Superheavy hidden sectors and the vacuum energy density

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Abstract

In the present work a quintessence like mechanism is presented, which models a considerable fraction of the critical energy density today \( \rho_c \simeq 10^{-47} \) GeV\(^4\). By assuming that the Quantum Field Theory vacuum energy is lowered down to zero by a suitable adjustment mechanism, the critical energy density is modelled in terms of a quintessence axion field \( a \). This axion is a pseudo-Goldstone boson arising due to a symmetry breaking mechanism in a hidden sector, corresponding to an SU(2) gauge interaction. The unification between the latter sector and QCD is produced at a very large energy scale, of the order of the GUT or even of the Planck energy. This theory is confining at a very low scale, of the order of a very light neutrino mass \( m_\nu \sim 10^{-2} \) eV. Due to the presence of non zero vacuum fermion condensates \( \langle \bar{f}f \rangle \), the axion acquires a small mass \( m_a \sim 10^{-32} \) eV. Since the inverse of this mass is of the order of the age of the universe, this axion acts as an energy density of the actual universe. The hidden sector contains very massive and very light fermions, and these masses arise due to a seesaw mechanism involving mass scales between \( 10^{-2} \) eV and \( M_{Pl} \). Assuming that these particles interact with the ordinary sector very weakly, we argue that some of these particles are stable and the products of their decays can be seen as events above the GKZ bound.

1. Introduction

In the last years several experimental results have been obtained whose interpretation inarguably demands new physics. One of these observed features is the cosmic acceleration. Since gravity is an attractive force, the velocity of the distant galaxies may be expected to slow down. Instead of doing this, astronomical observations support the fact that this velocity is indeed increasing. Another crucial phenomenon is the discrepancy between the luminous matter of several objects in the universe and their gravitational effects [1,2]. In fact, there is experimental evidence supporting a flat universe, which implies that the energy density of it should be of the order of the critical one, \( \rho_c \simeq 10^{-47} \) GeV\(^4\) [3]. This scenario does not agree with the contributions corresponding to non relativistic mass density dynamically measured, which are approximately \((0.1 - 0.3)\rho_c\).

Since the publication of these results several explanations have appeared. Some of them postulate the existence of dark matter; i.e an unknown matter sector whose contribution to the energy density compensates the difference between the critical and the observed densities. This hidden sector of particles interacts with the known particles weakly enough not to be detected by current accelerator technology [4,14]. Furthermore, the acceleration of the universe’s expansion suggests the presence of

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a cosmological constant. If this were to be interpreted as vacuum energy density, then its value would be a considerable fraction of the critical density $\rho_c$.

The presence of a non zero cosmological constant is an important observation by itself. What is striking is that the critical density is not natural from the point of view of the Standard Model. For example, the contribution of the quark condensate $\langle \bar{q}q \rangle$ to the vacuum energy is around 43 orders of magnitude larger than the critical density. If we focus our attention in gluon condensate $\langle G_{\mu\nu}G^{\mu\nu} \rangle$, the discrepancy is even bigger, 44 orders of magnitude. This implies there should be a mechanism for tuning down these contributions to zero. One possibility is the existence of supersymmetry which forces the vacuum energy to be identically zero. However, the presence of a small cosmological constant may signal that supersymmetry is broken; the spontaneously broken supersymmetric theories give contributions of at least 55 orders of magnitude larger than the critical density. For this reason, other scenarios are required to explain this problem. One of the approaches is to postulate that the vacuum energy does not gravitate by some unknown reason and that the effect of a cosmological constant is imitated by the so-called dark energy. The latter is an exotic fluid with negative pressure which drives the cosmic acceleration. Examples of this are the quintessence scenarios \cite{15}. In these models the vacuum energy is associated with a slowly rolling scalar field $\varphi$ under the influence of a nearly flat potential $V(\varphi)$. The nearly flat condition ensures that $\varphi$ is not at the minimum of its potential $V(\varphi)$ at present times. As a consequence, the vacuum energy is a temporary effect which disappears for large times.

The quintessence scenarios are able to describe the current energy density, but the assumption that the vacuum energy does not participate in the gravitational interaction is still to be studied further. If this assumption is relaxed, then other type of scenarios should be introduced, such as the so-called cancellation (or adjustment) mechanisms \cite{16–21}. These models assume the presence of an initial gravitating energy density, together with an unknown component which contributes to this density with opposite sign, in such a way that for large times the total energy density becomes very small. The cancellation mechanisms based on scalar fields suffer some potential problems, as noticed already in \cite{22, 23}. Nevertheless, there are some scalar models that may avoid these complications although they usually take place in modified theories of gravity \cite{17, 18}. The cancellation mechanisms for higher spin fields do not have these problems, since by construction they predict a very small vacuum energy. The possibility of them modelling the critical energy density without screening the Newton gravity constant is not excluded \cite{19, 21}.

In the present work, the existence of a suitable adjustment mechanism capable of lowering down the energy densities contributions mentioned above, will be assumed. Bearing this assumption in mind, the present energy density will be modelled in terms of a very light axion $a$, with a mass of $m_a \sim 10^{-32} - 10^{-33}$ eV. This axion arises due to a symmetry breaking mechanism in a hidden sector corresponding to an SU(2)$_L$ gauge interaction whose unification with QCD is produced at a very large energy scale of the order of the GUT or even of the Planck energy. This theory is confining at a very low scale, of the order of the light neutrino mass $m_\nu \sim 10^{-2} - 10^{-3}$ eV. The confining theory possess non zero vacuum fermion condensates $\langle \bar{f}f \rangle$, hence the axion acquires its small mass. The large scale of the spontaneously symmetry breaking implies that a hidden sector contains superheavy particles. These particles could be formed in an early stage of the universe and their large mass values may suggest that their mean lifetime is too short to be present in the actual universe. However, it will be argued below that this conclusion is not necessarily true. With this purpose in mind, an example of a hidden sector containing a gauge interaction and an axion of the characteristics described above will be constructed. In this scenario the right handed components interact weakly with the ordinary Standard Model particles. If some specific hierarchy of masses holds, all of these particles in this sector would be supermassive, with mass of at least $10^{13}$ GeV, and stable with mean lifetime equal or greater than the age of the universe, $\tau \sim 10^{10}$ yrs. As a result, this sector can arrange events above the Greisen-Zatsepin-Kuzmin (GKZ) limit \cite{24}, which is a very interesting fact to be taken into account.
Outline: The organization of this paper is as follows. In section 2 general properties of QCD axion models are discussed which are required for the construction of our model. In section 3 the Lagrangian for the hidden sector is presented. Section 4 contains an estimation of the mean lifetime of one of the Higgs composing this sector, and it is shown that this particle is stable. Section 5 contains a discussion of the results.

