Microscopic Cluster Models: application to the structure of the $^{16}$B nucleus

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Abstract. General aspects of microscopic cluster models based on the combination of the Generator-Coordinate-Method and of the $R$-matrix method are presented. The adequacy of such methods to describe the physics of exotic light nuclei is illustrated with the unbound $^{16}$B nucleus.

1. Introduction

Microscopic Cluster Models (MCM) based on the combination of the Generator-Coordinate-Method (GCM)[1] and of the Microscopic R-matrix method (MRM) [2] are very efficient tools to investigate the nuclear many-body problem.

The principle is to treat the wave function of the $A$-nucleon system at the cluster approximation. In other words, the $A$ nucleons are assumed to be divided in clusters, described by shell-model wave functions. The cluster approximation is at the origin of the Resonating Group Method (RGM) proposed by Wheeler [3]. A significant breakthrough in microscopic cluster theories has been achieved by the introduction of the GCM [1] which is equivalent to the RGM. The GCM leads to simpler and more systematic calculations. The RGM relative functions are expanded over Gaussian functions in the GCM. The Gaussian behaviour is corrected through the MRM for scattering states [2].

The MCM framework presents numerous advantages. The Pauli principle is exactly treated through the antisymmetrization of all nucleons. The center of mass is exactly separated and the quantum numbers associated with the colliding nuclei are restored. Furthermore, the GCM wave functions of the bound and scattering states present a correct asymptotic behaviour.

Microscopic cluster models present a wide range of applications. In nuclear structure, numerous calculations attest their adequacy to describe the physics of exotic nuclei, and in particular of halo nuclei. Molecular states, which are known to be strongly deformed, and present a marked cluster structure are successfully described by such investigations [4]. They are also well suited to describe nuclear reactions of astrophysical interest where antisymmetrization effects are expected to be important.

The present paper presents a short overview of the MCM theoretical framework (Section 1). For the sake of simplicity, we limit the expressions to a two-cluster model with identical oscillator parameters. More extensive presentations can be found in Refs. [2, 5]. The recent
study of the $^{16}$B resonances above the $^{15}$B+n threshold is chosen as a typical example of nuclear structure calculations well adapted to the microscopic cluster approach [6]. Indeed, the $^{16}$B being an unbound nucleus, only resonances are expected. In such conditions, wave-functions required a rigorous treatment of their asymptotic part that the MCM is able to performed in contrast to the traditional shell-model (SM) approaches (Section 2).

2. Short overview of the theoretical framework

In microscopic cluster models, the Hamiltonian of the $A$-nucleon system is usually written as

$$H = \sum_{i=1}^{A} T_i - T_{c.m.} + \sum_{i>j=1}^{A} (V_{ij}^C + V_{ij}^{SO} + V_{ij}^{Coul}),$$

(1)

where $T_i$ is the kinetic energy, $T_{c.m.}$ is the kinetic energy of the total center of mass, $V_{ij}^C$ is the central nucleon-nucleon part, $V_{ij}^{SO}$ is an effective spin-orbit interaction and $V_{ij}^{Coul}$ is the Coulomb force which is exactly treated. The central part is generally taken as a combination of Gaussian form factors. Typical interactions are the Volkov V2 and Minnesota interactions [7, 8].

In the following, we consider a typical process between a nucleus (1) and a nucleus (2) with $A_1$ and $A_2$ nucleons, respectively. The notation (1,2) refers to the unified nucleus with $A = A_1 + A_2$ nucleons. In the present framework, the wave functions of the unified nucleus are expressed as sums of multichannel functions. More precisely, in partial spin and parity $(J, \pi)$ wave, they can be written in RGM notation as

$$\Psi^{JM\pi} = \sum_{c \ell I} \mathcal{A} \, g_{c\ell I}^{J\pi} (\rho) \left[ (\phi_{c,1}^{I_{1}} \otimes \phi_{c,2}^{I_{2}})^{I} \otimes Y_{\ell}(\Omega_{\rho}) \right]^{JM},$$

(2)

where $\mathcal{A}$ is the $A$-nucleon antisymmetrizer, $c$ labels the various channels, $\ell$ is the angular momentum between (1) and (2), $I_1$ and $I_2$ are the spin of (1) and (2) respectively, $I$ is channel spin, and $g_{c\ell I}^{J\pi}(\rho)$ are the relative functions depending on the relative coordinate $\rho$.

