Vector Manifestation in Hot and/or Dense Matter *)

Masayasu HARADA

Department of Physics, Nagoya University, Nagoya 464-8602, JAPAN
School of Physics, Seoul National University, Seoul 151-742, KOREA

In this talk I summarize main features of the vector manifestation (VM) which was recently proposed as a novel manifestation of the Wigner realization of chiral symmetry in which the symmetry is restored at the critical point by the massless degenerate pion (and its flavor partners) and the $\rho$ meson (and its flavor partners) as the chiral partner. I show how the VM is realized in hot and dense QCD using the effective field theory of QCD based on the hidden local symmetry.

§1. Introduction

It is a very interesting subject to study the phase structure of QCD and the change of hadron properties in hot and/or dense matter. [See, for reviews, e.g., Refs. [6), 7), 8), 9), 10), 11), 12).] Especially, the light vector meson is important for analyzing the dilepton spectra in experimental facilities such as the BNL Relativistic Heavy Ion Colider (RHIC). In Refs. [13] and [8] it was proposed that the $\rho$-meson mass scales like the pion decay constant and vanishes at the chiral phase transition point in hot and/or dense matter (Brown-Rho scaling).

Recently, in Ref. [1], the vector manifestation (VM) was proposed as a novel manifestation of the Wigner realization of chiral symmetry in which the symmetry is restored at the critical point by the degenerate pion (pion and its flavor partners) and $\rho$ ($\rho$ meson and its flavor partners) as the chiral partner, in sharp contrast to the traditional manifestation à la the linear sigma model where the symmetry is restored by the degenerate pion and the scalar meson. It was shown that the VM is realized in the large flavor QCD by using the hidden local symmetry (HLS) model which is an effective field theory for $\pi$ and $\rho$ based on the chiral symmetry of QCD. Then, it was further shown that the VM can occur in the chiral restoration in hot QCD$^2$ and in dense QCD$^3$ where an essential role was played by the intrinsic temperature and density dependences of the parameters of the HLS Lagrangian determined by extending the Wilsonian matching between the HLS and the underlying QCD$^4$ to the one in hot and dense matter. Moreover, several predictions from the VM in hot matter are made in Refs. [4), 5).

In this talk I summarize main features of the VM, especially compared with the conventional manifestation à la the linear sigma model, and then show how the VM is realized in hot and/or dense matter by formulating the VM in the effective field theory for $\pi$ and $\rho$ based on the HLS.

*) Talk given at YITP-RCNP Workshop “Chiral Restoration in Nuclear Medium” (October 7-9,2002).

This talk is based on the works done in Refs. [1), 2), 3), 4), 5).
This report is organized as follows: In section 2, I summarize a difference between the VM and the conventional linear sigma model like manifestation in terms of the chiral representations of low-lying mesons. In section 3, I show the effective field theory of QCD based on the HLS and present the renormalization group equations for the parameters of the HLS Lagrangian. In section 4, I show the Wilsonian matching conditions which determine the intrinsic temperature and/or density dependences of the bare parameters of the HLS Lagrangian in terms of the parameters of the operator product expansion in QCD, and derive the constraints on the parameters at the critical point. Then, I show how the VM is realized in hot and dense matter in section 5. Finally, I give a brief summary in section 6. Several functions used in section 5 are summarized in Appendix A.

§2. Vector Manifestation

In this section I briefly explain some features of the vector manifestation (VM). The VM was first proposed in Ref. 1) as a novel manifestation of Wigner realization of chiral symmetry where the vector meson ρ becomes massless at the chiral phase transition point. Accordingly, the (longitudinal) ρ becomes the chiral partner of the Nambu-Goldstone boson π.

The VM is characterized by

\[
F_\rho^2 \rightarrow 0, \quad m_\rho^2 \rightarrow m_\pi^2 = 0, \quad F_\rho^2 / F_\pi^2 \rightarrow 1, \quad (2.1)
\]

where \( F_\rho \) is the decay constant of (longitudinal) ρ at ρ on-shell. This is completely different from the conventional picture based on the linear sigma model (I call this GL manifestation after the effective theory of Ginzburg–Landau or Gell-Mann–Levy.) where the scalar meson becomes massless degenerate with π as the chiral partner. Here, I discuss the difference between the VM and the GL manifestation in terms of the chiral representation of the mesons by extending the analyses done in Refs. 17), 18). Following Ref. 17), I define the axialvector coupling matrix \( X_a(\lambda) \) by giving the matrix elements at zero invariant momentum transfer of the axialvector current between states with collinear momenta as

\[
\langle \vec{q}\lambda'\beta' | (J_{5a}^0 + J_{5a}^3) | \vec{p}\lambda\alpha \rangle = 2E \delta_{\lambda\lambda'} [X_a(\lambda)]_{\beta\alpha}, \quad (2.2)
\]

where \( \alpha \) and \( \beta \) are one-particle states with momentum \( \vec{p} \) and \( \vec{q} \) in 3 direction, \( \lambda \) and \( \lambda' \) are their helicities. The quantity \( E \) is defined by writing the condition of zero invariant momentum transfer as

\[
|\vec{p}| + \sqrt{|\vec{p}|^2 + m_\alpha^2} = |\vec{q}| + \sqrt{|\vec{q}|^2 + m_\beta^2} \equiv E. \quad (2.3)
\]

