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Charged lepton electric dipole moments from TeV-scale right-handed neutrinos

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Abstract. We study the connection between charged lepton electric dipole moments, $d_l$ ($l = e, \mu, \tau$), and see-saw neutrino-mass generation in a simple two-Higgs doublet extension of the Standard Model plus three right-handed neutrinos (RHN) $N_a$, $a = 1, 2, 3$. For RHN with hierarchical masses and at least one with mass in the 10 TeV range, we obtain the upper bounds of $|d_e| < 9 \times 10^{-30}$ e-cm and $|d_\mu| < 2 \times 10^{-26}$ e-cm. Our scenario favours the normal mass hierarchy for the light neutrinos. We also calculated the cross section for $e^- e^- \rightarrow W^- W^-$ in a high luminosity collider with constraints from neutrinoless double beta decay of nuclei included. Among the rare muon-decay experiments we find that $\mu \rightarrow e \gamma$ is most sensitive and the upper limit is $<8 \times 10^{-13}$.

Contents

1. Introduction 2
2. A simple model with RHNs 3
3. EDMs of charged leptons 6
4. (0νββ) decay and its inverse 8
5. A trio of rare muon decays 10
6. Conclusions 11
Acknowledgments 12
References 12
1. Introduction

In the celebrated see-saw mechanism [1], the right-handed Majorana neutrinos are essential for generating small Majorana masses for the active left-handed neutrinos of the Standard Model (SM). These fields are singlets under the SM gauge group and the exact number required is open to debate. Since the light-neutrino masses are constrained to be less than an eV the masses of the right-handed neutrinos (RHNs) have to be heavier than $10^{12}$ GeV. This fits in well with expectations of the Grand Unified Theory (GUT) although the see-saw scale is lower than the GUT scale which is generally taken to be around $10^{16}$ GeV as required by proton stability. Moreover, the high scale also makes the see-saw mechanism impossible to test directly. The best we can hope for are indirect tests such as leptogenesis or renormalization effects. However, in order to make predictions in these latter studies, additional assumptions have to be made and the results become highly model-dependent. On the other hand, Majorana masses for the light active neutrinos can be tested in neutrinoless double beta decays of nuclei. Even in this case, one has to eliminate other possible sources of lepton number violating new physics such as exotic scalars. Recently, there have been attempts to lower the masses of the RHNs to the TeV in leptogenesis studies [2]. They are particularly useful in supersymmetric models [3]. Clearly, such low-scale Majorana neutrinos are of phenomenological and theoretical interests in their own rights. They can be detected in high-energy colliders and, due to their rich CP properties, effects in low-energy experiments can also be sought. The prominent example is the electric dipole moment (EDM) of a charged lepton denoted by $d_l$ where $l = e, \mu, \tau$. Already the experimental limit on $|d_e|$ is an impressive $<10^{-27}$ e-cm [4] and will be further improved in a new round of experiments. In contrast, the limit on $|d_\mu| < 10^{-19}$ e-cm is much less stringent and dedicated experiments are now being proposed.

In this paper, we investigate the contribution of TeV-scale Majorana RHNs, $N_R$, to $d_l$. An immediate issue is to decide whether they are involved in generating active neutrino masses. Naively, one expects that if they do so then the see-saw mechanism will restrict their Yukawa couplings to be very small, thereby making their contribution to $d_l$ minuscule. Thus, they can play an important role in the see-saw mechanism only in a subtle manner. Radiative effects on the left-handed part of the see-saw mass matrix was calculated in [5]. Previously, it was pointed out [6] that small Yukawa couplings can be avoided for more than one $N_R$. We shall display this in the context of a simple model which consists of the SM plus at least three right-handed Majorana neutrinos, $N_R$, and an additional Higgs doublet. In order not to be confused with possible CP violating phases from the scalar potential we assume that one Higgs doublet couples to $l_R$ and the other to $N_R$. This is the natural flavour conserving extended two-Higgs doublet studied in [7] and is also well known to be part of the minimal supersymmetric standard model. We shall see that one $N_R$ can be arranged to be heavy and is responsible for the see-saw and the other two can be much lighter. Furthermore, their Yukawa couplings can be of order unity. We do not attempt a detailed fit to the neutrino mixing data which can be a separate study but merely to demonstrate the possibility of such a scenario. This very simple set-up also gives rise to a non-vanishing $d_l$ at the two-loop level via a set of Feynman diagrams specific to Majorana fermions as pointed out by [8], a mechanism which has been checked by [9, 10]. The latter also contains a detailed discussion of the two-loop integrals. However, we will concentrate more on the structure of CP violation that appears and will be satisfied with order-of-magnitude estimates. We will be able to give a ‘natural’ order-of-magnitude estimate of the upper limit on $d_l$ coming from the possible existence of multi-TeV scale $N_R$. 

