Fokker-Planck simulation of non-local thermal smoothing due to non-uniform laser heating

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Abstract. A 1D Fokker-Planck code has been developed to investigate the thermal smoothing in direct-drive laser fusion. The results from Fokker-Planck simulations are compared with those from a 1D fluid code basing on Spitzer-Härm heat conduction theory. It is found that when the temperature profiles are provided by a sinusoidal modulating laser heating, the thermal smoothing is reduced from Fokker-Planck simulations due to strongly inhibited heat flux. We have compared with an extended heat flux formula and get a good consistency qualitatively.

1. Introduction

It is well known, that illumination uniformity is a crucial factor to the success of direct-drive laser fusion. Non-uniformities in spherical target illumination will lead to uneven energy deposition and the temperature corrugations in absorption region, which become a seed of Rayleigh-Taylor instability and deteriorate the implosion efficiency [1,2]. Longitudinal electron energy transport from underdense plasma with laser absorption via inverse bremsstrahlung (IB) process to high density solid surface has been extensively investigated in laser fusion target since it determines the implosion efficiency. Heat flux inhibition due to steep temperature gradient near cut-off density and enhanced heating near solid surface due to non-thermal tail components of electron energy distribution were found to play an important role. Alternatively, transverse (or lateral) energy transport to the direction of laser propagation and associated thermal smoothing of temperature corrugations are also a key ingredient which influences the implosion efficiency.

It is widely recognized that the electron energy distribution in laser fusion target is determined by the interplay between the effect of IB energy absorption and spatial energy transport with steep temperature gradient and generally exhibits non-Maxwellian nature. Epperlein \textit{et al.} [3] performed tow-dimensional Fokker-Planck (FP) simulations to investigate the effect of thermal smoothing. Contrary to an expectation that the non-local nature of heat transport may enhance the smoothing efficiency specifically in the regime such as \(l\) (inhomogeneity scale length) \(<80l_{\text{mfp}}\) (electron-ion collision mean free path), the smoothing efficiency is found to be weaker than that due to a flux limited Spitzer-Härm (SH) estimate. This suggests that the interplay between strong IB absorption process and its coupling to the spatial transport is a complex problem.
In this paper, in order to study the thermal smoothing problem originated from laser non-uniform illumination, we have developed a 1D fully kinetic FP code in the diffusive approximation by applying physical model and numerical methods implemented by Epperlein [4]. With a periodical boundary condition and an IB heating term [5], we have investigated the details of temperature corrugation due to laser non-uniform illumination with sinusoidal function form in the transverse direction and associated distribution functions. In order to compare with SH theory [6], we also program a one-dimensional fluid code to calculate the energy equation. We find that heat flux is strongly inhibited and the thermal smoothing is reduced as compared to the SH results.

2. Simulations of thermal smoothing

The physical model and numerical methods of this FP code are the same as Epperlein’s [4]:

\[
\frac{\partial}{\partial t} f_0^{(e)} = \nabla \cdot \left[ \chi (\nabla f_0 + a \alpha f_0) \right] + \frac{1}{v} \nabla \cdot \left[ \chi (a \cdot a \frac{\partial f_0}{\partial v} + \nabla \cdot a f_0) \right] + Y_\infty (C_0 f_0 + D_0 \frac{\partial f_0}{\partial v}) + \frac{n_e Z^2 Y_e v_0^2 \frac{\partial f_0}{\partial v}}{6v}
\]

(1)

The meanings of variables are given in Epperlein’s paper.

In our simulations, we assume a fully ionized plasma with \( Z = 4 \), electron number density \( n_e = 10^{21} \text{ cm}^{-3} \) and initial electron temperature \( T_e = 100eV \). We normalize the time and space by the electron-electron collision time \( \tau_{ee} \) and mean free path \( \lambda_{ee} \) with temperature \( 4T_0 \).

For simplicity, in the direction normal to that of laser propagation we consider a sinusoidal intensity modulation of laser beams given by \( I = I_0 [1 + \epsilon \sin(2\pi kx / L)] \), where \( I_0 \), \( \epsilon \), \( k \) and \( L \) represent the average laser intensity, perturbation factor, simulation box scale and modulated wave number respectively. We assume the periodical boundary condition in the x-direction as we set the perturbed laser heating to be a sinusoidal function.

2.1. Comparison between FP simulation and SH theory

We keep the laser heating for \( 10\tau_{ee} \) and record the temperature profile at this moment, then stop the heating and investigate the relaxation. A parameter scan is given for \( L = 200\lambda_{ee} \), \( I_0 = 1 \times 10^{14} - 2 \times 10^{16} \text{ W cm}^{-2} \), \( \epsilon = 0.1 - 1 \) and \( k = 1, 2, \) and \( 4 \). We also calculate the temperature evolution with the SH heat equation (2) predicted by SH local theory, in which the initial temperature profile is given as that recorded when time is \( 10\tau_{ee} \) in FP simulation. Figure 1 shows the time evolution of temperature profiles for the cases \( k = 2 \) and \( 4 \). The non-sinusoidal nature of temperature profile, specifically near low temperature region, results from the SH conductivity during the heating. It is clearly seen that the thermal smoothing of SH becomes faster than that of FP.

