Nucleon transversity generalized form factors with twisted mass fermions

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A Motivation

B Definition of Nucleon Parton Distributions

C Evaluation on the Lattice
  • Connected Diagram
  • Continuum Decomposition

D Renormalization
  • Non-perturbative
  • Physical point
  • Perturbative

E Results
  • Isovector FFs and GFFs
  • Isoscalar FFs

F Future work
A. Motivation

★ Introduced in 1979 by Ralston and Sober (Drell-Yan scattering)

★ Appeared again in 1990s by Artru and Mekhfi / Jaffe and Ji

★ Transversity distributions (TDs) are chirally-odd $\rightarrow$ fully inclusive DIS not useful

★ To measure TDs the chirality must be flipped twice:

1. hadron hadron collisions (2 hadrons in initial state)
2. semi-inclusive DIS (SIDIS) (1 hadron in initial state and 1 in final state)

★ Small contributions of tensor interactions in SM $\left(10^{-3}\right)$: future experiments are planned
B. Nucleon Generalized Parton Distributions (GPDs)

- Parametrization of off-forward nucleon matrix of a bilocal quark operator

\[ F_{\Gamma}(x, \xi, q^2) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p' | \bar{\psi}(-\lambda n/2) \mathcal{O} P e^{-\lambda/2} \int d\alpha \cdot A(\alpha) \psi(\lambda n/2) | p \rangle \]

where \( q = p' - p, \ P = (p' + p)/2, n: \) light-cone vector (\( \bar{P} \cdot n = 1 \)), \( \xi = -n \cdot \Delta/2 \)

- Choices of operators within LQCD: towers of local twist-2 operators

- Operators & hadron structure observables related via moments in the momentum fraction \( x \)

\[ f^n = \int_{-1}^{1} dx \ x^{n-1} f(x) \]
Twist-2 Parton Distributions:

complete set for describing the quark state inside the nucleon (leading-order hard processes)

\[ A \] unpolarized

\[ O^{\mu_1 \ldots \mu_n} = \bar{q} \gamma^{\{ \mu} iD^{\mu_1} \ldots iD^{\mu_{n-1}}\} q \]

\[ B \] helicity (polarized)

\[ \tilde{O}^{\mu_1 \ldots \mu_n} = \bar{q} \gamma_5 \gamma^{\{ \mu} iD^{\mu_1} \ldots iD^{\mu_{n-1}}\} q \]

Talk by C. Alexandrou

\[ C \] transversity

\[ O^{\mu_1 \ldots \mu_{n-1}} = \bar{q} \sigma^{\mu} \{ \nu iD^{\mu_1} \ldots iD^{\mu_{n-1}}\} q \]

net number of quarks with transverse polarization

in a transversely polarized nucleon

- Transversity distribution (scheme and scale dependent):

\[ \langle x^n \rangle_{\delta q} = \int_0^1 dx \, x^n \left[ \delta q(x) + (-1)^{n+1} \delta \bar{q}(x) \right] , \quad \delta q = q_T + q_\perp \]

- Nucleon case:

\[ H_T(x, \xi, q^2), E_T(x, \xi, q^2), \tilde{H}_T(x, \xi, q^2), \tilde{E}_T(x, \xi, q^2) \]
Nucleon transversity generalized form factors

Decomposition of matrix elements into GFFs: contain FFs, PDFs

Special cases for the tensor operator:

\( n = 0 \): \( A_{T10}, B_{T10}, \tilde{A}_{T10} \) quark helicity flip form factors

\[
\langle \bar{q}(0)i\sigma^{\mu\nu}q(0) \rangle = \langle i\sigma^{\mu\nu} \rangle A_{T10}(q^2) + \langle \frac{\gamma^{[\mu} \Delta^{\nu]}}{2m_N} \rangle B_{T10}(q^2) + \langle \frac{\vec{P}^{[\mu} \Delta^{\nu]}}{m^2_N} \rangle \tilde{A}_{T10}(q^2)
\]

- \( A_{T10}(0) \equiv \langle 1 \rangle \delta_q(x) \) (tensor charge \( g_T \))

