Correcting correlated errors for quantum gates in multi-qubit systems using smooth pulse control

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In multi-qubit system, correlated errors subject to unwanted interactions with other qubits are one of the major obstacles for scaling up quantum computers to be applicable. We present two approaches to correct such noise and demonstrate with high fidelity and robustness. We use spectator and intruder to discriminate the environment interacting with target qubit in different parameter regime. Our proposed approaches combines analytical theory and numerical optimization, and are general to obtain smooth control pulses for various qubit systems. Both theory and numerical simulations demonstrate to correct these errors efficiently. Gate fidelities are generally above 0.9999 over a large range of parameter variation for a set of single-qubit gates and two-qubit entangling gates.

Quantum error correction provides a path to large-scale universal quantum computers. But it is built on challenging assumptions about the characteristics of the underlying errors in quantum logic operations to be statistically independent and uncorrelated [1–3]. In realistic environments error sources could exhibit strong temporal or spatial correlations [4], cannot be corrected efficiently using close-loop quantum error correction. In a multi-qubit system, due to massive control lines and compact geometric structure, a qubit is unavoidably to interact with other quantum systems including qubits, couplers, neighbor control lines [5], etc. This introduces control crosstalk [6], leakage [7], parameter shift, decoherence and gate errors when operating on target qubit(s). Noises become spatially-correlated, making qubits more fragile and gate calibrations more complicated. The simultaneous gate error rate dramatically increases compared to isolated case as qubit system grows [2,7,9]. To better isolate the qubits and suppress the unwanted interactions, the state-of-art techniques rely on tunable qubits and tunable couplers [7,10–14]. The costs of introducing the tunability are that more control lines and more crowded spectral. Tuning over large regions of qubits’ or couplers’ frequency crosses more anti-crossing points, where diabatic dynamics causes additional errors [7,11–14]. These obstacles limit the applications of current qubit systems and slow down the development of scalable quantum computers.

In the NISQ era, open-loop quantum error correction (OQEC) technique is essentially important [15]. As an example of OQEC techniques, dynamical decoupling (DD) are successfully applied to further isolate target qubits from the environment [10,15], hence also corrects the correlated errors due to unwanted couplings. But it is difficult to apply DD sequences during a quantum gate operation to increase the fidelity [19]. Further, dynamically-corrected gate (DCG) are proposed to suppress small dephasing errors [18,20,21]. Traditional DCG relies on sequences of operation which takes the multiple times of gate depth increases as the expense for a better single gate. Similar to DD, use of simple pulses, such as square or $\delta$–function, could easily cause serious waveform distortion [22], which brings in additional operation errors and limits the realistic application of DD and DCG. Smooth pulse DCG has been proposed to implement single qubit gates [18,23,24] to enhance the robustness of quantum gates subject to local static noise. How to correct the errors induced during entangling two qubits is unexplored. Also, a most drawback of conventional DCG models is the assumption of small errors. When errors becomes significant and non-perturbative, DCG fails.

In this paper, we study how to dynamically suppress or correct the errors in multi-qubit system, especially the errors induced by unwanted interactions. We analyze these errors in two different coupling regime with reference to the control strength. For different regime, we propose different approaches to targeted correct the errors in the relevant regime. Our approaches are combinations of analytical theory and numerical methods, which could give unlimited solutions of smooth control pulses for universal gate sets. We present some examples of our solutions which are demonstrated numerically with high fidelities. Our approach could be applied to implement robust gates but also single qubit gate for qubit with always-on interaction with other qubits, as well as to resolve other issues in current stage of multi-qubit quantum computers. Our approaches provides a set of dynamical corrected gates for qubit even with always-on interaction. In such case, tunable couplers are not necessarily needed for scalable quantum processors any more.
Errors in Multi-qubit system. Qubits couple to each other in a multi-qubit system, target qubit spectrum is observed to be either split or broadened depending on the coupling strength, as Fig.1(b) shows. Its Hamiltonian in eigenbasis of a two-level subspace takes the form

$$H_{t}^{\text{diag}} = -\frac{\delta}{2}\sigma_{z}$$  \hspace{1cm} (1)

in rotating frame with \(\omega_{t}\), where \(t\) stands for ”target” and \(\delta\) corresponds to level-splitting or parameter fluctuation. \(\delta\) is generally nonvanishing in various qubit systems, including two-level system (TLS) and multi-level system (MLS) such as superconducting transmon qubits, as well as in various interaction regime. Analytical forms of \(\delta\) in different regime are derived in supplementary [25].

