History effect in inhomogeneous superconductors

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Abstract

A model was proposed to account for a new kind of history effect in the transport measurement of a sample with inhomogeneous flux pinning coupled with flux creep. The inhomogeneity of flux pinning was described in terms of alternating weak pinning (lower $j_c$) and strong pinning region (higher $j_c$). The flux creep was characterized by logarithmic barrier. Based on this model, we numerically observed the same clockwise V-I loops as reported in references. Moreover, we predicted behaviors of the V-I loop at different sweeping rates of applied current $dI/dt$ and magnetic fields $B_a$, etc. Electric transport measurement was performed in Ag-sheathed Bi$_{2-x}$Pb$_x$Sr$_2$Ca$_2$Cu$_3$O$_y$ tapes immersed in liquid nitrogen with and without magnetic fields. V-I loop at certain $dI/dt$ and $B_a$ was observed. It is found that the area of the loop is more sensitive to $dI/dt$ than to $B_a$, which is in agreement well with our numerical results.

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I. INTRODUCTION

Recently the so-called history effect (HE) of vortex matter has been frequently observed by a variety of methods such as electric transport measurement, dc magnetization hysteresis, ac susceptibility and torque techniques, and drawn much more attention \[1,2,3,4,5,6,7,8,9,10\]. HE refers to that critical current density $j_c$ of a superconductor has different values at a same field (or temperature) for either a thermal, magnetic or current cycles. The earlier observed hysteresis of R-T curves in the thermal cycle in fact is a kind of HE caused by first order phase transition of vortex matter (melting or freezing) \[11\]. Different mechanisms have been proposed to account for HE \[1,2,3,4,5,6,7,8,9,10\]. And a typical explanation contains two assumptions: relatively weak flux pinning and uniform small quenched disorders. Then HE is explained with a theory of disorder-order transition of vortex matter. The disordered vortex matter pinned more strongly (higher $j_c$) is supercooled to low temperature where it is in metastable state and then a process such as a bias current anneals the metastable disordered state into an ordered and stable one with lower $j_c$. An important point is that flux creep in the models is omitted for the low temperatures. In this case, the property of vortex matter is governed only by the competition between interactions of vortices-vortices and vortices-quenched disorders.

However, a different kind of HE has been observed \[12\]. The hysteresis loop of V-I curve of polycrystalline Bi$_{2-x}$Pb$_x$Sr$_2$Ca$_2$Cu$_3$O$_y$ (Bi2223) at liquid nitrogen temperature was measured there. See also Fig.6 in this paper. It is clear that such a V-I loop is a new kind of HE and very different from the V-I loop in low temperature superconductors reported before \[1\]. The difference between these two kinds of HE is that the direction of the V-I loop is anti-clockwise in reference 1 while clockwise in reference 12 and this paper. Despite the anti-clockwise loop has been explained very well, the clockwise one has not yet been understood up to our knowledge and interested us very much.

Note that this new kind of HE takes place in a system different from those with uniform and weak quenched disorders at low temperatures. First, at higher temperatures, such as
liquid nitrogen temperatures, thermal fluctuation should be important and flux creep could influence the property of vortex matter apparently [13]. In fact the influence of flux creep on the V-I curve and transport property of Bi2223 have been confirmed by earlier experimental and numerical results [14,15,16,17,18]. Second, the quenched disorders or flux pinnings are inhomogeneous in polycrystalline high temperature superconductors (HTS). There are weak links and grains in sintered samples such as polycrystalline Bi2223. And the flux pinning strength of the weak links at grain boundaries is very different from that of the grains [19]. In view of these two points, any models concerning the system with homogenous quenched disorders without flux creep could not be used to understand this novel hysteresis behavior in V-I curve.

A theoretical model describing irreversible electric-magnetic behaviors of a system with inhomogeneous quenched disorders has already been proposed [20]. Unfortunately, flux creep was omitted there. Moreover, this model has not been used to explain any HE, including the clockwise V-I loop. Experimentally, A.M. Campbell’s group did many important works on inhomogeneous superconductors. They did observe the hysteresis V-I curves before. However, only a qualitative explanation was given. And what interests us now is how to simulated the clockwise V-I loop.

