Exponential damping induced by random and realistic perturbations

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Given a quantum many-body system and the expectation-value dynamics of some operator, we study how this reference dynamics is altered due to a perturbation of the system’s Hamiltonian. Based on projection operator techniques, we unveil that if the perturbation exhibits a random-matrix structure in the eigenbasis of the unperturbed Hamiltonian, then this perturbation effectively leads to an exponential damping of the original dynamics. Employing a combination of dynamical quantum typicality and numerical linked cluster expansions, we demonstrate that our theoretical findings are relevant for the dynamics of realistic quantum many-body models. Specifically, we study the decay of current autocorrelation functions in spin-1/2 ladder systems, where the runs of the ladder are treated as a perturbation to the otherwise uncoupled legs. We find a convincing agreement between the exact dynamics and the lowest-order prediction over a wide range of interchain couplings, even if the perturbation is not weak.

Introduction. Understanding the dynamics of interacting quantum many-body systems is notoriously challenging. While their complexity grows exponentially in the number of degrees of freedom, the strong correlations between the constituents often prohibit any exact solution. Although much progress has been made due to the development of powerful numerical machinery [1] and the advance of controlled experimental platforms [2, 3], the detection of general (i.e. universal) principles which underline the emerging many-body dynamics is of fundamental importance [4]. To this end, a remarkably successful strategy in the past has been the usage of random-matrix ensembles instead of treating the full many-body problem. Ranging back to the description of nuclei spectra [5] and of quantum chaos in systems with classical counterparts [6], random-matrix theory also forms the backbone of the celebrated eigenstate thermalization hypothesis [7–9], which provides a microscopic explanation for the emergence of thermodynamic behavior in isolated quantum systems. More recently, random-circuit models have led to new insights into the scrambling of information and the onset of hydrodynamic transport in quantum systems undergoing unitary time evolution [10–12].

Concerning the out-of-equilibrium dynamics of quantum many-body systems, a particularly intriguing question is how the expectation-value dynamics of some operator is altered if the system’s Hamiltonian is modified by a (small) perturbation. Clearly, the effect of such a perturbation in a (integrable or chaotic) system can be manifold. In the context of prethermalization [13–18], the perturbation breaks a conservation law of the (usually integrable) reference Hamiltonian, leading to a separation of time scales, where the system stays close to some long-lived nonthermal state, before eventually giving in to its thermal fate at much longer times. Moreover, in the study of echo protocols, time-local perturbations have been shown to entail irreversible quantum dynamics [19], analogous to the butterfly effect in classical chaotic systems. Furthermore, the observation that some types of temporal relaxation, such as the exponential decay, are more common than others can be traced back to their enhanced stability against perturbations [20].

In this Letter, we build upon the success of random-matrix models and consider perturbations with a random matrix structure in the eigenbasis of the unperturbed Hamiltonian. Based on projection operator techniques, we unveil that, in such a case, the perturbation effectively leads to an exponential damping of the original expectation-value dynamics, similar to recent results in Ref. [21] (see also the examples [22–25] discussed therein). Employing state-of-the-art numerics, we demonstrate that our theoretical findings are readily applicable to realistic quantum many-body models.

Projection-operator approach. We consider a closed quantum many-body system $H_0$ which is affected by a perturbation $\mathcal{V}$, such that the total Hamiltonian reads $H = H_0 + \lambda \mathcal{V}$, where $\lambda$ denotes the strength of the perturbation. Moreover, we study how the expectation-value dynamics of some operator, $\langle \mathcal{O}(t) \rangle = \text{Tr} [\rho(t) \mathcal{O}]$, is altered due to the presence of the perturbation (compared to its bare dynamics under $H_0$), where $\rho(t) = e^{-i\mathcal{H}t} \rho(0) e^{i\mathcal{H}t}$, and $\rho(0)$ is a (mixed or pure) initial state. We tackle this question by means of a projection-operator approach, i.e., the so-called time-convolutionless (TCL) method (see [26, 27] for a comprehensive overview).

