Quantum beats and metrology in a rapidly rotating Nitrogen-Vacancy center

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In this paper, we study the dynamical behavior and quantum metrology in a rotating NV center system which is subject to an external magnetic field. Based on the recently realized rapid rotation of nano-rotor [J. Ahn, et. al., Phys. Rev. Lett. 121, 033603 (2018) and R. Reimann, et. al., Phys. Rev. Lett. 121, 033602 (2018)], the frequency of the rotation is close to that of the intrinsic frequency of the NV center system, we predict the quantum beats phenomenon in the time domain, which can be used to measure the angular velocity of rotation and strength of magnetic field. Furthermore, we also discuss the quantum Fisher information in our system, which is associated with the quantum metrology and precise measurement.

I. INTRODUCTION

The nitrogen vacancy (NV) center in diamond [1, 2] has attracted a lot of attentions both theoretically and experimentally. The potential applications of the NV-center include sensing biological cells [3], detecting the magnetic field [4], electric field [5], crystal strain [6], temperature [7] and so on. An NV center in a mechanically rotating diamond has brought the sensing modalities into the rotating frame [8] and it is found that the accumulate Berry phase can serve for the gyroscope [9, 10]. Also, an ensemble of NVs in rotating diamond have been used to explore the magnetic pseudofields generated in the rotating frame [11].

It is well known that two mechanical vibrations with almost frequency and amplitude can generate a periodic pulsation trend in displacement amplitude, named as beat phenomenon, which is widely used for frequency calibration. The quantum beats have been also predicted and observed in an ensemble of A three-level atoms with nearly degenerate excited-states and other quantum systems [12–19]. Moreover, the quantum beats have been reported to be observed in NV center system, which is subject to a periodical modulation [20–22].

In an alternative manner, we study the quantum beats phenomenon in NV center based on the realization of rapid rotation. With the recent experimental feasibility, the angular frequency of a nanomechanical rotor can achieve to the order of MHz [8] or even GHz [23, 24]. For a GHz rotation, its frequency is close to the intrinsic characterized frequency for the triplet ground states of the NV centers, which is induced by the zero splitting of 2.87GHz. Therefore, it is nature that the dynamics in time domain will exhibit a quantum beats phenomenon with the combined performance of the external rotation and intrinsic ground state energy spitting.

II. MODEL AND HAMILTONIAN

We consider a NV-center, which has a triplet ground state denoted by |m = 0⟩ and |m = ±1⟩, and will be shortened as |0⟩ and |±1⟩ in the rest of this paper. To simulate the magnetic field in the environment (for example, the magnetic field of the earth), we consider that the system is subject to a constant magnetic field in the x direction, with the strength in the order of 10−2 mT. Neglecting the strain-induced splitting, the Hamiltonian of the single NV center is written as (here and after, we
set $\hbar = 1$)

$$H = DS_z^2 + g_e \mu_B BS_x,$$

(1)

where $D/(2\pi) = 2.87$ GHz is the zero field splitting between the $|0\rangle$ state and $|\pm 1\rangle$ states. $S_x, S_y, S_z$ are the conventional Pauli spin-1 operators. $g_e = 2$ is the ground-state Lande factor and $\mu_B/(2\pi) = 14$ MHz/mT is the Bohr magneton, and $B$ is the amplitude of the magnetic field. In the presence of the magnetic field, the three energy levels can be obtained as

$$E_1 = \frac{D + \Delta}{2},$$

(2)

$$E_2 = D,$$

(3)

$$E_3 = \frac{D - \Delta}{2},$$

(4)

where $\Delta := \sqrt{D^2 + A^2}$ and $A = 2\mu_B g_e B$. In Fig. 1, we plot the curves of the energy levels as functions of the magnetic field. Governed by the Hamiltonian in Eq. (1), the corresponding free evolution operator is formally expressed as $U_1 = \exp(-iHt)$.

Recently, the NV center is becoming one of promising candidates to serve for quantum sensing and quantum metrology due to its long coherence time even in the room temperature and its feasibility in manipulation. Here, we apply the NV center to perform the quantum metrology on the gyroscope, that is, to estimate the angular velocity of the mechanically rotating. Without generality, we consider the system is rotated along the axis which is determined by the angles $\theta$ and $\phi$ as $n = (\sin \theta \sin \phi, \sin \theta \cos \phi, \cos \theta)$. The effect of the mechanical rotation on the ground states of the NV center is described by the unitary evolution operator

$$U_2 = \exp(-i\Omega S \cdot n),$$

(5)

with $S = (S_x, S_y, S_z)$, and $\Omega$ is the angular velocity of the mechanical rotation. Combining the free evolution and the effect from induced by the mechanical rotation, the dynamics of the system is described by

$$|\psi(t)\rangle = U_2 U_1 |\psi(0)\rangle,$$

(6)

where $|\psi(0)\rangle$ is the initial state.

