Adjoint Messengers and Perturbative Unification at the String Scale

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Abstract

We consider states in the adjoint representation of the Standard Model gauge group as messengers for mediation of supersymmetry (SUSY) breaking. These new messengers can shift the gauge coupling unification to the string scale at $O(5 \times 10^{17} \text{ GeV})$ if their masses are at $O(10^{14} \text{ GeV})$. The predicted SUSY mass spectrum at the electroweak scale is significantly different from those in other gauge-mediated or supergravity models, resulting in robust mass relations. The gravitino mass is predicted to be about $1 - 10 \text{ GeV}$. The heavy messenger sector could provide a superheavy dark matter candidate.

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Introduction: Weak scale supersymmetry (SUSY) is arguably the strongest candidate for physics beyond the Standard Model (SM). One of the most attractive features is that the supersymmetric $SU(5)$ theory provides a non-trivial coupling unification for the SM gauge group $SU_c(3) \times SU_L(2) \times U_Y(1)$, consistent with the experimental determination for the coupling constants at the electroweak scale ($M_Z$) [1]. In the minimal supersymmetric extension of the Standard Model (MSSM) with a SUSY mass scale near 1 TeV, the gauge coupling unification occurs at a scale $M_{GUT} \approx 2 \times 10^{16}$ GeV. On the other hand, heterotic string theories generically predict a perturbative string unification at a scale $M_{str} \approx 5 \times 10^{17}$ GeV. These two scales are mysteriously close (in relative value), yet significantly different (in absolute value). It is therefore extremely tempting to contemplate on physics scenarios to reconcile these two scales [2].

One of the possibilities to fulfill this idea is to introduce some states beyond the MSSM below the unification scale. The additional states modify the behavior of the gauge coupling evolution and may lift the unification scale from $M_{GUT}$ to $M_{str}$. An explicit example has been constructed by considering adjoint representations for an $SU_c(3)$ octet ($\Sigma_8$) plus an $SU_L(2)$ triplet ($\Sigma_3$) [3]. This scenario is particularly interesting since the new states could naturally arise from the non-Goldstone remnants of the Higgs multiplets $\Sigma_{24}$.

In spite of our ignorance about the SUSY breaking mechanism at high energy scale, one would like to at least explore how the SUSY breaking effects have been transmitted to the observable sector at the electroweak scale. A model with gauge mediation of SUSY breaking (GMSB) [4] is a simple and predictive version of the MSSM. In addition to the observable sector and a SUSY breaking hidden sector, the model also possesses messenger fields which mediate the SUSY breaking to the observable fields via the SM gauge interactions. The minimal model has a pair of messengers $\Phi_5 + \bar{\Phi}_5$ transforming under the $SU(5)$ representations $5 + \bar{5}$. By assumption, this minimal model of gauge-mediated SUSY breaking (mGMSB) contains messengers as complete $SU(5)$ representations. This construction automatically preserves the gauge coupling unification at $M_{GUT}$.

In this Letter, we propose a “marriage” of these two ideas: we introduce some states beyond the MSSM below $M_{GUT}$ (incomplete representations of the GUT group) as new messengers to mediate the SUSY breaking effects, and the scale of gauge coupling unification is lifted to $M_{str}$. This scenario may have profound theoretical implication: the gauge coupling unification at the string scale may be intimately connected with the gauge mediation of the SUSY breaking. It is important to note that the introduction of the new messengers predicts a different mass spectrum for SUSY particles (sparticles) from those in the GMSB models [3] and in the supergravity models (SUGRA) [5]. Thus, we should be able to test this idea once the SUSY mass parameters are measured at future collider experiments. There are also interesting cosmological consequences in this scenario that we will discuss in the later sections.
Adjoint Messengers and Gauge Coupling Unification: Following the proposal in Ref. [3] to resolve the string unification problem, we first introduce a pair of new messenger fields $\Sigma_8$ and $\Sigma_3$ with the following $SU_c(3) \times SU_L(2) \times U_Y(1)$ quantum number assignment

