Magnetic-field driven delocalization of in-gap-state BdG quasiparticles in a quasi-two-dimensional Fe-based superconductor

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Role of order-parameter fluctuations in Anderson transition of quasiparticles is a central question in revealing the microscopic mechanism of Anderson localization in the presence of spontaneous symmetry breaking. However, even correlations between the statistical information of such collective-mode fluctuations and that of (single) quasiparticle excitations have not been addressed well both experimentally and theoretically. Based on scanning tunneling microscopic (STM) measurements, we investigate how Bogoliubov-de Gennes (BdG) quasiparticles become Anderson-delocalized as a function of applied magnetic fields in a quasi-two dimensional disordered Fe-based superconductor. Two types of anomalous multifractal spectra from multiple moments of both local pairing gaps and local coherent peaks suggest that superconducting fluctuations play an essential role in delocalization of in-gap-state BdG quasiparticles. All these experiments are explained by real-space Hartree-Fock-BCS-Anderson simulations for superconductors under external magnetic fields in addition to our renormalization group analysis with superconducting random fluctuations. The present study not only proposes a novel measure to investigate Anderson transition of quasiparticles with symmetry breaking but also suggests a novel mechanism for delocalization of quasiparticle excitations in quasi-two dimensions beyond previously discussed well-known mechanisms.
Researches on superconductor to insulator transitions driven by disorders have deepened our understanding on quantum coherence in dynamics of Cooper pairs and Bogoliubov-de Gennes (BdG) quasiparticles\textsuperscript{1-19}. Generally speaking, two path ways of phase-disordered driven\textsuperscript{19,20} vs. amplitude-fluctuations governed\textsuperscript{30,31} have been proposed for these quantum phase transitions although strong inhomogeneity appears to mix these two path ways. The phase-disordered driven mechanism suggests Anderson localization of preformed Cooper pairs in the resulting insulating phase, which allows a pseudogap but without coherent peaks in the quasiparticle spectrum. On the other hand, the amplitude-fluctuations governed case results in Anderson localization of quasiparticles with gap closing, where breaking of Cooper pairs originates from enhanced effects of renormalized Coulomb interactions due to Anderson localization.

The present study focuses on the superconducting phase itself instead of the superconductor-insulator transition. More precisely, we investigate an Anderson transition of BdG quasiparticles inside the superconducting gap, applying magnetic fields to a disordered Fe-based superconductor\textsuperscript{32-34}. As far as we know, these in-gap quasiparticle states are not well generated by increasing the disorder strength only, confirmed by most real-space Hartree-Fock-type simulations\textsuperscript{26,27,29} including ours. On the other hand, time-reversal symmetry breaking perturbations give rise to strong pair breaking effects\textsuperscript{4,6}, responsible for such in-gap BdG quasiparticle states. In this respect, applying magnetic fields to quasi-two dimensional disordered superconductors serves as quite a unique opportunity to investigate the Anderson transition of in-gap-state quasiparticles within the superconducting phase. In particular, we show that the two path ways of phase-disordered driven\textsuperscript{19,20} vs. amplitude-fluctuations governed\textsuperscript{30,31} mechanisms are involved simultaneously with the Anderson transition of BdG quasiparticles inside the superconducting gap.

Anderson transitions of BdG quasiparticles have been studied for more than two decades, which serves as a central building block in the ten-fold way classification scheme of topological matter\textsuperscript{35,36}, referred to as the Altland-Zirnbauer symmetry class\textsuperscript{37}. Essential simplification in the ten-fold way classification scheme is that the superconducting order parameter remains to be fixed in the Anderson transition, where the dynamics of BdG quasiparticles is described by a non-interacting tight-binding lattice model. In this case the delocalization of in-gap-state BdG quasiparticles is allowed only at the zero energy, where other in-gap quasiparticle states are all localized in two dimensions. In this study, we observe the delocalization phenomenon even away from the zero energy and reveal importance of Cooper-pair fluctuations in the Anderson transition of in-gap-state BdG quasiparticles.