\( n = 1 \): \( A_{T20}, B_{T20}, \tilde{A}_{T20}, \tilde{B}_{T21} \) moments of parton distributions

\[
\langle \mathcal{O}_T^{\mu\nu\mu_1}(0) \rangle = \mathcal{A}_{\mu\nu} S_{\nu\mu_1} \left\{ \langle i\sigma^{\mu\nu} \vec{P}^{\mu_1} \rangle A_{T20}(q^2) + \langle \frac{\gamma^{[\mu} \Delta^{\nu]} \vec{P}^{\mu_1}}{m^2_N} \rangle B_{T20}(q^2) \right. \\
\left. \quad + \langle \frac{\vec{P}^{[\mu} \Delta^{\nu]} \vec{P}^{\mu_1}}{m^2_N} \rangle \tilde{A}_{T20}(q^2) + \langle \frac{\gamma^{[\mu} \vec{P}^{\nu]} \Delta^{\mu_1} \rangle \tilde{B}_{T21}(q^2) \right\}
\]

- \( A_{T20}(0) \equiv \langle x \rangle \delta_q(x) \) (tensor moment)
FFs / GFFs and GPDs

**FFs**

\[
A_{T10}(q^2) = \int_{-1}^{1} dx H_T(x, \xi, q^2)
\]

\[
B_{T10}(q^2) = \int_{-1}^{1} dx E_T(x, \xi, q^2)
\]

\[
\tilde{A}_{T10}(q^2) = \int_{-1}^{1} dx \tilde{H}_T(x, \xi, q^2)
\]

\[
0 = \int_{-1}^{1} dx \tilde{E}_T(x, \xi, q^2)
\]

**GFFs**

\[
A_{T20}(q^2) = \int_{-1}^{1} dxx H_T(x, \xi, q^2)
\]

\[
B_{T20}(q^2) = \int_{-1}^{1} dxx E_T(x, \xi, q^2)
\]

\[
\tilde{A}_{T20}(q^2) = \int_{-1}^{1} dxx \tilde{H}_T(x, \xi, q^2)
\]

\[
-2\xi \tilde{B}_{T20}(q^2) = \int_{-1}^{1} dxx \tilde{E}_T(x, \xi, q^2)
\]

**time reversal transformation properties**

- \( \overline{B}_{T10} \equiv B_{T10} + 2\tilde{A}_{T10} \)

- \( \overline{B}_{T10}(0) \equiv \kappa_T \) (tensor magnetic moment)\(^\dagger\)

\[^\dagger\text{[M. Burkardt, Phys. Rev. D72 (2005) 094020]}\]
C. Evaluation on the Lattice: Connected diagram

\[ \vec{q} = \vec{p}' - \vec{p} \]

2pt : \[ G(\vec{q}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f \cdot \vec{q}} \Gamma^1_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) \overline{J}_\beta(0) \rangle \]

3pt : \[ G^{T, DT}(\Gamma, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) O^{T, DT}(\vec{x}, t) \overline{J}_\beta(0) \rangle \]

Operator Insertion:

\[
\begin{align*}
O^T &= \sigma^{\mu \nu} \\
O^{DT} &= \sigma[\mu \{\nu\} \overrightarrow{D} \rho]
\end{align*}
\]

- No mixing with lower dimension operators
- Isovector combinations: No disconnected diagrams
- DT: antisymm., symm. and subtraction of the traces
• Sequential inversion “through the sink”: fix sink-source separation $t_f - t_i$
• Smearing techniques (Gaussian/APE): improvement of ground state dominance in 3pt correlators

**Types of projectors:**

- $\Gamma^1 = (1 + \gamma_0)$  
  (2pt & 3pt)
- $\Gamma^k = (1 + \gamma_0) i \gamma_5 \gamma_k$  
  (3pt)  
  ($Q^2 = 0$ FFs and GFFs)

