HIGH TEMPERATURE QCD

Edmond Iancu

Service de Physique Théorique, CE-Saclay
91191 Gif-sur-Yvette, France

Abstract

I discuss finite-temperature gauge theories as a framework to describe the quark-gluon plasma in the regime of high temperature where the gauge coupling is small, \( g \ll 1 \). I review recent progress in the understanding of the long-range physics, with emphasis on the collective phenomena and their consequences for the screening of the gauge interactions. I consider some of the infrared divergences of the perturbation theory, and discuss the physical mechanisms which remove these divergences.

Invited talk given at the XXXIth Rencontres de Moriond
QCD and High Energy Hadronic Interactions
March 23 – 30, 1996, Les Arcs, France
1 Introduction

It is generally accepted that, under sufficiently high temperatures and densities, the hadronic matter undergoes a phase transition to a deconfined phase, the quark-gluon plasma (QGP). That such a transition exists, it is suggested by the asymptotic freedom of QCD, and by the fact that, in a plasma phase, the confining color forces may be screened by many body effects, much alike as the ordinary electric charges get screened in electromagnetic plasmas. This expectation is further confirmed by lattice calculations which predict a phase transition at a critical temperature $T_{cr} \sim 200$ MeV, which is accessible to the nowadays experiences of relativistic heavy-ion collisions.

In the high temperature limit $T \gg T_{cr}$, where asymptotic freedom allows us to expect a weak coupling regime $g(T) \ll 1$, we can study the QGP in the framework of finite-temperature field theory and rely, at least to lowest orders, on a perturbative expansion in powers of $g$. The resulting description is, in many respects, complementary to the one offered by lattice calculations, since it allows us to study off-equilibrium evolution or dynamical properties, like the ones which may provide plasma signatures (e.g., particle production rates). On the other hand, the comparisons with the lattice results, whenever possible, are useful in order to verify to which extent the structures and the properties identified in perturbation theory do subsist in the lower temperature ($T \gtrsim T_{cr}$) and strong coupling ($g \gtrsim 1$) regime, which is the regime of direct phenomenological relevance.

In what follows, I shall review briefly some recent progress in the field of high temperature gauge theories, and also mention some of the open problems.

2 Collective excitations and screening

At very high temperatures $T \gg m_f$, we can ignore the fermion masses $m_f$ and speak about ultrarelativistic plasmas, either abelian (e.g., a QED plasma made by electrons, positrons and photons) or non-abelian (the quark-gluon plasma, as described by QCD). Indeed, the particles have typical momenta $k \sim T$, and therefore an ultrarelativistic dispersion relation, $E(k) = k$. Since particles can be produced or annihilated by thermal fluctuations, the particle number density $\rho$ is not an independent quantity, but it is rather related to the temperature as $\rho \sim T^3$. Then, the typical thermal wavelength $\lambda_T = 1/k \sim 1/T$ is of the same order as the mean interparticle distance $\bar{\rho} \sim \rho^{-1/3} \sim 1/T$, and quantum effects, like the Pauli principle, play an important role. In particular, in thermal equilibrium, we have to use the quantum distribution functions, namely $N(E) = 1/(e^{\beta E} - 1)$ for bosons and $n(E) = 1/(e^{\beta E} + 1)$ for fermions, where $\beta \equiv 1/T$. Thus, in contrast to what happens for non relativistic many body systems, the high temperature limit of an ultrarelativistic
plasma does not correspond to a naïve classical limit.

The analysis of the ultrarelativistic plasmas in the weak coupling limit \( g \ll 1 \) (in QED, \( g = e \) is the electric charge) reveals the emergence of collective phenomena over a typical space-time scale \( \lambda \sim 1/gT \), which is large with respect to both \( \bar{r} \) and \( \lambda_T \). Correspondingly, the collective excitations carry momenta \( \sim gT \), and are referred as “soft”, as opposed to the “hard” momenta \( \sim T \) of the single particle excitations. Since \( \lambda \gg \lambda_T \), such collective phenomena show quasi-classical features and admit a simple theoretical description which generalize the kinetic theory for ordinary non-relativistic plasmas.

