moment-curvature relationships given in the tables, the fracture types and behaviors of the column sections were examined.

![Graph showing moment-curvature relationships for different axial load levels (transverse reinforcement Φ8/50 mm)](image)

**Figure 2.** Moment-curvature relationships for different axial load levels (transverse reinforcement Φ8/50 mm)

| Table 3. Critical moment and curvature values calculated for (A) columns |
|---|---|---|---|---|
| No | \( N/N_{max} \) | Reinforcement Steel | Cover Concrete | Core Concrete |
| | | Yield | Hardening | Cracking | Destruction | Cracking | Destruction | M | C | M | C | M | C |
| A1 | 0.1 | | | | | | | | | | | | | | |
| A2 | 0.2 | | | | | | | | | | | | | | |
| A3 | 0.3 | | | | | | | | | | | | | | |
| A4 | 0.4 | | | | | | | | | | | | | | |

| Table 4. Critical moment and curvature values calculated for (B) columns |
|---|---|---|---|---|
| No | \( N/N_{max} \) | Reinforcement Steel | Cover Concrete | Core Concrete |
| | | Yield | Hardening | Cracking | Destruction | Cracking | Destruction | M | C | M | C | M | C |
| B1 | 0.1 | | | | | | | | | | | | | | |
| B2 | 0.2 | | | | | | | | | | | | | | |
| B3 | 0.3 | | | | | | | | | | | | | | |
| B4 | 0.4 | | | | | | | | | | | | | | |

| Table 5. Critical moment and curvature values calculated for (C) columns |
|---|---|---|---|---|
| No | \( N/N_{max} \) | Reinforcement Steel | Cover Concrete | Core Concrete |
| | | Yield | Hardening | Cracking | Destruction | Cracking | Destruction | M | C | M | C | M | C |
| C1 | 0.1 | | | | | | | | | | | | | | |
| C2 | 0.2 | | | | | | | | | | | | | | |
| C3 | 0.3 | | | | | | | | | | | | | | |
| C4 | 0.4 | | | | | | | | | | | | | | |
Table 6. Critical moment and curvature values calculated for (D) columns

| No | N/N_{max} | Reinforcement Steel | Cover Concrete | Core Concrete |
|----|------------|---------------------|---------------|--------------|
|    |            | Yield               | Hardening     | Cracking     | Destruction  | Destruction  |
|    |            | M       | C       | M       | C       | M       | C       | M       | C       |
| D1 | 0.1        | 247.3  | 0.0096 | 320.9  | 0.0327 | 291.1  | 0.0151 | 315.2  | 0.0423 | 339.4  | 0.1696 |
| D2 | 0.2        | 292.3  | 0.0108 | 337.6  | 0.0374 | 310.1  | 0.0122 | 338.7  | 0.0342 | 359.2  | 0.1283 |
| D3 | 0.3        | 329.8  | 0.0120 | 349.7  | 0.0423 | 307.1  | 0.0105 | 355.1  | 0.0284 | 366.1  | 0.1112 |
| D4 | 0.4        | 355.8  | 0.0134 | 356.4  | 0.0475 | 294.8  | 0.0088 | 363.1  | 0.0257 | 366.5  | 0.1005 |

Table 7. Critical moment and curvature values calculated for (E) columns

| No | N/N_{max} | Reinforcement Steel | Cover Concrete | Core Concrete |
|----|------------|---------------------|---------------|--------------|
|    |            | Yield               | Hardening     | Cracking     | Destruction  | Destruction  |
|    |            | M       | C       | M       | C       | M       | C       | M       | C       |
| E1 | 0.1        | 272.3  | 0.0097 | 354.9  | 0.0342 | 297.1  | 0.0113 | 341.3  | 0.0219 | 378.1  | 0.1596 |
| E2 | 0.2        | 316.3  | 0.0108 | 369.7  | 0.0374 | 310.8  | 0.0105 | 380.1  | 0.0284 | 395.8  | 0.1253 |
| E3 | 0.3        | 354.3  | 0.0120 | 381.2  | 0.0423 | 328.0  | 0.0105 | 385.7  | 0.0284 | 400.1  | 0.1112 |
| E4 | 0.4        | 380.3  | 0.0134 | 387.7  | 0.0475 | 341.6  | 0.0088 | 393.8  | 0.0257 | 400.1  | 0.1005 |

