Abstract

This is a brief review of three strongly related topics. In recent years, significant progress was reached in understanding of vacuum and hadronic structure: quantitative role of non-perturbative tunneling, described semiclassically by “instantons” was clarified. We review recent works on the point-to-point correlation functions, comparing those obtained from phenomenology, “instanton liquid” models and on the lattice. The second topic is physics of chiral symmetry restoration, which may lead to observable hadron modification and unusual event-per-event fluctuations. We discuss collective flow and phenomena related with a remarkable “softness” of the equation of state near the phase transition, as well as phenomena at small $p_t$. We also describe recent ideas on the mechanism of chiral restoration, based on formation of polarized instanton-antiinstanton molecules. The third part, related to quark-gluon plasma, is related mainly with the issue of initial equilibration of QGP in high energy collisions. We discuss different “parton cascade” approaches and argue that multi-gluon processes dominate them. We discuss predictions for RHIC and LHC energies.

1. Introduction

Studies of high-temperature (high-density) hadronic matter has grown into a wast field during the last decade. In 70’s there were just few theorists dreaming about phase transitions in QCD, and now we have conferences with hundreds of participants, most of them experimentalists. If in early 80’s all attempts to preserve ISR from destruction failed, now the nuclear physics community in US has united and pushed forward a dedicated complex, RHIC, now under construction. In Europe, heavy-ion experiments are planned at the largest accelerator to be made at CERN, LHC.

When our colleagues from other fields ask for a brief explanation of the major motivations of this program, we usually say that high temperature matter is relevant for Big Bang, remind them that high density exist in compact stars, and conclude that in quark-gluon plasma is a new state of matter and it is interesting in its own rights. It is all true, but it is far from being all of that.

A very important part of it is related with hopes to understand better the world we are living in. We know that QCD is the fundamental theory of strong interactions, but its ground state, the QCD vacuum, is not understood. It is important to realize that
it is in fact a very complicated and quite dense matter by itself. When all perturbative infinities are taken care of, one is left with the so called non-perturbative energy density $\epsilon_{\text{vac}} \approx -1\text{GeV/fm}^3$. The minus sign is important: it means that the physical vacuum is below the naive “normal” one, in which only small zero point fluctuations take place. So to say, we all live in a kind of a superconductor, and only by producing a tiny fireball of QGP we can learn about existence of this “normal” phase. Clearly, even such crude information as the value of the critical temperature $T_c$ is very important to understand of the phenomena underlying this energy density and excitations in our world. By studying how how hadrons “melt” we may learn more about their structure.

Large value of the vacuum energy density explains why we need really high energy accelerators: we have to melt it first, before filling the space with thermal quarks and gluons. And only when we will be able to do so, the physical reality of this “vacuum energy” (and the “vacuum pressure” $p_{\text{vac}} = -\epsilon_{\text{vac}}$) will become obvious. (This situation resembles very much what have happened when people has realised the reality of atmospheric pressure.)

But where this big negative energy comes from? An answer is not completely clear yet, but I will argue below that it comes from tunneling phenomena, between certain topologically different configurations of the glue. (As we know from quantum mechanics, if one has several potential wells with a bound state, tunneling from one to another does lower the ground state energy.) Of course, one would need a truly quantitative theory of those phenomena and a confirmation by the experiment in order to be quite sure about it. So far, we are at least sure that tunneling contribution to the vacuum energy shift has right order of magnitude.

In chapter 2 of this review I will describe a dramatic progress which took place during the last 2-3 years which has resulted in quantitative understanding of tunneling phenomena in QCD. We knew since mid-70’s, that one can use semiclassical theory based on the “instanton” solution to describe them, provided the action along the tunneling path is much larger than the Plank constant $S \gg 1$. Unfortunately, it is not generally clear whether it is the case, or other competing tunneling paths take over. A real breakthrough was related with studies of the QCD correlation functions (see ref. for a review), which has demonstrated quite clearly that instanton-related forces between quarks are indeed there and are indeed very very important. To mention one of those (quite unexpected) findings: the nucleons are actually bound mostly by those forces, not by a confining ones, as we thought before. Furthermore, the nucleon-delta (spin) splitting also seems to be mainly an instanton effect, not the gluo-magnetic spin interactions we used to believe in.

At this point of the introduction, let me make some general remark on our most powerful theoretical tool, the lattice gauge theory. In the last couple of years we are witnessing now a qualitative new stage of its development. Ten years ago, the main con-

* According to Standard model, Higgs fields create its own version of a superconductor, which can also be “melted” at $T_c$ about a thousand times that for QCD: but we would not touch the electroweak physics in this review.
cern in the field was whether the results make sense: people have checked universality, scaling, etc. Few years ago lattice community became convinced that it may not only reproduce experimentally known masses and other parameters (still with much worse accuracy), but go ahead and make some quantitative predictions of unknown parameters (e.g. $\Lambda_{QCD}$.) A new step are attempts to identify the most important structures in a very complicated vacuum. Two types of very interesting structures were identified so far: (i) instantons and (ii) paths of the monopoles. In the first case, it was the so called “cooling” (see details in ref. 2), in the second one it is the so called “abelian projections”. In both cases a kind of “radical surgery” was used, eliminating all fields except of those of an interesting structure. And in both cases it was demonstrated that the main phenomenon under consideration (chiral physics and confinement, respectively) do survive this operation. What should be strongly encouraged at this point, is a continuation of these efforts, and their generalization for non-zero temperatures (and densities, if the working method of doing it will be found).

The next chapter is related with chiral phase transitions and its possible manifestations in experiment. In subsection 3.1 we discuss the fate of two chiral symmetries, related with $SU(N_f)$ and $U(1)$ groups. Then we consider possible experimental approaches to search for signatures of the phase transition. Those potentially include the whole range of ideas, from simple utilization of the “softness” of equation of state and flow, to mass and width modification of hadronic modes, and eventually to very complicated questions related with critical fluctuations. We also consider new ideas, relating chiral restoration to formation of instanton-antiinstanton molecules.

In the last chapter we briefly address some topics related with QGP itself. We do not consider neither the progress in partial resummation of perturbation theory, nor those concerning the non-perturbative phenomena at very-high T. Instead, we have concentrated on QGP production and equilibration, at RHIC/LHC energies. We compare different theoretical tools used, as well as some of the results.

2. Correlators in the QCD vacuum and instantons

2.1. Vacuum and hadronic structure

Let us start with brief recollection of the history of hadronic physics. After many hadronic states were discovered in 50’s, it became clear that they should be some composite objects. In 1964, we have learned from Gell-Mann and Zweig that hadrons are made of quarks. The first model on the market was (i) constituent quark model, in which a nucleon was viewed as a non-relativistic bound state of three separate massive quarks. It works surprisingly well in some cases, e.g. for baryon magnetic moments. However, as it became obvious already by the end of 60’s, “true” quarks are actually nearly massless, so that the chiral symmetry is nearly exact. The first part of this

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† Here and below we discuss only hadrons made of light (u,d,s) quarks. Of course, things are very different in the world of heavy quarks: say $J/\psi$ and $\Upsilon$ physics is well described by the non-relativistic potential, a combination of a Coulomb and confining ones.
idea (light quarks) were well incorporated into the favorite model of 70’s, (ii) the MIT bag model. The second part (chiral symmetry) was badly violated in this model, and therefore a completely different model for a nucleon became popular, in 80’s, known as (iii) the skyrmion, in which nucleon is entirely made of a pion field. Later, another model was suggested, connecting the bag and the chiral symmetry together, into (iv) the chiral bag model. Looking at these models together, one could not be surprised that our students are confused: these models suggest drastically different pictures of hadronic structure. While the MIT bag model relates all dimensional quantities to a “bag constant”, representing a confining force, the skyrmion picture has no place for confining forces at all!

Such situation clearly reflects the fact, that this field is still not mature enough, in spite of its age. All those models cannot be true, and one has to work harder to tell which ones are wrong. My point here is that the information used so far (contained in the spectra of the low-lying states) is simply insufficient to make this choice. The same was true for nuclear forces: deuteron data are important, but only complete set of scattering phases has clarified them in sufficient details.

Unfortunately, one cannot do quark-antiquark scattering: but one is still able to connect the hadronic structure to the fundamental theory, QCD, by calculating Euclidean correlation functions for all distances and all channels. Hopefully, this will be enough.

My second general remark: all models of hadronic structure mentioned above ignore one general principle, which we have learned solving numerous problems of quantum physics. This principle is: solve the ground state first, then it is easier to understand the excitations. All these models try to avoid the question about the vacuum structure, in one way or another. The MIT bag model, for example, does acknowledge existence of non-perturbative effects in QCD, but only outside the hadrons.

In fact, in the list of models mentioned above one important model of hadronic structure (actually, the oldest one) was missing. This is the Nambu-Jona-Lasinio (below NJL) model, which was inspired by the BCS theory of superconductivity. This one actually has the right strategy: assuming existence of some short-range attraction between quark and antiquark (in the present-day notations), and moves first to the discussion of the ground state.

Let me briefly sketch here a picture of light-quark hadrons, as I see it today. Ironically enough, Nambu-Jona-Lasinio model is actually right in assuming that strong short-range attraction is the main effect. It does exist in QCD, generated by the tunneling phenomena we are going to discuss below. Those forces make pion light, $\eta'$ heavy and build the quark condensate. “Constituent quarks” are relatively heavy, with $M > 400\,\text{MeV}$, while some hadrons (in particularly, the nucleon) are deeply bound. Even $\rho$ and $\Delta$ baryon probably remain (weakly) bound, if one switches off confining

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*Although the exact effective interaction is not exactly the same, of course, and even have different symmetries.*
forces.

2.2. Instanton history in brief

Tunneling phenomena in gauge theories, discovered by Polyakov and collaborators,[4] were soon followed by the fascinating semiclassical treatment by ’t Hooft.[5] It was pointed out, that tunneling lead to completely new type of effective interaction between light quarks. This interaction actually explains how chiral anomalies work. The first applications to QCD problems such as ref.[6] have attracted a lot of attention in late 70’s. However, as no explanation for “diluteness” and validity of semiclassical approximation were suggested from the first principles, optimism has soon died out and most people left the field.

Next period has mainly focused on phenomenological manifestations of instanton-induced effects. Starting from QCD sum rules and its problems, the so called “instanton liquid model”[7] has emerged. It suggested relative diluteness and large action per instanton due to their relatively small size, but also emphasized significant interaction in the ensemble as the origin of density stabilization. Attempts to describe interacting instantons were initiated by the variational approach.[8] For the simplest “sum ansatz” for the gauge fields it was shown that there appears repulsive interaction at small distances, which may stabilize the density and lead to qualitatively correct instanton liquid. Further numerical studies of this problem[9] have allowed to get rid of many approximations and eventually included fermionic effects to all orders in ’t Hooft effective Lagrangian.[5] We return to discussion of this approach below, and now let me jump directly to several important steps made during the last years.

About 40 correlation functions were calculated in the framework of the simplest ensemble of the kind, the Random Instanton Liquid Model (RILM).[10] Agreement with data is generally good, and in some cases (including π, N etc) it is really astonishing. Recently, glueballs were added to the list,[11] and therefore the instanton liquid is certainly by far the best model of hadronic and vacuum structure, available today.

On the theoretical side, however, many important questions are left open, though. Is the classic repulsive interaction really there, or another explanation (e.g. related to confinement or charge renormalization) is need for density stabilization? Is this “random liquid” really a reasonable approximation? What is the role of interaction between instantons, especially of quark-related one? What happens for larger number of colors, or flavors?

Few words about instantons on the lattice. First of all, lattice calculation of point-to-point correlation functions[12] also show good agreement with experiment,[13] and the RILM results.[10] Furthermore, experimentation with “cooled” lattice configurations[16] has essentially confirmed parameters of the original “instanton liquid” model. However, a lot of work is still needed. In particularly, both lattice and instanton studies mentioned are some sense “quenched” so far, and inclusion of dynamical quarks should be done. This is very important at finite temperature, especially near the QCD phase transition.

