A Monte-Carlo Implementation of the SAGE Algorithm for Joint Soft Multiuser and Channel Parameter Estimation

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Abstract—An efficient, joint transmission delay and channel parameter estimation algorithm is proposed for uplink asynchronous direct-sequence code-division multiple access (DS-CDMA) systems based on the space-alternating generalized expectation maximization (SAGE) algorithm. The marginal likelihood of the unknown parameters, averaged over the data sequence, as well as the expectation and maximization steps of the SAGE algorithm are derived analytically. To implement the proposed algorithm, a Markov Chain Monte Carlo (MCMC) technique, called Gibbs sampling, is employed to compute the a posteriori probabilities of data symbols in a computationally efficient way. Computer simulations show that the proposed algorithm has excellent estimation performance. This so-called MCMC-SAGE receiver is guaranteed to converge in likelihood.

Index Terms— Asynchronous DS-CDMA, space-alternating generalized expectation maximization (SAGE), Markov Chain Monte Carlo (MCMC), Gibbs sampling.

I. INTRODUCTION

The performance of direct-sequence code-division multiple-access (DS-CDMA) transmission over mobile fading channels depends strongly on the reliability of channel parameter and quality of synchronization for each user: state-of-the-art detection algorithms that exploit multiple-access-interference and inter-symbol-interference require very powerful estimation algorithms.

Substantial amount of relevant references appeared in the literature on delay estimation. Namely, a new prospective is presented in [1] for the maximum likelihood (ML) time-delay estimation. Code timing estimation in a near-far environment for DS-CDMA systems was introduced in [2]. Joint symbol detection, time-delay and channel parameter estimation problems for asynchronous DS-CDMA systems have been investigated in several previous works (e.g., [3], [4]). Most of these works either work on one signal at a time and treat the other signals as interference, or employ a training sequence to obtain a coarse estimate of the channel parameters which is consequently used to detect data. It is clear that these approaches have disadvantages of having higher overhead and additional noise enhancement.

Some other proposed approaches for joint blind multiuser detection and channel estimation for DS-CDMA systems are subspace-based and linear prediction-based methods. Subspace-based method usually require singular value decomposition or eigenvalue decomposition which is computationally costly and does not tolerate mismatched channel parameters. Another drawback of this approach is that accurate rank determination may be difficult in a noisy environment [5], [6]. Moreover, it is not clear how these methods can be extended to include the estimation of the transmission delays jointly with the channel parameters.

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The expectation maximization (EM) and space alternating EM (SAGE) algorithms are ideally suited to these kind of problems as they are guaranteed to converge in likelihood. Earlier work related with delay estimation based on the EM algorithm has appeared [7], [8]. Efficient iterative receiver structures are presented in [9], [10], performing joint multiuser detection and channel estimation for synchronous as well as asynchronous coded DS-CDMA systems operating over quasi-static flat Rayleigh fading channels, under the assumption that the transmissions delays are known. The Bayesian EM/SAGE algorithm can be used for joint soft-data estimation and channel estimation but the computational complexity of the resulting receiver architecture is non-polynomial in the number of users [11]. To overcome this draw-back, Hu et al. applied the Variational Bayesian EM/SAGE algorithms to joint estimation of the distributions for channel coefficients, noise variance, and information symbols for synchronous DS-CDMA in [12]. Our work may be considered to be a twofold extension of the work by Gallo et al. in [11]: First, the proposed receiver performs joint channel coefficient and transmission delay estimation within the SAGE framework. Secondly, the implication of the Monte-Carlo method in the SAGE framework makes it possible to compute soft-data estimates for all users at polynomial computational complexity, as well. Here, an efficient Markov chain Monte Carlo (MCMC) technique [13] called Gibbs sampling is used to compute the a posteriori probabilities (APP) of data symbols [14]. The APP’s can be computed exactly with the MCMC algorithm, which is significantly less complex than a standard hidden Markov model approach. The resulting receiver architecture works in principal fully blind and is guaranteed to converge. For uncoded transmission, a few pilot bits must be inserted, though, to resolve the phase ambiguity problem.

The theoretical framework for the joint transmission delays and channel estimation as well as the data detection algorithms can easily be extended to coded transmission.

