Stabilized Stepwise Orthogonal Matching Pursuit for Sparse Signal Approximation

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Abstract. Orthogonal Matching Pursuit (OMP) algorithm is equipped with the capability to decompose any signal into a linear expansion of waveforms, which are selected from a redundant functional dictionary. Nevertheless, classical OMP algorithm suffers a heavy computational burden due to its single element selection strategy in each repetition. Recently, an accelerated implementation called stage wise orthogonal matching pursuit (StOMP) algorithm has been proposed through exploiting a multiple elements selection scheme based on an iterative threshold. However, as the defined threshold is a function of an empirical and undetermined parameter, such a reconstruction scheme is not optimal and the algorithm may get obstructed in some specific conditions. This manuscript presents an adaptive threshold selection strategy which takes signal structure into consideration and furthermore, a regularized iterative framework for sparse signal approximation is suggested. Compared with classical StOMP approach, these efforts can provide robust and more attractive approximation performance for sparse signal recoveries. Experimental results present the substantial improvements of these optimizations.

1. Introduction

Sparse signal decompositions[1] have received considerable attentions in recent years and applications are found in a wide range of field such as source coding[2] and signal acquisition[3]. Typically, a signal decomposition model approximates observed signal using a linear combination of elementary waveforms which are selected from a large collection. Nowadays, a variety of algorithms can be utilized for sparse signal approximation such as convex optimization tools[4-5], and various iterative methods[6-8]. While convex optimization techniques offer valuable procedures for computing sparse representations and L1-minimization solutions, which can be commonly found in the literature of Compressed Sensing(CS)[9-11], some more powerful solvers for such problems can be accomplished by a great number of greedy algorithms[12-16]. Greedy methods rely on gradual approximation progresses, either by iteratively identifying the support of signal until a convergence criterion is met, or alternatively by obtaining an improved estimate of sparse signal in each iteration. Among these competitive greedy algorithms, a fundamental implementation is Orthogonal Matching Pursuit (OMP)[17], which repeats by finding the column of measurement matrix most correlated with current residual and then updating estimated coefficients through seeking a least square solution (LS)[18-19] until the algorithm reaches certain stop criterion.

Nevertheless, while OMP can provide good approximation performances and stable recoveries, one crucial problem when applying the algorithm to large-scale data is that it suffers a slow convergence rate as the algorithm only selects one atom each time. D.L.Donoho in [20] proposed an innovative elements selection strategy in order to speed up the pursuit progress of OMP, called Stagewise Orthogonal Matching Pursuit (StOMP) algorithm. The implementation intelligently selects multiple
elements through introducing a threshold-based elements selection strategy into each iterative step and can substantially accelerate the convergence rate of pursuit progress. However, as the threshold suggested in[20] depends on an undetermined parameter and the selection of the unresolved parameter just relies on some empirical values, implementation in[20] is not robust and recovery progresses may get obstructed under certain conditions. The absence of guidance to robust parameter selection and unstable recovery capacity make the implementation rather inconvenient, thus dim the capability of StOMP algorithm in some practical applications.

This manuscript presents an optimized stepwise orthogonal matching pursuit scheme with the aim of providing stable and more appealing recovery performances for high dimensional sparse signal approximations. Similarly, we mainly concentrate on two aspects of conventional greedy algorithms, elements selection and coefficient update. On the one hand, this literature suggests an adaptive threshold concerning the internal structure of original signal, which can constantly provide robust recoveries with favorable convergence under general circumstance. On the other hand, a regularized framework for selected coefficients estimation is further developed, which carries greedier and more touching approximation performance compared with current StOMP algorithm. The brief review of this paper is organized as follows. The main part starts with the framework of classical greedy pursuit strategies, and restrictions of current OMP and StOMP algorithm are also investigated in Section II. In Section III, a robust elements selection scheme based on an “adaptive” threshold is firstly suggested. Then we turn to selected coefficients update step and a regularized coefficients update framework is further devised in this part. Section IV provides numerical results to demonstrate the attractive improvements of proposed algorithm and finally, discussion and conclusions are presented in Section V.

