Two stage unit commitment considering multiple correlations of wind power forecast errors

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Abstract
When the correlation of wind power output among wind farms is not considered, the integrated stochastic characteristics of wind power will not be captured accurately. Using this inaccurate feature may lead to an impractical even a failing result of unit commitment (UC). Therefore, this paper proposes a multiple correlations model for wind power forecast errors (WPFEs), and to capture this multiple correlation feature in UC problem, a two-stage chance-constrained interval UC (CIUC) model is proposed. First, an analytical expression of multiple correlations, including spatial, temporal and conditional correlations, is presented to improve the description accuracy of stochastic WPFEs. To strike a balance between risk and operational cost, a chance-constrained decision method is developed to optimize the time-varying interval of wind power output in the first stage. Subsequently, an interval UC model is established to determine the optimal operational schedule in the second stage. Finally, the proposed CIUC model is solved using a solution strategy that combines column-and-constraint generation and sample average approximation. The effectiveness and practicality of the proposed method are verified via the numerical results for IEEE 39-bus and 118-bus systems.

1 INTRODUCTION
The stochastic nature of wind power makes it difficult to maintain controllable power output in the same way as traditional power generation. And the impact of uncertainty of wind power is considerable with the increasingly large scale of wind power integration. Therefore, day-ahead unit commitment (UC) considering wind power usually requires high-quality representation of wind power uncertainty and extra reserve to mitigate the uncertainty of wind power.

According to the expression of wind power uncertainty, UC models are generally classified into two categories. One kind is stochastic method which characterizes uncertainty by establishing scenarios or distributions of wind power, including Monte Carlo simulations (MCS) [1], scenario-based stochastic programming [2], point estimate method [3] and chance-constrained programming (CCP). The other can be described as interval method which only needs to know the up and down limit of wind power, including robust optimization [4, 5], interval programming, information gap decision theory (IGDT) [6], and risk-based optimization etc. Different methods are extensively examined in the existing literature. Ref. [7] presents multi-horizon scenario trees for uncertainty on the strategic decision level. In [8], chance-constrained programming is used to trade-off between the risk and the cost of reserve capacity. Robust optimization requires the decision-maker to choose the optimal solutions according to the worst-case realization, so the decision results are relatively conservative [9]. Although there are more and more methods to reduce the conservativeness [10, 11], the degree of conservativeness is not easy to measure. According to a given fixed wind power output interval, interval optimization-based UC was studied, and the interval results of the generation output and minimal operation cost were obtained with fewer computations compared to scenario-based UC [12]. For IGDT, the decision-maker must make the following value judgments: how much robustness against the harmful uncertain
parameter is needed and how much opportunities should be facilitated from uncertainty [13, 14]. In summary, about the two kinds of methods above, interval methods can improve the computation efficiency, while stochastic methods can well capture the characteristics of wind power and handle more flexible and accurate uncertainties.

In UC problems, integrating renewable power and its uncertainty in a manner that is sound and accurate is crucial. However, the correlations of renewable power are often neglected when considering uncertainty. Due to the similar winds and temperatures in geographically adjacent areas, wind power or wind power forecast errors (WPFEs) of wind farms integrated into the same regional power grid are often spatiotemporally correlated [15]. It is proved that the correlation characteristics of wind power affect power utilization [16–19]. In [18], a short-term spatial-temporal wind power forecast method was proposed which reduced the cost of robust look-ahead dispatch. In [19], a robust security-constrained UC was proposed using the spatio-temporal relationship among multiple bands. Furthermore, the correlation between WPFEs and the corresponding forecast power, which is defined as a conditional correlation, also affects the description of stochastic characteristics for wind power [20]. In [21, 22], the wind forecast power was divided into different ranges, and the error distributions for each forecast power range were obtained. Nevertheless, the number of ranges is limited by the amount of data obtained. Based on the sequence operation theory, the conditional distribution of WPFEs given the corresponding forecast power was analysed considering the spatial-temporal correlation in [23]. However, that approach is computationally burdensome, incurring difficulty in practical applications. Despite the aforementioned efforts, spatial, temporal, and conditional correlations cannot be easily modelled at the same time, because the joint distribution of the errors cannot be easily obtained. Therefore, this paper mainly studies the modelling of multiple correlations and corresponding UC model.

Accurately embedding the multi-correlation model and reducing the computational complexity caused by uncertainties is the key and a difficult problem of UC with wind power integration. Interval methods cannot capture correlation characteristic, in this paper, CCP is chosen to handle multiple correlations. As one of the stochastic programming approaches, CCP can effectively capture the stochastic characteristic using the error distribution and balance risk and cost, but solving the chance-constrained problem is more time consuming than interval optimization [24]. A hybrid UC approach that applies the CCP to the initial operating hours to make the operating costs lower, and switches to the interval formulation for the remaining hours to achieve robustness and reduce computation time was proposed in [25]. However, the results of interval optimization are sensitive to the wind power output interval, and thus, the issue of how large an interval should be set must be seriously considered. Therefore, this paper seeks a method to reduce the computation burden.

The main contributions of our paper are summarized as follows:

A statistic model of multiple correlations for WPFEs is proposed to represent the characteristic of spatial, temporal, and conditional correlations for multiple wind farms. Based on the copula theory, the detail expression of multiple correlation for WPFEs is derived to improve its practicability.

In order to accurately capture the multiple correlations in UC problems and reduce computation at the same time, a two-stage UC model is designed, which is fit to the physical mechanism of UC problems. In the first stage, CCP is chosen to accurately handle the correlation model and an interval of wind power is optimized when balancing the risk and costs. In the second stage, interval programming UC is built given the interval passed from the first stage. In such a manner, the number of constraints is reduced when dealing with uncertainty, and it is unnecessary to deal with the stochastic simulation when computing unit output, so the calculation can be saved.

