On Higgs and sphaleron effects during the leptogenesis era

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Abstract

We discuss the effects of various processes that can be active during the leptogenesis era, and present the Boltzmann equations that take them into account appropriately. A non-vanishing Higgs number asymmetry is always present, enhancing the washout of the lepton asymmetry. This is the main new effect when leptogenesis takes place at $T > 10^{12}$ GeV, reducing the final baryon asymmetry and tightening the leptogenesis bound on the neutrino masses. If leptogenesis occurs at lower temperatures, electroweak sphalerons partially transfer the lepton asymmetry to a baryonic one, while Yukawa interactions and QCD sphalerons partially transfer the asymmetries of the left-handed fields to the right-handed ones, suppressing the washout processes. Depending on the specific temperature range in which leptogenesis occurs, the final baryon asymmetry can be enhanced or suppressed by factors of order 20%–40% with respect to the case when these effects are altogether ignored.

1 Introduction

One of the most attractive scenarios to explain the origin of the baryon asymmetry of the Universe ($Y_B \equiv (n_B - \bar{n}_B)/s \simeq 8.7 \times 10^{-11}$) is leptogenesis [1, 2]. In this framework the decays of heavy electroweak singlet neutrinos (such as those appearing in see-saw models) into lepton and Higgs particles generate a lepton asymmetry, which is then partially reprocessed into a baryon asymmetry by anomalous electroweak processes mediated by sphalerons.

To compute in detail the lepton (and baryon) asymmetry at the end of the leptogenesis era, one has to take into account the various processes which can modify the particle densities. Some of these, such as the heavy neutrino decays or the various interactions that can washout the lepton number, occur slowly as compared to the expansion rate of the Universe, and hence are naturally accounted for via appropriate Boltzmann equations. Other reactions can be very fast (depending on the temperature...
considered) and their effect is to impose certain relations among the chemical potentials of different particle species that hold within specific temperature ranges. These include the Standard Model gauge interactions, some Yukawa interactions involving heavy fermions, and electroweak and strong non-perturbative ‘sphaleron’ processes. In the present work we analyze how all these ingredients concur to determine the precise impact of the washout processes and we discuss the effects this has on the final value of the baryon asymmetry.

A final important phenomenon has to do with the flavor composition of the states involved \[3\], and the decoherence effects induced by the leptonic Yukawa interactions in equilibrium, which essentially act as measuring devices that project the densities onto the flavor basis. The flavor interplay between the lepton number violating processes and the Yukawa interactions is very rich and can lead to dramatic consequences. Here, for the sake of clarity, we assume a simple flavor structure. We will discuss the full flavor picture in a separate publication \[4\].

The main point where our paper provides new insights is the combined effect of all spectator processes – QCD sphalerons, electroweak sphalerons and Yukawa interactions – on the washout processes for the various relevant temperature regimes. In particular, we emphasize the role of the Higgs number asymmetry. The issue of Higgs processes has been raised and analyzed in ref. \[5\]. We improve upon their analysis at several points and, in the end, are led to opposite conclusions regarding the direction of the effect, at least for some temperature regimes. Spectator processes in the low temperature regime (region 6 of section 4.2 below) were appropriately taken into account in ref. \[6\].

The plan of this paper is as follows. In Section 2 we present our framework and we enumerate and parametrize the various relevant washout processes. In Section 3 we discuss the Boltzmann equations. In Section 4 we present our main results. Equilibrium conditions in the various temperature regimes are imposed, and the implications for the Boltzmann equations are analyzed. Results are obtained for various representative flavor-alignment structures. In Section 5 we explain how the leptogenesis bound on the absolute scale of neutrino masses is affected by our considerations.

2 The Basic Framework

We consider for simplicity the scenario in which right handed neutrino masses are hierarchical, \( M_1 \ll M_{2,3} \), and consequently the lepton asymmetry is mainly generated via the CP and lepton number violating decays of the lightest singlet neutrino \( N_1 \). Even in this case, the task of including a general flavor structure within the Boltzmann equations can be quite complicated. In the mass eigenstate basis of the heavy neutrinos \( N_\alpha \ (\alpha = 1, 2, 3) \) and of the charged leptons \((i = e, \mu, \tau)\), the Yukawa interactions for the leptons read

\[
\mathcal{L}_Y = -h_{i\alpha} \bar{N}_\alpha \ell_i \tilde{H} + h_{i} \bar{\ell}_i H + h.c.,
\]  

(1)
where $\ell_i$ and $e_i$ denote the $SU(2)$ lepton doublets and singlets, $H = (H^+, H^0)^T$ is the Higgs field ($\tilde{H} = i\tau_2 H^*$) and the couplings $h$ for $e_i$ and $N_\alpha$ can be easily distinguished by the presence of one or two indices.

It is convenient to define a lepton doublet state $\ell_D$ as follows:

$$\ell_D = \frac{h_{i1}}{\sqrt{(h^\dagger h)_{11}}} \ell_i.$$  \hspace{1cm} (2)

The state $\ell_D$ is the one appearing (at tree level) in the following relevant processes:

- $N_1$ decays and inverse decays, with rate $\gamma_D = \gamma(N_1 \leftrightarrow \ell_D H)$;
- $\Delta L = 1$ Higgs-mediated scattering processes with rates such as $\gamma_{S_s} = \gamma(\ell_D N_1 \leftrightarrow Q_3 t)$ and $\gamma_{S_t} = \gamma(\ell_D Q_3 \leftrightarrow N_1 t)$, where $Q_3$ and $t$ are respectively the third generation quark doublet and the top $SU(2)$ singlet, as well as those involving gauge bosons, such as in $\ell_D N_1 \rightarrow HA$ (with $A = W_i$ or $B$).
- The on-shell $N_1$-mediated $\Delta L = 2$ scattering processes contributing to the rate $\gamma_{N_s} = \gamma(\ell_D H \leftrightarrow \ell_D \tilde{H})$.

