String Gas Baryogenesis

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Abstract
We describe a possible realization of the spontaneous baryogenesis mechanism in the context of extra-dimensional string cosmology and specifically in the string gas scenario.

1 Introduction

One of the most peculiar features of the Universe is the observed baryon asymmetry, the difference between the density of baryons and anti-baryons, quantitatively described by the dimensionless number

\[ \frac{n_B}{s} \equiv \eta \simeq 10^{-10}, \tag{1} \]

where \( n_B \equiv n_b - n_{\bar{b}} \) is the difference between the baryon and anti-baryon density and \( s \) is the entropy density. The primordial nucleosynthesis, which is one of the most consistent and precise results in the standard model of cosmology, requires this value for \( \eta \) at the time of light elements \((^3\text{He}, ^4\text{He}, ^7\text{Li})\) production and it is believed not to have changed since.

The necessary conditions for generating the baryonic asymmetry were formulated by Sacharov in 1967 [1] (see also Ref. [2]) as:

1. Different interactions for particles and antiparticles, or, in other words, a violation of the \( C \) and \( CP \) symmetries;
2. Non-conservation of the baryonic charge;
3. Departure from thermal equilibrium.

The last condition results from an application of the CPT theorem. In fact, CPT invariance ensures that the energy spectra for baryons and anti-baryons are identical, leading consequently to an identical distribution in thermal equilibrium. This explains why the baryon number asymmetry was required to be generated out of thermal equilibrium.

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The so called spontaneous baryogenesis mechanism [3] uses the natural CPT non-invariance of the Universe during its early history to bypass this third condition, thus allowing the baryonic asymmetry to be generated in thermal equilibrium. We know, in fact, that an expanding Universe at finite temperature violates both Lorentz invariance and time reversal, and this can lead to effective CPT violating interactions [4]. Thus the cosmological expansion of the early Universe leads us naturally to examine the possibility of generating the baryon asymmetry in thermal equilibrium. The main ingredient for this mechanism is a scalar field $\phi$ with a derivative coupling to the baryonic current. If the current is not conserved and the time derivative of the scalar field has a non-vanishing expectation value, an effective chemical potential with opposite signs for baryons and anti-baryons is generated leading to an asymmetry even in thermal equilibrium.

We applied this mechanism in the context of brane cosmology, by identifying the scalar field with the radion field (the size of the extra-dimension) and by using its natural coupling to the trace of the four-dimensional stress energy tensor [5]. Subsequently this idea was applied in the context of four-dimensional standard cosmology by coupling the baryonic current directly to the trace of the stress-energy tensor [6].

Let us now turn to describe the string gas cosmology framework: string gas cosmology is an approach towards studying the effects of superstring theory on early universe cosmology and is based on new symmetries and new degrees of freedom of string theory. Within this context, it appears possible to stabilize the moduli which describe the size and shape of the extra spatial dimensions without the need of introducing many extra tools such as warping and fluxes. In fact, in the absence of a non-perturbative formulation of string theory, the approach to string cosmology is to focus on symmetries and degrees of freedom which are new to string theory (compared to point particle theories) and which will be part of any non-perturbative string theory, and to use them to develop a new cosmology. In particular the symmetry is T-duality and the new degrees of freedom are the string oscillatory modes and the string winding modes.

String gas cosmology is based on coupling a classical background which includes the graviton and the dilaton fields to a gas of strings (and possibly other basic degrees of freedom of string theory such as “branes”). If for simplicity, one takes all spatial directions to be toroidal and denote the radius of the torus by $R$, strings have three types of states: momentum modes which represent the center of mass motion of the string, oscillatory modes which represent the fluctuations of the strings, and winding modes counting the number of times a string wraps the torus. The second key feature of string theory upon which string gas cosmology is based is T-duality. To introduce this symmetry, let us discuss the radius dependence of the energy of the basic string states: The energy of an oscillatory mode is independent of $R$, momentum mode energies are quantized in units of $1/R$, i.e. $E_n = n/R$ and winding mode energies are quantized in units of $R$, i.e. $E_m = mR$ where both $n$ and $m$ are integers. Thus, a new symmetry of the spectrum of string states emerges: Under the change $R \to 1/R$ in the radius of the torus (in units of the string length $l_s$) the energy spectrum of string states is invariant if winding and momentum quantum numbers are interchanged $(n, m) \to (m, n)$. The above symmetry is the simplest element of a larger symmetry group, the T-duality symmetry group which in general also mixes fluxes and geometry. Postulating that T-duality extends to non-perturbative string theory leads to the need of adding D-branes to the list of fundamental objects in string theory. With this addition, T-duality is expected to be a symmetry of non-perturbative string theory.

