Zeeman Coupling and Abnormal Thermal Conductivities in BSCCO Superconductors

Qiang-Hua Wang\textsuperscript{1,2}, and Z. D. Wang\textsuperscript{2}

\textsuperscript{1}Department of Physics and National Laboratory of Solid State Microstructures, Nanjing University, Nanjing 210093, China
\textsuperscript{2}Department of Physics, University of Hong Kong, Pokfulam, Hong Kong, China
(September 27, 1999)

Using the path-integral formulation we derive microscopically a Ginzburg-Landau free energy with a Zeeman coupling between the magnetic field and the orbital angular momentum of the Cooper pairs in a superconductor with singlet pairing in the $d_{x^2-y^2}$- and the sub-dominant $d_{xy}$- channels. The Zeeman coupling induces a time-reversal-symmetry-breaking pairing state. Based on careful examinations of the energy gain due to the Zeeman coupling, the energy lost due to the kinetic energy of the excess superfluid, and the Doppler energy shift for quasi-particle excitations, we present a coherent interpretation for the puzzling and conflicting thermal conductivities observed at above 5K (K. Krishana, et al. Science \textbf{277}, 83(1997)) and at sub-Kelvins (H. Aubin, et al, Phys. Rev. Lett. \textbf{82}, 624 (1999)) in BSCCO superconductors.

74.20.De, 74.25.Fy, 74.60.Ex

Recently, the anomalous low-temperature thermal conductivity versus the magnetic field observed in BSCCO superconductors \cite{1,2} has been stimulating considerable interest in the study of the pairing states in the cuprates \cite{3,4}. Although it is widely accepted that the pairing is singlet, and that the dominant pairing channel is the $d_{x^2-y^2}$-channel \cite{5}, it is still interesting to investigate whether there is a bulk time-reversal-symmetry ($T$) breaking pairing states involving the $d_{x^2-y^2}$-channel and a sub-dominant channel, such as the $s$- or $d_{xy}$-channel \cite{6}. The abnormal field dependence of the low temperature (but above 5K) thermal conductivity $\kappa_e$ in BSCCO \cite{3} is remarkable, in that $\kappa_e$ is a bulk quantity, and the observed plateau in its field dependence suggests a possible bulk $T$-breaking states \cite{7}, so that a full gap is opened at the Fermi surface for quasi-particle excitations. On the other hand, at even lower (sub-Kelvin) temperatures, another anomaly arises: Instead of decreasing with increasing magnetic field $B$, $\kappa_e \propto \sqrt{B} \\textsuperscript{2}$. This behavior implies a pure $d_{x^2-y^2}$-wave pairing state, with which quasiparticle states are popularized at the Fermi surface along the nodal direction by the supercurrent around the vortices, with the induced density of states $\propto \sqrt{B} \\textsuperscript{2}$. Thus these two sets of $\kappa_e$ data seemingly point towards conflicting pairing states, and for a long time, remain puzzling.

It is now clear in this Letter that a Zeeman coupling between the magnetic field and the internal motion of Cooper pairs may shed remarkable lights on the abnormal thermal conductivities \cite{3}. Physically, the Zeeman coupling is expected based on general footing \cite{4}, in that a Cooper pair carrying an internal angular momentum also carries an internal magnetic moment, which is coupled to the magnetic field. This effect may be expected to cancel out after averaging over the Fermi surface, but will be examined seriously in this work. The main findings in this Letter are: (i) A sound microscopic derivation of the Zeeman coupling term in the GL theory; (ii) With the Zeeman coupling and the Doppler energy shift due to the superfluid for quasi-particle excitations, a coherent interpretation for both sets of the $\kappa_e$ data mentioned above is presented. The seemingly conflicting pictures regarding the pairing states relevant to $\kappa_e$ are unified.

