Two methods for the determination of beam energy spread near $\Lambda_c^+\Lambda_c^-$ production threshold

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Abstract: Two techniques are introduced to measure the beam energy spread in collision experiments of high energy physics, based on the threshold truncation effect and the beam-constrained mass spectra resolution, respectively. Both techniques are verified by Monte Carlo simulation, and take advantage of the BESIII collected data near the $\Lambda_c^+\Lambda_c^-$ production threshold. The measured results are consistent with each other, and also accord with the extrapolation from the value measured at the $J/\psi$ resonance.

Keywords: Beam dynamics; Spectrometers; Detector modelling and simulations I (interaction of radiation with matter, interaction of photons with matter, interaction of hadrons with matter, etc)
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## 1 Introduction

Precision measurement is crucial for the continuous progress in high-energy, nuclear, and accelerator physics. Particularly for experiments based on accelerator, the precise knowledge of the beam energy $E_{\text{beam}}$ is extremely important in order to achieve the highest possible accuracy of the desired physical results. The accuracy of beam energy is the result of intrinsic and external effects on the collision beam, e.g. quantum emission, space charge effect, Touschek effect, synchrotron radiation, etc. Beam energy spread $\sigma_B$ is one of the features reflecting the property of $E_{\text{beam}}$, which should be addressed in electron-positron collision experiments. For example, exactly-known $\sigma_B$ can help to obtain more reliable initial-state radiation (ISR) factors and detection efficiencies in the cross section measurement of hadron pair production process, especially when the center-of-mass (c.m.) energy is close to the production threshold of the final state hadrons [1, 2].

In principle, any observable quantity sensitive to $\sigma_B$ can be used to measure the energy spread, although not all of them are equally suitable. There are several methods to measure $\sigma_B$. In accelerator, chromaticity of storage ring causes synchrotron sideband peaks in a spectrum of beam betatron oscillation, where the amplitudes of the peaks are related to energy spread and thus can be used to measure $\sigma_B$ [3]. The energy spread can also be extracted by comparing the measured beam betatron motion with the theoretical curve [4, 5]. Compton back scattering is another way to measure the beam energy spread [6]. Since the maximal energy of scattered photons is strictly coupled with the beam energy, the width of the maximal energy edge is determined by the energy spread.

The Beijing Spectrometer (BESIII) is a general composite detector operating at the Beijing Electron Positron Collider (BEPCII) [7, 8], whose physical goals involve charmonium physics, $D$-physics, spectroscopy of light hadrons and $\tau$-physics. Accurate beam energy is essential to the control of systematic uncertainties for precise measurements at BESIII, e.g. $\tau$ mass measurement [9].
At BEPCII, $\sigma_B$ is usually measured by scanning the width of narrow resonances, typically $J/\psi$ and $\psi(2S)$. Once measured at one energy, $\sigma_B$ can be extrapolated to other c.m. energy $\sqrt{s}$, assuming it is proportional to $s$, i.e. $\sigma_B \propto s$ [10]. However, the extrapolation may not be suitable due to changes of accelerator status at different energies and data taking periods, therefore development of new methods for energy spread measurement is necessary. Here, we introduce two techniques to measure the energy spread via the $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ process utilizing the data collected at BESIII. One of them measures the center-of-mass energy spread $\sigma_E$ based on threshold truncation effect, while another one estimates beam energy spread $\sigma_B$ using beam-constrained mass ($M_{BC}$) spectra resolution. The center-of-mass energy is twice as the beam energy at a symmetric collider, and therefore the spread of $\sqrt{s}$ is larger than the beam energy spread by a factor of $\sqrt{2}$, i.e. $\sigma_E = \sqrt{2}\sigma_B$.

