MAXIMAL SUBGROUPS OF THE MATHIEU GROUP $M_{23}$ AND SYMPLECTIC AUTOMORPHISMS OF SUPERSINGULAR K3 SURFACES

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ABSTRACT. We show that the Mathieu groups $M_{22}$ and $M_{11}$ can act on the supersingular K3 surface with Artin invariant 1 in characteristic 11 as symplectic automorphisms. More generally we show that all maximal subgroups of the Mathieu group $M_{23}$ with three orbits on 24 letters act on a supersingular K3 surface with Artin invariant 1 in a suitable characteristic.

1. INTRODUCTION

Let $X$ be a K3 surface defined over an algebraically closed field. By definition, the irregularity of $X$ vanishes and there exists a unique (up to constants) non-zero regular 2-form on $X$. An automorphism $g$ of $X$ is called symplectic if $g$ fixes a non-zero regular 2-form on $X$. In case of complex K3 surfaces, Mukai [M] showed that any finite group of symplectic automorphisms of a K3 surface is a subgroup of the Mathieu group $M_{23}$ with at least five orbits in its natural action on 24 letters. However in case of positive characteristic, this does not hold. For example, the projective unitary group $PU(4, \mathbb{F}_9)$ acts on the Fermat quartic surface in characteristic 3 as projective transformations. By comparing their orders we can see that the group $PU(4, \mathbb{F}_9)$ is not a subgroup of $M_{23}$. Note that the Fermat quartic surface in characteristic 3 is a supersingular K3 surface with Artin invariant 1 (Shioda [S]). Also Dolgachev and the author [DKo] proved that the group $L_3(4) : 2$ acts on a supersingular K3 surface with Artin invariant 1 in characteristic 2. In this case $L_3(4) : 2$ is not a subgroup of $M_{23}$, too. Recently Dolgachev and Keum [DKe1], [DKe2] studied the details in case of positive characteristic. In particular they are trying to extend Mukai’s theorem to the case of positive characteristic.

In this note, inspired by Dolgachev and Keum [DKe2], we shall show that each maximal subgroup of $M_{23}$ with three orbits on 24 letters can act as automorphisms on a supersingular K3 surface with Artin invariant 1 by using Ogus’s Torelli type theorem for supersingular K3 surfaces (Ogus [O1], [O2]) (see Theorem 3.1). The simpleness of $M_{22}$ and $M_{11}$ imply that these actions are symplectic (Corollary 3.3). The idea of the proof comes from Mukai’s one in the appendix of [K]. Let $N$ be the Niemeier lattice with root sublattice $A_24^1$. Here we consider the negative definite one as $N$. The Mathieu group $M_{24}$ naturally acts on the set of 24 positive roots of $A_24^1$ as permutations and $M_{23}$ is the stabilizer of a fixed positive root. Let $G$ be a maximal subgroup of $M_{23}$ with 3 orbits on 24 letters. We can consider $G$ as a subgroup of the orthogonal group of $N$. Let $N^G$ be the invariant sublattice. Then by assumption $N^G$ is of rank 3, and hence the orthogonal complement $N_G$ of $N^G$ in $N$ is an even negative definite lattice of rank 21 and $N_G$ contains no $(-2)$-vectors. We can see that there exists an even positive definite lattice $<h>$ of rank 1 such that $<h> \oplus N_G$ can be embedded into the Néron-Severi lattice $S_X$ of a supersingular K3 surface $X$ with Artin invariant 1 in a suitable characteristic $p$. The action of $G$ on $N_G$ can be extended to the one on $S_X$ acting trivially on $<h>$.

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Since $N_G$ contains no $(-2)$-vectors, we may assume that $G$ preserves the ample cone of $X$. Moreover $G$ acts trivially on $N_G^*/N_G \cong (N^G)^*/N^G$ and hence acts trivially on $S_X^*/S_X$. This implies that $G$ preserves the "period" of $X$. Therefore it follows from Ogus’s Torelli theorem [O2] that $G$ is realized as a subgroup of $\text{Aut}(X)$.

