Network dynamics of ongoing social relationships

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Many recent large-scale studies of interaction networks have focused on networks of accumulated contacts. In this paper we explore social networks of ongoing relationships with an emphasis on dynamical aspects. We find a distribution of response times (times between consecutive contacts of different direction between two actors) that has a power-law shape over a large range. We also argue that the distribution of relationship duration (the time between the first and last contacts between actors) is exponentially decaying. Methods to reanalyze the data to compensate for the finite sampling time are proposed. We find that the degree distribution for networks of ongoing contacts fits better to a power-law than the degree distribution of the network of accumulated contacts do. We see that the clustering and assortative mixing coefficients are of the same order for networks of ongoing and accumulated contacts, and that the structural fluctuations of the former are rather large.

I. INTRODUCTION

The recent development in database technology has allowed researchers to extract very large data sets of human interaction sequences. These large data sets are suitable to the methods and modeling techniques of statistical physics, and thus, the last years has witnessed the appearance of an interdisciplinary field between physics and sociology (1, 7, 13). More specifically these studies have focused on network structure—in what ways the networks of social interaction deviates from completely random networks, and how this structure can emerge from individual behavior. Most of these recent large-scale social network studies have focused on networks of accumulated relationships. In many cases, the social network of interest is rather the network of ongoing social relationships: The dynamics of the spreading of diseases (2), opinion formation (3), and fads (4) are often rather fast compared to the evolution of the network—in such cases inactive relationships have no relevance. In social search processes (20), distant acquaintances can be helpful, but not all acquaintances a person has ever had. We also believe the network of ongoing contacts lies conceptually closer to the colloquial idea of a network of friends, than what the network of actors and their accumulated contacts do. Furthermore, traditional social network studies (e.g. Refs. (5, 12, 17) based on interviews or field surveys has mapped out ongoing contacts. The complication, and probably the reason earlier studies have focused on the network of accumulated contacts, is that the time of a tie’s cessation is less clear-cut than its beginning. However, if the sampling time of the data set is very large compared to the network dynamics; then one can, at a time \( t \) in the interior of the sampling time span, approximate the network of ongoing relationships by the network of contacts that has occurred and will occur again. In the present paper we use this method to study the structure and structural fluctuations of networks of ongoing relationships. To justify that the sampling time is long enough compared to the time evolution of the network, we investigate the temporal structure of the relationships. The data sets we use are obtained from scientific collaborations (11), email exchange (5) and interaction within an Internet community (10).

II. NOTATIONS AND NETWORK CONSTRUCTION

All our data sets take the form of lists of triples, or contacts, \((v_A, v_B, t)\) meaning that \(v_A\) and \(v_B\) has interacted at time \(t\). For the scientific collaboration networks the two first arguments are unordered, for the other two networks the interaction is directed. We call the set of contacts with the same two first elements (neglecting the order) a relationship between \(v_A\) and \(v_B\). Our approximation of the graph of ongoing contacts at time \(t\) is then defined as \(G(t) = \{V(t), E(t)\}\), where \(V(t)\) is the set of vertices (or actors) that occur in a contact at a time earlier than \(t\), and \(E(t)\) is the set of unordered pairs of vertices \((v_A, v_B)\) where there exist contacts between \(v_A\) and \(v_B\) at times \(t'\) and \(t''\) such that \(t' < t < t''\).

For the network of scientific collaborations we use similar data as used in Ref. (11) (but sampled one year longer). This data is extracted from the preprint repository arxiv.org where scientists themselves can upload manuscripts. An edge between \(v_A\) and \(v_B\) means that \(v_A\) has appeared as a coauthor of a preprint together with \(v_B\). The time the manuscript is uploaded is the time we say the collaboration has occurred.

The email network is the same data set as presented in Ref. (5) and consists of all in- and out-going email traffic to a server handling undergraduate students’ email.

