We investigate the statistics of work performed on generic disordered, non-interacting nanograins during quantum quenches. The time evolution of work statistics as well as the probability of adiabaticity are found to exhibit universal features, the latter decaying as a stretched exponential. In slowly driven systems, the most important features of work statistics are understood in terms of a diffusion of fermions in energy space, generated by Landau-Zener transitions, and are captured by a Markovian symmetrical exclusion process, with the diffusion constant identified as the absorption rate. The energy absorption is found to exhibit an anomalous frequency dependence at small energies, reflecting the symmetry class of the underlying Hamiltonian. Our predictions can be experimentally verified by calorimetric measurements performed on nanoscale circuits.

Introduction. — While energy transfer, heat, and work are fundamental concepts in thermodynamics and statistical physics, it is far from trivial to extend their concepts to generic non-equilibrium quantum systems [4, 2], where energy becomes a fluctuating statistical quantity even in pure quantum states, and energy transfer can only be understood in terms of a precise measurement protocol. Recent experimental developments allow, however, the investigation of these notions in quantum systems ranging from individual molecules subject to mechanical forces [3–5], through nuclear spins in a magnetic field [6], to mesoscopic grains [7, 8], and allow even to extract the forces [3–5], through nuclear spins in a magnetic field [6], ranging from individual molecules subject to mechanical col. Recent experimental developments allow, however, to understand in terms of a precise measurement protocol statistics [24, 25], and can be described in terms of time, disorder, and quantum statistics.

In slowly driven systems, the most important features of work statistics are understood in terms of a diffusion of fermions in energy space, generated by Landau-Zener transitions, and are captured by a Markovian symmetrical exclusion process, with the diffusion constant identified as the absorption rate. The energy absorption is found to exhibit an anomalous frequency dependence at small energies, reflecting the symmetry class of the underlying Hamiltonian. Our predictions can be experimentally verified by calorimetric measurements performed on nanoscale circuits.

We find that energy is absorbed by the system via particle diffusion in energy space, and that the statistical properties of work depend crucially on the speed $v$ of the quench as well as on underlying symmetries. For slow, almost adiabatic changes, in particular, $P_t(W)$ displays a universal structure, with surprisingly large work fluctuations, $\langle \delta W^2 \rangle \sim \langle W \rangle^{3/2}$. Energy absorption at small frequencies, $\omega \sim v$, reflects the symmetry (universality class) of the Hamiltonian and is predicted to scale as, $\langle W \rangle \sim \omega^{1+\beta/2}$, with the random matrix parameter $\beta = 1, 2$ and 4 corresponding to orthogonal, unitary, and symplectic symmetries.
FIG. 1. Quantum quenches for electrons in a generic disordered grain. The non-interacting fermions occupy levels of a random Hamiltonian. External time dependent gate voltages and fields move the levels and induce transitions between them. For slow changes, these happen through Landau-Zener transitions between neighboring levels (black arrow), while for faster changes electron-hole excitations between remote levels dominate (red dashed arrow).

Following Ref. [29], we apply a quench protocol, $\mathcal{H}(t) = \mathcal{H}_1 \cos \lambda(t) + \mathcal{H}_2 \sin \lambda(t)$, with $\lambda$ set to constant, and $\mathcal{H}_{1,2}$ independent $N \times N$ matrices, drawn from a Gaussian random matrix ensemble, $\mathcal{P}(\mathcal{H}) \sim e^{-\beta \text{Tr} \mathcal{H}^2}$ [24, 31]. Here we focus on Gaussian orthogonal ($\beta = 1$) and Gaussian unitary ($\beta = 2$) ensembles, corresponding to integer spin time-reversal-invariant systems, and systems with broken time-reversal symmetry, respectively.

