OBSERVATIONAL CONSEQUENCES OF MANY-WORLDS QUANTUM THEORY *

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Abstract

Contrary to an oft-made claim, there can be observational distinctions (say for the expansion of the universe or the cosmological constant) between “single-history” quantum theories and “many-worlds” quantum theories. The distinctions occur when the number of observers is not uniquely predicted by the theory. In single-history theories, each history is weighted simply by its quantum-mechanical probability, but in many-worlds theories in which random observations are considered, there should also be the weighting by the numbers or amounts of observations occurring in each history.

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Quantum mechanics is so mysterious that its precise content or interpretation is not agreed upon even by leading physicists. Although the number of versions or interpretations of quantum mechanics is huge, here I wish to focus upon two main classes of interpretations, which I shall call “single-history” versions and “many-worlds” versions, and show how they might be distinguished observationally. (Similar observational distinctions can be made between analogous “single-history” and “many-worlds” versions of classical physics, but since we know that the universe is quantum, here I shall focus on quantum theories.)

In single-history versions, the quantum formalism gives probabilities for various alternative sequences of events, but only one choice among the possible alternatives is assumed to occur in actuality. For example, a wavefunction that gives nonzero amplitudes for many different alternative events may be assumed to undergo a sequence of collapses to give a single sequence of actually occurring events, which may be considered to be a unique history.

On the other hand, the many-worlds versions began with Everett’s relative-state formalism [1] in which the wavefunction never collapses. In a suitable basis each component of the wavefunction may be considered to be a different “world,” leading to this interpretation’s being labeled the “many-worlds” interpretation.

The consistent or decohering histories formulation of quantum mechanics [2] does not by itself imply whether only a single coarse-grained history actually occurs, or whether many do, and the probabilities of histories that it gives do not depend on whether only one, or instead many, of the histories are actual rather than merely possible. However, I am considering probabilities for observations rather than merely probabilities for histories, so the consistent or decohering histories formalism needs to be extended in order to calculate these probabilities of interest here. The extension then depends on whether many, or only one, of the histories are actual.

It is often claimed that there is no observational distinction between many-world and single-history versions of a quantum theory [3], but here I shall refute that.

In processes with fixed observers that remember their observations, it does seem to be true that there is generally no distinction that a single observer can make between single-history and many-worlds quantum theories that are otherwise identical. This is because then the measure for each observation in a many-world theory is proportional to the probability of that observation in the corresponding single-history theory. This result depends upon the lack of interference between “worlds” in which different observations are made, which is assured if the memory records of the different observations are orthogonal.

To circumvent this no-observable-distinction result, David Deutsch [4] has proposed an experiment in which an observer “splits” into two copies which make different observations and remember the fact of observation, but not the distinct
observations themselves, and so can in principle be rejoined coherently back into a single copy. However, doing this in practice appears to be technologically extremely challenging.

On the other hand, what I wish to demonstrate here is that if different “worlds” do not have the same number of observers, then the measures for observations in the many-worlds theory can be different from being merely proportional to the probabilities in the corresponding single-history theory. Then what an observer would be typically expected to observe in the two theories can be distinct.

Consider a theory of quantum cosmology that gives a quantum state for the universe in which there are different “worlds” with greatly different numbers of observers. For calculating how typical various observations are, in a single-history theory one should weight the “worlds” purely by how probable they are, but in a many-worlds theory, one should weight the “worlds” not only by their quantum mechanical measures (the analogue in a deterministic many-worlds theory of the probabilities in an indeterministic single-histories theory), but also by how much observation occurs within each “world.” This distinction leads to different predictions as to which observations would be typical within the two types of theories.

As a grossly oversimplified illustration, consider the example in which a quantum cosmology theory gave a quantum state (before any possible collapse) that had one “world” with observers, and a second one with none. Suppose that the first “world” had a measure of 0.0000000001 and the second one had a measure of 0.9999999999.

In the single-history version of this theory, these two normalized measures would be the probabilities for the two “worlds,” so the probability would be extremely low that this theory led to any observers. A non-null observation would thus have such a low likelihood within this single-history theory that it would be strong evidence against this theory.

On the other hand, in the many-worlds version of this theory, both “worlds” would exist, with the measures indicating something like the “amount” by which they exist. But since the observations that occur in the first “world” definitely exist within this many-worlds theory as realities and not just as possibilities, the existence of an observation is not evidence against this many-worlds theory.

To put it another way, for considering observations within a many-worlds theory, one must multiply the measure for each world by a measure for the observations within that world. (Crudely, one may use the number of observations within the world, though in a final theory I would expect a refinement, so that, for example, a human’s observation is weighted more heavily than an ant’s). When one does this for the example above, the first “world” makes up the entirety of the weighting in the many-worlds theory, even though in the single-history theory that “world” has an extremely low probability and would be quite unexpected.
Now consider a second toy theory in which there are two “worlds” that both have observers, but their numbers and observations differ. For example, let World A last just barely long enough for it to have $10^{10}$ observers, all during the recontracting stage fairly near a big crunch, and let World B last much longer than the age range at which observers occur and have $10^{90}$ observers when the universe is expanding. Suppose World A has measure almost unity and World B has measure $10^{-30}$.

