APPLICATION OF SURVIVAL THEORY IN MINING INDUSTRY

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(Communicated by Kok Lay Teo)

Abstract. The paper deals with an application of survival theory in mineral processing industry. We consider the problem of maximizing copper recovery and determine the best operating conditions based on survival theory. The survival of the system reduces to a problem of maximizing a radius of a sphere inscribed into a polyhedral set defined by the linear regression equations for a flotation process. To demonstrate the effectiveness of the proposed approach, we present a case study for the rougher flotation process of copper-molybdenum ores performed at the Erdenet Mining Corporation (Mongolia).

1. Introduction. In recent years, mineral processing has attracted particular attention of various research communities on account of the emergence of new advanced production and control technologies. The efficiency of the copper flotation process is primarily determined by the content of nutrients in the final product concentrate and the amount of copper recovered. Moreover, it is also a natural area of applications of modern mathematical modeling and optimization techniques. The industry in turn takes a keener interest in such new technologies due to new economic and environmental challenges as well as the issue of the depletion of high-grade deposits. Optimization models and techniques naturally arise when dealing with the problem of optimal flotation process. According to Mendez et al.[16], all known optimization approaches applied to flotation process can be classified into four general groups: mathematical techniques based methods without integer variables, mathematical techniques based methods with integer variables, heuristic methods, and genetic algorithms based evolutionary methods. There are many works devoted to optimization of ore-processing and flotation processes[10, 12, 15, 17, 7, 18]. Maximization of concentrate grade formulated as a global optimization problem was considered in [10]. The optimization problem was maximizing a quadratic function

2010 Mathematics Subject Classification. Primary: 90C05, 90C25; Secondary: 90C99.
Key words and phrases. Mining, survival theory, flotation process, maximization.
This work was supported by the project of Business School of National University of Mongolia.
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with a indefinite matrix over the box constraints. The problem of maximizing both of concentrate grade and metal recovery was formulated in [12] as bi-objective DC program, and the solution method based not only the global optimality conditions by Strekalovsky but also on an epsilon-constraint method, an exact penalty method, and (more precisely) the global optimality conditions for penalized DC minimization problem, proposed by Strekalovsky [19].

In this paper, we consider the problem of maximizing copper recovery in flotation process from a viewpoint of survival theory. Our goal is to determine the best operating conditions leading to the maximization of copper recovery. In recent years, survival theory is found to be useful in economics, management, and industry [5, 6, 20, 14, 11, 13]. In [5], it has been shown that the survival theory and methodology can be applied to taxation.

The problem of survival is a new branch of optimization theory. Survival is the ability of any system to cope with unfavorable situations and effects. Formalization of this theory was first given in [2]. Further development of the survival theory for a system described by optimal control formulation has been done in [2]. Applications of survival theory related to ecology issues and economics have been considered in [1]. In general mathematical model of improving survival of any system is defined by control parameter \( u \in U \), perturbation parameter \( v \in V \) and state parameter \( x \in X \). Implementation of survival theory can be considered in several steps.

First, the target-oriented purpose of a system is defined as

\[ F(x, u, v) \in Q, \]

where, \( F : X \times U \times V \to \mathbb{R}^s, Q \) is a set of admissible solutions. Then a set of safe system perturbation can be defined in the following.

\[ K(x, u) = \{ v \in V | F(x, u, v) \in Q \} \]

for any \( x \in X \) and \( u \in U \). Finally, the survival of the system is defined by

\[ J(x, u) = \frac{\mu(K(x, u))}{\mu(V)}, \]

where \( J : X \times U \to [0, 1], \mu(V) \) is a measure of the set \( V \).

The higher the value \( J(x, u) \) is the higher survival of the system. This means that in order to improve the survival of a system one has to solve the following optimization problem

\[ \max J(x, u) \]

\[ (x, u) \in X \times U. \]

The paper is organized as follows. In section 2, a sphere packing problem as an auxiliary problem has been considered. Section 3 is devoted to a mathematical model of a flotation process. In Section 4, numerical experiments of the survival theory are provided.

2. Auxiliary Problem. We first consider the sphere packing problem which is to pack a ball of maximum total volume in a given bounded polyhedral set \( D \subset \mathbb{R}^n \).

We introduce the following sets. Denote by \( B(x^0, r) \) a ball with a center \( x^0 \in \mathbb{R}^n \) and a radius \( r \in \mathbb{R} \):

\[ B(x^0, r) = \{ x \in \mathbb{R}^n | \| x - x^0 \| \leq r \}. \] (1)
A bounded polyhedral set \( D \subset \mathbb{R}^n \) is given by
\[
D = \{ x \in \mathbb{R}^n | \langle a^i, x \rangle \leq b_i, a^i \in \mathbb{R}^n, b_i \in \mathbb{R}, i = 1, m \},
\]
(2)
here \( \langle , \rangle \) denotes the scalar product of two vectors in \( \mathbb{R}^n \), \( \| \cdot \| \) is the Euclidean norm, and \( \text{int} D \neq \emptyset \).

