Supersymmetric origin of neutrino mass

M Hirsch and J W F Valle
AHEP Group, Instituto de Física Corpuscular—C.S.I.C./Universitat de València
Edificio Institutos de Paterna, Apt 22085, E-46071 Valencia, Spain
E-mail: valle@ific.uv.es

New Journal of Physics 6 (2004) 76
Received 12 May 2004
Published 8 July 2004
Online at http://www.njp.org/
doi:10.1088/1367-2630/6/1/076

Abstract. Supersymmetry with breaking of $R$-parity provides an attractive way to generate neutrino masses and lepton mixing angles in accordance with the present neutrino data. We review the main theoretical features of the bilinear $R$-parity breaking (BRpV) model, and stress that it is the simplest extension of the minimal supersymmetric standard model (MSSM), which includes lepton number violation. We describe how it leads to a successful phenomenological model with hierarchical neutrino masses. In contrast with see-saw models, the BRpV model can be probed at future collider experiments, such as the Large Hadron Collider or the Next Linear Collider, since the decay pattern of the lightest supersymmetric particle provides a direct connection with the lepton mixing angles determined by neutrino experiments.
1. Introduction

A combination of solar, atmospheric, reactor and accelerator neutrino experiments [1]–[3] have now firmly established the existence of neutrino masses and therefore the incompleteness of the standard model of electroweak interactions. The determination of neutrino oscillation parameters presented in [4] uses the most recent data and state-of-the-art solar and atmospheric neutrino fluxes;¹ see [5]–[8] for previous reviews and references. We have now learnt that the atmospheric oscillations involving $\nu_\mu \leftrightarrow \nu_\tau$ are characterized by a nearly maximal mixing, whereas the solar neutrino mixing angle is large and significantly non-maximal. With the recent standard solar model fluxes, there is a unique range for the solar mass splitting $\Delta m^2_{\text{sol}}$, determined from the data to be about 30 times smaller than the atmospheric mass splitting $\Delta m^2_{\text{atm}}$.

The discovery of neutrino mass constitutes the only solid hint we currently have of physics beyond the standard model. There are theoretical arguments based on the stability of the gauge hierarchy which suggest the existence of physics at the TeV scale. Supersymmetry [9, 10] provides an answer to both these issues which fits well with unification and string theory ideas [11].

Prompted by these data, there has been a rush of theoretical and phenomenological papers on models of neutrino masses and mixings. The most popular idea is to ascribe neutrino masses to physics at a large mass scale to implement some variant of the see-saw mechanism [12]–[15]. Broken $R$-parity supersymmetry provides a theoretically interesting and phenomenologically viable alternative to the origin of neutrino mass and mixing [16]–[22]. Here we focus on the case of supersymmetry with bilinear $R$-parity breaking (BRpV) [23]. This is the simplest of all $R$-parity violating models. It also provides the simplest extension of the minimal supersymmetric standard model (MSSM) [23] to include the violation of lepton number, as well as a calculable framework for neutrino masses and mixing angles in agreement with the experimental data [24]–[27]. In this model, the atmospheric neutrino mass scale is generated at the tree level through

¹ The results presented in [4] were based on a previous paper published in Phys. Rev. D 68, 113010 (2003), (hep-ph/0309130), which also contains a more extensive list of references.
the mixing of the three neutrinos with the neutralinos, in an effective ‘low-scale’ variant of the see-saw mechanism [17]. In contrast, the solar mass and mixings are generated radiatively [26]. BRpV can be considered either as a minimal extension of the MSSM [28]–[31] (with no new particles) valid up to some very high unification energy scale, or as the effective description of a more fundamental theory in which the breaking of \( R \)-parity occurs in a spontaneous way by minimizing the scalar potential [32]–[34].

This short review is mainly devoted to the generation of neutrino masses and lepton mixing, both the tree-level atmospheric neutrino mass scale as well as a description of the main features of the full one-loop calculation of the neutrino–neutralino mass matrix and its various analytic approximations which, in some cases, can be rather simple. For definiteness we will stick to the case of explicit BRpV only.

However, in contrast with the see-saw mechanism, in BRpV neutrino masses are generated at the electroweak scale [24, 26, 27]. Such low-scale schemes for neutrino masses have the advantage of being testable also outside the realm of neutrino experiments. Although neutrino properties cannot be predicted from first principles, their fit to the data allows for unambiguous tests of the theory at accelerator experiments [27], [35]–[43]. Indeed, the measured lepton mixing angles lead to well-defined predictions of the decay properties of the lightest supersymmetric particle (LSP). This is a very general and robust feature of these theories, which holds irrespective of the nature of the LSP. Here we will illustrate possible phenomenological scenarios by discussing some examples of measurements of decay properties of different LSP candidates.

This paper is organized as follows. In section 2 we introduce the main features of the model, discuss the soft supersymmetry breaking terms, as well as the relevant fermion mass matrices and the main features of the corresponding diagonalizing matrices. In section 3 we discuss the generation of the atmospheric neutrino mass scale at the tree level, whereas in section 4 we analyse the main features of the one-loop-induced solar neutrino mass scale, including a discussion of the relevant Feynman graph topologies. We also give simplified approximation formula for the solar mixing angle. We then turn briefly to collider phenomenology and how the model under discussion could be tested in LSP decays in section 5 before we conclude and summarize our results in section 6.

2. Formalism

In this section, we introduce the main features of the model and the relevant mass matrices. The superpotential of the model and the soft SUSY breaking terms are given, and approximate solutions to the tadpole equations discussed.

2.1. The superpotential and the soft breaking terms

The minimal BRpV model we are working with is characterized by the presence of three extra bilinear terms in the superpotential analogous to the \( \mu \) term present in the MSSM. Using the conventions of Akeroyd et al [31] it may be given as

\[
W = \varepsilon_{ab}[h^{ij}_{U} \tilde{Q}^{i}_{L} \tilde{U}^{b}_{L} \tilde{H}^{a}_{u} + h^{ij}_{D} \tilde{Q}^{i}_{L} \tilde{D}^{b}_{L} \tilde{H}^{a}_{d} + h^{ij}_{E} \tilde{L}^{i}_{L} \tilde{R}^{b}_{L} \tilde{H}^{a}_{d} - \mu \tilde{H}^{a}_{u} \tilde{H}^{b}_{u} + \epsilon_{L} \tilde{L}^{a}_{L} \tilde{H}^{b}_{u}],
\]  

(1)
where the first three terms are the usual MSSM Yukawa terms, $\mu$ is the Higgsino mass term of the MSSM and $\epsilon_i$ are the three new terms which violate lepton number in addition to $R$-parity. The couplings $h_U$, $h_D$ and $h_E$ are $3 \times 3$ Yukawa matrices and $\mu$ and $\epsilon_i$ are parameters with units of mass. The smallness of the bilinear term $\epsilon_i$ in equation (1) may arise from a suitable symmetry. In fact, any solution to the $\mu$ problem [44] potentially explains also the ‘$\epsilon_i$ problem’ [45]. A common origin for the $\epsilon_i$ terms that account for the neutrino oscillation data, and the $\mu$ term responsible for electroweak symmetry breaking can be ascribed to a horizontal family symmetry of the type suggested in [46].

The smallness of $\epsilon_i$ could also arise dynamically in models with spontaneous breaking of $R$-parity [32]–[34], where it is given as the product of a Yukawa coupling times a singlet sneutrino vacuum expectation value.

