THE TEMPERATURE–DENSITY RELATION IN THE INTERGALACTIC MEDIUM AT REDSHIFT \((z) = 2.4^*\)

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ABSTRACT

We present new measurements of the temperature–density \((T–\rho)\) relation for neutral hydrogen in the \(2.0 < z < 2.8\) intergalactic medium (IGM) using a sample of \(\sim 6000\) individual H\(_i\) absorbers fitted with Voigt profiles constrained in all cases by multiple Lyman series transitions. We find model-independent evidence for a positive correlation between the column density of H\(_i\) \((N_{\text{HI}})\) and the minimum observed velocity width of absorbers \((b_{\text{min}})\). With minimal interpretation, this implies that the \(T–\rho\) relation in the IGM is not “inverted,” contrary to many recent studies. Fitting \(b_{\text{min}}\) as a function of \(N_{\text{HI}}\) results in line-width–column-density dependence of the form \(b_{\text{min}} = b_0(N_{\text{HI}}/N_{\text{HI},0})^{\Gamma-1}\) with a minimum line width at mean density \((\rho/\bar{\rho} = 1, N_{\text{HI},0} = 10^{13.6}\, \text{cm}^{-2})\) of \(b_0 = 17.9 \pm 0.2\, \text{km s}^{-1}\) and a power-law index of \((\Gamma - 1) = 0.15 \pm 0.02\). Using analytic arguments, these measurements imply an “equation of state” for the IGM at \((z) = 2.4\) of the form \(T = T_0(\rho/\bar{\rho})^{\gamma-1}\) with a temperature at mean density of \(T_0 = [1.94 \pm 0.05] \times 10^4\, \text{K}\) and a power-law index \((\gamma - 1) = 0.46 \pm 0.05\).

Key words: intergalactic medium – quasars: absorption lines

Online-only material: color figures

1. INTRODUCTION

The “equation of state” of the low-density intergalactic medium (IGM) is believed to be controlled by two principal processes: photo-heating and adiabatic cooling. The cooling is most directly tied to the expansion of the universe, while the heating is expected to be a complicated mixture of relic effects from the reionization of hydrogen and helium plus the current heating, predominantly from the UV background (Hui & Gnedin 1997; Schaye et al. 1999). This naturally imposes a relationship between the temperature and density of intergalactic gas. Denser gas is expected to trace larger overdensities, for which adiabatic cooling is suppressed because such regions are more bound against the expansion of the universe. At the same time, denser gas has a much faster recombination timescale and thus presents a larger cross-section for photo-ionization which leads to denser gas experiencing greater heating.

Recently, this simple picture has been called into question on the basis of several statistical studies of the transmitted flux observed in individual pixels within the Ly\(\alpha\) forest (Becker et al. 2007; Bolton et al. 2008; Viel et al. 2009; Lidz et al. 2010; Calura et al. 2012; Garzilli et al. 2012). These studies require comparison with large numerical simulations in order to interpret the physical implications of the detailed shape of these flux distributions. Further, the shape of the distribution may be sensitive to the full thermal history of the gas, not just the present temperature–density \((T–\rho)\) relation (Peeples et al. 2010).

Becker et al. (2007) considered the transmitted flux probability distribution function (PDF) and compared it to the numerical models of Miralda-Escudé et al. (2000), finding that none of the models could explain the observations. They found that a \(T–\rho\) relation in which lower density regions were hotter (an inverted \(T–\rho\) relation) provided a better fit to the data. Since this study, several other authors have obtained similar results using transmitted flux PDF analysis (Bolton et al. 2008; Viel et al. 2009; Calura et al. 2012), wavelet analysis (Lidz et al. 2010), or a combination of both (Garzilli et al. 2012).

In contrast, early work on the \(T–\rho\) relation and its evolution with redshift was performed via Voigt profile fitting of individual H\(_i\) absorption lines. These early studies found a monotonic \(T–\rho\) relation with a positive power-law index (Schaye et al. 2000; Ricotti et al. 2000; Bryan & Machacek 2000; McDonald et al. 2001).

