Modeling the Internet

M. Ángeles Serrano, 1 Marián Boguñá, 2 and Albert Díaz-Guilera 2

1School of Informatics, Indiana University,
  Eigenmann Hall, 1900 East Tenth Street, Bloomington, IN 47406, USA

2Departament de Física Fonamental, Universitat de Barcelona,
  Martí i Franquès 1, 08028 Barcelona, Spain

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Abstract

We model the Internet as a network of interconnected Autonomous Systems which self-organize under an absolute lack of centralized control. Our aim is to capture how the Internet evolves by reproducing the assembly that has led to its actual structure and, to this end, we propose a growing weighted network model driven by competition for resources and adaptation to maintain functionality in a demand and supply “equilibrium”. On the demand side, we consider the environment, a pool of users which need to transfer information and ask for service. On the supply side, ASs compete to gain users, but to be able to provide service efficiently, they must adapt their bandwidth as a function of their size. Hence, the Internet is not modeled as an isolated system but the environment, in the form of a pool of users, is also a fundamental part which must be taken into account. ASs compete for users and big and small come up, so that not all ASs are identical. New connections between ASs are made or old ones are reinforced according to the adaptation needs. Thus, the evolution of the Internet can not be fully understood if just described as a technological isolated system. A socio-economic perspective must also be considered.

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I. INTRODUCTION

In an attempt to bring nearer theory and reality, many researchers working on the new and rapidly evolving science of complex networks [1] have recently shifted focus from unweighted graphs to weighted networks. Commonly, interactions between elements in network-representable complex real systems—may they be communication systems, such as the Internet, or transportation infrastructures, social communities, biological or biochemical systems—are not of the same magnitude. It seems natural that the first more simple representations, where edges between pairs of vertices are quantified just as present or absent, give way to more complex ones, where edges are no longer in binary states but may stand for connections of different strength.

Weight is just one of the relevant ingredients in bringing network modeling closer to reality. Others come from the fact that real systems are not static but evolve. As broadly recognized, growth and preferential attachment are also key issues at the core of a set of recent network models focusing on evolution under an statistical physics approach [2, 3, 4, 5, 6, 7, 8, 9]. These models have been able to approximate some topological features observed in many real networks—specifically the small-world property or a power-law degree distribution—as a result of the organizing principles acting at each stage of the network formation process. Although a great step forward in the understanding of the laws that shape network evolution, these new degree driven models cannot describe other empirical properties. Further on, in order to achieve representations that closely match reality, it is necessary to uncover new mechanisms.

Following this motivation, we believe that the general view of networks as isolated systems, although possibly appropriate in some cases, must be changed if we want to describe in a proper way complex systems which not generate spontaneously but self-organize within a medium in order to perform a function. Many networks evolve in an environment to which they interact and which usually provides the clues to understand functionality. Therefore, rules defined on the basis of internal mechanisms alone, such as preferential attachment that acts internally at the local scale to connect nodes through edges, are not enough. When analyzing the dynamics of network assembly, the interlock of its constituents with the environment cannot be systematically obviated.

With the aim of approaching applicability, in this work we blend all ideas above to present
a growing network model in which both, nodes and links, are weighted \[10\]. The dynamical evolution is driven by exponential growth, competition for resources and adaptation to maintain functionality in a demand and supply “equilibrium”, key mechanisms which may be relevant in a wide range of self-organizing systems, in particular those where functionality is tied to communication or traffic. The medium in which the network grows and to with it interacts is here represented by a pool of elements which, at the same time, provide resources to the constituents of the network and demand functionality, say for instance users in the case of the Internet \[11\] or passengers in the case of the world-wide airport network \[12\].

Competition is here understood as a struggle between network nodes for new resources and is modeled as a rich get richer (preferential attachment) process. For their part, this captured elements demand functionality so that nodes must adapt in order to perform efficiently. This adaptation translates into the creation of weighted links between nodes.

