DOES CLASSICAL MECHANICS ALWAYS ADEQUATELY DESCRIBE "CLASSICAL PHYSICAL REALITY"?

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Abstract

The article is dedicated to discussion of irreversibility and foundation of statistical mechanics "from the first principles". Taking into account infinitesimal and, as it seems, neglectful for classical mechanics fluctuations of the physical vacuum, makes a deterministic motion of unstable dynamic systems is broken ("spontaneous determinism breaking", "spontaneous stochastization"). Vacuum fluctuations play part of the trigger, starting the powerful mechanism of exponent instability. The motion of the dynamic systems becomes irreversible and stochastic. Classical mechanics turns out to be applicable only for a small class of stable dynamic systems with zero Kolmogorov-Sinay entropy $h = 0$. For alternative "Stochastic mechanics" there are corresponding equations of motion and Master Equation, describing irreversible evolution of the initial distribution function to equilibrium state.

In 1893 A. Poincare published a short article "Le mechanisme et l’expérience" [1]. The article sum up a violent discussion on the problem of time irreversibility between L. Boltzmann on the one part and J. Loschmidt and E. Zermelo on the other part. Poincare's conclusion was, naturally, categorical and far-seeing. He concluded that paradoxes of irreversibility mean, evidently, that there is a serious failure of mechanics, and solving it will take revision of some basic rules, building up some theory, more general than the mechanics. The community of physicists, evidently, has not accepted these true in their nature conclusions. Anyway W. Tomson (Lord Kelvin) in the end of the 19 century in his famous report summing up the development of theoretical physics does not mention these important conclusions of Poincare, though speaking about notorious two "clouds" in the sky of physics: experiment of Michelson-Morley and difficulties of description of radiation of absolutely black body.

1Internet adress: www.shemizadeh.narod.ru
The problem of reversibility-irreversibility has not by now found adequate solution. V.L. Ginzburg in his famous program article "What Problems of Physics and Astrophysics Seems Now to be Especially Important and Interesting" [2] sets it among three most important "great problems".

General notion about the state of the matter, about different points of view of the problem, possible ways of solving it can be found in the following works of [3].

Nowadays it is principally clear that appearance of irreversibility is somehow connected with instability of classical dynamical systems. This point of view is supported by most of the researchers.

Below we will consider the central question: how this mechanism of local instability is started, how the deterministic motion is broken, how stochasticity appears in the behavior of dynamic systems, how the behavior of dynamic systems becomes time irreversible. Meanwhile the area of correctness of classical mechanics becomes more narrow, as well as the areas of adequate description of classical dynamic systems. We will come to the need of generalization of classical mechanics and formulate the basic equations of the new stochastic mechanic. In other words we will offer the solution of the paradox of reversibility-irreversibility.

As introduction let us consider the behavior of knowingly classical material point with mass of m=1 g in three situations: free state, in potential pit and on the top of potential hill.

a) Free mass is in the free state. What is its further behavior at strict consideration? By strict consideration we mean consideration of the mass’s behavior not in ideal mathematic vacuum, but in real physical one under the influence of vacuum fluctuations to our classical mass.

We suppose that, as in quantum mechanics, fluctuations of coordinate $\delta x$ and conjugated momentum $\delta p$ (to make it simple we consider only one-dimensional case), for which the dispersions meet the relation

$$<\delta x^2><\delta p^2>=\hbar^2/4. \quad (1)$$

Due to isotropy the mean values $<\delta x>=0, <\delta p>=0$.

For instant value of coordinate for a small period of time $\tau$ we can write:

$$\delta x_\tau = \frac{\delta p_\tau}{m}. \quad (2)$$

Taking the square of expression (2), taking mean value and excluding $<\delta p^2_\tau>$ by means of (1) we get

$$<\delta x^2_\tau>=\frac{\hbar}{2m} \tau. \quad (3)$$

It coincides with the formula for mean square root deviation of Brownian particle in the medium with diffusion coefficient $\hbar/4m$:

$$<\delta x^2>=2D\tau.$$

In the terms of stochastic calculs [4] we can record the equations of motion of the classical mass:

$$dx = \left(\frac{\hbar}{2m}\right)^{1/2} dw. \quad (4)$$
where \( dw \) is the differential of Wiener process with attributes \( < dw >= 0, \; < dw^2 >= dt \).

