Optimal Resource Allocation for a Single-Cell Multicast Transmission Scheme with a Supplementary Multicast Channel

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Abstract: Multicast transmission is an attractive solution when a large number of users receive the same content in a wide area, for example, as with mobile TV. Ever since the multimedia broadcast multicast service (MBMS) was introduced in the 3rd Generation Partnership Project (3GPP), continuing work on the multicast transmission has been done and its importance is growing in the fifth generation (5G) cellular networks. The use cases of multicast transmission have been enlarged from mobile TV and public safety to vehicular-to-everything (V2X) and unmanned aerial vehicles (UAV). Recently, for group communications in public safety networks and for geographical information sharing in automotive, airborne and social networks, multicast transmission has been targeted at fewer users in a relatively small area, which has stimulated extensive research on the single-cell multicast transmission scheme. In the proposed single-cell multicast transmission scheme, a supplementary multicast channel is additionally assigned in a single-cell multicast transmission scheme to exploit channel diversity. The resource allocation is adaptive to the channel variations of the users (responsive to users QoS needs), using channel feedback from the users, in contrast with previous approaches where resources were determined conservatively. An optimal resource allocation problem to minimize the required bandwidth while enabling every user to obtain the target multicast rate is formulated as a convex problem and an iterative algorithm is proposed in a computationally efficient way. Performance is evaluated mathematically and through intensive simulations, where other cell interference is considered using a fluid model. The proposed single-cell multicast transmission scheme provides benefits in comparison to existing multicast schemes in the simulations, under a set of various parameters including the number of multicast users and channel correlation between the multicast channels.

Keywords: single-cell multicast transmission; radio resource allocation; optimal problem; supplementary multicast channel; public safety networks

1. Introduction

Ever since the multimedia broadcast multicast service (MBMS) was introduced in the 3rd Generation Partnership Project (3GPP), continuing work on the multicast transmission has been done and its importance is growing in the fifth generation (5G) cellular networks [1]. The use cases of multicast transmission have been enlarged from mobile TV and public safety to multimedia and entertainment, internet of things (IoT), automotive, public warning, airborne communications, mmWave communications and so on [2,3]. To meet the stringent latency and reliability requirements in vehicular-to-everything (V2X), many studies have been done so far; for example, a low-latency multicast scheme in Reference [4], reliability improvement via user equipment (UE) acknowledgement...
feedback for hybrid automatic repeat request (HARQ) in Reference [5] and evaluation of latency and packet reception ratio in V2X scenarios in Reference [6].

Multicast transmission is an attractive solution when a large number of users receive the same content in a wide area, for example, as with mobile TV. Allocating an individual unicast channel for each user in a multicast service wastes resources, since the same content is delivered through different unicast channels. Obviously, as more people use the multicast service, multicast transmission consumes fewer resources than unicast transmission, since it uses a single, common multicast channel instead of as many unicast channels as the number of users. Unlike unicast transmission, however, conventional multicast transmission receives no feedback from users on channel quality information and therefore, the well-known link adaptation cannot be applied, that is, modulation level and coding rate are fixed [7]. Therefore, bandwidth is determined conservatively so that even the user having the worst channel quality, herein called the worst user, can obtain a target multicast rate. This conservative resource allocation, however, may consume excessive resources since the minimum required bandwidth varies depending on the channel quality of the worst user. For example, resources can be conservatively allocated so that even the user located at the cell edge can achieve the target multicast rate. Since the users are placed randomly within a cell, most of them lie closer to the base station (BS) and the worst user in a multicast service is located at the cell edge with a very low probability. Therefore, the worst user experiences better channel quality in most cases than when it is at the cell edge, which, in turn, requires fewer resources. Therefore, the worst user experiences better channel quality, which in turn, requires fewer resources. In Reference [8], to overcome this problem, unicast channels were additionally provided, in addition to the multicast channel, for users having poor channel quality. However, channel quality feedback is still not supported for the multicast channel and therefore, the multicast channel is not adaptive to the channel variations of the users.