Resorting to scanning tunneling microscopic (STM) measurements, we obtain statistical information from both local density of states and local superconducting gaps, schematically shown in Fig. 1a. First, we measure both distribution functions for the local density of states $P(\rho; \mathbf{H})$ and the local pairing gaps $P(\Delta; \mathbf{H})$, and reveal how they evolve as a function of the applied magnetic field. Furthermore, we obtain the distribution function of the coherent-peak height $P(Z_\Delta; \mathbf{H})$ to investigate the localization property of Cooper pairs as discussed in the first paragraph. Second, we ‘evaluate’ the multifractal spectrum based on these measurements\textsuperscript{38-40}. In particular, we introduce a novel multifractal spectrum $f_\Delta(\alpha; \mathbf{H})$ given by multiple moments of the local pairing gaps in addition to the conventional multifractal spectrum $f_\rho(\alpha; \mathbf{H})$ of the local density of states. We also discuss the multifractal spectrum $f_{Z_\Delta}(\alpha; \mathbf{H})$ of the coherent-peak height. These two ‘anomalous’ multifractal spectra
of \( f_A(\alpha; H) \) and \( f_{2A}(\alpha; H) \) with the conventional multifractal spectrum \( f_\mu(\alpha; H) \) suggest how the in-gap-state BdG quasiparticles become delocalized as a function of the external magnetic field, shown in the schematic phase diagram Fig. 1a. We claim that this study proposes a novel mechanism for delocalization of quasiparticle excitations in quasi-two dimensions beyond the two well-known mechanisms of the ten-fold way classification without electron correlations\(^{35,36,41}\) and the appearance of strong ferromagnetic fluctuations\(^{42,43}\).

To verify the magnetic-field driven delocalization of the in-gap-state BdG quasiparticles, we have taken local spectroscopic maps of the disordered quasi-2D Fe-based superconductor \( \text{Sr}_2\text{VO}_3\text{FeAs} \) at 0 T and 7 T. These maps are taken at 4.8 K well below the superconducting \( T_c \) of \( \sim 30 \) K, and the maximum magnetic field 7 T is well below \( H_{c2} (\gg 10 \) T) at 4.8 K\(^{32,34}\). Due to the significant separation of the adjacent FeAs layers by insulating \( \text{Sr}_2\text{VO}_3 \) bi-layers, as evidenced by the conductance in \( c \) direction 1/100 times smaller than those in the ab plane\(^{33,34}\), we can model the system with an ideal 2D FeAs superconductor, with the tunneling matrix effect of the thick \( \text{Sr}_2\text{VO}_3 \) layer contributing to the overall asymmetry of the tunneling spectra. The topograph simultaneously taken with the 0 T map is shown in Fig. 1b and Ext. Data Fig. 1. The zero-bias conductance map at 0 T (Fig. 1e) shows strong long-range modulations not visible in the same-area map taken at 7 T (Fig. 1f) with an identical tip. The field-dependence of the tunneling spectra is confined near the Fermi level as shown in Figs. 1h-i. The significantly enhanced correlation length of the zero-bias tunneling conductance at zero field (Fig. 1g), its confinement within a narrow energy range \( |E| < 3 \) meV (Fig. 1h), and its disappearance by application of the magnetic field (Fig. 1i) indicate that almost localized or critical in-gap-state BdG quasiparticles without magnetic fields become delocalized by such applied magnetic fields. Below, we discuss how this delocalization phenomenon is correlated with strong fluctuations of local pairing gaps.

