N=1 Heterotic-Supergravity Duality and Joyce Manifolds

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Abstract

We construct the heterotic dual theory in four dimensions of eleven dimensional supergravity compactified on a particular Joyce manifold, \( J \). In particular \( J \) is constructed from resolving fixed point singularities of orbifolds of the seven-torus in such a way that one is forced to consider a generalised orbifold compactification on the heterotic side. We conjecture that a heterotic dual exists for all the compact 7-manifolds of \( G_2 \) holonomy constructed by Joyce.
1 Introduction

Our best candidates, to date, of fundamental descriptions of nature, now all seem to be connected by the duality conjectures \[1, 2, 3, 4\]. The emerging picture is one of an underlying, even more fundamental, theory in which the other theories emerge as one approaches various limits in the moduli space.

Much effort has gone into understanding the $N = 4$ \[2, 3\], $N = 2$ \[5, 6, 7, 8\] and more recently the $N = 1$ \[11, 9, 10\] cases. In particular the $K3$ manifold plays a central role \[3, 4, 12, 13, 7, 8, 9, 14, 16, 15\] in many scenarios.

Compactification of $d = 11$ supergravity on a compact manifold of $G_2$ holonomy gives an effective four dimensional $N = 1$ theory. Joyce has given many examples of such manifolds \[17, 18\]. It is natural to conjecture that these manifolds play a crucial role in the $N = 1$ truncations of the conjectured dualities between the Type IIA superstring on $K3$, the heterotic string on $T^4$ and $d = 11$ supergravity on $K3\times S^1$.

In \[10\] a manifold of $G_2$ holonomy was constructed by considering freely acting involutions of a $K3\times T^3$ orbifold and resolving the singularities. These involutions translate directly to actions on the heterotic side giving an example of $N = 1$ duality. However, freely acting involutions of this type are very limited, and the case when the involutions act with fixed points on the seven coordinates on the supergravity side is less well understood. In fact, all of the manifolds of \[17, 18\] are constructed by resolving fixed point singularities, and if these manifolds are to play a role in $N = 1$ duality then this situation needs to be understood. This work aims to shed some light on this problem. We work at a generic point in moduli space throughout the following.

We focus on the simplest example given in \[17\], which we denote by $J$, constructed with three non-freely acting involutions of the seven torus, the singularities of which are then resolved. In section two a brief outline of Joyce’s construction is given. Then in section three we present the heterotic dual compactification. In particular, we are forced to consider a new kind of ‘overlapping orbifold’ on the heterotic side of the duality map. We end with some conclusions.
2 Joyce’s Construction

In [17, 18] Joyce gave explicit constructions of 7-manifolds of $G_2$ holonomy. When eleven-dimensional supergravity is compactified on these manifolds, the resulting theory is four dimensional N=1 supergravity with $b_2$ vector multiplets and $b_3$ chiral multiplets, where $b_2$ and $b_3$ are the non-trivial Betti numbers of the 7-manifold. The examples of [17] are all constructed by orbifolding the seven-torus by various discrete isometries and then resolving the singularities by replacing them with non-compact Eguchi-Hanson geometries, a process that is now more than familiar to string theorists.

The simplest example given in [17], which we denote by $J$ is constructed as follows:

Define the seven-torus coordinates as $(x_1, \ldots, x_7)$. Three $Z_2$ isometries of $T^7$ are defined by:

\begin{align}
\alpha(x_1, \ldots, x_7) &= (-x_1, -x_2, -x_3, -x_4, x_5, x_6, x_7) \\
\beta(x_1, \ldots, x_7) &= (-x_1, 1/2 - x_2, x_3, x_4, -x_5, -x_6, x_7) \\
\gamma(x_1, \ldots, x_7) &= (1/2 - x_1, x_2, 1/2 - x_3, x_4, -x_5, x_6, -x_7)
\end{align}

Obviously each of these $Z_2$’s has 16 fixed singular $T^3$’s and each one defines an orbifold limit of a particular $K3\times T^3$. There are thus 48 fixed singular $T^3$’s of the surface. However one must ask how the other two isometries act on the singular set of each $Z_2$. In particular, the $Z_2\times Z_2$’s generated by $(\beta, \gamma)$, $(\alpha, \gamma)$ and $(\alpha, \beta)$ act freely on the singular sets of $\alpha$, $\beta$ and $\gamma$ respectively. This implies that each isometry has only four singularites of the original sixteen, giving a total of 12 singular $T^3$’s. Resolving each of these is crucial for the consistency of the supergravity theory. This is done in the usual way for $K3$ singularities - by inserting Eguchi-Hanson geometries, [19]. The Betti numbers of the singular $T^7$ are $b_2 = 0$ and $b_3 = 7$. The resolution of each singularity adds 1 to $b_2$ and 3 to $b_3$. The Betti numbers of this Joyce manifold are thus $b_2 = 12$ and $b_3 = 43$. 

