Influence of strain on anisotropic thermoelectric transport of Bi$_2$Te$_3$ and Sb$_2$Te$_3$

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(Dated: February 17, 2022)

On the basis of detailed first-principles calculations and semi-classical Boltzmann transport, the anisotropic thermoelectric transport properties of Bi$_2$Te$_3$ and Sb$_2$Te$_3$ under strain were investigated. It was found that due to compensation effects of the strain dependent thermopower and electrical conductivity, the related powerfactor will decrease under applied in-plane strain for Bi$_2$Te$_3$, while being stable for Sb$_2$Te$_3$. A clear preference for thermoelectric transport under hole-doping, as well as for the in-plane transport direction was found for both tellurides. In contrast to the electrical conductivity anisotropy, the anisotropy of the thermopower was almost robust under applied strain. The assumption of an anisotropic relaxation time for Bi$_2$Te$_3$ suggests, that already in the single crystalline system strong anisotropic scattering effects should play a role.

PACS numbers: 31.15.A-, 71.15.Mb, 72.20.Pa, 72.20.-i

I. INTRODUCTION

Thermoelectric (TE) materials are used as solid state energy devices which convert waste heat into electricity or electrical power directly into cooling or heating\(^1\)\textsuperscript{–}\textsuperscript{3}. Telluride based thermoelectrics, e.g. the bulk materials bismuth (Bi$_2$Te$_3$) and antimony telluride (Sb$_2$Te$_3$) and their related alloys, dominate efficient TE energy conversion at room temperature for the last 60 years\(^4\)\textsuperscript{,}\textsuperscript{5}. The materials TE efficiency is quantified by the figure of merit

$$ZT = \frac{\sigma S^2}{\kappa_{el} + \kappa_{ph}} T,$$ (1)

where $\sigma$ is the electrical conductivity, $S$ the thermopower, $\kappa_{el}$ and $\kappa_{ph}$ are the electronic and phononic contribution to the thermal conductivity, respectively. From Eq. 1 it is obvious, that a higher ZT is obtained by decreasing the denominator or by increasing the numerator, the latter being called powerfactor PF = $\sigma S^2$. While bulk Bi$_2$Te$_3$ and Sb$_2$Te$_3$ show ZT values smaller 1 and applications have been limited to niche areas, a break-tough experiment of Venkatasubramanian et al. showed a remarkable ZT = 2.4/1.5 for p-type/n-type superlattices (SL) composed of the two bulk tellurides\(^6\)\textsuperscript{–}\textsuperscript{7}. With the availability of high-ZT materials, many new applications will emerge\(^2\). The idea of thermoelectric SL follows the idea of phonon-blocking and electron-transmitting at the same time. It suggests that cross-plane transport along the direction perpendicular to the artificial interfaces of the SL reduces phonon heat conduction while maintaining or even enhancing the electron transport\(^3\). While some effort in experimental research was done\(^8\)\textsuperscript{–}\textsuperscript{13}, only a few theoretical works discuss the possible transport across such SL structures\(^14\)\textsuperscript{,}\textsuperscript{15}. While Park et al.\(^14\) discussed the effect of volume change on the in-plane thermoelectric transport properties of Bi$_2$Te$_3$, Sb$_2$Te$_3$ and their related compound, Li et al.\(^15\) focussed on the calculation of the electronic structure for a Bi$_2$Te$_3$/Sb$_2$Te$_3$-SL, stating changes of the mobility anisotropy estimated from effective masses.

Superlattices are anisotropic by definition and even the telluride bulk materials show intrinsic anisotropic structural and electronic properties. However, investigations of Venkatasubramanian et al. found a strong decrease for the mobility anisotropy and the thermoelectric properties for the Bi$_2$Te$_3$/Sb$_2$Te$_3$-SL at certain periods. The reason for this behaviour is still on debate and could be related to strain effects which are induced by the epitaxial growth of the Bi$_2$Te$_3$/Sb$_2$Te$_3$-SL. To extend previous works\(^16\)\textsuperscript{–}\textsuperscript{18} and to clarify the open question on the reduced anisotropy, we are going to discuss in this paper the anisotropic electronic transport in bulk Bi$_2$Te$_3$ and Sb$_2$Te$_3$ and the possible influence of strain in epitaxially grown SL on the TE properties.

For this purpose the paper will be organized as follows. In section II we introduce our first principle electronic structure calculations based on density functional theory and the semi-classical transport calculations based on the solution of the linearized Boltzmann equation. With this, we discuss the thermoelectric transport properties, that is electrical conductivity, thermopower and the related powerfactor, of unstrained Bi$_2$Te$_3$ and Sb$_2$Te$_3$ with a focus on their directional anisotropies. While in epitaxially grown Bi$_2$Te$_3$/Sb$_2$Te$_3$-SL the atoms near the interfaces may be shifted from their bulk positions due to the lattice mismatch and the changed local environment, we modelled Bi$_2$Te$_3$ with the experimental lattice parameters and interatomic distances of Sb$_2$Te$_3$, and vice versa. We assume that from these two limiting cases one could estimate the effect of the interface relaxation on the electronic and transport properties in Bi$_2$Te$_3$/Sb$_2$Te$_3$-SL. With that structural data we first analyse in section III the anisotropic thermoelectric properties of the unstrained bulk systems, while in section IV a detailed view on the influence of strain, which may occur in Bi$_2$Te$_3$/Sb$_2$Te$_3$-SL, on the electronic transport of these tellurides is given. Throughout the paper we quote Bi$_2$Te$_3$ (Sb$_2$Te$_3$) as strained, if it is considered in the lattice structure of Sb$_2$Te$_3$ (Bi$_2$Te$_3$). As in the SL p-type,
as well as n-type, transport was reported, we discuss the concentration dependence for both types of carriers on the transport properties.

