Frequency performance analysis of proportional integral-type active disturbance rejection generalized predictive control for time delay systems

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ABSTRACT
To improve the performance of the Active Disturbance Rejection Control (ADRC) in systems which have large time delay, reduce online computation for Proportional Integral-type Generalized Predictive Control (PI-GPC) method, the Proportional Integral-type Active Disturbance Rejection Generalized Predictive Control (PI-ADRGPC) based on Controlled Auto Regression and Moving Average (CARMA) model is designed. The frequency domain analysis method is used to analyse the stability of the PI-ADRGPC based on CARMA model. By using the open-loop transfer function of the PI-GPC discrete form, the influence of parameter changes on the PI-ADRGPC performance is analysed. The performance of the PI-ADRGPC and ADRC algorithm is compared through the application in a second-order time delay system and distillation column system. The research results show that compared with the ADRC algorithm, the PI-ADRGPC method has a shorter rise time and better performance.

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1. Introduction
The Active Disturbance Rejection Control (ADRC) is an advanced method proposed by Han in 1988 (Han, 2009). It can estimate and overcome the disturbance (Cao et al., 2020; Huang & Xue, 2014; Xue & Huang, 2016). By simplifying the structure of ADRC, Gao proposed the Linear Active Disturbance Rejection Control (LADRC or simplified as ADRC). It has a wide range of applications (Hu et al., 2021; Luo et al., 2021; Yao et al., 2021). Xiang used ADRC in the electrostatic suspension system, and its performances well (Xiang et al., 2021). Zhu designed an electric current driver based on ADRC and combined it with parameter estimation (H. T. Zhu et al., 2021). For the nano-positioning worktable, Wei created a Phase Leading Active Disturbance Rejection Control (Wei et al., 2021).

The Model Predictive Control has a wide range of applications (Afisi et al., 2021; Qi et al., 2020; Yang & Ding, 2020). Based on it, Proportional Integral-type Generalized Predictive Control (PI-GPC) is introduced by Chen to solve the system interference problem. Through the multiple step prediction of the output, the PI-GPC algorithm has good dynamic performance and robustness and is widely used in industrial production (Forouz et al., 2021; Shui et al., 2021; Zhao et al., 2021).

These researches show that ADRC can overcome internal and external disturbances of the system, and PI-GPC has good performance on complex industrial processes. Despite the above advantages, their shortcomings should not be ignored. The ADRC algorithm is difficult to control large time delay systems (Madonski et al., 2020; Patelski & Dutkiewicz, 2020; Wang et al., 2020; H. Q. Zhu & Gu, 2020). The PI-GPC algorithm has a large amount of online calculation, and it is unsuitable in fast systems (Ai et al., 2021; Simek & Valouch, 2021).

Therefore, combined the advantages of these two algorithms, we have designed the Proportional Integral-type Active Disturbance Rejection Generalized Predictive Control (PI-ADRGPC). It observes and compensates the total disturbance through the Extended State Observer (ESO), compensates the system into an approximate series integral form with time delay, and then uses the PI-GPC method to control the system. In this way, the Diophantine equation only needs to be calculated once, and the online calculation is not required. Change the Proportion Differential (PD) controller in the original ADRC to PI-GPC, which can better deal with the time-delay system problem.

The main contribution of this paper is: (1) proposed PI-ADRGPC method; (2) for the second-order time delay system, derived its closed-loop feedback structure and gave the stable conditions of the closed-loop system; (3) used the frequency characteristics of the open-loop transfer...
function in the discrete domain to analyze the algorithm performance changing with parameters, and obtained the principles of controller parameter adjustment; (4) through the simulation experiment of the second-order time delay system and distillation column system, verified the superiority of PI-ADRGPC.

The remainder of this paper is given as follows. Section 2 introduced the PI-ADRGPC method. The discrete form of PI-ADRGPC is introduced in Section 3. Section 4 described the influence of PI-ADRGPC parameter changes on system performance. Second-order time delay system test and distillation column system simulation are described in Sections 5 and 6.

2. PI-type active disturbance rejection generalized predictive control introduction

Based on the advantages of ADRC and PI-GPC algorithms, this work proposes a PI-ADRGPC algorithm.

- The ADRC algorithm makes the proposed method does not need the precise model of the plant. During the online computation, the system parameters are not required to be identified, and the total disturbance can also be compensated, thereby the system can be simplified into the form of integrator series. The analytical solution of the Diophantine equation can be derived offline, so the online calculation amount is greatly reduced.
- The PI-GPC can realize the multi-step prediction of the process output and rolling optimization, hence, the system can overcome the large time delay.

