Dynamical generation of scalar mass in two Higgs doublet model under Yukawa interactions

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Abstract

Light scalars are among the expected particles in nature. If they indeed exist, dynamical generation of masses becomes an important phenomenon to investigate scalar interactions from the perspective of understanding new physics and the accompanying features, such as critical couplings. A two Higgs doublet model containing two complex doublet scalar fields, conveniently called the standard model Higgs and the second Higgs fields, interacting with each other via a real singlet scalar field by a modelled Yukawa interactions is studied to explore the extent of dynamical mass generation and the field propagators in the model at different cutoff values. The model is studied for various renormalized masses of the second Higgs field at various coupling constants. The renormalized mass of the standard model Higgs boson is kept at 125.09 GeV. The model has strong indications of existence of critical coupling between $10^{-6}$ GeV and $10^{-3}$ GeV. The observed dynamical masses are generally within 200 MeV. The two Higgs propagators are found to be considerably stable compared to scalar singlet propagators despite cutoff effects. No phase structure in the parameter space was observed. The model is a non-trivial theory in scalar sector.

1 Introduction

Dynamical mass generation (DMG) \[1-12\] is, by definition, a non-perturbative phenomenon. Historically, it has mostly been relevant to QCD physics \[13\], particularly to the lightest family of quarks due to limited capacity of QCD interactions to generate large dynamical masses. In the new physics, the phenomenon is equally relevant to masses considerably lighter than the electroweak scale. As existence of scalar sector in nature has become a possibility after the experimental finding of the Higgs boson \[14-18\], ascertaining the
extent of dynamical masses of new scalar fields due to different interactions can not be underestimated.

Yukawa interaction vertex is among the phenomenologically interesting vertices. A model containing the vertex and the standard model (SM) Higgs field serves as an important avenue to explore the scalar sector. Two Higgs doublet models (2HDM) are among widely studied models for various reasons. The model has its own strength in scalar sector, i.e. it presents two important scenarios once the SM Higgs is introduced in the theory. First, the model presents a mock up scenario of fermions interacting with the SM Higgs. Second, the model presents an opportunity to investigate how same particles from two different families interact with each other. A highly interesting aspect in both scenarios is that the physics of dark matter in scalar sector arising from possible Higgs-ultralight scalar interactions can also be studied in the model. It immediately justifies the investigation related to dynamical mass generation in the model.

This paper is an extension of Wick Cutkosky (WC) model to incorporate two complex doublet fields, termed as the SM Higgs and the second Higgs fields for convenience, which interact with each other only by a real singlet scalar field. The paper is a continuation of the study of a two Higgs doublet model. One of the motives for the study is to understand extent of generation of dynamical masses for a scalar singlet field in different regions of the parameter space of the model. The scalar singlet acts as a mediator field between the two different, and otherwise non-interacting, complex doublet fields under the Yukawa interactions. The renormalized masses of the two complex doublet scalars are kept at their physical masses in order to study the phenomenon of DMG from the perspective of phenomenology. The physical mass of the SM Higgs is kept at its experimentally known value while the renormalized mass of the second Higgs is chosen for different scenarios mentioned above. A certain advantage of fixing masses is reduction of the parameter space to be explored. Hence, It effectively renders the model only a 3 dimensional parameter space in the Lagrangian. It greatly facilitates in training an algorithm, which is not a novel approach in quantum field theory, over the sample of calculated dynamical masses in different regions of the parameter space, the details are given in the next section.

The approach of Dyson-Schwinger equations (DSEs) is used for the study. The DSEs for the three field propagators are considered while the interaction vertices are fixed at their tree level form up to certain renormalization dependent terms, the details are given in the next section.

There exist other renormalizable vertices to further extend the model

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\[ m_s = O(10^{-22}) \text{ eV} \]

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The terms renormalized mass and physical mass are interchangeably used throughout the paper for the SM Higgs. The term scalar field is reserved only for the singlet scalar.
However, such extensions may not be suitable for investigations using the approach of DSEs due to possible effects of further truncations and ansatz in the model. An extended model is to be reported somewhere else for which another non-perturbative method is employed.

