On the detectability of the hydrogen 3-cm fine-structure line from the epoch of reionization

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ABSTRACT
A soft ultraviolet radiation field, $10.2 < h\nu < 13.6$ eV, that permeates neutral intergalactic gas during the epoch of reionization (EoR) excites the 2p (directly) and 2s (indirectly) states of atomic hydrogen. Because the 2s state is metastable, the lifetime of atoms in this level is relatively long, which may cause the 2s state to be overpopulated relative to the 2p state. It has recently been proposed that for this reason, neutral intergalactic atomic hydrogen gas may be detected in absorption in its 3-cm fine-structure line ($2s_{1/2} \rightarrow 2p_{3/2}$) against the cosmic microwave background out to very high redshifts. In particular, the optical depth in the fine-structure line through neutral intergalactic gas surrounding bright quasars during the EoR may reach $\tau_{FS} \sim 10^{-5}$. The resulting surface brightness temperature of tens of $\mu$K (in absorption) may be detectable with existing radio telescopes. Motivated by this exciting proposal, we perform a detailed analysis of the transfer of Ly$\beta$, $\gamma$, $\delta$, $\ldots$ radiation, and reanalyse the detectability of the fine-structure line in neutral intergalactic gas surrounding high-redshift quasars. We find that proper radiative transfer modelling causes the fine-structure absorption signature to be reduced tremendously to $\tau_{FS} \lesssim 10^{-10}$. We therefore conclude that neutral intergalactic gas during the EoR cannot reveal its presence in the 3-cm fine-structure line to existing radio telescopes.

Key words: line: profiles – radiative transfer – cosmology: theory – diffuse radiation – radio lines: general.

1 INTRODUCTION
It has recently been proposed that it may be possible to observe neutral atomic hydrogen gas during the epoch of reionization (EoR), in absorption against the cosmic microwave background (CMB) in its 3-cm fine-structure line ($2s_{1/2} \rightarrow 2p_{3/2}$, Sethi, Subrahmanyan & Roshi 2007, hereafter S07).

The possible detectability of the 3-cm line derives from the fact that the 2s state of atomic hydrogen is metastable (Breit & Teller 1940). That is, the average lifetime of an undisturbed atom in the 2s state is $t_{2s} = 1/A_{2s1s} \sim 0.1$ s, after which it decays into the 1s state by emitting two photons (Spitzer & Greenstein 1951). For comparison, the mean lifetime of atoms in the 2p state is $t_{2p} = 1/A_{2p1s} \sim 10^{-9}$ s. Hence, when atoms are excited into their 2s and 2p states at comparable rates, the 2s level is expected to be highly overpopulated relative to the 2p level. This may result in a non-negligible optical depth in the $2s_{1/2} - 2p_{3/2}$ fine-structure line (see e.g. Wild 1952).

The detectability of this line has been discussed in the context of H$\text{II}$ regions in the local Universe (e.g. Dennison, Turner & Minter 2005, for a more detailed discussion). In local H$\text{II}$ regions, the 2s level is populated following recombination, while both recombination and photoexcitation by Ly$\alpha$ photons populate the 2p level. The Ly$\alpha$ excitation rate of the 2p state is limited by dust, and the 2s state is typically heavily overpopulated in H$\text{II}$ regions which result in 3-cm optical depths of the order of $\tau_{FS} \sim 10^{-5}$ (Ershov 1987; Dennison et al. 2005, where the absorption is measured relative to the free–free continuum that is produced in the H$\text{II}$ region). S07 have argued that during the EoR, bright quasars that are luminous in the rest-frame soft ultraviolet (UV) ($10.2 < E_{\gamma} < 13.6$ eV) may indirectly photoexcite the 2s state of neutral hydrogen gas in the low-density intergalactic medium (IGM) to levels such that $\tau_{FS} \sim 10^{-5}$ for CMB photons passing through these regions.$^1$ Absorption of these CMB photons would

