Multi-strategy Gaussian Harris hawks optimization for fatigue life of tapered roller bearings

Ahmad Abbasi1 · Behnam Firouzi1 · Polat Sendur1 · Ali Asghar Heidari2 · Huiling Chen3 · Rajiv Tiwari4

Received: 18 January 2021 / Accepted: 4 June 2021 / Published online: 3 August 2021
© The Author(s), under exclusive licence to Springer-Verlag London Ltd., part of Springer Nature 2021

Abstract
Bearing is one of the most fundamental components of rotary machinery, and its fatigue life is a crucial factor in designing. The design optimization of tapered roller bearing (TRB) is a complex design problem because various arrays of designing parameters and functional requirements should be fulfilled. Since there are many design variables and nonlinear constraints, presenting an optimal design of TRBs poses some challenges for metaheuristic algorithms. The Harris hawks optimization (HHO) algorithm is a robust nature-inspired method with unique exploitation and exploration phases due to its time-varying structure. However, this metaheuristic algorithm may still converge to local optima for more challenging problems such as the design of TRBs. Therefore, this study aims to improve the accuracy and efficiency of the shortcomings of this algorithm. The performance of the proposed algorithm is first evaluated for the TRB optimization problem. The TRB optimization design has nine design variables and 26 constraints because of geometrical dimensions and strength conditions. The productivity of the proposed method is compared with diverse metaheuristic algorithms in the literature. The results demonstrate the significant development of dynamic load capacity in comparison to the standard value. Furthermore, the enhanced version of the HHO algorithm presented in this study is benchmarked with various well-known engineering problems. For supplementary materials regarding algorithms in this research, readers can refer to https://aliasgharheidari.com.

Keywords Optimization · Swarm-intelligence algorithms · Harris hawks optimization · Constrained optimization · Tapered roller bearing · Fatigue life

1 Introduction
The tapered roller bearing (TRB) has been utilized for various applications since it can be employed to convey motions, such as rotation, oscillation, and linear motion of systems [1]. Since the tapered roller bearing has conical rollers capable of running on conical races, it can resist a huge amount of radial and thrust loads [2]. To evaluate the life of roller bearing, several factors should be taken into consideration, the most dominant factors of which are the heat treatment of the bearing, surface coating, and lubrication system [3, 4]. Consequently, to boost the performance of TRB, enhance the fatigue life...
and decrease the amount of maintenance/replacement costs, it is significant to present the optimal design of TRB. Therefore, the optimal design of TRB has been studied in a vast body of literature (Table 1 shows some of the previous works related to the roller bearing design). In the optimal design of TRBs proposed by Tiwari et al. [2] using a genetic algorithm, the objective function was fatigue life which must be maximized, and the constraints contained the geometrical parameters and strength. Furthermore, sensitivity analysis was conducted by the authors to investigate the influence of various parameters on the design parameters. Tiwari et al. [6] analyzed the thermal behaviour of TRBs. In another study, design optimization was presented for the cylindrical roller bearing with the logarithmic profile by Kumar et al. [7]. A robust optimization study was presented by Verma and Tiwari [8] for minimizing the variation and maximizing the performance of tapered roller bearing. Similarly, Kaylan et al. [9] used a multi-objective optimization methodology to maximize the dynamic capacity, minimize the film thickness, and maximize the bearing temperature. In this regard, an NSGA-II was used, and sensitivity analysis was performed to evaluate the sensitivity of objectives with the design variables. Genetic algorithm (GA) was used by Choi and Yoon [10] to present and optimize a design that enhanced the system life of a double row angular contact ball bearings used in an automobile wheel. Chakraborty et al. [11] proposed the optimal design of a deep groove ball bearing using GA and showed that the optimized design variables led to superior performance to the parameters presented in the standard catalog. Dandagwahal et al. [12] optimized the design of cylindrical roller bearings and deep groove ball bearings. In this study, the modified version of the optimization algorithm based on teaching–learning was used to achieve the best design for bearings. Related to the design on deep groove ball bearings, the performance of particle swarm optimization (PSO) and GA was evaluated by Panda et al. [13] to find the best design. Kang et al. [14] optimized the geometric parameters of an angular contact ball bearing to enhance its performance using a robust optimization analysis. Tiwari and Vaghole [15] hybridized the artificial bee colony and the grid search method to improve the performance of spherical roller bearing. Moreover, a sensitivity analysis was performed to evaluate the influence of design parameters on the objective function.

The optimization process in engineering cases is obligated to satisfy the decision-maker’s requests [16–19]. This target should be done within the decision-making procedure reasonably and efficiently [20–24]. Such complex problems can be within any engineering domain [25–27]. Some examples are parameters identification, prediction scenarios, electro-mechanical systems [28, 29], expert systems [30–35], and clustering problems [36]. Nowadays, the use of optimization algorithms such as PSO [37] and variants of differential evolution (DE) [38, 39] and ant colony optimizer (ACO) [40] has an undeniable role in engineering problems to address the challenging requirements of engineering systems [41, 42]. There are recently a good set of swarm-based optimizers, including slime mould algorithm (SMA)1 [43], hunger games search (HGS)2 [44], gradient-based optimizer (GBO) [45], and Runge–Kutta optimizer.

---

Table 1 Previous work-related to optimum design of roller bearing

| Type of bearing          | Description                                                                 |
|-------------------------|-----------------------------------------------------------------------------|
| Spherical roller [1]    | Multi-objective optimization related to maximization of dynamic capacity and minimize the elasto-hydrodynamic film thickness using non-dominated sorting genetic algorithm (NSGA-II) |
| Tapered roller [2]      | The maximization of the fatigue life using a genetic algorithm (GA)          |
| Tapered roller [4]      | Quasi-static analysis of tapered roller bearings for different roller surface profiles |
| Tapered roller [6]      | Optimum design based on the thermal behaviour of tapered roller bearing using an evolutionary algorithm |
| Crowned cylindrical roller [7] | Obtained optimal design to increase the life of cylindrical roller bearings using genetic algorithm (GA) |
| Tapered roller [8]      | Robust optimum design of tapered roller bearings using evolutionary algorithm |
| Tapered roller [9]      | Multi-objective optimization of tapered roller bearing design based on fatigue, wear, and thermal considerations through genetic algorithm (GA) |
| Contact ball bearing [10] | Maximize system life though filling geometrical and operational restrictions devoid of expanding mounting space |
| Ball bearing [11]       | Maximization of fatigue life through genetic algorithm (GA)                  |
| Deep groove ball bearing [12] | Optimization of fatigue life using teaching–learning-based algorithm |
| Angular contact ball bearing [14] | Robust design optimization under manufacturing tolerance |
| Spherical roller [15]   | Optimum design using artificial bee colony algorithm and grid search method  |

---

1 https://aliasgharheidari.com/SMA.html.
2 https://aliasgharheidari.com/HGS.html.
The HHO algorithm was improved by many researchers to enhance its performance. Zhang et al. [25] applied adaptive cooperative and dispersed foraging strategies to improve the position update. These changes improved diversity and avoided local optima. Abdol-Basset et al. [64] hybridized the HHO algorithm with simulated annealing to improve HHO performance for the feature selection.

Survival exploration strategies applied successfully to the structure of the HHO, which resulted in efficient results compared to other competitors [65]. Authors developed a Gaussian bare bone HHO in [66] for predicting entrepreneurial intentions. A multi-population DE-based version was also proposed that can show excellent exploratory patterns [67]. HHO and its progressive variants also applied to parameters identification of photovoltaic cells [60, 68], image segmentation [69, 70], web service composition [71], diagnosing coronavirus disease [72], predicting di-2-ethylhexyl phthalate toxicity [65], parameter estimation of photovoltaic models [73, 74], real-world engineering optimization problem [75], and feature selection [76, 77]. For a review of recent works on HHO, please refer to work in [78].

According to the reviewed papers, the optimization-based designs available in the literature are mainly based on classical algorithms like genetic algorithm (GA). While the classical algorithms can be efficient for some of the problems, there is still room for novel algorithms to present an optimal roller bearing design. This paper proposes an improved version of the HHO algorithm that shows superiority in roller bearing design. A multi-strategy algorithm based on the Harris hawk optimization algorithm is designed with some advantages compared to the standard HHO. At the beginning of standard HHO, a chaotic technique is performed to distribute agents equally in the search space. In the exploration phase, new strategies are added to increase the power of the exploration phase. Finally, a chaotic local search is added to avoid local optima. The optimization of TRBs demonstrates the efficiency of the proposed algorithm. Furthermore, to evaluate the accuracy and efficiency of the proposed algorithm, it is benchmarked on several famous engineering problems. The rest of this paper is organized as follows: In Sect. 2, the geometry of TRBs is given, and stress analysis is conducted on the roller bearing. An optimization methodology for TRB maximization, including the objective function, design variables, and the associated constraints, is elaborated on comprehensively in Sect. 3. An overview of the proposed EHHO algorithm is given in Sect. 4. The results are addressed in Sect. 5. Finally, Sect. 6 is reserved for conclusions.
2 Tapered roller bearing

2.1 Geometrical structure of TRB

There are four critical elements in this type of bearing, which are shown in Fig. 1: (1) shortened cones with the number of rollers inside this, (2) a cage to carry rollers, (3) cone (internal ring), and (4) cap or external ring. The TRB has a larger lip at the cone’s back to reinforce the axial force from the set of rollers and a smaller lip near the cone, the function of which is to provide the consistency of the rollers.

The measurement parameters of TRB, based on standard catalog [79], consists of (d) as a bore diameter, (T) as the width of the bearing, (D) appears for outer diameter, (C) stands for the width of the cup, (α) and (β) represent the contact angle and the cone’s width, respectively, as demonstrated in Fig. 1.

The semi-taper angle (β) is formulated according to two other separate parameters, named pitch diameter ($D_m$) and mean diameter ($D_r$), as follows:
The minimum thickness of the front-face of the cup, shown in Fig. 2, is calculated using Eq. (2), and the one related to the cone is determined using Eq. (3):

\[ S_{2\text{min}}^c = \frac{1}{2}D - HI = \frac{1}{2}D - \left( \frac{1}{2}D_m \cot(\alpha - \beta^o) + \frac{1}{2}t \right) \sec \beta^o \sin \alpha, \]

(2)

\[ S_{2\text{min}}^i = FJ - \frac{1}{2}d = \left( \frac{1}{2}D_m \cot(\alpha - \beta^o) - \frac{1}{2}t \right) \sec \beta^o \sin(\alpha - 2\beta^o) - \frac{1}{2}d. \]

(3)

The minimum width of the back-face and the front-face of the cup, \( C_{1\text{min}} \) and \( C_{2\text{min}} \), respectively, can be expressed as [2]:

\[ C_{2\text{min}} = (C + AX) - AI = \left[ C + \frac{1}{2}D_m \cot(\alpha) \right] - \left[ \left( \frac{1}{2}D_m \cot(\alpha - \beta^o) + \frac{1}{2}t \right) \sec \beta^o \right]. \]

(4)

\[ C_{1\text{min}} = C - C_{2\text{min}} - \frac{I \cos \alpha}{\cos \beta^o}. \]

(5)

Moreover, the total width is calculated as follows:

\[ C = C_{2\text{min}} + C_{1\text{min}} + \frac{I \cos \alpha}{\cos \beta^o}. \]

(6)

Similar to the cup, there is a back-face related to the cone, the minimum of which can be derived from Fig. 2 as follows:

\[ B_{2\text{min}} = (T + AX) - AY = \left[ T + \frac{1}{2}D_m \cot(\alpha) \right] - \left[ \left( \frac{1}{2}D_m \cot(\alpha - \beta^o) + \frac{1}{2}t \right) \sec \beta^o \cos(\alpha - 2\beta^o) \right]. \]

(7)

\( B_{1\text{min}} \) is related to internal dimensions and is given as

\[ B_{1\text{min}} = B - B_{2\text{min}} - \frac{1}{\cos \beta^o} \cos(\alpha - 2\beta^o). \]

(8)

Moreover, the thickness of the back-face of the cup is determined using Eq. (9):

\[ S_{1\text{min}}^c = \frac{1}{2}D - MN = \frac{1}{2}D - \left( \frac{1}{2}D_m - \frac{1}{2}t \sin(\alpha - \beta^o) \right) \]

\[ + \left( \frac{1}{2}D_r - \frac{1}{2}t \beta^o \right) \cos(\alpha - \beta^o) \right) \].

