Chiral Structure in Baryon Magnetic Moments

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We highlight recent advances in finite-range regularised chiral effective field theory. Application to baryon magnetic moments show an unambiguous signal of quenched chiral physics in FLIC fermion simulations.

1. INTRODUCTION

The issue of chiral extrapolation has been highlighted as an important problem facing modern lattice QCD simulations [1,2], if one is to make any comparison with experiment. Extrapolations in quark mass are nontrivial because of nonanalytic contributions which arise as a consequence of dynamically broken chiral symmetry in the QCD vacuum [3].

At typical lattice quark masses there is little evidence of nonanalytic variation in quark mass. Recently, the Kentucky group has reported observation of chiral nonanalytic behaviour in quenched simulations with overlap fermions by reaching pion masses as low as 180 MeV [4].

The lack of rapid, nonanalytic curvature at moderate quark masses is well understood through the use of a finite-range regulator (FRR) in chiral effective field theory. Finite-range regularisation provides a resummation of the chiral expansion which features the suppression of chiral loop effects with increasing quark mass and Goldstone boson source plays a role.

We first highlight the features of chiral extrapolation with the use of FRR chiral perturbation theory (χPT) in the case of the nucleon mass in dynamical QCD. We discuss the quenched chiral extrapolation of the magnetic moments of both nucleon and Delta baryons.

2. FRR CHIRAL EFFECTIVE THEORY

Chiral perturbation theory is a low-energy effective field theory (EFT) for QCD. This EFT provides a systematic expansion of QCD about the chiral limit. The key example here is that one can generate a quark mass expansion for the behaviour of the nucleon mass. It is therefore essential, for chiral extrapolations from moderate quark masses, to incorporate the rigorous properties of QCD near the chiral limit.

In the naive application of χPT one obtains a truncated Taylor series of the form

\[ m_N = c_0 + c_2 m_\pi^2 + c_3 m_\pi^3 + c_4 m_\pi^4 + c_4 L m_\pi^4 \log m_\pi + \ldots \]  \hspace{1cm} (1)

The terms nonanalytic in the quark mass \( m_\pi \propto m_\pi^2 \) are those arising strictly from pion loop integrals. They have coefficients which are model-independent constraints of QCD. The remaining analytic terms are determined by matching to non-perturbative QCD. In principle, it is the goal of lattice QCD to be able to constrain all coefficients — including the reproduction of the correct nonanalytic contributions. Until now, lattice simulations have not observed the rapid nonanalytic variation and one must fix the coefficients of these terms to their QCD values. For modern calculations, determination of the analytic terms enables extrapolation to the physical pion mass.

The application of the formal expansion in Eq. (1) has questionable applicability in the regime of modern QCD simulations, because the coefficients of higher order terms are very large and oscillate in sign. This lack of convergence can be overcome by using a FRR in chiral EFT. Chiral loop integrals are evaluated with a finite cutoff in momentum space. This procedure incorporates two additional light energy scales in the expansion — the N–∆ mass
splitting and the finite size of the source of the pion field. By incorporating both these scales and resumming the chiral expansion one then has an enhanced range of applicability of the effective field theory \[5\]. This resummation improves the convergence of the expansion to the point where the chiral extrapolation of modern lattice simulations can now be performed reliably \[9\].

By investigating a number of functional forms for the momentum-space cutoff of loop integrals in chiral EFT it is found that the extrapolated results show negligible sensitivity to the chosen form of regulator. Results of the extrapolation of CP-PACS lattice data \[10\] for four different regulators — sharp, monopole, dipole and gaussian — are shown in Figure \[11\] \[9\].

In summary, the resummation of the chiral series by the use of a FRR provides a remarkably robust chiral extrapolation for lattice simulations performed at moderate quark masses.

3. MAGNETIC MOMENTS

The study of magnetic moments on the lattice provides an enhanced opportunity to directly study chiral nonanalytic physics. In QCD the leading nonanalytic contribution to the nucleon magnetic moment is linear in \(m_\pi\) — compare \(m_\pi^2\) for the mass expansion, Eq. \[11\]. This contribution is large, nearly one third of the physical neutron’s magnetic moment. Such is the magnitude of the pion loop effects that one should expect dramatic results in lattice calculations as the light quark mass regime is probed.

Lattice simulations of the electromagnetic structure of baryons have so far been restricted to the quenched model of QCD \[11\] \[12\]. Because of the absence of \(q \bar{q}\) pair-creation, pion loop effects are typically suppressed in the quenched approximation \[13\] \[14\] \[15\]. The inclusion of the flavour singlet \(\eta'\) does offer the opportunity for detection of enhanced chiral behaviour in quenched simulations. As a result of double-hairpin loops the leading nonanalytic contribution is \(\log m_\pi\) — i.e. magnetic moments diverge in the chiral limit \[13\].