2. A brief review of axion mechanisms in QCD

The present section gives a description of some axion models of QCD with a two-folded motivation. On the one hand, the axion scenarios will be useful to construct our model. On the other, they can be considered as an example of a cancellation mechanism in which an extremely small parameter \( \theta_{eff} \) is interpreted in terms of a dynamical component, the axion. This cancels the effect of the bare parameter \( \theta \), resulting in an extremely small effective value \( \theta_{eff} \).

As is well known in ordinary QCD, the \( \theta \) term associated with the instantons solutions and related quantum effects of the theory \[ L_\theta = \frac{\theta}{32\pi^2} G^a_{\mu \nu} \tilde{G}^a_{\mu \nu}, \] violates CP invariance when the fermions of the theory are massive. In the latter expression, \( G^a_{\mu \nu} \) is the gluon strength field and \( \tilde{G}^a_{\mu \nu} \) its dual expression. For massless QCD instead, the chiral transformation

\[ \psi \rightarrow e^{i\gamma_5\alpha} \psi \quad (2.2) \]

on the fermions wave functions \( \psi \) of the theory, is a classical symmetry of the Lagrangian. On the contrary, at a quantum level there is an anomaly in the chiral current \( J_{\mu 5} \) given by

\[ \partial_{\mu} J_{\mu 5} = \frac{g^2}{16\pi^2} G^a_{\mu \nu} \tilde{G}^a_{\mu \nu}. \]

(2.3)

For this reason, if the fermions were massless, the chiral transformation would modify the \( \theta \) parameter in the following way:

\[ \theta \rightarrow \theta - 2\alpha. \]

This means that for massless QCD all the theories with different \( \theta \) would be equivalent. Thus, it is the mass term of the fermions which spoils the chiral symmetry and simultaneously the CP invariance.

The value of \( \theta \) is not fixed by the theory itself and should be determined by the experiments. The experimental known bound is \( \theta < 10^{-9} \). This value does not satisfy the majority of the scientific community, which regards the introduction of such small parameter in the theory as unnatural. For this reason in [27] an alternative to explain this lack of naturalness problem was introduced. They consider the \( \theta \) parameter as a dynamical field, the axion, which runs to the value zero regardless of its initial value. The effective Lagrangian describing the axion \( a \) and its interaction with the gluons is

\[ L_{eff} = L_{QCD} + L_k(a) + \left( \theta + \frac{a}{f_a} \right) \frac{\alpha_s}{8\pi} G^a_{\mu \nu} \tilde{G}^a_{\mu \nu}, \]

where \( f_a \) stands for the axion constant; \( L_k(a) \) its kinetic term and \( \alpha_s \) a constant. Grouping the \( \theta \) term with the coupling of the axion to the boson gauge fields of QCD, it is possible to redefine an effective \( \tilde{\theta} \) term

\[ \tilde{\theta} = \theta + \frac{a}{f_a}. \]

After shifting the field as follows \( a \rightarrow a - f_a \theta \), the \( \theta \) parameter can be discarded. This implies the theory will be CP invariant if we can find some mechanism which forces the axion to take the value
Figure 1: Diagram of the axion effective coupling to the gluons of QCD through the new heavy quark $Q$. As in the Standard Model, the gluons interact as usual with the ordinary quarks.

$a = 0$. This is in fact precisely what happens, since the axion is under the influence of an effective quantum potential $V(a)$, due to the effect of the quarks and gluons inside the Feynman path integral whose minimum is $a = 0$. This potential is given by

$$\exp\left[-\int d^4x V(a)\right] = \int DA_\mu \prod_i Dq^i D\bar{q}^i \exp\left[-\int d^4x \left(\mathcal{L}_{QCD} + \frac{\alpha_s}{8\pi f_a} a G^a_{\mu\nu} \tilde{G}^{a\mu\nu}\right)\right],$$

and its explicit expression has been presented in [28] as follows

$$V(a) \sim f_\pi^2 m_\pi^2 \left[1 - \cos\left(\frac{a}{f_a}\right)\right]; \quad (2.4)$$

where $f_\pi$ and $m_\pi$ are the pion’s coupling constant and mass respectively. Expression (2.4) implies that the minima is at $a = 0$, solving the CP problem. At the same time, the lack of flatness of the potential causes the axion to develop a mass of order

$$m_a \sim \frac{f_\pi m_\pi}{f_a}. \quad (2.5)$$

Although results given above take into account the colour interaction, they should be supplemented with the CP violating terms of the weak interaction. This produces a very tiny but non zero value $\theta_{eff}$.

There are several axion scenarios discussed in the literature [29–38]. In some of them, the axion does not interact directly with the ordinary quarks, but through the gluons and the coupling $aG\tilde{G}$ is interpreted as an effective interaction. In the diagram presented in figure 1, the triangle is composed by a new heavy quark $Q$, giving rise to an effective interaction of the form $f_a^{-1} aG\tilde{G}$. In this case, the axion parameter $f_a$ is a function of the mass $m_Q$ of this new quark.

