Simulation of wave impact on shore protection structures using 3D model of wave processes

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Abstract. The article is devoted to the research of the effects of wave processes on shore protection structures using mathematical model of wave processes based on the system of Navier-Stokes equations, which includes three equations of motion in areas with dynamically changing geometry of the computational domain. The pressure correction method is used for the approximation of a hydrodynamic model. The difference schemes describing the mathematical model of the wave propagation towards the shore are constructed on the basis of the integro-interpolation method using the scheme with weights. An adaptive alternating-triangular iterative method is used to solve the system of grid equations. The model presented in the article gives more realistic description of the physical wave process near the coastline. The practical significance of the numerical algorithms and program complex for their implementation is the possibility of their application in the study of hydrophysical processes in the coastal water systems, as well as to build the velocity and pressure fields of the water environment and the evaluation of hydrodynamic effects on shore protection structures and coastal structures.

1. Introduction
The construction and study of three-dimensional wave models in coastal marine systems makes it possible to use 3D models for studying hydrophysical processes in coastal water systems, as well as for assessing the hydrodynamic effects on shore protection structures and coastal structures in the presence of gravity surface waves.

The 3D model presented in the article gives more realistic description of the physical wave process near the coastline. Let us consider various works related to the analysis of 3D wave motion for modeling the engineering problems of the ocean and coast.

D. M. Hutnans (National Oceanographic Center in the Encyclopedia of Ocean Sciences) examined in detail the issue of coastal waves. He considers waves passing through the continental shelf and having periods of the order of 1 day or more. Cross wave displacements alter the depth and relative vorticity of the water column, causing transverse movements of adjacent water layers.

Waves propagate faster on wide profiles of the slopes of the shelf, the speed can be affected by flows along the shelf and the reverse movements of the lower slope. Large amplitudes and sharp changes in the topography of the coast cause distortion of the wave modes. Waves affect the movement of the shelf and slope on a daily scale and the width of the shelf, they are important for predicting changes in the geometry of the shelf and slope due to the effects of tides [1].

Jun Tang, Egan Liu, Yongming Shen, Minling Zhang, Meirong Su (State Laboratory of Coastal and Marine Engineering of Dalian University of Technology) conducted a numerical study of impact of the
location of breakwater on coastal waves, wave currents, sediment transport and the morphological evolution of the beach. Scientists have described a numerical model for studying the influence of the location of breakwaters on coastal waves caused by flow waves, sediment transport and the morphological evolution of the beach near the breakwater. The numerical model is developed on the basis of submodels for coastal waves, wave flow, sediment transport and the morphological evolution of the beach.

Coastal waves are modeled on the basis of a parabolic equation taking into account the effects of refraction, diffraction and wave destruction. The numerical model has been experimentally tested on coastal waves, coastal currents, sediment transport and the morphological development of the beach around a breakwater from a large-scale sediment transport facility at the US Army Corps of Engineering Research and Development Center [2].

Chinese scientists from Southwestern Jiaotong University Bo Huang, Bin Zhu, Shengai Tsui, Lunliang Duan, Zavei Zhang conducted experimental and numerical simulations of the impact of wave forces on coastal bridges. As a result of experimental tests and numerical simulations, the wave impact on the coastal bridge induced by a hurricane was studied. After the experiments, a 2D numerical model is used to study the wave action, in which the Reynolds-averaged Navier-Stokes equations in combination with the k-ω turbulence model are used to model the waves. The VOF method is used to track the interface between air and water [3].

Pedro J. Martinez-Ferrer, Lin Qian, Zhihua Ma, Derek M. Causon, Clive G. Mingham have performed accurate and efficient modeling of the generation, propagation and interaction of waves. Their works are devoted to advanced numerical wave generation for modeling ocean and coast engineering problems, scientists introduced a dynamic-boundary procedure for generating numerical waves, designed to model the interaction of the wave structure (WSI), typical of ocean and coast engineering problems.

This implementation relies on a dynamic grid that deforms to replicate wave motion, and is integrated into wsiFoam: a multi-regional communication strategy applied to two-phase Navier-Stokes solvers [4]. The combination of the dynamic boundary method with a grid of several regions counteracts the increase in computational costs, which is inherent in modeling using dynamic regions. This approach leads to the creation of a high-performance computational strategy for wave generation, which can be used to accurately and efficiently model the generation, propagation, and interaction of waves with fixed structures and floating bodies [5].