Table 8. Critical moment and curvature values calculated for (F) columns

| No | N/N_{max} | Reinforcement Steel | Cover Concrete | Core Concrete |
|----|------------|---------------------|---------------|--------------|
|    |            | Yield               | Hardening     | Cracking     | Destruction  | Destruction  |
|    |            | M       | C       | M       | C       | M       | C       | M       | C       |
| F1 | 0.1        | 299.1  | 0.0099 | 392.8  | 0.0342 | 350.9  | 0.0141 | 389.6  | 0.0374 | 418.6  | 0.1531 |
| F2 | 0.2        | 341.9  | 0.0109 | 403.8  | 0.0374 | 363.2  | 0.0122 | 405.4  | 0.0327 | 434.3  | 0.1225 |
| F3 | 0.3        | 380.5  | 0.0120 | 414.7  | 0.0423 | 350.4  | 0.0105 | 418.4  | 0.0284 | 436.4  | 0.1112 |
| F4 | 0.4        | 406.4  | 0.0133 | 421.1  | 0.0475 | 329.5  | 0.0088 | 429.6  | 0.0244 | 436.1  | 0.1005 |

The effect of longitudinal reinforcement ratio on the moment-curvature relationship in reinforced concrete column sections is given in Figures 3 and 5 comparatively. In Figure 3, the effect of the change of longitudinal reinforcement ratio under constant axial load for Φ8/50 mm transverse reinforcement on moment-curvature relationship is summarized. In Figure 5, the effect of the change of longitudinal reinforcement ratio under constant axial load for Φ10/50 mm transverse reinforcement on moment-curvature relationship is summarized.

![Figure 3](image1.png)

Figure 3. Moment-curvature relationships for different longitudinal reinforcement ratio (transversae Reinforcement Φ8/50 mm)
Table 9 was prepared to compare the effects of longitudinal reinforcement ratio and axial load levels on moment-curvature behavior of circular sections for constant transverse reinforcement spacing (Φ8/50 mm). In Table 9, by using the moment-curvature relationships given in Figure 3, maximum moment (\(M_u\)) and maximum curvature values (\(C_u\)) are summarized.

| No | N/N_max | Destruction | No | N/N_max | Destruction | No | N/N_max | Destruction | No | N/N_max | Destruction |
|----|---------|-------------|----|---------|-------------|----|---------|-------------|----|---------|-------------|
| A1 | 0.10    | 236.9 0.2015 A2 | 263.0 0.1467 A3 | 277.6 0.1112 A4 | 279.5 0.1005 |
| B1 | 0.20    | 269.0 0.1870 B2 | 292.8 0.1404 B3 | 304.8 0.1112 B4 | 306.2 0.1005 |
| C1 | 0.30    | 303.3 0.1800 C2 | 324.8 0.1343 C3 | 334.2 0.1112 C4 | 335.1 0.1005 |
| D1 | 0.40    | 339.4 0.1696 D2 | 359.2 0.1283 D3 | 366.1 0.1112 D4 | 366.5 0.1005 |
| E1 | 0.50    | 378.0 0.1596 E2 | 395.8 0.1253 E3 | 400.1 0.1112 E4 | 400.1 0.1005 |
| F1 | 0.60    | 418.6 0.1531 F2 | 434.3 0.1225 F3 | 436.4 0.1112 F4 | 436.0 0.1005 |