Large interaction between qubits gives benefit to entangling operations but harms single qubit operations. As what is used to be done in quantum dots and superconducting qubits, detuning target qubit from the interacted qubits could reduce the effective coupling strength but can’t decouple them [26, 27]. While effective coupling gets much stronger than de-coupling strength but can’t decouple them [26, 27].

Besides Stark effect, inhomogeneous driving strength and control crosstalk errors could be induced via unwanted couplings to intruders or spectators. The thicker of the link between qubits, the stronger residual coupling remains. (b) simulates the observed absorption versus \(\Delta\) and effective ZZ-coupling \(\delta\), for non-correlated decoherence rate \(\gamma = 1\) MHzx2\(\pi\). At \(\delta_{t}\approx 0.7\) MHzx2\(\pi\) split peaks combine as a single peak. Hence it is the transition point between the spectated regime intruded regime. When coupling strength is small \(\delta < \delta_{t}\), the doublet’s absorption overlap with each other and combined as one broadened peak, resulting in enhanced dephasing rate. Green line is the fitting curve of the resultant absorption profile at \(\delta = 0.5\) MHzx2\(\pi\). Numerical fitting of decoherence rate \(\gamma_{\text{fit}} = 1.341\) MHzx2\(\pi\), using a combination line shape of Lorentzian and Gaussian. The inset shows the fitting decoherence rate enhancement versus \(\delta\).

Current trending technique is to apply tunable couplers [7, 11–14] to connect qubits with the cost of more control lines and complexity of chip design. The tunability could be designed to realize very high on/off ratio, which lead to successful demonstration of quantum supremacy [7]. But with the realistic limit to the tunability and imperfection of geometrical isolation, residual coupling is still a problem for precise gate operations [11, 29, 29] citations. Small couplings compared to decoherence makes level-splitting invisible but broadens absorption peak, meaning a measured enhancement of decoherence rate, see Fig.1(b). The inset shows the increased decoherence rate subject to the broadening of measured absorption. Also with the tunable coupler, parameter fluctuations in the coupler could introduces correlated noise to the two operated qubits during an entangling gate. We call this regime the spectated regime. The coupled quantum systems are called spectators.
The entangling dynamics is only affected in the two-level subspace \( \text{span}\{01\}, |00\rangle \) \[\text{33}\], hence the effective Hamiltonian again takes the same form as Eq.1 excluding the control field \( \Omega(t) = g(t) \).

**Targeted-correction gates in intruded regime.** In this regime, the level splitting \( \delta \) is large, seen as split spectrum as Fig.1(b). The Hamiltonian in the 4-by-4 computational subspace \( \text{span}\{|\text{intruder}\rangle, |\text{target}\rangle\} \) is diagonalized as \( H^{\text{diag}} = \text{Diag}\{-\frac{\delta}{2}, \frac{\delta}{2}, g(t), -g(t)\} \). Control-rotational error between the intruder and the target qubit is significant if using simple pulses, see the green dashed line in Fig.2(b). Besides, a quantum gate targeted-correcting such errors needs to consider inhomogeneous dipole moments as shown in Eq.2. As pointed out above, implementing two-qubit entangling gates becomes relatively trivial in this regime compared to single qubit gates. A large ZZ interaction directly generates a CZ gate while large XX interaction implements an iSWAP gate. Here we present our targeted-correction gate (TCG) approach for given \( \delta \) and dipole moment inhomogeneity to implement single qubit operations. First we decompose the \( SU(4) \) dynamics to \( SU(2) \otimes SU(2) \) dynamics. A control field \( H_{st}^d = (\Omega^*(t) \tilde{\sigma}_+ e^{i\omega_d} + h.c.) \) is added to the target qubit. Drive amplitude \( \Omega(t) \) could be complex for IO control \[\text{citations}\] meaning driving both X and Y directions.

Transformed to interaction picture with \( \tilde{H}_0 \), the unitary operator

\[
U^I(t) = \left[ \begin{array}{cc} U_1 & B \\ -B^\dagger & U_2 \end{array} \right]
\]

Because \( B \) includes fast oscillating phases \( e^{i2\omega_d t} \) by taking \( \omega_d = \omega \), it could be demonstrated that \( B = 0 \) under the constraints \( \delta \ll \omega \) \[\text{25}\]. So the evolution operator is block diagonalized \( U^\text{int}(t) = \text{Diag}\{U_1, U_2\} \).

