Time-Varying Dark Energy Constraints From the Latest SN Ia, BAO and SGL

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ABSTRACT

Based on the latest SNe Ia data provided by Hicken et al. (2009) with using MLCS17 light curve fitter, together with the Baryon Acoustic Oscillation (BAO) and strong gravitational lenses (SGL), we investigate the constraints on the dark energy equation-of-state parameter $w$ in the flat universe, especially for the time-varying case $w(z) = w_0 + w_z z/(1 + z)$. The constraints from SNe data alone are found to be: (a) $(\Omega_M, w) = (0.358, -1.09)$ as the best-fit results; (b) $(w_0, w_z) = (-0.73^{+0.23}_{-0.97}, 0.84^{+1.66}_{-10.34})$ for the two parameters in the time-varying case after marginalizing the parameter $\Omega_M$; (c) the likelihood of parameter $w_z$ has a high non-Gaussian distribution; (d) an extra restriction on $\Omega_M$ is necessary to improve the constraint of the SNe Ia data on the parameters $(w_0, w_z)$. A joint analysis of SNe Ia data and BAO is made to break the degeneracy between $w$ and $\Omega_M$, and leads to the interesting maximum likelihoods $w_0 = -0.94$ and $w_z = 0$. When marginalizing the parameter $\Omega_M$, the fitting results are found to be $(w_0, w_z) = (-0.95^{+0.45}_{-0.18}, 0.41^{+0.79}_{-0.96})$. After adding the splitting angle statistic of SGL data, a consistent constraint is obtained $(\Omega_M, w) = (0.298, -0.907)$ and the constraints on time-varying dark energy are further improved to be $(w_0, w_z) = (-0.92^{+0.14}_{-0.10}, 0.35^{+0.47}_{-0.54})$, which indicates that the phantom type models are disfavored.

Keywords: dark energy, type Ia supernova, cosmological parameters
1. INTRODUCTION

Analysis of the distance modulus versus redshift relation of type Ia supernova (SNe Ia) provides a direct evidence that the universe expansion is accelerating in the last few billion years (e.g., Perlmutter et al. 1999, Riess et al. 1998, 2004, 2007; Astier et al. 2006; Wood-Vasey et al. 2007; Kowalski et al. 2008; Hicken et al. 2009; Kessler et al. 2009). This cosmic image is also supported by many other cosmological observations, like the Cosmic Microwave Background (CMB) (Hinshaw et al. 2009; Komatsu et al. 2009), the Baryon Acoustic Oscillation (BAO) measurement (Eisenstein et al. 2005; Cole et al. 2005; Huetsi 2006; Percival et al. 2007) and the weak gravitational lenses (Weinberg and Kamionkowski 2002; Zhan and Knox 2006). Based on the Friedmann equation, the acceleration can be explained through introducing a negative pressure component in the universe, named dark energy, which is nearly spatially uniform distribution and contributes about 2/3 critical density of universe today. To reveal the property of dark energy, the most of studies, either theoretical models or experiment data analysis, are focused on its equation-of-state parameter $w = p/\rho$. Here we shall utilize the latest SNe Ia data provided by Hicken et al. (2009) with using MLCS17 light curve fitter, together with the splitting angle statistic of strong gravitational lenses (SGL; Zhang et al. 2009) and the baryonic acoustic oscillations (BAO; Eisenstein et al. 2005) to investigate the constraint for the parameter $w$ of dark energy in the flat cosmology, especially for the time varying case.

By far, all observed data are consistent with the $\Lambda$CDM cosmology, with dark energy in the form of a cosmological constant $\Lambda$. However, this model raises theoretical problems related to the fine tuned value (see e.g. Padmanabhan 2003). Many other theoretical models, like quintessence models and phantom model (Ratra and Peebles 1988; Caldwell, Davé, and Steinhardt 1998; Caldwell 2002), reveal that dark energy might be a dynamical component and evolves with time. It is usual to parametrize dark energy as an ideal liquid with its equation-of-state (EOS) parameter $w(z) = w_0 + w_z z/(1 + z)$ (Chevallier et al. 2001; Linder 2003). Conveniently, it includes the case of a constant EOS with $(w_0 = w, w_z = 0)$, and the $\Lambda$CDM model $(w_0 = -1, w_z = 0)$. Then theoretical models can be classified in a phase diagram on the $(w_0, w_z)$ plane (see e.g. Barger et al. 2006; Biswas and Wandelt 2009). Thus the accurate measurement of the parameters $(w_0, w_z)$ is helpful for testing a certain theoretical models. The current allowed regions of $(w_0, w_z)$ given by the most observations or their combinations remain surrounding the crucial point $(w_0 = -1, w_z = 0)$, which is the common point in the phase diagram of different classified models. Therefore, the final judgment of models can not be made and more careful works are still needed.

