Hybrid RANS/LES Method for High Reynolds Numbers, Applied to Atmospheric Flow over Complex Terrain

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Abstract. The use of Large-Eddy Simulation (LES) to predict wall-bounded flows has presently been limited to low Reynolds number flows. Since the number of computational grid points required to resolve the near-wall turbulent structures increase rapidly with Reynolds number, LES has been unattainable for flows at high Reynolds numbers. To reduce the computational cost of traditional LES a hybrid method is proposed in which the near-wall eddies are modelled in a Reynolds-averaged sense. Close to walls the flow is treated with the RANS-equations and this layer act as wall model for the outer flow handled by LES. The well-known high Reynolds number two-equation $k-\epsilon$ turbulence model is used in the RANS layer and the model automatically switches to a two-equation $k-\epsilon$ subgrid-scale stress model in the LES region. The approach can be used for flow over rough walls.

To demonstrate the ability of the proposed hybrid method, simulations of the wind flow over a complex terrain near Wellington in New Zealand are presented. Under certain conditions unsteady flow features have been measured at the site - flow features that could lead to high structural loads on a planned wind farm. These transient flow phenomena are reproduced with the new RANS/LES method. Additionally, the results from the hybrid method are compared with pure RANS results.

1. Introduction
Today, an increasing number of wind farms are erected at sites of complex terrain with the hope of a large energy production. By placing wind turbines in hilly terrain, along ridges and even in mountainous areas, wind phenomena like flow separation and recirculation can, however, greatly increase the structural loads on the wind turbines. Reliable predictions of such wind features are therefore important for siting of wind turbines in complex terrain and is the subject of the present paper.

When simulating wind over terrain using computational fluid dynamics (CFD) state-of-the-art is to solve the incompressible RANS-equations (Reynolds-averaged Navier-Stokes) together with the high Reynolds number $k-\epsilon$ turbulence model, using a finite-volume code [7, 16, 20, 22]. The solution of the RANS equations provides information on the mean wind and the mean level of turbulent kinetic energy at all locations. For many complex sites, however, simulating the
inherent unsteady features of the flow can improve wind predictions. Large-eddy simulation (LES) has in recent years become a useful tool especially to investigate a wide range of industrial flows at relative low Reynolds numbers. LES resolves the turbulent structures larger than the filter-scale applied and is therefore able to capture the important unsteadiness of the flow. LES resolves most of the turbulent kinetic energy when computational grids are fine, and should become insensitive to the particular turbulence model used. Because of these advantages, this paper proposes the use of LES to predict wind over complex terrain.

The major downside of using LES compared to RANS is the increased computational cost of boundary layer flow simulations. Grid-resolution requirements for a properly resolved LES has been analyzed by several authors e.g. Chapman [4] and Spalart et al. [23]. A boundary layer flow can in general be divided into an inner near-wall region, where viscous effects are important, and an outer region away from the wall. The resolution requirements of the outer region is relatively small and is essentially independent of Reynolds number. The inner-layer, however, is much more demanding - resolving all the small inner-layer eddies with LES at high Reynolds number is simply too computationally demanding. In order to avoid resolving all the inner-layer structures, inner-layer modelling is of great importance for high Reynolds number LES. To complicate matters more, the surface of the atmospheric boundary layer is not smooth but consists of roughness elements, which cannot be resolved by a computational mesh. Ultimately, we must choose a technique where the calculation of the near-wall flow using LES is abandoned, and instead rely on accurate near-wall modelling.

