New constraints on the distance duality relation from the local data

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ABSTRACT

The cosmic distance duality relation (DDR), which connects the angular diameter distance and luminosity distance through a simple formula $D_A(z)(1+z)^2/D_L(z) = 1$, is an important relation in cosmology. Therefore, testing the validity of DDR is of great importance. In this paper, we test the possible violation of DDR using the available local data including type Ia supernovae (SNe Ia), galaxy clusters and baryon acoustic oscillations (BAO). We write the modified DDR as $D_A(z)(1+z)^2/D_L(z) = \eta(z)$, and consider two different parameterizations of $\eta(z)$, namely $\eta_1(z) = 1 + \eta_0 z$ and $\eta_2(z) = 1 + \eta_0 z/(1+z)$. The luminosity distance from SNe Ia are compared with the angular distance from galaxy clusters and BAO at the same redshift. Two different cluster data are used here, i.e. elliptical clusters and spherical clusters. The parameter $\eta_0$ is obtained using the Markov chain Monte Carlo methods. It is found that $\eta_0$ can be strictly constrained by the elliptical clusters + BAO data, with the best-fitting values $\eta_0 = -0.04 \pm 0.12$ and $\eta_0 = -0.05 \pm 0.22$ for the first and second parametrizations, respectively. However, the spherical clusters + BAO data couldn’t strictly constrain $\eta_0$ due to the large intrinsic scatter. In any case studied here, no evidence for the violation of DDR is found.

Key words: cosmological parameters – distance scale – supernovae: general

1 INTRODUCTION

The cosmic distance duality relation (DDR) plays an important role in cosmology and astronomy. According to this relation, the luminosity distance $D_L(z)$ is strictly correlated to the angular diameter distance by a simple formula, i.e. $D_A(z)(1+z)^2/D_L(z) = 1$ (Etherington 1933, 2007). The DDR holds in any metric theory of gravity such as the general relativity, as long as the photons travel along null geodesics and the photon number is conserved during the propagation (Ellis 1971, 2007). The violation of DDR may be caused by e.g. the coupling of photon with unknown particles (Bassett & Kunz 2004), the extinction of photon by intergalactic dust (Corasaniti 2006), the variation of fundamental constants (Ellis et al. 2013), and so on. The modern cosmology is strongly dependent on the validity of DDR. Any violation of DDR would imply that there are new physics beyond the standard cosmological model. Therefore, testing the validity of DDR is of very importance and has aroused great interests in recent years.

A straightforward way to test the DDR is to measure the luminosity distance $D_L(z)$ and angular diameter distance $D_A(z)$ at the same redshift. A lot of works have already been done along this line (de Bernardis, Giusarma & Melchiorri 2000; Holanda, Lima & Ribeiro 2010; Piorkowska et al. 2011; Yang et al. 2013; Santos-da-Costa, Busti & Holanda 2014; Holanda et al. 2016; Ma & Corasaniti 2016; Holanda, Busti & Alcaniz 2018; Li & Lin 2018; Hu & Wang 2018). The luminosity distance is usually measured from type-Ia supernovae (SNe Ia), which are perfect standard candles and are widely used to measure the cosmological distance (Perlmutter et al. 1999; Riess et al. 1998). However, there is no optimal way to measure the angular diameter distance. The possible methods to determine $D_A$ include (1) by using the combined data of the X-ray and Sunyaev–Zeldovich (SZ) effect of galaxy clusters (De Filippis et al. 2007; Bonamente et al. 2008), (2) by measuring the baryon acoustic oscillations (BAO) signal in the galaxy power spectrum (Beutler et al. 2011; Anderson et al. 2014; Kazin et al. 2014; Delubac et al. 2015), (3) by measuring the angular size of ultra-compact radio sources based on the approximately consistent linear size (Kellermann 1993; Gurvits 1994; Gurvits, Kellermann & Frey 1999).
All of these methods to measure the angular diameter distance have their own advantage and shortcoming. The distance of cluster obtained using SZ effect is available in the full redshift range \( z < 1 \), but it strongly depends on the mass profile of cluster hence induces large uncertainty. The BAO method has a high accuracy, but the number of available data points is very limited. The radio source sample is large and spans a wide redshift range, but the low redshift sources confront serious evolution effect thus are not suitable to be used as standard rulers. The strong gravitational lensing can confront serious evolution effect thus are not suitable to be used as standard rulers. The strong gravitational lensing can reach to a relatively high redshift, but can only give the ratio of lens-source distance to source-observer distance, and thus signals of DDR violation, if really exists, may be partially canceled out. Until now, no evidence for the violation of DDR. If both \( D_A \) and \( D_L \) have been measured at the same redshift, then the parameter \( \eta_0 \) can be constrained.