(9)

One of the internal dimensions \( S_{2\text{min}} \) relates to the surface of the cone, which is shown in Fig. 2 can be calculated as

\[ S_{2\text{min}}^i = \left( \frac{1}{2}D_m - \frac{D_r}{2} \cos(\alpha - \beta^o) + \frac{1}{2}t \beta^o \right) - \frac{d}{2}. \]

(10)

### 2.2 Stress analysis of TRB

Every equipment during its operation experiences loads and stresses in terms of normal and shear types. For TRB, these loads act on the bearing’s flange. To determine such forces, the free body diagram shown in Fig. 3 is taken into consideration. The forces which act on the bearing components are obtained according to the static equilibrium formula of bearing. \( Q_o \) and \( Q_f \) are loads on the cup and spherical face of TRB roller, respectively, which are written in Eq. (11) as follows:

\[ Q_f = Q_i \cos \alpha_i \left( \frac{\sin(\alpha - \tan(\cos \alpha \alpha_i))}{\sin(\alpha + \alpha_i)} \right), \]

\[ Q_o = Q_i \cos \alpha_i \left( \frac{\sin(\alpha + \tan(\cos \alpha \alpha_i))}{\sin(\alpha + \alpha_i)} \right), \]

(11)

where force \( Q_i \) is determined based on stribeck’s equation [80], \( \alpha \) is roller contact angle, \( \alpha_i \) is named as cone contact angle.
angle and calculated as \( \alpha - 2\beta \), and Flange angle \( (\alpha_f) \) is
given as Eq. (12)[8]:

\[
\alpha_f = \sin^{-1} \left( \frac{\frac{3}{2}D_r - (AB - R)\sin\beta}{R} \right) + \alpha - 2\beta.
\]

(12)

\[
AB = \frac{D_m}{2\sin(\alpha - 2\beta)} - \frac{D_r\cos(\alpha - \beta)}{2\sin(\alpha - 2\beta)} + \frac{l}{2\cos\beta}.
\]

where the value of \( R \) is 95% of the length \( AB \) and \( \xi = 0.125 \).

Load \( (Q_f) \) causes bending stress \( (\sigma_{bf}) \) and shear stress \( (\tau_f) \). Also, tensile stress \( (\sigma_f) \) happens due to flange loads and is formulated in Eq. (13).

\[
\sigma_f = \frac{Q_f}{A_f} \sin \alpha_f,
\]

(13)

\[
A_f = \frac{\pi \left( \frac{1}{2}d + S_{2_{\text{max}}}^i \right) B_{2_{\text{max}}}}{Z}.
\]

The maximum shear stress occurring in TRB’s flange shape is written by

\[
\tau_f = 1.5 \frac{Q_f \cos \alpha_f}{A_f}.
\]

(14)

Similarly, the other components of stresses which are created by bending moment are given by

\[
s_{bf} = \frac{Q_f}{EI} \frac{h_y}{A_f} \cos^2 \alpha_f
\]

(15)

\[
h_y = \frac{\frac{1}{2}d - \left( \frac{1}{2}d + S_{2_{\text{max}}}^i \right)}{2 \cos \alpha_f},
\]

\[
I = \frac{\pi \left( \frac{1}{2}d + S_{2_{\text{max}}}^i \right) B_{2_{\text{max}}}^2}{12Z}.
\]

Table 2: Input parameters for tapered roller bearings [79]

| Bearing number | Standard boundary dimensions | Standard internal dimensions | Standard chamfering dimensions | Dynamic load rating |
|----------------|-----------------------------|-----------------------------|-------------------------------|--------------------|
|                | \( D \) mm | \( d \) mm | \( C \) mm | \( B \) mm | \( T \) mm | \( d_{1_{\text{max}}} \) mm | \( d_{2_{\text{max}}} \) mm | \( a \) degree | \( r_{1_{\text{max}}} \) mm | \( r_{2_{\text{max}}} \) mm | \( r_{3_{\text{max}}} \) mm | \( r_{4_{\text{max}}} \) mm | \( C_d \) kN |
| 30,204         | 47   | 20   | 12   | 14   | 15.25 | 33.20 | 37.304 | 12.9527 | 1.0 | 1.0 | 1.0 | 1.0 | 0.5 | 27.5 |
| 30,205         | 52   | 25   | 13   | 15   | 16.25 | 37.40 | 41.135 | 14.0361 | 1.0 | 1.0 | 1.0 | 1.0 | 0.5 | 30.80 |
| 32,205         | 52   | 25   | 15   | 18   | 19.25 | 40.20 | 37.555 | 21.2500 | 1.0 | 1.0 | 1.0 | 1.0 | 0.5 | 35.80 |
| 322/28         | 58   | 28   | 16   | 19   | 20.25 | 43.90 | 42.436 | 20.5666 | 1.0 | 1.0 | 1.0 | 1.0 | 0.5 | 41.80 |
| 32,206         | 62   | 30   | 17   | 20   | 21.25 | 45.20 | 48.982 | 14.0361 | 1.0 | 1.0 | 1.0 | 1.0 | 0.5 | 50.10 |
| 30,207         | 72   | 35   | 15   | 17   | 18.25 | 51.80 | 58.844 | 14.0361 | 1.5 | 1.5 | 1.5 | 1.5 | 0.5 | 51.20 |
| 32,207         | 72   | 35   | 19   | 23   | 24.25 | 52.40 | 57.087 | 11.8597 | 1.5 | 1.5 | 1.5 | 1.5 | 0.5 | 56.10 |
| 32,208         | 80   | 40   | 19   | 23   | 24.75 | 54.50 | 65.769 | 11.8597 | 2.0 | 2.0 | 1.5 | 1.5 | 0.5 | 66.00 |
| 32,209         | 72   | 35   | 18   | 21   | 22.75 | 52.40 | 57.087 | 14.0361 | 1.5 | 1.5 | 1.5 | 1.5 | 0.5 | 66.00 |

where \( Z, I, \) and \( E \) indicate the number of rollers, area moment of inertia, and Young’s modulus, respectively. As the result of determining all the components of stresses, the maximum principal stress is achieved in the flange shape part of the roller using Eq. (16):

\[
\sigma_{f_{\text{max}}} = \frac{\sigma_{bf} + \sigma_f}{2} + \sqrt{\left( \frac{\sigma_{bf} + \sigma_f}{2} \right)^2 + \tau_f^2}.
\]

(16)

The computation of the above parameters exploited as the constraints for the optimization process is explained below.

### 3 Formulation of the optimization problem

In this part of the paper, the formula to achieving the best performance of TRB is explained in detail. Design parameters and structural constraints are set to maximize the objective function (fitness function) as shown below:

Objectives : Maximize \( f(x) \)

Variable bounds : \( x_j^{(L)} \leq x_j \leq x_j^{(U)}, x_j \in \mathbb{R}, j = 1, 2, 3, \ldots, n \)

Constraints : \( g_j(x) \text{onstr}(x) = 0, \quad j = 1, 2, 3, \ldots, k \)

(17)

In Sects. 3.1 to 3.3, the method of obtaining three parts of an optimization problem, including variables, objective functions, constraints, is defined.

### 3.1 Fitness function

Fatigue, corrosion, and creep are among the main factors for fracture in roller bearings. These kinds of failures can be diminished or removed by a proper design [2, 80]. The fatigue life in the bearing is distinguished as one of
Table 3 Material properties of the bearing (steel)

| Description      | Value  |
|------------------|--------|
| Safe contact stress | 4000 MPa |
| Young’s modulus  | 210 GPa |
| Yield strength    | 600 MPa |
| Poisson’s ratio   | 0.3    |

The crucial design considerations and can be calculated as follows:

\[ L_{10} = \left( \frac{C_d}{P} \right)^{10^6} \]  \hspace{1cm} (18)

where \( C_d \) is known as dynamic load, \( L_{10} \) is defined as the life of bearing with 90% reliability, \( n \) is defined as an exponent of load life, and \( P \) is specified as radial load \([80]\). The fatigue life of bearing and dynamic load is related to each other. Thus, maximizing dynamic capability is formulated as a fitness function to improve the bearing’s performance. The fitness function is written as shown below:

Maximize \( f(x) = C_d \)  \hspace{1cm} (19)

The dynamic load capacity for roller bearing is formulated as \([14]\):

\[ C_d = b_m f_c \left( \frac{d_e \cos \alpha}{r} \right)^{\gamma} Z^2 D_r^{3\gamma}, \]

where

\[ f_c = 207.9 \lambda y^2 \frac{(1 - \lambda)^{29/27}}{(1 + \lambda)^{1/2}} \times \left[ 1 + \left\{ 1.04 \left( \frac{1 - \lambda}{1 + \lambda} \right)^{16} \right\}^{9/2 - 7/9} \right], \]

\[ \gamma = \frac{D_{r\text{min}}}{D_m} \text{ and } D_{r\text{max}} = \frac{1}{2} \left( D_{r\text{lle}} + D_{r\text{lle}} \right). \]  \hspace{1cm} (20)

where \( Z, l_c, D_r \) are the number of rollers, adequate length, and mean diameter of rollers, respectively. Also, \( \lambda \) is equal to 0.65 and is taken as a reduction factor into account. \( v \) is related to edge loading, which is 1.2 for TRB \([80]\). Finally, \( b_m \) is taken as 1.1 \([81]\).

3.2 Optimization variables

Nine variables are used for optimizing the design of TRB. The internal geometry of bearing, including sufficient length \((l_c)\), mean and peach diameter \((D_r, D_m)\), and the number of rollers \((Z)\) influences the dynamic load of bearing. Also, there are other types of variables that are used as constraints. \( K_{D_{\text{min}}} \) and \( K_{D_{\text{max}}} \) are minimum and maximum roller diameter, respectively. The remaining three variables, i.e., \( e, \epsilon, \text{ and } \beta \), are described as mobility parameters, the outer ring strength, and semi-taper angle in the bearing, respectively. The nine design variables are defined as follows:

\[ X = \{ D_m, D_r, l_c, Z, K_{D_{\text{min}}}, K_{D_{\text{max}}}, e, \epsilon, \beta \}. \]  \hspace{1cm} (21)

All the design variables are positive integers for this problem.