$^1$ It may be surprising that the sparsely populated $n = 2$ levels of atomic hydrogen can produce a detectable opacity. One can estimate $\tau_{FS}$ by comparing it to the Gunn–Peterson optical depth $\tau_{GP} = \frac{n_{2s}}{n_{H}} \frac{A_{2s1s} \Phi_{Ly\alpha}}{\pi \nu_{Ly\alpha} \Delta \nu_{Ly\alpha}} \sim 20 \frac{n_{2s}}{n_{H}}$. That is, $\tau_{FS} = 10^{-5}$ only requires that $\frac{n_{2s}}{n_{H}} \sim 10^{-12}$. It will be shown in Section 5.1 that these levels can be reached in regions around high-redshift quasars.
result in an absorption feature with a brightness temperature of a few tens of μK at 1.4 GHz, which may be detectable with existing dish antennas such as the Green Bank Telescope, Parkes and the most compact configuration of the Australia Telescope Compact Array (e.g. Carilli 2008). This result is important, because it implies that neutral intergalactic atomic hydrogen gas that exists during EoR may reveal itself in a line that is different (and possibly easier to detect) than the well-studied 21-cm hyperfine transition (e.g. Scott & Rees 1990; Loeb & Zaldarriaga 2004; Furlanetto, Oh & Briggs 2006). Motivated by this exciting result, and by observational efforts to detect this transition, we reanalyse the detectability of the fine-structure line during the EoR. We perform more detailed modelling of the transfer of Lyβ radiation than originally discussed by S07. We will show that (unfortuantely) proper modelling of Lyβ (and Lyγ, δ, ...) radiative transfer radically lowers the value of τ_{FS} to levels that cannot be probed with existing radio telescopes.

The outline of this paper is as follows. We review the dominant excitation mechanisms of the 2s and 2p levels in Section 2. In Section 3, we compute the effective excitation rate of the 2s state during the EoR, by properly accounting for radiative transfer, and discuss the detectability of the 3-cm fine-structure line in Section 4. The validity of our model assumptions, and the implications of this work are discussed in Section 5. Finally, we present the conclusions in Section 6. The parameters for the background cosmology used throughout are (\Omega_m, \Omega_X, \Omega_b, h) = (0.27, 0.73, 0.042, 0.70) (Komatsu et al. 2008).

2 THE EXCITATION MECHANISM OF THE 2S LEVEL

Fig. 1 depicts the first three principle energy levels of the hydrogen atom. On the left-hand side, fine- and hyperfine-structure splitting

![Schematic diagram of the energy levels of a hydrogen atom.](image)

**Figure 1.** Schematic diagram of the energy levels of a hydrogen atom. On the left-hand side, we show the first three energy levels and their splitting into levels with different angular momentum. The notation for each level is nL, where n is the principle quantum number and L denotes the orbital angular momentum number (L = S corresponds to l = 0, L = P corresponds to l = 1 and L = D corresponds to l = 2). The dotted lines show the allowed transitions through the emission or absorption of a single photon. In a three-level atom, the selection rules only allow the 2s state to be populated by absorbing a Lyβ photon, and by subsequently emitting an Hα photon. On the right-hand side, the fine- + hyperfine-structure splitting of the 2s and 2p levels is shown in more detail. The Lamb–Rutherford shift causes the 2s1/2 level to lie above the 2p1/2 level by ΔE/hν \sim 1.1 GHz (e.g. Wild 1952). The three allowed transitions between the 2s1/2 and 2p1/2 levels are indicated with dashed lines, and the frequency (in MHz) of each transition is indicated (also see Dennison et al. 2005). The relative strength of each transition 9852:9876:10030 is 1:5:2 (Wild 1952).

of the lines are ignored. The notation that is used is nL, where n is the principle quantum number and L denotes the orbital angular momentum number (L = S corresponds to l = 0, L = P corresponds to l = 1 and L = D corresponds to l = 2). In most astrophysical conditions, the vast majority of hydrogen atoms are in the 1s state. Atoms can be excited out of the ground state through the following processes: (i) collisional excitation, (ii) recombination following photoionization or collisional ionization and (iii) photoexcitation. Following S07, processes (i and ii) are ignored (see Section 5.1 – under roman numeral I – for a more detailed motivation).

