Relativistic Corrections to Astrometric Shifts Due to Gravitational Microlensing

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Gravitational microlensing has proved to be a versatile astrophysical tool. Recently, the question of whether higher order relativistic corrections can influence the observable properties of microlensing has been addressed. This paper extends earlier studies, investigating higher order relativistic terms in astrometric shifts during microlensing events, revealing that they produce no effects. The magnitude of the relativistic corrections is examined in an astrophysical context and is found to be negligible.

1. Introduction

Over the past decade, the field of gravitational microlensing within the Local Group has grown rapidly. Several hundred microlensing events have been detected by large monitoring surveys, including MACHO,\(^1\) EROS\(^2\) and OGLE.\(^3\) These have provided important clues to the mass distribution of the Galaxy, as well as unique view of detailed surface features of distant stars.\(^4\)\(^{-}\)\(^6\) Microlensing also holds great promise in the identification of planetary systems beyond the range of other observational approaches.\(^7\)\(^{-}\)\(^9\) The details of gravitational microlensing are given in several recent reviews\(^10\) and will not be reproduced here.

The standard formalism employed in the study of gravitational microlensing considered the relativistic deflection up to first order, assuming higher order correction are small and can be neglected. Recently, Ebina, Osuga, Asada and Kasai\(^11\) have questioned this assumption and considered second order relativistic corrections to the currently employed formalism. Intriguingly, they found that while both the image positions and magnifications are modified by the addition of higher order terms, the sum of the magnifications, which is the observed quantity during the photometric monitoring of a microlensing event, does not suffer any second order correction.

Here, the study of Ebina et al.\(^11\) is extended to investigate the magnitude of the second order relativistic corrections to the image astrometric shift during a microlensing event.\(^12\)

2. Background

All angles are normalized to the natural scale length of gravitational microlensing, the angular Einstein radius,

\[ \theta_E = \sqrt{\frac{4GM}{c^2D_dD_s}}, \]

where \(M\) is the mass of the compact microlensing object and \(D_s, D_d\) and \(D_{ds}\) repre-

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sents the relative observer, deflector and source distances. For source at an angle $\theta_s$ from a lensing mass, two images are produced at angles:

$$\theta_{1,2} = \frac{1}{2} \left( \theta_s \pm \sqrt{\theta_s^2 + 4} \right).$$

(2)

For stellar mass lenses within the Local Group, the separation of these images is typically milliarcseconds, below the resolution of current optical observations. These images are, however, magnified with respect to the unlensed source, the magnifications given by

$$\mu_{1,2} = \frac{\theta_s^2 + 2}{2 \theta_s \sqrt{\theta_s^2 + 4}} \pm \frac{1}{2}, \quad \mu_{\text{tot}} = \mu_1 + \mu_2 = \frac{\theta_s^2 + 2}{\theta_s \sqrt{\theta_s^2 + 4}}.$$  

(3)

The magnification can be substantial, resulting in the characteristic ‘bell-shaped’ light curve observed during a gravitational microlensing event.

Combining the individual image positions and magnifications, the centroid of the light distribution of a gravitationally microlensed source is given by

$$\theta_{\text{cent}} = \frac{\theta_1 \mu_1 + \theta_2 \mu_2}{\mu_1 + \mu_2}.$$  

(4)

During a microlensing event, therefore, as a source brightens and fades it should also exhibit a shift in its light centroid. This will be small, again the order of milliarcseconds, but such centroid shifts are detectable with the next generation of space interferometers, which will have an accuracy of microarcseconds (see http://sim.jpl.nasa.gov/).

