What is "system" : the arguments from the decoherence theory

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Abstract: Within the decoherence theory we investigate the physical background of the condition of the separability (diagonalizability in noncorrelated basis) of the interaction Hamiltonian of the composite system, "system plus environment". It proves that the condition of the separability may serve as a criterion for defining "system", but so that "system" cannot be defined unless it is simultaneously defined with its "environment". When extended to a set of the mutually interacting composite systems, this result implies that the separability conditions of the local interactions are mutually tied. The task of defining "system" (and"environment") via investigating the separability of the Hamiltonian is a sort of the inverse task of the decoherence theory. A simple example of doing the task is given.

PACS number: 03.65Bz

1. Introduction

There is considerable interest in the theory of decoherence, and particularly in the "environment-induced superselection rules (EISR)" theory [1, 2]. This theory has been criticised by Machida and Namiki [3], particularly by posing the next question : "why environment would be so 'clever' as to recognize the 'pointer basis'?". This is really a substantial point in foundations of the EISR (decoherence) theory, for appointing the specific role of the "environment" in this theory. Actually, the role of the "environment" in the EISR theory is to meet some requirements (e.g., to "recognize the 'pointer basis'"), while itself being an almost ill-defined quantum system (its degrees of freedom are usually considered to be unobservable, while energetically the "environment" is usually considered [4] to be equivalent with the "bath" of the harmonic oscillators in the thermodynamical equilibrium). Thus, relative to the (open quantum) "system (S)" , the "environment (E)" is of the "secondary" importance in the decoherence theory (its degrees of freedom always being "traced out" in the corresponding calculations).

Recently [5, 6] it was pointed out existence of the necessary conditions for the occurrence of the "environment-induced superselection rules" (decoherence). Rigorously speaking, these are the effective necessary conditions but (cf. Appendix I below) one may forget about this "effectiveness". The conditions are: (i) Separability of the interaction Hamiltonian of the composite system "system plus environment (S+E)”, \( \hat{H}_{\text{int}} \), and (ii) "Nondemolition" character of \( \hat{H}_{\text{int}} \): \( [\hat{H}_{\text{int}}(t), \hat{H}_{\text{int}}(t')] = 0 \). Let us emphasize (for more details see Section 2): the separability means that there exists a (orthonormalized) basis in \( H^{(S)} \) which diagonalizes \( \hat{H}_{\text{int}} \), and that there exists a basis in \( H^{(E)} \) which also diagonalizes \( \hat{H}_{\text{int}} \). \( H^{(S)} \) and \( H^{(E)} \) represent the Hilbert state spaces of the system (S), and of the environment (E), respectively. [If \( \hat{H}_{\text{int}} \) proves nondiagonalizable in \( H^{(S)} \) and/or in \( H^{(E)} \),
we say that such interaction Hamiltonian is of the nonseparable kind.] If any of the two conditions, (i) and/or (ii), does not prove valid, one obtains nonoccurrence of decoherence, i.e. nonexistence of the "pointer basis" of the system S. Therefore, one obtains the answer to the question of Machida and Namiki: environment needs not to be so "clever" as to recognize the "pointer basis".

Still, this is a mathematical result which, by itself, hardly can be considered physically very transparent. And this especially in the context of the Zurek’s phrase [2] "no system no problem". Actually, this phrase refers to the "requirement for classicality" [2], which considers nonoccurrence of decoherence (nonexistence of the "pointer basis") as physically not very interesting issue.

Bearing this phrase in mind, in this paper we prepare an analysis of the condition of the separability. The analysis particularly refers to the next question: whether the cannonical transformations in the composite system (S+E) can help in transforming a nonseparable (separable) interaction Hamiltonian into a separable (nonseparable) form (thus eventually overcoming the predicted nonoccurrence of decoherence)? This way one obtains some interesting results, while making connection to the problem [2] "what is 'system'?"; by "system" we mean a given set of the "degrees of freedom". Actually, the analysis distinguishes the condition of the separability as a criterion for defining "system", but so that "system" is not defined unless it is simultaneously defined by its "environment". This is a new role of the "environment" in the EISR theory which has been anticipated by Machida and Namiki [3].

