Looking for pentaquarks in Lattice QCD

George T. Fleming
Sloane Physics Laboratory, Yale University, 217 Prospect St, New Haven, CT 06520-8120 USA
E-mail: George.Fleming@Yale.edu

Abstract. Pentaquark states in lattice QCD probably lie close in energy to two particle scattering states. Correctly identifying the resonant state is a challenging, yet tractable, problem given the terascale computing facilities available today. We summarize the initial round of exploratory lattice calculations and discuss what should be accomplished in the next round.

1. Introduction

The experimental evidence both for and against the existence of a narrow $S=1$ baryonic resonance around 1540 MeV is summarized elsewhere in these proceedings [1, 2]. If confirmed, this resonance lies 5–10% above the $K\bar{N}$ scattering threshold with an allowed strong decay. The implication for Lattice QCD theorists is that proper identification of this resonance in Lattice QCD simulations (even in the quenched approximation) should be substantially more difficult than the standard calculation of the low-lying mass spectrum of hadrons stable against strong decays. Fortunately, techniques are being developed to study hadronic excited states, scattering states and decays on the lattice. That this narrow resonance lies so close to the scattering threshold may make this one of the easier scattering problems to study.

Initial Lattice QCD simulations studied the $S=1$ ground state energies for various choices of total angular momentum $J$, isospin $I$ and parity $P$ [3, 4, 5, 6, 7, 8, 9, 10]. From table 1, we see that the lowest angular momentum ($J=\frac{1}{2}$) isosinglet and isovector channels have been studied, but not isotensor ($I=2$). In all these studies, the ground state energies were generally consistent with a $K\bar{N}$ threshold scattering state. If the first excited state energies were extracted and did not seem consistent with the next higher scattering state, this indicated evidence for a possible pentaquark state.

The pentaquark operators used to create these quantum numbers on the lattice generally had all five quarks at a single spatial point. In this case, the spatial symmetry enables a continuum-like operator construction with a reasonable expectation of correctly identifying the $J^P$ of the operator. Table 1 indicates various strategies for constructing the operators.

2. Hadronic scattering states on the lattice

Hadronic correlation functions in Lattice QCD behave as a sum of decaying exponentials

$$ C_{ij}(t) = \langle \mathcal{O}_i(t) \mathcal{O}^\dagger_j(0) \rangle = \sum_{n=0}^{\infty} \frac{Z_n^i}{\sqrt{2E_n}} \exp(-E_n t) \frac{Z_n^j}{\sqrt{2E_n}}, \tag{1} $$

where the operators $\mathcal{O}_i$ and $\mathcal{O}_j$ have the same quantum numbers $(I, I_z, J, J_z, P, S, \cdots)$. Since the statistical noise grows exponentially in $t$ it is very difficult to reliably estimate the contributions
Table 1. Current summary of lattice pentaquark spectroscopy (adapted from [11]).

| Ref. | I; J^P | signal | I=0 parity | operator |
|------|--------|--------|------------|----------|
| [3]  | 0; 1/2 ± | Yes    | negative   | other    |
| [4]  | 0; 1/2 ± | Yes    | negative   | diquark  |
| [5]  | 0; 1/2 ± | No     | N/A        | KN       |
| [6]  | 0; 1/2 ± | Yes    | positive   | diquark  |
| [7]  | 0; 1/2 ± | No     | N/A        | diquark  |
| [8]  | 0; 1/2 ± | Yes    | negative   | diquark  |
| [9]  | 0; 1/2 ± | Yes    | negative   | many     |
| [10] | 0; 1/2 ± | Yes    | negative   | KN, other |

of even the first excited state (n=1) unless its energy $E_1$ is not much larger than the ground state $E_0$ and a judicious choice of operators $O_i$ can be found which have much better overlap with the excited state ($|Z_{i1}| \gg |Z_{i0}|$). This has been the basic approach of most of the studies in table 1.

