Analysis of the Operation of Kinematic Seismically Isolated Foundations Taking into Account the Vertical Component of the Seismic Impact

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Abstract. The analysis of the work of kinematic foundations under seismic impact is carried out. In this case, the seismic action is set by two harmonic functions, one of which describes the vertical component of the movement of points on the day surface during an earthquake. The equation of motion of the considered seismically isolated supports is reduced to the form of the well-known Mathieu-Hill equation, the solutions of which can be both bounded and infinitely increasing functions. Thus, the possibility of occurrence in the considered system of seismic isolation of the phenomenon of parametric resonance has been proved and the parameters of the system and the parameters of seismic impact, on which the occurrence of this phenomenon depends, have been identified. The values of the damping parameters in the system and the amplitude of the vertical component of the seismic action, at which the movement of the kinematic supports is unstable, have been established. The construction of the zones of instability of oscillations of supports on the plane of change of the coefficients of the Mathieu-Hill equation is carried out. Also, the value of damping in the seismic isolation system is obtained, which is necessary to ensure the stability of motion under the combined action of the vertical and horizontal components of the seismic action.

1. Introduction

The study was carried out by reducing the equation of motion of a building on a kinematic foundation to the well-known Mathieu equation. The zones of instability of oscillations in the plane of variation of the coefficients of the Mathieu equation were built in the MathCad program [1-2].

Statement of research tasks. It is known that during an earthquake, a structure is exposed first to longitudinal waves, then transverse and surface waves. There are two types of surface waves: Rayleigh waves and Love waves (Fig. 1). When Rayleigh waves propagate in the ground, its particles move in a vertical plane. Thus, during an earthquake, buildings and structures experience loads from the combined action of vertical and horizontal ground displacements [3-4].
One of the principles of modern earthquake-resistant construction is the control of the seismic response of an object through the purposeful assignment of its dynamic characteristics. In this regard, the provision of seismic resistance of buildings and structures is currently carried out in two directions.

1) Traditional seismic protection - provides for the possibility of perceiving seismic loads by increasing the cross-sections of structural elements. The disadvantage of implementing this direction is a serious rise in the cost of construction with a seismicity of 8-9 points [5-6].

2) Special seismic protection - provides for a decrease in the level of seismic loads by assigning a rational dynamic scheme of the structure of a building or structure. Two methods of special seismic protection are possible:
   a) seismic isolation - ensuring a reduction in the share of energy that is transmitted to the building (structure) by seismic waves;
   b) seismic suppression - the redistribution of the mechanical energy of seismic vibrations between parts of the structure and the transition of this energy into thermal energy in special (damping) devices.

Special methods are the most effective. But for such buildings and structures, there is no design experience proven by past earthquakes, which makes the calculation justification extremely important.

An extensive literature is devoted to questions of special seismic protection. In fig. 2 shows the modern classification of special seismic protection systems.

Seismic damping systems by name provide for the presence of damping devices in the system, which are used either independently or in dynamic vibration dampers together with a damping mass.

**Figure 1.** Scheme of propagation of surface waves: (a) - Love waves, (b) - Rayleigh waves.
Figure 2. Classification of special seismic protection systems.

In the world practice of earthquake-resistant construction, the most widespread is seismic isolation using rubber-metal bearing parts and kinematic pendulum supports. In the world practice of seismic isolation, the most widespread are kinematic foundations with pendulum ball bearings [7-8].

The general view and diagram of such a support part is shown in Fig. 3.

Figure 3. General view and diagram of the ball bearing part.

The pendulum support consists of a lower plate (3), on the upper concave spherical surface of which there is a chamber in which either a sliding material in the form of a plate or a polished sheet of austenitic steel (5) is installed. A spherical segment (2) with either a polished chrome surface or a sliding material located in the chamber rests on the supporting spherical surface with the specified antifriction coating. The upper part of the spherical segment (2) has a convex spherical surface with a sliding material fixed in the chamber - (4), on which the upper sliding plate rests. On its inner
spherical (or cylindrical) surface there is a chamber in which a polished austenitic steel sheet is installed.

