Regular black holes and self-gravitating solitons replacing naked singularities

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Abstract. We present a systematic review of the basic properties of regular black holes (RBH) and self-gravitating solitons replacing naked singularities including electrically charged RBHs and electromagnetic spinning solitons, predicted by analysis of regular solutions to dynamical equations with metrics of the Kerr-Schild class governing their behavior. We briefly outline observational cases which display and verify their fundamental generic features which are the de Sitter vacuum interiors and relation of their masses to spacetime symmetry breaking.

1. Introduction

In this report we overview the results on the basic generic properties of regular black holes and solitons, obtained during many years research initiated by the pioneering ideas on the de Sitter vacuum replacing a singularity, originated in the A.F. Ioffe Physico-Technical Institute [1, 2].

The algebraic classification of stress-energy tensors provides a way to introduce a general model-independent definition of a vacuum as a medium by the algebraic structure of its stress-energy tensor [3], and implies the existence of vacua whose symmetry is reduced as compared with the maximally symmetric de Sitter vacuum $T_{\mu}^{\nu} = \rho_{vac} g_{\mu}^{\nu}$ ($p = -\rho$) associated with the Einstein cosmological term ($8\pi G \rho_{vac} = \Lambda$), to $T^{\mu}_{\alpha} = T_{\alpha}^{\mu}$ ($p_{\alpha} = -\rho$) with $\alpha = 1, 2$. This allows to describe a vacuum in general setting by essentially anisotropic vacuum dark fluid [4] which can be evolving and clustering. Vacuum dark fluid is presented by a time-evolving and spatially inhomogeneous cosmological term [5] and provides for the unified description of dark energy and dark matter based on the spacetime symmetry [4] (for a review [6]).

The regular solutions to the Einstein equations for the vacuum dark fluid, belong to the Kerr-Schild class ([7] and references therein) and have the de Sitter centers provided that the weak energy condition (WEC) is satisfied which guarantees non-negativity of density as measured by any local observer on a time-like curve [8]. They describe regular black holes (RBHs) and self-gravitating solitons G-lumps, non-singular non-dissipative particle-like structures replacing naked singularities, with the de Sitter vacuum interiors [9, 8].

Quantum evaporation of RBHs involves a 2-nd order phase transition followed by quantum cooling and resulting in a thermodynamically stable remnant [9, 10, 11] (for a review [12]). Primordial RBHs, their remnants and G-lumps can form gravitons binding electrically charged particles [13], and can be considered as heavy dark matter (DM) candidates generically related to a vacuum dark energy (DE) via their de Sitter vacuum interiors (for a review [6]).
Rotating RBHs and spinning G-lumps described by the axially symmetric solutions of the Kerr-Schild class have interior de Sitter vacuum disks and two types of interiors determined by the energy conditions, one of which can contain a phantom energy [14] (for a review [15]).

Electrically charged RBHs and spinning electromagnetic solitons are governed by regular solutions to self-consistent equations of nonlinear electrodynamics coupled to gravity (NED-GR). Nonlinear electromagnetic fields are described by the source-free NED-GR dynamical equations, and the source term in the Einstein equations is presented by the stress-energy tensor of these nonlinear electromagnetic fields. For a NED-GR object its internal de Sitter vacuum disks is confined by a superconducting ring current which serves as the non-dissipative electromagnetic source of its electromagnetic fields [16] and of intrinsic magnetic momentum [17].

The mass of objects with the de Sitter interiors are generically related to breaking of spacetime symmetry from the de Sitter group [8, 18]. This provides for the intrinsic relation between gravity, spacetime symmetry and the the Higgs mechanism for a particle mass generation [19].

In Section 2 we present spherically symmetric RBHs and G-lumps including the global structure of an RBH spacetime and its remarkable possibilities, and the observational signatures of the related heavy DM candidates. Section 3 is devoted to rotating RBHs and spinning G-lumps including their specific observational signatures as DM candidates with DE interiors, and the observational case for electromagnetic spinning soliton. Section 4 contains conclusions.

