Experimental demonstration of tripartite entanglement and controlled dense coding for continuous variables

Jietai Jing, Jing Zhang, Ying Yan, Fagang Zhao, Changde Xie, Kunchi Peng

The State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan, 030006, P.R.China

A tripartite entangled state of bright optical field is experimentally produced using an Einstein-Podolsky-Rosen entangled state for continuous variables and linear optics. The controlled dense coding among a sender, a receiver and a controller is demonstrated by exploiting the tripartite entanglement. The obtained three-mode position correlation and relative momentum correlation between the sender and the receiver and thus the improvements of the measured signal to noise ratios of amplitude and phase signals with respect to the shot noise limit are 3.28dB and 3.18dB respectively. If the mean photon number $\bar{n}$ equals 11 the channel capacity can be controllably inverted between 2.91 and 3.14. When $\bar{n}$ is larger than 1 and 10.52 the channel capacities of the controlled dense coding exceed the ideal single channel capacities of coherent and squeezed state light communication.

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Quantum entanglement shared by more than two parties is the essential base for developing quantum communication network and quantum computation. The three-particle entangled states for discrete variables, called also Greenberger-Horne-Zeilinger (GHZ) states, have been proposed and then experimentally realized with optical system consisting of nonlinear ($\chi^2$) crystal, pulse laser and linear optical elements and with nuclear magnetic resonance. The controlled dense coding for discrete variables using a three-particle entangled state has been proposed. Recently, under the motivation of the successful experiments on continuous-variable quantum teleportation and quantum dense coding, the schemes demonstrating quantum teleportation and controlled dense coding for quantum variables with a continuous spectrum using multipartite entanglement have been theoretically proposed. So far to the best of our knowledges, the experimental report on the generation of multipartite entangled states for continuous variables and its application has not been presented.

In this paper we report the first experimental demonstration of quantum entanglement among more than two quantum systems with continuous spectra. The tripartite entangled state is produced by distributing a two-mode squeezed state light to three parties using linear optics. The obtained tripartite entangled optical beams are distributed to a sender (Alice), a receiver (Bob) and a controller (Claire) respectively. The information transmission capacity of the quantum channel between Alice and Bob is controlled by Claire. The channel capacity accomplished under Claire’s help is always larger than that without his help. For the large mean photon number ($\bar{n} > 10.52$), the channel capacity of the controlled dense coding communication exceeds that of ideal squeezed state communication.

Fig.1 is the schematic of the experimental setup for tripartite entanglement generation and controlled dense coding. A semimonolithic nondegenerate optical parameter amplifier (NOPA) involving an $\alpha$–cut type-II KTP crystal and pumped by an intracavity frequency-doubled and frequency-stabilized Nd:YAP/KTP laser serves as the initial bipartite entanglement source. The configuration and operation principle of this source have been detailedly described in our previous publications. The output optical modes with horizontal and vertical polarizations from the NOPA operating at deamplification, $\hat{b}_1$ and $\hat{b}_2$, are a pair of bright Einstein-Podolsky-Rosen(EPR) entangled beams with anticorrelated amplitude quadratures and correlated phase quadratures. The polarizations of $\hat{b}_1$ and $\hat{b}_2$ are rotated by a half-wave plate ($\lambda/2$) the optical axis of which is in $\theta = 45^\circ - \frac{1}{2}\arcsin(1/\sqrt{6})$ relative to the horizontal direction, then the beams pass through a polarizing-beam-splitter (PBS) with horizontal and vertical polarizations. The output beam $\hat{b}_2$ is splitted again by a 50/50 beam-splitter consisting of a half-wave plate ($\lambda/2$) and a PBS to modes $\hat{c}_2$ and $\hat{c}_3$. In Ref.[8] we have proved theoretically that the modes $\hat{c}_1$, $\hat{c}_2$ and $\hat{c}_3$ are in a tripartite entangled state which is a "three-mode position eigenstate" with the quantum correlations of total position quadratures ($\hat{X}_{\hat{c}_1}$, $\hat{X}_{\hat{c}_2}$ and $\hat{X}_{\hat{c}_3}$) and relative momentum quadratures ($\hat{Y}_{\hat{c}_1}$, $\hat{Y}_{\hat{c}_2}$ and $\hat{Y}_{\hat{c}_3}$) (see Fig.4 of Ref.8 for the case of $r_2 = 0$). The outgoing tripartite entangled state for continuous variables, a bright "GHZ-like" state is utilized to implement the controlled dense coding.

