Polarization-universal rejection filtering by ambichiral structures made of indefinite dielectric-magnetic materials

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Abstract
An ambichiral structure comprising sheets of an anisotropic dielectric material rejects normally incident plane waves of one circular polarization (CP) state but not of the other CP state, in its fundamental Bragg regime. However, if the same structure is made of a dielectric-magnetic material with indefinite permittivity and permeability dyadics, it may function as a polarization-universal rejection filter because two of the four plane-wave components of the electromagnetic field phasors in each sheet are of the positive-phase-velocity type and two are of the negative-phase-velocity type.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction
This communication combines two topics of recent interest in electromagnetics: (i) the ambichiral structure that replicates the circular-polarization-sensitive filtering properties of cholesteric liquid crystals and chiral sculptured thin films, and (ii) the dielectric-magnetic material with indefinite permittivity dyadic and indefinite permeability dyadic.

The ambichiral structure is a structurally chiral pile of identical sheets that was conceived by Reusch [1] to transmit normally incident circularly polarized (CP) plane waves of one handedness but reflect CP plane waves of the other handedness, in a certain free-space-wavelength regime. This conceptualization influenced early theoretical research on the optical response characteristics of cholesteric liquid crystals [2, 3]. Following a systematic study in 2004, Reusch’s wavelength regime was identified as merely the first of a potentially infinite number of Bragg regimes [4]. For optical applications, since then the ambichiral structure has been experimentally realized using sculptured-thin-film technology [4, 5], and electro-optic versions have been suggested as electrically controlled CP filters [6, 7].

A real symmetric dyadic is said to be indefinite if some of its eigenvalues are positive but the remaining ones are negative. Artificial materials with indefinite permittivity and permeability dyadics came into prominence a few years ago, as such materials can exhibit negative refraction [8, 9]. Due to hyperbolic, instead of the usual elliptic, dispersion relations for planewave propagation in these materials [10, 11], several electromagnetic phenomenons—including surface-wave propagation [12], the Goos–Hänchen shift [13], and diffraction by surface-relief gratings [14, 15]—are exhibited by these materials in uncommon ways.

Motivated by these reports, an investigation was undertaken on the response to a normally incident plane wave of an ambichiral structure made of a dielectric–magnetic material with indefinite permittivity dyadic and indefinite permeability dyadic. True to expectation, the usual CP-filtering response of ambichiral structures was not obtained. Instead, a polarization-universal rejection response emerged, indicating thereby the existence of a polarization-universal bandgap [16, 17].

The plan of this communication is as follows: section 2 contains a description of the ambichiral structure comprising orthorhombically anisotropic dielectric–magnetic sheets. Section 3 provides a succinct description of the boundary-value problem to be solved in order to determine the response characteristics of the ambichiral structure to a normally
incident plane wave. Finally, numerical results are presented and discussed in section 4.

A note on notation: vectors are underlined and dyadics are double-underlined; the cartesian unit vectors are represented by $\mathbf{\hat{u}}_x$, $\mathbf{\hat{u}}_y$, and $\mathbf{\hat{u}}_z$; symbols for column vectors and matrices are decorated by an overbar; and an \text{exp}(\text{i}\omega t)$ time-dependence is implicit with $\omega$ as the angular frequency. The wavenumber, the wavelength and the intrinsic impedance of free space are denoted by $k_0 = \omega / \sqrt{\mu_0 \varepsilon_0}$, $\lambda_0 = 2\pi / k_0$ and $\eta_0 = \sqrt{\mu_0 / \varepsilon_0}$, respectively, with $\mu_0$ and $\varepsilon_0$ being the permeability and permittivity of free space.