We use the following symbols of finite groups in this paper:

- $n$: a cyclic group of order $n$.
- $n^k$: an $n$-elementary abelian group of order $n^k$.
- $S_n (A_n)$: a symmetric (alternating) group of degree $n$.
- $L_n(q)$: the projective special linear group $\text{PSL}(n,q)$.
- $M_k (k=11,12,22,23,24)$: the Mathieu group.

We shall say that a group $G$ is a group $N \cdot H$ when we mean that $G$ has a normal subgroup $N$ whose quotient is isomorphic to $H$. We denote by $N : H$ the semi-direct product.

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## 2. LATTICES

### 2.1. Preliminaries. A lattice is a a free $\mathbb{Z}$-module $L$ of finite rank endowed with a $\mathbb{Z}$-valued symmetric bilinear form $\langle \cdot, \cdot \rangle$. If $L_1$ and $L_2$ are lattices, then $L_1 \oplus L_2$ denotes the orthogonal direct sum of $L_1$ and $L_2$. Also we denote by $L^m$ the orthogonal direct sum of $m$-copies of $L$. An isomorphism of lattices preserving the bilinear forms is called an isometry. For a lattice $L$, we denote by $O(L)$ the group of self-isometries of $L$. A sublattice $S$ of $L$ is called primitive if $L/S$ is torsion free.

A lattice $L$ is even if $\langle x, x \rangle$ is even for each $x \in L$. A lattice $L$ is non-degenerate if the discriminant $d(L)$ of its bilinear form is non zero, and unimodular if $d(L) = \pm 1$. If $L$ is a non-degenerate lattice, the signature of $L$ is a pair $(t_+, t_-)$ where $t_\pm$ denotes the multiplicity of the eigenvalues $\pm 1$ for the quadratic form on $L \otimes \mathbb{R}$.

Let $L$ be a non-degenerate even lattice. The bilinear form of $L$ determines a canonical embedding $L \subset L^* = \text{Hom}(L, \mathbb{Z})$. The factor group $L^*/L$, which is denoted by $A_L$, is an abelian group of order $|d(L)|$. We denote by $l(L)$ the number of minimal generators of $A_L$. We extend the bilinear form on $L$ to the one on $L^*$, taking value in $\mathbb{Q}$, and define

$$q_L : A_L \to \mathbb{Q}/2\mathbb{Z}, \quad q_L(x + L) = \langle x, x \rangle + 2\mathbb{Z} \quad (x \in L^*)$$

We call $q_L$ the discriminant quadratic form of $L$.

Let $S$ be an even lattice. Let $L$ be an even lattice containing $S$ as a sublattice of finite index. We call $L$ an overlattice of $S$. Note that $L$ is determined by the isotropic subgroup $L/S$ in $A_S$ with respect to $q_S$.

We denote by $U$ the even lattice defined by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and by $A_m$, $D_n$ or $E_l$ the even negative definite lattice defined by the Dynkin matrix of type $A_m$, $D_n$ or $E_l$ respectively.
2.2. The Néron-Severi lattice of a supersingular $K3$ surface. A supersingular $K3$ surface is a $K3$ surface with the Picard number 22. A supersingular $K3$ surface exists only in positive characteristic $p$. Let $X$ be a supersingular $K3$ surface in characteristic $p$ and let $S_X$ be the Néron-Severi lattice of $X$. It is known that $\det(S_X) = -p^{2\sigma}$, $(1 \leq \sigma \leq 10)$ where the number $\sigma$ is called Artin invariant of $X$ (Artin [A]). A generic supersingular $K3$ surface has Artin invariant 10 and a supersingular $K3$ surface with $\sigma = 1$ is unique. Moreover the Néron-Severi lattice $S = S_X$ is uniquely determined by $\sigma$ (Rudakov-Shafarevich [RS], Ogus [O1]). For example,

