Exact Calculation of Ring Diagrams and the Off-shell Effect on the Equation of State

Jinfeng Liao, Xianglei Zhu and Pengfei Zhuang
Physics Department, Tsinghua University, Beijing 100084, China

The partition function with ring diagrams at finite temperature is exactly calculated by using contour integrals in the complex energy plane. It contains a pole part with temperature and momentum dependent mass and a phase shift part induced by off-shell effect in hot medium. The thermodynamic potentials for $\phi^4$ and $\phi^3$ interactions are calculated and compared with the quasi-particle (pole) approximation. It is found that the off-shell effect on the equation of state is remarkable.

PACS: 11.10.Wx, 24.10.Pa

I. INTRODUCTION

The thermodynamic properties of a system are fully characterized by its thermodynamic potential, with which one can derive all the thermodynamic functions such as pressure, energy density and entropy density from the well-known thermodynamic relations. With finite temperature field theory [1,2], the partition function can be calculated perturbatively for weakly coupling systems such as high temperature phase of Quantum Chromodynamics (QCD). However, the normal perturbation method can not describe the collective effect which is now believed to play a very important role [3] in understanding the quark-gluon plasma (QGP) [4] possibly formed in relativistic heavy ion collisions [5]. It is therefore necessary to perform resummation to include all high-order contributions in the equation of state of the system in hot and dense medium. In a quasi-particle description, the in-medium effect is reflected in a temperature and density dependent mass only. This effective mass is used to explain [6] the difference between the lattice calculation [7] of the equation of state of QGP and the corresponding Stefan-Boltzmann limit: the particle mass goes up with increasing temperature and then cancels partly the high temperature effect. However, the in-medium effect changes not only the particle mass but also its width. Most discussions concerning thermal width are focused on its relation to particle decay, while how it contributes to equation of state is still unclear. To study the off-shell effect on the equation of state is the main goal of this paper.

Ring diagrams are usually considered in the calculation of partition functions [1,2] to avoid infrared divergences and in the quark models [11] to form mesons at RPA level. Normally the ring diagrams are calculated only for static mode [1,2], namely in the frequency sum only the term with $n = 0$ is considered. We will calculate the thermodynamic potential with ring diagrams exactly and investigate the off-shell contribution to the equation of state. We first perform in general case the contour integration instead of the frequency sum in the ring diagrams, and derive the thermodynamic potential which contains the contributions from the quasi-particle with temperature dependent mass determined by the pole equation and from the scattering phase shift between the retarded and advanced particle self energy due to off-shell effect. To illustrate the quasi-particle and off-shell contributions to the equation of state related
to the study of QGP, we apply our formulas to the popular models, the $\phi^4$ and $\phi^3$ theory.

II. FORMULAS

The thermodynamic potential of a system with ring diagrams can be written as

$$\Omega = \Omega_0 + \Omega_1 + \Omega_{\text{ring}},$$

where

$$\Omega_0 = \frac{1}{\beta} \int \frac{d^3 k}{(2\pi)^3} \ln \left(1 - e^{-\beta E_0}\right)$$

is the free particle contribution with $E_0 = \sqrt{m^2 + k^2}$ being the particle energy and $\beta = 1/T$ the inverse temperature, and $\Omega_{\text{ring}}$ is defined in Fig.(1) [1,2]. To make the calculation definite we consider in the following meson ring diagrams only. However, the method can be straightly extended to fermions. A shaded circle in Fig.(1) means the particle self-energy $\Pi$. To use the standard definition of $\Omega_{\text{ring}}$ we have separated $\Omega_1$ with only one self energy on the ring from $\Omega_{\text{ring}}$. After the summation over the rings with different number of shaded circles, $\Omega_{\text{ring}}$ can be expressed as [1,2]

$$\Omega_{\text{ring}} = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\beta} \sum_n \left[ \ln \left(1 + A(i\omega_n, k)\right) - A(i\omega_n, k) \right],$$

