Hydrodynamics of Newtonian and power-law fluids
in microchannel with superhydrophobic wall

S A Vagner¹ and S A Patlazhan²

¹ Institute of Problems of Chemical Physics of the Russian Academy of Sciences, Academician Semenov Avenue 1, Chernogolovka, Moscow Region 142432, Russia
² Semenov Institute of Chemical Physics of the Russian Academy of Sciences, Kosygina 4, Moscow 119991, Russia
E-mail: sapat@yandex.ru

Abstract. The flow peculiarities of the Newtonian and Carreau–Yasuda power-law fluids in a microchannel with the striped superhydrophobic wall is studied numerically. The driving forces leading to deviation of streamlines from the channel axis are analyzed.

1. Introduction
Flow patterns in the macroscopic channels (pipes, extruders, etc) are defined by the Reynolds number and fluid rheological properties. In case of simple liquids (Newtonian media), the increase in average velocity may lead to various flow transitions from the laminar to eddy and then to turbulent flow. The last-named regimes of flow are particularly advantageous for mixing multiphase fluids and for increasing the efficiency of chemical reactors as well as heat transfer [1–4]. In rectilinear channels the vortex flows occur at high Reynolds numbers, \( \text{Re} = \frac{\rho U H}{\eta} \gg 1 \) (\( U \) and \( H \) are the characteristic flow rate and cross-section of the channel, while \( \eta \) and \( \rho \) are the fluid viscosity and density, respectively). However, to reach these values in microchannels is rather problematic. Meanwhile, in the last decade, the microfluidic devices have found a variety of applications in academic and biological research, production of the monodisperse microcapsules for the pharmaceutical industry, phonon and photonic crystals [5–8], etc. Microfluidic chips possess a number of advantages as compared with macroscopic analogs due to the use of small amount of liquid and high speed analysis. At the same time, the flow in microchannels has a number of peculiarities that should be taken into account when designing the microfluidic equipment. The first thing to mention is the high hydrodynamic resistance \( R_h \). In the rectangular channel, for instance, it sharply increases as the channel thickness \( H \) decreases, \( R_h \sim H^{-3} \). For this reason, the flow in microchannels is normally slow and characterized by low Reynolds numbers, \( \text{Re} \ll 1 \). On the other hand, it impedes to the efficient mixing of liquids that may hinder some physicochemical processes such as the synthesis of macromolecules under confinement conditions.

One way of solving these problems is the use of superhydrophobic (SHP) surfaces [9] which make it possible to significantly reduce the hydrodynamic resistance through the effective slip. Such effect becomes noticeable due to a microrelief of a solid wall which prevents penetration of a fluid into the sockets filled with gas (Cassie–Baxter state, figure 1) [10]. It results in
the inhomogeneous interface with alternating slip and stick boundary conditions over the gas-filled and solid areas, respectively. The slip intensity is characterized by the so-called slippage length defined as an extrapolated distance from the SHP wall to the point where the velocity vanishes [11].

There are isotropic and anisotropic SHP surfaces [12]. In contrast to the first case, the effective slippage length of the anisotropic coatings changes with flow direction. We shall consider the periodic SHP striped texture consisted of parallel stripes and grooves. The highest effective slippage length takes place along the texture symmetry axis, while the lowest one is in perpendicular direction [13]. This difference makes it possible to solve the second of the above mentioned problems—to implement passive fluid mixing in a microchannel at low Reynolds numbers. Indeed, a deviation of stripe orientation from the channel axis will cause deflection of fluid velocity from the main direction and form the helical secondary flow [14]. The effectiveness of passive mixing depends on the period of the helical flow. It is dependent of streamlines deflection angle $\gamma$: the larger is $\gamma$, the smaller is the period of the helix and better is the fluid mixing.

Along with Newtonian fluids, polymeric solutions and melts with viscosity dependent of shear rate are used in microfluidic devices. Such non-Newtonian rheological properties may result in a modification of the streamlines in microchannels with an anisotropic SHP wall. This assumption makes sense because viscosity of non-Newtonian fluids may undergo significant changes due to large fluctuations of the velocity gradient in the transition zones between stripes and grooves and thus can affect flow characteristics. In this paper we consider an origin of these processes as applied to Newtonian and non-Newtonian power-law fluids. The driving forces leading to deviation of the streamlines from the main flow direction in a microchannel are analyzed.

