Quantization of dilaton cosmology in two dimensions

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Abstract

The generalized kinetic term of a dilaton gives the classical superinflation without recourse to any potential, and the quantum version of the dilaton gravity exhibits the finite curvature and graceful exit. For \( p = 2 \) case, the model corresponds to the RST quantization of the s-wave sector of the four-dimensional Einstein cosmology. Further, the de Sitter universe is realized for \( p = 8 \) and the smooth transition to the Minkowski space-time is possible. Even in the accelerating contraction case of the universe for \( 4 < p < 0 \), the curvature singularity does not appear in a certain branch.
I. Introduction

There has been much interest in the cosmological problems in the low-energy string theory. Recently, it has been shown that the scale factor duality \[1\] of the string theory motivated the inflation scenario \[2\]. The natural graceful exit \[3\] of superinflation to the decelerating expansion phase has not been realized in the classical string cosmology. The classical curvature singularity separates two classical solutions corresponding to accelerating expansion and decelerating expansion, respectively. However, the finite quantum transition between two cosmological phases is possible \[3\]. The explicit resolution of the graceful exit problem was suggested by considering the quantum back reaction of the geometry in the two-dimensional string theory \[3\] and the one-loop superstring cosmology and the dilaton cosmology \[3\]. In fact, the curvature singularity can be mild by the quantum back reaction of the geometry of the black hole in the spherical symmetric part of the Einstein gravity \[7\]. So it seems to be very plausible to reconcile the singularity problems with the help of the quantum back reaction in the gravity theory.

On the other hand, the Callan-Giddings-Harvey-Strominger (CGHS) \[8\] and the Russo-Susskind-Thorlacius (RST) \[9\] models have been extensively studied in order to study the final state of the black hole and the various aspects of the quantum back reaction \[10\]. The exactly soluble gravity models are crucial to determine the correct evolution of the geometry \[11\]. At the same time, they are more amenable to quantum treatments than their four-dimensional counterparts.

In this paper, we shall study the exactly soluble dilaton-gravity model which has a generalized coefficient in the kinetic term of a dilaton. This model contains the s-wave sector of Einstein gravity for \(p = 2\). Further, for \(p < 0\), the kinetic part has the right sign which corresponds to the two-dimensional version of the Brans-Dicke theory. However, the range of coefficient is restricted to \(4 < p < 8\) in the first branch and \(p > 0\) in the second branch to obtain the finite curvature scalar in the quantized theory. In this range, the duality-related solutions are possible only for the positive definite \(p\). For the special case
of \( p = 8 \), the classical de Sitter universe is realized. The quantum correction modifies the geometry and the de Sitter universe appears only asymptotically. In sect. II, the motivation is presented in terms of the spherical symmetric reduction of the Einstein gravity. The detailed analysis of our model is given in sect. III. The case of \( p = 8 \) is briefly analyzed in sect. IV. Finally the discussion is given in sect. V.

II. Motivation

The most general spherically symmetric metric can be expressed in the form of

\[
\text{ds}^2 = \text{ds}^2_{(2)} + \frac{1}{2} e^{2} \text{d}^2 \; ;
\]

where \( e^{2} = \frac{2}{G_{N}} \) and \( G_{N} \) is a Newton's constant \([12]\). The spherical symmetric reduction of the Einstein-Hilbert action leads to a two-dimensional dilaton gravity,

\[
S_{cl} = \frac{1}{2} \int_{\Sigma} \text{d}^2 x \sqrt{\text{g}} \; e^{2} \text{R} + 2 \text{ln} (\text{r}^3 + \text{R}^2) \; ;
\]

where the cosmological constant in two dimensions is inversely proportional to the Newton constant. For simplicity, let us consider the very strong coupling limit, i.e., the vanishing cosmological constant. The classical equations of motion yield cosmological solutions with two classical branches in the conformal gauge,

\[
\text{g} = \frac{1}{2} e^{2} \; ; \quad \text{g} = 0;
\]

The first branch corresponding to the accelerating expansion (pre-big-bang phase) is

\[
\text{(t)} = \frac{1}{2} \text{ln} A \; ;
\]

\[
e^{2} \; \text{(t)} = t \; ;
\]

where \( A \) and are integration constants. In the comoving time, the scale factor and the curvature are given by

\[
a(\text{t}) \; (\text{t})^{3} \; ; \quad \text{R} (\text{t}) \; (\text{t})^{2} \; ;
\]
where the range of comoving time is \( 1 < \tau < 0 \). This branch exhibits the superinflation behavior and the curvature scalar diverges at \( \tau = 0 \). The other branch (post-big-bang phase) is at space-time,

\[
a(\tau) = R(\tau) = 0;
\]

where \( 0 < \tau < +1 \). The above two branches are not connected smoothly because of the curvature singularity at \( \tau = 0 \). This feature already has been shown by Rey [5] in the CGHS model with vanishing cosmological constant. The only difference comes from the power of scale factor, where \( a(\tau) \) in the classical low energy string theory. As a matter of fact, the cosmological constant is generically non-zero. So it is unclear whether the classical superinflation of the 1st branch in our model is possible or not even in the presence of the cosmological constant. In the quantized theory which will be shown in the next section, however, the effective cosmological constant can be set to zero.

