Input output linearization with non-minimum phase boost DC-DC converters

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Abstract: This paper presents the theory and implementation of the non-linear control method of Input to Output Linearization (IOL) for boost DC-DC switching power converters operating in continuous inductor current mode. The paper examines the non-linear and non-minimum phase nature of boost converters. The limitations of IOL control on boost converters have been outlined. It has been shown that these limitations are overcome by addressing the non-minimum phase nature in boost converters. Using analytical and experimental methods, it is demonstrated that IOL control scheme compensates the non-linear boost converter to achieve wide range of output voltages with excellent response to load and duty cycle variations.

Key Words: boost converters, switching, input to output linearization, non-minimum phase, non-linear

1. Introduction

There has been research over the past two decades on the control of switching power converters to achieve fast transient performance, efficiency, stability over a wide operating range etc. A major target for research has been boost and buck-boost type of DC-DC converters which present a non-linear transfer characteristic from duty cycle control input to the output voltage. Various non-linear control methodologies have been proposed to address the non-linear behavior and achieve stability and performance over a wide operating range of input and output voltages and load currents. [1] and [2] provide a comparison of the various non-linear control techniques proposed for switching power converters.

In [3], the non-linear control method of Input to Output Linearization (IOL) has been applied on boost and buck-boost switching power converters which exhibit a non-linear and non-minimum phase nature from control input to voltage output. It has been shown that the control technique of IOL is beneficial in addressing the non-linear dynamics in such power converters, and hence provides robust
control over wide range of operating points. But, due to the non-minimum phase nature of these converters, it has been shown that considering the output voltage of the converter as the output of the plant for applying the IOL control technique results in instability in the internal dynamics of the converter. Hence, using the inductor current as the output of the plant for applying IOL has been recommended. However, due to the indirect control involved, the choice of inductor current as the output of the plant requires the dynamic generation of the current reference to achieve the desired output voltage.

[4] shows the implementation of the IOL technique by addressing the non-minimum phase property of the boost converters using output redefinition technique. The technique results in a error between the desired and achieved output voltages and also requires dynamic generation of the reference current to achieve the desired output voltage. Also, the redefinition of the output used for applying IOL technique has been made on a trial and error basis and does not establish a concrete boundary between the non-minimum phase nature and the minimum phase nature of the boost converter using the output redefinition technique. [5] describes the application of IOL control on three phase boost rectifiers. The technique presented in this paper also involves the use of inductor current as the output for applying IOL control method, requiring the dynamic generation of reference current to achieve the desired steady state values of the inductor current and hence the desired steady state output voltage. In [6], the method of input to output linearization has been applied to permanent magnet synchronous generator with a boost chopper converter. The non-linear characteristics of the plant have been divided into several piecewise non-linear models which are further linearized using IOL. Due to the piecewise approximation, the method proposed in [6] does not result in true linearization over the operating range. Moreover, the technique requires tuning for the feedback linear controller. In [7], a method of feedback linearization known as Dynamic Linearizing Modulator is proposed for differential boost converters in order to generate high frequency sine waves. Differential boost converters do not contain a right half plane zero in the control-to-output transfer function and hence, application of feedback linearization does not lead to instability in the internal zero dynamics of the converter.

This paper presents a novel technique of applying IOL control technique on boost switching power converters. Output voltage of the converter is used as the output of the plant for applying IOL, without the need for dynamic generation of the reference inductor current. Section 2 describes the non-linear nature of boost switching converters along with the non-minimum phase property. Section 3 describes the general theory of the IOL control technique. Subsection 3.1 explores the limitations of IOL on non-minimum phase systems, by tracing the internal zero dynamics of the closed-loop system with IOL control. Section 3.2 discusses the technique used to eliminate the Right Half Plane (RHP) zero from the control loop of the boost converter and move it to the left half plane. Section 4 proposes the method of implementation of the IOL control technique on boost converters operating in continuous inductor current mode (CCM), choosing the output voltage of the boost converter as the output for applying IOL. In Section 5, the proposed technique has been implemented using a microprocessor and the observed results have been documented.