In this work, we apply those ideas to the Internet. In the realm of complexity theory, the Internet is a paradigmatic example and significant efforts has been devoted to the development of models which reproduce the topological properties observed in its maps \[11\]. Candidates run from topology generators \[13, 14\] to degree driven growing networks models \[15, 16\] or Highly Optimized Tolerance (HOT) models \[17\]. Some of them reproduce heavy-tailed degree distributions and small-world properties, but perform poorly when estimating correlations or other characteristic properties, such as the rich-club phenomenon or the k-core structure. By contrast, we will show that our model nicely reproduces an overwhelming number of observed topological features: the small-world property, the scale-free degree distribution \(P(k)\), high clustering coefficient \(c_k\) that shows a hierarchical structure, disassortative degree-degree correlations, quantified by means of the average nearest neighbors degree of nodes of degree \(k\); \(\bar{k}_{nn}(k)\) \[18\], the scaling of the higher order loop structure recently analyzed in \[19\], the distributions of the betweenness centrality, \(P(b)\), and triangles passing through a node, \(P(T)\), and, finally, the k-core decomposition uncovering its hierarchical organization \[20, 21\].

We will consider the Internet evolution at the Autonomous System (AS) level. ASs are defined as independently administered domains which autonomously determine internal communications and routing policies \[11\] and, as a first approximation, we can assign each AS to an Internet Service Provider (ISP). This level of description means a coarse grained representation of the Internet. Nevertheless, further detail is not necessary when aiming to
explain and predict the large-scale behavior. Thus, the network will be made up of ASs as nodes connected among them with links which can be of different strength or bandwidth. On the side of the environment modeling, we place hosts on the level as users.

In the next sections we analyze the growth of the Internet over the last years. Then we present the model. Working in the continuum approximation, we find analytically the distribution of the sizes (in number of users) of ASs and the degree distribution. Then, we introduce an algorithm in order to simulate network assembly. At this stage, we also make a first attempt to the consideration of geographical constraints. Finally, the synthetic networks are compared to the real maps of the Internet through a variety of different measures.

II. THE GROWTH OF THE INTERNET

Let \( W(t) \) be the total number of users in the environmental pool at a given time \( t \), measured as hosts. \( N(t) \) and \( E(t) \) stand for the number of ASs and edges among them in the network, respectively. Empirical measures for the growth in the number of users have been obtained from the Hobbes Internet Timeline \[22\]. The growth of the network is analyzed from AS maps collected by the Oregon route-views project, which has recorded data since November 1997 \[23\]. According to those observations, shown in Fig. 1, we will assume exponential growths for these quantities, \( W(t) \approx W_0 e^{\alpha t} \), \( N(t) \approx N_0 e^{\beta t} \), and \( E(t) \approx E_0 e^{\delta t} \). These exponential growths, in turn, determine the scaling relations with the system size: \( W \propto N^{\alpha/\beta} \), \( E \propto N^{\delta/\beta} \) and \( \langle k \rangle \propto N^{\delta/\beta - 1} \).

The rates of growth can be measured to be \( \alpha = 0.036 \pm 0.001 \), \( \beta = 0.0304 \pm 0.0003 \), and \( \delta = 0.0330 \pm 0.0002 \) (units are month\(^{-1}\)), where \( \alpha \gtrsim \delta \gtrsim \beta \). These three rates are quite close to each other but they are not equal. In fact, the inequality \( \alpha \gtrsim \beta \) must hold in order to preserve network functionality. When the number of users increases at a rate \( \alpha \), there are two mechanisms capable to compensate the demand they represent: the creation of new nodes and the creation of new connections by nodes already present in the network. When both mechanisms take place simultaneously, the rate of growth of new nodes, \( \beta \), as well as the rate for the number of connections, \( \delta \), must necessarily be smaller than \( \alpha \). Any other situation would lead to an imbalance between demand and supply of service in the system. On the other hand, in a connected network, \( \delta \) must be equal or greater than \( \beta \). If \( \delta \) equals \( \beta \) the average number of connections per node, or average degree, remains constant.
FIG. 1: Temporal evolution of the number of hosts, autonomous systems and connections among them from November 1997 to May 2002. Solid lines are the best fit estimates. Each point for the number of ASs and connections is an average over one month of daily measurements. Error bars are of the order of the symbol size.

in time, whereas it increases when $\delta \gtrsim \beta$. This increase could correspond to a demand per user which is not constant but grows in time, probably due to the increase of the power of computers over time and, as a consequence, to the ability to transfer bigger files or to use more demanding applications.

