Analysis on Error Compensation for Integrated Navigation based on forgotten Kalman Filter

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Abstract. Aiming at the error caused by accelerometer bias and gyro drift in SINS/GPS integrated navigation system, an error compensation algorithm based on forgetting Kalman filter is proposed. This algorithm can not only effectively estimate the constant error of inertial devices, but also compensate the error to obtain the optimal estimation output of integrated navigation system. Experiments show that for integrated navigation system the algorithm can achieve relative ideal navigation accuracy, verify the correctness of the integrated system and is superior to the subsystem in reliability. It has a very good application value.

1. Introduction
With the development of computer technology and modern control technology, various integrated navigation systems are widely used in aviation, aerospace, navigation and other fields. Strap-down Inertial Navigation System (SINS) and Global Position System (GPS) are the most commonly used integrated navigation systems. SINS/GPS integrated navigation system realizes the complementary advantages of the two independent navigation systems and improves the reliability as well as the adaptability of the system. Adaptive Kalman Filter (KF) is usually used in integrated navigation system. According to the dynamic characteristics of SINS/GPS integrated system, KF outputs the optimal error estimation of system state variables, compensates the system error so as to improve the accuracy of integrated navigation system.

2. Mathematical Model of SINS/GPS Integrated Navigation
Completely SINS relies on the motion carrier equipment to complete the navigation task, without the limitation of meteorological conditions, and can provide relatively complete navigation parameters in a short period. However, the accuracy of SINS system mainly depends on inertial measurement elements. The errors of navigation parameters accumulate with time. It can not meet the requirements of long-distance, high-precision navigation and rapid response. GPS can provide high-precision three-dimensional position, velocity and time information in real time and continuously all over the world without accumulating systematic errors. However, GPS system is easy to be vulnerable to external environmental impact, limited bandwidth, difficult to extract carrier attitude information and low data update rate. SINS/GPS integrated navigation system integrates the advantages of each subsystem, improves the system performance and environmental adaptability.

SINS/GPS integrated navigation system is divided into tight combination and loose combination. The tight combination implementation model is widely used since it is simple and accurate. The system establishes state equation with indirect method and the state equation is built with 15-dimensional state parameters. The Northeast Sky (ENU), i.e. the geographical coordinate system, is
selected as the navigation coordinate system. Its state space is described as:

\[
\begin{align*}
X &= FX + GW^b \\
Z &= HX + V
\end{align*}
\]  
(1)

Where:

\[
X = [\Phi \ \delta V \ \delta P \ \epsilon \ \nabla]
\]  
(2)

\( \Phi \) represents three misalignment angles in the northeastern sky direction, \( \delta V \) velocity error, \( \delta P \) position error, \( \epsilon \) denotes gyro drift, \( \nabla \) accelerometer bias.

3. Adaptive Kalman Filter

3.1 Standard Kalman Filter

Error Compensation is an important part of integrated navigation system. The accuracy and speed of Error Compensation determine the performance of the system. Kalman Filter (KF), based on prediction and correction, is the most commonly used optimal estimation algorithm in parameter processing of integrated navigation system. KF uses state variables to establish statistical mathematical models to describe the dynamic characteristics of the system, so as to estimate and compensate the attitude parameters and related error sources in real time.

The standard KF estimates the optimal state of the system by combining the system state equation with the measurement equation. It can be divided into two processes: prior prediction and posterior correction. It is assumed that the state equation and measurement equation of the discrete linear system are as follows:

\[
\begin{align*}
X_{k} &= \Phi_{k/k-1}X_{k-1} + \Gamma_{k/k-1}W_{k-1} \\
Z_{k} &= H_{k}X_{k} + V_{k}
\end{align*}
\]  
(3)

where \( X_{k} \) is the state vector and \( Z_{k} \) is the measurement vector. \( W_{k-1} \) is the system noise vector and \( V_{k} \) is the measurement noise vector. The two are unrelated zero-mean Gauss white noise vector series with variances of \( Q_{k} \) and \( R_{k} \), respectively. \( \Phi_{k/k-1} \), \( \Gamma_{k/k-1} \) and \( H_{k} \) are known structural parameters of the system.

KF error estimation is divided into five steps as follows.

One-step state prediction.

\[
\hat{X}_{k/k-1} = \Phi_{k/k-1}\hat{X}_{k-1}
\]  
(4)

Mean square error of one-step state prediction.

\[
P_{k/k-1} = \Phi_{k/k-1}P_{k-1}\Phi_{k/k-1}^{T} + \Gamma_{k/k-1}Q_{k-1}\Gamma_{k/k-1}^{T}
\]  
(5)

Filtering gain.

\[
K_{k} = P_{k/k-1}H_{k}^{T}(H_{k}P_{k/k-1}H_{k}^{T} + R_{k})^{-1}
\]  
(6)

State estimation.

\[
\hat{X}_{k} = \hat{X}_{k/k-1} + K_{k}(Z_{k} - H_{k}\hat{X}_{k/k-1})
\]  
(7)

Mean square error of state estimation.

\[
P_{k} = (I - K_{k}H_{k})P_{k/k-1}
\]  
(8)

SINS/GPS integrated navigation system has abrupt noise change and poor system observability, which affects the error estimation performance of KF and even leads to filtering divergence. Adaptive Kalman Filter (AKF) is usually used. AKF uses the results of the filter parameters obtained at the previous time point and then automatically adjust the filter parameters at the current time point in order to adapt to the statistical characteristics of the signal and unknown noise changing with time, so as to achieve the optimal filtering.

