Violating $R$-Parity at the $B$-Factory

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Abstract

Supersymmetry without $R$-parity contains new tree-level contributions to $B$-decays. A brief summary is provided of the current experimental bounds on those tree-level contributions which are relevant to $B$ physics. Signals to look for are outlined in the context of CP violation and rare decays. For the first time, the signature of general $R$-parity violating models is described. $B$-factory experiments will provide an opportunity to look for this signal.
The Minimal Supersymmetric Standard Model (MSSM) \cite{1} does not contain the accidental symmetries of baryon number (B) and lepton number (L) which grace the Standard Model (SM). Therefore, an ad-hoc symmetry called \( R \)-parity is often imposed to keep these global symmetries intact. This symmetry assigns a charge of \((-1)^{3B+L+2S}\) to each particle, where \( S \) is the particle’s spin. Particles of the SM are even under this symmetry, while their superpartners are odd. No compelling theoretical arguments exist for such a symmetry. Therefore, it behooves us to examine both the limits that current experimental data puts on \( R \)-parity violating \( (\not{R}_p) \) couplings and the effects they could have on the upcoming experiments on \( B \)-mesons (BaBar, BELLE, HERA B, CLEO, RUN II at FNAL) \cite{2}. While it has already been pointed out that a specific \( R_p \) model could effect CP violation measurements \cite{3}, this article is a comprehensive study of the effects of general models and points out new signals not yet discussed in the literature.

The gauge-invariant \( (\not{R}_p) \) terms that could be added to the MSSM are

\[
\lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k,
\]

in which bilinear terms are assumed to be rotated away \cite{4}. The couplings \( \lambda \) and \( \lambda' \) are antisymmetric in their first two and last two (flavor) indices, respectively. Thus, there are forty-five new independent terms and couplings. Bounds on the proton lifetime do not allow for reasonably-sized couplings for all of the terms \cite{5,6}. However, if we impose \( B \) or \( L \) conservation, the remaining couplings are much more weakly bounded. From these bounds, we can calculate the maximum possible effect on \( B \)-meson decays and proceed to look for these effects at the \( B \)-factories \cite{2}.

In the last few years, upper limits have been put on many of the \( R_p \) couplings using current experimental data. Relatively strong bounds have been put on both \( \lambda \) and \( \lambda' \) couplings, but few similar bounds have been put on the \( \lambda'' \) coupling. This is due mainly to the difficulty in measuring exclusive hadronic decays. The upper bounds on all \( \lambda \) couplings range between .04 and .1 for slepton masses of 100 GeV. They could lead to small amounts of CP violation, and contribute to rare leptonic decays. I will not focus on them \cite{7}. The majority
of the $\lambda'$ couplings are most strongly bound by their contribution to the decay $K \to \pi\nu\nu$. The remaining couplings are bounded less strongly by a number of different experimental phenomena. In the case of the $\lambda''$ couplings, only two are bounded significantly below unity. These are $\lambda''_{112} \leq 10^{-6}$ and $\lambda''_{113} \leq 10^{-4}$, where the former is due to non-observation of double-nucleon decay and the latter to non-observation of $n-\bar{n}$ oscillations. Table 1 in [11] includes a complete list of these bounds and their sources.

More stringent bounds have been put on some products of these couplings [8–10]. Bounds associated with $K-\bar{K}$ mixing, $B-\bar{B}$ mixing and neutrinoless double beta decay limit possible effects of $\tilde{R}_p$ terms on $B$-meson decays. These bounds are as follows:

$$\text{Re} \left[ \sum_{i,j,j'} \left( \frac{100 \text{ GeV}}{m_{\tilde{\nu}_i}} \right)^2 \lambda^{*}_{ij3} \lambda'_{ij1} V_{j1}^* V_{j3} \right] \leq 3 \times 10^{-8} \quad \text{(from $\beta\beta_0$, and $\Delta m_B$)} \quad (2)$$

$$\text{Re} \left[ \sum_{i,j,j'} \left( \frac{100 \text{ GeV}}{m_{\tilde{\nu}_i}} \right)^2 \lambda'_{ij2} \lambda'_{ij1} V_{j1}^* V_{j2} \right] \leq 4.5 \times 10^{-9} \quad \text{(from $\Delta m_K$),} \quad (3)$$

where $V_{jk}$ are Cabibo-Kobayashi-Maskawa (CKM) matrix elements. The bounds are shown in the usual mass basis of the right-handed and up-type left-handed quarks. Since these bounds are not rigorous ones on individual products, I shall take only their orders of magnitude and assume no 'conspiratorial' cancellations in the sum. Despite these bounds, dramatic phenomenological effects are not ruled out. Table 1 provides a list of bounds on the products of $\lambda'$ couplings that directly affect hadronic $B$-meson decays. Individual $\lambda''$ bounds are also shown.

