A short-time range-angle-decoupled beam pattern synthesis for frequency diverse arrays

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Abstract

Frequency diverse array (FDA), by virtue of its range-angle-dependent beam pattern, has drawn a substantial attention in recent years. However, as far as the time-variant range-angle-coupled beam pattern is concerned, such inherent flaw has become a bottleneck of its further development. To address this issue, herein, a logarithm-based optimised static non-linear frequency offset (LOSNFO) for FDA is proposed, which can successfully alleviate its inherent flaw, thus producing a short-time range-angle-decoupled beam pattern with low sidelobe levels (SLLs) and narrow half-power beam widths (HPBWs). Moreover, for practical applications, the proposed LOSNFO considers the propagation effect, which is always neglected in its counterpart. Numerical results are reported to demonstrate the effectiveness and superiority of the proposed LOSNFO-FDA.

1 | INTRODUCTION

Phased array antenna, with an attractive characteristic of flexible beam steering ability, has been widely used in radar, remote sensing, navigation, and wireless communication systems. However, the beam pattern of phased array is only angle-dependent, which limits its further development and applicability. Recently, frequency diverse array (FDA) has been proposed [1, 2]. By virtue of the flexible beam scanning ability, FDA has attracted considerable interests. Unlike the conventional phased array that has an angle-dependent-only beam pattern, FDA employs a tiny frequency offset between the adjacent array elements, thus forming a range-angle-dependent beam pattern. In Ref. [3], Seemen et al. discussed the periodic beam pattern in regard to the time, range, and angle of FDA. The additional degrees-of-freedom (DOF) in range domain enables FDA to have promising potentials in array design. Therefore, great efforts have been made for the research of FDA. In Refs. [4, 5], the transmit sub-aperture FDA radar and FDA-MIMO radar were utilised for cognitive target tracking and joint range and angle estimation, respectively. In Ref. [6], Wang proposed a beam pattern synthesis method with retrodirective FDA antenna for long-range simultaneous wireless information and power transfer. In Ref. [7], Hong et al. proposed a frequency diverse subarray-based fixed region beamforming method for secure mmWave wireless communications.

Though FDA shows encouraging potential in many applications [4–7], it has an inherent flaw in its time-variant range-angle-coupled beam pattern. This extremely limits its further development. To overcome this issue, plenty of research efforts have been devoted to achieving a focussing beam pattern of FDA at the target location, and holding its state for certain periods of time. To overcome the FDA’s range-angle-coupled beam pattern, some non-linear distributed frequency offsets, such as logarithmically increasing frequency offsets [8], non-uniform logarithmic frequency offset [9], hammering window based tapering frequency offsets [10], piecewise trigonometric frequency offset [11] and randomly assigned frequency offsets [12], were proposed to generate a single-maximum beam pattern towards the desired...
range and angle region. In Ref. [13], an overlapping subarray-based FDA using logarithmic increasing frequency offset was proposed to produce a dot-shaped beam pattern for linear and planar FDA. In Ref. [14], Xu et al. devised a flat-top beam pattern synthesis method for FDA based on the second-order cone programming to achieve a desired spatial region coverage. Although the beam patterns obtained by [8–14] were spatial focusing, they were still variant in the time dimension. To cope with this time-variant beam pattern, Xu et al. put forward a pulsed-FDA with the purpose of obtaining a quasi-static beam pattern in Ref. [15] and Khan et al. proposed a time-dependent frequency offset-based FDA in Ref. [16]. However, the beam patterns obtained by Refs. [15, 16] were still range-angle-coupled. In Ref. [17], Yao et al. utilised the time-modulated logarithmically increasing and optimised frequency offsets, that is TMLFO and TMOFO, to produce a time-invariant spatial fine focusing beam pattern. Then the work in Ref. [17] was further extended in the short-range and multi-targets scenarios [18, 19]. Based on the work in Refs. [17–19], some improved methods were applied in Refs. [20–22]. However, it is pointed out in Refs. [23, 24] that the designs of the FDA in Refs. [16–22] neglected the propagation process of the transmitted signals and the beam pattern could not focus at the target location, let alone to last for certain periods of time.

Herein, we propose a novel logarithm-based optimised static non-linear frequency offset (LOSNFO) for FDA to alleviate its intrinsic drawback. The potential of the proposed LOSNFO is that it only depends on a frequency offset control function and the duration of a short-time interval rather than introducing the time variable. Thus, it is a static frequency offset, which greatly eases the overload in the design of frequency offset. More importantly, with the proposed LOSNFO, a short-time range-angle-decoupled beam pattern with low sidelobe levels (SLLs) and narrow half-power beam widths (HPBWs) are achieved. During the short time interval, the beam pattern of the LOSNFO-FDA is decoupled at the target point, and the energy at the target point only has a small attenuation. Therefore, the energy is focused in the desired target location. Moreover, for practical applications, the propagation of transmitted signals can be considered using the proposed LOSNFO-FDA. Numerical results are provided to validate its effectiveness.

Herein, the fundamentals of FDA are presented in Section 2. The analysis of the TMLFO-FDA with the consideration of the propagation effect and the proposed beam pattern synthesis method are presented in Section 3. Then, simulation experiments for evaluation are carried out in Section 4. Finally, conclusions are drawn in Section 5.

2 | FUNDAMENTALS OF FREQUENCY DIVERSE ARRAY

Consider an N-element uniformly distributed linear FDA with a uniform amplitude weighting, as shown in Figure 1. The inter-element spacing between adjacent elements is \( d \). Assuming
a far-field observation point at angle $\theta$ and range $r$, for a continuous-wave FDA (CW-FDA), its electric field at point $(r, \theta)$ can be expressed as

$$E(t, r, \theta) = \sum_{n=0}^{N-1} e^{i(2\pi f_n (t-r_n)+\phi_n)}$$  \hspace{1cm} (1)$$

where $t$ is the time. $\phi_n$ is the phase weighting of the $n$-th ($n = 0, 1, \ldots, N-1$) element. $f_n$ is the carrier frequency of the $n$-th element. $r_n$ represents the range between the $n$-th element and the observation point $(r, \theta)$, and $r_n = r - n d \sin \theta / c$ represents the time delay with $c$ being the speed of light.

