New 3-D Combined Inversion Scheme Using Response Functions Free From Galvanic Distortion

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Research Article

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New 3-D Combined Inversion Scheme Using Response Functions Free from Galvanic Distortion

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Abstract

The combined inversion using distortion-free response functions is an effective approach to robustly estimate the 3-D electrical resistivity structure against the distortions caused by near-surface resistivity anomalies. However, previous combined inversion analyses have presented a significant dependency of the inversion results on initial and prior models. Therefore, in this study, we evaluated the effectiveness of the following two new types of 3-D combined inversion using distortion-free response functions: one uses the phase tensor and the vertical and inter-station horizontal magnetic transfer functions, while the other uses the Network-MT response functions, in addition to the former. Because long dipoles are used, the Network-MT response function is negligibly affected by galvanic distortion. To access the combined inversion approach, we developed a novel 3-D inversion scheme combining the response functions of the usual magnetotelluric measurements and the Network-MT response function. The synthetic inversion analysis demonstrated that both of the proposed combined inversions can recover the characteristic resistivity distributions of the target model without a significant dependence on the initial models, at least in the shallow
part. These results demonstrate that the combined inversions using only distortion-free response functions have the potential to estimate subsurface resistivity more robustly than what was previously thought. Furthermore, we confirmed that the combined inversion using the Network-MT response function can make the resultant resistivity structure closer to the actual one and enhance the stability of the inversion. This result suggests that the combined use of the Network-MT response function is the preferred approach.

**Keywords**

magnetotelluric inversion, Network-MT method, galvanic distortion, finite element method, phase tensor, magnetic transfer function, tetrahedral mesh

**1. Introduction**

The three-dimensional inversion of magnetotelluric (MT) data has become common in recent decades. This is because of the increase in computational resources and the widespread use of practical 3-D MT inversion codes, such as WSINV3DMT
(Siripunvaraporn et al. 2005; Siripunvaraporn and Egbert 2009) and ModEM (Egbert and Kelbert 2012; Kelbert et al. 2014). However, the galvanic distortion of the MT impedance tensor has long been recognized as a major problem in estimating subsurface electrical resistivity structures. Galvanic distortion, which is caused by lateral small-scale inhomogeneities near the Earth’s surface, distorts the electrical field and, hence, the estimated impedance tensor (e.g., Ogawa 2002; Simpson and Bahr 2005).

One effective approach to robustly estimate the subsurface electrical resistivity structure against the galvanic distortion is to use the response functions free from the galvanic distortion as datasets in the MT inversion (e.g., Patro et al. 2013; Tietze et al. 2015; Campanyà et al. 2016). Patro et al. (2013) developed a 3-D inversion scheme using the phase tensor (PT) (Caldwell et al. 2004) as the input data. They demonstrated that the PT inversion can provide the proper electrical resistivity structure by performing synthetic inversion tests. Later, Tietze et al. (2015) developed a 3-D combined inversion using PT and the vertical magnetic transfer function (VMTF) and demonstrated that the integration of VMTF improves the recovery of the actual resistivity structure. In
addition, Campanyà et al. (2016) assessed the sensitivity of the impedance tensor, VMTF, and the inter-station horizontal magnetic transfer function (HMTF) to subsurface resistivity anomalies by performing synthetic inversions with different combinations of response functions. Campanyà et al. (2016) demonstrated that the combined use of VMTF and HMTF with the impedance tensor recovers the resistivity structure more accurately than using the impedance tensor alone. However, no study has yet examined the combined inversion of PT, VMTF, and HMTF.

The previous studies using distortion-free response functions presented a deficiency in robustness. Patro et al. (2013) and Tietze et al. (2015) showed that the resultant electrical resistivity structure of the inversion using either or both PT and VMTF depends significantly on the initial and prior models. They concluded that a sensible selection of the prior and initial models is essential for recovering a reliable resistivity structure. Campanyà et al. (2016) demonstrated that the inversion only using VMTF and HMTF failed to recover the subsurface resistivity variation in the vertical direction and the absolute resistivity values. They suggested that the inversion using either VMTF or
HMTF is reliable only when additional information defining the background resistivity structure is provided. As discussed in the previous studies, the dependency on the prior and initial models should be caused by the low sensitivity of PT, VMTF, and HMTF to the absolute values of the subsurface electrical resistivity.

