A mean-field approach for an intercarrier interference canceller for OFDM

A Sakata\textsuperscript{1}, Y Kabashima\textsuperscript{1} and Y Peleg\textsuperscript{2}

\textsuperscript{1} Department of Computational Intelligence and Systems Science, Tokyo Institute of Technology, Midori-ku, Yokohama 226-8502, Japan
\textsuperscript{2} Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel

E-mail: ayaka@sp.dis.titech.ac.jp, kaba@dis.titech.ac.jp and yitzhak.peleg@gmail.com

Received 22 May 2012
Accepted 9 June 2012
Published 16 July 2012

Online at stacks.iop.org/JSTAT/2012/P07006
doi:10.1088/1742-5468/2012/07/P07006

Abstract. The similarity of the mathematical description of random-field spin systems to the orthogonal frequency-division multiplexing (OFDM) scheme for wireless communication is exploited in an intercarrier interference (ICI) canceller used in the demodulation of OFDM. The translational symmetry in the Fourier domain generically concentrates the major contribution of ICI from each subcarrier in the subcarrier’s neighbourhood. This observation in conjunction with the mean-field approach leads to the development of an ICI canceller whose necessary cost of computation scales linearly with respect to the number of subcarriers. It is also shown that the dynamics of the mean-field canceller are well captured by a discrete map of a single macroscopic variable, without taking the spatial and time correlations of estimated variables into account.

Keywords: analysis of algorithms, heuristics, communication, supply and information networks
1. Introduction

Wireless communication technologies play a significant role in the modern information society. As of the end of 2010, there are more than 4.6 billion mobile-cellular subscriptions in the world [1], and the use of wireless devices (such as personal digital assistants and GPS units) is ever increasing. To keep up with the accompanying rapid growth in data traffic, the communication efficiency of today’s wireless communication systems must thus be constantly improved.

A decade has passed since a fruitful connection between wireless communications and statistical mechanics was introduced by a seminal work by Tanaka [2]. On the basis of an analogy between the demodulation problem of wireless communications and statistical mechanics of disordered Ising spin systems, he successfully clarified the potential efficiency of a wireless communication scheme known as code-division multiple access (CDMA), which is employed in the third generation cellular phone systems. Later, this analogy was also utilized in developing practically feasible and efficient demodulation algorithms for CDMA [3, 4].

In a recent study [5], the connection to statistical mechanics was extended to another wireless communication scheme, namely, orthogonal frequency-division multiplexing (OFDM), which is today employed in the fourth generation cellular phones and the latest Wi-Fi systems. According to this scheme, the available frequency domain is divided into subdomains, and data are transmitted by the subcarriers associated with those subdomains [6]. Because of the orthogonality between the subcarriers, they can be closely placed in the frequency domain, thereby attaining high-rate data transmission. However, in a mobile radio environment, relative movement brings about a Doppler
A mean-field approach for an intercarrier interference canceller for OFDM spread, which destroys the orthogonality between the subcarriers. This destruction leads to the occurrence of intercarrier interference (ICI), which rapidly deteriorates the bit error rate. An efficient ICI cancellation scheme is, therefore, indispensable in use of the OFDM scheme in such environments. In [5], a Monte Carlo based ICI cancellation scheme was developed on the basis of mapping an OFDM model to a variant of random-field Ising spin systems. Numerical experiments indicated that the developed scheme can achieve significantly better performance than existing standard methods in terms of bit error rate. However, the cost of computation, which grows with the square of the number of subcarriers, and the technical difficulty in implementing electrical circuits, prevent the scheme from being practically significant.

The purpose of this study is to develop an approximate ICI cancellation scheme for resolving the above-mentioned drawbacks. For this purpose, a mean-field approximation (MFA) is utilized in conjunction with the analogy between OFDM and random-field Ising spin systems. Naive MFA requires as much computational cost as the Monte Carlo based cancellation scheme. We show that utility of translational invariance in the Fourier domain, which is intrinsic in OFDM, makes it possible to develop an ICI cancellation scheme whose computational cost is proportional to the number of subcarriers. We also show that the performance of the developed algorithm based on MFA is well captured by a discrete map of a single variable and that the fact is supported by numerical experiments.

This paper is organized as follows. In section 2, we explain the OFDM model studied in this paper. In section 3, we propose an MFA based ICI cancellation scheme. In section 4, we explore the performance of the proposed method and its time evolution, and derive the approximated expression of the proposed canceller. The derivations of the approximated expression and its higher order description are explained in appendix A and B, respectively. Finally, section 5 is devoted to the conclusion and summary.