To reveal the mutual correlation between the zero-bias conductance and the local pairing gap, we extract out the local pairing-gap map and the coherence peak-height map as shown in Figs. 2a-f and calculate the cross-correlation of every map with respect to the zero-bias conductance map (Fig. 2a) as shown in the insets of Figs. 2a-f. The coherence peak-height map has been discussed to play the role of the local Cooper-pair density map\(^{18,29,44}\). Excluding the QPI modulation length scale visible in Figs. 2a-f as concentric rings, there seems to be two length scales in the cross-correlation of the zero-bias conductance map and the local pairing-gap map (Fig. 2b), where they are negatively correlated in short range (< 2 nm) while positively correlated in long range (>2 nm). On the other hand, the coherence peak-height map shows negative correlation with the zero-bias conductance map in all length scales (Fig. 2c). The negative short-range correlation between the zero-bias conductance map and the local pairing-gap map looks natural to indicate local pair breaking effects. The positive long-range correlation seems consistent with the increased electron-phonon coupling coefficient near O-vacancy defects\(^{45}\). The O-vacancy defects tend to increase not only the local pairing-gap map but also the zero-bias conductance map (Ext. Data Fig. 1) due to its positive charge attracting negatively charged carriers.

It is one of our main experimental observations that the distributions of the local pairing-gap map (Fig. 2e) and the coherence peak-height map (Fig. 2f) are all anti-correlated with application of magnetic fields, while the distribution of the zero-bias conductance map (Fig. 2d) tends to increase with the magnetic field. These anti-correlation effects between the zero-bias conductance and the local pairing gap with respect to applied magnetic fields can be more quantified by investigating both their distribution functions (Figs. 2g-i) and corresponding
multifractal spectra (Figs. 2j-l). Although the shape of the distribution function for the zero-bias conductance does not change much with applied magnetic fields, it is shifted by the magnetic field to show clear enhancement of the LDOS near the zero energy (Fig. 2g). On the other hand, the distribution function of the local pairing gap shows that the distribution peak around 3meV is suppressed to move toward the zero energy with respect to external magnetic fields, regarded to be pair breaking effects (Fig. 2h). Accordingly, the coherence peak-height distribution function displays that these pair breaking effects give rise to strong suppression of the coherent peak, where the peak-height distribution function is more pronounced at zero height (Fig. 2i).

Based on these distribution functions, we obtain three types of multifractal spectra as a function of the external magnetic field (Figs. 2j-l). The conventional multifractal spectrum \( f_{\rho}(\alpha) \) of the zero-bias conductance shows enhanced metallicity with respect to applied magnetic fields as shown in Fig. 2j, where it becomes narrower. On the other hand, the anomalous multifractal spectrum \( f_{\Delta}(\alpha) \) of the local pairing gap indicates stronger fluctuations with respect to applied magnetic fields as shown in Fig. 2k, where it becomes broader. The other anomalous multifractal spectrum \( f_{\rho}(\alpha) \) of the coherence peak height also suggests stronger inhomogeneity of local pairing gaps with respect to applied magnetic fields as shown in Fig. 2l, implying more ‘localized’ Cooper pairs. These multifractal spectra imply that the magnetic-field driven delocalization phenomenon of the in-gap-state BdG quasiparticles is strongly correlated with the magnetic-field driven ‘localization’ phenomenon of the local superconducting gap.

One may point out that it is not clear to observe the signature of vortices, where the LDOS near the zero energy is more or less homogeneously distributed and enhanced by external magnetic fields. We suspect that the penetration depth of this quasi-two dimensional superconductor is much longer than the coherence length of the superconducting order parameter\(^{46}\). This gives rise to overlapping between vortices, being responsible for rather uniformly distributed and enhanced LDOS near the zero energy. We would like to refer some discussions involved with this issue to ref. \([47]\).\(^{47}\)

