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PAM-4 Eye-Opening Monitor Technique Using Gaussian Mixture Model for Adaptive Equalization*

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SUMMARY  To meet the increasing demand for high-speed communication in VLSI (very large-scale integration) systems, next-generation high-speed data transmission standards (e.g., IEEE 802.3bs and PCIe 6.0) will adopt four-level pulse amplitude modulation (PAM-4) for data coding. Although PAM-4 is spectrally efficient to mitigate inter-symbol interference caused by bandwidth-limited wired channels, it is more sensitive than conventional non-return-to-zero line coding. To evaluate the received signal quality when using adaptive coefficient settings for a PAM-4 equalizer during data transmission, we propose an eye-opening monitor technique based on machine learning. The proposed technique uses a Gaussian mixture model to classify the received PAM-4 symbols. Simulation and experimental results demonstrate the feasibility of adaptive equalization for PAM-4 coding.

key words: multi-valued signaling, PAM-4, eye-opening monitor, machine learning, Gaussian mixture model

1. Introduction

The data rates through wirelines in VLSI (very large-scale integration) systems have steadily increased to meet the requirements of aggregate input/output bandwidth for chip-to-chip links, backplanes, and data-center transmission. However, signal distortion due to inter-symbol interference (ISI) caused by the finite bandwidth of wired channels, reflection, and crosstalk substantially limits the input/output bandwidth and the overall VLSI system performance.

To reduce the signal bandwidth, four-level pulse amplitude modulation (PAM-4) signaling can be employed. This coding scheme transmits two binary non-return-to-zero bits (00, 01, 10, 11) using one symbol (0, 1, 2, 3) at half the symbol rate. Thus, the spectrally efficient PAM-4 can halve the channel bandwidth limitation compared with non-return-to-zero signaling. By conforming to next-generation high-speed data transmission standards such as IEEE 802.3bs, PAM-4 is a promising approach to support 400 Gb/s data transmission (i.e., 8-lane transmission at 50 Gb/s) [1].

Although PAM-4 allows data transmission at the same baud rate using half of the symbol rate, its four-level signaling is three times more sensitive (1/3 = −9.5 dB) to the noise amplitude than non-return-to-zero signaling. Therefore, PAM-4 requires a sophisticated waveform shaping technique using an equalizer and a four-level symbol detection circuit. A feedforward equalizer (FFE), pre-emphasis, continuous-time linear equalizer (CTLE) and decision feedback equalizer (DFE) are typical signal processing techniques [2], [3] to mitigate ISI. As an FFE can be implemented as a digital circuit, its parameters can be adjusted according to the transmission line characteristics. However, actual transmission line characteristics are complex and difficult to determine due to the effects of adjacent wiring and peripheral components such as connectors. Moreover, the transmission environment may vary over time. The conventional eye-opening monitor (EOM) technique [4], [5] allows to evaluate the quality of received signals under adaptive equalizer parameters to handle these difficulties. However, this technique fails to suitably operate when the eye is entirely closed.

We propose a machine learning method for statistical symbol detection of PAM-4 signals under adaptive equalizer parameters. The parameter optimization for unknown transmission characteristics can be formulated as an unsupervised learning problem. Specifically, the PAM-4 symbols correspond to four Gaussian distributions at the receiver side, and these distributions are mixed due to ISI. Thus, we adopt an EOM technique using a Gaussian mixture model (GMM) to estimate the effect of waveform distortion [6]. Then, GMM fitting can be used to correctly classify the mixed distributions of each transmitter PAM-4 symbol. Moreover, classification enables tuning the equalizer parameters to eliminate the ISI effects even if the eye diagram is closed and/or the transmitter presents nonlinearities.

The remainder of this paper is organized as follows. In Sect. 2, the effects of waveform distortion are explained, and a waveform shaping circuitry using analog/digital signal processing is adopted to suppress ISI. In Sect. 3, the proposed GMM-based EOM technique is introduced. In Sect. 4, the effectiveness of the proposed technique is verified, and the Bayesian information criterion (BIC) is used to obtain the likelihood of the proposed GMM fitting. To demonstrate the feasibility of the proposed method, we report the simulation results of transmitter nonlinear effect estimation and equalization adjustment using GMM fitting in Sect. 5. Finally, we summarize the main findings of this study in Sect. 6.