One possible technique to overcome the problem by adding information regarding the absolute values of the subsurface resistivity structure for inversion is the use of the response function of the Network-MT (NMT) measurement (Uyeshima 2007), where metallic telephone line networks are used to measure the electrical potential difference between two distant electrodes (Uyeshima 2007). Because dipoles longer than several kilometers are used in the NMT measurement, the response functions of the NMT measurement (NMTRF) are negligibly affected by the galvanic distortion (Uyeshima et al. 2001; Uyeshima 2007). In addition, NMTRF is significantly sensitive to the absolute values of the subsurface resistivity structure, like the impedance tensor. Thus, it is expected that the combined inversion with NMTRF allows the estimation of the electrical resistivity structure with the correct scale. Recently, Usui et al. (2021)
developed a 2-D combined inversion method using the usual MT response functions (apparent resistivity and phase) and NMTRF. Usui et al. (2021) confirmed that the combined inversion provides the correct resistivity structure despite the response functions of the usual MT stations being significantly affected by galvanic distortion.

In addition, the combined use of NMTRF is advantageous because it enhances the sensitivity to deep structures. Long dipole measurements of the NMT method make the signal-to-noise ratio of the observed electrical field higher than that of the standard MT measurement. In addition, well-maintained telephone lines enable us to perform long-term observations from several months to several years (Kinoshita et al. 1989; Uyeshima et al. 1989), allowing us to increase the signal-to-noise ratio by stacking the data. Consequently, we can estimate NMTRF with small errors, including periods longer than several thousand seconds, which assists in estimating a reliable electrical resistivity down to the upper mantle. Siripunvaraporn et al. (2004) developed a 3-D inversion scheme using NMTRF. However, no one has proposed a 3-D inversion scheme that combines the response functions of the standard MT measurements and
In this study, we assessed the effectiveness of two new combined 3-D inversions using response functions free from galvanic distortion, one of which is the combined inversion of the PT, VMTF, and HMTF. All the response functions can be estimated from the electromagnetic field observed by standard MT measurements. The second inversion combines the PT, VMTF, HMTF, and NMTRF. To assess the second combined inversion, we developed a novel 3-D inversion scheme that uses both NMTRF and the response function of the standard MT measurements because no such 3-D inversion scheme exists. We performed synthetic inversions utilizing the same model used in a previous study (Tietze et al. 2015) and compared the inversion results.

In the following sections, we first present the algorithm of the combined inversion scheme used in this study, including the treatments of NMTRF in the inversion. Next, we describe the synthetic inversion tests to evaluate the performance of the combined inversion approaches and discuss the results of the combined inversions.
2. Algorithm of the combined inversion scheme

We used the inversion scheme based on the 3-D MT inversion code of Usui (2015) and Usui et al. (2017). This section summarizes the basic inversion algorithm and the treatments of NMTRF in the forward calculation and inversion. The finite element method with a tetrahedral mesh is used to obtain the electromagnetic field of the computational domain. The governing equation is the vector Helmholtz equation for the electric field with $e^{-i\omega t}$ time dependence.

$$\nabla \times \nabla \times \mathbf{E} = i\omega \mu_0 \sigma \mathbf{E}$$

In (1), $\mathbf{E} \in \mathbb{C}^3$ denotes the electric field, $\omega$ is the angular frequency, $\mu_0$ is the magnetic permeability of free space, and $\sigma$ is the electrical conductivity. The computational domain $\Omega$ indicates a rectangular geometry, and all the boundary planes $\partial \Omega$ of $\Omega$ are perpendicular to the orthogonal coordinate axes. The Dirichlet boundary condition is set on the entire boundary, $\partial \Omega$. On the top of $\Omega$, a uniform source electric field is specified, while the tangential electric field is designed to be zero at the bottom of $\Omega$, which corresponds to the condition that the perfect conductor locates below the bottom. The electric field calculated by the 2-D forward calculation is provided on the
two sides parallel to the source field. The 2-D forward problems (TM mode) are solved by the Galerkin FEM using the triangular edge-based element with the lowest order (Jin 2002). On the other hand, the tangential electric field is forced to be zero on the sides perpendicular to the source field. The boundary value problem is solved by the Galerkin FEM (Jin 2002) using the edge-based tetrahedral element. Within each element, the electric field can be approximated as follows:

\[ \mathbf{E}^e = \sum_{k=1}^{6} \mathbf{N}^e_i E^e_i, \]  

(2)

where \( \mathbf{N}^e_i \in \mathbb{R}^3 \) and \( E^e_i \in \mathbb{C} \) denote the vector basis function (Jin 2002) and the tangential component of the electric field on the \( i \)-th edge of the element, respectively.

