Nonperturbative signatures in pair production for general elliptic polarization fields

Z. L. Li, D. Lu, B. S. Xie, B. F. Shen, L. B. Fu and J. Liu

Key Laboratory of Beam Technology and Materials Modification of the Ministry of Education, College of Nuclear Science and Technology, Beijing Normal University - Beijing 100875, China
Beijing Radiation Center - Beijing 100875, China
Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences - Shanghai 201800, China
National Laboratory of Science and Technology on Computational Physics, Institute of Applied Physics and Computational Mathematics - Beijing 100088, China

received 17 April 2015; accepted in final form 1 June 2015
published online 17 June 2015

PACS 12.20.Ds – Specific calculations
PACS 11.15.Tk – Other nonperturbative techniques
PACS 32.80.-t – Photoionization and excitation

Abstract – The momentum signatures in nonperturbative multiphoton pair production for general elliptic polarization electric fields are investigated by employing the real-time Dirac-Heisenberg-Wigner formalism. For a linearly polarized electric field we find that the positions of the nodes in momentum spectra of created pairs depend only on the electric-field frequency. The polarization of external fields could not only change the node structures or even make the nodes disappear but also change the thresholds of pair production. The momentum signatures associated to the node positions in which the even-number photon pair creation process is forbidden could be used to distinguish the orbital angular momentum of created pairs on the momentum spectra. These distinguishable momentum signatures could be relevant for providing the output information of created particles and also the input information of ultrashort laser pulses.

Copyright © EPLA, 2015

Introduction. – Electron-positron (EP) pair creation in strong electric fields [1–3] is a fundamentally important exploration to probe the quantum vacuum. The experimental observation of nonperturbative tunneling pair production is still absent due to the fact that present available laser fields (laser intensity \( I \sim 10^{22}\) W/cm\(^2\)) are far below the Schwinger critical electric field \( E_{cr} = m^2/e \sim 1.32 \times 10^{16}\) V/cm (laser intensity \( I_{cr} \sim 10^{29}\) W/cm\(^2\)), where \( m \) is the electron rest mass and \(-e\) is the electron charge (we use \( \hbar = c = 1\)). However, with the rapid development of laser technology, the stronger fields will be achieved in the near future through some laser facilities [4–6]. This improves greatly the hopes to realize the observation of EP pair production from vacuum [7–12]. In general two different pair production mechanisms are identified as nonperturbative pair creation process (\( \gamma < 1\)) and perturbative multiphoton pair production process (\( \gamma \gg 1\)), where \( \gamma = m\omega/eE_0\) is the well-known Keldysh adiabatic parameter [13], \( \omega \) and \( E_0 \) are the frequency and strength of the external electric fields, respectively. Fortunately, the perturbative multiphoton process has been experimentally observed at the Stanford Linear Accelerator Center (SLAC) [14].

Compared to the cases of \( \gamma < 1\) and \( \gamma \gg 1\), obviously the case of \( \gamma \sim \mathcal{O}(1)\) is seldom studied because of the difficulty to get simple asymptotic formulae theoretically. However, more and more evidence indicates the importance and appeal of pair production in this regime because it contains the signatures of multiphoton process and nonperturbative mechanism. Recently many novel phenomena have been discovered [15–18], for instance, the dynamically assisted Schwinger mechanism [16,17] and the effective mass signatures in the nonperturbative threshold regime [18].

For a rotating circular field, pair production are studied numerically with the real-time Dirac-Heisenberg-Wigner (DHW) formalism [19–21] in ref. [22] and semiclassically using WKB-like approximation in ref. [23]. In this letter, our studies are focused on the momentum signatures in the nonperturbative multiphoton pair creation for arbitrarily polarized fields by using the DHW formalism. This study,
on the one hand, is valuable to deepen the understanding of both the nonperturbative mechanism and the perturbative multiphoton process in arbitrary polarization fields. On the other hand, it is also helpful to understand other relevant physical processes, such as cosmological pair production [24], heavy-ion collisions [25], and ionization of atoms and molecules [26,27], especially the above-threshold ionization with elliptically polarized laser pulses [28,29].