Recently, for group communications in public safety networks and for geographical information sharing in social networks, multicast transmission has been targeted at fewer users in a relatively small area. In this case, the conventional multicast transmission scheme is no longer efficient, since it is designed for multicast transmission over a wide area where a large number of BSs are involved. Thus, a single-cell multicast transmission scheme has been studied by the 3GPP [9–11]. A representative one is the so-called single-cell point-to-multipoint (SC-PTM) scheme. The extended cyclic prefix (CP) is a special CP with a longer duration than the normal CP, which is designed to mitigate the inter-symbol interference due to the longer-delayed signals from neighboring cells in a multi-cell operation of the conventional multicast transmission scheme [2]. Owing to single-cell operation, however, the single-cell multicast scheme is able to use the normal CP, that is, the same frame structure as in the unicast transmission. This can not only reduce the overhead but also provides flexibility in resource allocation by allowing multiplexing multicast and unicast traffic at the same time interval. In addition, uplink feedback is available in the form of a channel quality indicator (CQI) and/or HARQ. Despite the overhead increase in the uplink, uplink feedback is a crucial function to meet the stringent reliability requirement in many use cases such as V2X and unmanned aerial vehicle (UAV). A low-overhead feedback scheme was proposed in Reference [12] to reduce the overhead while mitigating the limitation of the conventional multicast service. And HARQ-based feedback schemes were investigated for V2X and UAV cases in References [5,13], respectively. Furthermore, owing to uplink feedback, the bandwidth can be adaptively allocated based on the channel quality of the worst user. When the number of users in a multicast service, that is, the multicast group size, is 1, its operation is the same as unicast. As the multicast group size increases, more users with good channel quality suffer a disadvantage, since they have to use a multicast channel where bandwidth is allocated based on the worst channel quality. This is wasteful for them since a smaller bandwidth is enough to provide the target multicast rate. To reduce this resource waste, Kwon et al. proposed providing multicast transmission with two multicast channels, that is, a supplemental multicast channel in addition to the basic multicast channel [14]. The performance of the proposed multicast scheme was analyzed through simulations in terms of spectral efficiency. However, two multicast channels were not utilized
optimally due to partial channel quality feedback, where only the channel quality of the worst user is available at the base station.

In this paper, the channel quality information of individual users is assumed to be available at the base station, called full channel quality feedback. An optimal resource allocation method is proposed to minimize the required bandwidth with the target multicast rate, $R_T$, which is found to be a convex problem depending on the channel quality of the basic multicast channel (BMCH) and the supplementary multicast channel (SMCH) of the users in a multicast service. Also, a computationally efficient algorithm is proposed to solve the convex problem, which is a pointwise minimum of a family of affine functions. A fluid model is used to take into account other cell interference. Performance is evaluated in terms of the required bandwidth and is compared with existing multicast schemes by varying the multicast group size and the number of users assigned to the BMCH.

2. System Model

Figure 1 shows an exemplary scenario for the single-cell multicast transmission scheme with a supplementary multicast channel, where six users are placed randomly in a circular cell with a radius of $R$. One BS under consideration, indexed 0, is located at the origin of the $\mathbb{R}^2$ domain and it services the users in a multicast group, $\Omega$, with a fixed target multicast rate, $R_T$. Other-cell interference is accounted for by applying a fluid model where neighboring base stations (BSs) are distributed uniformly with a density of $\lambda [\text{BS} / \text{km}^2]$ in a region distant from BS 0 by at least $D_{\text{min}}$ [15]. We also assume that all the BSs use constant transmission power over the entire bandwidth and we let $p_b$ be the transmission power density of BS $b$. We also consider a single-input single-output system. We assume all the wireless links experience path loss, Rayleigh fading and additive white Gaussian noise (AWGN). Then, the signal-to-interference-plus-noise ratio (SINR) per hertz of user $j$, $\gamma_j$, is given as:

$$\gamma_j = \frac{P_0 g_{j,0}}{\sum_{b \neq 0} p_b g_{j,b} + \sigma_0^2}$$

where $g_{j,b}$ is the channel gain between user $j$ and BS $b$, which accounts for path loss and fading and $\sigma_0^2$ is the noise power density for the users of BS 0.