The quantum mechanical selection rules state that the only atomic transitions that are allowed by emitting or absorbing a photon are those in which f changes by Δf = ±1. Hence, absorption only results in transitions of the form 1s \rightarrow np, while subsequent emission only results in n p \rightarrow m s. The allowed transitions among the first three levels of the H atom are indicated in Fig. 1 as dotted lines. In a three-level atom (the impact of using a multilevel atom is discussed in Section 5.1 under roman numeral II), the selection rules only allow the 2s state to be populated by absorbing a Lyβ photon, and by subsequently emitting an Hα photon. Hence, the level population of atoms in the 2s state is determined by the effective Lyβ scattering rate, denoted by \(P_{\beta, eff}\), which is defined as the rate at which Lyβ photons absorbed, and subsequently converted into Hα + 2 continuum photons. This rate is lower than the ‘ordinary’ Lyβ scattering rate, \(P_{\beta}\), which is the rate at which Lyβ photons are absorbed.

3 THE 2S AND 2P LEVEL POPULATIONS

As mentioned above, the effective Lyβ scattering rate, \(P_{\beta, eff}\), is crucial in calculating the number of atoms in the 2s state. The goal of this section is to evaluate \(P_{\beta, eff}(r)\). In Section 3.1, we describe the Monte Carlo (MC) technique that was used to compute the general and effective Lyβ scattering rates, and in Section 3.2 we present our results.

3.1 Outline of Monte Carlo calculation

We compute the general and effective Lyβ scattering rate by performing MC calculation of the Lyβ radiative transfer in the neutral gas surrounding a spherical H I region of radius RH = 5 Mpc (Appendix A1). This MC approach to radiative transfer, in which the trajectories of individual photons are followed, is common for Lya radiation (e.g. Loeb & Rybicki 1999; Ahn, Lee & Lee 2000; Zheng & Miralda-Escudé 2002). We use the Lyα MC code originally described in Dijkstra, Haiman & Spaans (2006), which is easily adapted for Lyβ radiative transfer. The calculation is performed as follows.

(i) A photon is emitted with a frequency that is drawn randomly from the range \(v \in [0.95\nu_0, 1.05\nu_0]\). The transfer calculation therefore focuses on photons in a frequency range \(\Delta\nu_p = 0.1\nu_0\) centred on \(\nu_0\), similar to the optically thin calculation that was originally performed by S07 (and which is discussed in Appendix A1).

(ii) The photon propagates an optical depth \(\tau = -\log R\), where \(R\) is a random number between 0 and 1. The optical depth translates to a physical distance via \(\tau = \int_0^L ds n_H(s)\sigma_p(v(1 - H_\nu/c))\).

(iii) Each time a Lyβ photon is absorbed, there is a \(p_{\beta, abs} = 0.88\) probability for the atom to decay directly back to the ground state by emitting a Lyβ photon (Hirata 2006; Pritchard & Furlanetto 2006). This is simulated in the MC calculation by generating a
The scattering process is assumed to be partially coherent, and described by a Rayleigh phase function, (see Dijkstra & Loeb 2008 for a more detailed discussion of the scattering phase function).

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rate (black solid histogram) as extracted from the MC simulation agrees best with the analytic solution given by equation (A6) with \( \tau = 0 \). Naturally, here the effective scattering rate is suppressed by a factor of \( (1 - \sigma_{\text{scat}}) = 0.12 \).

