Indistinguishability and collection efficiency of transition metal dichalcogenides single photon emitters embedded in silicon nitride photonic integrated circuits

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Abstract. This work explores the impact of waveguide design in photon collection efficiency and indistinguishability of single photon emitters embedded in photonic integrated circuits. Transition Metal Dichalcogenides (TMDC) materials have been selected as single photon emitters because their prominent properties for single photon emission: their giant oscillator strength promotes a stronger Purcell effect and their short exciton lifetimes enhances the indistinguishability of photons. We have calculated the photon extraction efficiency and the indistinguishability of a TMDC point-source of photons with arbitrary orientation embedded at an arbitrary location within a SiN waveguide. For the calculation we propose an analytical model based in the Green dyadic of the Helmholtz equation for different geometries of the waveguide, position of the source and orientation. Calculations have been numerically evaluated through finite-difference time-domain (FDTD) simulations showing consistent results. We have found a maximum coupling up to 81% to the fundamental mode when the quantum emitter is placed in the centre of the waveguide and a maximum indistinguishability of 81% when the emitter is placed 10 nm away from the edge of the waveguide. The results help for a better understanding of the coupling of quantum emitters to nanophotonic devices and photonic integrated circuits (PICs).

1. Introduction

Indistinguishability of single photons generated by point defects is the central topic of quantum photonic integrated circuits for quantum information applications such as linear optical quantum computing, quantum teleportation, or quantum cryptography. Indistinguishable photons were usually generated by parametric down-conversion and, more recently, directly from a single two-level quantum emitter in a solid state environment. Enhancement and control of light–matter interaction through engineered dielectric environments is crucial to guarantee efficient collection and generation of truly identical single photon Fock states that implies indistinguishability. Over the past decade a variety of material systems have been investigated to create on-chip single photon emitters (SPEs) including III-V quantum dots, carbon nanotubes and crystal colour centres such as the NV[2] or SiV centres in diamond[3]. For most of those solid-state SPEs the intrinsic indistinguishability at room temperature is almost zero because pure dephasing rates are typically several orders of magnitude larger than the population decay rate (typically ranging from 3 to 6 orders of magnitude)[4]. Improvement of this efficiency can be achieved by low temperature operation or/and by reducing the radiative lifetime of the SPE using an optical cavity that takes advantage of the Purcell
effect [5]. More recently, SPEs were discovered in monolayer transition metal dichalcogenides (TMDC) [6] and monolayer and multilayer hexagonal boron nitride (hBN) [7]. It has been shown that nanoscale strain engineering can be used to scale up the creation of such 2D-SPEs [8]. TMDC are ideally suited as the active material in cavity-quantum electrodynamics because they exhibit pronounced exciton resonances even at room temperature. This feature is due to their great oscillation strength that leads to an absorption of up to 20% per monolayer [9] and radiative exciton lifetimes on the order of few 100 fs to several ps [10]. This extraordinary short lifetime may lead to indistinguishability values valid for quantum information tasks [11]. In addition, TMDC materials provide advantages in terms of extraction efficiency because flakes or monolayers can be integrated in waveguides of the photonic integrated circuits (PIC) using surface processing [35]. Therefore, improvement in the collection efficiency and indistinguishability through waveguide design is especially relevant.

The aim of this work is to present a method for waveguide design taking photon extraction efficiency and photon indistinguishability as figures of merit. We will present an analytical treatment of light radiation from a TMDC point-source at an arbitrary location and with arbitrary orientation coupled to a SiN waveguide. We will explore how the position of the source and its orientation affects the coupling to the waveguide mode and the indistinguishability of the photons. The formalism used will depart from a single solution of the Helmholtz equation using the Green dyadic. For the treatment of the indistinguishability we will calculate the Purcell factor from the evaluation of the imaginary part of the Green dyadic. We will also explore the effect on the indistinguishability of the width of the waveguide and the position and the orientation of the point-source. FDTD simulations will be used to compare with the analytical model for the coupling and for the calculation of the Purcell effect. The results show remarkable differences between the orientation of the SPE giving a maximum extraction efficiency of 80% when the source is placed at the centre of the waveguide and a value for the indistinguishability of 81% when placed 10 nm away from the edge of the waveguide.

2. Near field effects

Instead taking the dipole radiation analytical formula as the starting point for computing the Hertz vector [12,13,14,15,16,17,18,19,20] we calculate the Green function. By doing this we will implement the effect of the near-field of the point-source [21]. The electromagnetic description of a quantum emitter (two level system) can be given by the dipole source approximation introduced in the Maxwell equations [21]. The approximation considers that the emission wavelength is several orders of magnitude bigger than the size of the source. The lowest order of the quadrupole expansion for the distribution of the current has the following form [22]:

\[ j(r) = -i\omega \mu \delta[r - r_0] \]

Where \( \omega \) is the angular frequency of the emission and \( \mu \) is the dipole moment of the system.
The fields of an arbitrarily oriented electric dipole located at \( r = r_0 \) are determined by the Green dyadic which satisfies the dyadic Helmholtz equation [23]:

\[
\nabla \times \nabla \times \vec{G}(r - r_0) - k^2 \vec{G}(r - r_0) = i \delta[r - r_0]
\]

(1)

The Green’s function has terms in \((kR)^{-1}\), \((kR)^{-2}\) and \((kR)^{-3}\) with \( R = r - r_0 \). In the farfield, where \( r >> \lambda \) (with \( r = |r| \)) only the terms with \((kr)^{-1}\) survive. In the near-field \((r << \lambda)\) the terms with \((kr)^{-3}\) dominate the radial decay. The terms with \((kr)^{-2}\) dominate the intermediate-field at \( r \approx \lambda \). Close to the position of the dipole \((r_0)\) the intensity in the near-field is elongated along the dipole axis [21]. At larger distances (far-field) the intensity spreads transverse to the dipole axis. It can be shown that only the far-field of the dipole contributes to the net energy transport [21]. However, this does not mean that there is no energy contained in the near field. Near field components can be converted into propagating radiation if they interact in the proximity with subwavelength structures [21]. Therefore a quantum emitter placed at an arbitrary location within a symmetric dielectric waveguide can show different coupling to a waveguide mode depending on the waveguide geometry in the near field region.