## 2 Technical Details

The Euclidean version of the Lagrangian with the counter terms is given by

\[
L = \frac{1}{2}(1 + A)\partial_\mu \phi(x)\partial^\mu \phi(x) + \frac{1}{2}(m_s^2 + B)\phi^2(x) + (1 + \alpha)\partial_\mu h^\dagger(x)\partial^\mu h(x) + (m_h^2 + \beta)h^\dagger(x)h(x) + (1 + a)\partial_\mu H^\dagger(x)\partial^\mu H(x) + (m_H^2 + b)H^\dagger(x)H(x) + (\lambda_1 + C_1)\phi(x)h^\dagger(x)h(x) + (\lambda_2 + C_2)\phi(x)H^\dagger(x)H(x)
\]

where \(A, B, \alpha, \beta, a, b, C_1, \) and \(C_2\) are coefficients due to the counter terms in the Lagrangian. The real singlet scalar field is represented by \(\phi(x)\), \(h(x)\) is designated for the SM Higgs boson while \(H(x)\) represents the second Higgs boson. The resulting DSEs for the field propagators are given by:

\[
D_h^{-1}(p) = (1 + \alpha)p^2 + m_h^2(1 + \alpha) + 2(1 + A)(1 + \alpha)(1 + a)\sigma_h + (\lambda_1 + C_1) \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \Gamma_1(-p, q) D_h(q - p)
\]

\[
D_H^{-1}(p) = (1 + a)p^2 + m_H^2(1 + a) + 2(1 + A)(1 + \alpha)(1 + a)\sigma_H + (\lambda_2 + C_2) \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \Gamma_2(-p, q) D_H(q - p)
\]

\[
D_s^{-1}(p) = (1 + A)p^2 + m_s^2(1 + A) + 2(1 + A)(1 + \alpha)(1 + a)\sigma_s + (\lambda_1 + C_1) \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_h(q) \Gamma_1(q, -p) D_s(q - p) + (\lambda_2 + C_2) \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_H(q) \Gamma_2(q, -p) D_H(q - p)
\]

where the following definitions are used:

\[
\begin{align*}
\beta &= \alpha m_H^2 + 2(1 + A)(1 + \alpha)(1 + a)\sigma_h \\
b &= \alpha m_H^2 + 2(1 + A)(1 + \alpha)(1 + a)\sigma_H \\
B &= \alpha m_H^2 + 2(1 + A)(1 + \alpha)(1 + a)\sigma_s
\end{align*}
\]

\(^2\)The technical details, a significant part of which can also be found somewhere else, are generously included to keep the section self-contained.

\(^3\)The bare scalar singlet mass \((m_s)\) is kept in the equation for the sake of clarity.
\(\sigma_h, \sigma_H, \sigma_s\) are the terms to be determined during a computation. Due to their nature, above definitions do not impose any constraints on the equations. The definition of the two vertices during computations is given below:

\[
\begin{align*}
\Gamma_1(u,v) &= (1 + A)(1 + \alpha)(1 + a)\tilde{\Gamma}_1(u,v) \\
\Gamma_2(u,v) &= (1 + A)(1 + \alpha)(1 + a)\tilde{\Gamma}_2(u,v)
\end{align*}
\]

(6a) \hspace{1cm} (6b)

Hence, the DSEs for the three field propagators become

\[
\begin{align*}
D_{h}^{-1}(p) &= (1 + \alpha)[ p^2 + \frac{m_{h,r}^2}{(1 + \alpha)} + (\lambda_1 + C_1)(1 + A)(1 + a) \\
& \quad \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q)\tilde{\Gamma}_1(-p,q)D_h(q - p)] \\
D_{H}^{-1}(p) &= (1 + \alpha)[ p^2 + \frac{m_{H,r}^2}{1 + a} + (\lambda_2 + C_2)(1 + A)(1 + \alpha) \\
& \quad \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q)\tilde{\Gamma}_2(-p,q)D_H(q - p)] \\
D_{s}^{-1}(p) &= (1 + A)[ p^2 + 2(1 + a)(1 + \alpha)\sigma_s + (\lambda_1 + C_1)(1 + a)(1 + \alpha) \\
& \quad \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_h(q)\tilde{\Gamma}_1(q,-p)D_h(q - p) + (\lambda_2 + C_2)(1 + a)(1 + \alpha) \\
& \quad \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_H(q)\tilde{\Gamma}_2(q,-p)D_H(q - p)]
\end{align*}
\]

(7) \hspace{1cm} (8) \hspace{1cm} (9)

where in equation (7) the renormalized mass for the SM Higgs \(m_{h,r}\) is fixed at 125.09 GeV during the entire study, while the renormalized mass of the second Higgs boson is fixed during each computation. Equations (7-9) are the three DSEs considered for the study. The bare mass of the scalar field is set to zero in (9) in order to investigate the dynamical masses of the singlet scalar field. Lastly, the quantities \(\tilde{\Gamma}_1(u,v)\) and \(\tilde{\Gamma}_2(u,v)\) are fixed at \(\lambda_1\) and \(\lambda_2\), respectively [54]. However, in the current investigation the vertices can still change depending upon the contributions from the coefficients in the counter terms, see equations (6).

