Kernel Adaptive Filtering Multiple-model Actuator Fault Diagnostic For Multi-effectors Aircraft

Peng Zhu*, Wenhan Dong

1 Aeronautics Engineering College, Air Force Engineering University, Xi’an, 710038, P. R. China

*Corresponding author’s e-mail: 1499029789@qq.com

Abstract. In the traditional Multiple Model Adaptive Estimation (MMAE) algorithm, the extended Kalman filter has theoretical limitations, and the establishment of accurate aircraft mathematical model is almost impossible. In this paper, the Kernel Adaptive Filter (KAF) is introduced to replace the Kalman filter, a new multi-model adaptive estimation fault diagnosis method is proposed. Based on the kernel methods, the adaptive filter is designed in the high-dimensional feature space without the need to know the system model in advance. After training of KAF using the offline input control signal and output flight states measurement with noise, the estimation of real flight states values and actuator fault detection and isolation can be realized online. The simulation results show good performance of new fault diagnosis method in actuator fault diagnosis.

1. Introduction
With the increasing requirement for flight safety and maneuverability of modern aircraft, the design of multi-effectors has become the future direction of development [1]. The redundancy control with multi-effectors can improve the flight performance, however, which also inevitably causes the increase of actuator’s fault probability. At this point, it is of great importance to realize effective fault-tolerant control via control allocation techniques, and the key to that is the realization of fault detection in real time and rapid isolation of fault.

At present, there are two main methods for fault diagnosis, including model-based methods and parametric approach [2][3][4][5]. The analytical model-based method is the most commonly used method for fault diagnosis. If the system mathematical model is known, state estimation method can realize the effective fault diagnosis. On the basis of multiple model adaptive estimation which belongs to state estimation method, Ducard et al. proposed the Extended Multiple Model Adaptive Estimation (EMMAE) by introducing the Extended Kalman Filter (EKF) to detect and isolate typical faults of the aircraft’s control surface, the results can reach expected effect. However, the calculation of accurate aircraft model is essential for EKF in [6]. Due to the complexity of aircraft with multi-effectors, accurate mathematic model of aircraft is difficult to be calculated, which makes the application of EKF is restricted.

Meanwhile, many new methods have been proposed to realize fault diagnosis in recent years. Among them, the algorithm based on kernel method have attracted much attention, which have the following advantages [7][8]:
1. The kernel method can solve nonlinear problems in high-dimensional space using traditional linear algorithms; 2. No need for accurate analytical model of system; 3. The simple structure of algorithm can reduce the computation time.
Therefore, this paper introduces the Kernel Recursive Least Square (KRLS) filter into the MMAE method to detect the typical fault of the aircraft's actuator without the need to calculating precise aircraft mathematical model. The adaptive filter is trained by the kernel method, and the filter parameters are optimized by the genetic algorithm. By testing with fault data online, the flight state of aircraft can be estimated online using the trained kernel adaptive filter, then the mature and accurate fault judgement method is used to complete the fault detection and isolation of the actuator fault. Finally, simulation results show the effectiveness of the new method.

2. Multi-effectors Aircraft Modeling
The dynamic equations of the multi-effectors aircraft in this paper are as follows [6]:

\[
\begin{align*}
    x_{i+1} &= f(x_i, u_i) + w_i, \\
    y_i &= h(x_i) + v_i
\end{align*}
\]  

(1)

where \( x \) denotes the flight state variables, and \( u \) denotes the input control states. \( [p, q, r, \alpha, \beta, \delta_1, \delta_2, F_e] \) represent roll rate, pitch rate, yaw rate, angle of attack, sideslip angle, left aileron, right aileron, left elevator, right elevator, rudder and engine thrust, separately. \( y \) denotes the sensor measurement of aircraft. \( w, v \) are zero mean white noise whose covariance \( Q = E[ww^T] \) and \( R = E[vv^T] \), respectively.

