Gluelumps and Confinement in QCD

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Abstract

Confinement is explained via field correlators, and the latter are calculated via gluelumps. Behavior of gluelump Green’s function at small and large distances yields gluonic condensate and vacuum correlation length respectively and allows to check the consistency of the whole picture.

1 Introduction

Field theory based on Abelian and nonabelian gauge fields is now the foundation of physics of strong and electroweak interaction. Here the gauge covariance in general and gauge invariance of physical amplitudes is the cornerstone and the most general principle, which allows to fix the form of equations and resulting amplitudes even before any dynamical input. In the case of QCD this fact is even more important, as was stressed in [1,2]. Below we show, that confinement makes gauge invariance and all relations, connected to it, the most important property sine qua non, and in turn, gauge invariance principle allows to build theory of confinement in good agreement with all existing data.

It will be shown below, that gauge invariance requires each particle travelling from point x to point y to develop its own gauge factor \( \Phi_C(x, y) \) (the Schwinger line, or parallel transporter) pertaining to the path \( C(x, y) \). For neutral (color-singlet) systems the set of paths \( \{ C_i(x, y) \} \) forms a closed loop,
a simple Wilson loop $W(C)$ for two opposite charges, or more complicated closed loop with $N_c$ branches for the $N_c$ charges in the group $SU(N_c)$.

As will be shown (see also [3] and [4] for the review) all dynamics in the neutral system is given by the vacuum-averaged Wilson loop $\langle W(C) \rangle$, and since all physical amplitudes correspond to the gauge invariant and hence neutral systems, therefore $\langle W(C) \rangle$ contains all the clues to confinement and in general to the dynamics of strong interaction.

Here one can distinguish confining theories, like QCD, and nonconfining ones, like electroweak (EW) theory. In the first case gauge invariance and confinement make it impossible to consider only one part of color neutral system, e.g. propagator of quark or gluon. Formally, gauge invariant vacuum average of this colored object is zero, physically, the confining string, which appears in $\langle W(C) \rangle$ and connects this object to its neutralizing partner, strongly governs its motion and cannot be disregarded. Therefore all models, based on vacuum averages of gluon or quark propagators in any gauge, are irrelevant to the nature.

In nonconfining theories the situation is different. One can compose a system of an electron here and proton on the moon and consider the motion of electron in the gauge invariant language of $\langle W(C) \rangle$, but the latter factorizes into a product of propagators, however some initial and final conditions of electron should be fixed to avoid divergencies.

A similar situation occurs in the deconfined phase of QCD [5] (see [6] for a review), where (apart from magnetic confinement) $\langle W(C) \rangle$ also factorizes and dynamics is given by Wilson lines rather than Wilson loops.

Thus the basic problem is first the analysis of $\langle W(C) \rangle$ and, second, self-consistent calculation of $\langle W(C) \rangle$ or its ingredients. The first part is done using the gauge invariant cumulant expansion of $\langle W(C) \rangle$ in terms of field correlators. As was shown in [3, 4, 7], for QCD the lowest term (quadratic) $\langle F(x)F(y) \rangle$ saturates $\langle W(C) \rangle$ with few percent accuracy, which is supported by the Casimir scaling measurements on the lattice both in the confined [8] and deconfined phase [9]. The same Casimir scaling helps to put a strong upper limit on the presence of topological charges or adjoint fluxes [7] as a possible source of confinement. At the same time, Casimir scaling is well supported by the Gaussian quadratic term $\langle F(x)F(y) \rangle$ and the latter yields

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Note, that magnetic monopoles in QCD contribute largely to higher cumulants and hence violate Casimir scaling (see [10]) while in the weakly-coupled dilute 3d $SU(N)$ Georgi-Glashow model the Casimir scaling holds to a good accuracy [11], implying that higher cumulants are suppressed at low density.
a beautiful picture of linear confinement in good agreement with all lattice measurements and hadron physics.

Here comes the second part of problem: which field configurations are behind the Gaussian term, and can one calculate these correlators self-consistently, so that all QCD can be derived from the one constant, fixing the scale of our world. We show below, following [12], that this is in principle possible, and one can reexpress some physical observables, like string tension $\sigma$, through others like $\Lambda_{QCD}$ (however still with low accuracy). More importantly, one can establish the objects, which are behind confinement and characterize full vacuum structure: those are known as gluelumps\footnote{The name and the first calculations in $SU(2)$ lattice theory are due to I.H.Jorysz and C.Michael [13]. The detailed $SU(3)$ calculations have been done in [14]} and consist of one or two gluons propagating in the field of adjoint static source. Mass of these objects was calculated analytically before [15] and its inverse gives the correlation length of the vacuum, while the short distance behavior yields gluonic condensate, and finally, the integral of gluelump Green’s function yields string tension. Thus confinement is connected to gluelumps, and all other vacuum properties can be calculated via gluelumps. Moreover, the small correlation length $\lambda$ (due to large gluelump mass $M, \lambda \sim 1/M$) explains, why the Casimir scaling holds in QCD – the cumulant expansion is actually a series in powers of the dimensionless parameter $\sigma \lambda^2 \approx 0.05$, and the first (quadratic) term is dominant. Below we give a short exposition of Wilson loop and cumulant expansion in section 2, gluelumps are introduced in section 3, and exploited in section 4 to display consistency of the whole picture. Summary and conclusions are given in section 5.