$$\Sigma_8 : (8, 1, Y = 0); \quad \Sigma_3 : (1, 3, Y = 0).$$

They are in adjoint representations and thus anomaly-free, which will be referred as “adjoint messengers”. We consider a general model which includes $n_\Phi$ pairs of $\Phi_5 + \overline{\Phi}_5$ and $n_\Sigma$ pairs of $\Sigma_8 + \Sigma_3$ states. The renormalization group equations (RGEs) at one-loop level for the SM gauge couplings, $\alpha_i = g_i^2 / 4\pi$ ($i = 1, 2, 3$), up to $M_{\text{str}}$ are given by

$$\frac{d\alpha_i}{dt} = \left(b_i^{\text{MSSM}} + N_i\right) \frac{\alpha_i^2}{2\pi},$$

where $t = \ln Q$ and the $b$-coefficients in the MSSM are $b_i^{\text{MSSM}} = +33/5, +1, -3$ respectively, and $N_i$ the new state counting

$$N_1 = n_\Phi, \quad N_2 = n_\Phi + 2n_\Sigma, \quad N_3 = n_\Phi + 3n_\Sigma \quad \text{(above } \Phi_5, \Sigma_8 \text{ and } \Sigma_3 \text{ threshold } M).$$

Since $\Phi_5 + \overline{\Phi}_5$ form complete $SU(5)$ representations, they preserve the unification at the GUT scale and their masses can be arbitrary between the MSSM threshold and $M_{\text{GUT}}$. In contrast, the adjoint messengers can change the running behavior of the couplings and shift the unification scale around depending upon the number of states and their masses. We find that as long as we take the same number of states for $\Sigma_8$ and $\Sigma_3$, a unification can be achieved. This justifies our choice for a single $n_\Sigma$. The evolution of the couplings from $M_Z$ to $M_{\text{str}}$ is illustrated in Fig. 1, where we have evolved the couplings at two-loop level, including SUSY threshold corrections at the electroweak scale. The solid curves demonstrate the string scale unification with $n_\Phi = n_\Sigma = 1$, and the dashes show the unification with $n_\Phi = 1$ only. To reach a successful unification at $M_{\text{str}} \approx 5 \times 10^{17}$ GeV and accommodate the strong coupling constant $\alpha_s(M_Z) = 0.118$, the masses of the adjoint messengers need to be

$$M_8 \approx 2.5 \times 10^{13} \text{ GeV,} \quad M_3 \approx 1.2 \times 10^{14} \text{ GeV,}$$

for which the gauge couplings unify to $\alpha_{\text{GUT}}(M_{\text{str}}) \approx 1/20$. Following Eq. (4), we will generically identify the messenger scale ($M$-scale) for $\Phi_5 + \overline{\Phi}_5$ and $\Sigma_8 + \Sigma_3$ as

$$M \approx 10^{14} \text{ GeV.}$$

Perturbativity requirement for the gauge couplings up to $M_{\text{str}}$ leads to a bound on the numbers of the messenger states, $n_\Phi, n_\Sigma \leq \mathcal{O}(10)$, for their masses $M \leq \mathcal{O}(10^{14}$ GeV). Such a loose bound is due to the smaller running effects between the rather close scales $M$ and $M_{\text{str}}$. On the other hand, many pairs of the adjoint messengers
Figure 1: Evolution of the SM gauge couplings from the electroweak scale $M_Z$ to the string unification scale $M_{\text{str}}$ with a generic messenger scale $M \approx 10^{14}$ GeV. The solid curves show the gauge coupling unification at $M_{\text{str}}$ with the help of the adjoint messengers. The dashes give the unification at $M_{\text{GUT}}$ without the adjoint messengers, but with $\Phi_5 + \Phi_\bar{5}$ at $M$. The inner panel shows the blowup in the unification region. We have taken $\tan \beta = 2$, $\mu < 0$ and $\alpha_s(M_Z) = 0.118 \pm 0.003$.

would push their mass scale too close to $M_{\text{str}}$ for the unification. Furthermore, to avoid a too heavy sparticle spectrum, especially for the gravitino mass ($m_{3/2}$) as we will discuss later, $n_\Sigma = 1$ is strongly favored. For concreteness, we also choose $n_\Phi = 1$ in the rest of our studies, which can be easily generalized to other values of $n_\Phi$.