2. Model

When a time sequence of a signal, \( x = \{x_t\} \ (t = 1, \ldots, N) \), is transmitted in a mobile radio environment, the received symbol, \( y = \{y_t\} \), in the multipath channel is expressed by

\[
y_t = \frac{1}{\sqrt{M}} \sum_{p=1}^{M} h_p \exp \left( \sqrt{-1\frac{2\pi \epsilon_p t}{N}} \right) x_t + \eta_t.
\]

The time delay of each path is assumed to be zero for simplicity, and the channel noise, \( \{\eta_t\} \), is independent with respect to the time domain. The number of paths is \( M \), the amplitude of each path, \( \{h_p\} \), is distributed according to the Rayleigh distribution \([7, 8]\) as

\[
P(h_p) = h_p \exp \left( -\frac{h_p^2}{2} \right),
\]

and \( \{\epsilon_p\} \) is the Doppler shift, which is assumed to be distributed uniformly in the region \( \epsilon_p \in [0, \epsilon_{\text{max}}] \). With this model, the difference between the maximum and minimum values of the Doppler shift is significant irrespective of the sign of the Doppler shift.

By applying the discrete Fourier transformation to equation (1), the frequency-domain representation of the transmitted signal, \( X = \{X_k\} \), and the received signal, \( Y = \{Y_k\} \),

doi:10.1088/1742-5468/2012/07/P07006
A mean-field approach for an intercarrier interference canceller for OFDM

where \( k = 0, \ldots, N - 1 \) and \( N \) is the number of subcarriers, is given by

\[
Y_k = \sum_{l=1}^{N} W_{kl} X_l + n_k,
\]

(3)

where \( n_k \) is the discrete Fourier transform of the channel noise. We assume that \( n_k \) is characterized as an additive white Gaussian noise (AWGN) of mean zero and variance \( \sigma_0^2 \).

The component of the \( N \times N \) matrix \( W \), called a frequency-domain matrix, is given by

\[
W_{kl} = \sum_{p=1}^{M} h_p \sin(\pi(l - k + \epsilon_p)) e^{\sqrt{-1}(1-1/N)\pi(l-k+\epsilon_p)} \sqrt{MN} \sin(\pi(l + \epsilon_p)/N),
\]

(4)

where \( W_{kl} \) represents the intensity of the interference from subcarrier \( l \) to \( k \). When \( \{\epsilon_p\} = 0 \), the matrix is diagonal, namely \( W_{kl} = \delta_{k,l} \), where \( \delta \) is Kronecker’s delta, and there is no ICI between any subcarriers. In general, the frequency-domain matrix has translation symmetry, so the value of each component \( W_{kl} \) only depends on the difference of the indices, namely \( k - l \). This fact is a result of the Fourier representation.

The Doppler shift, \( \epsilon_p \), is normalized by the frequency separation of subcarriers, \( \Delta f = 1/N \), as \( \epsilon_p = f_{D,p}/\Delta f \), where \( f_{D,p} \) is the Doppler frequency at the \( p \)th path. The parameter \( \epsilon_p \) indicates the influence of the Doppler effect on ICI with a given alignment of subcarriers.

The transmitted bits, channel noise, and received bits are represented as complex numbers. Their real and imaginary parts are denoted by \( X^R \) and \( X^I \), \( n^R \) and \( n^I \), and \( Y^R \) and \( Y^I \), respectively. Hereafter, they are represented as vectors consisting of \( 2N \) elements: \( \mathbf{X} \equiv [X^R, X^I]^T \), \( \mathbf{n} \equiv [n^R, n^I]^T \), and \( \mathbf{Y} \equiv [Y^R, Y^I]^T \), respectively. \( T \) denotes the operation of the matrix transpose. The corresponding frequency-domain matrix is redefined as a \( 2N \times 2N \) matrix, \( \mathbf{W} \equiv \begin{pmatrix} W^R & -W^I \\ W^I & W^R \end{pmatrix} \), where \( W^R \) and \( W^I \) are the real and imaginary parts of the matrix, respectively [5].

