Global, and Local Optimization Beamforming for Acoustic Broadband Sources

Armin Goudarzi

German Aerospace Center (DLR), 37083 Göttingen, Germany

Abstract

This paper presents an extension to global optimization beamforming for acoustic broadband sources. Given, that properties such as the source location, spatial shape, multipole rotation, or flow properties can be parameterized over the frequency, a CSM-fitting can be performed for all frequencies at the same time. A numerical analysis shows that the non-linear error function for the standard global optimization problem is similar to a Point Spread Function and contains local minima, but can be improved with the proposed broadband optimization. Not only increases the broadband optimization process the ratio of equations to unknown variables, but it also smooths out the cost function. It also simplifies the process of identifying sources and reconstructing their spectra from the results. The paper shows that the method is superior on synthetic monopoles compared to standard global optimization and CLEAN-SC. For real-world data the results of broadband global optimization, standard global optimization, and CLEAN-SC are similar. However, the proposed method does not require the identification and integration of Regions Of Interest. It is shown, that by using reasonable initial values the global optimization problem reduces to a local optimization problem with similar results. Further, it is shown that the proposed method is able to identify multipoles with different pole amplitudes and unknown pole rotations.

Keywords: acoustics, beamforming, global optimization, local optimization

1. Introduction

Multiple noise-generating phenomena and mechanisms exist. For the localization and estimation of the sound power of complex source geometries, beamforming is a reliable method \cite{8}. Since for a sound-field observed with a finite amount of sensors, there exists an infinite amount of possible source configurations \cite{6}, beamforming methods rely on several assumptions. The main assumptions generally include one or all of these: spatially compact sources, monopole sources, incoherent sources, and independent sound radiation for each frequency. While these assumptions are typically violated in real-world scenarios, naive algorithms such as Conventional Beamforming (CB) are still widely popular due to their robustness and known limitations \cite{8}. More sophisticated approaches exist, such as inverse methods \cite{15}, deconvolution methods such as CLEAN-SC \cite{11}, and DAMAS \cite{2} where the true source distribution is reconstructed from beamforming maps. However, inverse methods and advanced deconvolution methods are often computationally expensive and still include assumptions about the source.

Additionally, most methods are grid-, or subspace-based. Global Optimization \cite{7} \cite{14} (GO) addresses this by introducing a small number of focus points, that are no longer fixed in space. Compared to an inverse method this implies that not only the amplitudes for each focus point have to be determined, but also the locations of the focus points. Thus, the propagation problem becomes non-linear, but the number of unknown variables is reduced by multiple orders of magnitude.

A problem that each beamforming method so far suffered from is, that results are obtained for each frequency independently. Thus, a post-processing step such as Source Identification based on Spatial
Normal Distribution [3] (SIND) is necessary that identifies Regions Of Interest (ROI), assigns the correct beamforming results to them, and creates spectra.

This paper presents an extension to GO that addresses this issue by introducing source objects. Source objects have multiple properties, such as a location, and a spectrum. This allows a source object to having a single location for all frequencies, which makes the post-processing mostly obsolete. It also allows the optimization of other parameterized variables in the propagation equation, such as the speed of sound, the spatial source shape, length, and coherence length of distributed sources [4]. Thus, the method has the potential to identify physical properties that have been neglected by naive beamformers.

2. Source description and propagation operator

GO is a Cross Spectral Matrix (CSM) fitting method. Thus, the free variables are optimized in such a way that the estimated CSM coincides with the measured CSM. For this process, a description of the source and the propagation is necessary. The acoustic propagation can be formulated as

$$T_q = c$$

with $c$ being the vectorized CSM, and $T$ being the propagation operator. The propagation operator is derived from the Green’s Matrix $H$ with

$$T = H^* \odot H$$

where $\odot$ is the column-wise Khatri-Rao product[12]. The Green’s matrix $H$ is given by

$$H_{nm} = h(x_n, y_m) \quad n = 1, \ldots, N \quad m = 1, \ldots, M$$

where $h$ describes the acoustic propagation from a source location $y$ to a microphone $x$. For a free-field monopole the propagation function $h$ is given by the Green’s Function

$$h_{\text{mono}}(x, y) = \frac{\exp(-jkd)}{4\pi d}$$

with the wavenumber $k = w/c$, $c$ is the speed of sound, and $d = |x - y|$. For a dipole, the propagation function is given by

