Effects of high-intensity Lasers on the Entanglement fidelity of quantum plasmas

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The dynamics of entanglement during the low energy scattering processes in bi-partite systems at the presence of a laser field is studied, using the Kramers-Henneberger unitary transformation as the semiclassical counterpart of the Block-Nordsieck transformation, in the quantized field formalism. The Stationary-state Schrodinger equation for quantum scattering process is obtained for such systems. Then, by using partial wave analysis, we introduce new form of entanglement fidelity containing high-intense laser field. Therefore, the effective potential of hot quantum plasmas including plasmon and quantum screening effect is used to show entanglement fidelity ratio as a function of the laser amplitude, plasmon parameter and the Debye length parameter for elastic electron-ion collisions. It is shown that the amplitude of laser beam or free electron oscillation play important roles in the evolution of entanglement of the system.

I. INTRODUCTION

The quantum correlation among distinct quantum systems, (or entanglement) is a complex and powerful phenomenon in the framework of quantum mechanics which nowadays is widely used in all branches of science [1–3]. Produced entanglement due to the Lorentz invariance violation in high energy physics processes [4], expansion of universe [5,10], variation of entanglement due to the environmental interactions [11–13] effects of observer acceleration in entanglement degradation (the Unruh effect) [14,17], the Quantum self-organization applied in physics [18] and biophysics [19] are some examples in this research area.

However, there have been presented many works in theoretical features of entanglement [20,21], but there exist few experimental based researches in this area [22,23], because of many difficulties in measuring the quantum entanglement in laboratory and needing very high technological tools. One of successful experimental methods is measuring the entanglement in bi-particle interaction. Saraga et. al. [21] observed creation of EPR pairs [25] during the 2-dimensional Coulomb scattering in an electron gas. They found that created entanglement is very sensitive to the potential energy function of bi-electron system. Measuring of entanglement in terms of wave packet localization has been investigated by Fedorov et al. [20], while Mishima and colleagues [27] have found a suitable measure for entanglement during the scattering interactions which is now called as Fidelity Entanglement (EF). The EF is now widely used because of its effectiveness in realization of quantum entanglement and quantifying information processes [28,29].

Several inter-particle interactions which are described by different functions of potential energies, produce different degrees of entanglement. By considering such descriptions, we can clearly understand that plasma is a very ideal media for creation and measuring some features of entanglement. It is because of the presence of high density charged particles. It is clear that in such media collective effects via long-range electromagnetic interactions play an important role and thus, quantum correlations can be appeared [30]. Presenting a successful definition of an appropriate potential which includes all important features on interaction is the first and also most important step in studying the quantum entanglement in plasmas. There are several theoretical and experimental investigations which provide more accurate knowledge about the nature of potential energy function in classical, semi-classical and quantum plasmas.

The simplest description for interaction of charged particles in plasma is presented using the Debye-Huckel screened potential which is applied in ideal plasmas where the energy if inter-particle interactions is smaller than the average kinetic energy of plasma constituents [31,32].

The effective interaction potential between projectile electrons and dressed ions by considering the strong quantum recoil effects in quantum plasmas has been investigated in [33]. Shukla and Eliasson [34] has presented a practical definition for the interaction potential function in degenerate electron-ion quantum plasmas by considering the screening and electron exchange effects. The effective potential in electron-ion interaction by considering the standard Debye potential as well as the effective Friedel far-field interaction term between particles for collisional degenerate quantum plasmas have been presented in [35]. An acceptable form for interaction potential in strongly coupled semi-classical plasmas can be constructed by effective pseudo-potential [36] interaction between different kinds of plasma particles based on the dielectric response function analysis. The inter-particle interaction potential for dressed electron-ion scattering in hot quantum plasmas [37] as well as in dense quantum plasmas with different initial conditions has been investigated [38–40]. Quantum mechanical effects based on the definition of effective potential functions are widely investigated in dense plasmas appeared in the core of giant.