The SNe Ia has homogeneity and extremely high intrinsic luminosity of peak magnitude and thus is widely used to measure the cosmological parameters (e.g. Riess et al. 2004;
Wood-Vasey et al. 2007; Kowalski et al. 2008; Hicken et al. 2009; Kessler et al. 2009). With given density of dark energy in the universe today \( \rho(z = 0) \), the change of equation-of-state parameter \( w \) will bring the change of its density \( \rho(z) \) at the redshift \( z \) and then the distance \( d(z) \). Inversely, the measurements of the redshift \( z \) of SNe Ia and corresponding distance \( d(z) \) can constrain the dark energy. In spite of the high accuracy of SNe Ia measurement, its potential of constraining dark energy is not very strong due to the degeneracy of \( w \) and \( \Omega_M \). Therefore, a combining analysis with other observations is often useful.

We shall also use the summary parameters of the baryonic acoustic oscillations as reported in previous studies (Eisenstein et al. 2005). The large-scale correlation function of a large sample of luminous red galaxies has been measured in the Sloan Digital Sky Survey and a well-detected acoustic peak was found to provide a standard ruler by which the absolute distance of \( z = 0.35 \) can be determined with 5% accuracy, which is independent of the Hubble constant \( h \). This ruler is a \( \Omega_M \) prior and can be used to constrain dark energy (e.g. Porciani and Madau 2000; Huterer and Ma 2004; Chae, 2007). We are going to show that the strong gravitational lensing statistic observation can also provide us a useful probe of dark energy of the universe. This is because the dark energy affects, mainly through the comoving number density of dark halos described by Press-Schechter theory and the background cosmological line element, the efficiency with which dark-matter concentrations produce strong lensing signals. Then by comparing the observed number of lenses with the theoretical expected result as a function of image separation and cosmological parameters, it enables us to determine the allowed range of the parameter \( w \). The constraint process also depends on the density profile of dark halos. Here we will use the two model combined mechanism to reproduce the observed curve of lensing probability to the image splitting angle (Sarbu, Rusin and Ma 2001; Li and Ostriker 2002; Zhang et al. 2009). The redshift of CMB is above 1000 and far larger than 1, and there is no other observation to fill up this redshift gap, thus we would not adopt the CMB data in the present analysis and limit our study on dark energy to the redshift region of \( z \sim 1 \), which is the characteristic redshift scale of SNe Ia, BAO and SGL statistic.

In our recent work (Zhang et al. 2009), we have present the constraint on the dark energy from the SGL splitting angle statistic. In this paper, by taking the latest analyzed SNe Ia data (Hicken et al. 2009), the baryonic acoustic oscillations (Eisenstein et al. 2005) and the CLASS statistical sample (Browne et al. 2003), we shall make a joint analysis to constrain the dark energy equation of state parameters \( w \), especially for the time-varying parameterization \( w(z) = w_0 + w_z z/(1 + z) \). We mainly highlight two issues which have not previously been illuminated. First, we carefully study based on the latest SNe Ia data the constraints for the dark energy EOS \( w(z) \) and the influences of \( \Omega_M \). Second, we investigate the joint analysis of SNe Ia data, BAO and SGL statistic in detail. Our paper is organized as follows: Sect. 2
shows the constraint by the latest SNe Ia data on dark energy. Sect. 3 describes the joint analysis of the SNe Ia, the BAO and the SGL statistic, more stringent constraints on dark energy are resulted. The conclusions are presented in the last section.

2. DARK ENERGY CONSTRAINTS BY THE LATEST SNe Ia DATA

As the standard candles of the cosmology, the SNe Ia is used to study the geometry and dynamics of the universe with redshift \( z \leq 1.7 \). In determinations of cosmological parameters about the accelerating expand and dark energy, the SNe Ia remains a key ingredient. In 1998, the SNe Ia measurement provided the first direct evidence for the presence of dark energy with the negative pressure. Then many SN Ia observations have been done and the total number of SNe Ia sample increases quickly. The SN Ia compilations are often consist of high-redshift \((z \simeq 0.5)\) data set and low-redshift \((z \simeq 0.05)\) sample at the same time (e.g. Riess et al. 1998; Perlmutter et al. 1999; Wood-Vasey et al. 2007). When combining several independent group’s SNe Ia data sets into one compilation, the consistent analysis method of light curves and the selection of supernova are crucial. For a certain sample, the different light curve fitter and corresponding different selection of supernova can lead to different constraints on the cosmological parameters (e.g. Hicken et al. 2009).

