Modeling of soil dumping based on a modified Upwind Leapfrog difference scheme

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Abstract. The difference scheme is developed for the diffusion-convection problem numerical solution at large grid Peclet numbers. The scheme based on linear combination of the Upwind and Standard Leapfrog difference schemes (LCUSLDS) with weight coefficients obtained by minimizing the approximation error at small Courant numbers. The proposed approach takes into account the function of cell fullness, which allows to increase the accuracy of modeling on the area with complex geometry. The proposed modification of the Upwind Leapfrog difference scheme for the diffusion-convection problem numerical solution has better accuracy compared to other schemes considered in the article for grid values of Peclet numbers in the range $2 \leq Pe \leq 20$. The problem of substances transport between two coaxial cylinders for large and small values of grid Peclet numbers is considered to test the proposed difference scheme. This movement of the water medium in hydrodynamics is called the Couette-Taylor flow. The steady fluid flow between two infinitely long coaxial circular cylinders is considered.

1. Introduction
The research scientific and practical significance is determined by the navigation safety problems, such as forecasting shipping routes silting, as well as predictive modeling of the man-made disasters consequences. The catastrophic storm in the Kerch Strait in November 2007 led to more than 20 ships wreck. Oil spills have led to the coastline and bottom sediments pollution with oil products and other harmful substances. Compounds of oil products in the form of bitumen and tar were found on the coast of the Black and Azov Seas, more than 200 km long in 2008-2011 as one of the consequences of this disaster. On September 24-25, 2014, the coastal areas of the Azov Sea were flooded as a result of a storm surge, the water level in the Taganrog port area rose by more than 4 meters, which caused significant damage to the region’s economy. The industrial enterprises located on the coastline can uncontrollably drain industrial waters that have not undergone the necessary treatment, which leads to a dangerous phenomenon, the poisoning of reservoir waters, this causes significant harm to living organisms. The processes of transport of pollutants and suspended particles are described using the diffusion-convection equation. It becomes necessary to grow grids at high flow velocities when solving this class of problems by grid methods based on central-difference schemes. Using the upstream scheme entails a loss of accuracy. The paper considers the use of difference schemes for solving the problem of transport of polluting and suspended particles at high flow rates.
In the numerical solution of the problems of transport of suspensions in hydraulic systems, which are based on central difference schemes, the problem of a drop in accuracy is created for large values of the grid Peclet number. One of the options for solving this problem is proposed to decrease the step along the spatial coordinate. However, this step entails an increase in labor intensity. Another solution to this problem is the use of other difference schemes, including hybrid ones. For example, such difference schemes as the Upwind and the Standard Leapfrog have similar properties, namely, they are dissipative-free and have the second order of accuracy with respect to steps in spatial and temporal coordinate directions. However, it should be noted that both difference schemes have low accuracy. The Standard Leapfrog design with solution limiters showed greater accuracy compared to the classic designs described above. This paper describes the use of a hybrid scheme, which is a LCUSLDS [1]. As a result of minimizing the error, weights of 2/3 and 1/3 were obtained, respectively.

2. Modified Upwind Leapfrog difference scheme

Consider the transfer equation

\[ q_t + u q_x' = 0, \]  

where \( t \in [0, T], x \in [0, L], \) \( q(0, x) = q^0(x), q(t, 0) = 0, u = \text{const}. \)

Let us introduce the uniform computational grid:

\[ \omega = \varpi_h \times \omega_\tau, \]

where \( \varpi_h = \{ x_i | x_i = i h, \ i = 0, N \}, \ N h = L, \) \( \omega_\tau = \{ t^n | n = 0, T \}, \) \( \tau = t^{n+1} - t^n = \text{const}. \)

Let consider the Upwind Leapfrog difference scheme (for the case \( u \geq 0 \) [2]):

\[ \frac{q_i^{n+1} - q_i^n}{2 \tau} + \frac{q_{i-1}^{n} - q_{i-1}^{n-1}}{2 \tau} + u \frac{q_i^n - q_{i-1}^n}{h} = 0. \]  