So our mass, despite the stipulations of the classical mechanics does not keep the quiescence, but is moving chaotically, diffusing according to (4). But is there any ground for refusal from the principles of classical mechanics? The mean square root deviation of mass \( m=1g \) from the initial position for the age of the Universe (13 billion years) will make

\[
< \delta x^2 >^{1/2} \sim 10^{-5} \; cm,
\]

That is infinitesimal value, not giving ground for revision of classical mechanics.

b) The next example is the same mass \( m \) in potential pit of harmonic oscillator.

\[
V(\delta x) = \frac{1}{2} m \omega^2 \delta x^2.
\]

The coordinate and momentum as we suppose fluctuate here too. The mean value of energy

\[
< E > = \frac{1}{2m} < \delta p^2 > + \frac{1}{2} m \omega^2 < \delta x^2 >,
\]

Considering (1) has minimum value at

\[
< \delta x^2 > = \frac{\hbar}{2m\omega}, \; < \delta p^2 > = \frac{1}{2} \hbar m\omega,
\]

That finally giving the formula we know

\[
< E > = \frac{1}{2} \hbar \omega.
\]

And here the mean energy and oscillations amplitude are infinitesimal and are of no practical interest.

c) The last example is the mass in the state of unstable equilibrium on the top of potential

\[
V(\delta x) = -\frac{1}{2} m \lambda^2 \delta x^2.
\]

For \( \delta x_\tau \) we have from the motion equations

\[
\delta x_\tau = \delta x_0 \text{ch}(\lambda \tau) + \frac{\delta p_0}{\lambda m} \text{sh}(\lambda \tau).
\]

Here for times \( \tau >> \lambda^{-1} \):

\[
< \delta x^2_\tau > = \frac{\hbar}{4m \lambda} e^{2\lambda \tau}.
\] (5)

Here as we see the situation is principally different. Infinitesimal noise plays the part of trigger and start the powerful mechanism of instability. The material point spontaneously falls from its unstable position of equilibrium to the left or to the right.

So we are ready to consider of the main matter, the spontaneous breaking of deterministic motion of unstable dynamical systems, stochastization and occurrence of irreversibility.

Let us consider some dynamic system with Hamiltonian \( \mathcal{H}(x, p) \).
Motion equations

\[
\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i}
\]  \hspace{1cm} (6)

Liuville equation

\[
\frac{\partial \rho}{\partial t} + \mathcal{L} \rho = 0,
\]

Where \( \mathcal{L} \) is the Liuville operator.

Stability of some trajectory can be estimated by the motion of trajectory close to the initial one \( \overline{x}_i = x_i + \delta x_i, \overline{p}_i = p_i + \delta p_i \). Or by the solutions of linearized equation for variations \( \delta x_i \) and \( \delta p_i \):

\[
\delta \dot{x}_i = \sum_j \frac{\partial^2 H}{\partial p_i \partial p_j} \delta p_j, \quad \delta \dot{p}_i = -\sum_j \frac{\partial^2 H}{\partial x_i \partial x_j} \delta x_j,
\]

In the matrix form

\[
\begin{pmatrix}
\delta \dot{p} \\
\delta \dot{x}
\end{pmatrix} =
\begin{pmatrix}
0 & A \\
B & 0
\end{pmatrix}
\begin{pmatrix}
\delta p \\
\delta x
\end{pmatrix},
A_{ij} = -\frac{\partial^2 H}{\partial x_i \partial x_j}, B_{ij} = \frac{\partial^2 H}{\partial p_i \partial p_j}.
\]

The matrix to the right can be led by means corresponding transformation \( 2 \times 2 \) to the view with matrixes on the diagonal. The matrixes can be of three types specified below (a,b,c).

\[
\begin{pmatrix}
0 & 0 \\
1/m_i & 0
\end{pmatrix},
\begin{pmatrix}
0 & -m_i \omega_i^2 \\
1/m_i & 0
\end{pmatrix},
\begin{pmatrix}
0 & m_i \lambda_i^2 \\
1/m_i & 0
\end{pmatrix},
\]

Actually, the first matrix shows that the corresponding degree of freedom does not interplay with the other ones, the corresponding index of Lepunov is equal to zero, the degree of freedom is stable (case a).

The second matrix (case b) also corresponds to the stability of the freedom degree, which oscillates with infinitesimal amplitude near the initial trajectory.

