Possible Duality of CBE and Penrose’s CCC Cyclic Cosmologies

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Abstract

In a cyclic entropy model in which the extroverse is jettisoned at turnaround with a Come Back Empty (CBE) assumption, we address matching of the contraction scale factor \( \dot{a}(t) = f(t_T) a(t) \) to the expansion scale factor \( a(t) \), where \( f(t_T) \) is the ratio at turnaround of the introverse to extroverse radii. Such matching is necessary for infinite cyclicity and fixes the CBE period at \( \sim 2.6 T_y \). We then compare such a CBE model with alternative cyclic cosmologies including Penrose’s Conformal Cyclic Cosmology (CCC) of period \( \gtrsim 10^{100} y \) and speculate that the CBE model may be related to the CCC model by a highly nontrivial isomorphism.
# 1 Introduction to the CBE model

In physics there exist simple-to-state questions which are difficult to answer. For example, here is a plausible examination question: Show how to construct an infinitely cyclic cosmological model which is consistent with the second law of thermodynamics.

This has been studied since 1931 when a no-go theorem [1] of Tolman stated that, subject to certain assumptions, there cannot exist any solution to creating the cosmological sequence

\[ \text{Expansion} \rightarrow \text{Turnaround} \rightarrow \text{Contraction} \rightarrow \text{Bounce} \rightarrow \text{etc.} \]  \tag{1}

where the entropy of the universe obeys the second law of thermodynamics. Fortunately in this case, one can be guided by a more recent discovery about Nature.

The most important questionable assumption implicit in the no-go theorem [1] of 1931 was pointed out not by a physicist but by Nature herself in 1998 when observers discovered [2,3] that the expansion rate of the universe is accelerating. To my knowledge, nobody had questioned prior to 1998 that the expansion rate was decelerating. Once one knows that it is accelerating it is very enlightening with respect to the second law of thermodynamics because the superluminal accelerated expansion creates, starting at the onset of dark energy domination, the extroverse into which entropy built up from irreversible processes during expansion can be jettisoned with impunity from a retained introverse. The CBE (Comes Back Empty) assumption is that the retained introverse contains energy of radiation, dark energy and curvature but no matter, luminous or dark, including no black holes.

The superluminal accelerated expansion is surely the most important discovery in observational cosmology since Hubble [4] established the expansion of the universe. In terms of entropy it made it possible to distinguish the two parts of the universe after the dark energy domination began at \( t_{DE} \sim 9.8 \text{Gy}. \) Those two parts which play a crucial role in the CBE model are the introverse and extroverse which we shall now define. Note that although the CBE assumption was first introduced in [5,6] the only CBE model discussed in the present paper is the recently improved CBE model [7] which eschews the use of phantom dark energy.

The introverse is the same as the visible universe, or particle horizon, whose radius \( R_{IV}(t) \) is given by

\[ R_{IV}(t) = c \int_0^t \frac{dt}{a(t)}, \]  \tag{2}

where \( a(t) \) characterizes the expansion history of the universe, being the scale factor in the FRLW metric which assumes homogeneity and isotropy

\[ ds^2 = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - k(t)r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right], \]  \tag{3}
where $k(t)$ is the curvature.

Inserting the well-known expansion history and normalizing the scale factor to $a(t_0) = 1$ at the present time $t_0 = 13.8Gy$ (all times are measured relative to the would-have-been big bang) one finds that at the end of the radiation-dominated era $a(t_m = 47ky) = 2.1 \times 10^{-4}$, a value which will be important in the CBE matching condition to be discussed in this paper. At the commencement of the superluminal accelerated expansion at $t_{DE} = 9.8Gy$ one finds $a(t_{DE}) = 0.75$. After this time the scale factor ($t \geq 9.8Gy$) is

$$a(t) = 0.75 \exp[H_0(t - t_{DE})],$$  \hspace{1cm} (4)

where the observed value of the Hubble constant is $H_0 \simeq (13.8Gy)^{-1}$.