2. Model for numerical calculations
We consider the simple shear flow of incompressible fluid between two parallel plates. The bottom wall is covered by the periodic SHP striped texture in the Cassie–Baxter state [10]. The comparison of theoretical and experimental data showed that the curvature of the fluid meniscus over the grooves has virtually no effect on the flow velocity profile [15, 16]. For this reason, the SHP surface can be considered as a plane interface with alternating stick and slip boundary conditions. The translational symmetry allows us to restrict the consideration to the computational domain of length $L$ corresponding to period of the striped texture. The surface fraction of the slippery areas is equal to $\varphi = \delta/L$, where $\gamma$ is the groove width. The periodic conditions for the velocity and pressure fields are imposed on the lateral boundaries of the computational domain shown in figure 2. The upper wall of the channel is moving with a rate $U$ directed at the angle $\theta$ to the texture axis while the basic shear rate $\dot{\gamma} = U/H$ is kept constant at all values of the microchannel thickness $H$. 

![Figure 1. Fluid in microchannel with SHP wall at the Cassie–Baxter state.](image)
The flow of incompressible fluid is governed by the Navier–Stocks equation and incompressibility condition:

$$
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = - \nabla p + \nabla (2\eta \mathbf{D}) \quad \text{and} \quad \nabla \cdot \mathbf{u} = 0, 
$$  \hspace{1cm} (1)

where $\mathbf{u} = (u, v, w)$ is the fluid velocity; $\mathbf{D} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$; $\rho$ and $\eta$ are the fluid density and viscosity, respectively. The stick boundary condition $\mathbf{u}_{L-S} = 0$ imposed on the solid parts of the channel walls while the Navier boundary conditions $\frac{d\mathbf{u}}{dz} \bigg|_{L-G} = 0$ at the infinite slippage length take place over the liquid–gas.

The viscosity of the Newtonian fluid is kept constant $\eta_0$ whereas viscosity of the non-Newtonian power-law fluid is considered to change with shear rate in an accordance with the Carreau–Yasuda rheological model [17]:

$$
\eta (\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty) \left[1 + (\dot{\gamma})^a\right]^{\frac{n-1}{a}}, 
$$  \hspace{1cm} (2)

where $\eta_0$ and $\eta_\infty$ are the fluid viscosities at low and high shear rates, respectively. At the intermediate values of shear rates viscosity $\eta(\dot{\gamma})$ the power-law fluid decreases with increase of $\dot{\gamma}$ at $n < 1$. This reflects the shear thinning effect inherent to polymeric liquids due to stretching of macromolecules along the flow direction. Figure 3 shows the rheological curve of the aqueous polyacrylamide solution of concentration 500 ppm, which is drawn according to equation 2 for the following parameters: $\eta_0 = 1.08 \text{ Pa}\cdot\text{s}$, $\eta_\infty = 0.0023 \text{ Pa}\cdot\text{s}$, $\lambda = 5 \text{ s}$, $n = 0.2$ and $a = 1$ [18]. These data are used in our calculations. For arbitrary strain rate tensor $\mathbf{D}$, the viscosity of the Carreau–Yasuda fluid takes the following form

$$
\eta (\mathbf{D}) = \eta_\infty + (\eta_0 - \eta_\infty) \left[1 + \left(2\lambda^2 I_2 \right)\right]^{\frac{n-1}{a}}, 
$$  \hspace{1cm} (3)

where $I_2 = \mathbf{D} : \mathbf{D} = \sum_{i,j} D_{ij} D_{ij}$ is the second invariant of the strain rate tensor. Equation 3 is relevant in solving the hydrodynamic problem under the inhomogeneous slip–stick boundary conditions when the local $\mathbf{D}$ is different from the basic strain rate tensor.

Equations 1 and 3 were solved numerically by means of the finite volume method and PISO algorithm on the basis of the CFD package OpenFoam [19]. The calculations were performed.
on the non-uniform computational mesh with the orthogonal cells with the density varied as approaching the SHP wall. 50 cells of the mesh were accounted for by one period of the striped texture. The analysis showed that further increase in the mesh density does not lead to a noticeable change in results. Therefore, the considered sampling of the computational domain was found to be optimal.