III. THE MODEL

We define our model according to the following rules: (i) At rate $\alpha W(t)$, new users join the system and choose node $i$ according to some preference function, $\Pi_i(\{\omega_j(t)\})$, where $\omega_j(t)$, $j = 1, \cdots, N(t)$, is the number of users already connected to node $j$ at time $t$. The function $\Pi_i(\{\omega_j(t)\})$ is normalized so that $\sum_i \Pi_i(\{\omega_j(t)\}) = 1$ at any time. (ii) At rate $\beta N(t)$, new nodes join the network with an initial number of users, $\omega_0$, randomly withdrawn from the pool of users already attached to existing nodes. Therefore, $\omega_0$ can be understood as the minimum number of users required to keep nodes in business. (iii) At rate $\lambda$, each user changes his AS and chooses a new one using the same preference function $\Pi_i(\{\omega_j(t)\})$. Finally, (iv) each node tries to adapt its number of connections to other nodes according to its present number of users or size, in an attempt to provide them an adequate functionality.

With all specifications above, we will work in the continuum approximation to find some analytic results, specifically the distribution of the sizes of ASs and the degree distribution.
A. Analytic results

The resource dynamics of single nodes is described by the following stochastic differential equation

$$\frac{d\omega_i}{dt} = A(\omega_i, t) + [D(\omega_i, t)]^{1/2} \xi(t),$$

where $\omega_i$ is the number of users attached to AS $i$ at time $t$. The time dependent drift is $A(\omega_i, t) = (\alpha + \lambda)W(t)\Pi_i - \lambda \omega_i - \beta \omega_0$, and the diffusion term is $D(\omega_i, t) = (\alpha + \lambda)W(t)\Pi_i + \lambda \omega_i + \beta \omega_0 - 2\lambda \omega_i \Pi_i$. Application of the Central Limit Theorem guaranties the convergence of the noise $\xi(t)$ to a gaussian white noise in the limit $W(t) \gg 1$. The first term in the expression for the drift is a creation term accounting for new and old users that choose node $i$. The second term represent those users who decide to change their node and, finally, the last term corresponds to the decrease of users due to introduction of newly created nodes. To proceed further, we need to specify the preference function $\Pi_i(\{\omega_j(t)\})$. We assume that, as a result of a competition process, nodes bigger in resources get users more easily than small ones. The simplest function satisfying this condition corresponds to the linear preference, that is, $\Pi_i(\{\omega_j(t)\}) = \omega_i/W(t)$, where $W(t) = \omega_0 N_0 \exp(\alpha t)$. In this case, the stochastic differential equation (1) reads

$$\frac{d\omega_i}{dt} = \alpha \omega_i - \beta \omega_0 + [(\alpha + 2\lambda)\omega_i + \beta \omega_0]^{1/2} \xi(t).$$

Notice that reallocation of users (i.e. the $\lambda$-term) only increases the diffusive part in Eq. (2) but has no net effect in the drift term, which is, eventually, the leading term. The complete solution of this problem requires to solve the Fokker-Planck equation corresponding to Eq. (2) with a reflecting boundary condition at $\omega = \omega_0$ and initial conditions $p(\omega_i, t_i | \omega_0, t_i) = \delta(\omega_i - \omega_0)$ ($\delta(\cdot)$ stands for the Dirac delta function). Here $p(\omega_i, t | \omega_0, t_i)$ is the probability that node $i$ has a number of users $\omega_i$ at time $t$ given that it had $\omega_0$ at time $t_i$. The choice of a reflecting boundary condition at $\omega = \omega_0$ is equivalent to assume that $\beta$ is the overall growth rate of the number of nodes, that is, the composition of the birth and dead processes ruling the evolution of the number of nodes.

Finding the solution for this problem is not an easy task. Fortunately, we can take advantage of the fact that, when $\alpha > \beta$, the average number of users of each node increases exponentially and, since $D(\omega_i, t) = \mathcal{O}(A(\omega_i, t))$, fluctuations vanishes in the long time limit. Under this zero noise approximation, the number of users connected to a node introduced
at time $t_i$ is
\[
\omega_i(t|t_i) = \frac{\beta}{\alpha}\omega_0 + (1 - \frac{\beta}{\alpha})\omega_0 e^{\alpha(t-t_i)}.
\]
(3)
The probability density function of $\omega$ can be calculated in the long time limit as
\[
p(\omega, t) = \beta e^{-\beta t} \int_0^t e^{\beta t_i} \delta(\omega - \omega_i(t|t_i)) dt_i
\]
(4)
which leads to
\[
p(\omega, t) = \tau(1 - \tau)\frac{\tau \omega_0^\tau}{(\omega - \tau \omega_0)^{1+\tau}} \Theta(\omega_c(t) - \omega),
\]
(5)
where we have defined $\tau \equiv \beta/\alpha$ and the cut-off is given by $\omega_c(t) \sim (1 - \tau)\omega_0 e^{\alpha t} \sim W(t)$. Thus, in the long time limit, $p(\omega, t)$ approaches a stationary distribution with an increasing cut-off that scales linearly with the total number of users. The exponent $\tau$ depends on the relative values of $\beta$ and $\alpha$, which can be different but typically would stay close so that $\tau$ would value around 2.