To introduce this description, I consider the simplest case of an ultrarelativistic QED plasma, and study the propagation of a slowly varying electromagnetic wave \( A_\mu(x) \) (with wavelength \( \lambda \sim 1/eT \)) as coupled to fluctuations in the phase-space densities of the charged particles, to be denoted by \( n_{\pm}(k, x, t) \) for electrons (charge \(-e\)) and positrons (charge \(e\)), respectively. The Maxwell equation

\[
\partial_\nu F^{\nu\mu}(x) = j^\mu(x),
\]

involves the induced current

\[
j_\mu(x) = 2e \int \frac{d^3k}{(2\pi)^3} v_\mu [n_+(k, x) - n_-(k, x)],
\]

where the factor of 2 accounts for the spin degrees of freedom, \( x^\mu = (t, \mathbf{x}) \), \( v^\mu = (1, \mathbf{v}) \) and \( \mathbf{v} = k/k \) is the velocity of the ultrarelativistic fermions. The single-particle distribution functions obey the Vlasov equation

\[
(v \cdot \partial_x)n_\pm \pm e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial n_\pm}{\partial \mathbf{k}} = 0,
\]

which together with eqs. (2.1) and (2.2) form a closed system of equations. In the absence of the electromagnetic field, the plasma is in thermal equilibrium, so that \( n_{\pm}(k, x) \rightarrow n(k) \). For small fields, and therefore small off-equilibrium perturbations, we write \( n_{\pm}(k, x) \equiv n(k) + \delta n_{\pm}(k, x) \), and linearize the Vlasov equation to get

\[
(v \cdot \partial_x)\delta n_{\pm}(k, x) = \mp e\mathbf{v} \cdot \mathbf{E}(x) \frac{dn}{dk}.
\]

The contribution of the magnetic field dropped out in the right hand side because of the isotropy of the equilibrium state. Eq. (2.4) can be easily integrated with, e.g., retarded boundary conditions, and the resulting current may be written in momentum space as

\[
j^\mu(q) = \Pi^{\mu\nu}(q)A_\nu(q),
\]

with the polarisation tensor

\[
\Pi_{\mu\nu}(q_0, q) = m_D^2 \left\{ -\delta_{\mu0}\delta_{\nu0} + q_0 \int \frac{d\Omega}{4\pi} \frac{v_\mu v_\nu}{q_0 - \mathbf{v} \cdot \mathbf{q} + i\eta} \right\}.
\]

2
where $m_D^2 = e^2 T^2 / 3$ and the small imaginary part in the denominator, $i \eta$ with $\eta \to 0^+$, reflects the retarded boundary conditions. The angular integral $\int d\Omega$ runs over all the orientations of the unit vector $v$.

Note that in the above, seemingly classical, description of the polarization phenomena, quantum effects entered explicitly, via the Fermi-Dirac occupation factor $n(k)$. To reassure the reader about this apparently hybrid description, let me emphasize that eqs. (2.1)–(2.3) can be rigorously derived from quantum field theory. They represent the leading order in a systematic expansion in powers of $e$ of the Dyson-Schwinger equations for thermal Green’s functions [1]. In this expansion, the electric charge controls not only the strength of the interactions, but also the soft gradients, since $\partial_x A_\mu \sim e T A_\mu$, and similarly $\partial_x n(k, x) \sim e T n(k, x)$. Thus the long-wavelength, collective degrees of freedom may be treated as classical, in contrast to the single-particle, hard degrees of freedom, which are always quantum. Genuine quantum effects, such as pair production, only enter the kinetic theory at the next-to-leading order in $e$, on the same footing as the collision terms in the right hand side of the Vlasov equation (2.3).

Being transverse, $q^\mu \Pi_{\mu\nu}(q) = 0$, the polarization tensor (2.5) is determined by only two independent scalar functions, which we choose as the electric ($\Pi_t$) and the magnetic ($\Pi_l$) components, respectively:

$$\Pi_t(q_0, q) \equiv -\Pi_{00}(q_0, q), \quad \Pi_l(q_0, q) \equiv \frac{1}{2} (\delta^{ij} - \hat{q}^i \hat{q}^j) \Pi_{ij}(q_0, q).$$  \hspace{1cm} (2.6)

This choice is natural since the medium effects distinguish between the electric (or longitudinal) and the magnetic (or transverse) sectors of the gauge interactions: indeed, the thermal bath involves electric charges, but not magnetic monopoles.

