Vehicle routing problem with time windows using natural inspired algorithms

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Abstract. Process of distribution of goods needs a strategy to make the total cost spent for operational activities minimized. But there are several constrains have to be satisfied which are the capacity of the vehicles and the service time of the customers. This Vehicle Routing Problem with Time Windows (VRPTW) gives complex constrains problem. This paper proposes natural inspired algorithms for dealing with constrains of VRPTW which involves Bat Algorithm and Cat Swarm Optimization. Bat Algorithm is being hybrid with Simulated Annealing, the worst solution of Bat Algorithm is replaced by the solution from Simulated Annealing. Algorithm which is based on behavior of cats, Cat Swarm Optimization, is improved using Crow Search Algorithm to make simplier and faster convergence. From the computational result, these algorithms give good performances in finding the minimized total distance. Higher number of population causes better computational performance. The improved Cat Swarm Optimization with Crow Search gives better performance than the hybridization of Bat Algorithm and Simulated Annealing in dealing with big data.

1. Introduction
Distribution is one of marketing activities in aim to facilitate the producers in sending goods to the customers. In the distribution process, a set of customers is served by a producer where located in an area. A set of crews operates a set of vehicles which is located in the producer place called a depot to fulfill the each customer demand. As a producer, for minimizing their total operational transportation cost of distribution, the depot have to find an appropriate road network for the vehicles. This problem is known as the Vehicle Routing Problem (VRP). The solution of VRP about determination of a set of routes where each performed by a vehicle that starts and ends on a depot, the demand of the customers are fulfilled, all the operational constrains are satisfied, and the total operational transportation cost is minimized [1]. One of the extension of VRP is VRP with Time Windows (VRPTW). In the VRPTW, each customer corresponds to service interval time which represents that the customer can only be served at the time interval given. A vehicle is allowed to serve the customer between known time intervals.

The road network is generally described as a graph. Given $G=(V,A)$ is a complete graph, where $V = \{0, 1, ..., L\}$ is the vertex set and $A$ is the arc set. Vertex 0 is corresponds to the depot and vertices $i = 1, ..., L$ correspond to customers where $L$ is number of customers. Each arc has a weighted value which represents the travel cost spent from vertex $i$ to vertex $j$, $c_{ij}$. Each customer has a known nonnegative demand, $d_i$. Given a vertex set $S \subseteq V$, the total demand is described as $d(S) = \sum_{S \subseteq S} d_i$. In a depot is available a set of $K$ vehicles where each has capacity $C$. To ensure feasibility, assume that $d_i \leq C$ for
each \( i = 1, \ldots, L \). And each customer is served by a vehicle in a time interval, between earliest time and latest time \([e_i, l_i]\).

Many optimization methods for solving VRPTW are studied recently. Meta heuristic methods have been applied inspired by the behavior of natures. In solving VRPTW, Yaseen et. al proposes a Meta-Harmony Search [2]. Simulated Annealing is also an effective algorithm for solving VRPTW [3,4]. Bat algorithm is natural inspired algorithm based behavior of bats. Bat algorithm shows better perform than Particle Swarm Optimization, Genetic Algorithm and Harmony Search [5]. Improved Bat Algorithm has outperformed significantly all other alternatives, which are Evolutionary Simulated annealing, Genetic Algorithm and Firefly Algorithm, in most cases for symmetric and asymmetric routing problem, Traveling Salesman Problem [6]. In this paper, Bat Algorithm is studied to solve VRPTW. For increasing its performance, Bat Algorithm is combined with Simulated Annealing. This paper also presents VRPTW using Cat Swarm Optimization and Crow Search Algorithm. Cat Swarm Optimization Algorithm is adopted from behavior of cats where has two modes, seeking mode and tracing mode [7]. The complexity of Cat Swarm Optimization is helped by Crow Search Algorithm. Crow Search Algorithm has fewer parameter which based on the intelligent of crows [8]. For performance simulation, the algorithms given are tested and compared. This paper is organized as follows: the description of VRPTW is in the next section of this section. Next, the proposed natural inspired algorithms are explained. The comparison performances of each algorithm simulation is presented on the next section. Last, the result is concluded on last section.