II. METHODOLOGY

For both bismuth and antimony telluride we used the experimental lattice parameters and relaxed atomic positions as provided for the rhombohedral crystal structure with five atoms, i.e. one formula unit, per unit cell belonging to the space group $D_3d$ ($R3m$). The related layered hexagonal structure is composed out of three formula units and has the lattice parameters $a_{\text{hex}}^{\text{BiTe}} = 4.384\,\text{Å}$, $c_{\text{hex}}^{\text{BiTe}} = 30.487\,\text{Å}$, and $a_{\text{hex}}^{\text{SbTe}} = 4.264\,\text{Å}$, $c_{\text{hex}}^{\text{SbTe}} = 30.458\,\text{Å}$, for Bi$_2$Te$_3$ and Sb$_2$Te$_3$, respectively. In fact, the main difference between the lattices of Bi$_2$Te$_3$ and Sb$_2$Te$_3$ is a decrease of the in-plane lattice constant with an accompanied decrease in cell volume. So, a change between the two lattice constants can be related to either compressive or tensile in-plane strain. This is very similar to the approach by Park et al., while omitting computational relaxation of internal atomic positions.

Our electronic structure calculations are performed in two steps. In a first step the detailed band structure of the strained and unstrained Bi$_2$Te$_3$ ans Sb$_2$Te$_3$ was obtained by first principles density functional theory calculations (DFT), as implemented in the fully relativistic screened Korringa-Kohn-Rostoker Greens-function method (KKR). Within this approach the Dirac-equation is solved self-consistently and with that spin-orbit-coupling is included. Exchange and correlation effects were accounted for by the local density approximation (LDA) parametrized by Vosco, Wilk, and Nusair. A detailed discussion on the influence of strain on the band structure topology of Bi$_2$Te$_3$ and Sb$_2$Te$_3$ is recently published. With the well converged results from the first step we obtain the thermoelectric transport properties by solving the linearized Boltzmann equation in relaxation time approximation (RTA) within an in-house developed Boltzmann transport code. Boltzmann transport calculations for thermoelectrics have been carried out for quite a long time and show reliable results for metals as well as for wide- and narrow gap semiconductors. TE transport calculations for bulk Bi$_2$Te$_3$ and Sb$_2$Te$_3$ were presented before. Here the relaxation time $\tau$ is assumed to be constant with respect to wave vector $k$ and energy on the scale of $k_BT$. This assumption is widely accepted for metals and highly doped semiconductors. Most of the presented results are in this high-doping regime. Within the RTA the transport distribution function $\mathcal{L}^{(0)}_{\nu,\mu}(\mu, T)$ (TDF) and with this the generalized conductance moment $\mathcal{L}^{(n)}_{\nu,\mu}(\mu, T)$ are defined as

$$\mathcal{L}^{(n)}_{\nu,\mu}(\mu, T) = \frac{\tau_\nu}{(2\pi)^2} \sum_{\nu'} \int d^3k \left( v_{\nu,\mu}^{\nu'}(k) \right)^2 (E_{k\nu'} - \mu)^n \left( -\frac{\partial f_{\mu,T}}{\partial E} \right)_{E=E_{k\nu'}}.$$

$\nu$, $\nu'$ denote the group velocities in the directions in the hexagonal basal plane and perpendicular to it.

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FIG. 1: (color online) Band structures of (a) Bi$_2$Te$_3$ and (b) Sb$_2$Te$_3$ along symmetry lines for both unstrained (black solid lines) and strained (red dashed lines) lattices. Energies are given relative to the valence band maximum.
respectively. Within here the group velocities were obtained as derivatives along the lines of the Blöchl mesh in the whole Brillouin zone\(^{22}\). A detailed discussion on implications and difficulties on the numerical determination of the group velocities in highly anisotropic materials is currently published elsewhere\(^{37}\). As can be seen straight forwardly, the electrical conductivity \(\sigma\) in- and cross-plane is then given by

\[
\sigma_{\perp, \parallel} = 2e^2 L_{\perp, \parallel}^{(0)}(\mu, T) \tag{3}
\]

and the temperature- and doping-dependent thermopower states as

\[
S_{\perp, \parallel} = \frac{1}{eT} \frac{L_{\perp, \parallel}^{(1)}(\mu, T)}{L_{\perp, \parallel}^{(0)}(\mu, T)} \tag{4}
\]

for given chemical potential \(\mu\) at temperature \(T\) and extrinsic carrier concentration \(N\) determined by an integration over the density of states \(n(E)\)

\[
N = \int_{\mu - \Delta E}^{\mu + \Delta E} dE n(E) [f(\mu, T) - 1] + \int_{\text{CB}^{\text{min}}}^{\text{VB}^{\text{max}}} dE n(E) f(\mu, T) \tag{5}
\]

where \(\text{CB}^{\text{min}}\) is the conduction band minimum and \(\text{VB}^{\text{max}}\) is the valence band maximum. The energy range \(\Delta E\) has to be taken sufficiently large to cover the tails of the Fermi-Dirac distribution function \(f(\mu, T)\) and to ensure convergence of the integrals in eq. 2 and 5\(^{25}\). The k-space integration of eq. 2 for a system with an intrinsic anisotropic texture is quite demanding. In previous publications\(^{22,37}\) we stated on the relevance of adaptive integration methods needed to reach convergence of the energy dependent TDF. Especially in regions close to the band edges the anisotropy of the TDF requires a high density of the k-mesh. Here, convergence tests for the transport properties showed that at least 150 000 k-points in the entire BZ had to be included for sufficient high doping rates \((N \geq 1 \times 10^{19} \text{cm}^{-3})\), while for energies near the band edges even more than 56 million k-points were required to reach the analytical values for the conductivity anisotropies at the band edges\(^{54}\). Within the RTA, from comparison of the calculated electrical conductivities (eq. 3) with experiment it is possible to conclude on the directional anisotropy of \(\tau\). For the thermopower \(S\) (eq. 4) the dependence of the TDF on the energy is essential. That is, not only the sloop of the TDF, moreover the overall functional behaviour of the TDF on the considered energy scale has to change to observe an impact on the thermopower. The calculations in this paper aim to cover band structure effects and not scattering specific impacts by an energy- and state-dependent relaxation time.

