In the descent phase of the hypersonic target flight trajectory, the hypersonic speed of the target makes the reaction time shorter. Due to overload restrictions, it is challenging for the interceptor to achieve successful interception. To address this problem, a predictive differential game guidance approach based on dynamic optimization algorithm is proposed to relax the interceptor acceleration requirements. Two-dimensional kinematics and dynamics engagement mode for hypersonic target interception is formulated as a nonlinear differential control model in the presence of matched uncertainties. The nonlinear differential model is transformed into a differential game model by combining a performance index. The performance index is positively correlated with the Line-Of-Sight (LOS) rate and control effort of interceptor and negatively correlated with the target maneuver. The Chaotic Quantum Particle Swarm Optimization (CQPSO) algorithm is utilized to address the nonlinear differential game problem to generate the instantaneous and predicted guidance commands for the interceptor and target. Numerical examples are given to verify the performance of the proposed guidance law in various engagement scenarios, including different target maneuvers, initial heading angles and target overload. Moreover, the performance of the algorithm is validated comparing with traditional and state-of-the-art guidance laws.

1. Introduction

The guidance algorithm for interceptors is a key technology for ensuring accurate interceptions. Proportional navigation (PN) and its variants [1, 2] are commonly used guidance methods for interceptors because of their simplicity, effectiveness, and ease of implementation [3].

Owing to the increasing guidance requirements and application of modern advanced control theories, guidance methods have developed rapidly, and state-of-the-art guidance methods continue to arise. These guidance methods not only decrease the miss distance but also study the interception of maneuvering targets, terminal angle constraints, energy optimization, and other issues. For example, the sliding mode guidance laws [4] solve the uncertainty of the system model and external disturbance suppression problem. Optimal guidance laws [5] solve the control problem in terms of time and energy optimization. Differential game guidance regards the target as an intelligent agent in which the target executes the best maneuver strategy to avoid interception [6]. Moreover, heuristic algorithms such as genetic algorithms, ant colony algorithm, and particle swarm optimization (PSO) algorithm [7–13] have been utilized to calculate the guidance commands, which further improve the performance of both traditional and modern guidance laws.

The engagement of hypersonic targets has recently become a new challenge. When the hypersonic target reenters the atmosphere and reaches the descent stage of the hypersonic flight trajectory, its speed is so fast that the remaining time for the defense system is so short that the interceptor no longer has an advantage in speed compared to the target [14, 15]. Traditional guidance methods cannot intercept hypersonic targets to lag behind the target in some engagement scenarios [16]. Thus, the application and improvement of modern guidance laws for hypersonic target interception has been investigated. Suboptimal midcourse guidance with a terminal-angle constraint for interceptors against near-space hypersonic targets was introduced in
which can provide handover conditions for intercepting hypersonic targets. The results of [18] exploited a fourth-order sliding mode controller in an augmented integrated guidance and control model using an arbitrary order robust exact differentiator to estimate the high-order derivatives of the sliding manifold. An integrated guidance and control algorithm based on the integral backstepping method was presented in [19], which eliminates the target maneuver on the miss distance and improves the robustness of the control system. A partially integrated guidance and control method that provides a quick and stable response to the target maneuver was introduced in [20]. It should be noted that when using these technologies, the normal acceleration required by the interceptor is much greater than the target acceleration in some combat situations, which results in reduced guidance accuracy or even misses the target in some engagement scenarios, especially at the descent stage of the hypersonic flight trajectory when the interceptor’s response time is shorter. It is well known that the best trajectory for a missile to intercept a hypersonic target is the inverse trajectory or nearby head-on [15]. Owing to the low closing speed with reduced energy requirements, the so-called head-pursuit guidance [21, 22] was developed for hypersonic target interception. However, in the orbit change stage of head-pursuit interception, the rail control engine of the interceptor is used to change the direction of the trajectory by approximately 180°. This guidance method is unachievable because of the short reaction time and overload limitation during the descent phase of the hypersonic target flight trajectory.

To date, extensive studies have been conducted on differential game guidance for subsonic and supersonic target interception. In [23], a guidance law based on the pursuit-evasion game was formulated for a maneuvering target, where the performance index is the intercept time. To achieve a zero missdistance, a new guidance law based on a differential game with bounded controls was derived [24]. Another differential game guidance law considering the estimation error in the target maneuvering acceleration was proposed in [25]. The paper [26] presents a new concept for deriving improved differential-game-based guidance laws that make use of information about the target orientation. However, there have been no studies on the application of differential game guidance to the interception of hypersonic targets in the descent phase of the flight trajectory.

