Preheating curvature perturbations with a coupled curvaton

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Abstract

We discuss the potentially important rôle played by preheating in certain variants of the curvaton mechanism in which isocurvature perturbations of a D-flat (and F-flat) direction become converted to curvature perturbations during reheating. We analyse the transition from inflation to reheating in some detail, including the dynamics of the coupled curvaton and inflation fields during this transition. We discover that preheating could be an important source of adiabaticity where parametric resonance of the isocurvature components amplifies the super-horizon fluctuations by a significant amount. As an example of these effects we develop a particle physics motivated model which we recently introduced in which the D-flat direction is identified with the usual Higgs field. Our new results show that it is possible to achieve the correct curvature perturbations for initial values of the curvaton fields of order the weak scale. In this model we show that the prediction for the spectral index of the final curvature perturbation only depends on the mass of the curvaton during inflation, where consistency with current observational data requires the ratio of this mass to the Hubble constant to be $\leq 0.3$. 

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1 Introduction

In a recent paper we proposed a new mechanism for generating curvature perturbations during reheating in hybrid inflation [1]. The idea was that, during inflation, one of the slowly rolling scalar fields, other than the inflaton, could develop large isocurvature perturbations, and that this could become converted to curvature perturbations during the process of reheating. This is quite different to the curvaton mechanism as originally proposed [2,3] since the scalar with large isocurvature perturbations is not late decaying\(^1\). Instead the mechanism in [1] relies on the simple observation that just after the end of inflation, at the onset of reheating, the fields of hybrid inflation [8,9] tend to share out the vacuum energy which dominates during inflation. This typically leads to hybrid fields with energy densities of the same order of magnitude during the epoch of reheating, allowing efficient conversion of isocurvature to curvature perturbations during reheating. This mechanism should not be confused with the late decaying curvaton mechanism, whose physics is not related to reheating. However for ease of terminology we shall continue to call the hybrid field whose isocurvature perturbations during inflation become converted to curvature perturbations during reheating the “curvaton”, even though our mechanism is different from the curvaton mechanism as originally proposed.

The rôle of our curvaton will be played by a D-flat (and F-flat) direction\(^2\) during inflation, coupled to the waterfall field \(N\) of hybrid inflation. Therefore it is a “coupled curvaton”, which again is different from the original curvaton idea. Furthermore, being a D-flat direction, typically it will decay through its gauge interaction before the singlets whose decay rate is controlled by small Yukawa couplings. We gave an explicit example of the general mechanism in which the the Higgs fields of the supersymmetric standard model played the rôle of a coupled curvaton, i.e. developed isocurvature perturbations during inflation which subsequently become converted to curvature perturbations [1]. We discussed the nature and evolution of the Higgs perturbations during the epoch of inflation, and described the evolution of the perturbations during reheating after the Higgs decayed. After Higgs decay, the Universe consists of a mixture of matter (the oscillating singlet fields) and radiation (Higgs decay products), and we were able to show that the isocurvature perturbations are converted into curvature perturbations in this framework. However in the previous paper we did not analyse the transition from inflation to reheating before

\(^1\)For recent discussion on several scenarios for the late decaying curvaton see Refs. [4–7]).

\(^2\)In supersymmetric models, a flat direction has to be both D-flat and F-flat. For the inflaton and the \(N\) mediating field, being singlets, the D-flatness condition is trivial fulfilled. By calling the curvaton a D-flat direction what we want to stress is its non-singlet nature.
the Higgs curvaton decayed, and we also neglected the very important effects of preheating.

The purpose of the present paper is to discuss both of the above effects in detail, with surprising results (at least to us). Due to the presence of the coupled curvaton, the isocurvature perturbation (entropy) between the inflaton and the $N$ waterfall field is converted into the adiabatic curvature during the transition from inflation to reheating. The value of the total curvature perturbation at the end of the transition from inflation to reheating is then given in terms of the value of the coupled curvaton field rather than the inflaton. As it turns out, with the coupled curvaton in hybrid inflation, the initial value at horizon crossing of the isocurvature perturbation decreases during the transition from inflation to reheating. To be precise, we find that during the transition from inflation to reheating, before the Higgs curvaton decays, the curvature perturbation drops by several orders of magnitude from our estimate in [1]. Fortunately when preheating is taken into account the curvature perturbation may subsequently be increased by a similar number of orders of magnitude, resulting in an acceptable final amplitude of curvature perturbations, consistent with COBE and WMAP.

The layout of the remainder of the paper is as follows. In section 2 we briefly review the model, and in section 3 we describe the inflationary trajectory and the transition from inflation to reheating. In section 4 we describe the evolution of the fluctuations during inflation, and we carefully analyse the effect of the transition from inflation to reheating. It is during this transition that the curvature perturbation first changes. In section 5 we discuss the effects of preheating on the curvature perturbation, while allowing the Higgs to decay to radiation. Section 6 contains a discussion of the prediction of the spectral index and its correlation with the parameters which are consistent with giving correct structure. Section 7 concludes the paper.

2 The inflationary model

The supersymmetric hybrid inflation model is based on the superpotential [10]:

$$W = \lambda N H_u H_d - \kappa \phi N^2 ,$$

(1)

where $N$ and $\phi$ are singlet superfields, $H_{u,d}$ are the Higgs superfields, and $\lambda, \kappa$ are dimensionless couplings. Other cubic terms in the superpotential are forbidding by imposing a global $U(1)_{PQ}$ Peccei-Quinn symmetry. The superpotential in Eq. (1) includes a linear superpotential for the inflaton field, $\phi$, typical of hybrid inflation, as well as the singlet $N$ coupling to Higgs doublets as in the NMSSM. We notice that in standard hybrid inflation [8,9] the singlet $N$ will
quickly settle to its false vacuum value at zero, leaving the inflaton flat direction to slow-roll until it reaches the critical value. This means that during inflation the $\mu$ term generated by the coupling of $N$ to the Higgses in (1) vanishes, and the Higgses become an F-flat direction. Therefore, we can have an additional flat direction made by a combination of the Higgs fields which satisfies the D-flatness condition [11], i.e., such that D-term contribution to the potential $\left(g_2^2 + g_1^2\right)\left(H_u^2 - H_d^2\right)/8$ vanishes; for example taking them to be equal, $H_u = H_d = h$. Therefore, a more general inflationary trajectory is given by allowing the Higgs fields to take non-vanishing, although as we will see, small values.

Inflation takes place below the SUSY breaking scale. Including the soft SUSY breaking masses, $m_\phi$, $m_N$ and $m_h$, and trilinears $A_\kappa$, $A_\lambda$, the potential for the real part of the fields reads

$$V = V(0) + \frac{\kappa^2}{4}N^4 + \frac{\lambda^2}{4}h^4 + \kappa^2(\phi - \phi_c^+)(\phi - \phi_c^-)N^2 + \frac{\lambda^2}{2}N^2h^2 + \lambda N h^2 \left(\frac{A_\lambda}{\sqrt{2}} - \kappa \phi\right) + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_h^2h^2 + \frac{1}{2}m_N^2N^2,$$

(2)

The constant $V(0)$ has been added to ensure the necessary vanishing of the potential at its global minimum, and the critical value of $\phi$, $\phi_c^\pm$, are given below in terms of the soft SUSY parameters:

$$\phi_c^\pm = \frac{A_\kappa}{2\sqrt{2}\kappa} \left(1 \pm \sqrt{1 - \frac{4m_N^2}{A_\kappa^2}}\right).$$

(3)

As we will see in the next section, during inflation (when $\phi > \phi_c^+$) the inflaton field, $\phi$, is slowly rolling along the potential and also the fields $N$ and $h$ get small values but such that the $N$ field dependent mass squared, $\bar{m}_N^2$, is dominated mainly by the term $\kappa^2(\phi - \phi_c^+)(\phi - \phi_c^-)$. Once at the critical value, $\phi = \phi_c^+$, $\bar{m}_N^2$ changes sign and both singlets, $\phi$ and $N$, roll down towards the global minimum $\phi_0$ and $N_0$ ending inflation.

$$\phi_0 \approx \frac{\phi_c^+ + \phi_c^-}{2} = \frac{A_\kappa}{2\sqrt{2}\kappa},$$

(4)

$$N_0 \approx \frac{\phi_c^+ - \phi_c^-}{\sqrt{2}} = \frac{A_\kappa}{2\kappa} \sqrt{1 - \frac{4m_N^2}{A_\kappa^2}}.$$

(5)

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3The axionic (imaginary) part of the complex fields can be set consistently to zero, and it will not affect the evolution of the real scalar fields either during inflation or the reheating period.

4We shall see that $h \ll N_0, \phi_0$ and also that $m_\phi^2, m_N^2 \ll \kappa^2N_0^2$. Therefore either contributions from the Higgs field, $h$, and from the bare masses, $m_\phi^2, m_N^2$, can be neglected in order to calculate the global minima, $\phi_0$ and $N_0$. 

3
The Higgs field, $h$, will oscillate once inflation ends around the minima which is zero as long as the Electroweak Symmetry is still preserving.