Supersymmetry breaking is parametrized with a set of soft supersymmetry breaking terms. In the MSSM these are given by

$$L^{MSSM}_{\text{soft}} = M_Q^{ij} \bar{Q}_i^a Q_j^a + M_U^{ij} \bar{U}_i^a U_j^a + M_D^{ij} \bar{D}_i^a D_j^a + M_L^{ij} \bar{L}_i^a L_j^a + M_R^{ij} \bar{R}_i^a R_j^a$$

$$+ m_{H_d}^2 H_d^a H_d^a + m_{H_u}^2 H_u^a H_u^a - \left[ \frac{1}{2} M_s \lambda_s \lambda_s + \frac{1}{2} M \lambda \lambda + \frac{1}{2} M_\lambda \lambda' + \text{h.c.} \right]$$

$$+ \epsilon_{ab} [A_U^{ij} \bar{Q}_i^a \tilde{U}_j^b + A_D^{ij} \bar{Q}_i^a \tilde{D}_j^b + A_L^{ij} \bar{L}_i^a \tilde{L}_j^b + A_R^{ij} \bar{R}_i^a \tilde{R}_j^b - B_i H_u^a H_u^b] .$$

(2)

In addition to the MSSM soft SUSY breaking terms in $L^{MSSM}_{\text{soft}}$, the BRpV model contains the following extra terms:

$$V_{\text{soft}}^{BRpV} = -B_i \epsilon_i \epsilon_{ab} \bar{L}_i^a H_u^b,$$

(3)

where the $B_i$ have units of mass. In what follows, we neglect inter-generational mixing in the soft terms in equation (2).

The electroweak symmetry is broken when the two Higgs doublets $H_d$ and $H_u$, and the neutral component of the slepton doublets $\tilde{L}_i^1$ acquire non-zero vacuum expectation values (vevs). These are calculated via the minimization of the effective potential or, in the diagrammatic method, via the tadpole equations. The full-scalar potential at tree level is

$$V^0_{\text{total}} = \sum_i \left| \frac{\partial W}{\partial z_i} \right|^2 + V_D + V^{MSSM}_{\text{soft}} + V^{BRpV}_{\text{soft}},$$

(4)

where $z_i$ is any one of the scalar fields in the superpotential in equation (1), $V_D$ are the $D$ terms and $V^{BRpV}_{\text{soft}}$ is given in equation (3).

The tree-level scalar potential contains the following linear terms:

$$V^0_{\text{linear}} = t^0_d \sigma^0_d + t^0_u \sigma^0_u + t^0_1 \tilde{v}^R_1 + t^0_2 \tilde{v}^R_2 + t^0_3 \tilde{v}^R_3 ,$$

(5)

where the different $t^0_i$ are the tadpoles at tree level, and are given by

$$t^0_d = (m_{H_d}^2 + \mu^2) v_d + v_d D - \mu (B v_u + v_i e_i) ,$$

$$t^0_u = -B u v_d + (m_{H_u}^2 + \mu^2) v_u - v_u D + v_i B e_i + v_u e^2 ,$$

$$t^0_1 = v_1 D + e_1 (-\mu v_d + v_u B_1 + v_i e_i) + \frac{1}{2} (v_i M_{L1}^2 + M_{L1}^2 v_i) ,$$

$$t^0_2 = v_2 D + e_2 (-\mu v_d + v_u B_2 + v_i e_i) + \frac{1}{2} (v_i M_{L2}^2 + M_{L2}^2 v_i) ,$$

$$t^0_3 = v_3 D + e_3 (-\mu v_d + v_u B_3 + v_i e_i) + \frac{1}{2} (v_i M_{L3}^2 + M_{L3}^2 v_i) .$$

(6)
where we have introduced the notation
\[ H_d = \begin{pmatrix} H_0^d \\ H_d^- \end{pmatrix}, \quad H_u = \begin{pmatrix} H_0^u \\ H_u^0 \end{pmatrix}, \quad \tilde{L}_i = \begin{pmatrix} \tilde{L}_0^i \\ \tilde{E}_i^- \end{pmatrix}, \]
(7)
and shifted the neutral fields with non-zero vevs as
\[ H_0^d \equiv \frac{1}{\sqrt{2}} [\sigma_0^d + v_d + i \phi_0^d], \quad H_0^u \equiv \frac{1}{\sqrt{2}} [\sigma_0^u + v_u + i \phi_0^u], \quad \tilde{L}_0^i \equiv \frac{1}{\sqrt{2}} [\tilde{\nu}_R^i + v_i + i \tilde{\nu}_I^i]. \]
(8)
The five vacuum expectation values can be expressed in spherical coordinates as
\[ v_d = v \sin \theta_1 \sin \theta_2 \sin \theta_3 \cos \beta, \]
\[ v_u = v \sin \theta_1 \sin \theta_2 \sin \theta_3 \sin \beta, \]
\[ v_3 = v \sin \theta_1 \sin \theta_2 \cos \theta_3, \]
\[ v_2 = v \sin \theta_1 \cos \theta_2, \]
\[ v_1 = v \cos \theta_1, \]
(9)
which preserves the MSSM definition \( \tan \beta = v_u/v_d \) with the \( W \) boson mass given as \( m_W^2 = \frac{1}{4} g^2 (v_2^2 + v_3^2 + v_4^2 + v_3^2) \). We have also defined \( D = \frac{1}{4} (g^2 + g'^2) (v_1^2 + v_2^2 + v_3^2 + v_d^2 - v_u^2) \) and \( \epsilon^2 = \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 \). A repeated index \( i \) in equation (6) implies summation over \( i = 1–3 \). The five tree-level tadpoles \( t_0^\alpha \) are equal to zero at the minimum of the tree-level potential, and from there one can determine the tree-level vevs.

2.2. Radiative breaking of the electroweak symmetry

A reliable description of electroweak symmetry breaking and Higgs boson physics in supersymmetry requires the inclusion of radiative corrections. In the BRpV model, the full-scalar potential at one-loop level, called effective potential, is
\[ V_{\text{total}} = V_{\text{total}}^0 + V_{\text{RC}}, \]
(10)
where \( V_{\text{total}}^0 \) is given in equation (4) and \( V_{\text{RC}} \) include the quantum corrections. Following Diaz et al [24] and Hirsch et al [26] we use the diagrammatic method, incorporating the radiative corrections through the one-loop corrected tadpole equations. The one-loop tadpoles are
\[ t_\alpha = t_\alpha^0 - \delta t_\alpha^{\text{DR}} + T_\alpha(Q) = t_\alpha^0 + \tilde{T}_\alpha^{\text{DR}}(Q), \]
(11)
where \( \alpha = d, u, 1, 2, 3 \) and \( \tilde{T}_\alpha^{\text{DR}}(Q) \equiv -\delta t_\alpha^{\text{MS}} + T_\alpha(Q) \) are the finite one-loop tadpoles. At the minimum of the potential we have \( t_\alpha = 0 \), and the vevs calculated from these equations are the renormalized vevs.

Neglecting inter-generational mixing in the soft masses, the five tadpole equations can be conveniently written in matrix form as
\[ [t_u^0, t_d^0, t_1^0, t_2^0, t_3^0]^T = M_{\text{lad}}^2 [v_u, v_d, v_1, v_2, v_3]^T, \]
(12)
where the matrix \( M_{\text{lad}}^2 \) is given in [26] and depends on the vevs only through the \( D \) term defined above.