3.3 Constraints

In this subsection, design constraints of the optimization problem are present. To decrease the stress concentration of the cup and cone of rollers, the lower and upper limits of the pitch diameter should be set as the following equation, which is defined as Constraints 1 and 2:

\[ (d + 2 r_{m_{\text{min}}}) \leq D_m \leq (D - 2 r_{m_{\text{min}}}). \]  \hspace{1cm} (22)

Hence, these constraints are written in the following form:

\[ G_1(X) = D_m - (d + 2 r_{m_{\text{min}}}) \geq 0, \hspace{1cm} G_2(X) = (D - 2 r_{m_{\text{min}}}) - D_m \geq 0. \]  \hspace{1cm} (23)

For restricting the contact stress, the mean diameter should be arranged as follows:

\[ D_{r_{\text{lle}}} \leq D_r \leq D_{r_{\text{lle}}}. \]

\[ D_{r_{\text{lle}}} = 212.43 \sqrt{\frac{Q_{\text{max}}}{\sigma_{\text{max}}}}, \hspace{1cm} D_{r_{\text{lle}}} = \frac{1}{2} \left( (D - 2 r_{m_{\text{min}}}) - (d + 2 r_{m_{\text{min}}}) \right), \]  \hspace{1cm} (24)

where \( Q_{\text{max}} \) is the largest value of the contact load and \( D_{r_{\text{lle}}} \) and \( D_{r_{\text{lle}}} \) are minimum and maximum ranges obtained for

Table 4 The optimization parameters

| Optimization method  | Parameters | Value |
|----------------------|------------|-------|
| EHHO and HHO         | Population | 80    |
| \( \beta \)          |            | 1.5   |
| Number of iterations |            | 10,000|
| WOA                  | Population | 80    |
| \( b \)              |            | 1     |
| Number of iterations |            | 10,000|
| SCA                  | Population | 80    |
| \( a \)              |            | 2     |
| Number of iterations |            | 10,000|
mean diameter. According to the above explanation, Constraints 3 and 4 are derived as shown below:

\[ G_3(X) = D_r - 212.43 \sqrt{\frac{D_{\text{max}}}{\sigma_{\text{safe}}}} \geq 0, \]
\[ G_4(X) = \frac{(D - 2r_{3\text{min}}) - (d + 2r_{1\text{min}})}{2\cos \alpha} - D_r \geq 0. \]  

(25)

According to the internal geometry of TRB, the length of the roller \( l_e \) should have the range in Eq. (26) to project from both faces of the cup.

\[ D_{\text{RL}} \leq l_e \leq l_{\text{RL}}. \]  

(26)

Therefore, Constraints 5 and 6 can be expressed as follows:

\[ G_5(X) = l_e - D_{\text{RL}} \geq 0, \]
\[ G_6(X) = \left( C - r_{2\text{min}} - r_{4\text{min}} \right) - l_e \geq 0. \]  

(27)

The limit area for the number of roller is specified according to Eq. (28), which contains lower and upper limits of pitch diameter.

\[ \frac{\pi (d + 2r_{1\text{min}})}{D_{\text{RL}}} \leq Z \leq \frac{\pi (D - 2r_{3\text{min}})}{D_{\text{RL}}}. \]  

(28)

Thus, Constraints 7 and 8 take the following form:

\[ G_7(X) = Z - \frac{\pi (d + 2r_{1\text{min}})}{D_{\text{RL}}} \geq 0, \]
\[ G_8(X) = \frac{\pi (D - 2r_3)}{D_{\text{RL}}} - Z \geq 0. \]  

(29)

The following design criteria are chosen for the roller diameter:

\[ K_{D_{\text{min}}} \frac{(D - d)}{2\cos \alpha} \leq D_r \leq K_{D_{\text{max}}} \frac{(D - d)}{2\cos \alpha}, \]
\[ 0.3 \leq K_{D_{\text{min}}} \leq 0.4 \text{ and } 0.5 \leq K_{D_{\text{max}}} \leq 0.6. \]  

(30)

The ranges for \( K_{D_{\text{min}}} \) and \( K_{D_{\text{max}}} \) are selected from a survey on TRBs [82]. \( d \) and \( D \) taken as bore and outer diameter, respectively. As a result, Constraints 9 and 10 are expressed as follows:

\[ G_9(X) = D_r - K_{D_{\text{min}}} \frac{(D - d)}{2\cos \alpha} \geq 0, \]
\[ G_{10}(X) = K_{D_{\text{max}}} \frac{(D - d)}{2\cos \alpha} - D_r \geq 0. \]  

(31)

Constraints 11 and 12 related to mobility factor are represented in the following form:

\[ G_{12}(X) = (0.5 + e)(D + d) - D_{\text{m}} \geq 0, \]
\[ G_{11}(X) = D_{\text{m}} - (0.5 - e)(D + d) \geq 0, \]
\[ 0.01 \leq e \leq 0.07. \]  

(32)

The criteria for value \( e \) are obtained from the study on TRBs [82].

Constraint 13 is associated with the width of the bearing cup, which is expressed as

\[ G_{13}(X) = \frac{0.5(D - D_{\text{m}} - D_{r})}{\cos \alpha} - \varepsilon D_r \geq 0, \]
\[ 0.4 \leq \varepsilon \leq 0.5. \]  

(33)

Constraint 14 comes from the periphery of bearing. Width of the cup \( S_{2\text{min}}^r \), should have the following condition in the internal geometry of the bearing:

\[ G_{14}(X) = S_{2\text{min}}^r - \left\{ \frac{1}{2}D - \left( \frac{1}{2}D_{D_{\text{max}}} + \text{Tan} \alpha \right) \right\} \geq 0. \]  

(34)

The higher stress level should be avoided in the cone of TRB. Thus, Constraint 15 is designed according to the cone of TRB, which is represented as

\[ G_{15}(X) = S_{1\text{min}}^i - S_{2\text{min}}^r \geq 0, \]  

(35)

where \( S_{1\text{min}}^i \) and \( S_{2\text{min}}^r \) are minimum thickness of cup and cone of TRB, respectively.

The difference between \( C_{1\text{min}} \), as the lowest thickness of the front-face of the cup, and value \( r_{3\text{min}} \) should be positive. Therefore, Constraint 16 can be formulated as Eq. (36):

\[ G_{16}(X) = C_{1\text{min}} - r_{3\text{min}} \geq 0. \]  

(36)

For secure operation of the bearing, there should be a clearance between back-face of the cup and angle of the roller’s short end. Constraint 17 is given as

\[ \text{Fig. 4 Energy factor} \]
The contact between the chamfer of the back-face of the cone and the roller’s corner of the big end should be avoided. Hence, adequate space between them is considered. Therefore, Constraint 18 is given as

\[ G_{18}(X) = B_2 - r_2 \geq 0. \tag{38} \]

An appropriate distance must be considered among the chamfer of the front-face of the cone’s and edge of the tiny end of the roller. This gap can be reflected as Constraint 19, which is written as:

\[ G_{19}(X) = B_1 - r_5 \geq 0. \tag{39} \]

Constraints 20 and 21 are associated with resistance of the cone’s lips and thickness of the cup’s front-face. For Constraint 20, because the larger lip in comparison to the smaller one is subjected to superior load, Eq. (40) is proposed as the constraint; for Constraint 21, the thickness of the front-face of the cup should have conditions based on Eq. (41).

\[ G_{20}(X) = B_2 - B_1 \geq 0, \tag{40} \]

\[ G_{21}(X) = C_1 - C_2 \geq 0. \tag{41} \]

Constraint 22 related to the sufficient length, \( l_r \), is expressed as Eq. (42):

\[ G_{22}(X) = \frac{\beta C}{\cos \alpha} - l_r \geq 0, \tag{42} \]

\[ 0.8 \leq \beta \leq 0.95. \]

Constraint 24 is in correlation with stress at the flange of the cone, which is given by Eq. (44):

\[ G_{24}(X) = \sigma_y - \sigma_{f_{max}} \geq 0, \tag{44} \]

where \( \sigma_{f_{max}} \) is the largest stress at the flange and \( \sigma_y \) is yield stress.

A restriction for contact stress is taken into account because the stress should not exceed 4000 MPa. Therefore, Constraint 25 is given as follows:

\[ G_{25}(X) = \sigma_{safe} - \sigma_{l_{max}} \geq 0, \tag{45} \]

where \( \sigma_{safe} \) is the allowable contact stress and \( \sigma_{l_{max}} \) is the actual contact stress between the cone and the roller when the roller is loaded with \( Q_{max} \) over the cone.

There should be sufficient spacing between the rollers to have the secure operation of the bearing. The relevant Constraint (26) is written as follows:

\[ G_{26}(X) = 2\pi - 2Z \sin^{-1}\left( \frac{D_p \cos \alpha}{D_m} \right) \geq Z \cdot \frac{\pi}{180}. \tag{46} \]
4 Optimization methodologies

In this section, the optimization process of TRB’s design is described in great detail. From bearing catalog [83] and bearing standard [79], different TRB cases are introduced, the fatigue life of which is improved by the proposed methods. To perform the optimization process, some constant inputs reported in Table 2 are needed. These inputs show some characteristics related to different case bearings.

The optimization algorithms are so sensitive to their parameters, and choosing proper parameters can guarantee good convergence toward the optimal solution. Thus, to calibrate the parameters of the optimization algorithm, several different cases are considered and performed on bearing number 30204. The best performance of the parameters is reported in Table 4. Also, 10,000 iterations are chosen for executing the optimization process.

For having the fair comparison of results [84–87], two other effective algorithms (whale optimization algorithm [88, 89], and sine cosine algorithm (SCA) [90–93]) are added to execute the simulated experiments. The coding of each algorithm is written in Matlab software, and the final results are based on ten separate runs. The best result of dynamic capacity for each algorithm is reported for the final comparison.

4.1 Harris hawks optimization algorithm (HHO)

Heidari et al. [50] have recently developed a novel and robust optimization algorithm that mimics Harris hawks birds’ cooperative behavior. These birds can capture the prey using several strategies. These strategies can be simulated as the main structure of the Harris hawk optimization algorithm (HHO). Like every other optimizer, the Harris hawk optimizer has exploration and exploitation phases to find the final optimum solution of the objective function. All details related to the mechanism of the Harris hawk optimizer are explained in the following subsections.

4.1.1 Exploration (observation) stage

In this stage, the birds search and track the variables’ space for detecting the prey’s position as the optimum solution. Two equations are represented in this stage based on the strike tactics of Harris hawks: perching strategy and crouching strategy. These two strategies are expressed in the following form:

\[
X(t + 1) = \begin{cases} 
X_{\text{rand}}(t) - r_1 |X_{\text{rand}}(t) - 2r_2 X(t)| & q \geq 0.5 \\
(X_{\text{tackle}}(t) - X_{\text{m}}(t)) - r_3 (LB + r_4 (UB - LB)) & q < 0.5
\end{cases}
\]

where \(X_{\text{tackle}}(t)\) is the position of the prey, \(X(t + 1)\) is the place of search agents for the next round of algorithm, \(X(t)\) is the position of the search agents in the current iteration, and \(X_{\text{rand}}(t)\) is selected arbitrarily based on the present search agents. \(r_1\) through \(r_4\) produce a random number in the interval of 0 and 1. \(X_{\text{m}}(t)\) is formulated as follows:

\[
X_{\text{m}}(t) = \frac{1}{N} \sum_{i=1}^{N} X_i(t),
\]

where \(X_i(t)\) is the location of each search agent, and \(N\) is the number of search agents.

4.1.2 Energy factor (transition factor)

The energy factor controls the changing phases between exploration and exploitation behaviours. The factor is described in Eq. (49). When the factor \(E\) is > 1, the exploration stage is performed, and when \(E\) is < 1, the exploitation phase is carried out.

\[
E = 2E_0 (1 - t/T),
\]

where \(E_0\) is the initial energy which is randomly chosen between \((-1, 1)\), \(T\) is the maximum iteration number, and \(t\) is the current iteration. The energy factor is plotted against the iteration number in Fig. 4.
4.1.3 Exploitation (intensification) stage

In this stage, the Harris hawks birds utilize diverse hunting strategies. These exploitation strategies contain 4 main attacking movements: soft besiege, hard besiege, soft besiege with progressive rapid dives, and hard besiege with progressive rapid dives. These four exploitation techniques are defined as follows:

4.1.3.1 Exploitation’s technique 1 (Soft besiege) In this situation, the prey is confused and looks for a way to escape. This strategy is performed when \( r \geq 0.5, |E| \geq 0.5 \), which means that the prey is very exhausted. This tactic is modelled as

\[
X(t + 1) = \Delta X(t) - E[JX_{\text{rand}}(t) - X(t)] ,
\]

\[
\Delta X(t) = X_{\text{rand}}(t) - X(t) ,
\]

\[
J = 2 (1 - r_5) ,
\]

where \( \Delta X(t) \) denotes the location of the tired prey. \( J \) is a factor to show the prey’s behaviour, and \( r_5 \) is randomly chosen from \([0,1]\).

4.1.3.2 Exploitation’s technique 2 (Hard besiege) This tactic happens when \( r \geq 0.5, |E| \geq 0.5 \). A killer method as a
4.1.3.3 Exploitation's technique 3 (Soft besiege with progressive rapid dives) When $r < 0.5$, and $|E| \geq 0.5$, the Harris hawk birds perform their next action based on the following expression:

$$X(t+1) = X_{\text{rabbit}}(t) - E |\Delta X(t)|,$$  \hspace{1cm} \text{(51)}

where $LF$ is known as the levy flight function, $S$ is the random vector, $D$ is the number of search agents. $u$ and $v$ are chosen from $[0,1]$ and $\beta$ is equal to $1.5$. To update the positions of search agents, the following expression proposed:

$$X(t+1) = \begin{cases} 
Y & \text{if } F(Y) < F(X(t)) \\
Z & \text{if } F(Z) < F(X(t))
\end{cases},$$  \hspace{1cm} \text{(54)}

where $Y$ and $Z$ are computed from Eq. (52) and Eq. (53).

