Educational engineering as research methodology in learning the concepts of limit and continuity

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Abstract. Attached to the differential calculus concepts, particularly the of limit and continuity usually difficult environment to its conceptual construction, since it is usually oriented to the domain of algebraic processes, restricting the understanding and thus limiting learning and appropriation of the same. This research was developed by students of the hotbed of research in mathematics education, aiming to promote the correct conceptual apprehension of the themes of the differential calculus, through the design and implementation of various didactic sequences, based on didactic engineering as research methodology. Sequences used different registers of semiotic representation such as graphical, algebraic, natural and tabular. The activities were implemented in students belonging to the “Facultad de Ingeniería, Universidad Francisco de Paula Santander”, Colombia. Through the raised situation a better apprehension on the understanding of the infinitesimal of the meaning of the boundary (related to the continuity of this) and a better insight on the graphic understanding of limit and continuity, has strengthened since they had the opportunity to visualize the object of study in a direct way.

1. Introduction

Study the form and methods required for the cognitive development of humans, is the most frequent activity in the majority of scientific communities, educational developments, which analyzed and postulating possible theories of learning-oriented. They help to facilitate the process of training academic and social one of them is called engineering teaching, from didactic situations Brousseau and the theory of the didactic transposition of Chevallard. This theory applied especially in the field of mathematics education is intended to form or shape knowledge by means of the sequential application of activities that are designed to facilitate understanding. According Faria E. the research methodology consists of four phase’s preliminary analysis; conception and analysis a priori of the didactic situations; experimentation; subsequent analysis and evaluation [1].

With respect to the foregoing, these phases involved sequentially, with emphasis on the historical significance of knowledge and its development, analysis of its concept and the intentional creation of semiotic representations. According to Duval are structured different activities such as training, transformation and conversion of external representations related to a central theme [2]. Thus, in the area of Mathematics examines your understanding through different representations such as graphs, tables, algebraic calculations and natural language.

However, it has to the differential calculus as one of the subjects of mathematics with greater complexity and abstraction for students throughout the learning process, according to [3] the causes that generate such complexity are described in three large categories:
(i) Those associated with the complexity of the Basic objects of the calculation (actual numbers, successions, functions).
(ii) Those associated with the conceptualization and the formalization of the notion of limit.
(iii) Those linked with necessary ruptures in relation to purely algebraic modes of thinking.

According to the third category [4] expressed the importance that has to work math concepts, specifically those of the differential calculus from different representations that lead students to visualize and understand widely the reality of each of the concepts, meaning since, in this way demonstrates to the student to understand the theme already is limits or continuity is not mechanically operated on the solution of some exercises [5].

In addition to the foregoing as evidenced [6] in the State of the art, on the limits and continuity of a function the difficulty in the study area has not been properly addressed in different aspects, one of them being the pedagogical, that is based on the lack of initiative on the part of teachers to introduce limits from a contextual approach, i.e. situations where students can understand the significant nature of the same. On the other hand, [7] inferred on epistemological errors that students have previous concepts, since they tend to justify them under practical schemes concerning solution of exercises during the development of different tasks individuals that distort the formal definitions of each.

Finally, [8] and [9] establish the importance of guiding learning through the application of a cycle of activities enabling the student to formalize the concept of limit and continuity during its implementation in different representations in order to improve the understanding of students both in the issues prior to the limit as in each of the sub-themes which make part of its conceptual structure, such as (lateral limits, limits to infinity and infinite asymptotes, continuous functions and discontinues in one interval, among others). According to what was said, in the present work proposes the construction of semiotic representations that shape the development of mathematical thinking for the understanding of limit and continuity of a function.

2. Methodology
The referential framework underpinning the research methodology is so-called didactic engineering, defined by [3] the didactic engineering is characterized first of all by a pilot scheme based on the educational achievements in class, is say, on the conception, realization, observation and analysis of sequences of teaching. On the basis of the above, this research initially was tracking at documentary level from a State of the art around the understanding on the concepts of limit and continuity of real functions; built with the aim of identifying the strengths and weaknesses that often students presented in these two themes.

Having intended to promote a better understanding in learning, were structured didactic sequences that strengthen these shortcomings, presented the students activities were organized using different records of semiotic representations (graphical, algebraic, tabular and natural language), these make reference, according to [10] to a system of signs which can carry out the functions of communication, treatment and objectivity. The evaluation instrument was quantitative and was applied at the end of each of the above topics. This research was conducted in the first semester 2018, taking as a population to 29 students from the “Facultad de Ingenieria” who were studying the subject of calculus differential at the Universidad Francisco de Paula Santander.

3. Results
Teacher instructor, first applied a series of didactic sequences, in the middle which is time proportionally curricular content attached to the limit and continuity of real functions, these were structured under methodology so-called didactic engineering. Each activity was aimed at promoting a better apprehension in terms of graphic and conceptual understanding of the above-mentioned issues, on the basis of different semiotic representations, such as natural language, chart (becoming a greater focus this), algebraic and tabular. At the end of the implementation of the activities in each of the interventions, used a test to
determine the assimilation developed by students. In the construction of the instrument was a greater emphasis on assessing the graphic understanding in each of the two themes named above.

