Cosmological Evolution of a Scalar-Charged Degenerate Cosmological Plasma with Higgs Scalar Fields

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Abstract—A mathematical model of the cosmological evolution of statistical systems of scalarly charged particles with Higgs scalar interaction is formulated and investigated. Examples are given of numerical modeling of such systems, revealing their very remarkable properties, in particular, the formation of paired bursts of cosmological acceleration.

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1. INTRODUCTION

In [1], a complete mathematical model of the cosmological evolution of the Higgs classical scalar vacuum field was formulated and studied, both by methods of qualitative analysis and by numerical simulation. In such models, transitions of the cosmological evolution from the stage of expansion to the stage of compression (and, conversely, for the phantom field) become possible.1 Earlier, a comprehensive study of incomplete cosmological models was carried out under the assumption that the Hubble parameter is nonnegative for the cases of the classical Higgs vacuum field [3, 4], the Higgs phantom field [5–10] and an asymmetric scalar doublet [5–10]. If we discard a number of incorrect results of these works, which are just related to the assumption that the Hubble parameter is nonnegative, then one of the results of these works can be summarized as follows: at late stages of the evolution, the cosmological model always passes on to inflation. The same result was confirmed by studies of the complete [1] model, in which the assumption that the Hubble parameter was nonnegative, was removed. Thus, it can be argued that cosmological models based on the vacuum scalar fields contradict the observational data on invariant cosmological acceleration in the late Universe with $w < 1$.

On the other hand, in a number of earlier works based on the theory of statistical systems of scalar charged particles, [18, 19], in which the cosmological evolution of such systems was investigated, the possibility of four types of behavior of the corresponding cosmological models was shown, among which were models with an intermediate ultrarelativistic stage and a final nonrelativistic one [21, 22]. However, these studies were based, firstly, on an incomplete mathematical model, secondly, on the quadratic potential of scalar fields, and thirdly, on a scalar singlet. In this connection, arises the problem of formulating a complete mathematical model of cosmological systems of scalar charged particles with Higgs scalar fields, including an asymmetric scalar doublet. Note that the phantom scalar field, due to negativity of its kinetic energy, should be considered only as a part of the usual components of matter. However, as will be seen from what follows, it is the presence of a phantom field in the system that ensures the correct behavior of the cosmological model. In this article, we formulate a mathematical model of a cosmological statistical system of scalar charged particles with Higgs scalar fields, examine its basic properties and show examples of numerical modeling.

2. MATHEMATICAL MODEL

2.1. Mathematical Model of a Degenerate Scalarly Charged Plasma

The foundations of the general-relativistic kinetic and statistical theory were laid in the 60s in the papers by Tauber and Weinberg [11], Chernikov (see, e.g., [12]), Vlasov [13] and others. Scalar fields in general-relativistic statistics and kinetics were introduced at the beginning of the 80s in the works by the author [14–17]. Further, in [18–20], a mathematical model of the statistical system of scalarly charged particles was formulated, based on a microscopic...
such a conservation law),

\[ m_* = m_a + \sum_{r=1}^{N} q_r^a \Phi_r, \quad (1) \]

where \( m_a \) is some bare particle rest mass, which it may be zero, \( \Phi_r \) is a scalar field of type \( r \), \( q^r \) is the scalar charge of a particle with respect to this field, \( (r = T, N) \).

Strict macroscopic consequences of the kinetic theory are the transport equations, including the conservation laws of the energy-momentum tensor (EMT) of particles, \( \sigma^r \) is the density of scalar charges with respect to the field \( \Phi_r \) [20], so that

\[ \nabla_i \sum_a g^a n^i_a = 0, \quad (2) \]

as well as the conservation laws of the energy-momentum of statistical systems:

\[ \nabla_k T^i_k - \sum_r \sigma^r \nabla^i \Phi_r = 0, \quad (3) \]

where \( n^i_a \) is a numerical vector, \( T^i_k \) is the energy-momentum tensor (EMT) of particles, \( \sigma^r \) is the density of scalar charges with respect to the field \( \Phi_r \) [20], so that

\[ T^i_k = \sum_a T^i_k^a; \quad \sigma^r = \sum_a \sigma^r_a. \quad (4) \]