The key point now is to construct a bridge between the competition and the adaptation mechanisms, in other words, to see how to relate the number of users attached to an AS with its degree and bandwidth. Our basic assumption is that vertices are continuously adapting their strength or bandwidth, the total weight of its connections, to the number of users they have. However, once a node decides to increase its bandwidth it has to find a peer who, at the same time, wants to increase its bandwidth as well. The reason is that connection costs among nodes must be assumed by both peers. This fact differs from other growing models in which vertices do not ask target vertices if they really want to form those connections. Our model is, then, to be thought of as a coupling between a competition process for users and adaptation of vertices to their current situation, with the constraint that connections are only formed between “active” nodes, that is, those ASs with a positive increment of their number of users. Let $b_i(t|t_i)$ be the total bandwidth of a node at time $t$ given that it was introduced at time $t_i$. This quantity can include single connections with other nodes, i.e. the topological degree $k$, but it also accounts for connections which have higher capacity. This is equivalent to say that the network is, in fact, weighted and $b_i$ is the weighted degree. To simplify the model we consider that bandwidth is discretized in such a way that single connections with high capacity are equivalent to multiple connections between the same nodes. Then, when a pair of nodes agrees to increase their mutual connectivity the connection is newly formed if they were not previously connected or, if they were, their mutual bandwidth increases by
one unit, reinforcing in this way their connectivity. Now, we assume that, at time \( t \), each node adapts its total bandwidth proportionally to its number of users, or size, following a lineal relation. Thus, we can write

\[
b_i(t|t_i) = 1 + a(t) (\omega_i(t|t_i) - \omega_0).
\]  \hspace{1cm} (6)

Summing Eq. (6) for all nodes we get \( a(t) = (2B(t) - N(t))/(W(t) - \omega_0N(t)) \approx 2B(t)/W(t) \), where \( B(t) \) is the total bandwidth of the network. \( B(t) \) is, obviously, an upper bound to the total number of edges of the network. This suggests that \( B(t) \) will grow according to \( B(t) = B_0e^{\delta t} \). As the number of users grows, the global traffic of the Internet also grows, which means that nodes do not only adapt their bandwidth to their number of users but to the global traffic of the network. Therefore, \( a(t) \) must be an increasing function of \( t \), which, in turn, implies that \( \delta' > \alpha \) and, thus, \( \delta' > \delta \). As a consequence, the network must necessarily contain multiple connections. This can be explicitly seen by inspecting the scaling of the maximum bandwidth, which reads \( b_c(t) \propto N(t)^{\delta'/\beta} \), that is, faster than \( N(t) \). Therefore, the topological degree of a node cannot be proportional to its bandwidth. Nevertheless, it is clear that \( k_i \) and \( b_i \) are positive correlated random variables. We then propose that degree and bandwidth are related, in a statistical sense, through the following scaling relation

\[
k(t|t_i) = [b(t|t_i)]^\mu, \quad \mu < 1,
\]  \hspace{1cm} (7)

which implies that all nodes can form multiple connections, regardless of their size. This scaling behavior has recently been observed in other weighted networks \[12, 24\]. The superlinear behavior of \( b_c(t) \), combined with this scaling relation, ensures that rich ASs will connect to a macroscopic portion of the system, so that the maximum degree will scale linearly with the system size. Empirical measurements made in \[4\] showed such linear scaling in the AS with the largest degree. This sets the scaling exponent to \( \mu = \beta/\delta' \).

