The Physics of Galactic Winds Driven by Cosmic Rays I: Diffusion

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ABSTRACT
The physics of Cosmic ray (CR) transport remains a key uncertainty in assessing whether CRs can produce galaxy-scale outflows consistent with observations. In this paper, we elucidate the physics of CR-driven galactic winds for CR transport dominated by diffusion. A companion paper considers CR streaming. We use analytic estimates validated by time-dependent spherically-symmetric simulations to derive expressions for the mass-loss rate, momentum flux, and speed of CR-driven galactic winds, suitable for cosmological-scale or semi-analytic models of galaxy formation. For CR diffusion coefficients \( \kappa \ll r_0 c_t \) where \( r_0 \) is the base radius of the wind and \( c_t \) is the isothermal gas sound speed, the asymptotic wind energy flux is comparable to that supplied to CRs, and the outflow rapidly accelerates to supersonic speeds. By contrast, for \( \kappa \gg r_0 c_t \), CR-driven winds accelerate more slowly and lose most of their energy to gravity, a CR analogue of photon-tired stellar winds. Given CR diffusion coefficients estimated using Fermi gamma-ray observations of pion decay, we predict mass-loss rates in CR-driven galactic winds of order the star formation rate for dwarf and disc galaxies. The dwarf galaxy mass-loss rates are small compared to the mass-loadings needed to reconcile the stellar and dark matter halo mass functions. For nuclear starbursts (e.g., M82, Arp 220), CR diffusion and pion losses suppress the CR pressure in the galaxy and the strength of CR-driven winds. We discuss the implications of our results for interpreting observations of galactic winds and for the role of CRs in galaxy formation.

Key words: Galaxies; Winds; Cosmic Rays

1 INTRODUCTION
Galactic winds play a key role in galaxy evolution, shaping the galaxy stellar mass function, the mass-metallicity relation, and affecting the morphology of galaxies over cosmic time (e.g., Somerville & Davé 2015). Despite their importance, many puzzles persist. These include the acceleration mechanism for the cool atomic and molecular gas seen in emission and absorption from rapidly star-forming galaxies across the universe (e.g., Veilleux et al. 2020), the very large mass fluxes and low star formation efficiencies inferred for dwarf galaxies, and the apparent need for massive outflows from normal star-forming galaxies in the local universe in order to match some models of Galactic chemical evolution (e.g., Andrews et al. 2017).

Among the physical mechanisms suggested for driving large amounts of cool gas from star-forming galaxies, cosmic rays (CRs) are of particular interest because their pressure is dynamically important with respect to gravity in the Milky Way disc (Boulares & Cox 1990). In this picture, CRs are injected into the disc of the galaxy by supernovae and stellar processes, and scatter off of magnetic fluctuations in the ISM, slowly diffusing (or streaming) away from the disc. This process sets up a pressure gradient that can in principle accelerate gas away from the host galaxy.

Many treatments of CR driven winds exist in the literature, starting with the foundational work of Ipavich (1975) who computed time-steady solutions for CR driven winds assuming streaming transport at the Alfvén velocity. More detailed models including CR streaming and hydromagnetic wave pressure were computed by Breitschwerdt et al. (1991), deriving a mass-loss rate of order \( M_\odot \ yr^{-1} \) from the Galaxy. Everett et al. (2008) further developed a hybrid thermal and CR driven wind model for the Galaxy. More generally, on the basis of momentum conservation, Socrates et al. (2008) argued that CRs could drive significant mass fluxes from star-forming galaxies.

In parallel to these more analytic and steady-state treatments, there is a large and growing body of work on multi-dimensional simulations of galaxy formation with CRs, which suggest that they can play a number of important roles: e.g., driving cold-gas outflows from galaxies (e.g., Booth et al. 2013), modifying the thermal and ionization state of the CGM (e.g., Ji et al. 2020), and heating gas in galaxy groups and clusters, suppressing cooling flows (e.g., Ruszkowski et al. 2017). These numerical models vary in
their treatment of CR transport, including isotropic diffusion (e.g., Ullig et al. 2012; Simpson et al. 2016), anisotropic diffusion along magnetic fields (e.g., Pakmor et al. 2016; Chan et al. 2019; Hopkins et al. 2020), and/or CR streaming along magnetic fields (e.g., Ruszkowski et al. 2017; Chan et al. 2019; Hopkins et al. 2020).

As suggested by the diversity of approaches in the literature, one of the significant uncertainties in modeling the properties of CRs in galaxy formation is that the physical mechanism(s) coupling CRs to the thermal plasma are not fully understood (e.g., Amato & Blasi 2018). Small-scale fluctuations in the magnetic field scatter CRs, setting their mean free path. On scales larger than this mean free path, the CRs can be approximated as a fluid (Skilling 1971). However, it is not clear whether the fluctuations that scatter CRs are self-excited by the CRs themselves (e.g., the streaming instability; Kulsrud & Pearce 1969) or produced by background turbulent fluctuations in the interstellar/circumgalactic-medium (ISM/CIM) cascading to small-scales where they couple to the CRs (e.g., Yan & Lazarian 2002). Nor is it clear whether the dominant scattering mechanism depends on the thermodynamic phase of the ISM/CIM, the local magnetic field strength, or other physical properties. The distinction between self-excited versus background turbulence as the source of scattering that sets the CR mean free path is at the core of the streaming/diffusion dichotomy that dominates CR transport modeling.

Wiener et al. (2017) explored the difference between galactic winds driven with CR diffusion and CR streaming in simulations, finding that diffusive CR transport results in much larger overall mass-loss rates than in CR streaming models. Chan et al. (2019) and Hopkins et al. (2020) found similar results and further argued that models with CR transport dominated by diffusion were required for CRs to efficiently escape Milky-way, M31, and Magellanic Cloud-like galaxies, as is required by gamma-ray observations of pion decay produced by CRs interacting with the ISM (Abdo et al. 2010b,a; Ackermann et al. 2012). These results highlight that a better understanding of CR microphysics is critical for understanding the role of CRs in galaxy formation.

Together with the numerical and analytic treatments of CR-driven winds in the literature, there is also a body of phenomenological work interpreting the non-thermal radio and gamma-ray emission from star-forming galaxies. This directly informs our understanding of the underlying CR population and their role in driving galactic winds. The far-infrared-radio correlation and the gamma-ray emission from star-forming galaxies, including those with strong winds like M82, NGC 253, and Arp 220 can be used to constrain the average CR injection rate per unit star formation, the gas density seen by CRs as they propagate, the magnetic field strength, and the CR escape timescale (e.g., Pavlidou & Fields 2001; Torres 2004; Lacki et al. 2010; Lacki & Thompson 2013; Yoast-Hull et al. 2013, 2014; Buckman et al. 2020; Crocker et al. 2021, 2020). These works thus provide benchmarks for understanding how the inner or "base" boundary conditions for a putative CR-driven wind vary as a function of galaxy properties.

This paper is the first in a series that aims to elucidate the physics of galactic winds driven by CRs using a combination of analytic calculations and idealized numerical simulations. In this paper we focus on the case of CR diffusion. A companion paper discusses the case of CR streaming, which turns out to be much physically richer than the diffusion limit considered here. In §2 we present analytic estimates of the mass-loss rate and terminal velocity of galactic winds driven by CRs in the diffusion approximation. In §3 we validate these estimates with time-dependent numerical simulations, which further show that the winds reach a laminar steady state. §4 calibrates CR diffusion coefficients and CR pressures in galaxies using Fermi gamma-ray observations of pion decay, and calculates the resulting implications for CR-driven galactic winds. We summarize and discuss the implications of our results in §5. Appendix A presents a linear stability analysis and shows that although sound waves are formally unstable in CR-driven winds with diffusion (Drury & Falle 1986), the growth rates are too slow for the instability to be dynamically important in almost all cases (the exception is extremely low gas sound speeds). We also derive the linear WKB dispersion relation for the two-moment cosmic-ray transport model simulated in §3 and show that entropy and sound waves are linearly stable in the WKB limit for CR transport by diffusion in the two-moment CR transport model.

2 ANALYTIC APPROXIMATIONS FOR COSMIC RAY DRIVEN WINDS WITH DIFFUSION

In this section we focus on analytic approximations to the spherical steady state galactic wind problem in the presence of cosmic rays. Following these analytical approximations, in Section 3 we treat the more complete numerical problem and solve the time-dependent spherically symmetric CR-driven wind problem using the numerical scheme of Jiang & Oh (2018). We show that the analytic approximations in this section do an excellent job of reproducing the more complete numerical results.

The general equation of motion, including gas pressure $p$, CR pressure $p_\pi$, a general gravitational potential $\Phi$, and magnetic fields in the limit of magnetohydrodynamics can be written as

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = -\frac{1}{\rho} \nabla p - \frac{1}{\rho} \nabla p_\pi - \nabla \Phi + \frac{1}{4\pi p} (\nabla \times B) \times B. \quad (1)$$

The energy equation for the CRs in the absence of CR sources and pionic losses, and including diffusion and streaming at the Alfvén velocity down the CR pressure gradient, i.e., $v_\pi = -v_\pi |\nabla p_\pi|/\nabla p_\pi$, with $v_\pi = B/(\rho p)^{1/2}$, can be written as

$$\frac{\partial E_\pi}{\partial t} + v \cdot \nabla E_\pi = (v + v_\pi) \cdot \nabla p_\pi \frac{\partial \rho}{\partial t}. \quad (2)$$

where the "equilibrium" CR flux $F_\pi$ is

$$F_\pi = 4p_\pi (v + v_\pi) - \kappa n (n \cdot \nabla E_\pi). \quad (3)$$

$E_\pi$ is the CR energy density, $p_\pi = E_\pi/3$, $\kappa$ is the diffusion coefficient, and $n = v_\pi/v_\pi$. Equation 2, and indeed the scalar CR pressure in equation 1, is formally valid only on scales larger than the mean-free-path of the ~GeV energy CRs that dominate the total energy of the CR population (e.g., Skilling 1971).\footnote{The two-moment CR model we solve numerically in §3 evolves $F_\pi$ as an independent variable and the flux reduces to equation 3 only when time variations are sufficiently slow (see eq. 43).}

Even given this restrictive assumption, the diffusion term in equation 3 in general depends on local plasma conditions. It can also depend on the cosmic ray energy density itself, in which case the 'diffusion' term in eq. 3 is not even truly diffusive (e.g., Skilling 1971; Wiener et al. 2013). We do not consider this complication in the present work.

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a hydrodynamic model which drops the field-aligned diffusive flux in equation 3 in favor of an effective isotropic diffusion equation with a spatially constant diffusion coefficient $\kappa$. We also consider a simple model for the gravity of a galaxy and its host dark matter potential: an isothermal sphere for which $\Phi = 2V_r^2 \ln r$ where $\sqrt{2V_r^2}$ is the circular velocity of the potential. Finally, we simplify the thermodynamics of the gas by using an isothermal equation of state with sound speed $c_s$, as would be appropriate for a warm gas in ionization equilibrium, or which might approximate the effects of turbulence in the atmosphere of the host galaxy. In addition, we consider a single phase flow and do not consider possible variation of CR transport with, e.g., the ionization state of the gas.

With these approximations, the steady state equations of motion are

$$M_\nu = 4\pi r^2 \rho v = \text{const},$$

$$\frac{dv}{dr} = -\frac{dp}{\rho} \frac{1}{r} - \frac{dp_p}{\rho} \frac{2V_r^2}{r^2},$$

and

$$\frac{d\rho_p}{dr} = -\frac{4p_p}{3\rho^2} \frac{dv^2}{dr} + \frac{1}{r^2} \frac{d}{dr} \left( \frac{r^2}{\pi} \frac{dp_p}{dr} \right).$$

A key simplification can be made for the purposes of analytic estimates by noting that the order of magnitude of the diffusion term in equation 6 relative to both of the other terms (which describe CR advection and adiabatic energy changes) is

$$\text{Diffusion} \quad \kappa \sim \left( \frac{\kappa}{10^{24} \text{ cm}^2 \text{ s}^{-1}} \right) \left( \frac{30 \text{ km s}^{-1} \text{ kpc}}{V} \right),$$

which suggests that in the limit of rapid diffusion, and near the base of the outflow where the gas velocity is small, the advective and adiabatic terms in the CR energy equation can simply be neglected. For reference, the diffusion coefficient in the Milky Way is estimated to be $\sim 10^{29} \text{ cm}^2 \text{ s}^{-1}$ (e.g., Trotta et al. 2011), although there is significant uncertainty in this estimate because of a degeneracy between the diffusion coefficient and the size of the CR propagation region (the ‘halo’, Linden et al. 2010; Trotta et al. 2011). In §4, we return to constraints on the diffusion coefficient in galaxies using observations of gamma-ray emission from pion decay. For now, we proceed under the assumption that for sufficiently large diffusion coefficients $\kappa$, one can neglect the advective and adiabatic terms in equation 6 and focus solely on the diffusion term, for which the steady state solution is

$$\rho = \rho_0 + \frac{E_c}{12\pi \kappa} \left( 1 - \frac{1}{r_0} \right)$$

where $\rho_0$ is the base CR pressure at radius $r_0$ and

$$E_c = -12\pi \kappa \frac{dp}{dr} \bigg|_{r_0},$$

is the energy per unit time supplied to the CRs by supernovae and other processes in the galaxy. The assumption that diffusion is much faster than advection near the base of the wind implies that $E_c$ is independent of radius. In our numerical wind solutions in §3 we shall see that this assumption is valid at small radii near the sonic point (as assumed here to estimate $M_\nu$), but that advection then takes over as the dominant energy transport mechanism at larger radii (see Fig. 4 discussed below).