When it comes to the pulsed FDA, its electric field at point $(r, \theta)$ is expressed as

$$E(t, r, \theta) = \sum_{n=0}^{N-1} U(t - t_n) e^{i(2\pi f_n (t-r_n)+\phi_n)}$$  \hspace{1cm} (2)$$

where $U(t) = \begin{cases} 1 & 0 \leq t \leq T_d \\ 0 & \text{otherwise} \end{cases}$ is a rectangular pulse. The pulse duration and pulse repetition of $U(t)$ are $T_d$ and $T_r$, respectively, with $T_d \ll T_r$.

From (1) and (2), it is obvious that the pulsed FDA has an electric field similar to that of the CW-FDA. But the difference is that its effective electric field is characterised by $U(t)$ and $T_d$, which means that in the range dimension, it is always a $c \cdot T_d$ truncation of the CW-FDA. Unless otherwise indicated, herein, the corresponding transmit beam patterns are derived by employing the pulsed FDA. It is worth noting that for a CW-FDA, $T_d$ in the pulse can be regarded as infinite [25].

To be more specific, the radiation frequency of the $n$-th array element is

$$f_n = f_0 + \Delta f_n$$  \hspace{1cm} (3)$$

where $f_0$ is the carrier frequency and $\Delta f_n$ is the frequency offset of the $n$-th element, and $\Delta f_n \ll f_0$.

With the far-field assumption, that is $r_n \approx r$ for the amplitude term and $r_n \approx r - n d \sin \theta$ for the phase term, (2) can be rewritten as

$$E(t, r, \theta) = \frac{e^{i2\pi f_0 (t-r/c)+\phi_0}}{r} \sum_{n=0}^{N-1} U(t - t_n) \left[ 2\pi f_0 n d \sin \theta / c + 2\pi \Delta f_n (t-r/c) \right] . e^{i2\pi \Delta f_n n d \sin \theta / c + \phi_n}$$  \hspace{1cm} (4)$$

It can be seen from (4) that the terms inside the summation sign are determined by the array geometry and the frequency offsets of the FDA, which have the ability to characterise the inherent radiation characteristics of the FDA. Thus the array factor of FDA can be expressed as

$$AF(t, r, \theta) = \sum_{n=0}^{N-1} U(t - t_n) \left[ 2\pi f_0 n d \sin \theta / c + 2\pi \Delta f_n (t-r/c) \right] . e^{i2\pi \Delta f_n n d \sin \theta / c + \phi_n}$$  \hspace{1cm} (5)$$

Since $\Delta f_n \ll f_0$ always holds for FDA, $(2\pi \Delta f_n n d \sin \theta / c) \ll (2\pi f_0 n d \sin \theta / c)$ is satisfied for the two exponential terms in (4). Thus, by neglecting the term $(2\pi \Delta f_n n d \sin \theta / c)$, the array factor of FDA becomes

$$AF(t, r, \theta) = \sum_{n=0}^{N-1} U(t - t_n) e^{i2\pi f_0 n d \sin \theta / c + 2\pi \Delta f_n (t-r/c)+\phi_n}$$  \hspace{1cm} (6)$$

Consequently, the transmit beam pattern of FDA is expressed as

$$P(t, r, \theta) = \left| \sum_{n=0}^{N-1} U(t - t_n) e^{i2\pi f_0 n d \sin \theta / c + 2\pi \Delta f_n (t-r/c)+\phi_n} \right|^2$$  \hspace{1cm} (7)$$

FIGURE 2 Transmit beam pattern of a standard CW-FDA ($N = 10, f_0 = 10$ GHz, $T_d = 1$ ms, $\Delta f = 2$ kHz, $(r_n, \theta_n) = (150$ km, 30°)). (a) Projection of the range-angle beam pattern when $t = r/c$. (b) Projection of the range-time beam pattern at $\theta = \theta_0$. [Page 857]
In a standard CW-FDA, $\Delta f_n = n \Delta f$ and $\phi_n = 2 \pi n \Delta f_0 / c - 2 \pi f_0 \, n d \sin \theta_0 / c$ with $\Delta f$ being a small frequency offset and $(r_0, \theta_0)$ being the target point. The transmit beam pattern of the standard FDA is shown in Figure 2, which reveals that the standard FDA has an inherent drawback in its time-variant and range-angle-coupled beam pattern. This inherent drawback is so serious that greatly limits the development of FDA. Therefore, it is urgently desired to find a beam pattern synthesis strategy to deal with this problem.

$$AF_{\text{TMLFO}}(t, r, \theta) = \sum_{n=0}^{N-1} \left\{ 2 \pi f_0 \frac{\left(n d \sin \theta \right)}{c} \right\} + 2 \pi \left( \Delta f_{n, \text{TMLFO}} (t) \cdot \frac{r_n}{c} \right) \left( \frac{t - r}{c} \right)$$

$$= \sum_{n=0}^{N-1} \left\{ \ln(n + 1) \left( \frac{t - r}{c} \right) - \frac{r_n}{c} \right\} , t \in [0, T_d]$$