2 Wilson loop and field Correlators

The Green’s function (propagator) of quark can be written in the Fock-Feynman-Schwinger Representation (FFSR) (see [16] for a review) as the quantum mechanical path integral (we consider Euclidean space-time throughout the paper)

$$S(x,y) = (m+\hat{D})^{-1} = (m-\hat{D}) \int_{0}^{\infty} ds (Dz)_{xy} e^{-K} P_{A} \exp(ig \int_{y}^{x} A_{\mu} dz_{\mu}) p_{\sigma}(x,y,s),$$

(1)
where \( K = m s + \frac{1}{4} \int_0^s \left( \frac{d s}{d \tau} \right)^2 d \tau \) and \( W_\sigma = P_A \exp(i g \int_A A_\mu dz_\mu) p_\sigma \) is the Schwinger line (parallel transporter) with spin insertions, and

\[
p_\sigma(x, y, s) = P_F \exp \left[ g \int_0^s \sigma_{\mu\nu} F_{\mu\nu}(z(\tau)) d\tau \right],
\]

The last factor contains spin operators

\[
\sigma_{\mu\nu} F_{\mu\nu} = \begin{pmatrix} \sigma \cdot B & \sigma \cdot E \\ \sigma \cdot E & \sigma \cdot B \end{pmatrix},
\]

and will be important for calculation of gluonic condensate in section 4; it also yields all spin-dependent interactions in hadrons, see [17] for a review and references.

In an analogous way one can construct FFSR for a gluon in the external and vacuum fields

\[
G(x, y) = \int_0^\infty ds \left( D_z \right)_{xy} e^{-K} P_A e^{ig \int x_y \hat{A}_\mu dz_\mu} p_\Sigma
\]

where \( \hat{A}_\mu \) implies \( A_\mu \) in the adjoint representation and the spin factor \( p_\Sigma \) can be written either in the form (3) with the gluon spin operator \( \mathbf{S} \) instead of \( \sigma \), or simpler, as

\[
p_\Sigma = [P_F \exp(2ig \int_0^s F_{\lambda\sigma}(z(\tau)) d\tau)]_{\mu\nu}.
\]

We omit in this section the spin parts of quark and gluon Green’s function, since they do not define the confinement picture at this stage.

For a white hadron, consisting of \( q\bar{q} \) or two gluons one can write the total Green’s function as

\[
G_{q\bar{q},gg}(\bar{C}_{xy}) = \int d\Gamma_{q\bar{q},gg}(\bar{C}) W(\bar{C}_{xy})
\]

and the Wilson loop operator for the closed loop \( \bar{C}_{xy} \) is

\[
W(\bar{C}) = \langle P_A \exp(i g \int_\bar{C} A_\mu dz_\mu) \rangle,
\]

where the brackets imply vacuum averaging. The use of the nonabelian Stokes theorem and cluster (cumulant) expansion [18] yields

\[
W(\bar{C}) = tr_c \exp \left\{ \sum_{n=1}^\infty \frac{(ig)^n}{n!} \int ds(1) \int ds(n) \langle \{ F(1) \ldots F(n) \} \rangle \right\},
\]
where \( ds(k) = ds_{\mu_k \nu_k} \) is the surface element, and

\[
F(k) \equiv F_{\mu_k \nu_k}(u_k, x_0) = \Phi(x_0, u_k)F_{\mu_k \nu_k}(u_k)\Phi(u_k, x_0)
\]  

(9)

while \( \Phi(x, y) \) is the parallel transporter. Field correlators \( \langle F \rangle \) \( \Phi \) depend in general on the chosen point \( x_0 \), which is convenient to choose somewhere on the minimal surface bounded by \( \bar{C} \). The great simplification for QCD is that \( FC \) decrease so fast with distance, as we shall see later, that \( \Phi(u_k, x_0)\Phi(x_0, u_j) \) in (8,9) can be replaced by straight line transporter \( \Phi(u_k, u_j) \), and moreover, the whole series is fast converging as \( (\sigma \lambda^2)^n \), where \( \lambda \) is the correlation length, \( \langle F(x)\Phi(y)\Phi \rangle \sim \exp \left(-\frac{|x-y|}{\lambda}\right) \). Therefore one can keep only the Gaussian correlator \( \langle \Phi \rangle \), and moreover, since \( \lambda \) is much smaller than all other characteristic quantities, e.g. hadron sizes, inverse masses of lowest hadrons etc., one can say that the QCD vacuum configurations correspond to the Gaussian white noise.

