Beyond S, T and U

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Abstract

The contribution to precision electroweak measurements due to TeV physics which couples primarily to the $W^\pm$ and $Z$ bosons may be parameterized in terms of the three ‘oblique correction’ parameters, $S$, $T$ and $U$. We extend this parameterization to physics at much lower energies, $\gtrsim 100$ GeV, and show that in this more general case neutral-current experiments are sensitive to only two additional parameters. A third new parameter enters into the $W^\pm$ width.

1. Introduction

The standard electroweak theory has recently come of age, with experiments now probing its predictions with sufficient accuracy to test its radiative corrections in some detail. Besides providing a detailed test of the model, these precision measurements are also very useful for the constraints they impose on any potential new physics that might exist at energies higher than those that have been hitherto experimentally explored.

A particularly interesting class of new physics that is constrained by these measurements consists of models which satisfy the following three criteria [1]:

1. Introduction
• (1): The electroweak gauge group must be $SU_L(2) \times U_Y(1)$, with no new electroweak
gauge bosons apart from the photon, the $W^\pm$ and the $Z$.

• (2): The couplings of the new physics to light fermions are suppressed compared to its
couplings to the gauge bosons.

• (3): The intrinsic scale, $M$, of the new physics is large in comparison with $M_W$ and $M_Z$.

These criteria are particularly interesting principally for two reasons. First, their
implications for low-energy observables may be completely described by three parameters,
denoted $S$, $T$ and $U$ in Ref. [1]. This allows these models to be meaningfully constrained
as a group by fitting for these parameters once and for all using the presently-available
precision electroweak data [2], [3]. Second, they include a large class of well-motivated
theories, such as technicolour models, models with extra generations, multi-Higgs models,
etc.

It is the purpose of this note to extend this analysis to theories which satisfy the
first two of the above criteria, but not the third. Since this third item is used to neglect
corrections to precision measurements that are $O(M_Z^2/M^2)$, there are two situations for
which this type of extension is necessary. The most obvious case is where the new physics
is comparatively light. Depending on the properties of the hypothetical new particles, they
could have masses as light as $M \lesssim 100$ GeV and yet still have escaped direct detection at
LEP or the Tevatron. In this case they are best constrained through their contributions
through loops to precision electroweak measurements, and we present here a formalism
which may be used to do so.

The second type of application is to the scenario where new particle masses are com-
paratively large, but where low-energy measurements become sufficiently accurate that
$M_Z^2/M^2$ corrections are no longer negligible. This is the case that must be considered
when using loop effects to constrain anomalous three-gauge boson interactions, as has
been done in Refs. [4] and [5]. As we show in more detail elsewhere [6], the small size,
$O(M_W^2/M^2)$, that is to be expected for these couplings implies that they can only be de-
tected in precision experiments to the extent that $O(M_Z^2/M^2)$ cannot be neglected. In
this case the usual analysis in terms of $S$, $T$ and $U$ [5] does not apply, requiring our more
general procedure.

We show that the advantages of the previous formalism survive, and that the impli-
cations of any such model for neutral-current data may in practice be parameterized in
terms of four parameters: Peskin and Takeuchi’s $S$ and $T$, as well as two additional ones,
which we call $V$ and $X$. If the mass, width and low-energy couplings of the $W^\pm$ boson are
also included, only two more parameters are required: the quantity $U$ of Ref. [1], and one
new variable, $W$. For practical applications $W$ often need not be considered, since it only arises in absolute measurements of the $W^\pm$ widths, but cancels in its branching ratios.

Although this economy in the number of parameters required to parameterize the data is similar to the economy that was found for physics at very high scales, its origins here are very different. For physics at very high scales only three parameters are possible \textit{a-priori}, since in this case the large value for $M$ permits an effective-lagrangian description in which only the first few lowest-dimension effective interactions need be considered. The same is not true in the present case, where the new degrees of freedom can be comparatively light. The relatively few parameters which do arise in this case reflect the fact that, at present, precision measurements are made at only a very few scales, $q^2 \approx 0$ and $q^2 = M_Z^2$ or $M_W^2$. As a result the number of independent probes of new physics is limited by the few scales at which this physics is sampled with sufficient precision. This will certainly change in the future, such as at LEP-200, once other scales become available for more detailed scrutiny.

Our results reduce to the previous analyses in the limit that the new physics is heavy. Since our treatment applies to both large and comparatively small values for $M$, we are able to more quantitatively identify the boundaries of applicability of the previous description. We do so here by explicitly working through an example, in which we take the new physics to consist of a degenerate doublet of new heavy fermions. We also present this example as a sample of the type of diagnostic calculation that will become necessary should a deviation from standard physics ever be detected in the future, using these precision experiments. In this happy event, a comparison between the sizes of the new parameters $V$, $W$ and $X$ relative to the size of $S$ and $T$ can be used to infer the mass scale that is associated with the underlying new physics.

