Effective cosmological constant from TeV–scale physics:  
Simple field-theoretic model

F.R. Klinkhamer

Institute for Theoretical Physics, University of Karlsruhe,  
Karlsruhe Institute of Technology,  
76128 Karlsruhe, Germany

Abstract

Adopting the $q$–theory approach to the cosmological constant problem, a simple field-theoretic  
model is presented which generates an effective cosmological constant (remnant vacuum energy  
density) of the observed order of magnitude, $\Lambda_{\text{eff}} \sim (\text{meV})^4$, if there exist new TeV–scale ultra-  
massive particles with electroweak interactions. The model is simple, in the sense that it involves  
only a few types of fields and two energy scales, the gravitational energy scale $E_{\text{Planck}} \sim 10^{15}$ TeV  
and the electroweak (new-physics) energy scale $E_{\text{ew}} \sim 1 - 10$ TeV.

PACS numbers: 95.36.+x, 12.60.-i, 04.20.Cv, 98.80.Jk

Keywords: dark energy, models beyond the standard model, general relativity, cosmology

*Electronic address: frans.klinkhamer@kit.edu
I. INTRODUCTION

The main cosmological constant problem (CCP1) can be phrased as follows (see, e.g., the review [1]): why do the quantum fields in the vacuum not produce naturally a large absolute value for the cosmological constant $\Lambda$ in the Einstein field equations or, practically, why is the measured value of $|\Lambda|^{1/4}$ very much smaller than the known energy scales of elementary particle physics? One possible solution relies on the so-called $q$–theory approach [2–4], which provides a compensation-type solution of CCP1 by self-adjustment of the $q(x)$ field. This $q$ field, considered to describe ultrahigh-energy microscopic degrees of freedom, must be of a very special type, being relativistic and conserved in the equilibrium state (Minkowski spacetime). In fact, the equilibrium value $q_0$ is constant over spacetime. This property allows for the study of the macroscopic equations in terms of $q_0$, which, in particular, give a vanishing gravitating vacuum energy density (provided there is no external pressure): $\rho_V(q_0) \equiv \Lambda_0 = 0$.

In this way, the zero-point energies of the standard-model fields in the equilibrium vacuum state can be compensated exactly by contributions from the microscopic degrees of freedom at a higher energy scale, without need to know the detailed microscopic theory. Having provided a possible explanation of the vanishing cosmological constant $\Lambda_0$ in the equilibrium state, the next task is to explain the measured small but nonzero value of the effective cosmological constant $\Lambda_{\text{eff}}$ in the present expanding (nonequilibrium) Universe. The search for the explanation of this last number, $\Lambda_{\text{eff}}$, has been called the second cosmological constant problem (CCP2), even though the term ‘puzzle’ is perhaps more appropriate.

In the early phase of the history of the Universe (close to the Planck epoch), the $q$–theory dynamical equations [3, 4] show that the gravitating vacuum energy density $\rho_V[q(t)]$ is rapidly relaxed to zero. What happens next depends on the details of the particle-physics theory, in particular, the theory at the TeV energy scale [3]. If there exist new ultramassive TeV–scale particles with electroweak interactions, the presence of these ultramassive particles affects the expansion rate of the Universe, and this change of the expansion rate “kicks” $\rho_V(t)$ away from zero [6, 7]. The maximal value of $\rho_V(t)$ is of the order of $(\text{meV})^4$, consistent with the value suggested by observational cosmology. The problem, however, is that $\rho_V(t)$ can be expected to drop to zero again if the ultramassive particles ultimately disappear.

Possible quantum-dissipative effects [6] may lead to a freezing of the gravitating vacuum energy density $\rho_V$, but this does not guarantee an asymptotic approach to a standard de-Sitter universe (consistent with the $\Lambda$CDM model of the present Universe). In fact, there appears to be a potential mismatch [7] between the $q$–theory dynamical equations and those of standard general relativity with a nonzero cosmological constant. It is possible to modify
the relevant \(q\)-theory dynamical equation by hand \cite{7}, but such a procedure is unsatisfactory.

In this article, it is shown that it is possible to remain entirely within the framework of \(q\)-theory by allowing for a nontrivial interaction between, on the one hand, the \(q\)-field and, on the other hand, the matter and gravitational fields. The model is remarkably simple and has one crucial ingredient, which, ultimately, needs to be derived from the underlying microscopic theory (assuming that the model is relevant). The purpose of this article is to present the simple model and to perform a numerical calculation in order to make sure that there is indeed a nonzero remnant vacuum energy density, leaving an extensive discussion of the systematics to a future publication.

In order to place the present work in context, the reader is referred to a recent review article on \(q\)-theory and the evolution of vacuum energy density in cosmology \cite{8}. At this moment, it may also be helpful to comment on how the results of the present article compare with those of so-called quintessence models from a dynamic scalar field \cite{9, 10} (see, e.g., Sec. 8 of the review \cite{11} for further discussion and references). There are, at least, two basic differences.