Since the first subspace \( \text{span}\{|00\rangle, |01\rangle\} \) is driven resonantly \( \omega_d = \omega \), \( U_1(t) = e^{\frac{i}{\hbar} \int_0^t \Omega(t) \sigma_x dt} \).

For the second subspace \( \text{span}\{|10\rangle, |11\rangle\} \), \( U_2(t) = T e^{\frac{i}{\hbar} \int_0^t \Omega(t) \sigma_x dt} \).

\[
\Omega(t) = \sum C_n \cos(2\pi n \frac{nt}{T} + \phi_n)
\]

To numerically search for appropriate waveforms for TCG of \( \theta_{\text{ideal}} \) rotations. Results are shown in Fig.2. On the Bloch spheres, the quantum trajectories \( |0_i\rangle \rightarrow |1_i\rangle \) and \( |0_i\rangle \rightarrow \cos \frac{\Omega t}{T} |0_i\rangle + \sin \frac{\Omega t}{T} |1_i\rangle \) in different subspaces \( \text{span}\{00, 01\} \) and \( \text{span}\{10, 11\} \). The numbers \( \{941, 1.833, -3.701, 0.628, 1.580, 7.874, 0.3506, 1.346, 2.128\} \), \( T = 98.6 \text{ns} \). The fidelity is 0.998. While for \( \pi/8 \) gate, \( C_n \in \{4.8, -17.7, -0.6, -1.1, -0.5, -0.4, -0.3, -0.2, -0.1, -0.1\} \). MHz×2π, and \( \phi_n \in \{0.941, 1.833, -3.701, 0.628, 1.580, 7.874, 0.3506, 1.346, 2.128\} \), \( T = 35 \text{ ns} \). The fidelity is 0.999. While allowing more harmonic components, both optimized fidelities are demonstrated to be above 0.9999.

Errors robust gates in the spectated regime. -When the errors are small compared to decoherence rate \( \gamma \), \( \delta \) is close to the background fluctuations, hence the \( \delta \) spreads over a range of \([-\Delta, \Delta]\). Instead of correcting a targeted value of \( \delta \), now the quantum gates should be robust to smooth pulse ansatz

\[
\Omega(t) = \sum C_n \cos(2\pi \frac{nt}{T} + \phi_n)
\]
The generated evolution operator \( U = U_0 U_e \), where \( U_0 = e^{-i \int_0^T H_0(\tau) d\tau} \) is the ideal evolution operator generated by \( H_0 = \Omega \cos \theta \sigma_x - \Omega \sin \theta \sigma_y - \frac{\delta}{2} \sigma_z \). And \( U_e = U_0^U \) is the ideal evolution operator generated by \( H_e = \Omega \cos \theta \sigma_x - \Omega \sin \theta \sigma_y - \frac{\delta}{2} \sigma_z \). The difference here is that this sphere represents the geometry of an operator instead of qubit states. So we parametrize \( H(t) = H(t) - \frac{\delta}{2} \sigma_z \), where \( T \) is an analog of Bloch sphere. The change of \( H(t) = H(t) - \frac{\delta}{2} \sigma_z \) in terms of time could be characterized as a directional trajectory \( \mathbf{T}(t) \) on the parametrisation sphere, starting from \( \mathbf{T}(0) \) and ending at \( \mathbf{T}(T) \) as finishing a gate. So \( \mathbf{T}(t) = 1 \), which defines time \( t \) in terms of the spherical parameters

\[
t = \int_0^T d\mathbf{T}'.
\]

Also, since \( \mathbf{T}(t) \cdot \mathbf{T}' = \frac{1}{2} \mathbf{T}(t) \mathbf{T}(t) \), the curvature of the curve \( \mathbf{T}(t) \) is proven to define the control field

\[
\mathbf{T}(t) = \mathbf{T}(t).
\]

Further, the torque of \( \mathbf{T} \)

\[
\alpha = \left| \mathbf{T} \times \mathbf{T}' \right| \mathbf{T}'
\]

defines the rotational angle for the gate operation. Therefore, the overall evolution trajectory \( U_0 (t) \) generated by time-dependent Hamiltonian \( H_0 (t) \) could be fully determined in terms of Eq[4,9]. Furthermore, the pulse constraints for the ERG pulses could be determined. Since the gate fidelity subjects to errors \( F(U_0 U_e, U_0) = \frac{1}{2} | \text{Tr}(U e) | \), an ERG requires that \( t = T, \frac{\partial}{\partial \mathbf{T}(U e)} T \rightarrow 0 \). The first order correction gives \( \frac{\partial}{\partial \mathbf{T}(U e)} T \) paves the way to look for any ERG pulses up to arbitrary order of correction. Our related work ref.[30] presents systematic construction of the ERG pulses.