II. SYSTEM DESCRIPTION

We consider an asynchronous single-rate DS-CDMA system with $K$ active users using binary phase shift keying (BPSK) modulation sharing the same propagation channel. The signal transmitted by each user experiences flat Rayleigh fading, which is assumed to be constant over the observation frame of $L$ data symbols. Each user employs a random signature waveform for transmitting symbols of duration $T_b$, such that each symbol consists of $N_c$ chips with duration $T_c = T_b/N_c$, where $N_c$ is an integer. The received signal is the noisy sum of all user’s contribution, delayed by the propagation delays $\tau_k \in [0, T_s/2)$, where the subscript $k$ denotes the label of the $k$th user.

After down-converting the received signal to baseband and passing it through an integrate-and-dump filter with integration time $T_s = T_c/Q$, $Q \in \mathbb{Q}^+$, $Q N_c (L + 1)$ samples over an observation frame of $L$ symbols are stacked into a signal column vector $r \in \mathbb{C}^{Q N_c (L + 1)}$. Note that sampling is chip-synchronous without knowledge of the
individual transmission delays. It can therefore be expressed as
\[ r = S(\tau) A d + w. \]  
(1)

In this expression the matrix \( S(\tau) \in \mathbb{C}^{Q N_c (L+1) - 1 \times L K} \) contains the signature sequences of all the users
\[ S(\tau) = [ S_1(\tau_1), S_2(\tau_2), \ldots, S_K(\tau_K) ] \]
where \( S_k(\tau_k) \in \mathbb{C}^{Q N_c (L+1) - 1 \times L} \) has the form
\[ S_k(\tau_k) = \begin{bmatrix} S_k(\tau_k, 0) & S_k(\tau_k, 1) & \cdots & S_k(\tau_k, L - 1) \end{bmatrix} \]
and the spreading code vector \( S_k(\tau_k, \ell) \in \mathbb{C}^{Q N_c (L+1) - 1 \times 1} \) is given by
\[ S_k(\tau_k, \ell) = \begin{bmatrix} 0_{Q N_c, \ell + \tau_k - \ell, 1} & s_k(\tau_k, \ell) \end{bmatrix}. \]
The vector \( s_k(\tau_k, \ell) \) contains the spreading code of user \( k \) having support \( [\ell N_c T_c, (\ell + 1) N_c T_c] \) with energy \( s_k^2(\tau_k, s_k(\tau_k, \ell)) = 1 \). Finally, \( 0_{M \times 1} \) denotes the \( M \times 1 \)-dim. all-zero column vector.

The block diagonal channel matrix \( A \in \mathbb{C}^{L K \times L K} \) in (1) is given by \( A = \text{diag}\{A_1, \ldots, A_K\} \). The channel matrix for user \( k \), \( A_k \in \mathbb{C}^{L \times L} \), is given by \( A_k = I_L \otimes a_k \) where \( I_L \) is the \( L \)-dim. identity matrix, and the symbol \( \otimes \) denotes the Kronecker product. The \( k \)th user’s channel coefficient \( a_k \) is a circularly symmetric complex Gaussian random variable with zero mean and variance \( \sigma_k^2 \). The \( k \)th user’s transmission delay is assumed to be uniformly distributed.

The symbol vector \( d \in \mathbb{C}^{L K} \) takes the form \( d = \{ d_1, \ldots, d_K \} \) where the vector \( d_k \in \mathbb{C}^{L} \) contains the \( k \)th user’s symbol vector, i.e. \( \{ a_k(0), \ldots, a_k(L-1) \} \) with \( d_k(\ell) \in \{-1, +1\} \) denoting the symbol transmitted by the \( k \)th user during the \( \ell \)th signalling interval. Finally, the column vector \( w \in \mathbb{C}^{Q N_c (L+1) - 1} \) contains complex, circularly symmetric white Gaussian noise having covariance matrix \( N_0 I \). We assume that the vectors \( a \triangleq \{ a_1, a_2, \ldots, a_K \} \), \( \tau \triangleq \{ \tau_1, \tau_2, \ldots, \tau_K \} \), \( d \) and \( w \) and their components are independent.

The receiver does not know the data sequences, the (complex) channel coefficients, or the transmission delays.