4.1.3.4 Exploitation's technique 4 (hard besiege with progressive rapid dives) In the last tactic of the exploitation phase, the Harris Hawks, as search agents for the optimizer, are very close to the prey and kill it. This strategy is explained according to Eq. (55) and Eq. (56):

$$X(t+1) = \begin{cases} 
Y & \text{if } F(Y) < F(X(t)) \\
Z & \text{if } F(Z) < F(X(t))
\end{cases},$$  \hspace{1cm} \text{(55)}
The flowchart of the HHO process is given in Fig. 5.

4.2 Proposed enhanced Harris hawks (EHHO)

The mathematical formulation and procedure of the enhanced version of the Harris hawk optimization algorithm are elaborated in this part. Based on the no free lunch (NFL) theory, not every algorithm applies to all problems. For instance, initialization in the HHO algorithm is based on arbitrary numbers, generating a mature population. Furthermore, the proposed strategies used in HHO for updating the population are limited for several applications. Plus, this algorithm is trapped in the local optimum. To enhance the performance of the HHO algorithm and improve the exploitation and exploration phases of the main algorithm, the following methods are added to the HHO algorithm: (1) chaotic method, (2) update besiege strategy 3, (3) update besiege strategy 4, (4) Gaussian mutation, and (5) CLS with a shrinking mode. The first two techniques are presented to assist the HHO algorithm in generating a different mature population; the third technique is added to the HHO algorithm to enhance its performance for updating the population, and the last two techniques are added to the main HHO algorithm to prevent it from being trapped in the local optimum.

4.2.1 Chaotic method

The initialization in the optimization algorithms is responsible for spreading the design variables in the design space, which can lead to the convergence of the algorithm to the global optimum. The chaotic initialization approach is a powerful procedure that helps the algorithm to generate a more diverse population. The chaotic method has been added to the HHO algorithm:

\[Y = X_{\text{rabbit}}(t) - E[X_{\text{rabbit}}(t) - X_m(t)],\]
\[Z = Y + S \times LF(D).\]

The flowchart of the HHO process is given in Fig. 5.

Table 7 Optimization results for TRB design

| Bearing number | Rank | Optimization method | Optimum parameters | Cost |
|----------------|------|---------------------|--------------------|------|
|                |      |                     | \(D_m\) (mm) | \(D_r\) (mm) | \(l_c\) | \(Z\) | \(K_{D_{\text{min}}}\) | \(K_{D_{\text{max}}}\) | \(\epsilon\) | \(\epsilon\) | \(\beta\) | \(C_f\) (N) |
| 30207          | 1    | EHHO                | 53.8069 9.1539 13.0691 | 18 | 0.356722 0.5213 0.4974 | 0.0514 0.9142 | 62,084.4 |
|                | 2    | HHO                 | 53.3063 9.6201 12.5616 | 17 | 0.361756 0.5476 0.4572 | 0.0101 0.8666 | 60,868.1 |
|                | 4    | WOA                 | 56.1596 6.9678 11.9946 | 24 | 0.300363 0.5006 0.4860 | 0.0339 0.8010 | 53,606.8 |
|                | 3    | SCA                 | 53.3906 9.5264 12.2235 | 17 | 0.36438 0.5359 0.4689 | 0.0188 0.8322 | 58,961.8 |
|                | 5    | GA [2]              | 56.117 6.89 13.0 | 23 | 0.3305 0.5027 0.4666 | 0.0699 0.9417 | 54,510 |
| 30306          | 1    | EHHO                | 51.5299 10.4971 14.3968 | 15 | 0.371458 0.5463 0.4084 | 0.0605 0.9085 | 63,557.4 |
|                | 2    | HHO                 | 52.2482 9.8508 14.0246 | 16 | 0.325396 0.56 0.4572 | 0.0101 0.8976 | 65,748 |
|                | 4    | WOA                 | 52.1721 10.0208 13.2669 | 15 | 0.378517 0.5463 0.4084 | 0.0605 0.9085 | 63,557.4 |
|                | 3    | SCA                 | 51.8496 10.3815 13.2908 | 15 | 0.330331 0.5463 0.4084 | 0.0605 0.9085 | 63,557.4 |
|                | 5    | GA [2]              | 54.221 7.74 14.0 | 20 | 0.3451 0.5597 0.4999 | 0.0699 0.9498 | 59,350 |
| 32207          | 1    | EHHO                | 53.2245 9.0549 17.5122 | 18 | 0.397334 0.5969 0.4998 | 0.0556 0.9463 | 77,162.2 |
|                | 4    | HHO                 | 55.4309 6.9632 16.9182 | 24 | 0.300001 0.5000 0.4630 | 0.0687 0.8663 | 70,135.9 |
|                | 2    | WOA                 | 53.0241 9.5731 16.2293 | 17 | 0.315465 0.5536 0.4555 | 0.0548 0.8974 | 73,982.6 |
|                | 3    | SCA                 | 53.2925 9.1628 16.7300 | 17 | 0.311826 0.5757 0.4844 | 0.0666 0.95 | 72,252 |
|                | 5    | GA [2]              | 54.381 7.99 16.0 | 20 | 0.3516 0.5968 0.4678 | 0.0678 0.9204 | 69,810 |
| 30307          | 1    | EHHO                | 58.0431 11.8241 16.0144 | 15 | 0.309913 0.5864 0.4378 | 0.07 0.95 | 84,483.3 |
|                | 5    | HHO                 | 60.1575 10.1419 13.8516 | 18 | 0.367968 0.5022 0.4115 | 0.0574 0.9387 | 73,329.2 |
|                | 3    | WOA                 | 60.4662 9.6153 14.6739 | 20 | 0.371041 0.5549 0.4623 | 0.0438 0.8433 | 75,411.3 |
|                | 2    | SCA                 | 56.9436 12.8484 15.2605 | 13 | 0.3776 0.6 0.4 | 0.0688 0.95 | 79,927.4 |
|                | 4    | GA [2]              | 60.875 8.77 15.0 | 20 | 0.3545 0.5823 0.4853 | 0.0643 0.9043 | 74,940 |
| 32208          | 1    | EHHO                | 59.7637 10.1674 17.5122 | 18 | 0.399854 0.5965 0.4987 | 0.0303 0.95 | 87,259.1 |
|                | 4    | HHO                 | 60.2563 9.6762 15.6860 | 20 | 0.30076 0.5012 0.4891 | 0.0693 0.8020 | 79,052.7 |
|                | 2    | WOA                 | 60.4622 9.7165 16.4699 | 19 | 0.4 0.6 0.5 | 0.0607 0.9498 | 82,458.4 |
|                | 3    | SCA                 | 59.5876 10.4600 16.1836 | 17 | 0.308451 0.6 0.4 | 0.0479 0.9116 | 81,068.9 |
|                | 5    | GA [2]              | 60.800 9.06 15.0 | 20 | 0.3847 0.5988 0.4517 | 0.0390 0.8626 | 75,420 |
employed in a wide range of engineering problems, such as feature selection and chaos control [94, 95]. A large number of chaotic maps [96], such as the Chebyshev map, circle map, and intermittency map, are available in the literature. Herein, a prominent logistic map is used, which is described as follows:

\[
\beta_{i+1} = \mu \beta_i \times (1 - \beta_i), \quad i = 1, 2, \ldots, S - 1, \tag{57}
\]

where \( \mu = 4 \) is a controlling factor, \( \beta_i \) is an arbitrary number between 0 and 1, and \( S \) is the number of search agents. To achieve a population with better quality, after the first arbitrary initialization in the main HHO algorithm, chaotic mapping is performed. This modification improves the convergence of the algorithm [97]. This disturbance can then be obtained using the following formulation:

\[
X_i^\alpha = \beta_i X_i, \tag{58}
\]

where \( X_i^\alpha \) is the position of the \( i \)th Harris hawk with chaotic disturbance, and \( \beta_i \) is the \( i \)th value in the chaotic sequence.

### 4.2.2 Update besiege strategy 3

To effectively update the location in Strategy 3 of the standard HHO algorithm, instead of utilizing a soft besiege with rapid progressive dives, a formulation from the flower pollination optimization algorithm is utilized [20]. This approach can be developed as follows [98]:

\[
Y = X(t) + LF(X_{rabbi}(t) - X(t)),
\]

\[
Z = X(t) + \varepsilon (X_j(t) - X_k(t)), \tag{59}
\]

\[
X(t + 1) = \begin{cases} 
Y & \text{if } F(Y) < F(X(t)) \\
Z & \text{if } F(Z) < F(X(t))
\end{cases}
\]

where \( \varepsilon \) denotes an arbitrary number between 0 and 1, and \( X_j(t) \) and \( X_k(t) \) represent pollens from the \( j \)th and \( k \)th flowers of the identical population, respectively. This technique can enhance the performance of the HHO algorithm to present a more diverse solution in the next iteration.

### Table 8: Optimum internal geometry obtained by different optimization algorithms

| Bearing number | Optimization method | Optimum parameters of internal geometry |
|----------------|---------------------|----------------------------------------|
|                |                     | \( S_{s_{min}} \) (mm) | \( S_{i_{min}} \) (mm) | \( B_{1_{min}} \) (mm) | \( B_{2_{min}} \) (mm) | \( C_{1_{min}} \) (mm) | \( C_{2_{min}} \) (mm) | \( \alpha_{f} \) | \( \beta_{o} \) |
| 30204          | EHHO                | 2.2030                     | 2.2030                     | 1.0400                     | 2.3337                     | 1.0339                     | 0.5                      | 8.7777                     | 2.3134                     |
|                | HHO                 | 2.2030                     | 3.0515                     | 1.2920                     | 2.4685                     | 1.4004                     | 0.5                      | 9.2694                     | 2.0416                     |
|                | WOA                 | 2.2030                     | 3.4253                     | 1.0686                     | 2.5396                     | 1.2424                     | 0.5                      | 9.5160                     | 1.9047                     |
|                | SCA                 | 2.3026                     | 2.7863                     | 1.2104                     | 2.8651                     | 1.2823                     | 0.9332                   | 9.1177                     | 1.2161                     |
|                | GA [2]              | 2.437                      | 3.665                      | 1.509                      | 3.603                      | 1.703                      | 1.520                    | 9.757                      | 1.693                      |
| 30205          | EHHO                | 2.3075                     | 2.3745                     | 1.0144                     | 2.3113                     | 1.0004                     | 0.5                      | 10.3736                    | 2.0283                     |
|                | HHO                 | 2.3075                     | 3.5346                     | 1.6393                     | 2.5202                     | 1.7984                     | 0.5                      | 10.9930                    | 1.6864                     |
|                | WOA                 | 2.3075                     | 2.5903                     | 1.0648                     | 2.3517                     | 1.0850                     | 0.5                      | 10.4966                    | 1.9603                     |
|                | SCA                 | 2.3536                     | 2.4301                     | 1.8304                     | 2.4816                     | 1.7893                     | 0.6843                   | 10.3245                    | 2.0569                     |
|                | GA [2]              | 2.352                      | 3.534                      | 1.475                      | 2.710                      | 1.642                      | 0.681                    | 10.79                      | 1.676                      |
| 32205          | EHHO                | 1.5837                     | 1.6465                     | 1.4182                     | 2.6248                     | 1.0000                     | 0.5                      | 16.0377                    | 2.8668                     |
|                | HHO                 | 1.5837                     | 2.3463                     | 1.5138                     | 2.8495                     | 1.2640                     | 0.5                      | 16.6748                    | 2.5172                     |
|                | WOA                 | 1.6002                     | 2.2189                     | 2.5027                     | 2.7563                     | 2.1172                     | 0.5423                   | 16.2961                    | 2.7283                     |
|                | SCA                 | 1.6829                     | 1.8344                     | 1.6553                     | 2.9283                     | 1.2598                     | 0.7550                   | 16.1557                    | 2.8034                     |
|                | GA [2]              | 1.766                      | 2.493                      | 2.192                      | 3.307                      | 1.901                      | 0.969                    | 16.71                      | 2.558                      |
| 322/28         | EHHO                | 1.9662                     | 1.9662                     | 1.6443                     | 2.4794                     | 1.0782                     | 0.5                      | 15.5429                    | 2.7660                     |
|                | HHO                 | 1.9870                     | 2.3064                     | 1.8310                     | 2.6277                     | 1.3302                     | 0.5554                   | 15.7802                    | 2.6362                     |
|                | WOA                 | 1.9662                     | 2.3211                     | 1.8718                     | 2.5717                     | 1.3707                     | 0.5                      | 15.7831                    | 2.6346                     |
|                | SCA                 | 1.9841                     | 2.2846                     | 2.3161                     | 2.5713                     | 1.7631                     | 0.5476                   | 16.5758                    | 2.7048                     |
|                | GA [2]              | 2.197                      | 2.627                      | 2.222                      | 3.272                      | 1.763                      | 1.116                    | 16.00                      | 2.541                      |
| 322/28         | EHHO                | 2.3840                     | 2.3840                     | 1.3077                     | 2.952969                   | 1.0039                     | 0.5                      | 20.1800                    | 2.1123                     |
|                | HHO                 | 2.3840                     | 2.9840                     | 2.9680                     | 3.013218                   | 2.6907                     | 0.5                      | 10.3759                    | 2.0271                     |
|                | WOA                 | 2.3840                     | 2.8116                     | 2.2291                     | 3.004961                   | 1.9558                     | 0.5                      | 10.3532                    | 2.0387                     |
|                | SCA                 | 2.3893                     | 2.9304                     | 2.2828                     | 3.047944                   | 2.0271                     | 0.5212                   | 10.4075                    | 2.0088                     |
|                | GA [2]              | 2.401                      | 4.154                      | 1.897                      | 3.358                      | 1.873                      | 0.568                    | 10.80                      | 1.654                      |
4.2.3 Update besiege strategy 4