(2)

Let use the decompositions of the functions \( q_i^{n+1} \) and \( q_i^{n-1} \) in the Taylor series in the neighborhood of points \( (i, n) \) and \( (i-1/2, n) \), respectively:

\[ \frac{q_i^{n+1} - q_i^n}{2 \tau} + \frac{q_{i-1}^{n} - q_{i-1}^{n-1}}{2 \tau} + u \frac{q_i^n - q_{i-1}^n}{h} = (q_i)^n_{i-1/2} + u (q_x)^n_{i-1/2} - \]

\[ - \frac{u h^2}{12} (q_{xxx})_{i-1/2} + \frac{v^2 \tau h}{4} (q_{xxx})_{i-1/2} - \frac{v^3 \tau^2}{6} (q_{xxx})_{i-1/2} + O (h^4). \]

Taking into account the (1):

\[ q_t = -u q_x, \quad q_u = u^2 q_{xx}, \quad q_{uu} = -u^3 q_{xxx}, \quad q_{ttt} = u^4 q_{xxxx} \]

(4)

and the quality (3) will take the form:

\[ \frac{q_i^{n+1} - q_i^n}{2 \tau} + \frac{q_{i-1}^{n} - q_{i-1}^{n-1}}{2 \tau} + u \frac{q_i^n - q_{i-1}^n}{h} = \]

\[ = (q_t + u q_x)^n_{i-1/2} - \left( \frac{c - 1}{12} \right) (2c - 1) u h^2 (q_{xxx})_{i-1/2} + O (h^4). \]

Let consider the Standard Leapfrog difference scheme

\[ \frac{q_i^{n+1} - q_i^n}{2 \tau} + u \frac{q_i^n - q_{i-1}^n}{2 h} = 0. \]  

(6)
Let use the decompositions of the function \( q_i^{n+1} \) in the Taylor series in the neighborhood of points \((i, n)\):

\[
\frac{q_i^{n+1} - q_i^n}{2\tau} + u \frac{q_i^{n+1} - q_i^n}{2h} = (q_i^n)^n + \frac{\tau^2}{6} (qu_i^n) + u (q_xi^n) + u \frac{h^2}{6} (q_{xxx}i^n) + O \left( \tau^4 + h^4 \right). \quad (7)
\]

Taking into account the 4, the quality 7 will take the form:

\[
\frac{q_i^{n+1} - q_i^n}{2\tau} + u \frac{q_i^{n+1} - q_i^n}{2h} = (q_i^n + u q_xi^n) + \frac{1 - c^2}{6} u h^2 (q_{xxx}i^n) + O \left( h^4 \right). \quad (8)
\]

Consider the difference scheme, which is a LCUSLDS with weights 2/3 and 1/3, respectively

\[
\frac{q_i^{n+1} - q_i^n}{\tau} + 2 \frac{q_i^{n+1} - q_i^n}{3\tau} + \frac{q_i^n - q_i^{n-1}}{3\tau} + u \frac{q_i^{n+1} + 4q_i^n - 5q_i^{n-1}}{3h} = 0, \quad u \geq 0; \quad (9)
\]

\[
\frac{q_i^{n+1} - q_i^n}{\tau} + 2 \frac{q_i^{n+1} - q_i^n}{3\tau} + \frac{q_i^n - q_i^{n-1}}{3\tau} + u \frac{5q_i^{n+1} - 4q_i^n - q_i^{n-1}}{3h} = 0, \quad u < 0.
\]

According to the 4, 5 and 8, the scheme 9 has a local approximation error equaled to \( c(1-c) u h^2 q_{xxx} + O \left( h^3 \right) \) relative to the fictional node \((i-1/3, n)\).