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1 To our knowledge, the only type of large (relatively) instantaneous network figuring in recent physics literature is networks of corporate directors sitting in the same board, Ref. (5).
TABLE I  Statistics of the networks. Date notations have the format year-month-day hour:minute:second GMT. The number of ties does not include self-communication (e.g. self-addressed e-mails) but in “number of contacts” such communication is included.

|                               | e-prints       | e-mail         | pussokram.com |
|--------------------------------|----------------|----------------|---------------|
| start of sampling              | 1995-01-01 06:00:00 | 2001-07-29 03:11:33 | 2001-02-13 14:39:25 |
| end of sampling                | 2001-01-01 05:31:00 | 2001-11-18 02:06:28 | 2002-07-10 15:28:00 |
| sampling duration, \( t_{stop} \) | 2192.0 days     | 112.0 days      | 512.0 days    |
| number of actors, \( N \)      | 58342           | 64370          | 29341         |
| number of ties, \( M \)        | 294901          | 97425          | 115684        |
| number of contacts             | 530481          | 447543         | 536276        |
| relationship duration, \( t_{dur} \) | 1532(30) days  | 187(5) days    | 129(10) days |

accounts in Kiel, Germany.

The Internet community network is constructed from the same data set as in Ref. (10). Here an edge represents any of four different ways of contacts between users of the Swedish Internet community pussokram.com. This community is intended for romantic communication among adolescents and young adults.

For the email and pussokram.com networks one can define a direction for the contacts. In the study of network structure, however, we will consider the contacts as bidirectional. Statistics for the networks are presented in Table I.

III. RELATIONSHIP DYNAMICS AND THE SPEED OF NETWORK CHANGE

Before we investigate the approximate network of ongoing contacts, as defined above, we discuss the speed of interaction and the validity of the approximation. First, we focus on the distribution of response times \( \tau \)—times between consecutive contacts of different direction within a relationship.\(^2\) For the undirected e-print data we simply define \( \tau \) as the time between consecutive uploads of e-prints within a relationship. We measure the \( \tau \)-distribution of the data sets, \( p' \), and also a quantity \( p \) where the effects of the finite size effects are compensated for. An earlier study (9) has found a power-law like \( \tau \)-distribution. As shown in Fig. 1(a) this picture is confirmed in the large scale. This stretched functional form makes the finite sampling time a problem as it imposes a cut-off on the recorded distribution \( p' \).

To compensate for this and construct a better approximation \( p \) to the real distribution, we use the formula

\[
P(B_\tau) = P(A \cap B_\tau)/P(A)
\]

where \( A \) is the event that a response interval that starts within the sampling interval \( I_t = [0,t_{stop}] \) also ends within \( I_t \), and \( B_\tau \) is the statement that the response time is \( \tau \). Now \( P(A \cap B_\tau) \) is just the frequency distribution of interval length as measured during the sampling. To find \( P(A|B_\tau) \) we note that, if we assume that contacts occurs with a constant rate (which is reasonable in a long term perspective for a system of a relatively constant number of actors), then a response interval ends within \( I_t \) with probability

\[
1 - \frac{\tau}{t_{stop}}.
\]

However, the sizes of the communities need not to be time independent. For response intervals involving an

\(^2\) As mentioned we will focus on undirected networks later, but for comparison with other works we use directed contacts in this definition. The conclusion from an undirected definition would be the same.
actor that enters the system at time $t$ Eq. (1) becomes $1 - (\tau + t)/t_{stop}$. Now we approximate the time an actor $v$ enters the system with the first time $t_v$ that $v$ is involved in a contact, and get the formula:

$$p(\tau) = P(A | B_v) = a \sum_{v \in V} \Theta(t_{stop} - t_v - \tau) \left[ 1 - \frac{t_v + \tau}{t_{stop}} \right]$$