3. Coupling efficiency

In this section we will show how the coupling efficiency of a TMDC point-source to a SiN waveguide changes with the geometry of the waveguide and with the position and orientation of the source. We will show that the effect of the orientation of the source gives a much stronger coupling for the horizontal position (i.e. when the source is parallel to the x-axis in Figure 1) than vertically (i.e. parallel to y-axis). We will calculate the fraction of light coupled to free radiation (non guided modes). This will allow us to calculate the decay rate enhancement of the source and an approximated estimation of the indistinguishability. The analytical model uses the explicit representation of the Green dyadic of an infinite 3D-rectangular waveguide filled with a linear homogeneous medium and considering arbitrary oriented 3D point sources. We will develop a partial eigenfunction expansion of the Green function involving the complete set of eigenfunctions of the transverse Laplacian operator (see Appendix 1). Those expressions will allow separating the energies of the field into each of the guided modes and obtaining their dependence with the width of the waveguide and the position and orientation of the source. Since we will explore the orientation of the dipole the effect of the waveguide thickness is an identical problem due to the symmetry of the system.

3.1 Analytical model
**Figure 1** shows an infinite (z-axis) waveguide with rectangular section enclosed by a perfect conducting surface S and filled with a linear homogeneous medium with electromagnetic constants $\varepsilon$ and $\mu$.

The electric Green’s dyadic is the solution of the dyadic Helmholtz equation (1) with the boundary conditions of the first and second kind on the surface $S$:[24]:

\[
\begin{align*}
\mathbf{n} \times \vec{G}^{(1)}(\mathbf{r} - \mathbf{r}_0) &= 0, & \mathbf{r} \in S \\
\mathbf{n} \times \nabla \times \vec{G}^{(2)}(\mathbf{r} - \mathbf{r}_0) &= 0, & \mathbf{r} \in S \\
\mathbf{n} \cdot \vec{G}^{(2)}(\mathbf{r} - \mathbf{r}_0) &= 0, & \mathbf{r} \in S
\end{align*}
\]

Where $\mathbf{n}$ is the normal vector of surface S. Each of the components of the Green’s dyadic $G_{\mu\nu}^{(1,2)}$ can be expressed as a double-series expansion over the complete system of eigenfunctions of the transverse Laplacian operator in the waveguide cross-section:[24]:

\[
G_{\mu\nu}^{(1,2)}(\mathbf{r} - \mathbf{r}_0) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \phi_{mn}^{(1,2)\mu}(x,y)\phi_{mn}^{(1,2)\nu}(x_0,y_0)f_{mn}^{(1,2)\mu\nu}(z,z_0)
\]

Where $\phi_{mn}$ represent the normalized value of the field in the cross section of the waveguide corresponding to each $mn$ mode. The explicit form of those eigensolutions is shown in Appendix A. Looking at the first factor in the right hand of (2) we can see that the profile of the Green dyadic components in the cross section is that of the contributing modes $\phi_{mn}(x,y)$. The second factor introduces the overlap between the point source and each mode (by multiplying the whole expression by the normalized field value of each mode at the position of the point source $x_0, y_0$). The contribution for the coupling of a point source inside the waveguide to each mode and its dependence with the width is given by the $f_{mn}$ “weight-elements”. Those elements represent the one dimensional characteristic Green function given by the following expressions:[24]:

\[
\begin{align*}
f_{mn}^{xx} &= \eta \frac{\sqrt{k^2 - \left(\frac{(m+1)\pi}{a}\right)^2}}{k^2}, & f_{mn}^{xy} &= \eta \frac{\frac{(m+1)\pi}{a}}{k^2} = f_{mn}^{yx}, \\
\frac{f_{mn}^{yy}}{f_{mn}^{xx}} &= \eta \frac{\frac{(n+1)\pi}{b}}{k^2}, & \eta &= \frac{1}{2\sqrt{k^2 - \left(\frac{(m+1)\pi}{a}\right)^2 - \left(\frac{(n+1)\pi}{b}\right)^2}} \times \sqrt{\frac{\varepsilon_0 m \varepsilon_0 n}{ab}}
\end{align*}
\]
With \( k \) defined as \( k = \omega \sqrt{\varepsilon \mu} \) being \( \omega \) the angular frequency of emission and \( \varepsilon_{0m}\varepsilon_{0n} \) the Neumann indexes such that \( \varepsilon_{00} = 0 \) and \( \varepsilon_{0m} = 2 \) for \( m \neq 0 \). The denominator in the first of \( \eta \) represents the propagation constant in the \( z \)-axis for each solution.

To calculate numerically the coupling we need to obtain the electric and magnetic fields from the previous expressions of the Green dyadic [21]:

\[
\mathbf{E}(\mathbf{r}) = i\omega \mu \int \mathbf{G}(\mathbf{r} - \mathbf{r}_0)\mathbf{\delta}[\mathbf{r} - \mathbf{r}_0]dV, \quad \mathbf{H}(\mathbf{r}) = \int \nabla \times \mathbf{G}(\mathbf{r} - \mathbf{r}_0)\mathbf{\delta}[\mathbf{r} - \mathbf{r}_0]dV
\]

Taking the \( z \)-component of the Pointing vector and integrating over the cross section of the waveguide we obtain the coupling value \( C_{hv} \) and its dependence with the waveguide width and with the position of the source for vertical \( (\mathbf{\mu} = (010)) \) and horizontal \( (\mathbf{\mu} = (100)) \) point-sources:

\[
C_{h}^{(1,2)} = \int (G_{xx}^{(1,2)}(\mathbf{r} - \mathbf{r}_0)[\nabla \times G_{yx}^{(1,2)}(\mathbf{r} - \mathbf{r}_0)] - [\nabla \times G_{xx}^{(1,2)}(\mathbf{r} - \mathbf{r}_0)] G_{yx}^{(1,2)}(\mathbf{r} - \mathbf{r}_0))dxdy
\]

\[
C_{v}^{(1,2)} = \int (G_{yy}^{(1,2)}(\mathbf{r} - \mathbf{r}_0)[\nabla \times G_{xy}^{(1,2)}(\mathbf{r} - \mathbf{r}_0)] - [\nabla \times G_{yy}^{(1,2)}(\mathbf{r} - \mathbf{r}_0)] G_{xy}^{(1,2)}(\mathbf{r} - \mathbf{r}_0))dxdy
\]

3.2 Quantum emitter inside the waveguide core

The explicit dependence of the coupling with the waveguide width, \( C_{hv}^{(1,2)} \), is contained in the terms \( f_{mm}^{xx} \) and \( f_{mm}^{yy} \) for the horizontal (\( x \)-oriented) and vertical (\( y \)-oriented) sources respectively. Using those expressions we have computed the coupling as a function of the waveguide width for horizontal and vertical point sources emitting at a wavelength of 750 nm (typical for SPEs in WSe2). The source is placed at the center of the waveguide cross-section \( (x_0=a/2, y_0=b/2) \). The waveguide thickness is \( b=250 \) nm. The refractive index of the waveguide is \( n_1=2 \) (similar to the SiN). The index of the surrounding cladding (air) is \( n_2=1 \). Figure 2 shows the coupling of a horizontal and vertical emitter to the \( \text{TE}_{10} \) and \( \text{TM}_{00} \) modes respectively.

