Minimal extension of tri-bimaximal mixing and generalized $Z_2 \times Z_2$ symmetries

Shivani Gupta$^a$, Anjan S. Joshipura$^b$ and Ketan M. Patel$^c$

Physical Research Laboratory, Navarangpura, Ahmedabad-380 009, India.

Abstract

We discuss consequences of combining the effective $Z_2 \times Z_2$ symmetry of the tri-bimaximal neutrino mass matrix with the CP symmetry. Imposition of such generalized $Z_2 \times Z_2$ symmetries leads to predictive neutrino mass matrices determined in terms of only four parameters and leads to non-zero $\theta_{13}$ and maximal atmospheric mixing angle and CP violating phase. It is shown that an effective generalized $Z_2 \times Z_2$ symmetry of the mass matrix can arise from the $A_4$ symmetry with specific vacuum alignment. The neutrino mass matrix in the considered model has only three real parameters and leads to determination of the absolute neutrino mass scale as a function of the reactor angle $\theta_{13}$.

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$^a$ shivani@prl.res.in
$^b$ anjan@prl.res.in
$^c$ kmpatel@prl.res.in
I. INTRODUCTION

Recent $\nu_e - \nu_\mu$ oscillation observations by T2K [1] and MINOS [2] and double CHOOZ [3] have led to a search of alternatives to the Tri-bimaximal (TBM) leptonic mixing [4] pattern among neutrinos.

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (1)$$

The above pattern corresponds to $\sin^2 \theta_{12} = \frac{1}{3}$, $\sin^2 \theta_{23} = \frac{1}{2}$ and $\sin^2 \theta_{13} = 0$ for the three mixing angles involved in neutrino oscillations. It is theoretically well founded and can be obtained using flavour symmetries in the leptonic sector, see [5] for a review and original references. While the predicted values of the $\theta_{12}$ and $\theta_{23}$ in TBM agree nearly within 1σ of the latest global analysis [6, 7] of the neutrino oscillation data, prediction $\theta_{13} = 0$ is at variance with T2K [1] (MINOS [2]) results by 2.5σ (1.6σ) and with the global analysis [6, 7] by about 3σ. This suggests that one should look either for perturbations to TBM affecting mainly $\theta_{13}$ or try to look for alternative flavour symmetries which imply non-zero $\theta_{13}$. Recently, several attempts [8–10] have been made in this directions. Some of these works [8, 9] discuss the possible schemes of perturbations to TBM while some [10] provide the models also. The minimal scheme would be the one in which $\theta_{13}$ gets generated but $\theta_{23}$ and $\theta_{12}$ remain close to their predictions in the TBM scheme. We show that this can be achieved by generalizing the $Z_2 \times Z_2$ symmetry of the TBM mass matrix and identify appropriate flavour symmetry which can lead to the modified pattern.

The paper is organized as follows. In the next section, we discuss the generalized $Z_2 \times Z_2$ symmetry and leptonic mixing angles.

II. GENERALIZED $Z_2 \times Z_2$ SYMMETRY AND LEPTON MIXING ANGLES

A well-known property of the TBM pattern is the presence of a specific $Z_2 \times Z_2$ symmetry [11] enjoyed by the corresponding neutrino mass matrix $M_{\nu f}$ in the flavour basis. This symmetry is defined in general by the operators $S_i$, $i = 1, 2, 3$:

$$(S_i)_{jk} = \delta_{jk} - 2U_{ji}U_{ki}^* \quad (2)$$

where $U$ is the matrix diagonalizing $M_{\nu f}$. Each $S_i$ defines a $Z_2$ group and satisfies

$$S_i S_j = -S_k \quad , \quad i \neq j \neq k \quad (3)$$

The $S_i$ also leave the neutrino mass matrix invariant

$$S_i^T M_{\nu f} S_i = M_{\nu f} \quad . \quad (4)$$
as can be verified from the Eq. (2) and the property $\mathcal{M}_{\nu f} = U^* D_\nu U^\dagger$, $D_\nu$ being the diagonal neutrino mass matrix.

The explicit forms for $S_2$ and $S_3$ in the TBM case are given by:

$$S_2 = \frac{1}{3} \begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix} \quad \text{and} \quad S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (5)$$

$S_2$ and $S_3$ respectively are determined by the second and the third column of the TBM mixing matrix $U$ using Eq. (2). The element $S_1$ can be obtained using relation (3). In particular, $S_3$ corresponds to the well-known $\mu$-$\tau$ symmetry which is responsible for two of the three predictions namely $\theta_{13} = 0$, $\theta_{23} = \frac{\pi}{4}$ of the TBM pattern.

