Exploring the Lattice Gas Model for isoscaling

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Abstract

Isotopic spin dependent lattice gas model is used to examine if it produces the isoscaling behaviour seen in intermediate energy heavy ion collisions. Qualitative features are reproduced but quantitative agreement with experiments is lacking.

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I. INTRODUCTION

A very interesting feature of heavy ion collisions at intermediate energy is that provided the experimental set ups are identical, the ratio of isotope yields from two different reactions, 1 and 2, \( R_{21}(N,Z) = \frac{Y_2(N,Z)}{Y_1(N,Z)} \) exhibits an exponential relationship as a function of the isotope neutron number \( N \) and proton number \( Z \):

\[
R_{21}(N,Z) = \frac{Y_2(N,Z)}{Y_1(N,Z)} = C \exp(\alpha N + \beta Z)
\]  

This is called isoscaling and \( \alpha \) and \( \beta \) are called isoscaling parameters. Various theoretical models have been examined for at least an approximate emergence of this law. A grand canonical model for multifragmentation naturally leads to this law but it is also seen to emerge as an approximate formula from a canonical model [6], an expanding excited source model [2] and even anti-symmetrised molecular dynamics calculations [7]. In this note we investigate how well isoscaling is obeyed in the lattice gas model (LGM). Part of what we do has already been looked at in a recent publication [8] but both our perspectives and the lessons we derive from our study are different. Although highly schematic, LGM has been profitably utilised for investigating phase transition properties and many features of inclusive cross-sections. One might expect that isoscaling is a general feature, not depending upon finer details of models and hence should be seen in the LGM multifragmentation model. The advantage of LGM is (1) it takes into account the interaction between nucleons, both Coulomb and nuclear exactly (although the nuclear interaction is very schematic); (2) it takes into account excluded volume effects exactly and (3) decay of hot nuclei need not be considered as the cluster algorithm recognises only particle stable clusters. Many other models can not handle the above three issues easily although there are other many virtues in these other models.

The methods for calculating fragments in an isotopic spin dependent LGM have been described in many places (see, for example, [9, 10, 11, 12]). The nearest neighbor bond between unlike particles \( \epsilon_{np} \) is set at -5.33 MeV (to produce binding energy of 16 MeV per particle); and the bonds between like particles \( \epsilon_{pp} \) and \( \epsilon_{nn} \) are set at 0. The justifications for these choices are explained in [10]. Coulomb energy between protons is included.

These specifications are enough to allow one to calculate populations of all fragments given the lattice size, the mass and charge numbers of the dissociating system and the temperature. But for later purposes we need to estimate the value of symmetry energy
implied in the model. The reason for this is the following. The parameter $\alpha$ (and also $\beta$ of eq.(1)) depends upon the value of $N/Z$ of the dissociating systems in reactions 1 and 2. Moreover, it also depends upon the value of symmetry energy which adds a term $c_s(N-Z)^2/A$ to binding energy in the liquid-drop formula. Indeed it is a standard practice to use a relation \[2\] between $\alpha$ and the symmetry energy coefficient $c_s$, as:

$$\alpha = \frac{4c_s}{T}[(Z/A)^2_1 - (Z/A)^2_2].$$  \hspace{1cm} (2)

It is therefore imperative to estimate a value of $c_s$ implied in the LGM.

II. ESTIMATION OF $c_s$ IN LGM

We obtain ground state energies of a large number of nuclei by Monte-Carlo sampling at zero temperature. For a given nucleus with mass number $A$ and charge $Z$ this ground state energy is denoted by $BE(LGM)$. Let the value of the Coulomb energy which is also available from this calculation be $E_c$. Since by choice the volume energy is $-16A$ MeV we can try to deduce the value of surface energy coefficient $a_s$ and the symmetry energy coefficient $c_s$ by setting

$$BE(LGM) - E_c + 16A = a_sA^{2/3} + c_s(N-Z)^2/A$$  \hspace{1cm} (3)

Ideally the values of $a_s$ and $c_s$ should be the same for all nuclei. But because there is no good reason why $BE(LGM)$ should obey this parametrisation exactly, values of $a_s$ and $c_s$ deduced from the above relation will change from nucleus to nucleus. We can now try to get the "best" values by minimising the sum of the squares of the deviation. We chose isotopes of some arbitrarily chosen $Z$’s. The fit with the best values of $a_s$ and $c_s$ is shown in Fig.1.

The fit in Fig.1 appears to be very good but some words of caution are needed. Of the two constants we are trying to get, $a_s$ is by far the most important one; $a_sA^{2/3}$ dominates over $c_s(N-Z)^2/A$ which is a smaller perturbation. But it is $c_s(N-Z)^2/A$ which is presumed to be the deciding factor for isoscaling. What is left after subtracting the surface term can not be fitted by the parametrisation $c_s(N-Z)^2/A$ very accurately. This is demonstrated in Fig.2.

