Distributed $H_{\infty}$ Filtering Over Sensor Networks Subject to Randomly Occurred Nonlinearity and Time-Varying Topology

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This work was supported in part by the National Natural Science Foundation of China under Grant NSFC-61503126, in part by the Heilongjiang Natural Science Foundation under Grant F2018024, in part by the Basic Research Fund of Heilongjiang University under Grant RCYJDJD201806, in part by the Open Fund of Shanghai Key Laboratory of Multidimensional Information Processing, East China Normal University under Grant 2019MIP003.

ABSTRACT This article is concerned with the distributed $H_{\infty}$ filtering problems with consensus for a class of systems subject to randomly occurred nonlinearity and time-varying topology in sensor networks where the nonlinearity considered can include the cyber attacks as a special case. A group of Bernoulli distributed random variables are utilized to describe the varying topology and missing measurements in a unified framework. In terms of two-dimensional (2-D) system theory in Rosser model, a nonlinear two-step filter is designed with consensus protocol where a multiple data strategy is used to improve the system performance. The desired filter parameters are obtained by LMI technique such that the filtering error system is asymptotically stable in the sense of mean square and meets a prescribed average $H_{\infty}$ performance level. Finally, a simulation example demonstrates the effectiveness of the proposed algorithm.

INDEX TERMS Sensor networks, distributed $H_{\infty}$ filtering, time-varying topology, randomly occurred nonlinearity.

I. INTRODUCTION

In recent years, sensor networks have been brought into the limelight due to the wide application prospects in urban management, remote medical services, environmental monitoring etc. [1]. For example, in environmental monitoring, sensor networks can be utilized to detect various changes of environment, such as temperature, humidity, etc. So we can take effective measures to reduce the level of pollution. In the military, sensor networks can be used to monitor specific areas and observe the enemy’s whereabouts. Distributed estimation in sensor networks has been paid more and more attention by researchers at home and abroad [2]–[5]. Consensus problem is a basic requirement for the final result of whole network data fusion [6]. Large amounts of studies try to combine the classical filtering methods with the consensus filtering to ensure that the output estimates of all nodes in the networks can reach a consensus [7]–[11]. During the last two decades, consensus-based $H_{\infty}$ distributed filtering has been paid more and more attention because $H_{\infty}$ filtering does not make any assumptions on the statistical properties of the process and measurement noises compared with the classical Kalman filtering method [12]–[17]. In terms of the complex network environment, such as the node failure, limited energy resources, communication capacity, sudden environment changes and so on, the topology may change or vary over time randomly. However, in most existing work, the topology of communication networks is assumed to be fixed or switched according to a Markov chain [18]–[22].

In other views, there inevitably exist random phenomena such as packet dropouts, time delays during the data transmission. There are two main types for data dropouts [23]: missing measurements [24], [25] and communication link failures [26], [27]. Missing measurements refer to the loss of data packets during transmission from the target plant to the sensor nodes. In practical application, battery supplies...
power to nodes in sensor networks. Due to battery capacity limitations, the signal transmission can be disrupted, which will lead to communication link failures. Communication link failure not only results in packet dropouts but also gives rise to the varying topology. Different from Markov chain described the packet dropouts or the switching communication topology, we model the randomly occurred packet dropouts and varying topology in a unified framework by adopting a group of Bernoulli distributed random variables. The main advantage of varying topology described by Bernoulli sequence is that it can meet many scenarios rather than a few cases of switching topology.

As another research frontiers, nonlinearity is recognized to exist universally in practical systems [28]. Almost all real-world systems are influenced by nonlinear disturbances and the research for nonlinear system has attracted great attention for several decades [29], [30]. At present, more and more scholars have paid their attention to the sector-bounded nonlinearity because it can cover Lipschitz and norm-bounded conditions [31], [32]. Also, norm-bounded nonlinearity often be used to represent the randomly occurred cyber attacks [33], [34]. Hence, the sector-bounded nonlinearity with randomly occurrence considered here can either to represent the nonlinear disturbance or to describe cyber attacks.

Compared with the existing results, the distributed $H_{\infty}$ filtering over sensor networks with the randomly occurred nonlinearity and time-varying topology is investigated in this article. The main contributions can be summarized as follows:

1. A fairly comprehensive circumstance is considered which includes missing measurements, communication link failures, randomly occurred nonlinearity and time-varying communication topology in sensor networks.
2. A nonlinear two-step filter which is composed of the measurement updates and the consensus updates is designed by a novel scheme based on two-dimensional (2-D) system theory.
3. To reduce the conservativeness, the Lyapunov matrices are partitioned in a new way and the slack variable is introduced to separate the coupled Lyapunov matrices and filter parameter matrices. Sufficient condition is obtained to ensure that the filtering error system is asymptotically stable and achieves the prescribed average $H_{\infty}$ performance.

II. PROBLEM FORMULATION

The topology of sensor networks is represented by a directed graph $\mathbb{C} = (\nu, \varsigma, A_i)$ of order $n$ with the set of nodes $\nu = \{1, 2, \cdots, n\}$, the set of edges, $\varsigma \subseteq \nu \times \nu$, and the adjacency matrix $A_i = [a_{ij}]_{n \times n}$ with nonnegative adjacency element $a_{ij}$. An edge of $\mathbb{C}$ is denoted by an ordered pair $(i, j)$. The adjacency elements associated with the edges are positive, i.e., $a_{ij} > 0 \iff (i, j) \in \varsigma$. The node $j$ is called an in-neighbor of node $i$ if $(i, j) \in \varsigma$. The set of in-neighbors of node $i$ is denoted by $N_i$. Assume that $\mathbb{C}$ is strongly connected.

A. SYSTEM MODEL

Consider the following discrete-time nonlinear system in sensor networks as shown in Fig. 1:

$$
\begin{align*}
\dot{x}(k + 1) &= Ax(k) + \chi(k)f(x(k)) + Bw(k) \\
z(k) &= Mx(k)
\end{align*}
$$

(1)

where $x(k) \in \mathbb{R}^m$ is the system state vector; $z(k) \in \mathbb{R}^r$ is the signal to be estimated, $w(k) \in \mathbb{R}^q$ is the process noise belonging to $l_2[0, N - 1]$. The Bernoulli distributed random variable $\chi(k)$ is introduced to describe the random nature of nonlinearity’s occurrence which is with following statistical characteristics:

$$
\begin{align*}
\text{Prob}\{\chi(k) = 1\} &= E\{\chi(k)\} = \bar{\rho}, \\
\text{Prob}\{\chi(k) = 0\} &= 1 - \bar{\rho} \\
E\{\chi(k) - \bar{\rho}\}^2 &= \bar{\rho}(1 - \bar{\rho}) = \sigma_\chi^2
\end{align*}
$$

In this article, the nonlinear function $f(\cdot)$ is assumed to satisfy the following sector-bounded conditions:

Assumption 1: $f(0) = 0$.

Assumption 2: For $\forall x, y \in \mathbb{R}^m$, there exist matrices $U_1, U_2$, such that

$$
[f(x) - f(y) - U_1(x - y)]^T [f(x) - f(y) - U_2(x - y)] \leq 0
$$

where $U_1 \in \mathbb{R}^{m \times m}$, $U_2 \in \mathbb{R}^{m \times m}$ are known real constant matrices, and $U_1 - U_2 > 0$.

For every $i (0 \leq i \leq n)$, the measurement of sensor networks node $i$ is given as:

$$
y_i(k) = C_i x(k)
$$

(2)

where $y_i(k) \in \mathbb{R}^p$ is the measurable output of the node $i$, and $A, B, M, C_i$ are known constant matrices with appropriate dimensions.
B. NETWORK-INDUCED PHENOMENA

Assumption 3: There exists the buffer at each sensor node and all the sensors are clock-driven.

