NMSSM from generalized deflected mirage mediation

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ABSTRACT: We propose to generate a realistic Next-to-Minimal Supersymmetric Standard Model (NMSSM) soft SUSY breaking spectrum from a generalized deflected mirage mediation scenario, in which additional gauge mediation contributions are included to deflect the renormalization group equation (RGE) trajectory. Additional contributions to $m_\text{S}^2$ can possibly ameliorate the stringent constraints from the electroweak symmetry breaking (EWSB) and 125 GeV Higgs mass. Based on the Wilsonian effective action after integrating out the messengers, we find that the new mixed gauge-modulus mediation contributions, which are missed (albeit trivial) in previous studies with ordinary form of messenger sector, can possibly arise and play a role in the generation of the NMSSM soft SUSY breaking spectrum. The relevant fine-tuning and the dark matter constraints are also discussed. The Barbieri-Giudice type fine-tuning in our scenario can be as low as 10, which indicates that our scenario is fairly natural.
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1. Introduction

Low energy supersymmetry (SUSY), which can elegantly solve the gauge hierarchy problem of the Standard Model (SM) and provide a viable dark matter (DM) candidate, has been regarded by many physicists as one of the most appealing candidates for the TeV-scale new physics. However, the reported data of the Large Hadron Collider (LHC) agree quite well with the SM predictions and no significant deviations have been observed in the electroweak precision measurements as well as in flavor physics. Besides, the lack of SUSY signals at the LHC \[1, 2\] and the difficulty to accommodate the discovered 125 GeV Higgs \[3, 4\] seem to indicate that the low energy SUSY spectrum should display an intricate structure. As the low energy soft SUSY breaking spectrum is mainly determined by the SUSY breaking mechanism, it is interesting to survey the phenomenology related to the SUSY breaking and mediation mechanism from a top-down approach.

A typical SUSY breaking mechanism from flux compactification of type IIB string theory can lead to some interesting consequence. In the generalized Kahler-modulus dominated scenarios, the dilaton and complex structure moduli fields can be stabilized by background NS and RR 3-form fluxes and be eliminated from the low energy effective theory. The remained Kahler moduli can be stabilized by non-perturbative effects, such as the gaugino condensation. To generates SUSY breaking in the observable sector and tune the cosmological constant to a tiny positive value, Kachru et al propose to add an anti-D3 brane at the tip of the Klebanov-Strassler throat to explicitly break SUSY and lift the AdS universe to obtain a dS one \[5\]. Consequently, the F-component of the light Kahler moduli fields could mediate the SUSY breaking effects to the visible sector and result in a mixed modulus-anomaly mediation SUSY breaking scenario \[6, 7\]. It is interesting to note that the involved modulus mediated SUSY breaking contributions can be comparable to that of the anomaly mediation \[8\]. With certain assumptions on the Yukawa couplings and the modular weights, the SUSY breaking contributions from the renormalization group running and anomaly mediation could cancel each other at a ”mirage” messenger scale, leading to a compressed low energy SUSY breaking spectrum \[9\]. Such a mixed modulus-anomaly mediation SUSY breaking mechanism is dubbed as ”mirage mediation”.

Anomaly mediation contribution is a crucial ingredient of such a mixed modulus-anomaly mediation. It is well known that the anomaly mediation, including its deflected version with messengers, needs an additional mechanism to tackle the $\mu - B\mu$ problem. In fact, in the mixed modulus-anomaly mediation mechanism, a straightforward extension of $\mu$ sector to predict a small $B\mu$ term needs some fine tuning \[9\]. Such a $\mu - B\mu$ problem can naturally be solved in the framework of the next-to-minimal supersymmetry standard model (NMSSM). The realization of NMSSM in TeV mirage mediation has been discussed in the literature \[10, 11\]. However, it was found that only a small portion of the allowed parameter space can be consistent with the EWSB condition and at the same time accommodate the 125 GeV Higgs \[10\]. So it is rather interesting to seek new ways to generate a realistic NMSSM spectrum.

Additional gauge contributions in the mirage mediation \[12\] can deflect the RGE trajectory and change the low energy soft SUSY predictions. In this work we propose to
generate the NMSSM spectrum by a generalized deflected mirage mediation mechanism [13] with additional gauge mediation contributions. We find that the existence of the messenger sector can possibly alleviate the stringent constraints from the 125 GeV Higgs and the EWSB. Besides, we argue that the possible mixed gauge-modulus contributions could arise in certain realizations of such a general deflected mirage mediation.

This paper is organized as follows. A general discussion on the analytical expressions for the soft SUSY parameters in the deflected mirage mediation is given in Section 2. The numerical results are discussed in Section 3. Section 4 contains our conclusions.

The analytical expressions of the soft SUSY parameters for the NMSSM are given in the appendix.

2. NMSSM from deflected mirage mediation

The mirage mediation of SUSY breaking is a typical mixed modulus-anomaly mediation mechanism, within which the tree-level modulus mediation contributions are comparable to anomaly mediation terms at the boundary scale. It is well known that the pure anomaly mediation is bothered by the tachyonic slepton problem [14]. One of its non-trivial extensions with messenger sectors, namely the deflected anomaly mediation SUSY breaking (AMSB), can elegantly solve such a tachyonic slepton problem through the deflection of the RGE trajectory [15, 16, 17]. Such a messenger sector can also be present in the mirage mediation and play an important role in giving a preferable low energy SUSY spectrum.

The NMSSM, which is a typical singlet extension of the MSSM, can not only solve the $\mu$-problem elegantly, but also naturally accommodate the observed 125 GeV Higgs mass with additional tree-level contributions or through a mixing between $H_S$ and $H_{SM}$ [18]. Motivated by such virtues, it is interesting to survey the possibility of realizing the low energy NMSSM spectrum from the mirage mediation. However, such a scenario is stringently constrained by accommodating the 125 GeV Higgs mass and at the same time achieving a successful EWSB condition with suppressed trilinear couplings $A_\kappa, A_\lambda, m_S^2$. If additional gauge mediation contributions from the messenger sector are included in mirage mediation, such a deflected mirage mediation scenario may lead to a more realistic NMSSM spectrum. Besides, the tension between the theoretical predictions and experiment data for the muon $g-2$ can possibly be ameliorated in this scenario.