In summary, the above techniques used to intercept hypersonic targets may be ineffective at the descent stage of the hypersonic flight trajectory because of the short reaction times and overload saturation. Moreover, there has been no research on the application of differential game guidance to the interception of hypersonic targets. Consequently, the above contents motivate this study to provide a heuristic guidance algorithm called predicted differential game guidance based on chaotic quantum particle swarm optimization (CQPSO-PDGG) for hypersonic target interception. The main contributions of this study can be summarized as follows:

1. To the best of our knowledge, the guidance algorithm proposed in this study is the first to apply the differential game guidance algorithm to intercept the hypersonic target in the descent phase of the flight trajectory.
2. The differential game guidance model is solved by CQPSO to obtain instantaneous and predicted guidance commands, which reduces the overload requirements of the interceptor and computational cost.
3. CQPSO-PDGG can achieve lateral interception instead of tail-chase in different engagement scenarios, so that the interceptor will not miss the target owing to speed disadvantages.

The remainder of this paper is organized as follows. The two-dimensional kinematics and dynamic model of engagement are described in Section 2. The differential game problem and mathematical foundations are presented in Section 3. The CQPSO-PDGG algorithm is presented in Section 4. The numerical simulation results are presented in Section 5. Finally, the conclusions are summarized in Section 6.

2. Kinematics and Dynamics

Engagement Model

The interception geometry in two-dimensional space is shown in Figure 1; hypersonic target tends to strike in a top-down manner at the descent stage of the flight trajectory. \( X_f - O_f - Y_f \) is the inertial reference frame. The engagement kinematic equations are written as follows [27].

\[
\dot{r} = -V_T \cos (\theta + \beta) - V_p \cos (\alpha - \theta),
\]

\[
\dot{\theta} = \frac{V_T \sin (\theta + \beta) - V_p \sin (\alpha - \theta)}{r},
\]

\[
\dot{\beta} = \frac{a_p}{V_p},
\]

\[
\dot{\alpha} = \frac{a_T}{V_T},
\]

where the relative distance and LOS angle are represented as \( r \) and \( \theta \) and \( \alpha \) and \( \beta \) are the heading angle of the interceptor and the target. \( V_p \) and \( V_T \) represent the velocity; \( a_p \) and \( a_T \) represent the acceleration. Assuming a first order lag for the interceptor and target, we can obtain that

\[
\dot{a}_p = -\frac{1}{\tau_p} a_p + \frac{1}{\tau_p} u,
\]

\[
\dot{a}_T = -\frac{1}{\tau_t} a_T + \frac{1}{\tau_t} v,
\]

where \( \tau_p \) and \( u \) represent the time-constant and the guidance command of the interceptor and \( \tau_t \) and \( v \) represent the time-constant and the guidance command of the target. Differentiating Equations (1) and (2) with respect to time
produces,

$$\dot{r} = r \dot{\theta}^2 + a_p \sin (\alpha - \theta) + a_T \sin (\theta + \beta),$$

(6)

$$\dot{\theta} = \frac{1}{r} \left( -2r \dot{\theta} - a_p \cos (\alpha - \theta) + a_T \cos (\theta + \beta) \right).$$

(7)

Equations (1)–(7) are formulated as nonlinear control model of the guidance system that can be rewritten as

$$\dot{x} = f(x(t)) + g(x(t))u(t) + h(x(t))\nu(t),$$

(8)

where

$$x = (r, \dot{r}, \dot{\theta}, a_p, a_T, \alpha, \beta, \theta)^T = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)^T$$

is the state of the guidance system and $f$, $g$, and $h$ are functions of $x(t)$ with the following form:

$$f(x(t)) = \begin{bmatrix}
-V_T \cos (\theta + \beta) - V_p \cos (\alpha - \theta) \\
r \dot{\theta}^2 + a_p \sin (\alpha - \theta) + a_T \sin (\theta + \beta) \\
\frac{1}{r} (-2r \dot{\theta} - a_p \cos (\alpha - \theta) + a_T \cos (\theta + \beta)) \\
\frac{1}{r} \frac{a_p}{r_T} \\
\frac{1}{r} \frac{a_T}{r_T} \\
a_p \frac{V_T}{V_p} \\
a_T \frac{V_T}{V_p} \\
V_T \sin (\theta + \beta) - V_p \sin (\alpha - \theta)
\end{bmatrix},$$