All four growth rates in the model are not independent but can be related by exploring the interplay between bandwidth, connectivity, and traffic of the network. Summing Eq. (7) for all vertices, the scaling of the total number of connections is \( E(t) \propto N(t)^{2-\alpha/\delta'} \), which leads to \( \delta' = \alpha\beta/(2\beta - \delta) \). Combining this relation with Eqs. (5), (6) and (7), the degree distribution reads

\[
P(k) \approx \frac{\tau(1-\tau)^{\tau} [\omega_0a(t)^{\tau} 1 \Theta(k_c(t) - k)}{\mu k^{\gamma}}.
\]  \hspace{1cm} (8)
for \( k \gg 1 \), where the exponent \( \gamma \) takes the value \( \gamma = 1 + 1/(2 - \delta/\beta) \). Strikingly, the exponent \( \gamma \) has lost any direct dependence on \( \alpha \) becoming a function of the ratio \( \delta/\beta \). Using the empirical values for \( \beta \) and \( \delta \), the predicted exponent is \( \gamma = 2.2 \pm 0.1 \), in excellent agreement with the values reported in the literature [18, 25]. Of course, this does not mean that the exponent \( \gamma \) is independent of \( \alpha \), since both, \( \beta \) and \( \delta \), may depend on the growth of the number of users. Anyway, our model turns out to depend on just two independent parameters which can be expressed as ratios of the rates of growth, \( \beta/\alpha \) and \( \delta/\beta \).

B. Simulations

So far, we have been mainly interested in the degree distribution of the AS map but not in the specific way in which the network is formed. To fill this gap we have performed numerical simulations that generate network topologies in nice agreement with real measures of the Internet. Although ASs are distributed systems, we assume they follow the same spatial distribution as the one measured for routers, so that we are able to define a physical distance among them to take into account connection costs [9]. Our algorithm, following the lines of the model, works in four steps:

1. At iteration \( t \), \( \Delta W(t) = \omega_0 N_0 (e^{\alpha t} - e^{\alpha (t-1)}) \) users join the network and choose provider among the existing nodes using the linear preference rule.

2. \( \Delta N(t) = N_0 (e^{\beta t} - e^{\beta (t-1)}) \) new ASs are introduced with \( \omega_0 \) users each, those being randomly withdrawn from already existing ASs. Newly created ASs are located in a two dimensional plane following a fractal set of dimension \( D_f = 1.5 \) [9].

3. Each AS evaluate its increase of bandwidth, \( \Delta b_i(t|t_i) \), according to Eq. [9].

4. A pair of nodes, \( (i, j) \), is chosen with probability proportional to \( \Delta b_i(t|t_i) \) and \( \Delta b_j(t|t_j) \) respectively, and, whenever they both need to increase their bandwidth, they form a connection with probability \( D(d_{ij}, \omega_i, \omega_j) \). This function takes into consideration that, due to connection costs, physical links over long distances are unlikely to be created by small peers. Once the first connection has been formed, they create a new connection with probability \( r \), whenever they still need to increase their bandwidth. This step is repeated until all nodes have the desired bandwidth.
It is important to stress the fact that nodes must be chosen with probability proportional to their increase in bandwidth at each step. The reason is that those nodes that need a high bandwidth increase will be more active when looking for partners to whom form connections. Another important point is the role of the parameter $r$. This parameter takes into account the balance between the costs of forming connections with new peers and the need for diversification in the number of partners. The effect of $r$ in the network topology is to tune the average degree and the clustering coefficient by modulating the number of multiple connections. The exponent $\gamma$ is unaffected except in the limiting case $r \to 1$. In this situation, big peers will create a huge amount of multiple connections among them, reducing, thus, the maximum degree of the network. Finally, we chose an exponential form for the distance probability function $D(d_{ij}, \omega_i, \omega_j) = e^{-d_{ij}/d_c(\omega_i, \omega_j)}$, where $d_c(\omega_i, \omega_j) = \omega_i \omega_j / \kappa W(t)$ and $\kappa$ is a cost function of number of users per unit distance, depending on the maximum distance of the fractal set. All simulations are performed using $\omega_0 = 5000$, $N_0 = 2$, $B_0 = 1$, $\alpha = 0.035$, $\beta = 0.03$, and $\delta' = 0.04$. The final size of the networks is $N \approx 11000$, which approximately corresponds to the size of the actual maps for 2001 that we are considering in this work.

IV. TESTING THE MODEL

To test the model we construct synthetic networks from our algorithm with and without taking into consideration the geographical distribution of ASs, and we contrast several measures on those graphs to those of real maps, more specifically, the AS map dated May 2001 from data collected by the Oregon Route Views Project [23], and the AS extended (AS+) map [26] which completes the previous one with data from other sources. Let us note that all the measures presented here are performed over the same synthetic networks. The parameters of the model are fixed once and for all before generating the networks so that they are not tuned in order to approach different properties.