The two-dimensional dilaton gravity model is analyzed in [13] by generalizing the central charge of the conformal anomaly and the coefficient of the local counter term. In the next section, we shall consider the two-dimensional dilaton-gravity model with the generalized dilaton kinetic term.

### III. Dilaton Cosmology

Let us now start with the effective action given by

\[
S = S_{cl} + S_{qt};
\]

\[
S_{cl} = \frac{1}{2} \int d^2x \rho \left[ e^{2a} \left( R + p(r) \right)^2 + 2 \right] \frac{1}{2} e^{2a} X_{i}^{2} \mid (r f_{i})^{2} \]

\[
S_{qt} = \frac{1}{2} \int d^2x \rho \left[ \frac{1}{4} R \frac{1}{2} R - \frac{1}{2} R \right] \]

where \( i = 1; 2 \); \( N \), and \( a = 0 \) for the scalar fields and \( a = 1 \) for the bosonized fields of s-wave fermions in the four-dimensional point of view. We generalize the coefficient of the kinetic part of dilaton. The conformal anomaly coefficient is not positive definite and...
= \frac{p^2}{4} is determined to solve exactly up to the quantum level. Further, the local ambiguity of conform anomaly is chosen to cancel the classical cosmological constant, \( = 2 \). Hence we naturally obtain the exactly solvable model.

In the conformal gauge, the gauge fixed action is written as
\[
S = \frac{1}{2} \int d^2x \left( \frac{1}{2} \left( \partial \Theta \partial \Theta - \frac{p}{4} \right) + \frac{1}{2} e^{2a} \right) \Theta + \frac{1}{2} \Theta - \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} \Theta \Theta + \Theta \Theta \Theta \Theta
\]
\[(11)\]

The equations of motion with respect to the metric, dilaton, and matters are given by
\[
\begin{align*}
T_+ &= e^2 \left( \partial \Theta \partial \Theta - \frac{p}{4} \right) + \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} e^{2a} \Theta \Theta = 0; \\
\Theta_0 &= \frac{1}{2} \partial \Theta \partial \Theta + \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} e^{2a} \Theta \Theta = 0; \\
\Theta_i &= \frac{1}{2} \partial \Theta \partial \Theta + \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} e^{2a} \Theta \Theta = 0;
\end{align*}
\]
\[(12)\]

with two constraints implemented by the conformal gauge fixing,
\[
T_+ = e^2 \left( \partial \Theta \partial \Theta - \frac{p}{4} \right) + \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} e^{2a} \Theta \Theta + \frac{1}{2} e^{2a} \Theta \Theta = 0;
\]
\[(13)\]

The truncates the non-locality of the induced gravity of the conform anomaly. Without the classical matter, \( f_i = 0 \), defining new fields as follows \([14],[15]\),
\[
\begin{align*}
\Theta_0 &= \frac{p}{8} e^2 + 1; \\
\Theta_i &= \frac{p}{8} e^2 + 1;
\end{align*}
\]
\[(14)\]

the gauge fixed action \([11]\) is obtained in the simple form
\[
S = \frac{1}{2} \int d^2x \left( \frac{1}{2} \partial \Theta \partial \Theta + \frac{1}{2} \Theta \Theta + \frac{1}{2} \Theta \Theta \right)
\]
\[(15)\]

and Eqs. \((12)\), \((13)\) and \((15)\) yield
\[
\begin{align*}
0 &= \Theta_0, \Theta_i = \Theta_0, \Theta_i; \\
T &= \frac{1}{2} \Theta \Theta + \frac{1}{2} \Theta \Theta + \Theta \Theta + \Theta \Theta + \Theta \Theta
\end{align*}
\]
\[(16)\]
respectively. In the homogeneous space, all fields depend only on the conformal time. The solutions are easily obtained as

\[ = \frac{P}{8} + e^2 = \varphi t + B; \quad (21) \]
\[ = \frac{P}{8} + e^2 = \varphi t + A; \quad (22) \]

with the following constraint,

\[ t \left( \frac{1}{4} \right) (\varphi) (\varphi + \varphi) = 0; \quad (23) \]

where \( \varphi, \varphi, A, \) and \( B \) are constants. The function \( t \) depends on the quantum matters, and it is naturally fixed \( t = 0 \) from the boundary condition for the at Minkowski space time. Hereafter, we set \( A = B = 0 \) without loss of generality, which has no essential effect.