Figure 2 shows the time evolution of the temperature variation \( \Delta T = T_{\text{max}} - \bar{T} \), where \( T_{\text{max}} \) is the maximum of the temperature corrugation and \( \bar{T} \) is the average value of temperatures in this simulation box. The dotted curves in Figure 2 show the SH results with a fluid code, which are much different from those results calculated by FP codes (solid curves). The relaxation mainly depends on the initial temperature profile, such as different k number from Figure 2.

In order to understand the underlying physical mechanism, we investigate the heat flux \( Q \) normalized by the local free streaming value \( Q_{\phi} = n_e k_B T_e (k_B T_e / m_e)^1 / 2 \) with respect to \( L_T / \lambda_{ee} \), where \( L_T = T_e / (dT_e / dx) \) is the local temperature gradient scale length and \( \lambda_{ee} = (k_B T_e)^2 / (4\pi n_e e^2 \ln \Lambda) \) is the e-e collision mean free path. The SH heat flux formula [6] provide \( \frac{Q_{\text{SH}}}{Q_{\phi}} = (128 / 4\sqrt{2\pi} Z^2) (L_T / \lambda_{ee})^{-1} \), leading to \( \frac{Q_{\text{SH}}}{Q_{\phi}} \approx 6.6 \times (L_T / \lambda_{ee})^{-1} \) for \( Z = 4 \). In Figure 3 and 4, the blue lines represent the SH heat flux, whereas the red dotted curves represent the results of FP simulations. A group of 3 plots for a single k, i.e. Figure 3(a)-(c), show the time
evolution of heat flux. The last plot of a group such as (c) of (a)-(c) presents the result of a relative long time relaxation. These plots show a strong phenomena of flux inhibition especially when the wave number \( k \) is larger. It is interesting that the difference between two heat fluxes is relative to \( k \), seeing Figure 4(a)-(c).

Qualitatively, we can estimate the heat flux from an extension theory of SH by Kishimoto et al. [7,8]:

\[
Q = -\kappa_{SH} \left\{ \frac{dT}{dx} \right\} \left[ \delta_1 \frac{\lambda}{T_e} \frac{d^2T}{dx^2} + \delta_2 \frac{\lambda^2}{T_e} \frac{d^3T}{dx^3} + \delta_3 \frac{\lambda}{T_e} \frac{d^2T}{dx^2} \right]
\]  

(2)

where \( \delta_1 = 52.12 \times 10^3 \), \( \delta_2 = 32.14 \times 10^3 \), and \( \delta_3 = 1.95 \times 10^3 \) given in reference [8]. If we consider a sinusoidal temperature profile \( T_e = T_0 \left[ 1 + \varepsilon \sin(kx) \right] \), we can get

\[
Q = -\kappa_{SH} \left\{ \frac{dT}{dx} \right\} \left[ (\lambda \varepsilon^2 \cos^2(kx) - \delta_3 \varepsilon \sin(kx) - \delta_1) \right]
\]  

(3)

Substituting the above values, we can derive \( \delta_3 \varepsilon^2 \cos^2(kx) - \delta_1 < 0 \) when \( \varepsilon < 0.2 \) and \( -\delta_3 \varepsilon \sin(kx) - \delta_1 < 0 \) when \( \varepsilon < 0.06 \) for any value of \( x \), which means \( |Q| < |Q_{SH}| \) when \( \varepsilon \) is small enough (it is satisfied for a long time relaxation). At the same time, the difference \( \Delta Q \) between \( Q \) and \( Q_{SH} \) is proportional to \( k^2 \) shown as Figure 4. So the reduction of heat flux takes place and the imprinting effect survives in small perturbation limit.

2.2. Influences of different initial distribution functions on relaxation
Compared with the above IB heating case, we set up the recorded temperature profile when time is $10\tau_{ee}$ with an initial Maxwellian distribution and make a relaxation. In Figure 2, solid curves refer to IB heating results and dash curves are Maxwellian results. The relaxations of Maxwellian cases are a little faster than those of IB heating cases that the distribution is a super-Gaussian, but the difference between them is very small.

This small difference can be explained by the plots of heat flux. For example, when $k=2$, shown in Figure 3(a) and (d), only at a very initial time during relaxation, the heat flux of Maxwellian case is larger than that of IB case. Soon the difference disappears during relaxation, shown in Figure 3(b), (e) and (c), (f).

The electron distribution functions at the places where the temperatures are the maximum (A) and the minimum (C) in Figure 1(a) are given in Figure 5. For IB case in Figure 5(a), the distribution functions with less high energy tail depart much from Maxwellian initially, due to which the heat flux in Figure 3(a) is much smaller than that of Figure 3(d). During relaxation process, e-e collision and non-local heat transport make the distribution functions become Maxwellian soon, shown in Figure 5(b), so that the thermal smoothing with different initial distribution functions becomes nearly no different. This kind of influence by initial distribution functions seems much less than our expectation. The thermal smoothing mainly depends on the initial temperature profile but not distribution functions.

3. Conclusions

We have investigated the thermal smoothing in transverse direction in laser fusion. A 1D FP code and a 1D fluid code for SH heat conduction have been used in numerical modeling of sinusoidal form temperature perturbations. For this kind of temperature perturbations, the magnitude of the heat flux is strongly inhibited and the thermal smoothing is reduced as compared to the SH results. A possible reason has been given by analyzing a modified heat flux theory.

References

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