New Journal of Physics 6 (2004) 76 (http://www.njp.org/)
In the MSSM limit, where $\epsilon_i = v_i = 0$, the angles $\theta_i$ are equal to $\pi/2$. In addition to the above MSSM parameters, our model contains nine new parameters, $\epsilon_i$, $v_i$ and $B_i$. Considering we have three tadpole equations one can take either the three $B_i$ as input and derive the three sneutrino vevs or vice versa, such that we have in total just six new parameters (compared with the MSSM).

To have approximate solutions for the tree-level vevs, consider the following rotation among the $H_d$ and lepton superfields:

$$M_{\text{lad}}^2 = RM_{\text{lad}}^2 R^{-1},$$

(13)

where the rotation $R$ can be split as

$$R = \begin{bmatrix} c_3 & 0 & 0 & -s_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_3 & 0 & 0 & c_3 \end{bmatrix} \times \begin{bmatrix} c_2 & 0 & 0 & -s_2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ s_2 & 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_1 & 0 & -s_1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ s_1 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

(14)

where the three angles are defined as

$$c_1 = \frac{\mu}{\mu'}, \quad s_1 = \frac{\epsilon_1}{\mu'}, \quad \mu' = \sqrt{\mu^2 + \epsilon_1^2},$$

$$c_2 = \frac{\mu'}{\mu''}, \quad s_2 = \frac{\epsilon_2}{\mu''}, \quad \mu'' = \sqrt{\mu'^2 + \epsilon_2^2},$$

$$c_3 = \frac{\mu''}{\mu'''}, \quad s_3 = \frac{\epsilon_3}{\mu'''}, \quad \mu''' = \sqrt{\mu''^2 + \epsilon_3^2}.$$

(15)

It is clear that this rotation $R$ leaves the $D$ term invariant. The rotated vevs are given by

$$[v'_u, v'_d, v'_1, v'_2, v'_3]^T = R[v_u, v_d, v_1, v_2, v_3]^T,$$

(16)

and under the assumption that $v'_1, v'_2, v'_3 \ll v$, these three small vevs have the approximate solution

$$v'_1 \approx -\frac{\mu \epsilon_1}{M_{L_1}^2 + D} \left[ \frac{m_{H_d}^2 - M_{L_1}^2}{\mu' \mu'''} v'_d + \frac{B_1 - B'}{\mu'} v'_u \right],$$

$$v'_2 \approx -\frac{\mu' \epsilon_2}{M_{L_2}^2 + D} \left[ \frac{m_{H_d}^2 - M_{L_2}^2}{\mu'' \mu'''} v'_d + \frac{B_2 - B''}{\mu''} v'_u \right],$$

$$v'_3 \approx -\frac{\mu'' \epsilon_3}{M_{L_3}^2 + D} \left[ \frac{m_{H_d}^2 - M_{L_3}^2}{\mu''' \mu'''} v'_d + \frac{B_3 - B'''}{\mu'''} v'_u \right],$$

(17)
where we have defined the following rotated soft terms:

\[
\begin{align*}
    m_{H_d}^2 &= \frac{m_{H_u}^2 \mu^2 + M_{L_1}^2 \epsilon_1^2}{\mu^2}, \\
    m_{H_d}'' &= \frac{m_{H_u}^2 \mu^2 + M_{L_2}^2 \epsilon_2^2}{\mu^2}, \\
    m_{H_d}''' &= \frac{m_{H_u}^2 \mu^2 + M_{L_3}^2 \epsilon_3^2}{\mu^2}, \\
    B' &= \frac{B \mu^2 + B_1 \epsilon_1^2}{\mu^2}, \\
    B'' &= \frac{B' \mu^2 + B_2 \epsilon_2^2}{\mu^2}, \\
    B''' &= \frac{B'' \mu^2 + B_3 \epsilon_3^2}{\mu^2}, \\
    M_{L_1}^2 &= \frac{m_{H_u}^2 \epsilon_1^2 + M_{L_1}^2 \mu^2}{\mu^2}, \\
    M_{L_2}^2 &= \frac{m_{H_u}^2 \epsilon_2^2 + M_{L_2}^2 \mu^2}{\mu^2}, \\
    M_{L_3}^2 &= \frac{m_{H_u}^2 \epsilon_3^2 + M_{L_3}^2 \mu^2}{\mu^2}.
\end{align*}
\]

As can be seen from equation (17), the approximation \( v'_1, v'_2, v'_3 \ll v \) is justified if either (i) \( \epsilon_i \ll \mu \) and/or (ii) \( (m_{H_u}^2 - M_{L_i}^2) / \mu^2 \ll 1 \) and \( (B_i - B) / \mu \ll 1 \). The latter holds automatically (to some extent) in many models of supersymmetry breaking as, for example, in minimal supergravity [23].

As in the MSSM, the electroweak symmetry is broken because the large value of the top quark Yukawa coupling drives the Higgs mass parameter \( m_{H_u}^2 \) to negative values at the weak scale via its RGE [47]. In the rotated basis, the parameter \( \mu''^2 \) is determined at one loop by

\[
    \mu''^2 = \frac{1}{2} \left[ m_Z^2 - \tilde{A}_{ZZ}(m_Z^2) \right] + \frac{(m_{H_d}^2 + \tilde{F}_{v_d}^{\text{DR}}) - (m_{H_u}^2 + \tilde{F}_{v_u}^{\text{DR}}) t_{\beta}^2}{t_{\beta}'^2 - 1},
\]

where \( t_{\beta}' = v'_d / v_d \) is defined in the rotated basis and is analogous to \( \tan \beta \) in equation (9) defined in the original basis. The finite \( \text{DR} \) Z-boson self-energy is \( \tilde{A}_{ZZ}(m_Z^2) \), and the one-loop tadpoles \( T_{v_d}^{\text{DR}} \) and \( T_{v_u}^{\text{DR}} \) are obtained by applying the rotation \( \mathbf{R} \) defined in equation (14) to the original tadpoles in equation (11). The radiative breaking of the electroweak symmetry is valid in the BRpV model in the usual way: the large value of the top quark Yukawa coupling drives the parameter \( m_{H_u}^2 \) to negative values, breaking the symmetry of the scalar potential.

2.3. Neutral fermion mass matrix

We consider the tree-level structure of the fermion mass matrices in this model. For a more complete discussion of different mass matrices in BRpV see the appendix in [26]. In the basis \( \psi^0 = (-i\lambda', -i\lambda^3, \tilde{H}_d^1, \tilde{H}_u^2, v_e, v_\mu, v_\tau) \) the neutral fermion mass matrix \( \mathbf{M}_N \) is given by

\[
    \mathbf{M}_N = \begin{bmatrix}
        \mathcal{M}_{\psi^0} & m^T \\
        m & 0
    \end{bmatrix},
\]

where

\[
    \mathcal{M}_{\psi^0} = \begin{bmatrix}
        M_1 & 0 & -\frac{1}{2} g' v_d & \frac{1}{2} g' v_u \\
        0 & M_2 & \frac{1}{2} g v_d & -\frac{1}{2} g v_u \\
        -\frac{1}{2} g' v_d & \frac{1}{2} g v_d & 0 & -\mu \\
        \frac{1}{2} g' v_u & -\frac{1}{2} g v_u & -\mu & 0
    \end{bmatrix},
\]
is the standard MSSM neutralino mass matrix \((M_2 \text{ and } M_1 \text{ are the SU}(2) \text{ and U}(1) \text{ gaugino soft masses})\) and

\[
m = \begin{pmatrix}
-\frac{1}{2} g' v_1 & \frac{1}{2} g v_1 & 0 & \epsilon_1 \\
-\frac{1}{2} g' v_2 & \frac{1}{2} g v_2 & 0 & \epsilon_2 \\
-\frac{1}{2} g' v_3 & \frac{1}{2} g v_3 & 0 & \epsilon_3
\end{pmatrix}
\] (22)