Test was developed according to analyze the performance of the students around the concept of limit in their different ways of understanding, in this respect were set out eight exercises, four evaluated different forms of representation, semiotics, and the remaining had the particularity of rating the transition from one record to another. Below is the Table 1 shows the type of representation used in each fiscal year.

| Exercise | Semiotic representations |
|----------|--------------------------|
| 1        | Natural language and graphic |
| 2 and 3  | Graphic and tabular, algebraic language |
| 4 and 8  | Natural language |
| 5        | Algebraic language |
| 6        | Graphic language |
| 7        | Graphic and algebraic language |

When applying the test, students had the possibility of selecting four exercises of the eight proposed (in total 276), thus allowing them to choose which agreed it, i.e., those who believed that they could solve. Figure 1 shows the frequency of selection in each exercise.

Figure 1 shows the tendency in the election of exercises1, exercises4, exercises5 and exercises7 characterized by representations of natural, algebraic and graphic language, that is, students that are considered to have a greater dominance in the previous registers. In addition, only 10% of students chose exercise2, had the singularity of relating the algebraic - tabular - graphic language. In addition, it is observed that the frequency of students who chose to exercise5 is 83%, this point has the characteristic of using the algebraic record, such findings are similar to those obtained in the research of [11] on the preference between three records (graphical, natural and symbolic procedure), since there is evidence that 27 of 33 students of industrial and electronic engineering opted for the symbolic registration of procedure.
Based on the results obtained on the preference of the students, established intervals depending on the level of performance acquired in each language representation. The performance levels were determined quantitatively, whereas the numeric value set in each financial year as shown in Table 2.

| Performance | Range     |
|-------------|-----------|
| Low         | 0.00 to 0.42 |
| Medium      | 0.43 to 0.85 |
| High        | 0.86 to 1.25 |

In this sense, categories in order to recognize the apprehension of the contents by the students from the use of the semiotic representations were established for the analysis of the information obtained. A category was determined for each type of record.

- Category 1. Natural language and graphic (exercise 1).
- Category 2. Algebraic, tabular and graphic (exercise 2 and 3).
- Category 3. Natural language (exercise 4 and 8).
- Category 4. Algebraic language (exercise 5).
- Category 5. Language graphic (exercise 6).
- Category 6. Graphic language and algebraic (exercise 7).

For the presentation of the results were formed in Figure 2, Figure 3, and Figure 4, which set out the level of performance obtained in each of the categorizations, in order to compare the performance of students in the exercises.

![Figure 2](image_url)

**Figure 2.** Low performance level.

Figure 2 performance level low, there is evidence that the students had greater difficulty in the solution of exercises related to the category 2, category 4 and category 5, although there was a tendency in the choice of exercise 5, 58% of them fail to use language algebraic correctly to calculate limits of functions raised, contrary to what happened with the choice of exercise 1, which presented a greater choice, but there was a lower percentage equal to 8%.
Looking at the percentages obtained in graphical Figure 3 average performance-related, can reveal that categories 3, categories 4, and categories 5 were presented a regular understanding, these periods presented were characterized by evaluating to the individual records of algebraic, graphical, and natural language understanding.

As shown in Figure 4 of the high-performance level, there is considerable variation in their percentages, on the basis of this it can be inferred that in categories 1 and categories 6 had better performance in the resolution of exercises 1 and exercises 7, which should pass the language natural to the chart and graph to the algebraic. While category 6 has a percentage higher than category 1 is that there were many students who understood the change in the limit of the natural to the graphic language, since the frequency of exercise preference is 26 and 18 respects selectively.

The results obtained through the application of the test on the use of semiotic representation in learning about the limit and continuity issues show us that the engineering students who were studying calculus differential had better understanding of the concepts of limit and continuity when they had to make a transition from natural language-graph-algebraic and graphic you can see that in two previous cases of transition, prevails the graphic application, that allows us to affirm that the direct visualization
of such concepts facilitates the apprehension of knowledge, particularly on the topics treated in this research.

On the other hand, when representations were used individually as in the case of 4, 5, 6 and 8 exercises, students failed to have a correct notion of knowledge workers. While in the 6 performance is made use of the graphic record, it did not have a significant arrest since [12] points out that the understanding (inclusive) of a conceptual content rests on at least two records of representation and coordination manifested by the speed and the spontaneity of the cognitive activity of conversion.

4. Conclusions
The limits sometimes teaching is characterized by the algebraic definition of these, thus neglecting the understanding graphic that should be built environment to the understanding of it, accordingly to the above disadvantages are presented at the time make an apprehension to the subject being treated, thus leaving empty concept that they limit the understanding of subsequent contents.

Analyzing the instrument applied there is evidence there was little understanding by students when asked that determine it the limit of a function, from purely algebraic records category 4 was that the lowest level arose from performance, on the other hand, to compare these results with those obtained in category 1, where requested move from a natural language to the graph, is inalienable to mention that there is a better assimilation.

In terms of the continuity of functions found a better insight when asked the language conversion chart to the algebraic language (category6), contrary to when calling for interpretation exclusively graphic (category5), in this sense Although a lack of knowledge to analyze the continuity with a single representation, some students showed a substantial improvement to raise them in two languages the understanding of this concept in two languages.

Finally it can demonstrate that the articulation of different registers semiotic representations are an important tool for the instruction of concepts related to differential calculus, more specifically in the limit and continuity of features real, in this sense it is concluded that the use of didactic sequences for instruction these topics allows a conceptual advance when these are related to the graph, although not individually register, since the results show us that it is necessary to move from a representation to another. While different studies claim that the use of algebraic language generates difficulties of conceptualization, existed a high performance in the resolution of situations which were log graph to the algebraic therefore suggests that relates to overall representation graphic and not individually.

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