The fitting results of cosmological parameters from different SN Ia compilations have moderate differences. To obtain the consistent and more powerful constraint, researchers have made many efforts to deal with the cross-calibration uncertainties when combining the different SNe samples. There is a conventional method to combine several group’s SNe Ia compilations, namely, by introducing an extra nuisance parameter in the \( \chi^2 \) statistic of every used SNe Ia sample and marginalizing them over in the fit, all \( \chi^2 \) statistics of samples can be summed into one total statistics (see e.g. Barger et al. 2006). The nuisance parameters are considered as analysis-dependent global unknown constants in the distances. Although this combined mechanism is wildly adopted, the so-called analysis-dependent unknown constant is just an averaged effect of analysis-dependent uncertainties.

Kowalski et al. (2008) provided the Union data set, a compilation of 307 SNe Ia discovered in different surveys. The heterogeneous nature of the data set have been reflected and all SNe Ia sample are analyzed with the same analysis procedure. In the Union data set, all SNe Ia light curve are fitted by using the spectral-template-based fit method of Guy et al. (2005) (also known as SALT). There are other light curve fitters used in literatures, such as SALT2 (Guy et al. 2007), MLCS2k2 (Jha, Riess, and Kirshner 2007) with \( R_V = 3.1 \) (MLCS31) and MLCS2k2 with \( R_V = 1.7 \) (MLCS17). Hicken et al. (2009) compared these light curve fitters and found that SALT produces high-redshift Hubble residuals with sys-
tematic trends versus color and larger scatter than MLCS2k2, and MLCS31 overestimates host-galaxy extinction while MLCS17 does not. For a certain SNe Ia, the analysis outcomes of different light curve fitters are not equal. Here we choose the SNe Ia compilation provided by using MLCS17 light curve fitter with the best cuts $A_V \leq 0.5$ and $\Delta < 0.7$ to constrain the dark energy.

In the flat universe, the Friedmann equation are given by

\[
\frac{H(z)}{H_0} = \sqrt{\Omega_M (1 + z)^3 + \Omega_{DE}(z)}
\]

\[
\Omega_{DE}(z) = \begin{cases} 
(1 - \Omega_M)(1 + z)^3(1+w) & \text{for constant } w, \\
(1 - \Omega_M)(1 + z)^3(1+w_0+w_1) e^{-3w_1 z/(1+z)} & \text{for } w(z) = w_0 + w_1 \frac{z}{1+z}, 
\end{cases} 
\]

with Hubble constant $H_0 = 100\ h\ km\ s^{-1}\ Mpc^{-1}$. The influence of cosmological parameter $w$ is focused on the dark energy density $\Omega_{DE}(z)$ and then the Luminosity distance $d_L$, which is defined as

\[
d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z')}
\]

Analysis of the distance modulus versus redshift relation of SNe Ia can give us the information about the cosmological parameters. Distance estimates of SNe Ia are derived from the luminosity distance, $d_L = (L/4\pi F)^{1/2}$ where $L$ and $F$ are the intrinsic luminosity and observed flux of the SNe Ia, respectively. It is usual to introduce the apparent magnitude $m$ and absolute magnitude $M$. From the definition of the distance moduli $\mu = m - M$, we have

\[
\mu = 5 \log d_L/\text{Mpc} + 25.
\]

Using Equations (1), (2) and (3), we can relate the parameter $w$ with the measured redshift $z$ and distance moduli $\mu(z)$ of SNe Ia data. Since parameter $H_0$ is irrelevant for the SN only data, the likelihood of the SNe Ia analysis can be determined from a $\chi^2$ statistic

\[
\chi^2(\Omega_M, w) = \sum_i \frac{(\mu^T_i(z_i; \Omega_M, w) - \mu^O_i)^2}{\sigma_i^2}
\]

where subscript $i$ denotes the $i$th SNe Ia data and $\sigma_i$ is the observed uncertainty. $\mu^O$ and $\mu^T$ are the observed and theoretical distance moduli, respectively.

Let us first discuss the constraints for the constant $w$ case. Using the Powell minimization method (Press, et al 1992), we minimize the likelihood function of the parameters $(\Omega_M, w)$ and find that the coordinate of the best-fit point is $(\Omega_M, w) = (0.358, -1.09)$. 
Though the results of SNe Ia data and WMAP observation are consistent in the statistical meaning, it is noted that this best-fit value of $\Omega_M$ is large, in comparison with $\Omega_M = 0.258$ of the concordance cosmology provided by WMAP five year data (Komatsu et al. 2009). Figure 1 shows the likelihoods of parameter $\Omega_M$ and $w$, in which the maximum likelihood points are located at $\Omega_M = 0.36$ and $w = -0.88$, respectively. It is interesting to notice that the parameter $w$ is restricted to be from $-2.0$ to $-0.5$ and $\Omega_M$ is less than 0.5.