It was stressed 17) that the definition of the axialvector couplings in Eq. (2.2) can be used for particles of arbitrary spin, and in arbitrary collinear reference frames, including both the frames in which \( \alpha \) is at rest and in which it moves with infinite momentum. As was done in Ref. 17), considering the forward scattering process \( \pi_a + \alpha(\lambda) \rightarrow \pi_b + \beta(\lambda') \) and requiring the cancellation of the terms in the t-channel, I obtain

\[
[X_a(\lambda), X_b(\lambda)] = if_{abc} T_c, \quad (2.4)
\]
where $T_c$ is the generator of $SU(N_f)_V$ and $f_{abc}$ is the structure constant. It should be noticed that Eq. (2.4) tells us that the one-particle states of any given helicity must be assembled into representations of chiral $SU(N_f)_L \times SU(N_f)_R$. Furthermore, since Eq. (2.4) does not give any relations among the states with different helicities, those states can generally belong to the different representations even though they form a single particle such as the longitudinal $\rho (\lambda = 0)$ and the transverse $\rho (\lambda = \pm 1)$. Thus, the notion of the chiral partners can be considered separately for each helicity.

Let me consider the algebraic relation (2.4) for zero helicity ($\lambda = 0$) states and saturate it by low-lying mesons; the $\pi$, the (longitudinal) $\rho$, the (longitudinal) axialvector meson denoted by $A_1$ ($a_1$ meson and its flavor partners) and the scalar meson, and so on. It should be noticed that, in the broken phase of chiral symmetry, the Hamiltonian matrix defined by the matrix elements of the Hamiltonian between states does not commute with the axialvector coupling matrix. Then, the algebraic representations of the axialvector coupling matrix do not coincide with the mass eigenstates: There occur representation mixings. Actually, $\pi$ and the longitudinal $A_1$ are admixture of $(8, 1) \oplus (1, 8)$ and $(3, 3^*) \oplus (3^*, 3)$:

$$|\pi\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \sin \psi + |(8, 1) \oplus (1, 8)\rangle \cos \psi,$$

$$|A_1(\lambda = 0)\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \cos \psi - |(8, 1) \oplus (1, 8)\rangle \sin \psi,$$

where the experimental value of the mixing angle $\psi$ is given by approximately $\psi = \pi/4$. On the other hand, the longitudinal $\rho$ belongs to pure $(8, 1) \oplus (1, 8)$ and the scalar meson to pure $(3, 3^*) \oplus (3^*, 3)$.

When the chiral symmetry is restored at the phase transition point, the axialvector coupling matrix commutes with the Hamiltonian matrix, and thus the chiral representations coincide with the mass eigenstates: The representation mixing is dissolved. From Eq. (2.5) one can easily see that there are two ways to express the representations in the Wigner phase of the chiral symmetry: The conventional GL manifestation corresponds to the limit $\psi \to \pi/2$ in which $\pi$ is in the representation of pure $(3, 3^*) \oplus (3^*, 3)$ together with the scalar meson, while the VM to the limit $\psi \to 0$ in which the $A_1$ goes to a pure $(3, 3^*) \oplus (3^*, 3)$, now degenerate with the scalar meson in the same representation, but not with $\rho$ in $(8, 1) \oplus (1, 8)$. Namely, the degenerate massless $\pi$ and (longitudinal) $\rho$ at the phase transition point are the chiral partners in the representation of $(8, 1) \oplus (1, 8)$.

§3. Effective Field Theory

In this section I show the effective field theory in which the vector manifestation is formulated. I should note that, as is stressed in Ref. [19], the VM can be formulated only as a limit by approaching it from the broken phase of chiral symmetry. Then, for the formulation of the VM I need an effective field theory (EFT) including $\rho$ and $\pi$ in the broken phase which is not necessarily applicable in the symmetric phase. One of such EFTs is the model based on the hidden local symmetry (HLS) which includes $\rho$ as the gauge boson of the HLS in addition to $\pi$ as the NG boson associated with the chiral symmetry breaking in a manner fully consistent with the chiral symmetry of QCD. It should be noticed that, in the HLS, thanks to the gauge
invariance one can perform the systematic chiral perturbation with including $\rho$ in addition to $\pi$ [20], [21], [22], [16], [19]. In subsection 3.1, I will explain the model based on the HLS, and then summarize the renormalization group equations for the parameters of the HLS Lagrangian in subsection 3.2.