characterizes the breaking of \(R\)-parity. The full \(7 \times 7\) neutrino/neutralino mass matrix \(M_N\) is diagonalized as

\[
N^* M_N N^{-1} = \text{diag}(m_{\chi_0}^i, m_{\nu_j}),
\] (23)

where \(i = 1, \ldots, 4\) for the neutralinos and \(j = 1, \ldots, 3\) for the neutrinos,

\[
N^* M_{\nu} N^{-1} = M_{\nu}^{\text{diag}}
\] (24)

and the eigenvectors are given by

\[
F_{i0} = N_{ij} \psi_j
\] (25)

using the basis \(\psi = (-i \lambda', -i \lambda^3, \tilde{H}_d^1, \tilde{H}_u^2, \nu_e, \nu_\mu, \nu_\tau)\). As discussed in more detail below, to a very good approximation, the rotation matrix can be written as

\[
N^* \approx \begin{pmatrix}
N^* & N^* \xi^\dagger \\
-\xi^\dagger V^T & V^T
\end{pmatrix}.
\] (26)

Here, \(N\) is the rotation matrix that diagonalizes the \(4 \times 4\) MSSM neutralino mass matrix, \(V_\nu\) is the rotation matrix that diagonalizes the tree-level neutrino \(3 \times 3\) mass matrix and \(\xi_{ij} \ll 1\) are the relevant small expansion parameters which characterize the violation of \(R\)-parity and whose form will be given in section 3.

### 2.4. Charged fermion mass matrix

The chargino/lepton mass matrix is given by

\[
M_C = \begin{pmatrix}
M_2 & \frac{1}{\sqrt{2}} g v_d & 0 & 0 & 0 \\
\frac{1}{\sqrt{2}} g v_d & \mu & -\frac{1}{\sqrt{2}} (h_E)_{11} v_1 & -\frac{1}{\sqrt{2}} (h_E)_{22} v_2 & -\frac{1}{\sqrt{2}} (h_E)_{33} v_3 \\
\frac{1}{\sqrt{2}} g v_1 & -\epsilon_1 & \frac{1}{\sqrt{2}} (h_E)_{11} v_d & 0 & 0 \\
\frac{1}{\sqrt{2}} g v_2 & -\epsilon_2 & 0 & \frac{1}{\sqrt{2}} (h_E)_{22} v_d & 0 \\
\frac{1}{\sqrt{2}} g v_3 & -\epsilon_3 & 0 & 0 & \frac{1}{\sqrt{2}} (h_E)_{33} v_d
\end{pmatrix}.
\] (27)

We note that the chargino sector decouples from the lepton sector in the limit \(\epsilon_i = v_i = 0\). As in the MSSM, the chargino mass matrix is diagonalized by two rotation matrices \(U\) and \(V\)
defined by

\[ U^* M_F V^{-1} = M_{F^+}^{\text{diag}} \]  

(28)

with the eigenvectors satisfying

\[ F_{Ri}^+ = V_{ij} \psi_j^+ \quad \quad F_{Li}^- = V_{ij} \psi_j^- \]  

(29)

in the basis \( \psi^+ = (-i \lambda^+, \tilde{H}_2^1, e_R^+, \mu_R^+, \tau_R^+) \) and \( \psi^- = (-i \lambda^-, \tilde{H}_1^2, e_L^-, \mu_L^-, \tau_L^-) \), with the Dirac fermions being

\[ F_i^+ = \left( \begin{array}{c} F_{Ri}^+ \\ F_{Li}^- \end{array} \right). \]  

(30)

To first order in the \( R \)-parity violating parameters we have

\[ V \approx \left( \begin{array}{cc} V & V_L \xi_R^T \\ -V_R^T \xi_R & V_R^T \end{array} \right), \quad \quad U \approx \left( \begin{array}{cc} U & U_L \xi_L^T \\ -V_L^T \xi_L & V_L^T \end{array} \right), \]  

(31)

where \( V_{L,R}^T \) and \( V_{L,R}^* \) diagonalize the charged lepton mass matrix according to \( V_{L,R}^* M_{L,R} V_{L,R} = M_{L,R}^{\text{diag}} \). For most purposes it is sufficient to take \( \xi_R = 0_{2 \times 3} \), since it is smaller than \( \xi_L \), typically by a factor of \( m_1/m_{SUSY} \). Note that we can choose \( V_{L,R}^* = V_{L,R} = 1_{3 \times 3} \). We then have

\[ \xi_{1L}^I = a_{1L} \Delta_i, \quad \xi_{2L}^I = a_{2L} \Delta_i + be_i \]  

(32)

and

\[ a_{1L} = \frac{g}{\sqrt{2} \Delta_+}, \quad a_{2L} = \frac{g^2 v_u}{2 \mu \Delta_+}, \]  

(33)

where \( \Delta_+ \) is the determinant of the \( 2 \times 2 \) chargino mass matrix and

\[ \Delta_i = \mu v_i + v_d \epsilon_i \propto v'_i \]  

(34)

are the alignment parameters.

3. Tree-level neutrino mass: the atmospheric scale

The tree-level contribution to neutrino masses from broken \( R \)-parity supersymmetry has a long history [48]. Owing to the Super-K findings [1], we are interested only in the case where the neutrino mass, determined at the tree level, is small to account for the atmospheric neutrino data. The above form for \( M_N \) is especially convenient in this case to provide an approximate analytical discussion valid in the limit of small \( R_{vp} \) violation parameters. Indeed, in this case, we perform a perturbative diagonalization of the neutral mass matrix, defining

\[ \xi = m \cdot M_{\chi^0}^{-1}. \]  

(35)
Since the effective RpV parameters are smaller than the weak scale, we can work in a perturbative expansion defined by $\xi \ll 1$, where $\xi$ denotes a $3 \times 4$ matrix given as [49]

$$\begin{align*}
\xi_{i1} &= \frac{g' M_2 \mu}{2 \det(\mathcal{M}_{\tilde{c}})} \Lambda_i, \\
\xi_{i2} &= -\frac{g M_1 \mu}{2 \det(\mathcal{M}_{\tilde{c}})} \Lambda_i, \\
\xi_{i3} &= -\frac{\epsilon_i}{\mu} + \frac{(g^2 M_1 + g'^2 M_2) v_u}{4 \det(\mathcal{M}_{\tilde{c}})} \Lambda_i, \\
\xi_{i4} &= -\frac{(g^2 M_1 + g'^2 M_2) v_d}{4 \det(\mathcal{M}_{\tilde{c}})} \Lambda_i.
\end{align*}$$

(36)

From equations (36) and (34) one can see that $\xi = 0$ in the MSSM limit where $\epsilon_i = 0, v_i = 0$. If the elements of this matrix satisfy

$$\forall \xi_{ij} \ll 1,$$

(37)
then one can use it as expansion parameter to find an approximate solution for the mixing matrix $N$.

