Nuclear matter to strange matter transition in holographic QCD

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ABSTRACT: We construct a simple holographic QCD model to study nuclear matter to strange matter transition. The interaction of dense medium and hadrons is taken care of by imposing the force balancing condition for stable D4/D6/D6 configuration. By considering the intermediate and light flavor branes interacting with baryon vertex homogeneously distributed along $R^3$ space and requesting the energy minimization, we find that there is a well defined transition density as a function of current quark mass. We also find that as density goes up very high, intermediate (or heavy) and light quarks populate equally as expected from the Pauli principle. In this sense, the effect of the Pauli principle is realized as dynamics of D-branes.

KEYWORDS: Gauge/gravity duality, Dense matter
1. Introduction

One of the challenging current problems in hadron physics is to elucidate the behavior of dense matter under extremely high-density environments, for a review, e.g. see [1]. It is theoretically expected that, at very high baryon densities (even at low temperatures), chiral symmetry is likely to be restored, and that baryon matter can be converted into quark matter. Various studies also suggest the possible formation of a kaon condensate at high densities. The existence of quark matter and/or a kaon condensate can have important consequences for the structure of compact stars and for the cooling behavior of a remnant star after supernova explosion and the subsequent formation of a neutron star. Thorough understanding on dense nuclear matter is also important in the physics of relativistic heavy-ion collisions.

In this study, we focus on a specific aspect of dense matter: transition from nuclear matter to strange matter. This transition is essential to understand relevance of kaon condensation in neutron star. This is because the presence of strangeness matter tends to hinder the formation of kaon condensation basically due to the Pauli exclusion principle. Here kaon condensation means $K^{-}$ condensation. Since $K^{-}$ is composed of $\bar{u}$ and $s$, strange matter will expel the strange quark and so the kaon condensation. Moreover, according to previous studies, the critical baryon number density for the nuclear matter to strange matter and that for the onset of kaon condensation are not very different from each other, $\sim (2 - 4)\rho_0$, where $\rho_0$ is the normal nuclear matter density $\simeq 0.17 \, \text{fm}^{-3} \sim m^3_{\pi}/2$. In conventional approaches, however, when one estimates the transition to strange matter, a bit large uncertainty comes in due to lack of robust information on hyperon coupling constants, see [2] for a review. In this study, we take a first step towards this direction in holographic QCD. Recent developments based on AdS/CFT [3, 4, 5] renders a new tool to study dense matter in the framework of a holographic model of QCD [6, 7], see [9] for a review. There have been many studies on dense nuclear matter [10] in the holographic QCD.
To study the transition we introduce two flavor D6 branes which correspond to light (u or d) and intermediate (strange) quarks respectively and spherical D4 brane with $N_C$ fundamental strings. The fundamental strings can be attached on a light quark D6 brane and/or intermediate mass quark (strange) D6 brane. By solving DBI action and applying the force balance condition, we find stable configuration of D4/D6/D6 system. After finding minimum energy configuration, we calculate the ratio of up and strange quarks in the system as a function of baryon density.

2. Baryon vertex

In this section we discuss baryon vertex (spherical D4 brane with $N_C$ fundamental strings) in confining background following [11]. The non-supersymmetric geometry for confining background of D4 in Euclidean signature is given by

$$ds^2 = \left( \frac{U}{R} \right)^{3/2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + f(U) dx_4^2 \right) + \left( \frac{R}{U} \right)^{3/2} \left( \frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right)$$

$$e^\phi = g_s \left( \frac{U}{R} \right)^{3/4}, \quad F_4 = \frac{2\pi N_c}{\Omega_4} \epsilon_4, \quad f(U) = 1 - \left( \frac{U_{KK}}{U} \right)^3, \quad R^3 = \pi g_s N_c l_s^3. \quad (2.1)$$

