Kinetic energy driven superconductivity in doped cuprates

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Within the $t$-$J$ model, the mechanism of superconductivity in doped cuprates is studied based on the partial charge-spin separation fermion-spin theory. It is shown that dressed holons interact occurring directly through the kinetic energy by exchanging dressed spinon excitations, leading to a net attractive force between dressed holons, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the superconducting ground state. The electron superconducting transition temperature is determined by the dressed holon pair transition temperature, and is proportional to the concentration of doped holes in the underdoped regime. With the common form of the electron Cooper pair, we also show that there is a coexistence of the electron Cooper pair and antiferromagnetic short-range correlation, and hence the antiferromagnetic short-range fluctuation can persist into the superconducting state. Our results are qualitatively consistent with experiments.

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Since the discovery of high-temperature superconductivity (HTSC) in doped cuprates, much effort has concentrated on the superconducting (SC) mechanism. Much experimental evidence, including the factor of 2e occurring in the flux quantum and in the Josephson effect, as well as the electrodynamic and thermodynamic properties, supports the pairing theory. The single common feature of cuprate superconductors is the presence of the two-dimensional (2D) CuO$_2$ plane, then it is believed that the relatively high SC transition temperature $T_c$ is closely related to doped CuO$_2$ planes. The undoped state of cuprate superconductors is a Mott insulator with antiferromagnetic (AF) long-range order (AFLRO), then changing the carrier concentration by ionic substitution or increasing the oxygen content turns these compounds into the SC state leaving the AF short-range correlation still intact. Moreover, the superfluid density in the underdoped regime vanishes more or less linearly with doping, and the SC transition temperature $T_c$ is proportional to a positive power of the concentration of doped holes $\delta$ ($T_c \propto \delta$ in doped CuO$_2$). Therefore there is a general consensus that the HTSC to holes interaction via a magnetic medium, and short-range AF correlation coexists with the SC state.

In conventional superconductors, the electrons interact by exchanging phonons. Since this interaction leads to a net attractive force between electrons, the system can lower its potential energy by forming electron Cooper pairs. These electron Cooper pairs condense into a coherent macroscopic quantum state and can move freely without resistance. As a result, pairing in the conventional superconductors is always related to an increase in kinetic energy which is overcompensated for by the lowering of the potential energy. On the contrary, it has been argued that the SC transition in doped cuprates is determined by the need to reduce the frustrated kinetic energy, where the driving attractive force between holes may be attributed to the fact that by sharing a common link two holes minimize the loss of the energy related to breaking AF links, and is therefore mediated by the exchange of spin excitations. Within the 2D $t$-$J$ model, robust indications of superconductivity have been found by using numerical techniques. Moreover, by solving a model for alkali doped fullerenes within dynamical mean-field (MF) theory, it has been argued recently that the strong electron correlation does not suppress superconductivity, but rather seems to favor it because the main ingredient was identified as a pairing mechanism not involving the charge density operator, but other internal degrees of freedom, like the spin, unveiling a kind of the charge-spin separation. These scenarios are consistent with recent optical experiments. The normal-state above $T_c$ exhibits a number of anomalous properties which are due to charge-spin separation, while the SC state is characterized by charge-spin recombination.

Recently, we have developed a partial charge-spin separation fermion-spin theory to study the physical properties of doped cuprates, where the electron operator is decoupled as the gauge invariant dressed holon and spinon. Based on this theory, we have discussed the unusual normal-state properties of the underdoped cuprates, and the results are good consistent with the experiments. It is shown that the charge transport is mainly governed by the scattering from the dressed holons due to the dressed spinon fluctuation, while the scattering from the dressed spinons due to the dressed holon fluctuation dominates the spin response. In this paper, we apply this approach to discuss the mechanism of HTSC. Within the $t$-$J$ model, we show that dressed holons interact occurring directly through the kinetic energy by exchanging dressed spinon excitations, leading to a net attractive force between dressed holons, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground state. The SC transition temperature is identical to the dressed...
holon pair transition temperature, and is proportional to the concentration of doped holes in the underdoped regime. With the common form of the electron Cooper pair, we also show that there is a coexistence of the electron Cooper pair and AF short-range correlation, and hence the AF short-range fluctuation can persist into the SC state.