The remainder of this paper is organized as follows. The analytical expression for the multiple correlations of WPFEs is derived in Section 2. The two-stage CIUC model is proposed in Section 3. The solution strategy framework and the transformation of the proposed model are described in Section 4. The numerical test results on 39-bus and modified 118-bus systems are presented in Section 5. Conclusions are given in Section 6.

2  |  MULTIPLE CORRELATIONS OF THE FORECAST ERROR

Based on the spatial-temporal correlation of WPFEs, the multiple correlations of WPFEs are studied using copula theory, and their analytical expression is exploited to accurately capture the stochastic characteristics of the error.

2.1  |  Spatial-temporal correlation of WPFEs

As a powerful method used to address statistical dependencies, the copula function can establish the correlation structure and marginal distributions separately [26]. For the high-dimensional joint distribution of WPFEs in this paper, the multivariate normal copula and t-copula can be applied to establish the joint cumulative distribution function (JCDF). Although the multivariate normal copula is relatively simple, the multivariate t-copula can capture the fat tail of the error distribution better. Therefore, the multivariate t-copula function is chosen to describe the spatial-temporal correlation of WPFEs in this paper.

If the WPFEs of \( N \) wind farms of \( T \) scheduling periods are expressed as \( [\varepsilon_{11}, \ldots, \varepsilon_{1T}, \varepsilon_{21}, \ldots, \varepsilon_{2T}, \ldots, \varepsilon_{N1}, \ldots, \varepsilon_{NT}] \), its JCDF can be described as follows:

\[
F(\varepsilon_{11}, \ldots, \varepsilon_{1T}, \varepsilon_{21}, \ldots, \varepsilon_{2T}, \ldots, \varepsilon_{N1}, \ldots, \varepsilon_{NT}) = C(F_{11}(\varepsilon_{11}), \ldots, F_{ij}(\varepsilon_{ij}), \ldots, F_{NT}(\varepsilon_{NT}))
\]

\[
= t_{NT}(t^{-1}(F_{11}(\varepsilon_{11})), \ldots, t^{-1}(F_{ij}(\varepsilon_{ij})), \ldots, t^{-1}(F_{NT}(\varepsilon_{NT})))
\]

(1)

where \( C \) is a copula function and \( t_{NT} \) is a t-copula function with \( NT \) dimensions.

Then, the joint probability density function (JPDF) of WPFEs considering spatial-temporal correlation can be
described as follows:

\[ f_E(t_1, \ldots, t_j, \ldots, t_N) = C(F_{t1}(t_1), \ldots, F_{tj}(t_j), \ldots, F_{tN}(t_N)) \prod_{i=1}^{N} \prod_{j=1}^{T} f_{tij}(t_{ij}) \]

(2)

where \( f_{tij}(t_{ij}) \) is the marginal distribution of \( t_{ij} \).

### 2.2 Conditional distribution of WPFEs given the forecast power

For any random sequence \( x \) and \( y \), the JPDF can be obtained according to Equation (2). In this work, we assume that \( y = \hat{p} \) denotes the point forecast power, \( x \) denotes the corresponding actual power, \( \epsilon \) represents WPFE, and thus, \( x = \hat{p} + \epsilon \). The conditional probability density function (CPDF) of the actual power output \( x \) given forecast power \( \hat{p} \) can be described as follows:

\[ f_{X|Y}(x|y = \hat{p}) = f_{XY}(x, \hat{p}) / f_Y(\hat{p}) \]

\[ = C(F_{X}(x), F_{Y}(\hat{p})) \cdot f_{X}(x) / f_{Y}(\hat{p}) \]

\[ = C(F_{X}(x + \epsilon), F_{Y}(\hat{p})) \cdot f_{X}(x + \epsilon + \epsilon) \]  

(3)

If the random sequence \( \epsilon \) and \( \hat{p} \) are extended to \( NT \) dimensions and the spatial and temporal correlations are considered together, the conditional distribution of the high-dimensional WPFEs given the corresponding forecast power can be exploited as follows [23].

\[ f_{X_{11}, \ldots, X_{NT}|Y_{11}, \ldots, Y_{NT}}(t_{11}, \ldots, t_{NT}|\hat{p}_{11}, \ldots, \hat{p}_{NT}) \]

\[ = \frac{C_{2NT}(F_{X_{11}}(t_{11} + \hat{p}_{11}), \ldots, F_{X_{NT}}(t_{NT} + \hat{p}_{NT}))}{C_{NT}(F_{Y_{11}}(\hat{p}_{11}), \ldots, F_{Y_{NT}}(\hat{p}_{NT}))} \]

\[ \times \prod_{i=1}^{N} \prod_{j=1}^{T} f_{tij}(t_{ij} + \hat{p}_{ij}) \]

(4)

where \( \hat{p}_{11}, \ldots, \hat{p}_{NT} \) are wind power forecasts of wind farm \( i \) in period \( j \), \( X_{ij} \) denotes the corresponding actual power, and \( \epsilon_{ij} \) represents the forecast error. Then, the marginal distribution during time period \( j \) can be given as follows:

\[ f_{X_{ij}|Y_{11}, \ldots, Y_{NT}}(t_{ij}|\hat{p}_{11}, \ldots, \hat{p}_{NT}) = \frac{C_{2NT}(F_{X_{ij}}(t_{ij} + \hat{p}_{ij}), F_{Y_{11}}(\hat{p}_{11}), \ldots, F_{Y_{NT}}(\hat{p}_{NT}))}{C_{NT}(F_{Y_{11}}(\hat{p}_{11}), \ldots, F_{Y_{NT}}(\hat{p}_{NT}))} \cdot f_{\epsilon_{ij}}(\epsilon_{ij} + \hat{p}_{ij}) \]

(5)

Although a detailed expression of the high-dimensional conditional distribution is given in Equations (4) and (5), they cannot be used directly because of the abstract and complex nature of the expression. In fact, [24] proved that the conditional distribution of the multivariate \( t \)-distribution also follows a multivariate \( t \)-distribution. That is, if the correlation model using the \( t \)-copula is built as in Equation (1), the conditional distribution in Equations (4) and (5) can be obtained from the original multivariate \( t \)-distribution via position and scale transformation.

