A periodic table for the excited $N^*$ and $\Delta^*$ spectrum in a relativistic chiral quark model

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A possibility of the construction of a periodic table for the excited baryon spectrum is shown in the frame of a relativistic chiral quark model based on selection rules derived from the one-pion exchange mechanism. It is shown that all the $N^*$ and $\Delta^*$ resonances appearing in the $\pi N$ scattering data and strongly coupling to the $\pi N$ channel are identified with the orbital configurations \((1S_{1/2})^2(nlj)\). Baryon resonances corresponding to the orbital configuration with two valence quarks in excited states couple strongly to the $\pi\pi N$-channel, but not to the $\pi N$ channel.

At low energy scale up to 2000 MeV, the obtained numerical estimations for the SU(2) baryon states (up to and including F-wave $N^*$ and $\Delta^*$ resonances) within the schematic periodic table are mostly consistent with the experimental data. It is argued that due to the overestimation of the ground state $N(939)$ and Roper resonance $N(1440)$ almost by the same amount and that the Roper resonance is a radial excitation of the $N(939)$, the "lowering mechanism" for the both baryon states must be the same. The same mechanism is expected in the $\Delta$ sector. At higher energies, where the experimental data are poor, we can extend our model schematically and predict seven new $N^*$ and four $\Delta^*$ resonances with larger spin values.

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I. INTRODUCTION

It is believed during the long period that the baryon spectrum can be described with a good accuracy in the Constituent Quark Models (CQM) based either on the Goldstone-boson exchange (GBE) \cite{1}, or one-gluon exchange (OGE) \cite{2} or (and) instanton induced exchange (IIE) \cite{3} mechanisms between non-relativistic constituent quarks. However, there are still some serious problems which cannot be avoided until now. The most important issue is the problem of "missing resonances": the CQMs predicted too many states even at low energies which are not observed at the experimental facilities \cite{4,5}. The situation is so serious, that "a modern view questions the usefulness of quarks to describe the nucleon excitation spectrum" \cite{6}.

Fortunately, there are also strong optimistic views on the problem. The recent review \cite{5} shows all the difficulties in baryon spectroscopy and concluded that the all photo-, pion- and hadron-induced reactions will be important to understand the excited baryon spectroscopy. From the theoretical side, essential developments are being done in the Lattice QCD \cite{7,8}, Dyson-Schwinger equations \cite{9}, effective field-theoretical methods \cite{10,12} and in the theoretical coupled channel approaches to meson-baryon scattering within the Juelich \cite{13} and Sato-Lee \cite{14} models.

In \cite{15,16,17} we have developed a relativistic chiral quark model \cite{18} for the lower excitation spectrum of the nucleon and delta. For the first time the mass values of the Roper resonance and lowest negative parity nucleon and delta resonances have been estimated in a relativistic chiral quark model. The splitting of the Roper resonance from the $N(939)$ was reproduced with a reasonable accuracy.

The aim of this paper is to show that in a way to extend the relativistic chiral quark model to the higher excitation spectrum, it is possible to construct a periodic table for the excited $N^*$ and $\Delta^*$ states based on the one-pion exchange mechanism between valence quarks. In the model the SU(2)-baryons (Nucleon and $\Delta$ states) are assumed to correspond to the orbital configuration \((1S_{1/2})^2(nlj)\) with two $S$-valence quarks and a single valence quark in the excited state. We will prove that such baryon states are identified with all the $N^*$ and $\Delta^*$ resonances appearing in the $\pi N$ scattering process. Then we can compare our theoretical construction with the experimental data. Our analysis is common for all relativistic chiral quark models describing the baryons as bound states of three valence quarks with a Dirac two-component structure and surrounded by the cloud of $\pi$-mesons, as required by the chiral symmetry \cite{15,23}.
II. MODEL

