Introduction.—The Rayleigh limit of resolution ($\lambda/2$), when using a light field of wavelength $\lambda$ for imaging or for lithographic fabrication, primarily arises due to the choice of the optical design that is limited by diffraction. It has been shown that the ultimate limit of resolution is the Heisenberg limit and within its bounds, in principle, one can obtain sub-Rayleigh resolution [1]. In general, sub-Rayleigh resolution can be achieved by employing diffraction-free techniques. Several different avenues are being explored on widely different fronts for obtaining sub-Rayleigh resolution based on near-field optics [2], quantum entanglement [3], monitoring fluorescence from atoms or molecules [4], and atomic coherence effects [5–8]. Nanoscale spatial resolution is useful for applications to the fields of lithography [9], imaging [10], atom localization [11], high precision interferometry [12] and it offers precise spatial selectivity for atomic qubits necessary for quantum information processing [13].

We have shown recently that coherent population trapping (CPT) [7] can be used to localize atoms to subwavelength regimes [6]. The idea has been taken further by others to theoretically propose schemes for subwavelength microscopy [14], subwavelength patterning of Bose-Einstein condensates [15], nanoscale trapping potentials for atoms [16], and two dimensional localization by coupling the atom with two spatially dependent fields [17]. While the CPT based super-resolution techniques continue to be developed; we note that a popular technique that attains nanometer resolution is the confocal fluorescence microscopy that makes use of the saturation of a two level transition. In another well-known, Stimulated Emission Depletion (STED), method one cuts down emission from outer regions by using strong fields [4]. Fujita et al. [18] use the nonlinear relation between the excitation and fluorescence to demonstrate improved spatial resolution in three dimensions beyond the diffraction limit.

Thus the nanoscale resolution is becoming rather common in microscopic and other applications. The question is how to break this barrier. In this letter we show how CPT can be adopted to reach sub-nanoscale resolution. We use the physics behind CPT and Electromagnetically-Induced Transparency (EIT) [8]. The sharp dispersion offered by EIT allows unprecedented control over the group velocity of the probe field. To take advantage of this steep dispersion we propose the use of amplitude modulated probe field and show that, in the best case scenario, the resolution can be further improved by a factor of 20 beyond that offered by the unmodulated CPT.

We present a coherent population trapping based scheme to attain sub-nanoscale resolution for atom localization, microscopy and lithography. Our method uses three-level atoms coupled to amplitude modulated probe field and spatially dependent drive field. The modulation of the probe field allows us to tap into the steep dispersion normally associated with electromagnetically induced transparency and offers an avenue to attain sub-nanometer resolution using just optical fields. We illustrate application of the techniques to the area of microscopy and lithography and show how multilevel schemes offer the possibility of improving resolution further.

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the given Hamiltonian (1). Once the steady state is reached, such a system demonstrates coherent population trapping signified by the form of the steady state given by \( |\Psi\rangle = (\Omega_p |2\rangle - \Omega_c |3\rangle)/\Omega \), where \( \Omega = \sqrt{\Omega_p^2 + \Omega_c^2} \); this state does not evolve dynamically as \( \mathcal{H} |\Psi\rangle = 0 \). Thus, if the atom is initially prepared in the state \( |3\rangle \), it will end up in the state \( |\Psi\rangle \) at the steady state, as long as the two-photon resonance is maintained. As the population in the state \( |\Psi\rangle \) can not escape, it is termed as the trapping state and the phenomena is called coherent population trapping. As shown in our earlier paper [6], taking the drive field to be a standing wave field, \( \Omega_c(x) = \Omega_c \sin kx \), the atomic dynamics becomes position dependent and possesses smaller-than-wavelength \( (\lambda = 2\pi/k) \) features at the nodes of the standing wave field as witnessed by monitoring the population of the level \( |2\rangle \):

\[
\rho_{22} = \frac{1}{1 + |\Omega_c/\Omega_p|^2} = \frac{1}{1 + \mathcal{R} \sin^2 kx}. \tag{3}
\]

This function has the same structure as the transmission function of the Fabry-Perot cavity [19], where the ratio, \( \mathcal{R} = |\Omega_c|^2/|\Omega_p|^2 \), of the effective field intensities plays the role of the cavity finesse. The full width at half maximum (FWHM) of the curve is given by \( k\Delta x = 2/\sqrt{\mathcal{R}} \). In fig. 1 (b) we compare the feature-size measurable by the Rayleigh limit \( (\lambda/2) \) with the one measurable by the 2D version (see Fig. 5(a) for the corresponding levelscheme) of the CPT based scheme \( (\lambda/\pi\sqrt{\mathcal{R}}) \). For a moderate value of \( \mathcal{R} = 1000 \), CPT-based scheme offers resolution improvement by a factor of 50. Further improvement is possible by increasing the value of \( \mathcal{R} \).