![FIG. 2: (color online) Anisotropic thermopower for bulk (a) Bi\(_2\)Te\(_3\) and (b) Sb\(_2\)Te\(_3\) in their unstrained bulk lattice constants. Electron doping refers to the blue (thick) lines in the lower part of the figure, while red (thin) lines refer to hole doping and positive values of the thermopower. Solid lines show the in-plane part \(S_{\perp}\) of the thermopower, while dashed lines show the cross-plane part \(S_{\parallel}\). The extrinsic charge carrier concentration of Bi\(_2\)Te\(_3\) and Sb\(_2\)Te\(_3\) was fixed to \(N = 1 \times 10^{19} \text{cm}^{-3}\) and \(N = 1 \times 10^{20} \text{cm}^{-3}\), respectively. Experimental data (squares, diamonds, circles, triangles) from Ref. 41–43 are given for comparison.](image-url)

III. ANISOTROPIC THERMOELECTRIC PROPERTIES OF UNSTRAINED Bi\(_2\)Te\(_3\) AND Sb\(_2\)Te\(_3\)

In order to understand the experimental findings on the in-plane and cross-plane transport of the Bi\(_2\)Te\(_3\)/Sb\(_2\)Te\(_3\)-SL, in the following section the anisotropies of the electrical conductivity, the thermopower and the related powerfactor of bulk Bi\(_2\)Te\(_3\) and Sb\(_2\)Te\(_3\) are discussed. Even though the behaviour of Sb\(_2\)Te\(_3\) is strongly p-type with an extrinsic carrier concentration of \(N = 1 \ldots 10 \times 10^{20} \text{cm}^{-3}\)\(^{38}\), we also discuss the related n-doped case, as in Bi\(_2\)Te\(_3\)/Sb\(_2\)Te\(_3\)-SL n- as well as p-doping was reported. Bulk Bi\(_2\)Te\(_3\) is known to be inherent electron conducting, while hole doping is experimentally achievable for bulk systems\(^{4,39–41}\). Figure 2 shows the variation of the anisotropic thermopower for unstrained Bi\(_2\)Te\(_3\) and Sb\(_2\)Te\(_3\) in a wide temperature range. The extrinsic charge carrier concentration of Bi\(_2\)Te\(_3\) and Sb\(_2\)Te\(_3\) was fixed to \(N = 1 \times 10^{19} \text{cm}^{-3}\) and \(N = 1 \times 10^{20} \text{cm}^{-3}\), respectively. As a reference experimental values for both single crystalline materials at the same doping conditions are given and an excellent agreement can be stated. It is worth noting, that within eq. 4 the calculation of the thermopower is completely free of parameters. For Bi\(_2\)Te\(_3\) the in-plane thermopower reaches a maximum of \(S_{\parallel} \sim -200 \mu \text{V/K}\) at 300 K, while the maximum for the hole-doped case is shifted to slightly higher temperatures of 350 K with a maximum values of \(S_{\parallel} \sim 225 \mu \text{V/K}\). We note, that the
temperature of the maximum is slightly overestimated. This might be caused by the missing temperature dependence of the energy gap, which was determined as $E_g = 105$ meV for unstrained Bi$_2$Te$_3$. The anisotropy of the thermopower is more pronounced for the p-doped case. Here the cross-plane thermopower $S_{\perp}$ is for the given doping always larger than the in-plane part $S_{\parallel}$. The anisotropy $S_{\parallel}/S_{\perp}$ is about 0.64 at 100K, evolving to $S_{\parallel}/S_{\perp} \approx 0.79$ and $S_{\parallel}/S_{\perp} \approx 0.55$ at 300K and 500K, respectively. The sole available experimental data show no noticeable anisotropy for the thermopower in the hole-doped case$^{42}$. For the electron-doped case the situation is more sophisticated. While up to 340K the overall anisotropy is rather small, with values $S_{\parallel}/S_{\perp} \approx 0.9$, a considerable decrease of $S_{\perp}$ at higher temperatures leads to high values of $S_{\parallel}/S_{\perp}$ for temperatures above 400K. This tendency could also be revealed by experiments$^{44,45}$. The crossing point of $S_{\parallel}$ and $S_{\perp}$ near room temperature could explain the fact of varying measured anisotropies for the thermopower at 300K. Here anisotropy ratios of $S_{\parallel}/S_{\perp} = 0.97 \ldots 1.10$ were reported$^{41,45}$. The maximum peak of the thermopower near room temperature can be explained by the position of the chemical potential $\mu$ as a function of temperature at a fixed carrier concentration. For $T$ much smaller than 300K the chemical potential is located in either the conduction- or valence band with the tails of the Fermi-Dirac-distribution in eq. 2 only playing a subsidiary role. For rising temperatures the chemical potential shifts towards the band edges and $S$ maximizes. At these conditions the conduction is mainly unipolar. For higher temperatures the chemical potential shifts into the bandgap and conduction becomes bipolar leading to a reduced thermopower. For the case of Sb$_2$Te$_3$, shown in fig. 2(b), the situation is different. Due to the ten times higher inherent doping and the smaller energy gap of $E_g = 90$ meV, the chemical potential is located deeply in the bands for the whole relevant temperature range. Therefore the functional behaviour can be understood in terms of the well known MOTT relation, where equation 4 qualitatively coincides with $S \propto T \frac{d \ln \sigma(E)}{dE} |_{E=E_\mu}$ for the thermopower in RTA$^{46}$. With increasing temperature the thermopower increases almost linearly, showing values of $S_{\parallel} \sim 87 \mu V/K$ and $S_{\perp} \sim -72 \mu V/K$ at 300K for p- and n-doping, respectively. The anisotropy of the thermopower for the hole-doped case is around $S_{\parallel}/S_{\perp} = 0.91$, almost temperature-independent and slightly underestimates the available experimental values$^{47,48}$. While for the electron-doped case the absolute values of the in-plane thermopower are comparable to those of the hole-doped case, the anisotropies are rather large. The anisotropy varies only weakly on temperature showing $S_{\parallel}/S_{\perp} = 0.48 \ldots 0.52$ over the hole temperature range. While bulk Sb$_2$Te$_3$ states a strong p-character due to inherent defects, we note here again, that n-doping is available in heterostructures combining Bi$_2$Te$_3$ and Sb$_2$Te$_3$.$^5$