The specific process of the algorithm design is as follows:

We take the system external disturbance and the internal unmodeled dynamics as the total disturbance and expand it into a new state $x_{n+1}$. Use the ESO to estimate and compensate it, then convert the plant into a series integrator form.

The ESO structure of the second-order system is

$$\begin{aligned}
\dot{z}_1 &= z_2 - \beta_1' e \\
\dot{z}_2 &= z_3 - \beta_2' e + b_0 u \\
\dot{z}_3 &= -\beta_3' e \\
\dot{e} &= z_1 - x_1
\end{aligned}$$

where $\beta_1', \beta_2'$ and $\beta_3'$ are the adjustable gains of ESO.

Then design the control law $u$ for the series integrator form, the control goal is achieved. The PI-ADRGPC method structure is shown in Figure 1.

![Figure 1. PI-ADRGPC algorithm structure.](image)

Give the discrete Single Input Single Output (SISO) systems, called Controlled Auto Regression and Moving Average (CARMA) model:

$$A(z^{-1})y(k) = B(z^{-1})u(k - 1) + C(z^{-1})\zeta(k),$$

where, $u(k)$ and $y(k)$ are the input and output, respectively. $z^{-1}$ is a backshift operator, $\zeta(t)$ is a random disturbance. $A(z^{-1}), B(z^{-1}), C(z^{-1})$ are shown in (2).

$$\begin{aligned}
A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n_a} \\
B(z^{-1}) &= b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n_b} \\
C(z^{-1}) &= 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_n z^{-n_c}
\end{aligned}$$

Let $C(z^{-1}) = 1$.

The second-order time delay system can be simplified as:

$$G(s) = \frac{1}{s^2} e^{-\tau s}.$$  

Ignore the time delay and (3) is:

$$G(s) = \frac{1}{s^2}.$$  

Its pulse transfer function is:

$$G(z^{-1}) = (1 - z^{-1})Z \left[ \frac{G(s)}{s} \right] = (1 - z^{-1})Z \left[ \frac{1}{s^2} \right]$$

$$= z^{-1} \frac{T^2}{2!} + \frac{z^{-1}}{1 - z^{-1}}^2.$$  

Ignore the $\zeta(k)$ of (1), then the $G(z^{-1})$ is pulse transfer function:

$$G(z^{-1}) = z^{-1} \frac{B(z^{-1})}{A(z^{-1})}.$$  

Correspond (4) and (5):

$$\begin{aligned}
A(z^{-1}) &= (1 - z^{-1})^2, \\
B(z^{-1}) &= \frac{T^2}{2} (1 + z^{-1}).
\end{aligned}$$
Consider the Diophantine Equation:
\[
\begin{align*}
1 &= E_j(z^{-1})A(z^{-1}) + z^{-1}F_j(z^{-1}) \\
E_j(z^{-1})B(z^{-1}) &= G_j(z^{-1}) + z^{-1}H_j(z^{-1}), \\
E_j(z^{-1}) &= e_1 + e_2 z^{-1} + \cdots + e_z z^{-(j-1)} \\
F_j(z^{-1}) &= f_1^j + f_2^j z^{-1} + \cdots + f_{n+1}^j z^{-n} \\
G_j(z^{-1}) &= g_1 + g_2 z^{-1} + \cdots + g_z z^{-(j-1)} \\
H_j(z^{-1}) &= h_1^j + h_2^j z^{-1} + \cdots + h_{n+1}^j z^{-(n+2)},
\end{align*}
\]  

(7)

where \(e_j\) is the \(j\)th element for the coefficient matrix of \(E_j(z^{-1})\), \(f_{n+1}^j\) is the \(n + 1\)th column vector for the coefficient matrix of \(F_j(z^{-1})\), \(g_j\) is the \(j\)th row vector for the coefficient matrix of \(G_j(z^{-1})\), \(h_{n+1}^j\) the \(n + 1\)th column vector for the coefficient matrix of \(H_j(z^{-1})\).

Substituting (6) into (7), we obtain the solution:
\[
\begin{align*}
e_j &= j, \\
f_1^j &= j + 1, \quad f_2^j = -j, \\
g_j &= \frac{T^2}{2!}(2j - 1), \\
h_1^j &= \frac{T^2}{2!}.
\end{align*}
\]

Consider the multistage cost function as follows:
\[
J = E \left\{ \sum_{j=1}^{N} [K_p (\Delta e(k+j))^2 + K_i e(k+j)^2] + \lambda \sum_{j=1}^{N_u} u(k+j-1)^2 \right\},
\]

(8)

where \(K_p \geq 0\) and \(K_i > 0\) are the scale and integral factor, respectively. \(u(k+j-1) = 0\) if \(j = N_u, \ldots, N\). \(N\) and \(N_u\) are the prediction and control step, respectively. And \(\lambda(\lambda > 0)\) is control weighting factor.

\[
\Delta e(k+j) = e(k+j) - e(k+j-1),
\]

where \(j = 1, 2, \ldots, N\).