The effective Lagrangian of our model $\mathcal{L}(x)$ contains the quark core part $\mathcal{L}_Q(x)$, the quark-pion $\mathcal{L}_{q\pi}^I(x)$ interaction part, and the kinetic parts for the pion $\mathcal{L}_\pi(x)$:

$$\mathcal{L}(x) = \mathcal{L}_Q(x) + \mathcal{L}_{q\pi}^I(x) + \mathcal{L}_\pi(x)$$

where $c$ and $m$ are constants. The quark-pion interaction $\mathcal{L}_{q\pi}^I(x)$ is given by

$$\mathcal{L}_{q\pi}^I(x) = \bar{\psi}(x)[i \not\partial - S(r) - \gamma^0V(r)]\psi(x) - 1/f_\pi\bar{\psi}(x)\gamma^5\tau^i\phi_i\psi(x) + \frac{1}{2}(|\partial_\mu\phi_i|^2 - m_\pi\phi_i^2].$$

(1)

Here, $\psi(x)$ and $\phi_i, i = 1, 2, 3$ are the quark and pion fields, respectively. The matrices $\tau^i(i = 1, 2, 3)$ are for the isospin. The pion decay constant $f_\pi = 93$ MeV. The scalar part of the static confinement potential is given by

$$S(r) = cr + m$$

(2)

where $c$ and $m$ are constants.

At short distances, transverse fluctuations of the string are dominating, with some indication that they transform like the time component of the Lorentz vector. They are given by a Coulomb type vector potential as

$$V(r) = -\alpha/r$$

(3)

where $\alpha$ is approximated by a constant. The quark fields are obtained from solving the Dirac equation with the corresponding scalar plus vector potentials

$$[i\gamma^\mu\partial_\mu - S(r) - \gamma^0V(r)]\psi(x) = 0$$

(4)

The respective positive and negative energy eigenstates as solutions to the Dirac equation with a spherically symmetric mean field, are given in a general form as

$$u_\alpha(x) = \left(\begin{array}{c} g_{N\kappa}^+(r) \\ -if_{N\kappa}^-(r) \hat{x} \end{array}\right) \mathcal{Y}_{\kappa}^{m_j}(\hat{x}) \chi_m \chi_{m_c} e^{-iE_\alpha t}$$

(5)

$$v_\beta(x) = \left(\begin{array}{c} g_{N\kappa}^-(r) \\ if_{N\kappa}^+(r) \hat{x} \end{array}\right) \mathcal{Y}_{\kappa}^{m_j}(\hat{x}) \chi_m \chi_{m_c} e^{iE_\beta t}$$

(6)

The quark and anti-quark eigenstates $u$ and $v$ are labeled by the radial, angular, azimuthal, isospin and color quantum numbers $N, \kappa, m, m_1, m_2$ and $m_c$, which are collectively denoted by $\alpha$ and $\beta$, respectively. The spin-angular part of the quark field operators

$$\mathcal{Y}_{\kappa}^{m_j}(\hat{x}) = [Y_i(\hat{x}) \otimes \chi_{1/2}]_{m_j} = |\kappa| - 1/2.$$ (7)

The quark fields $\psi$ are expanded over the basis of positive and negative energy eigenstates as

$$\psi(x) = \sum_\alpha u_\alpha(x)b_\alpha + \sum_\beta v_\beta(x)d_\beta^\dagger.$$ (8)

The expansion coefficients $b_\alpha$ and $d_\beta^\dagger$ are operators, which annihilate a quark and create an anti-quark in the orbits $\alpha$ and $\beta$, respectively.

The free pion field operator is expanded over plane wave solutions as

$$\phi_j(x) = (2\pi)^{-3/2}\int \frac{d^3k}{(2\omega_k)^{1/2}} \sum \left[ a_{jk} \exp(-ikx) + a_{jk}^\dagger \exp(ikx)\right]$$

(9)

with the usual destruction and creation operators $a_{jk}$ and $a_{jk}^\dagger$, respectively. The pion energy is defined as $\omega_k = \sqrt{k^2 + m_\pi^2}$. The expansion of the free zero mass gluon field operators is of the same form.