We now focus on the time-varying model $w(z) = w_0 + w_z z/(1+z)$. Figure 2 shows the contours of $(w_0, w_z)$, the best-fit point is $(w_0, w_z) = (-0.73, 0.84)$ after marginalizing the parameter $\Omega_M$. It is seen that the SNe Ia data alone have a poor constraint power on the parameter $w_z$. In figure 3 we plot the contours of two parameters $(\Omega_M, w_0)$ after marginalizing $w_z$, the best-fit point is found to be $(\Omega_M, w_0) = (0.45, -0.68)$. It is shown that when $\Omega_M$ increases from 0.3 to 0.45, the allowed region of parameter $w_0$ is enlarged quickly. For a smaller $\Omega_M < 0.3$, it leads to a much better constraint for the parameter $w_0$: $w_0 \sim -1.4 \sim -0.6$. Figure 4 gives the contours of $(\Omega_M, w_z)$ after marginalizing $w_0$, the best-fit point is $(\Omega_M, w_z) = (0.44, -4.63)$. It is noticed that when $\Omega_M$ increases from 0.34 to 0.5, the allowed region for the parameter $w_z$ is enlarged rapidly. For a smaller $\Omega_M < 0.34$, it also leads to a much better constraint for the parameter $w_z$: $w_z \sim -3.0 \sim 2.5$. From the figure 3 and figure 4, it indicates that an extra restriction on $\Omega_M$ is necessary to improve the constraint of the SNe Ia data on the parameters $w_0$ and $w_z$. Figure 5 shows the likelihoods of parameters $w_0$ and $w_z$, in which the maximum likelihood points are located at $w_0 = -0.8$ and $w_z = 0.4$, respectively. It can be seen that the parameter $w_0$ is limited in the region $-2.5 < w_0 < 0.5$ and the likelihood of parameter $w_z$ has a high non-Gaussian distribution.

3. JOINT ANALYSIS OF SNe Ia DATA, BAO AND SGL STATISTIC

As we have shown in the previous chapter, over 300 SNe Ia observed so far are not sufficient for determining the cosmological parameters, especially for $w_0$ and $w_z$. Many surveys (e.g. the Dark Energy Survey and Pan-STARRS) are proposed to obtain the SNe Ia sample with enlarged number and improved precision. Here we are going to constrain the dark energy through the combination of the SNe Ia data, the baryon acoustic oscillations as well as the SGL splitting angel statistic.
3.1. Baryon Acoustic Oscillations

In the relativistic plasma of the early universe, ionized hydrogens (protons and electrons) are coupled with energetic photons by Thomson scattering. The plasma density is uniform except for the primordial cosmological perturbations. Driven by high pressure, the plasma fluctuations spread outward at over half the speed of light. After about $10^5$ years, the universe has cooled enough and the protons capture the electrons to form neutral Hydrogen. This decouples the photons from the baryons, which dramatically decreases the sound speed and effectively ends the sound wave propagation. Because the universe has a significant fraction of baryons, these baryon acoustic oscillations leave their imprint on very large scale structures (about 100Mpc) of the Universe.

The measurement of baryon acoustic oscillations was first processed by the Sloan Digital Sky Survey (SDSS; York et al. 2000) and Eisenstein et al. (2005) studied the large-scale correlation function of its sample, which is composed of 46,748 luminous red galaxies over 3816 square degrees and in the redshift range 0.16 to 0.47. The typical redshift of the sample is at $z = 0.35$. The large-scale correlation function is a combination of the correlations measured in the radial (redshift space) and the transverse (angular space) direction (Davis et al. 2007). Thus, the relevant distance measure is modeled by the so-called dilation scale, $D_V(z) = \left[ D_A^2(z)/H(z) \right]^{1/3}$, with comoving angular diameter distance $D_A(z) = \int_0^z dz'/H(z')$. The dimensionless combination $A(z) = D_V(z)\sqrt{\Omega_M H_0^2}/z$ has no dependence on the Hubble constant $h$ and is found to be well constrained by the SDSS data at $z = 0.35$. A standard ruler is provided as (Eisenstein et al. 2005)

$$A = \frac{\sqrt{\Omega_M}}{[H(z_1)/H_0]^{1/3}} \left[ \frac{1}{z_1} \int_0^{z_1} \frac{dz}{H(z)/H_0} \right]^{2/3} = 0.469 \pm 0.017 ,$$  

where $z_1 = 0.35$. This ruler is a $\Omega_M$ prior and can be used to constrain dark energy. Then the statistic is given by

$$\chi^2(\Omega_M, w) = \frac{[A(\Omega_M, w) - 0.469]^2}{0.017^2}$$  

3.2. SGL Splitting Angle Statistic

The CLASS statistical sample has provided a well-defined statistical sample with $N = 8958$ sources. Totally $N_l = 13$ multiple image gravitational lenses have been discovered and all have image separations $\Delta \theta < 3''$ (Browne et al. 2003). The SGL statistics are sensitive to the equation-of-state parameter $w$ of dark energy, which influences the number density of lens galaxies and the distances between the sources and lens. The probability with image
separations larger than $\Delta \theta$ for a source at redshift $z_s$ on account of the galaxies distribution from the source to the observer can be obtained by (Schneider et al. 1992)