In Type IIB string theory compactified on a Calabi-Yau (CY) manifold, the presence of background fluxes can fix the dilaton and the complex structure moduli, leaving only the Kahler moduli in the four-dimensional Wilsonian effective supergravity action (defined at the boundary scale $\Lambda$) after integrating out the superheavy complex structure moduli and dilaton. The low energy effective theory in terms of compensator field and a single Kahler modulus parameterizing the overall size of the compact space [9] is given as

$$e^{-1}L = \int d^4\theta \left[ \bar{\phi}^\dagger \phi \left( -3 e^{-K/3} \right) - (\phi^\dagger \phi)^2 \bar{\theta}^2 \theta^2 P_{\mu \nu \lambda} \right] + \int d^2 \theta \phi^3 W + \int d^2 \theta \frac{f}{4} W_i W_i^a W_i^a$$  \hspace{1cm} (2.1)
with a holomorphic gauge kinetic term

\[ f_i = \frac{1}{g_i^2} + i \frac{\theta}{8\pi}. \] (2.2)

The Kahler potential involves not only the ‘no-scale’ kinetic form for the Kahler modulus but also additional kinetic terms for messenger fields

\[ K = -3\ln(T + T^\dagger) + Z_X(T^\dagger, T)X^\dagger X + Z_\Phi(T^\dagger, T)\Phi^\dagger \Phi \]
\[ + \sum_i Z_{P_i, \bar{P}_i}(T^\dagger, T)\left( P_i^\dagger \bar{P}_i + \bar{P}_i^\dagger P_i \right), \] (2.3)

with \('P_i' denoting the messenger superfields and \('\Phi' the NMSSM superfields. The Kahler metric for matter fields and messengers as well as holomorphic gauge kinetic functions are assumed to depend non-trivially on the Kahler moduli \(T, \Phi \)

\[ Z_X(T^\dagger, T) = \frac{1}{(T^\dagger + T)^n_X}, \quad Z_\Phi(T^\dagger, T) = \frac{1}{(T^\dagger + T)^n_\Phi}, \]
\[ f_i(T) = T^{l_i}, \quad Z_{P_i, \bar{P}_i}(T^\dagger, T) = \frac{1}{(T^\dagger + T)^{n_P}}. \] (2.4)

The choice of \(n_X, n_\Phi, n_P, l_i\) depends on the location of the fields on the D3/D7 branes. Besides, the universal \(l_i = 1\) are adopted in our scenario so that the gauge fields reside on the D7 brane.

The superpotential takes the most general form involving the KKLT setup and the messenger sectors

\[ W = (\omega_0 - Ae^{-aT}) + W_M + W_{NMSSM}, \] (2.5)

where the first term is generated from the fluxes and the second term from non-perturbative effects, such as gaugino condensation or D3-instanton. The modulus \(T\), which is not fixed by the background flux, can be stabilized by non-perturbative gaugino condensation. The resulting potential from KKLT can stabilize the Kahler moduli with

\[ a \Re\langle T \rangle \approx \ln\left( \frac{A}{\omega_0} \right) \approx \ln\left( \frac{M_{Pl}}{m_{3/2}} \right) \approx 4\pi^2 \] (2.6)

up to \(O(\ln[M_{Pl}/m_{3/2}]^{-1})\). Note that in the KKLT setup, the flux-induced SUSY breaking is dynamically cancelled by the non-perturbative SUSY breaking that stabilizes the Kahler moduli \(T\), leading to a SUSY-preserving solution. In order to obtain a vacuum with a positive cosmological constant and break SUSY, KKLT proposed to add a \(D3\) brane to provide an uplifting operator given by

\[ \mathcal{P}_{lift} = D(T + T^\dagger)^{n_P} \] (2.7)

with a positive constant \(D \sim O(m_{3/2}^2, M_{Pl}^2)\). The uplifting operator, which represents the low energy consequence of the sequestered SUSY-breaking brane, is independent of visible matter fields and \(T\) (with \(n_P = 0\) in the minimal KKLT set-up. Explicit SUSY breaking
via anti-D3 branes can be generalized to the traditional D-term or F-term soft SUSY breaking mechanisms [19].

With the uplifting low energy effective potential, we have the leading order results [9] on F-terms for compensator field \( F_\phi \) and Kahler moduli \( F_T \)

\[
F_\phi \approx m_{3/2} \approx \frac{\omega_0}{M_{Pl}^2 (T + T^*)^{3/2}},
\]

\[
M_0 \equiv \frac{F_T}{T + T^*} \approx \frac{2}{a(T + T^*)} m_{3/2} \approx \frac{m_{3/2}}{\ln \left( \frac{M_{Pl}}{m_{3/2}} \right)}.
\]

The light modulus \( T \) can develop a sizable \( F_T \sim m_{3/2} \). The non-zero F-term VEV of the heavy moduli \( H \) is approximated by

\[
F_H \sim \frac{m_{3/2}}{m_U} \ll m_{3/2} \quad \text{and thus it gives negligible contributions to SUSY breaking [20].}
\]

In the mirage mediation we have \( m_{3/2} \approx (4\pi^2)M_0 \) numerically, which means that the loop induced anomaly mediation contribution is comparable to the modulus contribution. The importance of the anomaly mediation contributions relative to that of the modulus mediations can be parametrized by

\[
\alpha' \equiv \frac{m_{3/2}}{M_0 \ln \left( \frac{M_{Pl}}{m_{3/2}} \right)}.
\]

So \( \alpha' \ll 1 \) corresponds to the limit of pure modulus-mediation, while \( \alpha' \gg 1 \) corresponds to the pure anomaly mediation. Although the minimal KKLT predicts \( \alpha' \approx 1 \), other values of \( \alpha' \sim \mathcal{O}(1) \) can be obtained with typical form of uplifting operator and potential for Kahler moduli [9]. So we leave the value \( \alpha' \) as a free parameter in the following discussions.

In the AMSB, the decoupling theorem states that the anomaly mediation contribution from heavy SUSY thresholds will cancel that of the gauge mediation contributions [21] at leading order. One elegant way to evade the decoupling theorem in AMSB is the deflected AMSB scenario within which a light singlet field is introduced to determine the messenger threshold by its VEVs. The deflected mirage mediation mechanism includes all the essential ingredients of the deflected AMSB.

The \( Z_3 \) symmetric NMSSM superpotential is

\[
W_{NMSSM} = \lambda SH_u H_d + \frac{1}{3} \kappa S^3 + W_{MSSM},
\]

with the messenger terms

\[
W_M = \sum_m \left[ \lambda_X^T X \bar{X}_m X_m + \lambda_Y^T Y \bar{Y}_m Y_m + \lambda_P^T S \bar{X}_1 X_2 + \lambda_{\bar{P}}^T S \bar{Y}_1 Y_2 \right] + W(X).
\]

Here \( W(X) \) is the superpotential for the pseudo-moduli field \( X \) whose VEV determines the messenger threshold. The \( 2m \) family of messengers can be fitted in terms of \( 5, \bar{5} \) representation of \( SU(5) \) GUT group

\[
P_m(5) = X_m(3, 1)_{-1/3} \oplus Y_m(1, 2)_{1/2},
\]

\[
\bar{P}_m(\bar{5}) = \bar{X}_m(3, 1)_{1/3} \oplus \bar{Y}_m(1, 2)_{-1/2}.
\]
The purpose to introduce double messenger family is to guarantee that no kinetic mixing between \( X \) and \( S \) is present, otherwise the tadpole terms for \( S \) would destabilize the weak scale \([22]\). The discrete \( Z_3 \) breaking by EWSB may generate domain walls in the early universe which may lead to an unacceptably large anisotropy of CMB. To avoid such a problem, the \( Z_3 \) symmetry is assumed to be broken by some higher dimensional operators.

