Mathematical Planning When Choosing Rational Dosages of Ingredients for Adjusting the Composition of Bakery Products

Tatyana Suhareva, Inna Sergienko, Alla Kutsova, Alexander Ratushny

Abstract: The article provides a mathematical solution to the problem of choosing the dosages of ingredients in regulating the baking properties of wheat flour with the use of soy protein isolate, which allows not only adjusting the process but also improving the biological value of bakery products. The chosen main factors that influenced bread quality were the following: \( x_1 \) – the dosage of soybean isolate, %; and \( x_2 \) – dough moisture, %. These factors were compatible and uncorrelated. The central-composite rotatable design was chosen. The research program was laid out in the experiments planning matrix. The order of experiments was randomized using the table of random numbers, which excluded the influence of uncontrollable parameters on the results of the experiment. For statistical processing of the experiment results, the following statistical criteria were used: the significance of regression equations coefficients – Student's t-test, adequacy of equations – Fisher's variance ratio. As a result of the experimental data statistical processing according to the known methods, regression equations that adequately describe at the significance level of 5 % the dependence of the specific volume of bread and dimensional stability on the studied factors were obtained. 

Index Terms: choice of rational dosages, correction of bakery product's structure, mathematical planning of an experiment, soy isolate.

I. INTRODUCTION

The quality of baking products depends on the quality of primary raw material – flour. Therefore, meeting the requirements of the baking and confectionery industry requires adjustment and stabilization of flour properties in terms of gluten content and quality. One of the ways to adjust the baking properties of wheat flour is using soybean flour that features lipoxygenase activity. The lipoxygenase enzyme acts on polyunsaturated fatty acids forming peroxides that have two functions: whitening of wheat flour pigments and stabilization of gluten. The maximum dosage of the enzymatically active soybean flour is 0.5 % of the weight of wheat flour [1], [2].

II. PROBLEM STATEMENT

For choosing the rational dosages of soybean isolate and dough humidity for adjusting the "strength" of wheat flour and dough humidity, mathematical methods of experiment planning were used. The chosen main factors that influenced bread quality were the following: \( x_1 \) – the dosage of soybean isolate, %; and \( x_2 \) – dough moisture, %. These factors are compatible and did not correlate with each other. The limits of variation of the studied factors are shown in Table I. The following criteria for assessing the effect of the selected factors on bread quality indicators were chosen: \( y_1 \) – specific bread volume, cm\(^3\)/100 g; and \( y_2 \) – dimensional stability of hearth bread (H/D). The central-composite rotatable design was chosen. The research program was laid out in the experiments planning matrix. The order of experiments was randomized using the table of random numbers, which excluded the influence of uncontrollable parameters on the results of the experiment [3], [4].

III. METHODS

Block Diagram

For statistical processing of the results of the experiment, the following statistical criteria were used: the significance of regression equations coefficients – Student's t-test, adequacy of equations – Fisher's variance ratio.

### Table I. Design characteristics for wheat flour with gluten of group II (satisfactorily weak)

| Parameters          | \( x_1 \), % | \( x_2 \), % |
|---------------------|--------------|--------------|
| Main level          | 8.0          | 46.0         |
| Variation interval  | 2.0          | 1.0          |
| Upper level         | 10.0         | 47.0         |
| Lower level         | 6.0          | 45.0         |
| Lower star point    | 5.17         | 44.59        |
| Upper star point    | 10.83        | 47.47        |

As a result of the experimental data statistical processing according to the known methods [5], the following regression equations that adequately describe at the significance level of 5 % the dependence of the specific volume of bread and dimensional stability on the studied factors were obtained:
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\[ y_1 = 238.35 - 9.68X_1 + 7.06X_2 - 2.25X_1X_2 + 1.87X_1^2 - 4.45X_2^2; \quad (1) \]

\[ y_2 = 0.5 - 0.006X_1 + 0.04X_2 + 0.01X_1X_2 - 0.008X_1^2 - 0.013X_2^2. \quad (2) \]

The response surfaces described by equations (1) and (2) are shown in Fig. 1; the centers of the response surfaces \( y_1 \) and \( y_2 \) are located in the center of experiment design.