Before describing the inflationary trajectory, we briefly discuss the values of the parameters of the potential in Eq. (2). They are uniquely determined by embedding this model into an extra-dimensional framework with just one fundamental (“string”) scale $M_s \sim 10^{13}$ GeV \[13\]. Embeding all the Higgs fields and singlets $\phi$ and $N$ in the bulk, while all the matter fields live on the brane, it is possible to show that \[13,12\]

$$\lambda \sim \kappa \sim \left(\frac{M_s}{m_P}\right)^2 \sim 10^{-10},$$

$$A_\kappa \sim A_\lambda \sim M_s \left(\frac{M_s}{m_P}\right)^2 \sim 10^3 \text{GeV},$$

$$m_\phi \sim m_N \sim m_h \sim \frac{1}{4\pi} M_s \left(\frac{M_s}{m_P}\right)^3 \sim 1 \text{MeV},$$

$$V(0)^{1/4} \sim M_s \left(\frac{M_s}{m_P}\right) \sim 10^8 \text{GeV},$$

where $m_P = M_P/\sqrt{8\pi} = 2.4 \times 10^{18} \text{GeV}$ is the reduced Planck mass. Using Eqs. (4) and (5) we obtain:

$$\phi_0 \sim N_0 \sim M_s \sim 10^{13} \text{GeV},$$

and the Hubble parameter during inflation is of the order of

$$H = \frac{V(0)^{1/2}}{\sqrt{3} m_P} \sim 10 \text{MeV}.$$  

Notice that $m_\alpha < H$ for $\alpha = \phi, N, h$.

We will use those values parameters in the following sections when presenting numerical results. However the mechanism is general for a potential like Eq. (2) independently of parameter values, and the analytical estimations are presented in general in terms of the model scales, $H$, $\phi_c$, $\kappa\phi_c$. We will refer in the following to the “Higgs” like the coupled curvaton, D-flat direction during inflation, included in Eq. (2).

### 3 Evolution of the background fields.

Including the Higgs terms in the potential Eq. (2), the term proportional to $\lambda N h^2$ can induce a non-zero value for $h$ and $N$ during inflation providing that the coefficient of this term is negative.

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\[5\] In a future work \[12\] we will show how the electroweak symmetry breaking is implemented in this model and their cosmological consequences. However here we are going to consider that the electroweak symmetry is preserved all the time, since our principal conclusions will not be affected by this issue.

\[6\] For order of magnitude estimations, we will take $\phi_c \sim \phi_c^\pm \sim \phi_0 \sim N_0$, and $\lambda \simeq \kappa$. 

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\[ \frac{A}{\sqrt{2}} - \kappa \phi < 0. \] However, in order to ensure a flat direction for the Higgs, and therefore allowing it to slow-roll during inflation, its effective field dependent mass must be smaller than \( H^2 \). The latter receives contributions from all the fields,

\[ \frac{\partial^2 V}{\partial h^2} \simeq \lambda^2 (3h^2 + N^2) + 2\lambda N \left( \frac{A}{\sqrt{2}} - \kappa \phi \right) + m_h^2 < H^2. \] (12)

This implies that both the fields \( N \) and \( h \) must take small values during inflation, with

\[ \frac{N}{\phi_c} < \left( \frac{H}{\kappa \phi_c} \right)^2 \approx 10^{-10}, \quad \frac{h}{\phi_c} < \frac{H}{\lambda \phi_c} \approx 10^{-5}, \] (13)

so that \( N \ll h \ll \phi \). These values are small enough not to disturb the inflaton slow-roll,

\[ \frac{\partial^2 V}{\partial \phi^2} \simeq \kappa^2 N^2 + m_\phi^2 \simeq m_\phi^2 < H^2. \] (14)

The Higgs and inflaton fields evolve independently of each other and \( N \), following the approximate slow-roll trajectories given by:

\[ \dot{\phi}(t) \simeq -\eta_\phi H \phi(t), \quad \dot{h}(t) \simeq -\eta_h H h(t), \] (15)

where \( \eta_h = m_h^2/(3H^2), \eta_\phi = m_\phi^2/(3H^2) \). Inflation as such is still controlled by the inflaton \( \phi \), and will end when \( \phi \) reaches one of its critical values \( \phi_c^\pm \).

On the other hand, the \( N \) field follows the evolution equation:

\[ \ddot{N} + 3H \dot{N} + V_N = 0, \] (16)

with

\[ V_N = \frac{\partial V}{\partial N} = (\kappa^2 N^2 + 2\kappa^2 (\phi - \phi_c^+)(\phi - \phi_c^-) + \lambda^2 h^2)N + \lambda h^2 \left( \frac{A}{\sqrt{2}} - \kappa \phi \right) \]

\[ \approx \omega_N^2(\phi)N + \lambda h^2 \left( \frac{A}{\sqrt{2}} - \kappa \phi \right), \] (17)

where in the second line \( \omega_N^2(\phi) \gg H^2 \) is the field dependent effective squared \( N \) mass, where we have dropped the small term \( \kappa^2 N^2 + \lambda^2 h^2 \) using Eq. (13). During inflation we can take the slow-rolling fields \( h \) and \( \phi \) to be approximately constant in Eq. (17). Thus, the \( N \) field will oscillate with an amplitude damped by the expansion as \( \propto a(t)^{-3/2} \) and a frequency of the order of \( \omega_N(\phi) \approx O(\kappa \phi_c) \gg H \), but around a non vanishing value given by the quasi-constant contribution in Eq. (17), i.e,

\[ N(t) \approx \frac{\lambda h^2}{\omega_N^2(\phi)} (\kappa \phi - \frac{A}{\sqrt{2}}) + \frac{1}{a(t)^{3/2}} F_{osc}[\omega_N(\phi)], \] (18)
Figure 1: We show the evolution of the background fields (solid lines) and time derivatives (dashed lines) during inflation up to the critical value $\phi = \phi^+_c$ (LHS), and from the critical value up to the global minimum (RHS). We have taken the values of the parameters: $\phi_0 = 10^{13}$ GeV, $\phi^+_c = 2\phi_0$, $\kappa \phi_0 = 1$ TeV, $A_\lambda = 1.5\sqrt{2}\kappa \phi_0$, $m_\phi = m_h = 0.1H$, and the value of the Higgs field $h(0) = 247$ GeV. The numerical integration starts around 60 e-folds before the end of inflation.

where $F_{osc}[\omega_N(\phi)]$ represents the oscillating part. Given that in any case the amplitude of the oscillations quickly decays due to the exponential expansion, after some e-folds they become completely negligible, and Eq. (18) gives the trajectory:

$$N(t) \simeq \frac{\lambda h^2}{\omega_N^2(\phi)} \left( \kappa \phi - \frac{A_\lambda}{\sqrt{2}} \right),$$

that is, the motion of $N$ effectively follows the valley of minima in the potential given by $V_N = 0$. This inflationary trajectory for $N$ is independent of the initial conditions, provided that the other fields are already slowly rolling. Although in origen $N$ is a “heavy” field (mass larger than the Hubble rate), the dynamics is such that it behaves effectively as a “light” one. The quasi-exponential expansion during inflation settles it to the minimum, but the minimum is a time-dependent, slow-rolling one. This has implications also from the point of view of its quantum fluctuations: they will behave in a similar way as those of a light field.

The evolution of the background fields and their time derivatives can be seen in Fig. (1). We have chosen to plot the evolution against the difference $|\phi - \phi^+_c|$ in order to detail the changes around the critical point, which we analyse in the remaining of this section. In the left panel, we have plotted the evolution during inflation up to the critical point, and in the right one that from the critical point to the global minimum. Time flows in both from left to right, and we
have started the numerical integration around 60 e-folds before the end of inflation. Although the $N$ field is also “evolving” during inflation, this is standard hybrid inflation in the sense that it is the $N$ field which triggers its end as can be seen in RHS plot in Fig. (1), when $\phi$ leaves the slow-rolling trajectory $\dot{\phi} \approx \text{Constant}$.

In the LHS plot in Fig. (1), it can be seen that close to but before $\phi$ reaches the critical value, as $\omega_N^2(\phi) \to 0$, the values of the derivatives of $N$ increase very rapidly and the effective “slow roll” of the $N$ field given in Eq. (19) is violated. In particular, $\ddot{N}$ becomes now the dominant term to cancel out the Higgs and $\phi$ quasi-constant contribution in the equation of motion, such that

$$\ddot{N} + 3H \dot{N} + V_N = 0 \Rightarrow \ddot{N} \simeq \lambda h^2 (\kappa \phi - \frac{A_\lambda}{\sqrt{2}}).$$

We want to estimate the ratio $(\dot{N}/N)$ at the critical value $\phi = \phi_c^+$, which will be of use later when computing the metric perturbations. The transition around the critical value, and from there to the global minimum, takes a very short interval of time with $H \Delta t \ll 1$, so that we can safely neglect the effect of the expansion in the analytical estimations. By matching the above trajectory Eq. (20) with the inflationary one Eq. (19), we see that the transition between trajectories occurs when

$$\kappa(\phi - \phi_c^+) \simeq \left(\frac{\dot{\phi}}{\phi}\right)^{2/3} \left(\kappa(\phi_c^+ - \phi_c^-)\right)^{1/3},$$

and we have denoted this point by a subindex “$-$”. Given the short interval of time lapsed between this point and the critical value, practically the values of $N$ and $\dot{N}$ at $\phi_c^+$ are given by those derived from Eqs. (19) but evaluated at $\kappa(\phi - \phi_c^+) \simeq \eta_\phi H(\phi_c^+ - \phi_c^-) \phi_c^+)^{1/3}$, which gives the ratio,

$$\left(\frac{\dot{N}}{N}\right)_c \simeq \frac{\eta_\phi H \phi_c^+}{(\phi - \phi_c^+)} \simeq \left(\eta_\phi H \kappa^2(\phi_c^+ - \phi_c^-)\phi_c^+\right)^{1/3},$$

where we have used the slow-roll approximation $^7 \dot{\phi}/\phi \simeq \eta_\phi H$.