In the next few years, $B$-factory experiments will be on-line, allowing for testing of many SM predictions. One such test is of the predicted CP violation associated with the phase in the CKM matrix. The matrix is often parametrized by triangles in the complex plane. Measurements of angles and lengths of sides of these triangles can, with varying degrees of accuracy, be extracted from experimental data.

For example, some angles of the triangle shown in Figure 2 of [12] can be extracted from the asymmetry of $B^0_d$ and $\bar{B}^0_d$ decays into the same CP eigenstate, $f_{CP}$. This asymmetry is

$$a_{f_{CP}}(t) \equiv \frac{\Gamma(B^0_{phys}(t) \to f_{CP}) - \Gamma(\bar{B}^0_{phys}(t) \to f_{CP})}{\Gamma(B^0_{phys}(t) \to f_{CP}) + \Gamma(\bar{B}^0_{phys}(t) \to f_{CP})}$$

3
\[
(1 - |r_{f_{CP}}|^2) \cos \Delta M t - 2 \text{Im}(r_{f_{CP}}) \sin \Delta M t \over 1 + |r_{f_{CP}}|^2,
\]

where \( B^0_{\text{phys}}(t)(\bar{B}^0_{\text{phys}}(t)) \) is the state at \( t = 0 \) which is the pure flavor eigenstate, \( B^0_d(\bar{B}^0_d) \), and \( \Delta M \) is the difference in masses of the two mass eigenstates. Moreover, \( r_{f_{CP}} \equiv qA_{f_{CP}}/pA_{\bar{f}_{CP}} \), where \( A_{f_{CP}}(A_{\bar{f}_{CP}}) \) is the total decay amplitude of \( B^0_d(\bar{B}^0_d) \) into \( f_{CP} \), and \( p \) and \( q \) are \( B-\bar{B} \) mixing parameters for which \( |q/p| \simeq 1 \). Each diagram that contributes to the total decay amplitude has the form \( Ae^{i(\psi + \phi)} \), where \( A \) is the magnitude of the amplitude and \( \phi \) and \( \psi \) are the weak and strong phases respectively. The weak phase comes from the weak couplings in the diagram and the strong phase comes from final-state rescattering effects. The strong part of the Hamiltonian is CP-symmetric; the weak part is not. Thus, the CP conjugate of the above decay amplitude is \( Ae^{i(\psi - \phi)} \). This analysis and notation follows that of [12].

When only one diagram completely dominates a given decay amplitude, \( |A_{R_p}| = |\bar{A}_{R_p}| \) and \( |r_{f_{CP}}| \simeq 1 \). This simplifies \( a_{f_{CP}} \) and allows for immediate extraction of one of the phases in the CKM matrix. Certain decays give angles in the triangle. For example, the CKM angle \( \beta \) can be extracted from the asymmetry of decays into \( f_{CP} = J/\psi \pi^0 \), which are proportional to

\[
\text{Im}(r_{f_{CP}}) = \text{Im} \left( \frac{V_{tb}^*V_{td}V_{cb}^*V_{cd}}{V_{tb}V_{td}^*V_{cb}V_{cd}^*} \right) = - \sin 2\beta. \tag{5}
\]

This is an example of CP violation due to interference between mixing and decay. If, however, more than one diagram contributes to the same process, and the diagrams have different strong and weak phases, then (e.g., for two contributions)

\[
|r_{f_{CP}}| \simeq \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\psi_1 - \psi_2 + \phi_2 - \phi_1)} \neq 1. \tag{6}
\]

Now the relationship between the weak phase and \( a_{f_{CP}} \) is not so straightforward. This is a case of direct CP violation. In such a case, asymmetries could be detected in decays into non-CP eigenstates (i.e., in \( B^0 \rightarrow f \) and \( \bar{B}^0 \rightarrow \bar{f} \)). Asymmetries could also be seen in the corresponding \( B^\pm \) decays.
The $R_p$ terms will give new tree-level contributions to three-body $b$-decays. The upper limits on contributions to the quark subprocess shown in Table 2 come from the current bounds on $\lambda'$ and $\lambda''$. The $|A_{R_p}|/|A_{SM}|$ in the last two columns is the ratio of the amplitudes of the dominant quark diagrams, and not of the full hadronic processes, which include strong matrix elements of the type:

\[
\frac{|M_{R_p}|}{|M_{SM}|} = \frac{|\langle \Psi K_S | (\bar{b}_R^\alpha c_L^\alpha |(\bar{c}_L^\beta s_R^\beta) |B_0^d)\rangle|}{|\langle \Psi K_S | (\bar{b}_R^\alpha c_L^\alpha |(\bar{c}_L^\beta s_R^\beta) |B_0^d)\rangle|} \quad (7)
\]

It is not known how to calculate these ratios accurately. I shall assume them to be of $O(1)$ and limit bounds on contributions to order of magnitude estimates.