We start from the 2D $t$-$J$ model,

$$H = -t \sum_{ij} C^d_{i\sigma} C_{i+\eta\sigma} + \mu \sum_{i\sigma} C^d_{i\sigma} C_{i\sigma} + J \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_{i+\eta},$$

(1)

acting on the Hilbert space with no doubly occupied site, i.e., $\sum_{\sigma} C^d_{i\sigma} C_{i\sigma} \leq 1$, where $\eta = \pm \hat{x}, \pm \hat{y}$, $C^d_{i\sigma} (C_{i\sigma})$ is the electron creation (annihilation) operator, $S_i = C^\dagger_i \Phi C_{i/2}$ is spin operator with $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ as Pauli matrices, and $\mu$ is the chemical potential. In the $t$-$J$ model (1), the strong electron correlation manifests itself by a single occupancy local constraint, and therefore the crucial requirement is to impose this local constraint. It has been shown that this constraint can be treated properly in analytical calculations within the partial charge-spin separation fermion-spin representation, as described in Ref.\textsuperscript{12}, the phase factor $\Phi_{i\sigma}$ is separated from the bare spinon operator, therefore it also is an operator, and describes a spinon cloud. It has been shown that these dressed holons and spinons are gauge invariant, and in this sense, they are real. This dressed holon $h_{i\sigma}$ is a spinless fermion $h_i$ (bare holon) incorporating the spinon cloud $e^{-i\Phi_{i\sigma}}$ (magnetic flux), and is a magnetic dressing. In other words, the gauge-invariant dressed holon carries some spinon messages, i.e., it shares some effects of the spinon configuration rearrangements due to the presence of the hole itself\textsuperscript{13}. Although in common sense $h_{i\sigma}$ is not a real spinful fermion, it behaves like a spinful fermion. The spirit of the partial charge-spin separation fermion-spin theory is that the electron operator can be mapped as a product of the spin operator and spinful fermion operator, this is very similar to those of bosonization in one-dimensional interacting electron systems, where the electron operators are mapped onto the boson (electron density) representation, and then the recast Hamiltonian is exactly solvable. In this partial charge-spin separation fermion-spin representation, the low-energy behavior of the $t$-$J$ model (1) can be expressed as\textsuperscript{12},

$$H = -t \sum_{ij} \langle h_{i\tau} S^+_{i\tau} h^\dagger_{i+\eta\tau} S^-_{i+\eta\tau} + h_{i\tau} S^-_{i\tau} h^\dagger_{i+\eta\tau} S^+_{i+\eta\tau} \rangle + \mu \sum_{i\sigma} h^\dagger_{i\sigma} h_{i\sigma} + J_{\text{eff}} \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_{i+\eta},$$

(3)

with $J_{\text{eff}} = (1 - \delta)^2 J$, and $\delta = \langle h_{i\tau} h_{i\sigma} \rangle = \langle h^\dagger_{i\tau} h^\dagger_{i\sigma} \rangle$ is the hole doping concentration. As a consequence, the kinetic energy ($t$) term in the $t$-$J$ model has been expressed as the dressed holon-spinon interaction, which dominates the essential physics of doped cuprates, while the magnetic energy ($J$) term is only to form an adequate dressed spinon configuration\textsuperscript{7}. The SC state is characterized by electron Cooper pairs, forming SC quasiparticles\textsuperscript{7}, and the order parameter for the electron Cooper pair can be expressed as,

$$\Delta = \langle C^\dagger_{i\downarrow} C^\dagger_{j\uparrow} C_{j\downarrow} C_{i\uparrow} \rangle = \langle h_{i\tau} h_{j\tau} S^+_{i\tau} S^-_{j\tau} - h_{i\tau} h_{j\tau} S^-_{i\tau} S^+_{j\tau} \rangle.$$  

(4)

In the doped regime without AFLRO, the dressed spinons form the disordered spin liquid state, where the dressed spinon correlation function $\langle S^+_{i\tau} S^-_{j\tau} \rangle = \langle S^-_{i\tau} S^+_{j\tau} \rangle$, then the order parameter for the electron Cooper pair in Eq. (4) can be written as $\Delta = -\langle S^+_{i\tau} S^-_{j\tau} \rangle h_{i\tau}$, with the dressed holon pairing order parameter $\Delta_h = \langle h_{i\tau} h_{j\tau} - h_{j\tau} h_{i\tau} \rangle$. In this case, the physical properties of the SC state are essentially determined by the dressed holon pairing state. However, in the extreme low doped regime with AFLRO, where the dressed spinon correlation function $\langle S^+_{i\tau} S^-_{j\tau} \rangle \neq \langle S^-_{i\tau} S^+_{j\tau} \rangle$, the condensate is disrupted by AFLRO. Therefore in the following discussions, we only focus on the case without AFLRO.