Given the outlined framework of String Cosmology, the question we would like to address in this article is whether the mechanism of spontaneous baryogenesis might be at work in such
models.

In Section 2 we describe the spontaneous baryogenesis mechanism, in Section 3 we show the basic equations of String Gas Cosmology and in Section 4 we show how the spontaneous baryogenesis mechanism might effectively be at work in the String Gas Cosmology scenario.

2 Spontaneous Baryogenesis

To illustrate the mechanism of spontaneous baryogenesis (see e.g. Refs. [3, 7, 8]) let us consider a theory in which a neutral scalar field \( \phi \) is coupled to the baryonic current \( J^\mu_B \) by the Lagrangian density

\[
L_{\text{int}} = \frac{\lambda'}{M_c} J^\mu_B \partial_\mu \phi ,
\]

where \( \lambda' \) is a coupling constant and \( M_c < M_{Pl} \) is a cut-off mass scale in the theory. Let us assume that \( \phi \) is homogeneous, so that only the time derivative term contributes,

\[
L_{\text{int}} = \frac{\lambda'}{M_c} \dot{\phi} n_B \equiv \mu(t) n_B ,
\]

where \( n_B = J^0_B \) is the baryon number density and \( \mu(t) \) is to be regarded as an effective time-dependent chemical potential. This interpretation (see Ref. [9]) is valid if the current \( J^\mu_B \) is not conserved (otherwise one could integrate the interaction term away) and if \( \phi \) behaves as an external field which develops a slowly varying time derivative \( \langle \dot{\phi} \rangle \neq 0 \) as the Universe expands. Since the chemical potential \( \mu \) enters with opposite signs for baryons and anti-baryons, we have a net baryonic charge density in thermal equilibrium at the temperature \( T \),

\[
n_B(T; \xi) = \int \frac{d^3 k}{(2\pi)^3} \left[ f(k, \mu) - f(k, -\mu) \right] ,
\]

where \( \xi \equiv \mu/T \) is regarded as a parameter, and

\[
f(k, \mu) = \frac{1}{\exp \left[ \left( \sqrt{k^2 + m^2} - \mu \right)/T \right] \pm 1}
\]

is the phase-space thermal distribution \(^1\) for particles with rest mass \( m \) and momentum \( k \). For \( |\xi| \ll 1 \) we may expand Eq. (5) in powers of \( \xi \) to obtain

\[
n_B(T; \mu) = g \frac{T^3}{6} \xi + O\left(\xi^2\right) ,
\]

where \( g \) is the number of degrees of freedom of the field corresponding to \( n_B \). Upon substituting in for the expression of \( \mu \), one therefore finds

\[
n_B(T; \mu) \simeq \frac{\lambda' g}{6 M_c} T^2 \langle \dot{\phi} \rangle .
\]

Whatever the mechanism of baryon number violation, we assume there is a temperature \( T_F \) at which the baryon number violating processes become sufficiently rare so that \( n_B \) freezes out (we will call \( T_F \) the freezing temperature). Once this temperature is reached as the universe cools down, one is left with a baryon asymmetry whose value is given by Eq. (7) evaluated at \( T = T_F \). The value of the parameter \( \eta \) remains unchanged in the subsequent evolution.

\(^1\)The plus sign is for fermions and the minus sign for bosons.
3 String Gas Cosmology

The equations of String Gas Cosmology are based on coupling an ideal gas of string and brane modes, described by an energy density $\rho$ and pressures $p_i$ in the $i$'th spatial direction, to the background space-time of dilaton gravity. They follow from the 10-dimensional string frame action