3. Second order lensing correction

In the weak gravitational fields of typical astrophysical regimes, it is usual to consider the influence of general relativity to low order. The question of higher order corrections to the gravitational lensing have been considered previously, and recently Ebina et al. examined the magnitude of second order relativistic effects to the gravitational microlensing equations. Considering this higher order term to be a perturbation to the solutions of the lensing equation, demonstrated the image positions become

$$\theta'_{1,2} = \theta_{1,2} + \frac{\lambda}{\theta_{1,2} \sqrt{\theta_s^2 + 4}} + \mathcal{O}(\lambda^2), \quad \lambda = \frac{15 \pi D_s \theta_E}{64 D_{ds}}.$$  

(5)

Here, $\lambda$ is a dimensionless parameter, the magnitude of which is considered in 5. Similarly, the inclusion of the second order term modifies the image magnifications, such that

$$\mu'_{1,2} = \mu_{1,2} + \lambda (\theta_s^2 + 4)^{-3/2} + \mathcal{O}(\lambda^2), \quad \mu'_{\text{tot}} = \frac{\theta_s^2 + 2}{\theta_s \sqrt{\theta_s^2 + 4}} + \mathcal{O}(\lambda^2) = \mu_{\text{tot}}.$$  

(6)

Surprisingly, while the magnification of the individual images is modified by the inclusion of the second order term, the total magnification is not, demonstrating that the inclusion of the higher order relativistic correction produces no modification of microlensing light curves.
4. Astrometric shifts  While the total magnification is independent of the second order corrections, the previous section demonstrates that the image positions and magnifications change. Considering Eq. (4), these changes can potentially modify the path of the image centroid during a microlensing event and result in detectable consequences of the higher order relativistic corrections.

Again defining the image centroid to be (Eq. (4))

$$\theta'_{\text{cent}} = \frac{\theta'_1 \mu'_1 + \theta'_2 \mu'_2}{\mu'_1 + \mu'_2}$$

and expanding this expression using Eqs. (5) and (6), this becomes

$$\theta'_{\text{cent}} = \theta_{\text{cent}} + \left( \frac{\lambda}{(\theta^2 + 4)^{3/2}} (\theta_2 - \theta_1) + \frac{\lambda}{(\theta^2 + 4)^{1/2}} \left( \frac{\mu_1}{\theta_1} + \frac{\mu_2}{\theta_2} \right) + O(\lambda^2) \right).$$

A closer examination reveals that the first two terms within the brackets are equivalent, but of opposite sign, and so cancel. Hence, when accounting for second order relativistic corrections, the centroid shift is

$$\theta'_{\text{cent}} = \theta_{\text{cent}} + O(\lambda^2),$$

and, like the total magnification, the position of the image centroid is independent of these higher order terms. Hence, consideration of the second order relativistic correction to the equations of gravitational microlensing produces no deviation to the observational properties available to us during a microlensing event.

5. How big is $\lambda$?  The previous sections have demonstrated how the inclusion of second order relativistic corrections to the formalism of gravitational microlensing produces no observable consequences to the total image magnification and centroid shift. With Eqs. (5) and (6), the individual image magnifications and positions are, however, modified by the higher order terms. It is prudent, therefore, to ask that, if observational techniques improve to allowing the resolution of the individual microlensed images, would the consequences of the relativistic corrections be then observable? This question depends entirely on the magnitude of the parameter $\lambda$.

In considering Eq. (5), the numerical value of the second order parameter $\lambda$ for a typical microlensing scenario within the Local Group is

$$\lambda = 3.6 \times 10^{-9} \frac{1}{[q(1 - q)]^{1/2}} \left[ \frac{M}{M_\odot} \right]^{1/2} \left[ \frac{D_s}{8kpc} \right]^{-1/2},$$

where $q = D_d/D_s$, the ratio of the distances to the lens and the source, $M_\odot$ is the mass of the Sun and 8kpc is distance to the Galactic centre. It must be remembered that the units presented in this paper are normalized with respect to the angular Einstein radius, hence the deviations due to the second order relativistic corrections are of order $\lambda \times \theta_E$. Given the above, it must be concluded that the deviations of the individual image positions and magnification due to such a correction is astrophysically negligible and beyond detection.

6. Conclusion  This paper has examined the influence of second order relativistic corrections on the magnitude of astrometric shift during a gravitational microlensing
event. It is demonstrated that this quantity, like the total magnification, is independent of this higher order term. It is also shown that the second order correction for gravitational microlensing within the Local Group is typically extremely small, and hence has negligible astrophysical impact.

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