Applying logarithmic density function to develop a segmentation model for intensity and noisy inhomogeneity images: towards informed decisions in computational and applied mathematics

Shahed Ali Hamil & Abdulrahman H. Majeed

1,2 Department of Mathematics
College of Science, University of Baghdad
Baghdad, Iraq.

1Email: shahedahamil@gmail.com

Abstract-- This study examined the how a logarithmic intensity function could be used to develop a segmentation model for intensity and noisy homogeneity images. Whereas inhomogeneous images demand the use of local image data, it remains notable that it remains defective for noisy images. The eventuality is that active contour motions tend to be misguided by the local information. In the proposed model, the logarithmic function was able to capture minute details contained in selected images and also counter or ignore the perceived noise. As such, the model proved effective and robust, worth applying to such images. To verify the effectiveness of the model, its results were compared to the experimental outcomes previously reported after employing local Chan-Vese Model. Indeed, the proposed model exhibited superior performance in relation to the treatment of intensity and noisy inhomogeneity images.

1. Introduction
In the fields of computer vision and image processing, one of the demanding tasks entails image segmentation [1]. The complexity has been felt due to the fields’ wide applications in areas such as medical sciences and engineering sciences [2]. Notably, the objective of image segmentation lies in the division of images into various parts [3], making it easier to analyze and also meaningful [4]. Hence, various algorithms have been developed to solve the problem of image segmentation [3, 4]. It is also notable that several techniques have been employed to enhance the results of previous image segmentation algorithms [5]. One of such algorithms entails the LCV model, which has gained application to both global and local image-information for segmentation [6]. The model has gained increasing application because it demands little iteration, hence proving efficient for image segmentation in the context of intensity inhomogeneity [7, 8]. It is also notable that for traditional level set techniques, the re-initialization procedure demands a lot of time [9]. This dilemma has attracted growing scholarly attention regarding the need to develop new techniques [10].

From the documentation above, it can be inferred that several algorithms have been developed to optimize the process of image segmentation. However, most of the contemporary real-world scenarios are characterized by noisy images, which have presented a new problem to the practice of image segmentation. As such, this study strived to establish a new model for image segmentation through which more efficient and better results could be obtained – relative to application on noisy images. The proposed model was that which would capture additional data in relation to noisy image segmentation, ensuring that the noise is countered or avoided when the final segmented results are reported. The implication is that the proposed model strived to enhance or improve on traditional frameworks that had aided in noisy image segmentation but also captured the noise simultaneously, compromising the accuracy and precision of the segmentation results. Of importance to note is that the study’s quest to improve on previous frameworks was achieved by combining ideas from the non-linear diffusion model and Chan-Vese models of noisy image segmentation. Overall, the objective of the investigation
was to establish an image segmentation framework that would reduce the number of noise in the final segmented image outcomes.

2. Methodology

Deviating from the LVC model, the proposed framework had a new algorithmic density function defined. Furthermore, little statistical information was used to ensure that the image segmentation problem linked to noise is reduced. For the new proposed model, the energy function was established as:

\[ F(v_j, v_k, k_j, \sigma^2_j, \sigma^2_k, k_j^2, \phi) = \lambda_1 \int_{\Omega \setminus \partial \Omega} -\log P_1(I) \, ds + \int_{\partial \Omega} \log P_1(I) \, ds \]
\[ + \lambda_2 \int_{\Omega \setminus \partial \Omega} -\log P_2(I) \, ds \]
\[ + \lambda_2 \int_{\partial \Omega} \log P_4(I) \, ds \]

In this case,

\[ P_1 = \frac{1}{\sqrt{2\pi \sigma_1}} \exp\left(-\frac{(I(u,v)-v_j)^2}{2\sigma_1^2}\right), \quad P_2 = \frac{1}{\sqrt{2\pi \sigma_1}} \exp\left(-\frac{(I(u,v)-v_j)^2}{2\sigma_1^2}\right) \]
\[ P_3(\phi) = \frac{1}{\sqrt{2\pi \sigma_3}} \exp\left(-\frac{(I(u,v)-k_j)^2}{2\sigma_3^2}\right), \quad P_4(\phi) = \frac{1}{\sqrt{2\pi \sigma_4}} \exp\left(-\frac{(I(u,v)-k_j)^2}{2\sigma_4^2}\right) \]

The energy function was, in turn, minimized. With variation calculus applied, variances and the constant functions responsible for energy function reduction translated into:

\[ \gamma_1(\phi) = \frac{\int_{\Omega} I(u,v) H(\phi) \, dudv}{\int_{\Omega} H(\phi) \, dudv}, \quad \gamma_2(\phi) = \frac{\int_{\Omega} I(u,v) (1-H(\phi)) \, dudv}{\int_{\Omega} (1-H(\phi)) \, dudv} \]
\[ k_1(\phi) = \frac{\int_{\Omega} I'(u,v) H(\phi) \, dudv}{\int_{\Omega} H(\phi) \, dudv}, \quad k_2(\phi) = \frac{\int_{\Omega} I'(u,v) (1-H(\phi)) \, dudv}{\int_{\Omega} (1-H(\phi)) \, dudv} \]