2. Non linear nature of boost converters
The non-linear nature of boost converters could be examined from the point of view of the transfer functions internal to the boost converter and also from the state space representation of the boost
converter. Figure 1 shows a standard boost converter where \( L \) is the boost inductor, \( C \) is the output capacitor, \( R_c \) is the effective series parasitic resistance (ESR) of the output capacitor, \( R \) is the load resistor. A n-channel MOSFET has been used for the switch. \( u_0 \) is the input voltage source, \( x_1 \) is the inductor current, \( x_2 \) is the voltage across the output capacitor, \( y \) is the output voltage, \( d \) is the control input duty cycle (PWM wave). \( d \) is comprised of a steady state duty cycle \( D \) and a small perturbation \( \hat{d} \). Similarly, the output voltage is comprised of a steady state value \( y_0 \) and a small signal response \( \hat{y} \). The transfer functions internal to the boost converter include the control input \( d \) to output voltage \( y \) transfer function, represented by \( G_{yd} \), and the control input \( d \) to inductor current \( x_1 \) transfer function represented by \( G_{x1d} \), as given by Eq. (1) and Eq. (2) respectively. The second order LC filter manifests as double poles in the transfer functions whose frequency is dependent on the steady state value of the control input duty cycle \( D \). The transfer functions also contain left half plane (LHP) zeros formed due to the combination of resistor elements and output capacitor in the boost converter. In addition to the above mentioned dynamics, \( G_{yd} \) contains a zero in the right half plane (RHP) whose frequency is also dependent on \( D \), the steady value of the duty cycle. The RHP zero, also known as positive zero, leads to the non-minimum phase nature of the boost converter. The positive zero in a system forces an initial response of the output in the opposite direction of perturbation. Hence, when the zero is not accounted for, it could potentially lead to instability in the control loop of the converter.

\[
G_{yd} = \frac{\hat{y}}{\hat{d}} = \frac{u_0 \left( 1 - s \frac{L}{R D^2} \right) \left( 1 + s \frac{1}{R_c C} \right)}{L C s^2 + \frac{L}{2 R} s + D^2} \tag{1}
\]

\[
G_{x1d} = \frac{\hat{x}_1}{\hat{d}} = \frac{2 u_0 \left( 1 + s \frac{(R + R_c) C}{2} \right)}{R D^2 \left( L C s^2 + \frac{L}{2 R} s + D^2 \right)} \tag{2}
\]

An example to show the dependence of the poles and zeros of the transfer function \( G_{yd} \) on the steady state duty cycle \( D \) is shown in Fig. 2. The steady state duty cycle has been varied from \( D = 0.15 \) (corresponds to \( y_0 = 14V \)) to \( D = 0.7 \) (corresponds to \( y_0 = 40V \)) for a prototype boost switching converter. Double poles at 2kHz with \( D = 0.15 \) have shifted to 500Hz when steady duty cycle is increased to \( D = 0.7 \). Similarly, RHP zero at 20kHz with \( D = 0.15 \) has shifted to 3kHz when \( D = 0.7 \). The LHP zero formed due to the output capacitor and its ESR are not affected by changes in the steady state duty cycle. It is noted that a change in the steady state duty cycle results in changes in the steady state inductor current \( (x_{1s}) \) and steady state output voltage \( (y_0) \). Change in
steady state values of variables of the boost converter are referred to as change in the steady state operating point of the converter.

The dependence of the locations of poles and zeros of a circuit on its steady state operating point is an indication of the non-linear behavior of the circuit. Since poles and zeros of the internal transfer functions of the boost converter is dependent on the steady state value of the control input $D$, the boost converter exhibits a non-linear nature.

