Hydraulic modeling of surface vortex funnels

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Abstract. The peculiarities of the operation of entrance portals for concrete and corrugated metal road pipes, water intake pipelines, spillway structures and hydroelectric power plants include the formation of surface vortex funnels. The formation of surface funnels reduces the water consumption in the pipeline, provokes the destruction of the fastening of the slopes at the inlet, the suction of sludge, ice and debris into the pipeline. It is desirable to avoid these phenomena. The article is devoted to the issues of physical modeling of surface funnels. The aim of the research is to determine the criteria for the breakthrough of the air core of the surface funnel into the pressure pipeline. As a result of research, it was found that the formation of vortex funnels in the upper basin is determined by the structure of the flow in the local area adjacent to the water intake. It is shown that the profile of the free surface of the vortex funnel and its depth on the axis of rotation are determined by the intensity of the circulation generating the funnel. A formula is obtained for calculating the limiting depth of a surface vortex funnel with an accuracy sufficient for engineering calculations. It is shown that in physical modeling according to Froude's defining criterion, the depth of the funnel obtained on the model should be recalculated for a full-scale object with a scale factor $m^{-1.5}$, where $m$ is the linear scale of the model. Another method for obtaining the depth of the funnel on the model, corresponding to its conversion to a full-scale object on a linear scale, is forcing the current velocities by $m^{-3/14}$ times in relation to their value according to the Froude criterion.

1. Introduction

The peculiarities of the operation of the entrance portals of tubular structures (concrete and corrugated metal road pipes, water intake and spillway structures, pipelines of hydroelectric power plants, etc.) include the formation of surface vortex funnels. The formation of surface funnels reduces the flow rate of water in the pipeline, provokes the destruction of the fastening of slopes at the inlet, suction of slush, ice, debris, etc. into the pipeline [1].

The circulating flow in the surface vortex funnel in front of the pipeline water intake (Figure 1) is a widespread phenomenon in nature. It is desirable to avoid it or prevent the burst of the funnel air core into the pressure water conduit, which occurs when the depth of the funnel exceeds the depth of the intake wall of the water intake ($Z_0 > h$).

Today, the methods for calculating this flow are far from perfect, therefore, the conditions for the formation of vortex funnels at the designed objects are studied on large-scale hydraulic models. But the physical modeling of vortex funnels is not an easy task, because such a flow is a multifactorial...
process determined by a number of dynamic similarity criteria, which cannot be simultaneously ensured for a model and a full-scale object due to their scale incompatibility.

Figure 1. Diagram showing surface vortex funnel.

2. Results and Discussion
In the general case, the hydrodynamics of a steady flow of a viscous fluid in a surface vortex funnel is described by the equation

\[
\text{rot} \ V \times V = -\text{grad} \left( \frac{P}{\rho} + \frac{V^2}{2 \Pi} - \frac{\varepsilon}{2} \text{rot} \left( \text{rot} \ V \right) \right),
\]

where \( \rho \) and \( \varepsilon \) are density and molecular kinematic viscosity of the liquid; \( P \), \( V \) and \( \Pi \) are pressure, local velocity vector and potential of external mass forces.

Moreover, the pressure on the surface of the air core of the vortex funnel can be assumed constant and equal to the atmospheric, then

\[ \text{grad}(P/\rho) = 0. \]

The local velocity should be considered as the sum of time-averaged \((\Bar{V})\) and pulsation \((V')\) components

\[ V = \Bar{V} + V', \]

then, when averaged over a finite time interval

\[ \text{rot} \ V \times V = \text{rot} \ (\Bar{V} \times V) + \text{rot} \ (V' \times V'), \]

and the potential of external mass forces on the surface of the air core, if the \( Z \) axis of the coordinate system is directed vertically upward, can be written as

\[ \Pi = -gZ - K \sigma / \rho \]

where \( g \) is the gravitational acceleration; \( Z \) is current coordinate of the surface of the air core from the horizon of the pool in advance of the water intake; \( \sigma \) is the surface tension; \( K \) is the curvature of the surface of the air core.