As shown in Usui (2015), we can obtain a linear equation for each element by using the Galerkin method as follows:

\[ \mathbf{K}^e \mathbf{x}^e = \mathbf{b}^e, \]  

(3)

where \( \mathbf{x}^e = (E^e_1, ..., E^e_6)^T \in \mathbb{C}^6 \) is the solution vector, \( \mathbf{b}^e \in \mathbb{C}^6 \) denotes the right-hand-side vector including the boundary condition, and the \( i \)-th row and \( j \)-th column of the coefficient matrix \( \mathbf{K}^e \in \mathbb{C}^{6 \times 6} \) is expressed as follows:

\[ \int_{V_e} \left[ (\nabla \times \mathbf{N}^e_i) \cdot (\nabla \times \mathbf{N}^e_j) - i \omega \mu_0 \sigma \mathbf{N}^e_i \cdot \mathbf{N}^e_j \right] \mathrm{d}V. \]  

(4)
In Equation (4), $V^e$ denotes the volume occupied by the $e$-th element. By assembling all the element equations, we can obtain the following linear equation.

$$Kx = b$$  \hspace{1cm} (5)

The dimension of (5) is the number of unknown tangential components of the electric field at the element edges, and the coefficient matrix $K$ is a symmetrical sparse complex matrix. MKL PARDISO is used to solve the linear equation. To calculate the response functions as indicated below, forward computation is performed twice for each frequency by appending a source field in the x-direction ($E_x$-polarization) and y-direction ($E_y$-polarization). After obtaining the electric field by solving the linear equation above, the magnetic field $H$ is calculated from the electric field using Faraday’s law.

$$\nabla \times E = i\omega \mu_0 H$$  \hspace{1cm} (6)

From the results of the $E_x$-polarization and $E_y$-polarization, we can calculate the frequency response function at each observation station. VMTF $T_v \in \mathbb{C}^2$ is calculated as follows:
\[ T_v = \begin{pmatrix} T_{zx} \\ T_{zy} \end{pmatrix} = \begin{pmatrix} \frac{H_x^{E_x-pol} E_x^{E_x-pol}}{H_x^{E_y-pol} E_y^{E_y-pol}} \\ \frac{H_y^{E_x-pol} E_y^{E_x-pol}}{H_y^{E_y-pol} E_y^{E_y-pol}} \end{pmatrix}^{-1} \begin{pmatrix} \frac{H_x^{E_x-pol} E_x^{E_x-pol}}{H_x^{E_y-pol} E_y^{E_y-pol}} \\ \frac{H_y^{E_x-pol} E_y^{E_x-pol}}{H_y^{E_y-pol} E_y^{E_y-pol}} \end{pmatrix}. \] 

(7)

Meanwhile, HMTF \( T_h \in \mathbb{C}^{2 \times 2} \) is calculated from the horizontal magnetic field components at an observation station and those at a reference station as follows:

\[ T_h = \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix} = \begin{pmatrix} \frac{H_x^{E_x-pol} E_x^{E_x-pol}}{H_x^{E_y-pol} E_y^{E_y-pol}} \\ \frac{H_y^{E_x-pol} E_y^{E_x-pol}}{H_y^{E_y-pol} E_y^{E_y-pol}} \end{pmatrix} \begin{pmatrix} \frac{H_x^{E_x-pol} E_x^{E_x-pol}}{H_x^{E_y-pol} E_y^{E_y-pol}} \\ \frac{H_y^{E_x-pol} E_y^{E_x-pol}}{H_y^{E_y-pol} E_y^{E_y-pol}} \end{pmatrix}^{-1}, \] 

(8)

where the characters with subscript \( r \) denote the magnetic field components at the reference station. Further, PT \( \Phi \in \mathbb{R}^{2 \times 2} \) is calculated from the real and imaginary parts of the impedance tensor as follows:

\[ \Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} = \begin{pmatrix} \text{Re}(Z_{xx}) & \text{Re}(Z_{xy}) \\ \text{Re}(Z_{yx}) & \text{Re}(Z_{yy}) \end{pmatrix}^{-1} \begin{pmatrix} \text{Im}(Z_{xx}) & \text{Im}(Z_{xy}) \\ \text{Im}(Z_{yx}) & \text{Im}(Z_{yy}) \end{pmatrix}. \] 