2. Formulation of the Problems

The main objective of VRPTW is minimization of the transportation cost but still satisfy all the constraints. The routes on VRPTW can be represented as a graph theoretic problem [1]. Let \( G = (V, A) \) be a complete graph where \( V \) is the vertex set and \( A \) is the arc set, \( V = \{0, 1, \ldots, L\} \). A depot is represented as vertex 0 and customers are represented as vertices \( i \), where \( i = 1, \ldots, L \). Since \( G \) is a weighted graph then there is a travel cost from vertex \( i \) to vertex \( j \) which is denoted as \( c_{ij} \). And \( K \) number of vehicles are available on depot. Each customer has time window \([e_i, l_i]\). When a vehicle comes before the upper bound, earliest time \( e_i \), then the vehicle have to wait until time \( e_i \). When a vehicle comes after the lower bound, latest time \( l_i \), then the vehicle have to get back to the depot [9]. The VRPTW can be stated as follows:

\[
\begin{align*}
\text{Min } Z, Z &= \sum_{k=1}^{K} \sum_{i=1}^{L} \sum_{j=1}^{L} c_{ij} x_{ijk} \\
\sum_{i=1}^{L} q_i y_{ik} &\leq Q, \quad \forall k \\
\sum_{k=1}^{K} y_{ik} &= 1, \quad \forall i \\
\sum_{j=1}^{L} x_{ijk} &= y_{ik}, \quad \forall j, k \\
\sum_{j=1}^{L} x_{ijk} &= y_{ik}, \quad \forall i, k \\
\sum_{i,j \in \mathcal{S}} x_{ijk} &\leq |\mathcal{S}| - 1, \quad \mathcal{S} \subset \{1,2,\ldots,L\}, \quad \forall k \\
s I_i + t_{ij} &\leq s e_j, \quad \forall i, j \\
e_i &\leq s e_j \leq l_i, \quad \forall i
\end{align*}
\]

where
3. Natural Inspired Algorithms

Meta-heuristic algorithms which are based on the natural inspired algorithms are proposed to solve VRPTW. This section describes Hybrid Bat Algorithm and Simulated Annealing, and Improved Cat Swarm Optimization using Crow Search.

3.1. Hybrid Bat Algorithm and Simulated Annealing

A hybrid algorithm between bat algorithm and simulated annealing for solving VRPTW is begun with generating positions of bats population randomly as the initial solutions of VRPTW. For easier understanding of this hybrid algorithm, the overview of the bat algorithm and also simulated annealing is given.

3.1.1. Overview of Bat Algorithm. Bat is natural algorithm that inspired by micro-bats behavior when become predators, find locations and avoid obstacles when flies, using ecolocation [5]. Bats emit a very loud sound pulse where have a constant frequency. The loudness of the emitted pulse also varies. For each bat $i$th ($i = 1, 2, ..., N$), the procedure as follows:

1. Define pulse frequency ($f_i$), and loudness ($A_i$).
2. Initialize the bats population with position $x_{i,d}$, velocities ($v_i$), and pulse rates ($r_i$).
3. Evaluate the objective value and obtain the best solution so far.
4. Update the position of each bat by adjusting the new velocities and frequencies using Eq. (9)-(11) at time step $t$ and $D$-dimensional space.

\[
\begin{align*}
    f_{i}^{t} &= f_{min} + (f_{max} - f_{min}) \times \text{rand} \\
    v_{i,d}^{t+1} &= v_{i,d}^{t} + (x_{i,d}^{t} - x_{best,d}) \times f_i \\
    x_{i,d}^{t+1} &= x_{i,d}^{t} + v_{i,d}^{t+1}
\end{align*}
\]

5. Generate randomly $\text{rand}$. Check the termination if $\text{rand} > r_i$ then select a solution among the best solution so far and generate a local solution using :

\[x_{i,d}^{t+1} = x_{i,d}^{t} + \text{rand} \times A_i\]

6. If $\text{rand} < A_i$ and the objective value of position $x_{i,d}^{t+1} <$ the objective value of position $x_{i,d}^{t}$, then accept the new solutions, reduce $A_i$ and increase $r_i$. Otherwise, accepting the $x_{i,d}^{t}$ as the new position.