Traditionally, two classes of wall models for LES are used: methods that completely bypass the near-wall region by employing wall-functions and hybrid methods that simulate the near-wall region using RANS or RANS-like methods. The first approach assumes that the near-wall flow can be described by a simple generalized law-of-the-wall that is applied at the first near-wall computational grid cell. Since effects of surface roughness can be build into the wall-function this method has been applied to simple cases of atmospheric boundary layer flows with success [1, 10, 24]. A downside of the approach is that the derivations of wall-functions are based on simple flow cases and their use for complex flows are questionable. The second approach to avoid the near-wall resolution requirements of LES, is to solve the near-wall region using RANS, which then act as wall model for the outer flow handled by LES. Rather than using a simple wall-function to achieve the velocity at the wall-layer edge, these hybrid methods actually solves the RANS-equations inside the wall-layer, thereby resolving a greater part of the mean near-wall velocity profile and avoid the use of wall-functions. Since the near-wall resolution requirements for RANS are considerably lower than LES, computational cost is greatly reduced. The hybrid method has recently been applied by Nikitin et al. [13], Piomelli et al. [15] based on the formulation of Detached-Eddy Simulation (DES) proposed by Spalart et al. [23].

The method proposed in this work combines the hybrid approach and a wall-function. This might seem contradictory at first, however, for flows at very high Reynolds numbers and over rough walls - wall-functions are unavoidable. By combining the two methods the advantage of resolving the mean near-wall velocity profile is maintained and the method becomes applicable for flows over rough walls. The hybrid formulation proposed in the following is based on the high Reynolds number $k − \epsilon$ RANS model in a formulation similar to DES. Thereby the wall-function, build into the $k − \epsilon$ RANS model, can be used directly to model the effect of rough walls. Contrary to earlier LES-models with wall-functions, the new method does not change to LES immediately above the wall-function edge. Instead a RANS region is generated, and the model switches to LES above this - when the grid resolution is adequate to support LES. The actual distance from the wall to the LES region is similar to earlier wall-function methods. However, instead of resolving this near-wall region using only one computational grid cell, a finer resolved RANS region can now be achieved.

The present work is divided into two parts. First, the hybrid modelling technique and the
numerical approach is thoroughly described. Secondly, the simulation procedure is illustrated by an actual case, where a complex site is analyzed. At specific incidences, measurements at the site show very high levels of turbulence - turbulence levels that could prove severe for a planned wind farm. Using the hybrid approach, the complex unsteady interaction between the terrain and the flow that causes the high turbulence level is illustrated.

2. Model description

2.1. Basic equations

The momentum equations and the continuity equation for an incompressible newtonian fluid of constant density can be stated in Cartesian coordinates as,

\[ \frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + f_i, \]  
\[ \frac{\partial u_i}{\partial x_i} = 0, \]

where the Einstein summation notation is used. \( u_i \) (i=1, 2 and 3) denote the velocity components, \( p \) is the dynamic pressure and \( \nu \) is the kinematic molecular viscosity. \( f_i \) represents body forces, which act on the mass of the fluid. If we, from a numerical point of view, decompose the pressure, the body forces and the velocity components into resolved parts denoted by overbar \( \bar{\cdot} \) and unresolved parts denoted by tilde \( \tilde{\cdot} \), i.e.

\[ u_i = \bar{u}_i + \tilde{u}_i, \]

and consider the effects of molecular viscosity negligible due to the high Reynolds numbers, then the equations for the resolved motions can be written as,

\[ \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \bar{\tau}_{ij}}{\partial x_j} + \bar{f}_i, \]

where

\[ \bar{\tau}_{ij} = - (\bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j). \]

The decomposition operation applied is only evident by the stress term \( \bar{\tau}_{ij} \) that has replaced the viscous term of Eq. (1). The actual operation applied has remained unspecified. However, if Eq. (3) is interpreted as the Reynolds decomposition of a variable into a time-averaged and a fluctuating component then Eq. (4) is the RANS-equations and \( \bar{\tau}_{ij} \) is the usual Reynolds-stress term. Though formally incorrect, the transient term of Eq. (4) has been retained and thereby the RANS-equations can actually resolve transient behaviors of flows, when instationarities are on a large timescale compared to the averaging time (this is often denoted Unsteady RANS).