Substituting Eq.(4) in Eq.(2) one finds that the radius of the introverse $R_{IV}(t)$ grows from an initial value at $t = t_{DE}$

$$R_{IV}(t_{DE}) = 39Gly, \hspace{1cm} (5)$$

to its present value

$$R_{IV}(t_0) = 44Gly, \hspace{1cm} (6)$$

and reaches an asymptotic value at $t \sim 50Gy$ so that

$$R_{IV}(50Gy \leq t < \infty) \simeq 58Gly. \hspace{1cm} (7)$$

The extroverse starts to form after $t = t_{DE}$ and its radius $R_{EV}(t)$ is defined initially as equal to the introverse radius

$$R_{EV}(t_{DE}) = R_{IV}(t_{DE}) = 39Gly, \hspace{1cm} (8)$$

and thereafter for $t_{DE} \leq t$ is

$$R_{EV}(t) = \frac{a(t)}{a(t_{DE})} R_{EV}(t_{DE}). \hspace{1cm} (9)$$

This leads to the present value of the extroverse radius

$$R_{EV}(t_0) = 52Gly, \hspace{1cm} (10)$$

which is significantly above $R_{IV}(t_0)$ given by Eq.(6), as discussed in [7].

Future values of $R_{EV}(t)$ can be illustrated by examples. When the introverse radius approaches its asymptotic value, the extroverse radius is already much larger than $R_{IV}(t = 50Gy)$ given by Eq.(7) namely

$$R_{EV}(t = 50Gy) = 720Gly. \hspace{1cm} (11)$$
Two other interesting values of $R_{EV}(t)$, one very large and one extraordinarily large, are

$$R_{EV}(t = 1T y) = 5.6 \times 10^{32} Gly,$$  \hspace{1cm} (12)

and

$$R_{EV}(t = 10^{100} y) = 10^{3.1 \times 10^{98}} Gly,$$  \hspace{1cm} (13)

where this last is at a time which occurs in an alternative cyclic cosmology, Penrose’s CCC model, to be discussed later.

In the CBE model, at a turnaround time $t = t_T$ to be determined in the next subsection, the scale factor for the contracting universe is $\hat{a}(t) = f(t_T)a(t)$ with the fraction $f(t_T)$ given by

$$f(t_T) = \frac{R_{IV}(t_T)}{R_{EV}(t_T)}.$$  \hspace{1cm} (14)

As shown in [7], this reduction in size of the adiabatically contracting universe with low entropy explains, without any need for an inflationary era, the flatness observed for the present universe and further predicts that the present flatness is accurate to many decimal places.

## 2 Matching of the Scale Factor

An important requirement for infinite cyclicity is that the scale factor $a(t)$ for the expansion era be matched correctly to that of the previous contracting era. Recall the the scale factor is redefined as $\hat{a}(t) = f(t_T)a(t)$ at turnaround so that, at first sight, it might appear that an inverse transformation $a(t) = f(t_T)^{-1}\hat{a}(t)$ might be necessary. However, this is not the case because the subluminal decelerating contraction rate is far more gradual than the superluminal accelerating expansion rate.

The contraction is radiation dominated throughout so that the relevant matching condition is at the transition time, $t_m \sim 47\, ky$, between radiation domination and matter domination of the expansion era, namely

$$\dot{a}(t_m) = a(t_m) = 2.1 \times 10^{-4}.$$  \hspace{1cm} (15)

This matching condition allow us to fix the turnaround time $t_T$ of the CBE model and hence its cyclic period, as follows.