What is reported here is similar to but not identical with the extraction of $a_s$ and $c_s$ in \[2\]. However there the symmetry energy derived had a volume part (like here) but also a
surface part. We have absorbed here all the effects of symmetry energy using a volume term only in order to test how well LGM calculations follow eq.(2) which is based on a volume symmetry energy.

III. COMPARISON WITH SOME DATA

We will compare our calculations with two sets of data. We first consider $^{112}$Sn+$^{112}$Sn (reaction 1) and $^{124}$Sn+$^{124}$Sn (reaction 2) central collision data. Experimental data are given in Fig.1 of [3]. Isoscaling is seen to be well satisfied with a value of $\alpha = 0.361$. In Fig.3 we show calculated results for $R_{21}$ where this is the ratio of $\langle n_2(N, Z) \rangle$ and $\langle n_1(N, Z) \rangle$; $\langle n_{N,Z} \rangle$ is the average multiplicity of the composite with $N$ neutrons and $Z$ protons. The dissociating systems are taken to be $A=168$, $Z=75$ for reaction 1 and $A=186$, $Z=75$ for reaction 2. These are the recommended values [3] after allowing for losses like pre-equilibrium emissions etc.. The average multiplicity is calculated from 100,000 Monte-Carlo events. We try 10,000 switches between two events. Metropolis algorithm is used. The slopes of the ratios of the average multiplicities should correspond to the measured value of the slope of experimental $Y_2(N, Z)/Y_1(N, Z)$ (Fig.1 of [3]).

No basic calculation with the grand canonical model which computes the value of $\alpha$ has been reported. Canonical model calculations are quite successful [13]. SMM calculations before decay of hot primaries show isoscaling quite well with $\alpha = 0.46$; but after decay of primaries isoscaling is not obeyed to the same precision and the approximate $\alpha$ value changes slightly to 0.44 [3].

Results from LGM with different lattice sizes (N) and at different temperatures (T) are shown in Fig.3. Isoscaling is obeyed very well though it is not as good as in experimental data. In LGM there is no correction due to secondary decay. The clusters calculated are all particle stable. Even though isoscaling is only approximately obeyed we deduce an average value of $\alpha$. It is about 0.20 for $8^3$ lattice and temperature 5 MeV (compared to 0.36 in experiment). A notable feature of Fig.3 is that for $Z=1$, $N$ can be as high as 6 (Fig.3 shows results up to 5); $Z=1$ and $N=6$ as a stable composite happens with the proton in the central cube and six neutrons at the six faces. The binding energy per particle for this “nucleus” is 5.33(6/7). Of course in the real world such a nucleus does not exist.

We now examine how well eq.(2) is obeyed in LGM. We can write eq.(2) as $\alpha = \ldots$
where \(g(1,2)\) is just a property of the two dissociating systems and \(f\) includes all the effects of symmetry energy. Examining the validity of \(\alpha\) can be ambiguous in our case as isoscaling is not equally good for different \(Z\)'s. Fig.3 shows that \(Z=2\) obeys isoscaling quite well. Let us limit ourselves to \(\alpha\) value for \(Z=2\). We consider \(T=4\) MeV, lattice size \(8^3\), keep \(Z_1\) fixed at 75, \(A_1\) fixed at 168. For \(Z_2\) fixed at 75 we vary \(A_2\) from 168 to 186 and calculate \(\alpha\) as \(A_2\) is varied. As a function of \((Z_2/A_2)^2\) the value of \(\alpha\) is quite linear (Fig.4) suggesting that the functional form of \(g(1,2)\) of eq.(2) is accurate for LGM. The same can not be said about \(f(c_s, T)\). If we use eq.(2), \(Z_1 = Z_2 = 75\) and \(A_1 = 168, A_2 = 186\) and \(c_s = 23.4\) MeV as obtained from the least square fit in section II, the predicted value of \(\alpha\) would be 0.86 (as contrasted with \(\alpha \approx 0.22\) as actually given by the LGM calculation). A different point of view is sometimes taken. One takes the value of \(\alpha\) and deduces the value of \(c_s\) using eq.(2). In our case from the value of \(\alpha = 0.22\) we would then be lead to believe that the value of \(c_s\) is way lower, about 6 MeV. We see no reason why a value of \(c_s\) would so drastically change from about 23 MeV for isolated nuclei to this low value at 4/5 MeV temperature in an expanded volume in co-existence with other hot nuclei. It is specially hard to understand this in LGM. The composites in LGM can not be squished or expanded. At most, some of the composites may be in excited states with slightly different number of bonds. But such widely different value for symmetry energy appears unlikely. We remind the reader that there are two parts to the calculations. One is: given the lattice size, the number of neutrons and protons, the temperature and the bond strengths \(\epsilon_{np}, \epsilon_{pp}\) and \(\epsilon_{nn}\), calculation of thermodynamic properties and many particle correlations at all levels. Monte-Carlo simulations solve this many-body problem of LGM correctly though numerically. Next comes the question: how does one define clusters, given that the physics upto this point has been done correctly. We follow the prescription, first formulated for LGM in [14], subsequently reformulated with the same result in [11], shown to be closely equivalent to the one derived in [15] and is now universally used. A practical, reasonable but different prescription is not known. Given that the choice of the values of \(\epsilon_{np}, \epsilon_{pp}\) and \(\epsilon_{nn}\) is very restrictive, we have no freedom to alter anything. We find it easier to believe that the function \(f(c_s, T)\) as implied by eq.(2) is not correct for LGM.