The decomposition operation of Eq. (3) can also be interpreted as a spatial filtering operation - separating the large and the small turbulent scales. This is the idea of LES. Using this concept Eq. (4) governs the large turbulent scales while the stress term, \( \bar{\tau}_{ij} \), represent the effect from the small scales on the large scales. No explicit filter has actually been applied in deriving Eq. (4). Instead, the finite-volume discretization of the flow equations on a numerical grid, can be interpreted as an implicit filter tied to the numerical resolution, and the computed velocity field can be associated with a filtered velocity. Turbulent scales smaller than the numerical grid spacing are unresolved, whereas, the larger scales are resolved. Since the spatial filter is fixed to the numerical grid we denote the flow structures that are unresolved, subgrid-scales (SGS).

Following Leonard [8] the term \( \bar{\tau}_{ij} \) may be expanded into various stress terms. The closure model presented in the following section, however, will model \( \bar{\tau}_{ij} \) as a whole without regard to
the individual stress components. Doing this the RANS- and the LES-equations can be written in the same form (Eq. 4). formally, for LES the overbar (\(\overline{\cdot}\)) denotes a resolved large-scale motion, whereas in RANS it denotes a time-averaged quantity, however, the RANS- and LES-equations are identical. The difference between RANS and LES is therefore only represented in how \(\tilde{\tau}_{ij}\) is modelled.

2.2. Turbulence model
In order to close Eq. (4) a turbulence model, often termed SGS-model in LES terminology, is necessary for the turbulent stress tensor \(\tilde{\tau}_{ij}\). In deriving the equations for the resolved motions (Eq. 4), the molecular viscosity term was replaced by the turbulent stress term, it therefore seems logical to model the SGS-stresses as a viscosity term. Most turbulence models for both RANS and LES follow the eddy-viscosity hypothesis by Boussinesq [3] where the turbulent stresses are assumed to be the product of the fluid strain and an eddy-viscosity, \(\nu_T\). We adopt this concept of Boussinesq,

\[
\tilde{\tau}_{ij} = 2\nu_T S_{ij} + \frac{\delta_{ij}}{3} \tilde{\tau}_{kk}; \quad S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),
\]

(6)

here \(S_{ij}\) is the strain rate tensor and the eddy-viscosity is a positive scalar. The eddy-viscosity concept is completely dissipative - it removes energy from the large resolved scales, an mimics the energy drain associated with the energy cascade (introduced by Richardson [18]).

In order to obtain the eddy-viscosity for RANS simulations we use the classical two-equation high Reynolds number \(k-\epsilon\) model [7] - a fine-tuned model found in most commercial CFD-solvers, which is widely used for atmospheric flows. Based on dimensional ground the eddy-viscosity may be described as the product of a length scale and a velocity scale. Determining the two scales, which should be characteristic for the modelled turbulence, is the essence of turbulence modelling. The \(k-\epsilon\) model is a two-equation model that solves transport equations for the turbulent kinetic energy, \(\tilde{k}\), and for the dissipation of turbulent kinetic energy, \(\tilde{\epsilon}\). From these two quantities a length scale \((l = \tilde{k}^{3/2}/\tilde{\epsilon})\) and a velocity scale \((v = \tilde{k}^{1/2})\) are constructed and the eddy-viscosity calculated,

\[
\nu_T = C_\mu \frac{k^2}{\bar{\epsilon}},
\]

(7)

where \(C_\mu\) is a model constant. The two transport equations are not presented here.

For RANS, the turbulence parametrization encompasses all the turbulent scales and the characteristic ”mixing-length” is therefore comparable to the largest flow scales. For LES, the largest turbulent structures are resolved, therefore the ”mixing-length” only represents the small unresolved structures - that scale with the spacing of the computational grid. A RANS and a LES length scale is introduced,

\[
l_{LES} = C_\Delta \Delta; \quad l_{RANS} = \frac{\tilde{k}^{3/2}}{\bar{\epsilon}},
\]

(8)

where \(\Delta = \max(\Delta x, \Delta y, \Delta z)\) is the local maximum grid spacing over the three directions and \(C_\Delta\) is a model constant similar to \(C_{DES}\) used in DES [23]. Following the methodology used by Travin et al. [25] the length scales are incorporated into the turbulence model by modifying the dissipative term of the \(k\)-transport equation,

\[
D_{k-Eq} = \frac{\tilde{k}^{3/2}}{l},
\]

(9)
where $\tilde{l}$ is either $l_{\text{RANS}}$ or $l_{\text{LES}}$. Switching between the two length scales it is possible to switch between a two-equation LES model and a two-equation RANS model very simply.