The non-linear nature of the boost converter could also be observed by examining the state space equations in the state space representation of the boost converter. Equations (3), (4) and (5) represent the state space model of the boost converter. These state space equations are obtained by averaging the corresponding state equations over the ON and the OFF intervals of the boost converter switch [8].

$$\dot{x}_1 = \frac{u_0}{L} - \frac{RR_e}{L(R + R_c)} x_1(1 - d) - \frac{R}{L(R + R_c)} x_2(1 - d)$$  \hspace{1cm} (3)  

$$\dot{x}_2 = \frac{R}{C(R + R_c)} x_1(1 - d) - \frac{1}{C(R + R_c)} x_2$$  \hspace{1cm} (4)  

$$y = \frac{RR_e}{R + R_c} x_1(1 - d) + \frac{R}{R + R_c} x_2$$  \hspace{1cm} (5)  

The above state space equations contain product terms of state variables $x_1$ and $x_2$ with control input $d$, as an indication of the non-linearity present in the state space representation of the boost converter. In order to obtain a linearized model of the boost converter plant, the product of small signal terms on the state variables ($\hat{x}_1$, $\hat{x}_2$) with the control input perturbation ($\hat{d}$) are necessary to be ignored.

**3. Input output linearization**

Consider a $n$ dimensional Single Input Single Output (SISO), non-linear system. In order to perform Input to Output Linearization (IOL) on a SISO system, the output of the system is differentiated until the control input appears in the resulting equation. The number of times the output is differentiated is represented by $r$ and is called the relative degree of the system. Applying co-ordinate changes further results in a $r$ dimensional linear subsystem. The linear subsystem is stabilized with linear state feedback. The dynamics of the remaining subsystem with a dimension of $n - r$ is known as internal dynamics or zero dynamics. The zero dynamics represent the system dynamics when the $r$ states of the linear subsystem are at rest [9].

![Fig. 3. Block diagram.](image_url)

Figure 3 shows a block diagram representation of IOL on a $n$ dimensional plant. The plant is a $n$ dimensional non-linear system from control input $u$ to the output of the plant $y$. When IOL is applied on the non-linear plant using the method described above, a $r$ dimensional linear subsystem is formed from $f$ to the output of the plant $y$. $f$ is called the auxiliary control input of the linear subsystem. Hence, the linear subsystem from auxiliary control input $f$ to output of the plant $y$ is stabilized using linear control methods such as proportional control, integral control or linear state feedback.

422
3.1 Internal dynamics of a system with IOL

The internal dynamics or zero dynamics are seen in the remaining subsystem of dimension \( n - r \) when the \( r \) states of the linear subsystem are at rest. These dynamics are a function of the location of zeros present in the original \( n \) dimensional non-linear system. The presence of a positive zero in the non-linear system results in unstable internal dynamics when IOL is applied on such a system.

Consider implementation of IOL on a boost switching converter. The boost converter is a two dimensional non-linear system. As shown in Eq. (1), the converter from control input \( d \) to output voltage \( y \) has two poles corresponding to a two dimensional system. It also has two zeros, one in the right half plane (positive zero) and another in the left half plane. These zeros are represented by \( \omega_{z1} \) and \( \omega_{z2} \).

When IOL is applied for the boost converter considering \( d \) as the input to the non-linear plant and output voltage \( y \) as the output of the non-linear plant, the remaining dynamics is seen in the inductor current state variable \( x_1 \). As the output voltage \( y \) approaches its steady state value \( y_0 \), the inductor current \( x_1 \) is expected to approach its steady state value \( x_{1o} \). The zero dynamics or the internal dynamics in the case of IOL applied for boost converters is defined as the dynamics of the inductor current approaching its steady state value. It is given by the following equation:

\[
x_1 = A_1 e^{\omega_{z1} t} + A_2 e^{\omega_{z2} t} + ... \tag{6}
\]

From the above equation, it can be seen that if \( \omega_{z1} \) or \( \omega_{z2} \) is positive, the dynamics of the inductor current may never converge to its steady state value \( x_{1o} \) and would eventually approach infinity. Hence, applying IOL on a boost converter by considering the output voltage \( y \) as the output of the non-linear plant, renders unstable internal dynamics in the inductor current \( x_1 \) due to the positive zero present in the control to output of the non-linear plant.

