Self-consistent equilibrium of a two-dimensional electron system with a reservoir in a quantizing magnetic field: Analytical approach

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Abstract

An analytical approach has been developed to describe grand canonical equilibrium between a three dimensional (3D) electron system and a two dimensional (2D) one, an energy of which is determined self-consistently with an electron concentration. Main attention is paid to a Landau level (LL) pinning effect. Pinning means a fixation of the LL on a common Fermi level of the 2D and the 3D systems in a finite range of the magnetic field due to an electron transfer from the 2D to the 3D system. A condition and a start of LL pinning has been found for homogeneously broadened LLs. The electronic transfer from the 3D to the 2D system controls an extremely sharp magnetic dependency of an energy of the upper filled LL at integer filling of the LLs. This can cause a significant increase of inhomogeneous broadening of the upper LL that was observed in recent local probe experiments.

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I. INTRODUCTION

A model of grand canonical equilibrium of a 2D system with a reservoir was proposed to explain the integer quantum Hall effect (IQHE) more than twenty years ago\(^1\). This so-called reservoir hypothesis can describe main features accompanied to the IQHE such as plateaus in a Hall resistance\(^1\), the Shubnikov-de Hass effect\(^2\) and oscillations of a magnetization and a thermoelectric power\(^3\). Key points of the reservoir model are following: a 2D chemical potential is fixed by the reservoir and it doesn’t depend on the magnetic field, a carriers concentration and 2D subband energies oscillate in the magnetic field. There was a wide discussion about a relevance of the hypothesis to conventional samples where the IQHE observed and it is still not clear. But there are structures where a reservoir is created nearby to a 2D system and one should take into account the carriers exchange. These are tunnel and resonant tunnelling diodes\(^4\). For these devices it was shown that a carriers transfer between 2D and 3D systems takes place and a 2D subband energy oscillates while a magnetic field perpendicular to the 2D plane sweeps\(^5\). Moreover the 2D subband energy oscillations can be so strong that a pinning effect of a partial filled Landau level (PFLL) takes place\(^6,7\). Usually authors employ self-consistent calculations to get magnetic dependencies of a carriers concentration and a 2D subband energy and to my knowledge there is no any thorough analytical investigation of the LL pinning effect. In this paper I would like to consider thoroughly pinning of a homogeneously broadened PFLL with aims to draw analytical expressions, to find a PFLL pinning condition and to make other estimations.

Recently a new local probe technique so-called a subsurface charge accumulation (SCA) imaging has been developed by S. H. Tessmer, R. C. Ashoori et al\(^8\) to investigate a local compressibility of a 2D electron system (2DES). Extraordinarily sensitive to a magnetic field features have been revealed in the images at the magnetic fields close to the integer quantum Hall state of the 2DES. The SCA is measured as a charge on a tip induced by an applied ac voltage between the metal tip and a contact to the 2DES. A frequency of the signal is chosen quite low to provide the charge accumulation without delay. In this case the SCA is proportional to a capacitance between the tip and the 2DES that is determined by the local compressibility of the 2DES. In other words all applied ac voltage drops between the tip and the 2DES, namely, Fermi levels of the 2DES and the contact are equal. This means equilibrium between the 2DES and the contact. Hence at the SCA image analysis
one should take into account effects of this equilibrium.

An analytic approach of cause requires some simplifications but the all made assumptions are appeared to be relevant to real tunnel structures with 2DESs. So I start in Sec. II with a $\delta$-profile of a quantum well that allows me to avoid many subbands in a 2DES. As usual an electron reservoir is considerably away from the 2DES that permits to neglect a variation of a width of a localized wave function and to use only the Poisson equation in Sec. III instead of self-consistent calculations of the Poisson and the Shrödinger equations. The next assumption is in homogeneous broadening of the LLs. In this case a shape of the LL can be described analytically for a large LL number. Relevance of this approach is also discussed in Sec. III. In Sec. IV the subband energy oscillations in the magnetic field have been considered when PFLL pinning takes place and a starting value of the magnetic field has been determined for PFLL pinning. In Sec. V I conclude this paper.