**Predicted SUSY Mass Spectrum and Physical Consequences:** In GMSB models, the messengers couple to gauge singlet fields $S_i$ through a superpotential

$$W = \lambda_5 S_5 \Phi_5 \Phi_\bar{5} \Phi_\bar{5} + \lambda_8 S_8 \Sigma_8 \Sigma_8 + \lambda_3 S_3 \Sigma_3 \Sigma_3.$$  \hspace{1cm} (6)

For simplicity, we take the singlets to be the same $S_5 = S_8 = S_3$, which acquires non-zero vacuum expectation values for both its scalar component ($S$) and auxiliary component ($F_S$).

One of the important features for this model is that all sparticle masses are determined by two dimensionful parameters: the messenger scale $M = \lambda S$ and the effective SUSY breaking scale $\Lambda = F_S/S$. The gaugino and scalar soft masses are given, at one- and two-loop level respectively, by

$$M_i(M) \approx N_i \frac{\alpha_i(M)}{4\pi} \Lambda, \quad i = 1, 2, 3.$$  \hspace{1cm} (7)
where $C_i$'s are 4/3, 3/4 for the fundamental representations of $SU_c(3)$, $SU_L(2)$ and $3Y^2/5$ for $U_Y(1)$.

Equation (7) implies a gaugino mass relation

$$\frac{M_1(M)}{\alpha_1(M)} : \frac{M_2(M)}{\alpha_2(M)} : \frac{M_3(M)}{\alpha_3(M)} = N_1 : N_2 : N_3.$$  

(9)

Alternatively, we can rewrite the mass ratio relation, independent of $n_\Phi$ and $n_\Sigma$ as

$$\left( \frac{M_2(M)}{\alpha_2(M)} - \frac{M_1(M)}{\alpha_1(M)} \right) : \left( \frac{M_3(M)}{\alpha_3(M)} - \frac{M_1(M)}{\alpha_1(M)} \right) = 2 : 3.$$  

(10)

At the $M$-scale, the gauge couplings are $\alpha_{1,2,3} \approx 30.7, 25.6, 22.9$. Equations (9) and (10) are one-loop RGE invariant so they approximately hold at the electroweak scale as well.

From the boundary condition Eq. (7) and the RGE evolution, we obtain a gaugino mass relation at the $M_Z$-scale, compared with those in the mGMSB or mSUGRA models,

$$m_{\tilde{g}} : m_{\tilde{\chi}_2^0, \tilde{\chi}_1^+} : m_{\tilde{\chi}_1^0} \approx \begin{cases} 22 : 6 : 1 & \text{for } n_\Phi = n_\Sigma = 1, \\ 6 : 2 : 1 & \text{for mGMSB or mSUGRA}. \end{cases}$$

(11)

In our scenario, $\tilde{\chi}_1^0$ is basically $\tilde{B}$, and $\tilde{\chi}_1^+ , \tilde{\chi}_2^0$ are $\tilde{W}^\pm, \tilde{Z}^0$.

