Identification of the elastic characteristics of a composite material based on the results of tests for the stability of panels made from it

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Abstract The article considers the problem of identification of the stiffness characteristics of a fiber-reinforced composite (FRC) from which, by applying at an angle to the edge, the cylindrical panels or plates are made. The problem was solved based on the analysis of the results of their tests, bringing up to a loss of stability. By means of the numerical experiments it was shown the solution of the problem in the proposed extended formulation is stable to variations in the initial data. The proposed approach makes it possible to obtain the calculated mechanical characteristics of the composite close to true even in the case of a considerable spread in the experimental data in determining the critical load and a spread in the stiffness characteristics of the composite from the sample to.

1. Introduction
Let us consider a thin elastic shell made by differently oriented appliance of the layers of a fiber-reinforced composite (FRC). The basic equations describing its behavior are calculated by the known data.

The equilibrium equations can be written in the following general form:

\[ F(w, P, g, k) = 0, \tag{1} \]

here \( w \) = \( \{w_x, w_y, w_z \} \) and \( P = \{p_x, p_y, p_z \} \) are displacement and load vectors in the axes \( x, y, z \); \( k \) is a vector, made up of design parameters; \( g \) is a vector of independent elastic constants of FRC, to be determined.

We write the boundary conditions in the following form:

\[ f(w, P, g, k) = \psi. \tag{2} \]

The mechanical characteristics of FRC belong to a predetermined space of functions, i.e. they must satisfy certain restrictions and conditions. We write them in the form:

\[ B(g) = 0, \quad H(g) \geq 0. \tag{3} \]

We will assume that we can determine the forms of the stability loss and the critical load at which the stability loss occurs, using known methods [1].

Thus, we assume that the expression for the calculated critical load can be written in the form
\[ P_{\text{calc}} = P_{\text{calc}}(g, k). \] (4)

Let us consider the problem of determining the constants \( g \) based on minimizing the residual error of the calculated and experimental data of the critical loads.

Let \( N \) series of tests of the elastic elements be carried out, bringing them up to a stability loss. Let \( P_i^{\text{exp}} \) \( (i = 1, N) \) denote the critical loads found from the experiment, and let \( k_i^{\text{exp}} \) denote the vectors of experimental samples.

We substitute \( k_i^{\text{exp}} \) in the relation (4) and put the problem of determining the elastic characteristics of FRC from the following relations:

\[ P_i^{\text{exp}} = P_i^{\text{calc}}(g, k_i^{\text{exp}}). \] (5)

In the general case, the system (5) cannot be solved exactly. That is why, in order to obtain the best solution of the system, this problem is usually reduced to the problem of mathematical programming, i.e. to the definition of the minimum of the target function, which is some measure of the residual error of the equations (5).

The traditional problem of identification usually corresponds to the following objective function:

\[ \delta^2 = \sum_{i=1}^{N} \left( \frac{P_i^{\text{calc}} - P_i^{\text{exp}}}{P_i^{\text{exp}}} \right)^2 n_i, \] (6)

where \( n_i \) is weighting coefficients.

When minimizing (6), restrictions should be taken into account (3). Minimizing the function (6) can lead us to a solution that does not satisfy the equations (1) and (2) with acceptable accuracy (due to the presence of the errors in determining the initial data and the errors of the models used, or the strong sensitivity of the problem to these errors).

If the solution of the problem in the traditional formulation is unstable to variations in the initial data, then it is necessary to apply any methods of regularization. Sometimes increasing the stability of the problem can be achieved using the formulation of the identification problem in the extended formulation [2].

With reference to this problem, its essence lies in the following. First, we believe that when being produced the composite changes its properties from sample to sample, and secondly, there are both errors in the measurement of critical loads \( P_i^{\text{exp}} \), and the error in measuring the design parameters \( k_i^{\text{exp}} \) of experimental samples.

