Development of novel mathematical models of plate heat exchanger for the task of optimization main parameters of the aviation gas turbine engine with heat recovery

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Abstract. Nowadays, the task of increasing the economic and technical efficiency of the gas turbine engine and decrement of the specific weight and specific fuel consumption is one of the main targets in the development of the aviation gas turbine engine. A promising approach for decrement of specific fuel consumption and obtaining high thermal efficiency of the gas turbine engine (30% and above) is based on the concept of the recuperative cycle of the gas turbine engine. The article presents the computer-aided calculation for analysing the heat exchanger surface and developing novel mathematical models of heat exchanger in terms of weight goodness, flow passage goodness, and optimum weight and flow passage, furthermore, the study presents the novel mathematical models of the pressure drops in the air and gas channels of the heat exchanger. 18 different heat transfer surfaces for heat exchanger cores have been evaluated and the results are presented. The same hydraulic diameter assumed for all heat transfer surfaces and only their thermo-hydraulic performances evaluated during the calculation design. To assess the reliability of the obtained models, the results of the design calculations of the developed models compared with the data of other authors and with the data of the existing gas turbine engine with heat exchanger. The obtained models focus on mathematical design calculation for optimizing the main thermodynamic parameters of a gas turbine engine coupled with a heat exchanger at the stage of conceptual design of aviation gas turbine engine with heat recovery of exhaust gas.

1. Introduction
Aviation gas turbine engines reached a high degree of perfection. Nevertheless, the task of improving the technical and economic efficiency of the engine by reducing the specific fuel consumption and reducing the specific weight is still one of the main trends in the development of the aviation industry. A promising approach for reducing specific fuel consumption and obtaining high thermal efficiency of the gas turbine engine (30% and above) based on the concept of the regenerative cycle of the gas turbine engine [1-3]. However, the predicament of technical implementation for this development is relevant to the intricacy of the design, increasing the size and weight of the engine due to the installation of the heat exchanger. Hence, the design of the gas turbine engine with the heat exchanger is mandatory to take consideration not only the increase in fuel efficiency also takes to consideration of the deterioration of weight characteristics since these factors have the opposite influence on the efficiency of the power unit as a whole. Through the viewpoint of the weight goodness configuration and acceptable performance indicators demand further going to improvement of the methods for
calculating compact heat exchangers, analyzing the conditions for rational matching the parameters of the heat exchanger and the engine, researching new highly efficient types of heat exchange surfaces, optimizing the operating parameters of the engine with heat exchangers. The main requirements of the heat exchangers are minimum weight and volume, structural integrity, low hydraulic pressure loss, and high thermal efficiency and high reliability and durability [4–6].

The most preferred characteristic of aviation gas turbine heat exchangers is the plate-type recuperators (compact heat exchanger) [4,8]. According to the previous research, McDonald, C.F. et.al [4, 10–12], summarized the characteristics of the types of heat exchangers to the required heat exchanger characteristics for the application of gas turbine engines. Min, J.K. et al. [9], studied the proposed potential design solutions for the recuperator of aero-engine. According to their results, the primary surface of the heat exchanger has the advantage of relatively larger effectiveness. Hence, it is more possible to achieve minimum volume and weight on the high heat exchanger effectiveness.

Compact heat exchangers are relatively uncomplicated to manufacture and sufficiently promising in terms of the opportunity of obtaining good overall weight parameters both whereas utilized in modern aircraft aviation gas turbine engine with heat exchanger and the further development of engines through the implementation of high-temperature cycles. Accordingly, the analysis of the possibilities for utilizing heat recovery in aircraft engines, in this study chosen compact heat exchangers.

Increase the heat exchanger effectiveness $\theta$ leads to decrease the specific fuel consumption (SFC) of the engines with the recuperation of exhaust gas, however, with an increase the heat exchanger effectiveness, the weight of the heat exchanger $W_{hex}$ also increased. Moreover, the weight of the heat exchanger has more intensively increased associated with increasing heat exchanger effectiveness. In evaluating the engine's efficiency in an aircraft system, it is necessary to take account of the decrease in specific fuel consumption and the increase in the weight of the engine $W_{engine}$. For this purpose, a criteria such as the total weight of power plant$W_{engine, fuel}$ is used, which includes the weight of the engine with heat exchangers, and the weight of fuel is necessary for flying at a given range. Reducing the specific fuel consumption and increase the weight of the heat exchanger with increase the heat exchanger effectiveness leads to the formation of a minimum of the total weight of the power plant. (fig.1). Accordingly, it is necessary to simultaneously optimize both the parameters of the working process $r^*, m, T_i$ and the heat exchanger effectiveness $\theta$ while choosing the parameters of the working process of the engine with the heat exchanger.

