Lorentz-violating corrections on the hydrogen spectrum induced by a non-minimal coupling

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The influence of a Lorentz-violating fixed background on fermions is considered by means of a torsion-free non-minimal coupling. The non-relativistic regime is assessed and the Lorentz-violating Hamiltonian is determined. The effect of this Hamiltonian on the hydrogen spectrum is determined to first-order evaluation (in the absence of external magnetic field), revealing that there appear some energy shifts that modify the fine structure of the spectrum. In the case the non-minimal coupling is torsion-like, no first order correction shows up in the absence of an external field; in the presence of an external field, a secondary Zeeman effect is implied. Such effects are then used to set up stringent bounds on the parameters of the model.

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I. INTRODUCTION

Since the pioneering work by Carroll-Field-Jackiw \cite{1}, Lorentz-violating theories have been extensively studied and used as an effective probe to test the limits of Lorentz covariance. Nowadays, these theories are encompassed in the framework of the Extended Standard Model (SME), conceived by Colladay and Kostelecký \cite{2} as a possible extension of the minimal Standard Model of the fundamental interactions. The SME admits Lorentz and CPT violation in all sectors of interactions by incorporating tensor terms (generated possibly as vacuum expectation values of a more fundamental theory) that account for such a breaking. Actually, the SME model sets out as an effective model that keeps unaffected the $SU(3) \times SU(2) \times U(1)$ gauge structure of the underlying fundamental theory while it breaks Lorentz symmetry at the particle frame.

Concerning the gauge sector of the SME, many studies have been developed that focus on many different respects \cite{2,3}. The fermion sector has been investigated as well, initially by considering general features (dispersion relations, plane-wave solutions, and energy eigenvalues) \cite{2}, and later by scrutinizing CPT-violating probing experiments conceived to find out in which extent the Lorentz violation may turn out manifest and to set up upper bounds on the breaking parameters. The CPT theorem, valid for any local Quantum Field Theory, predicts the equality of some quantities (life-time, mass, gyromagnetic ratio, charge-to-mass ratio) for particle and anti-particle. Thus, in the context of quantum electrodynamics, the most precise and sensitive tests of Lorentz and CPT invariance refer to comparative measurement of these quantities. A well-known example of this kind of test involves high-precision measurement of the gyromagnetic ratio \footnote{Electron and positron confined in a Penning trap for a long time.} for electron and positron confined in a Penning trap for a long time. The unsuitability of the usual figure of merit adopted in these works, based on the difference of the g-factor for electron and positron, was shown in...
refs. [8], in which an alternative figure of merit was proposed, able to constrain the Lorentz-violating coefficients (in the electron-positron sector) to 1 part in $10^{20}$. Other interesting and precise experiments, also devised to establish stringent bounds on Lorentz violation, proposed new figures of merit involving the analysis of the hyperfine structure of the muonium ground state [9], clock-comparison experiments [10], hyperfine spectroscopy of hydrogen and anti-hydrogen [11], and experiments with macroscopic samples of spin-polarized matter [12].

The influence of Lorentz-violating and CPT-odd terms specifically on the Dirac equation has been studied in refs. [13], with the evaluation of the nonrelativistic Hamiltonian and the associated energy-level shifts. A similar investigation searching for deviations on the spectrum of hydrogen has been recently performed in ref. [14], where the nonrelativistic Hamiltonian was derived directly from the modified Pauli equation. Some interesting energy-level shifts, such as a Zeeman-like splitting, were then reported. These results may also be used to set up bounds on the Lorentz-violating parameters.

In another paper involving the fermion sector [15], the influence of a non-minimally coupled Lorentz-violating background on the Dirac equation has been investigated. It was then shown that such a coupling, given at the form $\epsilon_{\mu\nu\alpha\beta}\gamma^\mu v^\nu F^\alpha\beta$, is able to induce topological phases (Aharonov-Bohm and Aharonov-Casher [16]) at the wave function of an electron (interacting with the gauge field and in the presence of the fixed background). Lately, in connection with this particular effect, it has been shown that (non-minimally coupled) particles and antiparticles develop opposite A-Casher phases. This fact, in the context of a suitable experiment, may be used to constrain the Lorentz-violating parameter [17]. In these papers, however, it was not addressed the issue concerning the nonrelativistic corrections induced by this kind of coupling in an atomic system.