3. Mean-field ICI canceller

The problem of the OFDM system is to recover the original signal by cancelling out the intercarrier interference. The Bayesian framework offers various ICI cancelling strategies on the basis of the posterior distribution,

\[
P(\mathbf{X}|\mathbf{Y}) = \frac{P(\mathbf{Y}|\mathbf{X})P(\mathbf{X})}{\sum_{\mathbf{X}} P(\mathbf{Y}|\mathbf{X})P(\mathbf{X})},
\]

(5)

where the likelihood \( P(\mathbf{Y}|\mathbf{X}) \) is given by the distribution of the channel noise [9]. At the receiving side, it is assumed that the channel noise is described by the Gauss distribution with mean zero and variance \( \sigma^2 \),

\[
P(\mathbf{Y}|\mathbf{X}) = \left(\frac{1}{2\pi\sigma^2}\right)^{N/2} \exp\left(-\frac{(\mathbf{WX} - \mathbf{Y})^2}{2\sigma^2}\right),
\]

(6)
and prior distribution $P(\mathbf{X})$ is the uniform distribution. The posterior probability can therefore be expressed as

$$P(\mathbf{X} \mid \mathbf{Y}) = \frac{1}{Z} \exp \left\{ \frac{1}{\sigma^2} \left( \frac{1}{2} \sum_{i,j} J_{ij} X_i X_j - \sum_{i=1}^{2N} h_i X_i \right) \right\}$$

$$= \frac{1}{Z} e^{-\beta \mathcal{H}(\mathbf{X} \mid \mathbf{J}, \mathbf{h})}$$

(7)

where $\sigma^{-2}$ is identified with the ‘inverse temperature’ $\beta$, the interaction matrix and the external field are given by $\mathbf{J} = \mathbf{W}^T \mathbf{W}$ and $\mathbf{h} = \mathbf{Y}^T \mathbf{W}$, respectively, and $Z$ corresponds to the partition function. The interaction matrix, $\mathbf{J} = (J_{11} J_{12} J_{21} J_{22})$, also has translational invariance; the $(k,l)$-components of $N \times N$ sub-matrices $\mathbf{J}_1$ and $\mathbf{J}_2$ only depend on $k - l$.

The Hamiltonian of the OFDM system, $\mathcal{H}(\mathbf{X} \mid \mathbf{J}, \mathbf{h})$, defined in equation (7), can be regarded as that for a random-field spin model. Unlike typical random-field models, the random field of the OFDM model is determined by the ICI between transmitted bits and the channel’s properties. In particular, when the number of paths, $M$, is equal to 1, the value of the off-diagonal element is much smaller than that of the diagonal element; hence, the system can be regarded as a single-body problem with random fields.

The maximum a posteriori probability (MAP) strategy

$$\hat{\mathbf{X}} = \arg \max_{\mathbf{X}} P(\mathbf{X} \mid \mathbf{Y})$$

(8)

where $\arg \max$ is the argument giving the maximum value of the function, is guaranteed to minimize the block-wise error probability. This strategy corresponds to the search for the ground state of Hamiltonian $\mathcal{H}(\mathbf{X} \mid \mathbf{J}, \mathbf{h})$. However, the numerical cost of exactly obtaining the maximizer $\hat{\mathbf{X}}$ increases exponentially with increasing number of subcarriers. Zero temperature ($\beta \rightarrow \infty$) synchronous dynamics of $N$ symbols based on the mean-field approximation (MFA) is a practically feasible approximate scheme for finding the MAP solution of equation (8) [10]. In the case of the quadrature-phase-shift-keying (QPSK) modulation, where the components of $\mathbf{X}$ take one of the two values, $\pm 1$, the scheme is expressed as

$$\hat{X}_k^{t+1} = \text{sgn} \left( h_k - \sum_{l \neq k} J_{kl} \hat{X}_l^t \right),$$

(9)

where $\text{sgn}(u)$ denotes the sign of $u$ and $\hat{X}_k^t$ is the tentative decision of the $k$th symbol after $t$ iterations. It is assumed that the configuration $\{\hat{X}_k^t\}$ is invariant at $\{\hat{X}_k^t\}$ after sufficient updates, and the fixed configuration is regarded as the final decision of the transmitted bits. Synchronous update schemes similar to equation (9) have been introduced for evaluating the minimum mean square error (MMSE) estimator of Gaussian priors $P(\mathbf{X}) \propto \exp(-|\mathbf{X}|^2/(2\sigma_X^2))$, in which the transmitted bits are estimated as $\hat{X}_{\text{MMSE}} = ((\sigma^2 / \sigma_X^2) I + \mathbf{J})^{-1} \mathbf{h}$ in the current case, where $I$ is the identity matrix [11]–[13].

The computational cost of equation (9) increases as $O(N^2)$, and it may reduce the practical feasibility of equation (9). To reduce this cost increase, the proposed algorithm utilizes the fact that the absolute value of $J_{1kl}$ and $J_{2kl}$ decreases as $|k-l|$ increases, which indicates that the major contributions to ICI (to which each subcarrier is subject) are concentrated on the subcarrier’s neighbourhood. Based on this observation, the strategy
A mean-field approach for an intercarrier interference canceller for OFDM proposed here considers only a part of the ICI among subcarriers in the frequency domain at each stage of the cancellation. A similar strategy was also proposed for a Gaussian MMSE estimator [14].

