Search algorithm for a gravitational wave signal in association with Gamma Ray Burst GRB030329 using the LIGO detectors

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Abstract. One of the brightest Gamma Ray Burst ever recorded, GRB030329, occurred during the second science run of the LIGO detectors. At that time, both interferometers at the Hanford, WA LIGO site were in lock and acquiring data. The data collected from the two Hanford detectors was analyzed for the presence of a gravitational wave signal associated with this GRB. This paper presents a detailed description of the search algorithm implemented in the current analysis.

PACS numbers:

Submitted to: Class. Quantum Grav.

1. Introduction

On 29 March 2003, one of the closest and brightest ever Gamma Ray Burst, GRB030329 [1], was detected by instruments aboard the HETE-2 satellite. Subsequent afterglow observations fixed the redshift of the host galaxy at $z \approx 0.17$ [2]. At the time when the GRB030329 was detected, the two LIGO interferometers at the Hanford site were in lock and taking data. The data in the vicinity of the GRB trigger time was analyzed for the presence of a gravitational wave burst signal. The results from the analysis as well as relevant details about the trigger itself will be reported elsewhere [3]. In this paper we present details of the search algorithm used in this analysis.

The distinction between a full analysis pipeline and a search algorithm should be emphasized. The analysis pipeline is the entire process that starts with the raw interferometric data output and ends with a statistical inference, such as a detection or no-detection, or a confidence interval estimate.

The search algorithm is the component which acts on pre-processed data and produces a list of candidate events or the value of a statistic which is then used in drawing a statistical inference. A first version of a full analysis pipeline was reported in [3] and the pipeline used for the present analysis will be described in [5].

The paper is structured as follows. Section 2 presents the line of reasoning that leads to the current form of the search algorithm. Section 3 gives a brief outline of
Search algorithm for GWs from GRB030329

2. Justification of the algorithm

The majority of present day astrophysical models and theories of GRBs suggest that the associated gravitational wave signal is a short burst lasting for tens of milliseconds contemporaneous with the GRB. There are also some models which predict signals lasting over tens of seconds. The analysis carried out in [3] focuses on short duration (< 150 msec) bursts.

At the time GRB030329 occurred, only the two co-located and co-aligned Hanford detectors were acquiring data. The spectral responses of these detectors are also very similar. Therefore, the arrival time and measured waveforms of a gravitational wave signal should be essentially the same in both detectors. Ergo, it is natural to consider cross-correlation of the two data streams as the basis of a search algorithm. This conclusion can also be reached via a more formal argument based on the maximum log-Likelihood ratio test.

Let us use a simple, albeit unrealistic, model for detector noise to illustrate the rationale behind the method. Assume that the two detectors have uncorrelated noise. Further assume that the noise in each is zero mean, Gaussian and stationary. Without loss of generality, this is equivalent to the noise being white with unit variance. Let the GW signal waveform in the output of H1 be

$$h = \{h[0], h[1], \ldots, h[N - 1]\},$$

where

$$N$$ is the length of the data segment being analyzed. The signal waveform in the output of H2 will be essentially the same modulo an overall amplitude factor that we will ignore for the present argument.

Let

$$x_1 = \{x_1[0], x_1[1], \ldots, x_1[N - 1]\}$$

and

$$x_2 = \{x_2[0], x_2[1], \ldots, x_2[N - 1]\}$$

denote data segments from H1 and H2 respectively. For the case outlined above, the Logarithm of the Likelihood Ratio [5] is given by,

$$\Lambda(x_1, x_2 | h) = \ln \left( \frac{p(x_1, x_2 | h)}{p(x_1, x_2 | 0)} \right),$$

where

$$p(x_1, x_2 | h)$$ is the joint probability density of the data in the presence of a signal $$h$$.