$$P(> \Delta \theta) = \int_0^{z_s} \int_0^\infty \frac{dD_L}{dz}(1 + z)^3 n(M, z) \sigma(> \Delta \theta) dMdz,$$

where $M$ is the mass of a dark halo, $D_L$ is the proper distance from the observer to the lens, $n(M, z)$ is the comoving number density of dark halos virialized by redshift $z$ with mass $M$ and $\sigma$ is the cross section for two images with a splitting angle $> \Delta \theta$.

According to the Press-Schechter theory, the comoving number density with mass in the range $(M, M + dM)$ is given by

$$n(M, z) dM = \frac{\rho_0}{M} f(M, z) dM,$$

with the matter density of universe today $\rho_0 = \Omega_M \rho_{\text{crit},0}$ and the critical matter density at present $\rho_{\text{crit},0} = 3H_0^2/(8\pi G)$. $f(M, z)$ is the Press-Schechter function, and we shall utilize the modified form by Sheth and Tormen (1999)

$$f(M, z) = -\frac{0.383}{\sqrt{\pi}} \frac{\delta_c}{\Delta} \frac{d\Delta}{dM} \left[ 1 + \left( \frac{\Delta^2}{0.707\delta_c^2} \right)^{0.3} \right] \times \exp \left[ -\frac{0.707}{2} \left( \frac{\delta_c}{\Delta} \right)^2 \right],$$

$$\Delta^2(M, z) = \int_0^\infty \frac{dk}{k^2} \Delta(k, z) W^2(kr)$$

where $\Delta$ is the variance of the mass fluctuations (Eisenstein and Hu 1999) and parameter $\delta_c(z)$ is the linear overdensity threshold for a spherical collapse (Wang and Steinhardt 1998; Weinberg and Kamionkowski 2002).

For different density profile of dark halo, the lensing cross section $\sigma$ can be calculated out based on the lensing equation. We shall use the combined mechanism of SIS and NFW model to explain the whole experimental curve of strong gravitational lensing statistic. For that a new model parameter $M_c$ was introduced by Li and Ostriker (2002): lenses with mass $M < M_c$ have the SIS profile, while lenses with mass $M > M_c$ have the NFW profile. Then the differential probability is given by

$$\frac{dP}{dM} = \frac{dP_{\text{SIS}}}{dM} \vartheta(M_c - M) + \frac{dP_{\text{NFW}}}{dM} \vartheta(M - M_c)$$

where $\vartheta$ is the step function, $\vartheta(x - y) = 1$, if $x > y$ and 0 otherwise. As the splitting angle $\Delta \theta$ is directly proportional to the mass $M$ of lens halos, the contribution to large $\Delta \theta$ of SIS profile is depressed by $M_c$. The lens data require a mass threshold $M_c \sim 10^{13} h^{-1} M_\odot$, which is consistent with the halo mass whose cooling time equals the age of the universe today.
The likelihood function of the SGL splitting angle statistic is defined as

\[ L(w) = (1 - p(w))^{N_{\text{obs}}} \prod_{i=1}^{N_{\text{obs}}} q_i(w). \]  
(11)

\( p(w) \) and \( q_i(w) \) represent the model-predicted lensing probabilities and the differential lensing probabilities, respectively. They are related to \( P \) in Equation (7) by an integration

\[ p(w) \equiv P_{\text{obs}}(> \Delta \theta) = \int \int B \frac{dP(> \Delta \theta)}{dz} \varphi(z_s)dzdz_s, \]  
(12)

and

\[ q(w) \equiv \frac{dP_{\text{obs}}(> \Delta \theta)}{d\Delta \theta} = \int \int B \frac{d^2P(> \Delta \theta)}{d\Delta \theta dz} \varphi(z_s)dzdz_s. \]  
(13)

\( B \) is the magnification bias and can be found in our previous work (Zhang et al. 2009). \( \varphi(z_s) \) is the redshift distribution of sources. Here we take the Gaussian model by directly fitting the redshift distribution of the subsample of CLASS statistical sample provided by Marlow et al. (2000), which is given by (Zhang et al. 2009)

\[ g(z_s) = \frac{N_s}{\sqrt{2\pi}\lambda} \exp \left[ -\frac{(z_s - a)^2}{2\lambda^2} \right], \]  
(14)

with \( N_s = 1.6125; a = 0.4224; \lambda = 1.3761. \)

