Physics of heavy neutrinos

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Abstract

Theoretical and experimental situation in physics of heavy neutrinos ($M_N > M_Z$) is briefly presented. Various experimental bounds on heavy neutrino masses and mixings are shortly reviewed. Special attention is paid to possibility of detecting heavy neutrinos in future lepton linear colliders.

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1 Introduction

After the discovery of the top quark, neutrinos (and Higgs particle - not observed yet) remain the most elusive particles. Since very beginning neutrinos have played an important part in our understanding of the laws of particle physics. They are the only particles which interact by only one type of fundamental interaction, the weak one. The weak interaction of the other particles is suppressed by their electromagnetic and strong ones. To understand how important the weak interaction of neutrinos is in Nature (especially for us) let’s mention just the mechanism in which the Sun is shining. Without any doubt investigation of properties of these particles can reveal many interesting, hidden until now physical phenomena or explain many hypothetical ideas. For instance existence of nonzero mass of neutrinos, besides theoretical interest, could explain some astrophysical or cosmological problems [1]. This feature of neutrinos would boost particle physics beyond the Standard Model (SM). Until now we know only that three known neutrinos are very light ($m_{\nu_e} \leq$ few eV, $m_{\nu_\mu} \leq 270$ keV, $m_{\nu_\tau} \leq 24$ MeV).

Many models beyond the SM predict massive neutrinos. Moreover, except the light neutrinos they predict very heavy ones: this is for instance in the case of so-called ‘see-saw’ models [2].

In this talk we give the short review of theoretical and experimental situation connected with heavy neutrino physics. In the next Section we specify areas where heavy neutrinos could reveal themselves. Section 3 will be devoted specially to future linear $e^+e^-$ and $e^-e^-$ colliders and possibility of finding heavy neutrinos there. These considerations will be given in the frame of the simplest extension of the SM including right-handed neutrinos.

Conclusions and outlook will be given in the Summary.
2 Where to hunt for a very heavy neutrino?

Heavy neutrinos have been looked for since the early seventies [3]. From the negative search of new neutral states and from the measurement of Z decay width at the LEP we know that there are no neutrinos with a standard coupling to Z and mass below $M_Z/2$ [4] or even below $M_Z$ if $BR(Z^0 \rightarrow \nu N) > 3 \cdot 10^{-5}$ [5]. The lack of detection of new neutrino states at the LEPI indicates that if they exist, they will generally have large masses ($\geq M_Z$).

Let’s describe shortly where such heavy neutrinos could be found.

2.1 Influence of heavy neutrino states on observables measured at LEP and low energy experiments

Even if they can not be directly produced at LEP now, it is still possible that with increasing precision of measurements their effects could be indirectly detected as small deviations of couplings of light neutrinos from their standard values. This could happen for any new neutrino states if they mix with the ordinary ones. As an example let us mention the observation made by C. Jarlskog [6] and discussed by other authors [7].

In the SM with $n$ left-handed lepton doublets and $n' = 1, 2, \ldots$ right-handed neutrinos the effective number of neutrino species $\eta_{\text{exp}}$ measured at the $Z^0$ peak and defined by ($\Gamma_0$ is the SM $Z^0$ decay width to the pair of massless neutrinos)

$$\Gamma(Z^0 \rightarrow \text{neutrinos}) = \Gamma_0 \eta_{\text{exp}}$$

fullfills relation

$$\eta_{\text{exp}} \leq n.$$

That means that any measured value $\eta_{\text{exp}}$ slightly below 3 (number of left-handed lepton doublets in the SM) would indicate existence of right-handed neutrinos (at the moment the best fit for $\eta_{\text{exp}}$ is $\eta_{\text{exp}} = 2.991 \pm 0.016$ [4]).

Similar phenomena could be observed for other LEP observables as $\Gamma_Z$, $Z$ partial decay widths, asymetries measured at the $Z$ resonance, W-mass and
low energy experiments as $\beta$, $\tau$ and $\pi$ decays, $\nu$–scattering, atomic parity violation, polarized e-D scattering, etc. Global analysis of fermion mixings with new neutral states can be found in [3,8].