2. Regular spherical black holes and solitons
The Einstein equations admit the class of regular spherical solutions with metrics of the Kerr-Schild class generated by stress-energy tensors with the algebraic structure $T^t_t = T^r_r$:

$$T^r_r = T^t_t \implies ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2(d\theta^2 + \sin^2 \theta d\phi^2).$$ (1)

In the case of two de Sitter vacuum scales, for the interior de Sitter vacuum and for a background cosmological constant $\lambda$ (as, e.g., in our Universe), a stress-energy tensor evolves between two de Sitter vacua, its $T^t_t$ component monotonically decreases since WEC requires $\rho' \leq 0$ [8] and can be presented as $T^t_t = \rho(r) + (8\pi G)^{-1}\lambda$, then the metric function $g(r)$ in (1) reads [20]

$$g(r) = 1 - \frac{\mathcal{M}(r)}{r} - \frac{\lambda}{3} r^2; \quad \mathcal{M}(r) = 4\pi \int_0^r \rho(x)x^2 dx$$ (2)

and asymptotically tends to the Schwarzschild-de Sitter metric $g(r)_{Schw-deS} = 1 - 2GM/r - \lambda r^2/3$, where $\rho$ related to the interior de Sitter vacuum gives the total mass $M = \mathcal{M}(r \to \infty)$.

The basic condition $T^t_t = T^r_r$ and conservation equation $T^\mu_\nu_{\text{cons}} = 0$ define the radial pressure $p_r$ and the transversal pressure $p_\perp$ as $p_r(r) = -\rho(r) - \rho_\lambda$; $p_\perp(r) = -\rho(r) - \rho_\lambda - r\rho'(r)/2$ [20].

Spacetime can have at most 3 horizons determined by zeros of the metric function $g(r)$, shown in figure 1 (Left). An observer in the region $r_b < r < r_c$ can observe a cosmological RBH confined by the event horizon $r_b$ and by the internal horizon $r_c$ in the universe with the cosmological horizon $r_c$. The horizon $r_a$ is the cosmological horizon for an observer in the region $0 \leq r < r_a$. This inspires an assumption on the possible existence of some universe(s) inside a black hole [21]. The global structure of spacetime shown in figure 1 (Right) [20] supports this conjecture. It represents an infinite sequence of regular black and white holes, $BH$ and $WH$, confined by the future and past horizons $r_+, r_-$; the future and past de Sitter cores $\mathcal{RC}$ approaching time-like regular surfaces $r = 0$, and parallel (not causally connected) universes $\mathcal{U}_1$, $\mathcal{U}_2$ with their future and past asymptotically de Sitter regions $\mathcal{CC}$ related to the background $\lambda$ and confined by horizons $r_{++-}$. The surfaces $\mathcal{F}^-$ and $\mathcal{F}^+$ are the null boundaries of the manifold. Inter-universe travels are possible towards a future regions of $\mathcal{U}_1$ (upwards the diagram).

Quantum temperature $T_h$ [22] and specific heat $C_h$ [11] for a horizon $r_h$ are given by
Figure 1. The metric function $g(r)$ for RBH (Left) and global structure of spacetime (Right).

$$kT_h = \frac{hc}{4\pi |g'(r_h)|}; \quad C_h = \frac{dE_h}{dT_h} = \frac{2\pi r_h}{g'(r_h) + g''(r_h)r_h}. \quad (3)$$

Temperature from the event horizon as seen by an observer shown in figure 1 (Left), is plotted in figure 2 (Left) [11]. Evaporation from all horizons proceeds with decreasing mass $M$ and goes towards the double-horizon thermodynamically stable remnant $r_a = r_b$ with $M = M_{cr1}$, zero temperature and positive specific heat [11] shown in figure 2 (Right, the curve $M = M_{cr1}$) [20]).

Figure 2. Temperature of an RBH event horizon (Left) and the metric function $g(r)$ (Right).

Temperature curve in figure 2 (Left) is generic [9, 11] since $T_h(r_h)$ evolves between two zeros, at $r_a = r_b$ and $r_b = r_c$, hence it must have a maximum in between, in which the specific heat $dE_h/dT_h = (dE_h/dr_h)(dr_h/dT_h)$ is broken and changes sign testifying for the 2-nd order phase transition and quantum cooling [9, 10, 11] (for a review [12]).