Pendulum insulators can also be installed in the opposite design, with a spherical sliding plate located below (it is advisable, for example, for metal bridge spans).

Pendulum insulators, due to the movement arising in them along curved surfaces and the reactive force arising in this case, reduce the amplitude of vibrations of structures to the value required by the project and strive to return them to their original position [9-10].

Currently, pendulum bearing parts are produced in sizes from 15 cm to one and a half meters. In fig. 4 shows a diagram of the installation of pendulum support parts and the displacement of a seismically isolated building under load.

![Figure 4. Installation diagram of spherical bearing parts.](image)

It was noted that the vertical component of seismic vibrations can affect the operation of kinematic supports, the analysis of the vibrations of a building on such supports under the combined action of the horizontal and vertical components of the seismic action is known, and a numerical solution of the problem posed for particular cases of the foundation and impact is given [11-12].

2. **Practical application and results**

The equation of vibrations of a building with a kinematic foundation (1) is reduced to the Mathieu-Hill equation.

\[
\ddot{\varphi} + \frac{g}{d} \dot{\varphi} = \frac{\Phi(t)}{d},
\]

where \(\varphi\) – the angle of rotation of the support, \(d\) – the characteristic of the support, has the dimension of length (different for different types of kinematic foundations), \(g\) – the acceleration of gravity, \(\Phi\) – the horizontal acceleration of the base.

Equation (1) is valid only in the absence of the vertical component of vibrations. If we add a vertical component, then equation (1) takes the form
\[ \ddot{\phi} + \frac{g + \ddot{z}(t)}{d} \phi = \frac{\Phi(t)}{d}, \tag{2} \]

where \( \ddot{z}(t) \) – vertical acceleration of the structure foundation.

The harmonic functions were taken as the horizontal and vertical accelerations of the base:

\[ \Phi(t) = Ag \sin \omega t \]
\[ \ddot{z}(t) = Ag \cos \omega t. \]

Here \( A, \dot{A} \) – respectively, the vertical and horizontal acceleration of the base in fractions of the acceleration of gravity. Then equation (2) is transformed to the form:

\[ \ddot{\phi} + k^2 (1 + A \cos \omega t) \phi = k^2 A \sin \omega t, \tag{3} \]

where \( k^2 = \frac{g}{L} \) – the square of the frequency of free vibrations of the system.

Then the transition to dimensionless time was performed: \( \tau = kt \);
\[ d\tau = kd\tau, \]
\[ \dot{\phi} = \frac{d\phi}{dt} = \frac{kd\phi}{d\tau} = k\dot{\phi}, \]
\[ \ddot{\phi} = \frac{d^2\phi}{dt^2} = \frac{k^2 d^2\phi}{d\tau^2} = k^2 \ddot{\phi}. \]

After the transition to dimensionless time, equation (3) takes the form (4).

\[ \ddot{\phi} + (1 + A_{wpm} \cos \chi\tau) \phi = \sin \Omega \tau, \tag{4} \]

where \( \chi = \frac{\omega}{k} \), \( \Omega = \frac{\omega}{k} \).

Equation (4) refers to the Mathieu equations, which are known, and has the form

\[ \ddot{\phi} + (L + 2q \cos \chi \tau) \phi = 0. \tag{5} \]

The left side of equation (5) coincides with the left side of equation (4):
\[ L = 1 \text{ and } 2q = A_{wpm}. \]

The nature of the solutions to the Mathieu equation depends on the values of the coefficients \( L \) and \( q \); they determine the stability of motion. The plane of variation of these coefficients can be divided into regions corresponding to stable (bounded) and unstable (infinitely increasing) solutions of equation (5), i.e. stable and unstable movements. Periodic solutions of Eq. (5) correspond to the boundaries between the zones of stability and instability or periodic oscillations of the system [13-14].

The construction of the approximate boundaries of the first, most important, zone of instability has been completed. In this case, viscous friction in the system was taken into account and an equation of the form

\[ \ddot{\phi} + \gamma k \dot{\phi} + (1 + A \cos \chi \tau) \phi = 0, \tag{6} \]

where \( \gamma \) – coefficient of inelastic resistance.