In the upcoming analysis, we will consider as presented in [37,38], the addition of the following terms corresponding to the wave function $\psi$ of a supermassive quark $Q$ coupled to a scalar field $\varphi$, to the Lagrangian of the theory of QCD

$$\mathcal{L}_{add} = i\bar{\psi} \not{D} \psi - (\delta \bar{\psi}_R\varphi \psi_L + \delta^* \bar{\psi}_L\varphi^* \psi_R) + (\partial_\mu \varphi^*) (\partial_\mu \varphi) + m^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2. \quad (2.6)$$

The first term $i\bar{\psi} \not{D} \psi$ includes the kinetic energy of the new quark and its coupling with the gluons; the parameters $\lambda$, $m$ and $\delta$ are to be determined. Since the new scalar field shows a non-vanishing
vacuum expectation value $|\langle \varphi \rangle| = m/\sqrt{2\lambda} \equiv \varphi_0$, it acquires a mass $\sqrt{2}m$. Furthermore, the last-mentioned v.e.v. rises a mass for the heavy quark given by $m_\psi = \delta \varphi_0$. In addition, there is a massless pseudoscalar $a$ defined by

$$\varphi = (\varphi_0 + \rho) \exp \left( i \frac{a}{\varphi_0 \sqrt{2}} \right).$$

\[ (2.7) \]

The $\rho$ field describes the radial excitations and $a$ the angular ones. The pseudoscalar $a$ is identified with the axion and it is the Goldstone boson associated to the breaking of the U(1)$_{PQ}$ symmetry transformation of the Lagrangian (2.6):

$$\psi \rightarrow e^{i\gamma_5 \alpha} \psi, \quad \varphi \rightarrow e^{-2i\alpha} \varphi,$$

\[ (2.8) \]

This axion field does not interact at the tree level with the light quarks and gluons, but acquires an effective interaction with the last field due to the diagram shown in figure 1. The resulting interaction, then, is determined by the quark loop and thus, the effective Lagrangian has the form

$$\frac{\alpha_s}{8\pi \sqrt{2}\varphi_0} a G^a_{\mu\nu} \tilde{G}^{a\mu\nu}.$$

From here it follows that the axion coupling constant is related to the vacuum expectation value according to $f_a = \sqrt{2}\varphi_0$. This result implies that the mass of the quark $\psi$ is proportional to $f_a$: so the heavier the quark is, the lighter the axion will be.

Whether an axion mechanism which solves the CP problem in QCD exists or not is an open question, but the particular scenario presented above will be helpful for constructing our model.

3. Proposed model

3.1 Preliminaries

In order to describe the present energy density of the universe, it is of interest to find mechanisms giving rise to extremely light particles whose characteristic time $t_m$ is of the order of the universe’s age. As is well known in classical cosmology, the Hubble parameter is related to the critical density $\rho_c$ and to the Newton constant $G_N$ by means of the Friedmann classical equation

$$H^2 = \frac{8\pi}{3} G_N \rho_c.$$  \[ (3.9) \]

Furthermore, the Hubble constant today can be parametrized as

$$H_0 \simeq \frac{M^2}{M_{Pl}}$$  \[ (3.10) \]

with $M \sim 10^{-2} - 10^{-3}$ eV, which is a value not far from the mass of a light neutrino $m_\nu$. This equation is expressed in natural units $\hbar = c = 1$; making explicit the factor $c/\hbar^2$ on the right hand side, the equation manifests its quantum behaviour. The left hand side of (3.10) is the result of experimental cosmological observations, while the right one involves the lightest and the heaviest scale known in physics. A dynamical interpretation of this numerical relation may be given by modelling the energy density of the universe with an extremely light particle $a$ with mass $m_a \simeq H_0$. Let us assume that this particle represents a large fraction of the dark energy. The numerical value of (3.10) in natural units is

$$m_a \sim 10^{-32} - 10^{-34} \text{ eV}.$$  \[ (3.11) \]

\[ ^1 \text{In fact, interesting relations between } H_0 \text{ and the pion mass } m_\pi \text{ have been pointed out in } [39] \text{ and worked out further in } [40]. \]
The characteristic time scale, \( t_a = 1/m_a \sim 10^9 - 10^{10} \) yrs for such particle is close to the estimated age of the universe. Thus, if such particle exists, together with a suitable component which cancels the QFT vacuum contribution, it will be the predominant component of the energy of the universe.

A dynamical interpretation of these relations can be found in terms of a hidden sector of the type described in section 2. Scenarios like those, implies the existence of a pseudoscalar \( a \), the axion, under the influence of an effective potential of the form

\[
V(a) \sim M^4 \left[ 1 - \cos \left( \frac{a}{f_a} \right) \right],
\]

(3.12)

which is analogous to (2.4). In the following, the notation \( a \) for this hidden axion will be employed, although this pseudoscalar should not be identified with the QCD invisible axion. The quantity \( M \) has energy density units and can hence be considered as \( M \sim (\rho_c)^{1/4} \). The axion mass \( m_a \) is related to the scale \( M \) appearing in (3.12) through the relation

\[
m_a^2 \sim \frac{M^4}{f_a^2} \sim 10^{-64} \text{eV}^2.
\]

(3.13)

In the last step (3.11) was taken into account. The combination of \( M \sim (\rho_c)^{1/4} \) and (3.13) gives a value for \( f_a \) of the order of the Planck mass, that is, \( f_a \sim 10^{19} \) GeV. In fact, \( (\rho_c)^{1/4} \) is not far from a light neutrino mass \( m_\nu \). This values has already been considered in the context of the solar neutrino problem \[41,42\] and it is also taken into account in \[43\]. In any case, a field with such characteristics is frozen by the Hubble constant along almost all the cosmic history and at the present times will be rolling to the minimum of its potential \[44\]. The initial value of the field, of course, determines the dynamics today. Although this may imply a fine tuning for the initial conditions, the fact that the potential is periodic softens the problem, since there is an appreciable fraction of the range \( (0, 2\pi a) \) of \( a \) which gives a viable dynamic at the present era.