2. Greedy Pursuit Framework

Greedy pursuit methods build up an approximation through making locally optimal choices at each step. Nowadays, there is a large and growing family of greedy pursuit techniques for signal approximation with a long and multi-rooted history. The main idea of these approaches can be stated as projecting the data in a given direction and then testing the derivation of estimated signal from measurements. A typical candidate for these techniques is OMP algorithm, which was first taken up by Mallat and Zhang [17] and used for signal approximation. This section will briefly cover the pursuit procedures of OMP and then touch upon the kernel of StOMP algorithm in the following part.

2.1 Overview of Orthogonal Matching Pursuit

OMP aims at estimating original signal by iteratively projecting data onto different directions. The algorithm begins with finding the column of measurement dictionary most correlated with the measurements, and then repeats this step by correlating the columns with evaluated residual, which is obtained by subtracting the contribution of a partial estimate from original measurement vector.

\[ y = Ax + e \]  

Consider a general noisy measurement model in CS, where \( A \in \mathbb{R}^{M \times N} \) is the measurement matrix \( (M < N) \). For a given \( y \in \mathbb{R}^M \), we want to recover an \( K \)-sparse vector \( x \in \mathbb{R}^N \) under the assumption that the error \( e \in \mathbb{R}^M \) is bounded and the measurement matrix \( A \) satisfies the restricted isometric property (RIP)[21]:

\[
(1 - \delta_K) \| x \|_2^2 \leq \| Ax \|_2^2 \leq (1 + \delta_K) \| x \|_2^2
\]  

Then the solution of Eq.(1) generally leads to a \( l_1 \) minimization [22-24] problem and the evaluation of \( x \) can be effectively estimated with the assistance of OMP algorithm when the measurement matrix \( A \) satisfies RIP condition [25]. For a given vector \( y \) and a matrix \( A \), OMP devotes to identifying a small
support set $\Lambda$ and an estimated vector $\hat{x}$ to approximate $y$ using:

$$\hat{y} = A_\Lambda \hat{x}_\Lambda$$

(3)

The algorithm is initialized with setting the first residual $r^{[0]} = y$, setting $x^{[0]} = 0$ and $\Lambda^{[0]} = \emptyset$. Each repetition it updates these three quantities. The iterative pursuit progress can be briefly summarized as the following steps:

a) Initialize the residual $r^{[0]} = y$, the sparse estimate $x^{[0]} = 0$ and initialize the index set of selected columns $\Lambda^{[0]} = \emptyset$. Set the iteration counter $i = 1$.

b) Calculate the correlation of column vectors of $A$ with the current residual $r^{(i)}$, $h_j = \langle A_j, r^{(i)} \rangle$ and find the column best correlated with current residual $j^{[i]} = \mathbf{arg max}_j \langle A_j, r^{(i)} \rangle$. Update the index set $\Lambda^{[i]} = \Lambda^{[i-1]} \cup \{ j^{[i]} \}$.

c) Compute new estimation by projecting $y$ onto the linear space spanned by the columns indexed by $\Lambda^{[i]}$, i.e., $\hat{x}_{\Lambda^{[i+1]}} = A_{\Lambda^{[i+1]}}^\dagger y$. Update the residual $r^{(i)} = y - A_{\Lambda^{[i]}} \hat{x}_{\Lambda^{[i]}}$.

d) If certain stopping criterion is satisfied, then stop. Otherwise, set $i = i + 1$ and turn to step 2.

While OMP enjoys superior approximation capability in the context of CS, one crucial problem the algorithm encounters is that it suffers a slow convergent performance for large scale application. The pursuit algorithm has to operate as many cycles as the number of atoms needed to be selected to approximate measurement $y$, as it selects only one atom at each repetition. When the sparsity $K$ gets large, the pursuit progress becomes impractically slow and inefficient.

2.2 Technique of Stagewise Orthogonal Matching Pursuit

Paper [20] first proposed an inventive elements selection strategy based on a defined threshold with the motivation of keeping the algorithm a low computational time consuming. Instead of choosing the element that is maximally correlated with the residual $j^{[i]} = \mathbf{arg max}_j \langle A_j, r^{(i)} \rangle$ in OMP, the stagewise strategy replaces the maximum with a threshold criterion and selects all the indexes satisfying the following condition in each iterative step:

$$T^{[i]} = \{ j : \langle A_j, r^{(i)} \rangle \geq \lambda^{[i]} \}$$

(4)

where $\lambda^{[i]}$ is the defined threshold at iteration $i$. While some choices for $\lambda^{[i]}$ are suggested in several literatures, most of these approaches remain to be dissatisfactory and undesirable. For example, a simple non-iterative threshold was presented in paper [26]. Although it’s by far the most computationally simple procedure, this suggestion has limited recovery guarantees. Another technique employing a dynamic threshold in the iterative framework is presented in StOMP. In this approach the threshold is calculated concerning current residual $r$ and an undetermined parameter $t$:

$$\lambda^{[i]}_{\text{omp}} = \frac{t \| r^{[i-1]} \|_1}{\sqrt{M}}$$

(5)