Considering flux creep and inhomogeneous flux pinning, we proposed a new model to explain this new kind of HE. We carried out numerical simulation with this model. Upon explaining the clockwise loop of V-I curve reported in reference 12, we further predicted some new phenomena. To test the predictions, electric transport measurement was conducted for characteristic V-I curve of silver-sheathed Bi$_{2-x}$Pb$_x$Sr$_2$Ca$_2$Cu$_3$O$_y$ tapes (Ag-Bi2223) immersed in liquid nitrogen with or without applied fields $B_a$. Clockwise loop of V-I curve at certain $dI/dt$ and $B_a$ was observed and is dependent on $dI/dt$ and $B_a$, confirming the numerical result.
II. MODEL

The model consists of two basic assumptions: there exists flux creep in a sample and the flux pinning is inhomogeneous. The complex inhomogeneity of flux pinning is simplified by periodic strong pinning region (S) with large $j_c$ ($j_{cS}$) and weak pinning region (W) with small $j_c$ ($j_{cW}$), see Fig.1. The flux diffusion is characterized by collective creep of vortex glass, which has been discussed widely [21]. To simplify the calculation, we use logarithmic $U(j)$ relationship, which is the special case of vortex glass model with very small glass exponent $\mu$ [22].

We should emphasize that our model is not a simplified version of the "brickwall" model, which has been widely discussed before [23]. There are some resemblances between them. However, there are at least two points that distinguish our model from the previous one. First, the "brickwall" model is a static critical state model without flux creep. Obviously, in any static models, the transport properties studied here will be independent of the sweeping rate of applied current $dI/dt$, contrasting with the experimental observation. Second, in the "brickwall" model the S and W regions are connected in series along the direction of current. While in our model, they are connected in parallel.

In our one-dimensional model, flux lines firstly enter the system from the surfaces into the weak pinning channel (W) and then diffuse into the strong pinning region (S). So W surrounds S. In other words, S is inside the slablike sample.

As an example, the model may be proper to polycrystalline samples of HTS, such as silver-sheathed Bi$_{2-x}$Pb$_x$Sr$_2$Ca$_2$Cu$_3$O$_{y}$ (Ag-Bi2223). The grain boundaries in polycrystalline HTS form weak link network of flux pinning and thus can be considered as W, whereas the intra-grain region is S. It is well known that the weak link network suppresses substantially the critical current density of HTS such as Ag-Bi2223. When current or magnetic field is applied, flux density will enter inter-grain (W) at first and then penetrate gradually into the intra-grain (S) by means of flux creep. Another example the model may be applicable to is the low temperature superconductors, such as Nb$_3$Sn metallic compounds, where flux lines
may be pinned by grain boundary (S) whereas the intra-grain is weaker pinning region (W) \[24\].

Certainly, one cannot expect that numerical result by such a simplified phenomenological model can quantitatively represent experimental data of a sample. For example, the one-dimensional geometry assumption in the model is not fulfilled for a real sample. And, the flux pinning is more complex by far than the two model parameters, \(j_{cS}\) and \(j_{cW}\). Nevertheless, our model is a new one accounting for the experimental clockwise V-I loop, which was observed very recently. The effectiveness of this model was demonstrated by the electric transport measurement of hysteresis V-I loop.

III. SIMULATION

A. Basic Equation

Consider a slab with infinite length along y-axis, thickness \(d\) along the x-axis and width \(w\) along the z-axis. The current is applied along the y-axis. In view of \(w \gg d\), we focus on the one-dimensional case for simplicity. The non-linear diffusion equation describing the macroscopic \(B\) or \(j\) is well known \[25,26\]. With a logarithmic dependence of \(U\) on \(j\):
\[
U(j) = U_0 \ln \frac{|j_c/j|}{n}
\]
the diffusion equation of flux can be written as:
\[
\frac{\partial B}{\partial t} = \frac{v_0}{(\mu_0 j_c)^{n+1}} \frac{\partial}{\partial x} \left[ \frac{\partial B}{\partial x} \right] \left( \frac{\partial B}{\partial x} \right) B
\]

where \(n = U_0/kT\) and \(v_0 = u \omega_m\). \(v_0\) is the attempt velocity of the thermal-activated vortex motion, \(u\) is the hopping distance and \(\omega_m\) is the microscopic attempt frequency.

