Research on intelligent vehicle trajectory tracking based on model predictive control

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Abstract. Intelligent vehicle trajectory tracking lateral control is the key technology to realize intelligent driving. Aiming at the problems that the existing control methods have a small effective control area and poor control effect under cornering conditions or small road adhesion coefficients, a model predictive control (MPC) algorithm is proposed for trajectory tracking control. Based on the two-degree-of-freedom vehicle model, a state space equation is established and discretized to obtain a predictive model that meets the conditions. Then, the corresponding objective function is designed based on the ideal yaw angular velocity under the optimal preview control theory, and the input-output constraints and side-slip angle constraints are designed according to the control and state quantities. The controller model is established in MATLAB / Simulink, and the road model is built in CarSim for joint simulation. Compared with CarSim's own algorithm, the changes in side acceleration and steering wheel angle are smoother and less oscillating. The maximum lateral deviation and lateral acceleration are also smaller, and the lateral stability of the car is improved. It has better adaptability on pavement with lower adhesion coefficient, and can also maintain control stability. The real vehicle test results are consistent with the simulation results, with a maximum lateral error of 0.44m and a maximum heading angle deviation of 1.43°. Therefore, the trajectory tracking control algorithm based on model predictive control designed in this paper has significantly improved control accuracy and stability, and can maintain control stability even when the road surface adhesion coefficient is low.

1. Introduction

Autonomous vehicles have become a research hotspot in recent years, and trajectory tracking control of autonomous vehicles is the key technology to achieve unmanned driving. Therefore, research on improving the accuracy and stability of control has important practical significance [1-3].

For trajectory tracking control, domestic and foreign scholars have conducted a series of in-depth studies. Commonly used lateral control methods include classic PID control methods[4], optimal control methods[5], adaptive control methods[6], sliding mode control methods[7], robust control methods[8], and fuzzy control methods[9]. Reference [10] uses nested PID steering control to maintain the path of autonomous vehicles. The control strategy consists of two nested control blocks C1 and C2. C1 tracks the yaw rate based on the yaw Angle under constant interference and parameter uncertainty. The yaw angular velocity reference signal makes the lateral deviation approach the zero
expected value. Reference [11] uses LQ to optimize the control of the front wheel angle based on the PD control frame, so that the steering wheel can track the desired angle. Although this method can ensure that the controlled state is stable near the system equilibrium point, it cannot guarantee the control effect of the system under uncertain parameters. Reference [12] proposed a robust control method for unmanned vehicle trajectory tracking based on conditional integration algorithm. It is composed of kinematics controller and dynamics controller, which enables unmanned vehicle to track a given reference trajectory. However, it does not consider the influence of road adhesion coefficient. The above methods have a small effective control area, and the control effect is not good under certain working conditions. Especially for mixed working conditions where the road curvature changes, problems such as failure to solve and large control errors may occur.

In view of the above problems, a smart pure electric SUV is taken as the research object. Based on model predictive control, the trajectory tracking lateral control method of intelligent cars is studied, and the trajectory tracking lateral controller is designed. The controller builds an algorithmic prediction model based on a two-degree-of-freedom vehicle model, tracks the ideal yaw angular velocity under the optimal preview control theory, and designs a tire cornering constraint to ensure that the vehicle does not lose stability. The effectiveness of the control method is verified by simulation and real vehicle tests to ensure accurate trajectory tracking during the driving of the smart car, and to ensure the stability of lateral control under the conditions of the tire nonlinear region.