![Figure 1. An exemplary scenario for single-cell multicast transmission with a supplementary multicast channel ($\Omega = \{1, 2, \ldots, 6\}$, $\Omega_B = \{1, 4\}$).](image)

3. Single-Cell Multicast Transmission Scheme with Partial Channel Quality Feedback

As in Reference [14], the two multicast channels are assigned for a single-cell multicast transmission scheme where some of the users in $\Omega$ receive data only through a basic multicast channel (BMCH),...
where the other users receive data from a supplementary multicast channel (SMCH) together with the BMCH. The set of those users who receive data only via the BMCH, $\Omega_B$, is given as

$$\Omega_B = \{ j \in \Omega | \gamma_j \geq \gamma_{(|\Omega_B|)} \}$$

(2)

where $\gamma_k$ is the $k^{th}$ highest SINR on the BMCH. In Figure 1, $\Omega_B = \{1, 4\}$ and $|\Omega_B| = 2$. Given $\Omega_B$, since the BMCH is common, the channel capacity density of the BMCH is decided by the worst user in $\Omega_B$, written as $\log_2(1 + \gamma_j)$.

Then, the amount of resource (the amount of resource is interchangeably referred to as bandwidth herein) assigned to the BMCH, $B_B$, is determined to be in the following range:

$$\frac{R_T}{\log_2(1 + \gamma_j)} \leq B_B \leq \frac{R_T}{\log_2(1 + \gamma_{(|\Omega_B|)})}$$

(3)

Note that if $B_B$ is set to the maximum, that is, $B_B = B_{|\Omega_B|}$, all the users in $\Omega$ can successfully obtain $R_T$ via the BMCH. However, if $B_B$ is set in between $B_{|\Omega_B|}$ and $B_{|\Omega|}$, only those users belonging in $\Omega_B$ can successfully obtain $R_T$ and the other users in $\Omega \setminus \Omega_B$ might not be able to obtain $R_T$ from the BMCH. Hence, the SMCH should also be assigned to them.

Note that by combining the signals received from the two channels, the aggregated channel capacity equals the sum of the two respective channel capacities [8]. Then, the required channel capacity of the SMCH for user $j$ in $\Omega \setminus \Omega_B$ is expressed as $R_T - B_B \log_2(1 + \gamma_j)$, where the second term corresponds to the channel capacity achieved via the BMCH. Then its additionally required bandwidth, $B_{S,j}$, is obtained by dividing it by the capacity density, which can be written as

$$B_{S,j} = \max \left\{ \frac{R_T - B_B \log_2(1 + \gamma_j)}{\log_2(1 + \gamma_{S,j})}, 0 \right\}$$

(4)

where $\gamma_j$ is the SINR of the SMCH of user $j$. Since the SMCH is also common to all the users in $\Omega \setminus \Omega_B$, the bandwidth for the SMCH, $B_S$, is determined as

$$B_S = \max_{j \in \Omega \setminus \Omega_B} B_{S,j}$$

(5)

As seen in (5), with the partial channel quality feedback where the channel quality information is only from the worst user in $\Omega_B$, the minimum of $B_S$ is $B_{|\Omega_B|}$. In this way, if a certain user suffers a very low SINR on the SMCH, the bandwidth of the SMCH, $B_S$, might be undesirably large, which results in a large total bandwidth of $B_B + B_S$. This situation can be avoided with full channel quality feedback. Now, the bandwidth of the BMCH, $B_B$, can be determined by considering the channel quality of the SMCH as well, that is, $B_B$ can be lower than $B_{|\Omega_B|}$. Let us assume that user $j$ is the worst user in $\Omega_B$. When the SMCH channel quality, $\gamma_j$, is better than the BMCH channel quality, $\gamma_j$, $B_B$ can be decreased and conversely, $B_S$ is increased to minimize the total bandwidth, which is herein called channel diversity.