### 2.3. Model Constants

The model constants of the $k-\epsilon$ equations has been determined from experimental data and by considering a wide variety of flows. The originally proposed constants by Launder and Spalding [7] were established for industrial flows, while slightly different values have been found for atmospheric flows [14, 16, 26]. The presented model is capable of dealing with both atmospheric and industrial flows, therefore, model constants for both cases are presented. The standard $k-\epsilon$ model constants $C_\mu, \sigma_k, \sigma_\epsilon, C_{\epsilon_1}, C_{\epsilon_2}$ and the new constant $C_\Delta$ need to be determined. The constant $C_\mu$ is found by considering equilibrium turbulence near a wall, which leads to,

$$C_\mu = \left(\frac{u^*_0}{k}\right)^2 = \left(\frac{\tau_0}{k}\right)^2,$$

(10)

$\tau_0$ is the wall stress and $u^*_0$ is the friction velocity. For industrial flows the value of $C_\mu = 0.09$ is well established, while a typical value for atmospheric flows is $C_\mu = 0.03$. At times, a different value of $C_\mu$ is used for the turbulent kinetic energy, $\tilde{k}$, to match specific measurements.

In order to derive a relationship between the standard model constants and the new $C_\Delta$ we compare the new LES model to the well known Smagorinsky model [19]. The Smagorinsky model is derived from the hypothesis of balance between shear production and dissipation of turbulent kinetic energy and reads,

$$\nu_T = (C_s \Delta)^2 |S|,$$

(11)

where $C_s$ is a model constant and $S$ is the local strain rate defined by $S = (2S_{ij}S_{ij})^{1/2}$. If the unresolved scales are located in the Kolmogorov inertial subrange it can be shown that $C_s \approx 0.17$ [9]. Unlike the Smagorinsky model the proposed model explicitly solves the $k$-equation so no equilibrium assumptions between shear production and dissipation is needed. To compare, however, we assume equilibrium, by equating the production and dissipation term of the turbulent kinetic energy equation, $\tilde{\epsilon} = \nu_T |S|^2$. Using this with Eq. (7) and estimating that $\tilde{\epsilon} \approx (C_{\epsilon_1}/C_{\epsilon_2}) \tilde{k}^{3/2}/l_{\text{LES}}$ the Smagorinsky model is recovered and $C_\Delta$ can be obtained,

$$\nu_T = (\Delta C_s)^2 |S| \approx \left(\Delta C_\Delta C_\mu^{3/4} \frac{C_{\epsilon_2}}{C_{\epsilon_1}}\right)^2 |S|,$$

(12)

$$C_\Delta = C_s C_\mu^{-3/4} \frac{C_{\epsilon_1}}{C_{\epsilon_2}}.$$

(13)

For numerical simulations the optimum value of $C_s$ may differ from the theoretically obtained value of $C_s \approx 0.17$ and the value can change depending on the numerical schemes applied. Based on simulations of decaying isotropic turbulence using centered differencing schemes [2], $C_s = 0.14$ was found. Furthermore, it was shown that this value is independent of $C_\mu$. The proposed model constants are listed in table 1.

| Flow     | $C_\mu$ | $\sigma_k$ | $\sigma_\epsilon$ | $C_{\epsilon_1}$ | $C_{\epsilon_2}$ | $C_\Delta$ | $C_s$ |
|----------|---------|-------------|-----------------|-----------------|-----------------|---------|-------|
| Industrial | 0.09    | 1.00        | 1.30            | 1.42            | 1.92            | 0.65    | 0.14  |
| Atmosph.  | 0.03    | 1.00        | 1.30            | 1.21            | 1.92            | 1.26    | 0.14  |
2.4. Hybrid RANS/LES Method