(10)

$$\|
\begin{align*}
\sum_{n=0}^{N-1} U(t) e^{-j 2 \pi} & \left\{ \frac{n d f_0}{c} \left( \sin \theta - \sin \theta_0 \right) \right\} \\
& = \sum_{n=0}^{N-1} U(t) e^{-j 2 \pi} \left\{ \ln(n + 1) \left( \frac{t - r}{c} \right) - \frac{r_n}{c} \right\} , t \in [0, T_d] 
\end{align*}
\|$$

3 | BEAM PATTERN SYNTHESIS OF FDA

3.1 | Correction of beam pattern of TMLFO-FDA

Previously, to achieve a time-invariant spatial focusing beam pattern for FDA, a time-modulated logarithmically increasing frequency offsets, that is TMLFO, was proposed in Ref. [17], which can be expressed as

$$\Delta f_{n, \text{TMLFO}} (t) = \frac{\ln(n + 1)^\kappa - f_0 \, n d \sin \theta_0 / c}{t - r_0 / c}$$

To steer the main beam of the transmit beam pattern to the target point $(r_0, \theta_0)$, the corresponding phase weighting $\phi_{n, \text{TMLFO}}$ is

$$\phi_{n, \text{TMLFO}} = -2 \pi \ln(n + 1)^\kappa$$

Thus, the array factor of the TMLFO-FDA in Ref. [17] is expressed as

However, after transmitting the signal from the antenna array, the corresponding frequency offset, that is TMLFO, in (10) was still assumed to be variable. Moreover, the beam pattern was calculated during $t \in [0, T_d]$, which is contrary to practical situation. In other words, the propagation process of transmitted signals is not considered. Actually, considering the transmitted signal propagation process, $\Delta f_{n, \text{TMLFO}}$ is a function of $t_0 = t - r_0 / c$ rather than $t$.

From (11), it can be observed that the maximum of the beam pattern of TMLFO$^C$-FDA no longer stays at the target point $(r_0, \theta_0)$. For illustration purpose, the transmit beam patterns of TMLFO-FDA and TMLFO$^C$-FDA are provided in Figure 3, which reveal that TMLFO$^C$-FDA no longer has the time-invariant and the spatial focusing ability. Moreover, since the TMLFO-FDA transmits pulse signals, the beam pattern should be plotted over $[c(t-T_c), cT]$ in range dimension, as shown in Figure 3(c) and (d).

Actually, Ref. [26] pointed out that a range-dependent time-invariant beam pattern was extremely difficult to establish, and Ref. [23] pointed that it is impossible for the FDA to achieve a beam pattern only focusing on some specific spatial regions and lasting for some specific time. Therefore, it is of great importance to find out a beamforming method that can greatly alleviate the time-variant beam pattern of the FDA.
3.2 Proposed beam pattern synthesis method

In this subsection, we propose a logarithm-based optimised static non-linear frequency offset, that is, LOSNFO, to achieve a short-time range-angle-decoupled beam pattern. The short-time range-angle-decoupled beam pattern is described as the range-angle beam pattern of FDA is decoupled and the energy at the target point \((r_0, \theta_0)\) only has a small attenuation within the time interval \(t\in[t_1, t_2]\). Thus, it can be ensured that within the time interval \(t\in[t_1, t_2]\), the energy is focused in the desired target location.

As discussed in Section 3.2, due to the introduction of \(t\) in the frequency offset design process, the TMLFO failed to achieve the time-invariant spatial focussing beam pattern. Naturally, it is desirable to employ a static frequency offset to replace the time-variant TMLFO. For this purpose, the proposed LOSNFO is designed as

\[
\Delta f_n^{\text{LOSNFO}} = \ln(n + 1)\kappa(n) \Delta - r_0/c \quad (12)
\]

where \(\kappa(n)\) is a frequency offset control function that needs to be optimised by an optimisation algorithm. \(\Delta = (t_1 + t_2)/2\) is a constant, which represents the average of the start and the end time of the time interval \(t\in[t_1, t_2]\), to ensure a short-time range-angle-decoupled beam pattern during \(t\in[t_1, t_2]\). Comparing (8) and (12), it can be seen that, unlike the time-variant TMLFO, the proposed LOSNFO employs a constant \(\Delta\) within the time interval \(t\in[t_1, t_2]\) to replace the time \(t\). By doing so, the LOSNFO is not a time-modulated frequency offset but a static frequency offset, thus effectively avoiding the time delay induced by the transmitted signal propagation process as reported in Ref. [23].

Observing that it takes a time \(t = r_0/c\) to transmit the signal from the FDA to the target point \((r_0, \theta_0)\), the time interval to achieve the short-time range-angle-decoupled beam pattern is considered as \(t\in[t_1, t_2]\) with \(t_1 = r_0/c\) and \(t_2 = r_0/c + T_d\). For a pulsed FDA, \(T_d\) can be taken as the pulse duration. When it comes to the CW-FDA, \(T_d\) represents the time duration that the short-time range-angle-decoupled beam pattern holds. Therefore, \(\Delta = (t_1 + t_2)/2\) can be written as

\[
\Delta = \frac{(r_0/c) + (r_0/c + T_d)}{2} \quad (13)
\]

Substituting (13) into (12), the LOSNFO is further expressed as

\[
\Delta f_n^{\text{LOSNFO}} = \frac{\ln(n + 1)\kappa(n)}{T_d/2} \quad (14)
\]

Substituting (14) into (6), the array factor of the LOSNFO-FDA is expressed as

\[
AF_{\text{LOSNFO}}(t, r, \theta) = \sum_{n=0}^{N-1} U(t - \tau_n) \exp \left[ \frac{2\pi \ln(n + 1)\kappa(n)}{T_d/2} \left( \frac{t - r/c}{T_d/2} \right) \right] + 2 \pi f_0 \frac{n d \sin \theta}{c} + \phi_n^{\text{LOSNFO}} \quad (15)
\]