$$\Lambda(x_1, x_2 | h)$$ is parametrized by the signal sample values $$h[k]$$. In the absence of any prior knowledge about the signal waveform, we shall simply maximize $$\Lambda(x_1, x_2 | h)$$ over each $$h[k]$$ independently. The maximum value, $$\lambda(x_1, x_2)$$, of $$\Lambda(x_1, x_2 | h)$$ occurs for $$h = (x_1 + x_2)/2$$, and

$$\lambda(x_1, x_2) = \frac{1}{4} \sum_{i=0}^{N-1} x_1[i]^2 + \frac{1}{4} \sum_{i=0}^{N-1} x_2[i]^2 + \frac{1}{2} \sum_{i=0}^{N-1} x_1[i]x_2[i].$$

Thus, the test statistic obtained from the Likelihood Ratio prescription involves the second moments of the individual data segments and their cross-correlation.

Our simplified assumptions do not hold for real data: interferometer noise is non-Gaussian and non-stationary; H1 and H2 are not completely uncorrelated. Among
sources of deviation from Gaussianity, stationarity and independence are high power narrowband noise (also called lines) and bursts of instrumental origins.

Lines in the two data streams with nearby frequencies can produce large spurious correlations, especially in short data segments. The presence of bursts will adversely affect the performance of \( \lambda(x_1, x_2) \) since even if such a non-GW burst were to occur in only one detector, \( \lambda(x_1, x_2) \) may cross the detection threshold because of the variance terms.

Lines are probably the easier problem to address, since most high power features seems to be persistent in their carrier frequencies and bandwidths. Therefore, they are easily identified and can be removed using notch filters (see Section 3). Lines that are not present at all times pose a more complex problem but this is still easier to trace than instrumental bursts. Therefore, it makes sense to use only the cross-correlation term as the basis of a detection strategy. (A cross-correlation based waveform consistency test has also been independently developed for use with untriggered burst searches.)

We know that GW signals must have about the same time of arrival in the output of the two Hanford detectors but we do not know a priori the delay between the GRB and the GW signal. Consequently, we do not know the absolute time of arrival of the GW signal itself. However, based on most present astrophysical models and known observational uncertainty in the onset of a GRB, we can make an educated guess about the start and stop times of the data segment to be analyzed. In our analysis, the start time is chosen to be 120 sec before the GRB arrival time and the stop time is fixed at 60 sec after it. We also verify the insensitivity of the search for 20-30% perturbation of these boundaries. This interval (\( \sim 180s \)), called the on-source segment, is much larger than the expected duration of GW signal (\( \sim O(10-100ms) \)).

The optimum length of the data segments to use for computing the cross-correlation depends on the duration of the signal and its signal-to-noise ratio, both of which are not known a priori. The combination of these two facts suggests that the cross-correlation of H1 and H2 outputs should be computed on segments whose start and lengths vary over a range covering the allowed range of arrival time and the expected duration of the GW burst signal.

An artificial time shift, much larger than the expected signal duration of \( \sim O(10-100ms) \), between the two data streams before computing cross-correlations suppresses the average contribution from the GW signal. This fact can be used to make a local estimation of noise properties, thus mitigating the effects of non-stationarity in the interferometer outputs.

The line of reasoning outlined above naturally leads to a search algorithm that processes the data as follows. (1) A three dimensional random field is constructed, consisting of cross-correlations for a range of start times, integration lengths and time shifts. A signal must produce a local change in the distribution of samples in this random field. (2) The signature of the signal is extracted in a systematic manner. First, the three dimensional field is reduced to a two dimensional image, called a corrgram. The value of a pixel in the corrgram indicates the significance of cross-correlations near zero time shift relative to large time shift cross-correlations. (3) Finally, a list of events is found by identifying significant regions in the corrgram image, each event being characterized by a peak time, optimal integration length and an event strength. The strength of each event can be compared with a preset detection threshold or can be translated into a confidence interval/upper limit for the strength of the underlying GW signal.
The detection threshold and the confidence intervals are obtained via extensive Monte Carlo simulations using sections of the interferometer output, called *off-source* data, that are well outside the on-source segment.

