Online Active Model Selection for Pre-trained Classifiers

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Abstract

Given k pre-trained classifiers and a stream of unlabeled data examples, how can we actively decide when to query a label so that we can distinguish the best model from the rest while making a small number of queries? Answering this question has a profound impact on a range of practical scenarios. In this work, we design an online selective sampling approach that actively selects informative examples to label and outputs the best model with high probability at any round. Our algorithm can be used for online prediction tasks for both adversarial and stochastic streams. We establish several theoretical guarantees for our algorithm and extensively demonstrate its effectiveness in our experimental studies.

1 INTRODUCTION

Model selection from a set of pre-trained models is an emerging problem in machine learning and has implications in several practical scenarios. Industrial examples include cases in which a telecommunication company or a flight booking company have multiple ML models trained over different sliding windows of data and hope to pick the one that performs the best on a given day. For many real-world problems, unlabeled data is abundant and can be inexpensively collected, while labels are expensive to acquire and require human expertise. Consequently, there is a need to robustly identify the best model under limited labeling resources. Similarly, one often needs reasonable predictions for the unlabeled data while keeping the labeling budget low.

Depending on data availability, one can consider two settings: the pool-based setting assumes that the learner has access to a pool of unlabeled data, and she tries to select informative data samples from the pool to achieve her task. The online (streaming) setting works with a stream of data, where the data arrives one at a time, and the learner decides to ask for the label of the data samples on the go or to just throw the sample away. While offering less options on which data to label next, this streaming setting alleviates the scalability challenge of storing and processing a large pool of examples in the pool-based setting.

Another important aspect is the nature of the data: the instance/label pairs might be sampled i.i.d. from a fixed distribution, or chosen adversarially by an (oblivious) adversary. While sometimes the i.i.d. assumption is reasonable, there are practical scenarios where this...
assumption fails to hold. These include cases where there are temporal or spatial dependencies or non-stationarities in the dataset. In these situations, it may be safer not to make assumptions on the data and rather consider worst-case data examples.

Contributions We develop a novel, principled and efficient approach – Model Picker – for the online setting, where the data examples arrive in a stream, and the learner actively decides to query the label per each example. Our query strategy is randomized and leverages hypothetical (possible) query answers to decide which data examples are likely to be informative for identifying the best model. We prove that our algorithm has no regret for adversarial streams, i.e., its performance for sequential label prediction is close to the best model for that stream in hindsight. Our bounds match (up to constants) those of existing online algorithms that have access to all labels. We also establish bounds on the number of label queries, as well as the confidence on the output of Model Picker and its accuracy. We furthermore conduct extensive experiments, comparing our method with a range of other methods. To reach the same accuracy, competing methods can often require up to $2.5 \times$ more labels. Apart from the relative performance, on the ImageNet dataset, Model Picker requires a mere 13% labeled instances to select the best among 102 pre-trained models with $90\%$, while having up to $1.3 \times$ reduction in regret. These results establish Model Picker as the state-of-the-art for this problem. We also make everything open and reproducible.\footnote{The code is available at https://github.com/DS3Lab/online-active-model-selection}

2 RELATED WORK

Our approach relates to several bodies of literature, but differs from each of them. For each related area, we reference similar works that match the objective of our paper.

Active Model Selection Madani et al. [27] develop an approach for the streaming setting. They seek to identify the best model via probing models, one at a time, with i.i.d. samples, while having a fixed budget for the number of probes. In contrast, our approach applies even to adversarial streams, and allows to make predictions online, while minimizing the number of queries made. Most other previous works [35, 18, 2, 34, 23, 24, 25] focus on pool-based sampling of informative instances, where the learner ranks the entire pool of unlabeled data and greedily selects the most informative examples. This setting substantially differs from the streaming setting, and we focus on the latter for reasons of scalability and applicability to many real-world situations.

Active Learning Active learning aims to select instances (and asks their label) for improving the training of classifiers, rather than selecting among pre-trained models. Here we review those methods that can potentially be adapted for model selection. The celebrated query-by-committee paradigm [40] forms a committee of classifiers to vote on the labeling of incoming examples. The query decision is made based on the degree of disagreement the incoming instance creates among the committee members. The strategy is to query those instances that help the learner to prune the committee. There are other query-by-committee approaches in active learning, such as [28, 1, 29, 39, 48]. One limitation of these algorithms is that they often focus on pool-based sampling [13], which limits their scalability. Several other approaches consider active learning in the streaming setting. The seminal work of [6], followed by [15, 8, 47] uses disagreement based strategies [20, 19]. The idea of using importance weights in active learning is studied by a series of works including [7, 42, 9, 5], where importance weights are introduced to correct sampling
bias and achieve statistically consistent convergence to the optimal classifier in the PAC learning setting. All the above approaches on stream-based active learning focus on i.i.d. streams and try to improve supervised training of classifiers, whereas our approach applies to the more general adversarial streams and performs no training.

**Online Learning and Bandit Algorithms** Sequential label prediction is an important problem in online learning. The setting closest to ours is label-efficient prediction [12]. The main difference is that Cesa-Bianchi et al. [12] query the label with a fixed probability at each round, and that probability also appears in the regret bound. However, we use the side-information of the model predictions to adapt the probability of querying to the information content of that instance, thereby significantly reducing the required labels in practice and lowering the regret, as demonstrated theoretically and in our experiments. Moreover, there is no study of the quality of the model output at the end of the stream, for neither adversarial nor stochastic streams. Another problem similar to ours is consistent online learning [22, 3], where the learner seeks to minimize the number of switches of her actions. Here, the learner observes the loss every round, even if she does not update her strategy. In our setting, in contrast, we do not know the true loss at the rounds we do not query. Similar challenges arise in the multi-armed bandit literature. In a way, our setup lies between the usual prediction with experts advice and multi-armed bandit problems. Our algorithm is related in spirit to the EXP3 algorithm [4] for adversarial bandits. The key difference, however, is that EXP3 uses the probability of selecting an arm to construct an unbiased loss estimator, whereas we consider the probability of observing the whole loss. While similar in spirit, the standard EXP3 analysis fails to yield a regret bound, as discussed in the footnote of page 6.

### 3 PROBLEM STATEMENT AND BACKGROUND

Assume that we have $k$ pre-trained classifiers (or experts). Let $\mathcal{X}$ be the set of all possible inputs and $\mathcal{C}$ be the set of all classes. Our sequential prediction problem is a game played in rounds. Let us consider a stream of data $\{(x_t, c_t) \in \mathcal{X} \times \mathcal{C}\}_{t \geq 1}$, generated by an unknown mechanism. At each round $t$, $x_t$ together with all classifiers’ predictions $p_t \in \mathcal{C}^k$ is revealed to the learner and she selects one of the experts $I_t \in [k]$ and incurs a loss of 1 if that expert misclassifies $x_t$. Then, the learner decides whether to query the label $c_t$. If no query is made, then $c_t$ remains hidden, otherwise, the learner observes $\ell_t \in \{0, 1\}^k$ with $\ell_{t,i} = 1_{\{p_t(i) \neq c_t\}}$. Note that $I_t$ can only depend on the past inputs and the observed labels. The goal of the learner is to select $I_t$ in such a way that up to any round $T$, the total misclassifications she makes is close to the total mistakes of the best expert up to time $T$ in hindsight. This performance measure is formalized as the regret of the learner,

$$R_T = \sum_{t=1}^T \ell_{t,I_t} - \min_{i \in [k]} \sum_{t=1}^T \ell_{t,i}.$$ 

A prediction strategy satisfying $\limsup_{t \to \infty} R_t/t \leq 0$, is called a no-regret algorithm.