Let us define the set of indices of subcarriers that are considered to be contributed on ICI of subcarrier $k$ as $\partial_k(\omega) \equiv \{ x, x+N | x = \text{mod}(k+N-1 \pm y, N)+1, y \in \mathbb{N}, 1 \leq y \leq \omega \}$, where $\text{mod}(k, N)$ means the remainder of $k/N$ as an integer. In general, $\partial_k(\omega)$ is composed of $4\omega$ elements; for instance, $\partial_1(1) = \{ 2, N, N+2, 2N \}$. The update rule is then given as

$$
\hat{X}_{k+1} = \text{sgn} \left( h_k - \sum_{l \in \partial_k(\omega)} J_{kl} \hat{X}_l \right).
$$

The algorithm (9) corresponds to the case for which $\omega = \lfloor N/2 \rfloor$ in equation (10), where $\lfloor x \rfloor$ denotes the largest integer not greater than real number $x$. The numerical cost per iteration of equation (10) is $O(N)$ as long as parameter $\omega$ is $O(1)$. When $\omega < N/2$, fixed point $\{ \hat{X}_i^* \}$ does not correspond to the maximizer of the posterior probability, but it is expected to be a good approximation of the maximizer.

4. Results

4.1. Performance of ICI canceller

We observe the bit error rate (BER), which is defined by

$$
\text{BER} = \frac{1}{2N} \sum_{i=1}^{2N} \langle \hat{X}_i^* X_i \rangle,
$$

where $\langle \cdots \rangle$ represent the average over the frequency-domain matrix $W$ and over the channel noise and transmitted symbol, respectively. The BER performance of the decoder is bounded from below by that for a single bit transmitted through the AWGN channel, which is given by

$$
\text{BER}^{\text{opt}} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right),
$$

because there is no ICI involved when only a single bit is transmitted. The BER of the ICI canceller approaches $\text{BER}^{\text{opt}}$ as the elimination of ICI becomes successful. In the current model, the signal to noise ratio (SNR) is given by

$$
\text{SNR} = \frac{1}{N} \sum_{i,j=1}^{2N} \frac{|W_{ij}|^2}{\sigma_0^2}.
$$

In the QPSK modulation case, the number of bits per symbol is two, so $E_b/N_0$ corresponds to $\text{SNR}/2$ [7].

We check the performance of the matched filter, whose mathematical manipulation corresponds to multiplying the received symbol by the Hermitian conjugate matrix of $W$. The matched filter minimizes the power of the channel noise, but it cannot reduce the error due to the intercarrier interference. Therefore, as a canceller of intercarrier interference, the performance of the proposed algorithm should be better than that of the matched
A mean-field approach for an intercarrier interference canceller for OFDM

Figure 1. Dependence of BER on Eb/N0 at (a) $N = 32, M = 3, \epsilon_{\text{max}} = 0.5$ and (b) $N = 32, M = 15, \epsilon_{\text{max}} = 0.5$. Results for $\omega = 1$ and 6 are shown and ‘Full’ corresponds to $\omega = 16$. The dashed line and dashed–dotted line represent the optimal and matched filter cases, respectively. GA means the BER performance under the Gaussian approximation. The data points are averaged over $10^6$ samples of $W$.

Figure 2. Dependence of the BER performance on $\omega$ at $M = 3, \epsilon_{\text{max}} = 0.5$, and Eb/N0 = 8.69 dB. Results for numbers of subcarriers $N = 64, 128, 256$ are shown, and the dashed line shows the BER of the optimal limit.

filter. In the proposed algorithm, $\omega = 0$ corresponds to the demodulation by the matched filter, and as $\omega$ increases the performance is expected to improve.

The Eb/N0 dependence of BER is shown in figure 1 for numbers of paths (a) $M = 3$ and (b) $M = 15$, respectively. The number of subcarriers is $N = 32$, and the maximum value of the Doppler shift is $\epsilon_{\text{max}} = 0.5$. BER$^\text{OPT}$ and BER for the matched filter are shown by the dashed line and dashed–dotted line, respectively. The BER performance of the canceller given by equation (10) with $\omega = 1$, which is the simplest case, is better than that of the matched filter, and it improves as the number of interactions increases. When all interactions between the subcarriers are taken into account, the BER performance of the canceller given by equation (9) almost coincides with the optimal performance BER$^\text{OPT}$.

doi:10.1088/1742-5468/2012/07/P07006
Figure 3. The time evolution of BER at $N = 128, M = 15, \epsilon_{\text{max}} = 0.5$, and $E_b/N_0 = 7.45$ dB. Results for $\omega = 3$ and the full case ($\omega = 64$) are shown, and GA means the corresponding Gaussian approximation. The trajectory is averaged over $10^4$ samples of $W$.