The present work has as its main goal to examine the effects of the Lorentz-violating background, whenever non-minimal coupled as in ref. [15], on the Dirac equation, with special attention to its nonrelativistic regime and possible implications on the hydrogen spectrum. The starting point is the Dirac Lagrangian supplemented by Lorentz and CPT-violating terms. The investigation of the nonrelativistic limit is performed and the Lorentz-violating Hamiltonian is written down. The effect of the background on the spectrum of hydrogen atom is then evaluated by considering a first-order perturbation. In the absence of external magnetic field, it is verified that three different corrections are attained, able to modify the fine structure of the spectrum. For the case of the torsion-like non-minimal coupling, $g_\alpha\epsilon_{\mu\nu\alpha\beta}\gamma^\mu v^\nu F^\alpha\beta$, no correction is found out. In the presence of an external field, this term yields a Zeeman splitting proportional to the background magnitude. The theoretical modifications here obtained are used to set up stringent bounds on the magnitude of the corresponding Lorentz-violating coefficient.

This paper is outlined as follows. In Sec. II, it is analyzed the influence of the non-minimal coupling on the nonrelativistic limit of the Dirac equation, focusing on the possible corrections induced on the spectrum of the hydrogen atom. This is done both for a torsion-free and torsion-like non-minimal coupling. In Sec. III, we present our Final Remarks.

II. NON-MINIMAL COUPLING TO THE GAUGE FIELD AND BACKGROUND

The non-minimal coupling of the particle to the Lorentz-violating background is here considered in two versions: a torsion-free and a torsion-like coupling. We begin by analyzing the torsion-free case, which is implemented by defining a covariant derivative with non-minimal coupling, as below:

$$D_\mu = \partial_\mu + ieA_\mu + igv^\nu F^*_{\mu\nu},$$

(1)

where $F^*_{\mu\nu}$ is the dual electromagnetic tensor ($F^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$). In this situation, the additional term sets a non-minimal coupling of the fermion sector to a fixed background $v^\mu$, responsible for the breaking of Lorentz symmetry [3] at the particle frame. The mass dimensions of the gauge field and the coupling constant are: $[A_\mu] = 1, [g] = -2$. The Dirac equation with such a coupling,

$$(ih\gamma^\mu D_\mu - m_e c)\Psi = 0,$$

(2)
is the starting point to investigate the influence of this background on the dynamics of the fermionic particle. Working with the Dirac representation\(^1\) of the \(\gamma\)-matrices, and writing \(\Psi\) in terms of two-component spinors, 
\[
\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix},
\]
there follow two coupled equations for \(\phi\) and \(\chi\) in momentum space:
\[
\begin{align*}
\left(\frac{E}{c} - m_e c - e A_0/c + g \vec{v} \cdot \vec{B}\right) \phi - \vec{\sigma} \cdot \left(\vec{p} - e \vec{A}/c + g v_0 \vec{B} - g \vec{v} \times \vec{E}\right) \chi &= 0, \\
- \left(\frac{E}{c} + m_e c - e A_0/c + g \vec{v} \cdot \vec{B}\right) \chi + \vec{\sigma} \cdot \left(\vec{p} - e \vec{A}/c + g v_0 \vec{B} - g \vec{v} \times \vec{E}\right) \phi &= 0.
\end{align*}
\]
(3)
(4)

To investigate the low-energy behavior of this system, the natural option is to search for its nonrelativistic limit, where the energy is given as \(E = m_e c^2 + H\), with \(H\) being the nonrelativistic Hamiltonian. Writing the weak component (\(\chi\)) in terms of the strong one (\(\phi\)), the following equation for \(\phi\) holds:
\[
\left(\frac{H}{c} - e A_0/c + g \vec{v} \cdot \vec{B}\right) \phi = \frac{1}{2 m_e c} \left(\vec{\sigma} \cdot \vec{\Pi}\right) \left(\vec{\sigma} \cdot \vec{\Pi}\right) \phi,
\]
where the generalized canonical moment is defined as 
\[
\vec{\Pi} = \left(\vec{p} - e \vec{A}/c + g v_0 \vec{B} - g \vec{v} \times \vec{E}\right).
\]
After some algebra, the nonrelativistic Hamiltonian for the particle comes out:
\[
H = \left\{\frac{1}{2 m_e} \left(\vec{p} - e \vec{A}/c\right)^2 + e A_0 - \frac{e \hbar}{2 m_e c} \left(\vec{\sigma} \cdot \vec{B}\right) + \frac{g \hbar}{2 m_e} (\vec{v} \times \vec{B})^2 + \frac{\hbar}{2 m_e} g v_0 \vec{\sigma} \cdot (\vec{\nabla} \times \vec{B})
- \frac{g \hbar}{2 m_e} \vec{\sigma} \cdot \left[\vec{\nabla} \times (\vec{v} \times \vec{E})\right] + \frac{g v_0}{m_e} \left(\vec{p} - e \vec{A}/c\right) \cdot \vec{B} - \frac{g}{m_e} \left(\vec{p} - e \vec{A}/c\right) \cdot (\vec{v} \times \vec{E}) - \frac{g v_0}{m_e} \vec{B} \cdot (\vec{v} \times \vec{E})\right\}.
\]
(5)