III. MONTE-CARLO SAGE JOINT PARAMETER ESTIMATION

A. The SAGE Algorithm

In previous applications, the SAGE algorithm [15] has been extensively used to iteratively approximate the ML/MAP estimate of a parameter vector \( \theta \) with respect to the observed data \( r \). To obtain a receiver architecture that iterates between soft-data and channel estimation, one might choose the parameter vector as \( \theta = \{ \theta_1, \ldots, \theta_K \} \) and encode the parameter vector as \( \bar{\theta}_k = \theta_k \). \( \theta_k \) are kept fixed. In the SAGE framework \( r \) is referred to as the incomplete data. The so-called admissible hidden data \( x_k \) with respect to \( \theta \) is selected to be \( x_k = \{ r, d \} \). Notice that \( x_k \) can only be partially observed. Applying the SAGE algorithm to MAP parameter estimation, yields the expectation (E)-step
\[ Q_k(\theta_k, \bar{\theta}_k) = E_d \left\{ \log p \left( r, d, a_k, \tau_k, a_k^{[\bar{\theta}_k]}, \tau_k^{[\bar{\theta}_k]} \right) \mid r, \bar{\theta}_k \right\}. \]  
(2)
The maximization (M)-step computes a value of the argument \( \tau_k \) in (2) to obtain the update \( \bar{\theta}_k \). The objective function is non-decreasing at each iteration.

B. The Monte-Carlo SAGE algorithm

We will see that direct computation of the expectation in (2) requires a non-polynomial number of operations in the number of users \( K \) and thus becomes prohibitive with increasing \( K \). To make the computation of the expectation in (2) feasible though, we propose to use the technique of Markov chain Monte Carlo (MCMC) to obtain the Monte-Carlo SAGE algorithm. MCMC is a statistical technique that allows generation of ergodic pseudo-random samples \( d_k^{[\bar{\theta}_k]}, \ldots, d_k^{[\bar{\theta}_k]} \) from the current approximation to the conditional pdf \( p(d|r, \theta_k) \). These samples are used to approximate the expectation in (2) by the sample-mean. The Gibbs sampler and the Metropolis-Hastings algorithm are widely used MCMC algorithms. Here we describe only the Gibbs sampler [16], [14], as it is the most commonly used in applications. With initialized \( d_0^{[\bar{\theta}_k]} \) randomly, the Gibbs sampler iterates the following loop at SAGE iteration \( i \):

- Draw sample \( d_i^{[\bar{\theta}_k]} \) from \( p(d_i|d_{i-1}^{[\bar{\theta}_k]}, \ldots, d_{i-1}^{[\bar{\theta}_k]}, r, \theta_k) \)
- Draw sample \( d_i^{[\bar{\theta}_k]} \) from \( p(d_i|d_{i-1}^{[\bar{\theta}_k]}, \ldots, d_{i-1}^{[\bar{\theta}_k]}, r, \theta_k) \)
- Draw sample \( d_i^{[\bar{\theta}_k]} \) from \( p(d_i|d_{i-1}^{[\bar{\theta}_k]}, \ldots, d_{i-1}^{[\bar{\theta}_k]}, r, \theta_k) \)

Following this approach, we have
\[ Q_k(\theta_k, \theta_k) = \frac{1}{N_i} \sum_{i=1}^{N_i} \left\{ \log p \left( r, d^{[\bar{\theta}_k]}, a_k, \tau_k, a_k^{[\bar{\theta}_k]}, \tau_k^{[\bar{\theta}_k]} \right) \right\}. \]

Notice that with increasing \( N_i \), the Monte-Carlo SAGE algorithm converges to the MAP solution \( \theta = \theta^* \) up to random fluctuations around \( \theta^* \) [17].

C. Receiver design

This subsection is devoted to the derivation of a receiver architecture for joint estimation of parameters within the Monte-Carlo SAGE framework. Discarding terms independent of \( a \) and \( \tau \), we obtain
\[ \log p(r, a, \tau) = \log p(r|d, a, \tau) + \log p(d) + \log p(a) + \log p(\tau). \]  
(3)
From (1), it follows that
\[ \log p(r|a, \tau, d) \propto \text{Re}\{r^H S A d\} = \frac{1}{2} \mu(\theta, d)^H \mu(\theta, d), \]  
(4)
where \( \mu(\theta, d) \triangleq \sum_{k=1}^{K} \sum_{\ell=0}^{L-1} S_k(\ell, \tau_k) a_k d_k(\ell) \) and \( \cdot^H \) is the conjugate transpose of the argument.

