Reviving the interference: framework and proof-of-principle for the anomalous gluon self-interaction in the SMEFT

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Interferences are not positive-definite and therefore they can change sign over the phase space. If the contributions of the regions where the interference is positive and negative nearly cancel each other, interference effects are hard to measure. In this paper, we propose a method to quantify the ability of an observable to separate an interference positive and negative contributions and therefore to revive the interference effects in measurements. We apply this method to the anomalous gluon operator in the SMEFT for which the interference suppression is well-known. We show that we can get constraints on its coefficient, using the interference only, similar to those obtained by including the square of the new physics amplitude.

Introduction The Standard Model Effective Field Theory (SMEFT) explores the deviations in SM couplings due to interactions among Standard Model (SM) particles and new states, too heavy to be produced at the LHC or any other considered experiment. Nonetheless, those new states affect the interactions between the SM particles and accurate measurements of their strengths should, thus, reveal or constrain the presence of new physics. In this framework, heavy new degrees of freedom are integrated out and the new physics is parametrised by higher-dimensional operators \cite{1}\cite{2},

\begin{equation}
\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i}{\Lambda^2} \mathcal{O}_i + \mathcal{O}(\Lambda^{-4}),
\end{equation}

where $\Lambda$ is the new physics scale. As a results, observables such as differential cross-sections display the same expansion,

\begin{equation}
\frac{d\sigma}{dX} = \frac{d\sigma^{SM}}{dX} + \sum_i \frac{C_i}{\Lambda^2} \frac{d\sigma}{dX} + \mathcal{O}(\Lambda^{-4})
\end{equation}

where $X$ is a generic name for a measurable variable. While constraints should ideally come from the second term, i.e. the term linear in the coefficients, they often come in practice from the term quadratic in $C_i$ or from terms of even higher power of $C_i$. This phenomenon mainly originates from the fact that the linear term is an interference between the SM amplitudes and the amplitudes linear in $C_i$, and this interference has been shown to be suppressed \cite{3} for $2 \to 2$ processes. As it will be illustrated below, this suppression occurs also in higher multiplicity processes. An interference suppression can have two origins: either the interference matrix element is small all over the phase space, or it changes sign over the phase space. This letter aims, in the second case, to revive the interference using differential measurements and to assess the efficiency of the reviving procedure. Although we will focus on a single operator in the rest of the letter, the method is generic and can be applied for any interference suppressed by a sign flip in the phase space, including interference unrelated to the SMEFT. Another obvious application in the SMEFT is the CP-violating operators \cite{4}. Their interference do not contribute to the total cross-section of $C$-even processes by symmetry, but they can probed using CP-violating observables.

Framework In this work we concentrate on the dimension-6 operator

\begin{equation}
O_G = g_\ast f_{abc} G_{\mu\nu}^a G_{\rho\sigma}^b G_{\mu\rho}^c G_{\nu\sigma}^d,
\end{equation}

with $G_{\mu\nu}$ the gluon field strength. While this operator is expected to contribute to multijets and top-pair production, its interference vanishes for dijet and is strongly suppressed for the other processes. Constraints on this operator affect the sensitivity over other operators involved, for example, in top quark production \cite{5}. High-multiplicity jet measurements strongly constrain this operator but mainly from the $\mathcal{O}(\Lambda^{-4})$ or even higher order terms \cite{6, 7}.

The stricter bound on this operator comes from the $\mathcal{O}(\Lambda^{-4})$ in dijet measurements \cite{8} and reads

\begin{equation}
\frac{C_G}{\Lambda^2} < (0.031 \text{ TeV})^{-2}
\end{equation}

at 95\% confidence level (CL).

We use the SMEFT@NLO \cite{9} Universal FeynRules Output (UFO) \cite{10}, written from a FeynRules model \cite{11} containing the dimension-six operators, to generate the LO partonic events needed for our study. All the operators coefficients are set to zero but the $O_G$ one, which is taken equal to 1 with $\Lambda = 5$ TeV. Madgraph@NLO \cite{12} is then used to generate events for the SM, the square of the $1/\Lambda^2$ amplitudes and their interference. Throughout this paper, we truncature the amplitude at $\mathcal{O}(1/\Lambda^2)$ and therefore $\mathcal{O}(1/\Lambda^4)$ terms always come from the square of the $1/\Lambda^2$ amplitudes. Namely, multiple insertions of the dimension-six operators are not allowed. We use the NNPDF2.3 parton distribution function (PDF) set \cite{13} and the results are given for LHC at 13 TeV at the partonic level. We leave the study of the effect of NLO
corrections, parton shower and detector effects for future studies.