(9)

$$g(x(t)) = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},$$

(10)

$$h(x(t)) = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}. $$

In real combat, the velocity of interceptor is affected by drag $D$ and thrust $T$; the time derivative of interceptor velocity can be calculated as [13]

$$\dot{V}_p = \frac{T - D}{m_p} - g \sin (\alpha),$$

(11)

where $g$ is the acceleration due to gravity and $m_p$ is the interceptor mass. The aerodynamic drag is modeled as [28]

$$D = D_h + D_p,$$

$$D_h = C_{D_h} Q s,$$

$$D_p = \frac{K a_p^2 m_p^2}{Q s},$$

where $Q$ is the dynamic pressure, $\rho$ is the air density, and $s = 1\text{m}^2$ is the reference area. The zero-lift coefficient $C_{D_h}$ and the induced drag coefficient $K$ are

$$C_{D_h} = \begin{cases}
0.02, M < 0.93, \\
0.02 + 0.2(M - 0.93), 0.93 \leq M < 1.033, \\
0.04 + 0.06(M - 1.03), 1.03 \leq M < 1.1, \\
0.0442 - 0.007(M - 1.1), 1.1 \leq M,
\end{cases}$$

(12)

$$K = \begin{cases}
0.02, M < 1.15, \\
0.02 + 0.245(M - 1.15), M \geq 1.15,
\end{cases}$$

(13)

where $M$ is Mach number, modeled as

$$M = \frac{V_p}{\sqrt{403.2 \text{Tem}}},$$

(14)

where variation of temperature Tem with altitude $z_p$ is related as

$$\text{Tem} = \begin{cases}
288.16 - 0.0065z_p, z_p < 11000\text{m}, \\
216.66, z_p \geq 11000\text{m}.
\end{cases}$$
The thrust and the mass of the interceptor are

\[
T = \begin{cases} 
33000 \text{N}, & 0 \leq t < 1.5 \text{s}, \\
7500 \text{N}, & 1.5 \text{s} \leq t < 8.5 \text{s}, \\
0, & t \geq 8.5 \text{s}, 
\end{cases}
\]

\[
m_p = \begin{cases} 
135-14.53 \text{tkg}, & 0 \leq t < 1.5 \text{s}, \\
113.205 - 3.331 \text{tkg}, & 1.5 \text{s} \leq t < 8.5 \text{s}, \\
90.035 \text{kg}, & t \geq 8.5 \text{s}.
\end{cases}
\]

The atmosphere density is given by

\[
\rho = \frac{P}{0.2869(\text{Tem} + 273.1)},
\]

where the atmospheric pressure, \(P\), is calculated as

\[
P = \begin{cases} 
101.29 \times \left(\frac{T + 273.1}{288.08}\right)^{5.256}, & 0 \leq z_p < 11000 \text{m}, \\
22.65 \times e^{0.73 - 0.000157}, & 11000 \text{m} \leq z_p < 25000 \text{m}, \\
2.488 \times \left(\frac{T + 273.1}{216.6}\right)^{-11.388}, & z_p \geq 25000 \text{m},
\end{cases}
\]

where \(h\) represents latitude.

Definition 1. The quantity Zero Effort Miss (ZEM) distance at an instant \(t\) is defined as the closest distance between the interceptor and the target that would result if both the interceptor and the target do not maneuver from that instant onwards. The value of ZEM, \(r_{\text{miss}}(t)\), comes out to be

\[
r_{\text{miss}}(t) = \frac{r^2 \dot{\theta}}{r^2 + (r \dot{\theta})^2}.
\]

From the expression of \(r_{\text{miss}}(t)\) in Equation (18), it can be seen that regulating \(\dot{\theta}\) to zero implies regulation of \(r_{\text{miss}}(t)\) to zero value, and regulation of \(\dot{\theta}\) to zero while keeping \(r < 0\) leads to interception. Therefore, the LOS rate will be considered in the performance index; the detail will be shown in the following sections.