The error sequence is:
\[
e(k+j) = w(k+j) - y(k+j).
\]

(9)

Then, the predicted output after \(j\) steps is:
\[
y(k+j) = G_j u(k+j-1) + F_j y(k) + H_j u(k-1).
\]

Assume that:
\[
y_0(k+j) = F_j y(k) + H_j u(k-1).
\]

Then,
\[
y(k+j) = y_0(k+j) + G_j u(k+j-1).
\]

(11)

Write (11) as:
\[
\mathbf{\bar{V}} = \mathbf{\bar{V}}_0 + \mathbf{G}_j \mathbf{\bar{U}}, \quad \mathbf{\bar{V}}_0 = [y_0(k+1), \ldots, y_0(k+N)]^T,
\]

(12)

where,
\[
\mathbf{\bar{V}} = [y(k+1), \ldots, y(k+N)]^T,
\]

According to (9) and (12):
\[
\mathbf{\bar{E}} = \mathbf{\bar{W}} - \mathbf{\bar{V}} = \mathbf{\bar{W}} - \mathbf{\bar{Y}}_0 - \mathbf{G}_p \mathbf{\bar{U}},
\]

\[
\Delta \mathbf{\bar{E}} = \Delta \mathbf{\bar{W}} - \Delta \mathbf{\bar{V}} = \Delta \mathbf{\bar{W}} - \Delta \mathbf{\bar{Y}}_0 - \mathbf{G}_p \mathbf{\bar{U}}.
\]

\[
\mathbf{\bar{E}} = [e(k+1), \ldots, e(k+N)]^T.
\]

(13)

\[
\mathbf{G}_p = \begin{bmatrix}
g_0 & 0 \\
g_1 & g_0 \\
\vdots & \vdots \\
g_{N_u-1} & g_{N_u-2} & \cdots & g_0 \\
\vdots & \vdots & \cdots & \cdots \\
g_N & g_{N-2} & \cdots & g_{N-N_u} \\
0 & \cdots & \cdots & \cdots & \cdots \end{bmatrix}_{N \times N_u}
\]

(14)

Equation (8) can be converted to:
\[
\mathbf{J} = K_p \mathbf{\bar{E}}^T \Delta \mathbf{\bar{E}} + K_i \mathbf{\bar{E}}^T \mathbf{\bar{E}} + \lambda \mathbf{\bar{U}}^T \mathbf{\bar{U}}
\]
\[
= K_p [\Delta \mathbf{\bar{W}} - \Delta \mathbf{\bar{Y}}_0 - \mathbf{G}_p \mathbf{\bar{U}}]^T [\Delta \mathbf{\bar{W}} - \Delta \mathbf{\bar{Y}}_0 - \mathbf{G}_p \mathbf{\bar{U}}]
\]
\[
+ K_i [\mathbf{\bar{W}} - \mathbf{\bar{Y}}_0 - \mathbf{G}_p \mathbf{\bar{U}}]^T [\mathbf{\bar{W}} - \mathbf{\bar{Y}}_0 - \mathbf{G}_p \mathbf{\bar{U}}] + \lambda \mathbf{\bar{U}}^T \mathbf{\bar{U}}.
\]
When the $J$ reaches the minimum:

$$
\mathbf{U} = (\lambda I + K_p G_p^T G_p + K_i G_i^T G_i)^{-1} \cdot [K_p G_p^T (\Delta W - \Delta Y_0) + K_i G_i^T (W - Y_0)].
$$

Assume that $K = [1, 0, \ldots, 0]^T_{n \times 1}$.

$$
u(k) = K^T (\lambda I + K_p G_p^T G_p + K_i G_i^T G_i)^{-1} \cdot [K_p G_p^T (\Delta W - \Delta Y_0) + K_i G_i^T (W - Y_0)].
$$

In which, $u(k)$ is the first element of $\mathbf{U}$:

$$
u(k) = R_p^T (\Delta W - \Delta Y_0) + R_i^T (W - Y_0).
$$

where,

$$R_p = K_p K^T (\lambda I + K_p G_p^T G_p + K_i G_i^T G_i)^{-1} G_p^T,$$

$$R_i = K_i K^T (\lambda I + K_p G_p^T G_p + K_i G_i^T G_i)^{-1} G_i^T.$$

After that we can obtain the control law.