When actuator fault happen, it can be regarded as the desired control input \( \delta_n \) is replaced by fault control input \( \delta_n' \). The actual input of the aircraft can be written as:

\[
u_n(i) = \delta_n'(i) + \sigma_{am}(\delta_n(i) - \delta_n'(i))
\]

(2)

\[
\sigma_{am} = \begin{cases} 
1, & \text{when } m^{th} \text{ actuator fails} \\
0, & \text{otherwise}
\end{cases}
\]

(3)

3. Kernel Adaptive Filter
A typical lateral linear adaptive filter is shown in Fig. 1:

\[d(i) = \omega(i)u(i) + e(i)\]

(4)
The basic principle of the kernel method is to transform the input data $\mathbf{u}(i)$ from the original data space to the high-dimensional feature space $\mathbf{\varphi}(\mathbf{u}(i))$ (denoted as $\mathbf{\varphi}(i)$) via the kernel function. In essence, the new feature space belongs to the Reproducing Kernel Hilbert Space. In this feature space, the inner product operation of $\mathbf{\varphi}(i)$ can be expressed as (5) using the kernel function which satisfies the Mercer theorem:

$$\kappa(\mathbf{u}(i), \mathbf{u}(j)) = \langle \mathbf{\varphi}(i), \mathbf{\varphi}(j) \rangle$$  \hspace{1cm} (5)

In the KRLS algorithm, with input of new input-output pair $\{\mathbf{u}(i), d(i)\}$, the weighting coefficient $\mathbf{\omega}(i)$ which is the minimizer of (6) at each iteration

$$\min_{\mathbf{\omega}} \sum_{j=1}^{l} \left| d(j) - \mathbf{u}(j)^T \mathbf{\omega}(j) \right|^2$$  \hspace{1cm} (6)

can be computed as:

$$\mathbf{\omega}(i) = \left[ \mathbf{\Phi}(i)^T \mathbf{\Phi}(i) \right]^{-1} \mathbf{\Phi}(i)^T d(i)$$

$$= \mathbf{\Phi}(i) \mathbf{a}(i)$$  \hspace{1cm} (7)

where $d(i) = [d(1), \ldots, d(i)]^T$, $\mathbf{\Phi}(i) = [\mathbf{\varphi}(1), \ldots, \mathbf{\varphi}(i)]$, and $\mathbf{a}(i) = \left[ \mathbf{\Phi}(i)^T \mathbf{\Phi}(i) \right]^{-1} d(i) = \mathbf{K}^{-1} d(i)$, $\mathbf{K}$ is called kernel matrix. It can be seen from (7) that the weighting coefficient is defined as a linear combination of input data.

When input data $\mathbf{u}(i)$ becomes available, the output of filter can be calculated as:

$$Y_{i+1}(\mathbf{u}(i)) = \sum_{j=1}^{i} \mathbf{a}_j (i-1) \kappa(\mathbf{u}(j), \mathbf{u}(i))$$  \hspace{1cm} (8)

It can be seen from equation (7) that there is matrix inversion computation in each updating of coefficient matrix. In order to simplify the calculation procedure, the $\mathbf{a}(i)$ is generally calculated using the following iterative method:

$$\mathbf{a}(i) = \begin{bmatrix} a(i-1) \cdot z(i) \cdot r(i)^T e(i) \\ r(i)^T e(i) \end{bmatrix}$$  \hspace{1cm} (9)

where:

$$\begin{align*}
\mathbf{h}(i) &= [\kappa(\mathbf{u}(i), \mathbf{u}(1)), \ldots, \kappa(\mathbf{u}(i), \mathbf{u}(i-1))]^T \\
\mathbf{Q}(i) &= r(i)^{-1} \begin{bmatrix} \mathbf{Q}(i-1) r(i) + z(i) z(i)^T & -z(i) \\ -z(i)^T & 1 \end{bmatrix} \\
z(i) &= \mathbf{Q}(i-1) \mathbf{h}(i) \\
r(i) &= \mathbf{\varphi}(i)^T \mathbf{\varphi}(i) - z(i)^T \mathbf{h}(i)
\end{align*}$$  \hspace{1cm} (10)

Prediction error $e(i)$ is defined as:

$$e(i) = d(i) - Y_{i+1}(\mathbf{u}(i))$$  \hspace{1cm} (11)

