ROBUST GRAPH-FILTER IDENTIFICATION WITH GRAPH DENOISING REGULARIZATION

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ABSTRACT

When approaching graph signal processing tasks, graphs are usually assumed to be perfectly known. However, in many practical applications, the observed (inferred) network is prone to perturbations which, if ignored, will hinder performance. Tailored to those setups, this paper presents a robust formulation for the problem of graph-filter identification from input-output observations. Different from existing works, our approach consists in addressing the robust identification by formulating a joint graph denoising and graph-filter identification problem. Such a problem is formulated as a non-convex optimization, suitable relaxations are proposed, and graph-stationarity assumptions are incorporated to enhance performance. Finally, numerical experiments with synthetic and real-world graphs are used to assess the proposed schemes and compare them with existing (robust) alternatives.

Index Terms— Robust graph signal processing, graph denoising, robust filter identification.

1. INTRODUCTION

Data is becoming not only pervasive but also more heterogeneous and intricate. One way to handle the more complex structure present in many contemporary applications is to model the structure of the data as a graph and, then, using the graph to analyze, process, and learn from the data. That is precisely the goal of graph signal processing (GSP) [1-4], which in recent years has made contributions in several tasks involving signals and filters defined over irregular domains. Those include sampling and reconstruction of graph signals [5,6], graph-signal denoising [7-9], graph-filter design [10-12] or graph-filter identification [13-15], to name a few. Applications range from setups where the graph is explicit—as in communication, power and social networks [3,16]—to those where the network is implicit and must be learned from the data itself—as in gene-regulatory data or brain networks [16,17], motivating the development of new (GSP-based) algorithms that learn the graph [18,19].

Since GSP is a relatively recent area of research, it is not surprising that (almost all) existing works assume that the graph topology is perfectly known, focusing on how to leverage the graph structure to process and learn from the data. Nonetheless, in many practical cases the graph contains errors, including perturbations and observation noise in setups dealing with explicit networks (e.g., link failures in a power or wireless network [20]) or imperfections associated with the limitations of the method used to learn the graph from the data in implicit networks (e.g., the thresholding operation employed in correlation networks [18]). Equally important, it is clear that those errors, if ignored, will have a negative impact on the performance of the subsequent GSP tasks.

Despite their theoretical and practical relevance, the number of works dealing with robust GSP approaches is limited [21,24]. Using a small perturbation analysis, [21] first studies how link imperfections affect the spectrum of the graph Laplacian and then proposes a Bayesian framework. The focus of [22] is on postulating (graphon-based) perturbation models and analyzing how those perturbations affect graph-signal operators. Differently, [23] combines structural equation models (SEMs), which can be viewed as a particularization of the GSP framework, with total least squares (TLS) for graph signal inference while also inferring the perturbations of the observed graph. The limited number of works is in part due to the fact that, while many GSP algorithms are based on spectral tools, characterizing how the errors on the graph translate to its spectrum is a challenging task [21,24].

Motivated by the previous discussion, this paper presents a (non-spectral) robust GSP approach for the problem of identifying a graph filter from input-output pairs given an imperfect (perturbed) graph. Apart from its theoretical interest, graph-filter identification has been shown to be practically relevant in, e.g., explaining the structure of real-world datasets [14,25] as well as understanding the dynamics of network diffusion processes [4,13,14]. We approach the robust estimation by recasting the problem as a graph filter identification augmented with a graph-denoising regularizer, solving the resultant optimization jointly. The proposed approach works entirely in the vertex-domain, bypassing the challenges associated with robust spectral graph theory, and yields an a posteriori estimate of the graph. Since the joint estimation leads to a non-convex optimization, we first propose suitable relaxations along with low-complexity algorithms, and then, we modify the schemes to incorporate additional graph-structure potentially present in the data. Finally, we provide numerical experiments testing the proposed schemes with synthetic and real-world graphs and comparing them with existing alternatives.