The $\omega$-dependence of BER performance at $M = 3, \epsilon_{\text{max}} = 0.5$, and $E_b/N_0 = 8.69$ is shown in figure 2 for $N = 64, 128, and 256$. This graph indicates that BER does not depend on the number of subcarriers, $N$. BER rapidly decreases as $\omega$ increases from zero, and it gradually approaches $\text{BER}^{\text{OPT}}$ as $\omega$ further increases. At $\omega \gtrsim 16$, the differences between the BER of the proposed method and the optimal limit are less than 10% of the value of $\text{BER}^{\text{OPT}}$. This result indicates that $\omega \sim 16$ is sufficient to practically achieve the BER of the original MFA based canceller (9) irrespective of $N$; thus, the required numerical cost per bit does not increase as $O(N)$. In this model, the unit width of the frequency domain is given by $1/N$; therefore, the proposed algorithm will provide more effective use of the frequency domain without increasing the numerical cost.

4.2. ICI dynamics

To determine the validity of the proposed method as a realistic canceller, the time evolution of the BER of the proposed algorithm, decoding algorithm at $N = 128, M = 15$, and $\epsilon_{\text{max}} = 0.5$, is plotted in figure 3. The initial condition, $\{\hat{X}_k^0\}$, is obtained by the matched filter, so BER at the zeroth step corresponds to that at the fixed point for $\omega = 0$. The horizontal axis represents the number of time steps, and the vertical axis the BER with respect to the tentative decision at the time step. As can be seen in the figure, the BER performances at $\omega = 3$ and $\omega = 64$ (full) converge to equilibrium values after two updates. The time steps required to reach the fixed point are quite short and only slightly depend on $N$, and the proposed algorithm is useful for an implementation as an ICI canceller.

4.3. Macroscopic description

To analyse the time evolution of the canceller given by equation (10), we attempt to describe the dynamics by using a finite number of macroscopic variables [4, 15]. The overlap between the transmitted bit and the predicted bit at step $t$ under a given
realization of $W$ is defined as

$$m^t = \frac{1}{2N} \sum_{k=1}^{2N} \langle \hat{X}_k^t X_k \rangle,$$

and the BER at step $t$ is given by $0.5 \times (1 - m^t)$. The simplest description of the time evolution of equation (14) is provided by ignoring all spatial/time correlations among the subcarrier symbols, which leads to a discrete map of $m^t$:

$$m^{t+1} = \int Dz \ \text{sgn}(z_0 + \Sigma^t z),$$

$$z_0 = \frac{1}{2N} \sum_{k=1}^{2N} J_{kk}, \quad \Sigma^t = \sqrt{2A(1 - m^t) + B + \sigma^2 C},$$

where $Dz = dz \ e^{-z^2/2}/\sqrt{2\pi}$, and the coefficients for the fixed sample of $W$ are given by

$$A = \frac{1}{2N} \sum_{k=1}^{2N} \sum_{l \in \partial_k(w)} J_{kl}^2,$$

$$B = \frac{1}{2N} \sum_{k=1}^{2N} \sum_{l \notin \partial_k(w)} J_{kl}^2,$$

$$C = \frac{1}{2N} \sum_{k=1}^{2N} \sum_{l} W_{lk}^2 = z_0.$$

The derivation is given in detail in appendix A. These expressions indicate that the ICI is approximated by Gaussian noise, and noise variance $\sigma^2$ is effectively increased by $B/C$ due to the insufficiency of the ICI cancellation of the MFA canceller (10). The macroscopic equation corresponding to equation (9) is obtained by setting $\omega = N/2$.

BERs defined at fixed points given by equation (15), denoted by GA (Gaussian approximation), are compared to the real BER curve in figure 1, where $\text{BER} = (1 - m^t)/2$. BERs at the fixed point are in good accordance with the experimental data irrespective of the value of $\omega$. The time evolution of BER is also well described by equation (15), as shown in figure 3, in which the initial condition, $\{m^0\}$, is chosen to correspond to the BER of the matched filter. One can push the current description to a higher order approximation by taking into account the time correlation of the crosstalk noise [15], which, however, does not lead to significant improvement of the fit to the experimental data (appendix B). These results differ substantially from that of the random spreading codes, where the time correlation plays a significant role in the macroscopic dynamics [4]. The difference implies that the orthogonality between the subcarriers in OFDM reduces the time correlation and enables the ICI dynamics to converge within a few steps.