A strongly enhanced cross-plane thermopower $S_{\perp}$ could lead to a strongly enhanced powerfactor $PF_{\perp}$, if the cross-plane electrical conductivity $\sigma_{\perp}$ is maintained at the bulk value. For this purpose the anisotropy of the electrical conductivity in dependence on the in-plane conductivity $\sigma_{\parallel}$ for unstrained Bi$_2$Te$_3$ and Sb$_2$Te$_3$ is shown in Figure 3. The temperature is fixed at 300K, blue and red lines refer to electron- and hole-doping, respectively. From comparison with experimental data$^{53}$, the in-plane relaxation time is determined to be $\tau_{\parallel} = 1.1 \times 10^{-14}$ s and $\tau_{\perp} = 1.2 \times 10^{-14}$ s for Bi$_2$Te$_3$ and Sb$_2$Te$_3$ , respectively. With that we find strong anisotropies for the electrical conductivity $\sigma_{\parallel}/\sigma_{\perp} \gg 1$, clearly preferring the in-plane transport in both bulk tellurides. For the strongly suppressed cross-plane conduction p-type conduction is more favoured than n-type conduction. For Bi$_2$Te$_3$ the pure band structure effects (solid lines in Figure 3(a)) overestimate the measured anisotropy ratio$^{39}$ of the electrical conductivity. With an assumed anisotropy of

![Image](image_url)
the relaxation time of $\tau / \tau_\perp = 0.47$ the experimental values are reproduced very well. That means, scattering effects strongly affect the transport and electrons travelling along the basal plane direction are scattered stronger than electrons travelling perpendicular to the basal plane. The origin of this assumed anisotropy has to be examined by defect calculations and resulting microscopic transition probabilities and state dependent mean free path vectors. It is well known, that in Bi$_2$Te$_3$ mainly anti-site defects lead to the inherent conduction behaviour 38,45,50. We have shown elsewhere 37, that the integration of the transport integrals eq. 2 in anisotropic k-space requires large numeric effort. Tiny regions in k-space close to the band gap have to be scanned very carefully and the texture in k-space has a drastic influence on the obtained anisotropy values, if integrals are not converged with respect to the k-point density. As shown, some integration methods tend for the given k-space symmetry to underestimate the ratio $\sigma / \sigma_\perp$ in a systematic manner and therefore would shift anisotropy closer to the experimental observed values, without representing the real band structure effects. For unstrained Bi$_2$Te$_3$ the electrical conductivity anisotropy is highest for low values of $\sigma$, i.e. small amounts of doping and bipolar conduction. For larger charge carrier concentrations, i.e. the chemical potential shifts deeper into either conduction or valence band, the in-plane conductivity $\sigma_\parallel$ increases and the ratio $\sigma / \sigma_\perp$ decreases. Values for $\sigma / \sigma_\perp$ will lower from 7 to 2 for p-type conduction and 9 to 3 for n-type conduction. However, cross-plane electrical transport is always more suppressed for n-type carrier conduction, which also holds for unstrained Sb$_2$Te$_3$. As shown in Figure 3(b) $\sigma / \sigma_\perp$ is almost doping independent for hole-doping, showing an anisotropy of around 2.7 in very good agreement with experiment (circle and triangles in fig. 3 from Ref. 47–49). In this case no anisotropic relaxation times had to be assumed. For electron doping the ratio $\sigma / \sigma_\perp$ is clearly higher, evolving values of 3.5 to 6 for rising in-plane conductivity. The dependence of the anisotropy ratio on the applied doping, i.e. changing $\sigma$, can be directly linked to the functional behaviour of the TDF near band edges, which is crucially influenced by the topology of the band structure 22.

![Graph](image_url)

**FIG. 4:** (color online) Conductivity ratio $\sigma / \sigma_\perp$ of the electrical conductivities at 300K for bulk (a) Bi$_2$Te$_3$ in the Sb$_2$Te$_3$ structure and (b) Sb$_2$Te$_3$ in the Bi$_2$Te$_3$ structure. Electron doping refers to blue lines, while red lines refer to hole doping. Isotropic relaxation times of $\tau = 1.1 \times 10^{-14}$ s and $\tau = 1.2 \times 10^{-14}$ s for $\sigma_\parallel$ and $\sigma_\perp$ are assumed for Bi$_2$Te$_3$ and Sb$_2$Te$_3$, respectively.