The entangled beams $\hat{c}_1$, $\hat{c}_2$ and $\hat{c}_3$ are sent to Alice, Bob and Clarie, respectively. Alice modulates two sets of classical signals, which she wants to send to Bob, on the amplitude and phase quadratures of her mode $\hat{c}_1$ by amplitude and phase modulators AM and PM. For example, a specific encoding scheme of binary pulse code modulation, in which the data are independently encoded as two trains of 1 and 0 pulse signals at some radio frequency (rf) on the amplitude and phase quadratures, can be applied. The modulations on mode $\hat{c}_1$ lead to a displacement of $a_s$:

$$\hat{c}_1 = \hat{c}_1 + a_s, \quad (1)$$

where $a_s = X_s + iY_s$ is the sent signal via the quan-
tum channel. The outgoing mode \( \tilde{c}_1 \) is sent to Bob who imposes a phase difference of \( \pi/2 \) between \( \tilde{c}_1 \) and himself mode \( \tilde{c}_2 \) with a phase-shifter (PS), and then combines the two modes on a 50/50 beam-splitter consisting of two PBS and a 1/2 wave-plate. The two bright output beams from BS2 are directly detected by photodiodes D1 and D2. The each photocurrent of D1 and D2 is divided into two parts with power splitters RF1 and RF2. Through analogous calculation with that used in Ref.[8] but taking into account the imperfect detection efficiency of the detectors (\( \eta < 1 \) for D1, D2 and D3) and the nonzero losses of optical systems (\( \xi_1 \neq 0 \) for \( \tilde{c}_1 \) and \( \tilde{c}_2 \), \( \xi_2 \neq 0 \) for \( \tilde{c}_3 \)) the noise power spectra of the sum and difference photocurrents are expressed by [10]:

\[
\langle \delta s_{\tilde{c}_1}^2 \rangle = \frac{\eta_1^2 \xi_1^2 (e^{2r} + 4e^{-2r} - 4) \eta_1^2 \xi_2^2}{4} + \frac{V_X}{2},
\]

\[
\langle \delta s_{\tilde{c}_2}^2 \rangle = \frac{\eta_2^2 \xi_1^2 (e^{2r} + 4e^{-2r} - 4) \eta_1^2 \xi_2^2}{4} + \frac{V_Y}{2},
\]

where \( r \) is the squeezing parameter of the EPR beams (0 \( \leq r < \infty \)), \( V_X \) and \( V_Y \) are the fluctuation variances of the modulated signals (\( X_s, Y_s \)). We can see from Eq.(2), for \( r > 0 \), the quantum fluctuation \( \langle \delta s_{\tilde{c}_1}^2 \rangle \) is always larger than \( \langle \delta s_{\tilde{c}_2}^2 \rangle \), and the larger the \( r \) is, the bigger the deviation between \( \langle \delta s_{\tilde{c}_1}^2 \rangle \) and \( \langle \delta s_{\tilde{c}_2}^2 \rangle \) is. If \( r \rightarrow \infty \), Bob only can gain the phase signal with high accuracy beyond the shot noise limit (SNL), however, he can not gain the amplitude signal that is submerged in large noise. For extracting the amplitude signal Bob have to have the Claire’s result of the amplitude-quadrature detection. Claire detects the amplitude quadrature of mode \( \tilde{c}_1 \) with photodiode D3 and sends the measured photocurrent to Bob. Bob displaces the Claire’s result on the sum photocurrent:

\[
\beta_{\tilde{c}_1} = \hat{\beta}_{\tilde{c}_1} + g\hat{\beta}_{\tilde{c}_2} = \frac{\eta_1^2 \xi_1^2 (\hat{X}_{\tilde{c}_1} + \hat{X}_{\tilde{c}_2}) + \eta_1 \sqrt{1 - \xi_1^2} (\hat{X}_{\tilde{c}_1})}{2} + \frac{\eta_1 \sqrt{1 - \xi_1^2} \hat{Y}_{\tilde{c}_1}}{2} + \frac{\eta_1 \sqrt{1 - \xi_1^2} \hat{Y}_{\tilde{c}_2}}{2} + \frac{\eta_1 \sqrt{1 - \xi_1^2} \hat{Y}_{\tilde{c}_3}}{2} + \frac{\eta_1 \sqrt{1 - \xi_1^2} \hat{Y}_{\tilde{c}_4}}{2} + \frac{\eta_1 \sqrt{1 - \xi_1^2} \hat{Y}_{\tilde{c}_1} \hat{Y}_{\tilde{c}_2}}{2}.
\]

Eq.(4) shows that Bob can also gain the amplitude signal with a sensitivity beyond SNL under the help of Claire.