4. Single-Cell Multicast Transmission Scheme with Full Channel Quality Feedback

Now, we assume all the users in $\Omega \setminus \Omega_B$ feed their channel quality information back for both the BMCH and the SMCH, while all the users in $\Omega_B$ feed their channel quality information back for the BMCH. This assumption of full channel quality feedback makes it possible to allocate resources more efficiently at the cost of a feedback overhead increase on uplink. The overhead of channel quality information feedback depends on the periodicity of feedback reporting, the resolution and the number of users in $\Omega_B$ and $\Omega \setminus \Omega_B$. For example, four bits are assigned for the CQI, with a periodicity of 10 msec and 10 users belong in $\Omega$ with half of them in $\Omega_B$. The feedback overhead comes to 6 kbps.
Now, given $|Ω_B|$, the optimal resource allocation problem is formulated to minimize $B_B + B_S$, while the users in $Ω_B$ can obtain a channel capacity higher than or equal to $R_T$ only with the BMCH (see (6b) below) and the other users in $Ω\setminus Ω_B$ can do it with both channels (see (6c) below). Then the problem is expressed as

$$\min(B_B + B_S)$$

subject to

$$B_B \log_2(1 + \gamma_j) \geq R_T, \forall i \in Ω_B$$

$$B_B \log_2(1 + \gamma_j) + B_S \log_2(1 + γ_j^S) \geq R_T, \forall j \in Ω\setminus Ω_B$$

$$B_S \geq 0$$

Let us denote $f_j(B_B)$ as the total bandwidth of user $j \in Ω\setminus Ω_B$ needed to obtain target multicast rate $R_T$ with a given $B_B$. By rewriting (6c), $f_j(B_B)$ can be expressed as

$$f_j(B_B) = α_jB_B + β_j, \quad B_{Ω_B} \leq B_B \leq B_{Ω_B}$$

where $α_j = 1 - \frac{\log_2(1+γ_j)}{\log_2(1+γ_j^S)}$ and $β_j = \frac{R_T}{\log_2(1+γ_j^S)}$. Since the multicast channel is common to all assigned users, the total bandwidth, $B_B + B_S$, should be $\max_{j \in Ω\setminus Ω_B} \{f_j(B_B)\}$. Then, the minimum total bandwidth ((6a) above) can be rewritten as

$$\min(B_B + B_S) = \min \left[ \max_{j \in Ω\setminus Ω_B} \{f_j(B_B)\} \right]$$

Figure 2 illustrates $f_j(B_B)$ and how the proposed algorithm works with six users in $Ω$ when $Ω_B = \{1, 4\}$. Note that $f_j(B_B)$ is an affine function that is both convex and concave. Then, by applying Jensen’s inequality, we can easily prove that $\max_{j \in Ω\setminus Ω_B} \{f_j(B_B)\}$ is also convex [16], which corresponds to the shaded feasible region. Note, because $\max_{j \in Ω\setminus Ω_B} \{f_j(B_B)\}$ is the pointwise minimum of a family of affine functions, the optimal point is one of the coordinates of the intersections of the lines given in (7) or boundaries. Instead of examining all the continuous values of $B_B$ in the range of $(B_{Ω_B}, B_{Ω_B})$, the optimal point can be found iteratively, as shown in Algorithm 1.

In the proposed algorithm, the user index, $j^{(k)}$, which maximizes the required bandwidth in the $k$-th iteration, is defined as

$$j^{(k)} = \lim_{\epsilon \to 0} \max_{j \in Ω\setminus Ω_B} \left\{ f_j\left(B_B^{(k)} + \epsilon\right) \right\}$$

where $B_B^{(k)}$ is the bandwidth of the BMCH at the leftmost point of intersection in the $k$-th iteration. Note that if user $j^{(k)}$ has better channel quality on the BMCH than on the SMCH, the slope of $f_j(B_B)$, that is, $α_{j^{(k)}}$, is negative; then, allocating more bandwidth to the BMCH can decrease the total bandwidth. Otherwise, allocating more bandwidth to the BMCH is disadvantageous. This implies that it is the optimal point where the sign of $α_{j^{(k)}}$ changes from negative to positive, for example, the 3rd iteration in Figure 2. Then, the total bandwidth of the proposed multicast scheme, $B_{prop}$, is $f_{j^{(3)}}(B_B^{(3)})$. Meanwhile, $f_2(B_{Ω_B})$ is the total bandwidth under the scheme in Reference [14], $B_{Ω_B}$ in Figure 2.
With a given \( 2019 \) Electronics the search set for \( j \) sought for \( R \) \( \gamma \) coherence bandwidth is smaller than the system bandwidth in multipath fading environments. We also consider a case when the two channels are correlated. The performance was evaluated in terms of loss. Regarding Rayleigh fading on the BMCH and the SMCH, independent and identically distributed (i.i.d.) Rayleigh fading is assumed as the ideal case, which is reasonable when the signal-to-noise ratio (SNR) is high. However, in practical scenarios, the channels may be correlated, and this correlation can affect the system performance. Therefore, it is important to consider the correlation between the channels.