Finally, about a separate brunch of the instanton studies. Few years ago the
instanton-induced processes has attracted much attention after works by Ringwald, Espinosa and others about baryon number violation in weak interactions. A hope was expressed for some time, that it would be possible to observe it experimentally, in multi-TeV collisions, but it looks that the relevant cross section are orders of magnitude below the level reachable by any experiments. However, ideas developed in this context may be used in the QCD context. Interesting examples are the instanton-induced deep-inelastic scattering, or jet production, see more in refs.\cite{14}

2.3. Why instantons?

We have already mentioned that the main reason why instantons are so important for physics of light fermions: it is related to the anomaly phenomenon, which means that each tunneling event leads to rearrangement of quark chiralities. Since that happens for all three light quarks u,d,s at the same time, one gets generally a 6-fermion effective interaction.

Let us however change the language, and outline another possible approach, which is the one actually used in calculations. Anomalies are related to famous ’t Hooft zero modes, the localized solutions of the Dirac equation

\[ D_\mu \gamma_\mu \phi_0(x) = 0 \]  

where D is the covariant derivative containing the instanton field. Evaluating the (Euclidean) quark propagator \( S = -1/[iD_\mu \gamma_\mu + im] \) for \( m \to 0 \) and large distances one has to deal mainly with small eigenvalues. The fermionic states which are “made out of” zero modes naturally have this property, and thus are “prime suspect” for being the relevant ones.

It is therefore convenient to look at the instanton as a trap for quarks, something like a “receptor atom” in a semiconductor, capable to create a new state in otherwise forbidden place. In metals an electron can propagate far, just by hopping from one atom to another. The same is true for finite instanton density, leading to a “zero mode zone” of collectivize quark states. That is the mechanism leading to the non-zero quark condensate, or chiral symmetry breaking.

Of course, in order to make these statements completely convincing, one should be able to check whether the “zero mode zone” does indeed dominate in the density of states at “virtuality” \( \lambda = 0 \). The QCD vacuum has a lot of different fluctuations of the color fields (e.g., the monopole loops): and one may imagine that most of them may contribute somehow to it. This can be done by tracing the nature of all lowest-\( \lambda \) fermionic states.

Furthermore, if more than one quark is travelling in the QCD vacuum (\( \bar{q}q \) for mesons and \( qqq \) for baryons), they “hop” over the same instantons. This fact leads to an effective interaction, which, if attractive enough, may in fact bind quarks together.

We have claimed that instantons are more important than any other fluctuations of the gauge field. To prove that phenomenologically, one has to look more specifically into the chiral and flavor structure of the instanton-induced interaction. As shown by
't Hooft, at tunneling quarks with one chirality “dive into the Dirac sea” while those with the opposite chirality “emerge” from it. Therefore instanton-induced forces should be strong in scalar and pseudoscalar channels, and absent (in first order) in vector or axial ones. Looking at phenomenological correlators at small distances, one finds that it is exactly right.

Furthermore, consider signs of the instanton-induced corrections to spin-zero channels. The correlation functions at small distances are essentially the free propagators squared (for mesons, and cubed for baryons). If the instanton-induced corrections are relatively small, one may use the 't Hooft interaction in the lowest order. There are 4 such channels for 2 flavors and it is a simple matter to see that correction is positive (or attractive) for \( \pi, \sigma \) channels and negative (or repulsive) for \( \delta, \eta' \) ones.

Thus the same mechanism leads to both light pion and heavy \( \eta' \)! Both splitting from a typical meson like \( \rho \) are large, which is a very strong hint. Moreover, looking at masses \( m_\pi^2 \approx 0, m_\rho^2 \approx 0.5 \text{GeV}, m_\eta'^2 \approx 1 \text{GeV} \) one finds the splittings to be even comparable!

2.4. Correlators in the instanton vacuum

Let me recall here very deep intuitive thoughts by Yukawa: in order to recognize existence of a particle, one should not necessarily find it in the detector, or observe a pole in a propagator. In fact one may just observe an amplitude, exponentially decaying with distance to recognize existence of a virtual particle. This is exactly how all hadronic masses are measured nowadays on the lattice.

One may exploit this idea further and consider a correlation of two local operators, separated by the space-like distance \( r \). For example, the operators can create a quark-antiquark pair. At not-very-large \( r \), the correlator is not falling exponentially with a single mass, and it cannot be described in terms of one propagating meson; but it still provides a lot of information about the quark-antiquark interaction. If such correlator is known, it allows us to tell the effect of the short-range forces (e.g. instanton-induced ones, to be much discussed below) from the long-range ones (e.g. confinement-related).

Now we proceed from qualitative hints to quantitative calculations. We have to evaluate a quark propagator in the multi-instanton field configuration, which can be done as follows:

\[
S(x, y) = \sum_{ZM} \frac{\phi_\lambda(x) \phi_\lambda^+(y)}{\lambda - im} + iS_{NZM}(x, y) \tag{2}
\]

where the first term is the sum over states belonging to the “zero mode zone”. The non-zero modes (analogs of unbound atomic states) are taken into account by the last term, see details in 10.

\[ \text{Note at this point, that many other models (e.g. those based on confining forces) for chiral symmetry breaking lead to light pions as well: Goldstone theorem simply demands it. However, for those models there is no hope to get the } \eta' \text{ right.} \]
We have first calculated correlators for the simplest ensemble possible, the random instanton liquid model (RILM), in which: (i) all instantons have the same size $\rho_0 = 0.35\text{fm}$; (ii) they have random positions and orientations; (iii) instanton and anti-instanton densities are equal, and in sum it is $n_0 = 1\text{fm}^{-4}$. These are the parameters suggested a decade ago in, the density comes from the gluon condensate and size from various other things, say from the quark condensate value. The main step in the calculation is inversion of the Dirac operator, written in the zero-mode subspace. (We typically use in sum 256 instantons and anti-instantons, which tells the dimension of this matrix and the volume of the box.)

Although the quark propagators are gauge dependent, we have looked at them first in order to see whether they can be reproduced by any simple model, say by “constituent quarks” with a constant mass. We have found that chirality-non-flipping part of the propagator indeed looks as if quark get a mass about 300-400 MeV, but the chirality-flipping part does not look like that at all. None of many calculated correlators in fact follow a constituent quark model.

Our results for $\pi, \rho, N, \Delta$ channels are shown in Fig.1,2. All correlators are plotted in a normalized way, divided by those corresponding to free quark propagation: that is why all of them converge to 1 at small distances. Solid lines correspond to experiment, while the long-dashed and short-dashed curves correspond to QCD sum rule predictions respectively.

The agreement for the pion curve is as perfect as it can be: both the mass ($142\pm12\text{MeV}$) and the (pseudoscalar) coupling are reproduced correctly, inside the error bars! Large deviations from perturbative behavior happens at very small distances for the $\pi$ channel, while exactly the opposite is observed in the $\rho$ case, the plotted ratio remains close to 1 up to a very large $x$. Both RILM and lattice has reproduced that non-trivial observation.

Proceeding to baryonic channels, let me mention that we have actually measured all 6 nucleon correlators and 4 delta ones, and have fitted them all. Again, agreement between RILM and lattice results is surprisingly good, literally inside the error bars. Both display a qualitative difference between the nucleon and the delta correlators: this can be traced to attractive instanton-induces forces for the spin-isospin-zero diquarks. Without any one-gluon exchange the RILM predicts the $N - \Delta$ splitting (actually, we have found that in RILM $m_N = 960\pm30\text{MeV}$ and $m_\Delta = 1440\pm70\text{MeV}$, so the splitting is in fact somewhat too large).

There is no place here to discuss other channels in details. Let me only mention here that the most difficult case proved to be the isosinglet scalar $\sigma$, for which one should not only evaluate the double quark loop term, but also subtract the disconnected $|<\bar{q}q>|^2$ part. Curiously enough, we have found dominance of a light state, with $m\sim500\text{MeV}$, reminiscent of the sigma meson of 60’s. For several reasons such measurements are now beyond the reach of lattice calculations, although existence of attractive interaction at $x \sim 1/2\text{fm}$ can probably be seen.

Let me also mention recent studies of the the so called “wave functions” (known...
also as Bethe-Salpeter amplitudes), also done for both RILM and (quenched) lattice simulations. The main qualitative features (e.g.: π, N are more compact than ρ, Δ) are also reproduced. They have shown with even greater clarity, that instantons do lead to quark binding in most channels involved, even without confining forces. The shape of the wave function is however not the same.

Clearly, RILM discussed above cannot be but a crude approximation: at least, the very phenomenon studied above, the quark’s hopping from one instanton to another, should lead to strong correlation between them. Another obvious source of interaction is the non-linear gluonic Lagrangian: a superposition of instantons and anti-instantons have the action different from the sum of the actions. Furthermore, at least in two “repulsive” channels (pseudoscalar isoscalar η′ and scalar isovector δ, or a₀) RILM leads to phenomenologically unacceptable results, showing too strong repulsion induced by an instanton. Only the presence of nearby anti-instantons may help, and this is precisely what “unquenching” of quarks is suppose to do.

Generally speaking, the ensemble of instantons should be described by a partition function of the type

$$Z = \int dΩ \exp(-S_{\text{glue}}) |\text{det}(i\hat{D} + im)|^{N_f}$$

(3)

(where dΩ is the measure in space of collective coordinates, 12 per instanton).

It is a problem similar to those traditionally studied in statistical mechanics, with the main complication being the non-local fermionic determinant. As shown in, if the fermionic determinant is calculated in the “zero mode zone” subset of fermionic states, it includes all diagrams with ’t Hooft effective interactions. For a simulations involving N/2 instantons and N/2 anti-instantons, one has to deal with a N*N matrix. However, it is still orders and orders of magnitude simpler than the lattice gauge theory!

The simulation done already in 80’s have shown this statistical sum describes a liquid, in which chiral symmetry is broken. Recent studies of correlation functions with interacting ensemble have shown other significant improvement over RILM. In particular, in recent paper it was shown how the global fluctuations of the topological charge are screened in the m → 0 (chiral) limit. In more practical terms, it have fixed incorrect behaviour observed in RILM for the η’ channel.

2.5. Glueballs and instantons

We have argued above, that the bulk of hadronic physics, including chiral symmetry breaking and properties of all major mesons and baryons can be reproduced using even the simplest instanton ensemble, the RILM. In this section we consider new development concerning glueballs, based on the recent work.

Let us first briefly summarize what is known about glueballs. Experimental evidences are too uncertain and subtle to be discussed here. The large-scale lattice efforts are still needed to get reliable results, but a few statements seem to be however established: (i) The lightest glueball is the scalar, and its mass is in the 1.6-1.8 GeV
range; (ii) The tensor glueball is significantly heavier $m_{2^{++}}/m_{0^{++}} = 1.4^{21,20}$ with the $0^{--}$ probably heavier still. (iii) The sizes of scalar and tensor glueballs were found to be drastically different. This can be inferred from the different magnitude of finite size effects (see e.g. $^{23}$), or seen directly in glueball wave functions. The scalar glueball seems to be very compact, with a size (to be defined below) $r_{0^{++}} \simeq 0.2 \text{fm}$, while the tensor is huge with $r_{2^{++}} \simeq 0.8 \text{fm}$. Clearly, this picture is very different from naive expectations, and such drastic difference between the glueballs cannot come from confining forces alone.

In $^{11}$ we propose an explanation to these phenomena based on small-size instantons. The main point here is that the QCD vacuum contains small spots of very strong gluon fields, with a specific chirality structure.

As a result, relatively heavy states, with specific spin splittings, are produced. Specifically, instantons generate attraction in the scalar channel, repulsion in the pseudoscalar one and no interaction in the tensor case. (The last case is a consequence of the fact that the stress tensor of the selfdual field of the instanton is zero.) The splittings are much stronger than in normal mesons, because large instanton action $S_0 = 8\pi^2/g(\rho) \sim 10$ enters (quadratically) here. (For mesons, the role of classical field is played by fermionic zero modes, which are however normalized to 1 instead of to the action.)

The results for correlation functions are shown in Fig.3. Note that scalar channel is not changed much if dynamical quarks are included, while the pseudoscalar one does, showing strong $\eta'$ signal. Our fitted glueball mass is around 1.6 GeV, and the threshold in the pseudoscalar channel is approximately 3 GeV.

