Effective Ginzburg-Landau free energy functional for multi-band isotropic superconductors

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It has been shown that interband mixing of gradients of two order parameters (drag effect) in an isotropic bulk two-band superconductor plays important role - such a quantity of the intergradients coupling exists that the two-band superconductor is characterized with a single coherence length and a single Ginzburg-Landau (GL) parameter. Other quantities or neglecting of the drag effect lead to existence of two coherence lengths and dynamical instability due to violation of the phase relations between the order parameters. Thus so-called type-1.5 superconductors are impossible. An approximate method for solving of set of GL equations for a multi-band superconductor has been developed: using the result about the drag effect it has been shown that the free-energy functional for a multi-band superconductor can be reduced to the GL functional for an effective single-band superconductor.

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I. INTRODUCTION

Two-band superconductors are a specific class of superconductors essentially differing in their properties from single-band superconductors. Their typical representatives are magnesium diboride MgB$_2$, strontium ruthenate Sr$_3$RuO$_4$, nonmagnetic borocarbides LuNi$_2$B$_2$C, YNi$_2$B$_2$C and ferropnictides. In this article we will consider only isotropic bulk (polycrystalline) s-wave superconductors. One of the main feature of these materials is the presence of two energy gaps $\Delta_1$ and $\Delta_2$ which, however, vanishes at the same temperature $T_c$ (Fig.1). According to microscopic theory $\Delta_1 \neq \Delta_2$, presence of the two gaps is explained by the fact that in each band $i$ an own coupling constant $g_{ii}$ exists - the intraband constant. In the same time, the interband coupling constant $g_{ij}$ exists too, which, on the one hand, enhances pairing of electrons, on the other hand, leads to the single critical temperature $T_c$. BCS gap equations for a two-band superconductor are [1–3]:

$$
\begin{align*}
\Delta_1 &= \sum_k \frac{g_{11}\Delta_1 \tanh(E_{1,k}/2k_BT)}{2E_{1,k}} + \sum_k \frac{g_{12}\Delta_2 \tanh(E_{2,k}/2k_BT)}{2E_{2,k}}, \\
\Delta_2 &= \sum_k \frac{g_{22}\Delta_2 \tanh(E_{2,k}/2k_BT)}{2E_{2,k}} + \sum_k \frac{g_{21}\Delta_1 \tanh(E_{1,k}/2k_BT)}{2E_{1,k}},
\end{align*}
$$

(1)

where $E_{i,k}$ is the quasiparticle’s energy in a band $i$. Unlike single-band BCS theory a superconducting state can exist both attractive interband coupling constant $g_{12} > 0$ and repulsive $g_{12} < 0$, moreover the gaps are nonzero if even the intraband couplings are absent $g_{11} = g_{22} = 0$. In the case of the attractive interband interaction the gaps have the same phases on both Fermi surfaces, while for the repulsive interaction the phases will be opposite. Thus the phase difference of the order parameters $|\Delta_1| e^{i\varphi_1}, |\Delta_2| e^{i\varphi_2}$ are:

$$
\cos(\varphi_1 - \varphi_2) = \begin{cases} 
1 & \text{if } g_{12} > 0 \\
-1 & \text{if } g_{12} < 0
\end{cases}
$$

(2)

For example, in absence of magnetic field we can suppose $\Delta_1 > 0$, then we will have $\Delta_2 > 0$ for $g_{12} > 0$ and $\Delta_2 < 0$ for $g_{12} < 0$. From Eq.(1) we can see the important property of a two-band superconductor: if we violate the phase relation (2) then suppression of the energy gaps $\Delta_1, \Delta_2$ will take place (extremely strong suppression if the intraband couplings are absent $g_{11} = g_{22} = 0$). For the suppression of the order parameters the violation of the phase-locked

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states $\varphi \equiv \varphi_1 - \varphi_2 = 0$ or $\pi$ must be macroscopic, constant in time and not small (for example, when there are two different coherence lengths in a system with two gaps), unlike Leggett’s mode, which is collective mode of small fluctuations of relative phase $\varphi(\mathbf{r}, t)$ and behaves like the Anderson plasmons in Josephson junctions [3, 6]. In addition, we assume that current in a two-band superconductor is less than some a critical current $J < J_c$ over which interband phase breakdown occurs, resulting in spontaneous phase solitons in $\varphi(\mathbf{r}, t)$ [7], which is nonequilibrium state. In phenomenological theory the coupling between the bands is represented by Josephson-like coupling term:

$$\varepsilon \left( \Psi_1^+ \Psi_2 + \Psi_1 \Psi_2^+ \right)$$

in a free energy functional, where $\Psi_1$ and $\Psi_2$ are order parameters for band 1 and 2 accordingly.

Currently there are two opinions about the properties of two-band superconductors:

1) In papers [8, 9] it has manifested about a new type of superconductivity in MgB$_2$ - a novel "type-1.5 superconductor", contrary to type-I and type-II superconductors. In papers [10, 11, 12] a two-band superconductor was studied, where they considered GL parameters $\kappa_i = \lambda / \xi_i$ ($i = 1, 2$) in two different regimes to produce type-I ($\kappa_1 < 1/\sqrt{2}$) and type-II ($\kappa_2 > 1/\sqrt{2}$) materials, that corresponds to different coherence lengths $\xi_1 = \hbar v_F / \pi \Delta_1(0)$ and $\xi_2 = \hbar v_F / \pi \Delta_2(0)$. That is each correlation length is sorted with a corresponding band, where Fermi velocities $v_{F1}, v_{F2}$ and energy gaps $\Delta_1, \Delta_2$ are different. Their prediction leads to what they call a "semi-Meissner state". Instead of homogeneous distribution, the vortexes form aperiodic clusters or vortexless Meissner domains, arising out of short range repulsion and long range attraction between vortexes.