2. ‘Oblique’ Corrections

We start with the observation that, at present, precision electroweak measurements exclusively involve the two-particle scattering of light fermions. Given that the new physics is too heavy to be directly produced in these experiments, there are three ways for it to indirectly contribute. It can contribute to: (a) the propagation of the gauge bosons that can be exchanged by the fermions, (b) the three-point fermion – boson couplings, and (c) the four-point direct fermion – fermion interactions (or ‘box’-diagram corrections).

The importance of criteria (1) and (2) above is that when these are satisfied then only corrections of type (a) — the so-called ‘oblique’ corrections — are dominant. (More precisely, we may use the freedom to perform field redefinitions to put all of the new physics into the vacuum polarizations. In fact, any new gauge-fermion or four-fermion vertex in
which the fermions appear only through linear combinations of the total SM currents may be re-expressed in this way. This freedom to recast the effective lagrangian is exploited in Ref. [6].) Under these circumstances the complete impact of any new heavy degrees of freedom arises through their contributions to the gauge boson vacuum polarizations, \( \Pi_{\mu \nu}^{ab}(q) = \Pi_{\mu \nu}^{ab}(q^2) g_{\mu \nu} + (q^\mu q^\nu \text{ terms}) \), with \( a, b = \gamma, W^\pm, Z \). We therefore supplement the standard model (SM) by adding a new-physics contribution to the gauge boson vacuum polarizations:

\[
\Pi_{\mu \nu}^{ab}(q^2) = \Pi_{\mu \nu}^{ab\SM}(q^2) + \delta \Pi_{\mu \nu}^{ab}(q^2).
\]

The first contribution represents the SM contribution, including all appropriate radiative corrections, while all new-physics effects are contained in the second term.

In Refs. [1]-[3], it was assumed that the \( \delta \Pi_{\mu \nu}^{ab}(q^2) \) are due to new physics at a very high mass scale, and are thus well-described by a Taylor expansion to linear order in \( q^2 \): \( \delta \Pi_{\mu \nu}^{ab}(q^2) \approx A_{ab} + B_{ab} q^2 \). Under this assumption it is straightforward to determine the number of independent parameters required to describe all new-physics effects. The reasoning goes as follows. There are eight quantities describing the new physics: \( A_{ab} = \delta \Pi_{ab}(0) \) and \( B_{ab} = \delta \Pi_{ab}'(0) \), with the pair \( (ab) \) taking the four independent values: \( (ab) = (\gamma\gamma), (Z\gamma), (ZZ) \) and \( (WW) \). (The prime denotes differentiation with respect to \( q^2 \): \( \delta \Pi' \equiv d \delta \Pi/dq^2 \).) Two of these—\( \delta \Pi_{\gamma\gamma}(0) \) and \( \delta \Pi_{Z\gamma}(0) \)—are automatically zero by gauge invariance. Three other linear combinations can be eliminated when the three input parameters—say, \( \alpha, M_Z \) and \( G_F \)—are renormalized. Thus, all new physics effects can be described by three combinations of the \( \delta \Pi \)'s, denoted \( S, T \) and \( U \) in Ref. [1] (the precise definitions of these parameters are given later).

In the more general case we cannot assume any specific form for \( \delta \Pi_{\mu \nu}^{ab}(q^2) \), and so there are potentially many more parameters that can enter into physical observables. Our purpose in this section is to compute the dependence of well-measured quantities on these oblique corrections, and to show that in fact only a few additional parameters arise beyond the three that were introduced by Peskin and Takeuchi. This remarkable simplification reflects the fact that current precision measurements sample the vacuum polarizations only at a few values of \( q^2 \): \( q^2 \approx 0 \) and \( q^2 = M_W^2 \) or \( M_Z^2 \).

We first compute the experimental implications of the \( \delta \Pi_{\mu \nu}^{ab}(q^2) \). This task is made much simpler by the very success of the standard model itself. The agreement between the data and the SM predictions, including radiative corrections, implies that the \( \delta \Pi_{\mu \nu}^{ab}(q^2) \) cannot be larger than at most \( O(1\%) \) of the size of their tree-level SM counterparts. It follows that we may simply perturb in the oblique corrections — with one important exception which we treat in detail below — and stop at linear order. Furthermore, SM
radiative corrections to any new-physics contribution may also be ignored to within the accuracy we require. We may therefore simply work to tree-level in the oblique corrections, \( \delta \Pi_{ab} \), and then simply add the result to the corresponding SM contribution, including potential radiative corrections.