The potential for the pseudo-modulus \( W(X) \) can determine the deflection parameter \( 'd' \) of either sign

\[
dF_\phi = \frac{F_X}{X} - F_\phi ,
\]

that characterizes the deviation from the ordinary AMSB trajectory. Similar to the deflected AMSB, a positive deflection parameter in the deflected mirage mediation, which can also be realized by a carefully chosen superpotential \([16]\) or from some strong dynamics \([17]\), may be preferable because less messenger species are needed so as to evade strong gauge couplings below the GUT scale.

There are two approaches to obtain the low energy SUSY spectrum in the deflected mirage mediation:

- In the first approach, the mixed modulus-anomaly mediation soft SUSY spectrum is still given by the expressions in \([9]\) at the boundary scale. Such a spectrum will receive additional contributions towards its RGE running to low energy scale, especially the threshold corrections related to the appearance of messengers. The relevant analytical expressions are given in \([23]\).

- In the second approach which we will adopt, the soft SUSY spectrum at low energy scale is derived directly from the (low energy) effective action. The SUGRA description in eq.(2.1) can be seen as a Wilsonian effective action after integrating out the complex structure moduli and dilaton field. After the pseudo-modulus acquires a VEV and determines the messenger threshold, the messenger sector can be integrated out to obtain a low energy effective action below the messenger threshold scale. So we anticipate the Kahler metric \( Z_\Phi \) and gauge kinetic \( f_i \) will depend non-trivially on the messenger threshold \( M_{mess}^2/\phi^\dagger\phi \) and \( M_{mess}/\phi \), respectively. The resulting soft SUSY spectrum below the messenger threshold can be derived from the wavefunction renormalization approach \([24]\).

Now we use the second approach in our analysis. Below the messenger scale, the soft gaugino mass at a scale \( \mu \) is given by

\[
M_i(\mu) = g_i^2 \left( F_T \frac{\partial}{\partial T} - F_\phi \frac{\partial}{\partial \ln \mu} + dF_\phi \frac{\partial}{\partial \ln X} \right) \frac{1}{g_i}(\mu, X, T) ,
\]

with

\[
\frac{\partial}{\partial \ln X} g_i(\alpha; X) = \frac{1}{2} \frac{\Delta b_i}{16\pi^2} g_i^3 .
\]

In the previous expression, the wavefunction renormalization approach is used, in which the messenger threshold \( M_{mess}^2 \) is replaced by the spurious chiral fields \( X \) with \( M_{mess}^2 = X^\dagger X \).
The trilinear soft terms will also be given by the wavefunction renormalization and the form of the superpotential

\[ A_{ijk} = y_{ijk} / \sqrt{e^{-K_0}Z_i Z_j Z_k} \]  

(2.17)

Note that the holomorphic cubic Yukawa couplings \( y_{ijk} \) are independent of \( T \).

After integrating out the messenger fields, the trilinear soft terms are given by

\[
A_{ijk}^0(\mu) \equiv \frac{A_{ijk}}{y_{ijk}} = \sum_i \left( \frac{F_T}{2} \frac{\partial}{\partial T} - \frac{F_\phi}{2} \frac{\partial}{\partial \ln \mu} + dF_\phi \frac{\partial}{\partial \ln X} \right) \ln \left[ e^{-K_0/3}Z_i(\mu, X, T) \right]
\]

(2.18)

with the 'no scale' Kahler potential \( K_0 = -3 \ln(T + T^*) \). The expression at the messenger scale is given as

\[
A_{ijk}^0 = \sum_{i,j,k} \left( m_i M_0 - \frac{F_\phi}{2} G_i^- + dF_\phi \frac{\Delta G_i}{2} \right)
\]

(2.19)

where \( m_i \equiv 1 - n_i \), the anomalous dimensions in the holomorphic basis given by [25]

\[
G^i = \frac{dZ_{ij}}{d\ln \mu} \equiv - \frac{1}{8\pi^2} \left( \frac{1}{2} d^{ikl}_{a} \lambda^{*}_{ikl} \lambda_{jmns} Z_{kms}^{-1} Z_{lns}^{-1} - 2e^i \partial \vec{g} \right)
\]

(2.20)

The details of the expression involving the derivative of \( \ln X \) can be found in [26, 27, 25].

We can see that there are several origins of the contributions from the interference terms:

- The modulus-anomaly interference contributions given by

\[
\frac{\partial^2}{\partial T \partial \ln \mu} Z_i^- = \frac{\partial}{\partial T} G_i^-(Z_a, g_m) = \left[ \frac{\partial Z_a}{\partial T} \frac{\partial}{\partial Z_a} + \frac{\partial g_m}{\partial T} \frac{\partial}{\partial g_m} \right] G_i^-(Z_a, g_m)
\]

(2.22)

- The modulus-gauge interference contributions given by

\[
\frac{\partial^2}{\partial T \partial \ln X} Z_i^- = \frac{\partial}{\partial T} \frac{\Delta G_i(Z_a, g_m)}{2} = \left[ \frac{\partial Z_a}{\partial T} \frac{\partial}{\partial Z_a} + \frac{\partial g_m}{\partial T} \frac{\partial}{\partial g_m} \right] \frac{\Delta G_i(Z_a, g_m)}{2}
\]

(2.23)
In previous discussions in deflected mirage mediation[12], the modulus-gauge interference contributions are missed albeit trivial for the form of the messenger sector given there. We should note that such contributions should in general be present and may give a contribution in certain circumstances.

The remaining terms are the sum of pure modulus and deflected anomaly mediation contributions.

3. Numerical Results

In the NMSSM the successful EWSB and the solution to the \( \mu \)-problem in general require a large VEV for the singlet \( S \). So it is preferable to introduce a negative \( m_S^2 \) and/or large trilinear terms \( A_\lambda, A_\kappa \) for the singlet superpotential interactions. A negative \( m_S^2 \) obviously prefers vanishing new positive modulus contributions in our generalized mirage mediation.

The choice of modulus weight for \( H_u, H_d \) can be understood from the EWSB conditions in NMSSM. We know that an important EWSB condition in NMSSM is

\[
\frac{M_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2. 
\]

(3.1)

It is clear from this EWSB condition that \( m_{H_u}^2 \) should be light to avoid a too large fine-tuning. So it is natural to choose vanish modulus contribution for \( H_u \). On the other hand, \( m_{H_u}^2 \ll m_{H_d}^2 \) for \( \tan \beta \gg 1 \). So we can set \( m(H_d) = 1 \) or \( 1/2 \).

