Scalable solid-state quantum processor using subradiant two-atom states

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We propose a realization of a scalable, high-performance quantum processor whose qubits are represented by the ground and subradiant states of effective dimers formed by pairs of two-level systems coupled by resonant dipole-dipole interaction. The dimers are implanted in low-temperature solid host material at controllable nanoscale separations. The two-qubit entanglement either relies on the coherent excitation exchange between the dimers or is mediated by external laser fields.

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The main stumbling blocks en route to the realization of useful quantum computers, comprised of many qubits, are (i) fidelity loss due to decoherence, which grows with the amount of single- and two-qubit operations and requires large redundancy for the application of error-correction methods; (ii) scalability of the quantum processor (QP), which restricts the choice of candidate systems and gives preference to solid-state structures. QP proposals and realizations have thus far predominantly involved optical manipulations of atoms in ion traps, high-Q cavities, and optical lattices. Yet, the decoherence caused by radiative (spontaneous emission) and nonradiative processes, as well as difficulties with the scalability, cast doubts on the suitability of these schemes for truly large-scale quantum computation. Solid-state QP realizations appear to be more promising, both principally and technologically.

Here we propose a combined optical/solid-state approach that can significantly enhance the speed, fidelity and scalability of a QP. The crux of this approach is the hitherto unexplored concept of a “subradiant dimer” (SD) qubit: two similar two-level systems (atoms or the hitherto unexplored concept of a “subradiant dimer” (SD) qubit: two similar two-level systems (atoms or

The atomic Hamiltonian, with $\omega_{eg}$ being the resonant frequency and $\gamma$ the radiative decay rate on the atomic transition $|e\rangle \rightarrow |g\rangle$, and $V_{\text{RDDI}} = \hbar (\Delta - \gamma_{12}/2) \langle |e_1\rangle |g_2\rangle + |g_1\rangle |e_2\rangle$ describes the interatomic RDDI potential, whose real part $\Delta$ is equal to the rate of coherent excitation exchange (hopping) between the atoms, and the imaginary part $\gamma_{12}$ is responsible for the cooperative radiative decay of the system. Both $\Delta$ and $\gamma_{12}$ are functions of the normalized distance $\zeta$, and $\gamma_{12} = |r_1 - r_2|$. The diagonalization of Hamiltonian (1) yields the “dimer” eigenstates $|G\rangle = |g_1g_2\rangle$, $|\pm\rangle = \sqrt{2} (|e_1g_2\rangle \pm |g_1e_2\rangle)$, and $|E\rangle = |e_1e_2\rangle$, with the energy eigenvalues $\lambda_G = 0$, $\lambda_{\pm} = \omega_{eg} \pm \Delta - i\Gamma_{\pm}/2$, and $\lambda_E = 2\omega_{eg} - i\Gamma_E/2$, respectively [Fig. 2b]. At small separations $\zeta \ll 1$, the symmetric $|+\rangle$ and doubly-excited $|E\rangle$ eigenstates are superradiant, having the corresponding decay rates $\Gamma_+ = \gamma + \gamma_{12} \approx \Gamma_E = 2\gamma$, while the antisymmetric eigenstate $|\mp\rangle$ is subradiant, with the decay rate $\Gamma_- = \gamma - \gamma_{12} \approx \gamma^2/5 \ll \gamma$. The energy levels of states $|\pm\rangle$ are then shifted from that of state $|e\rangle$ by $\pm \Delta$, with $|\Delta| \approx 3\gamma/(4\zeta^3) \gg \gamma$.

The coupling strength of a laser field $\mathcal{E}$, having fre-
During the qubit flip-time $T$ (rotation) can be performed by the laser field from the subradiant state $|−⟩$. In the limit of small interatomic separations, $Ω_+ ≃ 21/2Ω$ and $Δ \approx i21/2Ω\cos(\phi)$, where $\phi$ is the angle between the vectors $k$ and $r_{12}$. Hence, $Ω_- \parallel r_{12}$ identically vanishes if the propagation direction of the field is perpendicular to the interatomic axis, $k \perp r_{12}$, while it is maximized in the $k \parallel r_{12}$ configuration, for $ζ ≪ 1$. In physical terms, the subradiant $|G⟩ → |−⟩$ transition exhibits a quadrupolar behaviour and dipole-moment suppression, due to destructive interference of the two-atom interactions with the field, as opposed to their constructive interference in the superradiant $|G⟩ → |+⟩$ transition.