For Bi$_2$Te$_3$ the compressive in-plane strain causes an increase of the the band gap by around 23% yielding $E_g = 129$ meV. While the anisotropy $\sigma / \sigma_\perp$ for hole doping (red lines in fig. 4(a)) decreases to around 4 and is almost constant under varying doping level, the ratio considerably raises under electron doping to values up to 13 for $\sigma / \sigma_\perp \sim 100$…1000 (Ω cm)$^{-1}$, corresponding to electron charge carrier concentrations of $N = 3 \ldots 30 \times 10^{19}$ cm$^{-3}$. This concludes, that the cross-plane electrical conductivity of Bi$_2$Te$_3$ under compressive in-plane strain will be noticeably enhanced for p-doping, but drastically suppressed for n-doping. Such a compressive in-plane strain could be introduced by either a substrate with smaller in-plane lattice constant, e.g. GaAs-[111] with $a = 3.997$ Å, or a considerable amount of Sb$_2$Te$_3$ in the Bi$_2$Te$_3$/Sb$_2$Te$_3$-SL. For tensile in-plane strained Sb$_2$Te$_3$ the impact on the electrical conductivity ratio $\sigma / \sigma_\perp$ is less prominent. As shown in figure 4(b) at

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**IV. ANISOTROPIC THERMOELECTRIC PROPERTIES OF STRAINED Bi$_2$Te$_3$ AND Sb$_2$Te$_3$**

Before the influence of in-plane strain on the resulting powerfactor will be discussed, we will first note on the strain induced changes of the components electrical conductivity and thermopower. In Figure 4 the anisotropy of the electrical conductivity $\sigma / \sigma_\perp$ is shown for both Bi$_2$Te$_3$ in the lattice constant of Sb$_2$Te$_3$, i.e. under biaxial compressive in-plane strain (Figure 4(a)), and Sb$_2$Te$_3$ in the lattice constant of Bi$_2$Te$_3$, i.e. under biaxial tensile in-plane strain (Figure 4(b)).
hole doping $\sigma_{1}/\sigma_{\perp} \sim 2.5$ is only marginally altered compared to the unstrained case (comp. fig. 5(b)). Meanwhile $\sigma_{1}/\sigma_{\perp}$ decreases noticeably for n-type doping yielding about 3 at low $\sigma_{1}$ and low electron charge carrier concentrations, and slightly higher values of $\sigma_{1}/\sigma_{\perp} \sim 4$ for higher doping. Overall, the tensile strain reduces the electrical conductivity anisotropy by a factor of about 1.5, directly leading to an enhanced electrical conductivity along the z-axis of single crystal Sb$_{2}$Te$_{3}$. We note, that tensile in-plane strain opens the gap remarkably by around 56% compared to the unstrained case to a value of $E_{g} = 140$ meV. Furthermore, such tensile strain could be incorporated by using either a substrate with larger in-plane lattice constant, e.g. PbTe-[111] with $a = 4.567\text{Å}$, or a higher fractional amount of Bi$_{2}$Te$_{3}$ in Bi$_{2}$Te$_{3}$/Sb$_{2}$Te$_{3}$-SL. In Figure 5(a), (d) ((b), (c)) the doping dependent anisotropic thermopower of unstrained (strained) Bi$_{2}$Te$_{3}$ and Sb$_{2}$Te$_{3}$ at room temperature is shown, respectively. Blue thick (red thin) solid lines represent the in-plane thermopower $S_{\parallel}$ under electron doping (hole doping). The corresponding cross-plane thermopower $S_{\perp}$ is shown as a dashed line. The black dashed-dotted lines in fig. 5(d) emphasize the expected doping dependent behaviour of the thermopower for parabolic bands, following the Pisarenko-relation. For both tellurides we found, that the anisotropy of the thermopower shows a weak dependence on the strain state. However, for strained Bi$_{2}$Te$_{3}$ (see fig. 5(b)) the thermopower anisotropy under hole doping almost vanishes, leading to $S_{\parallel} \sim S_{\perp}$. It is worth noting, that the anisotropy of the thermopower is less pronounced for hole doping, than for electron doping for Bi$_{2}$Te$_{3}$ and Sb$_{2}$Te$_{3}$ in both strain states. As shown by the black dashed-dotted lines in fig. 5(d), the dependency of the thermopower on the charge carrier concentration differs from the Pisarenko-relation under sufficient high electron doping. This indicates, that the nonparabolicity of the energy bands has a noticeable impact in the investigated doping regime and should not be omitted by applying parabolic band models.

Actually, changes for the absolute values of the thermopower can be found for both telluride systems under applied strain. In fig. 6 the relative change for the in-

FIG. 5: (color online) In-plane (solid lines) and cross-plane (dashed lines) doping-dependent thermopower at 300K for (a) Bi$_{2}$Te$_{3}$ in the Bi$_{2}$Te$_{3}$ structure, (b) Bi$_{2}$Te$_{3}$ in the Sb$_{2}$Te$_{3}$ structure, (c) Sb$_{2}$Te$_{3}$ in the Bi$_{2}$Te$_{3}$ structure and (d) Sb$_{2}$Te$_{3}$ in the Sb$_{2}$Te$_{3}$ structure. Electron (hole) doping is presented as blue thick (red thin) line. The black (dashed-dotted) line in panel (d) shows the Pisarenko-dependence of the thermopower expected for parabolic bands. Experimental data (circles) from Ref. 51 is given for comparison. The charge carrier concentration is stated in units of $e$/uc $(1$/cm$^3$) at the bottom (top) x-axis.

FIG. 6: (color online) Change of the in-plane thermopower $S_{\parallel}$ under applied strain for (a) Bi$_{2}$Te$_{3}$ and (b) Sb$_{2}$Te$_{3}$. Given is the ratio of $S_{\parallel}$ in the "smaller" lattice of Sb$_{2}$Te$_{3}$ divided by $S_{\parallel}$ in the "larger" lattice of Bi$_{2}$Te$_{3}$. The doping was fixed to $N = 1 \times 10^{19}$ cm$^{-3}$ for Bi$_{2}$Te$_{3}$ and $N = 1 \times 10^{20}$ cm$^{-3}$ for Sb$_{2}$Te$_{3}$. Solid blue (dashed red) lines refer to electron (hole) doping, respectively.
plane component \( S_S \) for both tellurides under in-plane strain is given. To compare the changes with the lattice constant, we relate the in-plane thermopower \( S_S \) at the smaller lattice constant \( a_{SBT} \) to the value at the larger lattice constant \( a_{BiTe} \) for both compounds. The doping was fixed to \( N = 1 \times 10^{19} \text{cm}^{-3} \) for \( Bi_2Te_3 \) and \( N = 1 \times 10^{20} \text{cm}^{-3} \) for \( Sb_2Te_3 \) as done for fig. 2. Figure 6(a) shows, that in the relevant temperature range between 350K and 450K the thermopower increases for \( Bi_2Te_3 \) under compressive strain for both p and n doping by about 15-20%. For \( Sb_2Te_3 \) a decrease is expected under tensile strain at electron doping and nearly no change under hole doping (see Figure 6(b)). With nearly all values above 1 for \( Bi_2Te_3 \), as well as for \( Sb_2Te_3 \), it is obvious, that higher values of the thermopower require a smaller unit cell volume. One can expect, that the volume decrease causes a larger density of states and thus a shift of the chemical potential towards the corresponding band edge, connected with an increase of the thermopower \( S \). However Park et al.\textsuperscript{14} reported an unexpected increase of 16% for the in-plane thermopower \( S_S \) of \( Sb_2Te_3 \) under p-doping (T=300K, \( N = 1.32 \times 10^{19} \text{cm}^{-3} \)) if the material is strained into the \( Bi_2Te_3 \) structure. In the same doping and temperature regime we find a slight decrease of 4% for \( S_S \).