II. MODEL OF STRUCTURE

Let us consider a quantum well (QW) separated from a metal surface with a spacer barrier of thickness $d$ (see Fig. 1). To simplify calculations a QW potential profile is supposed to be of the following form:

$$U(z) = -\alpha \delta(z).$$

It is well known that such QW has only one localized state at the energy $E_0 = -m\alpha^2/2\hbar^2$ that is also valid in this situation because I suppose

$$d \gg \hbar^2/m\alpha.$$  

Here $2\hbar^2/m\alpha$ is an effective width of the localized wave function; $m$ is an effective mass of the electron and $\hbar$ is the Planck’s constant. It should be mentioned that a depletion effect inherent to a metal-semiconductor junction is out of the consideration and the potential profile of the structure at a magnetic field is shown in Figure II. As the next step I consider a dirty metal so that a magnetic field applied alone $z$ axis doesn’t influence on the metal density of states (DOS) and creates the Landau levels (LLs) in to the 2DES. This approximation is quite usual for semiconductor devices especially if one considers metal contacts. A temperature is supposed to be zero.
FIG. 1: Potential profile of the model structure with the Landau staircase and the 2D subband level $E_0$ in the quantizing magnetic field. Dashed arrows show the charge transfer and the 2D subband shift at the magnetic field increase while the PFLL is pined at the common Fermi level.

III. PINNING OF LANDAU LEVEL

A magnetic field applied alone the $z$ axis quantizes the 2D states into the Landau levels (LLs). It is well known that a Fermi energy of a 2DES oscillates in a quantizing magnetic field for a constant electron concentration of a 2DES. In my case the 2DES Fermi level is fixed by the metal thus a constant concentration $n$ is not relevant. Carried out in Ref. [5], thorough calculations show that $E_0$ and $n$ are oscillating functions of the magnetic field. My task here is to demonstrate that there is a situation when the subband energy $E_0$ changes in the magnetic field in such manner that the PFLL is almost fixed or pinned on the common Fermi level of the junction. Let me start from a definition of a 2D density of states (DOS) $G(\varepsilon, B)$:

$$n = \int_{\mu - E_0}^{\mu} G(\varepsilon, B) d\varepsilon, \quad (3)$$

where $\mu$ is the common Fermi energy. Applying a variation $\delta B$ of the magnetic field one should expect the following variation of the electron concentration:

$$\delta n = -G(\mu - E_0, B) \delta E + \int_{0}^{\mu - E_0} \frac{\partial G(\varepsilon, B)}{\partial B} d\varepsilon \delta B. \quad (4)$$

Now I would like to substitute on the place of the $\delta B$ a variation of the cyclotron energy $\delta \varepsilon_c = \hbar e \delta B / m$ and consider a derivative $\partial G(\varepsilon, B) / \partial \varepsilon_c$. To determine it’s value let me consider the DOS in detail. This value is found as a sum of spectral functions of the
broadened LLs:

\[ G(\varepsilon, B) = \sum_{i=0}^{\infty} \beta \varepsilon_c \varphi_i(\varepsilon, B). \]  

(5)

Here \( \beta \varepsilon_c \) is a degeneracy of the LL per unit area of the 2DES (\( \beta = m/\pi \hbar^2 \) is the DOS at zero magnetic field). As for the spectral function it has the following dependence on \( \varepsilon_c \):

\[ \varphi_i(\varepsilon, B) = \frac{1}{\Gamma} \psi \left[ \left( \frac{\varepsilon - (i + 1/2)\varepsilon_c}{\Gamma} \right)^2 \right]^{1/2}. \]  

(6)

\( \Gamma \) is a width of the LL, \( i \) is a LL number, the function \( \psi \) describes a shape of the LL. Here I neglect a LL spin splitting and an electron-electron interactions inside the 2DES.

A shape of the LLs was a subject of an intensive study during the last three decades\(^9,11,12\). The complexity of the problem arises from the fact that in the absence of the disorder the energy spectrum is discrete. As a result, the self-energy of an electron appears to be real in any finite order of the perturbation theory. Therefore, obtaining of a finite width of the LL requires summation of the entire diagram expansion. It was demonstrated\(^9,11\) that such a summation is possible when the number of the LL is large. The simplifications, arising in this limit, are different in the case of a short-range and a smooth disorder. In the former case the correlation radius \( R_c \) of the disorder less than the magnetic length \( \ell \) and only a subsequence of diagrams without self-intersections contributes to the self-energy, or, in other words, the self-consistent Born approximation\(^9\) becomes asymptotically exact. The shape of the LL in this case is close to semielliptical. For a smooth disorder, with \( \ell \ll R_c \ll R_L \approx i^{1/2} \ell \) (\( R_L \) is the Larmour radius) the LL shape had been found Gaussian. In this case the random potential can be renormalized to effective one with \( R_c \geq R_L \) and hence it becomes a long-range potential\(^11\). In the case of the long-range disorder \( R_c \gg R_L \) the semiclassical approach is valid when the disorder simply modulates the subband energy \( E_0 \) and LL staircase follows this modulation. This means an inhomogeneous broadening of the LL and it will be considered somewhere else. Thus only the short-range disorder leads to the homogeneous LL broadening.

