Summary: In this paper we present a computer-assisted procedure for proving the existence of transverse heteroclinic orbits connecting hyperbolic equilibria of polynomial vector fields. The idea is to compute high-order Taylor approximations of local charts on the (un)stable manifolds by using the parameterization method and to use Chebyshev series to parameterize the orbit in between, which solves a boundary value problem. The existence of a heteroclinic orbit can then be established by setting up an appropriate fixed-point problem amenable to computer-assisted analysis. The fixed point problem simultaneously solves for the local (un)stable manifolds and the orbit which connects these. We obtain explicit rigorous control on the distance between the numerical approximation and the heteroclinic orbit. Transversality of the stable and unstable manifolds is also proven.

MSC:
- 34C37 Homoclinic and heteroclinic solutions to ordinary differential equations
- 47N20 Applications of operator theory to differential and integral equations
- 34C45 Invariant manifolds for ordinary differential equations
- 37M99 Approximation methods and numerical treatment of dynamical systems

Keywords:
heteroclinic orbits; validated computations; computer-assisted proof

Software:
AUTO-07P; Taylor; AUTO; Matlab; INTLAB; MATCONT

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