We will make frequent use of the CR scale height near the base of the wind:

$$H_c \equiv \left( \frac{-d \ln \rho_c}{dr} \right) \bigg|_{r_0} = r_0 h_c$$

where we have defined $h_c$ as the dimensionless CR pressure scale-height at $r_0$. With this definition, $E_c = 2\pi \rho_0 h_c^{-1} k p_{c0}$ and equation 8 becomes $\rho_c = \rho_0 [1 + h_c^2 (r_0/r - 1)]$. Note that solutions with $h_c < 1$ have $\rho_c \rightarrow 0$ at a finite radius $r = r_0/(1 - h_c)$. If CR diffusion were the only energy transport mechanism, $h_c = 1$ would be the physical solution extending to large radii. However, we will see below that $h_c \lesssim 1$ is typically the appropriate approximate solution at small radii (where our analysis here applies), because advection of CR energy eventually takes over from diffusion as the dominant transport mechanism.

Equations 4 and 5 can be combined to yield a wind equation

$$\frac{1}{\rho} \frac{dv}{dr} (v^2 - c_i^2) = \frac{2c_i^2}{r} - 1 \frac{dp}{\rho} \frac{2V_r^2}{r}$$

With equation 8, i.e., assuming $E_c$ is a constant because diffusion is rapid, the wind equation becomes

$$\frac{1}{\rho} \frac{dv}{dr} (v^2 - c_i^2) = \frac{2c_i^2}{r} + \frac{E_c}{12\pi^2 \rho k} - \frac{2V_r^2}{r}.$$  

In the portion of the flow where $v \lesssim c_i$, an approximation to the density profile can be derived by assuming hydrostatic equilibrium so that

$$\rho \rho_0 \frac{dV}{dx} = \frac{E_c}{12\pi^2 \rho k} - \frac{2V_r^2}{r},$$

Equation 13 has the solution

$$\rho(r) = \rho_0 \left[ x^{\xi} + A(x^{-1} - x^{\xi}) \right]$$

where $x = r/r_0$ and $\xi$ and $A$ are the two key dimensionless parameters in this problem, which measure the strength of gravity relative to the gas and CR pressures at the outflow’s base:

$$\xi = \frac{2V_r^2}{c_i^2},$$

and

$$\xi \lesssim 1\xi$$

where we assume $\xi \approx 1$ in the approximations after the first equality, and where the last equality defines the base CR sound speed $c_{s,0} = (\rho_{c,0}/\rho_0)^{1/2}$. The conclusion that $A \lesssim 1$ follows from the fact that $A$ is roughly the ratio of the CR pressure force to the gravitational force at the base of the outflow. If that ratio is $\gtrsim 1$, then the initial assumption of HE is invalid and the gas distribution would expand out until $A \approx 1$.

The two power-laws in equation 14, namely $\rho \sim r^{-\xi}$ and $\rho \sim r^{1-\xi}$, correspond to the gas pressure and CR pressure dominated phases of the solution, respectively. The transition between the two happens at a radius $r_0 \approx r_0 A^{-1/(1-\xi)}$. The CR-dominated hydrostatic $\rho \propto \rho_{c,0}/r$ solution in eq. 14 was derived by Ji et al. 2020 for the CGM in their cosmological zoom-in simulations with CRs. Hopkins et al. (2021a) further used this solution to estimate
the properties outflows on CGM scales driven by CRs. The key difference between the solutions here and their estimates is that we self-consistently match our CR-dominated solution onto the gas-pressure dominated solution near the galaxy in order to calculate the properties of galaxy-scale winds. We do not include a CGM at larger radii in our calculations.

The mass-loss rate in the wind is set by the conditions at the critical (sonic) point of equation 12, which we denote as $r_c$. Setting the numerator and denominator of the wind equation to zero simultaneously yields two conditions:

$$v(r_c) = c_i$$
$$\frac{r_c \rho(r_c)}{\rho(r_c)} = \left(\frac{c_i^2}{2\eta_i V_{\text{eff}}^2}\right).$$

(17)

where

$$V_{\text{eff}}^2 = V_g^2 - c_i^2.$$  

(18)

These conditions allow for a compact expression for the wind mass-loss rate:

$$M_w = 4\pi r_c^2 \rho(r_c) v(r_c) = \frac{2\pi r_c \rho(r_c) c_i^2}{h_i} V_{\text{eff}}^2. $$

(19)

To estimate the radius of the sonic point $r_c$, one can numerically solving the 2nd critical point equation in eq. 17 using the analytic density profile in equation 13, i.e.,

$$\frac{c_i^2}{2\eta_i (V_g^2 - c_i^2)} = s_x^{\epsilon_{s+1}} + \frac{c_i^2}{h_i (2V_g^2 - c_i^2)} (1 - s_x^{\epsilon_{s+1}})$$

(20)

A good approximate solution to equation 20 can be found by expanding for $c_i^2 \ll 2V_g^2$ which yields

$$r_c \approx r_0 \left(\frac{4h_i V_g^4}{c_i^2 V_{\text{eff}}^2}\right)^{1/2}.$$  

(21)

As an example, if $c_i = c_{i,0} \approx 10 \text{ km s}^{-1}$ (as in the Milky Way), equation 21 predicts $r_c/r_0 \approx 1.38, 1.13, \text{ and } 1.05$ for $h_i = 1$ and $V_g = 30, 60, \text{ and } 100 \text{ km s}^{-1}$, while numerical solution of equation 20 yields nearly identical results of $r_c/r_0 \approx 1.39, 1.13, \text{ and } 1.05$ respectively. The sonic point is quite close to the base of the wind, i.e., $r_c \approx r_0$, unless $c_i \sim V_g$.

Using equation (21) for the sonic point in equation (19), we obtain an expression for the mass-loss rate:

$$M_w \approx \frac{2\pi r_0^2 \rho_0 c_i}{h_i} V_{\text{eff}}^2 \left(\frac{4h_i V_g^4}{c_i^2 V_{\text{eff}}^2}\right)^{1/2}.$$  

(22)

In the limit that $2V_g \gg c_i$,

$$M_w \approx \frac{2\pi r_0^2 \rho_0 c_i}{h_i} V_g^2 \left(\frac{c_i^2}{V_g^4}\right).$$  

(23)

We stress that for equation 23 to be applicable, CR diffusion must dominate over advection out to at least the sonic point since the mass-loss rate is set by the flow properties at and interior to the sonic point. This, together with equation 7, implies that the relevant criterion for the validity of our analytics is roughly $\kappa \gg n_c$. We shall see that this is borne out by the numerical simulations in §3. Diffusion coefficients satisfying this constraint are also strongly suggested by gamma-ray data on pion decay in nearby star-forming galaxies, as we show in §4.

Note that equations 22 and 23 imply that the mass-loss rate does not explicitly depend on the diffusion coefficient $\kappa$ at fixed base CR pressure $p_{i,0}$ (though there is an implicit dependence via $h_i$ as we will see below). However, for a fixed CR injection power $E_\gamma$, equations 22 and 23 imply $M_w \propto 1/h_i$ because the base CR pressure itself scales as $p_{i,0} \propto p_{i,0} c_i^2 \propto E_\gamma/k$ (see eq. 8). Substituting into equation (22), we find that

$$M_w \approx \left(\frac{r_0 c_i}{6k}\right) \frac{E_\gamma}{V_{\text{eff}}^2} \left(\frac{4h_i V_g^4}{c_i^2 V_{\text{eff}}^2}\right)^{1/2}.$$  

(24)

which makes the $k$ dependence, and the competition between diffusion and advection at the base of the outflow, explicit.

We now present an order of magnitude estimate of the scale-height $h_i$ by determining the radius $r_{\text{adv}} \approx r_0(1 + h_i)$ at which the advective flux $F_{\text{adv}} \sim 4p_{i,c} \mathcal{M}_w p_{i,c}/(\pi r_c^2)$ is comparable to the diffusive flux $F_{\text{diff}} = -3k dp_{i,c}/dr$. This occurs when the velocity of the outflow is given by

$$v(r_{\text{adv}}) \approx -\frac{d \ln p_{i,c}}{dr} \approx \left(\frac{c_i}{h_i c_{i,0}^5}\right)$$  

(25)

where we have dropped a factor of $3/4$ consistent with the rough nature of the estimates that follow. Since $h_i \lesssim 1$ and our analytics assumes $\kappa \gtrsim c_{i,0}$, equation 25 implies that the transition from diffusion to advection happens exterior to the sonic point. Gas pressure is then negligible and the momentum equation becomes $\rho dv/dr \approx -d p_{i,c}/dr - 2V_g^2/r$. At the sonic point, the two terms on the right-hand-side of $dv/dr$ are comparable (see eq. 17). Since the gas density scale-height is smaller than the CR scale-height, somewhat exterior to the sonic point the equation $\rho dv/dr \approx -d p_{i,c}/dr$. Multiplying by $4\pi r^2$ gives

$$M_w \frac{dv}{dr} \propto 4\pi r_{\text{adv}}^2 \frac{p_{i,c}}{V_g} \sim v(r_{\text{adv}}) \sim h_i V_g^2$$  

(26)

where in the second expression we have used the approximate version of $M_w$ from equation 23 and have taken $r_{\text{adv}} \approx r_0$, consistent with $h_i \lesssim 1$. Equations 25 and 26 then yields $h_i \sim (c_i/V_g)(k/r_{\text{adv}})^{1/2}$. For $k \to \infty$ this gives $h_i \gg 1$, inconsistent with the local approximation used here; the solution should be $p_{i,c} \propto 1/r$, i.e., $h_i \approx 1$, so that

$$h_i \sim \min \left(1, \left(\frac{c_i}{V_g}\right) \sqrt{\frac{k}{r_{\text{adv}}}}, \right).$$  

(27)

As an example, if $k \sim 10^{29} \text{ cm}^2 \text{ s}^{-1} \approx 10 (r_{\text{adv}}/30 \text{ km s}^{-1})$, $c_i \sim 10 \text{ km s}^{-1}$, and $V_g \sim 100 \text{ km s}^{-1}$, $h_i \sim 1/3$. In our analytic scalings that follow in this section we primarily normalize $h_i$ to a value of $1/4$ motivated by these fiducial parameters, but in several plots we will use equation 27. In §3, we also compare equation 27 to our numerical solutions and find good agreement.

Our final expression for the mass-loss rate in equation 22 can be written as

$$M_w \approx \frac{4\pi r_0^2 \rho_0 c_i}{h_i} V_g^2 \left(\frac{c_i^2}{V_g^4}\right)^{1/2} \left(\frac{r_0}{V_g/1 \text{ kpc}}\right)^{1/4} \left(\frac{4h_i V_g^4}{c_i^2 V_{\text{eff}}^2}\right)^{1/2} \left(\frac{n_0}{1 \text{ cm}^{-3}}\right) \left(\frac{100 \text{ km s}^{-1}}{10 \text{ km s}^{-1}}\right)^{2}$$  

(28)

where $n_0 = \rho_0/m_p$. For reference, if we scale for parameters appropriate to a galaxy like the Milky Way (with $V_g \approx 150 \text{ km s}^{-1}$, $c_i \approx 10 \text{ km s}^{-1}$, $n_0 \approx 1 \text{ cm}^{-3}$, and $r_0 \approx 5 \text{ kpc}$), equation
28 yields \( M_\nu \approx 1 \, M_\odot \, \text{yr}^{-1} \), comparable to the star formation rate. Equation 28 also predicts \( M_\nu \propto p_{0,0} / \rho_0 c_i \), which scales \( \propto p_{0,0} / \rho_0 c_i \) or \( \propto p_{0,0} c_i^2 \), depending on which regime of equation 27 is appropriate (these scalings assume \( r_0 = r_c \) for simplicity). To the (uncertain) extent that the CR pressure is comparable in different phases of the ISM, the outflow is thus likely to be dominated by the warmer ISM phases, though only by a factor of a few.

Figure 1 shows the mass-loss rate (eq. 28 with \( h_t \) from eq. 27 taking \( \kappa = 10 r_0 c_i \)) as a function of the two key dimensionless parameters in the problem, namely the strength of gravity (\( V_g / c_i = (\xi / 2)^{1/2} \); see eq. 15) and the base CR sound speed \( (p_{0,0} / \rho_0 c_i^2) \). The mass-loss rate is normalized in units of \( \dot{M}_0 = 4 \pi r_0^2 \rho_0 c_i \equiv 3.2 \, M_\odot \, \text{yr}^{-1} (r_0 / \text{kpc})^2 (n_0 / 10^3 \, \text{cm}^{-3}) (c_i / 10 \, \text{km s}^{-1}) \).

