Electron tunneling in double quantum dots and rings

I Filikhin, S G Matinyan and B Vlahovic
North Carolina Central University, 1801 Fayetteville St. Durham, NC 27707, USA,
E-mail: ifilikhin@nccu.edu

Abstract. The electron tunneling in double quantum dot (DQD) is studied by changing of inter-dot distance. Localization of an electron is calculated for each level of the whole spectrum of the confinement states of DQD. For identical QDs the tunneling goes consecutively from high energy levels to the ground state. It is changed essentially when the QDs of the system are non-identical. We show that violation of symmetry of the DQD geometry makes it difficult for the tunneling. For double quantum ring (DQR) we demonstrate that the electron states with the same type of rotation symmetry play an important role for tunneling.

1. Introduction
The dynamical tunneling in double quantum ring and dot (DQR [1] and DQD) is related to charge transport through the QD array. We study correlation between electron localization and energy spectrum of single electron confinement states in DQD and DQR with dependence on distance between QDs. When distance between the objects is large, the electron is localized in one object. When the distance decreases, the electron is tunneling; its wave function is spread over the whole double system. The cases of identical and non-identical QDs in DQD are considered. DQR may be considered as a variant of DQD with non-identical QDs. The tunneling may be changed essentially when the QDs of the system are non-identical. Generally, a type of the tunneling is defined by a dependence of confinement energy on quantum numbers due to the anti-crossing mechanism. Violation of symmetry of the DQD geometry leads to the change of such dependence for QDs of DQD and changes the type of the tunneling.

In the presented work we restrict the consideration by two dimensional quantum objects. This restriction allows us to use low computer resources for the numerical calculations of the whole spectrum of electron confine states (more than hundred states) with good accuracy. A 3D model has no principal difficulties in these studies. The 3D calculations are in progress.

2. Formalism
We study quantum dots and rings composed of InGaAs in a GaAs substrate utilizing a kp-perturbation single sub-band approach with the effective potential approach [2,3]. Two dimensional (2D) objects are considered. To describe tunnelling of a single electron in doublet quantum objects we made some definition. Probability of localization of electron into region \( \Omega_\gamma \) \( (\gamma = 1, 2) \) is defined by

\[
N_{n,\gamma} = \int_{\Omega_\gamma} |\Phi_n(x,y)|^2 \, dx \, dy,
\]

where \( \Phi_n(\rho,z) \) is wave function of electron, \( n \) is the number of the
state. $\Omega_{\gamma}$ ($\gamma = 1, 2$) are dictated by the QD shapes. Let us define as tunnelling measure parameter $\sigma = \frac{N_{n,1} - N_{n,2}}{N_{n,1} + N_{n,2}}$, with the range of $[-1, 1]$. Obviously, when $\sigma = 0$, the electron will be in QD1 ($\Omega_1$) or in QD2 ($\Omega_2$) with equal probability (here we assume, that the QD1 and QD2 have the same shape). This happens when electron is tunneling between QD1 and QD2. The case $\sigma = 1$ ($\sigma = -1$) corresponds to the case when the election is strongly located in QD1 (QD2).

3. InAs/GaAs double quantum dots

3.1 Identical QDs

The circle shaped InAs/GaAs dots compose the double quantum system. The DQD is considered to be two dimensional. The radius of each QD is 40 nm. The inter-dot distance is changed in this study. Visualization for the tunnelling is given In Fig. 1 where the parameter $\sigma$ is presented along the energies of single electron spectrum in the InAs/GaAs DQD for different inter-dot distances. When distance between the objects is large enough, an electron is localized in one of the objects $\sigma \approx 1$ ($\sigma \approx -1$); when the distance decreases, the electron is tunneling so that its wave function is spread over the whole double system and $\sigma = 0$. Tunnelling occurs in all the states for inter-dot distances less than 15 nm. There are no tunnelled states when $a$ is larger 40 nm. One can see that the obtained picture (in Fig. 1) is symmetric relative axis of $\sigma = 0$. From Fig. 1, one can see that for each energy level (or very close energy levels) there are four or two values of $\sigma$. There are quasi quadruplets or quasi doublets of the DQD spectrum. The spectrum has degeneracy order of two for each level when the QDs are independent. This degeneracy due to the rotation symmetry of each QDs (which are identical). Energy of the states can be approximated by the relation: $E_{n,l} \approx \hbar^2 / 2m * (n + |l|) / R^2$, where $n$ and $l$ are radial and orbital quantum numbers, $R$ is radius of QD. The state with $l$ and $-l$ are degenerate. There are quasi-doublet and quadruplet states. The states with $\sigma$ between $|\sigma| = 1$ and $\sigma \approx 0$ are tunnelled states. The symmetry of pictures in Fig. 1 reflects the symmetry between QDs in DQD due to their identity.

![Figure 1](image_url)

**Figure 1.** The parameter sigma for energies of the confinement single electron states in InAs/GaAs DQD. Inter-dot distances are left) 14 nm, 40 nm; right) 22 nm.

3.2 Non-identical QDs

Let us consider non-identical DQDs. Firstly we return to circle shaped DQDs discussed above (see Fig. 1). Initial radii of the QDs are 40 nm. The shape one of the QDs (right QD) was gradually decreased. The inter-dot distance was fixed with the value of 3 nm. The effect of non-identity on the
tunnelling parameter $\sigma$ be seen in Fig. 2. We used ratio $\xi = R_2 / R_1$ ($R_1 = 40$ nm) of the radii of the QDs for measure of the non-identity.