2. Reference signal generation module

2.1. Vehicle dynamics model

In the process of intelligent vehicle path tracking control, a four-wheeled vehicle is often simplified into a two-wheeled vehicle model. In this paper, a two-degree-of-freedom vehicle model is selected as a reference model [13]. Its state equation is:

\[
\begin{align*}
\dot{\omega}_y &= \frac{a^2 C_f + b^2 C_r}{I_z v} \omega_x + \frac{a C_f - b C_r}{I_z} \beta - \frac{a C_f}{I_z} \delta \\
\dot{\beta} &= \left(\frac{a C_f - b C_r}{Mv_x^2} - 1\right) \omega_x + \frac{C_f + C_r}{Mv_x} \beta - \frac{C_f}{Mv_x} \delta
\end{align*}
\]

\(M\) is the mass of the vehicle; \(\beta\) is the side angle of centroid; \(v_x\) is the longitudinal speed; \(a, b\) is the distance from the center of mass to the front and rear axles; \(\omega_x\) is the yaw angular velocity; \(\delta_f\) is the front wheel turning angle; \(I_z\) is the vehicle's moment of inertia about the Z axis; \(C_f\) and \(C_r\) are the equivalent sideslip stiffness of the front and rear wheels, respectively.

2.2. Optimal preview driver model

![Figure 1. Trajectory prediction under the assumption of constant yaw rate.](image)
As shown in Figure 1, \(x_{OY}\) is the vehicle coordinate system; point G is the current center of mass position of the vehicle; P is the target point on the target path; point C is the preview point of the vehicle; \(\beta\) is the vehicle's centroid skew angle. \(x_{GP}\) is Preview distance; \(\Delta f\) is Lateral deviation of car from track centerline [14].

Assuming that the car travels from the current position point G to the target point P at a constant yaw angular velocity \(\omega_d\) during the car preview time, according to the steady-state circular motion rule, we can know that the car's trajectory is an arc \(GP\) with R as the radius, and E is the center of the arc. At the same time:

\[
v = \sqrt{v_x^2 + v_y^2}
\]

\(t_p\) is the preview time of the car, then the longitudinal distance \(x_{GP}\) of the car can be expressed as:

\[
x_{GP} = v_x t_p
\]

During the steady running of the car, the vehicle's lateral speed \(v_x\) is greater than its longitudinal speed \(v_y\). Lateral speed and longitudinal speed are considered to remain unchanged. Under this assumption, the movement of the car body can be regarded as an arc with a radius R travel. Therefore, the steady-state yaw rate \(\omega_d\) of the car can be expressed as:

\[
\omega_d = \frac{\theta}{t_p}
\]

At the same time, the direction of the speed \(v\) of the car when driving is tangent to the arc GP. The values of \(\angle PGB\) and \(\angle GEA\) can be obtained:

\[
\angle PGB = \angle GEA = \frac{\theta}{2}
\]

The angle \(\angle PGC\) of the right triangle PGC in the figure can be expressed as:

\[
\angle PGC = \frac{\theta}{2} + \beta
\]

According to the nature of the right triangle, the relationship between \(\angle PGC\) and the right side PC and right side OC can be obtained:

\[
\tan(\angle PGC) = \frac{v_y}{x_{GP}} = \frac{\Delta f}{v_x t_p}
\]

By combining the above equations, we can find the \(\theta\):

\[
\theta = 2 \arctan\left(\frac{\Delta f}{v_x t_p}\right) - 2\beta
\]

Substituting equation (8) into equation (4), the ideal yaw angular velocity is:

\[
\omega_d = \frac{2 \arctan\left(\frac{\Delta f}{v_x t_p}\right) - 2\beta}{t_p}
\]

3. Design of model predictive controller

3.1. Principle of model predictive control

Model predictive control (MPC) is a computer control algorithm which originated in the 1970s. Its predecessor was industrial process control. MPC uses this model to predict the future behavior of the process output as the input changes [15-16]. MPC uses the measured values of the model and the current process to calculate the future actions of the manipulated variables, ensuring that all inputs and
outputs meet the constraints. Model predictive control mainly includes three aspects: prediction model, moving optimization and feedback correction [17].