Using the effective Lagrangian and the time-ordered perturbation theory one can develop a calculation scheme for the excitation spectrum of the nucleon and delta. At the zero-th order, the quark core result ($E_Q$) is obtained by
solving Eq. (1) for the single quark system numerically. The corresponding quark core energy is evaluated as the sum of single quark energies:

\[ E_Q = 2E(1S_{1/2}) + E(nlj) \]

with an appropriate correction on the center of mass motion [16]. Since we work in the independent particle model, the bare three-quark state of the SU(2)-flavor baryons is assumed to correspond to the orbital structure \((1S_{1/2})^2(nlj)\) in the non-relativistic spectroscopic notation. This assumption is very natural, since an orbital or radial excitation of two or three valence quarks in the baryon state must be suppressed strongly. However, we remember that a single valence quark state is described with the two-component Dirac wave function. The orbital momenta corresponding to these two components differ by one unity. Further we will see that the two-component Dirac structure of the quark wave function plays a crucial role in the derivation of the selection rules for the baryon state total momentum.

The second order perturbative corrections to the energy spectrum of the SU(2) baryons due to the pion field \((\Delta E^{(\pi)})\) are calculated on the basis of the Gell-Mann and Low theorem:

\[ \Delta E = \sum \langle \Phi_0 | \frac{(-i)^n}{n!} \int i\delta(t_1) \ldots d^4x_n \mathcal{T}[\mathcal{H}_f(x_1) \ldots \mathcal{H}_f(x_n)] | \Phi_0 \rangle \]

with \(n = 2\), where the relevant quark-pion interaction Hamiltonian density is

\[ \mathcal{H}_I^{(\pi)}(x) = \frac{i}{f_\pi} \bar{\psi}(x) \gamma^5 \tau \phi(x) S(r) \psi(x), \]

(10)

The stationary bare three-quark state \(| \Phi_0 \rangle\) is constructed from the vacuum state using the usual creation operators:

\[ | \Phi_0 >_{\alpha\beta\gamma} = b_{\alpha}^\dagger b_{\beta}^\dagger b_{\gamma}^\dagger | 0 >, \]

(11)

where \(\alpha, \beta\) and \(\gamma\) represent the quantum numbers of the single quark states, which are coupled to the respective baryon configuration. The energy shift of Eq. (10) is evaluated up to second order in the quark-pion interaction, and generates self-energy and exchange diagrams contributions.

The self-energy terms contain contribution both from intermediate quark \((E > 0)\) and anti-quark \((E < 0)\) states. These diagrams correspond to the case when a single pion is emitted and absorbed by the same valence quark which is excited to the intermediate quark and anti-quark states. The convergence of the self-energy diagrams was shown explicitly for the lowest \(1S, 2S, 1P_{1/2}\) and \(1P_{3/2}\) valence quark states [17].

The second-order one-pion exchange diagrams at one loop yield an additional correction to the mass spectrum of the SU(2) baryon state. The estimation of the lowest excitation spectrum of the Nucleon and \(\Delta\) in Ref. [16] shows that is possible to reproduce the main properties of the excited baryon spectrum based on the one-pion loop corrections.

### III. SELECTION RULES FOR THE QUANTUM NUMBERS OF THE EXCITED \(N^*\) AND \(\Delta^*\) STATES

Now we begin to analysis the excited \(N^*\) and \(\Delta^*\) spectrum based on the relativistic description of the one-pion exchange mechanism (see [16]). We do not write down here explicitly the expression for the pion-exchange operator, but only remember that it couples the upper and lower components of the two interacting valence quarks, respectively. In this way we can derive the selection rules for the quantum numbers of the baryon states with the fixed orbital configuration.