As a result one can use only the lowest correlator

\[
D_{\mu\nu,\lambda\sigma} \equiv \langle tr \frac{g^2}{N_c} F_{\mu\nu}(x)\Phi(x,y)F_{\lambda\sigma}(y)\Phi(y,x) \rangle,
\]

(10)

which can be expressed in terms of two scalar functions \( D(z) \) and \( D_1(z) \)

\[
D_{\mu\nu,\lambda\sigma}(z) = D(z)(\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\lambda}) + \frac{1}{2} \left[ \frac{\partial}{\partial z_\mu}(z_\lambda\delta_{\nu\sigma} - z_\sigma\delta_{\nu\lambda}) + \frac{\partial}{\partial z_\nu}(z_\sigma\delta_{\mu\lambda} - z_\lambda\delta_{\mu\sigma}) \right] D_1(z).
\]

(11)

Note, that (10), (11) is written for fundamental charges, for charge of irrepr. \( j \) one should multiply by the ratio \( \frac{C(j)}{C(\text{fund})} \), where \( C(j) \) is the quadratic Casimir operator.

Now all the spin-independent dynamical properties can be calculated from static potentials, the confining Lorentz scalar \( V(r) \),

\[
V(r) = 2\frac{C(j)}{C(\text{fund})} \int_0^r (r - \lambda)d\lambda \int_0^\infty d\nu D(\sqrt{\lambda^2 + \nu^2})
\]

(12)

and the Lorentz vector \( V_1(r) \)

\[
V_1(r) = \int_0^r \lambda d\lambda \int_0^\infty d\nu D_1(\sqrt{\lambda^2 + \nu^2})
\]

(13)
and the string tension (and confinement) is due to $D(z)$:

$$\sigma = \frac{1}{2} \frac{C(j)}{C(\text{fund})} \int \int d^2 z D(z).$$

(14)

Spin-dependent potentials can be found as well for quarks of any mass or gluons without $1/M_Q$ expansion, keeping in [8] also spin terms, see [17] for a review. The interaction (12), (13) was used in the framework of the relativistic string Hamiltonian [19] and spectra of mesons [20], glueballs [21], hybrids [22] have been computed in excellent agreement with lattice and experiment with minimal, input (current quark masses, $\sigma$ and $\alpha_s$ without any additional parameters) see [23] for reviews.

At this point one could wonder how to check the notion of Gaussian vacuum? The answer is very simple: Gaussian correlators $D$ and $D_1$ are proportional to $(gF)^2$, i.e. to the charge squared, and hence for higher color representations $j$ they are proportional to the Casimir operator $C_2(j)$, known for the group $SU(N_c)$, $C_2(\text{fund}) = \frac{N_c^2 - 1}{2N_c}$, $C_2(\text{adj}) = N_c$ etc..

This constitutes the Casimir scaling, and the numerical check was done on the lattice [8, 9] and compared with FC in [7]. results of the lattice measurements in the confined and deconfined phases of QCD are shown in Fig. 1 (taken from [8]) and Fig. 2 respectively. One can see an excellent agreement of $V_{Q\bar{Q}}(j) = (\frac{1}{2} T \ln W_j(C))$ computed on the lattice for different $j = 3, 6, 8, ...$ with the Casimir scaled potential $\frac{C_2(j)}{C_2(3)} V_{Q\bar{Q}}(3)$. (Note in Fig.1 $V$ and $r$ are units of $r_0^{-1}$ and $r_0$ respectively, $r_0 \approx 0.5$ fm.)

In the deconfined phase only $V_1(r,T)$ is nonvanishing (apart from color-magnetic correlators, see [6]) and it enters the Polyakov lines, $L_j(T) = \exp \left( -\frac{C_2(j)}{C_2(3)} \frac{V_1(\infty,T)}{2T} \right)$. One can see in Fig.2 the lattice measured values $L_{\text{fund}}(T), L_{\text{adj}}(T)$ as functions of temperature (points) vs our analytic calculations with Casimir factors.

Agreement is impressive. (Note, that at $T < T_c$ in confinement phase $L_{\text{fund}} \equiv 0$, formally since one should replace $V_1 \to V_1(\infty,T) + V(\infty,T)$ and the latter is infinite).

3 Gluelumps and Field Correlators

To express FC through gluon propagators, one should consider gluons in $D_{\mu\nu,\lambda\sigma}$, i.e. gluons emitted in $F_{\mu\nu}(x)$ and absorbed in $F_{\lambda\sigma}(y)$ and moving in
Figure 1: (from [8]). The potentials $V_D$ in units of $1/r_0$ for all measured representations $D = 3, 8, 6, ..., 15s$, obtained at $\beta = 6.2(r_0 = 0.5$ fm, $a = r_0/6.1$). The line for a representation $D$ is the fundamental potential, multiplied by the ratio $C_2(D)/C_2(\text{fund})$.