**Example 1.** It is required to find the solution of the transfer equation:

\[
\frac{\partial q}{\partial t} + u \frac{\partial q}{\partial x} = 0, \quad u=0.5 \text{ m/s, } 0 \leq t \leq T, 0 \leq x \leq L, q(t, 0) = 0
\]

with initial conditions \( q^0(x) = \theta(20-x) - \theta(10-x) \), where \( \theta(x) \) is the Heaviside function.

Fig. 1 shows the numerical solutions of Example 1 (1 – accurate solution; 2 – numerical solution) based on the schemes (9). The time step \( \tau \) is \( 0.02 \text{ s} \). The space step \( h \) is \( 1 \text{ m} \). The lengths of time interval \( T \) are equal to \( 100 \text{ s} \) and \( 500 \text{ s} \).

![Figure 1](image)

**Figure 1.** Solutions of Ex. 1 for different lengths \( T \) of time interval: (a) \( T = 100 \text{ s} \), (b) \( T = 500 \text{ s} \).

Fig. 2 shows the errors of the the numerical solution of Example 1 based on the difference scheme (9) (1), the Upwind Leapfrog difference schemes with limiters (2) and Standard Leapfrog difference schemes with limiters (3) depending on the values of the Courant numbers.

Time interval \( T = 100 \text{ s} \). The time step \( \tau \) is from \( 0.02 \text{ s} \) to \( 2 \text{ s} \). The Courant numbers range from \( 0.01 \) to \( 1 \). The error of the numerical solution is calculated by \( \Psi = \sqrt{\sum_i (\tilde{q}_i - q_i)^2 / \sum_i q_i^2} \), where \( q_i \) is the accurate solution in node \( i \), \( \tilde{q}_i \) is the numerical solution depending on time step length.
3. 2D convection-diffusion equation approximation

Consider the two-dimensional convection-diffusion equation:

\[ c_t^i + uc_x^i + vc_y^i = (\mu c_x^i)_x + (\mu c_y^i)_y + f \]  (10)

with boundary conditions \( c_n(x, y, t) = \alpha_n c + \beta_n \), where \( u, v \) are components of the 2D velocity vector; \( \mu \) is the coefficient of turbulent exchange, \( f \) is the function of sources intensity and distribution.

The computational domain is inscribed in a rectangle. For the numerical implementation of a discrete mathematical model of the problem, we introduced a uniform grid:

\[ w_h = \{ t^n = n\tau, x_i = ih_x, y_j = jh_y; n = 0, N_t, i = 0, N_x, j = 0, N_y \} , \]

where \( \tau \) is the time step; \( h_x, h_y \) are space steps; \( N_t \) is the upper time boundary; \( N_x, N_y \) are space boundaries.

3.1. Approximation of the one-dimensional convection-diffusion equation

Consider the nonstationary convection-diffusion equation [3, 4]

\[ c_t^i + uc_x^i = (\mu c_x^i)_x , \]  (11)

where \( t \in [0, T], x \in [0, L] \), with boundary and initial conditions:

\[ c(0, x) = c^0(x) , c(t, 0) = c(t, L) = 0, u = \text{const} . \]

For approximation of the convection operator, we use the difference scheme (9) obtained as a result of a LCUSLDS:

\[
\begin{align*}
\frac{c_i^{n+1} - c_i^n}{\tau} + 2c_i^{n+1} - c_i^{n-1} + 4c_i^n - 5c_i^{n-1} + 3\mu c_{i+1}^{n+1} - 2c_{i+1}^n + 3h
\end{align*}
\]

\[
\begin{align*}
\frac{c_i^{n+1} - c_i^n}{\tau} + 2c_i^{n+1} - c_i^{n-1} + 4c_i^n - 5c_i^{n-1} + 3\mu c_{i+1}^{n+1} - 2c_{i+1}^n + 3h
\end{align*}
\]