First, the standard quintessence models have a fundamental canonical scalar field \(\phi(x)\) and no natural explanation of CCP1 (see, e.g., Sec. VI of Ref. \cite{1}), whereas the (pseudo-)scalar \(q(x)\) field is nonfundamental and special, in order to provide a possible solution of CCP1 as explained in Ref. \cite{4}. (Remark that quintessence models may still provided valuable insights into CCP2, if CCP1 can be assumed to be solved.) Second, the “dark energy” from dynamic scalar fields \(\phi\) has generally an equation-of-state (EOS) parameter \(w \equiv P/\rho \neq -1\), whereas the vacuum energy density of \(q\)-theory in its simplest form has \(w = -1\) exactly. As regards \(q\)-theory, both points are clarified by considering explicit realizations, see Refs. \cite{2–4} and Sec. II A.

In short, the ultimate goal is to find an explanation of both CCP1 and CCP2. This article tries to make a modest contribution towards reaching that goal.

II. FIELD-THEORETIC MODEL

A. General properties

Consider two real scalars: an ultramassive scalar field \(\sigma\) with mass \(M = E_{\text{ew}} \sim \text{TeV}\) and a strictly massless scalar field \(\psi\). Let \(\sigma\) now exists in \(N_1\) identical copies and \(\psi\) in \(N_2\) copies. Then, the scalars \(\psi_c\), for \(c = 1, \ldots, N_2\) and \(N_2 \sim 10^2\), may correspond to the particles of
the Standard Model\(^1\) and the scalars \(\sigma_b\), for \(b = 1, \ldots, N\), to the particles of new TeV-scale physics (perhaps with \(N_1 = N_2 \sim 10^2\) resulting from broken supersymmetry [12, 13]). From now on, the indices \(b, c\) will be kept implicit by using an inner-product notation with \(\sigma \cdot \sigma \equiv \sum_b \sigma_b \sigma_b\) and \(\psi \cdot \psi \equiv \sum_c \psi_c \psi_c\).

This article employs the framework of \(q\)-theory, possibly viewed as an effective theory incorporating quantum effects of vacuum-matter interactions. Following Refs. \([3, 4]\), the \(q\)-theory is considered to be realized via a three-form gauge field \(A\) [14, 15]. The macroscopic effective action of the relevant “ultraviolet” fields (here, \(A\)) and the “infrared” fields (here, \(g, \sigma,\) and \(\psi\)) is taken to be of the following form:

\[
S_{\text{eff}, T}[A, g, \sigma, \psi] = \int_{\mathbb{R}^4} d^4x \sqrt{-\det(g)} \left(K_T(q) R[g] + \epsilon_V(q) + \mathcal{L}^{(M)}_{\text{eff}, T}[\sigma, \psi, g]\right), \tag{2.1a}
\]

\[
q = -\frac{1}{24} \epsilon^{\alpha\beta\gamma\delta} \nabla_{\alpha} A_{\beta\gamma\delta} / \sqrt{-\det(g)}, \tag{2.1b}
\]

where \(R[g]\) is the Ricci curvature scalar obtained from the metric \(g_{\alpha\beta}\), \(\epsilon_{\alpha\beta\gamma\delta}\) the Levi-Civita tensor density, and \(\nabla_{\alpha}\) the standard covariant derivative. The effective gravitational-coupling parameter \(K_T\) in (2.1a) is an even function of the vacuum variable \(q = q[A, g]\) and an indirect function of the matter fields manifesting itself as a dependence on the temperature \(T\) (see below). The energy density \(\epsilon_V(q)\) is assumed to be a generic even function of the vacuum variable \(q\), that is, a function different from the quadratic \(\frac{1}{2} q^2\) corresponding to a Maxwell-type theory [14, 15]. Here, and in the following, set \(\hbar = c = k = 1\) and take the metric signature \((-1, 1, 1, 1)\).

The \(q\)-dependence of \(K\), first introduced in Ref. \([3]\), allows for a continuous time dependence of \(q\) in a cosmological context. This behavior differs from the stepwise evolution of \(q\) in the Brown–Teitelboim mechanism which operates via quantum tunneling [16, 17]. The physical motivation for having \(K(q)\) is that \(q\) is considered to be one of the variables which characterize the quantum vacuum and that, therefore, \(q\) can be expected to determine all “constants” of the low-energy theory, one of which controls the gravitational coupling. The possible \(q\)-dependence of the “constants” in the matter Lagrange density \(\mathcal{L}^{(M)}\) of (2.1a) is neglected for simplicity. As it stands, the action (2.1a) corresponds to a type of Brans–Dicke model [18] with a special nonfundamental pseudoscalar field \(q\).

The nonfundamental pseudoscalar field \(q(x)\) from (2.1b) has been called the “quintaeessentialia” field in Ref. \([4]\), in order to distinguish it from the fundamental scalar field \(\phi(x)\)

\(^1\) In principle, it is no problem to extend the theory of \(\psi_c\) scalars to the one of the standard model with gauge and Higgs bosonic fields and quark and lepton fermionic fields. Actually, the fermions give the largest contribution to the total number of degrees of freedom in the standard model, hence the suggestive notation \(\psi_c\) for the corresponding scalars of the simplified model.
of typical quintessence models \[9–11\]. The explicit realization of \( q \) via (2.1) clarifies the two points mentioned in the penultimate paragraph of Sec. I, \( q \) being nonfundamental and giving rise to a dynamical vacuum energy density with EOS parameter \( w_V = -1 \). Note that, for this particular realization of \( q \)-theory, the mass dimension of \( q \) equals 2. For different realizations, \( q \) may have different mass dimensions and intrinsic parities.