For definiteness, we consider a superconductor with pairing in the $d_{x^2-y^2}$- and $d_{xy}$-channels. The pairing function is assumed to be $\Delta_{\mathbf{q} \mathbf{k}} = D_{\mathbf{q}} \cos 2 \theta_{\mathbf{k}} + D'_{\mathbf{q}} \sin 2 \theta_{\mathbf{k}}$, where $\mathbf{k}$ and $\mathbf{q}$ describe the internal motion and center-of-mass motion of the Cooper pair, and $D_{\mathbf{q}}$ and $D'_{\mathbf{q}}$ are the $\mathbf{q}$-modes of the order parameters in the $d_{x^2-y^2}$- and $d_{xy}$-channels, respectively. Here $\theta_{\mathbf{k}}$ is the angle between $\mathbf{k}$ and, say, the $a$-axis of the $ab$-plane. As is well accepted, the singlet pairing with a dominant $d_{x^2-y^2}$-channel is present in high temperature superconductors. The pairing interaction responsible for the pairing function is assumed to be $V_{\mathbf{k},\mathbf{k}'} \equiv V_D \cos 2 \theta_{\mathbf{k}} \cos 2 \theta_{\mathbf{k}'} + V_D' \sin 2 \theta_{\mathbf{k}} \sin 2 \theta_{\mathbf{k}'}$. The Bardeen-Cooper-Shrieffer (BCS) effective Hamiltonian reads,

$$H = \int_{\mathbf{x},\mathbf{x}'} \Psi_\uparrow^\dagger(\mathbf{x}) h_0(\mathbf{x}) \Psi_\uparrow(\mathbf{x}) + \Psi_\downarrow^\dagger(\mathbf{x}) V(\mathbf{x} - \mathbf{x}') \Psi_\downarrow(\mathbf{x}') \Psi_\downarrow(\mathbf{x}),$$

(1)

where $h_0(x) = (-i \nabla - eA)^2/2m$ is the single-particle Hamiltonian (with $\hbar = c = 1$) and $V$ is the pairing interaction (in real space). Repeated indices imply summation. The other notations are standard. Using the coherent Fermion path integral formulation of the BCS theory, the Ginzburg-Landau free energy can be obtained by a loop expansion at the saddle-point of the effective action after performing the standard Hubbard-Stratonovich transform that decouples the pairing interaction. It can be written as, up to the forth-order in the order parameters and symbolically,

\begin{align*}
\left(\Psi_\uparrow^\dagger(\mathbf{x}) \Psi_\uparrow(\mathbf{x}) \right)^4 \left(\Psi_\downarrow^\dagger(\mathbf{x}) \Psi_\downarrow(\mathbf{x}) \right)^4 \left(\Psi_\uparrow^\dagger(\mathbf{x}) \Psi_\downarrow(\mathbf{x}) \right)^4 \\
\frac{1}{m^2} \int_{\mathbf{x},\mathbf{x}'} \Psi_\uparrow^\dagger(\mathbf{x}) \Psi_\downarrow(\mathbf{x}') \Psi_\downarrow^\dagger(\mathbf{x}) \Psi_\uparrow(\mathbf{x}) + \Psi_\uparrow^\dagger(\mathbf{x}) \Psi_\downarrow(\mathbf{x}') \Psi_\downarrow^\dagger(\mathbf{x}) \Psi_\uparrow(\mathbf{x}) \Psi_\uparrow(\mathbf{x}) \Psi_\downarrow^\dagger(\mathbf{x}) \Psi_\uparrow(\mathbf{x}) \Psi_\downarrow(\mathbf{x}') \Psi_\downarrow^\dagger(\mathbf{x}) \Psi_\uparrow(\mathbf{x}),
\end{align*}
\[ F \approx \int \Delta^* V^{-1} \Delta - TT(r)(\Delta g^* \Delta^*) \]
\[ + TT(r)(\Delta g^* \Delta g^* \Delta^*), \]

where \( g \) is the normal state single-particle Matsubara Green's function and \( V^{-1} \) denotes the inverse operator of \( V \). In detail,