3.2. Design of prediction model

The model predictive control state space equation is designed based on the two-degree-of-freedom vehicle model state space equation [18-20]. The front wheel rotation angle \( \delta \), and the vehicle's center of mass lateral position coordinates \( Y_0 \), the center of mass side deflection angle \( \beta \), the yaw angle \( \psi \), and the yaw angular velocity \( \omega \) are used as the control system's state variables. It can be expressed as:

\[
X_c = \begin{bmatrix} Y_0 & \beta & \psi & \omega \end{bmatrix}^T
\]  

Rewriting it into the form of equation of state, we can get:

\[
\begin{align*}
\dot{X}_c &= A_cX_c + B_cu \\
Y_c &= C_cX_c
\end{align*}
\]  

Where:

\[
A_c = \begin{bmatrix} 0 & \frac{v_x}{mv} & 0 & 0 \\
0 & \frac{C_f + C_r}{mv} & 0 & \frac{-aC_f - bC_r}{mv} \\
0 & 0 & 0 & 1 \\
aC_f - bC_r & 0 & \frac{a^2C_f + b^2C_r}{I_z} & 0 \\
\end{bmatrix},
B_c = \begin{bmatrix} 0 \\
\frac{-C_f}{mv} \\
0 \\
-aC_f & 0 \\
\end{bmatrix},
C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

The two-degree-of-freedom vehicle model is a linear time-varying single-input multi-output system, but the model predictive controller designed in this paper needs discrete data. In order to apply the above state-space equation to the design of model predictive controller, the discretization of (11) is carried out:

\[
\begin{align*}
X_d(k + 1) &= A_dX_d(k) + B_du(k) \\
Y_d(k) &= C_dX_d(k)
\end{align*}
\]  

\( A_d, B_d \) are the state control matrices of the discrete state space equation (12). Which can be expressed in the following form by Euler method:

\[
A_d = e^{A_dT}, B_d = \int_{kT}^{(k+1)T} e^{A_d(T - \tau)}B_c d\tau
\]

Where, \( T \) is the sampling time of the discrete system.

\[
Y_d(k) = \begin{bmatrix} Y_0 \\
\beta \\
\omega \end{bmatrix}, C_d = C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

3.3. Design of objective function

In the trajectory tracking control of intelligent vehicle, the objective function is to obtain the optimal control value on the model predictive controller by taking the minimum value of the objective function, so that it can accurately and quickly track the target and ensure that the smart car can follow the trajectory [21]. Therefore, the control quantity, state quantity and reference signal of the model predictive controller have great influence on the design of the objective function. The predicted values of \( Y_0, \beta, \omega \) in the future \( k + N_p \) time are calculated according to the vehicle's previous \( k \) time in
the objective function by the prediction model. The difference between \( Y_0, \beta, \omega \) and the corresponding reference signal is also calculated at the same time[22]. The objective function is designed as follows:

\[
J = \sum_{i=1}^{N_f} \left\| Y_m(k+i|k) - R_{ref}(k+i|k) \right\|_2^2 + \sum_{i=0}^{N_r-1} \left\| \Delta u(k+i|k) \right\|_2^2 \tag{15}
\]

Where,

\[
\sum_{i=1}^{N_f} \left[ Y_0(k+i|k) - Y_{ref}(k+i|k) \right] Q_1 \left[ Y_0(k+i|k) - Y_{ref}(k+i|k) \right]^T + \\
\sum_{i=1}^{N_r} \left[ \beta(k+i|k) - \beta_{ref}(k+i|k) \right] Q_2 \left[ \beta(k+i|k) - \beta_{ref}(k+i|k) \right]^T + \\
\sum_{i=1}^{N_r} \left[ \omega(k+i|k) - \omega_{ref}(k+i|k) \right] Q_3 \left[ \omega(k+i|k) - \omega_{ref}(k+i|k) \right]^T + \\
\sum_{i=0}^{N_r-1} \left\| \Delta u(k+i|k) \right\|_2^2 = \sum_{i=0}^{N_r-1} \left[ u(k+i|k) - u(k+i-1|k) \right]^T R \left[ u(k+i|k) - u(k+i-1|k) \right]
\]

The objective function designed in this paper contains multiple variables. The weight matrix configuration of multiple variables is relatively difficult. Therefore, to solve this type of constrained optimization problem, the objective function needs to be transformed into a standard quadratic form, and solved by effective set method and interior point method.