Under the conditions of local thermodynamic equilibrium (LTE), the statistical system is isotropic and is described by local equilibrium distribution functions

\[ f^0_a = \frac{1}{e^{(-\mu_a + (n,p)/\theta) \pm 1}}, \quad (5) \]

where \( \mu_a \) is the chemical potential, \( \theta \) is the local temperature, \( u^i \) is the unit timelike vector of the dynamic velocity of the statistical system, the sign “+” corresponds to fermions, “-” to bosons. Furthermore, the kinematic momentum of the particle \( p^i \) lies on the effective mass surface:

\[ (p, p) = m_*^2 \Rightarrow p^4 = \sqrt{m_*^2 + p^2}, \quad (6) \]

where \( p^4 \) are reference projections of the momentum vector, and \( p^2 \) is the squared physical momentum. In this case, the macroscopic moments take the form of the corresponding moments of the ideal fluid for each of the components [18]:

\[ n^i_a = n_a u^i, \quad (7) \]

\[ T^i_k = (\varepsilon_a + p_a) u^i u^k - p_a g^i_k, \quad (8) \]

while

\[ (u, u) = 1. \quad (9) \]

The normalization relation (9) implies the well-known identity:

\[ u^k_i u^i_k \equiv 0. \quad (10) \]

Therefore, the conservation laws (3) can be reduced to the form

\[ (\varepsilon_p + p_p) u^i_k u^k = (g^i_k - u^i u^k) \left( p_{p,k} + \sum_r \sigma^r \Phi_{r,k} \right), \quad (11) \]

\[ \nabla_k [ (\varepsilon_p + p_p) u^k ] = u^k \left( p_{p,k} + \sum_r \sigma^r \Phi_{r,k} \right), \quad (12) \]

and the conservation law of the fundamental charge \( G (2) \) becomes:

\[ \nabla_k n_r u^k = 0; \quad n_r \equiv \sum_a q^a_r n_a. \quad (13) \]

Macroscopic scalars under LTE conditions have the form [20]²

\[ n_a = \frac{2S + 1}{2\pi^2} m_*^3 \int_0^\infty \frac{\sinh^2 x \cosh x dx}{e^{-\gamma_a + \lambda_* \cosh x} \pm 1}, \quad (14) \]

\[ \varepsilon_p = \sum_a \frac{2S + 1}{2\pi^2} m_*^4 \int_0^\infty \frac{\sinh^2 x \cosh^2 x dx}{e^{-\gamma_a + \lambda_* \cosh x} \pm 1}, \quad (15) \]

\[ p_p = \sum_a \frac{2S + 1}{6\pi^2} m_*^4 \int_0^\infty \frac{\sinh^2 x dx}{e^{-\gamma_a + \lambda_* \cosh x} \pm 1}, \quad (16) \]

\[ T_p = \sum_a \frac{2S + 1}{2\pi^2} m_*^2 \int_0^\infty \frac{\sinh^2 x \cosh x dx}{e^{-\gamma_a + \lambda_* \cosh x} \pm 1}, \quad (17) \]

\[ \sigma^r = \sum_a \frac{2S + 1}{2\pi^2} q^r_a m_*^3 \int_0^\infty \frac{\sinh^2 x dx}{e^{-\gamma_a + \lambda_* \cosh x} \pm 1}, \quad (18) \]

where \( T_p \) is the trace of particles’ EMT,

\[ \varepsilon_p = \sum_a \varepsilon_a, \quad p_p = \sum_a p_a, \quad \sigma^r = \sum_a \sigma^r_a, \]

\[ \lambda_* = [m_*] / \theta, \quad \gamma_a = \mu_a / \theta, \quad \text{and} \quad S \text{ is the spin of particles.} \]

Thus, under LTE conditions, formally on \( 5 + N \) macroscopic scalar functions \( \varepsilon_p, p_p, n_r \) and 3 independent components of the velocity vector \( u^i \), the

²To reduce the record, we omit the particle sort index in some places.
macroscopic conservation laws give $4 + N$ independent Eqs. (11)–(13). However, not all indicated macroscopic scalars are functionally independent, since all of them are determined by local equilibrium distribution functions (5). With a solved set of chemical equilibrium conditions, when only one chemical potential remains independent, with solved equation of the mass surface (see details in [18, 20]) and specified scalar potentials and the scale factor, the $4 + 2r$ macroscopic scalars $\varepsilon_p, p_p, n_r, \sigma^r$ are determined by two scalars: some chemical potential $\mu$ and the local temperature $\theta$. Thus the set of Eqs. (11)–(13) turns out to be completely determined.