In terms of $\ell_D$, the neutrino Yukawa interaction of $N_1$ in eq. \cite{11} reads $-\sqrt{(h^\dagger h)_{11}} N_1 \ell_D \tilde{H}$. Consequently, all the rates above depend on a single combination of neutrino Yukawa couplings, and are often parametrized in terms of a dimensional parameter $\tilde{m}_1$:

$$\tilde{m}_1 \equiv (h^\dagger h)_{11} \frac{v^2}{M_1},$$  \hspace{1cm} (3)

where $v = \langle H \rangle$. There is one additional class of relevant lepton number violating processes:

- Off-shell contributions to $\Delta L = 2$ scattering processes modify $\gamma_{N_s}$ and induce $\gamma_{N_t} = \gamma(\ell \ell \leftrightarrow \bar{H}H)$.

These processes are mediated by heavy neutrino exchanges, the first in the $s$- and $u$-channels while the second in the $t$-channel. The amplitude for the off-shell contributions to $\Delta L = 2$ washout is essentially proportional, in the limit $T < M_1$, to the light neutrino mass matrix, $M_{ij} = h_{i\alpha} \frac{v^2}{M_\alpha} h_{j\alpha}$. Consequently, the fastest rate couples to the lepton doublet containing the heaviest light neutrino state $\nu_3$. We see then that in general it is not only the state $\ell_D$ which is involved in these contributions, further complicating the flavor structure of the problem.

If the resonant contribution to the scattering is properly subtracted \cite{17}, one finds that these off-shell pieces are generally sub-dominant, so that the $\Delta L = 2$ washout processes are in general dominated by the on-shell $N_1$ exchange (with a possible exception if $M_1 \gg 10^{12}$ GeV, in which case some Yukawa couplings can become of order unity). In this case, the only direction in flavor space that appears in the lepton number violating
processes is that of $\ell_D$. In the following, we make the assumption that this is indeed the case and we often use the simplified notation $\ell \equiv \ell_D$ for this special direction. This assumption simplifies things considerably, because otherwise it becomes necessary to follow the evolution of the asymmetries in an approach based on Boltzmann equations for the density matrix \[2\] and keeping track of coherence effects.

Another issue related with flavor becomes quite important at temperatures when the processes induced by the Yukawa couplings of the charged leptons, $h_{\tau,\mu,e}$ of eq. (1), are no more negligible. The lepton Yukawa interactions define a flavor basis, and the density matrix for the lepton asymmetry is projected onto this basis. If the state $\ell$ is not aligned with a specific flavor $\ell_\tau$, $\ell_\mu$ or $\ell_e$ then the lepton asymmetry gets distributed between all the different flavors. Such misalignment has many interesting consequences, which we will present in \[4\].

### 3 The Boltzmann equations

In this section we present the Boltzmann equations that are relevant to the washout effects in leptogenesis, focusing on the case when $\ell$ is aligned along one specific flavor direction. This case can be treated more easily and allows us to understand in detail the flavor-independent effects that we want to explore in this paper.

With the simplifying alignment conditions discussed above, the (linearized) Boltzmann equations can be written as:

\[
\frac{dY_N}{dz} = -\frac{1}{sH_\zeta} \left( \frac{Y_N}{Y_N^{eq}} - 1 \right) \left( \gamma_D + 2\gamma_{Ss} + 4\gamma_{St} \right),
\]

\[
\frac{dY_L}{dz} = \frac{1}{sH_\zeta} \left\{ \epsilon \left( \frac{Y_N}{Y_N^{eq}} - 1 \right) \left( \frac{Y_N}{Y_N^{eq}} + 1 \right) \right\} \gamma_{St}
- \left( \frac{Y_N}{Y_N^{eq}} y_t + y_t - y_{Q_3} \right) \gamma_{Ss} - 2 \left( y_t + y_H \right) \left( \gamma_{N_s} + \gamma_{N_t} \right) \right\} + \frac{dY_{EW}^L}{dz},
\]

\[
\frac{dY_B}{dz} = \frac{dY_{EW}^B}{dz},
\]

where the standard notation $z \equiv M_1/T$ is used. Here $Y_N \equiv n_N/s$ is the density of the lightest heavy neutrinos (with two degrees of freedom) relative to the entropy $s$, $Y_L$ and $Y_B$ are the total lepton and baryon number densities, also normalized to the entropy, $y_X \equiv (n_X - n_{\bar{X}})/n_{eq}$ denote the asymmetries for the relevant different species $X = \ell$, $H$, $t$, $Q_3$ and all the asymmetries are normalized to the Maxwell-Boltzmann equilibrium densities. The reaction rates are summed over initial and final

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\[1\] Actually, the state $\ell_D$ produced in $N_1$ decays differs from the one in eq. \[2\] at one-loop \[3\]. Moreover, at one loop level the anti-lepton produced in $N_1$ decays $\bar{\ell}_D$ is not necessarily the conjugate of $\ell_D$, and this can have interesting effects, which will be explored in \[4\].
state quantum numbers, including the gauge multiplicities. In the asymmetries $y_X$, $X = \ell, H$ or $Q_3$ label any of the two doublet components, not their sum, and hence we normalize $y_X$ to the equilibrium densities with just one degree of freedom. This is different from the convention in [7], and allows us to keep the proportionality $y_X \propto \mu_X$ in terms of the chemical potentials, with the usual convention that e.g. $\mu_\ell_i$ is the chemical potential of each one of the two $SU(2)$ components of the doublet $\ell_i$.

In our analysis, we make two further simplifications:

1. In Eqs. (4)–(6) and in what follows we ignore finite temperature corrections to the particle masses and couplings [7], so that we take all equilibrium number densities $n_X^{eq}$ equal to those of massless particles.

2. We ignore scatterings involving gauge bosons, for whose rates no consensus has been achieved so far [7, 8]. They do not introduce qualitatively new effects and, moreover, no further density asymmetries are associated to them.

We would like to emphasize the following points concerning eq. (5):

• The CP violating parameter $\epsilon$ gives a measure of the $L$ asymmetry produced per $N_1$ decay [9].

• $y_\ell$ is the asymmetry for one component of the relevant $SU(2)$-doublet $\ell \equiv \ell_D$ (relative to equilibrium density), while the total lepton asymmetry is $Y_L = \sum_i Y_{Li} = \sum_i(2y_{\ell_i} + y_{e_i})Y^{eq}$, with $Y^{eq} = n^{eq}/s$.