Due to the smart characteristic of wireless sensors, we use multiple data in the buffer to be a local filter input of sensor \( i \). Thus, the measurement input \( \tilde{y}_i (k) \) is given by the local filter \( i \) can be described as follows:

\[
\tilde{y}_i (k) = \xi_i^{(0)} (k) y_i (k) + \xi_i^{(1)} (k) y_i (k - 1) + \cdots
\]

\[
+ \xi_i^{(d)} (k) y_i (k - d) + v_i (k)
\]

where \( v_i (k) \) is the measurement noise of the node \( i \) belonging to \( I_2 \) \( \{ 0, N - 1 \} \). \( \xi_i^{(b)} (k) \) \( (0 \leq i \leq n, 0 \leq b \leq d) \) are a group of Bernoulli distributed random variables which are uncorrelated with each other and also uncorrelated with \( \chi (k) \). The probabilities of \( \xi_i^{(b)} (k) \) \( (1 \leq i \leq n, 0 \leq b \leq d) \) satisfy that \( \text{Prob} \{ \xi_i^{(b)} (k) = 1 \} = \xi_i^{(b)} \) and \( \text{Prob} \{ \xi_i^{(b)} (k) = 0 \} = 1 - \xi_i^{(b)} \), where \( \xi_i^{(b)} \in [0, 1] \) are known constants, and the maximal number of data packets we used in the buffer is dependent on \( d \). Moreover, we can obtain the statistical properties of stochastic variable \( \xi_i^{(b)} (k) \) \( (1 \leq i \leq n, 0 \leq b \leq d) \) are as follows:

\[
\mathbb{E} \left\{ \xi_i^{(b)} (k) \right\} = \xi_i^{(b)}
\]

\[
\mathbb{E} \left\{ \xi_i^{(b)} (k) = \xi_i^{(b)} \right\} ^2 = \xi_i^{(b)} (1 - \xi_i^{(b)}) = \sigma_i^{2(b)}
\]

C. NONLINEAR TWO-STEP FILTER CONSTRUCTION

Consider the following nonlinear two-step filter on node \( i \) for system (1) using the measurement (3):

\[
\begin{align*}
\dot{x}_i (k + 1) &= A \hat{x}_i^+ (k) + \tilde{f} \left( \hat{x}_i^+ (k) \right) + K_i \left( \tilde{y}_i (k) \\
- \sum_{b=0}^{d} \xi_i^{(b)} C_i \hat{x}_i^+ (k - b) \right) \\
\hat{x}_i^+ (k) &= \hat{x}_i (k) + G \sum_{j \in N_i} a_{ij} \left( \hat{z}_j (k) - \theta_j (k) \hat{z}_j (k - \tau_k) \right) \\
\hat{z}_i (k) &= M \hat{x}_i (k)
\end{align*}
\]

where \( \hat{x}_i (k) \) is the estimation of system state \( x (k) \), \( \hat{x}_i^+ (k) \) is the consensus update of \( \hat{x}_i (k) \) through information exchange with the in-neighbors of node \( i \), \( \hat{z}_i (k) \) is the estimation of \( z (k) \), \( \hat{z}_i (k) \) is the signal received from node \( j (j \in N_i) \). A positive integer \( \tau_k \) denotes the time-varying delays and satisfies \( 0 \leq \tau \leq \tau_k \leq \bar{\tau} \), where \( \tau \) and \( \bar{\tau} \) are known positive integers representing the minimal and maximal delays, respectively. \( K_i, G \) are the filter parameter matrices to be designed. \( \theta_j (k) \) are Bernoulli distributed random variables, which are uncorrelated with each other and also mutually independent of \( \xi_i^{(b)} (k) \) and \( \chi (k) \). The statistical properties are given by

\[
\begin{align*}
\text{Prob} \{ \theta_j (k) = 1 \} &= \mathbb{E} \{ \theta_j (k) \} = \tilde{\mu}_{ij} \\
\text{Prob} \{ \theta_j (k) = 0 \} &= 1 - \tilde{\mu}_{ij} \text{ } \\
\mathbb{E} \{ (\theta_j (k) - \tilde{\mu}_{ij})^2 \} &= \tilde{\mu}_{ij} (1 - \tilde{\mu}_{ij}) = \sigma_{\theta_j}^2
\end{align*}
\]

Remark 1: It is worth mentioning that the local filter designed in (4) is based on nonlinear function \( f (\cdot) \). In practice, the nonlinear function \( f (\cdot) \) may be unknown. When the nonlinear function \( f (\cdot) \) cannot be known exactly, we can design a nonlinear disturbance observer to estimate \( f (\cdot) \) and then the nonlinear filter can be utilized, which remains our future research direction. In addition, we can also design linear filter without nonlinear term, i.e. \( f (\hat{x}_i^+ (k)) = 0 \), but it may make the accuracy of the filtering worse.

Remark 2: To fuse the signals received by node \( i \), consensus protocol is employed as local information fusion strategy which is frequently utilized in the research of consensus distributed filtering and cooperative control in multi-agent system. If \( (i, j) \in \xi \), \( \theta_{ij} \) can represent the communication link state between node \( i \) and \( j \) in the local communication. Obviously, \( \theta_{ij} (k) = 0 \) means that the communication link failures are happened at the time instant \( k \). It is worth mentioning that when the random variables \( \theta_{ij} (k) \) are changed, the topology can switch from one state to another.

Remark 3: Here, we take an example to compare the switching topology with time-varying topology proposed in this article which are shown in Fig. 2 and Fig. 3, respectively. If \( \theta_{21} (k) = 0, \theta_{23} (k) = 0, \theta_{34} (k) = 0 \) and \( \theta_{14} (k) = 0 \) in Fig. 3, the Fig. 3 is transformed into the first state of Fig. 2. If \( \theta_{21} (k) = 0, \theta_{23} (k) = 0, \theta_{34} (k) = 0 \) and \( \theta_{41} (k) = 0 \) in Fig. 3, the Fig. 3 is transformed into the second state of Fig. 2. It is obvious that time-varying topology is more general than switching topology.

Remark 4: In [35], the distributed \( H_{\infty} \) filtering problem is investigated in sensor networks for discrete-time systems with missing measurements and communication link failures. Compared with [35], the filter designed in this article considers nonlinear disturbance and time-varying transmission delays when the node \( i \) is exchanging information with its adjacent nodes. When \( \xi_i^{(b)} (k) = 0 \) \( (b = 1, \cdots, d) \), the model (3) is degraded to the model presented in [35].
It is clear that the model (3) is more general than the model addressed in [35] where only one data packet is be used.

Remark 5: Subsequently, we will deal with time-varying transmission delays by constructing a delay-dependent Lyapunov-Krasovskii functional [36], [37]. In this article, we assume that the time-varying transmission delays between adjacent nodes are identical, which leaves the conservativeness. If the time-varying transmission delays between adjacent nodes are different, the consensus update of mode (4) can be rewritten as the following form

\[ \hat{x}_i^+(k) = \hat{x}_i(k) + G_i \sum_{j \in \mathcal{N}_i} a_{ij} (\tilde{u}_{ij}(k) - \theta_{ij}(k) \hat{x}_j(k - \tau_{ij}(k))) \]

where \( \tau_{ij}(k) \) means the varying delays between node \( i \) and node \( j \). In this case, we can adopt persistent dwell-time scheme to deal with \( \tau_{ij}(k) \), then the delay problem can be converted to a switched system problem [38].