The first quantum branch defined by \( \varphi = \varphi (4 < \varphi < 8) \) is given by

\[ e^{\frac{\varphi}{8}} + \frac{1}{2} = \varphi t; \quad (24) \]
\[ e^2 + \frac{P}{8} = \varphi t; \quad (25) \]

where we assume \( \varphi < 0 \) so as to obtain the cosmological expansion solution. For convenience, by using the following relation

\[ (t) = \frac{P}{4} (t) = \frac{P}{8} e^{2} \varphi e^{2} \frac{P}{16} ; \quad (26) \]

which can be calculated from Eqs. (16), (17), and (19), the scalar curvature can be expressed in the form of

\[ R = 2e^{2} (t) = \frac{P}{4} e^{2} \varphi e^{2} \frac{P}{16} ; \quad (27) \]

Note that the curvature scalar is nonsingular when \( p < 0 \) which is our another restriction. We now obtain the extremum condition of curvature scalar,
From Eq. (28), the extremum condition is $e^{2 \pi} \frac{p}{8} \frac{4!}{16} - 0$ and the reality condition of the dilaton field gives the assumed restriction: $4 < p < 8$.

In fact, the curvature has a maximum value for $p > 0$ (a minimum value for $p < 0$) at a certain conformal time. In terms of the comoving time defined by $R = \frac{R}{dt} (1)$, the metric is recast in the form at the far past $t = 1$,

$$
\frac{ds^2}{-e^{2 \pi}} (\frac{dt}{1})^2 \frac{dx^2}{2} = d^2 + \left(1 \frac{p}{8}\right) \frac{2 \pi}{p} \frac{dx^2}{2}.
$$

through $\frac{8}{p} (\frac{\pi}{8})^2$. The curvature scalar is asymptotically at

$$
R = \frac{P}{4} (1 \frac{p}{8})^2
$$

and increases as time goes on and reaches the maximum value. The time evolutions of the scale factor are

$$a(t) = \frac{P}{8} \left(1 \frac{p}{8}\right)^{\frac{1}{p}} \text{sgn}(p); \quad (31)$$

$$a(t) = \frac{P}{8} \left(1 \frac{p}{8}\right)^{\frac{16}{4+1}} \text{sgn}(p); \quad (32)$$

and the expansion or contraction depends on the sign of $p$. If $p > 0$, the behavior of the space-time is accelerating expansion while it is accelerating contraction for $p < 0$.

On the other hand, in the far future $t = +1$, the metric and the curvature scalar are written as

$$
\frac{ds^2}{-e^{2 \pi} (\frac{dt}{1})^2 \frac{dx^2}{2}} = d^2 + \left(1 \frac{p}{8}\right)^{\frac{16}{4+1}} \frac{2 \pi}{p} \frac{dx^2}{2}.
$$

$$R(t) = \frac{P}{4} \left(1 \frac{p}{8}\right)^{\frac{16}{4+1}} \frac{2 \pi}{p} \frac{dx^2}{2}.
$$
where \( \frac{1}{2} e^2 \sigma t \). In this limit, the comoving time is \( \sigma + 1 \) for \( p > 0 \) while \( \sigma 0 \) for \( p < 0 \). It is interesting to note that all cases approach the Minkowski space-time asymptotically without encountering any divergent curvature. From the time evolution of the scale factor \( a(\sigma) \), \( \text{sgn}(p) \), \( a(\sigma) \), \( \text{sgn}(p) \), we note that for \( p > 0 \), the universe exhibits accelerating expansion and for \( p < 0 \), it does accelerating contraction.

As a result, the first branch exhibits the acceleration expansion for \( 0 < p < 8 \) with the bounded curvature which is asymptotically at in the far past and future. For \( 4 < p < 0 \), the universe starts at the at space-time and exhibits the accelerating contraction and reaches the at space-time.

Let us now analyze the second case, \( \sigma = 0 \) \( (p > 0) \). From Eqs. (21) and (22), the solutions are given by

\[
e^\frac{\sigma}{p} + \frac{e^\frac{16}{p} \sigma}{2} = 0;
\]

\[
e^2 + \frac{p}{8} \sigma = 0; \tag{35}
\]

\[
e^2 + \frac{p}{8} \sigma = 0; \tag{36}
\]

The extremum condition of the scalar curvature yields

\[
R(t) = \frac{p}{3} \sigma e^2 \exp \frac{4}{p} e^2 \sigma \frac{p}{16} e^4 \frac{p}{16} e^2 \frac{p}{64} e^2 #
\]

\[
= 0 \tag{37}
\]

when \( e^2 = \frac{h}{32} p + 8 \frac{p}{p^2 + 32 p + 64} \).

