THREE-DIMENSIONAL ADAPTIVE MESH REFINEMENT SIMULATIONS OF LONG-DURATION GAMMA-RAY BURST JETS INSIDE MASSIVE PROGENITOR STARS

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Received 2012 August 21; accepted 2013 February 17; published 2013 March 21

ABSTRACT
We present the results of special relativistic, adaptive mesh refinement, 3D simulations of gamma-ray burst jets expanding inside a realistic stellar progenitor. Our simulations confirm that relativistic jets can propagate and break out of the progenitor star while remaining relativistic. This result is independent of the resolution, even though the amount of turbulence and variability observed in the simulations is greater at higher resolutions. We find that the propagation of the jet head inside the progenitor star is slightly faster in 3D simulations compared to 2D ones at the same resolution. This behavior seems to be due to the fact that the jet head in 3D simulations can wobble around the jet axis, finding the spot of least resistance to proceed. Most of the average jet properties, such as density, pressure, and Lorentz factor, are only marginally affected by the dimensionality of the simulations and therefore results from 2D simulations can be considered reliable.

Key words: gamma-ray burst: general – hydrodynamics – supernovae: general

Online-only material: animations, color figures

1. INTRODUCTION

Long-duration gamma-ray bursts (GRBs) are produced by collimated relativistic outflows (Sari et al. 1999) ejected in the core of massive stars at the end of their evolution (Woosley 1993; Hjorth et al. 2003; Stanek et al. 2003; Woosley & Bloom 2006). Since their relativistic outflows have to propagate through their progenitor star material and exit the star before producing the gamma-ray photons, an outstanding issue with this scenario is to understand the mechanisms that prevent the entrainment of baryons in the light, hot jet (MacFadyen & Woosley 1999; Aloy et al. 2000).

On the other hand, even if the jet–star interaction cannot slow down the jet, it has a strong impact on its dynamics (Morsony et al. 2007) and can supply enough energy to explode the star as a supernova (SN; Khokhlov et al. 1999; MacFadyen et al. 2001; Wheeler et al. 2002; Maeda & Nomoto 2003; Lazzati et al. 2012). In most cases, the study of the jet–star interaction has been performed numerically, with analytic models used only for guidance (Aloy et al. 2002; Gómez & Hardee 2004; Morsony et al. 2007; Matzner 2003; Bromberg et al. 2011). Even so, studying the propagation of a relativistic outflow that is continuously shocked by a much denser environment is not trivial since the length scale of features in the relativistic material is typically $\sim R/\Gamma$ and therefore a large dynamical range is involved. When possible, adaptive mesh refinement (AMR) codes have been adopted (Morsony et al. 2007, 2010; Lazzati et al. 2009, 2010, 2011b; Nagakura et al. 2011), and the simulations have been limited to two dimensions (MacFadyen & Woosley 1999; Aloy et al. 2000; MacFadyen et al. 2001; Zhang et al. 2003; Mizuta et al. 2006; Morsony et al. 2007, 2010; Lazzati et al. 2009, 2010, 2011b; Mizuta & Aloy 2009; Nagakura et al. 2011). These studies have shown that even though the jet material is relativistic, the jet head propagates sub-relativistically inside the star, thereby allowing causal contact between the bow shock at the head of the jet and the star. The shocked star material therefore drains at the sides of the jet, producing a hot cocoon (Ramirez-Ruiz et al. 2002; Lazzati & Begelman 2005) instead of being entrained in the jet.

Two-dimensional (2D) simulations can provide important answers to the outstanding questions listed above. However, they are plagued by artifacts due to the presence of a symmetry axis in the center of the jet. First, a plug of dense material accumulates in front of the jet head, slowing down its propagation and creating plumes of hot plasma at wide angles (see Figure 1 in Lazzati et al. 2010 for an example). Second, recollimation shocks coming from the sides of the jet bounce strongly off the jet axis in 2D simulations, while they could dissipate more efficiently in a simulation at the natural dimensionality. Finally, the role of turbulence and instabilities cannot be properly explored in 2D simulations. Wang et al. (2008) found that in some cases a three-dimensional (3D) relativistic jet would break apart and not be able to produce a successful GRB (while in 2D it would produce a successful GRB).

While 3D simulations of GRB jets have been attempted in the past (Zhang et al. 2004), they were performed with a fixed grid code, casting doubt on their capability to resolve the required small scales. A 3D test case with AMR was presented by Wang et al. (2008), but since the jet-progenitor evolution varied drastically as a function of the numerical resolution (unlike our study), not much could be inferred from their study. Thus, in this paper we present, for the first time, 3D AMR simulations of GRB jets crossing a pre-SN progenitor and then flowing through the interstellar medium (ISM).