In leading order in $\xi$ the mixing matrix $N$ is given by

$$N^* = \left( \begin{array}{cc} N^* & 0 \\ 0 & V^T \end{array} \right) \left( \begin{array}{cc} 1 - \frac{1}{2} \xi \xi^\dagger & \xi^\dagger \\ -\xi & 1 - \frac{1}{2} \xi \xi^\dagger \end{array} \right).$$

(38)

The second matrix above block-diagonalizes the mass matrix $M_N$ approximately to the form

$$\text{diag}(\mathcal{M}_{\tilde{c}}, m_{\text{eff}}),$$

where

$$m_{\text{eff}} = -m \cdot \mathcal{M}_{\tilde{c}}^{-1} m^T = \frac{M_1 g^2 + M_2 g'^2}{4 \det(\mathcal{M}_{\tilde{c}})} \begin{pmatrix} \Lambda_\epsilon^2 & \Lambda_\epsilon \Lambda_\mu & \Lambda_\epsilon \Lambda_\tau \\ \Lambda_\epsilon \Lambda_\mu & \Lambda_\mu^2 & \Lambda_\mu \Lambda_\tau \\ \Lambda_\epsilon \Lambda_\tau & \Lambda_\mu \Lambda_\tau & \Lambda_\tau^2 \end{pmatrix}.$$  

(39)

The submatrices $N$ and $V_\nu$ diagonalize $\mathcal{M}_{\tilde{c}}$ and $m_{\text{eff}}$

$$N^* \mathcal{M}_{\tilde{c}} N^\dagger = \text{diag}(m_{\tilde{c}}),$$

(40)

$$V_\nu^T m_{\text{eff}} V_\nu = \text{diag}(0, 0, m_\nu),$$

(41)

where

$$m_\nu = \text{Tr}(m_{\text{eff}}) = \frac{M_1 g^2 + M_2 g'^2}{4 \det(\mathcal{M}_{\tilde{c}})} |\vec{\Lambda}|^2.$$  

(42)

The special form of the neutralino/neutrino mass matrix implies that the effective neutrino mass matrix $m_{\text{eff}}$ generated after diagonalizing out the heavy neutralinos has a projective form, a
Figure 1. $\Delta m^2_{\text{atm}}$ versus the BRpV alignment parameters.

feature common to many spontaneous $R$-parity violating models [48]. This implies that only one neutrino acquires a tree-level mass, the other two remaining massless [48]. As a result, at the tree approximation, one can rotate away one of the three angles in the matrix $V_\nu$, leading to

$$V_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & -\sin \theta_{23} \\ 0 & \sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \times \begin{pmatrix} \cos \theta_{13} & 0 & -\sin \theta_{13} \\ 0 & 1 & 0 \\ \sin \theta_{13} & 0 & \cos \theta_{13} \end{pmatrix},$$

(43)

where the mixing angles can be expressed in terms of the alignment vector $\Lambda_1$ as follows:

$$\tan \theta_{13} = -\frac{\Lambda_e}{(\Lambda_\mu^2 + \Lambda_\tau^2)^{1/2}}.$$  

(44)

$$\tan \theta_{23} = -\frac{\Lambda_\mu}{\Lambda_e}.$$  

(45)

The non-zero tree-level eigenvalue of the neutrino mass matrix is identified with the atmospheric mass scale. The calculated $\Delta m^2_{\text{atm}}$ can be expressed as a function of the alignment parameter $\Lambda_1$ (left panel in figure 1) or as function of $|\Lambda_1|/(\sqrt{M_{\mu} \mu})$ (right panel in figure 1), all of these expressed in GeV. The figure shows that equation (42) can be used to fix the relative size of $R$-parity breaking parameters to obtain the correct $\Delta m^2_{\text{atm}}$. On the other hand, as shown in figure 2, the atmospheric angle can be expressed in terms of $\Lambda_\mu/\Lambda_\tau$. Its maximality is obtained for $\Lambda_\mu \simeq \Lambda_\tau$ if $\Lambda_e$ is smaller than the other two. Let us stress once again that there is no solar mass splitting in the tree approximation so that, as a result, the ‘solar angle’ is not defined, as it can be rotated away by redefining the two degenerate neutrinos [14].

4. One-loop-induced neutrino mass: the solar scale

As we have just seen in the BRpV model the atmospheric mass scale and mixing arises at the tree level. We now discuss the determination of solar neutrino masses and mixings, which are both generated radiatively. One-loop corrections to the neutrino masses can be calculated numerically.
[26] or analytically [24]. Whereas the numerical approach can give ‘exact’ results (exact in the sense of being correct up to higher-order effects), the analytic approach, while being less accurate, gives a better understanding about which parameters control the loops and thus the solar neutrino mass and angles in our model. The discussion will therefore mainly concentrate on the analytical calculations.

In principle, to find the correct neutrino mixing angles one has to diagonalize the one-loop corrected neutralino/neutrino mass matrix. We define

\[ M_{ij}^{\text{pole}} = M_{ij}^{\text{DR}}(Q) + \Delta M_{ij}, \tag{46} \]

where \( M_{ij}^{\text{DR}}(Q) \) is the tree-level pole mass and \( \text{DR} \) indicates the dimensional reduction scheme we used in the numerical calculation. One-loop corrections are

\[ \Delta M_{ij} = \frac{1}{2} [\Pi_{ij}^V(m_i^2) + \bar{\Pi}_{ij}^V(m_j^2)] - \frac{1}{2} [m_{\chi i} \tilde{\Sigma}_{ij}^V(m_i^2) + m_{\chi j} \tilde{\Sigma}_{ij}^V(m_j^2)], \tag{47} \]

where the symmetrization is necessary to achieve gauge invariance and consistency with the Pauli principle. Here \( \Pi^V \) and \( \tilde{\Sigma}^V \) are the renormalized self-energies. They contain products of couplings and the usual Passarino–Veltman functions [50].

Diagonalizing the tree-level neutrino mass matrix first and adding then the one-loop corrections before re-diagonalization one finds that the resulting neutrino/neutralino mass matrix has non-zero entries in the neutrino/neutralino, the neutrino/neutralino and in the neutralino/neutralino sectors. We have found [24] that the most important part of the one-loop neutrino masses derives from the neutrino/neutralino sector and that the one-loop induced neutrino/neutralino mixing is usually negligible.

The relevant topologies for the one-loop calculation of neutrino masses are then illustrated in figure 3. Here our conventions are as follows: open circles with a cross inside indicate genuine mass insertions which flip chirality. On the other hand, open circles without a cross correspond to small \( R \)-parity violating projections, indicating how much of an Rp-even/odd mass eigenstate is present in a given Rp-odd/even weak eigenstate. In the actual numerical calculation, these projections really belong to the coupling matrices attached to the vertices. However, given the
smallness of Rp-violating effects, the prediagonalization ‘insertion method’ proves to be a rather useful tool to develop an analytical perturbative expansion and to acquire a simple understanding of the results.

These topologies have then to be ‘filled’ with all relevant combinations of particles/sparticles. Here we will concentrate on discussing the loop involving bottom/sbottom quarks, since (i) it is in large parts of parameter space the numerically most important one, and (ii) other loops can be calculated in a very similar way [24], although they are more complicated.