Then simplify the control law. Firstly, define:

$$
\mathbf{S} = 
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
$$

Then $G_p = S G_i$, $\Delta W = S \Delta W$, $\Delta Y_0 = S \Delta Y_0$. Use these to reduce (13):

$$
\mathbf{U} = [\lambda I + K_p G_i^T S^T G_i + K_i G_i^T S^T (S \Delta W - \Delta Y_0) + K_i G_i^T (S \Delta W - \Delta Y_0)]
$$

$$
= (\lambda I + G_i^T \Omega G_i)^{-1} G_i^T \Omega (S \Delta W - \Delta Y_0),
$$

$$\Omega = K_i + K_p S^T S.
$$

The control law of PI-ADRGPC based on CARMA model can be secured, and $u(k - \tau)$ is input.

### 3. Discrete form of PI-type active disturbance rejection generalized predictive control

According to the above analysis:

$$
u(k) = h^T (W - Y_0),
$$

where $h^T = [1, 0 \ldots 0](\lambda I + G_i^T \Omega G_i)^{-1} G_i^T \Omega$. Substituting (10) into (14):

$$
u(k) = h^T (F_a y_r(k) - (F - F_a) y(k) - H u(k - 1)).
$$

$$
T(z^{-1}) u(k) = R y_r(k) - S(z^{-1}) y(k).
$$

where $R = h^T F_a, S(z^{-1}) = h^T (F - F_a), T(z^{-1}) = 1 + z^{-1} h^T H(z^{-1}).$

$$
y(k) = G(z^{-1}) u(k).
$$

Multiply $T(z^{-1})$ to both sides of (16) and substitute it into (15):

$$
y(k) = \frac{G(z^{-1}) D(z^{-1})}{1 + G(z^{-1}) H(z^{-1})} y_r(k).
$$

where $D(z^{-1}) = \frac{R}{1 - z^{-1}}, H(z^{-1}) = \frac{S(z^{-1})}{1 - z^{-1}}$.

The PI-GPC algorithm can be transformed into the form of a closed-loop discrete system, and the structure is in Figure 2.

The internal model structure of Linear Extended State Observer (LESO) is shown in Figure 3.

The structure of closed-loop discrete system under PI-ADRGPC is shown in Figure 4.

According to Figure 3:

$$
\phi(s) = \frac{y(s)}{u_0(s)}
$$

$$
= \frac{b(s + \omega_0^2) e^{-ts}}{b_0(s^2 + 2\omega_0 s)(s^2 + a_1 s + a_2) + b_0 \omega_0^2 s^2 e^{-ts}},
$$

where $u_0(s)$ is the virtual control variable in PI-ADRGPC.

The characteristic equation is:

$$
D(s) = b_0 (s^2 + 2 \omega_0 s)(s^2 + a_1 s + a_2) + b_0 \omega_0^2 s^2 e^{-ts}.
$$

**Figure 2.** Closed-loop feedback structure under the control of PI-GPC algorithm.

**Figure 3.** Internal model structure of LESO.

**Figure 4.** Closed-loop feedback structure of PI-ADRGPC.
We perform Taylor expansion on the time delay part:
\[ e^{-\tau s} = 1 - \tau s + \frac{1}{2} \tau^2 s^2. \]

The (17) is transformed into the following form:
\[
D(s) = \left( b_0 + \frac{1}{2} b \tau^2 \omega^2 \right) s^4 + (a_1 b_0 + 2 \omega_0 b_0 - b \tau \omega_0^2) s^3 \\
+ (a_2 b_0 + 2 a_1 \omega_0 b_0 + b \omega_0^2) s^2 + 2 a_2 \omega_0 b_0 s.
\]

That is:
\[
G(s) = \frac{b(s + \omega_0)^2 e^{-\tau s}}{D(s)}.
\]

Let:
\[
G(z^{-1}) = Z \left[ \frac{1 - e^{-\tau s} b(s + \omega_0)^2 e^{-\tau s}}{s D(s)} \right].
\]

Then the secular equation of the closed-loop system is shown in Figure 4. It can be represented as follows:
\[
1 + G(z^{-1})H(z^{-1}) = 0.
\]

Hence, the PI-ADRGPC based system open-loop transfer function is:
\[
G(z^{-1})H(z^{-1}) = \frac{z^{-1} B(z^{-1}) S(z^{-1})}{A(z^{-1}) T(z^{-1})}.
\]

4. The influence of PI-ADRGPC parameter changes on system performance

Give the following second-order time delay plant:
\[
G_p(s) = \frac{1}{(2s + 1)(s + 1)} e^{-3s}.
\] (18)

Increasing \( b_0 \) can improve the stability of the time delay system. Let \( b_0 = 35 \). The sampling period \( T = 0.1, \omega_0 = 10 \).

