Geographical issues and physics applications of “very” long NF baselines

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We discuss several potential applications of “very” long neutrino factory (NF) baselines, as well as potential detector locations for these applications.

Neutrino factories \cite{1} are usually discussed in combination with baselines up to about 3 000 km. However, one could think about potential applications of a second, much longer baseline, which could be operated simultaneously or subsequently. In the following, we refer to “very” long baselines as baselines much longer than 3 000 km.

For long-baseline beam experiments, the electron or muon neutrino appearance probability \( P_{\text{app}}(P_{\mu\nu}, P_{\mu e}, P_{\nu e}, \text{or } P_{\mu \bar{e}}) \) is very sensitive to matter effects. It can be expanded in the small hierarchy parameter \( \alpha \equiv \Delta m^{2}_{31}/\Delta m^{2}_{21} \) and the small sin \( 2\theta_{13} \) up to the second order as (see Ref. \cite{2} and references therein):

\[
P_{\text{app}} \simeq \Delta^2 \cdot \sin^2 2\theta_{13} \cdot \sin^2 \theta_{23} \cdot f^2_1 \\
\pm \alpha \cdot \sin 2\theta_{13} \cdot \sin \delta_{\text{CP}} \cdot \sin \Delta \cdot \xi \cdot f_1 \cdot f_2 \\
+ \alpha \cdot \sin 2\theta_{13} \cdot \cos \delta_{\text{CP}} \cdot \cos \Delta \cdot \xi \cdot f_1 \cdot f_2 \\
+ \alpha^2 \cdot \cos^2 \theta_{23} \cdot \sin^2 (2\theta_{12} \cdot f_2^2). \tag{1}
\]

Here \( \Delta \equiv \Delta m^{2}_{31} L/(4E) \), \( \xi = \sin 2\theta_{12} \cdot \sin 2\theta_{23} \),

\[
f_1 = \frac{\sin[(1-\bar{A})\Delta]}{[(1-\bar{A})\Delta]}, \quad f_2 = \frac{\sin(\bar{A}\Delta)}{(\bar{A}\Delta)}, \tag{2}
\]

and \( \bar{A} = \pm(2\sqrt{2} G_F n_e L)/\Delta m^{2}_{31} \). Note that the matter effect in Eq. (1) enters via the matter potential \( \bar{A} \), where the equation reduces to the vacuum case for \( \bar{A} \rightarrow 0 \). In addition, the combination \( \bar{A}\Delta = \sqrt{2}/2 G_F n_e L \) does not depend on energy or oscillation parameters.

From Eq. (1), we can read off a special structure in terms of the factors \( f_1 \) and \( f_2 \). The first term is proportional to \( f_1^2 \), the second and third to \( f_1 \cdot f_2 \), and the fourth to \( f_2^2 \). In addition, we have pulled out a factor of \( \Delta^2 \propto L^2 \) from the equation, which means that the \( 1/L^2 \) geometrical drop of the flux is compensated by this factor, and that \( f_1 \) and \( f_2 \) determine the individual weights of the four terms as function of baseline, energy, and \( \Delta m^{2}_{31} \). However, note that the relative weight between the second and third CP terms is in addition given by the vacuum oscillation phase \( \Delta \). We show in Fig. (1) these two factors as function of \( L \) for different values of the energy. For this figure, \( \Delta m^{2}_{31} = 0.0025 \text{ eV}^2 \) and \( \rho = 4.3 \text{ g/cm}^3 \) is used.

\begin{figure}[ht]
\centering
\includegraphics[width=0.8\textwidth]{figure1.png}
\caption{Factors \( f_1 \) (solid curves) and \( f_2 \) (dashed curve) from Eq. (1) as function of \( L \) for different values of the energy. For this figure, \( \Delta m^{2}_{31} = 0.0025 \text{ eV}^2 \) and \( \rho = 4.3 \text{ g/cm}^3 \) is used.}
\end{figure}

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This “magic baseline” \cite{3,4} is the first application from Eq. (1): If we choose \( f_2 \equiv 0 \), the condition \( \sin(\bar{A}\Delta) = 0 \) evaluates to \( \sqrt{2} G_F n_e (L) L = 2\pi \) or \( L_{\text{magic}} \sim 7250–7500 \text{ km} \) for the first root. Obviously, the suppression of \( f_2 \) independent of en-
ergy and oscillation parameters allows an almost correlation- and degenerate-free measurement of $\sin^2 2\theta_{13}$ and the mass hierarchy, whereas no information on $\delta_{CP}$ can be obtained. As it has been demonstrated in Ref. [4] for a NF, the combination of the magic baseline with $L = 3000$ km has excellent capabilities for the $\sin^2 2\theta_{13}$, mass hierarchy, and $CP$ violation sensitivities down to $\sin^2 2\theta_{13} \approx 10^{-4}$. In addition, the magic baseline can be used for a risk-minimized precision measurement of $\delta_{CP}$ [5]. Potential magic baseline detector locations for three of the major potential NF laboratories can be found in Fig. 2 on the dashed curves.

Another application with a different purpose can be read off from Eq. (1) for $\sin^2 2\theta_{13} = 0$: In this case, all but the last term vanish. It is an interesting feature that $f_2$, which dominates the magnitude of the remaining “solar” term, does not drop in vacuum ($f_2 \to 1$), but is very small in matter ($f_2 \to 0$ for $L \to \infty$). Thus, the solar term is suppressed by the matter effect. One can use this effect for a direct high confidence level verification of the MSW effect in Earth matter: For a $5\sigma$ signal, a NF baseline $L > 6000$ km is required [6]. As the most important observation, this result does (compared to the mass hierarchy determination) not depend on $\sin^2 2\theta_{13}$ and even holds for $\sin^2 2\theta_{13} = 0$. Note that there are many potential detector locations for $L > 6000$ km – in particular, the “magic baseline” satisfies this requirement (cf., Fig. 2).

Finally, in the limit of large $\sin^2 2\theta_{13}$, Eq. (1) reduces to the first term as a first approximation. As one can read off from Fig. 1 the in this case dominating factor $f_1$ does not drop close to the matter resonance $A \to 1$ even for very long baselines. Note that the $1/L^2$ drop of the flux is already factored out, which means that the probability is proportional to $f_1^2$. However, the further off the resonance, the stronger is the change in the probability. Therefore, this factor becomes very sensitive to the matter density. In principle, it allows a per cent level measurement of the absolute density of the Earth’s core using a vertical NF baseline, as it has been demonstrated in Ref. [7] for $\sin^2 2\theta_{13} > 0.01$ including the correlations with the neutrino oscillation parameters. In addition, as shown in Fig. 2 for the baselines crossing the Earth’s inner core, there are potential locations on land on the other sides of many of the major potential NF laboratories.

In summary, we have demonstrated that there are several potential applications of “very” long NF baselines, and additional ones, such as the mass hierarchy determination for $\sin^2 2\theta_{13} = 0$, are under investigation [8]. Therefore, we conclude that possible muon storage ring configurations should be studied which allow for the simultaneous or subsequent operation of such a baseline in combination with a shorter baseline. Furthermore, the decay tunnel slopes would be a major challenge for such an application.

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