In addition, network size and dimension of kernel matrix will increase with the number of training data. In order to reduce the complexity of matrix inversion computation, a number of dimensional-reduction methods have been proposed. This paper uses the Approximate Linear Dependency (ALD)
proposed in [9]. The ALD can simplify the complexity of KRLS by calculating the linear dependency between new input-output pair \( \{ u_i, d_i \} \) with present dictionary. The ALD test function is written as follow:

\[
\delta_i = \min_k \left\| \sum_{j \in D_{\text{old}}} b_j \phi(c_j) - \phi(u_i) \right\|_2^2
\]  

(12)

Suppose the dictionary is \( D_{\text{old}} = \{ c_j \}_{j=1}^n \) at time \( i \), where \( n \) is the cardinality and \( c_j \) denotes the \( j \)th center of data. \( b \) denotes an arbitrary vector, and \( \sum_{c_j \in D_{\text{old}}} b_j \phi(c_j) \) is the linear combination of present data in dictionary. The least squares solution of (12) is:

\[
b = L_{x,i}^{-1}H_{x,i}
\]

where:

\[
H_{x,i} = \left[ \kappa(u_i, c_1), \ldots, \kappa(u_i, c_n) \right]^T
\]

\[
K_{x,i} = \left[ \begin{array}{c} \kappa(c_1, c_1), \ldots, \kappa(c_1, c_n) \\ \vdots \\ \kappa(c_n, c_1), \ldots, \kappa(c_n, c_n) \end{array} \right]
\]

\[
L_{x,i}^{-1} = K_{x,i}^{-1}
\]

The new data will be inserted into dictionary if \( \delta_i \) is larger than preset threshold \( v \) in ALD. If \( \delta_i \) is smaller than \( v \), indicating that there is obvious linear correlativity between new data and old data in dictionary, so that new data should be rejected. As a result, the sparsification of training data will be realized with moderate computational complexity of the algorithm.

4. KAF Multiple-Model Fault Diagnosis Method

The new method needs to train \( m+1 \) KAF, corresponding to no fault scenario and \( m \) th type of actuator faults. The weight coefficient of kernel adaptive filter will be locked through training offline.

For KAF, the input for training is the controller signals of five actuators: \( \delta_{\text{cmd}}=[\delta_{a1}(i), \delta_{a2}(i), \delta_{a3}(i), \delta_{a4}(i), \delta_{a5}(i)]^T \), and the noisy measurement of flight states \( y=[p \ q \ r \ \alpha \ \beta]^T \) is the output signal of training data.

After training offline, the testing input \( \delta_{\text{cmd}}=[\delta_{a1}, \delta_{a2}, \delta_{a3}, \delta_{a4}, \delta_{a5}]^T \) data can be sent into the designed MMAE online, and the trained banks of kernel adaptive filter will produce the estimate output \( \hat{y}_m=[\hat{p} \ \hat{q} \ \hat{r} \ \hat{\alpha} \ \hat{\beta}]^T \) corresponding to different fault situation, then the corresponding fault probability can be calculated using (15) [6]:

\[
p_m[i] = \frac{p(y = y_i | (\theta = \theta_m, Y_{i-1})) \cdot p_s[i-1]}{\sum_{m=1}^{M}p(y = y_i | (\theta = \theta_m, Y_{i-1})) \cdot p_s[i-1]}
\]

(15)

where \( p(y = y_i | (\theta = \theta_m, Y_{i-1})) \) denotes the conditional probability for obtaining the \( m \) th estimate output \( \hat{y}_m \), under the condition of assuming the fault \( \theta_m \) exists with the sequence of last measurements \( Y_{i-1} \). And the equation of probability density function is defined as follow:

\[
f(y = y_i | (\theta = \theta_m, Y_{i-1})) = \lambda_m e^{-\frac{1}{2} \| y_i - \hat{y}_m \|^2}
\]

(16)
with $\hat{y}_m = \frac{1}{(2\pi)^{1/2} |\Sigma_n|^{1/2}} e^{-\frac{|\Sigma_n||\hat{y}_m - \hat{y}_n|^2}{2}}$, $r_m = y - \hat{y}_m$ denotes the residuals from KAF.