The EW naturalness prefers relatively light stops. In the MSSM, light stops below 1 TeV are disfavored because it is difficult to accommodate the observed 125 GeV Higgs. However, large loop corrections involving stops are not necessarily required to interpret the 125 GeV Higgs in NMSSM. So light stops are still preferable from the criterion of a low EW fine-tuning. Squarks of the first two generations should be heavy to avoid various SUSY CP and flavor constraints. We note that even pure AMSB contributions can guarantee the heaviness of colored SUSY particles. Besides, light sleptons and electroweakinos are preferable to interpret the \( g_\mu - 2 \) anomaly in the framework of SUSY. We are interested in the case where the visible gauge fields originate from D7 branes. With the modular weight \( l_i = 1 \) for gauge fields, a positive deflection parameter \( d \) can possibly guarantee the lightness of the electroweakinos.

The notorious tachyonic slepton problem in AMSB can be solved in our scenario. Positive slepton masses can be realized by introducing a proper deflection parameter \( d \) or by adding extra moduli mediated contributions. So we chose the following modular weights \( n_i \equiv 1 - m_i \) in our scenario:

- A deflection parameter \( d \) of either sign with \( m_{(L_L)^{1,2,3}} = m_{(E_L)^{1,2,3}} = 1/2 \).
- Modular weights for other matter and messenger fields are given by

\[
m_{H_u} = m_S = m_{Q_1^L} = m_{Q_2^L} = m_{Q_3^L} = 0, \\
m_a = \frac{1}{2}, \quad (a = Q_1^U, (U_L^c)^{1,2}, (D_L^c)^{1,2}) , \\
m_{H_d} = m_X = m_{X} = m_Y = m_{\tilde{Y}} = 1. 
\]

(3.2) (3.3) (3.4)
Note that the messenger modular weights also play a role and contribute to $m_S^2$. The modular weights $n_i = 0$ correspond to matter fields on D7 branes while $n_i = 1$ on D3 branes. Modular weights $n_i = 1/2$ corresponds to fields on the intersections of the D3-D7 branes.

We also note that these coefficients are determined by the modulus-dependence of the Kahler metric as well as the Yukawa couplings. As noted in [10], we can write the modular weights as

$$n_i = n_i(\text{tree}) + \delta n_i(\text{loop}),$$

with $n_i(\text{tree})$ being calculated from the tree-level Kahler metric of the matter fields. The high order corrections $\delta n_i(\text{loop})$ are obtained with the one-loop Kahler metric of the matter fields, but such loop corrections to the Kahler metric depend on the details of the ultraviolet-model and are hard to calculate. So $n_i$ is ambiguous at the one-loop level and such ambiguity is subdominant and less important in most cases [20].

After fixing the modular weights, the remaining free parameters in our scenario are

$$d, \alpha, M_{\text{mess}}, M_0, \lambda, \kappa, \lambda_P^D, \lambda_P^T, \lambda_X^D, \lambda_X^T$$

(3.6)

with $F_\phi/(16\pi^2) \approx \alpha M_0$ and the simplest choice $\lambda_P^D = \lambda_P^T = \lambda_X^D = \lambda_X^T = \lambda_0$ in our numerical study. Note that for later convenience, the definition of $\alpha$ is four times smaller than $\alpha'$ that appears in eq. (2.10). The ratio $\alpha$ between $F_\phi/(16\pi^2)$ and $M_0$ holds in the messenger scale and in general is different from its value in the GUT scale. We choose a positive $\alpha$ in our numerical study. For a negative $\alpha$, virtual mirage unification at super-GUT energy scales will appear.

We need to check if successful EWSB condition is indeed fulfilled. In fact, the soft SUSY mass $m_{H_u}^2, m_{H_d}^2, m_S^2$ can be reformulated into $\mu, \tan \beta, M_Z^2$ by the minimum condition of the scalar potential. Usually, $M_A$ can be used to replace $A_\kappa$

$$M_A^2 = \frac{2\mu_{\text{eff}}}{\sin 2\beta} B_{\text{eff}}, \quad \mu_{\text{eff}} = \lambda(s), \quad B_{\text{eff}} = (A_\lambda + \kappa(s)).$$

(3.7)

In order to transform $m_{H_u}^2, m_{H_d}^2, m_S^2$ into $\mu, \tan \beta, M_Z^2$, we use the following approximation

$$|\mu_{\text{eff}}|^2 = -\frac{M_A^2}{2} - m_{H_u}^2 + \frac{1}{\tan^2 \beta} (m_{H_d}^2 - m_{H_u}^2) + O(1/ \tan^4 \beta),$$

$$\sin 2\beta = \frac{2B_{\text{eff}}\mu_{\text{eff}}}{m_{H_u}^2 + m_{H_d}^2 + 2|\mu_{\text{eff}}|^2 + \lambda^2 v^2},$$

(3.8)

(3.9)

to obtain $\mu$ and $\tan \beta$ iteratively.

We use NMSSMTools5.2.0[29] to scan the whole parameter space. The parameters are chosen to satisfy:

$$10^{15}\text{GeV} > M_{\text{mess}} > 10^9\text{GeV}, \quad 100\text{TeV} > M_0 > 0.1\text{TeV},$$

$$16 > \alpha > 0, \quad 4 > d > -4, \quad 0.7 > \lambda, \kappa > 0, \quad \sqrt{4\pi} > \lambda_0 > 0.$$  

(3.10)

Double messenger species with $m = 1$ are introduced in our numerical study.

In our scan, we impose the following constraints:

$$10^{15}\text{GeV} > M_{\text{mess}} > 10^9\text{GeV}, \quad 100\text{TeV} > M_0 > 0.1\text{TeV},$$

$$16 > \alpha > 0, \quad 4 > d > -4, \quad 0.7 > \lambda, \kappa > 0, \quad \sqrt{4\pi} > \lambda_0 > 0.$$  

(3.11)
(I) The lower bounds on SUSY particles \([30, 31]\) from the LHC

- Gluino mass: \(m_{\tilde{g}} \gtrsim 1.8 \text{ TeV}\).
- Light stop mass: \(m_{\tilde{t}_1} \gtrsim 0.85 \text{ TeV}\).
- Light sbottom mass \(m_{\tilde{b}_1} \gtrsim 0.84 \text{ TeV}\).
- Degenerated first two generation squarks \(m_{\tilde{q}} \gtrsim 1.0 \sim 1.4 \text{ TeV}\).