**Theorem 2.1.** [9] \( B(x^0, r) \subset D \) if and only if
\[
\langle a^i, x^0 \rangle + r\|a^i\| \leq b_i, i = 1, m.
\]
(3)

**Proof. Necessity.** Let \( y \in B(x^0, r) \) and \( y \in D \). The point \( y \in B(x^0, r) \) can be easily presented as \( y = x^0 + rh, h \in \mathbb{R}^n, \|h\| \leq 1 \). It follows from the condition \( y \in D \) that \( \langle a^i, y \rangle \leq b_i, i = 1, m \) or, equivalently, \( \langle a^i, x^0 \rangle + r\langle a^i, h \rangle \leq b_i, i = 1, m \), \( \forall h \in \mathbb{R}^n \). Hence, we have
\[
\langle a^i, x^0 \rangle + r\max_{\|h\| \leq 1} \langle a^i, h \rangle \leq b_i, i = 1, m,
\]
or
\[
\langle a^i, x^0 \rangle + r\langle a^i, \frac{a^i}{\|a^i\|} \rangle \leq b_i, i = 1, m,
\]
which yields
\[
\langle a^i, x^0 \rangle + r\|a^i\| \leq b_i, i = 1, m.
\]

**Sufficiency.** Let the condition (3) be satisfied, and on the contrary, assume that there exists \( \tilde{y} \in B(x^0, r) \) such that \( \tilde{y} \not\in D \). Clearly, there exists \( \tilde{h} \in \mathbb{R}^n \) such that \( \langle a^j, \tilde{y} \rangle > b_j \). Since \( \tilde{y} \not\in D \), there exists \( j \in \{1, 2, \ldots, m\} \) for which \( \langle a^j, \tilde{y} \rangle > b_j \) or \( \langle a^j, x^0 + r\tilde{h} \rangle = \langle a^j, x^0 \rangle + r\langle a^j, \tilde{h} \rangle > b_j \). On the other hand, we have \( \langle a^j, x^0 \rangle + r\|a^j\| > b_j \) which contradicts (3).

Let \( u \in \mathbb{R}^n \) be a center of the sphere and \( r \) its radius. Then one sphere packing problem is formulated as
\[
\max r
\]
(4)
\[
\langle a^i, u \rangle + r\|a^i\| \leq b_i, i = 1, m,
\]
(5)
\[
r \geq 0.
\]
(6)
Conditions (5) define that the ball is inscribed into a polyhedral set. We note that problem (4)-(6) is a particular case of the general packing problem [9].

On the other hand, the problem can be also considered as one circle Malfatti’s problem [8].

3. **Mathematical Model of Flotation Process.** Metal recovery of copper of the flotation process depends on its technological parameters and it can be characterized by a linear regression function using a statistical data of flotation process for a given period.

\[
f = \sum_{j=1}^{n} a_j x_j + a_0,
\]
where \( x_j, j = 1, 2, ..., n \) are variables.
Let $\gamma$ be a given level of copper recovery. Now we define the set of safe system perturbation as:

$$K(a, \gamma) = \left\{ x \in \mathbb{R}^n \left| \sum_{j=1}^{n} a_j x_j + a_0 \geq \gamma, \right. \right.$$

$$x_{j}^{\min} \leq x_j \leq x_{j}^{\max}, j = 1, 2, ..., n \right\}.$$ 

Assume that $K(a, \gamma) \neq \emptyset$.

This set defines the best operating conditions for a given level of $\gamma$. The wider this set is, the higher survival of the system. In order to improve the survival of the system, due to Theorem 2.1, we need to solve the linear programming problem (4)-(6) which for our case $m = 3n + 1$ is:

$$\max \quad r \quad (7)$$

$$\sum_{j=1}^{n} a_j x_j - r \sqrt{\sum_{j=1}^{n} a_j^2} \geq \gamma - a_0, \quad (8)$$

$$-x_j + r \geq -x_{j}^{\min}, j = 1, 2, ..., n, \quad (9)$$

$$x_j + r \leq x_{j}^{\max}, j = 1, 2, ..., n, \quad (10)$$

$$x_j \geq 0, j = 1, 2, ..., n \quad (11)$$

$$r \geq 0. \quad (12)$$

4. **Numerical Experiments.** In our experiments, we used the following technological parameters of 9 variables.