A desirable replacement of the $\mu$-$\tau$ symmetry would be the one which retains maximality of $\theta_{23}$ but allows a non-zero $\theta_{13}$. Such a symmetry is already known [12] and is obtained by combining the $\mu$-$\tau$ symmetry with the CP transformation. The neutrino mass matrix gets transformed to its complex conjugate under the action of the generalized $\mu$-$\tau$:

$$S_3^T \mathcal{M}_{\nu f} S_3 = \mathcal{M}_{\nu f}^* . \quad (6)$$

A neutrino mass matrix with this property leads to two predictions [12]:

$$\sin^2 \theta_{23} = \frac{1}{2}, \quad (7)$$

$$\sin \theta_{13} \cos \delta = 0. \quad (8)$$

One needs a non-zero $\theta_{13}$ in which case, the above equation leads to a prediction $\delta = \frac{\pi}{2}$ for the CP violating Dirac phase. Eq. (6) does not put any restrictions on the solar angle. In order to do this, we would like to combine the generalized $\mu$-$\tau$ symmetry with the “magic symmetry” corresponding to invariance under $S_2$ and define a generalized $Z_2 \times Z_2$ symmetry. This can be done in two independent ways.

**Case I:** Let us first assume that the neutrino mass matrix in flavour basis simultaneously satisfies

$$S_{1,3}^T \mathcal{M}_{\nu f} S_{1,3} = \mathcal{M}_{\nu f}^*. \quad (9)$$

Both these conditions together imply that

$$S_2^T \mathcal{M}_{\nu f} S_2 = \mathcal{M}_{\nu f}. \quad (10)$$

The above condition fixes the second column of the PMNS matrix $U$ to be $1/\sqrt{3}(1,1,1)^T$. This form of $U$ is studied before and known as tri-maximal mixing pattern [13, 14]. When compared with the standard form [15], it leads to the relation

$$| \sin \theta_{12} \cos \theta_{13} | = \frac{1}{\sqrt{3}} \implies \sin^2 \theta_{12} = \frac{1}{3}(1 + \tan^2 \theta_{13}). \quad (11)$$
which provides the lower limit $\sin^2 \theta_{12} \geq 1/3$. The neutrino mass matrix in the flavour basis $\mathcal{M}_{\nu f}$ that satisfies (9) can be written as

$$
\mathcal{M}_{\nu f} = \begin{pmatrix}
y + z - x & x + ix' & x - ix' \\
x + ix' & y - ix' & z \\
x - ix' & z & y + ix'
\end{pmatrix},
$$

where all the parameters are real. Note that $\Re(\mathcal{M}_{\nu f})$ is invariant under (4) and so it is in the TBM form while $\Im(\mathcal{M}_{\nu f})$ follows the condition

$$
S_{1,3}^T \Im(\mathcal{M}_{\nu}) S_{1,3} = -\Im(\mathcal{M}_{\nu f})
$$

The neutrino mass matrix in Eq. (12) can be diagonalized by the matrix

$$
U^f = U_{TBM} PR_{13}(\theta),
$$

where $P = \text{diag}(1, 1, i)$ and $R_{13}(\theta)$ denotes a rotation by an angle $\theta$ in the $1-3$ plane.

**Case II:** The second possibility is

$$
S_{2,3}^T \mathcal{M}_{\nu f} S_{2,3} = \mathcal{M}_{\nu f}^*,
$$

which leads to

$$
S_1^T \mathcal{M}_{\nu f} S_1 = \mathcal{M}_{\nu f}.
$$

This fixes the first column of $U$ to be $1/\sqrt{6}(2, -1, -1)^T$ which implies

$$
| \cos \theta_{12} \cos \theta_{13} | = \sqrt{\frac{2}{3}} \quad \Rightarrow \quad \sin^2 \theta_{12} = \frac{1}{3}(1 - 2 \tan^2 \theta_{13}).
$$

In contrast to the previous case, this provides an upper bound on the solar angle $\sin^2 \theta_{12} \leq 1/3$. The neutrino mass matrix in the flavour basis $\mathcal{M}_{\nu f}$ in this case can be written as

$$
\mathcal{M}_{\nu f} = \begin{pmatrix}
y + z - x & x + ix' & x - ix' \\
x + ix' & y + 2ix' & z \\
x - ix' & z & y - 2ix'
\end{pmatrix}.
$$