The cancellation over the phase space is efficient if the integrals of the interference in the phase space part where its matrix element is positive and negative are almost equal in absolute value. Those two integrals are obtained from the sum of the weights of events generated according to the interference, keeping respectively only positive or negative weighted events. In table I, we use the percentage of positive unweighted events to quantify the efficiency of this cancelation for top and jet processes. Since the strongest cancellation occurs for three-jets and this process has the large cross-section necessary for accurate differential measurement, in the remaining of this letter, we will restrict ourself to this process and leave the other for future analyses. The integral of the absolute valued interference differential cross-section,

\[ \sigma^{[\text{int}]} = \int d\Phi \left| \frac{d\sigma^{\text{int}}}{d\Phi} \right| \]  

is computed from the sum of the absolute values of the weights and is an upper bound of the total measurable effect of the interference over the whole phase space \( \Phi \). This quantity is given in table II together with the SM, and the interference and \( \mathcal{O}(1/A^4) \) total cross-sections. The comparison of those four quantities shows the strong suppression of the interference total cross-section, and how it is lifted by \( \sigma^{[\text{int}]} \). Unfortunately, \( \sigma^{[\text{int}]} \) is not a measurable quantity as it requires to measure not only the momenta of the jets, but also their flavours and helicities, as well as those of the incoming partons. Therefore, we define the measurable absolute value cross-section,

\[ \sigma^{[\text{meas}]} = \int d\Phi_{\text{meas}} \left| \sum_{\{um\}} \frac{d\sigma}{d\Phi} \right| \]  

where \( \{um\} \) is the set of unmeasurable quantities of the events. For other processes, the sum can be replaced, at least partially, by integrals over continuous unmeasurable quantities, such as the longitudinal momenta of a neutrino. This is the difference between the positive and negative contributions of the interference to the total cross-section using all the information experimentally available (and assuming perfect measurements of the jets momenta). As a result, this is an upper bound for any asymmetry build on one or a few kinematic variables aiming at restoring the interference, and therefore can be used to assess the efficiency of such asymmetry. \( \sigma^{[\text{meas}]} \) is estimated by

\[ \sigma^{[\text{meas}]} = \lim_{N \to \infty} \sum_{i=1}^{N} w_i \cdot \text{sign} \left( \sum_{\{um\}} ME (\vec{p}_i, um) \right) \]  

where ME is the part of the squared amplitude due to the interference and \( w_i \) and \( \vec{p}_i \) label the weight and the momenta of the jets of the event \( i \). Therefore, this can be seen as a matrix element method [14–19] at the partonic level to revive the interference. The values of \( \sigma^{[\text{meas}]} \) for the three-jet final state and different cuts are given in table II. The cancellation among positive and negative weighted events decreases with the \( p_T \) cut while the ratio \( \sigma^{[\text{meas}]} / \sigma^{[\text{int}]} \) remains roughly constant.

### Differential distributions

We tested the ability to separate positive and negative weight for various differential and double differential cross-sections. Tested distributions include the transverse momenta \( p_T \) and the pseudorapidities \( \eta \) of the jets, their angular distances \( \Delta R \), their invariant masses, the normalised triple product among the three-momenta of the jets, and some event-shape variables, including the transverse thrust, the jet broadening [20] and the transverse sphericity [21]. Several variables such as the \( p_T \) of the first jet, \( p_T[j_1] \), the transverse trust and the angular distance between the two lowest \( p_T \) jets, \( \Delta R[j_2,j_3] \) achieve an efficiency of about 40% compared to \( \sigma^{[\text{meas}]} \). For comparison, the efficiency of the total cross-section is about 2%. The best efficiency, however, is obtained for the transverse sphericity and is about 80%. Moreover, this efficiency barely varies with the global lower cut on each of the three \( p_T \) jets. The transverse sphericity \( Sph_T \) is defined by using the eigenvalues \( \lambda_i \) of the transverse momentum tensor:

\[ M_{xy} = \sum_{i=1}^{N_{jets}} \left( \begin{array}{cc} p_{x,i}^2 & p_{x,i} p_{y,i} \\ p_{y,i} p_{x,i} & p_{y,i}^2 \end{array} \right), \quad Sph_T = \frac{2\lambda_2}{\lambda_2 + \lambda_1}. \]  