$$S = U \oplus E_8 \oplus A_6 \oplus A_6 \quad (p = 7, \sigma = 1);$$

$$S = U \oplus A_{10} \oplus A_{10} \quad (p = 11, \sigma = 1).$$

In case $p = 5$ and $\sigma = 1$, $S$ is obtained as follows. Let $K = U \oplus E_7 \oplus A_4 \oplus A_9$. Then $K^*/K \cong (\mathbb{Z}/5\mathbb{Z})^2 \oplus (\mathbb{Z}/2\mathbb{Z})^2$. Let $x$ be a generator of $E_7^*/E_7$ and $y$ a generator of $A_9^*/A_9$. Then $q_S(x) = 1/2$ and $q_S(5y) = -1/2$. The isotropic vector $x + y$ of $K^*/K$ determines an even lattice $S$ which contains $K$ of index 2 and is with $\det(S) = -5^2$.

The discriminant form of the above $S$ is as follows:

$$\langle A_S, q_S \rangle = ((\mathbb{Z}/5\mathbb{Z})^2, (-2/5) \oplus (-6/5)) \quad (p = 5, \sigma = 1);$$

$$\langle A_S, q_S \rangle = ((\mathbb{Z}/7\mathbb{Z})^2, (-6/7) \oplus (-6/7)) \quad (p = 7, \sigma = 1);$$

$$\langle A_S, q_S \rangle = ((\mathbb{Z}/11\mathbb{Z})^2, (-10/11) \oplus (-10/11)) \quad (p = 11, \sigma = 1).$$

2.3. Niemeier lattices and Mathieu groups. A Niemeier lattice is an even negative definite unimodular lattice of rank 24. The isomorphism class of a Niemeier lattice is determined by the sublattice $R$ generated by all $(-2)$-vectors in it. It is known that there exists a Niemeier lattice $N$ with $R = A_{24}^1$. Moreover the orthogonal group $O(N)$ is isomorphic to $2^{24} : M_{24}$. The subgroup $2^{24}$ is generated by reflections associated to 24 positive roots in $A_{24}^1$ and $M_{24}$ naturally acts on the set of 24 positive roots of $A_{24}^1$. Then $M_{24}$ is the stabilizer of a fixed positive root. The following is the table of all maximal subgroups of $M_{23}$ ([C], page 71, [CS], Chap. 10).

| Maximal subgroup | Order | Orbit Decomposition |
|------------------|-------|---------------------|
| 1) $M_{22}$     | $2^7 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$ | [1, 1, 22] |
| 2) $L_3(4) : 2$ | $2^7 \cdot 3^2 \cdot 5 \cdot 7$ | [1, 2, 21] |
| 3) $2^4 : A_7$  | $2^7 \cdot 3^2 \cdot 5 \cdot 7$ | [1, 7, 16] |
| 4) $A_8$        | $2^6 \cdot 3^2 \cdot 5 \cdot 7$ | [1, 8, 15] |
| 5) $M_{11}$     | $2^4 \cdot 3^2 \cdot 5 \cdot 11$ | [1, 11, 12] |
| 6) $2^4 : (3 \times A_5) : 2$ | $2^7 \cdot 3^2 \cdot 5$ | [1, 3, 20] |
| 7) $23 : 11$    | $11 \cdot 23$ | [1, 23] |

Table 1.
2.4. Remark. The group $L_3(4) : 2$ in the Table 1 is different from the one mentioned in Introduction which appeared in the paper [DKo]. In the case of Table 1, the involution 2 is $2_2$ given in [C], page 71, and in the case of [DKo], the involution 2 is $2_1$ given in [C], page 80.

We recall that the Niemeier lattice $N$ is obtained from $A_1^{24}$ as follows. Let $C$ be the binary Golay code which is a subspace of $(A_1^*/A_1)^{24} \cong \mathbb{F}_2^{24}$ of dimension 12. Then

$$N = \{ x \in (A_1^*)^{24} \mid x \text{ mod } A_1^{24} \in C \}.$$

It is known that the length of non-zero entries of $x \in C$ is 8, 12, 16 or 24. The set of non-zero entries of length 8 is called an octad and one of length 12 a dodecad. In the case 3 on the Table 1, the union of orbits of length 1 and 7 is an octad. Also in case 4, the orbit of length 8 is an octad. In case 5, the orbit of length 12 and its complement are dodecad. For more details, we refer the reader to Conway-Sloane [CS].