This paper is organized as follows. We first describe the physics, initial setup, and the numerical simulations in Section 2, followed by our results and discussion in Section 3. Conclusions are given in Section 4.

2. PHYSICS, INITIAL SETUP, AND SIMULATIONS

2.1. Physics and Initial Setup

As what now seems to be the generic model used for long GRBs (Morsony et al. 2007, 2010; Lazzati et al. 2009, 2011a,
We considered the one-dimensional (1D) pre-SN 16TI model from Woosley & Heger (2006) as our initial stellar configuration. Initially (in the zero-age main sequence), model 16TI is a 16 $M_\odot$ Wolf–Rayet star with 0.01 $Z_\odot$ metallicity, and $3.3 \times 10^{52}$ erg s$^{-1}$ equatorial angular momentum. The final outcome of such a model is a pre-SN progenitor with 13.95 $M_\odot$ and nearly half the size of the Sun ($R_0 = 4.1 \times 10^{10}$ cm). Assuming spherical symmetry, the 1D density and pressure profiles were mapped onto a 3D configuration that we assumed to be initially without rotation. The internal energy and the temperature were calculated assuming a relativistic polytropic equation of state (EOS, $y = 4/3$). The pre-SN progenitor was immersed in an ISM with constant density ($n_{\text{min}} = 10^{-10}$ g cm$^{-3}$). Even though a wind environment would probably be more appropriate, we note that within the size of our simulation the dynamical role of the ambient medium is negligible and the results are therefore insensitive to the chosen ambient medium profile.

A relativistic jet commencing its flow at the center of the pre-SN progenitor was imposed at all times as a boundary inflow condition. The jet was launched at the center of the star (in fact slightly above it), flowing upward in the polar direction ($x = z = 0, y = R_1 = 10^7$ cm). The imposed jet had a half-opening angle of $\theta_0 = 10^\circ$, a constant luminosity of $L_0 = 5.33 \times 10^{50}$ erg s$^{-1}$, an initial Lorentz factor of $\Gamma_0 = 5$, and a ratio of internal to rest-mass energy equal to $\eta_0 = 80$ (Morsony et al. 2007, 2010; Lazzati et al. 2009). In order to break the 2D axisymmetry, the jet was slightly asymmetric. For the latter, we set the jet with a 1% density and pressure asymmetry on either side of a line in the XZ plane 40$^\circ$ from the X-axis. Differently from Wang et al. (2008; 3D numerical study in which a 2D symmetrical initial setup was assumed) our initial setup resembles that from model 3A in Zhang et al. (2004) enhanced with a small perturbation in the jet.

2.2. Numerical Simulations

In order to follow the temporal evolution of our initial setup, we solved the 3D gas-dynamic equations using the FLASH code (version 2.5) in Cartesian coordinates (Fryxell et al. 2000). The simulation domain covered the top half of the pre-SN progenitor star as well as the ISM it is immersed in (see, for example, panel (a) from Figure 1). The boundaries were set at $y_{\text{min}} = 10^9$ cm, $y_{\text{max}} = 2.4 \times 10^{11}$ cm, $x_{\text{max}} = -x_{\text{min}} = 6 \times 10^{10}$ cm, and $z_{\text{max}} = -z_{\text{min}} = 6 \times 10^{10}$ cm. Only the equatorial plane ($y = y_{\text{min}}$) was set with a reflective boundary condition; all the other boundaries were set with transmission conditions. We used a 10-level binary adaptive grid with square-shaped pixels ($\Delta x = \Delta y = \Delta z \equiv \Delta$). The highest refinement level (also referred to as the finest resolution level) was accessible only at the core of the pre-SN star where the jet is injected and initially propagates. Moving away from the stellar core, the maximum level of refinement was progressively decreased. In practice, the base of the jet had the finest resolution at all times and the next three finest levels followed the jet (and the polar part of the cocoon) as it drilled through the progenitor.

Two sets of simulations with a different value of $\Delta$ were performed. We will refer to the high-resolution model as “HR,” and the low-resolution as “LR.” The HR model had the finest resolution (covering the core of the star at all times) equal to $\Delta = 3.125 \times 10^7$ cm; the jet for this case was followed with a resolution of at least $\Delta = 1.25 \times 10^7$ cm. The LR model had the same setup but the value of the finest resolution was set equal to $\Delta = 6.25 \times 10^7$ cm, and the jet was followed with a resolution of at least $\Delta = 2.5 \times 10^8$ cm. The resolution with which we follow the jet is comparable to that from the 3D collapsar study of Zhang et al. (2004; where the maximum resolution was $\Delta \sim 10^8$ cm) and to the most recent 3D GRB jet study from Wang et al. (2008; where $\Delta = 7 \times 10^7$ cm). The resolution with which we resolve the core of the star is comparable to that from previous 2D GRB jet numerical studies (Zhang et al. 2003; Mizuta et al. 2006; Morsony et al. 2007; Nagakura et al. 2011). In order to understand the 3D effects properly, we also ran an extra 2D model.

