Background field quantization and non-commutative Maxwell theory

Ashok Das\textsuperscript{a}, J. Frenkel\textsuperscript{b}, S. H. Pereira\textsuperscript{b} and J. C. Taylor\textsuperscript{c}

\textsuperscript{a} Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627-0171 USA

\textsuperscript{b} Instituto de Física, Universidade de São Paulo, São Paulo, SP 05315-970, BRAZIL

\textsuperscript{c} Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, UK

Abstract

We quantize non-commutative Maxwell theory canonically in the background field gauge for weak and slowly varying background fields. We determine the complete basis for expansion under such an approximation. As an application, we derive the Wigner function which determines the leading order high temperature behavior of the perturbative amplitudes of non-commutative Maxwell theory. To leading order, we also give a closed form expression for the distribution function for the non-commutative $U(1)$ gauge theory at high temperature.
1 Introduction

Background field techniques are quite useful in the study of conventional non-Abelian gauge theories [1, 2]. In particular, they simplify calculations enormously, since in a background field gauge, invariance under background gauge transformations is manifest. On the other hand, canonical quantization of non-Abelian gauge theories within such a framework is highly nontrivial [3]. The difficulty comes from the fact that a complete basis for the field equation of the quantum gauge field is not easy to determine in general. The only known example of a successful canonical quantization is for the case where the background field strength is a constant [3]. However, there are many physical situations where the background field can be considered to be weak and the variations in the background field less rapid than the variations in the quantum field. In such a case, we show that canonical quantization in the background field gauge can be carried out and, in fact, we demonstrate this within the context of non-commutative Maxwell theory. (The quantization goes through unchanged even in the presence of fermion fields.)

The high temperature behavior of a plasma [4, 5, 6, 7] constitutes an example where the external field is assumed to be weak with slow variations compared with the quantum fields. Namely, in the hard thermal loop approximation, it is normally assumed that

\[ p \ll k \sim T, \]

where \( p \) represents a typical external momentum while \( k \) denotes an internal loop momentum. Our method finds a natural application in the study of such systems and we derive the Wigner function for non-commutative photons which, in turn, determines the leading high temperature behavior of amplitudes in this theory. In an earlier work [8], we had studied this question through the use of Wigner function (without the use of the background field method), where we had noted some peculiarity of noncovariance of the results under a gauge transformation and had argued for a covariant completion of a particular form. In the present approach, we show that the covariance of the results is manifest and that the particular covariantization found earlier, results from the proper quantization in the background field gauge. This, therefore, clarifies the meaning of the covariant completion found in [8]. When working within the framework of background field method, an alternate and simpler definition of Wigner function (to the leading order) is possible. Using this, we are also able to determine the leading order distribution function in a closed form at one loop.

The organization of our results is as follows. In section 2, we briefly recapitulate the basics of the background field method. Considering the leading order behavior of the equation of motion as well as the background gauge condition, in section 3 we determine a complete basis for the covariant D’Alembertian operator in this approximation which allows for an expansion of the quantum gauge field. As an application of our method, in section 4 we introduce an alternate, simpler definition of the Wigner function which describes the leading order behavior of the amplitudes through a transport equation and show that the calculations are manifestly covariant and agree with the perturbative results. Furthermore, using this Wigner function, we determine a closed form expression for the leading order distribution function at the one loop level.

2 Background field method in non-commutative Maxwell theory

In this section, we review very briefly the basics of background field method within the context of non-commutative Maxwell theory. For our conventions, notations and the definitions of star product
etc, we refer the reader to \[8\] as well as the vast literature on the subject of non-commutative field theories \[9, 10\].