New Journal of Physics 7 (2005) 65 (http://www.njp.org/)
One could wonder why we add one more Higgs doublet in our construction. With only one Higgs doublet, the two-loop contribution to $d_l$ from Majorana neutrinos is negligible [8, 10]. This is because now only the active neutrinos take part as they couple to the W bosons. Then $d_l$ is proportional to their mass-squared differences which are known to be small from neutrino oscillation data. Thus, in the SM extended to include see-saw neutrino mass, although $d_l$ happens at a two-loop it is still undetectably small. In contrast, the SM with massless neutrinos $d_l$ receives contribution at a four-loop or higher. With more than one Higgs doublet the physics changes. The RHNs do not decouple as we shall see later. This model can also serve as a prototype to study the interplay between scalars and Majorana fermions in EDMs.

In section 2, we describe in detail a model with three $N_R$’s. We show how it works to generate sub-eV neutrino masses with one of them having mass in the 10 TeV range while the others can be much higher. For early pioneering work on Majorana neutrinos in gauge theories see [11].

Next, we discuss the Majorana phases in the model and dive into the $d_l$ estimates. Since the physics scale we are interested in is relatively low, the expected renormalization group running of the parameters are not very significant and we shall ignore them. Another possible test of this mechanism can be done using the reaction $e^- e^- \rightarrow W^- W^-$; in the event that the RHNs are not kinematically accessible at LHC or the linear collider. This possibility has not been examined before and is discussed in section 4. No neutrino double beta decays (0$\nu$ββ) of nuclei are discussed here. Section 5 examines the possible tests in the rare decays $\mu^- \rightarrow e^- \gamma$ and $\mu^- \rightarrow 3e$, using the constraint we found previously. Discussions of other low-energy probes of Majorana phases can be found in [12]. Finally, we give our conclusions.

2. A simple model with RHNs

The model we study is the SM with two-Higgs doublet denoted by $\phi_1$ and $\phi_2$ and three RHNs $N_a$ with $a = 1, 2, 3$. The terms in the total Lagrangian of interest to us are given by

$$
\mathcal{L} = \frac{g_2}{\sqrt{2}} \overline{\nu_L} \gamma^\mu e_i W^+_{i\mu} + \text{H.c.} + y^{ij}_i \overline{L}_i \phi_1 e_{Rj} + \zeta^{ij}_i \overline{L}_i \tilde{\phi}_2 N_a + \text{H.c.}
- \frac{1}{2} (M_{ab} N_a N_b^* + \text{H.c.}) - V(\phi_1, \phi_2) + \cdots \; ,
$$

(1)

where $\tilde{\phi}_2 = i \sigma_2 \phi^*_2$ and indices $i, j = 1, 2, 3$. The hypercharge of Higgs doublets are $Y_1 = Y_2 = 1/2$. This is the simplest of two-Higgs doublet models (2HDM). The details of the scalar potential $V(\phi_1, \phi_2)$ is not important for us and will not be spelt out here. Note that a $Z_2$ symmetry can be applied to fields so that the Lagrangian is invariant under the transformation

$$
L_i \rightarrow L_i, \quad \phi_1 \rightarrow \phi_1, \quad \phi_2 \rightarrow -\phi_2, \quad N_a \rightarrow -N_a, \quad e_R \rightarrow e_R.
$$

(2)