Summarizing the above and taking in the first approximation a model of isotropic and homogeneous turbulence with constant vortex viscosity \( \varepsilon \), as a result of normalization Eq. (1) with respect to the characteristic velocity \( V_0 \) and dimension \( L_0 \), we obtain [2]

\[
\text{rot} \ \Bar{V} \times \Bar{V} = -\text{grad} \left( \frac{\Bar{V}^2 + \Bar{V}'^2}{2} + \frac{z}{\text{Fr}} + \frac{k}{\text{We}} \right) - \left( \frac{1}{\text{Re}} + \frac{1}{\text{Re}_f} \right) \text{rot} \left( \text{rot} \ \Bar{V} \right),
\]

where \( \Bar{V} = V/V_0 \) is the normalized value of the time-averaged local velocity; \( \Bar{V}'^2 \) is the standard of pulsations; \( z = Z/L_0 \) and \( k = KL_0 \) are current relative (normalized) values of the surface coordinate and curvature of the vortex air core; Froude number

\[
\text{Fr} = \frac{V_0^2}{gL_0},
\]

(3)
\[ \sigma = \rho V_0^2 L_0 / \sigma \] is Weber number, Reynolds number and the turbulent analogue of the latter
\[ \text{Re} = \frac{V_0 L_0}{\varepsilon} \quad \text{and} \quad \text{Re}_t = \frac{V_0 L_0}{\varepsilon_t}, \] (4)
here \( \varepsilon \) is the vortical viscosity.

It can be assumed that for real hydraulic engineering objects (concrete and corrugated metal road pipes, water intake and spillway structures, pipelines of hydroelectric power plants, etc.), the Weber and Reynolds numbers are so large that they do not affect the formation of vortex funnels, that is, the \( k/\text{We} \) and \( 1/\text{Re} \) complexes in the equation motions \((2)\) are negligible. It is also possible to consider these complexes to be insignificant if the flow simulation is performed in the zone of quadratic resistance, when the conditions of self-similarity are provided both by the Reynolds criterion and by the Weber criterion. However, this cannot be said about the \( 1/\text{Re}_t \) complex, because, as is known \([3]\), \( \varepsilon_t \gg \varepsilon \). As a result, the normalized equation of motion is reduced to the form
\[ \text{rot} \, \vec{v} \times \vec{v} = -\text{grad} \left( \frac{\vec{v}^2 + \vec{v}_r^2}{2} + \frac{z}{\text{Fr}} \right) - \frac{1}{\text{Re}_t} \text{rot(rot} \, \vec{v}) \].

Consequently, the criterion equation in the physical modeling of vortex funnels is
\[ f(\text{Fr}, \text{Re}_t) = 0, \] (5)

The Froude and Reynolds dynamic similarity criteria (turbulent analogue) are incompatible in scale. Under these conditions, they usually go to an approximate similarity according to the determining Froude criterion. However, the fact of discrepancy between the model and full-scale surface vortex funnels in hydraulic modeling according to Froude is well known. This fact is described in almost all works devoted to this problem. As an example, let us refer to the fundamental work published under the editorship of Knauss \([4]\).

In work \([2]\), devoted to mathematical and physical modeling of surface vortex funnels, we obtained the equations for the profile of the surface of the air core of the funnel and its depth on the axis of rotation \((Z_0)\), the latter is written in the form
\[ Z_0 / R_0 = \frac{\text{Fr}}{2} \left[ 1 + \Gamma_0^2 \right] + \frac{\text{Fr} \, \text{Re}_t \, \Gamma_0^2}{2 \left[ 1 - \exp \left( -\text{Re}_t / 2 \right) \right]^2} \left[ \text{Ei}(-\text{Re}_t) - \text{Ei} \left( -\frac{\text{Re}_t}{2} \right) - \ln(2) \right], \] (6)
where \( \text{Ei}(\ldots) \) is integral exponential function; \( R_0 \) is the outer radius of the funnel, beyond which the free surface of the liquid can be assumed horizontal; the Froude and Reynolds numbers from \((3)\) and \((4)\) contain as the characteristic velocity the average radial flow velocity to the funnel at its outside radius \( R_0 \). the outside radius of the funnel \( L_0 = R_0 \) is taken as the characteristic dimension; \( \Gamma_0 \) is relative circulation
\[ \Gamma_0 = V_{0R}/V_{00}, \] (7)
where \( V_{00} \) is the circumferential (tangential) component of the fluid velocity at the radius \( R_0 \). The number \((7)\) is also called the Rossby number.

