Plasma channel dynamics in sub- and microsecond discharges in water

A A Zherlitsyn, A V Kozyrev, N S Semeniuk, S S Kondratiev and V M Alexeenko

Institute of High Current Electronics, 634055, 2/3 Akademichesky Avenue, Tomsk, Russia

kozyrev@to.hcei.tsc.ru

Abstract. Simulation results of a fast electric discharge and a strong acoustic wave in the water is performed. A theoretical model of a high-current plasma channel is presented. The model accounts for the energy ratio between the input electric power and the plasma channel conductivity, and adiabatic expansion mechanism of this channel in water. It allows you to calculate the dynamics of the expansion of the channel and the generation of a radially diverging acoustic wave. The presented study makes it possible to estimate the probable parameters of the phenomenon: when electric energy is introduced into the channel, its expansion velocity reaches 1.9 km/s, electrons number density in the plasma is up to $2 \times 10^{20} \text{cm}^{-3}$. In this case, a strong acoustic wave propagates with a sonic speed (~ 1500 m/s), and the pressure amplitude in the vicinity of the plasma channel can reach 200 MPa. The stability of the model in relation to variations in the initial task parameters has been analyzed. The calculated data for the acoustic wave are in good agreement with the measurements.

1. Introduction

Pulsed high-current discharges in liquids and their associated shock waves have a high potential for practical application. Investigations of shock waves in water are carried out to achieve mineral liberation [1], refinement of hydrothermal quartz [2], destruction of concrete and reinforced concrete structures [3, 4], et cetera.

One of the main limitations of the extensive use of technologies based on shock waves is the low efficiency of transformation of the electrical energy in the capacitive storage of the supply generator into the mechanical energy of an acoustic wave, which in many cases does not exceed a few percent [5]. The acoustic efficiency can be significantly improved by using the electrical explosion of metal wires to initiate a discharge in water [6, 7].

Numerous experimental studies are aimed at increasing the intensity of shock waves and revealing their dependence on the electrical parameters of the generator and the discharge, such as the energy in the supplying capacitor bank at the time of breakdown [8], the energy transferred to the discharge channel [9], the length of the discharge channel [10].

Usually, on the basis of experimental data, a number of empirical dependencies are formulated that are subsequently the subject of theoretical research. For example, the identification of a number of functional dependencies was obtained in numerical calculations based on a 0-dimensional hydrodynamic model. For this, the empirical dependence of the channel resistance $R_{ch}(t)$ on its fixed length $l$ and the energy input from the discharge circuit $E(t)$ was used [11, 12]: $R_{ch} \propto l^2 / E(t)$. 

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In this work, which does not use any empirical dependencies, a mathematical model of the development of a plasma channel in a liquid medium is formulated in relation to the technical parameters of the experiment. The model is based on the hypothesis of an isothermal state of the plasma in the channel of a pulsed high-current discharge. A calculation of the dynamics of an acoustic wave is made on the basis of a theoretical description of the plasma channel. The divergence and reflection of the wave in the experimental design of the discharge chamber is taken into account. The model of the dynamics of the expanding channel makes it possible to calculate the intensity of the acoustic wave under specific experimental conditions.

2. Theoretical model of expanding channel

Let’s simplify the electrical circuit of the high-voltage pulse generator to an equivalent RLC circuit. Thus, the differential equation of the oscillatory circuit describes the discharge of the capacitance \( C \), charged to voltage \( U_0 \), through the active resistance \( R \), the inductance of the circuit \( L \) and the plasma channel:

\[
\frac{d^2U_C}{dt^2} + \frac{(R + R_{ch})}{L} \frac{dU_C}{dt} + \frac{U_C}{LC} = 0,
\]

where \( U_C \) is the voltage across the capacitor. The resistance of the plasma channel \( R_{ch}(t) \) is described within the framework of the model [13], outlined below. The current in the circuit is determined by the equation \( \dot{I}(t) = -C \frac{dU_C}{dt} \). Voltage applied to the plasma channel \( U(t) = R_{ch}(t)I(t) \). Electric power introduced into the channel, \( W(t) = R_{ch}(t)I^2(t) \).