As for the model under consideration the correlation radius \( R_c \) could not be longer than the distance between the 2DES and the metal \( d \) because of the metal screening\(^15\). Hence the short-range disorder approximation is adequate when \( \ell > d \). In this case the LL spectral function can be written as

\[ \varphi_i(\varepsilon, B) = \frac{2}{\pi \Gamma} \left[ 1 - \left( \frac{\varepsilon - (i + 1/2)\varepsilon_c}{\Gamma} \right)^2 \right]^{1/2}, \]  

(7)
where $\Gamma = \gamma \varepsilon_c^{1/2}$, $\gamma = (2\hbar/\pi \tau)^{1/2}$ and $\tau$ is a relaxation time at $B = 0$.

Other mechanisms of LL broadening such as phonon-electron, electron-electron interactions are negligible because the temperature is zero. LL broadening caused by electron tunnelling between the 2DES and the metal is possible but also negligible due to a very low tunnel transparency in accordance with Eq. (2).

Thus having the LL shape determined, one can make the next step. From Eqs. (4) and (5) one can get:

$$
\delta n = -G_F \delta E_0 + \int_0^{\mu - E_0} \sum_{i=0}^{\infty} \left( \beta \varphi_i(\varepsilon_c, \varepsilon_c) + \beta \varepsilon_c \frac{\partial \varphi_i(\varepsilon_c, \varepsilon_c)}{\partial \varepsilon_c} \right) d\varepsilon \delta \varepsilon_c.
$$

(8)

Here I have $G_F = G(\mu - E_0, B)$. In accordance with equations (4) and (5) one can simplify the equation in to the following:

$$
\delta n = -G_F \delta E_0 + \frac{n}{\varepsilon_c} \delta \varepsilon_c + \int_0^{\mu - E_0} \sum_{i=0}^{\infty} \beta \varepsilon_c \frac{\partial \varphi_i(\varepsilon_c, \varepsilon_c)}{\partial \varepsilon_c} d\varepsilon \delta \varepsilon_c.
$$

(9)

According to Eq. (7) the derivative in the last term of this equation can be expressed as:

$$
\frac{\partial \varphi_i(\varepsilon_c, \varepsilon_c)}{\partial \varepsilon_c} = -\frac{\varphi_i(\varepsilon_c, \varepsilon_c)}{2\varepsilon_c} - \left( \frac{i + 1/2}{2} + \frac{\varepsilon_c}{2\varepsilon_c} \right) \frac{\partial \varphi_i(\varepsilon_c, \varepsilon_c)}{\partial \varepsilon},
$$

(10)

or

$$
\frac{\partial \varphi_i(\varepsilon_c, \varepsilon_c)}{\partial \varepsilon_c} = -\left( \frac{i + 1/2}{2} \right) \frac{\partial \varphi_i(\varepsilon_c, \varepsilon_c)}{\partial \varepsilon} - \frac{1}{2\varepsilon_c} \frac{\partial [\varepsilon \varphi_i(\varepsilon_c, \varepsilon_c)]}{\partial \varepsilon}.
$$

(11)

Now one can easy perform integration in the last term of Eq. (9) and get the following:

$$
\delta n = -G_F \delta E_0 + \frac{n}{\varepsilon_c} \delta \varepsilon_c - \sum_{i=0}^{\infty} \beta \varepsilon_c \left( \frac{\mu - E_0}{2\varepsilon_c} + \frac{i + 1/2}{2} \right) \varphi_i(\mu - E_0, \varepsilon_c) \delta \varepsilon_c,
$$

(12)

where I have already used that $\Gamma \ll \varepsilon_c$ and set $\varphi_i(0, \varepsilon_c) = 0$. Following this approximation further one can get from Eq. (5) that $G_F = \beta \varepsilon_c \varphi_N(\mu - E_0, \varepsilon_c)$ ($N$ is a number of the PFL). In this case the Eq. (12) is modified to the following:

$$
\delta n = -G_F \delta E_0 + \frac{n}{\varepsilon_c} \delta \varepsilon_c - \sum_{i=0}^{\infty} \beta \varepsilon_c \left( \frac{\mu - E_0}{2\varepsilon_c} + \frac{N + 1/2}{2} \right) G_F \delta \varepsilon_c,
$$

