Production of three Vector Bosons
in $e^+e^-$ Annihilation
as a Test of $W^\pm$, $Z$, $\gamma$ Self-Interactions

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Abstract

We study the vector-boson production processes $e^+e^- \rightarrow W^+W^-Z$
and $e^+e^- \rightarrow W^+W^-\gamma$ which are directly affected by the trilinear and
quadrilinear self couplings of the $W^\pm$, $Z$ and $\gamma$. Our analysis is based upon
a single-parameter effective-Lagrangian model for these self interactions
which contains the standard model as a special case. Consequences for the
phenomenology at an $e^+e^-$ collider of 500 GeV (NLC) are discussed, and
fits of the free parameter around its standard model value are carried out.

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1 Introduction

The standard \(SU(2)_L \times U(1)_Y\) model of electroweak interactions \(^1\) has been confirmed by all experiments up to now. However, this empirical evidence is essentially restricted to vector-boson–fermion interactions. The vector-boson self interactions, which are a consequence of the non-Abelian structure of the gauge group \(SU(2)_L \times U(1)_Y\) (in its minimal realization), and which are essential for the renormalizability and unitarity of the theory, contribute to reactions realized at present collider energies only indirectly via vector-boson-loop corrections.

Future colliders like LEP II \((e^+e^- \text{ at } \sqrt{s} = 200 \text{ GeV})\) and NLC \((e^+e^- \text{ at } \sqrt{s} = 500 \text{ GeV})\) \(^2\) will make direct tests of \(W^\pm, Z, \gamma\) self interactions possible by the measurement of processes which get tree level contributions from these self couplings. The mainly studied process of this type is the two-vector-boson production process \(e^+e^- \rightarrow W^+W^-\) (e.g. \(^3\), \(^4\), \(^5\)) as a test of the trilinear self couplings. A full confirmation of the non-Abelian vector-boson sector of the model, however, needs a test of the quadrilinear couplings as well. One class of processes which is directly effected by tri- and quadrilinear self couplings, is vector-boson scattering \((V_1, V_2 \rightarrow V_3, V_4 \text{ with } V_i = W^\pm, Z, \gamma)\) \(^6\), \(^7\), but, unfortunately, it can only be measured in reactions of the type \(e^+e^- \rightarrow (e^+e^-, \bar{\nu}e^-, \nu e^+, \nu \bar{\nu}) + V_1V_2\) at CM-energies in the TeV region, which cannot be realized experimentally yet.

The reactions with direct contributions of tri- and quadrilinear self couplings which are of greatest phenomenological interest are the three-gauge-boson production processes \(e^+e^- \rightarrow W^+W^-Z\) and \(e^+e^- \rightarrow W^+W^-\gamma\) \(^8\), \(^9\), \(^10\), which will be measurable at the expected new linear collider (NLC) with a CM-energy of \(\sqrt{s} = 500 \text{ GeV}\) and a luminosity of \(20 \text{ fb}^{-1}\) a\(^{-1}\). The standard model cross sections for these processes were calculated by Barger, Han and Phillips \(^8\) and by Tofighi-Niafi and Gunion \(^9\). To examine the sensitivity of these cross sections to deviations of the self couplings from their standard model values and to find experimental bounds on such deviations, we need a more general parameter-dependent model which includes the standard model as a special case. The most general procedure would start from an effective Lagrangian containing all vector-boson self interactions which can be constructed in agreement with Lorentz invariance. Such an analysis was performed for trilinear self couplings and the \(W^+W^-\)-production process in \(^3\) and \(^4\). For three-vector-boson production, however, this seems to be a complicated procedure involving elaborate multi-parameter fits and large individual errors of the fit parameters as a consequence of limited future statistics.