In this example, the right-handed singlets only couple to $\phi_2$ which is the simplest way to enforce natural flavour conservation. Other assignments to accommodate flavon models can also be applied. However, such details are not necessary for us. All the fermions are in the weak eigenbasis. $M_{ab}$ is a complex symmetric $3 \times 3$ mass matrix for the right-handed singlets and $a, b = 1, 2, 3$. Without loss of generality, $M_{ab}$ can be chosen to be $\text{diag}(M_1, M_2, M_3)$, where the eigenvalues $M_a$ can be made real and positive. The Yukawa couplings $y^{ij}_i$ and $\zeta^{ij}_i$ are complex but the Higgs potential is taken to be real. In doing so the only source of possible CP violation comes from the Yukawa couplings.
It is worthwhile to examine the number of physical phases in this class of models. The discussion is clearest by first going into the charged-lepton-mass basis. In this basis, we denote the Yukawa coupling by $\zeta' \rightarrow \zeta$. Now consider the general case with $N$ right-handed singlets and $n$ left-handed active neutrinos coupling as above. The neutrino mass matrix is a $(n + N) \times (n + N)$ matrix in a block matrix form

$$
\begin{pmatrix}
0 & v\zeta \\
\bar{v}\zeta^T & M
\end{pmatrix},
$$

(3)

where $v\zeta$ is a $(n \times N)$ Dirac mass matrix and $v$ is a generic vacuum expectation value of the Higgs fields. For instance, for the Higgs doublet model of equation (1) the following substitution should be made: $v \rightarrow v\sin\beta$ where $\tan\beta = v_2/v_1$. Returning to equation (3), the lower block matrix, $M$, is the $(N \times N)$ Majorana mass matrix of the $N_R$s. It is symmetrical and complex and thus contain $N(N + 1)/2$ phases. These can be completely absorbed by the $U(N)$ mixing among the number $N$ of $N_R$ states. Another way to view this is to use the freedom of phase choice in $N_R$ to rotate away the $N$ complex phases in the eigenvalues. Once that is done, the phases of $N_R$ are fixed. The phases of the active $\nu_L$ are still free. A phase redefinition of the $\nu_L$ will then remove $n$ phases from $(n \times N)$ Yukawa terms. This leaves a total of $n(N - 1)$ physical phases [13]. For the case of $n = 3$ and $N = 3$ we have six physical phases. It is customary to assign one phase to the light-neutrino mixing matrix and leave the others in the mass eigenvalues. Moreover, the physics of EDM is seen more clearly using complex Yukawa couplings.

The light-neutrino Majorana mass matrix can be solved from equation (3):

$$
m_v = -v^2 \zeta M^{-1} \zeta^T.
$$

(4)

Explicitly, the matrix elements are

$$
m_{v,ij} = -v^2 \left( \frac{\zeta_i \zeta_j}{M_1} + \frac{\zeta_i \zeta_j}{M_2} + \frac{\zeta_i \zeta_j}{M_3} \right),
$$

(5)

where $i, j = e, \mu, \tau$. The standard see-saw mechanism is to assume $M_1 \sim M_2 \sim M_3 \sim 10^{14}$ GeV and the Yukawa couplings are all of order unity so as to get sub-eV masses for the active neutrinos. It is also noted by many that it is hard to obtain the observed bi-large mixing of the active neutrinos with inverse hierarchical masses. Without more assumptions we can extract one more result, i.e.

$$
|\det m_v| = \frac{v^6 (\det \zeta)^2}{M_1 M_2 M_3},
$$

(6)

which may be useful for constructing neutrino mass models.

On closer examination of equation (5), one discovers other ways of getting small neutrino masses. Firstly, we scale out the lowest of the three $N_R$ masses which we call $M_\prec$. Thus, equation (5) becomes

$$
m_{v,ij} = -\frac{v^2}{M_\prec} \left( \frac{\zeta_i \zeta_j}{r_1} + \frac{\zeta_i \zeta_j}{r_2} + \frac{\zeta_i \zeta_j}{r_3} \right),
$$

(7)

where $r_a = M_a/M_\prec$, $a = 1, 2, 3$ and $r_a \geq 1$ by construction. Each term in equation (7) can be viewed as a complex vector and it is a sum of three such vectors. If they form a triangle, then the
element vanishes. The smallness of the active neutrino masses can then be due to nearly closing of the complex triangle even with a value of $M_\nu$ in the TeV range. Similar techniques have been used to construct different hierarchies for the $N_R$ to yield the experimentally acceptable mass matrices for $m_\nu$ [14]. We leave aside the question of whether this is fine tuning or manifestation of approximate family symmetry of the heavy neutrinos. We take it to be purely phenomenologically motivated.

