Enabling Differentiated Services Using Generalized Power Control Model in Optical Networks
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Abstract—This paper considers a generalized framework to study OSNR optimization-based end-to-end link level power control problems in optical networks. We combine favorable features of game-theoretical approach and central cost approach to allow different service groups within the network. We develop solutions concepts for both cases of empty and nonempty feasible sets. In addition, we derive and prove the convergence of a distributed iterative algorithm for different classes of users. In the end, we use numerical examples to illustrate the novel framework.

I. INTRODUCTION
Reconfigurable optical Wavelength-Division Multiplexing (WDM) communication networks with arbitrary topologies are currently enabled by technological advances in optical devices such as optical add/drop MUXes (OADM), optical cross connects (OXC) and dynamic gain equalizer (DGE). It is important that channel transmission performance and quality of service (QoS) be optimized and maintained after reconfiguration. At the physical transmission level, channel performance and QoS are directly determined by the bit-error rate (BER), which in turn depends on optical signal-to-noise ratio (OSNR), dispersion and nonlinear effects, [1]. Thus, OSNR is considered as the dominant performance parameter in link-level optimization. Conventional off-line OSNR optimization is done by adjusting channel input power at transmitter (Tx) to equalize the dominant impairment of noise accumulation in chains of optical amplifiers. However, for reconfigurable optical networks, where different channels can travel via different optical paths, it is more desirable to implement on-line decentralized iterative algorithms to accomplish such adjustment.

Recently, this problem is addressed in many research works [2],[3],[4], and two optimization-based approaches are prevalently used: the central cost and the non-cooperative game approach. The goals and models of the two approaches are inherently different. Central cost approach satisfies the target OSNR with minimum total power consumption. The model embeds the OSNR requirements in its constraints and indirectly optimizes a certain design criterion. Such model yields a relatively simple closed-form solution; however, it doesn’t optimize OSNR in a direct fashion, and thus, channel performance can be potentially improved for users who need higher quality of transmission. On the other hand, the game approach is a naturally distributed model which directly optimizes OSNR based on a payoff function in a non-cooperative manner. Each user optimizes her own utility to achieve the best possible OSNR. The solution from this approach is given by Nash equilibrium. As a result, this solution concept yields best achievable OSNR levels for each user. Since the game approach involves a cost function arising from pricing, it gives an over-allocation of resources. Some users may wish to avoid such cost and only demand a basic level of transmission. Apparently, these two approaches are for two different type of users and different transmission purposes.

To make use of the advantages from each approach, we propose a generalized model that combines their features. Such a generalization allows to accommodate different types of users and also provides a novel mixed framework to study OSNR power control problem. We separate users into two different categories. One type of users are those who are willing to pay a price to fully optimize their transmission performance. Another type of users are those who are content with basic transmission quality, or OSNR level, set by the network. The quality of service (QoS) can be met for the former by a game-theoretically based optimization approach; and for the later by a mechanism similar to central cost approach.

The contribution of this paper lies in the capability of service differentiation of the generalized model. For simplicity, total capacity constraints are not considered. The paper is organized as follows. In section 2, we review the network OSNR model and the basic concepts about the two optimization-based approaches. In section 3, we establish a general framework and propose two solution concepts for two different cases of feasible sets. Section 4 gives an iterative algorithm to achieve such solutions in the framework. This is illustrated in section 5 by numerical examples. Section 6 concludes the paper and points out future directions of research.

II. BACKGROUND
A. Review of Optical Network Model
Consider a network with a set of optical links \( \mathcal{L} = \{1, 2, ..., L\} \) connecting the optical nodes, where channel add/drop is realized. A set \( \mathcal{N} = \{1, 2, ..., N\} \) of channels are transmitted, corresponding to a set of multiplexed wavelengths. Illustrated in Figure 1 a link \( l \) has \( K_l \) cascaded optically amplified spans. Let \( N_i \) be the set of channels transmitted over link \( l \). For a channel \( i \in \mathcal{N} \), we denote by \( R_i \) its optical path, or collection of links, from source (Tx) to destination (Rx). Let \( u_i \) be the \( i\)th channel input optical power (at Tx), and
requirement as a constraint, we can arrive at the following

be the target OSNR for each channel. By setting the OSNR
by allowing the minimum transmission power. Let
achieves the target OSNR predefined by each channel user
less communication networks \([6], [7]\), OSNR optimization