For each of the propagators, the following renormalization conditions are used.

\[
\begin{align*}
D_{h}^{ij}(p)|_{p^2=m_{h,r}^2} &= \frac{\delta^{ij}}{p^2 + m_{h,r}^2}|_{p^2=m_{h,r}^2} \\
D_{H}^{ij}(p)|_{p^2=m_{H,r}^2} &= \frac{\delta^{ij}}{p^2 + m_{H,r}^2}|_{p^2=m_{H,r}^2}
\end{align*}
\]

(10) \hspace{1cm} (11)

\footnote{The definition of the squared physical mass of the SM Higgs is \(m_{h,r}^2 = m_h^2 + \beta\), and the definition of the squared renormalized mass of the second Higgs is \(m_{H,r}^2 = m_{H}^2 + b\).}
\[ D_s(p)|_{p=1} = \frac{1}{p^2}|_{p=1} \quad (12) \]

The following two conditions are also imposed to numerically compute the correlation functions and the other quantities which are introduced for the counter terms.

\[ \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \tilde{\Gamma}_1(-p,q)D_h(q-p) \]  
\[ \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \tilde{\Gamma}_2(-p,q)D_h(q-p) \]  
\[ \left( \frac{1}{p^2} \right)^2 dp = 0 \quad (13) \]

\[ \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \tilde{\Gamma}_1(-p,q)D_h(q-p) \]  
\[ \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \tilde{\Gamma}_2(-p,q)D_h(q-p) \]  
\[ \left( \frac{1}{p^2} \right)^2 dp = 0 \quad (14) \]

Equations 13 - 14 are indeed the implementation of the least square method with errors \( E_1 \) and \( E_2 \) defined below.

\[ E_1 = \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \tilde{\Gamma}_1(-p,q)D_h(q-p) \]  
\[ \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \tilde{\Gamma}_1(-p,q)D_h(q-p) \]  
\[ \left( \frac{1}{p^2} \right)^2 dp \quad (15) \]

\[ E_2 = \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \tilde{\Gamma}_2(-p,q)D_h(q-p) \]  
\[ \int_{-\Lambda}^{\Lambda} \frac{d^4q}{(2\pi)^4} D_s(q) \tilde{\Gamma}_2(-p,q)D_h(q-p) \]  
\[ \left( \frac{1}{p^2} \right)^2 dp \quad (16) \]

With imposition of these constraints, the problem at hand becomes that of optimization in which solutions are sought which satisfy equations 13-14.

An additional condition given below is also imposed in order to ensure positivity of renormalized squared dynamical scalar mass and evade unwanted numerical fluctuations which may arise due to the difference between the fixed renormalized masses and the dynamically generated masses, and the ever-present limitation in momentum resolution.

\[ m_{s,r}^2 = (1 + A)(1 + \alpha)(1 + A)(1 + a) \sigma_s \geq 0 \quad (17) \]

In order to further suppress numerical fluctuations, the SM Higgs is expanded in the form given below:

\[ D_h^{ij}(p) = \delta^{ij} \frac{1}{c(p^2 + d + f(p))} \quad (18) \]
with \( f(p) \) given by

\[
f(p) = \sum_{l=0}^{N} a_{l} p^{2l} \frac{N}{\sum_{l=0}^{N} b_{l} p^{2l}}
\]  \tag{19}

In equations 18-19, \( c, d, a_{l}, \) and \( b_{l} \) are the coefficients to be determined during a computation. A similar expansion with different coefficients is used for the second Higgs propagator. Besides stability, these expansions are also time efficient while performing renormalization and updating the SM and the second Higgs propagators.

The computation starts with \( \sigma_{H} = \sigma_{s} = C_{1} = C_{2} = 0 \), i.e. with no contribution by the counter terms to the renormalized masses and the two Yukawa couplings. Both Higgs propagators are also rendered their respective tree level structures. For the SM Higgs in equations 18-19, \( c = 1 \) and \( d = m_{h}^{2} \) \(^5\) while all the other coefficients are zero. A similar setup of coefficients is used for the second Higgs. The terms \( 1 + \alpha \) and \( 1 + a \) are calculated from the renormalization conditions in equations 10 and 11. The scalar propagator assumes the values using the equation 4, and the quantity \( 1 + A \) is calculated by the renormalization condition 12 \(^6\).

An iteration involves updating of the correlation functions and parameters. During an iteration, first, the \( \sigma_{s}, C_{1}, \) and \( C_{2} \) are updated as in the mentioned ordered. The update of each of these quantities is performed using Newton Raphson’s method with the criteria imposed by the least square method in equations 13-14. The updated value is accepted only when both of the errors \( E_{1} \) and \( E_{2} \) reduce, see equations 15-16.