1) The E-step: Substituting (3) into (4) yields after some algebraic manipulations for the E-step of the Monte-Carlo SAGE algorithm:
\[ Q_k(\theta_k|\theta_k^{[\bar{\theta}_k]}) = \frac{2}{N_0} \sum_{\ell=0}^{L-1} \text{Re}\{a_k^H \Psi(\ell, \tau_k)\} - \frac{L}{N_0 \sigma_k^2} \left| a_k \right|^2 - \frac{1}{\sigma_k^2} \left| a_k \right|^2 \]  
(5)
with the branch definition
\[ \Psi(\ell, \tau_k) = S_k^H(\ell, \tau_k) \left( a_k^H(\ell) r - T_k^{[\bar{\theta}_k]}(\ell) \right). \]
and the interference term
\[ \mathcal{I}_k^i(\ell) \triangleq \sum_{k' \neq k} c_k^{i[i]} \left( S_{k'}(\ell + 1, \tau_{k'}^{i[i]}) (d_k(\ell)d_{k'}(\ell + 1))^2 \right) + S_{k'}(\ell, \tau_{k'}^{i[i]}) \left( d_k(\ell)d_{k'}(\ell) \right)^2 + S_{k'}(\ell - 1, \tau_{k'}^{i[i]}) \left( d_k(\ell)d_{k'}(\ell - 1) \right)^2. \]

Moreover,
\[ \bar{d}_k^{i[i]}(\ell) \triangleq \sum_{m \in \mathcal{S}} m P(d_k(\ell) = m | r, \tau^{i[i]}, a^{i[i]}). \]

and
\[ \left( d_k(\ell)d_{k'}(\ell') \right)^i \triangleq \sum_{m \in \mathcal{S}} \sum_{n \in \mathcal{S}} m n \times P(d_k(\ell) = m, d_{k'}(\ell') = n | r, \tau^{i[i]}, a^{i[i]}), \quad \text{for } k' \neq k. \]

Moreover,
\[ \bar{d}_{k}^{i[i]}(\ell) \triangleq \sum_{m \in \mathcal{S}} m P(d_k(\ell) = m | r, \tau^{i[i]}, a^{i[i]}). \]

(6)

From (1), we have \( p(r | D) \sim \exp(-\frac{1}{N_0} |r - G d|^2), \) with \( G \triangleq S(\tau) A \) and \( d = \{d_1, d_2, \ldots, d_K\}. \) After some algebra (11) can be expressed as
\[ \lambda^{i[i]} \triangleq \frac{4}{N_0} \Re \left\{ (g_q^i) \bar{d}_{k}(\ell) - (r - G_{q}^{i[i]} d_{k}^{i[i]}) \right\}, \]

(12)

where \( q \triangleq kL + \ell, \) and \( G_q \) is \( G \) with its \( q \)th column \( g_q \) removed. Similarly, \( d_q \) denotes the vector \( d \) with its \( q \)th component removed.

In summary, for each \( k = 1, 2, \ldots, K \) and \( \ell = 0, 1, \ldots, L - 1, \) to estimate the \textit{a posteriori} probabilities \( P(d_k(\ell)|r, \tau^{i[i]}, a^{i[i]} \in \mathcal{S})) \) in (9), the Gibbs sampler runs over all symbols \( N_t \) times to generate a collection of vectors \( \{\bar{d}_{k}^{i[i]}(\ell)\}_{t=1}^{N_t} \) which are used in (12) to estimate the desired quantities.

