Possible low energy manifestations of strings and gravity

I. Antoniadis
Department of Physics, CERN - Theory Division, 1211 Geneva 23, Switzerland
E-mail: ignatios.antoniadis@cern.ch

Abstract. Lowering the string scale in the TeV region provides a theoretical framework for solving the mass hierarchy problem and unifying all interactions. The apparent weakness of gravity can then be accounted for by the existence of large internal dimensions, in the submillimeter region, and transverse to a braneworld where our universe must be confined. I review the main properties of this scenario and its implications for observations at both particle colliders, and in non-accelerator gravity experiments. I also discuss the warped case and localization of gravity in the presence of infinite size extra dimensions.

1. Introduction
During the last few decades, physics beyond the Standard Model (SM) was guided from the problem of mass hierarchy. This can be formulated as the question of why gravity appears to us so weak compared to the other three known fundamental interactions corresponding to the electromagnetic, weak and strong nuclear forces. Indeed, gravitational interactions are suppressed by a very high energy scale, the Planck mass $M_P \sim 10^{19}$ GeV, associated to a length $l_P \sim 10^{-35}$ m, where they are expected to become important. In a quantum theory, the hierarchy implies a severe fine tuning of the fundamental parameters in more than 30 decimal places in order to keep the masses of elementary particles at their observed values. The reason is that quantum radiative corrections to all masses generated by the Higgs vacuum expectation value (VEV) are proportional to the ultraviolet cutoff which in the presence of gravity is fixed by the Planck mass. As a result, all masses are “attracted” to become about $10^{16}$ times heavier than their observed values.

Besides compositeness, there are two main theories that have been proposed and studied extensively during the last years, corresponding to different approaches of dealing with the mass hierarchy problem. (1) Low energy supersymmetry with all superparticle masses in the TeV region. Indeed, in the limit of exact supersymmetry, quadratically divergent corrections to the Higgs self-energy are exactly cancelled, while in the softly broken case, they are cutoff by the supersymmetry breaking mass splittings. (2) TeV scale strings, in which quadratic divergences are cutoff by the string scale and low energy supersymmetry is not needed. Both ideas are experimentally testable at high-energy particle colliders and in particular at LHC. Below, I discuss their implementation in string theory.
2. Strings and extra dimensions

The appropriate and most convenient framework for low energy supersymmetry and grand unification is the perturbative heterotic string. Indeed, in this theory, gravity and gauge interactions have the same origin, as massless modes of the closed heterotic string, and they are unified at the string scale $M_s$. As a result, the Planck mass $M_P$ is predicted to be proportional to $M_s$:

$$M_P = M_s/g,$$

where $g$ is the gauge coupling. In the simplest constructions all gauge couplings are the same at the string scale, given by the four-dimensional (4d) string coupling, and thus no grand unified group is needed for unification. In our conventions $\alpha_{GUT} = g^2 \simeq 0.04$, leading to a discrepancy between the string and grand unification scale $M_{GUT}$ by almost two orders of magnitude. Explaining this gap introduces in general new parameters or a new scale, and the predictive power is essentially lost. This is the main defect of this framework, which remains though an open and interesting possibility [1].

The other perturbative framework that has been studied extensively in the more recent years is type I string theory with D-branes. Unlike in the heterotic string, gauge and gravitational interactions have now different origin. The latter are described again by closed strings, while the former emerge as excitations of open strings with endpoints confined on D-branes [2]. This leads to a braneworld description of our universe, which should be localized on a hypersurface, i.e. a membrane extended in $p$ spatial dimensions, called $p$-brane (see Fig. 1). Closed strings propagate in all nine dimensions of string theory: in those extended along the $p$-brane, called parallel, as well as in the transverse ones. On the contrary, open strings are attached on the $p$-brane. Obviously, our $p$-brane world must have at least the three known dimensions of space.

![Diagram](https://via.placeholder.com/150)

**Figure 1.** In type I string framework, our Universe contains, besides our 3 spatial dimensions (denoted by the blue line), some extra dimensions ($d_\parallel = p - 3$) parallel to our world $p$-brane (green plane) where endpoints of open strings are confined, as well as some transverse dimensions (yellow space) where only gravity (described by closed strings) propagate.

But it may contain more: the extra $d_\parallel = p - 3$ parallel dimensions must have a finite size, in order to be unobservable at present energies, and can be as large as $\text{TeV}^{-1} \sim 10^{-18}$ m [3]. On the other hand, transverse dimensions interact with us only gravitationally and experimental
bounds are much weaker: their size should be less than about 0.1 mm [4]. In the following, I review the main properties and experimental signatures of low string scale models [5, 6].

2.1. Framework of low scale strings

In type I theory, the different origin of gauge and gravitational interactions implies that the relation between the Planck and string scales is not linear as (1) of the heterotic string. The requirement that string theory should be weakly coupled, constrain the size of all parallel dimensions to be of order of the string length, while transverse dimensions remain unrestricted. Assuming an isotropic transverse space of \( n = 9 - p \) compact dimensions of common radius \( R_{\perp} \), one finds:

\[
M_P^2 = \frac{1}{g^4} M_s^{2+n} R_{\perp}^n, \quad g_s \simeq g^2.
\]

(2)

where \( g_s \) is the string coupling. It follows that the type I string scale can be chosen hierarchically smaller than the Planck mass [7, 5] at the expense of introducing extra large transverse dimensions felt only by gravity, while keeping the string coupling small [5]. The weakness of 4d gravity compared to gauge interactions (ratio \( M_W/M_P \)) is then attributed to the largeness of the transverse space \( R_{\perp} \) compared to the string length \( l_s = M_s^{-1} \).