To confirm the strong anti-correlation between the dynamics of the in-gap-state BdG quasiparticles and that of the local superconducting order parameter, we perform the real-space Hartree-Fock-BCS-Anderson simulation\(^{48,49}\) for two-dimensional disordered superconductors with s-wave pairing symmetry. We introduce the following BCS Hamiltonian defined on a two-dimensional square lattice,

\[
H = - \sum_{\langle i,j \rangle \sigma} [t_{ij} + (\epsilon_i - \mu)\delta_{ij}]c_{i\sigma}^\dagger c_{j\sigma} + \sum_i (\Delta_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \text{H. c.})
\]

where \( \Delta_i = U(c_{i\uparrow}c_{i\downarrow}) \) is the superconducting order parameter and \( \epsilon_i \) is an on-site random potential uniformly given in \((-V,V)\). Here, we consider the strong disorder regime of \( V = 4 \), where all single-particle states are localized in the normal state. We also introduce an effective repulsive interaction \( U_r \), which reduces the effective random potential as \( \epsilon_i \rightarrow \epsilon_i + \frac{U_r}{2} (n_i)_0 \) and enhances the density of electrons around the zero energy. See the supplementary material for more details. \( \mu \) is the chemical potential to adjust the occupation number. \( t_{ij} \) is an effective hopping parameter between nearest neighbor sites. Considering an external uniform magnetic field applied in z-axis, this hopping parameter is modified as
$$t_{ij}c_{i\sigma}^\dagger c_{j\sigma} + h. c. = -t[c_{i+1\sigma}^\dagger c_{i\sigma} e^{i\phi x} + h. c. + c_{i\sigma}^\dagger c_{i+1\sigma} e^{i\phi y} + h. c.].$$

The phase factors are given by $\Phi_x = \left(\frac{e}{\hbar}\right) \int_{x_i}^{x_f} A(x) dx$ and $\Phi_y = (e/\hbar) \int_{y_i}^{y_f} A(y) dy$. Here, we consider the symmetric gauge $A_0 = (-y\phi_0 p/q, x\phi_0 p/q, 0)$.

We find that the distribution of the local pairing amplitude is Gaussian-like (Fig. 3h) without external magnetic fields, where the mean value of $\langle \Delta \rangle$ is smaller than that of the clean superconducting system. Applying external magnetic fields, the Gaussian distribution of Cooper pairs becomes broader and distorted. In particular, there appears finite population of the zero local pairing amplitude. Furthermore, the sign change of the local pairing amplitude, i.e., the $\pi$ phase shift of the Cooper pair is observed in the Cooper-pair distribution function. See Fig. 3h. The local-pairing multifractal spectrum becomes broader (Fig. 3k), which indicates existence of their strong fluctuations driven by external magnetic fields. On the other hand, the LDOS of the in-gap quasiparticles are enhanced and become more homogeneous (Fig. 3g). The density distribution function is narrower and shifted to larger mean values of the LDOS. The multifractal spectrum shows weaker multifractality by the magnetic field (Fig. 3j). The wavefunction distribution function (Fig. 3i) and the corresponding multifractal spectrum (Fig. 3l) show qualitatively similar features with those of the LDOS. All these simulation results are consistent with our STM measurements, which indicates that magnetic-field driven ‘localization’ of Cooper pairs occurs at the same time with delocalization of in-gap-state BdG quasiparticles.

The magnetic-field driven delocalization of the in-gap-state BdG quasiparticles is verified by the scaling behavior of the critical exponent $\alpha_2$ of the inverse participation ratio (IPR)\(^1\). The scaling behavior negatively correlated with enhancement of the system size indicates that the in-gap-state BdG quasiparticles are Anderson-localized without external magnetic fields (Fig. 4a). On the other hand, we find that the IPR critical exponent increases as enlarging the system size, driven by external magnetic fields, which confirms metallicity of the in-gap BdG quasiparticles (Fig. 4b). Furthermore, we identify the mobility edge of the in-gap BdG quasiparticle state inside the superconducting gap as a function of the applied magnetic field, given by the scale invariance of the IPR exponent. See Fig. 4. We point out that the mobility edge of the in-gap BdG quasiparticle state is quite close to the coherent peak position. We suspect that the superconductor-insulator transition would occur when the mobility edge crosses the coherent peak position.

