Pseudoscalar decay constants from $N_f = 2 + 1 + 1$ twisted mass lattice QCD

Federico Farchioni
Universität Münster, Institut für Theoretische Physik, Wilhelm-Klemm-Straße 9, 48149 Münster, Germany
E-mail: farchion@uni-muenster.de

Gregorio Herdoiza, Karl Jansen, Andreas Nube
DESY, Platanenallee 6, 15738 Zeuthen, Germany
E-mail: gregorio.herdoiza@desy.de, karl.jansen@desy.de, andreas.nube@desy.de

Marcus Petschlies
Humboldt Universität zu Berlin, Institut für Physik, Newtonstraße 15, 12489 Berlin, Germany
E-mail: marcus.petschlies@physik.hu-berlin.de

Carsten Urbach*
Helmholtz Institut für Strahlen und Kernphysik and Bethe Center for Theoretical Physics, Universität Bonn, Nussallee 14-16, 53115 Bonn, Germany
E-mail: urbach@hiskp.uni-bonn.de

We present first results for the pseudoscalar decay constants $f_K$, $f_D$ and $f_{Ds}$ from lattice QCD with $N_f = 2 + 1 + 1$ flavours of dynamical quarks. The lattice simulations have been performed by the European Twisted Mass collaboration (ETMC) using maximally twisted mass quarks. For the pseudoscalar decay constants we follow a mixed action approach by using so called Osterwalder-Seiler fermions in the valence sector for strange and charm quarks. The data for two values of the lattice spacing and several values of the up/down quark mass is analysed using chiral perturbation theory.

*Speaker.
The decay constants of the Pion, Kaon, the $D$- and the $D_s$-meson are phenomenologically interesting quantities, not least because the ratio $f_K/f_\pi$ together with the well known $|V_{ud}|$ can be used to determine $|V_{us}|$. While experimentally $f_\pi$ and $f_K$ are well known and $f_D$ and $f_{D_s}$ less so, lattice QCD is in principle able to provide calculations of all of these from first principles. And recent advances in the field allow now also for statistically precise determinations with $N_f = 2$ and $N_f = 2 + 1$ dynamical quark flavours [1, 2, 3, 4, 5], with systematic uncertainties more or less under control.

In this proceeding contribution we are going to present yet another determination of the aforementioned decay constants, however with a dynamical charm quark in place, i.e. for QCD with $N_f = 2 + 1 + 1$ quark flavours.

2. Set-up

We use gauge configurations as produced by the European Twisted Mass Collaboration (ETMC) with $N_f = 2 + 1 + 1$ flavours of Wilson twisted mass quarks and Iwasaki gauge action. The set-up is described in ref. [6] and the ensembles used in this investigation are summarised in table 1. The twisted mass Dirac operator in the light – i.e. up/down – sector reads [7]

$$D_\ell = D_W + m_0 + i\mu_\tau \gamma_5 \tau_3 \text{ , }$$

and in the strange/charm sector [8]

$$D_h = D_W + m_0 + i\mu_\sigma \gamma_5 \tau_1 + \mu_\delta \tau_3 \text{ , }$$

where $D_W$ is the Wilson Dirac operator. The value of $m_0$ was tuned to its critical value as discussed in refs. [9, 6] in order to realise automatic $O(a)$ improvement at maximal twist [10]. Note that the bare twisted masses $\mu_\sigma, \delta$ are related to the bare strange and charm quark masses via the relation

$$m_{c,s} = \mu_\sigma \pm (Z_P/Z_S) \mu_\delta \text{ , }$$

with pseudoscalar and scalar renormalisation constants $Z_P$ and $Z_S$.