### III. Quantum Beats Assisted by the Magnetic Field

In the above section, we have outlined the dynamical process for a single NV center yielding an external magnetic field and mechanical rotating. It is reported recently that the rotation frequency can be achieved by 1GHz, which is in the same order of the energy-level spacing of our system as shown in Fig. 1. Therefore, the free evolution governed by the Hamiltonian $H$ and the mechanical rotation will contribute two characterized frequencies to the dynamics of the system, which will naturally induce a quantum beat phenomenon. In this section, we will propose the scheme for performing the quantum metrology by use of the quantum beats phenomenon.

Considering the nowadays experimental feasibility, we prepare the system in the state $|0\rangle$ via the optical pumping technology initially. For the sake of simplicity, we assume that the system is rotated around the magnetic field, that is, the $x$ axis ($\theta = \pi/2, \phi = 0$). The case for an arbitrary rotating axis will be briefly discussed later. In such a situation, the dynamical evolution of the system is expressed by

$$|\psi(t)\rangle = U_2 U_1 |\psi(0)\rangle = \exp(-iD\Delta/2) [b(t)|0\rangle - ia(t)(|1\rangle + |-1\rangle)],$$

(7)

with

$$a(t) = \frac{1}{\sqrt{2}} \left( \cos \left( \frac{\Delta t}{2} \right) \sin (\Omega t) + \frac{1}{\Delta} \sin \left( \frac{\Delta t}{2} \right) [A \cos (\Omega t) + iD \sin (\Omega t)] \right),$$

(8)

$$b(t) = \cos \left( \frac{\Delta t}{2} \right) \cos (\Omega t) + \frac{1}{\Delta} \sin \left( \frac{\Delta t}{2} \right) [-A \sin (\Omega t) + iD \cos (\Omega t)].$$

(9)

It is obvious that the dynamics of the system is determined by the intrinsic zero field splitting, the external magnetic field as well as the mechanical rotating. The population of the system in state $|0\rangle$ is then obtained as

$$P(t) = |b(t)|^2 = \left[ \cos^2 \left( \frac{\Delta t}{2} \right) + \frac{D^2}{\Delta^2} \sin^2 \left( \frac{\Delta t}{2} \right) \cos^2 (\Omega t) \right].$$

(10)

To investigate the dynamics in a detail way, we here perform the Fourier transformation from the time domain to the frequency domain. It then yields

$$P(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} P(t) e^{-i\omega t} dt = \frac{1}{\sqrt{2\pi}} \left[ \frac{1}{\Delta} \delta (\omega - D) + \sum_{i=\pm 1} K_i \delta (\omega - 2\Omega) \right],$$

(11)
we can measure the angular velocity of the mechanically rotation and the magnetic field by the following steps. It is intuitively that the angular velocity can be measured by shutting off the magnetic field because there will be only one components with the frequency $\omega = 2\Omega$ (note that $K_{+1} = K_{-1} = 0$ for $B = 0$). So that, we can obtain the angular velocity of the rotation by measuring the oscillating frequency of the population $P(t)$. However, in a realistic situation, the external weak magnetic field cannot be avoided (for example, the magnetic field of from the earth), so that we here discuss the situation with non-zero magnetic field. As shown in Fig. 2 for an appropriate value of the magnetic field strength, we will have $|K_{+1}| \ll |K_0| = |K_{-1}|$, so that only two frequency components with $\omega_1 = 2\Omega$, and $\omega_2 = |\Delta - 2\Omega|$ (with same strength) can be observed. Measuring these two frequencies and solving the corresponding equations, we can obtain the magnetic field and the angular velocity simultaneously. From the dynamical view, when the angular velocity of the rotating is set as $\Omega = 10D$, which is in the order of 10GHz, we can also observe the quantum beats phenomenon clearly as shown in Fig. 3(a), where we plot the probability $P(t)$ as a function of the evolution time $t$. However, for a much more smaller angular velocity $\Omega = 0.05D$, the dynamical behavior is shown in Fig. 3(b), where we can not observe a quantum beat phenomenon. From the viewpoint of the frequency spectrum, we will obtain that $\omega_2 \approx 0.93\omega_1$ for $\Omega = 10D$, and $\omega_2 \approx 12\omega_1$ for $\Omega = 0.05D$ in the magnetic field strength satisfying $K_0 = K_{-1}$ as shown in Fig. 2. Since the beats will be formed only for two oscillations with same amplitudes and close frequencies, it is natural that a high speed rotation (in the order of GHz as reported in the recent literature [23, 24]) is beneficial to the quantum beats phenomenon.