4 THE SIGNAL

The change in brightness temperature for CMB radiation passing through the neutral gas with an enhanced population of atoms in the 2s state is given by

\[
\Delta T_b = \frac{\tau_{\text{FS}}}{1 + \nu} (T_{\text{ex}} - T_{\text{CMB}}),
\]

(1)

where \( T_{\text{CMB}} = 2.73(1 + z) \) is the CMB temperature at redshift \( z \), and \( T_{\text{ex}} \) is the excitation temperature of the 2s–2p transition, which is defined as \( T_{\text{ex}} \equiv \lambda_{\text{FS}}^2 S / 2 k_B \), where \( S = \frac{8 \pi j_{\text{fs}} \nu_{\text{ex}}}{c^2} \frac{n_e}{n_0} \) is the standard source function (see Rybicki & Lightman 1979, equation 1.79), \( n_1 \) (\( n_0 \)) denotes the number density of atoms in the 2s (2p) level and \( g_{\text{u}} \) (\( g_{\text{s}} \)) denotes the statistical weight of this level. Throughout, we ignore that hyperfine splitting of the 2s and 2p levels actually results in three possible 2s–2p transitions (see Fig. 1). We discuss the impact of hyperfine splitting on our results in Section 5.1 (under roman numeral V). Furthermore, the optical depth to 2s–2p absorption, \( \tau_{\text{FS}} \), is given by (Rybicki & Lightman 1979, equation 1.78)

\[
\tau_{\text{FS}}(\nu) = \int ds \sigma(\nu) \frac{\phi(\nu)}{\nu},
\]

where \( \Delta \nu_{\text{FS}} = \nu_{\text{ex}} \nu_{\text{FS}} / c \), \( B_{\text{fs}} \) is the Einstein B-coefficient for the fine-structure transition, and \( \phi(\nu) \) is the line profile function (see Rybicki & Lightman 1979). S07 move the factor \( \frac{\nu_{\text{ex}}}{\nu_0} \) outside of the integral and arrive at their equation (7). However, this is not allowed, because we found \( \Gamma_{\nu}(r) \), and therefore \( n_{\text{e}}(= n_{\text{H}}) \), to be a rapidly changing function of radius (see Fig. 2). This makes the evaluation of \( \tau_{\text{FS}}(\nu) \) somewhat trickier. Fortunately, we may simplify equations (1) and (2) by assuming that \( n_1 > n_0 \) (which is verified in Section 5.1, under roman numeral IV). Under this assumption, \( T_{\text{ex}} \ll T_{\text{CMB}} \) and \( \Delta \nu_{\text{FS}}(\nu) = -\tau_{\text{FS}}(\nu) T_{\text{CMB}} / (1 + \nu) \), and the optical depth to 2s–2p absorption at frequency \( \nu \) simplifies to

\[
\tau_{\text{FS}}(\nu) = \int ds \sigma_0(\nu) \sigma_{\text{FS}}(1 - H_0 / c),
\]

(3)

where \( \sigma_0(\nu) \) is the 3-cm fine-structure line from the EoR

Figure 3. This figure shows the optical depth in the 2s–2p fine-structure transition, \( \tau_{\text{FS}}(\nu) \), through neutral gas surrounding a quasar \( \text{H} \eta \) region at \( z = 6.5 \). The maximum optical depth is \( \tau_{\text{FS}, \text{max}} \sim 4 \times 10^{-10} \), which results in a maximum brightness temperature of \( \sim 10^{-10} \) K. The transitions are therefore more difficult to detect than previous estimates have suggested.

5 DISCUSSION

5.1 Validity of model assumptions

(1) In Section 2, we mentioned that the collisional excitation rate of – and the recombination rate into – the 2s state was negligibly small compared to the effective \( \text{Ly} \beta \) scattering rate. For example, the collisional excitation rate is given by \( C_{\text{ex}} = 2.24 \times 10^{-9} n_e T_{\text{gas}}^{-1/2} \exp (-10.2 \text{eV} / kT_{\text{gas}}) \) (Osterbrock 1989). If we assume that the gas is half-ionized \( (n_e = 0.5n_0) \), then we find that \( C_{\text{ex}} < 10^{-14} \text{ s}^{-1} \) for

\[
\begin{array}{c}
\text{The 3-cm fine-structure line from the EoR} \\
1433
\end{array}
\]
$T_{\text{rec}} < 3 \times 10^4$ K. This rate is orders of magnitude less than the effective Ly\(\beta\) scattering rates in the regions of interest. Similarly, we find that the recombination rate is $\tau_{\text{Ly} \beta} < 10^{-17}$ s\(^{-1}\), which is even smaller.