In this Letter, we return to the more direct test of the \(T–\rho\) relation in the IGM using Voigt profile fits to individual H\(_i\) absorbers. This method relies on the expected relationship between the column density of neutral hydrogen of an absorber, \(N_{\text{HI}}\), and its local overdensity, \(\rho/\bar{\rho}\) (Davé et al. 1999; Schaye 2001). If such a relationship exists, then one expects to observe a correlation between \(N_{\text{HI}}\) and the velocity widths of individual thermally broadened absorbers, \(b_{\text{q}}\). Absorbers having only thermal broadening\(^3\) are expected to have the smallest \(b_{\text{q}}\) at a given \(N_{\text{HI}}\), \(b_{\text{min}}\). Thus, by observing the behavior of the low-\(b_{\text{q}}\) “edge” of the distribution of \(b_{\text{q}}\) versus \(N_{\text{HI}}\), it is possible to infer the relationship between \(T\) and \(N_{\text{HI}}\) and through theory, the IGM \(T–\rho\) relationship.

Here, we present analysis of individual absorbers fitted in 15 high-resolution, high-S/N spectra of luminous QSOs at \(2.5 < z < 2.9\) from the Keck Baryonic Structure Survey (KBSS; Rudie et al. 2012). We discuss the data set used in this study as well as the line-fitting procedure in Section 2. In Section 3, the fit to the minimum \(b_{\text{q}}\) \((b_{\text{min}})\) as a function of \(N_{\text{HI}}\) is presented, followed by Section 4 in which the physical implications of the results in the context of the \(T–\rho\) relation are discussed. The results and their implications are summarized in Section 5.

\(^3\) Other sources of broadening of absorbers include bulk motions of the gas (generally parameterized as a turbulent component of \(b_{\text{q}}\)) and differential Hubble flow which broadens the absorption lines originating from the most diffuse and physically extended absorbers.
wavelength range pertaining to Ly\(\beta\). The average signal-to-noise ratio per pixel of the QSO spectrum in the range is given in Table 1. In this Letter, we use the path-length-weighted mean redshift of the sample \(z = 2.37\) as the fiducial redshift (Bahcall \& Peebles 1969).\(^5\)

The final H\(\alpha\) absorber catalog includes 5758 absorbers with \(12.0 < \log(N_{\mathrm{H}\alpha}/\text{cm}^{-2}) < 17.2\) and \(2.02 < z < 2.84\) over a total redshift path length of \(\Delta z = 8.27\). This H\(\alpha\) sample is the largest ever compiled at these redshifts and is more than an order of magnitude larger than previous samples that included constraints from higher-order Lyman lines.

### 3. The Temperature–Density Relation in the IGM

The equation of state of the IGM is expected to have the form

\[
T = T_0 \left( \frac{\rho}{\overline{\rho}} \right)^\gamma_{-1},
\]

where \(T_0\) is the temperature at the mean mass density \(\overline{\rho}\) (Hui \& Gnedin 1997).

Under the assumption of a relatively uniform ionizing radiation field, a power-law relationship between \(N_{\mathrm{H}\alpha}\) and \(\rho\) is also expected. Schaye (2001) presented a model for the low-density IGM in which “clouds” are in local hydrostatic equilibrium and therefore typically have sizes comparable to the local Jeans length. Employing this assumption, Schaye (2001) derived a relationship between \(N_{\mathrm{H}\alpha}\) and local overdensity, \(\rho/\overline{\rho}\). Using updated cosmology and evaluating at the path-length-weighted mean redshift of the sample,

\[
\rho_b/\overline{\rho}_b \approx \left( \frac{N_{\mathrm{H}\alpha}}{10^{13.6}} \right)^{2/3} T_4^{0.17} \left( \frac{\Gamma_{12}}{0.5} \right)^{2/3} \left( \frac{1+z}{3.4} \right)^{-3},
\]

where \(T_4\) is the gas temperature in units of 10\(^4\) K and \(\Gamma_{12}\) is the hydrogen photoionization rate in units of \(10^{-12}\) \(s^{-1}\) with the normalization suggested by Faucher-Giguère et al. (2008). Assuming this scaling,\(^6\) absorbers with \(\log(N_{\mathrm{H}\alpha}/\text{cm}^{-2}) \approx 13.6\) are expected to trace gas at the mean density of the universe \(z = 2.4\).