2. PRELIMINARIES

Let \( \mathcal{G} = (\mathcal{N}, \mathcal{E}) \) denote a (possibly directed) graph, where \( \mathcal{N} \) is the set of nodes, with cardinality \( N \), and \( \mathcal{E} \) is the set of edges, with \((i,j) \in \mathcal{E} \) if \( i \) is connected to node \( j \). The set \( \mathcal{N}_i := \{ j | (j,i) \in \mathcal{E} \} \) denotes the incoming neighborhood of node \( i \). For a given \( \mathcal{G} \), the adjacency matrix \( \mathbf{A} \in \mathbb{R}^{N \times N} \) is sparse with non-zero elements \( A_{ij} \) if and only if \((j,i) \in \mathcal{E} \). If \( \mathcal{G} \) is unweighted, the elements \( A_{ij} \) are binary; if not, the value of \( A_{ij} \) captures the strength of the link from \( j \) to \( i \). The focus of this paper is not on \( \mathcal{G} \), but on modeling data as graph signals defined on the nodes of \( \mathcal{G} \). Such signals can be represented as a vector \( \mathbf{x} = [x_1, \ldots, x_N]^T \in \mathbb{R}^N \) where the \( i \)th entry represents the signal value at node \( i \). Since the signal \( \mathbf{x} \) is defined on the graph \( \mathcal{G} \), the core assumption in GSP is that the

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The graph-shift operator (GSO). The GSO $S$ is defined as an $N \times N$ matrix whose entry $S_{ij}$ can be non-zero only if $i = j$ or $(i, j) \in \mathcal{E}$. Common choices for $S$ are $A$ and the graph Laplacian $L$, which is defined as $L := \text{diag}(A) - A$ [14]. The GSO accounts for the topology of the graph and, at the same time, represents a linear transformation that can be computed locally. Specifically, if $y = [y_1, \ldots, y_N]^T$ is defined as $y = Sx$, then node $i$ can compute $y_i$ provided that it has access to the values of $x_j$ at its neighbors $j \in \mathcal{N}_i$.

We assume that $S$ is diagonalizable so that there exists an $N \times N$ matrix $V$ and a diagonal matrix $\Lambda$ such that $S = V\Lambda V^{-1}$. The matrix $V^{-1}$ is adopted as the Graph Fourier Transform (GFT) for filters, and $V$ represents the graph Laplacian, which depends on the topology of the graph and, at the same time, represents a linear operator representing the frequency response of the graph filter $H$. The output of the graph filter is $y = Hx = \sum_{k=0}^K h_k(S^kx)$, where $S^kx$ can be viewed as a version of $x$ diffused across a k-hop neighborhood and $h_k$ are the coefficients of the linear combination [11].

Graph stationarity. A random graph signal $x$ with zero mean and covariance $C_x = \mathbb{E}[xx^T]$ is said to be stationary in the symmetric $S$ if its covariance matrix $C_x$ is diagonalized by $V$, the eigenvectors of $S$ [14]. Equivalent[1] a random graph signal is defined to be stationary in $S$ if $C_x$ can be written as a (positive-semidefinite) matrix polynomial of $S$.

Robust Structural Equation Models (SEMs). Given a graph signal $y$, sparse SEMs assume that the value $y$ at a particular node (say the $i$th one) can be explained based on: a) the values of $y$ at $\mathcal{N}_i$ and b) the value of an exogenous input signal $x$ at node $i$. More formally, let the $N \times M$ matrices $X = [x_1, \ldots, x_M]$ and $Y = [y_1, \ldots, y_M]$ collect $M$ exogenous and endogenous graph signals. Then, SEMs postulate that the following relation holds $y = Ay + Bx$, with $A_{ij} = 0$ and $B$ being a diagonal matrix whose $i$th entry $B_i$ accounts for the influence of $X_i$ on $Y_i$. Recently, [23] combined SEMs with TLS (TLS-SEMs) to account for possible perturbations on the graph topology. With $A$ denoting the perturbed adjacency matrix, $\Delta = \hat{A} - A$ denoting the perturbations, and setting $B = I$, the TLS-SEMs approach postulates that the graphs signals in $y$ satisfy