As simple examples demonstrating the performance of ERG, we set \( \alpha = 0 \). Now \( \mathbf{T}(t) \) reduces to a directional curve on a two dimensional plane. And the ideal rotational angle \( \phi \) is reduced to this constraint

\[
\phi = \alpha \left| \mathbf{T} \times \mathbf{T}' \right| = \Delta \phi + \pi \text{ or } \Delta \phi
\]

We then find a universal gate set of ERG to implement single qubit X rotation with angles \( \pi, \pi/2 \) and \( \pi/8 \), see Fig[5]. The fidelity is numerically calculated in a TLS with a range of fluctuation \( \delta / \sigma_z \). Wide range of high fidelity plateaus (\( F > 0.99999 \)) are demonstrated both for first order and second order ERG. These plateaus are never achieved in commonly used pulses, such as Gaussian, Cosine, etc. Here, Cartesian coordinates \( x = r \cos \theta, y = r \sin \theta \) are used to plot the parameterized curve, see the inset in Fig[5](a) and (b). Then we apply the \( \pi \) rotation curve to generate X gate as well as two qubit iSWAP gate for superconducting transmon qubits. Based on experimental parameters, the numerical results shown in Fig[5] also illustrate high fidelity plateaus for both single qubit operation and iSWAP. The ERG also exhibits certain robustness over pulse amplitude deviation (1 to 2% as shown in Fig[5](b) and (c)). The iSWAP fidelity plateau shifts to negative \( \delta \) because
FIG. 4: Application of a first order π RG pulse, see (a), is applied to superconducting transmon qubit system to implement X gate and iSWAP gate, with the numerical simulation results shown in (b) and (c), respectively. The first order RG pulse is resolved from the π parametrized curve in Fig[3](a). The parameters are $\omega_s = 5 \text{ GHz} \times 2\pi$, $\alpha_s = 230 \text{ MHz} \times 2\pi$, $\omega_I = 5.5 \text{ GHz} \times 2\pi$, $\alpha_I = 260 \text{ MHz} \times 2\pi$. The $XX + YY$ interaction strength $g$ varies in the range $-58 < g < 58 \text{ MHz} \times 2\pi$ and $-100 < g < 100 \text{ MHz} \times 2\pi$ to obtains the range of $\delta$ in (b) and (c). Four levels are simulated for the transmons.

of the leakage to transmon’s higher levels. Note that with the geometric formalism, finding appropriate pulses is still a challenging task and requires some techniques to systematical construction [36].

Discussion.—We have reported very promising approaches to correct spatial correlated errors in multi-qubit systems. We have firstly analyzed the mechanism of gate errors in multi-qubit system subject to spatial correlation. We have characterized two major regimes of couplings, and further studied in details the errors associated with spectators and intruders. To correct the errors induced by intruders, we propose targeted-correction gates to overcome not only level-splitting issue but also inhomogeneous control amplitudes. These two issues are by-passed by previous works. However, in some quantum computation or simulation tasks, operation qubit system in this regime provides lots of benefits. Now given our solutions, people could resolve these two issues potentially and go into a new paradigm of quantum computing with large always-on interactions. To correct the errors due to spectators, we propose error robust gates to produce high fidelity plateaus for a range of parameter variation. Compared to any conventionally-used pulses, our results lead great advantages on gate robustness for parameter variation. Also, the smoothness and limited-bandwidth of our pulses guarantee the waveform distortion under a controllable limit. For both scenarios, we have demonstrated exciting results with smooth and analytical pulses, high fidelities and powerful robustness. These results are supported with numerical simulations based on experimental parameters, which shows promising application in realistic multi-qubit quantum processors with current architecture or even allowing hardware simplifications.

Finally, we summarize some further observations of our approaches: 1. TCG provides an new paradigm for quantum computing or quantum simulation with always-on interaction, which is friendly for controlling fixed frequency qubits without increasing the complexity of control circuits; 2. Using ERG control, the residual coupling issue on current architecture doesn’t need to be eliminated; 3. TCG and ERG together opens up a new degree of control freedom; 4. Our approaches reduce the need for precise control of qubit frequency or tunable coupler. 5. Both TCG and ERG’s results have small number of parameters for further optimization, which is very friendly for experiments.

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