Differently from the 3D simulation, the 2D run was performed in cylindrical coordinates, the polar axis being coincident with the jet axis. The 2D model had an initial configuration akin to the $XY$ and the $ZY$ planes of the 3D model. The 2D model had the same input physics and resolution as that of the 3D HR model. A summary with the differences between the numerical models is shown in Table 1.

3. RESULTS AND DISCUSSION

3.1. Global Morphology

In Figure 1, we show the density stratification maps for the $XZ, XY,$ and $ZY$ planes for the 3D LR model. Each panel shows a different time frame: (a) $t_\text{bo} = 2.7$ s; (b) $t_\text{bo} = 4.2$ s; (c) $t_e = 5.3$ s; (d) $t_i = 7.3$ s; and (e) $t_e = 9.3$ s. These panels are arranged to illustrate the jet-progenitor-ISM temporal evolution (animations of the density stratification map, Lorentz factor, radial density, radial Lorentz factor, and Schlieren map in the $XY$ plane are linked to the online version of this journal). The morphology of our system is divided into two main phases: when the jet moving inside the progenitor and when the jet has broken out of the star and interacts with the ISM. Such temporal evolution is consistent with what has already been seen in previous numerical studies (Zhang et al. 2003; Morsony et al. 2007). Superimposed on the density stratification in Figure 1, we show the isocountour levels corresponding to $10^{-4}$, $10^{-2}$, 1, $10^2$, and $10^4$ (all in g cm$^{-3}$), these isocountour levels are shown in Figure 2.

The $t_\text{bo} = 4.2$ s breakout time is similar to (but somewhat shorter than) that already seen in previous collapsar studies (Zhang et al. 2003, 2004; Morsony et al. 2007, 2010). Depending on the progenitor that one chooses, and the particular characteristics of the jet, it takes $5$–$10$ s to cross the stellar envelope. Compared to power-law stellar models, the models from Woosley & Heger (2006) are more compact and dense and it takes less time for the jet to cross the realistic progenitors (Mizuta et al. 2006). Our $t_\text{bo}$ is very similar to the breakout time computed with the analytical model from Bromberg et al. (2011). Still, it must be stated that since the jet in our numerical simulations is launched at an inner radius which is at least $10^4$ times the gravitational radius ($R_\text{gr} \sim 10^4 R_\odot$ for a 1.4 $M_\odot$ black hole), the jet from the simulations is somewhat wider than that from the analytical model and thus it propagates slower.

| Model  | $\Delta$ in Core ($\times 10^7$ cm) | $\Delta$ in Jet ($\times 10^9$ cm) |
|--------|-----------------------------------|-----------------------------------|
| 3D LR  | 3.125                             | 1.25                              |
| 2D HR  | 3.125                             | 1.25                              |

Table 1: Model Characteristics
Figure 1. Density stratification maps (g cm⁻³) for different time frames ((a) 2.7 s; (b) 4.2 s; (c) 5.3 s; (d) 7.3 s; (e) 9.3 s) for model 3D LR. The isocontour levels correspond to 10⁴ g cm⁻³, 10² g cm⁻³, 1 g cm⁻³, 10⁻² g cm⁻³, and 10⁻⁴ g cm⁻³. In order to better visualize the internal structure in the pre-SN, the minimum value in all the density stratification plots was set to 10⁻⁵ g cm⁻³. Animations of this figure and of the Schlieren map (in the XY plane) are linked to the online version of the journal.

(Animations and a color version of this figure are available in the online journal.)

The $t_{bo}$ for our study implies that the average propagation velocity of the jet inside the star is $\sim 0.32c$. The jet, composed of low-density material, has its initial opening angle reduced by relativistic hydrodynamic collimation effects.