### 3.3. Joint Analysis and Numerical Results

In this section, we will investigate the constraint on the cosmological parameters from the joint analysis of (SNe + BAO) and (SNe + BAO + SGL), respectively. For the two (or three) independent observations, the likelihood function of a joint analysis is just given by

\[ L = L_{\text{SNe}} \times L_{\text{BAO}} \times L_{\text{SGL}} \]  
\[ = \exp(-\chi^2_{\text{SNe}}/2) \times \exp(-\chi^2_{\text{BAO}}/2) \times L_{\text{SGL}}. \]  
(15)

The statistic significance \( \chi^2_{\text{BAO}} \) and \( \chi^2_{\text{SNe}} \) can be obtained by using Equations (6) and (11), respectively. \( L_{\text{SGL}} \) is the likelihood function of SGL statistic and can be obtained by using Equation (11). For SGL data, we shall integrate the parameter \( h \) from 0.4 to 0.9.

Let us first discuss the constraints for the constant \( w \) case. In figure 6, we show the constraints on \( \Omega_M \) and the constant \( w \) from the joint analysis of (SNe + BAO + SGL). For a comparison, the results of (SNe + BAO) have been shown as dotted lines. The best fit result
is $(\Omega_M, w) = (0.29, -0.91)$. The 95% C.L. allowed regions of constant $w$ and $\Omega_M$ are found to be: $-1.06 \leq w \leq -0.77$ and $0.25 \leq \Omega_M \leq 0.34$. Comparing with the results of (SNe + BAO) case, it is seen that the results have only slight differences and the fitted $w$ is found to be slightly smaller after adding the SGL data.

Figure 7 plots the likelihoods for the parameters $\Omega_M$ and $w$ from the joint analysis of (SNe + BAO) and (SNe + BAO + SGL), respectively. The maximum likelihood points are located at $\Omega_M = 0.29$ and $w = -0.88$ for (SNe + BAO) and $\Omega_M = 0.296$ and $w = -0.91$ for (SNe + BAO + SGL). It is interesting to find that the parameters $w$ and $\Omega_M$ are restricted to the range: $-1.06 \leq w \leq -0.77$ and $0.25 \leq \Omega_M \leq 0.34$. Comparing with the results of (SNe + BAO) case, it is seen that the results have only slight differences and the fitted $w$ is found to be slightly smaller after adding the SGL data.

Figure 7 plots the likelihoods for the parameters $\Omega_M$ and $w$ from the joint analysis of (SNe + BAO) and (SNe + BAO + SGL), respectively. The maximum likelihood points are located at $\Omega_M = 0.29$ and $w = -0.88$ for (SNe + BAO) and $\Omega_M = 0.296$ and $w = -0.91$ for (SNe + BAO + SGL). It is interesting to find that the parameters $w$ and $\Omega_M$ are restricted to the range: $-1.06 \leq w \leq -0.77$ and $0.25 \leq \Omega_M \leq 0.34$. Comparing with the results of (SNe + BAO) case, it is seen that the results have only slight differences and the fitted $w$ is found to be slightly smaller after adding the SGL data.

After marginalizing the parameter $\Omega_M$, we obtain the constraint on $(w_0, w_z)$ in figure 8 from the joint analysis of (SNe + BAO) and (SNe + BAO + SGL), respectively. The crosshairs mark the best-fit point $(w_0, w_z) = (-0.95, 0.41)$ for the (SNe + BAO) case and $(w_0, w_z) = (-0.92, 0.35)$ for the (SNe + BAO + SGL) case. For the (SNe + BAO) case, the 95% C.L. allowed regions for the parameters $w_0$ and $w_z$ are found to be: $-1.22 \leq w_0 \leq -0.66$ and $-0.92 \leq w_z \leq 1.59$. For the (SNe + BAO + SGL) case, the 95% C.L. allowed regions for the parameters $w_0$ and $w_z$ are found to be: $-1.10 \leq w_0 \leq -0.72$ and $-0.55 \leq w_z \leq 1.32$. After adding the SGL data, the constraint on the parameter $w_0$ is improved moderately, but for the parameter $w_z$, the allowed region decreases by near half. The extra constraint power on the time-varying $w(z)$ obtained through adding SGL data is due to the larger redshift $0 < z < 3.0$ of the galaxies in CLASS observational sample, in comparison with the redshift range of SNe data $0 < z < 1.5$ and the redshift of BAO $z = 0.35$. It can be seen for the both cases that: (a) the most allowed region of $w_z$ is above $w_z = 0$; (b) in comparison with the cosmological constant $(w_0, w_z) = (-1.0, 0.0)$, the joint analysis for both cases favors more positive $(w_0, w_z)$; (c) in comparison with the results of SNe Ia data alone, the constraint on $w_z$ is much improved and $w_0$ also gets better constrained.