Thermally broadened absorbers are also expected to follow a power-law relation between \(b_{\gamma}\) and temperature: \(b_{\gamma} \propto T^{1/2}\). Combining with the expected \(T \rightarrow \rho\) relation and the conversion between \(N_{\mathrm{H}\alpha}\) and \(\rho\), the relationship between \(b_{\gamma}\) and \(N_{\mathrm{H}\alpha}\) would be a power law of the form

\[
b_{\gamma} = b_0 \left( \frac{N_{\mathrm{H}\alpha}}{N_{\mathrm{H}\alpha,0}} \right)^{\Gamma_{-1}},
\]

where \(b_0\) is the minimum line width of absorbers with \(N_{\mathrm{H}\alpha} = N_{\mathrm{H}\alpha,0}\). With this formalism, \((\gamma_{-1})\) is proportional to \((\Gamma_{-1})\) (see, e.g., Schaye et al. 1999).

More explicitly, for pure thermal broadening

\[
b = (2k_b T/m_p)^{1/2},
\]

where \(k_b\) is the Boltzmann constant and \(m_p\) is the mass of the proton. This suggests

\[
\log(T) = C + 2 \log \left( \frac{b}{\text{km s}^{-1}} \right),
\]

where \(C\) is a constant. \(\Delta X\) was defined by Bahcall \& Peebles (1969) such that absorbers with constant physical size and comoving number density would have a constant number per \(dX\).

We note here that with \(T_2 = 2\) as we find in this Letter, the \(N_{\mathrm{H}\alpha}\) corresponding to \(\rho = \log(N_{\mathrm{H}\alpha}/\text{cm}^{-2}) = 13.7\), a change of 0.1 dex. However, the uncertainty in other parameters (e.g., \(\Gamma_{12}\)) is large enough that we do not consider this small effect in this Letter.

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\(^5\) The comoving path length, \(dX\), was defined by Bahcall \& Peebles (1969) such that absorbers with constant physical size and comoving number density would have a constant number per \(dX\).

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| Name               | \(z_{\mathrm{QSO}}\) | \(z_{\mathrm{Ly}\beta}\) Range | \(S/N^a\) \(\mathrm{Ly}\alpha\) | \(S/N^a\) \(\mathrm{Ly}\beta\) |
|--------------------|----------------------|---------------------------------|---------------------------------|--------------------------|
| Q0100+130 (PHL957) | 2.721                | 2.0617–2.6838                   | 77                              | 50                       |
| HS0105+1619        | 2.652                | 2.1561–2.6155                   | 127                             | 89                       |
| Q0142–09 (UM673a)  | 2.743                | 2.0260–2.7060                   | 71                              | 45                       |
| Q0207–003 (UM402)  | 2.872                | 2.1532–2.8339                   | 82                              | 55                       |
| Q0449–1645         | 2.684                | 2.0792–2.6470                   | 73                              | 41                       |
| Q0821+3107         | 2.616                | 2.1650–2.5794                   | 50                              | 33                       |
| Q1009+29 (CSO 38)  | 2.652                | 2.1132–2.6031                   | 99                              | 58                       |
| SBS1217+499        | 2.704                | 2.0273–2.6669                   | 68                              | 38                       |
| HS1424+2931        | 2.660                | 2.0798–2.6237                   | 99                              | 47                       |
| HS1549+1919        | 2.843                | 2.0926–2.8048                   | 173                             | 74                       |
| HS1603+3820        | 2.551                | 2.1087–2.5066                   | 108                             | 58                       |
| Q1623+268 (KP77)   | 2.535                | 2.0544–2.4999                   | 48                              | 28                       |
| HS1700+64          | 2.751                | 2.0668–2.7138                   | 98                              | 42                       |
| Q2206–199          | 2.573                | 2.0133–2.5373                   | 88                              | 46                       |
| Q2343+125          | 2.573                | 2.0884–2.5437                   | 71                              | 45                       |

Notes:

\(^a\) The redshift of the QSO (Trainor \& Steidel 2012).

\(^b\) The average signal-to-noise ratio per pixel of the QSO spectrum in the wavelength range pertaining to \(\mathrm{Ly}\alpha\) and \(\mathrm{Ly}\beta\) absorption.