2 Measurement of center-of-mass energy spread based on threshold truncation effect

2.1 Method description

According to the energy and momentum conservation, the c.m. energy $\sqrt{s}$ is equal to the invariant mass of final states $X$ in the process $e^+e^- \rightarrow X$, i.e. $\sqrt{s} = M(X)$. In BESIII collaboration, the $e^+e^- \rightarrow \mu^+\mu^-$ process has been used to calibrate the $\sqrt{s}$ of the collected data samples [11], here $X = \mu^+\mu^-$. However, if the production threshold of $X$ is close to the c.m. energy $\sqrt{s}$, e.g. $X = \Lambda_c^+\bar{\Lambda}_c^-$ discussed here, the reconstructed invariant mass tends be higher than $\sqrt{s}$ on the average, due to the absence of some collisions below the production threshold caused by the energy spread. The observed invariant mass is expected to increase as the energy spread getting larger.

Therefore, $\sigma_E$ could be determined if its relation with the deviation between $\sqrt{s}$ and $M(X)$ is established. It is difficult to find a general analytical dependence, since plenty of factors can impact on it, including the calibration of $\sqrt{s}$, the energy spread $\sigma_E$, the behaviour of cross section line shape near the threshold, initial state radiation, final state radiation (FSR), the resolution of the detector and the like. Considering above effects, we utilize Monte Carlo (MC) simulation to reveal the dependency relation. After the simulation, the average $M(X)$ is not equal to the energy $\sqrt{s}$ as expected, i.e. the energy deviation $\Delta_E = M(X) - \sqrt{s} \neq 0$, due to the energy spread and the threshold truncation effect.

2.2 Application with data taken at BESIII

At BESIII, $\sqrt{s}$ measured with $\mu^+\mu^-$ events [11] is used as the standard value. The nominal energy of 4575 MeV is measured to be $\sqrt{s_0} = 4574.50 \pm 0.18 \pm 0.70$ MeV, where the first uncertainty is the statistical one and the second is the systematics. The uncertainty is simply summed as 0.72 MeV in the following texts, noting that the 0.72 MeV cited here is the uncertainty of the average $\sqrt{s}$, which is originated from the limited statistic of the data sample and imperfect calibration method, rather than the energy spread value. The $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ process is chosen to measure the energy spread, since the c.m. energy we are interested in is close to the $\Lambda_c^+\bar{\Lambda}_c^-$ production threshold, which is 4572.92 $\pm$ 0.28 MeV [12]. The $\Lambda_c^+$ and $\bar{\Lambda}_c^-$ baryons are unstable particles which decay with an average lifetime of 0.2 ps. The so-called Golden decay $\Lambda_c^+ \rightarrow pK^-\pi^+$ and its charge conjugate (c.c) decay are utilized to reconstruct the $\Lambda_c^+$ and $\bar{\Lambda}_c^-$, respectively. To improve the statistics, only
one \( \Lambda_c^+ \) or \( \bar{\Lambda}_c^- \) is required in the reconstruction of the \( e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^- \) process, and another one is obtained from the recoiling momentum of the reconstructed one in \( e^+e^- \) center of mass system with its mass constrained to the mass of \( \Lambda_c \) from Particle Data Group (PDG) [12]. Then the c.m. energy estimated from \( \Lambda_c^+\bar{\Lambda}_c^- \) pair is calculated with the invariant mass of total four-momentum, which is found to deviate from the mean value of collision energy \( \sqrt{s} \) as discussed above.

To estimate \( \sigma_E \) for the data, a toy MC is generated implementing the following features:

- The cross section has a sharp step at the \( \Lambda_c^+\bar{\Lambda}_c^- \) threshold [2] pursuant to the Coulomb enhancement factor, \( \pi \alpha / \beta \) [13], which cancels the phase space \( \beta \) and produces a nonvanishing cross section.
- Center-of-mass energy is subject to the Gaussian distribution with the mean value to be the nominal energy \( \sqrt{s} \) and the resolution \( \sigma_E \).
- ISR photon is simulated according to a probability density:
  \[
P(k) \sim \beta k^{\beta-1}(1-k^{1-\beta}+0.5k^{2-\beta}),
\]
  where \( k \) is the energy fraction taken by the photon and \( \beta = (4\alpha / \pi)[\ln(2E_{\text{beam}}/m_e) - 0.5] \) is the Touschek Bond factor [14].
- After the ISR, the residual effective collision energy \( \sqrt{s} \) is sampled according to a Crystal Ball function reflecting the detector effect, where the parameters of the function are extracted from experimental data in case of \( \Lambda_c^+\bar{\Lambda}_c^- \) pair production with \( \Lambda_c^+ \) and \( \bar{\Lambda}_c^- \) decaying to their Golden decay modes.