Instead of one unknown vector \( g \), we seek unknowns \( P_i^{\text{calc}}, g_i^{\text{calc}}, k_i^{\text{calc}}, \psi_i^{\text{calc}}, \) approximately satisfying the following system of equations:

\[ F\left(w_{i}^{\text{calc}}, P_{i}^{\text{calc}}, g_{i}^{\text{calc}}, k_{i}^{\text{calc}}\right) = 0, \quad f\left(w_{i}^{\text{calc}}, P_{i}^{\text{calc}}, g_{i}^{\text{calc}}, k_{i}^{\text{calc}}\right) = \psi_{i}^{\text{calc}}, \]

\[ P_{i}^{\text{calc}} - P_{i}^{\text{exp}} = 0, \quad g_{i}^{\text{calc}} - g = 0, \quad \psi_{i}^{\text{calc}} - \psi_{i}^{\text{exp}} = 0, \]

\[ k_{i}^{\text{calc}} - k_{i}^{\text{exp}} = 0, \quad i = 1, N. \] (7)

Here \( g_i^{\text{calc}} \) is vectors belonging to the supposed class of functions; \( P_i^{\text{calc}}, P_i^{\text{exp}} \) are calculated and experimentally found critical loads, respectively. Relations (3) will be assumed to be strictly satisfied. If this is not necessary, then we can similarly introduce (7) additional relations of the form

\[ B(g) = h^2, \quad b^2 \ll 1, \quad H(g) \geq -h^2, \quad h^2 \ll 1. \]

Further on we use the following functions:

\[ \Delta g_i = g_i^{\text{calc}} - g, \quad \Delta k_i = k_i^{\text{calc}} - k_i^{\text{exp}}, \]

\[ \Delta \psi_i = \psi_i^{\text{calc}} - \psi_i^{\text{exp}}, \quad \Delta P_i = P_i^{\text{calc}} - P_i^{\text{exp}}. \]

We will impose restrictions on \( \Delta g_i, \Delta k_i, \Delta \psi_i, \Delta P_i \) requiring their smallness

\[ |\Delta g_i| \ll |g|, \quad |\Delta k_i| \ll |k_i^{\text{exp}}|, \quad |\Delta \psi_i| \ll |\psi_i^{\text{exp}}|, \quad |\Delta P_i| \ll |P_i^{\text{exp}}|, \quad |\Delta \psi_i| \ll |\psi_i^{\text{exp}}|. \] (8)

We introduce a new target function (target function in the extended statement [2]), having the form:
\[ \Phi^2 = \delta_p^2 + \delta_m^2 + \delta_g^2 + \delta_{\Delta P}^2 + \delta_{\Delta \psi}^2 + \left( F \alpha \right)^2 + \left[ (f - \psi) \gamma \right]^2, \]

\[ \delta_p^2 = \sum_{i=1}^{N} \left( \frac{P_i^{calc} - P_i^{exp}}{P_i^{exp}} \right)^2 n_i, \quad \delta_m^2 = \sum_{i=1}^{3} \Delta P_i^2 \ L_i, \]

\[ \delta_g^2 = \sum_{i=1}^{N} \left( \Delta g_i \right)^2 \ z_i, \quad \delta_{\Delta P}^2 = \sum_{i=1}^{L} \sum_{j=1}^{L} \left( \Delta k_{ij} \right)^2 \ m_i, \quad \delta_{\Delta \psi}^2 = \sum_{i=1}^{N} \left( \left( \Delta \psi \right) \chi \right)^2, \]

(9)

\[ |\Delta g_i| \leq \beta_g \left| \sum_{j=1}^{M} g_j \right|, \quad \beta_g << 1; \quad |\Delta P| \leq \beta_p \left| P_i^{exp} \right|, \quad \beta_p << 1; \]

\[ |\Delta \psi| \leq \beta_{\psi} \left| \psi_{i, exp}^{calc} \right|, \quad \beta_{\psi} << 1; \quad |\Delta k_{ij}| \leq \beta_k \left| k_{ij}^{exp} \right|, \quad \beta_k << 1. \]

Here \( M \) is a number of stiffness characteristics of FRC; \( L \) is a number of design parameters of the sample; \( n, l, z, m \) are normalizing weighting coefficients; \( a, \gamma, \chi \) are vectors composed of normalizing weighting coefficients; \( P_i^{alc}, g_i^{calc}, \Delta g_i, \Delta P_i, \Delta \psi, \Delta k_{ij} \) are sought values.

