Chiral Corrections to Baryon Masses Calculated within Lattice QCD
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Consideration of the analytic properties of pion-induced baryon self energies leads to new functional forms for the extrapolation of light baryon masses. These functional forms reproduce the leading non-analytic behavior of chiral perturbation theory, the correct non-analytic behavior at the $N\pi$ threshold and the appropriate heavy-quark limit. They involve only three unknown parameters, which may be obtained by fitting lattice QCD data. Recent dynamical fermion results from CP-PACS and UKQCD are extrapolated using these new functional forms. We also use these functions to probe the limit of applicability of chiral perturbation theory.

1. Introduction

Chiral symmetry requires that the nucleon mass has the form

$$m_N(m_\pi) = m_N(0) + \alpha m^2_\pi + \beta m^3_\pi + \gamma m^4_\pi \ln m_\pi + \ldots,$$

for small $m_\pi$, where $m_N(0)$, $\alpha$, $\beta$, and $\gamma$ are functions of the strong coupling constant $\alpha_s(\mu)$. Recent work [1] has shown that using physical insights from chiral perturbation theory and heavy quark effective theory one can derive new functional forms which describe the extrapolation of light baryon masses as functions of the pion mass ($m_\pi$). These forms are applicable beyond the chiral perturbative regime and have been compared successfully with predictions from the Cloudy Bag Model [2] and recent dynamical fermion lattice QCD calculations.

2. Analyticity

By now it is well established that chiral symmetry is dynamically broken in QCD and that the pion is almost a Goldstone boson. It is strongly coupled to baryons and therefore plays a significant role in the $N$ and $\Delta$ self energies. In the limit where the baryons are heavy, the pion-induced self energies of the $N$ and $\Delta$, to one loop, are given by the
processes shown in Fig. 1(a–d). We label these by $\sigma_{NN}$, $\sigma_{N\Delta}$, $\sigma_{\Delta N}$, and $\sigma_{\Delta\Delta}$. Note that we have restricted the intermediate baryon states to those most strongly coupled, namely the $N$ and $\Delta$ states. Other intermediate states are suppressed by the baryon form factor describing the extended nature of baryons.

The leading non-analytic contribution (LNAC) of these self energy diagrams is associated with the infrared behavior of the corresponding integrals – i.e., the behavior as the loop momentum $k \to 0$. As a consequence, it should not depend on the details of a high momentum cut-off, or form factor. In particular, it is sufficient for studying the LNAC to evaluate the self energy integrals using a simple sharp cut-off, $u(k) = \theta(\Lambda - k)$ as the choice of form factor. The explicit forms of the self energy contributions for $\sigma_{NN}$, $\sigma_{N\Delta}$ and so on are given in [1]. Moreover, there is little phenomenological difference between this step function and the more natural dipole, provided one can tune the cut-off parameter $\Lambda$. The self energies involving transitions of $N \to \Delta$ or $\Delta \to N$ are characterized by a branch point at $m_\pi = \Delta M$.

2.1. Chiral Limit

The leading non-analytic (LNA) terms are those which correspond to the lowest order non-analytic functions of $m_q$ – i.e., odd powers or logarithms of $m_\pi$. By expanding the expressions given in [1], we find that the LNA contributions to the nucleon/delta masses are in agreement with the well known results of $\chi$PT [4,5].

Of course, our concern with respect to lattice QCD is not so much the behavior as $m_\pi \to 0$, but the extrapolation from high pion masses to the physical pion mass. In this context the branch point at $m_\pi^2 = \Delta M^2$ is at least as important as the LNA behaviour near $m_\pi = 0$.

2.2. Heavy Quark Limit

Heavy quark effective theory suggests that as $m_\pi \to \infty$ the quarks become static and hadron masses become proportional to the quark mass. In this spirit, corrections are expected to be of order $1/m_q$ where $m_q$ is the heavy quark mass. Thus we would expect the pion induced self energy to vanish as $1/m_q$ as the pion mass increases. The presence of a fixed cut-off $\Lambda$ acts to suppress the pion induced self energy for increasing pion masses. While some $m_\pi^2$ dependence in $\Lambda$ is expected, this is a second-order effect and does not
alter this qualitative feature. Indeed, in the large $m_\pi$ limit of the equations, we find that they tend to zero at least as fast as $1/m_\pi^2$.

The agreement with both the chiral limit and expected behaviour in the heavy quark limit suggests the following functional form for the extrapolation of the nucleon mass [1]:

$$M_N = \alpha_N + \beta_N m_\pi^2 + \gamma_N\Delta(m_\pi, \Lambda) + \sigma_N\Lambda(m_\pi, \Lambda).$$

(1)

3. Lattice Data Analysis

We consider two independent lattice simulations of the $N$ and $\Delta$ masses from CP-PACS [6] and UKQCD [7]. Both of these use improved actions to study baryon masses in full QCD with two light flavours. We find that the two data sets are consistent, provided one allows the parameters introducing the physical scale to float within systematic errors of 10%.

We begin by considering the functional form suggested in Section 2 with the cut-off $\Lambda$ fixed to the value determined by fitting CBM calculations. This is shown as the solid curve in Fig. 2. In order to perform model independent fits (i.e. with $\Lambda$ unconstrained), it is essential to have lattice simulations at light quark masses approaching $m_\pi^2 \sim 0.1$ GeV$^2$. This fit is illustrated by the dash-dot curve.

Common practice in the lattice community to use a polynomial expansion for the mass dependence of hadron masses. Motivated by $\chi$PT the lowest odd power of $m_\pi$ allowed is $m_\pi^3$:

$$M_N = \alpha + \beta\Delta + \gamma m_\pi^3$$

The result of such a fit for the $N$ is shown in the dashed curve of Fig. 2. The coefficient of the $m_\pi^3$ term, which is the leading non-analytic term in the quark mass, in the three parameter fit is $-0.761$. This disagrees with the coefficient of $-5.60$ known from $\chi$PT (which is correctly incorporated in Eq. (1), the solid and dash-dot curves) by almost an order of magnitude. This clearly indicates the failings of such a simple fitting procedure.

4. Summary

In the quest to connect lattice measurements with the physical regime, we have explored the quark mass dependence of the $N$ and $\Delta$ baryon masses using arguments based on analyticity and heavy quark limits. We have determined a method to access quark masses beyond the regime of chiral perturbation theory. This method reproduces the leading non-analytic behavior of $\chi$PT and accounts for the internal structure for the baryon under investigation. We find that the leading non-analytic term of the chiral expansion dominates from the chiral limit up to the branch point at $m_\pi = \Delta M \simeq 300$ MeV, beyond which $\chi$PT breaks down. The predictions of the CBM, and two-flavour dynamical-fermion lattice QCD results, are succinctly described by the formulae derived in [1]. The curvature around $m_\pi = \Delta M$, neglected in previous extrapolations of the lattice data, leads to shifts in the extrapolated masses of the same order as the departure of lattice estimates from experimental measurements.

Acknowledgments

This work was supported in part by the Australian Research Council.
Figure 2. A comparison between phenomenological fitting functions for the mass of the nucleon. The dotted curve corresponds to using Eq. (4) with $\gamma$ set equal to the value known from $\chi$PT. The three parameter fit (dashed) corresponds to letting $\gamma$ vary as an unconstrained fit parameter. The solid and dash-dot curves correspond to our preferred fit of the functional form of Eq. (1) with $\Lambda$ from the CBM and as a fit parameter respectively. The lattice data from are CP-PACS (solid) and UKQCD (open), each with a 5% scale change.

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