III. FILTER DESIGN BASED ON 2-D SYSTEM

Construct the following 2-D Rosser model:

\[
\begin{align*}
\dot{x}^f(k+1, h) &= A x^f(k, h) + \chi(k) f(x^c(k, h)) + Bw(k) \\
x^c(k, h+1) &= M x^f(k, h) \\
y_i(k, h) &= C x^c(k, h) \\
\tilde{y}_i(k, h) &= \tilde{e}^{(0)}_{ij}(k) y_i(k, h) + \tilde{e}^{(1)}_{ij}(k) y_j(k - 1, h) + \cdots + \tilde{e}^{(d)}_{ij}(k) y_j(k - d, h) + v_i(k)
\end{align*}
\]

Then, (5) can be rewritten in the following augmented form

\[
\begin{align*}
\dot{x}^f(k+1, h) &= \tilde{A}_1 \tilde{x}^c(k, h) + \tilde{\chi}(k) \tilde{f}(x^c(k, h)) + \Gamma \hat{x}_i(k, h) + B_1 \tilde{w}(k) \\
\tilde{x}^c(k, h+1) &= \tilde{M}_1 \tilde{x}^f(k, h) \\
\tilde{y}(k, h) &= \tilde{C} \tilde{x}^c(k, h) \\
\tilde{z}_i(k, h) &= M \tilde{x}_i^f(k, h)
\end{align*}
\]

where

\[
\begin{align*}
\tilde{x}^f(k, h) &= \text{vec}_n \left[ x^f(k, h) \right], \\
\tilde{x}^c(k, h) &= \text{vec}_n \left[ x^c(k, h) \right], \\
\tilde{f}(x^c(k, h)) &= \text{vec}_n \left[ f(x^c(k, h)) \right], \\
\tilde{y}(k, h) &= \text{vec}_n \left[ y(k, h) \right], \\
\tilde{z}_i(k, h) &= \text{vec}_n \left[ z_i(k, h) \right], \\
\tilde{\chi}(k) &= \text{diag}_n \{ \chi(k) \}, \\
\tilde{\tilde{\chi}}(k) &= \sqrt{\chi(k) - \tilde{\rho}}, \\
\tilde{\lambda}(k) &= \text{diag}_n \{ \lambda(k) \}, \\
\tilde{\Gamma} &= \text{diag}_n \{ \tilde{\sigma}_k \}.
\end{align*}
\]

The boundary conditions of the 2-D system are

\[
\begin{align*}
\dot{x}^f(0, h) &= x(0), \quad h = 0, 1, \ldots, N + 1 \\
x^c(0, k) &= 0, \quad k = 0, 1, \ldots, N - 1
\end{align*}
\]

Lemma 1 [35]: The 2-D Rosser model (5) satisfies the following conditions:

\[
\begin{align*}
x^f(k, h) &= x(k) \quad \text{for } h \geq k \\
x^c(k, 0) &= x(k) \quad \text{for } h \geq k + 1
\end{align*}
\]

where \( x(k), z(k) \) and \( y_i(k) \) are defined in (1) and (2), respectively.

Remark 6: In addition, two-dimensional (2-D) system has received considerable concern since 1970s because it can represent a wide range of practical systems, such as thermal processes, gas absorption, seismographic data processing, image processing and water stream heating, etc. [39], [40]. There are many fruitful results are obtained, such as [41] for sensor failure, [42] for transmission delays, [43] for measurement degradations and so on. Here, the 2-D system (5) is an assistant model for designing the two-step filter (4). The method of 2-D system can simplify the solving of filter parameters. It has been introduced in [35] where its relationship with (1) has been discussed in detail.

Next, we design the following filter under the consideration of the structure of (4) for the 2-D system (5) on node \( i \) using (6):

\[
\begin{align*}
\tilde{x}_i^f(k+1, h) &= A \tilde{x}_i^c(k, h) + \tilde{\rho} f(\tilde{x}_i^c(k, h)) + K_i \left( \tilde{y}_i(k, h) \\
&- \sum_{b=0}^{d} \tilde{e}^{(b)}_{ij} C_i \tilde{x}_i^c(k - b, h) \right) \\
\tilde{x}_i^c(k, h+1) &= \tilde{\tilde{x}}_i^f(k, h) + G_i \sum_{j \in \mathcal{N}_i} a_{ij} (\tilde{u}_{ij}(k) \tilde{x}_j(k - \tau_{ij}(k), h) \\
&- \theta_{ij}(k) \tilde{x}_j(k - \tau_{ij}(k), h)) \\
\tilde{z}_i(k, h) &= M \tilde{x}_i^f(k, h)
\end{align*}
\]

To conduct conveniently for the subsequent discussion, some new variables are introduced. Let

\[
\tilde{x}_i^f = x_i^f - \tilde{x}_i^f, \quad \tilde{x}_i^c = x_i^c - \tilde{x}_i^c, \quad f(\tilde{x}_i^c) = f(x_i^c) - f(\tilde{x}_i^c), \quad \tilde{z}_i(k, h) = z(k, h) - \tilde{z}_i(k, h)
\]

By combination of 2-D system (5) and filter (8), the filtering error system on node \( i \) can be obtained as follows:

\[
\begin{align*}
\tilde{x}_i^f(k+1, h) &= A \tilde{x}_i^c(k, h) + \tilde{\chi}(k) f(\tilde{x}_i^c(k, h)) \\
&+ \tilde{\rho} f(\tilde{x}_i^c(k, h)) - \sum_{b=0}^{d} \tilde{e}^{(b)}_{ij} C_i \tilde{x}_i^c(k - b, h) \\
&- K_i v_i(k) - \sum_{b=0}^{d} \tilde{e}^{(b)}_{ij} (k) \sigma_k \Sigma_{ij} C_i \\
&\times x_i^c(k - b, h) + Bw(k) \\
\tilde{x}_i^c(k, h+1) &= \tilde{\tilde{x}}_i^f(k, h) \\
&- G_i \sum_{j \in \mathcal{N}_i} \tilde{u}_{ij}(k) \tilde{x}_j(k - \tau_{ij}, h) - \tilde{x}_i^f(k, h) \\
&+ G_i \sum_{j \in \mathcal{N}_i} \tilde{u}_{ij}(k) \lambda_{ij} x_i^c(k, h) \\
&- G_i \sum_{j \in \mathcal{N}_i} \tilde{u}_{ij}(k) \lambda_{ij} \tilde{x}_j^c(k - \tau_{ij}, h) \\
\tilde{z}_i(k, h) &= M \tilde{x}_i^f(k, h)
\end{align*}
\]
where $\tilde{x} (k) = \frac{\tilde{z}(k) - \bar{z}}{\sigma_x}$, $\tilde{x}_i^{(b)} (k) = \frac{\tilde{z}_i^{(b)}(k) - \bar{z}_i^{(b)}}{\sigma_{\tilde{z}_i^{(b)}}}$, $\tilde{y}_i (k) = \frac{y_i - \bar{y}_i}{\sigma_{y_i}}$

Let

\[
\tilde{x}^f (k, h) = \text{vec}_n \left\{ \tilde{x}^c_i (k, h) \right\}, \\
\tilde{x}^c (k, h) = \text{vec}_n \left\{ \tilde{x}^c_i (k, h) \right\}, \\
\tilde{x}^c (k, h) = \tilde{x}^c (k, h) - \tilde{x}^f (k, h), \\
\tilde{z} (k) = \text{vec}_n \left\{ \tilde{z}_i (k, h) \right\}, \\
\tilde{z} (k, h) = \tilde{z} (k, h) - \tilde{z} (k, h), \\
\tilde{x}^c (k, h) = \text{vec}_n \left\{ \tilde{x}^c_i (k, h) \right\}, \\
\tilde{x}^c_i (k, h) = \text{vec}_n \left\{ \tilde{x}^c_i (k, h) \right\}, \\
\tilde{f} \left( \tilde{x}^c_i (k, h) \right) = \tilde{f} \left( \tilde{x}^c (k, h) \right) - \tilde{f} \left( \tilde{x}^c (k, h) \right), \\
\tilde{f} \left( \tilde{x}^c_i (k, h) \right) = \text{vec}_n \left\{ \tilde{f} \left( \tilde{x}^c_i (k, h) \right) \right\}.
\]