The quantum spin operators obey the Pauli spin algebra, and this problem can be discussed in terms of the two-time spin Green’s function within the Tyablikov scheme\textsuperscript{14}. In this case, we define the dressed holon diagonal and off-diagonal Green’s functions as,

$$g(i-j, t-t') = -i\theta(t-t') \langle [h_{i\tau}(t), h^\dagger_{j\tau}(t')] \rangle = \langle [h_{i\tau}(t), h^\dagger_{j\tau}(t')] \rangle,$$

(5a)

$$\mathfrak{A}(i-j, t-t') = -i\theta(t-t') \langle [h_{i\tau}(t), h_{j\tau}(t')] \rangle = \langle [h_{i\tau}(t), h_{j\tau}(t')] \rangle,$$

(5b)

$$\mathfrak{A}^\dagger(i-j, t-t') = -i\theta(t-t') \langle [h^\dagger_{i\tau}(t), h^\dagger_{j\tau}(t')] \rangle = \langle [h^\dagger_{i\tau}(t), h^\dagger_{j\tau}(t')] \rangle,$$

(5c)

and the dressed spinon Green’s functions as,

$$D(i-j, t-t') = -i\theta(t-t') \langle [S^+_{i\tau}(t), S^-_{j\tau}(t')] \rangle = \langle [S^+_{i\tau}(t), S^-_{j\tau}(t')] \rangle,$$

(6a)

$$D^\dagger(i-j, t-t') = -i\theta(t-t') \langle [S^+_{i\tau}(t), S^-_{j\tau}(t')] \rangle = \langle [S^+_{i\tau}(t), S^-_{j\tau}(t')] \rangle,$$

(6b)
respectively, where $\langle \ldots \rangle$ is an average over the ensemble. In the MF level, the dressed spinon system is an anisotropic away from the half-filling\textsuperscript{15}, therefore we have defined the two dressed spinon Green’s functions $D(i - j, t - t')$ and $\Delta_z(i - j, t - t')$ to describe the dressed spinon propagations. In the doped regime without AFLRO, i.e., $\langle S_z^\sigma \rangle = 0$, a MF theory of the $t$-$J$ model based on the fermion-spin theory has been developed\textsuperscript{16} within the Kondo-Yamaji decoupling scheme\textsuperscript{16}, which is a stage one-step further than Tyablikov’s decoupling scheme. In this MF theory\textsuperscript{15}, the phase factor $e^{i\phi_{\sigma,ss}}$ describing the phase part of the spin degree of freedom was not considered. Following their discussions\textsuperscript{15}, we can obtain the MF dressed holon and spinon Green’s functions in the present partial charge-spin separation fermion-spin theoretical framework as,

\begin{equation}
\tag{7a}
g^{(0)}(k) = \frac{1}{i\omega_n - \xi_k},
\end{equation}

\begin{equation}
\tag{7b}
D^{(0)}(p) = \frac{B_p}{(ip_m)^2 - \omega_p^2} = \frac{1}{2} \sum_{\nu=1,2} \frac{B_p}{\omega_{\nu}(p)} \frac{1}{ip_m - \omega_{\nu}(p)},
\end{equation}

\begin{equation}
\tag{7c}
D_z^{(0)}(p) = \frac{B_z(p)}{(ip_m)^2 - \omega_z(p)^2} = \frac{1}{2} \sum_{\nu=1,2} \frac{B_z(p)}{\omega_{\nu}(p)} \frac{1}{ip_m - \omega_{\nu}(p)},
\end{equation}

respectively, where the four-vector notation $k = (k, \omega_n)$, $p = (p, ip_m)$, $B_p = \lambda^2/(\xi + \chi - \epsilon)$, $B_z(p) = \lambda^2/(\chi + \chi - \epsilon)$, $\omega_{\nu}(p) = (1/2Z)\sum_{\eta} \epsilon_{\nu\eta}^p$, $\epsilon = 1 + 2t\phi/\Delta$, $Z$ is the number of the nearest neighbor sites, $\omega_1(p) = \omega_p$, $\omega_2(p) = -\omega_p$, $\omega_{\nu}(p) = \omega_{\nu}(p)$, $\omega_2(p) = -\omega_2(p)$, and the MF dressed holon and spinon excitation spectra are given by,