Non-commutative $U(1)$ gauge theory (Maxwell theory) is described by the action

$$S[A] = \int d^4x \left( -\frac{1}{4} F_{\mu\nu} \ast F^{\mu\nu} \right),$$  \hspace{1cm} (2)

where

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} - ie [A_{\mu}, A_{\nu}].$$  \hspace{1cm} (3)

Here, the commutator stands for the Moyal bracket of the fields. We now make the expansion

$$A_{\mu} = \bar{A}_{\mu} + a_{\mu}, \quad \langle a_{\mu} \rangle = 0,$$  \hspace{1cm} (4)

where $\bar{A}_{\mu}$ is the background field satisfying

$$F_{\mu\nu} = \bar{F}_{\mu\nu} + \bar{D}_{\mu} a_{\nu} - \bar{D}_{\nu} a_{\mu} - ie [\bar{A}_{\mu}, \bar{F}_{\mu\nu}] = 0,$$  \hspace{1cm} (5)

$a_{\mu}$ is the quantum field and $\langle a_{\mu} \rangle$ denotes the expectation value of the quantum field in any given state. Then, it follows that

$$F_{\mu\nu} = \bar{F}_{\mu\nu} + \bar{D}_{\mu} a_{\nu} - \bar{D}_{\nu} a_{\mu} - ie [a_{\mu}, a_{\nu}].$$  \hspace{1cm} (6)

In such a case, the action (2) can be expanded as

$$S[\bar{A} + a] = \int d^4x \left[ -\frac{1}{4} \bar{F}_{\mu\nu} \ast \bar{F}^{\mu\nu} - \frac{1}{2} (\bar{D}^{\mu} a^{\nu} - \bar{D}^{\nu} a^{\mu}) \ast \bar{D}_{\mu} a_{\nu} + ie \bar{F}_{\mu\nu} \ast a_{\mu} \ast a_{\nu} \right. \left. + ie (\bar{D}^{\mu} a^{\nu} - \bar{D}^{\nu} a^{\mu}) \ast a_{\mu} \ast a_{\nu} + \frac{e^2}{2} [a^{\mu}, a^{\nu}] \ast a_{\mu} \ast a_{\nu} \right],$$  \hspace{1cm} (7)

where linear terms in the quantum field do not occur by virtue of the equations of motion (5) for the background field.

The advantage of the background field method lies in the fact that the original gauge invariance of the theory under

$$\delta A_{\mu}(x) = \partial_{\mu} \epsilon(x) - ie [A_{\mu}(x), \epsilon(x)],$$  \hspace{1cm} (8)

can be viewed in one of two ways. First, this can be thought of as a quantum gauge invariance under the transformations,

$$\delta \bar{A}_{\mu}(x) = 0,$$
$$\delta a_{\mu}(x) = \partial_{\mu} \epsilon(x) - ie [\bar{A}_{\mu}(x) + a_{\mu}(x), \epsilon(x)],$$  \hspace{1cm} (9)

or as a background gauge invariance under

$$\delta \bar{A}_{\mu}(x) = \partial_{\mu} \epsilon(x) - ie [\bar{A}_{\mu}(x), \epsilon(x)],$$
$$\delta a_{\mu}(x) = -ie [a_{\mu}(x), \epsilon(x)].$$  \hspace{1cm} (10)

Namely, under a quantum gauge transformation, the background field is inert while the quantum field transforms like a gauge field. On the other hand, under a background gauge transformation,
the background field transforms like a gauge field while the quantum field transforms in the adjoint representation. It is, therefore, possible to take advantage of this and add the gauge fixing and the ghost actions

\[ S_{\text{GF}} + S_{\text{ghost}} = \int d^4x \left[ \frac{1}{2\xi} (\bar{D} \cdot a) \star (\bar{D} \cdot a) + \bar{D}^\mu \bar{c} \star (\partial_\mu c - ie [\bar{A}_\mu + a_\mu, c]) \right], \]  

(11)

which break the quantum gauge invariance \[ 9], but are invariant under the background gauge transformation \[ 10] with ghosts transforming in the adjoint representation. As a result, calculations carried out with such a background gauge fixing will lead to results that are manifestly invariant under background gauge transformations.