Then, we have

\[
\tilde{x}^f (k + 1, h) = \tilde{A}_1 \tilde{x}^c (k, h) + \tilde{x} (k) \Psi \tilde{f} \left( \tilde{x}^c (k, h) \right) \\
+ \tilde{B}_1 \tilde{w} (k) - K \tilde{v} (k) \\
- \sum_{b=0}^{d} \left( \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{x}_i^{(b)} (k) \left( \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{x}_i^{(b)} (k) \right) K \tilde{C} \tilde{x}^c (k, h) \\
- \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \tilde{y}_i (k) G \left( \tilde{A}_i \otimes \tilde{I} \right) \tilde{x} (k, h) \\
- \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \tilde{y}_i (k) G \left( \tilde{A}_i \otimes \tilde{I} \right) \tilde{x} (k, h) \\
+ \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{y}_i (k) G \left( \tilde{A}_i \otimes \tilde{I} \right) \tilde{x} (k, h) \\
- \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{y}_i (k) G \left( \tilde{A}_i \otimes \tilde{I} \right) \tilde{x} (k, h) \\
\tilde{z} (k) = \tilde{M} \tilde{x} (k, h)
\]

where

\[
\tilde{v} (k) = \text{vec}_n \left\{ v_i (k) \right\}, \quad K = \text{diag}_n \left\{ K_i \right\}, \quad G = \text{diag}_n \left\{ G_i \right\} \\
\tilde{A}_i = \left[ \tilde{a}_{ij} \right]_{n \times n}, \quad \tilde{d}_i = \sum_{j=1}^{n} \tilde{a}_{ij}, \quad \tilde{D} = \text{diag}_n \left\{ \tilde{d}_i \right\} \\
\sum^{(b)} = \text{diag}_n \left\{ \tilde{x}_i^{(b)} \right\} \\
\Sigma_i^{(b)} = \begin{bmatrix}
0 & \cdots & 0 \\
\cdots & \sigma_{\tilde{z}_i^{(b)}} & \cdots \\
0 & \cdots & 0
\end{bmatrix}
\]

Then we have

\[
\xi^f = \left[ \tilde{x}^f \right]_i, \xi^c = \left[ \tilde{x}^c \right]_i \\
f^f = \left[ \tilde{f} \left( \tilde{x}^f (k, h) \right) \right], \quad f^c = \left[ \tilde{f} \left( \tilde{x}^c (k, h) \right) \right].
\]

By setting

\[
\begin{align*}
\xi_{\tilde{b}+1} & = \begin{bmatrix}
\xi^f (k + 1, h) \\
\xi^c (k + 1, h)
\end{bmatrix}, \quad \xi_{\tilde{b}} = \begin{bmatrix}
\xi^f (k, h) \\
\xi^c (k, h)
\end{bmatrix} \\
\xi_{\tilde{b}+\tau} & = \begin{bmatrix}
\xi^f (k - \tau, h) \\
\xi^c (k - \tau, h)
\end{bmatrix}, \quad \xi_{\tilde{b}+\tau} = \begin{bmatrix}
\xi^f (k - \tau, h) \\
\xi^c (k - \tau, h)
\end{bmatrix} \\
\tilde{f} & = \begin{bmatrix}
f^f \\
f^c
\end{bmatrix}, \quad u (k) = \begin{bmatrix}
\tilde{u} (k) \\
0
\end{bmatrix}, \quad \tilde{z}_\tilde{b} = \tilde{z} (k, h)
\end{align*}
\]

We can obtain the 2-D filtering error system in a compact form as follows:

\[
\begin{align*}
\xi_{\tilde{b}+1} & = \begin{bmatrix}
\tilde{A} + \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{B}_i \tilde{w}_i (k) - K \tilde{v} (k) \\
- \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{x}_i^{(b)} (k) \left( \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{x}_i^{(b)} (k) \right) K \tilde{C} \tilde{x}^c (k, h) \\
- \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{y}_i (k) G \left( \tilde{A}_i \otimes \tilde{I} \right) \tilde{x} (k, h) \\
- \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{y}_i (k) G \left( \tilde{A}_i \otimes \tilde{I} \right) \tilde{x} (k, h) \\
- \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{y}_i (k) G \left( \tilde{A}_i \otimes \tilde{I} \right) \tilde{x} (k, h) \\
- \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{y}_i (k) G \left( \tilde{A}_i \otimes \tilde{I} \right) \tilde{x} (k, h)
\end{bmatrix} \\
+ \begin{bmatrix}
\tilde{f} \left( \tilde{x}^f (k, h) \right) \\
\tilde{f} \left( \tilde{x}^c (k, h) \right)
\end{bmatrix} \\
\tilde{z}_\tilde{b} = \tilde{M} \xi_{\tilde{b}}
\end{align*}
\]

\[
\begin{align*}
\tilde{A} & = \begin{bmatrix}
0 & A_1 \\
A_2 & 0
\end{bmatrix}, \quad \tilde{B} = \begin{bmatrix}
F_{ij} \\
F_{ij}
\end{bmatrix}, \quad \tilde{E} = \begin{bmatrix}
0 & 0 \\
0 & E^{(b)}
\end{bmatrix} \\
\tilde{D}^{(b)} & = \begin{bmatrix}
D^{(b)} \\
D^{(b)}
\end{bmatrix}, \quad \tilde{H} = \begin{bmatrix}
H_{ij} \\
H_{ij}
\end{bmatrix}, \quad \tilde{M} = \begin{bmatrix}
M_1 & 0 \\
M_1 & 0
\end{bmatrix} \\
\tilde{F} & = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}, \quad \tilde{M} = \begin{bmatrix}
M_1 & 0 \\
M_1 & 0
\end{bmatrix} \\
\tilde{B}_i & = \begin{bmatrix}
\tilde{B}_i \\
\tilde{B}_i
\end{bmatrix}, \quad \tilde{K} = \begin{bmatrix}
\tilde{K} \\
\tilde{K}
\end{bmatrix} \\
\tilde{E}^{(b)} & = \begin{bmatrix}
0 & 0 \\
0 & E^{(b)}
\end{bmatrix}, \quad \tilde{D}^{(b)} = \begin{bmatrix}
0 & 0 \\
0 & d^{(b)}
\end{bmatrix}
\end{align*}
\]
\[
\Phi = \begin{bmatrix}
-G (\hat{A}_{ij} \otimes I_m) & 0 \\
0 & 0
\end{bmatrix},
\]
\[
H_{ij} = \begin{bmatrix}
-G (A_{ij} \otimes I_m) & 0 \\
0 & 0
\end{bmatrix},
\]
\[
\tilde{\Gamma} = \begin{bmatrix}
\Gamma & 0 \\
0 & \Gamma
\end{bmatrix}, \quad \Psi = \begin{bmatrix}
0 & \Psi
\end{bmatrix}
\]

**Lemma 2 [35]:** The 2-D filtering error system (10) satisfies the following conditions:
\[
\begin{align*}
\xi^f (k, h) &= \xi^f (k, k) \quad \text{for} \ h \geq k \\
\xi^c (k, h) &= \xi^c (k, k + 1) \quad \text{for} \ h \geq k + 1
\end{align*}
\]

**Definition 1:** For a given scalar \( \gamma > 0 \), the filtering error system (10) is said to be asymptotically stable with an average \( H_\infty \) performance level \( \gamma \) if the following conditions hold:

1. (asymptotical stability) The filtering error system (10) with \( u (k) = 0 \) is said to be asymptotically stable in mean square if for any initial condition, the following holds
\[
\lim_{k+h \to \infty} \mathbb{E} \left\{ \| \xi (k, h) \|^2 \right\} = 0 \quad (11)
\]

2. (average \( H_\infty \) performance) Under zero initial condition, for any non-zero \( u (k) \), the filtering error \( \tilde{z}_d (k, h) \) satisfies
\[
\sum_{k=0}^{N-1} \sum_{h=0}^{N+1} \mathbb{E} \left\{ \sum_{i=1}^{n} \| \tilde{z}_d (k, h) \|^2 \right\} \leq \gamma^2 \sum_{k=0}^{N-1} \mathbb{E} \left\{ n \| w (k) \|^2 + \sum_{i=1}^{n} \| v_i (k) \|^2 \right\} \quad (12)
\]

**Remark 7:** Note that the sum of \( k \) is from 0 to \( N - 1 \) and the sum of \( h \) is from 0 to \( N + 1 \), only because the boundary conditions for the 2-D system (5), which is the same as [35]. When \( k, h \to \infty \), the filtering error system is asymptotically stable in [35], then we can also obtain when \( k+h \to \infty \), the 2-D filtering error system is asymptotically stable. That is, the value of \( N \) is not limited in this article.

**IV. FILTERING PERFORMANCE ANALYSIS**

**Theorem 1:** Let the filter parameters \( K, G \) be given. The 2-D filtering error system (10) with \( u (k) = 0 \) is asymptotically stable in the mean square, if there exist a given scalar \( \varepsilon > 0 \) and matrices \( P > 0, Q \geq 0 \) (\( l = 1, 2, \cdots, d \)), \( R > 0 \), such that the following LMI holds:
\[
\begin{bmatrix}
\Delta & P & 0 & 0 \\
-P & * & * & 0 \\
0 & P & * & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} < 0 \quad (13)
\]

where
\[
\Delta = \begin{bmatrix}
-P + V & * & * & * \\
0 & -\tilde{Q} & * & * \\
0 & 0 & -R & * \\
-\tilde{U} & 0 & 0 & -\varepsilon I
\end{bmatrix}
\]

\[
V = \sum_{l=1}^{d} Q_l + (\tilde{r} - \tau + 1) R - \varepsilon I \otimes \tilde{U}_1
\]
\[
\tilde{Q} = \text{diag} \{ Q_1, \cdots, Q_d \}, \quad \tilde{U} = \varepsilon I \otimes \tilde{U}_2^T
\]
\[
\Pi_1 = \begin{bmatrix}
\tilde{A} & \tilde{E}^{(1)} & \cdots & \tilde{E}^{(d)} & \Phi & \tilde{r}
\end{bmatrix}
\]
\[
\Pi_2 = \text{diag} \left\{ \sum_{i=1}^{n} \tilde{D}_i (0), \cdots, \sum_{i=1}^{n} \tilde{D}_i (d), 0, \tilde{\Psi} \right\}
\]
\[
\Pi_3 = \begin{bmatrix}
\sum_{i=1}^{n} \sum_{j \in \Omega_i} \tilde{F}_{ij} & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \sum_{i=1}^{n} \sum_{j \in \Omega_i} \tilde{H}_{ij} & 0
\end{bmatrix}
\]
\[
\tilde{A} = \tilde{A} + \tilde{E} (0), \quad \tilde{P} = P \otimes I_{d+3}
\]

**Proof:** Choose the Lyapunov functional as follows:
\[
V (\xi (k, h)) = \sum_{a=1}^{4} V_a (\xi (k, h))
\]
For convenience of later expression, let \( V_\delta (\xi_k) = V (\xi (k, h)) \). Then, we have
\[
V_\delta (\xi_k) = \sum_{a=1}^{4} V_a (\xi_k) \quad (14)
\]

Denote
\[
\eta_\delta = \begin{bmatrix}
\xi_{\delta}^T & \xi_{\delta - 1}^T & \cdots & \xi_{\delta - d}^T & \xi_{\delta - d - 1}^T & \xi_{\delta - d - 2}^T & \cdots & \xi_{\delta - \tau_k}^T
\end{bmatrix}^T
\]
and
\[
\mathbb{E} \{ \Delta V_\delta \} = \begin{bmatrix}
\eta_{\delta} & \cdots & \eta_{\delta - d} & \cdots & \eta_{\delta - \tau_k - 1} & \cdots & \eta_{\delta - \tau_k - d}
\end{bmatrix}^T
\]

Then calculating the difference of \( V_\delta \) along system (10) and taking the mathematical expectation, we have
\[
\mathbb{E} \{ \Delta V_\delta \} = \begin{bmatrix}
V_{\delta}^{(1)} (\xi_{\delta + 1} (k)) \eta_\delta - V_{\delta}^{(1)} (\xi_\delta) \\
V_{\delta}^{(2)} (k) \\
V_{\delta}^{(3)} (k) \\
V_{\delta}^{(4)} (k)
\end{bmatrix}
\]
\[
\mathbb{E} \{ \Delta V_\delta \} = \begin{bmatrix}
\xi_{\delta + 1}^T P \xi_\delta + P \xi_{\delta + 1} \\
\xi_{\delta} - \xi_{\delta - 1} \\
\xi_{\delta - d} - \xi_{\delta - d - 1} \\
\xi_{\delta - \tau_k} - \xi_{\delta - \tau_k - 1}
\end{bmatrix}
\]

By noticing that
\[
\begin{align*}
\mathbb{E} \{ \tilde{\chi} (k) \} &= 0, \quad \mathbb{E} \{ \tilde{\tilde{\xi}}_i (k) \} = 0, \quad \mathbb{E} \{ \tilde{\delta}_{ij} (k) \} = 0 \\
\mathbb{E} \{ \tilde{\tilde{\xi}}_i (k) \} &= 0, \quad \mathbb{E} \{ \tilde{\delta}_{ij} (k) \} = 0 \\
\mathbb{E} \{ \tilde{\tilde{\xi}}_i (k) \} &= 0, \quad \mathbb{E} \{ \tilde{\delta}_{ij} (k) \} = 0 \\
\end{align*}
\]
\[
\mathbb{E} \{ \tilde{\tilde{\xi}}_i (k) \}^2 = \frac{\tilde{\rho} (1 - \tilde{\rho})}{\alpha^2} = 1, \quad \mathbb{E} \{ \tilde{\tilde{\xi}}_i (k) \} = 0
\]
\[
\begin{align*}
E \left[ \xi_i^{(b)}(k)^2 \right] &= \frac{\xi_i^{(b)}(1-\xi_i^{(b)})}{\sigma_i^{2(b)}} = 1 \quad (b = 0, 1, \ldots, d) \\
E \left[ \delta_{ij}(k)^2 \right] &= \frac{\mu_{ij}(1-\mu_{ij})}{\sigma_{ij}^2} = 1
\end{align*}
\]

We can obtain
\[
E \left\{ \Delta V_{\delta}^{(1)} \right\} = \xi_\delta^T \left( \tilde{A}^T \tilde{P} \tilde{A} - P + \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{F}_{ij}^T \tilde{P} \tilde{F}_{ij} \right) \xi_\delta + 2 \sum_{b=1}^{d} \xi_\delta^T \tilde{B} \tilde{F}_{\delta-b} \xi_\delta + \sum_{i=1}^{n} \sum_{j=1}^{n} \xi_\delta^T \tilde{H}_{ij} \tilde{P} \tilde{H}_{ij} \xi_\delta
\]

Next, it can be derived that
\[
E \left\{ \Delta V_{\delta}^{(2)} \right\} = E \left\{ \sum_{b=1}^{d} \sum_{l=b+1}^{d} \xi_\delta^T Q_l \xi_\delta - \sum_{l=1}^{d-1} \sum_{b=l+1}^{d} \xi_\delta^T Q_l \xi_\delta \right\}
\]