\[ = 2\mu c_{i+1}^{n+1} - 2c_{i+1}^n + c_i^{n+1}, u \geq 0; \]  (12)

\[
\begin{align*}
\frac{c_i^{n+1} - c_i^n}{\tau} + 2c_i^{n+1} - c_i^{n-1} + 4c_i^n - 5c_i^{n-1} + 3\mu c_{i+1}^{n+1} - 2c_{i+1}^n + 3h
\end{align*}
\]

\[ = 2\mu c_{i+1}^{n+1} - 2c_{i+1}^n + c_i^{n+1}, u < 0. \]
Example 2. It is required to find the solution of the convection-diffusion equation (11) for

\[ u = 0.5 \text{m/s}, \quad \mu = \text{const}, \quad 0 \leq t \leq T, \quad 0 \leq x \leq L, \quad q(t,0) = q(t,L) = 0 \]

with boundary and initial conditions: \( c^0(x) = \theta(20 - x) - \theta(10 - x) \).

The accurate solution of Example 2 can be represented as:

\[
c(t,x) = \sum_{m=1}^{N-1} k_m^0 e^{-\mu \omega^2 t} \sin(\omega mx), \quad k_m^0 = \frac{2}{L} \int_0^L q^0(x + ut) \sin(\omega mx) dx, \quad \omega = \frac{\pi}{L}.
\]

Fig. 3 shows the accurate solution (1) and the numerical solutions of Example 2 based on the scheme (12) (2) and the Upwind Leapfrog difference scheme with limiters (3). The parameter \( \mu \) is equal to 0.025 \( \text{m}^2/\text{s} \) \( (a) \) and 0.0025 \( \text{m}^2/\text{s} \) \( (b) \). Grid Peclet numbers \( (Pe = u h / \mu) \) are equal to 20 and 200, respectively.

**Figure 3.** Solutions of Example 2: \( (a) \) Pe = 20; \( (b) \) Pe = 200.

![Figure 3](image)

**Figure 4.** The error values of the numerical solution of Example 2 in norm \( L_1 \) depending on the grid Peclet numbers: 1 – the scheme (12), 2 – the Upwind Leapfrog difference scheme.

Fig. 4 shows the errors \( \Psi^n \) of the numerical solution of Example 2 based on the scheme (12) and the Upwind Leapfrog difference scheme with limiters in norm \( L_1 \) depending on the grid Peclet numbers. Fig. 4 illustrates that the proposed difference scheme (12) for solving problem (11) has a small error in a wide range of the grid Peclet numbers.
3.2. Computational domain complex geometry

To approximate the operators of diffusion and convective transfer in the case of a complex geometry of the computational domain, finite-difference analogs are used, taking into account the filling of the cells. By the filling of the cells \( o_{i,j} \) to the node \((i, j)\) we mean the ratio of the area of the part of the cell filled with the medium to the total area of the cell. If the cell is completely filled, then the filling function \( o \) is equal to 1, if it is empty, then 0, if the cell is partially filled, then the filling function takes values from 0 to 1.

The control area \( D_0 \) is composed of cells adjacent to the node \((i, j)\). And the regions \( D_1, D_2, D_3, D_4 \) are composed of cells adjacent to the node \((i, j)\) and located to the left, to the right, above and below this node. The occupancy factors of the control areas will be denoted by \( q_0, q_1, q_2, q_3, q_4 \) respectively [5, 6].