(II) The CP-even component \(S_2\) in the Goldstone-'eaten' combination of \(H_u\) and \(H_d\) doublets corresponds to the SM Higgs. The \(S_2\) dominated CP-even scalar should lie in the combined mass range for the Higgs boson: \(122 \text{ GeV} < M_h < 128 \text{ GeV}\) from ATLAS and CMS data \([3, 4]\). Note that the uncertainty is 3 GeV instead of the default 2 GeV because a large \(\lambda\) may induce an additional 1 GeV correction to \(m_h\) at two-loop level \([32]\), which is not included in the NMSSMTools.

(III) The relic density of the neutralino dark matter should satisfy the Planck data \(\Omega_{DM} = 0.1199 \pm 0.0027\) \([33]\) in combination with the WMAP data \([34]\) (with a 10\% theoretical uncertainty). In our scenario, only the upper bound on DM relic density is used.

(IV) The electroweak precision observables \([35]\) and the lower bounds on neutralinos and charginos, including the invisible decay bounds for \(Z\)-boson. The most stringent constraints of LEP require \(m_{\tilde{\chi}} > 103.5\text{ GeV}\) and the invisible decay width \(\Gamma(Z \rightarrow \tilde{\chi}_0\tilde{\chi}_0) < 1.71 \text{ MeV}\), which is consistent with the 2\(\sigma\) precision EW measurement \(\Gamma^{non-SM}_{inv} < 2.0 \text{ MeV}\) \([36]\).

(V) Flavor constraints \([37]\) from B-meson rare decays

\[
1.7 \times 10^{-9} < Br(B_s \rightarrow \mu^+\mu^-) < 4.5 \times 10^{-9}, \\
0.85 \times 10^{-4} < Br(B^+ \rightarrow \tau^+\nu) < 2.89 \times 10^{-4}, \\
2.99 \times 10^{-4} < Br(B_S \rightarrow X_s\gamma) < 3.87 \times 10^{-4}.
\]

(3.12) \(3.13) \(3.14)

(VI) The tension between the theoretical prediction and the experiment value for the muon anomalous magnetic moment should be ameliorated by additional positive SUSY contributions. The E821 experimental result for the muon \(g - 2\) at the Brookhaven AGS \([38]\) is given by

\[
a_{\mu}^{\text{expt}} = 116592089(63) \times 10^{-11},
\]

(3.15)

which is larger than the SM prediction\([39]\)

\[
a_{\mu}^{\text{SM}} = 116591834(49) \times 10^{-11}.
\]

(3.16)

The deviation is about 3\(\sigma\)

\[
\Delta a_{\mu}(\text{expt} - \text{SM}) = (255 \pm 80) \times 10^{-11}.
\]

(3.17)

We adopt a conservative estimation \(4.7 \times 10^{-10} \lesssim \Delta a_{\mu} \lesssim 52.7 \times 10^{-10}\) in our numerical results.
We obtain the following numerical results:

- The low energy soft SUSY breaking spectrum of NMSSM determined from a top-down approach by a UV-completed theory is always bothered by the requirement to achieve the successful EWSB. In fact, EWSB within NMSSM in general requires a large VEV for the singlet. This prefers a negative $m^2_S$ and/or large $A_\lambda, A_\kappa$ for the singlet potential. The mirage mediation scenario always predicts a large positive value for $m^2_S$ and not very large $A_\lambda, A_\kappa$, suppressing the singlet VEV. In our scenario, because of the additional contributions to $m^2_S$ from gauge interaction, the stringent constraints from successful EWSB can be ameliorated. The observed 125 GeV Higgs mass, which is either the lightest or the second-lightest CP-even scalar in NMSSM, can also be successfully accommodated in our scenario.

**Figure 1:** The values of $(\lambda, \kappa)$ that satisfy the EWSB condition and at the same time accommodate the 125 GeV Higgs boson as the lightest (left panel) or second lightest (right panel) CP-even scalar. All points satisfy the constraints (I-VI).

We can see from Fig. 1 that many samples of $(\lambda, \kappa)$ can survive the EWSB conditions in NMSSM. In contrast to the numerical results in [10] within which the allowed $(\lambda, \kappa)$ only take values near $(0.7, 0.1)$, some portion of $(\lambda, \kappa)$ parameter space can survive all constraints in our scenario in case that the 125 GeV Higgs is the lightest CP-even scalar. Such differences are the consequence of additional deflection and different choices of energy scale at which the input parameters take values. Note that the vacuum stability bounds are also taken into account in our numerical studies, which impose stringent constraints on scenario in [10]. In case that the 125 GeV Higgs is the second lightest CP-even scalar, the allowed $(\lambda, \kappa)$ parameter space is also much bigger than that in [10]. The Higgs mass can be increased 8 GeV by the mixing with the singlet component for a large tan $\beta$ and $\lambda \lesssim 0.04$.

It can be seen from Fig. 2 that the modulus mediation contribution $M_0$ is bounded to lie between 180 GeV and 4 TeV for $H_{SM}$ being the lightest CP-even scalar. A small $M_0$ always prefers a large messenger scale $M_{mess}$. For $M_0$ less than 2 TeV, the Higgs mass can not exceed 124 GeV. A large $M_0$, which sets the whole soft SUSY breaking parameters to be heavy, can easily accommodate the SM-like Higgs mass because of large loop corrections from heavy stops in addition to the tree-level contributions.
involving $\lambda$. The value of $M_0$ is upper bounded to be less than about 5.5 TeV for $H_{SM}$ being the second lightest CP-even scalar, which sets an upper bound on the soft SUSY breaking parameters, especially for the gluino masses. A small separated region with $M_0 \in (70, 100)$ GeV, which predicts $H_{SM} \approx 122$ GeV, can also survive all constraints. Such region can give large SUSY contributions to $\Delta a_{\mu}$. The gluino mass, which is determined by the scale of $M_0$, is bounded to below 8 TeV for $H_{SM}$ being the lightest CP-even scalar and below 6 TeV for $H_{SM}$ being the second lightest CP-even scalar, respectively.

**Figure 2:** Scatter plots of the SM-like Higgs mass $m_h$ vs the modulus mediation parameter $M_0$ and gluino mass $m_{\tilde{g}}$ for $m_{SM}$ being the lightest (left panels) or second lightest (right panels) CP-even scalar. All samples satisfy the constraints (I-VI).

- The BG fine-tuning [40] with respect to a certain input parameter '$a'$ is defined as
  \[
  \Delta_a \equiv \left| \frac{\partial \ln M_Z^2}{\partial \ln a} \right|. \quad (3.18)
  \]
  The fine-tuning measure of our scenario is defined to be $\Delta = \max_a (\Delta_a)$ where '$a'$ stands for the set of parameters defined at the input scale.
  We can see from Fig.2 that the fine-tunings in our scenario can range from $\mathcal{O}(10)$ to $\mathcal{O}(1000)$. In fact, the lowest fine-tuning can reach 10 when $H_{SM}$ is the second lightest CP-even scalar. Such a low fine-tuning indicates that our scenario is fairly natural. It is known that the low fine-tuning needs light stops as well as a small effective $\mu$. 


which are determined by the only dimensional parameter $M_0$ that determines the whole soft SUSY spectrum. The lower the $M_0$ (consequently the lower gluino mass), the lower the BG fine-tuning.