The background of vacuum fields (consisting of gluon fields again and with the source in $\Phi(x, y)$). This is the basic idea of mean field approach. It is important, that we can express the interaction of propagating gluons with background field again in terms of FC, and thus one obtains a closed scheme, consistency of it will be discuss in the next section. Expanding $F_{\mu\nu}$ into Abelian and nonabelian parts, $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) - ig[A_\mu, A_\nu]$ one can write

$$D_{\mu\nu,\lambda\sigma}(x, y) = D_{\mu\nu,\lambda\sigma}^{(0)} + D_{\mu\nu,\lambda\sigma}^{(1)} + D_{\mu\nu,\lambda\sigma}^{(2)}$$ (15)
Figure 2: Shown on the figure are curves of $L_{\text{adj}}$ (lower curve for all $T/T_c$) and $L_{\text{fund}}$ (upper curve) compared to the lattice measured from [9]. In the $T < T_c$ region the $M(\bar{\alpha}_s = 0.195) = 0.982$ GeV gluelump mass was used. In the deconfinement region the fit was used with $T_c = 270$ MeV for $L_{\text{fund}}$ and the Casimir scaled value for $L_{\text{adj}}$.

where the superscript $0,1,2$ refers to the power of $g$, coming from the term $ig[A_\mu, A_\nu]$.

Here $D^{(0)}_{\mu\nu,\lambda\sigma}$ is connected to the one-gluon gluelump Greens’ function $G^{(1g)}_{\mu\nu}$,

\[ D^{(0)}_{\mu\nu,\lambda\sigma}(x, y) = \frac{g^2}{2N_c^2}\left\{ \frac{\partial}{\partial x_\mu} \frac{\partial}{\partial y_\lambda} G^{(1g)}_{\nu\sigma}(x, y) + \text{perm} \right\} + \Delta^{(0)}_{\mu\nu,\lambda\sigma}, \tag{16} \]

where the $1g$ gluelump Green’s function is

\[ G^{(1g)}_{\mu\nu}(x, y) = \langle \text{Tr}_a A_\mu(x) \bar{\Phi}(x, y) A_\nu(y) \rangle. \tag{17} \]

and $T_{ra}$ implies summation over adjoint indices.

$D^{(2)}_{\mu\nu,\lambda\sigma}$ is of basic importance, since it ensures confinement via $D(z)$ and is expressed via two-gluon gluelump Green’s function $G^{(2g)}(z)$.
where both one- and two-gluon gluelump Green’s functions can be written in terms of path integrals \[16\] and finally expressed via eigenvalues and eigenfunctions of relativistic string Hamiltonian \[19\]. For gluelumps this Hamiltonian was studied in \[15\] and results for lowest gluelump masses in comparison with lattice data \[14\] are (expressed via fundamental string tension \(\sigma_f = 0.18\) GeV\(^2\)).

\[
M^{(1g)} = 1.49 \text{ GeV}, \quad M^{(1g)}(\text{lat}) < 1.7 \text{ GeV},
\]
\[
M^{(2g)} = 2.61 \text{ GeV}, \quad M^{(2g)}(\text{lat}) = 2.7 \text{ GeV},
\]
(19)

where \(M^{(ig)}\) is the mass of gluelump with \(i = 1, 2\) gluons. The same Hamiltonian technic allows to find the asymptotics of Green’s functions \[12\]

\[
D_1(z) = \frac{2C_2(f)\alpha_s M_0^{(1g)} \sigma_{adj}}{|z|} e^{-M^{(1g)}|z|}, \quad |z| M^{(1g)} \gg 1.
\]
(20)

\[
D(z) = \frac{g^4(N_c^2 - 1)}{2} 0.1 \sigma_f^2 e^{-M^{(2g)}|z|}, \quad M^{(2g)}|z| \gg 1
\]
(21)

Here \(\alpha_s\) is the value of the strong coupling constant at large distance.

Correspondingly, one obtains the correlation length of \(D(z)\) and \(D_1(z)\)

\[
\lambda = \frac{1}{M^{(2g)}} \approx 0.08 \text{ fm}, \quad \lambda_1 = \frac{1}{M^{(1g)}} \approx 0.15 \text{ fm}.
\]
(22)

This should be compared with \(\lambda, \lambda_1\) calculated on the lattice previously \[24\], \(\lambda \approx \lambda_1 \approx 0.2\) fm, while a recent measurement \[25\], analyzed in \[17\] yields \(\lambda \approx 0.1\) fm, and the numerical HP\(^1\) formalism \[26\] yields \(\lambda \approx 0.13\) fm, \(\lambda_1 \approx 0.145\) fm.

Finally, a remarkable agreement with the values \[22\] has been found in Ref. \[27\], where the full calculation of the path integrals representing \(G^{(1g)}_{\mu\nu}\) and \(G^{(2g)}\) has been performed.

