Collider Implications of Kaluza-Klein Excitations of the Electroweak Gauge Bosons

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Abstract

It is possible for the SM fields to propagate into one or more large extra compact dimensions. Such fields have associated KK excitations that produce additional contributions to the SM processes. We calculate the effects that these KK excitations have on cross sections for $e^+e^-$ collider processes in a model in which the SM gauge bosons, and perhaps the Higgs, can propagate into one TeV$^{-1}$-size extra compact dimension. Specifically, we consider muon pair production, Bhabha scattering, dijet production, Higgs production, and single-photon production. At prospective high energy $e^+e^-$ colliders, we find significant deviations from the SM cross sections: For example, a 600 GeV collider produces a 20% reduction in the cross section for muon pair production as compared to the SM for a compactification scale of 3.5 TeV. The effect is much smaller at LEP2 energies, where the compactification scale must be lower than 2 TeV in order to produce a 6% effect.

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1. Introduction

There has been a flurry of recent interest in the low-energy phenomenology of string-inspired models where the presence of large extra compact dimensions produces a string scale that is quite small compared to the usual four-dimensional Planck scale \[ \Pi \].

In this class of models based on the approach of Arkani-Hamed, Dimopoulos, and Dvali (ADD) \[ \Pi \], \( n \) large extra dimensions are compactified at the same scale \( R^{-1} \), where the size \( R \) is related to the four-dimensional Planck scale via the relation

\[
M_P^2 = M_*^{n+2} R^n. \tag{1}
\]

This new scale \( M_* \) is the fundamental \((4+n)\)-dimensional Planck scale, which is of the same order as the string scale. It has been demonstrated that this relation (1) is phenomenologically viable \[ \Pi \] for \( n \geq 2 \), that \( R \) can be as large as a sub-millimeter, and that the string scale could be as small as a few tens of TeV. In the case where all six extra dimensions from the superstring theory are compactified at the same scale, then the compactification scale \( 1/R \) is about \( 10 \text{ MeV} \). Therefore, any Standard Model (SM) fields that propagate into these extra dimensions (the bulk) have Kaluza-Klein (KK) excitations with masses at the \( 10 \text{ MeV} \) scale. The present high-energy collider limit of about a TeV for such states thus implies, in these scenarios, that all SM fields are confined to a three-dimensional wall (D\(_3\) brane) of the usual three spatial dimensions. This is characteristic of the class of models in which the compactification is symmetric, i.e., all of the extra dimensions have the same compactification radius \( R \).

By contrast, in an asymmetric scenario, SM fields can propagate into a single extra dimension with a compactification radius as large as a TeV\(^{-1} \), for example, and thus have lowest-lying KK excitations at the TeV scale. Aside from the obvious collider implications that are just at the edge of the grasp of present high-energy colliders, the phenomenology of such a model includes an alteration in the evolution of the gauge couplings from the usual logarithmic to power law behavior \[ \Pi \]. The result is a unification scale that can be much smaller \[ \Pi \], perhaps as little as a few TeV. The simplest asymmetrical compactification scenario involves two distinct compactification scales. The recently analyzed case \[ \Pi \] of \( n \) extra dimensions of size \( R \sim \text{mm} \) and \( m \) of size \( r \sim \text{TeV}^{-1} \) was proposed as a unique model in which both the collider and gravitational \[ \Pi \] bounds are just beyond the reach of present experiments. In particular, the \( n = 1 \), \( m = 5 \) case was shown to satisfy all of the current astrophysical and cosmological constraints \[ \Pi \]. The scaling relation for this model is

\[
M_P^2 = M^8 R r^5, \tag{2}
\]

where \( M \) is the unification scale. As a numerical example, compactification scales of \( 1/R \sim 10^{-3} \text{ eV} \) and \( 1/r \sim 1 \text{ TeV} \) give \( M \sim 100 \text{ TeV} \).

*The bound is even lower if the fields propagate into fewer extra dimensions.*
The majority of the analyses on the collider phenomenology of extra dimensions [7] has been on the ADD scenario in which the graviton propagates in the bulk, but where the SM fields do not. Hence, the KK excitations of the graviton are the only additional source of contributions to collider processes. While the contributions of individual KK modes, with 4D gravitational strength, to collider processes is extremely small, the compactification scale $\mu$ is so small ($\mu \sim \text{mm}^{-1} \sim 10^{-3}$ eV) that a very large number of such modes contribute in a TeV-scale collider process. The net result can be a significant deviation from the SM results. Studies of various collider processes typically give a bound on the string scale that is the order of a TeV [7].