In order to simplify the following derivations, let us assume that $H_0$ has a very high and almost uniform density of states $\Omega \approx 1/\Delta \omega$, where $\Delta \omega$ is the mean
level spacing. Moreover, we shift the true eigenvalues of $H_0$ slightly, such that they result as $E_\omega = \Delta \omega - \omega$, with $\omega$ being an integer. For times $t < 2\pi/\Delta \omega$, the dynamics of this “shifted” $H_0$ should be indistinguishable from the original one. Furthermore, we define a Fourier component of the operator $O$ in the eigenbasis of $H_0$, $O_\omega = 1/\sqrt{\omega} \sum_q \langle \eta \rangle Q_{\eta, q + \omega}(\eta \omega) + \text{h.c.}$, with normalization $z_\omega = \sum_q |Q_{\eta, q + \omega}|^2$, and construct a set of corresponding projection operators $P_{\omega}$, which project onto the relevant part of the density matrix $\rho(t)$, $P_\omega \rho(t) = \text{Tr}(\rho(t)O_\omega)O_\omega$. Note that, from the definition of $O_\omega$, it immediately follows that $\langle O(t) \rangle = \sum_\omega \sqrt{\omega} \langle O_\omega(t) \rangle \cos(\Delta \omega \cdot \omega t)$. In particular, the dynamics of $O_\omega$ in the Schrödinger picture and in the interaction picture (subscript $I$) are related by $\text{Tr}(O_\omega \rho(t)) = \text{Tr}(O_\omega \rho_I(t)) \cos(\Delta \omega \cdot \omega t)$. Tactically focusing on initial states with $P_\omega \rho(0) = \rho(0)$ [28], the TCL framework then yields a time-local equation for $O_{1,t}(t) = \text{Tr}(O_\omega \rho_I(t))$, comprising a systematic perturbation expansion in powers of $\lambda$,

$$\dot{O}_{1,t}(t) = -\gamma_{\omega,t}(t)O_{1,t}(t); \; \gamma_{\omega,t}(t) = \sum_n \lambda^n \langle \omega_n(t) \rangle, (1)$$

where the $\gamma_{\omega,n}(t)$ are time-dependent rates of $n$-th order. Since the odd orders of this expansion vanish for many models, the leading-order term is $\gamma_{\omega,2}(t) = \int_0^t dt' \text{K}_{\omega,2}(t') = \int_0^t dt' \text{Tr} \{\text{i} [O_{\omega}, \mathcal{V}(t') \text{i}[O_{\omega}, \mathcal{V}] \}, \text{where } \mathcal{V}(t) = e^{iH_0t}V e^{-iH_0t}$, and the second-order kernel can be rewritten as $\text{K}_{\omega,2}(t) = \tilde{K}_{\omega,2}(t) + \tilde{K}_{\omega,2}(t)$, with

$$\tilde{K}_{\omega,2}(t) = \text{Tr} \left[ -O_{\omega} \mathcal{V}(t) O_{\omega} \mathcal{V} - \mathcal{V}(t) O_{\omega} \mathcal{V} O_{\omega} \right], \quad \tilde{K}_{\omega,2}(t) = \text{Tr} \left[ O_{\omega} \mathcal{V}(t) O_{\omega} \mathcal{V} + O_{\omega} \mathcal{V} \mathcal{V}(t) O_{\omega} \right]. \quad (2)$$

Let us now assume that $\mathcal{V}$ essentially is a random (and possibly banded) matrix in the eigenbasis of the unperturbed system $H_0$. In this case, the terms in (2) consist of sums in which each addend carries a product of two uncorrelated random numbers. If the random numbers have mean zero, these sums should be negligible, $\tilde{K}_{\omega,2}(t) \approx 0$. In contrast, the terms in (3) do contribute, and we find

$$\tilde{K}_{\omega,2}(t) = \frac{4}{\omega} \sum_{\eta, \kappa} |\mathcal{V}_{\kappa, \eta}|^2 |O_{\kappa, \eta + \omega}|^2 \cos((\kappa - \eta)\Delta \omega t) \approx \frac{4\Omega^2}{\omega} \sum_{\eta} \int_0^W |O_{\kappa, \eta + \omega}|^2 \cos(\lambda \eta) \text{d} \chi \quad (4)$$

$$= \frac{2\Omega^2}{\omega} \int_0^W \cos(\chi \omega) \text{d} \chi = 4\Omega^2 \frac{\sin(W\omega)}{W}. \quad (5)$$