An important property of these models is that gravity becomes effectively \((4+n)\)-dimensional with a strength comparable to those of gauge interactions at the string scale. The first relation of Eq. (2) can be understood as a consequence of the \((4+n)\)-dimensional Gauss law for gravity, with

\[
M_s^{(4+n)} = M_s^{2+n} / g^4
\]

(3)

the effective scale of gravity in \(4+n\) dimensions. Taking \( M_s \simeq 1 \) TeV, one finds a size for the extra dimensions \( R_{\perp} \) varying from \( 10^8 \) km, .1 mm, down to a Fermi for \( n = 1, 2, \) or 6 large dimensions, respectively. This shows that while \( n = 1 \) is excluded, \( n \geq 2 \) is allowed by present experimental bounds on gravitational forces [4, 8]. Thus, in these models, gravity appears to us very weak at macroscopic scales because its intensity is spread in the “hidden” extra dimensions. At distances shorter than \( R_{\perp} \), it should deviate from Newton’s law, which may be possible to explore in laboratory experiments (see Fig. 2).

![Figure 2](image.png)

**Figure 2.** Torsion pendulum that tested Newton’s law at 55 \( \mu \)m.

The main experimental implications of TeV scale strings in particle accelerators are of three types, in correspondence with the three different sectors that are generally present: (i) new compactified parallel dimensions, (ii) new extra large transverse dimensions and low scale quantum gravity, and (iii) genuine string and quantum gravity effects. On the other hand, there exist interesting implications in non accelerator table-top experiments due to the exchange of gravitons or other possible states living in the bulk.
3. Experimental implications in accelerators

3.1. World-brane extra dimensions

In this case $RM_s \gtrsim 1$, and the associated compactification scale $R^{-1}_\parallel$ would be the first scale of new physics that should be found increasing the beam energy [3, 9, 10]. The main consequence is the existence of Kaluza-Klein (KK) excitations for all SM particles that propagate along the extra parallel dimensions. Their masses are given by:

$$M^2_m = M^2_0 + \frac{m^2}{R^2_\parallel}; \quad m = 0, \pm 1, \pm 2, \ldots$$

where we used $d_\parallel = 1$, and $M_0$ is the higher dimensional mass. The zero-mode $m = 0$ is identified with the 4d state, while the higher modes have the same quantum numbers with the lowest one, except for their mass given in (4). There are two types of experimental signatures of such dimensions [9, 11, 12]: (i) virtual exchange of KK excitations, leading to deviations in cross-sections compared to the SM prediction, that can be used to extract bounds on the compactification scale; (ii) direct production of KK modes.

On general grounds, there can be two different kinds of models with qualitatively different signatures depending on the localization properties of matter fermion fields. If the latter are localized in 3d brane intersections, they do not have excitations and KK momentum is not conserved because of the breaking of translation invariance in the extra dimension(s). KK modes of gauge bosons are then singly produced giving rise to generally strong bounds on the compactification scale and new resonances that can be observed in experiments. Otherwise, they can be produced only in pairs due to the KK momentum conservation, making the bounds weaker but the resonances difficult to observe.

In addition to virtual effects, KK excitations can be produced on-shell at LHC as new resonances [11] (see Fig. 3). There are two different channels, neutral Drell–Yan processes

$$pp \rightarrow l^+l^-X$$

and the charged channel $l^+\nu$, corresponding to the production of the KK modes

![Figure 3](attachment:image.png)

Figure 3. Production of the first KK modes of the photon and of the Z boson at LHC, decaying to electron-positron pairs. The number of expected events is plotted as a function of the energy of the pair in GeV.
Table 1. Limits on $R_{\perp}$ in mm.

| Experiment | $n = 2$ | $n = 4$ | $n = 6$ |
|------------|--------|--------|--------|
| Collider bounds | | | |
| LEP 2 | $5 \times 10^{-1}$ | $2 \times 10^{-8}$ | $7 \times 10^{-11}$ |
| Tevatron | $5 \times 10^{-1}$ | $10^{-8}$ | $4 \times 10^{-11}$ |
| LHC | $4 \times 10^{-1}$ | $6 \times 10^{-10}$ | $3 \times 10^{-12}$ |
| NLC | $10^{-2}$ | $10^{-9}$ | $6 \times 10^{-12}$ |
| Present non-collider bounds | | | |
| SN1987A | $3 \times 10^{-4}$ | $10^{-8}$ | $6 \times 10^{-10}$ |
| COMPTEL | $5 \times 10^{-8}$ | - | - |

$\gamma^{(1)}$, $Z^{(1)}$ and $W_{\pm}^{(1)}$, respectively. The discovery limits are about 6 TeV, while the exclusion bounds 15 TeV.

On the other hand, if all SM particles propagate in the extra dimension (called universal), KK modes can only be produced in pairs and the lower bound on the compactification scale becomes weaker, of order of 300-500 GeV. Moreover, no resonances can be observed at LHC, so that this scenario appears very similar to low energy supersymmetry. In fact, KK parity can even play the role of R-parity, implying that the lightest KK mode is stable and can be a dark matter candidate in analogy to the LSP [13].

### 3.2. Extra large transverse dimensions

The main experimental signal is gravitational radiation in the bulk from any physical process on the world-brane. In fact, the very existence of branes breaks translation invariance in the transverse dimensions and gravitons can be emitted from the brane into the bulk. During a collision of center of mass energy $\sqrt{s}$, there are $\sim (\sqrt{s}R_{\perp})^n$ KK excitations of gravitons with tiny masses, that can be emitted. Each of these states looks from the 4d point of view as a massive, quasi-stable, extremely weakly coupled ($s/M^2$ suppressed) particle that escapes from the detector. The total effect is a missing-energy cross-section roughly of order:

$$\frac{(\sqrt{s}R_{\perp})^n}{M^2} \sim \frac{1}{s} \left(\frac{\sqrt{s}}{M_s}\right)^{n+2}.$$  

Explicit computation of these effects leads to the bounds given in Table 1.