Specifically, we consider two different parameterizations of \( \eta(z) \), i.e.

\[
\eta_1(z) = 1 + \eta_0 z, \quad \eta_2(z) = 1 + \eta_0 \left( \frac{z}{1 + z} \right),
\]

where \( \eta_0 \) is a free parameter representing the amplitude of violation of DDR. Any deviation of \( \eta_0 \) from zero would imply the violation of DDR. In both \( D_A \) and \( D_L \) have been measured at the same redshift, then the parameter \( \eta_0 \) can be constrained.

The first data we used to determine the angular diameter distance is the galaxy clusters. The distance of cluster can be obtained by measuring the SZ effect combined with the X-ray observation. This method to determine the angular diameter distance depends on the mass profile of clusters. De Filippis et al. (2003) have obtained the distance of 25 clusters in the redshift range \([0.023, 0.784]\) by assuming an elliptical profile, and we call this data cluster(E) hereafter. By assuming a spherical profile, Bonamente et al. (2006) have obtained the distance of 38 clusters in the redshift range \([0.142, 0.890]\), which we call cluster(S) hereafter.

The second \( D_A \) data we used come from the measurement of BAO. The BAO are regular, periodic fluctuations in the density of the visible baryonic matter, and it is widely used as the “standard ruler” to measure the distance in cosmology. Here we use three data points from the WiggleZ Dark Energy Survey at effective redshifts \( z = 0.44, 0.6 \) and 0.73 (Blake et al. 2012), one data point from the SDSS DR7 at effective redshift \( z = 0.35 \) (Xu et al. 2013), one data points from the SDSS-III Baryon Oscillation Spectroscopic Survey at effective redshift \( z = 0.57 \) (Samushia et al. 2014), and one data points from BOSS DR11 at effective redshift \( z = 2.34 \) (Delubac et al. 2015). These data points are listed in Table 1 and are plotted together with the cluster(E) and cluster(S) data points in Figure 1.

Since the cluster and BAO data have the same format, we combined these two datasets together and calculate the luminosity distance from equation (1).

\[
D_{\text{clus}} = \frac{(1 + z)^2 D_{A,\text{clus}}}{\eta(z)}.
\]

Then we convert the luminosity distance to the dimension-less distance modulus by

\[
\mu_{\text{clus}} = 5 \log_{10} \frac{D_{\text{clus}}}{\text{Mpc}} + 25.
\]

Table 1. The BAO data points used in this work.

| \( z \) | \( D_A \) [Mpc] | Reference |
|-------|--------------|-----------|
| 0.44  | 1205 ± 114   | Blake et al. (2012) |
| 0.60  | 1380 ± 95    | Blake et al. (2012) |
| 0.73  | 1534 ± 107   | Blake et al. (2012) |
| 0.35  | 1050 ± 38    | Xu et al. (2013)    |
| 0.57  | 1380 ± 23    | Samushia et al. (2014) |
| 2.34  | 1662 ± 96    | Delubac et al. (2015) |

2 DATA AND METHODOLOGY

In this section, we illustrate the method to test the DDR using the combined datasets of galaxy clusters, BAO and SNe Ia. We rewrite the possible violation of the standard DDR as

\[
\frac{D_A(z)(1 + z)^2}{D_L(z)} = \eta(z).
\]

SNe Ia.
possed by Marriner et al. (2011) but includes extensive sim-
ulations to correct the SALT2 light curve fitter. According
to the BBC method, the SNe data are binned into several
redshift bins, the nuisance parameters α and β are deter-
mined by fitting to a randomly chosen reference cosmology
with the cosmological parameters fixed. The key point is
that within each redshift bin, the local shape of the Hubble
diagram is well described by the reference cosmological
model. Marriner et al. (2011) has shown that the fitted α
and β will converge to consistent values which are indepen-
dent of the reference cosmology, as long as the bin number
is large enough. Once α and β are determined, a distance bias
correction term ΔD determined from simulation is added to
equation (6) for each SN. The simulation also depends on
an input cosmology, but the changes in the input cosmology
within typical statistical uncertainties have in general a neg-
ligible effect. The Pantheon dataset is calibrated using the
BBC method. Scolnic et al. (2017) reported the corrected
apparent magnitude $m_{\text{B,corr}} = m_B + \alpha X_1 - \beta C + \Delta B$ for
all the SNe. Therefore, to calculate the distance moduli we
just need to subtract $M_B$ from $m_{\text{B,corr}}$ and don’t need to
do the color and stretch corrections any more. The statisti-
cal uncertainty $D_{\text{stat}}$ and the matrix of systematics $C_{\text{sys}}$ are
also reported. The total uncertainty is the sum of $D_{\text{stat}}$ and
$C_{\text{sys}}$. The detailed information on the Pantheon dataset can
be found in Scolnic et al. (2017).