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The Eq. (1). Thus, we have:

\[
\partial_t \psi (\mathbf{r}, t) = \left[ \frac{-\hbar^2}{2\mu} \nabla^2 - i\hbar \frac{e}{2\mu} (\mathbf{A}(\mathbf{r}, t) \cdot \nabla + \nabla \mathbf{A}(\mathbf{r}, t)) + \frac{e^2}{2\mu} \mathbf{A}(\mathbf{r}, t)^2 - e\phi(\mathbf{r}, t) + V(\mathbf{r}) \right] \psi (\mathbf{r}, t),
\]

we consider the Coulomb gauge \[53\], such that \(\nabla \mathbf{A}(\mathbf{r}, t) = 0\) (and \(\phi(\mathbf{r}, t) = 0\)) in empty space and simplify the interaction term in Eq. 2 by applying gauge transformations within the framework of dipole approximation. In this approximation, for an atom which is located at the position \(r_0\), the vector potential can be considered independent of space, such that \(\mathbf{A}(\mathbf{r}, t) \approx \mathbf{A}(t)\). Moreover, term \(\mathbf{A}(\mathbf{r}, t)^2\) appearing in equation 2 is noticeable only in media with extremely high field strength and thus this term usually is very small and can be eliminated by extracting a time dependent phase factor from the wave function via \[58\]

\[
\varphi(\mathbf{r}, t) = \exp \left[ \frac{ie^2}{2\mu \hbar} \int_{-\infty}^{t} \mathbf{A}(t')^2 dt' \right] \psi(\mathbf{r}, t),
\]

Now we need to calculate the velocity gauge. According to equation \[3\], we can write:

\[
i\hbar \frac{\partial}{\partial t} \varphi(\mathbf{r}, t) = \left[ -\frac{\hbar^2}{2\mu} \nabla^2 - i\hbar \frac{e}{\mu} \mathbf{A}(t) \cdot \nabla + V(\mathbf{r}) \right] \varphi(\mathbf{r}, t),
\]

But, in order to find the state function of a system composed of two particles under intense high-frequency laser field we have to transform equation 1 into the Kramer-Henneberger (KH) accelerated frame \[59\]. The KH frame is a reference frame in which a free electron moves in the laboratory and \(\mathbf{A}(t)\) represents a shift to the accelerated frame of reference. It is indeed semi-classical counterpart of the BlockNordsieck transformation in the quantized field formalism, so that the coupling term \(\mathbf{A}(t) \cdot \nabla\) in the velocity gauge (i.e. equation 3) is eliminated. More explicitly, this can be done via

\[
i\hbar U^\dagger \frac{\partial}{\partial t} U \Psi(\mathbf{r}, t) = U^\dagger \left[ -\frac{\hbar^2}{2\mu} \nabla^2 - i\hbar \frac{e}{\mu} \mathbf{A}(t) \cdot \nabla + V(\mathbf{r}) \right] U \Psi(\mathbf{r}, t),
\]

Evaluation of terms in equation 6 are straightforward.
The term $U^\dagger V(\mathbf{r})U$ can be evaluated as following:

$$U^\dagger V(\mathbf{r})U = \exp\left(\frac{i}{\hbar}\xi(t)\cdot\mathbf{p}\right) V(\mathbf{r}) \exp\left(-\frac{i}{\hbar}\xi(t)\cdot\mathbf{p}\right)$$

$$= V(\mathbf{r}) + [\xi(t)\cdot\nabla] V(\mathbf{r}) + \frac{1}{2!}[\xi(t)\cdot\nabla]^2 V(\mathbf{r}) + ...$$

$$= V(\mathbf{r} + \xi(t)) \quad (7)$$

where $\xi(t)$ represents the displacement of a free electron under the influence of the incident laser field. Hence, equation 4 becomes

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2\mu} \nabla^2 + V(\mathbf{r} + \xi(t))\right] \Psi(\mathbf{r}, t) \quad (8)$$

Equation (8) is a space-translated version of the time-dependent Schrödinger wave equation with incorporation of $\xi(t)$ into the potential in order to simulate the interaction of atomic system with the laser field. Now, considering the steady field condition, the vector potential takes the form $A(t) = E_0 \omega^{-1} \cos(\omega t)$ with $\xi(t) = \xi_0 \sin(\omega t)$, where $\xi_0 = E_0/\mu \omega^2$ is the amplitude of oscillation of a free electron in the field (which is called the laser-dressing parameter), $E_0$ denotes the amplitude of electromagnetic field strength and $\omega$ is the angular frequency. Now, we consider a pulse of electric field with a steady amplitude, the wave function in the K-H frame represents by the following Floquet form [63]:

$$\Psi(\mathbf{r}, t) = e^{-iE_{KH}t} \sum_n \Psi_k^n(\mathbf{r}) e^{-in\omega t}, \quad (9)$$

where Floquet quasi-energy has been denoted by $E_{KH}$. The potential in the K-H frame can be expanded in Fourier series as [62] [64]:

$$V(\mathbf{r} + \xi(t)) = \sum_{m=-\infty}^{+\infty} V_m(\xi_0, r) e^{-in\omega t}, \quad (10)$$

$$V_m(\xi_0, r) = \frac{i}{\pi} \int_{-1}^{+1} V_m(\mathbf{r} + \xi_0\rho) \frac{T_n(\rho)}{\sqrt{1 - \rho^2}} d\rho, \quad (11)$$

where we have taken the period as $2\pi/\omega$ and introduced a new transformation of the form $\rho = \sin(\omega t)$ while $T_n(\rho)$ are Chebyshev polynomials. Substituting equations (9), (10) and (11) into (8) yields a set of coupled differential equations:

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V_m(\xi_0, r) - (E_{KH} + n\hbar\omega)\right] \Psi_k^n(\mathbf{r}) =$$

$$- \sum_{m=-\infty}^{+\infty} V_m(\mathbf{r} + \xi(t)) \Psi_k^n(\mathbf{r}), \quad (12)$$

Considering the lowest order of approximation ($n = 0$) and high frequency limit condition (which means $V_m$ vanishes when $m \neq 0$), equation (12) becomes:

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V_0(\xi_0, r) - E_{KH}\right] \Psi_k^0(\mathbf{r}) = 0, \quad (13)$$

while $V_0(\xi_0, r)$ can be expanded using the Fourier series by following coefficients:

$$V_0(\xi_0, r) = \frac{1}{\pi} \int_{-1}^{+1} V(r + \xi_0\rho) \frac{d\rho}{\sqrt{1 - \rho^2}}, \quad (14)$$

Employing the Ehlotzky approximation [65]:

$$V(r + \xi_0\rho) + V(r - \xi_0\rho) \approx V(r + \xi_0) + V(r - \xi_0), \quad (15)$$

and hence, by evaluating the integral in (14), we obtain

$$V_0(\xi_0, r) = \frac{1}{2} [V(r + \xi_0) + V(r - \xi_0)] \quad (16)$$

Equation (16) is the approximate expression to model the laser field. Now we can write the Stationary-state Schrodinger equation by considering laser effect for potential $V(r)$ as follows:

$$(\nabla^2 + k^2) \Psi_k^0(\mathbf{r}) = \frac{\mu}{\hbar^2} [V(r + \xi_0) + V(r - \xi_0)] \Psi_k^0(\mathbf{r}) \quad (17)$$

where $k = \sqrt{2\mu E_{KH}/\hbar^2}$ is wave number. The equation (17) represents motion for one spherically confined two particles exposed to linearly polarized intense laser field radiation.

### III. ENTANGLEMENT FIDELITY AT THE PRESENCE OF A LASER FIELD

In order to give a quantitative measurement for the entanglement, we introduce the entanglement fidelity (EF). First, we consider the scattering of a particle through the potential. The equation (17) describes the stationary-state Schrodinger equation containing the potential which characterizes the quantum collision processes and laser effect.

The final state wave function $\Psi_k^0(\mathbf{r})$ is represented by the partial wave expansion [66] [67] in the following form:

$$\Psi_k^0(\mathbf{r}) = \sum_{l=0}^{\infty} \frac{i^l}{\sqrt{l+1}} D_l(k) P_l(\cos \theta) R_l(r) \quad (18)$$

where $D_l(k)$ is the expansion coefficient, $i$ is the pure imaginary number and $R_l(r)$ is the solution of the radial wave equation:

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr}\right) - \frac{l(l+1)}{r^2} R_l(r) = 0 \quad (19)$$

$P_l(\cos \theta)$ is the Legendre polynomial of order $l$, while $l$ is the angular momentum quantum number. For a spherically symmetric potential, it has been shown that the radial wave equation $R_l(r)$ and the expansion coefficient $D_l(k)$ are given by [27] [30]:
Asymptotic form of the radial wave function can be achieved by the phase-shift $\delta_l$ such that $R_l(r) \propto (kr)^{-1} \sin(\pi l/2 + \delta_l)$. It has been shown that the collisional EF for the scattering process can be represented by $f_k \propto |d^2r \Psi_j^0(r)|^2$, which is the absolute square of the scattered wave function for a given interaction potential \cite{27}. In low energy collisions, the main contribution in the scattering process is related to the partial $s$-wave scattering ($l = 0$). Therefore, the EF (i.e. $f_k$) can be calculated through the expansion coefficient $D_l(k)$ and the radial wave equation $R_l(r)$, as follows:

$$f_k \propto \frac{|D_0(k)|^2 |f_0^2 kr^2 R_0(k)|^2}{1 + \left| \frac{\mu_k}{\hbar^2} \int_0^\infty \frac{drrj_l(kr)}{V(r + \xi_0) + V(r - \xi_0)} R_l(r) \right|^2}$$ \hspace{1cm} (22)

The above equation can explain most features of the col-lisional EF successfully. As a cross check, one can find that in the limit $\xi_0 \to 0$ (when the influence of laser field vanishes) the Eq. (22) reduces to the result (the equation (18) ) of the Mishima et al work \cite{27}.