First note that $t_T$ is necessarily in the asymptotic region of the introverse where

$$R_{IV}(t_T) \simeq 58Gly,$$  \hspace{1cm} (16)
and consequently
\[ \hat{a}(t_T) = f(t_T) a(t_T) = \frac{58 \text{Gly}}{R_{EV}(t_T)} a(t_T). \]  
(17)

We know also that
\[ R_{EV}(t_T) = a(t_T) R_{EV}(t_0) = a(t_T) \times 52 \text{Gly}, \]
which, when combined with Eq.(17), reveals that
\[ \hat{a}(t_T) = \frac{58 \text{Gly}}{52 \text{Gly}} = 1.11, \]
(19)
independent of the turnaround time \( t_T \) provided that it is in the asymptotic region \( t_T \gtrsim 50 \text{Gy} \).

The matching condition, Eq.(15) is now straightforward to implement because \( \hat{a}(t) \) contracts with the radiation-dominated behavior
\[ \hat{a}(t) = \hat{a}(t_T) \left( \frac{t}{t_T} \right)^{\frac{1}{2}}, \]
(20)
and the matching requirement is therefore
\[ \hat{a}(t_m) = 1.11 \left( \frac{47 \text{ky}}{t_T} \right)^{\frac{1}{2}} = a(t_m) = 2.1 \times 10^{-4}, \]
(21)
which has the unique solution \( t_T = 1.3 Ty \).

Only with this choice of turnaround time does the contracting universe match smoothly on to the time-reverse of the expansion radiation dominated era in such a manner that infinite cyclicity is achieved. The total cyclic period of the CBE model is thus
\[ \tau_{CBE} = 2t_T = 2.6 Ty. \]
(22)

### 3 Alternative Cyclic Cosmological Models

Other than the Comes Back Empty (CBE) model that we are discussing, there are two other cyclic cosmological models in the recent literature one [8] by Steinhardt and Turok (ST) which involves clapping branes moving according to an interbrane potential in an extra spatial dimension, the other [9] by Penrose which exploits conformal invariance and is called Conformal Cyclic Cosmology (CCC).
I shall first discuss the ST model which appears to be in a different universality class from both the CBE and CCC models for two reasons: firstly, it employs string theory ideas and an extra spatial dimension in which two branes can clap together and, secondly, the analysis of the ST model has not yet addressed consistency with the second law of thermodynamics. The first difference that it uses an extra spatial dimension is of course not a fatal flaw and in the prevailing intellectual climate the CBE and CCC models might well both be criticized for not using an extra dimension. Regarding the second difference, however, it seems a reasonable question to ask whether the ST model obeys the second law of thermodynamics. This highly nontrivial requirement has not yet been demonstrated in the literature. For the latter reason, no further discussion of the ST model will ensue.

Penrose’s CCC-model [9], unlike the CBE-model, does not jettison the extroverse, but retains all of the matter including black holes. There is no contraction era in the CCC-model which has instead a cosmological sequence

\[ \text{Expansion} \rightarrow \text{Crossover} \rightarrow \text{Expansion} \rightarrow \text{Crossover} \rightarrow \text{etc.} \quad (23) \]

where the crossover step entails, by conformal transformation, joining smoothly the end of one expansion era to the beginning of the subsequent one.

The lifetime of an isolated black hole with Bekenstein-Hawking radiation is proportional to the cube of its mass and and for a one solar mass black hole is \( \tau_{BH}(1M_\odot) = 2 \times 10^{67} \) years.

For general mass it is

\[ \tau_{BH}(M_{BH}) = 2 \times 10^{67} \left( \frac{M_{BH}}{M_\odot} \right)^3 \text{ years.} \quad (24) \]

One supermassive black hole, apparently formed surprisingly early, was recently discovered in 2015 in a quasar is named \((J0100 + 2802)\) with a mass \(1.2 \times 10^{10}\) solar masses [10] so its lifetime from Eq. (24) is

\[ \tau_{BH}(J0100 + 2802) = 3.5 \times 10^{97} \text{ years}, \quad (25) \]

which provides a lower limit of the period \( \tau_{CCC} \) of the CCC-model. Since there exist even slightly more massive black holes this period must be raised to at least one googol of years

\[ \tau_{CCC} \gtrsim 10^{100} \text{ years}, \quad (26) \]

in order to accommodate all supermassive black holes with \( M_{BH} \lesssim 7.9 \times 10^{11} M_\odot \).