(II) So far, we have ignored the fact that the 2s state of atomic hydrogen can also be accessed following the absorption of higher order Lyman series photons, e.g. via a scattering event of the form $\text{Ly} \beta \rightarrow \text{Ly} \alpha$. In principle, it is straightforward to expand the MC approach to include scattering of these higher order Lyman series photons, but this is not necessary. We found in Section 3.2 that the effective Ly\(\beta\) scattering rate was calculated reasonably well if one simply suppresses the incoming Ly\(\beta\) flux with a factor of $e^{-\tau}$. For simplicity, we also use this approach to calculate the effective scattering rates of the higher order Lyman series photons, and cast the total effective scattering rate in a form similar to that of equation (6):

$$P_{\text{eff,tot}} = \frac{L_n \pi e^2}{4 \sigma T^2 r^2 \pi m_e c} \sum_{n'=\gamma,\delta,...} \int \frac{f_{n'}}{h \nu_{n'}} dx \phi_n(x) e^{-\tau_{\text{Ly} \beta}(x)},$$

where $f_{n'}$ is the oscillator strength, $h \nu_n$ is the energy and $\phi_n(x_n)$ is the Voigt function,\(^4\) for the Lyman-$n$ transition (i.e. $1s \rightarrow n\pi$), where $x_n = (v - \nu_n) / \Delta v_n$. The first term of the series corresponds to equation (6). The result of summing the first 10 Lyman series transitions is shown as the green dot-dashed line in Fig. 2. Including these higher Lyman series photons, only boost the effective scattering rates by a factor of $\sim 1.5$ in the region that was optically thin to Ly\(\beta\) photons, which is due to the fact that the oscillator strength of $n\pi \rightarrow 1s$ transitions -- and hence the absorption cross-section -- decreases quite rapidly with $n$ (e.g. for $10 \rightarrow 1s, f_{10} = 0.0016$, which is $\sim 0.004f_{1s}$). Between $10^{-2}$ and $10^{-1}$ kpc from the edge of the H II region, the contribution from higher Lyman series photons is the largest and boosts the overall scattering rate by a factor of $\sim 15$. This larger boost is due the fact that at these distances the gas is not yet optically thick in these higher Lyman series lines. Note that we have accounted for this boost in our calculation of $\tau_{\text{Ly} \beta}$ (which is shown in Fig. 3).

(III) We have so far assumed that the edge of the H II region is infinitely sharp. In reality, the thickness of the H II region is determined by the mean free path of the ionizing photons emitted by the quasar. Especially, the X-rays emitted by the quasar can penetrate deeply into the neutral IGM and produce a low level of ionization. However, the presence of a partially neutral region does not affect our results at all: although the distance from the edge of the H II region at which the IGM becomes optically thick is increased by a factor of $1/\alpha_{\text{HI}}$ (where $\alpha_{\text{HI}}$ is the neutral fraction), $\tau_{\text{Ly} \beta}$ is reduced by the same factor.

(IV) The approximation $\Delta T_{\text{Ly} \beta}(v) = -\tau_{\text{Ly} \beta}(v) T_{\text{CMB}} / (1 + z)$ is valid only when $T_{\text{ex}} \ll T_{\text{CMB}}$, or when $n_1 > n_2$. The 2s and 2p populations are given by $n_2 = n_1 \Gamma_{\beta} / (\Delta z_{2s})$ and $n_2 = n_1 \Gamma_{\alpha} / (\Delta z_{2p})$, respectively. That is, $n_1 > n_2$ when $\Gamma_{\beta} > (\Delta z_{2s} / \Delta z_{2p}) \Gamma_{\alpha} \sim 10^{-1} \Gamma_{\alpha}$. Our assumption $n_1 > n_2$ therefore breaks down in the regions of interest only when $\Gamma_{\alpha} > 10^{-4}$ s\(^{-1}\), which is the orders of magnitude larger than the actual Ly\(\alpha\) scattering rate.