\[ g(x, x'; i\omega_n) \approx g_0(x, x'; i\omega_n) \exp[iA(x') \cdot (x - x')] \]  
for the relative motion in a Cooper pair (propagating in the imaginary time with a frequency \( \omega_n \)). Here \( g_0(x, x'; i\omega_n) \) is the Green's function at \( A = 0 \), and \( \omega_n = (2n + 1)\pi T \). The usual semi-classical approximation is used. The second term in Eq. (2) is, after some manipulations,

\[ F^{(2)} = -T \sum_{\mathbf{k}, \mathbf{k}'} \int_{\mathbb{R}^3} \left[ g_0(-\mathbf{k} + \mathbf{\Pi}/2 + e\delta \mathbf{A}; i\omega_n) \Delta^* (\mathbf{k}'; \mathbf{R}) \right] \]
\[ \times \left[ g_0(-\mathbf{k} + \mathbf{\Pi}/2 + e\delta \mathbf{A}; i\omega_n) \Delta (\mathbf{k}; \mathbf{R}) e^{i\mathbf{k} \cdot \mathbf{r}} \right] \]

where \( \mathbf{\Pi} = -i\mathbf{\nabla}_\mathbf{F}/2 - e\mathbf{A}(\mathbf{R}) \) is the gauge invariant gradient, \( \Delta(k; \mathbf{R}) = \Delta_0 e^{i\mathbf{q} \cdot \mathbf{R}} - \mathbf{\delta A}(\mathbf{R}) = \mathbf{r} \cdot \mathbf{\nabla} \mathbf{A}(\mathbf{R})/2 \approx \mathbf{B} \times \mathbf{r}/4 \) is a correction to the naive semi-classical approximation that is usually neglected. Here we have assumed \( \mathbf{A} = (B_1y/2, B_2x/2, 0) \) in the calculation of \( \delta \mathbf{A}(\mathbf{r}) \). This term generates the Zeeman coupling we discussed earlier. So let us first look into the energy in the first order of \( \delta \mathbf{A}(\mathbf{r}) \). After some algebra, we find it is,

\[ F_z = T \int_{\mathbb{R}^3} \left[ \psi_n^*(\mathbf{r}; \mathbf{R}) e\mathbf{B} \cdot \mathbf{\nabla}_\mathbf{r} \phi_n(\mathbf{r}; \mathbf{R}) + c.c. \right]/4, \]  

(4)
where \( \mathbf{L}(\mathbf{r}) = \mathbf{r} \times \mathbf{\nabla}_\mathbf{r}/i \) is the angular momentum operator for the internal motion of the Cooper pair, and \( \psi_n \) and \( \phi_n \) can be imagined as the dressed wave function for the relative motion in a Cooper pair (propagating in the imaginary time with a frequency \( \omega_n \)). Defined by \( \psi_n(\mathbf{r}; \mathbf{R}) = \alpha_n \Delta(\mathbf{r}; \mathbf{R}) \), \( \phi_n(\mathbf{r}; \mathbf{R}) = \beta_n \Delta(\mathbf{r}; \mathbf{R}) \), with

\[ \alpha_n = \int \frac{N(\epsilon) d\epsilon}{(i\omega_n + \epsilon)}; \quad \beta_n = \int \frac{N(\epsilon) d\epsilon}{(i\omega_n + \epsilon)^2}; \]

(5)
\[ \hat{\Delta}(\mathbf{r}; \mathbf{R}) \approx -J_2(kFr)/2[D(r) \cos 2\theta_r + D'(r) \sin 2\theta_r], \]  

(6)
where \( J_n \) is the Bessel function, \( N(\epsilon) \) is the normal state density of states, and \( \theta_r \) is the direction angle of \( \mathbf{r} \). After some further manipulations, we find

\[ F_z = \int_{\mathbb{R}^3} -i\hat{N}(0)(B/B_\ast)(D^* D' - c.c.), \]  

(7)
with \( B_\ast \) a characteristic magnetic field given by

\[ B_\ast^{-1} \sim \frac{m_F}{\pi}(N(0)/\rho_c)z(T) \int_{0}^{k_F r_0} (J_2(x))^2 dx, \]  