Let us to fix the orbital configuration as \((1S_{1/2})^2(nlj)\), with the intermediate spin coupling \(S_0 = S_1 + S_2 = \frac{1}{2}\sqrt{2} + \frac{1}{2}\) of the two 1S-valence quarks, where the last valence quark \((nlj)\) can be in the ground or an excited state. The upper and lower Dirac components of the last excited valence quark have orbital momenta \(l\) and \(l' = l \pm 1\), respectively. It is clear that these configurations must describe well the lowest baryon excited states. However, the question is, how successful are they in higher excitation spectrum? Our choice of the above orbital configuration is close to the limitation in the diquark-quark models [24], where some of the degrees of freedom are “frozen”. The corresponding baryon states are different from members of the SU(6)\(\otimes\)O(3) multiplets in the Constituent Quark Models.

The first two selection rules come from the coupling of the three valence quarks into the SU(2) baryon state with total momentum \(J\) and isospin \(T\):

\[ S_0 + \frac{\tilde{J}}{2} = \tilde{J}, \]
\[ S_0 + \frac{1}{2} = \tilde{T}, \]

(12)
where the symmetry property of the two S-quark coupling was used. The third rule comes from the pion exchange mechanism between the excited valence quark and the 1S quark. This mechanism couples the upper (lower) component of the 1S valence quark with the lower (upper) component of the excited \((nlj)\) valence quark. Since the upper component of the S-quark has zero orbital momentum, then for the orbital momentum of the exchanged pion we derive the equation

\[ L_\pi = l' = l \pm 1 \] (13)

The final selection rule is based on the assumption that the coupling of the last valence quark with quantum numbers \((nlj)\) to the 1S quark plus pion is the main component of the strong coupling of the excited baryon state to the \(N(939) + \pi\):

\[ \vec{L}_\pi + 1/2 = \vec{J} \] (14)

With this assumption, Eq. (13) can be used for the identification of the baryon resonance in the \(\pi N\)-scattering process. Namely, when \(l' = 0\) we have S-wave nucleon and delta resonances, when \(l' = 1\) we have P-wave resonances, etc.

An important consequence of the obtained selection rules is that all the \(N^*\) and \(\Delta^*\) resonances appearing in the \(\pi N\) scattering process and strongly coupling to the \(\pi N\) channel are identified with the orbital configurations \((1S_{1/2})^2(nlj)\) with two valence quarks in the ground state and a single valence quark in an excited state. A baryon resonance corresponding to the orbital configuration with two valence quarks in excited states \((1S_{1/2})(nlj)_1(nlj)_2\) couples strongly to the \(\pi N\)-channel, but not to the \(\pi N\) channel.

Using the obtained selection rules it is very natural to analysis the excited nucleon and delta spectrum. For the fixed orbital configurations \((1S_{1/2})^2(nlj)\) with the intermediate spin coupling of the two S-wave quarks \(S_0 = 0\) (the so-called instanton channel), Eq. (12) allows only a single \(N^*\) state with \(J = j\) and no any \(\Delta^*\) resonances.

Except the case, when the last valence quark is in the \(P_{1/2}\) orbit, the intermediate coupling \(S_0 = 1\), due-to the selection rule Eq. (11) yields two resonances in the both nucleon and delta sectors with the total momentum \(J = L_\pi \pm 1/2\). In this way one of the \(N^*\) resonances defined by the selection rules in Eq. (12) with \(J = j + 1\) or \(J = j - 1\) is ruled out. When the last valence quark is in the \(P_{1/2}\) orbit, i.e. has the lower S-component, the selection rules yield \(L_\pi = 0\) and \(J = 1/2\), and consequently, only single S-wave resonances in the both nucleon and \(\Delta\) sectors are allowed.

