Direct visualization of phase-locking of large Josephson junction arrays by surface electromagnetic waves

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Phase-locking of oscillators leads to superradiant amplification of the emission power. This is particularly important for development of THz sources, which suffer from low emission efficacy. In this work we study large Josephson junction arrays containing several thousands of Nb-based junctions. Using low-temperature scanning laser microscopy we observe that at certain bias conditions two-dimensional standing-wave patterns are formed, manifesting global synchronization of the arrays. Analysis of standing waves indicates that they are formed by surface plasmon type electromagnetic waves propagating at the electrode/substrate interface. Thus we demonstrate that surface waves provide an effective mechanism for long-range coupling and phase-locking of large junction arrays.

The creation of tunable, monochromatic, high-power and compact sources of electromagnetic (EM) waves in the 0.1-10 THz frequency range remains a serious technological challenge, colloquially known as the “THz gap” [1]. Josephson junctions (JJs) have a unique ability to generate EM radiation with tunable frequency \( f \) [1]. Josephson junctions (JJs) have a unique ability to generate EM radiation with tunable frequency \( f = \frac{2eV}{h} \), where \( h \) is the Planck constant, \( 2e \) is the charge of Cooper pairs and \( V \) is the dc-voltage across the JJs [2–7]. The record tunability range, 1-11 THz, [8] has been reported for intrinsic JJs in cuprate high-temperature superconductors (HTSC), for which the energy gap, determining the upper frequency limit, can be in excess of 20 THz [9, 10]. Emission power from a single JJ is small [7]. It can be amplified in the superradiant manner by phase-locking of many JJs [2–4, 11–14]. However, with an increasing number of JJs their synchronization becomes progressively more difficult due to a rapidly growing number of degrees of freedom.

Synchronization of large JJ arrays requires long-range interaction between JJs. Usually it is mediated by resonant cavity modes leading to formation of standing waves either inside [15–16], or outside [11–12] the JJs. With increasing array size cavity modes get damped by dissipation. For very large arrays, with sizes significantly larger than the wavelength of emitted radiation, an alternative, nonresonant mechanism of synchronization by traveling waves has been suggested and synchronization of up to 9000 JJs has been demonstrated recently [14]. Traveling waves in JJ arrays are essentially surface EM waves (SEMWs) propagating at electrode-substrate or vacuum interfaces. There is a great variety of SEMWs at metal/insulator interfaces, for a review see e.g., Ref. [17], including surface plasmons in the infrared range, which are being actively studied due to perspectives of the development of plasmonic components for optoelectronic devices [18–25]. SEMWs also exist in superconducting wires [26] and thin films [27]. Most interesting in the context of this work are leaky SEMWs [19] that facilitate emission of EM power into open space.

In this work we study Nb-based JJ arrays containing 1500 and 1660 JJs. We employ low-temperature scanning laser microscopy (LTSLM) for visualization of wave dynamics in the arrays. We observe that at certain bias voltages, corresponding to Josephson frequencies in the sub-THz range, two-dimensional standing-wave patterns appear in the arrays. Our analysis indicates that the standing waves represent interference patterns of leaky surface plasmon-type surface waves propagating in opposite directions along the electrode/substrate interface. The leakage of SEMW energy into open space facilitates both emission of EM waves and a long-range interaction between junctions in the array, which is needed for mutual phase-locking and superradiant emission.

We study arrays of serially connected Nb/Nb2Sn1−x/Nb (x~0.1) JJs. Fig. 1 shows layouts of two studied arrays, which we refer to as “linear” (a) and “meander” (b) arrays. JJs with sizes 8×8 \( \mu m^2 \) and a period of repetition of 15 \( \mu m \) are formed at the overlap between top and bottom Nb electrodes, as sketched in the inset. The linear array, Fig. 1(a), consists of five long parallel lines, containing 332 JJs each, thus yielding in total \( N_L = 1660 \) JJs. The meander array, Fig. 1(b), consists of 125 transverse strips with the length 290 \( \mu m \). The distance between strips is 40 \( \mu m \). Each strip contains 12 JJs yielding \( N_m = 1500 \) in total. The overall size of both arrays is 5 mm (from left to right in Fig. 1). More details about fabrication and characterization can be found in Refs. [28, 29]. Transport properties and emission characteristics of similar arrays can be found in Refs. [13–14] and in the Supplementary.

