The Cold Dark Matter crisis on galactic and subgalactic scales

Argyro Tasitsiomi

Department of Astronomy & Astrophysics, Center for Cosmological Physics, The University of Chicago, 5640 S. Ellis Ave., Chicago, IL 60637, USA

Abstract

The Cold Dark Matter (CDM/ΛCDM) model describes successfully our Universe on large scales, as has been verified by a wide range of observations. Nevertheless, in the last years, and especially with the advances in numerical simulations, a number of apparent inconsistencies arose between what is observed and what the CDM model predicts on small scales. In this work, the current status of observations on galactic and subgalactic scales is reviewed. Furthermore, theory and observation are brought together in order to reveal the nature of the inconsistencies and, consequently, to reveal their severity. Lastly, the progress towards the resolution of each one of these conflicts is briefly reviewed.
I. INTRODUCTION

There is something more in the Universe than luminous matter. This additional, missing mass component is named dark matter. Deciphering the nature of dark matter is among the most important cosmological problems seeking solution. Since ordinary matter is baryonic, the assumption that dark matter is baryonic as well is the obvious first assumption to make. Thus the diffuse, warm, intergalactic medium, cold $H_2$, Massive Compact Halo Objects (MACHOs), etc., have been proposed as baryonic dark matter candidates. Nevertheless, not all of the dark matter can be baryonic, due to constraints imposed on the contribution

---

1 MACHOs could be stellar evolution remnants (white dwarfs, neutron stars, black holes) of an early generation of massive stars (population III stars), or smaller objects that never initiated nuclear burning, e.g., brown dwarfs.
of baryons to the critical density ($\Omega_b$) by Big Bang Nucleosynthesis (Burles et al., 2001a, 2001b), as well as by the observed CMB anisotropies (Balbi et al., 2000; Lange et al., 2001; Pryke et al., 2002). For example, assuming a baryon density large enough to account for the dark matter will lead to CMB anisotropies that are larger than that observed. Thus, at least part of the dark matter has to be non-baryonic.

The first non-baryonic dark matter candidate studied was the (light) neutrino, a particle known to exist. Nevertheless, there is now significant evidence against neutrinos as the bulk of the dark matter. More specifically, neutrinos would be hot dark matter. Namely, they were relativistic when they decoupled. For our purposes, what is even more important is the fact that they were still highly relativistic when the horizon encompassed for the first time the amount of matter found in a typical galaxy (Primack, 2001, and references therein). It can be proven that in a universe dominated by ordinary, light neutrinos structure forms top-down, that is, large structures form first and smaller structures later by fragmentation of the larger objects. This is opposite to what is inferred from observation; for example, according to CMB and cluster abundance measurements, the power spectrum is that of the CDM model (see Fig. 1), and thus, structure has formed bottom-up, namely small structures formed first and then they merged towards the creation of larger structures.

Thus, a lot of effort went to the so-called Cold Dark Matter (CDM) (Blumenthal et al., 1982; Peebles, 1982). In the CDM model, dark matter consists of non-baryonic, collisionless, cold particles. These particles are said to be cold, since they decoupled while non-relativistic. In addition, they are assumed to have had small primeval (before structure formation) velocities, either because they were thermally produced (exactly as in the case of hot dark matter), but with a considerably high rest mass as, e.g., Weakly Interacting Massive Particles (WIMPS) are usually assumed to be, or because they were (non-thermally) created with momenta well below $k_B T$, e.g., axions [for a non-thermal WIMP production scenario, see Lin et al. (2001)]. The most well-known WIMP, the neutralino, is predicted by the Minimal Supersymmetric extension of the Standard Model (MSSM) of particle physics (Haber and Kane, 1985). It is, by definition, the Lightest Supersymmetric Particle (LSP) with a mass in the range of several tens of GeV up to a TeV. Axions, on the other hand, are predicted by extensions of the Standard Model that resolve the strong CP problem (Turner, 1990). They can occur in the early Universe in the form of a Bose condensate that never comes into thermal equilibrium. Axions formed in this way are non-relativistic and can be a significant
dark matter contribution if their mass is \( \simeq 10^{-5} \) eV. They can also be produced from the
decay of a network of axion strings and domain walls [for more on axions, see Kolb and
Turner (1994)].

An essential part of a cosmological model is the power spectrum of density fluctuations
that it assumes (see Fig. [3]). In the CDM model, structure grows out of random, Gaussian,
adibatic (curvature 2) fluctuations. These primordial density fluctuations are assumed to
have the following power-law spectrum

\[
P_k \propto k^n
\]

with \( n \) being the spectral index. There are several constraints on \( n \), and the most widely
used value is \( n = 1 \) that corresponds to a scale-invariant spectrum, known as the Harrison-
Zel’Dovich spectrum (Harrison, 1970; Zel’Dovich, 1972).

Numerous studies (e.g., Davis et al., 1985; Bardeen et al., 1986; Peebles, 1993, and
references therein) have been carried out regarding the evolved form of this power spectrum.
A simple form published, valid after matter-radiation equality and before the development
of nonlinear structures, is (Peebles, 1993, and references therein)

\[
P_k \propto \frac{k}{(1 + \alpha k + \beta k^2)^2}
\]

where

\[
\alpha = \frac{8}{(\Omega h^2)\text{Mpc}}, \quad \beta = \frac{4.7}{(\Omega h^2)^2}\text{Mpc}^2.
\]

This form, even though not very detailed, reveals the important feature of the spectrum:
the spectrum bends gently from \( P_k \propto k^{-3} \) (large \( k \), subgalactic scales) to \( P_k \propto k \) (small \( k \),
larger scales).

The model is definitely motivated by the inflation scenario, even though it is not uniquely
predicted by it. There are some relatively simple versions of inflation that predict exactly
the power spectrum that the CDM model has adopted. There are, however, other versions
of inflation that lead either to quite different forms for the spectrum, or to isocurvature
fluctuations [see, e.g., Efstathiou and Bond (1986)].

2 Curvature fluctuations are fluctuations in the energy density that can be characterized in a gauge-invariant
manner as fluctuations in the local value of the spatial curvature. The name adiabatic comes from the
fact that, in such perturbations, the corresponding fluctuations in the local number density of any species
with respect to the entropy density vanishes [see, e.g., Kolb and Turner (1994)].
Even so, cosmological models based on the paradigm of an inflationary universe with CDM, and either a cosmological constant or quintessence (Caldwell et al., 1998; Carroll, 2001a, 2001b), have recently enjoyed great success on large scales [for a review on CDM successes, see Bahcall et al. (1999), or Wang et al. (2000)]. Impressively diverse data like measurements of the CMB background radiation (e.g., Lee et al., 2001; Halverson et al., 2002; Netterfield et al., 2002), of the abundances of deuterium and other light elements (e.g., Burles et al., 2001a), of the absorption lines in the spectra of distant quasars (e.g., Efstathiou et al., 2000), surveys in the positions in redshift space of hundreds of thousands of galaxies (e.g., Peacock et al., 2001), measurements of the brightness of distant supernovae (e.g., Riess et al., 1998; Perlmutter et al., 1999), of the age of the Universe as measured from the oldest stars (e.g., Chaboyer et al., 1998), of the extragalactic distance scale as measured by distant Cepheids (e.g., Madore et al., 1998; Freedman et al., 2001), of the baryonic mass-fraction of galaxy clusters (e.g., White et al., 1993), of the present-day abundance of massive galaxy clusters (e.g., Eke et al., 1996; Carlberg et al., 1997; Bahcall and Fan 1998), of the shape and amplitude of galaxy clustering patterns (e.g., Wu et al., 1999), of the magnitude of large-scale coherent motions of galaxy systems (e.g., Zaroubi et al., 1997), etc., all point to a single new cosmology (Turner, 2002). In this cosmology ($\Lambda$CDM), matter makes up at present less than about one third ($\approx 0.3$) of the critical density, and a nonzero ($\approx 0.7$) cosmological constant ($\Lambda$) restores the flat geometry predicted by most inflationary models of the early Universe. The present rate of expansion is $H_0 \approx 70\text{km s}^{-1}\text{Mpc}^{-1}$, baryons make a very small fraction of the mass of the Universe ($\Omega_b \approx 0.0195h^{-2}$), and the present day rms mass fluctuation, $\sigma_8$, in spheres of radius $8h^{-1}\text{Mpc}$ is of order unity.

Nevertheless, an array of observations on galactic and subgalactic scales appears to be in conflict with both analytical calculations and numerical simulations done in the frame of the CDM model. These conflicts are what this article is about. More specifically, I focus on the discrepancy that exists between the theoretical predictions and the observations with respect to the central mass distribution of galaxies, to the shapes of galactic halos, to the existing substructure in these galactic halos, and to the angular momentum of the disks of galaxies. Throughout the paper, no distinction is made between the standard CDM and the $\Lambda$CDM models, unless their predictions differ interestingly.
II. MASS DISTRIBUTION IN THE INNER PARTS OF GALAXIES

Cold dark matter halos are the result of a complicated sequence of hierarchical mergers that lead to a global structure set primarily by violent relaxation. 3 Simulations of CDM halos (e.g., Navarro et al., 1995, 1996; Moore et al., 1998, 1999a, 1999b; Klypin et al., 1999; Ghigna et al., 2000; Jing and Suto, 2000) result in halo density profiles, for masses in the range $10^7 M_\odot - 10^{15} M_\odot$, that agree with a density profile of the general form introduced by Zhao (1996)

$$\rho(r) = \frac{\rho_o}{(r/r_o)^\gamma \left[1 + (r/r_o)^\alpha (\beta-\gamma)/\alpha \right]}$$

where $\rho_o$ is the characteristic density of the halo, and $r_o$ is its characteristic scale radius. The characteristic density is defined as follows,

$$\rho_o = \rho_{\text{crit}} \delta_c$$

where $\rho_{\text{crit}}$ is the critical density today, and $\delta_c$ is the dimensionless overdensity of the halo that depends on the collapse redshift and on the mass of the halo. The usual definition of $r_o$ is given in terms of the logarithmic slope $d \log \rho / d \log r$ of the density profile. It is defined as the distance from the center of the halo where the logarithmic slope of the profile equals $-(\beta+\gamma)/2$, with $\beta$ and $\gamma$ the same as in Eq. (4). Another quantity often used to characterize the dark matter halos is the concentration parameter $c$. This parameter is defined as the ratio of the virial radius, $r_{\text{virial}}$, of the halo to its scale radius, $r_o$, namely $c = r_{\text{virial}}/r_o$. The virial radius, on the other hand, is defined as the distance from the center of the halo that roughly encloses the region where matter is virialized. Often, $r_{\text{virial}}$ is taken to be equal to $r_{200}$, namely the radius of a sphere of a mean interior density equal to $200 \rho_{\text{crit}}$. Note that the characteristic overdensity $\delta_c$ and the concentration index $c$ are not independent parameters; they are linked by the requirement that the mean density within $r_{200}$ be equal to $200 \rho_{\text{crit}}$.

Originally, it was found that the dark matter density distribution in halos is well fitted by the specific case of Eq. (4) known as the Navarro, Frenk and White (NFW) (Navarro et al., 1995, 1996, 1997) density profile corresponding to $(\alpha, \beta, \gamma) = (1, 3, 1)$. Higher resolution simulations that followed (Fukushige and Makino, 1997, 2001; Moore et al., 1998, 1999b; Ghigna et al., 2000; Jing and Suto, 2000) found somewhat steeper profiles at small radii.

---

3 A collisionless relaxation process taking place in time-varying gravitational fields. For more details, see, e.g., Binney and Tremaine (1994).
with a behavior in agreement with the Moore et al. (1999b) density profile corresponding to \((\alpha, \beta, \gamma) = (1.5, 3, 1.5)\). The two profiles differ essentially only in their \(r \to 0\) \((r < r_o)\) behavior, with the Moore et al. being steeper, whereas at larger radii \((r > r_o)\) they both behave roughly as \(r^{-3}\).