$$y = (\hat{A} - \Delta)Y + X = (I - \Delta + \Delta)^{-1}X.$$  

Leveraging [2] and assuming that only noisy observations of $y$ (denoted as $\tilde{y}$) are available, TLS-SEMs address the robust estimation of $A$ and $Y$ by formulating a joint optimization over $Y$ and $\Delta$, enforcing [2] and promoting sparsity on $\Delta$ [23]. From the point of view of this paper, it is important to remark that while SEMs are popular in the non-GSP (statistics) literature, the (unperturbed version of the) problem in [2] can be reformulated using GSP tools by setting $S = A$ and defining the filter $H_{SEM} = (I - S)^{-1}$, so that $Y = H_{SEM}X$.

3. ROBUST GRAPH-FILTER IDENTIFICATION

In this section, we present our “robust graph-filter identification with graph denoising regularization (RFI)” approach. In contrast to existing approaches that try to recast the filter identification problem in the spectral domain, we address the design in the vertex domain and leverage the fact that graph filters are matrices that commute with the GSO. As will be clear next, formulating the problem in the vertex domain not only leads to more tractable formulations but also provides a natural way to account for the imperfections on the graph. To be mathematically precise, suppose that $M$ pairs of graph input-output signal pairs are available and use $X = [x_1, \ldots, x_M]$ to denote the $N \times M$ matrix collecting the inputs and $Y = [y_1, \ldots, y_M]$ the one collecting the outputs. We further assume that the $m$th output $y_m$ is related to the $m$th input $x_m$ via the graph filter represented by the $N \times N$ matrix $H$, which is a polynomial of the true GSO $S$. Generative models of the form $y = Hx$ (with, e.g., the filter $H$ being bandlimited, the inputs being white, or the inputs being sparse) have been shown to account accurately for several types of network diffusion processes as well as a number of real-world datasets [2]. With this notation at hand, we are ready to formulate our recovery approach.

Given $S$, a perturbed version of the true GSO $S$, and the (possibly noisy) input-output signals in $X$ and $Y$, we aim to estimate the graph filter $H$ that best explains the observations under the assumption that the graph perturbations are small. This is achieved by solving the RFI problem

$$\min_{S \in \mathcal{H}} \|Y - HX\|_F^2 + \lambda d(S, \hat{S}) + \beta \|S\|_0 \quad \text{s. t. } SH = HS,$$  

where $\mathcal{H}$ is the set of admissible GSOs (e.g., Laplacian matrices with zero-row sum, or adjacency matrices with no self-loops); and $d(\cdot, \cdot)$ is a distance function that measures the similarity between $S$ and $\hat{S}$ and must be chosen based on prior knowledge on the nature of the graph perturbations. The first term in the objective accounts for noise in the observations and/or modeling inaccuracies, while the $\ell_0$ norm in the third term accounts for the fact that the true GSO is sparse. Finally, the constrain $\text{SH} = \text{HS}$ captures the fact that $H$ is a polynomial of the true $S$. The main novelties of our approach are twofold.

• First, while most works formulate the filter recovery in the spectral domain, we put forth a vertex-domain formulation. Suppose for now that the GSO is free of errors. Since $H$ has the form in (1), a natural formulation of the graph identification problem is to find the $h = [h_0, \ldots, h_K]^T$ minimizing $\|Y - \sum_{k=0}^K h_kS^kX\|_F^2$ or, equivalently, that minimizing $\|Y - V\text{diag}(\Psi h)V^{-1}X\|_F^2$. However, those formulations involve high-order polynomials (either of matrix $S$ or of the entries of $A$ present in $\Psi$) that oftentimes give rise to numerically unstable problems [4]. As a result, the graph-identification task is typically reformulated in the spectral domain as finding the $h$ minimizing $\|Y - V\text{diag}(\Psi h)V^{-1}X\|_F^2$. Differently, thanks to the constraint $SH = HS$ in (3), we bypass the need to compute $V$ without facing the numerical problems of classical vertex-based approaches.