We further justify our real-space Hartree-Fock-BCS-Anderson simulation, investigating the energy dependence of the multifractal scaling exponent $\alpha_0$ (Figs. 4c-d) and the width of its spectrum (not shown here). Here, the former quantifies the uniformity of measures and the latter represents the strongness of multifractality\(^1\). According to our result, both quantities are positively correlated (in general not necessary). Both simulation and experimental results explain that the closer the zero energy, the weaker uniformity and the stronger multifractality are.

To reveal the role of spatial fluctuations of pairing fields in delocalization of in-gap BdG quasiparticle states, we perform a renormalization group (RG) analysis\(^{50}\). Considering the fact that the superconductor belongs to the class C in the tenfold way topological classification\(^{35,36}\), we write down a Lagrangian density of our minimal model as
$$L = \Psi^\dagger (-i \omega + v k \sigma_3 + \Delta_0 \sigma_z) \Psi + \Psi^\dagger \Delta_1 |k|^\alpha \sigma_3 \Psi + \sum_{\alpha=1}^{3} V_\alpha \Psi^\dagger \sigma_\alpha \Psi,$$

where \( \Psi = (\psi_\uparrow, \psi_\downarrow^\dagger) \) is a Nambu spinor field that describes BdG quasiparticle states. Here, \( \omega \) is an energy variable, \( k \) is a radial momentum expanded about the Fermi momentum, \( \Delta_0 \) is a constant pairing field, and \( \sigma_\alpha \) are the Pauli matrices. \( \Delta_1 |k|^\alpha \) is a non-uniform pairing field introduced phenomenologically to describe disorder-induced in-gap states [\( \alpha = 0.5 \) is taken without loss of generality; see Fig. 5a]. \( V_1, V_2 \) and \( V_3 \) stand for three disorder scattering terms, which originate from random pairing fluctuations in amplitude and phase, and a random potential, respectively. We assume a Gaussian white-noise distribution for each \( V_\alpha \), i.e., \( \langle V_\alpha(r)V_\alpha(r') \rangle = \delta(r-r') \Gamma_\alpha \), where the disorder parameter \( \Gamma_\alpha \) characterizes the variance of such a distribution. See Supplemental Material for more details about the construction of this model.

Taking into account quantum corrections in the one-loop order, we obtain RG flow equations, which are given in an approximate form as [see Supplemental Material for the full equations and computation details],

\[
\begin{align*}
\frac{d\Delta_0}{dl} &= \Delta_0 [1 + A(-\Gamma_1 + \Gamma_2)], \\
\frac{d\Delta_1}{dl} &= (1 - \alpha)\Delta_1, \\
\frac{d\Gamma_1}{dl} &= \Gamma_1 + B(\Gamma_1^2 + \Gamma_2^2), \\
\frac{d\Gamma_2}{dl} &= \Gamma_2 + 2B\Gamma_1\Gamma_2, \\
\frac{d\Gamma_3}{dl} &= \Gamma_3 - 2C\Gamma_1\Gamma_2,
\end{align*}
\]

where \( l \to \infty \) stands for the zero-temperature limit and the coefficients \( A, B, C \) are given by

\[
\begin{align*}
A &= \frac{1}{\omega^2 + (\Delta_0 + \Delta_1)^2}, \\
B &= \frac{(\Delta_0 + \Delta_1)^2}{[\omega^2 + (\Delta_0 + \Delta_1)^2]^2}, \\
C &= \frac{\omega^2}{[\omega^2 + (\Delta_0 + \Delta_1)^2]^2}.
\end{align*}
\]