It is followed by the SM Higgs propagator for which the coefficients in equations 18-19 are updated with the above-mentioned criteria of acceptance. Upon each change, the SM Higgs propagator is calculated from equations 18-19 and renormalized using equation 10.

Lastly, the second Higgs propagator is updated using the same procedure as described above for the SM Higgs, but using equation 11 for renormalization. Upon every change, the scalar propagator is calculated from equation 9 and renormalized using equation 12 as mentioned earlier.

A computation concludes only when there are either no further improvements in the quantities such that \( E_{1} \) and \( E_{2} \) are further reduced, or both of these errors are equal or below the preset value of tolerance. The value of tolerance is set at \( 10^{-20} \).

Gauss quadrature algorithm is used for numerical integration in the DSEs. The algorithms are developed in C++ environment.

\(^5\) \( m_{h}^{2} = m_{h_{s}}^{2} \), is used throughout the study.

\(^6\) The scalar propagator is calculated without the term \( 1 + A \) and then renormalization condition sets the value of the term \( 1 + A \).
The solutions presented are unique in the sense that order of updates performed on the propagators and other quantities practically does not effect the quantities being computed.

It is assumed that the model is not trivial. The assumption is supported by the fact that, despite that $\phi^4$ theory [58–60] is found trivial [61–63], Higgs interaction with gauge bosons does not render the model trivial [66, 67].

A peculiar feature of the model is the possibility of negative Hamiltonian due to certain paths [55]. However, it has been noticed in a different investigation of scalar interactions [56, 57] that in the presence of SM Higgs mass the corresponding histories do not contribute during simulations [7]. Presence of large $m_H$ in the model further diminishes the possibility that the potential term will cause negative Hamiltonian. Hence, it is assumed here that the results are not effected by the above mentioned feature of the potential term.

Once the scalar dynamical masses in the parameter space are calculated, knowledge from machine learning [47] is employed to represent the results in an attempt to understand how scalar dynamical masses manifest in the parameter space and estimate critical coupling value if possible. There are three free parameters in the model. Hence, scalar dynamical mass is expanded in terms of the second Higgs mass $m_H$, and the two Yukawa couplings ($\lambda_1$ and $\lambda_2$) as given below:

$$m_{s,f}(m_H, \lambda_1, \lambda_2) = \sum_{i=0}^{i=6} \sum_{j=0}^{j=6} \sum_{k=0}^{k=6} a_{ijk} m_H^i \lambda_1^j \lambda_2^k$$

(20)

where $m_{s,f}$ is the function representing scalar dynamical mass in the parameter space. The error value $E_f$ is described by the following expression:

$$E_f = \frac{1}{N} \sqrt{\sum_{i=0}^{i=N} (m_{s,r} - m_{s,f})^2}$$

(21)

where $N$ is the number of explored points in the parameter space. The model is studied on 64 points for each of the cutoff values set at 10 TeV and 100 TeV. The procedure of training at a fixed cutoff value starts with all weights $a_{ijk}$ set to zero. The results from the DSE calculations are successively introduced to the algorithm from higher to lower couplings.

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In the potential part of the model, the dominant contribution comes from the term containing the SM (squared) Higgs mass (125.09 GeV) while only $\phi(x)$ may force the potential term below zero. Hence, $\phi(x)$ must take significantly large values at all points of the spacetime to effectively compete the term containing $m_h$. Numerically, it is not favored in Monte Carlo simulations due to precision related issues and that the fields are rendered values from certain (usually Gaussian) distributions. Another argument is that a Monte Carlo simulation proceeds towards smaller action values which, depending upon the model, are favored by the fields with relatively lower values.
hence the results from DSE calculations being examined increase on every introduction. The weights are varied using Newton Raphson’s method\(^8\) such that the error value in equation 21 decreases. If the value does not decrease, the change in the weight is not accepted. All weights are examined during each iteration. Once, the algorithm has reached the point when it either cannot further improve the weights or the error value has reached its tolerance, further results are introduced to the algorithm and weights are reexamined, hence the algorithm is retrained. The tolerance is set at the value \(10^{-16}\). In short, scalar dynamical mass is fitted by the expansion in equation 20 while successively introducing the results from DSE calculations and readjusting the weights \(a_{ijk}\). Throughout the training, \(0 \leq m_{s,f}(m_H, \lambda_1, \lambda_2)\) is imposed. In order to remove any personal bias, no value of scalar dynamical mass was taken as outliers.

As is commonly known, training algorithms on finite data (and duration) may not render extreme accuracy. In addition, the scalar dynamical masses and other calculated quantities (from DSEs) may also suffer from numerical fluctuations. However, a relatively smoother description of the data immensely helps in understanding the overall features of the model and making nontrivial estimates. Hence, the algorithm training was employed to assist in our conclusions regarding the critical coupling value in the model.