Based on above considerations, two MC samples are generated setting \( \sigma_E \) to 2.2 MeV with and without threshold truncation effect, respectively. The comparisons of the \( M(\Lambda_c^+\bar{\Lambda}_c^-) \) distributions are shown in figure 1, including the original distributions and the reconstructed ones considering the smear of the detector. In the threshold truncation case, the reconstructed invariant mass \( M(\Lambda_c^+\bar{\Lambda}_c^-) \) is slightly shifted with respect to the one without the truncation. The deviation \( \Delta_E \) can be quantitatively described by the difference of the mean values or the peak positions.
Figure 2. The relation of $\sigma_E$ and $\Delta E$ from toy MC at $\sqrt{s} = 4574.50 \pm 0.72$ MeV. Dashed lines show the invariant mass deviation of data and the corresponding energy spread.

The numerical relation between the energy spread $\sigma_E$ and the deviation $\Delta E$ is extracted via a series of toy MC generated with assigning $\sigma_E$ from 1 to 4 MeV, 0.2 MeV step per MC sample. The relationship between $\sigma_E$ and $\Delta E$ is illustrated in figure 2 and turns out to be almost linear in a low order approximation. Figure 2 also shows that magnitudes of $\sigma_E$ and $\Delta E$ are in the same order in the case that $\sqrt{s}$ is 1.58 MeV above the production threshold of $\Lambda_c^+\bar{\Lambda}_c^-$. To measure the energy spread of the data sample at 4575 MeV, we should determine the deviation $\Delta E$ of experimental data. The invariant mass of $\Lambda_c^+\bar{\Lambda}_c^-$ is obtained by singly reconstructing the $\Lambda_c^+$ or $\bar{\Lambda}_c^-$, as shown in figure 3. The average invariant mass is measured to be $M(\Lambda_c^+\bar{\Lambda}_c^-) = 4575.28 \pm 0.55$ MeV by fitting it with a Crystal-Ball function, thus the $\Delta E$ is determined to be 0.78 MeV. Utilizing the relationship shown in figure 2, it is found $\sigma_E = 2.2 \pm 1.1$ MeV, where the uncertainty is obtained by fluctuating the measured $M(\Lambda_c^+\bar{\Lambda}_c^-)$ with its uncertainty. Here we only consider the statistical uncertainty, which is mainly originated from the determination of $M(\Lambda_c^+\bar{\Lambda}_c^-)$. If the systematical one is considered, the value of $\Delta E$ can fluctuate to a value close to or smaller than zero, where the relation between $\sigma_E$ and $\Delta E$ is nonlinear. This increases the difficulty in estimating the uncertainty of $\sigma_E$, and has to be carefully considered in a real measurement. The energy spread obtained here is consistent with the value estimated from the extrapolation of the spread at $J/\psi$ mass using the proportional relation between $\sigma_E$ and $s$, which is $\sigma_E \approx 0.9$ MeV at $J/\psi$ mass \[15\] and thus $\sigma_E \approx 2$ MeV at 4575 MeV.

BESIII has also taken data at $\sqrt{s} = 4600$ MeV, which is a little far away from the threshold of $\Lambda_c^+\bar{\Lambda}_c^-$. The invariant mass of $\Lambda_c^+\bar{\Lambda}_c^-$ is extracted with the same method as used at $\sqrt{s} = 4575$ MeV, which is $M(\Lambda_c^+\bar{\Lambda}_c^-) = 4599.3 \pm 0.2$ MeV as shown in figure 3. The value is consistent with that measured with $\mu^+\mu^-$ event in ref. [11], which is 4599.53 $\pm$ 0.07 $\pm$ 0.74 MeV. There is no significant deviation of $M(\Lambda_c^+\bar{\Lambda}_c^-)$ at $\sqrt{s}$ far away from threshold as expected.