This distinction is especially important when we consider the screening effects. The most familiar such effect is the Debye screening of the Coulomb interaction: the potential between two static pointlike sources $q_1$ and $q_2$ separated by $r$ reads

$$V(r) = q_1 q_2 \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 + \Pi_t(0, q)}.$$  \hspace{1cm} (2.7)

To leading order in $e$, eqs. (2.3) and (2.6) yield $\Pi_t(0, q) = m_D^2$, and eq. (2.7) exhibits exponential attenuation over a typical scale $\lambda_D = 1/m_D \sim 1/e T$: $V(r) \sim e^{-m_D r}/r$. The quantity $m_D$ is therefore known as the “Debye mass”.

The magnetic interactions, on the other hand, are not screened in the static limit $q_0 \to 0$: $\Pi_t(0, q) = 0$. For small, but non-vanishing, frequencies,

$$\Pi_t(q_0 \ll q) \simeq -i \frac{\pi}{4} \frac{q_0}{q} m_D^2$$  \hspace{1cm} (2.8)

is purely imaginary, and describes the attenuation of a time-dependent magnetic field via energy transfer toward the charged particles (“dynamical screening”). Microscopically,
this corresponds to the absorption of the space-like photons \((q_0^2 < q^2)\) by the hard thermal fermions (Landau damping) \[2\].

Before further discussing the consequences of the screening effects, let me just mention that a completely similar picture holds in QCD as well, to leading order in \(g\): the collective color oscillations of the hard thermal quarks and gluons are described by generalized Vlasov-type kinetic equations \[1\] which yield a polarization tensor of the form 
\[
\Pi_{\mu \nu}(q) = \delta_{ab} \Pi^{ab}_{\mu \nu}(q),
\]
where \(a\) and \(b\) are color indices for the adjoint representation, and \(\Pi^{ab}_{\mu \nu}(q)\) is given again by eq. (2.5), but with a Debye mass 
\[
m_D^2 = g^2 T^2 (N_f + 2N)/6
\]
for \(N\) colors and \(N_f\) number of flavors. Moreover, in QCD, the non-abelian gauge symmetry constrains the induced color current 
\[
 j^a_{\mu}(x) = \Pi^{ab}_{\mu \nu} A^\nu_b + \frac{1}{2} \Gamma^{abc}_{\mu \nu \rho} A^\nu_b A^\rho_c + ... 
\]
in symbolic notations. Finally, in ultrarelativistic plasmas, the bosonic and fermionic degrees of freedom play symmetrical roles, so that we also encounter collective excitations with fermionic quantum numbers, which can still be described by simple kinetic equations \[1\]. The thermal corrections which describe the collective behaviour at the scale \(gT\) — like the polarization tensor (2.5) and the vertex corrections in eq. (2.9) — are generally dubbed “hard thermal loops”. This reflects the fact that, in their original derivation, which is based on Feynman graphs for thermal QCD, they all arise from one-loop diagrams where the external line carry soft momenta \(\sim gT\), while the internal loop momentum is hard, \(\sim T\) \[3, 4, 5\].

3 The lifetime of the quasiparticles

Since the screening effects reduce the range of the gauge interactions, their resummation greatly improve the infrared (IR) behaviour of the perturbative expansion. To be more specific, let me consider the computation of the lifetimes of the plasma excitations (either hard, or soft). Information about the lifetime can be obtained from the retarded propagator \(S_R(t, p)\). In many cases, this decays exponentially in time, 
\[
S_R(t, p) \sim e^{-iE(p)t}e^{-\gamma(p)t},
\]
with a damping rate \(\gamma(p)\) which is essentially the total interaction rates of the excitation. The quasiparticle picture is consistent as long as \(\gamma \ll E\). Let me compute \(\gamma\) for a fermion with momentum \(p \sim T\) which scatters off the thermal particles (quarks and gluons). In the Born approximation (one gluon exchange), the interaction rate is simply 
\[
\gamma = \sigma \rho, \text{ where } \rho \sim T^3 \text{ is the density of the scatterers, and } \sigma = \int d^2q (d\sigma/dq^2), \text{ with } q \text{ denoting the momentum of the exchanged (virtual) gluon.}
\]
For a bare gluon, the Ruther-
ford formula yields $d\sigma/dq^2 \sim g^4/q^4$, so that $\gamma \sim g^4 T^3 \int (dq/q^3)$ is quadratically infrared divergent. Actually, the screening effects soften the IR behaviour, and distinguish between electric and magnetic scattering: $\gamma = \gamma_t + \gamma_l$. In the electric sector, we have Debye screening, i.e. $1/q^2 \to 1/(q^2 + m_D^2)$, and therefore a dynamical IR cut-off $m_D \sim gT$: $\gamma_t \sim g^4 (T^3/m_D^2) \sim g^2 T$. In the magnetic sector, on the other hand, the dynamical screening does not completely remove the divergence, which is just reduced to a logarithmic one:

$$\gamma_t \sim g^4 T^3 \int_0^\infty dq \int_0^q dq_0 |D_t(q_0, q)|^2$$

$$\sim g^4 T^3 \int_0^\infty dq \int_0^q dq_0 \frac{1}{q^4 + (\pi m_D^2 q_0/4q)^2} \sim g^2 T \int_0^{m_D} \frac{dq}{q}.$$  \hspace{1cm} (3.10)

In this equation, $D_t(q_0, q) = 1/(q_0^2 - q^2 - \Pi_t(q_0, q))$ is the propagator of the magnetic photon, and in writing the second line we used eq. (2.8) and retained only the leading, IR divergent, contribution to $\gamma_t$. With an IR cut-off $\mu$, $\gamma_t \sim g^2 T \ln(m_D/\mu)$. The remaining logarithmic divergence is due to collisions involving the exchange of very soft, *quasistatic* $(q_0 \to 0)$ magnetic photons, which are not screened by plasma effects. To see that, note that the IR contribution to $\gamma_t$ comes from momenta $q \ll gT$, where $|D_t(q_0, q)|^2$ is almost a delta function of $q_0$:

$$|D_t(q_0, q)|^2 \sim \frac{1}{q^4 + (\pi m_D^2 q_0/4q)^2} \sim q \to 0 \frac{4}{qm_D^2} \delta(q_0).$$  \hspace{1cm} (3.11)

In QCD, one generally expects the dynamical generation of a magnetic screening mass $\sim g^2 T$, by some non-perturbative mechanism. This is supported by lattice computations, and shows up through infrared divergences in perturbation theory. Then, the QCD damping rate is IR finite and $\sim g^2 T \ln(1/g)$ \[4\]. In QED, on the other hand, it is known that no magnetic screening can occur, so that the solution of the problem must lie somewhere else.

Let me concentrate on the abelian problem from now on. An analysis of the higher order corrections to eq. (3.10) reveals severe (power-like) IR divergences which signal the breakdown of the perturbation theory \[6\]. Because of the specific IR behaviour of the magnetic photon propagator, eq. (3.11), the leading divergences come from multiple collisions where all the exchanged photons are magnetic and quasistatic. They can be studied in the framework of an effective three-dimensional theory, which considers the interactions of the fermion with only *static* $(q_0 = 0)$ photons with propagator $D_t(0, q) = 1/\sqrt{q^2}$. By using the Bloch-Nordsieck approximation, it is possible to resum the leading IR divergences, and get the correct large-time $(t \gg 1/gT)$ behaviour of the fermion propagator $S_R(t)$ \[3\]. This is free of IR problems and, rather surprisingly, it shows a *non-exponential* decay in time:

$$S_R(t) \sim \exp\{-\alpha Tt \ln(m_D t)\},$$  \hspace{1cm} (3.12)
where $\alpha = e^2/4\pi$. Since at large times $S_R(t)$ is decreasing faster than any exponential, it follows that the Fourier transform

$$S_R(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} S_R(t)$$

exists for any complex energy $\omega$. Thus, the retarded propagator $S_R(\omega)$ has no singularity at the mass-shell. The associated spectral density $\rho(\omega) \propto \text{Im} S_R(\omega)$ retains the shape of a resonance strongly peaked around the perturbative mass-shell $\omega \sim E(p)$, with a typical width of order $\sim g^2T \ln(1/g)$ \[^6\]. Thus, the quasiparticles are well-defined, even if they do not correspond to the usual, exponential, time decay of the propagator.