As an example, we describe a scenario in which three $N_R$ can generate sub-eV active neutrino mass but possesses the features that some of them are light enough, say $\sim$ TeV, so that the see-saw mechanism can be amenable to testing in the near future. Assume that $\xi_{ia}$ obey the following relation:

$$\frac{\xi_{i1}}{\sqrt{r_1}} \left(1 - \frac{\delta_v}{2}\right) = \frac{\xi_{i2}}{\sqrt{r_2}} \exp(i\pi/3) = \frac{\xi_{i3}}{\sqrt{r_3}} \exp(i2\pi/3),$$

where $\delta_v$ is a small parameter we introduced. Then the see-saw mass contribution to the light neutrino from the three $N_R$s nearly cancel among themselves. In the limit $\delta_v = 0$, equation (8) has a geometrical interpretation, i.e. the three terms viewed as vectors in the complex plane form an equilateral triangle. Then a non-vanishing $\delta_v$ is a measure of the deviation from this configuration. In our example, the $\xi_1$ side is $(1 + \delta_v)$ longer than the sum of the other two and the small part left is responsible for the small value of $m_\nu$. As noted before, the model has six physical phases and there are nine complex Yukawa couplings. For definiteness, we will choose the three $\xi_{i1}$s to be real. Clearly, the deviation $\delta_v$ can be associated with either one of the $M_a$ and our choice is for simplicity of discussion. To obtain an acceptable mass matrix we need further assumptions. Taking a hint from the charged leptons, we assume the couplings $\zeta_{ea}$ are such that $\zeta_{ea} \ll \zeta_{ua} \sim \zeta_{ta} \cong \zeta_a$. Then a light-neutrino mass matrix of the normal-hierarchy type emerges

$$m_\nu \sim \frac{\delta_v \zeta_1^2 v^2}{r_1 M_\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \quad (9)$$

It is well known that equation (9) gives the observed bilarge mixing angles. Notice the scale of $m_\nu$ is set by $M_1$ and the parameters $\delta_v$ and $\zeta$ also play a crucial role in determining its magnitude. Moreover, it is important to note that the Yukawa couplings are complex as explicitly displayed in equation (8). These are physical phases which will enter into EDM considerations.

Interestingly, we can extract more information about possible hierarchies in $M_a$. There are three cases we can imagine:

I. $M_1 \sim 10^{12}$ GeV and it sets the scale for light active neutrinos. If we take $\delta_v \sim \xi_1 \sim 0.1$ then this is sufficient to ensure sub-eV neutrinos. As seen in equation (9), $M_2$ and $M_3$ play no role in determining $m_\nu$; thus they can be as light as a few TeV. However, equation (8) dictates that $\xi_{1,2}$ will be very small and hence will not give a detectable $d_l$.

II. $M_1 \ll M_2 \lesssim M_3$ with $M_1 \sim 10$ TeV. For light neutrino masses in the sub-eV range, equation (9) requires the product $\delta_v \zeta_1^2 \lesssim 10^{-9}$. Superficially one would expect that $\delta_v \sim \xi_1 \sim 10^{-3}$. Since we do not have a theoretical basis for the values of these two parameters it is prudent to use experimental constraints. We shall see later that neutrinoless double beta decays limit $\zeta_1 < 0.1$. From equation (8), we see that even if $M_1$ is of the order of $10^6$ GeV the corresponding $\zeta$ will be of order unity. This is the interesting case for $d_l$. Certainly, we can lower $M_1$ by simultaneously reducing $\delta_v$ or $\xi_1$. 

New Journal of Physics 7 (2005) 65 (http://www.njp.org/)
Figure 1. One-loop diagrams which may give non-zero EDM.

Figure 2. The topology of two-loop diagrams: the thick lines represent the fermion lines, and the thin ones could be scalars, gauge bosons, or fermion loop (sub-diagram (d)).

III. $M_1 \sim M_2 \sim M_3$ and they are in the TeV range. In this case, either all the Yukawa couplings are small or GIM-like cancellation with small mass splittings will have to take place. In either case, no interesting $d_l$ arises.

The above discussion is sufficient to illustrate the connection between the see-saw mechanism and $d_l$ we set forth to seek. Now we move on to the discussion of EDM.

3. EDMs of charged leptons

In models with charged scalars and Majorana neutrinos, $d_l$ can begin at one-loop. The relevant Feynman diagrams are given in figure 1. We shall employ the mass eigenstates in our discussions. The mixings between the light active neutrinos and the heavy $N_{R\bar{R}}$ are expected to be very small. Indeed, from the simple case of one family see-saw, this mixing is given by $\zeta \nu/M$. For the parameter values discussed in case II above, we estimate mixing between an active neutrino and the right-handed singlet which we generically called $\theta$ to be $\lesssim 10^{-3}$.