We use LTSLM combined with transport measure-
rature during the LTSLM measurements at the base temperature T ≃ 5 K, see Fig. 1(a). The resonant frequency, \( f_r = c/2L_4V\sqrt{\varepsilon^*} \), where c is the speed of light in vacuum, yields the effective dielectric permittivity \( \varepsilon^* \approx 6.9 \). The resonant frequency, \( f_r = c/2L_4V\sqrt{\varepsilon^*} \), where c is the speed of light in vacuum, yields the effective dielectric permittivity \( \varepsilon^* \approx 6.9 \). The resonant frequency, \( f_r = c/2L_4V\sqrt{\varepsilon^*} \), where c is the speed of light in vacuum, yields the effective dielectric permittivity \( \varepsilon^* \approx 6.9 \).

Fig. 2(b) represents LTSLM images \( \Delta U(x,y) \) of a part of the linear array at four bias currents, marked by arrows in Fig. 2(a). The horizontal scanning field, \( L_x \approx 0.91 \text{ mm} \), corresponds to \( \approx 18\% \) of the total array length. At the lowest bias point A, \( I = 2.0 \text{ mA} \) and \( V \approx 0.33 \text{ V} \), some junctions (pink and blue spots) are still in the superconducting state, due to some inhomogeneity of JJs in the array. Otherwise there is no well defined spatial variation of the array response. At a slightly higher bias point B, \( I = 2.17 \text{ mA} \) and \( V \approx 0.371 \text{ V} \), a certain alternation with maxima and minima of \( \Delta U(x,y) \) along the \( x \)-direction appears in the three middle lines. At point C, \( I = 2.37 \text{ mA} \) and \( V \approx 0.400 \text{ V} \), a clear standing-wave pattern develops in the whole array. It has an antisymmetric modulation with maxima in one line corresponding to minima in the neighbor lines. This is demonstrated in detail in Fig. 2(c), which represents averaged scans of \( \Delta U(x) \) along each of the three middle lines of the array (blue) together with fitting curves (red). An additional minor increment of the bias current to point D, \( I \approx 2.4 \text{ mA} \) and \( V \approx 0.412 \text{ V} \), leads to a visible reconstruction of the standing-wave pattern. Here it becomes almost symmetric with aligned maxima and minima in all the lines.

The increase of voltage leads to the growth of the Josephson frequency and a reduction of the wavelength of EM radiation. Indeed, such a tendency can be traced from Fig. 2(b). The Josephson frequencies at points B, C and D are: \( f(B) \approx 107.8 \text{ GHz} \), \( f(C) \approx 119.0 \text{ GHz} \) and \( f(D) \approx 119.9 \text{ GHz} \). The corresponding periods of standing waves are \( \Delta x(B) = 0.53 \text{ mm} \), \( \Delta x(C) = 0.47 \text{ mm} \), and \( \Delta x(D) = 0.46 \text{ mm} \). Since LTSLM probes only the EM amplitude, the EM wavelength is twice the period of LTSLM image, \( \lambda = 2\Delta x \). The observed decrease of \( \lambda \) with increasing \( f \) follows a simple relation \( \lambda = c/f\sqrt{\varepsilon^*} \) with \( \varepsilon^* \approx 6.9 \pm 0.2 \), consistent with the estimation above from the step voltages in the IVC.

The observation of a correlated two-dimensional standing-wave order indicates a global synchronization of the whole array. We did LTSLM scans over a broad bias range along the IVC. However, such correlated standing wave patterns were observed only in a limited bias range from slightly below the point B to slightly above the point D. This is qualitatively consistent with a narrow bias range in which significant EM wave emission occurs from such an array.

Fig. 3(a) shows the IVC of the meander array recorded during LTSLM measurements at \( T \approx 5 \text{ K} \). Similar to the linear array, the IVC of the meander array also has dis-
tinct resonant steps. Fig. 2(b) represents LTSLM images of the meander array at different bias points marked in Fig. 3(a). A: $I = 1.86 \text{ mA}$ and $V = 144 \text{ mV}$, B: $I = 1.9 \text{ mA}$ and $V = 174 \text{ mV}$, C: $I = 1.95 \text{ mA}$ and $V = 200 \text{ mV}$, D: $I = 2.18 \text{ mA}$ and $V \simeq 263 \text{ mV}$. Note that the IV curve of the meander array exhibits a hysteresis, presumably due to self-heating. Points A–C are measured at the reverse of the IV curve within the hysteretic area. LTSLM images are taken at the right end of the array with the same field of view as in Fig. 2(b), $L_x \simeq 0.91 \text{ mm}$, which encompasses 22 transverse strips.