B. Computation of the soft-value for the product of two data symbols in (12)

Similarly, a number of random samples \( \tilde{d}_{k}^{i[i]}(\ell) \triangleq \bar{d}_{k}^{i[i]}(\ell) + \epsilon_{k}, \ell \in (-1.0, +1) \) are drawn, using the Gibbs sampling technique, from the joint conditional posterior distribution, \( P(\tilde{d}_{k}^{i[i]} | r, \tau^{i[i]}, a^{i[i]}). \) Based on the samples \( \tilde{d}_{k}^{i[i]} \), \( d_{k}^{i[i]}(\ell)d_{k'}^{i[i]}(\ell') \) in (7) can be evaluated by
\[ \left( d_{k}^{i[i]}(\ell)d_{k'}^{i[i]}(\ell') \right)^i \triangleq \frac{1}{N_t} \sum_{t=1}^{N_t} m n P(d_k(\ell) = m, d_{k'}(\ell') = n | \tilde{d}_{k}^{i[i]}, r, \tau^{i[i]}, a^{i[i]}). \]

We need to evaluate the probability in the expression above. Following the same route taken in this section, we use some algebra, it can be expressed as
\[ P(\tilde{d}_{k}^{i[i]}(\ell) = m, d_{k'}^{i[i]}(\ell') = n | \tilde{d}_{k}^{i[i]}, r, \tau^{i[i]}, a^{i[i]} = \frac{1}{1 + \exp(-\tilde{d}_{k}^{i[i]}, r, \tau^{i[i]}, a^{i[i]}). \] (13)

The quantities \( \zeta^{i[i]} \) and \( \lambda^{i[i]} \) are given by
\[ \zeta^{i[i]} \triangleq \frac{4}{N_0} \Re \left\{ m(g_p^i) (r - G_{q}^{i[i]} d_{k}^{i[i]}) - mn(g_p^i) g_q^i \right\}, \]
\[ \lambda^{i[i]} \triangleq \frac{4}{N_0} \Re \left\{ m(g_p^i) (r - G_{q}^{i[i]} d_{k}^{i[i]}) \right\}, \]

where \( p \triangleq kL + \ell, \) and \( q \triangleq kL + \ell, \) \( G_{q}^{i[i]} \) is \( G \) with its \( p \)th and \( q \)th columns \( g_p, g_q \) removed. Similarly, \( d_{q}^{i[i]} \) denotes the vector \( d \) with its \( p \)th and \( q \)th components removed.

V. PERFORMANCE ANALYSIS

A. Modified Cramer-Rao Bounds for the Estimated Parameters

We now derive the modified Cramer-Rao lower bounds (MCRB) on the variances of any unbiased estimates \( \hat{\theta} \) of the parameter vector \( \theta. \) It is shown in [18] that for \( \theta_{p} \in \theta, \) \( \text{var}(\hat{\theta}_{p} - \theta_{p}) \geq \left| I^{-1}(\theta) \right|_{p} \), where \( I(\theta) \) is the \( 3K \times 3K \) Fisher information matrix whose \( (p, q) \)th component is defined by
\[ I(\theta)^{pq} \triangleq -E_{r,a} \left( \frac{\partial^2 \log p(r, a | \tau) }{\partial \theta_p \partial \theta_q} \right), \] for \( p, q = 1, 2, \ldots, 3K. \) For the joint likelihood function in (13), it is shown in [19] that the Fisher information matrix can be computed by
\[ I(\theta)^{pq} = \frac{2}{N_0} \text{E}_{d} \left\{ E_{a | d} \left\{ \Re \left[ \frac{\partial \mu^i(\theta, d)}{\partial \theta_p} \right] \right\} \right\}, \]

(14)

where \( p, q = 1, 2, \ldots, 3K. \)
Fig. 1 shows the mean-square-error (MSE) of the channel estimates with the short-cut normalized transmission delays. The individual powers are given by considering communication over AWGN (not known to the receiver). \( \tau \) considered subsequently. We refer to these scheme as "SAGE-JDE, transmission delays and hard-decision decoding has also been for joint data detection and channel estimation in [10] for known estimation (MMSE-SE). For comparison purpose, the SAGE-scheme on the interval between zero and the value on the abscissa. It can be seen that the MMSE-SDE scheme cannot handle transmission delays i.e., every user’s parameter vector is updated five times.

\[
[I(\theta)]_{pp} = \frac{2}{N_0} \left( \begin{array}{c} L; \\ L; \\ \sigma_p^2 \sum_{l=0}^{L-1} |S'(l)|^2; \quad p = K + 1, \ldots, 2K \end{array} \right)_{p = 1, \ldots, K}
\]

with the short-cut \( \sigma_p^2 = \sigma_q^2 / N_0 \) is the average SNR, \( B_{sk} \) is the Gabor bandwidth of the \( k \)th user’s spreading code waveform, \( s_k(t) \) i.e.,

\[
B_{sk} \triangleq \left( \int_{-\infty}^{\infty} f^2 |S_k(f)|^2 df \right)^{1/2},
\]

and \( S_k(f) \) is the Fourier transform of \( s_k(t) \), \( t \in [0, T_b] \). Note that the Gabor bandwidth \( B_{sk} \) tends to infinity for rectangular-shaped (continuous-time) chip waveforms.

