An Improved Network Motif Detection Tool

Luis A. A. Meira Vinicius R. Máximo Alvaro L. Fazenda
Arlindo F. da Conceição

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Abstract

Network motif provides a way to uncover the basic building blocks of most complex networks. This task usually demands high computer processing, specially for motif with 5 or more vertices. This paper presents an extended methodology with the following features: (i) search for motifs up to 6 vertices, (ii) multithread processing, and a (iii) new enumeration algorithm with lower complexity. The algorithm to compute motifs solve isomorphism in \(O(1)\) with the use of hash table. Concurrent threads evaluates distinct graphs. The enumeration algorithm has smaller computational complexity. The experiments shows better performance with respect to other methods available in literature, allowing bioinformatic researchers to efficiently identify motifs of size 3, 4, 5, and 6.

1 Introduction

Network Motifs, or simply motifs, correspond to small patterns that recurrently appear in a complex network [2]. They can be considered as the basic building blocks of complex networks and their understanding may be of interest in several areas, such as Bioinformatics [11], Communication [26], and Software Engineering [10].

Finding network motifs has been a matter of attention mainly after the 2002-seminal paper from Milo et al. [16], that proposed motifs as a way to uncover the structural design of complex networks. Nowadays, the design of efficient algorithms for network motif discovery is an up-to-date research area. Several surveys about motif detection algorithms were published in recent years [5,21,25].

Nowadays, the main tools available to network motif search are: Fanmod [24], Kavosh [7], NetMod [9] and acc-Motif [13]. Table 1 compares these tools, including information about the
motif size and the usage of Nauty\footnote{Nauty is an algorithm to isomorphism detection \cite{12}}. All these different methods share some features: uses direct graphs, perform a complete motif search, i.e., check for all possible isomorphic patterns and uses induced subgraphs. It is also possible to find other tools which can search for only one isomorphic pattern \cite{20}, or that deal with non induced subgraphs \cite{4}.

Table 1: Main features for the best well known motif search methods.

| Algorithm  | Motif size | Counting        | Parallel | Uses Nauty |
|------------|------------|----------------|----------|------------|
| FanMod     | 8          | Exact / Sampling | no       | yes        |
| Kavosh     | 12         | Exact          | no       | yes        |
| NetMod     | 6          | Exact          | yes      | no         |
| acc-Motif  | 6          | Exact          | yes      | no         |

This paper address the following problem (See \cite{13}):

**Problem 1 (Motifs-k(G))** Given a directed graph $G(V,E)$, the problem Motifs-k consists in counting the number of connected induced subgraphs of $G$ of size $k$ grouped by isomorphic distinct subgraphs of size $k$. The result is an histogram $H_k(G)$.

We deal with the problem for $k$ (vertices in a subgraph) equal to 3, 4, 5, and 6. It is important to notice the number of isomorphic distinct subgraphs of size 3, 4, 5, and 6 is 13, 199, 9,364 and 1,530,842, respectively. For $k = 7$ there are 880,471,142 distinct isomorphic patterns.

The algorithms for motif detection can be based into two main approaches: exact counting or heuristic sampling. As these names might suggest, the former approach performs a precise count of the isomorphic pattern frequency. The latter uses statistics to estimate frequency value. Several exact search-based algorithms and tools can be found in the literature, such as acc-Motif \cite{13,15}, NetMode \cite{9}, MAVisto \cite{22}, NeMoFinder \cite{4}, Kavosh \cite{7} and Grochow and Kellis \cite{6}. Sampling based algorithms examples are MFinder \cite{8,17}, Fanmod \cite{24} and MODA \cite{18}.

Exact algorithms to find network motifs are generally extremely costly in terms of CPU time and memory consumption, and present restrictions on the size of motifs \cite{7}. According to Ciriello and Guerra \cite{5}, motif algorithms typically consist of three steps: (a) list connected subgraphs of $k$ vertices in the original graph and in a set of randomized graphs; (b) group them into isomorphic classes; and (c) determine the statistical signif-
ificance of the isomorphic subgraph classes by comparing their frequencies to those of an ensemble of random graphs. The core of this paper focus in items (a) and (b).

In 2012, we proposed optimized methods for motif detection of size 3, 4 and 5 \cite{13,15}. This paper describes an extension for acc-Motif including motif detection for k=6, multithread and a smaller complexity in enumeration algorithm.

**Contribution:** We’ve made an algorithm for detecting motifs faster than the state of the art for motifs of size 3 up to 6.