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left[ \hat{R} + 4 \partial^\mu \phi \partial_\mu \phi \right] + S_m,$$

where $g$ is the determinant of the metric, $\hat{R}$ is the Ricci scalar, $\phi$ is the dilaton, $\kappa$ is the reduced gravitational constant in ten dimensions, and $S_m$ denotes the matter action. Let us thus first consider radion stabilization in the string frame [19]. For this purpose, the ansatz for the metric is

$$ds^2 = dt^2 - e^{2\lambda} dx^2 - e^{2\nu} dy^2,$$

where $x$ are the coordinates of the three large dimensions and $y$ the coordinates of the internal dimensions. The variational equations of motion which follow from the dilaton gravity action (8) for the above metric are

$$-3\ddot{\lambda} - 3\dot{\lambda}^2 - 6\dot{\nu} - 6\nu^2 + 2\ddot{\phi} = \frac{1}{2} e^{2\phi} \rho$$

$$\ddot{\lambda} + 3\lambda^2 + 6\dot{\lambda} \dot{\nu} - 2\lambda \dot{\phi} = \frac{1}{2} e^{2\phi} p_\lambda$$

$$\ddot{\nu} + 6\nu^2 + 3\dot{\lambda} \dot{\nu} - 2\nu \dot{\phi} = \frac{1}{2} e^{2\phi} p_\nu$$

$$-4\ddot{\phi} + 4\dot{\phi}^2 - 12\lambda \dot{\phi} - 24\nu \dot{\lambda} + 3\lambda^2 + 6\lambda^2 + 6\nu^2 + 21\nu^2 + 18\lambda \nu = 0$$

where $\rho$ is the energy density and $p_\lambda$ and $p_\nu$ are the pressure densities in the non-compact and compact directions, respectively.

Let us consider, for simplicity, a superposition of several string gases, one with momentum number $M_3$ about the three large dimensions, one with momentum number $M_6$ about the six internal dimensions, and a further one with winding number $N_6$ about the internal dimensions. The states considered here are massive, and would not be expected to dominate the thermodynamical partition function if there are states which are massless. However, for the purpose of studying radion stabilization in the string frame, the use of the above naive string gas is sufficient.

In the symmetric case $M_6 = N_6$ it follows from (16) that the equation of motion for $\nu$ is a damped oscillator equation, with the minimum of the effective potential corresponding to the self-dual radius. The damping is due to the expansion of the three large dimensions, driven by the pressure from the momentum modes $N_3$. We thus see that the naive intuition that the competition of winding and momentum modes about the compact directions stabilizes the radion degrees of freedom at the self-dual radius generalizes to this anisotropic setting.
3.1 Radion Stabilization in the Einstein Frame

In order to make contact with observational cosmology, it is important to consider the issue of radion stabilization when the dilaton is frozen, or, more generally, in the Einstein frame. As discussed in [17, 18] (see also earlier comments in [19]), the existence of string modes which are massless at the self-dual radius is crucial in obtaining radion stabilization in the Einstein frame.

Let us then consider the equations of motion which arise from coupling the Einstein action to a string gas. In the anisotropic setting the metric is taken to be

\[ ds^2 = dt^2 - a(t)^2 dx^2 - \sum_{\alpha=1}^{6} b_\alpha(t)^2 dy^\alpha, \]  

(17)

where the \( y_\alpha \) are the internal coordinates and the equation of motion for \( b_\alpha \) becomes

\[ \ddot{b}_\alpha + (3H + \sum_{\beta=1,\beta\neq \alpha}^6 \frac{\dot{b}_\beta}{b_\beta})b_\alpha = \sum_{n,m} 8\pi G \frac{\mu_{m,n}}{\sqrt{g} \epsilon_{m,n}} S \]  

(18)

where \( \mu_{m,n} \) is the number density of \((m, n)\) strings, \( \epsilon_{m,n} \) is the energy of an individual \((m, n)\) string, and \( g \) is the determinant of the metric. The source term \( S \) depends on the quantum numbers of the string gas, and the sum runs over all momentum numbers and winding number vectors \( m \) and \( n \), respectively (note that \( n \) and \( m \) are six-vectors, one component for each internal dimension).

Let us take the special case where all \( b_\alpha = b \) and write the metric as

\[ ds^2 = dt^2 - a(t)^2 dx^2 - b(t)^2 dy^2, \]  

(19)

We will analyse the evolution in a four-dimensional effective field theory, where we replace the radion field \( b(t) \) by a scalar field \( \varphi(t) \). In order that \( \varphi \) be canonically normalized when starting from the higher dimensional action of General Relativity, \( \varphi \) and \( b \) must be related as ([14])

\[ \varphi = M_p \sqrt{2d \log b} \]  

(20)

where \( M_p \) is the four-dimensional Planck mass. If the bulk size starts out at the string scale, then \( b(t_i) = 1 \), where \( t_i \) is the initial time. With these normalizations, \( \varphi = 0 \) corresponds to string separation between the branes.