Analysis of domestic and foreign experience shows that the effectiveness of coast protection largely depends on compliance with a number of conceptual principles. To determine the necessity of protection the shore it is requires to know what is the state of the seashore, outside its dynamic equilibrium, what are the reasons for the violation of this equilibrium and whether they require change. Any unsuccessful attempt to strengthen the shore can cause undesirable consequences, in particular, the transformation of the shore into a morphosystem with even worse properties relative to the initial state.

It is also necessary to choose an effective method of coastal protection that ensures the maintenance of the coastal zone in a dynamically stable state. A rational solution to this problem requires an integrated approach to the seashores as a complex natural system. Intervention in coastal processes in order to protect the coast from destruction implements the mandatory coordination of natural and technical elements in the form of a single optimized system. The design and construction of coastal protection complexes should cover lithodynamic systems that have an autonomous mode of development dynamics, as well as their own balance of bottom sediments.

The breakwater walls as a method of protection from the waves were used since ancient times. Modern breakwater walls are a complex hydraulic structure made of reinforced concrete with a deeply laid foundation, multi-stage profile and shaped visor.

The search for better coastal protection structures led to the emergence of underwater breakwaters. Underwater breakwaters are designed to break the wave, that is, to force it to collapse at some distance from the shore. Structurally, they are ordinary flooded walls of vertical or inclined profile. Breakwaters
can be fixed and mobile, permeable and impermeable. An important quality of breakwaters is the ability to accumulate under their protection sediments.

Buna is a transverse to the shoreline structure that helps to hold sediments and preserve a natural or artificial beach in the inter-buna bays. Buna is an active coastal protection structure, which on the one hand (windward) delays the beach-forming sediments moving along the shore, and on the other (leeward) causes a grassroots erosion. In addition, due to the erosion of leeward areas, it is necessary to continue construction and stretch the coast protection section, which leads to an increase in anthropogenic, engineering load, to a decrease in the stability of the surrounding landscapes. Properly designed bunas can slow down and even completely stop the movement of beach sediments along the shore. As a result, the beach accumulates in the inter-lagoon compartments, which protects the shore from the waves.

2. Statement of 3D wave hydrodynamics problem
Spatially inhomogeneous three-dimensional mathematical model of the wave hydrodynamics of a shallow water body includes [6-8]:

– equations of motion (Navier-Stokes):

\[
\begin{align*}
\rho' u_t' + (\rho u u')_x + (\rho u v')_y + (\rho u w')_z &= -\frac{1}{\rho} P'_x + \left( \mu u' \right)_x + \left( \nu u' \right)_y + \left( \nu u' \right)_z, \\
\rho' v_t' + (\rho v u')_x + (\rho v v')_y + (\rho v w')_z &= -\frac{1}{\rho} P'_y + \left( \mu v' \right)_x + \left( \mu v' \right)_y + \left( \nu v' \right)_z, \\
\rho' w_t' + (\rho w u')_x + (\rho w v')_y + (\rho w w')_z &= -\frac{1}{\rho} P'_z + \left( \mu w' \right)_x + \left( \mu w' \right)_y + \left( \nu w' \right)_z + g;
\end{align*}
\]

(1)

– continuity equation:

\[
\rho' + \left( \frac{\rho u}{V} \right)_x + \left( \frac{\rho v}{V} \right)_y + \left( \frac{\rho w}{V} \right)_z = 0,
\]

(2)

where \( V = \{u, v, w\} \) is the velocity vector of the water flow of a shallow water body; \( \rho \) is the density of the aquatic environment; \( P \) is the hydrodynamic pressure; \( g \) is the gravitational acceleration; \( \mu, \nu \) are coefficients of turbulent exchange in the horizontal and vertical directions; \( n \) is the normal vector to the surface describing the boundary of the computational domain.