In this section, to improve the performance of the HHO algorithm, a mutation vector extracted from the 2-Opt algorithm and differential evolution algorithm (DE) [99, 100] is replaced by hard besiege with a rapid progressive diving technique. This approach can be expressed as follows:

\[
X(t + 1) = \begin{cases} 
X_s(t) + F(X_p(t) - X_s(t)) & f(X_s(t)) < f(X_p(t)), \\
X_p(t) + F(X_s(t) - X_p(t)) & \text{otherwise}
\end{cases}
\]

where \( F \) is a parameter that strikes an equilibrium between the local and the global capacity of the enhanced version of the HHO algorithm. \( X_s(t), X_p(t), \) and \( X_r(t) \) are selected from the population. \( \beta^r \) is an arbitrary number between 0 and 1. This method aims to restrict the rising exploitation and prevents being trapped in local optima [60].

### Table 9 Optimum internal geometry obtained by different optimization algorithms

| Bearing number | Optimization method | Optimum parameters of internal geometry | Optimum parameters of internal geometry |
|----------------|---------------------|----------------------------------------|----------------------------------------|
|                | \( S_{s_{\min}}^- \) (mm) | \( S_{s_{\min}}^+ \) (mm) | \( B_{s_{\min}}^- \) (mm) | \( B_{s_{\min}}^+ \) (mm) | \( C_{s_{\min}}^- \) (mm) | \( C_{s_{\min}}^+ \) (mm) | \( \alpha_f \) | \( \beta^o \) |
| 30207          | EHHO                | 3.0310                                 | 3.7907                                 | 2.0602                                 | 2.0602                                 | 1.5009                                 | 0.8121                                 | 10.3803                                 | 2.0283                                 |
|                | HHO                 | 3.1125                                 | 3.3767                                 | 2.3059                                 | 2.3059                                 | 1.6669                                 | 1.1380                                 | 10.1922                                 | 2.1331                                 |
|                | WOA                 | 3.0625                                 | 6.0366                                 | 2.6099                                 | 2.6099                                 | 2.4212                                 | 0.9380                                 | 11.2649                                 | 1.5385                                 |
|                | SCA                 | 3.1576                                 | 3.4895                                 | 2.4410                                 | 2.5059                                 | 1.8146                                 | 1.3186                                 | 10.2299                                 | 2.1126                                 |
|                | GA [2]              | 3.000                                  | 5.955                                  | 1.863                                  | 2.370                                  | 1.695                                  | 0.688                                  | 10.90                                   | 1.524                                  |
| 30306          | EHHO                | 3.5805                                 | 4.6056                                 | 1.4645                                 | 3.2652                                 | 1.5018                                 | 0.4                                    | 8.2577                                  | 1.9991                                 |
|                | HHO                 | 3.5805                                 | 5.2806                                 | 1.7469                                 | 3.3612                                 | 1.8673                                 | 0.4                                    | 8.4846                                  | 1.8735                                 |
|                | WOA                 | 3.6121                                 | 5.2185                                 | 2.3656                                 | 3.4910                                 | 2.4582                                 | 0.5508                                 | 8.4326                                  | 1.9031                                 |
|                | SCA                 | 3.5919                                 | 4.8920                                 | 2.4862                                 | 3.3418                                 | 2.5307                                 | 0.4543                                 | 8.3110                                  | 1.9706                                 |
|                | GA [2]              | 3.643                                  | 7.219                                  | 1.175                                  | 3.988                                  | 1.595                                  | 0.698                                  | 8.797                                   | 1.474                                  |
| 32207          | EHHO                | 2.8315                                 | 3.1627                                 | 2.0059                                 | 3.7359                                 | 1.5000                                 | 0.5                                    | 10.3718                                 | 2.0283                                 |
|                | HHO                 | 2.8315                                 | 5.2111                                 | 2.2368                                 | 4.1453                                 | 2.0808                                 | 0.5                                    | 11.2268                                 | 1.5556                                 |
|                | WOA                 | 2.8315                                 | 2.9474                                 | 3.3453                                 | 3.6495                                 | 2.7443                                 | 0.5                                    | 10.1836                                 | 2.1339                                 |
|                | SCA                 | 2.8392                                 | 3.2167                                 | 2.7584                                 | 3.7521                                 | 2.2280                                 | 0.5309                                 | 10.3400                                 | 2.0467                                 |
|                | GA [2]              | 2.962                                  | 4.331                                  | 2.787                                  | 4.471                                  | 2.447                                  | 1.023                                  | 10.69                                   | 1.786                                  |
| 30307          | EHHO                | 3.5035                                 | 4.5974                                 | 1.6982                                 | 3.4281                                 | 1.5178                                 | 0.8                                    | 8.2579                                  | 1.9992                                 |
|                | HHO                 | 3.5035                                 | 6.5665                                 | 3.6045                                 | 3.6882                                 | 3.6380                                 | 0.8                                    | 8.7935                                  | 1.7035                                 |
|                | WOA                 | 3.5254                                 | 6.9000                                 | 2.6154                                 | 3.8705                                 | 2.7292                                 | 0.9043                                 | 8.9430                                  | 1.6198                                 |
|                | SCA                 | 3.6216                                 | 3.6367                                 | 2.0072                                 | 3.8521                                 | 1.6919                                 | 1.3625                                 | 7.9418                                  | 2.1752                                 |
|                | GA [2]              | 3.556                                  | 7.325                                  | 2.069                                  | 4.101                                  | 2.264                                  | 1.050                                  | 8.739                                   | 1.531                                  |
| 32208          | EHHO                | 3.0175                                 | 3.3879                                 | 1.7375                                 | 4.0043                                 | 1.5                                  | 0.5                                    | 10.3753                                 | 2.0283                                 |
|                | HHO                 | 3.2353                                 | 4.0084                                 | 2.5722                                 | 4.9797                                 | 2.4025                                 | 1.3710                                 | 10.5577                                 | 1.9299                                 |
|                | WOA                 | 3.0175                                 | 4.0227                                 | 2.6850                                 | 4.0948                                 | 2.5127                                 | 0.5                                    | 10.5523                                 | 1.9312                                 |
|                | SCA                 | 3.1223                                 | 3.2859                                 | 2.6658                                 | 4.3794                                 | 2.3700                                 | 0.9190                                 | 10.2781                                 | 2.0834                                 |
|                | GA [2]              | 3.348                                  | 4.611                                  | 2.685                                  | 5.554                                  | 2.615                                  | 1.825                                  | 10.70                                   | 1.809                                  |

4.2.4 Gaussian mutation

Another approach employed in this study to enhance the diversity of the solution is the Gaussian mutation. This strategy has been adopted in a wide range of metaheuristic algorithms to create diversity in solutions [97, 101]. The main purpose of the Gaussian mutation is to boost the global search. Gaussian mutation makes a slight arbitrary change in the group of search agents to prevent trapping in the local optima, leading to more exploitation ability and better convergence. Considering the average of 0 and the standard deviation of 1, a random variable is formed. The Gaussian density function can be formulated as follows [97]:

\[
\mathcal{f}_{\text{gaussian}}(\alpha) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\alpha^2}{2\sigma^2}},
\]

where \( \sigma^2 \) indicates the variance, and \( \alpha \) represents an arbitrary Gaussian value between [0,1]. The standard deviation is considered to be equal to 1. The Gaussian function with various standard deviation rates is demonstrated in Fig. 6. In the proposed algorithm, firstly, the population is updated according
to the energy escaping factor and besiege methods; then, a new population of Harris hawks is created based on Eq. (62):

$$X'(t) = X(t) * (1 + G(a)),$$

(62)

where $X'(t)$ denotes the location of the new population in each iteration. $G(a)$ represents the Gaussian step vector according to the Gaussian density presented in Eq. (61).

### 4.2.5 CLS with shrinking mode

Local search (LS) is one of the most efficient strategies for preventing algorithms from being trapped in local optima. It is essential to scour the final solution’s vicinity since, most of the time, the solution is in the neighbourhood of local optima, and the algorithm cannot detect it. Consequently, adding LS to the main HHO algorithm can significantly enhance the performance of the algorithm. Note that, sometimes, LS is not sufficient and would not lead to desirable results. Thus, a chaotic local search (CLS) can be utilized. Since there is randomicity in chaos, CLS can lead to immature convergence [97]. The CLS strategy is added in the last step of the algorithm to detect the best solution. The CLS approach can be formulated as follows:

$$X_k^{(c)} = (1 - \lambda)X^* + \lambda(LB + \beta_k(UB - LB)),$$

$$\lambda = 1 - \left| \frac{t - 1}{m} \right|,$$

(63)

where $X_k^{(c)}$ is the $k$th new location created by CLS, $X^*$ shows the best rabbit found so far, $\beta_k$ is the signal generated in the $k$th chaos, and $LB$ and $UB$ represent the lower and upper limits of the search space, respectively. $\lambda$ denotes the shrinking scale factor, and $t$ is the current iteration. $m$ is used to handle the shrinking rate and is equal to 1500. Figure 7 depicts

---

**Fig. 8** The convergence curve for different bearing numbers **a** 30204, **b** 30205, **c** 32205, and **d** 322/28
the flowchart of the enhanced version of the Harris hawk algorithm (EHHO).

5 Results and discussions

In this section of the study, first, the simulated experiment’s outcome is presented, and the manner every element of the proposed algorithm can affect the promotion of the final result is examined in detail. Also, the results are compared with the value from the existing catalog. For checking the proposed algorithm’s capability, some of the famous engineering problems are chosen to be examined under the proposed algorithm.

5.1 Optimum design of TRB

For performing the optimization process, every variable’s limit should be identified in the first calculation stage not to exceed the reasonable number. Maximum and minimum values of each variable used in the design of TRB are computed based on Constraints 1–8 (Table 5). The reported limitation of variables in Table 5 is used as the boundaries of the design variables in optimization algorithms, which include enhanced Harris hawk (EHHO), standard Harris hawk (HHO), whale algorithm (WOA), and sine cosine algorithm (SCA). Different algorithms, including the best solution with their optimum variables, are summarised in Tables 6, 7 for ten different tapered roller bearings. In Tables 6, 7, results are ranked based on the value of dynamic load capacity. It is seen from the results that the introduced enhanced Harris hawk optimization algorithm has superior capability in finding the maximum dynamic load capacity as the optimum outcome and ranks first among the other algorithms. After the enhanced Harris hawk algorithm, the standard Harris hawk has better convergence toward the maximum solution among the rest of the algorithms. It is noteworthy that all values of dynamic load capacity are significantly improved, among which bearing number 30205 has higher improvement (29.4%), and bearing number 32208 has the lowest improvement (16.6%).