In comparison, the asymmetric scenario, in which SM fields, in addition to the graviton, may propagate into one or more extra dimensions of TeV^{-1}-size, will have a more direct effect in high-energy collider processes. Originating with the suggestion by Antoniadis [8], some work has also been done for the collider phenomenology of the scenario in which the SM gauge bosons can propagate into the bulk, but where the SM fermions can not [9]. This includes the effects on EW precision measurements [10], Drell-Yan processes in hadronic colliders [11], $\mu^+\mu^-$ pair production in electron-positron colliders [11], and recently multijet production in very high-energy hadronic colliders [12]. Typically, the bound on the compactification scale is 1–2 TeV. More recently, the case where all of the SM fields, including the fermions, propagate into the extra dimensions has been investigated [13, 14]. These “universal” extra dimensions result in a much lower bound on the compactification scale of a few hundred GeV. The reason is that, in this scenario, the tree-level couplings involving one KK mode and two zero modes are not allowed. Thus, the vertices involve a pair of KK modes, and the KK modes need to be pair-produced, which results in the suppression of the production cross sections.

In this work, we study the asymmetric compactification scenario proposed in Ref. [4], in which only the SM gauge bosons (and perhaps the Higgs boson) propagate into one of the TeV^{-1}-size extra dimensions [4]. More specifically, we investigate the effects that the KK excitations of the electroweak (EW) gauge bosons have on various $e^+e^-$ collider processes. We calculate the modifications to the SM cross sections which arise from the direct production and exchanges of KK excitations of the EW gauge bosons. Included in our study are dijet production, associated Higgs production, single-photon production, and muon pair production and Bhabha scattering. Although the compactification scale must be quite small ($\lesssim 2$ TeV) for a 6% effect to be observed at the LEP2 energies, we find substantial deviations from the SM cross sections for a future collider with $\sqrt{s} = 600$ GeV (e.g., a compactification scale of 3.5 TeV produces an effect of 20% for muon pair, dijet, and Higgs production), and an even greater effect is predicted at higher energies. Our paper is organized as follows. We begin by discussing our formalism in Section 2. In Section 3, we consider charged lepton pair production, including Bhabha scattering and muon pair production. We

*However, our results apply to any compactified string model in which the SM gauge bosons propagate into one such extra dimension.
investigate dijet production in Section 4. In Section 5, we study Higgs production, both for the case of SM as well as supersymmetric (SUSY) Higgs doublets. We discuss the production of a single photon with a neutrino pair in Section 6. Section 7 contains our conclusions.

2. Formalism

As it is unlikely that a significant number of lowest-lying KK excitations will be directly produced in the near future at $e^+e^-$ colliders, we focus here on tree-level processes involving the exchanges of KK excitations of the EW gauge bosons. This necessarily restricts us to the case where the initial and final state fermions are each confined to the SM D$_3$ brane since otherwise, as we shall see shortly, the couplings for the processes of interest here are either zero or highly suppressed when KK number is not conserved, depending on which fields see the extra dimensions. Thus, for simplicity, we consider the case in which the EW gauge bosons can propagate into a single extra TeV$^{-1}$-size dimension, but where the SM fermions are restricted to lie on the SM wall (at the same location in the extra dimension). As for the Higgs boson, we also restrict it to lie on the SM D$_3$ brane when we investigate associated Higgs production (else there is either no effect or a suppressed effect).

The terms in the 5D Lagrangian density that involve fermion fields contain a delta function to constrain the SM fields to the usual 4 spacetime dimensions, and similarly for terms involving the Higgs field in the case in which the Higgs does not propagate in the bulk. The relevant parts of the 5D Lagrangian density can be expressed as

$$\mathcal{L}_5 = \left[ |\Xi_M H|^2 + ig_\gamma \bar{f} \gamma^\mu D_\mu f \delta(y) \right],$$

(3)

where $f$ is a SM fermion field, $H$ represents the Higgs doublet(s), $D_M$ with 5D spacetime index $M \in 0, 1, \ldots, 4$ is the 5D generalization of the usual covariant derivative $D_\mu$ with 4D spacetime index $\mu$, the factor $|\Xi_M H|^2$ denotes $|D_M H|^2$ for a Higgs propagating in the bulk and $|D_\mu H|^2 \delta(y)$ for a Higgs localized to the SM boundary, and the compactified extra dimension coordinate $y$ is related to the radius of the extra dimension $r$ by $y = r\phi$. We consider compactification on a $S^1/Z_2$ orbifold with the identification $\phi \rightarrow -\phi$. In terms of the compactified dimension $y$, a field $A_\mu(x, y)$ can then be Fourier expanded as

$$A_\mu(x, y) = \frac{1}{\sqrt{\pi r}} \left[ A_{\mu 0}(x) + \sqrt{2} \sum_{n=1}^{\infty} A_{\mu n}(x) \cos(n\phi) \right].$$