Now we are in a position to introduce the concept of the “subradiant dimer” (SD) qubit. The two subradiant states correspond to the ground $|G⟩$ and subradiant $|−⟩$ states of the dimer. An arbitrary single-qubit operation (rotation) can be performed by the laser field $E_σ$ with wave vector $k_σ$ and frequency $ω_σ = ω_{eg} − Δ$ that is resonant with the qubit transition $|G⟩ → |−⟩$ [Fig. 1(b)]. During the qubit flip-time $T_{flip} = \pi/(2Ω^+)$, the probability of error $P_{err}^+ \leq 2Ω_+|T_{flip}|/(2Δ)^2 = 8\sqrt{2π}Ω_+ζ^5/(9γ)$). As an example, for the parameters $ζ ≃ 0.02$ and $Ω_+ ≤ 500$, the decay rate of the antisymmetric state is $Γ_− ≃ 8 \times 10^{−5}\gamma$, while the probability for population transfer from the ground state $|G⟩$ to the subradiant state $|+⟩$ satisfies $P_{err} < 1/(Ω_+) < 10^{−5}$, and the error probabilities during the flip-time of a SD qubit are $P_{err}^+ ≤ 3 \times 10^{−5}$ and $P_{err}^+ < P_{err}^+$, as compared to the corresponding error probability for a single atom, $P_{atom}^+ ≤ 2\gamma/(2Ω_+) ≃ 0.05$. Such small errors of the SD qubit are amenable to error correction [4].

In order to read-out (measure) the state of the qubit, we may use a modification of the electron-shielding technique [4]. Let us apply for a time $T_{swap}$ a probe field $E_p$ at a frequency $ω_p = ω_{eg} + Δ$ that is resonant with the dimer transition $|G⟩ → |+⟩$. Since the Rabi frequency on this transition is much larger than on the qubit transition $|G⟩ → |−⟩$, from which the probe field is detuned by $Δ$, the presence or absence of fluorescence from $|+⟩$ would indicate whether the qubit state is $|G⟩$ or $|−⟩$, respectively. However, since the frequency $ω_p$ exactly matches that of the transition $|−⟩ → |E⟩$, the dimer in state $|−⟩$ can first be excited to $|E⟩$ by absorbing a probe photon, then decay to $|+⟩$, subsequently producing the same fluorescence signal as if it were initially in state $|G⟩$. Therefore, for a reliable measurement, the condition $γ−T_{swap} < 1$ should be satisfied, where $γ = |Ω_p|^2ζ^2/γ$ is the rate of transition $|−⟩ → |+⟩$. This leads to the condition $Ω_p/γ < \sqrt{2ζ}/Ω_−$, where $ζ ≪ 1$ is the detection efficiency. With $ζ ≃ 0.3$, $Ω_p/γ ≃ 5$, and $ζ ≃ 0.02$, we obtain 98% measurement reliability. If, however, the probe laser is applied for a time $T_{swap} ≃ γ−$, it will initialize the state of the qubit to its ground state $|G⟩$.

We next consider the RDDI-induced entanglement between two neighboring dimers of size $ζ$, labeled as $A$ and $B$, whose normalized separation $ξ = qτ_{AB}^2$ satisfies the condition $ζ ≪ ξ ≪ 1$ [Fig. 3(a)]. The rate of coherent excitation exchange between the dimers on the qubit transitions $|G⟩_{AB} → |−⟩_{AB}$ is given by $Δ_{AB}^2 ≃ 2Γ_−/(4ζ/3)$, if the difference in the qubit transition frequencies of the two dimers exceeds $Δ_{AB}^2$ (as is usually the case in a solid host), then their excitation exchange is effectively switched off. To switch their interaction on, one can apply an off-resonant, intense, standing-wave field, such that dimers $A$ and $B$ are exposed to different field amplitudes and therefore undergo different ac Stark shifts [Fig. 3(a)]. The standing-wave pattern is then shifted along the $A − B$ axis until the qubit transitions of the two dimers become resonant. Then, during the time $T_{swap} = π/(2Δ_{AB}^2)$, the swap transformation takes place, $|−⟩_{A(B)}|G⟩_{B(A)} → −i|G⟩_{A(B)}|−⟩_{B(A)}$, while other initial states of the two qubits, $|−⟩_{A}|-⟩_B$ and $|G⟩_A|G⟩_B$, remain unaffected [Fig. 3(b)]. In the same way, one can realize the square-root of swap $\sqrt{SWAP}$ gate between two qubits. By switching on the interaction for time $T_{\sqrt{SWAP}} = π/(4Δ_{AB}^2)$, one fully entangles the two qubits, attaining an equally-weighted superposition of SWAP and no-SWAP,