**Optimized Ratios:** Leading $t$ dependence cancels

$$R(\Gamma, \vec{q}, t) = \frac{G(\Gamma, \vec{q}, t)}{G(\vec{0}, t_f)} \times \sqrt{\frac{G(-\vec{q}, t_f-t)G(\vec{0}, t)G(\vec{0}, t_f)}{G(\vec{0}, t_f-t)G(-\vec{q}, t)G(-\vec{q}, t_f)}}$$

$$\lim_{t_f - t \to \infty} \lim_{t - t_i \to \infty} R(\Gamma, \vec{q}, t) \to \Pi(\Gamma, \vec{q})$$

$R(\Gamma, \vec{q}, t)$ depends on operator indices

![Graph showing optimized ratios $R(\Gamma^k, Q^2 = 0)$, $R(\Gamma^k, Q^2 = 1)$, and $R^{DT}(\Gamma^k, Q^2 = 0)$ with $t/a$ on the x-axis.](image)
Continuum Decomposition (Euclidean)

Ultra-local Tensor

\[ \Pi_{T}^{\mu \nu} (\Gamma^{1}) = \left( \frac{4i}{m} A_{T10} + 16im B_{T10} + 16i \tilde{A}_{T10} (E + m) \right) \left[ \delta_{\nu 0} p_{\mu} - \delta_{\mu 0} p_{\nu} \right] \]

\[ \Pi_{T}^{\mu \nu} (\Gamma^{k}) = 4 A_{T10} \left[ \epsilon_{\mu \nu k 0} - \frac{i \epsilon_{\mu \nu k \rho} p_{\rho}}{m} \right] + 8 B_{T10} \left[ (i ( -m \delta_{\mu 0} - m \delta_{\nu 0}) \epsilon_{\mu \nu k \rho} + \epsilon_{\nu k 0 \rho} p_{\mu} - \epsilon_{\mu k 0 \rho} p_{\nu}) p_{\rho} \right] \]

1-D Tensor

\[ \Pi_{DT}^{\mu \neq \nu \neq \rho \neq 0} (\Gamma^{1}) = \frac{-3}{m} (A_{T20} m + B_{T20} (E + m) + C_{T20} m) (\delta_{\mu (2), 0} p_{\mu} - \delta_{\mu, 0} p_{\nu}) p_{\rho} \]

\[ \Pi_{DT}^{0 0 \neq \rho} (\Gamma^{k}) = A_{T20} \left[ i \delta_{\nu k} \left( 2E + \frac{E^{2}}{m} + m \right) + p_{k} \left( -\frac{1}{2} - \frac{E}{2m} \right) \right] + C_{T20} i \left[ p_{k} \left( \frac{-E^{2}}{2m^{2}} - \frac{E}{2m} \right) + i \left( \delta_{\nu k} \left( \frac{-E}{2} + \frac{E^{3}}{2m^{2}} + \frac{E^{2}}{2m} - \frac{m}{2} \right) + p_{\nu} p_{k} \left( \frac{-E}{2m^{2}} - \frac{1}{2m} \right) \right) \right] + D_{T20} i \left[ p_{k} \left( -\frac{E^{2}}{m^{2}} + \frac{E}{m} \right) + i \left( \delta_{\nu k} \left( -E + \frac{E^{3}}{m^{2}} - \frac{E^{2}}{m} + m \right) + p_{\nu} p_{k} \left( -\frac{E}{m^{2}} + \frac{1}{m} \right) \right) \right] \]

★ Combination of 2 projectors: FFs, GFFs disentanglement (via SVD)
Ensembles

- $N_f = 2+1+1$ twisted mass gauge configurations
- $N_f = 2$ twisted mass/Clover gauge configurations
- Iwasaki gluon action

| $N_f$ | $\beta$ | $a$ (fm) | $a\mu_0$ | $c_{SW}$ | $m_\pi$ (MeV) | $L^3 \times T$ | Stat. |
|-------|---------|----------|----------|----------|----------------|----------------|-------|
| 2+1+1 | 1.95    | 0.082*   | 0.0055   | 0        | 373            | $32^3 \times 64$ | 770   |
| 2+1+1 | 2.10    | 0.064*   | 0.0015   | 0        | 213            | $48^3 \times 96$ | 425   |
| 2     | 2.10    | 0.097*   | 0.0009   | 1.57551  | 126            | $48^3 \times 96$ | 420   |