4.1. The impact of prediction time domain \( N \) change on system performance

When selecting \( N \) as 15, 20, 25, 30, 35, 40, 45, let \( Nu = 1, \omega_0 = 10, \lambda = 0.5, b_0 = 35, \alpha = 0.9, K_p = 1, K_i = 0.5 \). The bode diagram is in Figure 5. The cut-off frequency and phase margin under different \( N \) are shown in Table 1.

The experimental results show that when \( Nu = 1 \) and \( N \) is small. The cut-off frequency of the system is low. The response speed is slow. The phase angle margin is small,

| \( N \) | Phase margin (°) | Cut-off frequency (rad/s) |
|-------|------------------|--------------------------|
| 15    | 67.8564          | 0.0885                   |
| 20    | 55.8777          | 0.1462                   |
| 25    | 47.8481          | 0.1936                   |
| 30    | 40.3808          | 0.2436                   |
| 35    | 42.3890          | 0.2486                   |
| 40    | 43.0840          | 0.2595                   |
| 45    | 44.9003          | 0.2640                   |

Figure 5. Bode diagram when \( N \) changes.
and the stability is poor. We should choose a larger prediction time domain $N$, to get a faster response speed and a better stability.

In the PI-GPC algorithm, $N$ should exceed the time delay part of the plant impulse response. That is $N > \frac{\tau}{T}$. When $N < \frac{\tau}{T}$, the PI-ADRGPC output is fast and has no overshoot. Therefore, when other parameters are constant, the smaller $N$ makes PI-ADRGPC has better results.

4.2. The impact of control time domain $N_u$ change on system performance

When selecting $N_u$ as 1, 2, 4, 6, 8, 10, 12, let $N = 15, \omega_o = 2, \lambda = 0.5, b_0 = 35, \alpha = 0.9, K_p = 1, K_i = 0.5$. The bode diagram is in Figure 6. The cut-off frequency and phase margin under different $N_u$ are shown in Table 2.

In PI-GPC, $N_u \leq N$. When $N_u$ is small, the cut-off frequency is high, and the response speed is fast. The phase angle margin hardly changes with the change of $N_u$, which means that $N_u$ has little effect on system stability. From the algorithm principle, the increase of $N_u$ will increase the $G$ matrix dimension, thereby increasing the online computation and reducing real-time performance. Therefore, when choosing $N_u$, we should consider speed and stability.

4.3. The impact of observer bandwidth $\omega_o$ change on system performance

When selecting $\omega_o$ as 2, 5, 10, 15, 20, 25, 30, let $N = 15, N_u = 1, \lambda = 0.5, b_0 = 35, \alpha = 0.9, K_p = 1, K_i = 0.5$. The bode diagram is in Figure 7. The cut-off frequency and phase margin under different $\omega_o$ are shown in Table 3.

For time delay systems, the change of $\omega_o$ has a significant impact on the system response speed and stability. When $\omega_o$ is large, the cut-off frequency is high, and the system response speed is fast. But the phase angle margin is small, so the stability is poor. The accuracy of ESO is related to $\omega_o$. When the $\omega_o$ is larger, the ESO accuracy is higher, and the system performance is better. But the $\omega_o$ increase makes the system input and output more sensitive to noise. Therefore, $\omega_o$ should be limited to a suitable range to ensure observation accuracy.

4.4. The impact of control weighting factor $\lambda$ change on system performance

When selecting $\lambda$ as 0.005, 0.01, 0.05, 0.1, 0.5, 1, 1.1, let $N = 15, N_u = 1, \omega_o = 2, b_0 = 35, \alpha = 0.9, K_p = 1, K_i = 0.5$. The bode diagram is in Figure 8. The cut-off frequency and phase margin under different $\lambda$ are shown in Table 4.

![Figure 6](image-url) Bode diagram when $N_u$ changes.

**Table 2. Cut-off frequency and phase margin under different $N_u$.**

| $N_u$ | Phase margin (°) | Cut-off frequency (rad/s) |
|-------|------------------|--------------------------|
| 1     | 86.2046          | 0.0180                   |
| 2     | 86.5137          | 0.0165                   |
| 4     | 86.9242          | 0.0145                   |
| 6     | 87.1302          | 0.0135                   |
| 8     | 87.3131          | 0.0127                   |
| 10    | 87.3880          | 0.0123                   |
| 12    | 87.4231          | 0.0121                   |

![Figure 7](image-url) The bode diagram and cut-off frequency and phase margin under different $\omega_o$.