• For the temperature regimes in which the charged lepton Yukawa couplings become non-negligible ($T \lesssim 10^{13}$ GeV), the corresponding interactions define a lepton flavor basis. We assumed, for simplicity, that the state $\ell$ is aligned with (or orthogonal to) one of the lepton flavor states singled out by the Yukawa interactions. Then the difference in the rates $\Gamma \equiv \Gamma(N_1 \rightarrow \ell h)$ and $\bar{\Gamma} \equiv \Gamma(N_1 \rightarrow \bar{\ell} \bar{h})$ for the $N_1$ decays into $\ell$ leptons and $\bar{\ell}$ antileptons gives the total $CP$-asymmetry $\epsilon = (\Gamma - \bar{\Gamma})/(\Gamma + \bar{\Gamma})$, while the evolution of total lepton number is determined by the Boltzmann equation [5] solely in terms of one leptonic asymmetry $y_\ell$. However, in the general case of no alignments, the decay rates of $N_1$ into the specific flavors $\ell_i$ and anti-flavors $\bar{\ell}_i$ have to be considered, and the Boltzmann equations should track the evolution of all the relevant single-flavor asymmetries [3, 4].

• The thermally averaged reaction rate $\gamma_{N_1}$ is the $\Delta L = 2$ $s$-channel rate without subtraction of the real intermediate state, and thus it takes into account also the on-shell heavy neutrino exchanges [7]. Since we consider here only the tree level scatterings, there is no double counting of the CP violating one loop contribution included in the direct and inverse decay terms.
• The factor $dY_{EW}^L/dz$ is included to account for the lepton number violation induced by electroweak anomalous processes. This term is proportional to the electroweak sphaleron rate $\Gamma_{EW}$ and to the amount of $(B + L)$ asymmetry contained in the left-handed fields. It becomes relevant at temperatures below $10^{12}$ GeV. Since electroweak sphalerons preserve $B - L$, one has

$$\frac{dY_{B}^{EW}}{dz} = \frac{dY_{L}^{EW}}{dz}. \quad (7)$$

Hence, by subtracting eqs. (5) from (6), one can combine them into a single equation for $Y_{B-L}$. The resulting equation takes into account all the relevant washout processes, and has the advantage of being independent of the (poorly known) sphaleron rate. Note the the electroweak sphalerons preserve not only $B - L$, but also the three lepton-flavor related quantities

$$\Delta_i \equiv B/3 - L_i. \quad (8)$$

Equation (8) takes into account the fact that the heavy neutrino decays, besides producing an asymmetry in the left-handed leptons, also produce an asymmetry in the Higgs number density. The Higgs number is not conserved by Yukawa interactions, but its asymmetry is only partially transferred into a ‘chiral’ asymmetry between $Q_3$ and $t$ by the top quark Yukawa interactions (as well as into asymmetries for other fermions when their corresponding interactions with the Higgs enter into equilibrium). Indeed, the equilibrium conditions enforce $y_H \neq 0$, and hence $y_H$ acts as a source of washout processes. Similarly, $y_t - y_{Q_3}$ acts as a source for the $\Delta L = 1$ washout processes involving Higgs boson exchanges (washout processes involving standard model Yukawa couplings different from the top one can be safely neglected). These additional contributions to the washout of lepton number are often ignored in the literature and omitted from the Boltzmann equations for leptogenesis, yet they play a role that is similar, qualitatively and quantitatively, to the role of $y_t$.\(^2\)

The system of equations that now has to be solved corresponds to eq. (4) for $Y_N$ and the equation derived from subtracting eq. (3) from eq. (8) for $Y_{B-L}$. This system can be solved after expressing the densities $y_t$, $y_H$ and $y_t - y_{Q_3}$ in terms of $Y_{B-L}$ with the help of the equilibrium conditions imposed by the fast reactions, as described in the next section. The value of $B - L$ at the end of the leptogenesis era obtained by solving the Boltzmann equations remains subsequently unaffected until the present epoch. If electroweak sphalerons go out of equilibrium before the electroweak phase

\(^2\)The additional washout terms in eq. 6 were considered before in [5]. We improve this analysis by giving a proper treatment of the $B - L$ conserving electroweak sphaleron processes, by coupling the washout terms only to the relevant lepton doublet asymmetry, and by accounting for the QCD sphalerons as well as for all the other processes that enter into equilibrium at the different temperature regimes. This leads to results that differ from those of ref. [5].
transition, the present baryon asymmetry (assuming a single Higgs doublet) is given by the relation \[10\]

\[n_B = \frac{28}{79}n_{B-L}.\]  

(9)

If, instead, electroweak sphalerons remain in equilibrium until slightly after the electroweak phase transition (as would be the case if, as presently believed, the electroweak phase transition was not strongly first order) the final relation between \(B\) and \(B - L\) would be somewhat different \[11\].

4 The equilibrium conditions

In this section we discuss the equilibrium conditions that hold in the different temperature regimes which can be relevant for leptogenesis. Since leptogenesis takes place during the out of equilibrium decay of the lightest heavy right-handed neutrino \(N_1\), \(i.e.\) at temperatures \(T \sim M_1\), the relevant constraints that have to be imposed among the different asymmetries depend essentially on the value of \(M_1\). We use the equilibrium conditions to express \(y_\ell, y_H\) and \(y_t - y_Q\) in terms of \(Y_{B-L}\).

4.1 General considerations

The number density asymmetries for the particles \(n_X\) entering in eq. (5) are related to the corresponding chemical potentials through

\[n_X - n_{\bar{X}} = g_X T^3 \begin{cases} \frac{\mu_X}{T} & \text{fermions,} \\ \frac{2\mu_X}{T} & \text{bosons,} \end{cases} \]

(10)

where \(g_X\) is the number of degrees of freedom of \(X\). For any given temperature regime the specific set of reactions that are in chemical equilibrium enforce algebraic relations between different chemical potentials \[10\]. In the entire range of temperatures relevant for leptogenesis, the interactions mediated by the top-quark Yukawa coupling \(h_t\), and by the \(SU(3) \times SU(2) \times U(1)\) gauge interactions, are always in equilibrium. This situation has the following consequences:

- Equilibration of the chemical potentials for the different quark colors is guaranteed because the chemical potentials of the gluons vanish, \(\mu_g = 0\).