Theoretical formalism. – Here we consider the field in an antinode of the standing wave formed by two counterpropagating laser pulses with appropriate polarizations. Since the spatial scales of the EP pair production are smaller than the spatial focusing scales of the laser pulse, the spatial effects are not significant and the magnetic field is absent, therefore, the general polarization field can be written as

$$E(t) = \frac{E_0}{\sqrt{1 + \delta^2}} \exp\left(-\frac{t^2}{2\tau^2}\right)[\cos(\omega t + \phi), \delta \sin(\omega t + \phi), 0]^T,$$

where $E_0$ is the maximal field strength, $\tau$ defines the pulse duration, $\omega$ is the laser frequency, $\phi$ is the carrier phase, and $-1 \leq \delta \leq 1$ represents the polarization of the electric field, the factor $\sqrt{1 + \delta^2}$ is used to ensure the same laser intensity for different polarization fields. For convenience, we set $\tau = 100/m$, $\phi = 0$ and $0 \leq \delta \leq 1$ throughout this paper.

Our numerical results are based on the DHW formalism [19–21]. In ref. [20] the authors showed that the DHW formalism would be recovered to the quantum Vlasov equation (QVE) for a spatially homogeneous and time-dependent electric field with linear polarization. However, one of the advantages of the DHW formalism is that it can also solve more complex electric fields such as (1). In order to precisely obtain the momentum distribution function of created EP pairs $f$ for the spatially homogeneous electric field (1), we employ the method used in [22] to map the DHW formalism to

$$\dot{f} = 1/2 \delta \hat{\tau} \mathcal{F} \mathcal{w}_9,$$
$$\mathcal{w}_9 = \mathcal{H}_9 \mathcal{w}_9 + 2(1 - f)G \mathcal{e}_1.$$  \hspace{1cm} (2)

Here the dot denotes a total time derivative, $\mathcal{e}_1 = (m/\Omega(p), p/\Omega(p), 0, 0)^T$, $\Omega(p) = (m^2 + p^2)^{1/2}$ is the total energy of electrons with the kinetic momentum $p = q - eA(t)$, $q$ is the canonical momentum, $A(t)$ is the vector potential; $\mathcal{w}_9$ represents an auxiliary 9-component vector; $\mathcal{F} = \left(-p^T/m_0 \right)$ is a $10 \times 9$ matrix, $\mathcal{G} = (0 I_9)$ is a $9 \times 10$ matrix, and

$$\mathcal{H}_9 = \begin{pmatrix} e p \cdot E^T/\omega^2(p) & -2p \times -2m \\ -2p \times 0 & 0 \\ 2(m^2 + p \cdot p^T)/m & 0 & 0 \end{pmatrix}$$

is a $9 \times 9$ matrix. Thus, we can get the one-particle momentum distribution function $f(q, \epsilon)$ by numerically solving eq. (2) with the initial conditions $f(q,-\infty) = \mathcal{w}_9(q,-\infty) = 0$.

Results. – As a set of ordinary differential equations, eq. (2) can be solved with the fourth-order Runge-Kutta method. In our numerical computations, the calculation step of momentum components is $\Delta q_{x,y,z} < 0.01[m]$. To ensure the reliability of our numerical results, we repeat to get the same result of fig. 4 in ref. [18] by setting $\delta = 0$. We also get the momentum distribution which has the same order of magnitude as that of fig. 6 in ref. [22] by setting $\delta = 1$ even if field profiles are different but field parameters are the same.
In fig. 1, we plot the momentum spectra of created EP pairs for linearly polarized electric field (δ = 0) with different Keldysh parameters, from (a) to (f), γ = 2, 3, 4, 0.5, 0.75, and 1. One can see node structures in the rings of the momentum spectra of the created particles by using the field (1). From fig. 1(f) where γ = 1, we find that: for an even-number photon pair creation, the nodes are located at the positions where the momentum qz is an even multiple of half the frequency (cf. the 8-photon ring with 10 nodes); however, for an odd-number photon process, the nodes are located at the positions where qz is an odd multiple of half the laser frequency (cf. the 7-photon ring with 8 nodes). This result can be explained via the factor \[1 + (1 - n)^{2s} \cos(4qz/\omega \cdot \arctan \gamma)\] (where s is the spin of created particles, s = 0 for bosons and s = 1/2 for fermions, n is the absorbed photon number), in the approximate analytical solution of the pair production probability for a sinusoidal field with an infinite period [30].