Once the jet crosses the stellar envelope and breaks out of the surface, the cocoon (which surrounds the jet and is present since its formation) expands through the ISM (Ramirez-Ruiz et al. 2002; Lazzati & Begelman 2005), differently from when the jet is drilling through the progenitor when the cocoon is bound inside the star and close to the jet. When the jet breaks out of the progenitor it becomes uncollimated and the cocoon moves out in the polar direction (moving parallel to the jet), also expanding sideways on top of the stellar surface. Such spreading (see panels (c), (d), and (e) from Figures 1 and 2) was predicted by the analytic solution from Bromberg et al. (2011). By this time not only does the cocoon present zones where variability is clearly present, but the jet also presents turbulent-like structures. The variability in the cocoon is due to the fact that the jet–cocoon system is at least five orders of magnitude denser than the surrounding ISM. Hence, any instability that forms on the cocoon’s boundary or that travels upwind from the jet into the cocoon is not dissipated. Due to the location of the outer boundaries, we are not able to follow the jet–cocoon–ISM system entirely after approximately 10 s. By this time the cocoon has crossed the outer boundaries (the jet crosses the top boundary at approximately 13 s). Also, it must be noted that as time passes the inner isocontour ($\rho = 10^4$ cm g⁻³) disappears. This is due to the reverse shock, which is expanding and pushing the dense material outward. Such behavior has already been seen in the study of Lazzati et al. (2010).

3.2. Symmetry Loss

To understand when the cylindrical symmetry is broken, in Figure 3 we plot the radial density distribution as well as the energy density ($U$, in erg cm⁻³) for four different paths which move along a cone of 2° half-opening angle (with its origin set at $x = y = z = 0$) for model 3D LR. One of these paths moves radially ($R$) along the “(X, Z)” quadrant; another moves in
the “(+X, −Z)” quadrant; another in the “(−X, +Z)” quadrant; and finally a path which moves along the “(−X, −Z)” quadrant.

Consistently with previous 2D and 3D collapsar simulations (Zhang et al. 2003, 2004; Mizuta et al. 2006; Nagakura et al. 2011), we see that as the jet drills through the stellar envelope a complex shock system forms, characterized by a forward and a reverse shock at the head of the jet and by a series of conical recollimation shocks. The first recollimation shock, visible almost at the base of the jet, seems to be static (at $R \sim 2 \times 10^9$ cm), but this is due to the fact that it moves at relativistic speed in the rest frame. Since the study of the shock structure is not our goal, we do not focus on the nature of these shocks, nor do we need to know where the contact discontinuity is set. For the sake of our study all we need to be able to discern is the stellar and jet material that has and has not been shocked.

Specifically, the regions which we will be addressing in the rest of the discussion will be the shocked (SJ) and unshocked (UJ) parts of the jet. The UJ material maintains its initial density profile, while the SJ material breaks the symmetry in the 3D numerical simulations. The density profile can vary up to two orders of magnitude for different locations at the same distance from the progenitor center; on the other hand, the UJ varies less than an order of magnitude. As the jet crosses through the progenitor star its density decreases as a function of time (see Figure 4). Before the jet breaks out from the stellar surface, the density profile inside the progenitor follows a quasi-constant profile which for $t_{bo}$ is $\sim 10^{-1}$ g cm$^{-3}$. Then, when the jet breaks out of the stellar surface, it recovers a decaying radial density profile that for 9.3 s reaches density values as low as $10^{-5}$ g cm$^{-3}$.

### 3.3. Lorentz Factor Evolution

In Figures 5 and 6, we show the temporal evolution for the Lorentz factor (with the velocity field also present) and the radial Lorentz factor profile along the 2$^\circ$ radial path for model 3D LR. Before the breakout time only a relativistic jet (with $\Gamma \sim 10$) is present (see panel with $t = 4.2$ s from Figure 5). In Figure 6, we see that the SJ material for $t < t_{bo}$ (blue, red, and green lines) reaches values close to $\Gamma = 15$, and the UJ Lorentz factor remains practically the same as the initial Lorentz factor.

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**Figure 2.** Density stratification map for different isocontour levels (red = $10^4$ g cm$^{-3}$, yellow = $10^2$ g cm$^{-3}$, green = 1 g cm$^{-3}$, cyan = $10^{-2}$ g cm$^{-3}$, and blue = $10^{-3}$ g cm$^{-3}$) for model 3D LR. The time frames are the same as those indicated in Figure 1. (Animations and a color version of this figure are available in the online journal.)
Figure 3. Radial profiles for different $2^\circ$ paths and times for model 3D LR. Panels (a) through (d) show different radial density profiles (g cm$^{-3}$); panel (e) shows different energy density profiles (erg cm$^{-3}$). Each of the radial paths commences in the origin and runs through different quadrants: (+X, +Z) quadrant (red line); (+X, −Z) quadrant (green line); (−X, +Z) quadrant (blue line); (−X, −Z) quadrant (black line). Each panel corresponds to a different time frame: (a) 2.7 s, (b) 3.5 s, (c) 3.9 s, (d), and (e) 4.2 s. The forward and reverse shock Mach number for each time frame are also indicated.

