DESIGN THINKING ON $\delta$- DYNAMIC COLORING OF CENTRAL VERTEX JOIN OF GRAPHS

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Abstract: An $r$-dynamic coloring of a graph $G$ is a proper coloring $c$ of the vertices such that $|c(N(v))| \geq \min \{r, d(v)\}$, for each $v \in V(G)$. The $r$-dynamic chromatic number of a graph $G$ is the minimum $k$ such that $G$ has an $r$-dynamic coloring with $k$ colors. In this paper, we obtain the $\delta$-dynamic chromatic number of the central vertex join of two graphs.

Keywords: Central graph, Design thinking, $\delta$-dynamic coloring and Central vertex join of graph.

1. Introduction

Design thinking [9], [10] is a non-linear, iterative process that teams use to understand users, challenge assumptions, redefine problems and create innovative solutions to prototype and test. It is most useful to tackle problems that are ill-defined or unknown. Design thinking’s value as a world-improving, driving force in business (global heavyweights such as Google, Apple and Airbnb have wielded it to notable effect) matches its status as a popular subject at leading international universities. With design thinking, teams have the freedom to generate ground-breaking solutions. Design thinking is described as a five-stage process namely Empathize, Define, Ideate, Prototype and Test.

The $r$-dynamic chromatic number was first introduced by Montgomery [3]. An $r$-dynamic coloring of a graph $G$ is a proper coloring and it maps $c$ from $V(G)$ to the set of colors such that (i) if $uv \in E(G)$, then $c(u) \neq c(v)$ and (ii) for each vertex $v \in V(G)$, $|c(N(v))| \geq \min \{r, d(v)\}$, where $N(v)$ denotes the set of vertices adjacent to $v$, $d(v)$ its degree and $r$ is a positive integer. The $r$-dynamic chromatic number of a graph $G$, written $\chi_r(G)$, is the minimum $k$ such that $G$ has an $r$-dynamic proper...
k-coloring. In this paper we consider only the graphs which are simple, finite, loopless and connected. For all terms and definition which are not specifically defined in this paper, we refer to [2]. The r-dynamic chromatic number has been studied by many authors, for instance in [1], [2], [5], [6], [7], [8]. In this paper, we find the δ-dynamic chromatic number of the central vertex join of path with path, complete, complete bipartite and fan graphs.

2. Empathy

Graph theoretical concepts are widely used to study and model various applications in different areas. Graph coloring especially used in various research areas of computer science such as data mining, image segmentation, image capturing, networking etc. In graph theory, graph coloring is a special case of graph labeling; it is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints. In its simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices are of the same color; this is called a vertex coloring. Vertex coloring is usually used to introduce graph coloring problems, since other coloring problems can be transformed into a vertex coloring instance. Graph coloring problem is a NP complete problem. We use Graph coloring to optimize the resource we need and the graph is an abstract of the problem.

3. Define

The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color. The minimum number of colors used is called as the chromatic number of the graph. We have many coloring for vertex coloring of graphs. Some of the which are listed as below.

- b coloring
- Equitable coloring
- Acyclic coloring
- Irregular coloring
- r-dynamic coloring
- Star coloring
- Fall coloring

4. Ideate

In this paper, we find the δ-dynamic chromatic number of the central vertex join of path with path, complete, complete bipartite and fan graphs.

For a given graph G = (V, E), we do an operation on G by subdividing each edge exactly once and joining all the nonadjacent vertices of G. The graph obtained by this process is called central graph [6] of G denoted by C(G).

Let G₁ and G₂ be any two graphs. The central vertex join [4] of G₁ and G₂ is the graph G₁ ∨ G₂ is obtained from C(G₁ ) and G₂ by joining each vertex of G₁ with every vertex of G₂.
An r-dynamic coloring of a graph G is defined as a proper coloring c of the vertices such that |c(N(v))| ≥ min {r, d(v)}, for each v ∈ V(G). The r-dynamic chromatic number of a graph G is the minimum k such that G has an r-dynamic coloring with k colors. For δ-Dynamic coloring, we consider r = δ.

5. Prototype and Test

**Theorem 5.1**

Let m, n ≥ 2 and < n, the δ-dynamic chromatic number of \( P_m \circ P_n \) is
\[
\chi_\delta(P_m \circ P_n) = m + 2
\]

**Proof:** The maximum and the maximum degrees of \( P_m \circ P_n \) are \( \Delta(P_m \circ P_n) = n + m - 1 \) and \( \delta(P_m \circ P_n) = 2 \) respectively.

Let \( V(P_m \circ P_n) = \{v_i : 1 \leq i \leq m\} \cup \{u_j : 1 \leq j \leq n\} \cup \{x_i : 1 \leq i \leq m-1\} \)

\( E(P_m \circ P_n) = \{v_1, x_1, v_2, ..., v_{m-1}, u_1\} \cup \{u_1, u_2, u_3, ..., u_{n-2}, u_n\} \cup \{v_i, u_j, \forall i, j\} \cup \{v_1, u_2, v_3, u_2, v_3, ..., v_2, v_3, ..., v_{m-2}, u_m\} \)

Define a the color function \( c_1 : V(P_m \circ P_n) \rightarrow \{c_1, c_2, ..., c_{m+2}\} \) as follows.
\[
c(v_i) = c_1, \quad i = 1 \text{ to } m

c(x_i) = c_{i+1}, \quad \forall i = 1 \text{ to } m-2

c(x_{m-1}) = c_1

c(u_i) = \{c_2, c_{m+2}, c_2, c_{m+2}, \ldots\}
\]

Now, it is easy to check that δ-adjacency condition is fulfilled

Hence \( \chi_\delta(P_m \circ P_n) = m + 2 \)
Theorem 5.2

Let \( m, n \geq 3 \) and \( n < m \), the \( \delta \)-dynamic chromatic number of \( P_m \diamond K_n \) is

\[ \chi_\delta(P_m \diamond K_n) = m + n \]

Proof: The maximum and the maximum degrees of \( P_m \diamond K_n \) are \( \Delta(P_m \diamond K_n) = n + m - 1 \) and \( \delta(P_m \diamond K_n) = 2 \) respectively.