To steer the main beam of the transmit beam pattern to the target point \((r_0, \theta_0)\), the corresponding phase weighting \(\phi_n^{\text{LOSNFO}}\) should satisfy

\[
\phi_n^{\text{LOSNFO}} = -2 \pi f_0 n d \sin \theta_0 / c \quad (16)
\]

Substituting (16) into (15), the array factor of the LOSNFO-FDA finally becomes

\[
AF_{\text{LOSNFO}}(t, r, \theta) = \sum_{n=0}^{N-1} U(t - \tau_n) \exp \left[ \frac{2\pi f_0 \ln(n + 1)\kappa(n)}{c} + (n + 1)\kappa(n) \left( \frac{t - r/c}{T_d/2} \right) \right] \quad (17)
\]

Obviously, the transmit beam pattern of the LOSNFO-FDA can be completely separated into the angle- and range-component, which indicated that it is totally decoupled in the angle and range dimensions.

Next, we will focus on discussing the short-time range-angle-decoupled ability of the LOSNFO-FDA beam pattern.
According to (17), at the target point \((r, \theta) = (r_0, \theta_0)\), the array factor of the LOSNFO-FDA is

\[
AF_{\text{LOSNFL}}(r, r_0, \theta_0) = \sum_{n=0}^{N-1} U(t - t_n) e^{i\Phi_{\text{LOSNFL}}} \tag{18}
\]

where \(\Phi_{\text{LOSNFL}} = 2\pi \ln((n+1)\kappa(n)) (t - r_0/c/T_d/2)\). The maximum phase difference within the time interval \(t \in [t_1, t_2]\) is

\[
\Delta \Phi_{\text{LOSNFL}} \bigg|_{t_1=\frac{2\pi}{c}+T_d}^{t_2=\frac{2\pi}{c}+T_d} = 4\pi \ln(n + 1)\kappa(n) \tag{19}
\]

For the LOSNFL-FDA, if \(\Delta \Phi_{\text{LOSNFL}}\) is small enough, \(\Phi_{\text{LOSNFL}}\) can be regarded as almost unchanged within \(t \in [t_1, t_2]\). Thus, the beam pattern of the LOSNFL-FDA can be considered as short-time range-angle-decoupled. For this purpose, the constraint \(\ln((n+1)\kappa(n)) \ll 1\) should be satisfied. In addition, it can be seen from (14) that the LOSNFL depends on not only \(\kappa(n)\) but also \(T_d\). Since the frequency offset applied across each array element of the FDA should be much smaller than the carrier frequency \(f_0\), an appropriate value of \(T_d\) should be selected so that \(\Delta \Phi_{\text{LOSNFL}} \ll \Phi_{\text{LOSNFL}}\). Based on the above analysis, under the condition that the constraints in (20) are satisfied during the optimisation process, the short-time range-angle-decoupled beam pattern at the target location will be achieved.

\[
\begin{align*}
\ln(n + 1)\kappa(n) &\ll 1 \\
\Delta \Phi_{\text{LOSNFL}} &\ll f_0
\end{align*} \tag{20}
\]

Since \((n+1)\leq N\), \(\kappa(n)\) satisfies

\[
\kappa(n) \ll \frac{1}{\ln(N)} \tag{21}
\]

According to (14) and (20), \(T_d\) satisfies

\[
T_d \gg 2\pi \frac{n=0, \ldots, N-1}{f_0} (\ln(n + 1)\kappa(n)) \tag{22}
\]

By introducing the LOSNFL, the inherent flaw of the time-variant and range-angle-coupled beam pattern of the FDA can be greatly alleviated and a short-time range-angle-decoupled transmit beam pattern can be acquired.

As mentioned above, to determine the distribution of the frequency offset control function \(\kappa(n)\), one specific optimisation algorithm is required. In order to ensure that the distribution of \(\kappa(n)\) is global best, the genetic algorithm (GA) is selected, thanks to its global searching ability for the complex discontinuous multidimensional engineering problems [27–29]. To achieve a spatial focusing transmit beam pattern with low SLLs and narrow HPBWs, corresponding cost function for optimising the distribution of \(\kappa(n)\) is:

\[
\begin{align*}
\text{fit} &= \omega_1 \cdot |\text{BW}_{\text{BW}}^\theta - \text{BW}_{\text{BW}}^{\theta_0}| + \omega_2 \cdot |\text{BW}_{\text{BW}}^\theta - \text{BW}_{\text{BW}}^\theta| \\
&+ \omega_3 \cdot |\text{BW}_{\text{BW}}^\theta - \text{BW}_{\text{BW}}| + \omega_4 \cdot |\text{SLL}_{\text{BW}}^\theta - \text{SLL}_{\text{BW}}^\theta| \\
&+ \omega_5 \cdot |\text{SLL}_{\text{BW}}^\theta - \text{SLL}_{\text{BW}}^\theta| \tag{23}
\end{align*}
\]

where \(\text{BW}_{\text{BW}}^\theta\) and \(\text{BW}_{\text{BW}}^\theta\) are respectively the achieved HPBW and the desired HPBW of the range-angle beam pattern in the angle dimension when \(t_1 = r_0/c\). \(\text{BW}_{\text{BW}}^\theta\) and \(\text{BW}_{\text{BW}}^\theta\) are respectively the achieved HPBW and the desired HPBW of the range-angle beam pattern in the range dimension when \(t_1 = r_0/c\). \(\text{BW}_{\text{BW}}^\theta\) and \(\text{BW}_{\text{BW}}^\theta\) are the achieved HPBW and the desired HPBW of the range-time beam pattern in the range dimension at \(\theta = \theta_0\), respectively. Similarly, \(\text{SLL}_{\text{BW}}^\theta\) and \(\text{SLL}_{\text{BW}}^\theta\) are, respectively, the achieved SLL and the desired SLL of the range-angle beam pattern when \(t_1 = r_0/c\). \(\text{SLL}_{\text{BW}}^\theta\) and \(\text{SLL}_{\text{BW}}^\theta\) are the achieved SLL and the desired SLL of the range-time beam pattern at \(\theta = \theta_0\), respectively, and \(\omega_1\) to \(\omega_6\) are the corresponding weight coefficients.