3.1. Hidden Local Symmetry

Let me describe the HLS model based on the $G_{\text{global}} \times H_{\text{local}}$ symmetry, where $G = SU(N_f)_L \times SU(N_f)_R$ is the global chiral symmetry and $H = SU(N_f)_V$ is the HLS. The basic quantities are the gauge boson $\rho_\mu$ and two variables

$$\xi_{L,R} = e^{i\sigma/F_\sigma} e^{i\pi/F_\pi},$$

where $\pi$ denotes the pseudoscalar NG boson and $\sigma$ the NG boson absorbed into $\rho_\mu$ (longitudinal $\rho$). $F_\pi$ and $F_\sigma$ are relevant decay constants, and the parameter $a$ is defined as $a \equiv F_\sigma^2/F_\pi^2$. The transformation properties of $\xi_{L,R}$ are given by

$$\xi_{L,R}(x) \rightarrow \xi'_{L,R}(x) = h(x)\xi_{L,R}(x)g^\dagger_{L,R},$$

where $h(x) \in H_{\text{local}}$ and $g_{L,R} \in G_{\text{global}}$. The covariant derivatives of $\xi_{L,R}$ are defined by

$$D_\mu \xi_L = \partial_\mu \xi_L - ig_{\rho_\mu} \xi_L + i\xi_L \mathcal{L}_\mu,$$

$$D_\mu \xi_R = \partial_\mu \xi_R - ig_{\rho_\mu} \xi_R + i\xi_R \mathcal{R}_\mu,$$

where $g$ is the HLS gauge coupling, and $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ denote the external gauge fields gauging the $G_{\text{global}}$ symmetry.

The HLS Lagrangian at the leading order is given by

$$\mathcal{L} = F_\pi^2 \text{tr} \left[ \hat{\sigma}_\perp \alpha_\mu^\perp \right] + F_\sigma^2 \text{tr} \left[ \hat{\pi}_\parallel \alpha_\mu^\parallel \right] + \mathcal{L}_{\text{kin}}(\rho_\mu),$$

where $\mathcal{L}_{\text{kin}}(\rho_\mu)$ denotes the kinetic term of $\rho_\mu$ and

$$\hat{\alpha}_\perp^\mu = (D_\mu \xi_R \cdot \xi^\dagger_R + D_\mu \xi_L \cdot \xi^\dagger_L)/(2i).$$

When the kinetic term $\mathcal{L}_{\text{kin}}(\rho_\mu)$ is ignored in the low-energy region, the second term of Eq. (3.4) vanishes by integrating out $\rho_\mu$ and only the first term remains. Then, the HLS model is reduced to the nonlinear sigma model based on $G/H$.

In Refs. [8], [12], it was pointed out that the quasiquark picture is appropriate near the phase transition point, and that quasiquark mass approaches zero. Then, following Refs. [8], [9], I introduce the quasiquark field $\psi$ near the critical point into the Lagrangian in addition to $\rho$ and $\pi$. A chiral Lagrangian for $\pi$ with the constituent quark (quasiquark) was given in Ref. [23]. In Ref. [15] the quasiquark field $\psi$ is introduced into the HLS Lagrangian in such a way that it transforms homogeneously under the HLS: $\psi \rightarrow h(x) \cdot \psi$ where $h(x) \in H_{\text{local}}$. In Ref. [3] the Lagrangian of Ref. [13] was extended to a general one with which a systematic derivative expansion

\[^{\ast}\) Note that this $\sigma$ is different from the scalar meson in the linear sigma model.
can be performed. Since the model is introduced near the chiral phase transition point where the quasiquark mass is expected to become small, the quasiquark mass \( m_q \) is counted as \( \mathcal{O}(p) \). Furthermore, \( \mathcal{O}(p) \) is assigned to the chemical potential \( \mu \) or the Fermi momentum \( P_F \) as well as to the temperature \( T \), as the cutoff is considered to be larger than \( \mu \) and \( T \) even near the phase transition point. By using this counting scheme the systematic expansion can be performed in the HLS with the quasiquark included. I should note that this counting scheme is different from the one in the model for \( \pi \) and baryons given in Ref. 24) where the baryon mass is counted as \( \mathcal{O}(1) \). The leading order Lagrangian including one quasiquark field and one anti-quasiquark field is counted as \( \mathcal{O}(p) \) and given by

\[
\delta \mathcal{L}_Q = \bar{\psi}(x)(iD_\mu \gamma^\mu + \mu \gamma^0 - m_q)\psi(x) + \bar{\psi}(x) \left( \kappa \gamma^\mu \alpha_{\parallel \mu}(x) + \lambda \gamma^5 \gamma^\mu \alpha_{\perp \mu}(x) \right)\psi(x) \tag{3.6}
\]

where \( D_\mu \psi = (\partial_\mu - ig\gamma_5)\psi \) and \( \kappa \) and \( \lambda \) are constants to be specified later.

3.2. Renormalization Group Equations

At one-loop level the Lagrangian [3,4] plus (3.6) generates the \( \mathcal{O}(p^4) \) contributions including hadronic thermal/dense-loop effects as well as divergent effects. The divergent contributions are renormalized by the parameters, and thus the RGEs for three leading order parameters \( F_\pi \), \( a \) and \( g \) (and parameters of \( \mathcal{O}(p^3) \) Lagrangian) are modified from those without quasiquark field. In addition, I need to consider the renormalization group flow for the quasiquark mass \( m_q \) [4]. By calculating one-loop contributions for RGEs in an energy scale \( \mathcal{M} \) for a given temperature \( T \) and chemical potential \( \mu \), the RGEs are expressed as

\[
\mathcal{M} \frac{dF_\pi^2}{d\mathcal{M}} = C \left[ 3a^2 g^2 F_\pi^2 + 2(2 - a) \mathcal{M}^2 \right] - \frac{m_q^2}{2\pi^2} \lambda^2 N_c ,
\]