### 5.1. Simulation Setup

In this section, we present the simulation setup used to evaluate the performance of the proposed multicast scheme. The simulation parameters and settings are described in detail to ensure reproducibility of the results.

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**Algorithm 1:** Finding the minimum required bandwidth

1. Initialize
2. \( k \leftarrow 0 \)
3. \( B_B^{(k)} \) is set to \( B_{|\Omega_B|} \)
4. Find \( j^{(k)} \) according to (9).
5. while \( a_{j^{(k)}} < 0 \) do
6. Find the leftmost point of intersection with \( f_{j^{(k)}}(B_B) \) and \( f_j(B_B) \) \( j \neq j^{(k)} \) in \( B_B^{(k)}, B_{|\Omega|} \).
7. if there is no point of intersection then
8. \( B_B^{(k)} \) is set to \( B_{|\Omega|} \).
9. break
10. else
11. \( k \leftarrow k + 1 \).
12. \( B_B^{(k)} \) is set to the \( x \) coordinate of the leftmost point.
13. Update \( j^{(k)} \) according to (9).
14. end if
15. end while
16. return \( B_{\text{prop}} = f_{j^{(k)}}(B_B^{(k)}) \).

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It should be noted that the construction method in (2) is optimal to solving problem (6) if \( |\Omega_B| > 0 \). With a given \( |\Omega_B| \), let us assume that user \( p \) with \( \gamma_p < \gamma_{|\Omega|} \) is included in \( \Omega_B \) and user \( q \) with \( \gamma_q \geq \gamma_{|\Omega|} \) is excluded from \( \Omega_B \). Then, \( B_{|\Omega_B|} \) is increased to \( B_{B,p} \), which is enough for user \( p \) to obtain \( R_T \) only through the BMCH. Since \( \gamma_q \geq \gamma_{|\Omega_B|} \), \( B_{B,p} \) is enough for user \( q \) as well. So, the solution to (8) is sought for \( j \in \Omega \setminus \Omega_B \) in the range \( B_B \geq B_{B,p} \) instead of \( B_B \geq B_{|\Omega_B|} \). Note that \( B_{B,p} \geq B_{|\Omega_B|} \) and the search set for \( j \) is the same, except for \( p \) and \( q \), since they were already assigned enough BMCH.

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**Figure 2:** The proposed iterative algorithm and \( f_j(B_B) \).
bandwidth with $B_{B,P}$. Now, it is obvious that the original set, that is, the construction method in (2), can provide the minimum total bandwidth.

5. Performance Evaluation

5.1. Simulation Setup

All the simulation results were obtained through MATLAB. Simulation parameters are summarized in Table 1. Two multicast channels, that is, BMCH and SMCH, suffer from the same path loss. Regarding Rayleigh fading on the BMCH and the SMCH, independent and identically distributed (i.i.d.) Rayleigh fading is assumed as the ideal case, which is reasonable when the coherence bandwidth is smaller than the system bandwidth in multipath fading environments. We also consider a case when the two channels are correlated. The performance was evaluated in terms of the bandwidth required to obtain $R_T = 1$ Mbps with an outage probability of 5%, where a user experiences an outage when failing to meet the target multicast rate.

### Table 1. Simulation parameters.

| Parameters                      | Values                                                        |
|---------------------------------|---------------------------------------------------------------|
| Path-loss constant              | $-24$ dB                                                      |
| Path-loss exponent              | 3.4                                                           |
| Fading                          | Rayleigh                                                      |
| Neighboring BS density, $\lambda$ | 3.54 [BSs/km$^2$]                                           |
| Cell radius, $R$                 | 300 m                                                         |
| Minimum distance to the neighboring BS, $D_{\text{min}}$ | 600 m                                                        |
| Transmission power spectral density, $p_0$ | 43 dBm/10 MHz                                                |
| Noise spectral density, $\sigma_0^2$ | $-174$ dBm/Hz                                               |