To express the mass-loss rate in CR driven winds in a more intuitive form we take \( V_g \gg c_i \) and use the simplified expression for the mass-loss rate in equation 23. We then write the CR energy injection at the base of the outflow as

\[
\dot{E}_c = \epsilon_c M_\nu c_i, \quad (30)
\]

where \( M_\nu \) is the star formation rate and \( \epsilon_c \equiv 10^{-6} \) is related to the fraction of SNe energy that goes into CRs; for \( 10^4 \) ergs per SNe and 1 SNe per 100 \( M_\odot \) of stars formed, \( \epsilon_c = 10^{-6} \) if 10% of the SNe energy goes into primary CRs. Equation 23 can then be approximately rewritten as (taking \( r_0 = r_c \) to simplify eq. 21)

\[
\frac{M_\nu}{M_\nu} \approx 0.8 \epsilon_c r_0 \left( \frac{c_i}{V_g} \right) \left( \frac{100 \, \text{km s}^{-1}}{V_g} \right)^2 \approx 0.08 \epsilon_c \left( \frac{c_i}{V_g} \right) \left( \frac{10^9 \, \text{cm}^2 \, \text{s}^{-1}}{c_i} \right) \left( \frac{100 \, \text{km s}^{-1}}{V_g} \right)^2. \quad (31)
\]

Per equation 7 and the associated discussion, \( \kappa \sim 10 r_0 c_i \) is plausible; we have scaled to representative values in the second line. This expression shows that significant wind mass loading relative to the global star formation rate is in principle possible, and that it should grow strongly with decreasing \( V_g \). However, if \( \kappa \sim 10^9 \) cm\(^2\) s\(^{-1}\) is appropriate, the mass-loading factors in dwarf galaxies are relatively modest compared to the values \( M_\nu \gg M_\nu \) needed to reconcile the stellar and dark matter mass functions. We return in Section 4 to an observational calibration of the diffusion coefficients in other star-forming galaxies.

A second instructive expression for the wind mass-loss rate can be obtained by comparing the mass-loss rate estimated here to the star formation rate predicted by feedback-regulated models of star formation in galaxies, namely \( M_\nu \equiv \pi r_0^2 \Sigma \), where the surface density of star formation is (e.g., Thompson et al. 2005)

\[
\Sigma_s \approx \frac{\sqrt{8 \pi G \Sigma_s^2 \phi}}{\nu_c}, \quad (32)
\]

where \( \phi = 1 + \Sigma_0 / \Sigma_s \) describes the contribution of the stellar disc with surface density \( \Sigma_s \) to the local gravitational potential and \( \nu_c = \rho \sigma / m_* \approx 3000 \) km s\(^{-1}\) is the momentum per unit mass of star formed associated with stellar feedback, which supports the disc against its own self-gravity (Ostriker & Shetty 2011). Equation 23 for the mass-loss rate can then be recast as

\[
\frac{M_\nu}{M_\nu} \approx 8 \left( \frac{r_0}{h_0} \right) \left( \frac{c_i \nu_c}{3 \times 10^5 \text{km}^2 \text{s}^{-1}} \right) \left( \frac{100 \, \text{km s}^{-1}}{V_g} \right)^2 \frac{p_{0,0}}{\pi G \Sigma_s^2 \phi}. \quad (33)
\]

Figure 1. Analytic mass-loss rates for CR driven galactic winds in the limit of rapid diffusion (eq. 28 with \( h_t \) from eq. 27), as a function of the strength of gravity relative to the gas sound speed in the disc (\( V_g / c_i \)) and the base CR pressure (\( p_{0,0} / \rho_0 c_i^2 \)). The mass-loss rates are normalized by equation (29). Labeled values of \( V_g / c_i \) on the color bar are logarithmically distributed and correspond to the curves on the plot.

Equation 33 again shows that for Milky-way like conditions in which the CR pressure is comparable to that needed for hydrostatic equilibrium in the galactic disc (\( p_{0,0} \approx 1/3 \pi G \Sigma_0^2 \phi \); Boulares & Cox 1990), the wind mass-loss rate driven by diffusing CRs can be of order or larger than the star formation rate. As we discuss in Section 4, equation (33) also shows that for dense starburst galaxies, in which \( p_{0,0} \ll \pi G \Sigma_0^2 \phi \) (Lacki et al. 2010, 2011), the mass-loss rate in CR driven winds is significantly reduced.

2.1 Wind Energies, Momentum Flux, and Velocity

For a steady state solution, the momentum equation (eq. 5) and CR energy equation with diffusion (eq. 6) can be combined to yield a total conserved energy outflow rate, namely

\[
\dot{E}_c = M_\nu c_i^2 \left( \frac{1}{2} \right) \rho \dot{\nu}_c \rho + 4 \dot{\epsilon}_c + \Phi) + \dot{E}_c = \text{constant} \quad (34)
\]

where \( \dot{E}_c = -12 \pi r_0^2 \rho d p //dr \) (as before) and where the four terms in parentheses in equation 34 correspond to the gas kinetic energy flux (\( \dot{E}_k \)), the gas enthalpy/advective flux (assuming our isothermal equation of state for the gas), the CR enthalpy/advective flux (\( \dot{E}_c \)), and the gravitational energy flux (\( \dot{E}_g \)), respectively. Note that equation 34 no longer assumes that diffusion dominates over advection at all radii.

The total energy flux and terminal velocity of the wind can be estimated as follows. Under our assumption of \( \kappa \gg r_0 c_i \), at small radii near the base of the wind, the energy flux at small radii is almost entirely due to CR diffusion, so that \( \dot{E}_c = \dot{E}_c \). This neglects gravity near the base of the wind, which we return to below. The CR diffusive flux near the base can be related to the mass-loss flux using equation 24 where in this expression we now identify \( \dot{E}_c \) near the base as the total wind luminosity \( \dot{E}_c \) in equation 34. At large radii, the energy flux will be dominated by the gas with \( \dot{E}_g = 0.5 \dot{M}_w v_w^2 \). Equating our expressions for the total wind luminosity at small and large radii yields the terminal velocity of the
This quantity can be compared with the total momentum rate carried by photons from star formation \( \dot{p}_\gamma \), where

\[
\dot{p}_\gamma = L_{\gamma} / c = \epsilon_s M \epsilon_{\gamma} c,
\]

where \( \epsilon_{\gamma} \sim 10^{-3} \) for steady-state star formation and a standard IMF. We then have that

\[
\frac{\dot{p}_n}{\dot{p}_*} \approx \frac{\epsilon_n}{\epsilon_*} \left( \frac{r_0 c}{V_0} \right)^{1/2} \left( \frac{r_{0c}}{r_0 c} \right)^{1/2}.
\]

where the second expression assumes \( V_0 \gg c_1 \). Recall that we require \( \kappa \geq r_{0c} \), given our assumptions used in deriving the approximate wind solutions here. In this case, equation 35 shows that \( v_{\text{out}} \gtrsim v_{\text{eff}} \), with \( v_{\text{out}} \sim 1 - 10 V_0 \) plausible. We show below (Fig. 7) that equation 35 agrees very well with our numerical simulations.

Using equations (28) and (35) we can write down an expression for the asymptotic total momentum loss rate carried by the wind,

\[
\dot{p}_n = \dot{M}_n v_{\text{esc}} \approx 2 V_{\text{eff}} \left( \frac{3 \kappa}{r_{0c}} \right)^{1/2} \left( \frac{4 \hbar V_i}{c_{\text{eff}}^3} \right) \left( \frac{V_i}{100 \text{km s}^{-1}} \right)^{1/2} \left( \frac{10^{29} \text{cm}^2 \text{ s}^{-1}}{30 \text{ kpc km s}^{-1}} \right)^{1/2}.
\]

Note that the asymptotic kinetic energy loss rate \( \dot{E}_n = 0.5 \dot{M}_n v_{\text{esc}}^2 \) is \( \dot{E}_n = \epsilon_n M c^2 / \kappa \). CR-driven winds in the rapid diffusion approximation are thus energy-conserving in the sense that the wind kinetic energy power at large radii is of order that supplied to the CRs at small radii. As in the discussions following equations (31) and (33), we reiterate that this conclusion assumes that there are no pionic losses in the host galaxy (see §4).

\[ 2.2 \text{ Maximum Mass-Loss Rate} \]

There is a strict upper limit to the mass-loss rate associated with CR-driven winds that is set by energy conservation. This is the regime in which gravity significantly modifies the energetics of the outflow and cannot be neglected as we did in deriving equation 35. If the asymptotic speed of the wind vanishes then \( \dot{E}_n(r_0) + \dot{M}_\text{max} \Phi(r_0) \approx 0 \), i.e., \( \dot{M}_\text{max} \approx 2 \dot{E}_n(r_0)/v_{\text{esc}}^2(r_0) \) where \( v_{\text{esc}}(r_0) \) is the escape speed from the base of the wind.\(^3\) This regime is analo-
gous to photon-tired winds in stellar wind theory (Owocki & Gayley 1997). Using \( \dot{E} = \epsilon c M^2 \) (eq. 30), we can write

\[
\frac{\dot{M}}{M_\star} \approx 9 \epsilon \frac{100 \text{ km s}^{-1}}{v_{\text{esc}}(r_0)} \left( \frac{100 \text{ km s}^{-1}}{v_{\text{esc}}(r_0)} \right)^2, \tag{39}
\]

which has an “energy-driven” velocity scaling (Murray et al. 2005). Alternatively, using \( \dot{E}(r_0) \approx 12\pi r_0^2 \rho c_i / h_i \), we find

\[
\dot{M}_{\text{max}} \approx \frac{24 \pi r_0^2 \rho c_i}{v_{\text{esc}}(r_0)} \left( \frac{r_0}{h_i} \right)^2 \left( \frac{c_i}{v_{\text{esc}}(r_0)} \right)^2 \left( \frac{n_0}{\text{cm}^{-3}} \right) \cdot \left( \frac{100 \text{ km s}^{-1}}{v_{\text{esc}}(r_0)} \right)^2 \tag{40}
\]

The maximum cosmic ray-driven wind momentum will be realized for mass-loss rates a bit below \( \dot{M}_{\text{max}} \) when \( \epsilon \sim \epsilon_{n-6.3} \) (at \( \dot{M}_{\text{max}} \), \( \epsilon \approx 0 \) and so \( \rho_{c_i} = 0 \). A rough estimate is

\[
\frac{\rho_{c_i}}{\rho} \approx 3 \epsilon_{n-6.3} \left( \frac{100 \text{ km s}^{-1}}{v_{\text{esc}}(r_0)} \right)^2. \tag{41}
\]

but more detailed calculations are required to precisely determine the numerical pre-factor in this expression.

It is straightforward to show that the mass-loss rate estimated in equation 28 is

\[
\dot{M}_{\epsilon} \approx \dot{M}_{\text{max}} \left( \frac{r_0 c_i}{3 \lambda} \right), \tag{42}
\]

where we have assumed \( v_{\text{esc}}(r_0) \approx 41 v_i^2 \). Equation 42 shows that the wind mass-loss rate is in general \( \epsilon \dot{M}_{\text{max}} \) given our restriction to \( \kappa \gtrsim r_0 c_i \). Our analysis so far does not preclude that winds exist for larger radii (and/or for a computational domain of reasonable size even with a In(r) potential).

Figure 2. Left: CR pressure and gas density as a function of radius for \( V_\rho = 10 \) and several diffusion coefficients (in units of \( r_0 c_i \); see Table 2). The dotted profile is the analytic hydrostatic solution in equation 14 for \( h_c = 1/4 \). The analytic density profile is reasonable interior to the sonic point (and thus for estimating \( M_r \)) but not at larger radii for the higher \( \kappa \) solutions. Right: Flow velocity and CR sound speed for the same solutions, in units of gas sound speed \( c_i \). The vertical green dotted line in both panels is the analytic estimate of the location of the critical point (eq. 21). For the \( \kappa = 3 - 30 \) solutions, the flow accelerates very rapidly and the velocity exceeds the escape velocity just exterior to the base of the wind. For \( \kappa = 1 \) the flow is significantly slower, and the density profile is closer to hydrostatic. For \( \kappa = 0.33 \) the velocities are subsonic and well below the escape speed at all radii shown here. We return to the \( \kappa = 0.33 \) solution in Figure 6 and show that it does drive a wind but one whose properties are very different from the high \( \kappa \) simulations.

3 NUMERICAL SIMULATIONS

3.1 Equations

We solve the time-dependent cosmic ray hydrodynamic equations based on the two moment approach as developed by Jiang & Oh...
in one dimensional spherical polar coordinates. This algorithm has been implemented in the magneto-hydrodynamic code Athena++ (Stone et al. 2020). As in §2, we use an isothermal equation of state for the gas with isothermal sound speed $c_i$ and take the gravitational potential to be $\Phi = 2V_i^2 \ln r$.

Neglecting CR streaming, the full set of equations for gas density $\rho$, flow velocity $v$, cosmic ray energy density $E_c$ and flux $F_c$ in 1D spherical polar coordinates are

$$
\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho v \right) = 0,
$$

$$
\frac{\partial (\rho v)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho v^2 \right) = -\rho \frac{\partial \Phi}{\partial r} - c_i^2 \frac{\partial \rho}{\partial r} + \sigma_c [F_c - v(E_c + p_i)].
$$

$$
\frac{\partial E_c}{\partial t} + \frac{1}{V_i^2} \frac{\partial}{\partial r} \left( r^2 \rho v E_c \right) = -v \sigma_c [F_c - v(E_c + p_i)],
$$

$$
\frac{1}{V_i^2} \frac{\partial F_c}{\partial r} + \frac{\partial p_i}{\partial r} = -\sigma_c [F_c - v(E_c + p_i)].
$$

Here the cosmic ray pressure is $p_c = E_c/3$. The reduced speed of light is $V_{\infty}$, which is chosen to be much larger than $v$ in the whole simulation box. For CR transport by spatially independent diffusion, $\sigma_c$ is a constant and is related to the diffusion coefficient used elsewhere in this paper by $\kappa \equiv 1/(3\sigma_c)$.