If the stream is generated by sampling $(x_t, c_t)$ i.i.d. from a fixed distribution, it is called a stochastic stream, otherwise we call it adversarial, as if an (oblivious) adversary has chosen the stream for the learner. It is known [21] that if the learner utilizes a deterministic strategy, she can be forced by the adversary to have linear regret. Hence, the learner should randomize and select $I_t$ according to a distribution over the experts $w_t \in \Delta_k$. Intuitively, $w_t$ reflects how good the learner thinks the experts are. In this case, one is interested in the expected regret $\mathbb{E}[R_T]$, where the expectation is w.r.t. the (possible) randomness in the stream, as well as the randomness of the learner.
Algorithm 1: Model Picker

Set $\hat{L}_{0,i} = 0$ for all $i \in [k]$
for $t = 1, 2, \ldots$ do
  \[ \eta_t := \sqrt{\log k/(2t)} \]
  Compute the distribution $w_t$ over models, with $w_{t,i} \propto \exp\{-\eta_t \hat{L}_{t-1,i}\}$
  Get predictions $p_t$ of models for the observed data instance $x_t$
  Recommend $\pi_t := \arg \max_{i \in [k]} w_{t,i}$ as the best model up to round $t$
  Sample $I_t \sim w_t$ and output $p_t(I_t)$ as the predicted label for this instance
  Compute $q_t$ as in (1) and sample $Q_t \sim \text{Ber}(q_t)$
  if $Q_t = 1$ then
    Query the true label $c_t$
    $\hat{L}_{t,i} = \hat{L}_{t-1,i} + \frac{1}{q_t} \mathbb{I}_{\{p_t,i \neq c_t\}}, \forall i \in [k]$
  end
end

On top of the preceding task, it is often desirable that at each round $t$, the learner recommends an expert $\pi_t$ as the best expert so far. This recommendation is suited for model selection tasks, where one needs not only the predictions per round, but also a recommendation about which classifier is the best one. We measure the quality of $\pi_t$ in two ways: the probability of returning the true best model of the stream so far (identification probability), and the gap between the accuracies of the recommended model and the best one (accuracy gap). The choice of measure depends on the application: if one is interested only in identifying the best model, then the first measure, and if one just cares about getting a model that has an accuracy close to the best classifier, then the second measure is more important.

4 ALGORITHM AND ANALYSIS

4.1 The Algorithm

At any round $t$, our algorithm, based on the predictions $p_t$ and current distribution $w_t$, decides to query the label with probability $q_t$ (to be determined later). Let $Q_t \in \{0, 1\}$ be the indicator of querying ($Q_t = 1$ if we query and $Q_t = 0$ otherwise). Our algorithm then constructs an unbiased estimate $\hat{\ell}_t = \ell_t/q_t : Q_t$ of the loss $\ell_t$.

Query Probability Instead of observing $c_t$ with a constant probability (as done in [12]), we adaptively set this probability according to the predictions $p_t$ and our current distribution over the models $w_t$. Notice that based on the predictions, we know that the true loss vector is among $\{\ell^{(c)}_t : c \in C\}$, where $\ell^{(c)}_t$ is simply the hypothetical loss vector if the true label was $c$. We define $v(p_t, w_t) = \max_{c \in C} \mathbb{V}_{I_t \sim w_t} \ell^{(c)}_t = \max_{c \in C} \langle w_t, \ell^{(c)}_t \rangle(1 - \langle w_t, \ell^{(c)}_t \rangle)$ to be the maximum possible

Indeed, the estimated loss is not unbiased when $\ell_{t,i} = 1$ for all $i \in [k]$. We will, however, remove the rounds where the loss is either all ones or all zeros.
variance among different possible losses with respect to the distribution \( w_t \), and set

\[
q_t = \begin{cases} 
    v(p_t, w_t) \lor \eta_t & \text{if } v(p_t, w_t) \neq 0 \\
    0 & \text{otherwise}
\end{cases}
\]  

(1)

When \( v(p_t, w_t) \) is not zero, as seen above, we utilize a lower bound on \( q_t \) to prevent unboundedness issues. This lower bound, however, decreases over time.

The intuition behind the definition of \( v(p_t, w_t) \) is as follows. Hypothetically, if the true label is \( c \) and we observe it, the distribution \( w_t \) over the models would be updated to \( w_t + 1 \) according to the loss \( \ell(c)_t \). If we miss this update, as we show in Appendix A.1, the amount of regret we accumulate (due to not updating \( w_t \) to \( w_t + 1 \)) is proportional to the variance of \( \ell(c)_t \). Hence, the maximum variance among all hypothetical losses is a measure of the importance of the instance at hand, and we use this value as our query probability. Note that, e.g., if all models make the same prediction, observing the true label has no effect on the regret, and this behaviour is also reflected in (1), as it would be equal to zero in this case.

In what follows, we first tackle the general case of adversarial streams and prove bounds on regret, number of queries, and accuracy gap of Model Picker. We then strengthen our results for the stochastic setting and give improved bounds as well as a bound for the identification probability. All the omitted proofs can be found in Appendix A.

4.2 Guarantees for Adversarial Streams

We first prove that our algorithm has no regret. It is known (c.f. [11]) that the regret of any online algorithm that observes all of the labels, is at least \( \Omega(\sqrt{T \log k}) \). Our regret bound matches this lower bound up to constants, even though we do not see all the labels.

**Theorem 4.1** (Regret Bound). The expected regret of Algorithm 1 is bounded above by

\[
E[R_T] \leq 2\sqrt{2T \log k}.
\]

Compared to [12], our regret bound is smaller: they prove that for a fixed query probability \( \varepsilon \), the regret is bounded by \( \sqrt{2T \log k/\varepsilon} \), and for getting a regret of \( O(\sqrt{T \log k}) \) one has to set \( \varepsilon \) to be a constant. However, in our regret bound there are no additional terms, as the probability of querying is adapted to the situation.

**Proof of Theorem 4.1.** First note that the expected regret of the algorithm for the loss sequence \( \{\ell_t\} \) is less than the expected regret for \( \{\hat{\ell}_t\} \): the reason is that \( \hat{\ell}_t \) is an unbiased estimate of \( \ell_t \) and

\[
E[\min_{i \in [k]} \hat{L}_{T,i}] \leq \min_i E[\hat{L}_{T,i}] \leq \min_i L_{T,i}.
\]

Hence, it suffices to bound the expected regret for \( \{\hat{\ell}_t\} \). Let \( m_t = -\frac{1}{m} \log(w_t, e^{-\eta \hat{\ell}_t}) \) be the mix loss and \( M_t := \sum_{s \leq t} m_s \). For brevity, we write \( \hat{L}_{T,*} := \min_{i \in [k]} L_{T,i} \). The regret can be decomposed as

\[
R_T = M_T - \hat{L}_{T,*} + \sum_{t=1}^T \left[ \langle w_t, \hat{\ell}_t \rangle - m_t \right].
\]

(2)

Bounding \( M_T \) is standard and by Lemma A.2 we have \( M_T \leq \hat{L}_{T,*} + \log k/\eta T \). For the second term in the regret decomposition we prove the following lemma, which heavily relies on our
Lemma 4.1. For any $\eta > 0$ it holds that

$$\mathbb{E} \left[ \log \langle w_t, e^{-\eta \hat{\ell}_t} \rangle + \eta \langle w_t, \hat{\ell}_t \rangle \right] \leq \eta^2.$$  

Proof. If the predictions are all the same, there is nothing to prove, as $\hat{\ell}_t \equiv 0$ and the expectation vanishes.

Let $w_c = \sum_{i: y_{t,i} = c} w_{t,i}$ and set $c^*$ to be the true label of this round. We can then rewrite $q_t$ as $q_t = \max_{c \in C} w_c (1 - w_c) / \eta$. Observe that

$$\log \langle w_t, e^{-\eta \hat{\ell}_t} \rangle = \log \left[ (1 - w_{c^*}) \exp \left\{ -\frac{1}{q_t} Q_t \right\} + w_{c^*} \right].$$

It is clear that $\mathbb{E} \langle w_t, \hat{\ell}_t \rangle = \langle w_t, \ell_t \rangle = 1 - w_{c^*}$. Hence, the expected value in the lemma is equal to

$$\eta (1 - w_{c^*}) + q_t \cdot \log[(1 - w_{c^*}) \exp\{-\eta/q_t\} + w_{c^*}]. \quad (3)$$

Our desired result follows from Lemma A.4 by setting $x = 1 - w_{c^*}$ and noticing that $q_t \geq w_{c^*} (1 - w_{c^*}) / \eta$. \hfill $\Box$

Our next result concerns the number of queries. We show that in the adversarial setting, this number depends linearly on the total mistakes of the best model (not taking into account rounds where all models make the same mistake). For example, in the case that the best model is perfect, the query count is of $O(\sqrt{T})$.