Fig. 4 shows the cross-section for graviton emission in the bulk, corresponding to the process $pp \rightarrow jet + graviton$ at LHC, together with the SM background [14]. There is a particular energy and angular distribution of the produced gravitons that arise from the distribution in mass of KK states of spin-2. This can be contrasted to other sources of missing energy and might be a smoking gun for the extra dimensional nature of such a signal.

In Table 1, there are also included astrophysical and cosmological bounds. Astrophysical bounds [15, 16] arise from the requirement that the radiation of gravitons should not carry on too much of the gravitational binding energy released during core collapse of supernovae. The best cosmological bound [17] is obtained from requiring that decay of bulk gravitons to photons do not generate a spike in the energy spectrum of the photon background measured by the COMPTEL instrument. Bulk gravitons are expected to be produced just before nucleosynthesis due to thermal radiation from the brane. The limits assume that the temperature was at most 1 MeV as nucleosynthesis begins, and become stronger if temperature is increased.
Figure 4. Missing energy due to graviton emission at LHC, as a function of the higher-dimensional gravity scale $M_*$, produced together with a hadronic jet. The expected cross-section is shown for $n = 2$ and $n = 4$ extra dimensions, together with the SM background.

3.3. String effects

At low energies, the interaction of light (string) states is described by an effective field theory. Their exchange generates in particular four-fermion operators that can be used to extract independent bounds on the string scale. In analogy with the bounds on longitudinal extra dimensions, there are two cases depending on the localization properties of matter fermions. If they come from open strings with both ends on the same stack of branes, exchange of massive open string modes gives rise to dimension eight effective operators, involving four fermions and two space-time derivatives [18, 19]. The corresponding bounds on the string scale are then around 500 GeV. On the other hand, if matter fermions are localized on non-trivial brane intersections, one obtains dimension six four-fermion operators and the bounds become stronger: $M_s \sim 2 - 3$ TeV [19, 6]. At energies higher than the string scale, new spectacular phenomena are expected to occur, related to string physics and quantum gravity effects, such as possible micro-black hole production [20, 21, 22]. Particle accelerators would then become the best tools for studying quantum gravity and string theory.

Direct production of string resonances in hadron colliders leads generically to a universal deviation from Standard Model in jet distribution [23]. In particular, the first Regge excitation of the gluon has spin 2 and a width an order of magnitude lower than the string scale, leading to a characteristic peak in dijet production; similarly, the first excitations of quarks have spin 3/2. The dijet cross-section is shown in Fig. 5 for LHC energies. The reason for the universal behavior is that tree $N$-point open superstring amplitudes involving at most two fermions and gluons are completely model independent from the details of the compactification, including the number of supersymmetries that are left unbroken in four dimensions (even if all are broken). Such tree-level amplitudes do not receive contributions from KK, string winding or closed string graviton modes, but are given as a universal sum over exchanges of open string Regge excitations lying in Regge trajectories with masses

$$M_n^2 = M_s^2 n \quad ; \quad n = 0, 1, \ldots$$

and maximal spin $n + 1$.

The relevant partonic cross-sections for dijet production, involving at most two quarks
Figure 5. Production of the first Regge excitations at LHC in the dijet channel, for $M_s = 2$ TeV. The cross-section is plotted as a function of the dijet invariant mass $M$.

are $|\mathcal{M}(gg \rightarrow gg)|^2$, $|\mathcal{M}(gg \rightarrow q\bar{q})|^2$, $|\mathcal{M}(q\bar{q} \rightarrow gg)|^2$ and $|\mathcal{M}(qq \rightarrow gg)|^2$, where $g$ and $q$ denote gluons and quarks, respectively. They can be obtained from the first two, up to crossing symmetries, at the full (tree) string level and they are model independent [24]. The low energy expansion of these amplitudes reproduce the usual QCD expressions, while higher order terms describe the string corrections due to the exchange of Regge excitations. Besides these amplitudes, there are also those involving four quarks, such as $|\mathcal{M}(q\bar{q} \rightarrow q\bar{q})|^2$ and $|\mathcal{M}(qq \rightarrow qq)|^2$. These are model dependent, because the details of the compactification do not decouple. The reason is that they involve four vertices containing twist fields that describe quark states arising from open strings stretched in brane intersections. However, taking into account QCD color factors, their contribution is suppressed because parton luminosities in proton-proton collisions at TeV energies favor at least one gluon in the initial state. As a result, the dominant contribution comes from the model independent cross-sections described above, leading to the effect of Fig. 5.

We finish this section with some comments related to the possible micro-black hole production. Independently on the unresolved issue of the convergence of string perturbation theory in the kinematic region relevant to micro-black hole formation, there is a simple argument showing that at least within the perturbative TeV string framework, the energy threshold for black hole production is far above the LHC reach. Indeed, a string size black hole has a horizon radius $r_H \sim 1$ in string units, while the $d$-dimensional Newton’s constant behaves as $G_N \sim g_s^2$. It follows that the mass of a $d$-dimensional black hole is [25]:

$$M_{\text{BH}} \sim r_H^{d/2-1}/G_N \simeq 1/g_s^2.$$ 

(7)

Thus, for a weakly coupled theory, this energy threshold is much higher than the string and the higher dimensional Planck scales $M_s$ and $M_s$ of eq. (3). Comparing this energy threshold with the mass of Regge excitations (6), one finds $n \sim 1/g_s^4$ which is actually compatible with the relation one obtains by identifying the black hole entropy $S_{\text{BH}} \sim 1/G_N \sim 1/g_s^2$ with the perturbative string entropy $S_{\text{string}} \sim \sqrt{n}$. Using now relation (2), and the value of the Standard
Model gauge couplings \( g_s \approx g^2 \sim 0.1 \), one finds that the energy threshold \( M_{BH} \) of micro-black hole production is about four orders of magnitude higher than the string scale, implying that one would produce \( 10^4 \) string states before reaching \( M_{BH} \).