3.2 Addressing non-minimum phase nature of boost converters

Consider the standard boost switching converter shown in Fig. 1. During the switch ON time, the input voltage \( u_0 \) is applied across the inductor \( L \) and hence, the inductor is charging. The current in the inductor has a positive slope. The output capacitor is discharging into the load resistor and hence, the slope of the voltage across the capacitor is negative. During the switch OFF time, the inductor discharges into the load resistor and the output capacitor. Since the inductor is discharging, the slope of the inductor current is negative while the output capacitor is charging and hence, the voltage across it has a positive slope.

The amount of inductor current charge/discharge is called inductor current ripple and is represented by \( \Delta x_1 \). The amount of charging/discharging capacitor voltage is called capacitor voltage ripple and is represented by \( \Delta x_2 \). In order to maintain steady state (inductor flux balance and capacitor charge balance inherent to a switching converter), inductor current ripple \( \Delta x_1 \) during switch ON time is equal to \( \Delta x_2 \) during switch OFF time. The same behavior also applies to capacitor voltage ripple \( \Delta x_2 \).

The ripple at the output of the boost converter is shown in Fig. 4. It is a contribution of the voltage across the capacitor \( C \) and the voltage across the ESR of the capacitor \( R_c \). During switch OFF time, the discharging inductor current forms a voltage across the ESR of the output capacitor \( R_c \Delta x_1 \). This, in turn, results in the voltage across the ESR to have a negative slope.

Hence, during the switch OFF time, the slope of the voltage across the capacitor (\( \Delta x_2 \)) is positive while the slope of the voltage across the ESR of the capacitor (\( R_c \Delta x_1 \)) is negative. The resulting slope of the output ripple of the boost converter is dependent on the magnitudes of its two contributing slopes. Figure 5 shows the variations in the slope of the output ripple of a boost converter during switch OFF time. It can be seen that the slope of the resulting output ripple is not only dependent on the inductor current ripple and capacitor voltage ripple but also on the ESR of the output capacitor.

[10] and [11] show that, in a closed loop system, the sampling instant of the output ripple of the boost converter by the feedback circuit changes the location of the zeros of the control to output voltage transfer function as seen by the feedback loop. In other words, when the output voltage ripple of the boost converter is sampled during the switch OFF time, the positive zero in the control to
output voltage of the boost plant would be seen at a different location by the feedback circuit as compared to when the boost converter output ripple is sampled during the switch ON time. The transfer function from control input \( d \) to inductor current \( x_1 \) (Eq. 2) contains a zero in the left half plane whereas the transfer function from \( d \) to capacitor voltage \( x_2 \) contains a zero in the right half plane. It is therefore desirable that, during the switch OFF time, the contribution of inductor current ripple to the output voltage ripple of the boost converter be greater than the contribution of the capacitor voltage ripple. This can be expressed by the inequality:

\[
R_c \Delta x_1 > \Delta x_2
\]  

(7)

For a boost switching converter operating in steady state, the inductor current ripple \( \Delta x_1 \) and capacitor voltage ripple \( \Delta x_2 \) are given by:

\[
\Delta x_1 = \frac{u_0}{L} DT_s
\]  

(8)

\[
\Delta x_2 = \frac{y_0}{RC} DT_s
\]  

(9)

where \( T_s \) is the switching time period. Substituting the above equations for the inductor current ripple and capacitor voltage ripple in Eq. (7), the inequality condition for changing the location of the zero as seen by the feedback loop when the output voltage ripple is sampled during the switch OFF time, is given by:

\[
R_c C > \frac{L}{RD'}
\]  