Ongoing debate centers on the radial dependence of the logarithmic slope, as well as on whether this slope indeed converges to a well defined central value. Several opinions exist. For example, taking into account the conventional way of defining the center of a halo, namely by finding the most tightly bound particle, and given that the number of particles per halo in the simulations is finite, it becomes clear that, eventually, the choice of the center of the halo may be somewhat arbitrary; any departure from the true center of the halo, which is very probable given the currently feasible resolution, will give the impression of a halo less cuspy, namely less rapidly divergent at \(r \to 0\), than it actually is (Wandelt et al., 2000). This is however only one simple way of analyzing the problem. In general, resolution and discreteness effects can do either, namely they can lead to artificially low central densities, or to the formation of artificially dense central cusps (Navarro, 2001). To solve these problems, several convergence studies (e.g., Klypin et al., 2001; Navarro, 2001; Power et al., 2002) have been conducted. These studies consider the effects of the numerical parameters of the simulations – such as the timestep, the number of particles, and the gravitational softening – on the density profile of the simulated halos. As expected, the two models were found to be similar at large radii (above 1% of the halo virial radius); in the case of the NFW profile and for small \(r\) where the profile takes the form \(r^{-\alpha}\), it was found that the asymptotic slope \(\alpha = 1\) is not obtained in the simulations even at radii less than 1% – 2% of the virial radius, whereas the Moore et al. profile was found to be a very accurate fit for radii larger than 0.5% of the virial radius (Klypin et al., 2001). Navarro (2001) found that the radial dependence of the slope differs from that of the NFW profile, that the NFW profile underestimates the density at radii just inside \(r_o\), and that the Moore et al. profile describes the simulated halos better than the NFW profile in the range 0.15 < \(r/r_o\) < 0.5, even though it deviates systematically at smaller radii. Recently, Power et al. (2002) found that the logarithmic

---

4 Each particle in N-body simulations stands for a large number of less massive particles. For the system to be collisionless, one must ensure that there will be no close encounters among the massive particles. To prevent these collisions, one softens the force below a certain distance, that is sets the gravitational force equal to a constant, instead of \(\propto r^{-2}\).
slope of the spherically-averaged density profile is less than 1.2 and that the profile becomes increasingly shallow – has smaller and smaller logarithmic slope – inwards, with little sign of convergence to an asymptotic value in the inner regions. From the above discussion, if nothing else, it becomes clear that things are anything but settled.

No matter what the exact value of $\alpha$ is, it is a fact that so far CDM models predict a structure for the halo density profiles that behaves as $r^{-\alpha}$ at small radii, with $\alpha \geq 1$; this profile diverges as $r \to 0$, leading to the existence of a central cusp. This is to be expected since CDM particles are by definition moving slowly, and therefore there are no primordial phase space constraints $^5$ that could impose a cosmologically significant scale (Moore, 1994) [as opposed, for example, to the case of Warm Dark Matter particles (Alam et al., 1990)]. This also implies that CDM is consistent with cusp - and not with core - dominated halos, $^6$ since the core radius will be negligible (Moore 1994; van Albada, 1982).

This is what one expects from the CDM model regarding the central mass distribution of halos. It is time to see what observation has to say about this topic.

**A. Rotation Curves**

As ironic as it is, the potential problems with the structure of CDM halos were first highlighted by the observations this theory was initially designed to resolve: the flat rotation curves.

More specifically, the analysis of rotation curves of some nearby dwarf galaxies $^7$ has indicated that the steep rotation curves implied by CDM are hard to reconcile with the observed shallow rotation curves. $^8$ Flores and Primack (1994) found that the rotation curve

---

$^5$ This arises from an argument given by Tremaine and Gunn (1979). The basis for this argument is that the maximum coarse-grained phase space density of halo dark matter cannot exceed the maximum primordial fine-grained phase space density, which is conserved for collisionless matter. Estimating the maximum phase space mass-density, one finds that it only depends on the mass of the particle. In the case of a cold, massive particle, this quantity is large, and thus in practice there is no constraint.

$^6$ Halos with a central, essentially, constant density core.

$^7$ Dwarf (elliptical or spiral) galaxies are galaxies with considerably fewer stars and smaller sizes than their normal counterparts. For instance, Leo II is a dwarf elliptical galaxy with approximately one million stars, which means that its luminosity is about equal to the luminosity of the brightest individual stars, and a diameter of about 1.5 kpc. Note that the majority of elliptical galaxies are dwarf galaxies (Morrison et al., 1995).

$^8$ Steep and shallow rotation curves mean rapidly rising and slowing rising rotation curves, respectively (see
data of the well-studied, gas rich dwarf spirals DDO 154 and DDO 168 are inconsistent with a density profile singular at the center. Burkert (1995) derived similar conclusions after studying seven dwarf spiral galaxies. Both studies determined that these dark matter halos have a constant density core, namely that they are core- and not cusp-dominated. Another study of nine low-luminosity, late-type galaxies, in addition to the aforementioned, concluded that the constant density inner region (the core) was comparable to, or even greater in size, than the galaxy optical extent (Borriello and Salucci, 2001; Salucci and Borriello, 2001). Recently, Bolatto et al. (2002) have achieved high resolution combining CO and Hα observations in order to construct the rotation curve of the dwarf spiral NGC 4605; nevertheless, their results were similar to the previous cases and thus did not support the CDM model.

The so-called Low Surface Brightness galaxies (LSB) present yet another opportunity for studying the dark matter distribution by means of rotation curves – even though with larger uncertainties in the observational data compared with dwarf galaxies (Firmani et al., 2000, and references therein). These galaxies span a range of masses and have a large fractional amount of dark matter inside their optical region (Persic et al., 1996; de Blok and McGaugh, 1997). Thus, the contribution of the luminous matter – that is of the disk, bulge, and bar (if present) – to the gravitational potential is small. This, along with the fact that their stellar distribution is almost a perfect exponential, simplifies considerably their mass modeling (Borriello and Salucci, 2001).

The analysis of 19 LSB HI rotation curves by de Blok and McGaugh (1997) indicated that they rise less steeply than their High Surface Brightness (HSB) counterparts of similar luminosity, contrary to CDM predictions. After this analysis, other studies (e.g., Moore et al., 1999b; Firmani et al., 2000) came to verify that the observed rotation curves in LSB galaxies rise less steeply than predicted by CDM models (see Fig. 2). Furthermore, the behavior of the observed LSB rotation curves requires a constant mass density in the central

---

9 LSB galaxies are conventionally defined as galaxies that have an extrapolated central disk surface brightness in the blue band fainter than 23 mag/arcsec$^2$ (Freeman, 1970). In practice, they are a mixed group including objects as diverse as giant gas-rich disks and dwarf spheroidals.

10 Galaxies with central disk surface brightness $\mu_B \simeq 21.65 \pm 0.30$ mag/arcsec$^2$ [Freeman’s law, (Freeman, 1970)]. Note that the distribution in surface brightness from HSB to LSB galaxies is continuous, and the apparent gap is only because of the convention used to define the LSB galaxies.
region (Moore et al., 1999b; Firmani et al., 2000; de Blok, McGaugh, and Rubin, 2001; Moore, 2001), and this implies core-dominated halos; in fact, de Blok, McGaugh, and Rubin (2001) found that the density distribution profiles inferred from observations appear to be fitted accurately by pseudo-isothermal profiles, \(^{11}\) which are core-dominated, whereas the cusp-dominated CDM profile systematically deviates from the data, often having a small statistical probability of being the appropriate model. Recently, de Blok and Bosma (2002) via their study of high resolution rotation curves for 26 LSB and dwarf galaxies verified the aforementioned, finding in addition that the constant density cores are kpc-sized.

Moore et al. (1999b) reached similar conclusions about a constant density central region when they compared the scale-free shape of the observed rotation curves with simulation data. For \(\rho(r) \propto r^{-\alpha}\), results from high quality measurements of rotation curves indicate a best fit of \(\alpha \simeq 0.5\), with significant scatter, which is less divergent than the CDM profiles (Wandelt et al., 2000, and references therein). For a large sample of LSBs it was found that the distribution of inner slopes is peaked around \(\alpha = 0.2\), whereas \(\alpha \simeq 1\) was found to be the case only for the worst resolved cases (de Blok et al., 2001). Furthermore, only extremely well-measured rotation curves constrain simultaneously both the slope \(\alpha\) and the concentration parameter (van den Bosch et al., 2000). Thus, in practice, the majority of observations nowadays do not resolve these two parameters independently. This means that we can rephrase the problem in terms of the concentration parameter: CDM models yield a wide range of possible concentration indices [with values 10-20 (Navarro et al., 1996; Bullock et al., 2001b)], but most fits to rotation curves yield values well below (with values 6-8) the predicted range in all types of galaxies, even in the case of the Milky Way (Navarro and Steinmetz, 2000a).

Another related issue is the value of the central density and the way it varies from dwarf galaxies to clusters of galaxies. In fact, an analysis of data for dwarf and LSB galaxies, as well as some clusters of galaxies, found that the halo central density is nearly independent of the mass from galactic to galaxy cluster scales with an average of 0.02 M\(_{\odot}\) pc\(^{-3}\) (Firmani et al., 2000), whereas in the frame of CDM the central density is roughly proportional to the mean cosmic density at the time of collapse; this means that small mass galaxies, which collapsed

\(^{11}\) Profiles that exhibit the singular isothermal behavior \(r^{-2}\) at large distances, but have a constant density core, namely they are described by the expression: \(\rho(r) = \rho_0 [1 + (r/r_{\text{core}})^2]^{-1}\), with \(\rho_0\) and \(r_{\text{core}}\) the core density and radius, respectively.
sooner, when the cosmic density was high, are expected to have much higher densities than, e.g., clusters which formed much later, when the Universe was less dense. Thus, according to Firmani et al. (2000), there is an additional discrepancy, that of the central density dependence on the mass of the object. However, Shapiro et al. (2002) showed that when more and better data are used, then CDM and observation are generally in agreement.

In addition, the comparison of observations of LSB galaxies with CDM predictions can be often dismissed because of the limited resolution of the observed HI rotation curves. The early LSB rotation curves were obtained using the Very Large Array (VLA) and the Westerbork Synthesis Radio Telescope (WSRT). The relatively large beams of these instruments resulted in rotation curves with only a limited resolution, and the effect of this on the obtained rotation curves is known as beam smearing (de Blok and McGaugh, 1997; de Blok, McGaugh, and Rubin, 2001). Re-examining the rotation curves for a sample of LSBs, taking this time into account the beam smearing, van den Bosch et al. (2000) concluded that the observations are of such low resolution that they place little, or no constraints on the inner shapes of the dark matter profiles. The only galaxy for which they had very well-resolved data, and which consequently was their most reliable object with respect to constraining the dark matter profile, was found to have an inner slope exactly in agreement with the NFW density profile. Similar conclusions were reached for dwarf galaxies. More specifically, van den Bosch and Swaters (2001) analyzed and fitted the rotation curves of 20 late-type dwarf galaxies by mass models with different cusp slopes, ranging from constant density cores to $r^{-2}$ cusps. Due to the large uncertainties, no unique mass decompositions were found, and it was thus concluded that the rotation curves cannot be used to discriminate between halos that at their central parts have a constant density or are cuspy. Furthermore, Swaters et al. (2000) verified that optical data (H$_\alpha$) sometimes indicate a steeper rise of the LSB rotation curves compared with what was inferred for the same galaxies via HI observations. The general shapes of the rotation curves of these LSB galaxies were found to be almost identical to the shapes of the rotation curves of HSB galaxies, in agreement with CDM predictions.

---

12 Almost identical shapes in the sense that the same fraction of the maximum rotation velocity – the velocity of the flat part of the rotation curve – is reached at the same distance from the center of the galaxy, with the distance measured in units of the optical disk scale-length of the galaxy (which is, in principle, different for different galaxies).
Regarding other types of galaxies, apart from dwarf and LSB ones, the rotation curves of a large number of normal late-type galaxies are consistent with a fixed initial halo shape, characterized by a significant core inner region (Hernandez and Gilmore, 1998) with core radii much larger than a disk scale-length (Salucci, 2001). Work on barred galaxies (Debattista and Sellwood, 1998, 2000; Weiner et al., 2000) has shown that any dark matter halo makes a negligible contribution to the inner rotation curve, even after the formation of the disk and the bulge. Furthermore, there is empirical evidence against a systematic difference between barred and unbarred galaxies, at least in the case where these galaxies are HSB ones (e.g., Mathewson and Ford, 1996; Debattista and Sellwood, 1998, 2000, and references therein; Weiner et al., 2000, and references therein). They have similar overall HI properties, they appear to lie on the same Tully-Fisher relation and to have similar total mass-to-light ratios. These similarities, in addition to that halo dark matter does not appear to be the controlling factor with respect to whether a galaxy is barred or unbarred, imply that the dark matter distribution in HSB barred and unbarred galaxies is similar, and thus the conclusions derived regarding the dominance of the luminous matter in the central parts of barred galaxies may well be applicable in the case of their unbarred counterparts. A similar generalization is sometimes done starting from what has been concluded for LSB galaxies and apply it to HSB galaxies. The basis for such a generalization is the fact that LSB galaxies are abundant, span a wide range of masses, show the same diversity as HSB galaxies (from giant disks to dwarf spheroidals), have normal baryon fractions (at least as many as HSB galaxies), lie on the same Tully-Fisher relation, have similar structure with HSB galaxies – namely their light distribution falls of exponentially – and finally, they have properties that connect smoothly to the properties of HSB galaxies. That they have lower surface brightness may result from larger than average angular momentum in the dark matter component. On the basis that angular momentum is uncorrelated with both the environment and the halo structure, a generalization from the LSB to the HSB galaxies can be valid (see, e.g., Moore et al., 1999b).