There exist two stable fixed-point (FP) solutions for these equations [see Fig. 5b]:

\[
\begin{align*}
\text{FP1: } \left(\frac{\Gamma_1}{\omega}, \frac{\Gamma_2}{\omega}, \frac{\Gamma_3}{\omega}\right) &\to (\infty, \infty, \infty), \\
\text{FP2: } \left(\frac{\Gamma_1}{\omega}, \frac{\Gamma_2}{\omega}, \frac{\Gamma_3}{\omega}\right) &\to (\infty, \infty, 0),
\end{align*}
\]

where FP1 (FP2) indicates that an in-gap BdG quasiparticle state for given \( \omega \) is localized (delocalized). If \( \Gamma_1 \)
and $\Gamma_2$ are so weak that FP1 is stabilized for all $\omega$, the system belongs to the conventional disordered superconducting phase$^{1-18}$ where all in-gap states are localized. On the other hand, if $\Gamma_1$ and $\Gamma_2$ are strong enough that FP2 is stabilized for certain $\omega$, the system belongs to a novel anomalous metallic phase with superconductivity where delocalized in-gap states are developed. In the latter case, the delocalized in-gap BdG quasiparticles may constitute metallic charge carriers in the superconducting gap, which could be reminiscent of the failed superconductor$^{15}$. A quantum phase transition between two phases occurs when the disorder parameters are controlled (Fig. 5c).

Our detailed RG analysis taking the energy dependence demonstrates the delocalization nature of the quantum phase transition [See Supplemental Material for the computation details]. The energy dependent phase diagram in Fig. 5c explicitly shows the emergence of delocalized in-gap states as $\Gamma_1$ (the strength of superconducting fluctuations) increases. When $\Gamma_1$ is small, only localized in-gap states exist in an energy window (green region). When $\Gamma_1$ becomes larger, however, delocalized in-gap states also appear in a higher energy range (red region). A mobility edge (dashed line) appears in the latter case, which is comparable to that found in the numerical simulation [see Fig. 4]. The phase diagram in Fig. 5c also exhibits the sharp transition of the energy range for delocalized in-gap states across the phase boundary in the disorder parameter space. The RG analysis suggests that our samples may undergo this delocalization transition, which is unveiled by the multifractality spectrum analysis in Fig. 2 (Exps.) and Fig. 3 (Sims.). When a magnetic field is absent, the pairing fluctuations are weak, which corresponds to small $\Gamma_1$ and $\Gamma_2$ in the effective field theory. When the magnetic field is applied, however, the pairing fluctuations become strong, which corresponds to large $\Gamma_1$ and $\Gamma_2$. The RG analysis suggests that the former and latter belong to the conventional disordered superconducting phase and the anomalous metallic phase with superconductivity, respectively.

We summarize essential aspects of STM experiments, real-space Hartree-Fock-BCS-Anderson simulations, and disorder RG analyses. Both STM measurements and real-space mean-field simulations confirmed that external magnetic fields enhance Cooper-pair fluctuations rather strongly, evidenced by both the Cooper-pair multifractal spectrum and the coherent-peak height-distribution function. The disorder RG analysis implies the existence of a fixed point characterized by relatively large variance of Cooper pairs although this large-variance fixed point cannot be justified within the perturbative RG method. All these experimental and theoretical results suggest an anomalous insulator-metal transition of in-gap-state BdG quasiparticles in a quasi-two dimensional Fe-based superconductor, which may serve as a novel mechanism for Anderson transitions in quasi-two dimensions.

We believe that the metal-insulator transition of in-gap BdG quasiparticles is not limited in the superconducting state only. This phenomenon would be ubiquitous in the respect that the corresponding order-parameter field for spontaneous symmetry breaking is strongly fluctuating near the metal-insulator transition, expected to show the multifractal spectrum of the collective mode. On the other hand, the quasiparticle state can be delocalized by such enhanced fluctuations of order parameter fields, discussed in the present study, although the fundamental mechanism is not fully clarified in this study. It is natural to expect a two-parameter scaling theory for this Anderson transition, corresponding to the quasiparticle localization length and the order-parameter correlation length. STM measurements would reveal the mutual interplay between the symmetry-breaking field and the corresponding quasiparticle state by the multifractal analysis of the collective spectrum.
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Figure 1 | Schematic phase diagram, LDOS, and correlation length as a function of external magnetic fields. 