Several comments are in order. Since $\mathcal{V}$ is a random matrix, we have approximated in Eq. (4) all squared individual matrix elements by their averages $\langle \omega \rangle$, i.e., $|\mathcal{V}_{\kappa, \eta}|^2 \approx \langle \eta \rangle$. Furthermore, we have used an index shift $\kappa \to \chi + \eta$ and converted the original sum over $\kappa$ to an integral, where $W$ denotes the half-bandwidth of $\mathcal{V}$. From (4) to (5), we have exploited that the half-bandwidth can be evaluated independently and used the definition of $z_\omega$. Inserting (5) into the definition of $\gamma_{\omega,2}(t)$ then yields $\gamma_{\omega,2}(t) \approx 4\Omega^2 \frac{\int_0^W \cos(W\eta) \text{d} \eta}{W} \approx \frac{2\pi \Omega^2}{\omega}$, for times $t \gg \pi/\omega$. Thus, $\gamma_{\omega,2}(t)$ is essentially a constant (Fermi’s Golden Rule) rate, and we abbreviate $\gamma = 2\pi \Omega^2$. Moreover, it follows from Eq. (1) that $O_{1,t}(t) = \langle O_{\omega}(0) \rangle e^{-\lambda^2 \gamma t}$. Transforming back into the Schrödinger picture and comparing the expressions for $\langle O_{\omega}(t) \rangle$ and $\langle O_{\omega}(t) \rangle_{H_0}$ [above Eq. (1)], we find

$$\langle O(t) \rangle = \langle O(t) \rangle_{H_0} e^{-\lambda^2 \gamma t}. \quad (6)$$

This exponential damping of expectation-value dynamics induced by random perturbations is a central result of the present Letter, and consistent with recent results in Ref. [21]. While our derivations are rigorous up to second order in the perturbation strength [28], the evaluation of higher order corrections is a daunting task in practice [29] and, for random matrices, these corrections are expected to be irrelevant [30, 31]. Hence, apart from this truncation, the most crucial point is whether or not a random-matrix description is indeed justified for realistic physical perturbations (see also [32–34]). Therefore, let us exemplify that our findings are indeed applicable to generic quantum many-body systems.

**Numerical Illustration.** As an example, we study a (quasi-)one-dimensional spin-1/2 lattice model with ladder geometry [35–39], where the rung part of the ladder is treated as a perturbation to the otherwise uncoupled legs, i.e., the Hamiltonian reads $H = J_\parallel H_\parallel + J_\perp \mathcal{V}$, with

$$H_0 = \sum_{l=1}^L \sum_{k=1}^2 S_{l,k} \cdot S_{l+1,k}; \; \mathcal{V} = \sum_{l=1}^L S_{l,1} \cdot S_{l,2}. \quad (7)$$

Here, $S_{l,k} = (S_{l,k}^x, S_{l,k}^y, S_{l,k}^z)$ are spin-1/2 operators, $J_\parallel$ ($J_\perp$) is the coupling constant on the legs (rungs), and $L$ denotes the length of the ladder. While, for $J_\perp = 0$, $H$ consists of two separate Heisenberg chains and is integrable, this integrability is broken for any $J_\perp \neq 0$.

Let us study the current autocorrelation function $C(t) = \langle j(t) j(t) \rangle / L = \text{Tr} \rho_{eq} \langle j(t) j(t) \rangle / L$ [40], where $\rho_{eq} = e^{-\beta H}/Z$ is the canonical density matrix, $\beta = 1/T$ denotes the inverse temperature, and $j(t) = e^{iH_0t} j e^{-iH_0t}$. The spin-current operator $j$ follows from a lattice continuity equation [43], and is given by $j = J_\parallel \sum_k S_{l,k}^x S_{l+1,k}^y - S_{l,k}^y S_{l+1,k}^x$. Importantly, $j$ is independent of the perturbation $\mathcal{V}$. The correlation function $C(t)$ is an important quantity in the context of transport. Despite the integrability of $H_0$, the dynamics of $C(t)$ is nontrivial even for $J_\perp = 0$ [44]. While $C(t)$ has been numerically studied by various methods [38, 45, 46], we here rely on a powerful combination of dynamical quantum typicality (DQT) [47–57] and numerical linked cluster expansions (NLCE) [58, 59], recently put forward by two of us [60].
On the one hand, the concept of DQT relies on the fact that a single pure quantum state can imitate the full statistical ensemble. Specifically, \( C(t) \) can be written as a scalar product with the two pure states \( |\psi_\beta(t)\rangle = e^{-iHt}|\varphi_\beta\rangle \) and \( |\varphi_\beta(t)\rangle = e^{-iHt}|\varphi_\beta\rangle \), i.e.,

\[
C(t) = \langle \psi_\beta(t) | j | \psi_\beta(t) \rangle / L \langle \varphi_\beta | \varphi_\beta \rangle + \epsilon \quad [55, 56],
\]