$$h_{\text{dip}}(x, y) = (\epsilon_{\text{dip}} \cdot \epsilon_n) \frac{\exp(-jkd)}{4\pi} \left( \frac{1}{d^2} + \frac{jk}{d} \right)$$

with $c$ being the normalized direction vectors of the dipole and the microphone position $n$. The normalized vectors are given in spherical coordinates, which makes it easier to control the rotation and strength of the dipole independently with

$$e = [\sin(\theta) \cos(\phi), \sin(\theta) \sin(\phi), \cos(\theta)]^T$$

In typical inverse methods we find $q$ with

$$q = T^{-1}c$$

In GO we need to find $q$ and $T$. For incoherent monopoles the free variables in $T$ are the source positions $y_m$; for dipoles there are the additional dipole orientation angles $\theta$, and $\varphi$. Thus, there are four free variables per monopole, and six free variables for dipoles. Additionally, there are $N_f$ free variables in $q$ for each pole, where $N_f$ is the number of employed frequencies.

3. Error metric and optimization

To optimize $q$ and $T$ we need an error metric $E$. Von den Hoff et al. [14] suggest the MSE of the CSM, that is

$$E(f) = \sum_i \left( |c_{\text{mod},i}(f) - c_{\text{meas},i}(f)|^2 \right).$$

Only elements $i$ that correspond to the upper triangular CSM without diagonal should be evaluated to exclude uncorrelated self-noise. Since we will observe the errors for a varying number of frequencies and microphones, we replace the sum by an average over all employed frequencies and upper CSM entries. An additional error normalization for each frequency is added to account for different maximum SPL at different frequencies. The final error function then is

$$E = \left\langle \frac{|c_{\text{mod},i}(f_j) - c_{\text{meas},i}(f_j)|^2}{\max_j(|c_{\text{meas},i}(f_j)|^2)} \right\rangle_{i,j}.$$}

To make the error more robust for real-world applications, and since eq. [1] is no longer linear, it is also possible to choose the L1 norm over the L2 norm (which is typically chosen to ensure a convex error) which does not prefer single large CSM deviations in the optimization process. Alternatively, a weighted L2 norm, based on the standard deviation of the CSM Welch averages [9], can improve the error estimation.
4. Error for a single synthetic monopole

Due to the non-linearity of the propagation operator, the non-convex error, and non-uniqueness (the solution is at least permutation invariant), we will explore the error numerically. First, a synthetic monopole at \( y = [0.5, 0.5, 0]^T \text{m} \) with \( p^2 = 1 \text{Pa}^2 \) is generated. An equidistant 1D-array at \(-0.5 \text{m} \leq x_1 \leq 0.5 \text{m}, x_2 = 0 \text{m}, x_3 = 0 \text{m}\) detects the sound-field with \( M = 5 \) microphones. We perform standard single-frequency GO as case 1a) at \( f = 5000 \text{Hz} \), and multi-frequency GO as proposed in this paper at \( 100 \text{Hz} \leq f \leq 20 \text{kHz} \) with \( \Delta f = 100 \text{Hz} \) as case 1b).

![Figure 1](image1.png)

Figure 1: Case 1a), \((x_1,p^2)\) error from eq. 9 for \( x_2 = 0.5 \text{m}, x_3 = 0 \text{m}, \) and \( f = 5000 \text{Hz} \).

![Figure 2](image2.png)

Figure 2: Case 1b), \((x_1,p^2)\) error from eq. 9 for \( x_2 = 0.5 \text{m}, x_3 = 0 \text{m}, \) and \( 100 \text{Hz} \leq f \leq 20 \text{kHz}, \Delta f = 100 \text{Hz} \).

Figure 1 shows the \((p^2, x_1)\) log-error for case 1a) for \( x_2 = 0.5 \text{m}, x_3 = 0 \text{m} \). Multiple local minima are visible for case 1a), so that for a given \( p^2 \) the error looks like a reciprocal Point Spread Function (PSF) with grating lobes. With the averaging over the frequency, the PSF is also averaged, which smears out local minima in case 1b), see Figure 2. Thus, the error becomes mostly smooth and the local minima disappear.

5. Error for multiple synthetic monopoles

We will now examine the error for a two-source problem as case 2), with \( y_1 = [0.8, 0.6, 0]^T \text{m}, y_{11} = [0.3, 0.5, 0]^T \text{m}, p_{1}^2 = 1 \text{Pa}^2 \text{Hz}^{-1}, p_{11}^2 = 0.5 \text{Pa}^2 \text{Hz}^{-1}, \) and \( 100 \text{Hz} \leq f \leq 20 \text{kHz}, \Delta f = 100 \text{Hz} \). Since the error space is six-dimensional (three coordinates per source, one amplitude per source), we show only 2D \((x_1,x_2)\)-slices of the error-space. The slices are chosen so that all variables are correct except for the two being displayed.