3 Measurement of beam energy spread based on the resolution of beam-constrained mass spectrum

3.1 Method description

In high-energy physics experiments, Monte Carlo simulations are required to describe the experimental data as well as possible to determine reliable detection efficiencies and extract correct
signal shapes. However, the incorrect beam energy spread $\sigma_B$ assigned in the simulations will lead discrepancies between the experimental data and the MC samples. In this case, if we vary the $\sigma_B$ in the simulations to accomplish best consistency between data and MC, then corresponding $\sigma_B$ should be regarded as the actual value of the beam energy spread. To achieve this, an observable quantity which is sensitive to $\sigma_B$ should be proposed and its difference between data and MC can be quantitatively reflected. In this procedure, the hypothesis is held that the discrepancy between data and MC is completely originated from the incorrect input $\sigma_B$ in simulation. This is reasonable since the simulations are performed based on the geant4-based simulation package used in the BESIII experiment [16, 17], since geant4 is widely used to describe the interactions between final state particles and the detector materials.

### 3.2 Determination of beam energy spread

The $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ process is chosen to perform the beam energy spread measurement, and the beam-constrained mass $M_{BC}$ is used to reflect the difference between data and MC simulations. The $M_{BC}$ is defined by replacing the particle energy by the beam energy in definition of the invariant mass, i.e. $M_{BC} \equiv \sqrt{E_{\text{beam}}^2/c^4 - |p|^2/c^2}$, where $p$ is the momentum of the $\Lambda_c^+$ or $\bar{\Lambda}_c^-$ which is reconstructed from final state particles using the Golden decay channel. Thus, the $M_{BC}$ spectrum is sensitive to the $\sigma_B$ due to the containing of the constant variable $E_{\text{beam}}$ instead of the reconstructed energy of $\Lambda_c^+$ or $\bar{\Lambda}_c^-$ by definition. On the other hand, since the final states of the $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^-$ process consists of two symmetric particles, the $E_{\text{beam}}$ should be equal to the energy of $\Lambda_c^+$ or $\bar{\Lambda}_c^-$. Therefore, the $M_{BC}$ spectrum is expected to peak at the nominal mass of $\Lambda_c^+$. Figure 4 shows $M_{BC}$ distribution in data and MC with different beam energy spread $\sigma_B$. In the figure, the smooth curves are the MC shapes of $M_{BC}$ distributions, which is actually the probability density functions extracted from the $M_{BC}$ spectra of corresponding MC samples. According to the figure, different assignments of $\sigma_B$ in MC simulations will obviously result in different MC shapes of $M_{BC}$. Since the reconstruction of $\Lambda_c^+$ is based on a few final particles, the resolution of $M_{BC}$ is also a mixture of several resolutions and a combination effect. It is out of this paper’s scope to evaluate the difference between MC simulation and experimental result which is originated from to imperfect description.
of the detector behaviour. In the following text, the uncertainties are the pure statistical ones, and no evaluation of systematic uncertainties is performed.

To determine the beam energy spread $\sigma_B$, a series of MC samples are generated with the $\sigma_B$ increasing from 0.5 to 2.5 MeV by a step of 0.2 MeV. After that, the MC shapes of $M_{BC}$ spectra extracted from the MC samples are utilized to perform the unbinned maximum likelihood fits on the $M_{BC}$ distributions of experimental data. It is anticipated that the fit quality will become better when the $\sigma_B$ goes closer to the actual value. Naturally, the $\chi^2$ of the fit is regarded as the indicator of the $\sigma_B$ discrepancy between MC sample and experimental data. The resulting $\chi^2$ of each likelihood fit at 4575 MeV are shown in figure 5. To find the reasonable beam energy spread value, a simple fit using the quadratic function is performed on the $\chi^2$ distribution with respect to the $\sigma_B$, in which the fit function takes the form