The output of the prediction model can be expressed in the form of a matrix:

\[
Y_{out}(k) = \Theta(k)X_{e}(k) + \Phi(k)\Delta U(k) \tag{16}
\]

Where,

\[
\Theta(k) = \begin{bmatrix} C_a A_a & C_a A_{a}^2 & \cdots & C_a A_{a}^{N_r} \end{bmatrix}^T, \Phi(k) = \begin{bmatrix} C_a B_a & 0 & \cdots & 0 \\
C_a A_a B_a & C_a B_a & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
C_a A_{a}^{N_r-1} B_a & C_a A_{a}^{N_r-2} B_a & \cdots & C_a B_a \end{bmatrix}
\]

By combining 16 and 19, to ensure that each sampling point has a solution, the weight coefficient \( \rho \) and relaxation factor \( \varepsilon \) are added.

\[
J = [Y_{out}(k) - R_{ref}(k)]^T Q [Y_{out}(k) - R_{ref}(k)] + \Delta U(k)^T R \Delta U(k) + \rho \varepsilon^2 \tag{17}
\]

Where \( R_{ref}(k) \) is an array of \( R_{ref}(k+i|k) \).

Supposing

\[
E(k) = R_{ref}(k+i|k) - \Theta(k)X_{e}(k) \tag{18}
\]

By combining 17 and 18:

\[
J(k) = \frac{1}{2} \chi(k)^T H(k) \chi(k) + g(k) \chi(k) \tag{19}
\]

Where, \( H(k) = \begin{bmatrix} 2(\Phi(k)^T Q \Phi(k) + R) & 0 \\
0 & 2\rho \end{bmatrix}, g(k) = \begin{bmatrix} -2E(k)^T Q \Phi(k) \ 0 \end{bmatrix}, \chi(k) = \begin{bmatrix} \Delta U(k)^T \end{bmatrix} \)

Since the lateral-driver model has the ability to simulate the trajectory prediction of real driver, the yaw velocity calculated by the lateral-driver model at each \( k \) moment can be used as the reference
value of the output, and the value is also set as the output reference value at \( k+1, k+2 \cdots k+N_p \). The calculation method of yaw angular velocity reference value is shown in the equation.

\[
\omega_{ref} = \frac{2 \arctan \left( \frac{\Delta f}{v_t p} \right) - 2 \beta}{t_p}
\]  

At moment \( k \), the predicted value of vehicle lateral position can be calculated according to the vehicle running trajectory curve. Under the assumption of constant yaw angular velocity, the curve is approximately an arc. The reference value is the corresponding value of the lateral position of the center line of the road at that time. In active lane keeping control, the vehicle's centroid side Angle should be equal to zero ideally, so the reference value of centroid side Angle is set to zero at all times in this paper.

3.4. Constraint condition
In trajectory tracking control, while the vehicle is affected by wind direction, road curvature and some unknown disturbances, the control system is also limited by the actuator. The steering wheel Angle limit and the angular velocity limit have great influence on the controller design. At the same time, the trajectory tracking control system should ensure the safety and stability of the vehicle, ensure the vehicle tire side angle in the linear area, prevent sideslip, instability such as the occurrence of such situations.

In order to ensure the effectiveness and smoothness of the steering actuators during trajectory tracking control, this paper firstly constrains the front wheel angle and the front wheel angle increment. Constraints of the front wheel Angle and its increment are expressed as follows:

\[
u_{\text{min}}(k+t) < u(k+t) < u_{\text{max}}(k+t)
\]

\[
\Delta u_{\text{min}}(k+t) < \Delta u(k+t) < \Delta u_{\text{max}}(k+t)
\]

The sideslip phenomenon of vehicle is closely related to the wheel side Angle. When \( \alpha \leq \alpha_{\text{max}} \), the wheel underwent elastic deformation but no sideslip occurred. When \( \alpha > \alpha_{\text{max}} \), the tire forces exceed the limit value and the vehicle occurs sideslip. In order to ensure the driving safety, this paper will restrict the wheel side Angle.