Substituting the right-hand-side of the fourth of equations 43 into the 3rd term on the right-hand-side of the 2nd of equations 43 produces the usual $\partial \rho / \partial r$ cosmo-ray pressure gradient in the momentum equation, along with another term related to the time variation of the CR flux. Likewise, the term on the right-hand-side of the 3rd of equations 43 becomes the usual $\nu \partial p_i / \partial r$ term related to CR p$dv$ work, again with another term related to the time variation of the CR flux. The steady state versions of equations 43 are thus equivalent to the steady state equations 4-6 solved in §2.

### 3.2 Initial and Boundary Conditions

For each simulation, we pick gas density $\rho_0$ and cosmic ray pressure $p_{c,0}$ at the bottom boundary $r_0$ and then initialize the gas density and

$$
\frac{\partial p_i}{\partial r} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho v E_c \right) = -v \sigma_c [F_c - v(E_c + p_i)].
$$

Table 2.

| Quantity                  | Symbol | Units |
|---------------------------|--------|-------|
| Radial Velocity           | $v$    | $c_i$ |
| ‘Isothermal’ CR Sound Speed | $c_i \equiv \left( \frac{v}{c} \right)^{1/2}$ | $c_i$ |
| Gravitational Velocity    | $V_i$  | $c_i$ |
| Density                   | $\rho$ | $\rho_0$ |
| CR Pressure $p_c$         | $p_c$  | $p_{c,0}$ |
| CR Flux $F_c$             | $F_c$  | $c_i p_{c,0}$ |
| CR Diffusion Coefficient  | $\kappa$ | $c_i / \rho_0$ |

### 3.3 Overview of Simulations

Table 1 summarizes our suite of simulations. The key physical parameters of each simulation are $V_{\infty}/c_i$ and $p_{c,0}/p_{c,\infty}^2$, as in the analytics of §2, as well as the diffusion coefficient $\kappa$ (in units of $r_0/c_i$). The key numerical parameters are the resolution, reduced speed of light, and box size.

The units for the results of our numerical simulations are summarized in Table 2: gas density is in units of the base density $\rho_0$, speeds are in units of the gas isothermal sound speed $c_i$, CR pressure is in units of the base CR pressure $p_{c,0}$, and CR fluxes are in units of $p_{c,0}/c_i$.

Nearly all of our numerical solutions reach a steady state that does not evolve significantly in time once we run for a few box crossing times. Because the solutions reach a steady state, they are independent of the reduced speed of light $V_{\infty}$, which only enters into the time-dependent terms in equations 43.

Drury & Falle (1986) showed that rapid CR diffusion drives sound waves unstable in the presence of a background CR pressure.
gradient, so it is not a priori obvious that our wind solutions should reach a steady state. We show in Appendix A that the growth rate of this instability is too slow to grow significantly in galactic winds, unless the gas sound speed is extremely small (the latter is not possible given the divergence of the logarithmic potential used in our simulations, we define $E_{\text{grav}}$ using $\Phi = 2V_i^2 \ln(r/r_0)$ so that $E_{\text{grav}} > 0$.

3.4 Numerical Solutions For $\kappa \gtrsim r_0 c_i$

Figure 2 shows the resulting density, CR pressure, fluid velocity, and CR sound speed profiles for $V_i/c_i = 10$, varying $\kappa$ from 0.33 – 30 (in units of $r_0 c_i$; Table 2). The vertical dotted line in Fig 2 (as well as Figs. 3 & 4 below) is the critical point predicted in equation 21, namely $r_i \approx 1.047 r_0$.

The left panel of Figure 2 shows that for all of the solutions, the CR pressure falls off much more slowly than the density near the base. This is because the gas density scale-height is initially quite small, ~ $r_0 (c_i / V_i)^2 \approx 10^{-2} r_0$ while the CR pressure scale-height is significantly larger due to CR diffusion $\sim h_i r_0 \sim 0.25 r_0$ (eq. 27). For all of the solutions, the hydrostatic equilibrium approximation for the density profile (eq. 14) is reasonable at very small radii. There is, however, a bifurcation in the density profiles as a function of $\kappa$. The solutions with $\kappa = 0.33$ and 1 are closer to hydrostatic over the entire radial range while for $\kappa \gtrsim 1$ the density falls off much more rapidly at large radii. The right panel of Figure 2 shows that this bifurcation in density profiles corresponds to a bifurcation in velocity profiles. The solutions with $\kappa \gtrsim 3$ drive strong winds that accelerate to a velocity larger than the escape speed by $r \sim 1.2 r_0$. By contrast, for $\kappa = 1$ the velocities are much lower and for $\kappa = 0.33$ the solution is subsonic for all of the radii shown in Figure 2. We return to the lowest $\kappa$ simulation in §3.5 and show that it does produce a wind, but one whose character is very different from the higher $\kappa$ simulations focused on here.

Figure 3 quantifies the extent to which the numerical solution satisfies the time steady critical point conditions, for the $\kappa = 10$ numerical simulation. Specifically, from equation 11 we define the denominator and numerator of the critical point equation to be

$$D \equiv v_i^2 - c_i^2 \quad \text{and} \quad N \equiv \frac{2c_i^2}{r} - \frac{1}{\rho} \frac{d\rho}{dr} - \frac{2V_i^2}{r}$$

and we further define

$$N_2 \equiv \frac{2c_i^2}{r} + \frac{p_i v_i r_0}{\rho^2 h_i} - \frac{2V_i^2}{r}$$

(45)

(46)

to be the numerator of the critical point equation under the approximation that $E_i$ is constant as a function of radius, which was used in our analytic derivations in §2. Figure 3 shows that $N$ and $D$ pass through zero at the same point, as expected for a time steady solution that passes through the critical point. The location of the critical point is close to the prediction in equation 21, which is shown by the vertical dotted line in Fig 3; the deviation is because the analytics makes the approximation that the hydrostatic density profile (eq. 14) is valid all the way to the sonic point. Equation 46 (used in our analytics) is also a reasonable, though not perfect, approximation to the true numerator in the critical point equation.

Figure 4 quantifies the radial profiles of the mass-loss rate $\dot{M}$ and various contributions to the wind energy flux in the numerical solution for $\kappa = 10$. The mass-loss rate $\dot{M}$ is independent of radius as expected for a steady state solution. The contributions to the energy flux shown in Figure 4 are defined in and below equation 34 (we do not show the gas enthalpy/advective flux, which is negligible). We note that for the logarithmic potential used here, we define $\Phi = 2V_i^2 \ln(r/r_0)$.

The qualitative features of Figure 4 are the same for $\kappa = 3$ – 30. For $\kappa = 1$, however, most of the energy of the outer edge of the domain is in CR enthalpy rather than gas kinetic energy.

For $\kappa = 0.33$, the energetics of the flow is yet more different from that in Figure 4, as we discuss in §3.5.

Figure 5 shows how the $\kappa = 10$ solutions depend on the depth of the gravitational potential $V_r$ (in units of the isothermal sound speed $c_i$). The solutions are qualitatively very similar. The primary difference is that weaker gravity (lower $V_r$) implies a larger density scale-height at the base of the wind. Correspondingly, the wind accelerates significantly more slowly.

3.5 Low $\kappa \leq r_0 c_i$ Solutions

The analytic derivations in §2 primarily assumed $\kappa \gtrsim r_0 c_i$. Our numerical simulations with $\kappa \gtrsim r_0 c_i$ are consistent with the properties

6 This implies that $v(r_{\text{cut}}) = 0.5 V_i = 3c_i$ in Table 1 is a lower limit. If all of the CR enthalpy is converted to kinetic energy, the final velocity would be $1.8 V_i$, close to the analytic estimate in equation 35.
of these analytic solutions, as we discuss further in the next section. Here we present numerical wind models with $\kappa \leq r_0\sigma_i$, which differ dramatically from their high $\kappa$ counterparts. We stress that the higher $\kappa$ solutions are likely the most astrophysically relevant, given estimates of diffusion coefficients from MW CR data ($\S$2) and gamma-ray data from pion decay ($\S$4).

Figure 6 shows the kinematics (top) and energetics (bottom) of the wind for our $\kappa = 0.33$, $V_g = 10$ simulation, which differs significantly from the higher $\kappa$ simulations. The velocity profile in Figure 6 is shown over a larger range of radii than was plotted in Figure 2. The flow accelerates very gradually reaching the sonic point only at $r_s \sim 8r_0^7$, compared to $r_s \sim r_0$ for the high $\kappa$ solutions. The most striking property of the energetics is that $E_{\text{gen}} \approx E_u$ (see eq. 34) at large radii while at the base of the wind, nearly all of the energy is carried by CR diffusion ($\dot{E}_u$); recall that for our logarithmic potential $\Phi = 2V_c^2 \ln(r/r_0)$ and so $E_{\text{gen}} > 0$. The fact that $E_{\text{gen}}(r_0) \approx \dot{E}_u(r_0)$ implies that most of the energy supplied to CRs at the base goes into lifting (almost hydrostatically) the gas out to large radii. As a result, the gas kinetic energy is a minor contribution to the energy flux at large radii, in marked contrast to the high $\kappa$ simulation shown in Figure 4. The condition $\dot{E}_i \approx E_{\text{gen}}$ is precisely the condition for the CR-analogue of photon-tired winds; this is also the regime in which $\dot{M}_w \sim M_{\text{max}}$ (eq. 40). For our simulations, we can define the escape velocity from the base of the wind to be the velocity needed to reach the outer edge of the box, i.e., $v_{\text{esc}}(r_0) = 2V_c \sqrt{\ln(r_{\text{out}}/r_0)}$. Equation 40 then becomes

$$m_{\text{max}} \approx M_0 \frac{3}{2h_i \ln(r_{\text{out}}/r_0) r_0 \sigma_i} \frac{\kappa}{V_c^2}$$

(47)

Table 1 gives the simulation mass-loss rates in units of $M_{\text{max}}$. For $\kappa = 0.33 r_0\sigma_i$, $M_w \approx 0.78 M_{\text{max}}$, close to the maximum value allowed by energy conservation. This is consistent with the energetics in Figure 6. For comparison, we note that $M_w/M_{\text{max}} = 0.85, 0.78, 0.75, 0.42, 0.15, 0.054, 0.018$ for our simulations with $\kappa/r_0\sigma_i = 0.11, 0.33, 1, 2, 3, 3, 10, 30$, respectively. This is consistent with the expectation from equation 42 that $M_w/M_{\text{max}} \propto 1/\kappa$ for $\kappa \geq \sigma_i$. The bifurcation in density and velocity profiles in Figure 2 at $\kappa \approx r_0\sigma_i$ thus corresponds to solutions with $M_w \sim M_{\text{max}}$ ($\kappa \leq r_0\sigma_i$, nearly hydrostatic, slower acceleration) vs. those with $M_w \ll M_{\text{max}}$ ($\kappa \geq r_0\sigma_i$, rapid acceleration). More generally, the last column in Table 1 shows that for solutions with $\kappa > r_0\sigma_i$ nearly all of the CR energy supplied at the base goes into kinetic energy or enthalphy of the wind at large radii. By contrast, for solutions with $\kappa < r_0\sigma_i$, the asymptotic kinetic and enthalphy flux is small compared to the CR energy flux at the base because most of the energy is lost escaping the gravitational potential.

The importance of gravity for the low $\kappa$ solution in Figure 6 implies that some of the details of the solution – though not the fact that $M_w \approx M_{\text{max}}$ – will likely be sensitive to the form of the gravitational potential. In particular, we suspect that the exact acceleration profile for the gas, including the final terminal speed (which depends on the small residual energy not lost to work against gravity), will depend on the details of the potential.

### 3.6 Synthesis of Analytics vs. Numerics

Figure 7 compares our numerical and analytic solutions for the mass-loss rate, terminal velocity, and base CR scale-height. We focus on the $\kappa > r_0\sigma_i$ solutions for which we have the most detailed analytic estimates. For $\kappa < r_0\sigma_i$, $M_w \approx M_{\text{max}}$ (eq. 47), but we do not have a prediction for the terminal velocity in this regime.

Overall, Figure 7 shows that the analytics and numerics agree well using our analytically estimated base CR scale-height (eq. 27). The latter in turn agrees well with the full numerical simulations (bottom panel of Fig. 7). There is a slight systematic offset in the...
mean analytical estimates of \( h_c \) and \( \dot{M} \) that could be removed by multiplying equation 27 by \( \approx 0.75 \).

The agreement in Figure 7 holds over a factor of \( \sim 30 \) in CR diffusion coefficient, \( \sim 100 \) in base CR pressure, and \( \sim 10^4 \) in the ratio of gravity to gas pressure \( (\propto V_c^2) \) which is also a proxy for galaxy mass; see Table 1 for the full range of simulations. Figure 7 also compares the analytic estimate of the wind terminal velocity (eq. 35) to the speed in the simulations at the top of the box. There is again reasonably good agreement, though the analytic speeds tend to be somewhat (\( \sim 30\% \)) larger than the simulations. This is primarily because some of the energy remains in the CRs in the simulations given the finite outer radius of the computational domain, while the analytic estimate assumes that all of the CR energy has been transferred to the gas.