**Full Amplitude Modulation.**—Now we consider full modulation of the probe field with simple standing-wave coupling field and once again monitor the population of level \( |2\rangle \), that is \( \rho_{22} \) as a measure of the point spersad function. The profiles of the coupling and modulated probe fields can be taken as \( \Omega_p(x,t) = \Omega_p(x) \cos vt \). Thus, the population of the state \( |2\rangle \) can be determined to be

\[
\rho_{22}(t) = \frac{|\Omega_p(x)|^2 \cos^2 vt}{|\Omega_p(x)|^2 \cos^2 vt + |\Omega_c(x)|^2} = \sum_{j=1}^{\infty} f_{2j} \cos (2jvt), \tag{4}
\]

where

\[
f_{2l} = \sum_{m=\ell}^{\infty} (-1)^{m-\ell} \frac{(\Omega_p(x) \Omega_c(x))^{2m}}{m!} \frac{(2m)!}{(m - \ell)!}. \tag{5}
\]

Some of the lower order terms are given by

\[
f_0 = \rho_{22}^{(0)} = 1 - \frac{1}{1 + |\Omega_p(x)/\Omega_c(x)|^2},
\]

\[
f_2 = \rho_{22}^{(1)} = \frac{2 + 4 [\Omega_c(x)/\Omega_p(x)]^2}{\sqrt{1 + |\Omega_p(x)/\Omega_c(x)|^2}} - 4 [\Omega_c(x)/\Omega_p(x)]^2,
\]

\[
f_4 = \rho_{22}^{(2)} = - \frac{2[1 + 8 |\Omega_c(x)/\Omega_p(x)|^2 (1 + |\Omega_c(x)/\Omega_p(x)|^2)]}{\sqrt{1 + |\Omega_p(x)/\Omega_c(x)|^2}} + 8 |\Omega_c(x)/\Omega_p(x)|^2 (1 + 2 |\Omega_c(x)/\Omega_p(x)|^2).
\]

The functions \( f_{2l} \), which are functions of the transverse coordinates \( x \) and also of \( y \) in the 2D case, are identified as the point spread functions of the proposed scheme for super-resolution. The FWHM of the point spersad function, which is a measure of how a point object is perceived by the method employed for microscopic detection, is a good measure of the resolution. We now consider the case where the drive field is a standing-wave, \( \Omega_c(x) = \Omega_c \sin(kx) \), and the probe field has no position dependence \( \Omega_p(x) = \Omega_p \). The corresponding point spread functions (6) are plotted in Fig. 2 (a), with the general result that the higher-order Fourier components offer better resolution. The scheme can be extended to two dimensions (see Fig. 5 (a) for the corresponding level scheme) to obtain the plot of the FWHM of the corresponding point spread functions in 2D as in Fig. 2 (b). Taking the Fourier transform of the measured fluorescence signal from level \( |2\rangle \) one can obtain the
Rabi frequencies can be written as:

\[ \rho \]

Reduce position dependence in the atomic dynamics. The field amplitude modulated with a perturbative harmonic component.

Perturbative Amplitude Modulation.—We conclude the paper by examining a more traditional method of performing modulation spectroscopy. In this case the probe field is amplitude modulated with a perturbative harmonic component \( \sin \nu t \) where \( \nu \) is the frequency of the harmonic term. Whereas the drive field is spatially modulated along the \( x \) direction such that it takes the familiar form of a standing wave field to introduce position dependence in the atomic dynamics. The field Rabi frequencies can be written as:

\[ \Omega_x(x) = \Omega_x \sin kx, \quad \Omega_p(t) = \Omega_p (1 + a \sin \nu t), \quad (a \ll 1). \]  

(7)

Using a power series expansion in \( a \) for the density matrix elements \( \rho_{ij} = \rho_{ij}^{(0)} + a \rho_{ij}^{(1)} + a^2 \rho_{ij}^{(2)} + \cdots \), the steady state solution of the density matrix equations of motion can be obtained. Once again the population of state \( |2 \rangle \) turns out to be position dependent and is given by:

\[ \rho_{22} = \left[ \frac{1}{1 + (\Omega_x/\Omega_p)^2} \right] + \left[ \frac{2(\Omega_x/\Omega_p)^2}{(1 + (\Omega_x/\Omega_p)^2)^2} \right] (a \sin \nu t) \]

with the beam profiles in one dimension. It is clear that the spatial profiles offered by CPT are much finer than the spatial modulation of the light beams. This scheme can be easily extended to two dimensions in a straightforward manner as the transverse beam profiles are naturally two dimensional. In the part (c) of the figure Fig. 3 we plot the FWHM as the size of the point spread function with the background showing the contour plot of the drive beam profile. The FWHM of the beam profile is too large to fit in the scale shown. When one monitors the second order term, in comparison with the unmodulated CPT results the resolution can be improved by a factor of about 3 and in comparison with the Rayleigh limit by a factor of about 150 for the chosen value of \( \mathcal{R} = 1000 \). Higher values of \( \mathcal{R} \), which is the ratio of the intensities of the drive and probe fields offers better resolution.
verse spatial profile of the beams.