Add boundary conditions to system (1)-(2):

– the entrance (left border): \( V = V_0, P'_x = 0 \),

– the bottom border: \( \rho \mu (V_t)'_n = -\tau, V_n = 0, P'_n = 0 \),

– the lateral border: \( (V_t)'_n = 0, V_n = 0, P'_n = 0 \),

– the upper border: \( \rho \mu (V_t)'_n = -\tau, w = -\omega P'_t / \rho g, P'_n = 0 \),

– the surface of the structure: \( \rho \mu (V_t)'_n = -\tau, w = 0, P'_n = 0 \),

(3)

where \( \omega \) is the intensity of evaporation of a liquid, \( V_n, V_t \) are the normal and tangential component of the velocity vector, \( \tau = \{\tau_x, \tau_y, \tau_z\} \) is the vector of tangential stress. Figures 1, 2 show the geometry of the water body.
Figure 1. Depth map of the computational domain.

Figure 2. Isolines of the depth function of the bottom surface and the coastline.

Let \( \mathbf{\tau} = \rho_a C_d \mathbf{w} \) is the vector of tangential stress for the free surface, \( C_d = 0.0026 \), \( \mathbf{w} \) is the wind velocity relative to water, \( \rho_a \) is the atmosphere density, \( C_d \) is the dimensionless surface resistance coefficient, which depends on wind speed, is considered in the range of 0.0016-0.0032.

Let us set the tangential stress vector for the bottom taking into account the movement of water as follows: 
\[
\mathbf{\tau} = \rho C_d \mathbf{V} \mathbf{V}, \quad C_d = g k^2 / h^{1/3},
\]
where \( k = 0.04 \) is the group roughness coefficient in the Manning formula, considered in the range of 0.025 – 0.2; \( h = H + \eta \) is the depth of the water area, [m]; \( H \) is the depth to the undisturbed surface, [m]; \( \eta \) is the elevation of the free surface relative to the geoid (sea level), [m].

We will use an approximation that allows us to build a non-uniform in depth vertical turbulent exchange coefficient on the basis of measured pulsations of the water flow velocity [9]:

\[
\nu = C^2_\nu \Delta^2 \frac{1}{2} \sqrt{\left( \frac{\partial U}{\partial z} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2}, \quad (4)
\]
where $C_s$ is the dimensionless empirical constant, determined on the basis of the calculation of the attenuation process of homogeneous isotropic turbulence; $\Delta$ is the characteristic scale of the grid; $\bar{U}, \bar{V}$ are the time-averaged ripple components of the velocity of the water flow in the horizontal direction.

3. The discrete model of hydrodynamics

The computational domain inscribed in a parallelepiped. For the numerical realization of the discrete mathematical model of the hydrodynamic problem posed, a uniform grid is introduced:

$$\bar{w}_h = \left\{ \tau^n, x_i = ih_x, y_j = jh_y, z_k = kh_z; \ n = 0..N_x, i = 0..N_y, j = 0..N_z, k = 0..N_z; \right\},$$

where $\tau$ is the step by the time, $h_x, h_y, h_z$ are steps in space, $N_x$ is the number of time layers, $T$ is the upper bound on the time coordinate, $N_y, N_z$ are the number of nodes by spatial coordinates, $l_x, l_y, l_z$ are the boundaries along the parallelepiped in the direction of the axes $Ox, Oy$ and $Oz$ accordingly.

The method of correction to pressure was used to solve the hydrodynamic problem. The variant of this method in the case of a variable density will take the form [10-11]:

$$\frac{\bar{u} - u}{\tau} + u\bar{v} + v\bar{w} = (\mu\bar{u}_x)'_x + (\mu\bar{u}_y)'_y + (\mu\bar{u}_z)'_z,$$

$$\frac{\bar{v} - v}{\tau} + u\bar{v} + v\bar{w} = (\mu\bar{v}_y)'_y + (\mu\bar{v}_z)'_z,$$

$$\frac{\bar{w} - w}{\tau} + u\bar{w} + v\bar{w} = (\mu\bar{w}_z)'_z + (\mu\bar{w}_x)'_x + (\mu\bar{w}_y)'_y + g,$$

$$p_{a}^* + p_{b}^* + p_{c}^* = \frac{\bar{\rho} - \rho}{\tau^2} + \frac{(\bar{\rho}\bar{u})'}{\tau} + \frac{(\bar{\rho}\bar{v})'}{\tau} + \frac{(\bar{\rho}\bar{w})'}{\tau},$$