The next goal is to make explicit a scenario with these characteristics. As a preliminary step, consider a hidden sector that contains two hidden fermions \( u \) and \( d \), that are not necessarily the ordinary quarks. It is assumed that these particles constitute an SU(2) doublet

\[
D_1 = \begin{pmatrix} u \\ d \end{pmatrix},
\]

which is invariant under an SU(2) local gauge symmetry. This implies the introduction of three hidden “gluons” \( G_i \) with \( i = 1, 2, 3 \). The masses \( m_u \) and \( m_d \) of these particles are assumed to be very tiny, of the order of an ordinary neutrino mass \( m_\nu \). In addition we add another doublet

\[
D_2 = \begin{pmatrix} s \\ t \end{pmatrix},
\]

composed by two heavy fermions \( s \) and \( t \) with masses \( m_s \) and \( m_t \) of the order of \( M_{GUT} \) or even \( M_{Pl} \). At this point we are working in analogy with the KSVZ axion \[37,38\], but contrary to what we are proposing, in this axion mechanism the heavy fermions are playing the role of the heavy quark and the light fermions the role of the ordinary quarks. The gluons \( G_i \) are assumed to be massless and the masses of the fermions are obtained through an spontaneously symmetry breaking mechanism with two different Higgs like particles \( \Phi_1 \) or \( \Phi_2 \). These two Higgs do not participate in the SU(2) gauge interaction, otherwise the SU(2) gluons would become massive. As a consequence, the two pseudoscalars \( a \) and \( b \) are defined as

\[
\Phi_1 = (v_1 + h_1) e^{i \frac{a}{f_a}} \quad \text{and} \quad \Phi_2 = (v_2 + h_2) e^{i \frac{b}{f_b}}.
\]
and can not be gauged out, so they become physically relevant. The resulting Lagrangian that holds the properties listed above is

\[ \mathcal{L} = i \overline{D}_1 \nabla D_1 + i \overline{D}_2 \nabla D_2 + f_u \overline{D}_1 L \Phi_1 u_R + f_d \overline{D}_2 L \Phi_1 d_R + f_s \overline{D}_2 L \Phi_2 s_R + f_t \overline{D}_2 L \Phi_3 t_R + h.c. \]

\[ + V(\Phi_1) + V(\Phi_2) + \frac{1}{4\pi} G_{\mu\nu} \cdot G^{\mu\nu} + \frac{\theta_2}{4\pi} G_{\mu\nu} \cdot \tilde{G}^{\mu\nu}. \]  

(3.14)

The covariant derivative is defined as \( \nabla_\mu = \partial_\mu + i g \sigma_\mu \cdot G \) and \( f_i \) and \( g \) are dimensionless coupling constants. A potential term of the form

\[ V(\Phi_i) = \frac{\lambda_i}{4} (\Phi_i \Phi_i^*)^2 - \frac{m_i^2}{2} \Phi_i \Phi_i^* \]  

(3.15)

is taken into account, where \( \lambda_i \) and \( m_i \) are the corresponding Higgs coupling constant and mass respectively. The notation \( \Phi_i = i \sigma_2 \Phi_i^* \) has been introduced. The mass terms for the fermions are all of Dirac type and \( \theta_h \) is due to the effect of the instantons of the non-Abelian theory.

The SU(2) gauge interaction will be assumed so as to be unified with ordinary QCD at a very large scale \( M_0 \), of the order of \( M_{GUT} \) or \( M_{PL} \). Scenarios with different confining scales were considered in the literature of schizons, for example \cite{41,42}. Renormalization group arguments show that this interaction confines at a scale \( \Lambda_h \) given by

\[ \Lambda_h \sim \tilde{M} \left( \frac{\Lambda_{QCD}}{M} \right)^{\alpha}, \]

with \( \alpha \) an exponent which is close to 3/2. Taking into account that \( \Lambda_{QCD} \sim 0.1 \) GeV, if \( \tilde{M} \) is of the order of \( M_{GUT} \) or even of the Planck scale, then it follows that the confining scale is comparable to \( \Lambda_h \sim 0.01 \) eV. Clearly this result resembles the mass of a light neutrino \( m_\nu \). The \( \theta_h \) term is cancelled by the Goldstone pseudoscalar \( a \) as in ordinary QCD axion models. Moreover, in these models, the mass of the axion is usually given by

\[ m_a \sim \frac{m_{u,d} (\overline{u} + \overline{d})}{f_a}, \]

where it is a reasonable to assume that the mean values \( (\overline{u} + \overline{d}) \) are close \( \Lambda_h \). Note that both this scale and \( m_u \) or \( m_d \) are of the order of a light neutrino. The axion thus gets a small mass:

\[ m_a \sim \frac{m_{\nu}^2}{f_a} \sim 10^{-32} \text{ eV}. \]

Henceforth, the condition (3.13) is reproduced by this scenario. This means that the Compton wavelength associated to this mass is comparable to the radius of the observable universe and that this axion describes a considerable fraction of the energy density of the present universe.

**3.2 The full Lagrangian**

The next step is to modify this model in order to include stable superheavy particles. This type of particles should be stable and may act as ultra massive dark matter. Consider an analogy of the previous model (3.14), but now with a local gauge symmetry SU(2)\(_L\). The right handed part is, for the moment, inert. We include Majorana type of mass terms, which result in a seesaw mechanism giving rise simultaneously to a very small and very large mass eigenvalues for these fermions. A preliminary Lagrangian is

\[ \mathcal{L} = i \overline{D}_1 \nabla D_1 + i \overline{D}_2 \nabla D_2 + i \overline{\pi}_R \partial_\mu \pi_R + i \overline{\sigma}_R \partial_\mu \sigma_R + i \overline{\tau}_R \partial_\mu \tau_R + f_u \overline{D}_1 L \Phi_1 u_R + f_d \overline{D}_2 L \Phi_1 d_R + f_s \overline{D}_2 L \Phi_2 s_R + f_t \overline{D}_2 L \Phi_3 t_R + h.c. \]

\[ + V(\Phi_1) + V(\Phi_2) + \frac{1}{4\pi} G_{\mu\nu} \cdot G^{\mu\nu} + \frac{\theta_2}{4\pi} G_{\mu\nu} \cdot \tilde{G}^{\mu\nu} + \delta_\Phi \Phi_3 \Phi_4 C_R \]

\[ + \frac{1}{4\pi} G_{\mu\nu} \cdot G^{\mu\nu} + \frac{\theta_4}{4\pi} G_{\mu\nu} \cdot \tilde{G}^{\mu\nu}. \]  