5.2. Simulation Results

Figure 3 compares the different multicast transmission schemes in terms of the average bandwidth by varying $|\Omega_B|$ when $|\Omega| = 6$. In the conventional multicast transmission scheme operating in a single cell, the required bandwidth, $B_c$, is constant and given as

$$B_c = \frac{R_T}{\log_2(1 + \gamma_o)}$$

where $\gamma_o$ is a threshold satisfying $\Pr(\gamma_j < \gamma_o) = 0.05$. The single-cell multicast scheme with only a single multicast channel is denoted as BMCH-only and its bandwidth can be determined as $\frac{R_T}{\log_2(1 + \gamma_0)}$. The scheme proposed in Reference [8] is denoted as Multi + Unis. Since the required bandwidth varies according to channel quality, its average value is used as a performance measure except for the conventional multicast transmission scheme.

As expected, the average bandwidth remains constant regardless of $|\Omega_B|$ for BMCH-only and Multi + Unis since only a single multicast channel is used. In the scheme in Reference [14], with $|\Omega_B| = 0$ or 6, only a single multicast channel exists and hence, its performance is the same as that of BMCH only. And the minimum average bandwidth can be obtained at $|\Omega_B| = 3$. The proposed scheme, however, can obtain the minimum average bandwidth with $|\Omega_B| = 0$. As seen from (7) and Figure 2, the optimal bandwidth is sought in the range $B_B \geq B_{\Omega_B}$ and therefore, as $|\Omega_B|$ increases, the search range shrinks and the required bandwidth increases. Note that the users in $\Omega_B$ need to send channel quality feedback for the BMCH, while feedback from the users in $\Omega \setminus \Omega_B$ is required for both the BMCH and the SMCH. In addition, the algorithm works with more $f_j(B_B)$’s with a smaller $|\Omega_B|$. Thus, there is a compromise between the performance and feedback overhead/computational complexity; as $|\Omega_B|$ increases, the feedback overhead and computational complexity can be reduced at the cost of performance degradation. The minimum average bandwidth can be reduced by approximately
proposed scheme in more detail, the channel quality of the users at sample indexes 1, 11 and 20 are given in Table 2. In the other sample indexes, both the BMCH and the SMCH are allocated. In order to explain the operation of the proposed scheme in more detail, the channel quality of the users at sample indexes 1, 12, 15 and 18, only BMCH is allocated, while at sample indexes 2, 3, 7 and 20, only SMCH is allocated. In the other sample indexes, both the BMCH and the SMCH are allocated. In order to explain the operation of the proposed scheme in more detail, the channel quality of the users at sample indexes 1, 11 and 20 are given in Table 2.

To better understand the operation of the proposed multicast transmission scheme, snapshots of the bandwidths, \( B_B \), \( B_S \) and \( B_B + B_S \) of the proposed scheme as well as that of BMCH-only, are shown in Figure 4 when \( |Ω| = 0 \) and \( |Ω| = 6 \). It is interesting to note that at sample indexes 1, 12, 15 and 18, only BMCH is allocated, while at sample indexes 2, 3, 7 and 20, only SMCH is allocated. In the other sample indexes, both the BMCH and the SMCH are allocated. In order to explain the operation of the proposed scheme in more detail, the channel quality of the users at sample indexes 1, 11 and 20 are given in Table 2.

![Figure 3](image-url) **Figure 3.** Average bandwidth versus \( |Ω_B| \) (|Ω| = 6).

![Figure 4](image-url) **Figure 4.** Snapshots of the required bandwidth (|Ω| = 6, |Ω_B| = 0)
Table 2. Channel qualities of the basic multicast channel (BMCH) and the supplementary multicast channel (SMCH) for six users at three sample points (in decibels).

| Sample 1 | Sample 11 | Sample 20 |
|----------|-----------|-----------|
| $\gamma_j$ | $\gamma^S_j$ | $\gamma_j$ | $\gamma_j$ | $\gamma^S_j$ | $\gamma_j$ | $\gamma^S_j$ |
| User 1 | 15.29 | 6.58 | 13.70 | −0.98 | 31.14 | 15.12 |
| User 2 | 11.59 | 12.60 | −1.05 | 7.09 | 7.85 | 7.19 |
| User 3 | 13.07 | 13.16 | 6.82 | 3.02 | 1.89 | 7.30 |
| User 4 | −1.56 | −5.37 | 9.72 | 12.11 | 3.67 | 5.68 |
| User 5 | 3.78 | 6.92 | 15.58 | 15.03 | 18.44 | 12.59 |
| User 6 | 32.50 | 20.06 | 10.39 | 8.05 | 13.72 | 12.76 |