B. Boundary and initial conditions

In the V-I curve measurement, the current is always applied with a certain sweeping rate \(dI/dt\). The boundary condition of Eq.1 can be obtained from the current conservation equation:
\[
\frac{\partial}{\partial t} \int_0^w \int_0^d j \, dx \, dz = \frac{dI}{dt}
\]
Substituting \(\partial B/\partial x = -\mu_0 j\) into it, one comes \(B(d, t)\) –
\( B(0, t) = -\mu_0 I/w. \) Due to antisymmetry, one has \( B(d, t) = -B(0, t) \), thus the boundary condition is

\[
B(0, t) = \frac{\mu_0 I}{2w} = \left( \frac{\mu_0 dI}{2w \, dt} \right) t
\]  

(2)

As for the initial condition, there is no current at \( t=0 \) in the sample, hence

\[
B(x, 0) = 0
\]  

(3)

C. Numerical method and the choice of parameters

Such kind of non-linear diffusion equation can be numerically solved with the finite difference method. First, the temporal and spatial variances are discretized with different steps respectively. Since the temporal and spatial steps are very important for the stability of the calculation, we take pains to choose suitable steps to avoid a divergence problem. Second, the equation is discretized and rewritten in an implicit difference scheme, which is better than the explicit difference scheme in the stability of calculation. Finally, based on the initial condition, the calculation by iteration will obtain the temporal and spatial magnitude of physical quantity, namely \( B(x, t), j(x, t) \) and \( E(x, t) \). The voltage can be obtained by integration of \( E(x, t) \). Note that the boundary condition plays a role of constraint in every time step during current ascending and descending, see Eq.2.

It is easy to see that \( U(j) = U_0 \ln |j_c/j| \) combined with \( E = E_0 \exp(-U/kT) \) leads to the power dependence of the E-j curve \( E = E_0 (j/j_c)^n \) with \( n = U_0/kT \). This power law E-j curves are observed frequently in transport measurements and can be used to determine the parameter \( n \) [15]. Certainly, \( n \) is dependent on flux pinning strength, magnetic field and temperature [18,27]. The weaker flux pinning and higher temperature bring about smaller \( n \). In fact, many experiments including the present work show that for the Ag-Bi2223 in liquid nitrogen temperature, \( n = 6 \) is a typical value.
As for the velocity of the thermal-activated vortex motion and the critical current density, typical and reasonable magnitudes were employed based on the earlier theoretical and experimental work, for example we took $v_0 = u\omega_m = 1\text{m/s}$ (if $u \sim 10^{-6}\text{m/s}$, $\omega_m \sim 10^6\text{s}^{-1}$) and $j_{cW} = 2 \times 10^8 \text{A/m}^2$. It should be pointed out that our numerical result is not sensitive to the choice of $n$, $v_0$ and $j_{cW}$.

**D. Numerical results and discussions**

1. **Effect of $j_{cS}/j_{cW}$ ratio on the hysteresis loop**

Since $j_c$ represents the pinning strength, the ratio $j_{cS}/j_{cW}$ can be considered as a parameter reflecting the ratio of the two pinning strengths. Shown in Fig.2 is our numerical result of the dependence of HE on $j_{cS}/j_{cW}$ at fixed $j_{cW}$, $dS/dW$ and $dI/dt$. It is very clear that the V-I loop is clockwise as reported in reference 12. To understand how $j_{cS}/j_{cW}$ affects the V-I loop, the corresponding current distributions in sample were also calculated.

Note that the velocity of flux diffusion, namely the velocity of current diffusion in transport measurements, can be written as: $v = v_0 \exp(-U/kT) \propto (j/j_c)^n$ for logarithmic $U(j)$. Hence, for the same $j$, the larger the $j_c$, the smaller the $v$. Generally speaking, the average speed of current diffusion in S region ($\overline{v_s}$) is smaller than that of W region ($\overline{v_w}$). In our calculation, $j_{cW}$ is fixed, i.e. $\overline{v_w}$ is fixed. By changing $j_{cS}/j_{cW}$ ratio, namely $j_{cS}$, we adjust $\overline{v_s}$ only.