This background is related to the black hole solution of D4 brane by the double Wick rotation. The Kaluza-Klein mass is defined as inverse radius of the $x_4$ direction:

$$M_{KK} = \frac{3}{2} \frac{U_{KK}^{1/2}}{R^{3/2}}. \quad \text{While } U_{KK}, g_s, \text{ and } R \text{ are bulk parameters, } M_{MM} \text{ and } g_{YM}^2 \text{ are the parameters of the gauge theory. These are related by}$$

$$g_s = \frac{\lambda}{2\pi l_s N_c M_{KK}}, \quad U_{KK} = \frac{2}{9} \lambda M_{KK} l_s^2, \quad R^3 = \frac{\lambda l_s^2}{2 M_{KK}}, \quad \lambda = g_{YM}^2 N_c. \quad (2.2)$$

Introducing a dimensionless coordinate $\xi$: $\frac{ds^2}{\xi^2} = \frac{dt^2 + dx^2}{U^2 f(U)}$, we obtain the background geometry

$$ds^2 = \left( \frac{U}{R} \right)^{3/2} \left( dt^2 + dx^2 + f(U) dx_4^2 \right) + \left( \frac{R}{U} \right)^{3/2} \frac{U^2}{\xi^2} \left( d\xi^2 + \xi^2 d\Omega_4^2 \right). \quad (2.3)$$

Here $U$ and $\xi$ are related by $\left( \frac{U}{U_{KK}} \right)^{3/2} = \frac{1}{2} \left( \xi^{3/2} + \frac{1}{\xi^{3/2}} \right)$.

A baryon in three-dimensional theory corresponds to the D4 brane wrapping $S^4$ on which $N_c$ fundamental strings terminate. In this configuration, the background four-form field strength couples to the world volume gauge field $A_{(1)}$ via Chern-Simons term.

The background metric (2.3) can be written as

$$ds^2 = \left( \frac{U}{R} \right)^{3/2} \left( dt^2 + dx_3^2 + dx^2 \right) + R^{3/2} \sqrt{U} \left( \frac{d\xi^2}{\xi^2} + d\theta^2 + \sin^2 \theta d\Omega_3^2 \right). \quad (2.4)$$

We take $(t, \theta, \phi)$ as a world volume coordinate of D4 brane and turn on the $U(1)$ gauge field on it, $F_{t\theta} \neq 0$. For simplicity, we assume that the position of D4 brane and the gauge
field depends only on \( \theta \) i.e. \( \xi = \xi(\theta) \), \( A_t = A_t(\theta) \), where \( \theta \) is the polar angle in spherical coordinates. The induced metric on the compact D4 brane is

\[
ds_{D4}^2 = \left( \frac{U}{R} \right)^{3/2} dt^2 + R^{3/2} \sqrt{U} \left[ \left( 1 + \frac{\xi'^2}{\xi^2} \right) d\theta^2 + \sin^2 \theta \, d\Omega_3^2 \right],
\]

where \( \xi' = \partial \xi / \partial \theta \). The DBI action for single D4 brane with \( N_c \) fundamental strings is given by [12]

\[
S_{D4} = -\mu_4 \int e^{-\phi} \sqrt{\det(g + 2\pi \alpha' F)} + \mu_4 \int A_{(1)} \wedge G_{(4)} = \tau_4 \int dt \sin^3 \theta \left[ -\sqrt{\omega_+^{4/3}(\xi^2 + \xi'^2) - \tilde{F}^2} + 3\tilde{A}_t \right] = \int dt \mathcal{L}_{D4},
\]

where

\[
\tau_4 = \mu_4 \Omega_3 g_s^{-1} R^3 \frac{U_{KK}}{2^{2/3}} = \frac{N_c U_{KK}}{2^{8/3}(2\pi l_s^2)}
\]

\[
\tilde{F} = \frac{2\pi \alpha' F_{kk}^{2/3}}{U_{kk}}, \quad \tilde{A}_t = \frac{2^{2/3}}{U_{KK}} \cdot 2\pi \alpha' A_t
\]

with \( \omega_\pm = 1 \pm \xi^{-3} \). The dimensionless displacement is defined as follows;

\[
\frac{\partial \mathcal{L}_{D4}}{\partial \tilde{F}} = \frac{\sin^3 \theta \tilde{F}}{\sqrt{\omega_\pm^{2/3}(\xi^2 + \xi'^2) - \tilde{F}^2}} \equiv -D(\theta).
\]