5. Conclusion

A practically feasible ICI canceller for the OFDM model, which can be regarded as a variant of the random-field spin model, was developed. The cancellation scheme was derived by applying the mean-field approximation to the maximization of the posterior
probability, which corresponds to the search for the ground state of the spin model. The properties of the frequency-domain matrix, i.e. translation symmetry and smallness of off-diagonal elements compared to diagonal elements, were focused on, and only a part of the ICI among the subcarriers located within a distance $\omega$ in the frequency domain for each bit was considered. The numerical cost of the ICI canceller is thus controlled by the parameter $\omega$. When $\omega = \lfloor N/2 \rfloor$, the ICI between all subcarriers is considered, and the ICI canceller corresponds to the approximated MAP demodulator.

The BER of the proposed algorithm used for QPSK modulation is better than the matched filter even if $\omega = 1$, and it practically approaches the optimal limit as $\omega$ increases further. The BER performance is saturated near the optimal limit around a certain $\omega \sim O(1)$ in the whole $Eb/N0$ region. Furthermore, the performance under a given $\omega$ only slightly depends on $N$. This result means that the required numerical cost per iteration to achieve a feasible performance level is $O(N)$.

The number of time steps required to reach a fixed point of the ICI canceller is $O(1)$ and only slightly depends on the value of $N$ and $\omega$. The total numerical cost to eliminate ICI and demodulate the transmitted bits is therefore $O(N)$. The proposed algorithm will be practical to implement by virtue of its low computational cost.

The fixed point of the ICI canceller and the dynamics to reach there are well described by a discrete map of a single macroscopic variable under the approximation of ICI for each bit as independent Gaussian noise. It is considered that the orthogonality between the subcarriers prevents a time correlation being induced, and is a mathematical background of the high accuracy of the proposed ICI canceller.

The proposed algorithm is efficient for the parameter region where the Doppler shift causes large intercarrier interference. By introducing this algorithm as an ICI canceller, the OFDM scheme is useful when subcarriers are closely arranged in the frequency domain and a mobile object moves at high speed. More efficient use of a given frequency domain and enhanced accuracy in satellite communications are also expected.

Acknowledgments

We would like to thank Ido Kanter for his helpful comments and discussions. This work was supported by JSPS Fellowship No 23–4665 (AS) and KAKENHI No 22300003 (YK).

Appendix A. Derivation of macroscopic dynamics

According to the definition of $\{h_k\}$, equation (10) can be transformed as follows,

$$
\hat{X}_i^{t+1} = \text{sgn} \left[ - \sum_{j \in \partial_i(\omega)} J_{ij}\hat{X}_j^t + \sum_{j=1}^{2N} J_{ij}X_j + \sum_{\mu=1}^{2N} W_{\mu i}n_i \right].
$$

(A.1)

By separating the summation of the first and second terms into three parts, $j \in \partial_i(\omega)$, $j \notin \partial_i(\omega)$ and $j = i$, the time evolution of $m_i^t$ can be written as

$$
m_i^{t+1} = \left\langle \text{sgn}\left[ J_{ii} + X_i \sum_{j \in \partial_i(\omega)} J_{ij}(X_j - \hat{X}_j^t) + X_i \sum_{j \notin \partial_i(\omega)} J_{ij}X_j + X_i \sum_{\mu} W_{\mu i}n_i \right] \rightangle
$$

$$
\equiv \left\langle \text{sgn}\left[ J_{ii} + X_i z_i^t \right] \right\rangle.
$$

(A.2)

doi:10.1088/1742-5468/2012/07/P07006
The right-hand side of equation (A.2) depends on the randomness; the transmitted symbol and the channel noise, through $z_i^t$. The average over the randomness can therefore be replaced by the average over $z_i^t$ according to an appropriate distribution. The distribution of $z_i^t$ is approximated by a Gauss distribution. The first and second moments of $z_i^t$ are given by

$$\langle z_i^t \rangle = 0,$$  \hspace{1cm} (A.3)

$$\langle z_i^t \rangle^2 = 2 \sum_{j \in \partial_i(\omega)} J_{ij}^2 (1 - m_j^t) + 2 \sum_{j \notin \partial_i(\omega)} J_{ij}^2 + \sigma^2 \sum_{\mu=1}^{2N} W_{\mu i}^2,$$ \hspace{1cm} (A.4)

where it is assumed that $X_j = \hat{X}_j^t$ with probability $(1 + m_j^t)/2$, and $X_j = -\hat{X}_j^t$ with probability $(1 - m_j^t)/2$ for any $j$. With these quantities, the following expression can be obtained:

$$m_{i}^{t+1} = \int Dz \: \text{sgn} \left[ J_{ii} + X_i \sqrt{\Sigma_i^t} \right] z,$$ \hspace{1cm} (A.5)

where $\Sigma_i^t = \langle z_i^t \rangle^2 - \langle z_i^t \rangle^2$, and coefficient $X_i$ can be ignored because the function is invariant against translation $z \to -z$. The approximated expression for the full case can be obtained by setting $\omega = N/2$, so $\partial_i(\omega)$ contains all bits except $i$. Further, the translational invariance, which guarantees that $J_{ii}$, $\sum_{j \in \partial_i(\omega)} J_{ij}^2$, and $\sum_{j \notin \partial_i(\omega)} J_{ij}$ do not depend on the site index $i$, allows us to replace $m_i^t$ and $\Sigma_i^t$ with macroscopic variables $m^t = (2N)^{-1} \sum_{i=1}^{2N} m_i^t$ and $\Sigma^t = (2N)^{-1} \sum_{i=1}^{2N} \Sigma_i^t$, respectively.

### Appendix B. A higher order description of the macroscopic dynamics

The accuracy of the macroscopic description based on Gaussian approximation should be improved by taking into account the time correlation of the crosstalk noise $z_i^t$. Although we here show only a way of introducing the effects of the correlation between two contiguous time steps to the approximation, extending the treatment for handling correlations among general $k$ (=2, 3, …) consecutive time steps is also possible [15]. According to the definition of $z_i^t$, the difference between $z_i^{t+1}$ and $z_i^t$, denoted as $u_i^t$, can be defined as

$$z_i^{t+1} = z_i^t + X_i \sum_{j \in \partial_i(\omega)} \mathbb{I}(\hat{X}_j^{t+1} \neq \hat{X}_j^t) J_{ij} X_j \{ (1 - X_j \hat{X}_j^{t+1}) - (1 - X_j \hat{X}_j^t) \}$$

$$= z_i^t + X_i \sum_{j \in \partial_i(\omega)} \mathbb{I}(\hat{X}_j^{t+1} \neq \hat{X}_j^t) 2J_{ij} X_j \hat{X}_j^t$$

$$\equiv z_i^t + u_i^t,$$  \hspace{1cm} (B.1)

where $\mathbb{I}(x)$ is an indicator function that takes the value 1 when the condition $x$ is satisfied, and 0 otherwise. The transformation in equation (B.1) is based on the fact that $\hat{X}_j^{t+1} \neq \hat{X}_j^t$ means $X_j \hat{X}_j^{t+1} = -X_j \hat{X}_j^t$ when $\hat{X}_j^{t+1}$ and $\hat{X}_j^t$ take one of the values ±1. The first and second moments of $u_i^t$ and the covariances between $z_i^t$ and $u_i^t$ are given as

$$\langle u_i^t \rangle = 0$$  \hspace{1cm} (B.2)

$$\langle u_i^t \rangle^2 = 2 \sum_{j \in \partial_i(\omega)} J_{ij}^2 (1 - C_j^{t,t+1})$$  \hspace{1cm} (B.3)
A mean-field approach for an intercarrier interference canceller for OFDM

Figure B.1. The comparison between the Gaussian approximations with and without time correlation at \( N = 128, M = 15, \epsilon_{\text{max}} = 0.5, \) and \( \text{Eb}/N_0 = 7.45 \text{ dB} \) is shown for full \( (w = 64) \) and \( w = 3 \) cases.

\[
\langle z_t^i u_t^i \rangle = 2 \sum_{j \in \partial_i(\omega)} \langle \mathbb{I}(\hat{X}_{t+1}^j \neq \hat{X}_t^j) J_{t}^{2j} \hat{X}_t^j (X_j - \hat{X}_t^j) \rangle \\
= \sum_{j \in \partial_i(\omega)} J_{t}^{2j} (1 - C_{t+1}^{t+1}) (m_t^j - 1) \\
\equiv -\Theta_t^i, \quad (B.4)
\]

where we introduced an auto-correlation function \( C_{t+1}^{t+1} = \langle \hat{X}_{t+1}^j \hat{X}_t^j \rangle \), and with this quantity the probability that \( \hat{X}_{t+1}^j \neq \hat{X}_t^j \) is expressed as \( (1 - C_{t+1}^{t+1})/2 \). In equation (B.4), the independence of \( \mathbb{I}(\hat{X}_{t+1}^j \neq X_t^j) \) from \( \hat{X}_t^j \) is assumed. To summarize, the time evolution of the variance \( \Sigma_{t}^{2} \) is given by

\[
\Sigma_{t+1}^{2} = \Sigma_{t}^{2} + 2 \sum_{j \in \partial_i(\omega)} J_{t}^{2j} (1 - C_{t+1}^{t+1}) m_t^j. \quad (B.5)
\]