Thus, for the fixed \((1S_{1/2})^2(nlj)\) orbital configuration with \((nlj) \neq (nP_{1/2})\) there must be three \(N^*\) and two \(\Delta^*\) resonances. The lightest \(N^*\) state corresponds to the intermediate spin coupling \(S_0 = 0\) due-to strong attraction in this "instanton channel". The other two \(N^*\), as well as the two \(\Delta^*\) resonances correspond to the spin coupling \(S_0 = 1\) and must be close each to other.

In the case when the last quark is in the \(P_{1/2}\) orbit, there are two \(N^*\) states (not close each to other) and a single \(\Delta^*\) resonance appearing in the S-wave of the \(\pi N\) scattering.

**IV. NUMERICAL RESULTS**

In Table 1 we compare our numerical estimations of the excited \(N^*\) and \(\Delta^*\) spectrum within the developed schematic periodic table with the last experimental data from [8] and [9]. The calculations were done up-to and including F-wave baryon resonances in the frame of the developed chiral quark model. Details of the calculations were given in [14, 15]. In the Table we give the center of mass (CM) corrected quark core results (zero order estimation) (second column) together with the second order pion field contributions corresponding to the self energy (3-th column) and exchange diagrams (second order corrections) (4-th column). The final estimations are given in the 5-column with the fixed value of the weak decay coupling constant \(f_\pi = 93\) MeV. The results in the 6-column were obtained with the renormalized coupling constant \(f_\pi (ex) = 65.22\) MeV for the exchange diagrams, while keeping the standard value \(f_\pi = 93\) MeV for the self-energy terms.

We first fix the orbital configuration \((1S_{1/2})^2(nS_{1/2})\). In the data there are four \(N^*\) with \(J^* = 1/2^+\) (\(P_{3/2}\) resonances) and two \(N^*\) with \(J^* = 3/2^+\) (\(P_{1/2}\) resonances). With the above rules, we can find easily that \(N^*(1440)\), \(N^*(1710)\), and \(N^*(1720)\) resonances belong to the orbital configuration \((1S_{1/2})^2(2S_{1/2})\) with the radially excited 2S valence quark state, while the other three \(N^*(1880)\), \(N^*(1900)\) and \(N^*(2100)\) resonances correspond to the orbital configuration \((1S_{1/2})^2(3S_{1/2})\). In the \(\Delta\) sector there are two resonances with \(J^* = 3/2^+\) at 1600 MeV and 1920 MeV, and two states with \(J^* = 1/2^+\) at 1750 MeV and 1910 MeV which belong to the orbital configuration with the radially excited valence quark in consistency with our results.

The orbital configuration \((1S_{1/2})^2(1D_{3/2})\) is not presented in the data, since it would give two \(N^*\) resonances with \(J^* = 3/2^+\) and a single \(N^*\) resonance with \(J^* = 1/2^+\).
For the orbital configurations \((1S_{1/2})^2(nP_{1/2})\) there are four nucleon and three delta resonances with \(J^P = 1/2^-\) and they are not close to others. Each of the nucleon bands \(n = 1\) and \(n = 2\) contains two resonances, while \(\Delta^*\) resonances correspond to the three bands including \(n = 3\).

The orbital configuration \((1S_{1/2})^2(nP_{3/2})\) with \(n = 1\) yields three \(N^*\) resonances \(3/2^-\) (1520), \(5/2^-\) (1675) and \(3/2^-\) (1700), the first of which is less than other two states in accordance with our prediction. The band with \(n = 2\) yields next group of the D-wave Nucleon resonances \(3/2^-\) (1860), \(3/2^-\) (2080) and \(5/2^-\) (2200).

In the Delta sector there are four D-wave resonances, however only two of them \(\Delta(5/2^-)(1930)\) and \(\Delta(3/2^-)(1940)\) are close to each other. Since other D-wave resonances \(\Delta(3/2^-)(1700)\) and \(\Delta(5/2^-)(2350)\) are far, then we can predict possible new \(\Delta(5/2^-)\) (around 1700 MeV) and \(\Delta(3/2^-)\) (around 2350 MeV) resonances.