![Fig. 1: Response surfaces](image)

To solve the system of equations (7) with subsequent calculation of the values of the response function, the integrated package MAPLE 8 was used. The calculation was made with changing the radius of the sphere in the range from 1.4 to 0 (Table II).

| No. | \( \rho \) | \( X_1 \) | \( X_2 \) | \( \lambda \) | \( y_1 \) cm\(^3\)/100 g | \( y_2 \) |
|-----|-----|-----|-----|-----|----------------|-----|
| 1   | 0.2 | 0.17 | 0.10 | -30.9 | 240.78          | 0.50 |
| 2   | 0.4 | 0.26 | 0.3  | 15.19 | 233.58          | 0.48 |
| 3   | 0.6 | -0.54| 0.26 | -11.38| 245.98          | 0.51 |
| 4   | 0.6 | 0.34 | -0.49| 10.80 | 231.07          | 0.47 |
| 5   | 1.0 | -0.92| 0.38 | -7.56 | 251.73          | 0.51 |
| 6   | 1.0 | 0.39 | -0.92| 7.8   | 225.39          | 0.44 |
| 7   | 0.8 | -0.73| 0.32 | -8.98 | 248.78          | 0.51 |
| 8   | 0.8 | 0.38 | -0.71| 8.85  | 228.37          | 0.45 |
| 9   | 1.41| -1.32| 0.48 | -5.53 | 258.26          | 0.50 |
| 10  | 1.41| 0.38 | -1.35| 6.73  | 218.00          | 0.41 |

IV. RESULTS

Algorithm. As the optimization results show, promotion of the response surface \( y_1 \) (steps 1 – 7) results in increasing optimization parameters \( y_1 \) and \( y_2 \); further promotion (steps 8 – 10) results in increased response function \( y_2 \), but with that, the value of specific volume (response function \( y_1 \)) reduces. In this regard, the optimum values of independent variables should be \( X_1 = -0.92 \) (-1.32) and \( X_2 = 0.38 \) - 0.48, obtained at the 7th step of optimization; with that, the extremum of the function response \( y_1 = 251.7 - 258.26 \) cm\(^3\)/100 g and \( y_2 = 0.5 - 0.51 \) is achieved.

Moving from the coded factors to the natural ones, the optimum values of soybean isolate dosage \( x_1 \), \% and dough humidity \( x_2 \), \% are obtained (Table III).

| Soybean isolate dosage \( x_1 \), \% | Dough humidity \( x_2 \), \% | Specific volume of bread \( y_1 \), cm\(^3\)/100 g | Dimensional stability \( y_2 \) |
|-------------------------------|-------------------------------|---------------------------------|-------------------------------|
| 5.0                          | 46.5                          | 251                            | 0.51                          |
| 6.0                          | 46.5                          | 258                            | 0.50                          |

To solve the system of equations

\[
\begin{align*}
    dF(X_1, X_2, \lambda) \over dX_1 &= -9.68 - 2.25X_1 + 3.74X_2 + 2\lambda X_1 = 0 \\
    dF(X_1, X_2, \lambda) \over dX_2 &= -7.06X_1 - 2.25X_2 - 8.9X_2 + 2\lambda X_2 = 0 \\
    dF(X_1, X_2, \lambda) \over d\lambda &= X_1^2 + X_2^2 - \rho^2 = 0. \\
\end{align*}
\]

(7)
As a result of the experimental data statistical processing according to the known methods, the following regression equations that adequately describe at the significance level of 5% the dependence of the specific volume of bread and dimensional stability on the studied factors were obtained:

\[
y_1 = 229.39 - 31.14X_1 + 2.28X_2 - 4.01X_1^2 + 16.74X_2^2; \quad (8)
\]

\[
y_2 = 0.54 + 0.09X_1 + 0.006X_2 - 0.007X_1X_2 - 0.05X_1^2 - 0.027X_2^2. \quad (9)
\]

The response surfaces described by equations (8) and (9) are shown in Fig. 2, the centers of the response surfaces \(y_1\) and \(y_2\) are located in the center of experiment design.

![Fig. 2. Response surfaces of the specific volume \(y_1\), cm\(^3\)/100 g (a) and dimensional stability \(y_2\) (b) of the dosage of soybean isolate \(x_1\), % and dough humidity \(x_2\), %](image)

To search for optimal parameters \(x_1\) and \(x_2\), the optimization problem was formulated similar to the case of flour with III group gluten (unsatisfactorily weak):

\[
\begin{align*}
y_1(X_1, X_2) &\rightarrow \text{max} \\
0.35 &\leq y_2(X_1, X_2) \leq 0.5; \\
X_1^2 + X_2^2 &\leq \rho_2. 
\end{align*}
\]

To solve the problem, the Lagrangian multiplier method was used. For this purpose, the target function was made in the form of

\[
F(X_1, X_2, \lambda) = y_1(X_1, X_2) + \lambda(\rho_1 - X_1) + \lambda(\rho_2 - X_2^2). \quad (11)
\]

where \(\lambda\) was the indefinite Lagrange multiplier.