For simplification we base our analysis on the less extended KMSS model \(^5\), \(^6\), which rests upon global \(SU(2)_{W_1}\) symmetry broken by electromagnetism. Electromagnetic interactions are assumed to be \(P-\) and \(C-\) invariant and only dimensionless coupling constants are allowed, thus suppressing dimension-six quadrupole terms. These assumptions imply a four-parameter Lagrangian; the free parameters can be reduced to a single one, which is chosen as the anomalous magnetic...
moment $\kappa$ of the $W^\pm$ boson if, in addition, one imposes the requirement that those terms in the vector-boson scattering amplitudes which grow most strongly with energy (as $s^2$) are absent (BKS model) [3].

It turns out, that this single-parameter model (extended by appropriate couplings of the Higgs boson to the vector bosons) can be derived by adding one extra $SU(2)_L \times U(1)_Y$ invariant dimension-six interaction term to the standard model Lagrangian. This extension of the BKS model provides a mechanism which protects LEP I observables against (strong) deviations from their standard model values.

In Sect. 2, we explain the KMSS model and the reduction to a single parameter effective Lagrangian (BKS model). In Sect. 3 we give an $SU(2)_L \times U(1)_Y$ invariant derivation of the model and discuss its sensitivity to LEP I experiments. In Sect. 4, the calculated cross sections are presented and discussed. We show the dependence of the cross sections on $\kappa$. A fit of the free parameter $\kappa$ on the basis of the expected luminosity and systematic error of future data to be taken at the NLC is performed. Sect. 5 contains our final conclusions.

## 2 The effective Lagrangian

As mentioned, our investigations are based on the KMSS model for vector-boson self interactions [3, 4]. The effective Lagrangian is constructed in the following way:

1. One starts with a $SU(2)_{W1}$ triplet field $\vec{W}_\mu$ and an (unphysical) photon field $\vec{A}_\mu$. Kinetic terms and a mass term for the $\vec{W}$ field are introduced.

2. The most general self interaction of $\vec{W}_\mu$ in conformity with global $SU(2)_{W1}$ symmetry and the restriction to dimension-four couplings is constructed.

3. The photon field $\vec{A}_\mu$ is coupled to the $\vec{W}_\mu$ field by minimal substitution in the $\vec{W}_\mu$ kinetic term. This assures electromagnetic gauge invariance and breaks global $SU(2)_{W1}$.

4. Mixing between the neutral weak vector boson $W_{3\mu}$ and $\vec{A}_\mu$ is added.

5. Finally, an anomalous magnetic-moment term for the interaction of the electromagnetic field with the charged vector bosons is added.

The Lagrangian obtained by this procedure is given in [3]. It has four free parameters (of which only two affect the trilinear couplings).

The next step is the analysis of the vector-boson scattering processes $V_1, V_2 \rightarrow V_3, V_4$ with $V_i = W^\pm, Z$ performed in [4]. The requirement of vanishing of the most strongly unitarity violating terms of order $s^2$ in the tree amplitudes (which is equivalent to the demand of vanishing quartic divergences in one-loop
corrections leads to three conditions on the four free parameters, so they can be expressed in terms of a single one, namely the $W^\pm$ anomalous magnetic moment $\kappa$. The (higgsless) standard model is obtained by choosing $\kappa = 1$.

The final Lagrangian $L_{SI}$ for the self interactions is then given by

$$L_{SI} = i e \left[ A_\mu(W^{-\mu\nu}W^\nu_\nu - W^{+\mu\nu}W^-_\nu) + \kappa F_{\mu\nu}W^{+\mu}W^{-\nu} \right]$$

$$+ i e \frac{\kappa - \sin^2 \theta_W}{\sin \theta_W \cos \theta_W} \left[ Z_\mu(W^{-\mu\nu}W^\nu_\nu - W^{+\mu\nu}W^-_\nu) + \frac{\kappa \cos^2 \theta_W}{\kappa - \sin^2 \theta_W}Z_{\mu\nu}W^{+\mu}W^{-\nu} \right]$$