![Figure 2](image)

**Figure 2.** (a) Radiation from a horizontal source coupled to the \( \text{TE}_{10} \) mode versus width of the waveguide (b) Vertical orientation (coupled to \( \text{TM}_{00} \) mode). The normalization is respect to the power radiated by the same source in a homogeneous environment.
For the horizontal source the coupling vanish until the width reaches the cut-off of the TE$_{10}$ mode (a=170nm). From this point, the $f_{10}^{xx}$ function increases as the propagation constant decreases with $(1/a)$. This increase reaches a maximum of 79% for a=220 nm where the light confinement is maximum. If the waveguide becomes wider (a>220 nm) the modes start to spread out with lower intensity producing a decay of the coupling that scales with $1/a$. There is no coupling to the lowest order TE$_{00}$ and TM$_{00}$ modes because the $G_{xy}^{(1,2)}$ term vanish at the position of the source for this orientation. This is also expected since the x-components of the fundamental modes are antisymmetric respect to the source. For the vertical orientation the term $G_{xy}^{(1,2)}$ does not vanish at the position of the source for the lowest order TM$_{00}$. The width for the cut-off frequency of the TM$_{00}$ mode is above 100 nm. In this case the $f_{00}^{yy}$ function follows a similar trend than the $f_{10}^{xx}$. We find an optimal coupling of 63% for a width a=200 nm. When the width increases the coupling exponentially decays in a different way for each orientation due to the different $m,n$ values. The overlapping of the horizontal source is about 20% stronger than the vertical source because the value of the transverse electric field component of the TE$_{10}$ is bigger than the TM$_{00}$, at the position $(a/2,b/2)$ [25].

We have evaluated the analytical calculations through a series of FDTD simulations in Lumerical FDTD. Figure 4 shows the geometrical arrangement and set-up parameters used for the FDTD calculations.

![Figure 4. Geometrical setup of FDTD simulation. A dipole source placed inside the core of a SiN waveguide](image)

The waveguide core is represented in the blue region of thickness b=250 nm, length of 3µm and a material index $n_1=2.0$ at the wavelength of 750 nm. The numbers $(x_0,y_0,z_0)$ represents the position of the source. The source is configured as an oscillating point charge and situated at the centre of the waveguide cross-section at a distance $z = 2.5$µm from the origin. The source is orientated along the x-axis with the following emission
parameters: pulse duration of 100 ns, spectral width 10 MHz and central emission wavelength of 750 nm. Two xy-planes are placed at x=0 and x=3 with PML boundary conditions in order to avoid undesired interference effects and simulating an infinite rectangular waveguide. For the coupling to guided modes we integrate the Pointing vector Fourier transform over the surface \( S_1 \) with xy-dimensions equal to core’s thickness and width and we normalize with respect to the total emission in an homogeneous environment. For the coupling to non-guided modes we integrate over surface \( S_2 \) which is placed parallel to the xz-plane at a distance of 300 nm from the top of the core with a N.A.=0.55. The meshing in the region close to the source is set to \( \lambda/100 \), while for the rest of the structure is set to \( \lambda/10 \). For the computation of \( \Gamma/\Gamma_0 \), the Fourier transformed Pointing vector is integrated over the surfaces of a 10x10x10 nm squared box surrounding the source and then normalized with respect the total emission in an homogeneous environment.

![Figure 5](image.png)

**Figure 5.** Radiation coupled to guided modes, normalized with respect to power radiated by the source in a homogeneous environment, versus core width value in nm. (a) Horizontal, (b) Vertical.

Ensuring indistinguishability implies that no other modes have a non-vanishing overlap with the source. That is the reason why we have computed the coupling for all possible modes in the vicinity of the width value where we find the maximum coupling to the fundamental mode. **Figure 5** shows the analytical calculations (lines) for the radiation coupled to the two first contributing modes. For the horizontal source the closest (in frequency) contributing mode to the first order mode TE10 is the second order TE\(_{20}\) mode. This mode has its maximum coupling at a=380nm and disappears (it reaches its cut-off frequency) for a<320 nm. This value is far from the width value for maximum coupling to the fundamental mode (a=200nm). Figure 5 shows also the FDTD simulations (squares). For the horizontal source the coupling decreases as the width approaches to the cut-off value. We observe also a 1/a decrease until the activation of the second order mode (TE\(_{20}\)) at a=315 nm. The maximum coupling of 81% is achieved for a=218 nm. The FDTD results match the analytical calculations within an error of 4% for the coupling value and 1% for the width value. For the vertical source the closest neighbour to the fundamental mode shows up for a>340 nm. The maximum coupling is
63% when a=200 nm. The FDTD simulation gives a small discrepancy with the analytical model of 0.8% for the coupling and 0.1% for the width value. As an example of modal correspondence between analytical and FDTD calculations Figure 6 shows the transversal $E_y$ and $H_z$ field profiles captured by a Fourier Transform monitor in the cross section of the waveguide when there is an optimal overlapping between the horizontal source and the lowest order activated mode. The profile corresponds to the first order TE$_{10}$ mode as it was expected from the analytical calculation.