### B. Specific properties

As explained in the Introduction, the main focus is on the freezing mechanism of the vacuum energy density and the scalar Lagrange density is kept as simple as possible, only asking that it produces massive \( \sigma \) scalars at an appropriate epoch in the history of the Universe (temperatures of order \( E_{\text{ew}} \)). Specifically, take the following Lagrange density with a single quartic coupling term between the two types of scalars:

\[
L_{\text{eff},T}[\sigma,\psi,g] = \frac{1}{2} \partial_\alpha \psi \cdot \partial^\alpha \psi + \frac{1}{2} \partial_\alpha \sigma \cdot \partial^\alpha \sigma + \frac{1}{2} M^2 (\sigma \cdot \sigma) + g_T (\psi \cdot \psi) (\sigma \cdot \sigma),
\]  
(2.2a)

\[
M = E_{\text{ew}},
\]  
(2.2b)

\[
g_T = \begin{cases} 
0 & \text{for } T > T_{c,g}, \\
g_0 \left(1 - \left(T/T_{c,g}\right)^2\right) & \text{for } T \leq T_{c,g},
\end{cases}
\]  
(2.2c)

where \( T_{c,g} \) is a critical temperature of order \( E_{\text{ew}} \), above which the scalar interactions are suppressed. The nontrivial temperature behavior of (2.2c) may effectively result from an interaction term \( (\psi \cdot \psi) (\sigma \cdot \sigma) \tilde{\chi}^2/(E_{\text{ew}})^2 \) in an extended theory where a single \( \tilde{\chi} \) scalar picks up an expectation value at temperatures below a second-order continuous phase transition (see, e.g., Sec. 4.4 in Ref. [19]). But, most likely, this phase-transition explanation of the postulated behavior (2.2c) does not need to be taken literally: \( g_T \) may very well have an entirely different origin, provided the model is relevant at all.\(^2\)

The gravitating vacuum energy density near equilibrium \( (q = q_0) \) is taken to be quadratic\[6, 7\]

\[
\rho_V(q) = \epsilon(q) - \mu_0 q = \frac{1}{2} (q - q_0)^2,
\]  
(2.2d)

\(^2\) The quartic interaction term in (2.2a) leads to radiative corrections for the low-energy theory of the scalars \( \psi_c \) (which mimic the standard model particles as mentioned in Ftn. [11]), but these corrections are suppressed by the large masses of the \( \sigma_b \) scalars (in the simple model, all masses are taken to be equal to \( M \)). Still, radiative corrections may provide a valuable alternative to direct searches if all masses of the new \( \sigma \)-type particles are of the order of several TeV or more.
with $\epsilon = \epsilon_V$ for the scalar theory (2.2a) as it stands and an appropriate constant value $q_0$ of the dynamic $q$-field (or an appropriate value $\mu_0$ of the corresponding chemical potential $\mu$; see Refs. [2, 4, 8] for further discussion).

The really new input for model (2.1a) is the following Ansatz for the effective gravitational-coupling parameter:

$$K_T(q) = \frac{1}{2} q_0 + \frac{1}{2} (q - q_0) \theta \left[ T/T_{c,K}^{(+)} - 1 \right] = \begin{cases} q/2 & \text{for } T > T_{c,K}^{(+)} , \\ q_0/2 & \text{for } T \leq T_{c,K}^{(+)} , \end{cases} \quad (2.2e)$$

with the step function

$$\theta[x] = \begin{cases} 1 & \text{for } x > 0 , \\ 0 & \text{for } x \leq 0 . \end{cases} \quad (2.2f)$$

For the model universe to be discussed in Sec. III having a temperature decreasing with cosmic time, $K_T$ is a nontrivial function of $q$ above a critical temperature $T_{c,K}^{(+)}$ and a constant below $T_{c,K}^{(+)}$. A possible physical realization of the corresponding “first-order phase transition” will be given in Sec. II C. Here, two brief remarks suffice. First, there may be hysteresis-type effects, and the suffix ‘(+)' on the critical temperature is to indicate that the transition is approached from the high-temperature side. Second, there are the following assumptions on the critical temperatures entering (2.2c) and (2.2e):

$$T_{c,g} = O(E_{ew}) , \quad T_{c,K}^{(+)} = O(E_{ew}) , \quad T_{c,g} > T_{c,K}^{(+)} . \quad (2.2g)$$

The constant $q_0$ in (2.2c), relevant at zero temperature, is proportional to the inverse of Newton’s gravitational constant, specifically

$$q_0 = 1/(8\pi G_N) \equiv (E_{Planck})^2 \approx (2.44 \times 10^{18} \text{ GeV})^2 . \quad (2.3a)$$

Given the energy scale $E_{ew}$ from (2.2b) and (2.2g), a single dimensionless parameter characterizes the theory, namely, the ratio of the two energy-density scales,

$$\xi \equiv (E_{Planck}/E_{ew})^4 , \quad (2.3b)$$

which is approximately $10^{60}$ for $E_{ew} \approx 2.44 \text{ TeV}$.