\[
\mathcal{M} \frac{da}{d\mathcal{M}} = -C(a - 1) \left[ 3a(1 + a)g^2 - (3a - 1) \mathcal{M}^2 \right] + a \left[ \frac{\lambda^2 m_q^2}{2\pi^2 F_\pi^2} \right] N_c ,
\]

\[
\mathcal{M} \frac{dg^2}{d\mathcal{M}} = -C \left[ \frac{87}{6} - a^2 \right] g^4 + \frac{N_c}{6\pi^2} g^4 (1 - \kappa)^2 ,
\]

\[
\mathcal{M} \frac{dm_q}{d\mathcal{M}} = -m_q/8\pi^2 \left[ (C_\pi - C) \mathcal{M}^2 - m_q^2 (C_\pi - C_\sigma) + M_\rho^2 C_\sigma - 4C_\rho \right] ,
\]

where \( C = N_f/2(4\pi)^2 \) and

\[
C_\pi = \frac{\lambda^2 N_f^2 - 1}{2N_f} , \quad C_\sigma = \frac{\kappa^2 N_f^2 - 1}{2N_f} , \quad C_\rho = \frac{g^2 (1 - \kappa)^2 N_f^2 - 1}{2N_f} . \tag{3.7}
\]

It should be noted that the point \((g, a, m_q) = (0, 1, 0)\) is the fixed point of the RGEs in Eq. (3.7) which plays an essential role to realize the VM in the following analysis of the chiral restoration in hot and/or dense QCD.

\[\text{---\textsuperscript{a)} The constants \( \kappa \) and \( \lambda \) will also run. The running will be small near the critical point, so I will ignore their running here.}\]
§4. Wilsonian Matching and Intrinsic Temperature/Density Dependence

The Wilsonian matching proposed in Ref. [16] is done by matching the axial-vector and vector current correlators derived from the HLS with those by the operator product expansion (OPE) in QCD at the matching scale $\Lambda$. This was extended to non-zero temperature [12] and density [3] and it was shown that the parameters of the HLS Lagrangian have the intrinsic temperature and/or density dependences. In this section I summarize the Wilsonian matching conditions and the resultant conditions (VM conditions) for the bare parameters of the HLS at the phase transition point.

In general there is no longer Lorentz symmetry in hot and/or dense matter, and the Lorentz non-scalar operators such as $\bar{q}\gamma_{\mu}D_{\nu}q$ may exist in the form of the current correlators derived by the OPE. [see, e.g., Ref. [26]] However, we neglect these contributions since they give a small correction compared with the main term of the form $1 + \alpha_s \pi$. In this approximation the axial-vector and vector correlators in the OPE are expressed by the same form as used in Ref. [27] with putting possible temperature and/or density dependences on the gluonic and quark condensates:

$$G_{(QCD)}^A(Q^2; T, \mu) = \frac{1}{8\pi^2} \left[ -\frac{1}{\pi} \ln \frac{Q^2}{M_Q^2} + \frac{\pi^2}{3} \frac{(\frac{\alpha_s}{\pi} G_{\mu\nu})_{T,\mu}}{Q^4} + \frac{\pi^3}{3} \frac{1408 \alpha_s \langle \bar{q}q \rangle_{T,\mu}^2}{27 Q^6} \right],$$

$$G_{(QCD)}^V(Q^2; T, \mu) = \frac{1}{8\pi^2} \left[ -\frac{1}{\pi} \ln \frac{Q^2}{M_Q^2} + \frac{\pi^2}{3} \frac{(\frac{\alpha_s}{\pi} G_{\mu\nu})_{T,\mu}}{Q^4} - \frac{\pi^3}{3} \frac{896 \alpha_s \langle \bar{q}q \rangle_{T,\mu}^2}{27 Q^6} \right], \tag{4.1}$$

where $M_Q$ is the renormalization point of QCD. Consistently with the approximation adopted for the above current correlators in the OPE, I use the Lorentz invariant form of the HLS Lagrangian. It should be noted that the quasiquark does not contribute to the current correlators at bare level. Then, the axial-vector and the vector current correlators in the HLS around the matching scale $\Lambda$ are well described by the same forms as those at $T = \mu = 0$ with the bare parameters having the intrinsic temperature and/or density dependences:

$$G_{A}^{(HLS)}(Q^2; T, \mu) = \frac{F_A^2(A; T, \mu)}{Q^2} - 2z_2(A; T, \mu),$$

$$G_{V}^{(HLS)}(Q^2; T, \mu) = \frac{F_V^2(A; T, \mu)[1 - 2g^2(A; T, \mu)z_3(A; T, \mu)]}{M_{\rho}^2(A; T, \mu) + Q^2} - 2z_1(A; T, \mu). \tag{4.2}$$

The Wilsonian matching conditions are obtained by setting the above correlators to be equal to those in Eq. (4.1) at $\Lambda$ up until the first derivative. Then the resultant forms of the Wilsonian matching conditions at non-zero temperature and/or non-zero
density are expressed as

\[
\frac{F_\pi^2(A;T,\mu)}{A^2} - \frac{F_\sigma^2(A;T,\mu)}{A^2} \left[ 1 - 2g^2(A;T,\mu)z_3(A;T,\mu) \right] = 2 \left[ z_2(A;T,\mu) - z_1(A;T,\mu) \right]
\]

\[
= \frac{32\pi}{9} \frac{\alpha_s(q\bar{q}/T,\mu)}{A^6},
\]