![Figure 6](image)

Figure 6. (a) FDTD values for the electric field x-component profile in the cross section of the waveguide. (b) FDTD values for the magnetic field z-component profile in the cross section of the waveguide

3.3 *Quantum emitter outside the waveguide core*

There has been several experimental works placing colloidal quantum dots, molecules or TDMC flakes as quantum emitters on top of some nanophotonic devices including[27][35]. This has been done in part because those materials are easily integrated with PICs using hybrid integration. In this section we will explore the coupling of point sources placed in regions outside the waveguide. Figure 7 shows a layout of a source placed out of the waveguide and contained inside a rectangular box of index $n_3$. In this case the thickness of the box plays a relevant role in the coupling of the emission to the waveguide so we will calculate its effect.
This geometry is similar to the proposed in other works [27] where authors studied the coupling of molecular quantum emitters to SiN waveguides. The rectangular box where the source is placed has an index n3=1.8 and dimensions 500x180x1000nm. The box is at the top of a SiN rectangular waveguide with dimensions 500x320x9000nm. The source is centered in the XZ plane of the box at a distance of 10 nm of Region 2 in the positive direction of the y-axis. The orientation of the source is parallel to the x-axis.

The pulse duration, its spectral width and its center wavelength are the same than in Section 3.2. We chose PML boundary conditions surrounding all the structure (i.e., we simulate an infinite waveguide in the z-direction). We will calculate the radiation coupled to the guided modes and the radiation coupled to free space (Region 2.2) when the box thickness varies from 0 to 500 nm. The coupling to the guided modes is calculated by integrating the Pointing vector Fourier transforms along plane (A). The coupling to free space radiation is calculated by integrating in a surface (B) with an N.A. of 0.55. We normalize those values with respect to the transmission through a small cube surrounding the source.

**Figure 7.** Layout of the waveguide and source (out of the waveguide and inside a box of index n3) used for FDTD simulations.

**Figure 8.** Power flux of an emitter situated inside a box of index n3 placed outside of the waveguide AND normalized with respect to power in a homogeneous environment. Red dots: power transmitted from the box to the waveguide through the plane in contact between them. Black squares: power coupled to guided modes. Blue triangles: power coupled to non-guided modes in region 2.3.
**Figure 8** shows the power flux through the surfaces A (black dots) and B (blue triangles) versus the thickness of the box. The box behaves as a low-Q cavity in the y-coordinate. As we increase its dimension in this coordinate (i.e. the height of the box) the light emitted outside of the box shows an oscillating behaviour. The maxima of the oscillations will correspond to values of the thickness equal to multiples of half of the wavelength of emission. This can be seen in Figure 2 where the light transmitted outside the box (red dots) is plotted versus the box thickness. Therefore, changing the box thickness changes the amount of light (i.e. intensity) that we are injecting in the waveguide (Region 2). Black and blue points in Figure 2 show the light coupled to guided modes and non-guided modes, respectively. As expected, both couplings show an oscillatory behaviour with the box thickness. However, the phase of the oscillations is shifted from the phase of the power transmitted outside of the box (red dots). Also, the oscillations of the power coupled to guided modes are 90° phase-shifted with the power coupled to free-radiation. All of this suggest that there is other parameter different from the intensity of the light transmitted outside the box that is changing with the box thickness. Lets get a closer look on the effect of the near field. When evaluating the field at points very close to the source the different components of the electric field show different radial decay5,27. While both the transverse (i.e., parallel to θ coordinate in spherical coordinates) and the longitudinal (i.e. parallel to r coordinate) components contribute to the near-field only the transverse field survives in the far-field. When the thickness of the box \( << \lambda \) the light emitted by the source reflects inside the box and arrives at the waveguide having travelled a distance \( << \lambda \). In this situation the longitudinal field component of the near field is dominant over the transversal component. As we increase the thickness, the field arriving at the waveguide travels distances comparable to \( \lambda \) and the transversal far field becomes dominant. To check this point one can calculate the change of the \( Ex/Ez \) components of the field inside the box when the thickness of the box is varied.

![Figure 9](image)

**Figure 9.** Change in \( \theta = \arctan \frac{Ex}{Ez} \) of the field inside the box versus thickness.

**Figure 9** shows the change of the angle \( \theta = \arctan \frac{Ex}{Ez} \) (i.e. the angular coordinate of the field vector projected in the XZ plane) with the thickness of the box. A change in \( \theta \) of 57° is calculated when the thickness of the box varies from \( y=20\text{nm} \) to \( y=115\text{nm} \).
The calculation gives a variation of the coupling efficiency of about 40% with the box thickness. This change confirms what we could expect since the fundamental guided mode (TE₀₁) has its electric field parallel to the x-axis and the coupling with the guided modes scales with \( \cos \theta \).

### 3.4 Position and orientation of the source

Let's explore now the effect of the position and orientation of the source using our analytical model. **Figure 10** shows the coupling of a source placed inside the waveguide as a function of the x-position and its orientation. The position of the source in the x-axis is varied from the centre of the waveguide (x=0) to x=100 nm. The waveguide width is 210 nm corresponding to the maximum coupling with the TE₁₀ mode. As the horizontal source is separated from the centre, the coupling to symmetric modes starts to decrease and the coupling to antisymmetric modes becomes more efficient. This decreases the coupling to about 70%. The opposite behaviour is obtained for a source oriented parallel to the y-axis. In this case the minimum overlapping is obtained at the centre of the waveguide. The coupling reaches a maximum of 74% (to an antisymmetric mode) at 58 nm from the centre. FDTD simulations (blue squares) matched the analytical results within an error of 1%.

![Figure 10](image.png)

**Figure 10.** Radiation coupled to guided modes versus x-position of the point source inside the waveguide. (a) Horizontal source, (b) Vertical source.

The effect of the orientation of the source in the coupling can be evaluated as follows. There is a \( \cos \alpha \) contribution of the \( f_{mn}^{xx} \) function and a \( \sin \alpha \) contribution of the \( f_{mn}^{yy} \) function to the coupling. Here \( \alpha \) is the angle between the dipole vector and the x axis. The power coupled to guided modes shows a maximum of 80% at the angle \( \alpha = 0^\circ \) (i.e., parallel to the x-axis, horizontal) followed by a cosine decay until \( \alpha = 90^\circ \) (coupling of about 60%). As a consequence, we can state that is not possible to have coupling to a single mode if the source is not accurately oriented parallel to the x or y-axes. As we mentioned before, this is relevant for indistinguishability effects. In principle, according to Figures 2 and 5, one could choose a value for the waveguide width between 100nm and 170nm so only the mode TM₀₀ can contribute to the
coupling. Nevertheless in this range of widths the coupling of a vertical source with the TM00 is small because the maximum happens for a=220nm. Therefore, there is a trade-off between coupling efficiency and indistinguishability for sources arbitrarily oriented.

