The Mass Spectrum of a Static Adjoint Particle

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The bound states of fermions in the adjoint representation are of interest in supersymmetric models. We investigate the energy spectrum of the simplest – the gluino-gluon bound states – on several lattices in the quenched approximation. We use a static approximation for the gluino propagator. We find continuum limits of the splitting between the few lowest states, with the energy difference between the two lowest states of 354 ± 9 MeV.

Motivation

The supersymmetric standard model enriches the conventional particle spectrum in all sectors. There is motivation to study states in strongly coupled N=1 SQCD, where the particle mass spectrum must be determined by non-perturbative techniques. As well their relevance for supersymmetric phenomenology, these bound states, in R-parity conserving theories, are of cosmological interest as dark matter candidates. In addition to this, recent detector searches for such particles have sought to eradicate the window for theories with a light gaugino sector [1].

In a low temperature regime we do not expect gluino loops to contribute significantly to the propagation of a state. We study states containing gluinos using quenched lattices and a static gluino propagator approximation. The systematic error from quenching the quarks will be dominant. One of the simplest of these states is the bound states of gluons and a gluino. They can be constructed by measuring the propagation of a single heavy gluino coupled to the Yang Mills vacuum.

The Static Approximation

The contribution to the QCD action of a heavy fermion is given by

\[ S = \int d^4x \bar{\psi} i\gamma^\mu (\partial_\mu + igA_\mu) \psi - M_Q \bar{\psi} \psi. \]

The propagator for such fields is the Greens function of the corresponding wave equation. In the limit \( M_Q \rightarrow \infty \), we can discard space-like components of the covariant derivative and note that the group contribution is given by the phase evolution of the fermion as it moves. This provides us with a propagator for a static fermion;

\[ K = \delta^3(x^i - x'^i) \Pi(U^4) P_\pm e^{\mp M_Q(x^4 - x'^4)}, \]

where \( P \) is the helicity projection operator and the sign depends on the direction of particle evolution.

We see that this can be extended to encompass fermions in the adjoint representation simply by demanding the replacement of the link variables \( U_{ij} \) with elements of SU(3) in the adjoint representation; the real \( 8 \times 8 G_{ab}^4 \) matrices. These we construct by observing that the combination of fundamental elements with the generators \( \text{Tr}(UX^4U^\dagger X^b) \) satisfy all the group requirements.

The Cubic Group

We need to create a gluon field coupled to the static gluino propagator in order to measure the energy from the correlation \( C(t) \) with Euclidean time:

\[ < C >= \Sigma_m < X^\dagger(t)|n><n|X(0)> e^{-E_m t}. \]

Here \( X \) is the creation operator for the state, and \( n \) is a complete set of intermediate propagating states.

For maximal signal and minimal contamination from other states we choose \( X \) to be irreducible representations of the cubic group
such that $|< X|n >|^2$ is both large and orthogonal to other states.

In the continuum limit states are labelled by $J^{PC}$. In a lattice calculation the spatial rotational symmetry is broken down to that of $O$ and we recover the spin content of $O$ by subduction. The spin content of $O$ is given in Table 1.

| Representation | Dimension | Spin Content |
|----------------|-----------|--------------|
| $A_1$          | 1         | 0, 4, 6, 8... |
| $A_2$          | 1         | 3, 6, 7, 9... |
| $E$            | 2         | 2, 4, 5, 6... |
| $T_1$          | 3         | 1, 3, 4, 5, 5... |
| $T_2$          | 3         | 2, 3, 4, 5... |

We chose for simplicity the product of links in a square as the fundamental object in constructing operators. This shape does not allow all $J^{PC}$ combinations due to cancellations arising from the symmetries of the square. Combining the product of links to the adjoint propagator by a group generator, diagrammatically the correlation in a typical $O$ representation looks like that in figure 1.