(4.3)

\[
\frac{F_\pi^2(A;T,\mu)}{A^2} = \frac{1}{8\pi^2} \left[ 1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \left( \frac{\alpha_s}{\pi} g^2 \right)_T,\mu + \frac{\alpha_s}{A^4} \frac{1408}{27} \frac{\alpha_s(q\bar{q}/T,\mu)}{A^6} \right],
\]

(4.4)

\[
\frac{F_\sigma^2(A;T,\mu)}{A^2} \left[ 1 - 2g^2(A;T,\mu)z_3(A;T,\mu) \right] = \frac{1}{8\pi^2} \left[ 1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \left( \frac{\alpha_s}{\pi} g^2 \right)_T,\mu - \frac{\alpha_s}{A^4} \frac{896}{27} \frac{\alpha_s(q\bar{q}/T,\mu)}{A^6} \right].
\]

(4.5)

Through these conditions the dependences of the quark and gluonic condensates on the temperature and/or the chemical potential determine the intrinsic temperature and/or density dependences of the bare parameters of the HLS Lagrangian, which are then converted into those of the on-shell parameters through the Wilsonian RGEs.

As a result the parameters appearing in the hadronic thermal/dense-loop corrections have the intrinsic temperature and/or density dependences: \( F_\pi, a \) and \( g \) appearing there should be regarded as

\[
F_\pi \equiv F_\pi(\mathcal{M} = 0; T, \mu),
\]

\[
a \equiv a(\mathcal{M} = M_\rho; T, \mu),
\]

\[
g \equiv g(\mathcal{M} = M_\rho; T, \mu),
\]

(4.6)

where the parametric mass \( M_\rho \) is determined from the on-shell condition:

\[
M_\rho^2 \equiv M_\rho^2(T, \mu) = a(\mathcal{M} = M_\rho; T, \mu) g^2(\mathcal{M} = M_\rho; T, \mu) F_\pi^2(\mathcal{M} = M_\rho; T, \mu).
\]

(4.7)

Now, let me consider the Wilsonian matching near the chiral symmetry restoration point with assuming that the quark condensate becomes zero continuously for \((T, \mu) \to (T_c, \mu_c)\). First, note that the Wilsonian matching condition (4.4) provides

\[
\frac{F_\pi^2(A;T_c,\mu_c)}{A^2} = \frac{1}{8\pi^2} \left[ 1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \left( \frac{\alpha_s}{\pi} g^2 \right)_T,\mu_c \right] \neq 0,
\]

(4.8)

which implies that the matching with QCD dictates

\[
F_\pi^2(A;T_c,\mu_c) \neq 0
\]

(4.9)

*) One might think that there appear corrections from \( \rho \) and/or \( \pi \) loops in the left-hand-sides of Eqs. (4.4) and (4.5). However, such corrections are of higher order in the present counting scheme, and thus I neglect them here at \( Q^2 \sim A^2 \). In the low-energy scale I incorporate the loop effects into the correlators through the renormalization group equations.
even at the critical point where the on-shell π decay constant vanishes by adding the quantum corrections through the RGE including the quadratic divergence and hadronic thermal/dense-loop corrections. Second, we note that the axialvector and vector current correlators $G_\pi^{(QCD)}$ and $G_V^{(QCD)}$ derived by the OPE agree with each other for any value of $Q^2$. Thus we require that these current correlators in the HLS in Eq. (4.2) are equal at the critical point for any value of $Q^2$ around $\Lambda^2$. By taking account of the fact

$$F_\pi^2(\Lambda; T_c, \mu_c) \neq 0$$

derived from the Wilsonian matching condition given in Eq. (4.8), the requirement $G_A^{(HLS)} = G_V^{(HLS)}$ is satisfied only if the following conditions are met:

$$g(\Lambda; T, \mu) \rightarrow (T, \mu) \rightarrow (T_c, \mu_c) 0 \quad (4.10)$$

$$a(\Lambda; T, \mu) \rightarrow (T, \mu) \rightarrow (T_c, \mu_c) 1 \quad (4.11)$$

$$z_1(\Lambda; T, \mu) - z_2(\Lambda; T, \mu) \rightarrow (T, \mu) \rightarrow (T_c, \mu_c) 0 \quad (4.12)$$