When extended to analysing a (macroscopic) system S, which consists in many composite systems \( S = \bigcup_i (S_i + E_i) \), this result establishes that the conditions of the separability of the local interactions in the system S are mutually tied. This gives an interesting and consistent picture in the EISR theory which represents a more rigorous formulation of the "requirement for classicality", which is otherwise an intuitive and only plausible statement.

The plan of this paper is as follows. In Section 2 we give the different definitions of the separability, which, as a necessary condition for decoherence, is confronted with the "requirement for classicality". In Section 3 this situation is elaborated, thus leading to all the afore mentioned results. Section 4 is discussion. Section 5 is conclusion.

2. Nonseparability : nondivisibility of "system" and "environment"

In Ref. [6] it was proved that each (time independent) interaction Hamiltonian, \( \hat{H}_{\text{int}} \), can (nonuniquely) be (re)written in the "linear" form:

\[
\hat{H}_{\text{int}} = \sum_k \hat{C}_{Sk} \otimes \hat{D}_{Ek},
\]

but so that both sets of the observables, \( \{ \hat{C}_{Sk} \} \) of the "system (S)", and \( \{ \hat{D}_{Ek} \} \) of the "environment (E)", consist in linearly independent observables; i.e., \( \sum_k \alpha_k \hat{C}_{Sk} = 0 \Rightarrow \alpha_k = 0, \forall k \), and \( \sum_k \beta_k \hat{D}_{Ek} = 0 \Rightarrow \beta_k = 0, \forall k \).

Along with the proof of existence of the form Eq.(1), it was developed a method [6] for obtaining a particular form of \( \hat{H}_{\text{int}} \) of the type Eq. (1). This is a basis of the, so-called, "operational definition" of the separability.
The next four definitions of the separability are mutually equivalent:

(i) $\hat{H}_{\text{int}}$ is of the separable kind (i.e., it represents a separable interaction) if there exists a basis $\{|\phi_{Si}\rangle\}$ in the Hilbert state space of the "system", $H^{(S)}$, which diagonalizes $\hat{H}_{\text{int}}$, and if there exists a basis $\{|\chi_{Ej}\rangle\}$ in the Hilbert state space of the "environment", $H^{(E)}$, which also diagonalizes $\hat{H}_{\text{int}}$.

(ii) $\hat{H}_{\text{int}}$ is of the separable kind if there exists a noncorrelated basis in the Hilbert state space of the composite system (S+E), $\{|\phi_{Si}\otimes|\chi_{Ej}\rangle\}$, which diagonalizes $\hat{H}_{\text{int}}$.

(iii) $\hat{H}_{\text{int}}$ is of the separable kind if its spectral form is of the next type:

$$\hat{H}_{\text{int}} = \sum_{p,q} \gamma_{pq} \hat{P}_p \otimes \hat{\Pi}_q,$$

where $\gamma_{pq}$ represent the eigenvalues of $\hat{H}_{\text{int}}$, while $\hat{P}_p$ and $\hat{\Pi}_q$ being the projectors onto the subspaces of $H^{(S)}$ and of $H^{(E)}$, respectively.

(iv) (the "operational definition") $\hat{H}_{\text{int}}$ is of the separable kind if and only if, for a particular form of $\hat{H}_{\text{int}}$ of the type Eq. (1), one may state:

$$[\hat{C}_{Sk}, \hat{C}_{Sk'}] = 0, \forall k, k', \quad (3a)$$

and

$$[\hat{D}_{Ek}, \hat{D}_{Ek'}] = 0, \forall k, k', \quad (3b)$$

If $\hat{H}_{\text{int}}$ is not of the separable kind, we say that it is of the nonseparable kind.

Mutual equivalence of the first three definitions is rather obvious, while their equivalence with the "operational definition" is proved in Ref. [6]; the "operational definition" will be of special interest below.

It is important to note that conclusion concerning the (non)separability of a particular $\hat{H}_{\text{int}}$ uniquely and directly follows from a particular form of $\hat{H}_{\text{int}}$ of the type Eq. (1), being completely independent on the definitions of the observables $\hat{C}_{Sk}$ and $\hat{D}_{Ek}$. Being a characteristic of $\hat{H}_{\text{int}}$, the (non)separability puts specific limitations on the possible forms of $\hat{H}_{\text{int}}$. For instance, for the separable interaction, if one would obtain a "linear" form in which appear mutually incompatible observables of the "system" and/or of the "environment", it follows that the set(s) of the observables bears linear dependence. Further, if $\hat{H}_{\text{int}}$ is of the nonseparable kind, then each form of $\hat{H}_{\text{int}}$ bears incompatibility in the set of the observables of the "system" and/or of the "environment".