Figure 4. Moment-curvature relationships for different axial load levels (transverse reinforcement: Φ10/50 mm)
### Table 10. Critical moment and curvature values calculated for (J) columns

| No | $N/N_{\text{max}}$ | Reinforcement Steel | Cover Concrete | Core Concrete |
|----|---------------------|---------------------|----------------|---------------|
|    |                     | Yield | Hardening | Cracking | Destruction | M | C | M | C | M | C | M | C |
| J1 | 0.1                 | 183.5 | 0.0092   | 230.3    | 0.0312      | 215.6 | 0.0172 | 227.2 | 0.0512 | 247.0 | 0.2651 |
| J2 | 0.2                 | 231.9 | 0.0105   | 263.1    | 0.0358      | 249.9 | 0.0132 | 261.8 | 0.0374 | 279.9 | 0.2015 |
| J3 | 0.3                 | 269.9 | 0.0119   | 280.1    | 0.0406      | 256.4 | 0.0105 | 281.9 | 0.0312 | 300.4 | 0.1531 |
| J4 | 0.4                 | 297.5 | 0.0134   | 293.2    | 0.0458      | 256.6 | 0.0088 | 297.3 | 0.0257 | 308.7 | 0.1343 |

### Table 11. Critical moment and curvature values calculated for (H) columns

| No | $N/N_{\text{max}}$ | Reinforcement Steel | Cover Concrete | Core Concrete |
|----|---------------------|---------------------|----------------|---------------|
|    |                     | Yield | Hardening | Cracking | Destruction | M | C | M | C | M | C | M | C |
| H1 | 0.1                 | 203.2 | 0.0093   | 259.3    | 0.0312      | 238.8 | 0.0162 | 252.2 | 0.0475 | 281.9 | 0.2484 |
| H2 | 0.2                 | 250.8 | 0.0106   | 288.7    | 0.0358      | 265.7 | 0.0122 | 288.7 | 0.0358 | 311.4 | 0.1942 |
| H3 | 0.3                 | 289.1 | 0.0119   | 305.4    | 0.0406      | 272.8 | 0.0105 | 306.3 | 0.0312 | 329.9 | 0.1467 |
| H4 | 0.4                 | 316.6 | 0.0134   | 318.2    | 0.0458      | 269.4 | 0.0088 | 321.5 | 0.0257 | 336.6 | 0.1343 |

### Table 12. Critical moment and curvature values calculated for (I) columns

| No | $N/N_{\text{max}}$ | Reinforcement Steel | Cover Concrete | Core Concrete |
|----|---------------------|---------------------|----------------|---------------|
|    |                     | Yield | Hardening | Cracking | Destruction | M | C | M | C | M | C | M | C |
| I1 | 0.1                 | 224.7 | 0.0095   | 289.9    | 0.0327      | 266.1 | 0.0162 | 284.4 | 0.0458 | 319.6 | 0.2403 |
| I2 | 0.2                 | 271.4 | 0.0107   | 316.6    | 0.0358      | 282.2 | 0.0122 | 316.6 | 0.0358 | 345.1 | 0.1870 |
| I3 | 0.3                 | 309.8 | 0.0119   | 332.9    | 0.0406      | 290.8 | 0.0105 | 335.1 | 0.0298 | 361.8 | 0.1467 |
| I4 | 0.4                 | 337.5 | 0.0133   | 344.8    | 0.0440      | 283.5 | 0.0088 | 347.7 | 0.0257 | 366.8 | 0.1343 |

### Table 13. Critical moment and curvature values calculated for (G) columns

| No | $N/N_{\text{max}}$ | Reinforcement Steel | Cover Concrete | Core Concrete |
|----|---------------------|---------------------|----------------|---------------|
|    |                     | Yield | Hardening | Cracking | Destruction | M | C | M | C | M | C | M | C |
| G1 | 0.1                 | 247.9 | 0.0096   | 323.3    | 0.0327      | 292.3 | 0.0151 | 318.9 | 0.0440 | 358.6 | 0.2322 |
| G2 | 0.2                 | 293.8 | 0.0108   | 346.6    | 0.0374      | 312.4 | 0.0122 | 346.9 | 0.0342 | 381.2 | 0.1800 |
| G3 | 0.3                 | 332.5 | 0.0119   | 362.7    | 0.0406      | 310.3 | 0.0105 | 363.9 | 0.0298 | 396.1 | 0.1404 |
| G4 | 0.4                 | 360.1 | 0.0133   | 374.1    | 0.0440      | 298.9 | 0.0088 | 376.1 | 0.0257 | 399.5 | 0.1343 |

