Z_{L} associated pair production of charged Higgs bosons in the littlest Higgs model at 
\(e^+e^-\) colliders

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The production of single and doubly charged Higgs bosons associated with standard model gauge 
bozon \(Z_{L}\) in \(e^+e^-\) colliders are examined. The sensitivity of these processes on the littlest Higgs 
model parameters in the range of compatibility with electroweak precision observables are analyzed. 
The possibility of detecting lepton flavor violation processes are also discussed.

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I. INTRODUCTION

One of the unsolved problems of the Standard Model(SM) is the hierarchy problem. The little Higgs models[1, 
2, 3, 4] are introduced to solve the hierarchy problem by stabilizing the Higgs mass by a collective symmetry 
breaking mechanism due to the cancelation of divergent loops by appearance of new particles as a consequence of 
extra symmetries. The phenomenology of the little Higgs models are widely discussed in literature (for reviews see 
5, 6, 7), and constraints on little Higgs model parameters are studied 8, 9, 10, 11, 12, 13. The little Higgs 
models are also expected to give new significant signatures in future high energy colliders and studied in references 
14, 15, 16, 17, due to the new particles which are predicted by these models. Also the \(Z_{L}\) associated production 
of SM Higgs boson at \(e^+e^-\) colliders is studied in 18, 19.

In this work we examined the production of single and doubly charged scalars associated with \(Z_{L}\) boson at future linear \(e^+e^-\) colliders, namely, International Linear Collider (ILC) 20 and Compact Linear Collider (CLIC) 21 in 
the context of littlest Higgs model[1]. We examined the dependance of total and differential cross sections of the 
processes to the littlest Higgs model parameters at the range allowed by electroweak presicion measurements. Also 
we analyzed the decays of scalars in the context of lepton flavor violation. We found that the production rates of 
the single charged scalar pairs associated with \(Z_{L}\) are less than the \(Z_{L}\) associated production of doubly charged 
scalars, but both channels will be achieved at \(e^+e^-\) colliders at \(\sqrt{s} \geq 2 TeV\).

The layout of this paper is as follows: In section 2 first we give a brief review of the littlest Higgs model and then 
we calculate the pair production of charged scalars at \(e^+e^-\) colliders. In this section we also calculate the total and 
differential cross sections of the productions of charged scalars with \(Z_{L}\). Section 3 contains our numerical results 
and discussions.

II. THEORETICAL FRAMEWORK

Before examining the pair production of charged scalars with \(Z_{L}\), we introduce a few words on the littlest Higgs 
model based on reference [6]. In the littlest Higgs model global symmetry \(SU(5)\) is broken spontaneously to \(SO(5)\) 
at an energy scale \(f \sim 17 TeV\) leaving 14 nambu goldstone bosons(NGB) corresponding to broken symmetries 
represented by the goldstone boson matrix of non linear sigma model (nlsm) such as:

\[
\Pi(x) = \sum_{a=1}^{14} \Pi^a(x) X^a
\] (1)
where $X^a$ are broken generators of $SU(5)$. The vacuum bases triggering the symmetry breaking can be chosen as

$$
\Sigma_0 = \begin{pmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{pmatrix},
$$

and the $\Sigma$ field is defined as

$$
\Sigma(x) = e^{i\Phi/\Sigma_0}e^{i\Phi/\Sigma_0} = e^{2i\Phi/\Sigma_0},
$$

and the effective lagrangian of the $\Sigma$ field is:

$$
\mathcal{L}_\Sigma = \frac{f^2}{8} \text{Tr}[D_\mu \Sigma]^2.
$$

In the littlest Higgs model $SU(5)$ contains the gauged subgroup $[SU(2)_1 \otimes U(1)_1] \otimes [SU(2)_2 \otimes U(1)_2]$ of $SU(5)$, defining the covariant derivative such as:

$$
D_\mu \Sigma = \partial_\mu \Sigma - i \sum_{i=1}^{2} (g_i(W_{\mu i} \Sigma + \Sigma W^T_{\mu i}) + g_i^*(B_{\mu i} \Sigma + \Sigma B^T_{\mu i})),
$$

where $B_{\mu i}$ and $W_{\mu i}$ are the gauge fields, $g_i$ and $g_i^*$ are the corresponding couplings of $U(1)_i$ and $SU(2)_i$ respectively.