3. Differential Game Guidance Algorithm of the Guidance Problem

Consider the nonlinear system with uncertainties:

\[
\dot{x} = f(x(t)) + g(x(t))u(t) + g(x(t))a(x(t)),
\]

where \(x(t) \in \mathbb{R}^n\) represents the system state, \(u(t) \in \mathbb{R}^n\) represents the control variable, and \(a(x(t))\) denotes the uncertainties. The uncertainties should satisfy the following

\[
\text{Figure 2: Flowchart of the CQPSO-PDGG algorithm.}
\]
inequality constraint:

\[ \|a(x(t))\| \leq a_{\text{max}}(x(t)), \]  \hspace{1cm} (20)

where \( a_{\text{max}}(x(t)) \geq 0 \) denotes a function of state \( x(t) \). In the presence of uncertainties, the control variable \( u(t) \) should make the system reach an asymptotically stable, that is, the nonlinear system with uncertainties can be regarded as the robust control system. Moreover, the robust control system can be transformed into the optimal control systems as follows [29]:

\[ \dot{x} = f(x(t)) + g(x(t))u(t), \]  \hspace{1cm} (21)

\[ \int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t) + a_{\text{max}}^T(x(t))a_{\text{max}}(x(t)))dt. \]  \hspace{1cm} (22)

Here, Equation (22) is the performance index that considers the uncertainties; \( Q \) and \( R \) represent the state and control weighting matrices.

Considering the uncertainties, Equation (8) can be rewritten as a nonlinear differential guidance model with uncertainties as follows:

\[ \ddot{x} = f(x(t)) + g(x(t))(u(t) + a(x(t))) + h(x(t))(v(t) + b(x(t))). \]  \hspace{1cm} (23)

In Equation (23), \( a(x(t)) = [a_1(x(t)), \ldots, a_m(x(t))]^T \) and \( b(x(t)) = [b_1(x(t)), \ldots, b_m(x(t))]^T \) denote the uncertainties of interceptor and hypersonic target’s guidance commands, which satisfy the following conditions:

\[ a_i(x(t)) \leq a_{i,M}(x(t)), \quad i = 1, \ldots, m, \]  \hspace{1cm} (24)

\[ b_j(x(t)) \leq b_{j,M}(x(t)), \quad j = 1, \ldots, k, \]  \hspace{1cm} (24)

where \( a_{i,M}(x(t)) \) and \( b_{j,M}(x(t)) \) denote a function of state \( x(t) \). Also, we can transform the robust control model, described in Equation (23), into an optimal control model as follows:

\[ \dot{x} = f(x(t)) + g(x(t))u(t) + h(x(t))v(t), \]  \hspace{1cm} (25)

\[ J = \int_{t_0}^{t_T} (x^T(t)Qx(t) + u^T(t)R_1u(t) - v^T(t)R_2v(t) + S)dt, \]  \hspace{1cm} (26)

where \( S = a_{\text{max}}^T(x(t))a_{\text{max}}(x(t)) + b_{\text{max}}^T(x(t))b_{\text{max}}(x(t)) \), \( Q \) has to remain positive semidefinite for all \( x(t) \), and \( R_1 \) and \( R_2 \) have to remain positive-definite for all \( x(t) \) as well. \( T_p \) is the prediction horizon, and \( t_0 \) is the start time.

The optimal control model defined by Equation (25) and Equation (26) is essentially a differential game model. The objective of interceptor and target is to find one control
input $u(t)$ minimizing the infinite time performance index and another control input $v(t)$ maximizing performance index, that is, the saddle-point of the performance index.

4. Differential Game Guidance Algorithm Based on the CQPSO

As can be seen from the previous section, the differential game guidance model described by Equations (25) and (26) can be regarded as a dynamic optimization problem. In this section, we hope to find solutions to the dynamic optimization problem based on the CQPSO in the prediction horizon. CQPSO has faster convergence speed and higher convergence accuracy compared with Adaptive Particle Swarm Optimization (APSO) Algorithm [30] and Quantum Particle Swarm Optimization (QPSO) algorithm [31].

The general structure of the CQPSO-PDGG algorithm is shown in Figure 2. CQPSO-PDGG has two loops: the external loop iterates when the interceptor encounters the hypersonic target, and the internal loop iterates to calculate the instantaneous guidance commands and predicted guidance commands. More explanation is provided as follows:

Step 1. Initialization
The parameters of the CQPSO-PDGG are set. These parameters include the initial state of the interceptor and the target, the prediction horizon, the initial value and initial velocity of the particles, and the parameters of the CQPSO.

The initial value of particle satisfies the uniform distribution, and the elements of particle are mutually independent. The $j$th particle of CQPSO in this paper is defined as

$$X_{l}^{j}(k) = [u_{l}^{j}(k) u_{l}^{j}(k+1) \cdots u_{l}^{j}(k+T_{p} - 1) v_{l}^{j}(k) v_{l}^{j}(k+1) \cdots v_{l}^{j}(k+T_{p} - 1)] , j = 1, \cdots, N.$$  \hfill (27)

Here, the $N$, $k$, and $l$ are the number of particles, start time of the prediction horizon, and iteration number.