• Second, rather than considering the graph filter as the only optimization variable, we also optimize over the actual graph. From a practical point of view, there are many applications where denoising the graph is as useful as identifying the generative graph filter. From a technical point of view, considering the true graph for spectral-based approaches is challenging. It can be rigorously shown that small perturbations in $S$ can lead to significant perturbations in $V^{-1}$ [21,24] and, even when not large, characterizing how

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[1] A small technical condition must hold for these two statements to be equivalent; see [25].
those perturbations translate to the eigenvectors $V$ and incorporating
that into the optimization is not an easy task. The consideration of the
constraint $SH = HS$ circumvents these problems and opens
the door to introduce prior knowledge of the nature of the perturba-
tions by tailoring the selection of the matrix distance $d(\cdot, \cdot)$.

**Modeling graph perturbations.** A worth discussing topic is the
postulation and analysis of graph-perturbation models that combine
practical relevance with analytical tractability \[22\]. Unfortunately,
space limitations prevent us from engaging in that discussion and
we limit ourselves to describe one model, which will motivate the
formulation in the next section. With $\Delta = S - S$ denoting the per-
turbation matrix, we focus on perturbations that create and destroy
Bernoulli distribution with the creation and destruction probabili-
ties more generically, the perturbation can be rendered depend-
tent across

\[\text{Setting } 3.1. \text{ Algorithmic approaches} \]

Setting $d(S, S^*)$ in \[3\] to $||S - S||_0$, so that perturbations are infre-
quent and independent across links, yields

\[
\min_{S \in S, H} \|Y - HX\|^2_F + \lambda \|S - \hat{S}\|_1 + \beta \|S\|_1 + \gamma \|SH - HS\|^2_F \quad \text{s. t. } SH = HS, \quad (4)
\]

which is a non-convex problem due to the presence of the $\ell_0$ norms
and the bilinear constraint. A simple approach to deal with the $\ell_0$
loss is to replace it with its $\ell_1$ convex counterpart (iterative re-
weighted formulations are also possible). Regarding the constraint
$SH = HS$, we relax it and rewrite it as a regularizer, which is better
suited to an alternating minimization approach. The relaxed version
of \[4\] can then be written as

\[
\min_{S \in S, H} \|Y - HX\|^2_F + \lambda \|S - \hat{S}\|_1 + \beta \|S\|_1 + \gamma \|SH - HS\|^2_F \quad (5)
\]

While still non-convex due to presence of the bilinear terms HS
and SH, the problem in \[5\] is amenable to an (efficient) alternating
optimization approach \[23\] where we iterate between two steps:

**Step 1: Filter Identification.** Given $\hat{S}$, the current estimate of the
GSO, we substitute $S = \hat{S}$ into \[5\] and solve \[5\] with respect to $H$.

This yields

\[
\hat{H} = \arg \min_H \|Y - HX\|^2_F + \gamma \|SH - HS\|^2_F, \quad (6)
\]

which is a least-squares problem whose closed-form solution is

\[
\text{vec}(\hat{H}) = (XX^T \otimes I + \gamma (\hat{S}\hat{S}^T \otimes I + I \otimes \hat{S}\hat{S}^T \hat{S} - \hat{S}\hat{S}^T \otimes S - \hat{S}\otimes \hat{S}))^{-1} \times (X \otimes I)\text{vec}(Y),
\]

where $\otimes$ is the Kronecker product and $I$ has size $N \times N$.

**Step 2: Graph Denoising.** Given $\hat{H}$, the current estimate of the
filter, we substitute $H = \hat{H}$ into \[5\] and solve \[5\] with respect to $S$.

This yields

\[
\hat{S} = \arg \min_{\hat{S} \in S} \lambda \|S - \hat{S}\|_1 + \beta \|S\|_1 + \gamma \|SH - HS\|^2_F, \quad (7)
\]

which, provided that $S$ is convex (e.g., $S$ being the set of symmetric
adjacency matrices with $A_{ii} = 0$), can be handled using efficient
variants of the lasso algorithm \[29\].