3. Results and discussion

Figure 4 shows the calculated streamlines of the Newtonian (black) and power-law (red) fluids flowing through the rectangular channel making an angle \( \theta = 45^\circ \) with the stripes at the lower wall at \( H/L = 0.3, \varphi = 0.5 \). It can be seen that in both cases the double helical flow is formed. The period of the microhelix is of the same order as period \( L \) of the SHP texture, whereas period of macrohelix depends on the deflection angle \( \gamma \) of the average streamlines and on the width of microchannel. It is important to note that period of macrohelical flow of the Newtonian fluid is less than that of the power-law fluid.

To understand an origin of this phenomenon, consider the physical mechanism favoring a deviation of streamlines from the channel axis. Figure 5a shows streamlines and coloured pressure field near the SHP wall. It is evident that the pressure field is not uniform: the low pressure zones are formed at the transition from domains with stick to slip boundary conditions (blue bands), while the high pressure zones appear during transitions from the slippery to sticky domains (orange bands). Such pressure field increases the flow velocity over the liquid–gas interfaces but decreases it over the domains with stick boundary conditions. In the transversal directions lying in the horizontal (figure 5a) and vertical planes (figure 5b), the pressure distribution is also inhomogeneous. This generates secondary flows resulting in deviations of streamlines in different directions with regard to the channel axis thus inducing the microhelical flow. Note that a similar pattern holds for the simple shear flow.

On the other hand, the averaged streamline also deviates from the channel axis near the SHP wall (figure 6). This is the root cause of the subsequent formation of the macroscopic helical flow. The corresponding deflection angle \( \gamma \) is determined by the local deviation angles \( \alpha \) and \( \beta \) over domains with slip and stick boundary conditions, respectively:

\[
\gamma = \theta - \arctan \left( \frac{\tan \alpha \tan \beta}{(1 - \varphi) \tan \alpha + \varphi \tan \beta} \right).
\]
Figure 4. The streamlines in a rectangular microchannel with SHP striped texture (gray) on the bottom wall at $H/L = 0.3$, $\varphi = 0.5$, $\theta = 45^\circ$. Black and red streamlines correspond to the Newtonian and Carreau–Yasuda power-law fluids, respectively.

Figure 5. The streamlines (solid curves) and the pressure field near the SHP wall (a) and in the cross sections 1 and 2 (dashed lines) (b). The colors correspond to pressure scales.

Figure 6. The mechanism of formation of the deflection angle $\gamma$ of the average streamline (dashed line). The bright and dark domains correspond to liquid–gas and liquid–solid interfaces, respectively.

The angles $\alpha$ and $\beta$ were calculated by simulating the shear flow in the computational domain representing by figure 2. Substituting $\alpha$ and $\beta$ into equation 4, we found that the deflection
angle $\gamma$ coinciding with the slope angle of the dotted line which corresponds to the average streamline. It is interesting to consider the dependence of the deflection angle $\gamma$ of the average streamlines on the relative thickness $H/L$ of microchannel. The obtained results are presented in figure 7a for the anisotropic SHP texture with the equal portions of solid stripes and grooves, $\varphi = 0.5$, at the slope angle $\theta = 45^\circ$. It is seen that the deflection angle of the Newtonian fluid is substantially greater than that of the Carreau–Yasuda power-law medium at any $H/L$ values. This is consistent with the results presented in figure 4: the period of the helical flow of the non-Newtonian fluid is larger than that of Newtonian liquid, which is associated with the difference in the deflection angles of the average streamlines. In either case the deflection angle increases with the channel thickness and then reaches the limiting value. For the power-law fluid, the angle levels off at higher $H/L$ value than the Newtonian one. Note also that passes through minimum for both of the fluids at $H/L < 1$.