2.2 Heavy neutrinos in hadronic colliders

This possibility for detecting heavy neutrinos was examined in many papers [9-13]. The especially big hope was connected with construction of the Superconducting Super Collider (SSC) [10]. After cancellation of this project the possible options which remain are $e^-p$ and $pp$ at HERA, HERA upgrade and LEP+LHC colliders. According to [11] masses up to $\sim 160$ GeV, $320$ GeV and $700$ GeV can be tested in ep collisions at HERA, HERA upgrade and LEP+LHC, respectively. Let’s note however that such optimistic results for $e^-p$ collider are predicted for very large mixing angle $\xi = 0.1$ which is much above up-to-date constraints on $\xi$ (see the next section). $pp$ super-colliders could give detectable results through $pp \rightarrow W_R \rightarrow l^+N_l \rightarrow l^+l^+q\bar{q}'$ reaction (quark fusion) [12] or through the gluon fusion mechanism with off-shell Z gauge boson ($gg \rightarrow Z^* \rightarrow N\bar{N}$) [13].

2.3 Induced heavy neutrino loop effects

Some authors [14] indicate that significant rates are in general possible for one-loop-induced rare processes as $\mu - e$ conversion in nuclei, $\mu(\tau) \rightarrow 3e$, $\mu \rightarrow e\gamma$ due to exchange of virtual heavy neutrinos. The possibility of detecting such lepton number violating processes could arise when heavy neutrinos do not decouple in low energy processes. This can happen in other than ‘see-saw’ models [3,15].

2.4 Neutrinoless double-beta decay

The search for neutrinoless double-$\beta$ decay ($\langle \beta\beta \rangle_{0\nu}$)

$$(A, Z) \rightarrow (A, Z \pm 2) + 2e^-$$
is the most promising method for the discovery of light Majorana masses. The reaction is also sensitive to heavy neutrinos’ contribution. There are about 40 different experiments being carried out now in which people are looking for this type of reaction.

2.5 Indirect detection of heavy neutrinos in neutrino oscillation experiments

This interesting possibility was discussed in [16]. It was shown that if there is a see-saw type mixing between light and heavy Majorana particles and the mixing matrix is complex then the $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ transition probabilities could be different and indicate (indirectly) presence of heavy neutrinos.

None of the processes including heavy neutrinos described above have been discovered till now and only some constraints on allowed heavy neutrino masses and mixings can be derived from them. The most important constraints will be presented in the next section when we shall deal in details with the last, very important area where heavy neutrinos can be found - future linear lepton colliders. To complete the review let’s mention that a heavy neutrino is naturally highly unstable so no discrepancy with cosmology appears here.

3 Heavy neutrinos in the Next Linear Colliders

Recently hadron colliders gave spectacular results when W and Z bosons and the top quark were discovered. Nevertheless, in the meantime a lepton collider, LEPI, has reached spectacular results too, specially these connected with excellent precision with which the SM has been tested. The next planned $e^+ e^-$ colliders with energy up to 2 TeV [17] can become even more important as a tool in looking for new physics beyond the SM, for instance
connected with detection of heavy neutrinos. The last part of this talk will be devoted to the physics of heavy neutrinos in these future colliders.

We’ll focus on two reactions: the direct heavy neutrino $e^+e^- \rightarrow \nu N$ production and the indirect process with heavy neutrino exchange $e^-e^- \rightarrow W^-W^-$. This latter process is possible as the $e^-e^-$ option of the next linear accelerators and is seriously taken into account [18]. The $e^-e^-$ environment is much cleaner than the $e^+e^-$ one. There is much less SM activity and that is why it allows to explore even very weak signals of flavour violating processes as this given above. The values of cross sections which we are going to find depends on the model in which we calculate them. So called ‘see-saw’ models belong to the most popular ones as they can give an theoretical explanation for a smallness of known neutrino masses [2].

As an illustration let’s take the simplest model with massive neutrinos - the SM with additional right-handed neutrinos (RHS model). In the RHS model there are 3 left-handed and $n_R (=1,2,...)$ right-handed weak neutrino states transforming under $SU_L(2)$ gauge group as doublets and singlets, respectively. The neutrino mass matrix has $3 + n_R$ dimensions

$$M_\nu = \begin{pmatrix} 3 & n_R \\ 0 & M_D \\ M_D^T M_R \end{pmatrix} \begin{pmatrix} 3 \\ n_R \end{pmatrix}.$$ (1)

Without Higgs triplet fields the $3 \times 3$ dimension part $M_L$ of $M_\nu$ equals zero

$$M_L = 0.$$ (2)

Using $(3 + n_R)$ dimensional unitary matrix

$$U = \begin{pmatrix} K^T \\ U_R \end{pmatrix}$$ (3)

which acts on the weak neutrino states, we can diagonalize matrix $M_\nu$ ($U^T M_\nu U = M_{diag}$) and get the physical states.