Fig.2(a) is the measured noise power spectra of the amplitude sums \( \langle \delta s_{\tilde{c}_1}^2 \rangle \) (trace 3) and \( \langle \delta s_{\tilde{c}_2}^2 \rangle \) (trace 2). In trace 2 the modulated amplitude signal at 2MHz is submerged in the noise floor of \( \langle \delta s_{\tilde{c}_1}^2 \rangle \) and in trace 3 the modulated signal emerges from the reduced noise floor of \( \langle \delta s_{\tilde{c}_1}^2 \rangle \) due to the help of Claire. The measured noise floor of trace 3 is 1.57dB lower than that of trace 2. After the correction to the electronics noise floor (trace 4) which is \( \sim 7.83dB \) below the SNL (trace 1), the noise reductions of \( (X_{\tilde{c}_1} + X_{\tilde{c}_2} + X_{\tilde{c}_3}) \) and \( (X_{\tilde{c}_1} + X_{\tilde{c}_2}) \) relative to SNL should actually be 3.28dB and 1.19dB. (The difference of 1.57dB should be corrected to 2.09dB). The results definitely confirm the existence of the quantum correlation among three amplitude quadratures of modes \( \tilde{c}_1, \tilde{c}_2, \tilde{c}_3 \). The signal modulated on the phase quadrature at 2MHz is detected by Bob directly [10]. The noise power spectrum of phase quadratures between the pair of \( \tilde{c}_1 \) and \( \tilde{c}_2 \) modes is shown in Fig.2(b). Trace 2 is the measured noise power spectrum of \( (\hat{Y}_{\tilde{c}_1} - \hat{Y}_{\tilde{c}_2}) \) which is 2.66dB below the SNL (trace 1). Accounting for the electronics noise (trace 3), it should be 3.18dB below the SNL actually. Substituting the measured noise power of \( \langle \delta s_{\tilde{c}_1}^2 \rangle, \langle \delta s_{\tilde{c}_2}^2 \rangle \) and \( \langle \delta s_{\tilde{c}_3}^2 \rangle \) from Fig.2 into the Eqs.(2) and (4), we calculate the squeezing parameter \( r_{exp} = 0.674(5.85dB \text{ squeezing after the correction}) \). (The parameters \( \langle \delta s_{\tilde{c}_1}^2 \rangle = 0.76, \langle \delta s_{\tilde{c}_2}^2 \rangle = 0.47, \langle \delta s_{\tilde{c}_3}^2 \rangle = 0.48, \xi_1^2 = 98.7\%, \xi_2^2 = 93.7\%, \eta_1^2 = 95.0\% \) are taken in the calculation according to the experimental values).

Fig.3 is the functions of the normalized fluctuation variances of \( \langle \delta s_{\tilde{c}_1}^2 \rangle, \langle \delta s_{\tilde{c}_2}^2 \rangle \) and \( \langle \delta s_{\tilde{c}_3}^2 \rangle \) versus the squeezing parameter \( r \), where \( \xi_1, \xi_2, \eta_1 = \frac{1}{2} \) are the values for the experimental system. \( \langle \delta s_{\tilde{c}_1}^2 \rangle_{(\text{opt})} \) is the fluctuation variance of the amplitude sum of three modes when the optimal gain \( g_{\text{opt}} \) is applied. We can see, the difference between \( \langle \delta s_{\tilde{c}_1}^2 \rangle_{(\text{opt})} \) and \( \langle \delta s_{\tilde{c}_1}^2 \rangle \) is quite small (0.035) for the experimental squeezing \( r_{exp} = 0.674 \) and the difference tends to zero for larger \( r \). \( \langle \delta s_{\tilde{c}_1}^2 \rangle \) is smaller than \( \langle \delta s_{\tilde{c}_2}^2 \rangle \) and increasing \( r \), \( \langle \delta s_{\tilde{c}_1}^2 \rangle \) increases, however \( \langle \delta s_{\tilde{c}_1}^2 \rangle \) decreases. The results clearly exhibit the tripar-
tite entanglement among $\tilde{c}_1$, $\tilde{c}_2$ and $\tilde{c}_3$ modes. When $r_{\text{exp}} = 0.674$, $\langle \hat{\delta}^2 \hat{I}_+ \rangle$ is 0.29 lower than $\langle \hat{\delta}^2 \hat{I}_+ \rangle$. As shown in Eq.(2) and Fig.(3), generally, the noise floor of amplitude signal is not equal to that of phase signal, which is because the discussed state is not a maximally GHZ state and decoding amplitude signal must have the aid of Claire’s classical information and phase signal only needs the joint measurement of $\tilde{c}_1$ and $\tilde{c}_2$ modes.