For a large ratio ($j_{cS}/j_{cW}=10$), i.e. high inhomogeneity of flux pinning, $\overline{v_s}$ is so small comparing with $\overline{v_w}$. And in such a case, there is no enough time for current diffusion in response to both the ascending branch of V-I curve (Fig.2.1(b)) and the descending one (Fig.2.1(c)). Consequently, no obvious V-I loop can be detected (Fig.2.1(a)).

For a medium ratio ($j_{cS}/j_{cW}=4$), i.e. a medium inhomogeneity of flux pinning and a larger $\overline{v_s}$, it is shown that the area of the loop is very large (Fig.2.2(a)). In this case, although current has already penetrated into S region during the ascending branch (Fig.2.2
(b)), much more current penetrates into S during the descending branch (Fig.2.2(c)). That is to say, the current diffusion of S region has more time in response to the descending branch than to the ascending one. As a result, the V-I loop is obvious.

For a small \( \frac{j_{cS}}{j_{cW}} \) ratio, low inhomogeneity of flux pinning, \( \overline{v_s} \) is very large and approaches \( \overline{v_w} \). And there is little difference between the distributions of current during the two branches (Fig.2.3(b), (c)). The minor difference is only seen when the current is small and the corresponding voltage is too small to be detected. Hence, there is almost no hysteresis loop (Fig.2.3 (a)). Especially, at the uniform case \( (\frac{j_{cS}}{j_{cW}} = 1, \overline{v_s} = \overline{v_w}) \), there will be no loop at all.

2. Effect of \( d_S/d_W \) ratio on the hysteresis loop

With a fixed total thickness, the spatial distribution of S and W region, i.e. the thickness ratio of them is also important for the V-I loop. The numerical V-I loops with different \( d_S/d_W \) at fixed \( dI/dt \) and \( j_{cS}/j_{cW} \) were displayed in Fig.3. It is found that the smaller the \( d_S/d_W \), the smaller the area of the loop.

For \( d_S/d_W = 0.857 \) and other parameters given here, the thickness of S region i.e. the distance for current diffusion in S region is so long that current diffusion in S region has less time to respond the ascending branch (Fig.3.1(b)) than to respond the descending one (Fig.3.1(c)). So the area of the V-I loop is large (Fig.3.1(a)).

For \( d_S/d_W = 0.222 \), S region is thinner, i.e. the distance of current diffusion in S region is shorter, resulting that comparing with the \( d_S/d_W = 0.857 \) case, the current diffusion in S region has more time in response to the ascending current (Fig.3.2(b)). In other words, for such parameters given here, the response time of current diffusion in S region to the two branches of the V-I loop shows little difference. As a result, the area of the V-I loop becomes smaller (Fig.3.2(a)).

As for \( d_S/d_W = 0.105 \), S region is too thin and the distance of current diffusion in S region is too short. Hence there is enough time for current diffusion in response to both
current ascending and descending at the given parameters (Fig.3.2(b), (c)). That is to say, there is no obvious V-I loop (Fig.3.3(a)). As $d_s/d_W$ approaches 0, namely the uniform flux pinning case, the hysteresis loop will disappear thoroughly.

3. Current sweeping rate ($dI/dt$) dependent V-I loop

The numerical result of current sweeping rate dependent V-I loop at fixed $j_{cS}/j_{cW}$ and $d_s/d_W$ was illustrated in Fig.4.

It is seen that if $dI/dt$ is very small, such as 0.01A/s, flux lines will have enough time to respond the changing applied current and thus can penetrate into both W and S regions. The difference of current distribution during the ascending (Fig.4.1(b)) and descending branch (Fig.4.1(c)) is very small. As a result, the area of the V-I loop is also small (Fig.4.1(a)). As $dI/dt$ reaches a moderate value, say $dI/dt=2A/s$, the difference between the current distributions during the ascending and descending branch is larger. And the area of the V-I loop becomes larger too (Fig.4.2(a)). Nevertheless, when $dI/dt$ is too large, say $dI/dt=10A/s$, the area of the V-I loop decreases dramatically (Fig.4.3(a)). It is easy to understand that in such a case, comparing with $dI/dt$, $\tau_s$ is so small that the current cannot penetrate into the S region during both the ascending and descending branch. And the corresponding current distributions during the two branches do not show visible difference (Fig.4.3(b), (c)). So there is no obvious V-I loop.

