Differential Control Strategy based on an Equal Slip Rate for an All-wheel Electric-drive Underground Articulated Dumping Truck

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Abstract

A differential control strategy based on equal slip rates is introduced to improve the steering stability of an all-wheel-electric-drive underground articulated dumping truck. Steering kinematic and dynamic models of the truck are derived to describe the movement relationship and force of the driving wheels. In consideration of the difficulty of obtaining the absolute velocity for an all-wheel-drive truck, an acceleration sensor was set on a test truck, and a kalman filter was applied to obtain the actual value for the truck body. Simulation results for an equal-slip control strategy were compared with experimental results for an equal-torque control strategy. In the simulation, the four-wheel slip rate was 0.08 and the steering system of the truck was stable. The results verify that the equal-slip control strategy makes better use of the ground adhesion coefficient, is able to reasonably distribute drive power, notably reduces tire wear, and improves the use of driving power.

Keywords: Articulated Dumping Truck Steering, Wheeled Electric Drive, Electric Differential, Slip Rate, Control Strategy

1. Introduction

Compared with a traditional truck, an articulated dumping truck (ADT) has two obvious features. First, it is driven by all its wheels. Second, rather than a trapezoidal steering mechanism, an ADT has a vertical pivot joint between the front and rear parts of its frames, which acts as its steering mechanism. The front and rear parts move with respect to each other in a plane around the pivot joint when a steering handle is rotated \cite{1}. The ADT thus has better traffic ability and a small turning radius, and is especially suited to narrow, wet and slippery roads.

All-wheel drive increases the system complexity and reduces the reliability of the mechanical drivetrain of a truck. For example, a 6x6 ADT needs five differentials and five no spin devices to have good traffic ability. However, considering reliability, ease of manufacture and costs, a part-time drive system is employed usually and no spin configured on the axles and the differential between the two rear axles is omitted.

A wheeled drive vehicle, without a final drive and differential, driven by an in-wheel motor reducer, has the advantages of a simple structure and low failure rate \cite{2}, \cite{3}, and is thus widely used in the mining industry. A wheeled electric-drive underground ADT is especially designed for underground mines, and has double power, articulated steering and four-wheel independent drive, as shown in Fig. 1.

As the wheels of the truck are controlled independently, a differential control strategy should be applied to improve engine efficiency and the ground driving force and reduce tire wear. Therefore, the differential control strategy is a key problem to be solved for the truck. Present electrical-drive mining trucks, such as the EH5000 and Liebherr T282, apply equal torque control, which is obviously not suitable for some conditions. Independent wheel torque control has mainly been investigated for passenger cars. Some employed neural networks and four PID controllers to coordinate distribute torque to four in-wheel motors and realize electronic differential speed steering \cite{4}. Some input different torque to drive wheels, monitored the change of tire deflection angles and determined the targets slip rate \cite{5}. Researchers developed a Matlab/Simulink platform and presented the control of torque for an electric vehicle with in-wheel motors under complex conditions \cite{6}. Some proposed an integration of a steering system and traction...
system to use the differential drive torque of the rear axle and thus assist steering and improve the steering performance of a vehicle [7], using a vehicle model with two degrees of freedom, some designed a differential braking control law to improve the lateral stability and safety of a vehicle [8]. Others proposed the use of differential braking torque to correct steering maneuvers [9], [10], [11]. However, all the above research focused on control methods, and ignored the key point of differential control, namely how to get the absolute velocity of the vehicle. Additionally, an ADT has front and rear frames, articulated steering, and thus dynamic characteristics that are different from those of passenger cars.

The aim of this research is to establish an active differential control strategy by calculating the slip rates of wheels of an ADT. The paper is organized as follows. Section 2 describes the building of the truck kinematics and dynamic mathematical model. Section 3 describes the truck multi dynamics model constructed with Automatic Dynamic Analysis of Mechanical Systems (ADAMS) software. Section 4 presents the slip-rate control strategy. Section 5 presents the experimental method of testing a prototype, and estimation of the absolute velocity employing Kalman filtering.