As shown in (2) is commonly asymmetric and is more complicatedly
the system matrix used in wireless case, the matrix
system matrix which characterizes
the coupling between channels. System matrix \(\Gamma\) encapsulates
the basic physics present in optical fiber transmission and
implements an abstraction from a network to an input-output
system. This approach has been used in \([5]\) for the wireless
case to model CDMA uplink communication. Different from the
system matrix used in wireless case, the matrix \(\Gamma\) given in \([2]\)
is commonly asymmetric and is more complicatedly
dependent on parameters such as spontaneous emission noise,
noise, wavelength-dependent gain, and the path channels take.

\[
\Gamma_{i,j} = \sum_{l=1}^{L} \sum_{k=1}^{K_{l}} \frac{G_{l,j}}{G_{l,i}} \left( \prod_{q=1}^{l-1} \frac{T_{q,j}}{T_{q,i}} \right) \frac{ASE_{l,k,i}}{P_{o,l}}, \forall j \in N_{l}. \tag{2}
\]

where \(G_{l,k,i}\) is the wavelength dependent gain at \(k\)th span
in \(l\)th link for channel \(i\); \(T_{l,i} = \prod_{q=1}^{l} G_{l,k,i} L_{l,k}\) with \(L_{l,k}\)
being the wavelength independent loss at \(k\)th span in \(l\)th link;
\(ASE_{l,k,i}\) is the wavelength dependent spontaneous emission noise;
\(P_{o,l}\) is the output power at each span.

B. Central Cost Approach

Similar to the SIR optimization problem in the wireless
communication networks \([6], [7]\), OSNR optimization
achieves the target OSNR predefined by each channel user
by allowing the minimum transmission power. Let \(\gamma_{i}, i \in N\)
be the target OSNR for each channel. By setting the OSNR
requirement as a constraint, we can arrive at the following
central cost optimization problem (CCP):

\[
\text{(CCP)} \quad \min_{\mathbf{u}} \sum_{i \in N} u_{i} \text{ subject to } OSN R_{i} \geq \gamma_{i}, \forall i \in N. \tag{3}
\]

Under certain conditions, it has been shown in \([2]\) that the feasible set of (CCP) is nonempty and the optimal solution
is achievable at the boundary of the feasible set.

The formulated optimization problem can be extended to
incorporate more constraints such as

\[
\begin{align*}
\quad u_{i,\min} \leq u_{i} \leq u_{i,\max}, \tag{4}
\end{align*}
\]

where \(u_{i,\min}\) is minimum threshold power required for transmission
for channel \(i\) and \(u_{i,\max}\) is maximum power channel
\(i\) can attain. In the central cost approach, power \(u_{i}\) are the
parameters to be minimized and the objective function is
linearly separable. In addition, the constraints are linearly
coupled. These nice characteristics in central cost approach
leads to a relatively simple optimization problem.

C. Non-cooperative Game Approach

Let’s review the basic game-theoretical model for power
control in optical networks without constraints. Consider a
game defined by a triplet \(\langle N, (A_{i}), (J_{i}) \rangle\). \(N\) is the index
set of players or channels; \(A_{i}\) is the strategy set \(\{ u_{i} \mid u_{i} \in [u_{i,\min}, u_{i,\max}] \}\); and, \(J_{i}\) is the cost function. It is chosen in a way that minimizing the cost is related to maximizing OSNR
level. In \([3]\), \(J_{i}\) is defined as

\[
J_{i}(u_{i}, u_{-i}) = a_{i} u_{i} - \beta_{i} n \ln \left( 1 + a_{i} \frac{u_{i}}{X_{-i}} \right), i \in N \tag{5}
\]

where \(a_{i}, \beta_{i}\) are channel specific parameters, that quantify the
willingness to pay the price and the desire to maximize its
OSNR, respectively, \(a_{i}\) is a channel specific parameter, \(X_{-i}\) is defined as \(X_{-i} = \sum_{j \neq i} \Gamma_{i,j} u_{j} + n_{i}\). This specific choice of
utility function is non-separable, nonlinear and coupled.
However, \(J_{i}\) is strictly convex in \(u_{i}\) and takes a specially
designed form such that its first-order derivative is linear with
respect to \(u\).

The solution from the game approach is usually character-
ized by Nash equilibrium (NE). Provided that \(\sum_{j \neq i} \Gamma_{i,j} < a_{i}\),
the resulting NE solution is uniquely determined in a closed
form by

\[
\tilde{\mathbf{u}}^{*} = \tilde{b}, \tag{6}
\]

where \(\tilde{\Gamma}_{i,j} = a_{i}, \text{ for } j = i; \tilde{\Gamma}_{i,j} = \Gamma_{i,j}, \text{ for } j \neq i\) and
\(\tilde{b} = \frac{a_{0}}{\alpha_{i}} - n_{0,i}\).