(3.16)
where $C$ is the charge conjugation matrix; a typical mass term of the theory is for instance

$$\overline{M} = \begin{pmatrix} 0 & m_u \\ m_u & M_u \end{pmatrix},$$

where $m_u = f_u v_1$ and $M_u = \delta_u v_3$, with $v_1$ the minimum of the potential for $\Phi_i$. The terms for $d$, $s$ and $t$ fermions have a similar form. If $M_u \gg m_u$ the approximate eigenvalues are

$$\lambda_1^u \sim M_u \quad \text{and} \quad \lambda_2^u \sim \frac{m_u^2}{M_u},$$

and the mass eigenstates are

$$u_1 = \frac{1}{\sqrt{1 + \left(\frac{M_u}{m_u}\right)^2}} u_L + \frac{M_u}{m_u} \frac{1}{\sqrt{1 + \left(\frac{M_u}{m_u}\right)^2}} u_R,$$

$$u_2 = \frac{1}{\sqrt{1 + \left(\frac{M_u}{m_u}\right)^2}} u_R - \frac{M_u}{m_u} \frac{1}{\sqrt{1 + \left(\frac{M_u}{m_u}\right)^2}} u_L.$$

Correspondingly, if the previous mass limit is satisfied then $u_1 \sim u_R$ and $u_2 \sim u_L$. As it will be shown below, this will be the case for all the fermions of our scenario $u$, $d$, $s$ and $t$.

The model constructed above possess essentially an inert right handed sector, since the mixing effect between left and right components is not appreciable. A more attractive possibility is that the latter sector is weakly interacting with the ordinary one. At first sight, the particles which constitute these particles may arise as events above the GKZ boundary. With this goal in mind let us include SU(2)$_R$ local gauge symmetry in (3.16), which implies the inclusion of three new hidden gauge bosons $W_i$. These particles are assumed to acquire masses of the order of $M_{GUT}$ through a spontaneously symmetry breaking mechanism. The Lagrangian (3.16) is then rewritten as

$$\mathcal{L} = i D_{1L}^G \overline{Y}^G D_{1L} + i D_{2L}^G \overline{Y}^G D_{2L} + i D_{1R}^W \overline{Y}^W D_{1R} + i D_{2R}^W \overline{Y}^W D_{2R}$$

$$+ f_u D_{1L} \Phi_1 u_R + f_d D_{1L} \Phi_1 d_R + f_s D_{2L} \Phi_2 s_R + f_t D_{2L} \Phi_2 t_R + V(\Phi_1) + V(\Phi_2)$$

$$+ \delta_u \Phi_3 u_R C_u R + \delta_d \Phi_3 d_R C_d R + \delta_s \Phi_3 s_R C_s R + \delta_t \Phi_3 t_R C_t R + V(\Phi_3) + V(\Phi_4)$$

$$+ \frac{1}{4\pi} G_{\mu \nu} \cdot G^{\mu \nu} + \frac{\theta_\mu}{4\pi} G_{\mu \nu} \cdot \tilde{G}^{\mu \nu} + \frac{1}{4\pi} W_{\mu \nu} \cdot W^{\mu \nu} + \frac{\theta_\mu}{4\pi} W_{\mu \nu} \cdot W^{\mu \nu} + \mathcal{L}_{W_{\mu \nu}}(W_i) + \mathcal{L}_{W_{gM_i}}(W_i).$$

The last two terms are the symmetry breaking terms, which give rise to the hidden bosons masses $m_{W_i}$ and the interaction terms between the latter and the ordinary sector of the Standard Model. This interaction is assumed to be very weak and hence difficult to observe. The covariant derivatives $\nabla_{\mu}^G$ and $\nabla_{\mu}^W$ correspond to the hidden gluons $G_i$ and the hidden massive bosons $W_i$ respectively.

The next step is to fix the scales of the model. The masses will be partially determined by the requirement that the Higgs particle corresponding to $\Phi_3$ has the lowest value in hidden massive sector, $m_{h_3}$. For this reason its decaying channel will be the one shown in figure 2. So as to achieve this purpose, it is convenient to choose $m_3 \sim 10^{13}$ GeV and $\lambda_3 \sim 1$ as the parameters in the potential (3.15) for the chosen Higgs. Then $m_{h_3} \sim 10^{13}$ GeV, which is close above the GKZ cut-off [24]. If we choose $\delta_u \sim \delta_d \sim 1$ then $M_u \sim M_d \sim 10^{13}$ GeV. By further adjusting of $\Phi_1$ we get $m_u \sim m_d \sim 10$ GeV. In these terms we obtain the mass eigenvalues

$$\lambda_1^u \sim \lambda_1^d \sim 10^{13} \text{ GeV}, \quad \lambda_2^u \sim \lambda_2^d \sim m_\nu.$$
Figure 2: The main decay channel of the hidden Higgs. The gauge fields $W_\mu$ correspond to a very weak interaction with the ordinary matter. This will be the lowest order decay diagram only if the masses corresponding to the fermion triangle and the $W_i$ bosons are larger than the $m_{h_3}$.

Consequently, in this case there is a very light eigenstate paired with an extremely large one. The Higgs particle corresponding to $\Phi_3$ has a mass of the order of the right sector masses $\lambda_u^i \sim \lambda_d^i$, but the parameters $\delta_u$ and $\delta_d$ can be chosen so that the value of $m_{h_3}$ do not exceed $\lambda_u^i$ and $\lambda_d^i$ but remain still of the same order. If this condition is not satisfied, the main decay diagram will be the one presented in figure 3 which gives rise to an unwanted (unobserved) short lived particle. There is also a constrain for the gauge bosons masses $m_W > m_{h_3}$, for instance $m_W \gtrsim M_{GUT}$; otherwise the main decay channel would be the ABJ diagram. This type of diagram, analogous to the one presented in the figure 1 but with $W_\mu$ bosons as the decaying product, would give rise to short mean lifetime Higgs particles as well. Taken everything into account, the decaying diagram presented in figure 2 is what we are looking for. For the second doublet we choose $m_s \sim m_t \sim M_{GUT}$ and $M_s \sim M_t \sim M_{Pl}$, so that the mass eigenvalues will be

$$\lambda_s^1 \sim \lambda_t^1 \sim M_{Pl}, \quad \lambda_s^2 \sim \lambda_t^2 \sim 10^{13} \text{ GeV}.$$ 

Thus, both components of this doublet are supermassive.