Discrete forms of the convective \( u c'_x \) and diffusion transfer operators \((\mu c'_x)^'\) in the case of partial filling of cells:

\[
(q_0)_{i,j} u c'_{x} \simeq (q_1)_{i,j} u_{i+1/2,j} \frac{c_{i+1,j} - c_{i,j}}{2h_x} + (q_2)_{i,j} u_{i-1/2,j} \frac{c_{i,j} - c_{i-1,j}}{2h_x},
\]

\[
(q_0)_{i,j} (\mu c'_x)^' \simeq (q_1)_{i,j} \mu_{i+1/2,j} \frac{c_{i+1,j} - c_{i,j}}{h_x} - (q_2)_{i,j} \mu_{i-1/2,j} \frac{c_{i,j} - c_{i-1,j}}{h_x} - \left| (q_1)_{i,j} - (q_2)_{i,j} \right| \mu_{i,j} \frac{\alpha_x c_{i,j} + \beta_x}{h_x}.
\]

The study of the approximation error showed that the suspension transport model has a second order of error in spatial coordinates and a second order of error in a time variable.

3.3. Suspension transport problem at large grid Peclet numbers numerical solution

To approximate the homogeneous equation (10) for large grid Peclet numbers, we use the space splitting schemes:

\[
\frac{c^{n+1/2} - c^n}{\tau} + u \left( c^n \right)'_x = \left( \mu \left( c^n \right)'_x \right)_x,
\]

\[
\frac{c^{n+1} - c^{n+1/2}}{\tau} + v \left( c^{n+1/2} \right)'_y = \left( \mu \left( c^{n+1/2} \right)'_y \right)_y.
\]

To approximate the system of equations (13)-(14) we will use the scheme (12), while taking into account the VOF of cell. The difference scheme for the equation (13) describing the transfer along the \( Ox \) direction:

\[
\frac{2q_{2,ij} + q_{0,ij} c_{ij}^{n+1/2} - c_{ij}^n}{3} + \frac{u_{i-1/2,j} q_{2,ij} c_{ij}^n - c_{ij}^n}{3h_x} + u_{i+1/2,j} \min(q_{1,ij}, q_{2,ij}) \frac{c_{i+1,j}^n - c_{ij}^n}{3h_x} +
\]

\[
\frac{2\Delta x c_{i-1,j}^n q_{2,ij} + \Delta x c_{ij}^n q_{0,ij}}{3} = 2\mu_{i+1/2,j} q_{1,ij} \frac{c_{i+1,j}^n - c_{ij}^n}{h_x} - 2\mu_{i-1/2,j} q_{2,ij} c_{ij}^n \frac{\alpha_x c_{ij}^n + \beta_x}{h_x}, u_{i,j} \geq 0;
\]

\[
\frac{2q_{1,ij} + q_{0,ij} c_{ij}^{n+1/2} - c_{ij}^n}{3} + \frac{u_{i+1/2,j} q_{1,ij} c_{ij}^n - c_{ij}^n}{3h_x} + u_{i-1/2,j} \min(q_{1,ij}, q_{2,ij}) \frac{c_{ij}^n - c_{ij}^{n-1}}{3h_x} +
\]

\[
\frac{2\Delta x c_{i+1,j} q_{1,ij} + \Delta x c_{ij}^n q_{0,ij}}{3} = 2\mu_{i+1/2,j} q_{1,ij} \frac{c_{i+1,j}^n - c_{ij}^n}{h_x} - 2\mu_{i-1/2,j} q_{2,ij} c_{ij}^n \frac{c_{ij}^n - c_{ij}^{n-1}}{h_x} -
\]

\[
\frac{2\Delta x c_{i-1,j} q_{2,ij} + \Delta x c_{ij}^n q_{0,ij}}{3} = 2\mu_{i+1/2,j} q_{1,ij} \frac{c_{ij}^n - c_{ij}^{n-1}}{h_x}.
\]
\[ \alpha_{i,j} \frac{c_{i,j}^{n} + \beta_{i,j}}{h_{x}} - |q_{1,i,j} - q_{2,i,j}| \mu_{i,j} < 0, \text{ where } \Delta_{x}c_{i,j}^{n} = \frac{c_{i,j}^{n-1/2} - c_{i,j}^{n-1}}{\tau}. \]

The difference scheme for the equation (14) describing the transfer along the \( Oy \) direction is written similarly to (15).