The additional contribution from the adjoint messengers to scalar masses generally yields heavier scalars in this model. However, the masses of the right-handed sleptons do not receive any correction from them. Equation (8) gives a mass relation for the sfermion soft masses at the $M$-scale

$$m_{\tilde{E}}^2 \approx \begin{cases} (15.6n_\Sigma + 5.8n_\Phi) : (12n_\Sigma + 4.5n_\Phi) : (12n_\Sigma + 4n_\Phi) : (3.6n_\Sigma + 2n_\Phi) : n_\Phi. \end{cases}$$  

(12)

Squark masses receive a large contribution from the gluino soft mass $M_3$ via the RGE evolution. At the $M_Z$-scale, we obtain a very simple relation among the masses for the first two generation squarks and sleptons, also compared with that in the mGMSB,

$$m_{\tilde{u}_{L,R}, \tilde{d}_{L,R}} : m_{\tilde{e}_{L,R}} : m_{\tilde{\nu}_{L,R}} \approx \begin{cases} 9 : 3 : 1 & \text{for } n_\Phi = n_\Sigma = 1, \\ 6 : 2 : 1 & \text{for mGMSB with } M \approx 100 \text{ TeV}. \end{cases}$$

(13)

In the case $n_\Phi = n_\Sigma = 1$, the ratios of the $M_3, M_2, M_1$ to $m_{\tilde{E}}$ turn out to be $4.7 : 3.2 : 0.9 : 1$ at the $M$-scale. Evolving to the $M_Z$-scale, we find the mass ratio

$$m_{\tilde{e}_R} : m_{\tilde{\chi}_1^0} \approx \begin{cases} 2.4 & \text{for } n_\Phi = n_\Sigma = 1, \\ 1.4 & \text{for mGMSB with } M \approx 100 \text{ TeV}. \end{cases}$$
Since the adjoint messengers carry no $U_Y(1)$ charge, they do not contribute to $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\ell}_R}$. The above mass difference comes entirely from the RGE evolution at the two different messenger scales, namely $10^5$ GeV for mGMSB and $10^{14}$ GeV for our model. This mass ratio thus provides a direct measure in extracting the underlying messenger mass scale.

If $\Lambda \sim \mathcal{O}(30 - 100$ TeV), the sparticles can have a desirable mass spectrum of $\mathcal{O}(100$ GeV). The predicted sparticle mass spectra at the $M_Z$-scale are presented in Fig. 2, with (a) for $\Lambda = 30$ TeV and (b) for $\Lambda = 100$ TeV, where we have implemented the two-loop evolution for the RGEs, and have properly taken the radiative electroweak symmetry breaking into account. The mass spectrum exhibits apparent hierarchy relations as noted in the previous discussions. The exceptions happen for the third generation squarks and sleptons, where the Yukawa contributions in the RGE evolution are also important. In particular, the lightest tau-slepton $\tilde{\tau}_R$ could be significantly lighter than other scalar particles, as also known from the mGMSB models. The gluino happens to be the heaviest, and next come the squarks. There are three curves for the squarks right below the gluino mass. The lower two are for the lighter top-squarks $\tilde{t}_{1,2}$, and the upper one is for the other nearly degenerate squarks.

From Eq. (5), we estimate that $F_S \approx \Lambda M \approx 10^{18} - 10^{19}$ (GeV)$^2$. This implies a relatively heavy gravitino

$$m_{3/2} = \frac{F_S}{\sqrt{3} M_{Pl}} \approx 1 - 10 \text{ GeV},$$  

(12)
where \( M_{p_1} = 2.4 \times 10^{18} \) GeV is the reduced Planck scale. This rather heavy stable gravitino may form significant amount of dark matter. As long as the reheating temperature after the inflation does not exceed about \( T_{RH} \approx 10^8 \) GeV, their relic density would not be too high to overclose the Universe [8]. It has been argued that in realistic string theories, the light moduli remnants (like the gravitino here) may distort the observed X-ray spectrum by radiative decay through gravitational effects [9]. It turns out that it would not destroy the observed spectrum if \( m_{3/2} \geq \mathcal{O}(1 \text{ GeV}) \). On the other hand, if \( m_{3/2} \geq 10 \text{ GeV} \), the scalar mass universality might be violated by the large gravitational contribution and the unwanted flavor-changing neutral currents (FCNC) may be reintroduced [4]. Although our estimate on \( m_{3/2} \) is essentially on the safe side for the FCNC problem, this consideration may serve as a criterion for favoring \( n_{\Sigma} = 1 \), as noted earlier.