The parameters of the conditional distribution can be deduced as follows. Based on the analysis above, the vectors of actual power \( x_{1j} \) and forecast power \( y_j \) can be denoted as \( Z_x = [x_{1j}, \ldots, x_{NT}]^T \) and \( Z_y = [y_{1j}, y_{NT}]^T \), respectively. If \( [Z_x^T, Z_y^T] \) is considered to follow the \( 2NT \) dimensional multivariate \( t \)-distribution with scale matrix \( \Sigma \), then this matrix (the corresponding covariance matrix) can be obtained as follows:

\[
\Sigma = \begin{bmatrix}
1 & \cdots & \rho(x_{1j}, y_{NT}) & \rho(x_{1j}, y_{1j}) & \cdots & \rho(x_{1j}, y_{NT}) \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\rho(y_{1j}, x_{1j}) & \cdots & 1 & \rho(y_{1j}, y_{1j}) & \cdots & \rho(y_{1j}, y_{NT}) \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\rho(y_{NT}, x_{1j}) & \cdots & \rho(y_{NT}, x_{NT}) & 1 & \cdots & \rho(y_{NT}, y_{NT}) \\
\end{bmatrix}
\]

where \( \rho \) is the correlation coefficient between variables.

Thus, \( Z_x \) and \( Z_y \) follow an \( NT \)-dimensional multivariate \( t \)-distribution, and their conventional versions can be written as \( Z_x \sim t_N(\mu_x, \Sigma_{11}, \nu_1) \) and \( Z_y \sim t_N(\mu_y, \Sigma_{22}, \nu_2) \), respectively, where \( \mu_x \) and \( \mu_y \) are the location parameters of \( Z_x \) and \( Z_y \), and \( \nu_1 \) and \( \nu_2 \) are the degrees of freedom of \( Z_x \) and \( Z_y \), respectively. According to the proof in [27], the conditional distribution of \( Z_x \) given \( Z_y \) can be derived as follows:

\[
Z_x|Z_y \sim t_{NT} \left( \mu_{12} + \frac{\nu_2}{\nu_2 + NT} \sum_{11} \nu_2 \right)
\]

(7)

where

\[
\begin{align*}
\mu_{12} &= \mu_1 + \sum_{12} \sum_{22}^{-1} (z_y - \mu_2) \\
\sum_{11} &= \sum_{11} - \sum_{12} \sum_{22}^{-1} \sum_{21}
\end{align*}
\]

(8)

and where \( d_2 = (Z_y - \mu_2)^T \Sigma_{22}^{-1} (Z_y - \mu_2) \) is the squared Mahalanobis distance of \( Z_y \) from \( \mu_2 \) with scale matrix \( \Sigma_{22} \).

Accordingly, the high-dimensional JCDF of Equation (4) can be calculated directly by Equation (8) if the historical data for forecast power and the corresponding WPFE are given. The marginal distribution of Equation (5) can also be calculated directly. Based on the distribution calculated above, the stochastic characteristics of WPFEs can be described more...
First stage: Optimal interval decision of TWO-STAGE CIUC FORMULATION

The time-varying interval is shown in Figure 1. The time-varying interval can also be determined. The concept of the error distributions is developed and the corresponding correlations and CVaR, the time-varying confidence level of the original WPFE data.

Evaluating the features of the stochastic fluctuations is easy to obtain, the given fixed interval encounters difficulty in accurately capturing the features of the stochastic fluctuations of the original WPFE data.

Therefore, by combining the analytical expression of multiple correlations and CVaR, the time-varying confidence level of the error distributions is developed and the corresponding time-varying interval can also be determined. The concept of the time-varying interval is shown in Figure 1.

In Figure 1, the interval of wind power output at time $t_0$ on the left is primarily influenced by the distribution of the error at time $t_0$, shown as the curve on the right. When the interval for each time period is optimized by the corresponding quantiles of the error distribution, the characteristics of multiple correlations in the distribution can be reflected.

The relationship between the wind power output interval and the confidence level of the error at time period $t$ can be expressed as follows:

$$p_{\alpha_f}^{\max} = \min \left(p_{\alpha_f}^{\max}, P_{w,t}^{\max} + f_{t-1}^{-1}(1-\alpha_{f,t}) \right)$$ (9)

$$p_{\alpha_f}^{\min} = \max \left(0, P_{w,t}^{\min} + f_{t-1}^{-1}(\alpha_{f,t}) \right)$$ (10)

where $p_{\alpha_f}^{\max}$ and $p_{\alpha_f}^{\min}$ are the upper and lower bounds of the wind power output interval for wind farm $w$ at time $t$, respectively, $P_{w,t}^{\max}$ is the rated capacity of wind farm $w$, $P_{w,t}^{\min}$ is the forecast power of wind farm $w$ during time period $t$, and $\alpha_{f,t}$ and $\alpha_{f,t}$ indicate the probability of wind curtailment and load shedding. The terms $f_{t-1}^{-1}(1-\alpha_{f,t})$ and $f_{t-1}^{-1}(\alpha_{f,t})$ represent the quantiles of the corresponding error distribution at quantiles $1-\alpha_{f,t}$ and $\alpha_{f,t}$ of the distribution at time $t$, respectively.

