Study on Biphasic Material Model and Mechanical Analysis of Knee Joint Cartilage

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Abstract A material model of articular cartilage is formulated, and fundamental problems are analyzed. The soft tissue is assumed to comprise two phases: solid and fluid. The biphasic theory proposed by Spilker and Suh (1990) to deal with such materials is reviewed, and some new additional analyses are carried out on the basis of this theory. Assuming the elasticity for the solid phase and introducing the pressure, which is defined by the product of the volume change and penalty coefficient, it is shown that the viscoelastic property of the soft tissue can be reproduced. A preferable solution is obtained for the solid phase by using the reduction integral, even if a high-order interpolation function is used. However, the high-order element cannot satisfactorily capture the velocity distribution of fluids. The pressure distribution is studied by assuming the change in the surface characteristics of the cartilage tissue with the progress of osteoarthritis. The pressure is strongly related to the lubrication conditions, i.e., perfect lubrication, perfect adhesion, and partial adhesion.

1. Introduction

The surface of a joint is covered by articular cartilage, which possesses a high strength but low friction/abrasion. The cartilage tissue is composed of extracellular matrix (ECM), which surrounds a small amount of cartilage cells. The fraction of moisture of the ECM is 70-80%, and collagen fibers and proteoglycans are the other main components. The mechanical property of the articular cartilage is mainly determined by the microscopic physical properties of the cartilage cells, the abovementioned components, the water content, etc.[1]

Cartilage is the tissue without blood vessels and nerves. Therefore, it is known that the active transport mechanism for nutrition supply does not exist in it, and once damaged, self healing is difficult [2]. Osteoarthritis (OA) is a major knee disease, and it appears to be caused by the degeneration of the articular cartilage with aging. The degeneration change in the articular cartilage surface seems to be related to the origin of OA, although the actual mechanism has not been clarified thus far[3]. The mechanical properties of cartilage tissues change with the progress of OA[4]. In particular, a decrease in the proteoglycans effects a decrease in the elastic modulus and an increase in the moisture permeability. It is reported that once the cartilage surface is damaged, the stress of the
cartilage inside shifts to the severe direction, thereby supporting the concept that there is a relation between the generation of OA and the damage of the cartilage surface.

In the mechanical analysis of a soft tissue with such an internal structure, it is important to model the interaction between each component[5]. In this study, a material model of articular cartilage is formulated, and fundamental problems are analyzed. The soft tissue is assumed to comprise two phases: solid and fluid. In this study, the biphasic theory proposed by Spilker and Suh (1990) to deal with such materials is reviewed. The biphasic theory is a powerful tool to predict the phenomenological response of tissues comprising a fluid and solid phase; it also helps in understanding the internal state and mechanism of changes in the internal structure. In this study, we focus on the fact that the cartilage tissue is a mixture of fluid and solid phases and adopt the framework of the biphasic porous viscoelasticity theory for analyzing the mechanical behavior under compression to obtain a fundamental understanding of the mechanism of generation and progress of OA.

2. Biphasic Theory

The biphasic theory is one of the simplest mixture theory [6]. This theory can capture complex phenomena and the key mechanisms of such phenomena that cannot be treated by using conventional monophasic theories. The biphasic theory is formulated by assuming the soft tissue to be a mixture of solid and fluid phases. The solid phase is a compressible porous matrix and the fluid phase is incompressible water[7]. In the extended framework based on the biphasic viscoelasticity model (BPVE model), the solid phase is modeled as viscoelastic media. Subsequently, the equation of continuity, equation of momentum balance, and constitutive equation are written, respectively, as follows[8].

\[
\nabla \cdot (\phi^s \mathbf{v}^s + \phi^f \mathbf{v}^f) = 0
\]

\[
\nabla \cdot \mathbf{\sigma}^s + \pi^s = 0 \quad \text{and} \quad \nabla \cdot \mathbf{\sigma}^f + \pi^f = 0
\]

\[
\mathbf{\sigma}^s = -\phi^s pI + \mathbf{\tilde{\sigma}}^s, \quad \mathbf{\sigma}^f = -\phi^f pI
\]

\[
\pi^s = -\pi^f = \left(\frac{\phi^f}{\kappa}\right)^2 (\mathbf{v}^f - \mathbf{v}^s)
\]

where \(\pi\) and \(\phi\) are the momentum and volume fraction, respectively. The superscripts \(s\) and \(f\) represent the variables of the solid and fluid phases, respectively.

\[
\mathbf{\sigma} = \mathbf{\sigma}^s - pI \mathbf{e}^s
\]

Here, \(\mathbf{e}^s\) denotes the effective stress tensor; \(p\), the hydrostatic pressure; and \(I\), the unit tensor.