where $\omega_n = 2\pi n/\beta$ with $n = 0, \pm 1, \pm 2, \cdots$ are the Matsubara frequencies of meson field, and the function $A(i\omega_n, k)$ is related to the self-energy $\Pi(i\omega_n, k)$,

$$A(i\omega_n, k) = \frac{\Pi(i\omega_n, k)}{-(i\omega_n)^2 + E_0^2}.$$
\[ \lim_{|z| \to \infty} |z^2 A(z, k)| < \infty, \]
\[ \lim_{|z| \to \infty} |z^2 \ln (1 + A(z, k))| < \infty, \] \tag{6}

the frequency summation is finally written as an integration along the positive real axis \([11]\),

\[ \Omega_{\text{ring}} = \frac{1}{4\pi i} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty d\omega \left( 1 + \frac{2}{e^{\beta \omega} - 1} \right) \times \left( \ln \frac{1 + A_R(\omega, k)}{1 + A_A(\omega, k)} - (A_R(\omega, k) - A_A(\omega, k)) \right), \] \tag{7}

where the retarded and advanced functions \(A_R\) and \(A_A\) are defined by

\[ A_R(\omega, k) = \frac{\Pi_R(\omega, k)}{(\omega + i\eta)^2 - E_0^2}, \]
\[ A_A(\omega, k) = \frac{\Pi_A(\omega, k)}{(\omega - i\eta)^2 - E_0^2}, \] \tag{8}

with the retarded and advanced self-energies \(\Pi_R(\omega, k) = \Pi(\omega + i\eta, k)\) and \(\Pi_A(\omega, k) = \Pi(\omega - i\eta, k)\) and \(\eta\) being an infinitesimal positive constant.

Taking integration by parts for the logarithm term of (7) and using

\[ A_R(\omega, k) - A_A(\omega, k) = \frac{i}{2E_0} \left( 2\pi \delta(\omega - E_0)\Pi(\omega, k) - \frac{4E_0}{\omega^2 - E_0^2} Im\Pi_R(\omega, k) \right), \] \tag{9}

we have (the other \(\delta\) function \(\delta(\omega + E_0)\) is neglected since the integration over \(\omega\) is along the positive real axis)

\[ \Omega_{ring} = -\frac{1}{4\pi i} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty d\omega \left( \omega + \frac{2}{\beta} \ln \left( 1 - e^{-\beta \omega} \right) \right) \times \frac{d}{d\omega} \ln \left( \frac{1 + A_R(\omega, k)}{1 + A_A(\omega, k)} \right) - \frac{1}{4} \int \frac{d^3k}{(2\pi)^3} \left( 1 + \frac{2}{e^{\beta E_0} - 1} \right) \frac{\Pi(E_0, k)}{E_0} \]
\[ + \frac{1}{4} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty d\omega \frac{\Pi(E_0, k)}{\pi} \frac{2 \ln \Pi_R(\omega, k)}{E_0(\omega^2 - E_0^2)}, \] \tag{10}

\[ \text{FIG. 2. The integration contours in the complex } z \text{ plane.} \]
From the separation of the logarithm,
\[
\frac{\ln 1 + A_R(\omega, k)}{1 + A_A(\omega, k)} = \ln \frac{(\omega - i\eta)^2 - E_0^2}{(\omega + i\eta)^2 - E_0^2} + \ln \frac{(\omega + i\eta)^2 - E_0^2 - \Pi_R(\omega, k)}{(\omega - i\eta)^2 - E_0^2 - \Pi_A(\omega, k)},
\]
its derivative can be expressed as a free particle part
\[
\frac{d}{d\omega} \ln \frac{(\omega - i\eta)^2 - E_0^2}{(\omega + i\eta)^2 - E_0^2} = 2i\pi \delta(\omega - E_0),
\]
and a self-energy-dependent part
\[
\frac{d}{d\omega} \ln \frac{(\omega + i\eta)^2 - E_0^2 - \Pi_R(\omega, k)}{(\omega - i\eta)^2 - E_0^2 - \Pi_A(\omega, k)}
\]
\[
= -2i \frac{d\tilde{\phi}(\omega, k)}{d\omega}
\]
characterized by the phase shift \(\tilde{\phi}\) resulted from the difference between the retarded and advanced self-energies.