Similar to the wireless case \([5]\), we are able to construct
iterative algorithms to achieve the Nash equilibrium. A simple
deterministic first order parallel update algorithm is:

\[
u_{i}(n+1) = \frac{\beta_{i}}{a_{i}} - \frac{1}{OSN R_{i}(n)} - \Gamma_{i,i} u_{i}(n). \tag{7}
\]

As proved in \([3]\), the algorithm \([7]\) converges to Nash equi-
librium \(\mathbf{u}^{*}\) provided that \(\frac{1}{a_{i}} \sum_{j \neq i} \Gamma_{i,j} < 1, \forall i\).

III. GENERALIZED MODEL

In this section, we consider a game designed to allow
service differentiation by separating users into two groups:
one group seeking a minimum OSNR target and another group
participating in a game setting for OSNR optimization. The
minimum OSNR for target seekers is set by the network to
ensure the minimum quality of service. However, the game
players can submit their parameters and optimize their service accordingly, but they have to pay a price set by the network for unit power consumption. This concept is illustrated in Figure 2. Let’s denote set \( N_1 = \{1, 2, \ldots, N_1\} \) as the set of competitors, i.e. users who wish to compete for an optimal OSNR. Let set \( N_2 = \{N_1+1, \ldots, N_2\} \) be the group of users with target OSNR given by \( F_1 = \gamma_1 \). Users with target OSNR shall have \( u_i \) satisfy OSNR \( \geq F_1 \) for some \( \gamma_1 \), \( \forall \ \in N_2 \), or equivalently from (1),

\[
\Gamma_i, u_i \leq b_i, \quad \forall \ \in N_1
\]

and thus, \( u_i \) is chosen as \( \gamma_1 \geq \gamma_i, \forall \in N_1 \), and \( \Gamma_i \) is nonsingular, there exist a unique \( u_i \in R^N \) for every nonnegative \( \mu \). Therefore \( \Gamma_i \) is non-empty.

Suppose conditions in Theorem 3.1 hold and \( \Gamma_i \) is non-empty. We consider an appropriate solution in \( F \) that satisfies a certain design criteria. Thus, we formulate (DSNP) in which we minimize total power consumption subject to the conditions arising from the different service requirements.

\[
(\text{DSNP}) \quad \min \sum_i u_i \quad \text{s.t.} \quad \Gamma_i u = b_i, \Gamma_i u \geq b_i
\]

The constraints of (DSNP) can be relaxed and augmented into

\[
\Gamma_i u \geq \bar{b}_i
\]

where \( \Gamma_i = [\Gamma_i, \bar{b}] \in R^{N \times N} \) and \( \bar{b} = [\bar{b}] \in R^N \).

According to the fundamental theorem of linear programming [8], if (DSNP) is realistic, the solution is obtained at the extreme point of the feasible set \( F \). Since \( F \) has only one extreme point when \( \Gamma_i \) is non-singular, the solution is uniquely given by

\[
u_i = \Gamma^{-1}_i \bar{b}_i
\]

To further characterize the solution \( u_i \), we assume strict diagonal dominance of matrix \( \Gamma_i \) [9], which leads to non-singularity of the matrix and uniqueness of the solution.

**Theorem 3.2**: Suppose OSNR targets \( \gamma_i, i \in N_2 \) are chosen such that \( \gamma_i < \frac{1}{\sum_{i \in N_2} \Gamma_i} \), \( i \in N_2 \). In addition, parameters \( a_i \) are chosen as \( a_i > \sum_{i \in N_2} \Gamma_i \), \( \forall \ \in N_1 \). The matrix \( \Gamma_i \) is strictly diagonally dominant. And thus, a unique solution to (DSNP) is given by (12).

**Proof**: From the assumption that \( \gamma_i, i \in N_2 \) is non-empty, it is apparent that \( \gamma_i < \frac{1}{\sum_{i \in N_2} \Gamma_i} \) and \( 1 - \gamma_i \Gamma_i \) is strictly diagonally dominant. Using Gershgorin theorem in [9], we conclude that there exists a unique solution to (DSNP).

The assumption of strict diagonal dominance in Theorem 3.2 is reasonable because typical values of \( \Gamma_i \) are found to be on the order of \( 10^{-3} \) and desirable levels of OSNR are 20-30dB.