Comprising the statements on the electrical conductivity and the thermopower, the related powerfactor for both tellurides in their bulk lattice and in the strained state are compared in fig. 7. It is well known, that optimizing the powerfactor \( \sigma S^2 \) of a thermoelectric always involves a compromise on the electrical conductivity \( \sigma \) and the thermopower \( S^2 \).\textsuperscript{52} Due to the interdependence of \( \sigma \) and \( S \) it is not advisable to optimize the powerfactor by optimizing its parts. In Figure 7(a) and (d) the doping dependent anisotropic powerfactor of unstrained \( Bi_2Te_3 \) and \( Sb_2Te_3 \) at room temperature is shown, respectively. Blue thick (red thin) solid lines represent the in-plane powerfactor \( PF_\parallel \) under electron doping (hole doping). The corresponding cross-plane powerfactor \( PF_\perp \) is shown as a dashed line. Under p-doping both unstrained materials show a maximum powerfactor near carrier concentrations of \( N \sim 4 \times 10^{19} \text{cm}^{-3} \). Absolute values of 35 \( \mu \text{W/cmK}^2 \) and 33 \( \mu \text{W/cmK}^2 \) were found for unstrained \( Bi_2Te_3 \) and \( Sb_2Te_3 \), respectively, which is in good agreement to experimental and theoretical findings.\textsuperscript{3,4,51} Under electron doping the absolute values of \( PF_\parallel \) (thick blue lines in fig. 7) were found to be distinctly smaller. This is due to smaller absolute values of the thermopower for electron doping compared to hole doping (see Figure 2) and apparently smaller in-plane electrical conductivities \( \sigma_\parallel \) at fixed carrier concentrations. As a result, a powerfactor of 18 \( \mu \text{W/cmK}^2 \) and 8 \( \mu \text{W/cmK}^2 \) can be stated for unstrained \( Bi_2Te_3 \) and \( Sb_2Te_3 \), respectively, under optimal electron doping. We notice, that the powerfactor for unstrained \( Sb_2Te_3 \) is monotonically increasing for electron carrier concentrations of \( N \sim 6 \ldots 30 \times 10^{19} \text{cm}^{-3} \). This behaviour can be linked to a deviation of \( S_S \) from the PIŠARENKO-

\[ S \] relation under electron doping. While it is expected, that the thermopower will decrease for increasing carrier concentration, \( S_S \) was found to be almost constant in an electron doping range of \( N \sim 6 \ldots 30 \times 10^{19} \text{cm}^{-3} \) (see fig. 5(d)). For the investigated electron doping range of \( N \sim 6 \ldots 30 \times 10^{19} \text{cm}^{-3} \) the chemical potential \( \mu \) at 300K is located around 300 \ldots 450 meV above the VBM. As can be seen from the band structure for unstrained \( Sb_2Te_3 \) in fig. 1(b) (black, solid lines) flat non-parabolic bands near the high symmetry point Z dominate in this energy region and most likely lead to an increased thermopower. This feature is more pronounced for unstrained \( Sb_2Te_3 \), than for strained \( Sb_2Te_3 \) (red, dashed lines in fig. 1(b)). Similar statements can be done for strained and unstrained \( Bi_2Te_3 \) (see fig. 1(a)). We note, even though this picture is convincing, it is difficult to link such specific anomalies to the band structure on high symmetry lines, as the underlying TDF is an integral quantity over all occupied
states in the BZ.

Under applied in-plane compressive strain for Bi$_2$Te$_3$ (ref. Figure 7(b)) and tensile strain for Sb$_2$Te$_3$ (ref. Figure 7(c)) the obtained changes in the powerfactor are noticeable different for both tellurides. While for Bi$_2$Te$_3$ a decrease of the maximal powerfactor $PF_{||}$ of about 27% and 23% for n-doping and p-doping was found, the strain shows nearly no influence on the powerfactor for Sb$_2$Te$_3$. At a carrier concentration of about $N \sim 3 \times 10^{19} \text{cm}^{-3}$ the decrease in $PF_{||}$ for Bi$_2$Te$_3$ is about 17% and 28% for n- and p-doping, respectively, while in the work of Park et al.\textsuperscript{14} a slight increase of $PF_{||}$ under strain and hole doping is reported. Obviously this tendency has to be understood by analyzing the constituent parts $\sigma_{||}$ and $S_{||}$. For compressively strained Bi$_2$Te$_3$ at a hole carrier concentration of about $N \sim 3 \times 10^{19} \text{cm}^{-3}$ the electrical conductivity decreases by about 39% to 330 (Ω cm s)$^{-1}$. At the same time $S_{||}$ increases by about 9%, as shown in Figure 6(a). This results in the overall decrease of about 28% for $PF_{||}$. Under electron doping of $N \sim 3 \times 10^{19} \text{cm}^{-3}$ no influence of strain could be found for $S_{||}$ at room temperature (see solid blue lines in Figure 6(a)). Thus, the decrease of $PF_{||}$ under electron doping can be largely related to a decrease of the electrical conductivity under applied compressive strain. By detailed evaluation of the effective mass eigenvalues and eigenvectors we found a decrease of about 15% for the in-plane electrical conductivity of Bi$_2$Te$_3$ under applied strain in the low-temperature and low-doping limit\textsuperscript{22,37}. The discussion can be made in the same manner for Sb$_2$Te$_3$\textsuperscript{22,47}. The fact, that strain-induced effects in $\sigma$ and $S$ tend to compensate each other was already reported for the case of silicon\textsuperscript{25}.