In the expression above, there appears the Pauli Hamiltonian (between brackets) corrected by the terms that compose the Lorentz-violating Hamiltonian, \(H_{LV}\), which truly constitutes our object of interest. The purpose is to investigate the contribution of the Hamiltonian with Lorentz-violation \((H_{LV})\) in the states of the hydrogen atom. Such a calculation will be initially performed for the case of a free hydrogen atom (without external field, \(\vec{A} = 0\)), for which only three terms contribute. For all the terms that do not involve the spin operator, we shall use the hydrogen 1-particle wave functions \((\Psi)\) labeled in terms of the quantum numbers \(n, l, m, \Psi_{nlm}(r, \theta, \phi) = R_{nl}(r)\Theta_{lm}(\theta)\Phi_m(\phi)\), whereas the evaluation of the terms involving \(\vec{\sigma}\) requires the use of the wave function \(\Psi_{nljm, m_z}\), with \(j, m_j\) being the quantum numbers suitable to deal with the addition of angular momentum. Here, \(r, \theta, \phi\) are spherical coordinates.

As our initial evaluation\(^2\), we consider the first-order correction induced by the term \(g^2 \langle \vec{v} \times \vec{E} \rangle^2 / 2 m_e\), namely:
\[
\Delta E_1 = \frac{g^2 \hbar}{2 m_e} \int \Psi_{nlm}^* (\vec{v} \times \vec{E})^2 \Psi_{nlm} d^3r.
\]
(6)

To solve it, we write \(\langle \vec{v} \times \vec{E} \rangle^2 = v^2 E^2 - (\vec{v} \cdot \vec{E})^2\) and take the Coulombian electric field given by \(\vec{E} = -e \hat{r} / r^2\), so that the result is:
\[
\Delta E_1 = \frac{g^2 e^2}{2 m_e} \left[v^2 \langle nlm|1/r^4|nlm\rangle - \langle nlm|(\vec{v} \cdot \hat{r})^2/r^4|nlm\rangle - \langle nlm|\vec{v}^2/r^4|nlm\rangle\right].
\]
(7)

In spherical coordinates, \(\vec{v} \cdot \hat{r} = v_z \sin \theta \cos \phi + v_y \sin \theta \sin \phi + v_x \cos \theta\), which leads us to:
\[
\Delta E_1 = \frac{g^2 e^2}{4 m_e} \left[\left(\frac{1}{r^4}\right)(v_x^2 + v_y^2 + 2 v_z^2) + (v_x^2 + v_y^2 - 2 v_z^2) \left\langle nlm|\frac{\cos^2 \theta}{r^4}|nlm\rangle\right\rangle\right].
\]

\(^1\) In the such a representation, the Dirac matrices are written as: \(\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\), where \(\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)\) are the Pauli matrices.