7. Rank the bats and find the current best.
3.1.2. **Overview of Simulated Annealing.** Simulated Annealing is known as global search optimization inspired by annealing physical process of solids [11]. The idea of Simulated Annealing is that this algorithm not only accepts changes that improve the objective function, but also keeps changes that are not ideal. Annealing process needs temperature and cooling schedule controlling in aims to get the hard structures. The Simulated Annealing Algorithm procedure for time step $t$, $D$-dimensional spaces and $i (i=1,2,...,N)$ is described below [12]:

1. Define initial value for initial temperature, $T_0$ and generate randomly one initial solution $x_{i,d}^0$.
2. Set final temperature $T_f$ and define cooling schedule using $\alpha$ where $0 < \alpha < 1$.
3. Evaluate the objective value of the initial solution.
4. Get modification on the solutions then obtain the new solutions $x_{i,d}^{t+1}$.
5. Evaluate the objective value of the new solutions $f(x_{i,d}^{t+1})$.
6. Calculate $\Delta f = f(x_{i,d}^{t+1}) - f(x_{i,d}^{t})$.
7. If the new solution $x_{i,d}^{t+1}$ is better than $x_{i,d}^{t}$, then accept the new solution and go to step 9. Otherwise calculate probability of acceptance solution $p$.

   $$ p = e^{-\frac{\Delta f}{T}} \tag{13} $$

8. Generate randomly rand. Check the acceptance condition if $p > \text{rand}$ then accept the new solutions.
9. Update the temperature $T_{t+1} = \alpha T_t$ where $\alpha$ is the reducing temperature factor.
10. Check the termination condition. If the condition unsatisfied then go to step 4.

3.1.3. **Proposed Hybrid Bat Algorithm-Simulated Annealing.** For increasing the performances of Bat Algorithm, this paper proposes a hybrid algorithm. The worst solution while using Bat Algorithm procedure is set as the initial solution for Simulated Annealing Algorithm. After obtain the best solution of Simulated Annealing, this solution will replace the solution before. The flowchart of the proposed hybrid is shown in figure 1.

3.2. **Improved Cat Swarm Optimization with Crow Search**

Cat Swarm Optimization Algorithm is a nature algorithm inspired by behavior of cats which have two modes, seeking mode and tracing mode [7]. Cat Swarm Optimization Algorithm has complex procedure which requires higher computational time. In this paper, Cat Swarm Optimization Algorithm performance is improved by the Crow Search Algorithm to help gaining faster convergences. Crow Search Algorithm has fewer parameter which easier to hybrid with other algorithm. The intelligence of crows inspiring this evolutionary algorithm [8]. Crows are known have good memorized and also have a good self-awareness. Crows will move their foods from hiding places for avoid being pilfered. The overview of Cat Swarm Optimization and Crow Search is given.

3.2.1. **Overview of Cat Swarm Optimization.** In Cat Swarm Optimization, there are two modes which are seeking mode and tracing mode [7]. Ratio number of cats distributed is determined by a mixture ratio ($mr$). Flag is given to help labelling whether a cat is in a seeking or tracing mode. For $D$-dimensional spaces, and $N$ cats where $i = 1,2,...,N$, the procedure of Cat Swarm Optimization Algorithm is explained as follows:

1. Initialize the position of each cat and the velocity randomly.
2. Based on $mr$, distributed cats randomly into seeking and tracing mode.
3. Evaluate the objective values of position $x_{i,d}^{t}$ for time step $t$.
4. Apply seeking mode if the cat is in seeking mode, otherwise apply tracing mode.
5. Evaluate the objectives values of position $x_{i,d}^{t+1}$.
6. If the objective value of position $x_{i,d}^{t+1}$ is better than $x_{i,d}^{t}$, then update the position.
7. Check the termination condition. If the condition unsatisfied then go to step 2.
3.2.2. Overview of Crow Search. Crows are the most intelligent birds [8]. Crows have good memories, they can move their food from other thieves by flying and then hiding it on hiding places. This good self-awareness is one of distinctions of crows. The procedure of Crow Search Algorithm for $N$ crows is described as follows:

1. Initialize the position of each $i$th bat where $i = 1, 2, ..., N$ and the memory randomly for $D$-dimensional spaces. In this step, the initial memory is assumed by the current position.
2. Evaluate the objective values of the position $x_{i,t}$ where $t$ is the time step.
3. Get modification on the position.
4. Evaluate the objective values of the new positions, $x_{i,t+1}$. 