Step 2. Prediction of the future states
From Equation (25), the future states can be predicted as

$$x_{l}^{j}(k+i) = x_{l}^{j}(k+i-1) + \left( f(x_{l}^{j}(k+i-1)) + g(x_{l}^{j}(k+i-1)) u_{l}^{j}(k+i-1) \right) + h(x_{l}^{j}(k+i-1)) v_{l}^{j}(k+i-1) \times \Delta t , i = 1, \cdots, T_{p}.$$  \hfill (28)

where $\Delta t$ is the sampling interval.

Step 3. Computation of the performance index
The performance index of the $j$th particle $J_{l}^{j}(k)$ can be calculated as

$$J_{l}^{j} = J_{1}^{j} + J_{2}^{j} + J_{3}^{j} + J_{4}^{j}.$$  \hfill (29)
particle Gl aims to reduce the state tracking error, control uncertainty of interceptor, and increase the target effort, so as to obtain a smaller performance index value.

The local optimal particles $j_l,i_{\text{max}}$,best, $j_l,i_{\text{1}}$, and the corresponding performance index value and global performance index value are also updated.

Step 5. Generation of chaotic mapping sequence

The weighted average of the local optimal particles and the global optimal particles can be calculated as:

$$H_j^l(k) = \phi H_{j,\text{best}}^l(k) + (1 - \phi) H_{j,\text{1}}^l(k), \quad j = 1, \ldots, N,$$

where $\phi \in (0, 1)$. Also, taking the first element of jth particle $u_j^l(k)$ as an example, the chaotic mapping relationship is established by calculating the distance between $u_j^l(k)$ and $H_j^l(k)$, and the chaotic search range is dynamically adjusted during $l$ iterations. The search range of $u_j^l(k)$ is stated as

$$u_j^l(k) = \begin{cases} 
H_j^l(k) & \text{if } H_j^l(k) \neq H_j^l(k) \\
H_j^l(k) + |H_j^l(k) - u_j^l(k)| & \text{if } H_j^l(k) \neq u_j^l(k) \\
H_j^l(k) + |H_j^l(k) - u_j^l(k)| & \text{if } H_j^l(k) \neq u_j^l(k) \\
\frac{H_j^l(k) + H_j^l(k) - u_j^l(k)}{\alpha} & \text{if } H_j^l(k) \neq u_j^l(k), \\
\frac{H_j^l(k) + H_j^l(k) - u_j^l(k)}{\alpha} & \text{if } H_j^l(k) \neq u_j^l(k), \\
H_j^l(k) + H_j^l(k) - u_j^l(k) & \text{if } H_j^l(k) \neq u_j^l(k), \\
H_j^l(k) + H_j^l(k) - u_j^l(k) & \text{if } H_j^l(k) \neq u_j^l(k), 
\end{cases}$$

where $\alpha, \beta \in (0, 1)$. By normalizing $u_j^l(k)$, the initial value of
the chaotic sequence is given as
\[
G_{l,j}^i(k) = \frac{u_j^i(k) - u_j^i(k)_{\text{min}}}{u_j^i(k)_{\text{max}} + u_j^i(k)_{\text{min}}}. \tag{36}
\]

This paper exploits the Cat mapping function as chaotic sequence generator \[32, 33\]. A new chaotic sequence \(G_{l,j}^i(k) = [G_{l,j,1}^i(k), G_{l,j,2}^i(k), \cdots, G_{l,j,m}^i(k)]\) can be generated through this mapping function, where \(m\) is the length of the chaotic sequence. The same operation can be performed on the other elements of the \(j\)th particle and on the other particles, so that we can get new particles.

Step 6. Computation of the performance index

For each new particle, compare its current performance index value with the value of \(L_{l,j,\text{best}}^i(k)\) and \(G_{l,j,\text{best}}^i(k)\). If current value is better (with smaller value of performance
index), then update the particle and its performance index value.

Step 7. Updating the velocity and position of particles

The velocity and position of particles are updated as

\[ V_{l+j}^{i+1}(k) = \omega V_{l+j}^i(k) + c_1 r_1 (l_{j, \text{best}}^i(k) - X_{l+j}^i(k)) + c_2 r_2 (G_{j, \text{best}}^i(k) - X_{l+j}^i(k)), \]

\[ X_{l+j}^{i+1}(k) = X_{l+j}^i(k) + V_{l+j}^{i+1}(k). \]

(37)

Here, \( V_{l+j}^i(k) \) is the velocity, \( \omega \) is the inertia weight, \( c_1 \) and \( c_2 \) are constants, and \( r_1 \) and \( r_2 \) satisfy uniform distribution between \([0,1]\).  