It can be seen that in the analytical calculation, it is required to know the values of the radius \( R_0 \), runoff \( V_{0R} \) and tangential \( V_{00} \) velocities, which are determined by the structure of the flow in the water intake zone. The values of these parameters can be found by the methods of computational hydromechanics, however, even in this case, the combination of numerical and analytical calculations, which makes it possible to close the problem and obtain the desired solution, does not exclude the need to check it on a physical model, just as projected hydraulic structures and hydraulic calculation methods cannot do without laboratory studies. This allows us to speak of physical modeling as the most important stage in substantiating the adopted technical solutions, at which the main problem is the reliable interpretation of the data obtained. There is a need to find such a combination of scale-independent criteria of dynamic similarity included in equation \((5)\), which achieves a reliable
recalculation of results from model to real object. Note that there are two of these criteria, since the appearance of the Rossby number (7) in equation (6), which determines the depth of penetration of the air core of the vortex funnel into the water column in front of the water intake, does not change the form of criterion equation (5), because it is not a dynamic, but a kinematic criterion, which, subject to the geometric similarity of the model and real object, must be performed unconditionally. But in (4), which is included in equations (5) and (6), the value of the vortex viscosity $\varepsilon$, remains unknown. A very close correspondence to the conditions of turbulent flow in surface vortex funnels takes place in swirling submerged jets, and such flows have been studied in sufficient detail [5, 6]. In these works, the following relationship was recommended, linking the vortex and molecular viscosity

$$\varepsilon_{t} = \varepsilon = 0.185 \left( \frac{V_{0}/R_{0}}{\varepsilon} \right)^{2/3}.$$  

(8)

Verification (6) showed that the use of formula (8) makes it possible to obtain the calculated values that most accurately agree with the observed experimental and field data.

In the process of modeling according to the Froude criterion, the velocity field on the approach to the vortex surface funnel is determined by the general structure of the flow in the water intake zone. If the geometric similarity of the model and the full-scale object is observed, as a consequence, we can assume the kinematic similarity of the model and full-scale flows. Thus, if the Froude number and the normalized value of circulation on the model and on a full-scale object are equal, then the turbulent Reynolds number on the model will not correspond to its value on a full-scale object. This will distort the scale conversion according to the Froude criterion of the model dimensions of the surface funnel to the dimensions of the surface funnel on a natural object. To clarify this position, we substitute the value of the eddy viscosity according to (8) into the turbulent Reynolds number according to (4), as a result, we obtain

$$Re_{t} = \frac{1}{C} Re^{1/3},$$

(9)

where $Re = V_{R0} R_0 / \varepsilon$ is the Reynolds number, and $C = 0.185 (V_{0}/V_{R0})^{2/3}$ is a constant, the value of which will be equal to that of model and full-scale entity when the kinematic similarity $V_{0}/V_{R0} = \text{idem}$ is observed.

That is, the turbulent Reynolds number grows in proportion to the cubic root of the Reynolds number, calculated from the molecular kinematic viscosity. But Reynolds (molecular) and Froude’s criterion are scaled incompatible, therefore, when modeling by the prevailing Froude criterion, the parameters of the model and natural flows in surface funnels will also be scaled incompatible. So, if the linear scale of the model

$$m = L_{M}/L_{F},$$

(10)

where $L_{M}$ and $L_{F}$ are the characteristic linear dimensions of the model and full-scale entity, is equal, for example, $m = 1:25$, then when modeling according to Froude, the molecular Reynolds number in the transition from model to nature will increase by

$$\frac{Re_{F}}{Re_{M}} = \frac{1}{m^{\sqrt{m}}} = 125$$

while the turbulent Reynolds number increases only by

$$\frac{Re_{tF}}{Re_{tM}} = \left( \frac{Re_{F}}{Re_{M}} \right)^{1/3} = \left( \frac{1}{m^{\sqrt{m}}} \right)^{1/3} = \frac{1}{\sqrt[m]{m}} = 5,$$

(11)

here, and below the subscripts “M” and “F” refer to the model and full-scale (natural) object, respectively.