Now, it is clear what the effect of $dI/dt$ on the V-I loop means. With fixed distance and average speed of the current diffusion, to change $dI/dt$ is equivalent to shift our observing time window. When we choose time window by using $dI/dt$ as 10A/s, the observing time is so early that there is no enough current inside S region by means of diffusion even the applied current sweeping has been finished. Of course there is no detectable V-I loop. If we use $dI/dt=0.01A/s$, we shift our time window so late that there are always enough current diffused into S region during both the ascending and descending branch. It is natural that no obvious V-I loop exists at all. Only when we choose a proper observing window, i.e. a
proper $dI/dt$ such as $2A/s$, is there much more current in S region during the descending branch than the ascending one. In other words, only in such case, can the current diffusion in S region has more time to respond the descending branch than the ascending one. And the V-I loop will be obvious.

In conclusion, if there is more time for current diffusion of S region to respond the descending branch than to respond the ascending one, hysteresis loop in V-I curve, i.e. the HE will be obvious. On the other hand, if the current diffusion time of S region is similar or the same for the ascending and descending branch, there will be no obvious V-I loop, namely no HE. The observation of HE can be controlled by adjusting $dI/dt$ (the observing time window) or $d_S/d_W$ (the distance of current diffusion) or $j_{cS}/j_{cW}$ (the average speed of current diffusion).

4. Magnetic field dependent V-I loop

It is well known that $j_c$ is dependent on the magnetic field $B_a$. Commonly, $j_c$ decreases with increasing $B_a$. Therefore, to study the effect of $j_c$ on the V-I loop is equivalent to study the effect of $B_a$ on it. For the inhomogeneous pinning case, $j_{cW}$ is more sensitive to $B_a$ than $j_{cS}$, especially when $B_a$ is not very high [19]. Thus, we simulated the V-I loops in different applied fields $B_a$, namely different $j_{cW}$ at fixed $j_{cS}$, $dI/dt$ and $d_S/d_W$. Shown in Fig.5 are the numerical results. It is clearly seen that the loop shifts towards smaller current with decreasing $j_{cW}$ while the area of the loop is insensitive to $j_{cW}$. In other words, the area of the loop is weakly affected by magnetic field.

As mentioned above, the existence of HE depends on that the current diffusion of S region has different responding time for the two sweeping branches of V-I loop. Only changing $j_{cW}$, i.e. $B_a$, would not affect the responding time of S region. As a result, the area of V-I loop will not change acutely.
IV. EXPERIMENTAL

Because our numerical simulation has obtained more results than the experimental one reported in reference 12, such as the effect of \( dI/dt \) on the area of loops, we measured V-I curves of Ag-Bi2223 samples in various applied magnetic fields \( B_a \) with different \( dI/dt \) to confirm the numerical prediction. It has been pointed out above that the Ag-Bi2223 sample may be proper to test our model.

The Ag-Bi2223 samples cut from long silver sheathed tapes come from the National Center for R&D on Superconductivity of China. All the samples are the c-axis textured. The c-axis is perpendicular to the wider surfaces of the samples. Including the outer silver sheath, the average size of the samples is approximately: \( 4mm \times 1mm \times 4cm \). Between each two neighbor points of four (two voltage contacts and two current contacts) is 1cm in length. The transport measurement was carried out for the sample immersed in liquid nitrogen. The external magnetic field with range 0 - 200 Gs was applied in the superconducting state. The current range of the supply source is 0 - 40 A and sweeping rate range 0.02 - 1 A/s. The measurements were made in the case of \( B_a \) parallel to the c-axis of the samples. More details of the sample preparation and the measurements can be found in reference 18.

The V-I loops were measured by ascending current first and then descending it. Displayed in Fig.6 are the experimental V-I loops with different \( dI/dt \) and fixed \( B_a \). The spottiness of the experimental data came from the limited picking rate of our measurement system. As reported in reference 12, clockwise loop of V-I curve was observed. It is clearly seen that \( dI/dt \) affected not only the appearance of V-I curve but also the area of the loop. For example, the area of loop with \( dI/dt = 1 \) A/s is larger than the area of loop with \( dI/dt = 0.5 \) A/s, which is in agreement well with the numerical result.