As long as one is concerned with the time independent interactions, the occurrence of decoherence relies only upon the condition of the separability of $\hat{H}_{\text{int}}$. [Note : in general, for a time dependent interaction, the expression Eq. (1) refers to a particular instant, $t$.] Further, we shall be concerned only with the time independent interactions. Finally, it is worth stressing that the separability cannot be considered as a sufficient condition for the occurrence of decoherence. This is somewhat a more subtle issue, which here will not be elaborated.

It is probably obvious (cf. Ref. [5]) that the nonseparability of $\hat{H}_{\text{int}}$ implies mutual indistinguishability ("indivisability" [2]) of the "system (S)" and its "environment (E)". That is, the nonseparable interactions in the composite system S+E do not allow for
putting a definite "border line" between S and E. In the light of the Zurek's phrase [2] "no system no problem", one might pose the question of physical relevance and usefulness of the notion of existence of the necessary conditions for the occurrence of decoherence.

Actually, the requirement of divisability of S and E is the "requirement for classicality" [2], without which "the problems with the correspondence between quantum physics and the familiar everyday classical reality cannot be even posed" [2]. In other words, the requirement for classicality appears as a sort of a necessary condition in the decoherence and in the quantum measurement theory.

Since the nonseparability cannot meet this requirement, one may wonder if the nonseparability represents just a pathology of the EISR theory, without any nontrivial physical meaning. However, as it will be shown below, even in the context of the requirement for classicality (further : RC), the formal existence of the nonseparable interactions provides us with some interesting physical notions.

3. Separability as a criterion for defining "system"

Let us put RC in a more tractable form. For this purpose we are concerned with a system $S$, which is a set of (many-particle) quantum systems, $S_i$ ($S = \bigcup_i S_i$). Each subsystem $S_i$ is an open system, interacting with its environment, $E_i$. The Hamiltonian of the system $S$ is given by:

$$\hat{H} = \hat{H}_o + \sum_i \hat{H}^{(i)}_{int} + \sum_{i \neq j} \hat{H}^{(S)}_{ij} + \sum_{i \neq j} \hat{H}^{(E)}_{ij}, \quad (4)$$

where $\hat{H}_o$ represents the Hamiltonian of noninteracting systems, $\hat{H}^{(i)}_{int}$ denotes interaction in the pair $S_i + E_i$, while by the superscript "S" denoting the interactions between the "systems" (for instance $S_i$ and $S_j$), and by the superscript "E" denoting the interactions of the "environments" (for instance, of $E_i$ and $E_j$). By definition, each interaction $\hat{H}^{(i)}_{int}$ is of the separable kind.

For this composite system, the RC can be formulated as follows: an interaction $\hat{H}^{(i)}_{int}$, which is of the separable kind, cannot be changed into a nonseparable interaction, either spontaneously, or by an action from outside.

What is going to be shown is that RC can be proved, while providing us with some interesting notions within the decoherence theory.

3.1 What is "system"?

Let us go back to the expression Eq. (1). What is implicit in this expression is that each observable of the "system" and of the "environment" represents an analytical function of the corresponding degrees of freedom; that is, $\hat{C}_{Sk} = C_k(\hat{x}_{Si}, \hat{p}_{Sj})$, and $\hat{D}_{Sk} = D_k(\hat{X}_{E\alpha}, \hat{P}_{E\beta})$, where the degrees of freedom satisfy $[\hat{x}_{Si}, \hat{p}_{Sj}] = i\hbar\delta_{ij}$, and $[\hat{X}_{E\alpha}, \hat{P}_{E\beta}] = i\hbar\delta_{\alpha\beta}$.

Now one may wonder if the canonical (and particularly the linear) transformations of the observables, $\hat{x}_{Si}, \hat{p}_{Sj}, \hat{X}_{E\alpha}, \hat{P}_{E\beta}$, can help in transforming $\hat{H}_{int}$ which is in a particular form of the separable kind, into a form which is of the nonseparable kind, and vice versa.
Without any loss of generality we shall further be concerned with the transformations of the nonseparable interactions.