Therefore, sign flip occurs between the events that are more two-jets like \( (Sph_T \sim 0) \) and those that are three well separated and balanced jets \( (Sph_T \sim 1) \). This explains why the phase space cancellation is lower with the high \( p_T \) cut, as strong hierarchy between the jets becomes then unlikely. The separations of the negative and positive contributions for some of those variables are illustrated in figure 1, where the full distributions as well as those of the positive and negative weighted events are drawn separately. Contrarily to inefficient variables, the
distribution of the positive and negative weighted events are different, resulting in a non-zero and changing sign distribution for the full interference.

Using the transverse sphericity to split the positive and negative contributions, we now estimate the limits that could be obtained on $\frac{C_G}{\Lambda^2}$, either for the interference only or including the $\mathcal{O}(1/\Lambda^4)$ contribution, too. The bounds are obtained, for each double distribution, from the following $\chi^2$-squared

$$
\chi^2 = \sum_i \left( \frac{x_i^{\exp} - x_i^{th}}{\sigma_i} \right)^2 = \sum_i \left( \frac{C_G x_i^{1/\Lambda^2}}{\sigma_i} \right)^2 
$$

where $x_i^{\exp}$ and $x_i^{th} = x_i^{SM} + C_G x_i^{1/\Lambda^2}$ are respectively the measured and predicted content of each bin. Since the experimental results for the distributions we are interested in have not been published yet, we assume that the experimental data will follow the SM distributions for the considered quantities (resulting in the last step of Eq. (9)) and that the uncertainty, $\sigma_i$, for the $i$th bin is 10% of its SM content. This estimate of the uncertainty seems consistent with available experimental results [22]. We choose our binning such that each bin would contain enough events, assuming the SM only to ensure that the statistical errors are below 10%, for a luminosity of 100 fb$^{-1}$. The best results are displayed in table III.

Finally, to assess the validity of the SMEFT with our approach, we display in figure 2 how the limits on $\Lambda$ varies if a cut on the center-of-mass energy is applied, assuming $C_G = 1$. In principle, the EFT is valid if $\sqrt{s} < \Lambda$, which is only satisfied for $C_G$ slightly bigger that 1 with the low $p_T$ cuts. The situation improve for the stronger constraints derived with higher cuts. In both cases, the constraints barely change when the events with $\sqrt{s} \gtrsim 6$ TeV are included. The bounds, obtained through the interference only, grow faster than the ones which involve the $\mathcal{O}(\Lambda^{-4})$ contribution too, as it is expected because of their different dependency on $\Lambda$. The bounds obtained by using the $S_T$ variable, defined in [6], are also shown for comparison. As expected, our distribution shows a nice improvement for the bounds at $\mathcal{O}(\Lambda^{-2})$.

**Conclusions** We used the sign of the measurable matrix element as a tool to revive the interference and to quantify the efficiency of differential distributions to separate negatively and positively contributing regions of

| $p_{T,\text{min}}$ [GeV] | SM $\sigma$ [pb] | $\mathcal{O}(1/\Lambda^2)$ $\sigma$ [meas] [pb] | $\mathcal{O}(1/\Lambda^4)$ $\sigma$ [int] [pb] | $\mathcal{O}(1/\Lambda^4)$ $\sigma$ [pb] |
|-------------------------|-----------------|----------------|----------------|----------------|
| 50                      | 9.70$\times$10$^5$ | 4.08 50.4% 7.83$\times$10$^2$ | 1.05$\times$10$^3$ | 3.93$\times$10$^3$ |
| 200                     | 8.96$\times$10$^2$ | 2.92$\times$10$^{-1}$ 51.4% 3.5$\times$10$^3$ | 5.02$\times$10$^3$ | 2.73 |
| 500                     | 3.10            | 1.69$\times$10$^{-2}$ 54.0% 6.04$\times$10$^{-1}$ | 8.96$\times$10$^{-1}$ | 1.48$\times$10$^{-1}$ |
| 1000                    | 9.08$\times$10$^{-3}$ | 4.56$\times$10$^{-4}$ 60.1% 1.46$\times$10$^{-3}$ | 2.29$\times$10$^{-3}$ | 3.05$\times$10$^{-3}$ |