- Equilibration of the chemical potentials for the two members of an \(SU(2)\) doublet is guaranteed by the vanishing, above the electroweak phase transition, of \(\mu_{W^+} = -\mu_{W^-} = 0\). This condition was implicitly implemented in eq. (5) where we used \(\mu_Q \equiv \mu_{uL} = \mu_{dL}, \mu_\ell \equiv \mu_{eL} = \mu_{\nu_L}\) and \(\mu_H = \mu_{H^+} = \mu_{H^0}\) to write the particle number asymmetries directly in terms of the number densities of the \(SU(2)\) doublets.
Hypercharge neutrality implies
\[ \sum_i (\mu_{Q_i} + 2\mu_{u_i} - \mu_{d_i} - \mu_{\ell_i} - \mu_{e_i}) + 2\mu_H = 0, \]  \(11\)
where \(u_i, d_i\) and \(e_i\) denote the \(SU(2)\) singlet fermions of the \(i\)-th generation.

The equilibrium condition for the Yukawa interactions of the top-quark \(\mu_t = \mu_{Q_3} + \mu_H\) yields:
\[ y_t - y_{Q_3} = \frac{y_H}{2}, \]  \(12\)
where the factor 1/2 arises from the relative factor of 2 between the number asymmetry and chemical potential for the bosons, see eq. (10).

Using this relation, one can recast the Boltzmann equation for the \(B-L\) asymmetry in the aligned case as
\[ \frac{dY_{B-L}}{dz} = -\frac{1}{sHz} \left\{ \left( \frac{Y_N}{Y_{eq}^N} - 1 \right) \left[ \epsilon \gamma_D + \left( c_\ell \gamma_{Ss} + \frac{c_H}{2} \gamma_{St} \right) \frac{Y_{B-L}}{Y_{eq}} \right] + \left( 2c_\ell + c_H \right) \left( \gamma_{St} + \frac{1}{2} \gamma_{Ss} \right) + 2 \left( c_\ell + c_H \right) \left( \gamma_{Ns} + \gamma_{Nt} \right) \right\} \frac{Y_{B-L}}{Y_{eq}}, \]  \(13\)
where we have defined \(c_\ell\) and \(c_H\) through \(y_\ell \equiv -c_\ell Y_{B-L}/Y_{eq}\) and \(y_H \equiv -c_H Y_{B-L}/Y_{eq}\) while their numerical values are determined, within each temperature range, by the constraints enforced by the fast reactions that are in equilibrium. This equation is general enough to account for all the effects of the relevant spectator processes (Yukawa interactions, electroweak and QCD sphalerons), while to take into account the lepton flavor structure, a generalization of eq. (13) is required.

### 4.2 Specific temperature ranges and flavor structures

Let us now discuss the different temperature ranges of interest. At each step, we take into account all the relevant processes that enter into equilibrium. In order to understand and disentangle the various effects involved, we examine a rather large number of temperature windows, and for each window we also impose, when relevant, various conditions of flavor alignment.

Our main results can be understood on the basis of the examples presented in Table I that cover six different temperature regimes. For each regime, different possibilities of flavor alignments are considered. To do that, we define a parameter \(K_i\) (\(i = e, \mu, \tau\)):
\[ K_i \equiv |\langle \ell_i | \ell_D \rangle|^2. \]  \(14\)

The simple flavor structures that we investigate here correspond to either alignment with a specific flavor direction, \(K_i = 1\), or orthogonality, \(K_i = 0\). The more general case of \(K_i \neq 0, 1\) will be discussed in [4].
For each aligned case, we give in the table the coefficients $c_\ell$ and $c_H$ that relate the asymmetries $y_\ell$ and $y_H$ to $Y_{B-L}$. Note that $c_\ell$ and $c_H$ give a crude understanding of the impact of the respective asymmetries: $c_H/c_\ell$ gives a rough estimate of the relative contribution of the Higgs to the washout, while $c_\ell + c_H$ gives a measure of the overall strength of the washout. The last column gives the resulting $B-L$ asymmetry for each case. To disentangle the impact of the various processes from that of the input parameters, the $B-L$ asymmetry is calculated in all cases with fixed values of $\tilde{m}_1 = 0.06$ eV and $M_1 = 10^{11}$ GeV. The $\tilde{m}_1$ parameter was defined in eq. (3). It determines the departure from equilibrium of the heavy neutrino $N_1$, as well as the strength of the washout processes. For $\tilde{m}_1 < 10^{-3}$ eV, the departure from equilibrium is large and washout effects are generally negligible. Hence, in this case, there is no need to solve any detailed Boltzmann equations. In contrast, for $\tilde{m}_1 \gtrsim 0.1$ eV, washout processes become so efficient that, in general, the surviving baryon asymmetry is too small. We therefore consider the intermediate value $\tilde{m}_1 = 0.06$ eV, which is also suggested by the atmospheric neutrino mass-squared difference if neutrino masses are hierarchical. As concerns $M_1$, it is clear that the relevant temperature range is actually determined by it, yet – as explained above – we fix the value at $M_1 = 10^{11}$ GeV in order to have a meaningful comparison of the various effects of interest. Namely, since in each regime considered the same asymmetries are produced in the decay of the heavy neutrinos, a comparison between the final values of $B-L$ for the different cases can be directly interpreted in terms of suppressions or enhancements of the washout processes. Anyhow, the overall effects of the washouts turn out to be essentially independent of the values of $M_1$, as long as $M_1 (\tilde{m}_1/0.1 \text{ eV})^2 < 10^{14}$ GeV [9, 11], and hence the values of $Y_{B-L}$ obtained would not change significantly had we adopted smaller $M_1$ values. We start with vanishing initial values for $Y_N$ and for all the asymmetries, but notice that for $\tilde{m}_1 > 10^{-2}$ eV the results are insensitive to the initial values.

In the six different temperature regimes we will consider, additional interactions will enter into equilibrium at each step as the temperature of the thermal bath decreases:

1) *Only gauge and top-Yukawa interactions in equilibrium* ($T > 10^{13}$ GeV).
Since in this regime the electroweak sphalerons are out of equilibrium, no baryon asymmetry is generated during leptogenesis. Moreover, since the charged lepton Yukawa interactions are negligible, the lepton asymmetry is just in the left-handed degrees of freedom and confined in the $\ell = \ell_D$ doublet, yielding $Y_L = 2 y_\ell Y_{eq} = -Y_{B-L}$. As concerns $y_H$, although initially equal asymmetries are produced by the decay of the heavy neutrino in the lepton and in the Higgs doublets, the Higgs asymmetry is partially transferred into a chiral asymmetry for the top quarks ($y_t - y_{Q3} \neq 0$) implying $y_\ell \neq y_H$.