The part of the total action quadratic in the quantum fields is responsible for the one loop results and has the form

\[ S_q = \int d^4x \left[ -\frac{1}{2} (\bar{D}^\nu a^\nu - \bar{D}^\nu a^\nu) \star \bar{D}_\mu a_\nu + ie \bar{F}^{\mu\nu} \star a_\mu \star a_\nu - \frac{1}{2\xi} (\bar{D} \cdot a) \star (\bar{D} \cdot a) + \bar{D}^\mu \bar{c} \star \bar{D}_\mu c \right]. \]  

(12)

Here \( \xi \) represents the gauge fixing parameter and in the limit \( \xi \to 0 \) we have the background field gauge condition,

\[ \bar{D} \cdot a = \bar{D}_\mu a^\mu = \partial_\mu a^\mu - ie [\bar{A}_\mu, a^\mu] = 0. \]  

(13)

In this gauge, the equation of motion for the quantum field (at the one loop level) follows from (12) to be

\[ \bar{D}_\mu \bar{D}^\nu a^\mu = 2ie [\bar{F}^{\mu\nu}, a_\nu]. \]  

(14)

It can be easily checked that (13) and (14) are compatible.

### 3 Quantization

To quantize the gauge field, we have to determine a basis satisfying both (13) and (14). This is, in general, a very hard problem and a solution in a factorizable form may not always exist. We, therefore, look for a solution of these equations in the approximation that the background fields are weak and slowly varying compared to the quantum fields. As we have pointed out in the introduction, there are various physical phenomena of interest that satisfy such conditions and in the next section we will have an application to such a physical situation. With these assumptions, Eq. (13) reduces in the leading order to

\[ \bar{D}^2 a^\mu = \bar{D}_\nu \bar{D}^\nu a^\mu = 0. \]  

(15)

Thus, in this approximation, it is essential to determine a basis for the covariant D’Alembertian operator in order to quantize the gauge field. Furthermore, since \( a_\mu \) transforms covariantly under a background gauge transformation \[ 10], the basis function must reflect this. We know that the plane waves \( e^{ik \cdot x} \) represent a basis for the D’Alembertian operator in the absence of any background gauge fields. Correspondingly, let us denote the basis in the presence of background gauge fields as \( e^{ik \cdot X} \).
To determine this basis, let us define (see also [8])
\[ \tilde{A}_\mu = \bar{A}_\mu + \frac{1}{k \cdot D} \vec{F}_{\mu\nu} k^\nu = \frac{1}{k \cdot D} \partial_\mu (k \cdot \tilde{A}). \] (16)

It is clear that this transforms like a background gauge field under (10). We note from (16) that, in general,
\[ k \cdot \tilde{A} = k \cdot \bar{A}. \] (17)

Furthermore, we see from (16) that in the gauge
\[ k \cdot \bar{A} = 0, \] (18)
\[ \tilde{A}_\mu \] vanishes so that it must be a pure gauge field satisfying
\[ k \cdot \tilde{A} = \frac{i}{e} \Omega^{-1} \ast k \cdot \partial \Omega. \] (19)

This can also be equivalently written as
\[ k \cdot \partial \Omega + ie\Omega \ast k \cdot \tilde{A} = 0, \]
\[ k \cdot \partial \Omega^{-1} - iek \cdot \tilde{A} \ast \Omega^{-1} = 0. \] (20)