Let \( V(P_m \diamond K_n) = \{ v_i : 1 \leq i \leq m \} \cup \{ u_j : 1 \leq j \leq n \} \cup \{ x_i : 1 \leq i \leq m-1 \} \)

\( E(P_m \diamond P_n) = \{ v_1, v_2, \ldots, v_{m-1}v_m \} \cup \{ u_j, v_k, j, k=1 \) to \( m, j \neq k, j < k \} \) \cup \{ v_i, u_j, \forall i, j \} \cup \{ v_1v_3, v_1v_4, \ldots, v_{m-2}v_m \} \)

Define a color function \( c_1 : V(P_m \diamond K_n) \rightarrow \{ c_1, c_2, \ldots, c_{m+n} \} \) as follows.

\[ c(v_i) = c_i, \quad i = 1 \) to \( m \]
\[ c(x_i) = c_{i+1}, \forall i = 1 \) to \( m-2 \]
\[ c(x_{m-1}) = c_1 \]
\[ c(u_i) = c_{m+i} \]

Now, it is easy to check that \( \delta \)-adjacency condition is fulfilled

Hence \( \chi_\delta(P_m \diamond K_n) = m + n \)

Theorem 5.3

Let \( m, n \geq 3 \) and \( n < m \), the \( \delta \)-dynamic chromatic number of \( P_m \diamond K_{1,n} \) is

\[ \chi_\delta(P_m \diamond K_{1,n}) = m + 2 \]

Proof: The maximum and the maximum degrees of \( P_m \diamond K_{1,n} \) are \( \Delta(P_m \diamond K_{1,n}) = n + m \) and
\[ \delta(P_m \hat{\lor} K_{1,n}) = 2 \] respectively.

Let \( V(P_m \hat{\lor} K_{1,n}) = \{ v_i : 1 \leq i \leq m \} \cup \{ u, u_j : 1 \leq j \leq n \} \cup \{ x_i : 1 \leq i \leq m-1 \} \)

\[ E(P_m \hat{\lor} K_{1,n}) = \{ v_1 x_1, x_1 v_2, \ldots, x_{m-1} v_m \} \cup \{ u u_j : j = 1 \text{ to } n \} \cup \{ v_i u, v_i u_j \forall i, j \} \cup \\
\{ v_1 v_3, v_1 v_4, \ldots, v_1 v_m, v_2 v_4, \ldots, v_{m-2} v_m \} \]

Define a color function \( c_1 : V(P_m \hat{\lor} K_{1,n}) \to \{ c_1, c_2, \ldots, c_{m+2} \} \)

\[ c(v_i) = c_i, \text{ for } 1 \text{ to } m \]
\[ c(x_i) = c_{i+2}, \forall i = 1 \text{ to } m-2 \]
\[ c(x_{m-1}) = c_1 \]
\[ c(u) = c_{m+1} \]
\[ c(u_j) = c_{m+2}, \forall i = 1 \text{ to } n \]

Now, it is easy to check that \( \delta \)-adjacency condition is fulfilled

Hence \( \chi_\delta(P_m \hat{\lor} K_{1,n}) = m + 2 \)

\[ \text{Theorem 5.4} \]

Let \( m, n \geq 3 \) and \( < n \), the \( \delta \)-dynamic chromatic number of \( P_m \hat{\lor} F_n \) is

\[ \chi_\delta(P_m \hat{\lor} F_n) = m + 3 \]

\[ \text{Proof:} \] The maximum and the maximum degrees of \( P_m \hat{\lor} F_n \) are \( \Delta(P_m \hat{\lor} F_n) = m + 2n \) and \( \delta(P_m \hat{\lor} F_n) = 2 \) respectively.

Let \( V(P_m \hat{\lor} F_n) = \{ v_i : 1 \leq i \leq m \} \cup \{ u, u_j : 1 \leq j \leq 2n \} \cup \{ x_i : 1 \leq i \leq m-1 \} \)

\[ \text{Diagram of } (P_m \hat{\lor} K_{1,1}) = 6 \]
Define a color function \( c_1 : V(P_m \ast P_n) \rightarrow \{c_1, c_2, \ldots, c_{m+3}\} \) as follows.

\[
c(v_i) = c_1, \quad i = 1 \text{ to } m \\
c(x_i) = c_{i+1} \quad \forall i = 1 \text{ to } m-2 \\
c(x_{m-1}) = c_1 \\
c(u) = c_{m+1} \\
c(u_k) = \begin{cases} m + 2, & \text{if } k \text{ is odd} \\ m + 3, & \text{if } k \text{ is even} \end{cases}
\]

Now, it is easy to check that \( \delta \)-adjacency condition is fulfilled.

Hence \( \chi_{\delta}(P_m \ast P_n) = m + 3 \)

6. Conclusion:

Many challenges in the world are not solved because we try to focus too much on the problem statement. We also get much stress to find out the solution of the problem. The tool Design thinking helps us to gain the knowledge to balance problem statement and solution developed. Design thinking not only helps to come up with innovative solutions, but also helps to address the exact problem and target the solution in the best possible manner. In this paper we obtained the find the \( \delta \)-dynamic chromatic number of the central vertex join of path with path, complete, complete bipartite and fan graphs.

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