In order to understand these features of the deflection angle $\gamma$, consider behavior of the streamlines deviation angles $\alpha$ and $\beta$ (figure 6) of the Newtonian and power-law fluids as functions of the relative thickness of microchannel. Figure 7b shows that deviation angles $\beta$ over the regions with stick boundary conditions coincide for both of the fluids at any $H/L$ values. On the other hand, for the Newtonian and power-law fluids deviation angles $\alpha$ over the

Figure 7. Deviation angles $\gamma$ (a) and $\alpha$ and $\beta$ (b) versus relative microchannel thickness $H/L$ for Newtonian (1) and Carreau–Yasuda power-law (2) fluids.
Figure 8. Viscosity profiles of the Carreau–Yasuda power-law fluid over the central points of the solid stripe (a) and groove (b) of the SHP texture at different values of the relative thickness of microchannel: $H/L = 0.5$ (1), $1.0$ (2), $2.5$ (3), $5.0$ (4).

Figure 9. Variations of the deflection angle of the average streamline with distance from the bottom wall for the Newtonian (a) and Carreau–Yasuda power-law (b) fluids at different values of the relative thickness of microchannel: $H/L = 0.5$ (1), $1.0$ (2), $2.5$ (3), $5.0$ (4).

Liquid–gas interfaces of the SHP texture differ significantly. Thus, the difference between the deflection angles $\gamma$ of the considered media is governed by the distinction in the respective local angles $\alpha$. The maxima on the curves $\alpha(H)$ at $H/L < 1$ explain the respective minima on $\gamma(H)$ dependences for both of fluids.

The difference in the behavior of the local deviation angle over the slippery domains for the Newtonian and power-law fluids can be explained by peculiarities of viscosity variations of the latter fluid in a vicinity of the striped texture. Figure 8 shows viscosity profiles of the Carreau–Yasuda power-law fluid over the center points $x = L/4$ and $x = 3L/4$ of sections with stick (figure 8a) and slip (figure 8b) boundary conditions, respectively. It is seen that due to dependence of viscosity of the non-Newtonian fluid on the local shear rate, the more viscous
domain is formed near the liquid–gas interface of the SHP texture, whereas the low-viscous domain appears over the region with stick boundary conditions. The relative thickness of these domains was found to increase with decreasing of the microchannel thickness. If we compare the dependences of deviation angles $\alpha$ and $\beta$ of the relative thickness $H/L$ (figure 7) with the corresponding behavior of the viscosity profiles (figure 8), it can be seen that a reduction in viscosity of the power-law fluid above the solid stripes is not considerable and therefore has no significant impact on the behavior of streamlines of the Carreau–Yasuda fluid relative to the Newtonian one. For this reason, the local deflection angles $\beta$ of the non-Newtonian and Newtonian fluids are practically coincident. At the same time, viscosity of the power-law fluid over the sliding zones of the SHP texture is increased. In turn, this leads to a noticeable increase in the local deviation angle of streamlines as compared with that of the Newtonian medium.

To explain the reasons for the slower growth of the deflection angle $\gamma$ of the non-Newtonian power-law medium with the increase in the relative channel thickness $H/L$ as compared with the Newtonian fluid, let us consider the behavior of $\gamma$ along the $z$-axis as is shown in figures 9a and 9b. In fact, these plots correlate with the distribution of perturbations of the velocity vector induced by the SHP striped texture. It is seen that for a sufficiently thick channel ($H/L \geq 5$) the deflection angle $\gamma$ of the power-law fluid is larger than that of Newtonian one. This indicates that the perturbations in the velocity vector of the non-Newtonian medium decay slower than that of Newtonian fluid. Therefore the deflection angle $\gamma$ of the power-law fluid reaches its limit value at a larger thickness of microchannel.

4. Conclusions
The foregoing mathematical modeling of flow peculiarities of the Newtonian and non-Newtonian power-law fluids allowed one to find a number of regularities related to mechanisms of passive mixing in a microchannel with the anisotropic superhydrophobic wall. It was found that inhomogeneous stick–slip boundary conditions caused by the tilted striped SHP texture induce a nonuniform distribution of pressure both in the longitudinal and in the transversal sections of the channel. The consequence is a deviation of the streamlines from the channel axis followed by formation of the micro- and macrohelical flow. The deflection angle of the average streamlines of Newtonian fluid is substantially larger than that of Carreau–Yasuda power-law medium at any thickness of microchannel. It results in an increase in the period of the macroscopic helix of streamlines of the power-law fluid which in turn leads to a decrease of the mixing efficiency as compared to the Newtonian fluid.

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