We have also determined glueball Bethe Salpeter amplitudes (or ‘wave functions’). The scalar one is indeed found to be strongly decreasing function: it can be described by $\exp(-\delta/R)$ with $R = 0.2$ fm. The tensor wave functions has a much larger size, $R \approx \text{fm}$. Further studies of the decay modes of these glueballs, both on the lattice and in the instanton models. The suggested hierarchy of sizes, from very small scalar to large tensor, is of course of great relevance to phenomenological efforts to locate these states among the observed candidates.

3. Chiral symmetry restoration

3.1. Phase transitions and two chiral symmetries

The QCD undergoes a phase transition at high temperatures, to the so called quark-gluon plasma phase. Let me present at Fig.4 a sample of recent data $^{27}$ from MILC lattice collaboration. One can see, that the transition is very rapid, the energy density is rising very rapidly in a narrow region, of only few MeV width.

More specifically, since this theory have some continuous parameters, the quark masses $m_u, m_d, m_s$ (we can safely ignore charm and heavier flavors), and thus the phase diagram can be plotted in the corresponding 3-dimensional space. Taking $m_u = m_d$ one gets a plot $^{33}$ shown in Fig.5. It was found from lattice simulations, that on this diagram there exist two (seemingly disconnected) regions of strong first order transi-
tions: one includes pure gluodynamics (all masses very large), and another including
the point at which all masses are zero. By tradition, the former one is referred to as the
deconfinement transition, and the latter one as chiral symmetry restoration. Of course,
these first order transitions are separated by lines at which the transition is second
order. Other second order transitions are expected on ‘sides’, when only one of the
masses is nonzero.

One major physical question remains open. First, calculations with Kogut-Susskind
fermions were done by a Columbia group,\textsuperscript{33} indicated that the critical strange quark
mass is rather small and therefore we should not have a phase transition in the real
world. However, recent results obtained with Wilson fermions\textsuperscript{34} give the opposite re-
sult: even for $m_s \simeq 400$ MeV with $m_u = m_d \simeq 0$ clear two state signals are observed,
suggesting a first order QCD phase transition in the real world. Clearly we have to wait
for few years till next generation of simulations with dynamical fermions will clarify
the situation.

I have already mention a “hunt” for abelian-projected monopoles, as a source of
confinement. Naturally, it was checked whether deconfinement transition can be ex-
plained by those objects. In ref.\textsuperscript{3} it was shown that the longest monopole loop (which
is responsible for the string tension) indeed disappears in the deep deconfinement re-
gion. Furthermore, its behaviour reproduces T-dependence of the string tension. (At
the same time, the so called “spatial string tension” remains non-zero even at high T,
as it should.)

Before we concentrate below on the chiral transition let me add, that there is no
such thing as the impenetrable barrier between deconfinement and chiral restoration.
Whatever is the order of the phase transition, and whatever name we give to it, for
any quark masses there exist a rather narrow transition region $\Delta T << T_c$ in which
the energy density changes rapidly, from small value characteristic to few hadronic
degrees of freedom to a large one, ascribed to quasi-free quarks and gluons. We should
understand why it is so. No doubt, both instantons and monopole loops significantly
change their properties in this region. Thus, one can easily predict, that lattice people
will find a lot of interesting things in this area in the next few years.

For simplicity, we ignore all effects due to the non-zero quark masses, and consider
QCD in the chiral limit, with $N_f$ massless quarks. In this case the QCD Lagrangian
is just a sum of two separate terms, including right- and left-handed quarks, which
implies two chiral symmetries: $SU(N_f)\Lambda$ and $U(1)\Lambda$.

Their fate is well known to be different. The former one is spontaneously broken
in the QCD vacuum but it is restored at high temperatures, above some critical point,
denoted as $T = T_c$. The $U(1)\Lambda$ chiral symmetry is not related to Goldstone bosons
(as Weinberg has first pointed out) because this symmetry simply does not exist at
quantum level, being violated by the ‘chiral anomaly’ and instantons.\textsuperscript{5}

However, at high temperatures the instanton-induced amplitudes are suppressed
due to the Debye-type screening\textsuperscript{22,23} and therefore (at some accuracy level) we expect
this symmetry to be ‘practically restored’ at high T. Let us denote the point where it
happens with some *reasonable accuracy* as $T_{U(1)}$. The question to be discussed now (see more in) is the interrelation of the two temperatures, $T_c$ and $T_{U(1)}$. Let us refer as ‘scenario 1’ to the case $T_c \ll T_{U(1)}$ in which the complete $U(N_f)_A$ chiral symmetry is restored only well inside the quark-gluon plasma domain. Another possible case $T_c \approx T_{U(1)}$ which implies significant changes in many hadronic channels around this phase transition point. As we will discuss below, these two scenarios lead to quite different predictions.

Pisarski and Wilczek\footnote{The case $T_c >> T_{U(1)}$ does not seem to be possible.} have considered this question in connection with the order of the chiral phase transition. They have pointed out that in the special case $N_f = 2$ the ‘scenario 1’ is likely to lead to the *second order* transition. The reason is an effective Lagrangian describing the softest modes is essentially the Gell-Mann-Levy sigma model, same as for the O(4) spin systems. The most straightforward way to test these ideas is to compare the critical behaviour in both cases, testing whether the $N_f = 2$ QCD and the O(4) spin system do or do not belong to the same universality class.

The first critical index to compare is the one for the order parameter, for which the analogy\footnote{As far as I know, it remains unknown whether the coefficient is positive or negative: thus one can have a dip or a peak.} suggests

$$< \bar{\psi} \psi > \sim |(T - T_c)/T_c|^{38 \pm 0.01}$$

(4)

Recent analysis\footnote{Now particle data table denote notations $f_0$ and $a_0$ to I=0,1 scalars: however particular resonances listed there under these names hardly have anything to do with correlators under consideration.} has concluded, that the data are consistent with O(4) critical exponents, although say O(2) ones are not also excluded.

The second obvious issue is the behaviour of global thermodynamical quantities. The O(4) spin system has an amusing behaviour, with *positive* power for specific heat\footnote{Under SU(2)$_A$ transformations, $\sigma$ is an isoscalar I=0 scalar channel and $\delta$ one $^*^*$.}

$$C(T) \sim |(T - T_c)/T_c|^{19 \pm 0.06}$$

(5)

It means that the singular contribution of the soft modes *vanishes* at the critical point, and in order to single it out the 3-ed derivative of the free energy should then be calculated. Nevertheless, lattice data for the $N_f = 2$ QCD actually do show a *huge peak* in the specific heat around $T_c$. It certainly implies, that many new degrees of freedom become available (or are significantly changed) in this region. What these degrees of freedom are, both in hadronic language and in the quark-gluon one, remains the major open problem in the field. (Of course, there is no logical contradiction here: apart of large but smooth peak one may eventually find a small 'kink', which is truly singular.

Now we return to U(1) symmetry. For simplicity, we consider only two light flavors and use the old-fashioned notations, calling the isoscalar I=0 scalar channel a $\sigma$ one, and isovector $I = 1$ scalar channel a $\delta$ one $^*^*$. Under SU(2)$_A$ transformations, $\sigma$ is
mixed with $\pi$, thus restoration of this symmetry at $T_c$ require identical correlators for these two channels. Another chiral multiplet is $\delta, \eta_{\text{non-strange}}$, where the last channel is the SU(2) version of $\eta'$; at $T=0$ those are very heavy and are not considered in chiral Lagrangians, or course. $U(1)_A$ transformations mix e.g. $\pi, \delta$ type states, and thus its 'practical restoration' should imply that such type of correlators should become similar. Finally, if both chiral symmetries are restored, a simpler statement follows: left-handed quarks never become right-handed, therefore all $\pi, \eta_{\text{non-strange}}, \sigma, \delta$ correlators should become the same.

In their original paper Pisarski and Wilczek have actually argued\textsuperscript{††} in favor of the 'scenario 2'. Their argument was as follows: 'if instantons themselves are the primary chiral-symmetry-breaking mechanism, then it is very difficult to imagine the unsuppressed $U(1)_A$-breaking amplitude at $T_c$'. However (as will show below) instantons do not seem to be very strongly suppressed at $T \sim T_c$.

In ref\textsuperscript{25} I have argued that U(1) should be "practically restored" right above the transition region: the reason is instantons are forming molecules, rather than being suppressed. Let me now show that the latest lattice data indeed support this conclusion. The argument will need some preliminary discussion.

Suppose one is willing to measure masses of $\sigma$ and $\delta$ scalars on the lattice. If so, one should evaluate the so called "connected" and "disconnected" quark diagrams. $\delta$ correlator has only connected one (For its charged component e.g. $\bar{u}d$ it is trivial, because two quarks have different flavor, a one-line algebra shows why it is so for neutral component as well.) while disconnected diagram contributes to the difference between them.

The available set of data\textsuperscript{28} is shown in Fig.6, as the so called "susceptibilities", the second derivatives of the free energy over quark masses. In other words, it is the integrated point-to-point scalar correlation function: it is important, that the contribution of each scalar state is therefore inversely proportional to its mass squared.

One can see that both of them show strong $T$-dependence in the vicinity of $T_c$. The "disconnected" one, being a difference between $\sigma$ and $\delta$ correlators first rises sharply (because sigma mass goes to zero at $T_c$) and then rapidly drops. Unfortunately, there is no more data points, and therefore we do not know how small it actually become. If it is really small, one may say that both chiral symmetry are practically restored, and (if it happens), it is most probably at the temperature only few MeV above $T_c$. We return to consequences of this observation below, when we will discuss fluctuations in the critical region.

3.2. How to get experimental evidences for the phase transition?

In this subsection we jump from theoretical considerations and numerical experiments to a discussion of issues relevant for "real" experiments. All event generators,

\textsuperscript{††} They have even mentioned that this amplitude should be at $T_c$ at least an order of magnitude smaller than at $T=0$, although no details of this estimate were given.
hydro and other models agree that at AGS (10-14 GeV*A) and SPS (200 GeV*A) energies we should have reached the “mixed phase” region (with large and small baryon density, respectively). At the same time, “pure” QGP is either not there, or it exists for so small time that we (so far) cannot figure out how to find its direct manifestations‡‡. That is why eventually RHIC and LHC heavy ion experiments will be performed, where QGP will be created way above the critical region and therefore be there for significant time.

Nevertheless, one should be able to find some manifestations of the QCD phase transition in current experiments, performed at Brookhaven AGS and CERN SPS. A very direct signature is transverse collective flow, directly related with EOS. As noticed long ago‡‡ near the QCD phase transition the EOS is especially soft. As an illustration, we show Fig.7 from, where a conventional parametrization of EOS is presented in an unconventional way. We have eliminated temperature and plotted instead the hydrodynamically relevant ratio $p(\epsilon)/\epsilon$ versus $\epsilon$, thus emphasizing the existence of a minimum at $\epsilon = \epsilon_{max} \approx 1.5$ GeV/fm$^3$. This minimum is referred to below as the softest point of the EOS.

For long time only central collisions were discussed, with an axially symmetric (cylindrical) flow. Generally speaking, the AGS/SPS data agree well with soft EOS because no significant growth of the average transverse momentum $< p_t >$ with the multiplicity (or transverse energy) was observed. However, it is difficult to separate reliably thermal and collective components of the particle momenta, and therefore really quantitative analysis of flow in central collisions is still missing.

Fortunately, recent results from E814‡‡ have revealed flow for non−central collisions, as asymmetry in the reaction plane, similar to what was seen previously at BEVALAC. One may expect to reach quantitative understanding of the transverse expansion soon enough, leading to restrictions for the EOS.