### 3.4. Example of solving the suspension transport problem at large grid Peclet numbers

**Example 3.** It is required to find the solution of the suspension transport problem (10) between two coaxial cylinders. The flow field is defined by the function

\[
\begin{align*}
u(x, y) &= -\frac{5y}{x^2 + y^2}, \\
v(x, y) &= \frac{5x}{x^2 + y^2}, \\
P(x, y) &= P(r_1) - \frac{12.5\rho}{x^2 + y^2} + \rho/2
\end{align*}
\]

with the initial conditions: 

\[
c^0(x, y) = (\theta(1 - x) - \theta(0.5 - x))(\theta(-8.5 - y) - \theta(-9 - y)).
\]

Fig. 5 shows the results of numerical solution of the suspension transport problem (Example 3) at large grid Peclet numbers ((a) is the initial distribution of the concentration field; (b) is the result of the concentration field based on the Central difference scheme; (c) is the result of the concentration field based on the difference scheme (15)). The coefficient of turbulent exchange is equalled to 0.01 \( m^2/s \). In this case, the diffusion exchange is practically absent.

**Figure 5.** The numerical solution of the suspension transport problem between two coaxial cylinders.

The proposed difference schemes, based on the LCUSLDS with weight coefficients 2/3 and 1/3, respectively, were calculated as a result of minimizing the order of the approximation error. Fig. 5 shows that the proposed difference schemes have a lower grid viscosity for diffusion-convection problem. Therefore, they more accurately describe the behavior of solution in the case of large grid Peclet numbers.

### 4. Soil dumping problem

#### 4.1. Model description

To describe the transport of suspended particles, we use the diffusion-convection equation in the following form [7, 8]:

\[
c_t' + (uc)_x' + (vc)_y' + ((w + w_g)c)_z' = \mu(c''_{xx} + c''_{yy}) + (vc_z)'_z + f,
\]

(16)

with boundary conditions

\[
C_n' = 0, \ (\vec{V}, \vec{n}) \leq 0, \ \mu C_n' + w_g V_n = 0, \ (\vec{V}, \vec{n}) > 0,
\]
where \( c \) is the suspended solids concentration; \( \vec{V} = \{u, v, w\} \) are components of the 3D velocity vector; \( w_g \) is the hydraulic size or rate of settling of the suspension in the vertical direction; \( \mu \) is the horizontal coefficient of turbulent exchange; \( \nu \) is the vertical coefficients of turbulent exchange; \( f \) is the function source.

To construct a three-dimensional discrete model of lifting, transfer and sedimentation of suspended matter, the grid method is used and it was assumed that the computational domain is inscribed in a parallelepiped. The region of continuous change of arguments is replaced by a discrete set of points (nodes). Instead of functions of a continuous argument, functions of a discrete argument are considered, the values of which are set at the nodal points of the grid [9, 10].

4.2. Results of numerical solution of soil dumping problem

The problem of suspension sedimentation is considered. A marked section of a water body in the form of a parallelepiped was considered as the object under study. The boundary conditions are set in the form of an impermeable wall at the bottom, the side boundaries are permeable and describe the "water-water" section, and a free boundary is set on the surface. The program sets: flow rate; size, shape, location and intensity of the particulate matter source; the size of the computational domain; the coefficients of turbulent exchange in the vertical and horizontal coordinate directions; bottom topography and estimated time, water body depth, coordinates, parameters of suspended matter particles. Table 1 shows the initial data and parameters of the calculated area.