3 Field Propagators

Unlike the vertices in the model, the field propagators are rendered considerable freedom to have momentum dependence while conforming to the renormalization conditions. Scalar singlet field propagator has the highest freedom since it takes contributions from interactions with both Higgs field. However, such freedom may also allow contributions from numerical fluctuations of both Higgs propagators as well as other involved quantities in the model.

The SM Higgs propagators are given in figures 1-2. An immediate observation is that cutoff effects are not significant throughout the explored parameter space, particularly for the cases with \(\lambda_1 < 1\). The deviations are mostly due to the multiplicative constant which is computed using the renormalization condition [10].

In the parameter space with \(\lambda_1\) as low as \(10^{-6}\), there is no significant dependence of the propagators. However, for higher couplings slight enhancement is observed. Considerable changes occur only at \(\lambda_1 = 1.0\). The propagators are found to have similar qualitative behavior for all the renormalized masses of the second Higgs which indicates that the second Higgs field does not influence the SM propagators which could have been possible

\(^8\)Other methods are also used for this purpose, see [47] for instance.
Figure 1: The SM Higgs propagators with $m_H = 0.001$ GeV and $m_H = 1.0$ GeV are plotted for cutoff values at 10 TeV and 100 TeV. The parameters in the legend are given as $(m_H, \lambda_1, \lambda_2)$ with all of the parameters mentioned in GeV. For the same cutoff, every two consecutive propagators are 1.0 TeV apart on the momentum axis for the sake of clarity.

Figure 2: The SM Higgs propagators with $m_H = 100$ GeV and $m_H = 1000$ GeV are plotted for cutoff values at 10 TeV and 100 TeV. The parameters in the legend are given as $(m_H, \lambda_1, \lambda_2)$ with all of the parameters mentioned in GeV. For the same cutoff, every two consecutive propagators are 1.0 TeV apart on the momentum axis for the sake of clarity.
Figure 3: The second Higgs propagators with $m_H = 0.001$ GeV and $m_H = 1.0$ GeV are plotted for cutoff values at 10 TeV and 100 TeV. The parameters in the legend are given as $(m_H, \lambda_1, \lambda_2)$ with all of the parameters mentioned in GeV. For the same cutoff, every two consecutive propagators are 1.0 TeV apart on the momentum axis for the sake of clarity.

Figure 4: The second Higgs propagators with $m_H = 100$ GeV and $m_H = 1000$ GeV are plotted for cutoff values at 10 TeV and 100 TeV. The parameters in the legend are given as $(m_H, \lambda_1, \lambda_2)$ with all of the parameters mentioned in GeV. For the same cutoff, every two consecutive propagators are 1.0 TeV apart on the momentum axis for the sake of clarity.
Figure 5: The scalar propagators with $m_H = 0.001$ GeV and $m_H = 1.0$ GeV are plotted for cutoff values at 10 TeV and 100 TeV. The parameters in the legend are given as $(m_H, \lambda_1, \lambda_2)$ with all of the parameters mentioned in GeV. For the same cutoff, every two consecutive propagators are 1.0 TeV apart on the momentum axis for the sake of clarity.

Figure 6: The scalar propagators with $m_H = 100$ GeV and $m_H = 1000$ GeV are plotted for cutoff values at 10 TeV and 100 TeV. The parameters in the legend are given as $(m_H, \lambda_1, \lambda_2)$ with all of the parameters mentioned in GeV. For the same cutoff, every two consecutive propagators are 1.0 TeV apart on the momentum axis for the sake of clarity.
through the scalar propagators. Overall, there is no significant changes in the SM Higgs propagators in the parameter space of the model. Hence, it is deduced that if there is any non-trivial structure in the phase space, it does not influence the SM Higgs propagators and possibly the other field propagators since such effects can also translate to other quantities by the coupled DSEs.

In the model, the second Higgs field differs from the SM Higgs field due to different renormalized masses and couplings. Hence, particularly for masses in the vicinity of the SM Higgs mass, it is expected to have similar behavior within numerical fluctuations. The propagators are shown in figures 3-4. There are no considerable cutoff effects, as is the case with the SM Higgs propagators. However, a certain dependence on \( m_H \) is observed in the propagators which is relatively stronger for \( m_H < m_h \) and weakens as \( m_H \) approaches \( m_h \). The two Higgs propagators have similar behavior for \( m_H \simeq m_h \) which validates the implementation of the algorithms. For \( m_H = 1 \) TeV dependence on couplings is lost since the bare mass of the second Higgs dominates the contributions in the propagators, hence, rendering it a tree level structure up to the renormalization dependent term. The propagators are suppressed for such a large renormalized mass which is expected for the case of dominant tree level contribution in the propagator.