Assuming that an excited state energy can be reliably estimated, a resonance can be distinguished from a scattering state by varying the spatial volume. In a periodic box of size $L^3$, momentum components are quantized: $p_i = n_i 2\pi/L$. By varying $L$, energies of isolated moving hadronic resonances vary predictably: $E_{KN}^2(\vec{p}, \vec{q}) = E_{KN}^2(\vec{p}) + (n_2 2\pi/L)^2$. The energy of a two-particle scattering state will also vary according to the scattering length $a_0$:

$$E_{KN}(\vec{p}, \vec{q}) = [E_K(\vec{p}) + E_N(\vec{q})] \left[1 - \frac{2\pi a_0}{E_K(\vec{p})E_N(\vec{q})L^3} \right] \left[1 - 2.834 \frac{a_0}{L} + 6.375 \left(\frac{a_0}{L}\right)^2\right]$$

Figure 1 illustrates the expected volume dependence of $KN$ scattering for various isospin $I$ and orbital angular momentum $L$ channels where the scattering lengths have been experimentally determined. However, the energies of isolated resonances at rest do not vary in sufficiently large volumes where finite volume effects are not important. Thus, scattering states and resonances can be distinguished in principle, although a sufficiently wide range of volumes are seldom available in practice.

### 3. Group-theoretic methods for hadron spectroscopy

A better method for isolating hadronic excited states is a variational method where a large basis of $N$ different operators $O_i$ are identified and an $N \times N$ Hermitian matrix of correlators $C(t)$ is constructed. The matrix elements of $C(t)$ are given by equation (1). If we make a Schur decomposition of $C(t) = Z \Lambda^t Z^\dagger$ and truncate the sum in equation (1) to the first $N$ terms, then we can make an ansatz that the $N$ eigenvalues of $\Lambda$ determine the $N$ lowest energy levels: $\lambda_n = \exp(-E_n)$. In practice, it is better to find the $N$ solutions to the following equation, called a matrix pencil:

$$C(t_0) z_n = \lambda_n^{t-t_0} C(t) z_n, \quad z_m^\dagger C(t_0) z_n = \lambda_m^{t_0} \delta_{mn}. \quad (3)$$
The problem of isolating excited states is now reduced to finding a sufficiently large basis of operators such that some linear combination will overlap strongly with each state of interest.

Over the past few years, we have developed a group-theoretic technique for constructing baryon operators that transform irreducibly under the lattice symmetry group [12, 13, 14, 15]. We have extended our technique to pentaquarks by constructing operators that are linear superpositions of gauge invariant terms of the form

\[
\Phi_{ABCDE}^{ABCDE} = C_{abcde} \left( \tilde{D}^{(p)} \tilde{\psi} \right)^{A}_{\alpha i} \left( \tilde{D}^{(p)} \tilde{\psi} \right)^{B}_{\beta j} \left( \tilde{D}^{(p)} \tilde{\psi} \right)^{C}_{\gamma k} \left( \tilde{D}^{(p)} \tilde{\psi} \right)^{D}_{\mu m} \left( \tilde{D}^{(p)} \tilde{\psi} \right)^{E}_{\nu n}
\]

where \(A, B, C, D, E\) indices label quark flavor; \(a, b, c, d, e\) indices label color; \(\alpha, \beta, \gamma, \mu, \nu\) are Dirac indices; \(\tilde{\psi}\) (\(\tilde{\psi}\)) indicates a smeared quark (antiquark) field; and \(\tilde{D}^{(p)}\) denotes the \(p\)-link covariant displacement operator in the \(j\)-th direction. The tensor \(C_{abcde}\) indicates one of three linearly independent color contractions needed to produce a color singlet. For the displacements considered here, the simplest choice \(C_{abcde} = \epsilon_{abc} \delta_{de}\) was sufficient to produce a complete set of operators.

For pentaquarks at rest, our operators transform as irreps of the double-covered octahedral group \(O_h\). There are four two-dimensional irreps \(G_{1g}, G_{1u}, G_{2g}, G_{2u}\) and two four-dimensional irreps \(H_g, H_u\). Continuum spin assignments \(J\) for lattice states must be deduced from degeneracy patterns across different \(O_h\) irreps [16].

So far, we have constructed complete sets of two types of operators. The first type, called single-site, has all four quarks and the antiquark at a single site. The second type, called singly-displaced, has all four quarks at one site and the antiquark displaced by \(p\) sites along one of the six spatial directions. Table 2 shows the maximal number of linearly independent \(I_z=0, S=1\) pentaquark operators (\(uudd\)) for each isospin \(I\) and operator type.

| Operator Type         | \(I=0\) | \(I=1\) | \(I=2\) |
|-----------------------|---------|---------|---------|
| single-site           | 240     | 460     | 180     |
| singly-displaced      | 1440    | 2760    | 1080    |

Table 3 shows the maximal number of distinct irreducible representations of \(O_h\) that can be constructed for each operator type and isospin \(I\). Irreps of opposite parity occur with equal frequency, so the number of \(G_1\) irreps is the same for \(G_{1g}\) and \(G_{1u}\), etc. Thus, if the observed \(S=1\) resonance at 1540 MeV is \(I=0\) and \(J=\frac{1}{2}\) then we can use up to 19+76=95 different \(G_1\) operators of either parity to construct our correlator matrices. However, if neither the single-site nor the singly-displaced type operators have a significant overlap with the resonance state, then even more operators will need to be constructed using different displacement patterns.