In fact, results similar to those obtained for LSB galaxies were obtained for a large number of bright spiral galaxies, namely, the inner part of these galaxies was found to be dominated by luminous matter. The impressive match between the inner rotation curves and the predictions from luminous matter alone often suggests the model of the maximum disk, at least in the case of the large spirals (Kent, 1986; Palunas and Williams, 2000). In this
model, the rotation curve of the stellar component is scaled to the maximum value allowed by the observational rotation curve, with the requirement that the dark matter density be positive at all radii, in order to avoid a so-called hollow halo. The fact that large spirals were found to be described adequately by the maximum disk model indicates that the light distribution is an excellent predictor of the shape of the inner part of the rotation curve, namely that dark matter might even be virtually absent from the central parts of large spiral galaxies.

Another possible problem regarding the rotation curves is the disk-halo conspiracy (Bahcall and Casertano, 1985); it refers to that if, according to the aforementioned, the luminous matter is dominant in the central regions of galaxies, one expects a feature in the galaxy rotation curves signifying the passage from luminous to dark matter as the dominant source of gravity. Many galaxies are known nowadays, in which the rotation curve drops somewhat at the edge of the visible disk, but very rarely the drop exceeds about 10% [see, e.g., Casertano and van Gorkom, (1991)]. Given that the observed feature is very weak, it is accurate enough to say that observation verifies that the orbital circular speed from the luminous matter at the center is similar to the orbital circular speed from dark matter at larger radii. A conspiracy comes into existence on the basis that unless the dark and luminous matter are related, the production of a flat rotation curve requires that the initial conditions for both components, dark and luminous, be finely tuned. However, before concluding with respect to the existence of such a conspiracy, one must take into account the compression of the dark matter by the baryons during baryonic infall. This process provides a coupling between the baryons and the dark matter and thus might result in a total density distribution without any particular features or wiggles to signify the transition from the one component to the other. Actually, this was what Klypin, Zhao, and Somerville (2001) found when they applied the standard theory of disk formation within ΛCDM cuspy halo models, including the effects of adiabatic compression. In their models such a conspiracy is natural and anything but surprising.

Summarizing, for several types of galaxies – with LSB galaxies the most numerous – it appears that their rotation curves constitute observational evidence for a smaller central

---

13 In the simple case of a spherical system of particles on circular orbits, adiabatic compression means compression conserving both the mass and the angular momentum.
mass concentration than predicted by CDM. This is inferred by the fact that the central parts of the galaxy rotation curves rise less steeply than predicted. These pieces of evidence would prove the existence of a problem if, for example, a unique mass decomposition could be obtained from a rotation curve, or if observations were reliable, neither of which is necessarily true. Lastly, the “obvious” existence of a disk-halo conspiracy, might well be the result of incomplete understanding and treatment of the physical processes involved.

B. The Tully-Fisher relation

The Tully-Fisher relation (TFR) (Tully and Fisher, 1977) predicts that the luminosity, L, of a spiral galaxy correlates with its rotation velocity, \( v_{\text{rot}} \). This correlation is, essentially, a correlation between the mass of the luminous galactic components with the rotation velocity. Ideas regarding its origin may be grouped in two broad categories: the one that sees the TFR as a result of self-regulated star formation in disks of different mass (e.g., Silk, 1997), and the other that sees the TFR as a direct consequence of the cosmological equivalence between mass and rotation velocity (e.g., Mo et al., 1998). The second idea, which is more related to our approach, is based on the existence of the virial radius, which in turn, relates to the finite age of the Universe. More specifically, on dimensional grounds \( v^2_{\text{virial}} \propto GM_{\text{virial}}/r_{\text{virial}} \), and since in the CDM context \( r_{\text{virial}} \propto M_{\text{virial}}^{1/3} \) (Navarro et al., 1997), then \( M_{\text{virial}} \propto v^3_{\text{virial}} \) where \( v_{\text{virial}} \) and \( M_{\text{virial}} \) are the halo circular velocity and mass, respectively. Under the assumption that the disk rotation velocity, \( v_{\text{rot}} \), is proportional to \( v_{\text{virial}} \), and that the total stellar mass is proportional to \( M_{\text{virial}} \), the total luminosity of the galaxy will scale approximately as \( v^3_{\text{rot}} \) [also see Mo et al. (1998), and Dalcanton et al. (1997)].

The expression for the observational TFR is of the form

\[
L = A v^\beta_{\text{rot}}
\]

where \( \beta \) is the slope and \( A \) is the zero-point. The observed values of \( \beta \) range from about 2.5 to about 4, whereas both \( \beta \) and \( A \) may depend on the waveband (Mo and Mao, 2000, and references therein). The TFR systematically steepens from the blue to the red passbands and is surprisingly tight, especially at longer wavelengths (e.g., Kauffmann et al., 1993; Kauffmann et al., 1993; 14 Note that the TFR is plotted usually in a log \( v_{\text{rot}} \)-absolute magnitude diagram, hence the zero-point denotes the intersection with the log \( v_{\text{rot}} \) axis (see Fig. 3).
Sprayberry et al., 1995; Willick et al., 1997). For a fixed $v_{\text{rot}}$ the dispersion in absolute magnitude is less than 1 magnitude. It is believed that observational errors and intrinsic dispersion are, approximately, equally contributing to this dispersion (Willick et al., 1997; Sakai et al., 2000; Verheijen, 2001). The choice of the location where $v_{\text{rot}}$ is measured is very important. The best measure of the (optical) TFR $v_{\text{rot}}$, on the basis of the repeatability of its measurement, the minimization of the TFR intrinsic scatter, and the match with the radio (HI) linewidths (which is important given that HI linewidths have defined the standard for most TFR calibrations to date) is the rotation velocity measured at $2.2 \times$ the exponential scale length of the galactic disk. Note also that this is where the contribution of a pure exponential disk to the circular velocity attains its maximum, and it is at that radius that TFR velocities are typically measured (Courteau, 1997).

With respect to the TFR there are three major observational features a model must reproduce so that it be considered successful: the slope, the small dispersion, and the zero-point. CDM numerical simulations (Mo et al., 1998; Steinmetz and Navarro, 1999; Mo and Mao, 2000; Navarro and Steinmetz, 2000a, 2000b) manage to reproduce a slope in fairly good agreement with observational data in many passbands. The dispersion of the numerical TFR is about half of that observed (see Fig. 3-upper panel). Given that numerical calculations are free of observational dispersion, and taking into account the aforementioned regarding the contributions to the dispersion in the observational TFR, it seems that the model reproduces well the observed dispersion. The problem is the zero-point. The numerical studies result in one and the same conclusion: the zero point is offset by 1 to 2 magnitudes, depending on the cosmological parameters. More specifically, for a specific $v_{\text{rot}}$ the model galaxies in the numerical simulations appear to be 1 to 2 magnitudes fainter than the observed galaxies (Mo et al., 1998; Steinmetz and Navarro, 1999; Mo and Mao, 2000; Navarro and Steinmetz, 2000a, 2000b). This has also been hinted at by some semianalytic CDM models of galaxy formation (e.g., Kauffmann et al., 1993; Cole et al., 1994). In other words, the numerically obtained $v_{\text{rot}}$ at a given luminosity is about 40% to 60% higher than observed (Steinmetz and Navarro, 1999). This result seems to be independent of the disk mass-to-light ratio, and even with the extreme assumption that all baryons in a dark halo are turned into stars, the resulting disks are still about 2 magnitudes fainter than observed, for a given $v_{\text{rot}}$ (Navarro and Steinmetz, 2000a). This, along with the fact that 70% − 90% of all baryons inside the virial radius are confined within the luminous extend of the disk, at zero redshift (Steinmetz
and Navarro, 1999), mean that the model galaxies are already almost as bright as they can be; thus, making the model galaxies even brighter to match the observations is not an option. Note though that the correct zero point can be reproduced when $v_{\text{virial}}$ is used instead of $v_{\text{rot}}$ (e.g., Steinmetz and Navarro, 1999; also see Fig. 3-lower panel). Apparently, according to the aforementioned, $v_{\text{virial}}$ is 40% to 60% lower than $v_{\text{rot}}$.

The large difference between the two velocities is expected in CDM models. The mass of the central galaxy in a halo-galaxy system depends on the efficiency of gas cooling which, in turn, depends on the mass of the halo; the less massive the halo, the more efficient in assembling baryons and dark matter, which is drawn by the baryons, into galaxies. Systems that collect a large fraction of the available baryons into a central galaxy have their rotation speeds increased substantially over and above the circular velocity of their surrounding halo (Mo et al., 1998; Steinmetz and Navarro, 1999; Mo and Mao, 2000; Navarro and Steinmetz, 2000a, 2000b). Given that $v_{\text{virial}}$ depends on the total halo mass and that $v_{\text{rot}}$ depends on the total mass, both luminous and dark, that lies inside the radius where $v_{\text{rot}}$ is measured, it seems inevitable that in order to lower $v_{\text{rot}}$ so that it approaches $v_{\text{virial}}$ and thus reproduces the correct TFR zero-point, one must reduce the mass of the halo concentrated in the central regions. Namely, the zero-point discrepancy translates into a problem with the large central concentrations of the dark matter halos predicted in the CDM context. For the numerically derived TFR to coincide with the observational one, the dark matter in the innermost few kpc of galaxies must be reduced by a factor of 2 to 3 (Navarro and Steinmetz, 2000b) in terms of the concentration parameter, a reduction by a factor of 3 to 5 (Navarro 1998; Navarro and Steinmetz, 2000b) compared with the CDM values [$c = 15 - 20$ (Navarro et al., 1996)], and a value of $\simeq 3$, referring to the NFW profile (Mo and Mao, 2000), have been reported.

It is noteworthy, however, that there are many points that need to be clarified. For example, Eke et al. (2001) reported that the results regarding the TFR zero-point presented by Navarro et al. (2000a) are wrong due to the high power spectrum normalization used. A higher normalization ($\sigma_8 > 1.14$) means a higher collapse time, and consequently, a higher density and concentration which can be the source of the zero-point discrepancy. Indeed,

\[15\] This is true according to expectations from theoretical models where the mass of the central galaxy is determined by the efficiency of gas cooling (Navarro and Steinmetz, 2000b).
Eke et al. using $\sigma_8=1.14$, managed to reduce the offset to about 0.5 mag with respect to the observational TFR. They attributed that remaining offset to the fact that in their simulations star formation occurred sooner than it should – this was hinted by the fact that the simulated galaxies were slightly more red than their observed TFR counterparts – and claimed that correcting for this effect can eliminate the offset entirely. This is an example of how crucial including a realistic modeling can be.

Another aspect of this topic is the redshift evolution of the TFR. Sample selection and details of the observational technique used make the situation unclear. Claims that the TFR either brightens or dims at modest redshifts ($z \leq 1$) can be found in literature (Vogt et al., 1996, 1997; Rix et al., 1997; Hudson et al., 1998; Simard and Pritchet, 1998). Furthermore, it appears that whether the TFR dims or brightens depends upon the parameters used. For example, Vogt et al. (1996, 1997) reported that the TFR barely brightens only for the low value of $q = 0.05$, whereas the same data for $q = 0.5$ are consistent with a slight dimming, with $q$ being the deceleration parameter. The CDM model TFR brightens at $z = 1$ by $\simeq 0.7$ or 0.2 mag in the B-band, depending on whether $v_{\text{rot}}$ or $v_{\text{virial}}$ is used, respectively. Whether this is inconsistent with observation or not, depends again on the parameter values and observational data one uses. Adopting $q = 0.5$ and the data from Vogt et al. (1996), there appears to exist an inconsistency, which, nevertheless, has been attributed so far to the specific star formation algorithm used (e.g., Steinmetz and Navarro, 1999). In addition, it appears that the model TFR follows the observed dimming at passbands that are insensitive to star formation, e.g., the K-band (Steinmetz and Navarro, 1999). At present, there are many things that need to be understood with respect to the redshift evolution of the TFR. In the future, however, the TFR redshift evolution will be a powerful tool that can be used to test the CDM model.

Last, but not least, with respect to the TFR, one should mention what in literature appears as the surface brightness conspiracy (e.g., Evans, 2001), causing problems not only in the CDM model, but also in the very existence of dark matter in general. The observational fact is that the rotation velocity, $v_{\text{rot}}$, is similar for all galaxies of a given luminosity, no matter how widely spread the luminous material is. Namely, LSB galaxies lie on the same TFR as HSB galaxies (e.g., Sprayberry et al., 1995; Zwaan et al., 1995), even though with somewhat greater scatter (e.g., Sprayberry et al., 1995; McGaugh et al., 2000). On dimensional grounds $v_{\text{rot}}^2 \propto GM/D$, where $M$ is the mass (both luminous and dark) of the galaxy and $D$ is its
characteristic size. For two galaxies, one HSB and one LSB, with the same luminosity $L$, and with characteristic sizes $D_{HSB}$ and $D_{LSB}$,\textsuperscript{16} respectively, to have the same $v_{rot}$, the overall $M/L$ must increase with decreasing surface brightness in just the right way\textsuperscript{17} so that the tight $v_{rot} - L$ correlation observed be justified. This can be done by assuming either that the $M/L$ of the stellar population varies with surface brightness, which seems unlikely to be the case (de Blok and McGaugh, 1997), or that the dark matter fraction increases with decreasing surface brightness. This in not a problem in the case of the LSB galaxies, given that they are dominated by dark matter. The potential problem has to do with the bright galaxies whose inner parts are dominated by the luminous matter; eliminating the surface brightness dependance in the case of these bright galaxies requires some fine tuning.