\begin{equation}
\tag{8a}
\xi_k = Zt\chi \gamma_k - \mu,
\end{equation}

\begin{equation}
\tag{8b}
\omega_p^2 = A_1 \gamma_p^2 + A_2 \gamma_p + A_3,
\end{equation}

\begin{equation}
\tag{8c}
\omega_z(p)^2 = \epsilon \gamma^2(A - \alpha \gamma_p)(1 - \gamma_p),
\end{equation}

respectively, with $A_1 = \alpha \chi^2 + \chi + 2$, $A_2 = -\epsilon \gamma^2[\alpha(\chi^2 + \chi)/2 + (\alpha \gamma^2 + (1 - \alpha)/4Z) - \alpha \chi^2/(2Z)] + (\alpha C + (1 - \alpha)/4Z - \alpha \chi^2/2)], A_3 = \lambda^2/(\chi + \chi - \epsilon)$, $\alpha = \epsilon/(\alpha C + (1 - \alpha)/2Z - \alpha \chi^2/2), A_2 = \epsilon \gamma^2[\alpha C^2 + (1 - \alpha)/4Z - \alpha \chi^2/(2Z)], C = \chi = \langle S_z^+ S_z^- \rangle, C = \langle 1/Z^2 \rangle \sum_{\eta,\eta'} \langle S_{\eta}^z S_{\eta'}^z \rangle \langle S_{\eta}^+ S_{\eta'}^+ \rangle, \text{ and } C^2 = (1/Z^2) \sum_{\eta,\eta'} \langle S_{\eta}^+ S_{\eta'}^+ \rangle \langle S_{\eta}^+ S_{\eta'}^+ \rangle$. In order not to violate the sum rule of the correlation function $(S_z^+ S_z^-) = 1/2$ in the case without AFLRO, the important decoupling parameter $\alpha$ has been introduced in the above MF calculation\textsuperscript{15,16}, which can be regarded as the vertex correction.

In the $t$-$J$ model (3), the dressed holon-spinon coupling occurring in the kinetic energy term is quite strong. This interaction (kinetic energy) can induce the dressed holon pairing state (then the electron pairing state and superconductivity) by exchanging dressed spinon excitations in a higher power of the hole doping concentration $\delta$. For a discussion of this problem, we follow Eliashberg’s strong coupling theory\textsuperscript{17}, and obtain the self-consistent equations in terms of the equation of motion method\textsuperscript{18,12} which is satisfied by the full dressed holon diagonal and off-diagonal Green’s functions as,

\begin{equation}
\tag{9a}
g(k) = g^{(0)}(k) + g^{(0)}(k) \sum (\xi^{(h)}(k)g(k) - \xi^{(h)}(-k)\xi^{(s)}(k)),
\end{equation}

\begin{equation}
\tag{9b}
\xi^{(s)}(k) = g^{(0)}(-k) \sum (\xi^{(h)}(-k)\xi^{(s)}(-k) + \xi^{(h)}(k)g(k)),
\end{equation}

with the self-energies are evaluated as,

\begin{equation}
\tag{10a}
\xi^{(h)}(k) = (Zt)^2 \sum_{p, p'} \gamma^{2} \frac{1}{Z} \sum \frac{1}{ip_m} D(p') D(p' + p),
\end{equation}

\begin{equation}
\tag{10b}
\xi^{(s)}(k) = (Zt)^2 \sum_{p, p'} \gamma^{2} \frac{1}{Z} \sum \frac{1}{ip_m} D(p') D(p' + p).
\end{equation}

The pairing force and dressed holon gap function have been incorporated into the self-energy $\Sigma^{(h)}(k)$, therefore the self-energy $\Sigma^{(h)}(k)$ is called as the effective dressed holon gap function. Moreover, this self-energy $\Sigma^{(h)}(k)$ is an even function of $i\omega_n$, while the other self-energy $\Sigma^{(s)}(k)$ is not. As we\textsuperscript{12,19} have known from the discussion of the normal-state properties, the self-energy $\Sigma^{(h)}(k)$ renormalizes the MF dressed holon spectrum, and therefore it dominates the charge transport of the systems. As a qualitative discussion, we neglect $\Sigma^{(h)}(k)$ in this paper, and only study the static limit of the effective dressed holon gap function, i.e., $\Sigma^{(h)}(k) = \Delta_h(k)$. In this case, we obtain dressed holon diagonal and off-diagonal Green’s functions as,