$$|−⟩_{A(B)}|G⟩_{B(A)} → \frac{1}{\sqrt{2}}[|−⟩_{A(B)}|G⟩_{B(A)} − i|G⟩_{A(B)}|−⟩_{B(A)}].$$ (2)

The main source of error in this scheme is the cooperative spontaneous decay of the excited states of the qubits,
inset). The Rabi frequencies of this field on the transition $|G_A G_B\rangle$ undergo the Rabi cycle from $|G_A G_B\rangle$ to $|M\rangle$ and back, resulting in the $\pi$-phase-shift

$$|G_A G_B\rangle \rightarrow -|G_A G_B\rangle.$$

This transformation corresponds to the cphase logic gate, since all other initial states, such as $|\rangle_A |\rangle_B$ and $|\rangle_A |G_B\rangle$, remain unaffected, due to the fact that the RDDI between the dimers is present only if their combined state is either $|G_A |\rangle_B$ or $|\rangle_A |G_B\rangle$, otherwise the coupling field is off-resonant with the system. The error during this gate operation is due to the spontaneous emission from the state $|M\rangle$, with the probability $P^\text{err}_{\text{cphase}} \leq \Gamma_M T_{\text{cphase}} = 2\pi\gamma\xi/5\Omega_c$, as well as due to population transfer from the state $|G_A G_B\rangle$ to the state $|P\rangle$, with the probability $P^\text{tr}_{\text{cphase}} \leq \Gamma_P |\Omega^{c}_P|^2 T_{\text{cphase}}/(2\Delta^{+}_{AB})^2 = 16\pi\xi/9\gamma$. With $\Omega/\gamma \simeq 30$ and $\xi \simeq 0.1 > \zeta$, we obtain $P^\text{err}_{\text{cphase}} \leq 4 \times 10^{-4}$ and $P^\text{tr}_{\text{cphase}} \ll P^\text{err}_{\text{cphase}}$. This error probability is exactly the same as for a SWAP gate with similar $\xi$, but the cphase gate is then 25 times faster than the SWAP gate, since $T_{\text{SWAP}}/T_{\text{cphase}} = 10\Omega_c/14(3\gamma)^2$.

Having established all the basic principles of the proposed QP, we now describe its possible realization. We envision a solid-state host doped with active atoms having non-degenerate ground state, so as to avoid mixing of various degenerate atomic states, which would invalidate our simple two-level atomic model. Possible candidate systems include sulphur-doped silicon, rare-earth (Yb or Nd) doped crystals [15], or semiconductor based nanostructures (quantum dots) [10]. The implantation of dopants and dots with controllable separations of few nanometers is achievable with reasonable accuracy [14, 17].

With the arrangement of dopants shown in Fig. 4 inset, our scheme is capable of implementing arbitrary one-qubit rotations and two-qubit logic gates, so as to obtain any desired unitary transformation $\mathbf{1}$. (a) Individual SD qubits would be rotated or read-out (and initialized) by laser fields with frequency $\omega$, or $\omega_p$, respectively (see above), and wave vector parallel to the interatomic axis, using the “near-field” technique (Fig. 4 inset). The polarization of these fields can be chosen such that they act only on the atomic transition from the nondegenerate ground state to one of the magnetic sublevels of the excited state, consistently with our two-level description of the atoms. (b) The cphase gate between a chosen pair of qubits $A$ and $B$ is executed by a coupling field with frequency $\omega$, and wave vector $\mathbf{k}_c \parallel r_{AB}$ that are specific for that pair. (c) The SWAP action between neighboring qubits can be used to convey the information in the QP, step-by-step, over large distances for which

$$P^\text{err}_{\text{SWAP}} \leq 2T_{\text{SWAP}} = 4\pi\xi^3/3.$$
the direct RDDI vanishes. To neutralize the swap, one can flip the qubits at time intervals short compared to $[\Delta_{AB}]^{-1}$, which is equivalent to the spin echo technique used in NMR [13]. Alternatively, the $\sqrt{\text{SWAP}}$ gate between two qubits $A$ and $B$ can be switched on and off via external ac Stark fields.