According to the minimal model, the plasma channel is a three-component non-isothermal plasma (neutral atoms, singly charged ions and electrons). The temperature is constant, \( T_e = 15000 \) K for electrons and \( T_i = 5000 \) K for atoms and ions. The channel has the shape of a cylinder of length \( l \), radius \( r(t) \) and cross-sectional area \( S(t) = \pi r^2(t) \). The resistance of the plasma channel in a wide range of number density is given by the equation:

\[
R_{ch} = \frac{\eta l}{S} = \frac{m}{q^2 n_e} \left( \frac{8kT_e}{\pi m} \right)^{1/2} \frac{l}{S},
\]

where \( \eta \) is plasma resistivity, \( q \) is an elementary charge, \( m \) is an electron mass, \( k \) is Boltzmann constant, \( n_e(t) \) is electron number density, \( n_i(t) \) is atom number density, \( \sigma_{ea} \) is cross section for elastic collisions of electrons with atoms \( 4.4 \cdot 10^{-15} \) cm\(^2\), \( \sigma_{ei} \) is cross section for Coulomb collisions of charged particles, which has the form:

\[
\sigma_{ei} = \pi \left( \frac{Zq^2}{6\pi\epsilon_0 kT_e} \right)^2 \ln \Lambda, \quad \ln \Lambda = \ln \left[ \left( \frac{\epsilon_0 kT_e}{q^2 n_e} \right)^{1/2} \right] \left[ \frac{Zq^2}{12\pi\epsilon_0 kT_e} \right],
\]

where \( Z \) is ion charge multiplicity, \( \epsilon_0 \) is electrical constant, \( \ln \Lambda \) – Coulomb logarithm.

According to (2) channel resistance strongly depends on the electron and atom number densities. The electron number density in the channel changes under the influence of two factors: ionization of water molecules and diffusion of electrons onto the walls of the cylindrical channel

\[
\frac{dn_e}{dt} = \frac{b}{\epsilon_{ion}} \frac{R_{ch}I^2}{S l} - \left( \frac{8kT_e}{\pi m} \right)^{1/2} \left[ \frac{2.4}{r} \right] n_e,
\]

where \( \epsilon_{ion} \) is the ionization energy of an atom or molecule of a gas, in the model it is equal to 12.6 eV [14]. Dimensionless coefficient \( b \) describes part of the energy, put into gas, which goes to ionization processes. For example, for a discharge in molecular nitrogen \( b = 0.04 \) [13]. For water, we assumed
$b = 0.035$. We limited ourselves to taking into account only the diffusion escape of electrons from the channel in order to simplify as much as possible a workable model of the expansion of the plasma channel. Taking into account the recombination channel of the loss of free electrons would make the model unnecessarily complex due to the poorly defined kinetics of the recombination process.

The gas pressure in the channel is described by the ideal gas law $P = n_k T_s$. The change in the number density of neutral particles in the plasma is associated with the evaporation of water molecules from the channel walls as a result of heating due to thermal radiation of the plasma [15]

$$\frac{d}{dt}(nS) = 2\pi r l \frac{\sigma_{SB} T_s^4}{D_{ev}} \rightarrow \frac{d}{dt} PS = \frac{\sigma_{SB} k T_s^5}{D_{ev}} 2\pi r,$$

(5)

where $D_{ev}$ is evaporation energy per molecule (0.42 eV), $\sigma_{SB}$ is Stefan-Boltzmann constant. Below we show how the gas temperature in the channel affects the dynamics of its expansion.

The pressure in the expanding channel is related to the cross-sectional area according to the formula [15]

$$P = \rho_0 \frac{d^2 S}{2\pi d^2} \ln \left(\frac{\pi^{1/2} l}{S^{1/2}}\right) - \frac{\rho_0}{8\pi S} \left(\frac{dS}{dt}\right)^2,$$

(6)

where $\rho_0$ is density of water ($10^3$ kg/m$^3$).

