Translation by W. De Baere of the paper:

M. Renninger, “Zum Wellen–Korpuskel–Dualismus”, Zeitschrift für Physik 136, 251-261 (1953).

Translator’s note:

In the original, german, version of the paper, the numbering of footnotes and references starts with 1 on each page. Because the text in the original paper and in the translation, does not start and end with the same sentence on each page, we have chosen a consecutive numbering of the footnotes and the references.

It is the purpose of this translation, to bring Renninger’s important work available to a broader audience in the field of the foundations of quantum mechanics, in particular to those concerned with, and interested in, the significance and interpretation of the quantummechanical wave function.

Acknowledgment

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On Wave–Particle Duality*

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With 2 figures in text
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By means of a thought–experiment, consisting of an interference experiment with two interfering beams, it is shown that it can be demonstrated experimentally that with one single particle a wave can be associated which propagates in space and time as a physical reality, i.e. that it should not merely be considered as a distribution of probabilities. The notion “physical reality” should be understood such that, when this physical reality is considered in a particular space at a particular time, it should be experimentally possible to influence this reality in such a way that future results of experiments show unambiguously that this reality has been causally influenced by the experimental act in this space and at that time.

In modern physics it has become more and more the custom, to discuss the last questions about quantum theory, especially the wave–particle dualism, only by means of purely mathematical considerations, and to consider the visualization, or even the mere desire for it, as rudimentary and naive modesty. It is, however, the purpose of the following discussion, to point to some very precise conclusions, which follow merely from purely experimental physical aspects, without any previous knowledge of the mathematical quantum formalism, which to my knowledge have never been obtained in this way before. At the same time the aim is to warn that in considering such issues, the visualization should not be given up too early, but should be kept on as long as possible. Clearly there

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*This paper has been accepted for publication, after the correspondence of the author with leading quantum physicists has convinced the Editors of the fundamental importance of the subject.

†The manuscript was submitted already on 10.4.53 to “Naturwissenschaften”, but was accepted for publication only later.
is a limit in so doing, but this limit should not be artificially set so as to exclude interesting possible developments. It is precisely the purpose of this work to present such a development.

In general one speaks of the two different “pictures” or “aspects”, the “wave”– and “particle–aspect”. Detailed studies report the availability of a series of experiments evidencing the wave–like nature, and another series evidencing the particle–like nature of light and matter. But it has always been stressed, that there exists no experiment in which \textit{at the same time} both wave–like and particle–like properties are observed, and in which the wave–like behaviour of \textit{one single} photon or electron can be shown. According to P. Jordan\footnote{Jordan, P.: Anschauliche Quantentheorie, p. 115. Berlin 1936.} this were “logically and mathematically nonsense”. “It is impossible, that one single indivisible experimental act shows up as well the one as the other appearance of light”. Or as stated by W. Heisenberg\footnote{Heisenberg, W.: Die physikalischen Prinzipien der Quantentheorie, 2nd Ed., p. 7, 107. Leipzig 1941.} “Particle– and wave–picture are two different appearances of one and the same physical reality”. “Now it is clear, that matter cannot consist simultaneously of waves and particles …” “The seeming double nature has its origin in the inadequacy of our language”. “Atomic processes don’t have a visual representation. Fortunately for the mathematical description of these processes such a visualization is not necessary altogether; a mathematical formalism of quantum theory is at our disposal, which accounts for all experiments …”

In contrast with these statements I propose the following 3 propositions, the proof of which will be given thereafter; they are first formulated for light in the visual range, but they are also valid, in appropriately modified form, for radiation of other wavelength and for matter radiation:

1. It is possible to demonstrate experimentally, that the \textit{energy} of each photon moves \textit{across space and time} on a single continuous path, and concentrated in the form of a particle, i.e. at each instant it occupies a connected region in space\footnote{No specification is given about the form and extension of this region of space, i.e. the “particle”\textsuperscript{3}.}.

2. It can also be demonstrated experimentally, that \textit{with each single photon} there may be associated a \textit{guiding wave} (without energy but “causally” influencable), which obeys precisely the rules for the propagation of an electromagnetic wave (spatial extension, absorption, diffraction, interference, splitting by reflection, refraction, double refraction etc., except energy), hence has unambiguously a \textit{wave nature} which propagates likewise \textit{in space and time}, i.e. as long as the photon is on its way it occupies at each instant a specific region of space which is \textit{not} necessarily single connected.
3. The connection *in space* of the particle of energy and of the wave field is such, that the former can be found in a region where the intensity of the wave is different from zero, with a probability which is proportional to this intensity. Its propagation speed is equal to the group velocity of the wave within the limits set by Heisenberg’s uncertainty relations.