### Table 14. Critical moment and curvature values calculated for (K) columns

| No | $N/N_{\text{max}}$ | Reinforcement Steel | Cover Concrete | Core Concrete |
|----|---------------------|---------------------|----------------|---------------|
|    |                     | Yield | Hardening | Cracking | Destruction | M | C | M | C | M | C | M | C |
| K1 | 0.1                 | 273.1 | 0.0097   | 358.1    | 0.0342      | 319.7 | 0.0141 | 355.6 | 0.0406 | 399.6 | 0.2166 |
| K2 | 0.2                 | 317.9 | 0.0108   | 379.3    | 0.0374      | 338.3 | 0.0122 | 379.2 | 0.0342 | 419.5 | 0.1730 |
| K3 | 0.3                 | 357.0 | 0.0119   | 394.6    | 0.0406      | 331.4 | 0.0105 | 394.9 | 0.0298 | 432.6 | 0.1404 |
| K4 | 0.4                 | 384.6 | 0.0132   | 405.5    | 0.0440      | 315.7 | 0.0088 | 406.8 | 0.0257 | 434.6 | 0.1343 |

### Table 15. Critical moment and curvature values calculated for (L) columns

| No | $N/N_{\text{max}}$ | Reinforcement Steel | Cover Concrete | Core Concrete |
|----|---------------------|---------------------|----------------|---------------|
|    |                     | Yield | Hardening | Cracking | Destruction | M | C | M | C | M | C | M | C |
| L1 | 0.1                 | 299.9 | 0.0099   | 396.7    | 0.0342      | 352.3 | 0.0141 | 394.4 | 0.0390 | 442.4 | 0.2015 |
| L2 | 0.2                 | 343.7 | 0.0109   | 413.9    | 0.0374      | 351.2 | 0.0113 | 414.3 | 0.0327 | 460.1 | 0.1662 |
| L3 | 0.3                 | 383.2 | 0.0119   | 428.5    | 0.0406      | 353.9 | 0.0105 | 428.9 | 0.0284 | 470.9 | 0.1404 |
| L4 | 0.4                 | 410.7 | 0.0132   | 439.1    | 0.0440      | 333.6 | 0.0088 | 439.4 | 0.0257 | 471.9 | 0.1343 |
Table 16 was prepared to compare the effects of longitudinal reinforcement ratio and axial load levels on moment-curvature behavior of circular sections for constant transverse reinforcement spacing (Φ10/50 mm). In Table 16, by using the moment-curvature relationships given in Figure 5, maximum moment ($M_u$) and maximum curvature values ($C_u$) are summarized.

Table 16. Maximum moment ($M_u$) and maximum curvature ($C_u$) values at the moment of destruction in circular column sections (transverse reinforcement: Φ10/50 mm)

| No  | N/N$_{max}$ | Destruction | No  | N/N$_{max}$ | Destruction | No  | N/N$_{max}$ | Destruction |
|-----|-------------|-------------|-----|-------------|-------------|-----|-------------|-------------|
| J1  | 0.10        | 247.0       | 0.20| 279.9       | 300.4       | 0.30| 361.8       | 396.1       |
| H1  | 281.9       | 0.2484      |     | 311.4       | 329.9       |     | 361.8       | 396.1       |
| J2  | 319.6       | 0.2403      |     | 345.1       | 329.9       |     | 361.8       | 396.1       |
| G1  | 358.6       | 0.2322      |     | 381.2       | 329.9       |     | 361.8       | 396.1       |
| K1  | 399.6       | 0.2166      |     | 419.5       | 329.9       |     | 361.8       | 396.1       |
| L1  | 442.4       | 0.2015      |     | 460.1       | 329.9       |     | 361.8       | 396.1       |