In the single-history version of this theory, these (normalized) measures are probabilities, so with near certainty, we can deduce that we should be in World A and see a contracting universe in this theory. Our actual observation of an expanding universe would then be strong evidence against this single-history theory.

On the other hand, in the many-worlds version of this theory, all of the observations actually exist. To calculate which observations are typical, one needs the measures for the observations themselves. Presumably these are given by the expectation values of certain operators associated with the corresponding observations [5]. Crudely one might suppose the total for all the observations within one “world” is roughly proportional to the number of observers within that “world,” multiplied by the measure for the “world.” At this level of approximation, the total measure for the observations in the many-worlds version of this second toy model is thus $10^{10}$ for World A and $10^{60}$ for World B. Therefore, an observation chosen at random in this many-worlds theory is $10^{50}$ more likely to be from World B, with the universe observed to be expanding, than from World A with the universe seen to be contracting. Our actual observation of an expanding universe would then be consistent with this theory.

Thus in this second toy cosmological model, we can reject its single-history version because of the low probability it gives, not for our existence this time, but for whether we see the universe expanding. In this way observations can in principle be used to distinguish between many-worlds and single-history quantum theories.

In these examples, the statistical predictions of what a random observer should be expected to observe would be the same for a many-worlds theory and for the corresponding single-history theory if the latter had its quantum-mechanical probability for each history also weighted by the number of observers in that history, but I am assuming that this is not the case. Note that one could still get observable distinctions even if the single-history theory had a sequence of wavefunction collapses, each of which had the weighting by the number of observers in each branch at the time of the collapse, but I shall not consider further this possibility either.

There is the challenge that at present we apparently do not know enough about the quantum state of the universe to say with certainty whether our observations favor a many-worlds theory or a single-history theory. Nevertheless, I can summarize some highly speculative evidence that gives a preliminary suggestion that a many-
histories theory might be observationally favored.

This evidence starts with the Hartle-Hawking ‘no-boundary’ proposal for the quantum state of the universe \(\text{[3]}\), which of course is quite speculative but seems to me to be the most elegant sketch so far of a proposal (certainly not technically complete at present) for the quantum state of the universe. Under certain unproven assumptions and approximations, in a homogeneous, isotropic three-sphere minisuperspace toy model with a single massive inflaton scalar field, the no-boundary proposal leads in the semiclassical regime to a set of “worlds” or macroscopic classical spacetimes that are Friedmann-Robertson-Walker universes with various amounts of inflation and hence various total lifetimes and maximum sizes, and with measure approximately proportional to \(e^{\pi a_0^2}\), where \(a_0\) is the radius of the Euclidean four-dimensional hemisphere where the solution nucleates \(\text{[6]}\).

This nucleating radius \(a_0\) is inversely proportional to the initial value of the inflaton scalar field, multiplied by its mass \(m\) in Planck units, whereas the growth factor during inflation, and the lifetime of the resulting Friedmann-Robertson-Walker universe, go exponentially with the square of the initial value of the inflaton scalar field. Therefore, if one works out the quantum measure in terms of the volume of the universe at the end of inflation (say \(V\) in Planck units), one finds that at the tree level it is very roughly proportional to \(\exp[(4.5\pi/m^2)/(\ln m^3V + 1.5 \ln \ln m^3V)]\) for large values of \(m^3V\) (the universe volume in units of the cube of the reduced Compton wavelength of the inflaton scalar field).

Since the inflaton mass \(m\) is very small in Planck units, say roughly \(10^{-6}\) \(\text{[7]}\), the factor of \(4.5\pi/m^2\) is very large, say roughly \(10^{13}\), and one gets an utterly enormous exponential peak in the measure at relatively small values of \(m^3V\). (There is a cutoff in \(m^3V\) at a value of order unity, below which there is no inflationary solution \(\text{[8]}\), so the measure distribution does not actually have a divergence.) The expression above for the measure rapidly decreases with increasing volume and then flattens out to become asymptotically constant when \(m^3V\) gets large in comparison with \(\exp(4.5\pi/m^2)\).

If one takes at face value the expression above for the measure for all values of \(m^3V\) above its lower cutoff (at some number of order unity), then although the measure has an utterly enormous exponential peak at small values of \(m^3V\), this is in turn overwhelmed by the divergence one gets when one integrates the measure (actually a measure density) to infinite values of \(m^3V\). Then the total measure would be completely dominated by universes with arbitrarily large amounts of inflation. This means that with unit normalized probability, our universe would be arbitrarily large and arbitrarily flat when one ignores density fluctuations from corrections to the homogeneous isotropic minisuperspace model \(\text{[9]}\).