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2. Waveform Shaping

2.1 Waveform Distortion during High-Speed Data Transmission

Eye diagrams are widely used to evaluate the transmission characteristics of a wireline. These diagrams allow to measure the quality of signals received through serial links. Figure 1 shows the frequency characteristics of the S21 parameters of microstrip lines (MSLs) on an FR4 substrate fabricated for experimental evaluation and the corresponding PAM-4 eye diagrams. As frequency increases, attenuation also increases, resembling a lowpass filter. In addition, attenuation also increases with the length of the transmission line. Wave distortion narrows the eye opening according to the frequency-dependent insertion loss. At 4 Gb/s (−8.29 dB at 1 GHz), the PAM-4 eye diagram completely closes.

The measurement setup used in this study to obtain the eye diagrams is shown in Fig. 2. Figures 3 a and 3 b show the measured PAM-4 eye diagrams and corresponding histograms of the symbol distributions along the vertical region at 2 and 4 Gb/s, respectively, for a 1 m MSL. When the eye diagram is open (Fig. 3 a), the vertical histogram present four distinct and separate distributions. Each symbol follows a Gaussian distribution due to ISI. For a higher data rate (Fig. 3 b), the ISI increases, thus broadening the Gaussian distribution of each symbol. Consequently, the four Gaussian distributions overlap, hindering symbol detection and impeding eye diagram evaluation based on the eye height and width. Thus, the conventional EOM technique cannot be used to distinguish symbols from histograms with overlapping distributions, as shown in Fig. 3 b.

2.2 Waveform Shaping Using Analog/Digital Signal Processing

Besides PAM-4 coding, signal processing techniques are required to compensate for the signal deterioration in high-speed serial links. An FFE at the transmitter boosts the highfrequency components of the signal to counteract the lowpass channel characteristic. Figure 4 a illustrates a conventional FFE that can be implemented by digital filter circuits comprising adders, multipliers, and delay circuits at symbol rate \( T_s \). The operating frequency and number of taps determine the performance of waveform shaping. Figures 4 b and 4 c show eye diagrams obtained from simulation of received PAM-4 signals without and with a 3-tap FFE over a 1 m MSL at 4 Gb/s. The FFE allows to eliminate ISI, resulting in an open eye diagram.
Although the FFE can suppress ISI, modifying the eye shape is challenging, especially when aiming to improve the complicated rising and falling edges of PAM-4 signals. Double-rate equalization employing fractional delay \( \frac{1}{2} T_s \) can expand the PAM-4 eye, and the eye shape can be more flexibly controlled compared with a conventional FFE by doubling the symbol rate [7]–[9]. However, the equalizer requires a high-speed operation that increases the hardware costs due to the implementation of high-frequency circuits.

To reduce the required operation speed, we propose an analog/digital hybrid FFE transmitter [10], whose diagram is shown in Fig. 5a. Transmitter signals are generated by subtracting the FFE output signals from the PAM-4 input signals. Then, the PAM-4 input passes through digital-to-analog converter DAC1, and the FFE data at \( T_s \) pass through digital-to-analog converter DAC2. The output signal is generated by subtracting the output signals with delay \( T_s/2 \) using an analog subtractor. Although the proposed circuit requires two digital-to-analog converters, it can reduce operation frequency by half. In addition, DAC1 only needs a 2-bit resolution. Therefore, at speeds beyond gigabits per second, the half-frequency operation can reduce the hardware cost compared with the conventional double-rate circuit.

The proposed double-rate FFE transmitter can improve the rising and falling edges of the received signal to increase the eye width compared with the conventional FFE (Figs. 5b and 5c). However, if the transmission characteristics are not available, the FFE cannot effectively suppress ISI. Moreover, to handle process, voltage, and temperature variations as well as changes in the channel configuration, adaptive FFE parameter adjustment should be implemented.

### 3. Proposed EOM Technique

EOM techniques can be used to evaluate the quality of received signals after FFE parameter adjustment. However, as PAM-4 has three eyes, the corresponding EOM algorithm is complicated. Furthermore, if the initial eye is entirely closed, the EOM algorithm fails to operate correctly.