![Figure 1](image)

**Figure 1.** Effect of the heat exchanger effectiveness on the weight of the heat exchanger and fuel consumption.

2. Problem description
The study presents a computational calculation of a heat exchanger for estimating the mathematical models of a compact heat exchanger in the purpose of optimizing the main thermodynamic parameters
of a gas turbine engine coupled with a heat exchanger at the stage of conceptual design of aviation gas turbine engine with heat recovery of exhaust gas. The design calculations developed in the ASTRA computer-aided system [7] and three mathematical models of compact heat exchanger in terms of weight goodness, flow passage goodness and optimum weight and flow passage present in this study.

3. Results and discussion

3.1. Evaluation and selection the types of heat surfaces for compact heat exchanger

In this study, the design calculations of various types of heat surfaces carried out in terms of the specific weight of the heat exchanger $\gamma_{hex} = W_{hex}/G_a$, the pressure loss in the air and gas channel of the heat exchanger surfaces (respectively $\Delta p_a / p_a$ and $\Delta p_g / p_g$).

The types of heat transfer surfaces of the compact heat exchanger [13,17,18] according to their design specificity are shown in Fig.2.

1. Prime surface (PS)
   1. Smooth channels
   2. Channels with heat transfer intensification
2. Plate-fine surface
   1. Smooth fin (SF)
   2. Offset strip-fin (OSF)
   3. Wavy fin (WF)
   4. Louvre fin (LF)
   5. Perforated fin (PF)
6. Bunk finning with heat transfer intensification (BFI)

![Fig.2](image-url)

**Figure 2.** (a) Plain rectangular (b) plain trapezoidal (c) wavy fin (d) offset strip fin (e) louvred (f) perforated (g) Prime surface smooth channel (h) prime surface with heat transfer intensification [14 - 16].

For evaluating heat transfer surfaces of heat exchanger core, chose different types of heat transfer surfaces. The same hydraulic diameter selected for all heat transfer surfaces, only their thermohydraulic performances are under evaluation. The surfaces divided into 5 unique categories: 1. prime surface (PS), 2. Smooth fin (SF), 3. Offset strip-fin (OSF), 4. Wavy fin (WF) and 5. Bunk finning with heat transfer intensification (BFI).

The initial data of the Fanning friction factor $f$ in the channels of gas and air and parameters characterizing of the heat transfer (Nusselt number $Nu$) for design calculations analysis chose from open literature [7,8]. Comparison of heat surfaces carried out under the following parameter conditions shown in table 1. The following assumptions are the same for all heat surfaces while calculating:

- The plate thickness, air and gas fin thickness equal 0.12 mm and the thickness of welding alloy for pate-fin surface is equal 0.02 mm
• Flow schemes passage section of heat carriers Z=2
• Thermal conductivity and density of the material of the fin and walls $\lambda_p = 20.8 \text{ W/m/K}$, $\rho_p = 7900 \text{ kg/m}^3$, respectively

Table 1. Initial parameter conditions for design calculation of compact heat exchanger

| Parameter                                           | Value          |
|-----------------------------------------------------|----------------|
| Heat exchanger effectiveness $\theta$               | 0.7            |
| Airflow rate $G_a$                                  | 43 kg/s        |
| Inlet heat exchanger air temperature                | 660 K          |
| Inlet heat exchanger gas temperature                | 1000 K         |
| Inlet heat exchanger air pressure                   | 1497000 Pa     |
| Inlet heat exchanger gas pressure                   | 111000 Pa      |
| Hydraulic diameter of gas side                      | 1.5 mm         |
| Hydraulic diameter of air side                      | 0.75 mm        |

The calculation results are shown in Fig.3, 4, 5 and in table 2. The results compared with the available open literature [4-6, 19] to assess the reliability.

![Image 1](https://example.com/image1)

**Figure 3.** Comparison of various heat exchanger surfaces by specific weight.

![Image 2](https://example.com/image2)

**Figure 4.** Comparison of various heat exchanger surfaces for pressure losses in the air channel.

From the calculation results are shown fig. 3,4 and 5, the most preferable heat exchanger surface in terms of the specific weight is the prime surface (PS-4), and the most preferable heat exchanger surface in terms of the pressure loss in the air and gas channel is the prime surface (PS-2). For further investigation three models of the compact heat exchanger will be developed, the first model is compact heat exchanger with the prime surface (PS-4) for both air and gas side channels (weight goodness). The second model is compact heat exchanger with prime surface (PS-2) for both air and
gas side channels (flow passage goodness) and the third one model is the combined compact heat exchanger with prime surface (PS-2) for the gas side channel and prime surface (PS-4) for airside channels (optimum weight and flow passage).