For the fluctuations within this time-varying interval, the power system or wind farm operators could pay for sufficient reserve capacity. Nevertheless, wind curtailment or load shedding occurs when the actual wind power is greater than the upper bound or less than the lower bound. Accordingly, the risk values of wind curtailment and load shedding can be calculated using the CVaR theory as follows:

$$VoWC_t = \sum_{w=1}^{NW} \int_{p_{\alpha_f}^{\max}}^{P_{w,t}^{\max}} (x - P_{w,t}^{\max}) \cdot f_t(x)dx$$ (11)

$$VoLS_t = \sum_{w=1}^{NW} \int_{0}^{p_{\alpha_f}^{\min}} (P_{w,t}^{\min} - x) \cdot f_t(x)dx$$ (12)

where VoWC$_t$ and VoLS$_t$ represent the risk value of wind curtailment and the value of load shedding, respectively, and $f_t(x)$ is the error distribution during period $t$. Equations (11) and (12) can be transformed into linear form by piecewise linearization method and coded in CPLEX.

3.2 First stage: Optimal interval decision of wind power output

In the first stage, the aim is to minimize wind curtailment, load shedding and reserve cost according to characteristics of wind power uncertainty, including multiple correlations. So, the chance constraints and time-varying confidence levels are used in the model of this stage to capture the stochastic characteristic of the error distribution. The optimal interval of wind power output is determined by making a trade-off between risk costs and reserve cost. Therefore, minimizing the total costs of wind curtailment risk, the load-shedding risk and reserve is selected as the objective function to establish a pre-scheduling model. The wind power output interval is optimized, and the corresponding reserve requirement is obtained.

At the same time, considering the feasibility and efficiency of the solution, selected operation constraints, such as the...
minimum up- and down-time constraints, are relaxed and properly simplified. The unit on or off status is obtained from the second stage. The two-stage solution methodology is described in detail in Section 4.

The objective function in the first stage is given as follows:

$$F = \min \sum_{t=1}^{T} \left( c_{\text{W}} \text{VoW}_{t} + c_{\text{LS}} \text{VoLS}_{t} + c_{R}(R_{t}^{\text{up}}, R_{t}^{\text{down}}) \right) \quad (13)$$

subject to

$$\sum_{g=1}^{\text{NG}} p_{g,t} + \sum_{u=1}^{\text{NW}} p_{u,t}^{\text{pre}} = \sum_{d=1}^{\text{ND}} p_{d,t} \quad (14)$$

$$u_{g,t} p_{g,t}^{\text{min}} \leq p_{g,t} \leq u_{g,t} p_{g,t}^{\text{max}} \quad (15)$$

$$R_{t}^{\text{up}} \geq f^{-1}(1 - \alpha_{1,t}) \quad (16)$$

$$R_{t}^{\text{down}} \geq f^{-1}(\alpha_{2,t}) \quad (17)$$

$$\Pr \left\{ \sum_{d=1}^{\text{ND}} p_{d,t} \geq \sum_{g=1}^{\text{NG}} p_{g,t} + \sum_{u=1}^{\text{NW}} p_{u,t}^{\text{pre}} + R_{t}^{\text{down}} \right\} \geq 1 - \alpha_{1,t} \quad (18)$$

$$\Pr \left\{ \sum_{d=1}^{\text{ND}} p_{d,t} \leq \sum_{g=1}^{\text{NG}} p_{g,t} + \sum_{u=1}^{\text{NW}} p_{u,t}^{\text{pre}} + R_{t}^{\text{up}} \right\} \geq 1 - \alpha_{2,t} \quad (19)$$

Equation (13) is the objective function of the first-stage problem, minimizing the total costs of reserve and risk. Constraint Equation (14) is the power balance constraint. Constraint Equation (15) expresses the total generation limit. Constraints Equations (16) and (17) are the reserve capacity constraints that ensure sufficient reserve for the wind power fluctuation interval and system security, respectively. Constraints Equations (18) and (19) are chance constraints for avoiding the risks of load shedding and wind curtailment.

3.3 Second stage: Interval optimization-based UC formulation

After the decision variables $R_{t}^{\text{up}}$ and $R_{t}^{\text{down}}$ and optimal wind power output interval are determined in the first stage, interval optimization is used to build the UC model of the second stage.

Based on the results of wind power interval and unit state variables decided in the first stage, minimization of operation costs and start-stop costs is taken as the objective function. By combining the two stages, the CIUC model is completed, and the overall economy of the two-stage model is ensured. The objective function in the second stage is written as follows:

$$\min_{\delta_{g,t}} \sum_{t=1}^{T} \sum_{g=1}^{\text{NG}} \left( z_{g,t} S_{g} + F_{g}(p_{g,t}, u_{g,t}) \right)$$

The objective function of Equation (20) minimizes the total cost of fuel costs and start-up costs, denoted as $S_{g}$ and $F_{g}$, respectively. And the quadratic function $F_{g}$ can be transformed into a linear form by the piecewise linearization method. Decisions $u_{g,t}$ and $z_{g,t}$ represent the binary UC status and start-up decisions, respectively.

Subject to

$$\sum_{g=1}^{\text{NG}} p_{g,t} + \sum_{u=1}^{\text{NW}} [p_{u,t}^{\text{pre}}, p_{u,t}^{\text{up}}] = \sum_{d=1}^{\text{ND}} p_{d,t} \quad (21)$$

$$- \sum_{g=1}^{\text{NG}} G_{g} - \sum_{d=1}^{\text{ND}} G_{g} - \sum_{d=1}^{\text{ND}} p_{d,t} - F_{g}^{\text{max}} \leq \left[ \sum_{u=1}^{\text{NW}} G_{u} - \sum_{u=1}^{\text{NW}} G_{u}^{\text{up}} p_{u,t}^{\text{up}} \right] \quad (22)$$

$$\sum_{g=1}^{\text{NG}} G_{g} - \sum_{d=1}^{\text{ND}} G_{g} - \sum_{d=1}^{\text{ND}} p_{d,t} - F_{g}^{\text{max}} \leq \left[ - \sum_{u=1}^{\text{NW}} G_{u} - \sum_{u=1}^{\text{NW}} G_{u}^{\text{up}} p_{u,t}^{\text{up}} \right] \quad (23)$$