Many-body wave function. — To compute the distribution [2], we first need to determine the many-body wave function of our electron system after the quench, $|\Psi(t)\rangle$. To do that, we exploit the fact that our Hamiltonian is non-interacting, and – the initial state of the system being a Slater determinant – the state $|\Psi(t)\rangle$ can also be expressed as a Slater determinant at any time in terms of the time-evolved single particle wave functions. Alternatively, $|\Psi(t)\rangle$ can be written as

$$|\Psi(t)\rangle = \prod_{m=1}^{M} \tilde{c}^\dagger_{m,t} |0\rangle$$

with the operators $\tilde{c}^\dagger_{m,t}$ creating single particles in the states $\varphi^m(t)$: $\tilde{c}^\dagger_{m,t} |0\rangle \equiv |\varphi^m(t)\rangle$. Here the vectors $\varphi^m(t)$ satisfy the single particle Schrödinger equation

$$i \partial_t \varphi^m(t) = \mathcal{H}(t) \varphi^m(t)$$

with the boundary conditions $\varphi^m_0(t) = \delta^m$.

To solve Eq. (4), we use the adiabatic approach. We introduce the instantaneous eigenvalues of $\mathcal{H}(t)$, $\eta_t^m$, satisfying $\mathcal{H}(t) \eta_t^m = \varepsilon_m(t) \eta_t^m$, and corresponding fermionic creation operators, $|\eta_t^m\rangle \equiv b^\dagger_{m,t} |0\rangle$. We then expand the $\varphi^m(t)$’s in the instantaneous basis as

$$\varphi^m(t) = \sum_k \alpha_k^m(t) \eta_t^k,$$

determine the expansion coefficients by solving the corresponding equation of motion,

$$i \partial_k(t) = \varepsilon_k(t) \alpha_k(t) + \sum_l A_{kl}(t) \alpha_l(t)$$

with $A_{kl} = -i \eta_t^k \cdot \partial_i \eta_t^l$ the Berry connection [31].

Since $\tilde{c}^\dagger_{m,t} = \sum_k \alpha_k^m(t) b^\dagger_{m,t}$, knowledge of the coefficients $\alpha_k^m(t)$ allows us to express the many-body state $|\Psi(t)\rangle$ in the instantaneous basis. Observing further that the many-body Hamiltonian assumes a particularly simple form in the instantaneous basis, $\hat{H}(t) = \sum_m \varepsilon_m(t) \tilde{b}^\dagger_{m,t} \tilde{b}_{m,t}$, we can express the characteristic function of the work distribution as [32]

$$G_t(u) = \langle \langle \Psi(t)| e^{iu(M(t) - E_G(t))} |\Psi(t)\rangle \rangle_{\text{RM}}$$

with the $M \times M$ matrix $g_t(u)$ incorporating all information on the overlap between initial and final states,

$$[g_t(u)]^{nm'} = \sum_k [\alpha_k^m(t)]^* e^{iu \varepsilon_k(t)} \alpha_k^{m'}(t).$$

Eq. (7) thus establishes a connection between the work statistics of the many-body system and the time evolution of individual single particle states [33].

Quantum statistics of work. — To determine the full distribution, $P_t(W)$, we utilized Eqs. (6) and (7). We generated random matrices $\mathcal{H}_1$ and $\mathcal{H}_2$, determined the expansion coefficients $\alpha_k^m(t)$ and the determinant $\det g_t(u)$ numerically, averaged over the random matrix ensemble, and finally determined $P_t(W)$ by taking the Fourier transform of Eq. (7). Our results are summarized in Figs. 2 and 3.