2) However, in review [13] an opposite opinion has been suggested in respect of existence of the type-1.5 superconductivity in two-band superconductors. It was shown that for the real superconductor MgB$_2$ which possesses a single transition temperature, the assumption of two independent order parameters with separate penetration depths and separate coherence lengths is unphysical. In particular, in the above-mentioned works [8, 9] numerical estimates for $\xi_i$ are obtained by using the one-band BCS formula. On the other hand, in works [3, 14] it has been shown that in a two-band superconductor there are two coherence lengths which are not related to the concrete bands involved in the formation of the superconducting ordering in a system with interband interaction: one of the lengths is diverges at the critical temperature $\xi_1(T \rightarrow T_c) \rightarrow \infty$, the second of them is a nearly constant at all temperatures $\xi_2(T) \approx const$. Besides it is necessary to be more accuracy at calculations of interaction between vortexes - many corrections to the simple GL or London theories are expected to modify the monotonically decreasing interaction potential at large distances, $V(r) \sim \exp(-r/\alpha)$, such that $\alpha$ becomes complex. This, in principle, causes an oscillating potential, whose first minimum may occur at large distances where the amplitude of the potential is small. Generally, as discussed in [13], it should be taken into account dependence on the material, its purity, magnetic history, and temperature. In a paper [15] it was shown that coherence length is the same for both order parameters $\Delta_1, \Delta_2$, moreover the ratio of the order parameters is $T$-independent in the GL domain, $\Delta_1(r, T)/\Delta_2(r, T) = const$, with the constant depending on interactions responsible for superconductivity - thus the type-1.5 superconductivity is absent. In a paper [16] it was demonstrated that close to the transition temperature, where the GL theory is applicable, the two-band problem maps onto an effective single-band problem with a GL parameter $\kappa_i^2 = \kappa_1^2 + \kappa_2^2$, a penetration depth $\lambda^2 = \lambda_1^2 + \lambda_2^2$ and a coherence length $\xi_i = (\xi_1^2 + \xi_2^2)^{-1/2}$ where $\kappa_i, \lambda_i, \xi_i$ are quantities corresponding to a band $i$. Similar effective single-band GL approach also was applied in papers [17, 18]. The two-band GL theory has been developed in works [19, 21] where it was shown that the presence of two order parameters leads to a nonlinear temperature dependence of the upper and lower critical fields $H_{c2}(T), H_{c1}(T)$ and thermodynamic magnetic field $H_{cm}(T)$ unlike single-band GL theory. In [22] the temperature dependence of the London penetration depth $\lambda(T)$ has been determined. These results are in good agreement with the experimental data for bulk MgB$_2$ and borocarbides without any hypothesis about "type-1.5 superconductor" and "semi-Meissner state".

In this paper we study two problems which, in our opinion, are important for GL theory of isotropic bulk multi-band superconductors:

1) The coupling between the bands is represented by both the term of proximity effect Eq.(3) and the term of drag effect - interband mixing of order parameters’ gradients:

$$\eta \left( \nabla \Psi_1^+ \nabla \Psi_2 + \nabla \Psi_1 \nabla \Psi_2^+ \right).$$

Since electron from different bands are interacting, hence, if in some a band the order parameter is spatially inhomogeneous $\Psi_1(\mathbf{r})$ then in other band the order parameter must be inhomogeneous too $\Psi_2 = \Psi_2(\mathbf{r})$. If a current exists in one band then it drags Cooper pairs in other band. Therefore the coefficient $\eta$ must be function of carriers’ mass in each band $m_1, m_2$ and the coupling $\varepsilon$ between the order parameters. As a rule the drag effect is neglected or the coefficient $\eta$ is considered as an adjustable parameter. However in a work [23] where they considered Little-Parks effect for two-band superconductors, it has been found that the coefficient $\eta$ is not an arbitrary quantity and a relation between the coefficient and effective masses of carriers exists to ensure the existence of the absolute minimum of the
free energy functional. In present paper we show that the drag effect plays important role in two-band superconductors. Accounting of the drag effect leads to single coherence length $\xi$ for a two-band superconductor unlike the papers \[14\]. Moreover the ratio of the order parameters is $T$-dependent $\Delta_1(r,T)/\Delta_2(r,T) = \text{const}(T)$, unlike the work \[15\]. Neglecting of the drag effect leads to dynamical instability of the two-band superconductor due to violation of the phase relations \[2\]. Thus type-1.5 superconductors are impossible. Unlike previous works we have found the coefficient $\gamma$ as a function of $m_1, m_2, \varepsilon$.

2) GL equation for a single-band superconductor (in absence of a magnetic field) is a nonlinear second-order differential equation. Phenomenological theory for bulk isotropic two-band superconductors has been developed in works \[19\]–\[21\], where GL equations are a set of two nonlinear second-order differential equations. Exact GL theory for multi-band superconductors will be extremely complicated. Therefore approximate methods are required. In this paper we show that, using the result about the drag effect, the GL theory for a two-band superconductor can be reduced to the GL theory for an effective single-band superconductor. Generalizing this result we develop an algorithm which allows to reduce the free energy functional of a multi-band superconductor to the GL free energy functional of an effective single-band superconductor.

II. TWO-BAND SUPERCONDUCTOR

In presence of two-order parameters in a bulk isotropic s-wave superconductor, the GL free energy functional can be written as \[19\]–\[23\]:

$$
F = \int d^3r \left[ \frac{\hbar^2}{4m_1} |D\Psi_1|^2 + \frac{\hbar^2}{4m_2} |D\Psi_2|^2 + \frac{\hbar^2}{4} \eta \left( D^+\Psi_1^* D\Psi_2 + D\Psi_1 D^+\Psi_2^* \right) \right. \\
\left. + a_1 |\Psi_1|^2 + a_2 |\Psi_2|^2 + \frac{b_1}{2} |\Psi_1|^4 + \frac{b_2}{2} |\Psi_2|^4 + \varepsilon \left( \Psi_1^* \Psi_2 + \Psi_2^* \Psi_1 \right) + \frac{\hbar^2}{8\pi}, \right]
$$