2.2. Scalar Fields

In contrast to [14–17], in this paper we will consider Higgs scalar fields with the Lagrange function

$$L_s = \frac{1}{8\pi} \left( \frac{\varepsilon_r}{2} g^{ik} \Phi_{(r),i} \Phi_{(r),k} - V_r(\Phi_r) \right),$$

(19)

with the indicator $\varepsilon_r = +1$ for the classical scalar field and $\varepsilon_r = -1$ for the phantom scalar field, $V_r(\Phi_r)$ is the potential energy of the scalar field ($V = \sum_r V_r$):

$$V_r(\Phi_r) = -\frac{\alpha_r}{4} \left( \Phi_r^2 - \frac{m_r^2}{\alpha_r} \right)^2,$$

(20)

$\alpha_r$ is the self-action constant, $m_r$ is the mass of scalar bosons, $L_s = \sum_r L_s^r$.

Furthermore,

$$T_{r}^{ik} = \frac{1}{8\pi} \left( \varepsilon_r \Phi_{r,i} \Phi_{r,k} - \frac{\varepsilon_r}{2} g^{ik} \Phi_{r,j} \Phi_{r,j} \right. + \left. g^{ik} V_r(\Phi_r) \right)$$

(21)

is the EMT of the $r$-th scalar field, $T_{r}^{ik} = \sum_r T_{r}^{ik}$. Next, we omit the constant term in the Higgs potential (20) since it leads to a simple redefinition of the cosmological constant $\lambda$.

The scalar fields $\Phi_r$ are determined by the equations for charged scalar fields with the source [19]\(^4\)

$$\varepsilon_r \Box \Phi_r + V_{\Phi_r} = -8\pi \sigma^r,$$

(22)

where $\Box \psi$ is the d'Alembert operator on the metric $g_{ik}$. It can be shown that, due to (3) and (22), the conservation law for the complete MET system “plasma + charged scalar fields” is identical

$$\nabla_i T_r^{ik} = \nabla_i (T_p^{ik} + T_s^{ik}) \equiv 0.$$  

(23)

\(^3\)One of the equations (11) depends on the other ones due to the identity (10).

\(^4\)In connection with the normalization of the Lagrange function of the scalar field, different from normalization in [19], the scalar source function on the right-hand side is multiplied by 2.

2.3. Unperturbed Isotropic Distribution

As a background, we consider the spatially flat Friedmann metric

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2),$$

(24)

and as a background solution, we consider a homogeneous isotropic distribution of matter, where all thermodynamic functions and scalar fields depend only on time. It is easy to verify that $u^i = \delta^i_0$ converts Eq. (11) into identities, and the set of Eqs. (12)–(13) reduces to 1 + $N$ equations

$$\dot{\varepsilon}_p + 3\frac{\dot{a}}{a}(\varepsilon_p + p_p) = \sum_r \sigma^r \dot{\Phi}_r,$$

(25)

$$\dot{n}_r + 3\frac{\dot{a}}{a}n_r = 0.$$  

(26)

Thus there remain two differential equations for two thermodynamic functions $\mu$ and $\theta$. When passing to the limit $\mu \rightarrow 0$ or $\theta \rightarrow 0$, we get a set of two equations for one function, and the problem of inconsistency of these equations arises; moreover, this problem does not depend on the presence of a scalar field. However, in [20] it is shown that this problem is seeming, and in fact, no contradictions in Eqs. (11)–(13) also arise in the case of a degenerate Fermi system. As it turns out, in this case, the charge conservation laws (26) are a direct consequence of the energy conservation law (25).