The function $P_t(W)$ can be disentangled into an adiabatic (ground state) and a regular part as

$$P_t(W) = P_{QS}(t) \delta(W) + P_{reg}(W; t).$$

These functions depend not only on time, but also on the velocity $v$, by which we drag the Hamiltonian through the random matrix manifold, on the position of the Fermi level, and on the symmetry class, too. However, once expressed in terms of appropriate dimensionless quantities, $\tilde{W} = W/\Delta$, $\tilde{t} = t \Delta$, and $\tilde{v} = v/\Delta$, measured in units of the average single particle level spacing $\Delta$ at the Fermi energy, they become universal functions in the limit $N \rightarrow \infty$. $P_{reg} \rightarrow P_{reg}(\tilde{W}; \tilde{t})$ and $P_{QS} \rightarrow P_{QS}(\tilde{W})$, displayed in Figs. 2 and 3 for the orthogonal ensemble. They depend implicitly on $\tilde{v}$ but, supposedly, they are independent of all microscopic details (see also the Supplemental Material [32]), and depend just on the symmetry of the underlying Hamiltonian. Remarkably, these functions simplify even further in the slow quench limit, $\tilde{v} \lesssim 1$, and become functions only of the average work, $\langle \tilde{W} \rangle$, rather than time and velocity. This is demonstrated for the probability of adiabatic transitions in Fig. 3 within the orthogonal ensemble.

The functions $P_{reg}(\tilde{W}; \tilde{t})$ show a rather slow evolution towards a Gaussian distribution, as we increase ($\tilde{W}$). As closer numerical investigation reveals,
δ\(\tilde{W}^2 \sim \tilde{t}^{3/2} \sim \tilde{W}^{3/2}\). The behavior of \(P_{GS}(\tilde{t})\) is also somewhat unexpected: the probability of an adiabatic transition is found to decay as a stretched exponential, \(P_{GS}(\tilde{t}) \sim \tilde{t}^{1/4} e^{-C\sqrt{\tilde{t}}}\). As we discuss and demonstrate below, both are particular features of a symmetrical exclusion process which governs the dynamics in energy space.

Energy space diffusion and average work.— Fig. 3 shows the average of the occupation of each level \(f_k(t) = \langle (\tilde{n}_{k,t}) \rangle_{RM}\) for \(k = 1\). The occupation profile exhibits a clear diffusive character, and is very precisely described by a diffusively broadened Fermi-sea, \(f_k(t) \approx \left[ 1 - \text{erf} \left( \Delta k / (4D\tilde{t})^{1/2} \right) \right]/2\), where \(\Delta k = k - M\) is the distance of level \(k\) from the Fermi energy, and \(D\) denotes a dimensionless diffusion constant in energy space. The diffusive \(\sim \sqrt{\tilde{t}}\) broadening of the Fermi surface immediately implies a linear internal energy absorption, \(\sim \tilde{t}\),

\[
\langle \tilde{W}(t) \rangle \approx D(\tilde{v}) \tilde{t},
\]

as clearly demonstrated in the inset of Fig. 3. The diffusion constant \(D\) can thus be interpreted as an overall dimensionless energy absorption rate.

The constant \(D\) depends on the universality class of the system as well as on the velocity, \(\tilde{v}\). We can distinguish two distinct regimes: in the “fast limit”, \(\tilde{v} \gg 1\), electron-hole transitions between remote levels dominate energy absorption, and we find \(D(\tilde{v}) \sim \tilde{v}^2 \sim \omega^2\), as expected in metals. However, in the “slow limit”, \(\tilde{v} \ll 1\), nearest neighbor transitions mediated by Landau-Zener transitions dominate. The statistics of these latter have been studied thoroughly \cite{29, 35, 36}, and it has been shown that the parameters of Landau-Zener transitions display universal distributions (see also Supplementary Material \cite{32} for details). A simple calculation making use of this universal statistics then yields an energy absorption \(D(\tilde{v}) \sim \tilde{v}^{1+\beta/2} \sim |\omega|^{1+\beta/2}\) for \(\tilde{v} \ll 1\), as indeed confirmed by our detailed numerical simulations (see Ref. \cite{32}).