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http://www.ast.cam.ac.uk/~rfc/vpfit.html; \(\odot\) 2007 R. F. Carswell, J. K. Webb
where
\[ C = \log \left( \frac{m_p (\text{km s}^{-1})^2}{2k_B} \right) = 1.78. \] (6)

Rearranging the above equations, we expect the conversion between the index of \( b_{\text{min}}(N_{\text{HI}}) \) and the \( T-\rho \) relationship to be roughly
\[ \gamma - 1 \approx 3(\Gamma - 1). \] (7)

In the sections that follow, we fit to the trend of the \( b_{\text{min}} \) as a function of \( N_{\text{HI}} \) for the data sample. We then use Equations (5) and (7) to estimate the \( T-\rho \) relation in the \( \langle \gamma \rangle = 2.4 \) IGM.

3.1. A “Normal” \( T-\rho \) Relation

The measured values of \( b_3 \) and \( N_{\text{HI}} \), considered in this Letter are presented in Figure 1. Examination of Figure 1 already suggests the main result of this Letter; focusing on the low \( b_3 \) edge of the distribution, one can easily see that the lower envelope increases monotonically with \( N_{\text{HI}} \). Such behavior is expected for a normal temperature–density relationship in which denser regions are hotter.

3.2. Outlier Rejection

Before fitting to the “ridge-line” of \( b_{\text{min}} \) versus \( N_{\text{HI}} \), we exclude those absorbers that have large errors in their measured parameters. Schaye et al. (1999) suggest a simple algorithm for the outlier rejection in which absorbers with relative error in \( b_3 \) or \( N_{\text{HI}} \) larger than 50% are excluded.8 This method is expected to primarily reject lines that originate in blends and are thus unlikely to have accurate line-width measurements. Additionally, we exclude those absorbers with \( b_3 = 8 \text{ km s}^{-1} \) and \( b_3 = 100 \text{ km s}^{-1} \). These line widths correspond to the allowed line-width limits input to VPFIT and as such are artificial. The absorbers which remain after those exclusions are shown in Figure 1.

Considering Figure 1, we note a small set of points at very low \( b_3 \) which seem to lie significantly below the main locus of points. These absorbers are likely to be unidentified metal line contaminants, and thus we consider a \( \sigma \)-rejection algorithm to exclude them.

The \( \sigma \)-rejection algorithm is applied as follows. Only absorbers with \( b_3 < 40 \text{ km s}^{-1} \) are considered. The data are sorted by their \( N_{\text{HI}} \). The absorbers are placed into \( N_{\text{HI}} \) bins of width 0.25 dex and the mean and the standard deviation of \( b_3 \) are computed for the bin. Absorbers with \( b_3 \) larger than one mean absolute deviation above the fit are discarded. The power law is refit to the remaining absorbers and the rejection and refitting are repeated.

In the following section, we proceed with the measurement of the \( b_{\text{min}}-N_{\text{HI}} \) relation. We show that the application of the \( \sigma \)-rejection algorithm does not produce significant changes to the results. In the conclusions of the Letter, however, we prefer the combined error and \( \sigma \)-rejection method as it excludes the majority of the poorly measured absorbers as well as those which lie far below the \( b_{\text{min}}-N_{\text{HI}} \) trend.

5 In the following equation, we have neglected the minor dependence of \( \rho_{\text{HI}}/\rho_0 \) on \( T \) from Equation (2).

6 The relative error considered is computed from the formal error bars output by VPFIT for both \( b_3 \) and \( N_{\text{HI}} \).

Figure 1. Doppler widths of absorbers \( (b_3) \) vs. their column density \( (N_{\text{HI}}) \) for all the absorbers in the \( \text{H} \text{I} \) sample with relative errors in \( N_{\text{HI}} \) and \( b_3 \) less than 50%. Point-density contours are overplotted to guide the eye in the bottom panel. Note that the minimum value of \( b_3 \) at each \( N_{\text{HI}} \) increases with increasing \( N_{\text{HI}} \), suggesting a normal \( T-\rho \) relation.

(A color version of this figure is available in the online journal.)