As previously mentioned, the basic idea of the hybrid RANS/LES method is to model the near-wall turbulent structures using unsteady RANS while the outer flow is modeled using LES. In the RANS layer the grid resolution requirements of LES can be alleviated and wall-functions are more easily implemented. The Hybrid approach of combining time-averaged RANS and spatial filtered LES is not rigorously justified mathematically, but is computational appealing. Though the RANS and LES equations are identical the interpretation of the resolved quantities is not: RANS deals with averaged quantities whereas LES deals with filtered quantities. Nevertheless, in this work we view the RANS layer as a very large eddy simulation in which a large filtering length scale is used - a RANS layer that work as an advanced wall-model for LES.

Since the only difference between the $k-\epsilon$ RANS model and the presented $k-\epsilon$ LES model is the turbulent length scale used in the turbulence model, Eq. (9), changing between the two methods is very simple. The switch from RANS to LES is controlled by the turbulent length scale using a universal switch, similar to the one used in DES [25],

$$\tilde{l} = \min (l_{RANS}, l_{LES}) = \min \left( \frac{k^{3/2}}{\epsilon}, C_\Delta \Delta \right). \tag{14}$$

The model is solved on a single grid and the turbulent length scale is the only parameter that separates the RANS region from the LES region. The switch from RANS to LES is locally determined with no wall distance criteria. Near the wall $l_{RANS}$ is smaller than $l_{LES}$ and a RANS region is generated. Away from the wall $l_{LES}$ is small and the model switches to LES. In the RANS region the logarithmic law-of-the-wall is used as wall-function to determine the wall stress based on the instantaneous tangential velocity of the first off the wall grid-point. Since the position of a rough surface is unknown the logarithmic law is used down to $z/z_0 = 1$ where the tangential velocity is zero ($z_0$ is the roughness length). The method is implemented so that flow over both smooth and rough walls can be simulated and so that there are no restrictions on the height of the first near-wall grid cell [20]. A simple backscatter model that smoothen the RANS-LES transition region is presented in [2].

Since turbulent structures from the LES region are mixed into the RANS region, the precise height of the switch is determined as part of the solution, however, by equating the RANS and LES length scales the height, $z_{ml}$, can be estimated [2],

$$z_{ml} \approx \frac{C_s C_{e1}}{\kappa C_{e2}} \Delta \approx \Delta / 4. \tag{15}$$

As seen, $z_{ml}$ is dependent on $\Delta$ but is almost independent on $C_\mu$ ($C_{e1}$ is slightly dependent on $C_\mu$). This is an advantage since the same numerical mesh can be used for different $C_\mu$ values.

2.5. Numerical implementation

The CFD code EllipSys3D developed by Michelsen [11, 12] and Sørensen [20] has been used in all calculations presented. It is a multiblock finite-volume discretisation of the incompressible Navier-Stokes (NS) equations (Eq. 4). The multi-block facilities allow for large parallel computations and the exchange of information between processors is handled using Message Passing Interface, MPI.

The code is formulated in general curvilinear coordinates that can accurately describe the terrain and the code uses non-staggered grids with all variables stored in cell centers. In all simulations the PISO algorithm [5] has been used to solve the equation system and pressure/velocity decoupling is avoided by applying the Rhie/Chow interpolation technique [17]. The TDMA solver (Tri-Diagonal Matrix Algorithm) is used in altering directions to solve the transport equations and pressure solution is accelerated using a multigrid method.
The unsteady simulations presented are advanced in time using a second order iterative time-stepping method where the global time-step, $\Delta t$, is chosen to give a maximum CFL-number (the Courant-Friedrich-Levy-number) of no more than 0.25. For both pure RANS and hybrid RANS/LES simulations, solution time is minimized by using a three-level grid sequence. More than accelerating solution time, the grid sequence provides solutions on coarser mesh levels, which are important when validating that results are grid independent.