(V) Our calculation has ignored the hyperfine splitting of the 2s and 2p levels that was illustrated in Fig. 1. Hyperfine splitting results in three allowed 2s–2p transitions at $v = 9.852, 9.876$ and 10.030 Ghz, and the fine-structure absorption cross-section has maxima at these frequencies. The most prominent of these three maxima reaches $75$ per cent of the value that one obtains when hyperfine-structure splitting is ignored (Dennison et al. 2005). Including hyperfine structure therefore lowers the maximum observable brightness temperature by a factor of 0.75.

5.2 General detectability of the fine-structure line during the EoR

We have shown that the optical depth in the 2s–2p fine-structure line around quasar H II regions to be negligible because of the low-effective Ly\(\beta\), $\gamma$, $\delta$, ... scattering rates. These strongly reduced effective scattering rates apply more generally during the EoR. It is well established that Ly\(\alpha\) scattering rate plays an important role in determining the excitation temperature of the 21-cm hyperfine transition via the Wouthuysen–Field effect (see e.g. Furlanetto et al. 2006, for a review). However, each soft UV photon that redshifts towards (and through) the Ly\(\alpha\) resonance frequency scatters, on average, $N_{\text{scat}} \sim \tau_{\alpha} \sim 10^{-6}$–$10^{-8}$ times. In sharp contrast, each higher Lyman series photon only excites one atom into the 2s state. The effective Ly\(\beta\) (and total) scattering rate is, therefore, lower than the Ly\(\alpha\) scattering rate by a factor of $\sim \tau_{\alpha}$. Since the Ly\(\alpha\) scattering rate in the IGM during the EoR is typically $P_{\alpha} \sim 10^{-16}$–$10^{-14}$ s\(^{-1}\), the effective Ly\(\beta\) scattering rate is of the order of $P_{\text{eff}} \sim 10^{-18}$–$10^{-16}$ s\(^{-1}\), which barely competes with the recombination rate. From this we conclude that wherever the Wouthuysen–Field effect operates in the low-density neutral IGM, the optical depth in the fine-structure line is expected to be $\tau_{\text{Ly} \beta} < 10^{-10}$.

6 CONCLUSIONS

It has recently been proposed that neutral intergalactic atomic hydrogen gas may be detected in absorption in its 3-cm fine-structure line (2s-1s → 2p\(\gamma\)) against the CMB out to very high redshifts. In particular, bright quasars that are luminous in the rest-frame soft UV ($10.2 < E_{\gamma} < 13.6$ eV) may indirectly photoexcite the 2s state of neutral hydrogen gas in the IGM during the EoR to levels such that $\tau_{\text{Ly} \beta} \sim 10^{-6}$ for CMB photons passing through these regions (Sot07). The resulting brightness temperature of $\Delta T_{\text{Ly} \beta} \sim$ tens of $\mu$K could be detected with existing radio telescopes (e.g. Carilli 2008).

Motivated by this proposal, and by observational efforts to detect this transition, we have performed a detailed analysis of the transfer of Ly\(\beta\), $\gamma$, $\delta$, ... radiation, and have reanalysed the detectability of the fine-structure line in neutral intergalactic gas surrounding bright quasars during the EoR. We have found that radiative transfer radically complicates the detectability of this transition.

The main reason for this negative result is the large opacity of the (partially) neutral IGM to Lyman series photons. In such a medium, Ly\(\alpha\) scattering proceeds completely different than Ly\(\beta\), $\gamma$, $\delta$, ... scattering: Ly\(\alpha\) photons scatter $\tau_{\alpha} \sim 10^{-6}$–$10^{-7}$ times. However, higher Lyman series photons have a finite probability for being converted into Balmer, Paschen, ... etc. photons at each scattering event, which subsequently propagate to the observer unobstructed. These higher Lyman series photons therefore scatter, on average, only five to eight times (Section 3.2) before being destroyed (Pritchard & Furlanetto 2006). Furthermore, only one of these five to eight scattering events – defined as an effective scattering event – indirectly excites atoms into their 2s state (e.g. Ly\(\beta\) scattering corresponds to a sequence $1s \rightarrow 3p \rightarrow 1s$, and does not affect the level population of the 2s state). For these reasons, we found that the effective Ly\(\beta\) scattering rate was calculated reasonably well if one simply suppresses the incoming Ly\(\beta\) flux by a factor.