### IV. QUANTUM FISHER INFORMATION

In the above section, we have demonstrated how to measure the angular velocity and the magnetic field with the assistance of the quantum beats phenomenon. Motivated by the potential application for sensitive gyroscope and magneto-meter based on our system, we will discuss the accuracy of the parameter estimation by studying the QFI with respective to the angular velocity and the magnetic field.

The QFI is a central quantity in the field of quantum metrology and quantum estimation theory. It is introduced by extending the classical Fisher information to the quantum regime, and can characterize how sensitive of a parameter estimation can be achieved by use of the quantum source in a system. According to the quantum Cramer-Rao inequality, the uncertainty $\delta x$ in the estimation for the physical parameter $x$ is bounded by $\delta x \geq 1/\sqrt{\nu F_x}$, where $\nu$ is the times of the independent measurements and $F_x$ is the QFI. A larger QFI corresponds to a more accurate estimation to the parameter.
For a general quantum state described by the density matrix $\rho$, with the spectral decomposition $\rho = \sum_{i=1}^{M} p_i |\psi_i\rangle \langle \psi_i|$, where $M$ denotes the number of nonzero $p_i$, the QFI is given by \[ F_x = \sum_{i=1}^{M} \left( \partial_{x_s} p_i \right)^2 + 4 \sum_{i=1}^{M} p_i \langle \partial_{x_s} \psi_i | \partial_{x_s} \psi_i \rangle - \sum_{i,j=1}^{M} \frac{8p_i p_j}{p_i + p_j} | \langle \psi_i | \partial_{x_s} | \psi_j \rangle |^2. \]

In this paper, we consider a pure state as the initial input state, according to the unitary evolution governed by Eq. (13), the final state is also a pure state. As a result, the QFI in Eq. (13) can be further reduced to

\[ F_x = 4(\langle \partial_{x_s} \psi | \partial_{x_s} \psi \rangle - | \langle \psi | \partial_{x_s} (\psi \rangle |^2). \]

Firstly, we investigate the QFI to the angular velocity $\Omega$. For a pure state, according to the unitary evolution governed by Eq. (8), the final state is also a pure state. As a result, the QFI in Eq. (13) can be further reduced to

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FIG. 7. (Color online) The QFI $\mathcal{F}_B$ versus the time $t$. The parameters are set as $A = 2D/\sqrt{5}$, $\theta = \pi/2$, $\phi = 0$.

FIG. 8. (Color online) The QFI $\mathcal{F}_B$ versus magnetic field $B$. The parameters are set as $t = 10\text{ ns}$, $\theta = \pi/2$, $\phi = 0$.

by

$$\mathcal{F}_B = (g_e \mu_B)^2 \times \begin{cases} \frac{8D^2(\Delta^2+A^2)+64A^4}{A^4} & t \to 0 \\ \frac{16^32A^4}{\Delta^4} & t \to \infty \end{cases}.$$ (17)

Also, the dependence of $\mathcal{F}_B$ on the magnetic field is shown in Fig. 8 for a fixed time $t = 10\text{ ns}$, and it shows that a strong field will lead to a large QFI, that is a more accurate estimation to the strength of the magnetic field.

V. CONCLUSION

In this paper, we have investigated the dynamics and quantum metrology in a NV center system which is subject to a mechanical rotating and external magnetic field. Benefited from the realization of rapid rotation of nano-mechanical rotor, the frequency of the rotation is achieved by GHz, which is in the same order with the intrinsic characteristic frequency of the NV center, and it forms a quantum beats phenomenon. We claim the such beats phenomenon can be used to obtain the information about the angular velocity and the strength of the magnetic field. Furthermore, we also discuss the quantum metrology by calculating the QFI of the system, and it shows a quadratic function versus the evolution time. We hope that our studies will be applicable in designing the next generation quantum gyro and magneto-meter based on solid-spin system.