where $\Theta(\cdot)$ denotes the Heaviside function, and $a$ is a normalizing constant. $p$ is plotted, along with the $p'$ values that differs most from $p$, in Fig. (a) (here $a$ is chosen to make $p$ coincide with $p'$ for small $\tau$ values rather than to normalize $p$). We note that $p$ is straighter than $p'$ in the log-log scale (for at least the e-mail and pussokram.com curves), which suggests a power-law like behavior over a considerable range. (Of course there is eventually a cut-off—from the human life time, if nothing else.) The e-print curve has a peculiar bend as it seems to shift exponent around $\tau = 300$ days, an observation we hope future studies can explain. There is a conspicuous irregularity around $\tau = 1$ day for the e-mail and pussokram.com curves. This was also observed in Ref. (c) and explained as an effect of people’s everyday routines—the Kiel students read and reply their e-mails at the same hour as the day before, the pussokram.com members log in after school or work, and so on. This effect is more visible in a linear scale, see Figs. (b) and (c). For the e-mail curve the peak at seven days is larger than the surrounding peaks, indicating that some emails are associated with weekly routines among the Kiel students and their contacts. This one-week-peak can not be seen in the pussokram.com curve; possibly reflecting that business (or university studies) has more weekly scheduled routines than leisure do.

Now we turn to the more central question about the speed of relationship cessation. Our central quantity is the number of relationships existing at time $t_0$ that still remains at time $t$ (we assume $0 \leq t_0 < t < t_{stop}$), $\mu(t_0, t)$. This quantity can crudely be approximated with the number of relationships at $t$ that existed at $t_0$ that will occur again before $t_{stop}$, $\mu'(t_0, t)$. The error in the approximation will be rather large for $t$ close to $t_{stop}$. But, just as above, one can improve the approximation considerably. If one assumes that the response time distribution $p(t)$ applies to all relationships regardless if the relationship is new or old; then, during a time interval $\Delta t$, the change of $\mu$ can be written:

$$\Delta \mu = \Delta \mu' + \mu \Delta \pi$$

where $\pi(t)$ is the probability that a relationship, that has its last recorded contact at time $t$, actually continues after $t_{stop}$:

$$\pi(t) = \sum_{\tau = t_{stop} - t}^{\infty} p(\tau) \Delta \tau ,$$

where the sum is over the bins of the $p(t)$ histogram. A change of variables gives:

$$\Delta \pi(t) = \Delta t p(t_{stop} - t) ,$$

and finally a formula for integrating $\mu$:

$$\mu(t + \Delta t) = \frac{\Delta \mu'(t + \Delta t) + \mu(t)}{1 - \Delta t p(t_{stop} - t)}. \quad (6)$$

We also need the factor $a$ of Eq. (2) which is hard to estimate since we don’t exactly know $p(t)$’s long term behavior. However, we note that for certain $a$ the $\mu(t)$ curves are rather straight in a lin-log plot, see Fig. (as opposed to the e-mail and pussokram.com curves the e-print curve decays so slowly that a power-law form of $\mu(t)$ cannot be ruled out). This means that the characteristic duration time is well-defined—fitting to an exponential $A \exp(-t/t_{dur})$ ($A$ and $t_{dur}$ are the two degrees of freedom) gives the characteristic durations $t_{dur}$ of relationships displayed in Table I. To be able to approximate the network of ongoing contacts with the network of contacts that have happened and will happen again one would like $t_{dur} \ll t_{stop}$ to hold. We see that for the pussokram.com data $t_{stop}$ is about four times as large as $t_{dur}$ which enables us to draw some conclusions using this approximation. The effective sampling times of the e-print and e-mail data are, however, so short that we exclude these for the latter section of this paper.