\(^2\) It is worthwhile to mention here that all calculations have been carried out in the Gaussian unit system, adopted through this work.
Considering the intermediate result,
\[ \left\langle nlm | \frac{\cos^2 \theta}{r^4} | nlm \right\rangle = \frac{1}{r^4} \left[ \frac{(l^2 - m^2)}{(2l-1)(2l+1)} + \frac{(l^2 - m^2 + 2l + 1)}{(2l+3)(2l+1)} \right], \]
the following energy correction is obtained for the case the background is aligned along the z-axis (\( \vec{v} = v_z \hat{z} \)):
\[ \Delta E_1 = \frac{g^2 e^2 v_z^2}{4m_e} \left( \frac{1}{r^4} \right) \left[ 1 - \left( \frac{(l^2 - m^2)}{(2l-1)(2l+1)} + \frac{(l^2 - m^2 + 2l + 1)}{(2l+3)(2l+1)} \right) \right]. \tag{8} \]
where \( (1/r^4) = \langle nlm|1/r^4|nlm \rangle = 3|1-l(l+1)/3n^2|/|n^3a_0^3(l+3/2)(l+1)(l+1/2)(l-1/2)| \) is a well-known result for the hydrogen system. Here, \( a_0 = \hbar^2/e^2m_e \) is the Bohr radius \( (a_0 = 0.0529nm) \). This result shows that the non-minimal coupling is able to remove the accidental degeneracy, regardless the spin-orbit interaction. This effect, therefore, implies a modification on the fine structure of the spectrum. The order of magnitude of this correction is given by the ratio \( g^2 e^2 v_z^2/(m_e a_0^3) \), which is numerically \( 2 \times 10^{53} (gv_z)^2 \) eV. Considering that spectroscopic experiments are able to detect effects of one part in \( 10^{10} \) in the spectrum, the correction \( \Delta E_1 \) may not be larger than \( 10^{-10} \) eV, which implies an upper bound for the product \( gev_z \), namely: \( gev_z \leq 10^{-32} \).

In the absence of an external magnetic field, the next term to be taken into account is \( \vec{A}(\vec{r} - e \vec{A}) \cdot (\vec{v} \times \vec{E}) \), whose non-trivial part is \( \frac{q}{m_e} \vec{A} \cdot (\vec{v} \times \vec{E}) \). Hence, the first-order energy correction is:
\[ \Delta E_2 = -i\hbar m_e \int \Psi_{nlm}^* \nabla \cdot (\vec{v} \times \vec{E}) \Psi_{nlm} d^3r = -i\hbar \frac{q}{m_e} \int \Psi^* \nabla \cdot (\vec{v} \times \vec{E}) \Psi d^3r = -i\hbar \frac{q}{m_e} \int \Psi^* (\vec{v} \times \vec{E}) \cdot \nabla \Psi d^3r. \tag{9} \]
Taking the gradient of \( \Psi \) in spherical coordinates, and the scalar product with \( (\vec{v} \times \vec{E}) \), many terms are obtained that depend linearly on \( \sin \phi \), \( \cos \phi \) or \( \sin 2\phi \), except for two of them. These are the ones that survive after the angular integration is performed. The remaining expression is:
\[ \Delta E_2 = \frac{egv_z m_e}{m_e} \int R_{nl}(r) \Theta_{nlm}(\theta) \frac{1}{r^l} R_{nl}(r) \Theta_{nlm}(\theta) r^2 \sin \theta d\theta = \frac{egv_z m_e}{m_e} \left( \frac{1}{r^3} \right). \]
which can be explicitly written as:
\[ \Delta E_2 = \frac{egv_z m_e}{m_e} \frac{m}{a_0^3 l(l+1/2)(l+1)}, \tag{11} \]
where the well-known result \( (1/r^3) = [a_0^3 l(l+1/2)(l+1)]^{-1} \) has been used. A previous superficial examination of eq. \( 9 \) could lead to the misleading expectation of a vanishing result, once it consists of the average of a linear function of the momentum \( (p) \) on the state \( \Psi \). Yet, in the development of this expression, there arise the angular momentum, \( \vec{L} = \vec{r} \times \vec{p} \), whose expectation value in a bound state is generally non-vanishing, justifying the result of eq. \( 11 \). The order of magnitude of this correction is \( ehgv_z/(m_e a_0^3) \), whose numerical value is \( 2 \times 10^{27} (gv_z) eV \). Taking into account the possibility of detection of one part in \( 10^{10} \), we arrive at the following bound for the parameters: \( gev_z \leq 10^{-19} \).