The vev of $\Sigma$ field in the lagrangian breaks $(SU(2) \otimes U(1))^2$ symmetry to diagonal subgroup $(SU(2) \otimes U(1))$ of SM. As a consequence symmetry breaking, gauge bosons gain mass by eating the four of the NGBs. The mixing angles between the $SU(2)$ subgroups and between the $U(1)$ subgroups are defined respectively as:

$$
s = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \quad s' = \frac{g_2^*}{\sqrt{g_1^2 + g_2^2}}.
$$

The usual electroweak symmetry breaking occurs by the vev of the Higgs potential written by Coleman Weinberg method for scalars. By EWSB vector bosons get extra mixings due to vacuum expectation values of $h$ doublet and $\phi$ triplet. Again by diagonalizing the mass matrices, the final masses of vector bosons to the order of $v^2$ are expressed as:

$$
M^2_{W^\pm} = m_w^2 \left[ 1 - \frac{v^2}{f^2} \left( \frac{1}{6} + \frac{1}{4} (c^2 - s^2)^2 \right) + \frac{4 v^2}{v^2} \right],
$$

$$
M^2_{W^0} = \frac{f^2 g^2}{4 s^2 c^2} - \frac{1}{4} g^2 v^2 + \mathcal{O}(v^4/f^2) = m_w^2 \left( \frac{f^2}{s^2 c^2 v^2} - 1 \right),
$$

$$
M^2_{A_1} = 0,
$$

$$
M^2_{Z_2} = m_z^2 \left[ 1 - \frac{v^2}{f^2} \left( \frac{1}{6} + \frac{1}{4} (c^2 - s^2)^2 \right) + \frac{5}{4} (c^2 - s^2)^2 \right] + 8 v^2, \quad v^2,
$$

$$
M^2_{A_2} = \frac{f^2 g^2}{20 s^2 c^2} - \frac{1}{4} g^2 v^2 + g^2 v^2 \frac{x_H}{4 s^2 c^2} = m_z^2 s_w^2 \left( \frac{f^2}{5 s^2 c^2 v^2} - 1 + \frac{x_H c_w^2}{4 s^2 c^2 v^2} \right),
$$

$$
M^2_{Z_2} = \frac{f^2 g^2}{4 s^2 c^2} - \frac{1}{4} g^2 v^2 - g^2 v^2 \frac{x_H}{4 s^2 c^2} = m_z^2 s_w \left( \frac{f^2}{s^2 c^2 v^2} - 1 - \frac{x_H c_w^2}{s^2 c^2 v^2} \right),
$$

where $m_w = g v/2$, $m_z = g v/(2 c_w)$ and $x_H = \frac{1}{2} g g' x c' c (c^2 s^2 + s^2 c^2)$. In these equations $s_w$ and $c_w$ are the usual weak mixing angles:

$$
s_w = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad c_w = \frac{g}{\sqrt{g^2 + g'^2}}.
$$

The parameters $v$ and $v'$ in equation [7] are the vacuum expectation values of scalar doublet and triplet given as:

$$
\langle h^0 \rangle = v/\sqrt{2} \text{ where } v = 246 \text{GeV} \quad \text{and} \quad \langle i \phi^0 \rangle = v' \leq \frac{0.5}{2},
$$

bounded by electroweak precision data. Also diagonalizing the mass matrix for scalars the physical states are found to be the SM Higgs scalar $H$, the neutral scalar $\phi^0$, the neutral pseudo scalar $\phi^P$, and the charged scalars $\phi^+$ and $\phi^{++}$. The masses of these scalars are degenerate, and in terms of Higgs mass can be expressed as:

$$
M^2_\phi \approx \frac{\sqrt{2} M_H f}{v}.
$$
The scalar fermion interactions in the model are written in yukawa lagrangian preserving gauge symmetries of the model for SM leptons and quarks, including the third generation having an extra singlet, the $T$ quark. In this work, leptons are charged under both $U(1)$ groups, with corresponding hypercharges of $Y_1$ and $Y_2$. The restriction for $Y_1$ and $Y_2$ is that $Y_1 + Y_2$ should reproduce $U(1)_Y$ hypercharge $Y$ of SM, thus $Y_1 = xY$ and $Y_2 = (1-x)Y$ can be written. Due to gauge invariance, $x$ can be taken as $3/5[6,13]$. Also for light fermions, a Majorana type mass term can be implemented in yukawa lagrangian $\phi C^{-1}L_j + h.c.$, which results in lepton flavor violation by unit two, such as:

$$\mathcal{L}_{LFV} = Y_{ij}^2 L_i^c \phi C^{-1} L_j + h.c.,$$

where $L_i$ are the lepton doublets ($l_i \nu_l$), and $Y_{ij}$ is the yukawa coupling with $Y_{ii} = Y$ and $Y_{ij}(i \neq j) = Y'$. The values of yukawa couplings $Y$ and $Y'$ are restricted by the current constraints on the neutrino masses[22], given as; $M_{ij} = Y_{ij}v' \approx 10^{-10}$GeV[22]. Since the vacuum expectation value $v'$ has only an upper bound; $v' < 1$GeV, $Y_{ij}$ can be taken up to order of unity without making $v'$ unnaturally small.

In the littlest Higgs model the symmetry breaking scale $f$, and the mixing angles $s, s'$ are not restricted by the model. These parameters are constrained by observables of electroweak precision data[8,9,10,11,12,13]. For the values of the symmetry breaking scale $1$TeV $\leq f \leq 2$TeV, mixing angles are in the range $0.75 \leq s \leq 0.99$ and $0.6 \leq s' \leq 0.75$, for $2$TeV $\leq f \leq 3$TeV they have acceptable values in the range $0.6 \leq s \leq 0.99$ and $0.6 \leq s' \leq 0.8$, for $3$TeV $\leq f \leq 4$TeV they are in the range $0.4 \leq s \leq 0.99$ and $0.6 \leq s' \leq 0.85$, and for the higher values of the symmetry breaking scale, i.e. $f \geq 4$TeV, the mixing angles are less restricted and they are in the range $0.15 \leq s \leq 0.99$ and $0.4 \leq s' \leq 0.913$.

After these preliminary remarks lets start to calculate the cross sections of the pair productions for the charged scalars with $Z_L$ at $e^+e^-$ colliders. In the littlest Higgs model there are four neutral vector bosons; the SM Z boson $Z_L$, massless photon $A_L$, and two new bosons $Z_H$ and $A_H$. Their couplings to fermions are written as $i\gamma_{\mu}(g_\nu + g_A \gamma_5)$ where $i = 1, 2, 3, 4$ corresponds to $Z_L, Z_H, A_H$ and $A_L$ respectively. The couplings of gauge vector to electron positron pairs are given in table $I$ where $e = \sqrt{4\pi}\alpha, \gamma_e = \frac{1}{2}$ for anomaly cancelations, $x_Z^{W'} = -\frac{1}{2\sqrt{2}}sc(c^2 - s^2)$ and $x_Z^{l'} = \frac{1}{2\sqrt{2}}sc(c^2 - s^2)$. It is seen that vector and axial vector couplings of SM $Z_L e^+e^-$ vertex also gets contributions from littlest Higgs model. As a result total decay widths of SM vector bosons also gets corrections of the order $\frac{x_Z^{W'}}{\sqrt{s}}$, since the decay widths of vectors to fermion couples are written as; $\Gamma(V_i \rightarrow f f) = \frac{x_i^{W'}}{\sqrt{s}}(g_\nu^2 + g_A^2)M_{V_i}$, where $N = 3$ for quarks, and $N = 1$ for fermions. The total decay widths of the new vectors are given as[22]:

$$\begin{align*}
\Gamma_{A_H} &\approx \frac{g^2 M_{A_H}(21 - 70s^2 + 59s^4)}{48\pi s^2(1 - s^2)}, \\
\Gamma_{Z_H} &\approx \frac{g^2 (193 - 388s^2 + 196s^4)}{768\pi s^2(1 - s^2)}M_{Z_H}, \\
\Gamma_{W_H} &\approx \frac{g^2 (97 - 196s^2 + 100s^4)}{384\pi s^2(1 - s^2)}M_{W_H}.
\end{align*}$$

The new scalars and pseudo scalars also contribute to the analysis done in this study. Since these new scalars have lepton flavor violating decay modes, their total widths will depend on the yukawa couplings $Y_i = Y$ and $Y_{ij}(i \neq j) = Y'$ if the flavor violating term in the yukawa lagrangian is considered. The decay widths of scalars are given as[22]:

$$\Gamma_\phi \approx \frac{\nu'^2 M_\phi^3}{2\pi v'^4} + \frac{3}{8\pi}|Y|^2 M_\phi + \frac{3}{4\pi}|Y'|^2 M_\phi,$$

where we neglected the decays into quarks since they are very small compared to other terms. The first term in the decay width proportional to $|Y|^2$ corresponds to decays into same family of leptons and the second term proportional to $|Y'|^2$ correspond to the decays into different families. The branching ratios of scalars decaying to same family of leptons is denoted as $BR[Y]$ and to leptons of different flavor is $BR[Y']$. In this work the values of the yukawa mixings are taken to be $10^{-3} \leq Y \leq 1$ and $Y' \leq 10^{-4}$.