Then the equation of motion for the gauge field is

\[
\partial_\theta D(\theta) = -3 \sin^3 \theta.
\]

By integrating above equation, we get

\[
D(\theta) = 2(2\nu - 1) + 3(\cos \theta - \frac{1}{3} \cos^3 \theta),
\]

where the integration constant \( \nu \) determines the number of fundamental sting; \( \nu N_c \) strings are attached at south pole and \( (1 - \nu) N_c \) strings at north pole.

After Legendre transformation, we obtain ‘Hamiltonian’ as

\[
\mathcal{H}_{D4} = \tilde{F} \frac{\partial \mathcal{L}_{D4}}{\partial \tilde{F}} - \mathcal{L}_{D4} = \tau_4 \int d\theta \sqrt{\omega_+^{4/3}(\xi^2 + \xi'^2)} \sqrt{D(\theta)^2 + \sin^6 \theta},
\]

We solve the equation of motion for \( \mathcal{H}_{D4} \) numerically. We set \( \nu = 0 \) since we assume that all fundamental strings are attached at north pole \( \theta = \pi \). Then we impose smooth boundary
condition $\xi'(0) = 0$ and $\xi(0) = \xi_0$ at $\theta = 0$. Numerical solutions are parameterized by initial value of $\xi_0$. The solutions corresponding to different $\xi_0$’s are drawn in Fig. 1.

If we denote the position of the cusp of D4 brane by $U_c$, the force at the cusp due to the D4 brane tension can be obtained by varying the Hamiltonian of D4 brane with respect to $U_c$ while keeping other variables fixed;

$$F_{D4} = \left. \frac{\partial H}{\partial U_c} \right|_{\text{fix other values}} = N_c T_F \left( \frac{1 + \xi_c^{-3}}{1 - \xi_c^{-3}} \right) \frac{\xi_c'}{\sqrt{\xi_c^2 + \xi_c'^2}}, \quad \text{(2.12)}$$

where $T_F = \frac{2^{5/3} T_4}{N_c U_{KK}}$ is tension of fundamental string. The tension at the cusp of D4 brane is always smaller than the tension of the $N_C$ fundamental strings. Therefore, if there are no other object, the cusp should be pulled up to infinity and the final configuration of D4 brane would be ‘tube-like’ shape as in [12].

3. Holographic transition to strange matter

We begin with an simplified description of the nuclear matter to strange matter transition. Figure shows a simple view of the transition. Here we set aside issues like charge neutrality and $\beta$-equilibrium of the dense matter and consider two light quarks ($u, d$ quarks) and one intermediate mass quark ($s$ quark). The vertical axis of Fig. is roughly the number of quarks in the ground state since $n_q \sim k_F^3$, where $n_q$ is the quark number density. In low-density regime, we would have only $u$ and $d$ in our system since the mass of the strange quark, $m_s$, is roughly a few ten times bigger than that of light quarks, $m_s/m_u \sim 50$, $m_s/m_d \sim 20$, and so cost too much energy to be piled up in the ground state. As we increase the number density, the chemical potential of light quarks ($\mu_{u,d}$) become comparable with the mass of the strange quark, and then system could lower its ground state energy by piling up some strange quarks if $\mu_{u,d} > m_s$. This argument goes also with baryons. Instead of transition from $u, d$ quark matter to $u, d$, and $s$ matter, it could be a transition from
nuclear matter composed of nucleon to strange matter of nucleon and hyperons like Λ. As it stands, this is too simple. To be realistic we have to include interaction energy, charge neutrality of the matter, β-equilibrium, etc.