Discretization of the convective nonlinear terms in the NS-equations should in general be done with schemes with no or low numerical dissipation - especially for LES. For simulations performed in this work molecular viscosity is neglected why energy dissipation comes from the SGS-model and from numerical dissipation. To reduce numerical dissipation the convective terms for LES should only be solved using central schemes, however, due to the unboundedness of central schemes unphysical velocity fluctuations or wiggles can be generated. To avoid wiggles a scheme is used that combines 10% of the third order upwind scheme, QUICK, and 90% of a fourth-order central scheme, implemented using the deferred correction approach first suggested by Khosla and Rubin [6].

3. Simulation of complex terrain

3.1. Site description

To illustrate the hybrid approach, simulation results of the wind over a complex terrain are presented. The specific terrain is located near Wellington in New Zealand where a wind turbine farm is planned. Measurements of wind speed, direction and standard deviation are available from two masts: one is a 40 meter mast located at (2651683E; 5993941N) and the other is a 60 meter mast located at (2647798E; 5992586N). In this work the 40 meter mast is used as reference mast and termed M1 while the 60 meter mast is termed M2 (see figure 1). A third mast, M3, is shown on figure 1, however, measurements from this mast are not presented.

The measured wind rose from the Wellington site, figure 2, shows two dominating wind directions: around 345 degrees from north and around 150 degrees from south. During the measuring campaign high turbulence levels was measured at M2 for the 330 wind direction. In order to investigate this effect three wind directions has been selected (330, 345, 360) and is subject for further investigation using both hybrid RANS/LES and pure RANS simulations - these results are presented in the following.

![Figure 1. Overview of the Wellington site. The three measuring mast are marked (M1, M2 and M3) and the three simulated wind direction shown.](image1)

![Figure 2. The wind rose from the Wellington site is dominated by two relatively narrow wind directions, one from the north and one from the south.](image2)
For the selected wind directions, the wind approaches the terrain from the sea, where equilibrium wind conditions are considered. Mast M1 is located on a hill top (z=290m) while M2 is located in a small "pocket" (z=215m) between hills of higher elevation. The wind measured at M1 is relatively undisturbed and the wind direction measured here is considered representative for the equilibrium wind. Figure 3 shows the average turbulence intensity at M1 and M2 as function of the undisturbed wind direction measured during a four month period (4 August - 6 December 2006). At M1 the turbulence intensity is relatively constant ($I \approx 0.1$) while it increases at M2 when the wind changes from 360 to 300 degrees. On figure 4 the wind direction and the turbulence intensity at M2 measured during a single day is shown. Again, high turbulence intensities are observed for northwest winds - at times much higher than the averaged data of figure 3. The high turbulence intensity at M2 is probably generated by the ridge located upstream of the mast. The aim of the simulations is to reproduce this effect.

3.2. Simulation method
For the three wind directions selected (360, 345, 330) the wind approaches the terrain from the sea, where it is assumed to be in a state of equilibrium. This "undisturbed" wind need to be specified at the computational inlet for the terrain simulations. To provide a realistic turbulent inflow for both RANS and hybrid simulations, a separate simulation, a precursor, over flat terrain with a roughness length corresponding to rough sea ($z_0 = 0.001 m$) is performed (see figure 5). The generated wind fields, that are steady for RANS and unsteady for the hybrid method, are subsequently used as inflow for the terrain simulations. The precursor flow is driven by a constant pressure gradient in the flow direction that corresponds to a friction velocity of $u_{*0} = 0.42 m s^{-1}$. This value has been chosen together the standard model constants for atmospheric flows i.e. $C_{\mu} = 0.03$ (see table 1) in order to reproduce the measured turbulence intensity of about 0.1 at M1. Velocity profiles for both the hybrid RANS/LES and for the RANS precursor simulation are shown on figure 6 - as seen, both the RANS and the hybrid velocity profiles follow the logarithmic profile well.

3.2.1. Computational mesh
Since the Ellipsys3D code uses terrain-following coordinates it is possible for the lower boundary of the computational mesh to follow the topography. To generate the computational mesh, a map file (contour lines of terrain height) is first digitized. Secondly, based on this gridded file three surface meshes (x,y,z) are constructed using an in-house 2D
Figure 5. A precursor simulation for both RANS and hybrid RANS/LES is run to generate velocity field that can be used as input for the actual terrain simulations.

