Bose-Einstein correlations in multiple particle production

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Abstract

Bose-Einstein correlations are studied in the framework of a class of independent particle production models. This generalizes the studies for a variety of models proposed previously. It is shown that the Bose-Einstein correlations lead for this class of models to Einstein’s condensation at sufficiently high density. They also enhance unusual charge distributions and may explain the centauro and antcentauro events reported by cosmic ray physicists. For typical models the correlations cause a shrinking of the momentum distribution of the produced identical particles and an apparent shrinking of the production region.

1 INTRODUCTION

This short communication is based on three papers done in collaboration with Andrzej Bialas [1],[2],[3]. Further details and references to earlier work can be found in these papers. Here only the main assumptions and conclusions will be presented. Bose-Einstein correlations are an apparent attraction

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in momentum space between identical bosons. This effect is due to the symmetrization of the state vectors with respect to exchanges of identical particles, which is required by Bose-Einstein statistics. Let us mention two reasons why, among the many known correlations, these correlations are considered particularly interesting. The first reason is related to the fact that the Bose-Einstein attraction between particles is strong if and only if these particles are close to each other in momentum space. Thus, the effect becomes more and more important, when the density of (identical) particles increases. The RHIC heavy ion collider will begin taking data soon and there the particle densities will be very high. Consequently, the Bose-Einstein correlations are likely to be very significant. Since, as we shall see in the following, they give a number of effect, which are not intuitively expected, their understanding will be important for a correct interpretation of the data. Another reason is that from the Bose-Einstein corrections to the correlation functions of identical particles it is possible to get information about the size and shape of the region, where the particles are produced. Similarly, Hanbury-Brown and Twiss have successfully used the Bose-Einstein correlations among photons coming from stars to find the radii of these stars. Unfortunately the simplifying assumptions, which are well justified in the case of photons emitted by a star, are not always plausible for particles produced in a high energy scattering process. Thus the results for particles are rather model dependent. Since, however, no better method of measuring the particle production region is known, and since information about its size is necessary, e.g. to answer the question, whether the energy density in this region is big enough to expect that the quark-gluon plasma is formed there, the study of Bose-Einstein correlations attracts much interest as a source of information about the particle production region.

2 ASSUMPTIONS AND A SIMPLE MODEL

We make two assumptions. One is physical and the other is technical, chosen to make the problem easy to solve. The physical assumption is that intuition is applicable to the production of distinguishable particles, or perhaps more realistically to the production of particles at low density, where the Bose-Einstein correlations are not important. According to this assumption we guess for distinguishable particles a density matrix \( \rho_0(q, q') \). Here \( q(q') \)
is the set of all the momentum components of all the particles being pro-
duced. Thus for the production of \(N\) particles vector \(q\) has \(3N\) components.
The diagonal elements of the density matrix give, as usual, the momentum
distribution

\[ \Omega_0(q) = \rho_0(q, q). \]  

(1)

It is assumed that the trace of the density matrix \(\rho_0\), or equivalently the inte-
gral over the momentum distribution over all the \(3N\)-dimensional momentum
space, is one. In order to make numerical predictions it is necessary to as-
sume some functional form for \(\rho_0(q, q')\), but here we will be interested mostly
in general results, thus no specific function is introduced. For the production
of \(N\) identical bosons formula (1) should be replaced by the symmetrized
formula

\[ \Omega_N(q) = \sum_P \text{Re} \rho_{0N}(q, q_p), \]  

(2)

where \(q_P\) is the \(3N\) dimensional momentum vector, which arises from the
vector \(q\), when permutation \(P\) is applied to the ordering of the \(N\) particles.
The summation extends over all the \(N!\) permutations of the \(N\) identical
bosons.

Formula (2) has important implications. As our first example let us in-
tegrate both sides of equality (2) over \(q\). Thus we obtain the integrated
cross-section for the \(N\) identical bosons:

\[ \sigma_N = \int dq \, \Omega_N(q) = 1 + \ldots. \]  

(3)

According to formula (2) the integral is a sum of \(N!\) terms. The first term
corresponding to the identical permutation \(P = 1\) gives one because of our
normalization condition. The other terms, however, change \(\sigma_N\). Since the
change depends on the particle number \(N\), the multiplicity distribution of the
produced particles is also changed. In order to go further, we must introduce
some technical assumption to make the calculation feasible. According to
our general philosophy, we make the assumptions for distinguishable particles
and then we go over to the indistinguishable particles by symmetrizing. We
assume that the particles are produced independently in the sense that the
multiplicity distribution is poissonian
\begin{equation}
P_0(N) = \frac{\nu^N}{N!} e^{-N}.
\end{equation}