The next crucial step is to invoke a mechanism to stabilize the radius of the extra dimensions at a fixed radius. Such modulus stabilization mechanisms have recently been extensively studied both in the context of string theory models of inflation (see e.g. [23] for recent reviews) and in string gas cosmology [22]. We will make use of the mechanism developed in the latter approach.

String modes which carry momentum about the extra dimensions will generate an effective potential for the radion which is repulsive. These repulsive effects will dominate for values of the radion smaller than the self-dual radius. Since these modes are very light at large values of the radion, it is likely that they will be present in great abundance. Even if they are not, the subset of such modes which are massless at enhanced symmetry points will be copiously produced when the value of the radion approaches such points [15, 16].
The induced potential will lead to a source term in the equation of motion for the scale factor $b(t)$ which is of the form [17, 18]

$$\ddot{b} + 3H\dot{b} = 8\pi G n(t) \left[ \left( \frac{1}{b} \right)^2 - b^2 \right] + \ldots ,$$

(21)

where the dots indicate extra source terms from other string modes, as well as terms quadratic in $\dot{b}$. Note that $n(t)$ is given by the number density of the modes. Translating to the scalar field $\varphi$, and neglecting terms quadratic in $\dot{\varphi}$, the above equation becomes

$$\ddot{\varphi} + 3H\dot{\varphi} = 8\pi G n(t) e^{-\varphi/M_p} M_p \left( e^{-2\varphi/M_p} - e^{2\varphi/M_p} \right).$$

(22)

Thus, it follows that after approaching the self-dual radius, $b(t)$ will perform damped oscillations about $b(t) = 1$, or, in other words, $\varphi(t)$ will undergo damped oscillations about and get trapped at $\varphi = 0$ (which corresponds to string scale separation between the orbifold fixed planes). Interestingly this is the same kind of motion obtained in the string frame.

4 Baryogenesis

Following the discussion on [5, 7] let us now assume that the high energy Lagrangian contains an interaction term of the form of Eq. (3),

$$L_{\text{int}} = \frac{\lambda}{M_c} \phi n_B ,$$

(23)

where (see [25])

$$\phi = \sqrt{6} M_p \log b$$

(24)

is the canonically normalized scalar field describing the size of the extra-dimensions. From the results of Section 2 one has

$$\eta = \frac{n_B}{s} = \frac{\lambda}{6 M_c g_s} \frac{\langle \dot{\phi} \rangle}{T}$$

(25)

Let us now consider the stabilization of the extra-dimension in this context: neglecting small corrections, the evolution of $\sigma(t)$ is that of damped harmonic oscillator as previously described, with damping given by the Hubble parameter

$$\frac{\dot{b}}{b} = \langle \dot{\phi} \rangle \simeq 3H$$

(26)

One can straightforwardly obtain an expression for the baryonic asymmetry at temperature $T$

$$\eta \simeq \lambda \frac{g M_p}{g_s M_c} \frac{H}{T}$$

(27)

with $H(T)$ evaluated at the freezing temperature $T_F$. In a phase of radiation domination the request that this value be compatible with the observed baryonic asymmetry, would imply $T > 10^{-10} M_c$. On the other hand for a gas of strings one has (see [26])

$$H(T) \simeq c_s T$$

(28)
where \( c_s \) is a constant. This implies that the ratio \( H/T \) is a constant and the constraint for this mechanism to be compatible with the experimental data is translated in a constraint on the constant \( c_s \)

\[
c_s \, M_p \geq 10^{-10} M_c
\]  

Finally in the case of brane gas the Hubble parameter is proportional to the square root of the temperature

\[
H(T) \simeq c_b \sqrt{T}
\]

where \( c_b \) is a constant. One then has a constraint

\[
T_F \leq 10^{20} \left( c_b \frac{M_p}{M_c} \right)^2
\]

This case is peculiar in the sense that it gives an upper bound on the freezing temperature whereas one usually obtains lower bounds.

5 Conclusions

We have shown that the evolution and stabilization of extra-dimensions within the framework of string gas cosmology provides a natural scenario for the realization of the spontaneous baryogenesis mechanism. This allows us to extract bounds on the temperature at which the baryonic asymmetry might be generated.

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