Theorem 4.2 (Number of Queries). Assume that in every round there were at least two models that disagree. Also assume that the total number of mistakes of the best model satisfies $L_{T,*} \leq (|C|-1)T / \eta + \varepsilon T$ for some $\varepsilon > 0$. Then, for $T \geq 4 \log k / \varepsilon^2$ the expected number of queries we make up to round $T$ is at most

$$\mathbb{E} \left[ \sum_{t=1}^{T} q_t \right] \leq 5\sqrt{T} \log k + 2L_{T,*}.$$  

Proof Sketch. The main idea is to relate the number of updates to the regret. First we bound $q_t$ from above by $\eta + \sum_{c \in C} w_{t,c} (1 - w_{t,c})$, as maximum is smaller than the sum. Then, using concavity of $a(1-a)$ and Jensen’s inequality we further bound the sum over classes by $r_t \left( 2 - \frac{|C|}{|C|-1} r_t \right)$, where $r_t = w_{c^*} = \langle w_t, \ell_t \rangle$. The proof finishes by summing over $t$ and carefully invoking Jensen’s inequality again. \hfill $\Box$

Remark 1. If all the models are bad, that is, if $L_{T,*} \approx T$, then our algorithm will query a lot of times, and the bound above is not loose. A simple example is illustrated in Appendix B. Better bounds on the number of queries are possible with more assumptions on the stream, e.g., when the stream is stochastic.

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3The attentive reader familiar with OMD/FTRL might have realized that the proof deviates from the usual proofs. In a nutshell, if we consider a general regularizer, following the usual proofs, one has to bound the stability of the algorithm, which boils down to bounding $\|\hat{\ell}_t\|_\eta^2$, by a constant, where $\| \cdot \|_\ell$ is the local norm at round $t$ induced by the inverse Hessian of the regularizer. As $\hat{\ell}_t \in [0,1/\eta]$, it can scale up to $O(\sqrt{T})$ and there is no trivial way to bound the norm, as the norms are equivalent in $\mathbb{R}^k$.
We now consider the quality of Model Picker’s recommendations for model selection. In the full generality of the adversarial setting, one cannot say much about the identification probability. However, if we restrict the adversary and assume that after some round $t_0$, the cumulative losses that the models incur start to diverge and keep a minimal gap of $\Delta$, we can give a sharp lower bound on the identification probability, as well as a stronger bound on accuracy gap.

**Theorem 4.3 (Accuracy Gap).** Assume $\tau \in [t]$ is selected uniformly at random and consider Algorithm 1 with recommendation $\pi_t = I_\tau$. Then, to reach an expected accuracy gap of at most $\varepsilon$, it is enough to have a stream length at least

$$T \geq 8 \cdot \log k / \varepsilon^2.$$  

Moreover, if the adversary is restricted, in the sense that for all $t \geq t_0$, $L_{t,j} \geq L_{t,i^*} + \Delta t$ for all $j \neq i^*$, one needs to have

$$T \geq \min \left\{ 31 \cdot \frac{\log k}{\Delta^2 \log^2(\frac{1}{\varepsilon})}, t_0 \right\},$$

while recommending $\pi_t = \arg\max_{i \in [k]} w_{t,i}$.

The proof of the first part is based on our regret bound and is standard. The second part is a simple corollary of Theorem 4.4. The difference between the two guarantees is twofold: while the first guarantee is instance independent, its dependence on $1/\varepsilon$ is quadratic. However, the second guarantee comes with poly-logarithmic dependence on $1/\varepsilon$, but with a (possibly large) instance dependent constant $1/\Delta^4$.

**Theorem 4.4 (Identification Probability).** In the restricted adversarial setting, the probability that we misidentify the best model at round $T \geq t_0$ is at most

$$\Pr \{ \pi_T \neq i^* \} \leq k \cdot e^{-0.18 \Delta^2 \sqrt{T \log k}}.$$  

This theorem, together with Theorem 4.6 below, clearly shows why Model Picker is successful in model selection tasks, as the probability of misidentifying the best model decreases (close to) exponentially fast, even if the stream is (restricted) adversarial. The proof is similar to Theorem 4.6 and is based on martingale inequalities.

### 4.3 Guarantees for Stochastic Streams

In this section, we assume that the stream is i.i.d. and provide stronger results. Let $i^* \in [k]$ be the model with the highest expected accuracy, and define $\Delta_j = \mathbb{E} [\ell_{t,j} - \ell_{t,i^*}]$ for all $j \in [k]$ to be the gap between the accuracies of model $j$ and the best model. Also define $\theta_j = \Pr \{ \ell_{t,j} \neq \ell_{t,i^*} \}$ to be the probability that exactly one of $j$ and $i^*$ correctly classify a sample. Define

$$\lambda = \min_{j \in [k] \setminus \{i^*\}} \frac{\Delta^2_j}{\theta_j}.$$  

Intuitively, $\lambda$ measures the hardness of the instance for our algorithm. Set $\Delta = \min_{j \neq i^*} \Delta_j$ and assume that $\Delta > 0$, that is, there is a unique best model. To simplify the exposition, we always assume, w.l.o.g., that at in all rounds at least two models disagree, as it is clear that the rounds in which all models agree do not contribute to the regret or to the number of queries. Also the pseudo-regret is defined as $R_t = \mathbb{E} \sum_{t} \langle w_t, \ell_t \rangle - T \Delta$.

We first improve Theorem 4.2, and shows on average Model Picker asks $O(\sqrt{T \log k} \cdot |C| / \Delta)$ labels. The dependence on $1/\Delta$ has the following intuition: like a coin with bias $\Delta$, it takes on average $1/\Delta$ rounds to observe an instance where the best model performs better than the rest. The bound shows that Model Picker needs no more than $O(\sqrt{T \log k})$ of these instances to build up sufficient confidence in the best model.
Theorem 4.5 (Number of Queries). The expected number of queries up to round $T$ is bounded by

$$E\left[\sum_{t=1}^{T} q_t\right] \leq \sqrt{2T\log(1 + 4k|C|)}.$$  

Proof. First, notice that the expected regret can be lower bounded by the pseudo-regret $R_T$. Our adversarial regret bound (Theorem 4.1) implies that

$$\sum_{t=1}^{T} (1 - w_{t,i^*})\Delta \leq R_T \leq E[R_T] \leq 2\sqrt{2T\log k}.$$  

Hence, $\sum_{t=1}^{T} (1 - w_{t,i^*}) \leq \frac{2\sqrt{2T\log k}}{\Delta}$. This means that $w_{t,i^*} \geq \frac{1}{2}$ most of the times: if $N$ is the number of rounds such that $w_{t,i^*} \leq \frac{1}{2}$, we have

$$\frac{1}{2}N \leq E\sum_{t=1}^{T} (1 - w_{t,i^*}) \leq \frac{2\sqrt{2T\log k}}{\Delta}.$$  

Now, by the definition of $q_t$, we have $q_t \leq \eta_t + \sum_{c \in C} w_{t,c}(1 - w_{t,c})$. For a class $c$ that is present among the models predictions at round $t$, we can write $w_{t,c}(1 - w_{t,c}) = (w_{t,i^*} + a) \cdot b$, for some $a, b \geq 0$ with $b \leq 1 - w_{t,i^*}$. When $w_{t,i^*} \geq \frac{1}{2}$, we have $q_t \leq b \leq 1 - w_{t,i^*}$. If $w_{t,i^*} \leq \frac{1}{2}$ we bound $q_t$ by $\frac{1}{4}$. Using the bound on $N$, we finish the proof. \qed

The next three results are parallel to the ones in the previous section. By adopting careful martingale arguments, we first show that the probability of misidentifying the best model decreases (close to) exponentially, with rate depending on $\lambda$.

Theorem 4.6 (Identification Probability). For $T > 2\log k$, the probability that we misidentify the best model at round $T$ is at most

$$\Pr\{\pi_T \neq i^*\} \leq k \cdot e^{-0.18\lambda \sqrt{T\log k}}.$$  

Proof Sketch. Notice that $\xi_t = \Delta_j - \hat{\ell}_{t,j} + \hat{\ell}_{t,i^*}$ is a martingale difference sequence. Using a variation of Freedman’s inequality for martingales and a careful analysis, one arrives at the theorem. \qed

Bounds on accuracy gap follow easily. The idea is that by Theorem 4.6, the best arm is always recommended, except for a constant number of rounds.