Figure 6. The precursor velocity profiles follow the logarithmic profile \( u_{+0} = 0.42 \text{ms}^{-1} \) closely near the ground.

durface grid generator. The surface meshes are generated for each intended mean flow direction and are aligned with this direction (see figure 7). This is done in order to minimize numerical diffusion and ensures that the precursor wind field can be used for different wind directions. The surface meshes covers areas of 13.5km×10.5km using near equidistant grid spacing of 31m.

Finally, having generated the surface meshes the final volume meshes are generated. The height of the first near-wall grid cell is set equal to the roughness length of the terrain, \( z_0 = 0.03m \). From this height the mesh is stretched in the vertical, using the 3D hyperbolic mesh generator HypGrid3D based on [21]. The stretching is performed so that near-cubic grid cells are achieved at a certain height (see figure 8). Furthermore, in order to avoid spatial interpolation, the inflow boundary of the terrain meshes exactly matches the outlet boundary of the precursor mesh. The height of the three computational domains are \( H=3700m \) and the total number of grid cells for each are \( 336 \times 336 \times 144 \).

Figure 7. Three computational meshes are generated each aligned with the intended direction of the mean flow. The figure shows the three surface meshes.

Figure 8. The figure shows a section of the computational mesh for the 345 wind direction. The generated surface mesh and a slice of the hyperbolic stretching is seen.
3.2.2. Boundary conditions At the top of the computational domains, symmetry boundary conditions are used \((\partial u/\partial z = \partial v/\partial z = 0, w = 0)\). The symmetry condition inhibit the turbulent normal motions and therefore resembles an inversion layer (a layer where temperature increase with height at top of the atmospheric boundary layer). The transverse horizontal boundaries are specified as periodic (cyclic condition) while the traditional Neumann boundary condition (zero normal gradient) is used at the outlet. The wall boundary condition is implemented using the traditional wall-function of the high Reynolds number \(k – \epsilon\)-model [7, 20] that assumes a logarithmic velocity gradient in the vicinity of the wall.

3.3. Results - mean flow

43 minutes (5100 timesteps) of turbulence from the precursor simulations was saved and used as inflow for the terrain simulations. The RANS and hybrid simulations were allowed 12 minutes of simulation time before results were sampled and averaged (over the final 31 minutes).

In order to assure grid independent results, simulations have been performed on three grid levels. Grid level 1 (L1) is the finest resolved mesh with \(\Delta_1 = 31m\) while level 2 and level 3 (L2 and L3) has been coarsened by removing every second grid point in all directions \((\Delta_2 = 62m\) and \(\Delta_3 = 124m)\). Figure 9 shows a comparison between the measured (averaged over 4 month) and computed turbulence intensity for the different grid levels. The x-axis shows the wind direction at M1 while the y-axis shows the increase in turbulence intensity from M1 to M2. The first thing to notice is that the hybrid results are converging and that grid level 1 seems to provide sufficient resolution. Furthermore, the RANS results on the fine mesh are seen to compare well with the hybrid results. Comparing simulations with measurements, however, the agreement is not especially good. The simulations captures the overall trend that turbulence intensity increases when the wind direction changes from 360 to 330, but the overall intensity level at M2 is overestimated. One explanation for this deviation is that the measured data consist of 10 min averaged time series that again are averaged over a four month measuring period. The measurements thus represent a wider range of flow directions and velocities than the simulations, which are computed for specific directions and velocities. This inherent smoothing of measured data is not present in simulations. Inspecting intensities measured during a single day (figure 4) high turbulence levels can be found that are close to the simulated values. Figure 10 compares the computed and measured velocity speed-up, \((U_{m2} – U_{m1})/U_{m1}\), from M1 to M2. Again, the quantitative agreement is far from perfect, but the overall trend is captured by the simulations.