4. Supersymmetry in the bulk and short range forces

4.1. Sub-millimeter forces

Besides the spectacular predictions in accelerators, there are also modifications of gravitation in the sub-millimeter range, which can be tested in “table-top” experiments that measure gravity at short distances. There are three categories of such predictions:

(i) Deviations from the Newton’s law \( 1/r^2 \) behavior to \( 1/r^{2+n} \), which can be observable for \( n = 2 \) large dimensions of sub-millimeter size.

(ii) New scalar forces in the sub-millimeter range, related to the mechanism of supersymmetry breaking, and mediated by light scalar fields \( \phi \) with masses \( m_\phi \sim m^2_{susy}/M_P \), for a supersymmetry breaking scale \( m_{susy} \sim 1 - 10 \) TeV. They correspond to Compton wavelengths of 1 mm to 10 \( \mu \)m. \( m_{susy} \) can be either \( 1/R_\parallel \) if supersymmetry is broken by compactification [26], or the string scale if it is broken “maximally” on our world-brane [5]. A universal attractive scalar force is mediated by the radion modulus \( \phi \equiv M_P \ln R \), with \( R \) the radius of the longitudinal or transverse dimension(s). For \( n = 2 \), there may be an enhancement factor of the radion mass by \( \ln R \) decreasing its wavelength by an order of magnitude [27].

The coupling of the radius modulus to matter relative to gravity can be easily computed and is given by:

\[
\sqrt{\alpha_\phi} = \frac{1}{M} \frac{\partial M}{\partial \phi} ; \quad \alpha_\phi = \begin{cases} \frac{\partial \ln \Lambda_{QCD}}{\partial \ln R} \simeq \frac{1}{3} & \text{for } R_\parallel \\ \frac{2\pi}{n+2} = 1 - 1.5 & \text{for } R_\perp \end{cases}
\]

where \( M \) denotes a generic physical mass. Such a force can be tested in microgravity experiments and should be contrasted with the change of Newton’s law due the presence of extra dimensions that is observable only for \( n = 2 \) [4, 8]. The resulting bounds from an analysis of the radion effects are [28]: \( M_\phi \gtrsim 6 \) TeV.

(iii) Non universal repulsive forces much stronger than gravity, mediated by possible abelian gauge fields in the bulk [15, 29]. Such fields acquire tiny masses of the order of \( M^2_\phi/M_P \), as in (8), due to brane localized anomalies [29]. Although their gauge coupling is infinitesimally small, \( g_A \sim M_\phi/M_P \sim 10^{-14} \), it is still bigger that the gravitational coupling \( E/M_P \) for typical energies \( E \sim 1 \) GeV, and the strength of the new force would be \( 10^8 - 10^{10} \) stronger than gravity.

In Fig. 6 we depict the actual information from previous, present and upcoming experiments [8, 27]. The solid lines indicate the present limits from the experiments indicated. The excluded regions lie above these solid lines. Measuring gravitational strength forces at short distances is challenging. The horizontal lines correspond to theoretical predictions, in particular for the graviton in the case \( n = 2 \) and for the radion in the transverse case. These limits are compared to those obtained from particle accelerator experiments in Table 1.

5. Standard Model on D-branes

The gauge group closest to the Standard Model one can easily obtain with D-branes is \( U(3) \times U(2) \times U(1) \). The first factor arises from three coincident “color” D-branes. An open string with one end on them is a triplet under \( SU(3) \) and carries the same \( U(1) \) charge for all three components. Thus, the \( U(1) \) factor of \( U(3) \) has to be identified with gauged baryon number.
Figure 6. Present limits on new short-range forces (yellow regions), as a function of their range $\lambda$ and their strength relative to gravity $\alpha$. The limits are compared to new forces mediated by the graviton in the case of two large extra dimensions, and by the radion.

Similarly, $U(2)$ arises from two coincident “weak” D-branes and the corresponding abelian factor is identified with gauged weak-doublet number. Finally, an extra $U(1)$ D-brane is necessary in order to accommodate the Standard Model without breaking the baryon number [30].

It turns out that there are two possible ways of embedding the Standard Model particle spectrum on these stacks of branes [30], which are shown pictorially in Fig. 7. The quark doublet $Q$ corresponds necessarily to a massless excitation of an open string with its two ends on the two different collections of branes (color and weak). As seen from the figure, a fourth brane stack is needed for a complete embedding, which is chosen to be a $U(1)_b$ extended in

Figure 7. A minimal Standard Model embedding on D-branes.
the bulk. This is welcome since one can accommodate right handed neutrinos as open string states on the bulk with sufficiently small Yukawa couplings suppressed by the large volume of the bulk [31]. The two models are obtained by an exchange of the up and down antiquarks, $u^c$ and $d^c$, which correspond to open strings with one end on the color branes and the other either on the $U(1)$ brane, or on the $U(1)_b$ in the bulk. The lepton doublet $L$ arises from an open string stretched between the weak branes and $U(1)_b$, while the antilepton $\bar{L}$ corresponds to a string with one end on the $U(1)$ brane and the other in the bulk. For completeness, we also show the two possible Higgs states $H_u$ and $H_d$ that are both necessary in order to give tree-level masses to all quarks and leptons of the heaviest generation.