Before continuing, let's make a review of the main ideas presented in the model above: the left sector imitates de KSVZ invisible model and the resulting axion is an extremely light particle which acts as a false vacuum at present times. On the other hand, the right sector is supermassive with mass values between $10^{13}$ GeV and the Planck scale. This particles interacts with the ordinary Standard Model sector by the interchange of $W_\mu$ hidden vector bosons. We turn now our attention to the calculation of the Higgs mean lifetime to show that it is, in fact, extremely large. This suggest that the hidden Higgs $h_3$ decay is compatible with the generation of late high-energy cosmic rays above the GKZ limit.
Figure 3: Diagram corresponding to a short-lived scalar particle \( h_3 \). The decay of such particle gives rise to two fermions \( \psi \).

4. Hidden Higgs lifetime

4.1 Estimation of the decay amplitude

As has been exposed in the previous section, the leading decay mechanism for the hidden Higgs is the diagram of the figure 2. The amplitude for such a decay is given as follows

\[
\mathcal{M} = \frac{\delta_F (g_W)^4}{2\pi^8} \sum_{ijk} \int d^4k_1 d^4k_2 \frac{\eta_0 d\eta_0 (\gamma^d)^m j (\gamma^j)^k \bar{\psi}_e \psi_m (\bar{k}_1 + m)_{jk}}{[k_2^2 - m_{F_i}^2] [k_1^2 - m^2] (p_2 - k_1)^2 (p_1 - k_1)^2} \\
\times \left[ \frac{tr\{[\not{p}_1 - \not{k}_1 - \not{k}_2 + m_{F_i}] \gamma^\nu [\not{p}_2 - \not{k}_1 - \not{k}_2 + m_{F_i}] (\not{k}_2 + m_{F_k}) \gamma^\mu \}}{(p_1 - k_1 - k_2)^2 - m_{F_i}^2} \right].
\]

(4.18)

In the integrand, \( m \) is a typical mass of a light fermion in the usual Standard Model and \( m_{F_i} \) is the mass of a left hidden heavy fermion of the proposed model. The latter masses are at least of the order of \( 10^{13} \) GeV. \( \delta_F \) represents the coupling constant between the Higgs and the fermions of the hidden sector and \( g_W \), the interaction strength between the two sectors: the hidden and the visible one. As we have discussed previously, the interaction between sectors is weak enough so as not to be observed up to the present times. Nevertheless, we will show that with reasonable values it leads to a stable Higgs particle. At this stage it has to be pointed out that the large masses \( m_W \) of the mediating fermions have been neglected. The main reason is that the mean lifetime of the decay is expected to increase as long as the internal masses grow, since the decay becomes less probable. This means that \( \mathcal{M} \) must decrease as \( m_W \) is increased. Thus, the expression at zero mass (4.18) is an upper bound for the original decay amplitude. With this simplification, the estimation becomes less cumbersome. Hereinafter, under this assumption, the mean lifetime to be computed will be a lower limit, so that if this value turns to be larger than the age of the universe, the decay of such particles will not have been produced yet.

At first sight, the integral (4.18) is logarithmically divergent. The numerator is quartic and the loop contributes with two 4-dimensional momentum integrals, while the denominator is of degree twelve. However, when the trace is expanded it is found that

\[
tr \left[ (\not{p}_1 - \not{k}_1 - \not{k}_2 + m_{F_i}) \gamma^\mu (\not{p}_2 - \not{k}_1 - \not{k}_2 + m_{F_i}) (\not{k}_2 + m_{F_k}) \gamma^\nu \right] = (\not{p}_1 - \not{k}_1 - \not{k}_2) \xi (\not{p}_2 - \not{k}_1 - \not{k}_2) \eta \times \left[ \gamma^\xi \gamma^\mu \gamma^\eta \gamma^\delta \gamma^\nu \right] + m_F (\not{p}_1 - \not{k}_1 - \not{k}_2) \xi (\not{p}_1 - \not{k}_1 - \not{k}_2) \eta tr \left[ \gamma^\xi \gamma^\mu \gamma^\eta \gamma^\nu \right] + ...
\]
where the dots denote terms with lower powers in the momentum. The first term of this expansion is cancelled out, because it is multiplied by the trace of a product of an odd number of gamma matrices. This means that the numerator is cubic and not quartic in momenta and the integral is convergent instead of logarithmically divergent. Before making the estimation of this integral, it is be convenient to compare our situation with some results of the literature.

Comparison with an axion mechanism due to Kim \[38\]: In order to understand the behaviour of (4.18) with respect to the mass parameters of the model, consider first the situation in which the masses corresponding to the fermion triangle are very large values \(m_F \to \infty\). Then it is reasonable to expect that the decay probability will decrease to zero and therefore the mean lifetime \(\tau\) will becomes infinite in this limit. In other words, we expect

\[
\lim_{m_F \to \infty} \tau \to \infty .
\]

(4.19)

On the other hand, it is also feasible to expect that when \(m_{h_3}\) increases to \(m'_{h_3}\), the decay will becomes more probable and the mean lifetime will decreases, that is

\[
m_{h_3} < m'_{h_3} \implies \tau(m_{h_3}) > \tau(m'_{h_3}) .
\]

(4.20)

Instead, the behaviour of the amplitude with respect to the mass \(m\) of the products of the decay is complexer. A decrease of this mass is equivalent to an increase of the masses \(m_F\) and \(m_{h_3}\) with their ratio \(R = m_F/m_{h_3} > 1\) fixed. Following (4.20), the increase of the mass of the Higgs results in a shorter lifetime. However (4.19) implies that the increment on the loop triangle mass enlarges the lifetime of the boson. As a consequence, in the fixed ratio limit it is not clear which of the two effects prevails, if any. For instance, the work of Kim considers essentially the same diagram than the one presented in figure 4 which bears several analogies with our channel decay presented in figure 2. In the present case, an axion replaces the hidden Higgs, a heavy quark \(Q\) replaces our hidden fermions and the \(W_\mu\) bosons are replaced by the QCD gluons. The charge conservation in the outgoing vertices forces to replace the internal quark line for a mediating boson \(W_\mu\). Kim’s diagram induces an effective coupling between the decaying axion and the ordinary quarks whose schematic behaviour is

\[
g_{eff} \simeq g_k \frac{m_q}{m_Q} \ln \left( \frac{m_Q}{m_q} \right) .
\]