While the threshold provides performance guarantees for some specific coefficient distributions, similar guidance is not available for other general cases. From a practical point of view, the selection of parameter $t$ required in this method is critical for its performance. Despite the suggestion considering a value $2 \leq t \leq 3$ in [20], there do not seem to be any other guidelines available. The indeterminacy and fluctuation of parameter selection make the implementation rather inconvenient and unstable. Actually, specific cases may be encountered when using this improper threshold is that
the algorithm will terminate and get obstructed permanently when all inner products fall below the casual threshold, thus lead to an inaccurate recovery.

3. Stabilized Stepwise Orthogonal Matching Pursuit Scenario

This section presents a stable elements selection program through introducing an adaptive threshold, which takes signal structure into consideration. Furthermore, we employ a regularized framework at the coefficient update step to provide a more appealing performance for sparse signal approximation applications. The removal of undetermined factor and the introduction of regularized coefficients update scheme maximally stabilize the proposed algorithm, and thus can constantly provide an accurate even better recovery compared with classical StOMP approach.

Consider a signal to be recovered is $K$ sparse, theoretically, there are at most $K$ columns of $A$ contributing to the residual at $i$th iteration (for simplicity, we assume the first $K$ columns here), that is:

$$r^{(i)} = r_1^{(i)} A_1 + r_2^{(i)} A_2 + \cdots + r_K^{(i)} A_K$$

Then the $l_2$ norm of the residual $r^{(i)}$ at current iteration can be calculated as:

$$\|r^{(i-1)}\|_2^2 = \|r_1^{(i)} A_1 + r_2^{(i)} A_2 + \cdots + r_K^{(i)} A_K\|_2^2$$

$$\leq \sum_{k=1}^{K} (r_k^{(i)})^2 \|A_k^{(i)}\|_2^2$$

$$\leq K (r_j^{(i)})^2 \|A_{j_{\text{max}}}^{(i)}\|_2^2$$

where $r_j^{(i)}$ is the projection coefficient of $r^{(i)}$ on the direction of $j$th column vector $A_j$, and $j_{\text{max}}$ is the column index which remains $\|r_j \cdot A_j\|_2$ to be largest at current step. Then, a preferred reference for threshold can be acquired from Eq. (7):

$$\lambda^{(i)} = \frac{\|r^{(i-1)}\|_2}{\sqrt{K} \cdot \|A_{j_{\text{max}}}^{(i)}\|_2} \leq |r_j^{(i)}|$$

Setting Eq. (8) as the criterion of iterative threshold, we suggest selecting all the elements that satisfy the following condition at $i$ the iteration:

$$T^{(i)} = \{ j | r_j^{(i)} | \geq \lambda^{(i)} \}$$

Although this referential value operates similarly to that in [20], essential difference lies in the fact that this factor is adaptive and based on the correlation of the current residual and the intrinsic information of signal structure. The elimination of indeterminate parameter $t$ makes this choice to be constantly robust and preferable. Thus obstruction hazards concealed in classical StOMP pursuit progress will vanish and never occur in this scheme as there are always competitive candidates coming into the subset at each iterative step.

Once qualified elements are selected, coefficients update follows in subsequent procedure. StOMP provides an approximate estimation by taking orthogonal projections. The estimation for $x$ is acquired by projecting the measurement orthogonally onto the selected columns of $A$.
\[ X_{y,i} = A_{y,i}^\dagger y \]  \hspace{1cm} (10)\]

where \( \dagger \) represents pseudo-inverse operator. Then the algorithm subtracts off the contribution of the estimated vector \( \hat{x}_{y,i} \), and iterate on the residual. In contrast with StOMP, this manuscript suggests a regularization framework for the coefficient update step with the aim of offering a more fascinating approximation capacity for sparse signals. Similarly, at the beginning of each iterative step, an intermediate estimation \( \hat{x}_{y,i} \) is accomplished by finding a least-square solution:

\[ \hat{x}_{y,i} = \arg\min_{\tilde{x}_{y,i}} \| y - A_{y,i} \tilde{x}_{y,i} \|_2 \]  \hspace{1cm} (11)\]