Summarized briefly and drastically: *Each quantum consists of a particle of energy, “carried” or “guided” by an energy free wave.*

This image is not new. It was already formulated by de Broglie in 1927 in terms of the by him introduced notion of “pilot wave”, but abandoned again because of “des objections très graves”. According to a citation from N. Bohr, also Einstein once speaks of “ghost fields, guiding photons”.

On the other hand the 3 preceding propositions are, however, *neither intended as an explanation, nor as a mere visualization.* Instead, both the energy particle and the guiding field associated with the single photon are considered *simply as experimental facts.* Attempts for explanation should follow *after* observation of the facts!

These statements will be proved by means of a thought experiment, consisting of an elementary interference set–up. In my opinion the usual thought experiments for the analysis of the wave–particle duality (mostly refraction at a single or double slit) fail to penetrate to the deepest possible level, and are not able to reveal the most basic characteristics. Mostly they concern refraction set–ups with a continuous intensity distribution. The main characteristics of our set–up will be: *two* interfering rays, detection in *two* possible places, *two* discrete intensities (probability distributions) which may be chosen as either 1 or 0. This set–up is shown in Fig. 1. (In an Appendix a more detailed description of the set–up is given in order to prove its essential realizability.)

In the proposal it is assumed that two facts are experimentally proven:

1. The outcome of the interference experiment is the same whether many

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4Broglie, L. de: Rapport au V. Congres Solvay 1927, p. 105.

5Broglie, L. de: Ondes et Corpuscules, p. 34. Paris 1930 and Introductions à l’Etude de la Mécanique Ondulation, p. 132. Paris 1930. – Remark during the correction. Only after submission of the present manuscript, the Editor became aware of the fact that de Broglie took up again these former pictures since one year, in connection with relativistic considerations, together with J.-P. Vigier. In an article entitled: “La Physique Quantique restera–t–elle Indeteminististe?” containing all his publications about this issue so far, he reports in an impressive way about his motivation, which lead him to abandon 25 years ago his former concepts and to take up it again presently. – In the same way the Editor was not aware of the work by D. Bohm [Phys. Rev. 85, 166, 180(1952)], in which de Broglie’s concept of “pilot–wave” is developed theoretically, and which was the first main reason for de Broglie to develop further his former picture.

6Schilpp, P.A.: Albert Einstein, Philosopher–Scientist, p. 206. Evanston, Illinois 1949.
photon operators contribute in a short time or in a long time: each photon interferes only with itself (see e.g. Dirac, Principles of Quantum Mechanics, p. 9).

Fig. 1. Interference setup, schematic

2. Several coherent subrays of a light source remain coherent, i.e. are able to interfere with each other after having traveled arbitrary long distances along separated paths. This has been verified experimentally up to distances of 2 km by Michelson and Gale.

A parallel ray (1) is split by (2) in 2 rays A and B which move, in separate spaces, through a system of pipes towards (3). There they interfere with each other, e.g. after transmission through and reflexion by a beam-splitter S, and arrive in places (4) and (5) where they can be detected.

The path difference between A and B can be set up (e.g. by displacing one of the mirrors $S_A$ or $S_B$) such that it is either $+\lambda/4$ or $-\lambda/4$, resulting either in brightness in (4) and darkness in (5), or the other way round (the ray which is reflected by the beam splitter S gets a phase shift of $\pi/2$, the phase of the transmitted ray remains unchanged).

The light source should:

1. Allow the transition to such weak intensities, that never (i.e. with a probability tending to zero), more than one photon at the same time is present between (2) and (3).

2. Each time inform the observer by means of a signal the instant – taken each time as $t = 0$ –, when a photon has passed the point (2), within an uncertainty $\pm \Delta t$, which is small compared with the time of passage between (2) and (3).

The perfection of the optical set-up [parallelism of the ray, reflection and precision of the mirrors, perfection of the detectors in (4) and (5) etc.] should be such that each photon that passes (2) will be observed either in (4) or in (5) (e.g. by means of scintillation or phototube) with a probability very close to 1 (i.e. almost with certainty). Thus if the path difference between A and B is set up such that brightness will be observed in (4), darkness in (5), then each photon entering (1) will be observed in (4).

At different places, such as (6), (7), (8) and (9) it should be possible, to insert in the paths A and B a slide, either an opaque screen with detector, which – as in (4) and (5) – registrates each photon which is absorbed by the screen, or a $\lambda/2$ plate. The paths A and B should be long enough (say some light–minutes), in

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7Michelson and Gale: Nature, Lond. 115, 556 (1925).
8With respect to this point, see the Appendix for the essential possibility of such a signal.
order to allow the insertion of such slides on different instants while a photon is moving between (2) and (3)\textsuperscript{9}.