![Diagram of quark states](a) and (b)

**Figure 2:** Schematic picture for the nuclear matter (a) to strange matter (b) transition.

Now, we delve into the transition in terms of a holographic QCD. For simplicity, we will again ignore the charge neutrality and β-equilibrium of the matter at hand, relegating them to a future study. Now we consider the system with two flavors, one light and one intermediate mass quarks. To introduce two flavors, we put two probe D6 branes in the background. The the bulk metric (2.3) can be written as

\[
ds^2 = \left(\frac{U}{R}\right)^{3/2} (dt^2 + d\vec{x}^2) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{U}{\xi}\right)^2 \left( d\rho^2 + \rho^2 d\Omega_2^2 + dy^2 + y^2 d\phi^2 \right),
\]

where D6 brane world volume coordinates are \((t, \vec{x}, \rho, \theta)\). The embedding ansatz is that only \(y\) depends on \(\rho\) (we set \(\phi = 0\)). The induced metric on a single D6 brane is

\[
ds_{D6}^2 = \left(\frac{U}{R}\right)^{3/2} (dt^2 + d\vec{x}^2) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{U}{\xi}\right)^2 \left[ (1 + \dot{y}^2) d\rho^2 + \rho^2 d\Omega_2^2 \right],
\]

where \(\dot{y} = \partial y / \partial \rho\).

We also introduce \(U(1)\) gauge field on D6 brane, which is coupled to the string end point. The DBI action for the single D6 brane is

\[
S_{D6} = -\mu_6 \int \sqrt{\det(g + 2\pi \alpha' F)}
\]

where

\[
\tau_6 = \frac{1}{4} \mu_6 V_3 \Omega_2 g_s^{-1} U_{KK}^{-3}, \quad \tilde{F} = \frac{2^{2/3} \pi \alpha' F_{t\rho}}{U_{KK}}.
\]

We define dimensionless quantity \(\tilde{Q}\) from the equation of motion for \(\tilde{F}\):

\[
\frac{\partial S_{D6}}{\partial \tilde{F}} = \frac{\rho^2 \omega_+^{4/3} \tilde{F}}{\sqrt{\omega_+^{4/3} (1 + \dot{y}^2) - \tilde{F}^2}} \equiv \tilde{Q}.
\]

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The Hamiltonian is connected to the number of point sources (number of fundamental strings) $Q$ by

$$\tilde{Q} = \frac{U_{KK} Q}{2 \cdot 2^{2/3} \pi \alpha' \tau_6}. \quad (3.6)$$

The Hamiltonian can be obtained by the Legendre transformation;

$$H_{D6} = F \frac{\partial S_{D6}}{\partial F} - S_{D6}$$

$$= \tau_6 \int d\rho \sqrt{\omega_+^{4/3} \left( \tilde{Q}^2 + \rho^4 \Omega_+^{8/3} \right)} \sqrt{1 + \dot{y}^2}$$

$$= \tau_6 \int d\rho V(\rho) \sqrt{1 + \dot{y}^2} \quad (3.7)$$

To solve the equation of motion, we impose appropriate initial condition to D6 brane. We are considering two D6 branes that are connected to a D4 brane with fundamental strings. As discussed in [11], the tension of fundamental strings is always larger than that of D-branes. Therefore, two D6 branes are pulled down and spherical D4 brane pulled up until length of fundamental strings becomes zero. Finally, the position of the cusp of D6 branes should be located at the same position of the cusp of D4 brane $\xi_c$. We consider $Q_1$ fundamental strings attached on one D6 brane and $Q_2$ strings attached on another D6 brane. We also denote the slope at cusp of each brane as $\dot{y}_c^{(1)}$ and $\dot{y}_c^{(2)}$. The force at the cusp of D6 branes can be obtained as

