Two-Stage Eagle Strategy with Differential Evolution

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Abstract

Efficiency of an optimization process is largely determined by the search algorithm and its fundamental characteristics. In a given optimization, a single type of algorithm is used in most applications. In this paper, we will investigate the Eagle Strategy recently developed for global optimization, which uses a two-stage strategy by combining two different algorithms to improve the overall search efficiency. We will discuss this strategy with differential evolution and then evaluate their performance by solving real-world optimization problems such as pressure vessel and speed reducer design. Results suggest that we can reduce the computing effort by a factor of up to 10 in many applications.

Keywords: eagle strategy; bio-inspired algorithm; differential evolution; optimization.

Reference to this paper should be made as follows:

Yang, X. S. and Deb, S., (2012). ‘Two-Stage Eagle Strategy with Differential Evolution’, Int. J. Bio-Inspired Computation, Vol. 4, No. 1, pp.1–5.

1 Introduction

Metaheuristic optimization and computational modelling have become popular in engineering design and industrial applications. The essence of such paradigm is the efficient numerical methods and search algorithms. It is no exaggeration to say that how numerical algorithms perform will largely determine the performance and usefulness of modelling and optimization tools (Baeck et al.,1997; Yang, 2010).

Among all optimization algorithms, metaheuristic algorithms are becoming powerful for solving tough nonlinear optimization problems (Kennedy and Eberhart, 1995; Price et al., 2005; Yang, 2008; Cui and Cai, 2009). The aim of developing modern metaheuristic algorithms is to enable the capability of carrying out global search, and good examples of nature-inspired metaheuristics are particle swarm optimisation (PSO) (Kennedy and Eberhart, 1995) and Cuckoo Search (Yang and Deb, 2010a). Most metaheuristic algorithms have relatively high efficiency in terms of finding global optimality.

The efficiency of metaheuristic algorithms can be attributed to the fact that they are designed to imitate the best features in nature, especially the selection of the fittest in biological systems which have evolved by natural selection over millions of years. In real-world applications, most data have noise or associated randomness to a certain degree, some modifications to these algorithms are
Objective functions $f_1(x), \ldots, f_N(x)$
Initialization and random initial guess $x^{t=0}$

\begin{algorithm}
\textbf{while} (stop criterion)

Global exploration by randomization
Evaluate the objectives and find a promising solution
Intensive local search around a promising solution via an efficient local optimizer
\textbf{if} (a better solution is found)
Update the current best
\textbf{end}

Update $t = t + 1$
\textbf{end}

Post-process the results and visualization.
\end{algorithm}

Figure 1: Pseudo code of the eagle strategy.

often required, in combination with some form of averaging or reformulation of the problem. There exist some algorithms for stochastic optimization, and the Eagle Strategy (ES), develop by Yang and Deb, is one of such algorithms for dealing with stochastic optimization (Yang and Deb, 2010b).

In this paper, we will investigate the Eagle Strategy further by hybridizing it with differential evolution (Storn, 1996; Storn and Price, 1997; Price et al., 2005). We first validate the ES by some multimodal test functions and then apply it to real-world optimization problems. Case studies include pressure vessel design and gearbox speed reducer design. We will discuss the results and point out directions for further research.

2 Eagle Strategy

Eagle strategy developed by Xin-She Yang and Suash Deb (Yang and Deb, 2010b) is a two-stage method for optimization. It uses a combination of crude global search and intensive local search employing different algorithms to suit different purposes. In essence, the strategy first explores the search space globally using a Lévy flight random walk, if it finds a promising solution, then an intensive local search is employed using a more efficient local optimizer such as hill-climbing and downhill simplex method. Then, the two-stage process starts again with new global exploration followed by a local search in a new region.

The advantage of such a combination is to use a balanced tradeoff between global search which is often slow and a fast local search. Some tradeoff and balance are important. Another advantage of this method is that we can use any algorithms we like at different stages of the search or even at different stages of iterations. This makes it easy to combine the advantages of various algorithms so as to produce better results.

It is worth pointing that this is a methodology or strategy, not an algorithm. In fact, we can use different algorithms at different stages and at different time of the iterations. The algorithm used for the global exploration should have enough randomness so as to explore the search space diversely and effectively. This process is typically slow initially, and should speed up as the system converges (or no better solutions can be found after a certain number of iterations). On the other hand, the algorithm used for the intensive local exploitation should be an efficient local optimizer. The idea is to reach the local optimality as quickly as possible, with the minimal number of function evaluations. This stage should be fast and efficient.
3 Differential Evolution

Differential evolution (DE) was developed by R. Storn and K. Price by their nominal papers in 1996 and 1997 (Storn, 1996; Storn and Price, 1997). It is a vector-based evolutionary algorithm, and can be considered as a further development to genetic algorithms. It is a stochastic search algorithm with self-organizing tendency and does not use the information of derivatives. Thus, it is a population-based, derivative-free method. Another advantage of differential evolution over genetic algorithms is that DE treats solutions as real-number strings, thus no encoding and decoding is needed.