We measured correlations for a given state using four paths at both source and sink. These were constructed using two fuzzing levels on clover sums built from two sizes of squares. We then employed a variational technique the resulting matrix to determine the lowest eigenvalue (See for example [2]). We used a bootstrap analysis to determine the mass and statistical variation at a time separation of 2 or 3 lattice units, where by inspection of the error on the signal, a plateau had been reached.

We measured correlations from all sites on various quenched lattices, the statistics to date are shown in Table 2. The fuzzing levels and specific sizes were tuned according to the lattice spacing to give the best signal.

| Lattices used in calculation |
|-----------------------------|
| $\beta$ | Size | Number |
| 5.7   | $8^3 \times 16$ | 20    |
| 5.7   | $12^3 \times 24$ | 20    |
| 5.9   | $12^3 \times 24$ | 10    |
| 6.0   | $16^3 \times 48$ | 202   |
| 6.2   | $24^3 \times 48$ | 60    |

The Adjoint Static Spectrum and Continuum Limit Extrapolation

Figure 2 shows the spectrum of states calculated at $\beta = 6.0$. We have used a value $r_0/a(\beta = 6.0) = 5.272$ to scale the right hand axis independently of the lattice spacing $a$. The points marked by circles are the ten measured representations and are labelled as such. They are plotted assuming the lowest spin contained in the $O$ representation.

Note we confirm that the most symmetric state ($A^{++}$) is not the ground state.

If we consider a variational basis of large dimension, and consider the restoration of rotational symmetry, then we may expect some of the higher eigenvalues of a variational analysis to
correspond to higher spin content in the $O$ representation. We plot some of the excited states: they are marked by triangles and labelled according to their expected (continuum) spin content. Where two states are expected to coincide in spin content they have been grouped. The spin assignments shown are in qualitative agreement with this degeneracy. With excited states added in this way we see clear Regge trajectories beginning to emerge in the spectrum.

A direct extrapolation of the continuum mass spectrum is not feasible as the self energy of the static propagator has an ultraviolet divergence. The energy differences between states, however, have a well defined continuum limit and we extrapolate to $a=0$ the difference between the second, third and $A^{++}$ states and the lowest $T^{+-}_1$. These results are shown in table 3 and the extrapolation shown in figure 3. The $r_0$ used to remove explicit $a$ dependence from observables is also measured in the quenched approximation and introduces normalization errors $O(10\%)$.

Table 3

| Transition       | $\Delta(M_0 r_0)$ | Mev  |
|------------------|-------------------|------|
| $T^{++}_1 - T^{++}_2$ | 0.898 ± 0.022     | 354 ± 9 |
| $T^{--}_2 - T^{--}_1$ | 1.426 ± 0.023     | 562 ± 9 |
| $A^{++} - T^{--}_1$  | 2.667 ± 0.068     | 1053 ± 27 |

As well as $O(a^2)$ errors in measurements that we seek to remove in the continuum extrapolation, we also note that there may exist $O(m^2 a^2)$ corrections also. We see in figure 3 that the $T^{++}_2$ and $E^{++}$ states are measured at different energies. We expect, from their spin content, that in the continuum limit these two states become degenerate. The extrapolation to $a=0$ of these states is unclear with our present data, but a common value is within statistical error.

Conclusions

The measured spectrum is in agreement with previous calculations and phenomenology suggesting that the $1^{+-}$ and $1^{--}$ states of the gluon field are the lowest lying. The lowest energy difference is higher than a previous SU(2) calculation which obtained a value of $200 \pm 70$ MeV [3].

We note that this is a measurement of the $J^P C$ of the gluonic fields only. The lower lying states correspond to the simplest magnetic and electric modes of excitation of these fields. With explicit gluino spin included, these states split into degenerate $J=\frac{1}{2}, \frac{3}{2}$ states.

We see that finite size effects are under control and that extrapolation to the continuum limit of energy differences is well defined.

The mass spectrum and structure of such states can act as phenomenological tool to aid understanding of other gluonic states.

REFERENCES

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