The Feynman diagrams for the $Z_L$ associated scalar pair production processes are given in figure $I$. The first ten of the Feynman diagrams in figure $I$ are for the both processes $e^+e^- \rightarrow Z_L \phi^+\phi^-$ and $e^+e^- \rightarrow Z_L \phi^+\phi^-$. The last four ones only contribute to the associated production of single charged scalars. Now we present the amplitudes for these Feynman diagrams. The relevant Feynman rules for the vertices for single charged scalars are given in tables $I$, $III$ and $IV$ and for the doubly charged scalars in tables $V$ and $VI$. The amplitudes for the associated production of doubly charged scalars are get by replacing the couplings with the primed ones. In the calculations of cross sections, terms are expanded up to order of $\nu'^2/f^2$.

The first four Feynman diagrams contributing to both processes contain only the $e^+e^-$ vertices. The amplitudes for the first two diagrams in figure $I$ are written as:

$$M_1 = \bar{u}[-p_2]i\gamma_{\mu}(g_{\nu} + g_A \gamma_5)e^\mu[p_3]i\frac{\bar{q}_2}{q_2^2}i\gamma_{\nu}g_{\nu}[q_1](i)\frac{\nu'^2}{q_2^2}iE_4^\phi(p_4 - p_5)\alpha.$$
TABLE I: The vector and axial vector couplings of $c\bar{c}$ with vector bosons. Feynman rules for $c\bar{c}V_i$ vertices are given as $\gamma_\mu (g_{V_i} + g_{A_i} \gamma_5) \bar{v}$.

| i | vertices | $g_{V_i}$ | $g_{A_i}$ |
|---|----------|-----------|-----------|
| 1 | $e\bar{c}Z_L$ | $-\frac{g}{2v_w} \left( -\frac{1}{2} + 2s_w^2 + \frac{c_w x_w^2}{s_w} \right) c / 2s$ | $-\frac{g}{2v_w} \left( \frac{1}{2} - \frac{c_w x_w^2}{s_w} \right) c / 2s$ |
| 2 | $e\bar{c}Z_H$ | $-gc/As$ | $gc/As$ |
| 3 | $e\bar{c}A_H$ | $\frac{g_3}{2v_w c^2} (2y_c - \frac{2}{3} + \frac{1}{3}c^2)$ | $\frac{g_3}{2v_w c^2} (\frac{1}{3} - \frac{1}{3}c^2)$ |
| 4 | $e\bar{c}A_L$ | $e$ | $0$ |

$\phi^+ \phi^-$ channel processes with $q = p_2 - p_3$, $q' = p_4 + p_5$ and coefficients $E_i^{\phi \phi}$ are given in table III $E_i^{\phi \phi}$ for doubly charged scalars given in table IV. The amplitudes for diagrams 3 and 4 in figure II are written as:

$$M_2 = \sum_{i=1}^{3} \bar{u}[-p_2]i\gamma_\mu (g_{V_i} + g_{A_i} \gamma_5) \epsilon^\nu [p_3] \frac{i}{q^2} \gamma_\nu (g_{V_i} + g_{A_i} \gamma_5) u[p_1] (i)$$

$$g^{\mu \nu} - g^{\mu \nu} - \frac{q^{\mu} q^{\nu}}{M_i^2} \frac{i E_i^{\phi \phi} (p_4 - p_5)_{\alpha}}{q^2 - M_i^2 + iM_i \Gamma_i},$$

where $q = p_2 - p_3$, $q' = p_4 + p_5$ and coefficients $E_i^{\phi \phi}$ are given in table III $E_i^{\phi \phi}$ for doubly charged scalars given in table IV. The rest of the amplitudes from 5 to 14 in figure II are $\alpha$ channel processes with $q = p_1 + p_2$. For the diagrams 5 to 8 in figure II corresponding amplitudes 5 to 8 have a heavy scalar as a propagator, and proportional to the square of the couplings of two vectors with a charged scalar. These amplitudes are written as:

$$M_3 = \bar{u}[-p_2]i\gamma_\mu (g_{V_i} + g_{A_i} \gamma_5) \frac{i g^{\mu \nu}}{q^2} E_4^{\phi \phi} (p_4 - p_5)_{\nu} \gamma_\alpha (g_{V_i} + g_{A_i} \gamma_5) \epsilon^\alpha [p_3] u[p_1],$$

$$M_4 = \sum_{i=1}^{3} \bar{u}[-p_2]i\gamma_\mu (g_{V_i} + g_{A_i} \gamma_5) i \frac{g^{\mu \nu} - g^{\mu \nu}}{M_i^2} \frac{i E_i^{\phi \phi} (p_4 - p_5)_{\nu}}{q^2 - M_i^2 + iM_i \Gamma_i} i E_i^{\phi \phi} (q', -p_3) \epsilon^\alpha [p_3] u[p_1],$$

$\phi^+ \phi^-$ vertices are given as $\bar{v} (g_{V_1} + g_{A_1} \gamma_5) v$.

| i | vertices | $E_i^{\phi \phi} Q_{\nu}$ |
|---|----------|-----------------|
| 1 | $\phi^+ \phi^- Z_L$ | $\frac{g_3}{2v_w} (p_1 - p_2)_{\alpha}$ |
| 2 | $\phi^+ \phi^- Z_H$ | $O(u^2 / f^2) \sim 0$ |
| 3 | $\phi^+ \phi^- A_H$ | $ig^{\nu \alpha} (\epsilon^\nu (p_1 - p_2))_{\alpha}$ |
| 4 | $\phi^+ \phi^- A_L$ | $-i \epsilon (p_1 - p_2)_{\alpha}$ |

TABLE III: Feynman rules for $\phi^+ \phi^- V_i$ vertices [5].
Their amplitudes are given as:

\[ (18) \]

\[
M_6 = \bar{u}[-p_2]i\gamma_{\mu}g_{\nu 4}u[p_1]i\frac{g^{\mu\nu}}{q^2}iE_1^{\phi}(p_5 + q')_{\nu} \frac{i}{q^2 - M_{\phi}^2}iE_1^{\phi}(-q' - p_3)_{\sigma} \epsilon^\sigma[p_3],
\]

\[
M_7 = \sum_{i=1}^3 \bar{u}[-p_2]i\gamma_{\mu}(g_{\nu 4} + g_{A,\gamma 5})u[p_1]i\frac{g^{\mu\nu} - \frac{q^\nu q^\mu}{M_{\phi}^2}}{\frac{q^2 - M_{\phi}^2 + iM_\Gamma i}{q^2 - M_{\phi}^2 + iM_\Gamma i}}(iE_1^{\phi})(p_4 - q')_{\nu} \frac{i}{q^2 - M_{\phi}^2}(iE_1^{\phi})(-q' - p_3)_{\sigma} \epsilon^\sigma[p_3],
\]

\[
M_8 = \sum_{i=1}^3 \bar{u}[-p_2]i\gamma_{\mu}(g_{\nu 4} + g_{A,\gamma 5})u[p_1]i\frac{g^{\mu\nu} - \frac{q^\nu q^\mu}{M_{\phi}^2}}{\frac{q^2 - M_{\phi}^2 + iM_\Gamma i}{q^2 - M_{\phi}^2 + iM_\Gamma i}}(iE_1^{\phi})(p_5 + q')_{\nu} \frac{i}{q^2 - M_{\phi}^2}(iE_1^{\phi})(-q' - p_3)_{\sigma} \epsilon^\sigma[p_3],
\]

where \( E_1^{\phi} \) are given in table III[VII], and \( q' = q - p_5 \) for amplitudes 6 and 8 and \( q' = q - p_4 \) for 5 and 7. The diagrams 9 and 10 in figure II are sub processes in which the four point couplings of vectors and scalars contribute. Their amplitudes are given as:

\[
M_9 = \bar{u}[-p_2]i\gamma_{\mu}g_{\nu 4}u[p_1]i\frac{g^{\mu\nu} - \frac{q^\nu q^\mu}{M_{\phi}^2}}{\frac{q^2 - M_{\phi}^2 + iM_\Gamma i}{q^2 - M_{\phi}^2 + iM_\Gamma i}}(iC_{\gamma 1})g_{\nu \sigma} \epsilon^\sigma[p_3],
\]

\[
M_{10} = \sum_{i=1}^3 \bar{u}[-p_2]i\gamma_{\mu}(g_{\nu 4} + g_{A,\gamma 5})u[p_1]i\frac{g^{\mu\nu} - \frac{q^\nu q^\mu}{M_{\phi}^2}}{\frac{q^2 - M_{\phi}^2 + iM_\Gamma i}{q^2 - M_{\phi}^2 + iM_\Gamma i}}(iC_{\gamma 1})g_{\nu \alpha} \epsilon^\alpha[p_3],
\]
The last four diagrams only contribute to the single charged scalars. 

where $C_{ij}^{\phi \phi}$ ($C_{ij}^{\phi W}$) are four point couplings given in table IIIV. The amplitudes 11 to 14 corresponds to diagrams 11 to 14 in figure IV are only for single charged scalars, where an electric charge is carried by a $W$ propagator. These sub processes are proportional to the couplings of a vector with a charged scalar and a charged vector $W_j$, where $j = 1, 2$ corresponds to $W_L$ and $W_H$ respectively. Their amplitudes are given as: 

\begin{equation} 
M_{11} = \sum_{j=1}^{2} \bar{u}_{p \bar{2}} i \gamma_{\mu} (g_{V\alpha} u_{p \bar{1}}) \frac{i g^{\mu \nu}}{q^2} (i B_{1j}^{\phi W}) g_{\nu \alpha} \frac{g^{\alpha \beta} - \frac{i g^{\alpha \mu}}{m_{\tilde{W}_j}}}{q^2 - M_{W_j}^2} g_{\beta \epsilon} [p_3], 
\end{equation} 

\begin{equation} 
M_{12} = \sum_{j=1}^{2} \bar{u}_{p \bar{2}} i \gamma_{\mu} (g_{V\alpha} u_{p \bar{1}}) \frac{i g^{\mu \nu}}{q^2} (i B_{1j}^{\phi W}) g_{\nu \alpha} \frac{g^{\alpha \beta} - \frac{i g^{\alpha \mu}}{m_{\tilde{W}_j}}}{q^2 - M_{W_j}^2} g_{\beta \epsilon} [p_3], 
\end{equation} 

\begin{equation} 
M_{13} = \sum_{i,j=1,1}^{3,2} \bar{u}_{p \bar{2}} i \gamma_{\mu} (g_{V\alpha} + g_{A_i \alpha}, \gamma_5) u_{p \bar{1}} i \frac{g^{\mu \nu} - \frac{i g^{\mu \alpha}}{m_{\tilde{W}_j}}}{q^2 - M_{i}^2} g_{\beta \epsilon} [p_3] (i B_{1j}^{\phi W}), 
\end{equation} 

\begin{equation} 
M_{14} = \sum_{i,j=1,1}^{3,2} \bar{u}_{p \bar{2}} i \gamma_{\mu} (g_{V\alpha} + g_{A_i \alpha}, \gamma_5) u_{p \bar{1}} i \frac{g^{\mu \nu} - \frac{i g^{\mu \alpha}}{m_{\tilde{W}_j}}}{q^2 - M_{i}^2} g_{\beta \epsilon} [p_3] (i B_{1j}^{\phi W}). 
\end{equation}
where $B^{\ell \nu W}_{ij}$ are given in table IV and $q' = q - p_5$ for amplitudes 11 and 13 and $q' = q - p_4$ for 12 and 14.

The numerical calculations of cross sections of the pair production of charged scalars are performed by CalChep generator [28].

III. RESULTS AND DISCUSSIONS

In this section we present and discuss our results for the $Z_L$ associated pair production of charged scalars. In performing the numerical calculations, we take the electromagnetic coupling constant $\epsilon = \sqrt{4\pi\alpha} = 0.092$, the Higgs mass $M_H = 120$GeV and the mass of the standard model bosons $M_{Z_L} = 91$GeV, $M_W = 80$GeV and the SM mixing angle $s_W = 0.47$ using the recent data [27]. In the calculations, we ignored $v^2/f^2$ terms in the couplings, since we are not dealing with the corrections to a SM process.