\begin{equation}
\tag{11a}
g(k) = \frac{i\omega_n + \xi_k}{i\omega_n} - \frac{1}{E^2} \sum_{\nu=1,2} \left(1 + \frac{\xi_k}{E^2} \right) \frac{1}{i\omega_n - E^2},
\end{equation}

\begin{equation}
\tag{11b}
\xi^{(s)}(k) = -\frac{\Delta_h(k)}{i\omega_n} - \frac{1}{E^2} \sum_{\nu=1,2} \frac{\Delta_h(k)}{i\omega_n - E^2},
\end{equation}

with $E_1(k) = E_k, E_2(k) = -E_k$, and the dressed holon quasiparticle spectrum $E_k = \sqrt{\xi_k^2 + \Delta_h(k)^2}$. It has
been shown\textsuperscript{20} from angle resolved photoemission spectroscopy (ARPES) measurements that in real space the gap function and pairing force have a range of one lattice spacing, this indicates that the effective dressed holon gap function can be expressed as $\Delta_h(h) = \Delta_h^{(a)}(a)$. On the other hand, some experiments seem consistent with an s-wave pairing\textsuperscript{21}, while other measurements gave evidence in favor of d-wave pairing\textsuperscript{22,2}, therefore in the following discussions, we consider the cases of $\Delta_h^{(a)} = \Delta_h^{(s)}$, and $\gamma_k^{(a)} = \gamma_k^{(s)} = \gamma_k = (\cos k_x + \cos k_y)/2$, for s-wave pairing, and $\Delta_h^{(a)} = \Delta_h^{(d)}$, $\gamma_k^{(a)} = \gamma_k^{(d)} = (\cos k_x - \cos k_y)/2$, for d-wave pairing, respectively. Furthermore, we limit ourselves to the MF level for the dressed spinon part, since the normal-state charge transport obtained at this level can well describe the experimental data\textsuperscript{12,19}. In this case, we find from Eq. (10b) that the effective dressed holon gap parameter satisfies the equation,

$$1 = -(Zt)^2 \frac{1}{N^3} \sum_{k, q, p} \gamma_k^{(a)} (1 - \frac{\xi_k}{\beta E_k} \text{th} \frac{1}{2} \beta E_k) + \sum_{\nu', \nu''} \frac{B_{\nu' \nu''}(q)}{\nu' \nu''(p)} \frac{B_{\nu' \nu''}(q)}{\nu' \nu''(p)} + \frac{1}{2 \nu' \nu''},$$

(12)

where $B_{\nu' \nu''}(k, q, p) = n_F [E_{\nu''}(k)] (n_B [\omega_{\nu'}(q)]) - n_B [\omega_{\nu'}(p)] (1 + n_B [\omega_{\nu'}(q)])$, with $n_B [\omega_{\nu'}(p)]$ and $n_F [E_{\nu'}(k)]$ are the boson and fermion distribution functions, respectively. This gap equation must be solved simultaneously with other self-consistent equations,

$$\phi = \frac{1}{2N} \sum_k \chi_k \left(1 - \frac{\xi_k}{\beta E_k} \text{th} \frac{1}{2} \beta E_k\right),$$

(13a)

$$\delta = \frac{1}{2N} \sum_k \chi_k \left(1 - \frac{\xi_k}{\beta E_k} \text{th} \frac{1}{2} \beta E_k\right),$$

(13b)

$$\chi = \frac{1}{N} \sum_k \gamma_k \frac{B_k}{2\omega_k} \text{coth} \left(\frac{1}{2} \beta \omega_k\right),$$

(13c)

$$C = \frac{1}{N} \sum_k \gamma_k^2 \frac{B_k}{2\omega_k} \text{coth} \left(\frac{1}{2} \beta \omega_k\right),$$

(13d)

$$\frac{1}{2} C = \frac{1}{N} \sum_k \gamma_k \frac{B_k}{2\omega_k} \text{coth} \left(\frac{1}{2} \beta \omega_k\right),$$

(13e)

$$\chi_s = \frac{1}{N} \sum_k \gamma_k \frac{B_s(k)}{2\omega_s(k)} \text{coth} \left(\frac{1}{2} \beta \omega_s(k)\right),$$