Moreover, in the results obtained by the enhanced Harris hawk optimization algorithm, it can be seen that although some bearing cases have lower roller numbers, they show better efficiency in terms of fatigue life. Therefore, having a higher number of rollers does not guarantee better fatigue life. Some parameters related to internal geometry which are obtained during the optimization process are summarized in Tables 8, 9.

The convergence curve of the optimization algorithms shows how to obtain the final result (Fig. 8). Better performance of EHHO than other algorithms is seen obviously. One of the main reasons for this high-quality result is to
have a mature population at the beginning of the optimization process, which is attributed to the chaotic initialization. Another reason for the improved efficiency is due to using the flower pollination and 2-Opt algorithms. Using two other techniques, Gaussian mutation and chaotic local search, can facilitate the optimization process for finding the optimum solution at the end of the algorithm. Local optimum is a significant obstacle experienced by most algorithms; having an efficient structure for avoiding local optima is one of the advantages of suitable algorithms. Using a chaotic local search technique can assist the algorithm in preventing falls in the local optima. It is noteworthy that the structure of SCA is not appropriate for this TRB problem and has minimum convergence result compared to the other algorithms.

To see the proficiency of the algorithms in more detail, a statistical analysis is performed for some cases of tapered roller bearings. Table 10 shows the result of this analysis in terms of standard deviation (STD), worst solution, and best mean. The result of statistical analysis demonstrates that the EHHO has the lowest STD among the other algorithms. The lowest STD demonstrates that EHHO can easily find the maximum solution in every run of the optimization process and proves the robustness of this algorithm. Also, the best, mean, and worst solutions belong to the EHHO algorithm.

After the EHHO algorithm, the SCA ranks second in terms of the lowest STD; however, SCA obtains the lowest best result.

**LL** lower limit, **UL** upper limit

Figure 9 compares the design variables from the enhanced Harris hawk algorithm (EHHO), genetic algorithm (GA), and the standard catalogue. The graph in Fig. 9 shows that the EHHO algorithm successfully maximizes the fatigue.
life of bearing with a lower number of rollers. Also, Fig. 9 shows that the EHHO algorithm uses a higher mean diameter value and has sufficient length compared to GA and bearing standard.

Stability evaluation is one of the vital steps of performance verification [102, 103]. For quality evaluation metrics [104], we considered the average of solutions as the primary metric to judge the accuracy of the performance. We fixed fair judgments as per references [24, 85, 105, 106]. For having an exhaustive vision of the proposed algorithm, different algorithms are created with separate techniques used in EHHO. These algorithms are created based on techniques including chaotic initialization, Gaussian mutation, differential evaluation, pollination algorithm, and chaotic local search mentioned in the optimization methodology section. The optimization process is performed for these algorithms in 500 iterations, and the results are summarized in Table 11. The convergence curves are shown in Fig. 10, in which using strategies of chaotic local search, chaotic initialization, and Gaussian mutation with standard HHO does not affect the accuracy significantly. By replacing the pollination algorithm and differential evolution formulation with the hunting strategies of the Harris hawk algorithm, the results improve significantly. The EHHO, which is a combination of all the mentioned techniques, has the best results compared to the other ten algorithms in terms of precision, and its performance is not trapped to local optima. The STD result of EHHO also illustrates the performance of every element of this algorithm.

### 5.2 Engineering problems

There are many problems that their feature space is more complex than the assessed benchmark spaces [107–111]. Despite benchmark cases, engineering problems always involve some variables that are constrained [28, 112–115]. For testing and benchmarking the proposed algorithm, some popular and perplexing engineering functions are common in the literature. These engineering functions can challenge every optimization algorithm with their complexity in their structure. Most of these problems have more than three variables and constraints with many local optima. In this part of the paper, five well-known and challenging engineering problems are evaluated by the EHHO algorithm to examine the effectiveness and capability of this algorithm. The aspects of the engineering problems are described in the next subsections.
Fig. 11 Cantilever beam design

Table 12 Comparison results for cantilever beam design

| Optimization method | Optimum variables | Optimum cost | Weight |
|---------------------|-------------------|---------------|--------|
|                     | $h_1$, $h_2$, $h_3$, $h_4$, $h_5$ |               |        |
| EHHO                | 6.0143, 5.3029, 4.4964, 3.5053, 2.1548 | **1.33995825** |        |
| HHO                 | 6.1016, 5.343, 4.4237, 3.4533, 2.1582 | 1.3403595 |        |
| MFA [116]           | 5.98487, 5.3167269, 4.49733, 3.5136165, 2.161620 | 1.3399881 |        |
| CS [117]            | 6.0089, 5.3049, 4.5023, 3.3077, 2.1304 | 1.33999 |        |
| SOS [118]           | 6.01878, 5.30344, 4.49587, 3.49896, 2.15564 | 1.33996 |        |

Bold value indicates the best results.

Table 13 Comparison results for pressure vessel design

| Optimization method | Optimum variables | Optimum cost | Fabrication cost |
|---------------------|-------------------|--------------|------------------|
|                     | $T_s$, $T_h$, $r$, $L$ |               |                  |
| EHHO                | 0.77817, 0.38465, 40.3196, 200 | **5885.36355** |                |
| HHO [50]            | 0.81758, 0.40729, 42.09174, 176.75873 | 6000.46259 |                |
| CMVHHO [120]        | 0.849756, 0.421472, 43.900722, 155.517156 | 6039.6918 |                |
| ADHPO [121]         | 0.87015, 0.43114, 45.01254, 143.5317 | 6072.56 |                |
| CCMWOA [97]         | 0.77966, 0.38561, 40.34738, 199.6141 | 5895.2039 |                |
| WOA [88]            | 0.81250, 0.43750, 42.0892, 176.6389 | 6059.7410 |                |
| VPLSCA [122]        | 0.8152, 0.4265, 42.085, 176.73154 | 6042.71935 |                |
| UBSCIW [123]        | 0.7798, 0.3866, 40.3844, 199.0685 | 5889.2305 |                |
| ESSA [124]          | 0.781463, 0.386278, 40.4903, 197.63744 | 5890.9885 |                |

Bold value indicates the best results.

Table 14 Results for tension/compression spring design

| Optimization method | Optimum variables | Optimum cost | Weight |
|---------------------|-------------------|--------------|--------|
|                     | $d$, $D$, $N$ |               |        |
| EHHO                | 0.0516751748, 0.356383766, 11.30857249 | **0.012665236** |        |
| HHO                 | 0.05179, 0.3593, 11.13885 | 0.012665443 |        |
| MHHO [125]          | 0.051654, 0.355881, 11.33883 | 0.01266619 |        |
| CCMWOA [97]         | 0.051843, 0.360444, 11.07410 | 0.0126660 |        |
| WOA [88]            | 0.051207, 0.345215, 12.0043032 | 0.0126763 |        |
| ESSA [124]          | 0.051719, 0.357434, 11.247123 | 0.0126653 |        |
| GCHHO [59]          | 0.0516479, 0.355729, 11.3471231 | 0.012665264 |        |

Bold value indicates the best results.
5.2.1 Cantilever beam

Figure 11 shows the structure of this benchmarked problem. This beam is exposed to the load at the right end. Five variables of beam design contain the vertical length of the connected boxes. The range of the variables is from 0 to 100.

The optimization scheme of this problem is given in Eq. (64) below:

\[
\begin{aligned}
\vec{x} &= [h_1, h_2, h_3, h_4, h_5], \\
\text{Minimize} : \quad f(\vec{x}) &= 0.0624(x_1 + x_2 + x_3 + x_4 + x_5), \\
\text{Subject to} : \quad G(\vec{x}) &= \frac{61}{x_1^2} + \frac{37}{x_2^2} + \frac{19}{x_3^2} + \frac{7}{x_4^5} + \frac{1}{x_5} - 1 \leq 0, \\
0 &\leq x_i \leq 100.
\end{aligned}
\]

(64)

The optimum solution of beam design is reported in Table 12. Moreover, Table 12 contains the optimum design results of other algorithms such as the modified firefly algorithm [116], crow search (CS) [117], and symbiotic organisms search (SOS) [118], and the standard Harris Hawk on the beam structure. The EHHO found 1.33995825 as the optimal solution of weight for the cantilever beam design, indicating the efficiency of this algorithm compared to the modified firefly algorithm, crow search algorithm, and symbiotic organisms algorithm. This case may be handy for building structures and how engineers can deal with a component [119].

5.2.2 Pressure vessel

The cost of construction of every piece of equipment is substantial for manufacturers, and the minimization of expenses can be

Table 15 Results for welded beam design

| Optimization method | Optimum variables | Optimum cost | Fabrication cost |
|---------------------|------------------|--------------|------------------|
|                     | h    | l    | t    | b    |             |               |
| EHHO                | 0.2057003 | 3.4711372 | 9.03668181 | 0.20572935 | **1.72490231** |
| HHO                 | 0.204039  | 3.531061  | 9.027463   | 0.206147   | 1.73199057   |
| CMVHHO [120]        | 0.205331  | 3.4787    | 9.039544   | 0.205723   | 1.726023     |
| WOA [88]            | 0.205396  | 3.484293  | 9.037426   | 0.206276   | 1.730499     |

Table 16 Results for speed reducer design

| Optimization method | Optimum variables | Optimum cost | Weight |
|---------------------|------------------|--------------|--------|
|                     | x_1  | x_2  | x_3  | x_4  | x_5  | x_6  | x_7  |                |
| EHHO                | 3.5   | 0.7   | 17   | 7.3  | 7.3  | 3.352023 | 5.2866 | **2994.4710** |
| HHO                 | 3.50253 | 0.7   | 17   | 7.3  | 7.3  | 3.3538   | 5.2867 | 3000.9479     |
| m-HHO [62]          | 3.5   | 0.7   | 17   | 7.3  | 7.3  | 3.352023 | 5.2866 | 2996.6162     |
| GLF-GWO [126]       | 3.5000091 | 0.7   | 17   | 7.3  | 7.3  | 3.3502335 | 5.2866856 | 2996.3680    |

Bold value indicates the best results.
formulated as an interesting problem. The variables' vector of the pressure vessel problem contains some critical parameters such as the thickness of the head and shell \((T_r\) and \(T_h\), respectively) and cylindrical curvature radius \((r)\) and distance \((L)\). The limitation of the first and second variables is between 0 and 99, and the criteria for the third and fourth variables are between 10 and 200. The layout for this structure is as follows:

\[
\vec{x} = [x_1, x_2, x_3, x_4] = [T_r, T_h, R, L],
\]

Minimize : \( f(\vec{x}) = 0.6224x_1x_2x_3 + 1.7781x_1^2x_4 + 3.1661x_4^2 + 19.84x_2x_1^2 \),

Subject to : \( h_1(\vec{x}) = -x_1 + 0.0193x_3 \leq 0, \)
\( h_2(\vec{x}) = -x_2 + 0.00954x_3 \leq 0, \)
\( h_3(\vec{x}) = -\pi x_4x_3^2 - \frac{4}{3}\pi x_3^3 + 1296000 \leq 0, \)
\( h_4(\vec{x}) = x_4 - 240 \leq 0, \)
\( 0 \leq x_1 \leq 99, 0 \leq x_2 \leq 99, 10 \leq x_3 \leq 200, 10 \leq x_4 \leq 200. \)

(65)

The result for this problem is presented in Table 13. It is obvious that the fabrication cost of the pressure vessel design related to EHHO is better than that of other algorithms mentioned in Table 13. Note that EHHO has improved the optimal solution by 2% compared to the standard HHO.