3. Wild symplectic automorphisms

In this section we shall prove the following:

3.1. Theorem. Let $G$ be a maximal subgroup of $M_{23}$ with three orbits. Then there exists a prime number $p$ such that $G$ acts as automorphisms on a supersingular $K3$ surface with Artin invariant 1 in characteristic $p$.

First we shall show the following Lemma.

3.2. Lemma. Let $G$ be a maximal subgroup of $M_{23}$ with three orbits. Then there exists a prime number $p$ such that $G$ acts on the Néron-Severi lattice $S$ of a supersingular $K3$ surface with Artin invariant 1 in characteristic $p$. Moreover $G$ acts trivially on $S^*/S$ and the orthogonal complement of the invariant sublattice $S^G$ in $S$ contains no $(-2)$-vectors.

Proof. Let $N$ be the Niemeier lattice with the root sublattice $A_1^{24}$ on which $M_{23}$ naturally acts. Let $N^G$ be the invariant sublattice. Since $G$ has three orbits, $\text{rank}(N^G) = 3$. Let $N_G$ be the orthogonal complement of $N^G$ in $N$. Then $\text{rank}(N_G) = 21$. For each $G$ in the Table 1, we shall show the following: First we calculate the discriminant forms $q_{N_G} = -q_{N^G}$. Next we take a vector $h$ with $h^2 = |\text{det}(N_G)|$ and consider the lattice $< h > \oplus N_G$. Then we shall show that there exists an over lattice $S$ of $< h > \oplus N_G$ which is isomorphic to the Néron-Severi lattice of a supersingular $K3$ surface $X$. Moreover the action of $G$ on $N_G$ can be extended to the one on $S$ acting trivially on $< h >$. Since $G$ acts on $< h >^* / < h > \oplus N_G^*/N_G$ trivially, $G$ acts on $S^*/S$ trivially.

Note that $N$ contains exactly 24 positive roots ($(-2)$-vectors) and $G$ acts on the set of positive roots as permutations. Hence $N_G$ contains no $(-2)$-vectors.

In the following we denote by $\{ x_1, \ldots, x_{24} \}$ the set of positive roots of $A_1^{24}$.

Case 1: $G = M_{22}$.

We assume that $x_1, x_2, x_3 + \cdots + x_{24}$ are invariant under the action of $G$. Then $N^G$ is generated by $x_1, x_2, x_3 + \cdots + x_{24}$ and $(x_1 + x_2 + x_3 + \cdots + x_{24})/2$. Hence $\text{det}(N^G) = 2 \cdot 2 \cdot 44/2^2 = 44$. By using Nikulin [N1], Proposition 1.5.1, we can easily see that $q_{N_G} = (5/4) \oplus (-4/11)$. Hence $q_{N_G} = (5/4) \oplus (4/11)$ (Nikulin [N1], Corollary 1.6.2). Take a vector $h$ with $h^2 = 44$. Consider the subgroup $H$ of order 4 in $< h >^* / < h > \oplus N_G^*/N_G$ generated by $h/4 + \theta$, where $\theta$ is a generator of 2-Sylow subgroup of $N_G^*/N_G$. Since $H$ is totally isotropic with respect to $q_{h^2} \oplus q_{N_G}$, it determines the overlattice $S$ with the discriminant form $q_S = (10/11) \oplus (10/11)$ (Nikulin [N1],}
Case 2: $G = L_3(4) : 2$.