As in genetic algorithms, design parameters in a d-dimensional search space are represented as vectors, and various genetic operators are operated over their bits of strings. However, unlike genetic algorithms, differential evolution carries out operations over each component (or each dimension of the solution). Almost everything is done in terms of vectors. For example, in genetic algorithms, mutation is carried out at one site or multiple sites of a chromosome, while in differential evolution, a difference vector of two randomly-chosen population vectors is used to perturb an existing vector. Such vectorized mutation can be viewed as a self-organizing search, directed towards an optimality. This kind of perturbation is carried out over each population vector, and thus can be expected to be more efficient. Similarly, crossover is also a vector-based component-wise exchange of chromosomes or vector segments.

For a d-dimensional optimization problem with d parameters, a population of n solution vectors are initially generated, we have \( x_i \), where \( i = 1, 2, ..., n \). For each solution \( x_i \) at any generation \( t \), we use the conventional notation as

\[
 x_i^t = (x_{1,i}^t, x_{2,i}^t, ..., x_{d,i}^t),
\]

which consists of d-components in the d-dimensional space. This vector can be considered as the chromosomes or genomes.

Differential evolution consists of three main steps: mutation, crossover and selection.

Mutation is carried out by the mutation scheme. For each vector \( x_i \) at any time or generation \( t \), we first randomly choose three distinct vectors \( x_p, x_q \) and \( x_r \) at \( t \), and then generate a so-called donor vector by the mutation scheme

\[
 v_{i}^{t+1} = x_p^t + F(x_q^t - x_r^t),
\]

where \( F \in [0, 2] \) is a parameter, often referred to as the differential weight. This requires that the minimum number of population size is \( n \geq 4 \). In principle, \( F \in [0, 2] \), but in practice, a scheme with \( F \in [0, 1] \) is more efficient and stable. The perturbation \( \delta = F(x_q^t - x_r^t) \) to the vector \( x_p^t \) is used to generate a donor vector \( v_i^t \), and such perturbation is directed and self-organized.

The crossover is controlled by a crossover probability \( C_r \in [0, 1] \) and actual crossover can be carried out in two ways: binomial and exponential. The binomial scheme performs crossover on each of the \( d \) components or variables/parameters. By generating a uniformly distributed random number \( r_i \in [0, 1] \), the \( j \)th component of \( v_i^t \) is manipulated as

\[
 u_{j,i}^{t+1} = v_{j,i}^t \quad \text{if} \quad r_i \leq C_r
\]

otherwise it remains unchanged. This way, each component can be decided randomly whether to exchange with donor vector or not.

Selection is essentially the same as that used in genetic algorithms. It is to select the most fittest, and for minimization problem, the minimum objective value.

Most studies have focused on the choice of \( F \), \( C_r \) and \( n \) as well as the modification of (2). In fact, when generating mutation vectors, we can use many different ways of formulating (2), and this leads to various schemes with the naming convention: DE/x/y/z where x is the mutation scheme (rand or best), y is the number of difference vectors, and z is the crossover scheme (binomial or exponential). The basic DE/Rand/1/Bin scheme is given in (2). For a detailed review on different schemes, please refer to Price et al. (2005).
4 ES with DE

As ES is a two-stage strategy, we can use different algorithms at different stage. The large-scale coarse search stage can use randomization via Lévy flights. In the context of metaheuristics, the so-called Lévy distribution is a distribution of the sum of $N$ identically and independently distribution random variables (Gutowski, 2001; Mantegna, 1994; Pavlyukevich, 2007).

This distribution is defined by a Fourier transform in the following form

$$F_N(k) = \exp[-N|k|^\beta]. \quad (4)$$

The inverse to get the actual distribution $L(s)$ is not straightforward, as the integral

$$L(s) = \frac{1}{\pi} \int_0^\infty \cos(\tau s) e^{-\alpha \tau^\beta} d\tau, \quad (0 < \beta \leq 2), \quad (5)$$

does not have analytical forms, except for a few special cases. Here $L(s)$ is called the Lévy distribution with an index $\beta$. For most applications, we can set $\alpha = 1$ for simplicity. Two special cases are $\beta = 1$ and $\beta = 2$. When $\beta = 1$, the above integral becomes the Cauchy distribution. When $\beta = 2$, it becomes the normal distribution. In this case, Lévy flights become the standard Brownian motion.