- It is known that a positive deflection parameter $'d'$ is favored by solving the tachyonic slepton problem in the deflected AMSB for fewer messenger species. In the deflected mirage mediation, if the modulus contribution is subdominant, a realistic model still prefers a positive deflection parameter $'d'$ with less double messenger species. In our scenario with double messenger species, we find that all survived samples have a positive deflection parameter $'d'$. Negative values of $'d'$ are mostly ruled out by the EWSB condition and tachyonic sfermions. The relevant results are displayed in Fig.3. As the parameter $\alpha$ determines the relative size between the anomaly mediation and the modulus mediation, a large value of $\alpha$, which indicates less modulus mediation contributions, needs a larger positive deflection parameter $'d'$ to avoid tachyonic sleptons. In the anomaly dominated regions with a large $\alpha$, the deflection parameter should lie between 1 and 2 to tune the tachyonic slepton masses to positive values by additional gauge mediation contributions. It can also be seen that the modulus mediation dominated scenario, that is small $\alpha$ with $d = 0$, can also lead to a realistic NMSSM model. From Fig.3, we can see that most survived regions require $d \neq 0$, especially for scenarios with $H_{SM}$ being the lightest CP-even scalar. For scenarios with $H_{SM}$ being the second lightest CP-even scalar, the allowed values of $\alpha$ will also be increased. So, additional deflection, which is characterized by a non-vanishing $'d'$, plays an very important role in predicting a realistic NMSSM spectrum.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure3a.png}
\includegraphics[width=0.4\textwidth]{figure3b.png}
\caption{The allowed regions for the deflection parameter $'d'$ versus $\alpha$, which parametrize the relative size between the anomaly mediation and the modulus mediation. All samples satisfy the constraints (I-VI).}
\end{figure}

- From eq.(A.1) in the appendix, we can see the gaugino ratio at the EW scale

$$M_3 : M_2 : M_1 \approx 6 \cdot \left[ \frac{1}{g_3^2} + \alpha(-3 - 2d) \right] : 2 \cdot \left[ \frac{1}{g_2^2} + \alpha(1 - 2d) \right] : \left[ \frac{1}{g_1^2} + \alpha(6.6 - 2d) \right]$$

where $g_1, g_2, g_3$ take values at the messenger scale. On the other hand, the singlino mass is determined by $\kappa$ and $\langle s \rangle$, which rescales the $\mu$ parameter by a factor $2\kappa/\lambda$. 

\[ \text{(3.19)} \]
In general, a pure singlino-like LSP tends to have a too large relic density due to a comparatively small annihilation cross section because of its small couplings to SM particles.

Our numerical results indicate that most parameter space predicts a wino-like LSP in case that $H_{SM}$ is the lightest CP-even scalar. It is well known that the DM relic abundance requires a wino DM mass to lie near 3 TeV. As the allowed wino DM mass is too light (see Fig. 3), other DM components are necessary to provide a large enough $\Omega_{DM}$.

In case that $H_{SM}$ is the second lightest CP-even scalar, the DM is mostly singlino-like with a small higgsino component. With the presence of small higgsino components, the singlino-like LSP annihilates via the t-channel $\chi_0^1$ exchange into pairs of mostly singlet-like $H_1$ and $A_1$ by enhanced $\chi_0^0\chi_0^0 H_1(A_1)$ couplings. Co-annihilation with heavier $\chi_2^0$ (for $\Delta m \lesssim 10$ GeV) will also efficiently reduce the singlino relic abundance to a proper $\Omega_{DM}$.

It can be seen from the lower panels of Fig. 4, a large portion of parameter space can survive the spin-independent (SI) DM direct detection constraints from the LUX [41] and PANDAX [42]. Because of the heavy squarks, the tree-level SI-cross section from the s-channel squark processes are suppressed. It is obvious that a wino-like DM, which is the case for $H_{SM}$ being the lightest CP-even scalar, can easily escape the SI direct detection bounds because the coupling of the $\chi_0^1$ to Z or Higgs bosons scales with gaugino-higgsino mixing. For a singlino-like DM, the exchange of a light $H_1$ can possibly lead to a large direct detection cross section which will be accessible in the present generation of detectors.

• Fig. 5 shows the SUSY contributions to the muon $g - 2$. It is known that the required SUSY contributions to $\Delta a_\mu$ can be achieved only if the relevant sparticles($\tilde{\mu}, \tilde{\nu}_\mu, \tilde{B}, \tilde{W}, \tilde{H}$) are lighter than 600 $\sim$ 700 GeV for $\tan\beta \sim 10$ in the MSSM [43]. The inclusion of the singlino in the NMSSM can not give sizable contributions to $\Delta a_\mu$ because of the suppressed couplings of singlino to MSSM sector. Although the two loop contributions involving the Higgs are negligible in the SM, the new Higgs bosons in the NMSSM could have an important impact on $\Delta a_\mu$ if the lightest neutral CP-odd Higgs scalar is very light [44]. In fact, a positive two-loop contribution is numerically more important for a light CP-odd Higgs being a bit heavier than 3 GeV and the sum of both one-loop and two-loop contributions is maximal around $m_{a_1} \sim 6$ GeV. In our scenario, the lightest CP-odd Higgs $a_1$ is always not light enough to give sizable contributions to $\Delta a_\mu$. The main contribution is thus similar to that in the MSSM.

4. Conclusions

We proposed to generate a realistic NMSSM soft SUSY breaking spectrum from a generalized deflected mirage mediation scenario, in which additional gauge mediation contributions
Figure 4: The upper panels show the plots of DM mass vs the DM components in case that $H_{SM}$ is the lightest (left panel) or second lightest (right panel) CP-even scalar. Similarly, the lower panels show the plots of DM mass versus the Spin-Independent(SI) direct detection bounds. All samples satisfy the constraints from (I-VI).

Figure 5: The SM-like Higgs mass versus the SUSY contributions to the muon anomalous magnetic moment $\Delta a_\mu$. All samples satisfy the constraints (I-VI).

are included to deflect the renormalization group equation trajectory. Additional contributions to $m_2^S$ can possibly ameliorate the stringent constraints from electroweak symmetry breaking (EWSB) and 125 GeV Higgs mass. We also noted that the new mixed gauge-modulus mediation contributions, which are missed in previous studies with ordinary forms of messenger sector, can possibly arise and play a role in the generation of NMSSM soft
SUSY breaking spectrum. The relevant fine-tuning and dark matter constraints were also discussed. The Barbieri-Giudice fine-tuning in our scenario can be as low as 10, which indicates that our scenario is fairly natural. Our discussion is based on the Wilsonian effective action after integrating out all heavy fields including the messengers. The numerical inputs, which take values at the messenger scale, were in general different from the inputs of the ordinary mixed modulus-anomaly mediation scenarios which takes values at the GUT scale.