## 2 Notation and definitions

This work is an extension of \cite{14}. In this way, more attention will be given to the improvements made. The reader can refer to the previous work for more details about the algorithms.

Let $G(V,E)$ be a directed graph with $n = |V(G)|$ vertices and $m = |E(G)|$ edges. Assume that $m \geq n-1$. If $(u,v) \in E(G)$ and $(v,u) \in E(G)$, we say it is a bidirected edge. Alternatively, if only $(u,v) \in E(G)$, we say it is a directed edge.

Given a vertex $v \in V$, we partitioned the graph in four disjoint sets: $\mathcal{A}(v)$, $\mathcal{B}(v)$, $\mathcal{C}(v)$ and $\mathcal{N}(v)$, as follows:

\[
\begin{align*}
    u \in \begin{cases} 
        \mathcal{A}(v), & \text{if } (u,v) \in E(G) \text{ and } (v,u) \in E(G) \\
        \mathcal{B}(v), & \text{if } (v,u) \in E(G) \text{ and } (u,v) \notin E(G) \\
        \mathcal{C}(v), & \text{if } (u,v) \in E(G) \text{ and } (v,u) \notin E(G) \\
        \mathcal{N}(v), & \text{if } (u,v) \notin E(G) \text{ and } (v,u) \notin E(G)
    \end{cases}
\end{align*}
\]

It means that $\mathcal{A}(v)$ are the vertices with a bidirected edge to $v$. The vertices with edges directed from $v$ are in $\mathcal{B}(v)$ and the vertices with edges directed to $v$ are in $\mathcal{C}(v)$. The set $\mathcal{N}(v)$ represent the vertices with no relationship with vertex $v$.

Let us define $\delta(v) = \mathcal{A}(v) \cup \mathcal{B}(v) \cup \mathcal{C}(v)$. We define the adjacency of a set of vertices $\mathcal{X} \subseteq V$ as $\text{adj}(\mathcal{X}) = \{ \cup_{v \in \mathcal{X}} \delta(v) \} \setminus \mathcal{X}$. The induced graph $G[\mathcal{X}]$ is connected in this context.

Let $\text{Part}(\text{adj}(\mathcal{X}))$ be a partition of $\text{adj}(\mathcal{X})$, for $|\mathcal{X}| \leq 4$, defined as follow. If $\mathcal{X}$ is a single vertex $v \in V$, then $\text{Part}(\text{adj}(v)) = \{ \mathcal{A}(v), \mathcal{B}(v), \mathcal{C}(v) \}$ as shown in Figure 1. Note that, by definition, $\mathcal{N}(v)$ does not belongs to $\text{adj}(v)$.

For each pair of vertices $\{v_1, v_2\}$, the set $\text{adj}(v_1, v_2)$ is partitioned in $\text{Part}(\text{adj}(v_1, v_2)) = \{ \mathcal{A}(v_1, v_2), \mathcal{B}(v_1, v_2), \mathcal{C}(v_1, v_2), \mathcal{N}(v_1, v_2) \}$.
Figure 1: Four sets definition representing a possible relationship with vertex $v$.

$\{AA, AB, AC, AN, BA, BB, BC, BN, CA, CB, CC, CN, NA, NB, NC\}$, where $AA = A(v_1) \cap A(v_2)$, $AB = A(v_1) \cap B(v_2)$ and so on. The size of $|Part(adj(v_1, v_2))| = 4^2 - 1 = 15$.

Note that $N(v_1) \cap N(v_2)$ is the only possible combination that does not belongs to the set $adj(v_1, v_2)$. See Table 1 of [14].

For sets containing three vertices $\{v_1, v_2, v_3\}$, $Part(adj(v_1, v_2, v_3)) = \{AAA, AAB, AAC, AAN, ABA, ..., NN B, NNC\}$ where $AAA = A(v_1) \cap A(v_2) \cap A(v_3)$, $AAB = A(v_1) \cap A(v_2) \cap B(v_3)$ and so on. The size of $|Part(adj(v_1, v_2, v_3))| = 4^3 - 1 = 63$ because the set $N(v_1) \cap N(v_2) \cap N(v_3)$ does not belongs to the adjacency.

For sets $\{v_1, v_2, v_3, v_4\}$, $Part(adj(v_1, v_2, v_3, v_4)) = \{AAAA, AAAB, AAAC, AAAN, AABA, ..., NNNB, NNNC\}$, where $AAAA = A(v_1) \cap A(v_2) \cap A(v_3) \cap A(v_4)$, $AAAB = A(v_1) \cap A(v_2) \cap A(v_3) \cap B(v_4)$ and so on. The size $|Part(adj(v_1, v_2, v_3))|$ is $4^4 - 1 = 255$, since $N(v_1) \cap N(v_2) \cap N(v_3) \cap N(v_4)$ dont belong to $adj(v_1, v_2, v_3)$.