The total phase shift can be separated into two parts, a pole part \(\phi_0\) which leads to the pole equation for quasiparticle and a scattering phase shift \(\phi_s\) defined in the region \([-\pi/2, \pi/2]\),
\[
\tilde{\phi} = \phi_0 + \phi_s,
\]
\[
\phi_0 = \pi \theta(\omega^2 - E_0^2 - \Pi),
\]
\[
\phi_s(\omega, k) = \arctan \left( \frac{iM \Pi_R(\omega, k) - 2\omega \eta}{\omega^2 - E_0^2 - \Pi(\omega, k)} \right),
\]
where we have made use of the property \(Im \Pi_R(\omega, k) \leq 0\), which can be observed from the relation between \(Im \Pi_R(\omega, k)\) and the decay rate [1,2]
\[
\omega \frac{d\Gamma}{d^3k} = -\frac{Im \Pi_R(\omega, k)}{(2\pi)^3(e^{\beta\omega} - 1)}. \tag{15}
\]

Substituting (12) and (13) into (10) and considering the fact that the derivative of a \(\theta\) function is a \(\delta\) function, we obtain
\[
\Omega_{ring} = \int \frac{d^3k}{(2\pi)^3} \left[ \frac{1}{\beta} \ln \frac{1 - e^{-\beta E}}{1 - e^{-\beta E_0}} - \frac{1}{e^{\beta E_0} - 1} \Pi(E_0, k) \right] - \int \frac{d^3k}{(2\pi)^3} \int_0^\infty d\omega \frac{1}{\pi} \frac{\phi_s(\omega, k) - Im \Pi_R(\omega, k)}{e^{\beta\omega} - 1}, \tag{16}
\]
where we have neglected the zero-point energy in the vacuum to avoid the ultraviolet divergence. The effective energy \(E = \sqrt{m^s + k^2}\) in (16) is related to the effective meson mass \(m^s\) determined through the pole equation,
\[
m^s = m^2 + \Pi(E, k). \tag{17}
\]
The first term in the first square bracket of (16) is the contribution from quasiparticles subtracting the corresponding free particles which will later be cancelled in the total thermodynamic potential \( \Omega \) by \( \Omega_0 \), and the rest in this bracket is an extra term resulted from the free meson propagator between two self-energies (lines between shaded circles in Fig.(1)). The second line of (16) is due to the imaginary part of the retarded self energy which is reflected in the scattering phase shift \( \phi_s \) and an extra term resulted also from the free meson propagators in the ring diagrams in Fig.(1). The connection between the phase of the scattering amplitude and the grand canonical potential was first made by Dashen [8] and continued in [9] and [10].

Now we turn to considering the lowest order correction to the thermodynamic potential, namely \( \Omega_1 \) in (1). Since \( \Omega_1 \) differs from the second term in (3) with only a sign and a symmetry factor, it can be written as

\[
\Omega_1 = \gamma_s \int \frac{d^3k}{(2\pi)^3} \frac{1}{\beta} \sum_n A(i\omega_n, k) \tag{18}
\]

where the factor \( \gamma_s \) is due to the difference between the symmetry factor for self-energy and that for partition function. The value of \( \gamma_s \) is determined by the interaction, it is \( 1/4 \) for \( \phi^4 \) theory and \( 1/6 \) for \( \phi^3 \) theory with the lowest order self-energy.