(10)

The corresponding location of the left half plane zero in the control to output voltage transfer function as seen by the feedback loop is given by:

\[
\omega_z = \frac{D'}{L RD'} - R_c C
\]  

(11)
It has to be noted that during the switch ON time interval, the output voltage ripple is only comprised of capacitor voltage ripple and does not have a contribution from the inductor current ripple. Due to this, sampling of the output voltage ripple of the boost converter by the feedback circuit during the switch ON time, always results in a right half plane zero in the feedback loop. On the same note, averaging of the samples of the output voltage ripple sampled during the switch ON and OFF times results in two zeros as shown in the original control to output transfer function (Eq. (1)).

With switch OFF time sampling and ensuring the inequality condition in Eq. (10) is satisfied, it is possible to implement input output linearization (IOL) for a boost converter and maintain stable internal dynamics.

4. Input output linearization for boost converters

The boost converter state space equations shown in Eqs. (3), (4) and (5) in Section 2 were obtained by averaging the individual state space equations for the switch ON and OFF time intervals. Eq. (3) and Eq. (4) represent the state space model of the boost converter (Fig. 1) and define the dynamics of the boost converter. Although these equations remain unmodified, there is flexibility in choosing the output equation for the state space model. Equation (5) is the averaged equation obtained by average sampling of the output voltage switching ripple during the switch ON and OFF time intervals.

As discussed in Section 3.2, it is desirable for the feedback circuit to sample the output voltage ripple of the boost converter during the switch OFF time interval in order to move the positive zero to the left half plane. The output equation representing sampling of the output voltage ripple only during the switch OFF time interval is given by:

\[ y = \frac{RR_c}{R + R_c}x_1 + \frac{R}{R + R_c}x_2 \quad (12) \]

Due to the two storage elements inductor and capacitor present in a boost converter, the order of the system is two \((n = 2)\). In order to implement input output linearization for the boost converter, the output (sampled during switch OFF time) needs to be differentiated once \((r = 1)\) for the control input \(d\) to appear in the resulting equation. Differentiating Eq. (12) once, the resulting equation is given by:

\[ \dot{y} = \frac{R}{R + R_c}(\dot{x}_2 + R_c\dot{x}_1) \quad (13) \]

Substituting for \(\dot{x}_1\) and \(\dot{x}_2\) from Eq. (3) and Eq. (4) in Eq. (13) and substituting for \(\dot{y}\) by \(f\), the equation for the control input \(d\) is algebraically solved and is given by:

\[ d = \frac{(RR_cC + L)y - Lx_1 - RR_cu_0 + (R + R_c)L Cf}{R(R_cCy - \frac{Lx_1}{R + R_c})} \quad (14) \]

Equation (14) represents the equation for implementing IOL on a boost converter by considering output voltage of the converter as the output of the plant for applying IOL. The equation creates a linear subsystem of one dimension \((r = 1)\) from the auxiliary control input \(f\) to output of the non-linear boost plant \(y\). A proportional control is used to stabilize the established linear subsystem from \(f\) to \(y\). Hence, the equation for the auxiliary input \(f\) becomes:

\[ f = -k(y - y_0) \quad (15) \]

where \(y_0\) is the desired steady state value of the output voltage and \(k\) is the proportional gain term that multiplies the error voltage. The implementation of IOL on a boost plant is represented as a block diagram in Fig. 6. The linear subsystem formed from auxiliary control input \(f\) to output of the boost plant \(y\) is represented as an integrator and is governed by the equation \(\dot{y} = f\).