\(^{4}\)The oscillator strengths can be found in chapter 10.5 of Rybicki & Lightman (1979). The Einstein coefficients, and therefore the absorption cross-sections, can be derived from the oscillator strength using equation (10.79) of Rybicki & Lightman (1979).
of $e^{-\tau}$, in which $\tau$ is the (frequency dependent) optical depth of the IGM to Ly$\beta$ radiation (see Fig. 2). The same applies to Ly$\gamma$, $\delta$, . . . radiation.

Because of the reduced effective scattering rates, we found a substantial lower column of neutral atoms in the 2s state. This, in combination with the fact that the profile of the fine-structure absorption cross-section is dominated by the damping wings (Section 4), results in $T_{FS} \sim 10^{-11} - 10^{-10}$. This is five to six orders of magnitude smaller than what was originally found by S07, and we conclude that the 3-cm fine-structure absorption line from neutral intergalactic gas surrounding high-redshift quasars is presently undetectable.

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APPENDIX A: ANALYTIC CALCULATIONS OF Ly$\beta$ SCATTERING RATES

A1 The optically thin limit

In the regime where the gas is optically thin to Ly$\beta$ radiation, the Ly$\beta$ scattering rate is given by

$$P_\beta = 4\pi \int \frac{d\nu}{h\nu} \frac{J(\nu)}{h\nu} \sigma_\beta(\nu),$$  (A1)

where $J(\nu)$ is the mean intensity in the UV radiation field near the Ly$\beta$ resonance (in erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$ sr$^{-1}$), and $\sigma_\beta(\nu)$ is the Ly$\beta$ absorption cross-section. The mean intensity $J(\nu)$ relates to the luminosity of the central source (in erg s$^{-1}$ Hz$^{-1}$) as $J(\nu) = \frac{L_\nu}{4\pi r^2 h\nu}$, where $r$ is the distance to the central source. Assuming that $J(\nu)$ is constant across the Ly$\beta$ resonance, which is extremely narrow in frequency, equation (A1) simplifies to

$$P_\beta = \frac{L_\nu f_\beta e^2}{4\pi^2 h\nu m_e c},$$  (A2)

where the photon energy $h\nu$, that was originally in the integrand, was taken outside of the integral since it barely varies over the limited range of frequencies where the integrand is not negligibly small. Using that $\int d\nu \sigma_\beta(\nu) = f_\beta \frac{4\pi}{3} \frac{e^2}{m_e c}$, where $f_\beta = 0.079$ is the Ly$\beta$ oscillator strength, $c$ is the speed of light, $e$ and $m_e$ are the electron charge and mass, respectively (see e.g. Rybicki & Lightman 1979, their equation 3.66), the total Ly$\beta$ scattering rate simplifies further to

$$P_\beta = 2.4 \times 10^{-11} \frac{\text{photons s}^{-1}}{r},$$  (A4)

$^5$ This assumption corresponds to a flux density that is $6 \times 10^{32}$ erg s$^{-1}$ Hz$^{-1}$, which corresponds to an absolute AB magnitude of $M_{AB} = -30.45$. For comparison, the brightest $z = 6$ quasars have $M_{1450} \sim -27.8$ (Fan et al. 2006). Since the emitted flux by quasars is not expected to vary by an order of magnitude between $\lambda = 1450$ Å (rest frame), we conclude that the quasar luminosity assumed by S07 is likely too high by about a factor of $\sim 10$. 