(5)

where the differential operator are $D = \nabla - \frac{2\pi i}{\Phi_0} A$ ($\Phi_0 = \pi h c / e$ is a magnetic flux quantum, $H = \text{rot} A$ is a vector potential), $m_{1,2}$ denotes the effective mass of carriers in the correspond band, the coefficient $a_i$ is given as $a_i = \gamma_i(T - T_c)$, $\gamma$ is constant, the coefficients $b_{1,2}$ are independent on temperature, the quantities $\varepsilon$ and $\eta$ describe interband mixing of two order parameters (proximity effect) and their gradients (drag effect), respectively. If we switch off the interband interaction $\varepsilon = \eta = 0$ then we will have two independent superconductors with the different critical temperatures $T_{c1}$ and $T_{c2}$ because the intraband interactions can be different $g_{11} \neq g_{22}$. There is another form of the coefficients $a_i$ \[15\]–\[25\]: they acquire constant parts $\text{const}_i + \gamma_i(T - T_c)$ such that $\text{const}_1 \text{const}_2 = \varepsilon^2$ and $T_{c}$ is critical temperature of a two-band superconductor. However in this case if we switch off the interband interaction $\varepsilon = 0 \Rightarrow \text{const}_{1,2} = 0$, then we will have two independent superconductors with the same critical temperatures $T_c$.

![Figure 1: Superconductor gap parameters $\Delta_1$ and $\Delta_2$ if the interband interaction is absent ($\varepsilon = 0$) (dash lines) and if the interband interaction takes place ($\varepsilon \neq 0$) (solid line).](image)

Minimization of the free energy functional with respect to the order parameters, if $\nabla \Psi_{1,2} = 0$ and $A = 0$, gives

$$
\begin{align*}
\{ a_1 \Psi_1 + \varepsilon \Psi_2 + b_1 \Psi_1^3 &= 0 \\
 a_2 \Psi_2 + \varepsilon \Psi_1 + b_2 \Psi_2^3 &= 0 \}
\end{align*}
$$

(6)
Near critical temperature $T_c$ we have $\Psi^2 \rightarrow 0$, hence we can find the critical temperature as a solvability condition of the linearized Eqs. (6):

$$a_1 a_2 - \varepsilon^2 = \gamma_1 \gamma_2 (T_c - T_{c1})(T_c - T_{c2}) - \varepsilon^2 = 0.$$  \hspace{1cm} (7)

Solving this equation we find $T_c > T_{c1}, T_{c2}$, moreover the solution does not depend on sign of $\varepsilon$. The sign determines the phase difference of the order parameters $|\Psi_1|e^{i\varphi_1}, |\Psi_2|e^{i\varphi_2}$:

$$\cos(\varphi_1 - \varphi_2) = 1 \quad \text{if} \quad \varepsilon < 0$$

$$\cos(\varphi_1 - \varphi_2) = -1 \quad \text{if} \quad \varepsilon > 0,$$  \hspace{1cm} (8)

that follows from the Eqs. (6) and is an analogue of Eq. (2): the case $\varepsilon < 0$ corresponds to attractive interband interaction $g_{12} > 0$, the case $\varepsilon > 0$ corresponds to repulsive interband interaction $g_{12} < 0$. It should be noted that the interband mixing of two-order parameters $\varepsilon$ ensures the single critical temperature $T_c$ of a two-band superconductor whilst each band has own critical temperature $T_{c1}$ and $T_{c2}$ if the interband interaction is absent. This fact is illustrated in Fig. (1), where it is given the qualitative picture of calculations in [3].

Phase relations (8) imposes restrictions on the coefficient $\eta$. For temperatures near $T_c$ and magnetic fields smaller than $H_{c1}$, the influence of the field on modulus of the order parameters can be neglected and we assume $|\Psi_1| = const, |\Psi_2| = const$. Then the wave function can be written as $\Psi = |\Psi_j| \exp(i\varphi_j(r))$, where $\varphi_j(r)$ are the phases of the order parameters. The GL free energy functional (9) can be rewritten as

$$F = \int d^3 r \frac{\hbar^2}{8 m_1} n_1 \left( \nabla \varphi_1 - \frac{2 \pi A_0}{\Phi_0} \right)^2 + \frac{\hbar^2}{8 m_2} n_2 \left( \nabla \varphi_2 - \frac{2 \pi A_0}{\Phi_0} \right)^2$$

$$+ \frac{\hbar^2}{4 \eta \sqrt{n_1 n_2}} \left( \nabla \varphi_1 - \frac{2 \pi A_0}{\Phi_0} \right) \left( \nabla \varphi_2 - \frac{2 \pi A_0}{\Phi_0} \right) \cos(\varphi_1 - \varphi_2)$$

$$+ \varepsilon \sqrt{n_1 n_2} \cos(\varphi_1 - \varphi_2) + \frac{H^2}{8 \pi},$$  \hspace{1cm} (9)

where $n_1 = 2|\Psi_1|^2$ and $n_2 = 2|\Psi_2|^2$ are the densities of superconducting electrons for the corresponding bands. Phase relations (8) must be satisfied over the entire volume of a superconductor: $\varphi_1(r) - \varphi_2(r) = \text{const}$, otherwise superconducting state will be destroyed - Eqs. (12). Hence the phases must change equally:

$$\nabla \varphi_1(r) = \nabla \varphi_2(r).$$  \hspace{1cm} (10)

Minimizing the free energy functional (9) with respect to the vector potential $A$ we find the current $J = \frac{e}{4 \pi} \nabla \times H$:

$$J = \frac{2 \pi e}{\Phi_0} \left( \frac{\hbar^2}{4 m_1} n_1 \left( \nabla \varphi_1 - \frac{2 \pi A_0}{\Phi_0} \right) + \frac{\hbar^2}{4 m_2} n_2 \left( \nabla \varphi_2 - \frac{2 \pi A_0}{\Phi_0} \right) \right)$$

$$+ \frac{\hbar^2}{4 \eta \sqrt{n_1 n_2}} \left( \nabla \varphi_1 - \frac{2 \pi A_0}{\Phi_0} \right) \left( \nabla \varphi_2 - \frac{2 \pi A_0}{\Phi_0} \right) \cos(\varphi_1 - \varphi_2).$$  \hspace{1cm} (11)