To clarify the effect of the increase in the Reynolds number when switching from model to full-scale entity, let us analyze how this is reflected in the value of the main parameter of the funnel - its depth on the axis of rotation $Z_0$. The calculated turbulent Reynolds numbers, as practice shows, are
quite high and even for small hydraulic models, for example, with a scale of \( m = 1: 100 \), are at a level not lower than \( \text{Re}_t = 25 \). Taking this into account, it is possible to exclude from (6) small high orders
\[
\exp(-\text{Re}_t/2) \rightarrow 0,
\]
\[
\text{Ei}(-\text{Re}_t) = -\exp(-\text{Re}_t)/\text{Re}_t \rightarrow 0,
\]
\[
\text{Ei}(-\text{Re}_t/2) = -2\exp(-\text{Re}_t/2)/\text{Re}_t \rightarrow 0,
\]
this will reduce it to the form
\[
Z_0 = R_0 \frac{\text{Fr}}{2} \left[ 1 + \text{Re} \frac{2}{\text{Fr}} \right] \left[ 1 - \text{Re}_t \ln(2) \right] = -0.347 \cdot R_0 \cdot \text{Fr} \cdot \text{Re}_t \cdot \text{Re}^2.
\]

The formula gives reason to believe that it is circulation that has an overwhelming effect on the depth of the surface vortex funnel, and in general it allows us to conclude that the change in the velocity field in time, that is, the dynamics associated with an unsteady flow regime, or velocity pulsations, will affect the shape and the depth of the funnel to a degree greater than the square of the velocity. At a high level of pulsation, the depth of the funnel will be subject to significant fluctuations, which is often observed in experiments.

According to (12), for model and full-scale funnels, we can write
\[
Z_{0M} = -0.347 \cdot R_{0M} \cdot \text{Fr}_{M} \cdot \text{Re}_{OM} \cdot \Gamma_{OM}^2,
\]
\[
Z_{0F} = -0.347 \cdot R_{0F} \cdot \text{Fr}_{F} \cdot \text{Re}_{OF} \cdot \Gamma_{OF}^2.
\]

Since according to the conditions of Froude modeling \( \text{Fr}_{OM} = \text{Fr}_{F} \) and \( \Gamma_{OM} = \Gamma_{OF} \) then, dividing (14) by (13), taking into account (10) and (11), we find
\[
\frac{Z_{0F}}{Z_{0M}} = \frac{R_{0F} \cdot \text{Re}_{OF}}{R_{0M} \cdot \text{Re}_{OM}} = \frac{1}{m^{3/4}}.
\]

Consequently, the ratio of the depths of the full-scale and model funnels in the physical modeling according to Froude is \( 1/(m\sqrt{m}) \), and not \( 1/m \), as one would expect from a linear scale conversion. The graph of dependence (15) is shown in Figure 2. It can be seen that in our example, the depth of the funnel measured on the model under the Froude simulation should be increased for full-scale entity not \( 1/m = 25 \), but \( 1/(m\sqrt{m}) = 125 \) times; that is, 5 times higher.

![Figure 2](image)

**Figure 2.** \( Z_{0F}/Z_{0M} = m^{-1.5} \) and \( m_Y/m_{F} = m^{-3/14} \) functions.

Considering the fact that the physical modeling of vortex funnels does not meet the Froude
criterion, underestimating the values of their parameters by scale conversion from model to full-scale entity, many researchers use the method of increasing (forcing) the flow velocities on the model. Let us answer the question, what kind of velocity boost is necessary to obtain the depth of the funnel on the model, corresponding to the depth on the full-scale entity. Obviously, this is achieved subject to condition (10), according to which

$$\frac{Z_{0F}}{Z_{0M}} = \frac{R_{0F}}{R_{0M}} = \frac{1}{m}. \quad (16)$$

If the conditions of kinematic similarity ($\Gamma_0 = V$) are preserved, then (16), taking into account (13) - (14), is reduced to the equality

$$\text{Fr}_M \cdot \text{Re}_M = \text{Fr}_F \cdot \text{Re}_F. \quad (17)$$

where the turbulent Reynolds numbers and Froude numbers for the model and full-scale entity are written in accordance with (9) and (3) as