4. Indistinguishability

Our analytical model allows to obtain from the Green dyadic the analytical expressions for the indistinguishability of single photons as a function of waveguide geometry and position/orientation of the source. This section will show the results of the indistinguishability for a TMDC point-source inside and outside a SiN waveguide. We will also explore, as before, the effects of the geometry of the waveguide and the orientation and position of the source. FDTD simulations will be used again to assess the analytical results. We will show that the orientation plays a strong role and that a maximum of ≈80% of indistinguishability is obtained for optimal conditions.

4.1 Indistinguishability in TMDC materials

The indistinguishability of single-photon s in a two-level system can be defined in the context of two-photon interference in a Hong-Ou-Mandel experiment [36]. Two consecutive emissions from the source are characterized by the same parameters T1 and T2 being T1 the radiative lifetime of the emitter and T2 the coherence time of the emitted radiation. However, the fields radiated from two consecutive excitations have no correlation in phase, since the emitter couples at each event separately to the environment which causes them to dephase separately. [28] The dephasing of the subsequent excitations produces a phase diffusion of the emitted fields and transforms the interference of the two photons dissolving in some degree the phenomenon of coalescence. In this scheme, the two-level system is subject to random fluctuations of its energy that can be described by a stationary stochastic process characterized by a dephasing rate $\Gamma^\star$. This is related to the characteristic time for pure dephasing according to $T_2^\star = 2/\Gamma^\star$ [28]. Without dephasing, every time the photons arrive at the same moment on the beamsplitter they generate a two-photon state leaving the system by the same output port like photons generated by parametric down conversion. However, in the presence of dephasing, the depth and the width of the $g^{(2)}$ function dip are reduced, denoting that photons just mix in a perfect two photon state if they arrive within a time interval corresponding to their coherence time $T_2$. In this situation the quality of photon coalescence and the indistinguishability value $I$ is reduced to [11]:

$$I = \frac{T_2}{2T_1} = \frac{\Gamma}{\Gamma + \Gamma^\star} \tag{3}$$

Where $\Gamma$ is the population decay rate of the emitter. This reduction of the indistinguishability can also be read viewing the emission process with dephasing as the emission of photon wave-packets of duration $T_2/2$. This emission takes place randomly
within a time interval corresponding to the excited state lifetime and leads to a temporal irregularity of the order of $T_1$ [28]. All of this reduces and widens the dip at the origin in the $g^{(2)}$ correlation function.

The pure dephasing rates of solid-state quantum emitters like color centres, quantum dots or organic molecules are about 3 to 6 orders of magnitude larger than their radiative decay rates [11]. For example in InAs quantum dots, where decay rates are typically on the order of $T_1 = 1.6 \text{ ns}[29]$ and $T_2^* = 600 \text{ ps}[30]$ the indistinguishability is only $I \approx 0.19$. For any practical implementation in quantum information processing $I > 50\%$ [28]. Improvement of this efficiency can be achieved increasing the radiative decay rate of the emitter using an optical cavity that takes advantage of the Purcell effect [28]. More recent developments implementing new technics like adiabatic rapid passage have shown $I \approx 0.95$ for semiconductor quantum dots [37].

The situation is radically different for the case of TMDC materials which manifest huge oscillator strengths $f = \Gamma_0/\omega_0 \gtrsim 10^{-3}$, with $\omega_0$ the exciton resonance frequency. This results in optimal light-matter interactions in monolayer structures [31] and radiative lifetimes on the order of few hundreds fs to several ps[10]. The short radiative lifetime yields a largecoupling constant $g$ with electromagnetic modes in diverse dielectric environments where the radiative decay can rival the pure dephasing values in optimal conditions ($\Gamma \approx \Gamma^*$)[32]. While radiation from TMDC deposited on $\text{SiO}_2$ is broadened by substrate-induced effects, recent advances in fabrication (like encapsulation strategies based on graphene solutions or hexagonal BN) yield values close to the homogeneous limit[32]. Since the pure dephasing and the radiative decay rate may have similar values ($\Gamma \approx \Gamma^*$) the indistinguishability can change significantly if the TMDC emitter is inside a cavity and even if it has a relatively small Purcell factor.

### 4.2 Analytic model for the indistinguishability

We can calculate the indistinguishability using (3) and calculating the value of $\Gamma$ from the Purcell factor. The rate of energy dissipation of an emitting dipole in an inhomogeneous environment, compared to that of a homogeneous environment, is equal to the ratio between the power emitted in the inhomogeneous case between the power in a homogeneous environment. In the language of decay rates [21]:

$$\frac{\Gamma}{\Gamma_0} = \frac{P}{P_0}$$

Therefore we can obtain the radiative decay rate enhancement by integrating the power emitted by the source inside the waveguide and normalizing respect to the power in a homogeneous surrounding. From expression (4) we can obtain a dependence on the Green dyadic where the decay rate is related with the imaginary part of the Green dyadic [21]:
\[ \Gamma = \frac{4\omega^2}{\pi c^2 \varepsilon_0} \left[ \mu \cdot \text{Im}\{\mathcal{G}(\mathbf{r}_0, \mathbf{r}_0)\} \cdot \mu \right] \]  

(5)

Where \( \omega \) is the frequency of emission of the source, \( \varepsilon_0 \) is the vacuum dielectric constant, \( c \) the speed of light in vacuum and \( \hbar \) the reduced Planck constant.

Figure 11. Enhancement of the radiative decay rate as a function of the waveguide width from analytical calculation (lines) and FDTD simulations (blue squares). The decay is normalized with respect to a homogeneous environment. (a) Horizontal orientation (b) Vertical.

Figure 11 shows the \( \frac{\Gamma}{\Gamma_0} \) enhancement as a function of the waveguide width calculated using (4) (lines) and simulated by FDTD (blue squares). FDTD details of the calculation can be seen in Appendix B. Both plots of Figure 11 have some similarities in shape to those obtained for the coupling efficiency shown in Figure 5. This is expected since both calculations come from the Green dyadic. The differences between them arise from the different components of the Green dyadic used for the present calculation. For the horizontal source and \( a = 220 \text{ nm} \) we obtain a maximum enhancement \( \frac{\Gamma}{\Gamma_0} \approx 1.6 \). According to (3) and taking \( \Gamma_0 \sim \Gamma^* \sim 100 \text{ ps}^{-1} \) the indistinguishability is 61%. Since the indistinguishability of the source not coupled to the waveguide is as much as 50% the enhanced indistinguishability produced by the waveguide is 11%. A deviation from the optimal width of about 100 nm can lead to a decrease of indistinguishability of about 10%. For the vertical source the maximum radiative enhancements are very close to unity when \( a=200\text{nm} \) and \( a=400\text{nm} \). In this case the indistinguishability remains at 50% even for optimal conditions and it is not strongly dependent of the width. This shows that a good coupling (see Figure 5) is not sufficient condition for a high indistinguishability.
Figure 1. Enhancement of the radiative decay rate as a function of the position of the point source inside the waveguide. Results from analytical calculation (lines) and FDTD simulations (blue squares) are shown. (a) Horizontal (b) Vertical.