Let us consider a superconductor with an inner cavity. We integrate Eq. (11) along a closed path lying within the superconductor around the cavity at a distance from the cavity’s surface larger than magnetic penetration depth $\lambda$. Hence on the path we have $J = 0$ and integral on the right-hand is equal to zero. Then

$$\int \left( \frac{n_1}{m_1} + \eta \sqrt{n_1 n_2} \cos(\varphi_1 - \varphi_2) \right) \int \nabla \varphi_1 dl + \int \left( \frac{n_2}{m_2} + \eta \sqrt{n_1 n_2} \cos(\varphi_1 - \varphi_2) \right) \int \nabla \varphi_2 dl$$

$$= \frac{2 \pi \Phi}{\Phi_0} \left( \frac{n_1}{m_1} + 2 \eta \sqrt{n_1 n_2} \cos(\varphi_1 - \varphi_2) + \frac{n_2}{m_2} \right),$$  \hspace{1cm} (12)

where $\Phi = \oint A dl$ is a magnetic flux. Taking into account the functions $\varphi_1$ and $\varphi_1$ must be single-valued $\oint \nabla \varphi_1 dl = \oint \nabla \varphi_2 dl = 2 \pi n_1$, we find that the magnetic flux through the cavity takes a discrete series $\Phi = n \Phi_0$ like in single-band superconductors [19, 24].

Let us analyze the functional (9). The term $\sqrt{n_1 n_2} \cos(\varphi_1 - \varphi_2) < 0$ always because Eq. (8). This lowers the free energy. For stability of the superconducting state it is necessary that a spatial inhomogeneity of the order parameters enlarges the free energy. Since we have $\nabla \varphi_1 \nabla \varphi_2 = (\nabla \varphi_1)^2 = (\nabla \varphi_2)^2 > 0$ from Eq. (10) then the stability condition is

$$\frac{n_1}{m_1} + \frac{n_2}{m_2} + 2 \eta \sqrt{n_1 n_2} \cos(\varphi_1 - \varphi_2) > 0.$$  \hspace{1cm} (13)
From the Eq.(11) we find the London penetration depth in the following form
\[
\lambda^{-2}(T) = \frac{4\pi e^2}{c^2} \left[ \frac{n_1(T)}{m_1} + \frac{n_2(T)}{m_2} + 2\eta \sqrt{n_1(T)n_2(T)} \cos(\varphi_1 - \varphi_2) \right].
\] (14)

From this formula we can see the condition $\lambda^2(T) > 0$ when $n_1(T), n_2(T) \neq 0$. Thus the condition $n_1(T), n_2(T) \neq 0$ restricts the possible quantities of the parameter $\eta$. Let $\nabla \varphi_1 = \nabla \varphi_2 = 0$, that is a paramagnetic current is absent. Then the free energy functional takes the form
\[
F = \frac{1}{8\pi} \int d^3r [H^2 + \lambda^2 (\text{rot} H)^2],
\] (15)

Let the field $H_0$ is directed along the axis Oz and a superconductor are in a halfspace $x > 0$ then the magnetic field within the superconductor are $H(x) = H_0 \exp(-x/\lambda)$. Substituting this field in Eq.(15) and integrating we have the free energy per unit of square:
\[
F = \frac{H_0^2}{8\pi} \lambda.
\] (16)

We can see the smaller London penetration depth $\lambda$ the smaller free energy. Then from Eq.(14) it follows that such quantities of the parameter $\eta$, when
\[
\eta \cos(\varphi_1 - \varphi_2) > 0 \Rightarrow \eta \varepsilon < 0,
\] (17)
lower the free energy.

Let us consider a case when a two-band superconductor in a normal state ($a_1a_2 > \varepsilon^2$) has contact with a metal in a superconducting state. Let the superconductor in a normal state occupies a halfspace $x > 0$. Since in a normal region the order parameters are small, then minimization of the free energy functional leads to the order parameters gives
\[
\begin{align*}
\frac{\hbar^2}{4m_1} \frac{d^2 \Psi_1}{dx^2} + \frac{\hbar^2}{4} \eta \frac{d^2 \psi_1}{dx^2} - a_1 \psi_1 - \varepsilon \Psi_1 &= 0, \\
\frac{\hbar^2}{4m_2} \frac{d^2 \Psi_2}{dx^2} + \frac{\hbar^2}{4} \eta \frac{d^2 \psi_2}{dx^2} - a_2 \psi_2 - \varepsilon \Psi_2 &= 0,
\end{align*}
\] (18)

Eq.(18) are a set of linear equations with constant coefficients. Hence we must seek a solution in a form $\Psi_1 = \psi_1 e^{kx}$, $\Psi_2 = \psi_2 e^{kx}$, where the quantity $k$ has physical sense of an inverse coherence length: $k = 1/\xi$. Then we have
\[
\begin{align*}
\left( \frac{\hbar^2}{4m_1} - a_1 \right) \psi_1 + \left( \frac{\hbar^2}{4} \eta - \varepsilon \right) \psi_2 &= 0, \\
\left( \frac{\hbar^2}{4} \eta - \varepsilon \right) \psi_1 + \left( \frac{\hbar^2}{4m_2} - a_2 \right) \psi_2 &= 0.
\end{align*}
\] (19)

The characteristic equation are
\[
k^4 \left( \frac{\hbar^2}{4} \right)^2 \left( \frac{1}{m_1m_2} - \eta^2 \right) - k^2 \frac{\hbar^2}{4} \left( \frac{a_2}{m_1} + \frac{a_1}{m_2} - 2\eta \varepsilon \right) + a_1a_2 - \varepsilon^2 = 0.
\] (20)