Figure 9 plots the likelihoods of parameters $w_0$ and $w_z$ from the joint analysis of (SNe + BAO) and (SNe + BAO + SGL), respectively. For (SNe + BAO) case, the maximum likelihood points are located at $w_0 = -0.94$ and $w_z = 0$. Note that $w_z = 0$ implies a constant equation-of-state of dark energy. For (SNe + BAO + SGL) case, the maximum likelihood points are found to be $w_0 = -0.91$ and $w_z = 0.34$. We see that the parameters $w_0$ and $w_z$ are restricted to be: $-1.20 \leq w_0 \leq -0.67$ and $-1.0 \leq w_z \leq 2.0$. After marginalizing the parameter $\Omega_M$, we obtain the constraint on $(w_0, w_z)$ in figure 8 from the joint analysis of (SNe + BAO) and (SNe + BAO + SGL), respectively. The crosshairs mark the best-fit point $(w_0, w_z) = (-0.95, 0.41)$ for the (SNe + BAO) case and $(w_0, w_z) = (-0.92, 0.35)$ for the (SNe + BAO + SGL) case. For the (SNe + BAO) case, the 95% C.L. allowed regions for the parameters $w_0$ and $w_z$ are found to be: $-1.22 \leq w_0 \leq -0.66$ and $-0.92 \leq w_z \leq 1.59$. For the (SNe + BAO + SGL) case, the 95% C.L. allowed regions for the parameters $w_0$ and $w_z$ are found to be: $-1.10 \leq w_0 \leq -0.72$ and $-0.55 \leq w_z \leq 1.32$. After adding the SGL data, the constraint on the parameter $w_0$ is improved moderately, but for the parameter $w_z$, the allowed region decreases by near half. The extra constraint power on the time-varying $w(z)$ obtained through adding SGL data is due to the larger redshift $0 < z < 3.0$ of the galaxies in CLASS observational sample, in comparison with the redshift range of SNe data $0 < z < 1.5$ and the redshift of BAO $z = 0.35$. It can be seen for the both cases that: (a) the most allowed region of $w_z$ is above $w_z = 0$; (b) in comparison with the cosmological constant $(w_0, w_z) = (-1.0, 0.0)$, the joint analysis for both cases favors more positive $(w_0, w_z)$; (c) in comparison with the results of SNe Ia data alone, the constraint on $w_z$ is much improved and $w_0$ also gets better constrained.

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4. CONCLUSIONS

We have carefully investigated, based on the latest SNe Ia data, BAO and SGL statistic, the constraint on the equation-of-state parameter \( w \) of dark energy, especially for the time varying cases in the flat universe. The influences of the matter density \( \Omega_M \) on the fitting results are carefully demonstrated. The typical redshift measured by the three kinds of observations is \( z \sim 1 \) and far smaller than the redshift of CMB involved, their constraints on the parameter \( w \) are effective and significant only for the redshift region \( z < 1.5 \).

The influence of the equation-of-state parameter \( w \) on the density \( \rho(z) \) of dark energy in the universe and the distance \( d(z) \) makes SNe Ia data a powerful probe of dark energy. In this paper, we have utilized the latest 324 SNe Ia data provided by Hicken et al. (2009) using MLCS17 light curve fitter with the best cuts \( A_V \leq 0.5 \) and \( \Delta < 0.7 \) to carefully investigate the constraint on the equation-of-state parameter \( w \) of dark energy. For the constant \( w \) case, the best-fit results for the two correlated parameters are found to be \((\Omega_M, w) = (0.358, -1.09)\). It is seen that \( \Omega_M \) is somewhat large in comparison with \( \Omega_M = 0.26 \) of the concordance cosmology provided by WMAP five year data (Komatsu et al. 2009); note that using a different parameterization of dark energy, an alternative analysis (Huang et al. 2009) presented a best-fitted result \( \Omega_M = 0.446 \) from SNe Ia data, which is even larger but still consistent with our result at 95% C.L. For the time-varying case, after marginalizing the parameter \( \Omega_M \), we have obtained the fitting results \((w_0, w_z) = (-0.73^{+0.23}_{-0.97}, 0.84^{+1.66}_{-1.03})\), which indicates that (a) the SNe Ia data alone have only a poor constraint power on the parameter \( w_z \), an extra restriction of \( \Omega_M \) is necessary, so that the constraint of SNe Ia on the parameters \( w_0 \) and \( w_z \) can be much improved; (b) the likelihood of parameter \( w_z \) has a high non-Gaussian distribution.