The estimation of received symbols under unknown transmission characteristics resembles unsupervised learning in machine learning. Therefore, by applying GMM fitting for estimating symbol distributions for an unknown transmission line, we propose a novel PAM-4 EOM technique based on statistical classification.

#### 3.1 Overview of Gaussian Mixture Model

In probability and statistics, the linear summation of several probability distributions establishes a mixture distribution. In particular, the linear summation of multiple Gaussian distributions can be modeled as a GMM [11], which is expressed as

\[
p(x) = \sum_{k=0}^{K-1} \pi_k N(x | \mu_k, \sigma_k)
\]

where \( N(x | \mu, \sigma) \) follows a Gaussian distribution with mean \( \mu \) and standard deviation \( \sigma \), and \( \pi_k \) is a mixing coefficient that weights each Gaussian distribution. This coefficient is normalized as follows:

\[
\sum_{k=0}^{K-1} \pi_k = 1
\]

A GMM is a probabilistic model that assumes that all data points are generated from a mixture of Gaussian distributions with unknown parameters. By fitting the GMM from the distribution of observations, we can estimate the mean and standard deviation of each Gaussian distribution constituting the mixture. Accordingly, the received PAM-4 symbol can be distinguished and estimated. For PAM-4 symbol estimation based on a GMM, as PAM-4 has four symbols, we can assume that the received symbols correspond to the following mixture of four Gaussian distributions:

\[
p(x) = \pi_0 N(x | \mu_0, \sigma_0) + \pi_1 N(x | \mu_1, \sigma_1) + \pi_2 N(x | \mu_2, \sigma_2) + \pi_3 N(x | \mu_3, \sigma_3)
\]

where \( \mu_i \) and \( \sigma_i \) are the mean and standard deviation of the distribution for symbol \( i \) (\( i = 0, 1, 2, 3 \)).

We use the expectation–maximization algorithm to estimate the mean, standard deviation, and weight of each Gaussian distribution. This algorithm iteratively calculates the solution that maximizes the likelihood of the observations through the following procedure:

1. Initial value setting
2. Expectation step
3. Maximization step
4. Go to step 2 until convergence

Generally, the initial value is set by using k-means clustering, which provides appropriate initial groups from the observations. Then, steps 2–4 are repeated until convergence.
3.2 GMM-Based EOM Technique for Received PAM-4 Signals

Figure 6 illustrates the proposed GMM-based EOM technique for PAM-4. To evaluate the eye-opening ratio at the receiver, the histogram of the obtained PAM-4 signals is determined. Subsequently, the PAM-4 transmitter symbol distributions are assumed to be Gaussian due to ISI, and the distributions are clustered assuming a GMM.

To evaluate the eye diagram, the histograms are obtained from observations over a period. In PAM-4 data transmission, the mean ($\mu_0$, $\mu_1$, $\mu_2$, and $\mu_3$) and standard deviation ($\sigma_0$, $\sigma_1$, $\sigma_2$, and $\sigma_3$) of each symbol can be estimated by fitting the symbol distributions into four Gaussian distributions from the observations at the receiver. Hence, we can evaluate the ISI effects on each symbol at the receiver, even if measuring the eye aperture rate is difficult. Using the proposed GMM evaluation, the level of each symbol at the receiver can be estimated from its mean ($\mu_0$~$\mu_3$). Moreover, the ISI effects on each symbol can be estimated from the standard deviation ($\sigma_0$~$\sigma_3$). If ISI increases and the received signals are distorted, the standard deviation of the GMM also increases.

4. Evaluation of Proposed EOM Technique

4.1 Simulation and Experimental Results from PAM-4 Using GMM-Based EOM Technique

Figure 7 shows simulated eye diagrams for 2 Gb/s PAM-4 signal transmission over a 1 m MSL and the histogram of the observations at the receiver. In the simulation, the received signals were recovered using the impulse response, which is determined by the frequency characteristics of the measured MSL S-parameters. The received signals were simulated as the convolution between the transmitter signals and the impulse response, and the eye diagram was constructed using the received signals. In Fig. 7b, the received values are distinguished considering the transmitter symbols, with the histograms of the received signals being colored according to transmitter symbols (0, 1, 2, 3). As shown in the histograms, the 2 Gb/s PAM-4 received signals corresponding to each symbol are spread due to the ISI effect. Although the eye height in the amplitude direction decreases (Fig. 7a), the eye is open because the symbol distributions do not overlap. Thus, it is sufficient to distinguish the symbols using three threshold levels.