First, we analyze a first category of measures which include the features of traditional interest when aiming to reproduce the Internet topology. The small world effect becomes clear when analyzing the distribution of the shortest path lengths, as seen in the left side graph of Fig. 2 with an average shortest path length very close to the real one. The graph on the right of Fig. 2 shows simulation results for the cumulative degree distribution, in
FIG. 2: Distribution of the shortest path lengths (left) and cumulative degree distribution \( P_c(k) = \sum_{k' \geq k} P(k') \) (right) for the extended AS map compared to simulations of the model, \( r = 0.8 \). Inset (right): Simulation results of the AS’s degree as a function of AS’s bandwidth. The solid line stands for the scaling relation Eq. (7) with \( \mu = \beta/\delta' = 0.75 \).

nice agreement to that measured for the AS+ map. The inset exhibits simulation results of the AS’s degree as a function of the AS’s bandwidth, confirming the scaling ansatz Eq. (7). Clustering coefficient and average nearest neighbors degree are showed in Fig. 3. Dashed lines result from the model without distance constraints, whereas squares correspond to the model with distance constraints. Interestingly, the high level of clustering coming out from the model arises as a consequence of the pattern followed to attach nodes, so that only those AS willing for new connections will link. As can be observed in the figures, distance constraints introduce a disassortative component by inhibiting connections between small ASs so that the hierarchical structure of the real network is better reproduced.

Now, we turn our attention to new measures, which run from the scaling of higher orders loops to the k-core structure. Not only two-point correlations are well approximated by our model, but it is also able to reproduce the scaling behavior of the number of loops of size 3, 4 and 5. This has been recently measured for the Internet at the AS level in [19], and it is seen to follow a power of the system size of the form \( N_h(N) \sim N^{\xi(h)} \), with exponents that are closely reproduced by our synthetic networks, see Fig. 4 and table I. In Fig. 5 we observe on the left the cumulative distribution of betweenness centrality as proposed by Freeman [27], a measure of the varying importance of the vertices in a network. On the right, the cumulative distribution of triangles passing by a node (for a discussion of the relevance of \( P(T) \) see, for instance, [28]).
FIG. 3: Clustering coefficient, $c_k$, (left), and normalized average nearest neighbors degree, $\bar{k}_{nn}(k)/\langle k \rangle^2$, (right), as functions of the node's degree for the extended autonomous system map (circles) and for the model with and without distance constraints (red squares and dashed line, respectively).

TABLE I: Values for the exponents $\xi(h)$ for $h = 3, 4, \text{ and } 5$ for the Internet and the models with and without distance constraints (after Bianconi et al. [19]).

| System               | $\xi(3)$       | $\xi(4)$       | $\xi(5)$       |
|----------------------|----------------|----------------|----------------|
| Internet AS map      | $1.45 \pm 0.07$| $2.07 \pm 0.01$| $2.45 \pm 0.08$|
| Model with distance  | $1.60 \pm 0.01$| $2.20 \pm 0.03$| $2.70 \pm 0.03$|
| Model without distance| $1.59 \pm 0.03$| $2.11 \pm 0.03$| $2.64 \pm 0.03$|

Finally, we also show the k-core decomposition of the actual and the synthetic maps. The k-core decomposition is a recursive reduction of the network as a function of the degree, which allows the recognition of hierarchical structure and more central nodes [20]. A very good agreement between real measures and our models can be appreciated in Fig 6. In the case of the model with distance constraints, even the coreness, the maximum number of layers in the $k$-core decomposition, is almost the same as in the Internet map. These visualizations have been produced with the tool LANET-VI [21].

V. CONCLUSIONS

In summary, we have presented a simple weighted growing network model for the Internet, based on evolution, environmental interaction and heterogeneity. The dynamics is driven
by two key mechanisms, competition and adaptation, which may be relevant in other self-organizing systems. Beyond technical details, many empirical features are nicely reproduced but open questions remain, perhaps the more important one being whether the general ideas and mechanisms exposed in this work could help us to better understand other complex systems.

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FIG. 6: k-core decompositions for the AS extended map of the Internet (left) and for the maps generated from our model with and without distance (center and right respectively). These visualizations have been produced with the tool LANET-VI [21].

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