(13f)

$$C_s = \frac{1}{N} \sum_k \gamma_k \frac{B_s(k)}{2\omega_s(k)} \text{coth} \left(\frac{1}{2} \beta \omega_s(k)\right),$$

(13g)

and therefore all the above order parameters, decoupling parameter $\alpha$, and chemical potential $\mu$ are determined by the self-consistent calculation\textsuperscript{15}, then the dressed holon pair order parameter is obtained in terms of the off-diagonal Green’s function (11b) as,

$$\Delta_h^{(a)}(k) = \frac{2}{N} \sum_k \gamma_k^{(a)} \frac{\Delta_h^{(a)}(k)}{E_k} \text{th} \left(\frac{1}{2} \beta E_k\right).$$

(14)

The dressed holon pairing state originating from the kinetic energy term by exchanging dressed spinon excitations will also lead to form the electron Cooper pairing state as mentioned in Eq. (4). For a discussion of the physical properties of the SC state, we need to calculate the electron off-diagonal Green’s function $\Gamma^\dagger(i - j, t - t') = \langle \langle C_{i\dagger}(t); C_{j\dagger}(t') \rangle \rangle$, which is a convolution of the dressed spinon Green’s function $D(p)$ and off-diagonal dressed holon Green’s function $\Im(k)$ in the framework of the partial charge-spin separation fermion-spin theory, and can be expressed as,

$$\Gamma^\dagger(k) = \frac{1}{N} \sum_p \frac{1}{\beta} \sum_{\nu, \nu''} D(p) \Im(p - k).$$

(15)

This convolution of the dressed spinon Green’s function and off-diagonal dressed holon Green’s function reflects the charge-spin recombination\textsuperscript{7}, and can be evaluated in terms of the dressed spinon Green’s function (7b) and off-diagonal dressed holon Green’s function (11b) as,

$$\Gamma^\dagger(k) = \frac{1}{N} \sum_{p, \nu, \nu''} \frac{\Delta_h^{(a)}(p - k)}{2 \nu \nu''} \text{th} \left(\frac{1}{2} \beta p_k - \frac{1}{2} \beta E_p \right) \frac{B_p}{2 \omega_p},$$

(16)

With the help of this electron off-diagonal Green’s function, we can obtain the SC gap function as,

$$\Delta^\dagger(k) = \frac{1}{N} \sum_{p, \nu, \nu''} \frac{\Delta_h^{(a)}(p - k)}{2 \nu \nu''} \text{th} \left(\frac{1}{2} \beta p_k - \frac{1}{2} \beta E_p \right) \frac{B_p}{2 \omega_p},$$

(17)

which shows that the symmetry of the electron Cooper pair is essentially determined by the symmetry of the dressed holon pair, and therefore the SC gap function can be written as $\Delta^\dagger(k) = \Delta^\dagger(\gamma^a_k)$, with the SC gap parameter evaluated in terms of Eqs. (17) and (14) as $\Delta^\dagger = -\chi \Delta_h^{(a)}$. It has been shown that the AF fluctuation is dominated by the scattering of dressed spinons\textsuperscript{12,23}, while in the present case, this AF fluctuation has been incorporated into the electron off-diagonal Green’s function (and hence the electron Cooper pair) in terms of the dressed spinon Green’s function. Since the form of the electron Cooper pair (4) is common, and the off-diagonal electron Green’s function always is a convolution of the dressed spinon Green’s function and dressed holon off-diagonal Green’s function in the framework of the partial charge-spin separation fermion-spin theory, there is a coexistence of the electron Cooper pair and short-range AF correlation\textsuperscript{24}, and hence the short-range AF fluctuation can persist into superconductivity, which is consistent with the experiments\textsuperscript{3}. In Fig. 1, we plot the dressed holon (a) and SC (b) gap parameters in the s-wave symmetry (solid line) and d-wave symmetry
Green’s function (10b), we have replaced the full dressed spinon spectrum function. In the above calculation of the effective dressed holon (then SC) gap function (10b), we have replaced the full dressed spinon Green’s function $D(p)$ by the MF dressed spinon Green’s function $D(0)(p)$, which leads to favor the s-wave pairing state, $\Delta_h^{(s)}(k) \big|_{k_c} \sim \Delta_h^{(s)}$, and the d-wave pairing state is suppressed, $\Delta_h^{(d)}(k) \big|_{k_c} \sim 0$, since in the MF level the main contribution for the weight of the dressed spinon spectrum function comes from dressed spinon excitations around wave-vector $k_c \sim [\pi, \pi]$. However, the s-wave pairing state is suppressed, and the d-wave pairing state is enhanced if the full dressed spinon Green’s function $D(p)$ is considered. Since in this case, the dressed spinon self-energy renormalization due to the dressed holon-spinon interaction leads to the incommensurate spin fluctuation and the main contribution for the weight of the dressed spinon spectrum function comes from the renormalized dressed spinon excitations around wave-vectors $k_c \sim [(1 \pm \xi)\pi, \pi]$ and $k_c \sim (\pi, (1 \pm \xi)\pi)$, which is favorable for the d-wave pairing state, $\Delta_h^{(d)}(k) \big|_{k_c} \sim \Delta_h^{(d)}[1 + \cos(\xi\pi)]$, and not for the s-wave pairing state, $\Delta_h^{(s)}(k) \big|_{k_c} \sim \Delta_h^{(s)}[1 - \cos(\xi\pi)]$. In fact, the d-wave gap function $\Delta^{(d)}(k) \propto k_x^2 - k_y^2$ belongs to the same representation $\Gamma_1$ of the orthorhombic crystal group as does the s-wave gap function $\Delta^{(s)}(k) \propto k_x^2 + k_y^2$, the two perhaps can mix at will, and there is some evidence from experiments to support this symmetrical picture. This is also why some experiments can be fitted well by both s-wave and d-wave symmetries.