(13)

The next step is to find a relation between $\delta n$ and $\delta E_0$. The rigorous way is to solve self-consistently the Poisson and Schrödinger equations but I have inequality (2) that allows me to neglect the variations of the wave function and QW widths in compare with that of the QW depth. The depth variation can be easy evaluated from the variation of the electric
field between the metal and the 2DES. In this case the subband level $E_0$ follows the QW depth. Hence one can get the following:

$$\delta E_0 = e^2 \delta n/C_0. \quad (14)$$

$C_0$ is a geometric capacity per unit area, i.e., $C_0 = \kappa/d$, where $\kappa$ is a permittivity of the structure material$^{16}$. Combining Eqs. (13) and (14) one can determine the derivative $dE_0/d\varepsilon_c$ as:

$$\frac{dE_0}{d\varepsilon_c} = \frac{2e^2n - [\mu - E_0 + (N + 1/2)\varepsilon_c]G_Fe^2}{2e^2\varepsilon_cG_F + 2\varepsilon_cC_0}. \quad (15)$$

According to this equation the derivative of a central energy of the PFLL $dE_{LL}/d\varepsilon_c$ can be calculated as

$$\frac{dE_{LL}}{d\varepsilon_c} = N + \frac{1}{2} + \frac{dE_0}{d\varepsilon_c} = \frac{2e^2n + 2\varepsilon_cC_0(N + 1/2) + [(N + 1/2)\varepsilon_c - \mu + E_0]e^2G_F}{2\varepsilon_cG_F + 2\varepsilon_cC_0}. \quad (16)$$

PFLL pinning takes place when the derivative is zero or negligibly small, i.e., $dE_{LL}/d\varepsilon_c \ll 1$. Hence the pinning condition is the following:

$$G_F \gg n/\varepsilon_c + NC_0/e^2, \quad (17)$$

here I have used that in the case of the large $G_F$ the Fermi level is very close to the LL center and the following inequation is justified $\mu - E_0 - (N + 1/2)\varepsilon_c < \Gamma \ll \varepsilon_c$. Also the $N \gg 1$ is supposed.

IV. RESULTS AND DISCUSSION

Since the PFLL is pined on the Fermi level the subband energy $E_0$ should have a magnetic dependence. From Eqs. (15), (17) one can get a quite expectable result for this dependence:

$$dE_0/d\varepsilon_c = -(N + 1/2). \quad (18)$$

This means that under the pining condition a basement of the LL staircase $E_0$ shifts down on the energy scale and this shift compensates the PFLL energy increase due to the increase of the LL staircase step with the magnetic field (see Fig. 1). For integer filling of the LLs $G_F$ is zero and from Eqs. (15) and (16) one can get: $dE_0/d\varepsilon_c = e^2n/(\varepsilon_cC_0)$ and $dE_{LL}/d\varepsilon_c = $$
\[ N + 1/2 + e^2 n/(\varepsilon_c C_0). \] In the case of integer filling factor the electron concentration can be expressed as \( n = N\beta\varepsilon_c \) then one can get

\[ \frac{dE_0}{d\varepsilon_c} = \frac{e^2\beta N}{C_0}, \quad (19) \]

and

\[ \frac{dE_{LL}}{d\varepsilon_c} = (1 + \frac{e^2\beta}{C_0}) N - 1/2. \quad (20) \]

Here I suppose the \( N \)-th LL as just emptied. Let me make evaluations of the derivatives for the typical structures parameters. For the Al\(_x\)Ga\(_{1-x}\)As/GaAs heterostructures I have \( \beta = 0.28 \times 10^{11} \text{ (meV cm}^2)^{-1} \) and \( d = 100 \text{ nm} \). Hence I get \( e^2\beta/C_0 \approx 46 \). This means that the LL shift becomes 47 times faster in the magnetic field at the integer filling factor. As for the subband energy \( E_0 \) it also has the large derivative that signifies a rapid increase with the magnetic field. Thus the central energy \( E_{LL} \) demonstrates the step-like behavior while \( E_0 \) has a saw-tooth oscillation on the magnetic field (see Fig. 2). This is qualitatively consistent with experimental observations and numerical calculations.