We assume that the expansion of the channel is an adiabatic process (gas adiabatic exponent $\gamma = 1.26$). Then the energy balance equation takes the form

$$P \frac{dS}{dt} + \frac{1}{\gamma - 1} \frac{d}{dt} PS = \frac{W}{I}.$$

(7)

Using the equation (5) and taking into account the dependence of power on current and resistance, we obtain

$$PS \frac{dS}{dt} = \eta I^2 - \frac{1}{\gamma - 1} \frac{\sigma_{SB} k T_s^5}{2\pi r S}.$$

(8)

The system of differential equations (1)-(8) is supplemented with initial conditions.

3. Experimental setup

The theoretical model is compared with experiment, the setup and the equivalent electrical circuit are shown in figure 1.

![Figure 1. Scheme of the experiment with a discharge in water: 1 – high-voltage electrode; 2 – grounded electrode; 3 - discharge channel; 4 – pressure sensor.](image_url)

The crushing chamber is filled with water with a specific electrical resistance of $\sim 10^5$ Ohm·cm. Equivalent parameters of the discharge circuit of the $RLC$ generator are as follows: $C = 8$ nF,
$L = 2120 \text{ nH}, \quad R = 1.5 \text{ Ohm}$. A voltage pulse from a submicrosecond generator is applied to flat electrode 1. The charging voltage $U_0 = 210 \text{ kV}$ gives 176 J of energy in the capacitive storage. A plasma channel is formed between the flat electrode and the grounded electrode made in the form of a thin-walled cylinder 25 mm in diameter. As a result of the expansion of the channel, a shock wave is generated, which is recorded using a PS-02 sensor (Globaltest, Russia) with a diameter of 8 mm. The location of the sensor is marked with 4 in figure 1. Also in the experiments, the current $I$ and the discharge voltage $U$ are recorded.

An example of a typical waveform is shown in figure 2. An energy of $\sim 70 \text{ J}$ is injected into the channel at a peak power of $\sim 300 \text{ MW}$. The sensor records a pressure pulse with an amplitude of $\sim 15 \text{ MPa}$ and a half-amplitude duration of $\sim 6 \mu\text{s}$.

![Figure 2](image1.png)

**Figure 2.** Waveforms of the discharge voltage and current (a), their conversion into power and input energy (b), and the profile of the pressure pulse of the acoustic sensor (c).

### 4. Influence of the parameters of the expanding channel on its dynamics

The presented model makes it possible to calculate the dynamics of the plasma channel. Our goal is to find out how a change in the initial conditions affects the development of the discharge. If the variation of any parameter is not specified, then the initial conditions of the problem are assumed to be equal: $r(0) = 0.1 \text{ mm}, \quad l = 20 \text{ mm}, \quad n_e(0) = 10^{14} \text{ cm}^{-3}, \quad P(0) = 3 \text{ atm}, \quad T_e = 15000 \text{ K}, \quad T_i = 5000 \text{ K}$. The initial current in the channel $I(0) = 0.07 \text{ A}$. The parameters of the discharge circuit were taken the same as in the experiment: $C = 8 \text{ nF}, \quad L = 2120 \text{ nH}, \quad R = 1.5 \text{ Ohm}, \quad U_0 = 210 \text{ kV}$.

The choice of these initial conditions is dictated both by the features of the experimental setup and by reasonable physical considerations. Thus, in the initial period of breakdown, gas bubbles with a diameter of about 10–30 $\mu\text{m}$ are formed in the channel. The pressure in these bubbles is definitely higher than the atmospheric pressure, however, it is limited by the pressure of saturated water vapors.

#### 4.1. Variation of gas temperature

In the presented model, the temperature of neutral particles is considered constant. However, with a significant energy input of 70 J and a current pulse duration of 450 ns, it is obvious that as a result of collisions of electrons with atoms, the gas heats up from room temperature to several thousand degrees. In figure 3, we show the temporal dynamics of some discharge parameters with variations in the temperature of neutral particles.

Note that a change in the temperature of the gas in the channel in a wide range from 500 to 10000 K leads to a change in the rate of expansion of the channel by only 14.5%. The maximum pressure in the channel varies within $13\% \times (13.5–15.2) \times 10^9 \text{ Pa}$. The greatest dependence on the gas temperature is shown by the number density of charged particles in the channel, the differences are more than 12 times, when the temperature changes 20 times. The power introduced into the channel essentially depends on the gas temperature (changes of about 36%).