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[1] M. W. Doherty, N. B. Manson, P. Delaney, F. Jelezko, J. Wrachtrup, and L. C. L. Hollenberg, Phys. Rep. 528, 1 (2013).
[2] R. Schirhagl, K. Chang, M. Loretz, and C. L. Degen, Ann. Rev. Phys. Chem. 65, 83 (2014).
[3] L. P. McGuinness, Y. Yan, A. Stacey, D. A. Simpson, L. T. Hall, D. Maclaurin, S. Prawer, P. Mulvaney, J. Wrachtrup, F. Caruso, R. E. Scholten, and L. C. L. Hollenberg, Nat. Nanotechnol. 6, 358 (2011).
[4] L. Rondin, J.-P. Tietjenn, T. Hingant, J.-F. Roch, P. Maletinsky, and V. Jacques, Rep. Prog. Phys. 77, 056503 (2014).
[5] F. Dolde, H. Fedder, M. W. Doherty, T. Nobauer, F. Rempp, G. Balasubramanian, T. Wolf, F. Reinhard, L. C. L. Hollenberg, F. Jelezko, and J. Wrachtrup, Nat. Phys. 7, 459(2011).
[6] M. W. Doherty, V. V. Struzhkin, D. A. Simpson, L. P. McGuinness, Y. Meng, A. Stacey, T. J. Karle, R. J. Hemley, N. B. Manson, L. C. L. Hollenberg, and S. Prawer, Phys. Rev. Lett. 112, 047601 (2014).
[7] V. M. Acosta, E. Bauch, M. P. Ledbetter, A. Waxman, L.-S. Bouchard, and D. Budker, Phys. Rev. Lett. 104, 070801 (2010).
[8] A. A. Wood, E. Lillette, Y. Y. Fein, N. Tomke, L. P. McGuinness, L. C. L. Hollenberg, R. E. Scholten, and A. M. Martin, Sci. Adv. 4, eaar7691 (2018).
[9] D. Maclaurin, M. W. Doherty, L. C. L. Hollenberg, and A. M. Martin, Phys. Rev. Lett. 108, 240403 (2012).
[10] M. A. Kowarsky, L. C. L. Hollenberg, and A. M. Martin, Phys. Rev. A 90, 042116 (2014).
[11] A. A. Wood, E. Lillette, Y. Y. Fein, V. S. Perunicic, L. C. L. Hollenberg, R. E. Scholten, and A. M. Martin, Nat. Phys. 13, 1070 (2017).
[12] M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge university press, 1997).
[13] W. Chow, M. O. Scully, and J. Stoner, Phys. Rev. A 11, 1380 (1975).
[14] R. Herman, H. Grotch, R. Kornblith, and J. Eberly, Phys. Rev. A 11, 1389 (1975).
[15] T. H. Jeys, F. B. Dunning, and R. F. Stebbings, Phys. Rev. A 29, 379 (1984).
[16] M. Mitsunaga and C. L. Tang, Phys. Rev. A 35, 1720 (1987).
[17] G. C. Hegerfeldt and M. B. Plenio, Phys. Rev. A 47, 2186 (1993).
[18] T. Legero, T. Wilk, M. Hennrich, G. Rempe, and A. Kuhn, Phys. Rev. Lett. 93, 070503 (2004).
[19] D. G. Norris, L. A. Orozco, P. B.-Blostein, and H. J. Carmichael, Phys. Rev. Lett. 105, 123602 (2010).
[20] X.-F. He, P. T. H. Fisk, N. B. Manson, and J. Lumin. 60&61, 739 (1994).
[21] S. C. Rand, A. Lenef, and S. W. Brown, J. Lumin. 53, 68 (1992).
[22] K. Fang, V. M. Acosta, C. Santori, Z. Huang, K. M. Itoh, H. Watanabe, S. Shikata, and R. G. Beausoleil, Phys. Rev. Lett. 110, 130802 (2013).
[23] J. Ahn, Z. Xu, J. Bang, Y.-H. Deng, T. M. Hoang, Q. Han, R.-M. Ma, and T. Li, Phys. Rev. Lett. 121, 033603 (2018).
[24] R. Reimann, M. Doderer, E. Hebestreit, R. Diehl, M. Frimmer, D. Windey, F. Tebbenjohanns, and L. Novotny, Phys. Rev. Lett. 121, 033602 (2018).
[25] S. L. Braunstein and C. M. Caves, Phys. Rev. Lett. 72, 3439 (1994).
[26] S. L. Braunstein, C. M. Caves, and G. J. Milburn, Ann. Phys. NY 247, 135 (1996).
[27] Y. M. Zhang, X. W. Li, W. Yang, and G. R. Jin, Phys. Rev. A 88, 043832 (2013).
[28] W. Zhong, Z. Sun, J. Ma, X. Wang, and F. Nori, Phys. Rev. A 87, 022337 (2013).
[29] J. Liu, H.-N. Xiong, F. Song, and X. Wang, Physica A 410, 167(2014).
[30] J. Liu, X.-X. Jing, W. Zhong, and X.-G. Wang, Commun. Theor. Phys. 61, 45 (2014).