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A. Analytical expressions of soft SUSY parameters

From the analytical expressions for the generalized mirage mediation given in Section 2, the soft SUSY breaking parameters in NMSSM at the messenger scale after integrating out the messenger fields can be given explicitly. The gaugino masses are given as

\[ M_i = l_i M_0 + F_\phi \frac{\alpha_i}{4\pi} (b_i - d \Delta b_i) , \]

with \( l_i = 1 \) and

\[ (b_1 , b_2 , b_3) = \left( \frac{33}{5}, 1, -3 \right) , \]

\[ \Delta(b_1 , b_2 , b_3) = (2, 2, 2). \]

Within the expression, the relative size between the anomaly and modulus mediation contribution at the messenger scale is determined by the free parameter

\[ \alpha = \frac{F_\phi}{(16\pi^2)M_0} . \]

The trilinear soft terms at the messenger scale are given by

\[ A_t = (m_{Q_{L,3}} + m_{H_u} + m_{t_L}) M_0 + \frac{F_\phi}{16\pi^2} \left[ 6y_t^2 + y_b^2 + \lambda^2 - \left( \frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2 \right) \right] , \]

\[ A_b = (m_{Q_{L,3}} + m_{H_d} + m_{b_L}) M_0 + \frac{F_\phi}{16\pi^2} \left[ y_b^2 + 6y_b^2 + \lambda^2 - \left( \frac{16}{3} g_3^2 + 3g_2^2 + \frac{7}{15} g_1^2 \right) \right] , \]
\[ A_r = (m_{L_L} + m_{H_u} + m_{Y_L}) M_0 + \frac{F_\phi}{16 \pi^2} \left[ 3g_y^2 + \lambda^2 - (3g_y^2 + \frac{9}{5} g_1^2) \right], \]

\[ A_\lambda = (m_S + m_{H_d} + m_{Y_u}) M_0 + \frac{F_\phi}{16 \pi^2} \left[ 4\lambda^2 + 2\kappa^2 + 3y_t^2 + 3y_b^2 - (3g_y^2 + \frac{3}{5} g_1^2) \right] + \Delta A_\lambda, \]

\[ A_\kappa = 3m_S M_0 + \frac{F_\phi}{16 \pi^2} [6\lambda^2 + 6\kappa^2] + \Delta A_\kappa, \]

with new contributions due to non-vanishing \( \Delta G_S \)

\[ \Delta A_\lambda = -d \frac{F_\phi}{16 \pi^2} \left[ 3(\lambda P)^2 + 2(\lambda P)^2 \right], \]

\[ \Delta A_\kappa = -3d \frac{F_\phi}{16 \pi^2} \left[ 3(\lambda P)^2 + 2(\lambda P)^2 \right]. \]

The expressions for scalars are rather complicated. We parameterize the contribution

\[ m_{\text{soft}}^2 = \delta_m + \delta_d + \delta_I, \]

with each part is given in the following:

- The pure modulus contribution part

\[ \delta_m = m_i M_0^2 = (1 - n_i) M_0^2. \]

- The deflected anomaly mediation part

\[ \delta^d_{Q_{L,1,2}} = \frac{F_\phi^2}{16 \pi^2} \left[ \frac{8}{3} G_3 \alpha_3^2 + \frac{3}{2} G_2 \alpha_2^2 + \frac{1}{30} G_1 \alpha_1^2 \right], \]

\[ \delta^d_{U_{L,1,2}} = \frac{F_\phi^2}{16 \pi^2} \left[ \frac{8}{3} G_3 \alpha_3^2 + \frac{8}{15} G_1 \alpha_1^2 \right], \]

\[ \delta^d_{D_{L,1,2,3}} = \frac{F_\phi^2}{16 \pi^2} \left[ \frac{8}{3} G_3 \alpha_3^2 + \frac{2}{15} G_1 \alpha_1^2 \right], \]

\[ \delta^d_{L_{L,1,2,3}} = \frac{F_\phi^2}{16 \pi^2} \left[ \frac{3}{2} G_2 \alpha_2^2 + \frac{3}{10} G_1 \alpha_1^2 \right], \]

\[ \delta^d_{E_{L,1,2,3}} = \frac{F_\phi^2}{16 \pi^2} \frac{6}{5} G_1 \alpha_1^2, \]

\[ \delta^d_{H_u} = \frac{F_\phi^2}{16 \pi^2} \left[ \frac{3}{2} G_2 \alpha_2^2 + \frac{3}{10} G_1 \alpha_1^2 \right] \]

\[ + \frac{F_\phi^2}{(16 \pi^2)^2} \lambda^2 \left( 4\lambda^2 + 2\kappa^2 + 3y_t^2 + 3y_b^2 - (3g_y^2 + \frac{3}{5} g_1^2) \right) \]

\[ + \frac{F_\phi^2}{(16 \pi^2)^2} 3y_t^2 \left( 6y_t^2 + y_b^2 + \lambda^2 - \frac{16}{3} y_b^2 - 3g_y^2 - \frac{13}{15} g_1^2 \right) \]

\[ - 2d \frac{F_\phi^2}{(16 \pi^2)^2} \lambda^2 \left[ 3(\lambda P)^2 + 2(\lambda P)^2 \right], \]

\[ \delta^d_{H_d} = \frac{F_\phi^2}{16 \pi^2} \left[ \frac{3}{2} G_2 \alpha_2^2 + \frac{3}{10} G_1 \alpha_1^2 \right]. \]
So the gauge mediation contribution is

\[ G_i = Nd^2 + 2Nd - b_i, \]  

(A.21)

\[ (b_1, b_2, b_3) = \left( \frac{33}{5}, 1, -3 \right). \]  

(A.22)

For the third generation \( \tilde{Q}_{L,3}, \tilde{U}_L \), we need to include the \( y_t \) Yukawa contributions

\[
\begin{align*}
\delta_{Q_{L,3}}^d &= \delta_{Q_{L,1,2}}^d + \frac{F_0^2}{(16\pi^2)^2} \frac{1}{2} \left( 6y_t^2 + y_b^2 + \lambda^2 - \frac{16}{3} g_t^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right) \quad \text{(A.23)} \\
\delta_{U_{L,3}}^d &= \delta_{U_{L,1,2}}^d + \frac{F_0^2}{(16\pi^2)^2} \frac{1}{2} \left( 6y_t^2 + y_b^2 + \lambda^2 - \frac{16}{3} g_t^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right) \quad \text{(A.24)} \\
\delta_{D_{L,3}}^d &= \delta_{D_{L,1,2}}^d + \frac{F_0^2}{(16\pi^2)^2} \frac{1}{2} \left( 6y_t^2 + y_b^2 + \lambda^2 - \frac{16}{3} g_t^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right) \quad \text{(A.25)}
\end{align*}
\]