The alternating algorithm is initialized with $\hat{S} = S$ and then
iterations between steps 1 and 2 are run until convergence (to a lo-
cal optimum) is obtained \[28\]. The number of iterations required to
converge will be sensitive to the value of $\gamma$. When $\gamma$ is very close
to zero, the two problems decouple and the solution converges quickly
to that of two separate problems [cf. \[6\] and \[7\] with $\gamma = 0$]. If $\gamma$
is too large, the filter $\hat{H}$ obtained in the first iteration will be an (al-
most exact) polynomial of $S$ so that the algorithm will converge in
one iteration to the same solution as that of the (non-robust) design
that assumes that the true GSO is $S$ [cf. \[4\] with $S = \hat{S}$]. Hence,
$\gamma$ must be tuned so that the algorithm can explore a larger set of
solutions. In this context, adoption of schemes that start with a small
$\gamma$ (encouraging exploration during the warm-up phase) and then in-
crease $\gamma$ as the iteration index grows (guaranteeing that the final $H$
is a polynomial of $S$) is a suitable alternative for the setup at hand.

**3.2. Additional structure: Stationary observations**

Up to this point, we have used the fact that $Y$ and $X$ are related via
a polynomial of the GSO. However, the input/output signals can ex-
hibit additional properties that depend on the supporting graph. No-
table examples include signals being graph-bandlimited \[1\], diffused
sparse graph signals \[13\, 25\] or graph stationary. If that is indeed the
case, this additional information can be incorporated into the opti-
mization, opening the door to an enhanced recovery performa-
ance. Due to space limitations, we focus our discussion on the case
where the observations are stationary in the symmetric GSO $S$. Since
the covariance matrix of a graph stationarity signal can be writ-
ten as a polynomial of the GSO (cf. Section \[3\]), we update \[5\] as

\[
\min_{S \in S, H} \|Y - HX\|^2_F + \lambda \|S - \hat{S}\|_1 + \beta \|S\|_1 + \gamma \|SH - HS\|^2_F \quad (8)
\]

with $CV$ being $C_X$ the (exact or estimated) covariances of $Y$ and
$X$, respectively. If the covariance is perfectly known, then the cor-
responding $\epsilon$ can be set to zero. Differently, when the covariance
is estimated from the observations the value of $\epsilon$ must be selected based
on the quality of the estimator (accounting, e.g., for the number of
available observations). The constraints in \[8\] involve the true GSO.
This implies that, when an alternating optimization is adopted, such
constraints must be considered in the graph-denoising step. Altema-
tively, since $C_Y$, $C_X$ and $H$ are all polynomials of $S$, the equalities

Fig. 1: Estimation performance for “Test case 1”. Normalized me-
dian error of $\hat{H}$ for different levels of noise in the observed signals.
“FI” stands for the filter-identification algorithm ignoring graph per-
turbations, “RFI iter” is the two-step iterative algorithm solving \[5\],
and “RFI-D” is a low-complexity approximation to \[8\].

| Normalized noise power | Median error |
|------------------------|-------------|
| 0.05                   | 0.25        |
| 0.1                    | 0.15        |
| 0.15                   | 0.10        |
| 0.20                   | 0.05        |
| 0.25                   | 0.01        |

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where the observations are stationary in the symmetric GSO $S$. Since
the covariance matrix of a graph stationarity signal can be written as a
polynomial of the GSO (cf. Section \[3\]), we update \[5\] as

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\min_{S \in S, H} \|Y - HX\|^2_F + \lambda \|S - \hat{S}\|_1 + \beta \|S\|_1 + \gamma \|SH - HS\|^2_F \quad (8)
\]

with $CV$ being $C_X$ the (exact or estimated) covariances of $Y$ and
$X$, respectively. If the covariance is perfectly known, then the cor-
responding $\epsilon$ can be set to zero. Differently, when the covariance
is estimated from the observations the value of $\epsilon$ must be selected based
on the quality of the estimator (accounting, e.g., for the number of
available observations). The constraints in \[8\] involve the true GSO.
The code, along with additional experiments, can be found in the repository [https://git.io/JIGQN](https://git.io/JIGQN).