Theorem 4.7 (Accuracy Gap). For the recommendation $\pi_T$ to reach an expected accuracy gap of at most $\epsilon$, it is enough for the stream to have length at least

$$T \geq 31 \cdot \frac{\log^2(k \max_i \Delta_i)}{\lambda^2 \log k} \log^2(\frac{1}{\epsilon}).$$  

To bound the regret, Theorem 4.1 is still applicable. Additionally, if one predicts according to $I_t = \pi_t$ (a.k.a. Follow The Leader strategy [21]), the following theorem shows that the regret is bounded by a constant.\footnote{In full information, when one observes all the labels, the FTL strategy fails to have the no-regret property in adversarial setting. However, it has been shown that it favors a constant regret bound in stochastic settings. We show that our algorithm has the same behaviour.}

Theorem 4.8 (Regret Bound). If in Algorithm 1 one sets $I_t = \pi_t$ for all $t$, then the pseudo-regret is of $O(1)$, and is at most

$$R_T \leq 62 \cdot \frac{k}{\lambda^2 \log k} \max_i \Delta_i.$$
Figure 1: Performance of Model Picker (M-Picker) and other adapted baselines on four datasets \{CIFAR-10, ImageNet, Drift, EmoContext\}. Model Picker is able to return the true best model with high probability, while querying up to 2.6× fewer labels than the best competing method.

5 EXPERIMENTS

We conduct an extensive set of experiments to demonstrate the practical performance of Model Picker for online model selection and online sequential label prediction. We first run experiments on common data sets where the examples come i.i.d. from a fixed data distribution. This setting allows us to empirically assess the performance in the stochastic setting. We then consider a more challenging scenario, where examples come from a drifting data distribution, which we treat as an adversarial stream. For both sets of experiments, we examine the capability of Model Picker and other baselines with the same labeling budget, upon having seen the entire stream of examples.

Datasets and Model Collection. We conduct our experiments using various models trained on common datasets used in the natural language processing domain, such as the SemEval 2019 dataset (EmoContext) for emotion detection [37] as well as the long-term gas sensor drift dataset (Drift) from the UCI Machine Learning Repository [45, 46], and on more complex datasets of natural images such as CIFAR-10 [33] and ImageNet [43]. These datasets cover a wide range of scale: CIFAR-10, EmoContext and Drift are of smaller scale while ImageNet is a large scale dataset. For each dataset, we collected a set of pre-trained models. We provide a detailed explanation on the characteristics of the datasets and our model collections in Appendix C.1.

For CIFAR-10, we trained 80 classifiers varying in machine learning model, architecture and parameter setting [33]. The ensemble contains models having accuracies between 55-92%. The size of the test set for CIFAR-10 dataset is 10000 images. The ImageNet dataset poses...
a 1000-class classification problem. We collected 102 image classifiers that are available on TensorFlow Hub [43]. The accuracy of these models is in the range 50-82%. For test sets, we use the whole official test set with 50,000 images. For the EMOContext dataset, we collected 8 pre-trained models that are the development history of a participant in SemEval 2019. The accuracy of models varies between 88-92% on the test set of size 5,509. Lastly, for the DRIFT dataset, we trained an SVM classifier on each of 9 batches of gas sensor data that were measured in different months. We use the last batch as a test set, which is of size 3,000. Due to the drift behaviour of sensor data among different time intervals, the accuracy of the models on the test set is relatively low, and lies between 25-60%. We further collect a set of pre-trained models on CIFAR-10 dataset whose accuracies are relatively low, i.e., ranging from 40 to 70%. We defer further analysis on this to the Appendix C.3.

Baselines To compare with existing methods, we implement variations of query-by-committee, namely vote entropy (Entropy) [14] and structural query by committee (S-QBC) [44], label efficient prediction (Efficient) [12], and importance weighted active learning (Importance) [7, 9], as described below and depicted in Figure 1.

Label Efficient Prediction/Passive Learning. We implement [12] by querying the label of each round randomly with a fixed probability $u_t = \varepsilon$. For a fair comparison, we restrict our interest merely to the data instances in which at least two models disagree, as others are non-informative in the ranking of models. In our evaluation, for having an expected number of $b$ queries in a stream of size $T$, we set the query probability to $\varepsilon = b/T$. Note that our way of setting $\varepsilon$ depends on the whole stream in order to make results comparable in terms of the number of queries, as we shall drop the non-informative samples first.

Query by Committee/Vote Entropy. We use [14] and adapt it to the streaming setting as a disagreement-based selective sampling baseline. Upon seeing each instance, we measure the disagreement between the model predictions $p_t$ to compute the query probability. In our implementation, we consider every pre-trained model as a committee member and use vote entropy [14] as the disagreement measure.

Structural Query by Committee. The (interactive) structural query by committee algorithm [44] is built upon the query-by-committee principle, and its sampling probability is specified via the disagreement between competing structures that are drawn from a posterior distribution $\pi_t$. After each new query, the posterior is updated as $\pi_t \propto \pi_{t-1} \exp(-\beta \ell_t)$, where $\beta$ is a fixed constant. In our implementation, at each time instance $t$, we draw two models $i$ and $j$ from $\pi_t$ and set the query probability to be the fraction of disagreement between $i$ and $j$ up to round $t$, that is,

$$u_t = \frac{1}{t} \sum_{s \leq t} I_{\{p_s(i) \neq p_s(j)\}}.$$

Importance Weighted Active Learning. We implemented the importance weighted active learning algorithm, introduced by [7] as well as its variant for efficient active learning [8, 9]. Among these two adaptations of importance weights, we only focus on the superior [7] in our empirical evaluation and leave the other to Appendix C.2.

It is crucial to note that none of the methods above are tailored for the task of ranking pre-trained models and, with the exception of [12], for sequential label prediction, yet we consider them as selective sampling baselines; see Appendix C.2 for further discussions about our baselines.

5.1 Experimental Setup

Evaluation Protocol and Tuning For a fair comparison, we focus on the following protocol. In order to mimic the streaming setting, we sequentially draw $T$ i.i.d. instances uniformly at random from the entire pool of test instances, then input it into each method, and call it a realization. For each realization we declare the model with the highest accuracy on that stream as the true winner of the stream.
For each realization and up to any round $t$, we declare the winner of Model Picker as $\arg\max_i x_{t,i}$, and of other methods as the one having the highest accuracy on the queried labels. Upon exhausting the stream ($t = T$), we evaluate the performance of each method based on the model that is returned. We realize this process many times to have an estimation of the expected performance of methods.

For comparing the methods under the same budget constraint, we tune the (hyper-)parameter of each method to query the same number of instances, and perform comparison of their average performance under various labeling budgets. For structural query by committee, we treat the posterior update probability $\beta$ as the hyperparameter. For query by committee, importance weighted approach and Model Picker, we introduce a hyperparameter $\beta$ to scale the sampling probability according to given labeling budget.\footnote{It is straightforward to see that by scaling the value of $q(p_t, w_t)$ by some constant, one still gets similar theoretical results. The regret bounds, as well as the bounds on the confidence and accuracy gap will be scaled accordingly.} We perform hyperparameter selection via a grid search. The hyperparameters used for each budget, together with a large range of hyperparameters and their respective budgets can be found in Appendix C.4.

Performance Metrics For a given labeling budget, we consider the following key quantities as performance measures: Regret, for a fixed labeling budget, Accuracy gap between the returned model and the true best model, and Identification Probability, the fraction of realizations that methods return the true best model.

Scaling and Computation Cost We conduct our experiments on different stream sizes. We choose sizes of 5000, 10000, 1000 and 2500 for CIFAR-10, ImageNet, EmoContext and Drift test sets, respectively. We implement Model Picker, along with all other baseline methods in Python. All the baseline methods combined, each realization takes between 1 second (for EmoContext) and 4 minutes (for ImageNet) when executed on a single CPU core. Model Picker alone takes between 75 milliseconds (for EmoContext) and 47 seconds (for ImageNet). For all datasets we run 500 independent realizations for each budget constraint. To improve overall runtime, we run the realizations in parallel over a cluster with 400 cores.

5.2 Experimental Results

We review our numerical results for each of the metrics we introduced earlier. We refer to Appendix C.3 for an extensive discussion of our findings. For each of our metrics, we observe the following:

Regret We measure the regret across all rounds and for those budgets where Model Picker returns the best model with high confidence. Namely, we set the budget to 1250, 1200, 130 and 1000 for the CIFAR-10, ImageNet, EmoContext and Drift datasets, respectively. The regret behaviour is shown in Figure 1(a). In all cases, the regret grows sub-linearly for all of the algorithms. The regret of our algorithm in all cases is smaller up to a factor of $1.3 \times$, which shows that Model Picker can be used for sequential label prediction tasks as well as model selection.