6. Non-compact extra dimensions and localized gravity

There are several motivations to study localization of gravity in non-compact extra dimensions: (i) it avoids the problem of fixing the moduli associated to the size of the compactification manifold; (ii) it provides a new approach to the mass hierarchy problem; (iii) there are modifications of gravity at large distances that may have interesting observational consequences. Two types of models have been studied: warped metrics in curved space [32], and infinite size extra dimensions in flat space [33]. The former, although largely inspired by stringy developments and having used many string-theoretic techniques, have not yet a clear and calculable string theory realization [34]. In any case, since curved space is always difficult to handle in string theory, in the following we concentrate mainly on the latter, formulated in flat space with gravity localized on a subspace of the bulk. It turns out that these models of induced gravity have an interesting string theory realization [35] that we describe below, after presenting first a brief overview of the warped case.

6.1. Warped spaces

In these models, space-time is a slice of anti de Sitter space (AdS) in $d = 5$ dimensions while our universe forms a four-dimensional (4d) flat boundary [32]. The corresponding line element is:

$$ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2; \quad \Lambda = -24M^3k^2,$$

where $M, \Lambda$ are the 5d Planck mass and cosmological constant, respectively, and the parameter $k$ is the curvature of AdS$_5$. The fifth coordinate $y$ is restricted on the interval $[0, \pi r_c]$. Thus, this model requires two ‘branes’, a UV and an IR, located at the two end-points of the interval, $y = 0$ and $y = \pi r_c$, respectively. The vanishing of the 4d cosmological constant requires to fine tune the two tensions: $T = -T' = 24M^3k^2$. The 4d Planck mass is given by:

$$M_P^2 = \frac{1}{k} (1 - e^{-2\pi kr_c}) M^3.$$

On the other hand, there is an exponential hierarchy of the mass scales in the IR relative to the UV brane due to the warped factor in the metric (10). One can then identify the electroweak scale $M_W \sim M_P e^{-2\pi kr_c}$.

By studying the corresponding wave functions, one can show that 4d gravity is localized on the UV-brane, while KK gravitons on the IR. It follows that the main prediction of this model is the existence of spin-2 resonances at the TeV scale coupled with (TeV)$^{-1}$ strength. Their masses are given by:

$$m_n = c_n k e^{-2\pi kr_c}; \quad c_n \simeq \left(n + \frac{1}{4}\right) \text{for large } n$$

and they are weakly coupled for $m_n < Me^{-2\pi kr_c}$, i.e. for $k < M$. Viable models can be constructed when Standard Model gauge bosons propagate in the 5d bulk, fermions are localized near the UV brane, while the Higgs near the IR [36]. Using AdS/CFT correspondence, they are
dual to strongly coupled 4d field theories with composite Higgs; their gauge coupling $g_{YM} \sim M/k$ is strong when spin-2 KK modes are weakly coupled in the gravity dual description.

6.2. The induced gravity model

The DGP model and its generalizations are specified by a bulk Einstein-Hilbert (EH) term and a four-dimensional EH term [33]:

$$M^{2+n} \int_{M_{4+n}} d^4 x d^n y \sqrt{G} R_{(4+n)} + M_P^2 \int_{M_4} d^4 x \sqrt{G} R_{(4)}; \quad M_P^2 \equiv r_c^n M^{2+n} \tag{13}$$

with $M$ and $M_P$ the (possibly independent) respective Planck scales. The scale $M \geq 1$ TeV would be related to the short-distance scale below which UV quantum gravity or stringy effects are important. The four-dimensional metric is the restriction of the bulk metric to the $n$-brane.

6.2.1. Co-dimension one

In the case of co-dimension one bulk ($n = 1$) and $\delta$-function localization, it is easy to see that $r_c$ is a crossover scale where gravity changes behavior on the world. Indeed, by Fourier transform the quadratic part of the action (13) with respect to the 4d position $x$, at the world position $y = 0$, one obtains $M^{2+n}(p^{2-n} + r_c^2 p^2)$, where $p$ is the 4d momentum. It follows that for distances smaller than $r_c$ (large momenta), the first term becomes irrelevant and the graviton propagator on the "brane" exhibits four-dimensional behavior $(1/p^2)$ with Planck constant $M_P = M^3 r_c$. On the contrary, at large distances, the first term becomes dominant and the graviton propagator acquires a five-dimensional fall-off $(1/p)$ with Planck constant $M$.

An important consequence of this model is the existence of self-accelerating solutions for the Universe at late times [37]. Indeed, the Friedman equations become:

$$H^2 + \frac{H}{r_c} = \frac{8\pi G_N}{3} \rho(t), \tag{14}$$

where $H$ is the Hubble constant and $\rho$ the 4d energy density. For the upper sign (+), one finds usual Robertson-Walker cosmologies extrapolating between 4d to 5d at large scales, while for the lower (negative) sign, one finds a de Sitter solution at late times. This can account for the present dark energy if $r_c \simeq 10^{28}$ cm. On the other hand, this model suffers from the van Dan-Veltman-Zakharov discontinuity of massive gravity, implying a strong coupling regime at distances shorter than the Vainshtein radius $r_V = (r_c^2 w)^{1/3}$, where $w$ is the UV cutoff, corresponding to the 'brane' thickness [38]. Moreover, imposing in general $r_c$ to be larger than the size of the universe, $r_c \gtrsim 10^{28}$ cm, one finds $M \lesssim 100$ MeV, which is in conflict with experimental bounds.

