Mathematical modeling of non-stationary gas flow in gas pipeline

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Abstract. An analysis of the operation of the gas transportation system shows that for a considerable part of time pipelines operate in an unsettled regime of gas movement. Its pressure and flow rate vary along the length of pipeline and over time as a result of uneven consumption and selection, switching on and off compressor units, shutting off stop valves, emergence of emergency leaks. The operational management of such regimes is associated with difficulty of reconciling the operating modes of individual sections of gas pipeline with each other, as well as with compressor stations. Determining the grounds that cause change in the operating mode of the pipeline system and revealing patterns of these changes determine the choice of its parameters. Therefore, knowledge of the laws of changing the main technological parameters of gas pumping through pipelines in conditions of non-stationary motion is of great importance for practice.

1. Purpose of study
To solve differential equations describing non-stationary motion in trunk gas pipelines, where pumping, sampling or emergency sources are present, it is necessary to have initial and boundary conditions that correspond to specific technological modes of operation of gas pipelines.

2. Carrying out the research
The system of difference equations:

\[
\left( (\rho u)_{j+1}^{i+1/2} - (\rho u)_j^{i+1/2} \right) \Delta t + \left( \rho j_{j+1/2}^{i+1} - \rho j_{j+1/2}^i \right) \Delta x = 0, \tag{1}
\]

\[
\left( (\rho u)_{j+1}^{i+1/2} - (\rho u)_j^{i+1/2} \right) \Delta x + \left( (\rho v^2 + P)_j^{i+1/2} - (\rho v^2)_j^{i+1/2} \right) \Delta t + \left( \rho g \sin \alpha + \frac{1}{2d} \rho v^2 \right)_{j+1/2}^{i+1/2} \Delta x \Delta t = 0, \tag{2}
\]

\[
\Delta t - \left( \rho v \left( \frac{v^2}{2} + h + + gH \right)_j^{i+1/2} \right) \Delta t + \left( \frac{1}{d} K(T - T_0) \right)_{j+1/2}^{i+1/2} \Delta x \Delta t = 0, \tag{3}
\]

approximates the original system of differential equations:
The statement of the initial boundary value problem is as follows. At the beginning of the simulation, it is necessary to specify the distribution of the unknown functions entering the equation. In this case, unknown functions are pressure, velocity and temperature of gas, which depend on time and coordinate \([1]\). The temperature enters the system of the equation through the equation of state and the thermodynamic potentials of gas. Instead of gas velocity, it is more convenient to use a mass flow of gas:

\[ G = \rho v F. \]  

When the gas pipeline is in steady state, the equation of continuity is transformed into an algebraic equation:

\[ G = \text{const.} \]  

At compressor stations, measurements of temperature, pressure and gas flow are carried out. Therefore, at the boundaries of pipeline, it is necessary to set pressure, temperature and mass flow for a solution. Not all possible statements of the boundary value problem are correct \([2]\).

Mathematically, this leads to the fact that the algebraic system of equations (1), (2), (3) will have an infinite set of solutions or not one solution.

For example: Let pressure and mass flow be set at the beginning of the gas pipeline. In this case, there is only one correct stationary solution. If we take the case of the operation of gas pipeline under non-stationary conditions, the following situation will arise. At the end of the pipeline, there is a significant increase in pressure, but the maximum pressure has not yet reached the gas pipeline. Obviously, there is no way to obtain such solution if one sets all the boundary conditions at the beginning of the pipeline since, from the point of view of setting the problem, it does not differ in any way from the stationary operating mode condition.

Mathematically, this means that the difference system of equations (1), (2), (3) will have an infinite set of solutions. There are at least two solutions that satisfy these limiting and initial conditions and hence there are infinitely many of them.