New possibilities.— The essentially 1D model presented in Fig. 1 can be extended to 2D by taking advantage of the multilevel nature of atoms as shown in Fig. 5(a). Two standing waves in \( x \) and \( y \) directions induce sharp features in \( \rho_{33}(x,y) \) allowing sub-nanoscale localization in the \( x-y \) plane. Further, considering the lower level states \( |2\rangle \) and \( |3\rangle \) as hyperfine sublevels of a real atom with \( F \geq 2 \) one can obtain multiple \( \Lambda \) systems in a single atom that are simultaneously excited; such systems have been shown to be useful for generation of skyrmions in the context of BEC and Laguerre-Gaussian beam interaction [20]. In the case shown in Fig. 5(b) the result with unmodulated CPT gets modified to a function that can be approximated by the square of the Fabry-Perot transmission function (3) and hence is sharper by a factor of about 1.5 in the case of unmodulated CPT. Adding the probe field modulation on top of this CPT in the coupled \( \Lambda \) system one can, in principle, improve the resolution further. The ideas presented so far can also be extended to three dimensions by employing standing waves in the three directions much on the same lines as techniques for optical trapping of neutral atoms in 3D [21].

Conclusions.— We offer sub-nanoscale resolution by modulating probe fields in standard CPT. By modulating the CPT we take advantage of the dispersion accompanying sharp resonances in EIT that is closely related to the phenomena of CPT. The improvement in resolution obtained in comparison with the Rayleigh limit is by a factor of about 1000 in the best case scenario discussed here which gives sub-nanometer resolution with the use of optical fields.

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[1] J. M. Vigoureux and D. Courjon, App. Opt. 31, 3170 (1992).
[2] C. Girard and A. Dereux, Rep. Prog. Phys. 59, 657 (1996).
[3] A. N. Boto, P. Kok, D. S. Abrams, S. L. Abrams, C. P. Williams, and J. P. Dowling, Phys. Rev. Lett. 85, 2733 (2000).
[4] S. W. Hell, Science 316, 1153 (2007).
[5] E. Paspalakis and P. L. Knight, Phys. Rev. A 63, 065802 (2001); J. of Mod. Opt. 52, 1685 (2005).
[6] G. S. Agarwal and K. T. Kapale, J. Phys. B 39, 3437 (2006).
[7] E. Arimondo, in Progress in Optics, edited by E. Wolf (Elsevier Science B. V., Amsterdam, 1996), vol. XXXV, pp. 257–354; R. M. White and C. R. Stroud, Phys. Rev. A 14, 1498 (1976).
[8] S. E. Harris, Phys. Today 50, 36 (1997); M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. 77, 633 (2005).
[9] M. Kiffner, J. Evers, and M. S. Zubairy, Phys. Rev. Lett. 100, 073602 (2008); H. S. Park, S. K. Lee, and J. Y. Lee, Opt. Exp. 16, 21982 (2008).
[10] H. Li, V. A. Sautenkov, M. M. Kash, A. V. Sokolov, G. R. Welch, Y. V. Rostovtsev, M. S. Zubairy, and M. O. Scully, Phys. Rev. A 78, 013803 (2008).
[11] M. Sahrai, H. Tajalli, K. T. Kapale, and M. S. Zubairy, Phys. Rev. A 72, 03820 (2005); K. T. Kapale and M. S. Zubairy, Phys. Rev. A 73, 023813 (2006); M. Macovei, J. Evers, C. H. Keitel, and M. S. Zubairy, Phys. Rev. A 75, 033801 (2007).
[12] H. G. de Chatellus and J.-P. Pique, Opt. Lett. 34, 755 (2009).
[13] A. V. Gorshkov, L. Jiang, M. Greiner, P. Zoller, and M. D. Lukin, Phys. Rev. Lett. 100, 093005 (2008).
[14] D. D. Yavuz and N. A. Proite, Phys. Rev. A 76, 041802(R) (2007).
[15] J. Mompart, V. Ahufinger, and G. Birkl, Phys. Rev. A 79, 053638 (2009).
[16] D. D. Yavuz, N. A. Proite, and J. T. Green, Phys. Rev. A 79, 055401 (2009).
[17] L. Jin, H. Sun, Y. Niu, S. Jin, and S. Gong, J. of Mod. Opt. 56, 805 (2009).
[18] K. Fujita, M. Kobayashi, S. Kawano, M. Yamanaka, and S. Kawata, Phys. Rev. Lett. 99, 228105 (2007).
[19] M. Born and E. Wolf, Principles of Optics: Electromagnetic Theory of Propagation, Interference, and Diffraction of Light (Cambridge University Press, Cambridge, England, 1999).
[20] L. S. Leslie, A. Hansen, K. C. Wright, B. M. Deutsch, and N. P. Bigelow (2009), arxiv:0910.4918.
[21] H. J. Metcalf and P. van der Straten, Laser Cooling and Trapping (Springer, New York, NY, 2001).