CONSTRAINTS ON THE LORENTZ INVARIANCE VIOLATION WITH GAMMA-RAY BURSTS VIA A MARKOV CHAIN MONTE CARLO APPROACH

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ABSTRACT

In the quantum theory of gravity, for photons we expect the Lorentz Invariance Violation (LIV) and the modification of the dispersion relation between energy and momentum. The effect of the energy-dependent velocity due to the modified dispersion relation for photons was studied in the standard cosmological context by using a sample of gamma-ray bursts (GRBs). In this paper we mainly discuss the possible LIV effect of using different cosmological models for the accelerating universe. Due to the degeneracies among model parameters, the GRBs’ time delay data are combined with the cosmic microwave background data from the Planck first-year release, the baryon acoustic oscillation data at six different redshifts, and Union2 Type Ia supernovae data to constrain both the model parameters and the LIV effect. We find no evidence of the LIV.

Key words: dark energy – gamma-ray burst: general – gravitation

1. INTRODUCTION

Above the Planck energy scale $E_{\text{Pl}}$, we expect a quantum theory of gravity in place of Einstein’s theory of general relativity. The quantization of spacetime will lead to the modification of the dispersion relation between the energy and the momentum of a particle with mass $m$ and the breakdown of Lorentz invariance. In some quantum gravity models, the consequence of the Lorentz Invariance Violation (LIV) is the energy dependence of the speed of light in a vacuum (Amelino-Camelia et al. 1998, 2001). In these scenarios, the energy-dependent velocity of light is $v = c(1 - \xi E/E_{QG})$, with an effective quantum gravity energy scale $E_{QG}$, the speed of high-energy photons is slower, and low-energy photons will reach us earlier. Therefore, the measurement of the light speed in a vacuum can be used to study the LIV effect.

In past years, both astrophysics and particle physics experiments have been proposed to test the LIV and quantum gravity effects (Sarkar et al. 2002; Mattingly 2005; Amelino-Camelia 2013). Because photons with different energies will reach us at different times and the energy of gamma-ray bursts (GRBs), which are the most luminous explosions in the universe with short duration ranges from KeV to GeV, GRBs with intense bursts of $\gamma$-ray photons from cosmological distances have been proposed to measure the difference in the arrival times of photons with different energies (Amelino-Camelia et al. 1998, 2002; Kowalski-Glikman & Nowak 2002). GRBs were also used to constrain LIV theories (Ellis et al. 2003, 2006; Rodriguez-Martinez & Piran 2006; Jacob et al. 2007). Biller et al. (1999) used the data for a TeV $\gamma$-ray flare coming from the active galaxy Markarian 421 to set a bound on the energy scale of quantum gravity as $E_{QG} > 6 \times 10^{16}$ GeV. Abdou et al. (2009a) used GRB 090510 to study the LIV effect and get $E_{QG} > 1.2E_{\text{Pl}}$, the energy scale of quantum gravity was found to be $E_{QG} > 1.3 \times 10^{18}$ GeV by using GRB 080916C (Abdo et al. 2009b). The LIV effects due to extra dimensions can also be constrained by GRBs (Baukh et al. 2007). Experiments with cold atoms have also been proposed to constrain the modified dispersion relation due to quantum gravity (Amelino-Camelia et al. 2009).

In order to measure the LIV, statistical and possible systematic uncertainties must be minimized. For this purpose, Ellis et al. (2003, 2006) developed a method for analyzing samples of GRBs with different redshifts and energy bands. This technique has the advantage of extracting time-dependent features from the signals of many GRBs. In their analysis Ellis et al. (2003) used both the BATSE and OSSE data from the Compton Gamma Ray Observatory.5 By adding larger samples of GRBs with known redshifts from HETE6 and SWIFT,7 the observed time delay was formulated in terms of linear regression where the intercept denoted intrinsic time delay and the linear term denoted LIV effect (Ellis et al. 2006; Jacob et al. 2008). By using the concordance $\Lambda$CDM model with $\Omega_\Lambda = 0.7$, they found no strong evidence of the LIV and they obtained $E_{QG} > 1.4 \times 10^{16}$ GeV (Ellis et al. 2006).