IV. EFFECTIVE POTENTIAL OF HOT QUANTUM PLASMAS

An analytical formulation for the effective interaction potential in hot quantum plasmas has been derived using the quantum approach including the influence of the effective plasma screening effects due to the collective plasma oscillations. The picture of a dressed Debye interaction between charged particles in hot quantum plasmas usually finds a complicated situation if we consider effective screening effects. Using the effective screening potential, the dressed electron-ion interaction potential $V_{eff}(r, \beta, \lambda_D)$ in hot quantum plasma can be written as \cite{37, 52, 68}:

$$V_{eff}(r, \beta, \lambda_D) = -\frac{Ze^2}{r} \frac{1}{4\sqrt{1 - \beta^2}} \left[ (4 - \beta) e^{-r/L_1(\beta, \lambda_D)} - 2 \left( 1 - \sqrt{1 - \beta^2} \right) e^{-r/L_2(\beta, \lambda_D)} \right]$$ \hspace{1cm} (23)

if the plasmon effects are neglected, the effective interaction potential $V_{eff}(r, \beta, \lambda_D)$ goes toward the classical Debye-Huckel potential: $V_{eff} \to V_{DH} = -\frac{Ze^2}{r} e^{-r/\lambda_D}$, since $L_1 \to \lambda_D$ and $L_2 \to 0$ as $\beta \to 0$.

V. EF OF HOT QUANTUM PLASMAS BY CONSIDERING A LASER FIELD

The fidelity ratio is a powerful measure for investigating the plasmon and plasma screening effects on the entanglement fidelity for the elastic collisions in hot quantum plasmas. The fidelity ratio $R_F = \frac{f_{\text{eff}}}{f_{\text{Coul}}}$ is calculated by the ratio of the entanglement fidelity $f_{\text{eff}}$ for the elastic electron-ion collision (using the effective interaction potential $V_{eff}(r, \beta, \lambda_D)$ in hot quantum plasmas) to the entanglement fidelity $f_{\text{Coul}}$ for the elastic
electron-ion collision using the pure Coulomb interaction $V_C = -\frac{Ze^2}{r}$. Thus, we have:

$$R_F = \frac{1 + \left| -\frac{2eZ^2}{\hbar^2} \int_0^\infty dr r^2 \sin(kr) \right|^2}{1 + \left| \frac{2eZ^2}{\hbar^2} \int_0^\infty dr r^2 \sin(kr) \right|^2}.$$  

We arrive at the following relation for the fidelity ratio:

$$R_F (E, \beta, \lambda_D, \xi_0) = \frac{1 + \frac{4\xi_0^2\mu^2 e^4}{\hbar^2 k^2}}{1 + \frac{4\xi_0^2\mu^2 e^4}{\hbar^2 k^2}|G_L|^2}$$  

We have used the following integration relation to calculate (26):

$$\int_0^\infty r e^{-\eta(r \pm \varepsilon)} \sin(kr) dr = \text{Im} \left\{ e^{\mp ik \xi} \times \right.$$  

$$\left[ \frac{\Gamma(1, \pm (\eta - ik) \varepsilon)}{\eta - ik(1 - \pm (\eta - ik) \varepsilon)} \mp \varepsilon Ei(1, \pm (\eta - ik) \varepsilon) \right]\}$$  

where

$$\Gamma(a, x) = \int_x^\infty e^{-t}dt; \text{Re}(a) > 0$$

$$Ei(1, x) = \int_x^\infty \frac{e^{-t}}{t}dt$$

It may be noted that $\Gamma(a, x)$ is called incomplete Gamma function and $Ei(1, x)$ is the Exponential integral [arfken]. In the limit $\xi_0 \to 0$, the Eq. (26) reduces to the derived equation by Jung [52] for a similar system but without considering the laser field.