2) *Strong sphalerons in equilibrium* ($T \sim 10^{13}$ GeV).
QCD sphalerons equilibration occurs at higher temperatures than for the corresponding
electroweak processes because of their larger rate \( \Gamma_{QCD} \sim 11(\alpha_s/\alpha_W)^5\Gamma_{EW} \). These processes are likely to be in equilibrium already at temperatures \( T_s \sim 10^{13} \text{ GeV} \) \( \ref{12,13,14} \) and yield the constraint

\[
\sum_i (2\mu_{Q_i} - \mu_{u_i} - \mu_{d_i}) = 0.
\] (15)

Direct comparison with the previous case allows us to estimate the corresponding effects: while the relation \( Y_L = 2y_bY^{eq} = -Y_{B-L} \), implying \( c_\ell = 1/2 \), holds also for this case, we see that switching on the QCD sphalerons reduces the Higgs number asymmetry by a factor of 21/23. This effect yields a suppression of the washout that does not exceed 5%.

3) Bottom- and tau-Yukawa interactions in equilibrium \( (10^{12} \text{ GeV} \lesssim T \lesssim 10^{13} \text{ GeV}) \).

Equilibrium for the bottom and tau Yukawa interactions implies that the asymmetries in the \( SU(2) \) singlet \( b \) and \( e_\tau \) degrees of freedom are populated. The corresponding chemical potentials obey the equilibrium constraints \( \mu_b = \mu_{Q_3} - \mu_H \) and \( \mu_{\tau} = \mu_{\ell_\tau} - \mu_H \). Possibly \( h_b \) and \( h_{\tau} \) Yukawa interactions enter into equilibrium at a similar temperature as the electroweak sphalerons \( \ref{13} \). However, since the rate of the non-perturbative processes is not well known, we first consider the possibility of a regime with only gauge, QCD sphalerons and the Yukawa interactions of the whole third family in equilibrium. This will also allow us to disentangle by direct comparison with the next case the new effects induced by electroweak sphalerons. As regards the flavor composition of the lepton asymmetry, we distinguish two alignment cases: first, when the lepton asymmetry is produced in a direction orthogonal to \( \ell_\tau \) \( (K_\tau = 0) \) and second, when it is produced in the \( \ell_\tau \) channel \( (K_\tau = 1) \). Since electroweak sphalerons are not yet active, \( L_\tau = 0 \) or \( L_\tau = L \) are conserved quantities in the respective cases. When \( L_\tau = 0 \), the lepton asymmetry is produced in one of the two directions orthogonal to \( \ell_\tau \) and therefore it does not ‘leak’ into the \( SU(2) \) singlet degrees of freedom, implying that \( c_\ell = 1/2 \) still holds. In the case when \( L = L_\tau \), the washout effects are somewhat suppressed, since the lepton asymmetry is partially shared with \( e_\tau \) that does not contribute directly to the washout processes. Our results for these two cases suggest that the effect on the final value of \( B - L \) associated to the \( \tau \) Yukawa interactions is of the order of 10%.

Equilibrium for both the top and the bottom quark Yukawa interactions enforces the constraint \( 2\mu_{Q_3} - \mu_{u_3} - \mu_{d_3} = 0 \) and therefore chemical potentials of the third generation are not constrained by the QCD sphaleron condition \( \ref{15} \). A similar statement holds for each generation when its quark Yukawa interactions \( (i.e. h_c \) and \( h_u \) and, at low enough temperature, also \( h_u \) and \( h_d \) enter into equilibrium. We conclude that the lower the temperature that is relevant to leptogenesis, the less important is the role played by QCD sphaleron effects.

4) Electroweak sphalerons in equilibrium \( (10^{11} \text{ GeV} \lesssim T \lesssim 10^{12} \text{ GeV}) \).

The electroweak sphaleron processes take place at a rate per unit volume \( \Gamma/V \propto \)}
Equilibrium processes, constraints, coefficients and $B - L$ asymmetry

| $T$ (GeV) | Equilibrium | Constraints | $c_\ell$ | $c_H$ | $Y_{B-L}$ [$10^{-5} \times \epsilon$] |
|----------|-------------|-------------|---------|-------|-------------------------------|
| $\gg 10^{13}$ | $h_t$, gauge | $B = \sum_i (2Q_i + u_i + d_i) = 0$ | $\frac{1}{2}$ | $\frac{1}{3}$ | 0.6 |
| $\sim 10^{13}$ | QCD-Sph | $\sum_i (2Q_i - u_i - d_i) = 0$ | $\frac{1}{2}$ | $\frac{7}{23}$ | 0.6 |
| $10^{12}\div 13$ | $h_b, h_\tau$ | $b = Q_3 - H$, $\tau = \ell_\tau - H$ | $K_\tau = 0$ | $\frac{1}{2}$ | $\frac{3}{16}$ | 0.7 |
| | | | $K_\tau = 1$ | $\frac{3}{8}$ | $\frac{1}{4}$ | 0.8 |
| $10^{11}\div 12$ | EW-Sph | $\sum_i (3Q_i + \ell_i) = 0$ | $K_\ell = 0$ | $\frac{49}{115}$ | $\frac{41}{230}$ | 0.8 |
| | | | $K_\ell = 1$ | $\frac{39}{115}$ | $\frac{28}{115}$ | 0.9 |
| $10^{8}\div 11$ | $h_\epsilon, h_\mu, h_\mu$ | $c = Q_2 + H$, $s = Q_2 - H$, $\mu = \ell_\mu - H$ | $K_\epsilon = 1$ | $\frac{151}{337}$ | $\frac{37}{538}$ | 1.0 |
| | | | $K_\tau = 1$ | $\frac{172}{337}$ | $\frac{26}{179}$ | 1.1 |
| $\ll 10^8$ | all Yukawas $h_i$ | $K_\epsilon = 1$ | $\frac{221}{771}$ | $\frac{8}{79}$ | 1.2 |

Table 1: The relevant quantities in the different temperature regimes. Chemical potentials are labeled here with the same notation used for the fields: $\mu_{Q_i} = Q_i$, $\mu_{\ell_i} = \ell_i$ for the $SU(2)$ doublets, $\mu_{u_i} = u_i$, $\mu_{d_i} = d_i$, $\mu_{e_i} = e_i$ for the singlets and $\mu_H = H$ for the Higgs. The relevant reactions in equilibrium in each regime are given in the second column and the constraints imposed on the third. The alignment conditions adopted for the $K_i$ are indicated. The appropriate constraints on the conserved quantities $\Delta_i = B/3 - L_i$ should also be imposed. The values of the coefficients $c_\ell$ and $c_H$ are given respectively in the fourth and fifth column while the resulting $B - L$ asymmetry (in units of $10^{-5} \times \epsilon$) obtained for $\bar{m}_1 = 0.06$ eV and $M_1 = 10^{11}$ GeV is given in the last column.