Since the distances from the sources to the observer depend on cosmological models, the conclusions regarding the LIV and quantum gravity effect may be affected by the cosmological model used in the analysis. However, only the concordance $\Lambda$CDM scenario ($\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$) was studied (Ellis et al. 2003). Therefore, it is necessary to consider other cosmological models. Recently Biesiada et al. (2009) studied the LIV by using the quintessence and Chaplygin Gas model and they found weak evidence of the LIV. In their analysis the degeneracies among model parameters were neglected, as they were taken as fixed values. In this paper we mainly discuss the possible LIV using different cosmological models that explain the present cosmic acceleration. Compared with previous work, in addition to the relevant parameters quantifying the LIV, the cosmological parameters are also treated as free parameters. We adopt the Markov Chain Monte Carlo (MCMC) technique to constrain the LIV and

5 ftp://legacy.gsfc.nasa.gov/compton/data/hatse/usccii_data/64ms/
6 http://space.mit.edu/HETE/Bursts/
7 http://swift.gsfc.nasa.gov/docs/swift/archive/
model parameters with the observational data. In order to derive tighter constraints on the model parameters, we combine the time delay data from GRBs with the cosmic microwave background (CMB) data from the Planck first-year release (Wang et al. 2013; Ade et al. 2014), the baryon acoustic oscillation (BAO) data (Percival et al. 2010; Beutler et al. 2011; Blake et al. 2011; Gong et al. 2014), and the 557 Union2 SNe Ia data (Amanullah et al. 2010). This paper is organized as follows. We introduce the LIV in Section 2. The GRBs’ time delay and other observational data are discussed in Section 3. The constraint results on the LIV and cosmological parameters with different cosmological models are presented in Section 4, and finally, the conclusions are drawn in Section 5.

2. THE LORENTZ INVARIANCE VIOLATION

The modified dispersion relation due to quantum gravity models can be parameterized as follows (Ellis et al. 2003; Biesiada et al. 2009):

\[ E^2 - p^2 c^2 - \xi_n E^2 \left[ \frac{E}{E_{QG}} \right]^\alpha, \]  

(1)

where \( \xi_n \) is a dimensionless parameter, \( \xi_1 = 1, \xi_2 = 10^{-7} \) (Jacob et al. 2007), \( \epsilon = +1 \), and the effective energy scale \( E_{QG} \) of quantum gravity is expected to be near the Planck scale. Because the LIV effects are small (Ellis et al. 2008; Biesiada et al. 2009) we only consider the \( n = 1 \) term in this study.

Since the arrival time for photons with energy \( E \) is equal to (Biesiada et al. 2007, 2009; Jacob et al. 2007, 2008)

\[ t_{LIV} = \int_0^z \left[ 1 + \frac{E}{E_{QG}} (1 + z') \right] \frac{dz'}{H(z')}, \]  

(2)

the arrival times of two photons with different energy will be different, and the difference in time for the energy difference \( \Delta E \) is

\[ \Delta t_{LIV} = \frac{\Delta E}{H_0 E_{QG}} \int_0^z \left( 1 + z' \right) \frac{dz'}{h(z')}, \]  

(3)

where the dimensionless Hubble parameter \( h(z) = H(z)/H_0 \), \( H_0 \) is the Hubble constant, and \( H(z) \) is the Hubble parameter at redshift \( z \). In order to account for the unknown intrinsic time lags, we fit the measured time lags by including a parameter \( b \) that is specified in the rest-frame of the source. The arrival time delays are fitted by the formula \( \Delta t_{obs} = \Delta t_{LIV} + b(1 + z) \) (Ellis et al. 2006). Therefore, the simple linear fitting function is

\[ \frac{\Delta t_{obs}}{1 + z} = a_{LIV} K + b, \]  

(4)

where the intercept represents the intrinsic time lags, the LIV effects are encoded in the slope \( a_{LIV} = \Delta E/(H_0 E_{QG}) \), which is related to the scale of Lorentz violation (Biesiada et al. 2007; Jacob et al. 2008), and \( K = (1 + z)^2 \int_0^z dz'/(1 + z')/h(z') \) is related to the measurements of cosmic distances. The model dependence is through the function \( K(z) \), which will be calculated for three popular cosmological models.