Accuracy Gap Next we consider the average accuracy gap over the realizations. Figure 1(b) shows that the accuracy gaps for Model Picker are much smaller than that of other adapted methods under the same budget constraints. Quantitatively, in both CIFAR-10 and ImageNet datasets, Model Picker achieves the same expected accuracy gap as Vote Entropy (Query by Committee) by querying nearly $2.5 \times$ less labels. For the Drift dataset, for instance, Model Picker returns a model that is within a 0.1%-neighborhood of the accuracy of best model after...
querying merely 11% of the entire stream of examples (when the budget is 270 for a stream of size 2500). Note that active learning over drifting data distribution is a very challenging task, and Label Efficient Prediction/Passive Learning is considered to be the strongest baseline [38]. Our experiments thus suggest that, even for small labeling budget, Model Picker returns a model whose accuracy is close to that of best model, if not the best. We conduct further numerical analysis on the accuracy of returned models over a large number of realizations in order to investigate if they are performing well with high probability. For the DRIFT, IMAGENET and CIFAR-10 datasets, for instance, in 90% of realizations Model Picker returns the true best model after querying merely 8%, 15% and 20% of the entire stream of examples, respectively. We defer to Appendix C.3 for a detailed analysis to demonstrate the robustness of our algorithm in ranking the models.

**Identification Probability** As illustrated in Figure 1(c), Model Picker achieves significant improvements of up to $2.6\times$ in labeling cost while returning the true best model, and requesting far fewer labels than other adapted methods. For CIFAR-10, IMAGENET, EMOCONTEXT and DRIFT datasets, Model Picker queries $2.5\times$, $2.5\times$, $1.2\times$ and $1.7\times$ fewer labels respectively than that of the best competing method (mainly, Vote Entropy) in order to reach confidence levels 95%, 97%, 92% and 97%, respectively. This shows that Model Picker is able to achieve the same identification probability as the adapted baselines at a much lower labeling cost.

## 6 CONCLUSIONS

We introduced an online active model selection approach to - Model Picker - selectively sample instances that are informative for ranking pre-trained models and sequentially predicting labels. Our framework is generic, easy to implement, and applies across various classification tasks. We derived theoretical guarantees and illustrate the effectiveness of our method on several real-world datasets.

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A.1 On the Choice of Query Probability

Here we elaborate on the discussion for (1). Let $w_t$ be the current distribution over experts, and $w_{t+1}^{(c)}$ be the hypothetical distribution over the experts having observed the loss if the true label is $c$.

First, according to [30, Lemma 1], the divergence $KL(w_t || w_{t+1}^{(c)})$ accumulates into the regret. This means that if we do not update $w_t$ accordingly, we miss this amount of information.
Second, the KL divergence between \( w_t \) and \( w_{t+1}^{(c)} \) computes
\[
\text{KL}(w_t \| w_{t+1}) = \eta r + \log(re^{-\eta} + 1 - r),
\]
where \( r = \langle w_t, \ell_t^{(c)} \rangle \). By Höffdings inequality, one can show that this quantity is between 0 and \( \eta^2/8 \). As the variance is between 0 and \( 1/4 \), it makes sense to scale the KL divergence to the range \([0, 1/4]\), by multiplying it by \( 2/\eta^2 \). The following lemma completes the comparison promised in Section 4.1. We drop the subscript \( t \) for readability.

**Lemma A.1.** For a distribution over the experts \( w \) and a fixed \( \ell \in \{0, 1\}^k \), define \( w_+ \propto w - \eta \ell \). Then
\[
\left| \frac{1}{2} \sum_{A \sim w} \frac{2}{\eta^2} \text{KL}(w \| w_+) \right| < \frac{1}{18\sqrt{3}} \eta + O(\eta^2).
\]

**Proof.** Define \( r = \langle w, \ell \rangle \). We have that
\[
\text{KL}(w \| w_+) = \eta r + \log(re^{-\eta} + 1 - r) = \log \mathbb{E} e^{-\eta(X - \mathbb{E}X)},
\]
where \( X \) is a Bernoulli random variable with \( \mathbb{E}X = r \). Note that the equation above is the cumulant generating function of \( X \) and has the Taylor series
\[
\log \mathbb{E} e^{-\eta(X - \mathbb{E}X)} = \frac{\eta^2}{2} \text{Var}(X) + \frac{\eta^3}{6} \kappa_3 + O(\eta^4),
\]
where \( \kappa_3 \) is the third cumulant. Note that by the relation between cumulants of a Bernoulli random variable, we have
\[
\kappa_3 = r(1 - r) \frac{d}{dx} \kappa_2 = r(1 - r)(1 - 2r).
\]

Easy algebra finds that for \( r \in [0, 1] \), we have \( \kappa_3 \in [-1/6\sqrt{3}, 1/6\sqrt{3}] \).

Summing all up, we find
\[
\frac{2}{\eta^2} \text{KL}(w \| w_+) = \text{Var}(X) + \frac{\kappa_3}{3} \eta + O(\eta^2),
\]
and the result of the lemma follows. \( \square \)

### A.2 Mix Loss Properties

**Lemma A.2.** The cumulative mix loss \( M_T \) is bounded above by \( \tilde{L}_{T,*} + \log k / \eta_T \).

Before stating the proof, first we bring a standard lemma:

**Lemma A.3 ([16]).** The cumulative mix loss \( M_T \) has the following properties for constant learning rates (\( \eta_t \equiv \eta \) for all \( t \geq 1 \)):

(i) \( M_T = -\frac{1}{\eta} \log \sum_{i \in [k]} e^{-\eta \tilde{L}_{T,i}} + \frac{1}{\eta} \log k \),

(ii) Defining \( \tilde{L}_T := \min_{i \in [k]} \tilde{L}_{T,i} \), one has \( M_T \leq \tilde{L}_T + \frac{1}{\eta} \log k \).

Moreover, for any sequence of decaying learning rates \( \{\eta_t\}_{t \geq 1} \), let \( M_T(\{\eta_t\}) \) be the corresponding cumulative mix loss, and set \( M_T(\eta_T) \) be the cumulative mix loss for fixed learning rate \( \eta_T \). Then, it holds that \( M_T(\{\eta_t\}) \leq M_T(\eta_T) \).
Proof. Define $W_t = \sum_{i \in [k]} e^{-\eta L_{t,i}}$. For part (i) observe that

$$\langle w_t, e^{-\eta t} \rangle = \sum_{i \in [k]} \frac{e^{-\eta L_{t-1,i}} e^{-\eta t_i}}{W_{t-1}} = \frac{W_t}{W_{t-1}}.$$ 

Hence, $\log \langle w_t, e^{-\eta t} \rangle = -\eta m_t = \log W_t - \log W_{t-1}$, and $M_T = \frac{1}{T} (\log W_0 - \log W_T)$. Observing that $W_0 = k$ gives (i).

Noticing that $W_T \geq e^{-\eta L_T^0}$ easily implies (ii).

For the last part of the lemma, first we prove that $M_T$ for constant learning rate $\eta$ is nonincreasing in $\eta$. This is shown by looking at the derivative of $M_T$ with respect to $\eta$ which is equal to

$$\frac{1}{\eta^2} \log \sum_{i \in [k]} e^{-\eta L_{t,i}} \leq \frac{1}{\eta^2} \sum_{i \in [k]} \eta L_{t,i} - \frac{1}{\eta^2} \log k \leq 0,$$

as $\log \sum_{i \in [k]} e^{-\eta L_{t,i}} \leq \log k$.

Now we can prove the last part of the lemma.

$$\sum_{t=1}^T m_t(\{\eta_t\}) = \sum_{t=1}^T M_t(\eta_t) - M_{t-1}(\eta_t) \leq \sum_{t=1}^T M_t(\eta_t) - M_{t-1}(\eta_{t-1}) \leq M_T(\eta_T). \quad \square$$

Proof of Lemma A.2. By the lemma above, we see that $M_T \leq M_T(\eta_T) \leq \tilde{L}_{T,*} + \frac{\log k}{\eta^2}$, where we lower bounded the sum over the models by the one that corresponds to $\tilde{L}_{T,*}$. \quad \square

A.3 Lemmas for Regret Bound

Lemma A.4. For all $x \in (0, 1)$ and all $\eta \in (0, 1]$, defining $u = x(1-x) \lor \eta$, one has

$$f(x, u, \eta) := \eta x + u \log \left[ x e^{-\eta x} + 1 - x \right] \leq \eta^2.$$ 

Moreover, for fixed $x$ and $\eta$, $f(x, u, \eta)$ is decreasing in $u$ for $u \geq \eta$.