For each multiplicity, moreover, we assume that the state of the produced particles is pure i.e. that \( \rho_{0N} = |\psi_N\rangle \langle \psi_N| \). This is a rather unrealistic assumption and soon we will replace it by something better, but as we shall see the present simple case illustrates important features of the more realistic model in a very simple way. Since the state vector of \( N \) particles does not change under permutations of identical bosons, we find for the integrated cross-section of producing \( N \) indistinguishable particles

\begin{equation}
\sigma_N = N! \sigma_{0N}.
\end{equation}

Consequently the corrected (unnormalized) probability for producing \( N \) particles is by a factor \( N! \) larger than for the distinguishable particles and, if the probability distribution for the distinguishable particles is given by the poissonian distribution (4), for the indistinguishable particles we have the (normalized) probability distribution

\begin{equation}
P(N) = (1 - \nu) \nu^N,
\end{equation}

which is a geometrical distribution, very different from the original poissonian one. Let us note two implications of this formula. When many pions are produced, usually about one third of them are \( \pi^0 \)-s. If the number of \( \pi^0 \)-s is governed by the poissonian distribution, large deviations from this average are very unlikely. For the geometric distribution, however, large deviations from the average are much more probable. This could explain the centauro and anticentauro events reported by cosmic ray physicists. In such events the \( \pi^0 \)-s are respectively either completely absent, or they are the only pions produced (in the region of momentum space accessible to experiment!). Another observation is that according to distribution (6) the average multiplicity of particles is

\begin{equation}
\overline{N} = \frac{\nu}{1 - \nu}.
\end{equation}

For small values of \( \nu \) this reproduces the result of the poissonian distribution as expected. At \( \nu = 1 \), however, there is a singularity. In the more realistic model introduced below this corresponds to the Einstein condensation.
3 A BETTER MODEL

A more realistic model can be obtained by replacing the assumption of a pure state by the assumption that at given \( N \) the density matrix for distinguishable particles is a product of single particle density matrices. Thus

\[
\rho_{0N} = \prod_{i=1}^{N} \rho_{01}(q_i, q'_i),
\]

where \( q_i(q'_i) \) is the momentum vector of particle \( i \). Also for this model all the calculations can be easily performed. This model is not yet realistic, but it is certainly much closer to reality than the previous one and some of its implications may be relevant for real experiments. We will present four predictions.

The single particle momentum distribution, the two-particle correlation function and all the \( n \)-particle correlation functions for \( n = 3, 4, \ldots \) can be expressed in terms of one function \( L(q_1, q'_1) \). Thus (if it is legitimate to neglect the variations of phase of the function \( L \)) one can measure the two-body correlation function and from that predict without further assumptions all the other correlation functions. Our model is too crude to expect more than qualitative agreement with experiment, but the formulae exist and can be checked.

The following two predictions do not hold for an arbitrary choice of the density matrix \( \rho_{01} \), but it is plausible that they hold for ”reasonable” choices. Firstly, the momentum distribution is expected to shrink as compared to the unsymmetrized distribution. Suppose that centauro, or anticentauro, events are found in accelerator experiments. This would mean that the Bose-Einstein correlations for these events are very large. We predict that the relevant groups of charged (or neutral) pions should have reduced relative momenta as compared e.g. to Monte Carlo calculations, which do not include symmetrization. Secondly, the radius of the production region, as calculated using standard methods, decreases. This can serve to illustrate our point of view that intuition should be applied to the uncorrelated case. Suppose that the pions are produced in a sphere the size of a nucleus. When the density of the produced particles in momentum space increases, the measured production radius decreases, although ”really” the particles are produced all the time from the same spherical region. For this reason we propose to
interpret the radius measured at low particle density as the "true" radius, while the radii measured at higher densities are only "effective" radii.

Finally, we expect the Einstein condensation at sufficiently high density. Out of the many ways of exhibiting this phenomenon let us look at the formula for the function $L(q, q')$, which can be rewritten in the form

$$L(q, q') = \frac{\psi_0(q)\psi_0^*(q')}{1 - \nu\lambda_0} + \tilde{L}(q, q').$$

(9)

Here $\psi_0$ is the eigenfunction of the single particle density operator, which corresponds to the largest eigenvalue $\lambda_0$. The parameter $\nu$ is that from the poissonian distribution $P_0(N)$. When $\nu\lambda_0 \to 1$, the term $\tilde{L}$ stays bounded, while the first term tends to infinity. At sufficiently high density, $\tilde{L}$ becomes negligible and almost all the particles are in the state $|\psi_0\rangle$. This is Einstein’s condensation and also the case considered in the framework of our first simplified model.

References

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