Figure 3 shows the error from eq. 9 for case 2). True source positions are marked with an x.

![Figure 3](image3.png)

Figure 3: Case 1a), \((x_1,x_2)\) error from eq. 9 for \( x_3 = 0 \text{m}, p^2 = 1 \text{Pa}^2, \) and \( f = 5000 \text{Hz} \).

![Figure 4](image4.png)

Figure 4: Case 1b), \((x_1,x_2)\) error from eq. 9 for \( x_3 = 0 \text{m}, p^2 = 1 \text{Pa}^2, \) and \( 100 \text{Hz} \leq f \leq 20 \text{kHz}, \Delta f = 100 \text{Hz} \).

![Figure 5](image5.png)

Figure 5: Slices of the six-dimensional error, see. eq. 9 for \( S_I \) and \( S_{11} \). All variables for each slice are chosen correctly, except for the shown combinations of \( x_1 \) and \( x_2 \).

There is a clear global minimum in the error, however, the error is not very smooth, and there exist local minima.
6. Global optimization on synthetic monopoles

To evaluate the proposed method and compare them to standard GO and CLEAN-SC [11] we propose two examples. First, case 3a) contains a monopole located at \( y_1 = [0.5, 0.5, 0]^T \text{m}, \) with \( P_1 = 100 \text{dB} \). Second, case 3b) contains two incoherent monopoles with \( y_{II} = [0.5, 0.5, 0]^T \text{m}, \) \( P_{II} = 95 \text{dB}, \) and \( y_{III} = [0.5, 0.6, 0]^T \text{m}, \) \( P_{III} = 105 \text{dB}. \) Thus, source \( S_2 \) is located behind source \( S_1 \). The array is an equidistant 1D-array at \(-0.5 \text{m} \leq x_1 \leq 0.5 \text{m}, \) \( x_2 = 0 \text{m}, \) \( x_3 = 0 \text{m} \) with \( M = 11 \) microphones. To acquire depth-information with conventional beamforming, we choose steering vector formulation IV [10].

The focus grid is \(-1 \text{m} \leq x_1 \leq 1 \text{m}, \) \( 0.3 \text{m} \leq x_2 \leq 0.7 \text{m} \) with \( \Delta x_1 = \Delta x_2 = 0.01 \text{m}, \) and \( x_3 = 0 \text{m}. \)

Figure 6 shows the resulting conventional beamforming result of the source configuration, top row case 3), bottom row case 4). It shows that the singular monopole is resolved well. However, the main-lobe width is large at low frequencies, and from around \( f \approx 2 \text{kHz} \) first side-lobes are visible. The same holds true for the two monopole configuration, case 3b). Additionally, we observe a shift of the main-lobe towards \( x_1 \approx 0.4 \) m at high frequencies, and that the low SPL source can only be distinguished in \( x_2 \) above around \( f \geq 6 \text{kHz}. \)

Figure 7 shows the corresponding CLEAN-SC result, top row case 3), bottom row case 4). The first column shows the spatial position of the reconstructed source-parts. A source-part is defined as a CLEAN-SC result in a single location and frequency with \( P(x, f) \geq 0 \text{Pa}^2 \text{Hz}^{-1}, \) that - once integrated through a ROI - gives a full source spectrum [3]. The source part color indicates their frequency, and their opacity indicates their log level \( \log(P + 1) \), normalized for each frequency to \( (0, 1). \) The ROI are chosen around the true source locations with case 3a) radii \( r_I = 0.1 \text{m}, \) and case 3b) \( r_{II} = 0.05 \text{m} \) so that the ROI do not overlap. Figure 7 shows in the right column the corresponding integrated spectra. Source-parts that lie outside of any ROI are integrated and labeled as noise. The position and SPL of the single monopole configuration in case 3) is estimated well, but CLEAN-SC is not able to resolve the two monopole configuration in case 4) below \( f \leq 3 \text{kHz}. \) This results in a SPL overestimation for ROI1.