The above $\mathcal{M}_{\nu f}$ can be diagonalized by the matrix

$$
U^{II} = U_{TBM} PR_{23}(\theta),
$$

where $R_{23}(\theta)$ denotes a rotation by an angle $\theta$ in the $2-3$ plane. The third possibility is to consider $S_{1,2}^T \mathcal{M}_{\nu f} S_{1,2} = \mathcal{M}_{\nu f}^*$ and this results into the $\mu-\tau$ symmetric $\mathcal{M}_{\nu f}$ which leads to $\theta_{13} = 0$, so it is not the case of our interest. Both the above scenarios predict small deviations in $\sin^2 \theta_{12}$ from its tri-bimaximal value, but in opposite directions. While both of them are consistent with the present $3\sigma$ ranges of $\theta_{12}$ and $\theta_{13}$ obtained from the global
fits to the recent neutrino oscillation data [7], prediction (16) is more favored if only 1σ is considered. Note that both these scenarios lead to the trivial Majorana phases (0 or π) and do not restrict the masses of neutrinos.

The mass matrices in Eqs. (12,17) based on the generalized $Z_2 \times Z_2$ symmetry are different and more predictive compared to most other proposed modifications of the TBM structure [8,10]. Let us emphasize the main differences:

- Eqs. (12,17) contain four real parameters and hence lead to five predictions among nine observables. These are two trivial Majorana phases, a Dirac phase $\delta = \pm \pi/2$, an atmospheric mixing angle $\theta_{23} = \pi/4$ and the solar mixing angle predicted by Eq. (11) or (16).

- Grimus and Lavoura [13] and He and Zee [9] proposed a mixing matrix similar to Eq. (13). The differences being the absence of $P$, presence of the Majorana phase matrix and the replacement of $R_{13}$ by a unitary transformation in the $1-3$ plane with an undetermined Dirac CP phase $\delta$. In the process, $\delta$ and Majorana phases become unpredictable and $\theta_{23}$ deviates from the TBM value by a term of $O(\theta_{13})$.

- Likewise, Ma in his classic paper [16] considered a modification to TBM analogous to Eq. (18). Here also $R_{23}$ gets replaced by a unitary transformation in the $2-3$ plane with an undermined phase resulting in less predictivity than the present case.

- A special case of Eq. (12) was considered by Grimus and Lavoura [14]. This corresponds to choosing

$$x' = -\frac{1}{\sqrt{3}}(z - x).$$

As a result, $M_{\nu_{eff}}$ contains only three parameters and allows determination of the absolute neutrino mass scale in addition to the five predictions mentioned above. It is also shown in [14] that such a mass matrix can arise in a model based on the $\Delta_{27}$ group. So far we have not appealed to any flavour symmetry at the Lagrangian level and considered only the effective $Z_2 \times Z_2$ symmetry of the neutrino mass matrix. We now propose to realize this effective symmetry from an underlying flavour symmetry. In the process, we find that the use of flavour symmetry also leads us to a three parameter neutrino mass matrix as in the case proposed by Grimus and Lavoura [14].

### III. MODEL AND PHENOMENOLOGY

We use the flavour symmetry $A_4$. Several versions of this symmetry are proposed [5] to obtain a neutrino mass matrix which exhibits the TBM mixing. Here we show that a simple modification of the existing $A_4$ schemes can lead to more predictive mass matrix given in Eq. (17). For definiteness, we concentrate on a specific $A_4$ model of He, Keum and Volkas [17]. We propose two possible schemes one based on the type-I seesaw and the other using a combination of both the type-I and type-II seesaw mechanisms.
Let us first outline the basic features of $A_4$ model proposed in [17]. Though it was proposed to explain both quark and lepton mixing patterns, we here discuss only the lepton sector of it. The matter and Higgs field content of the model with their assignments under the SM gauge group $G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ and $A_4$ group are given in Table I.

| $G_{SM}$ | $l_L$ | $e_R$ | $\mu_R$ | $\tau_R$ | $\nu_R$ | $\Phi$ | $\phi$ | $\chi$ |
|----------|-------|-------|---------|---------|--------|-------|-------|-------|
| $A_4$    | 3     | 1     | 1'      | 1''     | 3      | 3     | 1     | 3     |

TABLE I. Various fields and their representations under $G_{SM} \times A_4$.