In the strange and charm quark sector, we use a mixed action approach with Osterwalder-Seiler (OS) valence quarks by formally introducing a twisted doublet for valence strange and charm quark
Pseudoscalar decay constants from $N_f = 2 + 1 + 1 \text{tmLQCD}$

| $\beta$ | $\mu$ | $\mu'$ |
|---------|-------|--------|
| 1.95    | 0.0130, 0.0145, 0.0160, 0.0180, 0.0210 | 0.0110, 0.0120, 0.0130, 0.0150, 0.0180 |
| 2.10    | 0.0145, 0.0120, 0.0130, 0.0150, 0.0180 | 0.0110, 0.0120, 0.0130, 0.0145, 0.0160 |

Table 2: Bare values of the valence quark masses in the strange region for $\beta = 1.95$ and $\beta = 2.10$.

flavours \([11, 12]\), a set-up without flavour mixing artifacts in the valence sector. The two actions in the sea and the valence sector can be matched by tuning the bare values of valence strange and charm such that unitary Kaon and D-meson masses are reproduced. Unfortunately, it turned out that the unitary Kaon and D-meson masses were not exactly tuned to their physical values and currently we are still lacking ensembles which would allow to interpolate to the corresponding physical values. Hence, for the time being we vary the valence quark masses to interpolate the valence Kaon and D-meson masses to their physical values, as discussed below. This approach has been successfully applied to the ETMC $N_f = 2$ flavour gauge configurations in ref. \([4]\).

The determination of the (unitary) K-meson mass is described in ref. \([13]\). Its decay constant can be determined from

$$f_K = (m_\ell + m_s) \frac{\langle 0 | \tilde{P}_K | K \rangle}{m_K^2},$$

where $\tilde{P}_K$ represents the projection to the physical Kaon interpolating operator as discussed in ref \([13]\). $m_s$ is the strange quark masses defined in eq. (2.3). The pseudoscalar decay constant $f_{PS}$ in the valence sector is determined from

$$f_{PS} = \left( \mu^{(1)}_{val} + \mu^{(2)}_{val} \right) \frac{|\langle 0 | P | PS \rangle|}{m_{PS} \sinh m_{PS}},$$

where $P = \bar{q}_1 \gamma_5 q_2$ with quark fields $q_1, q_2$ suitably chosen for the desired quark content. The meson mass $m_{PS}$ and the matrix element $|\langle 0 | P | PS \rangle|$ entering eq. (2.5) have been extracted from a single state fit of the corresponding two-point pseudoscalar correlation function. The replacement of $m_{PS}$ with $\sinh m_{PS}$ in the lattice definition (2.5) of the decay constant helps in reducing discretisation errors for heavy meson masses \([4]\).

Any value for $f_{PS}$ and $m_{PS}$ depends on the mass-value of the light dynamical quark (sea strange and charm quark mass values are fixed for a given $\beta$-value) and two valence quark masses, which we denote by $f_{PS}(\mu^{sea}_{\ell}, \mu^{val}_{1}, \mu^{val}_{2})$. We have investigated several values for the valence quarks: $\mu^{val} = \mu^{sea} = \mu_{\ell}$ and five to six values in the strange and charm quark region $\mu^{val} = \mu_{s,c}$. The ensembles used are summarised in table \([4]\) and the bare values of the valence strange and charm quark masses in tables \([3]\) and \([3]\).

### 3. Results

One may expect large differences in between the unitary set-up and the mixed action set-up at finite values of the lattice spacing. In order to investigate this point we have determined $f_K$ in the
unitary and the mixed action set-up after matching the Kaon mass. The result is shown in figure 1, where we plot $f_K$ as a function of the squared pion mass. Both quantities are in units of $f_0$, the pion decay constant in the chiral limit as determined in ref. [6]. Within errors unitary and mixed set-up agree within errors. This indicates small lattice and small unitarity breaking effects at least in $f_K$. Note that the ratio $Z_P/Z_S$ used for the determination of the unitary $f_K$ has been determined by matching the bare value of the OS strange quark mass to $\mu_\sigma$ and $\mu_\delta$ via eq. (2.3).

In order to determine the physical value of $f_K$ we fit SU(2) $\chi$PT formulae to our pion and Kaon decay constants data [14, 3] simultaneously according to

$$f_{PS}(\mu_\ell, \mu_\ell, \mu_\ell) = f_0 \cdot (1 - 2 \, \xi_{\mu_\ell} \ln \xi_{\mu_\ell} + b \, \xi_{\mu_\ell}),$$

$$f_{PS}(\mu_\ell, \mu_\ell, \mu_\ell) = (f_0^{(K)} + f_m^{(K)} \, \xi_{\mu_\ell}) \cdot \left[ 1 - \frac{3}{4} \, \xi_{\mu_\ell} \ln \xi_{\mu_\ell} + (b_0^{(K)} + b_m^{(K)} \, \xi_{\mu_\ell}) \, \xi_{\mu_\ell} \right],$$