3. THE OBSERVATIONAL DATA

In this paper we use the BATSE data with 9 light curves whose time resolutions are 64 ms and whose redshifts span from \( z = 0.835 \) to \( z = 3.9 \); the HETE data with 15 light curves whose time resolutions are 164 ms and whose redshifts span from \( z = 0.168 \) to \( z = 3.572 \); and the SWIFT data with 11 light curves whose time resolutions are 64 ms and whose redshifts span from \( z = 0.258 \) to \( z = 6.29 \). The data was shown in Table 1 of Ellis et al. (2006). We use the data of time lags between different energy channels measured from the light curves of 35 GRBs with redshifts from \( z = 0.168 \) to \( z = 6.29 \) (Ellis et al. 2006). The spectral time lags are obtained from the light curves in the 115–320 keV energy band with respect to those in the lowest 25–55 keV energy band. To test the LIV effect, we apply the \( \chi^2 \) statistics. Here \( \chi^2 \) is

\[ \chi^2_{GRBs} = \sum_{i=1}^{N_{GRBs}} \left( \frac{\Delta t_i - \Delta t_{obs}}{\sigma_i} \right)^2, \]  

(5)

where \( \Delta t_i \) and \( \Delta t_{obs} \) respectively denote the theoretical and observational values of time delays of GRBs, and \( \sigma_i \) is the observational uncertainty.

In fitting the data, in addition to the slope \( a_{LIV} \) and the intercept \( b \), we also need to fit the model parameters. In previous studies, the model parameters were fixed and the degeneracies among parameters were neglected. To account for the degeneracies among these parameters, it would be better to treat the model parameters as free parameters and estimate their nominal values from the observational data. With this aim, we combine the CMB data from the Planck first-year release, the 557 Union2 SNe Ia data, and the BAO data with the data of spectral time lags from GRBs to constrain the model parameters. Since we have at least four parameters to fit, we take the MCMC (Lewis et al. 2002) technique to constrain the model parameters.

For the 557 Union2 SNe Ia data, the distance modulus \( \mu(z) \) is measured at different redshifts and the theoretical value of the distance modulus is

\[ \mu = 5 \log \frac{d_L}{\text{Mpc}} + 25 = 5 \log_{10} H_0 d_L - \mu_0, \]  

(6)

where \( \mu_0 = 5 \log_{10} [H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})] - 42.38 \), and the luminosity distance is \( d_L = (1 + z) H_0^{-1} \int_0^z dz' / E(z') \). To fit the 557 Union2 SNe Ia data, we calculate

\[ \chi^2_{\text{SNe}} = \sum_{i=1}^N \left[ \frac{\mu(z_i) - \mu_{\text{obs}}(z_i)}{\sigma_{\mu i}^2} \right]^2, \]  

(7)

where \( \mu(z_i) \) and \( \mu_{\text{obs}}(z_i) \) are the theoretical and observed distance moduli for the SNe Ia at redshift \( z_i \). The \( \sigma_{\mu i} \) is observational error. For the nuisance parameter \( H_0 \), we marginalize it with a flat prior.

The BAO A data consist of the measurements of \( A \) at three redshifts \( z = 0.44, z = 0.6, \) and \( z = 0.73 \) from the WiggleZ dark energy survey (Blake et al. 2011; Gong et al. 2014), and the parameter \( A \) is defined as

\[ A = \sqrt{\Omega_m H_0 D_V(z)} / z, \]  

(8)
where the effective distance is given by (Eisenstein et al. 2005)

\[ D_V(z) = \left( \frac{d_L(z)}{(1 + z)^2 E(z)} \right)^{1/3}. \]  

(9)

The \( \chi^2 \) value for this data set is

\[ \chi^2_{BAO1} = \Delta A_{BAO}^2 C_{BAO}^{-1} \Delta A_{BAO}, \]  

(10)

where \( C_{BAO}^{-1} \) is the corresponding inverse covariance matrix (See Table 1).

In addition to the above \( A \) parameter, we also consider the BAO distance ratio \( d_z \) at \( z = 0.2 \) and \( z = 0.35 \) from the SDSS data release 7 (DR7) galaxy sample (Percival et al. 2010) and \( d_{0.106} = 0.336 \pm 0.015 \) from the 6dFGS measurements (Beutler et al. 2011). The BAO distance ratio is

\[ d_z = \frac{r_s(z_d)}{D_V(z)}. \]  

(11)

where the comoving sound horizon is

\[ r_s(z) = \int_z^{\infty} \frac{c_s(z)dz}{E(z)}. \]  

(12)