Figure 8 shows the results of GMM estimation from the histogram. The bar graphs and curves show the frequency of the received symbols and the results of GMM fitting, respectively. The solid lines indicate the probability density per symbol. For this simulation, we assumed that the ISI effects on each symbol are almost the same, and GMM fitting considers equal standard deviation $\sigma$ for all the distributions. Applying GMM estimation, each symbol from the 2 Gb/s PAM-4 signals can be described by the following Gaussian
distributions: \(N_0(x|\mu_0, \sigma), N_1(x|\mu_1, \sigma), N_2(x|\mu_2, \sigma),\) and \(N_3(x|\mu_3, \sigma).\) The distributions have means \(\mu_0, \mu_1, \mu_2,\) and \(\mu_3\) of \(-0.5815, -0.1966, 0.1741,\) and \(0.5618\) V, respectively, and standard deviation \(\sigma\) of \(0.0443\) V. Using the mean values, we obtain symbol distances \(d_0-1, d_1-2,\) and \(d_2-3\) of \(0.3849, 0.3707,\) and \(0.3877\) V, respectively.

Figure 9 shows the simulation results for 4 Gb/s PAM-4 data transmission. At this transmission speed, the ISI effect is severe, and the eye diagram at the receiver is completely closed (Fig. 9 a). Moreover, the histograms in Fig. 9 b show that the distributions of the transmitter symbols are overlapping due to the closed eye diagram. Although the histograms do not allow to easily distinguish the symbols without information of transmitter signals, symbol classification from the observations is possible by using GMM estimation. Figure 10 shows that GMM fitting can be used to classify each symbol distribution \((N_0(x|\mu_0, \sigma), N_1(x|\mu_1, \sigma), N_2(x|\mu_2, \sigma),\) and \(N_3(x|\mu_3, \sigma))\), and then evaluate the ISI effect when the eye is closed. The fitting provides mean values \(\mu_0, \mu_1, \mu_2,\) and \(\mu_3\) of \(-0.4539, -0.1504, 0.1500,\) and \(0.4489\) V, respectively, a standard deviation \(\sigma\) of \(0.1001\) V, and symbol distances \(d_{0-1}, d_{1-2},\) and \(d_{2-3}\) of \(0.3035, 0.3004,\) and \(0.2989\) V, respectively. Although conventional EOM fails to evaluate the ISI effect when the eye is closed (Fig. 9 a), the proposed method enables such evaluation by using these parameters.

To experimentally evaluate the proposed GMM-based EOM technique for PAM-4, Fig. 11 shows the measured eye diagrams from the symbols in the time domain and corresponding histograms during actual 4 Gb/s PAM-4 transmission. Figures 11 a and 11 b show that the eyes are entirely closed, and it is difficult to distinguish each symbol distribution from the histograms of the observations. Although the eye is entirely closed, each symbol distribution can be estimated using GMM classification. To evaluate the GMM classification, the histogram of received symbols is divided into four histograms, as shown in Fig. 11 c. Each color denotes the frequency of received values according to transmitter symbols \((0, 1, 2, 3).\) As shown in Fig. 11 c,
the estimated curves are approximately equal to those of the symbols. Thus, the proposed GMM-based EOM technique can estimate PAM-4 symbol distributions without transmitter information even when the eye is closed. Therefore, this technique can be used to adjust the parameters of waveform shaping equalizers, such as FFEs and DEFs.