where \( |\varphi_\beta\rangle = e^{-iHt/2}|\varphi\rangle \), and \( |\varphi\rangle \) is randomly drawn (Haar measure [51]) from the full Hilbert space with dimension \( D = 4^L \). Importantly, the statistical error \( \epsilon = \epsilon(|\varphi\rangle) \) vanishes as \( \epsilon \propto 1/\sqrt{D} \) (for \( \beta = 0 \)), and the typicality approximation becomes practically exact already for moderate values of \( L \). Since the time evolution of pure states can be conveniently evaluated by iteratively solving the real-time Schrödinger equation, e.g., by means of fourth-order Runge-Kutta [55, 56] or a Trotter product formula [61], it is possible to treat large \( D \), out of reach for standard exact diagonalization (ED).

On the other hand, NLCE provides a means to obtain \( C(t) \) directly in the thermodynamic limit \( L \to \infty \). Specifically, the current autocorrelation is calculated as the sum of contributions from all connected clusters which can be embedded on the lattice [58], \( \langle j(t)j(t)\rangle_{\text{eq}} / L = \sum_c L_c W_c(t) \), where \( W_c(t) \) is the weight of cluster \( c \) with multiplicity \( L_c \). Moreover, the weights \( W_c(t) \) are obtained by the so-called inclusion-exclusion principle,

\[
W_c(t) = \langle j(t)j(t)\rangle_{\text{eq}}^{(c)} - \sum_{c' < c} W_{c'}(t), \quad \langle j(t)j(t)\rangle_{\text{eq}}^{(c)}
\]
denotes the (extensive) current autocorrelation evaluated on the cluster \( c \) (with open boundary conditions), and the sum runs over all subclusters of \( c \). In practice, this series has to be truncated to a maximum cluster size \( c_{\text{max}} \). This truncation in turn leads to a breakdown of convergence of \( C(t) \) at some time \( \tau \), where a larger \( c_{\text{max}} \) leads to a longer \( \tau \), see also [60, 62]. Thanks to DQT, we can evaluate \( \langle j(t)j(t)\rangle_{\text{eq}} \) on large clusters and obtain \( C(t) \) in the thermodynamic limit for long times. While we here focus on \( \beta = 0 \), both DQT and NLCE allow for accurate calculations of \( C(t) \) at \( \beta > 0 \) as well [56, 57, 60].

Unperturbed Dynamics. First, we study \( C(t) \) in the unperturbed system \( H_0 \), i.e., in the Heisenberg chain. (\( L \) now denotes the length of a single chain.) In Fig. 1 (a), \( \langle j(t)j(t)\rangle / L \) is shown for periodic boundary conditions (PBC), obtained by ED (\( L = 18 \)) and DQT (\( L = 32, 34, 36 \)) [56, 63]. While the curves for different \( L \) coincide at short times, the ED curve starts to deviate from the DQT data for \( t \gtrsim 8 \). Moreover, for \( t \gtrsim 20 \), \( C(t) \) takes on an approximately constant value which decreases with increasing \( L \) [56]. Next, Fig. 1 (b) shows \( C(t) \) for open boundary conditions (OBC), and additional system sizes \( L = 38, 39 \). Compared to the previous PBC data in Fig. 1 (a), the convergence towards the thermodynamic limit is considerably slower for OBC, and finite-size effects are more pronounced.

We now come to our NLCE results. In Fig. 1 (c), \( C(t) \) is shown for various expansion orders \( c_{\text{max}} \leq 39 \). For increasing \( c_{\text{max}} \), we find that \( C(t) \) is converged up to longer and longer times, until the expansion eventually breaks down. As a comparison, we also depict data obtained by the time-dependent density matrix renormalization group (tDMRG) [38, 64]. As becomes evident from Fig. 1 (c), tDMRG and NLCE agree perfectly for times \( t \lesssim 27 \). However, for the largest \( c_{\text{max}} = 39 \) considered, the NLCE data actually is converged up to significantly larger times \( t \approx 40 \). This fact demonstrates that the combination of DQT and NLCE provides a powerful numerical approach to real-time correlation functions, and in comparison to Fig. 1 (a), also outperforms standard finite-size scaling on short to intermediate time scales. This determination of the unperturbed dynamics is important for this Letter and also extends earlier results presented in Ref. [60]. We have checked that NLCE data of similar quality can be obtained for other integrable models as well [32].