Thus all data support the conclusion, that the correlation length is very small \(\lambda, \lambda_1 \sim 0.1\) fm and the QCD vacuum is therefore highly stochastic. Moreover, we have calculated FC in terms of nonperturbative parameters, like \(\sigma\), which can be calculated, as in \[14\] via FC themselves. This calls for the consistency check, to be done in the next section.
4 Consistency of the gluelump mechanism

We start with small distances and with $D_1(z)$. The 1g gluelump Green’s function can be written in FFSR \cite{16}

$$C_{\mu\nu}^{(1g)}(x,y) = \text{Tr}_a \int_0^\infty ds (Dz)_{xy} \exp(-K) \langle W^F_{\mu\nu}(C_{xy}) \rangle,$$  \hspace{1cm} (23)

where the spin-dependent Wilson-loop with insertion of operators $F_{\mu\nu}$ is

$$W^F_{\mu\nu}(C_{xy}) = PP_F \left\{ \exp(ig \int A_\lambda dz_\lambda) \exp F \right\}_{\mu\nu}$$  \hspace{1cm} (24)

where $\exp F$ is as in (5).

With zero background ($A_\mu \to 0$) this is easily calculated and yields the lowest order perturbative result, $D_1(z) = \frac{16\alpha_s}{3\pi z^4}$, then from \cite{13} $V_1(r) = -\frac{4\alpha_s}{3r}$.

The first nonperturbative (np) contribution comes, when one accounts for the small Wilson loop behaviour (second paper in \cite{3})

$$W^F_{\mu\nu} \sim \exp(\frac{-\pi}{8} G_2 S^2), \hspace{1cm} G_2 = \frac{\alpha_s}{\pi} \langle (F^a_{\mu\nu}(0))^2 \rangle, \hspace{1cm} S \text{ area}.$$  \hspace{1cm} (25)

As was shown in \cite{12}, insertion of (25) in (23) yields finally

$$D_1(z) = \frac{16\alpha_s + O(\alpha_s^2 \ln z)}{3\pi z^4} + \frac{\pi^2}{6N_c} G_2.$$  \hspace{1cm} (26)

Note two important facts; 1) perturbative and np contributions in (26) enter additively in this lowest order, this supports the original idea in \cite{28}; 2) the sign of np part is positive and this contributes to the np shift down of the vacuum energy density.

We turn now to the function $D(z)$, connected via (18) to the 2g gluelump Green’s function. Here the small distance region is more complicated. First of all, the same small-area term (25) yields for $D(z)$ a negative result!

$$D(z) = \Delta_1 D(z) + \Delta_2 D(z),$$

$$\Delta_1 D(z) = -\frac{g^4 N_c G_2}{4\pi^2}$$  \hspace{1cm} (27)

However the same spin-dependent Wilson loop (now with two gluons and one parallel transporter (Schwinger line)) in (24) contains paramagnetic contribution $\langle W^F(1)W^F(2) \rangle \sim \exp(-c \langle FF \rangle)$, which has positive sign (see Fig. 3).
Here $\lambda_0$ is the point, which is above the asymptotic free region of $\alpha_s(\lambda(z))$, $\lambda_0 \gtrsim 0.24$ fm (see Appendix of the last paper in [12]). Note an interesting feature of (28): for vanishing $z$, $g^2(z) \sim \frac{1}{\ln 1/z}$ and $\Delta_2 D(z)$ tends to a positive constant, so that the total contribution,

$$D(z \to 0) = \Delta_1 D(z) + \Delta_2 D(z) \to D(0) = \frac{N_c^2}{2\pi^2} D(\lambda_0) \left( \frac{2\pi}{\beta_0} \right)^2.$$  

For $N_c = 3$ this yields $D(0) \approx 0.15 D(\lambda_0)$, and implies the humpbacked form of $D(z)$, see Fig.4, since it decreases exponentially at large $z$, see Eq.(21).

Note, that this exponential behavior $D(z) = D_\sigma(0)e^{-|z|/\lambda}$, if continued to zero, yields string tension from Eq.(14) $\sigma \approx \pi \lambda^2 D_\sigma(0)$, and $D_\sigma(0)$ is the AF behavior of (28), illustrated by Fig.4.

Now let us discuss the selfconsistency at large distances. We start with the asymptotic at $|z| \gg \lambda$, $D(z) \approx D_\sigma(0) \exp(-|z|/\lambda)$, $\lambda = 1/M_0^{(2)}$ with $D_\sigma(0)$ given in (21), and assume, that this form can be extended to $z = 0$, at least in the integral $\sigma = \frac{1}{2} \int d^2z D(z)$, where small $z$ are less important.
Figure 4: Qualitative behaviour of the correlator $D(x)$ versus $x$, as predicted by (20) and (27), (28) at large and small distances $x$ respectively.