2. Ambichiral structure

The ambichiral structure is a structurally chiral pile of $N$ identical sheets, each of thickness $D$ and infinite transverse extent. The $n$th sheet, $1 \leq n \leq N$, occupies the region $(n-1)D < z < nD$; thus, the total thickness of the pile is $L = ND$. The sheets are not necessarily electrically thin, and the structure can therefore be considered as a $1-D$ photonic crystal [18, 19]. The halfspaces $z < 0$ and $z > L$ are vacuous.

The permittivity and permeability dyadics of the $n$th sheet are chosen as

$$\epsilon(z) = \epsilon_0 S_{\xi_n} (h_{\xi_n}) \cdot S_\varphi (\chi) \cdot \left( \epsilon_a \mathbf{\hat{u}}_z \cdot \mathbf{\hat{u}}_z + \epsilon_b \mathbf{\hat{u}}_{\perp} \cdot \mathbf{\hat{u}}_{\perp} + \epsilon_c \mathbf{\hat{u}}_{\parallel} \cdot \mathbf{\hat{u}}_{\parallel} \right) \times S_\varphi (-\chi) \cdot S_{\xi_n} (-h_{\xi_n})$$

(n - 1)D < z < nD

and

$$\mu(z) = \mu_0 S_{\xi_n} (h_{\xi_n}) \cdot S_\varphi (\chi) \cdot \left( \mu_a \mathbf{\hat{u}}_z \cdot \mathbf{\hat{u}}_z + \mu_b \mathbf{\hat{u}}_{\perp} \cdot \mathbf{\hat{u}}_{\perp} + \mu_c \mathbf{\hat{u}}_{\parallel} \cdot \mathbf{\hat{u}}_{\parallel} \right) \times S_\varphi (-\chi) \cdot S_{\xi_n} (-h_{\xi_n})$$

(n - 1)D < z < nD

respectively. Both dyadics indicate anisotropy of the orthorhombic symmetry [20, 21]. The dyadic

$$S_{\xi_n} (h_{\xi_n}) = \left( \mathbf{\hat{u}}_z \cdot \mathbf{\hat{u}}_z + \mathbf{\hat{u}}_{\perp} \cdot \mathbf{\hat{u}}_{\perp} \right) \cos(h_{\xi_n}) + \left( \mathbf{\hat{u}}_z \cdot \mathbf{\hat{u}}_z - \mathbf{\hat{u}}_{\perp} \cdot \mathbf{\hat{u}}_{\perp} \right) \sin(h_{\xi_n}) \cdot \mathbf{\hat{u}}_{\parallel} \cdot \mathbf{\hat{u}}_{\parallel}$$

indicates rotation about the $z$-axis by an angle $h_{\xi_n}$ with respect to the first sheet, with

$$\xi_n = (n - 1) \frac{\pi}{q}, \quad 1 \leq n \leq N,$$

the integer $q \geq 3$ [4], the ratio $N/q$ an even integer, and the parameter $h = 1$ for structural right-handedness and $h = -1$ for structural left-handedness. The number of structural periods in the ambichiral structure is $N/2q$.

The dyadic

$$S_{\chi} (\chi) = \left( \mathbf{\hat{u}}_x \cdot \mathbf{\hat{u}}_x + \mathbf{\hat{u}}_y \cdot \mathbf{\hat{u}}_y \right) \sin \chi + \left( \mathbf{\hat{u}}_z \cdot \mathbf{\hat{u}}_z - \mathbf{\hat{u}}_x \cdot \mathbf{\hat{u}}_x \right) \sin \chi \cdot \mathbf{\hat{u}}_{\parallel} \cdot \mathbf{\hat{u}}_{\parallel} \cdot \mathbf{\hat{u}}_{\parallel}$$

$\chi \in [0, \pi/2]$ indicates a tilt with respect to the $xy$-plane by an angle $\chi$.