The F-wave \(N^*\) resonances \(N^*(5/2^+)(1680), N^*(5/2^+)(1780)\) and \(N^*(7/2^+)(1990)\) belong to the orbital configuration \((1S_{1/2})^2(nD_{5/2})\) with \(n = 1\) together with delta states \(\Delta^*(5/2^+)(1905)\) and \(\Delta^*(7/2^+)(1950)\), while the \(\Delta^*(5/2^+)(2000)\) and \(\Delta^*(7/2^+)(2390)\) belong to the \(n = 2\) band.

We can continue our analysis at higher energies and predict in summary seven new \(N^*\) resonances with \(J^P = 7/2^-\) (2000 MeV), \(9/2^+\) (2100 - 2300 MeV), \(11/2^+\) (2100 - 2300 MeV), \(11/2^-\) (2500-2700 MeV), \(13/2^-\) (2500-2700 MeV), \(15/2^+\) (2600-2800 MeV) and four \(\Delta^*\) resonances with \(J^P = 5/2^-\) (around 1700 MeV), \(3/2^-\) (2350 MeV), \(11/2^-\) (2750 MeV), \(13/2^+\) (2950 MeV). These resonances are expected to be observed in current experimental facilities.

It is clear now that the remaining "missing \(N^*\) and \(\Delta^*\) resonances" predicted by the Constituent Quark Models must appear in the \(\pi N\) strong coupling sector, if they exist. As we have argued above, they will be assigned with the orbital configuration \((1S_{1/2})(nlj_1)(nlj_2)\) with two excited valence quarks and a single ground state valence quark. Now we can analysis the numerical values in Table 1 in comparison with the experimental data-analysis from Ref. 5. As can be seen from the Table, the mass spectrum of the Nucleon and \(\Delta\) is described reasonably well in the relativistic chiral quark model. The important observation is that one needs an additional exchange mechanism for the lowering both the ground state \(N(939)\) and Roper resonance \(N(1440)\) almost by the same amount. This fact indicate that the "lowering mechanism" for the both \(N(939)\) and Roper resonance \(N(1440)\) should be the same, since these states have identical quantum numbers except the radial quantum number. The same situation is in the \(\Delta \) sector. Additionally, most of the radially excited \(N^*\), except \(N^*(1710)(1/2^+)\) and \(N^*(2100)(1/2^+)\) and all the radially excited \(\Delta^*\) resonances are overestimated in our model. The exception is possible due to the experimental errors.

Contrary, the orbitally excited \(N^*\) resonances are mostly underestimated, except the states \(N^*(1650)(1/2^-), N^*(1860)(3/2^-)\) and \(N^*(1905)(1/2^-)\). The situation in the \(\Delta^*\) sector is different. The negative parity \(\Delta^*\) states are consistent with the experimental data: exception is here for the \(\Delta^*(1700)(3/2^-)\), which is underestimated by 70 - 170 MeV. The positive parity \(\Delta^*(1905)(5/2^-)\) and \(\Delta^*(1950)(7/2^-)\) resonances are also underestimated by some amount.

The use of the renormalized coupling constant \(f_c^x (cx) = 65.22\) MeV for the exchange diagrams yields a good estimation for energy values of the ground states \(N(939)\) and \(\Delta(1232)\) and their energy difference, but strongly moves down the lowest orbitally excited baryon resonances \(N(1520), N(1535), \Delta(1620)\) and \(\Delta(1700)\).