First, see sample index 20, because $|\Omega_B| = 0$, $B^{(0)}_B = 0$. Then, since user 4 has the worst channel quality on the SMCH, $j^{(0)} = 4$. Note that user 4 also experiences worse channel quality on the BMCH, that is, $a_{j^{(0)}} > 0$. Therefore, the proposed algorithm stops and only the SMCH is allocated. If BMCH-only is applied instead, however, the bandwidth will be determined by the worst user on the BMCH, that is, user 3 and the required bandwidth is larger, as seen in Figure 4. Secondly, at sample index 11, user 1 (who has the worst channel quality on the SMCH) has better channel quality on the BMCH, that is, $j^{(1)} = 1$ and $a_{j^{(1)}} < 0$. Therefore, the proposed algorithm enters an iteration stage. In the first iteration, $B^{(1)}_B$ satisfying $f_1(B^{(1)}_B) = f_3(B^{(1)}_B)$ is found and $j^{(1)}$ is updated to 3. Also, because user 3 has better channel quality on the BMCH, that is, $a_{j^{(1)}} < 0$, the iteration continues. In the second iteration, $B^{(2)}_B$ is found to satisfy $f_3(B^{(2)}_B) = f_2(B^{(2)}_B)$ and $j^{(2)}$ is updated to 2. Now, user 2 has worse channel quality on the BMCH, so the algorithm stops and both the BMCH and the SMCH are allocated. Third, at sample index 1, user 4 with the worst channel quality on the SMCH enters the iteration stage. Unlike sample 11, however, user 4 has the worst channel quality on the BMCH and on the SMCH, implying $B^{(1)}_B$ satisfying $f_4(B^{(1)}_B) = f(B^{(1)}_B)$ cannot be found in the range $[0, B|\Omega|]$. Then, $B^{(1)}_B$ is set to $B|\Omega|$ and only the BMCH is allocated.

It is worth investigating the effect of the correlation between Rayleigh fading on the BMCH and on the SMCH, since the proposed multicast transmission scheme exploits channel diversity and hence, the correlation might degrade performance. The fading generation scheme proposed in Reference [17] was adopted, where two correlated Rayleigh fading envelopes are generated by means of a coloring matrix obtained from Cholesky decomposition of the correlation matrix. Figure 5 shows the average bandwidth under the proposed scheme by varying correlation factor $\rho$, as well as BMCH-only for comparison. As expected, the proposed scheme requires a larger average bandwidth as $\rho$ increases. However, the proposed scheme still outperforms BMCH-only because the variation in channel quality is still meaningfully large, despite the correlation. We observed from the simulations that $\gamma^S_j$ is 3 dB higher than $\gamma_j$ with probabilities of 70% and 58% when $\rho = 0$ and 0.5, respectively. Interestingly, even with $\rho = 0.9$, the probability is still 24%. It can be said that this amount of variation in channel quality on the BMCH and the SMCH is large enough to get the channel diversity gain.
6. Conclusions

In a multicast transmission, resource allocation is important to minimize the required bandwidth while enabling every user to obtain the target multicast rate. However, owing to the constraints that the resource allocation should be subject to the channel quality of the worst user and as well as the lack of channel feedback information, the performances of the conventional multicast schemes were not satisfactory. In this paper, we solved an optimal resource allocation problem for a single-cell multicast transmission scheme with a supplementary multicast channel. With the aid of full channel quality feedback, it becomes a convex optimization problem that can be solved with a computationally efficient iterative algorithm. The proposed single-cell multicast transmission scheme outperforms the previous multicast schemes in terms of required bandwidth, in which every user satisfies a target multicast rate with a given outage probability of 5%. Our future research direction is to extend the current work with
dynamic seamless switching between unicast and multicast modes and efficient multiplexing of two transmission modes.

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