For the second stage, we can use differential evolution as the intensive local search. We know DE is a global search algorithm, it can easily be tuned to do efficient local search by limiting new solutions locally around the most promising region. Such a combination may produce better results than those by using pure DE only, as we will demonstrate this later. Obviously, the balance of local search (intensification) and global search (diversification) is very important, and so is the balance of the first stage and second stage in the ES.

5 Validation

Using our improved ES with DE, we can first validate it against some test functions which are highly nonlinear and multimodal.

There are many test functions, here we have chosen the following 5 functions as a subset for our validation.

Ackley’s function

$$f(x) = -20 \exp \left[ -\frac{1}{5} \sqrt{\frac{1}{d} \sum_{i=1}^{d} x_i^2} \right] - \exp \left[ \frac{1}{d} \sum_{i=1}^{d} \cos(2\pi x_i) \right] + 20 + e, \quad (6)$$

where $d = 1, 2, \ldots$, and $-32.768 \leq x_i \leq 32.768$ for $i = 1, 2, \ldots, d$. This function has the global minimum $f_* = 0$ at $x_* = (0, 0, \ldots, 0)$.

The simplest of De Jong’s functions is the so-called sphere function

$$f(x) = \sum_{i=1}^{d} x_i^2, \quad -5.12 \leq x_i \leq 5.12, \quad (7)$$

whose global minimum is obviously $f_* = 0$ at $(0, 0, \ldots, 0)$. This function is unimodal and convex.

Rosenbrock’s function

$$f(x) = \sum_{i=1}^{d-1} \left[ (x_i - 1)^2 + 100(x_{i+1} - x_i^2)^2 \right], \quad (8)$$
whose global minimum $f_\ast = 0$ occurs at $x_\ast = (1,1,...,1)$ in the domain $-5 \leq x_i \leq 5$ where $i = 1,2,...,d$. In the 2D case, it is often written as

$$ f(x,y) = (x-1)^2 + 100(y-x^2)^2, $$

which is often referred to as the banana function.

Schwefel’s function

$$ f(x) = \sum_{i=1}^{d} x_i \sin(\sqrt{|x_i|}), \quad -500 \leq x_i \leq 500, \quad (10) $$

whose global minimum $f_\ast \approx -418.9829n$ occurs at $x_i = 420.9687$ where $i = 1,2,...,d$.

Shubert’s function

$$ f(x) = \left( \sum_{i=1}^{K} i \cos \left( i + (i+1)x \right) \right) \cdot \left( \sum_{i=1}^{K} i \cos \left( i + (i+1)y \right) \right), \quad (11) $$

which has multiple global minima $f_\ast \approx -186.7309$ for $K = 5$ in the search domain $-10 \leq x, y \leq 10$.

Table I summarizes the results of our simulations, where 9.7% corresponds to the ratio of the number of function evaluations in ES to the number of function evaluations in DE. That is the computational effort in ES is only about 9.7% of that using pure DE. As we can see that ES with DE is significantly better than pure DE.

| Functions            | ES/DE |
|----------------------|-------|
| Ackley ($d = 8$)    | 24.9% |
| De Jong ($d = 16$)  | 9.7%  |
| Rosenbrock ($d = 8$)| 20.2% |
| Schwefel ($d = 8$)  | 15.5% |
| Shubert             | 19.7% |

6 Design Benchmarks

Now we then use the ES with DE to solve some real-world case studies including pressure vessel and speed reducer problems.

6.1 Pressure Vessel Design

Pressure vessels are literally everywhere such as champagne bottles and gas tanks. For a given volume and working pressure, the basic aim of designing a cylindrical vessel is to minimize the total cost. Typically, the design variables are the thickness $d_1$ of the head, the thickness $d_2$ of the body, the inner radius $r$, and the length $L$ of the cylindrical section (Coello, 2000; Cagnina et al., 2008). This is a well-known test problem for optimization and it can be written as

$$ \text{minimize } f(x) = 0.6224d_1rL + 1.7781d_2r^2 $$
$$ + 3.1661d_1^2L + 19.84d_1^2r, $$

subject to the following constraints

$$ g_1(x) = -d_1 + 0.0193r \leq 0 $$
$$ g_2(x) = -d_2 + 0.00954r \leq 0 $$
$$ g_3(x) = -\pi r^2L - \frac{4\pi}{3}r^3 + 1296000 \leq 0 $$
$$ g_4(x) = L - 240 \leq 0, $$