(9)

where \( Z_{xx}, Z_{xy}, Z_{yx}, \) and \( Z_{yy} \) are the components of the impedance tensor \( Z \in \mathbb{C}^{2 \times 2} \), which is computed as follows:

\[ \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} = \begin{pmatrix} \frac{E_x^{E_x-pol} E_x^{E_x-pol}}{E_y^{E_x-pol} E_y^{E_x-pol}} \\ \frac{E_x^{E_y-pol} E_y^{E_y-pol}}{E_y^{E_y-pol} E_y^{E_y-pol}} \end{pmatrix} \begin{pmatrix} \frac{H_x^{E_x-pol} E_x^{E_x-pol}}{H_x^{E_y-pol} E_y^{E_y-pol}} \\ \frac{H_y^{E_x-pol} E_y^{E_x-pol}}{H_y^{E_y-pol} E_y^{E_y-pol}} \end{pmatrix}^{-1}. \] 

(10)

NMTRF \( Y = (Y_x, Y_y)^T \in \mathbb{C}^2 \) is defined as the frequency response function between the voltage difference \( V \in \mathbb{C} \) for the dipole of an NMT station and the horizontal magnetic field components at a reference station (Uyeshima et al. 2001).
\[ V = Y_x H_x + Y_y H_y \]  \hspace{1cm} (11)

In the developed combined inversion scheme, voltage differences were computed using an algorithm similar to that proposed by Siripunvaraporn et al. (2004). Because long dipoles of the NMT method generally extend over multiple finite elements of the computational mesh, dipoles are divided into segments by element edges (Figure 1). Each segment is located within an element surface, which is triangular. The voltage difference along a dipole can be obtained by summing the voltage differences of the small segments. According to Faraday’s law with the \( e^{i\omega t} \) time dependence, the voltage difference \( dV \in \mathbb{C} \) along each segment (illustrated as a red dashed line in Figure 1) can be calculated as follows:

\[ dV = -E_1 L_1 - E_2 L_2 - i\omega \mu_0 H_n S, \]  \hspace{1cm} (12)

where \( E_1 \) and \( E_2 \) are the electric field components along the triangle edges, \( L_1 \) and \( L_2 \) are the lengths of the edges, \( i \) is the imaginary unit, \( H_n \) is the magnetic field component normal to the triangle, and \( S \) is the area of the triangle. The magnetic field component normal to triangle \( H_n \) is calculated from the electric field components of the element edges on the Earth’s surface using Faraday’s law (Equation (6)). Based on the
voltage differences and the horizontal magnetic field of the two polarizations, NMTRF can be calculated as follows:

$$
\mathbf{Y} = \begin{pmatrix}
Y_x \\
Y_y
\end{pmatrix} = \begin{pmatrix}
H_x^{E_x-pol} & H_y^{E_x-pol} \\
H_x^{E_y-pol} & H_y^{E_y-pol}
\end{pmatrix}^{-1}
\begin{pmatrix}
V_x^{E_x-pol} \\
V_y^{E_y-pol}
\end{pmatrix} = \mathbf{H}^{-1} \mathbf{V}.
$$

(13)

In the combined inversion, we find the model parameters (subsurface electrical resistivities) that minimize the objective function $\phi(\mathbf{m}) \in \mathbb{R}$:

$$
\phi(\mathbf{m}) = \| \mathbf{Wd} - \mathbf{WF(} \mathbf{m} \mathbf{)} \|^2 + \alpha^2 \| \mathbf{Rm} \|^2,
$$

(14)

where $\mathbf{W}$ is the diagonal matrix of the diagonals that are reciprocals of the errors in the observed data; $\mathbf{d}$ and $\mathbf{F(} \mathbf{m} \mathbf{)}$ are the real vectors containing the observed and calculated response functions, respectively; and $\mathbf{R}$ denotes the roughening matrix. Vectors $\mathbf{d}$ and $\mathbf{F(} \mathbf{m} \mathbf{)}$ contain the real and imaginary parts of the complex frequency response functions separately. The roughening matrix provides the differences in the log-resistivities of the adjacent parameter cells $\mathbf{r}$ as follows:

$$
\mathbf{Rm} = \mathbf{r} = \begin{pmatrix}
r_1 & \cdots & r_i & \cdots & r_M
\end{pmatrix}^T
= F_i m_i - \sum_{j=1}^{F_i} m_{j,i}
$$