In conclusion, two out of the three features of the observational TFR are easily reproduced in the frame of CDM. The zero-point appears to be more difficult to predict correctly. For a specific galaxy luminosity, the CDM predicted rotation velocity is higher than observed; this can be attributed to the high central mass concentration predicted by CDM. The lack of realistic modeling and other factors, such as inconsistencies in the calculations, may be held, at least partially, responsible for the zero-point discrepancy.

C. Barred galaxies

Bars are seen in optical images of roughly 30% of all disk galaxies (Sellwood and Wilkinson, 1993), and the fraction of strongly barred galaxies rises to over 50% in the near-IR (Eskridge \textit{et al.}, 2000). These numerous objects constitute a useful laboratory where the dynamics of the central regions of galaxies may be probed and, thus, where the CDM hypothesis can be tested.

A way to test CDM using this class of objects concerns the pattern speeds of the rotating bars. The usual way to characterize the rotation rate of a bar is the ratio $R = d/\alpha$, where $d$ is the corotation radius and $\alpha$ the bar semimajor axis. More precisely, $d$ is the distance from the center to the Lagrange point on the bar major axis where the gravitational attraction balances the centrifugal acceleration (in the frame rotating with the bar). Theoretical

\textsuperscript{16} Note that it has to be $D_{HSB} < D_{LSB}$ since on dimensional grounds $S \propto L/D^2$, with $S$ and $L$ denoting the surface brightness and luminosity, respectively, and given that $S_{HSB} > S_{LSB}$, by definition.

\textsuperscript{17} In a way so that the increment of $D_{LSB}$ be compensated by an increment in $M$. 

18
arguments on the basis of bar stability require $R > 1$ (Contopoulos, 1980), whereas that $R$ can also equal unity has been reported [for a review, see Sellwood and Wilkinson (1993)]. A bar can be classified as fast or slowly rotating based on its R value. Fast rotating bars are the ones with low values of $R$. For example, a classification of bars with respect to this criterion treats bars with $1 \leq R \leq 1.4$ as rapidly rotating and bars with $R > 1.4$ as slowly rotating ones (Debattista and Sellwood, 2000). More generally, characterizing a bar as slow means that it has an $R$ value substantially greater than 1.

Both numerical and analytical arguments exist that lead to the conclusion that if there is substantial dark matter in the bar region, the rotation of the bar is slowed down on a time scale of a few rotation periods (e.g., Weinberg, 1985; Debattista and Sellwood, 1998, 2000; Athanassoula and Misiriotis, 2002). Along the same lines with these arguments, this is to be expected since CDM, being dissipationless, rotates much more slowly than the galaxy baryons and does not participate fully in the galactic bar. The response of the dark matter to the bar is out of phase with it, and this provides a drag force that is usually referred to as dynamical friction. This dynamical friction is at its most severe when both dark matter and baryons provide comparable mass (Binney and Evans, 2001). This is, more or less, the conventional picture with respect to how the rotation of a bar is expected to change with time due to its interaction with the surrounding matter. This picture is true, as long as the bar is a solid rotator. In this case, a bar that loses angular momentum will inevitably slow down. However, as pointed out by Valenzuela and Klypin (2002), real bars may behave differently because they are not solid rotators, but they consist of particles; when the bar loses angular momentum it is the individual particles that suffer this loss and that move to smaller radius trajectories that, nevertheless, correspond to higher angular frequencies. The bar speed is expected to be proportional to the angular frequency of the particles, and since losses of angular momentum lead to higher orbital frequencies, it appears possible that a real bar can speed up while losing angular momentum, contrary to what is usually assumed on the basis of a solid bar rotator. In the same study, Valenzuela and Klypin also show that the results obtained via previous numerical simulations, which resulted in bars that slowed down fast (even though far slower than analytically predicted), may not be accurate due to the initial setup used and the numerical resolution limitations. More specifically, in the low resolution simulations they find that bars appear to form from the very beginning – long before they appear at higher resolution simulations – and that these bars slow down relatively fast, at
least compared with what happens in the higher resolution cases where the pattern speed of the bars appears to remain unchanged over billions of years. With respect to the initial setup, they point out the fact that the numerical simulations that the conventional picture for bar pattern speed evolution is based on, do not use realistic halos, neither with respect to the halo virial extent – usually halos in these simulations are truncated at radii well below the virial radius – nor with respect to the halo density profiles, since the profiles used were not the ones suggested by CDM. In terms of numbers, the disagreement is expressed as a 2.0 to 2.6 range for the $R$-values based on conclusions derived, e.g., by Debattista and Sellwood (1998, 2000), and a 1.2 to 1.7 range based on the study carried out by Valenzuela and Klypin (2002) which is, as we will see, partially in agreement with observations.

The number of galaxies with measured $R$ is quite small. The main difficulty is that it requires knowledge of the bar angular speed, $\Omega$, which is hard to determine from observational data, even though reliable methods have been proposed and been used (e.g., Tremaine and Weinberg, 1984; Dehnen, 1999). Among the most reliable results reported so far are the ones obtained using the Tremaine-Weinberg (1984) method. The method has yielded the values $R = 1.4 \pm 0.3$ for NGC 936 (Merrifield and Kuijken, 1995), and $R = 1.15^{+0.38}_{-0.23}$ for NGC 4596 (Gerssen et al., 1999). Indirect, but reliable estimates have been accomplished by matching hydrodynamical and stellar dynamical simulations with observations (e.g., Lindblad and Kristen, 1996; Lindblad et al., 1996; Englmaier and Gerhard, 1999; Weiner and Sellwood, 1999; Hafner et al., 2000). The typical range of values for $R$ was found to be from 1.1 to 1.3, approximately. Namely, the existing observations unanimously indicate that $R$ is small, which means that the corotation point lies typically just beyond the bar’s optical edge, and thus the bars are rotating fast. According to the conventional picture, bars can maintain the high pattern speeds observed only if the disk provides most of the central attraction in the inner regions (maximum disk, see Sec. I.A). It has been reported that at most a 10% contribution by spherically or axially-symmetrically distributed dark matter can be allowed within the bar region to avoid conflict with the observed kinematics of certain barred galaxies (Sellwood and Kosowsky, 2000, and references therein). In the case of our Galaxy, a barred galaxy, even at the Solar circle the halo contribution to the radial force has been calculated.

---

18 The results published are $69 \pm 15$ arcsec for the corotation distance and 50 arcsec for the bar extend from which I estimated $R$. 
to be only \( \approx 20\% \) (Englmaier and Gerhard, 1999). According to these results, again, there is no room for highly centrally concentrated dark matter halos. Moreover, as before, the generalization from HSB barred galaxies to their HSB unbarred counterparts seems rather attractive. Empirical evidence has been presented against a systematic difference with respect to the dark matter fraction of these two types of HSB galaxies (Mathewson and Ford, 1996; Debattista and Sellwood, 2000, and references therein). Thus, the conclusion that the aforementioned regarding the dark matter component hold for all bright galaxies, and not only for barred ones, has been presented by some authors, e.g., Debattista and Sellwood, 2000.

Note, however, that the 1.2 to 1.7 range for \( R \) found by Valenzuela and Klypin (2002) is, as already mentioned, in partial agreement with the above, measured \( R \) values. Furthermore, Klypin, Zhao, and Somerville (2001) found that ΛCDM models of, e.g., the Milky Way are able to reproduce the observed kinematics, while satisfying numerous constraints in the solar neighborhood (e.g., surface density constraints). They also find that none of their models appears to be dark matter dominated in the central region, in accordance with what one expects for a galaxy like the MW, and that the sustenance of a rapidly rotating bar appears to be possible, as long as some transfer of angular momentum from the baryons to the dark matter is included.

D. The microlensing optical depth to the galactic center

A crucial difference between baryonic and particle dark matter is that the former can cause microlensing events, whereas the latter cannot.\(^{19}\) This fact can be used to derive quantitative conclusions about the baryonic mass alone.

Extremely high microlensing optical depths towards the galactic bulge – either exactly towards Baade’s window \((l = 1^\circ \text{ and } b = -4^\circ)\), or towards directions very close to it – have been measured. The optical depths reported so far lie in the range \((1.4 - 4) \times 10^{-6}\) (e.g., Udalski et al., 1994; Alcock et al., 1995, 1997, 2000; Zhao et al., 1995; Klypin, Zhao, and Somerville, 2001, and references therein), with the most recent analysis to conclude that the optical depth is in the range \((1.4 - 2) \times 10^{-6}\) (Klypin, Zhao, and Somerville, 2001, and

---

\(^{19}\) Referring to the smooth component of the dark matter which, by definition, has no substructure and thus cannot cause microlensing.
references therein). All these observational estimates are nearly an order of magnitude higher than the values theoretically anticipated, which lie typically in the range \((0.1 - 4) \times 10^{-7}\) (e.g., Griest et al., 1991; Paczynski, 1991; Kiraga and Paczynski, 1994), except from the study carried out more recently by Klypin, Zhao, and Somerville (2001) that results in the conclusion that the microlensing optical depths for their model galaxies can be of the order of \(10^{-6}\).

The microlensing optical depth increases with both the surface density of bulge stars and the effective depth of the bulge along the line of sight. These dependencies can be used to derive the contribution to the rotation curve by the stellar disk and the interstellar medium. To find the contribution of the relatively unexplored bar, one must assume a bar massive enough that the total baryonic matter in the inner galaxy reproduces the measured optical depth. The other contribution to the rotation curve will come from the dark matter component. A cuspy CDM halo can be constructed in the case of the Galaxy, that should be normalized using the appropriate data, for example, the local surface density, which can be derived from kinematics of stars in the solar neighborhood. Thus, the contribution of the CDM component to the rotation velocity can be found, and the total rotation velocity, which is to be compared with the observed one, can be obtained by adding in quadrature the two contributions, the luminous and the dark matter ones.

This procedure in several levels of completeness has been followed, e.g., by Binney et al. (2000) and Binney and Evans (2001) for the Galaxy. A unanimous conclusion has been reached: almost all the matter along the lines of sight to the galactic bulge must be capable of causing microlensing, and thus it cannot be particle dark matter. Binney and Evans (2001) conclude that even the least concentrated CDM halo profile used \((\alpha = 0.3)\) is ruled out since it violates the constraint of the observed rotation curve, especially the inner part that appears to rise extremely steeply. In their study, no halo with a cusp as steep as or steeper than \(r^{-0.3}\) is viable. According to the same study, to obtain an optical depth of order \(10^{-6}\) using an NFW halo, one must push all the related parameters to their very extremes, i.e., one must use the smallest possible dark matter surface density consistent with the observed rotation curve. But, as pointed out by Klypin, Zhao, and Somerville (2001), these conclusions are based on a quite non-realistic treatment of both the dark matter and the bar. For example, Binney and Evans used the unmodified NFW profile, ignoring thus the effects that adiabatic compression has in the inner density profile. Furthermore, Klypin, Zhao, and Somerville
have modeled their bars in far greater detail, so that numerous observational constraints be satisfied, and have used more detailed treatments of microlensing events compared with Binney and Evans. All these led the two studies to totally different conclusions, Binney and Evans to the conclusion that cuspy CDM halos are inconsistent with microlensing data, and Klypin, Zhao, and Somerville to the conclusion that such an inconsistency is nonexistent.

Summarizing with respect to the apparent central mass discrepancy, there is a large number of observations indicating that the CDM model predicts a higher central mass concentration than observed. This translates into a steeper than observed rise of the CDM predicted inner rotation curves, a higher than observed zero-point of the TFR, into bars that rotate more slowly than observed, and into microlensing optical depths towards the galactic bulge an order of magnitude lower than measured. However, all these are pieces of evidence that there is something wrong with CDM, as long as 1. observations are reliable and are interpreted in the correct way, 2. the assumptions made to derive the CDM theoretical predictions are correct and consistent, and 3. the simulations are reliable with respect to both resolution and convergence, and with respect to their realistic and complete treatment of the related physical processes. We saw, using specific examples, that none of these prerequisites is necessarily true. Thus, not only there is no proof for a discrepancy, but also the strength of the existing evidence is questionable.