![Figure 2](image_url)

**Figure 2.** left) Parameter $\sigma$ for different coefficient of asymmetry $\xi$ in DQD with $R_1 = 40$ nm and $R_2 = \xi R_1$; right) Density function $D(\sigma)$ of $\sigma$ for various $\xi$.

We are comparing the cases $\xi = 1$, $\xi = 0.9975$ and $\xi = 0.9875$ in Fig. 2 left). One can conclude that when $\xi = 1$ the wave function is distributed uniformed between QDs, when $\xi = 0.9975$ there are few states of single electron whose wave functions are well localized in the left or right QDs. For $\xi = 0.9875$ all states have strong localization in the left or right QDs. The efficiency of tunnelling depends on the ratio of asymmetry of QDs shapes. The larger asymmetry gives larger number of non-tunnelled states. It is important to note that the small violation of the QD symmetry drastically affects the tunnelling. In Fig. 2 right) the last result is presented using the density functions $D(\sigma)$ of $\sigma$ shown for various $\xi$. The function $D(\sigma)$ is defined as a probability density for the variable $\sigma$ to have a value $\sigma$ with the normalization $\int D(\sigma)d\sigma = 1$. We conclude that the dependence of the tunnelling on $\xi$ is continuous, but the separation of the electron spectral states between two cases – “independent QDs” (no tunnelling states) to “full coupled QD” occurs for $\xi$ in small vicinity of 1.

Note also that the inter-dot distance when all states of the DQD spectrum are tunnelled is depend essentially on value of $\xi$. For non-symmetric DQD ($\xi < 1$) this inter-dot distance is decreased. Thus we can conclude that a violation of symmetry of the DQD geometry creates difficulties for the tunnelling.

![Figure 3](image_url)

**Figure 3.** $\sigma$-parameter for the asymmetric DQD with $R_1 = 40$ nm and $R_2 = 39.9$ nm for different inter-dot distances: a) $a = 0$, b) $a = 2$ nm, c) $a = 4$ nm.

In Fig. 3 the dependence of the tunneling on inter-dot distance is shown. Violation of the symmetry of this DQD system, defined by $\xi$, is 0.9975. For the distances decreasing to $a = 0$ there are
non-tunnelled states in the spectrum. Comparing the cases of symmetric and non-symmetric DQD tunnelling, one can conclude that there is possibility for \( a = 0 \) when the charge transport through non-symmetric DQD will be larger than through symmetric DQD due to the degeneracy (double) of the spectrum of symmetric DQD. In other words the number of electron level which allows the tunnelling for non-symmetric DQD may be larger than the number of such levels for symmetric DQD.

### 4. InAs/GaAs double quantum rings

The double concentric quantum ring is an example of the double quantum object with violation of the shape symmetry of constituent objects due to difference of median radii of the inner and outer rings. In Fig. 3 left) we show 2D DQR. Energy of a single electron in single QR of this DQR is related by the followed formula with radial and orbital quantum number: 

\[
E_{nl} \sim \frac{\hbar^2}{2m^*} \left( \frac{n^2}{W^2} + \frac{l^2}{R^2} \right), \quad D \ll R, \quad \text{where} \quad W \quad \text{is the width of the QR, and} \quad R \quad \text{is median radius.}
\]

The single electron levels in QR are separated by the \( n = 1,2,3,... \) bands, with in-band states with \( |l| = 0,1,2,... \). The \( \sigma \)-parameter for spectrum of concentric DQR is shown in Fig. 3. We describe approximately the states using the "good" radial quantum numbers of the each ring. The description corresponds to the case of "independent rings". In Fig. 3, the calculated points (in the coordinates \( (E, \sigma) \)) are connected for \(|l| = 0,1,2,...\) by a line. One can obtained some "traces" for each radial number \( n = 1,2,3 \) as it shown in Fig. 3. We conclude that tunnelling in DQRs occurs through the levels having the same type of wave function symmetry defined by \( l \) for the rings (see [4]). The tunnelling occurs when \( |\sigma| < 1 \). For these states the tunnelling is possible by means the anti-crossing mechanism [4].

![Figure 3. left) DQR shape; right) The \( \sigma \)-parameter for spectrum of concentric DQR. The traces are created by consequence increasing the orbital quantum number \(|l| = 0,1,2,...\) for states with fixed \( n = 1,2,3 \).](image)

### 5. Conclusion

We studied dynamical tunnelling in the DQDs changing of inter-dot distance. Identical and non-identical QDs in DQD were considered. The two types of the tunnelling were found. We show that the electron states with the same type of rotation symmetry play important role for the tunnelling in double nano-sized systems. Generally, violation of symmetry of the DQD geometry diminishes the tunnelling.

This work was supported by the NSF (HRD-0833184) and NASA (NNX09AV07A).

### References

[1] Kuroda T et al. 2005 Phys. Rev. B72, 205301
[2] Filikhin I, Suslov V M and Vlahovic B 2006 Phys. Rev. B73, 205332
[3] Filikhin I, Matinyan S G and Vlahovic B 2011 Phys. Lett. A375, 620
[4] Filikhin I, Matinyan S G and Vlahovic B 2012 J. Comp. and Theor. Nanoscience 9, 1033