The \( \chi^2 \) value of the BAO distance ratio is (Percival et al. 2010)

\[ \chi^2_{BAO2} = \Delta P_{BAO}^T C_{BAO}^{-1} \Delta P_{BAO}, \]  

(14)

where \( \Delta P_{BAO} = P_{th} - P_{obs} \), \( P_{obs} \) is the observed distance ratio, and \( C_{BAO} \) is the covariance matrix for the distance ratio. To use the BAO data, we minimize

\[ \chi^2_{BAO} = \chi^2_{BAO1} + \chi^2_{BAO2} + \frac{(d_{0.106} - 0.336)^2}{0.015^2}. \]  

(15)

For the CMB measurement, we use the derived parameters, including the acoustic scale \( l_a \), the shift parameter \( R \), and \( \Omega_b h^2 \) (Wang et al. 2013; Ade et al. 2014) from the Planck temperature measurements combined with lensing, as well as WMAP polarization data at low multipoles with \( l \leq 23 \). The acoustic scale is

\[ \Omega_k^{1/2} H_0^{-1} \sin (\Omega_k^{1/2} \int_0^{z_\text{sh}} dz/E(z)) \]  

\[ l_a = \pi \frac{r_s(z_{\text{sh}})}{\Omega_k^{1/2} H_0^{-1} \sin (\Omega_k^{1/2} \int_0^{z_\text{sh}} dz/E(z))}, \]  

(16)

where \( r_s(z_{\text{sh}}) = H_0^{-1} \int_0^{z_{\text{sh}}} c_s(z)/E(z)dz \) is the comoving sound horizon at the recombination. The shift parameter is

\[ R(z_{\text{sh}}) = \frac{\bar{\Omega}_m^{1/2}}{\Omega_k^{1/2} H_0} \sin (\Omega_k^{1/2} \int_0^{z_{\text{sh}}} dz/E(z)) \]  

(17)

The recombination redshift \( z_{\text{sh}} \) is fitted by (Hu et al. 1996)

\[ z_{\text{sh}} = 1048 \left[ 1 + 0.00124 (\Omega_b h^2)^{-0.738} \right] \left[ 1 + g_1 (\Omega_m h^2)^{g_2} \right]. \]  

(18)

The \( \chi^2 \) value for the CMB data is

\[ \chi^2_{CMB} = \Delta P_{CMB}^T C_{CMB}^{-1} \Delta P_{CMB}, \]  

(19)

where \( \Delta P_{CMB} = P_{th} - P_{obs} \), \( P_{obs} \) and \( P_{th} \) are the observed and the theoretical values of the derived parameters, respectively, and \( C_{CMB} \) is the covariance matrix for the derived CMB data (Wang et al. 2013).

4. CONSTRAINTS ON THE LIV PARAMETERS AND ANALYSIS

To explain the cosmic acceleration, an exotic energy component called dark energy with negative pressure was proposed (Riess et al. 1998; Perlmutter et al. 1999; Spergel et al. 2003, 2007; Pegram et al. 2004; Eisenstein et al. 2005; Astier et al. 2006; Hicken et al. 2009; Komatsu et al. 2009, 2011; Amanullah et al. 2010; Cao et al. 2011, 2012a, 2012b; Gong et al. 2013, 2014; Gao & Gong 2014). The most simple candidate for dark energy is the vacuum energy known as the cosmological constant \( \Lambda \). Because of the huge difference between the predicted and measured values of the vacuum energy, many other dynamical dark energy models have also been considered, including quintessence (Ratra et al. 1988; Caldwell et al. 1998), phantom (Caldwell 2002; Caldwell et al. 2003), k-essence (Armendariz-Picon et al. 2001; Chiba 2002), and quintom models (Feng et al. 2005a, 2006; Guo et al. 2005). In this paper, we consider three different dark energy models: the \( \Lambda \)CDM model, the dark energy model with constant equation of state parameter \( w \), and the Chevallier-Polarski-Linder (CPL) model (Chevallier et al. 2001; Linder 2003).