(13)
The simple bounds are
\[ 0.0625 \leq d_1, d_2 \leq 99 \times 0.0625, \tag{14} \]
and
\[ 10.0 \leq r, \ L \leq 200.0. \tag{15} \]

Table 2: Comparison of number of function evaluations

| Case study    | Pure DE | ES | ES/DE  |
|---------------|---------|----|--------|
| Pressure vessel | 15000   | 2625 | 17.7%  |
| Speed reducer  | 22500   | 3352 | 14.9%  |

Recently, Cagnina et al. (2008) used an efficient particle swarm optimiser to solve this problem and they found the best solution
\[ f_\ast \approx 6059.714 \]
at
\[ x_\ast \approx (0.8125, 0.4375, 42.0984, 176.6366). \tag{16} \]
This means the lowest price is about $6059.71.

Using ES, we obtained the same results, but we used significantly fewer function evaluations, comparing using pure DE and other methods. This again suggests ES is very efficient.

### 6.2 Speed Reducer Design

Another important benchmark is the design of a speed reducer which is commonly used in many mechanisms such as a gearbox (Golinski, 1973). This problem involves the optimization of 7 variables, including the face width, the number of teeth, the diameter of the shaft and others. All variables are continuous within some limits, except \( x_3 \) which only takes integer values.

\[
f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) \]
\[-1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^2 + x_7^2)\]
\[+0.7854(x_4x_6^2 + x_5x_7^2) \tag{17} \]
\[g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \leq 0, \tag{18} \]
\[g_2(x) = \frac{397.5}{x_1x_2^2x_3} - 1 \leq 0 \tag{19} \]
\[g_3(x) = \frac{1.93x_3^3}{x_2x_3x_6^2} - 1 \leq 0, \tag{20} \]
\[g_4(x) = \frac{1.93x_3^3}{x_2x_3x_7^2} - 1 \leq 0 \tag{21} \]
\[g_5(x) = \frac{1.0}{110x_6^2} \sqrt{\left(\frac{745.0x_4}{x_2x_3}\right)^2 + 16.9 \times 10^6 - 1} \leq 0 \tag{22} \]
\[g_6(x) = \frac{1.0}{85x_7^2} \sqrt{\left(\frac{745.0x_5}{x_2x_3}\right)^2 + 157.5 \times 10^6 - 1} \leq 0 \tag{23} \]
\[g_7(x) = \frac{x_2x_3}{40} - 1 \leq 0 \tag{24} \]
\[g_8(x) = \frac{5x_2}{x_1} - 1 \leq 0 \tag{25} \]
\[ g_9(x) = \frac{x_1}{12x_2} - 1 \leq 0 \]  
\[ g_{10}(x) = \frac{1.5x_9 + 1.9}{x_4} - 1 \leq 0 \]  
\[ g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0 \]

where the simple bounds are \( 2.6 \leq x_1 \leq 3.6, 0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 7.3 \leq x_4 \leq 8.3, 7.8 \leq x_5 \leq 8.4, 2.9 \leq x_6 \leq 3.9, \) and \( 5.0 \leq x_7 \leq 5.5. \)

In one of latest studies, Cagnina et al. (2008) obtained the following solution

\[ x_\ast = (3.5, 0.7, 17, 7.3, 7.8, 3.350214, 5.286683) \]  

with \( f_{\text{min}} = 2996.348165. \)

Using our ES, we have obtained the new best

\[ x_\ast = (3.5, 0.7, 17, 7.3, 7.8, 3.3433649, 5.285351) \]  

with the best objective \( f_{\text{min}} = 2993.7495888. \) We can see that ES not only provides better solutions but also finds solutions more efficiently using fewer function evaluations.

7 Discussions

Metaheuristic algorithms such as differential evolution and eagle strategy are very efficient. We have shown that a proper combination of these two can produce even better performance for solving nonlinear global optimization problems. First, we have validated the ES with DE and compared their performance. We then used them to solve real-world optimization problems including pressure vessel and speed reducer design. Same or better results have been obtained, but with significantly less computational effort.

Further studies can focus on the sensitivity studies of the parameters used in ES and DE so as to identify optimal parameter ranges for most applications. Combinations of ES with other algorithms may also prove fruitful. Furthermore, convergence analysis can provide even more profound insight into the working of these algorithms.

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