C. Additional remarks

The previous subsection has given the detailed description of the field-theoretic model (2.1a). Still outstanding is the promised physical realization of the gravitational-coupling
Ansatz \((2.2c)\). The particular realization relies on the possibility of having symmetry restoration at low temperatures and symmetry breaking at high temperatures, the opposite of what is the case in most systems. This possibility has been discussed in, e.g., Ref. \[20\] and it is the easiest to just follow Example 3 of Sec. IV of that article.

The argument, then, proceeds in four steps. First, start from a scalar theory with global \(O(n) \times O(n)\) symmetry \[20\], where the scalar fields are denoted as \(\chi_A\) and \(\eta_a\), respectively, with both indices \(A\) and \(a\) running over 1, \ldots, \(n\) [again, an inner-product notation will be used, \(\chi \cdot \chi \equiv \sum_A \chi_A \chi_A\) and \(\eta \cdot \eta \equiv \sum_a \eta_a \eta_a\)]. Second, take the parameters in the zero-temperature potential in such a way as to give the following symmetry-breaking pattern in a finite-temperature context:

\[
\begin{align*}
O(n) \times O(n) \Bigg|_{T=0} & \xrightarrow{\text{IPT}} O(n) \times O(n-1) \Bigg|_{T=\infty},
\end{align*}
\]

which implies that the \(\eta\) scalars develop an expectation value at high temperatures. As indicated by the superscript on the arrow in \((2.4a)\), the finite-temperature phase transition is arranged to be first-order.\(^3\) Third, consider the following hypothetical interaction term in the effective Lagrange density:

\[
K_0 R + \left(\frac{q}{2} - K_0\right) \frac{\chi \cdot \chi + \eta \cdot \eta}{(E_{\text{ew}})^2 + \chi \cdot \chi + \eta \cdot \eta} R, \quad (2.4b)
\]

where the fraction is close to 1 for field values \(\chi \cdot \chi\) or \(\eta \cdot \eta\) very much larger than \((E_{\text{ew}})^2\), which is the case relevant to temperatures far above the phase transition. Fourth, with a single ultraviolet parameter \(K_0 \equiv q_0/2\), the finite-temperature behavior \((2.4a)\) for the term \((2.4b)\) essentially gives the previous Ansatz \((2.2c)\). Again, the phase-transition explanation of the postulated behavior \((2.2c)\) does not need to be taken literally: the origin of \(K_T(q)\) may very well have an entirely different origin, assuming the model to be relevant at all.

It is clear that Eqs. \((2.1)-(2.2)\), as they stand, only provide a phenomenological model. The main ingredient is the discontinuous phase-transition-type behavior of \((2.2c)\). The model is simpler than the one used in Ref. \[7\] and, more importantly, entirely within the framework of \(q\)-theory (which provides a possible solution of CCP1).

\(^3\) Specifically, the potential \(P(\chi, \eta)\) is given on p. 3367, left column of Ref. \[20\] with mass-square parameters now positive and of order \((E_{\text{ew}})^2\), to which are added the following “cubic” terms: \(g_1 E_{\text{ew}} (\chi \cdot \chi)^{3/2}\) and \(g_2 E_{\text{ew}} (\eta \cdot \eta)^{3/2}\). The coupling constants are taken as in the fourth unnumbered equation on p. 3368, left column of the same reference, together with, for example, \(g_1 < 0\) and \(g_2 < 0\). These cubic terms and the finite-temperature corrections of order \(T \eta^3\) can give a first-order (discontinuous) phase transition.\(^\text{14}\) The first-order nature of the phase transition \((2.4a)\) may also have other origins, see Endnote \[24\] of Ref. \[20\].
III. COSMOLOGY

A. Setup

A spatially flat, homogeneous, and isotropic universe with scale factor \( a(t) \) will be considered \([19,21]\). For convenience, this cosmology will be called a Friedmann–Robertson–Walker (FRW) universe, even though, as will become clear in Sec. III C, the Friedmann equation is slightly modified when the electroweak kick sets in.

The homogeneous matter content of this model universe consists of two perfect fluids (called type 1 and type 2), with energy density \( \rho_{M1}(t) \) from massive \( \sigma \) scalars with an effective number of degrees of freedom \( N_1 \) and energy density \( \rho_{M2}(t) \) from massless \( \psi \) scalars with \( N_2 \) degrees of freedom (see the first paragraph of Sec. II A and the Appendix of Ref. \([7]\) for the physical motivation of having \( N_1 = N_2 = 10^2 \)). In thermal equilibrium and without energy exchange, the type–2 energy density is given by \( \rho_{M2} = (N_2 \pi/30) T^4 \). The temperature of the Universe can, therefore, be identified as approximately \( (\rho_{M2})^{1/4} \).

For \( \rho_{M1} = 0 \) and \( \rho_{M2} \sim (E_{ew})^4 \), the expansion of the Universe is governed by a Friedmann-type equation (see below) with a timescale set by

\[
t_{ew} \equiv E_{\text{Planck}}/(E_{ew})^2,
\]

in terms of the reduced Planck energy from \([2.3a]\). A value \( E_{ew} \sim \text{TeV} \) gives \( 1/t_{ew} \sim \text{meV} \).