$^7$This is only consistent if by that time the value of $N_c$ is still small enough not to disturb the others trajectories, i.e.,

$$N_c \simeq \lambda h^2 (\kappa \phi_c^+ - \frac{A_\lambda}{\sqrt{2}}) \left(\frac{\phi}{\phi_c^+}\right)^{2/3} \left(\frac{H}{\kappa \phi_c^+}\right)^2 < \left(\frac{H}{\kappa \phi_c^+}\right)^2 \phi_c^+$$

which implies

$$\lambda h < O(H \left(\frac{\phi}{\phi_c^+}\right)^{1/3} < H,$$

which we assume to hold hereafter unless otherwise stated. That is, end of inflation in the standard sense will proceed only after $\phi$ crosses the critical value. If Eq. (24) were not fulfilled, for example when the value of the Higgs at horizon crossing is very close to the upper limit given in Eq. (13), then inflation will terminate instead before $\phi$ reaches the critical value by the increasing values of $N$ and $h$. 

7
Then, as shown in the RHS plot in Fig. (1), as soon as $\phi$ crosses the critical value and the $N$ field dependent squared mass becomes negative, the value of the field starts growing exponentially,

$$N \sim N_c exp(\int \sqrt{2\kappa (\dot{\phi}_c^+ - \phi)(\phi - \phi_c^-)} dt),$$

until finally the large values of $N$ modify the slow-roll of the inflaton field, and this also moves fast towards the minimum. Afterwards, $N$ and $\phi$ will move towards a straight-line trajectory [14] given by

$$N = \sqrt{2}(\phi_c^+ - \phi).$$

Once $N$ reaches the trajectory in Eq. (26), the slow-roll regime for the inflaton (and the Higgs) ends. The Higgs also gets desestabilized due to the increasing $N$ field, and it will just become increasingly coupled to the others, but at a slower pace; i. e., the Higgs does not really affect the transition of the inflaton and the $N$ from the critical point to the global minimum.

## 4 Curvature perturbation during inflation and the subsequent phase transition.

Our final aim is to compute the spectrum of primordial curvature perturbations, initially originated during inflation. In the presence of several light (slow-rolling) scalar fields $\phi_\alpha$, the comoving curvature perturbation [15] can be written as [16,17]

$$\mathcal{R} = H \sum_\alpha \left( \frac{\dot{\phi}_\alpha}{\sum_\beta \dot{\phi}_\beta^2} \right) Q_\alpha,$$

in terms of the gauge-invariant scalar field amplitude perturbations, the Sasaki-Mukhanov variables $Q_i$ [18],

$$Q_\alpha = \delta \phi_\alpha + \frac{\dot{\phi}_\alpha}{H} \psi,$$

where $\delta \phi_\alpha$ is the quantum fluctuation of the background field $\phi_\alpha$, and $\psi$ the gauge-dependent metric perturbation (see Appendix A). The total curvature perturbation is then given as the weighted sum of “individual” curvature perturbations

$$\mathcal{R}_\alpha = H \frac{Q_\alpha}{\dot{\phi}_\alpha},$$

such that,

$$\mathcal{R} = \sum_\alpha \frac{\dot{\phi}_\alpha^2}{\sum_\beta \dot{\phi}_\beta^2} \mathcal{R}_\alpha = \sum_\alpha \frac{\rho_\alpha + P_\alpha}{\rho + P} \mathcal{R}_\alpha.$$
with \( \dot{\phi}_\alpha = \rho_\alpha + P_\alpha \) for a scalar field, \( \rho_\alpha \) being the energy density and \( P_\alpha \) the pressure. The RHS of Eq. (30) applies in general to a multi-component Universe, independently of the nature (scalar fields or other) of the components. In order to follow the evolution of the curvature perturbation from inflation to the reheating stage, we will follow in detail that of the individual components \( R_\alpha \) and how their relative contribution to \( R \) changes.

During inflation, nevertheless, the field that dominates the total curvature perturbation is the one with the largest velocity, this being the inflaton field \( \dot{\phi} \gg \dot{h} \gg \dot{N} \). From Eq. (30), we simply have \( R_{inf} \simeq R_\phi \). However, as we will see next, the curvature perturbations of the Higgs and \( N \), \( R_h \) and \( R_N \), are orders of magnitude larger than that of \( R_\phi \). Relative perturbations between different components are the origin of the so-called entropy or isocurvature perturbations, \( R_\alpha - R_\beta \). Therefore in our scenario, given the strong hierarchy between the values of the fields, there is a large “isocurvature” perturbation between the inflaton and the Higgs, and the inflaton and the \( N \). Those can later seed the total curvature perturbation, if one of the field (kinetic) energy density other than that of the inflaton becomes non-negligible [19–24]. This is what happens at the end of inflation: as soon as the inflaton field reaches the critical value, the field \( N \) evolution changes and moves fast towards the global minimum, with increasing kinetic energy. Therefore, during the transition from the critical point \( \phi_0^+ \) towards the minimum \( \phi_0 \), the field \( N \) with a large kinetic energy gives the dominant contribution to the total curvature perturbation in Eq. (30); i.e., the entropy perturbation between \( N \) and \( \phi \) becomes adiabatic curvature perturbation. This can be seen by writing the evolution equation for the total curvature \( \dot{R} \) in the presence of the scalar fields [23] \( \phi, N \) and \( h \):

\[
\dot{R} \simeq -3H \sum_{\alpha,\beta} (c^2_\alpha - c^2_\beta) \frac{h_\alpha h_\beta}{h_T^2} (R_\alpha - R_\beta) \\
\simeq -3H (c^2_\phi - c^2_N) \frac{h_\phi h_N}{h_T^2} (R_\phi - R_N) - 3H (c^2_N - c^2_h) \frac{h_N h_h}{h_T^2} (R_N - R_h) \\
- 3H (c^2_h - c^2_\phi) \frac{h_h h_\phi}{h_T^2} (R_h - R_\phi),
\]

(31)

where as a shorthand notation we have defined: \( h_\alpha = \rho_\alpha + p_\alpha = \dot{\phi}_\alpha \) and \( h_T \) is the total density energy, \( h_T = h_\phi + h_N + h_h \); \( c^2_\alpha = \dot{\phi}_\alpha / \dot{\rho}_\alpha \) is the sound speed for each component,

\[
c^2_\alpha = 1 + \frac{2V_\alpha}{3H \dot{\alpha}}.
\]

\( ^8 \)So called as far as the originating field or fluid component does not dominate the total energy density, and therefore its fluctuations make little contribution to the curvature one.

\( ^9 \)As we are only interested in large scale fluctuations, we neglect the term that goes like \((k/aH \ll 1)\) in the equation.
Clearly when $\dot{N} \simeq \dot{\phi}$, the first term in Eq. (31) will dominate, such that $R \simeq R_N$. Notice also that during the transition $\phi$ and $N$ behave as “different fluid” components with different sound speeds: the former is still slowly-rolling while $N$ is increasing exponentially.

For a multi-scalar system, rotating the fluctuations in field space, as done in Ref. [17], helps to identify all along the “adiabatic” mode responsible for the total curvature, and the “entropy” modes that influence its evolution. For comparison with this approach, we have included the rotated equations in Appendix B. Entropy modes source the adiabatic one when the fields follow a curved trajectory in field space. This matches the analytical behaviour in the next subsection: during the transition, the trajectory is curved by the motion of $N$, such that the curvature changes.

4.1 During inflation.

The equation of motion for the gauge invariant quantum fluctuation $Q_\alpha$, with comoving wavenumber $k$, is given by [25]

$$\ddot{Q}_\alpha + 3H\dot{Q}_\alpha + \frac{k^2}{a^2}Q_\alpha + \sum_\beta V_{\alpha\beta}Q_\beta = \left[\frac{1}{a^3m_p^2} \left(\frac{a^3}{H}\dot{\phi}_\alpha \dot{\phi}_\beta\right)\right] Q_\beta,$$

where $V_{\alpha\beta} = \partial^2 V/\partial \phi_\alpha \partial \phi_\beta$. For the inflaton and Higgs fields we have $V_{\phi\phi}, V_{hh}, V_{\phi h} \ll H^2$ during inflation; i.e., their fluctuations behave nearly as those of a massless field. Hence, $Q_\phi$ and $Q_h$ will be frozen to a constant value once outside the horizon, $k < aH$, given approximately by the value at horizon crossing $k = aH$ [26],

$$Q_{\phi^*} = Q_{h^*} = \frac{H_s}{\sqrt{2k^3}},$$

with $R_\phi = HQ_\phi / \dot{\phi} \ll R_h = HQ_h / \dot{h}$ during inflation.