The renormalizable $G_{SM} \times A_4$ Yukawa interactions of the model can be written as

$$-\mathcal{L}_Y = y_e(l_L \tilde{\Phi})_1 e_R + y_\mu(l_L \tilde{\Phi})_1^\mu \mu_R + y_\tau(l_L \tilde{\Phi})_1^\tau \tau_R$$

$$+ y_D(l_L \nu_R)_1 \phi + \frac{1}{2} M_{\nu_R} \nu_R^c + \frac{1}{2} B'(\nu_R \nu_R^c) \chi + h.c.,$$

(19)

where $\tilde{\Phi} \equiv i\tau_2 \Phi^*$ and $(..)_R$ denotes $R$-dimensional representation of $A_4$. Note that in [17], an additional $U(1)_X$ symmetry is also imposed so that an unwanted $G_{SM} \times A_4$ invariant term $l_L \nu_R \Phi$ can be forbidden when it is assumed that $l_L, e_R, \mu_R, \tau_R$ and $\phi$ carry $X = 1$ and other fields are chargeless under $U(1)_X$. Specific choice of the $A_4$ vacuum $\langle \Phi \rangle = v(1, 1, 1)^T$ leads to the charged lepton mass matrix:

$$M_l = \sqrt{3} v U(\omega) \text{Diag.}(y_e, y_\mu, y_\tau),$$

(20)

where

$$U(\omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

(21)

and $\omega = e^{2i\pi/3}$ is a cube root of unity. The Dirac neutrino mass matrix is proportional to the identity matrix

$$M_D = y_D v_\phi I,$$

(22)

where $v_\phi = \langle \phi \rangle$. Further, assuming that the field $\chi$ develops a vacuum expectation value (vev) in the direction $\langle \chi \rangle = v_\chi(1, 0, 0)^T$, the heavy neutrino mass matrix can be written as

$$M_R = \begin{pmatrix} A & 0 & 0 \\ 0 & A & B \\ 0 & B & A \end{pmatrix},$$

(23)

where $B = B' v_\chi$. After the seesaw, Eq. (22) and (23) lead to the light neutrino mass matrix

$$\mathcal{M}_\nu = -M_D M_R^{-1} M_D^T = \begin{pmatrix} (a^2-b^2) & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix},$$

(24)
where \(a = -\frac{y_D^2 v_\phi^2}{A^2 - B^2} A\) and \(b = \frac{y_D^2 v_\phi^2}{A^2 - B^2} B\).

As is well-known, Eqs. (21, 24) lead to an \(M_{\nu f} = U(\omega)^T M_{\nu} U(\omega)\) in the form exhibiting the TBM mixing:

\[
M_{\nu f} = \frac{1}{3a} \begin{pmatrix}
(a + b)(3a - b) & -b(a + b) & -b(a + b) \\
-b(a + b) & b(2a - b) & 3a^2 - b(a + b) \\
-b(a + b) & 3a^2 - b(a + b) & b(2a - b)
\end{pmatrix}.
\] (25)

We need to change the existing model in two ways to obtain more predictive form of Eq. (17). First, we require that all the Yukawa couplings in Eq. (19) as well as the vacuum expectation values are real. Eq. (25) then coincides with the real part of (17) with \(z = a - \frac{b}{3a}(a + b)\); \(y = \frac{b}{3a}(2a - b)\); \(x = -\frac{b}{3a}(a + b)\). (26)

We need to enlarge the model to introduce the imaginary part. This can be done either by adding additional \(SU(2)_L\) singlet or triplet fields transforming as an \(A_4\) triplet. Conventionally, one uses CP symmetry to obtain the real Yukawa couplings. The reality of Yukawa couplings follows if definition of CP is generalized in a manner analogous to [12]. This generalized CP combines the CP and \(\mu - \tau\) symmetry as follows:

\[
\begin{align*}
(l_l, \nu_R, \Phi, \chi) &\rightarrow S_3 (l^c_l, \nu^c_R, \Phi^\dagger, \chi^\dagger), \\
(e_R, \mu_R, \tau_R) &\rightarrow (e^c_R, \mu^c_R, \tau^c_R),
\end{align*}
\] (27)

where superfix \(c\) on a field denotes its CP conjugate and \(S_3\) is defined in Eq. (5). Note that the above symmetry behaves like ordinary CP on the \(A_4\) singlet right handed charged leptons and is thus slightly different from the generalized \(\mu - \tau\) symmetry introduced in [13].