With this set-up we make then the following observations [Reduced intensity as mentioned above, path lengths adjusted so that brightness is observed in (4), darkness in (5)]:

a) As long as nothing is inserted in the places (6), (7), (8) and (9), each photon passing (1) will be observed in (4), nothing in (5).

b) If in (6) a slide is inserted within the interval between \(t_6 - \Delta t\) and \(t_6 + \Delta t\), where \(t_6\) is the time the photon needs to move from (2) to (6), \(t_6 = l_6/c\) \([l_6=\text{distance between (2) and (6)}]\), then for the observation there are two possibilities, each of which is realized with probability 50%:

\(\alpha\) The photon is detected in (6). It is then absorbed and has disappeared, and no further experimental intervention can result in a further observation, e.g. revealing the existence in path B of a further existing physical reality.

\(\beta\) The photon is not detected in (6). Then it can be experimentally verified that with certainty it can be detected e.g. either in (4) at the time \(t_9 = l_9/c\), or, when this is not the case, to detect it \textit{either} in (4) \textit{or} in (5), with probability \(\frac{1}{2}\). Blocking path A has, when the quantum is not observed in it, yet a definite physical effect, namely that there, where in the case of both open paths no quantum may be observed, now a quarter of all entering quanta will be detected; and it is indifferent, whether path A or path B is closed.

c) Insert now in (6) a fully transparent \(\lambda/2\) plate instead of an absorbing slide, again within the time interval \(t \pm \Delta t\). The result is: \textit{All photons are now detected in (4) instead of in (5)}. The same result is obtained when the plate is inserted in (7) instead of in (6). This means, that it is possible, by this experimental intervention, to send the photon from (4) to (5), \textit{at will from within each of the paths A and B}, i.e. to influence it \textit{causally}. Yes, it is possible, to “cancel” an already decided “bending”, by inserting at corresponding later times \(t_8 \text{ resp. } t_9\) a further – or also the same – \(\lambda/2\) plate.

With these results the 3 abovementioned propositions are proven:

Proposition 1 by experiment b. The photon can always be absorbed, i.e. detected, only in \textit{one} of the paths A and B, never in both together, and further each time at a specific instant, determined by the propagation speed. The fact, that the \textit{certainty}, to “find” the photon in one of both paths – and when it is not found in \textit{one} path, the certainty to find it \textit{then} on a corresponding later instant in the \textit{other} one – moves with speed \(c\), leads to the compelling conclusion, that the

\textsuperscript{9}This does not require large signal speeds, because the paths A and B can be deflected as many times as necessary, so that the positions (2) to (9) are at the same time separated in space and in the immediate neighbourhood of the observer.
photon itself (i.e. its total energy) moves inside a confined self contained region on a continuous path through space and time. The conclusion of the localization of a system in a region, from the certainty to detect that system in that region, is undoubtedly allowed, and even required by the most elementary logic.

The validity of Proposition 2 follows from observation b, β and especially from c. By the intervention of inserting the phase plate in one of both paths, whatever one, the single photon can be steered at will between both possible detection places. This proves, that with each single photon there is associated a physical reality which simultaneously moves on both paths. Additional observations, which will not be discussed further, show, that the evolution of this “physical reality” obeys in all points the laws for the propagation of an electromagnetic wave, except the fact, proven by observation b, that it does not contain energy (the total energy moves on one of both paths; because the wave moves on both paths, it does not contain energy).

Proposition 3, which basically should be considered as a supplementary one, is proven by the combination of all observations, but in particular by the fact, that the experimental interventions of inserting slides or phase plates has the described result, when they happen within the mentioned time interval, which must be the same in each path, independent whether it is an intervention on the energy particle (by means of slides) or an intervention on the wave (by means of phase plates).

Are now, after all these considerations, the wave-“picture” and particle-“picture” really but “two forms of appearance of one and the same physical reality”? Is it not, on the contrary, the case that wave and particle are two separate physical realities associated with single photons? Did the observation c above not “reveal in one single indivisible experimental intervention both the one and the other picture of the nature of light”? (The result of inserting the phase plate proves the wave nature, the observation in one single point proves the particle nature of one and the same photon).

Of course one is free, to speak of the wave as a pure “probability”-wave. But one should be aware of the fact, that this probability wave propagates in space and time in a continuous way, and in a way that she can be influenced in a finite region of space – and only there! – and also at that time! –, with an unambiguous observable physical effect!