Markovian simulation and symmetrical exclusion process.— In the slow limit, \(\tilde{v} \ll 1\), dominated by
Landau-Zener transitions, most features of the work statistics can be understood in terms of a simple, classical model, the symmetrical exclusion process (SEP) in energy space. In this approach, we consider the occupation of each level as a classical statistical variable, taking values \( n_{k,t} = 1 \) and 0, and think of Landau-Zener transitions as random, Markovian events, transferring particles between neighboring levels with some probability \( p_{LZ} \). The probabilities \( P_{\{n_k\}} \) can then be obtained by performing Monte Carlo simulations, and then can be used to determine the work statistics as

\[
P_t(W) = \left\langle \sum_{\{n_k\}} P_{\{n_k\}} \delta \left[ W - (E_{\{n_k\}}(t) - E_{GS}(t)) \right] \right\rangle_{\text{RM}}
\]

with \( E_{\{n_k\}}(t) = \sum_k \varepsilon_k(t)n_k(t) \), and the random matrix average performed only on the final eigenenergies, \( \varepsilon_k(t) \).

As shown in Figs. 2, 3, and 4, apart from the very short time behavior, where quantum mechanics rules, the simple SEP model reproduces the results of our fully quantum mechanical computations with amazing accuracy for slow quenches. This result now allows us to use SEP computations to obtain predictions for the work distribution function in the regime of large injected work, \( \tilde{W} \gtrsim 20 \), inaccessible to our quantum mechanical simulations. These results are shown in the rightmost panel of Fig. 2.

Mean field description. — The SEP model thus provides an accurate description of energy absorption for the most interesting, universal regime, \( \tilde{v} \lesssim 1 \), but provides limited analytical understanding. We can construct, however, a simple mean field theory of work distribution, by assuming that the occupations \( n_k \) are classical binary variables with expectation values \( f_k(t) = \langle n_k \rangle \), and that they are independent – apart from overall particle number conservation. This simple mean field theory incorporates three important ingredients: Pauli principle, particle number conservation, and the diffusive character of Fermi surface broadening. With this approach, we obtain, e.g., the asymptotic estimate,

\[
P_{GS}(t) = (8\pi D\tilde{t})^{1/4} e^{-C\sqrt{D\tilde{t}}},
\]

yielding an excellent fit for \( \langle \tilde{W} \rangle > 1 \), as shown in Fig. 3.

As discussed in Ref. [22], the scaling \( \delta \tilde{W}^2 \sim \tilde{t}^{3/2} \) can also be easily understood in terms of this approach.

Conclusions. — We have derived and used a determinant formula to compute the distribution of work, \( P_t(W) \), in course of a quantum quench in generic, disordered, fermionic nano-systems, and have shown that it displays a large degree of universality, especially in the slow quench limit. Even if a complete experimental characterization of \( P_t(W) \) may still be challenging, many of our curious findings such as the diffusive broadening of the Fermi energy, the \( \sim \tilde{t}^{3/2} \) scaling of the variance of the energy absorbed, or the low frequency \( \sim \omega^{1+\beta/2} \) absorption rate, could be readily verified experimentally.

Our results also demonstrate that quantum work statistics in these systems is, after all, to a large extent classical. In the most interesting slow quench limit, we demonstrate a close connection to the symmetrical exclusion process (SEP), a classical diffusion of hard core particles in energy space. Quantum mechanics enters here through level collisions, giving rise to Landau-Zener transitions, the exclusion process, mirroring the Pauli principle and, finally, level statistics, reflecting the symmetry of the underlying quantum mechanical system. Our investigations represent only a first step. Inclusion of interactions, generalizations to the symplectic universality class as well as to finite temperatures are all exciting open questions for future research.