On the other hand, neglecting for simplicity the sub-dominant terms coming from metric contributions on the R.H.S in Eq. (33), the evolution equation for $Q_N$ reads

$$\ddot{Q}_N + 3H\dot{Q}_N + \left(\frac{k^2}{a^2} + V_{NN}\right)Q_N + V_{N\phi}Q_\phi + V_{Nh}Q_h \simeq 0.$$  \hspace{1cm} (35)

Like in the case of the evolution of the background field $N$, now the large mass term $V_{NN} \sim O(\kappa^2\phi^2) \gg H^2$ gives rise to oscillations with an amplitude decaying as $a^{-1}$, but displaced from zero due to the mixing with $Q_\phi$ and $Q_h$. Therefore, once the amplitude of the oscillations

\hspace{1cm}\footnotesize{\textsuperscript{10}}The metric contributions are of the order of $O(\phi^2/\rho)$, and therefore negligible during inflation.
becomes negligible, the $N$ field fluctuation will also tend to a quasi-constant value given by
\[ Q_N \simeq -\frac{V_{N\phi}}{V_{NN}} Q_\phi - \frac{V_{Nh}}{V_{NN}} Q_h, \tag{36} \]
and using Eq. (19) we have for $Q_N$ and the background velocity $\dot{N}$:
\[ Q_N \simeq N \left( 2 \frac{Q_h}{h} + C_\phi(\phi) \frac{Q_\phi}{\phi} \right) \simeq 2N \frac{Q_h}{h}, \tag{37} \]
\[ \dot{N} \simeq N \left( 2 \frac{\dot{h}}{h} + C_\phi(\phi) \frac{\dot{\phi}}{\phi} \right), \tag{38} \]
with
\[ C_\phi(\phi) = \frac{\kappa \dot{\phi}}{\kappa \dot{\phi} - A_\lambda/\sqrt{2}} - \frac{4\kappa^2 (\phi - \phi_0) \phi}{\omega_N^2(\phi)} \sim O(1), \tag{39} \]
and $Q_h/h \gg Q_\phi/\phi$. Then, using Eqs. (37) and (38), $R_N$ is given during inflation by:
\[ R_N \simeq 2 \left( \frac{N}{N} \right) \left( \frac{\dot{h}}{h} \right) R_h \tag{40} \]
\[ \simeq -\frac{H}{2\eta_h + C_\phi(\phi) \eta_\phi} \frac{Q_h}{h}, \tag{41} \]
which is of the same order of magnitude than $R_h$ during inflation, while $\phi$ is not too close to the critical value. We stress that it is the Higgs flat direction which holds the $N$ field to a non-vanishing, non-decreasing value during inflation, in a kind of “slow-roll” trajectory; in a similar way their quantum fluctuations $Q_N$ follow $Q_h$ instead of being redshifted away.

We do not require any special initial conditions for Eq. (40) to hold, apart from the fields being already in their slow-roll trajectories. Given that inflation will last more than this, the fields will have most probably reached that trajectory much before the largest observable scale today become super-Hubble, around say 60 e-folds before the end of inflation.

Therefore, up to near the critical value, $R_\phi$ and $R_h$ are frozen to their values at horizon crossing, as expected for nearly massless and decoupled fields, while $R_N$ in Eq. (40) decreases as the ratio $\dot{N}/N$ evolves in time. In order to estimate its value by the time inflation ends, we use the fact that the ratio $Q_N/N$ remains approximately constant, and given by Eq. (37) all along inflation. Therefore, the contribution $R_N$ at the critical value is given by Eq. (40) but evaluated using $(\dot{N}/N)_c$ in Eq. (22),
\[ R_{Nc} \simeq 2 \left( \frac{N}{N}_c \right) \left( \frac{\dot{h}}{h}_c \right) R_{hc} \simeq 2 \left( \frac{\eta_h}{\eta_\phi} \right) \left( \frac{H^2}{\kappa^2(\phi_c^+ - \phi_c^-)} \right)^{1/3} \frac{1}{\eta_\phi} R_{hc}. \tag{42} \]
Thus, by the end of inflation the value of $\mathcal{R}_N$ is suppressed with respect to its value at the time of horizon crossing by a factor

$$\mathcal{R}_{Nc} \approx \left(\frac{\eta_{\phi}H}{\kappa\phi_c^+}\right)^{2/3} \mathcal{R}_{N*} \approx \left(\frac{\eta_{\phi}\phi_c}{m_P}\right)^{2/3} \mathcal{R}_{N*},$$

which for $\eta_{\phi} \approx 0.01$ and $\kappa\phi_c \approx 1$ TeV, with $H \approx 10$ MeV ($\phi_c \approx 10^{13}$ GeV) would give a suppression factor approximately of the order of $10^{-5}$.

### 4.2 During the phase transition.

After crossing the critical value, the ratio $Q_N/N$ still remains practically constant until $N$ reaches the straight line trajectory Eq. (26). Both evolutions, that of $N$ and its fluctuation, are now governed by the tachyonic instability in their effective mass. This means that $\mathcal{R}_N \propto Q_N/\dot{N}$ diminishes again by a factor $\dot{N}/N$, before the fields reach the global minimum. Let us denote the point when they reach the trajectory Eq. (26) by a subindex “+”. Thus, the value of $\mathcal{R}_{N+}$ is given by

$$\mathcal{R}_{N+} = \left(\frac{\dot{N}/N}{N/N_+}\right)_{\mathcal{R}_{Nc}} = \left(\frac{\phi_c^+ - \phi}{\phi - \phi_c^+}\right)_{\mathcal{R}_{Nc}},$$

with $\mathcal{R}_{Nc}$ given by Eq. (42) above, and $(\phi - \phi_c^+)_{-}$ by Eq. (21). By using Eq. (25) and the condition Eq. (26), we have $(\phi_c^+ - \phi)_{+} \approx (\phi - \phi_c^+)_{-}$. Therefore, during the waterfall the value of $\mathcal{R}_N$ would only decrease at most by an order of magnitude. Numerically we found $(\phi_c^+ - \phi)_{+} \approx 0.2(\phi - \phi_c^+)_{-}$.

In the final part of the transition from $\phi_c$ to $\phi_0$, the evolution of the quantum fluctuations become coupled through the interaction in the potential. Given that $N = \sqrt{2}(\phi - \phi_c^+)$, and $\dot{N} = -\sqrt{2}\dot{\phi}$, for the fluctuations we have $Q_N \propto \dot{\phi}$ and $Q_{\phi} \propto \dot{\phi}$ such that $Q_N = -\sqrt{2}Q_{\phi}$ [14]; i.e., once Eq. (26) is fulfilled, $\mathcal{R}_N$ and $\mathcal{R}_{\phi}$ remain constant. By the time they reach the global minimum we simply have:

$$\mathcal{R}_{\phi_0} = \mathcal{R}_{N_0} = \mathcal{R}_{N_+}.$$  \hfill (45)

Because the background Higgs is still moving towards the minimum but with $\dot{h} \ll \dot{\phi}$, the total curvature perturbation is then given by the contributions of the 2 singlets, with

$$\mathcal{R}_0 \approx \mathcal{R}_{\phi_0} = \mathcal{R}_{N_0}. \hfill (46)$$

Using Eqs. (44) and (42), we then have:

$$\mathcal{R}_0 \approx 0.4 \left(\frac{\eta_{\phi}}{\eta_{\phi_{_{1/3}}}^2}\right) \left(\frac{H^2}{\kappa^2(\phi_c^+ - \phi_c^-)\phi_c^+}\right)^{1/3} \mathcal{R}_{h*}. \hfill (47)$$

12
And for the spectrum, \( P_\mathcal{R} \equiv k^3\langle |\mathcal{R}|^2 \rangle / (2\pi^2) \), we obtain
\[
P_{\mathcal{R}0}^{1/2} \simeq 0.4 \left(\frac{H_s^2}{\kappa^2 (\phi_c^+ - \phi_c^-) \phi_c^+} \right)^{1/3} \frac{H_s}{\eta_\phi^{1/3} h_*} \approx \left(\frac{\phi_c}{m_P} \right)^{2/3} \frac{H_s}{\eta_\phi^{1/3} h_*}.
\] (48)

This can be compared to the initial value of the amplitude of the spectrum, given by that of the inflaton at horizon crossing,
\[
P_{\mathcal{R}_*}^{1/2} \approx \frac{H_*}{\eta_\phi \phi_*},
\] (49)

with
\[
P_{\mathcal{R}_0}^{1/2} \approx \left(\frac{\eta_\phi \phi_c}{m_P} \right)^{2/3} \frac{\phi_*}{h_*} P_{\mathcal{R}_*}^{1/2}.
\] (50)

Clearly, Eq. (48) can be larger than the standard value Eq. (49), if \( h_*/\phi_* \ll (\phi_c \eta_\phi / m_P)^{2/3} \), which is required in any case to ensure slow-roll inflation. In this scenario, the total curvature at the end of inflation grows from its value at horizon crossing, as the fields dynamic changes and the relative contribution of the \( N \) field to the total curvature become non-negligible during the phase transition. As we have already mentioned, this can be interpreted as converting the isocurvature perturbation between \( N \) and \( \phi \) into the adiabatic one \( \mathcal{R} \), see Eq. (31). However, the presence during inflation of a large amplitude \( \mathcal{R}_N \) is initially controlled by the Higgs. Because of that we prefer to refer to the Higgs \( h \) field as “curvaton”, instead of the mediating field \( N \).

The Higgs does not contribute per se to the total curvature at this point, because its velocity is still negligible with respect to the others. Nevertheless, the evolution of \( h, \dot{h} \) and \( Q_h \) resembles that of the other fields. First, during the waterfall when \( N \) becomes larger than \( H^2/\phi \), the slow-roll approximation for the Higgs breaks down. The evolution of the Higgs is then dominated by its linear coupling to \( N \), such that
\[
\ddot{h} \approx 2\lambda h N (\kappa \phi - \frac{A_\lambda}{\sqrt{2}}),
\] (51)

and then \( \dot{h} \propto \sqrt{N} \). The fluctuation \( Q_h \) follows the background field, and as a result \( R_h \) decreases. Until the other fields reach the the trajectory Eq. (26); then again we recover the relation \( Q_h \propto \dot{h} \), and \( \mathcal{R}_h \) will tend to level with the others.