B. Frozen-electroweak-kick mechanism

With the field-theoretic model of Sec. II for an energy scale \( E_{ew} \sim \text{TeV} \), the basic steps of the frozen-electroweak-kick mechanism in a flat FRW universe are as follows:

(i) start from a standard radiation-dominated FRW universe at an ultrahigh temperature \( T \) with \( \rho_V = \rho_{M1} = 0 \) and \( \rho_{M2} \sim T^4 \) from the massless scalars \( \psi \) (the \( \psi \) scalars have standard electroweak interactions and are in thermal equilibrium, whereas the \( \sigma \) scalars may have nonstandard interactions and are assumed to be initially absent, \( \rho_{M1} = 0 \));

(ii) as the temperature \( T \) drops below \( T_{c,g} \) (with \( T_{c,g} \gtrsim E_{ew} \)), the \( \psi^2 \sigma^2 \) coupling of the scalar theory \([2.2a]\) generates a nonzero density of massive \( \sigma \) scalars at cosmic times \( t \) around \( t_{c,g} \) (with \( t_{c,g} \lesssim t_{ew} \));

(iii) the presence of massive scalars \( \sigma \) modifies the Hubble expansion rate \( H(t) \equiv a(t)^{-1} da(t)/dt \) at \( t \sim t_{ew} \);
(iv) the modified Hubble expansion rate kicks $\rho_V$ away from zero \(\rho_V(t) \sim H(t)^4 \sim (1/t_{ew})^4 \sim (\text{meV})^4\) at $t \sim t_{ew}$;

(v) a nonzero value of $\rho_V$ remains when $K_T$ from (2.2c) is frozen to the constant value $q_0/2$ at a temperature $T = T_{c,K}$ (with $T_{c,K} \lesssim E_{ew}$) or cosmic time $t = t_{c,K}$ (with $t_{c,K} \gtrsim t_{ew}$);

(vi) the subsequent evolution is that of a standard $\Lambda$–FRW universe (here, with relativistic scalars $\psi$, as the massive scalars $\sigma$ ultimately disappear).\(^4\)

The phenomenological model of Sec. II is, most likely, over-simplified, but may provide a benchmark calculation for a dynamically generated cosmological constant. Expanding on items (iv)–(vi) above, note that, at cosmic times $t \sim t_{ew}$, the frozen vacuum energy density $\rho_{V,\text{remnant}} \sim (1/t_{ew})^4$ is negligible compared to the matter energy density $\rho_{M2}(t_{ew})$ by a factor $\xi \sim 10^{60}$. With $\rho_{M2}(t) \propto 1/t^2$, the tiny (but constant) vacuum energy density $\rho_{V,\text{remnant}}$ only becomes dominant at very much later times, $t_{V,M-\text{equal}} \sim \sqrt{\xi} t_{ew} \sim 10^{18}$ s, suggesting a possible solution of the so-called cosmic coincidence puzzle (see, e.g., Ref. [5, 10, 11]).

In the next subsection, a preliminary numerical calculation is presented, which supports the scenario of the frozen-electroweak-kick mechanism outlined above. As this next subsection is rather technical, it may be skipped in a first reading, except for a quick look at the numerical results in Fig. II.

C. Numerical solution

With the timescale $t_{ew}$ from (3.1) and the hierarchy parameter $\xi$ from (2.3b), it turns out to be useful to introduce the following dimensionless variables for the cosmic time, the Hubble expansion rate, the energy densities, and the $q$ shift away from equilibrium \(^4\):

\[
\begin{align*}
\tau & \equiv (t_{ew})^{-1} t, & h & \equiv t_{ew} H, \\
\rho_{Mn} & \equiv \xi^{-1} (t_{ew})^4 \rho_{Mn}, & r_V & \equiv (t_{ew})^4 \rho_V = x^2/2, \\
x & \equiv \xi (q/q_0 - 1),
\end{align*}
\]

where $n$ stands for the matter-species label ($n = 1, 2$), and the $\rho_V$ Ansatz (2.2d) has been used. From now on, an overdot will denote differentiation with respect to $\tau$.

\[^4\] Strictly speaking, such a standard $\Lambda$–FRW universe would not allow for further large contributions to the vacuum energy density at temperatures $T \lesssim T_{c,K} \sim E_{ew}$. Naively, one expects a contribution of order \((100 \text{ MeV})^4\) from quantum chromodynamics at $T \sim 10^2$ MeV, but it has also been argued that this is not the case [22].
The relevant dimensionless ordinary differential equations (ODEs) for the model of Sec. II are then given by \[3, 7\]

\[
\begin{align*}
(\dot{h} + 2h^2) \left[ x^2/2 + \xi (r_{M1} + r_{M2} - 3h^2) - 3h^2 x \dot{\theta} \right] - h x \dot{x} &= 0, \\
\dot{r}_{M1} + (4 - \pi_{M1}) h r_{M1} &= \lambda_{21} r_{M2} - \lambda_{12} r_{M1}, \\
\dot{r}_{M2} + 4 h r_{M2} &= -\lambda_{21} r_{M2} + \lambda_{12} r_{M1}, \\
(3h \dot{x} + 3h^2 x) \dot{\theta} - \left[ x^2/2 + \xi (r_{M1} + r_{M2} - 3h^2) \right] &= 0,
\end{align*}
\]