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SUPPLEMENTARY MATERIAL

Statistics of avoided level crossing

At small velocities, \( \tilde{v} \lesssim 1 \), Landau-Zener transitions dominate the energy absorption process. The statistical properties of these transitions are universal \(^1\) and \(^2\) and depend on the particular universality class considered. In practice, we consider motions within the random matrix manifold along a ‘circle’,

\[
H(\lambda) = H_1 \cos \lambda + H_2 \sin \lambda,
\]

and investigate the separation of neighboring levels, \( \Delta(\lambda) \equiv \varepsilon_{k+1}(\lambda) - \varepsilon_{k}(\lambda) \). This function displays well-defined minima, \( \Delta_{\text{min}} \), at positions \( \lambda_0 \), close to which the level separation can be well-described in terms of an effective two-level Hamiltonian as

\[
\Delta(\lambda) \approx \sqrt{\Delta_{\text{min}}^2 + \gamma^2(\lambda - \lambda_0)^2}. \tag{14}
\]

The number of level crossings occurring between two selected neighboring levels is directly proportional to the distance covered within the random matrix manifold, \( N_{\text{cross}} \sim s_{\text{tot}} \sim \sqrt{N} \, d\lambda \). Therefore, we can define the probability distribution of \( \Delta_{\text{min}} \) as

\[
dN_{\text{cross}} = \rho(\Delta_{\text{min}}) \, ds \, d\Delta_{\text{min}}. \tag{15}
\]

Here the density function \( \rho(\Delta_{\text{min}}) \) depends implicitly on the size of the random matrices, \( N \), as well as on the universality class considered \(^3\). The dependence on \( N \), however, appears just in a trivial way in the large \( N \) limit, through the level spacing \( \Delta \) at the energy of the Landau-Zener transitions. This dependence can be scaled out, yielding a universal distribution function,

\[
\hat{\rho}(\tilde{\Delta}_{\text{min}}) \equiv \tilde{\Delta} \rho(\Delta_{\text{min}}), \tag{16}
\]

with \( \tilde{\Delta}_{\text{min}} = \Delta_{\text{min}}/\Delta \) denoting the dimensionless Landau-Zener gap.

Similarly, the distribution of the slopes \( \gamma \) is also universal, provided we measure it in its natural unit,

\[
\tilde{\gamma} = \gamma \sqrt{N}.
\]

The numerically computed statistics of the distributions \( \hat{\rho}(\Delta_{\text{min}}) \) and \( \hat{\rho}(\tilde{\gamma}) \) are displayed for the Gaussian orthogonal ensemble (GOE, \( \beta = 1 \)) and Gaussian unitary ensemble (GUE, \( \beta = 2 \)) in Fig. 4. The numerically extracted distributions fit perfectly the analytical predictions of Ref. \(^1\). In particular, the distribution of the slopes has a peaked structure, i.e., there exist a typical value of \( \tilde{\gamma} \), characterizing the transitions.

The distribution \( \hat{\rho}(\Delta_{\text{min}}) \) is, on the other hand, more peculiar. In particular, \( \hat{\rho}(\Delta_{\text{min}}) \sim \Delta_{\text{min}}^{\beta-1} \), and scales as

\[
\hat{\rho}(\Delta_{\text{min}} \ll 1) \approx \begin{cases} 
\frac{\pi^{3/2}}{\sqrt{18}} & \text{for } \beta = 1, \\
\frac{2}{\pi^{3/2}} \frac{\Delta_{\text{min}}}{3} & \text{for } \beta = 2,
\end{cases}
\]

in the orthogonal and unitary universality classes, investigated here in detail \(^4\). This curious behavior implies that avoided level crossings with very small gap are abundant in the orthogonal class, which leads to a breakdown of adiabatic perturbation theory for \( \beta = 1 \).