$x_1$: addition of collector agent VK-901 (in grams per ton) ,

$x_2$: consumption of foaming agent MIBK (in grams per ton) ,

$x_3$: content of $\geq 74$ micrometer grain class in the hydro cyclone overflow (in percentage of mass),

$x_4$: monflot-03 (in grams per ton) ,

$x_5$: total copper grade in the rougher (in percentage of mass),

$x_6$: total content of oxidized copper in the feed (in percentage of mass),

$x_7$: total content of primary copper in the feed (in percentage of mass) ,

$x_8$: $\pm 74$nm grinding level percentage ,

$x_9$: total copper grade in the feed, in percentage of mass ,

$f$: metal copper recovery.

A linear regression model constructed on a real data for the period (June 2019) of Erdenet Mining Corporation of Mongolia in coded variables in $[-1,1]$ was:

$$f = 0.86 - 0.004x_1 + 0.006x_2 - 0.012x_3 + 0.005x_4 + 0.011x_5 - 0.014x_6$$

$$+ 0.002x_7 + 0.006x_8 - 0.0005x_9$$

with the determination coefficient $R^2 = 0.75$ which is the proportion of the variance in the dependent variables that is predictable from the independent variables. In other words, 75 percent of the flotation process can be explained by the above linear equation. Increase of variable $x_j$ in one unit effects value of $f$ by $a_j$ unit. Then
problems (7)-(12) reduces to the following problem

\[
\max r \\
-0.004x_1 + 0.006x_2 - 0.012x_3 + 0.005x_4 + 0.011x_5 - \\
-0.014x_6 + 0.002x_7 + 0.006x_8 - 0.0005x_9 - 0.86385r \geq \gamma - 0.86,
\]

\[\begin{align*}
x_j + r &\leq 1, \ j = 1,2, ..., 9, \\
x_j + r &\leq 1, \ j = 1,2, ..., 9, \\
x_j &\geq 0, \ j = 1,2, ..., 9, \\
r &\geq 0.
\end{align*}\]

Since problems (13)-(18) are required to be solved for each period of shift at the industry, then the best performance of the flotation is found by the following algorithm ST.

**Algorithm ST**

**Step 1.** Set \( k := 0 \) and choose a value of \( \gamma_0=0.85 \) which is a current level of copper recovery and \( x^0 \) is its corresponding technological variable, and \( \delta = 0.001 \).

**Step 2.** Solve problem (13) - (18) for \( \gamma := \gamma_k \).

**Step 3.** If the problem has a no solution then \( \gamma_k \) is the best performance of the copper recovery and \( x^k \) is an optimal solution.

**Step 4.** If the problem has a solution \((r^k, x^k)\) then set \( \gamma_{k+1} := \gamma_k + \delta \), \( k := k + 1 \), return to **Step 2**.

After converting the coded variables into real variables, we get optimal solutions to problem (7) - (12) computed on Matlab as:

\[
\begin{align*}
x_1^* &= 4.25, \ x_2^* = 21.9735, \ x_3^* = 10.25, \ x_4^* = 9.9837, \ x_5^* = 0.5996, \ x_6^* = 2.51, \\
x_7^* &= 79.9389, \ x_8^* = 64.9858, \ x_9^* = 15.00, \ \text{and} \ r^* = 0.004.
\end{align*}
\]

The maximum value of \( \gamma^* = 0.893 \). It means that we can increase copper recovery up to 89.3%.

Note that for \( \gamma=0.894 \) problem (13)-(18) has a no solution.

Optimal range of original variables to ensure the level \( \gamma^* \) of copper recovery is the following: \( 4.2486 \leq x_1 \leq 4.2514, 21.9722 \leq x_2 \leq 21.9749, 10.2486 \leq x_3 \leq 10.2514, 9.9824 \leq x_4 \leq 9.9851, 0.5982 \leq x_5 \leq 0.6009, 2.5086 \leq x_6 \leq 2.5114, 79.9376 \leq x_7 \leq 79.9403, 64.9844 \leq x_8 \leq 64.9871, 14.9986 \leq x_9 \leq 15.0014. \)

5. **Conclusion.** Flotation performance such as a maximization of copper recovery and determination of the best operating conditions have been examined from a view point of survival theory. The problem reduces to finding a ball with the maximum radius inscribed into a set defined by a set of safe perturbation of the system. The proposed approach can be easily implemented numerically on computer software with frequent adjustment of given levels of copper recovery which depends on a concentrate of ore. The algorithm was numerically conducted on Matlab for a real data of Mongolian Mining company giving the best performance. The proposed methodology can be applied to other metallurgical industries.
Acknowledgements. This work was partially supported by a grant of Business School of NUM. The authors thank reviewers for their valuable and constructive comments which greatly improved an earlier version of the paper.

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Received April 2020; 1st revision June 2020; Final revision July 2020.

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