2. Establishment of a mathematical model

2.1 Kinematic model

![Fig. 2. Kinematic model of the ADT](image)

The kinematics parameters of the ADT are shown in Fig. 2. O’X’Y’Z’ is a coordinate system fixed on the ground, and OXYZ and O’X’Y’Z’ are dynamic coordinates fixed on the centers of mass of the front and rear frames. The X and X’ axes are respectively the longitudinal axes of the front and rear frames, B is the axle length, Lf is the distance between the front axle and the articulation point, Lr is the distance between the rear axle and the articulation point, h is the distance between the articulation point and the center of mass of the front frame, and h is the distance between the articulation point and the center of mass of the rear frame. \( \delta \) is the angle separating the orientations of the front and rear frames. \( \delta \) is the angle separating the orientations of the articulation and the articulation point.

\[
\dot{\delta} = r_1 + r_2
\]  

Considering the kinematic relationship of the wheels, and taking the steering angle as the known quantity, equation (2) is obtained.

Here, \( u_{fl}, u_{fr}, u_{rl}, \) and \( u_r \) are the longitudinal velocities of left front, right front, left rear and right rear wheels respectively. From equation (2), the slip rate is obtained as equation (3). Here, \( S_{fl}, S_{fr}, S_{rl}, , \) and \( S_r \) are the slip rates of the left front, right front, left rear and right rear wheels respectively, and \( \omega \) is the angular velocity of the wheels.

\[
\begin{align*}
&\begin{align*}
&u_{fl} = (1 - \frac{B}{2L_c \cot \delta + 2L_c \sqrt{1 + \cot^2 \delta}}) \cdot u_1 \\
&u_{fr} = (1 + \frac{B}{2L_c \cot \delta + 2L_c \sqrt{1 + \cot^2 \delta}}) \cdot u_1 \\
&u_{rl} = (1 - \frac{B}{2L_c \cot \delta + 2L_c \sqrt{1 + \cot^2 \delta}}) \cdot u_2 \\
&u_{r} = (1 + \frac{B}{2L_c \cot \delta + 2L_c \sqrt{1 + \cot^2 \delta}}) \cdot u_2
\end{align*}
\end{align*}
\]

\[
\begin{align*}
&\begin{align*}
&\frac{wR - (1 - \frac{B}{2L_c \cot \delta + 2L_c \sqrt{1 + \cot^2 \delta}}) \cdot u_1}{wR} = S_{fl} \\
&\frac{wR - (1 + \frac{B}{2L_c \cot \delta + 2L_c \sqrt{1 + \cot^2 \delta}}) \cdot u_1}{wR} = S_{fr} \\
&\frac{wR - (1 - \frac{B}{2L_c \cot \delta + 2L_c \sqrt{1 + \cot^2 \delta}}) \cdot u_2}{wR} = S_{rl} \\
&\frac{wR - (1 + \frac{B}{2L_c \cot \delta + 2L_c \sqrt{1 + \cot^2 \delta}}) \cdot u_2}{wR} = S_r
\end{align*}
\end{align*}
\]

2.2 Establishment of a dynamic model

As shown in Fig. 3, \( a_{x1} \) and \( a_{y1} \) are the accelerations of the front frame, \( a_{x2} \) and \( a_{y2} \) are the accelerations of the center of mass of the rear frame, \( F_{x1} \) and \( F_{y1} \) are respectively the longitudinal and lateral forces of the wheels(\( i = 1-4 \)), \( T_o \) is the torque acting between the front and rear frames when steering, and \( F_x \) and \( F_y \) are the forces acting on the articulation point.

![Fig. 3. Dynamic model of the ADT](image)
From the force relationship between the front and rear frames described in Figure 3, the following torque equations are obtained for the ADT.

\[ I_n \ddot{r} = T_0 + (F_{r3} - F_{r4})B/2 - (F_{b3} + F_{b4})(h_r - L_r) - F_r h_r \]  

(4)

\[ m_1(u_1 - r_1 v_1) = F_x + F_{x2} - F_x \]  

(5)

\[ m_1(u_2 + r_2 u_1) = F_y + F_{y1} + F_{y2} \]  

(6)

\[ I_n \ddot{r} = T_0 + (F_{r3} - F_{r4})B/2 - (F_{b3} + F_{b4})(h_r - L_r) + (F_y \cdot \cos \delta - F_x \cdot \sin \delta) h_r \]  

(7)

\[ m_2(u_1 - r_2 y_1) = F_x \cos \delta + F_y \sin \delta + F_{x3} + F_{y4} \]  

(8)

\[ m_2(v_2 + r_2 u_1) = F_y \sin \delta - F_x \cos \delta + F_{y3} + F_{x4} \]  

(9)

Eliminating intermediate variables \( F_x \) and \( F_y \) of equations (4)–(7), the following steering dynamic equations of the ADT are obtained.