Let me now switch to the following general question: which collision energy is most suitable for observations of this “softness” effect? The answer considered in a recent work‡‡ seems quite obvious: it is the collision energy at which matter is first produced close to the “softest point” indicated in Fig.7 above. It was found that in this case one can see significant effect not only the transverse, but also the longitudinal expansion. Hydrodynamical studies of central Au+Au collisions at varying energies were made, from the SPS to the AGS ones, 200 to 10 GeV/N. Radical changes are found: from (i) violent longitudinal expansion, close to scale-invariant solution at high energies, to (ii) a “slow burning” at the softest point; and then leading again to (iii) a noticeable longitudinal expansion of the hadronic gas at low energies. In Fig.8,9 we have shown how the picture of the expansion and some global parameters of the space-time picture

‡‡This point of view is not universally accepted. For example, H.Satz has repeatedly suggested that the observed $J/\psi, \psi'$ suppression cannot happen in hadronic phase, and thus some presence of QGP is needed to explain that. J.Rafelski thinks the same is true about observed enhancement of multi-strange baryons. I think both claims may well be right, but more data (especially with new lead beam) and more quantitative theory are needed to accept them.
depend on collision energy. The *maximal lifetime of the mixed phase* (measured at \(z = 0\)) \(\tau_{\text{mix}}\) has a clear peak, corresponding to initial conditions at which the pressure-to-energy ratio is the smallest. The total 4-volume of the mixed phase \(V_M\) (also shown in Fig.9) has only a “shoulder”; this is because longer lifetime is compensated by smaller spatial volume. Furthermore, these radical changes in the space-time evolution translates into experimentally observable quantities. The *penetrating probes*, \(\gamma\) and \(e^+e^-\) production are most relevant here. Using production rates\(^4\) for photons and\(^5\) for dileptons (which can be compared to existing SPS data), it was found that the total yield of dileptons \(dN_{e^+e^-}/dy(M,y = 0)\) have non-monotonous dependence on the collision energy, with a very sharp rise near the “softest point”, with more or less constant production level at higher energies. Equally dramatic is the energy dependence of the *width* (at half maximum) of dilepton rapidity distribution.

If the collision energy corresponding to the “softest point” is found, and the lifetime of the fireball is indeed factor 2-3 longer than elsewhere, one may suggest a whole list of interesting questions, related to modifications of all possible signals. To give few examples: Is there any additional strangeness enhancement, or \(J/\psi\) suppression in this case? Other phenomena sensitive to the *total* lifetime of the excited system\(^\wedge\) are the so called low-\(p_t\) enhancements. In my paper\(^15\) it was suggested that the collective potential may be able to *trap* pions and kaons, provided their transverse kinetic energy \(m_t - m\) is smaller than the attractive potential. (The phenomenon is similar, say, to total light reflection when it comes out of water.) The deviation from pure exponential (or pp collisions) was first seen for pions, leading to a series of different explanations. Direct observations of multiple \(\Delta\) resonances have supported the resonance interpretation. In Quark Matter 1993 the results of E814 experiment\(^2\) has been reported: similar enhancement was found in spectra of negative and positive kaons, which have no significant resonances near the threshold. What was also surprising, these first data, for SiAu collisions, have produced the unusually small inverse slope \(T \approx 15\,\text{MeV}\) while more recent AuAu data (see Fig.10) have shown much larger value. Why should this effect be *strongly projectile-dependent*?\(^1\)

Recall that for any amplitude, the energy derivative is related to duration of the process under consideration: this is basically uncertainty relation. Thus, spectrum modification at so small transverse energies as \(m_t - m = 10-20\,\text{MeV}\) implies that the system lives long enough. A particular kinetic model was used in\(^\wedge\) in order to explain these puzzling data. Indeed, it was found in the simulation that if the fireball is expanded slowly enough, the low-\(p_t\) kaons can be *trapped* in the fireball and “cooled”. In brief, the reason for cooling (the same in ordinary refrigerator) is that kaons are reflected from the wall moving *outward*, loosing energy.

\(^\wedge\) By the way, contrary to wide-spread opinion, the HBT correlations do not measure it: they are only sensitive to the *emission* time of pions. It is by no means the same thing: for example, fireball may exist for long time, and then emit pions in a short flash.

\(^\dagger\) Note that at very small momenta one can see a dip in \(K^+\) spectra, and also a noticeable difference for \(\pi^-\), \(\pi^+\) ones: it should be related to collective Coulomb field: a good clock by itself.
Let me present here a simple explanation, reproducing at least these simulations (if not data). No matter how deep is potential and how slow the motion is: if it is slow enough one may use conservation of adiabatic invariants, which accurately predict the behaviour of many similar phenomena, from atomic traps to expanding Universe. If a particle is trapped into an expanding potential well the following relation should hold $\langle p_t \rangle \sim R_t = \text{const}(\text{time})$, which for non-relativistic kaons translates into the following "cooling relation"

$$\frac{T_{\text{final}}}{T_{\text{initial}}} = \left( \frac{R_{\text{initial}}}{R_{\text{final}}} \right)^2 \quad (6)$$

The initial transverse size $R_{\text{initial}}$ is that of the projectile nuclei, while the final one should be that measured by the HPT interferometry. For Si Au their ratio is known to be about 2.5, so this formula indeed leads to nearly an order of magnitude drop in T. For AuAu collisions the transverse expansion is less strong, leading to the final/initial size ratio of about 1.2, with much smaller cooling ratio.

However, many features of this picture remains unexplained. In particular, large magnitude of the observed enhancement can only be the case if the K rescattering rate is miraculously reduced, so that they may remain ‘cool’ inside the hot fireball! Furthermore, it was found in $^{53}$ that in order to get that one needs fireball lifetimes at least 30-40 fm/c. It is several time more than event generators suggest, and only comparable to what we get above for the long-lived scenario at the “softest point”. Does it mean that such scenario may actually take place at AGS?

Another set of so far unexplained data is related with “penetrating probes”, photons and especially dileptons. Those observables clearly is the best ways to get information about the early hot stage of nuclear collisions $^{66}$. Unfortunately, such experiments are difficult, so only recently their first preliminary results were reported by 4 CERN experiments: WA80 (photons), NA34/3 and NA38 (dimuons) and CERES (dielectrons). (See $^{54}$ for recent review.) All of them see significant excess over the expected background effects due to hadronic decays: in the CERES $^{55}$ case at dilepton masses $M \approx 3 - 5 GeV$ it reaches one order of magnitude, as it is shown in Fig.11.

Attempts to explain these observations using conventional dilepton production mechanisms (such as the fundamental $\bar{q}q$ annihilation in the QGP phase, $\pi^+\pi^-$ annihilation in the hadronic and mixed phase, as well as other important reactions involving the $A_1$ meson) has been made by many groups, but they do not actually succeeded to get quantitative explanation of the effect. In ref. $^{56}$ we have suggested that again the reason might be much longer lifetime of the fireball, 30-40 fm/c instead of conventional 10. Unfortunately, after hydro calculations $^{57}$ were completed, it turns out that even the long-lived fireball produced at the “softest point” does not really help here, because larger lifetime is compensated by its smaller spatial volume.

3.3. Hadron modifications at finite temperatures

It is very natural to expect the elementary excitations (hadrons) to change their properties at non-zero temperatures/densities, especially close to the phase transition.
We have discussed above lattice data on some scalar masons, $\sigma, \delta$: do we have any experimental evidences from ‘real” experiments as well?

This question was studies for long time in normal nuclear matter, for nucleons. In short, large amplitude scalar and vector collective fields are found, which however nearly cancel each other in the nucleon case (and we do not understand why). What about other hadrons?

One of the most interesting case is that of $K$-mesons (we have discussed it just above). Recent analysis of $K^-$ atoms have demonstrated very large attractive potential about $-200$ MeV inside the heavy nuclei. If correct, it makes kaon condensation in stars unavoidable, and kaon “trapping” inside the fireball of expanding matter very easy. There are indirect evidences of $\rho$ modification as well: and this is about all.

There are several different ways how one can approach the problem of hadron modification theoretically, such as: (i) chiral perturbation theory; (ii) direct lattice calculations; (iii) rescattering corrections based on phenomenologically known scattering amplitudes; (iv) QCD sum rules; (v) effective Lagrangians; (vi) other models, including “instanton liquid”; etc. Let us discuss them subsequently.

**Chiral perturbation theory** can generally be used to describe interactions of soft pions: and at low $T$ this is the case. Classic examples include Gerber and Leutwyller expression for the quark condensate:

\[
\frac{\langle \bar{q}q(T) \rangle}{\langle \bar{q}q(0) \rangle} = 1 - \frac{T^2}{8f^2_\pi} - \frac{T^4}{384f^4_\pi} + \ldots
\]  

or the general theorem according to which $O(T^2)$ corrections to hadronic masses are absent. (In the next $O(T^4)$ order such shifts however appears.)

New result, obtained in similar way, is the low-$T$ correction to the instanton density, recently evaluated in. We will discuss it below.

**Lattice calculations** are covered in recent review. We have partly discussed those above, in connection with the softest modes, the pseudoscalars and scalars: those particles do indeed show large mass modification.

The most interesting channels from experimental point of view are vector ones, especially $\omega, \phi$, because their modification can be measured with high accuracy and also because their lifetime is comparable to that of the fireball, so they are, so to say, natural clocks. There are some preliminary studies of vector mesons, so far in the quenched approximation: from those one may conclude that they are not shifted noticeably till $T \approx 0.9T_c$. However, it remains unknown what happen with vector mesons in the “mixed phase”, where hadronic system produced in nuclear collisions spends most of its time.

Lattice people has invented another way of looking at hadronic modes, measuring hadronic correlation functions in spatial rather than temporal direction. They have some poles, known as “screening masses”. Those exist also well above $T_c$, and should not be confused with modification of hadrons discussed above. In Fig.12 we show a

*Propagation in time and space directions are related to electric and magnetic forces, which are
sample of lattice measurements compiled by Gocksch: at large T one can see that these masses tend to $2\pi T$ for mesons and $3\pi T$ for baryons. We will return to discussion of the screening masses below.

**Rescattering corrections based on phenomenologically known scattering amplitudes:** such way of calculation is quite traditional. For hadronic gas it was explored in papers where for $\pi\pi$ and many other channels the well known phase shifts can be explored. What is important, in this way one gets not just the mass or width shift, but actually modification of the whole dispersion curve $\omega(k)$ for excitations. It may be quite non-trivial: for example, pions (and other pseudoscalar mesons) are protected at small momenta by the Goldstone theorem: they do not interact much. However, at larger momenta it is no longer so, and one may expect much larger modifications.

In general, such approach leads to relatively small “collective potentials”, of the order of nuclear potential in nuclear matter. As a sample of the results, let me show the calculated optical potential (both real and imaginary parts) in Fig.13 for omega meson, versus its momentum $p$. This modification should be directly observable in the dilepton channel at RHIC by PHENIX detector, which has sufficient mass resolution.

**QCD sum rules at finite temperatures** Basically, this approach relates hadronic masses with the quark condensate: and as it is dropping near $T_c$, one expects hadronic masses to decrease as well. In generally, it seems to be very reasonable idea. However, in practice its implementations has met some problems.

First of all, the original papers have confused technical issues (see discussion in). A fictitious “T-dependent OPE coefficients” were introduced (even for unit operators), while many relevant operators (those which are not Lorentz scalars) were omitted. Later on, those points were corrected, see e.g.

Another general problem, undermining predictive power of QCD sum rules is very small (and not quite understood) region of validity. OPE provide the correlators at small distances only, to start with, and it is well known that in certain channels it does not work at all. Those are scalar and pseudoscalar channels, the nucleon one and others where “direct instantons” can contribute.

By the phase transition point $T \approx 150MeV$ the $\rho$ meson mass was found to be shifted by about 10% down, while the $\omega$ mass is predicted to remain the same. (This contrasts my estimates based on scattering amplitudes, which found similar shifts for both $\rho, \omega$: future dilepton experiments hopefully can tell the difference.) Asakawa and Ko has calculated the shift of $\phi$ mass, and again by the phase transition point they have predicted it to become reduced by about 10%. If so, experimentalist clearly have possibility to fond a “second peak” in dilepton spectra.

Let me also mention that in some cases one can prove exact relations, which should be obeyed by spectral densities at any $T$. Those are for examples analogs of

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affected by QGP in a different way. Electric ones are strongly screened in the lowest order, while magnetic ones are probably screened non-perturbatively. Also confinement effects work differently: for paths going in space directions there is no “deconfinement” till any $T$. 

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Weinberg sum rules, for vector minus axial correlators, derived in 44.