For GO we split the first case in two separate cases. Since we have to set a fixed number of sources, we first perform GO (dual annealing [13]) with the correct number of estimated sources \( N = 1 \) as case 3a), then we perform GO with an additional estimated source as case 3b). Since in real-world application we lack the information of sources, the idea is to simulate a case, where we will overestimate the number of true sources. In the same manner we choose \( N = 3 \) for the two monopole scenario. True sources will be denoted with Roman numbers (e.g. \( S_I \)), and estimated sources with Arabic numbers (e.g. \( S_1 \)).

Figure 8 shows the results of standard GO. The left column shows the estimated position of the source parts, the color encodes their frequency. Since for standard GO a source location is estimated for each frequency separately we also de-
For the proposed broadband GO method no ROI is needed. However, the solution is not unique, since any true source can be approximated using multiple estimated sources. Thus, multiple estimated sources may have to be integrated based on a spatial criterion, such as a minimum distance between them. Figure [9] shows the result of the proposed broadband GO method. For case 3a) the source location and amplitude is estimated correctly, like for standard GO, and CLEAN-SC. For case 3b) the true source is approximated by a superposition of $S_1$ and $S_2$, since their position is identical. The spectral density of $S_{III}$ is arbitrarily distributed between them, but the summation of both spectra reveals that their sum approximates the true PSD. For case 4) the spectra are reconstructed well throughout the frequency range, except for $S_1 f < 1$ kHz, where the SPL is slightly underestimated. However, the method outperforms CLEAN-SC and standard GO. Table [1] provides the spatial errors and frequency averaged SPL error.

| case | source | $|\Delta y|$ in [m] | $E_{\text{SPL}}$ in [dB] |
|------|--------|-----------------|-----------------|
| 3a   | $S_1$  | $1.38 \times 10^{-5}$ | $2.38 \times 10^{-5}$ |
| 3b   | $S_1$  | $2.52 \times 10^{-7}$ | $6.54 \times 10^{-9}$ |
|      | $S_2$  | $3.66 \times 10^{-7}$ |                |
| 4    | $S_1$  | $2.05 \times 10^{-5}$ | $1.97 \times 10^{-3}$ |
|      | $S_2$  | $1.78 \times 10^{-3}$ |                |
|      | $S_3$  | $5.74 \times 10^{-5}$ | $2.12 \times 10^{-1}$ |

Table 1: Error metrics of case 3a), 3b), and 4) for the proposed broadband GO method.

7. Global optimization on real monopole sources

To evaluate the proposed method on real data, we use the presented open wind tunnel data from the SIND method [3] at Mach $M = 0$ as case 3a). The data features an equidistant 7x7 microphone array $-0.27 \text{m} \leq x_{1,2} \leq 0.27 \text{m}$, $x_3 = -0.65 \text{m}$, with $\Delta x_1 = \Delta x_2 = 0.09 \text{m}$, and a generic monopole source (streamlined housing with a $r = 2.5 \text{mm}$ opening at the downstream end) in a reverberation-free environment. The source is moved to the locations $y_I = [-0.055, 0.105, 0]^T \text{m}$, $y_{III} = [0.105, 0.105, 0]^T \text{m}$, $y_{IV} = [0.255, 0.105, 0]^T \text{m}$ and uses uncorrelated white noise with three different band-pass frequencies to generate different source spectra. The CSM has a sampling frequency of $f_S = 2^{16}$ Hz, and uses a blocksize of $2^9$, which results in around $6 \times 10^4$ Welch averages and $\Delta f = \ldots$
As described in the paper [3], we estimate the ground truth source powers by using the CSM diagonals, and calculate the source emission using the distances between the source positions and microphones assuming a perfect monopole, see eq. [4]. We average the results over all diagonal CSM entries to obtain a mean ground truth and standard deviation. We then use a superposition of the three measurements (addition of their CSM) to obtain a problem that contains three sources.

Figure 10: CLEAN-SC results for case 5a). The left column shows the spatial distribution of source-parts, their frequency, and normalized amplitude. The right column shows the PSD reconstruction (dotted lines), based on the same colored ROI with $r_1 = r_2 = 0.075$ m, $r_3 = 0.3$ m. The ground truth is depicted in the same color with $1\sigma$ standard deviation (solid lines).