$$- e^2(A_\mu A_\nu W^{+\nu -\nu} - A_\mu A_\nu W^{+\mu}W^{-\nu})$$

$$- 2e^2 \frac{\kappa - \sin^2 \theta_W}{\sin \theta_W \cos \theta_W} \left[ A_\mu Z^{\mu\nu}W^{-\nu} - \frac{1}{2} A_\mu Z_\nu(W^{+\mu}W^{-\nu} + W^{-\mu}W^{+\nu}) \right]$$

$$- e^2 \left( \frac{\kappa - \sin^2 \theta_W}{\sin \theta_W \cos^2 \theta_W} (Z_{\mu\nu}W^{+\nu -\nu} - Z_{\mu\nu}W^{+\mu}W^{-\nu}) \right)$$

$$+ \frac{1}{2} e^2 \frac{1}{\sin^2 \theta_W} \kappa^2(W_{\mu}^{-\mu}W^{+\mu}W^{+\nu} - W_{\mu}^{-\mu}W^{+\mu}W^{+\nu})$$

(1)

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $W^{\pm}_{\mu\nu} = \partial_\mu W^{\pm}_\nu - \partial_\nu W^{\pm}_\mu$, etc., and a single free parameter $\kappa$. Considering small deviations from the SU(2)$_L \times$ U(1)$_Y$ value of $\kappa = 1$,

$$\kappa \equiv 1 + \Delta \kappa, \quad \Delta \kappa \ll 1,$$

(2)

the last two terms in (1) may be approximated by terms linear in $\Delta \kappa$, i.e.,

$$- e^2 \frac{\cos^2 \theta_W}{\sin^2 \theta_W} \left( 1 + \frac{2\Delta \kappa}{\cos^2 \theta_W} \right) (Z_{\mu\nu}Z^{\mu\nu}W^{+\nu -\nu} - Z_{\mu\nu}Z^{+\mu}W^{-\nu})$$

$$+ \frac{1}{2} e^2 \frac{1}{\sin^2 \theta_W} (1 + 2\Delta \kappa)(W_{\mu\nu}^{-\mu}W^{+\mu}W^{+\nu} - W_{\mu\nu}^{-\mu}W^{+\mu}W^{+\nu})$$

(3)

The Lagrangian is a single-parameter extension of the standard model Lagrangian and a reasonable reduction of the multi-parameter Lagrangian of a model with arbitrary self interactions, since it is constructed using the above-mentioned physical considerations, and it is the most general Lagrangian in this formalism in which the vector-boson scattering amplitudes do not contain $s^2$ terms. It can be used to analyze how changes of vector-boson self couplings influence the cross sections and to carry out a parameter fit.
3 Derivation of the Model from an $SU(2)_L \times U(1)_Y$-invariant Lagrangian via the Higgs mechanism

As mentioned, the interactions (1) of the vector bosons with one another are such that tree-level scattering amplitudes do not contain any (tree-)unitarity-violating terms growing as $s^2$ at high energies. Corresponding to the absence of such terms, there are no $\Lambda^4$ divergences at the one loop level (1). For the standard-model value of $\kappa = 1$, the remaining linear high-energy growth of the scattering amplitudes (proportional to $s$) is compensated by adding the Higgs scalar with sufficiently low mass $M_H$. Vector-boson loops depend logarithmically on $M_H$.

One may pose the question whether a simple convergence-producing mechanism also exists in case of the Lagrangian (1) for $\kappa \neq 1$. In this section we will show, that the quadratic loop divergences can indeed be removed. Addition of suitable non-standard interactions of the Higgs scalar with the vector bosons allows to embed the self interactions (1) into an $SU(2)_L \times U(1)_Y$ symmetric framework.

Even though the added Higgs-scalar–vector-boson interactions involve nonrenormalizable dimension-six interactions, they will nevertheless provide a sufficiently decent behaviour of the loop corrections to protect LEP I observables from violent deviations from standard predictions.