* Determination of lattice spacing from nucleon masses

We focus on:

- connected diagram: Isovector/Isoscalar nucleon transversity FFs/GFFs

since:

- disconnected diagram: Isoscalar computation ($O^T$): negligible contribution

Talk by A. Vaquero

Thursday 14:00
D1. Non-perturbative Renormalization

Ultra-local tensor operator

- Momentum source method† (high accuracy)
- RI’-MOM scheme
- Chiral extrapolation
- Continuum extrapolation
- Subtract perturbative $\mathcal{O}(a^2)$
- Conversion to $\overline{\text{MS}}$ at 2GeV

†[M. Gockeler et al., Nucl. Phys. B544 (1999) 699]

Perturbative $\mathcal{O}(a^2)$ terms: (Iwasaki gluons, Langau gauge)

(C. Alexandrou et al., Phys. Rev. D86 (2012) 014505)

$$a^2 \frac{g^2 C_F}{16 \pi^2} \left[ + \mu^2 \left( 0.2341 - 1.0950 c_{sw} - 0.4297 c_{sw}^2 \right) + \frac{\mu^4}{\mu^2} \left( 2.6676 + 0.1843 c_{sw} + 0.1203 c_{sw}^2 \right) + \log(a^2 \mu^2) \left( \left( \frac{7271}{60000} + \frac{c_{sw}}{2} + \frac{c_{sw}^2}{4} \right) \mu^2 - \frac{28891}{30000} \mu^4 \right) \right]$$

$$\mu^4 \equiv \sum_{i=1,4} \mu_i^4$$
D2. Renormalization at the physical point

- Same ensemble as for the FFs/GFFs computation
- $m_\pi$ dependence expected insignificant [C. Alexandrou et al., Phys. Rev D152 (1979) 109]
- Democratic momenta in the spatial direction

\[
\frac{1}{L^2} \ll \Lambda_{QCD}^2 \ll \mu^2 \ll \frac{1}{a^2}
\]

Reliable perturbation theory

Small $O(a)$ lattice effects

**Criterion for choosing the momenta**

[M. Constantinou et al., JHEP 1008 (2010) 068]

\[
\text{ultra} - \text{local} : \frac{\sum_\rho p_\rho^4}{\left(\sum_\rho p_\rho^2\right)^2} \leq 0.3 \quad \text{1} - \text{D} : \frac{\sum_\rho p_\rho^4}{\left(\sum_\rho p_\rho^2\right)^2} \leq 0.4
\]

⇒ Non-Lorentz invariant contributions under control
**D3. Perturbative Renormalization**

**One-Derivative tensor operator**

- 1-loop perturbation theory

\[
Z_q = \frac{1}{12} \text{Tr}
\left[
(S^L(p))^{-1} \ S^{(0)}(p)\right]
\bigg|_{p^2 = \bar{\mu}^2}
\]

\[
Z_{q}^{-1} \ Z_{DT}^{\mu\nu\rho} \ \frac{1}{12} \text{Tr}
\left[
\Gamma_{\mu\nu\rho}^L(p) \ \Gamma^{(0)}_{\mu\nu\rho}(p)\right]
\bigg|_{p^2 = \bar{\mu}^2} = \text{Tr}
\left[
\Gamma^{(0)}_{\mu\nu\rho}(p) \ \Gamma^{(0)}_{\mu\nu\rho}(p)\right]
\bigg|_{p^2 = \bar{\mu}^2}
\]

- RI’-MOM renormalization scheme

- Conversion to $\overline{\text{MS}}$ at 2GeV

\[\begin{array}{c}
\text{\large\textcolor{red}{\text{\vdots}}} \text{ gluon field} \\
\text{\large\textcolor{green}{\text{\vdots}}} : \text{ fermion field} \\
x: \text{ operator insertion}
\end{array}\]
Perturbative Results
(C. Alexandrou et al., Phys. Rev. D83 (2011) 014503)