The behavior of the spectrum of the different contributions \( R_\alpha \) during inflation and the phase transition, is plotted in Fig. (2). The evolution is shown in logarithmic scale versus the value of \( \phi - \phi_c^+ \), same than in Fig. (1). In the LHS plot, we follow them from horizon crossing \( k = aH \), taken to be 60 e-folds before the end, through inflation, up to the critical value (strictly speaking, at infinity outside the plot); and in the RHS plot we have their evolution during
the transition towards the global minimum. This plot summarises the previous findings: during inflation $\mathcal{R}_h$ and $\mathcal{R}_\phi$ remains constant, while the initial $\mathcal{R}_N \simeq \mathcal{R}_{h*}$ is suppressed as it approaches the critical point due to the increasing $\dot{N}$ (see Fig. (1)). When inflation ends, $N$ falls fast into the global minimum, dragging in the way also the inflaton and to some extent the Higgs (Fig. (1)). As a result, their quantum fluctuations will tend to follow each other, such that finally $\mathcal{R}_{h0} \simeq \mathcal{R}_{N0} = \mathcal{R}_{\phi0}$.

Nevertheless, given the model parameter values in section 2.1, it seems that the amplitude of the spectrum in Eq. (48) is too small to match the COBE value, $P_{\mathcal{R}}^{1/2} \simeq 5 \times 10^{-5}$, unless we fine-tune the value of $\eta_\phi$. However, this is the value at the end of inflation. This will again be modified by the subsequent evolution of the fields during reheating, in particular it can be amplified by “preheating” effects.

5 Curvature perturbation during reheating and preheating.

When the background fields oscillate around their global minimum values, all the three effective masses $\bar{m}_i$ become of the same order of magnitude, i.e., $\bar{m}_i \sim O(\kappa \phi_0) \sim O(1 \text{ TeV})$, this being the
typical value for the frequency of the three oscillating fields. Given the mixing in the potential, they will end oscillating with similar amplitudes. That is, the vacuum energy term $V(0)$ ends being equally redistributed among the three matter fields. Later on, they will decay and transfer that energy into radiation. This is the essence of the reheating process. In our scenario, the Higgs has the largest perturbative decay rate $\Gamma_h$ due to either its large top Yukawa coupling, or gauge couplings $\alpha_W$, i.e., $\Gamma_h \simeq \alpha_W(\kappa \phi_0)$. On the other hand, the singlet fields are very long-lived, due to their tiny Yukawa couplings, $\kappa \sim \lambda \sim 10^{-10}$, with $\Gamma_{N,\phi} \simeq \kappa^2(\kappa \phi_0)$. Therefore, at least perturbatively, we expect the Higgs to be the first in being converted into radiation almost immediately after the oscillations start. At the same time, Higgs perturbations are converted into those of the radiation fluid.

Nevertheless, previous to any perturbative decay, the evolution of the system might be dominated by non-perturbative effects as those of “preheating”, i.e., the parametric amplification of the fluctuations in a background of oscillating fields [27,28]. Through parametric resonance, induced by the time dependent effective mass term in the evolution equations, field mode amplitudes can grow exponentially in time within certain resonance bands in $k$-space. The parametric resonance can be present whenever there is a non-adiabatic change in the effective masses, $|\dot{m}_i(t)/m_i(t)| > 1$. Thus, preheating become a more effective mechanism of transferring energy than the standard perturbative decay of the fields, in this case from the oscillating background fields to their quantum fluctuations. The question is whether it also provides a different and efficient non-adiabatic source for the curvature perturbation, with super-Hubble ($k \ll aH$) fluctuations exponentially amplified during preheating [29–33]. This would translate into an exponential increase in the curvature perturbation. In hybrid inflation the effect can be stronger first due to the presence of negative effective squared masses at the end of inflation [14,34], which are per se a source of instabilities in the evolution equations for the fluctuations. In addition the “mixing” through the potential between the fields can also enhance the resonance [14,35].

Whether or not super-Hubble perturbations can be parametrically amplified during preheating is a model-dependent question [32,33] that has to be studied case by case. We have then numerically integrated the evolution equations for fields and fluctuations during the oscillatory period, in order to see the effect on the curvature. As already mentioned, the presence of tachyonic instabilities in the effective masses, combined with the fact that the fields are oscillating with large amplitudes and the effect of the expansion is negligible, could give rise to a strong parametric resonance effect just in a few oscillations. This can be seen in Fig. (3), where we have let the fields oscillate. It can be noticed that the resonance for the super-Hubble fluctuations
Figure 3: Shown is the amplitude of the spectrum of the field fluctuations versus $N_{\text{osc}} = \Delta t(\kappa \phi_0)/(2\pi)$, without taking into account the decay of the Higgs fields. The numerical integration starts when the fluctuations left the Hubble horizon, i.e., $k = H_a a$, roughly 60 e-folds before the end of inflation. We have taken the values of the parameters as before: $\phi_0 = 10^{13}$ GeV, $\phi_c^+ = 2\phi_0$, $\kappa \phi_0 = 1$ TeV, $A_\lambda = 1.5\sqrt{2}\kappa \phi_0$, $m_\phi = m_h = 0.1H$ and $h_\ast = 247$ GeV.

only sets in once the Higgs start oscillating with an amplitude comparable to that of the others, disturbing the straightline trajectory followed until then by $N$ and $\phi$. Within that trajectory, their large scale field fluctuations with $k \ll aH$ follow the same evolution equations as their background fields, without amplification [17].

Although it may seem that the amplitude of the curvature perturbation will increase beyond control, the above example was not realistic as we still need to take into account the effect of the Higgs “decay”. We remark that we used the term “decay” in an ample sense, meaning the transfer or dissipation of energy into radiation. In general, parametric resonance occurs within certain $k$-bands, and if not other, the backreaction of the quantum fluctuations will tend to shut off the resonance. Being a non-perturbative effect, it is very difficult to study the the whole process without resorting to numerical lattice calculations [34,36], which are far beyond the scope of this paper. Analytical or semi-analytical calculations in $k$ space have been carried out using the Hartree-Fock or large $N$ approximations [37]. Using those approximations, the overall effect of the quantum fluctuations would be analogous to introducing “friction” in the evolution
equations \[38,39\]. Therefore, we model this effect by using a constant parameter $\Gamma_h$, that is, a friction-like term in the Higgs evolution equation \[40,28\],

$$\ddot{h} + 3H \dot{h} + V_h = -\Gamma_h \dot{h}.$$ (52)

$\Gamma_h$ gives then the typical time interval needed for the conversion into radiation to take place. The assumption behind is that the quantum fluctuations are first produced by the Higgs, this being the field with the larger coupling to massless degrees of freedom. Although we are still working explicitly with the background “fields” and their perturbation variables, radiation is described as a perfect fluid component, with background energy density $\rho_R$ and pressure $p_R = \rho_R/3$. From the conservation of the total (fields plus radiation) stress-energy tensor, it follows

$$\dot{\rho}_R + 3H(\rho_R + P_R) = \Gamma_h \dot{h}^2 = \Gamma_h(\rho_h + P_h),$$ (53)

where the RHS is the source term from the Higgs.

Similarly for the Higgs fluctuations we have \[40\]:

$$\ddot{Q}_h + 3H \dot{Q}_h + \left(\frac{k^2}{a^2} + V_{hh}\right)Q_h + V_{h\phi}Q_\phi + V_{hN}Q_N = -\Gamma_h \dot{Q}_h;$$ (54)

here for simplicity we have neglected the sub-dominant metric contributions. In addition, we need to introduce their perturbations $\delta \rho_R$ and $\delta P_R$, in particular their corresponding gauge invariant variables. Gauge invariant perturbations can be defined in different ways \[40\], and a summary with our choice of gauge-invariant variables and their evolution equations is given in Appendix A. In order to compute the amplitude of the total curvature perturbation $\mathcal{R}$ we directly integrate the evolution equations for the scalar field fluctuations, and the contribution of the radiation to the curvature perturbation, $\mathcal{R}_R$ (see Eq. (80) in Appendix A), such that

$$\mathcal{R} = \sum_{\alpha} \frac{\rho_\alpha + P_\alpha}{\rho + P} \mathcal{R}_\alpha = \frac{1}{\rho + P} \left( \sum_{\alpha=\phi,N,h} H \dot{\phi}_\alpha Q_\alpha + (\rho_R + P_R)\mathcal{R}_R \right).$$ (55)

By changing the pattern of oscillations of the scalar fields, the parameter $\Gamma_h$ will control not only what fraction of the total energy density is transferred into radiation but how much “preheating” is allowed. As energy is transferred from the oscillating scalar fields to radiation and the amplitude of the oscillations decrease, this has the effect of shifting the resonance from the “broad” (large growth index for the fluctuations) regime towards the narrow (smaller growth) regime, until it finally dissapears. From Fig. (3) we can read that preheating built-in on a time interval of the order of $\Delta t_{preh}(\kappa \phi_0) \simeq O(100)$. Then, depending on the value of $\Gamma_h \Delta t_{preh}$ we
divide the parameter range into 3 different regimes, “large”, “medium” and “small”, in order to study the influence of the friction term in the final value of the curvature:

(a) **Large:** \( \Gamma_h \Delta t_{preh} \gg 1\) \(\Gamma_h \approx O(\kappa \phi_0) \approx O(\bar{m}_i)\).