Nearly all of the decays listed in Table 2 could have significant contributions from the new physics, but not simultaneously. Remarkably, a signal emerges. An $R_p$ theory could deviate notably from the SM in its predictions of decay rates into heavy mesons or decay rates into solely light mesons, but not both. Other theories which could allow for new tree contributions to $B$-decays (e.g., theories with extra Higgs or diquarks) need not satisfy this constraint. These deviations from the SM could be detected in a variety of ways:

- The SM predicts that the asymmetry in some decays are proportional to the same CKM angle. A difference in the asymmetries would imply an additional contribution to at least one of the decays. For example, the SM predicts that the asymmetries in $B^0_d \to \Psi K_S$ and $B^0_d \to D^+D^-$ are both proportional to the sine of the angle $\beta$ [12]. If $R_p$ terms contribute significantly to either process with a different weak phase, the asymmetries would differ, indicating new physics. Another example is the decays $B^0_d \to D^+D^-$ and $B^0_d \to \Psi \pi^0$, which have the same quark subprocesses. In this case, if there is a significant $R_p$ contribution with a weak phase different from the SM contribution, the asymmetries will be different as long as the ratio of strong matrix element contributions differ from each other, i.e., if

\[
\frac{\langle D^+D^- | (\bar{b}_L^\gamma \mu c_L^\gamma |(\bar{c}_L^\beta s_R^\beta) |B_0^d)\rangle}{\langle D^+D^- | (\bar{b}_R^\alpha c_L^\alpha |(\bar{c}_L^\beta s_R^\beta) |B_0^d)\rangle} \neq \frac{\langle \Psi \pi^0 | (\bar{b}_L^\gamma \mu c_L^\gamma |(\bar{c}_L^\beta s_R^\beta) |B_0^d)\rangle}{\langle \Psi \pi^0 | (\bar{b}_R^\alpha c_L^\alpha |(\bar{c}_L^\beta s_R^\beta) |B_0^d)\rangle} \quad (8)
\]
• The SM also predicts that certain decays will measure the CKM angles $\alpha$, $\beta$, and $\gamma$. The sum of these angles is $180^\circ$. If the sum of the measurements differs from $180^\circ$, this is a signal of new physics.

• The SM predicts very little direct CP violation in most tree-level $B$-decays. For example, there is a single diagram which dominates the SM contribution to $B_d^0 \to D_s^+D^-$ (and its excited modes), and therefore, the direct CP violation in this mode is expected to be negligible. A contribution to $\bar{b} \to \bar{c}c\bar{s}$ with phases different from the SM contribution could give a measureable asymmetry. This effect would also be seen in $B^\pm$ decays.

• The following quark subprocesses do not appear at tree-level in the SM: $\bar{b} \to \bar{d}d\bar{d}$, $d\bar{s}, s\bar{s}, d\bar{s}, \bar{d}s, s\bar{s}$, and $\bar{s}d\bar{s}$. The first four are dominated by penguin diagrams and the last two by 1-loop box diagrams. In a supersymmetric theory without $R$-parity, any of these decays could be allowed at tree-level. Within current bounds, $B \to \bar{K}^0 K^0$, $\phi\pi^0$, $\phi K^0$, $K^0\bar{K}^0$ and $\bar{K}^0\pi^0$ (and their excited states) could be seen in abundances greater than those predicted by the SM signaling the existence of new physics. The last two modes would only have to be seen, as they are predicted to be extremely rare by the SM. There is, of course, a complementary set of charged $B$-decays: $B^+ \to K^+\bar{K}^0$, $\phi\pi^+$, $\phi K^+$, $K^0\pi^+$, $K^+K^0$ and $\bar{K}^0\pi^+$. Again, the last two are greatly suppressed in the SM.

In addition to hadronic decays, leptonic and semileptonic $B$-decay rates could be increased greatly by $R_p$ effects. The process, $B \to \ell^+\ell^-$, is highly suppressed in the SM when the leptons are the same and forbidden by the SM when they are different [13]. The experimental bounds on this process [14] set some of the strongest limits on the relevant $R_p$ parameters [10]. The high statistics at the $B$-factories will make it possible to probe these limits and look for an enhanced signal or lepton flavor violation. The exisitance of $R_p$ couplings could result in lepton nonuniversality in $\bar{b} \to \bar{c}\ell^+\nu$ and $\bar{b} \to \bar{u}\ell^+\nu$. Current limits allow contributions of 5% and 100% of the SM rates respectively. Contributions to
one choice of $\ell$ at these levels would lead to detectable violation of universality at the $B$-factories. Enhancements of $B \to X \bar{\nu} \nu$ \cite{15} could also be large, but may be difficult to detect in experiments of the near future \cite{16}. See reference \cite{17} for a full analysis of $B$-decays.