Solutions of this equation corresponds to two coherence lengths. At $T \rightarrow T_c$ we have
\[
\begin{align*}
k_1^2 &= \frac{a_1a_2 - \varepsilon^2}{\frac{\hbar^2}{4} \left( \frac{a_2}{m_1} + \frac{a_1}{m_2} - 2\eta \varepsilon \right)}, \\
k_2^2 &= \frac{\left( \frac{a_2}{m_1} + \frac{a_1}{m_2} - 2\eta \varepsilon \right)}{\frac{\hbar^2}{4} \left( \frac{1}{m_1m_2} - \eta^2 \right)}, \quad \eta^2 \neq \frac{1}{m_1m_2}.
\end{align*}
\] (21)
(22)

The first of them is $k_1 = 0$ at the critical temperature (when $a_1a_2 - \varepsilon^2 = 0$). That is the coherence length $\xi_1 = 1/|k_1|$ is diverging $\xi_1(T \rightarrow T_c) \rightarrow \infty$. On the contrary $k_2(T = T_c) \neq 0$ and it varies little with temperature. These length scales are not related to the concrete bands involved in the formation of the superconducting ordering in a system
with interband interaction. This result corresponds to the results in works [3] [14] obtained by microscopic approach, however they suggested that the intergradient interaction is absent ($\eta = 0$).

According to the method for solving of a set of linear differential equations with constant coefficients we have to write solutions of Eq. (18) in a form

$$\Psi_1 = C_1 \psi_1^{(1)} e^{k_1 x} + C_2 \psi_1^{(2)} e^{k_2 x},$$

$$\Psi_2 = C_3 \psi_2^{(1)} e^{k_1 x} + C_4 \psi_2^{(2)} e^{k_2 x},$$

(23)

where coefficients $\psi_1^{(1)}, \psi_1^{(1)}$ correspond to the eigenvalue $k_1$ (they must be found from Eq. (19) substituting $k = k_1$), the coefficients $\psi_1^{(2)}, \psi_2^{(2)}$ correspond to the eigenvalue $k_2$. Solutions (23) corresponds to boundary conditions $\Psi_{1,2}(x \to \infty) = 0$, that is $k_1, k_2 < 0$. A case $\eta^2 > 1/m_1 m_2$, when the eigenvalue $k_2$ is complex (the solution $\Psi = e^{k_2 x}$ is oscillating), will be considered below. From the first equation of Eq. (19) we have:

$$\psi_2 = \frac{\hbar^2 k^2}{4m_1 \hbar^2 k^2} - \frac{a_1}{\eta - \varepsilon} \psi_1$$

(24)

For $k = k_1 = 0$ (at $T = T_c$) we have

$$\psi_2^{(1)} = -\frac{a_1}{\varepsilon} \psi_1^{(1)}.$$  

(25)

Eq. (25) conserves the phase relations (8): if $\varepsilon < 0$ the condensates in different bands are in a phase, if $\varepsilon > 0$ the condensates in different bands are in antiphase (we are in a temperature region $T_{c1}, T_{c2} < T < T_c$ hence $a_1, a_2 > 0$). For $k = k_2$ and taking into account the condition Eq. (17), which lowers the free energy of a superconductor in a magnetic field, we have:

$$\psi_2^{(2)} = -\frac{1}{\eta m_1} a_2 m_2 + 2\eta \varepsilon |m_1 m_2 + m_1^2 m_2 \eta^2 a_1] \psi_1^{(2)}.$$  

(26)

In the case when the drag-effect is neglected $\eta = 0$ we have:

$$\psi_2^{(2)} = \frac{a_2 m_2}{\varepsilon m_1} \psi_1^{(2)}.$$  

(27)

We can see that Eqs. (26, 27) are opposite to the phase relations (8): when $\varepsilon < 0$ then $\psi_2^{(2)} = const \cdot \psi_1^{(2)}, const < 0$ (because $\eta > 0$), for $\varepsilon > 0$ it is analogously. This fact leads to instability of a superconducting state in a spatial inhomogeneous medium: any spatial inhomogeneity violates the phase relations (8) and, consequently, suppress the superconducting state. For some quantities of $\eta$ in a case $\eta \varepsilon > 0$ the dynamical stability can perhaps exist, however in this case the London penetration depth (14) increases and, hence, the free energy (16) increases compared with a case $\eta \varepsilon < 0$. In a case $\eta^2 > 1/m_1 m_2$ the solution $\Psi = e^{k_2 x}$ is oscillating and it does not satisfy the boundary conditions $\Psi_{1,2}(x \to \infty) = 0$. The solution $\Psi = e^{k_2 x}$ could be removed supposing $C_2 = 0$. However the eigenvalues $k_1$ and $k_2$ are derived from the intrasystem interaction and corresponds to the different length scales in the system. Consequently their selection by the boundary conditions is unphysical (unlike symmetric solutions $k$ and $-k$ one of which can be selected according to the boundary conditions). Thus, to ensure stability of a superconducting state and minimality of the free energy, the solution $k_1$ must exist only. Then from Eq. (20) and Eqs. (14, 16) we can see that the coefficient of intergradient interaction must be

$$\eta^2 = \frac{1}{m_1 m_2}, \quad \eta \varepsilon < 0.$$  

(28)

In this case we have only one eigenvalue $k = k_1$ such that $k(T \to T_c) = 0$.

Usind Eqs. (21, 28) and Eq. (7) we can obtain the coherence length as

$$\xi^2 = \frac{\hbar^2}{4} \left( \frac{a_2}{m_1} + \frac{a_2}{m_2} + 2|\eta||\varepsilon| \right) \approx \sqrt{\frac{a_2}{a_1} \left( \frac{1}{m_1} + \frac{a_1}{a_2 m_2} + 2|\eta| \sqrt{a_2} \right)}.$$  

(29)
However the current through the junction between two superconductors is \( J = J_0 \sin \Delta \theta \), that is the condition \( J = 0 \) is satisfied by both \( \Delta \theta = 0 \) and \( \Delta \theta = \pi \). In [27] it has been shown the dependence of the current on the phase difference \( J \propto \sin \Delta \theta \) for the junction between a single-band superconductor and a two-band superconductor also takes place. Since the phase different in a two-band superconductor is either 0 or \( \pi \) - Eq. (8), then proximity of a single-band superconductor can not change the phase relation in a two-band superconductor.