The contribution to \( \delta_S^d \) is divided into three parts

\[ \delta_S^d = \Delta^A_P + \Delta^G_P + \Delta_P. \]  

(A.27)

We have the pure anomaly mediation part

\[ \Delta_P^A = \frac{F_0^2}{(16\pi^2)^2} \left[ 2\lambda^2 \left( 4\lambda^2 + 2\lambda^2 + 3g_t^2 + 3g_2^2 - 3 \frac{3}{5} g_1^2 \right) + 2\lambda^2 (6\lambda^2 + 6\lambda^2) \right] \]  

(A.28)

Besides, the \( m_S^2 \) term receives new contributions involving \( \lambda_P \) because \( G_S \) is not continuous across the messenger threshold

\[ \Delta G_S = \frac{1}{8\pi^2} \left[ 3(\lambda_P^T)^2 Z_{XX}^{-1} Z_{XX}^{-1} + 2(\lambda_P^D)^2 Z_{YY}^{-1} Z_{YY}^{-1} \right] \]  

(A.29)

So the gauge mediation contribution is

\[ \Delta_P^G = -\frac{d^2 F_0^2}{4(8\pi^2)} \left[ 3(\lambda_P^T)^2 \left( G_{XX}^+ \right) + 2(\lambda_P^D)^2 \left( G_{XX}^+ \right) \right] + \frac{d^2 F_0^2}{16\pi^2} \left( \lambda^2 \Delta G_X + \lambda^2 \Delta G_\lambda \right) \]  

(A.30)

with

\[ G_{XX}^+ = -\frac{1}{8\pi^2} \left[ 5(\lambda_P^T)^2 + 2(\lambda_P^D)^2 + 2\lambda^2 + 2\lambda^2 + 2(\lambda_P^T)^2 - \left( \frac{16}{3} g_t^2 + \frac{4}{15} g_1^2 \right) \right] \]  

(A.31)

\[ G_{XX}^+ = -\frac{1}{8\pi^2} \left[ 3(\lambda_P^T)^2 + 4(\lambda_P^D)^2 + 2\lambda^2 + 2\lambda^2 + 2(\lambda_P^T)^2 - \left( \frac{3}{5} g_t^2 + \frac{3}{5} g_1^2 \right) \right] \]  

(A.32)

\[ \Delta G_X = -\frac{1}{8\pi^2} \left[ 3(\lambda_P^T)^2 + 2(\lambda_P^D)^2 \right] \]  

(A.33)

\[ \Delta G_\lambda = -\frac{1}{8\pi^2} \left[ 3(\lambda_P^T)^2 + 2(\lambda_P^D)^2 \right] \]  

(A.34)
The anomaly-gauge mixing term is given by
\[
\Delta_P = -\frac{2dF^2_F}{(16\pi^2)^2} \left\{ 2\lambda^2 \left[ 3(\lambda_P^T)^2 + 2(\lambda_P^D)^2 \right] + 2\kappa^2 \right\} (A.35)
\]

- The interference terms involving the Kahler moduli ‘T’:

\[
\delta'_{Q_L} = \frac{M_0F_{\phi}}{8\pi^2} \left[ y_L^2 (m_{Q_L} + m_{H_u} + m_{t_R}) + y_R^2 (m_{Q_L} + m_{H_d} + m_{b_R}) \right.
\]
\[
- \left( 3g_3^2 + \frac{1}{2}g_2^2 + \frac{1}{2}g_1^2 \right) \right],
\]
(A.36)
\[
\delta'_{U_L} = \frac{M_0F_{\phi}}{8\pi^2} \left[ 2y_t^2 (m_{Q_L} + m_{H_u} + m_{t_R}) - \left( \frac{8}{3}g_3^2 + \frac{8}{15}g_1^2 \right) \right],
\]
(A.37)
\[
\delta'_{D_L} = \frac{M_0F_{\phi}}{8\pi^2} \left[ 2y_b^2 (m_{Q_L} + m_{H_d} + m_{b_R}) - \left( \frac{8}{3}g_3^2 + \frac{2}{15}g_1^2 \right) \right],
\]
(A.38)
\[
\delta'_{L_L} = -\frac{M_0F_{\phi}}{8\pi^2} \left[ \frac{3}{2}g_2^2 + \frac{3}{10}g_1^2 \right],
\]
(A.39)
\[
\delta'_{E_L} = -\frac{M_0F_{\phi}}{8\pi^2} \left[ \frac{1}{2}g_2^2 + \frac{3}{30}g_1^2 \right],
\]
(A.40)
\[
\delta'_{H_u} = \frac{M_0F_{\phi}}{8\pi^2} \left[ 3y_t^2 (m_{Q_L} + m_{H_u} + m_{t_R}) + \lambda^2 (m_S + m_{H_u} + m_{H_d}) \right.
\]
\[
- \left( \frac{3}{2}g_2^2 + \frac{3}{10}g_1^2 \right) \right],
\]
(A.41)
\[
\delta'_{H_d} = \frac{M_0F_{\phi}}{8\pi^2} \left[ 3y_b^2 (m_{Q_L} + m_{H_d} + m_{b_R}) + \lambda^2 (m_S + m_{H_u} + m_{H_d}) \right.
\]
\[
- \left( \frac{3}{2}g_2^2 + \frac{3}{10}g_1^2 \right) \right],
\]
(A.42)
\[
\delta'_{S} = \frac{M_0F_{\phi}}{8\pi^2} \left[ 2\lambda^2 (m_S + m_{H_u} + m_{H_d}) + 2\kappa^2 m_S \right] + \Delta^T_X(m_{\tilde{S}}).
\]
(A.43)

Note that the expressions for fermions are hold for the third generation, the first two generation can be obtained by setting $y_t \to 0$. Within the expressions, modular weight $l_i = 1$ for gauge couplings are used. The previous expressions are the interference part of anomaly-modulus mediation. Possible modulus-gauge interference part will also appear in our scenario.

All anomalous dimensions except $S$ are continuous across the messenger threshold, so the modulus-gauge interference contributions vanish. However, there is anomalous dimension discontinuity for $G_S$, so we have the new $T, X$ interference contributions to $m_S^2$

\[
\Delta^T_X(m_{\tilde{S}}) = -\frac{dM_0F_{\phi}}{8\pi^2} \left[ 3(\lambda_P^T)^2(m_X + m_{\tilde{X}} + m_S) \right.
\]
\[
+ 2(\lambda_P^D)^2(m_Y + m_{\tilde{Y}} + m_S) \right].
\]
(A.44)

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