As seen in figure 1(a) both heavy RHNs and light active neutrinos can enter. Moreover, the Yukawa couplings are conjugate of each other. Also the necessary helicity flip occurs in one of the external charged lepton lines; hence, there is no EDM from this diagram. For the $W$ boson exchange diagram, see figure 1(b), active neutrino exchanges are dominant with a small admixture of the RHNs entering. In either case, the two $W$ vertices are also conjugate to each other and clearly there is no EDM from this diagram.

At the two-loop level, there are four distinct topologies we have to consider as shown in figure 2. Figures 2(b)–(d) do not give rise to $d_l$ when the thin lines represent gauge bosons. This is well known form the SM. When they represent scalar particles, the Yukawa couplings involved come in conjugate pairs, thus negating their contributions. Since we have no phases in the scalar
sector we arrive at the result that figures 2(b)–(d) give no EDM. This leaves figure 2(a) as the only type that can lead to a non-vanishing $d_l$.

To see the physics more clearly we put the details in figure 3. The external photon can attach to any charged object in the loops. The Majorana mass insertions for the light neutrinos are indicated by the open box. These also flip helicity and change lepton numbers by two units. The corresponding insertions for the heavy $N_R$s are denoted by the filled boxes. The diagrams with the internal $\nu_j$ lines replaced by $N_a$ or vice versa are multiplied by the mixing $\theta$ alluded to in the discussions at the beginning of this section and will be suppressed.

It is instructive to examine the two $W$-boson diagram. It has two open-box insertions which involve light active neutrinos. Summations over different neutrino species and the three charged leptons $l_i$ are to be taken. The active neutrino mixing matrix elements at the incoming and outgoing lepton vertices can be different; thus leading to a non-vanishing $d_l$. The open-box insertions indicating the lepton number violating nature of the Majorana masses are mandatory for this to happen. They do not exist for Dirac neutrinos and thus $d_l$ cannot happen at the two-loop level with Dirac neutrinos. Explicit calculations \cite{8,10} show that this diagram gives a contribution proportional to $m_\nu^2$ and hence is completely negligible. Similarly, the diagrams figures 3(a) and (b) are suppressed by powers of $m_\nu/M_W$ and also $y_e \sim 10^{-6}$. These graphs can be neglected. This leaves only the two-charged Higgs exchange diagram. The lepton EDM can be estimated as

$$\frac{d_l}{e} \sim \sum_{a < b} \frac{m_i}{(16\pi^2)^2} \sum_{i=e,\mu,\tau} \text{Im} \left[ \zeta_{ia}^* \zeta_{ib} \right] \frac{M_a - M_b}{(M_a + M_b)^3} \ln \frac{M_H}{(M_a + M_b)}$$

in the limit that the $N_R$s are heavier than the charged Higgs boson which we assume to be of the weak scale. Strictly speaking, in the mass eigenbasis, the Yukawa couplings should be modified due to the mixing with the active neutrinos. These are expected to be small, i.e. $O(m_\nu/M)$, and can be neglected. Note that the imaginary part of the product of four Yukawa couplings flips sign when one exchanges indices $a \leftrightarrow b$. In other words, only the antisymmetric part in the loop integral yields the desired EDM operator. On the other hand, the factor $(M_a - M_b)$ also reflects

Figure 3. Two-loop diagrams which give non-vanishing EDM.
that the CP violating effects go away when the masses of two RHNs become degenerate. It is also interesting to note that the diagram with the photon attached to the internal charged lepton has no EDM contribution since the loop integral is completely symmetric in $a$ and $b$.

From equation (10), we see that the EDM scales linearly as the mass of the charged lepton.