with the EOS function \(\pi_{M1}(\tau)\) defined in Sec. A2 of Ref. \[7\] and the effective step function

\[
\dot{\theta}(\tau) \equiv \theta [r_{M2}(\tau) - r_{c,K}],
\]

using definition \(2.21\). Here, the energy density \(r_{M2}\) of the massless scalars \(\psi\) monitors the ambient temperature of the model universe and determines the moment when the \(q\)–dependence of \(K(q)\) changes from \(q/2\) to \(q_0/2\), as given by Ansatz \(2.2c\). For the benefit of the reader, the four ODEs in \(3.3\) trace back to Eqs. (4.1) and (4.2a) of Ref. \[3\] evaluated for the \(K_T\–Ansatz \(2.2c\) and Eqs. (A5b) and (A5d) of Ref. \[7\] adapted to the case considered.

As discussed in Sec. IIIB, the scalar interactions turn on below a certain critical temperature, which corresponds to \(r_{M2} \leq r_{c,g}\) in the cosmological context. The postulated behavior \(2.2c\) suggests the following coupling parameters in the ODEs \(3.3\):

\[
\begin{align*}
\lambda_{12}(\tau) &= \lambda \theta [r_{c,g} - r_{M2}] \left( 1 - \sqrt{r_{M2}/r_{c,g}} \right)^2, \\
\lambda_{21}(\tau) &= \lambda_{12}(\tau) \exp \left[ -\left( \frac{\pi N_2}{30 \, r_{M2}(\tau_{\text{min}})} \right)^{1/4} \frac{a(\tau)}{a(\tau_{\text{min}})} M / E_{\text{ew}} \right],
\end{align*}
\]

with \(\lambda \propto (g_0)^2\). The argument of the exponential in \(3.4c\) equals the negative inverse of the \(T/M\) expression \((A3d)\) from Ref. \[7\], and \(\tau_{\text{min}}\) is an arbitrary reference time before \(\tau_{\text{BCS}}\) to be introduced below. The exponential factor of \(3.4c\) ensures that the ODEs \(3.3b\) and \(3.3c\) for Minkowski spacetime \((H = 0)\) at a fixed temperature \(T < T_{c,g}\) \((r_{M2} < r_{c,g})\) give an equilibrium ratio \(r_{M1}/r_{M2} = \exp[-M/T]\).

The new physics from Eqs. \(2.2c\) and \(2.2d\) is assumed to operate in a temperature window set by \(r_{M2}(\tau)\) values between \(r_{c,g}\) and \(r_{c,K}\), with

\[
r_{c,g} > r_{c,K},
\]

according to \(2.2g\). In this regime, the square bracket in \(3.3d\) corresponds to the standard Friedmann equation \((H^2 \propto \rho_{\text{tot}})\), to which are added two terms (proportional to \(\dot{q}\) and \(q - q_0\)) tracing back to the \(q\)–dependence of the gravitational-coupling parameter \[3\].

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The ODEs (3.3) have two interesting analytic solutions. The first corresponds to a standard radiation-dominated FRW universe at high enough temperatures [with \( \lambda_{12}(\tau) = \lambda_{21}(\tau) = 0 \)],

\[
\begin{align*}
    h(\tau) &= (1/2) \tau^{-1}, \\
    x(\tau) &= r_{M1}(\tau) = 0, \\
    r_{M2}(\tau) &= (3/4) \tau^{-2} > r_{c,g},
\end{align*}
\]

(3.6a)

and the second to a standard de-Sitter universe with constant vacuum energy density and without matter,

\[
\begin{align*}
    h^2 &= \xi^{-1} x^2/6, \\
    \dot{h} &= \dot{x} = 0, \\
    r_{M1} = r_{M2} = 0.
\end{align*}
\]

(3.6b)

The numerical solution to be presented shortly will be seen to interpolate between these two analytic solutions.

The hypothetical TeV-scale physics has a very large hierarchy parameter \( \xi \sim 10^{60} \) from (2.3b), and four \( \xi = \infty \) equations turn out to be relevant for the phase of the electroweak kick [7]. Specifically, there are two differential equations,

\[
\begin{align*}
    (\hat{\theta}) \dot{h} \dot{x} & \left[ 3 (\dot{h} + 2 h^2) - \frac{1}{2} \kappa_{M1} (3 h^2 - r_{M2}) \right] = (1 - \hat{\theta}) h x \left\{ \dot{x} \right\}, \\
    \dot{r}_{M2} + 4 h r_{M2} + \lambda_{21} r_{M2} &= \lambda_{12} (3 h^2 - r_{M2}),
\end{align*}
\]

(3.7a)

and two algebraic equations,

\[
\begin{align*}
    x &= (\hat{\theta}) \left[ \frac{1}{2} \kappa_{M1} (3 h^2 - r_{M2}) \right] + (1 - \hat{\theta}) \left\{ \frac{1}{2} \kappa_{M1} (3 h^2 - r_{M2}) \right\}_{r_{M2}=r_{c,K}}, \\
    r_{M1} &= 3 h^2 - r_{M2},
\end{align*}
\]

(3.8a)

with \( \hat{\theta} \) defined by (3.4). Recall that \( \kappa_{M1} \) is a functional of \( h(\tau) \equiv a(\tau)^{-1} da(\tau)/d\tau \), explicitly given by Eqs. (A3a)–(A3d) in Ref. [7]. For the numerical analysis of Eqs. (3.7)–(3.8), the relevant parts of the equations for the \( \hat{\theta} = 1 \) (high-temperature) phase have been indicated by square brackets, and those for the \( \hat{\theta} = 0 \) (low-temperature) phase by curly brackets, keeping the two generally valid equations without such brackets.