The next step in our program is to extend our existing software which performs the needed Wick contractions to compute the elements of baryon correlation matrices to include pentaquark operators. We hope to report on initial quenched simulations in the near future. We expect these pentaquark resonances will have a relatively large spatial extent and heavy mass, so the use of improved actions on anisotropic lattices will be important. Once we have identified...
linear combinations of operators which overlap strongly with the desired resonances, it may be worthwhile to use the same operators to compute correlation matrices on dynamical isotropic lattices as well.

4. Conclusion
The lattice studies completed to date have most likely seen the threshold $KN$ scattering state in the $I=0,1 \ I_z=0 \ S=1$ channels. In some cases, there have been tantalizing hints of an excited state which have been interpreted as either scattering states or resonances. Until the lowest three or more excited states can be cleanly extracted over a range of volumes, it remains unlikely that lattice calculations can confirm or deny the existence of pentaquark resonances in QCD. The best method for extracting excited states is the calculation of large correlator matrices. The group-theoretic operator construction technique can be used to produce the necessary large basis of operators.

Acknowledgments
Portions of this work were performed in collaboration with members of the hadron spectrum working group of the Lattice Hadron Physics Collaboration, including S. Basak, I. Sato and S. J. Wallace (University of Maryland), R. G. Edwards and D. G. Richards (Jefferson Lab), H. R. Fiebig (Florida International University), U. H. Heller (American Physical Society) and C. Morningstar (Carnegie Mellon University). This work was supported in part by DOE contract DE-AC05-84ER40150 Modification No. M175, under which the Southeastern Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility.

References
[1] K. Hicks. An experimental review of the $\Theta^+$ pentaquark. hep-ex/0412048.
[2] A. R. Dzierba, C. A. Meyer, and A. P. Szczepaniak. Reviewing the evidence for pentaquarks. hep-ex/0412077.
[3] F. Csikor, Z. Fodor, S. D. Katz, and T. G. Kovacs. Pentaquark hadrons from lattice QCD. JHEP, 11:070, 2003.
[4] S. Sasaki. Lattice study of exotic $S=+1$ baryon. Phys. Rev. Lett., 93:152001, 2004.
[5] N. Mathur et al. A study of pentaquarks on the lattice with overlap fermions. Phys. Rev., D70:074508, 2004.
[6] Ting-Wai Chiu and Tung-Han Hsieh. Pentaquark baryons in quenched lattice QCD with exact chiral symmetry. hep-ph/0403020.
[7] N. Ishii et al. Penta-quark baryon in anisotropic lattice QCD. hep-lat/0408030.
[8] C. Alexandrou, G. Koutsou, and A. Tsapalis. The pentaquark potential, mass and density-density correlator. hep-lat/0409065.
[9] J. W. Negele, O. Jahn, and D. A. Sigaev. Calculations of the Quark Structure of Hadrons in Lattice QCD. May 2004. Plenary talk by J. W. Negele at QNP2004. Slides available online at http://www.qnp2004.org/.
[10] T. T. Takahashi, T. Umeda, T. Onogi, and T. Kunihiro. Lattice QCD study of the pentaquark baryons. hep-lat/0410025.
[11] S. Sasaki. Pentaquarks: Status and perspectives for lattice calculations. hep-lat/0410016.
[12] R. Edwards et al. Baryonic operators for lattice simulations. Nucl. Phys. Proc. Suppl., 129:236–238, 2004.
[13] S. Basak et al. Baryon operators and spectroscopy in Lattice QCD. Nucl. Phys. Proc. Suppl., 128:186–192, 2004.
[14] S. Basak et al. Baryonic sources using irreducible representations of the double-covered octahedral group. hep-lat/0409080.
[15] S. Basak et al. Group-theoretical construction of extended baryon operators. hep-lat/0409093.
[16] R. C. Johnson. Angular momentum on a lattice. Phys. Lett., B114:147, 1982.