For the \( \Lambda \)CDM model, the equation of state parameter \( w = p/\rho = -1 \) and the Friedmann equation is

\[ H^2(z) = \Omega_m (1 + z)^3 + \Omega_\Lambda, \]  

(20)

where \( \Omega_m \) is the matter energy density, and the dark energy density is \( \Omega_\Lambda \). Since \( \Omega_m + \Omega_\Lambda = 1 \) for a flat \( \Lambda \)CDM model, we have only one independent parameter \( \Omega_m \) in this model. By fitting the \( \Lambda \)CDM model to the above data, we get
Table 2

| Cosmological Model | Regression Coefficient $a$ and Intercept $b$ |
|--------------------|-----------------------------------------------|
| $\Lambda$CDM       | $a = -0.017^{+0.0717}_{-0.0718}, 0.1415$      |
|                    | $b = -0.00013^{+0.0154}_{-0.0155}, -0.0305$  |
| wCDM               | $a = -0.0168^{+0.0711}_{-0.0702}, 0.1392$    |
|                    | $b = -0.00015^{+0.0153}_{-0.0154}, -0.0304$  |
| CPL                | $a = -0.0183^{+0.0712}_{-0.0711}, -0.0141$   |
|                    | $b = 0.00018^{+0.0155}_{-0.0155}, -0.0301$   |

the 1σ and 2σ constraints: $a = -0.017^{+0.0717}_{-0.0718}, 0.1415$, $b = -0.00013^{+0.0154}_{-0.0155}, -0.0305$, $\Omega_m = 0.29^{+0.010}_{-0.011}, -0.022$, and $H_0 = 69.5^{+0.9}_{-0.9}, 1.1$. The results show that Lorentz invariance is consistent with the data. We show the results for $a$ and $b$ in Table 2. The one-dimensional (1D) probability distribution of each parameter and the two-dimensional (2D) confidence contours for the parameters are shown in Figure 1.

For the flat wCDM model, the Friedmann equation is

$$h^2(z) = \Omega_m(1 + z)^3 + \Omega_X(1 + z)^{3(1+w)},$$

where the dark energy density $\Omega_X = 1 - \Omega_m$. Using the MCMC method, the marginalized 1σ and 2σ constraints on the model parameters are: $a = -0.0168^{+0.0711}_{-0.0702}, 0.1392$, $b = -0.00015^{+0.0153}_{-0.0154}, -0.0304$, $\Omega_m = 0.288^{+0.011}_{-0.011}, -0.022$, $w = -1.05^{+0.04}_{-0.045}, -0.088$, and $H_0 = 70.2^{+1.1}_{-1.1}, -2.1$. There is no evidence of the LIV in the wCDM model. The results are shown in Table 2 and the marginalized plots are shown in Figure 2.

For the CPL model, the equation of state parameter is parameterized as $w(z) = w_0 + w_1z/(1 + z)$ and the Friedmann equation is

$$h^2(z) = \Omega_m(1 + z)^3 + \Omega_X(1 + z)^{3(1+w_0+w_1)} \exp\left(-\frac{3w_1z}{1 + z}\right).$$

In this model, $\Omega_m$, $w_0$, and $w_1$ are the model parameters. Fitting the CPL model to the combined data, we obtain the marginalized 1σ and 2σ constraints on the model parameters: $a = -0.0183^{+0.0712}_{-0.0711}, -0.140$, $b = 0.00018^{+0.0159}_{-0.0155}, -0.0306$, $\Omega_m = 0.288^{+0.012}_{-0.012}, -0.024$, $w_0 = -1.02^{+0.123}_{-0.122}, -0.237$, $w_1 = -0.203^{+0.593}_{-0.595}, -1.281$, and $H_0 = 70.3^{+1.27}_{-1.0}, -2.57$. We see no evidence of the LIV in the CPL model. The constraints on $a$ and $b$ are shown in Table 2 and the marginalized probability distributions and the 1σ and 2σ contours of $a$ and $b$ are shown in Figure 3.

In the previous analysis, the model parameters took fixed values and a weak indication of the LIV was found (Ellis et al. 2006; Biesiada et al. 2009). Here we consider the possible degeneracies among all the parameters and take all the parameters as free parameters, finding no evidence of the LIV and results that all are consistent with each other. The conclusion is independent of the background model. Although the uncertainties on the parameters $a$ and $b$ become larger due to more fitting parameters, the difference in $a$ for the different cosmological models is still relatively small. To better understand the effect of the background model, we show the re-scaled spectral time lags $\Delta t_{obs}/(1 + z)$ versus the $K(z)$ function for the three different cosmological models in Figures 4–6; the data from GRBs is also shown in the figures. Because the background cosmological model changes the value of $K(z)$, the value of $a$ also changes. Unlike the models considered in Biesiada et al. (2009), the wCDM model and the CPL model considered here are close to the $\Lambda$CDM model, as seen from the above best-fitting values of $w$, $w_0$, and $w_1$, so our results are all consistent with each other.