($\Gamma_0 = 5$). This behavior is consistent with what has already been seen in previous GRB jet numerical simulations where the initial Lorentz factor, prior to $t_{bo}$, reaches values close to 10 (Zhang et al. 2004; Mizuta et al. 2006). Once the jet breaks out of the stellar surface, the jet is accelerated. The high internal energy is able to accelerate material with Lorentz factors values of order $\Gamma \sim 100$ in some zones. If accelerated with no energy dissipation, the jet’s maximum Lorentz factor would be 400 ($\Gamma_{\infty} = \Gamma_0 \eta_0$; Morsony et al. 2007). In the panel with $t = 5.3$ s from Figure 5 (brown line in Figure 6), we show how the jet’s forward shock and the recently formed cocoon produce a “mushroom-like” high-$\Gamma$ structure. At later times ($t > t_{bo}$) (orange, cyan, and black lines in Figure 6) the mushroom-like structure grows bigger and its Lorentz factor $\Gamma$ increases significantly.

To see the high Lorentz factor material in the jet, in Figure 7 we plot the $\Gamma$ iscontours for $t = 9.3$ s. By this time the mushroom structure is evident and also certain regions in the polar axis reach Lorentz factor values as high as $\Gamma = 50$ (pink

Figure 4. Time evolution (0 s, 5.3 s, 7.3 s, 9.3 s) for the $2^\circ$ radial density profile (g cm$^{-3}$) from the (+X, +Z) quadrant (black line in Figure 3). The stellar surface is indicated by the cyan dashed line.

(An animation and a color version of this figure are available in the online journal.)

Figure 5. Lorentz factor stratification maps and velocity field for different time frames (4.2 s; 5.3 s; 7.3 s; 9.3 s) for model 3D LR. The brown dashed line indicates the stellar surface.

(An animation and a color version of this figure are available in the online journal.)
lorentz factor isocontours). Unfortunately, our numerical domain does not permit us to follow such high-$\Gamma$ regions and they escape the top boundary after approximately 10 s. This is once more congruent with the results from previous studies where after $t_{bo}$ the cocoon reaches values as high as $\Gamma \sim 15$, and the jet values of order $\Gamma \sim 100$ (Mizuta et al. 2006). It must also be noted that when the jet breaks out of the stellar surface, a low-speed wind forms. This wind expands isotropically from the point in the stellar surface where the jet drilled through, and moves at an average speed vastly inferior ($v \ll 0.01c$) to that of the jet.

3.4. Resolution Effects

In order to be able to evolve the initial setup up to integration times of order $\sim 10$ s and to resolve the jet-progenitor with a suitably fine grid, an AMR mesh was used. In Figure 8, we show the fraction of the volume that the three finest resolution levels occupied as a function of time. The finest grid level, the one with which the base of the jet was resolved ($\Delta \sim 10^7$ cm, red line in Figure 8), occupied less than $10^{-7}$ of the entire volume. Meanwhile, the two next finest levels, which followed the propagation of the jet through the progenitor ($\Delta \sim 10^8$ cm, blue and green lines in Figure 8), occupied less than $10^{-6}$ and $10^{-4}$ of the volume. Needless to say, if we had used a fixed mesh with a comparable resolution to that with which the base of the jet was resolved, it would have required $\sim 10^6$ more computational power (compared the computational power used in our simulations), and thus the benefit from using an AMR scheme.

To verify that the evolution of the jet from our results is not dependent on the numerical resolution, we ran a new model with the same setup and physics but with a maximum resolution two times finer than for the LR (see Section 2.2 for details). In Figure 9, we show the density profiles for the 3D HR and 3D LR case. In each case we show the two main phases already discussed in Section 3.1: prior to the breakout phase, the breakout, and the post-breakout phase. We must note that the selected time frames for each of the phases were chosen arbitrarily so that the LR and HR cases resemble each other, and hence their basic morphology characteristics can be compared. The main result is that the basic morphology characteristics from each phase (see discussion in Sections 3.1–3.3) are well reproduced independently of the numerical resolution. Unlike the results from the numerical 3D jet GRB study from Wang et al. (2008) where HR models gave qualitatively different jet dynamics, we obtain consistent jet behavior independently of the resolution.
Among the differences associated with the resolution are a higher level of turbulence and a slower advance of the jet head in the HR model. The latter’s jet moves ~20% slower than the LR case; hence, the breakout time for the HR case is $t_{bo} = 5.1$ s. The jet’s velocity resolution difference is due to the fact that the HR case has a wider jet (~5% wider than the LR case). Since we are powering both jets equally, the narrow-LR jet will move faster. The turbulence resolution difference is due to the fact that the LR simulation has higher diffusion, and thus suppresses the small-scale instabilities which are present in the HR model. The higher amount of turbulence in the HR model also slows it down (compared to the LR model), this is due to the fact that a larger fraction of the energy is converted into turbulence. We must note that these two resolution effects are consistent with what has already been seen in previous jet-collapsar simulations, for example, Morsony et al. (2007). Even though the latter study is a 2D one, it also presents more vortices in the HR than in the LR case.

