Two static charged masses in the external field

J Sánchez-Mondragón¹*, V S Manko¹ and E Ruiz²

1 Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, A.P. 14-740, 07000 México D.F., Mexico
2 Instituto Universitario de Física Fundamental y Matemáticas, Universidad de Salamanca, 37008 Salamanca, Spain
E-mail: jsanchez@fis.cinvestav.mx

Abstract. We consider a binary system of magnetically charged black holes in the external uniform magnetic field by applying the Harrison transformation to the magnetic analog of the double–Reissner–Nordström solution. The balance of subextreme components is only possible in the case of asymmetric black diholes – configurations composed of two non-extreme black holes possessing unequal masses and charges equal in magnitude but opposite in sign.

1. Introduction

In 2000 Emparan demonstrated [1] that the well–known Bonnor magnetostatic solution [2] can be interpreted as describing a system of two extreme black holes with opposite magnetic charges. He also suggested the name ‘black diholes’ for the two–body configurations consisting of separated equal black holes carrying opposite charges and showed that the extremal constituents in the Bonnor–type dihole can be balanced by placing the latter into the external uniform magnetic field [3, 4] via the Harrison transformation [5].

In the present communication we shall consider a system of two arbitrary magnetically charged static black holes. The external magnetic field will be shown to ensure equilibrium of the black–hole constituents only in the case of asymmetric black diholes – configurations composed of two black holes with unequal masses and charges equal in magnitude but opposite in sign.

2. The double–Reissner–Nordström solution in magnetostatics

The magnetic analog of the general double–Reissner–Nordström solution is defined by the Ernst potentials \( \mathcal{E} \) and \( \Phi \) [6] of the form [7]

\[
\mathcal{E} = \frac{A - B}{A + B}, \quad \Phi = \frac{iC}{A + B},
\]

\[
A = \sigma_1 \sigma_2 \left[ \nu (Q_+ + Q_-) (r_+ + r_-) + 4 \kappa (Q_+ Q_- + q_+ q_-) \right] - (\mu^2 - 2 \kappa^2) (R_+ - R_-) (r_+ - r_-),
\]

\[
B = 2 \sigma_1 \sigma_2 \left[ (\nu m_2 + 2 \kappa m_1) (Q_+ + Q_-) + (\nu m_1 + 2 \kappa m_2) (r_+ + r_-) \right] + 2 \sigma_2 \left[ \nu (q_1 - \mu) - 2 \kappa (q_1 + \mu q_2 - \mu^2) (Q_+ - Q_-) \right] + 2 \sigma_1 \left[ \nu (q_2 + \mu) + 2 \kappa (q_2 + \mu q_1 - \mu^2) (Q_+ - Q_-) \right],
\]

\[
C = 2 \sigma_1 \sigma_2 \left[ \nu (q_2 + \mu) + 2 \kappa (q_1 - \mu) (Q_+ + Q_-) + \nu (q_1 - \mu) + 2 \kappa (q_2 + \mu) (r_+ + r_-) \right] + 2 \sigma_2 \left[ \nu m_1 + 2 \kappa (m_2 - Q_1 + \mu Q) (Q_+ - Q_-) \right].
\]
in the case of the solution (1) have the following form involving the functions $A$, where $Q$, $f$

The metric coefficients $f$ and $\gamma$ from the Weyl line element

The corresponding magnetic potential $A_\varphi$ has the form [7]

while the corresponding magnetic potential $A_\varphi$ has the form [7]

3. Asymmetric black diholes

In analogy with the paper [1], the application of the Harrison transformation to the magnetostatic solution considered in the previous section yields the new functions $\tilde{f}$, $\tilde{\gamma}$ and $\tilde{A}_\varphi$ of the form

where $Q \equiv q_1 + q_2$ is the total charge of the system.
which describe two magnetically charged masses in the external uniform magnetic field determined by the constant $B$.

In order to see whether the black–hole constituents can be balanced by the external field, it is necessary to consider for them the balance conditions which reduce to the requirement of the regularity of the parts of the symmetry axis outside the location of the sources, i.e.,

\[ \gamma_{I,II,III}(A) = 0, \]

where the segments $I$, $II$ and $III$ of the symmetry axis are defined as $\rho = 0$, $z > \frac{1}{2}R + \sigma_2$ (the upper part), $\rho = 0$, $\frac{1}{2}R + \sigma_1 < z < \frac{1}{2}R - \sigma_2$ (the intermediate part), and $\rho = 0$, $z < -\frac{1}{2}R - \sigma_1$ (the lower part), respectively. From the equations (1), (4) and (6) it follows that the system of the balance equations takes the form

\[
A_{\varphi}^{(I)} = 0, \quad \left(1 + \frac{1}{2}BA_{\varphi}^{(II)}\right) e^{2\gamma_{III}} = 1, \quad A_{\varphi}^{(III)} = 0, \tag{7}
\]

where $A_{\varphi}^{(I,II,III)}$ denote the values of the magnetic potential at the respective parts of the $z$–axis. On the other hand, from (5) we find that

\[
A_{\varphi}^{(I)} = 0, \quad A_{\varphi}^{(II)} = 2q_2, \quad A_{\varphi}^{(III)} = 2Q, \tag{8}
\]

which implies vanishing of the total charge $Q \Leftrightarrow q_1 = -q_2$. Then, solving the second equation in (7) for $B$, we obtain the value of the external magnetic field leading to the equilibrium:

\[
B = \frac{1}{q_2} \left(\sqrt{\frac{\nu + 2\kappa}{\nu - 2\kappa}} - 1\right), \quad Q = 0. \tag{9}
\]