(4)

The normalization for the $n = 0$ field $A_{\mu 0}(x)$ is one-half that of the $n > 0$ field $A_{\mu n}(x)$. Integration over $y$ results in a tower of $A_{\mu n}(x)$ KK excitations. The $n = 0$ modes of the 5D photon, $W^\pm$, and the $Z$ are identified as the SM photon, $W^\pm$, and $Z$. The $n > 0$ KK modes of these fields are represented with a star ($\star$), as in $\gamma^\star_n$. As
it is convenient to refer to the \( n = 0 \) and \( n > 0 \) modes separately, unless specified otherwise, \( n \) shall henceforth imply strictly \( n > 0 \).

The detailed procedure for integrating over the fifth dimension \( y \) to obtain, in the effective 4D theory, the factors for the allowed vertices involving KK excitations of the EW gauge bosons is similar to the procedure given in Ref. \[12\] for the couplings of quarks and gluons to KK excitations of the gluons. The difference is that any Higgs fields confined to the boundary induce mixing terms \[15\] between the EW gauge bosons and their KK excitations. This causes a slight reduction in the couplings involving KK excitations compared to the case in which the Higgs fields propagate into the extra dimension; in addition, previously forbidden couplings are allowed with a suppression factor. However, because this mixing is highly suppressed (by a factor of \( \sim m_W^2/\mu^2 \)), we only use the couplings given by the case in which the Higgs fields propagate into the bulk, to a good approximation. The couplings for the vertices involving KK excitations of the \( A^\mu, B^\mu, \) and \( C^\mu \) fields are then given in terms of the corresponding couplings between SM fields by the factors given in Ref. \[12\]; the corresponding couplings for KK excitations of the photon, \( W^\pm, \) and \( Z \) are then related to the former via the usual mixing relations. In particular, there is a factor of \( \sqrt{2} \) relative to the analogous SM coupling for a vertex involving a single KK excitation and two SM fermions confined to the SM \( D_3 \) brane, which originates from the different rescaling of the \( n = 0 \) and \( n > 0 \) modes necessary to obtain canonically normalized kinetic energy terms in the effective 4D Lagrangian density \[15\]. Also worth noting is that a single \( X^*_n \) can not couple to two (or three) \( Y \)'s, where \( X, Y \in \{ \gamma, W^\pm, Z \} \) and \( X^*_n \) denotes a KK excitation of gauge boson \( X \) with mode \( n > 0 \). In fact, \( X^* \)'s can only couple to other \( X^* \)'s and \( Y^* \)'s if the modes \( n_1, n_2, \ldots, n_N \) of these KK excitations satisfy KK number conservation:

\[
|n_1 \pm n_2 \pm \cdots \pm n_{N-1}| = n_N.
\]  

Another difference between the Feynman rules for the EW gauge bosons \( \{ X \} \) and their KK excitations \( \{ X^*_n \} \) is that the KK excitations are considerably heavier. For a compactification scale \( \mu = 1/r \), the mass of the \( n^\text{th} \) KK excitation is:

\[
m_{X^*_n} = \sqrt{m_X^2 + n^2 \mu^2}.
\]  

The \( X^*_n \) propagator is that of a usual massive gauge boson:

\[
-i\Delta_{\mu\nu}(X^*_n, p^2) = -i \frac{g_{\mu\nu} - \frac{p_\mu p_\nu}{m_{X^*_n}^2}}{p^2 - m_{X^*_n}^2 + im_{X^*_n} \Gamma_{X^*_n}}.
\]  

At tree-level, the \( W^*_n \) and \( Z^*_n \) have the same decay rates as the \( W \) and \( Z \) except for a factor of \( 2 \frac{m_{W^*_n}}{m_W} \) for the \( W^*_n \) and similarly for the \( Z^*_n \); also, the KK excitations are heavy
enough to include the top quark in the decay rates. The $\gamma_n^*$ decays to fermion pairs with total width $\frac{1}{3} \alpha(m_{\gamma_n^*}) m_{\gamma_n^*}$. For diagrams where a $X_n^*$ exchanges between two fermion pairs (e.g., in $e^+e^- \rightarrow t\bar{t}$), there is the usual diagram with the $X$ propagator in addition to a tower of diagrams with $X_n^*$ propagators, or, equivalently, an effective propagator given by the sum

$$\Delta_{\text{eff}}(X, p^2) = c_{X_0} \Delta(X_0, p^2) + \sum_{n=1}^{\infty} c_{X_n} \Delta(X_n, p^2). \quad (8)$$