According to the two-degree-of-freedom vehicle model, the front and rear tire side Angle can be expressed as:

\[
\begin{aligned}
\alpha_f &= \beta + \frac{a \omega}{v_r} - \delta \\
\alpha_r &= \beta - \frac{b \omega}{v_s}
\end{aligned}
\]

Linearize it, and we can get:

\[
\alpha = EX_c + Fu
\]

Where:

\[
E = \begin{bmatrix}
0 & 1 & 0 \\
0 & 1 & 0
\end{bmatrix},
F = \begin{bmatrix}
a \\
b
\end{bmatrix},
X_c = \begin{bmatrix}
\alpha_f \\
\alpha_r
\end{bmatrix}
\]

4. Trajectory tracking simulation experiment
In order to verify the effectiveness of the above algorithm, based on the joint simulation platform of CarSim and Simulink, the vehicle model and controller model are built in the environment for joint simulation. The vehicle simulation parameters are shown in Table 1. The controller designed in this
paper is compared with the simple sliding mode controller to verify the control performance of the controller.

| Table 1. Vehicle simulation parameter table. |
|---------------------------------------------|
| Parameters                         | Values | Parameters                             | Values |
| Vehicle quality $m/kg$         | 1350   | Near point preview time $tp1/s$      | 0.8    |
| rotational $Iz/kg\cdot m^{-2}$ | 1950   | Far point preview time $tp2/s$      | 1.5    |
| Distance from center of mass to front axle $a/m$ | 1.18 | Side deflection stiffness of front wheel $C_f/N/\text{rad}$ | 48300 |
| Distance from center of mass to back axle $b/m$ | 1.31 | Side deflection stiffness of back wheel $C_r/N/\text{rad}$ | 39600 |
| Steering ratio $i$               | 15.9   |                                      |        |

The condition of IOS double shifting line is a closed loop test condition for evaluating lateral dynamic characteristics of vehicles. Therefore, it is often applied to the standard double-shift line in the closed-loop simulation experiment of vehicle stability test and driver model. During simulation, the vehicle tracks the double-shift line at a speed of 20 m/s. Respectively, the coefficient of road adhesion is 0.85. The simulation results are shown in the figure.

As shown in Figure 2, under the control of CarSim driver model and MPC, the lateral deviation are all under 0.8m. However, compared with the CarSim driver model, the MPC has significantly improved the path tracking ability, the maximum lateral deviation is reduced to 0.48m, and the steady-state error is stable between 0.2m. As shown in Figures 3 and 4, compared with the CarSim driver model, under the control of MPC, the changes of lateral acceleration and steering wheel angle are smoother, the vibration is less, and the maximum value of lateral acceleration is smaller. It means that the lateral stability of the vehicle is improved.

The above results show that MPC has better control effect on the road with higher coefficient of road adhesion than the self-contained control algorithm in CarSim. In order to verify the stability of the algorithm and its adaptability to different road surfaces, the Carsim simulation software was used to set the road adhesion coefficient to 0.3 and the vehicle speed to 20 m/s. The simulation results are compared with the above simulation results with the road adhesion coefficient of 0.8. The comparison diagram of relevant data is shown in Figures 5, 6 and 7.

It can be seen from Figure 5 that the control accuracy of the MPC controller decreases, and a small overshoot in the turn back timing when the coefficient of road adhesion is 0.3. When the adhesion coefficient is 0.8, the maximum lateral deviation is 0.53 m. When the adhesion coefficient decreases to 0.3, the maximum lateral deviation increases to 0.89 m. From Figure 6, we can see that the lateral acceleration is relatively small at low adhesion coefficient, which may be because the tire cannot provide enough lateral deflection force at low coefficient of road adhesion. It can be seen from Figure
7 that the steering wheel angle changes smoothly under different road adhesion coefficients. From these we can see that the designed MPC controller also has a good effect on the pavement with low adhesion coefficient.