$$\chi^2 = p_0 \cdot (x - p_1)^2 + p_2,$$

where $x$ denotes the value of beam energy spread and $p_1$ is expected to be the actual value of the $\sigma_B$. The fit results are shown in figure 5. Note that the $\chi^2$ values are not true data, since there are no uncertainties in them. Therefore the nominal uncertainty of $p_1$, which is output by the fit, is not reliable. Accordingly, we assign the deviation of the beam energy spread value, which enlarges corresponding $\chi^2$ by 1.0, to be the uncertainty. The fit result from the $\Lambda_c^+$ decay is $p_1^+ = 1.65 \pm 0.69$ MeV while the $\bar{\Lambda}_c^-$ decay gives that $p_1^- = 1.51 \pm 0.69$ MeV. The final beam energy spread value is obtained from weighting average of the resulted $p_1^+$ and $p_1^-$, i.e. $\sigma_B = \bar{p}_1 = 1.58 \pm 0.49$ MeV. The corresponding c.m. energy spread is $\sigma_E = \sqrt{2} \sigma_B = 2.23 \pm 0.69$ MeV, which is consistent with the first method.

The method is also applied to the data sample collected at 4600 MeV, where the beam energy spread is fitted to be $p_1^+ = 1.63 \pm 0.24$ MeV for the $\Lambda_c^+$ decay and $p_1^- = 1.50 \pm 0.24$ MeV for the $\bar{\Lambda}_c^-$ decay. The average beam energy spread at this energy point is $\sigma_B = 1.57 \pm 0.17$ MeV, corresponding to $\sigma_E = 2.22 \pm 0.24$ MeV. Since 4600 MeV is very close to 4575 MeV, the energy spread almost shares the extrapolation from the $J/\psi$ mass, $\sigma_E = 2$ MeV, which is also consistent with the value measured using current method.

![Figure 4. $M_{BC}$ distribution fitted with signal shape obtained from pure signal simulation with (red dashed line) and without (blue solid line) beam energy spread at 4575 MeV. The shapes are scaled by the number of events.](image-url)
Figure 5. The fit to $\chi^2$ values at $\sqrt{s} = 4575$ MeV (left) and 4600 MeV (right). The fit to $\chi^2$ values at $\sqrt{s} = 4575$ MeV via the function $p_0 \cdot (x - p_1)^2 + p_2$, where $\chi^2$ value is from fitting $M_{BC}$ distribution of data with MC shape under different beam energy spread, and the $x$ represents the value of beam energy spread and $p_1$ is expected to be the nominal value of beam energy spread. The red (blue) solid lines represent the fit functions for the $\Lambda_c^+ (\bar{\Lambda}_c^-)$ decay.

4 Discussion and summary

Two methods are introduced to determine the center-of-mass energy spread in the symmetric electron-positron collider based on the threshold truncation effect and the beam-constrained mass spectrum. The results of $\sigma_E$ at 4575 MeV are determined to be $2.20 \pm 1.10$ MeV and $2.23 \pm 0.69$ MeV, respectively. These two results are consistent with each other and also accord with the extrapolation from the energy spread measured at the $J/\psi$ resonance. Since the first method is motivated by the threshold truncation effect, it only works well for the data samples close to the production threshold of specific two-body final states. The beam-constrained mass distributions make the second method valid in a relative broad c.m. energy region. In this method, the simulation quality of the interaction between final state particles and the detector materials will directly affect the determination of the energy spread, and may limit its application. Although only the process $e^+ e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$ is utilized to perform the measurement, these two methods can be generalized to other two-body production processes, such as the production of $\Xi_c$ and $\Omega_c$, as long as corresponding near threshold data samples are collected. The methods described in this paper provide new techniques in the measurement of the energy spread for near threshold data samples, which will be helpful for the production behavior study of corresponding baryon pair.

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