We first consider a one-component statistical system of scalarly charged fermions under the conditions of complete degeneracy:

$$\theta \rightarrow 0.$$  

(27)

when the locally equilibrium distribution function of fermions (5) takes the form of a step function [20]:

$$f^0(x, p) = \chi_+(\mu - \sqrt{m^2 + p^2}),$$

(28)

where $\chi_+(z)$ is the Heaviside step function. In this case, however, we will admit the presence of several scalar fields with respect to which the same particle can have different scalar charges $q^r$.

The result of integrating the macroscopic densities (14)–(18) with respect to the distribution (28) is expressed in elementary functions [20]:

$$n = \frac{1}{\pi^2} p_F^3,$$

(29)

$$\varepsilon_p = \frac{m_p^4}{8\pi^2} F_2(\psi),$$

(30)

$$p_p = \frac{m_p^4}{24\pi^2} (F_2(\psi) - 4F_1(\psi)),$$

(31)

$$\sigma^r = \frac{q^r \cdot m_p^4}{2\pi^2} F_1(\psi).$$

(32)
where the dimensionless function \( \psi \) is introduced,
\[
\psi = \frac{p_F}{m_*},
\] (33)
being equal to the ratio of the Fermi momentum \( p_F \) to the effective mass of the fermion, and to reduce writing, the functions \( F_1(\psi) \) and \( F_2(\psi) \) were introduced:
\[
F_1(\psi) = \psi \sqrt{1 + \psi^2} - \ln(\psi + \sqrt{1 + \psi^2}),
\]
\[
F_2(\psi) = \psi \sqrt{1 + \psi^2}(1 + 2\psi^2) - \ln(\psi + \sqrt{1 + \psi^2}).
\] (34)
The functions \( F_1(x) \) and \( F_2(x) \), firstly, are odd:
\[
F_1(-x) = -F_1(x); \quad F_2(-x) = -F_2(x),
\] (35)
and, secondly, they have the following asymptotic behavior:
\[
F_1(x)_{|x \to 0} \approx \frac{2}{3} x^3, \quad F_2(x)_{|x \to 0} \approx \frac{8}{3} x^3,
\]
\[
(F_2(x) - 4F_1(x))_{|x \to 0} \approx \frac{8}{5} x^5,
\] (36)
\[
F_1(x)_{|x \to \infty} \approx x| x |
\]
\[
F_2(x)_{|x \to \infty} \approx 2x^3| x |
\] (37)
It is easy to verify the validity of the identity:
\[
\varepsilon_p + p_p \equiv \frac{m^4}{3\pi^2\psi^3} \sqrt{1 + \psi^2}.
\] (38)

Further, the EMT of the scalar field in the unperturbed state also takes the form of the EMT of a perfect isotropic fluid:
\[
T^{ik}_{s} = (\varepsilon_s + p_s)u^i u^k - p_s g^{ik},
\] (39)

The equation of scalar fields in the Friedmann metric takes the form:
\[
e_r \left( \ddot{\Phi} + 3\frac{\dot{a}}{a} \dot{\Phi} \right) + m^2 \Phi - \alpha_r \Phi \frac{\dot{\Phi}}{\Phi} = -8\pi \sigma \dot{a}(t),
\] (40)

where \( r = \Omega, N \).

2.4. Complete Set of Background Equations

We consider the standard Einstein equations with a cosmological constant
\[
G^i_k \equiv R^i_k - \frac{1}{2} R \delta^i_k = 8\pi T^i_k + \Lambda \delta^i_k.
\] (41)

We write Einstein’s independent background equations for the Friedmann metric (24):
\[
2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \sum_r \left( e_r \frac{\dot{\Phi}^2}{2} - \frac{m^2 \Phi^2}{2} + \frac{\alpha_r \Phi^4}{4} \right)
+ 8\pi p_p = \Lambda,
\] (42)
\[
3 \frac{\dot{a}^2}{a^2} - \sum_r \left( e_r \frac{\dot{\Phi}^2}{2} + \frac{m^2 \Phi^2}{2} - \frac{\alpha_r \Phi^4}{4} \right)
- 8\pi \varepsilon_p = \Lambda.
\] (43)