**Effective Lagrangians** are well known tools, and respectively there are many papers on the subject. The simplest proposal is Brown-Rho scaling, according to which for all particles:

\[
\frac{m(T)}{m(0)} = \left[\frac{\langle \bar{q}q(T) \rangle}{\langle \bar{q}q(0) \rangle}\right]^{1/3}
\] (8)

Next, there are extensions of sigma-model. Recent paper by Pisarski 51 is based on gauged version of it, so one can calculated shifts of \(\rho\) and \(A_1\). His predictions for \(M_\rho(T)\) are as follows: it first starts decreasing (in the chiral limit, proportional to \(T^4\), of course), but then grow again, reaching at \(T_c \sim 962\, MeV\). However, \(M_{A_1}\) does exactly the opposite, it grows and then decreases (of course, meeting the rho mass at \(T_c\)). The width of the thermal \(\rho - a_1\) peak is estimated to be about \(200 - 250\, MeV\) at this point. So, at low \(T\) the behaviour resembles simple repulsion of two levels which are being mixed: from that perspective one may think that effect of all unaccounted states (e.g. of \(\rho'(1600)\) on \(A_1\)) will be to push them downward.

All applications of effective theories have one basic problem: by using them one specifically assumes that all coupling constants (as well as ultraviolet cutoffs etc) to be T-independent. (Otherwise, there predictive power is lost.) But why should it be the case?

**Microscopic models** try to explain how hadrons are created, and thus can provide some insight into T-dependence of such parameters. For example, the “instanton liquid” model does generate NJL-type interactions, and we know that those have strength proportional to the instanton density, and cutoffs related to instantons sizes. As we will discuss below, we have some information about their T-dependence.

### 3.4. Critical fluctuations

Another general idea, discussed in many papers, is to try to find large-amplitude fluctuations in data, similar to critical fluctuations known in many other physical systems. Let me skip discussion of fluctuation in hadronic reactions (including the pp ones) as too vast subject, and only comment on specific proposal related to the QCD phase transition.

Evaluation of “nucleation rate” of hadronic bubbles in supercooled plasma was discussed by Czernai and Kapusta 57. Their main assumption is that the transition is of the first order and the main result is that supercooling cannot be too strong, so that the system returns back to the mixed phase as determined by Maxwell construction. At the same time, supercooling \(\Delta T\) seems to be large enough, more than 5 MeV or so in which lattice data seem to be safely confined: it means that this work would not be affected if the transition happen to be actually a rapid crossover. The major problem of this work (discussed in fact by its authors themselves) is a relative smallness of nucleation rate compared to realistic lifetime of the system.
Related discussion of nucleation, this time for *overheated* hadronic gas going into the plasma phase, was done in. Again, classic thermal excitation formulae give too small rate, suggesting probability for central AuAu AGS collisions to create QGP to be of the order of 1%. However, in real liquid-to-gas transitions we know that some small inhomogeneous perturbations (rough surface, ions or other dirt, etc) actually dominate nucleation, and we have to find their analog in heavy ion collisions. Work on such “seeds” is in progress and that now they hope to get much larger probability to produce QGP.

Still it is natural to imagine, that if we are somewhere in the mixed phase (which is certainly the case at AGS), not 100% of the events follow the same trajectory on the phase diagram. How can we separate those? I think a possibility is to think about “softness” of EOS and related flow (which is determined for non-central collisions on event-per-event basis, more or less). If the event with overheated mesonic gas is found, which suppose to have very different and hard EOS, therefore the flow might be much stronger.

The idea of *disoriented chiral condensate* (DCC) has created much theoretical work. In short, it suggests that the newly formed bubble of hadronic matter imbedded in QGP does not know the “politically correct” direction of the quark condensate in the isospin space (that is prescribe by relatively small quark masses) and can have random orientation instead. A consequence is large fluctuations in neutral-to-charged pion ratio, especially in the bin of small $p_t$.

The major proposal was put forward by Rajagopal and Wilczek who has shown that if cooling is very rapid (“quench”) one has instabilities of lowest modes, which may grow significantly. (That was indeed observed in numerical studies mentioned.) However, it remains completely unclear whether in fact such a quenched scenario can be the case in heavy ion collisions. In fact, very large jump in entropy and energy density at $T_c$ force the system to stay very long time near $T_c$. For example, the duration of the “mixed phase” is expected to be 20-30 fm/c at RHIC, to be compared to instability increment $1/m_\sigma \sim 0.3 fm$.

My own ideas about DCC are related with the “practical” U(1) restoration discussed above. Large fluctuations in $\pi, \sigma$ directions should exist right above $T_c$: but the same should be true for their U(1) partners, $\delta, \eta'$. Unfortunately, unlike vectors those excitations do not decay into something which is possible to see directly. However, if they are trapped inside the DCC bubble by their large masses outside, they may for example significantly enhance the bubble lifetime. Much more work is needed in order to figure out whether we have chances to see this phenomena.

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*Vischer and Kapusta, private communication.
†And even the FNAL dedicated experiment, lead by Bjorken himself.
‡In fact, it is not due to pion and sigma fields, of course, and was not included in simulations based on sigma model.
3.5. **Instantons at finite temperatures.**

We have claimed in chapter 2 that instantons dominate hadronic physics in the vacuum. If so, they should also produce important effects at non-zero temperatures, below and even above $T_c$. Unfortunately, this subject was not yet studied in sufficient details, and only first steps have been made, to be reviewed in this subsection. But before we do so, some technical points are needed.

First of all, a finite temperature is introduced in a remarkably simple way (invented by Matsubara long ago) in Euclidean formulation of quantum field theories: just “time” direction simply become finite, with the length $1/T$. Second, it is quite straightforward to generalize the instanton solution of Yang-Mills equations to this case: one should simply look for a periodic solution, or a periodic set of instantons. We would not show the corresponding formulae here, and only comment about the limiting cases.

Naturally, at small temperatures, when the box size is very large, it leads to relatively small deformation of instantons, compared to its original 4-dimensionally symmetric form at $T=0$. At high $T$, on the contrary, all traces of space-time symmetry are gone. At small spacial distances the solution develops a universal ($\rho$-independent) “dion” field, with static electric and magnetic fields.

It is useful to remind here, that in the rest of this chapter we will consider mostly the region close to the QCD phase transition. Therefore, the relevant box size is supposed to be about $1/T_c \approx 1.3 fm$, roughly 4 times the mean instanton radius, $\rho = 1/3 fm$. So, the instantons themselves can be well fitted into the box, without too much deformation.

How the *instanton density* depends on the temperature? Clearly, one should know that in order to understand the role they play in his case. At large $T$ it is well known\(^\text{22,23}\) that only the small-size instantons such that $\rho < 1/T$ can survive, because essentially of the Debye screening of their field. It was argued in ref.\(^\text{35}\) that such suppression of instantons should only exist above $T_c$, because it is essentially a Debye screening.

At small $T$ was worked out only recently, in\(^\text{36}\) using the PCAC methods. The result (for two massless flavors) is expressed via vacuum expectation values of two complicated 4-fermion operators, which have calculable $T$ dependence, namely $[1 - \frac{T^2}{6F_{\pi}^2}]$ and $[1 + \frac{T^2}{6F_{\pi}^2}]$ (our pion decay constant is $F_{\pi} = 93 MeV$). To start with, this dependence is weaker than for the condensate squared ($1 - \frac{T^2}{4F_{\pi}^2}$), and available estimates of the vacuum average values even suggest that it may even cancel out, when the two terms are added.

The relevant lattice data are very very crude so far, the best\(^\text{36}\) are shown in Fig.14. One can see that both conclusions seem to be justified, in particular suppression seems to agree with Pisarski-Yaffe formula, provided one substitutes $T^2 \to (T^2 - T_c^2)$. Also, in ref.\(^\text{36}\) there are evidences that instanton size is constant below $T_c$, but it decreases above it.

Now, how the instanton *interaction* changed with temperatures? This is the topic discussed in details in\(^\text{41}\) The most drastic changes happen to be with quark propaga-
tion. At high $T$, the corresponding zero modes look approximately as

$$\psi_0(\tau, r) = \sin(\pi T \tau) / \cosh(\pi T r)$$

where $\tau, r$ are distance from the center in time and space direction. Note a crucial difference between the dependence on time and space distance: oscillations in time versus exponential decay in space. The latter are due to the famous fact: the lowest Matsubara frequency $\pi T$ for fermions are non-zero, due to anti-periodic boundary conditions.

In our discussion above, we have compared ensemble of instantons with some “liquid” made of atoms, with quarks playing a role similar to electrons. Using this language further, one may say that our “atoms” becomes more and more anisotropic, as the temperature grows $^\ast$. As we will see below, such deformation will radically change properties of their ensemble.

The main phenomenon in this region is a strong “pairing” of instantons, leading to splitting of the instanton liquid into a set of $\bar{\Pi}$ molecules. The first (strongly simplified) discussion of chiral restoration transition at this angle was made in $^3$. New finding is strong and rapid “polarization” of these molecules in the critical region. We have already discussed the main anisotropy of the interaction, which comes from the quark-induced interaction. Consider an instanton at the origin, and an anti-instanton with a center placed at distance $\tau, r$ is time and space directions. If they form an isolated system (a “molecule”) and we are discussing the theory with $N_f$ type of massless quarks $^\ast$ then the fermionic determinant should be proportional to

$$\det D \sim |\sin(\pi T \tau) / \cosh(\pi T r)|^{2N_f}$$

because a pair of each quarks should travel from one to another. (We have used here an approximate form of the zero mode considered above.)

Note that the point $r = 0, \tau = 1/(2T)$ is a strongly peaked maximum of this function. It corresponds to “polarization” of the molecule in time direction, and a particular position in time direction, for which both centers are at the opposite sides of the torus.

In order to see how important is this configuration for a single molecule, we made a simulation (of course, with realistic masses and more accurate expressions). The results demonstrate that the degree of polarization rapidly grows in the vicinity of $T=150$ MeV, or exactly at the point of chiral phase transition.

Roughly speaking, the critical temperature is then defined as the size of the Matsubara box, such that an “molecule” can be nicely fitted in, in the time direction. In more strict sense, one can say that at the transitory region $T \sim T_c$ the “instanton liquid” is changed into a kind of “liquid crystal” of nematic type. Instantons are being

$^\ast$ That is similar to what happens with ordinary atoms in very strong magnetic field (e.g., on a pulsars), in which the Larmore radius is smaller than the Bohr one.

$^\ast$ The QCD can be considered as a case between $N_f = 2$ and $N_f = 3$, something like $2.5$. 
“married” into closed pairs, therefore they stop communicating with each other, which in turn leads to the disappearance of their common ground, the quark condensate.

Let me add a comment on the physical meaning of the polarization phenomenon in a less technical language. The $I\bar{I}$ molecules are a virtual (or failed) tunneling event, in which the gauge fields penetrate into a new classical vacuum only for a short period of time, and then return back. For that reason, they do not contribute to the quark condensate and other related quantities. “Polarization” of the $I\bar{I}$ molecules at $T \sim T_c$ means that at such temperatures the tunneling is concentrated in the vicinity of the same spatial point.

Before we consider this problem more quantitatively, let us make one more digression. Even if the contributions of “molecules” in the QCD vacuum and the phase transition is not as large as I think it is, there is an external parameter that can increase their role. The way of doing this is to increase the number of light flavors $N_f$. As the fermionic determinant is raised to a higher power $N_f$, the role of correlations induced by the determinant certainly increases. Thus, one may anticipate larger role of molecule-type correlations even at zero $T$, and smaller $T_c$ in this case. Lattice data do indeed suggest decrease of $T_c$ for larger $N_f$. Furthermore, there are (so far not very convincing) data about existence of some critical number of flavors $N_{f,cr}$ above which chiral symmetry breaking and confinement are absent even in the ground state. Whether it is indeed so, and whether this new phase of QCD-like theories can be described by the instanton ensemble dominated by molecules, still remains to be seen.

3.6. Instanton-induced interactions and the equation of state

The results discussed in the preceding section has significant impact on physics around the phase transition point $T_c$ and above it: if instantons do not disappear there, they generate non-prturbative forces. Furthermore, “molecules” generate forces quite different from those for random ensemble.