The solution of (19) (or equivalently of (20)) can be determined easily in terms of link operators and has the form
\[ \Omega(x) = U^{(\tilde{A})}(-\infty, x) = P \left( e^{-ie \int_{-\infty}^{0} du k \cdot \tilde{A}(x+ku)} \right), \] (21)

where the integration is along a straight path parallel to \( k^\mu \) from \(-\infty\) to \( x^\mu \). For simplicity, we have taken the reference point to be at \(-\infty\) although this is not necessary. We note here that open Wilson lines of the form in (21) play an important role in non-commutative gauge theories from various points of view at zero temperature [9, 10, 11] (including in the construction of gauge invariant observables [12]) as well as at finite temperature in the construction of effective actions [13]. From the form of the solution in (21) as well as the properties of star products, it immediately follows that
\[ \Omega^{-1}(x) \ast e^{ik \cdot x} \ast \Omega(x) = U^{(\tilde{A})}(x, -\infty) \ast U^{(\tilde{A})}(-\infty, x + \theta k) \ast e^{ik \cdot x} = U^{(\tilde{A})}(x, x + \theta k) \ast e^{ik \cdot x}, \] (22)

where we have introduced the notation
\[ (\theta k)^\mu = \theta^{\mu\nu} k_\nu, \] (23)
and used the property
\[ e^{ik \cdot x} \ast f(x) \ast e^{-ik \cdot x} = f(x + \theta k). \] (24)

From the transformation properties of the link operators, we see that the combination of factors on the left hand side of (22) transforms covariantly under (11).

From the definition in (19) (or (20)) as well as the fact that \( k^2 = 0 \), it follows that
\[ k \cdot D \left( \Omega^{-1}(x) \ast e^{ik \cdot x} \ast \Omega(x) \right) = 0 = k \cdot D \left( U^{(\tilde{A})}(x, x + \theta k) \ast e^{ik \cdot x} \right). \] (25)
With this, we now see that, to leading order in our approximation,

\[ \bar{D}^2 \left( U(\bar{A})(x, x + \theta k) \star e^{ik \cdot x} \right) = i k \cdot \bar{D} \left( U(\bar{A})(x, x + \theta k) \star e^{ik \cdot x} \right) = 0, \] (26)

where we have neglected the term where the covariant derivative acts on the link operator since the momentum of the background field is sub-leading compared to \( k \). Thus, we see that to leading order in our approximation, a basis for the covariant D’Alembertian operator can be written as

\[ f_k(x) = e^{i k \cdot X} = U(\bar{A})(x, x + \theta k) \star e^{ik \cdot x}. \] (27)

This transforms covariantly under (10) and reduces to ordinary plane waves when \( \bar{A} = 0 \) (and, therefore, \( \bar{A} = 0 \) or when \( \theta^{\mu\nu} = 0 \)). We also note that following the derivation in [11] (where an integrated form of the relation is obtained), it is easy to show that

\[ U(\bar{A})(x, x + \theta k) \star e^{ik \cdot x} = e^{i(k \cdot x + ek \times \bar{A}(x))}, \] (28)

where we have used the notation standard in non-commutative field theories,

\[ A \times B = \theta^{\mu\nu} A_{\mu} B_{\nu}. \] (29)

Equation (28) represents precisely the covariantization factor determined earlier in [8] from different considerations and this derivation clarifies the origin of such a factor, showing that we can think of \( X^\mu \) (see (27)) as the appropriate covariant coordinate for the problem at hand.

The basis for the covariant D’Alembertian operator allows us to make an expansion of a covariant (from the non-commutative \( U(1) \) point of view) scalar field. However, the expansion of a gauge field must, in addition, satisfy (13). To that end, we note that in the leading order of our approximation, Eq. (13) can be written as

\[ \partial_\mu a_\mu(x) = 0, \] (30)

which is the Landau gauge. Correspondingly, expansion in terms of the usual transverse polarization vectors is sufficient to satisfy the gauge condition in the leading order. We would like to emphasize that, in general, the polarization vector can have sub-leading terms that are not necessarily factorizable (which has been checked to lowest orders) which is another reason why quantization in the background field method is highly nontrivial in general. With all this, we can now expand the quantum fields (in the leading order of our approximation) as

\[ a_\mu(x) = \sum_s \int \frac{d^3k}{(2\pi)^3 \sqrt{2k_0}} \epsilon_\mu(\vec{k}, s) \left( a(\vec{k}, s)e^{-i\vec{k} \cdot X} + a^\dagger(\vec{k}, s)e^{i\vec{k} \cdot X} \right), \] (31)

where we have assumed without loss of generality that the polarization vector is real and

\[ \vec{k}^0 = |\vec{k}|, \quad \vec{k} \cdot \epsilon(\vec{k}, s) = 0. \] (32)