2.1) Shifting the Standard-Model Couplings

There are two distinct ways in which these quantities can enter into predictions for any particular observable. Besides contributing directly to predictions for the quantities of interest, they can also change the numerical values that are inferred from experiment for the various SM parameters, such as for the electric charge, \( e \), \( s_w \equiv \sin \theta_w \), etc. This change then shifts the SM prediction for all other quantities. We first compute this shift.

The standard electroweak interactions are parameterized by three variables, (in addition to other parameters, like fermion masses, which do not concern us here) which we denote as \( \tilde{e} \), \( \tilde{s}_w \), and \( \tilde{m}_Z \). The values for these are fixed by comparing SM predictions to the three best-measured observables, which we take to be: (i) the fine-structure constant, \( \alpha \), as measured in low-energy electron scattering, Fermi’s constant, (ii) \( G_F \), as measured in muon decay, and (iii) the Z boson mass, \( M_Z \), as measured at LEP. In order to calculate the change that the new physics implies for these parameters we must compute the contribution of the \( \delta \Pi_{ab} \)'s to these quantities.

Following the above reasoning, we compute the corrections to \( \alpha \) and \( G_F \) by computing the corrections to low-energy electron-scattering and to muon decay, working to tree level in the oblique corrections. The shift in the Z boson mass follows from its definition in terms of the pole of the propagator. This leads to the following expressions:

\[
\begin{align*}
\alpha &= \alpha_{SM}(\tilde{e}) \left[ 1 + \delta \Pi_{\gamma \gamma}'(0) \right], \\
G_F &= (G_F)_{SM}(\tilde{e}, \tilde{s}_w, \tilde{m}_Z) \left[ 1 - \frac{\delta \Pi_{WW}(0)}{M_W^2} \right], \\
M_Z^2 &= (M_Z^2)_{SM}(\tilde{e}, \tilde{s}_w, \tilde{m}_Z) \left[ 1 + \frac{\delta \Pi_{ZZ}(M_Z^2)}{M_Z^2} \right], \\
\end{align*}
\]

(2)

We may define ‘standard’ parameters by equating the left-hand sides of these expressions to their SM formulae. For example, we define \( e \) by requiring \( \alpha = \alpha_{SM}(e) \equiv 4\pi e^2 + \text{(loops)} \),
\[ G_F = \frac{e^2}{(4\sqrt{2}s_w^2c_w^2m_Z^2)} + \text{(loops), etc.} \]

This leads to the following expressions:

\[ \begin{align*}
\tilde{e} &= e \left( 1 - \frac{1}{2} \delta\Pi'_{\gamma\gamma}(0) \right), \\
\tilde{s}_w^2 &= s_w^2 \left( 1 - \frac{c_w^2}{c_w^2 - s_w^2} \left( \delta\Pi'_{\gamma\gamma}(0) - \frac{\delta\Pi_{zz}(M_Z^2)}{M_Z^2} + \frac{\delta\Pi_{WW}(0)}{M_W^2} \right) \right), \\
\tilde{m}_Z^2 &= m_Z^2 \left( 1 - \frac{\delta\Pi_{zz}(M_Z^2)}{M_Z^2} \right).
\end{align*} \]

Equation (3)

In our notation \( m_Z \) denotes the standard model parameter, as opposed to the physical quantity, \( M_Z \). We make this distinction, since these can differ once SM radiative corrections are included.

The prediction for any other observable, \( A \), may now be written \( A = A_{SM}(\tilde{e}, \tilde{s}_w, \tilde{m}_Z) + \delta A \), where the first term is the SM prediction, and where the second term is the ‘direct’ contribution of the new oblique corrections to the observable in question. In order to take advantage of the most precise radiatively corrected SM calculations, it is then useful to re-express \( A \) using eqs. (3), as \( A = A_{SM}(e, s_w, m_Z) + \delta A' \), where \( A_{SM}(e, s_w, m_Z) \) now takes the same numerical value as it does in the standard model in the absence of new physics.

2.2) Low-Energy Observables: \( S, T \) and \( U \)

The direct contributions to observables from the new physics are now easily computed. The first step comes in choosing which observables to compute. Here we meet a further simplification. The main point is that most, if not all, precision electroweak measurements are performed at very few scales. They are either at very low energies, \( q^2 \ll M_W^2 \), or at the \( Z \) resonance. To the extent that the \( W^{\pm} \) mass and widths are also regarded as being sufficiently precisely measured, then \( q^2 = M_W^2 \) is also relevant. This implies that the vacuum polarizations are only sampled at these very few scales. As a result, even though \( \delta\Pi_{ab}(q^2) \) can potentially depend on \( q^2 \) in a complicated way, its implications for precision measurements can be summarized in a few numbers.