**B. Numerical Examples**

To assess the performance of the proposed (non-linear) Monte-Carlo SAGE scheme, an asynchronous uncoded DS-CDMA system with \( K = 5 \) users, rectangular chip waveforms with processing gain \( N_c = 8 \), and \( L = 80 \) transmitted symbols per block is considered. The receiver processes \( Q = 12 \) samples per chip. For each data block, Gibbs sampling is performed over 50 iterations. A few, say \( L_p = 4 \) pilot symbols are embedded in each block to overcome the phase ambiguity problem. Each user’s strongest path from the MMSE estimate of \( a \) given the \( K Q (L_p + 1) - 1 \) samples of \( r \) and the pilot symbols, yield the initial estimates \( a^{[0]} \) and \( \tau^{[0]} \). The MMSE estimate of \( d \), given \( r \) and weighted by \( a^{[0]} \) yields the initial symbol estimate \( d^{[0]} \). We refer to this method as MMSE-separate estimation (MMSE-SE). For comparison purpose, the SAGE-scheme for joint data detection and channel estimation in [10] for known transmission delays and hard-decision decoding has also been considered subsequently. We refer to these scheme as "SAGE-JDE, \( \tau \) known".

To study the behavior of the proposed MCMC-SAGE scheme, we consider communication over AWGN (not known to the receiver). The individual powers are given by

\[
\sigma_1^2 = -4 \text{ dB}, \quad \sigma_2^2 = -2 \text{ dB}, \quad \sigma_3^2 = 0 \text{ dB}, \quad \sigma_4^2 = +2 \text{ dB}, \quad \sigma_5^2 = +4 \text{ dB},
\]

Fig. 1 shows the mean-square-error (MSE) of the channel estimates \( \hat{a}_1 \) (weakest user) and \( \hat{a}_3 \) (normal user) as a function of the normalized transmission delays \( \tau / T_b \), which are uniformly distributed on the interval between zero and the value on the abscissa. It can be seen that the MCMC-SAGE performs close to the MCRB over the entire range of \( \tau \). Not shown in the plot, convergence is achieved after around 25 iterations i.e., every user’s parameter vector is updated five times.

Fig. 2 depicts the MSE of the delay estimates \( \hat{\tau}_1 \) and \( \hat{\tau}_3 \). Notice that the MCRB for \( \tau \) tends to zero for time-continuous signature waveforms. It can be seen that user 3 does not encounter delay estimation errors for small transmission delays i.e., \( \tau / T_b \leq 0.2 \). This effect can be partially explained by the large number of samples per chip i.e., \( Q = 12 \). Though for higher transmission delays, var(\( \hat{\tau}_3 \)) is finite, because of the increasing residual interference in the receiver.

The bit-error-rate (BER) of the proposed receiver is plotted in Fig. 3 versus the effective SNR \( \frac{\gamma_k}{\sigma_k^2} \) for \( k = 1, \ldots, K \). The transmission delays are uniformly distributed on \([0, T_b / 2]\). It can be seen that the MMSE-SE scheme cannot handle delay estimation errors at all due to high correlations between the users’ signature sequences. The proposed MCMC-SAGE scheme and the "SAGE-JDE, \( \tau \) known" perform similar. The weakest user 1 performs close to the single-user (SU) bound. The normal user 3 has a multiuser efficiency of roughly 1 dB over the entire range of SNR values.
Fig. 3. BER-performance in near-far scenario.

VI. CONCLUSIONS

A computationally efficient estimation algorithm has been proposed for estimating the transmission delays and the channel coefficients jointly in a non-data-aided fashion via the SAGE algorithm. The a posteriori probabilities needed to implement the SAGE algorithm have been computed by means of the Gibbs sampling technique. Exact analytical expression have been obtained for the estimates of transmission delays and channel coefficients. At each iteration the likelihood function is non-decreasing.

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