Part of the equations in (7) can be satisfied exactly. Then we obtain a problem on the constrained extremum of the function \( \Phi^2. \)

We expound the proposed approach through the example of a hingedly fixed and pre-curved by compressive force thin elastic plate, produced by applying the layers of FRC at an angle \( \pm \varphi \) to the axis \( x \) and located under the action of a vertically applied concentrated load. An analytical solution of this problem is given in [3] (for other variants of edge fixing, eccentric application of an external load, and also for the case of a cylindrical panel for solving similar problems is shown in [4]). Let us consider the case of an asymmetric form of stability loss. An approximate expression for the centrally applied transverse load \( P_{KP} \), at which the pre-curved plate loses stability, has the form:

\[ P_{KP} = \frac{4a \pi^3}{2b^2} D, \]

(10)

where \( a \) is a boom of lift of the curved plate, \( b \) is its length, \( D \) is cylindrical stiffness of the plate.

Let us consider several model problems, some of which we will solve under the assumption that the critical load is found from the experiment with some error, while others - under the assumption that the influence of technological factors of the process of manufacturing elastic elements, that is, under the assumption that the stiffness characteristics of FRC vary from sample to sample.

As the theoretically accurate values of the stiffness characteristics of the FRC from which the plates are made, the following values are accepted given in [5] for carbonplast:

\[ E_1 = 180 \text{ hPa}; \quad E_2 = 6.2 \text{ hPa}; \quad G_{12} = G_{13} = 5 \text{ hPa}; \quad v_{12} = 0.007; \quad G_{23} = 4 \text{ hPa}. \]

(11)

As we will see from numerical experiments, the use of (9) instead of (6) makes it possible to reconstruct the stiffness characteristics of the layer with sufficient accuracy, both in the case of approximate values of the experimental data, and in the presence of a spread in values of stiffness characteristics from sample to sample.

2. Traditional approach

According to the mentioned above, the task of identification in the traditional formulation is based on minimizing the target function in the form (6):

\[ \delta_p^2 = \sum_{i=1}^{N} \left( \frac{P_i^{alc} - P_i^{exp}}{P_i^{exp}} \right)^2 n_i, \]

(12)

where \( P_i^{alc}, P_i^{exp} \) are calculated and experimentally found critical loads, respectively; \( n_i \) is weighting coefficients, \( N \) is a number of the conducted experiments.

To find the calculated critical load \( P_i^{alc} \), we use the simplified analytical formula (10).
The cylindrical stiffness of a pre-compressed plate formed by applying FRC of a ribbon-type at an angle $\pm \varphi$ to the axis $x$, has the following form [5]:

$$D = g_{11} \cos^4 \varphi + g_{22} \sin^4 \varphi + 2(g_{12} + 2g_{33}) \cos^2 \varphi \sin^2 \varphi,$$

where $g_{ij}$ is stiffness characteristics of a composite in the orthotropic axes of a unidirectional material.

When solving the problem of nonlinear programming, gradient (quasi-Newtonian) and search (deformable polyhedron) methods were used. The results obtained with the help of them do not have any significant difference, only the rate of convergence of the solution turned out to be different – the gradient method has a high speed in comparison with the search method.

The numerical analysis has shown that the technique makes it possible to determine for FRC two stiffness characteristics $g_{11}$ and $g_{22}$, as well as rigidity $D_3 = g_{12} + 2g_{33}$, what is more, for their determination, it is necessary to carry out at least three experiments with the plates at different angles of layer packing $\pm \varphi$.