The relevant Feynman rules for the bottom–sbottom loops are, in the case of left sbottoms:

$$O_{Rij}^{b\tilde{b}} = R_{\tilde{b}j}^i g \frac{\sqrt{2}}{2} (N_{i2} - \frac{1}{3} \tan \theta_W N_{i1}) - R_{\tilde{b}j}^i h_b N_{i3}^*.$$

(48)

After approximating the rotation matrix $\mathcal{N}$ we find that expressions with the replacement $\mathcal{N} \to N$ are valid when the neutral fermion is a neutralino. When the neutral fermion $F^0$ is a neutrino, the following expressions hold:

$$O_{Lij}^{b\tilde{b}} \approx R_{\tilde{b}j}^i h_b (a_3 |\tilde{A}| \delta_{i3} + b \bar{\epsilon}_i) + R_{\tilde{b}j}^i \frac{2g}{3 \sqrt{2}} \tan \theta_W a_1 |\tilde{A}| \delta_{i3},$$

$$O_{Rij}^{b\tilde{b}} \approx R_{\tilde{b}j}^i \frac{g}{\sqrt{2}} (\frac{1}{3} \tan \theta_W a_1 - a_2) |\tilde{A}| \delta_{i3} + R_{\tilde{b}j}^i h_b (a_3 |\tilde{A}| \delta_{i3} + b \bar{\epsilon}_i),$$

(49)

where $i' = i - 4$ label one of the neutrinos. $R_{\tilde{b}j}^i$ are the rotation matrices connecting weak and mass eigenstate basis for the scalar bottom quarks. In case of no inter-generational mixing in the squark sector, $R_{\tilde{b}j}^i$ can be parametrized by just one diagonalizing angle $\theta_b$.
Putting these couplings together one finds the simplest contribution to the radiatively induced neutrino mass from loops involving bottom quarks and squarks [26]:

$$\tilde{\Pi}_{ij}(0) = -\frac{N_c}{16\pi^2} \sum_r (O_{Rjr}^{bnb} O_{Lir}^{bm} + O_{Ljr}^{bm} O_{Rir}^{bnb}) m_b B_0(0, m_b^2, m_r^2),$$

(50)

where $B_0(0, m_b^2, m_r^2)$ is a Passarino–Veltman function [50] and can be written as follows:

$$\tilde{\Pi}_{ij} = -\frac{N_c m_b}{16\pi^2} 2 s_b c_b h_b^2 \Delta B_0^{\tilde{b}_1\tilde{b}_2} \left( \frac{\tilde{\epsilon}_i \tilde{\epsilon}_j}{\mu^2} + a_3 b (\delta_{i3} \tilde{\epsilon}_j + \tilde{\epsilon}_j \delta_{i3}) |\tilde{\Lambda}| + \left( a_3^2 + \frac{a_L a_R}{h_b^2} \right) \delta_{i3} \delta_{j3} |\tilde{\Lambda}|^2 \right).$$

(51)

This expression is proportional to the difference of two $B_0$ functions,

$$\Delta B_0^{\tilde{b}_1\tilde{b}_2} = B_0(0, m_b^2, m_{\tilde{b}_1}^2) - B_0(0, m_b^2, m_{\tilde{b}_2}^2).$$

(52)

Parameters $\Lambda_i$ have been defined above. The $\tilde{\epsilon}$ parameters are defined as $\tilde{\epsilon}_i = (V^T \nu)^{ij} \epsilon_j$, and are given by

$$\tilde{\epsilon}_1 = \frac{\epsilon_3 (\Lambda_\mu^2 + \Lambda_\tau^2) - \Lambda_\epsilon (\Lambda_\mu \epsilon_3 + \Lambda_\tau \epsilon_\tau)}{\sqrt{\Lambda_\mu^2 + \Lambda_\tau^2 \sqrt{\Lambda_\epsilon^2 + \Lambda_\mu^2 + \Lambda_\tau^2}}},$$

$$\tilde{\epsilon}_2 = \frac{\Lambda_\epsilon \epsilon_\mu - \Lambda_\mu \epsilon_\tau}{\sqrt{\Lambda_\mu^2 + \Lambda_\tau^2}},$$

(53)

$$\tilde{\epsilon}_3 = \frac{\tilde{\Lambda} \cdot \tilde{\epsilon}}{\sqrt{\Lambda_\epsilon^2 + \Lambda_\mu^2 + \Lambda_\tau^2}}.$$  

On the other hand, $a_{L,R}$ are defined as

$$a_R = \frac{g}{\sqrt{2}} \left\{ \frac{1}{3} t_W a_1 - a_2 \right\}, \quad a_L = \frac{g}{\sqrt{2}} \frac{2}{3} t_W a_1.$$

(54)

The different terms in equation (51) can be understood as coming from the graphs corresponding to the first topology of figure 3. They have been depicted in more detail in figure 4, where we have adopted the following conventions: (i) as before, open circles correspond to small $R$-parity-violating projections, indicating how much of a weak eigenstate is present in a given mass eigenstate, (ii) full circles correspond to $R$-parity-conserving projections and (iii) open circles with a cross inside indicate genuine mass insertions which flip chirality.

The open and full circles should really appear at the vertices since the particles propagating in the loop are the mass eigenstates. We have however separated them to better identify the origin of the various terms. There is another set of graphs analogous to the previous ones, which corresponds to the heavy sbottom. They are obtained from the previous graphs making the replacement $\tilde{b}_1 \rightarrow \tilde{b}_2$, $s_{\tilde{b}} \rightarrow c_{\tilde{b}}$ and $c_{\tilde{b}} \rightarrow -s_{\tilde{b}}$. Note that for all contributions to the $2 \times 2$ sub-matrix corresponding to the light neutrinos the divergence from $B_0(0, m_b^2, m_{\tilde{b}_1}^2)$ is cancelled.
by the divergence from $B_0(0, m^2_{b_1}, m^2_{b_2})$, making the contribution from bottom–sbottom loops to this sub-matrix finite, as it should be, since the mass is fully ‘calculable’.

The second most important contribution to the radiatively induced neutrino mass usually comes from charged-scalar/charged-fermion loops [26]. Since all possible topologies of figure 3 contribute to this loop the structure of the contribution from charged Higgs/slepton loops is more complex than that of the bottom–sbottom loop considered above. However, the same topology as for the sbottom–bottom loop also contributes to the charged scalar loop. It leads to a final expression similar to equation (51), with appropriate replacements, which is good enough for an order-of-magnitude estimate of the charged scalar loop.

4.1. Results for the solar mass scale

We give a discussion of the analytical versus numerical results of the solar mass scale first. In figure 5 we show a comparison of approximate and exact calculation for two different numerical data sets. In both figures, we show the ratio of the approximate-over-exact solar neutrino mass
parameter $m_{\nu\text{Appr}}/m_{\nu\text{Exc}}$ versus $\Delta m_{\text{sol}}^2$ in eV$^2$, where $m_{\nu\text{Appr}}$ is the approximate loop calculation involving the bottom–sbottom and the charged scalar loop, whereas $m_{\nu\text{Exc}}$ is the exact numerical computation taking into account all loops. The set to the left called ‘Ntrl’ contains neutralinos being the LSP, whereas the set to the right (Stau) has the charged scalar tau as LSP.

We have found numerically that the terms proportional to $\tilde{\epsilon}_i \times \tilde{\epsilon}_j$ in the self-energies in equation (51) give the most important contribution to $m_{\nu_2}$ in the bottom–sbottom loop calculation in most points of our sets. If these terms are dominant, one can find a very simple approximation for the bottom–sbottom loop contribution to $m_{\nu_2}$. It is given by

$$m_{\nu_2} \simeq \frac{3}{16\pi^2} \sin(2\theta_{\tilde{b}}) m_b \Delta B_0^{b_2h_1} (\tilde{\epsilon}_1^2 + \tilde{\epsilon}_2^2) \mu^2.$$  \hspace{1cm} (55)

Equation (55) works surprisingly well for almost all points in our data sets.

The more complicated structure of the charged scalar loop makes it difficult to give a simple equation for $m_{\nu_2}$, similar to equation (55) for the bottom–sbottom loop. However, we note that equation (55), with appropriate replacements, allows us to estimate the typical contributions to the charged scalar loop within a factor of $\sim 3$. However, such an estimate will be biased towards too small or too large $m_{\nu_2}$ depending mainly on which SUSY particle is the LSP [24].