At first glance, one obtains a straight answer to the above question: since separability is a definite characteristic of $\hat{H}_{int}$, one would expect that it does not depend on either particular form, or upon the choice of the degrees of freedom. However, this answer is only partially correct.

Actually, validity of the above answer substantially depends upon a sort of the canonical transformations. Particularly, one may prepare the two different sorts of the canonical transformations: (a) the transformations which "mix" the degrees of freedom of $S$, independently on the transformations which "mix" only the degrees of freedom of $E$, and (b) the transformations "mixing" the degrees of freedom of both, $S$ and $E$. So, the above answer refers only to the transformations of the sort (a), which can be proved as follows.

First, the canonical transformations of this sort lead to the new degrees of freedom of the system $S$, $\{\hat{\xi}_{Sk}, \hat{\pi}_{Sl}\}$, independently on the degrees of freedom of the system $E$; let us by $\{\hat{Q}_{E\gamma}, \Pi_{E\delta}\}$ denote the "new" degrees of freedom of the system $E$. Then the transformations of the sort (a) are presented by:

$$\hat{\xi}_{Sk} = \xi_k(\hat{x}_{Si}, \hat{p}_{Sj}),$$

$$\hat{\pi}_{Sl} = \pi_l(\hat{x}_{Si}, \hat{p}_{Sj}),$$

$$\hat{Q}_{E\gamma} = Q_{\gamma}(\hat{X}_{E\alpha}, \hat{P}_{E\beta}),$$

$$\hat{\Pi}_{E\delta} = \Pi_{\delta}(\hat{X}_{E\alpha}, \hat{P}_{E\beta}).$$

These transformations imply the transformations of the observables appearing in Eq. (1): e.g., $\hat{C}_{Sk} = C_k(\hat{x}_{Si}, \hat{p}_{Sj}) \rightarrow \hat{C}_{Sk}' = C'_k(\hat{\xi}_{Sk}, \hat{\pi}_{Sl})$, so giving rise to a new form of $\hat{H}_{int}$:

$$\hat{H}_{int} = \sum_m \hat{C}'_{Sm} \otimes \hat{D}'_{Em}.$$  \hspace{1cm} (6)

According to the "operational definition" of the separability one obtains: the nonseparability of $\hat{H}_{int}$ implies existence of at least two observables of the system, $\hat{C}_{Sk}$ and $\hat{C}_{Sk'}$, for which $[\hat{C}_{Sk}, \hat{C}_{Sk'}] \neq 0$, and/or analogously for the observables of the system $E$. Now, if the form Eq. (6) should be of the separable kind, the same definition of the separability implies:

$$[\hat{C}'_{Sm}, \hat{C}'_{Sm'}] = 0, \forall m, m',$$  \hspace{1cm} (7)

and analogously for the observables of $E$.

However, the canonical transformations of the sort (a) cannot provide the loss of incompatibility which justifies the above statement.

On the other side, however, as regards the transformations of the sort (b), there is no such obstacle in the same concern. Actually, one can think of the transformations of this sort, given by:

$$\hat{\xi}_{S'k} = \xi_k(\hat{x}_{Si}, \hat{p}_{Sj}, \hat{X}_{E\alpha}, \hat{P}_{E\beta}),$$

$$\hat{\pi}_{S'l} = \pi_l(\hat{x}_{Si}, \hat{p}_{Sj}, \hat{X}_{E\alpha}, \hat{P}_{E\beta}).$$

5
\[ \hat{Q}_{E'\gamma} = Q_\gamma(\hat{x}_{Si}, \hat{p}_{Sj}, \hat{X}_{E\alpha}, \hat{P}_{E\beta}), \quad (8c) \]
\[ \hat{\Pi}_{E'\delta} = \Pi_\delta(\hat{x}_{Si}, \hat{p}_{Sj}, \hat{X}_{E\alpha}, \hat{P}_{E\beta}), \quad (8d) \]

so as to the nonseparable interaction can be transformed into a separable form (and *vice versa*). But this is a substantial step, which is distinguished by the new subscripts, \( S' \) and \( E' \).

