Developing proportional reasoning problem like PISA with "choosing services" context

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Abstract. This study aims to produce the valid mathematic problems like proportional reasoning in PISA question. The research involved seventh-grade students in one of the junior high school in Palembang. The methodology of this research is a design research with development studies type that consist of preliminary and formative evaluation stage. This article just only discuss validation of problems process and data about student strategies in one to one process. The data collection techniques used walkthrough, documentation, observation and interview. The validity of the problems discovered from expert review in validation process. Problems validated based on content, problem has been accord to ratio and proportion learning in seventh-grade, construct, the problem require the student’s proportional reasoning and accord to abilities of the seventh-grade students; and language, the problems use of enhancing spelling, could be understood, and didn’t have variety of meanings. The result show that problems are designed using the context of choosing services can indicate how student’s proportional reasoning. Problems developed indicate that students had different strategies and do not refer to the proportion formula, but with reasoning and the use of a ratio table.

Keywords: Design Research, PISA, Proportion, Proportional Reasoning

1. Introduction

Based on the 2013 curriculum, ratio and proportion learning seventh-grade are intended so that students can use ratio concepts of value and turn value in solving real problems [1]. Based on [2] through ratio and proportion learning students can have an understanding of the concept of ratio and use proportional reasoning to solve real world problems and mathematical problems. Proportional Reasoning is also one of the abilities needed to solve the program for International Student Assessment (PISA) [3, 4, 5]. It can be concluded that in the problem solving process that uses the concept of ratio and proportion, proportional reasoning is needed.

Lack of students experiences in solving reasoning problems makes students only have a few alternative strategies if the first strategy they use does not work [6]. The result of the research show that many students who solve the converse proportion using the concept of inverse proportion [7, 8, 9, 10]. This is because students do not have much experience in solving converse and inverse proportion problems so that when they solve a problem that is different from what their teacher gives they feel difficulties [11].

The main purpose in learning ratios and proportions for students is to focus on developing proportional reasoning not just the ability to solve the problem of proportions [12]. Proportional thinking is developed through activities that involve comparing and determining the equivalence of
ratios and resolving proportions in contexts and situations based on variation of problems without referring to formulas [12]. Many traditional mathematics curricula focus on the problem of proportions and use cross multiplication procedures for solve the problems without helping students develop reasons why the procedure can be used [13]. It can also be concluded that in the process of solving problems that use the concept of ratio and proportion, it also requires reasoning ability called proportional reasoning. Evidence of the importance of proportional reasoning for one's mathematical literacy comes from observations of 11 of the 85 mathematical assessment questions in PISA 2003 involving proportional reasoning [14]. Based on the results of PISA related to the question of proportional reasoning, ranking among countries has been carried out. The results of the ranking indicate that in several questions proportional reasoning Indonesia is ranked between 1-4 from the bottom [14].

This research objective is to produce mathematics problems like proportional reasoning in PISA questions. The problems can be used as ratio and proportion teaching material that can support students' ability to recognize ratios in various situations which are one of the proportional reasoning indicators.

2. Method
This type of this research is a design research with development studies type. This research was conducted in two phases: preliminary and formative evaluation. Preliminary includes the study of literature on matters relating to research and designing proportional reasoning problems. Then in the formative evaluation includes self-evaluation, expert review, one to one, small group and field test [15, 16]. This research only discussed until one to one process as formative evaluation stage.

In the self-evaluation the problems that have been design in preliminary stage evaluated by the researcher it self. Problems are checked for suitability of the problems then the problems will be validated by the expert. The results of this stage are called prototype I.

In the expert review phase the problems that have been designed are validated by experts. In this research the problems was validated by several lecturers of Mathematics Education at Sriwijaya University and also held a panel discussion with students of the Sriwijaya University mathematics education master's program who had experience in developing teaching materials, as well as discussions with the mathematics teacher of SMP Srijaya Negara. This validation process is related to content, constructs and language in the problems that have been developed.

In the one-to-one the problems give to three students. The purpose of this stage is to get responses, comments, and find out how students respond when given a problems that has been designed. Students' responses and comments will then be used to revise the matter again. The revised results at this phase called prototype II.

Data collection techniques used was the walkthrough, document, observation, and an interview. The students strategies as a data from one to one phase then the data were analyzed by the qualitative descriptive method and describe the result of each step of development.

3. Result and Discussion
This objective of this research produced two mathematic problems like proportional reasoning in PISA question. However, in this article only discussed one problem that intertwined converse and inverse proportion.

3.1. Preliminary
In preliminary stage carried out is review some article about proportional reasoning to get the indicators of proportional reasoning and type of problem like PISA that require proportional reasoning. Then analyzed the curriculum and level of mathematical ability of students as research subject. The next steps taken are to design proportional reasoning problems like PISA and make alternative strategies used by students. The results of this phase were design lattices, some of proportional reasoning problem using “choosing service” context, and rubric assessment.
3.2. Formative evaluation

3.2.1. Self evaluation. In self-evaluation, proportional reasoning problems that designed evaluated based on content, construct, and language. The revision result called prototype I. Proportional reasoning problem using “choosing service” context can be see in figure 1.