3
3 The Heterotic Dual and Generalised Orbifold

Eleven-dimensional Supergravity on $K3 \times T^3$ is conjectured to be dual to the heterotic string on $T^6$. Orbifolding the $K3 \times T^3$ by symmetries such as the Enriques involution on the $K3$ part translates to a particular orbifold action on the heterotic side. This has been illustrated successfully in several examples [12, 15, 9, 10].

In particular, in [10] an example was considered in which $T^7$ was modded out by a particular $Z_2 \times Z_2 \times Z_2$, one of which defined a particular orbifold limit of $K3 \times T^3$. This is what has been called $\alpha$ in the construction of $J$.

The remaining $Z_2$'s acted freely, and we note that they are essentially 'freely acting versions' of what we denoted by $\beta$ and $\gamma$ in constructing $J$. This is so because the example of [10] contained half shifts of $S^1$'s defined by $x_i \rightarrow x_i + 1/2$. In that example there is only one $K3$ present in constructing the 7-manifold and it is straightforward to map the freely acting $Z_2 \times Z_2$ to the heterotic side. So what is the dual of the supergravity theory on $J$? $J$ was constructed by taking $T^7$, orbifolding by three $Z_2$'s (each of which defines a particular $K3 \times T^3$) and resolving all the singularities. In a sense, we have three overlapping $K3$'s and when the singularities are suitably resolved we give non-trivial holonomy to the whole $T^7$, promoting it to $G_2$. This implies that on the heterotic side, the dual theory should be 'an overlapping' of three $Z_2 \times Z_2$ orbifolds, because each of the three $K3$'s on the supergravity side should be treated on an equal footing.

Denote by $\alpha t$, $\beta t$ and $\gamma t$ the action of the $Z_2 \times Z_2$ generated by $(\beta, \gamma)$, $(\alpha, \gamma)$ and $(\alpha, \beta)$ respectively on the heterotic side. First we note that, if treated separately, the $Z_2 \times Z_2$ orbifolds given by $\alpha t$, $\beta t$ and $\gamma t$ each produce the same massless spectra as the model considered in [10] 4. Namely four vector multiplets and 19 chiral multiplets.

To find the massless spectrum of our 'overlapping orbifold', it is only necessary to note that we have three copies of the same entity on both sides of the duality map. On the supergravity side resolution of singularities of each $K3$ gave a specific massless spectrum. On the heterotic side, each of the three orbifolds involved in the 'overlapping' each give essentially the same

\footnote{Appropriate $S^1$ shifts are included in the duality map, so that the adiabatic argument of \cite{9} applies to each orbifold.
compactification. This suggests that we sum the massless spectra separately, which leads naturally to the definition of the ‘overlapping orbifold’ as one in which each orbifold involved in the process should be treated separately, and the spectra summed. However, we should only count the dilaton and six moduli once. This gives a spectrum with precisely 12 vector multiplets and 43 chiral multiplets.

4 Conclusions.

It is further interesting to note that all the examples of Joyce constructed from $Z_2$’s have the Betti numbers of the singular $T^7$ as $b_2 = 0$ and $b_3 = 7$. This should then always correspond to the seven chiral multiplets containing the dilaton and moduli. In our example, the number of singularities of each isometry then corresponded on the heterotic side to the number of surviving $N = 4$ vector multiplets, each of which give rise to one $N = 1$ vector multiplet and three $N = 1$ chiral multiplets on both sides.

Of the more general cases considered in [18], it turns out that for certain subsectors of the singularities there exists more than one topologically distinct ways of resolving. The different resolutions add different numbers to $b_2$ and $b_3$. This then should correspond on the heterotic side to subsets of $N = 4$ vector multiplets surviving the orbifold projection and possibly to extra massless states from the twisted sectors. It is thus natural to conjecture that there is a heterotic dual for each of the manifolds given in [18] and this deserves further investigation [20].

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