Finally, the final estimate output of MMAE can be written as the weighted sum of $\hat{y}_m$ as (17), the $p_n$ are defined as the corresponding weight value.

$$\hat{y}[i] = \sum_{m} \hat{y}_m[i] \cdot p_n[i]$$

(17)

5. Simulation

5.1. Simulation Conditions
The simulation model in this paper is an aircraft with a rudder, two aileron and elevator as shown in (1).

The traditional PID controller is designed to keep the flight state stable.

Zero mean white noise is introduced into the measurement of output flight states with covariance:

$$\Sigma_{pqr} = 0.0076 \times I_{rad/s} \sum_{\alpha} = 0.0012 \times I_{rad/s} \sum_{\alpha,\beta}$$

The simulation of training is set for 90s with the following excitation signal:

1. $t = 0s - 20s$, square signal with frequency 0.1 Hz and amplitude 15° for desired pitch angle;
2. $t = 30s - 50s$, square signal with frequency 0.08 Hz and amplitude 10° for desired roll angle;
3. $t = 60s - 80s$, square signal with frequency 0.06 Hz and amplitude 10° for desired roll angle;

5.2. KAF Parameters Selection and Training Results
The Gaussian kernel $\kappa(x, x') = \exp(-a\|x - x'\|^2)$ is chosen as the kernel function of KAF, where $a$ is called the kernel parameter needed to be specified.

The KRLS algorithm used in this paper has two parameters $a$ and the threshold $v$ of ALD that need to be adjusted. The traditional genetic algorithm is used to optimize these parameters. The searching scope of $a$ and $v$ are set as [0.01, 70] and [1E-7, 1E-2] respectively.

The leave-one-out cross-validation function [8] $LOOCV(i) = \sum_{j \neq i} \left( y_{ij} - f_{hj}(x_j) \right)^2$ is chosen as the fitness function of genetic algorithm, where $\{(x_j, y_j)\}_{j=1}^N$ denotes the training set and $f_{hj}$ is the estimated function of KAF.

Taking the KAF corresponding to no fault scenario as example. Because of different nonlinear relationship between control input $u = [\delta_a, \delta_e, \delta_a, \delta_e, \delta_e]^T$ and the five flight states $\dot{p}, \dot{q}, \dot{r}, \dot{\alpha}, \dot{\beta}$ respectively. When we train the KAF to estimate five flight states, there will be performance difference in the estimation results of different states using KAF with the same filter parameters.

It is necessary to use genetic algorithm to optimize the parameters of KAF corresponding to the five different flight states respectively. The result of optimization is shown in TABLE 1:

| Table 1. Optimizing result of KAF parameters |
|---------------------------------------------|
| Kernel Parameter   | ALD Threshold |
| roll rate          | 0.11          | 1.82e-3          |
| pitch rate         | 0.10          | 1.41e-3          |
| yaw rate           | 0.16          | 1.13e-3          |
| angle of attack    | 0.21          | 8.24e-4          |
| sideslip angle     | 0.28          | 9.41e-4          |

Fig. 2-3 shows the training results of KAF with the filter parameters in TABLE I. For simplicity, only two figures of training results are shown in this paper. The black line is the true value of the flight...
state, and the red line is the state estimation of KAF. It can be seen that after training, the estimate output of KAF can accurately track the real output of the aircraft. The trained KAF can effectively simulate the complex nonlinear model of multi-effector aircraft.

Figure 2. Training result of roll rate.

Figure 3. Training result of sideslip angle.

5.3. Simulation of Fault Diagnosis Testing

After training to determine weighting coefficient of KAF, the performance of the fault diagnosis can be tested via detecting the online fault signal. The simulation of testing is set for 300s with following fault signal:

1. \( t = 10s \sim 40s \), the left elevator is stuck at \(-20\); 
2. \( t = 70s \sim 100s \), the right elevator is stuck at \(-10\); 
3. \( t = 130s \sim 160s \), the rudder is stuck at \(3\); 
4. \( t = 190s \sim 220s \), the left aileron is stuck at \(-15\); 
5. \( t = 250s \sim 280s \), the right aileron is stuck at \(20\).

Figure 4. Probabilities of each KAF after a sequence of faults. (a) Probability of no fault (b) Fault probability of left aileron (c) Fault probability of right aileron (d) Fault probability of rudder (e) Fault probability of left elevator (f) Fault probability of right elevator.