The Delta baryon offers a unique opportunity to study chiral loop effects in quenched QCD. The \(\Delta \to N\pi\) loop integral appears with a negative sign, hence chiral curvature has the opposite effect to that of QCD \[8\]. The enhancement of the \(N-\Delta\) mass splitting at light quarks in the quenched approximation has been observed \[16\] \[17\].

The opposite sign on the \(\Delta \to N\pi\) loop means that such a vertex correction to the Delta baryon magnetic moment will also have a sign which is opposite to that of QCD. Finite volume effects cannot change this sign difference. The pion loop contributions to the magnetic moments are \(p\)-wave and hence the main effect of the finite volume is to create a gap in momentum space between 0 and \(2\pi/L\). Because the loop integrals are positive definite for both the proton and Delta when \(m_\pi + m_N > m_\Delta\), the finite volume can only suppress the magnitude of loop effects but cannot change their sign. Studying the nucleon and Delta magnetic moments together therefore provides a clear signal of quenched chiral physics independent of volume effects.

We have calculated the quenched chiral corrections to the Delta baryon magnetic moment using the diagrammatic method of Leinweber \[14\] \[15\]. The flavour symmetry of the decuplet baryon interpolating fields means that the magnetic moments are simply proportional to the charge in the quenched isospin symmetric limit. Consideration of the \(\Delta^{++}\) is therefore sufficient to determine the quark mass dependence of the whole isobar.
The photon vertex correction from the double hairpin $\eta'$ dressing $\Delta^{++} \rightarrow \Delta^{++} \eta'$ provides a logarithmic divergence. The correction from a photon coupling to the meson loop trivially vanishes in the quenched theory. In the case of the $\Delta^{++}$ a pion loop is always a neutral $u\bar{u}$ pair and cannot carry a charge current. It is therefore necessary to go to the next order where the photon couples to the intermediate baryon state with a meson loop in the air. The decuplet diagram, $\Delta^{++} \rightarrow \Delta^{++} \pi^0$, is actually increased from the analogous contribution in QCD by a factor $4/3$.

It is the decay channel to an intermediate octet state which gives rise to the most interesting behaviour in the quenched theory. There is a vertex correction from an intermediate $uuu$-octet state — a double-charge “proton”, $p^{++}$ [8]. As discussed by Cloet et al., the opening of the decay channel at $m_\pi = m_\Delta - m_N$ gives rise to dramatic curvature at pion masses near threshold [18].

We show preliminary results of FLIC fermion simulations [19] of baryon magnetic moments [20]. In particular, we plot together the $p$ and $\Delta^+$ magnetic moments in Figure 2. We observe that at moderate quark masses the expected heavy-quark theory result is observed, with the $\Delta^+$ lying slightly above the proton. However, in the light-quark regime chiral physics becomes apparent with the clear separation of the two signals.

The downward curvature in the $\Delta^+$ is dominated by the coupling to an intermediate octet baryon.

The opposite behaviour of the proton and Delta allows one to unambiguously identify quenched chiral physics at pion masses $\lesssim 500$ MeV.

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REFERENCES

1. C. Bernard et al., arXiv:hep-lat/0209086.
2. A. W. Thomas, arXiv:hep-lat/0208023.
3. W. Detmold et al., Pramana 57, 251 (2001) arXiv:nucl-th/0104043.
4. S. J. Dong et al., arXiv:hep-lat/0304005.
5. R. D. Young et al., Prog. Part. Nucl. Phys. 50, 399 (2003) arXiv:hep-lat/0212031.
6. J. F. Donoghue et al., Phys. Rev. D 59, 036002 (1999) arXiv:hep-ph/9804281.
7. D. B. Leinweber et al., Phys. Rev. D 61, 074502 (2000) arXiv:hep-lat/9906027.
8. D. B. Leinweber et al., arXiv:nucl-th/0308083.
9. D. B. Leinweber et al., arXiv:hep-lat/0302020.
10. A. Ali Khan et al. [CP-PACS Collaboration], Phys. Rev. D 65, 054505 (2002) arXiv:hep-lat/0105013.
11. D. B. Leinweber et al., Phys. Rev. D 43, 1659 (1991).
12. M. Gockeler et al. [QCDSF Collaboration], arXiv:hep-lat/0303019.
13. M. J. Savage, Nucl. Phys. A 700, 359 (2002) arXiv:nucl-th/0107038.
14. D. B. Leinweber, Nucl. Phys. Proc. Suppl. 109A, 45 (2002) arXiv:hep-lat/0112021.
15. D. B. Leinweber, arXiv:hep-lat/0211017.
16. R. D. Young et al., Phys. Rev. D 66, 094507 (2002) arXiv:hep-lat/0205017.
17. D. B. Leinweber et al., arXiv:nucl-th/0211014.
18. I. C. Cloet et al., Phys. Lett. B 563, 157 (2003) arXiv:hep-lat/0302008.
19. J. M. Zanotti et al. [CSSM Lattice Collaboration], Phys. Rev. D 65, 074507 (2002) arXiv:hep-lat/0110216.
20. J. M. Zanotti et al. these proceedings, arXiv:hep-lat/0309186.