VI. NUMERICAL DISCUSSION

Analytical discussion on the behavior of the system is very difficult because of complicated functions in which physical parameters are involved. Therefore, we have setup several numerical calculations in order to understand different features of the system. All numerical calculations have been done using the following dimensionless variables:

$$\bar{E} = a_Z^2k^2, \bar{L}_1 = \frac{L_1}{a_Z}, \bar{L}_2 = \frac{L_2}{a_Z}, = \frac{\lambda_D}{a_Z}, a_Z = \frac{\hbar^2}{\mu Ze^2}$$

The Amplitude of free electron oscillation in applied field, $\xi_0 = eE_0/\mu\omega^2$ is an important parameter. It is clear that $\xi_0$ is an increasing function of applied electric field and inversely proportional to the field angular frequency. In order to understand effects of $\xi_0$, we have plotted $R_F$ for several values of scaled collision energy $\bar{E}$. Figure [1] presents fidelity ratio $R_F$ as functions of $\beta$ (ratio of plasma energy to the thermal energy) for different values of $\xi_0$ and $\bar{E}$. Figures [1] show that, general behavior of $R_F$ is the same for all values of scaled collision energy $\bar{E}$ and the free electron oscillation amplitude ($\xi_0$). Fidelity rises to a maximum value and rapidly fall as $\beta$ increases. Maximum value of fidelity of the system increases when the scaled collision energy increases. This means that the entanglement between particles reduces, as the collision energy is increased. Variation of $R_F$ respect to $\xi_0$ is more noticeable with greater values of $\bar{E}$ as one can find from figures [2].

For finding better view about the behavior of $R_F$, we have plotted fidelity as functions of the scaled collision energy $\bar{E}$ for different values of $\beta$ and $\xi_0$ in figure [2].

Figures [2] indicate that Fidelity in the system destroys as collisional energy $\bar{E}$ increases. On the other hand, increasing the free electron oscillation amplitude $\xi_0$ also another reason of fidelity lost. In other words, both collision energy and free electron oscillation energy leading to decreasing the entanglement in the system. One can find from figures [2] that, we can optimize the fidelity ratio $R_F$. 
FIG. 1: The EFR $R_F$ including the plasmon, screening and laser effect as a function of the plasmon parameter $\beta$ when $\lambda_D = 15$ and a) $\bar{E} = 0.05$, b) $\bar{E} = 0.5$, c) $\bar{E} = 5$ and d) $\bar{E} = 10$. In all panels we have chosen, solid line ($\xi_0 = 0$), dashed line ($\xi_0 = 0.005$), dotted line ($\xi_0 = 0.01$), dash-dot line ($\xi_0 = 0.015$), long dash ($\xi_0 = 0.02$).

by taking suitable values for $\xi_0$ and $\bar{E}$ parameters, which is an interesting result. Another important parameter is the scaled Debye length $\lambda_D$. Effects of this parameter on the entanglement of the system can be explained using the figure 3.

Figures 3 show that fidelity ratio, slightly decreases when increases. In distances smaller than the Debye length, long range effects are screened. Therefore, it is expected that entanglement effects due to long range interactions decrease as the Debye length increases. On the other hand, the distance between entangled atoms are very smaller than the Debye length, thus screening effect in the entanglement between plasma particles is small as we learn from the figure 3.

The figures 4 provide better view to understand the effect of the amplitude of free electron oscillation ($\xi_0$), the collisional energy $\bar{E}$ and $\beta$ on the behavior of entanglement fidelity $R_F$. Figures 4 show that the $R_F$ generally is a periodic function of $\xi_0$ which its amplitude and frequency are functions of $\bar{E}$ and $\beta$. The oscillation frequency and its amplitude increase as collision energy increases. For a fixed value of $\xi_0$, variation of $R_F$ decreases as maximum value of $\beta$ increases.

VII. CONCLUSIONS

In this work, we have studied effects of electric field due to the applied laser beam on the effective potential, describing bi-partite systems. We have used the Kramers-Henneberger unitary transformation, in the quantized field formalism while the squared vector potential (which appears in the equation of motion) is
FIG. 4: The EFR $R_F$ including the plasmon, screening and laser effect as a function of the $\xi_0$, for $\bar{E} = 0.05$ (Solid line), $\bar{E} = 0.5$ (dashed line), $\bar{E} = 5$ (dotted line), $\bar{E} = 10$ (dash-dot line). Plasmon parameter is a) $\beta = 0.2$, b) $\beta = 0.5$, c) $\beta = 0.8$ while $\bar{\lambda}_D = 15$ in all case.

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