$$F_{D6} = \frac{\partial H(Q_1)_{D6}}{\partial U_c} \bigg|_0 + \frac{\partial H(Q_2)_{D6}}{\partial U_c} \bigg|_0$$

$$= \frac{Q_1}{2\pi\alpha'} \left( \frac{1 + \xi_c^{-3}}{1 - \xi_c^{-3}} \right) \frac{\dot{y}_c^{(1)}}{\sqrt{1 + \dot{y}_c^{(1)}^2}} + \frac{Q_2}{2\pi\alpha'} \left( \frac{1 + \xi_c^{-3}}{1 - \xi_c^{-3}} \right) \frac{\dot{y}_c^{(2)}}{\sqrt{1 + \dot{y}_c^{(2)}^2}}$$

$$\equiv F_{D6}^{(1)}(Q_1) + F_{D6}^{(2)}(Q_2). \quad (3.8)$$

To make the whole system stable, the force at the cusp of D4 brane should be balanced to force of D6 branes;

$$\frac{Q}{N_c} F_{D4} = F_{D6}^{(1)}(Q_1) + F_{D6}^{(2)}(Q_2), \quad (3.9)$$

where $Q = Q_1 + Q_2$.

Rewriting $Q_1$ and $\dot{y}_c^{(i)}$ by using new parameters $\alpha$ and $\beta$,

$$Q_1 = (1 - \alpha)Q , \quad Q_2 = \alpha Q_2 ,$$

$$\dot{y}_c^{(1)} = \dot{y}_c^{(1)} , \quad \dot{y}_c^{(2)} = \beta \dot{y}_c^{(1)}, \quad (3.10)$$

the force balance condition (3.9) becomes

$$\frac{\xi_c' \sqrt{\xi_c' + \xi_c^2}}{\sqrt{1 + \dot{y}_c^{(1)}^2}} \equiv \frac{\tau_6}{\sqrt{1 + \dot{y}_c^{(1)}^2}} + \frac{\alpha \beta \dot{y}_c^{(1)}}{\sqrt{1 + \dot{y}_c^{(1)}^2}}. \quad (3.11)$$

We show solutions that satisfy the force balance condition above in Figure 3(a).
Here we fix asymptotic values of D6 branes which are correspond to quark mass to be \( m_1 = 0.1 \) and \( m_2 = 5 \). We call these branes as up and strange quarks brane for convenience. We note here that \( m_i \) is dimensionless and it is related to the real quark mass by the following relation
\[
M_q = \frac{m_q \lambda M_{KK}}{9\pi}.
\] (3.12)
We will use this relation to discuss the parameter fixing.

The behavior of solutions change depending on the density and \( \xi_0 \). If we fix \( \xi_0 \), then from the equation of motion of D4 brane, the cusp point \( \xi_c \) and slope at cusp \( \xi'_c \) are automatically determined. However, from eq. (4.1), for given \( \xi_c \) and \( \xi'_c \), slopes of two D6 branes which give \( m_1 = 0.1 \), \( m_2 = 5 \) cannot be uniquely determined. In fact, there are infinite set of \( \alpha \) and \( \beta \). So we have to choose one embedding by minimum energy condition. The total energy of this system can be written as follows;

\[
E_{\text{tot}} = \frac{Q}{N_C} \mathcal{H}_{D4} + \mathcal{H}_{D6}(Q_1) + \mathcal{H}_{D6}(Q_2) = \tau_6 \left[ \frac{\tilde{Q}}{4} E_4 + E_6(\tilde{Q}_1) + E_6(\tilde{Q}_2) \right],
\] (3.13)
where \( E_i \) is numerical integration of each ‘Hamiltonian’ in (2.11) and (3.7) without overall constant \( \tau_4 \) and \( \tau_6 \). The \( \alpha \) dependence of total energy is drawn in Fig. 3(b). In this figure, we can see that the total energy of system is minimized at at \( \alpha = 0.361 \). In other words, for \( \tilde{Q} = 8 \) case, 36.1% of total quarks of ground state (or Fermi sea) are occupied by strange (intermediate mass) quarks. For each \( \tilde{Q} \), we find corresponding \( \alpha \) by imposing the energy minimum condition.