For the hierarchical mass of case II and taking $M_2 \sim M_3 \sim 10^8$ GeV and $M_H \sim 200$ GeV, we obtain for the electron EDM

$$|d_e| \sim 9.2 \times 10^{-31}(10 \text{ TeV} / M_1)^2 |\zeta_{e1}/0.1|^2 |\zeta_1|^2 \text{ e-cm.}$$

The above estimate is not very sensitive to the values of $M_H$ and $M_{2,3}$ since their dependence is logarithmic. Notice that we use a small value of $\zeta_{e1}$ as is required by the normal hierarchy solution and $0\nu\beta\beta$ (see next section). If the RHN masses are hierarchical such as $M_1 \ll M_2 < M_3$ then we have

$$|d_e| \sim 9.2 \times 10^{-31}(10 \text{ TeV} / M_1)^2 (M_2 / M_3) |\zeta_{e1}/0.1|^2 |\zeta_1|^2 \text{ e-cm}$$

and will be suppressed compared to equation (11). Our estimate of $d_e$ is three orders of magnitude below the current experimental limit [15] but is within reach of new plan experiments [16]. We note that equation (10) is a good approximation to the actual two Feynman integrals which cannot be given in analytical form. Our numerical investigations show that it is accurate for order-of-magnitude estimates.

We can also give an estimate of the muon EDM and it is

$$|d_\mu| \sim 1.8 \times 10^{-26}(10 \text{ TeV} / M_1)^2 |\zeta_1|^4 \text{ e-cm.}$$

In this case the internal $\tau$ and $\mu$ lines give important contributions. When combined, the coupling involved is $\zeta_1$ which need not be small. In contrast, $\zeta_{e1}$ which is small enters the calculation for $d_e$ (see equation (12)). Thus, it is possible that $d_\mu$ can be more enhanced than just the mass factor $m_\mu / m_e$ when compared to $d_e$. The earlier discussion on the mass scaling violation of muon EDM is given in [17]. This illustrates the importance of doing both types of measurements. We add that our estimate is six to seven orders of magnitude lower than the current limit [18] and will be a challenge even for the newly proposed dedicated $d_\mu$ measurements [19].

4. ($0\nu\beta\beta$) decay and its inverse

Here, we discuss how low-scale $N_R$ affects the decay rates of ($0\nu\beta\beta$) decays of nuclei. We will be concerned with the elementary quark level $dd \rightarrow eeuu$ transition, and not worry about the detailed nuclear physics. At the fundamental fermion level the amplitude is given by the diagrams in figure 4.

An estimate of the amplitude for the 2W exchange diagram is

$$A_a \sim g^4 \frac{1}{M_W^4} \frac{m_{v,ee}}{(p)^2},$$

where $\langle p \rangle$ is the average momentum of the exchanged light neutrino. The corresponding diagram with $N$ replacing the $v$ line is suppressed by $M$ and $\theta^2$. For the amplitude of figure 4(b) with $v$
exchange we estimate
\[ A_b \sim g^2 \frac{m_q m_e}{M_W^2 M_W^2 M_H^2} \frac{m_{\nu,ee}}{(p)^2} \sim 3 \times 10^{-11} A_a, \]  
(15)
where \( m_q \) represents the light quark mass, and with \( N \) exchange we obtain
\[ A_b \sim g^2 \frac{m_q}{M_W M_W M_H M_N} \frac{\zeta}{\theta} \sim 3 \times 10^{-4} A_a. \]  
(16)
Similarly, the dominant \( 2H \) exchange graph is associated with a \( N \)-line and it gives
\[ A_c \sim g^2 \frac{m_q^2}{M_W^2 M_H^4 M_N} \frac{\zeta^2}{\theta} \sim 6 \times 10^{-8} A_a. \]  
(17)

In arriving at the above estimation, we have used the following numerical numbers: \( M_N \sim 10 \text{ TeV} \), \( M_W \sim 100 \text{ GeV} \), \( M_H \sim 200 \text{ GeV} \), \( m_{\nu,ee} \sim 10^{-10} \text{ GeV} \), the quark mass \( m_q \sim 10^{-3} \text{ GeV} \) and \( \langle p \rangle \sim 0.1 \text{ GeV} \) [20]. This analysis suggests that WW exchange with active Majorana neutrino exchange is still the dominant tree-level contribution to \( (0\nu\beta\beta) \) decay.

Interestingly, for low scale right-handed Majorana neutrinos’ important contribution to \( (0\nu\beta\beta) \) decays can come from one-loop diagrams depicted figure 5.