We assume that $x_1, x_2 + x_3, x_4 + \cdots + x_{24}$ are invariant under the action of $G$. Then $N^G$ is generated by $x_1, x_2 + x_3, x_4 + \cdots + x_{24}$ and $(x_1 + x_2 + x_3 + \cdots + x_{24})/2$. Hence $\det(N^G) = 2 \cdot 2^2 \cdot 42/2^2 = 84$ and $q_{N^G} = (-3/4) \oplus (-2/3) \oplus (-6/7)$. Hence $q_{N_G} = (3/4) \oplus (2/3) \oplus (6/7)$. Take a vector $h$ with $h^2 = 84$. We consider the totally isotropic subspace $H$ of order 12 generated by $h/12 + \theta$ where $\theta$ is a generator of the subgroup of order 12 in $N_G^*/N_G$. Then as in the Case 1, $H$ determines the overlattice $S$ isomorphic to the Néron-Severi lattice of a supersingular $K3$ surface with Artin invariant 1 in characteristic 7.

Case 3: $G = 2^4 : A_7$.

We assume that $x_1, x_2 + \cdots + x_8, x_9 + \cdots + x_{24}$ are invariant under the action of $G$. Then $N^G$ is generated by $x_1, x_2 + \cdots + x_8, x_9 + \cdots + x_{24}, (x_1 + x_2 + x_3 + \cdots + x_{24})/2$ and $(x_2 + \cdots + x_9)/2$. Hence $\det(N^G) = 2 \cdot 14 \cdot 32/2^4 = 56$ and $q_{N^G} = (-1/8) \oplus (-2/7)$. Hence $q_{N_G} = (1/8) \oplus (2/7)$. Take a vector $h$ with $h^2 = 56$. We consider the totally isotropic subspace $H$ of order 8 generated by $h/8 + \theta$ where $\theta$ is a generator of the 2-Sylow subgroup of order 8 in $N_G^*/N_G$. Then as in the Case 1, $H$ determines the overlattice $S$ isomorphic to the Néron-Severi lattice of a supersingular $K3$ surface with Artin invariant 1 in characteristic 7.

Case 4: $G = A_8$.

We assume that $x_1, x_2 + \cdots + x_9, x_{10} + \cdots + x_{24}$ are invariant under the action of $G$. Then $N^G$ is generated by $x_1, x_2 + \cdots + x_9, x_{10} + \cdots + x_{24}, (x_1 + x_2 + x_3 + \cdots + x_{24})/2$ and $(x_2 + \cdots + x_9)/2$. Hence $\det(N^G) = 2 \cdot 16 \cdot 30/2^4 = 60$ and $q_{N^G} = (-1/4) \oplus (-4/3) \oplus (-6/5)$. Hence $q_{N_G} = (1/4) \oplus (4/3) \oplus (6/5)$. Take a vector $h$ with $h^2 = 60$. We consider the totally isotropic subspace $H$ of order 12 generated by $h/12 + \theta$ where $\theta$ is a generator of the subgroup of order 12 in $N_G^*/N_G$. Then as in the Case 1, $H$ determines the overlattice $S$ isomorphic to the Néron-Severi lattice of a supersingular $K3$ surface with Artin invariant 1 in characteristic 5.

Case 5: $G = M_{11}$.

We assume that $x_1, x_2 + \cdots + x_{12}, x_{13} + \cdots + x_{24}$ are invariant under the action of $G$. Then $N^G$ is generated by $x_1, x_2 + \cdots + x_{12}, x_{13} + \cdots + x_{24}, (x_1 + x_2 + x_3 + \cdots + x_{24})/2$ and $(x_2 + \cdots + x_{12})/2$. Hence $\det(N^G) = 2 \cdot 22 \cdot 24/2^4 = 66$ and $q_{N^G} = (-3/2) \oplus (-2/3) \oplus (-2/11)$. Hence $q_{N_G} = (3/2) \oplus (2/3) \oplus (2/11)$. Take a vector $h$ with $h^2 = 66$. We consider the totally isotropic subspace $H$ of order 6 generated by $h/6 + \theta$ where $\theta$ is a generator of the subgroup of order 6 in $N_G^*/N_G$. Then as in the Case 1, $H$ determines the overlattice $S$ isomorphic to the Néron-Severi lattice of a supersingular $K3$ surface with Artin invariant 1 in characteristic 11.

Case 6: $G = 2^4 : (3 \times A_5) : 2$.