Let us consider an example of determining the stiffness characteristics of carbonplastic by the results of three numerical experiments with various errors in determining the critical load and the angles $\varphi = 25^\circ$, $35^\circ$, $45^\circ$. As experimental values $P_{i}^{exp}$ we use the values of the critical loads obtained by formula (10). In this case, using the parameters (11), we get the following flexural characteristics of the plate:

$$g_{11} = 180.25 \text{ hPa}; \ g_{22} = 6.2088 \text{ hPa}; \ D_3 = 11.262 \text{ hPa}.$$ 

To solve the problem of restoring stiffness characteristics $g_{11}$, $g_{22}$, $D_3$ in the traditional formulation, the target function is minimized (12). The results of the calculations are presented in Table 1. The first column lists the "measurement errors" of the critical loads $P_{i}^{exp}$. These "errors" were simulated by specifying deviations $P_{i}^{exp}$ from the true values according to the normal distribution law of the random value.

Numerical analysis has shown that when solving a problem in a traditional formulation, there is sometimes a lack of precision established by default. For example, the third line of Table 1 shows the results obtained with the default accuracy of 16-digit numbers, and the second line – with the working accuracy of 40-digit numbers.

Further, the problem was solved in the formulation, in which, apart from the assumption that the critical loads $P_{i}^{exp}$ were measured with some error, there was also the assumption about the change in the stiffness characteristics of a composite tape (although not significantly) in the process of making a structure. The results of solving the identification problem using the target function (12) are presented in Table 2. Since the best results are obtained by holding 40-digit numbers, then in the solution of the following problems of this section this accuracy was used. The first column presents the "errors" $\Delta P$ in measuring the critical loads and changing the stiffness characteristics of FRC. Here, the spread of the initial data was also given by the normal distribution law of the random value.

The presented Tables 1-2 show that a small spread in the stiffness characteristics of a composite tape in the process of making a structure or small errors in the measurement of critical loads lead $P_{i}^{exp}$, with the use of the traditional approach, lead to a significant difference in the calculated values of small stiffness characteristics $g_{22}$, $D_3$ from true ones.

3. Use of the advanced target function

The technique of expanding the target function is initially demonstrated on the problem of finding the stiffness characteristics of carbonplastic also based on the tests of three plates with the angles of placing the layers $\varphi = 25^\circ$, $35^\circ$, $45^\circ$. First we only take into account the errors in measuring the critical load $\Delta P$. 


Taking into account the assumption about the smallness of the error in determining the critical load $\Delta P_i$ from the experiment, we seek the calculated critical load in the form:

$$P_i = P_i^{calc} \left(1 + \alpha \sin(\Delta P_i / P_i^{calc})\right),$$

where $\alpha \sin(\Delta P_i / P_i^{calc})$ simulates errors in load measurement; $\alpha$ is a dimensionless value which was assumed to be equal to 1 in the calculations (it gives the order of the error in measurement); $P_i^{calc}$ and $\Delta P_i$ are required parameters.

The target function is taken in the form:

$$\Phi^2 = \delta_p^2 + \delta_{\Delta p}^2,$$

where

$$\delta_p^2 = \sum_{i=1}^{K} \left(\frac{P_i - P_i^{exp}}{P_i^{exp}}\right)^2 n_i, \quad \delta_{\Delta p}^2 = \sum_{i=1}^{3} (\Delta P_i / P_i^{calc})^2 l_i .$$

The goal function is expanded due to new variable parameters $\Delta P_i$ – the "errors" in measuring the critical load; $n_i, l_i$ are normalizing weighting coefficients.

For comparison with the traditional method, let us consider the problem of finding the stiffness characteristics of carbon plastic at the same values $P_i^{exp}$, that were used when compiling Table 1. The results obtained in the solution of the problem are presented in Table 3.

The table shows the solution of the problem in the extended formulation is stable to the errors of the initial data and the proposed approach makes it possible to obtain the calculated mechanical characteristics close to the true ones even in the case of a large error in determining the critical load.