Shown in Fig.7 are the experimental V-I loops in different applied fields \( B_a \) at a fixed \( dI/dt \). It is found that the area of the loop is insensitive to magnetic field \( B_a \) whereas the transport critical current density \( j_c \) decreases with increasing applied field very sensitively. This experimental dependence confirms the above numerical results and thus supports
strongly our model. Furthermore, the agreement between the numerical and experimental V-I shifting by applied field implies that the critical current density $j_c$ of a sample with spatially non-uniform flux pinning strength, such as the silver sheathed Bi2223 tapes, is governed mainly by the weaker flux pinning region (the region with smaller $j_c$). Hence, for these HTS materials, which have potential for technical applications, the point to enhance their critical current density is to improve the flux pinning strength of weak pinning region.

We are unable to measure more V-I curves to test all the numerical curves for the limited ability of our instrument. For example, $dI/dt$ cannot be too small because Ohm heat at the current contacts will destroy the sample.

**V. SUMMARY**

We have proposed a model to account for a new kind of history effect in an inhomogeneous flux pinning superconductor at high temperatures. The model consists of alternating weak and strong flux pinning regions whose strength was depicted by different critical current densities $j_{cW}$ and $j_{cS}$, respectively. Based on the model, our numerical simulation successfully observed a hysteresis loop in the characteristic V-I curve as reported in references. Our numerical results also predicted some new characteristics of the V-I loop at different sweeping rates of applied current $dI/dt$, applied magnetic fields $B_a$, the ratio of the two regions’ pinning strength and their thickness.

Physically, due to the difference of flux pinning strength in S and W regions, the average speeds of flux (or current) diffusion in the two regions are different too. If $dI/dt$ (or $dS/dW$, $j_{cS}/j_{cW}$) is proper, the V-I loop, i.e. the HE is obvious. Otherwise, no obvious V-I loop can be observed.

To confirm the numerical results, electric transport measurement was conducted to get the characteristic V-I curve of Ag-sheathed Bi$_{2-x}$Pb$_x$Sr$_2$Ca$_2$Cu$_3$O$_y$ tapes immersed in liquid nitrogen with or without applied fields. Clockwise V-I loop was observed to be dependent on $dI/dt$ and $B_a$. Hence, we presented a new possible mechanism accounting for the new
kind of HE in an inhomogeneous flux pinning superconductor.

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15
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**Figure Captions:**

Fig.1. A schematic sketch of the one-dimensional model: a superconducting slab consisting of periodic strong (S) and weak (W) pinning regions. Vortex lines enter the system firstly from the surfaces into W region, and then diffuse from the W into S regions. So the W region surrounds the S region.

Fig.2. Numerical results, showing the effect of $j_{cS}/j_{cW}$ ratio on the V-I loop (a) and the distribution of current during the ascending (b) and descending branch (c) of the loop. The arrows indicate the increasing and decreasing of current, i.e. the time evolution. The dot lines represent the boundary between S and W regions. $d_S/d_W=1/2$, $dI/dt=1A/s$, $j_{cW}=2 \times 10^8$A/m$^2$.

(2.1) $j_{cS}/j_{cW}=10$, high inhomogeneity of flux pinning. Average speed of current diffusion in S region is too small comparing with that of W region, resulting that in S region there is no enough time for current diffusion in response to both the ascending sweeping and the descending one at the given parameters. No obvious V-I loop.

(2.2) $j_{cS}/j_{cW}=4$, medium inhomogeneity of flux pinning and larger average speed of current diffusion in S region, resulting that more current has penetrated into the S region during the descending branch than the ascending one at the given parameters. Obvious V-I loop is observed.

(2.3) $j_{cS}/j_{cW}=1$, uniform flux pinning, equal average speed of current diffusion in the two regions. There is little difference between the distributions of current during the two branches. No obvious V-I loop.

Fig.3. Numerical results, showing the effect of $d_S/d_W$ ratio on the V-I loop (a) and the distribution of current during the ascending (b) and descending branch (c) of the loop. $j_{cS}=4\times j_{cW}$, $dI/dt=1A/s$, $j_{cW}=2 \times 10^8$A/m$^2$.

(3.1) $d_S/d_W=0.857$, the thickness of S region is proper, resulting that in S region there is
enough time for current diffusion in response to current descending, but the current diffusion is too busy to respond to current ascending for the given parameters, and area of the V-I loop is large.