3.3. Fitting \( b_{\text{min}} \) versus \( N_{\text{HI}} \)

In this section, we measure the \( b_{\text{min}}-N_{\text{HI}} \) relation using the two data sets resulting from the outlier rejection methods described above. To fit the trend between \( b_{\text{min}} \) and \( N_{\text{HI}} \), we follow the iterative power-law method proposed by Schaye et al. (1999). A power-law relationship of the form shown in Equation (3) with \( \log(N_{\text{HI},0} \text{ cm}^{-2}) = 13.6 \) is fit to the data. Data points which have \( b_3 \) larger than one mean absolute deviation above the fit are discarded. The power law is refit to the remaining absorbers and the rejection and refitting are repeated until the power law converges. Then, absorbers more than one mean absolute deviation below the fit are removed once, and the power law is refit. This fit is taken as the final form of the minimum \( b_{\text{min}}-N_{\text{HI}} \) relation. Errors in the parameters of the fit are derived by applying the bootstrap resampling method to the data sample and following the above outlined procedure.
Equations (5) and (7), respectively. Notably, both outlier rejection algorithms produce statistically consistent results suggesting that the measurements are robust to changes in the rejection algorithm; however (as expected) slightly higher b0 values slightly higher than expected given purely thermal broadening. They suggested that all absorbers in the $N_{\text{HI}}$ range used to measure the $T-\rho$ relationship experienced mild Hubble broadening (also expected to scale as $T^{1/2}$ for clouds having sizes similar to the local Jeans scale). In this case, estimates made using the above equations would mildly overpredict the temperature at mean density. We prefer to report physical values tied closely to our measurements rather than rely directly on a specific set of simulations; however, our measurements of $b_0$ and $\Gamma - 1$ can be converted to physical parameters using a different set of assumptions. Our simplifying assumption of pure thermal line broadening for absorbers near $b_{\text{min}}(N_{\text{HI}})$ results in $T_0$ values slightly higher than some previous studies which used simulations as a reference (see Table 3).

In Table 3, we compare our results with those of previous studies. We find generally good agreement between our results and those of past studies which used line fitting. We further emphasize that our results differ significantly from those obtained using the transmitted flux PDF and similar statistical methods, calling into question the interpretation of such techniques.

4.1 He II Reionization

One of the expected results of He II reionization, believed to occur at $z \approx 3$, is a significant change in the $T-\rho$ relation due to excess photoionization heating (Hui & Gnedin 1997). These changes are not long lived, and the exact effects depend on the speed and patchiness with which reionization proceeds.
the sample with

In this Letter, we briefly mention that the absorbers in the KBSS et al. 2009). While detailed discussion is beyond the scope of and the spectral hardness of the ionizing sources (see McQuinn et al. 2009). While detailed discussion is beyond the scope of this Letter, we briefly mention that the absorbers in the KBSS sample show no strong evidence for a change in the slope of \( b_{\min}(N_{\text{HI}}) \) for higher-redshift absorbers, which (presumably) are temporally closer to the HeII reionization epoch. The bottom panels of Figure 2 show the iterative power-law fit to absorbers with, respectively, \( z < 2.6 \) (green) and \( z > 2.6 \) (orange) from the sample with \( \sigma \)-rejection. Notably, the \( b_{\min}-N_{\text{HI}} \) relation appears to be independent of redshift (left panel); however, the expected evolution in the value of \( N_{\text{HI}} \) at the mean density (Equation (2)) results in differing values of \( N_{\text{HI},0} \) for the two redshift bins. This in turn leads to expected variation in the values of \( b_0 \) and \( T_0 \) with \( z \). The implied parameters from the fit are listed in Table 2.

5. CONCLUSIONS

We have inferred the \( T-\rho \) relationship in the \( \langle z \rangle = 2.4 \) IGM using Voigt profile fitting of individual Lyman lines. Fitting the trend of the minimum line width (\( b_{\min} \)) as a function of column density (\( N_{\text{HI}} \)) with a power law, we find best-fit values of \( b_{\min} \) at mean density (which at \( \langle z \rangle = 2.4 \) corresponds to \( N_{\text{HI},0} = 10^{13.6} \) cm\(^{-2} \)) \( b_0 = 17.9 \pm 0.2 \) km s\(^{-1} \) and a power-law

### Table 2

| Outlier Rejection | \( z \) Range | \( (\gamma - 1) \) | \( (\Gamma - 1) \) | \( b_0 \) | \( T_0/10^4 \) K
|-------------------|-------------|----------------|----------------|--------|----------------
| Default           | 2.0–2.8     | 2.4            | 13.6           | 0.156 ± 0.032 | 0.47 ± 0.10 | 17.56 ± 0.40 | 1.87 ± 0.08 |
| \( \sigma \)-rej. | 2.0–2.8     | 2.4            | 13.6           | 0.152 ± 0.015 | 0.46 ± 0.05 | 17.90 ± 0.21 | 1.94 ± 0.05 |
| 2\( \sigma \)-rej. | 2.0–2.6     | 2.3            | 13.7           | 0.156 ± 0.016 | 0.47 ± 0.05 | 18.50 ± 0.22 | 2.07 ± 0.05 |
| 2\( \sigma \)-rej. | 2.6–2.8     | 2.7            | 13.4           | 0.171 ± 0.032 | 0.51 ± 0.10 | 17.39 ± 0.44 | 1.83 ± 0.09 |

**Notes.**

\( a \) (\( \gamma - 1 \)) is calculated from the measured value of \( (\Gamma - 1) \) using Equation (7).