3. Reflectances and transmittances

Suppose that an arbitrarily polarized plane wave is normally incident on the ambichiral structure from the halfspace $z < 0$. In consequence, a reflected plane wave must exist in the same halfspace and a transmitted plane wave in the halfspace $z > L$. The electric field phasors associated with the two plane waves in the halfspace $z \leq 0$ are stated as

$$E_{\text{inc}} (r) = \left( a_L \mathbf{\hat{u}}_x + a_R \mathbf{\hat{u}}_x \right) \exp (\text{i}k_0 z), \quad z \leq 0$$

and

$$E_{\text{refl}} (r) = \left( r_L \mathbf{\hat{u}}_x + r_R \mathbf{\hat{u}}_x \right) \exp (-\text{i}k_0 z), \quad z \leq 0,$$

where $\mathbf{\hat{u}}_x = (\mathbf{\hat{u}}_x \pm i \mathbf{\hat{u}}_y) / \sqrt{2}$. Likewise, the electric field phasor in the halfspace $z \geq L$ is represented as

$$E_{\text{trans}} (r) = \left( t_L \mathbf{\hat{u}}_x + t_R \mathbf{\hat{u}}_x \right) \exp [\text{i}k_0 (z - L)], \quad z \geq L.$$
shall have to be interchanged if either

\[ \epsilon_d = \epsilon_a \cos^2 \chi + \epsilon_b \sin^2 \chi, \]  \hspace{1cm} (16)

\[ \mu_d = \mu_a \mu_b / (\epsilon_a \cos^2 \chi + \epsilon_b \sin^2 \chi). \]  \hspace{1cm} (17)

The reflection amplitudes \( r_{LR} \) and the transmission amplitudes \( t_{LR} \) can be computed for specified incidence amplitudes \((a_L, a_R)\) by solving (9). Interest usually lies in determining the reflection and transmission coefficients entering the \( 2 \times 2 \) matrices in the following two relations:

\[ \begin{pmatrix} r_L \\ r_R \end{pmatrix} = \begin{pmatrix} r_{LL} & r_{LR} \\ r_{RL} & r_{RR} \end{pmatrix} \begin{pmatrix} a_L \\ a_R \end{pmatrix}, \]  \hspace{1cm} (18)

\[ \begin{pmatrix} t_L \\ t_R \end{pmatrix} = \begin{pmatrix} t_{LL} & t_{LR} \\ t_{RL} & t_{RR} \end{pmatrix} \begin{pmatrix} a_L \\ a_R \end{pmatrix}. \]  \hspace{1cm} (19)

Both \( 2 \times 2 \) matrices are defined phenomenologically. The co-polarized transmission coefficients are denoted by \( t_{LL} \) and \( t_{RR} \), and the cross-polarized ones by \( t_{LR} \) and \( t_{RL} \); and similarly for the reflection coefficients in (18). Reflectances and transmittances are denoted, e.g., as \( T_{LR} = |t_{LR}|^2 \).

4. Numerical results and discussion

Let us assume that the chosen material has negligible dispersion and dissipation over the free-space-wavelength range of interest, for the sake of simplicity. With the assumption that both the relative permittivity and the relative permeability dyadics are positive definite (i.e. \( \epsilon_{a,b,c} > 0 \) and \( \mu_{a,b,c} > 0 \), the fundamental Bragg regime of the ambichiral structure for normally incident plane waves has [4]

\[ \lambda_0^{\text{Br}} = qD \left( \sqrt{\epsilon_c \mu_d} + \sqrt{\epsilon_d \mu_c} \right) \]  \hspace{1cm} (20)

as its center-wavelength. This equation must also hold if both the relative permittivity and the relative permeability dyadics are negative definite (i.e. \( \epsilon_{a,b,c} < 0 \) and \( \mu_{a,b,c} < 0 \) ) [24, 25]. Even when the two dyadics are indefinite, we must ensure that the ambichiral structure is electromagnetically penetrable by normally incident plane waves, in general. Therefore, let us impose the twin restrictions [14, 15]

\[ \begin{cases} \epsilon_c \mu_d > 0 \\ \epsilon_d \mu_c > 0 \end{cases} \]  \hspace{1cm} (21)