$$\sum_{g=1}^{\text{NG}} u_{g,t} p_{g,t}^{\text{max}} \geq \sum_{d=1}^{\text{ND}} p_{d,t} + R_{t}^{\text{up}} \quad (24)$$

$$\sum_{g=1}^{\text{NG}} u_{g,t} p_{g,t}^{\text{min}} \leq \sum_{d=1}^{\text{ND}} p_{d,t} - R_{t}^{\text{down}} \quad (25)$$

$$- u_{g,t-1} + u_{g,t} - u_{g,k} \leq 0, \quad t \leq k \leq T_{g}^{\text{on}} + t - 1 \quad (27)$$

$$u_{g,t-1} - u_{g,t} + u_{g,k} \leq 1, \quad t \leq k \leq T_{g}^{\text{off}} + t - 1 \quad (28)$$

$$- u_{g,t-1} + u_{g,t} \leq z_{g,t} \leq 0 \quad (29)$$

$$p_{g,t} - p_{g,t+1} \leq u_{g,t+1} D_{g} + (1 - u_{g,t+1}) p_{g,t}^{\text{max}} \quad (30)$$

$$p_{g,t+1} - p_{g,t} \leq u_{g,t} D_{g} + (1 - u_{g,t}) p_{g,t}^{\text{max}} \quad (31)$$
where Equation (21) is the power balance constraint, \( p_{g,t} \) is the power output of unit \( g \) at time period \( t \), and \( p_{-w,t} \) and \( p_{+,w} \) are the lower and upper bounds of the output interval for wind farm \( w \) at time \( t \), respectively. Constraints Equations (22) and (23) ensure power flow on a transmission line within its capacity. Constraints Equations (24) and (25) are the reserve capacity constraints. Constraint Equation (26) represents the generation limit constraint. Constraints Equations (27), (28) and (29) are the minimum up- and down-time constraints. Constraints Equations (30) and (31) are the ramp rate constraints.

4 | SOLUTION METHODOLOGY

A two-stage chance-constrained interval UC model is built above. In order to make the two-stage model easy to solve, it is necessary to find a decomposition method to separate continuous variables from discrete variables. There are several decomposition methods to solve UC problem, such as Benders decomposition method and C&CG algorithm. C&CG has advantages in fewer iterations, and the number of iterations in the C&CG is insensitive to problem sizes [28]. Therefore, C&CG algorithm is chosen and the two-stage UC model is decomposed to solve the time-varying interval optimization of wind power output and discrete unit state decision iteratively.

In this section, a solution strategy based on the C&CG and SAA algorithms is proposed to solve the two-stage CIUC model. The SAA algorithm is applied to solve the chance-constrained model in the first stage. The C&CG algorithm is adopted to decompose the UC model and combine the two-stage models. The framework of the solution strategy for the two stages is presented in Figure 2.

In this process, via the master/slave solution mechanism of C&CG, the binary status and start-up decisions of the unit can be obtained from the master problem. The decision results are used to optimize the wind power output interval in the first stage. Then, the sub-problems of interval optimization in the second stage can be solved using the results above. Accordingly, the iterative solution strategy of the two stages is built. In addition, the interval optimization model in the sub-problems is transformed into deterministic models [29] for the solution, as in Subsection 4.2.

4.1 | Chance constraint transformation using SAA

SAA is an effective method for solving CCP. The basic idea of SAA is to obtain an empirical distribution of a random variable by sampling and take the approximate empirical distribution as the actual distribution. SAA usually contains scenario generation, convergence analysis, and solution validation parts. Because the error distribution that considers multiple correlations is a high-dimensional JPDF, for the sampling process, the traditional Monte Carlo sampling method has a low accuracy and a long computing time. To improve the efficiency and accuracy of sampling, the Gibbs sampling method based on Markov chain Monte Carlo (MCMC) is applied, which is more suitable for high-dimensional distribution sampling [30]. The convexity was studied and the convergence of SAA was proven in [31], and the solution validation was thoroughly studied in [32].

According to SAA, chance constraints Equations (18) and (19) of the load shedding and wind curtailment considering the wind power characteristics can be expressed as follows:

\[
N_s^{−1} \sum_{j=1}^{N_s} 1_{(0,\infty)} \left( \sum_{g=1}^{NG} p_{g,j} + \sum_{u=1}^{NW} p_{w,j}^u + R_{j}^{lw} - \sum_{d=1}^{ND} p_{d,j} \right) \leq \alpha_{1,j}
\]

\[
N_s^{−1} \sum_{j=1}^{N_s} 1_{(0,\infty)} \left( - \sum_{g=1}^{NG} p_{g,j} - \sum_{u=1}^{NW} p_{w,j}^u - R_{j}^{up} + \sum_{d=1}^{ND} p_{d,j} \right) \leq \alpha_{2,j}
\]

where \( N_s \) is the sampling number and \( 1_{(0,\infty)}(\cdot) \) is an indicator function to approximate the chance constraint. In this expression, \( 1_{(0,\infty)}(b) = 1 \) or 0 when \( b \geq 0 \) or \( b \leq 0 \), respectively. Regarding [33], Equations (32) and (33) can be transformed into a mixed integer programming (MIP) model as follows:

\[
\sum_{g=1}^{NG} p_{g,j} + \sum_{u=1}^{NW} p_{w,j}^u + R_{j}^{lw} - \sum_{d=1}^{ND} p_{d,j} \leq M \zeta_{3,j}
\]
where $\alpha_{k,t}$ is a binary variable indicating whether the left-hand side of constraints Equations (34) and (35) is less than 0, $\alpha_{k,t} \in \{0,1\}$, and $M$ is a large positive integer.