$$\frac{\bar{\rho} - \rho}{\tau} = -\frac{1}{\rho} \frac{\bar{p}_x'}{v}, \quad \frac{\bar{\rho} - \rho}{\tau} = -\frac{1}{\rho} \frac{\bar{p}_y'}{v}, \quad \frac{\bar{\rho} - \rho}{\tau} = -\frac{1}{\rho} \frac{\bar{p}_z'}{v},$$

where $V = \{u, v, w\}$ are the components of the velocity vector, $\{\bar{u}, \bar{v}, \bar{w}\}, \{\bar{\rho}, \bar{\bar{\rho}}, \bar{\bar{\rho}}\}$ are the components of the velocity vector fields on the «new» and intermediate time layers, respectively, $\bar{u} = (\bar{u} + u) / 2$. $\bar{\rho}$ and $\rho$ are the distribution of the density of the aqueous medium on the new and previous time layers, respectively.

In the construction of discrete mathematical models of hydrodynamics, the fullness of the control cells was taken into account, which makes it possible to increase the real accuracy of the solution in the case of a complex geometry of the investigated region by improving the approximation of the boundary.

Through $o_{i,j,k}$ marked the volume of fluid (VOF) of the cell $(i,j,k)$ [10]. VOF is determined by the pressure of the liquid column inside this cell. If the average pressure at the nodes that belong to the vertices of the cell in question is greater than the pressure of the liquid column inside the cell, then the cell is considered to be full ($o_{i,j,k} = 1$). In the general case, VOF can be calculated by the following formula:

$$o_{i,j,k} = \frac{P_{i,j,k} + P_{i-1,j,k} + P_{i,j-1,k} + P_{i-1,j-1,k}}{4\rho gh_l}.$$

We introduce the coefficients $q_0, q_1, q_2, q_3, q_4, q_5, q_6$, describing VOF of regions located in the vicinity of the cell (control areas). In the case of boundary conditions of the third kind
\( c_i'(x, y, t) = \alpha \epsilon + \beta \), the discrete analogs of the convective \( uc' \) and diffusion \( \left( \mu c' \right)' \) transfer operators, obtained with the help of the integro-interpolation method, taking into account the VOF, can be written in the following form [12]:

\[
\begin{align*}
(q_0)_t uc' &\equiv (q_1)_t u_{i+1/2} \frac{c_{i+1} - c_i}{2h_x} + (q_2)_t u_{i-1/2} \frac{c_i - c_{i-1}}{2h_x}, \\
(q_0)_t \left( \mu c' \right)' &\equiv (q_1)_t \mu_{i+1/2} \frac{c_{i+1} - c_i}{h_x^2} - (q_2)_t \mu_{i-1/2} \frac{c_i - c_{i-1}}{h_x^2} - \left( (q_1)_t - (q_2)_t \right) \mu \frac{\alpha c_i + \beta \epsilon}{h_x}.
\end{align*}
\]

Similarly, approximations for the remaining coordinate directions will be recorded. The error in approximating the mathematical model is equal to \( O\left( \tau + \|h\|^2 \right) \), where \( \|h\| = \sqrt{h_x^2 + h_y^2 + h_z^2} \). The conservation of the flow at the discrete level of the developed hydrodynamic model is proved, as well as the absence of non-conservative dissipative terms obtained as a result of discretization of the system of equations. A sufficient condition for the stability [13, 14] and monotony of the developed model is determined on the basis of the maximum principle [15-16], with constraints on the step with respect to the spatial coordinates: \( h_x < 2\mu / \|u\| \), \( h_y < 2\mu / \|v\| \), \( h_z < 2\nu / \|w\| \) or \( \text{Re} \leq 2N \), where \( \text{Re} = |V| \cdot l / \mu \) is the Reynolds number [10], \( l \) is the characteristic size of the region \( N = \max \{N_x, N_y, N_z\} \). Discrete analogs of the system of equations (5) are solved by an adaptive modified alternating-triangular method of variational type [17-19].

4. Parallel version of the algorithm for solving discrete equations

Consider parallel algorithm for calculating the correction vector: \((D + \omega R_1)D \cdot (D + \omega R_2)w^m = r^m\), where \( R_1 \) is the lower-triangular matrix, and \( R_2 \) is the upper-triangular matrix: \((D + \omega R_1)y^m = r^m\), \((D + \omega R_2)w^m = Dy^m\).