Single coherence length allows us to represent the orders parameters in a form \( \Psi_2(\mathbf{r}) = C(T)\Psi_1(\mathbf{r}) \), where the coefficient \( C \) is not function of spatial coordinates (as follows from the above, \( C > 0 \) if \( \varepsilon < 0 \) and \( C < 0 \) if \( \varepsilon > 0 \)). Hence the free energy functional of a two-band superconductor (5) can be rewritten in the form of GL functional of a single-band superconductor:

\[
F = \int d^3r \left[ \frac{\hbar^2}{4M} \left( \nabla - \frac{2\pi i}{\Phi_0} \mathbf{A} \right) \right] |\Psi|^2 + A |\Psi|^2 + \frac{B}{2} |\Psi|^4 + \frac{H^2}{8\pi},
\]

(30)

where the coefficients have a form

\[
A = a_1 + a_2 C^2 + 2\varepsilon C
\]

(31)

\[
B = b_1 + b_2 C^4
\]

(32)

\[
M^{-1} = \frac{1}{m_1} + \frac{a_2^2}{m_2} + \frac{2|C|}{\sqrt{m_1 m_2}},
\]

(33)

and we have redesignated \( \Psi \equiv \Psi_1 \). Thus the theory of a two-band superconductor is reduced to GL theory of a single-band superconductor. All characteristics (coherence length, magnetic penetrations depth, GL parameter, critical magnetic fields, magnetization, critical currents in a wire etc.) can be found by usual GL theory. However, unlike GL theory, the coefficient \( B \) and the effective mass \( M \) are functions of temperature since the coefficient \( C \) is a function of temperature.

Now we should find the coefficient \( C \). Let us substitute \( \Psi_2 = C\Psi_1 \) in Eq. (6):

\[
\left\{ \begin{array}{l}
a_1 + \varepsilon C + b_1 \Psi_1^2 = 0 \\
a_2 C + \varepsilon + b_2 C^3 \Psi_1^2 = 0
\end{array} \right\}
\]

(34)

If \( T \to T_c \) then the equations can be linearized. In this case we have solutions \( C = -a_1/\varepsilon \) or \( C = -\varepsilon/a_2 \). Near the critical temperature we can use Eq. (7), that is \( |\varepsilon| = \sqrt{a_1 a_2} \). Then the solution becomes unique:

\[
C = \sqrt{\frac{a_1}{a_2}} \quad \text{if} \quad \varepsilon < 0
\]

\[
C = -\sqrt{\frac{a_1}{a_2}} \quad \text{if} \quad \varepsilon > 0
\]

(35)

This approximation expresses the fact that relation between the order parameters is determined by the single-band critical temperatures \( T_{c1}, T_{c2} \); if \( T_{c1} > T_{c2} \) then \( \Delta_1 > \Delta_2 \) - Fig. (1).

Using Eqs. (31, 32, 33, 34) we can find main characteristics of a superconductor as in the usual GL theory. A coherence length:

\[
\xi^2 = \frac{\hbar^2}{4M|A|} = \frac{\hbar^2}{4} \left( \frac{1}{m_1} \sqrt{\frac{a_2}{a_1} + \frac{1}{m_2} \sqrt{\frac{a_2}{a_2}}} + \frac{2}{\sqrt{m_1 m_2}} \right),
\]

(36)

a magnetic penetrations depth:

\[
\lambda^2 = \frac{Mc^2 B}{8\pi e^2 |A|} = \frac{c^2}{8\pi e^2} \frac{b_1 /a_1 + b_2 /a_2}{2 \sqrt{a_1 a_2} - |\varepsilon|} \left( \frac{1}{m_1} \sqrt{\frac{a_2}{a_1} + \frac{1}{m_2} \sqrt{\frac{a_2}{a_2}}} + \frac{2}{\sqrt{m_1 m_2}} \right),
\]

(37)

a GL parameter:

\[
\kappa = \frac{\lambda}{\xi} = \frac{c}{2\sqrt{\pi} e} \frac{M \sqrt{B}}{m_1 /a_1 + b_2 /a_2} \left( \frac{1}{m_1} \sqrt{\frac{a_2}{a_1} + \frac{1}{m_2} \sqrt{\frac{a_2}{a_2}}} + \frac{2}{\sqrt{m_1 m_2}} \right),
\]

(38)

We can see that the GL parameter is a function of temperature unlike single-band GL theory. However this dependence is type \( \frac{T - T_{c1}}{T_{c1}} \); that is little varying function of temperature if \( T \gg T_{c1}, T_{c2} \). It should be noticed that this approximation
is correct if $T > T_{c1}, T_{c2}$ only. We can extrapolate the obtained expressions for all temperature. To do it we can suppose $a_i = \gamma_i(T_c - T_{ci}) = const$, then $M = const, B = const$, however it is necessary to expand the expression $\sqrt{a_1a_2} - |\varepsilon|$ in powers of $(T - T_c)$:

$$\sqrt{a_1a_2} - |\varepsilon| = \frac{\gamma_1\gamma_2(2T_c - T_{c1} - T_{c2})}{2\sqrt{a_1a_2}}(T - T_c)$$

$$+ \frac{\gamma_1\gamma_2}{2\sqrt{a_1a_2}}\left(1 - \frac{\gamma_1\gamma_2(2T_c - T_{c1} - T_{c2})^2}{4a_1a_2}\right)(T - T_c)^2 + \ldots$$

(39)

Thus in the functional for a two-band superconductor the coefficient $A$ is a power series of $(T - T_c)$ unlike the GL functional for a single-band superconductor. From this fact a nonlinear temperature dependence of the upper critical field follows (hear $f_1, f_2$ are some coefficients):

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2} \propto f_1(T_c - T) + f_2(T_c - T)^2 + \ldots$$

(40)

single-band GL theory

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2} \propto f_1(T_c - T) + f_2(T_c - T)^2 + \ldots$$