The other set of data we use is for \(^{58}\)Ni and \(^{64}\)Ni on \(^9\)Be. We assume that the much larger nucleus Ni engulfs Be so that for reaction 1 we have \(A_1 = 67, Z_1 = 32\) and for reaction 2 we have \(A_2 = 73, Z_2 = 32\). Experimental data can be found in [16, 17]. The subset of data
we use here can also be found in [6] (Figs.6 and 7). Experimental results for Ni on Be show a larger deviation from isoscaling compared to what is seen experimentally for Sn on Sn. Fig.5 shows the results of our calculation. There are significant deviations from isoscaling. The calculated value of \( \alpha \) hovers around 0.22 (these averages are always ambiguous since isoscaling deviations are more significant here) whereas experimental values are around 0.6. Thus, as in the case of Sn+Sn, theory underestimates the value of \( \alpha \).

In the case of Ni on Be, experimental values of \( Y_2(N,Z)/Y_1(N,Z) \) are available for a large range of \( Z \) from 1 to 28. Low and moderate values of \( Z \) have an effective \( \alpha \) which is much smaller than those belonging to \( Z \approx 22 \). (Data can be found in [6, 16].) Such details do come out in LGM as well. We demonstrate this in Fig.6. This is highly satisfying. Grand canonical model can not explain this difference although the canonical model model can and indeed fits the data very well [6].

IV. SUMMARY

Our first objective was to see if isoscaling is obtainable in LGM. It does appear that approximate isoscaling is obtained in the model. We confronted the calculations with two sets of experimental data. There are no free parameters in the model. The experimental values of \( \alpha \) are larger than what the model predicts, by about a factor of 2. But the model does reproduce some remarkable features. It did show a significant increase in the value of \( \alpha \) going from low \( Z \) isotopes to high \( Z \) isotopes as seen in measurements. It also gave linearity of \( \alpha \) with \( \Delta (Z/A)^2 \). In view of the schematic nature of the model and obvious drawbacks, these successes are quite pleasing. Underestimation of the value of \( \alpha \) remains a problem. We have checked that reasonable variations of lattice size or temperature will not correct this problem. Moderate variations in the values of the bond strengths do not provide enough corrections.

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FIG. 1: For selected isotopes we show plots of \( BE(LGM) - E_c + 16A \) (stars) and compare these with \( a_s A^{2/3} + c_s (N - Z)^2 / A \) where \( a_s = 17.0 \) and \( c_s = 23.4 \). These values are chosen from least squares fit. All energies are in MeV.
FIG. 2: To test the accuracy of the parametrisation $c_s(N - Z)^2/A$ we plot $BE(LGM) - E_c + 16A - 17A^{2/3}$ and compare that with $23.4 \times (N - Z)^2/A$. All energies are in MeV.
FIG. 3: These figures are calculated to compare with central collision data for Sn on Sn reported in Fig.1 of [3]. For reaction 2, the dissociating system is set at $A = 186, Z = 75$. For reaction 1 we take $A = 168, Z = 75$. Isoscaling is approximately obeyed but the deviations are not negligible. The “average” value of $\alpha$ is $\approx 0.2$. Experimentally for this case isoscaling is better obeyed and the value of $\alpha$ is $\approx 0.34$. 
FIG. 4: For $Z_1 = 75$ and $A_1 = 168$ we vary $A_2$ from 168 to 186 for fixed $Z_2=75$. The dots are from LGM calculation where $\alpha$ is calculated for $Z = 2$ as for this isotope isoscaling is well obeyed (see Fig.3)). The best linear fit is shown. This comparison tests the accuracy of the $[(Z_1/A_1)^2 - (Z_2/A_2)^2]$ factor of eq.(2).
FIG. 5: For reaction 2, the dissociating system is set at $A = 73, Z = 32$. For reaction 1, the dissociating system is set at $A = 67, Z = 32$. Experimental data for Ni on Be can be found in [16] and also in [6], Figs 6 and 7.
FIG. 6: Same case as in Fig.5, but we also plot the ratios for much higher Z isotopes. Note the much higher value of the slopes for large Z nuclei. Experimental data confirm this. The high Z cases are not shown in the lowest right panel as there was not enough statistics to get a dependable ratio.