To illustrate how in the HR model there is more variability than in the LR one, in Figure 10 we plot the radial density profiles for both resolution models. The upper panel of Figure 10 is the radial density profile for a path within the jet (specifically a $2^\circ$ path), while the lower panel is the radial density profile in the edge of the jet ($10^\circ$ path). Inside the jet, there is no major difference between the two resolution models. On the other hand, at the edge of the jet the numerical resolution clearly affects the density profile. Here the HR presents numerous depressions in the density radial profile, while the LR case has a radial profile which follows a smoother distribution with less depressions and variability.

We also analyzed the effects of numerical resolution on the jet’s Lorentz factor distribution. Before the jet breaks out, apart from the velocity with which the jet evolves, there is no clear difference between the LR and HR models. But at $t > t_{bo}$ there are morphological changes due to the resolution.
in the Lorentz factor seems to be due to the ability of the high-$\Gamma$ material to wobble around the star’s rotation axis and propagate through paths of least resistance (see below for more details). The observation of this effect is likely facilitated by the fact that we have a larger dynamic range inside the star ($\delta R = R_0/R_{\text{min}} = 41$) compared to the one from Zhang et al. (2004; $\delta R = 8.8$). Thus, there is enough range inside the star for 3D instabilities to develop. In addition, the individual grid pixels in the Zhang et al. (2004) simulations were not square. Rectangular pixels generate more diffusion in the longest of the pixels direction, and results are therefore not as robust as those from square pixel grid simulations (where the diffusion is the same for all three directions).

Finally, we must remark that we do not claim to reach convergence. If we take the number of grid cells across the jet diameter as an estimate of the Reynolds number (Re) of the simulation, we see that at the present time we can only reach Re $\sim 200$. Such Reynolds number is approximately two orders of magnitude finer are not feasible (due to technical difficulties) to date.

### 3.5. 2D versus 3D Simulations

Finally, we checked how the evolution of the jet through the stellar envelope varies in 2D and 3D simulations. For this we ran an extra 2D numerical model with the same resolution ($\Delta = 1.25 \times 10^8$ cm) and the same parameter values (luminosity, $R_i$, $\theta_0$, $\Gamma_0$, and $\eta_0$; see Section 2.2 for more details) as the 3D HR case. Apart from the dimension difference from the 3D models, we assumed polar axisymmetry in the 2D simulation; thus in reality we only simulated half of the $x$-axis domain (i.e., $x_{\text{min}} = 0$). The 2D simulation was carried out in cylindrical coordinates, with the polar axis coincident with the jet axis. In Figure 12, we show the density stratification maps for the 2D model at the two phases (as well as for the breakout time): (a) $t < t_{\text{bo}}$; (b) $t \approx t_{\text{bo}}$; and (c) $t > t_{\text{bo}}$. The time frames for each of the 2D phases shown in Figure 12 were chosen so that they resemble the correspondent time frames from the 3D HR case. The basic morphology in the 2D case resembles that from the 3D model. In both cases we see a collimated jet that manages to drill through the stellar envelope. Apart from the polar axisymmetry imposed...
in the 2D model, there are many subtle differences between the 2D and 3D results.

1. The jet moves slower in the 2D model than in the 3D one (congruent with Zhang et al. 2004). Thus, the 2D breakout time is larger ($t_{\text{bd}, 2D} = 7$ s) than for its correspondent 3D HR model. The reason for the slower jet’s motion in the 2D model is the imposed symmetry. Not only is the jet axisymmetric, but also the material in front of the jet has to remain symmetric at all times. The SJ material can only escape from the jets plug sideways, so a lot of energy ends up going into accelerating this stellar material. In 3D models, instead, the jet can deflect slightly and go around the plug (rather than continuing to accelerate it, see below). Finally, the 2D jet model has an SJ material mildly broader (≈20%) than its respective from the 3D model.

2. Even though the 2D model has the same resolution, the 2D jet presents less turbulent-like morphology than what is present in the 3D HR case. Also, the cocoon is less turbulent and broader in the 2D scenario, as is the case in the numerical study of Zhang et al. (2004).