(4.21)

\[\text{Figure 4: Diagram which describes the effective coupling between the KSVZ invisible axion and the QCD quarks } q.\]

The triangle lines corresponds to a hidden massive quark \(Q\), which is a singlet under the electroweak interaction.
Here $m_Q$ is the mass of the heavy quark and $m_q$ is the mass of any ordinary quark resulting from the decay. In this expression, it is found that $g_{\text{eff}} \to 0$ when $m_Q \to \infty$ which is what intuition suggests. However it is also be seen that $g_{\text{eff}} \to 0$ when the masses $m_q \to 0$, which is not obvious at first sight. These conditions imply that the mean lifetime of the axion becomes infinite in this limit, since there is no coupling and thus no decay. In other words, the lighter the ordinary quarks are, the larger the mean lifetime of the axion would become.

It should be stressed that the result of Kim’s calculation can not be applied directly to the present situation: the main difference between Kim’s decay model described above and the one we are presenting is that in the fist one, the axion is a pseudoscalar while in the latter the hidden Higgs is a scalar. Another discrepancy is the difference between the quark-boson internal line. However, it would be expected that these differences will not alter the amplitude’s behaviour on the mass $m$ of the products of the decay. In other words, we will postulate that

$$
\lim_{m \to 0} \tau \to \infty \quad (4.22)
$$

as in the Kim model. Owing to this technical problem, an independent estimation of the hidden Higgs lifetime will be done in the next subsection by taking into account the three conditions (4.19), (4.20) and (4.22) and the expression (4.21) as a partial guide.

## 4.2 Estimation of the mean lifetime of the hidden Higgs

In order to obtain a lower limit for the mean lifetime $\tau = \Gamma^{-1}$ of the Higgs boson $h_3$, a reasonable upper value of $|M|$ is needed. The decay width can be computed integrating with respect to the momentum of the outgoing particles $p_1$ and $p_2$ in a rest frame for $h_3$ in the subsequent equation

$$
d\Gamma = \frac{1}{m_{h_3}} \sum_{s_1,s_2} |M|^2 \frac{d^3p_1 d^3p_2}{(2\pi)^2 E_1 E_2} \delta^4(p_1 + p_2 - q) . \quad (4.23)
$$

The energies $E_i$ correspond to the outgoing quarks. On the other hand (4.18) indicates that $M$ is proportional to $\delta_F g_W^4$. Under the mass limit $m_{h_3} \gg m$ and taking into account the fact that $\Gamma$ has mass dimensions in natural units, it follows from (4.23) that

$$
\Gamma \simeq \alpha_F \alpha_W^4 m_{h_3} \mathcal{F}(m; m_F; m_{h_3}) . \quad (4.24)
$$

The decay constants verify $4\pi \alpha_F = \delta_F^2$ and $4\pi \alpha_W = g_W^2$ and the function $\mathcal{F}(m, m_F, m_{h_3})$ should be dimensionless, otherwise $\Gamma$ would have wrong mass units. This means that

$$
\mathcal{F}(m; m_{h_3}; m_F) = f(x_1; x_2)
$$

where the dimensionless parameters

$$
x_1 = \frac{m}{m_F} \quad \text{and} \quad x_2 = \frac{m}{m_{h_3}} \quad (4.25)
$$

have been introduced. Furthermore, the conditions (4.19) and (4.22) imply that

$$
\lim_{x_1 \to 0} f(x_1; x_2) \to 0 .
$$

At this point it is convenient to bear in mind Kim’s formula (4.21). The decay rate for the Kim’s axion is proportional to $g_{\text{eff}}^2$ and thus proportional to $m^2/m_F^2 \log^2(m_F/m)$. This is expected to occur in the present case, then

$$
f(x_1, x_2) = g(x_1; x_2) \log^2 \left( x_1^{-1} \right) , \quad (4.26)
$$
with \( g(x_1; x_2) \) a function of the dimensionless variables \((4.25)\). Since in the limit \( m_F \to \infty \) the parameter \( x_1 \to 0 \), as the decay rate should vanish by \((4.19)\) it follows that \( g(x_1; x_2) \to 0 \) faster than \( \log^{-2}(x_1^{-1}) \). Let us assume for the time being that this function is analytic with respect to \( x_2 \), although further on this assumption will be relaxed. Since \( x_1 \ll 1 \) a Taylor expansion of \( g(x_1; x_2) \) may be performed in \((4.26)\) and would lead us to the first non-vanishing term

\[
f(x_1, x_2) \simeq h \left( \frac{m}{m_{h_3}} \right) \frac{m}{m_F} \log^2 \left( \frac{m_F}{m} \right).
\]

Plugging this result back in \((4.24)\) we obtain

\[
\Gamma \simeq \alpha_F \alpha_W^4 m_{h_3} h \left( \frac{m}{m_{h_3}} \right) \frac{m}{m_F} \log^2 \left( \frac{m_{h_3} R}{m} \right).
\]

Still the new function \( h(x_2) \) can be further characterized. As we have discussed between \((4.19)\) and \((4.22)\), the limit \( m \to 0 \) is equivalent to take the masses \( m_{h_3} \) and \( m_F \) to infinite with their ratio fixed \( R = m_F/m_{h_3} > 1 \). The condition \((4.22)\) implies that in this limit the lifetime goes to infinite and thus \((4.27)\) should go to zero. Replacing \( m_F \) by \( m_{h_3} R \) in \((4.27)\), it follows that the expected limit is

\[
\lim_{m_{h_3} \to \infty} m \frac{h}{R} \left( \frac{m}{m_{h_3}} \right) \log^2 \left( \frac{m_{h_3} R}{m} \right) \to 0.
\]

This means that \( h(x_2) \to 0 \) when \( x_2 \to 0 \) faster than \( \log^{-2}(x_2^{-1}) \). If we suppose this function is analytic in the variable \( x_2 \) then, we fulfill the conditions to make a Taylor expansion in this parameter. Consequently, \((4.27)\) may be approximated by

\[
\Gamma \simeq c \alpha_F \alpha_W^4 \frac{m^2}{m_F} \log \left( \frac{m_F}{m} \right),
\]

with \( c = h'(0) \). It is important to remark that the expected dependence on \( m^2 \) has been obtained, namely the decay is less probable as \( m \to 0 \).