4.2. Analytical approximation for the solar mixing angle

In the basis where the tree-level neutrino mass matrix is diagonal the mass matrix at one-loop level can be written as

$$\tilde{m}_\nu = V^{(0)}_{\nu}^T m_\nu V^{(0)} = \begin{pmatrix} c_1 \tilde{\epsilon}_1 \tilde{\epsilon}_1 & c_1 \tilde{\epsilon}_1 \tilde{\epsilon}_2 & c_1 \tilde{\epsilon}_1 \tilde{\epsilon}_3 \\ c_1 \tilde{\epsilon}_2 \tilde{\epsilon}_1 & c_1 \tilde{\epsilon}_2 \tilde{\epsilon}_2 & c_1 \tilde{\epsilon}_2 \tilde{\epsilon}_3 \\ c_1 \tilde{\epsilon}_3 \tilde{\epsilon}_1 & c_1 \tilde{\epsilon}_3 \tilde{\epsilon}_2 & c_0 |\tilde{\Lambda}|^2 + c_1 \tilde{\epsilon}_3 \tilde{\epsilon}_3 \end{pmatrix} + \cdots,$$  \hspace{1cm} (56)

where the $\tilde{\epsilon}_i$ were defined before in equation (53). Coefficients $c_0$ and $c_1$ contain couplings and supersymmetric masses. Since they cancel in the final expression for the angle their exact definition is not necessary in the following. Dots stand for other terms which we will assume to be less important in the following. This matrix can be diagonalized approximately taking in account that

$$x \equiv \frac{c_1 |\tilde{\epsilon}|^2}{c_0 |\tilde{\Lambda}|^2} \ll 1.$$  \hspace{1cm} (57)

Then

$$\tilde{m}_\nu = c_0 |\tilde{\Lambda}|^2 \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & 1 + x \end{pmatrix}.$$  \hspace{1cm} (58)
Figure 6. \(\frac{\tan^2 \theta_{\text{sol}}^{\text{Appr}}}{\tan^2 \theta_{\text{sol}}^{\text{Exact}}}\) versus \(\tan^2 \theta_{\text{sol}}^{\text{Exact}}\). In the left panel, the darker region contains over 90% of the points in our sample; in the right panel the points in the region shown satisfy the cut \(\sin(2\theta_0)\Delta B_0^{\tau\tau_1} > 0.02\).

The rotation matrix that diagonalizes \(\tilde{m}_\nu\) in equation (58) can be written as

\[
\tilde{V}_\nu^T \tilde{m}_\nu \tilde{V}_\nu = \text{diag}(m_1, m_2, m_3),
\]

where

\[
\tilde{V}_\nu^T = \begin{pmatrix}
e_{1,1} & e_{1,2} & e_{1,3} \\
e_{2,1} & e_{2,2} & e_{2,3} \\
e_{3,1} & e_{3,2} & e_{3,3}
\end{pmatrix}.
\]

The lepton mixing matrix is then given by

\[
U = (V_\nu^T \tilde{V}_\nu^T)^T.
\]

The expression for the solar mixing angle can be obtained from

\[
\tan^2 \theta_{\text{sol}} = \frac{U_{e_2}^2}{U_{e_1}^2}.
\]

From the above equations we obtain the very simple expression for the solar mixing angle,

\[
\tan^2 \theta_{\text{sol}} = \frac{\epsilon_1^2}{\epsilon_2^2}.
\]

This formula is a very good approximation if the one-loop matrix has the structure \(\epsilon_i \times \epsilon_j\), as is the case of the bottom–sbottom loop if \(m_{\nu_3} \gg m_{\nu_2}\), as illustrated in figure 6.
In the left panel, we show a calculation comparing for all points the approximate to the exact solar angle in the set with neutralino LSP, whereas the right panel shows a subset of points with the cut $\sin(2\theta_b) \Delta B^0_\ell \tilde{\tau}_i > 0.02$. Note that this cut is designed so as to favour points in which there is a sizeable bottom–bottom loop contribution to the full one-loop neutrino mass. One sees from the right panel that for this case the true solar angle is well approximated by our analytical formula. Note finally that equation (63) will fail completely if $\Lambda_\mu \equiv \Lambda_\tau$ and $\epsilon_\mu \equiv \epsilon_\tau$, since then $\tilde{\epsilon}_2^2 = 0$; see equation (53). This is the origin of the ‘sign condition’ discussed in [26].

5. Testing neutrino properties at high-energy accelerators

Since $R$-parity is broken in our model, the lightest supersymmetric particle is unstable and decays. This leads to the exciting possibility to test the bilinear model at future colliders, such as the LHC or a possible Linear Collider.

The principal idea of such a collider test [39, 41, 43] is easily understood: bilinear $R$-parity breaking leads to mixing between particles and sparticles with the same quantum numbers, as discussed above extensively for the case of neutrinos/neutralinos. This mixing, however, is not arbitrarily different for each particle/sparticle species. In fact, the bilinear model has just six new parameters, which we choose to be $\epsilon_i$ and $\Lambda_1i$, compared with the MSSM. Essentially five of these six can be fixed from neutrino physics.

Thus, if the MSSM parameters were known, all mixing effects could be calculated and thus all decay properties of the LSP would be fixed—apart from the effects of the last unknown parameter. In reality, however, the MSSM soft SUSY breaking parameters are completely unknown. The approach taken in [39, 41, 43] therefore is to calculate ratios of branching ratios of different decays. By taking ratios, one essentially scales out the unknown MSSM parameters approximately and obtains observables which are proportional to either $\Lambda_i/\Lambda_j$ or $\epsilon_i/\epsilon_j$ (or some weird combination thereof). The ratio one measures depends of course on the final state and the LSP under consideration. Since ratios of $\Lambda_i$’s (or $\epsilon_i$’s) are correlated with the neutrino angles, as discussed above, fixing neutrino angles from experimental data therefore gives definite predictions for some ratios of branching ratios.

One example is shown in figure 7, where ratio of branching ratios of neutralino LSP decays are plotted. Note that $\text{Br}(\mu qq')/\text{Br}(\tau qq')$ is directly proportional to $\tan^2(\theta_{\text{am}})$, i.e. should be near $\sim 1$ according to current neutrino data. The spread in the points is due to the unknown MSSM parameters. Of course, once SUSY is discovered these unknowns will be measured allowing for much sharper tests of the model than that indicated in figure 7.

A second example is shown in figure 8, where we show $\text{Br}(\tilde{\tau}_1 \rightarrow \sum \nu \ell)/\text{Br}(\tilde{\tau}_1 \rightarrow \sum \nu \mu)$ as a function of $(\epsilon_1/\epsilon_2)^2$ (left panel) and as a function of $\tan^2(\theta_{\odot})$ (right panel). Obviously this ratio is strongly correlated with the solar angle and, thus, if scalar taus turn out to be the LSP, such a measurement would provide an excellent check of the bilinear model as the origin of the solar neutrino mass scale.

With the LSP unstable, in principle, any sparticle can be the LSP. In [43] the remaining candidates have been discussed: charginos, scalar quarks, gluinos and scalar neutrinos. The main conclusion in [43] is that whichever SUSY particle is the LSP, measurements of branching ratios at future accelerators will provide a definite test of bilinear $R$-parity breaking as the model of neutrino mass. We just mention that chargino LSPs would be more sensitive to atmospheric
neutrino physics (as are neutralinos), whereas the other LSP candidates mentioned above show more dependence on the solar neutrino angle.

