On the need to calculate the stability of the quantum frequency standard on the rubidium gas cell for navigation systems

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Abstract. The article presents the methods for calculating the stability of the rubidium frequency standards. To implement these methods, new software has been developed. Experimental measurements of telemetric parameters of the quantum frequency standard are processed using the developed software. The stability of the prototype of the rubidium standard has been determined using the proposed new algorithms.

1. Introduction

Currently, one of the urgent tasks in modern communication and navigation systems is improved the accuracy of synchronizing time scales between different devices [1-8]. Depending on the required accuracy of synchronization of time scales, different models of frequency standards are used in the systems [1-5, 9-13]. The greatest difficulties arise when synchronizing time scales in a device that uses moving objects (for example, satellites, etc.). This problem is especially acute when developing new navigation systems [1-3, 5, 9, 13-17]. In these systems, it is required to determine the position of the object with an error of not worse than 0.5 m. This requires synchronization of time scales with an error not worse than 10⁻¹⁴ seconds. The most optimal solution to this problem is the use of quantum frequency standards (QFS) [9-19]. Among the quantum frequency standards for various navigation systems the most popular were rubidium QFS, as they have small size and low cost compared to other types of QFS [1, 2, 5, 9, 20-29].

One of the most important characteristics of any QFS is the stability of its operation. According to this characteristic, determine the quality of QFS. In navigation satellite systems, QFS should work autonomously for a long time. In some cases, it is only possible to adjust the time scale of the standard with reliable communication with ground stations. Therefore, the stability of the QFS in satellite systems has increased requirements compared to ground-based systems [9, 20-24, 28-30].

Constant modernization of the constructions of rubidium QFS for solving various navigation tasks is accompanied by an increase in the requirements for their stability. Therefore, the development of new algorithms for calculating the stability of the QFS and software for their implementation is an urgent task. One of the possible solutions to this problem is presented in this paper.

2. Methods for assessing the stability of frequency standards

One of the main characteristics of the standard is the stability of the output frequency. The following methods are used to assess the stability of frequency standards: the standard deviation of the frequency (classical variance) is calculated and the Allan deviation is calculated. The standard deviation S of the group containing N measurement results is calculated by the formula:

\[ S = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N(N-1)}}. \]
where $x_i$ — $i$-th result of frequency measurements, $\bar{x}$ — the arithmetic mean of measurement results.

This characteristic is used to assess the stability of frequency standards, but its use can be difficult if there is a correlation between fluctuating values. In addition, as a result of various studies [5, 14-17, 28-31], it was found that with a large number of measurements, the use of standard deviation becomes ineffective in assessing the stability of frequency standards. Therefore, in most cases, Allan deviation is used. When calculating the deviation, it is necessary to use the difference between two adjacent frequency measurements, and not the measurement of the frequency deviation from the mean value, as in the classical case [9, 32]:

$$\sigma_y(\tau) = \sqrt{\frac{\sum_{i=1}^{N-1} \delta_{0i}^2}{2(N-1)}}$$  \hspace{1cm} (2)

where $\delta_{0i}$ — the relative frequency variation in the $i$-th measurement:

$$\delta_{0i} = \frac{f_{i+1} - f_i}{f_{rat}}$$  \hspace{1cm} (3)

$f_i$ — value of frequency measure at $i$-th measurement, $f_{rat}$ — rated frequency value, $N$ — number of measurements.

Currently, Allan deviation is the most convenient and more reliable measure to determine the stability of the frequency in the time domain. Its analogue in Russian standards is the mean square relative random variation $\sigma$ (SRRV), which differs from the Allan deviation by a constant $\sigma_y(\tau) = \frac{\sigma}{\sqrt{2}}$.

3. Experimental study of frequency stability and processing of measurement results

For figure 1 the scheme of the experimental setup for investigating the stability of the operation of the rubidium QFS. One of the outputs of the test sample of the rubidium frequency standard is connected to the measuring device, which is used as a frequency counter Pendulum CNT-91. At the same time, a signal from the reference standard is sent to the frequency counter. The hydrogen standard is used as a reference in the experiment. The frequency counter in this case acts as a comparator, comparing the signal of the studied standard with a more stable signal of the reference standard. The obtained data from the frequency counter is fed to a computer, where they are analyzed and further processed.

To process the frequency values, special software was developed, which implemented the above calculation methods: standard deviation, Allan deviation and SRRV. During the calculations, the interaction of two data streams was used: the first stream was used to obtain the primary frequency values from the device, and in the second stream, the required values were directly calculated.

![Figure 1. Block diagram of the experimental setup.](image-url)

This processing algorithm proposed by us in the implementation of calculations using (1), (2) and (3) in the developed program allowed us to assess the stability of standards in real time and significantly
accelerated the processing of experimental results. Figure 2 shows the calculation of SRRV in the program during the experiment for two days.

![Figure 2. Fragment of the program when calculating SRRV.](image)

When processing the results, the time series recommended by the state standard was used: 1, 2, 5, 10, 100... s, which can be set by the user in the program settings. In the settings of the developed program, there is also a special section "Connection to the device", which allows measurements using modern models of frequency counters from different manufacturers, which makes the methods proposed for assessing stability more universal.

In parallel with the current experiment, similar measurements were carried out to test the developed program and assess compliance with technical requirements. For this, the second output of the rubidium frequency standard was connected to the VCH-315 multichannel comparator, which includes its own software. Comparison of the results of research SRRV is presented in figure 3.

![Figure 3. Comparison of the results of SRRV calculations using different measuring equipment.](image)
The discrepancy between the results can be explained using different measuring instruments. The Pendulum CNT-91 frequency meter stores fewer significant digits than the VCH-315 comparator. As $\tau$ increases, the data are averaged over a larger interval and the deviation between the results gradually decreases. Therefore, additional analysis of the data is necessary. To do this, we compare the experimental results with the requirements for the stability of the rubidium standard (table 1).

**Table 1.** Comparison of experimental data with technical requirements for the standard model under study.

| Measurement time, t, s | SRRV(CNT-91) | SRRV (VCH-315) | SRRV (technical requirements), no more than | Stability |
|------------------------|--------------|----------------|---------------------------------------------|-----------|
| 1                      | 1.66*10^{-11}| 1.08*10^{-11}  | 1.5*10^{-11}                               | Satisfactorily |
| 100                    | 3.18*10^{-12}| 3.73*10^{-12}  | 1.0*10^{-11}                               | Satisfactorily |
| 1000                   | 1.12*10^{-12}| 1.29*10^{-12}  | 5.0*10^{-12}                               | Satisfactorily |

Comparing the values of the SRRV calculated during the experiment with the permissible values in the technical requirements, we can conclude that the tests of the rubidium standard are correct and that its stability is consistent with the required quality.

**4. Conclusion**

The results of experimental studies have shown that rubidium QFS reaches maximum stability at a measurement time $t=1000$ s. The new developed software allows us to increase the speed of processing the results of measurements of various parameters, to increase the speed of various automatic systems of automatic adjustment of the standard parameters and to receive prompt information about the quality of its work.

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