Now we consider the problem of finding the stiffness characteristics of carbon plastic by the results of testing three plates with the angles of placing the layers $\varphi = 25^\circ, 35^\circ, 45^\circ$, in which we will take into account both the technological factors of making (we believe that when making a composite it changes its properties from sample to sample), and the errors in measuring the critical load $\Delta P_i$. In this case, we expand the target function as follows:

$$\Phi^2 = \delta_p^2 + \delta_{\Delta p}^2 + \delta_{\Delta g_{11}}^2 + \delta_{\Delta g_{22}}^2 + \delta_{\Delta D_3}^2,$$

where

$$\delta_{\Delta g_{11}}^2 = \sum_{i=1}^{3} (\Delta g_{11} / g_{11})^2 k_i , \quad \delta_{\Delta g_{22}}^2 = \sum_{i=1}^{3} (\Delta g_{22} / g_{22})^2 z_i , \quad \delta_{\Delta D_3}^2 = \sum_{i=1}^{3} (\Delta D_3 / D_3)^2 m_i , \quad \delta_{\Delta p}^2 = \sum_{i=1}^{3} (\Delta P_i / P_i)^2 l_i .$$

We seek the calculated stiffness characteristics in the form:

$$g_{11} = g_{11}^{calc} (1 + \beta_1 \sin \Delta g_{11} / g_{11}), \quad g_{22} = g_{22}^{calc} (1 + \beta_2 \sin \Delta g_{22} / g_{22}), \quad D_3 = D_3^{calc} (1 + \beta_3 \sin \Delta D_3 / D_3), \quad g_{11}^{calc} > 0, \quad g_{22}^{calc} > 0, \quad D_3^{calc} > 0.$$

Here $\beta_i \sin()$ is a maximum possible relative value of the change in rigidity (in calculations $\beta_i \ll 1$ was accepted); $z_i, l_i, m_i, k_i$ are normalizing weighting coefficients, $g_{11}^{calc}, g_{22}^{calc}, D_3^{calc}, \Delta g_{11}, \Delta g_{22}, \Delta D_3$ are required parameters. The results of the calculations are presented in Table 4. Its analysis shows that in this case the extension of the target function leads to regularization of the problem, too.

In Tables 1-4 the first column shows the error in measuring the critical load and changing the stiffness characteristics of the FRC. The relative error in determining the stiffness characteristics is shown in the brackets.
Table 1

The results of numerical experiments on the identification of stiffness characteristics \( s_{11}^{\text{calc}}, s_{22}^{\text{calc}}, D_3^{\text{calc}} \) FRC in the traditional formulation for various errors \( \Delta P_i \) in determining the critical load.

| \( \Delta P_i \) | \( s_{11}^{\text{calc}} \) | \( s_{22}^{\text{calc}} \) | \( D_3^{\text{calc}} \) | \( \sqrt{\delta_P^2} \) | \( k_i \) |
|----------------|----------------|----------------|----------------|----------------|----------------|
| 0%**          | 180.25 (0%)    | 6.2088 (0%)   | 11.262 (0%)   | 0 (0%)         | 1 (0%)         |
| 0%*           | 179.96 (0.2%)  | 5.1682 (16.8%)| 11.909 (5.7%) | 6.06 \cdot 10^{-4} (5.7%) | 1 (5.7%) |
| 1%            | 185.52 (2.9%)  | 25.369 (308%) | 0 (0%)        | 0.179 (16.8%)  | 1 (16.8%)      |
| 5%            | 184.80 (2.5%)  | 31.919 (414%) | 0 (0%)        | 0.067 (5.7%)   | 1 (5.7%)       |
| 10%           | 181.84 (0.9%)  | 41.5 (569%)   | 0 (0%)        | 0.147 (5.7%)   | 1 (5.7%)       |
| 20%           | 168.91 (6.3%)  | 65.562 (955%) | 0 (0%)        | 0.307 (5.7%)   | 1 (5.7%)       |

Table 2

The results of numerical experiments on the identification of stiffness characteristics \( s_{11}^{\text{calc}}, s_{22}^{\text{calc}}, D_3^{\text{calc}} \) FRC in the traditional formulation for various errors \( \Delta P_i \) in determining the critical load and with the availability of technological changes.