$T^4 \alpha_W^5 \log(1/\alpha_W)$ [15, 16, 17], and are expected to be in equilibrium from temperatures of about $\sim 10^{12}$ GeV, down to the electroweak scale or below [13]. Electroweak sphalerons equilibration implies

$$\sum_i (3\mu_{Q_i} + \mu_{\ell_i}) = 0.$$  \hspace{1cm} (16)

As concerns lepton number, each electroweak sphaleron transition creates all the doublets of the three generations, implying that individual lepton flavor numbers are no longer conserved, regardless of the particular flavor direction along which the doublet $\ell_D$ is aligned. As concerns baryon number, electroweak sphalerons are the only source of $B$ violation, implying that baryon number will be equally distributed among the three
families of quarks. In particular, for the third generation, $B_3 = B/3$ is distributed between the doublets $Q_3$ and the singlets $t$ and $b$.

In Table 1 we give the coefficients $c_\ell$ and $c_H$ for the two aligned cases: (i) $\ell \perp \ell_\tau$ ($K_\tau = 0$) implying $\Delta_\tau = 0$, and (ii) $\ell = \ell_\tau$ ($K_\tau = 1$) implying $\Delta_e = \Delta_\mu = 0$. Again we see that the transfer of part of the lepton asymmetry to a single right handed lepton ($e_\tau$) can have a 10% enhancing effect on the final $B - L$.

5) Second generation Yukawa interactions in equilibrium ($10^8$ GeV $\lesssim T \lesssim 10^{11}$ GeV).

In this regime, the $h_e$, $h_\tau$ and $h_\mu$ interactions enter into equilibrium. We consider two cases of alignment: (i) $\ell = \ell_e$ ($K_e = 1$), implying $\Delta_\tau = \Delta_\mu = 0$, and (ii) $\ell \perp \ell_e$. To ensure a pure states regime we further assume complete alignment of $\ell$ with one of the two flavors with Yukawas in equilibrium, for definiteness $\ell_\tau$, and therefore we have $K_\tau = 1$ and $\Delta_e = \Delta_\mu = 0$. The difference in $c_\ell$ between the two aligned cases is larger than in the regimes 3 and 4; this, however, is well compensated by an opposite difference in $c_H$, keeping the effect on $B - L$ at the same level as in the cases in which just the third generation Yukawa couplings are in equilibrium.

6) All SM Yukawa interactions in equilibrium ($T \approx 10^8$ GeV).

In this regime, since all quark Yukawa interactions are in equilibrium (actually this only happens for $T < 10^6$ GeV), the QCD sphaleron condition becomes redundant. Hence ignoring the constraint of eq. (15), as is usually done in the literature, becomes fully justified only within this regime. If, however, leptogenesis takes place at $T \gg 10^6$ GeV, as favored by theoretical considerations, the constraint implied by the QCD sphalerons is non-trivial, even if the associated numerical effects are not large.

Due to the symmetric situation of having all Yukawa interactions in equilibrium we have just one possible flavor alignment (the other two possibilities being trivially equivalent). We take for definiteness $\ell = \ell_e$ ($K_e = 1$) implying $\Delta_\tau = \Delta_\mu = 0$. We see that in this case $c_\ell$ is reduced by a factor of almost two with respect to the case in which the spectator processes are neglected ($c_\ell = 1/2$) and the final value of $B - L$ is correspondingly enhanced. The reason for the reduction in $c_\ell$ can be traced mainly to the fact that a sizable amount of $B$ asymmetry is being built up at the expense of the $L$ asymmetry, and also a large fraction of the asymmetry is being transferred to the right handed degrees of freedom at the same time when inverse decays and washout processes are active, reducing the effective value of $y_\ell$ that contributes to drive these processes.

4.3 Discussion

The range of final asymmetries presented in Table 1 that correspond to the cases of flavor alignment, gives a measure of the possible impact of the spectator processes.
Figure 1: The heavy neutrino density $Y_N$ (solid) and equilibrium density $Y_N^{eq}$ (short-dashes), and the $B - L$ asymmetry $|Y_{B-L}/\epsilon|$ for three sets of values for $(c_\ell, c_H)$: (i) the long dashed curve corresponds to $c_\ell = 1/2$, $c_H = 1/3$ (first row in Table I); (ii) the dot-dashed curve corresponds to $c_\ell = 221/711$, $c_H = 8/79$ (last row in Table I); (iii) the dotted curve corresponds to $c_\ell = -1/2$, $c_H = 0$. We take $M_1 = 10^{11}$ GeV and $\tilde{m}_1 = 0.06$ eV.