The probability \( p_{\text{LZ}} \) of a Landau-Zener transition between two neighboring levels depends on the velocity at which the transition is approached, and is known analytically \(^3\), \(^5\),

\[
p_{\text{LZ}} = \exp \left( -\frac{\pi^2}{2} \frac{\Delta_{\text{min}}^2}{\tilde{v} \, \tilde{\gamma}} \right). \tag{17}
\]

Given the exponential sensitivity to \( \Delta_{\text{min}} \), small Landau-Zener gap transitions dominate in the limit of small velocities, \( \tilde{v} \ll 1 \). This leads to the estimate

\[
\tilde{D} \sim \langle d t N_{\text{cross}} \rangle p_{\text{LZ}} = \tilde{v} \int \tilde{\Delta}_{\text{min}} \hat{\rho}_{\beta}(\tilde{\Delta}_{\text{min}}) \langle p_{\text{LZ}} \rangle_{\gamma} \\
\sim \tilde{v}^{1+\beta/2}.
\]

The universal scaling of the many-body diffusion constant, as extracted from our simulations, is plotted in Fig. 5. For time reversal symmetry breaking (unitary) systems, the diffusion constant scales both at small and high velocities as \( \tilde{v}^2 \). In the orthogonal case, however, a clear crossover is demonstrated between a Landau-Zener dominated small velocity regime with \( \tilde{D} \sim \tilde{v}^{3/2} \), and a high velocity fast quench regime with \( \tilde{D} \sim \tilde{v}^2 \).

Time evolution of single particle states

To solve Eq. \(^6\), it is practical to make a simple gauge transformation, and eliminate the phase dependence gen-
We can now evaluate the expectation values using Wick’s theorem as

\[\langle 0 | \hat{b}_{k_1}^d \hat{b}_{k_2}^d \cdots \hat{b}_{k_M}^d | \hat{b}_{k_1}^\dagger \cdots \hat{b}_{k_M}^\dagger | 0 \rangle = \sum_p (-1)^p \delta_{\{k_1, k_2, \ldots, k_M\}, \{P k_1', P k_2', \ldots, P k_M'\}}.\]

In our numerics, we made use of the fourth order Runge-Kutta method to solve the single particle time-dependent Schrödinger equation. Dynamical phases in Eq. [19] were determined numerically using the Simpson-formula. Since the phases of the instantaneous states often make jumps, we enforced the choice \( A_{kk} = -i \eta_k^k : \partial_t \eta_k^k = 0 \) by requiring that the overlaps of two consecutive eigenstates remain close to 1, \( \langle \eta_k^k | \eta_{k+\delta t}^k \rangle \approx 1 \).

**Derivation of determinant formula**

In this subsection, we outline some of the steps leading to the determinant formula, Eq. [22]. We first use the linear relation between the operators \( \hat{c}_{m,t} \) and \( \hat{b}_{m,t} \) associated with the time evolved and instantaneous basis states, respectively, and rewrite \( \langle \Psi(t) | e^{iu\mathcal{H}(t)} | \Psi(t) \rangle \) as

\[\langle \Psi(t) | e^{iu\mathcal{H}(t)} | \Psi(t) \rangle = \sum_{\{k_1, k_2, \ldots, k_M\}}^{N=10, E=0} \sum_{\{k_1, k_2, \ldots, k_M\}}^{N=20, E=0.5} \sum_{\{k_1, k_2, \ldots, k_M\}}^{N=50, E=-1} e^{iu \sum_{m=1}^{M} \varepsilon_{k_m}(t) \prod_{m=1}^{M} \alpha_{km}^m(t) \left[ \alpha_{km}^m(t) \right]^\ast \langle 0 | \hat{b}_{k_1}^d \hat{b}_{k_2}^d \cdots \hat{b}_{k_M}^d | \hat{b}_{k_1}^\dagger \cdots \hat{b}_{k_M}^\dagger | 0 \rangle.\]
FIG. 7. Comparison of the regular part of the work statistics, $P_{\text{reg}}(\tilde{W}; \tilde{t})$ for the orthogonal (GOE, on the left) and the unitary matrix ensembles (GUE, on the right), with $\tilde{W}$ and $\tilde{t}$ referring to dimensionless work and time. Symbols: regular part for different values of the dimensionless average work, $\langle \tilde{W} \rangle = 1.5, 3, 5$, and a dimensionless quench velocity, $\tilde{v} = 0.8$, for two different matrix sizes, $N = 20$ and $N = 50$. The distributions show universal behavior and collapse into a single curve independent of the system size, $N$. Arrows indicate the positions of the first neighboring levels at $\tilde{W} = 1, 2, \ldots$. Solid lines: classical SEP simulations provide an excellent approximation for $\tilde{v} < 1$. In this limit, $P_{\text{reg}}(\tilde{W}; \tilde{t})$ depends only the average dimensionless work, $\langle \tilde{W} \rangle$, for both ensembles.