| \( \Delta P_i \) | \( s_{11}^{\text{calc}} \) | \( s_{22}^{\text{calc}} \) | \( D_3^{\text{calc}} \) | \( \sqrt{\delta_P^2} \) | \( k_i \) |
|----------------|----------------|----------------|----------------|----------------|----------------|
| 0%            | 180.25 (0%)    | 6.2088 (0%)   | 11.262 (0%)   | 0 (0%)         | 1 (0%)         |
| 1%            | 180.69 (0.2%)  | 6.1831 (0.4%) | 11.435 (1.5%) | 7.58 \cdot 10^{-3} (1.5%) | 1 (1.5%) |
| 5%            | 180.3 (0.03%)  | 6.1973 (0.2%) | 11.23 (0.3%)  | 2.78 \cdot 10^{-4} (0.3%) | 1 (0.3%) |
| 10%           | 180.27 (0.01%) | 6.3479 (2.2%) | 11.211 (0.4%) | 4.71 \cdot 10^{-7} (0.4%) | 1 (0.4%) |
| 20%**         | 180.3 (0.02%)  | 6.4461 (3.8%) | 11.156 (0.9%) | 5 \cdot 10^{-12} (0.9%) | 1 (0.9%) |

Table 3

The results of numerical experiments on the identification of stiffness characteristics \( s_{11}^{\text{calc}}, s_{22}^{\text{calc}}, D_3^{\text{calc}} \) FRC in the traditional formulation for various errors in determining the critical load.
The results of numerical experiments on the identification of stiffness characteristics \( g_{11}^{\text{calc}}, g_{22}^{\text{calc}}, D_3^{\text{calc}} \) FRC in the traditional formulation for various errors in determining the critical load and with the availability of spread in stiffness characteristics from sample to sample.

| \( \delta \)  | \( g_{11}^{\text{calc}} \)  | \( g_{22}^{\text{calc}} \)  | \( D_3^{\text{calc}} \)  | \( \sqrt{\sigma_p^2} \)  | \( n_i, l_i, m_i \)  | \( k_i \)  |
|------------|----------------|----------------|----------------|----------------|----------------|--------|
| 0%         | 180.25 (0%)    | 6.2088 (0%)    | 11.262 (0%)    | 0              | 1              | 1      |
| 1%         | 182.52 (1.2%)  | 6.0781 (2.1%)  | 11.523 (2.3%)  | 6.83\times10^{-5} | 10^4         | 1      |
| 5%         | 189.31 (5%)    | 6.1635 (0.7%)  | 11.663 (3.6%)  | 4.51\times10^{-8} | 10^7         | 1      |
| 10%        | 198.35 (10%)   | 5.8721 (5.4%)  | 12.241 (8.7%)  | 2.78\times10^{-4} | 10^7         | 1      |
| 20%        | 216.43 (20%)   | 5.4507 (12.2%) | 13.255 (17.7%) | 2.72\times10^{-4} | 10^8         | 1      |

The analysis of the results presented in the tables shows that the proposed approach makes it possible to obtain the calculated mechanical characteristics of a composite close to the true ones even in the case of a considerable spread in the stiffness characteristics of a composite (up to 20%) from the sample to the sample and a large error (up to 20%) in determining the critical load.

4. Conclusion

It is possible to point to one more advantage of the proposed approach when solving the problem of determining the mechanical characteristics by identification methods based on the analysis of the data of the tests for stability of shell structures in comparison with the traditional methods. It proves that in the experiment it is not necessary to measure the deformations or displacements. This greatly simplifies and makes the experiment much cheaper, reduces the time it takes. Besides, such tests can be made nondestructive. Finally, this approach allows to carry out the tests repeatedly and obtain practically any number of experimental data necessary to satisfy the requirements while performing their statistical processing.

The numerical analysis has shown that sometimes to solve the identification problem using the nonlinear programming methods it is necessary to keep a large number of -digit numbers (in the considered problems it turned out that there should be at least 40 of them).

The test problems show that when using the traditional approach, the moderate spread in the stiffness characteristics of a composite tape from the sample to the sample (used, for example, by the process of manufacturing a structure) and the small errors in measuring the critical load, lead in the identification problem to a significant difference in the design values of the small stiffness characteristics from the true ones (that is the problem is unstable to perturbation of the initial data). Thus, the application of the standard approach in real identification problems is most likely to lead to the results that are very different from the true ones. But using the method proposed in [2] allows us to regularize the problem and obtain a solution that is stable to perturbations of the original data.

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