In order to evaluate the correction associated with the terms involving the spin operator, it is necessary to work with the wave functions \( \Psi_{nljm,s} = \psi_{nljm,s} (r, \theta, \phi) \chi_{s,m_s} \), suitable to treat the situations where there occurs addition of angular momenta \( (J = L+S) \), with \( n, l, j, m \) being the associated quantum numbers. Considering the free hydrogen atom, the first non-null spin term is \( \vec{\sigma} \cdot (\vec{v} \times (\vec{v} \times \vec{E})) \), which implies the following first-order correction:
\[ \Delta E_3 = \frac{g\hbar}{2m_e} \langle nljm_s | \vec{\sigma} \cdot (\vec{v} \times (\vec{v} \times \vec{E})) | nljm_s \rangle. \tag{12} \]
For the case of the Coulombic electric field, \( \vec{\sigma} \cdot (\vec{v} \times (\vec{v} \times \vec{E})) = 2e(\vec{\sigma} \cdot \vec{v})/r^3 - e(\vec{v} \cdot \vec{E})(\vec{\sigma} \cdot \vec{r})/r^3 \). After some algebraic manipulations, one obtains:
\[ \Delta E_3 = \frac{ge\hbar}{m_e} \langle nljm_s | (\vec{\sigma} \cdot \vec{v})/r^3 - (\vec{f} \cdot \vec{v}) | nljm_s \rangle, \tag{13} \]
with: \( f_x = e(-v_x + 3v_x \sin^2 \theta \cos^2 \phi + 3v_y \sin^2 \theta \cos \phi \sin \phi + v_z \cos \theta \sin \theta \cos \phi) / r^3 \); \( f_y = e(-v_y + 3v_y \sin^2 \theta \sin^2 \phi + 3v_z \sin^2 \theta \cos \phi \sin \phi + v_x \cos \theta \sin \theta \cos \phi) / r^3 \); \( f_z = e(-v_z + 3v_z \cos^2 \theta + 3v_x \sin \theta \cos \phi \sin \phi + v_y \cos \theta \sin \phi \cos \phi) / r^3 \).

To complete this calculation, it is necessary to write the \(|jm_j\rangle\) kets in terms of the spin eigenstates \(|mm_s\rangle\), which is done by means of the general expression: \(|jm_j\rangle = \sum_{m,m_s} \langle mm_s | jm_j \rangle | mm_s \rangle\), where \(|mm_s | jm_j \rangle\) are the Clebsch-Gordan coefficients. Evaluating such coefficients for the case \( j = l + 1/2, m_j = m + 1/2 \), one has: \(|jm_j\rangle = \alpha_1 |m \uparrow \rangle + \alpha_2 |m + 1 \downarrow \rangle\): one the other hand, for \( j = l - 1/2, m_j = m + 1/2 \), one obtains: \(|jm_j\rangle = \alpha_2 |m \uparrow \rangle - \alpha_1 |m + 1 \downarrow \rangle\), with: \( \alpha_1 = \sqrt{(l + m + 1)(2l + 1)}, \alpha_2 = \sqrt{(l - m)(2l + 1)} \). Taking now into account the orthonormalization relation \( \langle m' m_s | mm_s \rangle = \delta_{m'm} \delta_{m_s m_s} \), it is possible to show that eq. (12) leads to:

\[
\Delta E_j = \pm \frac{3 e h \nu_z}{2 m_e} \frac{m_j}{a_0^2 n^2 l(l + 1/2)(l + 1)} \left\{ 1 - \left( \frac{(l^2 - m^2)}{(2l - 1)(2l + 1)} + \frac{(l^2 - m^2 + 2l + 1)}{(2l + 3)(2l + 1)} \right) \right\},
\]

(14)

where the positive and negative signs correspond to \( j = l + 1/2 \) and \( j = l - 1/2 \), respectively; it was also used: \( \langle nljm_s | nljm_{m_s} \rangle = \pm m_j/(2l + 1) \), \( \langle nljm_s | nljm_{m_s} \rangle = \langle nljm_s | nljm_{m_s} \rangle = 0 \), and the expression for \( (1/\alpha^3) \). The order of magnitude of this correction is \( g v_z e h / (m_e a_0^3) \), the same of the correction \( \Delta E_2 \).