The upper curve, $M < M_{cr1}$, in figure 1 (Right) presents a self-gravitating soliton G-lump, a non-dissipative particle-like structure without the event horizon taking itself together by self-interaction [9, 8]. It replaces the Schwarzschild naked singularity whose unpredictable influence on outside observers is undesirable (in default of screening a singularity by the event horizon).

Primordial RBHs, their remnants and G-lumps can arise during the early inflationary stage(s), and form graviatoms, gravitationally bound ($\alpha_G = GMm/hc$) quantum structures [13].

The specific observational signature of RBH remnants, G-lumps and graviatoms as heavy DM candidates [4, 23] is directly related to their de Sitter interiors. In their GUT scale interiors the baryon and lepton numbers are not conserved, as a result they can induce proton decay in an underground detector. In the $1km^3$ detector, like the IceCUBE, there could be expected up to 300 events per year [23]. The additional specific observational signature of graviatoms is their oscillatory electromagnetic radiation, whose frequencies depend on the energy scale of the interior de Sitter vacuum and fall into the range available for observations [13].
3. Regular spinning black holes and solitons

Rotating black holes and spinning solitons are described by the axially symmetric solutions obtained from the spherical solutions of the Kerr-Schild class with using the general model-independent approach developed by Gürses and Gürsey [7] on the basis of the complex Trautman-Newman translations which include the Newman-Janis algorithm typically applied for constructing the rotating metrics presented in the literature [24]-[32] (for a review [15]).

In the Boyer-Lindquist coordinates the Gürses-Gürsey metric reads [7]

\[ ds^2 = \frac{2f - \Delta}{\Sigma} dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 - \frac{4af \sin^2 \theta}{\Sigma} dtd\phi + \left( r^2 + a^2 + \frac{2fa^2 \sin^2 \theta}{\Sigma} \right) \sin^2 \theta d\phi^2 \]  

(4)
in the units \( c = G = 1 \). The Lorentz signature is \([+, +, +, +]\), and

\[ \Delta = r^2 + a^2 - 2f(r); \quad \Sigma = r^2 + a^2 \cos^2 \theta \]  

(5)

where \( a \) is the angular momentum. The master function \( f(r) = rM(r) \) relates an axial solution with an original spherical solution. In the asymptotically flat case outlined here the metric (4) coincides with the Kerr metric \( f(r) \rightarrow M r \) as \( r \rightarrow \infty \). The mass parameter \( M = M(r \rightarrow \infty) \) represents the finite positive mass generically related to breaking of spacetime symmetry from the de Sitter group in the origin for any object specified by \( T^t_t = T^r_r \) [8]. For the rotating objects this condition is satisfied in the co-rotating frame with the angular velocity \( \omega(r) = a/(r^2 + a^2) \) ([18, 15] and references therein). The surfaces of constant \( r \) are the oblate confocal ellipsoids \( r^4 - (x^2 + y^2 + z^2 - a^2)r - a^2z^2 = 0 \) which degenerate, for \( r = 0 \), to the equatorial disk

\[ x^2 + y^2 \leq a^2, \quad z = 0, \]  

(6)

bounded by the ring \( a^2 + y^2 = a^2, \quad z = 0 \) [33]. The Cartesian coordinates \( x, y, z \) are related to the Boyer-Lindquist coordinates \( r, \theta, \phi \) by \( x^2 + y^2 = (r^2 + a^2) \sin^2 \theta; \quad z = r \cos \theta \).

An asymptotically flat rotating RBH has at most two horizons and two ergospheres [14]. Horizons are defined by \( \Delta(r_+, r_-) = r^2_+ + a^2 - 2f(r_\pm) = 0 \) where \( r_- \) and \( r_+ \) is the internal and event horizons, respectively. Ergospheres defined by \( g_\alpha = r^2_e + a^2 \cos^2 \theta - 2f(r_e) = 0 \), confine ergoregions where extraction of energy can occur due to \( g_\alpha < 0 \). G-lumps can have one ergoregion between two ergospheres or one ergoregion involving the whole interior [14].