As a benchmark we perform CLEAN-SC with steering vector formulation IV [10], see Figure 10. The left column depicts the spatial setup and source-parts, their color indicates the frequency. Since the problem is three-dimensional, we show a $(x_1, x_2)$ and $(x_1, x_3)$ projection. There are three ROI defined based on the true source positions with a radius of $r_1 = r_2 = 0.075$ m, $r_3 = 0.3$ m. Due to the low array resolution in $x_3$, the ROI are elongated with $r_3 = 4r_{1,2}$. Every source-part within these radii is integrated for the estimated source spectrum, every source-part that is not contained in any ROI is integrated and classified as noise. The right column shows the corresponding spectra, the estimated ROI spectra are depicted with dashed lines, the ground truth is depicted with solid lines and standard deviation. Overall, CLEAN-SC is able to reconstruct the spectra up to a SNR $\approx 15$ dB. There is an underestimation of the source power of $S_2$ at $f \approx 15$ kHz. Additionally, there is some high frequency content above $f \geq 10$ kHz that does not correspond to any ROI.

Figure 11: Standard GO result for case 5a), three true sources, and $N_{\text{max}} = 4$.

Figure 12 shows the result of the proposed broadband GO for $N_{\text{max}} = 4$. On the left column the estimate source positions are depicted. The source $S_I$ is not identified, $S_{II}$ is approximated with $S_2$ and $S_4$, and $S_{III}$ is approximated with $S_1$ and $S_3$. Since the ordering of the sources is permutation invariant, these correspondences were identified based on the spatial distances to the true positions. Similar to the CLEAN-SC result, the sources are well localized in $x_1$ and $x_2$, and distributed in $x_3$ direction. The right column depicts the estimated source spectra by integrating all estimated sources that correspond to the true source positions. Source
$S_{II}$ is approximated well, except for the peak at $f \approx 15\text{kHz}$, where instead the missing $S_I$ is approximated. Source $S_{III}$ is approximated well up to a similar frequency in the CLEAN-SC results, above which it approximates mainly $S_{II}$. This result indicates, that the overall CSM reconstruction error is lower by approximating $S_{II}$, and $S_{III}$ with multiple, spatially scattered sources, than by approximating $S_I$ instead.

Figure 13: Broadband GO result for case 5a), three true sources, and $N_{\text{max}} = 6$.

To overcome this issue, we increase the maximum number of sources and perform broadband optimization for $N_{\text{max}} = 6$, which is chosen arbitrarily. Figure 13 shows the corresponding results. The spatial source distribution is similar to $N_{\text{max}} = 4$, with the two additional sources being located at $S_1$ and $S_{III}$ (with an $x_3$ offset). Again, the spatial source distribution is very similar to the CLEAN-SC, and standard GO results. Overall, the CLEAN-SC and broadband GO results are very similar with broadband GO outperforming CLEAN-SC at $f \approx 15\text{kHz}$. The main difference is, that the CLEAN-SC high frequency source-parts that are rejected as noise due to the ROI definition, are included in the approximation of $S_{III}$. The similar source distribution and high frequency content suggests, that they are not an artifact by the different methods, but a physical phenomenon that might be caused by reflections, the CSM superposition process, or by additional sound radiation from the speaker housing.

The main difference between the results is the post-processing. While CLEAN-SC requires manual ROI selections or a process such as SIND [3], broadband optimization requires the integration of source objects. While these are not well localized in $x_3$ in the given example, they are very well localized in $x_1$ and $x_2$, which allows an easy manual or automated detection based on a minimum source distance below which sources will be grouped and integrated.

Figure 14: Broadband LO result for case 5a), three true sources, and $N_{\text{max}} = 3$ with initial start positions close to the true positions, and PSD\textsubscript{start} = 50 dB.

Given we use a method like SIND to automatically obtain source positions and spectra, we can use these as initial guesses to transition from global optimization to local optimization (LO), since the error function is smooth in the amplitude dimension, see Section 3. To do so, we use the true source positions with a random normal error of $\sigma_x = 0.025$, and bound the optimizer with $\pm 4\sigma_x$. The initial amplitude is PSD\textsubscript{start} = 50 dB, and not bounded. Figure 14 shows the result of the LO process for $N_{\text{max}} = 3$. Again, the results are similar to CLEAN-SC and GO. However, local optimization is much faster than GO. The main reason to perform the optimization process after acquiring a CLEAN-SC result would be to include properties in the source object that are neglected by conventional beamforming and CLEAN-SC, such as distributed sources, and dipoles.