Without loosing the generality we can assume that the charged lepton mass matrix is diagonal, so then the physical neutrino $N = (N_1,...,N_{3+n_R})^T$
couplings to gauge bosons are defined by \((\hat{l} = (e, \mu, \tau)^T, P_L = \frac{1}{2}(1 - \gamma_5))\)

\[
L_{CC} = \frac{g}{\sqrt{2}} \bar{N} \gamma^\mu K P_L \hat{l} W_{\mu}^+ + h.c.,
\]

\[
L_{NC} = \frac{g}{2 \cos \theta_W} \left[ \bar{N} \gamma^\mu P_L (K K^\dagger) N \right].
\]

For instance for \(n_R = 3\) we get three light (known) neutrinos and three very heavy \(M_{1,2,3} \geq M_Z\) ones as \(M_R\) and \(M_D\) are proportional to different scales of symmetry breaking and \(|M_R|_{ii} > |M_D|_{lk}\). Then without any additional symmetry the matrix elements \(K_{ae}\) are proportional to \(<M_D>/M_a\). Typically \(<M_D> \sim 1\) GeV so \(K_{ae}\) is very small and very sensitive to the \(M_a\) mass. The process \(e^- e^+ \rightarrow \nu N\) is proportional to \(|K_{ae}|^2\) [19] and the \(e^- e^+ \rightarrow W^- W^-\) to \(|K_{ae}|^4\) [20] and typical cross sections as a function of \(M_N\) for different \(\sqrt{s}\) energies are given in Fig.1 (taken from [19]) and Fig.2. One can see that it is not possible to detect the \(e^- e^+ \rightarrow W^- W^-\) process (the ‘detection limit’ on the \(\sigma=0.1\) fb level is reasonable for this process [21]). The cross section for the \(e^- e^- \rightarrow \nu N\) process is small. However, the ‘see-saw’ mechanism is not the only scenario which explains small masses of the known neutrinos. There are models based on symmetry argument [15] where no simple relations connected \(M_a\) with \(K_{ae}\) are present. In this case the mixing matrix elements are independent parameters and as such are bounded only by experimental data. There are four different and important sources of constraints on heavy neutrino mixings coming from experiments.

(i) From LEPI we know that if neutrinos with masses below \(M_Z\) exist their couplings to \(Z_0\) should be such that \(Br(Z \rightarrow N\nu) \leq 3 \cdot 10^{-5}\) [5] (what is equivalent to \(K_{ae}^2 \leq 8 \cdot 10^{-5}\)). Because this mixing is very small we resign from study very tiny effects connected with neutrinos with \(M_N < M_Z\) and we’ll only study the case \(M_N \geq M_Z\).

(ii) Low energy experiments (e.g. lepton universality, the \(\mu\) decay) and LEPI give also information about heavy neutrinos with masses above \(M_Z\). The reason is that due to unitarity properties of the \(U\) matrix (Eq.(3)), the nonzero mixing elements \(K_{ae}\) slightly reduce the couplings of
light neutrinos from their SM values thus affecting all processes including neutrinos [3] (in the SM matrix $K$ in Eqs.(4,5) equals I). The up-to-date limit for RHS model is [22]

$$\kappa^2 = \sum_{a=\text{heavy}} K_{ae}^2 \leq 0.0054. \quad (6)$$

(iii) The lack of signal of neutrinoless double-$\beta$ decay ($\beta\beta_0\nu$) gives the bound for light neutrinos

$$\left| \sum_{\nu(\text{light})} K_{\nu e}^2 m_\nu \right| < \kappa_{\text{light}}^2 \quad (7)$$

where $\kappa_{\text{light}}^2 < 0.68$ eV [23].

(iv) From the ($\beta\beta_0\nu$) process it is also possible to get the bound for heavy neutrinos

$$\left| \sum_{N(\text{heavy})} K_{N e}^2 \frac{1}{M_N} \right| < \omega^2. \quad (8)$$

Typically the bound is: $\omega^2 < 5.6 \cdot 10^{-4} \text{ TeV}^{-1}$ [24].