Figure 5. Moment-curvature relationships for different longitudinal reinforcement ratio (transverse reinforcement Φ10/50 mm)

4. Research Results and Discussion

The examined behavioral effects of the parameters were evaluated by the curvature and moment carrying capacity of the cross-section. Moment and curvature values in case of destruction according to different parameters of reinforced concrete circular column sections are summarized in Table 17 comparatively. The behavior of the circular column sections and the types of refraction were interpreted according to the results obtained from the moment-curvature relationship of the section. The influence of different parameters on the moment bearing capacity and curvature of the reinforced concrete circular columns are given Figures 6 to 11 comparatively.
Table 17. Moment and curvature values in case of destruction according to different parameters

| No | N/N_{max} | Longitudinal Reinforcement | Transverse Reinforcement | Destruction | No | Transverse Reinforcement |
|----|------------|-----------------------------|--------------------------|-------------|----|--------------------------|
|    |            | 8Φ20 mm                     | Φ8/50 mm                 | Mu          | C_u | 8Φ10/50 mm               |
| A1 | 0.10       |                             |                          | 236.9       | 0.2015 |                         |
| A2 | 0.20       |                             |                          | 262.9       | 0.1467 |                         |
| A3 | 0.30       |                             |                          | 277.6       | 0.1112 |                         |
| A4 | 0.40       |                             |                          | 279.5       | 0.1005 |                         |
| B1 | 0.10       |                             |                          | 286.9       | 0.1870 |                         |
| B2 | 0.20       |                             |                          | 292.8       | 0.1404 |                         |
| B3 | 0.30       |                             |                          | 304.8       | 0.1112 |                         |
| B4 | 0.40       |                             |                          | 306.2       | 0.1005 |                         |
| C1 | 0.10       |                             |                          | 303.3       | 0.1800 |                         |
| C2 | 0.20       |                             |                          | 324.8       | 0.1343 |                         |
| C3 | 0.30       |                             |                          | 334.2       | 0.1112 |                         |
| C4 | 0.40       |                             |                          | 335.1       | 0.1005 |                         |
| D1 | 0.10       |                             |                          | 339.4       | 0.1696 |                         |
| D2 | 0.20       |                             |                          | 359.2       | 0.1283 |                         |
| D3 | 0.30       |                             |                          | 366.1       | 0.1112 |                         |
| D4 | 0.40       |                             |                          | 366.5       | 0.1005 |                         |
| E1 | 0.10       |                             |                          | 378.1       | 0.1596 |                         |
| E2 | 0.20       |                             |                          | 395.8       | 0.1253 |                         |
| E3 | 0.30       |                             |                          | 400.1       | 0.1112 |                         |
| E4 | 0.40       |                             |                          | 400.1       | 0.1005 |                         |
| F1 | 0.10       |                             |                          | 418.6       | 0.1531 |                         |
| F2 | 0.20       |                             |                          | 434.3       | 0.1225 |                         |
| F3 | 0.30       |                             |                          | 436.4       | 0.1112 |                         |
| F4 | 0.40       |                             |                          | 436.1       | 0.1005 |                         |

Figure 6. Influence of different parameters on the moment bearing capacity of the reinforced concrete circular columns

Figure 7. Influence of different parameters on the curvature of the reinforced concrete circular columns (ductility)
Figure 8. Influence of different longitudinal reinforcement ratios on the moment bearing capacity of the reinforced concrete circular columns

Figure 9. Influence of different longitudinal reinforcement ratios on the curvature of the reinforced concrete circular columns