By assuming a linear isotropic solid phase, the effective stress tensor is expressed as follows[9].

\[
\mathbf{\tilde{\sigma}}^s = \lambda \mathbf{e}^s I + 2\mu \mathbf{e}^s.
\]

Here, \(\mathbf{e}^s\) is the dilatation strain; \(\mathbf{e}^s\), the strain tensor; and \(\lambda\) and \(\mu\), Lamé’s constants.

3. Finite Element Formulation of Biphasic Theory

The weak forms of the momentum equations for solid and fluid phases are obtained using the method of weighted residuals as follows[9].
\[
\int_\Omega w^s \cdot (\nabla \cdot \sigma^s + \Pi^s) d\Omega + \int_{\Gamma} h^s \cdot (\vec{t}^s - \vec{t}) d\Gamma = 0
\]
\[
\int_\Omega w^f \cdot (\nabla \cdot \sigma^f + \Pi^f) d\Omega + \int_{\Gamma} h^f \cdot (\vec{p} - p) n d\Gamma = 0
\]

Here, \(w^s, h^s, w^f, \) and \(h^f\) are weighted functions. \(n\) is the unit normal vector to the boundary surface. \((\xi)\) means the specified value of a variable \(\xi\) on the boundary. \(p\) is defined by introducing a penalty parameter \(\beta\) as follows.

\[
p = -\beta \nabla \cdot (\phi^f v^f + \phi^s v^s)
\]

The problem is limited to be an axisymmetric problem; by using the 8-node quadrilateral isoparametric element and the 4-node linear element, the interpolation of the velocity and displacement of the solid phase and the velocity of fluid phase within the element is carried out by using the nodal values. Both the interpolation functions of the solid and fluid phases are identical.

The stress, strain, and volumetric strain within the element are related to the values of the node depending on the derivative of the interpolation function. By applying the Galerkin method, the global form of the differential equation is obtained as follows.

\[
[C] \{v\}_{m+1} + [K] \{d\}_{m+1} = \{F\}_{m+1}
\]

where \(\{d\}_n = \{\{d^s_n\}, \{d^f_n\}\}^T\) and \(\{v\}_n = \{\{v^s_n\}, \{v^f_n\}\}^T\). The subscript \(n\) refers to the variables of the \(n\)-th element. \([C]\) and \([K]\) are the attenuation matrix and stiffness matrix, respectively. \(F(t)\) refers to the external load. The discretization of time by a finite difference scheme is carried out by an increase in the time \(\Delta t\).

The subscript \(m\) refers to the value at time \(t_m\). The coefficient matrices \([C]\) and \([K]\) do not depend on time. By applying a trapezoidal rule, the equation for obtaining the values at time \(t_{m+1}\) is given as follows:

\[
[C] + \omega \Delta t [K] \{v\}_{m+1} = \{F\}_{m+1} - [K] \{\{d\}_m + \Delta t(1 - \omega) \{v\}_m\}
\]

where \(\omega\) is set to \(1/2\).

4. Analysis Examples

The small deformation problem for tissues with axisymmetric geometry is analyzed using the finite element method. Deformation analysis is carried out by using the confined compression test and unconfined compression test/relaxation test, which were employed by Spilker and Suh (1990).