Now we take a brief look at the time evolution of the network sizes—$n$, the number of active actors at time $t$, and $m$, the number of edges in our approximate network of ongoing relationships. If the number of active users increases during the sampling period, the time evolution of $n$ and $m$ should be right-skewed, and this is indeed true for the e-print data (as seen in Figs. (a) and (b)). The e-mail and pussokram.com curves are more symmetric (the pussokram.com curve is indeed slightly left-skewed). We note that for pussokram.com, $m$ is much less than $M$. The kinks of the e-mail curves are due to group or spam e-mails, the other quantities $m, p$ and so on, are not affected by this.

IV. NETWORK STRUCTURE AND STRUCTURAL FLUCTUATIONS

Now we turn to the structure of the network of ongoing contacts, and the fluctuations of the structural measures. In this Section we only use the pussokram.com data (due to, as mentioned above, the large effective sampling time for this data set). We focus on three quantities that recently have received much attention: The first structural measure is the distribution of degree (number of edges to a vertex). The first quantity is the clustering coefficient $C(G)$ where we use the traditional sociological definition

$$C(G) = c_3(G)/p_3(G) ,$$


where $c_3(G)$ is the number of representations of every 3-cycle (triangle) of $G^3$ and $p_3(G)$ is the number of representations of 3-paths. The second quantity is the assortative mixing coefficient \( r = \frac{\langle k_1 k_2 \rangle - \langle k_1 \rangle \langle k_2 \rangle}{\sqrt{\langle k_1^2 \rangle - \langle k_1 \rangle^2} \sqrt{\langle k_2^2 \rangle - \langle k_2 \rangle^2}} \) \( \text{(8)} \)

where averages are taken over $E$, and $k_1$ and $k_2$ are the degrees of an edge's first and second arguments as they appear in $E$.

The cumulative degree distribution of our approximate network of ongoing relationships, along with the corresponding data for the network of accumulated contacts is plotted in Fig. 4(a). Just as for the accumulated network, our approximate network of ongoing relationships has a fat tailed degree distribution; but the network of ongoing relationships fits better to a single power-law with. The stronger downward bend of $P(k)$ for accumulated social contacts has been observed earlier \cite{10,11}; maybe this larger correction to a power-law form is due to inactive edges. We note that even if the degree distribution fits very well to that of the Barabasi-Albert model \cite{4}, \cite{4} the central ingredient in the Barabasi-Albert model (the “preferential attachment”) does not apply directly to the pussokram.com community. Preferential attachment means that a vertex acquires new edges with a rate proportional to its degree, but in the pussokram.com community the degree of a member is invisible to others \cite{10}.

Next we turn to the time evolution of $C$ and $r$. In Figs. 4(b) and (c) these are displayed for the whole sampling time. The earliest and latest times can, of course, be affected by the proximity to the borders of the sampling time frame—for our discussion we focus on the interval

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3 If $(v_1, v_2, v_3)$ is a triangle, then $(v_2, v_3, v_1)$ another representation of the same triangle. So the number of distinct triangles is $c_3(G)/6$.

4 For a case study of papers citing Ref. \cite{4}, see Ref. \cite{15}.
of the two vertices, and can thus increase introduces a new triangle for every common neighbor between two of the most central actors. Such a new edge is too short, despite the fast pussokram.com dynamics, of centrality measures (18) are probably even more sensitive will a or disappearing between two of the most connected ver-
gree distributions, as the rare event of an edge appearing for networks of ongoing relationships with fat tailed de-
cay similar to \( t_{\text{dur}} \). W e can expect sudden jumps in many structural quantities and weekly routines. The distribution of relationship duration is consistent with an exponential decay. This indicates that there is a well-defined characteristic duration time of a relationship, \( t_{\text{dur}} \), and that if \( t_{\text{dur}} \) is much less than the sampling time \( t_{\text{stop}} \) the network of ongoing contacts can be reasonably well approximated by the network of contacts that have happened and will happen again. For one of our data sets—that of the Internet community pussokram.com—we have \( 4t_{\text{dur}} \approx t_{\text{stop}} \). For this data set we compare the approximate network of ongoing contacts with networks of accumulated contacts—the common way of constructing social networks from interaction data. W e find a degree distribution that fits much better to a power-law for the network of ongoing contacts than the network of accumulated contacts. The clustering coefficient and assortative mixing coefficients are of the same order; which, to some extent, justifies the use of network of accumulated contacts as a proxy for networks of ongoing contacts. The fluctuations in these quantities are, however, rather large. A fact that may have important consequences for dynamical systems. W e hope these results will inspire more extensive longitudinal studies of interaction networks with fast dynamics, as well-converged data for relationship duration distribution and autocorrelation functions of structural quantities are within reach. We also point out the interplay between dynamical systems on the networks and the structural fluctuations as an interesting area of future studies.