Putting together \( \Omega_0, \Omega_1 \) and \( \Omega_{\text{ring}} \), the total thermodynamic potential defined through (1) is now written as

\[
\Omega = \Omega_R + \Omega_I \tag{19}
\]

with the \( Re\Pi \)- and \( Im\Pi \)-dependent parts

\[
\Omega_R = \frac{1}{\beta} \int \frac{d^3k}{(2\pi)^3} \ln(1 - e^{-\beta E})
- \left( \frac{1}{2} - \gamma_s \right) \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\beta E_k} - 1} \frac{\Pi(E_0, k)}{E_0},
\]

\[
\Omega_I = -\int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} \frac{1}{e^{\beta \omega} - 1} \phi_s(\omega, k)
+ \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\pi} \frac{1}{e^{\beta \omega} - 1} \left( \frac{1}{2} - \gamma_s \right) \frac{2Im\Pi_R(\omega, k)}{\omega^2 - E_0^2} \tag{20}
\]

It is clear to see that \( \Omega_R \) contains not only a quasi-particle part with a temperature dependent mass \( m^* \) hidden in the particle energy \( E \), but also an extra term coming from the free meson propagator between two self-energies, and \( \Omega_I \) arises from the off-shell effect which leads to a phase shift \( \phi_s \) and a similar extra term. For NJL-type [11] interactions, there is no free propagator between two self-energies, thus the extra terms in \( \Omega_R \) and \( \Omega_I \) disappear. In this case, the in-medium effect is simple and clear: it results in a quasi-particle with a scattering phase shift. It is also necessary to note that the quasi-particle and phase shift are introduced only from the summation of all ring diagrams. For any ring diagram with fixed number of self-energy \( \Pi \), there are no such contributions to \( \Omega \).

III. EXAMPLES

With the formulas established in the last section, we can evaluate the equation of state including resummation effect for any given self-energy. We now consider some examples to illustrate the exact calculation of ring diagrams.
and compare it with the usually used quasi-particle approximation. The point we will focus on is the contribution from the off-shell effect included in $\Omega_I$.

Let us first consider $-\lambda \phi^4$ theory. The self-energy to the leading order shown in Fig.(3a) is evaluated analytically for massless particles ($m = 0$) [1,2]

$$\Pi = \lambda T^2 .$$  \hspace{1cm} (21)

![FIG. 3. The leading order self-energies for $\phi^4(a)$ and $\phi^3(b)$.](image)

Since the self-energy is $\omega$ and $k$ independent, the in-medium correction is reflected in the shift of the particle mass only, and there is no off-shell effect. With the formulas we develop in last section, it reads

$$\Omega_I = 0 ,$$

$$\Omega = \Omega_R = \Omega_{\text{quasi}} = \frac{1}{4} \int \frac{d^3k}{(2\pi)^3} \frac{\Pi}{k} \frac{1}{e^{k/T} - 1} ,$$

$$\Omega_{\text{quasi}} = \int \frac{d^3k}{(2\pi)^3} T \ln(1 - e^{-E/T}) ,$$

$$E = \sqrt{m^{*2} + k^2} , \quad m^{*2} = \Pi .$$  \hspace{1cm} (22)

After integrating over the angles and scaling the momentum $k$ by temperature $T$, the quasi-particle and total thermodynamic potentials are both proportional to $T^4$,

$$\Omega_{\text{quasi}} = a(\lambda) T^4 ,$$

$$\Omega = \left( a(\lambda) - \frac{\lambda}{48} \right) T^4 ,$$

$$a(\lambda) = \frac{1}{2\pi^2} \int_0^\infty dk k^2 \ln \left( 1 - e^{-\sqrt{\lambda+k^2}} \right) .$$  \hspace{1cm} (23)

Note that such temperature scaling arises from the $T^2$ dependence of the effective mass $m^*$. For any other temperature dependence, this scaling will be broken.

![FIG. 4. The quasi-particle and full thermodynamic potentials scaled by the Stefan-Boltzmann limit as functions of the coupling constant in $\phi^4$ theory.](image)
Fig.(4) shows the quasi-particle and full thermodynamic potentials scaled by the corresponding Stefan-Boltzmann limit as functions of the coupling constant $\lambda$. We see that both the quasi-particle and the total thermodynamics can not reach the Stefan-Boltzmann limit, and the deviation becomes more and more significant when the coupling constant increases. This phenomena is fully due to the quasi-particle mass behavior: The particles become heavy in the hot mean field and this mass transport becomes more and more strong with increasing temperature and coupling constant. If we take the coupling constant $\lambda$ to be about 0.3, the difference between the lattice calculation of the equation of state of QGP and the corresponding Stefan-Boltzmann limit (about 15 percent [7]) can be accounted. We are now certainly not handling QCD, but the spirit here and that used in [6] to fit successfully the lattice calculation are quite the same.