4.1 Analysis of Confined Compression Test

Finite element analyses of the confined compression test of the soft tissue are carried out, as shown in Fig.1. It is shown that the biphasic theory can express the viscoelastic response. The accuracy of the obtained displacement and velocity distribution is also examined for the different element selections.
As shown in Fig.1(a), the diameter of the tissue specimen is \( d = 6.35 \) mm, and the height is \( h = 1.78 \) mm. The geometry is that of a circular cylinder, and the side is assumed to be contact with the rigid wall with perfect lubrication. The inflow and outflow of the fluid occur through the upper porous wall, while there is no inflow or outflow through the rigid walls on the sides and bottom. As shown in Fig.1(b), the compression quantity gives nominally constant rate to the rigid plate of the upper surface. The nominal strain rate is the value of the reference strain \( \varepsilon = 0.05 \) divided by the reference time \( t_0 = 500 \) s. The material constants and parameters are set as follows: \( \phi_s = 0.2 \), \( \kappa = 0.76 \times 10^{-14} \text{m}^4/\text{N}s \), \( \lambda_s = 0.0143 \text{MPa} \), \( \mu_s = 0.343 \text{MPa} \), \( \nu_s = 0.02 \). For the context of the patch test, the analysis is carried out for three cases: the 6-element mesh (the 4-node and 8-node elements) indicated by continuous solid lines and 24-element mesh (the 4-node element) represented by a dashed lines (Fig.1(c)).

### 4.2 Unconfined Compression / Relaxation Tests
As shown in Fig.2, a finite element analysis of the unconfined compression test and relaxation test of the soft tissue is carried out, and the relaxation response under a compressive force load is analyzed. The analysis of the soft tissue held between two rigid plates is carried out in a half region by considering the symmetry shown in Fig.2(a).

As shown in Fig.2(b), the relative displacement between the rigid plates decreases linearly. The nominal strain rate is measured by the value \( \varepsilon_0 = 0.05 \) divided by the lamp time \( t_0 = 500 \) s. Subsequently, a constant relative displacement is maintained. The inflow and outflow of the fluid occur through the sides, and the rigid plates are non-permeable. The diameter and height of the circular cylinder are \( d = 6.35 \text{mm} \) and \( h = 2.5 \text{mm} \), respectively. Two extreme lubrication conditions for
the region between the contacting surfaces of the rigid plates and the soft tissue are taken into account, i.e., unconfined compression under a perfect lubrication condition (PLUC) and a perfectly adhesive condition. Moreover, the ring-shaped adhesion region between radii $r_1$ and $r_2$ is considered to have a partially lubricating and partially adhesive condition. Six cases are considered: $r_2 = 0.4(d/2)$, $0.6(d/2)$, $0.8(d/2)$ when $r_1 = 0$ and $r_1 = 0.6(d/2)$, $0.4(d/2)$, $0.2(d/2)$ when $r_2 = d/2$. The material constants and parameters are set as follows: $\phi_s = 0.2$, $\kappa = 0.76 \times 10^{-14} \text{ m/Ns}$, $\lambda_s = 0.1 \text{ MPa}$, $\mu_s = 0.2 \text{ MPa}$, and $\beta = 10^{14}$. The finite element analysis is carried out by using a non-uniform but rectangular mesh of 4-node linear elements, as shown in Fig. 2(c).

5. **Analysis Result**

5.1 **Result of Confined Compression Analysis**

Figure 3 shows the nodal values of displacement, and velocity of solid phase, and velocity of the fluid phase.

![Fig. 3 Nodal value of soft tissue in the confined compression test. (a)Displacement of the solid phase, (b)velocity of the solid phase, and (c)velocity of the fluid phase.](image-url)
According to the displacement distribution of the solid phase (Fig. 3(a)), the strain, which is the gradient of the displacement, increases in the part near the upper surface, and it is understood that viscoelastic features can be expressed. This nonlinear feature is different from that of the simple linear elastic solid for which the strain distribution is uniform and the displacement distribution becomes linear. According to the velocity distribution of solid phase(Fig.3(b)), the velocity distribution is not steady but temporarily changes. This feature is different from that of the simple linear elastic solid for which the velocity does not depend on time and becomes a linear distribution proportional to the coordinates along the thickness direction. In addition, the final velocity distribution is to converge to a straight line. From Figs.3(a) and (b), it is observed that accurate results are obtained by comparing the analytical results for the 8-node element of the 6-element mesh and 4-node element of 24-element mesh. According to the velocity distribution of the fluid phase (Fig.3(c)), the fluid permeates through the porous rigid body, which increases the deformation; moreover it is understood that the velocity distribution finally becomes a straight line, similar to the solid phase. The velocity distribution obtained using the 8-node element shows oscillatory characteristics, and the accuracy of the evaluated values is not sufficient. Because the accuracy of the values evaluated at element centers or points in the vicinity of integral points is better in comparison to the nodal values, which is not shown here, the result of the deformation modes includes some hourglass modes or zero-energy modes[10].