$$\text{Re}_M = \frac{1}{C} \left(\frac{\text{Re}_M}{\varepsilon}\right)^{1/3} = \frac{1}{C} \left(\frac{V_{R0M} R_{0M}}{\varepsilon}\right)^{1/3} \quad \text{and} \quad \text{Fr}_M = \frac{V_{R0M}^2}{g R_{0M}},$$

$$\text{Re}_F = \frac{1}{C} \left(\frac{\text{Re}_F}{\varepsilon}\right)^{1/3} = \frac{1}{C} \left(\frac{V_{R0F} R_{0F}}{\varepsilon}\right)^{1/3} \quad \text{and} \quad \text{Fr}_F = \frac{V_{R0F}^2}{g R_{0F}}. \quad (18)$$

Substituting these equalities into (17), as a result, we obtain

$$\left(\frac{V_{R0M}}{V_{R0F}}\right)^{7/3} = \left(\frac{R_{0M}}{R_{0F}}\right)^{2/3},$$

but $V_{R0M}/V_{R0F} = m_V$ is the scale of the velocity, and $R_{0M}/R_{0F} = m$ is the linear scale of the model, therefore,

$$m_V = m^{2/7}. \quad (19)$$

When simulating by Froude's criterion, the velocity scale is equal to $m_{\text{Fr}} = \sqrt{m}$; dividing expression (18) by it, we find

$$m_V/m_{\text{Fr}} = m^{-3/14}. \quad (19)$$

This is the necessary velocity boost when simulating by Froude's criterion. Function (19) of the required velocity boost depending on the linear scale of the model is shown in Figure 2. Thus, in our example, with a linear scale of the model $m = 1:25$, the velocity boost must be taken equal to $m_V/m_{\text{Fr}} = m^{-3/14} = (1/25)^{-3/14} = 1.99$, that is, the flow velocity should be increased in relation to its value by a large-scale recalculation according to Froude's rule is 2 times. This almost exactly corresponds to the calculation of the forcing according to the empirical version described in [7], but refutes the popular opinion that the velocity should be forced more significantly - up to full-scale values, for our example it is $m_V/m_{\text{Fr}} = 1/\sqrt{m} = 5$ times.

3. Conclusion

The circulating flow in the surface vortex funnel in front of the pipeline water intake (Figure 1) is a widespread phenomenon in nature. It is desirable to avoid it or prevent the burst of the funnel air core into the pressure water conduit, which occurs when the depth of the funnel exceeds the depth of the intake wall of the water intake ($Z_0 > h$).

It is shown that the profile of the free surface of the vortex funnel and its depth $Z_0$ on the axis of rotation are determined by the intensity of the circulation $\Gamma_0 = V_{00}/V_{R0}$ generating the funnel, the values of the Reynolds numbers $\text{Re}_t = V_{00} R_0/\varepsilon$, and Froude $\text{Fr}_t = V_{R0}^2/g R_0$.

It is established that, with an accuracy sufficient for engineering calculations, the depth of the surface funnel on the axis of its rotation can be found by the formula
\[ Z_0 = -0.347 \cdot R_0 \cdot Fr \cdot Re_f \cdot \Gamma_0^2, \]
showing that circulation has the most significant effect on the depth of the surface vortex funnel.

A change in the velocity field in time, that is, the dynamics associated with an unsteady flow regime, or velocity pulsations will be reflected in the shape and depth of the vortex funnel to a degree exceeding the square of the velocity, and for example, at a high level of pulsations, the depth of the funnel will be subject to significant fluctuations, which in practice, it is observed quite often.

Comparison of the funnels obtained on the models and taking place in natural conditions shows that the recalculation of the model parameters according to the Froude criterion gives significantly underestimated values of the depth of the full-scale surface funnel. Such a discrepancy between the model and full-scale funnel is determined by the scale independence of the Froude and turbulent Reynolds criteria.

In hydraulic modeling according to Froude's defining criterion, the depth of the funnel obtained on the model should be recalculated to a full-scale object with a scale factor \( \frac{1}{(m \sqrt{m})} \), where \( m \) is the linear scale of the model.

Another method for obtaining the depth of the funnel on a model corresponding to a linear recalculation in scale for a full-scale object is forcing the flow velocities in \( m^{-3/14} \) times in relation to their value according to the Froude criterion.

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