Due to the energy-momentum conservation law (23), (25) for the field equations (46), one of the Einstein equations (48), (49) is a differential algebraic consequence of the remaining equations. In [1] it is shown that to study a dynamic system it is more convenient to consider their difference instead of these Einstein equations, taking into account the identity for the Hubble parameter
\[
\dot{H} \equiv \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}.
\]
Thus we obtain the necessary equation
\[
\dot{H} + 4\pi (\varepsilon + p) = 0,
\] (44)
where \( \varepsilon = \varepsilon_p + \varepsilon_s \) and \( p = p_p + p_s \).

Further, according to [1], we introduce the total energy \( E \), of the cosmological matter:
\[
E = \frac{1}{8\pi}(3H^2 - \Lambda) - \varepsilon,
\]
with the help of which the Einstein equation (49) can be given the simple form
\[
E = 0, \tag{52}
\]
reflecting the fact that the total energy of the spatially flat Friedman universe is zero.

Differentiating in time the total energy (51), taking into account the identity (39), we obtain the energy conservation law
\[
\frac{d}{dt}E = 0 \Rightarrow E = E_0. \tag{53}
\]
Thus a consequence of the set of dynamic equations (25), (46) and (50) is the conservation law of total energy of the cosmological system (53), \( E = E_0 \). The Einstein equation (49) is a particular integral of this system \( E_0 = 0 \). A similar situation arises for the vacuum scalar fields [1]. This means that the first integral (52) can be considered as the initial condition in the Cauchy problem for the cosmological model.

In particular, for a degenerate Fermi system, taking into account (39), we obtain from (50) the equation:
\[
\dot{H} + \sum_r \varepsilon_r \dot{\Phi}_r^2 + \frac{4}{3\pi}m^4_\ast \psi^3 \sqrt{1 + \psi^2} = 0. \tag{54}
\]

The set of equations (25), (46) and (54), together with the definitions (30)–(32), describes a closed mathematical model of cosmological evolution of a completely degenerate Fermi system with scalar interaction.

Differentiating the energy density of the Fermi system (30), taking into account the identity (39), we bring the energy conservation law for the Fermi system (25) to the form of the equation
\[
\frac{d}{dt} \ln m_\ast \psi \alpha = 0. \tag{55}
\]
From this, taking into account the definition of the function \( \psi \) (33), we get
\[
apF = \text{const}. \tag{56}
\]
From here, taking into account (7) we obtain the law of conservation of the number of fermions (see [20]):
\[
a^3 n = \text{const}. \tag{57}
\]

Thus, despite the apparent complexity of Eq. (25), its solution is easy to find—from the energy conservation law of Fermi system, the conservation law for the number of particles is obtained. We can say that the scalar charge conservation law in the form (13) is, at least in our case, redundant. Note that, unlike the electric charge conservation law, this law does not follow from anywhere, but is nevertheless fulfilled.

Following [1], we also introduce a nonnegative effective energy of the cosmological system \( E_{\text{eff}} \) according to (51) and (52):
\[
E_{\text{eff}} = \varepsilon + \frac{\Lambda}{8\pi} \geq 0
\]
\[
\Rightarrow \frac{e}{2} \dot{\Phi}^2 + \frac{m^2 \Phi^2}{2} + \frac{\alpha \Phi^4}{4} + \frac{m^4_\ast F_2(\psi)}{\pi} \geq 0. \tag{58}
\]