3. Two low-density plumes are present in the 2D simulations (see right panel from Figure 12). It must be noted that due to the imposed axisymmetry, the plumes actually correspond to a low-density torus around the jet head (if it were a 3D domain and not 2D). Such low-density torus is not present in any of the 3D simulations. This is somewhat similar to the findings from Zhang et al. (2004) where the head of the jet is noticeably different depending on the simulations dimensionality (either 2D or 3D).

4. As shown in the upper panel of Figure 14, where we plot the density profile along the polar axis for the 2D and 3D models, the 2D density radial profile is less turbulent and the depressions are more profound than those in the 3D density radial profile. Also, the SJ material is denser by nearly two orders of magnitude in the 2D scenario close to the jet head.

In Figure 13, we show the Lorentz factor structure for the 2D case (once the jet has just broken out of the stellar surface). Even though the mushroom $\Gamma$ structure forms, the 2D Lorentz factor morphology is noticeably different to that from the 3D HR case. The 2D $\Gamma$ structure presents much less variability.

The 2D low-density regions have high $\Gamma$ values of up to 15–20, values which are in agreement with those obtained by other 2D GRB jet studies (Zhang et al. 2004; Nagakura et al. 2011). The 2D model also has an SJ which is broader than that from the 3D case. As was the case for the density map in the 2D model, the head front of the cocoon has less turbulent-like $\Gamma$ structures. In order to clarify this point, we show the radial Lorentz factor profile (along the polar axis) in Figure 14 (lower panel). Not only is a smoother radial profile present in the 2D, but also the high-density (low-$\Gamma$) relationship is present. The SJ from the 2D case is approximately two orders of magnitude more dense than from the 3D model, but has a rather smaller $\Gamma$ value (of at most 5).

In order to analyze how much the jet changes direction as it drills through the progenitor star (and later through the ISM once the jet has broken out of the star) in a 3D domain, we plot in Figure 15 the energy density ($U$) map in the XZ plane. The XZ planes shown for each time frame correspond to the position, where the $U$ centroid of the forward shock front was located at (see the caption of Figure 15 for more details). Panels (a) through (d) show the $U$ map for when the jet is drilling through the stellar progenitor. In these it is noticeable how the centroid of the forward shock (CFS) does not have a Gaussian-like profile (it may have turbulent-like behavior or even multiple spikes) and how the CFS wobbles around the polar axis finding the spot of least resistance to proceed. For example, note how just before the jet breaks out of the star (panel (d)), the CFS is located far from the polar axis ($x = -0.3 \times 10^{10}$ cm, $z = -0.6 \times 10^{10}$ cm). Panel (e) shows how once the jet has broken out of the star and the cocoon has expanded thoroughly around the progenitor star, its corresponding CFS also expands and also remains far from the polar axis.

To further understand the deflection of the jet inside the pre-SN progenitor, we show the temporal evolution of the angle between the CFS and the polar axis ($\theta$; black line in Figure 16).
Figure 15. Energy density (erg cm⁻³) AX stratification maps for the centroid of the head front of the jet-cocoon structure for different times. Panels (a) through (e) correspond to the 3D HR model, panel (f) corresponds to the 3D LR model. Panel (a) t = 1 s, (b) t = 2 s, (c) t = 4 s, (d) t = 5.1 s, (e) t = 5.7 s, and (f) t = 10 s have the XY plane located at Y/(10¹⁰ cm) = 1.3, 3.3, 18.9, 39.9, and 57.1 (respectively). In each panel the maximum and minimum values of the forward shock’s energy density centroid (in erg cm⁻³) are indicated. Note how some panels have different scales.

(A color version of this figure is available in the online journal.)

For a jet that is well aligned with the polar axis then the CFS displacement angle would yield θ = 0, clearly in Figure 16 this is not the case and the jet wobbles inside the star (with θ oscillating between 0° and 2°). Hence, the jet moves faster in 3D than in 2D because it is able to wobble and move along the path with least resistance (apart from having a narrower jet-cocoon). Note that θ is always within the relativistic collimation angle (1/θ, red line in Figure 16); thus, the relativistic jet is causally connected at all times.

3.6. Limitations and Comparison to Other Work

As with all numerical work, the choices made in carrying out the simulations reflect intentions and biases, and the current investigation is lacking in several aspects. For example, similar to Zhang et al. (2004), Morsony et al. (2007, 2010), and Lazzati et al. (2009) we assumed that the star was static at all times which is clearly not the real case as the pre-SN for long GRBs has very high angular momentum values (J > 10¹⁵ cm² s; Woosley & Heger 2006). We justify this by pointing out that the dynamical timescale of the pre-SN is of order close to hours. Then, since the integration time in our numerical simulations was of order 10 s, we were safe to assume that the pre-SN progenitor remained practically static at all times. In a previous study with a similar setup Lazzati et al. (2011b) found that after 10² s the pre-SN stellar envelope had only expanded 2% of its original size.