**Remark 3.1**: (DSNP) can be seen as a generalized approach that combines central cost approach in [2] and non-cooperative game approach in [3]. When \( N_1 = \emptyset, N_2 \neq \emptyset, (\text{DSNP}) \) reduces to the central cost approach. Similarly, when \( N_1 \neq \emptyset, N_2 = \emptyset, (\text{DSNP}) \) reduces to the game-theoretical approach and the given solution is Nash equilibrium accordingly. This framework allows to study two different types of users at the same time.

**Remark 3.2**: We illustrate a two-person (DSNP), where player 1 chooses to compete and optimize his utility and player

1 DSNP stands for “Differentiated Service N-person Problem”.

Fig. 2. Game players and target seekers in the network

\[
\begin{array}{c}
\text{Target Seeker} \\
\text{Game Players} \\
\text{Network}
\end{array}
\]

\( \hat{u}_i, a_i, a_i, b_i, a_i, b_i \)

\( \hat{u}_i, a_i, a_i, b_i, a_i, b_i \)

\( \hat{u}_i, a_i, a_i, b_i, a_i, b_i \)

\( \hat{u}_i, a_i, a_i, b_i, a_i, b_i \)

\( \hat{u}_i, a_i, a_i, b_i, a_i, b_i \)
2 chooses to meet a certain OSNR target $\gamma_2$. We form the 2by-2 matrix $\mathbf{F}$ and $\mathbf{B}$ as follows.

$$
\mathbf{F} = \begin{bmatrix}
-\alpha_1 \\
\Gamma_{22}
\end{bmatrix}
$$

The feasible set $F = F_1 \cap F_2$ is shown in Figure 3 by a dotted line. The relaxed (DSNP) has its relaxed feasible depicted in the shaded region. The solution is given by $u^* = \mathbf{F}^{-1}\mathbf{B}$, which is illustrated by the dark point in Figure 3. $u^*$ is nonnegative componentwise if network power $\alpha_1$ is set such that $s_2 > \frac{n_0\gamma_2}{\Gamma_{22}}$.

Based on Theorem 3.2, we can further investigate how parameters chosen by game players and target seekers influence the outcome of the allocation. The result is summarized in Theorem 3.3.

**Theorem 3.3:** Let $\kappa$ be the condition number of $\mathbf{F}$, $T_i = a_i + \sum_{j \in N} \Gamma_{ij}$, $\forall i \in N_1$ and $S_k = 2 - 2\gamma_k \Gamma_{kk}, \forall k \in N_2$. Suppose $\mathbf{F}$ is strictly diagonally dominant by satisfying conditions in Theorem 3.2. In addition, $T_i > S_k$ and $b_i > b_k, \forall i \in N_1, \forall k \in N_2$. The maximum allocated power allocated to users are bound as follows.

$$
\frac{\max_{i \in N_2} \gamma_n \alpha_i}{2a_i} \leq \|u\|_{\infty} \leq \kappa \max_{i \in N_1} \frac{\beta_i}{\alpha_i}
$$

**Proof:** Let $R_i$ denote the i-th row absolute sum of matrix $\mathbf{F}$, i.e.,

$$
R_i = \sum_{j \in N} |\Gamma_{ij}|
$$

Using conditions from Theorem 3.2, we arrive at

$$
R_i = \left\{ \begin{array}{ll}
1 + \gamma_i \sum_{j \in N} \Gamma_{ij} - 2\gamma_i \Gamma_{ii} < 1, & i \in N_2 \\
1 + \sum_{j \in N} \Gamma_{ij} < 2a_i, & i \in N_1
\end{array} \right.
$$

With the assumption that $a_i + \sum_{j \in N} \Gamma_{ij} > 2 - 2\gamma_k \Gamma_{kk}, \forall i \in N_1, \forall k \in N_2$, we obtain $\|\mathbf{F}\|_{\infty} = \max_{i \in N_1} R_i = \max_{i \in N_1} a_i + \sum_{j \in N} \Gamma_{ij}$. Using (14) and the fact that $\Gamma_{ij} \geq 0$, we obtain an upper and lower bound on $\|\mathbf{F}\|_{\infty}$, i.e.,

$$
\max_{i \in N_1} a_i \leq \|\mathbf{F}\|_{\infty} \leq \max_{i \in N_1} 2a_i.
$$