![Comparison of various heat exchanger surfaces for pressure losses in the gas channel.](image)

**Figure 5.** Comparison of various heat exchanger surfaces for pressure losses in the gas channel.

**Table 2.** Design calculation results of various heat exchanger surfaces

| №  | Type of heat exchanger surfaces | Heat exchanger weight $W_{hex}, kg$ | Specific weight $γ_{hex}$ $kg/kg/sec$ | $Δp_g/p_g$ % | $Δp_a/p_a$ % |
|----|---------------------------------|------------------------------------|----------------------------------------|---------------|---------------|
| 1  | WF-1                            | 1133                               | 26.35                                  | 2.51          | 0.88          |
| 2  | WF-2                            | 1310                               | 30.48                                  | 2.22          | 0.73          |
| 3  | WF-3                            | 1132                               | 26.34                                  | 2.34          | 0.79          |
| 4  | WF-4                            | 839                                | 19.52                                  | 2.08          | 0.74          |
| 5  | SF-1                            | 2256                               | 52.48                                  | 3.12          | 0.93          |
| 6  | SF-2                            | 2252                               | 52.38                                  | 3.12          | 0.93          |
| 7  | SF-3                            | 1667                               | 38.79                                  | 2.48          | 0.78          |
| 8  | BFI-1                           | 1472                               | 34.23                                  | 4.5           | 1.52          |
| 9  | BFI-2                           | 1552                               | 36.11                                  | 3.99          | 1.34          |
| 10 | BFI-3                           | 1642                               | 38.20                                  | 3.52          | 1.19          |
| 11 | PS-1                            | 1106                               | 25.73                                  | 2.1           | 0.67          |
| 12 | PS-2                            | 899                                | 20.91                                  | 2             | 0.65          |
| 13 | PS-3                            | 811                                | 18.87                                  | 2.25          | 0.75          |
| 14 | PS-4                            | 668                                | 15.54                                  | 3             | 1.10          |
| 15 | OSF-1                           | 1946                               | 45.27                                  | 2.9           | 0.91          |
| 16 | OSF-2                           | 1645                               | 38.26                                  | 2.75          | 0.86          |
| 17 | OSF-3                           | 1326                               | 30.85                                  | 2.55          | 0.81          |
| 18 | OSF-4                           | 1229                               | 28.59                                  | 2.87          | 0.92          |

3.2. **Estimating the model of the compact heat exchanger**

The mass model of heat exchanger (weight goodness) developed in this step of the work. The compact heat exchanger carried out for various degrees of heat exchanger effectiveness $\theta$ and gas velocity $V_g$ based on the design calculation algorithms of the compact heat exchanger developed in ASTRA software tool. The results of the design calculations and the initial design data are given in table 3. The study presents the dependence of the specific weight of the heat exchanger $γ_{hex}$ and gas velocity on the heat exchanger effectiveness $\theta$, as well as the dependence of the relative weight of the heat exchanger $\overline{γ}$ on heat exchanger effectiveness $\theta$ (Fig. 6, 7) based on the obtained results.

The results have been evidenced through investigated data in the available open literature [23], the data on plate heat exchangers of aircraft gas turbine engines [20-22], data on plate heat exchangers of stationary gas turbine plants [25,26]. As well as data published in the paper from MAI [24], the results
present in fig. 6 and 7. It can be seen from the figure 6 and 7, the influence of the specific weight of the heat exchanger according to the developed model and from various sources almost coincides, which proves the adequacy of the mathematical model.

### Table 3. Design calculation results of the specific weight heat exchangers

| №  | Plate heat exchangers nomination | airflow rate, $G_\alpha$, kg/sec | heat exchanger effectiveness $\theta$ | Gas velocity $V_g$, m/sec | Heat exchanger weight $W_{hex}$, kg | Specific weight $\gamma_{hex}$, kg/kg/sec | relative weight $\bar{\gamma}$ |
|----|---------------------------------|---------------------------------|----------------------------------|----------------------------|----------------------------------|--------------------------------|------------------|
| 1  | HE- A1                          | 43                              | 0.5                              | 30                         | 260                              | 6.04                          | 1                |
| 2  | HE-A2                          | 43                              | 0.6                              | 30                         | 477                              | 11.09                         | 1.83             |
| 3  | HE-A3                          | 43                              | 0.7                              | 30                         | 880                              | 20.46                         | 3.38             |
| 4  | HE-A4                          | 43                              | 0.8                              | 30                         | 1750                             | 40.69                         | 6.73             |
| 5  | HE-A5                          | 43                              | 0.9                              | 30                         | 4550                             | 105.81                        | 17.5             |
| 6  | HE-B1                          | 43                              | 0.5                              | 150                        | 100                              | 2.32                          | 1                |
| 7  | HE-B2                          | 43                              | 0.6                              | 150                        | 195                              | 4.53                          | 1.95             |
| 8  | HE-B3                          | 43                              | 0.7                              | 150                        | 400                              | 9.3                           | 4                |
| 9  | HE-B4                          | 43                              | 0.8                              | 150                        | 858                              | 19.95                         | 8.58             |
| 10 | HE-B5                          | 43                              | 0.9                              | 150                        | 1992                             | 46.32                         | 19.92            |

**Figure 6.** The influence of the specific weight of the various types heat exchangers on the heat exchanger effectiveness (logarithmic scale).