Proof. Note that the value of the LHS and RHS agree when $\eta = 0$, so we have to prove that for all $\eta \geq 0$, the derivative of the LHS is at most $\eta$. Fix some $x \in (0, 1)$. We prove this fact in two cases:

Case where $\eta \leq x(1-x)$. In this case, $u = x(1-x)$. For brevity, define $y := \frac{\eta}{x(1-x)}$. The derivative of $f$ with respect to $\eta$ becomes

$$\frac{(e^y - 1)x(1-x)}{e^y(1-x) + x},$$

and we are left with proving

$$\frac{(e^y - 1)}{e^y(1-x) + x} \leq 2y$$

As $0 < y \leq 1$, we have that $1 + y + y^2/2 \leq e^y \leq 1 + y + (e - 2)y^2$. Replacing these bounds in the equation above leaves us with proving that

$$\frac{1 + (e-2)y}{1 + (y + y^2/2)(1-x)} \leq 2.$$
For a fixed $y$, the left hand side is increasing in $x$, and hence, it is enough to prove that
\[ 1 + (e - 2)y \leq 2, \]
but this is true as $y \leq 1$ and $e - 1 < 2$. Thus, we are done with the proof of this case.

Case where $y > x(1 - x)$. In this case, $u = \eta$, and $f(x, \eta) = \eta x + \eta \log[xe^{-1} + 1 - x]$. To prove the claim, we have to show that $x + \log[xe^{-1} + 1 - x] \leq \eta$, or, as the left hand side does not depend on $\eta$, we shall prove
\[ x + \log[xe^{-1} + 1 - x] \leq x(1 - x), \]
or, equivalently,
\[ 1 - (1 - e^{-1})x \leq e^{-x^2}, \]
which is proven in Lemma A.5. Thus, in both cases, we have proved our inequality and we are done with the proof of the first part of lemma.

We now prove the monotonicity of $f$ with respect to $u$. For that, we show the derivative of $f$ with respect to $u$ is nonpositive. The derivative computes
\[ f'(u) = \log[xe^{-u}\eta + 1 - x] + \frac{xe^{-u}\eta/u}{xe^{-u} + 1 - x}. \]
Define $a = \eta/u$. The equation above is zero for $a = 0$. So it suffices to show that the derivative of above is nonpositive for $0 \leq a \leq 1$. Computing the derivative w.r.t. $a$ and setting it less than 0 is equivalent to
\[ xe^{-a} + 1 - x - xe^{-a} \geq 0, \]
which is true. Hence, we are done.

**Lemma A.5.** For all $x \in [0, 1]$ one has $1 - (1 - e^{-1})x \leq \exp(-x^2)$.

**Proof.** Note that $\exp(-x^2)$ is concave on $[0, \sqrt{1/2}]$ and convex on $[\sqrt{1/2}, 1]$. Also at $x = 0$ and $x = 1$, both sides are equal. Hence, we just have to show that at $x = \sqrt{1/2}$, the right hand side is bigger than the left hand side, which automatically shows the inequality for $x \in [0, \sqrt{1/2}]$, and we have to show that the derivative of the right hand side is smaller than the left hand side at $x = 1$, which automatically shows the inequality for the other half of the interval, as $\exp(-x^2)$ is convex there. For the first part, evaluate
\[ \exp(-1/2) - 1 + (1 - 1/e)\sqrt{1/2} = (1 - 1/\sqrt{e})\left(\frac{1}{\sqrt{2}} (1 + \frac{1}{\sqrt{e}}) - 1\right) \geq 0. \]
For the second part, note that $\frac{d}{dx} \exp(-x^2) = -2x \exp(-x^2)$, and at $x = 1$ it is equal to
\[ -2/e \leq -(1 - 1/e), \]
as $3/e > 1$. Hence, we are done.

**A.4 Proof of Theorem 4.2**

**Proof.** We assume that at all rounds we have $u_t > 0$, as there is no label request on the rounds that all models predict the same. First observe that
\[ \mathbb{E} \sum_{t=1}^T u_t \leq \mathbb{E} \left\{ \sum_{t=1}^T \eta_t + \sum_{c \in C} x_{t,c}(1 - x_{t,c}) \right\}, \]
as maximum of positive numbers is less than their sum. Next, at round $t$ suppose that the true label is $c_t$. As $x(1 - x)$ is concave and $\sum_{c \neq c_t} x_{t,c} = 1 - x_{t,c_t} = \langle \omega_t, \ell_t \rangle =: r_t$, using Jensen’s inequality we have

$$
\sum_{c \in C} x_{t,c}(1 - x_{t,c}) = x_{t,c_t}(1 - x_{t,c_t}) + \sum_{c \neq c_t} x_{t,c}(1 - x_{t,c}) \\
\leq x_{t,c_t}(1 - x_{t,c_t}) + (1 - x_{t,c_t}) \left( 1 - \frac{1 - x_{t,c_t}}{|C| - 1} \right) \\
= r_t \left( 2 - \frac{|C|}{|C| - 1} r_t \right).
$$

Using Jensen now for the concave function $x(2 - \frac{|C|}{|C| - 1} x)$, we get

$$
\sum_{t=1}^{T} \sum_{c \in C} x_{t,c}(1 - x_{t,c}) \leq T \cdot \left( \sum_{t=1}^{T} r_t \right) \left( 2 - \frac{|C|}{|C| - 1} \sum_{t=1}^{T} r_t \right).
$$

Now observe that if the expected total loss of the best model is $L^*$, by our regret bound in Theorem 4.1 we have

$$
\mathbb{E} \sum_{t=1}^{T} r_t \leq 2\sqrt{2T \log k} + L^*.
$$

Also note that for $x \leq \frac{|C|-1}{|C|-2}$, the function $x(2 - \frac{|C|}{|C|-1} x)$ is increasing. Hence, for large enough $T$ (as described in the theorem), $\frac{1}{t} \sum_{t=1}^{T} r_t \leq \frac{|C|-1}{|C|-2}$, and we have

$$
\mathbb{E} \sum_{t=1}^{T} x_{t,c}(1 - x_{t,c}) \leq \left( 2\sqrt{2T \log k} + L^* \right) \cdot \left( 2 - \frac{|C|}{|C|-1} \left( 2\sqrt{2 \log k/T + L^*/T} \right) \right).
$$

Noting that $\sum_{t} \eta_t \leq 2\sqrt{T \log k}$, one obtains the result.

\[\square\]

### A.5 Proof of Theorem 4.6

**Proof.** First, we remind the following martingale inequality, which is an improved version of McDiarmid’s:

**Lemma A.6 ([36]).** Let $\xi_1, \ldots, \xi_T$ be a martingale difference sequence with respect to the filtration $\{\mathcal{F}_t\}_{t \leq T}$, where each $\xi_t$ is integrable and bounded. Let $M_t := \sum_{s \leq t} \xi_s$ be the associated martingale. Define $\nu_T = \sum_{t \leq T} \mathbb{E} [\xi_t^2 | \mathcal{F}_{t-1}]$ and $c_T = \max_{t \leq T} \xi_t$. Then for any $\beta, \nu, c > 0$,

$$
\Pr \left\{ \left( M_T \geq \sqrt{2 \nu \beta T} + \frac{1}{3} c \beta T \right) \wedge (\nu_T \leq \nu) \wedge (c_T \leq c) \right\} \leq e^{-\beta T}.
$$

Remember that the weight of model $i$ at the end of round $t$ is proportional to $\exp\{-\eta t + 1\tilde{L}_t[i]\}$. Hence, identifying the best model $i^*$ after round $t$ reduces to the fact that $\tilde{L}_{t,i^*} = \min_{i \in [k]} \tilde{L}_{t,i}$. The probability of this event not happening can be bounded by a union bound on the models:

$$
\Pr \left\{ \exists j \neq i^* : \tilde{L}_{t,i^*} \geq \tilde{L}_{t,j} \right\} \leq \sum_{j \neq i^*} \Pr \left\{ \tilde{L}_{t,i^*} \geq \tilde{L}_{t,j} \right\} = \sum_{j \neq i^*} \Pr \left\{ \tilde{D}_{t,j} \leq 0 \right\}.
$$
where we define \( \tilde{D}_{t,j} = \tilde{L}_{t,j} \geq \tilde{L}_{t,i} \). From now on, we focus on a single model \( j \) and drop the index \( j \) from \( \Delta_j \) and \( \theta_j \). Set \( d_t := \ell_{t,j} - \ell_{t,i} \) and define
\[
\xi_t := \Delta - \frac{d_t Z_t}{u_t}.
\]