Returning to general values of \( \xi \), the numerical solution of the ODEs (3.3) has been obtained for the case with \( N_1 = N_2 = 10^2 \) and an equal mass \( M \) of all type-1 particles (i.e., the case-B mass spectrum in the terminology of Sec. A2 of Ref. [7]). As mentioned above, at temperatures above the critical temperature \( T_{c,g} \), it is possible to have a standard radiation-dominated FRW universe (3.6a) with all type-2 particles in thermal equilibrium (the type-1 particles are assumed to be absent in the early phase, see Sec. IIIB). Hence,
FIG. 1: Numerical solution of the dimensionless ODEs \((3.3)\) with EOS function \(\kappa_{M1}(\tau)\) defined in Sec. A2 of Ref. \([7]\) and further parameters \((3.4)\). The panels are organized as follows: the four basic dynamic variables \([h(\tau), r_{M1}(\tau), r_{M2}(\tau), \text{and } x(\tau)\) as defined by \((3.2)\) are shown on the top row and secondary or derived quantities on the bottom row. The dashes lines in the panels of the third column correspond to \(10^{-1} r_{M2}\) and \(\lambda_{21}/\lambda\). The main result is the nonzero remnant value of the dimensionless gravitating vacuum energy density \(r_V \equiv x^2/2\) shown in the top right panel.

The model parameters are \(\{\xi, \lambda, r_{c,g}, r_{c,K}\} = \{10^7, 10^4, 12, 3\}\). The ODEs are solved over the interval \([\tau_{\text{min}}, \tau_{\text{max}}] = [0.01, 10]\) with the following boundary conditions \((3.9)\) at \(\tau = \tau_{\text{bcs}} = 0.25\) corresponding to \(r_{M2}(\tau_{\text{bcs}}) = r_{c,g}\): \(\{x, h, a, r_{M1}, r_{M2}\} = \{0, 2, 1, 0, 12\}\).

The appropriate boundary conditions on the four dynamical variables at a time \(\tau = \tau_{\text{bcs}}\) are

\[
\begin{align*}
h(\tau_{\text{bcs}}) &= \frac{1}{2} (\tau_{\text{bcs}})^{-1}, \\
r_{M1}(\tau_{\text{bcs}}) &= 0, \\
r_{M2}(\tau_{\text{bcs}}) &= 3 [h(\tau_{\text{bcs}})]^2, \\
x(\tau_{\text{bcs}}) &= 0.
\end{align*}
\]

The precise value of \(\tau_{\text{bcs}}\) is irrelevant as long as it is sufficiently small, with \(r_{M2}(\tau_{\text{bcs}}) \geq r_{c,g}\) [physically interpreted as \(T(\tau_{\text{bcs}}) \geq T_{c,g}\)]. Furthermore, choose for \(\lambda\) the value \(10^4\), making \(r_{M1}(\tau)\) decrease significantly before the \(K_T\) transition at \(r_{M2} = r_{c,K}\) is reached (the new physics then operates in a relatively narrow temperature interval near \(T \sim E_{\text{ew}}\)). Other values \(\lambda \gtrsim 10^3\) give similar numerical solutions.

The numerical results are shown in Fig. 1. The \(r_V\) panel, in particular, shows the narrow new-physics window with the critical temperature \((2.2\text{c})\) at \(\tau = 0.25\) (from \(r_{M2} = r_{c,g}\)) and the freezing of the gravitational-coupling parameter \((2.2\text{c})\) at \(\tau \sim 0.45\) (from \(r_{M2} = r_{c,K}\)).
TABLE I: Asymptotic values of the dimensionless gravitating vacuum energy density $r_V(\tau)$ for various hierarchy parameters $\xi$ ranging from $10^4$ to $10^8$. All other parameters are given in the caption of Fig. 1. The entry for $\xi = \infty$ has been calculated from Eqs. (3.7) and (3.8). The numerical accuracy is estimated at $\pm 1$ in the last digit shown.

| $\xi$  | $10^3 \times r_V(\infty)$ |
|--------|---------------------------|
| $10^4$ | 7.310                     |
| $10^5$ | 5.380                     |
| $10^6$ | 2.028                     |
| $10^7$ | 2.376                     |
| $10^8$ | 2.376                     |
| $\infty$ | 2.376                    |

The calculated numerical value of $r_{V,\text{remnant}}$ is approximately $2.4 \times 10^{-3}$. For different values of $r_{c,K}$ than chosen in Fig. 1, while keeping the other parameters the same, the numerical values of $r_{V,\text{remnant}}$ will, of course, be less than the maximal value of $r_V(\tau)$ shown in the top right panel of the figure, that is, $r_{V,\text{remnant}} \lesssim 1.5 \times 10^{-2}$. Incidentally, the peak of the $r_V(\tau)$ curve at $\tau = \tau_{\text{peak}} \sim 0.275$ looks sharp in the plot but is really a concave parabola with a large negative second derivative, $r''_V(\tau_{\text{peak}}) \sim -10^2$.