5.2.3 Spring geometry

The objective function of this design problem is to minimize the weight of a tension/compression spring. Shear stress and deflection influence the design of the spring, which can be related to constraints of spring. Three variables of coil number \((N)\), cord diameter \((d)\), and mean diameter \((D)\) are chosen in the design vector to provide the lowest weight for the spring. The range for \((d)\) is 0.05–2, for \((D)\) is 0.25–1.3, and for the last variable \((N)\) is 2–15. The spring formulation is derived as below:

\[
\vec{x} = [x_1, x_2, x_3] = [d, D, N],
\]

Minimize : \( f(\vec{x}) = x_1^3x_2x_3 + 2x_1^2x_2, \)

Subject to : \( h_1(\vec{x}) = 1 - \frac{x_2x_3}{71785x_1^4} \leq 0, \)
\( h_2(\vec{x}) = \frac{x_3^3x_3}{12566(x_2x_3^3 - x_1^4)} + \frac{1}{5108x_1^4} - 1 \leq 0, \)
\( h_3(\vec{x}) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0, \)
\( h_4(\vec{x}) = \frac{x_1 + x_2}{1.5} - 1 \leq 0, \)
\( 0.05 \leq x_1 \leq 2, 0.25 \leq x_2 \leq 1.3, 2 \leq x_3 \leq 15. \)

(66)

Table 14 summarizes the value obtained from the optimization process. The EHHO diminishes the weight to an accuracy of 0.01266 kg compared to the other algorithms such as modified whale algorithm (CCMWOA) [97], enhanced salp swarm (ESSA) [124], whale algorithm (WOA) [88], and modified HHO (GCHHO) [59].

5.2.4 Welded beam

There are 4 design variables for the welded beam design optimization problem (thickness, \(b\), height, \(t\), length of the bar, \(l\), with weld \(h\)). There are 7 constraints, and most of them relate to load, stresses on the bar, and end deflection on the spring. The structural formulation, range of variables, and some constant parameters are provided by Eq. (67):

\[
\vec{x} = [x_1, x_2, x_3, x_4] = [h, l, t, b],
\]

Minimize : \( f(\vec{x}) = 1.10471x_2x_3^2 + 0.04811x_3x_4(14 + x_2), \)

Subject to : \( h_1(\vec{x}) = \tau(\vec{x}) - \tau_{\text{max}} \leq 0, \)
\( h_2(\vec{x}) = \sigma(\vec{x}) - \sigma_{\text{max}} \leq 0, \)
\( h_3(\vec{x}) = \delta(\vec{x}) - \delta_{\text{max}} \leq 0, \)
\( h_4(\vec{x}) = x_1 - x_4 \leq 0, \)
\( h_5(\vec{x}) = P - P_0(\vec{x}) \leq 0, \)
\( h_6(\vec{x}) = 0.125 - x_1 \leq 0, \)
\( h_7(\vec{x}) = 0.10471x_2^2 + 0.04811x_3x_4(14 + x_2) - 5 \leq 0, \)
\( 0.1 \leq x_1 \leq 2, 0.1 \leq x_2 \leq 10, 0.1 \leq x_3 \leq 10, 0.1 \leq x_4 \leq 2, \)

where
\[
\tau(\bar{x}) = \sqrt{\left(\tau'\right)^2 + 2\tau' \tau - \frac{2x^2}{R}} + \left(\tau'\right)^2,
\]
\[
\bar{\tau} = \frac{P}{\sqrt{2x_1x_2}}, \quad \tau = \frac{MRJ}{M} = P\left(\frac{x_2}{2}\right),
\]
\[
R = \left(\frac{x_3}{4} + \left(\frac{x_1 + x_1}{2}\right)^2\right)^2, \quad J = 2\left\{\sqrt{2x_1x_2}\left(\frac{x_3^2}{12} + \left(\frac{x_1 + x_1}{2}\right)^2\right)^2\right\}
\]
\[
\sigma(\bar{x}) = \frac{6PL}{x_4x_5}, \quad \delta(\bar{x}) = \frac{4PL^3}{E\times x_4x_5}, \quad P(\bar{x}) = \frac{4.013E\sqrt{\frac{x_3^4}{36}}}{\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)}
\]
\[
P = 6000\text{lb}, L = 14\text{in}, \delta_{max} = 0.25\text{in}, E = 30 \times 10^6\text{psi}, G = 12 \times 10^9\text{psi}, \tau_{max} 13600\text{psi}, \sigma_{max} = 30000\text{psi}.
\]

The results in Table 15 show the excellence of the proposed algorithm.

5.2.5 Speed reducer

The scheme of the speed reducer is depicted in Fig. 12. The speed reducer shaft is exposed to stress and transverse deflection, and the gear teeth tolerate stresses such as bending stress, which can be considered constraints. The optimization problem for the speed reducer problem has seven variables and 11 nonlinear constraints. The variables are explained in Fig. 12. The formulation for the optimization design of the speed reducer can be expressed in Eq. (68):

\[
\bar{x} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7],
\]

Min : \( f(\bar{x}) = 0.7854x_1x_2^2 (3.3333x_3^2 + 14.9334x_3 - 43.0934)
\]
\[- 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_6x_6^2 + x_7x_7^2),
\]

Subject to : \( h_1(\bar{x}) = \frac{27}{x_1x_2x_3} - 1 \leq 0, \)
\[ h_2(\bar{x}) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \leq 0, \quad h_3(\bar{x}) = \frac{1.93x_3^3}{x_2x_4^4x_5} - 1 \leq 0, \]
\[ h_6(\bar{x}) = \frac{1.1x_7 + 1.9}{x_3} - 1 \leq 0, h_7(\bar{x}) = \left[\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5 \times 10^6\right]^{1/2} \]
\[ \frac{85x_7}{x_5} \right) - 1 \leq 0, \]
\[ h_8(\bar{x}) = \frac{x_5x_4}{40} - 1 \leq 0, \quad h_9(\bar{x}) = \frac{5x_2}{x_1} - 1 \leq 0 \]
\[ h_{10}(\bar{x}) = \frac{x_4}{12x_2} - 1 \leq 0, \quad h_{11}(\bar{x}) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0, \]
\[ 2.6 \leq x_1 \leq 3.6, \quad 0.7 \leq x_2 \leq 0.8, \quad 17 \leq x_3 \leq 28, \quad 7.3 \leq x_4 \leq 8.3
\]
\[ 7.3 \leq x_5 \leq 8.3, \quad 2.9 \leq x_6 \leq 3.9, \quad 5 \leq x_7 \leq 5.5
\]

6 Conclusions and future works

In this paper, a novel algorithm is proposed to enhance the performance of the Harris hawk optimization algorithm (HHO) based on new features, which improve the exploration and exploitation phases of the original HHO algorithm. At first, chaotic initialization is used to explore the search area extensively to cover and generate all possible solutions equally. In this way, a mature population is created. After that, two of the Harris hawk pouncing strategies are changed to generate more appropriate agents for updating the population. Also, the Gaussian strategy makes the update of the population boosted. At the end of the proposed algorithm, a chaotic local search with the shrinking mode is exploited to avoid possible local optima. The proposed algorithm is tested on tapered roller bearings successfully. The objective function is related to the maximization of the fatigue life of TRB. It contains nine variables and 26 constraints. The
results show that the best result can be obtained by the application of the enhanced Harris hawk algorithm (EHHO) on the design optimization of the fatigue life of TRB. In addition, the mentioned algorithm is tested on some common engineering problems in the literature. Similarly, the optimization results are improved using the EHHO algorithm compared to the algorithms from the literature. Therefore, the proposed algorithm can be used in complex engineering problems where there are many design variables.

Acknowledgements This paper results from the MSc thesis of the first name that defended his thesis successfully within the revision of this research. We acknowledge the supports of Ozyegin University. We also acknowledge reviewers’ comments and the editor’s efforts, which significantly enhanced this research’s excellence.