We introduce the so-called static-mode correction [2], which is obtained by taking $n = 0$ only in the frequency sum in (3). The static mode plays an important role in some recent works [3] as it is related to the soft degrees of freedom in an effective theory dealing with high temperature properties of QGP. Fig.(5) shows the coupling constant dependence of the static-mode and full calculations of the ring diagrams. We see that only in the small $\lambda$ region the static-mode calculation is a good approximation, for strong interaction the contribution from the excited modes can not be neglected.

$$\Pi(\omega, k) = 18\lambda^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{4E_1E_2} [(1 + f(E_1) + f(E_2))$$
$$\times \left( \frac{1}{\omega - E_1 - E_2} - \frac{1}{\omega + E_1 + E_2} \right)$$
$$- (f(E_1) - f(E_2))$$
$$\times \left( \frac{1}{\omega - E_1 + E_2} - \frac{1}{\omega + E_1 - E_2} \right),$$

where the factor 18 is the symmetry factor of the $\phi^3$ theory, and $f(z) = 1/(e^{\beta z} - 1)$ is the boson distribution function. The imaginary part of the retarded self-energy

$$\text{Im}\Pi_R(\omega, k) = -18\pi\lambda^2 \int \frac{d^3q}{(2\pi)^3} \frac{1}{4E_1E_2} [(1 + f(E_1) + f(E_2))$$
\begin{align*}
\times (\delta(\omega - E_1 - E_2) - \delta(\omega + E_1 + E_2)) \\
-(f(E_1) - f(E_2)) \\
\times (\delta(\omega - E_1 + E_2) - \delta(\omega + E_1 - E_2))
\end{align*}

(25)
can be simplified as

\begin{align*}
Im\Pi_R(\omega, k) &= -\frac{9\lambda^2}{8\pi k} \int_0^\infty dq \frac{q}{E_1} \\
&\times [\epsilon(\omega - E_1)f(E_1) + f(|\omega - E_1|)] \\
&\times \theta(1 - x_1)\theta(1 + x_1) \\
&+ (f(E_1) - f(\omega + E_1)) \\
&\times \theta(1 - x_2)\theta(1 + x_2)]
\end{align*}

(26)
with

\begin{align*}
E_1 &= \sqrt{q^2 + m^2}, \quad E_2 = \sqrt{(k - q)^2 + m^2}, \\
x_1 &= \frac{k^2 - \omega^2 + 2\omega E_1}{2kq}, \quad x_2 = \frac{k^2 - \omega^2 - 2\omega E_1}{2kq},
\end{align*}

(27)
and \(\epsilon(x)\) being the sign function.

By substituting the self-energy and the imaginary part of the retarded self-energy into (20), we obtain a rather complicated expression for the thermodynamic potential, which is now impossible to get analytical result and can only be evaluated numerically. Since the coupling constant \(\lambda\) in \(\phi^3\) theory is dimensional, we scale it by a momentum cutoff \(\Lambda = 1\, GeV\) to get a dimensionless effective coupling constant \(g = \lambda/\Lambda\). For all the calculations below, we choose the meson mass in the vacuum to be \(m = 200\, MeV\). In general, the self-energy \(\Pi\) depends separately on the variables \(\omega^2\) and \(k^2\). In free space at \(T = 0\), it can only be a function of the relativistically invariant combination \(s = \omega^2 - k^2\). We find, from numerical calculations, that this is also approximately true for \(T \neq 0\), and, since it introduces large computational simplifications, we thus assume this form in the following numerical calculations.