The required imaginary part in \(M_{\nu f}\) can be generated in two ways:

**A. Type-II extension**

Add three copies of \(SU(2)_L\) triplet fields \(\Delta\) which form a triplet of \(A_4\) with the \(U(1)_X\) charge \(X = 2\). This modifies the Yukawa interaction by an additional triplet seesaw term

\[
-\mathcal{L}^\Delta_Y = y_L (\bar{l}_L l^c_L) \Delta + h.c.
\] (28)

\(y_L\) becomes real if \(\Delta \rightarrow S_3 \Delta^\dagger\) under the generalized CP. Let us now assume that \(\Delta\) takes vev along the following direction

\[
\langle \Delta \rangle = v_\Delta (0, -1, 1)^T
\] (29)

Such vacuum alignment can be achieved through some terms that breaks \(A_4\) softly and explicit example is discussed in [16]. Eq. (28) gives rise to a type-II contribution in neutrino
mass matrix. Combining this with the type-I contribution, Eq. (24), we get the following:

\[
\mathcal{M}_\nu = \begin{pmatrix}
\frac{(a^2-b^2)}{a} & c & -c \\
-c & a & b \\
-c & b & a
\end{pmatrix}
\] (30)

Parameters \(a, b, c\) are real but when transformed to the flavour basis one obtains a complex \(\mathcal{M}_{\nu f}\) coinciding exactly with Eq. (17) with \(x, y, z\) defined in Eq. (26) and

\[x' = -\frac{c}{\sqrt{3}}.\]

The generalized \(Z_2 \times Z_2\) symmetry emerges here as an effective symmetry. The type-II contribution (characterized by the parameter \(c\)) in the above neutrino mass matrix generates nonzero reactor angle and modifies the solar mixing angle as in Eq. (16).

### B. Type-I extension

Another viable extension of the model is obtained by adding \(A_4\) triplet, \(SU(2)_L\) singlet field \(\chi'\) in addition to \(\chi\) already present. \(\chi'\) introduces the following term in Eq. (19).

\[
-\mathcal{L}_{Y'} = \frac{1}{2}y_R (\nu_R \nu_R^c) \chi' + h.c.
\] (31)

\(y_R\) can be made real using the similar generalized CP symmetry defined in Eq. (27). Assuming that \(\chi'\) takes a vev along the same direction as \(\Delta\) in the previous case, i.e. \(\langle \chi' \rangle = v_{\chi'}(0, -1, 1)^T\), we get

\[
M_R = \begin{pmatrix}
A & C & -C \\
C & A & B \\
-C & B & A
\end{pmatrix},
\] (32)

where \(C = y_R v_{\chi'}\). After the seesaw the light neutrino mass matrix can be suitably written as

\[
\mathcal{M}_\nu = \begin{pmatrix}
\frac{(a^2-b^2+c^2)}{a} & c & -c \\
-c & a & b \\
-c & b & a
\end{pmatrix}.
\] (33)

This matrix also exhibits the generalized \(Z_2 \times Z_2\) symmetry and is determined by three real parameters as before. The only difference from the previous case is a small contribution of the \(\sim \mathcal{O}(a\theta_{13}^2)\) in 11 entry in \(\mathcal{M}_\nu\). As a result the phenomenology of neutrino masses in both cases are very similar and we now turn to this discussion.

### C. Phenomenology

We now derive the phenomenological consequences of the generalized \(Z_2 \times Z_2\) structures Eqs. (30,33) obtained using the \(A_4\) symmetry and specific vacuum alignment. While the
most general, $Z_2 \times Z_2$ invariant structure, Eq. (17) has four parameters, specific realization obtained here has only three parameters. This follows from Eq. (26) which shows that $x, y, z$ are not independent but are related by:

$$z = -y + \frac{x(x + 5y)}{2x + y}.$$  \hspace{1cm} (34)

The situation here is similar to the original $A_4$ models in which specific realizations of the TBM schemes lead to more constrained mass pattern than the most general one and lead to sum rules among neutrino masses [18]. Specifically, Eq. (26) leads to a mass sum rule [18, 19]

$$\frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1},$$  \hspace{1cm} (35)

where $m_i$ are the neutrino masses. Note that $m_i$ are real in our case since all the parameters in the neutrino mass matrix (24) are real. The phenomenological implications of this neutrino mass sum-rule are already considered in [18, 19]. Generalization introduced through Eq. (30) modify this sum rule to

$$\frac{2}{m_2 + 3(m_3 - m_2)s_{13}^2} + \frac{1}{m_3 + 3(m_2 - m_3)s_{13}^2} = \frac{1}{m_1}.$$  \hspace{1cm} (36)