Standing wave patterns in the horizontal direction can be seen in all images of Fig. 3(b). The periodicity $\Delta x$ is gradually decreasing with increasing voltage from A to D: in A it is about five transverse strips and in D about three. This is in a qualitative agreement with the expected decrease of the wavelength with increasing frequency. However, if we calculate the speed of EM wave propagation along the $x$-axis, $c_x = 2\Delta x f$, with $f = 2eV/hN_m$, we obtain unreasonably low values. For example, at bias point A the periodicity along the horizontal axis is $\Delta x \simeq 200 \mu \text{m}$, which yields $c_x \simeq 1.8 \times 10^7 \text{ m/s} \simeq 0.06c$. This would require a huge dielectric constant $\varepsilon^* \simeq 300$, which does not make sense. Therefore, the observed standing-waves in the meander array can not be caused by propagation of a volume EM wave in some media. This conclusion is also confirmed by the observation that with changing frequency the nodal areas (blue color) are shifting along the meander line and may start/end at an arbitrary place of the transverse strips. E.g. in A they usually start/end at the edges, but in C - in the middle of the strips. Thus, the standing wave pattern is not periodic along the horizontal $x$-axis, as would be expected for a straightforward EM-wave propagation in the substrate. Those inconsistencies, revealed by a specific meander geometry of the array, which has a large geometrical deceleration [40], provide a clue to understanding of the nature of EM waves in our arrays.

In Fig. 3(c) we plot the LTSLM response for the image C (top) and D (bottom panel) along the meander line, as a function of the overall length of the Nb electrode from the bottom-left to the bottom-right edges of the images in the Fig. 3(b). From this plot it becomes clear that there is a long-range standing wave order along the electrode. The periodicity is $\Delta l = 1.23 \text{ mm}$ for the image C and $\Delta l = 0.85 \text{ mm}$ for the image D. The corresponding phase velocities, $c^* = 2\Delta f$, are C: $\simeq 1.59 \times 10^8 \text{ m/s}$ and D: $\simeq 1.44 \times 10^8 \text{ m/s}$, which yield a reasonable $c^* \simeq 4.0 \pm 0.3$ [41]. The performed estimations unambiguously prove that EM waves, building the standing wave, are propagating along the electrode line and a significant value of $c^*$ indicates that they propagate at the Nb/substrate side. This is a signature of surface EM waves at metal/insulator interfaces [17][27].

Another confirmation of SEMW character of standing wave resonances in the meander array comes from transport measurements. At low temperatures we observe a very fine step structure in the IV curve (see the Supplementary [20]) with a typical voltage separation $\Delta V \simeq 7 \text{ mV}$. It yields a very low primary resonant frequency $f_r \simeq 2.3 \text{ GHz}$ corresponding to a long resonator length $\simeq 3 \text{ cm}$, which is consistent with the total length
FIG. 3. LTSLM analysis of the meander array at $T \sim 5$ K. (a) $I$-$V$ characteristic of the array, measured during LTSLM imaging. (b) LTSLM images at different bias points, indicated in (a). The length of scans along $x$-axis is $L_x = 0.91$ mm. Development of standing-wave pattern is clearly seen. Note that the standing wave is not periodic in the horizontal direction. Dot lines indicate the start and the end of track along the length of the meander line where the periodicity of response is observed. (c) LTSLM responses (blue lines) along the tracks indicated in (b) from the bottom-left to the bottom-right corner of the pattern at the bias points C (top) and D (bottom). The data are averaged over the width of strip. Red line represents fitting curves obtained by the method of least squares. Clear periodicity along the whole meander length indicates that standing waves are formed by plasmon-type surface waves propagating along Nb electrodes.

of the meandering line. This confirms that such resonances are formed by traveling surface waves bound to Nb electrodes.

There are many known modes of SEMWs [17]. While we cannot provide a decisive distinction of the SEMW mode in our case, we argue that the intermediate value of $\varepsilon^* \sim 6.9 - 4.0$ in between Si (11.9) and vacuum (1) suggests that these are leaky surface-plasmon-type SEMWs propagating along one interface and leaking energy at the opposite interface of the metallic film [19]. However, the actual sub-THz frequency is well below the plasma frequency. Consequently they correspond to the linear part of the dispersion relation for surface plasmons (for an additional discussion see the Supplementary [30]). Importantly, the leaky nature of the involved SEMW both facilitates long-range synchronization of a large array and enables EM wave emission into open space. All this is a prerequisite for creation of a high-power coherent THz oscillator.

To conclude, synchronization of large oscillator arrays is a challenging problem. In this work we performed simultaneous transport measurements and low-temperature scanning laser microscopy of large arrays with 1500 and 1660 Josephson junctions. Our main result was the observation of standing-wave patterns, indicating global phase-locking of the arrays. From an analysis of the evolution of standing wave patterns with changing Josephson frequency we deduced that those patterns are formed by plasmon-type surface EM waves propagating along electrodes at the superconductor/substrate interface. We conclude that such type of surface waves can facilitate both emission of power and phase-locking of very large oscillator arrays, which is required for creation of high-power THz sources.

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