In addition, from $b_i > b_k, \forall i \in N_1, \forall k \in N_2$, we obtain an upper bound and lower bound for $\|\mathbf{B}\|_{\infty}$, given by

$$
\max_{i \in N_2} \gamma_n \alpha_i \leq \|\mathbf{B}\|_{\infty} = \max_{i \in N_2} b_i \leq \max_{i \in N_1} \frac{\alpha_i}{\beta_i}
$$

Since $\mathbf{F}$ is strictly diagonally dominant, using matrix norm sub-multiplicativity, we obtain from (12)

$$
\|\mathbf{B}\|_{\infty} \leq \|u\|_{\infty} \leq \kappa \|\mathbf{B}\|_{\infty},
$$

where $\kappa$ is the condition number of $\mathbf{F}$ given by $\kappa = \|\mathbf{F}\|_{\infty} \|\mathbf{F}^{-1}\|_{\infty} \geq 1$.

Using (15), (16) and (17), we obtain

$$
\frac{\max_{i \in N_2} \gamma_n \alpha_i}{\max_{i \in N_1} 2a_i} \leq \|u\|_{\infty} \leq \kappa \frac{\max_{i \in N_1} \alpha_i \beta_i}{\alpha_i}
$$

It is easy to observe that the upper bound is dependent on the parameters of the game players and the lower bound is dependent on the OSNR levels of target seeker and parameter $a_i$ of the game players. In essence, game players control the outcome of the model and the choice of OSNR target can only affect the lower bound. Such relation describes a fair scenario in which game players, who pay for their power at $a_i$, have their choices of parameters $a_i$, $\beta_i$ to influence the network allocation.

**Remark 3.3:** Since $\|u\|_{\infty} \leq \|u\|_2 \leq \sqrt{N} \|u\|_{\infty}$, we can translate the result obtained in (18) directly into Euclidean norm, i.e.,

$$
B_{\infty}^L \leq \|u\|_2 \leq \sqrt{N} B_{\infty}^U
$$

where $B_{\infty}^U = \kappa \max_{i \in N_1} \beta_i \alpha_i$ and $B_{\infty}^L = \max_{i \in N_1} \gamma_n \alpha_i / 2a_i$. By (19), we can see that the network can encourage uniform channel power distribution by letting $B_{\infty}^U$ close to $\sqrt{N} B_{\infty}^L$ and provide incentive for differentiated services by letting them far apart. It can be implemented by the network by adjusting OSNR level $\gamma_i$ and pricing $\alpha_i$. Decreasing $\alpha_i$ encourages more users to be game players, giving rise to more competitions or service differentiation as a result of higher upper bound. On the other hand, increasing $\gamma_i$ raises the lower bound and encourages more users being target-seekers.
B. Empty Feasible Set

In this section, we consider the second case where feasible set \( F \) is empty. Instead of finding an appropriate feasible solution, we find the closest points between set \( F_1 \) and \( F_2 \). We use a quadratic program (DS2) to minimize the error norm subject to the constraint described by \( F_2 \).

\[
\text{(DS2)} \quad \min_u \|\Gamma u - \bar{b}\|_2 \\
\text{s.t.} \quad \Gamma u \geq \bar{b}
\]

We can turn the constrained problem \((20)\) into an unconstrained problem by studying its corresponding dual problem. Since \( \|\Gamma u - \bar{b}\|_2^2 = u^T \Gamma^T \Gamma u - 2(b^T \Gamma)u + b^T \bar{b} \), we denote \( H = \frac{1}{2} \Gamma^T \Gamma, d = -2(b^T \Gamma) \), \( D = -\Gamma(H^T \Gamma)^{-1} \Gamma^T \), \( e = \bar{b} + \Gamma(H^T \Gamma)^{-1} \Gamma^T d \); and form a Lagrangian from the original problem (DS2).

\[
D(\mu) = \min_u \mathcal{L}(u, \mu) = \min_u \left( \frac{1}{2} u^T Hu + d^T u + b^T \bar{b} + \mu^T (-\Gamma u + \bar{b}) \right)
\]

Since the objective function is convex, the necessary and sufficient condition for a minimum is that the gradient must vanish, i.e.,

\[
Hu + d - \Gamma^T \mu = 0.
\]

For \( n < N, \Gamma \) is not full rank. Therefore, \( H \) is singular and there exist multiple solutions to \((22)\). Using pseudoinverse \([9]\), we can find a solution to \((22)\) given by

\[
u = -(H^T H)^{-1} H^T (d - \Gamma^T \mu).
\]

Thus, after replacing into \((21)\), we obtain \( \mu \) as a solution to the dual problem (DS2).

\[
\text{(DS2)} \max_{\mu \geq 0} \frac{1}{2} \mu^T D \mu + \mu^T c - \frac{1}{2} d^T (H^T H)^{-1} H^T d + b^T b
\]

\[
\text{subject to } \mu_i \leq \gamma_i \quad \forall i \in N.
\]

The problem (LDS2) and dual problem (DS2) can be solved using unconstrained optimization algorithms in [10], [8].