Throughout this paper we have only dealt with the radiative relaxation of the excited atomic state $|e\rangle$. This is adequate provided the competing nonradiative relaxation process are strongly suppressed by working below the liquid helium temperature [19] and/or using fast ac Stark modulation of the vibrationally relaxing levels [20]. Another important consideration is the inhomogeneous broadening of the atomic resonances. Consider two atoms having slightly different resonant frequencies, $\omega_{eg}^{(2)} - \omega_{eg}^{(3)} = \delta$, due to the host inhomogeneity. This frequency mismatch results in an increase of the decay rate $\Gamma_\text{C}$ of the SD qubit in the amount $\gamma \delta^2/(8\Delta^2)$. If we require that this additional relaxation rate does not exceed $\Gamma_\text{C}$ for two resonant atoms, we obtain that the inhomogeneous width $\delta$ must be less than $\gamma/\zeta^2$, which, for $\zeta \simeq 0.02$, yields $\delta \lesssim 2.5 \times 10^3 \gamma$.

It is instructive to compare our scheme with previously considered optically-controlled single- and two-qubit quantum gates:

1) In a commonly used optical scheme [1, 3, 11], a Raman qubit is represented by two metastable ground states $|g_1\rangle$ and $|g_2\rangle$ that are manipulated by two laser fields detuned by the amount $\delta_e \gg \gamma_e$ from the intermediate excited state $|e\rangle$ having the spontaneous decay rate $\gamma_e$. The error probability during the qubit flip $P_{\text{err}} \lesssim \pi \gamma_e/(2\delta_e)$ is then an order of magnitude larger than for the SD qubit, given similar values of the single-photon $\Omega_R$ and the effective two-photon $\Omega_R^2/\delta_e$ Rabi frequencies.

2) A cphase logic gate between two closely spaced Raman qubits [see 1 above], $A$ and $B$, trapped in an optical lattice [1], is realized by an off-resonant “catalysis” field with Rabi frequency $\Omega_C$, which induces a RDDI-dependent ac Stark shift of the two-qubit states. One can then show that during the gate operation, the probability of error due to spontaneous decay of the excited states $|e\rangle_{A,B}$ is given by $P_{\text{err}}^{(\text{cphase})} \simeq 8\pi\xi^3/3$, where $\xi$ is the Lamb-Dicke parameter. With $\xi \lesssim 0.1$, we obtain that $P_{\text{err}}^{(\text{cphase})} \simeq 8 \times 10^{-3}$, which is twice worse than in our scheme with the same $\xi$. More dramatically, for similar field strengths, e.g., $\Omega_C/\gamma_e \simeq 30$ and $\delta_e \simeq 5\Delta_{AB}^{(R)} \gg \Omega_C$, where $\Delta_{AB}^{(R)}$ is the RDDI coupling strength between the atoms on the transitions $|g_2\rangle_{A,B} \rightarrow |e\rangle_{A,B}$, we find that the SD qubit implementation of the cphase gate is $\sim 30$ times faster.

3) A cphase gate in an ion trap [2] operates with speed and error probability similar to our scheme. The error in the ion trap QP is caused by the radiative decay of the auxiliary excited state, but one must also reckon with error due to the phonon-mode decoherence [1, 3]. The main limitations of ion trap schemes are related to difficulties with their scalability.

To conclude, our proposal for an optically-manipulated, solid-state quantum processor has no principal limitations on scalability. It allows us to suppress radiative decoherence and enhance the speed of photon-mediated quantum-logic gates, owing to the use of the ground and subradiant states of effective dimers formed by resonant dipole-dipole interacting two-level systems. These states constitute a physically realistic, simple and robust “decoherence-free subspace” [21], whose implementation draws efficiently upon the system resources (only two atoms per qubit). The highly challenging experimental realization of such a quantum computer requires nanofabrication techniques with nanometer precision of dopant or quantum dot implantation [14, 17].