In order to validate the realizability of GA in optimising the non-linear frequency offset control function \(\kappa(n)\), the time complexity of GA is analysed. The time complexity of GA for optimising \(\kappa(n)\) is expressed as \(O(g(N))\) [31, 32], where \(g(N)\) is the maximum number of generations, \(N_p\) is the population size and \(g(N)\) is the number of operation in fitness evaluation. In the fitness evaluation during each generation, three calculations are involved. One is the cost function calculation according to (23) and the other two are cumulative sum calculations to obtain the beam pattern of the LOSNFL-FDA in the range-angle.
F I G U R E 5 Flow chart of the optimisation of $\kappa(n)$

- Start
- Determine $N$ and $f_0$ of the LOSNFO-FDA
- Determine the optimization reference LOSNFO-FDA and select appropriate $\kappa_0$ and $T_d$ according to constraints (19)-(21)
- Obtain the desired SLLs and HPBWs in the cost function (22) according to the LOSNFO-FDA
- Initialize random population with $N_p = 20$, $P_c = 0.8$, $P_m = 0.025$
- Set $Gen_{max} = 50$ and $Gen = 1$
- Fitness calculation according to (22)
- Selection
- Crossover
- Mutation
- Produce new population
- $Gen > Gen_{max}$?
  - No
  - Gen = Gen + 1
  - Yes
- Output global best distribution of $\kappa(n)$
- End

dimension and range-time dimension. Thus $g(N) = 2N + 1$. The time complexity of GA for optimising $\kappa(n)$ finally becomes $O(\text{Gen}_{max}N_p(2N + 1))$, which is a polynomial of the array size of $N$. Figure 4 plots the variations of the time complexity of GA in each iteration, the linear complexity $O(N)$, logarithmic complexity $O(\log(N))$, log linear complexity $O(\log(N))$, quadratic complexity $O(N^2)$, and cubic complexity $O(N^3)$ with the input size $N$. As it can be seen, since GA is an algorithm based on an iterative mechanism, the use of loops makes the time complexity of GA in each iteration higher than the linear complexity $O(N)$, logarithmic complexity $O(\log(N))$ and log linear complexity $O(N\log(N))$. However, as the input size $N$ increases, the time complexity of GA increases linearly, which is much lower than that of the quadratic complexity $O(N^2)$ and cubic complexity $O(N^3)$. The flow chart of the optimisation of $\kappa(n)$ is demonstrated in Figure 5. Note that the CW-LOSNFO-FDA is employed during the optimisation process to obtain a beam pattern with the low SLLs and narrow HPBWs in the whole range and angle dimensions. It is indicated from (1) and (2) that the pulsed FDA has an effective transmit beam pattern which is a truncation form of that of the CW-FDA, thus the optimisation of the transmit beam pattern of the CW- LOSNFO- FDA ensures that the pulsed LOSNFO-FDA has a beam pattern with the low SLLs and narrow HPBWs.
4 | NUMERICAL AND EXPERIMENTAL RESULTS

In this section, to validate the effectiveness of the proposed LOSNFO in terms of achieving a short-time range-angle-decoupled beam pattern for FDA, numerical experiments are conducted first. In the simulations, unless otherwise indicated, a uniform linear FDA with $N = 10$ and $d = \lambda_0 / 2$ is considered, operating at a reference frequency of $f_0 = 10$ GHz. Without loss of generality, assume that a target is located at $(r_0, \theta_0) = (150 \text{ km}, 30^\circ)$.

Before optimizing, the frequency offset control function $\kappa(n)$, it is necessary to determine the desired HPBWs and the SLLs in the cost function (23), that is $BW_{\theta d}^{OR}$, $BW_{\theta d}^{TR}$, $BW_{\phi d}^{TR}$, $SLL_{\phi d}^{OR}$, and $SLL_{\phi d}^{TR}$. Thus, an optimisation reference should be determined. In order to generate a short-time range-angle-decoupled beam pattern with the low SLLs and narrow HPBWs, a CW-LOSNFO-FDA is considered, in which a specific LOSNFO with fixed $\kappa(n) = \kappa_0$ satisfying (21) is employed for all array elements, acting as the optimisation reference. For convenience, we denote the LOSNFO with $\kappa(n) = \kappa_0$ as logarithm-based static non-linear offset (LSNFO) [33]. From (21), it is calculated that $\kappa(n) \ll 1/\ln(10) = 0.43$. Herein, assuming that two real numbers $a$ and $b$ satisfy $(a \pm b)/a \approx 1$, it can be regarded that $\kappa \ll a$. Based on this criteria, $\kappa(n) = \kappa_0$ can be set within $[0, 0.043]$. Without loss of generality, we set $\kappa(n) = \kappa_0 = 0.03$ for the LSNFO. Then an appropriate value of $T_d$ should be determined. According to (22), $T_d \gg 1.38 \times 10^{-5}$ is calculated for the LSNFO. To achieve a beam pattern with low SLLs and narrow HPBWs, the variation of the SLLs and HPBWs of the LSNFO-FDA in the range-angle dimension and range-time dimension with different values of $T_d$ is plot in Figure 6. From Figure 6(a), we can see that as $T_d$ increases from 0.001 to 10 $\mu$s, both the SLLs in the range-angle dimension and range-time dimension gradually decrease. When $T_d > 10 \mu$s, the SLLs tend to remain unchanged. From Figure 6(b), we can see that with the increase of $T_d$, the HPBW in the range dimension widens while the HPBW in the angle dimension remains unchanged. This phenomenon can be explained by (17). It can be seen from (17) that the beam pattern can be completely separated into the angle- and range-component. For the angle-component, it is similar as the beam pattern of the conventional phased array, which is not affected by $T_d$. For the range-component, it is
dependent on the $\kappa(n)$ and $T_d$. Thus, the HPBW in the range dimension changes with the variation of $T_d$. In order to achieve a beam pattern with both.