The $SU(2)_L \times U(1)_Y$ invariant Lagrangian, which yields the vector boson interactions (1), is

$$\mathcal{L}_{SI} = \mathcal{L}_{SM} + \Delta \kappa \frac{g}{M_W^2} \mathcal{O}_{W\Phi}, \quad (4)$$

where $\Delta \kappa = \kappa - 1$ as in (4), $\mathcal{L}_{SM}$ denotes the standard-model Lagrangian, $g = e/\sin \theta_W$ and $\mathcal{O}_{W\Phi}$ is given by

$$\mathcal{O}_{W\Phi} = i(D_\mu \Phi)^\dagger \bar{\Phi} \cdot \mathcal{W}{}^{\mu\nu} (D_\nu \Phi) = -i \frac{\mu^2}{2} \text{tr}(W_{\mu\nu} (D^\mu U D^\nu U^\dagger)) \quad (5)$$

with $\bar{W}_{\mu\nu} = \partial_\mu \bar{W}_\nu - \partial_\nu \bar{W}_\mu - g \bar{W}_\mu \times \bar{W}_\nu$, $W_{\mu\nu} = \frac{\xi}{2} \cdot \bar{W}_{\mu\nu}$, $U = \sqrt{2} \left( \tilde{\Phi} : \Phi \right)$, $\tilde{\Phi} = i \tau_2 \Phi^*$ and $D_\mu U = \partial_\mu U + ig \frac{\xi}{2} \cdot \bar{W}_\mu U - ig' U \frac{\xi}{2} B_\mu$. Explicitly, upon introducing the $Z$ and photon fields via the usual diagonalization, $\mathcal{O}_{W\Phi}$ is given by

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1 We thank the referee of this paper whose questions initiated the present section.
\[
\frac{g}{M_W^2} \mathcal{O}_{W\Phi} = i e F_{\mu\nu} W^{\mu\nu} + \frac{1}{\sin \theta_W \cos \theta_W} \left[ Z_{\mu} (W^{-\mu\mu} W_{\nu} - W^{\mu\nu} W_{\nu}) + \cos^2 \theta_W Z_{\mu\nu} W^{\mu\nu} W_{\mu\nu} \right] \\
- 2 e^2 \frac{1}{\sin \theta_W \cos \theta_W} \left[ A_{\mu} Z_{\mu} W^{\nu} - \frac{1}{2} A_{\nu} Z_{\nu} (W^{\mu\nu} W_{\mu\nu} + W^{\mu\nu} W_{\mu\nu} - W^{\mu\nu} W_{\mu\nu}) \right] \\
- 2 e^2 \frac{1}{\sin \theta_W} \left( Z_{\mu} Z^{\mu\nu} W^{\nu} - Z_{\mu} Z_{\nu} W^{\mu\nu} \right) \\
+ e^2 \frac{1}{\sin^2 \theta_W} \left( W_{\mu} Z_{\nu} W^{\mu\nu} W_{\mu\nu} - W_{\mu\nu} W^{\mu\nu} W_{\mu\nu} \right) \\
+ \text{Higgs couplings.} \tag{6}
\]

Substituting this explicit form of \( \mathcal{O}_{W\Phi} \) into (4), one easily verifies that the gauge-boson self interactions contained in (4) are identical to those of the BKS Lagrangian (1) with the linear approximation (3) for the quadrilinear interactions. In the present paper we only consider small values of \( \Delta \kappa \), so we can identify the BKS self couplings (1) with those of the SU(2)_L \times U(1)_Y invariant model (4).

The effect of boson loops on LEP I observables due to non-standard interactions such as \( \mathcal{O}_{W\Phi} \) and other dimension-six terms was investigated in [12, 13]. Typically, in distinction from the SU(2)_L \times U(1)_Y spontaneously broken standard model, for \( \mathcal{O}_{W\Phi} \) one finds a quadratic dependence on the Higgs-boson mass, \( M_H^2 \), and a logarithmic dependence on the necessary cut-off, \( \ln \Lambda \). Anomalous couplings of the kind of extra dimension-six terms considered here are not restricted very much by LEP I data, as these data are not very sensitive to logarithmic cut-off-dependent loop corrections [13].