(8)
\[ z(T) = \int_{\epsilon_1, \epsilon_2} \frac{N(\epsilon_1) dN(\epsilon_2)/d\epsilon_2 f(\epsilon_1) + f(\epsilon_2) - 1}{2N(0)^2 \epsilon_1 + \epsilon_2}. \]  

(9)
Here \( m_F = ev_F \lambda_F/2 \) is a characteristic magnetic moment, \( \omega_c \) is the BCS energy cutoff, \( \rho_c \) is the density of charge carriers and \( r_0 \sim 2\pi v_F/\omega_c \) is a length cutoff because of the BCS truncation \( (\delta k \sim \omega_c/v_F \sim 2\pi/\delta r) \). Here \( v_F, \lambda_F \) and \( f(\epsilon) \) are the Fermi velocity, length and distribution, respectively. Although \( z(T) \) could not be evaluated exactly, an upper bound exists,

\[ |z(T)| < (W/8T) \int_{\epsilon} dN(\epsilon)/d\epsilon|N(0)|, \]

where \( W \) is the order of the band width. The fact that \( |z(T)| \) scales at most as \( T^{-1} \) is important to extrapolate it to low temperatures, as compared to the kinetic terms to be discussed below. Clearly, \( z(T) = 0 \) if \( N(\epsilon) \) is an even function of \( \epsilon \). Thus for a nonzero Zeeman coupling as described by Eq. (3), a necessary condition is the particle-hole asymmetry in the density of states \( [11,12] \). Such asymmetry might be related to strong coupling effects, although our derivation is in the weak coupling limit.

The other contributions to the free energy are usually \( z_3 \). For our purpose, we present explicitly the kinetic energy of the superfluid,

\[ F_k \approx \int_{\mathbb{R}^3} N(0)(v_F^2/16)\Gamma(\mathbf{PID}^2 + \mathbf{PID}'^2), \]  

(10)
where \( \Gamma = 7\zeta(3)/(8\pi^2 T^2) \) with \( \zeta \) being the Riemann zeta function. It is easy to see that the dimensionless factor \( \delta k = 8\Phi_0/(\pi v_F^2 B_\ast \Gamma) \) is a measure of the relative importance of the Zeeman energy as compared to the kinetic energy, recalling that \( F_z \) can also be cast into a similar form to that of \( F_k \) (i.e., in terms of gradients) because of the identity \( [\Pi_x, \Pi_y] = 2ieB \). Strictly speaking, the parameters in \( F_z \) and \( F_k \) are defined near \( T_c \) (\( = T_D \) here). But if extrapolated to low temperatures, we may expect that \( |\delta k| = T/T_c \) with an unknown constant \( T_c \) (applicable at \( T \lesssim T_c \)).

The complete GL free energy can be casted in the following form \( 3 \),

\[ F \approx F_c \int_{\mathbb{R}} [-|d|^2 - \alpha_d |d|^2 + |\pi d|^2 + |\pi d'|^2 - i\delta k b(d^d d' - c.c.) + |d|^4/2 + |d'|^4/2 + |d|^2 |d'|^2/3 + (d^d d' + c.c.)^2/6 + \kappa^2 b^2], \]  

(11)
where \( F_c = H_0^2 \xi^2/(4\pi) \) with \( H_0 \) and \( \xi \) being the thermodynamic critical field and the coherence length, respectively, when the \( d_{xy} \)-channel is absent. All quantities under the integration symbol are now dimensionless: \( \alpha_d = \ln(T_D/T)/\ln(T_D/T) \) with \( T_1 = (2\omega_c e^2/\pi)^{1/2} \) being the bare critical temperature for the \( i \)-th order parameter \( (i = D, D', \text{ and } \gamma \text{ is the Euler constant}) \); \( d = D/D_0 \) and \( d' = D'/D_0 \) are the normalized order parameters, with \( D_0 \) being the value of \( D \) at zero magnetic field and in the absence of \( D' \), \( \pi = \xi \mathbf{E} = -i\mathbf{\nabla} - \mathbf{a} \) is the dimensionless gauge invariant gradient, \( \kappa \) is the GL parameter,
\[ b = \nabla \times a = B / B_0 \] is the dimensionless magnetic field with \( B_0 = \Phi_0 / 2\pi \xi^2 \), and finally \( r = R / \xi \). The merit of the dimensionless form is to hide all irrelevant parameters, but we shall also use dimensionless and dimensioned forms interchangeably.