In the normal-state, the electron diagonal Green’s function $G(i - j, t - t’)$ = $(C_{i\sigma}(t), C_{j\sigma}^{\dagger}(t’))$, which is a convolution of the spinon Green’s function $D(p)$ and diagonal $\delta$ function (dashed line) as a function of hole doping concentration $\delta$ at $T = 0.005J$ and $t/J = 2.5$, where both dressed holon and SC gap parameters are increased with increasing dopings. Although there is a coexistence of the electron Cooper pair and short-range AF correlation, the value of the SC gap parameter is still suppressed by this AF fluctuation. Moreover, the range of $\Delta_h^{(s)}(\Delta^{(s)})$ in the s-wave symmetry is initial from doping $\delta \approx 0$, while $\Delta_h^{(d)}(\Delta^{(d)})$ in the d-wave symmetry from $\delta \approx 0.05$ in the present case of $t/J = 2.5$, and the value of $\Delta_h^{(s)}(\Delta^{(s)})$ is always larger than $\Delta_h^{(d)}(\Delta^{(d)})$ in the whole doped regime. The present result in Eq. (17) also shows that the SC transition temperature $T_c^{(a)}$ occurring in the case of $\Delta^{(a)} = 0$ is identical to the dressed holon pair transition temperature occurring in the case of $\Delta_h^{(a)} = 0$. In correspondence with the SC gap parameter, the SC transition temperature $T_c^{(a)}$ as a function of hole doping concentration $\delta$ in the s-wave symmetry (solid line) and d-wave symmetry (dashed line) for $t/J = 2.5$ is plotted in Fig. 2 in comparison with the experimental result $\delta$ (inset). Our results indicate that the underdoped regime $T_c^{(a)} = 0$ is proportional to concentration of doped holes $\delta$, in qualitative agreement with the experimental data. These results can also be understood from the properties of the dressed holon excitation spectrum $\xi_h$ in Eq. (8a). At $T = T_c^{(a)}$, $\Delta^{(a)} = \Delta_h^{(a)} = 0$ and $E_h = \xi_h$. Within the present partial charge-spin separation fermion-spin framework, the dressed holons and spinons move self-consistently, where $T_c^{(a)}$ and other order parameters are determined by the self-consistent equations (12) and (14) in the condition $\Delta_h^{(a)} = 0$. For small dopings, the dressed holons are concentrated around the wave vector $k \sim (0, 0)$, then from Eq. (13b) we find that $T_c^{(a)} \propto \rho^{(a)-1}(0)\delta$, with $\rho^{(a)}(0)$ is the dressed holon density of state at the corresponding chemical potential $\mu$. Although the SC state is characterized by the charge-spin recombination, the main physical properties of the SC state are dominated by charged holons. This is why the superfluid density in the underdoped regime vanishes more or less linearly with concentration of doped holes, and the doped cuprates are the hole-type gossamer superconductors.