4.2 Evaluation of GMM Fitness for PAM-4

To obtain the likelihood of the GMM-based evaluation of PAM-4 received signals, we employed the BIC (Bayesian information criterion), which reflects the probability of the true model. Figure 12 shows the results of the GMM-based evaluation and BIC according to the number of components (i.e., Gaussian distributions) for 4 Gb/s PAM-4 data transmission. The GMM fits the distributions of each PAM-4 symbol to a distribution at 4 Gb/s (Fig. 12 a). Therefore, the BIC for four components is the optimum (minimum) compared with the BIC for other number of components (Fig. 12 b). Hence, a GMM with four Gaussian distributions is the most correct representation for the PAM-4 observations, and the GMM allows to accurately estimate the PAM-4 symbol distributions. On the other hand, for severe ISI, the GMM-based EOM technique fails to correctly estimate the distributions of received PAM-4 symbols at 8 Gb/s, as shown in Fig. 13 a. Moreover, the BIC for four components is not the optimum in this case, as shown in Fig. 13 b, due to the highly overlapping symbol distributions.

5. Applications of Proposed EOM Technique

We applied the proposed GMM-based EOM technique to transmitter nonlinearity estimation and adaptive equalization.
to evaluate from the histogram without information for symbol identification at the receiver.

Figure 16 shows the result of the proposed GMM-based EOM technique depicting each Gaussian distribution and the histogram of each transmitter symbol. The GMM fitting allows to estimate the four Gaussian distributions reflecting the unbalanced symbol histograms with mean values $\mu_0, \mu_1, \mu_2,$ and $\mu_3$ of $-0.4384, -0.0398, 0.3036,$ and $0.4957$ V, respectively, a standard deviation $\sigma$ of $0.0934$ V, and symbol distances $d_{0-1}, d_{1-2},$ and $d_{2-3}$ of $0.3986, 0.3434,$ and $0.1921$ V, respectively. Therefore, in the proposed EOM technique, the nonlinear effect is correctly estimated as non-uniform symbol distances without training patterns available.

### 5.2 Adaptive Waveform Equalization Adjustment Using GMM Fitting

We also applied the proposed GMM-based EOM technique for parameter adjustment in the initial setting of a waveform shaping circuit without using training signals. For verification, we set the parameters of the double-rate FFE illustrated in Fig. 17 considering that neither information on transmission data nor transmission characteristics were available.

For this experiment, we assumed an FFE with two taps and two adjustment parameters and used the COBYLA algorithm from the Python SciPy library [12] for parameter optimization. The objective function for parameter optimization sets standard deviation $\sigma$, which is correlated to the ISI effects. Standard deviation $\sigma$ was calculated from 500 data points of a sampling signal by using the GMM. The ISI effect was evaluated by assuming that the standard deviations of the symbols ($\sigma_0$–$\sigma_3$) were equal.

To evaluate the EOM rate, we adopted sampling-point control. As shown in Fig. 18, the received symbols are asynchronously sampled on timing for optimization, randomly varying in $-40$% to $+20$% unit interval from the judgment timing. Then, the optimization algorithm searches the parameters that minimize the standard deviation to consequently minimize the ISI effect.

Figure 19 shows the eye diagrams at different parameter adjustment results for 4 Gb/s PAM-4 data transmission.

The optimization algorithm searches for the set of parameters that minimizes the objective function. The optimal value minimizes the intra-symbol variation due to the ISI effect at the receiver for the received symbols. Therefore, it is possible to realize adaptive waveform shaping without a training pattern for optimization.

The results confirm that the EOM ratio improves with the decreasing value of the objective function by conducting iterative GMM fitting. Even if the initial eye diagram is entirely closed, parameter adjustment improves the eye opening. Figure 20 shows the measured eye diagram for 4 Gb/s
PAM-4 data transmission using the optimal FFE parameters. In the eye diagram, the optimized double-rate FFE mitigates ISI, with the four-valued signal eyes being open to 58 % unit interval.

6. Conclusions

This paper presents a novel PAM-4 EOM technique based on GMM fitting. GMM-based classification is used to optimize the FFE parameters for unknown transmission characteristics without requiring training signals. Moreover, by GMM fitting, the PAM-4 EOM technique allows to estimate the ISI effects even when the eye diagram is closed. Therefore, the proposed EOM technique allows to optimally adjust the parameters of waveform shaping circuits, even when the channel conditions are not available. In addition, suitable EOM circuitry can be implemented with an optimized design.

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