For small value of \( \tilde{Q} \), \( \alpha \) dependence of total energy have different behavior Fig. 3(b) as shown in Figure. 4(a). In this figure, total energy monotonically decreases as \( \alpha \) decreases. Finally, the minimum energy configuration is at \( \alpha = 0 \), it means that at small density, strange quark cannot come into the system. The embedding in this case is drawn in

Figure 3: (a) Embedding of D6 branes for \( m_1 = 0.1 \) and \( m_2 = 5 \). Red circle denotes the singularity at \( U_{KK} = 1 \). (b) \( \alpha \) vs. total energy for \( \tilde{Q} = 8 \).
Figure 4: (a) $\alpha$ vs. total energy for $\tilde{Q} = 3$. (a) Embedding of D6 branes for $\alpha = 0$ with $m_1 = 0.1$ and $m_2 = 5$. Red circle denotes the singularity at $U_{KK} = 1$.

As we increase the density, at a certain density, $\alpha = 0$ is not a minimum energy embedding anymore. From $\tilde{Q} \sim 4.2$, non-zero $\alpha$ embedding has minimum energy, and the value increases as density increases. We also check the large density behavior of embedding: when $\tilde{Q}$ is large, the ratio of strange quark to up quark seems to go to 0.5, for example when $\tilde{Q} = 500$ the value of $\alpha$ is around 0.49. The final result is drawn in Figure. 5(a), which shows sharp transition from nuclear matter ($\alpha = 0$) to strange matter ($\alpha \neq 0$). It is interesting to note that similar tendency has observed in QCD-rooted model studies, for instance see [13].

Figure 5: (a) Density dependence of $\alpha$, the fraction of the strange quarks. (b) Density dependence of $r_0$, a measure of the baryon mass, for $m_1 = 0.1$ and $m_2 = 5$.

Next, we calculate the value of $r_0$ for each embedding which is proportional to the energy of spherical D4 brane. As we discussed in [11], this value can be interpreted as mass of baryon. For small value of $\tilde{Q}$, the behavior of $r_0$ is the same as [11] because in this region only one probe brane touch the baryon vertex - as $\tilde{Q}$ increase, $\rho_0$ decreases first and then increases. After $\tilde{Q} \sim 4.2$, two probe branes are attached to baryon vertex and $r_0$ is increase. The behavior of $r_0$ as a function of $\tilde{Q}$ is drawn in Figure. 5(b).
Intuitively as $m_2/m_1$ increases, the transition density $\tilde{Q}_c$ from zero $\alpha$ to finite $\alpha$, or from nuclear matter to strange matter, should increase. We check if our D4/D6/D6 system follows this expectation. For this we used three different values for $m_2$, $m_2 = 2, 3, 5$, with $m_1 = 0.1$. The results are summarized in Fig. 6 to show that our D4/D6/D6 system respects the intuition.

![Figure 6](image_url)  

**Figure 6:** (a) Density dependence of $\alpha$ with different $m_2/m_1$. All lines saturate $\alpha = 0.5$ for large $Q$; (b) $\rho_c$ vs. $m_2/m_1$.