One must consider also the particle theory after waiting for \(10^{100}\) years. Can we be confident that all protons have decayed? The answer is almost certainly yes, as follows. The present lower limits on proton partial lifetimes are [11]

\[ \Gamma(p \rightarrow e^+ \pi^0) > 8.2 \times 10^{33} \text{ years,} \quad (27) \]
\[ \Gamma(p \rightarrow \mu^+ \pi^0) > 6.6 \times 10^{33}\text{years}, \quad (28) \]

and \[ \Gamma(p \rightarrow \nu K^+) > 5.9 \times 10^{33}\text{years}, \quad (29) \]

while a reasonable upper limit from virtual black holes is \[ \tau_p \lesssim 10^{50}\text{years}, \quad (30) \]

so that after waiting more than \(10^{100}\) years, the period of the CCC model in Eq. (26), it is very safe to assume that every proton will have already decayed.

The situation for electrons and positrons is different since they cannot all annihilate and the stability of electrons is ensured by electric charge conservation which follows from local electromagnetic gauge invariance. In order to achieve the required conformal invariance, Penrose assumes that residual electrons lose their rest mass after \(10^{100}\) years using as argument that the relevant little group is the de Sitter group rather than the normally assumed Poincare group. In other words, the rest mass is cancelled against a scale associated with the cosmological constant.

Regarding entropy, a black hole of mass \(M_{BH} = \eta M_{\odot}\) has entropy

\[ S_{BH}(\eta M_{\odot}) \sim 10^{78} \eta^2, \quad (31) \]

so that the supermassive black hole (J0100 + 2802) has dimensionless entropy

\[ S(J0100 + 2802) \sim 1.4 \times 10^{98}, \quad (32) \]

which is ten orders of magnitude greater than for all the CMB radiation in the introverse.

According to the assumptions of the CCC model all of the black hole information and entropy is lost during the evaporation. This is in line with the original suggestion by Hawking [14] in 1974.

In 2003, it was argued in [15,16] that there is no information loss during the black hole evaporation. The statement is that if AdS/CFT duality is correct then black hole information loss is impossible [17]. On the other hand, despite a number of highly nontrivial checks, AdS/CFT duality has not been rigorously proven so that it is possible that the entropy of black holes can decrease to zero by evaporation.

There remains to discuss the crossover stage in Eq. (23). Although the extroverse has after \(10^{100}y\) apparently grown to the extraordinarily large radius given in Eq. (13) above, this is actually misleading because the universe is conformally invariant. This invariance allows that the scales of time and space can be redefined by a conformal transformation of the metric according to

\[ \hat{g}_{\mu\nu} = \Omega(x)^2 g_{\mu\nu}. \quad (33) \]
This freedom permits transformation to a space-time which matches smoothly on to a subsequent expansion era which begins from a new big bang. The detailed mathematics underlying the choice of $\Omega(x)$ in Eq. (33) is explained in Appendix B of [9].

4 Duality of CCC and CBE models

In the CBE model, gravitational interactions play a role in the overall behavior of the expansion and contraction of the inverse but not in its entropy because the gravitational entropy which is dominated by black holes is jettisoned at turnaround to the extroverse. This is to be contrasted with the CCC model where gravitational entropy is the major contributor to entropy until black holes evaporate.

The two models both solve the question of constructing a cyclic cosmological model which respects the second law of thermodynamics and it seems unlikely that such a difficult and highly constrained question can have two really different solutions. The relationship between the two solutions is reminiscent of AdS/CFT duality [18] which relates a gravitational theory to a non-gravitational one. It is possible therefore to speculate that the CBE and Penrose’s CCC models are themselves connected by a highly nontrivial isomorphism.

Acknowledgement

Thanks are due for useful discussions with F. Englert, J.M. Maldacena, R. Penrose and J.C. Taylor.

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