As in Refs. 8), 12), I expect that the quasiquark mass vanishes at the chiral restoration point. Since $m_q = 0$ itself is a fixed point of the RGE for $m_q$ in Eq. (3.7), $(g, a, m_q) = (0, 1, 0)$ is a fixed point of the coupled RGEs for $g$, $a$ and $m_q$. Then, the VM conditions $g = 0$ and $a = 1$ obtained for the bare parameters remain intact in the low-energy region including the on-shell of $\rho$:

$$g(M = M_\rho; T, \mu) \rightarrow (T, \mu) \rightarrow (T_c, \mu_c) 0 \quad (4.13)$$

and

$$a(M = M_\rho; T, \mu) \rightarrow (T, \mu) \rightarrow (T_c, \mu_c) 1 \quad (4.13)$$

where the parametric $\rho$ mass $M_\rho = M_\rho(T, \mu)$ is determined from the condition (4.7). The above conditions with Eq. (4.7) imply that the parametric mass $M_\rho(T, \mu)$ also vanishes:

$$M_\rho(T, \mu) \rightarrow (T, \mu) \rightarrow (T_c, \mu_c) 0 \quad (4.14)$$

§5. Vector Manifestation in Hot and Dense Matter

In the previous section I have shown that the parametric $\rho$ mass $M_\rho$ vanishes at the chiral restoration point due to the intrinsic temperature/density dependences obtained from the Wilsonian matching between the HLS and the OPE. For obtaining the pole mass of $\rho$, I need to included the hadronic thermal/dense-loop effects. So far, the hadronic thermal/dense-loop corrections were calculated for two cases; $T > 0$ with $\mu = 0$ and $\mu > 0$ with $T = 0$. In this section I summarize the resultant expressions of the hadronic thermal and dense loop corrections to the $\rho$ pole mass in two cases, and then show how the vector manifestation (VM) is realized in hot and dense matter.

5.1. Vector meson mass in hot matter

In Ref. 2) the VM in hot matter was shown to take place by using the hadronic thermal correction from the $\pi$ and $\rho$ to the $\rho$ pole mass calculated in the Landau
gauge.\cite{92,93} In Refs.\cite{92,93}, the background field gauge was adopted and the contribution from the quasiquark was included further. Here, following Refs.\cite{94,95,1}, I explain how the VM is realized in hot QCD.

I define pole masses of longitudinal and transverse modes of \( \rho \) from the poles of longitudinal and transverse components of the vector current correlator at rest frame. Hadronic thermal corrections from the \( \pi, \rho \) and quasiquark to the pole masses were calculated at one-loop level in Ref.\cite{96}, and it was shown that, at the rest frame, the longitudinal pole mass agrees with the transverse one. The resultant expression for the \( \rho \) pole mass is obtained as\cite{97}

\[
[m^L_\rho(T)]^2 = [m^T_\rho(T)]^2 \equiv m_\rho^2(T) \\
= M_\rho^2 + N_f g^2 \left[ \frac{a^2}{12} \tilde{G}_{2(B)}(M_\rho;T) + \frac{4}{5} \tilde{J}^2_{1(B)}(M_\rho;T) + \frac{33}{16} M_\rho^2 \tilde{F}^2_{3(B)}(M_\rho;M_\rho;T) \right] \\
+ N_c g^2 (1 - \kappa)^2 \left[ 4 \tilde{J}^2_{1(F)}(m_q;T) - \frac{7m_q^2 - M_\rho^2}{6} \tilde{F}^2_{3(F)}(M_\rho;m_q;T) \right],
\]  

(5.1)

where the explicit forms of the functions expressing the bosonic corrections \([\tilde{G}_{2(B)}, \tilde{J}^2_{1(B)}\) and \(\tilde{F}^2_{3(B)}\)] and fermionic corrections \([\tilde{J}^2_{1(F)}\) and \(\tilde{F}^2_{3(F)}\)] are listed in Appendix A.

Now, let me study the \( \rho \) pole mass near the critical temperature. As shown in section 4, the intrinsic temperature dependences of the parameters of the HLS Lagrangian determined from the Wilsonian matching imply that the parametric \( \rho \) mass \( M_\rho \) vanishes at the critical temperature. Furthermore, I expect that the quasiquark mass \( m_q \) also vanishes as was shown in, e.g., Refs.\cite{98,12}. Then, near the critical temperature I should take \( M_\rho \ll T \) and \( m_q \ll T \) in Eq. (5.1). By noting that

\[
\begin{align*}
\tilde{G}_{2(B)}(M_\rho;T) & \xrightarrow{M_\rho \rightarrow 0} \tilde{I}_{2(B)}(T), \\
\tilde{J}^2_{1(B)}(M_\rho;T) & \xrightarrow{M_\rho \rightarrow 0} \tilde{I}_{2(B)}(T), \\
M_\rho^2 \tilde{F}^2_{3(B)}(M_\rho;m_\rho;T) & \xrightarrow{M_\rho \rightarrow 0} 0, \\
\tilde{J}^2_{1(F)}(m_q;T) & \xrightarrow{m_q \rightarrow 0} \tilde{I}_{2(F)}(T), \\
m_q^2 \tilde{F}^2_{3(F)}(M_\rho;m_q;T) & \xrightarrow{M_\rho \rightarrow 0, m_q \rightarrow 0} 0, \\
M_\rho^2 \tilde{F}^2_{3(F)}(M_\rho;m_q;T) & \xrightarrow{M_\rho \rightarrow 0, m_q \rightarrow 0} 0,
\end{align*}
\]  

(5.2)

the pole mass of the vector meson at \( T \sim T_c \) becomes

\[
m^2_\rho(T) = M_\rho^2 + N_f g^2 \frac{15 - a^2}{144} T^2 + N_c g^2 (1 - \kappa)^2 \frac{1}{18} T^2.
\]  

(5.3)

This shows that the hadronic thermal effect gives a positive correction in the vicinity of \( a \simeq 1 \), and then the \( \rho \) pole mass is actually larger than the parametric mass \( M_\rho \). However, the intrinsic temperature dependences of the parameters obtained in
section 4 lead to \( g \to 0 \) and \( M_\rho \to 0 \) for \( T \to T_c \). Then, from Eq. (5.3) it was concluded that the \( \rho \) pole mass \( m_\rho \) vanishes at the critical temperature:

\[
m_\rho(T) \to 0 \quad \text{for } T \to T_c .
\] (5.4)

This implies that the VM is realized at the critical temperature.