The friction is too large to allow the Higgs to oscillate (the Higgs motion is overdamped). The motion of the other two singlets is practically undisturbed by the presence of the Higgs, and they oscillate in phase along the straight line trajectory Eq. (26). Thus, there is no parametric resonance on the large scale perturbations. In addition, only the energy density held by the Higgs during inflation would be transfer into radiation, negligible compared to the total energy density. Inflation ends with the vacuum energy being converted into that of “matter” (oscillating fields), and there is no further change in the curvature perturbation. The final value of the amplitude of the primordial spectrum is then given by the value at the end of inflation, Eq. (48).

(b) **Medium:** \( \Gamma_h \Delta t_{preh} \approx O(10^{-1})\).

The friction is large enough to quickly shift the resonance to the narrow regime, but allowing first the Higgs to oscillate with an amplitude similar to the others. In this range of values, the energy density in radiation is comparable to the one left in the singlets system. As an example, on the RHS in Fig. (4) where we have plotted the amplitude for the spectrum of the fields fluctuations for \( \Gamma_h = 0.1(\kappa \phi_0) \). The Higgs decays in a time interval comparable to that of the frequency of the oscillations, and comparable to the time needed to built the resonance for the
field fluctuations, so that indeed the latter hardly starts for the fields; Higgs fluctuations are just converted into radiation perturbations.

Therefore, very quickly we are left with a mixture of radiation and two oscillating singlets. We may think, quite correctly, that in any case given that the individual curvature perturbations were more or less the same at the end of inflation, and the first stage of the oscillations does not seem either to create large differences among them, there is no further large entropy/isocurvature perturbation to be converted into curvature one, so that the evolution of the latter would not be much altered. However, it is not only the presence of the radiation which make the cosmic fluid non-adiabatic and may affect the evolution of the curvature perturbation, but the fact that the fields are still oscillating not in phase with large amplitudes. Taking now the system to be made of 3 components, radiation, $N$ and $\phi$ field, the evolution equation for the total curvature can be written as$^{11}$ [40,19]:

$$
\dot{\mathcal{R}} \simeq H(1 - c_R^2) \frac{h_S h_R}{h_T^2} S_{SR} + 3H(1 - c_S^2) \frac{h_S h_R}{h_T^2} (\mathcal{R}_S - \mathcal{R}_R) - 3H(c_\phi^2 - c_N^2) \frac{h_\phi h_N}{h_T h_S} (\mathcal{R}_\phi - \mathcal{R}_N),
$$

(56)

where we use the same notation than in Eq. (31); the subscript “T” means total quantities, and “S” refers to the combination of the two scalar fields, with $h_S = h_\phi + h_N = \dot{\phi}^2 + \dot{N}^2$ and

$$
\mathcal{R}_S = \frac{h_\phi}{h_S} \mathcal{R}_\phi + \frac{h_N}{h_S} \mathcal{R}_N;
$$

(57)

$S_{SR}$ is the entropy perturbation between “scalars” and “radiation”, defined in Appendix A, Eq. (85); and $c_\alpha^2 = \bar{p}_\alpha/\bar{\rho}_\alpha$ are the sound speed for each component. For the scalar fields they are given in Eq. (32), and they are time-dependent, fast oscillating functions. Therefore, even if on average (over many oscillations) we could consider both fields as behaving like matter, with $\langle c_\alpha^2 \rangle = 0$, strictly speaking the last term in Eq. (56) only cancels out when $V_N \dot{\phi} - V_\phi \dot{N} = 0$, which is no more than the condition for the straight-line trajectory. Pushed away from that trajectory by the Higgs, the last term in Eq. (56) introduces another non-adiabatic source in the equation, different from the one in the first line due to the presence of the radiation. Both together can still enhance the curvature perturbation during the long reheating period that follows inflation. Until the radiation is redshifted away, we are left again only with “matter”, and the curvature remains constant thereon.

**Small:** $\Gamma_h \Delta t_{preh} < O(1)$.

$^{11}$We neglect for simplicity terms that go like $(k/aH)^2 \ll 1$.  

19
Figure 5: Amplitude of the spectrum of the curvature $P_{\mathcal{R}}^{1/2}$ (wiggly line) and that of the radiation $P_{\mathcal{R}_R}^{1/2}$ (solid line), for different values of $\Gamma_h$ given in units of $(\kappa\phi_0)$. Values of the other parameters as in Fig. (1).

There is still partial preheating, while the system continuously shifts from broad to a narrow resonance, and the fluctuations are initially amplified. In addition, small values of $\Gamma_h$ are more efficient in “extracting” energy from the oscillating fields, so that by the time the Higgs oscillations disappear we end practically with a radiation dominated Universe, with $\rho_R/\rho_T \simeq 0.9$. As an example, on the LHS in Fig. (4) where we have plotted the amplitude for the spectrum of the fields fluctuations\footnote{Numerically we found that for $\Gamma_h = 5 \times 10^{-4}(\kappa\phi_0)$, and the other value parameters as given in the figures, the resonance remains broad for too long, given too large amplitude fluctuations.} for $\Gamma_h = 6 \times 10^{-4}(\kappa\phi_0)$. Still there is enough energy left for the scalar fields to oscillate, so the last term in Eq. (56) cannot be neglected.

In Fig. (5) it is shown the spectrum of the total curvature perturbation and that of the radiation, for “medium” and “small” values of $\Gamma_h$, but using now a more cosmological time scale $\ln(a/a_0)$, with $a_0$ set when reheating/preheating (oscillations) starts. The smallest $\Gamma_h$, the larger the initial amplification of the fluctuations and then the curvature. After roughly half e-fold, the evolution follows a more regular pattern, with the curvature increasing as $\mathcal{R} \sim \ln(a/a_0)$, until the radiation is diluted away and we recover $\mathcal{R} \simeq \text{Constant}$. For example, for $\Gamma_h = 0.1(\kappa\phi_0)$, we have $(\rho_R/\rho_T)^{\text{max}} \simeq 0.78$, and then the radiation becomes subdominant by $\ln(a/a_0) \simeq 3.5$,
and the curvature perturbation levels off as can be seen in the plot. The spread in the value of $P_{R}^{1/2}$ is related to the oscillations: during those the kinetic energy of the fields passes close to cero, at which points the curvature is bounded by the contribution in radiation.

To summarise, we enter the reheating phase after inflation with a non-adiabatic mixture of oscillating fields and radiation, which will influence the evolution of the curvature perturbation (see Eq. (56)). On average, it will increase among one and two orders of magnitude before the radiation is redshifted away and becomes constant. In addition, for small values of $\Gamma_{h}$ there is further enhancement of the perturbations due to initial preheating effects, which are difficult to quantify analytically. The final spectrum would be given by that at the end of inflation, Eq. (48), but multiply by a function of $\Gamma_{h}$,

$$\mathcal{R} \sim F[\Gamma_{h}]_{r_{eh}} \left( \frac{\phi_{c}}{m_{P}} \right)^{2/3} H_{s} \frac{H_{s}}{\eta_{1/3} \phi_{h}},$$

(58)

which depending on the value of $\Gamma_{h}$ can be as large as $10^{5}$, and it could then compensate for the suppression of the initial spectrum of $N$ and $h$ during the phase transition at the end of inflation.

Before ending this section, some remarks about some of the assumptions involved in the final value of the curvature perturbation. First, as already remarked in section 2.3, the value of the Higgs vev is such that it never interferes with the evolution of the other fields during inflation, see Eq. (24). Its evolution plays an important rôle during the preheating stage, but not before. Were this not the case, and the value of the Higgs background field $h$ is such that the combination $\lambda h$ became larger than $H$ before the critical point this would signal the end of inflation, at which point we would have already $\mathcal{R} \simeq \mathcal{R}_{N} \simeq \mathcal{R}_{h} \simeq \mathcal{R}_{\phi}$, with a different (smaller) suppression factor than that given in Eq. (47). In this case $h$ would be the field with the increasing kinetic energy, so that it would be directly the isocurvature perturbation between the inflaton and $h$ which sources the total curvature perturbation. Nevertheless, because in this case the initial value of the Higgs vev would be larger, its initial curvature value at horizon crossing becomes smaller, and we do not obtain a larger amplitude for the curvature at the end of the phase transition.

For the range of values $h_{s}$ for which Eq. (48) holds, the final value of the spectrum will scale as the inverse of the Higgs’ vev, or almost. For “medium” values of $\Gamma_{h}$ this is the case, as reducing or increasing the value of $h_{s}$ has null or little effect on the pattern of oscillations. However, for “small” values of $\Gamma_{h}$ for which preheating is at hand, any change in the parameters may affect the final value, by shifting the resonance from one band to another, from broad
to narrow. This will modify the initial “boost” in the perturbations, but not the afterwards
behaviour $\mathcal{R} \propto \ln(a/a_0)$.