![FIG. 6. The effective mass \(m^*\) scaled by the free mass \(m\) as a function of temperature at \(k = 0\) for different coupling constant \(g\) in \(\phi^3\) theory.](image)

As in \(-\lambda\phi^4\) theory, the summation of ring diagrams in hot medium leads to the emergence of quasiparticles with a temperature-dependent mass determined by the pole equation (17). Fig.(6) shows the temperature dependence of
the effective mass $m^*$ for different coupling constant $g$. Unlike the $-\lambda\phi^4$ theory where the particles obtain mass from the hot medium, the particles in $-\lambda\phi^3$ theory lose mass in the hot medium. For any given coupling constant the effective mass drops down with increasing temperature, and at a high-enough temperature the system finally reaches the limit with $m^* = 0$. When the system is beyond this maximum temperature $T_m$ determined by $m^*(T_m) = 0$, there is no more real mass solution for the pole equation, and it leads to an unphysical jump in thermodynamic potential (19). For the coupling constant $g = 0.5$, the effective mass $m^*$ falls quite fast and reaches zero at $T_m = 275 \text{MeV}$. We show in Fig.(7) the maximum temperature as a function of the coupling constant $g$. We see that the maximum temperature increases with decreasing $g$.

The resummation of ring diagrams in $-\lambda\phi^3$ theory leads to not only an effective mass but also a scattering phase shift due to off-shell effect. Fig.(8) shows the scattering phase shift $\phi_s$ as a function of the Lorentz-invariant variable $s$ at $T = 250 \text{MeV}$ for different coupling constant. Since $s - m^2 - \Pi(s) < 0$ corresponding to $m^*/m < 1$ for the effective mass, the scattering phase shift is always in the region $0 < \phi_s < -\pi/2$ according to the definition of $\phi_s$ (14). It starts at the threshold energy $\sqrt{s} = 2m$ for the decay process, then reaches the maximum rapidly, and then gets damped slowly, and finally disappears. Because the off-shell effect in the medium arises from the thermal motion and the interaction of particles, the scattering phase shift increases with increasing temperature and coupling constant.

![FIG. 7. The maximum temperature $T_m$ for meson field as a function of coupling constant $g$ at $k = 0$ in $\phi^3$ theory.](image1)

![FIG. 8. The scattering phase shift $\phi_s$ as a function of the Lorentz-invariant variable $s$ at $T = 250 \text{MeV}$ and $k = 0$ for different coupling constant.](image2)
FIG. 9. The static-mode approximation ($\Omega_s$) and the full calculation ($\Omega_{\text{ring}}$) of the thermodynamic potential of ring diagrams as functions of the temperature for different coupling constant in $\phi^3$ theory.

The off-shell effect reflected in the imaginary part of the retarded self-energy dominates the decay process as shown in (15). For thermodynamics, the thermal motion of free particles, namely $\Omega_0$ is obviously the zeroth order contribution, and the effective mass and scattering phase shift result in leading order corrections. Fig.(9) shows the difference between the static-mode approximation and the full calculation of the ring diagrams for three values of coupling constant. While both $\Omega_s$ and $\Omega_{\text{ring}}$ increase with increasing coupling constant (note that the scales for $g = 0.1, 0.3$ and 0.5 are different in Fig.(9)), the deviation of $\Omega_{\text{ring}}$ from $\Omega_s$ is always significant, and even more important for weak couplings. This is quite different from the case in $\phi^4$ theory, see Fig.(5), where the calculation with only static mode is a good approximation in the region of weak coupling. This qualitative difference is mainly from the off-shell effect in the $\phi^3$ theory. It is the off-shell effect resulted from the frequency sum over the excited modes that leads to the $\text{Im}\Pi$-related part in $\Omega_{\text{ring}}$, see the second line of (16). Fig.(10) indicates directly the temperature and coupling constant dependence of $\Omega_I$ induced by the off-shell effect. To see how important the off-shell effect on the equation of state is, we plot in Fig.(11) the ratio between the thermodynamical potentials induced by the off-shell effect and by the quasi-particles on the mass-shell. Since the off-shell and quasi-particle effects are the same order correction to the thermodynamical potential, we plot the ratio $\Omega_I/(\Omega_R - \Omega_0)$ instead of $\Omega_I/\Omega_R$. It is interesting to note that while both $\Omega_I$ and $(\Omega_R - \Omega_0)$ get enhanced by strong coupling, the ratio is enlarged in weak coupling case. The reason is that the effective mass which dominates $(\Omega_R - \Omega_0)$ changes with coupling constant faster than the change in the
scattering phase shift which controls $\Omega_I$.