I am not sure, whether the facts and conclusions discussed in this work have some connection with the doubts of A. Einstein about the completeness of the description of elementary processes by quantum mechanics\(^\text{10}\). It is obvious, that

\(^\text{10}\)Summarized in: P.A. Schilpp, Albert Einstein, see footnote 6 page 4.
precisely the proven reality of the wave associated with the single particle, which
quantum mechanics, as it is stated explicitly, is unable to account for, may be con-
sidered precisely as an expression of the incompleteness maintained by Einstein\textsuperscript{11}. In any case was our definition of “physical reality” inspired by the definition of Einstein, Podolsky and Rosen\textsuperscript{12} of the same notion.

In the same way I cannot say, whether there is a connection with the new work by W. Weizel\textsuperscript{13}, in which it is tried to refute von Neumann’s proof of the impossibility of a causal model for quantumtheoretical processes.

For sure, the picture of a particle guided by a wave devoid of energy, which is intended to be more than a “picture”, namely “reality”, nowhere contradicts quantum mechanics, and may moreover be a valuable aid for the visual comprehension of elementary processes and for making exact prognoses about the outcome of experiments.

Heisenberg’s uncertainty relation, as far as it has something to do with the motion of a particle as treated here, may be given also a visual significance: it states, that after emission of the particle it is impossible to know where, i.e. \textit{at which point in the wave} the particle, the knot is situated [as well transversally, i.e. in which direction, as longitudinally, i.e. the distance] within the coherence length $\Delta x$ determined by the uncertainty relation (the expression of the uncertainty relation for light is: $\Delta x \cdot \Delta /\lambda \geq 1$). Yet, this relation says nothing about our additional statement, which restricts the uncertainty, that within the space made available by the uncertainty relation, the particles follow a continuous path in space and time, and that once a certain direction is taken, it keeps moving in this direction. The “coordinates” of the particle within the guiding wave should be considered as “hidden parameters” in v. Neumann’s sense.

It is clear that the concept of a “wave devoid of energy” is disturbing. However, in attempting to avoid this one should realize that the experimental results would allow only much less attractive alternatives: because the existence of the wave can be experimentally verified during the entire motion of the photon, and the existence of the energy only in two points, the emission and the absorption point, the assumption of the existence of a normal electromagnetic wave con-

\textsuperscript{11}Remark during the correction. As the articles which the Editor became aware of since the submission of the manuscript (see footnote 5 p. 4) show, and in accordance with a kind personal announcement by Mr. Einstein this is indeed the case. In this respect also the work by L. Jánossy [Ann. Phys. \textbf{11}, 323 (1953)] should be mentioned, which attempts to give an alternative deterministic account of quantummechanical processes, although there is a violation of the assumption of the coherence of separated rays over arbitrary distances, the validity of which we accepted in our work (see page 4).

\textsuperscript{12}Einstein, A., B. Podolsky and N. Rosen: Phys. Rev. \textbf{47}, 777 (1935).

\textsuperscript{13}Weizel, W.: Z. Physik \textbf{134}, 264 (1953).
taining energy would have the unavoidable consequence, that at the moment
of absorption the wave would contract with superluminal speed, and moreover
through closed walls. Such assumption would be completely unacceptable.

The author is fully aware of the fact, that the uneasiness of the idea of a wave
devoid of energy still increases, when the consequences are considered, of the es-
tablishment that the laws of propagation of this wave are in every respect those of
an electromagnetic wave. For instance there is refraction (which in the Maxwell–
Lorentz theory has its origin in energy exchanges with dipole oscillations of the
electrons), double refraction, polarization, scattering etc.; this are all manifesta-
tions which come about through interactions with matter. Could this uneasiness
be relaxed by the consideration, that also matter must have guiding waves, and
that these are the “spooky” entities, to use Einstein’s expression, similar with
“matter waves”? That the guiding waves “devoid of energy” associated with en-
ergy radiation should correspond with guiding waves devoid of matter associated
with matter radiation? And that the interactions between these spooky entities
should co–determine the events resultant from the energy interactions between
the lumps of energy, resp. matter? A directly unobservable world behind the
observable one, guided by the former? But, to stress it again, a world the reality
of which can in time and (three dimensional) space be experimentally followed
up, and which does not exist only in the form of abstract probabilities\textsuperscript{14}.

What happens to the wave devoid of energy of a photon after its absorption?
When it is absorbed for example in (6), and when in addition the detectors in
(4) and (5) are removed, what happens then with the wave in $B$? Does she move
further towards infinity, or does she disappear at the moment of absorption?
Of course this question cannot be answered principally. The former assumption
appears to be the more natural one, because it avoids the conclusion to the
existence of influences which propagate with infinite speed also through closed
walls, a conclusion which within the physical world is inconceivable. In any case
were such influences not associated with transport of energy.