As mentioned before (summarized in fig. 3 and fig. 4), we found a strong anisotropy in the electrical conductivity with $\sigma_{||}/\sigma_{\perp} \gg 1$. The clearly preferred in-plane transport in both bulk tellurides is also reflected in the cross-plane powerfactor $PF_{\perp}$ (dashed lines in Figure 7), which is clearly suppressed for all strain states. It is obvious that $PF_{\perp}$ is more suppressed for electron- than for hole-doping.

Nonetheless, we want to include experimental findings for the thermal conductivity to our calculations, to give an estimation for the figure of merit ZT in-plane and cross-plane. In Ref. \textsuperscript{49} $\kappa_{\parallel} = 2.2 \text{W/mK}$, $\kappa_{\perp} = 1.0 \text{W/mK}$, and $\kappa_{\parallel} = 7.5 \text{W/mK}$, $\kappa_{\perp} = 1.6 \text{W/mK}$ for unstrained Bi$_2$Te$_3$ and Sb$_2$Te$_3$ are given, respectively. With this we find maximal values for the figure of merit at room temperature and optimal hole doping of $ZT_{||} \sim 0.48$ and $ZT_{\perp} \sim 0.41$ for unstrained Bi$_2$Te$_3$ and $ZT_{||} \sim 0.13$ and $ZT_{\perp} \sim 0.23$ for unstrained Sb$_2$Te$_3$. We note, that the figure of merit ZT maximizes at slightly lower carrier concentration than the powerfactor $\sigma S^2$ shown in fig. 7. This can be linked directly to an increasing electronic part of the thermal conductivity $\kappa_{el}$ with increasing carrier concentration\textsuperscript{25,53}.

V. CONCLUSION

In the present paper the influence of in-plane strain on the thermoelectric transport properties of Bi$_2$Te$_3$ and Sb$_2$Te$_3$ is investigated. A focussed view on the influence of strain on the anisotropy of the electrical conductivity $\sigma$, thermopower $S$ and the related powerfactor $\sigma S^2$ could help to understand in-plane and cross-plane thermoelectric transport in nanostructured Bi$_2$Te$_3$/Sb$_2$Te$_3$-superlattices. Based on detailed \textit{ab initio} calculations we focussed mainly on band structure effects and their influence on the thermoelectric transport. For both tellurides no reasonable decrease of the anisotropy for $\sigma$ and $S$ could be found under strain, while in principle the anisotropy for $\sigma$ and $S$ is more pronounced under electron doping, than at hole doping. Thus a favoured thermoelectric transport along the z-direction of Bi$_2$Te$_3$/Sb$_2$Te$_3$-heterostructures due to superlattice-induced in-plane strain effects can be ruled out and a clear preference of p-type thermoelectric transport can be stated for Bi$_2$Te$_3$ and Sb$_2$Te$_3$ and their related epitaxial heterostructures. The absolute value of the in-plane thermopower $S_{||}$ was increased under reduced cell volume, which is in contrast to recent findings by Park et al.\textsuperscript{14}.

We found, that even if thermopower or electrical conductivity are enhanced or decreased via applied strain, they tend to compensate each other suppressing more distinct changes of the powerfactor under strain. We found the thermoelectrically optimal doping to be in the range of $N \sim 3 \ldots 6 \times 10^{19} \text{cm}^{-3}$ for all considered systems. Our assumption of an anisotropic relaxation time for Bi$_2$Te$_3$ states that already in the single crystalline system strong anisotropic scattering effects should play a role.

Acknowledgments

This work was supported by the Deutsche Forschungsgemeinschaft, SPP 1386 ‘Nanostrukturierte Thermoelektrika: Theorie, Modellsysteme und kontrollierte Synthese’. N. F. Hinsche is member of the International Max Planck Research School for Science and Technology of Nanostructures.

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\textsuperscript{1} B. Sales, Science \textbf{295}, 1248 (2002).

\textsuperscript{2} A. Majumdar, Science \textbf{303}, 777 (2004).

\textsuperscript{3} H. Böttner, G. Chen, and R. Venkataraman, MRS Bulletin \textbf{31}, 211 (2006).