where

$$\xi_{XY} = \frac{m_{PS}(\mu_X, \mu_Y)}{(4\pi f_0)^2}.$$

are expressed in our analysis as a function of meson masses$^1$. We correct our data for finite size effects using NLO $\chi$PT [13, 14]

$$f_{PS}(\mu_\ell, \mu_\ell, \mu_\ell; L) = f_{PS}(\mu_\ell, \mu_\ell, \mu_\ell) \cdot [1 - 2 \, \xi_{\mu_\ell} \, \bar{g}_1(L, \xi_{\mu_\ell})],$$

$$f_{PS}(\mu_\ell, \mu_\ell, \mu_\ell; L) = f_{PS}(\mu_\ell, \mu_\ell, \mu_\ell) \cdot \left[ 1 - \frac{3}{4} \, \xi_{\mu_\ell} \, \bar{g}_1(L, \xi_{\mu_\ell}) \right].$$

We fit to our $\beta = 1.95$ and $\beta = 2.10$ data simultaneously. We do not include lattice artifacts of $O(a^2)$ in our fit, because the amount of data is not sufficient to do so: including these effects lets the fits become unstable. Moreover, the data is fittable without these terms with $\chi^2$/dof = 50/30.

The physical input to our fits is $f_\pi = 130.7$ MeV, $m_\pi = 135$ MeV and $m_K = 497.7$ MeV. In figure 2 we show pion and Kaon decay constants as a function of the squared pion mass $m_\pi^2$. The corresponding Kaon decay constant is determined at a value of $m_{PS}^2(\mu_\ell, \mu_\ell, \mu_\ell) = 2m_K^2 - m_\pi^2$. For the figure we have interpolated our data linearly to these values. As a result we obtain $f_K/f_\pi = 1.224(13), f_K = 160(2)$ MeV and $\ell_4 = 4.78(2)$ with statistical errors only as determined from a bootstrap analysis.

$^1$We use the normalization in which $f_\pi = 130.7$ MeV.
Pseudoscalar decay constants from $N_f = 2 + 1 + 1$ tmLQCD

Carsten Urbach

Figure 1: $f_K/f_0$ as a function of the pion mass squared. Results for the unitary and the mixed action $f_K$ are shown for two values of the lattice spacing.

The fit also allows us to determine the values of the lattice spacings at $\beta = 1.95$ and $\beta = 2.10$ and the value of $f_0$. All these quantities agree very well with the results obtained in ref. [6].

Following the procedure described in ref. [4] we have also analysed the data for $f_D$ and $f_{D_s}$ using SU(2) heavy meson chiral perturbation theory [17]. We consider here the expansions of

$$f_{D_s}\sqrt{m_{D_s}} \quad \text{and} \quad \frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} \quad (3.5)$$

including terms proportional $a^2m_{D_s}^2$ and $1/m_{D_s}$. For details we refer to ref. [4]. Our first results for $f_D$ and $f_{D_s}$ are very encouraging, however, quoting results requires a better control of the systematics involved in this investigation.

4. Summary and Outlook

We have presented the first determination of the pseudoscalar decay constants $f_K$, $f_D$ and $f_{D_s}$ from lattice QCD with dynamical up, down, strange and charm quark flavours. We have used a mixed action approach with Osterwalder-Seiler valence quarks on a maximally twisted mass sea. The analysis indicates small lattice and unitarity breaking artifacts. The preliminary results are $f_K/f_\pi = 1.224(13)$ and $f_K = 160(2)$ MeV obtained with a SU(2) chiral perturbation theory fit. The errors are statistical only. Using the results of ref. [18] this translates to $|V_{us}| = 0.220(2)$.
A comparison of our results for $f_K$ and $f_K/f_\pi$ to results available in the literature shows that to the current level of accuracy there is no difference visible, neither to $N_f = 2$ flavour results [4] nor to $N_f = 2 + 1$ flavour results [3, 2, 1].