\[ (m_1 + m_2)v_1 - (m_1 + m_2)r_1 u_1 - m_2 h_1 \dot{r}_1 + m_2 h_2 \dot{r}_2 \sin \delta + m_2 h_2 \dot{r}_2 \cos \delta = F_{x3} + F_{y2} + (F_{x4} + F_{y3}) \cos \delta - (F_{y4} \cdot \sin \delta) h_r \]  

(10)

\[ I_n \ddot{r}_2 + m_2 h_1 \dot{u}_1 \sin \delta - m_2 h_2 \dot{r}_1 \sin \delta + m_2 h_2 \dot{r}_2 \sin \delta + m_2 h_2 \dot{r}_2 \cos \delta = F_{x3} + F_{y2} + (F_{x4} + F_{y3}) \cos \delta + (F_{y4} \cdot \sin \delta) h_r \]  

(11)

\[ (m_1 + m_2)u_1 - (m_1 + m_2)r_1 v_1 + m_2 h_2 \dot{r}_1 + m_2 h_2 r_2 \cos \delta \]  

(12)

Equations (10)–(12) show that the longitudinal and lateral velocities of the ADT are related to the driving force and articulation angle. However, it is difficult to describe the dynamic properties of the ADT, such as the change in wheel loads, with the equations and doing so would lead to an error in the slip rate. Multi-body dynamics software is therefore used to simulate the slip-rate control of the ADT.

3. Multi body dynamics model

The multi body dynamics model was constructed using ADAMS software. Various parts of the truck were developed and shaped as exactly as possible to coincide with the actual ADT. The handling performance and direction response of the ADT are greatly affected by the forces and moments generated by the tires and a correct tire model is thus important to the simulation. In this work, the tire model chosen was based on the UA.

The truck driving model is shown in Fig. 4.

4. Control strategy based on equal slip rates

In the control strategy of equal slip rates, the distributions of torque and speed are based on the change in slip rate under different conditions. In this paper, the mean value for the four wheels is presented as the target slip rate. The actual slip rate of the wheels and the target slip rate are transmitted in real time to the power distribution module. According to deviation between the actual and target slip rates, the power distribution module calculates the required torques and speeds, adjusts the wheel speeds through motor driving modules, and controls the slip rates of the wheels within a target range. The control flow chart is shown in Fig. 5.

Here, \( s \) is the calculated slip rate, \( \bar{s} \) is the mean value of the slip rate, \( p \) is the calculated power, and \( p' \) is the target power.

![Fig. 5. Flow chart of control of the slip ratio](image-url)
5. Analysis and discussion of the control strategy

5.1 Experimental method

| Assembly | Parameter       | Value     |
|----------|-----------------|-----------|
| Engine   | Rated power     | 399 kW    |
| Motor    | Rated power     | 90 kW     |
| Reducer  | Reduction torque| 1200Nm    |
| Tire     | 29.5R29         |           |
| Weight   | Loading eight   | 35T       |
|          | Curb weight     | 29T       |

Fig. 6. Sensor installation positions

The experiment employed an LMS SCADAS MOBILE SCM05 acquisition card and sensors for which the sampling frequency was 2560Hz. INS devices were fixed on the centers of the front and rear driving axes to measure the accelerations and angular velocities along the three orthogonal axes of each of the front and rear frames. A rotary encoder mounted on the steering column measured the angles of the steering wheels. An angular displacement sensor was fastened to the articulation point between front and rear frames. The installation positions are shown in Fig. 7. Data of the engine, generator, and motor were obtained from output of a CAN device, and collected through a USBCAN interface.

5.2 Absolute velocity estimation based on Kalman filtering

The absolute velocity is needed to control the slip rate. Theoretically, it can be obtained from the measurements of wheel velocity sensors; however, the wheel velocity signal must be filtered, after gross errors and system measuring errors are eliminated. If the direct integral of longitudinal acceleration was used, the signal-to-noise ratio would be lowered, resulting in imprecision and, even divergence, and traditional velocity estimation methods are thus not suitable in this research. Additionally, the method of measuring the speed of driving wheels is unsuited to an all-wheel-drive truck.