Next, we still consider an external fixed field and we evaluate the corrections induced by it. In principle, three terms of the Hamiltonian \( ^{15} \) might yield non-zero contributions in the presence of a magnetic field, namely: \( \Delta E_{1B} = \frac{q_{\mu}}{m_e} \langle nlm| (\overrightarrow{p} - e \overrightarrow{A}) \cdot \overrightarrow{B} | nlm \rangle \), \( \Delta E_{2B} = -\frac{e \nu_z}{m_e} \langle nlm| \overrightarrow{A} \cdot (\overrightarrow{v} \times \overrightarrow{E}) | nlm \rangle \), \( \Delta E_{3B} = -\frac{q_{\mu}}{m_e} \langle nlm| \overrightarrow{B} \cdot (\overrightarrow{v} \times \overrightarrow{E}) | nlm \rangle \). For a fixed magnetic field along the z-axis, \( \overrightarrow{B} = B_0 \hat{z} \), the vector potential in the symmetric gauge reads: \( \overrightarrow{A} = -B_0(y/2, -x/2, 0) \). Concerning the first term, only the product \( \overrightarrow{A} \cdot \overrightarrow{B} \) could provide a non-trivial contribution, once the evaluation of the product \( \overrightarrow{v} \cdot \overrightarrow{B} \) on the wave function obviously vanish. After a simple inspection, one gets \( \Delta E_{1B} = \frac{q_{\mu}}{m_e} \langle nlm| \overrightarrow{A} \cdot \overrightarrow{B} | nlm \rangle = 0 \).

In order to solve the second term, we should write \( (\overrightarrow{v} \times \overrightarrow{E}) = -\frac{q}{c}(v_x \cos \theta - v_z \sin \theta \sin \phi) \hat{\phi} + (v_z \sin \theta \cos \phi - v_x \cos \theta \sin \phi) \hat{\theta} \). The explicit calculation of this term yields a trivial result. Finally, it remains to evaluate the third term, which turns out to be also vanishing. We thus verify that the magnetic field does not yield any correction associated with the background; it only leads to the well-known Zeeman effect. This is the situation for the torsion-free coupling.

Another possible way to couple the Lorentz-violating background \((v^\mu)\) to the fermion field is by proposing a torsion-like non-minimal coupling,

\[
D_\mu = \partial_\mu + e A_\mu + i g a_n \gamma_5 v^\nu F^*_{\mu\nu},
\]

(15)

which has a chiral character, and has been examined in ref. \( ^{17} \) as well.

Writing the spinor \( \Psi \) in terms of the so-called small and large components in much the same way as it was done in the previous case, there exist two coupled equations for the 2-component spinors \( \phi, \chi \),

\[
\left[ (E/c - mc - eA_0/c) - ga \sigma \cdot (v^0 \overrightarrow{B} - \overrightarrow{v} \times \overrightarrow{E}) \right] \phi - \phi [\sigma \cdot (\overrightarrow{p} - e \overrightarrow{A}/c) - ga \overrightarrow{v} \cdot \overrightarrow{B}] \chi = 0,
\]

(16)

\[
[\sigma \cdot (\overrightarrow{p} - e \overrightarrow{A}/c) + ga \overrightarrow{v} \cdot \overrightarrow{B}] \phi - \left[ (E/c + mc - eA_0/c) - ga \sigma \cdot (v^0 \overrightarrow{B} - ga \overrightarrow{v} \times \overrightarrow{E}) \right] \chi = 0.
\]

(17)

from which we can read the weak component in terms of the strong one, \( \chi = \frac{1}{2m_e} \left[ \sigma \cdot \left( \overrightarrow{p} - \frac{e}{c} \overrightarrow{A} \right) + ga \overrightarrow{v} \cdot \overrightarrow{B} \right] \phi \).

It is then possible to write the Pauli equation,

\[
\left[ (H/c - eA_0/c - ga \sigma \cdot (v^0 \overrightarrow{B} - \overrightarrow{v} \times \overrightarrow{E}) \right) \phi = \left[ \sigma \cdot \left( \overrightarrow{p} - \frac{e}{c} \overrightarrow{A} \right) - ga \overrightarrow{v} \cdot \overrightarrow{B} \right] \frac{1}{2m_e} \left[ \sigma \cdot \left( \overrightarrow{p} - \frac{e}{c} \overrightarrow{A} \right) + ga \overrightarrow{v} \cdot \overrightarrow{B} \right] \phi.
\]

(18)
whose structure reveals as canonical generalized moment the usual relation, \( \Pi = (\vec{p} - e\vec{A}) \). Simplifying the equation above, the nonrelativistic Hamiltonian takes the form:

\[
H = \left( \frac{\vec{p} - e\vec{A}/c}{2m_e} \right)^2 + eA_0 - \frac{e\hbar}{2m_e} (\vec{\sigma} \cdot \vec{B}) + g_\alpha v_\alpha c \vec{\sigma} \cdot \vec{B} - g_\alpha c \vec{\sigma} \cdot (\vec{v} \times \vec{E}) - \frac{g_a}{2m_e}(\vec{\sigma} \cdot \vec{B})^2. \tag{19}
\]

This Hamiltonian has yet two additional terms, \((\vec{\sigma} \cdot \vec{p})(\vec{\sigma} \cdot \vec{B}) - (\vec{\sigma} \cdot \vec{B})(\vec{\sigma} \cdot \vec{p})\), which are equal (canceling each other) for the case of a uniform magnetic field. They will not be considered here.