**Figure 7.** The influence of the heat exchanger effectiveness on the relative weight of heat exchangers.
Depending on the given heat exchanger effectiveness and gas velocity through the heat exchanger, the specific weight of the heat exchanger is calculated by:
\[ \gamma_{hex} = (4.25/V_g + 0.025)e^{6.8\theta}, \text{kg/kg/s} \]

For a given air flow rate through the heat exchanger and the calculated specific weight, the weight of the heat exchanger is determined by:
\[ W_{hex} = G_a[(4.25/V_g + 0.025)e^{6.8\theta}] \text{, kg} \]

Furthermore, the study presents the dependence of the pressure losses in the air and gas channels of the heat exchanger (\(\Delta p_a / p_a\), \(\Delta p_g / p_g\) respectively) and gas velocity on the heat exchanger effectiveness \(\theta\), (Fig. 8, 9) based on the obtained results.

**Figure 8.** The dependence of the pressure losses in the air channels of the heat exchanger on the heat exchanger effectiveness (logarithmic scale).

**Figure 9.** The dependence of the pressure losses in the air channels of the heat exchanger on the heat exchanger effectiveness (logarithmic scale).

Depending on the given heat exchanger effectiveness and gas velocity through the heat exchanger, the pressure losses in the air and gas channels of the heat exchanger (\(\Delta p_a / p_a\), \(\Delta p_g / p_g\) respectively) are calculated:
\[ \Delta p_a / p_a = (0.0044 - 0.127/V_g)e^{3.25\theta} \]
\[ \Delta p_g / p_g = (0.0042 - 0.12/V_g)e^{4.65\theta} \]
The mathematical model of the pressure drop in the air and gas channels of heat exchanger can be presented as:

\[
\sigma_{\text{hex,a}} = 1 - \frac{\Delta p_a}{p_a}, \%
\]

\[
\sigma_{\text{hex,g}} = 1 - \frac{\Delta p_g}{p_g}, \%
\]

Moreover, the second and third mass models (flow passage goodness and optimum weight and flow passage) were developed and the investigation results shown in the fig.10.

Figure 10. The influence of the specific weight of various models of heat exchanger on the heat exchanger effectiveness.

4. Conclusion

One of the most effective possibilities of reducing specific fuel consumption and obtaining high thermal efficiency of gas turbine engines is the use of heat recovery at the exhaust gas. However, the utilizing of heat recovery in aviation gas turbine engines faces a contradiction: on the one hand, heat recovery can decrease the specific fuel consumption, and on the other increase the weight of the power unit due to the installation of a heat exchanger. To realize the desired effect, it is necessary to simultaneously optimize both engines working process parameters and the heat exchanger effectiveness confer to the flight- technical criteria for evaluating the power plant in the aircraft system. Herewith, it is necessary to have a mathematical model for estimating the weight of a highly efficient compact heat exchanger for aviation purposes.

Three mathematical models according to the weight of heat exchanger had been developed and presented in the article, the first mass model is compact heat exchanger with the prime surface (PS-4) for both air and gas side channels (weight goodness). The second mass model is compact heat exchanger with prime surface (PS-2) for both air and gas side channels (flow passage goodness) and the third one mass model is the combined compact heat exchanger with prime surface (PS-2) for the gas side channel and prim surface (PS-4) for airside channels (optimum weight and flow passage). Furthermore, the study presents the novel mathematical models of the pressure drops in the air and gas channels of the heat exchanger with the prime surface (PS-4) for both air and gas side channels.

These models can be used for optimizing the parameters of the working process and the heat exchanger effectiveness at the stage of conceptual design of aviation gas turbine engine with heat recovery of exhaust gas. Herewith, two way-cross scheme of the relative motion of the heat carriers in the heat exchanger were chosen as the most rational flow schemes passage of heat carriers. The dependence of the specific weight of the heat exchanger on the heat exchanger effectiveness at different gas velocity is determined for the selected surface type on the basis of a detailed calculation algorithm, which led to develop a correlation-regression model. A comparative analysis had been
made for the influence of the heat exchanger effectiveness on the specific weight of the heat exchanger to assess the reliability of the obtained model, with the data of other open literature and with the data of the existing heat exchanger.

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