5. Trajectory tracking real vehicle test
As shown in Figure 8, a real vehicle test is carried out based on the intelligent vehicle developed by our team. Using RTK (real-time kinematic) positioning and IMU inertial navigation as environment sensing system, the accuracy of GPS positioning system can reach centimetre-level. RTK positioning system includes two parts, mobile station and base station. RTK's base station is located in an open area near the test site, while the mobile station is installed near the vehicle's center of mass and communicated with each other via 4G network.

In the experiment, the control program is written in the LabVIEW environment, and the CAN signal is sent to the steering system directly by the OBD interface, and then the underlying operation is carried out by the executive controller. The experiment start, the driver controls the car to drive along the designed path. RTK system is used to collect the GPS position information of the vehicle, and the acquisition frequency is 10Hz. Then run the LabVIEW control program to control the vehicle and collect the real-time position of the vehicle. The speed of the experimental vehicle is 10 m/s. Comparing the data collected two times, the experimental results are shown in Table 2, Figures 9 and 10.

In Figure 9, the solid line is the driving path of the driver's car, that is, the target trajectory, and the dashed line is the driving path that the vehicle trajectory tracks during the experiment. It can be seen from the figure that the car can track the trajectory stably according to the planned path. In the east-west direction of the road, the trajectory tracking performance is good, the experimental trajectory and the target trajectory basically coincide, the maximum error is 0.2m. The maximum error is 0.36m when driving on north-south roads. When entering the curve, there will be a slight turning lag.
phenomenon, the error reaches 0.44m, but it can be corrected in time. In Figure 10, the comparison of the heading angle of the driving trajectory also confirms the correction ability of the designed algorithm. During straight driving, the maximum heading angle deviation is 1.43°. Heading angle lag appears at the turn, but can be corrected in time. The longest lag correction time appears around 40s. The target course was basically reached after 8 seconds. After several experiments, it was found that the road error in the north-south direction is always greater than the road error in the east-west direction. This is because the trees on both sides of the north-south road blocked the positioning satellite signals. Its position needs to be corrected using IMU inertial navigation. There is a certain impact on positioning accuracy. In addition, the algorithm can also ensure good tracking accuracy and stability when tracking circular roads, and it is in good agreement with the simulation results.

| Error type       | Maximum error |
|------------------|---------------|
| East-west $X_{\max}$ m | 0.20          |
| North-south $Y_{\max}$ m | 0.36          |
| Curve $C_{\max}$ m  | 0.44          |
| Heading angle $\gamma_{\max}$ deg | 1.43          |

Figure 9. Driving trajectory.  
Figure 10. Course angle.

6. Conclusion
Based on the two-degree-of-freedom vehicle model, a state-space equation is established and discretized to obtain a prediction model that meets the model's predictive control conditions. Then, the corresponding objective function is designed based on the ideal yaw angular velocity under the optimal preview control theory, and the input-output constraints and side-slip angle constraints are designed according to the control and state quantities. Thereby, the designed model predictive control algorithm is obtained. Simulation results show that the trajectory tracking control algorithm based on model predictive control has significantly improved control accuracy and stability compared with Carsim's own sliding film control algorithm. The maximum lateral deviation is 0.48m, and the steady-state error is stable at about 0.2m. The lateral acceleration and steering wheel angle change controlled by the MPC algorithm are smoother and less oscillating. The maximum lateral acceleration is also smaller, and the lateral stability of the car is improved. The MPC controller has better adaptability on pavement with lower adhesion coefficient, and can also maintain control stability. The actual vehicle test results are consistent with the simulation results, with a maximum lateral error of 0.44m and a maximum heading angle deviation of 1.43°. The simulation and real vehicle test results verify the
effectiveness of the control method, ensure the accurate trajectory tracking of the smart car during driving, and ensure the stability of the lateral control under the conditions of the tire nonlinear region.

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