It is still possible to consider the following issue: what happens to the particle
when the wave is splitted by reflection at a refracting surface, or at a partially
transparent mirror, or by double refraction, i.e. each time a wave is splitted in
more component waves with different propagation vector and mostly also different
polarization state? All these cases can be understood without contradiction with
the help of the plausible assumption, that each time the particle follows one of
the component waves, with a probability which is proportional to the intensity of
this component wave, in the same way as a particle in a splitting fluid current.

\textsuperscript{14}See footnote 15, page \pageref{footnote15}
It is not necessary to associate with the particle properties, such as polarization. These should be associated only with the guiding wave.

Appendix.

In order to anticipate objections against the principal (not the practical) feasibility of the described experiments, more precise technical details will be given which support their realizability. The set-up should have the characteristics as shown in Fig. 2:

An almost pointlike monochromatic light source $L$ is situated at the center of a small half spherical convex mirror $S_1$, which itself is large as compared to $L$. Furthermore $L$ is in the focus of a parabolic mirror $S_2$ which again is large with respect to $S_1$. In this way the spherical wave originating in $L$ is transformed in a parallel bundle (1), which contains the photons.

This ray (1) is split at (2) in 2 separated rays $A$ and $B$ by means of 2 mirrors which are inclined by $45^\circ$. In order to remove the negative effects of the angular divergence of the parallel rays (due to the finite dimension of $L$) the parabolic mirror may have a large section, i.e. the focus and diameter can be chosen arbitrary large. The following choices may be made: diameter of the light source: $10^{-6}$ (100 Å), $S_1$: 1 cm, $S_2$: $10^6$ cm (10 km), total light distance between (2) and (3): $10^{12}$ cm (30 light–seconds). As already mentioned, for performing experiments b and c it is required that the places (2) to (9) are close to each other. Therefore, the separated paths must come repeatedly in the neighbourhood of the observer, which is illustrated in Fig. 2c. In (6), (7), (8) and (9) the section

![Fig. 2a–c. Technical details of the set-up. a Light source with reflecting mirrors; b Complete set-up; c Bringing together paths A and B through multiple bending.](image-url)
of the rays may be decreased by optical arrangements, in order to facilitate the
insertion of slides and phase plates. – Both lenses $O_4$ and $O_5$ focus the outgoing
rays (4) and (5) on the ingoing gates of two photomultiplicators. The detectors
to be inserted in (6), (7), (8) and (9) equally consist of such aggregates (lenses
and counters).

An indication is still needed how the observer may get information of the time
of emission of a photon within an uncertainty interval which is small compared
to the time needed by the photon for moving between (2) and (3). This could be
resolved as follows: The light source $L$ consists of a small hollow sphere, which
contains a small amount of gas the atoms of which have for simplicity only one
excitation level above the ground level. A beam of electrons with known velocity
is sent through this sphere containing the gas, and with such an intensity that
only one atom is excited in a time interval which is large compared with the time
needed for a photon to move between (2) and (3), in our example of the order
of hours. A supplementary arrangement measures the energy of the transmitted
electrons and informs the observer as soon as an electron is detected which has
lost an energy corresponding to the excitation energy of the atoms. This signal
warns the observer, that one of the atoms is excited and a photon will be emitted
in the apparatus within the time needed to return to the ground state (of the
order of $10^{-8}$ sec).

The first compositions of the above ideas date already from 20 years ago. The
final explanation of the present formulation benefited in a substantial way from
a series of very exciting discussions with Dr. S.N. Bagghi and R. Hosemann\textsuperscript{15},
Fritz Haber–Institute, Berlin–Dahlem, in the course of the last half year.

\textit{Marburg a.d. Lahn}, Crystallographic Institute of the University.

\textsuperscript{15}Remark during the correction. In a work in preparation these authors investigate the present
issue in terms of their newly introduced “Algebra physikalisch beobachtbarer Funktionen mittels
Faltungsoperationen”, Part I. [Z. Physik \textbf{135}, 50 (1953)], and come, following a kind personal
announcement, to a confirmation of the guiding wave picture, in which the amplitude of the
guiding wave has the dimension of energy. In fact, the average \textit{in time and in space} of the
energy density of such a wave should be zero at any time and in each point. How far the related
assumption of “negative energy densities” of, resp. the necessity to attribute a kind of “level of
energy density” to, empty space can be carried through, will not be discussed here.