Numerical simulation of coupled diffuse radiation and natural convection in a cubic cavity heated from bottom

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Abstract. We present, a three-dimensional numerical simulation of coupled natural convection with diffuse radiation in a cubic cavity whose all four vertical walls are isothermal, the bottom wall is convectively heated and the top wall is insulated. All walls are treated as black, diffuse and opaque for radiation. The simulations are carried out for the fixed Rayleigh (\(Ra=10^5\)) and Prandtl numbers (\(Pr=0.71\)) for a transparent and participating medium. The flow visualization technique Q-criteria has been used for analysis of the flow structure. The isothermal surfaces inside the cavity form vertical co-axially convergent-divergent three-dimensional open and closed nozzles, while inside the cavity Q-criteria reveals the formation of Jellyfish like flow structure. The cavity contains four conical vortices whereas each vortex is occupied in tetrahedron space.

1. Introduction
The two-dimensional analysis of fluid flow and heat transfer is an approximation of more realistic three-dimensional problems. Whereas, most of the time, simple geometries like, cubes, rectangular cuboids etc., have been considered for the idealization of practical complex geometries to understand the phenomenon. However, with the advent of high-performance computing and numerical techniques, the analysis of fluid flow and heat transfer in a three-dimensional geometries becomes possible. Natural convection due to heating from the bottom has many practical applications like, the initial phase of boiling, furnaces heating, electric oven, cooking, etc. The natural convection coupled with radiation in cubic cavities was extensively by studied both experimentally and numerically [1-4] over the years. The works revealed that the flow inside the cavity was spiraling between the core and vertical walls. In addition, the simulation results indicated that the inner spiraling flows were sensible to the location and direction of radiative heat transfer. In contrast, the peripheral spiraling motion was qualitatively insensitive to a radiative mode of heat transfer.

2. Problem description
A cubic cavity as shown in figure 1, which is convectively heated (\(h_\infty = 50\,\text{W/m}^2\text{K}\) and \(T_\infty = 305\,\text{K}\)) from the bottom and a thermally insulated top wall and isothermal (296 K) vertical
walls is chosen to perform coupled natural convection and diffuse radiation simulations. The gravity acts in a downward direction (i.e., negative Y direction). All thermophysical properties are assumed to be constant while density variation is modeled by Boussinque approximation and buoyancy force corresponds to Rayleigh number $10^5$ and Prandtl number of fluid is 0.71. Further, the flow is incompressible, steady and laminar [5,6] and the walls are black, diffuse and gray for radiation.

![Figure 1. Schematic diagram of a cubic cavity which is convectively heated from the bottom](image)

3. Verification
For the present investigation, the buoyantBoussinesqSimpleFoam application of OpenFOAM [7] has been used. First, the application is validated for the coupled diffuse radiation and natural convection against the work by Kumar and Eswaran [6]. The average Nusselt number values by the present simulation match very well with the published results, as shown in Table 1. Further, the bottom heating problem with the present application is verified against the commercial software ANSYS-FLUENT [8]. The average Nusselt numbers match well with the ANSYS-FLUENT as presented in Table 2.

4. Results and Discussions
The results by numerical simulations of coupled diffuse radiation and natural convection in a cubic cavity have been discussed. The results are presented in non-dimensional form and the non-dimensionalizing scales for temperature and velocity are $(T_{\text{free}} - T_c)$ and $(\sqrt{g\beta(T_{\text{free}} - T_c)})$, respectively.

4.1. Temperature and Flow field characteristics
The cross-sectional views of temperature field and projections of velocity vectors are shown in figure 2. The isotherm surfaces are almost circular geometry and form like a co-axially convergent and divergent nozzle. These isosurfaces near to vertical walls forms an open nozzle, while the isosurfaces near the centre of the cavity form a closed nozzle at the top wall. The

Table 1. Average Nusselt number variation on hot wall for the Rayleigh number $Ra = 10^5$

| Optical thickness($\tau$) | Kumar and Eswaran[7] | Present simulation | Difference (%) |
|---------------------------|-----------------------|-------------------|----------------|
| 1                         | 63.18                 | 62.89             | 0.45           |
| 10                        | 27.26                 | 26.98             | 1.03           |
ends of the open nozzle surface are bend due to the adiabatic boundary condition on the top wall in the combined modes of heat transfer. These characteristics of temperature fields are same for transparent and participating medium (figure 2a and 2b) while only maximum value of temperature is different. The velocity vectors near to the isothermal walls (see figure 2c) reveal a smooth descending flow near the walls where viscous effects are more dominant. The rise of fluid from the centre and descends from the all four vertical walls causes development of four vortices where each vortex is occupied in tetrahedron space and its projection is shown at Z=0.5 in figure 2c.

4.2. Q Criterion Characteristics
The Q criterion is the second invariant of strain rate tensor. Its positive value shows the rotation dominates shearing over the fluid element. The isosurface of Q criterion for value of 0.03 is shown in figure 3. A Jellyfish kind of fluid structure is formed.

4.3. Nusselt number
The conduction and radiation Nusselt numbers on the bottom wall for optical thickness $\tau = 0$ and 5 are depicted in figure 4. The minimum conduction Nusselt number is found at the centre of the bottom wall because of the stagnation of the fluid and increases towards walls (figure 4(a) and (b)). While, the maximum radiative Nusselt number is at the centre for the transparent medium case (figure 4(b)) and decreases towards walls. In contrast, the minimum Nusselt number is at the centre and maximum near the corners for the optical thickness $\tau = 5$ (figure 4(d)). This is owing the fact that emission and absorption become almost equal for high optical thickness medium, thus minimizes the radiation Nusselt number.
Figure 4. Variation of conduction and radiation Nusselt numbers on the bottom wall for transparent (a,b) and participating (c,d) media

Table 2. Area average Nusselt numbers on different walls for Ra=10^5 and Pr = 0.71

| Nu/Wall   | τ = 0  | τ = 5  |
|-----------|--------|--------|
|           | Cond   | Rad    | Total  | cond   | Rad    | Tot    |
| Bottom    | 6.696 (6.707) | 1.544 (1.541) | 8.24 (8.248) | 6.941 (6.89) | 0.907 (1.015) | 7.848 (7.956) |
| Isothermal| -1.626 (-1.642) | -0.434 (-0.42) | -2.06 (-2.062) | -1.702 (-1.74) | -0.26 (-0.287) | -1.962 (-1.989) |

Note: Nusselt numbers shown in the parenthesis are from ANSYS-FLUENT [8]

The area average Nusselt numbers on different walls for the optical thickness τ = 0 and 5 are shown in Table 2. The conduction Nusselt number increases with an increase in optical thickness, while the radiative Nusselt number decreases on both the walls.

5. Conclusions
A numerical simulation of combined diffuse radiation and natural convection in a cubic cavity heated from the bottom has been performed for Ra=10^5 and Pr = 0.71 to study the fluid flow and heat transfer characteristics. The following conclusions are drawn from the present study:

1. The isothermal isosurface near to vertical walls form co-axially open nozzles while away from the vertical walls form closed nozzles inside the cavity.
2. The hottest point in the cavity is found at the mid point of the cavity at the bottom wall.
3. The cavity contains four conical vortices where each vortex is occupied in tetrahedron space.
4. The natural convection problem heated from the bottom generates a Jellyfish kind of fluid structure.
5. The total Nusselt number variation on the bottom and isothermal walls decreases with the increase of optical thickness of the medium.

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