The following four cases were chosen for numerical investigation:

- PosDef: \( \epsilon_c = 2.7, \epsilon_d = 3.2, \mu_c = 1.1, \mu_d = 1.2 \).
- NegDef: \( \epsilon_c = -2.7, \epsilon_d = -3.2, \mu_c = -1.1, \mu_d = -1.2 \).
- Indef-1: \( \epsilon_c = -2.7, \epsilon_d = 3.2, \mu_c = 1.1, \mu_d = -1.2 \).
- Indef-2: \( \epsilon_c = 2.7, \epsilon_d = -3.2, \mu_c = -1.1, \mu_d = 1.2 \).

All four cases satisfy the restrictions (21).

Computed spectra of the eight reflectances and transmittances are shown in figure 1 for case PosDef and the following structural parameters: \( h = 1, q = 3, N = 20q \) and \( qD = 200 \text{ nm} \). The fundamental Bragg regime in the figure as a high-reflectance feature for incident RCP plane waves is correctly predicted by (20). In the same free-space-wavelength regime, the transmission of incident LCP plane waves is very high. This CP-discriminatory phenomenon is called the circular Bragg phenomenon [22]. It occurs when the number of structural periods is sufficiently large, the sufficiency depending on the magnitude of the difference \( \sqrt{\epsilon_c \mu_d} - \sqrt{\epsilon_d \mu_c} \), which quantity may be called an effective local linear birefringence. The larger this birefringence in magnitude, the fewer are the structural periods required to observe (and exploit) the circular Bragg phenomenon.

The reflectances and transmittances for RCP and LCP plane waves in figure 1 shall have to be interchanged if either (i) the sign of \( h \) is changed, or (ii) the constitutive parameters for case NegDef were to be used instead of those for case PosDef.

The equivalence of the changes (i) and (ii) has been established analytically [25]. When the real parts of the permittivity and the permeability dyadics of a structurally chiral, magnetic-dielectric material are reversed in sign, the circular Bragg phenomenon displayed by the material in

**Figure 1.** Case PosDef: reflectances and transmittances of an ambichiral structure as functions of the free-space wavelength \( \lambda_0 \). The following structural parameters were used for these plots: \( h = 1, q = 3, N = 20q \) and \( qD = 200 \text{ nm} \). The constitutive parameters are as follows: \( \epsilon_c = 2.7, \epsilon_d = 3.2, \mu_c = 1.1, \mu_d = 1.2 \). Solid (red) lines are for \( R_{LL} \) and \( T_{LL} \), black dotted lines for \( R_{RR} \) and \( T_{RR} \), blue dashed lines for \( R_{RL} \) and \( T_{RL} \), and (green) dash-dotted lines for \( R_{LR} \) and \( T_{LR} \). Interchange the subscripts L and R in the reflectances and transmittances for either (i) \( h = -1 \) or (ii) Case NegDef (\( \epsilon_c = -2.7, \epsilon_d = -3.2, \mu_c = -1.1, \mu_d = -1.2 \)). The subscripts L and R must not be interchanged if both (i) and (ii) hold together.
The situation changes completely for case Indef-1. Spectra of the four reflectances are plotted in figure 2, and those of the four transmittances in figure 3, for $h = 1$, $q = 3$ and $q D = 200$ nm. These spectra are provided for ambichiral structures with 4, 6, 8 and 10 structural periods. When the number of structural periods is small, the responses to incident RCP and LCP plane waves are different. As the number of structural periods increases, the discrimination between the responses to CP plane waves of different handednesses decreases and virtually vanishes for $N = 20q$ in the two figures; concurrently, the transmittances become increasingly smaller. Qualitatively similar conclusions were drawn from the spectra of the reflectances and transmittances for case Indef-2, for which reason those spectra have not been provided here.
Furthermore, for both cases Indef-1 and Indef-2, the same conclusions emerged for all values of \( q \leq 50 \).