### Table 1. The initial data for the model of suspended particles’ motion and calculated area

| The initial data                  | Parameters of the calculated area                  |
|----------------------------------|---------------------------------------------------|
| The water depth 10 m             | Length 3 km                                      |
| The volume of loading 741 m\(^3\) | Width 1.4 km                                     |
| Flow rate 0.2 m/s                | Step on the horizontal spatial coordinate 20 m    |
| Deposition rate (Stokes) 2.042 mm/s | Step on the vertical spatial coordinate 1 m     |
| Soil density 1600 kg/m\(^3\)    | Calculated interval 2 h                          |
| Percentage of dusty particles \((d < 0.05 \text{ mm})\) in sandy soils 26.83 % |                                                   |

Fig. 6 shows the dynamics of changes in the suspended particles’ concentration (mg/l) over time. Values of suspension concentration field in the cross-section of the computational domain by a plane passing through the discharge point and formed by vectors directed vertically and along the flow. The calculated interval was 15 min, 30 min, 1 hour, and 2 hours, respectively. The currents are directed from left to right.

Based on the obtained materials, we calculated the total amount of contaminated water at soil dumps (see Table 2).

When modeling the transport of suspended particles, it is necessary to use three-dimensional models that take into account the stratification of the concentration of suspended matter in the vertical direction. Another feature of the developed software package is the use of difference schemes with low dissipative properties. The proposed difference scheme for solving diffusion-convection problems practically does not have grid viscosity and, as a consequence, more accurately describe the behavior of the solution in the case of large grid Peclet numbers, and also preserves the smoothness of the solution at the interface between media when solving hydrodynamic problems with a complex shape of the boundary surface (absent oscillations associated with the stepwise approximation of the boundaries).
Figure 6. Suspended particle concentration field values (mg/l) after: 15 min; 30 min; 1 h; 2 h after discharge of hold.

Table 2. The volume of contaminated water at the dumping ground

| Plot number | Total volume of polluted water at a single discharge, million m³ | including water with concentrations of pollutants, million m³ | Number of discharges | Total volume of water with concentrations of pollutants, million m³ |
|-------------|-------------------------------------------------------------|------------------------------------------------------------|----------------------|-------------------------------------------------------------|
|             |                                                              | >0.25 | >20 | >100 |                                                              | >0.25 | >20 | >100 |
| 1           | 1.285                                                       | 0.89  | 0.245 | 0.15  |
| 2           | 1.12                                                        | 0.813 | 0.202 | 0.105 |
| 3           | 1.279                                                       | 0.889 | 0.24  | 0.15  |

5. Conclusions
A difference scheme has been developed for modeling transport problems in the case of large grid Peclet numbers for a complex geometry of the computational domain. Test calculations are carried out on the basis of the proposed and central-difference scheme for solving the problem of the transport of substances at large and small values of the grid Peclet numbers. For the stability of the central difference scheme in the case of large grid Peclet numbers, artificial viscosity is used, which negatively affects the accuracy of calculations. It follows from the examples given that for small Courant numbers (0.1 and less) the proposed difference scheme is more accurate than the others considered in this article. Note that the proposed modification of the Upwind Leapfrog difference scheme for the numerical solution of the diffusion-convection problem has better accuracy compared to other schemes considered in the article for grid values of Peclet numbers in the range $2 \leq Pe \leq 20$.

To test the proposed difference scheme, the problem of transport of substances between two coaxial cylinders for large and small values of the grid Peclet numbers is considered. This movement of the water medium in hydrodynamics is called the Couette - Taylor flow. A steady flow of fluid between two infinitely long coaxial circular cylinders is considered. To compare the results of numerical calculations with the analytical solution for the inner cylinder with a radius of 5 meters, we set the rotation speed equal to 1 m/s, and for the outer cylinder with a
radius of 10 meters, we set the rotation speed equal to 0.5 m/s. The three-dimensional diffusion-convection equation is used to describe the transport of suspended particles. The equation of the suspension transport model is considered under the following boundary conditions. On a free surface, the flux is zero. At the bottom of the reservoir, the flow depends on the sedimentation rate and the concentration of suspended matter. On the lateral surfaces, the flow, depending on the direction of the flow, can be either equal to zero or specified through the normal component of the velocity vector and the concentration of suspended matter at the boundary.

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