Figure 12 shows the $\frac{\Gamma}{\Gamma_0}$ enhancement against the position of the source for the two orientations. For both orientations the enhancement has a similar profile than the coupling versus the position (Figure 10). For the horizontal source the maximum enhancement of 1.6 leads to an indistinguishability of 61% when the position of the source is in the centre ($a/2$, $b/2$) of a waveguide with optimal width ($a=220$nm). A deviation from the optimal position of the source of about 80 nm leads to a decrease of indistinguishability of 20%. For the vertical source the enhancement achieves similar values than the horizontal. The maximum indistinguishability for the vertical orientation is 60% when the position of the source is about 70 nm away from the centre (close to the edge of the waveguide). This means that the indistinguishability for both orientations can be almost the same if they are placed at the optimal position. FDTD simulations provide a maximum enhancement of 1.62 for $x_0=a/2$ matching the analytical calculations within an error of 0.2% for the enhancement and 0.3% for $x_0$.

Figure 13 shows the FDTD results of the decay rate enhancement for horizontal and vertical sources placed outside of the waveguide. The sources are placed at different positions ranging from $x_0=a/2$ to 100 nm away from the edge of the waveguide in the x-direction. The waveguide width is 220 nm in all cases. Due to the index contrast between air and waveguide the electric field feels a strong discontinuity at the interface with an amount comparable to the square of the index ratio at the interface [34]. This effect can lead to an alteration of the mode profile in the vicinity of the edge potentially incrementing the emission enhancement $\frac{\Gamma}{\Gamma_0}$. In this case, $\frac{\Gamma}{\Gamma_0}$ jumps from 0.6 to 3.8 for the horizontal source and from 1.1 to 2.5 for the vertical. Those values provide an indistinguishability close to 80% for the horizontal and 75% for the vertical source when the source is positioned 10 nm away from the edge of the core. The cost of this enhancement is an expected decrease in the coupling efficiency which for the optimal position (at 10 nm away from the edge) is close to 40% (50%) for the horizontal (vertical) sources.
Figure 13. Total radiated power emitted by the source normalized with respect to the power emitted in a homogeneous environment versus position of the point source inside and outside the core. (a) Horizontal source, (b) Vertical source.

4.3. Case of strong coupling

There has been reports on the prediction of strong coupling using TMDCs and waveguides [40]. Following this assumption, we consider a waveguide in the strong-coupling regime where \(2g \gg \Gamma + \Gamma^* + \kappa\) [11]. Here \(g\) is the coupling between the two level system and the electromagnetic field, and \(\kappa\) the cavity decay rate. The indistinguishability is then [11]:

\[
I = \frac{(\Gamma + \kappa)(\Gamma + \kappa + \Gamma^*)}{(\Gamma + \kappa + \Gamma^*)^2}
\] (5)

We can compute \(\kappa\) from the coupling of a source inside the waveguide to non-guided modes (i.e. free-radiation and evanescent modes) using the expansion of the Green dyadic in the continuous spectrum of solutions beyond the discrete subspace of guided modes. This expansion is given by [26]:

\[
G^{(1,2)}_{\mu\nu}(r - r_0) = \int_{\omega^2(n_1^2 - n_2^2)}^{+\infty} \frac{\phi^{(1,2)\mu}_{mn}(x, y)\phi^{(1,2)\mu}_{mn}(x_0, y_0)}{\lambda - \omega^2(n_1^2 - n_2^2)} \sqrt{\lambda - \omega^2(n_1^2 - n_2^2)} \phi_{mn}(a, b)^2 - \phi_{mn}(a, b)^2} d\lambda
\] (6)

Where \(\lambda\) represent the eigenvalue associated with each solution. The expression in the denominator originates a series of periodic maxima depending on the width of the waveguide. We can compute the coupling to free-radiation modes as a function of the width using (6) and integrating the Pointing vector over a surface parallel to the XZ-plane (at \(z=1\) um).
Figure 14. Free radiation coupled to non-guided modes versus normalized waveguide width. The coupling is normalized with respect to power radiated by the source in a homogeneous environment.

Figure 14 shows the radiation coupled to non guided modes versus the normalized width of the waveguide ($a/\lambda$). The average Full Half Width Maximum (FHWM) taken from this series of peaks can be used to estimate the value of $\kappa$ and the indistinguishability enhancement using (5). The resulting FHWM value is in $\lambda$ domain so we have to transform this result to $\omega$ domain. For that we need to use the Jacobian conversion [39]:

$$f(\omega) = f(\lambda) \frac{d\lambda}{d\omega} = -f(\lambda) \frac{c}{\omega^2}$$

This way we obtain our FHWM* value in the $\omega$ domain. Since $Q = \frac{\omega}{2(FHWM^*)}$ with $Q$ the quality factor and $\kappa = \frac{\omega}{2Q}$ [38] we have that $\kappa=$FHWM*. We obtain a cavity decay rate $\kappa = \frac{1}{557\, \text{ps}}$. Assuming optimal conditions where radiative decay rate can rival pure dephasing $\Gamma \approx \Gamma^*$ we obtain $I \approx 81\%$.

5. Conclusions

We have calculated the coupling and the indistinguishability of a point-source (a TMDC quantum emitter at 750nm) of arbitrary orientation at an arbitrary location of a dielectric waveguide (SiN). The analytical model used permits a fast computing of the coupling and indistinguishability from a set of simple analytical expressions coming from the same solution of the dyadic Helmholtz equation for different geometries of the core, source position and orientation. The model can be used in more complex dielectric environments typically utilized in single photon integration engineering such as photonic crystals or coupled cavities. The results of the model have been numerically evaluated through FDTD simulations with excellent agreement. The results show a maximum coupling of 81% to the fundamental mode of a horizontal source placed in the centre of the waveguide with optimal width. The coupling is maximum when the
source is placed in the centre of the waveguide and smoothly decays when separated from the centre due to the overlapping with antisymmetric modes. Maximum indistinguishability of 80% for optimal conditions is found for a source placed 10 nm away from the edge of the waveguide. We hope this work can help for an optimized design of PIC waveguides in quantum photonic circuits.