Then, the regularized coefficients update scheme differs in last step. Instead of keeping all the calculated coefficients as the current estimate, the regularized scheme only retains the largest \( K \) elements of intermediate estimation to define a new support \( T^{[i]}_{\text{opt}} \) and then decrease the new estimate contribution to measurement:

\[ r^{[i]} = \| y - A_{T^{[i]}_{\text{opt}}} \hat{x}_{y,i} \|_2 \]  \hspace{1cm} (12)\]

The regularization framework not only carries profits for the stability of pursuit progress, but also provides sufficient insurance for the qualification of selected coefficients at each step, thus benefits the accuracy of sparse signal recovery

**Algorithm 1.**

Stabilized Stepwise Orthogonal Matching Pursuit (Stabilized StOMP)

**Input:** \( y, A, K \)

**Initialization:**

\( r^{[0]} = y, \hat{x}^{[0]} = 0, T^{[0]}_{\text{opt}} = \phi, i = 0 \)

**For** \( i = 1, i := i + 1 \) until \( \| r^{[i]} \| \text{< threshold} \) is met do

**Step1:** \( g^{[i]} = A^T r^{[i-1]} \)

**Step2:** \( j_{\text{max}}^{[i]} = \arg\max_j | g^{[i]}_j | \| A_j \| \)

   \[ \lambda^{[i]} = \frac{\| r^{[i-1]} \|_2}{\sqrt{K \cdot \| A_{j_{\text{max}}^{[i]}} \|_2}} \]

   \[ T^{[i]} = T^{[i-1]}_{\text{opt}} \cup \text{supp}\{ j : | g^{[i]}_j | \geq \lambda \} \]

**Step3:** \( \hat{x}_{y,i} = A_{T^{[i]}_{\text{opt}}}^\dagger y \)

\[ \hat{x}^{[i]} = \hat{x}_{y,i}^{[i]}, T^{[i]}_{\text{opt}} = \text{supp}(\hat{x}^{[i]}) \]

**Step4:** \( r^{[i]} = r^{[i-1]} - A_{T^{[i]}_{\text{opt}}} \hat{x}^{[i]} \)

end

\( r = r^{[i]}, \hat{x} = \hat{x}^{[i]} \)

**Output:** \( r \) and \( \hat{x} \)
The full scenario of the stabilized stepwise Orthogonal Matching Pursuit algorithm (Stabilized StOMP) can be listed as the pseudo code in Algorithm.1 below:

4. Experimental Results Analysis
In order to evaluate the approximation capacity of the stabilized StOMP scheme, numerical experiments are presented in this section through comparisons with classical OMP and StOMP algorithms.

4.1 Robustness Analysis
This experiment is conducted on a sparse dataset with the aim of investigating the stability improvement for proposed method. Experiment setup is depicted as follows. A sparse signal $x$ is generated with a length of $n=1000$, and the sparsity of the signal is $s = 	ext{ceil}(1000 \times 0.075) = 75$ (blue plot in Figure.1). The magnitudes of all nonzero elements are normalized and their positions follow random distribution. After Fourier transform operation on signal $x$ (multiplication with a $n \times n$ Fourier transform matrix $F$), measurements are acquired through two time random down-sampling of the transformed signal. Then, subsequent signal recoveries are performed separately through OMP, StOMP and the proposed algorithm. The threshold parameter for StOMP is empirically chosen as $t_s = 2.5$. The relative recovery error at the $i$-th iteration is defined as $e = \|x - x^{(i)}\|_2 / \|x\|_2$, and the iterative convergence criterion is chosen as $\|r\|_2 \leq 0.1$. The recoveries and convergence rates for all three methods are presented in Figure.1 and Figure 2 below. It can be observed from Figure.1 that while OMP algorithm can reconstruct the sparse spectrum exactly with a relaxed convergence rate (red plot in Figure.1 and Figure.2), classical StOMP just fails to provide an accurate recovery (purple plot in Figure.1) due to sharp obstruction after 8 iterations. This is led by the fact that the empirical selection of parameter $t_s$ is inappropriate and an overlarge threshold assignment just blocks the update of support set in the elements selection step, thus induce a suspension with a large recovery error (pink curve in Figure 2). Nevertheless, through the introduction of an adaptive threshold concerning the signal structure, the proposed scenario can remain constantly robust and prevent the recovery progress from getting obstructed.