3. DYNAMIC SYSTEM ANALYSIS

3.1. A Dynamical System for a One-Component Degenerate Cosmological Plasma

In this article, we will consider a cosmological model based on a one-component degenerate Fermi system and a single scalar field \( \Phi \). An important circumstance is that the energy conservation law of the statistical system (25) for a degenerate Fermi system is completely equivalent to Eq. (55). Instead of choosing the nonnegative dynamic variable \( a(t) \geq 0 \), choosing \( \xi(t) \),
\[
\xi = \ln a, \quad \xi \in (-\infty, +\infty), \tag{59}
\]
\[
\dot{\xi} = \dot{H}, \tag{60}
\]
we use the integral (56) of the energy conservation law (55) to find the function \( \psi(t) \) (33):
\[
\psi = \frac{p^0_\ast e^{-\xi}}{m_\ast}. \tag{61}
\]
Thus, taking into account (40), we obtain the expression for the function \( \psi(t) \):
\[
\psi = \frac{p^0_\ast e^{-\xi}}{q \Phi} \equiv \frac{\beta}{\Phi} e^{-\xi}, \tag{62}
\]
where
\[
\beta = \frac{p^0_f}{q}, \quad p^0_f = p_f (\xi = 0). \tag{63}
\]
For the scalar charge density (32) we obtain the expression
\[
\sigma = \frac{q^4 \Phi^2}{2\pi^2} F_1(\psi). \tag{64}
\]
Assuming further
\[
\Phi = Z, \tag{65}
\]
we write the field Eq. (46) in these notations as
\[
\dot{Z} = -3HZ - e m^2 \Phi + e \Phi^3 \left( \alpha - \frac{4 e^4}{\pi} F_1(\psi) \right). \tag{66}
\]

Further, Eq. (54) takes the form:
\[
\dot{H} = -\frac{eZ^2}{2} + \frac{4}{3\pi} q^4 \Phi^4 \psi^3 \sqrt{1 + \psi^2}. \tag{67}
\]

Moreover, the first integral of Eq. (52) takes the form:
\[
\Sigma_E : 3H^2 - \Lambda - \frac{q^4 \Phi^4}{\pi} F_2(\psi) - \frac{eZ^2}{2} - \frac{m^2 \Phi^2}{2} + \frac{\alpha \Phi^4}{4} = 0. \tag{68}
\]

Equation (68) is an algebraic equation for the dynamic variables \( \Phi, \xi, Z, H \) and describes some hypersurface in the arithmetic space \( \mathbb{R}_4 = \{ \Phi, \xi, Z, H \} \), which we will call [1] the Einstein hypersurface. All phase trajectories of the dynamic system (65), (60), (66) and (71), as well as the starting points, must lie on the Einstein hypersurface. Since (68) is the first integral of a dynamical system, to solve the Cauchy problem, it suffices to require that the initial point of the dynamic trajectory belongs to the Einstein hypersurface.

The points of the phase space \( \mathbb{R}_4 \), at which the effective energy (58) is negative, are not available for the dynamic system. These points lie on the hypersurface of the phase space \( \Sigma_E \subset \mathbb{R}_4 \), which is a cylinder with the axis \( OH \):
\[
\Sigma_E : \Lambda + \frac{q^4 \Phi^4}{\pi} F_2(\psi) + \frac{eZ^2}{2} + \frac{m^2 \Phi^2}{2} - \frac{\alpha \Phi^4}{4} = 0, \tag{69}
\]

moreover, the hypersurface of zero effective energy (69) touches the Einstein hypersurface (68) on the hyperplane \( H = 0 \):
\[
\Sigma_E \cap \Lambda = H = 0. \tag{70}
\]

Further, as can be seen from Eq. (67) in the case of a scalar neutral statistical system (\( q \equiv 0 \)), the sign of the derivative of the Hubble parameter is completely determined by the indicator \( e \): for a classical scalar field (\( e = +1 \)) \( \dot{H} < 0 \), and for a phantom scalar field (\( e = -1 \)) always \( \dot{H} > 0 \). The play of these factors during cosmological evolution can also fine-tune the model parameters to ensure the desired behavior. In the presence of charged matter, its contribution to this game, as can be seen from (67), is determined by the sign of the scalar potential: for \( \Phi > 0 \) it contributes to an increase in the Hubble parameter, for \( \Phi < 0 \) it decreases. It should be noted that with a suitable Einstein hypersurface topology, cosmological models based on single vacuum scalar fields at the final stage of evolution will go either to the inflationary compression mode (classical field) or to the inflationary expansion mode (phantom field).