The Zeeman term violates both parity as well as time-reversal symmetry, leading to many nontrivial consequences. First, in the presence of a magnetic field, there will be a \( T \)-breaking \( d_{x^2−y^2} \pm id_{xy} \)-pairing state, with a minimum gap for quasi-particle excitations at the Fermi surface given by \( \min(|D_1|, |D_2|) \). Second, because the vector potential is coupled to the supercurrent, the Zeeman coupling should induce a spontaneous edge supercurrent. This effect may be used for an experimental verification of the Zeeman coupling. Third, there will also be a bulk first-order transition from the Meissner state to the mixed state with a finite density of vortices if \( \alpha_d' = 1 \), as discussed earlier.

Let us now discuss the relevance of the Zeeman coupling in the abnormal thermal conductivity observed in the cuprates. To be consistent with experiments, \( T_D \gg T_D' \). At low temperatures, \( D_0 \sim 2.13T_D \) is essentially independent of \( d \)-wave superconductors. From Eq. (11), there would be a zero-field \( T \)-breaking pairing transition at \( \alpha_d' = 1/3 \), or at a temperature \( T_\alpha = \sqrt{T_D^2 / T_D} \) if the above relation of \( \alpha_d \) were valid at all temperatures. Such a zero-field \( T \)-breaking pairing state posed the major difficulty in a previous theoretical study to explain the sub-Kelvin \( \eta \) data. In fact, the derived temperature dependence of the parameters in the GL theory is restricted near the superconducting transition temperature, and the above \( \alpha_d' \) is invalid at low temperatures. Instead, in the spirit of the two-fluid model for a general superconductor, the temperature dependence of \( \alpha_d' \) is better replaced by \( \alpha_d' \propto (1 − T^2 / T_D^2) / (1 − T^2 / T_D^2) \). The \( T \)-breaking transition (in the bulk) has not been observed experimentally yet, we demand that \( T_\alpha = 0 \), leading to \( \alpha_d' = (1 − T^2 / T_D^2) / 3 \). In this realistic case, \( \alpha_d' \) can only be induced by the Zeeman coupling (and is always out of phase with \( d \)). This permits a perturbative treatment of \( \alpha_d' \). At low fields and in the London limit, \( |d| \sim 1 \) at \( 1 \ll r \ll \kappa \) where \( r \) is the distance off a vortex core, while the field \( b \) is essentially uniform. Let us set \( \alpha_d' = −id \sgn(\delta k) \) (even in the vortex state), and find \( \eta \) variationally, which is at least a qualitative estimation. The dimensionless excess energy density due to the induced \( \alpha_d' \) is estimated as, to the second order in \( \eta \):

\[
\delta f = [1/3 − \alpha_d + b \ln(1/b)]\eta^2 − 2|\delta k|\eta, \tag{12}
\]

where the core energy of vortices is neglected, and the \( b \ln(1/b) \) term is the kinetic energy of \( \alpha_d' \). The latter is obtained as follows. In the absence of \( \alpha_d' \), we have a well-known magnetization curve at low and intermediate fields, \( H = B + (H_{c1} / \ln(B_0/B)) \) where \( H_{c1} \) is the lower critical field. The kinetic energy density of \( d \) due to its superfluid is roughly given by \( B(H − B) / 4\pi \) from the Virial theorem. The kinetic energy density of \( \alpha_d' \) is \( \eta^2 \) times that of \( d \), which enters our \( \delta f \) as it stands after proper normalization. Thus the optimum \( \eta \) is given by

\[
\eta = \frac{3|\delta k|b}{1 − 3\alpha_d + 3b \ln(1/b)} = \frac{3(T/T_x)b}{T^2/T_D + 3b \ln(1/b)} \tag{13}
\]