The SNe Ia and clusters or BAO are usually not mea-
ured at the same redshift. To solve this problem, we first use
the Gaussian processes to construct the
$\mu - z$ relation from the Pantheon data, then calculate the luminosity distance at the redshifts of clusters and BAO from the reconstructed
$\mu - z$ relation. Unlike the best fitting method which needs an
explicit fitting model, the Gaussian processes can construct a function from discrete data points without involving any
model. Seikel, Clarkson & Smith (2013). The Gaussian pro-
cesses only depends on the covariance function $k(x, \tilde{x})$, which
characterizes the correlation between the function value at
$x$ to that at $\tilde{x}$. There are many covariance functions avail-
able, but any covariance function should be positive definite and
monotonously decreasing with the increment of distance
between $x$ and $\tilde{x}$. Here we use the widely used squared-
exponential covariance function, which reads

$$k(x, \tilde{x}) = \sigma_f^2 \exp \left[ -\frac{(x - \tilde{x})^2}{2l^2} \right].$$

The hyperparameters $\sigma_f$ and $l$ characterize the “bumpiness”
of the function and should be properly chosen. We opti-
mize the hyperparameters by maximizing the marginal like-
lihood marginalized over function values $f$ at the whole loca-
tions $X$. We use the publicly available python package
GapP (Seikel, Clarkson & Smith 2012) to reconstruct the
corrected apparent magnitude $m_{\text{B,corr}}$ as a function of red-
shift. The results are plotted in Figure 2. In the range where
data points are sparse, the uncertainty of the reconstructed
function is large.

The reconstructed $\mu(z)$ function is then fitted to the
combined clusters and BAO data. We use the Markov chain
Monte Carlo methods (Foreman-Mackey et al. 2013) to cal-
culate the posterior probability distribution functions (pdf)
of free parameters. The likelihood is given by
\[
L(\text{Data}|\theta) = \prod \frac{1}{\sqrt{2\pi}\sigma_\mu} \exp \left[ -\frac{1}{2} \left( \frac{\mu_{\text{sn}} - \mu_{\text{clus/BAO}}}{\sigma_\mu} \right)^2 \right],
\]
(8)
where
\[
\sigma_\mu = \left( \sigma_{\mu_{\text{sn}}}^2 + \sigma_{\mu_{\text{clus/BAO}}}^2 + \sigma_{\text{int}}^2 \right)^{1/2}
\]
(9)
is the total uncertainty inherited from SNe and cluster/BAO, \(\theta = (\eta_0, M_B, \sigma_{\text{int}})\) is the set of free parameters, and the product runs over all the cluster and BAO data points. We have added an intrinsic scatter term to account for any other uncertainties. The posterior pdf is proportional to the product of likelihood and prior,
\[
P(\theta|\text{Data}) \propto L(\text{Data}|\theta) \times P_0(\theta).
\]
(10)
We assume a non-informative prior (namely the flat prior) on all the free parameters. To ensure that equation (2) is positive definite in the full redshift range of available data, we restrict \(\eta_0 > -0.4\). The intrinsic scatter is restricted to be \(\sigma_{\text{int}} > 0\), and no bounds on \(M_B\) are given.

3 RESULTS

We consider two different combination of datasets, i.e., cluster(E)+BAO and cluster(S)+BAO. Each combination of dataset is used to constrain the two different parametrizations of DDR. The publicly available python package emcee (Foreman-Mackey et al. 2013) is used to calculate the posterior pdf of free parameters. We report the median values and the 68% (1\(\sigma\)) confidence intervals of free parameters in Table 2. The marginalized likelihood distributions and the 2-dimensional confidence regions for the parameters are plotted in Figures 3–6.

The constraints on the first parametrization of DDR from cluster(E)+BAO data is shown in Figure 3. In this case, the best-fitting parameters and their 1\(\sigma\) uncertainties are \(\eta_0 = -0.04 \pm 0.12\), \(M_B = -19.44 \pm 0.13\), \(\sigma_{\text{int}} < 0.08\). Although \(\eta_0\) and \(M_B\) can be tightly constrained, only the
dimensional confidence regions for the parameters \( \eta \). The marginalized likelihood distributions and 2-

Figure 5. The marginalized likelihood distributions and 2-dimensional confidence regions for the parameters \( \eta_0 \), \( M_B \) and \( \sigma_{\text{int}} \) constrained from the cluster(E)+BAO data in the second parametrization \( \eta_2(z) = 1 + \eta_0 z / (1 + z) \).

Figure 6. The marginalized likelihood distributions and 2-dimensional confidence regions for the parameters \( \eta_0 \), \( M_B \) and \( \sigma_{\text{int}} \) constrained from the cluster(S)+BAO data in the second parametrization \( \eta_2(z) = 1 + \eta_0 z / (1 + z) \).