4 MODELS CALIBRATED TO GAMMA-RAY OBSERVATIONS

The analytic and numerical calculations in §2 and 3 show how the properties of galactic winds driven by CR diffusion depend on the diffusion coefficient. For example, winds lose most of their energy to gravity if \( \kappa \lesssim r_0c_i \) (Table 1), while if \( \kappa \gtrsim r_0c_i \), the terminal speed of the wind \( \propto \kappa^{1/2} \) (eq. 35) and the base CR pressure and the wind mass-loss rate depend on \( \kappa \) for a given star formation rate (eq. 24). To assess the implications of our results for CR driven galactic winds we must thus have a handle on \( \kappa \) in other galaxies. Unfortunately, however, there is still sufficient uncertainty in CR transport that it is non-trivial to determine from first principles if the transport is indeed diffusive (vs. streaming), let alone the value of the diffusion coefficient in different phases of the ISM and for the full range of conditions realized in galaxy formation. As a result, we appeal instead to observations to calibrate physically reasonable diffusion coefficients (see also Chan et al. 2019; Hopkins et al. 2020) and then use those, together with the results of §§2 and 3, to quantify the implications for CR-driven galactic wind models.

The non-thermal radio and gamma-ray emission from galaxies provide direct observational constraints on the properties of CRs in external galaxies. These in turn inform the CR pressure and diffusion coefficient that are critical for setting the strength of CR-driven galactic winds. In this section, we provide simple estimates to elucidate these constraints. We focus on the gamma-ray emission from pion decay in star forming galaxies observed by Fermi (Abdo et al. 2010b,a; Ackermann et al. 2012), rather than non-thermal radio emission, despite the fact that there are far more observations available for the latter (see Lacki et al. 2010). The reason is that the Fermi data directly constrains the properties of the GeV protons that dominate the CR energy density.

We consider a simple one-zone model in which a galaxy is characterized by its size \( r_0 \), gas surface density \( \Sigma_g \), and isothermal/turbulent velocity \( c_t \). The cosmic-ray scale-height is \( H_c \) while
the gas scale-height is \( H_g \). For CR diffusion we expect \( H_d > H_g \), as is indeed the case in our simulations in §3. The gamma-ray luminosity is set by the ratio of the timescale for pion losses \( \tau_\pi \) to the mass function \( \epsilon_8 \)

Note that per SN (\( \epsilon_8 \)) and is defined below) to the timescale for CRs to escape by diffusion, \( \tau_\text{diff} \simeq H_d^2/\kappa \). The effective density \( n_{\text{eff}} \) is the density averaged over the regions the CRs are diffusing through and so is given by \( \approx nH_d/H_c \), where \( n \) is the mid-plane density of the galaxy. For reasons that will become clear, we choose to express \( \epsilon_8 \) in terms of gas surface density by using \( \Sigma_g \approx 2\mu H_c \). The ratio of the diffusion timescale to the pion loss timescale is then

\[
\frac{\tau_\text{diff}}{\tau_\pi} \approx \frac{\sum_i H_i}{2\kappa_\text{p0} \rho_0} \tag{48}
\]

where \( \rho_0 = n_0 m_p \sim 1.67 \times 10^{-24} \text{cm}^{-3} \). Equation 48 shows that for a fixed ratio of hadronic losses \( \tau_\pi \) to CR escape \( \tau_\text{diff} \) there is a degeneracy between the CR scale-height and diffusion coefficient, with \( \kappa \propto H_d \). This is consistent with the known degeneracy between CR diffusion coefficient and ‘halo size’ in the literature (e.g., Fig. 3 of Trotta et al. 2011).

The gamma-ray emission from pion decay in a star-forming galaxy is given by \( L_\gamma \sim 1/3 \bar{E}_\gamma \) with \( \bar{E}_\gamma \) the energy per SN going into CRs and \( m_\gamma \sim 100 M_\odot \), the total stellar mass formed per core-collapse SN. We find (Lacki et al. 2011)

\[
L_\gamma \approx A_L L_c \min \left( \frac{\tau_\text{diff}}{\tau_\pi} \right) \tag{49}
\]

where \( L_c \equiv \epsilon_8 M_c c^2 \) is the total luminosity produced by star formation. The factor \( A_L \equiv 3.3 \times 10^{-4} \bar{E}_\gamma (\epsilon_8, m_\gamma M_c^{-1}) \) quantifies the maximum possible gamma-ray luminosity from pion decay, which is realized in the limit \( \tau_\pi < \tau_\text{diff} \) when all of the CR proton energy is lost to pion decay before the CRs can escape (Thompson et al. 2007; Lacki et al. 2011). Note that the factor \( \epsilon_8, m_\gamma M_c^{-1} \sim 1 \) is relatively independent of the stellar initial mass function because massive stars set the SN rate and the luminosity of a star-forming population. To good accuracy, we can thus take \( A_L \sim 3.3 \times 10^{-4} \bar{E}_\gamma 50 \).

Observations of star-forming galaxies by Fermi show that there is a correlation between gamma-ray luminosity and star formation rate (or infrared luminosity; Ackermann et al. 2012; Grif- fin et al. 2016; Linden 2017) and a correlation between gamma-ray luminosity and gas surface density (Lacki et al. 2011). Both correlations imply that high star formation rate and high gas surface density systems approach the ‘proton calorimeter’ limit (Portilh 1994; Thompson et al. 2007) in which most of the CR proton energy is lost to pion decay before the CRs escape the galaxy. Because the correlation between gamma-ray luminosity and infrared luminosity is better constrained than the correlation between gamma-ray luminosity and gas surface density, we use the former to calibrate the diffusion coefficients in our models: \( L_\gamma \sim 2.3 \times 10^8 L_{\text{TIR}} (10^9 L_{\odot})^{-25} \) (Griffin et al. 2016) where \( L_{\text{TIR}} \) is the total infrared luminosity from 0.1 – 1000 \( \mu \)m.

At high infrared luminosities, the infrared luminosity is linearly proportional to the star formation rate but this is not true for lower infrared luminosities where the ultraviolet radiation makes an increasingly important contribution to the total radiated starlight from galaxies. We correct for this using Bell (2003) who finds

\[
M_* \approx 0.12 M_\odot \text{yr}^{-1} \left( \frac{L_{\text{TIR}}}{10^9 L_{\odot}} \right)^{1} \left( 1 + \frac{10^8 L_{\odot}}{L_{\text{TIR}}} \right) \tag{50}
\]

Given \( L_{\gamma}(L_{\text{TIR}}) \) and \( L_{\text{TIR}}(M_*) \), equation 49 allows us to infer \( \tau_\text{diff} / \tau_\pi \) as a function of star formation rate. This is shown with the purple line in the left panel Figure 8: \( \tau_\text{diff} / \tau_\pi \simeq t_N \sim 10^4 M_\odot \text{yr}^{-1} \), while \( \tau_\pi / \tau_\text{diff} \) for much lower star formation rates. The latter is a direct consequence of the fact that \( L_{\gamma} \sim A_L L_c \), in systems like the Milky Way, M31, and the Magellanic clouds (e.g., Ackermann et al. 2012) so that CR protons escape before losing most of their energy to pion decay.

Given \( \tau_\text{diff} / \tau_\pi \) as a function of galaxy star formation rate (purple line Fig. 8), we now use equation 48 to constrain the CR diffusion coefficient. To do so, however, we need an estimate of the CR scale-height and gas surface density as a function of star formation rate. We model \( \kappa \equiv H_d / r_0 \) using equation 27 which assumes that the CR scale-height is set by diffusion in a CR-driven galactic wind. We model the gas surface density \( \Sigma_g \) using equation 32 which yields \( M_* \sim \pi r_0^2 \sqrt{\Sigma_g} \phi / c \), where \( r_0 \) is the size of the galactic disc. The free parameters of our model are thus the size of the star-forming galactic disc \( r_0 \), the galaxy circular velocity by \( V_g \), the gas isothermal sound speed \( c_s \), and the dimensionless stellar contribution to the gravitational potential \( \phi \) (as well as several ‘microphysics’ parameters such as the CR energy per supernovae). Given choices for these parameters, as well as the observed \( L_\gamma / L_c \) correlation, we solve equation 27, 32, and 48 for \( \kappa, \phi, \Sigma_g \). In what follows, we initially assume \( c_s = 10 \text{ km s}^{-1} \) and consider parameters approximating 3 different classes of galaxies. Results for these classes are given in Figure 8:

(i) Nuclear starbursts: black lines in Fig. 8; \( r_0 = 0.3 \text{ kpc}, V_g = 150 \text{ km s}^{-1}, \phi = 1 \). This model is meant to approximate the nuclear starbursts M82 and NGC 253 with \( M_* \sim 10^9 M_\odot \text{ yr}^{-1} \), as well as ultra-luminous starbursts like Arp 220 with \( M_* \sim 10^7 M_\odot \text{ yr}^{-1} \)

(ii) Dwarf galaxies: red lines in Fig. 8; \( r_0 = 1 \text{ kpc}, V_g = 50 \text{ km s}^{-1}, \phi = 5 \). This model is meant to approximate normal star-forming dwarf galaxies like the LMC or SMC and their potentially much more rapidly star-forming counterparts.

(iii) Star-forming disc galaxies: blue lines in Fig. 8; \( r_0 = 3 \text{ kpc}, V_g = 150 \text{ km s}^{-1}, \phi = 5 \). This model is meant to approximate the Milky Way, M31, and other star-forming spirals in the local (\( M_* \sim 10^9 M_\odot \text{ yr}^{-1} \)) and high-redshift (\( M_* \sim 10 – 100 M_\odot \text{ yr}^{-1} \)) universe.

We discuss below (Fig. 9 and eqs. 55-65) the scaling of our results to other gas sound speeds \( c_s \).

The dotted lines in the left panel of Figure 8 show the CR diffusion coefficient in units of \( 10^{23} \text{ cm}^2 \text{ s}^{-1} \) required to explain the gamma-ray luminosities of star-forming galaxies, per the method described in the previous paragraph. Figure 8 also shows the dimensionless CR scale-height \( h_\pi \) we derive (dashed lines). For the dwarf and starburst models in Figure 8, \( h_\pi \sim 1 \) and the diffusion coefficient is \( \sim 2 \times 10^{29} \text{ cm}^2 \text{ s}^{-1} \) \( \gg \kappa_0 c_s \) with only a modest factor of few variation with star formation rate. For the star forming disc model, however, \( h_\pi \sim 0.1 \text{ and } \kappa \sim \kappa_0 c_s \sim 10^{25} \text{ cm}^2 \text{ s}^{-1} \). The dimensionless scale-height for our ‘disc’ model is comparable.
to that inferred from synchrotron emission in nearby star-forming disc galaxies (e.g., Krause et al. 2018), though the latter depends on both the magnetic field and CR scale-heights. Our inferred values of \( h_t \sim 0.1 \) and \( \kappa \sim 10^{28} \) cm\(^2\) s\(^{-1}\) for disc galaxies in Figure 8 are, however, factors of \( \sim 3 \) smaller than preferred in phenomenological models constrained by Milky Way CR data (e.g., Trota et al. 2011). However, the estimated gamma-ray luminosity of the Milky-Way \( \sim 8 \times 10^{38} \) erg s\(^{-1}\) (Strong et al. 2010) is a factor of \( \sim 3 \) lower than what would be predicted by its infrared luminosity given the correlations from Ackermann et al. (2012) and Griffith et al. (2016) used here. \( h_t \) and \( \kappa \) would then be \( \sim 3 \) and 10 times larger, respectively (see eqs 55 & 61 below), in better agreement with detailed Milky-Way modeling. In addition, we show below that for \( h_t \lesssim 1 \), \( h_t \propto c_i \) and \( \kappa \propto c_i \) (eqs. 55 & 61). Our disc galaxy model in Figure 8 would thus have a larger scale-height and diffusion coefficient if we assumed that the CRs primarily coupled to the warm-hot phase of the ISM, as is quite plausible.

The key dimensionless number from §3 that determines the properties of CR driven galactic winds is \( \kappa/(r_0 c_i) \); winds accelerate rapidly and reach speeds significantly larger than \( V_g \) if \( \kappa/(r_0 c_i) \gtrsim 1 \) (Fig. 2). For our 3 galaxy models in Figure 8, we note that \( r_0 c_i \approx 10^{17} \) cm s\(^{-1}\) (starburst; black lines), \( r_0 c_i \approx 3 \times 10^{27} \) cm s\(^{-1}\) (dwarf; red lines), and \( r_0 c_i \approx 10^{25} \) cm s\(^{-1}\) (disc; blue lines). A key conclusion from Figure 8 is that the gamma-ray data on star-forming galaxies are consistent with diffusion coefficients that are in the regime of \( \kappa \gtrsim r_0 c_i \), though only marginally so for our star-forming disc model. This does not prove that such diffusion coefficients are correct, at a minimum because it is possible that CR escape is not set by diffusion but rather by streaming or advection in winds driven by other mechanisms (e.g., supernovae), in which case our constraint on the CR diffusion coefficient using equation 49 would not apply. But the diffusion coefficients inferred in Figure 8 are nonetheless a useful and instructive observational check on diffusive CR-driven galactic wind models.