5. Results

Input output linearization control method was implemented on a prototype boost converter specifically made, on a printed wire board. The circuit elements of the boost converter are as follows: inductor
$L = 330 \mu H$, output capacitor $C = 2200 \mu F$, estimated ESR of the output capacitor $0.1 \Omega$ (from data sheet), power MOSFET (Infineon IPP80N06S207) and power diode (Vishay MBR745-E3/45), load resistor $R = 75 \Omega$, input voltage $u_0 = 12 V$, switching frequency $f_{sw} = 50 \text{kHz}$. In order to satisfy the inequality condition of Eq. (10), an external resistor of $0.1 \Omega$ is placed in series with the output capacitor, hence making the total effective ESR to be $0.2 \Omega$. The additional external resistor ensures that the zero in the sampled output-voltage is always in the LHP by satisfying the condition in Eq. (10). The IOL equation (Eq. (14)) is implemented on a microprocessor from Texas Instruments (TMS320F28335). A keypad connected to the microprocessor is used to dynamically select the desired output voltage. The experimental setup is shown in Fig. 7.

The steady state duty cycle $D$ and the steady state output ripple of the boost converters for output voltages $y_0 = 14 V$ and $y_0 = 40 V$ are shown in Fig. 8 and Fig. 9 respectively. The shape of the output ripple during switch OFF time match the expected shape for the output ripple as shown in Fig. 5 for the case $R_c \Delta x_1 > \Delta x_2$. Hence, sampling of the output ripple by the feedback circuit (Analog to Digital converter of the microprocessor) during the switch OFF time results in a left plane zero in the control input $d$ to output voltage $y$ transfer function, as seen by the feedback loop.

The stability of the closed loop boost converter with IOL control is verified by its response to changes in operating points namely: load current and desired output voltage. The keypad selector is used to dynamically generate a reference voltage $y_0$ for boost converter control loop. Figure 10 shows the various output voltages achieved by the boost converter with IOL control. It can be seen that the converter is stable for a wide range of operating steady state duty cycles corresponding to a wide range of output voltages. Since the dynamic variation of the reference voltage was done with a constant resistor load $R$, the above test also includes disturbance rejection due to change in the load.
Fig. 8. Steady state duty cycle (blue) and output voltage ripple for $y_0 = 14\text{V}$(red).

Fig. 9. Steady state duty cycle (blue) and output voltage ripple for $y_0 = 40\text{V}$(red).

current ($= y_0/R$).

With a constant resistor load of $R = 75\Omega$, the maximum load current at $y_0 = 14\text{V}$ is 180mA and at $y_0 = 40\text{V}$ is 530mA. The resistor load was disconnected and an electronic load module was used to step the load at the output of the boost converter with IOL control from no load to full load for two steady state output voltages. Figure 11 shows the response of the control loop to a no load to full load transition of 180mA with output voltage $y_0 = 14\text{V}$. Figure 12 shows the response of the control loop to a no load to full load transition of 530mA with output voltage $y_0 = 40\text{V}$. It is to be noted that only the ac response of the output voltage is shown. The response of the boost converter with IOL control, to a load step, exhibits less than 10% undershoot in the output voltage. The response exhibits smooth asymptotic recovery from the load step disturbance. The response time is limited by the large output capacitor $2200\mu\text{F}$ which was chosen to satisfy the inequality condition (Eq. (10)) for all operating points.

6. Conclusions

The paper has demonstrated by theory and implementation, the application of the control method of Input to Output linearization on non-linear non-minimum phase boost switching converters. It discusses the non-linear nature of boost converter and unstability in its internal dynamics with IOL due to the non-minimum phase nature of the converter. It is shown that it is possible to control the boost converter with IOL when its non-minimum phase nature is addressed. An equation to
implement IOL by choosing output voltage of the boost converter as the output of the non-linear system has been proposed. A prototype boost converter has been built for experimental purposes, its control with IOL technique is implemented on a microprocessor. The operation of the control loop on the boost converter is verified through the response of the loop to step changes to the output voltage.

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and output load current. The robust operation of the non-linear boost converter with IOL control is proven over a wide range of operating points.

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