6. Discussion and conclusions

We have presented a brief review of the idea that supersymmetry with explicit bilinear breaking of $R$-parity is the origin of neutrino masses and lepton mixing. The bilinear $R$-parity breaking
(BRpV) model is the simplest extension of the minimal supersymmetric standard model (MSSM) which includes lepton number violation. We have seen how it leads to a successful phenomenological model for neutrino oscillations, in accordance to present neutrino data. The pattern of neutrino masses is hierarchical, with the atmospheric mass scale arising at the tree level, whereas the solar scale is induced from calculable loop corrections. We have seen how, in contrast with see-saw models, the BRpV model can be probed at future collider experiments, such as the LHC or the NLC. Indeed, we have discussed how, irrespective of the supersymmetric particle which is the lightest, its decay pattern will be directly related to the lepton mixing angles determined in low-energy neutrino experiments.

Acknowledgments

This work was supported by Spanish grant BFM2002-00345, by the European Commission RTN grant HPRN-CT-2000-00148 and the ESF Neutrino Astrophysics Network. MH is supported by a Ramon y Cajal contract.

References

[1] Super-Kamiokande, Fukuda Y et al 1988 Phys. Rev. Lett. 81 1562 (Preprint hep-ex/9807003)
[2] SNO, Ahmad Q R et al 2002 Phys. Rev. Lett. 89 011301 (Preprint nucl-ex/0204008)
[3] KamLAND, Eguchi K et al 2003 Phys. Rev. Lett. 90 021802 (Preprint hep-ex/0212021)
[4] Maltoni M, Schwetz T, Tortola M A and Valle J W F 2004 this volume (Preprint hep-ph/0405172)
[5] Pakvasa S and Valle J W F 2004 Proc. of the Indian National Academy of Sciences on Neutrinos, Part A ed D Indumathi, M V N Murthy and G Rajasekaran, vol 70A, pp 189–222
[6] Barger V, Marfatia D and Whisnant K 2003 Int. J. Mod. Phys. E 12 569
[7] Gonzalez-Garcia M C and Nir Y 2003 Rev. Mod. Phys. 75 34
[8] Fogli G L et al 2003 Talk at 10th Int. Workshop on Neutrino Telescopes, Venice, Italy, 11–14 March 2003 Neutrino Telescopes, pp 151–179
[9] Nilles H P 1984 Phys. Rep. 110 1
[10] Haber H E and Kane G L 1985 Phys. Rep. 117 75
[11] Witten E 2002 Preprint hep-ph/0207124
[12] Gell-Mann M, Ramond P and Slansky R 1979 Print-80-0576 (CERN)
[13] Yanagida T 1979 KEK Lectures ed O Sawada and A Sugamoto (KEK)
[14] Schechter J and Valle J W F 1980 Phys. Rev. D 22 2227
[15] Mohapatra R N and Senjanovic G 1981 Phys. Rev. D 23 165
[16] Aulakh C S and Mohapatra R N 1982 Phys. Lett. B 119 13
[17] Ross G G and Valle J W F 1985 Phys. Lett. B 151 375
[18] Ellis J R et al 1985 Phys. Lett. B 150 142
[19] Abada A, Davidson S and Losada M 2002 Phys. Rev. D 65 075010 (Preprint hep-ph/0111332)
[20] Grossman Y and Haber H E 1999 Preprint hep-ph/9906310
[21] Bednyakov V, Faessler A and Kovalenko S 1998 Phys. Lett. B 442 203 (Preprint hep-ph/9808224)
[22] Joshipura A S, Vaidya R D and Vempati S K 2002 Nucl. Phys. B 639 290 (Preprint hep-ph/0203182)
[23] Chang C H and Feng T F 2000 Eur. Phys. J. C 12 137 (Preprint hep-ph/9901260)
[24] Diaz M A, Romao J C and Valle J W F 1998 Nucl. Phys. B 524 23 (Preprint hep-ph/9706315)
[25] Chun E J, Jung D-W and Park J D 2003 Phys. Lett. B 557 233 (Preprint hep-ph/0211310)

New Journal of Physics 6 (2004) 76 (http://www.njp.org/)
[26] Hirsch M et al 2000 Phys. Rev. D 62 113008 (Preprint hep-ph/0004115)
    Hirsch M et al 2002 Phys. Rev. D 65 119901
[27] Romao J C et al 2000 Phys. Rev. D 61 071703 (Preprint hep-ph/9907499)
[28] de Campos F et al 1995 Nucl. Phys. B 451 3 (Preprint hep-ph/9502237)
[29] Banks T, Grossman Y, Nardi E and Nir Y 1995 Phys. Rev. D 52 5319 (Preprint hep-ph/9505248)
[30] de Carlos B and White P L 1996 Phys. Rev. D 54 3427 (Preprint hep-ph/9602381)
[31] Akeroyd A G et al 1998 Nucl. Phys. B 529 3 (Preprint hep-ph/9707395)
[32] Masiero A and Valle J W F 1990 Phys. Lett. B 251 273
[33] Romao J C, Santos C A and Valle J W F 1992 Phys. Lett. B 288 311
[34] Romao J C, Ioannisian A and Valle J W F 1997 Phys. Rev. D 55 427 (Preprint hep-ph/9607401)
[35] Mukhopadhyaya B, Roy S and Vissani F 1998 Phys. Lett. B 443 191 (Preprint hep-ph/9808265)
[36] R-Parity Working Group, Allanach B et al, 1999 Preprint hep-ph/9906224
[37] Choi S Y, Chun E J, Kang S K and Lee J S 1999 Phys. Rev. D 60 075002 (Preprint hep-ph/9903465)
[38] Bartl A et al 2001 Nucl. Phys. B 600 39 (Preprint hep-ph/0007157)
[39] Porod W, Hirsch M, Romao J and Valle J W F 2001 Phys. Rev. D 63 115004 (Preprint hep-ph/00111248)
[40] Restrepo D, Porod W and Valle J W F 2001 Phys. Rev. D 64 055011 (Preprint hep-ph/0104040)
[41] Hirsch M, Porod W, Romao J C and Valle J W F 2002 Phys. Rev. D 66 095006 (Preprint hep-ph/0207334)
[42] Bartl A et al 2003 JHEP 11 005 (Preprint hep-ph/0306071)
[43] Hirsch M and Porod W 2003 Phys. Rev. D 68 115007 (Preprint hep-ph/0307364)
[44] Giudice G F and Masiero A 1988 Phys. Lett. B 206 480
[45] Nilles H-P and Polonsky N 1997 Nucl. Phys. B 484 33 (Preprint hep-ph/9606388)
[46] Mira J M, Nardi E, Restrepo D A and Valle J W F 2000 Phys. Lett. B 492 81 (Preprint hep-ph/0007266)
[47] Ibanez L E and Ross G G 1982 Phys. Lett. B 110 215
[48] Santamaria A and Valle J W F 1987 Phys. Lett. B 195 423
    Santamaria A and Valle J W F 1988 Phys. Rev. Lett. 60 397
    Santamaria A and Valle J W F 1989 Phys. Rev. D 39 1780
[49] Joshipura A S and Nowakowski M 1995 Phys. Rev. D 51 2421 (Preprint hep-ph/9408224)
    Nowakowski M and Pilaftsis A 1996 Nucl. Phys. B 461 19 (Preprint hep-ph/9508271)
    Hirsch M and Valle J W F 1999 Nucl. Phys. B 557 60 (Preprint hep-ph/9812463)
[50] Passarino G and Veltman M J G 1979 Nucl. Phys. B 160 151