Given the diffusion coefficients inferred in the left panel of Figure 8, we can estimate the CR energy density in the galactic disc - which sets the base conditions for the wind - as follows: if the star formation rate per unit area is \( \dot{\Sigma}_*, \) the CR pressure in the galaxy is given by (e.g., Thompson & Lacki 2013)

\[
p_{\pi,0} \lesssim \frac{\dot{\Sigma} E_c}{6 \pi m} \min(t_{\text{diff}}, t_\pi)
\]

where \( \min(t_{\text{diff}}, t_\pi) \) determines the effective loss/escape time for CRs in the galaxy. Using equation 32 and that fact that \( t_{\text{diff}} \lesssim t_\pi \) even at the highest star formation rates in Figure 8, we find:

\[
p_{\pi,0} \lesssim \frac{\sqrt{2} E_c h_t r_0}{3 k \rho_*}
\]

The dashed lines in Figure 8 (middle panel) shows the resulting base CR pressures for our 3 galaxy classes/models. For Milky Way-like “star-forming disc” conditions with \( r_t \sim 3 \) kpc, we find that \( p_{\pi,0} \gtrsim 0.1 \pi G \Sigma_0 \rho_* \), a bit smaller than local measurements in the Milky Way (Boulares & Cox 1990), though we stress that our model is not intended to reproduce solar-circle measurements, but rather approximate the disc-averaged physical conditions. Figure 8 predicts that Milky Way-like galaxies have roughly the largest fraction of disc pressure support from CRs, though for a given value of \( r_0 \), the base CR pressure only varies by a factor of a few over a factor of \( \lesssim 10^4 \) in star formation rate. The base CR pressure is, however, sensitive to \( r_0 \), and is significantly smaller, \( \sim 10^{-3} \), for nuclear starbursts like M82 and Arp 220. Physically this is because for a given star formation rate, a smaller size for the star-forming disc implies higher gas densities and thus more rapid pion losses. In order to be compatible with the gamma-ray observations, the diffusion time must be correspondingly shorter as well and thus the CR pressure cannot build to as large a value. These conclusions are qualitatively similar to those of Lacki et al. (2010, 2011) (see also Crocker et al. 2021, 2020) who developed one-zone galaxy models with CRs in order to reproduce the far-infrared radio correlation.

\[\begin{align*}
\text{Figure 8.} & \quad \text{Empirically constrained cosmic-ray properties in star-forming galaxies inferred from gamma-ray observations of pion decay by Fermi, as a function of galaxy star formation rate. All calculations assume gas isothermal sound speed} \ c_i = 10 \text{ km s}^{-1}; \text{ scalings to other values of} \ c_i \text{ are shown in Figure 9 and discussed in the text. Star formation rates of example galaxies are indicated near the x-axis. Three examples are considered corresponding to nuclear starbursts (black lines; } r_0 = 0.3 \text{ kpc, } V_g = 150 \text{ km s}^{-1}, \text{ dwarf galaxies (red lines; } r_0 = 1 \text{ kpc, } V_g = 50 \text{ km s}^{-1}, \text{ and star-forming disc galaxies (blue lines; } r_0 = 3 \text{ kpc, } V_g = 150 \text{ km s}^{-1}). \text{ Left: } t_{\text{diff}}/t_\pi \propto L/L_\star \text{ is the ratio of the CR diffusion time to the pion loss time and sets the gamma-ray luminosity of the galaxy (eq. 49). We infer } t_{\text{diff}}/t_\pi \text{ for different galaxy star formation rates using the Fermi correlation between } L_\star \text{ and star formation rate. We then calculate } \kappa \text{ and } h_t \text{ are correct, at a minimum because it is possible that CR escape is not set by diffusion but rather by streaming or advection in winds driven by other mechanisms (e.g., supernovae), in which case our constraint on the CR diffusion coefficient using equation 49 would not apply. But the diffusion coefficients inferred in Figure 8 are nonetheless a useful and instructive observational check on diffusive CR-driven galactic wind models.} \\
\text{Given the diffusion coefficients inferred in the left panel of Figure 8, we can estimate the CR energy density in the galactic disc - which sets the base conditions for the wind - as follows: if the star formation rate per unit area is } \dot{\Sigma}_*, \text{ the CR pressure in the galaxy is given by (e.g., Thompson & Lacki 2013)} \end{align*}\]
They concluded that $p_{g,0} \sim \pi c S^2$ at low gas-surface densities, but that $p_{g,0} \sim \pi c S^2$ at the high gas densities of nuclear starbursts (see Fig. 15 of Lacki et al. 2010 and section 6.3 of Lacki et al. 2011).

Finally, if we combine equation 52 and equation 28 we arrive at a simple expression for the mass-loss rate in CR-driven galactic winds:

$$\frac{M_w}{M_*} \approx \frac{E_v/m_*}{V_{\text{eff}}^2} \frac{c}{3\kappa} \left( \frac{4h c v}{c^3 \pi c^2} \right)^{1/2}$$  \hspace{1cm} (53)

where $\sqrt{E_v/m_*}$ is a velocity scale associated with CR feedback, which is $\approx 220$ km s$^{-1}$ for $E_v = 10^{50}$ erg and $m_* = 100M_\odot$.

The solid lines in the middle panel of Figure 8 shows the mass-loading of CR-driven galactic winds $M_w/M_*$ from equation 53 using $c$ in the left panel of Figure 8 calibrated to gamma-ray observations. We again show results for our 3 fiducial galaxy models. The mass-loss rates in CR-driven winds are significant for a wide range of normal disc galaxy conditions with $M_w \sim M_*$, but are strongly suppressed ($M_w \sim 10^{-3}M_*$) in nuclear starbursts like M82, NGC 253, and Arp 220 because rapid CR diffusion and pion losses suppress the base CR pressure (Fig. 8) and thus the mass-loss rate in the wind. Dwarf galaxies lie somewhere in between with $M_w \sim 0.2M_*$.

The right panel of Figure 8 shows the terminal velocity (eq. 35) and momentum flux $\rho_v \approx M_w v_\infty$ (in units of the photon momentum flux $\rho_v = L/c$) for the same three galaxy models (compare with Lochhaas et al. 2020). For the disc and dwarf models $v_\infty \sim 1000$ km s$^{-1}$ and $\rho_v \sim \rho_c$ while for the starburst model $v_\infty \sim 10^7$ km s$^{-1}$ and $\rho_v \sim 0.1\rho_c$. The large velocities and correspondingly lower momentum fluxes for the starburst model are a consequence of $\kappa \gg r_g c_i$ needed to avoid overproducing the gamma-ray luminosities.

One of the uncertainties in assessing the implications of our results for observations is the appropriate value of the gas isothermal sound speed. Most of the mass in the ISM is in cooler phases but most of the volume is in warmer phases. As a result, it is plausible, though not guaranteed, that the warmer phases set the scattering rate and diffusion coefficient for the cosmic-rays. Figure 9 shows how the mass-loss rates, terminal velocities, and momentum fluxes we infer from gamma-ray data in our starburst and dwarf models depend on the assumed value of $c_i$. We do not show similar results for the normal star-forming disc model because that model is in the regime $h_i \lesssim 1$ where the mass-loss rate, terminal velocity, and momentum flux given gamma-ray inferred diffusion coefficients are a very weak function of $c_i$ (e.g., the results in Figure 8 for the 'disc' model apply to better than a factor of 2 accuracy for $c_i \lesssim 100$ km s$^{-1}$); this is derived analytically below. Figure 9 shows that the mass-loss rate increases notably with increasing $c_i$ for both our starburst and dwarf galaxy models. The momentum flux increases more slowly with $c_i$ and the terminal velocity of the wind decreases due to the larger mass-loadings. However, the qualitative conclusions drawn from Figure 8 remain robust. Namely, for the starburst models $M_w \ll M_*$ and for the dwarf models $M_w$ is at most $\sim 10^4 M_\odot$. The latter is, however, still smaller than the large mass-loadings in dwarf galaxies typically needed to reconcile the galaxy stellar mass and dark matter halo mass functions (e.g., Somerville & Davé 2015).

Luminous starbursts (e.g., Arp 220, and to a lesser extent M82) have gamma-ray luminosities approaching the calorimeter limit $L_\gamma \approx A_{\gamma} L_\nu$ due to $t_{\nu} \lesssim t_{\text{doff}}$ (e.g., Lacki et al. 2011; Ackermann et al. 2012; Griffin et al. 2016). In the calorimeter limit, the constraints on $\kappa$ in Figure 8 are best interpreted as upper limits, since the gamma-ray emission is roughly independent of $\kappa$ for $t_{\nu} \lesssim t_{\text{doff}}$. In this regime, the estimated mass-loss rate is independent of $\kappa$ because the base CR pressure is set by $t_{\nu}$ in equation 52 rather than $t_{\text{doff}}$. However, because the asymptotic wind speed is $\sim k^{1/2}$ (eq. 59), in the calorimeter limit, we can only empirically derive an upper limit on $v_\infty$ and the asymptotic wind kinetic energy and momentum flux. Thus, particularly for the more luminous starbursts in Figures 8 & 9, our results may be better interpreted as upper limits on the wind terminal velocity and momentum flux. This

Figure 9. Empirically constrained mass-loss rates, terminal velocities, and momentum fluxes in galactic winds, as a function of galaxy star formation rate, for different values of the gas isothermal sound speed $c_i$. The nuclear starburst model (top) assumes $r_g = 0.3$ kpc, $V_g = 150$ km s$^{-1}$, and $\phi = 1$ while the dwarf galaxy model assumes $r_g = 1$ kpc, $V_g = 50$ km s$^{-1}$, and $\phi = 5$. Note the different x-axis range and normalization of $v_\infty$ for the two panels. The results for the star-forming disc galaxy model shown in Figure 8 are nearly independent of $c_i$ (see text) and so are not plotted here. Starburst mass-loss rates are $\ll M_*$ independent of $c_i$ while dwarf galaxy mass-loss rates can reach $\sim M_*$ for larger values of $c_i$. For starbursts with high star formation rates, the terminal speed and momentum flux are best interpreted as upper limits (see §4 for details).
further strengthens our conclusion that winds due to cosmic-rays alone are weak in starburst galaxies and cannot drive their exceptional outflows (e.g., see Barcos-Muñoz et al. 2018).

We now derive analytic approximations to the results in Figures 8 and 9. These are valuable because they show how the results depend on all of the physical parameters of the problem. In the analytics we assume that \( r_s \sim r_0 \) (see eq. 21), i.e., that the factor \( \left(4h^2v_e^2/c_0^2 \right)^{1/(2k_A)} \sim 1 \). This is an excellent approximation for Figure 8 in which \( c_i = 10 \text{ km s}^{-1} \), but is less applicable for the largest values of \( c_i \) in Figure 9. In our analytic estimates, we also use a fit to our observational calibration of the CR diffusion timescale. An approximate fit to the results in Figure 8 is given by

\[
\frac{t_{\text{diff}}}{t_{\text{ff}}} \approx \alpha E_{\gamma,50}^{-1/2} \left( \frac{M_*}{1 \text{ M}_\odot \text{ yr}^{-1}} \right)^{0.5} \tag{54}
\]

with \( \alpha \approx 0.07 \alpha_{0.07} = 0.07 \). Equation 54 is accurate to better than a factor of 2 over the entire range of \( M_* \) shown in Figure 8. It is not asymptotically correct, however, at either high or low star formation rates. For high star formation rates, \( L_*/L_{\text{IR}} \gg M_*^{0.25} \). Thus \( L_*/L_{\text{IR}} = M_*^{0.25} \). For low star formation rates, however, \( L_* = M_*^{1.5} \). In practice, we find that equation 54 is a good compromise, particularly given its simplicity. In particular, following the derivations of \( h_*, \kappa, p_{cr}, \) and \( M_* / M_* \) per equations 27, 48, 52, and 54 we can show how the results in Figure 8 depend on the various micro (CR and star formation) and macro (global galaxy) parameters in the problem. Combining equations 27, 48, & 54 we find

\[
h_0 \approx \min \left[1, \frac{0.9 E_{\gamma,50} v_{8.5}^{1/2}}{c_i} \left( \frac{c_i}{10 \text{ km s}^{-1}} \right) \frac{\text{kpc}}{r_0} \left( \frac{100 \text{ km s}^{-1}}{v_g} \right)^2 \right] \tag{55}
\]

where \( v_{8.5} = v_c/3000 \text{ km s}^{-1} \).

There are two regimes depending on whether \( h_0 < 1 \) or \( h_0 = 1 \) in equation 55. For \( h_0 < 1 \), which is the regime appropriate for dwarf galaxies and nuclear starbursts in our models in Figs 8 & 9,

\[
\kappa \approx 3 \times 10^{29} \text{ cm}^2 \text{s}^{-1} \left( \frac{h_*E_{\gamma,50} v_{8.5}^{1/2}}{c_i} \frac{\text{kpc}}{r_0} \left( \frac{100 \text{ km s}^{-1}}{v_g} \right)^2 \right) \tag{56}
\]

\[
\frac{p_{cr}}{\pi G M_*^2 \phi} \approx 0.03 \alpha_{0.07} \phi^{1/2} v_{8.5}^{1/2} m_*^{-1/2} \left( \frac{r_0}{3 \text{ kpc}} \right) \tag{57}
\]

\[
M_* \approx 0.1 \alpha_{0.07} \phi^{1/2} v_{8.5}^{1/2} m_*^{-1/2} \left( \frac{100 \text{ km s}^{-1}}{v_g} \right)^2 \left( \frac{c_i}{30 \text{ km s}^{-1}} \right) \tag{58}
\]

where \( m_* = m_* / 100 M_* \). Equations 56 and 35 can also be combined to estimate the terminal velocity of the wind:

\[
v_w \approx 1600 \text{ km s}^{-1} \alpha_{0.07}^{-1/2} \left( \frac{E_{\gamma,50}}{M_*} \phi^{-1/4} v_{8.5}^{1/4} \right) \left( \frac{c_i}{30 \text{ kpc} \text{ km s}^{-1}} \right)^{1/2} \tag{59}
\]
5 SUMMARY AND DISCUSSION

The physics of cosmic ray (CR) transport in galaxies and in the circumgalactic medium remains a significant uncertainty in assessing the impact of CRs on galaxy formation. A central question is what determines the scattering mean free path of CRs, and how this depends on local plasma conditions (e.g., Amato & Blasi 2018; Hopkins et al. 2021b). In this paper, we have assumed that CR transport can be modeled by a spatially independent diffusion coefficient. The diffusion approximation for CR transport is particularly appropriate if ambient turbulence scatters the CRs (vs. scattering by fluctuations excited by the CRs themselves). A companion paper will consider the case of CR transport mediated by the streaming instability. These two mechanisms of CR transport differ dramatically in their predictions for how the CR pressure decreases away from a galaxy: in the limit of rapid CR diffusion, \( p_\text{CR} \propto r^{-1} \) (eq. 8), i.e., the CR pressure scale-height is of order the size of the system, while in the limit of rapid CR streaming, \( p_\text{CR} \propto \rho \Delta \nu^2 \) (for a split-monopole field geometry; e.g., Mao & Ostriker 2018) and so the CR pressure scale-height is tied to that of the gas. This difference in the dynamics of the CRs in general leads to significantly different wind properties for the two CR transport models, as has been highlighted previously in numerical simulations (e.g., Wiener et al. 2017; Chan et al. 2019). One aim of this paper and its companion is to understand these differences analytically and using idealized time-dependent numerical simulations, thus elucidating how the properties of CR-driven galactic winds depend on global galaxy properties and the physics of CR transport.