3.2 AKF algorithm based on forgetting factor estimation

The commonly used AKF algorithms are AKF algorithm based on forgotten factor estimation, AKF algorithm based on noise statistical characteristics estimation and AKF algorithm based on filter gain matrix estimation. In order to alleviate the over convergence and make the error estimation better adapt to the new measurement changes, an AKF based on forgotten factor estimation is proposed.
When there is a deviation between the selected mathematical model and the actual system, the weights of system noise and measurement noise are modified in the filtering process in order to reduce the weights of historical observation data and improve the weights of new information, which can better reflect the actual situation and achieve the purpose of reducing the inertia of the filter. Therefore, AKF based on forgotten factor estimation has stronger robustness to process parameters and better estimation accuracy than standard KF by utilizing the effective information in "new information".

Variable forgotten factors are used to adjust the mean square deviation matrix of state estimation errors in real time and the past data are gradually eliminated. The mean square error matrix of the state estimation error is as follows:

\[ P_{k-1}^* = \Phi_{k-1}^*(sP_{k-1}^T)\Phi_{k-1}^* + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^T \]

(9)

where \( s \) is a forgotten factor or a fading memory factor.

It can be observed that the mean square deviation matrix is formally independent of the current time \( N \) and it only needs to multiply the \( P_{k-1}^* \) of the previous time state mean square deviation matrix by the forgetting factor \( s \), which is equivalent to expanding the uncertainty of the state prediction and forgetting the previous estimates.

If the forgetting factor \( s > 1 \), the state vector is as follows

\[ X_k^* = (I - K_k^* H_k)X_{k-1}^* + K_k^* Z_k \]

(10)

The formula above reflects that AKF based on forgotten factor estimation enhances the weight of measurement \( Z_k \) and reduces the weight of state prediction \( X_{k-1}^* \) correspondingly, which reduces the impact of historical measurement data.

3.3 Determination of Forgotten Factor

The determination of forgotten factor is one of the key points of AKF algorithm based on the estimation of forgotten factor.

By introducing forgotten factor, the statistical characteristics of noise can be continuously corrected and predicted in real-time so that the algorithm achieves adaptive effect. However, in this data tracking algorithm the estimated value will adapt faster if the forgotten factor \( H \) is too small. But the deviation trend of tracking data will be larger. If the forgotten factor \( s \) is selected too big, it will reduce the adaptive speed of the estimated value greatly. So it is very important to select the appropriate forgotten factor for this algorithm. The determination of forgotten factor is one of the key points of AKF algorithm based on the estimation of forgotten factor. It is assumed that the initial values of \( Q_k \), \( R_k \) and \( P \) are positive definite symmetric matrices and the matrix \( H_k \) is full rank. Then the optimal forgotten factor can be determined as follows:

\[ s = \max\{1, tr[N_k]/tr[M_k]\} \]

(11)

where

\[
\begin{align*}
S_k &= [Z_k - H_k X_k/k-1][Z_k - H_k X_k/k-1]^T \\
M_k &= \Phi_{k-1} P_{k-1} H_k^T \\
N_k &= S_k - H_k Q_k H_k^T - R_k
\end{align*}
\]

(12)

4. Simulation Analyses

The white noise of the gyro is 0.01°/h, the white noise of the accelerometer is 10-4g, and the speed error is 0.1m/s. The test was carried out 300 s in total with a sampling time of 0.1 s. The update period of the adaptive Kalman filter is \( T=0.1s \), and the forgetting factor is 1.001. At this time, the height channel of the system is considered. The attitude error curve of the system after Kalman filtering is shown in Figure 1. It can be seen from the figure that the error results no longer diverge with time accumulation. Although the initial period is fluctuating, the northeast sky three-way attitude error is bounded overall and the curve is smooth.
In order to compare the performance of the SINS/GPS integrated navigation system, the SINS navigation system been tested statically at first. The navigation sampling time is 0.1s, and the three subsample rotation vector optimization algorithm is used. The test is performed for 3600s. A graph of attitude error, velocity error, and position error for the SINS navigation system is shown in Figure 2.

The integrated navigation system uses the navigation parameter error of SINS/GPS was inputted as the KF input based on the forgetting factor, the error of various combined navigation parameters was estimated by filters, the parameters such as the filtered output attitude of the combined navigation was to correct the navigation parameters. Thereby, the optimal estimated output of the attitude calculation result of the integrated navigation system could be executed. The three misalignment angles, speed error, position error, gyro drift and accelerometer bias curve of the SINS/GPS integrated navigation system in the northeast direction was showed in Figure 3 and Figure 4. The test is carried out for
3600s in total.

Figure 3. SINS/GPS integrated navigation system attitude error, speed error, position error

Figure 4. SINS/GPS integrated navigation system gyro random constant drift, accelerometer random constant bias

The error of the SINS/GPS integrated navigation system has been reduced according to time range, and the accumulated error in the previous period has few influence on the system operation, the accuracy of the system are improving hereby. However a certain degree of distortion occurs in the initial stage. The main reason of the initial value is selected in the KF process, but the curve tends to converge with time.

5. Conclusion
It can be seen from test data that the SINS/GPS integrated navigation system has the advantages of
high precision and good convergence. The advantages of GPS and INS was given full play which be designed by the SINS/GPS integrated navigation system in this paper, and Navigation parameter error was inputted in process to estimate navigation parameter based on forgotten Kalman filter to input signal which improved the navigation accuracy and suppressed the issue of Filter divergence. Overall the SINS/GPS integrated navigation system has high value of application on precision, reliability, wide application range.

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