Taken together, the foregoing results offer the following significant result: an ambichiral structure, made of a dielectric-magnetic material with indefinite permittivity dyadic and indefinite permeability dyadic and containing a sufficiently large number of structural periods, can function as a polarization-universal rejection filter in its fundamental Bragg regime. Despite its structural chirality, an ambichiral as a polarization-universal rejection filter in its fundamental dielectric-magnetic material with indefinite permittivity all four planewave components are of the NPV-type. The discriminatory rejection is possible in the fundamental Bragg regime when all four planewave components are of the NPV-type and two of (NPV)-type \([13]\).

The polarization-universal rejection in figures 2 and 3 occurs over a much larger bandwidth than the CP-discriminatory rejection in figure 1. However, this observation is subject to modification when both dissipation and dispersion are considered.

In order to understand the different response characteristics for cases PosDef and NegDef on the one hand and cases Indef-1 and Indef-2 on the other hand, the electromagnetic field phasors inside the ambichiral structure have to be examined. Since all sheets are identical, except for a rotation about the \( z \)-axis, it suffices to examine the fields in the sheet labeled \( n = 1 \). In this sheet, the electromagnetic field phasors may be represented in terms of four plane waves as

\[
E(r) = -\eta_0 \eta_{id} \left[ A^{(+)} e^{-i k_0 z + \omega t} \hat{u}_z + \eta_0 \eta_{dc} \left( B^{(+)} e^{-i k_0 z + \omega t} \hat{u}_z \right) \right], \\
0 \leq z \leq D,
\]

and

\[
H(r) = \left( A^{(\pm) (\mp)} e^{-i k_0 z + \omega t} \hat{u}_z \right), \\
0 \leq z \leq D,
\]

where \( A^{(\pm)} \) and \( B^{(\pm)} \) are coefficients of expansion, and

\[
\eta_{id} = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}, \quad k_{ed} = \sqrt{\mu \epsilon}, \quad \eta_{dc} = \frac{\sqrt{\mu}}{\sqrt{\epsilon}}, \quad k_{dc} = \sqrt{\mu \epsilon}.
\]

The projection of the wavevector \( \mp k_0 \hat{u}_z \) on the time-averaged Poynting vector \( \mp \eta_0 \eta_{dc} \left\{ A^{(+)} \right\}^2 \hat{u}_z \) of the planewave component associated with \( A^{(\pm)} \) is either (i) positive if \( \mu_\epsilon > 0 \) or (ii) negative if \( \mu_\epsilon < 0 \). Likewise, the projection of the wavevector \( \mp k_0 \hat{u}_z \) on the time-averaged Poynting vector \( \mp \eta_0 \eta_{dc} \left\{ B^{(+)} \right\}^2 \hat{u}_z \) of the planewave component associated with \( B^{(\pm)} \) is either (i) positive if \( \mu_\epsilon > 0 \) or (ii) negative if \( \mu_\epsilon < 0 \).

For case PosDef, all four plane-wave components inside each sheet are of the positive-phase-velocity (PPV)-type. For case NegDef, all four are of the negative-phase-velocity (NPV)-type \([26]\). For either case Indef-1 or Indef-2, two plane-wave components are of the NPV-type and two of the PPV-type. Both theory and experiment \([1–7]\) show that CP-discriminatory rejection is possible in the fundamental Bragg regime when all four plane wave components are of the PPV-type. From the conjugate invariance of the frequency-domain Maxwell equations \([27]\), it follows that CP-filtering must be possible in the fundamental Bragg regime when all four plane wave components are of the NPV-type. The polarization-universal rejection exemplified by figures 2 and 3 for cases Indef-1 and Indef-2—despite the pile of sheets being structurally chiral—must therefore be attributed to the fact that two plane-wave components out of four are of the PPV-type and two of the NPV-type.

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