In this paper, we analytically estimated the properties of galactic-winds driven by diffusion by assuming that the CR diffusion timescale is short compared to the flow time (or dynamical time) near the base of the wind; this requires CR diffusion coefficients \( \kappa \gtrsim r_0 c_s \) where \( r_0 \) is the size of the galaxy (i.e., the star-forming disc) and \( c_s \) is the gas sound speed. In this limit, the asymptotic kinetic energy flux carried by the wind is comparable to that supplied to the CRs at the base of the wind, i.e., the wind is energy conserving. The mass-loss rate of CR driven winds has the form \( M_\text{w} \sim 2\pi r_0^2 \rho_0 c_s / (\gamma V_\infty^2) \sim 2\pi r_0^2 \rho_0 c_s / V_\text{esc}^2 \) (eq. 28; see Fig. 1), and the asymptotic wind speed is \( V_\infty \sim 2V_\text{esc} (0.3\kappa / r_0 c_s) \) (eq. 35) where \( \rho_0, \rho_0 c_s \) and \( c_s \) are the gas density, CR pressure, and CR sound speed at the base of the outflow and \( \sqrt{2}\gamma V_\infty \) is the rotation velocity of the galaxy. Equation 31 compares this estimate of the mass-loss rate in CR-driven winds to the galaxy star formation rate, with \( M_\text{w} / M_* \sim 1/\kappa \). Physically, for a given rate of CR production, set by the star formation rate, the CR pressure in the galaxy, and thus the strength of the wind, decreases with increasing diffusion coefficient since the CRs escape the galaxy more rapidly.

In addition to our analytic estimates, we also carried out time-dependent spherically symmetric simulations of CR-driven winds using the two-moment CR transport scheme for Athena++ developed by Jiang & Oh (2018). The simulations show that, for \( \kappa \gtrsim r_0 c_s \), the analytic estimates for the mass-loss rate, terminal speed, and CR scale-height near the base of the wind are accurate to \( \sim 50\% \) over a factor of \( \sim 30 \) in CR diffusion coefficient, \( \sim 30 \) in base CR pressure, and \( \sim 100 \) in the ratio of the escape speed to the gas sound speed (Fig. 7; see Table 1 for the full range of simulations). In addition, the simulations show that there is a critical value of the CR diffusion coefficient \( \kappa \sim r_0 c_s \) below which the character of the solution changes considerably. For \( \kappa \lesssim r_0 c_s \), CR-driven winds accelerate much more slowly and are nearly hydrostatic over a very extended radial range. In this regime most of the energy supplied to CRs at the base of the wind goes into work against gravity expanding to large radii (Fig. 6); the asymptotic kinetic energy flux in the wind is only a small fraction of that initially supplied to the CRs (see the last column of Table 1). These low \( \kappa \) solutions are CR analogues of photon-tired stellar winds (Owocki & Gayley 1997). The mass-loss rate in this regime can be accurately estimated from global energy conservation as \( M_\text{w} \sim M_\text{gas} \sim 2E_\infty / \rho_0 V_\text{esc}^2 \) (eqs. 40 & 47), where \( E_\infty \) is the energy per unit time supplied to CRs at the base of the wind. This maximum possible mass-loss rate in CR-driven winds is quite large, \( \sim M_\odot (300 \text{ km s}^{-1} / V_\text{esc})^2 \) (eq. 39). For \( \kappa > r_0 c_s \), however, the actual outflow rate is much less than this maximal value (eq. 42).

A key difference between our treatment of CR-driven winds in this paper and analogous treatments of stellar winds driven by radiation in the diffusion approximation (e.g., Owocki et al. 2017) is that stellar wind theory is typically formulated in terms of a given photon-matter cross section \( \sigma \), which sets the Eddington luminosity. By contrast, here we are considering a fixed CR diffusion coefficient, equivalent to a fixed value of the mean-free path of CRs (\( 1/\kappa \)). This difference means that many solutions in stellar wind theory do not directly carry over to the CR problem, although many of the important concepts do.

In our models with \( \kappa \gtrsim r_0 c_s \), the properties of CR-driven winds are largely set close to the ‘base’ of the wind, i.e., near the galaxy. In particular, the sonic point - which sets the mass-loss rate - is close to the base of the wind (eq. 21) unless \( c_s \sim V_\text{esc} \) and the energy flux in the wind - which sets the terminal velocity - is set by the CR diffusive flux at the base (eq. 35 and associated discussion). As a result, we suspect that the properties of these solutions are unlikely to be that sensitive to spatial variation in the CR diffusion coefficient unless there are large variations at small radii near the sonic point. By contrast, our solutions with \( \kappa \lesssim r_0 c_s \) accelerate much more slowly (Fig. 6) and are likely much more sensitive to spatial variation in the microphysics of CR transport. In addition, because the low \( \kappa \) solutions have a kinetic energy \( E_\infty \) at large radii that is small compared to the cosmic ray power at the base of the wind, they are likely more sensitive to the ambient pressure in the CGM, which could confine lower \( E_\infty \) outflows.
Our time-dependent simulations allow us to study the stability of the analytic steady state wind solutions. Nearly all of our simulations reach a laminar steady state with no evidence of instability. This is at first glance surprising since Drury & Falle (1986) showed that CR diffusion in the presence of a background CR pressure gradient renders sound waves linearly unstable. We show in Appendix A, however, that the growth rate of the sound wave instability is not fast enough compared to the flow time in the wind for the instability to grow significantly; the one exception to this is our lowest gas sound speed simulation (the $V_g = 200$ km/s simulation in Table 1; see Figure A1). Appendix A also carries out a WKB linear stability calculation (neglecting the background cosmic-ray pressure gradient) for the two-moment CR transport scheme used in our simulations, and shows that sound waves and entropy modes are linearly stable in the presence of CR diffusion, consistent with the steady state solutions found in the simulations.

A key parameter that sets the strength of the galactic wind in our models is the CR pressure in the bulk of the ISM (with $M_w \propto p_{0,0}$). If $p_{r,0} \approx \pi G \Sigma r^2 \phi$ (the pressure required for hydrostatic equilibrium), CR-driven winds will have dynamically important mass-loss rates with $M_w \gtrsim M_*$. If, however, $p_{r,0} \approx \pi G \Sigma r^2 \phi$, then since $M_w \propto p_{0,0}$ (eq. 28), the mass-loss rates will be significantly smaller. The equilibrium CR pressure $p_{r,0}$ is in turn set by CR escape (i.e., the diffusion coefficient $\kappa$) and/or hadronic losses (eq. 51). To assess the implications of our results for the role of CRs in driving galactic winds, it is thus necessary to estimate the CR diffusion coefficient in other galaxies. This remains a daunting task from first principles, so we instead turned to observations (see §4). In particular, observations of the non-thermal emission from CRs in other galaxies provide direct constraints on CR diffusion coefficients and the CR pressure in galaxies (e.g., Lacki et al. 2010, 2011; Crocker et al. 2021). The non-thermal gamma-ray emission from neutral pion decay is particularly important in this regard because (1) it constrains the properties of CR protons (vs. synchrotron emission), and (2) observations at GeV energies by Fermi, though modest in number, directly constrain the CRs that dominate the total CR pressure. In §4 we developed a simple analytic model interpreting gamma-ray observations in the context of diffusive CR transport. This model essentially derives the theoretically and observationally uncertain diffusion coefficient as a function of the observed gamma-ray luminosity of galaxies. We find that a model with a diffusion coefficient $\sim 10^{26-28}$ cm$^2$ s$^{-1}$ (Fig. 8 and eqs. 56 & 61) is consistent with the Fermi data on gamma-ray emission from star-forming galaxies. This is consistent with similar estimates by Chan et al. (2019) and Hopkins et al. (2020) and their more detailed numerical calculations.

Our constraint on the diffusion coefficient in other galaxies also translates into an estimate of the CR pressure in galactic discs. For typical star forming galaxies with disc sizes $r_d \sim 3$ kpc, we find that the CR pressure is of order 10% of the pressure required for vertical hydrostatic equilibrium in the disc (Fig. 8 and eq. 57). This is reasonably consistent with Milky Way measurements. However, the fractional contribution of CRs to pressure support in the disc is $\propto r_d$ and is only $\sim 10^{-2-3}$ for typical nuclear starburst conditions (Fig. 8 and eq. 57). Physically, this is because in more compact star-forming regions, the gas densities are higher and thus pion losses are stronger. In addition, the CR diffusion time is shorter. There is thus less time for the CR pressure to build up and so the equilibrium CR pressure in the disc is lower. These conclusions are consistent with the earlier work of Lacki et al. (2010) based on modeling the far infrared-radio correlation.

Our results on the CR diffusion coefficient and CR pressure implied by gamma-ray observations can be used to estimate the properties of CR-driven galactic winds across a wide range of galaxies. The middle panel of Figure 8 plots the resulting ratio of the CR-driven mass-loss rate to the star formation rate for three fiducial galaxy models, while the right panel shows the terminal velocity and momentum flux of the wind. We find that for massive star-forming disc galaxies, the mass-loss rates are of order the star formation rate, momentum fluxes are of order $\dot{M} \sim L/c$, and terminal velocities are $\sim 500$ km s$^{-1}$ (a few times the circular velocity). For lower-mass dwarf galaxies, however, we find that CRs are somewhat less efficient at driving winds ($M_w \sim 0.2 M_*$ and $p_{w} \sim L/c$), primarily because the CR diffusion time is so short (to explain the gamma-ray data) that the CR pressure in the disc is comparatively low. This is even more true in nuclear starbursts: CRs become much less efficient at driving winds with $M_w/M_* \propto \tau_0$ (eq. 58) and $M_w/M_* \sim 10^{-2} - 10^{-3}$ for well-studied local starbursts like M82 and Arp 220 (Fig. 8). This conclusion fundamentally rests on our inference that $p_{r,0} \approx \pi G \Sigma r^2 \phi$ given CR diffusion coefficients and pion loss timescales motivated by gamma-ray observations. An independent observational probe of the CR proton pressure in other galaxies would be a valuable test of our models.

One of the uncertain parameters in applying our results to observations is the appropriate isothermal gas sound speed. This depends on the phase of the ISM that the cosmic-rays most effectively couple to. Figure 8 assumes $c_t = 10$ km s$^{-1}$, which is an appropriate mass-averaged value in the Milky Way. For typical star-forming disc galaxy parameters, we find that the properties of the winds using gamma-ray constrained diffusion coefficients are weakly dependent on $c_t$ (also derived analytically in equations 63-65). Our conclusion that cosmic-rays are a significant source of winds in normal disc galaxies is thus reasonably robust to the uncertainty of the phase of the ISM that primarily determines CR transport.

Figure 9 shows our gamma-ray inferred wind properties for dwarf galaxy and nuclear starburst models for larger values of $c_t$, appropriate if cosmic-rays primarily couple to volume filling warm-hot gas. For these galaxy models, the mass-loss rate can increase significantly for larger values of $c_t$, as does the momentum flux in the wind; the terminal speed of the wind is correspondingly smaller for larger $c_t$. However, our general conclusions are reasonably robust to uncertainties in $c_t$; the mass-loss rates due to CRs alone in starburst galaxies are $\propto M_*$ and in dwarf galaxies are at most $\sim$ few $\times M_*$. The latter is still below what is typically needed to reconcile the stellar and dark matter halo mass functions (see, e.g., Muratov et al. 2015 Table 3).

As noted earlier in the discussion, the maximum mass-loss rate in CR-driven winds allowed by energy conservation is appreciable, $M_{\text{max}} \sim \left[ \frac{M_1 (300 \text{ km s}^{-1})^2}{c_0} \right]^{-1}$ (eq. 39). Mass-loss rates $\sim M_{\text{max}}$ would be particularly important in dwarf galaxies. However, these large mass-loss rates are only realized when the outflow is very slow and most of the energy supplied to CRs by star formation goes into work leaving the gravitational potential of the galaxy (Fig. 6). This is true requires low CR diffusion coefficients. Such slow outflows would produce gamma-ray luminosities in dwarf galaxies and compact nuclear starbursts larger than are observed. This is the fundamental observational constraint that leads us to favor larger diffusion coefficients and modest mass-loss rates in dwarf and starburst galaxies. A corollary of this result is that in all of our models calibrated to explain gamma-ray luminosities well below the proton-calorimeter value, most of the CR proton energy is vented into the CGM. Even if the CR-driven mass-loadings on
galactic scales are modest, CRs may play an important ‘preventive’ feedback role on CGM scales and/or may significantly modify the dynamics and thermodynamics of the CGM (as was indeed found in the simulations of Ji et al. 2020).