The third matrix corresponds to the case of exponential instability. Here at motion of this freedom degree every moment there occurs a spontaneous jump into infinitesimal trajectory, there is Brownian motion of the showing points on the neighbor trajectories, the mechanism of exponential deviation of trajectories switches on, that leads to breaking of deterministic motion and to stochasticization.

Similar process takes place on all the instable degrees of freedom. In this case the equation for variations (letting alone the index)

\[
\begin{pmatrix}
\delta \dot{p} \\
\delta \dot{x}
\end{pmatrix} =
\begin{pmatrix}
0 & m \lambda^2 \\
1/m & 0
\end{pmatrix}
\begin{pmatrix}
\delta p \\
\delta x
\end{pmatrix},
\]

Or

\[
\delta \dot{p} = m \lambda^2 \delta x, \quad \delta \dot{x} = \frac{1}{m} \delta p.
\]

Performing analogous to (1)-(4) calculations we get:

\[
< \delta x_i^2 > = \frac{\hbar}{2m} \delta \tau, \quad < \delta p_i^2 > = \frac{1}{2} \hbar m \lambda^2 \delta \tau,
\]
Where
\[ \frac{\hbar}{4m}, \quad \frac{1}{4} \hbar m \lambda^2 \]
are the diffusion coefficients according to coordinate and impulse. Inputting as before the stochastic differentials \( \overline{dx} \) and \( \overline{dp} \), we have
\[ \overline{dx} = \left( \frac{\hbar}{2m} \right)^{1/2} dw_x, \quad \overline{dp} = \left( \frac{\hbar m \lambda^2}{2} \right)^{1/2} dw_p, \]

We have to revise Hamilton motion equations (6) as stochastic Hamilton equations:
\[ dx_i = \frac{\partial H}{\partial p_i} dt + \frac{\hbar}{2m} dt^{1/2} dw_x, \quad dp_i = -\frac{\partial H}{\partial x_i} dt + \frac{\hbar m \lambda^2}{2} dt^{1/2} dw_p. \quad (7) \]

These are typical Itoh’s stochastic differential equations. They describe stochastic and irreversible motion. Liouville’s equation appears to becomes irreversible:
\[ \frac{\partial \rho}{\partial t} + \mathcal{L} \rho = \frac{\hbar}{4} D \rho, \quad (8) \]

Where \( \mathcal{L} \)- is Liouville’s operator and \( D \)- the diffusion operator. This equation of Fokker-Plank type describes irreversible evolution of the initial distribution function \( \rho(0) \) to the equilibrium state. The diffusion operator in transformed (normal) coordinates is:
\[ D = \sum_i \left( \frac{\partial^2 H}{\partial p_i^2} \frac{\partial^2 H}{\partial x_i^2} + \frac{\partial^2 H}{\partial x_i^2} \frac{\partial^2 H}{\partial p_i^2} \right), \]

Where the summing is on all the unstable degrees of freedom.

Finally, returning to the initial coordinates by reverse transformation we have instead of reversible Liouville equation an irreversible equation of the Fokker-Plank type (8) with non-negative defined diffusion operator \( D \)-containing second derivatives on coordinates and momentums.

So we have modified classical mechanics and came to the new, more general theory, i.e. "Scholastic mechanics". All the dynamical systems with the unstable trajectories (where the entropy \( \hbar > 0 \)) cannot be adequately described by the classical mechanics. These systems are irreversible and have in their motion is stochastic random elements and are described by the above-mentioned stochastic mechanics.

Equation (8) can be named Master Equation. It, at stability of all the degrees of freedom \( (\hbar = 0) \) transforms into Liouville equation, in the same way with the stochastic equations of Hamilton (7) transforms into ordinary Hamilton equations of motion (6). These are the general features of solution of the problem of reversibility-irreversibility and the answer to the question about where stochastic and irreversible motion of dynamical systems is taken from. The answer to the question set in the title of the article must be as follows: "No, not always, but very seldom, classical mechanics adequately describes only a narrow class of stable dynamic systems”.

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References

[1] Poincare H. - Rev. Methaphys. et Morale, 1893, vol. 1, p. 534-537.

[2] Ginzburg V.L. Uspehi Fizicheskiih Nauk, 1999, 169, 4, p.420-441.

[3] Prigogine I. - From Beginning to Becoming: time and complexity in the physical sciences, W.H.Freeman and Company, San Francisco, 1980, 217.

[4] Gardiner C.W. - Handbook of Stochastic Methods for Physics, Chemistry and the Natural Science, Springer, Berlin, 1985, 526 p.