**TABLE II.** Cross-sections for three-jet production, for different values of the $p_T$-cut, $\Delta R > 0.4$, $\Lambda = 5$ TeV and renormalisation scales fixed respectively at 150, 250, 500, 1000 and 2000 GeV, with up to one $O_C$ insertion. The percentages of the total amount of positive-weighted events, the percentages of the positive and negative measurable matrix elements (mme) and $\sigma^{\text{int}}$ are shown for the interference.
TABLE III. Best bounds on the $C_G$ coefficient for different cuts on the $p_T$, for $\Lambda = 1$ TeV and 68% CL. The number of bins is reported, for each distribution; the cut on the sphericity is the value, between 0 and 1, in which we separated the two bins used for this variable. In the bounds columns, the first numbers are obtained through the $O(\Lambda^{-2})$ contribution only, the ones into brackets take into account the $O(\Lambda^{-4})$ data, too.

| $p_T,_{\text{min}}$ [GeV] | Distribution | Sph$_T$ cut | Bins | Upper bound on $C_G$ | Lower bound on $C_G$ |
|----------------------------|--------------|-------------|------|---------------------|---------------------|
| 50                         | $p_T [j_3] \text{ vs } Sph_T$ | 0.23        | 34   | 2.5·10$^{-1}$ (1.1·10$^{-1}$) | -2.5·10$^{-1}$ (-1.2·10$^{-1}$) |
| 200                        | $S_T \text{ vs } Sph_T$        | 0.27        | 34   | 7.5·10$^{-2}$ (2.3·10$^{-2}$) | -7.5·10$^{-2}$ (-2.4·10$^{-2}$) |
| 500                        | $M[j_2j_3] \text{ vs } Sph_T$  | 0.31        | 21   | 5.5·10$^{-2}$ (5.3·10$^{-2}$) | -5.5·10$^{-2}$ (-3.5·10$^{-2}$) |
| 1000                       | $M[j_2j_3] \text{ vs } Sph_T$  | 0.35        | 7    | 2.6·10$^{-2}$ (1.9·10$^{-2}$) | -2.6·10$^{-2}$ (-1.8·10$^{-2}$) |

FIG. 2. Upper bounds on $\Lambda$ (for $C_G = 1$) as functions of the upper cut over the center-of-mass energy $\sqrt{s}$, inferred from the best distribution for each $p_T$-cut. The red line shows the bounds from the $O(\Lambda^{-2})$ term only, which are symmetrical with respect to 0, while the blue line take into account the $O(\Lambda^{-4})$ one, too. The orange and purple lines reproduce the bounds, obtained through the $S_T$ variable, considered in [6], at $O(\Lambda^{-2})$ and $O(\Lambda^{-4})$. The axis on top of the plots quantifies the percentage of events, in the interference sample, that get lost form the cut on $\sqrt{s}$

the phase space. We used it to find efficient distributions to look for the interference effect of anomalous gluon interactions, as predicted by the SMEFT, and to put on the corresponding operators, for the first time, contraints which are dominated by the leading ($O(\Lambda^{-2})$) interference and not by the $O(\Lambda^{-4})$ term, coming from the new physics amplitude squared. Therefore, the observable would also be sensitive to the sign of the coefficient. In addition, the proposed measurement can be easily reinterpreted in other BSM scenarios if SMEFT assumptions turn out not to be valid, as they are purely kinematic distributions. While the method has been tested on this particular case, it is fully generic and can be applied for any interference suppression due to sign flips over the phase space.

Acknowledgements. We are grateful to Fabio Maltoni and Vincent Lemaitre for interesting discussions during the completion of this work. This work was funded by the F.R.S.-FNRS through the MISU convention F.6001.19.

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