Note that \( E\{\xi_t \mid \mathcal{F}_{t-1}\} = E\{E\{\xi_t \mid \ell_t, \mathcal{F}_{t-1}\} \mid \mathcal{F}_{t-1}\} = 0 \). Moreover, the following holds:
\[
\xi_t \leq \Delta + \frac{1}{u_t} \leq \Delta + \eta_t^{-1}, \quad E\{\xi_t^2 \mid \mathcal{F}_{t-1}\} = E\{\frac{d_t^2}{u_t} \mid \mathcal{F}_{t-1}\} - \Delta^2 \leq \theta \eta_t^{-1} - \Delta^2.
\]

The sum of the conditional variances up to \( T \) satisfies
\[
\sum_{t=1}^{T} E\{\xi_t^2 \mid \mathcal{F}_{t-1}\} \leq T \eta_T^{-1} \theta - T \Delta^2 =: \nu
\]
Also, set \( c = \Delta + \eta_T^{-1} \). By lemma above we have
\[
Pr\left\{ \tilde{D}_{F,j} = \sum_{t=1}^{T} \frac{d_t Z_t}{u_t} \leq T \Delta - \sqrt{2T \nu \beta} - \frac{1}{3} c \beta T \right\} \leq e^{-\beta T}.
\]

We will find the largest \( \beta \) such that the right hand side of the inequality above becomes positive. As it is a quadratic polynomial in \( \sqrt{\beta} \), we should have that
\[
\sqrt{\beta} \leq \sqrt{\frac{2 \nu / T + \frac{4}{3} c \Delta - \sqrt{2 \nu / T}}{\frac{5}{3} c}}.
\]

Now we lower bound the right hand side, and write \( \gamma := \eta_T^{-1} \) for brevity:
\[
\sqrt{\frac{2 \nu / T + \frac{4}{3} c \Delta - \sqrt{2 \nu / T}}{\frac{5}{3} c}} \geq \frac{4 \Delta \sqrt{2 \nu / T}}{8 \nu / T + \frac{4}{3} c \Delta} \geq \frac{3 \Delta \sqrt{2 \nu / T}}{6 \nu / T + c \Delta} = \frac{3 \Delta \sqrt{2 (\gamma \theta - \Delta^2)}}{6 \gamma \theta - 5 \Delta^2 + \gamma \Delta} = \frac{\sqrt{2}}{\sqrt{\gamma}} \frac{\Delta \sqrt{\theta - \Delta^2 / \gamma}}{\sqrt{\gamma} \theta + \Delta - 5 \Delta^2 / \gamma} \geq \frac{3 \Delta \sqrt{\theta}}{\sqrt{\gamma} \theta} \text{ as } \theta \geq \Delta \text{ and } \Delta^2 / \gamma \leq \theta / 2.
\]

Hence, we conclude that setting
\[
\beta = 0.18 \sqrt{\log k} \frac{1}{\sqrt{2T}} \frac{\Delta^2}{\theta}
\]
gives the desired property. The proof follows by plugging in the value of \( \beta \) and taking a union bound over the experts.\[\square\]
A.6 Proof of Theorem 4.4

The proof is very much similar to Theorem 4.6. The difference is that the conditional variance is bounded above by
\[ \mathbb{E}\{\xi^2_t \mid \mathcal{F}_{t-1}\} \leq \eta_t^{-1} - \Delta^2, \]
and the rest of the proof follows by setting \( \theta = 1 \).

A.7 Proof of Theorem 4.3

The first bound is standard and can be found in [41]. The argument is completed by noting that the expected accuracy gap will be bounded by
\[ \frac{1}{T} \mathbb{E}[L_{T,i} - L_{T,i^*}] \leq \sqrt{\frac{8 \log k}{T}}, \]
and setting the right hand side less than \( \epsilon \).

For the second part, we use Theorem 4.4. With probability at most \( k \cdot e^{-0.18 \Delta^2 \sqrt{T \log k}} \), the recommended expert is not the best, for which its accuracy gap is at most 1, and otherwise, the best expert is returned, with accuracy gap 0. Combining the two gives the result.

A.8 Proof of Theorem 4.7

The proof is very similar to Theorem 4.3, with the difference that here one upper bounds the accuracy gap by \( \max_i \Delta_i \) instead of 1.

A.9 Proof of Theorem 4.8

First we prove a lemma that help us proving the theorem:

**Lemma A.7.** The expected number of times that the recommendation \( \pi_t \) is not the best model is a constant up to any round and is bounded by \( \frac{62k}{\lambda^2 \log k} \).

**Proof.** By Theorem 4.6, we know that the probability of not recommending the best model at round \( t \) is upper bounded by \( k \cdot e^{-0.18 \lambda^2 \sqrt{T \log k}} \). Using integral approximation, one finds that
\[ \sum_{t=1}^{\infty} e^{-a \sqrt{T}} \leq \frac{2}{a^2} \text{ for all } a > 0. \]
This gives
\[ \mathbb{E}\left( \sum_{t=1}^{\infty} 1_{(\pi_t \neq i^*)} \right) = \sum_{t=1}^{\infty} \mathbb{P}\{\pi_t \neq i^*\} \leq \sum_{t=1}^{\infty} k \cdot e^{-0.18 \lambda^2 \sqrt{T \log k}} \leq \frac{62k}{\lambda^2 \log k}. \]

Using the lemma, over \( T \) rounds, we make at most \( \frac{62k}{\lambda^2 \log k} \) mistakes, for which we get at most \( \max_i \Delta_i \) added to the regret, and in other rounds, we make no mistakes, hence no regrets on those rounds. Adding up gives the result.

B Example for Large Number of Updates

Consider a binary classification scenario with two models. Set the loss sequence to be \((1, 0), (0, 1), (1, 0), \ldots\), that is, on the odd rounds the second model is correct and on the even rounds, the first one. One can see that the probability of querying the label is always \( \frac{1}{2} \) near \( \frac{1}{4} \), as the models weights are always around \( \frac{1}{2} \). Hence, the total number of queries is linear.
C Experiments

C.1 Details on the Model Collections

- **CIFAR-10**: As image classification dataset, we train 80 models on CIFAR-10 dataset [33] varying in machine learning models (ranging from DenseNet, Resnet to VGG), architecture and parameter setting. The ensemble of models have accuracies between 55-92% on a test set consists of 10,000 instances. We enclose further details in the supplementary material.

- **ImageNet**: This dataset consists of 102 image classification models (ranging from ResNet, Inception to MobileNet) pre-trained on ImageNet that are available on TensorFlow Hub [43]. The accuracy of models occupy the range in 50-80%. For each model we obtain the ImageNet validation dataset with 50,000 data examples, and furthermore normalize and resize them according to expected input format for each model, and finally conduct inference on the given model to produce predicted labels.

| Dataset       | #Classes | #Instances | #Models | Accuracy of Models |
|---------------|----------|------------|---------|--------------------|
| CIFAR-10      | 10       | 10,000     | 80      | 55-92%             |
| ImageNet      | 1,000    | 50,000     | 102     | 50-80%             |
| Drift         | 6        | 3,000      | 9       | 25-60%             |
| EmoContext    | 4        | 5,509      | 8       | 88-92%             |
| CIFAR-10 (worse models) | 10       | 10,000     | 80      | 40-70%             |

- **Drift**: For the Drift dataset, we trained models on the gas sensor drift data that is collected over a course of three years. The dataset has ten batches, each collected in different months. We trained an SVM classifier on each of the batch but the last one, and use the last batch of size 3,000 as test set. Although each model has good training accuracy on the batch it is trained on, namely above 90%, their test accuracy on the test set lies between 25-60%. This is due to the drift behaviour of sensor data among different time intervals, and can be treated as the adversarial setting.

- **EmoContext**: This dataset consist of pretrained models that are the development history of a participant on EmoContext task in SemEval 2019 [37]. The task aims to detect emotions from text leveraging contextual information which is deemed challenging due to the lack of facial expressions and voice modulations. We treat each development as an individual pretrained model where development stages differ in various word representations including ELMo [32] and GloVe [31]. The dataset consists of 8 pre-trained models whose accuracy varies between 88-92% on the test set of size 5,509.