We concentrate first on low-energy observables, for which we may take \( q^2 \approx 0 \). (We defer the treatment of quantities that are defined near the mass shell, \( q^2 = M_W^2 \) and \( M_Z^2 \) until later, since for these there is an added complication.) Examples of observables of this type include the low-energy electron scattering and muon decay processes of the previous section, as well as deep-inelastic neutrino scattering, atomic parity violation experiments etc. In this case, the new physics contributions may be straightforwardly worked out perturbatively in the \( \delta\Pi_{ab} \).
For example, low-energy measurement of parity-violating asymmetries, such as $A_{LR}$ and $A_{FB}$ in electron scattering, can be used to define an effective value for $\sin^2 \theta_w$. This is given by:

$$(s_w^2)_{\text{eff}}(q^2 \approx 0) = s_w^2 - s_w c_w \delta \Pi'_{Z\gamma}(0)$$

$$= s_w^2 \left[1 - \frac{c_w^2}{c_w^2 - s_w^2} \left(\delta \Pi'_{\gamma\gamma} - \frac{\delta \Pi_{zz}(M_Z^2)}{M_Z^2} + \frac{\delta \Pi_{ww}(0)}{M_W^2}\right) - \frac{c_w}{s_w} \delta \Pi'_{Z\gamma}(0)\right]. \quad (4)$$

Clearly the effects of the new physics on any such observable are given by the induced shifts in the SM parameters, as well as a linear combination of the various $\delta \Pi_{ab}$’s — or their derivatives — evaluated at $q^2 \approx 0$. As a result they never probe the oblique corrections beyond linear order in their expansions in powers of $q^2$, and so they are completely described in terms of the three Peskin-Takeuchi parameters, $S$, $T$ and $U$:

$$\frac{\alpha S}{4 s_w^2 c_w^2} = \left[\frac{\delta \Pi_{zz}(M_Z^2)}{M_Z^2} - \delta \Pi_{zz}(0)\right] - \frac{(c_w^2 - s_w^2)}{s_w c_w} \delta \Pi'_{Z\gamma}(0) - \delta \Pi'_{\gamma\gamma}(0), \quad (5)$$

$$\alpha T = \frac{\delta \Pi_{ww}(0)}{M_W^2} - \frac{\delta \Pi_{zz}(0)}{M_Z^2}, \quad (6)$$

$$\frac{\alpha U}{4 s_w^2} = \left[\frac{\delta \Pi_{ww}(M_W^2)}{M_W^2} - \delta \Pi_{ww}(0)\right] - c_w^2 \left[\frac{\delta \Pi_{zz}(M_Z^2)}{M_Z^2} - \delta \Pi_{zz}(0)\right] - s_w^2 \delta \Pi'_{\gamma\gamma}(0) - 2 s_w c_w \delta \Pi'_{Z\gamma}(0). \quad (7)$$

These definitions are deliberately cast in a way that does not assume that the $\delta \Pi_{ab}(q^2)$ are linear functions of $q^2$. Notice that although these expressions agree with the formulation of $S$, $T$ and $U$ given in Ref. [2], the analysis of these authors only applies when $\delta \Pi_{ab}(q^2) = A_{ab} + B_{ab} q^2$. With these definitions eq. (4) takes the usual form:

$$\frac{(s_w^2)_{\text{eff}}(q^2 \approx 0)}{(s_w^2)_{SM}} = 1 + \frac{\alpha S}{4 s_w^2 (c_w^2 - s_w^2)} - \frac{c_w^2}{c_w^2 - s_w^2} \alpha T. \quad (8)$$

Any other low-energy observable may be analyzed in a similar fashion.

2.3) Observables on Mass Shell: Beyond $S$, $T$ and $U$

A slightly different analysis is required when considering observables that are defined at $q^2 = M_Z^2$ or $M_W^2$, as is appropriate for the masses and widths of the gauge bosons themselves. This is because it is no longer a good approximation to simply work to linear
order in the two quantities $\delta \Pi_{zz}(M_Z^2)$ and $\delta \Pi_{WW}(M_W^2)$ when these quantities are inserted into boson lines which are on shell. This is because when these insertions are combined with the very small denominators of the corresponding gauge boson propagators in these lines (which are themselves $O(\delta \Pi)$) their contributions can be $O(1)$. In this case the vacuum polarization insertions must be summed to all orders in the usual way, using the Schwinger-Dyson equations.