References

1. Jat A, Tiwari R (2020) Multi-objective optimization of spherical roller bearings based on fatigue and wear using evolutionary algorithm. J King Saud Univ-Eng Sci 32(1):58–68
2. Tiwari R, Sunil KK, Reddy R (2012) An optimal design methodology of tapered roller bearings using genetic algorithms. Int J Comput Methods Eng Sci Mech 13(2):108–127
3. Senthil Kumar S, Srinivasan K (2020) A review on life increment of tapered roller bearings. J Crit Rev 7(6):764–775
4. Bhownick H, Choudhary RTG (2006) Quasi-static analysis of tapered roller bearings and comparison of bearing lives for different roller surface profiles. In: 2nd international congress on computational mechanics and simulation, 2006
5. Hu Y et al (2021) Corrosion fatigue lifetime assessment of high-speed railway axle EA4T steel with artificial scratch. Eng Fract Mech 245:107588
6. Tiwari R, Chandran R (2013) Thermal based optimum design of tapered roller bearing through evolutionary Algorithm. In: Gas turbine India conference, vol 35161. American Society of Mechanical Engineers, p V001T05A021
7. Kumar KS, Tiwari R, Prasad P (2009) An optimum design of crowned cylindrical roller bearings using genetic algorithms. J Mech Des. https://doi.org/10.1115/1.3116344
8. Verma SA, Tiwari R (2020) Robust optimum design of tapered roller bearings based on maximization of fatigue life using evolutionary algorithm. Mach Mech Theory 152:103894
9. Kalyan M, Tiwari R, Ahmad MS (2020) Multi-objective optimization in geometric design of tapered roller bearings based on fatigue, wear and thermal considerations through genetic algorithms. Sadhana. https://doi.org/10.1007/s12046-020-01385-3
10. Choi D-H, Yoon K-C (2001) A design method of an automotive wheel-bearing unit with discrete design variables using genetic algorithms. J Trib 123(1):181–187
11. Chakraborty I et al (2003) Rolling element bearing design through genetic algorithms. Eng Optimiz 35(6):649–659
12. Dandagwahal R, Kalyankar V (2019) Design optimization of rolling element bearings using advanced optimization technique. Arab J Sci Eng 44(9):7407–7422
13. Panda S et al (2018) Re-examination for effect of ball race conformity on life of rolling element bearing using Metaheuristic. Int J Adv Mech Eng 8(1):285–294
14. Kang K et al (2019) Robust design optimization of an angular contact ball bearing under manufacturing tolerance. Struct Multi-Discip Optim 60(4):1645–1665
15. Tiwari R, Waghole V (2015) Optimization of spherical roller bearing design using artificial bee colony algorithm and grid search method. Int J Comput Methods Eng Sci Mech 16(4):221–233
16. Zhou Y et al (2019) Video coding optimization for virtual reality 360-degree source. IEEE J Select Topics Signal Process 14(1):118–129
17. Wu C et al (2019) Differential received signal strength based RFID positioning for construction equipment tracking. Adv Eng Inf 42:100960
18. Xue X et al (2020) Affine transformation-enhanced multifactorial optimization for heterogeneous problems. IEEE Trans Cybernet. https://doi.org/10.1109/TCYB.2020.3036393
19. Ding L et al (2020) Definition and application of variable resistance coefficient for wheeled mobile robots on deformable terrain. IEEE Trans Rob 36(3):894–909
20. Wu C et al (2020) Critical review of data-driven decision-making in bridge operation and maintenance. Struct Infrastructure Eng. https://doi.org/10.1080/15732479.2020.1833946
21. Jiang Q et al (2017) Optimizing multistage discriminative dictionaries for blind image quality assessment. IEEE Trans Multimedia 20(8):2035–2048
22. Wang B et al (2021) A kind of improved quantum key distribution scheme. Optik 235:166628
23. Yang Y et al (2015) New pore space characterization method of shale matrix formation by considering organic and inorganic pores. J Nat Gas Sci Eng 27:496–503
24. Bo W et al (2021) Malicious URLs detection based on a novel optimization algorithm. IEICE Trans Inf Syst 104(4):513–516
25. Alam Z et al (2021) Experimental and numerical investigation on the complex behaviour of the localised seismic response in a multi-storey plan-symmetric structure. Struct Infrastructure Eng 17(1):86–102
26. Zuo X et al (2020) The modeling of the electric heating and cooling system of the integrated energy system in the coastal area. J Coast Res 103(S1):1022–1029
27. Zhu D et al (2019) Evaluating the vulnerability of integrated electricity-heat-gas systems based on the high-dimensional random matrix theory. CSEE J Power Energy Syst 6(4):878–889
28. Zhang Y et al (2017) Analysis of grinding mechanics and improved predictive force model based on material-removal and plastic-stacking mechanisms. Int J Mach Tools Manuf 122:81–97
29. Yin F et al (2021) Multifidelity genetic transfer: an efficient framework for production optimization. SPE J, https://doi.org/10.2118/205013-PA
30. Eshtay M, Faris H, Ala’M AZ, Aljarah I (2021) AutoRWN: automatic construction and training of random weight networks using competitive swarm of agents. Neural Comput Appl 33(11):5507–5524
31. Faris H et al (2019) An intelligent system for spam detection and identification of the most relevant features based on evolutionary Random Weight Networks. Inf Fus 48:67–83
32. Faris H et al (2019) Time-varying hierarchical chains of salps with random weight networks for feature selection. Expert Syst Appl 140:112898
33. Lin A et al (2019) Predicting intentions of students for master programs using a chaos-induced sine cosine-based fuzzy k-Near-est neighbor classifier. IEEE Access 7:67235–67248
34. Liu G et al (2020) Predicting cervical hyperextension injury: kernel extreme learning machines with orthogonal learning butterfly optimizer and broyden—Fletcher-Goldfarb-Shanno Algorithms. IEEE Access 8:119991–119930
35. Liu G et al (2020) Predicting cervical hyperextension injury: a covariance guided sine cosine support vector machine. IEEE access 8:46895–46908
36. Aljarah I et al (2020) Multi-verse optimizer: theory, literature review, and application in data clustering. In: Mirjalili S, Song-Dong J, Lewis A (eds) Nature-inspired optimizers: theories, literature reviews and applications. Springer International Publishing, Cham, pp 123–141
37. Bai B et al (2021) Application of adaptive reliability importance sampling-based extended domain PSO on single mode failure in reliability engineering. Inf Sci 546:42–59
38. Ma X, Zhang K, Zhang L, Yao C, Yao J, Wang H et al (2021) Data-driven niching differential evolution with adaptive parameters control for history matching and uncertainty quantification. SPE J 26(02):993–1010
39. Sun G, Li C, Deng L (2021) An adaptive regeneration framework based on search space adjustment for differential evolution. Neural Comput Appl. https://doi.org/10.1007/s00521-021-05708-1
40. Zhao D et al (2020) Chaotic random spare ant colony optimization for multi-threshold image segmentation of 2D Kapur entropy. Knowl-Based Syst. https://doi.org/10.1016/j.knosys.2020.106510
41. Hu J et al (2021) Orthogonal learning covariance matrix for defects of grey wolf optimizer: insights, balance, diversity, and feature selection. Knowl-Based Syst 213:106684
42. Shan W et al (2020) Double adaptive weights for stabilization of moth flame optimizer: balance analysis, engineering cases, and medical diagnosis. Know-Based Syst 214:106728
43. Li S et al (2020) Slime mould algorithm: a new method for stochastic optimization. Futur Gener Comput Syst 111:300–323
44. Yang Y et al (2021) Hunger games search: visions, conception, implementation, deep analysis, perspectives, and towards performance shifts. Expert Syst Appl 177:114864
45. Ahmadifar I, Bozorg-Haddad O, Chu X (2020) Gradient-based optimizer: a new Metaheuristic optimization algorithm. Inf Sci 540:131–159
46. Ahmadifar I et al (2021) RUN beyond the metaphor: an efficient optimization algorithm based on Runge Kutta method. Expert Syst Appl 181:115079
47. Mafarja M et al (2020) Dragonfly algorithm: theory, literature review, and application in feature selection. Nature-Inspired Optimizers. Springer, pp 47–67
48. Aljarah I et al (2020) Multi-verse optimizer: theory, literature review, and application in data clustering. Nat-Inspired Optimiz. https://doi.org/10.1007/978-3-030-12127-3_8
49. Heidari AA, Abbaspour RA, Chen H (2019) Efficient boosted grey wolf optimizers for global search and kernel extreme learning machine training. Appl Soft Comput 81:105521
50. Heidari AA et al (2019) Harris hawks optimization for parameters identification of single-diode solar cell models. Energy Convers Manag 209:112660
51. Barshandeh S, Piri F, Sangani SR (2020) HMPA: an innovative hybrid multi-population algorithm based on artificial ecosystem-based and Harris Hawks optimization algorithms for engineering problems. Eng Comput. https://doi.org/10.1007/s00366-020-01120-w
52. Gupta S et al (2020) Opposition-based learning Harris hawks optimization with advanced transition rules: principles and analysis. Expert Syst Appl 158:113510
53. Shi B et al (2020) An improved Harris’s hawks optimization for SAR target recognition and stock market index prediction. IEEE Access 8:65891–65910
54. Abdel-Basset M, Ding W, El-Shahat D (2020) A hybrid Harris Hawks optimization algorithm with simulated annealing for feature selection. Artif Intell Rev. https://doi.org/10.1007/s10462-020-09860-3
55. Shi B et al (2020) Predicting di-2-ethylhexyl phthalate toxicity: hybrid integrated harris hawks optimization with support vector machines. IEEE Access 8:161188–161202
56. Wei Y et al (2020) Predicting entrepreneurial intention of students: an extreme learning machine with Gaussian Barebone Harris hawks optimizer. IEEE Access 8:76841–76855
57. Chen H et al (2020) Multi-population differential evolution-assisted Harris hawks optimization: framework and case studies. Futur Gener Comput Syst 111:175–198
58. Chen H et al (2020) Parameters identification of photovoltaic cells and modules using diversification-enriched Harris hawks optimization with chaotic drifts. J Clean Prod 244:113778
59. Rodríguez-Esparza E et al (2020) An efficient Harris hawks-inspired image segmentation method. Expert Syst Appl 155:113428
60. Elaziz MA et al (2020) A competitive chain-based Harris Hawks Optimizer for global optimization and multi-level image thresholding problems. ApplSoft Comput J 95:106347
61. Li C et al (2021) Memetic Harris hawks optimization: developments and perspectives on project scheduling and QoS-aware web service composition. Expert Syst Appl 171:114529
62. Ye H et al (2021) Diagnosing coronavirus disease 2019 (COVID-19): efficient Harris hawks-inspired fuzzy k-nearest neighbor prediction methods. IEEE Access 9:17787–17802
63. Jiao S et al (2020) Orthogonally adapted Harris hawks optimization for parameter estimation of photovoltaic models. Energy 203:117804. https://doi.org/10.1016/j.energy.2020.117804
64. Liu Y et al (2020) Horizontal and vertical crossover of Harris hawks optimizer with Nelder-Mead simplex for parameter estimation of photovoltaic models. Energy Convers Manag 223:113211. https://doi.org/10.1016/j.enconman.2020.113211
Al-Betar MA et al (2020) Survival exploration strategies for Harris hawks optimizer. Expert Syst Appl. https://doi.org/10.1016/j.eswa.2020.114243

Thaëter T et al (2020) Binary Harris Hawks optimizer for high-dimensional, low sample size feature selection. Evolutionary machine learning techniques. Springer, pp 251–272

Zhang Y et al (2020) Boosted binary Harris hawks optimizer and feature selection. Eng Comput. https://doi.org/10.1007/s00366-020-01028-5

Alabool HM et al (2021) Harris hawks optimization: a comprehensive review of recent variants and applications. Neural Comput Appl. https://doi.org/10.1007/s00521-021-05720-5

IS 3824 (2003) In rolling bearings: dynamic load ratings and rating life. Bureau of Indian Standards, New Delhi, India. https://archive.org/details/gov.in.is.7461.1.1993

Harris TA (2001) Rolling bearing analysis. Wiley

IS 3824 (2003) In rolling bearings: dynamic load ratings and rating life. Bureau of Indian Standards, New Delhi, India. https://archive.org/details/gov.in.is.3824.2002

Rao BR, Tiwari R (2007) Optimum design of rolling element bearings using genetic algorithms. Mech Mach Theory 42(2):233–250

Group S (2005) SKF general catalogue 6000. AB SKF, Gothenburg

Zhang J, Qu G (2019) Physical unclonable function-based key sharing via machine learning for IoT security. IEEE Trans Industr Electron 67(8):7025–7033

Chen Y et al (2021) Large group activity security risk assessment with initial population strategy for symmetric TSP. Math Prob Eng. https://doi.org/10.1155/2015/212794

Xiong L et al (2016) Improved stability and H∞ performance for neutral systems with uncertain Markovian jump. Nonlinear Anal Hybrid Syst 19:13–25

Xu Z et al (2020) Orthogonally-designed adapted grasshopper optimization: a comprehensive analysis. Expert Syst Appl 150:113282

Yang M, Sowmya A (2015) An underwater color image quality evaluation metric. IEEE Trans Image Process 24(12):6062–6071

Zhang K et al (2021) History matching of naturally fractured reservoirs using a deep sparse autoencoder. SPE J. https://doi.org/10.2118/205340-PA

Shida H et al (2020) MRMD2.0: a python tool for machine learning with feature ranking and reduction. Curr Bioinf 15(10):1213–1221

Jiang Q et al (2018) Unified no-reference quality assessment of singly and multiply distorted stereoscopic images. IEEE Trans Image Process 28(4):1866–1881

Zuo C et al (2015) Transport of intensity phase retrieval and computational imaging for partially coherent fields: the phase space perspective. Opt Lasers Eng 71:20–32

Zuo C et al (2017) High-resolution transport-of-intensity quantitative phase microscopy with annular illumination. Sci Rep 7(1):1–22

Zhang J et al (2020) On a universal solution to the transport-of-intensity equation. Opt Lett 45(13):3649–3652

Liu M et al (2021) Walnut fruit processing equipment: academic insights and perspectives. Food Eng Rev. https://doi.org/10.1007/s12393-020-09273-6

Zuo J et al (2020) Efficient deployment with geometric analysis for mmWave UAV communications. IEEE Wirel Commun Lett 9(7):1115–1119

Xu S et al (2020) Computer vision techniques in construction: a critical review. Arch Comput Methods Eng. https://doi.org/10.1007/s11831-020-09504-3

Huang H et al (2020) Experimental investigation on rehabilitation of corroded RC columns with bsp and hpl under combined loadings. J Struct Eng 146(8):04020157

Sun Y et al (2020) Constraints hindering the development of high-rise modular buildings. Appl Sci 10(20):7159

Hatamilo A (2013) Black hole: a new heuristic optimization algorithm. Expert Syst Appl 40(5):1256–1265

Luo J et al (2019) Multi-strategy boosted mutative whale-inspired optimization approaches. Appl Math Model 73:109–123

Yang X-S (2012) Flower pollination algorithm for global optimization. In: Durand-Lose J, Jonoska N (eds) International conference on unconventional computing and natural computation. Springer

Kler D et al (2017) PV cell and module efficient parameters estimation using Evaporation Rate based Water Cycle Algorithm. Swarm Evol Comput 35:93–110

Croes GA (1958) A method for solving traveling-salesman problems. Oper Res 6(6):791–812

Deng Y, Liu Y, Zhou D (2015) An improved genetic algorithm for continuous engineering optimization: harmony search optimization approaches. Eng Appl Artif Intell 88:103370
121. Zhang X, Zhao K, Niu Y (2020) Improved Harris hawks optimization based on adaptive cooperative foraging and dispersed foraging strategies. IEEE Access 8:160297–160314

122. Moghdani R et al (2020) An improved volleyball premier league algorithm based on sine cosine algorithm for global optimization problem. Eng Comput. https://doi.org/10.1007/s00366-020-00962-8

123. Pathak VK, Srivastava AK (2020) A novel upgraded bat algorithm based on cuckoo search and Sugeno inertia weight for large scale and constrained engineering design optimization problems. Eng Comput. https://doi.org/10.1007/s00366-020-01127-3

124. Zhang H et al (2020) A multi-strategy enhanced salp swarm algorithm for global optimization. Eng Comput. https://doi.org/10.1007/s00366-020-01099-4

125. Zhang Y, Zhou X, Shih PC (2020) Modified Harris Hawks optimization algorithm for global optimization problems. Arab J Sci Eng 45(12):10949–10974

126. Gupta S, Deep K (2019) Enhanced leadership-inspired grey wolf optimizer for global optimization problems. Eng Comput. https://doi.org/10.1007/s00366-019-00795-0

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.