With $\kappa(n) = \kappa_0 = 0.03$ and $T_d = 10$ $\mu$s, the LSNFOs of each array element can be obtained from (14). The corresponding frequency offset control function $\kappa(n)$ and the frequency offsets, that is LSNFOs, are plotted in Figure 7(a) with the blue dashed line and red solid line, respectively. The obtained transmit beam pattern of the LSNFO-FDA is shown in Figure 8. As it can be seen from Figure 8(a), the maximum beam pattern of the LSNFO-FDA in the range-angle dimension is only at the target point $(r_0, \theta_0) = (150$ km, $30^\circ)$. The SLL is 0.56 in the range-angle dimension. The HPBWs are 19.1 km and 12.0$^\circ$ in the range and angle dimensions, respectively. From Figure 8(b), we can see that the beam pattern at $(150$ km, $30^\circ)$ is short-time range-angle-decoupled within the time interval $t \in [t_1, t_2]$. The SLL is 0.10 in the range-time dimension. And the HPBW is 19.1 km in the range dimension. Although the beam pattern of the LSNFO-FDA is totally decoupled in the range-angle dimension, it is worth noting that its SLLs are relatively high and HPBWs are relatively wide. Thus, it is desirable to employ the SLLs and HPBWs of the LSNFO-FDA as the optimisation reference to obtain a short-time range-angle-decoupled beam pattern with lower SLLs and narrower HPBWs by optimising the frequency offset control function $\kappa(n)$. Therefore, $BW_{\theta_0}^{OR}$, $BW_{\theta_d}^{OR}$, $BW_{\theta_d}^{TR}$, $SLL_{\theta_0}^{OR}$, and $SLL_{\theta_d}^{TR}$ in (23) are set to 12.0, 19.1, 19.1, 0.56, and 0.10, respectively.

Next, the beam pattern synthesis for LOSNFO-FDA will be carried out. GA is utilised to optimise the frequency offset control function $\kappa(n)$ according to the optimisation flow chart of Figure 5 in the following simulation. For the proposed LOSNFO-FDA, the performance of the range-angle-decoupled beam pattern in terms of low SLLs and narrow HPBWs should not be influenced too much by the control parameters of GA. This indicates that the insensitive control parameters of GA are expected to be obtained. Therefore, it is necessary to perform sensitivity analysis on the control parameters of GA to ensure the robustness of the proposed method. Generally, the population size $N_p$, crossover rate $P_c$ and mutation rate $P_m$ are the major factors.

**FIGURE 7** Distributions of the frequency offset control function $\kappa(n)$ and the frequency offset. (a) For the LSNFO-FDA. (b) For the LOSNFO-FDA. Low SLLs and narrow HPBWs, $T_d$ is selected as 10 $\mu$s.

**FIGURE 8** Transmit beam pattern of CW-LSNFO-FDA. (a) Projection of the range-angle beam pattern when $t_1 = r_0/c$. (b) Projection of the range-time beam pattern at $\theta = \theta_0$. 
influencing the performance of GA. Generally, the value of \( N_p \) is greater than 20, the value of \( P_c \) is greater than 0.5, and the value of \( P_m \) is within \([0.005, 0.05]\) [30]. Based on this, consider the value of \( N_p \) within \([20, 40, 60]\), and for each value of \( N_p \), the performance of GA using various parameter combinations with 100 independent runs are analysed, where \( P_c \) and \( P_m \) are randomly selected within \([0.5, 1]\) and \([0.005, 0.05]\), respectively. For each independent run, the maximum number of the generation \( \text{Gen}_{\text{max}} \) is set as 50. The SLLs and HPBW of the proposed LOSNFO-FDA obtained by GA with 300 independent runs are shown in Figure 9. It can be seen from Figure 9 that for all the values of \( N_p \), when \( P_c \) is within \([0.7, 0.9]\), it is easier to obtain lower SLLs in the range-angle and range-time dimensions and narrower HPBWs in the range dimension. While \( P_m \) within \([0.005, 0.05]\) has little influence on the performance of the SLLs and HPBW.
Therefore, it can be considered that the $P_c$ within $[0.7, 0.9]$ and $P_m$ within $[0.005, 0.05]$ are insensitive control parameters of GA. Based on this, herein, we set $P_c$ and $P_m$ as 0.8 and 0.025, respectively. When $N_p$ takes the value of 20, 40 or 60, satisfactory SLLs and HPBWs can be obtained. In order to make the proposed method computationally efficient, $N_p$ is set as 20. In the optimisation, $\kappa(n)$ is restricted in the region of $[0, 0.043]$ which is obtained from (21) to guarantee a short-time range-angle-decoupled transmit beam pattern. The weight coefficients $\omega_1, \omega_2$ are all set as 1.0.