### 4.2 Solution of the interval optimization model

In order to describe the transformation and solution of the interval optimization model, the compact formulation of the interval model in Subsection 3.2 can be expressed as follows:

$$
\begin{align*}
- \sum_{n=1}^{N_C} p_{u,i} - \sum_{n=1}^{N_D} p_{d,i} + R_{i}^{up} + \sum_{d=1}^{ND} p_{d,i} & \leq M \ z_{t,j} \\
\sum_{j=1}^{N_G} \ z_{t,j} & \leq \alpha_{k,t} N_a \ k = 1,2
\end{align*}
$$

(35)

(36)

shown in Equation (40).

$$
\begin{align*}
Z^- = & \min_{w,d} \min_{x,j} c^T_{d,j} y^- \\
\text{s.t.} & \quad w^- - D y^- \leq w^+ \\
& \quad l \leq y^- \leq u \\
& \quad F x + G y^- \leq b \\
& \quad \forall w \in [w^-,w^+], \forall d \in [d^-,d^+]
\end{align*}
$$

(39)

$$
\begin{align*}
Z^+ = & \max_{w,d} \max_{x,j} c^T_{d,j} y^+ \\
\text{s.t.} & \quad D y^+ = [w^-,w^+] \\
& \quad l \leq y^+ \leq u \\
& \quad F x + G y^+ \leq b \\
& \quad \forall w \in [w^-,w^+], \forall d \in [d^-,d^+]
\end{align*}
$$

(40)

In this formulation, sub-problem 1 can be transformed into a ‘min’ problem that can be solved directly. Sub-problem 2 is a ‘max-min’ bi-level problem in which the inner problem must be transformed into a ‘max’ problem using duality theory to obtain a single-level problem and is solved by the big-M linearization method.

### 5 CASE STUDY

In this section, the IEEE 39-bus system and a modified 118-bus system are studied to illustrate the proposed CIUC method. The algorithm is coded in MATLAB 2017b and solved with CPLEX 12.6. All studies are implemented on an ordinary 64-bit computer with a 2.40 GHz CPU and 8 GB of memory. The unit costs of wind curtailment, load shedding and reserve are assumed to be $30, $100 and $40/MWh, respectively. In the case study, the relative MIP gap tolerance is set to 1%, and the gap in C&CG is set to within 1%.

First, the proposed method is tested on the IEEE 39-bus system. The system network, unit and load data are derived from [34]. The capacities of wind farm A, B and C are 150, 100 and 150 MW, which are assumed to be located at buses 7, 8 and 9, respectively. Based on the 39-bus system, the analyses regarding the multiple correlations, wind power output interval, UC result and sensitivity analysis are conducted in subsections A, B, C and D, respectively. The modified IEEE 118-bus system is tested to further demonstrate the practical performance for the proposed method. The IEEE 118-bus system data are obtained from [34].
5.1 Analysis of multi-correlation

In this paper, the historical wind power data from 1 January, 2015 to 31 December, 2015 were obtained from three wind farms in Hebei Province, China. The distance between wind farms A and B is approximately 17 miles, and the distances between AC and BC are 8.94 and 9.76 miles, respectively. Using the actual data above, the forecast power data were forecasted by the SVM method in [18].

Firstly, we take a small example of multiple correlation model computation by using data of two wind farms (A and B) in hour 1. The correlation coefficient of spatial correlation can be easily computed by statistical method. And we can get that

$$\mu_1 = [-0.066, -0.076]^T, \mu_2 = [0.235, 0.224]^T,$$

then the data of forecast wind power of the two wind farms in hour 1 are used, and the correlation matrix of Equation (6) can be estimated as follows:

$$\Sigma = \begin{bmatrix}
1 & 0.91 & 0.08 & 0.01 \\
0.91 & 1 & 0.03 & 0.07 \\
0.08 & 0.03 & 1 & 0.93 \\
0.01 & 0.07 & 0.93 & 1
\end{bmatrix}$$

According to Equations (7) and (8), parameters of multivariate t distribution considering multiple-correlation are computed. Given forecast wind power is [0.4, 0.4], the result is as follows:

$$Z_x | Z_y \sim t_{NT} \left( [-0.115, -0.038]^T, \begin{bmatrix} 0.506 & 0.488 \\ 0.488 & 0.518 \end{bmatrix}, 4 \right)$$

The result can be applied to UC problem by SAA directly.

Considering the whole multiple-correlation model, according to Equations (6)–(8) in Section 2, the error distribution considering multiple correlations can be obtained. For presentation and comparison purposes, ten error distributions of wind farm A given different point forecasts at 10:00 a.m. are computed. In order to illustrate the differences for each distribution, the error distributions and their location parameters are shown in Figure 3.

The correlation between the forecast power and its error indicates that the error distribution is affected by the forecast value. When the forecast power is [0, 0.1] p.u. or [0.7, 1] p.u., where the forecast power is close to 0 or the rated power, the error distributions are thin and tall, which means that the mean error is smaller. In contrast, the error distributions given a forecast power of [0.1, 0.7] are short and fat, which means that the mean error is larger. The differences among different forecasts illustrate the impact of multiple correlations and necessity to consider multiple correlations in decision-making.

5.2 Analysis of the wind power output interval

Based on the two-stage solution strategy, the proposed two-stage CIUC model can be solved. To enable a better comparison, three groups of cases are defined as follows:

- Case 1: Ignore any correlations;
- Case 2: Only consider spatial-temporal correlation;
- Case 3: Consider the proposed multiple correlations.

Figure 4 shows wind power output interval results for each case in the first stage.