In the absence of magnetic field, only the term \(\vec{\sigma} \cdot (\vec{v} \times \vec{E})\) contributes for the energy, implying the following correction:

\[
\Delta E_\sigma = g_\alpha \langle nljm|m_s|\vec{\sigma} \cdot (\vec{v} \times \vec{E})|nljm|m_s \rangle. \tag{20}
\]

Considering that \(\vec{\sigma} \cdot (\vec{v} \times \vec{E}) = -\frac{e}{\hbar}[(v_y \cos \theta - v_z \sin \theta \sin \phi)\sigma_x + (v_z \sin \theta \cos \phi - v_x \sin \theta \sin \phi)\sigma_y + (v_x \sin \theta \cos \phi - v_y \sin \theta \sin \phi)\sigma_z]\), and the action of the spin operators on the kets \(|nljm|m_s\rangle\), it is easy to note that: \(\Delta E_\sigma = 0\). Hence, the non-minimal pseudoscalar coupling yields no background contribution for the energy levels.

Now, the presence of an external magnetic field shall be taken into account. In this case, there appears a non-zero new contribution associated with the term \(c g_a v_\alpha \vec{\sigma} \cdot \vec{B}\), which generates a Zeeman splitting of the levels, whose separation is linear on the product \(c g_a v_\alpha B\). For the case the magnetic field is aligned with the z-axis, the implied energy correction is \(\Delta E_{1B} = c g_a v_\alpha B_0 \langle nljm|m_s|\sigma_z|nljm|m_s \rangle\), which yields:

\[
\Delta E_{1B} = \pm g_\alpha v_\alpha c B_0 \frac{m_j}{2l + 1}. \tag{21}
\]

where the positive and negative signs correspond to \(j = l + 1/2\) and \(j = l - 1/2\), respectively. This is exactly the same pattern of splitting of the Zeeman effect, here with amplitude given as \(g_\alpha v_\alpha B_0\). Hence, besides the usual Zeeman effect, there occurs this secondary Zeeman splitting that implies a correction to the effective splitting. The last term of eq. (19) only implies a constant correction on all levels, which does not lead to any change in the spectrum. The magnitude of this correction is proportional to \(g_\alpha v_\alpha c B_0\). If such an effect is not detectable for a magnetic strength of 1 G, it should not imply a correction larger than \(10^{-10}\text{eV}\), so that the bound \(g_\alpha v_\alpha \leq 10^{-18}\) is attained.

### III. FINAL REMARKS

In this work, we have studied low-energy effects of a Lorentz-violating background (non-minimally coupled to the fermion and gauge fields) on a nonrelativistic system. Indeed, the nonrelativistic limit has been worked out and the Lorentz-violating Hamiltonian (derived from the non-minimal coupling) evaluated. The first-order corrections induced on the energy levels of the hydrogen atom have been determined. As a result, we have observed effective shifts on the hydrogen spectrum, both in the presence and absence of an external magnetic field. In the absence of the external magnetic field, the term \(\epsilon_{\mu\nu\alpha\beta} F^\mu\nu F^{\alpha\beta}\) induces three different corrections, all of them implying modifications on the fine structure of the spectrum. This result indicates the breakdown of the accidental degeneracy, with the energy depending on \(l, m\) quantum numbers. Stipulating \(10^{-10}\text{eV}\) as the magnitude of a maximally undetectable change in the spectrum, we have set up an upper bound on the product of parameters: \(g_\nu z \leq 10^{-32}\).

In the case of the torsion-like non-minimal coupling, no correction is implied in the absence of external magnetic field; on the other hand, in the presence of such a fixed field, a secondary Zeeman effect is obtained. Considering that such a correction should be smaller than \(10^{-10}\text{eV}\), an upper bound is set up for the product, namely: \(g_\nu v_\alpha \leq 10^{-18}\). These results show that Lorentz violation in the context of the non-minimal coupling regarded here turns out as a real negligible effect.
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