The result is simple to state, however. The shift in the gauge boson mass is simply given by the (real part of the) position of the pole in the corresponding propagator. We take as our first example the mass of the $W^\pm$, which is given in the SM by $(M_W^2)_{SM}(\tilde{e}, \tilde{s}_w, \tilde{m}_Z) = \tilde{m}_Z^2 \tilde{c}_w^2 + (\text{loop corrections})$. The oblique corrections change this to

$$M_W^2 = (M_W^2)_{SM}(\tilde{e}, \tilde{s}_w, \tilde{m}_Z) \left[ 1 + \frac{\delta \Pi_{WW}(M_W^2)}{M_W^2} \right].$$  \hfill (9)

Shifting to the ‘standard’ parameters then gives:

$$M_W^2 = (M_W^2)_{SM}(e, s_w, m_Z) \left[ 1 - \frac{c_w^2}{c_w^2 - s_w^2} \frac{\delta \Pi_{zz}(M_Z^2)}{M_Z^2} + \frac{s_w^2}{c_w^2 - s_w^2} \left( \delta \Pi'_{\gamma\gamma}(0) + \frac{\delta \Pi_{WW}(0)}{M_W^2} \right) + \frac{\delta \Pi_{WW}(M_W^2)}{M_W^2} \right].$$  \hfill (10)

Let us now consider the $W^\pm$ width. This is obtained by multiplying the lowest-order SM result by the renormalization factor, $Z_W = 1 + \delta \Pi_{WW}(M_W^2)$, that arises due to the use of the fully summed propagator. As a result:

$$\delta \Gamma(W \to e\nu) = \frac{e^2 M_W}{48\pi \tilde{s}_w^2} \delta \Pi'_{WW}(M_W^2).$$  \hfill (11)

Notice that we use the full mass, $M_W$, in this expression since this is what appears in the phase space integration. Transforming to ‘standard’ parameters leads to:

$$\frac{\Gamma(W \to \text{all})}{\Gamma_{SM}(W \to \text{all})} = \frac{\Gamma(W \to e\nu)}{\Gamma_{SM}(W \to e\nu)} = 1 + \frac{1}{c_w^2 - s_w^2} \left[ s_w^2 \delta \Pi'_{\gamma\gamma}(0) + c_w^2 \left( \frac{\delta \Pi_{WW}(0)}{M_W^2} - \frac{\delta \Pi_{zz}(M_Z^2)}{M_Z^2} \right) \right] + \delta \Pi_{WW}(M_W^2).$$  \hfill (12)
The $Z$ width into neutrinos may be computed in an identical way. Proceeding along precisely the same lines as for the $W^\pm$ width, and using $Z_Z = 1 + \delta \Pi_{ZZ}(M_Z^2)$, gives the following result:

$$\frac{\Gamma(Z \rightarrow \nu \overline{\nu})}{\Gamma_{SM}(Z \rightarrow \nu \overline{\nu})} = 1 - \frac{\delta \Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\delta \Pi_{WW}(0)}{M_W^2} + \delta \Pi_{ZZ}^\prime(M_Z^2).$$

(13)

A new feature enters into the calculation of the $Z$-boson widths into charged-particle final states, since these receive contributions from $\delta \Pi_{Z\gamma}(M_Z^2)$. Unlike the effects due to the insertion of $\delta \Pi_{ZZ}$, the contribution of this term may be computed perturbatively, since the gauge boson line is not forced to be on shell. The result is a contribution to the effective value for $s_w^2$ that is measured in $A_{LR}$, $A_{FB}$ at the $Z$ resonance. We find:

$$\left(\frac{s_w^2}{s_w^2}_{\text{eff}}(q^2 \approx M_Z^2) - \frac{s_w^2}{s_w^2}_{\text{eff}}(q^2 \approx 0)\right) = s_w c_w \left[\frac{\delta \Pi_{Z\gamma}(M_Z^2)}{M_Z^2} - \delta \Pi_{Z\gamma}^\prime(0)\right].$$

(14)

At this point a dramatic simplification occurs. All of the new physics then combines into compact expressions involving Peskin and Takeuchi’s three parameters, as well as the following three new ones, which we call $V$, $W$ and $X$:

$$\alpha V = \delta \Pi_{ZZ}^\prime(M_Z^2) - \left[\frac{\delta \Pi_{ZZ}(M_Z^2) - \delta \Pi_{ZZ}(0)}{M_Z^2}\right],$$

(15)

$$\alpha W = \delta \Pi_{WW}(M_W^2) - \left[\frac{\delta \Pi_{WW}(M_W^2) - \delta \Pi_{WW}(0)}{M_W^2}\right],$$

(16)

$$\alpha X = -s_w c_w \left[\frac{\delta \Pi_{Z\gamma}(M_Z^2)}{M_Z^2} - \delta \Pi_{Z\gamma}^\prime(0)\right].$$

(17)

Notice that these expressions would vanish if $\delta \Pi_{ab}(q^2)$ were simply a linear function of $q^2$.