Perturbed Dynamics. Next, we come to the actual discussion of \( C(t) \) in spin ladders. While Fig. 2 (a) shows the second-order kernel \( K_2(t) = \text{Tr}\{j[j, V_j(t)]\} / \text{Tr}\{j^2\} \), the respective decay rate \( \gamma_2(t) = \int_0^t \text{d}t' K_2(t') \) is shown in Fig. 2 (b) for various \( L \leq 15 \). On the one hand, for short times \( t \lesssim 1 \), we observe a linear growth \( \gamma_2(t) \propto t \). On the other hand, for \( t \gtrsim 2 \), we find that \( \gamma_2(t) \) first decreases slightly, but then starts to increase again. However, by comparing \( \gamma_2(t) \) for different \( L \), the latter apparently is a finite-size effect, and we expect the decay rate to be

![FIG. 1. (Color online) C(t) for J_\perp = 0 at β = 0, obtained by ED (L = 18) and DQT (L ≤ 39) for (a) PBC and (b) OBC. Here denotes the length of a single chain. (c) C(t) obtained by NLCE up to expansion order c_{max} ≤ 39. As a comparison, we depict tDMRG data [38].](https://www.example.com/fig1.png)
Comparing data for (c) and (b), respectively. Comparing data for two different system sizes, consistent with recent results in Ref. [32]. Let us now compare this temporal decay of $C(t)$ to our prediction of an exponential damping. To this end, the unperturbed correlation function $C_0(t)$ as obtained by NLCE [Fig. 1 (c)] is exponentially damped according to

$$C(t) = C_0(t) \exp \left[ -J_{\perp}^2 \int_0^t dt' \gamma_2(t') \right],$$  

where we have manually set $\gamma_2(t \geq 8) \approx 0.67$ (see discussion above). Due to the linear growth of $\gamma_2(t)$ at short times, Eq. (8) leads to a Gaussian damping $\propto e^{-J_{\perp}^2 t^2}$ for $t \lesssim 1$, and turns into a conventional exponential damping $\propto e^{-bJ_{\perp}^2 t}$ for longer $t$. Remarkably, we find that Eq. (8) agrees very well with the exact $C(t)$ for all values of $J_{\perp}$ shown here, even though the perturbation is not weak. In particular, let us stress that there is no free parameter involved. This is a central result and confirms our theoretical arguments based on random matrices.

In order to corroborate our findings further, let us study another but similar model. Namely, we consider the dynamics of $C(t)$ in the XX ladder, i.e., $H_0$ and $V$ are almost identical to Eq. (7), but the $S_i^x S_j^x$-terms are absent. Note that, in this case, $J$ is exactly conserved in the unperturbed system, $[H_0, J] = 0$, and the decay of $C(t)$ is solely due to $V$. Analogous to Fig. 2, the second-order kernel $K_2(t)$ and the decay rate $\gamma_2(t)$ are depicted in Figs. 3 (a) and (b), respectively. Comparing data for $L = 9, 15$, we observe that finite-size effects are negligible, and $\gamma_2(t) \approx 0.63$ becomes essentially constant for times $t \gtrsim 2$ [65]. Next, Fig. 3 (c) shows $C(t)$ for two different system sizes $L \leq 18$ (symbols), obtained by DQT for $J_{\perp}/J_{\parallel} = 1/4, 1/2, 1$ [66], as well as the the lowest-order prediction (8) (curves). (Recall that $C_0(t) = \text{const.}$ in this case.) Similar to Fig. 2 (c), we find that Eq. (8) describes the decay of $C(t)$ remarkably well, albeit the agreement is certainly better for smaller $J_{\perp}/J_{\parallel}$.

**Conclusion.** How does the expectation-value dynamics of some operator changes under a perturbation of the system’s Hamiltonian? Based on projection operator techniques, we have answered this question for the case of a perturbation with random-matrix structure in the eigenbasis of the unperturbed Hamiltonian. As a main result, we have unveiled that such a (small) perturbation yields an exponential damping of the original reference dynamics, consistent with recent results in Ref. [21].

In addition, we have numerically confirmed that our findings are readily applicable to generic quantum many-body systems. In particular, we have illustrated that even a truncation to second order in the perturbation still provides a convincing description of the exact dynamics, also in cases where the perturbation is not weak.
Since an entirely random matrix structure of realistic physical perturbations might be questionable, promising directions of research include the identification of relevant substructures, as well as a better understanding of the pertinent correlations between matrix elements [32, 34].