The present study collected the longitudinal acceleration signal and wheel speed signal measured for the prototype ADT as input data, and applied a Kalman filtering algorithm to estimate the velocities of the front and rear frames separately. The process of Kalman filtering began with known state initial values and the initial values of the state covariance matrix, filtering and estimating wheel velocity. The estimation of speed involved prediction and calibration using the state equation (13) and observation equation (14).

\[
x(k + 1) = A(k)x(k) + w(k)
\]

(13)

\[
z(k) = B(k)x(k) + n(k)
\]

(14)

The parameter matrix is given by equation (15).

\[
x(k) = \begin{bmatrix} a_x(k) \\ v_x(k) \\ \omega R(k) \end{bmatrix}; Z(k) = \begin{bmatrix} a_x(k) \\ \omega R(k) \end{bmatrix}
\]

\[
w(k) = \begin{bmatrix} w_x(k) \\ w_y(k) \end{bmatrix}; n(k) = \begin{bmatrix} n_x(k) \\ n_y(k) \end{bmatrix}
\]

(15)

\[
A(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \Delta t & 1 \end{bmatrix}; B(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

Here, \(a_x\) and \(v_x\) are the estimation values of longitudinal acceleration and angular speed of the wheels respectively, \(R\) and \(\Delta t\) are test values of longitudinal acceleration and angular speeds of the wheels respectively, \(R\) is the rolling radius, \(\Delta t\) is the time interval, \(w_x\) and \(w_y\) are system noise, and \(n_x\) and \(n_y\) are measurement noise. System noise and measuring noise are known to be Gaussian white noise. To describe the true rolling radius, Gaussian white noise was added according to the real fluctuation.

Fig. 8 shows the estimation and measurement of the longitudinal acceleration of the front frame after filtering. The difference between the estimation and measurement is shown in Fig. 9. Analysis shows large fluctuation in the difference between the estimation and measurement from 5 to 25s, reaching a maximum of 0.19m/s. The difference becomes small in steady-state steering. With subsequent acceleration, the speed reaches 4.5m/s. The speed then reduces. The estimation error is 0.2m/s. Generally, the difference is small, and nearly consistent in an even state.
phase. The filtered signal can thus be used to estimate the actual velocity directly.

5.3 Simulation of the control of an equal slip rate
To validate the multi-body dynamics simulation platform, an equal-torque control strategy was adopted for the prototype and simulation platform. Measurement and simulating results are shown as Fig. 11.

The data obviously fluctuated in the first 20s because the driver could not keep the acceleration stable. After 25s, the steering velocity became even. The data for the prototype show that the slip rate of the outside wheel is about 0.05, which corresponds to a slip state, and that of the inside wheel is –0.05, which corresponds to a slip-rotation state. Simulation data are consistent with the measurements. Because the path lengths of the inside and outside wheels are not equal after 25s, the outside wheel had slipped. The simulation and experimental results thus demonstrate that an equal-torque control strategy introduces a difference in the slip rate between the inside and outside wheels.

The changes in the driving torques of the inside and outside wheels are shown in Fig. 12. Under the equal-torque strategy, the torques of the inside and outside wheel motors of the prototype ADT remain consistent. The multi-body dynamics simulation model could quickly reach a stable state, and have values consistent with data for the prototype, finally reaching torque of 680Nm. The above results verify the multi-body dynamics simulation model.
Fig. 13 illustrates the slip rates of the inside and outside wheels under the equal-slip-rate control strategy. The slip rate is in a steady state after 11s, with the slip rates of the inside and outside wheels remaining at ~0.08. The variation in the slip rate at 8–10s is due to the change in motor torque.

Fig. 14 illustrates the input torques of the inside- and outside-wheel motors. The outside-wheel torque was greater than the inside-wheel torque, reaching 810Nm at a steady state. The slipping of the outside wheels was thus reduced.

Fig. 15 illustrates the yaw angular velocities of the front and rear frames. The front and rear frames move in opposite directions at 5s during steering. The yaw angular velocity begins to converge at 15s, and reaches a steady state after 15s.

6. Conclusions

A novel wheeled electric-drive underground ADT and control strategy were presented. This is the first time that the wheeled driving technique has been applied to an underground mining vehicle. Simulation results obtained with the multi-body dynamics model are consistent with experimental data for a tested prototype. The estimation of absolute velocity based on Kalman filtering is suitable for the truck, and can provide the slip rate with small errors and in real time. The application of equal-slip-rate steering control strategy to a wheeled electric-drive underground ADT is better than that of the equal-torque control strategy. The former strategy fully uses the ground adhesion coefficient, reduces tire wear and improves handling stability.

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