The density \( \rho(r, \theta) \) and transversal pressure \( p_\perp(r, \theta) \) (eigenvalues of a stress-energy tensor in the co-rotating frame) are given by \( \rho(r, \theta) = r^4 \tilde{\rho}(r)/\Sigma^2; \quad p_r = -\rho; \quad p_\perp(r, \theta) = (r^4 - r^2 \Sigma) \tilde{\rho}(r)/\Sigma^2 + r^2 \tilde{p}_\perp(r)/\Sigma \), where \( \tilde{\rho}(r) \) and \( \tilde{p}_\perp(r) \) refer to a spherical solution. The master equations applied to scrutinize an interior behavior by the energy conditions, read [34, 14]

\[ p_r = -\rho; \quad p_\perp(r, \theta) = \frac{r^4 |\tilde{\rho}|}{2\Sigma^2} S(r, z); \quad S(r, z) = r^4 - z^2 P(r); \quad P(r) = \frac{2a^2}{r|\tilde{\rho}|}(\tilde{\rho} - \tilde{p}_\perp). \]  

(7)

On the equatorial disk (6) the equation of state takes the form \( p_\perp = p_r = -\rho \) and describes the rotating de Sitter, \( p = -\rho \), in the co-rotating frame [18, 34, 14].

If a related spherical solution violates the dominant energy condition (DEC, which requires \( \rho \geq p_\alpha \)), then \( \tilde{\rho} - \tilde{p}_\perp < 0, \quad P(r) < 0 \), and the function \( S(r, z) \) in (7) zeroes out only at the de Sitter disk [34]. This 1-st type interior satisfies WEC (which requires \( \rho + p_\alpha \geq 0 \) and is shown in figure 3 (Left) together with the horizons and the ergosphere. If a spherical solution satisfies DEC, then \( P(r) \geq 0 \) and there can exist the closed de Sitter surface, \( S \)-surface \( p_\perp + \rho = 0 \), with the de Sitter disk embedded as a bridge [34]. In the cavities between the \( S \)-surface and the disk WEC is violated, \( p_\perp + \rho < 0 \), i.e. \( p_\perp < -\rho \), which is identified as a phantom fluid. This 2-nd type interior is shown in figure 3 (Right), where \( r_\epsilon \) is a model-dependent regularization parameter [34, 14].
The 1-st type interior, horizons and ergosphere (Left) and 2-nd type interior (Right).

Figure 3. The 1-st type interior, horizons and ergosphere (Left) and 2-nd type interior (Right).

For a G-lump with the 2-nd kind interior phantom energy is not screened by the horizon which makes possible extraction of a phantom energy and presents the additional observational signatures for G-lumps and gravitons with spinning G-lumps as heavy DM coordinates, providing a source of information about the scale and properties of a phantom energy.

Information on the interior content of a rotating RBH can be extracted from observation of its shadow whose shape depends on the interior density [35]. Comparison of the observed black hole shadow with the Kerr shadow can decide whether the black hole is singular or regular.

Electrically charged regular objects are described by the NED-GR equations ([18, 34] and references therein). Nonlinear electrodynamics proposed by Born and Infeld and intended to consider electromagnetic field and particles in the frame of one physical entity (electromagnetic field) and to avoid divergences of physical quantities, succeeded in keeping electromagnetic energy finite but failed in preventing appearance of a singularity in geometry [37].

The NED-GR spinning regular objects are made of a nonlinear electromagnetic field and described by the source-free equations $\nabla \mu (F_{\mu \nu}) = 0$; $\nabla_\mu F^{\mu \nu} = 0$ where $F^{\mu \nu} = \eta^{\mu \nu \alpha \beta} F_{\alpha \beta} / 2$; $\eta^{123} = -1 / \sqrt{-\eta}$; $\mathcal{L}_F = d\mathcal{L} / dF$, which can be written in the Maxwell form