Note that from Assumptions 1 and 2, it follows that
\[
\tilde{f} - (I \otimes U_1) \xi_\delta \leq 0
\]

One can immediately obtain
\[
E \left\{ \Delta V_{\delta}^{(4)} \right\} \leq (r - \tau) \xi_\delta^T R \xi_\delta - \sum_{b=1}^{d} \xi_\delta^T R \xi_\delta
\]

where
\[
\tilde{U}_1 = \left( U_1^T U_2 + U_2^T U_1 \right)/2 \\
\tilde{U}_2 = -\left( U_1^T + U_2^T \right)/2
\]

Subsequently, from (16), it follows that
\[
E \left\{ \Delta V_{\delta} \right\} \leq E \left\{ \sum_{d=1}^{4} \Delta V_{\delta}^{(d)} \right\} - \varepsilon \left( \tilde{f} - (I \otimes U_1) \xi_\delta \right)^T \left( \tilde{f} - (I \otimes U_2) \xi_\delta \right)
\]

By using Schur complement, we have the conclusion that (13) implies \( \Pi < 0 \). Therefore, \( E \{ \Delta V_{\delta} \} < 0 \). Furthermore, according to Lyapunov stability theory, it can be concluded that the 2-D filtering error system (10) with \( u(k) = 0 \) is asymptotically stable in mean square for given filter parameters. This proof is completed.

Remark 8: In this part, we utilize the 1-D system theory to analyze the filtering performance by means of defining the new variables. Moreover, we can also take advantage of the method adopted in [44], namely, making use of 2-D system theory directly to analyze the filtering performance, which will be our future research direction.

Theorem 2: Let the filter parameters \( K, G \) be given. For given scalars \( \gamma > 0, \varepsilon > 0 \), the 2-D filtering error system (10) is asymptotically stable in the mean square, and when \( u(k) \neq 0 \), the performance constraint is achieved, if there exists the matrices \( P > 0, Q_l > 0 (l = 1, 2, \ldots, d) \), \( R > 0 \), such that the following LMI holds:

\[
\begin{bmatrix}
\tilde{A} & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
\end{bmatrix}
\begin{bmatrix}
P & -P & * & * & * \\
\tilde{P} & 0 & -\tilde{P} & * & * \\
0 & 0 & -P & * & * \\
0 & 0 & 0 & -I_n & * \\
0 & 0 & 0 & 0 & -\gamma^2 I
\end{bmatrix}
< 0
\]
\[ \tilde{\Pi}_4 = \begin{bmatrix} \bar{M} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \bar{d} + 3 \end{bmatrix}, \quad \tilde{A} = \bar{A} + \bar{E}^{(0)}, \quad \tilde{P} = \bar{P} \otimes I_{d+4} \]

Proof: When \( u(k) \neq 0 \), let us analyze the \( H_\infty \) performance of the filtering error system (10). For this purpose, following performance index is defined as

\[ J = E \left\{ \sum_{k=0}^{N} \sum_{h=0}^{N+1} \left( \tilde{z}_k^T \tilde{z} - \gamma^2 u(k)^T u(k) + \Delta V_3 \right) \right\} \]

\[ = E \left\{ \sum_{k=0}^{N} \sum_{h=0}^{N+1} \left( \tilde{z}_k^T \tilde{z} - \gamma^2 u(k)^T u(k) + \Delta V_3 \right) \right\} \]

\[ = \tilde{\eta}_7^T \left( \tilde{A} + \tilde{\Pi}_1 P \tilde{\Pi}_1 + \tilde{\Pi}_2 P \tilde{\Pi}_2 + \tilde{\Pi}_3 P \tilde{\Pi}_3 + \tilde{\Pi}_4 P \tilde{\Pi}_4 \right) \tilde{\eta}_7 \]

where \( \tilde{\eta}_7 = \left[ \xi_7^T \xi_{7-1}^T \cdots \xi_{7-d}^T \xi_{7-n_k}^T \right]^T \).

Then, it can be concluded from (17) that \( J < 0 \). Also, it is obvious that \( \Pi < 0 \) under the condition that \( \tilde{\Pi} < 0 \), which means filtering error system (10) with \( u(k) = 0 \) is asymptotically stable in the mean square according to Theorem 1. Moreover, under zero initial condition, it is easy to see that

\[ \sum_{k=0}^{N-1} \sum_{h=0}^{N+1} E \left\{ \| \tilde{z}_k(k, h) \|^2 \right\} \leq \gamma^2 \sum_{k=0}^{N-1} E \left\{ \| u(k) \|^2 \right\} \]

(19)

Then \( \| \tilde{z}_k(k, h) \|^2 = \sum_{i=1}^{n} \| \tilde{z}_i(k, h) \|^2 + \sum_{i=1}^{n} \| v_i(k) \|^2 \), (19) can be rewritten as follows

\[ \sum_{k=0}^{N-1} \sum_{h=0}^{N+1} E \left\{ \sum_{i=1}^{n} \| \tilde{z}_i(k, h) \|^2 \right\} \]

\[ \leq \gamma^2 \sum_{k=0}^{N-1} E \left\{ \sum_{i=1}^{n} \| w_i(k) \|^2 + \sum_{i=1}^{n} \| v_i(k) \|^2 \right\} \]

which completes the proof of Theorem 2.

Remark 9: In terms of Lemma 1 and Lemma 2, as the proof process of [35], we have

\[ \sum_{k=0}^{N-1} \sum_{h=0}^{N+1} E \left\{ \sum_{i=1}^{n} \| \tilde{z}_i(k, h) \|^2 \right\} = \sum_{k=0}^{N-1} E \left\{ \sum_{i=1}^{n} \| \tilde{z}_i(k) \|^2 \right\} \]

Then, the constructed problem of 2-D system has returned to a problem of 1-D system about variable \( k \).
\[ Q_{t, 24} = \text{diag}_n \left \{ Q^{(t)}_{1, 24} \right \}, \quad \hat{Q} = \text{diag} \left \{ Q_1, \cdots, Q_d \right \}. \]

\[ \hat{U} = \varepsilon I \otimes \hat{U}_2 \]

\[ R = \begin{bmatrix} R_1 & \ast \\ 0 & R_2 \end{bmatrix}, \quad R_1 = \begin{bmatrix} R_{11} & \ast \\ R_{13} & R_{14} \end{bmatrix}, \]

\[ R_2 = \begin{bmatrix} R_{21} & \ast \\ R_{23} & R_{24} \end{bmatrix} \]

\[ R_{11} = \text{diag}_n \left \{ R^{(t)}_{11} \right \}, \quad R_{13} = \text{diag}_n \left \{ R^{(t)}_{13} \right \}, \]

\[ R_{14} = \text{diag}_n \left \{ R^{(t)}_{14} \right \}, \]

\[ R_{21} = \text{diag}_n \left \{ R^{(t)}_{21} \right \}, \quad R_{23} = \text{diag}_n \left \{ R^{(t)}_{23} \right \}, \]

\[ R_{24} = \text{diag}_n \left \{ R^{(t)}_{24} \right \} \]

\[ \Theta_2 = [\Omega_2 \quad \Omega_3 \quad \Omega_4 \quad \Omega_5 \quad \Omega_6], \quad \Omega_2 = \begin{bmatrix} 0 & \Omega_{212} \\ \Omega_{221} & 0 \end{bmatrix} \]

\[ \Omega_{212} = \begin{bmatrix} S_1 \tilde{A}_1 - \tilde{K} \hat{C} \left ( \Sigma^{(t)}_1 \otimes I_m \right ) & S_2 \tilde{A}_1 \\ S_3 \tilde{A}_1 - \tilde{K} \hat{C} \left ( \Sigma^{(t)}_3 \otimes I_m \right ) & S_4 \tilde{A}_1 \end{bmatrix}, \]