The quantum field can now be quantized and the physical Hilbert space identified in the standard manner [14].
4 Application

As an application of the background field quantization of the previous section, we will now derive the Wigner function \[15\] which determines the leading order high temperature behavior of the amplitudes in non-commutative QED. We note that conventionally the covariant Wigner function for the non-commutative photon is defined as (see \[8\])

\[
W_{\mu\nu}(x, k) = \int \frac{d^4y}{(2\pi)^4} e^{-iy \cdot k} G_{\mu\lambda}^{(+)}(x) \star G_{\nu\sigma}^{(-)}(x),
\]

where

\[
G_{\mu\nu}^{(\pm)}(x) = U^{(A)}(x, x_{\pm}) \star F_{\mu\nu}(x_{\pm}) \star U^{(A)}(x_{\pm}, x), \quad x_{\pm} = x \pm \frac{y}{2}.
\]

On the other hand, within the framework of the background field method an alternate and simpler definition of the covariant Wigner function \[16\], which describes the leading order behavior in the hard thermal loop approximation, is possible and has the form

\[
w_{\mu\nu}(x, k) = \int \frac{d^4y}{(2\pi)^4} e^{-iy \cdot k} G_{\mu}^{(+)}(x) \star G_{\nu}^{(-)}(x),
\]

\[
G_{\mu}^{(\pm)}(x) = U^{(A)}(x, x_{\pm}) \star a_{\mu}(x_{\pm}) \star U^{(A)}(x_{\pm}, x).
\]

There are several things to note from \[35\] and \[36\]. First, the Wigner function in \[35\] transforms covariantly under \[10\] independent of whether the link operator in \[36\] is defined with respect to the complete gauge field \(A_{\mu}\) or with respect to the background gauge field \(\bar{A}_{\mu}\). However, we have defined it with respect to the background field to avoid some problems that arise otherwise in a practical calculation. (Such problems also arise in the conventional definition and need various assumptions on the factorizability of thermal correlation functions. However, a definition such as in \[36\] avoids such assumptions.) Since the Wigner function \[35\] is already quadratic in the quantum fields, at one loop level, the gauge fields in the link operators would factor out of the thermal correlation functions as background fields even if we use the complete gauge field to define the link operators. However, keeping an eye on the potential difficulties that may arise at higher loop level from such terms (if defined with a complete gauge field), we have chosen the particular definition in \[36\]. Second, the Wigner function in \[35\] can be easily seen to be related to the one in \[33\] in the leading hard thermal loop approximation as

\[
\eta_{\mu\nu} \langle w_{\mu\nu}(x, k) \rangle = - \lim_{k^2 \to 0} \frac{\eta_{\mu\nu}}{2k^2} \left( \langle W_{\mu\nu}(x, k) \rangle - \bar{W}_{\mu\nu}(x, k) \right),
\]

where \(\langle .. \rangle\) denotes thermal average.

Following the derivation in \[8\], the transport equation for \(w_{\mu\nu}\) can now be derived. In fact, if we define the distribution function

\[
\mathcal{F}(x, k) = \eta^{\mu\nu} \langle w_{\mu\nu}(x, k) \rangle,
\]

then, it can be easily shown that (this also follows from equation \(33\) derived in \[8\] and the identification in \[37\])

\[
k \cdot \bar{D} \mathcal{F}(x, k) = \frac{e}{2} \frac{\partial}{\partial k_{\sigma}} k^{\rho} \left[ \bar{F}_{\rho\sigma} \star \mathcal{F} + \mathcal{F} \star \bar{F}_{\rho\sigma} - 2 \int \frac{d^4y}{(2\pi)^4} e^{-iy \cdot k} \langle G_{\mu}^{(+)} \star \bar{F}_{\rho\sigma} \star G_{\nu}^{(-)} \rangle \right].
\]