$\nabla \cdot D = 0$; $\nabla \times H = \partial D / \partial t$; $\nabla \cdot B = 0$; $\nabla \times E = -\partial B / \partial t$, with the fields vectors $E = \{ F_{\mu 0} \}$; $D = \{ \mathcal{L}_F F^{0 \beta} \}$; $B = \{ * F^{0 \beta} \}$; $H = \{ \mathcal{L}_F * F_{0 \beta} \}$. A source of geometry in the Einstein equations is provided by stress-energy tensor of electromagnetic fields which intrinsically has the algebraic structure $T^\text{T}_I = T^\text{T}_I$ [18]. The electric induction $D$ and magnetic induction $B$ are related with the field intensities $E$ and $H$ by $D^\text{a} = \epsilon_0^\beta E^\beta$; $B^\text{a} = \mu_0^\beta H^\beta$, where $\epsilon_0$ and $\mu_0$ are the tensor of the electric and magnetic permeability, respectively [18] $\epsilon_0 = (r^2 + a^2) \mathcal{L}_F / \Delta$; $\epsilon_0^\beta = \mathcal{L}_F$; $\mu_0^\beta = (r^2 + a^2) / (\Delta \mathcal{L}_F)$; $\mu_0^\beta = 1 / \mathcal{L}_F$. Asymptotic solutions obtained for the strongly nonlinear regime on the de Sitter disk (6) yield $\mathcal{L}_F = 2 \epsilon_0^\beta / (\Sigma^\text{a} (p_+ + \rho) )$ [18, 34], so that $\mathcal{L}_F \to \infty$, $\mu_0^\beta = 1 / \mathcal{L}_F \to 0$ and $\epsilon_0^\beta = \mathcal{L}_F \to \infty$, and the disk displays the properties of a perfect conductor and ideal diamagnetic [18, 34].

The surface current on the de Sitter disk is $j_0 = - c e / 2 \pi a \sqrt{1 + e^2 / a^2} \sin \xi \left( \mu / \cos^3 \xi \right)$ [16]. It vanishes throughout the disk due to $\mu = \mu_0^\beta = \mu_0^\beta = 0$, except the ring $\xi = \pi / 2$, where both terms in the fraction vanish independently. The ring current can be thus any and amount to a non-zero value, which is the general criterion for a superconductivity [38]. It flows without resistance and presents a non-dissipative source of the electromagnetic fields, which can in principle ensure an unlimited life time of an object [16]. The current $j_0$ produces an intrinsic magnetic momentum $\mu_{in} = -(ea / 2) \sqrt{T + e^2 / a^2 U}$ [17] where $U$ is an uncertain coefficient. When the intrinsic magnetic moment of an object is known, the coefficient $U$ can be restored.

For the soliton with the parameters of the electron, $mac = \hbar / 2$, this gives $j_0 = 79.277 \ A$ [17].

Generic properties of the electromagnetic soliton can explain appearance of a minimal length scale $l_e = 1.57 \times 10^{-17} \ cm$ revealed with the $5 \sigma$ significance in the annihilation reaction $e^+ + e^- \to \gamma \gamma(\gamma)$ at the energy $E = 1.253 \ TeV$ [39]. The length $l_e = 1.57 \times 10^{-17} \ cm$ can be identified as a distance of the closest approach of annihilated particles at which electromagnetic attraction is balanced by the gravitational repulsion of the interior de Sitter vacuum [39, 19].
4. Conclusions

RBHs and solitons replacing naked singularities specified by $T^4_r = T^r_r$ have obligatory de Sitter interiors. The predicted observational signatures of RBH remnants, G-lumps and gravitons as heavy DM candidates, include the induced proton decay in an underground detector, and the electromagnetic radiation of gravitons depending on the scale of the de Sitter vacuum.

Mass of all objects with the de Sitter interiors is genetically related to breaking of spacetime symmetry from the de Sitter group. The intrinsic relation of the Higgs mechanism to gravity and spacetime symmetry via the de Sitter vacuum [19] allows to explain the negative mass squares for neutrinos and to evaluate the gravito-electroweak scale [40].

The basic feature of any regular electrically charged NED-GR objects is the internal de Sitter vacuum disk which has properties of a perfect conductor and ideal diamagnetic and is confined by a superconducting ring current that serves as a source of its electromagnetic fields and of its intrinsic magnetic momentum. The repulsive gravity of the de Sitter vacuum allows to explain the appearance of the minimal length scale in the $e^+ + e^-$ annihilation.

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