However, the expression above for the measure is purely at the tree-level or zero-
loop approximation, ignoring prefactors that are expected to distort the measure distribution significantly for $m^3V$ large in comparison with $\exp(4.5\pi/m^2)$, because these enormous universes are generated by inflation that starts with the inflaton potential exceeding the Planck density, where one cannot trust the tree-level approximation or any other approximation we have at present.

If the correct quantum measure distribution diverges when one integrates to infinity the spatial volume shortly after the end of inflation, then the universe is most probably arbitrarily large and very near the critical density (spatially very flat), whether a many-worlds or a single-history quantum theory is correct, and so our observation of a universe near the critical density would not distinguish between the two possibilities.

However, if the correct quantum measure density is cut off or damped for large initial values of the inflaton energy density so that one does not get arbitrarily large universes with certainty, then the enormous exponential peak in the distribution at small universes is likely to dominate and (in a single-history version in which the quantum state collapses to a single macroscopic Friedmann-Robertson-Walker universe) make the universe most probably have only a small amount of inflation and a very short lifetime, not sufficient to produce observers, like the first world in the first example above. If one said that somehow the quantum state collapsed to a Friedmann-Robertson-Walker universe that gets large enough for observers, then the most probable universe history under this requirement would be one that lasts just barely long enough for observers before the final big crunch. In this case the observers would most likely exist only near the end of the universe, when it is recollapsing, like World A in the second example above, which is contrary to our observations of an expanding universe. Thus a single-history version of this theory with the quantum measure cut off to produce a normalizable probability distribution would most likely be refuted by our observations of an expanding universe.

On the other hand, if one took a many-worlds version of this quantum cosmology theory, one would have to weight the “worlds” (classical universes) by something like the number of observers within them. One would expect this number to be proportional to the volume of space at the time and other conditions when observers can exist (other factors being equal) \[^{10}\]. Therefore, in the many-worlds version one would multiply the quantum measure given above for the “worlds” (the “bare” probability distribution for universe configurations \[^{11}\]) by something like $V$ to get the measure for observations (the “observational” probability distribution \[^{11}\]).

The result, $V^{\exp\left[{(4.5\pi/m^2)/\left[\ln m^3V + 1.5 \ln \ln m^3V\right]}\right]}$, is then sufficiently rapidly rising with large $m^3V$ that the part with large $m^3V$, even if cut off at $m^3V$ of order $\exp(4.5\pi/m^2)$, dominates over the exponentially large peak near the minimum value of $m^3V$. There is thus enough space for the no-boundary proposal to be
consistent with our observations of a large and expanding universe \cite{11}, but this argument implicitly assumed a many-worlds version of the no-boundary proposal. A similar assumption had been made earlier in the broader context of eternal stochastic inflation \cite{10}. In a single-history version, it seems plausible that the Hartle-Hawking ‘no-boundary’ quantum state may collapse with nearly unit probability to a classical universe configuration that only lasts of the order of the Compton wavelength of the inflaton scalar field, presumably far too short to be consistent with our observations.

This suggestive evidence against a single-history quantum cosmology theory is, of course, not yet conclusive, since we do not yet know what the quantum state of the universe is. Indeed, the ‘tunneling’ wavefunction proposals of Vilenkin, Linde, and others \cite{12} predict that the “bare” quantum measure for small universes is exponentially suppressed, rather than enhanced as discussed above for the Hartle-Hawking ‘no-boundary’ proposal. The ‘tunneling’ proposals would thus apparently be consistent with our observations whether one used a many-worlds version or a single-history version. But the possibility is open that increased theoretical understanding of quantum cosmology may lead us to favor a quantum theory, such as the ‘no-boundary’ one may turn out to be when it is better understood, that is consistent with our observations only in its many-worlds version rather than in its single-history version.

Another tentative piece of observational evidence in favor of many-worlds quantum theory is a comparison with the calculation \cite{13} of likely values of the cosmological constant. If the assumptions of that paper are correct, and if the “subuniverses” used there are the “worlds” used here (“terms in the state vector” \cite{13}) rather than different spacetime regions within one “world” (“local bangs” \cite{13}), then our observational evidence of the cosmological constant is consistent with many-worlds quantum theory but not with single-history quantum theory. However, we need a better understanding of physics to know whether the assumptions are correct (such as the assumption that “the cosmological constant takes a variety of values in different ‘subuniverses’ ” \cite{13}).

Therefore, it may turn out, when we better understand fundamental physics and quantum cosmology, that the observational evidence of the expansion of the universe and of the cosmological constant may lead us to favor many-world quantum theories over single-history quantum theories.

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