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[1] U. Schollwöck, Ann. Phys. **326**, 96 (2011).
[2] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008).
[3] T. Langen, R. Geiger, and J. Schmiedmayer, Ann. Rev. Condens. Matter Phys. **6**, 201 (2015).
[4] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, Rev. Mod. Phys. **83**, 863 (2011).
[5] T. A. Brody, J. Flores, J. B. French, P. A. Mello, A. Pandey, and S. S. M. Wong, Rev. Mod. Phys. **53**, 385 (1981).
[6] O. Bohigas, M. J. Giannoni, and C. Schmit, Phys. Rev. Lett. **52**, 1 (1984).
[7] J. M. Deutsch, Phys. Rev. A **43**, 2046 (1991).
[8] M. Srednicki, Phys. Rev. E **50**, 888 (1994).
[9] M. Rigol, V. Dunjko, and M. Olshanii, Nature **452**, 854 (2008).
[10] C. W. von Keyserlingk, T. Rakovszky, F. Pollmann, and S. L. Sondhi, Phys. Rev. X **8**, 021013 (2018).
[11] A. Nahum, S. Vijay, and J. Haah, Phys. Rev. X **8**, 021014 (2018).
[12] V. Khemani, A. Vishwanath, and D. A. Huse, Phys. Rev. X **8**, 031057 (2018).
[13] J. Berges, Sz. Borsányi, and C. Wetterich, Phys. Rev. Lett. **93**, 142002 (2004).
[14] M. Moeckel and S. Kehrein, Phys. Rev. Lett. **100**, 175702 (2008).
[15] B. Bertini, F. H. L. Essler, S. Groha, and N. J. Robinson, Phys. Rev. Lett. **115**, 180601 (2015).
[16] T. Mori, T. N. Ikeda, E. Kaminishi, and M. Ueda, J. Phys. B **51**, 112001 (2018).
[17] P. Reimann and L. Dabelow, Phys. Rev. Lett. **122**, 080603 (2019).
[18] K. Mallayya, M. Rigol, and W. De Roeck, Phys. Rev. X **9**, 021027 (2019).
[19] M. Schmitt and S. Kehrein, Phys. Rev. B **98**, 180301(R) (2018).
[20] L. Knipschild and J. Gemmer, Phys. Rev. E **98**, 062103 (2018).
[21] L. Dabelow and P. Reimann, arXiv:1903.11881.
[22] A. Flesch, M. Cramer, I. P. McCulloch, U. Schollwöck, and J. Eisert, Phys. Rev. A **78**, 033608 (2008).
[23] S. Trotzky, Y-A. Chen, A. Flesch, I. P. McCulloch, U. Schollwöck, J. Eisert, and I. Bloch, Nat. Phys. **8**, 325 (2012).
[24] P. Barmettler, M. Funk, V. Gritsev, E. Demler, and E. Altman, Phys. Rev. Lett. **102**, 130603 (2009).
[25] K. Balzer, F. A. Wolf, I. P. McCulloch, P. Werner, and M. Eckstein, Phys. Rev. X **5**, 031039 (2015).
[26] S. Chaturvedi and F. Shibata, Z. Phys. B **35**, 297 (1979).
[27] H.-P. Breuer and F. Petruccione, The Theory of Open Quantum Systems (Oxford University Press, New York, 2007).
[28] Note that Eq. (1) strictly holds if there is no “inhomogeneity”, i.e., we have to demand \( \rho_p(t) = \rho(0) \). Even if this is not the case, the violation of (1) is expected to be minor for small \( \lambda \) [27].
[29] R. Steinigeweg and T. Prosen, Phys. Rev. E **87**, 050103(R) (2013).
[30] C. Bartsch, R. Steinigeweg, and J. Gemmer, Phys. Rev. E **77**, 011119 (2008).
[31] For time-dependent problems, a truncation to lowest order is in general (for nonrandom matrices) not meaningful a priori, even if the perturbation is small. For small perturbations, the relevant time scales are long and higher orders can become relevant on these time scales.
[32] See Supplemental Material for details on the matrix structure of the specific perturbation under consideration, as well as for additional NLCE data on the integrable Hubbard chain and the nonintegrable Heisenberg and XX ladders.
[33] J. Richter, J. Gemmer, and R. Steinigeweg, Phys. Rev. E **99**, 050104(R) (2019).
[34] L. Foini and J. Kurchan, Phys. Rev. E **99**, 042139 (2019).
[35] X. Zotos, Phys. Rev. Lett. **92**, 067202 (2004).
[36] P. Jung, R. W. Helmes, and A. Rosch, Phys. Rev. Lett. 96, 067202 (2006).
[37] M. Žnidarič, Phys. Rev. B **88**, 205135 (2013).
[38] C. Karrasch, D. M. Kennes, and F. Heidrich-Meisner, Phys. Rev. B **91**, 115130 (2015).
[39] R. Steinigeweg, J. Herbachy, X. Zotos, and W. Brenig, Phys. Rev. Lett. **116**, 017202 (2016).
[40] Here, the current autocorrelation \( C(t) \) can also be thought of as the expectation-value dynamics \( \langle j(t) \rangle \), resulting from an initial state \( \rho(0) \propto 1 + \varepsilon j \). Note that, in this case, there is no inhomogeneity [28, 41, 42].
[41] R. Steinigeweg and R. Schnalle, Phys. Rev. E **82**, 040103(R) (2010).
[42] R. Steinigeweg, Phys. Rev. E **84**, 011136 (2011).
[43] F. Heidrich-Meisner, A. Honecker, and W. Brenig, Eur. Phys. J. Spec. Top. **151**, 135 (2007).
[44] J. Sirker, R. G. Pereira, and I. Affleck, Phys. Rev. Lett. **103**, 216602 (2009).
[45] C. Karrasch, J. H. Bardarson, and J. E. Moore, Phys. Rev. Lett. **108**, 227206 (2012).
[46] J. Richter, F. Jin, L. Knipschild, J. Herbachy, H. De Raedt, K. Michielsen, J. Gemmer, and R. Steinigeweg, Phys. Rev. B **99**, 144422 (2019).
[47] J. Gemmer, M. Michel, and G. Mahler, Quantum Thermodynamics (Springer, Berlin, 2004).
[48] S. Popescu, A. J. Short, and A. Winter, Nat. Phys. **2**, 754 (2006).
[49] S. Goldstein, J. L. Lebowitz, R. Tumulka, and N. Zanghì, Phys. Rev. Lett. **96**, 050403 (2006).
[50] P. Reimann, Phys. Rev. Lett. **99**, 160404 (2007).
In fact, since the XX chain can be brought into a quadratic form, $K_2(t)$ and $\gamma_2(t)$ could in principle even be obtained analytically for this particular case (see also [42]).
SUPPLEMENTAL MATERIAL