Step 8. Stopping criteria of inner loop

The internal loop of the CQPSO-PDGG is terminated after the maximum number of iterations (\( I_{\text{max}} \)).

Step 9. Calculation of the guidance commands

After \( I_{\text{max}} \) iterations, the instantaneous guidance commands and predicted guidance commands can be estimated

\[
\begin{align*}
\frac{\partial X}{\partial t} & = V, \\
\frac{\partial V}{\partial t} & = \frac{1}{m} (U - cV - \beta X) \\
\frac{\partial \beta}{\partial t} & = \frac{1}{m} \left( \frac{\partial V}{\partial t} + c \frac{\partial V}{\partial X} \right) \\
\end{align*}
\]

\[
\begin{align*}
U & = \begin{cases} 
U_{\text{sat}} & \text{if } |U| > U_{\text{sat}} \\
\pm U_{\text{sat}} & \text{otherwise} 
\end{cases}, \\
\end{align*}
\]

where \( \beta \) is the angle of attack, \( V \) is the velocity, \( m \) is the mass, \( U \) is the control input, and \( U_{\text{sat}} \) is the saturation level.

Table 2: Simulation data based on references [6, 7, 12, 13, 27].

| Ref. | Interceptor x_{p0} (km) | y_{p0} (km) | z_{p0} (km) | V_{p0} (m/s) | \alpha_0 (\text{deg}) | x_{T0} (km) | y_{T0} (km) | z_{T0} (km) | V_{T0} (m/s) | \beta_0 (\text{deg}) | Maneuver |
|------|------------------------|-------------|-------------|-------------|---------------------|------------|------------|-------------|-------------|---------------------|-----------|
| [6]  | 0                      | 0           | 0           | 600         | 0                   | 2.5        | 0          | 0           | 400         | 0                   | 50 (step) |
| [6]  | 0                      | 0           | 0           | 600         | 0                   | 2.5        | 0          | 0           | 400         | 0                   | 100 sin (2\pi/2.5) |
| [13, 27] | 0                      | 0           | 0           | 600         | 30                  | 2.5        | 0          | 0           | 400         | 55                  | No maneuver |
| [12] | 0                      | 0           | 2.95        | 686         | 37                  | 5.9        | 4.4        | 3.1         | 275         | 10                  | 10 (step)  |
| [7]  | 0                      | 0           | 0           | 450         | 25                  | 4.2        | 0          | 2.4         | 200         | 10                  | 30 (step)  |

Figure 8: Different initial heading angles of interceptor.

Figure 9: Comparison with SDRE-DG and LQDG in the presence of step target maneuver.

(a) Engagement trajectories

(b) Lateral acceleration of the interceptor
based on optimal \( s \) particles as follows:

\[
\hat{X}(k) = \frac{1}{S_k} \sum_{j \in S_k} X^l_j(k) = \left[ \hat{u}^l_{\text{max}}(k) \hat{v}^l_{\text{max}}(k + 1) \cdots \hat{v}^l_{\text{max}}(k + T_P - 1) \hat{u}^l_{\text{max}}(k + 1) \cdots \hat{v}^l_{\text{max}}(k + T_P - 1) \right],
\]

where \( S_k \) represents the set of subscripts of the top \( s \) particles after the \( l_{\text{max}} \) iterations, \( \hat{u}^l_{\text{max}}(k) \) and \( \hat{v}^l_{\text{max}}(k) \) are the instantaneous guidance commands; \( \hat{u}^l_{\text{max}}(k + 1) \cdots \hat{v}^l_{\text{max}}(k + T_P - 1) \) and \( \tilde{u}^l_{\text{max}}(k + 1) \cdots \tilde{v}^l_{\text{max}}(k + T_P - 1) \) are the predicted guidance commands. The average value of the guidance commands from \( k \) to \( k + T_P - 1 \), input to the autopilot as the

![Figure 10: Comparison with SDRE-DG and LQDG in the presence of sinusoidal target maneuver.](image)

![Figure 11: Comparison with ADP and PSO-DGG.](image)

![Figure 12: Comparison with PN-IPSOG in the presence of step target maneuver.](image)
actual guidance command, can be calculated as

\[
\begin{align*}
    u(k+j-1) &= \frac{1}{T_p} \sum_{i=1}^{t_p} \bar{u}^{\max}(k+i-1), \\
    v(k+j-1) &= \frac{1}{T_p} \sum_{i=1}^{t_p} \bar{v}^{\max}(k+i-1),
\end{align*}
\]

where \( t_p < T_p \), \( u(k+j-1) \), and \( v(k+j-1) \) are the actual guidance commands input to the autopilot at \( k + j - 1 \) for \( j = 1, \ldots, t_p \). Also, \( \tilde{X}(k) \) is the initial value of particle at \( k + t_p \).