As a result of this insertion of $D(z)$ into (14), one obtains an interesting relation

$$0.1 \cdot 8\pi^2 \alpha_s^2(N_c^2 - 1)\sigma_f^2 = \frac{\sigma_f}{\pi\lambda^2}$$

(30)

where in $\alpha_s = \alpha_s(\mu)$, and the scale $\mu$ corresponds to the average momentum ($\approx$ inverse radius) of the two-gluon gluelump. One can take $\mu_0 \approx 1$ GeV, then from (30) one gets:

$$\alpha_s(\mu \approx 1 \text{ GeV}) \geq 0.4; \quad \alpha_s(\mu) \approx \frac{4\pi}{\beta_0 \ln \left( \frac{\mu^2 + M_B^2}{\Lambda_{QCD}^2} \right)}$$

(31)

where we have used the IR saturated form of $\alpha_s$, suggested earlier in [30], and derived for QCD with np background in [31] and tested in hadronic physics in [32] with the so-called background mass parameter $M_B \approx 1$ GeV. Eq. (31) is actually the “dimensional transmutation relation”, since it connects $\sigma = 0.18$ GeV$^2$, as a basic scale parameter of QCD, yielding all masses and $\lambda$, to $\alpha_s(\mu)$, i.e. $\Lambda_{QCD}$. One gets from (31) for three flavors $n_f$

$$\Lambda(n_f = 3) \geq 0.29 \text{ GeV.}$$

(32)

\footnote{Note the sign $\geq$ in (31) due to the fact, that the extension of exponential form of $D(z)$ to small $z$, overestimates the integral, cf. Eq(29)}
One should take into account, however, that we have used Eq. (31) in the $x$-space, where $\lambda$ should be approximately 1.3 times larger, than $\Lambda_{\text{QCD}}(n_f = 3) \approx 0.27$ GeV.

Thus this large distance consistency check is reasonably satisfied.

We end this section with two remarks:

1) Till now only np contribution to $D(z)$ have been discussed. Naively the 2g gluelump Green’s function yields perturbative result for $D(z) \sim \frac{\alpha_s}{z}$, however, as shown in [33], all perturbative contributions to $D(z)$ are exactly cancelled by those from higher correlators.

2) $D(z)$ and $D_1(z)$ have been obtained here in the leading approximation, when gluelumps of minimal number of gluons contribute: 2 for $D(z)$ and 1 for $D_1(z)$. In the higher orders of $O(\alpha_s)$ one has an expansion of the type

$$D(z) = D^{(2g)}(z) + c_1\alpha_s^3 D^{(1g)}(z) + c_2\alpha_s^3 D^{(3g)}(z) + ...$$

$$D_1(z) = D^{(1g)}(z) + c'_1\alpha_s^3 D^{(2g)}(z) + ...$$

Hence the asymptotic behavior for $D(z)$ will contain exponent of $M_0^{(1)} |z|$ too, but with a small preexponent coefficient.

5 Summary and discussion

We have shown, that every physical amplitude can be expressed through the corresponding Wilson loop, integrated as a path integral with the known weight. Therefore all perturbative and nonperturbative expressions for any amplitude can be derived from the corresponding expansions of the Wilson loop, e.g. the double-logarithm asymptotics of Sudakov type also follows from this representation [31]. On the other hand, Wilson loop can be exactly given by the cumulant series, containing all FC, and this is exact representation both for perturbative and np contributions.

We have also demonstrated, that the np part of the Wilson loop is saturated by the quadratic (Gaussian) FC, which is proved by Casimir scaling on the lattice and understood as an expansion in powers of $(\sigma \lambda)^2 \approx 0.04 \ll 1$.

Thus all observables can be expressed via Gaussian FC with accuracy of few percent. On the other hand we can calculate FC as gluelump Green’s functions, and the latter are computed again in terms of FC (or simply in terms of $\sigma$, which is integral of FC). Thus one obtains the closed scheme of calculation and approximations, which is subject to the selfconsistency...
check. This analysis reveals an interesting picture, explaining why gluonic condensate is relatively weak (as compared with hadron mass scale), and supporting quantitatively the idea of positive $G_2$ and hence negative vacuum energy density, yielding np stable vacuum. Finally, the idea of QCD as selfconsistent theory defined by the only one mass scale, is realized in this FC approach and the first connection between $\sigma$ and $\Lambda_{QCD}$, exemplifying dimensional transmutation, is obtained in (31).

The approach, discussed above, is universal and can be applied both to spectrum and scattering, in particular to high-energy processes, where np contributions are important. In all cases the problem reduces to the ground state or excited string dynamics, pertinent to the Wilson loop manifold with changeable (and integrated over) contours. Some first steps in this direction were done in [35, 16].