The Lagrangian for these new interactions was considered in. To model the “mixed phase”, a schematic “cocktail model” was used, containing both random component and some fraction of molecules $f = 2N_{molecules}/N_{all}$. We have first found that $\langle \bar{q}q \rangle$, depends on $f$ in a way similar to its $T$-dependence, measured on the lattice: it changes little first, and then rapidly vanishes at $f \to 1$. Different correlation functions depend on $f$ quite differently. For example, for $\pi, \rho$ channels one finds remarkable stability for $f = 0 - 0.8$, with subsequent strong drop toward $f = 1$. At the last point complete chiral symmetry gets restored, so the pion correlator coincides with its scalar partners $\sigma, \delta$.

In Fig.12 we show a sample of results for “screening” masses for pion, rho, nucleon, $a_1$ axial meson, and Delta. One may see that although the temperature was kept to be $T = T_c = 150MeV$, for all channels except pion one the high-$T$ limits $2\pi T$ and $3\pi T$, corresponding to lowest Matsubara frequencies, are actually reached. Similar calculations for interacting ensemble (where the fraction $f$ is determined by the statistical sum itself), and for “real” masses are now in progress.
Finally, let us return to a very difficult problem (mentioned already at the very beginning of the Introduction), that of non-perturbative vacuum energy density. Recall that, in terms of (Minkowski) field strengths, it is

\[ \epsilon = \frac{1}{2}(E^2 + B^2) + g^2 \left( \frac{11}{3} N_c - \frac{2}{3} N_f \right) \frac{128 \pi^2}{N_c} (E^2 - B^2) \quad (11) \]

The first (Maxwellian) term is classical, and the second is “anomalous" due to quantum corrections.

This expression can be compared to a bag-type expression \( \epsilon_{\text{vac}} = \epsilon_{\text{perturbative}} - B \). Perturbative energy density is related with both classical and anomalous terms, it is badly divergent but those infinite parts are just additive constant which can be subtracted. (In other words, we put “perturbative vacuum energy" to be zero by convention.) Instantons have imaginary E and real H which cancel each other in the classical term, but the second one contributes. This is the “vacuum energy" shift due to tunneling we mentioned in the Introduction.

At high \( T >> T_c \) the perturbative part gets its Stephan-Boltzmann \( \epsilon, p \sim T^4 \) contribution of the QGP (times the perturbative corrections). The nonperturbative phenomena also become T-dependent. In particular, for the “polarized” molecules one finds \( E^2 = -0.8B^2 \). Thus, the first classical term works, producing positive energy, which is actually large, of the order of 1 GeV/fm^3!

The issue is what happens in the vicinity of \( T_c \). On general grounds one can see, that certain scenarios are impossible. For example, if all instantons would disappear instantly, at \( T = T_c \), then thermal pressure of QGP should be sufficiently large to compensate that loss, because \( p(T) \) cannot be a decreasing function, \( s(T) = dp/dT > 0 \). Since for QCD with dynamical quarks the critical temperature is rather low, \( T_c \approx 150 MeV \), this condition is not actually fulfilled. Thus, the B term cannot instantly disappear!

As it was discussed in the preceeding sections, instantons do not disappear indeed, and generate important effects even above \( T_c \). How exactly it happens remains unknown. In reference a schematic model was developed, which provide some example, and also show whether the bag-type model makes any sense. In this model the instanton ensemble is described as a mixture of a molecular and a random component. The partition function for the two components are assumed to be

\[ dZ_m = C^2 d\rho_1 d\rho_2 d^4 R dU (\rho_1 \rho_2)^{b-5} \exp \left[ -\kappa(\rho_1^2 + \rho_2)^2 (\bar{\rho}_a^2 n_a + 2 \bar{\rho}_m^2 n_m) \right] \langle (T \bar{T}^\dagger) N_f \rangle \quad (12) \]

for the molecular component and

\[ dZ_a = 2C d\rho b^{-5} \exp \left[ -\kappa \rho^2 (\bar{\rho}_a^2 n_a + 2 \bar{\rho}_m^2 n_m) \right] \langle TT^\dagger \rangle N_f \quad (13) \]

for the random component. Here, \( n_a, n_m \) denote the densities of the random and the molecular components, \( \bar{\rho}_a^2, \bar{\rho}_m^2 \) are the average square radii of instantons in the two components, \( C \) is the normalization of the single instanton density, and \( b = \frac{11}{3} N_c - \frac{2}{3} N_f \) is the coefficient of the Gell-Mann-Low function.
The model uses a simplified gluonic interaction corresponding to an average repulsion \( \langle S_{int} \rangle = \kappa \rho_1^2 \rho_2^2 \) parameterized in terms of a single dimensionless constant \( \kappa \). The fermion determinant for the random component is approximated by the average \( \langle TT^\dagger \rangle \) of the overlap matrix element \( T_{ij} \) averaged over all positions and orientations. For the molecular component, on the other hand, the overlap matrix element is first raised to the \( N_f \) power and then it is averaged over all positions, whereas the relative orientation is kept fixed.

Thus, as one can see, the only element of the model depending on the temperature \( T \) is the quark-induced interaction. Remarkably enough, it is sufficient to generate the chiral phase transition, at about the right \( T \). It happens as follows: the average value for the quark determinant gradually decreases with temperature for the random component, whereas the determinant for the molecular component first increases (at \( T \sim T_c \)) and eventually, at larger \( T \), starts to decrease.

In Fig.16 we show a sample of results\(^3\) for the resulting thermodynamical quantities, and one can see that a significant portion of the jumps at \( T_c \) is due to instanton contributions.

4. Formation and equilibration of quark-gluon plasma

4.1. Main predictions for RHIC energies

Already the first papers where search and signals for quark-gluon plasma were suggested\(^6\) has actually addressed the issue of parton thermalization at high energies\(^\dagger\), and the main qualitative features of what later became known as “the hot gluon scenario” were proposed.

In particular, it was already recognised that going from low (AGS/SPS) energies to RHIC ones one enter new domain, in which soft hadronic physics play smaller role, while processes, which involve partons with momenta \( p \sim 1-3 \text{ GeV} \) (known also as ‘mini-jets’) are in fact dominant. Later those were studied in details\(^6\)-\(^7\) for pp collisions, and extrapolations to nuclear collisions were attempted. An additional complication compared to the pp case is that in heavy ion collisions they can no longer be considered as isolated rare events, but a part of a complicated “parton cascades”.

To make a benchmark, let us recall what was called a standard scenario\(^\ddagger\), which is simply based on Bjorken’s guess about the equilibration time \( \tau_0 = 1 \text{ fm} \). As one knows from pp,pA data the rapidity density of secondaries, one can extrapolate to nuclear collisions. Another guess is multiplicity extrapolation: we use for central AA collisions

\[
\frac{dN_{AA}}{dy} = A^\alpha 0.8 \ln E_{cm}
\]

Let me remind that at that time it was assumed that CERN ISR could become a major facility for high energy heavy ion studies: unfortunately those were later cut off by the decision of CERN leaders to destroy it.

\(\ddagger\) It was considered to be standard for about a decade, in 80’s.
with $\alpha = 1.1$. The entropy conservation leads then to the following initial entropy density:

$$s_i = \frac{3.6dN/dy}{\pi R_A^2 \tau_0}$$  \hfill (15)

(where 3.6 comes from the entropy/number density ratio for the pion gas at breakup) and conclude that in this scenario for central collisions at RHIC (Au Au $\sqrt{s} = 200$ GeV*A) and LHC (PbPb $\sqrt{s} = 6300$ GeV*A) the initial temperatures $T_i \approx 240; 290$ MeV at RHIC and LHC.

Moving forward, let us try to estimate the kinetic equilibration time, using partonic kinetics. The relevant cross sections in the lowest QCD order are known to be

$$\frac{d\sigma}{dt} = \frac{\pi \alpha_s^2}{s^2} M^2$$  \hfill (16)

$$M_{gg\rightarrow gg}^2 = \frac{9}{2} (3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}), \ M_{gg\rightarrow \bar{q}q}^2 = \frac{1}{6} \left( \frac{(u^2 + t^2)^2}{u^2 t^2} - \frac{3}{8} \frac{u^2 + t^2}{s^2} \right)$$  \hfill (17)

$$M_{gg\rightarrow g\bar{g}}^2 = -\frac{4}{9} \left( \frac{u^2 + s^2}{us} + \frac{u^2 + s^2}{t^2} \right), \ M_{q_1 q_2\rightarrow q_1 q_2}^2 = \frac{4}{9} \frac{s^2 + u^2}{t^2}$$  \hfill (18)

where the subscripts in the last formula mean that two quarks are of different kind, so the cross diagram is absent. The last expression also holds for $q_1\bar{q}_2$ scattering.

Large angle cross sections\$ are very different: at $90^0$ the $M^2$ for these 4 processes are related as

$$gg/gg \rightarrow \bar{q}q/qq/g\bar{g}/qq = 30.4/0.14/5.4/2.2$$  \hfill (19)

so the gg scattering\¶ significantly exceeds other processes, especially quark production. Thus one should expect a two-stage equilibration, first of gluons with noticeably higher $T_i$, and later of quarks, with smaller $T_i$.

The "equilibration time" can be defined in many ways, let it be the time during which each parton has been in average scattered once\∥. That leads to the following "selfconsistency equation"\footnote{Note that it is essentially the same condition as traditionally used for defining final (or breakup) parameters: the system size is comparable to constituent mean free path.}

$$\tau_0 = \frac{3.6dN/dy}{\pi R_A^2} \frac{1}{7.0 T_i^3} \approx \tau_g = \frac{1}{\text{const} T_i}$$  \hfill (20)

\footnote{Small angle ones are larger, but they contribute less to momentum equilibration.}

\footnote{In the gg case an extra factor 1/2 can be used, reflecting twice smaller t range: one should not take into account the same final state twice.}
(The factor 7.0 comes from the entropy of the gluonic plasma at \(T = T_i\), and the constant in the r.h.s. should be taken from the scattering rates discussed above.) Assuming the same total multiplicity as above, one gets the initial gluonic temperatures and the equilibration times
\[
T_g \approx 500\, \text{MeV} \quad \tau_g \approx 0.3\, \text{fm} \quad \text{(RHIC)}
\]
\[
T_g \approx 660\, \text{MeV} \quad \tau_g \approx 0.25\, \text{fm} \quad \text{(LHC)}
\]
(Here the effect of small-angle scattering is also included.) Note that these predictions are significantly different from the “standard scenario” mentioned above.

Observable consequences of this “hot glue” scenario include charm production. It was proposed as signature for high-T QGP in\(^6\). The mechanism is \(gg \rightarrow \bar{c}c\) reaction, and its implementations for nuclear collisions were studied later in great details. Direct (parton model) charm production results in about 1 \(\bar{c}c\)/event (RHIC), while ”thermal” production leads to \(\sim 10^{-2}, 1, 10\, \bar{c}c\)/event at \(T_i = 300, 400, 550\, \text{MeV}\). Therefore, one can expect significant increase of charm production\(\star\) compared to the scaled pp estimates.

Spectra of photons and dileptons produced in this scenario should also be significantly different from those in the ”standard” one: during the ”transitory time period” \((\tau_g < \tau < \tau_q)\) one has smaller number of quarks, but those are hotter: the \(gg \rightarrow \bar{q}q\) process is mainly active at small angles, so quarks simply have the temperature of gluons. As most photons and dileptons originate from the tails of the distribution functions, it is important that their relaxation to the equilibrium ones happens from above.

Our discussion above assumed the amount of entropy: and the treatment was made so to say backward in time. Of course, one should be able to evaluate how much entropy is produced directly, considering parton kinetics. Although qualitatively all approaches agree with the “hot glue scenario” outlined, the detailed predictions for the entropy produced are very different.

One approach, based on binary scattering processes at the first impact, is the HIJING model\(^7\) which was formulated as an event generator and therefore widely used. It was supplemented by consideration of radiative “energy losses” in medium, and in this sense contain some multi-gluon interaction in the small-angle (leading log) approximation. However, rescatterings and multi-parton processes are not yet included in this model.