We assume that $x_1, x_2 + x_3 + x_4, x_5 + \cdots + x_{24}$ are invariant under the action of $G$. Then $N^G$ is generated by $x_1, x_2 + x_3 + x_4, x_5 + \cdots + x_{24}$ and $(x_1 + x_2 + x_3 + \cdots + x_{24})/2$. Hence $\det(N^G) = 2 \cdot 6 \cdot 40/2^2 = 120$ and $q_{N^G} = (-9/8) \oplus (-2/3) \oplus (-5/8)$. Hence $q_{N_G} = (9/8) \oplus (2/3) \oplus (8/5)$. Take a vector $h$ with $h^2 = 120$. We consider the totally isotropic subspace $H$ of order 24 generated by $h/24 + \theta$ where $\theta$ is a generator of the subgroup of order 24 in $N_G^*/N_G$. Then as in the Case 1, $H$ determines the overlattice $S$ isomorphic to the Néron-Severi lattice of a supersingular $K3$ surface with Artin invariant 1 in characteristic 5.
Proof. (Theorem 3.1) Let $G$ and $S$ be as in Lemma 3.2. Let $X$ be the supersingular $K3$ surface with Artin invariant 1 in characteristic $p$ satisfying $S_X \cong S$. Since $N_G$ contains no $(-2)$-vectors, $h$ is contained in a fundamental chamber of the reflection subgroup $W(X)$ of $O(S_X)$ generated by $(-2)$-reflections. Hence there exists a $w \in W(X)$ so that $w(h)$ is an ample class. Thus $wGw^{-1}$ preserves the ample cone of $X$. Since both $G$ and $W(X)$ act trivially on $S^*/S$, so is $wGw^{-1}$, and hence $wGw^{-1}$ preserves the characteristic subspace ("Period") of $X$ (see Ogus [O2], page 366). Now the assertion follows from Ogus [O2], Corollary of Theorem II' (page 371).

3.3. Corollary. The Mathieu groups $M_{22}$, $M_{11}$ and the alternating group $A_8$ act as symplectic automorphisms on a supersingular $K3$ surface with Artin invariant 1.

Proof. Since automorphisms act on a regular 2-form on a $K3$ surface as a multiplicative group, the symplecticness follows from the simpleness of $M_{22}$, $M_{11}$, $A_8$.

We summarize the prime number $p$ and the degree $h^2$ of the invariant polarization $h$ under $G$ in the following Table 2:

| $G$        | $p$ | $h^2$ |
|------------|-----|-------|
| 1) $M_{22}$ | 11  | 44    |
| 2) $L_3(4):2$ | 7   | 84    |
| 3) $2^4:A_7$ | 7   | 56    |
| 4) $A_8$    | 5   | 60    |
| 5) $M_{11}$ | 11  | 66    |
| 6) $2^4:(3\times A_5):2$ | 5   | 120   |

Table 2.

It would be interesting to realize these actions geometrically.

3.4. Problem. Let $g$ be an automorphism of a $K3$ surface $X$. In case that $X$ is a complex $K3$ surface, if $g$ is symplectic, then $g$ acts trivially on the transcendental lattice of $X$ and hence trivially on the discriminant group $S^*_X/S_X$ of the Néron-Severi lattice of $X$ (Nikulin [N2], Theorem 3.1). Moreover if $G$ is a finite group of symplectic automorphisms of a complex $K3$ surface $X$, denote by $L_G$ the orthogonal complement of the invariant sublattice of $H^2(X, \mathbb{Z})$. Then

$$l(L_G) \leq 22 - \text{rank}(L_G)$$

where $l(L_G)$ is the number of minimal generator of $L^*_G/L_G$ ([K], Proposition 2). This means that if $G$ becomes bigger, then $L_G$ becomes bigger, too, and hence $l(L_G)$ becomes smaller.

In case of positive characteristic, does any symplectic automorphism of a supersingular $K3$ surface act trivially on the discriminant group of the Néron-Severi lattice $A_8$? And if $G$ becomes bigger, then does the Artin invariant become smaller?
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