(15)

where $M$ is the number of model parameters; $F_i$ and $m_i$ are the number of faces and the log-resistivity of the $i$-th parameter cell, respectively; and $m_{j,i}$ is the log-resistivity
of the parameter cell adjacent to the \(i\)-th parameter cell through its \(j\)-th face. It should be noted that the objective function (Equation (14)) does not include the prior model, unlike the previous inversion schemes that use the response functions free from galvanic distortion (Patro et al. 2013; Tietze et al. 2015; Campanyà et al. 2016).

Therefore, our inversion scheme can estimate subsurface electrical resistivity without dependency on the prior model. After linearizing Equation (14), the model parameters are calculated by the data-space Gauss–Newton method, as described in Usui et al. (2017).

The Gauss–Newton method requires a sensitivity matrix (Jacobian matrix), which comprises the partial derivatives of the response functions with respect to the model parameters. The sensitivity matrix is computed using the reciprocity property of the electromagnetic field (Rodi 1976). Because the voltage difference and the horizontal magnetic field used for calculating NMTRF can be expressed as linear combinations of the electric field along the element edges, which are the unknowns in Equation (5), the sensitivity matrix for NMTRF can be calculated using the reciprocity approach (Rodi
1976). While Siripunvaraporn et al. (2004) used only the derivatives of the voltage differences for calculating the sensitivity matrix, we consider the derivatives of the reference horizontal magnetic field components in addition to the derivatives of the voltage differences. Thus, the derivatives of NMTRF can be expressed as follows:

$$\frac{\partial V}{\partial m} = \frac{\partial H^{-1}}{\partial m} V + H^{-1} \frac{\partial V}{\partial m}$$

(17)

3. Synthetic inversion analysis and discussion

We performed inversions using two different datasets. The first dataset contains PT, VMTF, and HMTF, whereas the second dataset contains PT, VMTF, HMTF, and NMTRF. Hereon, we refer to the former and latter cases as Comb-A and Comb-B, respectively. We performed a synthetic inversion analysis to assess whether the two new types of the combined inversion approaches can reproduce the appropriate resistivity structure robustly. The target model used in this study was the oblique conductor (OC) model used by Tietze et al. (2015). However, we exchanged the x- and y-axes to have the strike of the regional 2-D structure of the model parallel to the x-axis. PT and VMTF were calculated for the 100 observation stations with the exact locations used by
Tietze et al. (2015). The HMTF was also calculated for the same observation stations, except for the bottom-right point in Figures 2 and 3 (the station at x= -18 km and y=18 km). We used the bottom-right point as the reference station to calculate the HMTF. In addition, we calculated NMTRF at the 36 NMT observation stations (Figure 3). The NMT observation stations are represented as broken lines with circular ends, as shown in Figure 3. The dipole length of each NMT station is 5.66 km. We used the horizontal magnetic field at the bottom-right point as the reference magnetic field to calculate NMTRF in the same manner as the HMTF. We calculated PT, VMTF, and HMTF for 16 logarithmically spaced periods from 0.01 to 1,000 s, as in Tietze et al. (2015). Meanwhile, we calculated NMTRF for ten logarithmically spaced periods from 10 to 10,000 s because it is usually difficult to obtain the response functions at short periods at NMT stations. Because NMTRF can be estimated with small errors at periods longer than several thousand seconds, as indicated in the Introduction section, we set the maximum period for NMTRF to be longer than the other response functions. Because it is difficult to incorporate a thin 0.1 km near-surface layer (100 Ωm) of the OC model using a tetrahedral mesh, which requires a huge number of elements to represent such a
thin layer, we used a forward program using hexahedral mesh (Usui 2020; Usui et al. 2020) to calculate the synthetic response functions. The algorithm for the forward calculation using the hexahedral mesh is the same as that described in Section 2. By using different meshes, we can also avoid the so-called “inverse crime.” Before calculating PT from the impedance tensor, we provided a galvanic distortion to the calculated impedance tensor by multiplying a distortion tensor because the impedance tensor observed in the real world is generally distorted. The distortion tensors were calculated by randomly generated values of the twist angle (within ±30°), shear angle (within ±30°), anisotropy (within ±0.5), and the gains (from 0.1 to 10.0 in the logarithmic axis) of the Groom–Bailey decomposition (Groom and Bailey 1989).