6.2.2. Higher co-dimension

The situation changes drastically for more than one non-compact bulk dimensions, $n > 1$, due to the ultraviolet properties of the higher-dimensional theories. Indeed, from the action (13), the effective potential between two test masses in four dimensions

$$\int [d^3 x] e^{-ip \cdot x} V(x) = \frac{D(p)}{1 + r_c^2 p^2} D(p) \left[ \hat{T}^{\mu \nu} T^\mu \nu - \frac{1}{2 + n} \hat{T}^\mu T^\nu \right], \tag{15}$$

$$D(p) = \int [d^n q] \frac{f_w(q)}{p^2 + q^2} \tag{16}$$

We avoid calling $M_4$ a brane because, as we will see below, gravity localizes on singularities of the internal manifold, such as orbifold fixed points. Branes with localized matter can be introduced independently of gravity localization.
is a function of the bulk graviton retarded Green’s function $G(x, 0; 0, 0) = \int [d^4p] e^{ipx} D(p)$ evaluated for two points localized on the world $(y = y' = 0)$. The integral (16) is UV-divergent for $n > 1$ unless there is a non-trivial brane thickness profile $f_w(q)$ of width $w$. If the four-dimensional world has zero thickness, $f_w(q) \sim 1$, the bulk graviton does not have a normalizable wave function. It therefore cannot contribute to the induced potential, which always takes the form $V(p) \sim 1/p^2$ and Newton’s law remains four-dimensional at all distances.

For a non-zero thickness $w$, there is only one crossover length scale, $R_c$:

$$R_c = w \left( \frac{r_c}{w} \right)^{\frac{1}{n}},$$

(17)

above which one obtains a higher-dimensional behaviour [39]. Therefore the effective potential presents two regimes: (i) at short distances $(w \ll r \ll R_c)$ the gravitational interactions are mediated by the localized four-dimensional graviton and Newton’s potential on the world is given by $V(r) \sim 1/r$ and, (ii) at large distances $(r \gg R_c)$ the modes of the bulk graviton dominate, changing the potential. Note that for $n = 1$ the expressions (15) and (16) are finite and unambiguously give $V(r) \sim 1/r$ for $r \gg r_c$. For a co-dimension bigger than 1, the precise behavior for large-distance interactions depends crucially on the UV completion of the theory.

At this point we stress a fundamental difference with the finite extra dimensions scenarios. In these cases Newton’s law gets higher-dimensional at distances smaller than the characteristic size of the extra dimensions. This is precisely the opposite of the case of infinite volume extra dimensions that we discuss here.

As mentioned above, for higher co-dimension, there is an interplay between UV regularization and IR behavior of the theory. Indeed, several works in the literature raised unitarity [40] and strong coupling problems [41] which depend crucially on the UV completion of the theory. A unitary UV regularization for the higher co-dimension version of the model has been proposed in [42]. It would be interesting to address these questions in a precise string theory context. Actually, using for UV cutoff on the “brane” the 4d Planck length $w \sim l_P$, one gets for the crossover scale (17): $R_c \sim M^{-1}(M_P/M)^{n/2}$. Putting $M \gtrsim 1$ TeV leads to $R_c \lesssim 10^{(6(n-2))}$ cm. Imposing $R_c \gtrsim 10^{28}$ cm, one then finds that the number of extra dimensions must be at least six, $n \geq 6$, which is realized nicely in string theory and provides an additional motivation for studying possible string theory realizations.

### 6.3. String theory realization

The gravity induced model (13) with $n = 6$ is realized as the low-energy effective action of type II string theory on a non-compact six-dimensional manifold $M_6$ [35], preserving $N = 2$ supersymmetry in four dimensions. The resulting expressions for the Planck masses $M$ and $M_p$ are:

$$M^2 \sim M_s^2 / g_s^{1/2}; \quad M_p^2 \sim \chi \left( \frac{c_0}{g_s^2} + c_1 \right) M_s^2,$$

(18)

with $c_0 = -2\zeta(3)$, $c_1 = \pm 4\zeta(2) = \pm 2\pi^2/3$ and $\chi$ is the Euler number of $M_6$.

It is interesting that the appearance of the induced 4d localized term preserves $N = 2$ supersymmetry and is independent of the localization mechanism of matter fields (for instance on D-branes). Localization requires the internal space $M_6$ to have a non-zero Euler characteristic $\chi \neq 0$. Actually, in type II $A/B$ compactified on a Calabi-Yau manifold, $\chi$ counts the difference between the numbers of $N = 2$ vector multiplets and hypermultiplets: $\chi = \pm 4(n_V - n_H)$ (where the graviton multiplet counts as one vector). Moreover, in the non-compact limit, the Euler number can in general split in different singular points of the internal space, $\chi = \sum I \chi_I$, giving rise to different localized terms at various points $y_I$ of the internal space. A number of conclusions (confirmed by string calculations in [35]) can be reached by looking closely at (18):
M_p \gg M requires a large non-zero Euler characteristic for M_6, and/or a weak string coupling constant g_s \to 0.

Since \chi is a topological invariant the localized R_{(4)} term coming from the closed string sector is universal, independent of the background geometry and dependent only on the internal topology. It is a matter of simple inspection to see that if one wants to have a localized \text{EH} term in less than ten dimensions, namely something linear in curvature, with non-compact internal space in all directions, the only possible dimension is four (or five in the strong coupling M-theory limit).