(ignoring flavor issues) in the different regimes. In Fig. 1 we show the results of integrating the Boltzmann equations with the two pairs of extreme values of $c_{\ell,H}$ given in the first and in the last row of Table I. We also show the results for the (incorrect) case in which only the asymmetry $y_\ell$ is included in the washout terms, and all the effects of the spectator processes discussed in this paper are ignored ($c_\ell = 1/2$ and $c_H = 0$). We learn that when the electroweak sphalerons are not active and flavor effects are negligible, the Higgs contribution enhances the washout processes, leading to a smaller final $B - L$ asymmetry. As more and more spectator processes become fast (compared to the expansion rate of the Universe), the general trend is towards reducing the value of the washout coefficients and hence increasing the final value of the resulting $B - L$ asymmetry. A rough quantitative understanding of our results can be obtained relying
on the fact that the surviving asymmetry is inversely proportional to the washout rate, as can be demonstrated along lines similar to those given in Appendix 2 of ref. [3]. Hence, the relative values for the final $B - L$ asymmetries obtained in the relevant temperature regimes in Table 1 can be roughly explained as being inversely proportional to $c_\ell + c_H$.\(^3\) The largest value for $B - L$ given in the table corresponds to the case in which we assumed all the Yukawa interactions in equilibrium during the leptogenesis era. This result is different from the one obtained in [5], where an order one enhancement of the washout processes was found for this same case, and hence a smaller final $B - L$ asymmetry. (In more general non-aligned flavor configurations this disagreement would be even more pronounced [4].) In [5] the washout term involving the leptonic asymmetry was taken to be proportional to the total asymmetry $Y_L$ rather than just to the asymmetry in the lepton doublet $\ell$, and we think that this may be the main cause of the discrepancy.

5 Implications for light neutrino masses

Leptogenesis, besides providing an attractive mechanism to account for the baryon asymmetry of the Universe, has interesting implications for low energy observables. In particular, assuming that leptogenesis is indeed the source of the baryon asymmetry, the observed value of this quantity then implies a strong upper bound on the absolute scale of the light neutrino masses. In this section we discuss the implications of our analysis for this bound. Note that we are concerned here with the high temperature regime $T \gg 10^{13}$ GeV for which flavor considerations are not relevant. Therefore, the simplifying flavor-related assumptions that we make in this work are fully justified for the purposes of this section.

The numerical value of the baryon to entropy ratio can be expressed as

$$\frac{n_B - \bar{n}_B}{s} = -1.38 \times 10^{-3} \epsilon \eta \approx 8.7 \times 10^{-11},$$

(17)

and the upper bound implied for the mass of the heaviest neutrino reads [18, 19, 20, 21]:

$$m_3 \lesssim 0.15 \text{ eV}.$$  

(18)

In eq. (17) the washout factor $\eta$ is related to the various lepton number violating processes of eq. (5) and depends on the coefficients $c_\ell$ and $c_H$. Then if the effect of the Higgs asymmetry, that in the high temperature regimes contributes to the washout reducing the value of $\eta$, is taken into account, this could result in strengthening the

\(^3\)The washout rate having the strongest impact on the final value of the asymmetry for $M_1 < 10^{14}$ GeV $\times (0.1 \text{eV}/\tilde{m}_1)^2$ is the $\Delta L = 2$ on-shell piece of $\gamma_{N_\nu}$, which has a Boltzmann suppression factor $\exp(-z)$, similar to the $\Delta L = 1$ rates. Hence the proof given in ref. [3] for the case of $\Delta L = 1$ washout dominance and small departure from equilibrium holds also for the cases considered in Table 1.
bound (18) on $m_3$. This bound lies in the region of quasi-degenerate light neutrinos, that is, $(m_3)_{\text{max}} \gg m_{\text{atm}} \equiv \sqrt{\Delta m_{\text{sol}}^2} \sim 0.05 \text{ eV}$. Thus, we can use in a self-consistent way the approximation

$$m_1 \simeq m_2 \simeq m_3 \quad \text{with} \quad m_3^2 - m_1^2 \simeq \Delta m_{\text{atm}}^2,$$

and neglect $\Delta m_{\text{sol}}^2 = m_2^2 - m_1^2 \ll \Delta m_{\text{atm}}^2$.

The maximal value of the CP asymmetry $\epsilon$, for quasi-degenerate light neutrinos (and hierarchical heavy neutrinos), is given by [19, 20, 22]

$$\epsilon_{\text{max}} = \frac{3}{32\pi} \frac{\Delta m_{\text{atm}}^2}{m_3^2} \frac{1}{1 - \frac{m_3^2}{m_1^2}}.$$  \hspace{1cm} (20)

In order to set an upper bound on the neutrino masses, the relation $m_3 = \max(m_{\text{atm}}, \tilde{m}_1)$ is often adopted [3]. With this plausible ansatz we see that, for $\tilde{m}_1 > m_{\text{atm}}$ one has $\epsilon_{\text{max}} \propto M_1/m_3^2$. (For $\tilde{m}_1 \leq m_{\text{atm}}$, one has instead $\epsilon_{\text{max}} \propto M_1/m_3$.)

As concerns the washout factor, the lower bound on the $\tilde{m}_1$ parameter, $\tilde{m}_1 \geq m_1$, implies that for quasi-degenerate neutrinos one is in the strong washout regime, defined by (see, for example, [21])

$$\tilde{m}_1 \gg \tilde{m}_1^* \equiv \frac{256g_{\text{SM}}v^2}{3M_{\text{Pl}}} \simeq 2.3 \times 10^{-3} \text{ eV},$$  \hspace{1cm} (21)

where $g_{\text{SM}} = 118$ is an effective number of degrees of freedom for $T \gg 100$ GeV. Within the strong washout regime, we distinguish between two regions:

(i) For $M_1 < 10^{14}$ GeV(0.1 eV/$\tilde{m}_1$)$^2$, $\eta$ is inversely proportional to the strength of the on-shell washout rates. More precisely, a fit to $\eta$ valid for $\tilde{m}_1 > \tilde{m}_1^*$ (small departure from equilibrium) and $M_1 < 10^{14}$ GeV(0.1 eV/$\tilde{m}_1$)$^2$ gives [3]:

$$\eta \simeq \frac{1}{\lambda} \left( \frac{\tilde{m}_1}{0.55 \times 10^{-3} \text{ eV}} \right)^{-1.16} \equiv \eta_l.$$  \hspace{1cm} (22)

Notice that we introduced here the factor $\lambda \equiv (c_{\tau} + c_H)/0.5$ to account for the scaling of the rates.