The quantitative treatment of subsequent rescattering was attempted in the “parton cascade model” (PCM) by Geiger and Mueller\(^8\) which aims to trace the partonic system evolution all the way, from the structure functions of colliding hadrons to final

\(\star\)Considerable confusion has been created in literature in relation with the so called “intrinsic charm excitation”. In particular, K.Geiger\(^9\) has predicted it to be the dominant mechanism of charm production, dominating by a significant factor over the thermal production. However, I think those calculations strongly overestimate the yield because most of the gluons are not virtual enough to resolve charmed pairs. Further work is needed to get quantitative results.
hadronization of emerging mini-jets. This model hives the highest numbers for produced entropy, because this model includes collisions with very soft gluons, \( x \sim 0.001 \), which also happens \textit{prior} to real first impact.

It is based on \textit{sequential branching of virtual} partons (e.g. \( g^* \to g^*g^* \), where star means non-zero invariant mass), described by the Lipatov-Altarelli-Parisi (LAP) branching functions. However, this approach is limited in general, because a virtual gluon is not a \textit{gauge invariant concept}. In fact, LAP approach can be used \textit{only} for soft gluons and for radiation at \textit{small-angle}: the term which is picked up is the leading power of the log.

However, the soft radiation is exactly the process which is strongly affected by the plasma screening effects: and when those are taken into account, the log is not large enough to keep the leading term only. Moreover, as we will show below, the leading phenomenon is not sequential radiation but simultaneous production of several gluons with comparable momenta and at large angles.

All perturbative approaches mentioned share the same general uncertainty, related to the so far uncertain \textit{infrared cut off} \( p_0 \), separating soft and perturbative physics. So far, none of them has not yet been able to derive its value theoretically, or even locate the specific phenomena responsible for it. Furthermore, it was phenomenologically determined for pp case, but for nuclear collisions it is expected to be different.

\subsection*{4.2. The multi-gluon processes}

Ref.\textsuperscript{92} have introduced \textit{the multi-parton processes} as a substitute for sequential branching. The specific problem addressed in that work is gluon \textit{chemical} equilibration, while \textit{kinetic} equilibration assumed. In this work the averaged matrix elements for the multigluon processes was used, which are known for the sum of \textit{tree} diagrams. They are defined \textit{on mass shell} and therefore they are manifestly gauge invariant. Furthermore, one can separate the 'short-time' processes (for which a cascade approximation is justified) from the 'long-time' ones (such as interaction with collective soft modes) by cutting off certain kinematical regions, in which none of the kinematical invariants is small. The role of these processes at first impact was discussed in the recent work\textsuperscript{93} (see below).

In this section we present some details about the multi-gluon QCD processes on which the previous estimate was based. This discussion is limited to gluons only, due to the following two reasons. First, the gluons do dominate the nucleon structure functions at small \( x \), as well as scattering or production cross sections we are dealing with. The second reason: matrix elements for higher order gluon multiplication processes \( gg \to (n - 2)g \) were clarified in the last few years only and similar general expressions for quarks are still unknown.

The so called “Parke-Taylor formula”\textsuperscript{82} to be used below is proven to be exact for \( n \)-gluon processes in the \textit{maximum helicity violation} case. The squared matrix elements
is

$$|M_{n}^{PT}|^2 = g_s^{2n-4} \frac{N_c^{n-2}}{N_c^2 - 1} \sum_{i>j} s_{ij}^4 \sum_{P} \frac{1}{s_{12}s_{23}...s_{n1}}$$  \hspace{1cm} (21)

In the above $s_{ij} = (p_i + p_j)^2$, the summation P is over the $(n-1)!/2$ non-cyclic permutation of $(1...n)$.

Unfortunately, the exact result for other chiral amplitudes remains unknown. However, assuming that they are of the same magnitude as the “Parke-Taylor” one, one gets some estimate for the n-gluon matrix element. This was proposed by Kunszt and Stirling\textsuperscript{83} who add the following factor in front of the “Parke-Taylor” formula

$$|M_{n}^{KS}|^2 = KS(n)|M_{n}^{PT}|^2, \quad \text{with} \quad KS(n) = \frac{2^n - 2(n + 1)}{n(n - 1)}$$  \hspace{1cm} (22)

It agrees with the exact results for $n = 4$ and $n = 5$, while for higher orders a number of authors have checked this expression up to $n = 10$ using the Monte-Carlo generators, evaluating diagrams numerically. They have found that Eq.(22) does a very reasonable job.

Evaluation of the total cross section is a matter of integration over the many-body phase space, which is difficult to do analytically. For symmetry, we introduce the universal cut off parameter $s_0$, in the following way: all binary invariants are subject to a condition

$$s_{ij} = (p_i + p_j)^2 \geq s_0$$  \hspace{1cm} (23)

including all incoming and outgoing particles. This condition corresponds to production of the resolved (=nonoverlapping) jets.

The exclusive cross sections should have the general form

$$\sigma_n(s) = \frac{1}{s_0} f_n\left(s_0 \frac{s}{s_0}\right)$$  \hspace{1cm} (24)

The functions $f_n$ are quite complex. It is analytically known in the case of $n = 4$

$$f_4(\epsilon) = \frac{9\pi\alpha_s^2}{2}\left[1 + \frac{17\epsilon}{12} - 3\epsilon^2 + \frac{\epsilon^3}{2} - \frac{\epsilon^4}{3} - \frac{\epsilon}{1 - \epsilon} - \epsilon \log \frac{1 - \epsilon}{\epsilon}\right].$$  \hspace{1cm} (25)

In the limit of $\epsilon = s_0/s \to 0$, it tends to a constant. For $n > 4$, the functions $f_n$ are the polynomials of the $\log(s/s_0)$, since each binary invariant happen to be present in denominator only once. The leading term of the total cross section should have the double log behavior $f_n(\epsilon) \sim [\log^2(\epsilon)]^{n-4}$. This can be best shown in the soft-gluon case, where the Parke-Taylor matrix element can be factorized as

$$|M_{n}^{PT}|^2 \approx (n - 1)g_s^{2n}N_c \frac{1}{p_n^2(1 - \cos \theta)}|M_{n-1}^{PT}|^2,$$  \hspace{1cm} (26)
where $p_n$ is the three momentum of the $n$-th gluon, $\theta$ is its orientation with respect to any one among the $n-1$ gluons. The total cross section then has the form

$$\sigma_n(\sqrt{s}) \approx \frac{n-1}{n-2} \frac{\alpha_s N_c}{4\pi} \int_{s_0}^{s-s_0} \frac{dM^2}{s-M^2} \sigma_{n-1}(M) \int \frac{d\cos \theta}{1-\cos \theta}$$ \hspace{1cm} (27)$$

When one proceeds iteratively, it is still true that each next particle gives an extra double log, so the answer should look as

$$\sigma_{gg \rightarrow (n-2)g} \approx \sigma_{gg \rightarrow gg} \left[ \alpha_s N_c \log^2(s/s_0) \right]^{n-4}$$ \hspace{1cm} (28)$$

The coefficient $C_n$ is however non-trivial to estimate. Under a series of approximations, it was shown to converge to

$$C_n \rightarrow \frac{1}{4\pi \sqrt{3}}$$ \hspace{1cm} (29)$$

Note that there is no factorial suppression of large $n$: it originated from constructive interference of $n!$ “strings” in the squared matrix element. Therefore, instead of exponential series, one has in fact a geometric one. This asymptotic behavior signals a warning to the eligibility of perturbative theory since squared log can overcome the coupling constant and the total cross section $\sigma_{tot} = \sum_n \sigma_n$ would diverge. We will return to this problem in the section, devoted to SSC energies.

The energy dependence of the exclusive cross sections was obtained numerically, and it can be parameterized by the following analytic expressions

$$S_0 \sigma_n(M) [\text{GeV}^2 \text{mb}] = (\alpha_s/\alpha_{s0})^{n-2} 10^{a_n + b_n (\log_{10} s^{\frac{M}{s_0}} )^c}. \hspace{1cm} (30)$$

The parameters are found to be

$$a_5, b_5, c_5 = 1.0175, -1.6675, -1.977 \hspace{1cm} (31)$$
$$a_6, b_6, c_6 = 2.1323, -3.6323, -1.688 \hspace{1cm} (32)$$
$$a_7, b_7, c_7 = 2.8426, -5.6426, -1.871 \hspace{1cm} (33)$$

One can see that large $n$ processes are more sensitive to $s/s_0$, and they also are more important for larger $s/s_0$.

The main point is that the leading log approximation is not reliable for the problems related with the minijets, because the typical $s$ is only several times larger than $s_0$; so picking only the leading $\log^{2n}(s/s_0)$ terms is not justified. Therefore, we study the parton multiplication processes considering all the kinematic regions and interference effects.

\(\dagger\dagger\) One may wander how square root of 3 can appear in expression for Feynman diagram: the answer is there are different coefficients for even and odd $n$, both without such roots: but their geometric average has it.
Now we move from cross sections to one-body (exclusive) distributions. We have found very peculiar consequences of the Parke-Taylor formula, which is significantly different from the picture one is used to in QED radiative processes. This is demonstrated in Fig. 17 where we show the transverse momentum $p_t$ and rapidity $y$ distributions of secondary gluons, for multigluon processes ($gg \rightarrow (n-2)g$). We have chosen a jet resolution $s_0/s = 0.02$.

Going from $n=4$ (elastic process) to larger $n$ one can see that the particle distribution begin to build up very rapidly at central rapidity. When $n = 5$, a soft gluon radiation is filling the gap between the two major outgoing gluons: the rapidity distribution becomes flat. This result is well known: not that it already significantly deviates from the QED case, where there exist a dip at mid-rapidity caused by destructive interference of radiation in initial and final states.

When $n$ is increase further, all the outgoing particles are piled up around $y = 0$. Its width is $O(1)$, so the angular distribution is in fact nearly isotropic. This can be traced to constructive interference of many diagrams: soft gluons are effectively emitting each other.

Let us now proceed to the $p_t$ spectrum. It is somewhat surprising to see, that for larger $n$ it becomes roughly exponential, in the large range of $p_t$. Moreover, the slope is almost universal for all $n$ processes, about 1% of the total energy. Thus, something like thermal $p_t$ distribution of gluons is produced already in one multigluon scattering event!

These distribution can be compared with the leading log or 'soft gluon approximations’, predicting flat rapidity distribution $d\omega/\omega = dy$ and power like $p_t$ spectra $dp_t^2/p_t^2$. Clearly, such approximation is qualitatively wrong for the kinematical region under investigation.

The inclusive cross section for $n$-jet production from hadronic collision can be calculated from convoluting the $n$-gluon matrix element with the luminosity function

$$E_1E_2...E_{n-2}\frac{d\sigma_n}{d^3p_1d^3p_2...d^3_{n-2}} = \int dxLum(x, s_0)\frac{1}{2x^2s}|M_n|^2(2\pi)^4\delta^4(\sqrt{s} - \sum_{i=1}^{n-2} p_i) \quad (34)$$

which is the probability that the initial two partons carries $x$ fraction of the total invariant mass and depends on the gluon structure function as

$$Lum(x, Q^2) = \int_{x}^{\sqrt{x}} dx_1 G(x_1, Q^2)G(x^2/x_1, Q^2)\frac{2x}{x_1} \quad (35)$$

We define a number

$$\delta \equiv \sqrt{s_0/s} \quad (36)$$

\footnote{One should not confuse this phenomenon neither with true thermalization of partons, for which their rescatterings are needed, nor with exponential $p_t$ spectrum observed in pp collisions due to soft hadronic processes.}
to be the ratio of the cutoff mass to the total (center of mass) energy and will use it for analysis later on. In this notation HIJING’s cut-off is $\delta = 0.014$.

The inclusive jet production in pp collisions was studied in multiple experiments: those can be well reproduced by gluon jets from gg-gg only. One should therefore consider exclusive multi-jet events, to test these formulae. Furthermore, formulae discussed above correspond to the tree-level diagrams, so the natural question is what higher-order corrections may do to them. Experience with binary processes suggests that even in the kinematic region where they should work, one gets the so called K-factors, changing the cross section by a factor 2 or so. For multi-gluon processes, with many kinematical variables, the radiative corrections are generally a very complicated functions of many of them, leading presumably to some sort of formfactors.