Furthermore, from the factor, one can see that the node positions depend on the Keldysh parameter.

To carefully analyze our results shown in fig. 1, we plot the momentum distribution function of n-photon pair creation \(f_n(q_x, +\infty)\) as a function of momentum \(q_x (q_y \geq 0, q_z = 0)\) in fig. 2. Obviously, (a) indicates that the nodes (the minimum values) of even-number photon pair creation are still located at the positions where the values of \(q_x\) are integral multiples of the frequency for different ω and the fixed \(E_0\). Furthermore, in (b), we find that for a given frequency the node positions will not be changed by the electric-field strength. These results imply that the positions of the nodes in the momentum spectra depend only on the field frequency rather than the values of γ. This still holds true for a sinusoidal field without Gaussian envelope. Considering this result and the effective mass signatures [18] in nonperturbative multiphoton pair production, the approximate analytical expression of particle momentum distributions [30] can be modified as

\[
f_n(q_x, +\infty) \sim \frac{2\pi^2}{\pi} w(q) \left[1 + (-1)^{n/2s} \cos \beta^*\right] \times \delta(2\Omega_{\text{rms}}(q) - n\omega),
\]

where \(w(q) = \exp(-\pi E_{\text{cr}}/E_0) \cdot [g(\gamma) + Qb_1(\gamma)(q_y^2 + q_z^2)/m^2 + 2b_2(\gamma)q_x^2/m^2]\) with \(b_2(\gamma) = -\gamma b_1(\gamma)\) and \(b_1(\gamma) = g(\gamma) + 2\gamma g'(\gamma)/2;\) the prime represents \(d/d\gamma,\) and \(g(\gamma) = \frac{1}{\pi} \int_0^\infty (1 - u^2)^{1/2}/[1 + (\gamma u)^2]^{1/2} du;\) \(Q\) is a parameter used to include the contributions of the high-order terms of the momentum; \(\beta^* = 2\pi q_z/\omega\) and \(\Omega_{\text{rms}}(q) = (q^2 + m^2)^{1/2}\) is the effective energy of the electrons with the effective mass \(m_*.\)

The semianalytical results (dashed blue lines) are shown in fig. 2(c) and compared with the DHW results (solid red lines). We can see that these two results are in good agreement with each other (see footnote 2).

In addition, our result is valuable to precisely measure the frequency of external fields with or without envelopes by analyzing the node positions in momentum spectra. For instance, the distance between any two adjacent nodes in a ring along the \(q_z\) direction is exactly the field frequency. Our findings can also explain why the spacing between the neighbouring peaks in the center of the longitudinal momentum distribution \(f(q_x, +\infty)\) is equal to the laser frequency (cf. fig. 2 in ref. [32]). When the electric-field parameters in [32] are chosen as \(E = 0.1E_{\text{cr}}, \tau = 100/\mu\) and \(\sigma = \omega \tau = 5,\) one can find that \(\gamma \sim 1,\) and the pair production belongs to the nonperturbative multiphoton regime. Therefore, the center-of-momentum spectra will be split according to eq. (3). A similar result can also be seen in fig. 1(d) or (e).

For arbitrarily polarized electric fields, the momentum spectra of created pairs in the \((q_x, q_y)\)-plane with \(q_z = 0\) are shown in fig. 3. It is found that the node structures in the particle momentum spectra gradually disappear as the polarization increases, and the rings become continuous and uniform for the circular polarization \(\delta = 1.\) This is because the rapidly rotating electric fields are more isotropic in the \((x, y)\)-plane. Additionally, we find that the number of created particles in the center-of-momentum spectra decreases as the polarization increases, and reaches a minimum at \(\delta = 1.\) Moreover, it is found that the rings

![Fig. 2: (Color online) Momentum distribution function of n-photon pair creation \(f_n(q_x, +\infty)\) as a function of momentum \(q_x (q_y \geq 0, q_z = 0)\). Panel (a) is for \(n = 8\) of fig. 1(c), 10 of fig. 1(b), and 14 of fig. 1(a), from top to bottom. Panel (b) corresponds to 8-photon process with the fixed frequency \(\omega = 0.4m\) and \(E_0/E_{\text{cr}} = 0.4, 0.3, 0.2,\) and 0.1, from top to bottom. Panel (c) shows the comparisons between the numerical results (solid red lines) and the approximate analytical ones (dashed blue lines) with the parameter \(Q = 0.88\) for \(n = 6, 7,\) and 8 in fig. 1(c), from top to bottom.](image-url)

1Note that in order to correctly predict the node positions in the case of \(\gamma = 1,\) the number 4 in the factor should be replaced by 8, refer to [31].