To generate synthetic data, we added Gaussian noise with a mean of zero to the response functions. The standard deviation of the noise for VMTF was 0.02, as used in Tietze et al. (2015). Similarly, the standard deviation of the noise for HMTF was 0.02. The standard deviations for NMTRF were 3% of their absolute values; that is, \(0.03|Y_x|\) or \(0.03|Y_y|\). For PT, we added Gaussian noise to the distorted impedance tensor and
then calculated the PT errors by error propagation, as indicated in Patro et al. (2013).

The standard deviation of the noise to the impedance tensor was 3% for each component, along with a floor of 3% of $|Z_{xy}Z_{yx}|^{1/2}$ for diagonals, following Tietze et al. (2015). We started the combined inversion from three different uniform half-space models (10, 100, and 1,000 Ωm) to investigate the dependency of the inversion results on the initial model. In all cases, the tradeoff parameter $\alpha$ was set to 10.

The resultant resistivity structures of Comb-A are shown in Figure 2. The estimated resistivity structures have characteristic resistivity anomalies in the OC model. Specifically, both the resistivity contrast along the y-direction and the oblique conductor could be adequately imaged. However, the estimated resistivity values tended to be larger than the actual values of the OC model. It should be noted that, despite using significantly different initial models, no significant differences were observed in the resultant resistivity structure down to 15 km below ground. Tietze et al. (2015) performed a combined inversion of PT and VMTF with different prior and initial models between 10 Ωm and 1000 Ωm (the prior model was identical to the initial model.
in each case). Although the target model (i.e., OC model) was the same as that used in this study, Tietze et al. (2015) indicated a more significant dependency of the resultant resistivity structure on the prior and initial models, which is inconsistent with our results. Even if we performed the combined inversion of PT and VMTF, just like Tietze et al. (2015), the resultant resistivity structure down to 15 km below ground presented no significant dependency on the initial models as the Comb-A results (Figure S1 in Supporting Information). Therefore, this inconsistency was not attributed to the use of HMTF. Because the inversion scheme of this study does not use the prior model, unlike Tietze et al. (2015), the robustness of the results of our combined inversions is caused by not using the prior model. Other studies using PT and/or the magnetic transfer functions demonstrated a deficiency of robustness owing to the low sensitivity to the absolute values of the resistivity structure (Patro et al. 2013; Tietze et al. 2015; Campanyà et al. 2016). However, the results of our synthetic inversions suggest that not using the prior model can increase the robustness of the inversion using PT and the magnetic transfer functions.
Figure 3 presents the resultant resistivity structures of Comb-B down to 15 km below ground. Similar to Comb-A (Figure 2), only a slight difference is observed among the resultant resistivity structures starting from the different initial models. Characteristic resistivity anomalies of the OC model can also be adequately recovered by Comb-B.

Upon comparing the results of Comb-A (Figure 2) and Comb-B (Figure 3), Comb-B recovered the resistivity values at depths of 5 km and 15 km closer to the actual values of the OC model. It is considered that the additional use of NMTRF increased the sensitivity to the absolute values of the resistivity structure, allowing us to obtain a more reliable resistivity structure.

Figure 4 illustrates the electrical resistivity variation down to a depth of 100 km at the two points of (x=10 km, y=-10 km) and (x=-10 km, y=10 km). The locations of the two points are symmetrical about the axis of the oblique conductor. The former point is located at the resistive side, whereas the latter is located at the conductive side.

Although both Comb-A and Comb-B were unable to recover sharp resistivity changes at a depth of 70 km, this was inevitable owing to the smoothing effect due to the
stabilizing term of the objective function (Rodi and Macki 2012). At depths greater than 15 km, apparent differences are observed between the resultant resistivity structures of Comb-A and Comb-B. Figure 4 demonstrates that the resistivity values obtained by Comb-A tend to be larger than the actual values of the OC model at the depths down to 100 km. Furthermore, the overestimation of the resistivity becomes more significant with an increase in the initial resistivity value. This overestimation demonstrates that the results of the combined inversion using PT and the magnetic transfer function depend on the initial model, especially in the deep parts, where the sensitivity of the data is relatively low. Compared to the Comb-A results, the resistivity values obtained by Comb-B are closer to the actual values. Although the dependency on the initial model can also be found in the Comb-B results, no significant bias exists from the actual model, unlike the results of Comb-A. Especially at depths greater than 70 km, the resistivity values of Comb-B converge to the actual value (100 Ωm) without depending on the initial model. It is believed that the use of NMTRF increased the robustness against the initial models and enabled us to estimate a more accurate resistivity structure because NMTRF has a significant sensitivity to the absolute values of the subsurface
resistivity as the impedance tensor.