On the other hand, the auto-correlation function \( C_{t+1}^{t+1} \) is given as

\[
C_{t+1}^{t+1} = \langle \text{sgn} [(J_{t}^i X_i + z_t^i)(J_{t}^i X_i + z_{t+1}^i)] \rangle \\
= \int Dz_1 Dz_2 Dz_3 \text{sgn} \left[ (J_{t}^i + \sqrt{\Theta_t^i} z_1 + \sqrt{\Sigma_{t}^{2} - \Theta_t^i} z_2) \right] \times \left( J_{t}^i + \sqrt{\Sigma_{t}^{2} - \Theta_t^i} z_3 + \sqrt{\Sigma_{t+1}^{2} - \Sigma_{t}^{2} + \Theta_t^i} z_2 \right), \quad (B.6)
\]

where we replaced \( z_t^i \) and \( z_{t+1}^i \) with three independent standard Gaussian random variables \( z_1, z_2, z_3 \) so as to satisfy the relationships \( \langle z_t^{2i} \rangle = \Sigma_{t}^{2}, \langle z_{t+1}^{2i} \rangle = \Sigma_{t+1}^{2}, \) and \( \langle z_t^{2i} z_{t+1}^{2i} \rangle = \Sigma_{t}^{2} - \Theta_t^i \). Further, the translational invariance allows us to replace \( m_t^i, \Sigma_t^i, \) and \( C_{t+1}^{t+1} \) with macroscopic variables \( m_t^i, \Sigma_t^i, \) and \( C_{t+1}^{t+1} = (2N)^{-1} \sum_{i=1}^{2N} C_{i}^{t+1} \), respectively.

A higher order description of the macroscopic variable \( m_t \) is obtained by inserting the time-correlated variance, equation (B.5), into equation (A.5). At each time step \( t \), the auto-correlation \( C_{t+1}^{t+1} \) and variance \( \Sigma_{t}^{2} \) are determined in a self-consistent manner from equations (B.5) and (B.6) for a given set of \( m_t^i, \Sigma_t^i, \) and \( C_{t+1}^{t+1} \).
A mean-field approach for an intercarrier interference canceller for OFDM

The time evolution of BER for the higher order description is shown in figure B.1 at $N = 128$, $M = 15$, $\epsilon_{\text{max}} = 0.5$, and $E_b/N_0 = 7.45$dB. This figure indicates that the fit to the experimental data is slightly improved by taking into account the time correlation, but its significance is reduced as $\omega$ is set smaller. Equation (B.4), which describes the strength of the time correlation, is given by a summation with respect to $j \in \partial_i(\omega)$, and hence it grows as $\omega$ increases. This means that when $\omega$ is small enough, there is little room for improvement by taking into account the time correlation.

References

[1] 2010 Robust Demand for Mobile Phone Service will Continue UN News Centre, 15 February
[2] Tanaka T, 2001 Europhys. Lett. 54 540
[3] Kabashima Y, 2003 J. Phys. A: Math. Gen. 36 11111
[4] Tanaka T and Okada M, 2005 IEEE Trans. Inform. Theory 51 700
[5] Efraim H, Peleg Y, Kanter I, Shental O and Kabashima Y, 2010 Phys. Rev. E 82 060101
[6] Chang R W and Gibby R A, 1968 IEEE Trans. Commun. COM-16 529
[7] Zhao Y and Haggman S-G, 1996 IEEE 46th Vehicular Technology Conf. ‘Mobile Technology for the Human Race’ vol 3, p 1564
[8] Zhao Y and Haggman S-G, 2001 IEEE Trans. Commun. 49 1185
[9] Li J, Letaief K B, Cheng R S and Cao Z, 2001 IEEE Int. Conf. on Communications vol 4, p 1152
[10] Varanasi M K and Aazhang B, 1990 IEEE Trans. Commun. 38 509
[11] Gorokhov A and Linnartz J-P, 2004 IEEE Trans. Commun. 52 572
[12] Hou W-S and Chen B-S, 2005 IEEE Trans. Wire. Commun. 4 2100
[13] Molisch A F, Toeltsch M and Vermani S, 2007 IEEE Trans. Veh. Technol. 56 2158
[14] Schniter P, 2004 IEEE Trans. Sign. Proc. 52 1002
[15] Okada M, 1995 Nural Netw. 8 833