Compared to case 1, the wind power output intervals of cases 2 and 3 increase obviously over time. In particular, the intervals increase more significantly after time period 16 than earlier periods. This trend occurs because the dependence in the time series of the WPFEs increasing with time is captured by considering the spatio-temporal correlations. To accommodate the larger WPFEs, additional reserves are needed to prevent the risk of wind curtailment or load shedding from increasing. By using CIUC model, spatio-temporal correlation is captured, and reserve and risk are balanced automatically.

A comparison between cases 2 and 3 illustrates that the interval decreases in certain time periods when considering multiple correlations. The decrease is distinct when the forecast power is small in time periods 4–7 or close to the rated capacity of the wind farm in time periods 12–14 because of the small WPFEs during these time periods. This feature reflected from
the optimal interval results is consistent with those from the multi-correlation analysis in Figure 3. As a result, the stochastic characteristic of WPFEs and multiple correlations are effectively captured by the wind power output interval determined in the first stage.

5.3 | Comparison of the CIUC results

In this section, the results of CIUC model under three cases is computed. And the conventional chance-constrained programming model is taken for comparison.

The interval of the total cost (including operation, reserve and risk costs) can be obtained using the two-stage CIUC model, which accurately captures the stochastic characteristics of WPFEs. This interval indicates the most optimistic and pessimistic results within the uncertain wind power output interval, and operators can decide on a generation schedule according to their preference of optimism. To discuss the advance of CIUC method proposed in this paper, a chance-constrained UC model with the same confidence interval and the same constraints is tested in three cases for comparison. The costs of two types of UC models are shown in Table 1. The results for case 2 tend to be more conservative, and the results for case 3 are more optimistic than case 1. The operational costs for both cases 2 and 3 are higher than those for case 1 because the wind power output interval determined in the first stage is decreased, especially the lower bound during the last few periods, as shown in Figure 4. However, the interval decreased to a lesser extent in case 3 compared with case 2, and thus, the operational cost of case 3 is lower. When added to the reserve and risk costs, the total cost of case 3 is the lowest because the reserve and risk cost considering multiple correlations is the lowest.

For a clearer comparison and more intuitive expression, all of the results in Table 1 are illustrated in Figure 5, in which the rectangle areas denote the intervals of total cost and the red point marks in the rectangles denote the costs using CCP in the same confidence interval. Comparing the results of the two types of UC models, all of the results using the CCP method are contained within the corresponding interval results of the CIUC model. Additionally, the total cost using CCP is closer to the lower bound of the cost interval in all cases. Furthermore, Figure 5 illustrates the differences of the interval costs among different cases and illustrates the optimistic or pessimistic degree of the solution using CCP in each case directly. Thus, the solution obtained using the proposed CIUC method can measure the performance of the solution for other methods, and even the performance of real-time economic dispatch can be estimated.

Case 4, in which the time-varying interval of CIUC model in case 3 is changed to a given fixed interval, is considered to illustrate the influence for the economy by capturing the characteristics of wind power and optimizing the time-varying interval of wind power output. To ensure the comparability of cases 3 and 4, the width of the fixed confidence interval is set to the mean width of the confidence intervals of case 3 over 24 time periods. The fixed confidence interval is [0.09, 0.91]. The corresponding interval result of the total cost in case 4 is [554893, 676132] $. The results are shown in Table 2, and the result of case 4 is also illustrated in Figure 5. Comparing the results of case 3 and case 4, the interval cost in case 3 with the optimized wind power output interval is more optimistic than that in case 4 with the fixed interval. Because the flexible time-varying confidence interval can capture the stochastic characteristic of the error distribution for each time period in a trade-off between the costs of reserve and risk. As demonstrated above, the two-stage CIUC model can improve the economy of the power system operation because the characteristic of the error distribution under the known corresponding forecast power is captured and the trade-off between cost and risk is optimized.

5.4 | Sensitivity analysis on cost penalties and reserve cost

To illustrate the impact of different risk levels on the CIUC result, the variations in reserve cost, risk cost and total cost are analysed under different unit costs of load shedding, as shown in Figures 6 and 7.

According to Figure 6, together with an increase in the unit cost of load shedding, the reserve cost increases and the risk cost decreases. Because when the unit cost of load shedding...
TABLE 2  Comparison of UC results for fixed and time varying interval

| Case | Total cost/($) | Operation cost/($) | Reserve cost/($) | Risk cost/($) |
|------|---------------|--------------------|-----------------|--------------|
| 3    | [554893, 676132] | [472708, 593947] | 77672           | 4513         |
| 4    | [539994, 654044] | [476239, 590289] | 61238           | 2517         |

FIGURE 6  Cost of reserve and risk under different load-shedding risk levels

FIGURE 7  Total cost under different load-shedding risk levels

increases, to ensure the economy, the power system increases the reserve to reduce the risk of load shedding in the process of balancing the costs of reserve and risk. When the unit cost of load shedding exceeds $200/MWh, both the costs of risk and reserve exhibit smaller changes. The trade-off mentioned above can also be observed in Figure 7 in which the interval result tends to be more optimistic with a higher unit cost of load shedding risk and changes less dramatically in the end.

5.5 IEEE 118-bus system implementation

The IEEE 118-bus system is tested to demonstrate the practicality of the CIUC method. Different levels of wind penetration are simulated in this system. It takes three iterations on average to obtain the solution of the CIUC model. Although the resolving time is approximately 40 min, it is acceptable for day-ahead UC problems.

Table 3 compares the results of the two cases under different penetration levels. Case 3 results in the lowest total costs for all wind penetration levels. Compared with case 1, the proposed method achieves savings of up to 5.2% for 30% wind generation in case 3. Case 3 also results in lower reserve costs, although both reserve and risk costs increase at higher levels of wind penetration.