There would be an induced full gap at the Fermi surface of the size \( \eta D_0 \) in the bulk (excluding the vortex cores). However, as found earlier, the Doppler energy shift \( E_{\text{Doppler}} \) turns out to be essential to explain the low-field sub-Kelvin \( \kappa_e \). Roughly speaking, \( E_{\text{Doppler}} \sim P_v \eta \sim (a / \sqrt{\ln \kappa}) D_0 \sqrt{b \ln(1/b)} \), where \( P_v \) is the Fermi momentum, \( v_s \sim h / 2m v_F \) is the characteristic superfluid velocity with an inter-vortex spacing \( R_v \), and \( a \) is a vortex-lattice dependent constant of order unity. Here we have included the logarithmic correction in \( b \), in the same spirit as for the superfluid kinetic energy, in order to take into account the suppression of the superfluid arising from surrounding vortices. In a nodal \( d_{x^2−y^2} \)-wave pairing state, \( E_{\text{Doppler}} \) is responsible for the vortex induced density of states at the Fermi surface, scaling as \( \sqrt{b} \) at low fields. Collecting both effects, the net gap at the Fermi surface is \( \Delta_{\text{min}} = \eta D_0 − E_{\text{Doppler}} \). Of course, \( \Delta_{\text{min}} \) blocks (promotes) quasi-particle excitations if it is positive (negative). \( \kappa_e(B,T)/\kappa_e(0,T) \propto (1/T) \exp(−\Delta_{\text{min}}/T) \) would develop a kink at \( \Delta_{\text{min}} \sim T \). In the absence of the Doppler effect, the kink field \( B_k \) is given by \( T = T_D \sqrt{3(B_k / B_0)[D_0 / T_x − \ln(B_0 / B_k) / T_x]} \). At \( B_k \gg B_\chi = B_0 e^{−D_0 / T_x} \), we would have the celebrated power law \( B_k \sim T^2 \). At \( T \to 0 \), \( \eta D_0 \propto T \) due to the emerging role of the kinetic energy in \( \eta \), and the power law is violated. The Doppler effect emerges at low fields, with \( E_{\text{Doppler}} \propto \sqrt{b} \) within logarithmic accuracy, which dominates over \( \eta D_0 \), and one would expect \( \kappa_e \propto \sqrt{b} \). However, the competition is complicated by the role of temperature in \( \eta \), and the value of \( a / \sqrt{\ln \kappa} \) in \( E_{\text{Doppler}} \). In order to proceed, we take reasonable parameters: \( T_D = 100K \), \( T_D' = 8K \), \( T_x = 10K \) and \( a / \sqrt{\ln \kappa} = 0.1 \). The field dependence of \( \Delta_{\text{min}} \) is shown in Fig.1(a) at the specified temperatures, where \( \Delta_{\text{min}} > 20K \) is disposed, as we are treating \( d' \) perturbatively. \( \Delta_{\text{min}} \) rises quasi-linearly at above \( 2K \), but \( \Delta_{\text{min}} < 0 \) for good at \( T = 0.1K \). This marks the Zeeman regime at above \( 2K \), the Doppler regime at sub-Kelvins, and the cross-over regime (grey zone) at intermediate temperatures, as highlighted in Fig.1(b). In the Zeeman regime, as we already declared, a kink in \( \kappa_e(B) \) develops at \( \Delta_{\text{min}} \sim T \). The kink points extracted from Fig.1(a) is plotted in Fig.1(b) (squares), where the dotted line is a fit to the power law \( B_k \sim T^2 \). In the Doppler regime, we naturally expect an increasing \( \kappa_e(B) \), since \( \kappa_e \propto N(0) \Delta_{\text{min}} \) (with \( \Delta_{\text{min}} < 0 \)). Moreover, the scaling.
law $\kappa_c \propto \sqrt{B}$ is already visible at $T = 0.1K$ in Fig.1(a),
as if in a pure $d_{x^2-y^2}$-wave state.