2Note that due to the effect of a Gaussian envelope and the finite period, the delta function in eq. (3) is too ideal to exactly describe the ring widths in momentum spectra. Therefore, the delta function is just used to estimate the positions of the maximum values of \(f_n(q)\). To compare with the numerical result, the strength of delta function is chosen as \(P = 923.\) Obviously \(P\) also includes the contributions of higher-order terms of momentum.
shrink with the polarization increasing. For example, the radius of the ring corresponding to 6-photon pair production (the smallest ring) is about 0.628 m for $\delta = 0$, while it is about 0.568 m for $\delta = 1$. These indicate that a large $\delta$ increases the thresholds of multiphoton pair creation for a fixed field frequency. Although the electric fields we used can give the same expression of the effective mass, $m^* = m\sqrt{1 + e^2E_0^2/(2m^2\omega^2)}$, for different polarizations according to [18], the thresholds still grow with the increase of $\delta$. This is beyond the effective mass model in [18].

In addition, we also present the momentum spectra for different polarizations in the $(q_x, q_z)$-plane with $q_y = 0$ and in the $(q_y, q_z)$-plane with $q_x = 0$, respectively, see fig. 3. In the $(q_x, q_z)$-plane, one can see that for a very small $\delta$, the node structures are pronounced and their positions can be determined approximately by eq. (3). However, when the $\delta$ becomes large, the field $E_y$ will be very strong, then the node structures will be disturbed by the combined effects of the field $E_x$ and $E_y$, and finally disappear for a large $\delta$. In the $(q_y, q_z)$-plane, it shows that the node structures are not obvious except near $q_y = 0$, because a small value of $\delta$ corresponds to a small value of $E_y$. With the polarization increasing, larger rings are present, and a channel along the $q_z$ direction is gradually opened up near $q_y = 0$.

To study the relation between the node structures in momentum spectra and the absence of even-number photon pair production, the distribution function $f(+\infty)$ as a function of the laser frequency is plotted in fig. 4. By analyzing fig. 4(a), we find that for a linearly polarized field $\delta = 0$, once the momentum component along the external field is zero, namely $q_x = 0$, the even-number photon pair creation will not occur. A similar result can also be seen for $\delta \neq 0$, i.e., the even-number photon pair creation vanishes at $q_x = q_y = 0$, see fig. 4(b) and (c). The former result can be explained quantitatively employing eq. (3). When $q_x = 0$, the expression becomes $f \propto [1 + (-1)^{n+1}]$ for EP pair production. This shows that the momentum distribution function has a minimum for even-number photon pair creation, i.e., the even-number photon process vanishes. Another result can also be interpreted qualitatively applying eq. (3). However, since there are two electric fields $E_x$ and $E_y$ in the $(x, y)$-plane,
nonperturbative signatures in pair production for general elliptic polarization fields.

Conclusions. – We have investigated momentum signatures in nonperturbative multiphoton pair production for general elliptic polarized electric fields by using the real-time DHW formalism. It is found that the node positions in momentum spectra depend only on the field frequency for linear polarization and will be changed by other polarizations. Furthermore, the thresholds of multiphoton pair creation grow with the increase of polarization under the same laser intensity. The momentum signatures not only possibly provide us with the OAM of created particles but also can present some signatures of ultrashort laser pulses, such as the frequency.

This work was supported by the National Natural Science Foundation of China (NSFC) under Grant Nos. 11475026, 11175023 and 11335013, and also supported partially by the Open Fund of National Laboratory of Science and Technology on Computational Physics at IAPCM and the Fundamental Research Funds for the Central Universities (FRFCU).