It is possible to construct a difference scheme which approximates this system of equations. Difference equations (1), (2) can be changed for the stationary case. For this, it is necessary that the network functions do not depend on from the time step:

\[
\begin{align*}
\frac{\partial}{\partial t} (\rho v^2) + \frac{\partial}{\partial x} \left( \rho v^2 \right) + \frac{\partial P}{\partial x} + \rho g \sin \alpha + \frac{1}{2d} \lambda \rho v |v| &= 0, \\
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v) &= 0, \\
\frac{\partial}{\partial t} \left( \rho e u + \frac{\rho v^2}{2} + \rho g H \right) + \frac{\partial}{\partial x} \left( \rho v \left( \frac{v^2}{2} + h + gH \right) \right) &= \frac{4}{d} K_m (T - T_0), \\
\begin{cases} 
h^u = e^u + \frac{\rho}{\rho} \\
h^u = \frac{1}{M} h(P, T) \\
\rho = \rho(P, T)
\end{cases}
\end{align*}
\]  

\begin{align*}
(\rho v^2 + P) - (\rho v^2 + P)_j + \left( \rho g \sin \alpha + \frac{1}{2d} \lambda \rho v |v| \right)_{j+1/2} \Delta x &= 0, \\
\left( \rho v \left( \frac{v^2}{2} + h + gH \right) \right)_{j+1} - \left( \rho v \left( \frac{v^2}{2} + h + gH \right) \right)_{j+1/2} + \left( \frac{1}{d} K (T - T_0) \right)_{j+1/2} \Delta x &= 0.
\end{align*}
The system of difference equations (6), (7) supplemented by boundary conditions and equation (5) has a solution. Having obtained the solution of the stationary problem, one can start dynamic simulation, using the temperature and flow pressure as the initial distribution, let us obtain a stationary solution [3,4].

Modeling of a non-stationary operation of the gas pipeline is reduced to solving the system of first-order partial differential equations, and the problem is reduced to solving the system of difference algebraic equations (1), (2), (3).

As the initial conditions for solving practical problems, laws governing the distribution of gas flow rate or pressure along the length of gas pipeline at a time are taken as the start of the process \(t = 0\). If the differential equation is compiled for the gas flow rate, then the mass flow distribution over the length of the gas pipeline is assumed as the initial condition: \(M(0,x) = f(x)\).

In the absence of gas extraction along length and steady motion of gas, mass flow rate will be constant throughout length: \(M(0,x) = M = \text{const.}\)

In gas pipelines, where constant and uniform track selection is present gas, its mass flow at any point of the pipeline will be determined by the expression [5,6]:

\[
M(0,x) = M_0 - m_1x,
\]  

where \(M_0\) – mass flow of gas at the beginning of the pipeline;
\(m_1\) – gas selection;
\(x\) – distance to the point of selection.

If the path selection of gas \(m_1\) depends on the distance, then the initial distribution of the discharge will be written as follows:

\[
M(0,x) = M_0 - m_1(x)x.
\]  

For gas pipelines with constant pumping of gas along the length under the conditions of expressions (8) and (9), the sign before the second term on the right side must be changed to the opposite one.

Concentrated gas sampling in the initial conditions are taken into account as follows:

\[
M(0,x) = M_0 - [m_1 \delta(x - x_1) + m_2 \delta(x - x_2) + \cdots + m_n \delta(x - x_n)],
\]  

where \(\delta\) – Dirac function.

In general, gas extraction path can be replaced by the sum of a constant and a concentrated selection. Then the initial conditions are written in the form:

\[
M(0,x) = M_0 - m_1(x)x - [m_1 \delta(x - x_1) + m_2 \delta(x - x_2) + \cdots + m_n \delta(x - x_n)].
\]  

In real gas pipelines, concentrated gas withdrawals are uniformly distributed over the entire length, and they can be replaced by a constant distributed selection. Selections with a large gas flow, which cannot be represented as distributed constant, are taken into account in conditions (11) as concentrated [7].

For differential equations describing the change in pressure, the initial conditions are the distribution of pressure over the entire length of the gas pipeline \(P(0,x) = \varphi(x)\).

In the stopped horizontal gas pipeline, gas pressure is equalized along its entire length. For this case, the initial conditions are written in this way: \(P(0,x) = P_0 = \text{const.}\) For an inclined gas pipeline, the initial pressure distribution can be described by the expression: \(P(0,x) = P_t \exp\left(-\frac{bx}{2l}\right)\).