On the other hand, the impurity scattering was indicated to be important in a pure $d_{x^2-y^2}$ superconductor in the Doppler regime to explain a monotonically increasing $\kappa_c(B)$ at low temperatures, and a nonmonotonic field dependence at higher temperatures [2,4], but without a $T$-breaking state at higher temperatures and fields, nothing could be said for the power law in the kink field. Therefore, the present theory nicely bridges the ‘gap’ between the existing conflicting pictures regarding the pairing states [2–5].

Finally, we remark that it was argued [16] that the competition between the quasi-particle density of states and the vortex scattering effect could be responsible for the plateau in $\kappa_c(B)$. However, this scenario alone is difficult to account for the celebrated power law $B_0 \propto T^2$ [1]. We think that including the vortex scattering effects in our theory would render even more promising agreement with the experiments. On the other hand, the role of a sub-dominant $s$-wave pairing can be ruled out, as it does not participate the Zeeman coupling by symmetry, so that it can only be induced locally by inhomogeneities [5]. In contrast, the Zeeman coupling is effective in the bulk because $B$ is essentially uniform in the high $\kappa$ limit.

This work was supported by National Natural Science Foundation of China and the RGC grant of Hong Kong under No. HKU7116/98P and HKU 7144/99P. We also thank Dr. Shun-Qing Shen for helpful discussions.

[1] K. Krishana, et al, Science 277, 83(1997).
[2] H. Aubin, K. Behnia, S. Ooi, and T. Tamegai, Phys. Rev. Lett. 82, 624(1999).
[3] R. B. Laughlin, Phys. Rev. Lett. 80, 5188(1998).
[4] C. Küber, and P. Hirschfeld, Phys. Rev. Lett. 80, 4963(1998).
[5] Qiang-Hua Wang, Z. D. Wang, and Q. Li, Cond-Matt/9906210; Phys. Rev. B 60 (in press).
[6] C. C. Tsuei, et al, Phys. Rev. Lett. 73, 593(1997); J. R. Kirtley, et al, Nature 373, 225(1995).
[7] M. Franz, and Z. Tesanovic, Phys. Rev. Lett. 80, 4763 (1998); I. Maggio-Aprile, et al, Phys. Rev. Lett. 75, 2754 (1995); M. Covington, et al, Phys. Rev. Lett. 79, 277 (1997).
[8] G. E. Volovik, Pis’ma Zh. Eksp. Teor. Fiz. 58, 457(1993), JETP Lett. 58, 469(1993).
[9] Such a coupling was included phenomenologically and emphasized in a recent study [5], where the $\kappa_c$ data at above 5K was explained successfully, but the sub-Kelvin data remained puzzling.
[10] M. Sigrist, and K. Ueda, Rev. Mod. Phys. 63, 239 (1991).
[11] T. Koyama, and M. Tachiki, Phys. Rev. B 53, 2662(1996).
[12] This same condition was discussed in Ref. [11] but we need no ‘ghost’ $p$-wave pairing interaction (that does not lead to a $p$-wave gap).
[13] Y. Ren, J. H. Xu, and C. S. Ting, Phys. Rev. Lett. 74, 3680(1995); R. Joynt, Phys. Rev. B 41, 4271 (1990); A. J. Berlinsky, A. L. Fetter, M. Franz, C. Kallin, and P. I. Soininen, Phys. Rev. Lett. 75, 2200(1995).
[14] Z. D. Wang, and Q. H. Wang, Phys. Rev. B 57, R724(1998); Q. H. Wang, and Z. D. Wang, ibid 57, 10307(1998).
[15] M. Tinkham, “Introduction to Superconductivity” (second edition), McGraw Hill 1996.
[16] M. Franz, Phys. Rev. Lett. 82, 1760 (1999).