a, Schematic figures of the local density of states and a phase diagram based on our experiments. Based on STM measurements for the local density of states, we obtain three kinds of distribution functions for the local density at zero energy, the local pairing gap, and the coherent-peak height, respectively, as a function of external magnetic fields. We obtain two novel multifractal spectra of the local pairing gap and the coherent-peak height, and one conventional multifractal spectrum of the local density from these distribution functions. As a result, we find the phase diagram for Anderson delocalization of the in-gap BdG quasiparticles inside the quasi-two dimensional superconducting phase. b, Topograph of cleaved Sr$_2$VO$_3$FeAs. c, Average tunneling spectra and the energy-dependent coefficients of variation (CoV’s, stddev./mean) over the field of view at 0T and 7T. d, Difference of the averaged spectra at 0T and 7T. Approximate positions of coherence peaks and mobility edges are marked with green and blue arrows respectively. e-f, Zero-bias tunneling conductance maps at 0 T and 7 T. The local density of states increases to become more homogeneous by the applied magnetic field. Based on these tunneling spectra, we obtain all the distribution functions and the multifractal spectra, mentioned above. See the supplementary material for our data analysis. g, Angle-averaged auto-correlations of e and f. The experimental data (scattered dots) and their fits (solid lines) to the model $Ae^{-r/\xi} + Be^{-r\cos(2k_Fr + \phi)}$ with the first term due to the spatial fluctuation directly affected by Anderson localization and the second term due to quasiparticle interference. Long-range density fluctuations are suppressed by the applied magnetic field ($\xi_{0T} = 6.618 \pm 0.099 \text{ nm} \rightarrow \xi_{7T} = 2.121 \pm 0.144 \text{ nm}$). h-i, Energy-dependent angle-averaged auto-correlations of tunneling conductance maps at 0 T (h) and 7 T (i). The ‘weakly-divergent’ length scale for density fluctuations, given by a sharp signature near the zero energy, disappears by the applied magnetic field.
Figure 2 | STM measurements for three types of distribution functions and multifractal spectra as a function of external magnetic fields. a, Zero-bias tunneling conductance map at 0 T. b, Gap map at 0 T. c, Coherence peak height map at 0 T. d, Zero-bias tunneling conductance map at 7 T. The local density of states is enhanced to be more uniform. e, Gap map at 7 T. Local pairing amplitude fluctuations are enhanced by the magnetic field. f, Coherence peak height map at 7 T. The coherence peak height is rather suppressed. Inset of a: auto-correlation map of a. Insets of b-f: cross-correlation maps of a and b-f. The main point is that local density fluctuations are negatively correlated with both local pairing amplitude fluctuations and coherence peak height ones while fluctuations of local pairing amplitudes are positively correlated with those of coherence peak heights. See the text for more details. g-i, Histograms of maps a-f. The distribution function of the local density of states is shifted to a larger average value of the local density of states by the applied magnetic field. The width of the distribution function also decreases by the magnetic field. The distribution function of the local pairing gap shows relatively huge suppressions around 3meV. These weights are transferred to those around the zero energy. The width of the distribution function is also enhanced by the magnetic field. The distribution function of the coherence peak height is suppressed by the magnetic field. j-l, Multifractal spectra $f(\alpha)$ of maps a-f. The multifractal spectrum of the local density of states is narrower by the magnetic field. On the other hand, both multifractal spectra of the local pairing gap and the coherence peak height are broader by the magnetic field.