In order to find the width $w$ of the localized term, one has to do a separate analysis. On general grounds, using dimensional analysis in the limit $M_p \to \infty$, one expects the effective width to vanish as a power of $l_P \equiv (M - 1)^{1/2}$: $w \sim l_P^{\nu}/l_5^{\nu-1}$ with $\nu > 0$. The computation of $\nu$ for a general Calabi-Yau space, besides its technical difficulty, presents an additional important complication: from the expression (18), $l_P \sim g_s l_5$ in the weak coupling limit. Thus, $w$ vanishes in perturbation theory and one has to perform a non-perturbative analysis to extract its behavior. Alternatively, one can examine the case of orbifolds. In this limit, $c_0 = 0$, $l_P \sim l_5$, and the hierarchy $M_p > M$ is achieved only in the limit of large $\chi$. In this case, one finds the power $\nu = 1$.

6.3.1. Summary of the results  Using $w \sim l_P$ and the relations (18) in the weak coupling limit (with $c_0 \neq 0$), the crossover radius of eq. (17) is given by the string parameters ($n = 6$)

$$R_c = \frac{r^3_c}{w^2} \sim g_s l_5^4 l_P^4 \sim g_s \times 10^{32} \text{ cm,}$$  \hspace{1cm} (19)

for $M_s \simeq 1 \text{ TeV}$. Because $R_c$ has to be of cosmological size, the string coupling can be relatively small, and the Euler number $|\chi| \simeq g_s^2 l_P^2 \simeq 10^{32}$ must be very large. The hierarchy is obtained mainly thanks to the large value of $\chi$, so that lowering the bound on $R_c$ lowers the value of $\chi$. Our actual knowledge of gravity at very large distances indicates [43] that $R_c$ should be of the order of the Hubble radius $R_c \simeq 10^{38} \text{ cm}$, which implies $g_s \geq 10^{-4}$ and $|\chi| \simeq 10^{24}$. A large Euler number implies only a large number of closed string massless particles with no a-priori constraint on the observable gauge and matter sectors, which can be introduced for instance on D3-branes placed at the position where gravity localization occurs. All these particles are localized at the orbifold fixed points (or where the Euler number is concentrated in the general case), and should have sufficiently suppressed gravitational-type couplings, so that their presence with such a huge multiplicity does not contradict observations.

The explicit string realization of localized induced gravity models offers a consistent framework that allows to address a certain number of interesting physics problems. In particular, the effective UV cutoff and the study of the gravity force among matter sources localized on D-branes. It would be also interesting to perform explicit model building and study in detail the phenomenological consequences of these models and compare to other realizations of TeV strings with compact dimensions.

Acknowledgments

Work supported in part by the European Commission under the ERC Advanced Grant 226371 and in part by the CNRS grant GRC APIC PICS 3747.