The effects outlined here are highly suppressed or completely forbidden in supersymmetric theories with $R$-parity, because they do not exist at tree-level. Theories which include additional tree-level contributions to $b$-quark decays, such as the SM with diquarks or additional Higgs scalars, could have similar effects. Because such theories are not restricted to the same signal pattern as $R_p$ theories, it may be possible to distinguish them experimentally.

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Bounds on $\sum_{i=1}^{3} |\lambda_{ijk}'|^{2}$

$$\begin{array}{|c|c|c|c|}
\hline
(j, k; j', k') & Upper \text{ Bound} & Source & (j, k; j', k') & Upper \text{ Bound} & Source \\
\hline
(1,3;1,1) & 2 \times 10^{-5} \left( \frac{m_{\tilde{B} R}}{100 \text{ GeV}} \right)^2 & (a) & (3,1;1,1) & 3 \times 10^{-3} & (b),(d) \\
(1,3;1,2) & 1.4 \times 10^{-4} \left( \frac{m_{\tilde{B} R} m_{\tilde{S} R}}{100 \text{ GeV}} \right)^2 & (b) & (3,1;1,2) & 6 \times 10^{-7} \left( \frac{m_{\tilde{B} R}}{100 \text{ GeV}} \right)^2 & (c) \\
(1,3;2,1) & 9 \times 10^{-7} \left( \frac{m_{\tilde{B} R}}{100 \text{ GeV}} \right)^2 & (a) & (3,1;2,1) & 3 \times 10^{-3} & (b),(d) \\
(1,3;2,2) & 1.4 \times 10^{-4} \left( \frac{m_{\tilde{B} R} m_{\tilde{S} R}}{100 \text{ GeV}} \right)^2 & (b) & (3,1;2,2) & 6 \times 10^{-7} \left( \frac{m_{\tilde{B} R}}{100 \text{ GeV}} \right)^2 & (c) \\
(2,3;1,1) & 8 \times 10^{-5} \left( \frac{m_{\tilde{B} R}}{100 \text{ GeV}} \right)^2 & (a) & (3,2;1,1) & 3 \times 10^{-6} \left( \frac{m_{\tilde{B} R}}{100 \text{ GeV}} \right)^2 & (c) \\
(2,3;1,2) & 1.4 \times 10^{-4} \left( \frac{m_{\tilde{B} R} m_{\tilde{S} R}}{100 \text{ GeV}} \right)^2 & (b) & (3,2;1,2) & 5 \times 10^{-3} & (b),(d) \\
(2,3;2,1) & 4 \times 10^{-6} \left( \frac{m_{\tilde{B} R}}{100 \text{ GeV}} \right)^2 & (a) & (3,2;2,1) & 1 \times 10^{-6} \left( \frac{m_{\tilde{B} R}}{100 \text{ GeV}} \right)^2 & (c) \\
(2,3;2,2) & 1.4 \times 10^{-4} \left( \frac{m_{\tilde{B} R} m_{\tilde{S} R}}{100 \text{ GeV}} \right)^2 & (b) & (3,2;2,2) & 5 \times 10^{-3} & (b),(d) \\
\hline
\end{array}$$

Table 1. Product bounds on $R_p$ couplings from the following sources: (a) $\beta\beta_0$ \cite{8} and $B^-\bar{B}$ mixing \cite{9}, (b) $K \rightarrow \pi\nu\nu$ \cite{18}, (c) $K^-\bar{K}$ mixing \cite{9}, (d) a combination of Atomic parity violation \cite{19}, $\nu_\mu$ deep-inelastic scattering \cite{19}, Z decay width \cite{20}, and top-decay. The bounds on $\lambda''$ come from: (w) double nucleon decay \cite{21}, (x) $n-\bar{n}$ oscillations \cite{21}, (y) Z decay width \cite{22}, (z) perturbative unitarity \cite{6,23}.

$^*$These terms are linearly dependent on different squark and/or slepton masses. The values shown are for $\tilde{m} = 100$ GeV

$^\dagger$This bound is proportional to the square of the squark masses, keeping the gluino mass fixed. The value shown is for $\tilde{m} = 100$ GeV

$^{\ddagger}$This bound grows exponentially with squark masses. The value shown is for $\tilde{m} = 100$ GeV. At $\tilde{m} = 400$ GeV, it reduces to $\sim 10^{-2}$. 

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Table 2. Bounds on $R$-parity violating contributions to $B$-decays. Sparticle masses are assumed to be 100 GeV where not shown.

*Compared with $\bar{b} \to \bar{u}u\bar{d}$ in the SM.

†Compared with penguin.

‡Bound dominates the 1-loop SM contribution.

§This bound becomes unity at $m_{\text{sparticles}} = 400$ GeV.