Influence of Roof Shape on Snow Accumulation

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Abstract. The paper describes the snow accumulation issues on the buildings and structures covers. The main factors influencing the nature of the snow load formation on random shape coverings are revealed. The holding capacity dependence of incoherent snow on the coating configuration and the snow friction coefficient on the roof surface is established. The individual factors influence degree on the snow accumulation nature is revealed. An analytical dependence of the snow capacity to be retained on the coating surface from the roof outline and the snow adhesion forces on the coating material surface is proposed. Recommendations for taking into account the coating structures outline in the buildings and structures design allowing to clarify the snow accumulation features that are characteristic of this roof shape are given. The dependence in the parametric shape of the cover inclination critical angle from the magnitude of the snow friction coefficient on the roof surface is established. The obtained results allow clarifying the snow load for incoherent snow depending on the friction coefficient magnitude.

1. Introduction
The collapse cases of the coatings structure in winter point out that the snow accumulation features are not fully taken into account in buildings and structures. One of the main collapse causes marked by special commissions investigating accidents is unusual constructive coating solution with increased snow accumulation on the roof [1]. There was also a mismatch between the adopted snows loading scheme, actual snow load for the surface coating given shape [2].

It is necessary to have the exact idea of the loading scheme for competent design which corresponds to the most possible snow load distribution on the buildings and structures roof with different coverage outlines. So, it is preferable to choose a more rational configuration of the coating structural solution which allows reducing the snow load to a greater degree.

2. Theoretical Part
The snow accumulation nature is mainly determined by the cover configuration and the adhesion forces between the snow and the roof material. The snow mechanical retention problems on pitched roofs are not sufficiently studied. Let us consider the incoherent snow fragment balance located on a curvilinear outline section (Fig. 1). We cut out a basic snow section by $dS$ length of unit width and thickness $h$. This part is affected by the snow weight $dG$ and the frictional force $T$ on the coating surface. The considered snow fragment weight $dG$ with the volume weight $\rho$ and the length of the section $dS=Rd\phi$ is $dG=\rho Rd\phi$. The fragment position is determined by the curvature radius of the coating $R$ and the inclination angle to the vertical $\phi$. The snow weight directed vertically downwards is decomposed into a sloping component and a normal component directed radially to the coating surface $dP$. The value of the normal pressure will be $dP=dG\cos\phi=\rho R\cos\phi$.
The sloping component tends to move the snow down and the normal component pushes the snow to the coating and causes the frictional force that keeps the snow on the roof. From the frictional force action $T$ directed along the tangent, there is a radially directed component of the force $dN$. Leonard Euler mentioned such a force existence produced by the diffraction of curved surfaces. He obtained terms for determining the force radial component in rope friction solving problem on a cylindrical surface according to which $dN=Td\phi$. When the snow friction coefficient on the roof surface $f$, the increase of the force friction on the elementary snow fragment will be

$$dT = f(dP + dN) = f(\rho h\cos \phi + T)d\phi.$$  \hfill (1)

We obtain a linear inhomogeneous differential equation with constant coefficients dividing the variables

$$\frac{dT}{d\phi} - fT = f\rho h\cos \phi.$$  \hfill (2)

The inhomogeneous equation solution for constant values $f, \rho, h, R$ is obtained as the total solution sum of the corresponding homogeneous and particular solution of the inhomogeneous differential equations

$$T = C\exp(f\phi) + \frac{\rho h R}{1+f^2} (\sin \phi - f \cos \phi).$$  \hfill (3)

The integration constant $C$ can be determined from the boundary condition. The frictional force will be absent $T=0$ at $\phi=0$ as the horizontal component of the snow weight is zero. Substituting the boundary condition in the formula (3), we have

$$C = \frac{\rho h R}{1+f^2}.$$  \hfill (4)

The final expression to determine the total strength holding the accumulated snow from rolling is

$$T = \frac{\rho h R}{1+f^2} [f \exp(f\phi) + \sin \phi - f \cos \phi].$$  \hfill (5)

Therefore, we can state that the snow retention capacity increases with coating curvature radius, the snow friction coefficient on the roof surface, volumetric weight characterizing the water content of snow. The diagram for different values of friction coefficient (fig. 2) to establish the nature impact on the total friction strength of a roof slope angle is made. The dependence of the conditional holding strength equal to the relative size $T/\rho h R$ from a roof slope angle is given in this schedule. According
to published data [3,4,5] the snow friction coefficient depends on covering surface roughness, temperature conditions, roof material and changes in rather wide range.

There is definition method of a large amount loadings calculation from accumulated snow for structures design. We consider the task to determine the loading maximum value matching to the specified parameters of physical and geometrical characteristics. The torque relating to the rotation center $O$ from the elementary fragment weight of snow length of $ds$ (fig. 1) is

$$dM = R \sin \varphi \rho h R d\varphi.$$  \hspace{1cm} (6)

So the expression for definition of the total moment from all accumulated snow weight on a roof we get

$$M = \int_0^\varphi \rho h x R^2 \sin \varphi d\varphi = \rho h R^2 (1 - \cos \varphi).$$  \hspace{1cm} (7)

Trying to move the snow from the roof of the moment from its own weight counteracts the holding torque due to friction forces taking into account obtained expression (5) will be determined from the following ratio

$$TR = \frac{\int \rho h R^2}{1 + f^2} [\exp(f \varphi) + \sin \varphi - f \cos \varphi].$$  \hspace{1cm} (8)

Figure 2. The conditional dependence of the holding force from the roof angle

The maximum snow load on the cover will comply with the equality holding and tending to move the snow from the roof points. Equating these two, we have

$$f \frac{\rho h R^2}{1 + f^2} [\exp(f \varphi) + \sin \varphi - f \cos \varphi] = \rho h R^2 (1 - \cos \varphi).$$  \hspace{1cm} (9)

After transformations we receive a parametric equation of the maximum snow load on the cover

$$f^2 e^{f \varphi} + f \sin \varphi + \cos \varphi - f^2 - 1 = 0.$$  \hspace{1cm} (10)

The obtained transcendental equation can be solved by numerical methods. A dependence graph of the tangent critical angle from the friction coefficient where will be a descent of incoherent snow is shown in Fig. 3.

The holding capacity will change due to adhesion forces of cohesive snow between the snow individual particles, which will increase the angle inclination at which there will be a descent of snow weight. This problem was considered with the snow toughness in tension [6] in determining snow load on a surface and curvilinear shapes.
3. Conclusion

The nature of incoherent snow loading distribution on buildings and constructions coverings is defined generally to a roof outlines and snow friction coefficient on a covering surface.

The received analytical expression allows to determine the size of the holding force from a roof slope angle and snow friction coefficient on a cover.

It is possible to define a critical angle of a roof inclination depending on friction coefficient of incoherent snow on a covering by the parametrical equation.

The received results allow to specify snow loading for incoherent snow depending on the friction coefficient size and to reduce probability of emergency situations for buildings and constructions with a non-standard covering configuration.

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