Following the theoretical calculations on the quantum channel capacity for dense coding in Ref.[8][11][12] we calculate the channel capacity of the presented experimental system. The channel capacities with and without Claire’s help can be deduced from Eqs.(2) and (4):

$$C_{n-c}^{\text{dense}} = \frac{1}{2} \ln \left(1 + \frac{\sigma^2}{\langle \hat{\delta}^2 \hat{I}_- \rangle} \right) \left(1 + \frac{\sigma^2}{\langle \hat{\delta}^2 \hat{I}_+ \rangle} \right)$$

$$C_{c}^{\text{dense}} = \frac{1}{2} \ln \left(1 + \frac{\sigma^2}{\langle \hat{\delta}^2 \hat{I}_- \rangle} \right) \left(1 + \frac{\sigma^2}{\langle \hat{\delta}^2 \hat{I}_+ \rangle} \right)$$

where, $\sigma^2$ is the average value of the signal photon number and the mean photon number per mode $\bar{n} = \sigma^2 + \sinh^2 r$. The dependences of the channel capacities for ideal single mode coherent state ($C_{\text{ch}} = \ln(1+ \bar{n})$) and squeezing state ($C_{sq} = \ln(1+ 2\bar{n})$) on the mean photon number $\bar{n}$ are given in Fig.4 to compare with that of controlled dense coding with ($C_{\text{dense}}$) and without ($C_{n-c}^{\text{dense}}$) Claire’s help according to Eq.5 and taking the experimental parameters. For the given squeezing ($r_{\text{exp}} = 0.674$), when the mean photon number $\bar{n}$ is larger than 1.00(1.31) and 10.52 the channel capacities of the controlled dense coding with(without) the help of Claire exceed that of coherent state and squeezed state communication. By increasing the average signal photon number $\sigma^2$, the channel capacity of quantum dense coding can be improved. The channel capacity with the help of Claire ($C_{c}^{\text{dense}}$) is always larger than that without his help ($C_{n-c}^{\text{dense}}$) which is just the result of three-partite entanglement. For example, when $\bar{n} = 11$, the channel capacity of the presented system can be controllably inverted between 2.91 and 3.14.

In conclusion, we experimentally obtained bright tripartite entangled state light and accomplished the quantum controlled dense coding for the continuous variables. We deduced the formulae designating the tripartite entanglement among amplitude and phase quadratures of three modes in the case of accounting for the influences of imperfect detection efficiency and the losses of optical system and using gain $g = \frac{1}{\sqrt{2}}$. The experiment shows that using the limited squeezing the channel capacity of the controlled dense coding can exceed that of coherent state and squeezed state communication when the signal photon number is larger than a certain value. The nature technique of optical parametric amplification, the simple linear optical system and the direct measurement for Bell state are used in the presented scheme, thus the experimental implementation is significantly simplified. The presented schemes generating tripartite entanglement and achieving controlled dense coding are valuable for developing future information network of quantum systems with continuous spectra.

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Caption
Fig.1 Experimental setup for tripartite entanglement generation and controlled dense coding.
Fig.2(a) The noise power spectra of amplitude sums $\langle \hat{\delta}^2 \hat{I}_+ \rangle$ (trace 3) and $\langle \hat{\delta}^2 \hat{I}_- \rangle$ (trace 4); Fig.2(b) The noise power spectra of phase difference $\langle \hat{\delta} \hat{I}_- \rangle$ (trace 2), trace 1—shot noise limit(SNL), trace 4—Electronics noise level(ENL), measured frequency range: 1.5MHz-2.5MHz, resolution bandwidth 30KHz, video bandwidth 0.1kHz.
Fig.3 The variances of amplitude sums $\langle \hat{\delta}^2 \hat{I}_+ \rangle$ and phase difference $\langle \hat{\delta} \hat{I}_- \rangle$ versus the squeezing parameter $r$ with beam propagation efficiency $\xi_2 = 98.7\%$, $\xi_3 = 93.7\%$, and the quantum efficiency of detector $\eta^2 = 95.0\%$. 

3
Fig. 4 Channel Capacities for the controlled dense coding with \( C_{\text{dense}} \) and without \( C_{n-c} \) Claire’s help, single-mode coherent state with heterodyne detection, and squeezed state \( C_{sq} \) communication. The parameters are same with Fig. 3.
Fig. 1
Fig. 2
\[ r_{\text{exp}} = 0.674 \]

\[
\begin{align*}
\langle \delta^2 i_+ \rangle \\
\langle \delta^2 i_- \rangle \\
\langle \delta^2 i'_+ \rangle \\
\langle \delta^2 i'_- \rangle \\
\end{align*}
\]

Fig. 3