The last constraint which we use comes from the fact that the mass term for the left-handed neutrinos is absent

(v) $$\sum_{\nu(\text{light})} K_{\nu e}^2 m_\nu + \sum_{N(\text{heavy})} K_{N e}^2 M_N = M_L \equiv 0. \quad (9)$$

This fact confronted with Eq.(7) gives

$$\left| \sum_{N(\text{heavy})} K_{N e}^2 M_N \right| < \kappa_{\text{light}}^2. \quad (10)$$

This relation includes an interesting information. To get meaningful values of cross sections for the studied processes we need the values of $K_{N e}$ as big as possible. As $\kappa_{\text{light}}^2$ in Eq.(10) is very small the only possibility to
reconcile these two facts is to assume that some $K_{Ne}$ matrix elements are complex numbers. If CP symmetry is conserved then complex $K_{Ne}$ numbers are equivalent to the fact that $\eta_{CP}$ parities of heavy neutrinos are not all equal.

Now we deduce that if CP parities of all heavy neutrinos are the same or we have only one right-handed neutrino ($n_R = 1$) then both considered processes are very small. Situation is different if $n_R = 2$. In agreement with our discussion let’s take heavy neutrinos with opposite CP parities $\eta_{CP}(N_1) = -\eta_{CP}(N_2) = i$ and masses $M_1 = M$, $M_2 = AM$ ($A \geq 1$). Then taking into account Eqs.(6)-(10) the biggest mixing angle $K_{N_1e}$ is for $A \to 1$ (for details see [20]). The result is shown in Fig.3 (taken from [20]) for the $e^+e^- \to \nu N$ process. The solid line represents the biggest result and does not change for $n_R > 2$. However the $e^-e^- \to W^-W^-$ process still remains below the detection limit. This is because for $A \to 1$ we have two degenerate Majorana neutrinos ($M_1 = M_2$) with opposite CP parities which is equivalent to one Dirac neutrino.

The case with $n_R = 3$ changes situation for the $e^-e^- \to W^-W^-$ process. In Fig.4 (taken from [20]) we show the most optimistic results for the $e^-e^- \to W^-W^-$ cross section. Taking $\eta_{CP}(N_1) = \eta_{CP}(N_2) = -\eta_{CP}(N_3) = i$ and $M_1 = M$, $M_2 = AM$, $M_3 = BM$ we found values A,B for which $\sigma(e^-e^- \to W^-W^-)$ reaches maximum. This situation takes place for the very heavy second ($A \gg 1$) and heavier third neutrino ($B \sim 2 - 10$). In this Figure we depict also the cross section for production of the lightest heavy neutrinos with the mass M in the $e^+e^- \to \nu N$ process taking exactly the same mixing angle $K_{N_1e}$ as for the $e^-e^- \to W^-W^-$ process.

We can conclude that

(i) everywhere in the possible region of phase space the production of heavy neutrinos in the $e^+e^-$ process has greater cross section than the lepton

\footnote{The biggest possible $K_{Ne}$ is [20,25] $(K_{Ne})_{max} \simeq \frac{e^{2}}{2} = 0.0027$, that is why $\xi = K_{Ne} = 0.1$ as mention in Section 2.2 is too big.}
violating process $e^-e^-$. It is impossible to find such mixing angles and masses that would show the opposite. The large values of $\sigma (e^+e^- \rightarrow N\nu)$ make this process a good place for the heavy neutrino searching and for future detailed studies (decay of heavy neutrinos, background from other channels [25]).

(ii) there are also regions of heavy neutrino masses outside the phase space region for $e^+e^-$ where the $\Delta L = 2$ process $e^-e^-$ is still a possible place to look for heavy neutrinos. It is a small region $1 \text{ TeV} < M < 1.1 \text{ TeV}$ for $\sqrt{s} = 1 \text{ TeV}$, $1.5 \text{ TeV} < M < 2 \text{ TeV}$ for $\sqrt{s} = 1.5 \text{ TeV}$ and $2 \text{ TeV} < M < 3.1 \text{ TeV}$ for $\sqrt{s} = 2 \text{ TeV}$ where the cross section $\sigma (e^-e^-)$ is still above the ‘detection limit’. There is no such place with the $\sqrt{s} = 0.5 \text{ TeV}$ collider. The experimental value of $\kappa^2$ (see Eq.(6)) would have to be below $\sim 0.004, \sim 0.003, \sim 0.002$ for $\sqrt{s} = 1, 1.5, 2 \text{ TeV}$ respectively to cause these regions to vanish.

