PLANETARY MIGRATION IN RESONANCE: 
THE QUESTION OF THE ECCENTRICITIES

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Abstract. The formation of resonant planets pairs in exoplanetary systems involves planetary migration inside the protoplanetary disc: an inwards migrating outer planet captures in Mean Motion Resonance an inner planet. During the migration of the resonant pair of planets, the eccentricities are expected to rise excessively, if no damping mechanism is applied on the inner planet. We express the required damping action to match the observations, and we show that the inner disk can play this role. This result applies for instance to the system GJ 876: we reproduce the observed orbital elements through a fully hydrodynamical simulation of the evolution of the resonant planets.

1 Introduction

Most of the known exoplanets are giant planets on short period orbits around their stars. In the core-accretion model, such planets should have formed much further away, beyond the so-called “snow-line”, where water is solid and contributes to the formation of a massive solid core. This suggests that inward planetary migration occurred in the protoplanetary disc. Through mutual gravitational interactions, a planet and the gaseous disc exchange energy and angular momentum. For a giant planet, this lead to the opening of a gap and to the migration of the planet \cite{Lin1986}.

A stronger evidence for migration is that a large fraction of multiplanet systems are engaged a Mean Motion Resonance (MMR): the ratio of the orbital periods of the outer to the inner planet, equals that of two small integers. This cannot happen by chance. It betrays a convergent migration of the planets and a dissipative process to enter this dynamical configuration.

For instance, in GJ 876, the two most massive planets are in a 2:1 MMR, with periods of \(\approx 30\) and \(\approx 60\) days. To model the formation of this system,
Lee & Peale (2002) performed 3-body simulations of a central host star and two planets, with additional (dissipative) forces that reproduced the effects of disc-planet interaction. Only the outer planet was forced to migrate inward (assuming that the inner planet has no ambient gas). Capture into resonance occurred when the outer planet crosses the location at which the mean orbital periods have a ratio of 2. After the resonant capture, however, the eccentricities of the two planets rise dramatically (except if an unrealistically high eccentricity damping is applied to the outer planet).

2 Migration of a pair of giant planets: analytical calculations

A planet of mass $M_p$, semi-major-axis $a$, and eccentricity $e$ has an energy $E = -GM_*M_p/2a$ and an angular momentum $H = M_p\sqrt{GM_*a(1-e)}$, where $M_*$ denote the sum of the masses of the central star and the planet. Denoting by $\tau_a = -\dot{a}/a$ and $\tau_e = -\dot{e}/e$, one finds:

$$\dot{E} = E/\tau_a$$  \hspace{1cm} (2.1)

$$\dot{H} = H/2 \left( \frac{2e^2}{1-e^2} \frac{1}{\tau_e} - \frac{1}{\tau_a} \right).$$  \hspace{1cm} (2.2)

If two planets are in resonance, the ratio between their energies is constant. We define $\varepsilon = E_2/E_1 = M_2a_1/M_1a_2$. The variation of the energy of the entire system (the pair of planets), $E = E_1 + E_2 = E_1(1 + \varepsilon)$, is the sum of the energy variations applied to both planets, such that $\dot{E} = E_1/\tau_a + E_2/\tau_e$.

The same holds for the angular momentum. If the planets are in MMR and their eccentricities are constant, the ratio between their angular momenta is also constant: $\eta = H_2/H_1$.

With some algebra, one finds a relation between $\tau_{a_1}$, $\tau_{a_2}$, $\tau_{e_1}$, and $\tau_{e_2}$ for the planets to evolve in resonance with given eccentricities. It reads:

$$\frac{1}{\tau_{e_1}} = \frac{1-e_1^2}{2e_1^2} \left( \frac{1}{\tau_{a_2}} \frac{\eta - \varepsilon}{1+\varepsilon} - \frac{1}{\tau_{e_2}} \frac{2e_2^2\eta}{1-e_2^2} \right) + \frac{1}{\tau_{a_1}} \frac{1-e_1^2}{2e_1^2} \frac{\varepsilon - \eta}{1+\varepsilon}$$  \hspace{1cm} (2.3)

The damping rate that should be applied to the inner planet ($1/\tau_{e_1}$) is the sum of two terms. The first one ($1/\tau_{e_1,\text{max}}$) is required to balance the action on the outer planet; it is not zero in general. This explains why the eccentricities rose too much in Lee & Peale’s simulations.