First, the vector \( y^m \) is calculated, and the calculation starts in the lower left corner. Then the calculation of the correction vector \( w^m \) begins from the upper right corner.

Figure 3 shows the calculation of the vector \( y^m \).

The results of calculating the acceleration and efficiency, depending on the number of processors for the parallel variant of the adaptive alternating-triangular method, are given in the table 1.
The table 1 shows that the algorithm of the alternating-triangular iterative method and its parallel implementation on the basis of decomposition in two spatial directions can be effectively applied to solve hydrodynamic problems for a sufficiently large number of cores ($p \leq 128$).

| Number of processors | Time, sec. | Acceleration | Efficiency |
|----------------------|------------|--------------|------------|
| 1                    | 7.490639   | 1            | 1          |
| 2                    | 4.151767   | 1.804        | 0.902      |
| 4                    | 2.549591   | 2.938        | 0.734      |
| 8                    | 1.450203   | 5.165        | 0.646      |
| 16                   | 0.882420   | 8.489        | 0.531      |
| 32                   | 0.458085   | 16.351       | 0.511      |
| 64                   | 0.265781   | 28.192       | 0.44       |
| 128                  | 0.171535   | 43.668       | 0.341      |

5. Description of the software package
A software package has been developed for implementing model hydrodynamic problems implemented in C++. Parallel algorithms implemented in the software package for solving model problems of the systems of grid equations arising in the process of discretization by a modified adaptive alternating triangular method of variational type were developed using MPI technology. Figure 4 shows the scheme of the algorithm of the program that numerically implements the developed 3D models of wave hydrodynamics.

![Algorithm of the software complex.](image_url)

The developed software package includes: a control unit (contains a cycle for a time variable, and the functions are called: calculating the velocity field without taking into account the elevation function, calculating the elevation function of the velocity, calculating the two-dimensional velocity field, checking the presence of the structure on the surface of the aquatic environment and data output); the input unit of the initial distributions for calculating the velocity of currents and the level elevation function (the initial distributions of the velocity field and the elevation functions of the level and the initial values of the degree of filling of the calculated cells are set); block of construction of grid equations for 3D fields of water flow rates without pressure; block of construction of grid equations for calculating the pressure field and the function of elevation of the level; a unit for checking the presence of the structure on the surface of the aquatic environment.
on the surface of the aquatic environment of the structure; the unit for calculating the velocity field with regard to pressure; a block for solving grid equations with a modified adaptive alternating triangular method of variational type; block output values of the velocity field and the pressure function (field elevation function level).

6. Results of numerical experiments based on wave-hydrodynamic model
The developed software package allows to set a complex configuration of a surface object, as well as the type and characteristics of the source of oscillations.

Developed numerical algorithms for solving model problems and implementing a set of programs can be used to study hydrophysical processes in coastal water systems [20], as well as to find the velocity field and pressure of the aquatic environment, to assess the possible negative impact on the coastline in the presence of surface waves on the basis of measurement of parameters of wave processes on the basis of field observations (table 2). The developed software tools allow to set the parameters of the source of oscillations, as well as the bathymetry of the reservoir.

| №  | Depth, cm | Wave length, s | Average wave height, cm | Maximum wave height, cm | Dispersion of elevation function | Correlation with normal distribution | Correlation with lognormal distribution |
|----|-----------|----------------|-------------------------|------------------------|-------------------------------|-------------------------------------|----------------------------------------|
| 1  | 12.734    | 3.181          | 1.434                   | 3.266                  | 3.384                         | 0.72622403                         | 0.72818161                            |
| 2  | 21.657    | 3.187          | 2.216                   | 5.127                  | 2.875                         | 0.71970734                         | 0.75497854                            |
| 3  | 34.296    | 3.257          | 2.673                   | 6.673                  | 2.587                         | 0.76756352                         | 0.80809736                            |
| 4  | 47.696    | 3.208          | 2.903                   | 7.278                  | 2.373                         | 0.80434285                         | 0.81516631                            |
| 5  | 50.221    | 3.238          | 3.408                   | 8.779                  | 2.465                         | 0.80072646                         | 0.82234947                            |
| 6  | 56.95     | 3.323          | 3.423                   | 10.05                  | 2.539                         | 0.82520735                         | 0.83499856                            |
| 7  | 58.256    | 3.094          | 3.538                   | 13.742                 | 2.468                         | 0.70451786                         | 0.75010325                            |
| 8  | 75.284    | 3.482          | 3.595                   | 12.716                 | 2.317                         | 0.80464887                         | 0.82816629                            |
| 9  | 83.353    | 3.056          | 4.472                   | 14.647                 | 2.498                         | 0.7677805                          | 0.80442466                            |
| 10 | 123.251   | 3.23           | 4.671                   | 15.749                 | 2.327                         | 0.78716382                         | 0.82809779                            |