If possible at all (see Section 4 and Appendix II), these transformations should lead to a new, separable form of \( \hat{H}_{int} \), but with respect to the *new sets of the degrees of freedom*: \( \hat{\xi}_{S'k}, \hat{\pi}_{S'l}, \hat{Q}_{E'\gamma}, \hat{\Pi}_{E'\delta} \). Therefore, to be meaningful, the transitions (8a-d) should define the new "system", \( S' \), and its (new) "environment", \( E' \).

Thus one comes to the next notion: if possible at all, and if meaningful, the transformations (8a-d) lead to the *redefining of the composite system*, \( S + E \) (relative to whose degrees of freedom the given \( \hat{H}_{int} \) appears to be of the nonseparable kind). Hence, *instead of the "old" system* \( S + E \), one obtains a new composite system, \( S' + E' \), whose degrees of freedom define a separable form of the given interaction Hamiltonian, \( \hat{H}_{int} \).

Once the new composite system is defined by the "new" degrees of freedom, \( (\hat{\xi}_{S'k}, \hat{\pi}_{S'l}; \hat{Q}_{E'\gamma}, \hat{\Pi}_{E'\delta}) \), there remains the task of the precise putting the "border line" (i.e., dividing this set onto the two subsets), which gives *precise definition* of the "system" \( S' \), and its "environment", \( E' \). Although this needs not to be unique, it is important to stress that, as a matter of principle, it always can be done. Actually, since the limit \( N \to \infty \) is legitimate (\( N \) being the number of "particles" in the "old" "environment" \( E \)), the analogous limit is automatically fulfilled for the new composite system (for no constraints of the degrees of freedom have been involved). The task of putting the "border line" is comparatively trivial in our considerations (this is just the exchange of the "particles" in the "new" composite system), so further we shall assume this task completed. Thus one reaches the point at which the application of the standard scheme of the EISR theory is straightforward, which allows for defining the "pointer basis" of the "system", \( S' \).

Everything told in this subsection can be formally summarized as follows: For a given set of the "degrees of freedom", \( (\hat{x}_{Si}, \hat{p}_{Sj}; \hat{X}_{E\alpha}, \hat{P}_{E\beta}) \), an interaction Hamiltonian, \( \hat{H}_{int} \), given in a particular form of the type Eq. (1), is of the nonseparable kind. However, with respect to the "new" set of the degrees of freedom, \( (\hat{\xi}_{S'k}, \hat{\pi}_{S'l}; \hat{Q}_{E'\gamma}, \hat{\Pi}_{E'\delta}) \), the same interaction Hamiltonian obtains a separable form (of the type of Eq. (1)):

\[ \hat{H}_{int} = \sum_p \hat{E}_{E'p} \otimes \hat{F}_{E'p}, \quad (9) \]

where \( \hat{E}_{E'p} = E_p(\hat{\xi}_{S'k}, \hat{\pi}_{S'l}) \), and \( \hat{F}_{E'p} = F_p(\hat{Q}_{E'\gamma}, \hat{\Pi}_{E'\delta}) \).

And this brings us to the next task: for an *a priori* given set of the degrees of freedom, the nonseparability of a given interaction Hamiltonian might be overcome by applying the canonical transformations of the sort (b), thus obtaining a definition of the "system" (above : \( S' \)), *but only simultaneously* with obtaining a definition of the corresponding "environment" (above : \( E' \)). [Note: then the "old systems", \( S \) and \( E \), remain mutually indivisible.]

**3.2 The proof of RC**
As regards the time independent interactions, the proof of RC relies upon the considerations of the actions from "outside" the composite system \( \cup_i (S_i + E_i) \).

The physical situation here to be analysed is the next one: one wonders if by an action from outside, a local separable interaction can be transformed into a nonseparable form. As above, we shall be concerned with the inverse transformations, bearing in mind the conclusion of subsection 3.1: that such transformations imply the redefining of an a priori given composite system.

Let us refer to a particular composite system, \( S_i + E_i \). Each action from outside assumes an interaction with an outer quantum system, \( A \). Since the task is to transform the nonseparable, local interaction \( \hat{H}_{S_i E_i} (\equiv \hat{H}^{(i)}_{\text{int}}) \), into a separable form, the system \( A \) must interact with the composite system \( S_i + E_i \) as a whole; let us denote this by \( (S_i + E_i) + A \). Finally, being a macroscopic system, the system \( A \) is an open quantum system, thus leading to the next physical situation: \( (S_i + E_i) + A + E_A \), where \( E_A \) denotes the environment of the system \( A \).