Given a graph $G$, a subset of vertices $\mathcal{X}$, with $|\mathcal{X}| \leq 4$, the partition set $Part(adj(\mathcal{X}))$, and two sets $Y,Z \in Part(adj(\mathcal{X}))$. Suppose there are no edge in $adj(\mathcal{X})$. The triple $\mathcal{X},Y,Z$ corresponds to a motif, named $motif(\mathcal{X},Y,Z)$. In other words, the subgraph induced $G[\mathcal{X} \cup y \cup z]$ corresponds to same isomorphic graph of size $|\mathcal{X}|+2$ for any $y \in Y$ and $z \in Z$. See an example (Figure 2).

Let $\mathcal{P}_k$ be the maximal set of distinct isomorphic connected subgraphs size $k$. For example, $\mathcal{P}_1 = \{\bullet\}$ has only one vertex, $\mathcal{P}_2 = \{\bullet \rightarrow \bullet, \bullet \leftrightarrow \bullet\}$, $\mathcal{P}_3$ is a set with 13 subgraphs and so on.

The histogram $\mathcal{H}_k$ as described in Problem 1 is a function $\mathcal{H}_k: \mathcal{P}_k \rightarrow \mathbb{N}$ associating the pattern $p \in \mathcal{P}_k$ to the occurrences number in $G(V,E)$. To a graph $G(V,E)$, $\mathcal{L}_r(G)$ be the
The motif \( \mathcal{X}, \mathcal{Y}, \mathcal{Z} \) for \( \mathcal{X} = v, \mathcal{Y} = \mathcal{A}(v) \) e \( \mathcal{Z} = \mathcal{B}(v) \).

set of all induced connected subgraphs from \( G \) with size \( r \). See an example in Figure 3.

\[ L_1 = \{ G[v_1], G[v_2], G[v_3], G[v_4] \} \]
\[ L_2 = \{ G[v_1, v_4], G[v_1, v_2], G[v_2, v_4], G[v_1, v_3] \} \]
\[ L_3 = \{ G[v_1, v_2, v_3], G[v_1, v_2, v_4], G[v_1, v_3, v_4] \} \]

Note that \( G[v_2, v_3, v_4] \) is not connected thus, by definition, not belong to \( L_3 \)

Figure 3: Example of \( L_r \) for a give graph.

3 Computing the Histogram \( \mathcal{H}_k \)

The work [14] solve Motifs-k(G) using \( L_{k-2} \) as a base. The set \( L_{k-2} \) contains all connected induced subgraphs with size \( k - 2 \). The histogram \( \mathcal{H}_k \) contains the frequency of elements in \( L_k \) grouped by isomorphic pattern. Thus, the algorithm to compute the histogram has complexity \( \Omega(|L_{k-2}|) \), since all elements in \( L_{k-2} \) are considered. Furthermore, complexity upper bounded by \( o(|L_k|) \), due the fact the algorithm computes the frequency for several induced subgraph in a constant time.

The diagram in Figure 4 shows the inputs and outputs in the enumeration procedure for motif of size \( k \) in acc-Motif method.

Algorithm [1] presents the motif counting process. First, it generates all induced subgraph of size \( k - 2 \). For \( k = 3 \), \( L_{3-2} = \{ G[v] \ | \ \forall v \in V \} \). For \( k = 4 \), \( L_{4-2} = \{ G[u, v] \ | \ \forall (u, v) \in \}

\[ \mathcal{B}(v) \] \( v \)
\[ \mathcal{A}(v) \]
Figure 4: The acc-Motif algorithm. Input: $G$, $\mathcal{L}_{k-2}$ and precomputed variables. Output: $\mathcal{H}_k$.

For $k = 5$, the set $\mathcal{L}_{5-2}$ was shown in [13]. For $k = 6$, the algorithm to compute $\mathcal{L}_4$ is adapted from [7]. The execution time to compute $\mathcal{L}_{k-2}$ is negligible in relation to the total time.