MATRIX STRUCTURE OF THE PERTURBATION

Let us study if the realistic physical perturbation from the main part of this Letter is indeed compliant with a random-matrix description. Referring to Eq. (7), we have considered a spin-1/2 lattice model with ladder geometry, where the rung part of the ladder is treated as a perturbation, i.e., the Hamiltonian reads $H = J_\parallel H_0 + J_\perp V$, with

$$
H_0 = \sum_{l=1}^{L} \sum_{k=1}^{2} S_{l,k} \cdot S_{l+1,k} ; \quad V = \sum_{l=1}^{L} S_{l,1} \cdot S_{l,2}.
$$

(S1)

In Fig. S1, the matrix representation of $V$ in the eigenbasis of $H_0$ is illustrated, where we restrict ourselves to a single symmetry subsector with magnetization $S^z = -1$, momentum $k = 2\pi/L$, and even parity $p = 1$. (Both $H_0$ and $V$ are entirely real in this case.) Moreover, we employ a suitable coarse graining according to

$$
g(E, E') = \frac{\sum_{mn} |V_{mn}|^2 D(E)}{D(E) D(E')},
$$

(S2)

where the sum runs over matrix elements $V_{mn}$ in two energy shells of width $2\delta E$, $E_n \in [E - \delta E, E + \delta E]$ and $E_m \in [E' - \delta E, E' + \delta E]$. $D(E)$, $D(E')$, and $D(E)$ denote the number of states in these energy windows. This rough structure of $V$ is shown in Figs. S1 (a) and (b), both for the Heisenberg ladder and the XX ladder. In both cases, we find that $V$ exhibits a banded matrix structure with more spectral weight close to the diagonal. However, especially in the case of the XX ladder, $g(E, E')$ is not homogeneous within this band, but rather exhibits some fine structure.

For a more detailed analysis, a close-up of $200 \times 200$ matrix elements $V_{mn}$ is shown in Figs. S1 (c) and (d). We find that there is a coexistence between regions where the $V_{mn}$ appear to be random, and regions where the $V_{mn}$ vanish (e.g. due to additional conservation laws). Moreover, in the case of the XX ladder, these regions are more extended.