In fig. 5 shows the approximate boundaries of the first region of instability at \( \gamma = 0 \) and \( \gamma = 0.2 \); as well as points corresponding to the values \( A = 0.4 \) and \( A = 0.2 \).
Thus, it becomes obvious that during earthquakes with certain dynamic parameters of the structure, the phenomenon of parametric resonance is possible or a situation when the solutions of Eq. (6) fall into the zone of dynamic instability and the amplitude of oscillations increases indefinitely with time.

But it should be taken into account that seismic impacts on structures are limited in time. Then the question arises of to what values the amplitude of fluctuations will increase during this time and whether this increase will be the cause of significant damage [15-16].

The amplitude of oscillations in the mode of the main parametric resonance, its change is described by the following law:

\[ \varphi(t) = a_0 \cdot \sin \frac{\alpha t}{2} + b_0 \cdot \cos \frac{\alpha t}{2}. \]  
(7)

\[ a = a_0 e^{ht}, \quad b = b_0 e^{ht}. \]

The characteristic index \( h \) is calculated by the formula:

\[ h = \frac{\mu k}{2} - n, \]  
(8)

where \( \mu = \frac{A}{2} \) – excitation coefficient; \( n = \frac{\gamma k}{2} = \frac{\delta k}{2\pi} = \frac{\delta}{T} = \zeta k \), \( \delta \) – logarithmic decrement,

\[ T = \frac{2\pi}{k} \] – oscillation period, \( \zeta \) – attenuation in fractions of the critical.

Hence it follows that the nature of the amplitude of the oscillations mainly depends on the value of the characteristic exponent \( h \). If \( h < 0 \), then the amplitude of the oscillations decreases (the oscillations are damped); if \( h = 0 \), then the vibration amplitude is constant; if \( h > 0 \), then the amplitude increases indefinitely, the phenomenon of parametric resonance arises [17-18].

Now it is possible to find such a value of \( \gamma \), at which the phenomenon of parametric resonance does not develop.
The designation \( n^* \) is introduced - such a value of the parameter \( n \), at which \( h = 0 \).

\[
n^* = \frac{\mu k}{2}.
\]  

(9)

Then \( h < 0 \), if \( n > n^* \) or \( \frac{\gamma k}{2} > \frac{\mu k}{2} \), from where

\[
\gamma > \mu = \frac{A_{ncpm}}{2}.
\]  

(10)

Obviously, if condition (10) is not satisfied, \( \gamma < \frac{A_{ncpm}}{2} \), then parametric resonance arises. In this case, it is possible to estimate how long the oscillation amplitude will increase by a factor of \( m \), from the simple condition

\[
m \cdot e^{ht} = e^{h(t + \theta)},
\]  

(11)

from where amplitude rise time

\[
\theta = \frac{1}{h} = \ln \left( \frac{m}{h} \right).
\]  

(12)

Formula (12) determines the damping in a system with kinematic seismic isolation, which is necessary to exclude the possibility of dangerous parametric oscillations. So, on sites with accelerations of more than 0.4g, the minimum damping should be at least 10% of the critical (~0.2). With less damping, a doubling of the vibration amplitudes can occur rather quickly. This is illustrated in Fig. 6. For example, with a damping of \( \gamma \approx 0.15 \), a doubling of the oscillation amplitude will occur every 2.5sec, which should be considered catastrophic for an intense earthquake phase of 10-15sec [19-20].

![Figure 6](attachment:image.png)

**Figure 6.** Dependence of the doubling time of the oscillation amplitudes on the parameter \( \gamma \).
3. Conclusion
An analysis of the operation of kinematic seismic-isolated foundations, taking into account the vertical component of the seismic action, showed that with certain parameters of the seismic action and structure, a parametric resonance is possible in the system. It has been established that the motion will be stable when the value of the viscous drag coefficient exceeds the value of half the acceleration of the vertical disturbance, expressed in fractions of the acceleration of gravity.

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