Another issue which can be improved is the ISM distribution. The pre-SN progenitor that we use as the initial setup has no hydrogen shell since during its stellar evolution it was lost due to the presence of a stellar wind (which will also affect the ISM surrounding the pre-SN star). So, in order to have full consistency the ISM should have a density profile which was affected by the pre-SN wind, i.e., a profile that follows a ∝R⁻² distribution (Zhang et al. 2003, 2004; Cannizzo et al. 2004; Nagakura et al. 2011). But since the jet–cocoon system is an ultrarelativistic flow, the density profile of the ISM will barely affect the jet once this has just broken out of the stellar surface (Morsony et al. 2007). In fact, the GRB jet needs to reach ~10¹⁴ cm for the ISM’s profile to play a key role in the flow (Blandford & McKee 1976; De Colle et al. 2012). Thus, we were secure in assuming that the ISM density was constant.

We use an adiabatic (γ = 4/3) as our EOS (Zhang et al. 2003, 2004; Mizuta et al. 2006; Morsony et al. 2007, 2010; Lazzati et al. 2009; Nagakura et al. 2011). We do not take into account the neutrino pressure, nor do we take into account the gravitational effects from the central compact object. Even though it has been shown that close to the pre-SN’s progenitor nucleus the neutrinos play an important role (López-Cámara et al. 2009), since the inner boundary was set so far away, Rᵢ ∼ 10⁹ cm, equivalent to approximately 10⁴ gravitational radii, from the region where neutrinos dominate (and where the compact object relativistic effects must be taken into consideration), the neutrino and relativistic effects were safely ignored.

Since the follow-up of newly formed elements was not the aim of this study and that the calculation of such new elements would have not permitted us to study the flow both at an adequate resolutions and for the long times desired, nuclear burning was not included. Also, even though magnetic fields will affect the emissivity of the jet (Mizuta et al. 2006), and could even give rise to variability in the light curve (Balbus & Hawley 1998), they were disregarded due to the technical difficulties when following a magnetized relativistic flow with an adaptive mesh code.

4. CONCLUSIONS

We present, for the first time, 3D AMR simulations of GRB jets expanding inside a realistic pre-SN progenitor and then
flowing through the ISM. Our numerical simulations confirm that relativistic jets can propagate and break out of the progenitor star while remaining relativistic.

The morphology is divided into two main phases.

1. Pre-$t_{\text{bo}}$. During this phase the jet head moves at mildly relativistic velocities ($\sim c/2$) inside the progenitor's stellar envelope.

2. Post-$t_{\text{bo}}$. Once the jet breaks out of the surface, it accelerates and reaches Lorentz factors of order $\Gamma \sim 50$.

The initial progenitor density profile is reshaped by the forward and reverse shocks. The material between the forward and reverse shocks breaks the 2D symmetry in the numerical simulations.

We obtain similar behavior independently of the numerical resolution. The resolution does not affect in great detail the flow, and the morphology in each phase is well reproduced. Still, the amount of turbulence and variability observed in the simulations is higher for higher resolutions. Also, for finer numerical resolutions the jet moves slower and regions with high Lorentz factors break up into smaller regions with lower $\Gamma$ values.

The propagation of the jet head inside the progenitor star is slightly faster in 3D simulations compared to 2D ones at the same resolution. This behavior is due to the fact that the jet in 3D simulations is narrower and can wobble around the jet axis, finding the spot of least resistance to proceed. Most of the jet properties, such as density, pressure, and Lorentz factor, are only marginally affected by the dimensionality of the simulations and therefore results from 2D simulations can be considered reliable. If, instead, more detailed properties such as variability are to be investigated, simulations carried out in the proper dimensionality (i.e., 3D) are required.

We thank S. E. Woosley and A. Heger for making their pre-SN models available, and the referee for comments, suggestions, and constructive criticism which helped improve the original version of the manuscript. The software used in this work was in part developed by the DOE-supported ASC/Alliance Center for Astrophysical Thermonuclear Flashes at the University of Chicago. This work was supported in part by the Fermi GI program grants NNX10AP55G and NNX12AO74G (D.L. and D.L.-C.). B.J.M. is supported by an NSF Astronomy and Astrophysics Postdoctoral Fellowship under award AST-1102796.

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