3. **Pre-processing and data conditioning**

To mitigate the effect of instrumental artifacts and uneven spectral sensitivity, the raw output from the detectors needs to be processed before using the search algorithm. A brief description of this part of the pipeline is provided here for the sake of completeness.

The output from each interferometer is broken into 330 sec long sections with a 15 second overlap between consecutive segments, providing a continuous tiling of the data with 300s long segments. In order to avoid edge effects, the on-source segment (180 sec long), lies in the middle of one such 330 sec long segment. The pre-processing steps are, as follows. For each 330 sec long segment:

(i) Time-domain, zero-phase notch filters are applied to remove a set of narrow frequency bands corresponding to known line features.

(ii) A simple IIR filter is applied to H2 data with a phase response that approximates the difference in the phase responses of H1 and H2 calibration transfer functions. This restores the proper phase and waveform relation between the possible gravitational wave signals in the two data streams. This is to compensate for the slightly different spectral response of the two Hanford interferometers.

(iii) Each section is bandpass filtered using time domain zero-phase filtering. The lower and upper frequencies of the pass band are 80 Hz and 2048 Hz respectively.

(iv) The bandpassed data is downsampled from 16384 Hz to 4096 Hz by dropping the appropriate subset of samples.

(v) A linear predictor filter (LPF) is trained on the data between 5 and 15 seconds at the beginning. The LPF is a procedure for constructing a high order FIR filter whose transfer function is the inverse of the square root of the Power Spectral Density (PSD) of the data.

(vi) The first and last 5 sec of the resulting data are dropped in order to avoid filter transients. Only very short portions of first and last 10 sec are processed by the search algorithm, in order to avoid edge effects at the beginning and the end of each 300s long tile.

The result of the pre-processing is a 320 sec segment consisting of noise with a flat PSD (*white noise*) between $\sim$80 Hz and $\sim$2048 Hz. In practice, only the middle 300 sec from each 320 sec segment are used to find events. The first and last 10 seconds are not used to make the final event list but are processed in order to take care of edge effects.

The pre-processing and data conditioning outlined above also modifies a signal present in the data. However, all the filters applied to the data, except for the H2 spectral shape correction and the LPF derived filter, are the same for both H1 and H2 and zero-phase filtering is used for all filters, including the LPF derived ones. The spectral shape of H1 and H2 were close enough that the LPF whitening filters did not have to be significantly different. This ensured that the signal start times in the pre-processed data had no significant shift and the signal phases and waveforms remained about the same in the pre-processed and conditioned data streams.
4. Detailed description of analysis algorithm

We will now describe the main analysis algorithm that is applied to pairs of preprocessed data segments from H1 and H2. The basic idea behind the algorithm has already been stated in Section 2.

First, we fix our notation.

(i) Let the pre-processed time series from H1 be \( H_1 = \{ H_1[0], H_1[1], \ldots, H_1[N-1] \} \) and from H2 be \( H_2 = \{ H_2[0], H_2[1], \ldots, H_2[N-1] \} \).

(ii) Let \( s_1[k, j] \) be a subsegment from \( H_1 \), \( s_1[k, j] = \{ H_1[k], H_1[k+1], \ldots, H_1[k+j-1] \} \). Similarly define \( s_2[k, j] \) be a subsegment from \( H_2 \).

(iii) The cross-correlation \( C_{k,m,j} \) between segments \( s_1[k, j] \) and \( s_2[m, j] \) is defined as,

\[
C_{k,m,j} = \sum_{i=0}^{j-1} H_1[k+i]H_2[m+i].
\]  

(Note that the ordering of the first two indices in the definition of \( C_{k,m,j} \) is important since they correspond to data from different detectors.)

(iv) The length \( j/4096 \) sec of the segment pair is the integration length and \( k/4096 \) sec is the offset.