The largest value of the mixing parameter $|K_{Ne}|$ for $n_R > 3$ is the same as in the $n_R = 3$ case and we do not obtain quantitatively new results in these cases.

To sum up, we have found the ‘maximum possible’ cross sections for production of the heavy neutrino ($e^+e^- \rightarrow N\nu$ process) and for the inverse neutrinoless double-β decay ($e^-e^- \rightarrow W^-W^-$ process) in the energy range interesting for future lepton colliders (0.5–2 TeV). The upper values for the cross sections are still large enough to be interesting from an experimental point of view. For the $e^+e^- \rightarrow N\nu$ process the cross section could be as large as $275 \text{ fb}$ for $\sqrt{s} = 1 \text{ TeV}$ and $M = 100 \text{ GeV}$. The $e^-e^- \rightarrow W^-W^-$ process could give indirect indication for larger massive Majorana neutrino existence, not produced in the $e^+e^-$ scattering.

4 Summary

In this talk we review the possibilities of detecting heavy neutrinos which are present in plenty of theoretical models beyond the SM. None of the nonstandard processes involving heavy neutrinos has ever been detected. However,
on theoretical ground, narrow windows are still open even after taking into account up-to-date stringent limits on heavy neutrino mixing angles and masses. The most promising are reactions with the ep hadron colliders and the $e^+e^-$ accelerators. Indirect signals of heavy neutrinos presence can be looked for in induced by them loop processes as $\mu \to e\gamma$, $\mu(\tau) \to 3e$ and in the future $e^-e^-$ accelerators.

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Figure Captions

Fig.1 The cross section for the $e^-e^- \rightarrow W^-W^-$ process as a function of the heavy neutrino mass for the ‘classical’ see-saw models, where the mixing angles between light and heavy neutrinos are proportional to the inverse of mass of the heavy neutrino. Solid (dashed) line is for the TLC (NLC) collider’s energy.

Fig.2 The cross section for the $e^-e^+ \rightarrow \nu N$ process in the frame of the ‘classical’ see-saw models. Solid, dashed lines and line with
stars are for 1 TeV, 500 GeV and 200 GeV CM energies of future colliders respectively.

**Fig.3** The cross section for the $e^+e^- \rightarrow N\nu$ process as a function of heavy neutrino mass $M_1 = M$ for $\sqrt{s} = 1$ TeV in the models with two heavy neutrinos ($n_R = 2$) for different values of $A = \frac{M_2}{M_1}$ (solid line with $A = 1.0001$, ‘●’ line with $A = 1.004$, dots line with $A=1.01$ and ‘*’ line with $A=100$). Only for very small mass difference $A \sim 1$ existing experimental data leave the chance that the cross section is large, e.g. $\sigma_{\text{max}}(M = 100 \text{ GeV}) = 275$ fb. If $M_2 \gg M_1$ then the cross section must be small, e.g. for $A = 100$, $\sigma_{\text{max}}(M = 100 \text{ GeV}) \simeq 0.5$ fb. The solid line gives also $\sigma_{\text{max}}(e^+e^- \rightarrow N\nu)$ for $n_R > 2$ (see the text).

**Fig.4** The cross sections for the $e^+e^- \rightarrow N\nu$ and $e^-e^- \rightarrow W^-W^-$ processes as a function of the lightest neutrino mass $M_1 = M$ for different CM energy (the curves denoted by F05, F10, F15 and F20 depicted the cross section for both processes for $\sqrt{s} = 0.5, 1, 1.5$ and 2 TeV respectively) for $n_R = 3$. The cross sections for the $e^-e^- \rightarrow W^-W^-$ process are chosen to be the largest. For the $e^+e^- \rightarrow N\nu$ reaction the cross section for each of neutrino masses is calculated using the same parameters as for $\sigma(e^-e^- \rightarrow W^-W^-)$ and is not the largest one (see the solid line in Fig. 3 for the maximum of $e^+e^- \rightarrow N\nu$). The solid line parallel to the $M$ axis gives the predicted ‘detection limit’ ($\sigma = 0.1$ fb) for both processes.
Figure 1:

Figure 2:
Figure 3:

\[ \sigma \text{ [fb]} \]

\[ e^+ e^- \rightarrow N \nu \text{ for } \sqrt{s} = 1 \text{ TeV} \]

Figure 4:

\[ \sigma \text{ [fb]} \]

\[ e^- e^- \rightarrow W^+ W^- \] \[ e^+ e^- \rightarrow N \nu \]