The calculation for \((4.28)\) given above was based on the assumption that \( g(x_1; x_2) \) is analytic in \( x_2 \). The fact that the resulting decay rate \((4.28)\) does not depend on the mass \( m_{h_3} \) of the decaying particle, forces us to analyse the case where the analyticity condition is relaxed without loosing sight of the \( m^2 \) dependence\(^2\). It may be reasonable to postulate that

\[
g(x_1; x_2) = c x_1^\beta x_2^\gamma = \frac{c m^2}{m_{h_3}^3 m_F^3},
\]

with \( \beta + \gamma = 2 \) since \( g(x_1; x_2) \) should be dimensionless. Note that the case \( \beta = \gamma = 1 \) is the one considered recently. Other values corresponds to \( \beta < 1 \) or to \( \gamma < 1 \), so that \( g(x_1; x_2) \) will not be analytic in the corresponding variable \( x_1 \) or \( x_2 \). With the assumption \((4.29)\) the decay rate \((4.24)\) becomes

\[
\Gamma \simeq c \alpha_F \alpha_W^4 \frac{m^2}{m_{h_3}^3 m_F^3} \log^2 \left( \frac{m_F}{m} \right).
\]

The exact values of the powers \( \beta \) and \( \gamma \) are not fix by the theory. Nevertheless, since it has been assumed that \( m_F \) and \( m_{h_3} \) are of the same order of magnitude one can use that \( m_F \simeq m_{h_3} \) in \((4.30)\). Recalling that \( \beta + \gamma = 2 \), if follows that the decay width given by \((4.30)\) is essentially the same as the one computed in \((4.28)\). There is no real constrain on the value of the slope \( c \), but it seems reasonable to expect that it will not take values much larger than the unity.

\(^2\)Note that higher powers such as \( m^4 \) will lead to a larger mean lifetime, since the powers of the denominator in \( \Gamma \) should also increase by dimensional arguments.
To conclude, let us give numerical values to the formula (4.30). To compute the mean lifetime limit that the formula predicts let us take the value $g_W \sim 10^{-3}$ for the coupling constant and $\delta_F \sim 1$ with the masses given by the values $m_F \sim m_{h_3} \sim 10^{13}$ GeV as in section 3. The result is

$$\tau \sim 10^{11} \text{ yrs}$$

which is of the order of the age of the universe. We should remark that this is an underestimation, the mean lifetime can be considerably larger. Note that the value of $g_W$ can be considerably smaller since we are suggesting a very weak interaction between the ordinary and the hidden sectors. In addition, the effect of the masses of the $W_\mu$ bosons has not been considered here. If this effect would have been taken into account, as we have discussed above, the mean lifetime would be even larger. In conclusion, the mean lifetime for the hidden Higgs $h_3$ predicted by this model is of the order or even larger than the age of the universe. This makes plausible the existence of supermassive particles, whose masses are of the order of $10^{13}$ GeV and are present at our era. The products of the decay of such particle may arise as late high-energy cosmic rays above the GKZ limit.

5. Discussions and open perspectives

In the present work the possibility of modelling the vacuum energy density of the present universe in terms of the potential energy of an axion pseudoscalar $a$ was considered. The hypothetical axion of the model presented here is not identified with a QCD invisible one; instead, it is a pseudo-Goldstone boson corresponding to a U(1) global symmetry which is spontaneously broken at a scale $f_a$ of the order of the Planck mass $M_{Pl}$. The hidden axion acquires a small mass due to non-zero fermion condensates $\langle \bar{f}f \rangle$ corresponding to a SU(2)$_L$ gauge interaction whose unification is produced with QCD at around the same large scale. It turns out that this theory is confining at a very low energy scale, of the order of the mass $m_\nu$ of an ordinary neutrino. The axion acquires a mass of around $10^{-32}$ eV, thus it becomes the lightest massive particle in the universe. Its Compton wavelength is of the order of the Hubble radius and therefore this component represents a considerable fraction of the critical energy density $\rho_c$, since its relaxation time is of the order of the age of the present universe.

The hidden scenario yielding this axion is composed by supermassive particles and very light particles, whose masses arise from an spontaneous symmetry breaking together with a suitable seesaw mechanism, which induces very large and very tiny mass eigenvalues for the fermion sector. In addition, this scenario gives rise to a stable superheavy particle, a hidden Higgs, with a mass of the order of $10^{13}$ GeV, which may be present at our era. The particles composing this hidden sector have mass values which match the ones described in [45], that is, weakly interacting matter with masses near or above the order of the GUT scale but with no need of considering transplanckian physics as in [46]. A determinant proof on whether such supermassive particles exist or not in the present universe is beyond the scope of this work. There are discussions in the context of supersymmetric models that suggest that they may exist [47]. However, our work is focused in characterizing the mechanisms which, if these hypothetical particles do exist, guarantee their stability. An interesting feature is that such particles may generate events above the GKZ bound at the present universe and this is a possibility to be analysed further.

The model presented here assumes that the quantum field theory vacuum energy contributions are not present, perhaps due to a suitable adjustment mechanism such as [16–21]. The motivation of this work is to propose a common scenario giving rise to high-energy cosmic rays with stable superheavy particles and simultaneously, modelling the universe’s energy density. It may be a valuable task to identify supersymmetric models or superstring compactifications which encompass these type of scenarios. We leave this for a future investigation.

This assumed value has the same order of the electroweak interaction, despite the fact that a lower value for the strength interaction is expected.
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