III. HALO SHAPES

Dissipationless halo formation leads to strongly triaxial halos, as revealed by numerical simulations (e.g., Frenk et al., 1988; White et al., 1990; Dubinski and Carlberg, 1991; Warren et al., 1992; Jing and Suto, 2002). In fact, triaxial modeling has been found to improve significantly the fit to the simulated profiles, at least for the relatively relaxed halos, compared to the usual spherical model (Jing and Suto, 2002). Both that there is a slight preference for prolate (e.g., Frenk et al., 1988; Warren et al., 1992) – especially in the inner regions – configurations, and that there are roughly equal numbers of halos with oblate and prolate forms 20 have been reported (e.g., Dubinski and Carlberg, 1991). These halos are highly flattened. The flattening is usually expressed using the ratio $c/a$, where $c$ and $a$

20 Oblate: $c/b < b/a$, prolate: $c/b > b/a$, with $a$, $b$, $c$ the major, intermediate, and minor axis of the triaxial ellipsoid, respectively.
stand for the minor and the major axis of the triaxial ellipsoid, respectively. The typical value of this ratio, as calculated by numerical simulations, is \( \simeq 0.5 \), leading to the above conclusion that CDM halos are highly flattened (Frenk et al., 1988; Dubinski and Carlberg, 1991; Warren et al., 1992). \(^{21}\) A mean value of the ratio \( b/a \simeq 0.71 \) has been reported (Dubinski and Carlberg, 1991; Dubinski, 1994), with \( b \) denoting the intermediate axis of the halo (see also Fig. 4). Including the dissipative infall of gas – which results in the formation of the luminous part of the galaxy – causes the prolate halos to transform from prolate triaxial to oblate triaxial, at least at the inner parts. The 2:1 flattening is approximately preserved. At their central parts these halos are spheroids, as opposed to their outer parts where, at least in a statistical sense, they become more triaxial and twisted (Dubinski, 1994; Sellwood and Kosowsky, 2001). Furthermore, in this case \( b/a \geq 0.7 \) (Dubinski, 1994). The shapes of the triaxial halos formed in the CDM model are supported by anisotropic velocity dispersion rather than by angular momentum, since their rotation alone is not sufficient to account for their flattening (Warren et al., 1992). In the ΛCDM context, the ratio \( c/a \) was found to be \( \simeq 0.70 \) to within \( 30h^{-1}\text{kpc} \) with a scatter of about \( \pm 0.17 \) about the mean, with a long tail skewed towards highly flattened objects (Bullock, 2001b).

From the very first CDM studies of halo shapes it became clear that there are some similarities between halos and elliptical galaxies. Elliptical galaxies are also believed to be triaxial bodies supported by pressure anisotropy (e.g., Binney, 1976; Frenk et al., 1988). Furthermore, it is believed that if the history of elliptical galaxy formation is one of a hierarchy, involving lumps consisting mostly of stars, then the dynamics of halo and elliptical galaxy formation are similar (e.g., Zurek et al., 1988), and their shape distribution is expected to be the same (e.g., van Albada, 1982; Aguilar and Merritt, 1990). Comparisons though, between real elliptical galaxies and CDM halo shapes, concluded that the former are much rounder (less elliptical) than the latter (e.g., Frenk et al., 1988; White et al., 1990; Dubinski and Carlberg, 1991). Warren et al. (1992) argued that this difference between the shape distributions of elliptical galaxies and halos is expected. Their argument is based on the dynamical friction that dense stellar systems undergo when moving through the dark matter during a merger, and which results in a one-way transport of angular momentum.

\(^{21}\) The \( c/a \simeq 0.5 \) case also appears in literature as 2:1 flattening. It is also worth noting that a halo of dark matter particles that is flattened more than about 3:1 could not survive and would puff up through dynamical bending instability (Merritt and Sellwood, 1994).
and kinetic energy from the orbits of the dense stellar systems to the less dense dark matter halo.

Even though the number of measurements is very small, mainly due to the lack of visible tracers that can probe the gravitational potential around galaxies, the predictions of the CDM model regarding the halo shapes, such as the 2:1 flattening, are nowadays generally thought to be consistent with constraints on halo shapes inferred from observation (Sackett, 1999; Merrifield, 2001; Hoekstra et al., 2002). Nonetheless, there are some exceptions. The halo of NGC 2403 seems to become more nearly axisymmetric at large radii (Sellwood and Kosowsky, 2001, and references therein). Sackett et al. (1994) found for the polar ring galaxy NGC 4650A that the inner dark matter halo is as flat as $0.3 \leq c/a \leq 0.4$. Olling (1996), on the assumption of an isotropic gas velocity dispersion tensor, estimated that the inner halo of NGC 4244 is as flat as $c/a \approx 0.2^{+0.3}_{-0.1}$. The halo of NGC 3198 was found to be closely axisymmetric at all radii outside the disk (Sellwood and Kosowsky, 2001, and references therein). The gas ring surrounding the early-type galaxy IC2006 was found to have an ellipticity equal to $0.012 \pm 0.026$, namely it was found to be essentially circular, whereas the departure from axisymmetry of the potential was estimated to be $\leq 1\%$ (Franx et al., 1994). For our Galaxy, Olling and Merrifield (2000) reported a value of $c/a$ approximately equal to 0.8 that was obtained, however, using galaxy parameters (distance to the galactic center and local galactic rotation speed) that differ considerably from those recommended by the IAU; thus, it is unclear whether the method followed for this derivation yields sensible results. In addition, van der Marel (2001) using the velocity ellipsoid of halo stars to probe the halo potential of the Galaxy estimated a lower limit of the ratio $c/a > 0.4$. The CDM expectations for the MW were also contested by the study carried out by Ibata et al. (2001). In this study, Ibata et al. managed to reproduce satisfactorily enough observations related to cool carbon giant stars in the galactic halo, assuming a standard spherical model for the galactic potential. Thus, they concluded that the galactic dark matter halo is most likely almost spherical, at least between the galactocentric radii of 16 kpc to 60 kpc. Flat halos with $c/a < 0.7$ were ruled out at very high confidence levels.

Lastly, lensing data for 20 strong lenses indicated that the mass distributions are aligned with the luminous galaxy; a $10^\circ$ upper limit on the rms dispersion in the angle between the major axes of the dark and the luminous components’ distributions has been found (Kochanek, 2001). This can be interpreted as that light traces the mass, and this light to
mass correspondence is sometimes used as evidence for that there is no need for dark matter.

IV. EXCESSIVE SUBSTRUCTURE

According to the hierarchical clustering scenario, galaxies are assembled by merging and accretion of numerous satellites of different sizes and masses (e.g., Klypin et al., 1999). In the frame of this scenario, smaller galaxies collapsed earlier (when the density of the universe was higher) and then they participated in the assembling of the new galaxy. This process is not 100% efficient in destroying these smaller objects, especially the ones with adequately large central densities whose central parts, at least, can survive the merging process and exist as subhalos within larger halos. Furthermore, some of the satellites may have been accreted by the galaxy at later stages. Both high-resolution N-body simulations (e.g., Klypin et al., 1999; Moore et al., 1999a; Springel, 2000) and semianalytic theory (Press and Schechter, 1974; Kauffmann et al., 1993) predict the existence of considerable substructure in CDM halos. This substructure has been calculated to amount to of order 10% of the halo mass and to continue down to objects of unresolved scales of $10^6 M_{\odot}$ or smaller (Silk, 2001). Furthermore, numerical simulations have revealed that the abundance of dark matter subhalos within a galaxy is the same as found within a scaled galaxy cluster (e.g., Moore et al., 1999a; also see Fig. 3).

Observation does not seem to verify these CDM predictions and this discrepancy appears in literature as the satellite catastrophe (e.g., Evans, 2001). The number of predicted dwarf-galaxy satellites exceeds that observed around the Milky Way or the Andromeda galaxy by at least one order of magnitude (Klypin et al., 1999; Moore et al., 1999a). Klypin et al. (1999) estimated that a halo the size of our Galaxy should have about 50 dark matter halos with circular velocities $> 20$ km/sec and mass greater than $3 \times 10^8 M_{\odot}$ within a 570 kpc radius. The same study found that satellites with circular velocities 10 km/sec-20 km/sec are approximately a factor of 5 more than the number of satellites actually observed in the vicinity of the Milky Way or the Andromeda galaxy. Moore et al. (1999a) found that the virialized extent of the Milky Way halo should contain about 500 satellites with masses $\geq 10^8 M_{\odot}$ and tidally limited sizes $\geq 1$ kpc. As observed, the Milky Way contains just 11 satellites within its virial radius with velocity $\geq 10$ km/sec (Mateo, 1998, and references therein). In the case of the Local Group, the number of dwarf galaxies is an order of magnitude
lower than predicted by simulations, with the discrepancy growing towards smaller masses (Klypin et al., 1999; Moore et al., 1999a). Both the numerical simulations mentioned and semianalytic theory (Press and Schechter, 1974; Kauffmann et al., 1993) predict that there should be roughly 1000 dark matter halos within the Local Group, whereas observations reveal about 40 (Mateo, 1998). Although more and more galaxies are being discovered, most of the new galaxies are very small and faint, making it thus unlikely that too many larger satellites have been missed.

Thus, either CDM is incorrect and the (different) nature of dark matter suppresses the formation of substructure or CDM is correct, but we are missing something. One of the first semianalytic scenarios that appeared in literature in order to resolve the apparent discrepancy assumes that the dynamical friction is as efficient as necessary (highly efficient – not many clumps could have survived) so that the abundance of low-mass satellites agrees with observations. Nevertheless, this solution comes at the expense of the total destruction of larger-mass satellites; a dynamical friction capable of destroying a large fraction of low-mass clumps would also make difficult the survival of any system of the size of the Magellanic Clouds (Kauffmann et al., 1993). Other scenarios use physical processes that may have operated during the early stages of galaxy formation and could have resulted in many dark, in the sense of invisible, satellites (e.g., Navarro and Steinmetz, 1997; MacLow and Ferrara, 1999; Gnedin, 2000; Somerville, 2002; Stoehr et al., 2002). Many of these scenarios, with the suppression of gas accretion in low-mass halos after the epoch of reionization the most complete and natural scenario so far (e.g., Bullock et al., 2000), manage to predict considerably less substructure, in good agreement with observations. The basic ingredient of these scenarios is a mechanism that leads to efficient mass loss prior to star formation. However, one can argue that star formation is a local process that occurs in localized inhomogeneities, and the free-fall time is short because the sound crossing time is short, whereas stripping is a global process that requires star formation to first occur (Silk, 2001), and thus it is expected that stars have formed before the gas had been entirely stripped, and thus the satellites cannot be completely dark.

Even if there is a mechanism that inhibits star formation in small satellites and thus makes them invisible, there is one more aspect that needs to be examined; that of a possible problem with respect to spiral disk formation and stability (Toth and Ostriker, 1992; Weinberg, 1998; Moore et al., 1999a). In the presence of large amounts of substructure, the strongly
fluctuating potential during clumpy collapses inhibits disk formation and has been shown to lead to the formation of elliptical galaxies instead (e.g., Steinmetz and Muller, 1995). But, old thin disks as well as cold stellar streams have been observed (e.g., Shang et al., 1998). As regards the stability of the disk, the factor that determines whether the disk is in danger or not is the amount of energy transferred from the numerous substructure clumps to the disk. Moore et al. (1999a) found that the substructure clumps are on orbits that take a large fraction of them through the stellar disk and consequently the passage of these lumps will heat the disk significantly. They estimated the energy input from encounters in the impulse approximation – which however is inadequate here – and found that this energy is a significant fraction of the disk binding energy. Waker et al. (1996) and Velazquez et al. (1999) on the other hand, have shown that disk overheating and stability become real problems only when interactions with numerous, large satellites (as for example the Large Magellanic Cloud) take place. Since such large satellites are rare, these two studies conclude that disk stability does not impose any constraint on the substructure predicted by CDM N-body simulations. More recently, Font et al. (2001) concluded as well that there is no conflict between the existence of numerous satellites and the existence of stellar disks. Contrary to what Moore et al. (1999a) concluded, Font et al. find that the orbits of satellites in present-day CDM halos very rarely take them near the disk, where tidal effects become extremely important. Thus, under the assumption that this was always the case, namely the effects of substructure at earlier times were similar to those in the case of present-day halos, the predicted substructure does not appear incompatible with stellar disks, since there appears to be no significant issues of disk overheating and stability.