Iwasaki gluon action:

\[ Z_{DT}(p = \bar{\mu}) = 1 + \frac{g^2 C_F}{16 \pi^2} (2.3285 - 2.2795 c_{sw} - 1.0117 c_{sw}^2 - 3 \log(a^2 \bar{\mu}^2)) \]

RI'-MOM scheme:
\[ \beta = 1.95 : \quad Z_{RI}^{DT}(p = \bar{\mu}) = 1.0891 \]
\[ \beta = 2.1 : \quad Z_{RI}^{DT}(p = \bar{\mu}) = 1.1177 \]
\[ \beta = 2.1, c_{sw} : \quad Z_{RI}^{DT}(p = \bar{\mu}) = 1.2059 \]

\[ Z_{RI}^{MS}(p = \bar{\mu}) = 0.9091 \]
\[ Z_{MS}^{MS}(p = \bar{\mu}) = 0.9900 \]
\[ Z_{MS}^{MS}(p = \bar{\mu}) = 1.1041 \]

Conversion factor:
\[ C_{RI, MS}^{RI, MS} = 1 - \frac{g^2 C_F}{16 \pi^2} (3\alpha + 7) \]

(J. Gracey, Nucl. Phys. B667 (2003) 242)

★ Note: \( Z_{DV/DA}^{pert}, Z_{DV/DA}^{nonpert} \cdot 10\% \) difference
E. Results: Tensor GFFs
Preliminary

Isovector:

![Graph showing A_T20 vs Q^2 (GeV^2) for TMF (N_f=2+1+1): 373MeV, TMF (N_f=2+1+1): 213MeV, TMF/Clover (N_f=2): 126MeV.]

Isoscalar:

![Graph showing A_T20 vs Q^2 (GeV^2) for TMF (N_f=2+1+1): 373MeV, TMF (N_f=2+1+1): 213MeV, TMF/Clover (N_f=2): 126MeV.]

E2. Isovector Tensor FFs

Zero-momentum transfer Tensor charge

Fundamental parameter that characterize properties of the nucleon

LQCD. points:
Agreement
Mild $m_\pi$ dependence

exp. point:

$$A_{T10}^{\exp}(0.8\text{GeV}^2) = 0.77^{+0.18}_{-0.36} \, \dagger$$
(at $\mu^2 = 110\text{GeV}^2$)

Global fit of HERMES, COMPASS, Belle $e^+ e^-$ data (9 parameters)

Running scale (3-loops):

$$R(110, 4) = 1.11034523$$

[T. Bhattacharya et al., Phys. Rev. D85 5 (2012)]

† (M. Anselmino et al., Nucl. Phys. Proc. Suppl. 191 (2009) 98)
Momentum dependence

\[ A_{T10} : \text{linearly decreasing in } Q^2 \]

\[ B_{T10}(0) \text{ from fitting} \]

\[ B_{T10}(0) \text{ Model dependent} \]
Comparison for $A_{T10}$, $\overline{B}_{T10}$

- Mild pion mass dependence
  - $A^u_{T10} > 0$ (decreasing)  $A^d_{T10} < 0$ (increasing)
  - $|A^u_{T10}| > |A^d_{T10}|$
  - $\overline{B}^u,^d_{T10} > 0$ (decreasing)

[QCDSF/UKQCD: M. Göckeler et al., Phys. Lett. B627 (2005) 113; Phys. Rev. Lett. 98 (2007) 222001]
E3. Isoscalar Tensor FFs

Tensor charge

exp. point:

\[ A_{T10}^{\text{exp}}(0.8 \text{GeV}^2) = 0.34^{+0.18}_{-0.36} \]

(at \( \mu^2 = 110 \text{GeV}^2 \))

[T. Bhattacharya et al., Phys. Rev. D85 5 (2012)]

† (M. Anselmino et al., Nucl. Phys. Proc. Suppl. 191 (2009) 98)
Future Work

★ Increase the statistics for $m_\pi = 126, 213$ MeV

★ Compute non-perturbative renormalization function for one-D tensor and subtract $\mathcal{O}(a^2)$ terms

★ Include a $3^{rd}$ projector in the 3-pt function (Stochastic Method)

Talk by K. Hadjigiannakou
Thursday 17:30

THANK YOU