In addition, we found notable differences in the stabilization of the inversion between Comb-A and Comb-B (Figure 5). Figure 5 compares the variations in the objective functions during the combined inversions. At the 0-th iteration, the objective functions of Comb-B varied according to the initial model, while those of Comb-A presented similar values without a dependency on the initial model. Because the objective function at the 0-th iteration was calculated from the initial model, the variation of the objective function at the 0-th iteration reflects the high sensitivity of NMTRF to the absolute values of the subsurface resistivity structure. In the Comb-B inversions, the objective functions decreased rapidly and remained unchanged up to the 7-th iteration. Meanwhile, in Comb-A, the convergence of the inversion significantly depended on the initial models. While the objective functions remained unchanged after the 5-th iteration when the initial resistivity was 10 Ωm, it required up to the 16-th iteration to converge when we used the initial model of 1,000 Ωm. When starting from the same initial model, Comb-A required more iterations than Case B. The relatively low sensitivity to
the absolute values of the subsurface resistivity apparently led to the instability in Comb-A. Figure 5 demonstrates that the addition of NMTRF can improve the inversion stability.

Because both Comb-A and Comb-B were able to recover the characteristic features of the target resistivity structure, we can conclude that both combined inversion approaches are effective in estimating the subsurface resistivity structure. This study indicates that combined inversions using distortion-free response functions have the potential to estimate subsurface resistivity more robustly than what was previously thought. However, an overestimation bias in the resultant resistivity structure and poor convergence of the Gauss–Newton iteration was found in the results of Comb-A. The low sensitivity to the absolute values of the subsurface resistivity is thought to be the cause of these problems. Because both issues were improved by Comb-B, we confirmed that the combined inversion using NMTRF can enhance the reliability of the resultant resistivity structure and the stability of the inversion. Therefore, we believe that the combined use of NMTRF is the preferred approach for robustly estimating the
resistivity structure.

4. Summary

To evaluate the effectiveness of the two new types of 3-D combined inversion using response functions free from galvanic distortion, we performed synthetic inversion analyses using a newly developed 3-D combined inversion scheme; this can include NMTRF as the datasets in addition to the response functions of the usual NMT measurements. By comparing the estimated resistivity structures and histories of the objective functions, we obtained the following results:

1. Despite using only PT, VMTF, and HMTF, the combined inversion recovered the characteristic resistivity anomalies of the target model in the shallow part without a significant dependence on the initial models. This suggests that the combined inversion using only PT and the magnetic transfer functions has the potential to recover the subsurface resistivity structure more robustly than what was previously thought if the prior model is not used.

2. When we used NMTRF in addition to PT and the magnetic transfer functions, the
resultant resistivity structure became closer to the actual model. Therefore, we confirmed that the additional use of NMTRF enabled us to estimate a more accurate resistivity structure.

3. The objective functions stably decreased when NMTRF is included as data. In contrast, the convergence of the inversion was worse and depending significantly on the initial model when we used only PT, VMTF, and HMTF. This difference demonstrates that the use of NMTRF can improve the stability of the combined inversion.

Because PT, VMTF, and HMTF can be calculated from the observed data of the MT measurements, we can apply the combined inversion proposed in this study to existing data. In addition, in many areas of Japan, NMT measurements have been conducted in the same survey area as usual MT measurements. This enables us to perform the inversion by combining NMTRF with other distortion-free response functions. The aforementioned results suggest that the combined inversion using only the response function free of galvanic distortion has the potential to robustly recover the target resistivity structure, especially by using NMTRF. Therefore, we expect that the
developed combined inversion approach will help in robustly recovering resistivity structures by avoiding the distortions caused by near-surface anomalies.