The condition $Q = 0$ naturally singles out the family of asymmetric black diholes as the only static subextreme two–black–hole configurations which can be balanced by the external magnetic field. In this case, by setting $q_1 = -q_2 = q$ in (2), we get

\[
\sigma_1 = \sqrt{m_1^2 - q^2(1 - 2\mu)}, \quad \sigma_2 = \sqrt{m_2^2 - q^2(1 - 2\mu)}, \quad \mu := \frac{m_1 + m_2}{R + m_1 + m_2}, \nonumber
\]

\[
\nu = R^2 - m_1^2 - m_2^2 + 2q^2(1 - \mu)^2, \quad \kappa = m_1m_2 + q^2(1 - \mu)^2, \tag{10}
\]

while formulae of the previous section for the metric functions $f$, $\gamma$ and for the magnetic potential $A_{\varphi}$ can be rewritten as

\[
f = \frac{A^2 - 4B^2 + 4q^2C^2}{(A + 2B)^2}, \quad e^{2\gamma} = \frac{A^2 - 4B^2 + 4q^2C^2}{K_{0}^2R_{+}R_{-}r_{+}r_{-}}, \quad A_{\varphi} = \frac{q(I - 2\zeta C)}{A + 2B};
\]

\[
A = \sigma_1\sigma_2[\nu(R_+ + R_-)(r_+ + r_-) + 4\kappa(R_+R_- + r_+r_-)]
- (q^2\mu^2 - 2\kappa^2)(R_+ - R_-)(r_+ - r_-),
\]

\[
B = \sigma_1\sigma_2[(m_1\nu + 2m_2\kappa)(R_+ + R_-) + (m_1\nu + 2m_2\kappa)(r_+ + r_-)]
+ q^2(\mu^2 - 2\kappa^2)(\nu - 2\kappa)[\sigma_2(R_+ - R_-) - \sigma_1(r_+ - r_-)]
- 2\kappa\sigma_1\sigma_2(R_+ - R_-) - m_2\sigma_1(r_+ - r_-)];
\]

\[
C = \sigma_1\sigma_2[(1 - \mu)(\nu - 2\kappa)(r_+ + r_-) - R_+ - R_-)
+ \sigma_2[m_2\nu + 2\kappa(m_1\nu - R + R\mu)](r_+ - r_-)
+ \sigma_1[m_2\nu + 2\kappa(m_1\nu - R + R\mu)](r_+ - r_-),
\]

\[
R_{\pm} = \sqrt{\rho^2 + \left(z + \frac{1}{2}R \pm \sigma_1\right)^2}, \quad r_{\pm} = \sqrt{\rho^2 + \left(z - \frac{1}{2}R \pm \sigma_2\right)^2},
\]

\[
K_0 = 4\sigma_1\sigma_2[R^2 - (m_1 - m_2)^2 + 4q^2(1 - \mu)^2], \tag{11}
\]
with

\[ I = 4\sigma_1\sigma_2[R(1 - \mu) - \mu(m_1 + m_2)](m_1R_+R_- + m_2r_+r_-) \\
+ \mu(\nu - 2\kappa)[(\kappa + \mu^2q^2)(R_+ - R_-)(r_+ - r_-) - \sigma_1\sigma_2(R_+ + R_-)(r_+ + r_-)] \\
-(1 - \mu)[(R^2 - (\sigma_1 + \sigma_2)^2)(m_1\sigma_2 + m_2\sigma_1)(R_+R_- - R_-r_+)] \\
+[R^2 - (\sigma_1 - \sigma_2)^2](m_1\sigma_2 - m_2\sigma_1)(R_+r_+ - R_-r_-)] \\
+ \sigma_1[m_2\mu\nu + 2m_1\mu\kappa - 2R\kappa(1 - \mu)][2\sigma_2(r_+ + r_-) - R(r_+ - r_-)] \\
+ \sigma_2[m_1\mu\nu + 2m_2\mu\kappa - 2R\kappa(1 - \mu)][2\sigma_1(R_+ + R_-) + R(R_+ - R_-)] \\
- \sigma_1\sigma_2R(1 - \mu)(\nu - 2\kappa)(R_+ + R_- + r_+ + r_-) \\
- 2(\nu - 2\kappa)[\sigma_2[m_1R\mu + (\sigma_1^2 + 2q^2\mu^2)(1 - \mu)](R_+ - R_-) \\
- \sigma_1[m_2R\mu + (\sigma_2^2 + 2q^2\mu^2)(1 - \mu)](r_+ - r_-) \\
+ \mu\sigma_1\sigma_2[(m_1 - m_2)(r_+ + r_- - R_+ - R_-) + 2(\nu + 2\kappa)]]. \quad (12) \]

Lastly, the explicit expression for the parameter \( B \) at which balance occurs assumes the form

\[ B = -\frac{1}{q} \left( \sqrt{\frac{R^2 - (m_1 - m_2)^2 + 4q^2(1 - \mu)^2}{R^2 - (m_1 + m_2)^2}} - 1 \right). \quad (13) \]

We remark in conclusion that in the future paper we will generalize the results of the present work by including into the consideration the additional dilatonic field.

References

[1] Emparan R 2000 Phys. Rev. D 61 104009
[2] Bonnor W B 1966 Z. Phys. 190 444
[3] Bonnor W B 1954 Proc. Phys. Soc. Lond. A 67 225
[4] Melvin M A 1964 Phys. Lett. 8 65
[5] Harrison B K 1968 J. Math. Phys. 9 1744
[6] Ernst F J 1968 Phys. Rev. 168 1415
[7] Manko V S, Ruiz E and Sánchez-Mondragón J 2009 Phys. Rev. D 79 084024