IV. Iterative Algorithm

In this section, we develop algorithm for the case of nonempty \( F \) set. Let \( u_i(n) \) denote the power at channel \( i \) at step \( n \). An iterative algorithm is given as follows.

\[
\begin{align*}
\forall i \in N_1: \quad u_i(n+1) &= \frac{1}{\alpha_i} - \frac{1}{\beta_i} \left( \frac{1}{\alpha_i} \sum_{i,j \in E(N_1)} \Gamma_{i,j} u_j(n) \right) u_i(n), \\
\forall i \in N_2: \quad u_i(n+1) &= \frac{1}{\alpha_i} - \frac{1}{\beta_i} \left( \frac{1}{\alpha_i} \sum_{i,j \in E(N_2)} \Gamma_{i,j} u_j(n) \right) u_i(n)
\end{align*}
\]

\[
\text{Theorem 4.1: } \text{Algorithm (24) converges provided that } a_i > \sum_{j \neq i, j \in N} \Gamma_{i,j} \text{ and } \gamma_i \text{ is chosen such that } \gamma_i < \frac{1}{\sum_{j \neq i, j \in N} \Gamma_{i,j}}.
\]

\[
\text{Proof: } \text{We use a similar approach from [8] to show the convergence of (24). Let’s define } e_i(n) = u_i(n) - u_i^*, \text{ where } u_i^* \text{ is given in (22). Since } \Gamma^* u = b, \Gamma_{i,j} u_j^* + \sum_{j \neq i} \Gamma_{i,j} u_j^* = b_i, \text{ for } i \in N_1; \text{ and, } \Gamma_{i,j} u_j^* + \sum_{j \neq i} \Gamma_{i,j} u_j^* = b_i, \text{ for } i \in N_2.
\]

\[
\text{Substitute the expression for } u_i^* \text{ into } e_i(n+1), \text{ and we obtain } e_i(n+1) = u_i(n+1) - u_i^* = -\frac{1}{\alpha_i} \left[ \sum_{j \neq i} \Gamma_{i,j} u_j(n) - u_j^* \right], \text{ for } i \in N_1; \text{ and } e_i(n+1) = u_i(n+1) - u_i^* = -\frac{1}{\alpha_i} \left[ \sum_{j \neq i} \Gamma_{i,j} u_j(n) - u_j^* \right], \text{ for } i \in N_2.
\]

V. Numerical Examples

In this section, we illustrate the concept by a MATLAB simulation. We consider an end-to-end link described in Figure 1 with 5 amplified spans. We assume channels are transmitted at wavelengths distributed centered around 1555nm with channel separation of 1nm. Suppose input noise power is 0.5 percent of the input signal power. The gain profile for each amplifier is identically assumed to be parabolic as in Figure 4, which is normalized with respect to \( G_{\text{max}} = 30.0 \text{dB}. \) Suppose 20dB is the target OSNR level for users who just want to meet a satisfactory level of transmission. We first show the case of 3 users, in which 2 users need better quality of service and one user is simply interested in meeting 20dB as a target. From Figure 5 we can observe that users who need better services reach an OSNR of 26.33dB and 29.20dB, respectively. With an appropriate choice of initial conditions, the algorithm quickly converges in 1-2 steps. In Figure 6 we similarly show the case of 30 users, in which 20 are game players and 10 are target seekers.
VI. CONCLUSION

In this paper, we examined a generalized power control model in optical networks, which combines features of central cost approach and game-theoretical approach. It enables two major service types in the network. One is game player, who pays for his power consumption and the other is target seeker, who is satisfied with a minimum service level set by the network. We discussed two different solutions concepts for nonempty and empty feasible set respectively and specifically designed an iterative algorithm that converges to a unique solution for the case of nonempty feasible set. The convergence of the algorithm was proved and illustrated by numerical examples of a WDM end-to-end optical link.

In this work, we didn't include capacity constraints for the sake of simplicity. We hope this work will lead to future investigations of more complicated cases where network constraints and nonlinear effects are considered. In addition, we expect this framework to be used to solve similar problems in other types of networks, for example, wireless networks.

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