The analysis shows that one needs an additional exchange mechanism between valence quarks to reproduce the whole SU(2) baryon spectrum. The new exchange forces must depend on the spin and flavor of valence quarks as well as on the quantum numbers of the baryon state. Of course, some part of the interaction comes from gluon fields and two-pion exchange mechanism.

\[ V. \ \text{CONCLUSIONS} \]

In summary, we have derived selection rules for the excited baryon state, assuming that it’s orbital configuration is of the form \((1S_{1/2})^2(nlj)\) with two valence quarks in the ground state and a single excited quark. These selection rules were derived on the basis of the one-pion exchange mechanism between valence quarks in the frame of the relativistic chiral quark model. An important consequence of the obtained selection rules is that all the \(N^*\) and \(\Delta^*\) resonances appearing in the \(\pi N\) scattering process and strongly coupling to the \(\pi N\) channel are identified with the orbital configurations \((1S_{1/2})^2(nlj)\). Baryon resonances corresponding to the orbital configuration with two valence quarks in excited states couple strongly to the \(\pi \pi N\)-channel, but not to the \(\pi N\) channel.

Based on obtained selection rules, we have constructed a schematic periodic table for the excited \(N^*\) and \(\Delta^*\) spectrum. The obtained numerical estimations for the energy positions of baryon resonances (up to and including F-wave) are consistent with the experimental data.

The important observation is that one needs an additional exchange mechanism for the lowering both the ground state \(N(939)\) and Roper resonance \(N(1440)\) almost by the same amount. This fact indicate that the "lowering
mechanism” for the both N(939) and Roper resonance $N^*(1440)(1/2^+)$ should be the same, since these states have identical quantum numbers except the radial quantum number. The same situation is in the $\Delta$ sector.

At higher energies, where the experimental data are poor, we can extend our model schematically and predict seven new $N^*$ and four $\Delta^*$ states with larger spin values. Of course, the number of "missing resonances" in our model is strongly suppressed due-to restriction of the configuration space to the orbits $(1S_\frac{1}{2})^2(nlj)$. However, as we have shown above, at lower energies this construction works reasonably well.

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TABLE I: Estimations for the energy values of the $N^*$ and $\Delta^*$ states in MeV up to and including F-wave resonances with the CM correction in the LHO method