Figure 1. Flowchart of Hybrid Bat Algorithm and Simulated Annealing.
5. Update the memory if the objective value of position $x_{i,d}^{t+1}$ is better than $x_{i,d}^{t}$.
6. Check the termination condition. If the condition unsatisfied then go to step 3.

3.2.3. Proposed Improved Cat Swarm Optimization with Crow Search. For time step $t$ and $D$-dimensional spaces, the procedure of Improved Cat Swarm Optimization with Crow Search is explained by the flowchart on figure 2 and more clearly is described as follows:

1. Initialize parameters: number of individuals ($N$), seeking memory pool ($smp$), count dimension to change ($cdc$), seeking range dimension ($srd$), acceleration coefficient ($c$), mixing ratio ($mr$), flight length ($fl$) and awareness probability ($AP$).
2. Initialize the position ($x_{i,d}^{t}$), velocity ($v_{i,d}^{t}$), and memory ($m_{i,d}^{t}$) of the individual randomly. It is assumed that the initial position is set as initial memory.
3. Evaluate the fitness of each individual.
4. Individual which as the best solution ($x_{best,d}$) is labeled the self-position consideration ($SPC$) = 1, otherwise $SPC = 0$.
5. Randomly picked individuals from population based on $mr$ and flagged in seeking mode. The flag remains is set to tracing mode.
6. For each individual in seeking mode, apply:
   a) If $spc$ of $j$th cat = 1 then generates ($smp$-1) copies and keeps the origin position. Otherwise, generates $smp$ copies.
   b) According $cdc$, apply a mutation operation to modify the $k$th dimension of each position.
   
$$x_{i,k}^{t+1} = x_{i,k}^{t} + (-1)^k * srd * x_{i,k}^{t}$$

   c) Evaluate the fitness.
   d) Store the position in archive.
   e) Apply Roulette Wheel selection for each copies cat [13].
7. For each individual in tracing mode, apply:
   a) Update the velocities.
   
$$v_{i,k}^{t+1} = v_{i,k}^{t} + c * rand * (x_{best,d}^{t} - x_{i,d}^{t})$$

   b) Update the position of $i$th cat.
   
$$x_{i,d}^{t+1} = x_{i,d}^{t} + v_{i,d}^{t+1}$$

   c) Evaluate the fitness.
   d) Store the position in archive.
8. Collect the new individual positions from seeking and tracing mode process in the archive.
9. If the fitness of new position is better than the memorized position then update the memory.
10. Generate $r$ randomly with uniform distribution $(0,1)$ and the new position from the chosen copy is modified using:

$$x_{i,d}^{t+1} = \begin{cases} 
    x_{i,d}^{t} + r * fl * (m_{j,d} - x_{i,d}) & \text{if } r \geq AP, \\
    \text{a random position, otherwise}
\end{cases}$$

11. Evaluate the fitness.
12. Update the memory when the fitness value of the modified position is better than the memorized position.
13. Check the termination criterion, if the termination is satisfied then the best position is as the best solution. Otherwise repeat step (4)-(13).
4. Computational Study

As an approximation for obtaining the optimal solution of VRPTW, the Bat Algorithm is being hybrid with Simulated Annealing and the Cat Swarm Optimization is being improved with Crow Search Algorithm. This proposed natural inspired algorithms are combined and gets to be compared in aims knowing which one is has a better performance for solving VRPTW. Both proposed algorithms have own technique for helps other weakness. Simulated Annealing is used to find better solution for

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**Figure 2.** Flowchart of Improved Cat Swarm Optimization with Crow Search.
replacing the worst solution of Bat Algorithm. The simplicity of Crow Search and how behavior of crows in memorizing hidden place helps the Cat Swarm Optimization to get its convergence faster.