And

\[ \sigma_1^2(\phi) = \frac{\int_{\Omega} (I(u,v)-v_j)^2 H(\phi) \, dudv}{\int_{\Omega} H(\phi) \, dudv}, \quad \sigma_2^2(\phi) = \frac{\int_{\Omega} (I(u,v)-v_j)^2 (1-H(\phi)) \, dudv}{\int_{\Omega} (1-H(\phi)) \, dudv} \]
\[ \sigma_3^2(\phi) = \frac{\int_{\Omega} (I(u,v)-k_j)^2 H(\phi) \, dudv}{\int_{\Omega} H(\phi) \, dudv}, \quad \sigma_4^2(\phi) = \frac{\int_{\Omega} (I(u,v)-k_j)^2 (1-H(\phi)) \, dudv}{\int_{\Omega} (1-H(\phi)) \, dudv} \]

Hence,

\[ \phi = \frac{\partial \phi}{\partial t} = \lambda_1 \left[ \frac{\log \sigma_1^2}{\sigma_1^2} - \frac{(I(u,v)-v_j)^2}{\sigma_1^2} + \frac{(I(u,v)-v_j)^2}{\sigma_1^2} \right] 
\[ + \lambda_2 \left[ \frac{\log \sigma_2^2}{\sigma_2^2} - \frac{k_j)^2}{\sigma_2^2} + \frac{(I'(u,v)-k_j^2)^2}{\sigma_2^2} \right] \]
3. Experimental Results

In the results, the objective was to determine the new proposed model’s performance and efficiency. The results were also compared to those that had been documented previously after applying LCV to noisy images. From the specific findings, the proposed model and LCV model exhibited robustness. However, the proposed model exhibited superior results compared to LCV model in terms of efficiency because, given the original image, it captured more information.

![Figure 1: LCV model outcomes versus the results from the new proposed model for noisy image segmentation](image1)

Another area where comparison was done between the performance of LCV model and the proposed model involved their application to a real noisy medical image. Based on the figures that follow, it is evident that both models exhibited a desirable degree of robustness in relation to noisy image segmentation. However, the proposed model’s outcomes were more superior in such a way that its efficiency was evidenced by the ability to capture less noise than the case of LCV framework.

![Figure 2: LCV model performance versus the proposed framework’s performance in relation to a real noisy medical image](image2)
Apart from noise reduction, the proposed model was also observed to capture more information of the original image than that which was captured by LCV framework. On this parameter, the study relied on an original satellite moon image. The performance of the two models, which aided in depicting their degree of robustness, is summarized in the figure below.

![Figure 3: Model performances in relation to an original satellite moon image](image)

4. Discussion, Summary, and Future Implications

Based on the experimental results demonstrated above, the proposed model combined features from the non-linear diffusion model and LCV model. The study’s objective was to discern the degree to which the proposed framework would support image segmentation for noisy intensity inhomogeneity. The results were also compared with those that had been reported after applying LCV model previously. Overall, it was evident that both LCV and the proposed model were robust. However, the proposed model yielded more superior results because it captured more information of the original images, and less noise.

From the previous scholarly investigations, LCV model, upon encountering noisy intensity inhomogeneity images, fails to give good and efficient results relative to image segmentation. The eventuality is that the model captures less information of the original images and also tends to be affected by noise, having failed to ignore it from the final segmented images. It is also notable that the majority of traditional segmentation models, LCV included, continue to face problems relative to inhomogeneity segmentation. This dilemma comes at a time when most of the real-world images are noisy. Hence, this experimental study strives to counter this problem that continues to be experienced in inhomogeneous intensity segmentation by proposing a new model. For the case of LCV framework, it has strived to counter the perceived challenge documented above through the combination of the global and local statistical information. However, the model continues to face drawbacks. Apart from those that were revealed in this study after comparing the performance of the new proposed model with that which had been documented for LCV (regarding parameters such as the capturing of less or more information of the original image and also the ability to counter (or otherwise) the noise in images to be segmented), other demerits with which LCV model has been associated include failure in images with low frequencies and low contrast [4-6]. The implication is that LCV model’s results remain unsatisfactory.

Therefore, this study strived to counter the perceived weaknesses with which LCV model is associated by proposing and examining the performance of a new model for image segmentation. The overall findings demonstrated that the new proposed model would not only use a given noise image and capture more information but also ensure that the noise associated with the image is avoided at the same time. Hence, the proposed model’s efficiency in performance and robustness was confirmed in such a way that the above features characterized the final segmented images or results. For LCV model, it was also able to
segment noisy images but ended up capturing some noise in the noisy images, hence compromising its robustness. The eventuality is that the results obtained after LCV model implementation were not good. Rather, they were defective. The implication for the future of noisy image segmentation processes is that the proposed model is worth adopting and implementing because it counters the perceived weaknesses associated with traditional frameworks, having combined features of non-linear diffusion model and LCV framework.

5. References

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