The diagrams generate the effective Lagrangian
\[ \mathcal{L} = F_i \left( \bar{e}^c \hat{L} e \right) \epsilon_1 \cdot \epsilon_2, \quad F_i = \frac{c g^2}{32 \pi^2} \sum_a \frac{e_a^2}{M_a} \ln \left( \frac{M_H}{M_a} \right), \]  
(18)
where \( c \) is an order-one constant which depends on the details of the 2HDM such as scalar mixings. The \( \epsilon_{1,2} \) are polarization four-vectors of the W bosons. We have also assumed \( M_\alpha \gg M_H \) with \( M_H \) denoting a common scalar mass. After dropping the \( m_\nu,ee \) part, the sum in the above equation yields

\[
F_i = \frac{\alpha}{8\pi \sin^2 \theta_W} \frac{\epsilon_{1i}^2}{M_i} \left[ \ln r_2 \exp\left( -\frac{2\pi i}{3} \right) + \ln r_3 \exp\left( -\frac{4\pi i}{3} \right) \right]
\]

(19)

by using equation (8) and assuming \( c = 1 \) for simplicity. If this is the dominant contribution to \((0v\beta\beta)\) decays, then we can set the limit \( \zeta_{e1} < 0.1 \) for \( M_1 = 10 \text{ TeV} \). In comparison, the low-energy effective Lagrangian from the exchange of \( N_1 \) is given by

\[
L_{\text{tree}} = F_t (\bar{e}^\gamma \gamma \gamma, \hat{L} e) \epsilon_{1i} \epsilon_{2r}, \quad F_t = \frac{4\pi \alpha}{\sin^2 \theta_W} \left( \frac{g^2}{M_1} \right).
\]

(20)

The numerical absolute value of the square bracket in equation (19) is \( \sim 5 \) for a large range of \( r_2 \) and \( r_3 \). For \( \zeta_{e1} = 0.1 \) and \( M_1 = 10 \text{ TeV} \) we get \( F_i = 6 \times 10^{-9} \), whereas \( F_t = 4 \times 10^{-11} \) in units of GeV\(^{-1}\). Thus, the loop diagram can be more important even when \( \zeta_{e1} \) is not that large.

The effective Lagrangian of equation (18) can also give rise to two W-boson production in \( e^-e^- \) colliders even when \( N_1 \) is too heavy to be directly produced. The cross section can be easily calculated to be

\[
\sigma(e\bar{e} \rightarrow WW) = \frac{F_i^2 s^2}{128\pi M_W^4}
\]

(21)

which is dominated by the longitudinal components of the W bosons. Although, we have neglected the energy dependence in \( F_i \), this is sufficient for a ball park estimate of the cross section. For a linear collider with \( \sqrt{s} = 2 \text{ TeV} \) and a luminosity of \( 10^{34} \text{ s}^{-1} \text{ cm}^{-2} \), we obtain 1.3 events in a 100 days running for 10 TeV Majorana neutrinos. As mandated by the transient high \( s \) behaviour of equation (21) one would require the highest available energy for a given high luminosity collider to probe this physics. It is easy to check that the usual tree-level \( t \)-channel \( N_R \) exchange mechanism in the see-saw scenario gives a even smaller cross section [21].

5. A trio of rare muon decays

The rare decays \( \mu \rightarrow e + \gamma, 3e \) and \( \mu - e \) conversion in nuclei have always been a favourite for testing models of lepton violations. If TeV-scale Majorana \( N_R \)S exist, one also expects that these processes will occur. A general up-to-date review is given in [22] and we shall follow the notations used there.

We begin with \( \mu \rightarrow e \gamma \). The most general Lorentz and gauge invariant \( \mu - e - \gamma \) interaction is given by

\[
\mathcal{M} = -e A_1 \bar{u}_\mu(p_\mu) \left\{ [f_{E0}(q^2) + f_{M0}(q^2) \gamma^5] \gamma_\mu q^\nu \left( g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \right.

+ [f_{M1}(q^2) + f_{E1}(q^2) \gamma^5] \frac{i\sigma_{\mu\nu} q_\nu}{m_\mu} \right\} u_e(p_e),
\]

(22)
where \( q^\lambda \) and \( A_\lambda \) are the photon four-momentum and polarization respectively and \( p_e = p_\mu - q \).