The above sum rule allows determination of the absolute neutrino mass scale as a function of $s_{13}^2$. This determination depends on the type of hierarchy and approximate analytic form for the lightest neutrino mass are given in the limit $s_{13} = 0$ by [18]

For normal hierarchy  \hspace{1cm} $|m_1| \approx \sqrt{\frac{\Delta m_{\text{sol}}^2}{3}} \left( 1 \pm \frac{4\sqrt{3}}{9} \sqrt{\frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}} \right)$,  \hspace{1cm} (37)

For inverted hierarchy  \hspace{1cm} $|m_3| \approx \sqrt{\frac{\Delta m_{\text{atm}}^2}{8}} \left( 1 + \frac{1}{3} \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2} \right)$.  \hspace{1cm} (38)

Using the values of $\Delta m_{\text{sol}}^2$ and $\Delta m_{\text{atm}}^2$ obtained from recent global fits [7] to the neutrino oscillation data, above equations imply

For normal hierarchy  \hspace{1cm} $|m_1| \approx 5.7 \times 10^{-3}$ eV  or  
  \hspace{2cm} $\approx 4.4 \times 10^{-3}$ eV,  \hspace{1cm} (39)

For inverted hierarchy  \hspace{1cm} $|m_3| \approx 0.0179$ eV.  \hspace{1cm} (40)

Further, the three mass dependent neutrino observables, namely (1) the sum of absolute neutrino masses $\Sigma m_i$, (2) the kinematic electron neutrino mass in beta decay $m_\beta$ and (3) the effective mass for the neutrinoless double beta decay $m_{\beta\beta}$ can also be obtained by their approximated expressions given in [18, 19]. The presence of a non-zero $\theta_{13}$ modifies the predicted values of observable compared to the models in [18, 19]. We determine the effect of non-zero $\theta_{13}$ numerically using Eq. (30). The results of such analysis are given in Fig. 1.
FIG. 1. Correlations between the reactor angle and different neutrino mass dependent observables implied by the neutrino mass matrix in Eq. (30). The black (red) points correspond to the normal (inverted) hierarchy in neutrino masses. The black horizontal line shows the mean value of $\sin^2 \theta_{13}$ obtained from the global fits. The unshaded and the shaded regions correspond to $1\sigma$ and $3\sigma$ ranges of $\sin^2 \theta_{13}$ respectively.

As can be seen from Fig. 1, all the mass dependent observables varies slightly with the reactor angle. These variations are smaller for inverted hierarchy compared to the normal hierarchy. Results of a similar numerical analysis for purely type-I extension, Eq. (33) are given in Fig. 2.

FIG. 2. Correlations between the reactor angle and different neutrino mass dependent observables arises in the neutrino mass matrix given by Eq. (33). The black (red) points correspond to the normal (inverted) hierarchy in neutrino masses. The black horizontal line shows the mean value of $\sin^2 \theta_{13}$ obtained from the global fits. The unshaded and the shaded regions correspond to $1\sigma$ and $3\sigma$ ranges of $\sin^2 \theta_{13}$ respectively.

IV. SUMMARY

The evidence of possible non-zero $\theta_{13}$ requires the modification of the TBM patterns motivated by $A_4$ and other discrete symmetries. We have proposed a minimal modification which retains the prediction of the maximality of $\theta_{23}$, allows non-zero $\theta_{13}$ and introduces small $\mathcal{O}(\theta_{13}^2)$ deviation from the $\theta_{12}$ predicted in the TBM. The basis of our proposal is the
$Z_2 \times Z_2$ symmetry of the TBM mass matrix. It is shown that combination of this symmetry with the CP gives rise to very predictive structure determined in terms of only four real parameters. The generalized $Z_2 \times Z_2$ can emerge from a simple extension of the standard $A_4$ schemes if Yukawa couplings and vev are real. The resulting neutrino mass matrix is quite predictive and is determined in terms of only three parameters making it one of the simplest modification of the TBM scheme consistent with the present information on neutrino masses and mixing angles.

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