For the pair production of heavy scalar associated with $Z_L$, the differential cross sections versus energy of the $Z_L$ boson graphs for fixed values of mixing angle parameters $s/s'$ at symmetry breaking scale $f = 1$TeV at total center of mass energy $\sqrt{s} = 3$TeV appropriate for CLIC are presented in figure 2(a). The total cross sections for these processes for different fixed values of the parameters $s/s' = 0.8/0.6, 0.8/0.7, 0.95/0.6, 0.5/0.1$ are presented in table VII. The dependance of the total cross section on $\sqrt{s}$ for these processes are presented in figure 3. The differential cross section gets its maximum value about $0.1 fb/GeV$ when $s/s' = 0.5/0.1$, correspondingly for total cross section we obtain that $59 fb$. This means that thousands of productions per year at high integrated luminosity of $100 fb^{-1}$ are expected. But this parameter set is not allowed when symmetry breaking scale $f = 1$TeV by electroweak precision data. For parameters $s/s' = 0.8/0.6, 0.8/0.7, 0.95/0.6$, the peak values of differential cross sections are obtained at the order of $10^{-4} fb/GeV$ for low $E_Z$ values, $E_Z \sim 100$GeV. The total cross section is calculated as $0.04 fb$. This result implies $1 \sim 10$ events per year accessible for a collider luminosity of $100 fb^{-1}$. The single charged scalar $\phi^-$ has leptonic decay modes $l_\nu l_\nu$ and $l_\nu \nu_j$ with branching ratios proportional to $|Y^2|$ and $|Y^{j2}|$ respectively. It also has decay modes to SM particles; $W^\pm H, W^\pm Z_L$ pairs. The SM decays dominates when the yukawa coupling $Y$ is either zero or very small ($Y \ll 1$). For higher values of yukawa coupling ($Y \sim 1$) the leptonic modes ($l_\nu l_\nu$) of $\phi^+ \phi^-$ couple dominates the final states, giving a signature of missing energy of the neutrino anti neutrino couple, and a lepton anti lepton pair of same or different flavor plus $Z_L$. These signals can be lepton flavor violating but the observation is challenging due to the low production rates of the single charged pair.

For the associated pair production of doubly charged scalars within $Z_L$ the differential cross sections with respect to $E_Z$ are plotted in figure 2(b) for different fixed values of mixing angles at $\sqrt{s} = 3$TeV, at symmetry breaking.

| $s/s'$ | $\sigma_{\phi^+ \phi^-}$ | $\sigma_{\phi^+ \phi^-}$ |
|-------|-----------------|-----------------|
| 0.8/0.6 | 0.042 | 0.48 |
| 0.8/0.7 | 0.031 | 0.44 |
| 0.95/0.6 | 0.043 | 0.78 |
| 0.5/0.1 | 59 | 84 |

TABLE VII: The total cross sections in fb for pair production of charged scalars associated with $Z_L$ for $f = 1$TeV and at $\sqrt{s} = 3$TeV.
FIG. 3: Total cross section vs. $\sqrt{s}$ graphs for $Z_L$ associated pair production of charged scalars at $f = 1\, TeV$ when parameters $s/s' : 0.8/0.7$.

scale $f = 1\, TeV$. The dependence of total cross section on the center of mass energy of this production process is plotted in figure 3 and the numerical values of total cross sections are presented in table VIII for parameters of interest. The differential cross section of the production process reaches its maximum when model parameters $s/s' = 0.5/0.1$ about $0.1 \frac{fb}{GeV}$ for $E_Z \sim 100\, GeV$, resulting a total cross section of $84\, fb$. This will give about 8000 events per year for high luminosities such as $100\, fb^{-1}$. But this remarkable event numbers are out of reach, since $s/s' = 0.5/0.1$ is out favored by electroweak precision data for $f = 1\, TeV$. For the electroweak allowed parameters $s/s' = 0.8/0.6, 0.8/0.7, 0.95/0.6$ at $f = 1\, TeV$ at $\sqrt{s} = 3\, TeV$, the differential cross section gets low

FIG. 4: Total cross section vs. $\sqrt{s}$ graphs for $Z_L$ associated pair production of charged scalars at (a) $f = 2.5\, TeV$, and at (b) $f = 3.5\, TeV$ when parameters $s/s' : 0.8/0.7$.