In summary, the BKS Lagrangian, originally derived from global SU(2)_W weak isospin symmetry broken by electromagnetism and vanishing of the most strongly (as \( s^2 \)) rising contribution to vector-boson-scattering amplitudes, can be embedded into an SU(2)_L \times U(1)_Y symmetric theory of the simple and compact form (4) with (dimension-six) non-standard Higgs self interactions. This embedding provides an example of how non-standard trilinear and quadrilinear vector-boson-scattering amplitudes can coexist with “standard” empirical LEP I results.

In the calculations of the present paper the Higgs sector of the theory is disregarded (although it contributes to three-gauge-boson-production processes) since we only want to study the effect of anomalous gauge-boson self interactions. This is justified if we assume that \( M_H \) lies above the energy region to be investigated in \( e^+ e^- \) annihilation at the NLC so that the contributions of Higgs exchange to the cross sections are negligible.

\(^2\text{So our cross sections for } \sqrt{s} = 2000 \text{ GeV should not be taken too seriously, as they would eventually be changed by the Higgs effects.} \)
4 Cross-Sections

Figure 1 shows schematically the tree-level Feynman diagrams which contribute to the three-vector-boson production processes $e^+e^- \rightarrow W^+W^-Z$ and $e^+e^- \rightarrow W^+W^-\gamma$. Altogether there are 15 diagrams. We calculated the cross-sections in the same way as Barger, Han and Phillips evaluating the amplitudes by employing helicity techniques and integrating numerically over the phase space. All standard-model couplings were derived via the 1-loop relations among the masses and coupling constants in the standard model (e.g. see [15]) from the Z-boson mass $M_Z$, the Fermi coupling $G_F$ and the electromagnetic fine structure constant $\alpha_{em}$. The latter was set equal to $\alpha_{em} = 1/128$, which is the high energy value of the running coupling constant. As in [8] we imposed the following transverse-momentum and pseudorapidity cuts for the photon produced in the reaction $e^+e^- \rightarrow W^+W^-\gamma$:

$$p_{t,\gamma} > 20 \text{ GeV}, \quad |\eta_\gamma| < 2.$$  \hfill (7)

First we study the total cross-sections for the production of unpolarized vector bosons in the reactions $e^+e^- \rightarrow W^+W^-Z$ and $e^+e^- \rightarrow W^+W^-\gamma$. Figure 2 shows these cross-sections as a function of the energy for different values of the free parameter $\kappa$ around the standard-model value of $\kappa = 1$. We find with $\kappa = 1$ at $\sqrt{s} = 500 \text{ GeV}$ the results $\sigma_{tot} = 39 \text{ fb}$ for $e^+e^- \rightarrow W^+W^-Z$ and $\sigma_{tot} = 135 \text{ fb}$ for $e^+e^- \rightarrow W^+W^-\gamma$. Beyond the threshold region standard model cross-sections decrease with increasing energy as a consequence of gauge cancellations, i.e., the cancellations of those parts of the amplitudes for the production of longitudinally polarized vector bosons which grow as nonegative powers of $s$. These are caused by the relations among the self couplings of the vector bosons and the couplings to fermions in the standard model[4]. If the free parameter $\kappa$ is set $\kappa \neq 1$ these relations are violated. Therefore, in those cases there are no complete cancellations, and $\sigma_{tot}$ is growing with energy, so that unitarity is violated at high energies and must eventually be restored by “new physics” contributions. That is why the deviations of the cross sections in the general BKS model from the standard model increase with the energy $\sqrt{s}$ and with $\Delta \kappa$, which characterizes the magnitude of the deviations of the self coupling constants from the standard model values. Although we find differences of some orders of magnitude from the standard model at the TeV energy scale, at the NLC energy of $\sqrt{s} = 500 \text{ GeV}$ there are just some % differences for our choices of $\kappa$ from 0.9 to 1.1. We will come back to this point later.