In we have used data on multijet measurement are obtained by the UA2 collaboration at CERN SPS collider energy $\sqrt{s} = 630\,\text{GeV}$. The data on 4 jet production have especially good statistics and each of the four jets are required to have $p_t$ larger than 15 GeV, it corresponds to $x_{\text{min}} \approx 0.05$. The first level of our investigation was studies of the transverse momentum distributions for different jet number. It actually agree with the universal exponent found in the previous section. The slope furthermore is indeed almost the same for different n’s. Thus, whatever $K$-factor may be, it is presumably not a strong function of $p_t$, but more or less constant in the whole kinematical domain. The second level is the absolute values for exclusive cross sections with different n, which is suppose to tell us what the magnitude of this K-factor might be. The authors of themselves have compared their data with the exact matrix elements calculation and with the “improved” Parke-Taylor formula in. They have reported agreements with data within an impressive 20%: so, this formula really works!

Assuming kinetic equilibrium of gluons, or momentum distribution $f(p_t) = \xi \exp(-p_t/T)$ at time $\tau_{\text{kinetic}} \sim 0.3\,\text{fm}$, one can consider gluon multiplication leading to chemical equilibration of glue, at which $\xi \to 1$. Time evolution of gluon fugacity and temperature is shown in Fig.18 for 3 scenarios, with intial values $(T, \xi) = (0.56, 0.06), (0.5, 0.25), (0.5, 0.5)$. The dashed curve is for gg into ggg only, while two others include multigluon processes in two different approximations. One may conclude from these results, that chemical equilibration of gluons proceed sufficiently rapid, and is concluded during the lifetime of QGP.

Let us now proceed to initial impact in AA collisions. As it was explained above, already after the first multi-gluon scattering the spectrum has momentum distribution of the type $f(p_t) = \xi \exp(-p_t/T)$. The slope $T$ is not very sensitive to the cut-off, for RHIC energy $T = 2\,\text{GeV}$. But the fugacity (and the total entropy) is strikingly sensitive to the cut-off.

* Note that in $\delta$ it is the total energy of collided hadrons, not partons, as for the parameter $\epsilon$ considered above.

† Note that this energy is factor 3.1 higher than the nominal RHIC one, therefore scaling by this factor down one gets “mini-jets” of about 5 GeV, which is not very far from those we consider in relation with QGP thermalization at RHIC.
It is important that in AA central collisions the situation is completely different from the pp case: partons are in a dense system of their neighbours. If anything, the lessons from finite-T QCD is that parton interaction in dense systems leads to \textit{density-dependent} screening, which makes the direct extrapolation from pp case impossible. Some guidance can probably be obtained from the equilibrium situation, for which lattice data exist. For \( T > 2 - 3T_c \) they tell us that \( M_{\text{eff}} \approx 2T \). If applied to gluon system \textit{after} equilibration, with \( T \approx 500 \text{MeV} \), one gets \( M_{\text{eff}} \approx 1 \text{GeV} \). It can probably be taken as some lower bound of the cut-off momenta.

The physical cutoff may be the screening mass of the parton system. In a non-equilibrated plasma, one expects that

\[
m^2 \approx g_s^2 \xi T^2
\]

So the cut-off should be larger for higher energy hadronic collisions, because the partonic system is denser. We have calculated the number of produced gluons using the self-consistent cut-off: it leads to initial temperature and fugacity at RHIC \( T \approx 2 \text{GeV}, \xi = 0.1 \). If so, the produced entropy is on the large side, roughly comparable to that predicted by PCM. The corresponding time scale is just time of the first collision, by uncertainty relation it is about .1 fm/c.

4.3. \textit{The LHC energies: limitations of the perturbation theory}

We have discuss the LHC case in a separate subsection, because here we seem to find a serious problem. But before we come to it, let us briefly consider the mini-jet production at LHC in general.

Kinematically, in this case one is dealing with \( x \sim 10^{-4} \), and we now know from HERA measurements that nucleon gluonic structure functions experience strong growth in this region\(^\ddagger\). In recent paper\(^9\) those were taken into account, and with \textit{binary} processes with the HIJING cutoff they have evaluated the number of mini-jets produced by binary scattering. The result suggests nearly chemically equilibrated system of gluons at the time \( \tau = 1/p_0 = 0.1 \text{fm/c}, \) with \( T \approx 1 \text{GeV} \).

Unfortunately, this calculation is completely destroyed\(^9\) if one includes the multiparton processes. Each subsequent subprocess \( 2 \rightarrow (n-2) \) lead to the estimate \textit{larger} than \((n-1)\)-st! The numbers we get are 0.35, 1.32, 3.35, 15.11, for \( n=4,5,6, \) and 7 at \( \delta = 0.00045 \), which corresponds to the HIJING cut-off \( p_0 = 2 \text{ GeV} \). The effect calculated in ref.\(^\ddagger\) is therefore just a beginning of a divergent\(^\ddagger\) geometrical series!

The reason this happens is that we are actually beyond \textit{applicability limits of the perturbation theory}. Although \( \alpha_s \ll 1 \), the powers of \( \log(s/s_0) \) has overcome it. The

\(^\ddagger\) Whether \textit{nuclear} structure function also grow, as the \textit{nucleon} does remains a matter of controversy. For heavy ions at LHC one may finally find the so called \textit{saturation}.

\(^\ddagger\) Mathematically, the series are of course limited by the applied cut off, so they are not really divergent. However, one can hardly take the scenario of rapid transformation of TeV gluons into thousand GeV ones for an answer.
only comment we may make here is that probably in so dense partonic system the cut off should be much larger. We have calculated and shown the number of gluons produced at LHC in the Table 1 examples, for few values of $p_0$. Thus, either (i) there exist some mechanism producing high enough cutoff $p_0 \sim 10\text{GeV}$ and the perturbation theory is then justified, in its reduced domain; or (ii) has to develop and apply some non-perturbative methods even for multi-GeV gluon jets.

5. Summary

The main lesson from the first part of this paper is as follows: instantons are the dominant dynamical phenomenon, as far as physics of light quarks and lowest hadrons is concerned. Random liquid and quenched lattice data give very consistent results for correlators in many channels, for various wave functions etc. Glueballs also can be qualitatively related to instantons, and one can see there direct evidences for strong classical fields and self-duality. The unquenched results for the instanton approach are under way, as well as their lattice analogs. One effect considered, the screening of the topological charge, is especially interesting, and it should be studied in much greater details.

The second part deals with various consequences of an idea, attributing the chiral phase transition in QCD to a rearrangement of the instanton liquid, going from a random phase (at low T) to a correlated phase of polarized $\Pi$ molecules at $T > T_c$. In this scenario, a significant number of instantons is present at temperatures $T = (1 - 2)T_c$, causing a variety of nonperturbative effects.

One of them is polarization of $\Pi$ molecules, at temperatures $T \simeq T_c$, related with their significant contribution to the energy density and pressure of the system near the phase transition region. The presence of $\Pi$ molecules above $T_c$ also produces quite specific interactions between light quarks, which is $U(2) \times U(2)$ symmetric. This results is in agreement with lattice simulations, in which the presence of an attractive interaction in the scalar channel (but not vector ones) has been established from an analysis of spacelike screening masses.

Finally, we have discussed a recently proposed strategy for the experimental search for the QCD phase transition. The usual view is that the QGP will reveal itself as we go to higher and higher energy density, since then the signals from QGP should outshine that of the hadronic background. We certainly have no dispute with this approach. However, it seems also possible to go down in energy from the nominal SPS one, looking for the “softest point”, at which evolution of the excited matter leads to especially long-lived fireball.

In the last chapter we have studied QGP properties, as it is expected to be at RHIC and LHC energies. Our main concern was the proper language which is needed to describe “parton cascades”. We have argued that it is the multi-gluon processes $gg \rightarrow (n-2)g$, which can be done using improved Parke-Taylor formula. First of all, we have found some new features of these processes, such as (i) piling up at mid-rapidity, and (ii) exponential $p_t$ spectrum of gluons, with nearly universal slope.
Using those processes, we have found that gluonic component of the plasma can be chemically equilibrated during its lifetime: but quarks cannot. Proceeding to the first impact in AA collisions, we have found that here the multi-gluon processes \( n,4 \) are much more important than lower-order ones. At RHIC energies we have proposed a self-consistent evaluation of initial conditions for a cascade. At LHC, the situation is more dramatic and (in contrast to what was reported earlier) perturbative predictions start to diverge, in the sense that each next process is more probable than the previous one.

6. References

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Fig. 1. Correlators for pseudoscalar (P) and vector (V) channels according to RILM (open points) and lattice results (closed points) versus distance $x$ in fm. The correlators are normalized to those corresponding to free quark propagation: therefore at small $x$ all of them tend to 1. The long-dash lines correspond to experimental data (other lines are used for fitting of lattice data).
Fig. 2. Similar to what is shown in Fig. 1, but now for N,Δ channels. Triangles are RILM, squares are lattice results. The lines are predictions by two works based on the QCD sum rules, by Belyaev and Ioffe (long-dashed) and Farrar et al (short-dashed).
Fig. 3. Point-to-point scalar, pseudoscalar and tensor glueball correlation functions (normalized to the corresponding free correlators) versus distance $\tau$, in fm. Stars, triangles and squares show results for random ensemble, that with gluon interactions, and full “unquenched” QCD interaction between instantons, respectively. Solid lines are parametrization used to extract masses and other parameters, the dashed lines are the one-instanton contribution.
Fig. 4. Equation of state for lattice simulations with 2 light quark flavors, from MILC collaboration. Upper and lower points are for energy density and pressure, respectively.
Fig. 5. Schematic phase diagram of QCD as a function of quark masses, from Columbia group.

Fig. 6. “Disconnected” and “connected” scalar susceptibilities (upper and lower points) for 3 different quark masses, measured by Bielefeld group. The peaks, marking the minimal $\sigma$ mass, correspond to $T_c$. 
Fig. 7. Equation of state, plotted as the pressure-to-energy-density ratio versus energy density. The minimum is the “softest point” of the QCD equation of state.
Fig. 8. Hydrodynamical evolution at two energies (from Hung and Shuryak) on time $t$ - longitudinal coordinate $z$ plane. Solid lines correspond to fixed temperatures, dotted ones to fixed longitudinal velocity. M and H mark the domains of mixed and hadronic phase. Fig.(b) corresponds to the long-lived fireball discussed in the text.
Fig. 9. Energy dependence of some observables, from Hung and Shuryak. Part (a) show lifetime of the mixed phase (dotted line and right scale) and 4-volume of the mixed phase (solid line and left scale). Part (b) show the height (dotted curve, left scale) and the width (solid line, left scale) of the dilepton rapidity distribution.
Fig. 10. Preliminary transverse energy spectra for $K^+$ mesons and pions, reported by E877 experiment at QM95.
Fig. 11. CERES preliminary data on dielectron mass spectrum, reported at QM95. Curves correspond to “cocktail” of known decays, they explain p-Au data but not the S-Au ones.

Fig. 12. Compilation of lattice results on “screening masses” versus temperature, from Gocksch.
Fig. 13. Real and imaginary optical potential for $\omega$ meson moving in the pion gas, versus its momentum $p$. Three curves correspond to $T=150,175,200$ MeV. Curves for other mesons look similar.

Fig. 14. Density of instantons as measured by the “cooling” method versus temperature $T$, from Chu and Schramm (see text). PCAC stands for Shuryak and Velkovsky low-$T$ limit, and P-Y for Pisarski-Yaffe high $T$ one.
Fig. 15. “Screening masses” for correlators with pion, rho, $a_1$, nucleon and delta quantum numbers in the instanton ensemble, possessing fraction $f$ of “paired” instantons. Note that at $f=1$ chiral symmetry is completely restored (e.g. $\rho, a_1$ masses are the same).
Fig. 16. Instanton density, pressure and energy density versus temperature $T$ in the schematic model described in the texts. Left and right panels are two variants of the model, showing uncertainties involved. For n panel, solid line is random component and the dashed line is the density of molecules. For p, solid line is total pressure, decomposed into quark-gluon one (dotted lines) and instanton contribution (dash-dotted one). For energy density (bottom), solid lines are total sums, and dash-dotted ones show the instanton contribution.
Fig. 17. Transverse momentum and rapidity distribution of gluons produced in multi-gluon processes with different number of participating gluons, from $n=4$ (elastic scattering) to $n=8$. 
Fig. 18. Time dependence for temperature and fugacity during “chemical equilibration” of gluons: see text for explanations.
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