With $\kappa(n)$ optimised by GA, the LOSNFOs are calculated from (14). The corresponding frequency offset control function $\kappa(n)$ and the frequency offsets LOSNFOs are plotted in Figure 7(b) with the blue dashed line and red solid line, respectively. Comparing LOSNFOs in Figure 7(b) with LSNFOs in Figure 7(a), it can be discovered that the variation trend of LOSNFOs is greatly improved, which indicates that LOSNFO-FDA has a more focussing beam pattern. The transmit beam pattern of CW-LOSNFO-FDA is plotted in Figure 10. From Figure 10(a), we can see that the SLL of the CW-LOSNFO-FDA is 0.46 in range-angle dimension, which is 0.1 lower than that of CW-LSNFO-FDA, and the HPBW in the range dimension is 16.3 km, which is 2.8 km narrower than that of the CW-LSNFO-FDA. Therefore, CW-LOSNFO-FDA has the ability to achieve a more focussing beam pattern at the target location with lower SLLs and narrower HPBWs compared with the CW-LSNFO-FDA. Figure 10(b) provides the range-time beam pattern projection at $\theta = \theta_0$. The SLL is 0.10 in range-time dimension. The HPBW is 16.3 km in the range dimension, which is 2.8 km narrower than that of the CW-LSNFO-FDA. Figure 11 provides the corresponding transmit beam pattern of the pulsed LOSNFO-FDA, which indicated that its effective transmit beam pattern spans in the range dimension at the speed of light. The width of the effective transmit beam pattern in the range dimension is $c\cdot T_d = 3$ km.

**FIGURE 10** Transmit beam pattern of CW-LOSNFO-FDA. (a) Projection of the range-angle beam pattern when $t = r_0/c$. (b) Projection of the range-time beam pattern at $\theta = \theta_0$.

**FIGURE 11** Transmit beam pattern of pulsed LOSNFO-FDA. (a) Projection of the range-angle beam pattern when $t = r_0/c$. (b) Projection of the range-time beam pattern at $\theta = \theta_0$. 

**Figure 12** Transmit beam pattern projected on range-time dimension of CW and pulsed LOSNFO-FDAs. (a) Detail view of CW LOSNFO-FDA at the target location when $\theta = \theta_0$. (b) Beam pattern of CW-LOSNFO-FDA in the range dimension when $t_1 = r_0/c$ and $t_2 = r_0/c + T_d$ at $\theta = \theta_0$. (c) Detail view of pulsed LOSNFO-FDA at the target location when $\theta = \theta_0$. (d) Beam pattern of pulsed LOSNFO-FDA in the range dimension when $t_1 = r_0/c$ and $t_2 = r_0/c + T_d$ at $\theta = \theta_0$.

**Table 1** SLLs, HPBW and optimisation time of LOSNFO-FDAs with different array sizes

| Array size | SLL (°) | SLL (°) | HPBW (°) | HPBW (°) | Optimisation time (s) |
|------------|---------|---------|----------|----------|----------------------|
| 10         | 0.46    | 0.1     | 12°      | 16.3 km  | 36.1 s               |
| 20         | 0.20    | 0.04    | 6°       | 15.0 km  | 58.4 s               |
| 30         | 0.19    | 0.03    | 4.5°     | 14.1 km  | 76.7 s               |
| 40         | 0.18    | 0.03    | 3.5°     | 13.3 km  | 97.6 s               |
| 50         | 0.15    | 0.02    | 2.5°     | 11.5 km  | 123.1 s              |

In order to further assess the short-time range-angle-decoupled beamforming ability of LOSNFO-FDA within the time interval $t \in [t_1, t_2]$, a detail view of the transmit beam patterns of CW and pulsed LOSNFO-FDAs projected on the range-time dimension are presented in Figure 12(a) and (c), which shows that there exists a range offset for both CW and pulsed LOSNFO-FDA beam patterns. Specifically, Figure 12(b) and (d) presents the transmit beam patterns of CW and pulsed LOSNFO-FDAs in the range dimension with $\theta = \theta_0$, at the times $t_1 = r_0/c$ and $t_2 = r_0/c + T_d$. Within the time interval $t \in [t_1, t_2]$, the pointing of the maximum of the range beam pattern shifts from $r_0 = 150$ km to $r_1 = 153$ km, which reveals that within $T_d = 10 \mu s$, the wave travels 3 km at the speed of light. From Figure 12(b), we can see that for the CW-LOSNFO-FDA, though a range offset of $\Delta r = 3$ km exists, $r_1$ is always within the HPBW ([141.9 km, 158.1 km]) of the range beam pattern at the start time point of $t \in [t_1, t_2]$, that is $t_1 = r_0/c$. Moreover, at the end time point, that is $t_2 = r_0/c + T_d$, the beam pattern value at the target point (150 km, 30°) is 0.914, which reveals only a $\Delta P = 0.086 (-0.39 \text{dB})$ attenuation of energy exists at the target point. For a pulsed LOSNFO-FDA, we can see a
similar phenomenon from Figure 12(d). Within the time interval \( t \in [t_1, t_2] \), a \( \Delta r = 3 \) km range offset exists and the attenuation of energy is \( \Delta P = 0.086 \). In addition, Figure 12(d) indicates that in the range dimension, the pulsed LOSNFO-FDA always has an effective transmit beam pattern which is a \( c \cdot T_d \) truncation of CW-LOSNFO-FDA. Therefore, both the CW and pulsed LOSNFO-FDAs have the ability to form a short-time range-angle-decoupled beam pattern within the time interval \( t \in [t_1, t_2] \), thus alleviating the time variant property of FDA.

Then the LOSNFO-FDAs with different array sizes ranging from 10 to 50 are synthesised to explore the complexity and realisability of the proposed approach for larger array synthesis. The corresponding SLLs and HPBWs are summarised in Table 1. It can be seen that as the array size increases, the SLLs in the range-angle dimension and range-time dimension of the LOSNFO-FDA gradually decrease and the HPBWs in the angle and range dimensions gradually narrow, which reveals that LOSNFO-FDAs with larger sizes have the ability to achieve more focussing beam patterns. Therefore, the proposed approach can be applied to FDAs with larger sizes. In addition, the optimisation time required for LOSNFO-FDAs with different array sizes is also provided in Table 1. It can be seen that as the array size increase from 10 to 50, the optimisation time increases from 36.15 to 123.1s, which is an acceptable time cost for large array synthesis. The above simulation experiments validate that when facing array synthesis of LOSNFO-FDA with a larger array size, the proposed approach is totally achievable with an acceptable optimisation time.