For the far past \( (1 (1) \), the metric is asymptotically written as

\[
d s^2 = \exp \frac{4}{e^\frac{16}{p} \sigma t} (d t^2 + d x^2)
\]

\[
= d t^2 + a^2(\sigma) d x^2; \tag{38}
\]

where the scale factor is

\[
a(\sigma) \exp \frac{2}{p} \sigma \frac{16}{p} \sigma 0 \tag{39}
\]

and \( t \). The curvature scalar asymptotically vanishes at \( \sigma + 1 \),

\[8\]
It has the maximum value at certain time similarly to the scalar curvature of the first branch. The time evolutions of the scale factor are

\[ a(\ ) \text{ sgn}\ (p); \quad (41) \]

\[ a(\ ) \text{ sgn}\ (p); \quad (42) \]

This shows that the scale factor starts at the finite value and increases, and the accelerating expansion is possible.

On the other hand, for \( t ! + 1 \) \( ( t ! + 1 \) ), the metric and the scale factor are

\[ ds^2 = ( \omega t ) \frac{2}{( \omega t )} e^{\frac{4}{\omega t}} (dt^2 - dx^2) \]

\[ = d^2 + a^2(\ )dx^2; \]

\[ a(\ ) = \frac{2}{0} - \ln \frac{2}{0} \left[ \frac{1}{8} \right] ; \quad (44) \]

where \( \frac{2}{0} e^{\frac{2}{0}} \). The asymptotic curvature is

\[ R = \frac{2}{4} \frac{2}{0} - \ln \frac{2}{0} \left[ \frac{1}{8} \right] ; \quad (45) \]

and it approaches the at Minkowskian space-time. As a comment, when \( p < 0 \) \( ( t ! > 0) \), the curvature diverges, and we cannot obtain the bounded curvature on the contrary to the first branch. For \( p > 0 \), the time evolution of the scale factors are given by

\[ a(\ ) = \frac{2}{0} > 0; \quad (46) \]

\[ a(\ ) = \frac{p}{16} \frac{1}{0} > 0; \quad (47) \]

As a result, the second branch \( ( t ! > 0) \) exhibits the accelerating expansion, and the curvature scalar is bounded and asymptotically at the far past and future.

IV. de Sitter Universe \( ( p = 8) \)
We briefly discuss the case of \( p = 8 \). From the beginning by setting \( p = 8 \), the de Sitter space is obtained as

\[
a(\tau) = e^A; \quad R = \text{const};
\]

(48)

where \( A \) and \( \tau \) are integration constants \((A < 0)\) and the range of comoving time is \( 1 < \tau < 0 \). Another solution (see the Eq. [49]) can be given by the at Minkowskian space-time for \( 0 < \tau < +1 \). These two solutions are not smoothly connected at \( \tau = 0 \). The de Sitter universe of the constant curvature abruptly changes to the at space-time. In the quantized theory, however, the metric is asymptotically de Sitter type at \( \tau \rightarrow 1 \) for the first quantum branch,

\[
ds^2 = \left( \frac{\alpha}{\tau} \right)^2 (d\tau^2 - dx^2);
\]

(49)

where \( \alpha \) and \( e^0 \) and follows the same behavior of the first quantum branch at \( \tau > +1 \) in sect. III. The quantum corrected maximum scalar curvature is

\[
R = 2(\alpha)^2
\]

(50)

at \( \tau = 1 \), and monotonically decreases and approaches zero. So the universe for the \( p = 8 \) case exhibits the accelerating expansion from the de Sitter space-time smoothly connected with the accelerating expansion which ends up with the at Minkowskian space-time in the first branch. There is no de Sitter phase in the second quantum branch.

V: Discussion

Especially we comment on the case \( p = 2 \) corresponding to the s-wave sector of Einstein gravity. Classically, it is not clear whether or not the superinflation is possible except for the case of the strong coupling constant. However, the nonvanishing cosmological constant can be canceled effectively by the induced cosmological constant in the quantized theory. So the quantized behaviors are very similar to those of the CGHS model [11]. Further,
we note that the radius of the two-sphere in four dimensional sense is proportional to $e^2$, and the two quantum branches are some kind of T-duality [16] related instead of the scale factor duality for $p = 4$ case.

In summary, we have showed that the s-wave sector of the Einstein gravity ($p = 2$) in the quantized theory exhibits the accelerating expansion and ends up with the small accelerating expansion ($a > 0$) in both branches. Especially, for $p = 8$, the asymptotic de Sitter space is smoothly connected with the at Minkowski space-time in the first branch of the quantized theory. Further, for $4 < p < 0$ in the first branch, the universe is accelerating contraction from the Minkowski space-time and has no curvature singularity. In this case, the dilaton plays a matter role and gives an attractive force which is of relevance to the contraction of the universe.

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