Table 2. The best-fitting parameters and 1\( \sigma \) uncertainties in two different parametrizations of \( \eta(z) \).

| par. | data      | \( \eta_0 \)   | \( M_B \)   | \( \sigma_{\text{int}} \) |
|------|-----------|----------------|-------------|---------------------------|
| \( \eta_0(z) \) | clus(E)+BAO | \(-0.04 \pm 0.12\) | \(-19.44 \pm 0.13\) | \(<0.08\) |
|      | clus(S)+BAO | \(0.13 \pm 0.19\) | \(-19.25 \pm 0.18\) | \(0.38 \pm 0.08\) |
| \( \eta_2(z) \) | clus(E)+BAO | \(-0.05 \pm 0.22\) | \(-19.44 \pm 0.16\) | \(<0.08\) |
|      | clus(S)+BAO | \(0.27 \pm 0.36\) | \(-19.21 \pm 0.21\) | \(0.38 \pm 0.08\) |

upper limits of \( \sigma_{\text{int}} \) can be obtained. This means that the intrinsic scatter is negligible, and the data is well consistent with DDR.

The constraints on the first parametrization of DDR from cluster(S)+BAO data is shown in Figure 4. In this case, the best-fitting parameters and their 1\( \sigma \) uncertainties are \( \eta_0 = 0.13 \pm 0.19 \), \( M_B = -19.25 \pm 0.18 \), \( \sigma_{\text{int}} = 0.38 \pm 0.08 \).

Similar to the cluster(E)+BAO data, the cluster(S)+BAO data are still well consistent with DDR. However, the intrinsic scatter is very large compared to the former.

The constraints on the second parametrization of DDR from cluster(E)+BAO data is shown in Figure 5. The best-fitting parameters are consistent with the first parametrization case: \( \eta_0 = -0.05 \pm 0.22 \), \( M_B = -19.44 \pm 0.16 \), \( \sigma_{\text{int}} < 0.08 \). The intrinsic scatter is still smaller enough to be negligible, and the DDR is well holds.

The constraints on the second parametrization of DDR from cluster(S)+BAO data is shown in Figure 6. The best-fitting parameters and their 1\( \sigma \) uncertainties are \( \eta_0 = 0.27 \pm 0.36 \), \( M_B = -19.21 \pm 0.21 \), \( \sigma_{\text{int}} = 0.38 \pm 0.08 \). The best-fitting \( \eta_0 \) value is a little larger than, but is still consistent with the first parametrization. As is in the first parametrization case, the intrinsic scatter of cluster(S)+BAO data is also large. Similarly, no evidence for the violation of DDR is found.

In summary, DDR can be tightly constrained by cluster(E)+BAO dataset in both parametrizations, and the intrinsic scatter is negligible. However, the cluster(S)+BAO dataset has large intrinsic scatter, and the uncertainty on the parameters is relatively large compared to the cluster(E)+BAO dataset. In any case, no evidence for the violation of DDR is found.

4 DISCUSSIONS AND CONCLUSIONS

In this paper, we used the SNe Ia, combined with the galaxy clusters and BAO data to constrain the possible violation of DDR. We first reconstructed the \( \mu - z \) relation from the most up-to-date Pantheon compilation of SNe Ia, and then used the reconstructed \( \mu - z \) relation to fit to the \( D_A \) data obtained from clusters and BAO. Since the angular diameter distance measured from clusters depends on the mass profile of cluster, two different mass profiles have been used, i.e. the elliptical cluster and spherical cluster. The former contains 25 data points and the latter contains 38 data points. It was showed that the cluster(E)+BAO data have a negligible intrinsic scatter and the DDR violation can be tightly constrained in both parametrizations, i.e. \( \eta_0 = -0.04 \pm 0.12 \) and \(-0.05 \pm 0.22\) in the first and second parametrizations, respectively. In both parametrizations, no signal of the DDR violation was found. On the other hand,
although the cluster(S)+BAO dataset is also consistent with the DDR, the intrinsic scatter of this dataset is large. This may imply that the spherical profile is not a good approximation to model the mass distribution of galaxy clusters.

We note that there is only one BAO data point at redshift \( z = 2.34 \), while the other BAO and cluster data points all locate at redshift \( z < 1 \). There is a wide redshift range between \( 1.0 < z < 2.3 \) lacking of data. Therefore, it is interesting to test if the \( z = 2.34 \) BAO data point has some influence on the results. We redo the previous calculations but omitted the \( z = 2.34 \) BAO data point. We find that the results are almost unaffected. This is not surprising, because the reconstructed \( \mu - z \) function has large uncertainty at \( z = 2.34 \), therefore the \( z = 2.34 \) data point has small weight in the fitting. We have also tried to fill the redshift gap between \( 1.0 < z < 2.3 \) with the binned ultra-compact radio source data from [Li & Lin (2018)]. However, adding the radio source introduces an additional parameter (the linear size of the radio source) so couldn’t help to improve the constraints.

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