**Input:** Graph $G(V,E)$ and an integer $k$

**Output:** $\mathcal{H}_k$, that is the histogram for motifs of size $k$

1. Compute $\mathcal{L}_{k-2}$.
2. **foreach** subgraph $G[\mathcal{X}] \in \mathcal{L}_{k-2}$ **do**
   3. Compute $\text{Part}(adj(\mathcal{X}))$
   4. **foreach** $Y \in \text{Part}(adj(\mathcal{X}))$ **do**
      5. **foreach** $Z \in \text{Part}(adj(\mathcal{X}))$ **do**
         6. **if** $Y = Z$ **then**
            7. Increment in $\mathcal{H}_k$ the motif $\text{motif}(\mathcal{X},Y,Y)$ with $\left\lceil \frac{|Y|}{2} \right\rceil$ units
         8. **end**
         9. **else**
            10. Increment in $\mathcal{H}_k$ the motif $\text{motif}(\mathcal{X},Y,Z)$ with $|Y| \cdot |Z|$ units
         11. **end**
     12. **end**
   13. **end**

14. **foreach** $(u,v) \in G[adj(\mathcal{X})]$ **do**
   15. Add one unit to motif $G[\mathcal{X} \cup u \cup v]$
   16. Subtract one unit from motif $\text{motif}(\mathcal{X},Y,Z)$ such that $u \in Y \in \text{Part}(adj(\mathcal{X}))$ and $v \in Z \in \text{Part}(adj(\mathcal{X}))$.
17. **end**
18. **end**

**Algorithm 1:** Enumerate all subgraphs of size $k$.

The first difference between the original algorithm [14] and this is that, in [14], each edge $(u,v) \in adj(\mathcal{X})$ increments once the variable $m_{YZ}$ for all $Y,Z \in \text{Part}(adj(\mathcal{X}))$. In this solution, each edge uses $\mathcal{H}_k$ directly, without to create a counter $m_{YZ}$.

Note that for a constant $k$, the size $|\text{Part}(adj(\mathcal{X}))|$ and the number of variables $m_{xy}$ are constant, not affecting the complexity of acc-Motif. However, the constants affect the final execution time.

Another difference is an improved algorithm to compute the edges in an induced graphs as described in Section 3.1.
3.1 Computing Induced Subgraph $G[S]$ efficiently

Given an oriented graph $G(V,E)$, with $n = |V|$ and $m = |E|$. Given a set of vertices $S$, let $E_S$ be the set of edges of $G[S]$. In this section we consider only the induced graph $G[S]$. This section shows algorithms to obtain $E_S$ efficiently. For this, it is assumed that the neighbors of $v$, $\delta(v)$, are already precomputed.

Several strategies are combined to produce an efficient algorithm to find the edges $E_S$. Let $d(v) = |\delta(v)|$ be the degree of $v$ and $D(S) = \sum_{v \in S} d(v)$ the sum of the degrees of the vertices in $S$.

**Algorithm 2:** $\theta(|S|^2)$.  
**Algorithm 3:** $\theta(|S|D(S))$.  
**Algorithm 4:** $\theta(\min\{|S|^2, |S|D(S)|\})$.

The first strategy consists of analyzing each $u,v \in S$, resulting in a complexity $|S|^2$. This strategy is efficient for small $|S|$. If, for example, $|S|$ is a constant, the algorithm will have complexity $O(1)$. The complexity of this algorithm regardless of whether the graph is dense or sparse.

The second strategy consists of analyzing the neighbors of $v$ for all $v \in S$. This strategy is efficient if the graph is sparse. For example, if each vertex $v \in S$ has degree limited by a constant, the complexity will be $O(|S|)$, even for large $S$.

The third strategy choose the most efficient algorithm between Algorithm 2 and 3 according to the values of $|S|$ and $D(S)$. The resulting complexity is the minimum between the two complexity.

The fourth strategy make a partition from $S$ into $S_1$ and $S_2$. The partition is made in such a way that vertices with smaller degree are in $S_1$ and those of greater degree are in $S_2$ (Algorithm 5).

In this strategy, every pair $u,v \in S_2$ is checked, with complexity $O(|S_2|^2)$. For the lowest degree vertices, all neighbors of $d(s)$, for all $s \in S_1$, are checked. Note that the edges in $\delta(S_1,S_2)$ are computed by sweeping the neighbors of $S_1$.