6 Discussion

Like in the original curvaton model, the final primordial amplitude of the spectrum is con-
trolled by the Higgs vev $h_*$ instead of the inflaton vev at the time of horizon crossing. This
is achieved through the $N$ field in hybrid inflation, which mediates between the inflaton and
the Higgs/curvaton flat direction. In addition, due to dynamics of the system, the curvature
perturbation changes first during the phase transition, which enhances the initial amplitude at
horizon crossing by a factor $\sim (\eta_\phi \phi_c/m_P)^{2/3}(\phi_* / h_*)$; and then through preheating effects, which
depend on the parameter $\Gamma_h$ but nevertheless give rise to an enhancement of the spectrum. For
small values of $\Gamma_h$, both effect can cancel each other such that one recover a similar curvaton
scenario result, with $P_R^{1/2} \approx H_*/\eta_\phi^{1/3} h_*$. However, even in this simplify estimation, the different
origin of the spectrum compared to the more standard curvaton scenario can be seen in the
factor $\eta_\phi^{1/3}$. That is, the amplitude of the spectrum depends to some extend on the inflationary
dynamics, in particular on the mass of the inflaton field instead of the mass of the Higgs field.

As far as the Higgs evolution does not backreact onto the others until the fields start oscillating
(see Eq. (24)), it is only its vev at horizon crossing which sets the scale for the amplitude.

On the other hand, the evolution of the Higgs field and not the others will control the spectral
index $n_S$ of the primordial spectrum,

$$n_S - 1 = \frac{d \ln P_R}{d \ln k}.$$

Although in the final value of the spectrum we have to take into account the function $F[\Gamma_h]_{reh}$
which numerically may depend on the value of $h_*$, this does not bring any additional $k$
dependence into the spectrum. That function parametrise the effects on the large scale spectrum
during the oscillatory phase of the fields, which depends largely on the patter of the oscillations
and modify equally all the wavelengths with $k \ll aH$ by the time of reheating. Given than in
this model the variation of the Hubble parameter during inflation is practically negligible, with
$2\dot{H}/H^2 \simeq \eta_\phi^2(\phi_c/m_P)^2$, and using $d \ln k \simeq H \, dt$, we simply have:

$$n_S - 1 \simeq 2\eta_h \simeq \frac{2m_H^2}{3H^2},$$

and practically no running of the spectral index. The recent WMAP data [41], combined with
other CMB experiments, prefers a red tilted spectrum with $n_S \simeq 0.96 \pm 0.02$. This leads to
the constraint $|\eta_h| \leq 0.03$, which would only require $m_h$ slightly smaller than $H$, $m_h \sim 0.3H$. Strictly speaking, the spectrum is red tilted when $\eta_h < 0$, i.e., $m_h^2 < 0$ during inflation. Although up to now we have been implicitly assuming the squared Higgs mass parameter in the potential to be positive, none of the results regarding the amplitude of the spectrum would change had we taken initially the opposite sign.

7 Summary

We have presented a variant of the curvaton scenario in SUSY hybrid inflation where: (a) it is coupled to the inflaton through the mediating field in hybrid inflation; (b) instead of being a late-decaying scalar, the curvaton decays before the inflaton, already at the beginning of the reheating period. The essence of the scenario is the same: isocurvature (relative) perturbations between the fields originated during inflation can be converted into adiabatic ones later on. Thus the primordial spectrum of perturbations is given in terms of the parameters related to the curvaton field instead of those of the inflaton. The conversion occurs when the curvaton energy density becomes dominant or comparable to that of the inflaton.

However, in our scenario it is the fact that the fields are coupled which first induced the conversion of the isocurvature perturbation between the inflaton and the $N$ field into the total one, previous to any decay. First, the coupling to the “curvaton” gives rise to an $N$ curvature perturbation given at the time of horizon crossing by the value of the curvaton one. During the phase transition from the false vacuum towards the global minimum, this contribution comes to dominate the total curvature when the kinetic energy of the $N$ field grows until becoming comparable to that of the inflaton (see Fig. (2)). Therefore, the initial amplitude of the total curvature perturbation at the time of horizon crossing, $P_{R^*}^{1/2} \simeq H_*/\eta_\phi \phi_*$, gets enhanced by a factor of the order of $(\eta_\phi \phi_c/m_P)^{2/3} (\phi_*/h_*)$, already at the end of inflation.

Second, the curvaton coupling to the other fields is relevant during the period of oscillations and it gives rise to preheating or parametric amplification of the large scale fluctuations. In standard SUSY hybrid inflation, the background fields oscillate along a practically straight-line trajectory. This regular behaviour prevent any amplification of the fluctuations with wavenumbers $k \ll H a$. Including the coupled curvaton directly perturb the background trajectory, which immediate effect is the amplification of the large scale fluctuations, and consequently the amplitude of the curvature perturbation. Without a detail numerical computation including backreaction and rescattering effects of the modes (large and small scale) produced, it is diffi-
cult to estimate how large could be this effect. However, heuristically we expect the curvaton to quickly decay in a few oscillations, and this would ensure that the resonance ends by the time the curvaton disappears, if not before. We model then this behaviour by introducing a “decay rate” term in the equation of motion of both curvaton background field and fluctuations, which simply parametrises the energy transfer rate between the curvaton and radiation. In a sense this is equivalent to take into account the backreaction effect of the fluctuations. Friction or dissipation effects are ultimately due to quantum fluctuations, which modify the propagation of the fields. The approximation behind our equations is that the dominant effects are due to the fluctuations generated by the curvaton, with the largest decay rate, but not directly coupled to the singlets, and then we do not consider any other direct backreaction effect in the evolution equations for the singlets. Within this approximation, the smaller the decay rate parameter is, the larger the initial preheating and the larger the final value of the amplitude of the curvature. Therefore, depending on the value of $\Gamma_h$, the amplitude enhancement during preheating can be larger than during the phase transition.

This general behaviour of the curvature perturbation does not depend on model parameters, but on the fact that we have coupled fields. However, the model introduced in Section 2 was motivated by particle physics considerations, which lead us to identify the “curvaton” D-flat direction with the Higgs of the MSSM. The $N$ field then originates the $\mu$ term in the superpotential, needed for electroweak symmetry breaking. We also impose an $U(1)$ Peccei-Quinn symmetry which solves the strong CP problem. Those physical requirements fixed uniquely the values of the parameters in the model, as given in Section 2. The numerical results in the Figures have been obtained using those values, mainly $\kappa \phi_0 \simeq \kappa \phi_c \simeq 1 \text{ TeV}$ and $\phi_0 \simeq \phi_c \simeq 10^{13}$ GeV. Although the vev of the Higgs D-flat direction during inflation can be considered a free parameter, it looks natural to assume a value not that far from its value at the electroweak symmetry breaking minimum, i.e., of the order of 1 TeV. For that choice, it is clear that the value of the spectrum at the end of inflation, Eq. (47), although larger than at the time of horizon crossing, it is still far below the observational bound $P_{R}^{1/2} \simeq 5 \times 10^{-5}$. However, as we have seen, preheating effects can further enhanced this value. Although numerically the final value will depend on the particular value of the parameter $\Gamma_h$, i.e., on how fast energy is transferred to the radiation, it can be seen in Fig. (5) that for example for $\Gamma_h \sim 1 \text{ GeV}$ the amplitude is of the correct order of magnitude.

In conclusion we have discussed the potentially important rôle played by preheating in certain variants of the curvaton mechanism in which isocurvature perturbations of a D-flat (and F-flat)
direction become converted to curvature perturbations during reheating. We have analysed the transition from inflation to reheating in some detail, including the dynamics of the coupled curvaton and inflation fields during this transition. We have discovered that preheating could be an important source of adiabaticity where parametric resonance of the isocurvature components amplifies the super-horizon fluctuations by a significant amount. As an example of these effects we developed a particle physics motivated model which we recently introduced in which the D-flat direction is identified with the usual Higgs field. Our new results show that it is possible to achieve the correct curvature perturbations for initial values of the curvaton fields of order the weak scale. In this model we have showed that the prediction for the spectral index of the final curvature perturbation only depends on the mass of the curvaton during inflation, where consistency with current observational data requires the ratio of this mass to the Hubble constant to be $\leq 0.3$.

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Appendix A

In this Appendix we summarise our conventions and notation for the perturbations and their evolution equations. We work with gauge invariant (GI) quantities, to first order in linear perturbation theory.

- Definitions: Linear scalar perturbations of the metric are given by the line element:

$$ds^2 = -(1 + 2A)dt^2 + 2B_i dx^i dt + a^2[(1 - 2\psi)\delta_{ij} + 2E_{ij}]dx^i dx^j,$$

where $B_i = i k_i/kB$, and $E_{ij} = (-k_i k_j/k^2)E$ for scalar perturbations; $\psi$ is the gauge-dependent curvature perturbation. On the other hand, the GI variable

$$\Phi = -\psi + Ha(B - a\dot{E}),$$

correspond to the curvature perturbation in the longitudinal (or zero-shear) gauge.
The perturbations of the stress-energy tensor are:

\[
\begin{align*}
\delta T^0_0 &= -\delta \rho , \\
\delta T^0_j &= (\rho + P)(v_j - B_j) , \\
\delta T^j_0 &= -\rho + P \nu^j , \\
\delta T^i_j &= \delta P \delta^i_j + P \Pi^i_j ,
\end{align*}
\]

where $\Pi_{ij}$ is the anisotropic stress taken to be zero hereon. The comoving curvature perturbation is then given by:

\[
\mathcal{R} = \psi + \frac{H a}{k} (v - B) = -\Phi + \frac{H a}{k} V ,
\]

where $V = v - ak\dot{E}$ is the GI velocity of the fluid.