**Test case 1.** We generate 100 ER graphs. For each graph we generate a graph filter with random coefficients drawn from a uniform distribution in the interval $[-1, 1]$, and for each $(S, H)$ pair, $M = 10$ observed signals following the graph-filter generative model assumed in Section 3. The inputs $X$ are generated as zero mean white Gaussian signals. The random vector $h$ is scaled to have unit norm and length $K = 4$. Signal observations are assumed to be corrupted by additive white Gaussian noise (AWGN). Fig. 1 plots the median normalized error of the estimated graph filters as the power of the observation noise increases. "FT" stands for the filter identification algorithm that ignores graph perturbations, equivalent to $S = S_0$; "RFI iter" is the two-step iterative algorithm proposed to solve (8), and "RFI-D" implements (8) by first estimating $S$ with $S = S_0$, "RFI iter" is the two-step iterative algorithm proposed to solve (8), and "RFI-D" implements (8) by first estimating $S$ with $S = S_0$; "RFI iter" is the two-step iterative algorithm proposed to solve (8), and "RFI-D" implements (8) by first estimating $S$ with $S = S_0$, "RFI iter" is the two-step iterative algorithm proposed to solve (8), and "RFI-D" implements (8) by first estimating $S$ with $S = S_0$, "RFI iter" is the two-step iterative algorithm proposed to solve (8), and "RFI-D" implements (8) by first estimating $S$ with $S = S_0$. The elimination of the bilinear terms renders (8) convex and separable in $S$ and $H$, decreasing the complexity and converging in two steps.

To gain insights, we start by representing $\|\hat{S} - S\|_1/(N(N-1))$, the error between the estimated and the actual graph, for the different algorithms and values of $M$; see Fig. 2(a). The results confirm that, as $M$ grows, the robust methods are able to find a more accurate $\hat{S}$. Regarding "FT", where no graph denoising is implemented, the normalized error coincides with the link perturbation probability. This enhanced graph-estimation performance is expected to help in the filter identification task, as confirmed in Fig. 2(b). The results reveal that as $M$ increases, the joint influence of the enhanced estimation of $S$ and the larger number of observations promote an improved estimation of $H$. Moreover, the two low-complexity schemes that leverage stationarity achieve good results. As expected, "RFI-D", which estimates $H$ relying on both $S$ and $C_Y$, outperforms "RFI-R", which only relies on $C_Y$.

**Test case 3.** Using 100 ER graphs with 200 observed signals each and setting the normalized power of the noise corrupting the observations to 0.1, this test case compares the estimation performance of the proposed algorithms with that of TLS-SEMs as the probability of perturbing a link increases; see Fig. 2(c). Two sets of input-output signals are considered: one generated using the SEMs in (8) (suffix "SEM" in the legend) and another one generated as in the previous experiments (suffix "H"). For the cases where the signals follow the SEMs, "TLS-SEM" outperforms our algorithms when the link perturbation probability is small (this is expected because our algorithms need to learn the particular form of the generating filter), but the results get similar to those of "RFI-R" and "RFI-D" as this probability increases. Differently, when data follows the model "H" the algorithms need to learn the particular form of the generating filter), but the results get similar to those of "RFI-R" and "RFI-D" as this probability increases. Differently, when data follows the model "H" the algorithms need to learn the particular form of the generating filter), but the results get similar to those of "RFI-R" and "RFI-D" as this probability increases. Differently, when data follows the model "H" the algorithms need to learn the particular form of the generating filter), but the results get similar to those of "RFI-R" and "RFI-D" as this probability increases. Differently, when data follows the model "H" the algorithms need to learn the particular form of the generating filter), but the results get similar to those of "RFI-R" and "RFI-D" as this probability increases.
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