A similar analysis has been performed for $f_D$ and $f_D$. In the case of these charmed quantities it will be in particular interesting to understand whether lattice artifacts proportional to $a^2 m_c^2$ are small enough in our lattice set-up to allow for a precise determination of the corresponding quantities. However, the $N_f = 2$ results presented in ref. [4] give rise to optimism that also $f_D$ and $f_D$ can be reliably determined in our set-up.

Clearly the results presented here need a better understanding of the systematic uncertainties. This includes in particular the dependence of the decay constants on the sea strange and charm quark mass values. ETMC is currently producing ensembles to investigate this point. These new ensembles should eventually allow to interpolate to the physical values of $m_K$ and $m_D$. Moreover, more ensembles with different light quark mass values are required for $\beta = 2.10$ to better control lattice artifacts, for which also a third value of the lattice spacing is desirable.

On the analysis site we are currently implementing different fit formulae (see for instance refs. [5, 19]) in order to better understand the extrapolation in the various quark masses.

Acknowledgements

We thank Marc Wagner for useful discussions. We thank the members of ETMC for the most enjoyable collaboration. The computer time for this project was made available to us by the John...
von Neumann-Institute for Computing (NIC) on the JUMP, Europa and Jugene systems in Jülich and apeNEXT system in Zeuthen, BG/P and BG/L in Groningen, by BSC on Mare-Nostrum in Barcelona (www.bsc.es) and by the computer resources made available by CNRS on the BlueGene system at GENCI-IDRIS Grant 2009-052271 and CCIN2P3 in Lyon. We thank these computer centers and their staff for all technical advice and help. This work has been supported in part by the DFG Sonderforschungsbereich TR9 Computergestützte Theoretische Teilchenphysik and the EU Integrated Infrastructure Initiative Hadron Physics (I3HP) under contract RII3-CT-2004-506078.

References

[1] MILC Collaboration, C. Aubin et al., Phys. Rev. D70, 114501 (2004), arXiv:hep-lat/0407028.
[2] HPQCD Collaboration, E. Follana, C. T. H. Davies, G. P. Lepage and J. Shigemitsu, Phys. Rev. Lett. 100, 062002 (2008), arXiv:0706.1726 [hep-lat].
[3] RBC-UKQCD Collaboration, C. Allton et al., Phys. Rev. D78, 114509 (2008), arXiv:0804.0473 [hep-lat].
[4] B. Blossier et al., JHEP 07, 043 (2009), arXiv:0904.0954 [hep-lat].
[5] S. Dürr et al., Phys. Rev. D81, 054507 (2010), arXiv:1001.4692 [hep-lat].
[6] ETM Collaboration, R. Baron et al., JHEP 06, 111 (2010), arXiv:1004.5284 [hep-lat].
[7] ALPHA Collaboration, R. Frezzotti, P. A. Grassi, S. Sint and P. Weisz, JHEP 08, 058 (2001), hep-lat/0101001.
[8] R. Frezzotti and G. C. Rossi, Nucl. Phys. Proc. Suppl. 128, 193 (2004), hep-lat/0311008.
[9] T. Chiarappa et al., Eur. Phys. J. C50, 373 (2007), arXiv:hep-lat/0606011.
[10] R. Frezzotti and G. C. Rossi, JHEP 08, 007 (2004), hep-lat/0306014.
[11] R. Frezzotti and G. C. Rossi, JHEP 10, 070 (2004), arXiv:hep-lat/0407002.
[12] ETM Collaboration, B. Blossier et al., JHEP 04, 020 (2008), arXiv:0709.4574 [hep-lat].
[13] ETM Collaboration, R. Baron et al., arXiv:1005.2042 [hep-lat].
[14] J. Gasser and H. Leutwyler, Ann. Phys. 158, 142 (1984).
[15] J. Gasser and H. Leutwyler, Phys. Lett. B188, 477 (1987).
[16] D. Becirevic and G. Villadoro, Phys. Rev. D69, 054010 (2004), arXiv:hep-lat/0311028.
[17] S. R. Sharpe and Y. Zhang, Phys. Rev. D53, 5125 (1996), arXiv:hep-lat/9510037.
[18] W. J. Marciano, Phys. Rev. Lett. 93, 231803 (2004), arXiv:hep-ph/0402293.
[19] V. Bernard, S. Descotes-Genon and G. Toucas, arXiv:1009.5066 [hep-ph].