FIG. S1. (Color online) Matrix structure of $V$ in the eigenbasis of $H_0$ in the symmetry subsector with magnetization $S^z = -1$, momentum $k = 2\pi/L$, and even parity $p = 1$, both for the Heisenberg ladder (top) and the XX ladder (bottom). We have $L = 9$ in all cases.
Thus, the realistic $V$ is certainly not an entirely random matrix. Nevertheless, as demonstrated in the main part, our theoretical arguments still apply remarkably well and $V$ causes to good quality a damping of the unperturbed dynamics according to lowest-order perturbation theory. This can be understood due to the fact that bare randomness is not necessarily crucial for a successful truncation within the TCL approach. Rather, $V$ is required to exhibit a so-called “Van Hove” structure [S2].

NLCE DATA FOR ANOTHER INTEGRABLE MODEL

In the main text, we have used a combination of DQT and NLCE to calculate the unperturbed dynamics of the spin-current autocorrelation in the integrable spin-1/2 Heisenberg chain. This combination has allowed us to obtain numerically exact information on rather long time scales, which cannot be reached in direct calculations in systems with periodic or open boundary conditions, due to significant finite-size effects. To illustrate that this combination of DQT and NLCE can yield also for other integrable models the reference dynamics with a similar quality, we show additional data for the Fermi-Hubbard chain, described by the Hamiltonian $H = \sum_{l=1}^{L} h_l$,

$$h_l = -t_h \sum_{s=\downarrow,\uparrow} (a_{l,s}^\dagger a_{l+1,s} + \text{H.c.}) + U(n_{l,\downarrow} - \frac{1}{2})(n_{l,\uparrow} - \frac{1}{2}),$$  \hspace{1cm} (S3)

where the operator $a_{l,s}^\dagger (a_{l,s})$ creates (annihilates) at site $l$ a fermion with spin $s$, $t_h$ is the hopping matrix element, and $L$ is the number of sites. The operator $n_{l,s}$ is the local occupation number and $U$ is the strength of the on-site interaction. For this model, we consider the particle current $j = \sum_{l=1}^{L} j_l$,

$$j_l = -t_h \sum_{s=\downarrow,\uparrow} (ia_{l,s}^\dagger a_{l+1,s} + \text{H.c.}),$$  \hspace{1cm} (S4)

and summarize our numerical results for the corresponding autocorrelation and $U=4$ in Fig. S2. Apparently, the situation is like the one in Fig. 1 of the main text. On the one hand, in direct calculations with open or periodic boundary conditions, strong finite-size effects set in at short times, even for quite large $L$. On the other hand, NLCE for the largest expansion order $c_{\text{max}}$ is converged to substantially longer times. Even though not shown explicitly, we have checked that a good convergence is also reached for $U=8$. We thus expect that a perturbative analysis, as presented in this work, can be carried out for a wide class of quantum many-body systems, which we plan to do in detail in future work.

NLCE DATA FOR NONINTEGRABLE MODELS

While it is certainly possible to use NLCE also for nonintegrable models, finite-size effects in direct calculations are much less pronounced in these models, as evident from Figs. 2 and 3 of the main text. This is why we have not shown corresponding NLCE data in these figures and instead relied on pure DQT data for systems with periodic boundary conditions. To demonstrate that these DQT data are indeed in excellent agreement with NLCE data, we show in Fig. S3 corresponding numerical results for the Heisenberg and XX ladder, in both cases for $J_\perp/J_\parallel = 1$.

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[S1] J. Richter, J. Gemmer, and R. Steinigeweg, Phys. Rev. E 99, 050104(R) (2019).
[S2] C. Bartsch, R. Steinigeweg, and J. Gemmer, Phys. Rev. E 77, 011119 (2008).
[S3] C. Karrasch, D. M. Kennes, and J. E. Moore, Phys. Rev. B 90, 155104 (2014).
FIG. S2. (Color online) Current autocorrelation $C(t)$ in the Fermi-Hubbard chain ($U = 4$), obtained by ED ($L = 8$) and DQT ($L \leq 19$) at $\beta = 0$ for (a) periodic boundary conditions (PBC) and (b) open boundary conditions (OBC). (c) $C(t)$ obtained by NLCE up to expansion order $c_{\text{max}} \leq 19$. As a comparison, we depict data from the time-dependent density matrix renormalization group (tDMRG) [S3].

FIG. S3. (Color online) Current autocorrelation $C(t)$ for the (a) Heisenberg ladder and (b) XX ladder, in both cases for $J_\perp / J_\parallel = 1$. DQT data for periodic boundary conditions, as shown in the main text, is compared to NLCE data for two expansion orders $c_{\text{max}} = 18$ and 19.