**Figure 9.** Difference in turbulence intensity between M1 and M2 as function of wind direction at M1.

**Figure 10.** Simulation results of the velocity speed-up between M1 and M2 is compared to measurements.
3.4. Results - unsteady effects
Since the hybrid RANS/LES method resolves a large part of the unsteady turbulent structures, the method can be used to illustrate the transient flow phenomena that causes the high turbulence intensity at M2 (numerical timestep $\Delta t = 0.5s$). Figure 11 shows three snapshots of velocity contours, at a plane through M2 parallel to the mean flow direction (dir 330) taken at three succeeding times. Picture A, shows the formation of a recirculation bubble behind the ridge upstream of M2 (M2 is shown as a black dot). On picture B, the low speed bubble is released and advected past M2. Finally, the process is repeated on picture C, where a new recirculation zone is generated. This sequence of pictures gives an explanation to the high turbulence levels also found in measurements.

To compare the unsteady effects of the hybrid simulations and the measurements in greater detail, normalized velocity timeseries are shown on figure 12. On the top of the figure velocity signals measured at a frequency of 5Hz are shown. The velocity signals are 30 minute timeseries, measured at August 19th, 2006 at 03:00, 07:30 and 13:00 (see figure 4). At these times the measured wind directions at M1 were 322, 335 and 345, which are reasonable close to the simulated values of 325, 338 and 352 (due to terrain effects the simulated wind direction at M1 has changed slightly from the undisturbed directions of 330, 345 and 360). In the middle of the figure the same time series are shown, however, a filter has been applied that reduces the sampling rate to 0.2Hz. Finally, in bottom of the figure the simulated timeseries are shown. The figure gives a visual impression of the frequencies resolved with the hybrid simulations.

A better impression of resolved frequencies are given on figure 13 that shows normalized one-dimensional spectra of the streamwise velocity component for measurements and simulation. As seen the simulated spectrum drops off at a dimensionless frequency of about 1. Because of the limited grid resolution the hybrid spectrum does not show the extended inertial range found in the measurements. However, for the low frequencies the spectral shape of the simulation follow measurements reasonably well.

4. Conclusions
Due to high computational cost, LES has been unattainable for wall bounded flows at high Reynolds numbers. To reduce the computational cost, this paper proposes a hybrid method, that solves the RANS-equations near the wall and switches to LES above this layer - where the grid resolution is adequate to support LES. Since a wall-function is build into the RANS model
Figure 12. Normalized velocity timeseries from measurements and simulation at M2.

Figure 13. Normalized one-dimensional spectra of the streamwise velocity component for measurements and simulation. A peak is observed for the simulated spectrum at a dimensionless frequency of $n=0.4 \ (z = 180 \text{m}, \ u = 9\text{ms}^{-1})$. This is the frequency at which separation bubbles are generated and released.
the method can be used to simulate atmospheric flow over rough terrain.

The presented model is based on the standard high Reynolds number two-equation $k - \epsilon$ RANS model [7] found in most commercial CFD-solvers. By simply changing the turbulent length scale build into the RANS model, a hybrid model is achieved that automatically switches to a two-equation $k - \epsilon$ LES model away from the wall. Model constants were suggested, based on simulations of decaying isotropic turbulence. The new turbulence model can be run as either hybrid RANS/LES or as pure RANS dependent on the level of detail necessary. Since the hybrid method is computational expensive it should primarily be used for complex terrain where inherent unsteady features characterizes the flow and need to be captured accurately.

To demonstrate the hybrid approach, the wind over natural complex terrain located near Wellington in New Zealand was simulated using both RANS and hybrid RANS/LES. Both methods gave similar predictions of the mean flow field, but compared to measurements deviations were found. Simulations did, however, capture the overall behavior of the flow measured at the two masts, both with respect to velocity and turbulence. Following this, using the hybrid approach, it was possible to illustrate the unsteady flow phenomena that is responsible for the high turbulence level measured at mast 2. Using the proposed hybrid method, detailed information on the atmospheric wind in complex terrain can be achieved.

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