![Figure 1](image-url)

It can be found from Figure.1 that the proposed algorithm can stably provide an accurate recovery (green profile) within a reduced computational time consuming and reach to convergence within only 20 iterations (green curve in Figure.2), while that for OMP need to perform at least 75 iterations. Note
that the final approximation residual for proposed method is 0.009242 while those for OMP and StOMP turn to be 0.009177 and 0.799, respectively.

4.2 Investigation of Approximation Capacity on Real Dataset

General approximation performance for proposed algorithm has been also explored on real image dataset. In the following experiment, the size of tested imagery data is 256*256 and it can be demonstrated that the image vector \( f \) has a sparse representation on DCT basis. Explicit settings are specified in illustration of Figure.3 and characters of running results for all three algorithms are listed in Table I. The parameter \( \text{psnr} \) is defined as \( \text{psnr} = 20 \log_{10} \frac{255}{\sqrt{\|f - \hat{f}\|}} \). It can be observed from Figure.3 that while the implementation can accomplish a blurred approximation with \( \text{psnr} = 26.2136\, \text{dB} \) when \( t_s \) is assigned to 2.0, the recovery for assignment \( t_s = 3.0 \) just reveals disordered, and the output \( \text{psnr} \) for this running has decreased to 19.5093dB due to an unexpected obstruction. In fact, a proper choice for \( t_s \) is rather difficult in practical applications. Although we can persistently alternate to a lower factor, these attempts seem unwise and inefficient. Figure.3 (e) shows the result of proposed algorithm. It can be found from this picture that the stabilized algorithm can provide a comparative approximation (\( \text{psnr} = 28.2890\, \text{dB} \)) concerning with classical OMP algorithm (\( \text{psnr} = 28.6253\, \text{dB} \)) while consumes a lower computation time. Statistics in Table I reveals that the introduction of regularized strategy in element update procedure can substantially benefit the approximation performance for sparse signals, as the \( \text{psnr} \) evaluation for proposed algorithm is better than classical StOMP (\( t_s = 2.0 \)).

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Convergence for OMP, StOMP and proposed algorithm.

| Time consuming (sec) | OMP  | StOMP(2.5) | StOMP(3.0) | Stabilized StOMP |
|----------------------|------|------------|------------|-----------------|
| PSNR (dB)            | 28.6253 | 26.2136  | 19.5093   | 28.2890         |

The original image vector is acquired from ‘Lena’ image datasets through DCT transformation, quantization and multiplication with an inverse DCT matrix. The sparsity of original image is 9340/(256*256) = 0.1579. (a) Original sparse image vector. (b) Approximation by OMP algorithm. (c) Recovery by StOMP algorithm (\( t_s = 2.0 \)). (d) Reconstruction by StOMP algorithm (\( t_s = 3.0 \)). (e) Recovery by stabilized StOMP algorithm of running results for all three algorithms are listed in Table I.
I. The parameter psnr is defined as 
\[ \text{psnr} = 20 \log_{10} \frac{255}{\| \tilde{x} - x \|} \].

![Figure 3. Image approximation under DCT bases for three methods with 50% samples.](image)

5. Discussion And Conclusions
Sparse signal approximation can be found in a variety of areas in modern signal processing. Classical StOMP algorithm can significantly decrease the iteration number of pursuit progress but suffers an unstable recovery performance due to the indeterminacy of parameter. This manuscript suggests a new guidance through the introduction of an adaptive threshold concerning original signal structure. Compared with classical StOMP algorithm, the elimination of undefined parameter can substantially stabilize the pursuit progress and keep the algorithm from getting obstructed. Furthermore, in order to improve the recovery accuracy of pursuit progress, this paper presents an additional regularization framework for coefficients update step. Compared with conventional StOMP, the regularized scenario can provide more fascinating approximation performances for sparse signal approximation applications. Experimental tests validate the benefits and efficiency of the proposed algorithm. Numerical results show that the new algorithm outperforms classical StOMP approach in both the robustness and the approximation accuracy when applied to high dimensional signals.

6. References
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