The scalar singlet propagators are shown in figures 5-6. As argued in above, cutoff effects are evident on the propagators. The overall behavior is that for higher couplings, particularly \( \lambda_2 \) since \( m_h \) is kept fixed, the propagators are enhanced depending upon the second Higgs mass \( m_H \). As \( m_H \) increases, this effect tends to disappear in favor of a tree level structure, as is the case for the other field propagators. The scalar propagators suffer strongly from the cutoff effects despite that both Higgs propagators are not effected to such an extent. It implies that the cutoff effects must show up in at least one of the other calculated quantities other than the two Higgs propagators.

4 Dynamical Scalar Masses

Since the scalar propagator also contains its dynamically generated mass, the arguments related to the cutoff effects and numerical artifacts translate into the dynamical masses. However, there are certain vivid features in the masses shown in the figures.

Considering the dynamical masses for 100 TeV cutoff in figure the extent of the mass production is relatively higher for the second Higgs mass in MeVs and high second coupling for a number of cases. For very small

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99There may also be contributions from the numerical interpolation performed during renormalization of the scalar propagators. In order to suppress these effects, resolution in momentum is kept at its highest feasible value.

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Figure 7: Dynamical scalar masses are plotted against second Higgs mass $m_H$ for various couplings at 10 TeV cutoff. The couplings (in GeV) are shown in the caption as $\lambda_1, \lambda_2$. Every consecutive pair of couplings is slightly displaced along x-axis for clarity. The error bars represent difference between the values of scalar dynamical masses obtained by computation (plotted values) and by training of the algorithms.

couplings, large difference between the computed value and $m_{s,f}$, possibly due to numerical fluctuations, suggests that these two points could have been taken as outliers. Hence, it is concluded with confidence that the dynamically generated scalar mass in the model is restricted well within 200 MeVs. It is clear that the Yukawa interactions in the scalar sector have a shortcoming when it comes to producing dynamical masses in GeVs.

It is also evident from figures 7-8 that cutoff effects are far less severe for the cutoff in hundreds of TeVs. For higher cutoff, the model produces scalar masses with less dependence on the second Higgs mass. Furthermore, the production of scalar mass is significantly lower if one of the couplings is as low as $10^{-6}$ GeV, unless the other coupling reaches 1.0 GeV, see figure 8.

Hence, the model does posses a demarcation over the coupling values in the vicinity of 1.0 and $10^{-6}$ GeV. For the case of both couplings at $10^{-6}$ GeV, scalar mass practically looses dependence on the two couplings. It is taken as a sign of existence of critical coupling value between $10^{-3}$ GeV and $10^{-6}$ GeV.

An interesting feature in the low coupling region is that for higher cutoff values the model produces smaller masses which also tend to decrease with the two couplings. One may expect that the behavior persists until the critical coupling value. If this is indeed the case, the model may be useful to study ultralight scalar interactions [33–40], though it may undoubtedly
Figure 8: Dynamical scalar masses are plotted against second Higgs mass $m_H$ for various couplings at 100 TeV cutoff. The couplings (in GeV) are shown in the caption as $\lambda_1, \lambda_2$. Every consecutive pair of couplings is slightly displaced along x-axis for clarity. The error bars represent difference between the values of scalar dynamical masses obtained by computation (plotted values) and by training of the algorithms.

Figure 9: Dynamical scalar masses obtained by training algorithms are shown against the two couplings $\lambda_1$ and $\lambda_2$, for various second Higgs masses $m_H$ in GeV and at cutoff $\Lambda = 10$ TeV indicated on the caption.

be a daunting task from the perspective of numerical precision.

Scalar dynamical masses obtained by the trained algorithm are plotted
Figure 10: Dynamical scalar masses obtained by training algorithms are shown against the two couplings $\lambda_1$ and $\lambda_2$ for various second Higgs masses at 100 TeV, for various second Higgs masses $m_H$ in GeV and at cutoff $\Lambda = 10$ TeV indicated on the caption.

in figures 9-10. The weights of the expansion in equation 20 are given in appendix A. An immediate observation is existence of a unique value of the critical coupling in the model below $10^{-3}$ GeV. Determining exact value is hampered by the ever-present limitation of the data in machine learning. However, it is evident that the model strongly favors a critical coupling in the region $10^{-6} < \lambda_i < 10^{-3}$ GeV.