The factors $\{c_{X_n}\}$ incorporate the different $f_1-f_2-X$ and $f_1-f_2-X_n^*$ vertex factors (i.e., $c_{X_0} = 1$, $c_{X_n>0} = 2$). This effective propagator can be generalized to the case of arbitrary vertices via adjustment of the $c_{X_n}$ factors (including setting $c_{X_n}$ equal to zero when either vertex is forbidden).

### 3. Muon Pair Production and Bhabha Scattering

Muon pair production is among the best prospects for the indirect observation of KK excitations of the EW gauge bosons in $e^+e^-$ processes because it can be measured with relatively high precision at present and upcoming colliders while still producing rather substantial deviations from the SM cross section in comparison with other processes. Muon pair production has already been investigated elsewhere in the literature [11] for the purpose of setting present and future $e^+e^-$ collider bounds. We primarily include this process here as a standard by which to compare our other results for other processes, but it also serves as a check on our calculations. We keep the first 100 KK excitations in our analysis for purely direct-channel processes and 25 KK excitations for processes with both direct- and cross-channel Feynman diagrams; in addition, we consider $e^+e^-$ collider energies from LEP2 energies to 1.5 TeV and compactification scales up to 10 TeV.

For charged lepton pair production, the KK excitations of the EW gauge bosons manifest themselves through a tower of diagrams with $\gamma_n^*$ and $Z_n^*$ propagators. The net tree-level effect of the $\gamma_n^*$’s and $Z_n^*$’s on charged lepton pair production is the replacement of the SM propagator by an effective KK propagator. We employ gauge invariance to drop the second term in Eq. (7) in our analysis of fermion pair production. The effective moduli-squared of the propagators for direct-channel $\gamma_n^*$ and $Z_n^*$ exchange and the corresponding direct-channel $\gamma_n^*Z_n^*$ interference are thus $\dagger$.

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*We neglect the top quark mass relative to the very heavy KK excitations.

†When generalizing to the case where the EW gauge bosons may propagate into more than one large extra dimension, the sum in the effective propagator is formally divergent. However, various solutions to this problem have been proposed in the literature [11].
\[ |D_{\text{eff}}(\gamma, s)|^2 = \frac{1}{2} \sum_{m,n=0}^{\infty} c_\gamma c_n \frac{s'_{\gamma m} s'_{\gamma n} + m_{\gamma m} \Gamma_{\gamma m} m_{\gamma n} \Gamma_{\gamma n}}{(s^2_{\gamma m} + m^2_{\gamma m} \Gamma^2_{\gamma m})(s^2_{\gamma n} + m^2_{\gamma n} \Gamma^2_{\gamma n})} \]

\[ |D_{\text{eff}}(Z, s)|^2 = \frac{1}{2} \sum_{m,n=0}^{\infty} c_Z c_n \frac{s'_{Z m} s'_{Z n} + m_{Z m} \Gamma_{Z m} m_{Z n} \Gamma_{Z n}}{(s^2_{Z m} + m^2_{Z m} \Gamma^2_{Z m})(s^2_{Z n} + m^2_{Z n} \Gamma^2_{Z n})} \]

\[ \left[ D_{\text{eff}}^*(Z, s) D_{\text{eff}}(\gamma, s) + D_{\text{eff}}(\gamma, s) D_{\text{eff}}^*(Z, s) \right] = \sum_{m,n=0}^{\infty} c_\gamma c_n \frac{s'_{\gamma m} s'_{Z n} + m_{\gamma m} \Gamma_{\gamma m} m_{Z n} \Gamma_{Z n}}{(s^2_{\gamma m} + m^2_{\gamma m} \Gamma^2_{\gamma m})(s^2_{Z n} + m^2_{Z n} \Gamma^2_{Z n})}. \] (9)