\[ \Omega_{221} = \begin{bmatrix} Z_1 + \hat{G} (\hat{D} \otimes I_m) & Z_2 \\ Z_3 + \hat{G} (\hat{D} \otimes I_m) & Z_4 \end{bmatrix}, \quad \Omega_3 = [Y_1 \cdots Y_d] \]

\[ Y_b = \begin{bmatrix} 0 & Y_{b12} \\ 0 & 0 \end{bmatrix} (b = 1, 2, \cdots, d), \]

\[ Y_{b12} = \begin{bmatrix} -\tilde{K} \hat{C} \left ( \Sigma^{(t)} \otimes I_m \right ) \end{bmatrix} \]

\[ \Theta_3 = H, \quad H = \begin{bmatrix} P_1 - S - S^T & 0 \\ 0 & P_2 - Z - Z^T \end{bmatrix} \]

\[ P_1 - S - S^T = \begin{bmatrix} P_{11} - S_1 - S^T_1 \\ P_{13} - S_3 - S^T_3 \end{bmatrix}, \quad P_{14} - S_4 - S^T_4 \]

\[ P_2 - Z - Z^T = \begin{bmatrix} P_{21} - Z_1 - Z^T_1 \\ P_{23} - Z_3 - Z^T_3 \end{bmatrix}, \quad P_{24} - Z_4 - Z^T_4 \]

\[ \Theta_4 = \text{diag} \left \{ \Omega_7, \Omega_8, 0, \Omega_9, 0 \right \}, \quad \Omega_7 = \begin{bmatrix} 0 & \Omega_{712} \\ 0 & 0 \end{bmatrix} \]

\[ \Omega_{712} = \begin{bmatrix} 0 & -\varphi^{(t)}_1 \\ 0 & -\varphi^{(t)}_2 \end{bmatrix} \]

\[ \varphi^{(t)}_1 = \text{vec} \left \{ \tilde{K} \hat{C} \left ( \Sigma^{(t)}_1 \otimes I_m \right ) \right \} i = 1, \cdots, n \]

\[ \varphi^{(t)}_2 = \text{vec} \left \{ \tilde{K} \hat{C} \left ( \Sigma^{(t)}_3 \otimes I_m \right ) \right \} i = 1, \cdots, n \]

\[ \tilde{\eta} = \text{diag} \left \{ U_1, \cdots, U_d \right \}, \quad U_b = \begin{bmatrix} 0 & U_{b12} \\ 0 & 0 \end{bmatrix} (b = 1, 2, \cdots, d), \]

\[ U_{b12} = \begin{bmatrix} 0 & -\varphi^{(t)}_1 \\ 0 & -\varphi^{(t)}_2 \end{bmatrix} \]

\[ \varphi^{(t)}_1 = \text{vec} \left \{ \tilde{K} \hat{C} \left ( \Sigma^{(t)}_1 \otimes I_m \right ) \right \} i = 1, \cdots, n \]

\[ \varphi^{(t)}_2 = \text{vec} \left \{ \tilde{K} \hat{C} \left ( \Sigma^{(t)}_3 \otimes I_m \right ) \right \} i = 1, \cdots, n \]

\[ \Theta_5 = \text{diag} \left \{ \tilde{H}, \tilde{H}, H, H, H \right \}, \quad \tilde{H} = \tilde{H} \otimes I_d \]

\[ \Theta_6 = \begin{bmatrix} \Omega_{10} & 0 & \cdots & 0 \\ 0 & \Omega_{11} & 0 & 0 \end{bmatrix}, \quad \Omega_{10} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \]

\[ \Omega_{11} = \begin{bmatrix} \vartheta_1 \\ 0 \end{bmatrix}, \quad \vartheta_1 = \begin{bmatrix} -\vartheta_1 \\ -\vartheta_2 \end{bmatrix} \]

\[ \Theta_7 = \tilde{H}, \quad \tilde{H} = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{11} \end{bmatrix}, \quad \Theta_8 = \begin{bmatrix} \Omega_{12} & 0 & 0 & 0 \\ 0 & \Omega_{12} & 0 & 0 \end{bmatrix} \]

\[ \Omega_{121} = \begin{bmatrix} 0 \end{bmatrix}, \quad \Theta_9 = -I_n \]

And other parameters are defined as in Theorem 2. Moreover, if the LMI (20) is feasible, the desired \( H_{\infty} \) filter in the form of (8) can be obtained as

\[ K = \begin{bmatrix} S_1 & \tilde{K} \\ S_3 & \hat{G} \end{bmatrix}, \quad G = \begin{bmatrix} Z_1 & \tilde{G} \end{bmatrix} \tag{21} \]

where \( \tilde{G} \) is the G matrix pseudo-inverse.

**Proof:** From Theorem 2, by employing Lemma 3 and Schur complement Lemma, it leads to the equivalence between (17) and following inequality:

\[ \begin{bmatrix} \tilde{\Delta} & * & * & * \\ \tilde{W} \tilde{\Pi}_1 & N & * & * \\ \tilde{W} \tilde{\Pi}_2 & 0 & \tilde{N} & * & * \\ \tilde{W} \tilde{\Pi}_3 & 0 & 0 & \tilde{N} & * \\ \tilde{\Pi}_4 & 0 & 0 & 0 & -I_n \end{bmatrix} < 0 \tag{22} \]

where \( \tilde{W} = W \otimes I_{d+4}, N = P - W - W^T, \quad \tilde{N} = \text{diag} \left \{ \tilde{N}, \tilde{N}, N, N \right \}, \quad \tilde{N} = \tilde{N} \otimes I_d, \quad \tilde{N} = \hat{P} - \hat{W} - \hat{W}^T \)

Since \( P \) is positive definite, \( W \) is a nonsingular matrix.

Without loss of generality, we partition \( W \) as \( W = \begin{bmatrix} S & 0 \\ 0 & Z \end{bmatrix} \).
Furthermore, partition $S$ and $Z$ as follows:

$$
S = \begin{bmatrix} S_1 & S_2 \\ S_3 & S_4 \end{bmatrix}, \quad Z = \begin{bmatrix} Z_1 & Z_2 \\ Z_3 & Z_4 \end{bmatrix}
$$

where

$$
S_c = \text{diag}_n \left\{ S_c^{(i)} \right\} \quad (i = 1, 2, \cdots, n) \quad (c = 1, 2, 3, 4)
$$

$$
Z_c = \text{diag}_n \left\{ Z_c^{(i)} \right\} \quad (i = 1, 2, \cdots, n) \quad (c = 1, 2, 3, 4)
$$

We have

$$
\begin{bmatrix} P_1 - S - S^T & 0 \\ 0 & P_2 - Z - Z^T \end{bmatrix}, \quad \hat{N} = \begin{bmatrix} \text{diag}_n \left\{ P_1 - S - S^T \right\} & 0 \\ 0 & \text{diag}_n \left\{ P_2 - Z - Z^T \right\} \end{bmatrix}
$$

Then, we can obtain (20) by defining $\tilde{K} = S_1K, \hat{K} = S_3K, \tilde{G} = Z_1G, \hat{G} = Z_3G$. Therefore, the filter parameters (21) can be given according to the definition of pseudo-inverse. The proof is completed.

**Remark 10:** Furthermore, the scalar $\gamma^2$ can be included as an optimization variable in LMI (20) to obtain a minimum attenuation level, which is given in the following convex optimization problem:

**Problem 1:** The optimal distributed $H_{\infty}$ filtering problem is

$$
\min_{\sigma > 0, \gamma > 0, \hat{K}, \tilde{G}} \sigma
$$

subject to (20) with $\sigma = \gamma^2$

The optimal distributed $H_{\infty}$ filtering performance level $\gamma$ can be given as $\gamma^* = \gamma_{\text{min}} = \sqrt{\sigma^*}$.