Since the optimization process depends on the goodness of the CSM estimation, it is of interest how it performs on data at a Mach number $M > 0$. Figure 15 shows the result of the LO process for $N_{\text{max}} = 3$ at $M = 0.03$, using a denoised CSM by subtracting a background-noise measurement (wind tunnel on, speaker off). Additionally, the Amiet open wind tunnel correction [4] was incorporated into the steering-vector. The result is similar to the LO $M = 0$ case, even the additionally $S_3$ high-frequency bumps are similar.
Figure 15: Broadband LO result for case 5b) for three true sources at $M = 0.03, N_{\text{max}} = 3$, with initial start positions close to the true positions, and PSD$_{\text{start}} = 50$ dB. This indicates that the LO process is robust at $M > 0$ for low background-noise measurements. High background-noise situations such as closed wind tunnel measurements are likely to require a noise model [5] either subtracted from the measurement or added to the fitted CSM.

8. Multipole source objects

As described in the previous section it could be of interest to generate complex source objects with properties that can be parameterized in the propagation operator. One example is a multipole that contains poles of any orders, such as a monopole, a dipole, and so on. We incorporate this by superposing the estimated multipole CSMs in eq. 9 with

$$\epsilon_{\text{mod}} = \epsilon_{\text{M}} + \epsilon_{\text{D}} + \ldots,$$

by using a shared location, and pole rotation angle for all frequencies and poles.

For proof of concept we perform LO for one true source with a monopole amplitude $P_M = 100$ dB Hz$^{-1}$, and a dipole amplitude of $P_D = 110$ dB Hz$^{-1} - 20 \log_{10}(k)$ as case 6). The frequency-dependent amplitude is chosen, so that it negates the frequency dependence in the dipoles Green’s function, see eq. [5] and has the same amplitude ratio to the monopole in the resulting CSM at all frequencies. The true source position is $y_1 = [0.5, 0.5, 0.5]$ m, the true dipole rotation angle is $\theta = 90^\circ, \varphi = 0^\circ$, so that the dipoles main lobes are parallel to the $x_1$-axis. The array setup from Sec. [6] with is chosen. For the local optimization process the initial positional start values are chosen according to the last section with $\sigma_x = 0.025$m. The start amplitude for the optimizer is PSD$_{\text{M, start}} = \text{PSD}_{\text{D, start}} - 20 \log_{10}(k) = 90$ dB Hz$^{-1}$. The start values for the dipole rotation angles are random, since there cannot be obtained reasonable start values from naive beamforming methods.

| $M$ | $\epsilon_y$ [$10^{-6}$m] | $\epsilon_y$ [$10^{-5}$m] | $\epsilon_y$ [$10^{-3}$] | $\epsilon_{\text{PSD}}$ [$10^{-2}$dB] |
|-----|-----------------|-----------------|-----------------|-----------------|
| D   | 1.22            | 1.36            | 1.87            | 1.94            |

Table 2: Spatial, and angular errors $\epsilon$ for the multipole reconstruction. The angular errors are $\pi$-invariant. For the reconstructed PSD the mean absolute error, averaged over the frequency, is displayed.

Figure 16: Broadband LO result for case 6) with one true multipole sources, $N_{\text{max}} = 1$, and initial starting positions close to the true positions. The left column shows the spectral estimation, the right column shows the mean (over frequency) absolute CSM reconstruction error in decibel.

The resulting errors are given in Table [2] and the resulting PSD reconstruction is shown in Figure [16] left. On the right the frequency averaged absolute error of the reconstructed CSM is displayed. The position, rotation, and PSD of the multipole source is well reconstructed.

9. Conclusion

This paper presented global and local optimization for acoustic broadband sources, by introducing source objects with shared parameters such as the spatial location or source orientation for all frequencies. The advantage over the single frequency optimization process is the increased ratio of equations to unknown variables, and the process of extracting the source positions and spectra from the results.

While the approach outperformed methods such as CLEAN-SC, and single-frequency global optimization on synthetic data, the results were similar on a real-world problem, containing three monopoles with little background noise.
One challenge for the global optimization process is the correct determination of the number of source objects. It was shown, that an overestimation might be necessary to account for reflections or when the source objects or Green’s functions do not include the true properties. Then, multiple source objects approximate a single source. However, they can be identified based on their spatial distance and integrated.

Since global optimization is computationally expensive, it was shown that using reasonable starting locations for the source objects the problem can be reduced to local optimization, since the combination of multiple frequencies reduces local minima in the error. A reason to perform local or global optimization after obtaining reasonable initial values might be to identify source properties that are neglected by grid-based beamformers, such as multipoles, distributed sources, etc. The paper proposed a synthetic multipole problem which was reconstructed with low errors.

For future work we propose the incorporation of a flow-induced, correlated sensor-noise model for the CSM reconstruction, to extend the area of application to closed wind tunnel measurements.

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