Here \( s'_{X_n} \) is shorthand for the subtraction of \( m^2_{X_n} \) from \( s \) (i.e., \( s'_{X_n} \equiv s - m^2_{X_n} \)) and \( c_{X_n} \) represents the fact that the \( f \- \bar{f}-X \) and the \( f \- \bar{f}-X^* \) vertex factors differ by a \( \sqrt{2} \) (i.e., \( c_{X_0} = 1, c_{X_{n>0}} = 2 \)). (We make an exception in the effective propagator equations by including the \( n = 0 \) and \( n > 0 \) modes together for more compact notation.) The effective propagator formulas for cross-channel exchanges and interference are the same as in Eq. 3 with the replacement of the direct-channel Mandelstam variable \( s \) by the cross-channel variable \( t \) (or \( u \)). The interference between direct-channel exchanges of \( \gamma^* \)'s and cross-channel exchanges of \( Z^* \)'s is described by

\[ \left[ D_{\text{eff}}^*(Z, t) D_{\text{eff}}(\gamma, s) + D_{\text{eff}}(\gamma, s) D_{\text{eff}}^*(Z, t) \right] = \sum_{m,n=0}^{\infty} c_\gamma c_n \frac{s'_{\gamma m} t'_{Z n} + m_{\gamma m} \Gamma_{\gamma m} m_{Z n} \Gamma_{Z n}}{(s^2_{\gamma m} + m^2_{\gamma m} \Gamma^2_{\gamma m})(t^2_{Z n} + m^2_{Z n} \Gamma^2_{Z n})}. \] (10)

Finally, the effective propagator formulas for direct-channel exchanges of \( Z^* \)'s and cross-channel exchanges of \( \gamma^* \)'s are identical to Eq. 10 with the replacement \( \gamma \leftrightarrow Z \). For collider energies in the range \( 100 \text{ GeV} \leq \sqrt{s} \leq 1 \text{ TeV} \) and compactification scales \( \mu > 1 \text{ TeV} \), the sums in the effective propagators converge quite rapidly. Thus, they can be truncated after only a few terms with negligible error.

The cross section for muon (or tau) pair production is easily obtained via replacement of the usual SM propagator terms by the effective propagator terms of Eq. 10. Because the compactification scale \( \mu \) is a TeV or more and feasible values of \( \sqrt{s} \) for colliders in the present and not-too-distant future range up to about a TeV, the primary effect of the KK excitations of the EW gauge bosons arises from the interference of the \( n = 0 \) (SM) mode terms with the \( n > 0 \) (KK) mode terms. Since \( (s - m^2_{X_{n>0}}) < 0 \) for the ranges of \( \sqrt{s} \) and \( \mu \) that we consider, the overall effect of the KK excitations is a reduction in the muon (or tau) pair production cross section as compared to the SM. This effect is illustrated in Fig. 1, where the ratio

\[ R \equiv \frac{\sigma}{\sigma_{\text{SM}}} = \frac{\sigma_{\text{SM}} + \sigma_{\text{KK}}}{\sigma_{\text{SM}}} \] (11)
Figure 1: The contributions of the exchanges of $\gamma^*_n$'s and $Z^*_n$'s to muon (or tau) pair production are illustrated as a function of the compactification scale $\mu$ for fixed values of the collider energy $\sqrt{s}$ (top), and as a function of $\sqrt{s}$ for specific choices of $\mu$ (bottom).
is plotted for variation of compactification scale $\mu$ and collider energy $\sqrt{s}$ in the ranges $1 \, \text{TeV} \leq \mu \leq 10 \, \text{TeV}$ and $0.3 \, \text{TeV} \leq \sqrt{s} \leq 1.5 \, \text{TeV}$. It is clear that a very precise measurement of the cross section ($\lesssim 6\%$ uncertainty) is needed in order to observe a KK effect at a LEP2 collider running at $\sqrt{s} = 200 \, \text{GeV}$ for $\mu \geq 2 \, \text{TeV}$. However, the effect increases significantly for larger collider energies: For example, a compactification scale of 4 TeV produces an effect of only a few percent at LEP2 energies, while it reduces the cross section by more than 30% at a TeV-scale $e^+e^-$ high-energy collider.

Bhabha scattering involves the cross-channel exchanges of the $\gamma$ and $Z$ as well as the direct-channel exchanges of muon pair production. Again the primary effect can be attributed to the interference of the $n = 0$ mode with the $n > 0$ modes. However, although this interference causes a reduction in the direct-channel cross section (i.e., muon pair production), it has the opposite effect in the cross-channel. These competing effects lead to a smaller overall effect of the KK excitations on Bhabha scattering as compared to muon pair production. This overall enhancement of the SM cross section is depicted in Fig. 2 for the same range of parameters as in the muon case. The enlargement of the SM cross section is less than 10% for $\mu > 3 \, \text{TeV}$ or $\sqrt{s} < 900 \, \text{GeV}$. This makes Bhabha scattering considerably less attractive for observable effects of the KK states as compared to muon production.