![Graph: The thermodynamical potential induced by the off-shell effect in hot medium as a function of the temperature for different coupling constant in $\phi^3$ theory.](image1)

FIG. 10. The thermodynamical potential induced by the off-shell effect in hot medium as a function of the temperature for different coupling constant in $\phi^3$ theory.

![Graph: The ratio between the thermodynamical potentials induced by the off-shell effect and by the quasi-particles in hot medium in $\phi^3$ theory.](image2)

FIG. 11. The ratio between the thermodynamical potentials induced by the off-shell effect and by the quasi-particles in hot medium in $\phi^3$ theory.

IV. CONCLUSIONS

We have presented an exact calculation of ring diagrams in the frame of finite temperature field theory in imaginary time formalism. The resummation of the ring diagrams not only changes the particle mass, but also generates a scattering phase shift, both are collective effects in the hot medium. Only for the system with self-energy in the mean field approximation, the thermodynamics can be described by quasi-particles. In general case, the corrections from the effective mass and from the scattering phase shift to the equation of state are introduced at the same time. We have applied our formulas to $\phi^4$ and $\phi^3$ theories. For the former with the self-energy to the first order, the static-mode can be considered as a good approximation for weak couplings, and the enhanced mass in the hot medium leads to the phenomena that the system can not reach the Stefan-Boltzmann limit at high temperatures. For the latter with the self-energy at Fock level, the suppressed mass in the hot medium results in a maximum temperature where the particles become massless and the system starts to collapse. The off-shell effect in $\phi^3$ theory generates a scattering phase shift simultaneously when the quasi-particles are formed. The total thermodynamic potential contains a quasi-particle part and an off-shell part. The off-shell part increases with increasing temperature and coupling constant, while its contribution to the total thermodynamics is enhanced in the case of weak couplings. For the coupling constant $g = 0.1$ the ratio of the off-shell part to the quasi-particle part reaches 25% at high temperatures. The formulas developed here will be extended to discuss more realistic systems.
Acknowledgments: This work was supported in part by the grants NSFC19925519, 10135030 and 10105005, and the national research program G2000077407.

[1] J.I. Kapusta, Finite-Temperature Field Theory, Cambridge University Press, 1989.
[2] M. Le Bellac, Thermal Field Theory, Cambridge University Press, 2000.
[3] J.P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. Lett. 83, 2906(1999);
       J.P. Blaizot and E. Iancu, Phys. Rept. 359, 355(2002).
[4] For instance, see Quark-Gluon Plasma, ed. R.C. Hwa (World Scientific, 1990).
[5] For instance, see Proceedings of QM’01, Nucl. Phys. A698(2002).
[6] A. Peshier, B. Kaempfer, O. P. Pavlenko and G. Soff, Phys. Rev. D54, 2399(1996);
       P. Levai and U. Heinz, Phy. Rev. C57, 1879(1998).
[7] For instance, see: F. Karsch, Nucl. Phys. A698, 199c(2002).
[8] R. Dashen, S. Ma, and H. J. Bernstein, Phys. Rev. 187, 345(1969).
[9] R. E. Norton, Ann. Phys. (N.Y.)170,18(1986).
[10] S. Jeon and P. J. Ellis, Phys. Rev. D58, 045013(1998).
[11] J. Hüfner, S.P. Klevansky, P. Zhuang and H. Voss, Ann. Phys. (N.Y.)234, 225(1994);
       P. Zhuang, J. Hüfner and S.P. Klevansky, Nucl. Phys. 576A, 525(1994).