In terms of these parameters our previous expressions for the $W^\pm$ and $Z$ widths, and $(s_w^2)_{\text{eff}}(M_Z^2)$, become:

$$\frac{\Gamma(W \rightarrow \text{all})}{\Gamma_{SM}(W \rightarrow \text{all})} = 1 - \frac{\alpha S}{2(c_w^2 - s_w^2)} + \frac{c_w^2 \alpha T}{4s_w^2 - s_w^2} + \alpha U + \alpha W,$$

(18)

$$\frac{\Gamma(Z \rightarrow \nu \overline{\nu})}{\Gamma_{SM}(Z \rightarrow \nu \overline{\nu})} = 1 + \alpha T + \alpha V,$$

(19)

$$\left(\frac{s_w^2}{s_w^2}_{\text{eff}}(q^2 \approx M_Z^2) - \frac{s_w^2}{s_w^2}_{\text{eff}}(q^2 \approx 0)\right) = \frac{\alpha S}{4s_w^2(c_w^2 - s_w^2)} - \frac{c_w^2 \alpha T}{c_w^2 - s_w^2} + \alpha X.$$
Notice that one of these new parameters, $W$, turns out to appear only in this expression for $\Gamma_W$. The $Z$ width into any fermion pairs may be expressed in terms of the remaining two parameters, $V$ and $X$, where $V$ describes a contribution to the overall normalization of the strength of the interaction, and $X$ acts to shift the effective value of $(s^2_w)_{\text{eff}}$.

2.4) Numerical Results

We tabulate the contributions to some precision measurements in Table (1). In preparing this table we use the following numerical values in obtaining these results: $\alpha(M_Z) = 1/127.8$, $s^2_w(M_Z) = 0.2323$, and $M_Z = 91.17$ GeV. For the SM predictions we choose the fiducial values, $m_t = 140$ GeV and $m_H = 100$ GeV. When appropriate our numbers clearly reduce to the results of Ref. [2], from whom we also have taken the experimental limits.

Several features come to light on inspection of Table (1). First, for neutral-current data at low energies and on the $Z$ resonance, only the four parameters $S$, $T$, $V$ and $X$ arise. Of these, only the standard two, $S$ and $T$, contribute to low-energy observables for which $q^2 \approx 0$, since $V$ and $X$ appear only in observables that are defined at $q^2 = M_Z^2$. As a result all of the predictions for low-energy quantities agree with earlier work [2]. In particular, the favouring of negative values for $S$ by the cesium atomic parity violation experiments is not affected by the introduction of the additional parameters.

Next, although the $Z$ results are the most precise, some $W^\pm$ properties, such as measurements of $M_{W^\pm}$, are sufficiently accurate to competitively bound the relative parameters. For these charged-current observables two more parameters enter: the usual quantity $U$ for quantities defined at $q^2 \approx 0$ (as well as $M_{W^\pm}$), and the parameter $W$ in the $W^\pm$-boson decay widths.

We next apply these results to an illustrative example.

3. ‘Technifermions’: An Example

New, massive fermions which carry electroweak quantum numbers, but which do not mix appreciably with ordinary light fermions, furnish an concrete model to which the above reasoning applies. If these fermions are sufficiently heavy, say $m \gtrsim 1$ TeV, their implications may be summarized as contributions to $S$, $T$ and $U$. We wish to explore here the much lighter mass range, $m \sim (\text{several hundred GeV})$, that can still be consistent with such particles not having been detected at current accelerators. Our goal is to show how

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1 The absence of $U$ in these expressions follows from our choice of three standard-model input observables.
the formalism just presented can be used to constrain the properties of such particles. As a bonus we can vary the fermion mass, and determine quantitatively at what point the usual three-parameter description becomes sufficiently accurate.