4. Dijet Production

The full dijet final state cross section is given by summing $e^+e^- \rightarrow q\bar{q}$ over all quark flavors for which $s > 4m_q^2$. Thus, top quark production is only included for $\sqrt{s} > 350 \, \text{GeV}$. We include the standard top quark corrections in the cross section formulae, since the top quark mass is significant compared to $\sqrt{s}$ for LEP energies, but note that the top quark mass is negligible in comparison to TeV-scale KK excitations. The KK excitations result in the same effective $s$-channel propagator expressions as in the case of muon pair production (Eq. 9). As a result, the ratio $R$ is virtually identical in the two cases. The KK effect on dijet final state production is plotted in Fig. 3. As for muon production, a compactification scale of 3.5 TeV results in a reduction by 50% at a TeV-scale collider, by 12% for $\sqrt{s} = 500 \, \text{GeV}$.

5. Higgs Production

First, we consider the associated SM production of the Higgs boson: $e^+e^- \rightarrow ZH$. Here, we take the $Z$ boson to be produced on-shell. As discussed previously, the KK contribution is either zero or strongly suppressed due to KK number non-conservation unless the Higgs boson is confined to the SM three-brane. The $Z-Z^*_n-H$ coupling is non-zero in this situation because the corresponding term in the 5D Lagrangian density contains a delta function to constrain the Higgs boson to the SM wall. Therefore, we focus on this scenario here. The effect of the KK excitations of the $Z$ is described by the replacement of the SM direct-channel $Z$ boson propagator by the effective
Figure 2: The same as Fig. 1, but for Bhabha scattering.
Figure 3: The same as Fig. 1 but for dijet production.
propagator given in Eq. [9]. Although the Higgs mass is important in limiting
the available collider energy range and determining the numerical value of the SM cross
section, it does not affect the ratio of the total cross section given by the sum of the
SM and KK contributions to the SM cross section. In fact, since there is only one
diagram involved, the ratio $R$ is given by the effective propagator from Eq. [9]:

$$R = \left| D_{\text{eff}}(Z, s) \right|^2 = \frac{1}{2} \sum_{m,n=0}^{\infty} c_{Z_m} c_{Z_n} \frac{s'_{Z_m} s'_{Z_n} + m_{Z_m} \Gamma_{Z_m} m_{Z_n} \Gamma_{Z_n}}{(s_{Z_m}^2 + m_{Z_m}^2 \Gamma_{Z_m}^2)(s_{Z_n}^2 + m_{Z_n}^2 \Gamma_{Z_n}^2)}$$

(12)

Thus, not only is the ratio $R$ independent of the Higgs mass, it is also independent
of the Higgs model (to the point where there are only exchanges of $Z$ or $Z_n^*$ bosons).
For example, the ratio $R$ is the same in the SUSY Higgs doublet case of $e^+e^- \rightarrow Ah$
as in the SM case of $e^+e^- \rightarrow ZH$. However, the total cross section depends on the
Higgs model, and this plays a strong role in determining if the total cross section is
significant enough to observe the Higgs boson(s) (and if so, if it is large enough to see
a KK effect).

The KK effect on Higgs production for processes (such as the SM and SUSY
Higgs doublet cases discussed above) in which there are only exchanges of $Z$ or $Z_n^*$
bosons is shown in Fig. [4], where $R$ is graphed as a function of the compactification
scale $\mu$ and collider energy $\sqrt{s}$ for the same range of parameters as in the case of
muon pair production. Although there are no photon exchanges in the Higgs case,
the effect of KK excitations of the $Z$ boson on Higgs production is almost identical
to the KK effect on muon pair production. Lowest-lying KK excitations of the $Z$
bozon with masses of about 5 TeV cause a 20% reduction compared to the SM cross
section for a collider energy of 1 TeV, whereas the reduction is only 5% at $\sqrt{s} = 500$
GeV, and only 2% at the LEP2 energies. However, a compactification scale of 3.5
TeV produces at least a 10% effect at collider energies beginning at 400 GeV; the
reduction is about half for a 1 TeV energy collider. This reduction in the overall cross
section as compared to the SM cross section also has a significant effect on the Higgs
mass bound, and this is Higgs model-dependent.