Suppose that $G(V,E)$ is a sparse graph where $Hub \subseteq V$ such that $v \in Hub$ if and only
Sort $S$ in $S' = (s'_1, \ldots, s'_k)$ such that the degree is increasing.
Let $p$ be the value that minimizes the function $\sum_{i=1}^{p} d(s_i) + (|S| - p)^2$.
Let $S_1 = (s'_1, \ldots, s'_p)$ be the $p$ vertices of smaller degree.
Let $S_2 = (s'_{p+1}, \ldots, s'_k)$ be the $k-p$ vertices of greater degree.

```plaintext
foreach $u \in S_2$ do
  for $v \in S_2$ do
    if $(u, v) \in E$ then
      $E_S \leftarrow E_S \cup (u, v)$
    end
  end
end
foreach $u \in S_1$ do
  for $v \in \delta(u)$ do
    if $v \in S$ then
      $E_S \leftarrow E_S \cup \{u, v\}$
    end
  end
end
```

Algorithm 5: Fourth strategy is $\Theta(\min_p \{|S| - p\}^2 + \sum_{i=1}^{p} d(s_i))$, where $s_i$ is sorted according to vertices degrees.

| Algorithm | Complexity |
|-----------|------------|
| Algorithm 2 $(G, S)$ | $\Theta(k^2)$ with worst case $\Theta(n^2)$ |
| Algorithm 3 $(G, S)$ | $\Theta(D(S))$ with worst case $\Theta(m)$ |
| Algorithm 4 $(G, S)$ | $\Theta(\min\{D(S), k^2\})$ with worst case $\Theta(\min\{m, n^2\})$ |
| Algorithm 5 $(G, S)$ | $\Theta(k)$ |

Table 2: Complexity of the calculation of $G[S]$ for graphs of celebrity $|S| = k$.

If $d(v) \notin O(1)$. $Hub$ is a set of vertices whose degree is not limited to $O(1)$. Assume that $|Hub| \in O(1)$, thus the number of vertices with degree not limited to $O(1)$ is constant.
Graphs in complex networks tend to respect the above conditions. Let’s call this graph class of celebrities.

Suppose the execution of a celebrity graph by Algorithm 5 $(G, S)$. The vertices in $Hub$ have degree greater than the vertices and $V \setminus Hub$. There is a $p$, not necessarily a minimum, such that $S_1 = V \setminus Hub$ and $S_2 = Hub$. For this $p$ we have that $(k-p)^2 + \sum_{i=1}^{p} d(s_i) = |Hub|^2 + \sum_{s \in V \setminus Hub} d(s) = \Theta(S)$. Thus, the complexity for calculating $G[S]$ for celebrity graphs is $O(|S|)$. 

8
3.2 Calculating Isomorphism in $O(1)$

The use of hash to calculate isomorphisms in $O(1)$ is not new \cite{9,14}. However, there are challenges to working with motifs of size 6, due to the large number of different isomorphic patterns.

The main challenge of acc-Motif for $k = 6$ is to compute isomorphism in $O(1)$, it is done by pre-processing. Given a oriented graph $G(V,E)$ of size 6, we need to compute the isomorphic pattern representation in a hash table. Let $\mathcal{A}_6$ be the set of all adjacency matrix of size 6 of $G$. The size of $\mathcal{A}_6$ is $2^{30}$ (≈ 1 billion possibilities). A hash table was generated where the key is an adjacency matrix $A \in \mathcal{A}_6$ and the value is a number $ID \in \{1, 2, \ldots, 1530842\}$ where $ID$ represents a motif. In other words, a function $ISO : \mathcal{A}_6 \rightarrow ID$. After computing this table, it is possible to solve isomorphism in $O(1)$ for any adjacency matrix.

4 Results

This section show the results of empirical evaluations. We compare acc-Motif with other tools present in the literature. The graphs evaluated were the same used in \cite{14}, Table 3 summarizes the data sets used in the experiments.

All experiments were performed using a processor IBM Power 755, of 3.3 GHz. The first experiment consider only one thread running.

In this experiment we compared the performance of acc-Motif to Fanmode \cite{23}, Kavosh \cite{7} and NetMode \cite{9} by varying $k \in \{3, 4, 5, 6\}$ and using the graphs described in the Table 3. In this experiment we used only one thread and the execution time was reported in milliseconds. The result of this experiment is presented in Table 4.