V. SUMMARY AND CONCLUSIONS

In this paper we investigate networks of ongoing contacts from three large sets of social interaction data. W e study the response time distribution and distribution of relationship duration. W e reanalyze these quantities to compensate for the finite sampling time by supposing that the response time distribution is the same for all relationships, and the same throughout the duration of the relationship. W e find a response time distribution that has a power-law like shape in the large scale, but has an informative small-scale structure reflecting the daily and weekly routines. The distribution of relationship duration is consistent with an exponential decay. This indicates that there is a well-defined characteristic duration time of a relationship, \( t_{\text{dur}} \), and that if \( t_{\text{dur}} \) is much less than the sampling time \( t_{\text{stop}} \) the network of ongoing contacts can be reasonably well approximated by the network of contacts that have happened and will happen again. For one of our data sets—that of the Internet community pussokram.com—we have \( 4t_{\text{dur}} \approx t_{\text{stop}} \). For this data set we compare the approximate network of ongoing contacts with networks of accumulated contacts—the common way of constructing social networks from interaction data. W e find a degree distribution that fits much better to a power-law for the network of ongoing contacts than the network of accumulated contacts. The clustering coefficient and assortative mixing coefficients are of the same order; which, to some extent, justifies the use of network of accumulated contacts as a proxy for networks of ongoing contacts. The fluctuations in these quantities are, however, rather large. A fact that may have important consequences for dynamical systems. W e hope these results will inspire more extensive longitudinal studies of interaction networks with fast dynamics, as well-converged data for relationship duration distribution and autocorrelation functions of structural quantities are within reach. We also point out the interplay between dynamical systems on the networks and the structural fluctuations as an interesting area of future studies.

\[ [t_{\text{dur}}, t_{\text{stop}} - t_{\text{dur}}] \approx [129, 383] \text{days.} \]

We see that both \( C \) and \( r \) are of the same order of magnitude for the networks of ongoing and accumulated contacts. These values of \( C \) and \( r \) are rather neutral in the sense that they can be expected from a random network with a skewed degree distribution (13). The fluctuations are rather large, especially for the clustering coefficient (with a standard deviation of around half the average value). An intriguing question for future studies is how dynamical systems on the networks are affected by strong structural fluctuations. Slightly outside our interval, at \( t \approx 395 \) there is an upward jump in both \( C \) (from 0.023 to 0.041) and \( r \) (from \(-0.057\) to \(-0.043\)) that is the result of a new contact between two of the most central actors. Such a new edge introduces a new triangle for every common neighbor of the two vertices, and can thus increase \( C \) substantially as two high-degree actors may have many neighbors in common. Such an event will, by definition, also give a large positive contribution to the assortative mixing. W e can expect sudden jumps in many structural quantities for networks of ongoing relationships with fat tailed degree distributions, as the rare event of an edge appearing or disappearing between two of the most connected vertices will affect many structural measures (various kinds of centrality measures (13) are probably even more sensitive to such events). Unfortunately the sampling time is too short, despite the fast pussokram.com dynamics, to get good statistics for the autocorrelation function of \( C(t) \) and \( k(t) \) (it is consistent with a characteristic time of decay similar to \( t_{\text{dur}} \)).
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