In the GCM, the $g_{c\ell I}^{J\pi}(\rho)$ functions are expanded over Gaussian functions $\Gamma_\ell(\rho, R)$

$$g_{c\ell I}^{J\pi}(\rho) = \int f_{c\ell I}^{J\pi}(R) \, \Gamma_\ell(\rho, R) \, dR,$$

(3)

where $R$ is the generator coordinate defined as the distance between the two clusters and $f_{c\ell I}^{J\pi}(R)$ are the generator functions obtained through the MRM (see Ref.[2] for more details). In practice, Eq. (3) is replaced by a finite sum over the generator coordinates $R$. It results a Gaussian behavior of the wave function which is corrected through the MRM.

The principle of the MRM is to divide the space into two regions delimited by a surface defined by $\rho = a$ where $a$ is the channel radius. In the internal region, the wave functions are given by the GCM. In the external region, radial functions are known to be the Coulomb functions. Matching conditions at $\rho = a$ provide the collision matrix and the cross section. Let us notice that the results do not depend on $a$. This method ensures the correct treatment of the asymptotic properties of the wave functions and leads to an unified description of bound states and resonances [2].

3. Application to the structure of the $^{16}$B

Experiments involving the $^{16}$B nucleus are difficult to perform. Its unbound nature has been confirmed by Kryger et al. [9]. Besides it exists mainly two experimental investigations of the low-lying structure. The first one performed by Kalpacheva et al. [10] shows the existence of a very narrow structure some 40 keV above the $^{15}$B+n threshold. Another peak is identified at
2.32 MeV. A second experiment performed more recently by Lecouey et al. [11] confirms the existence of the narrow structure at some 85 keV above the $^{15}$B+n threshold.

On the theoretical side, shell-model calculations have been performed [11]. However the unbound nature of the $^{16}$B nucleus raises the question of the accuracy of such an approach to describe states where the asymptotic behavior of the wave function cannot be neglected. MCM approach seems then better adapted.

In such a context, we have investigated the structure of $^{16}$B nucleus with an Extended Two-Cluster Model which includes many $^{15}$B+n channels [6]. The $^{16}$B wave functions are defined as a sum of $^{15}$B+n channel functions. $^{15}$B internal wave functions are obtained with 3 protons in the $p$ shells, the $s$ shell being filled for the proton part and 2 neutrons in the $sd$ shell, the $s$ and $p$ shells being filled for the neutron part. This description involves 1320 different Slater determinants leading to 87 $^{15}$B+n different channels corresponding to the $^{15}$B ground state and 86 $^{15}$B excited states. All the calculations are performed with the Volkov interaction.

Our calculations reproduce the $^{15}$B 3/2$^-$, 5/2$^-$ and 7/2$^-$ ground-state band in fair agreement with experiment. It is well known that a good description of $^{15}$B is important in order to get a good description of the $^{16}$B nucleus. $^{16}$B spectra are shown in Fig. 1. Experimental values of Kalpachieva et al. [10] and Lecouey et al. [11] are displayed together with the present GCM calculations. SM results [11] are also reproduced in order to compare both theoretical approaches. In the present work, the $^{16}$B resonance analysis is performed in terms of eigenphase shifts which are obtained from the diagonalization of the matrix of collision. The number of eigenphases is equal to the number of open channels at a given energy. To illustrate this point, we show typical curves obtained for the 0$^-$ and 1$^-$ states in Fig. 2. These curves are expected to provide information on the resonant structure for a given $J^\pi$ (see more details in Ref.[6]).

![Figure 1. $^{16}$B spectra.](image)

Several resonances are obtained at low energy with the GCM. SM calculations also lead to a similar result but the ordering of the states is different. The existence of a very narrow resonance with a $\ell = 2$ dominant component above the $^{15}$B+n threshold is confirmed by the GCM. The spin is assigned to 0$^-$. The corresponding value of the width is $\Gamma = 1.26 \times 10^{-2}$ keV in agreement with experiment.

All other resonances predicted by the GCM near the threshold present dominant $\ell = 0$ components. Let us notice that the 1$^-$ state is even slightly bound ($E_{c.m.} = -0.01$ MeV). Taking
Figure 2. $^{16}$B eigenphaseshift analysis of the $0^-$ and $1^-$ states as a function of the energy.

into account the accuracy of our calculations, one can just predict the existence of a $1^-_1$ state near the threshold which could be the ground state. All these results are not in agreement with the SM ones. In particular, the $1^-_1$ states are predicted at higher energy with the SM. The main origin of these differences is probably the fact that the SM is not adapted to describe broad resonances because of the non accurate asymptotic behavior of the wave functions. New experiments would be necessary to clarify the situation.

4. Conclusion

The MCM framework presents many advantages to solve the nuclear many body problem. It is particularly well adapted to the study the structure of light nuclei. In such context, we aim to apply this framework to the physics of states presenting a condensate structure.

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