(v) Let \( C_{k,j}[p], p = 0, \pm 1, \pm 2, \ldots \), be the sequence,

\[
C_{k,j}[p] = \left[ C_{k-j/2,k+p-j/2,j}^2 + C_{k-p-j/2,k-j/2,j}^2 \right]^{1/2}.
\]  

(\( j \) is always kept even.) \( C_{k,j}[p] \) contains the autocorrelation of the GW signal for samples at and near \( p = 0 \), while far away samples only have the signal-noise cross-correlation. The far from zero time shift samples of \( C_{k,j}[p] \) can be used to make an estimation of background mean and variance. This estimate uses detector noise that is localized near \( p = 0 \) and is, thus, advantageous in the presence of non-stationarity.

(vi) We divide the sequence \( C_{k,j}[p] \) into two parts:

\[
\begin{cases} 
-W/2 \leq p \leq W/2 & \text{core} \\
2.5j \leq |p| \leq 2.5j + W & \text{side lobes}
\end{cases}
\]

where \( W \) is an even integer. For a core size \( \mathcal{W} \) comparable to the effective duration of the signal autocorrelation function, a sum over core samples will have a comparable or larger signal to noise ratio than the sample at \( p = 0 \). In the analysis algorithm, the core size is \( \max\{20, j\} \leq \mathcal{W} \leq \min\{j, 40\} \). In practice, the analysis is found not to be sensitive to the exact choice of the core size around this point.

The steps in the algorithm are as follows. First, we fix a rectangular grid in offset and integration length space. Let the list of integration lengths be \( I \) and that of offsets be \( O \). Let the coordinates of the grid points be \( (k, j) \), with \( I[k]/4096 \) sec and \( O[j]/4096 \) sec being the integration length and offset respectively. The spacing, \( (O[k+1] - O[k])/4096 \), between grid points along the offset direction is 1 msec. The distribution of the integration lengths are logarithmic. Each consecutive integration length is 50% longer than the previous one.

(i) For each point \( (k, j) \), the sequence \( C_{O[j],I[k]}[p] \) is computed over a range of \( p \) covering the lobes.
(ii) The mean \( \hat{\mu}_{k,j} \) and standard deviation \( \hat{\sigma}_{k,j} \) of \( C_{O[j],I[k]}[p] \) in the lobe regions is computed.

(iii) The values of \( C_{O[j],I[k]}[p] \) in the core region are standardized by subtracting \( \hat{\mu}_{k,j} \) and then dividing by \( \hat{\sigma}_{k,j} \). Positive standardized values in the core region are summed. Let the sum be \( X_{k,j} \). We call this two dimensional random field the corrgram and each pixel contains the “excess correlation” of close-to zero versus non-zero time shifts. (A standardization using unphysical time shifts was also used in [9].)

(iv) For each \( j \),
   (a) the largest 3% of values are dropped from the sequence \( X_{k,j} \).
   (b) A Gamma distribution fit is found for the distribution of the uncorrelated portion of the remaining pixels. The mean \( \mu_\gamma[j] \) and variance \( \sigma_\gamma^2[j] \) are computed and stored.

(v) Each \( X_{k,j} \) is standardized by subtracting \( \mu_\gamma[j] \) and dividing by \( \sigma_\gamma[j] \).

The purpose of step (iv) is to bring all pixels of the corrgram on to the same footing. The purpose of step (iv)a is to prevent the biasing of the significance estimate by the presence of a strong signal or instrumental artifacts.

(vi) A very low threshold of 0.05 is used to partition the standardized corrgram into pixels with values above and below the threshold. The pixels in the latter set are assigned the value zero. We continue to denote each pixel value as \( X_{k,j} \).