### 4 Conclusions

The removal of the infrared divergences by physical mechanisms is an important self-consistency check for high temperature gauge theories. The computation of the damping rate illustrates both the power and the limits of the screening effects in this sense. They sensibly improve the infrared behaviour of the perturbation theory, and completely remove the IR problems from the electric sector. Still, IR divergences persist in the magnetic sector, due to the unscreened static magnetic gluons or photons. It has been pointed out by Baym et al. \[^7\] that the dynamical screening of the time-dependent magnetic fields, as illustrated by eq. (2.8), is sufficient to yield IR finite results for many quantities of physical interest, like transport coefficients or the collisional energy loss. This suggests that it may be possible to further develop the kinetic approach discussed previously in order to include collision terms and off-shell effects, thus leading to a consistent transport theory for the high temperature QCD plasma.

More generally, the resummation of the screening effects in the “hard thermal loop” approximation enables us with a consistent perturbative description of the physics at short ($\sim 1/T$) and intermediate ($\sim 1/gT$) scales. The resulting physical picture turns out to be quite similar for abelian or non-abelian plasmas, but important differences occur when going to even larger scales, $\gtrsim 1/g^2T$. Lattice simulations of hot QCD reveal traces of the confinement in the long-range correlations, and these may be associated with the infrared divergences encountered in perturbation theory. In QCD, one expects these divergences to be cured by new, non-perturbative, screening effects, which should manifest in the magnetostatic sector at momenta $\sim g^2T$ \[^8\]. In abelian theories, where there is no magnetic screening, the divergences are removed — as we have seen on the example of the fermion lifetime — by further resummations of soft photon processes to all orders in perturbation theory \[^8\]. Another IR problem, which is currently under investigation, is the appearance of collinear divergences, e.g., in the computation of the plasma production
rate for soft real photons [9]. This problem is currently under investigation [10].

Finally, one may wonder about the relevance of perturbative QCD for the phenomenology of heavy ion reactions. We have indeed evidence that in the temperature regime that is presumably accessible to these collisions, the coupling strength is not small, rather \( g \sim 2 - 3 \). Is this to say then that all the physics described here is irrelevant? I do not believe so. It is physically plausible, and partially supported by lattice calculations, that some of the structures identified at scale \( gT \), as the screening effects, may be sufficiently robust to survive even in a regime of parameters where the approximations made to derive them cannot be justified.

References

1. J.P. Blaizot and E. Iancu, Nucl. Phys. B390 (1993) 589; Phys. Rev. Lett.70 (1993) 3376; Nucl. Phys. B417 (1994) 608; Phys. Rev. Lett.72 (1994) 3317.
2. E.M. Lifshitz and L.P. Pitaevskii, Physical Kinetics, (Pergamon Press, Oxford, 1981).
3. V.V. Klimov, Sov. J. Nucl. Phys. 33 (1981) 934; Sov. Phys. JETP 55 (1982) 199; H.A. Weldon, Phys. Rev. D26 (1982) 1394; Phys. Rev. D26 (1982) 2789.
4. R.D. Pisarski, Phys. Rev. Lett.63 (1989) 1129; E. Braaten and R.D. Pisarski, Phys. Rev. Lett.64 (1990) 1338; Nucl. Phys. B337 (1990) 569.
5. J. Frenkel and J.C. Taylor, Nucl. Phys. B334 (1990) 199. J.C.Taylor and S.M.H.Wong, Nucl. Phys. B346 (1990) 115.
6. J.P. Blaizot and E. Iancu, Phys. Rev. Lett.76 (1996) 3080; Nucl. Phys. B459 (1996) 559.
7. G. Baym, H. Monien, C.J. Pethick, and D.G. Ravenhall, Phys. Rev. Lett.64 (1990) 1867.
8. E. Braaten, Phys. Rev. Lett.74 (1995) 2164; P. Arnold and L.G. Yaffe, Phys. Rev. D52 (1995) 7208.
9. R. Baier, S. Peigné and D. Schiff, Z. Phys. C62 (1994) 337; P. Aurenche, T. Becherrawy and E. Petitgirard, preprint ENSLAPP-A-452-93, [hep-ph/9403320](http://arxiv.org/abs/hep-ph/9403320).
10. F. Flechsig and A.K. Rebhan, Nucl. Phys. B464 (1996) 279; P. Aurenche, F. Gelis, R. Kobes and E. Petitgirard, preprint ENSLAPP-A-586-96, [hep-ph/9604398](http://arxiv.org/abs/hep-ph/9604398).