![Figure 8](image-url) The bode diagram and cut-off frequency and phase margin under different $\lambda$.
Figure 7. Bode diagram when $\omega_o$ changes.

| $\omega_o$ | Phase margin ($^\circ$) | Cut-off frequency (rad/s) |
|------------|--------------------------|---------------------------|
| 2          | 86.2046                  | 0.0180                    |
| 5          | 79.2888                  | 0.0448                    |
| 10         | 67.8564                  | 0.0885                    |
| 15         | 56.5192                  | 0.1320                    |
| 20         | 47.3283                  | 0.1680                    |
| 25         | 38.9044                  | 0.2027                    |
| 30         | 31.7036                  | 0.2342                    |

Figure 8. Bode diagram when $\lambda$ changes.

| $\lambda$ | Phase margin ($^\circ$) | Cut-off frequency (rad/s) |
|-----------|--------------------------|---------------------------|
| 1.1       | 88.1215                  | 0.0089                    |
| 1         | 87.9491                  | 0.0097                    |
| 0.5       | 86.2046                  | 0.0180                    |
| 0.1       | 77.5431                  | 0.0592                    |
| 0.05      | 74.0385                  | 0.0761                    |
| 0.01      | 67.7534                  | 0.1067                    |
| 0.005     | 66.6076                  | 0.1123                    |
Figure 9. Bode diagram when $\alpha$ changes.

$\lambda$ is used to limit the drastic change of the control increment $\Delta u$ to prevent a significant impact on the plant. It can be seen from Figure 8 that as $\lambda$ decreases, the cut-off frequency increases, and the response speed becomes faster. But at the same time, the phase angle margin is increased, and the stability is improved. Therefore, we select smaller $\lambda$.

Table 5. Cut-off frequency and phase margin under different $\alpha$.

| $\alpha$ | Phase margin (°) | Cut-off frequency (rad/s) |
|----------|------------------|---------------------------|
| 0.3      | 83.2850          | 0.0275                    |
| 0.4      | 83.3056          | 0.0275                    |
| 0.5      | 83.3515          | 0.0273                    |
| 0.6      | 83.3392          | 0.0275                    |
| 0.7      | 83.6998          | 0.0262                    |
| 0.8      | 84.3547          | 0.0240                    |
| 0.9      | 86.2046          | 0.0180                    |

Figure 10. Bode diagram when $b_0$ changes.
4.5. The impact of control softening factor $\alpha$ change on system performance

When selecting $\alpha$ as 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, let $N = 15, N_u = 1, \omega_o = 2, \lambda = 0.5, b_0 = 35, K_p = 1, K_i = 0.5$. The bode diagram is in Figure 9. The cut-off frequency and phase margin under different $\alpha$ are shown in Table 5.

It can be seen from Figure 9 that when $\alpha \leq 0.5$, with the increase of $\alpha$, the cut-off frequency and phase angle margin of the system are almost unchanged. When $\alpha \geq 0.5$, with the increase of $\alpha$, the cut-off frequency decreases, and the response speed becomes slower, but the phase angle margin increases and the stability improves. Therefore, to ensure system stability, $\alpha$ should choose a larger value, which is sufficiently close to one.

4.6. The impact of $b_0$ change on system performance

When selecting $b_0$ as 10, 15, 20, 25, 30, 35, 40, let $N = 15, N_u = 1, \omega_o = 2, \lambda = 0.5, \alpha = 0.9, K_p = 1, K_i = 0.5$. The bode diagram is in Figure 10. The cut-off frequency and phase margin under different $b_0$ are shown in Table 6.

The increase of $b_0$ reduces the system cut-off frequency and slows down the response speed. However, the phase angle margin is increased, and the stability of the system is improved. Therefore, we should consider $b_0$ comprehensively to meet the requirements of response speed and stable control.

4.7. The impact of $K_p$ change on system performance

When selecting $K_p$ as 0.02, 0.03, 1, 1.4, 1.5, 6, 7, let $N = 15, N_u = 1, \omega_o = 2, \lambda = 0.5, b_0 = 35, \alpha = 0.9, K_i = 0.5$. The bode diagram is in Figure 11. The cut-off frequency and phase margin under different $K_p$ are shown in Table 7.