To illustrate the effect of violation of the gauge cancellations, we show the cross-sections for the production of exclusively transversely and of exclusively

3Except for the Higgs boson contribution which we do not consider here as explained above. For Higgs boson effects see [8,8].

4In distinction to vector-boson scattering, in these processes the Higgs boson is not needed for good high energy behavior.
longitudinally polarized vector bosons in the reaction $e^+e^- \rightarrow W^+W^-Z$ (Fig. 3). Production of tranverse vector bosons yields a large contribution to the total cross section in the standard model. However, for $\kappa \neq 1$ the deviations from the SU($2)_L \times U(1)_Y$ predicts are very small, because the amplitudes of the different Feynman graphs do not grow with energy and no cancellations are necessary. In contrast, for the production of longitudinal vector bosons, the standard model cross section is very small, but the non-cancellation of the leading amplitudes leads to enormous deviations, when $\kappa$ departs from $\kappa = 1$.

We turn to the question of how well trilinear and quadrilinear self interactions will be measurable in future experiments. We determine the empirical limits which can be assigned to $\kappa$ if the standard model cross sections will be confirmed in experiments. First, we perform an analysis at the NLC energy of $\sqrt{s} = 500$ GeV. Our investigations are based on the cross-sections $e^+e^- \rightarrow W^+W^-Z$, $e^+e^- \rightarrow W^+W^-\gamma$ (production of unpolarized vector bosons) and $e^+e^- \rightarrow W^+_LW^-_LZ_L$ (production of longitudinal vector bosons). The expected luminosity of NLC is $20\text{fb}^{-1}\text{a}^{-1}$, so there would be 800 annual events of $e^+e^- \rightarrow W^+W^-Z$. Following the analysis of [8], 20% of them will be reconstructable from the decay products of the final vector bosons. If we determine the statistical error to 90% confidence level and assume 2% systematic error we find a total error of 10% after three years of collecting data. For the reaction $e^+e^- \rightarrow W^+W^-\gamma$ the statistical error is smaller because of the larger cross-section. By the same reasoning we find a total error of 5% after three years of running. For the process $e^+e^- \rightarrow W^+_LW^-_LZ_L$ the cross-section of 0.5 fb is extremely small and causes a large statistical error. The total error in this case is 70%. As an outlook we perform the same analysis at the energy of $\sqrt{s} = 2000$ GeV in order to see, how the precision of the parameter fit increases with energy. Since no precise parameters of a 2000 GeV machine are available, we assume an experimental error of 10% for production of unpolarized vector bosons and again 70% error for the reaction $e^+e^- \rightarrow W^+_LW^-_LZ_L$. Figure 4 shows the total cross sections for these processes at the two abovementioned fixed energies as a function of $\kappa$. We see, how small deviations of $\kappa$ from its standard model value of $\kappa = 1$ affect the total cross-sections. From these results, we now can find the interval around the standard model value of $\kappa = 1$ to which $\kappa$ is restricted if the deviations from the standard model do not exceed the experimental error. The results of these parameter fits are given in table 1. We see that NLC results can restrict $\kappa$ to a region of a few % around its standard model value of $\kappa = 1$. Accuracy increases by one order of magnitude if $\sqrt{s}$ is raised from 500 GeV to 2000 GeV. The production of exclusively longitudinal vector bosons $e^+e^- \rightarrow W^+_LW^-_LZ_L$ yields limits in the same order as production of unpolarized gauge bosons, because the effect

\[\text{\footnotesize We are aware of the fact that these estimates should eventually be refined by taking into account the effects of a Higgs scalar of unknown mass } M_H \leq 2\text{TeV. Our results have to be considered as crude exploratory values.}\]
of the larger deviations from the standard model is compensated by the larger experimental error.