VRPTW has capacities and time windows constraints. Because a route is only handled by one vehicle then the total customers demand in the same route is not exceed the capacity of the vehicle. And the service time of customers is according to the time windows of customer \( t_{e_i, l_i} \). If a vehicle come after time \( l_i \), then the vehicle have to back to the depot. And when a vehicle come before time \( e_i \), then the vehicle should wait until time \( e_i \). The proposed inspired algorithms are applied in test problems of VRPTW.

4.1. Test Problems

VRPTW will be more complex in a large number of customers. More customers in the problem, more computational time will be required. This paper uses data which is taken from http://neo.lcc.uma.es/vrp/vrp-instances/capacitated-vrp-with-time-windows-instances/. In the objective of performance comparison, this paper takes 25 customers, 50 customers and 100 customers. The data which available are the customers coordinate, demand of each customer, service duration of each customer, and time windows of each customer. The route of each vehicle is the solution of this problem. Each route must satisfy all the constraints and the solution can minimize the total travel cost spent of the vehicles.

4.2. Computational Results

In comparison the performance of the hybrid Bat Algorithm and Simulated Annealing, firstly shows the computational result using Bat Algorithm. Using vehicle capacity = 200, maximum number of vehicle = 25, number of bats (\( N \)) = (10, 50, 100), pulse rate = 0.9, loudness = 1, \( f_{\text{min}} = 0 \), and \( f_{\text{max}} = 100 \), the computation result of 1000 generations is shown in table 1. In table 1 also explain the computational result of hybrid Bat Algorithm and Simulated Annealing using the same parameters value with the Bat Algorithm, initial temperature = 1000000, final temperature = (905000, 369000, 43607), reducing temperature coefficient = 0.99, and maximum limits = 1000. From the comparison result, shows that the proposed hybrid Bat Algorithm and Simulated Annealing is performed better than the Bat Algorithm itself.

|                | Total Distance |
|----------------|----------------|
|                | 25 customers   | 50 customers   | 100 customers |
|                | \( N = 10 \)  | \( N = 50 \)  | \( N = 100 \) |
| CSO            | 905,1          | 712,0          | 688,1         |
| BA             | 623,8          | 568,1          | 599,0         |
| BA-SA          | 616,4          | 597,8          | 442,9         |
| CSO-CS         | 546,3          | 523,9          | 447,3         |

The Improved Cat Swarm Optimization is being improved with Crow Search Algorithm. Table 1 shows the computational result of Cat Swarm Optimization using \( smp = 5 \), \( mr = 0.4 \), \( srd = 0.5 \), \( c = 2 \), \( cdc = 0.2 \), with the same values of parameter VRPTW. Using \( fl = 2 \), \( AP = 0.5 \) for Crow Search parameters, table 1 shows the computational result of improved Cat Swarm Optimization with Crow Search. Comparison between the origin Cat Swarm Optimization and the improved Cat Swarm Optimization with Crow Search states that the improved algorithm gives better performance in minimizing the total distance of vehicle routes which is summary on table 1.

Overall computational results conclude that there are two algorithms which given good performance. A hybrid Bat Algorithm and Simulated Annealing and the improved Cat Swarm Optimization with Crow Search give a significant result. The number of population of the individuals (\( N \)) has high impact for the algorithm performances. Higher number of the population size gives higher performance of finding the
solution which minimized the total distance. Hybrid Bat Algorithm and Simulated Annealing perform well in small data with the high population, but a higher number of data makes the algorithm gets slower convergences. The improved Cat Swarm Optimization performs better than hybrid of Bat Algorithm and Simulated Annealing in higher number of data.

5. Conclusions
Vehicle Routing Problem with Time Windows has complex constraints which needs meta-heuristic approximation for solving the problem. The objective is finding the optimal routes which minimized the total cost spent when some vehicles travel around to serve the customers. Simulated Annealing which has been applied in VRPTW is combined with Bat Algorithm to improve the performance of the Bat Algorithm. In computational result using variance of number of the customers, the hybrid Bat Algorithm and Simulated Annealing gives better performance comparing with the Bat Algorithm. But the improved algorithm, Cat Swarm Optimization with Crow Search has the better performance than the hybrid of Bat Algorithm and Simulated Annealing in dealing with large number of data which is the number of customers.

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