It is important to note that, if the interaction of \( A \) and the "whole", \( S_i + E_i \), is of the separable kind, everything remains intact. Actually, as it is implicit in the subsection 3.1, the separable interaction keeps the degrees of freedom of the mutually interacting systems (here: of the system \( A \), and the "whole", \( S_i + E_i \)). It particularly means that the nonseparable interaction \( \hat{H}_{S_i E_i} \) would thus remain intact.

On the other side, and this proves RC, neither the nonseparable interaction \( \hat{H}_{A(S_i+E_i)} \) could change the nonseparability of \( \hat{H}_{S_i+E_i} \). The proof of this assertion is as follows.

According to the subsection 3.1, the nonseparable interaction \( \hat{H}_{A(S_i+E_i)} \), if possible at all, and if meaningful, would imply redefining of the complete system \( (S_i + E_i) + A \), but not only (as desired) the redefining of the system \( S_i + E_i \). This would (cf. Section 2) make the system \( A \) indivisible from the system \( S_i + E_i \). But this produces a contradiction. Actually, indistinguishability of the system \( A \) contradicts the separability of the interaction \( \hat{H}_{AE_A} \). The only way to overcome this contradiction is to be concerned with another the "whole", \( (S_i + E_i) + A + E_A \) (instead of the system \( (S_i + E_i) + A \)). But this is nothing else but extending the original task (which refers to \( S_i + E_i \)), onto the new "whole" \( (S_i + E_i) + A + E_A \), which proves impossibility of the change of the nonseparable (separable) local interaction to the separable (nonseparable) interaction via an action from outside.

### 3.3 A new physical role of the separability

Besides giving the proof of RC, the above subsections provide us with some interesting ideas. The separability condition concerning the two local interactions, \( \hat{H}^{(i)}_{\text{int}} \) and \( \hat{H}^{(j)}_{\text{int}} \) cannot be considered mutually independent (as one would plausibly expect). should be mutually USAGLASENE. That is, given \( \hat{H}^{(i)}_{\text{int}} \), an interaction \( \hat{H}^{(j)}_{\text{int}} \) cannot be of a completely arbitrary type.

This assertion follows from the previous subsection. First, the separability of \( \hat{H}^{(i)}_{\text{int}} \) simultaneously defines the "system" \( S_i \) and its environment \( E_i \). According to Eq. (4), there are the interactions between the "systems", e.g., \( S_i \) and \( S_j \) (likewise the interactions of their "environments"). The interaction between the two systems is defined by their degrees of freedom. But the degrees of freedom of \( S_j \) are determined by the condition of separability
of $\hat{H}_{\text{int}}^{(j)}$ - which gives the connection of the two local interactions, $\hat{H}_{\text{int}}^{(i)}$ and $\hat{H}_{\text{int}}^{(j)}$. It is interesting to note that these "connections" are transitive, thus leading to a new physical picture: the overall condition of the separability in a system consisting in many composite systems, assumes that the separability conditions of the local interactions are mutually tied, thus giving a consistent physical picture in the EISR theory - which is only plausibly and poorly stated by the "requirement for classicality" [2].

In general, everything told in this Section refers to a particular instant of time, $t$. The discussion concerning this issue here will be left out.

4. Discussion

Without any assumption coming from the outside of the separability considerations, we have obtained a new physical role of the separability in the EISR theory. (A) The condition of the separability of $\hat{H}_{\text{int}}$ may serve as a criterion for simultaneous defining of the "system" and of its "environment" (i.e., "system" is not defined unless it is simultaneously defined with its "environment"), and (B) In a set of the composite macroscopic systems, $S$, the separability conditions concerning the local interactions are mutually tied (i.e., the separability on one place determines the separability condition on another, "distant" place in the system $S$). Thus the points (A) and (B) represent an elaborated form of the only plausibly (and poorly) formulated the "requirement for classicality" [2].

The point (B) allows for extending the considerations to the complete system $S$ ($S = \cup_i(S_i + E_i) = \cup_iS_i + \cup_iE_i$) as an isolated system. Then one may refer to the system $S$ as to the "macroscopic piece of the Universe", for which the point (B) establishes consistency and "rigidity" of the definitions of its parts ($S_i$s and $E_i$s), which can be considered as a counterpart of the "conditions of consistency" in the cosmological considerations [7].