Fig. 4 shows the fault probability calculated by different fault diagnosis method. The black line denotes the real fault probability of test data, the blue line denotes the detected fault probability via EMMAE in [6] and the red line is the fault probability detected by KAF fault diagnosis method proposed in this paper. If the probability exceeds 90% in corresponding fault scenario, the
corresponding actuator is detected as fault. If the probability is less than 5%, the fault is considered as removed.

It can be seen from Fig. 4, the KAF fault diagnosis method proposed in this paper can accurately detect different actuator fault in a short time. Compared with the EMMAE method proposed in [6], the false alarm rate and diagnosis time of new method are both better than EMMAE.

Figure 5 shows the flight state estimation calculated by two fault diagnosis methods. The black line is the true value of the flight state, the green line is the measurement value with noise, the blue line is the state estimation value of EMMAE, and the red line is the output result of KAF method. It can be seen that there are both errors when comparing true value with the state estimation of two method, but overall, the result of KAF method is smoother than EMMAE, indicating that its filtering effect is better than EMMAE. However, in some parts, for example, from 130s to 190s in Figure 5(d), the error of the KAF method is larger than EMMAE. The reason maybe that the training of the kernel adaptive filter has not met the optimal result, it is mainly related to the filter parameters and the selected kernel adaptive filtering algorithm. Theoretically, the KRLS algorithm can perfectly fit the nonlinear model of aircraft by selecting appropriate filter parameters or having algorithm improvements, such as introducing the forgetting factor and selecting different dimensionality reduction methods. Therefore, if more suitable KAF algorithm for actuator fault diagnostic can be found, the estimation accuracy of filter will be further improved.
Figure 5. Final state estimate of different methods. (a) roll rate (b) pitch rate (c) yaw rate (d) angle of attack (e) sideslip angle.

6. Conclusion
In this paper, a new multi-model fault diagnosis method based on kernel adaptive filtering is proposed. The optimal filter parameter set of KAF has been found using the genetic algorithm. On the basis of the offline training and online fault diagnosis, the typical actuator faults of multi-effectors aircraft can be effectively detected in real time. The new fault diagnosis method has expanded the application range of KAF, improved performance of the traditional MMAE method, and avoided the calculation of accurate aircraft model. At the same time, the KAF-based method has also improved the accuracy and speed of fault diagnosis and reduced the calculation amount of the diagnosis process.

REFERENCES
[1] Bi, K., Zhang, W., Chi, C., & Zhang, J. (2014). Reconfigurable Control Allocation of Multi-Surfaces Aircraft Based on Improved Fixed Point Iteration. Foundations and Practical Applications of Cognitive Systems and Information Processing. Springer Berlin Heidelberg.
[2] Zolghadri, A., Cieslak, J., Efimov, D., Goupil, P., & Dayre, R. (2016). Practical design considerations for successful industrial application of model-based fault detection techniques to aircraft systems. Annual Reviews in Control, S1367578816300785.
[3] Zolghadri, A., Leberre, H., Goupil, P., Gheorghe, A., Cieslak, J., & Dayre, R. (2016). Parametric approach to fault detection in aircraft control surfaces. Journal of Aircraft, 1-10.
[4] Takizawa, H., Natsu, M., Aoyagi, Y., & Asakura, H. (2013). Model-based approaches for fast and robust fault detection in an aircraft control surface servo loop from theory to flight tests. Control Systems IEEE, 33(3), 20.
[5] Berdjag, D., Cieslak, Jérôme, & Zolghadri, A. (2012). Fault diagnosis and monitoring of oscillatory failure case in aircraft inertial system. Control Engineering Practice, 20(12), 1410-1425.
[6] Ducard, G. J. J. (2009). Fault-tolerant Flight Control and Guidance Systems. Springer London.
[7] Boser, B. E. (1992). A training algorithm for optimal margin classifiers. The Workshop on Computational Learning Theory (Vol.5, pp.144-152).
[8] Haykin, S. (2010). Kernel Adaptive Filtering: A Comprehensive Introduction.
[9] Engel, Y., Mannor, S., & Meir, R. (2004). The kernel recursive least-squares algorithm. IEEE Trans Signal Process, 52(8), 2275-2285.