Figure 10. Influence of different transverse reinforcement ratios on the moment bearing capacity of the reinforced concrete circular columns
As can be seen from Table 17, two different transverse reinforcement diameters (∅8 mm and ∅10 mm) have been chosen for reinforced concrete circular column section models, with constant transverse reinforcement spacing. In terms of the ratio of transverse reinforcement, by examining the bearing capacity and curvature of the reinforced concrete column section; the increase in the ratio of transverse reinforcement was found to be effective in terms of bearing capacity and the curvature (ductility) of the section. Increased axial load level for reinforced concrete column sections, fixed longitudinal reinforcement, transverse reinforcement diameter and spacing has been found to be effective in moment and curvature. Increasing the axial load value in reinforced concrete column sections increases the moment bearing capacity of the section and decreases its ductility (curvature value decreases). It was observed that the axial load level was the effective parameter for the moment bearing capacity and the moment-curvature relation of the reinforced circular columns (Figure 4 and Table 17). With the increase of the longitudinal reinforcement ratio, the bearing capacity of reinforced concrete column sections increases, but the section ductility decreases. Increasing the ratio of transverse reinforcement increases both bearing capacity and ductility of reinforced concrete column sections and affects the moment-curvature relationships of the sections significantly.
5. Conclusion

The following results were obtained from the moment curvature analyses of the circular columns:

When the analysis results are examined, it is observed that the variation of the axial load, longitudinal reinforcement diameter, transverse reinforcement diameter and transverse reinforcement spacing have an important effect on the moment-curvature behavior of the reinforced concrete circular columns. Axial load, transverse reinforcement diameter and spacing are very important parameters affecting the ductility of the cross-section. With increasing axial load values yield curvature, yield moment and ultimate moment values increase, however, the ultimate curvature and curvature ductility values decrease. As can be seen from the moment-curvature relationships, the axial load is a very important parameter affecting the ductility of the cross-section of the columns. Significant reductions in ductility capacities of the column sections under increasing axial force have been observed.

It is observed that the variation of the axial load, longitudinal reinforcement diameter, transverse reinforcement diameter and transverse reinforcement spacing have an important effect on the moment-curvature behavior of the reinforced concrete circular columns. The cross-section ductility decreases when the transverse reinforcement spacing is increased under constant axial load. As can be seen from the moment-curvature relationships, it is observed that the cross-section ductility and the curvature increase significantly with the reduction of the transverse reinforcement spacing. The ratio of transverse reinforcement is effective in cross-section behavior of reinforced concrete cross-section. The increase in transverse reinforcement diameter increases the ductility of the cross-section and the maximum moment bearing capacity. The increase in the transverse reinforcement diameter increases the ultimate moment, ultimate curvature and curvature ductility values, but yield moment and yield curvature values remain almost constant (transverse reinforcement spacing and axial load levels are the constant). Yield moment, yield curvature, ultimate moment and ultimate curvature values increases however, curvature ductility values decreases as the longitudinal reinforcement diameter increases while other parameters kept constant. Moreover, with the increase of the transverse reinforcement ratio, the more ductile behavior is achieved due to the increment of curvature ductility on reinforced concrete columns. In order to see the real behavior of a reinforced concrete cross-section, a concrete model that takes the transverse reinforcement ratio into consideration should be used.

As can be seen from the comparison of the limit values of the damage zones calculated from the moment-curvature relations of the reinforced concrete circular columns according to different parameters: the first damage occurs by cracking the cover concrete or yielding of the reinforcement. In cases where the axial load value applied to reinforced concrete column sections is small ($N/N_{\text{max}} = 0.1$ and 0.2), the first damage occurs with the yielding of the reinforcement. In case of increased axial load value ($N/N_{\text{max}} = 0.3$ and 0.4), first damages occur by cracking the cover concrete outside the core concrete of reinforced concrete columns. Under increasing deformations after the reinforcement yield, the cover concrete is cracked and the reinforced concrete column section is damaged. After cover concrete is cracked hardening in reinforcement occurs and moment bearing capacity is also increasing. The moment and curvature values increase up to the maximum axial load value that the reinforced concrete columns can bear. The load-bearing capacity of reinforced concrete column sections ends with the destruction of the core concrete. Reinforced concrete column sections damaged by reinforcement yield before crushing of cover concrete exhibit more ductile behavior.

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