Table 3: Summary of the data sets used in the experiments.

| Graph        | n   | m   |
|--------------|-----|-----|
| E.coli       | 418 | 519 |
| Levedura     | 688 | 1079|
| CSplad       | 1882| 1740|
| Roget        | 1022| 5074|
| Epa          | 4271| 8965|
| California   | 6175| 16150|
| Facebook     | 1899| 20296|
| ODLIS        | 2900| 18241|
| Grafo | $k$ | FanMod [24] | Kavosh [7] | NetMod [9] | acc-Motif |
|-------|-----|-------------|-------------|-------------|-----------|
| E.coli | 3   | 22,600      | 4,940       | **0.298**   | 0.372     |
|       | 4   | 319,139     | 118,416     | 3.525       | **2.710** |
|       | 5   | 7,726,250   | 3,400,500   | 206,188     | **82.547**|
|       | 6   | 164,933,500 | 76,107,000  | 5,072,250   | **1,892,261** |
| Levedura | 3   | 69,420      | 11,600      | **0.635**   | 0.742     |
|       | 4   | 980,733     | 262,050     | 8.208       | **4.075** |
|       | 5   | 21,336,313  | 7,278,625   | 268,625     | **119.602** |
|       | 6   | 408,474,250 | 133,385,000 | 8,056,500   | **2,646,239** |
| CSphd  | 3   | 34,287      | 8,971       | 2.463       | **1.727** |
|       | 4   | 166,792     | 53,475      | **2.634**   | 3.211     |
|       | 5   | 1,751,313   | 718,688     | 160,125     | **43.984** |
|       | 6   | 19,705,000  | 8,225,500   | 1,102,000   | **1,002,137** |
| Roget  | 3   | 164,321     | 24,774      | 6.550       | **3.933** |
|       | 4   | 1,727,228   | 380,396     | 21,406      | **16.509** |
|       | 5   | 32,436,375  | 9,524,438   | 510,375     | **329.866** |
|       | 6   | 647,152,750 | 215,113,000 | 28,856,000  | **6,450,374** |
| Epa    | 3   | 808,695     | 193,472     | **10.155**  | 14.261    |
|       | 4   | 41,931,931  | 13,008,337  | 390,139     | **110.174** |
|       | 5   | 2,688,902,875 | 1,029,417,875 | 16,457,875 | **6,239,431** |
|       | 6   | 151,426,226,500 | 55,472,966,500 | 2,548,300,750 | **364,114,721** |
| California | 3   | 1,508,350   | 300,445     | 40,313      | **33.707** |
|       | 4   | 64,086,535  | 17,245,257  | 602,535     | **263.317** |
|       | 5   | 4,105,529,375 | 1,490,712,188 | 29,181,625 | **9,188,792** |
|       | 6   | 259,409,634,500 | 111,734,296,500 | 5,743,212,000 | **566,674,477** |
| Facebook | 3   | 3,749,735   | 471,141     | 25,506      | **21.001** |
|       | 4   | 310,491,881 | 46,347,327  | 2,056,158   | **597.636** |
|       | 5   | 30,622,129,500 | 5,300,279,938 | 180,014,250 | **67,657,011** |
|       | 6   | > 648 $\times$ 10^6 | 604,958,138,500 | 68,980,139,250 | **9,122,144,244** |
| ODLIS  | 3   | 8,674,493   | 848,033     | 44,655      | **26.432** |
|       | 4   | 1,384,241,099 | 187,599,673 | 5,251,980   | **774.438** |
|       | 5   | > 162 $\times$ 10^6 | 54,086,528,125 | 756,913,375 | **146,503,080** |
|       | 6   | > 648 $\times$ 10^6 | > 648 $\times$ 10^6 | 454,180,808,500 | **28,835,680.726** |

Figure 5 shows the execution time obtained in Experiment I for the graph California. It is possible to verify that acc-Motif have performed better than the other algorithms. This result highlights the computational gain obtained by acc-Motif in relation to the best algorithms present in the literature.

According to the results presented in Table 4, the acc-Motif algorithm presented a performance inferior to NetMod only for instances whose computational cost is small. For larger instances, acc-Motif was superior to the other algorithms.

Motifs Detection consists in enumerating the isomorphic patterns of induced subgraphs in the original graph and in a group of random graphs. It is a paralleling problem, since each induced subgraph can be treated by a separate thread. This version of acc-Motifs multi-threaded. See the performance gain in tables 6 and 5.
Figure 5: Comparison between Fanmode, Kavosh, NetMode, acc-Motif by varying $k \in \{3, 4, 5, 6\}$ using the California graph.

Figure 6: Experiment varying the number of threads for $k = 6$, with 511 random graphs for the graph Roget.

### 5 Conclusion

In this work we present a tool to detect motifs of size up to 6. Computational experiments show that acc-Motifs the fastest tool compared to algorithms available in the literature.
We have proposed an efficient algorithm for calculating induced subgraphs. Finally, a multi-threaded version of the program was generated.

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