With regard to equations (10) and (11), the target function was obtained:

\[
F(X_1, X_2, \lambda) = 229.39 - 31.14X_1 + 2.28X_2 - 4.01X_1^2 + 16.74X_2^2 + \lambda(X_1^2 + X_2^2 - \rho_2^2). \quad (12)
\]

Let us formulate a system of equations

\[
\begin{align*}
\frac{dF(X_1, X_2, \lambda)}{dX_1} & = -31.14 - 4X_2 - 8.02X_1 + 2\lambda X_1 = 0 \\
\frac{dF(X_1, X_2, \lambda)}{dX_2} & = 2.28X_2 - 4X_1 - 33.48X_2 + 2\lambda X_2 = 0. \\
\frac{dF(X_1, X_2, \lambda)}{d\lambda} & = X_1^2 + X_2^2 - \rho_2^2 = 0. \quad (13)
\end{align*}
\]

To solve the system of equations (13) with subsequent calculation of the values of the response function, the integrated package MAPLE 8 was used. The calculation was made with changing the radius of the sphere in the range from 1.4 to 0 (Table V).

### Table V. Search for optimal input parameters values

| No. | \(\rho\) | \(X_1\) | \(X_2\) | \(\lambda\) | \(y_{1r}\), cm\(^3\)/100 g | \(y_2\) |
|-----|------|------|------|------|--------------------|-------|
| 1   | 0.2  | -0.19| 0.03 | -74.79| 235.49             | 0.54  |
| 2   | 0.2  | 0.19 | -0.007| 81.84 | 222.58             | 0.53  |
| 3   | 0.4  | -0.33| -0.39 | 264.39| 216.29             | 0.52  |
| 4   | 0.4  | 0.39 | -0.005| 42.91 | 261.38             | 0.52  |
| 5   | 0.6  | -0.54| 0.25 | -25.54| 247.34             | 0.52  |
| 6   | 0.8  | 0.64 | -0.48 | -21.79| 253.85             | 0.51  |
| 7   | 1.0  | -0.69| 0.71 | -20.29| 261.38             | 0.50  |
| 8   | 1.41 | -0.78| 1.17 | -19.03| 280.67             | 0.48  |
| 9   | 1.41 | -0.7 | -1.22| -14.66| 268.26             | 0.47  |
| 10  | 1.41 | 1.35 | -0.39| -6.90 | 263.63             | 0.45  |

As the optimization results show (Table V), promotion of the response surface \(y_1\) (steps 1 – 7) results in increasing optimization parameters \(y_1\) and \(y_2\); further promotion (steps 8 – 10) results in increased response function \(y_2\), but with that, the value of porosity (response function \(y_1\)) reduces. In this regard, the optimum values of independent variables should be \(X_1 = -0.35 - (-0.7)\) and \(X_2 = -0.39 - (-1.22)\), obtained at the 7th step of optimization; with that, the extremum of the function response \(y_1 = 263.37 - 268.0\) cm\(^3\)/100 g and \(y_2 = 0.47\) is achieved. Moving from the coded factors to the natural ones, the optimum values of soybean isolate dosage \(x_1\), % and dough humidity \(x_2\), % were obtained (Table VI).

### Table VI. Optimal modes of making bread from wheat flour with group III gluten (satisfactorily weak), and its quality indicators

| Soybean isolate dosage \(x_1\), % | Dough humidity \(x_2\), % | Specific volume of bread \(y_{1r}\), cm\(^3\)/100 g | Dimensional stability \(y_2\) |
|-------------------------------|------------------------|-----------------------------|--------------------------|
| 10.6                          | 47.0                   | 264                         | 0.45                     |
| 13.2                          | 47.0                   | 268                         | 0.47                     |

### V. CONCLUSION

The suggested by the authors mathematical solution to the problem of choosing the dosages of ingredients in regulating the baking properties of wheat flour with the use of soy protein isolate allows not only adjusting the process but also improving the biological value of bakery products.
REFERENCES

1. N. A. Baturina, Vliyanie dobavok muki bobovykh kultur na formirovanie kachestva khaeva iz pshenichnoi muki [The effect of adding legumes flour on the formation of wheat flour bread quality]. Diss. ... Cand. Tech. Sciences: 05.18.01. Saint Petersburg, 2006.

2. L. P. Pashechenko, Y. N. Ryabikina, I. V. Sergienko, and M. A. Tsygannik, Sposob uluchsheniya khlebopекarnykh svoistv muki slaboi o "sile" [The method of improving the baking properties of "weak" flour]. RF patent No. 2374844, 2009.

3. A. I. Boyarinov, and V. V. Kafarov, Metody optimizatsii v khimicheskoi tehnologii [Optimization methods in chemical technology]. Moscow: Chemistry, 2000.

4. I. V. Sergienko, T. N. Sukhareva, S. G. Selyaninov, “Vysokobelkovye ingredienty v stabilizatsii khlebopекarnykh svoistv muki [Protein-rich ingredients in stabilizing flour baking properties],” in Priority areas of food industry development. Collection of scientific articles. Stavropol: Stavropol State Agrarian University, 2016, pp. 525 – 528.

5. Y. P. Grachev, Matematicheskie metody planirovaniya eksperimenta [Mathematical methods of designing an experiment]. Moscow: DeLi, 2005.