(39)
By iteratively solving the transport equation (39), the distribution function (38) can be determined to any order in the leading approximation. This would, then, determine the current defined as

\[ J_\mu(x) = -e \int d^4k \theta(k^0) \{ k_\mu (F(x,k) - F(x,-k)) \} , \]  

(40)

which, in turn, would yield the leading order amplitudes of the theory through functional differentiation. (By leading order in this context, we refer to the class of terms inside the curly bracket in (40) which, apart from a \( \delta(k^2) \), are functions of zero degree in \( k \). In conventional QCD, this class yields all the dominant contributions, but in noncommutative QED, this may not give the complete contribution for arbitrary values of the noncommutative parameter \( \theta^{\mu\nu} \) as discussed in [13].) The anti-symmetrization in the definition in (40) is to ensure the correct charge conjugation property [17] of the current as discussed in [8].

With the field expansion given in (31), the thermal averages can be calculated in the physical space [5] satisfying

\[ \sum_{s=1,2} \epsilon_\mu(\vec{k},s) \epsilon^\mu(\vec{k},s) = -2 , \]  

(41)

and the current can be determined order by order. It is straightforward to check that this coincides with the results obtained in [8] (which also gives the appropriate perturbative amplitudes). However, unlike the earlier work, here the results are manifestly covariant at any order with the expansion in (31), without any need for covariantization. Furthermore, in this case, the distribution function in (38) can even be obtained in a closed form in the leading order at one loop and has the form

\[ F(x,k) = 2 \int \frac{d^4y}{(2\pi)^4} e^{-iy\cdot k} \int \frac{d^4\bar{k}}{(2\pi)^3} \delta(\bar{k}^2) n_B(|\bar{k}^0|) \times \left\{ \theta(\bar{k}^0) \left( e^{y \cdot \vec{D}/2} e^{i\bar{k} \cdot \vec{X}} \right) \ast \left( e^{\ast y \cdot \vec{D}/2} e^{\ast i\bar{k} \cdot \vec{X}} \right) + (\bar{k} \leftrightarrow -\bar{k}) \right\} , \]  

(42)

where \( n_B(|\bar{k}^0|) \) denotes the Bose-Einstein distribution and the covariant translation (for a covariant function under the non-commutative \( U(1) \)) is explicitly given by

\[ e^{\pm \frac{y \cdot \vec{D}}{2}} f(x) = U^{(A)}(x, x_\pm) \ast f(x_\pm) \ast U^{(A)}(x_\pm, x) . \]  

(43)

Using the transport equation (39), the distribution function in (42) can be systematically expanded order by order in the number of background fields (or powers of \( e \)) and substituted into the definition of the current in (40). We have verified explicitly, up to the three photon amplitude, that this reproduces correctly the perturbative results in the leading order in the hard thermal loop approximation.

### 5 Conclusion

In this work, we have quantized non-commutative \( U(1) \) gauge theory canonically in the background field method using the background field gauge for weak and slowly varying background fields. We have determined a (covariant) basis for the covariant D’Alembertian operator which indeed coincides with the particular covariantization factor determined earlier from different considerations. We
have applied our quantization method to study the high temperature behavior of non-commutative Maxwell theory in the leading order using the Wigner function. The calculations are manifestly covariant and agree with the perturbative results. We have also determined a closed form expression for the distribution function for the photon in the leading approximation at one loop. Although our discussion has been within the context of non-commutative Maxwell theory, this can be generalized to conventional QCD as well. In particular, a basis for the covariant D’Alembertian operator can again be constructed in terms of a pure gauge (background) field $\tilde{A}_\mu$ defined in (16). This can then be used to determine, in principle, the leading order distribution function in QCD. This is an interesting question that deserves further study.

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