Appendix A. Explicit form of transverse laplacian operator eigenfunctions

The eigenfunctions are determined by the solutions of [15]:

\[(\nabla^2_{xy} + \kappa_{mn}^2)\phi_{mn}^{(1,2)uv}(x,y) = 0\]

Subject to boundary conditions of first and second kind:

\[
\begin{align*}
\frac{\partial \phi_{mn}^{(1)x}}{\partial x} &= 0; & \phi_{mn}^{(1)y} &= 0; & \phi_{mn}^{(1)z} &= 0 \\
\frac{\partial \phi_{mn}^{(2)x}}{\partial x} &= 0; & \phi_{mn}^{(2)y} &= 0; & \phi_{mn}^{(2)z} &= 0 \quad \text{at } x = 0, b \\
\phi_{mn}^{(2)x} &= 0; & \frac{\partial \phi_{mn}^{(2)y}}{\partial y} &= 0; & \phi_{mn}^{(2)z} &= 0 \\
\phi_{mn}^{(1)x} &= 0; & \frac{\partial \phi_{mn}^{(1)y}}{\partial y} &= 0; & \phi_{mn}^{(1)z} &= 0 \quad \text{at } y = 0, b
\end{align*}
\]

Which leads to the following expressions:

\[
\begin{align*}
\phi_{mn}^{(1)x} &= \cos(m\pi/a) \sin(n\pi/b) \\
\phi_{mn}^{(1)y} &= \sin(m\pi/a) \cos(n\pi/b) \\
\phi_{mn}^{(1)z} &= \sin(m\pi/a) \sin(n\pi/b) \\
\phi_{mn}^{(2)x} &= \sin(m\pi/a) \cos(n\pi/b) \\
\phi_{mn}^{(2)y} &= \cos(m\pi/a) \sin(n\pi/b) \\
\phi_{mn}^{(2)z} &= \cos(m\pi/a) \cos(n\pi/b)
\end{align*}
\]

Where \(\phi_{mn}\) form an orthonormal and complete set of eigenfunctions of the transverse Laplacian operator.

Appendix B. FDTD simulations
We have evaluated numerically the results presented in the previous sections through a series of FDTD simulations (Lumerical FDTD) emulating similar conditions and collecting same type of data. The layout of the geometrical setup is shown in Figure 11.

Figure 11. Geometrical setup of FDTD simulation. A dipole source placed inside the core of a SiN waveguide

The waveguide is represented in the blue region with a thickness “b” of 250 nm, a length of 3μm and a material index n₁=2.0 at the wavelength of 750 nm. The numbers (x₀,y₀,z₀) represents the original position of the source which is configured as an oscillating point charge positioned at the centre of the cross section at a distance of z = 2.5μm from the origin. The source is orientated along the x-axis with the following emission parameters: pulse duration=100 ns, spectral width=10 MHz and central wavelength=750 nm. Two xy-planes are placed at x=0 an x=3 with PML boundary conditions in order to avoid undesired interference effects and simulating an infinite rectangular waveguide. For the simulation of the coupling to guided modes we integrate the pointing vector Fourier transform over the surface S₁ with xy-dimensions equal to the waveguide thickness and width. We normalize with respect the total emission in an homogeneous environment. For the coupling to non-guided modes we integrate over surface S₂ which is placed parallel to the xz-plane at a distance of 300 nm from the top of the core with a N.A.=0.55. The meshing in the region close to the source is set to λ/100 while for the rest of the structure is set to λ/10. For the computation of the Fourier transformed Pointing vector was integrated over the surfaces of a 10x10x10 nm squared box surrounding the source and then normalized with respect the total emission in an homogeneous environment.
References:

[1] Santori, C., Fattal, D., Vučković, J., Solomon, G. S., & Yamamoto, Y. (2002). Indistinguishable photons from a single-photon device. Nature, 419(6907), 594.

[2] Mouradian, S. L., Schröder, T., Poitras, C. B., Li, L., Goldstein, J., Chen, E. H., ... & Lipson, M. (2015). Scalable integration of long-lived quantum memories into a photonic circuit. Physical Review X, 5(3), 031009.

[3] Sipahigil, A., Evans, R. E., Sukachev, D. D., Burek, M. J., Borregaard, J., Bhaskar, M. K., ... & Camacho, R. M. (2016). An integrated diamond nanophotonics platform for quantum-optical networks. Science, 354(6314), 847-850.

[4] Kiraz, A., Ehrl, M., Hellerer, T., Müstecaplıoğlu, Ö. E., Bräuchle, C., & Zumbusch, A. (2005). Indistinguishable photons from a single molecule. Physical review letters, 94(22), 223602.

[5] Gérard, J. M., Sermage, B., Gayral, B., Legrand, B., Costard, E., & Thierry-Mieg, V. (1998). Enhanced spontaneous emission by quantum boxes in a monolithic optical microcavity. Physical review letters, 81(5), 1110.

[6] Tonndorf, P., Schmidt, R., Schneider, R., Kern, J., Buscema, M., Steele, G. A., ... & Bratschitsch, R. (2015). Single-photon emission from localized excitons in an atomically thin semiconductor. Optica, 2(4), 347-352.

[7] Tran, T. T., Bray, K., Ford, M. J., Toth, M., & Aharonovich, I. (2016). Quantum emission from hexagonal boron nitride monolayers. Nature nanotechnology, 11(1), 37.

[8] Kumar, S., Kaczmarchczyk, A., & Gerardot, B. D. (2015). Strain-induced spatial and spectral isolation of quantum emitters in mono-and bilayer WSe2. Nano letters, 15(11), 7567-7573.

[9] Li, Y., Chernikov, A., Zhang, X., Rigosi, A., Hill, H. M., van der Zande, A. M., ... & Heinz, T. F. (2014). Measurement of the optical dielectric function of monolayer transition-metal dichalcogenides: MoS 2, Mo S e 2, WS 2, and WS e 2. Physical Review B, 90(20), 205422.