Step 10. Stopping criteria of outer loop
The external loop is terminated as soon as the interceptor encounters the hypersonic target.

5. Simulation and Discussion

In this section, the effectiveness of CQPSO-PDGG law proposed in this paper would be investigated through numerical simulation. In the first part, the simulation results of the algorithm are presented in various engagement scenarios.

In the second part, the performance of the CQPSO-PDGG is compared with traditional and state-of-the-art guidance laws. In the following sections, the parameters of CQPSO-PDGG are set as follows: \( Q = \text{diag}(0, 0, 0, 150, 0, 0, 0, 0) \), \( R_1 = 1 \), \( R_2 = 1 \), \( F_1 = 1 \), \( F_2 = 1 \), \( T_p = 10 \), \( t_p = 5 \), \( \tau_1 = 0.1 \), \( \tau_p = 0.1 \), \( \omega = 0.73 \), \( c_1 = 1.49 \), \( c_2 = 1.49 \), \( N = 35 \), \( l_{\max} = 20 \), and \( \Delta t = 0.1 \) seconds. Therefore, \( \omega < 1 \) and \( c_1 + c_2 < 2(1 + \omega) \) are met; the stability and convergence of CQPSO are guaranteed [34, 35]. Moreover, the guidance command uncertainties are modeled according to [27], define \( a(x_3(t)) = \delta_1 x_3(t) \cos (\delta_2 x_3(t) + (\delta_3 x_3(t))^2) \) and \( b(x_3(t)) = \delta_4 \sin (x_3(t)) \), and Equation (7) can be further written as

\[
\begin{align*}
    \ddot{x}_3(t) &= \frac{1}{x_4(t)} (-2x_2(t)x_3(t) - (x_3(t) + a(x_3(t))) \cos (x_3(t) - x_7(t)) \\
    &\quad + (v + b(x_3(t))) \cos (x_4(t) + x_5(t))),
\end{align*}
\]

where \( \delta_1 \in [-10, 10] \), \( \delta_2 \in [-25, 25] \), \( \delta_3 \in [-15, 15] \), and \( \delta_4 \in [-1, 1] \).
5.1. Hypersonic Target Interception in Various Engagement Scenarios. The initial coordinates of the interceptor and the target are \((x_{p0}, \dot{x}_{p0}) = (0, 0)\) and \((x_{T0}, \dot{x}_{T0}) = (0, 30\text{km})\), respectively. The initial velocity of the interceptor and the target are \(V_{p0} = 1000\text{m/s}\) and \(V_{p0} = 1500\text{m/s}\). The initial heading angles of the interceptor and the target are \(\alpha_0 = 160^\circ\) and \(\beta_0 = 160^\circ\). Figures 3–5 show the engagement scenario against the nonmaneuvering target, step-maneuvering target, and sinusoidal-maneuvering target, respectively.

Simulation results are depicted in Figures 3–6. The LOS rate \(\dot{\theta}\) is regulated to zero within a period of 4 to 6 seconds as shown in Figures 3(a), 4(a), and 5(a), which leads the ZEM to zero according to Equation (17). Meanwhile, Figures 3(b), 4(b), and 5(b) show that a negative \(\dot{r}\) is maintained to ensure that the relative distance between the interceptor and the target decreases. Lateral interpolations are achieved successfully in Figures 3(c), 4(c), and 5(c) in the present of the model uncertainties. The acceleration of the interceptor is presented in Figure 6. Figures 6(a) and 6(b) show that the acceleration of the interceptor tends to zero when the target is nonmaneuvering or perform a step maneuver. In Figure 6(c), the target performs a sinusoidal maneuver; the acceleration of the interceptor is stable within an interval.