For time delay systems, the increase of $K_p$ makes the system response faster, but it has little influence on the system stability. So we should choose the right $K_p$.

| $b_0$ | Phase margin (°) | Cut-off frequency (rad/s) |
|-------|-----------------|--------------------------|
| 10    | 76.5822         | 0.0625                   |
| 15    | 81.0837         | 0.0418                   |
| 20    | 83.3349         | 0.0314                   |
| 25    | 84.5808         | 0.0256                   |
| 30    | 85.5716         | 0.0209                   |
| 35    | 86.2046         | 0.0180                   |
| 40    | 86.6838         | 0.0157                   |

| $K_p$ | Phase margin (°) | Cut-off frequency (rad/s) |
|-------|-----------------|--------------------------|
| 0.02  | 86.2593         | 0.0177                   |
| 0.03  | 86.2587         | 0.0177                   |
| 1     | 86.2046         | 0.0180                   |
| 1.4   | 86.1138         | 0.0184                   |
| 1.5   | 86.1768         | 0.0181                   |
| 6     | 85.9341         | 0.0194                   |
| 7     | 85.8778         | 0.0197                   |

Figure 11. Bode diagram when $K_p$ changes.
4.8. The impact of $K_i$ change on system performance

When selecting $K_i$ as 0.4, 0.5, 0.7, 1, 2, 4, 6, let $N = 15, N_u = 1, \omega_o = 2, \lambda = 0.5, b_0 = 35, \alpha = 0.9, K_p = 1$. The bode diagram is in Figure 12. The cut-off frequency and phase margin under different $K_i$ are shown in Table 8.

The experimental results show that the change of $K_i$ will affect the system phase angle margin and the cut-off frequency at the same time. When $K_i$ is small, the system cut-off frequency is low, and the response speed is slow. The phase angle margin is small, and the stability is poor. So we should choose a larger $K_i$.

5. Second-order time delay system test

We use PI-ADRGPC and ADRC algorithm to control the system (18). The parameters of two methods are shown in Table 9. The system response and performance indicators are shown in Figure 13 and Table 10.

Compared with ADRC, the PI-ADRGPC has faster response. It can reach the set value faster than ADRC.

6. Distillation column system simulation

To test the performance of PI-ADRGPC method, the high-purity distillation column model with the time delay is employed, and the performance is compared with ADRC.

The distillation column has 41 layers, including 39 trays, one reboiler and one condenser (Cheng et al., 2018, 2019, 2020). It is shown in Figure 14.

Table 11 introduces the parameter in the distillation column.

Reboiler: \[ \frac{dM_B}{dt} = \frac{L_i+1 - V_B - B_i}{V_B} \]

\[ \frac{dM_B x_B}{dt} = \frac{L_i+1 x_i+1 - V_B y_i - B x_i}{V_B} \]

where $i = 1, V_1 = V_B, x_1 = x_B, M_1 = M_B$. 

---

**Table 8.** Cut-off frequency and phase margin under different $K_i$.

| $K_i$ | Phase margin ($^\circ$) | Cut-off frequency (rad/s) |
|-------|-------------------------|--------------------------|
| 0.4   | 86.8601                 | 0.0149                   |
| 0.5   | 86.2046                 | 0.0180                   |
| 0.7   | 85.0148                 | 0.0236                   |
| 1     | 83.2441                 | 0.0320                   |
| 2     | 79.6511                 | 0.0491                   |
| 4     | 75.3905                 | 0.0695                   |
| 6     | 73.0642                 | 0.0807                   |

**Table 9.** Parameters of two methods.

|       | $N$ | $N_u$ | $\lambda$ | $\alpha$ | $\omega_o$ | $b_0$ | $K_p$ | $K_i$ | $\alpha_c$ |
|-------|-----|-------|------------|----------|------------|-------|-------|-------|------------|
| PI-ADRGPC | 20  | 10    | 0.01       | 0.9      | 3          | 6.5   | 1     | 15    | –          |
| ADRC   | –   | –     | –          | –        | –          | 8     | –     | –     | 0.5        |

Note: The ‘–’ represents the parameter that does not exist in PI-ADRGPC or ADRC.

The Integrated product of Time and Absolute Error (ITAE) and Integrated Absolute Error (IAE) of PI-ADRGPC are smaller. PI-ADRGPC has better performance on controlling second-order time delay system.
Figure 13. The response of two methods.

Table 10. Performance index of two methods.

| Method       | ITAE<sup>a</sup> | IAE<sup>b</sup> |
|--------------|-------------------|------------------|
| PI-ADRGPC    | 851.4832          | 109.7514         |
| ADRC         | 1860.9788         | 171.8145         |

<sup>a</sup>ITAE: Integrated product of time and absolute error. <sup>b</sup>IAE: Integrated absolute error.

Figure 14. Distillation column structure.