(40)

two-band GL theory

that is consistent with experimental data (in bulk LuNi$_2$B$_2$C, MgB$_2$) in [28, 30] and theoretical results in [17, 19], where it has been shown that the presence of two order parameters for two bands yields a nonlinear temperature dependence of $H_{c2}(T)$ in the vicinity of the critical temperature unlike the single-band s-wave BCS theory and GL theory. It should be noted that this difference can be a cause of strong enhancement of $H_{c2}(T)$ (up to ten-fold increase) in dirty two-gap superconductors, that, as noted in [31], is result from the anomalous upward curvature of $H_{c2}(T)$. For the lower critical field $H_{c1}$ and the thermodynamic magnetic field $H_{cm}$ we have analogous expansion because

$$H_{c1} = \frac{\Phi_0}{2\pi\lambda^2} \ln \kappa \propto |\varepsilon| - \sqrt{a_1a_2},$$

(41)

$$H_{cm} = \frac{\Phi_0}{2\sqrt{2}\pi\lambda^2} \propto |\varepsilon| - \sqrt{a_1a_2},$$

(42)

that demonstrates nonlinear temperature dependence too and correlates with theoretical results of [18, 19]. Let the carriers have different effective masses in different bands, for example $m_1 \gg m_2$. From Eq. (33) we can see that the two-band effective mass $M$ is determined mainly by the smaller mass $m_2$. From Eqs. (36,37,40,41) we can see the critical fields $H_{c1}$ and $H_{c2}$ depend on the effective mass as $H_{c1} \propto 1/M, H_{c2} \propto M$. Hence, as noted in [19, 21], the critical fields are determined mainly by the smaller mass $m_2$, while the contribution from the lager mass can be neglected.

III. MULTI-BAND SUPERCONDUCTOR

Using results of previous section we can generalize the above-described method for two-band superconductors to multi-band superconductors. In presence of $n$ order parameters in an isotropic s-wave superconductor, the free energy functional can be written as

$$F = \int d^3r \sum_{i=1}^{n} \left[ \frac{\hbar^2}{4m_i} |D\Psi_i|^2 + a_i |\Psi_i|^2 + \frac{b_i}{2} |\Psi_i|^4 + \sum_{j=2,j>i}^{n} \frac{\hbar^2}{4} \eta_{ij} (D^+\Psi_i^+D\Psi_j + D\Psi_iD^+\Psi_j^+) + \sum_{j=2,j>i}^{n} \varepsilon_{ij} (\Psi_i^+\Psi_j + \Psi_i\Psi_j^+) \right] + \frac{H^2}{8\pi} |\Psi|.$$  

(43)

Critical temperature can be found from an equation:

$$\begin{vmatrix}
    a_1 & \varepsilon_{12} & \ldots & \varepsilon_{1n} \\
    \varepsilon_{12} & a_2 & \ldots & \varepsilon_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    \varepsilon_{1n} & \varepsilon_{2n} & \ldots & a_n \\
\end{vmatrix} = 0,$$

(44)
which is analog of Eq.(7). However we should notice that in general case the symmetry $\varepsilon \leftrightarrow -\varepsilon$ for critical temperature, like in the two-band case, is absent. If all $\varepsilon_{ij} < 0$ but some $\varepsilon_{ij} > 0$, suppression of superconductivity is possible. We will consider a case of attractive interband interaction only, that is all $\varepsilon_{ij} < 0$.

Following our scheme we should find coefficients of the intergradients interaction $\eta_{ij}$ and the coherence length $\xi$. Equation for the coherence length $\xi^2 = 1/k^2$ is

$$
\begin{vmatrix}
\frac{\hbar^2}{M_i}k^2 - a_1 & \frac{\hbar^2}{4}\eta_{12}k^2 - \varepsilon_{12} & \cdots & \frac{\hbar^2}{4}\eta_{1n}k^2 - \varepsilon_{1n} \\
\frac{\hbar^2}{4}\eta_{12}k^2 - \varepsilon_{12} & \frac{\hbar^2}{4}k^2 - a_2 & \cdots & \frac{\hbar^2}{4}\eta_{2n}k^2 - \varepsilon_{2n} \\
\cdots & \cdots & \cdots & \cdots \\
\frac{\hbar^2}{4}\eta_{1n}k^2 - \varepsilon_{1n} & \frac{\hbar^2}{4}\eta_{2n}k^2 - \varepsilon_{2n} & \cdots & \frac{\hbar^2}{4}\eta_{mn}k^2 - a_n \\
\end{vmatrix} = f_n(m_i, \eta_{ij})k^{2n} + f_{n-1}(m_i, \eta_{ij})k^{2(n-1)} + \ldots + f_1(m_i, \eta_{ij})k^2 + f_0 = 0,
$$

(45)

which is analog of Eq.(20). At $T = T_c$ we have $f_0 = 0$. The coefficients $\eta_{ij}$ must be such that the functions $f_n = f_{(n-1)} = \ldots = f_2 = 0$, then the coherence length is

$$
1/\xi^2 = k^2 = f_0/f_1
$$

(46)

By analogy of (28) and using the condition $\varepsilon_{ij} < 0$ we can suppose

$$
\eta_{ij} = \frac{1}{\sqrt{m_i m_j}}. \tag{47}
$$

In the next step we should to represent the orders parameters in a form $\Psi_2 = C_2(T)\Psi_1, \Psi_3 = C_3(T)\Psi_1, \ldots, \Psi_n = C_n(T)\Psi_1$. Then the free energy functional of a multi-band superconductor (43) takes the form of the GL functional (30) of a single-band superconductor with coefficients

$$
A = a_1 + \sum_{i=2}^{n} a_i C_i^2 + 2 \sum_{i=2}^{n} \varepsilon_{1i} C_i + 2 \sum_{i=2}^{n} \sum_{j=i+1}^{n} \varepsilon_{ij} C_i C_j \tag{48}
$$

$$
B = b_1 + \sum_{i=2}^{n} b_i C_i^4 \tag{49}
$$

$$
M^{-1} = \frac{1}{m_1} + \sum_{i=2}^{n} \frac{C_i^2}{m_i} + 2 \sum_{i=2}^{n} \eta_{1i} C_i + 2 \sum_{i=2}^{n} \sum_{j=i+1}^{n} \eta_{ij} C_i C_j \tag{50}
$$