For \( \mu \to e\gamma \), only the form factors \( f_{M1} \) and \( f_{E1} \) contribute. They can be calculated from the dominant diagram given by figure (6). The transition rate is

\[
B(\mu \to e\gamma) = \frac{3\alpha}{64\pi} \frac{|\zeta_{e1}\zeta_{\mu1}^*|^2}{G_F^2 M_1^4} = 8.01 \times 10^{-11} |\zeta_{e1}\zeta_{\mu1}^*|^2 \left( \frac{10 \text{ TeV}}{M_1} \right)^4.
\]

(23)

In arriving at the last formula, we have ignored the small \( M_H/M_a \) term. The current experimental limit of \( < 1.2 \times 10^{-11} \) [23] sets a loose constraint on the mixings. Alternatively, we can take \( \zeta_{e1} < 0.1 \) as required by our model and \( \zeta_{\mu1} < 1 \) so as not to have strong Yukawa; then we get an upper limit of \( 8 \times 10^{-13} \) for a 10 TeV \( N_R \). We note in passing that similar decays for the \( \tau \) are sensitive to the mixing \( \zeta_{e1} \) which will be hard to obtain from other experiments.

For \( \mu - e \) conversion in nuclei, the see-saw model belongs to the class where the photonic penguin diagram as given in figure 6 with the photon off shell dominates the transition rate. An explicit calculation gives

\[
B_{\text{conv}} = \frac{m_\mu^5 G_F^2 F_p^2 \alpha^4 Z_{\text{eff}}^4 Z}{12\pi^3 \Gamma_{\text{capt}}} B(\mu \to e + \gamma).
\]

(24)

For \( ^{48}_{22}\text{Ti} \), \( F_p = 0.55 \), \( Z_{\text{eff}} = 17.61 \) and \( \Gamma_{\text{capt}} = 1.71 \times 10^{-18} \text{ GeV} \) which implies \( B_{\text{conv}}^{\text{Ti}} \sim 0.004 B(\mu \to e + \gamma) \).

Similarly, for \( \mu \to 3e \) the photonic penguin is the most important graph. The box diagram with two-charged Higgs exchange is completely negligible. The Z-penguin graphs are also sub-dominant. Thus we obtain simply

\[
B(\mu \to 3e) = \frac{2\alpha}{3\pi} \left( \ln \frac{m_\mu}{m_e} - \frac{11}{8} \right) B(\mu \to e\gamma)
\]

(25)

or \( B(\mu \to 3e) \sim 0.006 B(\mu \to e\gamma) \). Hence, \( \mu \to e\gamma \) and \( \tau \to \mu(e)\gamma \) are the most important processes to probe TeV-scale see-saw.

6. Conclusions

In this paper, we have given a detailed study of the connection between see-saw neutrino mass generation and charged-lepton EDMs. The 2HDM we employed is simple and it captures the
physics clearly and succinctly. It is expected to be a crucial part of any elaborate embedding of the see-saw mechanism into a grand unified picture. As noted previously, if all the RHNs have very high masses, i.e. \( > 10^{10} \text{ GeV} \), then \( d_l \) will be undetectably small. We found that it is crucial to have at least one \( N_R \) have a mass in 10 TeV or slightly lower range. In addition, not all the Yukawa couplings can be suppressed as in the charged leptons. The charged lepton EDM arises from two-loop diagrams involving Majorana neutrinos and the associated physical phases that have no counterparts in the SM with Dirac neutrinos. Under a favourable choice of parameters we estimated that the upper limits are \( |d_e| < 9 \times 10^{-30} \text{ e-cm} \) and \( |d_\mu| < 1.8 \times 10^{-26} \text{ e-cm} \) for a 10 TeV Majorana neutrino. The parameters involved are consistent with a normal mass hierarchy for the light active neutrinos.

Interestingly, a RHN in the 10 TeV mass range may also be required for a successful leptogenesis [3]. It is reasonable to expect that the mass of the lowest Majorana neutrino is in the 10 TeV range especially in the supersymmetry context. Moreover, the direct production of \( N_1 \) is out of reach for high-energy colliders under discussion. Even so we considered how a high luminosity \( e^- e^- \) collider in the TeV range can still probe their existence via the same sign \( 2W^- \) production. Completing this we calculated that the rare muon decay \( \mu \rightarrow e\gamma \) at the level of \( 10^{-13} \) is found to be sensitive test of the scenario we discussed. Similarly, one can contemplate the rare \( Z \) decays into \( \tau\mu \) or \( \tau e \). The signatures are clean and unmistakable. At one-loop this proceeds via a similar diagram as given in figure 6. We estimate this to give a branching ratio of \( < 10^{-16} \) which is too small even for a Z-factory.

Interestingly, the CP violation from the see-saw mechanism has negligible effect in the quark sector. This demonstrates clearly that the search of neutron, electron and muon EDMs are independent powerful probes of physics beyond the SM.

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