6. Neutrino Pair and Single Photon Production

We first consider the case of neutrino pair production, $e^+e^- \rightarrow \sum_\ell \nu_\ell \bar{\nu}_\ell$. The production
of muon or tau neutrino pairs only consists of direct-channel $Z$ and $Z_n^*$ exchanges
described by the effective propagator modulus-squared of Eq. [9], whereas the production
of electron neutrino pairs also includes the cross-channel exchanges of $W$’s and
$W_n^*$’s. The modulus-squared of the effective propagator for this $t$-channel production
involving $W$’s and $W_n^*$’s is the same as the $s$-channel production involving $Z$’s and
$Z_n^*$’s with the replacements $Z \rightarrow W$ and $s \rightarrow t$. Similarly, the $s$-$t$ interference is given
by Eq. [10] with the replacement $\gamma \rightarrow W$.

As in the case of Bhabha scattering, the effect of the direct-channel exchanges of
the $Z$’s and $Z_n^*$’s to reduce the cross section and the competing effect of the cross-
Figure 4: The contributions of the exchanges of $Z_n^*$’s to Higgs production are illustrated as a function of the compactification scale $\mu$ for fixed values of the collider energy $\sqrt{s}$ (top), and as a function of $\sqrt{s}$ for specific choices of $\mu$ (bottom).
channel exchanges of the $W$’s and $W_n^*$’s to increase the cross section as compared to the SM results in a considerably smaller effect than processes such as muon pair production and dijet production where there are only $s$-channel exchanges of the EW gauge bosons. Although the $s$-channel KK effect to increase the cross section is larger than the $t$-channel KK counter-effect percentage-wise, the SM $t$-channel is dominant for neutrino pair production, which causes a slight increase in the cross section as compared to the SM. This is illustrated in Fig. 3, where the same ranges of the collider energy $\sqrt{s}$ and compactification scale $\mu$ are employed as in the case of muon pair production. The KK effect is smaller for neutrino pair production than for Bhabha scattering; also, there appears to be far less dependence on the variation of the collider energy. The result is a KK effect of less than 7% even for a collider energy as high as a TeV.

Single-photon production via $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ is somewhat more complicated. Single-photon production was considered in Ref. [17] in the context of $Z'$ physics, where only the lowest-lying KK excitations of the $W$ and $Z$ bosons were included. Here, we extend their results to include the 25 lowest-lying states, but concede that the effect depends almost exclusively on the first few states, and primarily on the first. The diagrams for single-photon production are the same as for neutrino pair production with a photon radiating off the incoming electron or positron or the internal $W$ or $W_n^*$. The effect of the KK excitations results in the same direct-channel effective propagator as the neutrino production case, and the same cross-channel effective propagator when the photon radiates off the incoming electron or positron. However, for the case where the photon radiates off the internal $W$ or $W_n^*$, a difference arises from the coupling of the photon to $W$’s and $W_n^*$’s. The $\gamma-W-W_n^*$ and $\gamma-W_n^*-W_{n\neq m}^*$ couplings are forbidden due to KK number non-conservation, as discussed in Section 2. On the other hand, the diagram with the $\gamma-W_n^*-W_n^*$ coupling has two propagators with KK excitations of the $W$ boson, which are quite massive (TeV-scale). This suppresses the KK contribution from this diagram in comparison to the contributions of the diagrams in which the photon radiates off either of the incoming particles.

The overall KK effect on single-photon production is very much similar to the KK effect on neutrino pair production. Graphically, the total KK contribution is shown in Fig. 3. Again, the enlargement of the SM cross section is very small. The single-photon and neutrino pair production KK effects are almost identical for large collider energies $\sim$ 1 TeV, but the single-photon case is more dependent on the collider energy, resulting in an even smaller effect for LEP energies than the neutrino pair case. Again, the compactification scale must be quite small $\lesssim$ 2 TeV in order to see even a 5% effect for a collider with very high energy (TeV-scale).
Figure 5: The contributions of the exchanges of $W_n^*$'s and $Z_n^*$'s to neutrino pair production are illustrated as a function of the compactification scale $\mu$ for fixed values of the collider energy $\sqrt{s}$ (top), and as a function of $\sqrt{s}$ for specific choices of $\mu$ (bottom).
Figure 6: The same as Fig. 5, but for single-photon production.
7. Conclusions

We have investigated the phenomenology of the KK excitations of the EW gauge bosons for a class of string-inspired models in which the SM gauge bosons propagate into one TeV-scale compact extra dimension, but where the SM particles are confined to the usual SM three-brane. Specifically, we have examined the effects that these KK excitations have on the cross sections for various processes at present and future high energy $e^+e^-$ colliders. Included in our study were Bhabha scattering and muon pair production, dijet production, Higgs production, and neutrino pair and single-photon production. Exclusively direct-channel processes, namely, muon, dijet, and Higgs production, produced a considerably greater effect than processes with both direct- and cross-channel Feynman diagrams, i.e. Bhabha scattering and neutrino and single-photon production. This is due to the competing effects of the effective propagators for $s$-channel exchanges and $t$-channel exchanges: The primary effect of the KK excitations arises from the interference of the $n = 0$ (SM) mode exchanges with the $n > 0$ (KK) mode exchanges, which results in a reduction of the modulus-squared of the effective propagator and thus the corresponding amplitude-squared for direct-channel exchanges, and an opposing enlargement for cross-channel exchanges and the $s$-$t$ interference.