5.2. Vector meson mass in dense matter

In this subsection, following Ref. 3, I briefly review how the VM is realized in dense QCD.

In Ref. 3 the pole masses of longitudinal and transverse modes of \( \rho \) is defined from the poles of longitudinal and transverse components of the vector current correlator at rest frame, and it was shown that the hadronic dense loop corrections from the quasiquark to them agree with each other at one-loop level. The resultant expression for the \( \rho \) pole mass is expressed as

\[
\left[ m_\rho^L(\mu) \right]^2 = \left[ m_\rho^T(\mu) \right]^2 \equiv m_\rho^2(\mu)
= M_\rho^2 + \frac{2}{3} g^2 (1 - \kappa)^2 \left[ \bar{B}_S - (M_\rho^2 + 2m_q^2) \text{Re} \bar{B}_0(M_\rho) \right] ,
\] (5.5)

where the functions \( \bar{B}_S \) and \( \bar{B}_0(M_\rho) \) are defined in Eq. (A.5) in Appendix A. Near \( \mu \simeq \mu_c \) by taking \( M_\rho \ll \mu \) and \( m_q \ll \mu \), this expression becomes

\[
m_\rho^2(\mu) = M_\rho^2 + \frac{\mu^2}{6\pi^2} (1 - \kappa)^2 ,
\] (5.6)

which shows that the \( \rho \) pole mass \( m_\rho \) is larger than the parametric mass \( M_\rho \) due to the hadronic dense-loop correction. However, the intrinsic density dependences of the parameters derived from the Wilsonian matching in section 4 imply that \( g \to 0 \) and \( M_\rho \to 0 \) for \( \mu \to \mu_c \). Then, from Eq. (5.6), it was concluded that the vector meson pole mass vanishes at the critical density:

\[
m_\rho(\mu) \to 0 \quad \text{for } \mu \to \mu_c .
\] (5.7)

This implies that the VM is realized at the critical density.

\[\textbf{§6. Summary}\]

In this talk I first summarized a main feature of the vector manifestation (VM) proposed in Ref. 1 as a novel manifestation of Wigner realization of chiral symmetry in which the symmetry is restored at the critical point by the massless degenerate \( \pi \) (pion and its flavor partners) and \( \rho \) (\( \rho \) meson and its flavor partners) as the chiral partner, in sharp contrast to the traditional manifestation à la the linear sigma model where the symmetry is restored by the degenerate pion and scalar meson. Then, determining the intrinsic temperature and density dependences of the bare parameters of the Lagrangian of the hidden local symmetry (HLS) and including the hadronic thermal and dense loop corrections to the \( \rho \) pole mass, I have shown how the VM is realized in hot and dense matter. In this talk, I did not show the
details of the calculations on the hadronic thermal and dense loop corrections which can be seen in Refs. [3] and [4].

In Refs. [2], [3], [4], [5], based on the VM in hot matter, several predictions on the physical quantities were made: the value of the critical temperature is expressed in terms of the parameters in the OPE; the vector susceptibility actually agree with the axialvector susceptibility at the critical temperature taking non-zero value; the velocity of $\pi$ approaches the speed of light near the critical temperature; the vector dominance of the electromagnetic form factor of pion is largely violated at the critical temperature; and so on. I did not present those interesting results in this talk due to the lack of time.

Acknowledgments

I would like to thank Doctor Youngman Kim, Professor Mannque Rho, Doctor Chihiro Sasaki and Professor Koichi Yamawaki for collaboration in the works on which this talk is based. I am very grateful to organizers for giving me an opportunity to present this talk. This work was supported in part by the Brain Pool program (#012-1-44) provided by the Korean Federation of Science and Technology Societies.

Appendix A

--- Functions ---

In this Appendix, I list the integral forms of the functions which appear in the expressions of hadronic thermal/dense-loop corrections to the $\rho$ pole mass.