6  CONCLUSION

By deriving an analytical expression of multiple correlations, a novel two-stage CIUC model considering the multiple correlations of WPFEs is proposed in this paper. In this model, CCP is adopted to capture the characteristics of wind power uncertainty and determine the optimal time-varying interval of wind power output by balancing the cost and risk. The CIUC model is built and solved using an effective solution strategy that combines the SAA and C&CG algorithms. Case studies demonstrate that the stochastic characteristic of the error distribution can be impacted by the multiple correlations of WPFEs. And CIUC model can effectively cope with the multiple correlation model. Additionally, the CIUC model using the optimized time-varying interval of wind power output can improve the scheduling economy. Finally, the results of the case studies also demonstrate the advantages and effectiveness of the CIUC method.

The proposed model is not restricted to multiple wind farms and is widely applicable. For instance, it can also be applied to modelling the correlations of wind power and photovoltaic generation, or correlations of wind power and load. Even any correlated uncertain variables or parameters can be modelled. To derive a model that can be applied to UC with market problems is our further work.

TABLE 3  UC results for different wind penetration rates

| Wind penetration | Total cost/($) | Reserve cost/($) | Risk cost/($) |
|------------------|---------------|-----------------|--------------|
| Case 1           |               |                 |              |
| 10%              | [1405589, 1432799] | 38855           | 4475         |
| 20%              | [1304700, 1329160] | 76925           | 8265         |
| 30%              | [1183370, 1276290] | 106390          | 15411        |
| Case 3           |               |                 |              |
| 10%              | [1388120, 1412920] | 32106           | 3355         |
| 20%              | [1293950, 1337920] | 66408           | 8033         |
| 30%              | [1121560, 1245550] | 101940          | 7818         |
NOMENCLATURE

Constants and variables

\[ \epsilon_{ij} \] Wind power forecast errors of wind field \( i \) at time \( j \)

\[ f_\epsilon(\epsilon_{ij}) \] The marginal distribution of \( \epsilon_{ij} \)

\[ \Sigma \] The covariance matrix

\( u \) The degrees of freedom of \( Z \)

\[ d_G \] The squared Mahalanobis distance

\[ p_{\text{up}}^{\max} \] The upper bounds of wind power for wind farm \( w \) at time \( t \)

\[ p_{\text{min}}^{\max} \] The lower bounds of wind power for wind farm \( w \) at time \( t \)

\[ P_G^{\text{pr}} \] The rated capacity of wind farm \( w \)

\[ p_{\text{g}}^{\text{pr}} \] The forecast power of wind farm \( w \) during time period \( t \)

\[ C_R(\mathbf{R}_L^{\text{up}}, \mathbf{R}_L^{\text{down}}) \] The reserve cost function

\[ p_d \] The system load of node \( d \) at time \( t \)

\[ \epsilon_{ij} \] Wind power forecast errors of wind field \( i \) at time \( j \)

\[ f_\epsilon(\epsilon_{ij}) \] The marginal distribution of \( \epsilon_{ij} \).

\[ 1_{[0,\infty]}(\cdot) \] An indicator function

\( C \) A copula function

\( c_1,c_2,c_3,c_4 \) Constant coefficient matrix

\( c_{\text{CW}} \) The unit costs of wind curtailment

\( q_L \) The unit costs of load shedding

\( D_g,U_g \) The climbing-down and climbing-up rates of unit \( g \)

\[ F_G \] The total cost of start-up costs

\[ F/\max \] The transmission capacity of line \( l \)

\[ f_G(\mathbf{x}) \] The error distribution during period \( t \)

\[ G_{d/l} \] The shift distribution factor of load demand to line \( l \)

\[ G_{g/l} \] The shift distribution factor of unit \( g \) to line \( l \)

\[ G_{w/l} \] The shift distribution factor of wind farm \( w \) to line \( l \)

\( M \) A large positive integer

\( \text{ND} \) The numbers of load demands

\( \text{NE} \) The number of normal inequalities

\( N_{\text{g}} \) Number of generations that can be conducted

\( \text{NG} \) The numbers of generators

\( \text{NM} \) The number of interval inequalities

\( N_t \) The sampling number

\( \text{NW} \) The numbers of wind farms

\( p_{{\text{g}},t}^{\text{gen}} \) The generation of unit \( g \) during time period \( t \)

\( p_{{\text{g}},t}^{\max} \) The generation limits of unit \( g \)

\( p_{{\text{g}},t}^{\min} \) The generation limits of unit \( g \)

\( P_{\text{up}}^{\text{wind}} \) The generation of wind farm \( w \) at time period \( t \)

\[ R_{\text{d}}^{\text{spin}} \] The down-spinning reserve requirements at time \( t \)

\[ R_{\text{u}}^{\text{spin}} \] The up-spinning reserve requirements at time \( t \)

\[ S_f \] The total cost of fuel costs

\[ T \] Total scheduling period

\[ T_{\text{g}},T_{\text{off}} \] The minimum on and off time intervals of unit \( g \)

\[ T_{\text{g}}^{\text{on}} \] The minimum on and off time intervals of unit \( g \)

\( t_{\text{NT}} \) A t-copula function with \( NT \) dimensions

\( \mu_{\text{g}},t \) The on/off status of unit \( g \) during time period \( t \)

\( \nu \) A variable of cut

\( \text{VoLS}_t \) The risk value of load shedding

\( \text{VoWG}_t \) The risk value of wind curtailment

\( x \) The corresponding actual power

\( y_p \) The point forecast power

\( y_p^{\max} \) The point forecast power

\( z_{\text{d}},t \) Start-up decisions

\( z_{\text{u}},t \) A binary variable

\( \alpha_{1,t},\alpha_{2,t} \) The probability of wind curtailment and load shedding

\( \alpha \) The location parameters of \( Z \)

\( \rho \) The correlation coefficient between variables

Acronyms

CCP Chance-constrained program
CIUC Chance-constrained interval UC
CVaR Conditional value at risk
JCDF Joint cumulative distribution function
SAA Sample average approximation
WPFE Wind power forecast errors

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