Despite the limitations in training of algorithms and numerical fluctuations in the exposed data, the weights $a_{ijk}$ are found to have a particular behavior. Firstly there are strong contributions for the terms with $k = 0$ and $i, j \neq 0$, while there is practically no contributions by $a_{ijk}$ for $i, j = 0$ irrespective of the value of $k$. Furthermore, increasing the cutoff suppresses $a_{ijk}$ for most of the $i$ and $k$ values at $j = 0$. Since the model has significant cutoff effects which becomes milder as the cutoff is raised, it is expected that scalar dynamical mass may not receive significant contributions from terms involving $j = 0$ at high cutoff values. Since the mass of the SM Higgs boson is kept fixed throughout the investigation, these observations suggest that the (magnitude of) contributions to dynamical scalar mass favor interactions with the SM Higgs boson mass.

5 Conclusion

In the presence of an SM Higgs, the two Higgs doublet model has the capacity to dynamically produce scalar mass significantly larger than of the
lightest quarks and leptons. As the cutoff is raised above 100 TeV, stability in the mass with magnitude below 200 MeV ensues against the second Higgs mass. It is expected that at a cutoff much higher than 100 TeV, a single value of dynamical mass is favored by the model irrespective of the second Higgs mass.

However, the dynamical mass has sensitivity to the couplings which diminishes as the couplings are reduced below $10^{-3}$ GeV, with the minimum mass being produced. It strongly implies existence of a critical coupling between $10^{-3}$ GeV and $10^{-6}$ GeV. It presents an opportunity to investigate the model to understand new physics involving the particles considerably lighter than 1 GeV, such as ultra-light scalars. At the same time, it invites for further study of richer models which contain higher renormalizable vertices in an attempt to explore existence of critical couplings and extent of mass generation due to diversity in interactions.

The role of cutoff effects can not be neglected in the model. Scalar propagator and scalar dynamical mass suffer the most, while for masses larger than 100 GeV the field propagators are relatively less effected as was observed in the two Higgs propagators. Since dynamical masses are less than 1 GeV, numerical fluctuations and the cutoff effects hamper in finding an accurate description of how the masses behave in the parameter space.

However, considering sufficient number of points in the parameter space a mathematical description for dynamical masses was still found which concurs with the deduction regarding the critical coupling in the model.

A model with the capacity to dynamically produce masses considerably larger than the lightest quarks and leptons can certainly not be ruled out in scalar interactions. The study invites extensions, such as by including richer interactions or other fields, to have a better understanding of how (particularly light) scalars play role at the fundamental level in our universe.

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7 Appendix A

The values of the weights $a_{ijk}$ in equation 20 for cutoff values at $\Lambda = 10$ TeV and $\Lambda = 100$ TeV are given below:

| $i$ | $j$ | $k$ | $a_{ijk}$ (10 TeV) | $a_{ijk}$ (100 TeV) |
|-----|-----|-----|-------------------|-------------------|
| 0   | 0   | 0   | 0.062470119000000 | -0.005102152199999 |
| 1   | 0   | 0   | 0.432639286999998 | 0.067154687499999 |
| 0 | 1 | 0 | 0.431424060300000 | 0.276498924000000 |
| 0 | 0 | 1 | 0.000127544300000 | 0.000023271900000 |
| 2 | 0 | 0 | -0.196267245799999 | 0.826964182800007 |
| 1 | 1 | 0 | 3.483278381999988 | -2.646594633199990 |
| 1 | 0 | 1 | 0.00125607676799999 | 0.000786048000000 |
| 0 | 2 | 0 | -2.426385300799987 | -0.005208820599999 |
| 0 | 1 | 1 | -0.001780857000000 | -0.000148474000000 |
| 0 | 0 | 2 | -0.000000108300000 | -0.000000023100000 |
| 3 | 0 | 0 | -0.595301560000001 | -0.158007649599999 |
| 2 | 1 | 0 | 0.311572103699999 | 5.428065021000011 |
| 2 | 0 | 1 | 0.001414899099999 | 0.002876035999999 |
| 1 | 2 | 0 | -2.967471651800008 | -3.556474306000007 |
| 1 | 1 | 1 | -0.011901846299999 | 0.013548386599999 |
| 1 | 0 | 2 | -0.000001844100000 | -0.000000603000000 |
| 0 | 3 | 0 | 3.360127912999990 | -0.928262618399999 |
| 0 | 2 | 1 | 0.004300518299999 | -0.002201626699999 |
| 0 | 1 | 2 | 0.0000000999000000 | -0.000000029000000 |
| 0 | 0 | 3 | 0.0 | 0.0 |
| 4 | 0 | 0 | 1.045801962500008 | -0.402565733999999 |
| 3 | 1 | 0 | -1.420411216799999 | -0.572378273000001 |
| 3 | 0 | 1 | 0.003927822499999 | -0.0005545230599999 |
| 2 | 2 | 0 | -1.122774422700004 | -0.529377973400000 |
| 2 | 1 | 1 | 0.000976498800000 | -0.005964260699999 |
| 2 | 0 | 2 | 0.000000491500000 | 0.000001401500000 |
| 1 | 3 | 0 | 0.322474038600001 | 3.267544194000007 |
| 1 | 2 | 1 | 0.005696769799999 | -0.001591530000000 |
| 1 | 1 | 2 | 0.000009325400000 | -0.000008751100000 |
| 1 | 0 | 3 | 0.0 | 0.0 |
| 0 | 4 | 0 | -3.398481779199986 | 1.658322491499994 |
| 0 | 3 | 1 | -0.002802221199999 | 0.001725826300000 |
| 0 | 2 | 2 | -0.000000049500000 | 0.000000259100000 |
| 0 | 1 | 3 | 0.0 | 0.0 |
| 0 | 0 | 4 | 0.0 | 0.0 |
| 5 | 0 | 0 | -0.831871660400005 | 0.132814269100000 |
| 4 | 1 | 0 | 0.949120657999995 | -1.788138425800004 |
| 4 | 0 | 1 | -0.003439259299999 | 0.001325414999999 |
| 3 | 2 | 0 | -1.903333068200005 | -4.520420235500010 |
| 3 | 1 | 1 | -0.005681518600000 | 0.006113348800000 |
| 3 | 0 | 2 | 0.00000018543 | 0.000000181000000 |
| 2 | 3 | 0 | 5.737533950000011 | 2.994909360000007 |
| 2 | 2 | 1 | 0.004100454999999 | -0.003434122199999 |
| 2   | 1   | 2   | 0.0000045994 | -0.000004740900000 |
|-----|-----|-----|-------------|-------------------|
| 2   | 0   | 3   | 0.0         | 0.0               |
| 1   | 4   | 0   | -4.607335835299982 | 3.560205072699987 |
| 1   | 3   | 1   | -0.0017640611 | 0.000632370000000 |
| 1   | 2   | 2   | -0.000002936 | 0.000001718100000 |
| 1   | 1   | 3   | 0.0         | 0.0               |
| 1   | 0   | 4   | 0.0         | 0.0               |
| 0   | 5   | 0   | 2.476695333800004 | -1.43805615169995 |
| 0   | 4   | 1   | -0.001170340799999 | 0.000608526900000 |
| 0   | 3   | 2   | -0.0000001277 | 0.000000238100000 |
| 0   | 2   | 3   | 0.0         | 0.0               |
| 0   | 1   | 4   | 0.0         | 0.0               |
| 0   | 0   | 5   | 0.0         | 0.0               |
| 6   | 0   | 0   | 0.250543694400000 | -0.303845693199999 |
| 5   | 1   | 0   | -0.872865771300006 | -0.570790623500001 |
| 5   | 0   | 1   | -0.0014202491 | -0.000099293799999 |
| 4   | 2   | 0   | -2.6299926507000005 | -4.544265494000010 |
| 4   | 1   | 1   | -0.002726043499999 | 0.004424348300000 |
| 4   | 0   | 2   | 0.0000007263  | 0.000000419100000 |
| 3   | 3   | 0   | 4.539023046400008 | -1.631924930300001 |
| 3   | 2   | 1   | 0.004210737  | 0.000127814799999 |
| 3   | 1   | 2   | 0.0000038225 | -0.000004546200000 |
| 3   | 0   | 3   | 0.0         | 0.0               |
| 2   | 4   | 0   | -2.3617412604000014 | 7.959941882000007 |
| 2   | 3   | 1   | 0.0004258372  | -0.003437821899999 |
| 2   | 2   | 2   | -0.0000105292 | 0.000010411400000 |
| 2   | 1   | 3   | 0.0         | 0.0               |
| 2   | 0   | 4   | 0.0         | 0.0               |
| 1   | 5   | 0   | 2.659292929700001 | -3.122536580600026 |
| 1   | 4   | 1   | 0.007337953699999 | -0.008389092999999 |
| 1   | 3   | 2   | -0.0000050914 | 0.000004083200000 |
| 1   | 2   | 3   | 0.0         | 0.0               |
| 1   | 1   | 4   | 0.0         | 0.0               |
| 1   | 0   | 5   | 0.0         | 0.0               |
| 0   | 6   | 0   | -0.408070357499999 | 0.605073464699998 |
| 0   | 5   | 1   | 0.0012935581  | -0.001141694000000 |
| 0   | 4   | 2   | -0.0000006819 | 0.000000429700000 |
| 0   | 3   | 3   | 0.0         | 0.0               |
| 0   | 2   | 4   | 0.0         | 0.0               |
| 0   | 1   | 5   | 0.0         | 0.0               |
| 0   | 0   | 6   | 0.0         | 0.0               |
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