What sometimes is presented as another manifestation of excessive substructure in CDM models – and on the scales we are concerned with – is the failure, at high confidence level (> 99.9%) to account for the velocity dispersion distribution of $z > 1.5$ damped Lyman-α systems (Prochaska and Wolfe, 1997, 2001). A theory for these systems must be able to reproduce both their number density and their velocity dispersion distribution. So far,

---

22 See footnote 3, Sec. II.

23 Systems (clouds) that go up to column densities $\lesssim 10^{23}$ cm$^{-2}$ and that cause the Lyman-α forest observed in quasar spectra. They are called damped because for column densities higher than $10^{18}$ cm$^{-2}$ the optical depth becomes unity at a point where the line profile is dominated by the Lorentzian wings of the damped profile rather than the Gaussian core (see, e.g., Peacock, 2000).
with our current understanding of these systems, CDM appears to be unable to reproduce both; even when it comes close enough, it does so assuming a circular velocity dependence of the *cross section* (gaseous extent) of the systems that is different from the one obtained by CDM numerical simulations (Prochaska and Wolfe, 2001, and references therein). Note, however, that there are yet a lot to be understood about the gas dynamics in these systems, and it is premature to attribute any apparent inconsistency to the CDM model rather than to our incomplete understanding.

Recapitulating, contrary to what was initially believed, the survival of considerable substructure does not seem to be in conflict with the existence of disks. With respect to the substructure abundance, there is either a discrepancy, or there is no discrepancy and, e.g., numerous dark matter satellites exist, but have not been detected yet because they have no stars. Alternative ways of detecting these dark matter clumps (e.g., Tasitsiomi and Olinto, 2002) might help resolve this issue.

**V. THE ANGULAR MOMENTUM OF DISK GALAXIES**

Another inconsistency, between CDM predictions and observations, that is closely related to both the persistence of substructure and the TFR problems, exists. It is the one known as the *angular momentum catastrophe* (e.g., Evans, 2001). It pertains to the fact that the predicted angular momentum of disks in spiral galaxies is at least one order of magnitude less than that observed (e.g., Navarro *et al.*, 1995; Navarro and Steinmetz, 1997, 2000a, 2000b; also see Fig. 6).

The reason for this discrepancy between the model and the real galaxies is that during galaxy assembling, clumpiness induces strong angular momentum transfer from the dissipating baryons to the energy-conserving dark matter (Navarro and Steinmetz, 1997, 2000b; Steinmetz and Navarro, 1999). There are two distinct mechanisms contributing to this angular momentum exchange. The one is the dynamical friction acting on the orbiting gas clumps; the other, a global mechanism, is due to the gravitational torques exerted on the orbiting gas clumps by the non-spherical dark matter distribution (e.g., Katz and Gunn, 1991; Navarro and Benz, 1991). Note that if the specific angular momentum is conserved during baryon infall and accretion – an assumption that is common in analytic treatments – then the initial angular momentum of a typical protogalaxy when the halo first collapsed
could reproduce the angular momenta observed [see, e.g., Fall and Efstathiou (1980)].

The baryonic component of simulated disks retains, on average, less than 15%–20% of the specific angular momentum of their surrounding halos (Navarro et al., 1995; Navarro and Steinmetz, 1997). The connection between the angular momentum and the TFR problem is demonstrated clearly using the velocity-squared scaling relations that hold for both the halo and the disk angular momentum. These scaling relations imply that a disk will have retained about half of the available angular momentum during assembly, if its rotation speed is approximately the same as the circular velocity of its surrounding halo (Navarro and Steinmetz, 2000b). It has also been found that the disk transfers more than 50% of its original angular momentum to the halo (between 48% and 73%) (Katz and Gunn, 1991). The losses of angular momentum that take place seem too large for the model to be a viable mechanism for making real spiral disks (Navarro and White, 1994). Most of the gas populates extremely low angular momentum orbits, and only a mass fraction of about 10% has an angular momentum comparable to that of the collisionless dark matter particles (Navarro and Benz, 1991).

Another equivalent way to show that the angular momenta of the model disks are deficient, compared with the observed spiral galaxies, is to compare the sizes of the former with the luminous radii of the latter. The equivalence relies on the fact that the angular momentum losses cause a large contraction factor to be required for a model disk to reach centrifugal equilibrium (Navarro and White, 1994; Navarro et al., 1995). The scale-lengths of simulated disks are predicted to be too small by a factor of ≃ 5 compared with observation (see, e.g., Katz and Gunn, 1991; Steinmetz and Navarro, 1999). Again, if angular momentum were conserved during the merging process, the predicted and the observed sizes would be in agreement (Fall and Efstathiou, 1980).

To solve this problem we need a mechanism that can serve as a source of energy that would significantly reduce the amount of gas that can cool (and delay the cooling) and that would prevent the baryonic component from sinking in the deep cores of the nonlinear clumps at high redshifts, and from losing a large fraction of their angular momentum during subsequent mergers. For this purpose, for example, SNe feedback (e.g., Thacker and Couchman, 2001),

\[ \frac{j_{\text{disk}}}{j_{\text{halo}}} \simeq \left( \frac{v_{\text{rot}}}{v_{\text{virial}}} \right)^2, \]

with \( j \) denoting the specific angular momentum.

\[ \text{Compare to TFR, Sec. 1B.} \]
or a photoionizing UV background (Navarro and Steinmetz, 1997) have been proposed. The problem is that thus we are led to delayed disk formation. In particular, giant disks would form relatively late, possibly in conflict with the observational data at \( z \simeq 1 \), as well as with the ages in the outer parts of nearby disks (Silk, 2001, and references therein). Moreover, it is noteworthy that disks forming in the presence of a photoionizing UV background have angular momenta that are even lower than the angular momenta of disks forming without including the effects of such a background (Navarro and Steinmetz, 1997). Last, but not least, the loss of angular momentum appears to be a sensitive function of the numerical resolution used in simulations, becoming more intense with increasing resolution (Navarro and Steinmetz, 1997).

Nevertheless, the evidence about the existence of a discrepancy should be considered inconclusive. All this evidence is derived on the basis of a premature understanding and treatment of the systems at hand. Thus, there are often problems concerning the ways the CDM theoretical predictions are obtained. For example, until recently a usual assumption was that both dark matter and baryons have initially similar specific angular momentum distribution \(^{26}\) which – taking into account the predicted distribution for the dark matter (e.g., Bullock et al., 2001a) – means that there will be too much baryonic matter having a very low angular momentum to form the observed, rotationally supported disks. Better agreement with observation has been recently achieved assuming different angular momentum distributions for the two components (Vitvitska et al., 2001).

VI. EPILOGUE

Despite its astonishing successes, the CDM model appears to be problematic on galactic scales. Concluding with certainty whether the model simply appears, or indeed is irreparably problematic, is one of the big challenges of our times.

In order of increasing radicalness, the opinions appearing in literature regarding the way the apparent weaknesses of the CDM model should be handled can be classified as follows:

- *The CDM model is too compelling to be wrong.* Thus, to achieve agreement between

\(^{26}\) This was assumed based on the idea that the angular momentum will be similar across the halo since it arises from large scale tidal torques [see, e.g., Primack (2001), and references therein].
CDM and observation we must either add a feature in the initial power spectrum of inflation, e.g., a tilt in the spectrum (Alam et al., 1990; Bullock, 2001a) that favors structure formation on large scales while suppresses it on small scales, or elaborate on the astrophysics of galaxy formation, e.g., to assume the formation of Super-Massive Black Holes (SMBHs) at the centers of the dark halos (Gebhardt et al., 2000; Menou et al., 2001; Silk, 2001).

- There is something wrong with the CDM model. The way to solve the problem is by stripping the collisionless or the cold properties of the traditional CDM, or by considering additional exotic properties for it; thus a plethora of studies (e.g., Colin et al., 2000; Kaplinghat et al., 2000; Spergel and Steinhardt, 2000; Alam et al., 2001; Bode et al., 2001) assuming particles that are self-interacting, warm, fluid, annihilating, etc., have been carried out.

- The CDM model is wrong. This is the opinion of the most ardent adversaries not simply of the CDM model, but of dark matter itself. A solution here is, e.g., the Modified Newtonian Dynamics (MOND) scenario (e.g., McGaugh and de Blok, 1998; Sanders and Verheijen, 1998).

So far, none of these three alternative ways of thinking has been proven to be the panacea the problem is in need of. Each one of them solves some of the problems either leaving, in the best case, the other problems unsolved or generating, in the worst case, new problems.

Furthermore, not all the problems discussed in this paper are of the same nature. Thus, there are problems that might not only be problems of the CDM model, but also of the dark matter hypothesis itself. There are problems that appear to be closely related to the cold nature of the dark matter and its implications, as for example the central halo cusps or the excessive substructure. There are problems concerning the ways the CDM theoretical predictions are obtained, such as problems with the assumptions made or problems with the basic tool used to explore the CDM theoretical predictions: the simulations. Simulations are a huge chapter in the CDM debate. Apart from resolution and convergence issues, and from difficulties in comparing different simulations, due to different numerical techniques,
cosmological models, etc., the really important question is how close simulations are to reality. To make realistic predictions of galactic properties in cosmological theories one must take into account a series of hydrodynamical phenomena, such as baryonic infall, and must understand and model important processes such as star formation and SN feedback. Thus, an important issue is to understand whether it is the CDM model or the simplified treatment of physics in simulations that is the source of the controversies. It is also important to understand whether making simulations be very close to reality – which can be extremely costly – is a necessity or not, and whether, instead, we can benefit more through semianalytic treatments.

From what has been already mentioned, it is clear that understanding and modeling in the correct way the related physical processes might resolve, or might have already resolved, some of the CDM shortcomings, such as the angular momentum catastrophe, the zero-point of the TFR, or even the satellite catastrophe. The central mass concentration problem appears to be the most robust discrepancy. What is more, studies (e.g., Blumenthal et al., 1986; Kochanek et al., 2001) of the effect of baryonic infall on the dark matter distribution conclude that the dark matter is adiabatically compressed by the cooled baryons during the formation of the central galaxy and is drawn towards the center; as a result, it appears so far that baryonic infall acts towards more centrally concentrated halos than the ones predicted by collisionless N-body simulations and thus exacerbates the problem. But, this is not necessarily the end of the story; for example, in the case of a barred galaxy, after including baryonic infall, one must include the dynamical friction from the bar that may act in a way opposite to the way baryonic infall acts, and so forth.

Before concluding with respect to the fatality of the apparent problems of the CDM model, the problems pertaining to observations should be noted. Even though the amount of information that we obtain via observation increases considerably day-by-day, there are still important accuracy and resolution issues and quite often, there are issues of correct interpretation of the observations. Another big problem is the fact that observational constraints are strongest just where theoretical predictions are least trustworthy.

With respect to the severity of the problems discussed, it seems that if a problem proves to be fatal for CDM, it will most certainly be the central mass problem, mainly as it manifests itself via the rotation curves of LSB galaxies. As already mentioned, a lot of ideas have appeared that have already given, or have promised to give, solutions to the other problems,
or to the other manifestations of the central mass problem. Regarding the central mass problem, the outlook is not that promising. Nevertheless, a few attempts have been made. To solve this problem we need a mechanism that enables heat transport to the inner part of the halo. This will lead to the puffing up of the central region and to the flattening of the density cusp. Apart from the small number of studies and proposed scenarios, the generality of the scenarios proposed is an additional issue. For example, Weinberg and Katz (2001) have shown recently that a bar can produce cores in cuspy CDM density profiles within 5 bar orbital times, by means of angular momentum transfer from the bar to the cusp that occurs via an inner-Lindblad-like resonance. The problem, however, essentially remains since first, this scenario assumes the existence of very large bars at early epochs, and second, it would be applicable in the case of barred galaxies and thus not in the case of the dark matter dominated dwarf and LSB galaxies, that have small or nonexistent bars. A few studies that might be more general have been carried out and have promising results, even though some further investigation of their assumptions is necessary. For example, recently El-Zant et al. (2001) have found that provided the gas is not smoothly, but in the form of lumps distributed initially, the dynamical friction that acts on these lumps dissipates their orbital energy and deposits it in the dark matter; this energy was found to be enough to heat the halo and thus eliminate the cusp. However, issues such as the creation and survival of these lumps need further investigation. A scenario that is very promising with respect to the resolution of the central cusp problem is the one motivated by the observed correlation between the mass of the SMBH at the center of a galaxy and the mass of the galaxy spheroidal (e.g., Gebhardt et al., 2000). The idea is that SMBHs are formed at the center of the dark matter halos. The merging of the halos is accompanied by the merging of their SMBHs. The SMBH merger results in the heating of the cusp which thus, becomes more flat (e.g., Merritt et al., 2000).

If the central mass problem will not turn out to be fatal, one can be optimistic; valid possible solutions will be found (and have been found already), at least for each one of the problems separately. Eventually, the real challenge will be how to find a unique, complete, and consistent scenario that will complement and extend the CDM model starting from ideas and scenarios that were designed to resolve each one of the discrepancies. Namely, how to complement the CDM model with the correct processes, so that the it becomes a theory that can encompass all the related phenomena.
Acknowledgments

I would like to express my gratitude to Sean Carroll for carefully reading the manuscript, for his useful comments and remarks, for the stimulating conversations and for his inspiring encouragement. I wish to thank Andrey Kravtsov for critically reading the draft version and for his useful and constructive comments. I thank Angela Olinto for her encouragement, and Craig Tyler for reading parts of the initial version. Lastly, I would like to thank Dimitrios Zisoulis for his endless support, and for being an inexhaustible source of inspiration. This work was supported by the National Science Foundation grant NSF PHY-0114422 at the Center for Cosmological Physics at the University of Chicago.