Let me list the functions used in subsection 5.1 for expressing the hadronic thermal-loop corrections. The functions $\tilde{I}_{n(B,F)}(T)$ and $\tilde{J}_{m(B,F)}(M;T)$ ($n, m$: integers) are given by

$$
\tilde{I}_{n(B)}(T) = \int \frac{d^3k}{(2\pi)^3} \frac{|\vec{k}|^{n-3}}{e^{k/T} - 1} = \frac{1}{2\pi^2} \hat{I}_{n(B)} T^n ,
$$

$$
\tilde{I}_{n(F)}(T) = \int \frac{d^3k}{(2\pi)^3} \frac{|\vec{k}|^{n-3}}{e^{k/T} + 1} = \frac{1}{2\pi^2} \hat{I}_{n(F)} T^n ,
$$

$$
\hat{I}_{2(B)} = \frac{\pi^2}{6} , \quad \hat{I}_{4(B)} = \frac{\pi^4}{15} , \quad \hat{I}_{2(F)} = \frac{\pi^2}{12} ,
$$

(A.1)

$$
\tilde{J}_{m(B)}(M;T) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\omega/T} - 1} \frac{|\vec{k}|^{n-2}}{\omega^m} ,
$$

$$
\tilde{J}_{m(F)}(M;T) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\omega/T} + 1} \frac{|\vec{k}|^{n-2}}{\omega^m} ,
$$

(A.2)

where The functions $\tilde{F}_{3(B,F)}(p_0;M;T)$ and $\tilde{G}_{n(B)}(p_0;T)$ are defined as

$$
\tilde{F}_{3(B)}(p_0;M;T) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\omega/T} - 1} \frac{4|\vec{k}|^{n-2}}{\omega(4\omega^2 - p_0^2)} ,
$$

$$
\[
\tilde{F}^{n}_{3(F)}(p_0; M; T) = \int \frac{d^3k}{(2\pi)^3} e^{\omega/kT + 1 + \omega/(4\omega^2 - p_0^2)}, \quad (A.3)
\]

\[
\tilde{G}_{n(B)}(p_0; T) = \int \frac{d^3k}{(2\pi)^3} e^{k/T - 1} \frac{4|\vec{k}|^2}{|\vec{k}|^n - |\vec{k}|^3}, \quad (A.4)
\]

The functions appearing in the hadronic dense loop correction from the quasi-quark to the \(\rho\) pole mass are given by

\[
\tilde{B}_S = \frac{1}{4\pi^2} \left[ P_F \omega_F - m_q^2 \ln \frac{P_F + \omega_F}{m_q} \right],
\]

\[
\tilde{B}_0(M) = \frac{1}{8\pi^2} \left[ - \ln \frac{P_F + \omega_F}{m_q} + \frac{1}{2} \sqrt{4m_q^2 - M^2 - i\epsilon} \right. \\
\left. \times \ln \frac{\omega_F \left( \sqrt{4m_q^2 - M^2 - i\epsilon} + P_F \sqrt{-M^2 - i\epsilon} \right)}{\omega_F \left( \sqrt{4m_q^2 - M^2 - i\epsilon} - P_F \sqrt{-M^2 - i\epsilon} \right)} \right], \quad (A.5)
\]

where \(P_F\) is the Fermi momentum of the quasiquark and \(\omega_F \equiv \sqrt{P_F^2 + m_q^2}\). Note that, in the present analysis, I can take \(P_F = \sqrt{\mu^2 - m_q^2}\) and \(\omega_F = \mu\).

References

1) M. Harada and K. Yamawaki, Phys. Rev. Lett. 86, 757 (2001).
2) M. Harada and C. Sasaki, Phys. Lett. B 537, 280 (2002).
3) M. Harada, Y. Kim and M. Rho, Phys. Rev. D 66, 016003 (2002).
4) M. Harada, Y. Kim, M. Rho and C. Sasaki, arXiv:hep-ph/0207012.
5) M. Harada and C. Sasaki, in preparation.
6) T. Hatsuda and T. Kunihiro, Phys. Rept. 247, 221 (1994).
7) R. D. Pisarski, hep-ph/9503330.
8) G.E. Brown and M. Rho, Phys. Rept. 269, 333 (1996).
9) T. Hatsuda, H. Shiomizu and H. Kuwabara, Prog. Theor. Phys. 95, 1009 (1996).
10) R. Rapp and J. Wambach, Adv. Nucl. Phys. 25, 1 (2000).
11) F. Wilczek, hep-ph/0003183.
12) G. E. Brown and M. Rho, Phys. Rept. 363, 85 (2002).
13) G. E. Brown and M. Rho, Phys. Rev. Lett. 66, 2720 (1991).
14) M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. 54, 1215 (1985).
15) M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988).
16) M. Harada and K. Yamawaki, Phys. Rev. D 64 014023 (2001).
17) S. Weinberg, Phys. Rev. 177, 2604 (1969).
18) F. J. Gilman and H. Harari, Phys. Rev. 165, 1803 (1968).
19) M. Harada and K. Yamawaki, to appear in Phys. Rep.
20) H. Georgi, Phys. Rev. Lett. 63, 1917 (1989); Nucl. Phys. B 331, 311 (1990).
21) M. Harada and K. Yamawaki, Phys. Lett. B 297, 151 (1992).
22) M. Tanabashi, Phys. Lett. B 316, 534 (1993).
23) A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984).
24) U. G. Meißeuer, J. A. Oller and A. Wirzba, Annals Phys. 297, 27 (2002).
25) M. Harada and K. Yamawaki, Phys. Rev. Lett. 83, 3374 (1999).
26) T. Hatsuda, Y. Koike and S. Lee, Nucl. Phys. B 394, 221 (1993).
27) M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979); Nucl. Phys. B 147, 448 (1979).
28) M. Harada and A. Shibata, Phys. Rev. D 55, 6716 (1997).