Fingerprints of level repulsion and deviations from universality.

As discussed in the main text, the distribution of the work is universal, i.e., it is independent from the system size $N$ as well as the Fermi energy; it depends solely on the dimensionless time, the dimensionless velocity, and the symmetry class of the random Hamiltonians. This is illustrated in Fig. 7 where we have fixed the dimensionless quench velocity to $\tilde{v} = 0.8$, and the dimensionless times such that they correspond to an injected dimensionless work $\langle \tilde{W} \rangle = 1.5, 3, 5$ in both symmetry classes.

For relatively small injected internal energies, $W \lesssim 5 \Delta$, the discreteness of the energy levels becomes apparent, and fingerprints of level repulsion can be observed. First, $P(\tilde{W})$ vanishes continuously at zero (within SEP as $\tilde{W}^2$), since the first empty level is repelled from the last occupied level. Moreover, additional wiggles appear in $P(\tilde{W})$, reflecting the positions of first, second and third neighbors. These features are more pronounced in the Gaussian unitary ensemble, where level repulsions are stronger, and are expected to become even more pronounced for the symplectic ensemble, not studied here. The observed distributions are, however, independent of the matrix size, $N$, as stated above.

Remarkably, even for $\tilde{v} = 0.8$, SEP simulations (continuous lines in Fig. 7) provide an almost perfect description of the full quantum results. For larger velocities, however, deviations occur. This is demonstrated in Fig. 8. Clearly, for $\tilde{v} \gtrsim 1$ the role of Landau-Zener transitions is reduced, the SEP description loses its validity, and distributions depend explicitly on the velocity $\tilde{v}$.

Mean field approach

Probability of adiabaticity

Within the mean field approach, we consider each occupation number $n_k$ as a binary probability variable, having values $n_k = 0$ and 1 with weights

$$p_{k,\pm}(n_k) = n_k f_k(t) + (1 - n_k)(1 - f_k(t)),$$

but obeying a global constraint, $\sum_k n_k = M$. For simplicity here we focus on the half-filled case $M = N/2$, though calculations can easily be extended to any value of $M$. We enforce the constraint by inserting a Kronecker
\( P \left( \{ n_k \} \right) = \frac{1}{N!} \prod_{k=1}^{N} p_{k,t}(n_k) \delta_{N/2=\sum n_k} \)
\[ = \frac{1}{N!} \int_{-\pi}^{\pi} \frac{d\lambda}{2\pi} e^{i\lambda \sum_{k=1}^{N} (n_k-1/2)} \prod_{k=1}^{N} p_{k,t}(n_k). \]

Here \( N_t \) is a time dependent normalization factor, which can be estimated by first carrying out the summation over \( \{ n_k \} \), and then applying the saddle point approximation as

\[ N_t \approx \int_{-\pi}^{\pi} \frac{d\lambda}{2\pi} \left[ \cos^2(\lambda/2) + \sin^2(\lambda/2) \text{erf}^2 \left( \frac{j}{\sqrt{4Dt}} \right) \right] \approx \left( 8\pi \tilde{D}t \right)^{-1/4}. \]

The probability of staying in the ground state then reads

\[ P_{\text{GS}}(t) = \frac{1}{N!} \prod_{k=1}^{N/2} f_k(t) \prod_{k=N/2+1}^{N} (1 - f_k(t)) \approx \frac{1}{N!} e^{2\sqrt{4Dt} \int_0^\infty dx x \log(1+\text{erf}(x))/2] = \left( 8\pi \tilde{D}t \right)^{1/4} e^{-C\sqrt{\tilde{D}t}}, \]

where, again, an integral approximation has been made by assuming \( \tilde{D}t \approx \tilde{W} \gg 1 \), and a saddle point approximation has been carried out.