The attractive interaction between dressed holons in Eq. (10b) is induced by exchanging dressed spinon excitations, and therefore is determined by the weight of the dressed spinon spectrum function. In the above calculation of the effective dressed holon (then SC) gap function (10b), we have replaced the full dressed spinon Green’s function $D(p)$ by the MF dressed spinon Green’s function $D(0)(p)$, which leads to favor the s-wave pairing state, $\Delta_h^{(s)}(k) \big|_{k_c} \sim \Delta_h^{(s)}$, and the d-wave pairing state is suppressed, $\Delta_h^{(d)}(k) \big|_{k_c} \sim 0$, since in the MF level the main contribution for the weight of the dressed spinon spectrum function comes from dressed spinon excitations around wave-vector $k_c \sim [\pi, \pi]$. However, the s-wave pairing state is suppressed, and the d-wave pairing state is enhanced if the full dressed spinon Green’s function $D(p)$ is considered.
agonal holon Green’s function $g(k)$, has been calculated within the fermion-spin theory. With the help of this electron diagonal Green’s function, the physical properties of the electron spectral function $A(k) = - 2 \text{Im} G(k)$ have been discussed, and the results are qualitatively consistent with ARPES experiments. This electron spectral function has been used to extract the electron momentum distribution (then the electron Fermi surface) $n_k = \int_0^\infty d\omega A(k,\omega)n_F(\omega)/2\pi$, and the results show that $n_k$ for the 2D $t$-$J$ model does not follow the behavior expected from Luttinger’s theorem, in agreement with the numerical results. The holons center around $[0,0]$ in the Brillouin zone, then the charge-spin recombination from the convolution of the spinon Green’s function and holon Green’s function leads to form the electron Fermi surface, therefore the electron is a composite particle, and could account for the spread of low energy excitations throughout the Brillouin zone, these are consistent with the results found in ARPES experiments. The SC fluctuations at low temperatures in the 2D $t$-$J$ model from a higher temperature state which can not be described as a Fermi liquid. Since the electron Cooper pairing state is originated from the holon pairing state as mentioned above, then the electron gap is induced by the holon gap. In this case, both holon and electron gaps are responsible for the SC state.

Finally, we have noted that an obvious weakness of the present theoretical results is that $T_c^{(a)}$ is too high, and not suppressed in the overdoped regime. Recently, ARPES measurements have shown that $T_c$ in doped cuprates is dependent on both the gap parameter and weight of the coherent excitations in the spectral function $Z_A$, while this $Z_A < 1$ is closely related to the antisymmetric part of the self-energy function $\Sigma_1^{(h)}(k)$, and increases monotonically with increasing dopings in the underdoped and optimally doped regimes, and then slows down in the overdoped regime. This experimental result strongly suggests that the single particle coherence plays an important role in HTSC. In this case, it is then possible that the weakness perhaps due to dropping $\Sigma_1^{(h)}(k)$ in Eq. (10a) in the present case may be cured by considering this self-energy function, and these and other related issues are under investigation now.

In summary, we have discussed the mechanism of HTSC in doped cuprates based on the partial charge-spin separation fermion-spin theory. Within the $t$-$J$ model, it is shown that dressed holons interact occurring directly through the kinetic energy by exchanging dressed spinon excitations, leading to a net attractive force, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground state. The SC transition temperature $T_c^{(a)}$ is determined by the dressed holon pair transition temperature, and is proportional to the concentration of doped holes in the underdoped regime. With the common form of the electron Cooper pair, we also show that there is a coexistence of the electron Cooper pair and short-range AF correlation, and hence the short-range AF fluctuation can persist into the SC state. These results are qualitatively consistent with experiments in the underdoped regime. In the present picture, each lattice site is singly occupied by a spin-up or -down electron at the half-filling, then the spins are coupled antiferromagnetically without AFLRO. With dopings, dressed holons move in the dressed spinon liquid background, and form pairs by exchanging dressed spinon excitations at low temperature, then these dressed holon (then electron) pairs condense to the SC state, which is not far in spirit from the original resonant valence bond theory.

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1 See, e.g., P.W. Anderson, *The Theory of Superconductivity in the High-$T_c$ Cuprates* (Princeton, New Jersey, 1997).

2 See, e.g., C.C. Tsuei and J.R. Kirtley, Rev. Mod. Phys. 72, 969 (2000).