Condenser:

\[
\begin{align*}
\frac{dM_D}{dt} &= V_{i-1} - L_T - D, \\
\frac{d(M_Dx_D)}{dt} &= V_{i-1}y_{i-1} - L_Tx_i - Dx_i,
\end{align*}
\]

Feed layer:

\[
\begin{align*}
\frac{dM_i}{dt} &= L_{i+1} - L_i + V_{i-1} - V_i + F, \\
\frac{d(M_i x_i)}{dt} &= L_{i+1}x_{i+1} - L_ix_i + V_{i-1}y_{i-1} - V_iy_i + Fz_F,
\end{align*}
\]

where \( i = nt, L_{nt} = L_T, x_{nt} = x_D, M_{nt} = M_D \).

Other trays:

\[
\begin{align*}
\frac{dM_i}{dt} &= L_{i+1} - L_i + V_{i-1} - V_i, \\
\frac{d(M_i x_i)}{dt} &= L_{i+1}x_{i+1} - L_ix_i + V_{i-1}y_{i-1} - V_iy_i,
\end{align*}
\]

where \( i = 2, 3, \ldots, nt - 1, nt + 1, \ldots, n - 1 \).

Gas-liquid equilibrium equation:

\[ y_i = \frac{\alpha x_i}{1 + (\alpha - 1)x_i}. \]
Liquid flow equation:

\[ L_i = L_{0,i} + \frac{(M_i - M_{0,i})}{\tau} + \lambda(V_i - V_{i-1}), \]

\[ L_{0,i} = \begin{cases} L_{0,T}, & i > N_F, \\ L_{0,T} + q_F \cdot F, & i \leq N_F, \end{cases} \]

where \( L_{0,i} \) is the initial liquid phase velocity of the \( i \)th tray. \( M_{0,i} \) is the initial liquid phase content. \( L_{0,T} \) is the initial reflux \( L_T \). \( V_{0,i} \) is the initial steam flow rate \( V_i \).

Gas flow velocity equation:

\[ V_i = \begin{cases} V_B, & i < N_F, \\ V_B + (1 - q_F)F, & i \geq N_F, \end{cases} \]

where \( i \) is the number of layers.

Table 12. PI-ADRGPC method parameters.

| Parameters | \( x_B \) | \( x_D \) |
|------------|-----------|-----------|
| \( N \)    | 298       | 252       |
| \( N_u \)  | 48        | 30        |
| \( \lambda \) | 0.9951    | 0.9135    |
| \( \alpha \) | 19.212    | 13        |
| \( w_0 \)  | -0.0117   | 0.0153    |
| \( b_0 \)  | 3.0945    | 8.78      |
| \( K_i \)  | 1         | 1         |

Table 13. ADRC method parameters.

| Parameters | \( x_B \) | \( x_D \) |
|------------|-----------|-----------|
| \( w_D \) | 0.5378    | 1.9762    |
| \( w_D \) | 24.9108   | 26        |
| \( b_0 \) | -0.1038   | 1.017     |

At \( t = 10 \) min, the set value of \( x_D \) increases from 0.99 to 0.995. At \( t = 150 \) min, the set value of \( x_B \) changes from 0.01 to 0.005. Two input channels \( L_T \) and \( V_B \) has a time delay \( \tau = 1.2 \) min, and the parameters of two methods are shown in Tables 12 and 13.

Figure 15 is the output, and Table 14 are the performance indicators.

The response speed of the PI-ADRGPC algorithm is fast, and the shake of the output is small. The ITAE and IAE of PI-ADRGPC are small. For the distillation process, the performance of the proposed algorithm is better than the ADRC algorithm.

7. Conclusion

The PI-ADRGPC does not need to identify system parameters. Thus it can avoid parameter identification errors on algorithm performance. The PI-ADRGPC can overcome the limitations of the ADRC method in the large time delay system, and the Diophantine Equation can be solved offline. Compared with the PI-GPC algorithm, the real-time performance is improved.

Table 14. Performance Index of the PI-ADRGPC and ADRC method.

| Control algorithm | \( x_B \) | \( x_D \) | \( x_B \) | \( x_D \) |
|-------------------|-----------|-----------|-----------|-----------|
| PI-ADRGPC         | 5941.15   | 232.6404  | 77.73     | 8.6801    |
| ADRC              | 6079.45   | 289.6502  | 78.84     | 9.4091    |

Figure 15. The response of the PI-ADRGPC and ADRC method.
This paper derives the closed-loop feedback structure of the PI-ADRGPC algorithm. By analysing the frequency domain characteristics, we studied the influence of the parameters on the algorithm performance and obtained the parameter adjustment method. We compared the PI-ADRGPC and ADRC algorithm performance through the application in a second-order time delay system and distillation column system. The research results show that compared with ADRC, the PI-ADRGPC algorithm has a faster response speed and better performance.

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Data availability statement

Data sharing is not applicable to this article as no new data were created or analysed in this study.

Disclosure statement

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