Quantum scalar fields are decomposed as $\Phi_\alpha(x,t) = \phi_\alpha(t) + \delta \phi_\alpha(x,t)$, where $\delta \phi_\alpha(x,t)$ are the quantum fluctuations of the field around background values $\phi_\alpha$. For a multi-scalar field system, we have then:

\[
(\rho + P)(v - B) = \frac{k}{a} \sum_\beta \dot{\phi}_\beta \delta \phi_\beta ,
\]

and the comoving curvature perturbation:

\[
\mathcal{R} = \psi + \frac{H}{\rho + P} \sum_\beta \dot{\phi}_\beta \delta \phi_\beta = \frac{H}{\rho + P} \sum_\beta (\rho_\beta + P_\beta) Q_\beta ,
\]

where $Q_\beta$ are the Sasaki-Mukhanov GI variables:

\[
Q_\alpha = \delta \phi_\alpha + \frac{\dot{\phi}_\alpha}{H} \psi
\]

**Evolution equations:** Let us consider a multi-scalar system, with the fields interacting through the potential $V(\phi_\alpha)$. The derivatives of the potential with respect to the fields are denoted by $V_\alpha = \partial V/\partial \phi_\alpha$, and $V_{\alpha\beta} = \partial^2 V/\partial \phi_\alpha \partial \phi_\beta$. We also allowed source terms $S_\alpha$ in the evolution equation for the background fields,

\[
\ddot{\phi}_\alpha + 3H \dot{\phi}_\alpha + V_\alpha = -S_\alpha ,
\]

subject to the constraint $\sum_\alpha S_\alpha \dot{\phi}_\alpha = 0$, as required by the conservation of the total stress-energy tensor. The corresponding GI source perturbation is defined as:

\[
\mathcal{D} S_\alpha = \delta S_\alpha + \frac{\dot{S}_\alpha}{H} \psi ,
\]
and the evolution of the fields perturbations is given by [40]:

\[
\ddot{Q}_\alpha + 3H\dot{Q}_\alpha + \frac{k^2}{a^2}Q_\alpha + \sum_\beta V_{\alpha\beta}Q_\beta = -\mathcal{D}S_\alpha - \left(\frac{k}{a}\right)^2 \frac{\dot{\phi}_\alpha}{H} \frac{\Phi}{H} + \dot{\phi}_\alpha \dot{A} + 2(\ddot{\phi}_\alpha + 3H\dot{\phi}_\alpha)A,
\]

where

\[
\mathcal{A} = A + \left[\frac{\psi}{H}\right] = \frac{3\rho + P}{\rho} R.
\]

In the absence of source terms, the above equation reduces to the more familiar one [25]:

\[
\ddot{Q}_\alpha + 3H\dot{Q}_\alpha + \frac{k^2}{a^2}Q_\alpha + \sum_\beta V_{\alpha\beta}Q_\beta = -\Gamma_\alpha \dot{Q}_\alpha - 3H^2(\rho_R + P_R) = \Gamma_\gamma \dot{\phi}_\gamma - \Gamma_\gamma (\rho_R + P_R).
\]

The other particular case we have considered in this paper is when only one of the fields $\gamma$ decay into radiation $\rho_R$, such that

\[
\ddot{\phi}_\gamma + 3H\dot{\phi}_\gamma + V_\gamma = -\Gamma_\gamma \dot{\phi}_\gamma,
\]
\[
\dot{\rho}_R + 3H(\rho_R + P_R) = \Gamma_\gamma \dot{\phi}_\gamma^2 = \Gamma_\gamma (\rho_R + P_R).
\]

The GI radiation perturbations associated to the energy density, $\rho_R\Delta g_R$, and comoving curvature, $R_R$, are then:

\[
\rho_R\Delta g_R = \delta \rho_R + \dot{\rho}_R \frac{\psi}{H};
\]
\[
R_R = -\Phi + \frac{Ha}{k} V_R.
\]

The variables $Q_\alpha$ and $\rho_\alpha\Delta g_\alpha$ are the fluctuations of the field and energy density respectively on uniform curvature slices of space-time. For a scalar field, we have:

\[
\rho_\alpha\Delta g_\alpha = -\dot{\phi}_\alpha^2 A + \dot{\phi}_\alpha Q_\alpha + V_\alpha Q_\alpha.
\]

The evolution equations for the fluctuations can be written as:

\[
\ddot{Q}_\alpha + 3H\dot{Q}_\alpha + \frac{k^2}{a^2}Q_\alpha + \sum_\beta V_{\alpha\beta}Q_\beta = -\Gamma_\alpha \dot{Q}_\alpha
\]
\[
+ \frac{3H}{2\rho} \left\{ \sum_\beta \left[ (2\ddot{\phi}_\alpha + 3H\dot{\phi}_\alpha + \Gamma_\alpha \dot{\phi}_\alpha)\dot{\phi}_\beta + (2\ddot{\phi}_\beta + 3H\dot{\phi}_\beta + \Gamma_\beta \dot{\phi}_\beta)\dot{\phi}_\alpha - 2\frac{H}{H} \dot{\phi}_\alpha \dot{\phi}_\beta \right] \right\} Q_\beta
\]
\[
+ (2\dddot{\phi}_\alpha + \Gamma_\alpha \dot{\phi}_\alpha) \frac{h_R R_R}{H} - 2\frac{H}{H} \dot{\phi}_\alpha \frac{h_R R_R}{H} + \dot{\phi}_\alpha (c_R^2 - 1) \rho_R \Delta g_R + \dot{\phi}_\alpha \Gamma_\gamma \dot{\phi}_\gamma Q_\gamma \right\},
\]

27
where we use the shorthand notation $h_\alpha = \rho_\alpha + P_\alpha$. The equations for the radiation variables are given by:

$$
(h_R \mathcal{R}_R)' = -\frac{\dot{H}}{H} h_R (\mathcal{R} - \mathcal{R}_R) + H c^2_{\mathcal{R}R} \Delta_{gR} - 3H h_R \mathcal{R}_R + \Gamma_\gamma \dot{\omega}_\gamma^2 \mathcal{R}_\gamma
$$

(83)

$$
(\rho_{\mathcal{R}} \Delta_{g\mathcal{R}})' = -3H (1 + c^2_{\mathcal{R}R}) \rho_{\mathcal{R}} \Delta_{g\mathcal{R}} - \frac{k}{a} h_{\mathcal{R}V_R}
$$

$$
+ \Gamma_\gamma \dot{\omega}_\gamma (2 \dot{Q}_\gamma + \frac{\dot{H}}{H^2} \dot{\omega}_\gamma \mathcal{R}).
$$

(84)

The entropy perturbation $S_{\alpha\beta}$ introduced in Eq. (56) is defined as the difference between the energy density perturbations:

$$
S_{\alpha\beta} = \frac{\rho_\alpha \Delta_{g\alpha}}{h_\alpha} - \frac{\rho_\beta \Delta_{g\beta}}{h_\beta},
$$

(85)

which for non-interacting components is equivalent to

$$
S_{\alpha\beta} = -3H \left( \frac{\delta \rho_\alpha}{\bar{\rho}_\alpha} - \frac{\delta \rho_\beta}{\bar{\rho}_\beta} \right).
$$

(86)

**Appendix B**

Following Ref. [17], for a multi-scalar system the field perturbations can be rotated in field space into the “adiabatic” fluctuations along the background trajectory, which originates the total curvature perturbation, and the “entropy” components, which are orthogonal to the fields trajectory. Suppose we have $N$ numbers of fields and let us denote by $\sigma$ the adiabatic field and $\theta_i$ the $N-1$ angles that parametrize the trajectory. $v_\sigma$ will be the tangent vector to the field trajectory and $v_{\theta_i}$ the $N-1$ orthogonal vectors. Denoting by $Q$ the $N$-component vector with the fluctuations of the original fields, the adiabatic and the $N-1$ entropy fluctuations are given by

$$
Q_\sigma = v_\sigma \cdot Q, \quad Q_{\theta_i} = v_{\theta_i} \cdot Q,
$$

(87)

with

$$
\mathcal{R} = H \frac{Q_\sigma}{\dot{\sigma}},
$$

(88)

where $\dot{\sigma}^2 = \sum_{\alpha=1,N} \dot{\phi}_\alpha^2$. The evolution of the curvature for scales $k \ll aH$ can be written as

$$
\dot{\mathcal{R}} \simeq \frac{2H}{\dot{\sigma}} \sum_{i=1,N-1} \dot{\theta}_i Q_{\theta_i},
$$

(89)

This way, it is clear that the evolution of the total curvature depends only on the entropy fluctuation fields, and the curvature of the background trajectory in field space. As remarked
in [17], if the fields move in a straight line in field space, \( \dot{\theta}_i = 0 \), the curvature perturbation on super-Hubble scales remains constant\(^{13}\).

In our scenario, with 3 scalar fields, the change in the curvature during the phase transition can be related to the change in the trajectory in the \( \phi - N \) subspace, such that the corresponding \( \dot{\theta}_i \) becomes non-negligible and dominant. As far as the Higgs does not contribute, the inflaton and \( N \) will end moving in a (practically) straight line, the system effectively reduces to that of one scalar field, and \( \mathcal{R} \simeq \text{Constant} \). Later on, when the Higgs “curves” the trajectory, the evolution of \( \mathcal{R} \) is modified again.

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\(^{13}\)This strictly holds for scalar fields with minimal kinetic terms. Non-canonical kinetic terms typically appear in extended gravity theories, and can be the origin of the entropy source in the evolution of the curvature [23,22,6].
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