References

[1] For a review, see e.g. K. R. Dienes, Phys. Rept. 287 (1997) 447 [arXiv:hep-th/9602045]; and references therein.
[2] C. Angelantonj and A. Sagnotti, Phys. Rept. 371 (2002) 1 [Erratum-ibid. 376 (2003) 339] [arXiv:hep-th/0204089].
[3] I. Antoniadis, *Phys. Lett. B* 246 (1990) 377.
[4] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, *Phys. Rev. Lett.* 98 (2007) 021101.
[5] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *Phys. Lett. B* 429 (1998) 263 [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *Phys. Lett. B* 436 (1998) 257 [arXiv:hep-ph/9804398].
[6] For a review see e.g. I. Antoniadis, Prepared for NATO Advanced Study Institute and EC Summer School on Progress in String, Field and Particle Theory, Cargese, Corsica, France (2002); and references therein.
[7] J. D. Lykken, *Phys. Rev. D* 54 (1996) 3693 [arXiv:hep-th/9603133].
[8] J. C. Long and J. C. Price, *Comptes Rendus Physique* 4 (2003) 337; R. S. Decca, D. Lopez, H. B. Chan, E. Fischbach, D. E. Krause and C. R. Jamell, *Phys. Rev. Lett.* 94 (2005) 240401; R. S. Decca et al., arXiv:0706.3283 [hep-ph]; S. J. Smullin, L. A. Geraci, D. M. Weld, J. Chiaverini, S. Holmes and A. Kapitulnik, arXiv:hep-ph/0508204; H. Abele, S. Haedler and A. Westphal, in 271th WE-Heraeus-Seminar, Bad Honnau (2002).
[9] I. Antoniadis and K. Benakli, *Phys. Lett. B* 326 (1994) 69.
[10] K. R. Dienes, E. Dudas and T. Gherghetta, *Phys. Lett. B* 436 (1998) 55 [arXiv:hep-ph/9803466]; Nucl. Phys. B 537 (1999) 47 [arXiv:hep-ph/9806292].
[11] I. Antoniadis, K. Benakli and M. Quiros, *Phys. Lett. B* 331 (1994) 313 and *Phys. Lett. B* 460 (1999) 176; P. Nath, Y. Yamada and M. Yamaguchi, *Phys. Lett. B* 466 (1999) 100 T. G. Rizzo and J. D. Wells, *Phys. Rev. D* 61 (2000) 016007; T. G. Rizzo, *Phys. Rev. D* 61 (2000) 055005; A. De Rujula, A. Donini, M. B. Gavela and S. Rigolin, *Phys. Lett. B* 482 (2000) 195;
[12] E. Accomando, I. Antoniadis and K. Benakli, *Nucl. Phys. B* 579 (2000) 3.
[13] G. Servant and T. M. P. Tait, *Nucl. Phys. B* 650 (2003) 391.
[14] G. F. Giudice, R. Rattazzi and J. D. Wells, *Nucl. Phys. B* 544 (1999) 3; E. A. Mirabelli, M. Perelstein and M. E. Peskin, *Phys. Rev. Lett.* 82 (1999) 2236; T. Han, J. D. Lykken and R. Zhang, *Phys. Rev. D* 59 (1999) 105006: K. Benakli and M. Quiros, Phys. Lett. B 62 (1999) 69; T. Han, J. D. Lykken and R. Zhang, *Phys. Rev. D* 60 (2000) 112003; C. Balazs et al., *Phys. Rev. Lett.* 83 (1999) 2112; L3 Collaboration (M. Acciarri et al.), *Phys. Lett. B* 464 (1999) 135 and *470* (1999) 281: J. L. Hewett, *Phys. Rev. Lett.* 82 (1999) 4765.
[15] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Rev. D* 59 (1999) 086004.
[16] S. Cullen and M. Perelstein, *Phys. Rev. Lett.* 83 (1999) 268; V. Barger, T. Han, C. Kao and R. J. Zhang, *Phys. Lett. B* 461 (1999) 34.
[17] K. Benakli and S. Davidson, *Phys. Rev. D* 60 (1999) 025004; L. J. Hall and D. Smith, *Phys. Rev. D* 60 (1999) 085008.
[18] E. Dudas and J. Mourad, Nucl. Phys. B 575 (2000) 3 [arXiv:hep-th/9911019]; S. Cullen, M. Perelstein and M. E. Peskin, *Phys. Rev. D* 62 (2000) 055012; D. Bourilkov, *Phys. Rev. D* 62 (2000) 076005; L3 Collaboration (M. Acciarri et al.), *Phys. Lett. B* 489 (2000) 81.
[19] I. Antoniadis, K. Benakli and A. Lauinger, *JHEP* 0105 (2001) 044.
[20] P. C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B 411 (1998) 96 [arXiv:hep-th/9808138]; T. Banks and W. Fischler, arXiv:hep-th/9906038.
[21] S. B. Giddings and S. Thomas, *Phys. Rev. D* 65 (2002) 056010 [arXiv:hep-ph/0106219]; S. Dimopoulos and G. Landsberg, *Phys. Rev. Lett.* 87 (2001) 161602 [arXiv:hep-ph/0106295].
[22] P. Meade and L. Randall, arXiv:0708.3048 [hep-ph].
[23] L. A. Anchordoqui, H. Goldberg, D. L. Lust, S. Nawata, S. Stieber and T. R. Taylor, Phys. Rev. Lett. 87 (2001) 240003 [arXiv:hep-th/0010162].
[24] D. Lust, S. Stieber and T. R. Taylor, Nucl. Phys. B 808 (2009) 1 [arXiv:0807.3333 [hep-th]].
[25] G. T. Horowitz and J. Polchinski, Phys. Rev. D 55 (1997) 6189 [arXiv:hep-th/9612146].
[26] I. Antoniadis, S. Dimopoulos and G. Dvali, *Nucl. Phys. B* 516 (1998) 70; S. Ferrara, C. Kounnas and F. Zwirner, *Nucl. Phys. B* 429 (1994) 589.
[27] I. Antoniadis, K. Benakli, A. Lauinger and T. Maillard, *Nucl. Phys. B* 662 (2003) 40 [arXiv:hep-ph/0211409].
[28] E. G. Adelberger, B. R. Heckel, S. Hoedl, C. D. Hoyle, D. J. Kapner and A. Upadhye, *Phys. Rev. Lett.* 98 (2007) 131104.
[29] I. Antoniadis, E. Kiritsis and J. Rizos, *Nucl. Phys. B* 637 (2002) 92.
[30] I. Antoniadis, E. Kiritsis and T. N. Tomaras, *Phys. Lett. B* 486 (2000) 186; I. Antoniadis, E. Kiritsis, J. Rizos and T. N. Tomaras, *Nucl. Phys. B* 660 (2003) 81.
[31] K. R. Dienes, E. Dudas and T. Gherghetta, *Nucl. Phys. B* 557 (1999) 25 [arXiv:hep-ph/9811428]; N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali and J. March-Russell, *Phys. Rev. D* 65 (2002) 024032 [arXiv:hep-ph/9811448]; G. R. Dvali and A. Y. Smirnov, *Nucl. Phys. B* 563 (1999) 63.
[32] L. Randall and R. Sundrum, *Phys. Rev. Lett.* 83 (1999) 4690 and *Phys. Rev. Lett.* 83 (1999) 3370.
[33] G. R. Dvali, G. Gabadadze and M. Porrati, *Phys. Lett. B* 485 (2000) 208.
[34] H. Verlinde, *Nucl. Phys. B* **580** (2000) 264; S. B. Giddings, S. Kachru and J. Polchinski, *Phys. Rev. D* **66** (2002) 106006.

[35] I. Antoniadis, R. Minasian and P. Vanhove, *Nucl. Phys. B* **648** (2003) 69 [arXiv:hep-th/0209030].

[36] For a review see e.g. R. Contino and A. Pomarol, Comptes Rendus Physique **8** (2007) 1058; and references therein.

[37] C. Deffayet, G. R. Dvali and G. Gabadadze, *Phys. Rev. D* **65** (2002) 044023 [arXiv:astro-ph/0105068].

[38] M. A. Luty, M. Porrati and R. Rattazzi, arXiv:hep-th/0303116.

[39] G. R. Dvali and G. Gabadadze, *Phys. Rev. D* **63** (2001) 065007; G. R. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, *Phys. Rev. D* **64** (2001) 084004.

[40] S. L. Dubovsky and V. A. Rubakov, *Phys. Rev. D* **67** (2003) 104014 [arXiv:hep-th/0212222].

[41] V. A. Rubakov, arXiv:hep-th/0303125.

[42] M. Kolanovic, M. Porrati and J. W. Rombouts, *Phys. Rev. D* **68** (2003) 064018 [arXiv:hep-th/0304148].

[43] A. Lue and G. Starkman, *Phys. Rev. D* **67** (2003) 064002.