It is instructive to compare our results to related results in the literature. For example, Booth et al. (2013) assumed $\kappa = 3 \times 10^{27}$ cm$^2$ s$^{-1}$ in their simulations of the impact of cosmic-rays on star-forming galaxies. By contrast, Salem & Bryan (2014) considered a range of diffusion coefficients $\kappa = 3 \times 10^{27} - 10^{39}$ cm$^2$ s$^{-1}$ in a similar study. Neither work compared to gamma-ray observations. Our results strongly disfavor the low diffusion coefficient used by Booth et al. (2013) and favor the upper end of the values modeled in Salem & Bryan (2014). Chan et al. (2019) studied three-dimensional simulations of idealized galaxies with CRs and other forms of stellar feedback, and directly compared to gamma-ray observations. They also concluded that CR diffusion coefficients of $\sim 10^{29}$ cm$^2$ s$^{-1}$ were required for consistency with gamma-ray observations. Hopkins et al. (2020) reached similar conclusions using cosmological zoom-in simulations. Our analytics help firm up the conclusions drawn from these simulations and show how they depend on other stellar feedback parameters and the galaxy model (see, in particular, our analytic scalings in equations 56-65). Both Chan et al. (2019) and Hopkins et al. (2020) also found, as we do, that while CRs can drive winds in Milky-way mass galaxies, CRs are not very important wind-drivers in dwarf galaxies relative to other mechanisms.

A significant difference between our solutions and the cosmological zoom-in simulations with CRs of Hopkins et al. (2020), Ji et al. (2020), and Hopkins et al. (2021a) is that we find that advection of CR energy by the gas motion becomes the dominant CR transport mechanism relatively close to the base of the wind, with the gas kinetic energy flux taking over at yet larger radii (see Fig. 4 and eq. 25). By contrast, Hopkins et al. (2020), Ji et al. (2020), and Hopkins et al. (2021a) argue that diffusion sets up a $p \propto r^{-\gamma}$ profile throughout the CGM. A possible resolution of this difference is that diffusion would likely again be the dominant CR transport mechanism exterior to a termination shock between a galactic wind and the CGM, which is not included in our calculations. It is also worth noting that our simulations require high resolution to resolve the acceleration of the gas at small radii, particularly for colder phases (see Table 1). This is not achievable in cosmological simulations. If we take our fiducial simulation (Table 1) and reduce the resolution to $18$, there is no boundary in the galaxy.

Finally, we stress that our observational calibration of CR diffusion coefficients using Fermi gamma-ray data is based on a limited sample of galaxies, primarily those in the local group, M82, NGC 253, and Arp 220 (Ackermann et al. 2012; Griffin et al. 2016). It is thus entirely possible that there are physical correlations of CR transport with galaxy properties (gas density, metallicity, galaxy size, ...) that are not revealed by the current data. Despite this caveat, given the particularly large theoretical uncertainties in the microphysics of CR transport, we believe that observational calibration of the models is an important constraint, and one that will hopefully improve in the coming years.

**DATA AVAILABILITY**

The numerical simulation results used in this paper will be shared on reasonable request to the corresponding author.

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10 http://www.astropy.org
APPENDIX A: LINEAR STABILITY

In this Appendix, we study the linear stability of the CR magnetohydrodynamic equations. Since the simulations are one-dimensional we restrict ourselves to one-dimensional perturbations and also consider a local Cartesian approximation instead of the global spherical geometry used in §3. Physically, this system of equations admits longitudinal sound waves, in which both gas and CR pressure are the restoring force, as well as gas and CR entropy modes. In what follows, we show that ignoring background gradients, the two-moment CR system is linearly stable in the presence of cosmic-ray diffusion. There is, however, an instability driven by a background cosmic-ray pressure gradient that is present in the one-moment CR system (Drury & Falle 1986), i.e., the instability does not rely on the finite speed of light. We show, however, that this instability grows too slowly to be dynamically important in galactic winds, consistent with the laminar numerical solutions we found in §3. The only exception to this is if the gas sound speed is very low, as we show in Figure A1.

A1 Instabilities of the Two-Moment System

We assume here that perturbations are \( \propto \exp(-i\omega t + ikr) \) and that \( kH \gg 1 \) (where \( H \) is a characteristic length-scale in the equilibrium state) so that a WKB analysis is appropriate. For now, we neglect the background gradients in the problem.

The key frequencies in the problem are the isothermal gas sound wave frequency

\[ \omega_g = k\eta, \quad (A1) \]

the adiabatic CR sound wave frequency

\[ \omega_c = k\beta \equiv k \sqrt{4\rho_g/3\rho_i} \quad (A2) \]

and a characteristic frequency in the problem due to the finite speed of light, which we define as

\[ \omega_M = \frac{v_M^2}{k} \quad (A4) \]

Note that in the simulations described in §3, \( \kappa \sim 1 - 30c_f \) so that \( \omega_d/\omega_c \sim r(k/c_f) \gg 1 \). The same inequality holds for \( \omega_d/\omega_g \). By contrast, \( \omega_d/\omega_M \sim (k^2/M^2 - (k/l)^2 < 1 \) is required for the fluid approximation to the CR dynamics to be valid (where \( \ell \) here is the CR mean free path).

Working in the WKB limit, the one-dimensional linear dispersion relation for equations 43 is given by

\[
0 = \frac{3\omega^3}{\omega_M} + i\omega \left[ 1 + \frac{\omega_g^2}{\omega_M^2} \right] - \omega^2 \left[ \omega_c + 3\frac{\omega_g^2}{\omega_M} \right] - i\omega(\omega_g^2 + \omega_c^2) + \omega_c^2 \omega_d
\]

(A5)

In the rapid diffusion limit of \( \omega_M \gg \omega_c, \omega_g \) and \( \omega_M \gg \omega_d \), the 4 solutions to equation A5 are all stable:

\[
\omega \approx -i\frac{\omega_M}{3}
\]

\[
\omega \approx -i\omega_c
\]

\[
\omega \approx \pm \frac{\omega_c^2}{2\omega_d}
\]

(A6)

The first two solutions in equation A6 are strongly damped entropy modes. The last is a weakly damped gas sound wave. Physically, the latter wave arises because in the limit \( \omega_d \rightarrow \infty \), CR pressure gradients are completely wiped out by diffusion and the only restoring force for a sound wave is the gas pressure. At finite \( \omega_d \), there is a small residual CR pressure gradient, the diffusion of which leads to damping of the associated sound wave.

We reiterate that the rapid diffusion ordering used to derive equation A6 is the appropriate one for our simulations in §3. The absence of any growing modes in equation A6 is consistent with the numerical solutions which find laminar wind solutions.
In the limit of slow CR diffusion, $\omega_d \ll \omega_s, \omega_r \ll \omega_M$ the solutions of equation A5 are also damped, namely
\[
\omega \simeq -\frac{\omega_M}{3}
\]
\[
\omega \simeq -i \frac{\omega_d}{\omega_s^2 + \omega_r^2}
\]
\[
\omega \simeq \sqrt{\omega_s^2 + \omega_r^2} - i \frac{\omega_d}{2} \frac{\omega_s}{\omega_s^2 + \omega_r^2}
\]  
(A7)

A2 Instabilities of the One-Moment CR System with Background Gradients

Instabilities of the one-moment CR system for a homogeneous background can be derived using the results in §A1 by taking $v_M \to \infty$. The sound and entropy modes are both stable in this limit. Including background gradients in the calculation, however, leads to an instability of sound waves that was discussed by Drury & Falle (1986). We briefly summarize a derivation of this instability for completeness and then discuss its relevance to our galactic wind simulations. The Drury & Falle (1986) instability is present in the one-moment CR system and so we restrict our analysis to this limit for ease of algebra.

We consider an isothermal gas plus CR system that satisfies the following conservation laws
\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0
\]  
(A8)
\[
\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial z} = -c_s^2 \frac{\partial \rho}{\partial z} - \frac{\partial p_c}{\partial z} - \rho g
\]  
(A9)

We linearize equations A8 & A9. To start we assume that all perturbations, labeled by $\delta$, are $\propto \exp(-i\omega t)$ but we do not Fourier transform in $z$. We do the latter only at the end of the calculation to ensure that all background gradient terms are properly kept. The linearly perturbed equations are then
\[
i\omega \delta \rho = \frac{d(\rho \rho v)}{dz}
\]  
(A10)
\[
-i\omega \delta v = -c_s^2 \frac{\partial \rho}{dz} - \frac{\partial p_c}{dz} - \delta g
\]  
(A11)

Equations A10 and A11 can be combined to yield
\[
\omega^2 \delta \rho = -c_s^2 \frac{\partial^2 \rho}{\partial z^2} - \frac{\partial \rho}{\partial z} - \delta g
\]  
(A12)

In the limit of rapid CR diffusion, the linearized CR energy equation with diffusion (eqs. 2 and 3) simply becomes $\kappa \partial^2 \delta \rho / \partial z^2 \simeq 0$. Substituting this into eq A12, assuming perturbations $\propto \exp[i(kz - \omega t/2H)]$, where $H$ is the density scale-height, and using hydrostatic equilibrium in the background yields
\[
\omega = |k| c_s + i \frac{c_s^2}{2c_s} \frac{k}{|k|} \frac{d \ln p_c}{dz}
\]  
(A13)
to $O(1/H)$ ($c_s^2 = p_c/\rho$ as in the main text). Equation A13 is equivalent to the dispersion relation in Drury & Falle (1986) in the limit of rapid CR diffusion. Drury & Falle (1986) further show that the rapid diffusion approximation leading to equation A13 only applies if $\kappa \gtrsim 4/3 |d \ln p_c/|dt|^{-1} c_s$; otherwise the system is stable. Our lowest $\kappa$ simulations in Table 1 with $\kappa/10c_s = 0.33, 0.11$ are stable at most radii per this condition; otherwise, the rapid diffusion approximation is a good one in our simulations.

The number of e-foldings for the Drury & Falle (1986) instability can be estimated as $A(r) \approx \Im(\omega) H/r c_s$ where $H$ is the density scale-height on which the background structure changes and the flow accelerates. Using equations A13 and 27, we find
\[
A(r) \sim \sqrt{\frac{\rho_0 c_s^2}{\kappa}} \frac{c_s^2}{c_s V_F^2}
\]  
(A14)

near the base of the outflow where the instability derivative is applicable. For our fiducial simulation with $\kappa \sim 10 r_c c_s$ and $c_s \simeq 0.1V_F$ near the base, we find $\Delta \sim 0.03$, i.e., very little growth of the instability. This is consistent with our laminar numerical simulations. Fundamentally, the reason for this is that the CR pressure gradient that drives the Drury & Falle (1986) instability is very shallow in galactic winds driven by CR diffusion, with a CR pressure scale-height much larger than the density scale-height in the subsonic portion of the wind at small radii where equation A13 applies (see Fig. 2). The large CR pressure scale-height in the present context means that the the growth of the Drury & Falle (1986) instability is slow and is the key reason why nearly all of our simulations do not show any sign of this linear instability.

From equation A14, the Drury & Falle (1986) instability is most likely to grow when $c_s$ is small and/or $V_F$ is large (gravity is strong), both of which decrease the CR scale-height (eq. 27). Indeed, we find that our simulation with the smallest value of the gas isothermal sound speed does show evidence of an instability. In this case (the first row in Table 1), we predict $A(r) \simeq 0.5$ near the base of the wind, and a somewhat larger value at the sonic point where $c_s$ is larger. Figure A1 shows that there is indeed evidence of an instability that sets in at $r \sim 1.07$ in this simulation. This may be a manifestation of the Drury & Falle (1986) instability. However, the instability in Figure A1 sets in at radii well exterior to the sonic point and even exterior to where the flow speed equals the CR sound speed. We suspect that these are fluctuations generated by the Drury & Falle (1986) instability at small radii and advected out to large radii where they become nonlinear due to conservation of wave action. Despite the large density fluctuations, however, the wind mass-loss rate and terminal velocity in this simulations are still well-described by the analytic solution in §2. We note that the resolution of the simulation in Figure A1 decreases at $r \sim 1.14$ due to a change in mesh refinement, which likely is responsible for suppressing the short wavelength fluctuations exterior to that radius.
Figure A1. Density, velocity, cosmic-ray pressure profiles, and CR sound speed ($c_s = \sqrt{\frac{p_c}{\rho}}$) for our $V_g = 200$ simulation (see Table 1). For this plot, because of the very low base gas sound speed, we have normalized the velocity, CR sound speed, and CR pressure using $V_g$, $V_g$, and $\rho V_g^2$, respectively. Note the onset of an instability and strong fluctuations at $r \sim 1.07$ (the resolution decreases at $r \approx 1.14$ due to a change in mesh refinement, which likely is responsible for suppressing the short wavelength fluctuations exterior to that radius). Despite the large density fluctuations, the mass-loss rate and terminal velocity in the simulation are well-described by our steady state analytic solutions.