(ii) For $M_1 > 10^{14}$ GeV(0.1 eV/$\tilde{m}_1$)$^2$, contributions associated to off-shell $N_\alpha$ exchange become the dominant washout processes, and give rise to an exponential suppression of $\eta$, with exponent proportional to the square root of the $\Delta L = 2$ rates [3]:

$$\eta \simeq \exp \left[ -\frac{\tilde{m}_1}{\tilde{m}_1^*} \sqrt{\frac{M_1}{M_1^*}} X \right] \equiv \eta_h.$$  \hspace{1cm} (23)

Here $M_1^* \simeq 3.3 \times 10^{15}$ GeV and $X \geq 1$ is a parameter related to the flavor structure of the $\Delta L = 2$ off-shell processes, which can be taken as $X \simeq 1$ for $\tilde{m}_1 \simeq m_3$ (see [3]).
Since for $M_1 \gg 10^{12}$ GeV no leptonic Yukawa couplings are in equilibrium during the leptogenesis era, flavor alignment issues in the Boltzmann equations can be ignored and the effects of the $\Delta L = 2$ rates are just proportional to $\lambda$.

In the regime in which $\eta \simeq \eta_l$ one has that $n_B/s|_{\text{max}} \propto M_1/m_3^{16}$ and hence for a given value of $M_1$ and upper bound on $m_3$ results. For increasing values of $M_1$ the bound gets correspondingly relaxed, until for $M_1 \sim 10^{14}$ GeV $(0.1 \text{ eV}/\tilde{m}_1)^2$ we approach the regime in which $\eta \simeq \eta_h$. In this regime the maximal CP asymmetry $\epsilon_{\text{max}}$ still increases with $M_1$; however, due to the exponential suppression of the efficiency factor $\eta_h$, here the upper bound on $m_3$ gets strengthened with increasing $M_1$.

Nevertheless, in order to get some insight into the possible scaling behavior of the limit with $\lambda \neq 1$, let us proceed analytically by adopting the simple interpolation

$$\eta \simeq \left( \frac{1}{\eta_l} + \frac{1}{\eta_h} \right)^{-1}, \quad (24)$$

that gives a reasonable fit to the detailed numerical results obtained in [3] and [7] for $\tilde{m}_1 > \tilde{m}_1^\star$. In general the maximum value of $m_3$ leading to successful leptogenesis, i.e. to $n_B/s > 9 \times 10^{-11}$, is obtained by looking to the parametric curves $m_3(M_1)$ corresponding to $n_B/s = 9 \times 10^{-11}$, and requiring that $dm_3/dM_1 = 0$. It is easy to show that, at fixed $m_3$, the baryon asymmetry is then maximized for a value of $M_1$ satisfying:

$$\frac{d \ln \eta_h}{dM_1} = -\frac{\eta_h}{\eta M_1} \implies M_1 \simeq \frac{4}{\lambda} \left( \frac{\xi \tilde{m}_1^\star}{m_3} \right)^2 M_1^\star \equiv \tilde{M}_1, \quad (25)$$

where for convenience we have introduced the factor $\xi \equiv \eta_h/\eta > 1$. For $M_1 = \tilde{M}_1$ we have $\eta = \exp(-2\xi)/\xi$ and since the upper bound on $m_3$ is associated with values of $\eta \simeq 10^{-3}$, we can expect a typical value $\xi \simeq 3$, that yields $\tilde{M}_1 \simeq 6 \times 10^{13}\lambda^{-1}(0.1 \text{ eV}/m_3)^2$ GeV. Defining $\bar{\eta} \equiv \eta(\tilde{m}_1 = \tilde{m}_3, M_1 = \tilde{M}_1)$, the maximum value of $m_3$ that results then is

$$\tilde{m}_3 \lesssim 0.19 \text{ eV} \left( \frac{\bar{\eta}\xi^2}{\lambda 10^{-2}} \right)^{1/4}, \quad (26)$$

that is in reasonable agreement with the results of dedicated numerical analyses. Note, however, that the scaling behavior of this bound under a change in $\lambda$ can be different from what is implied by the explicit dependence $\lambda^{-1/4}$, since also the parameters $\bar{\eta}$ and $\xi$ depend, in general, on $\lambda$. Finding the real $\lambda$-dependence is a non-trivial problem. We
can study this behavior by performing an infinitesimal transformation \( \lambda \rightarrow (1 + \epsilon)\lambda \), and finding out how the relevant quantities \( X = \bar{m}_3, \bar{M}_1, \eta, \eta_h \) and \( \eta_l \) scale under this transformation:

\[
X \propto \lambda^{n_X}, \quad \text{with} \quad n_X = \frac{\mathrm{d} \ln X}{\mathrm{d} \epsilon}.
\]  

(27)

Relating the exponents \( n_X \) of the different quantities, it is then possible to show that for the particular interpolation we have adopted in eq. (24) \( \bar{m}_3 \propto \lambda^{-0.4} \) is obtained. Note however, that the fine details of the transition between \( \eta_l \) and \( \eta_h \) are important for a precise determination of the scaling exponent. For example, for the more general class of interpolating functions \( \eta = (\eta_l^a + \eta_h^a)^{-1/a} \) one would find \( n_{m_3} = -0.25 \) for \( a = 0 \) and \( n_{m_3} = -0.8 \) for \( a = \infty \). Still, in spite of this uncertainty that is intrinsic to the analytical approach, the final numerical results for the strengthening of the neutrino mass limit with increasing values of \( \lambda \) do not differ too much. In particular, in the regime corresponding to \( M_1 > 10^{13} \) GeV which is relevant for the neutrino mass bound we have \( c_\ell + c_H = 5/6 \) that corresponds to \( \lambda = 5/3 \). Hence the bound on \( m_3 \) will be smaller than what obtained assuming \( \lambda = 1 \) by a factor \( (5/3)^{n_{m_3}} \), whose likely range is in between 0.66 and 0.88. We can conclude that by taking properly into account \( y_H \) in the Boltzmann equations, a bound on \( m_3 \) stronger by something of the order \( \sim 20\% \) could be obtained.

To summarize, we have considered the combined effects of the spectator processes – Yukawa, strong- and electroweak-sphaleron interactions – on the \( B - L \) asymmetry generated by leptogenesis. The effects range between reducing the final asymmetry by order 40\%, if the lepton asymmetry is generated at temperatures higher than \( 10^{13} \) GeV, to enhancing it by order 20\%, if the relevant temperature is well below \( 10^8 \) GeV. (As will be discussed in \cite{4}, when misalignment in the lepton doublet flavor space between the combination to which \( N_1 \) decays and the direction defined by fast Yukawa interactions occurs, qualitatively different and much stronger effects can arise.) Spectator processes strengthen the leptogenesis bound on the light neutrino mass scale by order 20\%.

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