[10] Pöllmann, C., Steinleitner, P., Leierseder, U., Nagler, P., Plechinger, G., Porer, M., ... & Huber, R. (2015). Resonant internal quantum transitions and femtosecond radiative decay of excitons in monolayer WSe 2. Nature materials, 14(9), 889.

[11] Grange, T., Hornecker, G., Hunger, D., Poizat, J. P., Gérard, J. M., Senellart, P., & Auffèves, A. (2015). Cavity-funneled generation of indistinguishable single photons from strongly dissipative quantum emitters. Physical review letters, 114(19), 193601.
[12] Brueck, S. R. J. (2000). Radiation from a dipole embedded in a dielectric slab. IEEE Journal of Selected Topics in Quantum Electronics, 6(6), 899-910.

[13] Creatore, C., & Andreani, L. C. (2008). Quantum theory of spontaneous emission in multilayer dielectric structures. Physical Review A, 78(6), 063825.

[14] Alexandrov, O., & Ciraolo, G. (2004). Wave propagation in a 3-D optical waveguide. Mathematical Models and Methods in Applied Sciences, 14(06), 819-852.

[15] Bermel, P., Joannopoulos, J. D., Fink, Y., Lane, P. A., & Tapalian, C. (2004). Properties of radiating pointlike sources in cylindrical omnidirectionally reflecting waveguides. Physical Review B, 69(3), 035316.

[16] Schneider, P. I., Srocka, N., Rodt, S., Zschiedrich, L., Reitzenstein, S., & Burger, S. (2018). Numerical optimization of the extraction efficiency of a quantum-dot based single-photon emitter into a single-mode fiber. Optics express, 26(7), 8479-8492.

[17] Hoehne, T., Schnauber, P., Rodt, S., Reitzenstein, S., & Burger, S. (2019). Numerical Investigation of Light Emission from Quantum Dots Embedded into On-Chip, Low-Index-Contrast Optical Waveguides. physica status solidi (b), 256(7), 1800437.

[18] Verhart, N. R., Lepert, G., Billing, A. L., Hwang, J., & Hinds, E. A. (2014). Single dipole evanescently coupled to a multimode waveguide. Optics express, 22(16), 19633-19640.

[19] Chen, Y., Nielsen, T. R., Gregersen, N., Lodahl, P., & Mørk, J. (2010). Finite-element modeling of spontaneous emission of a quantum emitter at nanoscale proximity to plasmonic waveguides. Physical Review B, 81(12), 125431.

[20] Devaraj, V., Jang, Y., & Lee, D. (2016). Maximum photon extraction from a single quantum dot embedded in a metal/dielectric-cladded cylindrical structure. Journal of the Korean Physical Society, 68(8), 1014-1018.

[21] Novotny, L., & Hecht, B. (2012). Principles of nano-optics. Cambridge university press.

[22] De Wolf, D. A. (2001). Essentials of electromagnetics for engineering. Cambridge University Press.

[23] Dyadic Green’s Function: EECS 730 Winter 2009 c K. Sarabandi

[24] Hanson, G. W., & Yakovlev, A. B. (2013). Operator theory for electromagnetics: an introduction. Springer Science & Business Media.

[25] Chuang, S. L., & Chuang, S. L. (1995). Physics of optoelectronic devices.

[26] Santosa, F., & Magnanini, R. (2001). Wave propagation in a 2-D optical waveguide. SIAM Journal on Applied Mathematics, 61(4), 1237-1252.
[27] Lombardi, P., Ovvyyan, A. P., Pazzagli, S., Mazzamuto, G., Kewes, G., Neitzke, O., ... & Toninelli, C. (2017). Photostable molecules on chip: integrated sources of nonclassical light. ACS Photonics, 5(1), 126-132.

[28] Bylander, J., Robert-Philip, I., & Abram, I. (2003). Interference and correlation of two independent photons. The European Physical Journal D-Atomic, Molecular, Optical and Plasma Physics, 22(2), 295-301.

[29] J.M. G’erard, O. Cabrol, B. Sermage, Appl. Phys. Lett. 68, 3123 (1996)

[30] P. Borri, W. Langbein, S. Schneider, U. Woggen, R.L. Sellin, D. Ouyang, D. Bimberg, Phys. Rev. Lett. 87, 157401-1 (2001)

[31] Jakubczyk, T., Delmonte, V., Koperski, M., Nogajewski, K., Faugeras, C., Langbein, W., ... & Kasprzak, J. (2016). Radiatively limited dephasing and exciton dynamics in MoSe2 monolayers revealed with four-wave mixing microscopy. Nano letters, 16(9), 5333-5339.

[32] Schneider, C., Glazov, M. M., Korn, T., Höfling, S., & Urbaszek, B. (2018). Two-dimensional semiconductors in the regime of strong light-matter coupling. Nature communications, 9(1), 2695.

[33] Cadiz, F., Courtade, E., Robert, C., Wang, G., Shen, Y., Cai, H., ... & Manca, M. (2017). Excitonic linewidth approaching the homogeneous limit in MoS 2-based van der waals heterostructures. Physical Review X, 7(2), 021026.

[34] Majumder, S., & Chakraborty, R. (2013). Semianalytical method to study silicon slot waveguides for optical sensing application. Optical Engineering, 52(10), 107102.

[35] Peyskens, F., Chakraborty, C., Muneeb, M., Van Thourhout, D., & Englund, D. (2019). Integration of Single Photon Emitters in 2D Layered Materials with a Silicon Nitride Photonic Chip. arXiv preprint arXiv:1904.08841.

[36] Hong, C. K., Ou, Z. Y., & Mandel, L. (1987). Measurement of subpicosecond time intervals between two photons by interference. Physical review letters, 59(18), 2044.

[37] Wei, Y. J., He, Y. M., Chen, M. C., Hu, Y. N., He, Y., Wu, D., ... & Pan, J. W. (2014). Deterministic and robust generation of single photons from a single quantum dot with 99.5% indistinguishability using adiabatic rapid passage. Nano letters, 14(11), 6515-6519.

[38] Vuckovic, J. (2014). Quantum optics and cavity QED with quantum dots in photonic crystals (No. arXiv: 1402.2541).

[39] Mooney, J., & Kambhampati, P. (2013). Get the basics right: Jacobian conversion of wavelength and energy scales for quantitative analysis of emission spectra.

[40] Walker, P.M., Whittaker, C.E., Skryabin, D.V. et al. Spatiotemporal continuum generation in polariton waveguides. Light Sci Appl 8, 6 (2019)