6 Conclusion

The modern picture of confinement in QCD is a result of tedious efforts of many people during last 35 years, see [36, 37] for reviews. The basic source of confinement – the nonperturbative vacuum – has passed many stages during this period. From constant chromomagnetic vacuum of Savvidy [38], and selfdual of Leutwyler [39], via spaghetti-vacuum of Nielsen-Olesen [40], using general idea of stochastic vacuum of Ambjorn, Olesen, Peterson [41] and finally to the Gaussian stochastic vacuum of the Field Correlator Method [3, 4], discussed in this short review. Many people contributed to this field, suggesting models of confinement, based on different mechanisms (see discussion in reviews [36, 37]), which we could not include here, and this study will certainly go on. However, at present the only approach, which explains all details of confinement in experiment and lattice calculation, is the Field Correlator Method (FCM). The Gaussian correlators $D$ and $D_1$ (and their colormagnetic counterparts for nonzero temperatures) are sufficient to describe all data up to now. The fact, that it is possible to calculate both $D$ and $D_1$ selfconsistently, with the only one scale input (in addition to current quark masses), gives a strong hope to construct the analytic nonperturbative QCD ab initio, as a valuable counterpart of lattice QCD.

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References

[1] L.D.Faddeev, A.A.Slavnov, Gauge Fields: an Introduction to Quantum Theory, (Frontiers in Physics, Westview Press, 2nd edition, 1993, Boulder, USA).

[2] A.A.Slavnov, Theor. Math. Phys, 10, 99 (1972).

[3] H. G. Dosch, Phys. Lett. B 190, 177 (1987); H. G. Dosch, Yu. A. Simonov, Phys. Lett. B. 205, 339 (1988); Yu. A. Simonov, Nucl. Phys. B 307, 512 (1988).

[4] A.Di Giacomo, H.G.Dosch, V.I.Shevchenko and Yu.A.Simonov, Phys. rept. 372, 319 (2002).

[5] Yu.A.Simonov, Ann. Phys. 323, 783; E.V.Komarov, Yu.A.Simonov, Ann. Phys. 323 1230 (2008).

[6] A.V.Nefediev, Yu.A.Simonov, M.A.Trusov, Int.J. Mod. Phys. E 18, 549 (2009).

[7] V.I.Shevchenko and Yu.A.Simonov, Phys. Rev. Lett. 85, 1811 (2000), Int. J. Mod, Phys. A 18, 127 (2003).

[8] G.Bali, Phys. Rev. D 62, 114503 (2000).

[9] S.Gupta, K.Huebner, O.Kaczmarek, Nucl. Phys. A 785, 278 (2007); hep-lat/0608014; K.Huebner, C.Pica; PoSLATTICE 2008; 197, 2008.

[10] Yu.A.Simonov, Yad. Fiz. 50, 500 (1989).

[11] D. Antonov and L. Del Debbio, JHEP 12, 060 (2003); D. Antonov, L. Del Debbio and D. Ebert, JHEP 12, 022 (2004).

[12] Yu.A.Simonov, Phys. Atom. Nucl. 69, 528 (2006); Yu.A.Simonov, V.I.Shevchenko, Adv. High En. Phys., arXiv:0902,1405.

[13] I.H.Jorysz and C.Michael, Nucl. Phys. B 302, 448 (1988).

[14] M.Foster and C.Michael, Phys. Rev. D 59, 094509 (1999).

[15] Yu.A.Simonov, Nucl. Phys. B 592, 350 (2000).
[16] Yu.A.Simonov, J.A. Tjon, Ann. Phys. 300, 54 (2002); ibid 228, 1 (1993).

[17] A.M. Badalian, A.V.Nefediev, and Yu.A.Simonov, Phys. Rev. D78, 114020 (2008).

[18] M.B.Halpern, Phys. Rev. D 19, 517 (1979); N.E.Bralic, Phys. Rev. D 22, 3090 (1980); I. Ya.Aref’eva, Theor Math. Phys. 43, 111 (1980); Yu.A.Simonov, Yad.Fiz. 50 3213 (1989); M.Hirayama, M.Ueno, Prog. Theor. Phys. 103, 151 (2000); N.G.Van Kampen, Phys. Rept. 24, 171 (1976); Physica, 74, 239 (1974).

[19] A.Yu.Dubin, A.B.Kaidalov, and Yu.A.Simonov, Phys. Lett. B 323, 41 (1994); Yad. Fiz. 56, 213 (1993); V.L.Morgunov, A.V.Nefediev and Yu.A.Simonov, Phys. Lett. B 459, 653 (1999).