The second term ($K_1/\tau_{a_1}$) is proportional to $\tau_{a_1}^{-1}$, with $K_1 < 0$: if the inner planet is pushed outwards by an inner disk ($\dot{a}_1 > 0$), an additional damping is required.

3 Inner disc

We computed a few hydro-simulations of a giant planet, on a fixed orbit with various eccentricities. A gaseous disc is present inside the planetary orbit. We
Table 1. Parameters of the system GJ 876 as given by Laughlin et al. (2005) for GJ 876 b and GJ 876 c, and by http://exoplanets.eu/ for GJ 876 d.

| name | $M.\sin i$ ($M_{\text{Jup}}$) | period (days) | $a$ (AU) | $e$ |
|------|-----------------|-----------------|-------|-----|
| b    | 1.935           | 60.93 ± 0.03    | 0.20783 | 0.029 ± 0.005 |
| c    | 0.56            | 30.38 ± 0.03    | 0.13   | 0.218 ± 0.002 |
| d    | 0.023           | 1.94            | 0.020807 | 0. |

measure the torque and power of the force of the disc on the planet. We find that for $e > 0.1$, the disc damps the eccentricity of the planet. For $e < 0.3$, the expected migration if one releases the planet is directed outwards.

Thus, the inner disc can play a crucial role on the orbital evolution of a pair of planets in resonance. Note that the evolution of the inner disc depends crucially on the radius of its inner edge $R_{\text{inf}}$, with respect to $a$, the semi-major-axis of the orbit of the gap-opening planet (Crida et al., 2006). Thus, it was in type I migration, and most likely migrated all the way inwards, until it falls in the empty cavity between the star and $R_{\text{inf}}$. More precisely until the outermost resonance with the planet (the 2:1) lies in the cavity: the disc and the planet exchange angular momentum through the resonances. Therefore, we can say that in this system, $R_{\text{inf}}$ was located at the 2:1 MMR with GJ 876 d, that is at 0.033 AU.

4 Application to GJ 876

The two planets GJ 876 b and GJ 876 c are in 2:1 MMR (see table 1). A third, smaller planet orbits very close to the star. This planet is not massive enough to open a gap in the gas disc (see e.g. Crida et al., 2006). Thus it was in type I migration, and most likely migrated all the way inwards, until it falls in the empty cavity between the star and $R_{\text{inf}}$. More precisely until the outermost resonance with the planet (the 2:1) lies in the cavity: the disc and the planet exchange angular momentum through the resonances. Therefore, we can say that in this system, $R_{\text{inf}}$ was located at the 2:1 MMR with GJ 876 d, that is at 0.033 AU.

We use a code in which the whole disc is consistently simulated down to $R_{\text{inf}}$, thanks to the use of 1D grid coupled to the classical 2D grid (Crida et al., 2007). We compute a simulation of the two most massive planets of GJ 876 in a disc with a viscosity given by an $\alpha$-prescription with $\alpha = 10^{-2}$ and an aspect ratio of 0.07. First, the planets are on fixed circular orbits, and shape a gap in the disc. Then, they are released and they migrate freely. The results are shown on Figure 4. A resonant capture in the 2:1 MMR happens at $t = 110$; then, the eccentricities increase, but the presence of the inner disc prevents $e$ reaching too high values. In the end of the simulation, the semi-major-axes and eccentricities have almost exactly the observed values.
Fig. 1. Semi-major axis (blue, left y-axis) and eccentricity (red, right y-axis) evolution of GJ 876 b (light colour, $a_1$, $e_1$) and GJ 876 c (dark, $a_2$, $e_2$). The horizontal lines correspond to the observed values, as given by Table 1.

5 Conclusion

This work shows that the eccentricity of resonant giant exoplanets is governed by the presence of a gas disc inside the orbits of the innermost one. For given eccentricities, resonances and planet masses, the required action of the inner disc is provided analytically. In the GJ 876 system, the radius of the inner edge of the disc can be estimated thanks to the presence of a third planet; a numerical simulation of this system reproduces very well the observed parameters, which confirms the validity of the model.

In conclusion, the observed moderate eccentricities of the exoplanets in resonance suggest that in these systems, an inner disc was present during the migration.

This work has also led to the publication of an article in A&A (Crida et al., 2008).

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