Figure 5 shows the results of the prediction of changes in wave hydrodynamic processes during the flow of a surface body in an aqueous medium, taking into account the complex geometry of shore protection structures in a liquid. Figure shows the results of numerical experiments on modeling the propagation of wave hydrodynamic processes when a wave reaches the shore protection structure (single sea bune) taking into account the geometries of the bottom and shore protection structure located in a liquid and on the bottom of a reservoir at different points in time.

![Figure 5. The field of the vector of the velocity of movement of the aquatic environment (cut of the XOZ plane).](image-url)
Sea buns are transverse (normally located to the shoreline) structures intended for retaining beach-forming material from the natural long-shore sediment flow and preserve the natural or artificial beach in the inter-bune compartments. Linear buns can be single-row and double-row, pile and precast concrete, on columns-shells and prismatic blocks. Cape-shaped buns differ from linear ones, they form an artificial territory that can be used for recreational or technical purposes.

Buns disturb the natural landscapes of the sea coast and from modern aesthetic positions their use is undesirable. The presence of bun in itself makes a significant change in the dynamics of the coastal zone. First of all, they violate the longitudinal structure of the movement of mud and sand sediments, and therefore the leeward coast will almost always experience a shortage of material. The protective properties of buns have local character: they contribute to the accumulation of the beach and protect the shore only where built.

The development of a 3D model of wave hydrodynamic processes based on field data made it possible to describe the movement of the aquatic environment of shallow water body taking into account the presence of shore protection structures and wave propagation towards the shore. A modern software package has been created that is adapted for modeling hydrodynamic wave processes, is used in a wide range of parameters to calculate velocity fields and pressures of the aquatic environment, and to estimate the hydrodynamic impact on the coast and shore protection structures in the presence of surface waves.

![Graphs of the bottom relief.](image)

The source of perturbations is set at a given distance from the shoreline. The modeling area has dimensions of 50 by 50 m and a depth of 2 m, the peak point rises above sea level by 2 m.

Suppose that the liquid is at rest at the initial moment of time. When solving the posed model problem, a grid of 100 × 100 × 40 sizes of calculated nodes was used; the time step was 0.01 s.

Figure 6 demonstrates the field of the velocity vector of the motion of the aquatic environment when the wave rolls onto the coastline, the level elevation function dynamically changes, and flooding and shallowing zones are formed. Accounting for flooding and dehumidification of coastal areas was carried out at the expense of recalculation of the fullness of the calculated cells. The proposed approach allows solving problems in regions with complex and dynamically changed boundary geometry.

Figure 7 shows the results of a numerical experiment based on 3D model of wave processes in the presence of a technical object located below the level of the undisturbed surface of the reservoir.
The developed software was used for the numerical implementation of the proposed 3D model of wave hydrodynamics and building a forecast of the movement of the aquatic environment in the presence of a technical object located below the level of the undisturbed surface of the reservoir; hydrodynamic loads on the structure supports were calculated.

7. Conclusion
The article describes the development and study of 3D model of wave processes, taking into account shore protection facilities. The description of the developed software is given, it allows to change the characteristics and location of the source of oscillations, as well as take into account the shape of zones and the degree of intensity of drainage and flooding of coastal areas. The software package allows to calculate the hydrodynamic impact on shore protection structures such as buns. The protective properties of buns have a local character, they contribute to the accumulation of the beach and protect the shore only where built. The created software complex can be widely used for practical research of the calculation of the force effect of waves on the geometry of the bottom surface, as well as surface and coastal infrastructure objects.

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