The redshifts of the GRBs span from $z = 0.168$ to $z = 6.29$ so we also consider the effect of the redshift distribution on the results. We divide the GRBs into 4 groups with upper boundaries at $z = 1.0$, $z = 2.0$, $z = 3.0$, and $z = 6.3$. The first group has 11 GRBs with redshifts $0 < z < 1.0$, the second group has 21 GRBs with redshifts $1 < z < 2.0$, the third group has 27 GRBs with redshifts $2 < z < 3.0$, and the fourth group contains all 35 GRBs. We then fit the cosmological models to
each group of GRBs and the constraints on the slope $a$ normalized to the $\Lambda$CDM model are shown in Figure 7. The differences between different models are negligible in the first and the third redshift groups. For both the wCDM and CPL models, the effects of redshift distributions are similar although the deviation is a little larger for the CPL model. The biggest contribution comes from the GRBs with redshifts in the ranges $z_{12}<z<3$. As we discussed above, the CPL model deviates more from $\Lambda$CDM than the wCDM model does, so the value of the slope $a$ for the $\Lambda$CDM model is more negative. The reason for the redshift-dependence is due to the distributions of the GRBs. In the first and third groups, the average of the spectral time lags is close to zero. But we have more negative $\Delta t_{\text{obs}}/(1+z)$ data in the redshift intervals $z_{12}<z<2$ and $z>3$, therefore we see bigger deviations for the data up to redshift 2 and for the whole data set.

5. CONCLUSIONS

We test the LIV with the 35 GRBs, the Union2 SNe Ia, the CMB, and BAO data for three different cosmological models.
The slope $a$ and the intercept $b$ in the linear regression method, as well as the model parameters, are fitted to the combined data. For the $\Lambda$CDM model, the marginalized $1\sigma$ and $2\sigma$ results are: $a = -0.017^{+0.017}_{-0.017}$ and $b = -0.00013^{+0.0054}_{-0.0038}$. For the wCDM model, the marginalized $1\sigma$ and $2\sigma$ results are: $a = -0.0168^{+0.0711}_{-0.0139}$ and $b = -0.00015^{+0.0153}_{-0.0030}$. For the CPL model, we obtain the marginalized $1\sigma$ and $2\sigma$ results $a = -0.0183^{+0.0172}_{-0.0141}$ and $b = 0.00018^{+0.0159}_{-0.0155}$. These results are also summarized in Table 2. Because the slope $a$ is consistent with $a = 0$ for all three models, there is no evidence of the LIV. The results for all three models are also consistent. Our conclusion is in conflict with previous results reported by Biesiada et al. (2009). In their studies, the model parameters are fixed at certain values, so the degeneracies among parameters are neglected. The dynamical dark energy models with the fixed parameter values discussed in the previous work are very different from the $\Lambda$CDM model, so they found different results for different cosmological models. In our analysis, we treat the model parameters as free parameters, and the model parameters are fitted with the latest SNe Ia and BAO data. The best fitted wCDM and CPL models are close to the $\Lambda$CDM model. Although the uncertainties on the model parameters become almost double compared with the previous analysis (Biesiada et al. 2009) due to more fitting parameters, the differences between different models are relatively small and all the results are consistent. Because the GRBs’ data are not uniformly distributed, especially in the redshift intervals $1 < z < 2$ and $z > 3$, we see the small differences between different models come from the GRBs at the redshift intervals $1 < z < 2$ and $z > 3$. In conclusion, our results show no evidence of the LIV in the three cosmological models.

These results are also in good agreement with the recent neutrino analysis, which discussed the LIV using two cascade neutrino events with energies around 1 PeV recently detected by IceCube (Borriello et al. 2013). Yet it is worth noting that experimental probes of the LIV are limited by the scarcity of GRB data. Other high-energy astrophysics experiments such as the photon time delay measurements from objects like pulsars (Kaar et al. 1999) and active galactic nuclei (Albert et al. 2008) may provide complementary probes of the LIV effect. Further studies are still needed to draw a more quantitative conclusion.

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