Figure 3 | Hartree-Fock-BCS-Anderson simulations for distribution functions and multifractal spectra as a function of external magnetic fields. a, Local Density of states inside the superconducting gap (the energy window $E \sim [0.25,0.27]$) at $\frac{\Phi}{\Phi_0} = 0$. b, Gap map at $\frac{\Phi}{\Phi_0} = 0$ for a specified disorder configuration. c, The wavefunction amplitude for $E=0.26$ at $\frac{\Phi}{\Phi_0} = 0$. d, Local Density of states within the energy window $E \sim [0.25,0.27]$ at $\frac{\Phi}{\Phi_0} = 0.002$. The local density of states is enhanced to be more homogeneous by the magnetic field. e, Gap map at $\frac{\Phi}{\Phi_0} = 0.002$ for a specified disorder configuration. The region of vanishing local pairing amplitudes is enlarged by the magnetic field. f, Wavefunction amplitude for $E=0.26$ at $\frac{\Phi}{\Phi_0} = 0.002$. The wavefunction amplitude inside the superconducting gap is more enhanced to be uniform by the magnetic field. g-i, Histograms of maps. The distribution function of the local pairing amplitude is more broadened by the magnetic field. In particular, there appears finite population of the vanishing local pairing amplitude. Even negative sign of the local pairing gap is observed. On the other hand, the distribution function of the local density of states is narrowed and shifted to a larger average value of the local density of states. The distribution function of the wavefunction amplitude around $E = 0.26$ is consistent with that of the local density of states. j-l, Multifractal spectra $f(\alpha)$ of maps a-f. The multifractal spectrum of the local pairing gap is broadened by the magnetic field. On the other hand, both the multifractal spectra of the local density of states and the wavefunction amplitude at a given energy are narrowed by the magnetic field.
Figure 4 | Mobility edge of in-gap-state BdG quasiparticles as a function of external magnetic fields and comparison between experiments and simulations for the multifractal spectrum as a function of energy. a, Mobility edge of in-gap-state BdG quasiparticles in the Hartree-Fock-BCS-Anderson simulation. The scale-invariance behavior of the multifractal critical exponent, here that of the inverse participation ratio, identifies the mobility edge. b, Evolution of the mobility edge as a function of the applied magnetic field. Applied magnetic fields turn an Anderson insulating state of in-gap-state BdG quasiparticles into their metallic phase. There also exists a mobility edge at a high energy, not shown here and not affected by applied magnetic fields. It is interesting to observe that BdG quasiparticles near the coherent peak become more critical by applied magnetic fields. c, Exp. $\alpha_0$ for the LDOS of BdG quasiparticles VS. energy, d, Sim. $\alpha_0$ for the wavefunction VS. energy. Applied magnetic fields suppress the magnitude of $\alpha_0$ for in-gap-state BdG quasiparticles, consistent with the experiment.
Figure 5 | Renormalization group analysis for the delocalization transition of in-gap BdG quasiparticles. 

A schematic diagram of the density of states (DOS) showing the disorder-induced in-gap BdG quasiparticle states. Here, $\Delta_0$ and $\Delta_1$ stand for the pairing terms in the effective field theory in Eq. (1), which correspond to the energy gap and energy range of in-gap states, respectively. 

b, An example of the renormalization group flow for the effective field theory. Here, $\Gamma_1$ and $\Gamma_3$ are the disorder strength for random pairing fluctuations and random potentials for the in-gap BdG quasiparticles, respectively. At the stable fixed point of FP1 (FP2), $\Gamma_3$ grows (diminishes), indicating that the in-gap states are localized (delocalized).

c, An energy-dependent phase diagram of BdG quasiparticle states as a function of quasiparticle energy $\omega$ and $\Gamma_1$. When $\Gamma_1$ (the strength of superconducting fluctuations) is weak, only localized in-gap states appear (green region). On the other hand, when $\Gamma_1$ is strong, delocalized in-gap states emerge in higher energy states (red region). Here, $W$ and $B$ stand for the energy ranges of the full in-gap states and delocalized in-gap states, respectively. 

d-e, Phase diagrams showing $W$ and $B$, which are drawn in c, as a function of $\Gamma_1$ and $\Gamma_3$, respectively.