Typically, the next-to-lightest SUSY particle (NLSP) in GMSB models is either \( \tilde{\chi}_1^0 \) or \( \tilde{\tau}_R \) (for large \( \tan \beta \) and higher \( n_{\Phi} \)). With such a heavy gravitino, the NLSP would be very long-lived, with a decay length much larger than the size of the detectors. The NLSP would appear to be stable in the collider environment. This would imply that the standard missing-energy technique should be applicable for the searches if \( \tilde{\chi}_1^0 \) is the NLSP, while a heavy charged track in the detector would be the signal for \( \tilde{\tau}_R \) as the NLSP.

The mass spectrum scales linearly with the parameter \( \Lambda \). For \( \Lambda = 30 \text{ TeV} \), we have a relatively light mass spectrum, which can be accessed by next generation collider experiments, while for \( \Lambda = 100 \text{ TeV} \), most sparticles are probably not easy to be produced except for \( h, \tilde{\chi}_1^0 \) and \( \tilde{\ell}_R \). One can also interpolate the mass spectrum for the \( \Lambda \)-parameter in between.

**Further Remarks:** Concerning the schemes in preserving the gauge coupling unification beyond the MSSM particle contents at a high energy scale, we would like to re-emphasize that adding in more states in complete representations of the GUT group would automatically keep the unification without changing the GUT scale; while introducing the (matching pair) adjoint representations of the SM gauge group would also keep the unification, but generally shift the GUT scale, depending on the mass threshold of the adjoint states. This is applicable even beyond the specific model under discussion, namely, aiming only at \( M_{str} \). In principle, the gauge coupling unification could occur at a different scale, regardless of the heterotic string prediction \( M_{str} \). However, our idea with the adjoint states beyond the MSSM as messengers at a high \( M \)-scale, which help preserve the unification, can be tested against the distinctive sparticle spectrum prediction by future collider experiments.

Regarding the origin of the adjoint messengers \( \Sigma_8 \) and \( \Sigma_3 \), we noted that they may be identified as the remnants resulting from certain realistic string models as continuous moduli [3]. In fact, although highly model-dependent, there are often other vector-like representations which could provide the \( \Phi_5 + \bar{\Phi}_5 \) states in \( SU(5) \) as well [2]. Along the similar line, an attempt [11] has been made in which the messenger
sector consists solely of color triplets, arising from the Wilson-line breaking of unifying non-Abelian gauge symmetries in string models. However, this model predicted a very light sparticle spectrum that has been excluded by the LEP-II experiments.

Although typical GMSB models are generally lack of satisfactory cold dark matter candidates [11], a stable heavy particle associated with our messenger sector may provide a superheavy dark matter candidate with \( M = O(10^{14} \text{ GeV}) \) [12]. More investigation in this regard is needed before drawing a conclusion.

**Conclusion:** We have introduced the adjoint messengers \( \Sigma_8 \) and \( \Sigma_3 \) for gauge-mediation of SUSY breaking. These new messengers lift the gauge coupling unification to the string scale at \( O(5 \times 10^{17} \text{ GeV}) \) if their masses are at \( O(10^{14} \text{ GeV}) \). This proposed “marriage” may have profound implication: some remnant states in certain realistic string models may serve as the messengers for gauge-mediation of SUSY breaking. The model is highly predictive and restrictive. The predicted SUSY mass spectrum at the electroweak scale is significantly different from those in other GMSB and mSUGRA models, resulting in experimentally testable robust mass relations. The gravitino mass is predicted to be approximately \( 1 - 10 \text{ GeV} \). Consequently, the NLSP appears to be very long-lived and would only decay outside the detector in the collider environment. The very heavy stable particle associated with the messenger sector may also provide a superheavy dark matter candidate.

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