\textsuperscript{4} H. Goldsmid, A. Sheard, and D. Wright, British Journal
of Applied Physics 9, 365 (1958).
5 R. Venkatasubramanian, E. Siivola, and T. Colpitts, Nature 413, 597 (2001).
6 R. Venkatasubramanian, T. Colpitts, B. O’Quinn, S. Liu, N. El-Masry, and M. Lamvik, Applied Physics Letters 75, 1104 (1999).
7 R. Venkatasubramanian, Phys. Rev. B 61, 3091 (2000).
8 H. Beyer, J. Nurnus, H. Böttner, A. Lambrecht, E. Wagner, and G. Bauer, Physica E: Low-dimensional Systems and Nanostructures 13, 965 (2002).
9 H. Böttner, J. Nurnus, A. Gavrikov, and G. Kuhner, Journal of Micromechanical Systems 13, 414 (2004).
10 J. König, M. Winkler, S. Buller, W. Bensch, U. Schürmann, L. Kienle, and H. Böttner, Journal of electronic Materials 40, 1266 (2011).
11 C.-N. Liao, C.-Y. Chang, and H.-S. Chu, Journal of Applied Physics 107, 066103 (2010).
12 N. Peranio, O. Eibl, and J. Nurnus, Journal of Applied Physics 100, 114306 (2006).
13 M. N. Tonzellaev, P. Zhou, R. Venkatasubramanian, and K. E. Goodson, Journal of Applied Physics 90, 763 (2001).
14 M. S. Park, J.-H. Song, J. E. Medvedeva, M. Kim, I. G. Kim, and A. J. Freeman, Phys. Rev. B 81, 155211 (2010).
15 H. Li, D. Bile, and S. D. Mahanti, Mat. Res. Soc. Symp. Proc. 793, 837 (2004).
16 T. Scheidemantel, C. Ambrosch-Draxl, T. Thonhauser, J. Badding, and J. Sofo, Physical Review B 68, 125210 (2003).
17 T. Thonhauser, J. Scheidemantel, J. Sofo, J. Badding, and G. Mahan, Physical Review B 68, 085201 (2003).
18 B.-L. Huang and M. Kaviany, Phys. Rev. B 77, 125209 (2008).
19 Landolt-Börnstein New Series, group III/41C (Springer Verlag, Berlin, 1998).
20 M. Gradhand, M. Czernek, D. V. Fedorov, P. Zahn, B. Y. Yavorsky, L. Szunyogh, and I. Mertig, Phys. Rev. B 80, 224413 (2009).
21 S. H. Vosko, R. V. Wilk, and R. Venkatasubramanian, Phys. Rev. B 22, 3812 (1980).
22 B. Yu. Yavorsky, N. F. Hinsche, I. Mertig, and P. Zahn, arXiv:1109.0186 (2011).
23 I. Mertig, Reports on Progress in Physics 62, 237 (1999).
24 P. Zahn, I. Mertig, M. Richter, and H. Eschrig, Phys. Rev. Lett. 75, 2996 (1995).
25 N. F. Hinsche, I. Mertig, and P. Zahn, J. Phys.: Condens. Matter 23, 295502 (2011).
26 T. Vojta, I. Mertig, and R. Zeller, Phys. Rev. B 46 (1992).
27 J. Yang, H. Li, T. Wu, and W. Zhang, Advanced Functional Materials 18, 2880 (2008).
28 J. Barth, G. Fecher, B. Balle, S. Ouardi, T. Graf, C. Fels, A. Shkablo, A. Weidenkaff, K. Klaer, and H. Elmers, Physical Review B 81, 064404 (2010).
29 D. J. Singh, Physical Review B 81, 195217 (2010).
30 D. Parker and D. J. Singh, Physical Review B 82, 035204 (2010).
31 A. May, D. J. Singh, and G. J. Snyder, Physical Review B 79, 153101 (2009).
32 M.-S. Lee, F. Poudeu, and S. Mahanti, Physical Review B 83, 085204 (2011).
33 S. Lee and P. von Allmen, Applied Physics Letters 88, 022107 (2006).
34 M. Situmorang and H. Goldsmid, physica status solidi (b) 134, K83 (1986).
35 T. Thonhauser, Solid State Communications 129, 249 (2004).
36 G. Mahan and J. Sofo, Proceedings of the National Academy of Sciences 93, 7436 (1996).
37 P. Zahn, B. Yu. Yavorsky, N. F. Hinsche and I. Mertig, arXiv:1108.0023 (2011).
38 D. M. Rowe, ed., CRC handbook of thermoelectrics (CRC Press London, 1995).
39 R. Delves, A. Bowley, and D. Hazelden, Proceedings of the Phys. Society 78, 838 (1961).
40 H. Jeon, H. Ha, D. Hyun, and J. Shim, Journal of Physics and Chemistry of Solids 52, 579 (1991).
41 H. Kaibe, Journal of Physics and Chemistry of Solids 50, 945 (1989).
42 M. Stordeur and W. Kühnberger, physica status solidi (b) 69, 377 (1975).
43 M. Stordeur and W. Heiliger, physica status solidi (b) 78, K103 (1976).
44 M. Zhitinskaya, V. Kaidanov, and V. Kondratev, Soviet Physics Semiconductors 10, 1300 (1976).
45 E. Müller, Bandstruktur und Ladungsträgerstreuung in p-leitenden (Bands structure and carrier scattering in p-type semiconductors) (VDI Verlag, Düsseldorf, 1998).
46 M. Cutler and N. Mott, Phys. Rev. 181, 1336 (1969).
47 G. Simon and W. Eichler, phys. stat. sol. (b) 103, 289 (1981).
48 H. Langhammer, M. Stordeur, H. Sobotta, and V. Riede, physica status solidi (b) 109, 673 (1982).
49 A. Jacquot, N. Farag, M. Jaegle, M. Bobeth, J. Schmidt, D. Ebling, and H. Böttner, Journal of Electronic Materials 39, 1861 (2010).
50 S. Cho, Y. Kim, A. Divere, G. K. Wong, J. B. Ketterson, and J. R. Meyer, Applied Physics Letters 75, 1401 (1999).
51 J. Nurnus, Ph.D. thesis, Albert-Ludwigs-Universität Freiburg (2001).
52 A. F. Ioffe, Physics of Semiconductors (Academic, New York, 1960).
53 A. S. Snyder and E. Toberer, Nature Materials 7, 105 (2008).
54 The analytical value of the ratio $\sigma(\parallel) / \sigma(\perp)$ at the band edges was obtained by scanning the energy landscape near the conduction band minimum and valence band maximum fitting the dispersion relation in terms of an effective mass tensor. A detailed description is given in a recent publication by Ref. 22.
55 The calculated dependencies of the electrical conductivity on the thermopower and the electrical conductivity on the applied doping were matched to fit experiments from Ref. 4.39.48.