5.2 Result of Unconfined Compression/Relaxation Analyses
The result of a temporal change in the pressure of a point in the vicinity of the center on the relaxation response is shown in Fig.4; this result is obtained by increasing the pressure in the unconfined compression test and then holding it at a constant value. The relaxation responses for all cases show the characteristics of typical viscoelastic materials. These characteristics do not qualitatively depend on the lubrication condition between the soft tissue and the rigid plate.

From the temporal changes in the pressure near the center of the specimen, it is observed that the pressure for the PLUC is considerably larger than that for the perfectly adhesive condition (PAUC) during the compression. However, during the subsequent relaxation, the relaxation time of the PLUC is shorter than that of the PAUC. From the results for different surface lubrication conditions, the response of perfect adhesion is approached by that for the perfect lubrication with increasing area in the adhesive region.

In both cases \( r_1 = 0, r_2 = 0.8(d/2) \) and \( r_1 = 0.6(d/2), r_2 = d/2 \), the area fraction in the adhesive region is 0.64. The temporal changes in the pressure distribution of both the cases are in good agreement with the analytical results. Fig. 4 Pressure relaxation at \( r = 0.05(d/2) \) point for the unconfined compression test: PLUC, PAUC, and partial lubrication and adhesion. (a)Partial lubrication and adhesion from the center: \( r_1 = 0, r_2 = 0.4(d/2), 0.6(d/2) \) and \( 0.8(d/2) \). (b) Partial lubrication and adhesion from outside: \( r_2 = d/2, r_1 = 0.6(d/2), 0.4(d/2) \), and \( 0.2(d/2) \).
agreement. This can be stated on the basis of the application of St. Venant’s principle, and the internal stress in the partial adhesive problem may be expressed by the area in the adhesive region.

Fig. 5 Pressure relaxation for unconfined compression test. (a) PLUC, (b) PAUC, (c) partial lubrication and adhesion \((r_1 = 0, r_2 = 0.8(d/2))\), and (d) partial lubrication and adhesion \((r_1 = 0.6(d/2), r_2 = d/2)\).
The contour figures of the pressure distribution at \( t = t_0 \) and \( t = 4t_0 \) are shown in Fig. 5. In the case of perfect lubrication, the pressure distribution is a function of only the radial coordinate, and it does not depend on the coordinate along the thickness direction. On the other hand, it is understood that in the case of perfect adhesion, the pressure value at a deep position increases for the compression; however, the contrariety quickly released the pressure at a deep position in the relaxation. The release of the pressure near the surface is slightly delayed.

From the abovementioned fact, it is confirmed that the lubrication condition on the boundary surface is closely related to magnitude and temporal evolution of the pressure distribution inside the tissue. The adhesion occurs when the cartilage surface is damaged and when the friction coefficient increases; these features may yield a high-pressure situation. This fact is in good agreement with the clinical result of relating to promoting the progress of OA when the cartilage is damaged.

6. Concluding Remarks

The fundamental problem relating to the material model of articular cartilage based on the biphasic theory and mechanical analysis were studied through a review of the study of Spilker and Suh (1990); further, some new analyses were carried out. In the confined compression test for the soft tissue, considerable useful information concerning deformation mechanisms, such as the flow velocity distribution of the interior as well as macroscopic viscoelastic properties, was obtained by employing the biphasic formulation. The combination of a high-order interpolation function and reduced integration were useful in the solid phase analysis; however, it was inappropriate to obtain the velocity distribution of the fluid. The typical and viscoelastic relaxation response was shown without considering the lubrication condition of the compression plates in the unconfined compression test. Further, it was understood that the values and temporal changes in the pressure distribution inside the tissue strongly depended on the lubrication condition.

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