The strong field dependencies of the levels \( E_0 \) and \( E_{LL} \) can explain an extraordinarily field sensitivity of the SCA features. Actually let me imagine an inhomogeneous variation of the subband and the PFLL energies in the 2DES. This means different local Fermi densities of states \( G_F \) that corresponds to different derivatives \( dE_{LL}/d\varepsilon_c \) and \( dE_0/d\varepsilon_c \) in accordance with Eqs. (15) and (16). Due to the difference of the derivatives PFLL broadening or a dispersion of a \( E_{LL} \) distribution changes with a magnetic field variation. The grater are values of the derivatives \( dE_{LL}/d\varepsilon_c \) the grater is variation of the dispersion. Thus at low values of \( G_F \), namely, near integer LL’s filling the \( E_{LL} \) dispersion should be most sensitive to the magnetic field variation. Moreover since the derivative \( dE_{LL}/d\varepsilon_c \) increases at the \( C_0 \) decrease, the closer is the reservoir to the 2DES (the lesser is \( d \)) the lesser are the derivatives \( dE_{LL}/d\varepsilon_c \) [see Eq. (16)] and the lesser is the sensitivity of the \( G_F \) and the \( E_{LL} \) dispersions and the SCA image contrast. This was observed in SCA images measured on a 2DES tunnel coupled to a \( n^+ \) reservoir. For instance, one can compare Figure 3 in Ref. 14 and Figure 3 in Ref. 13. In former case the image contrast changed markedly at the magnetic field variation of \( \delta B_1 = 1 \text{ kOe} \) and in the last case it is of \( \delta B_2 = 50 \text{ Oe} \). According to Eq. (20) at low values of \( G_F \) \( dE_{LL}/d\varepsilon_c \) is inverse proportional to \( C_0 \) that can be estimated from parameters of investigated structures. Thus in the case of Ref. 14 the tunnel distance is 40 nm that corresponds to \( C_0 = \kappa/d = 2.8 \times 10^{-3} \text{ F/m}^2 \). The parameters of Ref. 13 were
FIG. 2: Magnetic dependencies of the subband energy $E_0$ and the central energy $E_{LL}$ of the $N$-th PFLL. 

thoroughly considered in Ref. 8 where a $C_{\text{stray}}$ can be considered as a capacity between the 2DES and the ohmic contact or the reservoir. Authors had also evaluated the stray capacity as 1 fF. At this case a charging area of the 2DES is a disk of $L = 90$ nm diameter under the tip. Hence the equivalent capacity $C_0s$ can be estimated as $C_{0s} = C_{\text{stray}} / L^2 = 0.12$ F/m$^2$. By this means one can compare the relation of the magnetic field variations $\delta B_1 / \delta B_2 = 20$ and that of the LL centers derivatives, which can be estimated as a relation of the capacities $C_{0s} / C_0 \approx 43$ in accordance with Eq. (20). One can see that the relations are of the same order and the model proposed in this paper can be applied for description of the local probe experiments. However to describe the SCA features in details some improvement of the model is necessary. First of all the in-plane charge transfer should be considered. In this case the $C_0$ dependence upon $G_F$ should be found. Many-body effects also can be included in the model. The derivative $dE_F/dn$ therewith should be locally found and substituted in the place of $G_F$. 


Analytic Eqs. (16) and (15) allow to determine a magnetic field value when PFLL pinning starts. This question is not simple because according to Eq. (16) the derivative \( dE_{\text{LL}}/d\varepsilon_c \) is always positive. Hence there is no a sharp LL pinning transition but some criterion can be found. To do this let me consider a discrepancy between a LL pinning picture and a LL behavior in a closed 2DES with a constant electron number. In the last case a subband energy \( E_0 \) is constant and \( dE_{\text{LL}}/d\varepsilon_c = N + 1/2 \). Under the LL pinning condition \( E_{\text{LL}} \) is constant and Eq. (18) is valid. Thus it seems to be reasonable to choose \( dE_{\text{LL}}/d\varepsilon_c = -dE_0/d\varepsilon_c = (N + 1/2)/2 \) as a starting point for LL pinning. Hence according to Eq. (15) and one can get the following:

\[
\frac{e^2 n + \varepsilon_{c0} C_0 (N + 1/2)}{\varepsilon_{c0} e^2 G_F + \varepsilon_{c0} C_0} = \frac{N + 1/2}{2},
\]

(21)

where \( \varepsilon_{c0} \) is the cyclotron energy at the LL pinning start. Since I’m interesting in the starting point I should consider \( G_F \) value in the LL center, i.e., \( G_F = 2\beta\varepsilon_{c0}^{1/2}/\pi\gamma \) in accordance with Eq.(7). Also taking in to account that \( N + 1/2 = \nu/2 - 1 \approx n/\beta\varepsilon_{c0} \) for the half populated PFLL the start cyclotron energy \( \varepsilon_{c0} \) can be found as

\[
\varepsilon_{c0} = \frac{\pi^2\gamma^2}{2} \left( 1 + \frac{C_0}{2\beta e^2} \right)^2.
\]

(22)

It worth to note that this value is one order larger than cyclotron energy of the LLs resolution, which is determined from \( \varepsilon_{c1} \approx \Gamma \) as \( \varepsilon_{c1} \approx \gamma^2 \).