\( b \) \( T_0 \) is calculated from the measured value of \( b_0 \) using Equation (5).

### Table 3

| Source              | Method                  | \( (\gamma - 1) \) | \( (\Gamma - 1) \) | \( b_0 \) | \( T_0/10^4 \) K | Comment                  |
|---------------------|-------------------------|-------------------|-------------------|--------|----------------|-------------------------|
| Schaye et al. (2000) | Line fitting            | 2.3               | 1.17              | 0.27   |                 | Their Figure 6           |
| Ricotti et al. (2000) | Line fitting            | 2.5               | 1.38              | 0.56   |                 | Their Table 6            |
| Bryan & Machacek (2000)\( a \) | Line fitting       | 2.7               | 1.65              | 0.29   |                 | Their Table 5, \( z_{\text{sim}} = 3 \) |
| McDonald et al. (2001) | Line fitting            | 1.90             | 1.77              | 0.32 ± 0.30 | |              |
| Zaldarriaga et al. (2001) | Transmission power spectra | 2.4           | 1.74 ± 0.19       | 0.52 ± 0.14 | |              |
| Becker et al. (2007) | Flux PDF                | 2–6              | ...               | 0±0.5  | |              |
| Bolton et al. (2008)\( c \) | Flux PDF                | 2.07             | ...               | −0.33  | |              |
| Viel et al. (2009)  | Flux PDF                | 3                | 1.9 ± 0.6         | −0.25 ± 0.21 | |              |
| Lidz et al. (2010)  | Wavelet analysis        | 2.6              | 1.6 ± 0.6         |         | |              |
| Becker et al. (2011) | Curvature analysis      | 2.4              | 1.11 ± 0.06       |         | |              |
| Garzilli et al. (2012)\( f \) | Wavelet analysis + Flux PDF | 2.1       | 1.7 ± 0.2         | 0.11 ± 0.11 | |              |
| Garzilli et al. (2012)\( f \) | Wavelet analysis        | 2.1              | 1.6 ± 0.5         | >−0.14 | |              |
| Calura et al. (2012) | Flux PDF                | 2.5              | 1.5 ± 0.3         | −0.01 ± 0.21 | |              |
| This Work           | Line fitting            | 2.37             | 1.94 ± 0.05       | 0.46 ± 0.05 | | Results from data set using both relative error and \( \sigma \)-rejection |

**Notes.**

\( a \) Based on the spectrum of HS 1946+7658.

\( b \) The authors suggest this is an upper limit.

\( c \) Measured by Kim et al. (2007), sample includes 18 QSOs (metals removed by hand to eliminate noise in flux PDF).

\( d \) Marginalized over \( \gamma - 1 = 0–0.6 \).

\( e \) Assuming \( \gamma - 1 = 0.56 \).

\( f \) Find that flux PDF results in lower values of \( \gamma \) than wavelet analysis in general.

\( f \) Joint analysis with Kim et al. (2007).
index \((\Gamma - 1) = 0.15 \pm 0.02\). Assuming a monotonic relation between \(N_{\text{H}i}\) and \(\rho/\bar{\rho}\), these data support the conclusion that the temperature–density \((T-\rho)\) relationship in the IGM is not inverted at \(\langle z \rangle = 2.4\) but instead has a power-law index \((\gamma - 1) = 0.46 \pm 0.05\). Further, our results suggest a temperature at mean density of \(T_0 = [1.94 \pm 0.05] \times 10^4\) K. Within our sample spanning the redshift range \(2.0 \lesssim z \lesssim 2.8\), there is no evidence for significant evolution in the \(b_{\text{min}}-N_{\text{H}i}\) relation or in the power-law index \((\gamma - 1)\) of the \(T-\rho\) relation.

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