![Figure 1. Proportional reasoning problem before revision](image)

3.2.2. Expert reviews and one-to-one. Expert reviews and one-to-one phase were conducted in parallel to see the validity of problem. prototype I was validated by experts based on existing criteria both regarding content, construct, and language. While the process of one-to-one involved three students of seventh-grade of the junior high school, namely SB (high-ability), LS (medium-ability), and RM (low-ability).

The students are asked to read and examine the problem so that the can find out the responses, constraints faced, and understanding of students in solving the problem. The responses and obstacles that observed focus on legibility and clarity of problems. Table 1 shows the validation result from expert reviews and one-to-one phases.

| Validation         | Comment/Response | Revise                        |
|--------------------|------------------|-------------------------------|
| Expert reviews     |                  |                               |
| • Convection works in teams so that the third information needs to be changed | • Change the term “pekerja” to “penjahit” |
| • Numbers used from information on working days and hours are too complicated | • Questions give students the freedom to determine selected convection |
| • What the meaning of “pcs” | • Change information about working days and working hours |
| • Students think that the working day on the information is that the convection completes the order | • Change “pcs” to “jaket” |
| Students           |                  | • Add note “ hari kerja”    |
Furthermore, the problem revised based on the results of expert validation and one-to-one. It can be concluded that the problems have designed are categorized as valid problems. It’s reflected in the comment provided by expert, then responses, expression, and student’s understanding when solving the problem [17].

The validity of the problems in term of content problem has been accord to ratio and proportion learning in seventh-grade; in terms of construct, the problem require the proportional reasoning for get the answer and accord to abilities of the seventh-grade students; and in terms of language, the problems use of enhancing spelling, could be understood, and didn’t have variety of meanings [9, 18, 19]. Then, the revised result called prototype II and used in the next phase. The problem in prototype II can be seen in Figure 2. Information that is given a red mark is the result of a revision made.

![Figure 2. Proportional reasoning problem after revision](image)

In one-to-one phase, the students strategies data in solving requested and analysed. From the three students with different cognitive levels (high, medium, low) researchers found that the strategies they used were different. From their answer show how the problems designed can be used to see students' proportional reasoning. SB’s answer (high-ability) can be seen in Figure 3.

![Figure 3. SB’s strategies solve the problem (convection 1)](image)

![Figure 4. SB’s use additive thinking](image)
Figure 3 shows that SB can recognize ratios in various situations. This is part of proportional reasoning [12]. Students also can describe proportional situations using multiplicative language. This shows that SB shows indicators of proportional reasoning [8]. Even though, SB uses additive thinking in the ratio situation between many working days and hours at convection 1 and 2. This is indicated by students looking for 6 added one by one until they get a score of 48. The same is shown in the other work in figure 4.

Strategi berbeda ditunjukkan oleh siswa dengan kemampuan rendah (TM). TM’s strategies can be seen in figure 5.

![Figure 5. TM's strategies solve the problem](image)

The strategy used by TM is different from the other two students. The first step he use ratio of many jacket with the finishing time. At first, TM use additive thinking, then TM looks to use multiplicative thinking to arrive at 144 jackets. This is because to get number 144 from number 3 is 48 times, so this require students to use proportional reasoning.

The second step TM use the ratio of many tailors with finishing time (inverse proportions). This strategy shows that TM can think that if work is done with more workers then the time taken will be less. From Figure 5 TM use division process.

Based on all the research phases the problem using “choosing service” context that has designed is categorized valid. The results of the description of the validation process from experts indicate that the problems developed are in accordance with proportional reasoning indicators. Based on the analysis results of student answers, students can identify the ratio of the information provided. From the strategy students also can be seen how their proportional reasoning abilities. Proportional reasoning appear when students able to use multiplicative thinking to solve problems [3, 5]. Multiplicative thinking in mathematics is the ability to see situations in a multiplicative sense rather than additive and flexibility in thinking about numbers and situations involving numbers [20].

Based on results of student answers, it can be seen that students still tend to use additive thinking. From this study the teacher can design learning activities that help students understand the ratio is a multiplication relationship. The activity used does not only focus on the use of formulas but uses reasoning activities. One informal activity to develop proportional reasoning is to help students distinguish between proportional situations and additive situations [12]. Dengan mendesain soal PISA semacam ini dapat dimanfaatkan sebagai potential learning resources for improving students’ mathematical literacy [21].

4. Conclusion
This research produce valid problem using choosing sevice as context to see how student’s proportional reasoning. The problem concluded as a valid problem based on all expert validator state. From the validation process said that the problems are well designed from content, construct and language point of view. Some students' answers can be used to identify who uses proportional reasoning to solve problems that have been developed. The student strategies for identify the ratio situation can be seen from the way they determine how long each convection finished the order. Calculations that students do also show they do not use some formule but rather than informal method. There is student still use additive thinking rather than multiplicative thinking as a characteristic of proportional thinker.

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