Note that instead of Eq. (67) we can consider an equivalent equation (see [1]), substituting the expression for \( Z^2 \) from (68) to (67):
\[
\dot{H} = -3H^2 + \Lambda + \frac{e q^4 \Phi^4}{\pi} F_2(\psi) - \frac{m^2 \Phi^2}{2} + \frac{\alpha \Phi^4}{4} + \frac{4}{3\pi} q^4 \Phi^4 \psi^3 \sqrt{1 + \psi^2}. \tag{71}
\]

3.2. Singular Points of the Dynamic System

The singular points of a dynamical system represented by a normal autonomous system of differential equations are determined by algebraic equations obtained by equating to zero the derivatives of all dynamic variables. Thus, from (60), (65), (66) and (71) we obtain the set of algebraic equations for finding the coordinates of singular points:
\[
Z = 0, \tag{72}
\]
\[
H = 0, \tag{73}
\]
\[
\Phi^3 \left( \alpha - \frac{4 e^4}{\pi} F_1(\psi) \right) - m^2 \Phi = 0, \tag{74}
\]
\[
\frac{4}{3 \pi} q^4 \Phi^4 \psi^3 \sqrt{1 + \psi^2} = 0. \tag{75}
\]

In addition, we must take into account the total energy integral (68), since the coordinates of the singular point must satisfy this equation, which, given (72), takes the form
\[
\Lambda + \frac{e q^4 \Phi^4}{\pi} F_2(\psi) - \frac{m^2 \Phi^2}{2} + \frac{\alpha \Phi^4}{4} = 0. \tag{76}
\]

From (75) it follows: (i) \( \psi = 0 \) or (ii) \( \Phi = 0 \).

We will first investigate the first possibility \( \psi = 0 \). Since \( F_1(0) = 0 \), according to (74) we get the equation for \( \Phi \)
\[
\alpha \Phi^3 - m^2 \Phi = 0, \tag{77}
\]
where we get the roots
\[
\Phi_0 = 0; \quad \Phi_{\pm} = \pm \sqrt{\frac{m^2}{\alpha}}. \tag{78}
\]

In this case, the remaining equation (76) gives a relation between the fundamental constants
\[
\Lambda = \Lambda_0 \equiv \frac{m^4}{4 \alpha}, \tag{79}
\]
at which there is a singular point \( M_\Phi \):
\[
M_\Phi^\pm : \left( \pm \sqrt{\frac{m^2}{\alpha}}, +\infty, 0, 0 \right), \quad \Lambda = \frac{m^4}{4 \alpha} > 0. \tag{80}
\]
This case corresponds to the singular points of the vacuum scalar field without charged fermions [1], and the cosmological constant $\Lambda_0$ is fully generated by the Higgs field.

Let us now investigate the second possibility $\Phi = 0$. In this case, Eq. (74) becomes an identity, and Eq. (76) gives $\Lambda = 0$. At that, $\psi \to \pm \infty$, while the dynamic variable $\xi_0$ can take any values:

$$M^\pm_\xi: \left( \xi_0, 0, 0, 0 \right), \quad (\forall \xi_0, \Lambda = 0). \quad (81)$$

Calculating the basic matrix of the dynamical system (65), (60), (66) and (71)

$$A = \left| \frac{\partial X_i}{\partial x_k} \right|, $$

it is easy to show that in both cases (80) and (81) this matrix is degenerate, therefore the qualitative theory of differential equations for our dynamic system, in contrast to a dynamic system with vacuum scalar fields, does not give anything. Let us therefore proceed to numerical modeling.

4. NUMERICAL MODELING
AND DISCUSSION OF RESULTS

First, we note that the topology of the Einstein hypersurface (68) can be quite complex, and at the same time, the phase trajectories of the system can be complex since they lie on this hypersurface. Bearing in mind a wide variety of dynamic system behavior models depending on the fundamental parameters of the model $P = [\epsilon, \alpha, \beta, m, q, \Lambda]$, in this paper we restrict ourselves to a study of some special cases, postponing a full study and general conclusions to a more detailed article, which we hope to present in the near future. In what follows, for brevity, we will describe the initial conditions for our model with the ordered set $I = (\Phi_0, \xi_0, Z_0, \epsilon)$, where the indicator $\epsilon = \pm 1$ takes the value $+1$ if the initial state of the dynamic system corresponds to an expansion phase $H_0 > 0$, and the value $-1$ if the initial state of the dynamic system corresponds to a compression phase $H_0 < 0$. Recall that the initial value of the Hubble parameter is determined from Eq. (68).