Therefore, for the purpose of defining "system", within the EISR theory appears the next task: For a given set of the degrees of freedom, the separability of a given $\hat{H}_{\text{int}}$ should be tested. If it would prove nonseparable, one should look for the canonical transformations of the sort "(b)", so as to provide (if possible at all) a separable form of $\hat{H}_{\text{int}}$, thus obtaining a definition of the new composite system. This is really the inverse task of the EISR theory, in which (likewise in the measurement theory), one constructs $\hat{H}_{\text{int}}$ for an a priori given set of the degrees of freedom.

However, this procedure needs not to lead to unique result. Actually, in general one may obtain the different results with respect to the next criteria: (i) if there appear (at least) the two different systems ($S_1, S_2$) and their environments ($E_1, E_2$), both referring to the separable forms of $\hat{H}_{\text{int}}$, and (ii) even for unique result (unique composite system $S + E$), one may pose the question of the choice of the canonical variables describing the "system" (and its "environment") - which is even more difficult problem of "what is 'object'?" [8]. Therefore, the above mentioned task is extended by the tasks corresponding to the points (i) and (ii). Yet, the elaboration of these tasks depends on the details of the model of $\hat{H}_{\text{int}}$, and here will be left out.

So far, we have been concerned only with the interaction Hamiltonian, without taking into account the other terms of the complete Hamiltonian (Eq. (4)). Certainly, so as to make the results of the above tasks complete, one must apply the same method (and
reasoning) to the complete Hamiltonian. *Only in this way* one may obtain the fully sensible definition of "system" (and of its "environment").

Finally, one may doubt about existence of the canonical transformations Eq. (8a-d), which should provide the transition from the nonseparable (separable), to the separable (nonseparable) form of the Hamiltonian. Again, this is rather a matter of the details in the model of the Hamiltonian, but for an example see Appendix II.

5. Conclusion

We have investigated the physical meaning of the separability of the interaction Hamiltonian. Actually, we have confronted the separability as a necessary condition for decoherence [5, 6], with the "requirement for classicality" [2], thus obtaining some interesting results.

When expressed in terms of the concept of the separability, the "requirement for classicality" can be proved; i.e., this plausible statement [2] appears as a corollary of the separability considerations in the context of the decoherence (EISR) theory. This way comes to scope a *physical role of the separability*. Particularly, the condition of the separability may serve as a criterion for defining "system", but so that the "system" is not defined unless it is simultaneously defined by its "environment". Second, in a set of mutually interacting open quantum systems, one meets mutual connections of the separability conditions concerning the local interactions. The later gives a physically richer formulation of the "requirement for classicality", which is otherwise only a plausible statement.

Appendix I

In Appendix II of Ref. [6] it was emphasized that the necessary conditions might break in some exceptional cases - the existence of which has not been proved but just not disproved (cf. Ref. [9]). Particularly, for some special models of the interaction Hamiltonian, *and* for some special the initial states of the environment, one eventually might obtain the occurrence of decoherence even if the necessary conditions are not fulfilled. However, one can forget about these exceptions, for many reasons. Probably the most striking one is the next one: a special choice of the initial state of the environment requires the preparation of the initial state. But then remains the question: who would provide this preparation? And the answer can be stated by making reference to Omnés [8]: that one can not consider the environment’s environment ("apparatus" acting on the environment) physically sound idea, thus removing the problem of the preparation (and also the question of the special choice) of the initial state of the environment.

Appendix II

We are interesting in the next two problems: First, whether the canonical transformations (8a-d) can provide the transformation of $\hat{H}_{\text{int}}$ of a nonseparable, to a separable form; Second (cf. the task (i) in Section 4), whether there exist the two different, both separable, forms of $\hat{H}_{\text{int}}$, which correspond to the different composite systems, $S_1 + E_1$, and $S_2 + E_2$?
In answering, we shall first refer to the second question by analyzing the interaction Hamiltonian. Then we shall refer to the first question, but by analyzing the complete Hamiltonian.