References

Alam, S. M. K., J. S. Bullock, and D. H. Weinberg, 2001, “Dark Matter Properties and Halo Central Densities ”, eprint astro-ph/0109392.
Alcock, C. et al., 1995, Ap.J. 445, 133.
Alcock, C., et al., 1997, Ap.J. 479, 119.
Alcock, C., et al., 2000, Ap.J. 541, 734; 2001, Ap.J. 557, 1035(erratum).
Aguilar, L. A., and D. Merritt, 1990, Ap.J. 354, 33.
Athanassoula, E., and A. Misiriotis, 2002, MNRAS 330, 35.
Bahcall, J. N., and S. Casertano, 1985, Ap.J. 293, L7.
Bahcall, N. A., and X. Fan, 1998, Ap.J. 504, 1.
Bahcall, N. A., J. P. Ostriker, S. Perlmutter, and P. J. Steinhardt, 1999, Science 284, 1481.
Balbi, A., et al., 2000, Ap.J. 545, L1B.
Bardeen, J. M., J. R. Bond, N. Kaiser, and A. S. Szalay, 1986, Ap.J. 304, 15.
Binney, J., 1976, MNRAS 177, 19.
Binney, J., N. Bissantz, and O. E. Gerhard, 2000, Ap.J. 537, L99.
Binney, J., and N.W. Evans, 2001, MNRAS 327, 27.
Binney, J., and S., Tremaine, 1994, Galactic Dynamics, (Princeton University, Princeton, New Jersey).
Blumenthal, G. R., S. M. Faber, R. Flores, and J. R. Primack, 1986, Ap.J. 301, 27.
Blumenthal, G. R., H. Pagels, and J. R. Primack, 1982, Nature 299, 37.
Bode, P., J. P. Ostriker, and N. Turok, 2001, Ap.J. 556, 93.
Bolatto, A. D., J. D. Simon, A. Leroy, and L. Blitz, 2002, Ap.J. 565, 238.
Borriello, A., and P. Salucci, 2001, MNRAS 323, 285.
Bullock, J. S., 2001a, “Shapes of dark matter halos”, eprint astro-ph/0106380.
Bullock, J. S., 2001b, “Tilted CDM versus WDM in the Subgalactic Scuffle”, eprint astro-ph/0111005.
Bullock, J. S., A. Dekel, T. S. Kolatt, A. V. Kravtsov, C. Porciani, and J. R. Primack, 2001a,
Ap.J. 555, 240.
Bullock, J. S., T. S. Kolatt, Y. Sigad, R. S. Sommerville, A. V. Kravtsov, A. A. Klypin, J. R.
Primack, and A. Dekel, 2001b, MNRAS 321, 559.
Bullock, J. S., A. V. Kravtsov, and D.H. Weinberg, 2000, Ap.J. 539, 517.
Burkert, A., 1995, Ap.J. 447, L25.
Burles, S., K. M. Nollett, and M. S. Turner, 2001a, Ap.J. 552, L1.
Burles, S., K. M. Nollett, and M. S. Turner, 2001b, Phys. Rev. D 63, 3512B.
Caldwell, R. R., R. Dave, and P. J. Steinhardt, 1998, Phys. Rev. Lett. 80, 1582.
Carlberg, R. G., S. M. Morris, H. K. C. Yee, and E. Ellingson, 1997, Ap.J. 479, L19.
Carroll, S., 2001a, Living Rev. Rel. 4, 1.
Carroll, S. M., 2001b, “Dark Energy and the Preposterous Universe”, eprint astro-ph/0107571.
Casertano, S., and J. H. van Gorkom, 1991, A.J. 101, 1231.
Chaboyer, B., P. Demarque, P. J. Kernan, and L. .M Krauss, 1998, Ap.J. 494, 96.
Cole, S. M., A. Aragon-Salamanca, C. S. Frenk, J. F. Navarro, and S. E. Zepf, 1994, MNRAS 271, 781.
Colin, P., V. Avila-Reese, and O. Valenzuela, 2000, Ap.J. 542, 622.
Contopoulos, G., 1980, A. & A. 81, 198.
Courteau, S., 1997, A.J. 114, 2402.
Dalcanton, J., D. N. Spergel, and J. J. Summers, 1997, Ap.J. 482, 659.
Davis, M., G. Efstathiou, C. S. Frenk, and S. D. M. White, 1985, Ap.J. 292, 371.
Debattista, V. P., and J.A. Sellwood, 1998, Ap.J. 493, L5.
Debattista, V. P., and J.A. Sellwood, 2000, Ap.J. 543, 704.
de Blok, W. J. G., and A. Bosma, 2002, A. & A. 385, 816.
de Blok, W. J. G., and S.S. McGaugh, 1997, MNRAS 290, 533.
de Blok, W. J. G., S. S. McGaugh, A. Bosma, and V. C. Rubin, 2001, Ap.J. 552, L23.
deblok, W. J. G., S. S. McGaugh, and V. C. Rubin, 2001, A.J. 122, 2396.
Dehnen, W., 1999, Ap.J. 524, L35.
Dubinski, J., 1994, Ap.J. 431, 617.
Dubinski, J., and R.G. Carlberg, 1991, Ap.J. 378, 496.
Efstathiou, G., and J.R. Bond, 1986, MNRAS 218, 103.
Efstathiou, G., J. Sehaye, and T. Theuns, 2000, Philos. Trans. R. Soc. Lond. A 358, 2049 (eprint astro-ph/0003400).
Eke, V. R., S. Cole, and C. S. Frenk, 1996, MNRAS 282, 263.
Eke, V. R., J. F. Navarro, and M. Steinmetz, 2001, Ap.J. 554, 114.
El-Zant, A., I. Shlosman, and Y. Hoffman, 2001, Ap.J. 560, 636.
Englmaier, P., and O.E. Gerhard, 1999, MNRAS 304, 512.
Eskridge, P. B., et al., 2000, A.J. 119, 536.
Evans, N. W., 2001, in IDM 2000: Third International Workshop on the identification of Dark Matter (eprint astro-ph/0102082), edited by N. Spooner (World Scientific, Singapore).
Fall, S. M., and G. Efstathiou, 1980, MNRAS 193, 189.
Firmani, C., E. D’Onghia, V. Avila-Reese, G. Chincarini, and X. Hernandez, 2000, MNRAS 315, L29.
Flores, R. A., and J.R. Primack, 1994, Ap.J. 427, L1.
Font, A. S., J. F. Navarro, J. Stadel, and T. Quinn, 2001, Ap.J. 563, L1.
Franx, M., J. H. van Gorkom, and T. de Zeeuw, 1994, Ap.J. 436, 642.
Freedman, W. L., et al., 2001, Ap.J 553, 47.
Freeman, K. C., 1970, Ap.J. 160, 811.
Frenk, C., S. D. M. White, M. Davis, and G. Efstathiou, 1988, Ap.J. 327, 507.
Fukushige, T., and J. Makino, 1997, Ap.J. 477, L9.
Fukushige, T., and J. Makino, 2001, Ap.J. 557, 533.
Gebhardt, K., et al., 2000, Ap.J 539, 13.
Gerssen, J., K. Kuijken, and M.R. Merrifield, 1999, MNRAS 306, 926.
Ghigna, S., B. Moore, F. Governato, G. Lake, T. Quinn, and J. Stadel, 2000, Ap.J. 544, 616.
Gnedin, N. Y., 2000, Ap.J. 542, 535.
Griest, K., et al., 1991, Ap.J. 372, L79.
Haber, H. E., and G. L. Kane, 1985, Phys. Rep. 177, 75.
Hafner, R. M., N. W. Evans, W. Dehnen, and J. J. Binney, 2000, MNRAS 314, 433.
Halverson, N. W., et al., 2002, Ap.J. 568, 38.
Harrison, E. R., 1970, Phys. Rev. D 1, 2726.
Hernandez, X., and G. Gilmore, 1998, MNRAS 294, 595.
Hoekstra, H., H. K. C. Yee, and M. D. Gladders, 2002, “Current status of weak gravitational lensing”, eprint astro-ph/0205205.
Hudson, M. J., S. D. J. Gwyn, H. Dahle, and N. Kaiser, 1998, Ap.J. 503, 531.
Ibata, R., G. F. Lewis, M. Irwin, E. Totten, and T. Quinn, 2001, Ap.J. 551, 294.
Jing, Y. P., and Y. Suto, 2000, Ap.J. 529, L69.
Jing, Y. P., and Y. Suto, 2002, “Triaxial Modeling of Halo Density Profiles with High-resolution N-body Simulations", eprint astro-ph/0202064.
Kaplinghat, M., L. Knox, and M. Turner, 2000, Phys. Rev. Lett. 85, 3335.
Katz, N., and J. E. Gunn, 1991, Ap.J. 377, 365.
Kauffmann, G., S. D. M. White, and B. Guiderdoni, 1993, MNRAS 264, 201.
Kent, S. M., 1986, A.J. 91, 1301.
Kiraga, M., and B. Paczynski, 1994, Ap.J. 430, L101.
Klypin, A. A., A. V. Kravtsov, J. S. Bullock, and J. R. Primack, 2001, Ap.J. 554, 903.
Klypin, A. A., A. V. Kravtsov, O. Valenzuela, and F. Prada, 1999, Ap.J. 522, 82.
Klypin, A., H. Zhao, and S. Somerville, 2001, “LCDM-based models for the Milky Way and M31 I: Dynamical Models”, eprint astro-ph/0110390.
Kochanek, C. S., 2001, “Mass Follows Light”, eprint astro-ph/0106495.
Kochanek, C. S., and M. White, 2001, Ap.J. 559, 531.
Kolb, E. W., and M. S., Turner, 1994, The Early Universe (Perseus, Cambridge, Massachusetts).
Lange, A. E., et al., 2001, Phys. Rev. D 63, 042001.
Lee, A. T., et al., 2001, Ap.J. 561, L1.
Lin, W. B., D. H. Huang, X. Zhang, and R. Brandenberger, 2001, Phys. Rev. Lett. 86, 954.
Lindblad, P. A. B., and H. Kristen, 1996, A. & A. 313, 733.
Lindblad, P. A. B., P. O. Lindblad, and E. Athanassoula, 1996, A. & A. 313, 65.
Madore, B. F., et al., 1998, Nature 395, 47.
MacLow, M. - M., and A. Ferrara, 1999, Ap.J. 513, 142.
Mateo, M. L., 1998, A.R.A.A. 36, 435.
Mathewson, D. S., and V. L. Ford, 1996, Ap.J.S. 107, 97.
McGaugh, S. S., and W. J. G. de Blok, 1998, Ap.J. 499, 66.
McGaugh, S. S., J. M. Schombert, G. D. Bothun, and W. J. G. de Blok, 2000, Ap.J. 533, 99.
Menou, K., Z. Haiman, and V. K. Narayanan, 2001, Ap.J. 558, 535.
Merrifield, M. R., 2001, “Halo Tracing with Atomic Hydrogen”, eprint astro-ph/0107291.
Merrifield, M. R., and K. Kuijken, 1995, MNRAS 274, 933.
Merritt, D., F., Cruz, and M., Milosavljevic, 2000, in Dynamics of Star Clusters and the Milky Way, ASP Conference Series, edited by S. Deiters, B. Fuchs, R. Spurzem, A. Just, and R. Wielen (San Fransisco: Astronomical Society of the Pacific), Vol. 228 (eprint astro-ph/0008497).
Merritt, D., and J. A. Sellwood, 1994, Ap.J. 425, 551.
Mo, H. J., and S. Mao, 2002, MNRAS 318, 163.
Mo, H. J., S. Mao, and S. D. M. White, 1998, MNRAS 295, 319.
Moore, B., 1994, “The Nature Of Dark Matter”, eprint astro-ph/9402009.
Moore, B., 2001, in 20th Texas Symposium on relativistic Astrophysics, Austin, Texas, AIP Conference Proceedings, edited by J. C. Wheeler and H. Mastel, (AIP, Melville, New York), Vol. 586, p. 73 (eprint 0103100).
Moore, B., S. Ghigna, F. Governato, G. Lake, T. Quinn, J. Stadel, and P. Tozzi, 1999a, Ap.J. 524, L19.
Moore, B., F. Governato, T. Quinn, J. Stadel, and G. Lake, 1998, Ap.J. 499, L5.
Moore, B., T. Quinn, F. Governato, J. Stadel, and G. Lake, 1999b, Ap.J. 310, 1147.
Morrison, D., S., Wolfe, and A., Fraknoi, 1995, Exploration of the Universe (Saunders College Publishing).
Navarro, J. F., 1998, “The Cosmological Significance of Disk Galaxy Rotation Curves”, eprint astro-ph/9807084.
Navarro, J. F., 2001, in Astrophysical SuperComputing using particles, IAU symposium 208, edited by J. Makino and P. Hut (eprint astro-ph/0110680).
Navarro, J. F., and W. Benz, 1991, Ap.J. 380, 320.
Navarro, J. F., C. S. Frenk, and S. D. M. White, 1995, MNRAS 275, 720.
Navarro, J. F., C. S. Frenk, and S. D. M. White, 1996, Ap.J. 462, 562.
Navarro, J. F., C. S. Frenk, and S. D. M. White, 1997, Ap.J. 490, 493.
Navarro, J. F., and M. Steinmetz, 1997, Ap.J. 478, 13.
Navarro, J. F., and M. Steinmetz, 2000, Ap.J. 528, 607.
Navarro, J. F., and M. Steinmetz, 2000, Ap.J. 538, 477.
Navarro, J. F., and S. D. M. White, 1994, MNRAS 267, 401.
Netterfield, C. B., et al., 2002, Ap.J. 571, 604.
Olling, R. P., 1996, A.J. 112, 481.
Olling, R. P., and M. R. Merrifield, 2000, MNRAS 311, 361.
Paczynski, B., 1991, Ap.J. 371, L63.
Palunas, P., and T. B. Williams, 2000, A.J. 120, 2884.
Peacock, J. A., 2000, Cosmological Physics (Cambridge University, Cambridge, United Kingdom).
Peacock, J. A., et al., 2001, Nature 410, 169.
Peebles, P. J. E., 1982, Ap.J. 263, L1.
Peebles, P. J. E., 1993, Principles of Physical Cosmology (Princeton University, Princeton, New Jersey).
Perlmutter, S., et al., 1999, Ap.J. 517, 565.
Persic, M., P. Salucci, and F. Stel, 1996, MNRAS 281, 27.
Power, C., J. F. Navarro, A. Jenkins, C. S. Frenk, S. D. M. White, V. Springel, J. Stadel, and T. Quinn, 2002, “The Inner Structure of LambdaCDM Halos I: A Numerical Convergence Study”, eprint astro-ph/0201544.
Press, W. H., and P. Schechter, 1974, Ap.J. 187, 425.
Primack, J. R., 2001, “The Nature of Dark Matter”, eprint astro-ph/0112255.
Prochaska, J. X., and A. M. Wolfe, 1997, Ap.J. 486, 73.
Prochaska, J. X., and A. M. Wolfe, 2001, Ap.J. 560, 33.
Pryke, C., N. W. Halverson, E. M. Leitch, J. Kovac, J. E. Carlstrom, W. L. Hopzapfel, and M. Dragovan, 2002, Ap.J. 568, 46.
Riess, A. G., et al., 1998, A.J. 116, 1009.
Rix, H. -W., P. Guhatakurta, M. Colless, and K. Ing, 1997, MNRAS 285, 770.
Sackett, P. D., 1999, in Galaxy Dynamics, ASP Conference Series 182, edited by D. Merritt, J. A. Sellwood, and M. Valluri, p. 393 (eprint astro-ph/9903420).
Sackett, P. D., H. -W. Rix, B. J. Jarvis, and K. C. Freeman, 1994, Ap.J. 436, 629.
Sakai, S., et al., 2000, Ap.J. 529, 698.
Salucci, P., 2001, MNRAS 320, L1.
Salucci, P., and A. Borriello, 2001, “Cold Dark Matter Halos Must Burn”, eprint astro-ph/0106251.
Sanders, R. H., and M. A. W. Verheijen, 1998, Ap.J. 503, 97.
Sellwood, J. A., and A., Kosowsky, 2000, in Gas and Galaxy Evolution, ASP Conference Proceedings, edited by J. E. Hibbard, M. Rupen, and J. H. van Gorkom, (San Fransisco, Astronomical Society of the Pacific), Vol. 240 (eprint astro-ph/0009074).
Sellwood, J. A., and A. Kosowsky, 2001, “Distinguishing Dark Matter from Modified Gravity”, eprint astro-ph/0109555.
Sellwood, J. A., and A. Wilkinson, 1993, Rep. Prog. Phys. 56, 173.
Shang, H., et al., 1998, Ap.J. 504, L23.
Shapiro, P. R., and I. T. Iliev, 2002, Ap.J. 565, L1.
Silk, J., 1997, Ap.J. 481, 703.
Silk, J., 2001, “Supermassive Black Holes and Galaxy Formation”, eprint astro-ph/0109325.
Sinard, L., and C. J. Pritchet, 1998, Ap.J. 505, 96.
Somerville, R. S., 2002, Ap.J. 572, 23.
Spergel, D. N., and P. J. Steinhardt, 2000, Phys. Rev. Lett. 84, 3760.
Sprayberry, D., G. M. Bernstein, C. D. Impey, and G. D. Bothun, 1995, Ap.J. 438, 72.
Springel, V., 2000, MNRAS 312, 859.
Steinmetz, M., and E. Muller, 1995, MNRAS 276, 549.
Steinmetz, M., and J. F. Navarro, 1999, Ap.J. 513, 555.
Stoehr, F., S. D. M. White, G. Tormen, and V. Springel, 2002, “The Milky Way’s satellite population in a LambdaCDM universe”, eprint astro-ph/0203342.
Swaters, R. A., B. F. Madore, and M. Trewhella, 2000, Ap.J. 531, L107.
Tasitsiomi, A., and A. V. Olinto, 2002, “The Detectability of Neutralino Clumps via Atmospheric Cherenkov Telescopes”, eprint astro-ph/0206040.
Thacker, R. J., and H. M. P. Couchman, 2001, Ap.J. 555, L17.
Toth, G., and J. P. Ostriker, 1992, Ap.J. 389, 5.
Tremaine, S., and J. E. Gunn, 1979, Phys. Rev. Lett. 42, 407.
Tremaine, S., and M. Weinberg, 1984, Ap.J. 282, L5.
Tully, R. B., and J. R. Fisher, 1977, A. & A. 54, 661.
Turner, M. S., 1990, Phys. Rep. 197, 67.
Turner, M. S., 2002, “Making Sense Of The New Cosmology”, eprint astro-ph/0202008.