[20] A.M. Badalian, B.L.G. Bakker, Phys. Rev. D 66, 034025 (2002); A.M. Badalian, V.L.Morgunov and B.L.G. Bakker, Phys. At. Nucl. 63, 1635 (2000); A.M. Badalian, B.L.G. Bakker, Phys. Rev. D 64, 114010 (2001); ibid D 67, 071901 (2003); A.M. Badalian, I.V.Danilkin, Phys. At. Nucl. 72, 1206 (2009); A.M. Badalian, A.I.Veselov, and B.L.G. Bakker, J. Phys. G 31, 417 (2005); A.M. Badalian, B.L.G. Bakker, Phys. At. Nucl. 70, 1764 (2007); Yu.S.Kalashnikova, A.V.Nefediev, and Yu.A.Simonov, Phys. Rev. D 64, 014037 (2001); S.M. Fedorov, Yu.A. Simonov, JETP Lett. 78, 57 (2003); Yu.A.Simonov, Z.Phys. C53, 419 (1992).

[21] A.B.Kaidalov and Yu.A.Simonov, Phys. Lett. B 477, 163 (2000); ibid B 636, 1 (2006); Phys. At. Nucl. 63, 1428 (2000).

[22] Yu.S.Kalashnikova and Yu.B.Yufryakov, Phys. Lett. B 359, 175 (1995); Yu.S.Kalashnikova, D.S.Kuzmenko, Phys. At.Nucl. 64, 1716 (2001), ibid. 66, 955 (2003); Yu.S.Kalashnikova, A.V.Nefediev;

[23] Yu.A.Simonov, “QCD and theory of hadrons”, in: QCD: Perturbative or Nonperturbative? L.Ferreira, P.Nogueira and J.L.Silva Marcos eds., World Scientific, Singapore, 2001; arXiv[hep-ph/9911237]; A.M. Badalian, V.I.Shevchenko, Yu.A.Simonov, Phys. At. Nucl. 69, 1781 (2006).

[24] A.Di Giacomo and H.Panagopoulos, Phys. Lett. B 285, 133 (1992); M.D’Elia, A.Di Giacomo, and E.Meggiolaro, Phys. Lett. B 408, 315
(1997); A.Di Giacomo, E.Meggiolaro and H.Panagopoulos, Nucl. Phys. B 483, 371 (1997); M.D’Elia, A.Di Giacomo, and E.Meggiolaro, Phys. Rev. D 67, 114504 (2003); G.S.Bali, N.Brambilla and A Vairo, Phys. Lett. B 42, 265 (1998).

[25] Y.Koma and M.Koma, Nucl. Phys. 769, 79 (2007).

[26] V.D.Orlovsky, V.I.Shevchenko, arXiv:0906.1340.

[27] D. Antonov, Phys. Lett. B 696, 214 (2011).

[28] M.A.Shifman, A.I.Vainshtein, and V.I.Zakharov, Nucl. Phys. B 147, 385 (1979).

[29] Yu.A.Simonov, M.A.Trusov, Phys. Lett. B650, 36 (2007); for a review see A.V.Nefediev, Yu.A.Simonov, M.A.Trusov, Int. J. Mod. Phys. E 18, 549 (2009), arXiv:0902.0125.

[30] J.M.Cornwall, Phys. Rev. D 26, 1453 (1982); G.Parisi and R.Petronzio, Phys. Lett. B 94, 51 (1980); A.C.Mattingly and P.M.Stevenson, Phys. Rev. D 49, 437 (1994).

[31] Yu.A.Simonov, Phys. At Nucl. 58, 107 (1995); arXiv:hep-ph/9311247; in: Lecture Notes in Physics v.479, p. 144; Springer, 1996; Phys. At. Nucl. 65, 135 (2002), ibid 66, 704 (2003); arXiv:hep-ph/0109081

[32] A.M. Badalian, D.S.Kuzmenko, Phys. Rev. D65, 016004 (2002), arXiv; hep-ph/0104097; A.M. Badalian, A.I. Veselov, Phys. At. Nucl. 68, 582 (2005).

[33] V.I.Shevchenko, Yu.A.Simonov, Phys. Lett. 437, 146 (1998).

[34] Yu.A.Simonov, Phys. Lett. B 464, 265 (1999).

[35] Yu.A.Simonov, QCD:Confinement, Hadron Structure and DIS in:I.Ya.Pomeranchuk and Physics at the Turn of the Century, A.Berkov, N.Narozhny and L.Okun eds., World scientific, 2003; arXiv:hep-ph/0310031

[36] Yu.A.Simonov, Phys. Uspekhi, 39, 313 (1996).
[37] D.S.Kuzmenko, V.I.Shevchenko, Yu.A.Simonov, Phys. Uspekhi, 47, 1 (2004).

[38] G.K.Savvidy, Phys. Lett. B 71, 133 (1977).

[39] H.Leutwyler, Nucl. Phys. B 179, 129 (1981).

[40] N.K.Nielsen and P.Olesen, Nucl. Phys. B 144, 376 (1978).

[41] P.Olesen, Nucl. Phys. B 200, 381 (1982); J.Ambjorn, P.Olesen, Nucl. Phys. B 170, 60, 265 (1980); J.Ambjorn, P.Olesen, C.Peterson, Nucl. Phys. B 240, 189, 533 (1984).