(3.2) \( \frac{d_S}{d_W} = 0.222 \), S region is thinner and the distance of current diffusion in S region is shorter, resulting that in S region there is much more time for current diffusion in response to current ascending at the given parameters, and the area of V-I loop becomes smaller.

(3.3) \( \frac{d_S}{d_W} = 0.105 \), S region is too thin and the distance of current diffusion in S region is too short, resulting that in S region there is enough time for current diffusion in response to both the current ascending and descending at the given parameters, no obvious V-I loop.

Fig.4. Numerical results. Current sweeping rate dependent V-I loop (a). The corresponding distributions of current during the ascending (b) and descending branch (c) were also shown. \( \frac{d_S}{d_W} = 1/2, \frac{j_{cS}}{j_{cW}} = 4, j_{cW} = 2 \times 10^8 \text{A/m}^2 \).

(4.1) \( \frac{dI}{dt} = 0.01 \text{A/s} \), the current sweeping rate is so slow that in S region there are enough time for current diffusion in response to both the ascending and descending current sweeping at the given parameters, no obvious V-I loop.

(4.2) \( \frac{dI}{dt} = 2 \text{A/s} \), the current sweeping is faster, resulting that in S region there is no enough time for current diffusion in response to current ascending at the given parameters, and the V-I loop is larger.

(4.3) \( \frac{dI}{dt} = 10 \text{A/s} \), the current sweeping is so fast that in S region there is no enough time for current diffusion in response to both the ascending and descending current sweeping at the given parameters. No obvious V-I loop.

Fig.5. Numerical results. Magnetic field dependent V-I loop, where the area of the loop is insensitive to \( j_{cW} \) while the transport \( j_c \) is strongly affected by magnetic field. \( \frac{d_S}{d_W} = 1/2, j_{cS} = 8 \times 10^8 \text{A/m}^2, \frac{dI}{dt} = 1 \text{A/s} \).

Fig.6. Experimental V-I loops of the Ag-Bi2223 tape with different current sweeping rates \( \frac{dI}{dt} \), which is qualitatively with the numerical curves shown in Fig.4 and is a conformation
Fig. 7. Experimental V-I loops of the Ag-Bi2223 tape at different applied magnetic fields, which is qualitatively with the numerical curves shown in Fig. 5. It is noted that the area of the loop is insensitive to $B_a$ whereas the value of the $j_c$ is strongly affected by magnetic field, which supports the present model further.
(a) $d_s/d_w = 0.857$

(b) $J/J_{cw}$

(c) $J/J_{cw}$
(a) $\frac{dl}{dt} = 0.01 \text{A/s}$

(b) $\frac{J}{J_{cw}}$

(c) $\frac{J}{J_{cw}}$
(a) $J_{cs}/J_{cw} = 4$

(b) $J/J_{cw}$

(c) $J/J_{cw}$
(a) $dl/dt = 2 \text{A/s}$

- Graph of $V$ vs. $I$ shows a nonlinear relationship.
- The graph highlights different curves, each representing a different condition.

(b) Graph of $J/J_{CW}$ vs. $x/d$ for various values of $J/J_{CW}$.
- The curves exhibit a peak at $x/d = 0.4$ and $x/d = 0.6$.

(c) Graph of $J/J_{CW}$ vs. $x/d$ for various values of $J/J_{CW}$.
- The curves show a peak at $x/d = 0.4$ and $x/d = 0.6$, with the peak becoming steeper as $J/J_{CW}$ increases.
(a) $J_{cs}/J_{cw} = 1$

(b) $J/J_{cw}$

(c) $J/J_{cw}$
(a) $d_s/d_w = 0.105$

(b) $\frac{J}{J_{CW}}$

(c) $\frac{J}{J_{CW}}$
(a) $dl/dt = 10\text{A/s}$

(b) $J/J_{CW}$

(c) $J/J_{CW}$
$T = 77K$

$\frac{dI}{dt} = 1 \text{A/s}$

$B_a (\text{Gs})$

- $\boxed{\bullet} \ 0$
- $\boxed{\circ} \ 50$
- $\boxed{\diamond} \ 120$
- $\boxed{\star} \ 200$

$V (\text{mV})$

$I (\text{A})$