The KK excitations of the EW gauge bosons would be particularly elusive for detection at LEP2 energies, where the largest effect for the processes that we examined is below 6% for even small compactification scales such as $\sim 2$ TeV and below 3% for compactification scales of $\sim 3.5$ TeV. Thus, quite precise measurements as well as very low compactification scales would be necessary for hints of KK excitations of the EW gauge bosons at LEP energies. However, the effects are considerably greater for prospective high energy colliders. For example, a 500 GeV collider can see about a 20% reduction in the cross section for muon pair production compared to the SM if the lowest-lying KK excitations have masses of $\sim 3$ TeV, and a 10% reduction for KK masses starting at $\sim 4$ TeV. A very high energy (TeV) collider could probe compactification scales up to 5 TeV and find a 20% effect, and up to 7 TeV with a 10% effect; meanwhile, a smaller compactification scale of 3 TeV reduces the cross section by half compared to the SM background.

We found that the KK excitations of the EW gauge bosons could play an important role on the discovery of the Higgs by enhancing the Higgs production cross section significantly. This is true for a Higgs boson that is confined to the SM three-brane, else the coupling of the Higgs to a single KK excitation of a gauge boson is zero.

Here we address the differences between KK excitations and other new physics that might produce a similar collider signal. In particular, $W'$ and $Z'$ physics produce the same effects as the lowest-lying KK excitations of the $W$ and the $Z$, except that the couplings of the $W'$ and $Z'$ to fermions can be different, and there are no restrictions on how many $W'$'s and/or $Z'$'s can couple to SM gauge bosons. Although the KK case involves an infinite tower of $W_n^*$'s and $Z_n^*$'s, the primary effect arises from the interference between the $n = 0$ (SM) and $n = 1$ (KK) modes, which is
exactly the effect of the $W'$ and $Z'$. In the case of multiple $Z'$'s, for example, if the various $Z'$'s have masses that are not integral multiples of the smallest $Z'$ mass, then this would clearly be different from the KK tower formed by a SM $Z$ boson that propagates into one extra dimension. Also, there has been abundant interest in $Z'$ models with restricted couplings to fermions, such as leptophobic $Z'$'s that couple to quarks but not to leptons, which seek to explain discrepancies between SM theoretical predictions and experimental measurements for particular processes without destroying fine agreement with processes such as charged lepton production at the $Z$ pole. In these cases, the couplings are different from the KK model that we consider here, where the KK excitations couple to all fermions with a $\sqrt{2}$ relative to the SM couplings. However, it is also possible to construct models in which some fermions see extra dimensions while others do not. For example, if the leptons see an extra dimension while the quarks are confined to the usual SM wall, then the situation will mimic the behavior of leptophobic $Z'$ physics for $e^+e^-$ processes. Finally, we considered a model in which all of the EW gauge bosons propagate into the same extra dimension. In this case, if there is a $Z^*_1$ with a mass of 3 TeV, then there are also $\gamma^*_1$'s and $W^*_1$'s with masses that are approximately 3 TeV as well. However, it is also possible that the various EW gauge bosons propagate into different extra dimensions with different compactification scales, or that some do not see extra dimensions at all. All in all, there is are several differences between the KK and $Z'$ effects that can be calculated for various processes, but the general behavior is quite similar. The chief test would come from very high energy colliders. If a $Z'$ or $Z^*$ is detected, then a search at twice that scale that fails to find a $Z^*_n$ would clearly reveal that it is a $Z'$ and not a $Z^*$, and a search that finds a $Z^*_2$ at the correct scale only leaves a small probability that this is coincidentally a second $Z'$ with twice the mass of the first (in which case a search at three times the first scale could reduce this probability even further or decide in favor of the $Z'$).

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*For this KK case, the $eq \rightarrow Z^*_n \rightarrow eq$ cross-channel process does not vanish, whereas a $Z'$ can not couple to the leptons.
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