We therefore require the vacuum polarization that is induced by such a collection of fermions. Evaluating the graph of Fig. (1) produces the following result:

\[
\delta \Pi_{ab}(q^2) = \frac{1}{2\pi^2} \sum_{ij} \int_0^1 dx \ f_{ab}(q^2, x) \ln \left[ \frac{m_{ij}^2(x) - q^2 x(1-x)}{\mu^2} \right], \tag{21}
\]

where \( m_{ij}^2(x) \equiv m_i^2(1-x) + m_j^2 x \), and

\[
f_{ab}(q^2, x) = \frac{g_{ab}^a g_{ab}^b}{2} \left[ x(1-x) q^2 - \frac{m_{ij}^2(x)}{2} \right] + \frac{g_{ab}^a g_{ab}^b}{2} \left( \frac{m_i m_j}{2} \right). \tag{22}
\]

In these expressions, \( m_i \) and \( m_j \) are the masses of the fermions which circulate in the loop, and \( g_{ab}^a \) and \( g_{ab}^b \) represent their left- and right-handed couplings to the gauge-bosons: \( a = \gamma, W^\pm, Z \). For a standard-model doublet \( g_L^g = g_R^g = e Q_i, g_L^z = (e/s_w c_w) [T_3 - Q_i s_w^2], g_R^z = (e/s_w c_w) [-Q_i s_w^2], g_L^\nu = e/\sqrt{2}s_w \) and \( g_R^\nu = 0 \). We have renormalized \( \delta \Pi_{ab} \) using \( \overline{\text{MS}} \), and \( \mu^2 \) is the associated renormalization scale.

For simplicity we consider only the specific case of one additional doublet of degenerate leptons, for which \( m_i = m_j \equiv m \). Then

\[
\delta \Pi_{\gamma \gamma}(q^2) = \frac{e^2 q^2}{2\pi^2} \int_0^1 dx \ x(1-x) \ln \left[ \frac{m^2 - q^2 x(1-x)}{\mu^2} \right], \tag{23}
\]

\[
\delta \Pi_{w \gamma}(q^2) = \frac{e^2 s_w q^2}{2\pi^2 c_w} \int_0^1 dx \ x(1-x) \ln \left[ \frac{m^2 - q^2 x(1-x)}{\mu^2} \right], \tag{24}
\]

\[
\delta \Pi_{ww}(q^2) = \frac{e^2}{8\pi^2 s_w^2} \int_0^1 dx \ \left( q^2 x(1-x) - \frac{m^2}{2} \right) \ln \left[ \frac{m^2 - q^2 x(1-x)}{\mu^2} \right], \tag{25}
\]

\[
\delta \Pi_{z z}(q^2) = \frac{e^2}{16\pi^2 s_w^2 c_w^2} \int_0^1 dx \ \left[ (1 - 2s_w^2 + 4s_w^4) q^2 x(1-x) - m^2 \right] \ln \left[ \frac{m^2 - q^2 x(1-x)}{\mu^2} \right]. \tag{26}
\]

Using these expressions in the definitions, eqs. (5) and (15), gives the parameters \( S \) through \( X \) as functions of the mass of the doublet. The parameter \( T \), which is a measure of custodial symmetry breaking, vanishes since the doublet is degenerate. We plot the behaviour of the remainder of these parameters against \( m \) in Fig. (2). We use \( \mu = m \) in this figure.
The curves in Fig. (2) verify the dominance of the parameter $S$ when $m$ is large. The parameters $V$, $W$ and $X$ fall quite quickly to zero, as might be expected. Interestingly, they are always larger than $U$, even for $m$ as large as 1 TeV. This is because in this particular example the custodial symmetry is unbroken [8]. In general, however, $U$ is not expected to be quite this small. Nevertheless, this does suggest that if fits which include $U$ are to be performed using precision data, then the same fits should also include $V$, $W$ and $X$.

4. Conclusions

The $S$-$T$-$U$ parameterization of oblique electroweak corrections, as presented in Ref. [1], has proven to be a useful tool for constraining new physics from above the electroweak scale. It is useful because it summarizes into a few quantities the implications for precision electroweak experiments of a broad class of interesting models. One of its limitations is that it can only be applied when the scale, $M$, of new physics is high enough to justify the neglect powers of $M_{Z}/M^{2}$. We have extended the analysis to the case where the threshold for new physics is too low to justify this approximation.

We find that even for the case of comparatively light new physics, precision electroweak measurements are only sensitive to a small number of independent parameters. Precisely three new ones are required, which we call $V$, $W$ and $X$. The dependence on these parameters of many of the well-measured observables is summarized in Table (1). Neutral-current data is completely described by four quantities, $S$, $T$, $V$ and $X$, of which the latter two only contribute to observables defined at the $Z$ resonance. A description of $W^{\pm}$ physics requires both the Peskin-Takeuchi $U$ parameter, and the additional variable $W$, although $W$ will only be relevant once measurements of the $W^{\pm}$ width considerably improve.