| Process                      | $\sqrt{s} = 500$ GeV (NLC) | $\sqrt{s} = 2000$ GeV |
|------------------------------|-----------------------------|------------------------|
| $e^+e^- \rightarrow W^+W^-Z$ | $0.95 \leq \kappa \leq 1.06$ | $0.997 \leq \kappa \leq 1.006$ |
| $e^+e^- \rightarrow W^+W^-\gamma$ | $0.98 \leq \kappa \leq 1.09$ | $0.997 \leq \kappa \leq 1.009$ |
| $e^+e^- \rightarrow W^+_LW^-_Z$ | $0.98 \leq \kappa \leq 1.07$ | $0.999 \leq \kappa \leq 1.003$ |

Table 1: Results from the fit of the free parameter $\kappa$ based on the total cross-sections for $e^+e^- \rightarrow W^+W^-Z$, $e^+e^- \rightarrow W^+W^-\gamma$ and $e^+e^- \rightarrow W^+_LW^-_Z$ at $\sqrt{s} = 500$ GeV and $\sqrt{s} = 2000$ GeV.

5 Conclusions

- The three vector-boson production processes $e^+e^- \rightarrow W^+W^-Z$, $e^+e^- \rightarrow W^+W^-\gamma$ supply the phenomenological easiest way to test the tri- and quadrilinear self interactions of the electroweak vector bosons directly.

- The BKS model reduces the most general form of vector-boson self interactions by some physically reasonable assumptions to a single parameter model, which can be used to study the sensitivity of the standard model cross sections to variations of the self couplings.

- The BKS model is not restricted very much by present LEP I data since its anomalous vetor-boson self couplings can be obtained from a simple one-parameter locally SU(2)$_L \times$ U(1)$_Y$-invariant interaction term. As a consequence, loop-contributions in this model (extended by appropriate Higgs-couplings) diverge at most logarithmically.

- The cross-sections of the three-gauge-boson-production processes are very sensitive to variations of the free parameter. At the NLC energy of $\sqrt{s} = 500$ GeV variations of $\kappa$ at the per-cent level lead to measurable differences from the standard model. Measurement of the polarisation of the final vector bosons yields only slightly stricter limits on $\kappa$.

- Although $\kappa$ can be determined in $e^+e^- \rightarrow W^+W^-$, even with stricter parameter limits due to better statistics (see Table 4.2), this does not imply the structure of the quadrilinear couplings. Indeed, the agreement of the values of $\kappa$ deduced from $e^+e^- \rightarrow W^+W^-$ and $e^+e^- \rightarrow W^+W^-Z,\gamma$ measurements is essential for establishing the full non-Abelian Yang–Mills structure.
In summary: Future $e^+e^-$ colliders at 500 GeV energy or more will supply good empirical possibilities to explore the self interactions of the vector bosons and to test the Yang–Mills structure of the electroweak standard model.

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Figure Captions

Figure 1: Feynman diagrams for three-vector-boson production

Figure 2: Total cross-sections for the reactions (a) $e^+e^- \rightarrow W^+W^-Z$ and (b) $e^+e^- \rightarrow W^+W^-\gamma$ as a function of the CM-energy for different values of $\kappa$. The solid lines show the results for the standard-model value of $\kappa = 1$.

Figure 3: Cross-sections for the production of (a) exclusively transversely polarized vector bosons $e^+e^- \rightarrow W^+_T W^-_T Z_T$ and (b) exclusively longitudinally polarized vector bosons $e^+e^- \rightarrow W^+_L W^-_L Z_L$ as a function of $\sqrt{s}$ for different $\kappa$.

Figure 4: Total cross-sections for (a) $e^+e^- \rightarrow W^+W^-Z$, (b) $e^+e^- \rightarrow W^+W^-\gamma$ and (c) $e^+e^- \rightarrow W^+_L W^-_L Z_L$ for fixed $\sqrt{s} = 500$ GeV and $\sqrt{s} = 2000$ GeV as a function of $\kappa$ (solid lines). The dashed lines border the regions where the results agree with the standard model value within the estimated experimental error.