For a horizontal operating gas pipeline with steady gas flow, the initial pressure distribution is characterized by a parabolic law:

\[
P(0,x) = \sqrt{P_1^2 - (P_1^2 - P_2^2)x/l}.
\]  

\(P_1\) and \(P_2\) are the gas pipeline ends pressures.
For an inclined gas pipeline under the same conditions, the pressure change is described by the formula:

\[ P^2(0, x) = P_t^2 \exp(-bx/l) - (P_1^2 \exp(-b) - P_2^2)\frac{1 - \exp(-b)}{1 - \exp(-bx/l)}, \]

where

\[ b = \frac{2g\Delta z}{ZRT}, \quad (1) \]

\( Z \) – coefficient of gas compressibility;
\( R \) – gas constant, J/kg ∙ K;
\( T \) – absolute gas temperature, K.

At relatively small pressure drops in gas pipelines, the pressure change along the length is close to linear. In this case, the initial conditions are written in the form:

\[ P(0, x) = P_1 - \frac{P_1 - P_2}{l} x, \quad (14) \]

where \( P_1 \) – pressure at the beginning of the pipeline;
\( P_2 \) – pressure at the end of the pipeline.

With frequent and large amounts of concentrated road gas withdrawals, the pressure change in the gas pipeline between sampling points should be described for each site by the appropriate law \[8,9\].

Then the initial conditions for such case can be written as follows:

\[ P(0, x) = [\sigma(x - x_1) - \sigma(x - x_2)]\sqrt{\frac{P_1^2 - P_2^2}{P_1^2 - P_3^2}(x - x_1)/(x_2 - x_1)} +
+ [\sigma(x - x_2) - \sigma(x - x_3)]\sqrt{\frac{P_2^2 - P_3^2}{P_2^2 - P_4^2}(x - x_2)/(x_3 - x_2)} + \ldots +
+ [\sigma(x - x_{n-1}) - \sigma(x - x_n)]\sqrt{\frac{P_{n-1}^2 - P_n^2}{P_{n-1}^2 - P_1^2}(x - x_{n-1})/(x_n - x_{n-1})}, \quad (15) \]

where \( \sigma \) – Heaviside function.

The initial conditions for a linear pressure distribution on each section can be written in the form \[10\]:

\[ P(0, x) = [\sigma(x - x_1) - \sigma(x - x_2)]\left[ P_1 \frac{P_1 - P_2}{x_2 - x_1}(x - x_1) \right] +
+ [\sigma(x - x_2) - \sigma(x - x_3)]\left[ P_2 \frac{P_2 - P_3}{x_3 - x_2}(x - x_2) \right] + \ldots +
+ [\sigma(x - x_{n-1}) - \sigma(x - x_n)]\left[ P_n \frac{P_n - P_{n-1}}{x_n - x_{n-1}}(x - x_{n-1}) \right]. \quad (16) \]

In main gas pipelines, the gas pressure at the beginning of each section depends on the characteristics of the compressor station, which can be analytically represented using the mathematical model in the form:

\[ P_1^2 = \bar{a}P_2^2 - \bar{b}M_1^2, \quad (17) \]

where \( \bar{a} \) and \( \bar{b} \) – coefficients depending on the number of aggregates, the scheme of their inclusion, the number of revolutions of the impellers of the centrifugal compressor, on the physical state and properties of gas.

Taking into account equation (18), the boundary conditions can be represented as follows:

\[ P(0, t) = P_1(t) = \sqrt{\bar{a}P_2^2 - \bar{b}M_1^2}, \quad (18) \]

At the end of the gas pipeline, calculating a mathematical model, the following conditions may exist:
With relatively small changes in the gas flow rate at the compressor station, its characteristics can be approximately represented by a linear relationship. In this case, instead of condition (17), one can obtain:

\[ P(0, x) = P_1(t) = a_1 P_{21} - b_1 M; \]

where \( a_1 \) and \( b_1 \) are constant coefficients.

Gas pressure in front of the compressor station during gas pumping can vary in time, depending on the mode of operation of the previous section of the pipeline. In this case, the boundary conditions at the beginning of the section should be recorded taking into account the pressure change at the inlet of the compressor station:

\[ P(0, t) = P_1(t) = \sqrt{a b^2_{21} (t) - \overline{b} M^2_1}; \]  
\[ P(0, t) = P_1(t) = a_1 P_{21} - b_1 M. \]

3. Conclusions

The dependence of gas extraction at the end of the gas pipeline on the injection time is formed in accordance with specific conditions in a mathematical form; such relationship can be either linear or non-linear. The change in inlet pressure can be described by a corresponding mathematical expression characterizing the actual process of pumping gas.

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