So few parameters suffice because at present precision experiments are confined to light-fermion scattering with four-momentum transfer that is either equal to, or much lower than, $M_{W}$ or $M_{Z}$. As a result the $q^{2}$-dependence of any oblique corrections is only sampled for two or three places. This emphasizes the importance of performing precision measurements at other values of $q^{2}$, such as will be possible at LEP-200, since these experiments can probe complementary facets of the underlying physics.

We have illustrated how the parameters $S$ through $X$ can arise using a particularly simple example of a degenerate doublet of heavy fermions. This permits us to quantitatively follow the heavy-mass dependence of all six quantities. We verify how $S$ comes to dominate as the doublet mass grows. In our example, however, the parameter $U$ is never
larger than $V$, $W$ and $X$. In the future such an analysis could prove useful if a discrepancy between the standard model and these measurements were ever to arise. In this case the comparison between the sizes of the new parameters with $S$ and $T$ can be used to infer the mass scale of the new physics that is involved.

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Figure Captions

- *Figure (1)*: The Feynman graph through which the heavy fermion doublet contributes to the gauge boson vacuum polarization.

- *Figure (2)*: A plot of the oblique-correction parameters $S$, $U$, $V$, $W$ and $X$ against the mass of the heavy doublet of degenerate fermions which generate them. The parameter $T$ vanishes identically because the doublet is degenerate.
5. References

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[8] We thank T. Takeuchi for correspondence on this point.
'Oblique' Correction Parameters

M (GeV) (10^2)

S

-\( S \)

-\( V \)

-\( W \)

-\( U \)

-\( X \)
| Observable                        | Prediction                                                                 | Present Constraint   |
|----------------------------------|-----------------------------------------------------------------------------|----------------------|
| Cs Parity Violation              | $Q_W(\beta^{133}\text{Cs}) = -73.20 - 0.8S - 0.005T$                      | $-71.04 \pm 1.58 \pm 0.8$ |
| W Mass                           | $M_W = 80.20 - 0.29S + 0.45T + 0.34U$ GeV                                   | $80.14 \pm 0.31$ GeV |
| $\Gamma(W \rightarrow \text{all})$ | $\Gamma/\Gamma_{SM} = 1 - 0.0073S + 0.011T + 0.0084U + 0.0078W$            | 1.02 $\pm 0.05^*$    |
| $\Gamma(Z \rightarrow \nu\bar{\nu})$ | $\Gamma/\Gamma_{SM} = 1 + 0.0078T + 0.0078V$                              | 0.992 $\pm 0.036$    |
| $\Gamma(Z \rightarrow e^+e^-)$   | $\Gamma/\Gamma_{SM} = 1 - 0.0021S + 0.0093T - 0.0044X + 0.0078V$           | 1.004 $\pm 0.011$    |
| $\Gamma(Z \rightarrow \text{all})$ | $\Gamma/\Gamma_{SM} = 1 - 0.0038S + 0.011T - 0.0082X + 0.0078V$           | 1.002 $\pm 0.008$    |
| $\Gamma(Z \rightarrow \text{all})$ | $\Gamma/\Gamma_{SM} = 1 - 0.0038S + 0.011T - 0.0082X + 0.0078V$           | 1.002 $\pm 0.008$    |
| $\Gamma(Z \rightarrow \text{all})$ | $\Gamma/\Gamma_{SM} = 1 - 0.0038S + 0.011T - 0.0082X + 0.0078V$           | 1.002 $\pm 0.008$    |
| $Z$ Asymmetries                  | $(s_\omega^2)^{\text{exp}}/(s_\omega^2)^{\text{SM}} = 1 + 0.016S - 0.011T + 0.034X$ | 0.978 $\pm 0.056$    |
| $eD$ Asymmetry                   | $(s_\omega^2)^{\text{exp}}/(s_\omega^2)^{\text{SM}} = 1 + 0.016S - 0.011T$ | 0.965 $\pm 0.086$    |
| $eC$ Asymmetry                   | $(s_\omega^2)^{\text{exp}}/(s_\omega^2)^{\text{SM}} = 1 + 0.016S - 0.011T$ | 0.86 $\pm 0.22$      |
| $R \equiv \sigma(\nu_\mu e)/\sigma(\overline{\nu}_\mu e)$ | $R/R_{SM} = 1 - 0.029S + 0.021T$                                          | 0.997 $\pm 0.11$     |

(*) CDF, K. Einsweiler, private communication.

Table (1)

The new physics contributions to various well-measured electroweak observables. We use top-quark and Higgs masses of 160 GeV and 100 GeV for the Standard Model numbers. Other quantities are as discussed in the text.