The summary parameter of BAO can provide a standard ruler by which the absolute distance of \( z = 0.35 \) can be determined with 5% accuracy. This ruler can be a \( \Omega_M \) prior and has been used to constrain dark energy. The strong gravitational lensing (SGL) statistic is a useful probe of dark energy. Through comparing the observed number of lenses with the theoretical expected result, it enables us to constrain the parameter \( w \). We have used the latest SNe Ia data together with the BAO (and the CLASS statistical sample) to constraint dark energy. For the constant \( w \) case, the results obtained from (SNe + BAO) and (SNe + BAO + SGL) only have a slight difference. We have shown that: (a) for the (SNe + BAO) case, the best fit results of the parameters \((\Omega_M, w)\) are \((0.287, -0.885)\) and for the (SNe + BAO + SGL) case, the best fit point is \((\Omega_M, w) = (0.298, -0.907)\); (b) the fitting results are found to be \( \Omega_M = 0.29^{+0.03}_{-0.03} \) and \( w = -0.91^{+0.19}_{-0.10} \) for the (SNe + BAO + SGL), which are consistent with the \( \Lambda \)CDM at 95% C.L.; (c) the most allowed region of parameter \( w \) is above the line \( w = -1 \). Comparing with the fitting results from the SNe Ia data alone, we
have found: (a) the allowed region at 95% C.L. for $\Omega_M$ is reduced to one-fifth; (b) the best fit value of $w$ is almost not changed but its variance is reduced very much.

For the time-varying case $w(z)$ after marginalizing ($\Omega_M$), we have obtained the fitting results $(w_0, w_z) = (-0.95^{+0.45}_{-0.18}, 0.41^{+0.79}_{-0.96})$ for the (SNe + BAO) case and $(w_0, w_z) = (-0.92^{+0.14}_{-0.16}, 0.35^{+0.47}_{-0.54})$ for the (SNe + BAO + SGL) case. It has been seen that the adding of the SGL data makes the constraints on parameter $(w_0, w_z)$ to be much improved. For both cases, the most allowed region of $w_z$ is above $w_z = 0$, which indicates that the data from the three observations (SNe + BAO + SGL) disfavor the phantom type models. Comparing with the fitting results from the latest SNe Ia data alone, we have observed that: (a) the best fit values for $w_0$ are decreased by over 0.2 and the variances are approximately reduced to one-fourth; (b) the best fit values of $w_z$ are decreased by 0.49 and the variances are reduced to one-twelfth.

In conclusion, the joint analysis of the latest MLCS17 data set given by Hicken et al. (2009), summary parameters of BAO and SGL data have provided an interesting constraint on the equation-of-state parameter $w$ of dark energy, especially for the time-varying case with parameters $(w_0, w_z)$. A large number of SNe Ia samples with reduced systematical uncertainties in the near future, together with possible new observations on BAO and SGL statistic, would be very useful to understand the properties of dark energy.

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Fig. 1.— The likelihoods of the parameters $\Omega_M$ and $w$. The maximum likelihood points are located at $\Omega_M = 0.36$, and $w = -0.88$, respectively.
Fig. 2.— The \((w_0, w_z)\) contours of SNe Ia data alone. The crosshairs mark the best-fit point \((w_0, w_z) = (-0.73, 0.84)\).
Fig. 3.— The \((\Omega_M, w_0)\) contours of SNe Ia data alone after marginalizing \(w_z\). The crosshairs mark the best-fit point \((\Omega_M, w_0) = (0.45, -0.68)\).
Fig. 4.— The $(\Omega_M, w_z)$ contours of SNe Ia data alone after marginalizing $w_0$. The crosshairs mark the best-fit point $(\Omega_M, w_z) = (0.44, -4.63)$. 
Fig. 5.— The likelihoods of the parameters $w_0$ and $w_z$. The maximum likelihood points are located at $w_0 = -0.8$ and $w_z = 0.4$, respectively. It can be seen that the likelihood of parameter $w_z$ has a high non-Gaussian distribution.
Fig. 6.—68% C.L. and 95% C.L. allowed regions of $(\Omega_M, w)$ from the joint analysis of (SNe + BAO + SGL) (solid lines) in comparison with the joint analysis of (SNe + BAO)(dotted lines). The best-fit result from (SNe+ BAO + SGL) is $(\Omega_M, w) = (0.29, -0.91)$. 
Fig. 7.— The likelihoods of the parameters $\Omega_M$ and $w$ from the joint analysis of (SNe + BAO) and (SNe + BAO + SGL), respectively. The maximum likelihood points are located at $\Omega_M = 0.29$ and $w = -0.88$ for (SNe + BAO) and $\Omega_M = 0.296$ and $w = -0.91$ for (SNe + BAO + SGL).
Fig. 8.—the 68% C.L. and 95% C.L. allowed regions of \((w_0, w_z)\) from the joint analysis of (SNe + BAO) and (SNe + BAO + SGL), respectively. The crosshairs mark the best-fit point \((w_0, w_z) = (-0.95, 0.41)\) for the (SNe + BAO) case and \((w_0, w_z) = (-0.92, 0.35)\) for the (SNe + BAO + SGL) case.
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