4.1. Example of a Model with a Classic Scalar Field

Figures 1, 2 show an example of a phase trajectory of a cosmological system based on a classical scalar field, with the parameters $P = [1, 0.1, 0.1, 1, 0.001, 0.01]$ and the initial conditions $I = [0.1, 1, 0.1, \pm 1]$.

Note that the topology of the Einstein surface in the case under consideration makes it impossible for the phase trajectory to pass from the region $H > 0$ to the region $H < 0$. Therefore, the phase trajectories in the upper half-plane $H > 0$ are collected at the minimum point $H_{\min} > 0$, and the trajectories in the lower half-plane, on the contrary, leave the maximum point $H_{\max} < 0$ and go “down” unlimitedly.

Figures 3–6 show the behavior of the basic physical parameters of the model under study with the classical Higgs scalar field: the Hubble parameter $H$, the effective energy $E_{\text{eff}}$ (58), the invariant cosmological acceleration $w$

$$w = 1 + \frac{\dot{H}}{H^2} \quad (82)$$
Fig. 3. Evolution of the Hubble parameter $H(t)$ in the system under study.

Fig. 4. Evolution of the effective energy $E_{\text{eff}}(t)$ in the system under study.

and the invariant curvature

$$\sigma = \sqrt{R_{ijkl}R^{ijkl}} = H^2 \sqrt{6(1 + w^2)}.$$ 

4.2. Example of a Phantom Scalar Field Model

Figures 7–12 show the results of numerical simulations for a cosmological statistical system with phantom interaction with the parameters $P = [-1, 0.1, 0.1, 1, 0.001, 0.01]$ and the initial conditions $I = [0.1, 1, 0.1, \pm 1]$.

Figures 8–9 show the phase trajectories of the model with a phantom scalar field on the Einstein surface in a three-dimensional section $\{\Phi, Z, H\}$, the first case corresponds to $\epsilon = +1$ and the second to $\epsilon = -1$. We see how in the first case the trajectory rises from the lower part of the left cavity of the Einstein surface to its upper point, starting from the points of the neck, and in the second case, the trajectory rises from the lower part of the right cavity of the Einstein surface to the upper point of the left plane, slipping through the neck.

Despite the seeming exoticism, the supergiant acceleration bursts are not dangerous for the cosmological model since they pass under the conditions of zero
cosmological expansion rate, $H \rightarrow 0$, and instantly flat space-time, $\sigma \rightarrow 0$. Nevertheless, these possible bursts are of certain interest for observational cosmology, so they can give evidence of a change in the compression mode to the expansion mode in the early Universe.

We will especially dwell on the behavior of the invariant cosmological acceleration $w$ for a statistical system with a phantom field (see Figs. 11 and 12). Here we observe two giant bursts of acceleration for a system that started from the expansion phase (Fig. 11) and the compression phase (Fig. 12), and the graphs in these figures are, in fact, mirror images of each other.

In Fig. 11, a giant surge of acceleration, $w \sim 10^2$, precedes a supergiant one, $w \rightarrow +\infty$, and in Fig. 12 the burst sequence changes. First, we note that supergiant acceleration bursts $w \rightarrow +\infty$ are associated with passage of the point $H = 0$ (see Eq. (82)) and are not observed in models with a quadratic interaction potential. Secondly, the giant bursts of $w \sim 10^2$ are characteristic of a theory with the quadratic potential [21, 22].

In the near future, we intend to publish more detailed studies of the models of cosmological evolution of scalarly charged statistical systems with Higgs scalar fields.

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Fig. 11. The evolution of invariant cosmological acceleration $w$ in the system under study $\epsilon = +1$.

Fig. 12. The evolution of invariant cosmological acceleration $w$ in the system under study $\epsilon = -1$.

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