Though our D4/D6/D6 system is surely far from a realistic dense QCD matter at this stage, we convert the transition $\tilde{Q}_c$ obtained in this study into a baryon number density in QCD. From (2.2) and (3.4), we can calculate the baryon number density in terms of $\tilde{Q}$ as follows:

$$
\rho \equiv \frac{N_b}{V_3} = \frac{Q}{N_c V_3} = \frac{2 \cdot 2^{2/3}}{81(2\pi)^3} \lambda M_{KK}^3 \tilde{Q}.
$$

(3.14)

For $m_1 = 0.1$ and $m_2 = 5$, we obtained $\tilde{Q}_c \sim 4.2$. Empirically the transition baryon number density is $\sim (2 - 4) \rho_0$, where $\rho_0$ is the normal nuclear density, from various model studies based on QCD. To compare our result with the empirical transition density, we choose $\lambda = 1.2$ and $M_{KK} = 1.5$ GeV. With this choice we obtain $\rho_c \sim 1.96\rho_0$ for $\tilde{Q}_c \sim 4.2$. From (3.12), corresponding quark masses are $M_1 \sim 6.4$ MeV and $M_2 \sim 318$ MeV, which are close to the mass of up and strange quarks in QCD. At the end of the day, however, on completion of our D4/D6/D6 model for realistic dense matter, we may have to fix the values of $\lambda$ and $M_{KK}$ ab initio by considering fluctuations of D6 branes and by doing meson spectroscopy.

So far we consider $m_1 = 0.1$ and $m_2 = 2, 3, 5$. For fixed ratio between $m_1$ and $m_2$, however, we can choose several different embeddings.

- **Case 1:** $m_1 \ll U_{KK} = 1 < m_2$. This is the case we discussed above.

- **Case 2:** both of $m_q$ are smaller than $U_{KK}$, such as $m_1 = 0.01$ and $m_2 = 0.5$. In this case, brane embedding is drawn in Figure 7(a). The density dependence of $\alpha$ is
drawn in Figure. (b). In this case, the value of $\alpha$ seems to saturate to 0.5 for any non-zero density. That is, it does not go to zero even in extremely small density. We can understand this behavior in geometrically. If both of $m_q$ is smaller than $U_{KK}$, the difference of geometry detected by each brane is very small. So, the each brane shares nearly equal number of quarks.

- Case 3: both of $m_q$ are larger than $U_{KK}$. Here we expect similar behavior as in case 2. Two quarks will populate evenly.

Finally we remark that in our model the value of $\alpha$ depends on not only the ratio of $m_1$ and $m_2$, but also each value of $m_1$ and $m_2$.

\begin{figure}[h]
\centering
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{a.png}
\caption{}
\end{subfigure}
\hfill
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{b.png}
\caption{}
\end{subfigure}
\caption{(a) Brane embedding for $m_1 = 0.01$ and $m_2 = 0.5$. (b) Density dependence of $\alpha$.}
\end{figure}

4. Summary and Discussion

In this paper, we constructed a simple holographic QCD model to study nuclear matter to strange matter transition. The interaction of dense medium and hadrons was taken care of by imposing the force balancing condition for stable D4/D6/D6 configuration. We considered the oversimplified model where only one intermediate and one light flavor branes are interacting with baryon vertex which is homogeneously distributed along $R^3$ space. We imposed the energy minimization condition and found that there is a well defined transition density as a function of current quark mass and that the transition density increases as the ratio $m_2/m_1$ increases. We also showed that at very high densities, intermediate (or heavy) and light quarks populate equally as expected from the Pauli principle. (add a few lines and lower the voice) So, we may conclude that in our study the effect of Pauli principle is realized as dynamics of D-branes.

We could lessen the oversimplification a little bit by considering three flavors with $m_u = m_d < m_s$, which could be realized by considering the light quark favor brane to have weight 2 relative to the intermediate one in the force balancing condition. That is, if we change it to

$$\frac{Q}{N_c} F_{D4} = 2 F_{D6}^{(1)}(Q_1) + F_{D6}^{(2)}(Q_2).$$  \hspace{1cm} (4.1)
In this case, it is expected that the asymptotic value of $\alpha$ would be $1/3$ instead of $1/2$. To confirm this we take $\tilde{Q} = 500$ and obtain $\alpha \sim 1/3$.

It will be interesting to consider fluctuations in our D4/D6/D6 system to study meson masses in iso-spin asymmetric matter, which will be reported elsewhere.

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