Thus, the variation in the initial gas heating during channel formation slightly corrects the model, increasing the power introduced into the channel and lengthening the pulse. In this case, the pressure
amplitude in the channel practically does not change, which is essential for verifying the adiabatic model.

**Figure 3.** Changes in the radius of the plasma channel, the electron number density and the power introduced into the channel as a function of time with a change in the gas temperature in the channel.

4.2. Variation of plasma channel radius
In the first moments of time, vapor bubbles are formed as a result of water breakdown. Later they grow, form a channel, the gas inside is ionized and plasma is generated. The presented model describes the dynamics of the channel at the stage of plasma formation. However, the initial radius of the channel is unknown. In figure 4 shows the effect of variation of the initial radius of the channel on the dynamics of the channel parameters.

**Figure 4.** Changes of pressure in the channel, the electron number density and the power introduced into the channel as a function of time with variations in the initial radius of the channel.

It can be seen from the figure 4 that the initial radius strongly affects the pressure dynamics; accordingly, the expansion rate of the channel also strongly depends on the initial radius (from 516 to 290 m/s). Large changes in the electron number density and in the power introduced into the channel: the amplitude values differ by a factor of 2.5. It can be argued that the initial radius of the channel is a key parameter in the simulation of the discharge.

4.3. Variation of initial channel pressure
The initial pressure and radius of the channel are not determined and depend on the moment from which we begin to follow the dynamics of the channel. As in the case of gas temperature variation, changes in the initial pressure by two orders of magnitude lead to insignificant changes in some parameters. So, the expansion rate of the channel changes by 15%, the pressure by 18%. Such parameters as the number density of charged particles (several times) and the power introduced into
the channel (40%) react most significantly (figure 5). However, within the most probable initial conditions (1-5 atm), the changes are not critical and, in general, will not affect the assessment of the shock wave formed in the working chamber.

Figure 5. Changes of plasma channel radius, electron number density and power introduced into the channel as a function of time with variation of the initial gas pressure in the channel.

5. Shock wave

The expanding plasma channel generates a shock wave that propagates through the water. Knowing the power introduced into the channel and its radius, we will use the Consol Multiphysics software package for the axisymmetric calculation of acoustic waves in the discharge chamber [16, 17]. Let us present the calculation results using the example of a discharge; its parameters are given in Section 4.

Figure 6. Instantaneous image of the acoustic wave (a) (right scale shows pressure in MPa). Local time profiles of the compression-rarefaction pressure (b) along the dashed line in figure 6 (a), radial coordinate (in mm) of the observation point is marked near each curve.

In figure 6 (a) shows the distribution of pressure in the chamber. An acoustic wave moves from the axis to the periphery of the chamber with the speed of sound \( c_s \approx 1500 \text{ m/s} \), reflecting from the walls. The width of the acoustic pulse corresponds to the time of energy input into the channel (about 400 ns). The amplitude decreases as the wave travels in the radial direction. In figure 6 (b) shows the change in the acoustic pulse at various points along the dashed line on figure 6 (a, the distance (in mm) is measured from the axis of symmetry. It can be seen that the pressure near the channel (2 mm from the axis of symmetry) reaches \( 2 \cdot 10^8 \text{ Pa} \), gradually decreasing. So, when the pressure sensor is reached (number 4 in figure 1 is also marked in figure 6 (a) with a circle), the amplitude is \( 2 \cdot 10^7 \text{ Pa} \).
6. Conclusions
An adiabatic model of the expansion of a high-current plasma channel in water is formulated. It is relatively simple and assumes a number of assumptions: the presence of only three types of particles (atoms, electrons, singly charged ions), uniformity of parameters over the channel cross section, and a fixed particle temperature. It is shown that, despite the strong variation of the plasma channel parameters depending on the choice of the initial conditions, the adiabatic approximation predicts well the characteristics of the acoustic wave. The model makes it possible to adequately describe the electrical and acoustic data obtained in the experiment.

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