- **CIFAR-10 V2**: We also train a set of models with relatively lower accuracies on CIFAR-10 and call it CIFAR-10 V2. The sole purpose behind creating such a collection is to investigate the performance of Model Picker on a practical scenario like this. Using similar model architectures to that of CIFAR-10, the pretrained models have accuracy between 40-70% on a test set of size 10,000.

The properties of model collections for all datasets are depicted in Table 1 on page 23 and Figure 2.
C.2 Note on the Baselines

In this section, we provide further details on some of the adapted baseline methods, namely, query by committee (Entropy), importance weighted active learning (Importance) and efficient active learning (EfAL).

- **Query by Committee**: As indicated in Section 5, we adapt the query-by-committee paradigm proposed in [14] for model selection in the online setting. The query by committee method consist of two sub-strategies (a) ensemble learning, and (b) determining a maximal disagreement measure. The ensemble learning indicates how the committee is formed from the candidate classifiers. This step is crucial to make the disagreement measure more reliable while aiming to form a set of classifiers those with high accuracy. In literature, there exist many ensemble learning methods including [1, 29, 10, 17]. Most, if not all, of these methods are either designed for pool-based sampling or for cases where observed data is stored. Bagging predictors [10] proposes to improve performance of a single predictor by forming a committee from multiple versions of it, where the versions are trained on the bootstrap replicates of training data. This is followed by [1] where diverse ensembles are generated using bagging and boosting techniques [17]. These strategies focus on a setting where the observed data is stored as opposed to our setting. Another popular ensemble learning algorithm, Active-Decorate relies on the existence of artificial training data to form a diverse set of examples. In our setting, however, we assume neither storing of previously seen data nor availability of artificial data. In the online setting, however, one could benefit from the strategy introduced in [17]. Upon seeing the label $y_t$, the authors propose to update the belief on the models such that $\pi_t \propto \pi_{t-1} e^{\beta t}$. We note that, this update rule very closely resembles that of the structural query by committee, which we include in our numerical analysis. In fact, it is identical when both of $\beta$ are tuned to query budget $b$ amount of label in average over many realizations.

As a disagreement measure, popular choices include vote margin, vote entropy and KL divergence between the label distributions of each committee member and the consensus in [39]. We first note that the latter two are equivalent for 0-1 loss functions $\ell$. The former, vote margin is measured by the difference between the votes of most voted and second most voted label. We omit this in our analysis motivated by the preliminary observation on the success of entropy over the vote margin.

- **Importance Weighted Active Learning**: As indicated earlier, we implement the importance weighted active learning algorithm, introduced by [7]. Formally, upon seeing a new instance $x_t$, the algorithm computes a rejection threshold $\theta_t$ using sample complexity bounds, and update the hypothesis space $H_t$ to contain only the models whose weighted error is $\theta_t$ greater than weighted error of the current best model at time $t$. The sampling probability $u_t$ is set to $\max_{i,j \in H_t, c \in [C]} \tilde{\ell}_i(c) - \tilde{\ell}_j(c)$. We use 0-1 loss. Therefore, adaptation in our setting becomes making query decision based merely on the disagreement between the “surviving” hypotheses at
time $t$. That is, we query the label $y_t$ if and only if the ‘surviving’ classifiers at time $t$ disagree on the labeling of $x_t$.

- **Efficient Active Learning**: We adapt the efficient active learning algorithm presented by [8, 9]. In a manner similar to the importance weighted approach, the efficient active learning algorithm also uses the importance weighted framework. Upon receiving a new instance $x_t$, the algorithm measures the weighted error estimate between two competing models, and specifies a sampling probability based on a threshold that is a function of $\frac{C_0 \log k}{t}$ for some parameter $C_0 > 0$. If the gap between the estimated weighted errors of two competing models is below this threshold, then the label $y_t$ is queried. Otherwise, the algorithm computes the sampling probability $p_t$ that is roughly $\min \{1, O(1/G_k^2 + 1/G_k) \frac{C_0 \log k}{t}\}$. We refer to Algorithm 1 of [8] for further details. In our implementation, we consider the threshold parameter $C_0$ as hyperparameter and tune for efficient active learning algorithm to request amount of labels not exceeding the labelling budget $b$. However, as indicated in Figure 3, it underperforms the importance weighted active learning algorithm. However, it is crucial to emphasize again that these methods are meant to improve supervised training of classifiers instead of ranking of pretrained models. We include them in our comparison for the completeness.

![Figure 3: Comparison of importance weighted methods {IMPORTANCE, EFAL} and on the EmoContext and CIFAR-10 datasets](image)

### C.3 Extended Experimental Results

#### C.3.1 Robustness of Model Picker: A worst-case analysis

As aforementioned in Section 5, we conduct further numerical analysis on the quality of returned model to investigate the robustness of Model Picker, that is, if it performs well with high probability. Towards that, we compute the 90th percentile of accuracy gap (the maximum accuracy gap which 90% of realizations fall in) illustrated in Figure 4. For the DRIFT dataset, for instance, Model Picker returns the true best model after querying merely 8% of the labels (when the budget is 200 with a stream size of 2,500). For the CIFAR-10 and IMAGENET datasets, Model Picker returns a model that is within a 0.1%-neighborhood of the accuracy of best model after querying nearly 12% of the entire stream of examples whereas the best computing method achieves this after querying 24% of the same stream of examples. Moreover, Model Picker returns the true best model after querying 15% and 20% of the entire stream of examples, respectively. These results clearly demonstrate the robustness of Model Picker.

#### C.3.2 Performance of Model Picker on models with low accuracies

As mentioned in Section 5, we conduct another numerical analysis on the performance of Model Picker when pretrained models have relatively lower accuracies. Towards that, we train...
Figure 4: The worst-case analysis on the returned models: 90th percentile accuracy gap

80 models on CIFAR-10 varying in machine learning models and parameters. The accuracy of pretrained models line in 40-70% over a test set of of size 10 000. We compare the model selection methods over this new model collection by following the exact same procedure as in the Section 5. We use a stream size of 5000 and average the results over 500 realizations. Figure 5 summarize the comparison. When the accuracy of premodels are low, the query by committee algorithm expectedly underperforms as the disagreement measure becomes noisy under the existence of models with low accuracies. Model Picker, on the other hand, noticeably outperforms in returning the true best model as well as the ranking of the models (Figure 5). The regret analysis in Figure 5 suggests that the structural query by committee method maintains a low regret throughout the streaming process as well as for different labeling budgets, and very closely followed by Model Picker.

Figure 5: Performance evaluation of model selection methods on CIFAR-10 V2 dataset that consist of pre-trained models with low accuracies.

C.4 Hyperparameters

The hyperparameter tuning is performed via grid search. For each grid point, we run the experiment for 100 realizations and compute the average number of requests. The grid search was performed over the following search space:

- **CIFAR-10**: Model Picker: [0, 3000], Entropy: [0, 20], S-QBC: [0, 10], Importance: [0, 0.9], EFAl: [0, 1.5e-2]
- **ImageNet**: Model Picker: [0, 135], Entropy: [0, 22], S-QBC: [0, 20], Importance: [0, 1]
- **Drift**: Model Picker: [0, 60], Entropy: [0, 4], S-QBC: [0, 4], Importance: [0, 0.5]
- **EmoContext**: Model Picker: [0, 60], Entropy: [0, 4], S-QBC: [0, 4], Importance: [0, 0.5], EFAl: [0, 1e-2]
- **CIFAR-10 V2**: Model Picker: [0, 1000], Entropy: [0, 3], S-QBC: [0, 10], Importance: [0, 0.9], EFAl: [0, 1e-1]
with grid size of 250 where grid points are equally spaced. The respective number of requests for each grid point can be found in our supplementary material.

Remark that the amount of requests by Model Picker saturates when Model Picker reaches at a high identification probability. Therefore, the update probability is upscaled with a very high value such that Model Picker queries large number of requests and comparison to other methods for large budget constraints are made possible. Practically, this would not be required as Model Picker itself decides when to stop requesting labels. For example, when the update probability is upscaled by a factor of 11 for CIFAR-10 V2 dataset, the number of requests made by Model Picker is 3,800 labels, whereas an upsizing of 835 is used to enable Model Picker requests nearly 4,800 labels.