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where we used that \( L = N_\beta \times h v_\beta = 1.9 \times 10^{47} \text{ erg s}^{-1} \), and \( r = 5 \text{ Mpc} \) based on the physical size of the \( \text{H}_\alpha \) region that possibly exist around \( z = 6 \) quasars (Mesinger & Haiman 2004; Wyithe & Loeb 2004, but see Bolton & Haehnelt 2007; Lidz et al. 2007). This scattering rate was plotted as the dotted line in Fig. 2.

### A2 Including radiative transfer: ignore scattering

When the assumption of working in the optically thin regime is dropped, the scattering rate (equation A1) modifies to

\[
P_\beta(r) = 4\pi \int \frac{J(v)}{h v} \sigma_\beta(v) [1 - Hr/c] e^{-\tau(v,x)},
\]

(A5)

where \( \tau(v, r) \) is the optical depth to photons emitted at frequency \( v \) to a hydrogen atom at a distance \( r \) from the central source. Furthermore, the factor of \( [1 - Hr/c] \) accounts for the fact that the Hubble expansion redshifts the photons by a factor of \( \Delta v = Hr \) over a distance \( r \). The equation (A5) does not account for the fact that individual Ly\( \beta \) photons are scattered multiple times, which modifies the actual value of mean intensity (see Section 3.2).

For illustration purposes, it is useful to rewrite equation (A5) into a form similar to equation (A1):

\[
P_\beta(r) = \frac{L_{Ly\beta} f_\beta \pi e^2}{4\pi^2 h v_\beta m_e c} \times S(r),
\]

(A6)

where \( S(r) \) (the ‘suppression factor’) is given by

\[
S(r) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx \phi(x) e^{-\frac{x^2}{2}}.
\]

(A7)

Here, \( x \) denotes a dimensionless frequency variable of the form \( x = (v - v_\beta)/\Delta v_\text{D} \), where \( \Delta v_\text{D} \equiv v_\text{th} \, \beta/c \) and \( v_\text{th} = \sqrt{2kT/m_p} \). Furthermore, \( \phi(x) \) is the Voigt function (see equation 10.77 of Rybicki & Lightman 1979. Note that this function obeys \( \frac{1}{\sqrt{\pi}} \int dx \phi(x) = 1 \). Hence, we recover equation A3 when we set \( \tau = 0 \). The function \( S(r) \) quantifies by how much the optically thin approach is off.

We again assume that the edge of the \( \text{H}_\alpha \) region is infinitely sharp (as in Section 4) and extends out to \( r = R_{\text{dis}} = 5 \text{ Mpc} \) (see Section 5.1, roman numeral III). The optical depth \( \tau(r, v) \) is then

\[
\tau(v, r) = \int_{R_{\text{dis}}}^{r} ds n_H(x) \sigma_\beta[v(1 - Hx/c)].
\]

(A8)

We plot the suppression factor \( S(r) \) in Fig. A1, which shows that \( S(r) < 10^{-2} \) for \( r > 10^{-3} \text{ Mpc} \). The reason that \( S(r) \) drops so rapidly can be understood as follows: regardless of their frequency, photons traverse by definition an average optical depth \( \tau = 1 \) into the neutral IGM before being absorbed. For Ly\( \beta \) photons, this implies that the vast majority of photons are absorbed well before they have redshifted into resonance. That is, \( \tau(x, r) \sim 1 \) when \( x \gg 1 \) and therefore \( \phi(x) \ll 1 \), in the notation of equation (A7). Only those photons that have redshifted into resonance at the very edge can scatter near resonance (i.e. \( \phi(x) = 1 \) while \( \tau(x, r) \ll 1 \), also see Section 3.2). This special region is very thin: the line-centre optical depth of neutral intergalactic gas in the Ly\( \beta \) line is \( \tau_0 \equiv \sigma_\beta n_{\text{HI}} r = 2.4 \times 10^3 (T_{\text{gas}}/10^5)^{-1/2} (r/\text{kpc}) \). That is, the line-centre optical depth approaches unity at \( r \sim 10^{-3} \text{ kpc} = 10^{-6} \text{ Mpc} \). The scattering rate that is obtained by combining equations (A6–A8) plotted as the dashed line in Fig. 2.

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