(vii) A clustering algorithm is used to identify groups of adjacent pixels with non-zero values. The steps in the clustering algorithm are as follows:
   (a) Each pixel \( X_{k,j} \) is assigned a binary score, 
      
      \[
      \text{score 1 if } X_{k\pm1,j\pm1} \neq 0, \tag{6}
      \]
      
      and score 0 otherwise. For pixels at the borders of the corrgram, only the five nearest neighbours are required to be non-zero.
   (b) For all \( (k,j) \) such that \( X_{k,j} \) has score 1, set \( X_{k,j} = 0 \) if \( X_{k,j} < 0.5 \). We thus drop a group of pixels whose innermost pixel has a low value. This reduces the number of small clusters due to background noise. The likelihood of rejecting a genuine signal is small since, for the corresponding cluster of pixels, higher pixel values are most likely to occur inside the cluster and not on its border.

(viii) Take \( \Lambda = \max_{k,j} X_{k,j} \) and let \( \Lambda = X_{m,n} \) (\( m,n \) thus being the location of the maximum).

(ix) Let \( G_{m,n} = \{X_{k,j} \mid n - b_0 \leq j \leq n + b_0\} \), where \( b_0 \) is such that \( |O[n \pm b_0] - O[n]|/4096 = 0.064 \) sec, that is, \( G_{m,n} \) is the set of all pixel values within 64 msec of the maximum. (Border pixels are handled as special cases.) Thus, we count pixels lying within 64 msec of each other as a single event. This choice is guided by Monte Carlo studies using a variety of waveforms.

(x) The average of the 5 highest pixels in \( G_{m,n} \) is computed. This quantity is called the strength of the event. The triplet of strength, integration length \( = I[m]/4096 \) sec and time of arrival \( = O[n]/4096 \) sec are stored. Each triplet constitutes an event.

(xi) Set \( X_{k,j} = 0 \) for \( n - b_0 \leq j \leq n + b_0 \). Repeat step (xi) until no more non-zero pixels are left.
(xii) Finally a list of events is obtained. Each event is a potential GW signal candidate. The list of events is used as the basis for statistical inferences which form a separate part of the analysis pipeline. In brief, two types of inferences are drawn, detection and, in the absence of detection, an upper limit on the root sum square of a possible gravitational wave signal. For the former, a detection threshold on the strength of each event, corresponding to a reliably known false alarm rate, is used. The threshold is estimated via Monte Carlo simulations using off-source data. The upper limit estimation is also performed via Monte Carlo simulations based on off-source data but this time with a variety of signal waveforms injected in software. The root sum square value of a signal that leads to, say, 90% detection probability for a given threshold is taken as the 90% confidence level upper limit on the root sum square of a possible GW signal. Details are provided in [3].

5. Conclusions

We have presented a detailed description of the algorithm used in a search for gravitational waves associated with GRB030329 using data from the two Hanford LIGO interferometers. The search algorithm involves a scan over all the three free parameters involved in the cross-correlation based search: integration length, offset and relative time-shifts. This is followed by a systematic extraction of the information contained in the resulting three dimensional phase space in the form of an event list.

The performance of the full analysis pipeline has been quantified using simulated data and the results are encouraging. As far as the search algorithm itself is concerned, more effort will be put towards the analytical characterization of its performance. We will also explore improvements in the computational performance of the code. This is required in order to reduce the time needed for the background studies and Monte Carlo simulations which dominates computational costs at the present.

Acknowledgments

We thank L. Cadonati for thoughtful comments on the text of the paper. The authors gratefully acknowledge the support of the United States National Science Foundation for the construction and operation of the LIGO Laboratory and the Particle Physics and Astronomy Research Council of the United Kingdom, the Max-Planck-Society and the State of Niedersachsen/Germany for support of the construction and operation of the GEO600 detector. The authors also gratefully acknowledge the support of the research by these agencies and by the Australian Research Council, the Natural Sciences and Engineering Research Council of Canada, the Council of Scientific and Industrial Research of India, the Department of Science and Technology of India, the Spanish Ministerio de Ciencia y Tecnologia, the John Simon Guggenheim Foundation, the David and Lucile Packard Foundation, the Research Corporation, and the Alfred P. Sloan Foundation.

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