In the simulations corresponding to Figures 8–12 and Table 1, the array element is considered as an ideal isotropic element to verify the effectiveness of the proposed LOSNFO-FDA in generating the short-time range-angle-decoupled beam pattern. However, in practical applications, it is necessary to consider the real arrays with mutual coupling effects in the synthesis process. For this purpose, a 10-element linear microstrip antenna array, as shown in Figure 13 is synthesised to obtain a short-time range-angle decoupled beam pattern including mutual coupling effects. The experiment is carried out by using the High Frequency Structure Simulator (HFSS) full-wave software. Each microstrip patch is on the substrate \( \varepsilon_r = 2.2 \) with a thickness 1.57 mm. According to the optimisation process shown in Figure 5, the resulting beam pattern of the microstrip LOSNFO-FDA is shown in Figure 14. From Figure 14(a), we can see that the SLL of microstrip LOSNFO-FDA is 0.51 in range-angle dimension. Although it is slightly higher than that of the ideal LOSNFO-FDA caused by the mutual coupling effects, it is 0.05 lower than that of ideal LSNFO-FDA and the HPBW in the range dimension is 16 km, which is 3.1 km narrower than that of the ideal LSNFO-FDA and is comparable to that of the ideal LOSNFO-FDA. From Figure 14(b), we can see that the SLL.
is 0.09 in range-time dimension. The practical simulation result reveals that the short-time range-angle-decoupled beam pattern is successfully achieved by the 10-element linear microstrip antenna array with mutual coupling effects. Therefore, the proposed synthesis method is not only applicable to the ideal arrays but also applicable to the real arrays with mutual coupling effects.

Next, in order to validate the superior spatial beam pattern focussing performance of the proposed LOSNFO-FDA, the SLLs in the range-angle dimension and HPBWs in the range dimension of the ideal and microstrip LOSNFO-FDA are compared with those of the FDA using logarithmically increasing frequency offset (Log-FDA) [8] and the FDA using non-uniform logarithmic frequency offset (NULog-FDA) [9]. For the Log-FDA and NULog-FDA, δ in [8, 9] is set as 2kHz. For a comprehensive comparison, two FDAs with different reference frequencies and array sizes are considered. Specifically, one FDA has 10 elements and works at 10 GHz as presented in the previous simulations. The other FDA has eight elements and works at 3.2 GHz as shown in Ref. [8]. The beam patterns of the Log-FDAs, NULog-FDAs, ideal LOSNFO-FDAs and microstrip LOSNFO-FDAs are plotted in Figure 15.

**FIGURE 15** Transmit beam patterns projected on range-angle dimension of Log-FDAs, NULog-FDAs, ideal LOSNFO-FDAs, and microstrip LOSNFO-FDAs. (a) 8-element FDAs operating at 3.2 GHz. (b) 10-element FDAs operating at 10 GHz

**TABLE 2** SLLs and HPBWs of Log-FDAs [8], NULog-FDAs [9], ideal LOSNFO-FDAs, and microstrip LOSNFO-FDAs with different reference frequencies and array sizes

| Array Size | Reference Frequency | SLL in range-angle dimension | HPBW in range dimension |
|------------|---------------------|------------------------------|--------------------------|
|            |                     | Log-FDA | NULog-FDA | Ideal LOSNFO-FDA | Microstrip LOSNFO-FDA | Log-FDA | NULog-FDA | Ideal LOSNFO-FDA | Microstrip LOSNFO-FDA |
| 8          | 3.2 GHz             | 0.52    | 0.65      | 0.51           | 0.52                      | 42.4 km | 21.0 km | 8.8 km         | 8.8 km         |
| 10         | 10 GHz              | 0.56    | 0.50      | 0.46           | 0.51                      | 57.4 km | 22.6 km | 16.3 km        | 16.0 km        |

**FIGURE 16** Manufactured 8-element linear microstrip antenna array
The corresponding SLs and HPBWs are summarised in Table 2. We can see that although all beam patterns of the Log-FDAs, NULog-FDAs, ideal LOSNFO-FDAs and microstrip LOSNFO-FDAs are decoupled in the range-angle dimension, for both the 8-element FDAs and 10-element FDAs, the proposed ideal and microstrip LOSNFO-FDAs have lower SLs and narrower HPBWs compared with the Log-FDAs and NULog-FDAs, which reveals that the proposed LOSNFO-FDA has a superior spatial focussing ability for both the ideal antenna array and the real antenna array considering mutual coupling effects.

Finally, as shown in Figure 16, the 8-element linear microstrip antenna array operating at 3.2 GHz discussed above was manufactured and measured. The simulated beam patterns and the measured beam patterns projected on the angle and range dimensions are plotted in Figure 17. As it can be seen, the measured results are in good agreement with the simulated results. The beam pattern of the 8-element microstrip LOSNFO-FDA is totally decoupled in the range and angle dimensions, which further validates the capability of the proposed LOSNFO-FDA.

5 | CONCLUSION

Herein, we propose a novel logarithm-based optimised static non-linear frequency offset, that is LOSNFO, for FDA in order to alleviate its inherent flaw in the time-variant and range-angle-coupled beam pattern. With the proposed LOSNFO, a short-time range-angle-decoupled beam pattern with low SLs and narrow HPBWs is successfully achieved for FDA. Theoretical analysis, simulation and experimental results demonstrate the effectiveness of the proposed LOSNFO-FDA.

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