**Variance of work**

For \( \tilde{W} \gg 1 \), we can estimate the variance of the work by neglecting the fluctuations of the individual energy levels, \( \varepsilon_k(t) \rightarrow \Delta k \) \cite{1}, while keeping track of the fluctuations of the occupation numbers. For a given realization of \( \mathcal{H}(t) \), this yields the approximate expression

\[ \delta \tilde{W}^2(t) \approx \left( \left( \sum_{k=1}^{N} \Delta k \tilde{n}_{k,t} \right)^2 \right) - \left( \sum_{k=1}^{N} \Delta k \tilde{n}_{k,t} \right)^2, \]

the average signs denoting here just quantum averages. By separating the ‘diagonal’ contributions, the average \( \langle \delta \tilde{W}^2 \rangle \) be rewritten as

\[ \langle \delta \tilde{W}^2(t) \rangle \approx \sum_k \Delta k \langle \langle \delta \tilde{n}_{k,t}^2 \rangle \rangle + \sum_{k \neq k'} \Delta k \Delta k' \langle \langle \delta \tilde{n}_{k,t} \delta \tilde{n}_{k',t} \rangle \rangle, \]

with \( \delta \tilde{n}_{k,t} \equiv \tilde{n}_{k,t} - \langle \tilde{n}_{k,t} \rangle \) denoting the deviation of the level occupation number. Since the \( \tilde{n}_{k,t} \) behave as binary variables, averages in the first term can be expressed as \( \langle \langle \delta \tilde{n}_{k,t}^2 \rangle \rangle = f_k(t) (1 - f_k(t)) \), with \( f_k(t) \approx (1 - \text{erf}(\Delta k/\sqrt{4Dt}))/2 \). The correlators appearing in this equation can be expressed in terms of the amplitudes \( \alpha^m(t) \) as \( \langle \Delta \tilde{n}_{k,t} \delta \tilde{n}_{k',t} \rangle = -\sum_{m=1}^{N/2} \alpha^m_k(t) \alpha^m_{k'}(t) \). Notice that this correction is negative, indicating that the level occupations are anticorrelated, as dictated by particle number conservation.

By neglecting for a moment these correlations and replacing sums by integrals, we obtain the estimate

\[ \langle \delta \tilde{W}^2(t) \rangle \approx \int_{-\infty}^{\infty} dx x^2 \frac{1 - \text{erf}^2(x/\sqrt{4\tilde{D}t})}{4} \sim \tilde{W}^{3/2}, \]

implying that \( \langle \delta \tilde{W}^2(t) \rangle \sim \langle \tilde{W} \rangle^{3/2} \). While neglecting the correlations explains the observed behavior of the work’s variance, it does not reproduce the correct prefactor. Indeed, a more careful mean field calculation along the lines of the previous subsection shows that correlations (the conservation of fermion number) cannot be entirely neglected, and they also give a similar but smaller contribution to the variance – without changing the overall scaling.

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[3] More generally, the density of avoided level crossings between any two neighboring levels depends also on the energy of the colliding levels, \( \varepsilon \). By concentrating on the middle of the spectrum, here we restrict our analysis to energy \( \varepsilon = 0 \).
[4] Our convention differs from the one used in Ref. \cite{1}, where \( \Delta \) is measured in units \( 1/N \equiv \Sigma_n \pi, \) leading to a factor of \( \pi \) difference in the density \( \rho_\pi \).
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