Linearized equations for $C_2, C_3, \ldots, C_n$ are

$$
\begin{cases}
a_1 + \varepsilon_{12} C_2 + \ldots + \varepsilon_{1n} C_n = 0 \\
\varepsilon_{12} + a_2 C_2 + \ldots + \varepsilon_{2n} C_n = 0 \\
\vdots \\
\varepsilon_{1n} + \varepsilon_{2n} C_2 + \ldots + a_n C_n = 0
\end{cases} \tag{51}
$$

which have to be solved taking into account Eq.(44) so that the solutions are unequivocal (as we have shown in the two-band case). However we can use an approximate method. In the two-band problem we supposed the coefficient $C = \sqrt{\frac{a_1}{a_2}} = \sqrt{\frac{T_1}{T_2}}$ for $\Psi_2 = C\Psi_1$, that is relation between the order parameters is determined by the single-band critical temperatures $T_{c1}, T_{c2}$: if $T_{c1} > T_{c2}$ then $\Delta_1 > \Delta_2$. This fact can be used for the coefficients $C_i$ in the multi-band problem, where we can suppose:

$$
C_2 = \sqrt{\frac{a_1}{a_2}}, \quad C_3 = \sqrt{\frac{a_1}{a_3}}, \ldots, \quad C_n = \sqrt{\frac{a_1}{a_n}}. \tag{52}
$$

Substituting Eq.(52) in Eq.(48) and reducing to a common denominator we have

$$
A = \frac{n \sqrt{a_1}}{\prod_{i=2}^{n} \sqrt{a_i}} f(a_i, \varepsilon_{ij}), \tag{53}
$$
where

\[ f(a_i, \varepsilon_{ij}) = \prod_{i=1}^{n} \sqrt{a_i} - \frac{2}{n} \sum_{i=2}^{n} \varepsilon_{1i} \prod_{k=2,k\neq i}^{n} \sqrt{a_k} + \frac{2}{n} \sum_{i=2}^{n} \sum_{j=3,j>i}^{n} \varepsilon_{ij} \prod_{k=2,k\neq i,k\neq j}^{n} \sqrt{a_k}, \]

(54)

The critical temperature is such a temperature when \( f(T = T_c) = 0 \). As in the two-band problem we can extrapolate the obtained expressions for all temperature. To do this we have to suppose \( a_i = \gamma_i (T_c - T_c^*) = \text{const} \), then \( M = \text{const}, B = \text{const} \), however it is necessary to expand the expression \( f(a_i, \varepsilon_{ij}) \) in powers of \( T - T_c \). Thus the multi-band problem is reduced to the single-band problem with the effective mass \( M \), however the coefficient \( A \) is power series of \( (T - T_c) \) unlike the GL free energy functional.

\section*{IV. RESULTS}

In this work we have shown that the term of the drag effect \( \eta (\nabla \Psi_1^* \nabla \Psi_2 + \nabla \Psi_1 \nabla \Psi_2^*) \) in the free energy functional of an isotropic bulk two-band superconductor plays important role and the restrictions for the coefficient \( \eta \) exist. If the coefficient is \( \eta^2 = \frac{1}{m_1 m_2} \) and it’s sign is opposite to the sign of the coefficient in the term of the proximity effect \( \varepsilon (\Psi_1^* \Psi_2 + \Psi_1 \Psi_2^*) \), that is \( \eta \varepsilon < 0 \), then this leads to a single coherence length \( \xi \), which diverges at the critical temperature \( \xi (T \rightarrow T_c^*) \rightarrow \infty \), and a single GL parameter. This quantity ensures the stability of a superconductor state and the least possible free energy in this case. Other quantities of the coefficient or neglecting of the drag effect \( \eta = 0 \) leads, at first, to the existence of two coherence lengths, where one of them diverges at the critical temperature while the second length is finite at all temperatures. Secondly, it leads to the dynamical instability (suppressing of a superconducting state if the order parameters are spatial inhomogeneous) due to violation of the phase relations \( \eta^{\geq 2} \). These results mean that the isotropic bulk type-1.5 superconductors are impossible.

It should be noticed that these results are obtained in the GL domain only. Hence it can be supposed that at low temperatures the disproportion \( \Psi_2(r, T) \neq C(T) \Psi_1(r, T) \) can takes place, that is there are two different coherence lengths \( \xi_1 \neq \xi_2 \). However this fact means that the order parameters have different gradients \( \nabla \Psi_1(r) \neq \nabla \Psi_2(r) \). Since the order parameters are \( |\Psi_1| e^{i\varphi_1}, |\Psi_2| e^{i\varphi_2} \), then the different gradients can lead to violation of the equality \( \eta^{\geq 2} \), hence to violation of the phase relations \( \eta^{\geq 2} \). Thus the state with different coherence lengths is dynamically unstable.

The approximate method for solving of set of GL equations for an isotropic bulk multi-band superconductor has been developed. Using the results about the drag effect we have shown that the free energy functional for a two-band superconductor can be reduced to the GL functional for an effective single-band superconductor. This effective superconductor is characterized with some an effective mass of carriers (as a function of \( m_1, m_2, \eta \)) and a coefficient at \( |\Psi|^2 \) as a power series of \( (T - T_c) \) in the vicinity of the critical temperature. This temperature dependence causes nonlinear dependence of upper and lower critical fields \( H_{c2}, H_{c1} \), thermodynamical magnetic fields \( H_{cm} \) on temperature unlike the single-band GL theory. Generalizing this result we have developed an algorithm which allows to reduce the free energy functional of a multi-band superconductor to the effective GL free energy functional of a single-band superconductor provided that all interband interactions are attractive.

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