Table I presents the numerical values for $r_V(\infty) \equiv \lim_{\tau \to \infty} r_V(\tau)$ for various hierarchy parameters $\xi$. The $r_V(\tau)$ solution for $\xi \lesssim 10^6$ has, in fact, significant oscillations superposed on the smooth curve shown in the bottom right panel of Fig. 1 which explains the somewhat erratic behavior of the first three entries in Table I. Based on the analysis of Ref. [7], the results for $\xi = \infty$ can be expected to give a close approximation to those for $\xi \sim 10^{60}$, which corresponds to the physically relevant case according to (2.3b).

D. Analytic result for $\xi = \infty$

The $\xi = \infty$ equations (3.7) and (3.8) immediately give an analytic expression for the asymptotic value of the dimensionless vacuum energy density,

$$
\lim_{\tau \to \infty} r_V(\tau) \big|_{\xi=\infty} = \frac{1}{8} \left( \bar{\kappa}_{M1}(\tau_{\text{freeze}}) \left[ 3 h(\tau_{\text{freeze}})^2 - r_{M2}(\tau_{\text{freeze}}) \right] \right)^2 \bigg|_{r_{M2}(\tau_{\text{freeze}}) = r_{c,K}},
$$

where the EOS function $\bar{\kappa}_{M1}(\tau)$ is determined by the dimensionless Hubble expansion rate $h(\tau)$ and $r_{Mn}(\tau)$ stands for the dimensionless energy density of matter component $n$ [all dimensionless variables are defined in (3.2)].
Expression (3.10) is formal, because the numerical solution of the two nonlinear ODEs (3.7a) and (3.7b) for $\hat{\theta} = 1$ is needed to determine the value of, for example, $h(\tau_{\text{freeze}})$. But the analytic expression (3.10) does clarify the essential physics involved: the EOS function $\kappa M_1(t)$ multiplied by the corresponding energy density of massive particles ($r_{M_1} = 3 h^2 - r_{M_2}$) and the freezing of $K(t) = q(t)/2$ to the constant value $q_0/2$ at a cosmic temperature of the order of $E_{\text{ew}} \sim \text{TeV}$.

**IV. CONCLUSION**

In this article, a simple field-theoretic model (2.1)–(2.2) has been presented, which remains entirely within the framework of $q$–theory [2–4] and does not require unnaturally small coupling constants (the energy scales $E_{\text{ew}} \sim \text{TeV}$ and $E_{\text{Planck}} \sim 10^{15} \text{TeV}$ are considered to be given [5]). As summarized in Sec. III B, the model generates via the electroweak-kick mechanism [6] an effective cosmological constant $\Lambda_{\text{eff}}$ (remnant vacuum energy density $\rho_V$), which is consistent with the value $\Lambda_{\text{obs}} \sim 1 \times 10^{-29} \text{ g cm}^{-3} \sim (2 \times 10^{-3} \text{ eV})^4$ from observational cosmology [11, 21]. In addition, having $\Lambda_{\text{eff}} \sim (E_{\text{ew}})^8/(E_{\text{Planck}})^4$ provides a natural explanation [3] of the fact that the orders of magnitude of the energy densities of vacuum, matter, and radiation are approximately the same in the present Universe (also known as the triple cosmic coincidence puzzle).

With the calculated dimensionless vacuum energy density $r_V(\infty) \sim 2.4 \times 10^{-3}$ from Table I, the required energy scale $E_{\text{ew}}$ of the new physics is approximately 4.7 TeV, according to Eq. (5.2) of Ref. [7]. However, the main focus of the present article is not on numerical estimates (plenty have been given in Ref. [7]), but rather on the physical content of a theory capable of generating the observed cosmological “constant” of our Universe.

In that spirit, the most interesting result of this article is the observation that the proposed model involves one crucial ingredient, namely, the discontinuous phase-transition-type behavior of the dependence of the gravitational coupling (2.2e) on the quinta-essentia field $q$. (The need for some form of singular behavior in order to freeze $\rho_V[q(t)]$ has also been emphasized in the second remark of Sec. III E of Ref. [7].) The main task is, therefore, to find the rationale for this phase-transition type of behavior or for a different effect with the same result of freezing part of the vacuum energy density below a certain cosmic temperature scale. Furthermore, the simple model (or a suitable generalization of it) needs to be embedded in the complete solution of the cosmological constant problem, which is still outstanding.
ACKNOWLEDGMENTS

The author gratefully acknowledges the hospitality of the Perimeter Institute for Theoretical Physics, Canada, where this work was initiated. He also thanks M. Guenther, G.E. Volovik, and the referee for helpful comments on an earlier version of this article.

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