Udalski, A., M. Szymanski, K. Z. Stanek, J. Kaluzny, M. Kubiak, M. Mateo, M. Krzeminski, B. Paczynski, and R. Venkat, 1994, Acta Astron. 44, 165.

Valenzuela, O., and A. Klypin, 2002, “Secular bar formation in galaxies with significant amount of dark matter”, eprint astro-ph/0204028.

van Albada, T. S., 1982, MNRAS 201, 939.

van den Bosch, F. C., B. E. Robertson, J. J. Dalcanton, and W. J. G. de Blok, 2000, A.J. 119, 1579.

van den Bosch, F. C., and R. A. Swaters, 2001, MNRAS 325, 1017.

van der Marel, R. P., 2001, “The Shapes of Galaxies and Their Halos as Traced by Stars: The Milky Way Dark Halo and The LMC Disk”, eprint astro-ph/0107248.

Velazquez, H., and S. D. M. White, 1999, MNRAS 304, 254.

Verheijen, M. A. W., 2001, Ap.J. 563, 694.

Vitvitska, M., A. A. Klypin, A. V. Kravtsov, J. S. Bullock, R. H Wechsler, and J. R. Primack, 2001, “The origin of angular momentum in dark matter halos”, eprint astro-ph/0105349.

Vogt, N. P., D. A. Forbes, A. C. Phillips, C. Gronwall, S. M Faber, G. D Illingworth, and D. C. Koo, 1996, Ap.J. 465, L15.

Vogt, N. P., A. C. Phillips, S. M. Faber, J. Gallego, C. Gronwall, R. Guzman, G. D. Illingworth, D. C. Koo, and J. D. Lowenthal, 1997, Ap.J. 479, L121.

Walker, I. R., J. C. Mihos, and L. Hernquist, 1996, Ap.J. 460, 121.

Wandelt, B. D., R., Dave, G. R., Farrar, P. C., McGuire, D. N., Spergel, and P. J., Steinhardt, 2000, in Sources and Detection of Dark Matter and Dark Energy in the Universe (eprint astro-ph/0006344), 4rth International Symposium, Marina del Rye, California, edited by D. B. Cline, (Springer, New York).

Wang, L., R. R. Caldwell, J. P. Ostriker, and P. J. Steinhardt, 2000, Ap.J. 530, 17.

Warren, M. S., P. J. Quinn, J. K. Salmon, and W. H. Zurek, 1992, Ap.J. 399, 405.

Weinberg, M., 1985, MNRAS 213, 451.

Weinberg, M., 1998, MNRAS 299, 499.

Weinberg, M. D., and N. Katz, 2001, “Bar-driven dark halo evolution: a resolution of the cusp–core controversy”, eprint astro-ph/0110632.

Weiner, B. J., and J. A. Sellwood, 1999, Ap.J. 524, 112.
Weiner, B. J., J. A. Sellwood, and T. B. Williams, 2000, “The Disk and Dark Halo Mass of the Barred Galaxy NGC 4123. II. Fluid-Dynamical Models”, eprint astro-ph/0008205.

White, S. D. M., J. F. Navarro, A. E. Evrard, and C. S. Frenk, 1993, Nature 366, 429.

White, S. D. M., and J. P. Ostriker, 1990, Ap.J. 349, 22.

Willick, J. A., S. Courteau, S. M. Faber, D. Burstein, A. Dekel, and M. A. Strauss, 1997, Ap.J.S. 109, 333.

Wu, K., O. Lahav, and M. Rees, 1999, Nature 397, 225.

Zaroubi, S., I. Zehavi, A. Dekel, Y. Hoffman, and T. Kolatt, 1997, Ap.J. 486, 21.

Zel’Dovich, Y. B., 1972, MNRAS 106, 1P.

Zhao, H., 1996, MNRAS 278, 488.

Zhao, H. S., D. N. Spergel, and R. M. Rich, 1995, Ap.J. 440, 13.

Zurek, W. H., P. J. Quinn, and J. K. Salmon, 1988, Ap.J. 330, 519.

Zwaan, M. A., J. M. van der Hulst, W. J. G. de Blok, and S. S. McGaugh, 1995, MNRAS 273, L35.
FIG. 1  [Taken from Bahcall et al. (1999).] The power spectrum for several variants of the CDM model: Standard CDM (SCDM), Tilted CDM (TCDM), Open CDM (OCDM), and ΛCDM. The shaded areas on the left represent COBE and CMB anisotropy measurements. The boxes on the right are measurements of the cluster abundance at $z \simeq 0$. The data points with open circles and $1\sigma$ error bars represent the APM galaxy redshift survey.
FIG. 2 [Taken from Moore et al. (1999b).] Rotation curves of high resolution CDM halos (solid curves) and LSB rotation curve data (dotted curves). The total rotational velocity and the baryonic contribution from the stars and gas for a typical LSB galaxy (UGC 128) are shown by open squares. Note the steeper rise of the model galaxy rotation curves compared with the observations.
FIG. 3 [Taken from Steinmetz and Navarro (1999).] I-band TFR at $z = 0$. Upper panel: simulations reproduce approximately the slope and dispersion, but not the zero-point of the observational TFR. Lower panel: when using $V_{200}$ (halo rotation velocity) all features of the observational TFR, even the zero-point, are well reproduced.
FIG. 4 [Taken from Dubinski and Carlberg (1991).] Distribution of axial ratios for CDM halos (collisionless simulations). Axial ratios within 25 kpc (asterisks), 50 kpc (circles), and 100 kpc (crosses) are displayed. The solid curves represent ellipsoids with $c/\alpha = 0.4$ and 0.5. In the inner regions (< 25 kpc) the halos are very flat and prolate. The shapes measured at larger radii represent oblate and prolate forms in approximately equal numbers.
FIG. 5 [Taken from Moore et al. (1999a).] Abundance of cosmic substructure within the MW, the Virgo cluster, and within a simulated cluster and a simulated galaxy. The dotted curve shows the distribution of satellites within the MW halo and the open circles with Poisson errors are data for Virgo. The dashed lines correspond to the substructure in a simulated galaxy today and 4 billion years ago. The solid line corresponds to a cluster mass halo. The agreement of theory and observation on large scales and the disagreement on galaxy scales is obvious. First, the simulation predicts comparable substructure on large and smaller scales; second, it predicts excessive substructure on galactic scales compared with observations.
FIG. 6  [Taken from Navarro and Steinmetz (2000b).] Specific angular momentum against circular velocity of model galaxies compared with observational data ($c_*$ and $\epsilon_\nu$ are parameters in the star formation and feedback algorithm used). The dotted line represents the (halo) velocity-squared scaling of the halo angular momentum. The solid line represents the (disk) velocity-squared scaling of the disk angular momentum. Note that the angular momenta resulting from the simulations are at least one order of magnitude lower than what is observed.