Declarations

Ethics approval and consent to participate

Not applicable

Consent for publication

Not applicable

List of abbreviations

MT, magnetotelluric; NMT, Network-MT; PT, phase tensor; VMTF, vertical magnetic transfer function; HMTF, inter-station horizontal magnetic transfer function; NMTRF, response functions of the NMT measurement; RMS, root mean square

Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.
Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

YU developed a combined inversion scheme and performed synthetic inversion tests. MU is the inventor of the combined inversion of the phase tensor, magnetic transfer functions, and Network-MT response function. All authors read and approved the final manuscript.

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In this study, we used the computer systems of the Earthquake and Volcano Information Center of the Earthquake Research Institute, the University of Tokyo. We illustrated a few figures using generic mapping tools (Wessel et al. 2013).

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Figure legends:

Figure 1. Schematic view illustrating how to calculate the voltage difference along each dipole of the NMT observation station. Here, \( E_1 \) and \( E_2 \) are the electric field components along the edges of the triangle, \( L_1 \) and \( L_2 \) are the lengths of the edges, \( i \) is the imaginary unit, \( \mu_0 \) is the magnetic permeability of vacuum, \( H_n \) is the magnetic field component normal to the triangle, and \( S \) is the area of the triangle. The solid lines indicate the element edges, while a thick dash line denotes a dipole of the NMT observation station. The black-filled circle denotes the endpoints of the dipole. The voltage difference along each dipole is computed by summing the voltage differences \( dV \) of the small segments.

Figure 2. Electrical resistivity structures obtained by Comb-A. (a) True resistivity structure (OC model). (b)-(c) Resultant resistivity structures obtained by the combined inversion with three different initial models. The lowermost panels are the vertical cross-sections of the profile along the major axis of the oblique conductor, which is shown as a white line in the upper-left panel. The inverted triangles indicate the
locations of observation stations. We used the bottom-right point (the station at $x= -18$ km and $y=18$ km) as the reference station for calculating HMTF.

Figure 3. Electrical resistivity structures obtained by Comb-B. (a) True resistivity structure (OC model). (b)-(c) Resultant resistivity structures obtained by the combined inversion with three different initial models. The lowermost panels indicate the vertical cross-sections of the profile along the major axis of the oblique conductor, which is shown as a white line in the upper-left panel. The inverted triangles indicate the locations of observation stations of PT, VMTF, and HMTF. The broken lines with circular ends indicate the dipoles of the NMT stations. We used the bottom-right point (the station at $x= -18$ km and $y=18$ km) as the reference station for calculating HMTF and NMTRF.

Figure 4. Electrical resistivity variation with a depth at (a) $(x, y) = (10$ km, -10 km) and (b) $(x, y) = (-10$ km, 10 km). The thick black lines indicate the resistivity variation of the true resistivity structure. The broken colored and solid-colored lines indicate the
resistivity variations of the resistivity structure obtained by Comb-A and Comb-B, respectively.

Figure 5. Objective function versus the iteration number for combined inversions with different initial models. The broken and solid lines indicate the objective function changes during Comb-A and Comb-B inversions.
**Figures**

**Figure 1**

Schematic view illustrating how to calculate the voltage difference along each dipole of the NMT observation station. Here, E1 and E2 are the electric field components along the edges of the triangle, L1 and L2 are the lengths of the edges, i is the imaginary unit, \(\mu_0\) is the magnetic permeability of vacuum, \(H_n\) is the magnetic field component normal to the triangle, and S is the area of the triangle. The solid lines indicate the element edges, while a thick dash line denotes a dipole of the NMT observation station. The black-filled circle denotes the endpoints of the dipole. The voltage difference along each dipole is computed by summing the voltage differences \(dV\) of the small segments.

**Figure 2**
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Figure 3

Electrical resistivity structures obtained by Comb-B. (a) True resistivity structure (OC model). (b)-(c) Resultant resistivity structures obtained by the combined inversion with three different initial models. The lowermost panels indicate the vertical cross-sections of the profile along the major axis of the oblique conductor, which is shown as a white line in the upper-left panel. The inverted triangles indicate the locations of observation stations of PT, VMTF, and HMTF. The broken lines with circular ends indicate the dipoles of the NMT stations. We used the bottom-right point (the station at x= -18 km and y=18 km) as the reference station for calculating HMTF and NMTRF.
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Figure 5

Objective function versus the iteration number for combined inversions with different initial models. The broken and solid lines indicate the objective function changes during Comb-A and Comb-B inversions.

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