**Corollary 1:** We can also adopt the approach proposed in [35] for the distributed $H_{\infty}$ filter design. Similar to the proof of Theorem 3, by letting $P_f = \begin{bmatrix} P_{f1} & 0 \\ 0 & P_{f2} \end{bmatrix}$ and $P_c = \begin{bmatrix} P_{c1} & 0 \\ 0 & P_{c2} \end{bmatrix}$, the parameters of the filter can be designed as $\hat{K} = P_{f1}^{-1}\hat{K}$ and $\tilde{G} = P_{c1}^{-1}\tilde{G}$, which are omitted here for conciseness.

**Remark 11:** It is easy to see that $P_f$ is equivalent to $P_1$ with the constraint $P_{13} = 0$ and $P_c$ is equivalent to $P_2$ with the constraint $P_{23} = 0$. Moreover, in Theorem 3, we introduce slack variables $S$ and $Z$ which are not required to be positive definite. So, it can increase the flexibility in determining the matrices. Obviously, the approach we adopted in Theorem 3 includes the approach of Corollary 1, in other words, Corollary 1 is a special case of Theorem 3. Therefore, the filter design method proposed in this article can reduce the design conservativeness.

**VI. SIMULATION EXAMPLE**

In this section, an example is presented to illustrate the effectiveness of the proposed method of this article.

In the example, we consider a sensor network with 4 nodes as shown in Fig. 4, whose topology is represented by a directed graph $\mathcal{G} = (\nu, \varsigma, A_{ij})$ with the set of nodes $\nu = \{1, 2, 3, 4\}$, the set of edges $\varsigma = \{(1, 4), (2, 1), (2, 4), (3, 2), (4, 2), (4, 3)\}$, and the adjacency matrix

$$
A_{ij} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}
$$

Consider the following discrete-time system from literature [46]:

$$
A = \begin{bmatrix} -0.2 & 0.9 \\ -0.4 & 0.8 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 \\ 0.6 \end{bmatrix}, \quad M = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}
$$

$$
C_1 = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.6 \\ 0.7 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.5 \\ 0.8 \end{bmatrix}, \quad C_4 = \begin{bmatrix} 0.9 \\ 0.7 \end{bmatrix}
$$

Moreover, the randomly occurred nonlinear function $f (x) = \left[ f_1 (x), f_2 (x) \right]^T$ is given by

$$
\begin{bmatrix} f_1 (x) \\ f_2 (x) \end{bmatrix} = \begin{bmatrix} \tanh (-x_1) + 0.3x_1 + 0.2x_2 \\ 0.1x_1 - \tanh (-x_2) + 0.2x_2 \end{bmatrix}
$$

It can be easily verified that

$$
U_1 = \begin{bmatrix} -0.7 & 0.2 \\ 0.1 & -0.8 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}
$$

The randomly varying time delay is $\tau_k = 1/2 \times (1 + (-1)^k)$, which means $\tau = 0$ and $\bar{\tau} = 1$.

For simulation purposes, we give the following assumptions:

1. The node 1 experiences randomly missing measurements.
2. The sensor network has imperfect communication link.
3. Communication link failures occur randomly between node 3 and node 4 and between node 2 and node 4.
4. The maximum data number we used is $\mu = 42$.

We choose $\bar{\mu} = 0.2, \xi_0^{(0)} = 0.8, \xi_0^{(1)} = 0.7, \xi_i^{(0)} = 1, \xi_i^{(1)} = 0 (i = 2, 3, 4), \mu_{43} = 0.7, \bar{\mu}_{24} = 0.5, \mu_{21} = 1, \bar{\mu}_{32} = 1, \bar{\mu}_{42} = 1, \bar{\mu}_{14} = 1$. The attenuation level is taken as $\gamma = 3.80$. By using MATLAB LMI Toolbox to solve the feasibility of LMI (20) in Theorem 3, the filter parameters can be obtained as follows:

$$
K = \text{diag} \{K_1, K_2, K_3, K_4\}, \quad G = \text{diag} \{G_1, G_2, G_3, G_4\}
$$
Let the disturbance \( w(k) = e^{-0.2k} \times \sin(k), \quad v_i(k) = e^{-0.2k} \times \sin(k) \) and the initial conditions are \( x(0) = \left[ 0.1 \quad 0.1 \right]^T \) and \( \hat{x}_i(0) = \left[ 0 \quad 0 \right]^T \) \((i = 1, 2, 3, 4)\), respectively. When the random variables \( \theta_{ij}(k) \) are changing based on probabilities \( \bar{\theta}_{ij} \), the time-varying topology is plotted in Fig.5, where \( N \) denotes the number of communication link failures. The estimation errors of \( \xi(k) \) of four nodes are plotted in Fig.6, where it can be confirmed that the proposed algorithm is effective.

**TABLE 1.** Comparison of \( \gamma^* \) for different communication link case.

| \( d \) | \( \frac{\bar{\gamma}^{(0)}}{\bar{\gamma}^{(1)}} \) | \( \frac{\bar{\gamma}^{(0)}}{\bar{\gamma}^{(2)}} \) | \( \gamma^* \) |
|---|---|---|---|
| 0 | 0.7 | 2.0054 |
| 1 | 0.7/0.6 | 1.9211 |
| 2 | 0.7/0.6/0.7 | 1.7826 |

In order to show the advantages of measurement model (3) proposed in this article, Table 2 gives the comparison for the number of data we used. From Table 2, we can see that the \( H_{\infty} \) performance level is becoming smaller when the used data is becoming more, which means that the performance of the system is getting better. Therefore, the multiple data packets approach proposed in this article is effective.

**TABLE 2.** Comparison of \( \gamma^* \) for different maximal number of used data packets.

In the end, Table 3 gives the comparison with measurement model proposed in [35]. Reference [35] considers that only one data is used, which is equivalent to \( \tilde{\xi}^{(b)}(k) = 0 \) \((b = 1, \cdots, d)\) of model (3). The simulation results are given in Table 3, where \( \gamma^*_1 \) means the results obtained in this article under the case of \( \tilde{\xi}_1^{(1)} = 0.6 \) and \( \gamma^*_2 \) means the results obtained by [35]. From Table 3, It can be seen that the \( H_{\infty} \) performance level is becoming lower, that is, the performance of the system is getting better when the value of \( \tilde{\xi}_1^{(1)} \) is becoming larger. And we can see that the values of \( \gamma^* \) by utilizing our algorithm are smaller than [35]. Obviously, multiple packets strategy proposed in this article is much better than [35], which can improve the performance of the system. In addition, the result by adopting the method of

**TABLE 3.** Comparison of \( \gamma^* \) with reference [35].
Corollary 1 to solve LMI (20) is infeasible due to the complex networked environment while the method of Theorem 3 is still effective. It verifies that our algorithm is much less sensitive than the method proposed in [35].

VII. CONCLUSION
In this article, the distributed $H_{\infty}$ filtering problem over the sensor networks is investigated where the randomly occurred nonlinearity and missing measurements as well as varying transmission delays and communication topology are considered simultaneously. A nonlinear two-step filter is constructed by means of 2-D system where a multiple packets strategy is used. Sufficient conditions are given to confirm that the filtering error system is asymptotically mean-square stable and the average $H_{\infty}$ performance constraint is guaranteed by using LMI technique. The simulation results clarify that the algorithm presented in this article is superior to [35]. Our future research topic is to investigate robust distributed $H_{\infty}$ filtering problem over sensor networks for T-S fuzzy systems by taking uncertain network-induced phenomena into consideration.

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