| N $(939) (1/2^+) \ (1S)^3$ | $E_Q (CM_{corrected})$ | $\Delta E_{s (s.e.)}$ | $\Delta E_{s (ex.)}$ | $E_Q + \Delta E_s$ | $E_Q + \Delta E_s^*$ | $\text{exp.} [5]$ |
|--------------------------|------------------------|-------------------|-------------------|-----------------|-----------------|------------------|
| $N(1440) (1/2^+) \ (1S)^2(2S)$ | 1200 | 603 | -113 | 1690 | 1573 | 1430 $\div$ 1470 |
| $N(1710) (1/2^+) \ (1S)^2(2S)$ | 1200 | 603 | -66 | 1737 | 1669 | 1700 $\div$ 1750 |
| $N(1720) (3/2^+) \ (1S)^2(2S)$ | 1200 | 603 | 1 | 1804 | 1805 | 1700 $\div$ 1760 |
| $N(1880) (1/2^+) \ (1S)^2(3S)$ | 1361 | 788 | -110 | 2039 | 1925 | 1840 $\div$ 1940 |
| $N(1910) (1/2^+) \ (1S)^2(3S)$ | 1361 | 788 | -39 | 2110 | 2070 | 2000 $\div$ 2200 |
| $N(1900) (3/2^+) \ (1S)^2(3S)$ | 1361 | 788 | -3 | 2146 | 2143 | 1900 $\div$ 2000 |
| $N(1950) (1/2^+) \ (1S)^2(1P_{3/2})$ | 1129 | 501 | -119 | 1511 | 1388 | 1528 $\div$ 1548 |
| $N(1650) (1/2^-) \ (1S)^2(1P_{3/2})$ | 1129 | 501 | 46 | 1676 | 1724 | 1640 $\div$ 1680 |
| $N(1905) (1/2^-) \ (1S)^2(2P_{3/2})$ | 1301 | 713 | -111 | 1903 | 1788 | 1850 $\div$ 1950 |
| $N(2090) (1/2^-) \ (1S)^2(2P_{1/2})$ | 1301 | 713 | 24 | 2038 | 2063 | 2100 $\div$ 2260 |
| $N(1520) (3/2^-) \ (1S)^2(1P_{3/2})$ | 1107 | 515 | -126 | 1496 | 1366 | 1518 $\div$ 1526 |
| $N(1700) (3/2^-) \ (1S)^2(1P_{3/2})$ | 1107 | 515 | -79 | 1543 | 1461 | 1675 $\div$ 1775 |
| $N(1675) (5/2^-) \ (1S)^2(1P_{3/2})$ | 1107 | 515 | 11 | 1633 | 1645 | 1670 $\div$ 1680 |
| $N(1860) (3/2^-) \ (1S)^2(2P_{3/2})$ | 1293 | 713 | -111 | 1895 | 1871 | 1810 $\div$ 1890 |
| $N(2070) (5/2^-) \ (1S)^2(2P_{3/2})$ | 1293 | 713 | -31 | 1975 | 1943 | 2045 $\div$ 2155 |
| $N(1680) (5/2^+) \ (1S)^2(1D_{5/2})$ | 1212 | 638 | -114 | 1736 | 1618 | 1680 $\div$ 1690 |
| $N(1870) (5/2^+) \ (1S)^2(1D_{3/2})$ | 1212 | 638 | -37 | 1813 | 1775 | 1840 $\div$ 1960 |
| $N(1990) (7/2^+) \ (1S)^2(1D_{3/2})$ | 1212 | 638 | 12 | 1862 | 1874 | 1860 $\div$ 2100 |
| $\Delta (1232) (3/2^+) \ (1S)^3$ | 966 | 380 | -36 | 1310 | 1273 | 1230 $\div$ 1234 |
| $\Delta (1600) (3/2^+) \ (1S)^2(2S)$ | 1200 | 603 | -23 | 1780 | 1756 | 1535 $\div$ 1695 |
| $\Delta (1750) (1/2^+) \ (1S)^2(2S)$ | 1200 | 603 | 1 | 1804 | 1805 | 1710 $\div$ 1780 |
| $\Delta (1910) (1/2^+) \ (1S)^2(3S)$ | 1361 | 788 | -3 | 2146 | 2143 | 1845 $\div$ 2025 |
| $\Delta (1920) (3/2^+) \ (1S)^2(3S)$ | 1361 | 788 | -18 | 2131 | 2112 | 1880 $\div$ 2020 |
| $\Delta (1620) (1/2^-) \ (1S)^2(1P_{3/2})$ | 1129 | 501 | -24 | 1606 | 1581 | 1603 $\div$ 1649 |
| $\Delta (1900) (1/2^-) \ (1S)^2(2P_{3/2})$ | 1301 | 713 | -24 | 1900 | 1965 | 1860 $\div$ 1960 |
| $\Delta (1700) (3/2^-) \ (1S)^2(1P_{3/2})$ | 1107 | 515 | -18 | 1604 | 1585 | 1670 $\div$ 1770 |
| $\Delta (5/2^-) \ (1S)^2(1P_{3/2})$ | 1107 | 515 | -35 | 1587 | 1551 | ... |
| $\Delta (1940) (3/2^-) \ (1S)^2(2P_{3/2})$ | 1293 | 713 | -9 | 1997 | 1988 | 1935 $\div$ 2055 |
| $\Delta (1930) (5/2^-) \ (1S)^2(2P_{3/2})$ | 1293 | 713 | -22 | 1984 | 1961 | 1900 $\div$ 1960 |
| $\Delta (1905) (5/2^+) \ (1S)^2(1D_{5/2})$ | 1212 | 638 | -12 | 1838 | 1826 | 1860 $\div$ 1940 |
| $\Delta (1950) (7/2^+) \ (1S)^2(1D_{3/2})$ | 1212 | 638 | -27 | 1823 | 1795 | 1915 $\div$ 1960 |