FIG. 5: Total cross section vs. $\sqrt{s}$ graphs for $Z_L$ associated pair production of charged scalars at $f = 5\, TeV$ when parameters (a) $s/s' : 0.8/0.7$ and (b) $s/s' : 0.6/0.4$. 
values at the order of $10^{-4} \frac{fb}{T eV}$. The resulting cross sections are calculated by integrating over $E_Z$, and found about $0.4 \sim 0.8 \times fb$ (table VII) resulting $40 \sim 80$ events per year for integrated luminosity of $100 fb^{-1}$. For $\sqrt{s} < 2 T eV$, this production channel is not reachable. In the littlest Higgs model, $\phi^{++} \rightarrow$ decays to charged vectors $W^+_L W^-_L$ and also to leptons $l^+_i l^-_j$ proportional to squares of the values of the yukawa couplings; $|Y^2|$ for same families and $|Y^2|$ for different lepton families when lepton violating modes are considered. So this channel provides final signals for doubly charged scalar discovery and lepton flavor violation. The final states of the doubly charged scalar pairs dominantly contain leptonic modes $l_i l_j l_i l_j$, semi leptonic modes $l_i l_j W^+_L W^-_L$ and to standard model charged vector pair $W^+_L W^-_L W^+_L W^-_L$ depending on $Y$ and $Y'$, while $Z_L$ dominantly decays to jets carrying the energy at the order of masses of the scalar pair. For $f = 1 T eV$ the leptonic branching ratio of doubly charged scalars can reach values close to 1 for $Y \rightarrow 1$, independent from $Y'$. If the value of the yukawa coupling $Y$ is high enough ($Y \sim 1$), the number of final state lepton flavor violating signals such as: $Z_L l_i l_j l_i l_j$, can reach up to 50 events per year for luminosities of $100 fb^{-1}$, which can be directly detectable free from backgrounds.

Finally, we have also analyzed the behavior of the production processes for higher values of $f$. The dependence total cross section on $\sqrt{s}$ when $s/s' = 0.8/0.7$ for $f = 2.5 T eV, 3.5 T eV, 5 T eV$ are plotted in figures 3(a), 3(b) and 3(c), respectively. It is seen that the maximum value of total cross sections, production rates and final lepton flavor violating signals for these $f$ values for both processes remain same, but the required energy is shifted to $\sqrt{s} = 3.8 T eV (5.5 T eV) \{7.5 T eV\}$ for $f = 2.5 T eV (3.5 T eV) \{5 T eV\}$, due to the increase in heavy scalar masses. For $f = 5 T eV$, we have also analyzed the case $s/s' = 0.6/0.4$, since the parameters are less constrained. The dependence of total cross sections for associated production of both single and doubly charged scalars are plotted in figure 5(b). It is seen that the total cross sections are increased by order one. For the single charged pair, the production cross section is calculated as $0.9 fb$ for $\sqrt{s} \gtrsim 10 T eV$, resulting 90 production events for luminosities $100 fb^{-1}$. For the double charged scalar pair, the total cross section is $5 \sim 9 fb$ for $\sqrt{s} \gtrsim 10 T eV$, giving $500 \sim 900$ productions at luminosities $100 fb^{-1}$. In this case the final number of four lepton signals ($l_i l_j l_i l_j$) will be around 600 for higher values of yukawa coupling ($Y \sim 1$). If the values of yukawa coupling is smaller, in the region $0.1 \leq Y \leq 0.3$, the double charged pair will decay into semi leptonic modes, such as $W^+_L W^-_L l_i l_i$, resulting 200 $\sim 400$ signals accessible for luminosities of $100 fb^{-1}$.

In conclusion, we found that at an $e^+e^-$ collider of $\sqrt{s} \gtrsim 2 T eV$ with a luminosity of $100 fb^{-1}$, the $Z_L$ associated production of charged scalars will be in the reach, being the single charged pair is quite challenging due to low production rates, and the production of doubly charged scalar pair more promising for the electroweak allowed parameters at $f = 1 T eV$. The final states will contain lepton flavor violating signals if the value of yukawa coupling $Y$ is close to unity. For larger values of $f$ the mixing angles $s/s'$ are less constrained, e.g. for $f = 5 T eV$ and $s/s' = 0.6/0.4$, the production rates increases allowing remarkable final lepton number violating events for $0.1 \leq Y \leq 1$, but for these set of parameters the center of mass energy of the colliders should be increased.

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