References

[1] Sauter F., Z. Phys., 69 (1931) 742.
[2] Heisenberg W. and Euler H., Z. Phys., 98 (1936) 714.
[3] Schwinger J., Phys. Rev., 82 (1951) 664.
[4] See http://www.extreme-light-infrastructure.eu/.
[5] See http://www.xfel.eu/.
[6] Ringwald A., Phys. Lett. B, 510 (2001) 107; Alkofer R., Hecit M. B., Roberts C. D., Schmidt S. M. and Vinnik D. V., Phys. Rev. Lett., 87 (2001) 193902; Roberts C. D., Schmidt S. M. and Vinnik D. V., Phys. Rev. Lett., 89 (2002) 153901.
[7] Di Piazza A., Müller C., Hatsagortsyan K. Z. and Keitel C. H., Rev. Mod. Phys., 84 (2012) 1177.
[8] Bell A. R. and Kirk J. G., Phys. Rev. Lett., 101 (2008) 200403.
[9] Di Piazza A., Lötstedt E., Milstein A. I. and Keitel C. H., Phys. Rev. Lett., 103 (2009) 170403.
[10] Bulanov S. S., Mur V. D., Narozhny N. B., Nees J. and Popov V. S., Phys. Rev. Lett., 104 (2010) 220404.
[11] Abdukerim N., Li Z. L. and Xie B. S., Phys. Lett. B, 726 (2013) 820.
[12] Li Z. L., Lu D. and Xie B. S., Phys. Rev. D, 89 (2014) 067701; Li Z. L., Lu D., Xie B. S., Fu L. B., Liu J. and Shen B. F., Phys. Rev. D, 89 (2014) 093011.
[13] Keldysh L. V., Sov. Phys. JETP, 20 (1965) 1307.
[14] Burke D. L. et al., Phys. Rev. Lett., 79 (1997) 1626; Bamber C. et al., Phys. Rev. D, 60 (1999) 092004.
[15] Ruf M., Mocken G. R., Müller C., Hatsagortsyan K. Z. and Keitel C. H., Phys. Rev. Lett., 102 (2009) 080402.
[16] Schützhold R., Gies H. and Dunne G., Phys. Rev. Lett., 101 (2008) 130404.
[17] Dunne G. V., Gies H. and Schützhold R., Phys. Rev. D, 80 (2009) 111301.
[18] Kohlfürst C., Gies H. and Alkofer R., Phys. Rev. Lett., 112 (2014) 050402.
[19] Bielynicki-Birula I., Görnicki P. and Rafelski J., Phys. Rev. D, 44 (1991) 1825.
[20] Hebendreit F., Alkofer R. and Gies H., Phys. Rev. D, 82 (2010) 105026.
[21] Hebendreit F., Alkofer R. and Gies H., Phys. Rev. Lett., 107 (2011) 180403.
[22] Blinne A. and Gies H., Phys. Rev. D, 89 (2014) 085001.
[23] Strobel E. and Xue S. S., *Phys. Rev. D*, **91** (2015) 045016.

[24] Parker L., *Phys. Rev. Lett.*, **21** (1968) 562.

[25] Kharzeev D., Levin E. and Tuchin K., *Phys. Rev. C*, **75** (2007) 044903.

[26] Borca B., Frolov M. V., Manakov N. L. and Starace A. F., *Phys. Rev. Lett.*, **87** (2001) 133001.

[27] Shafir D. et al., *Phys. Rev. Lett.*, **111** (2013) 023005.

[28] Bashkansky M., Buckbaum P. H. and Schumacher D. W., *Phys. Rev. Lett.*, **59** (1987) 274.

[29] Paulus G. G., Zacher F., Walther H., Lohr A., Becker W. and Kleber M., *Phys. Rev. Lett.*, **80** (1998) 484; Paulus G. G., Grasbon F., Dreischuh A., Walther H., Köpold R. and Becker W., *Phys. Rev. Lett.*, **84** (2000) 3791.

[30] Popov V. S., *Pis’ma Zh. Eksp. Teor. Fiz.*, **18** (1973) 435 (*JETP Lett.*, **18** (1973) 255); Popov V. S., *Yad. Fiz.*, **19** (1974) 1140 (*Sov. J. Nucl. Phys.*, **19** (1974) 584); Marinov M. S. and Popov V. S., *Fortschr. Phys.*, **25** (1977) 373.

[31] Mocken G. R., Ruf M., Müller C. and Keitel C. H., *Phys. Rev. A*, **81** (2010) 022122.

[32] Hebenstreit F., Alkofer R., Dunne G. V. and Gies H., *Phys. Rev. Lett.*, **102** (2009) 150404.