Table 1 illustrates the interceptor overload requirements of methods studied in other references for hypersonic target interception. The interceptor overload requirements is greater than the target acceleration, and the ratio between the two is greater than 1.5. Figure 7 describes the lateral acceleration of the interceptor \(a_p\) when the target performs a sinusoidal maneuver with different overload. When the hypersonic target is maneuvering with 5 g, 10 g, 20 g, and 40 g overloads, the maximum acceleration of \(a_p\) is 5.5 g, 10.7 g, 21.8 g, and 44.4 g, respectively. The maximum acceleration of \(a_p\) is slightly greater than the target acceleration, and the ratio between the two is less than 1.2. Therefore, the results manifest that the overload capability requirements of CQPSO-PDGG is much less compared to the techniques studied in the other literature.

As shown in Figure 6, the initial guidance commands are large at the beginning of interception. Figure 8 depicts that the missile intercepts the hypersonic target at different heading angles. It can be observed from these figures that when the deviation between the heading angle of interceptor and the line-of-sight angle is large, the initial overload required by the interceptor is large, when the deviation is small, the required overload is small, that is, the two variables are positively correlated.

5.2. Comparison with Multiple Guidance Law. In this section, the proposed CQPSO-PDGG algorithm is compared with LQDG [6], SDRE-DG [6], ADP [27], PSO-DGG [13], PN-IPSOG [12], and MCACC [7]. The parameters, required for the simulation, are selected according to the corresponding references as given in Table 2. The simulation results are shown in Figures 9–13 and Table 3.

Compared with the SDRE-DG [6], LQDG [6], PSO-DGG [13], and ADP [27], the lateral acceleration of interceptor and engagement trajectories are illustrated in Figures 9–11 and Table 3. Figures 9–11 are investigated with step-maneuvering target, sinusoidal-maneuvering target, and nonmaneuvering target, respectively. The values of the parameters used in simulation are shown in Table 2, which are the same as Ref. [6, 13, 27]. As seen in Table 3, the maximum acceleration and control effort of CQPSO-PDGG are less than the other algorithm. It can be observed from Figures 9(a), 10(a), and 11(a) that the CQPSO-PDGG can generate stable lateral acceleration and tend to zero, but the lateral acceleration of the SDRE-DG, LQDG, and ADP is unstable in final engagement, and the overload requirement of CQPSO-PDGG is lower than the values reported in [13, 27] in the initial stage.

Moreover, CQPSO-PDGG is compared with PN-IPSOG [12] and MCACC [7] as shown in Figures 12 and 13. The lateral acceleration of interceptor is presented in Figure 12. It can be seen from the figures that the CQPSO-PDGG is more stable than the PN-IPSOG guidance at the initial stage. Figure 13 reports the LOS rate and lateral acceleration of interceptor. One can see from Figure 13(a), the CQPSO-PDGG makes the LOS rate approach 0 faster than the MCACC. Also, the maximum acceleration and control effort of CQPSO-PDGG are less than the other algorithm as shown in Table 3. As a brief summary, the simulation results reveal that the CQPSO-PDGG had a better performance than the above guidance techniques.

5.3. Computational Requirement. The simulations of CQPSO-PDGG are implemented in a MATLAB environment, and the main configuration of the computer is 2.1 GHz CPU and 16 GByte RAM. The calculation cycle of each guidance command is approximately 0.08 seconds. Therefore, using C++ programming language will greatly improve the real-time performance of the algorithm.

6. Conclusion

In this paper, a CQPSO-PDGG method is proposed for hypersonic target interception in the descent phase of the hypersonic target flight trajectory. The kinematics and dynamics engagement model, namely, nonlinear guidance models, is established in the presence of uncertainties. Then, the nonlinear differential guidance problem is transformed into a differential game problem by introducing a performance index. Finally, the differential game guidance model is solved by CQPSO to obtain the instantaneous guidance commands and predicted guidance commands. The performance of the proposed algorithms is verified by the numerical simulations in different engagement scenarios such as various target maneuvers, heading angles of interceptor, and target acceleration. Moreover, comparisons with LQDG, SDRE-DG, ADP, PSO-DGG, PN-IPSOG, and MCACC show that the CQPSO-PDGG method can intercept the target with lower overload requirements and less control effort. Also, the CQPSO-PDGG is more stable than these guidance algorithm at the initial stage and final stages of interception. In the future, we will investigate the cooperative guidance law with multiple interceptors based on DG.
to further improve the performance of guidance system for hypersonic target interception.

**Data Availability**

All data and models generated or used during the study appear in the submitted article.

**Conflicts of Interest**

The authors declare that they have no conflict of interest.

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