To finish the section it will be pertinent to discuss real structure parameters and magnetic fields to those the developed model is applicable. First of all one should satisfy \( N \gg 1 \) and \( \varepsilon_c \gg \Gamma \) conditions. This definitely requires quite high mobility and concentration of electrons in a 2DES. Let me suppose a ten times larger than one as a much more than one. So for a quite usual mobility of the 2DES as \( \eta = 100 \text{m}^2/(\text{V}\times\text{s}) \) one can resolve LLs at \( B = 1/\eta = 0.01 \text{T} \). Since according to Eq. (22) PFFT pinning will start at \( B = B_0 \approx 0.1 \text{T} \). In this case one should provide at least ten LLs populated that corresponds to \( E_F \approx 2 \text{meV} \) or \( n \approx 6 \times 10^{10} \text{cm}^{-2} \). It is worth noting the larger is the electron concentration the better is the model applicability. Now one should provide homogeneous LL broadening that is to locate a metal at the distance \( d \ll \ell \) from the 2DES plane. At \( B = B_0 \) the magnetic length \( \ell \) is about 250 nm. Thus \( d = 25 \text{nm} \) will be sufficient to assure homogeneous broadening of the LLs up to \( B = 10 \text{T} \). At the last step one need to justify inequality (2). This means that QW width should be about 3 nm.
V. CONCLUSIONS

In conclusion it is noteworthy that some results can be expanded on a closed 2DES with a constant electron concentration $n$. In this case it would be more relevant to say about a Fermi level capture by the PFLL. Condition of the capture can be derived in similar way like that of LL pinning. It can be obtained from Eq. (17) setting $C_0 = 0$ that is equivalent to an infinitely far reservoir. The same setting should be apply to get a start of the Fermi level capture from Eq. (22).

Summarizing thermodynamic equilibrium between a 2DES and a 3D electron system in a quantizing magnetic field has been considered in the case of the homogeneous broadened Landau levels. Parameters of the proposed model have allowed to consider analytically such effects as PFLL pinning and the subband energy oscillations in the 2DES. A condition and a starting point of PFLL pinning have been defined and a starting value of a cyclotron energy has been determined. Moreover found dependencies of $E_{\text{LL}}$ and $E_0$ up on $C_0$ allow to describe qualitatively the extraordinary sensitivity of the SCA features observed recently in local probe experiments. All results are in a good qualitative agreement with experimental data.

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Strictly speaking this estimation is valid in the case of an electrostatic disorder formed by electric field modulations. Besides the electrostatic there is also a structural disorder originated from QW width or layer thickness modulations and QW depth or layer composition those. In this case $R_c$ is determined by a correlation radius of these modulations $R_m$ that depends on technological parameters. In this case to provide homogeneous LL broadening one needs satisfy $\ell \gg R_m$.

It is important to note that such simple expression of $C_0$ is a following of the homogeneous model. In the case of inhomogeneous LL broadening one should take into account an in-plane charge exchange that leads to a coordinate dependency of $C_0$ and in particularly it’s dependency upon the local value of $G_F$. The Eq. (14) therewith is valid locally.

A direct comparison of the figures is not eligible because different values were measured in in-phase and out-of-phase components of SCA signal. In particularly the data shown in Fig. 3 of Ref. 13 correspond to the in-phase SCA signal that is proportional to the local compressibility of the 2DES while those shown in Ref. 14 are proportional to a tunnel resistance of a barrier separated the 2DES and the $n^+$ reservoir. The direct comparison is possible only after finding of an appropriate explanation of the features observed in the SCA images. But here I would like to pay attention only on a sensitivity of the features to the magnetic field variation and
compare it with that of $E_0$ and $G_F$. This comparison can be justified by that the 2DES compressibility is proportional to $G_F$ at least in the single particle approximation and the tunnel barrier transparency is determined by $E_0$. 