Let us consider the hydrogen atom Hamiltonian:

\[ \hat{H} = \frac{\hat{p}_p^2}{2m_p} + \frac{\hat{p}_e^2}{2m_e} + V_{\text{Coul}}, \quad (\text{II.1}) \]

where the subscript "p" denotes the proton and "e" denotes the electron, while \( V_{\text{Coul}} \) represents the Coulomb interaction. Therefore, the interaction Hamiltonian for this "composite system", "electron + proton" is:

\[ \hat{H}_{\text{int}} \equiv V_{\text{Coul}}. \quad (\text{II.2}) \]

As it can be easily proved (cf. point (ii) of Section 2), this interaction is of the separable kind with respect to the noncorrelated basis \( |\vec{r}_p⟩ \otimes |\vec{r}_e⟩ \). But, as it is probably obvious, \( \hat{H}_{\text{int}} \) is of the separable kind also with respect to the "center of mass", and the "relative particle" degrees of freedom, \( \vec{R}_{\text{CM}}, \vec{r}_{\text{rel}} \), respectively. That is, \( \hat{H}_{\text{int}} \) is of the separable kind also with respect to the noncorrelated basis \( |\vec{R}_{\text{CM}}⟩ \otimes |\vec{r}_{\text{rel}}⟩ \). Note: here one meets the two different "composite systems", "proton + electron (\( S_1 + S_2 \))", and "center of mass + the relative particle (\( S_2 + E_2 \))", both referring to the separable forms of \( \hat{H}_{\text{int}} \); this is the answer to the above the second question.

However, as it was strongly emphasized in Section 5, in defining "system" one must take into considerations the complete Hamiltonian, which in this case, leads to unique definition of "system".

Actually, there are the two different forms of the complete Hamiltonian:

\[ \hat{H} = \frac{\hat{p}_p^2}{2m_p} \otimes \hat{I}_e + \hat{I}_p \otimes \frac{\hat{p}_e^2}{2m_e} - \frac{Ze^2}{4\pi\varepsilon_0|\vec{r}_p - \vec{r}_e|}, \quad (\text{II.3}) \]

and

\[ \hat{H} = \frac{\hat{p}_{\text{CM}}^2}{2M} \otimes \hat{I}_{\text{rel}} + \hat{I}_{\text{CM}} \otimes \left( \frac{\hat{p}_{\text{rel}}^2}{2\mu} - \frac{Ze^2}{4\pi\varepsilon_0\hat{r}_{\text{rel}}} \right), \quad (\text{II.4}) \]

As it easily follows from the "operational definition" of the separability (cf. point (iv) of Section 2), the expression (II.3) is of the nonseparable kind, while the expression (II.4) is of the separable kind, thus uniquely defining the composite system: it is the system "center of mass + the relative particle", defined by the separable form of the complete Hamiltonian, Eq. (II.4).

Note: the cannonical transformations, \((\vec{r}_p, \vec{p}_p, \vec{r}_e, \vec{p}_e) \to (\vec{R}_{\text{CM}}, \vec{P}_{\text{CM}}, \vec{r}_{\text{rel}}, \vec{p}_{\text{rel}})\), provide the transformation of the complete Hamiltonian from the nonseparable (Eq. (II.3)), to the separable form (Eq. (II.4)). (In the position representation this reads: \( \hat{H} \) is of the nonseparable form with respect to a noncorrelated basis \( \{\Psi_m(\vec{r}_p) \otimes \chi_n(\vec{r}_e)\} \), but is of the separable form in a noncorrelated basis \( \{\Phi_p(\vec{R}_{\text{CM}}) \otimes \phi_q(\vec{r}_{\text{rel}})\} \). This gives the answer to the above the first question.

Although this model does not refer to the decoherence theory, we feel it paradigmatic for the considerations of the many-particle quantum systems for which the results strongly depend upon the details in the model of the Hamiltonian.
References:

[1] W. H. Zurek, Phys. Rev. D26 (1982) 1862
[2] W. H. Zurek, Prog. Theor. Phys. 89 (1993) 281
[3] S. Machida and M. Namiki, in Proc. 2nd Int. Symp. Foundations of Quantum Mechanics, Tokyo, 1986, pp. 355-359; and references therein
[4] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) 149 (1983) 374
[5] M. Dugić, Physica Scripta 53 (1996) 9
[6] M. Dugić, Physica Scripta 56 (1997) 560
[8] R. Omnès, ”The Interpretation of Quantum Mechanics”, Princeton University Press, Princeton, 1994
[9] M. Dugić, J. Res. Phys. 27 (1998) 141.