Axial Currents in Electrodynamics

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Abstract

In the present work we argue that the usual assumption that magnetic currents possess the vector structure characteristic of electric currents may be the source of several difficulties in the theory of magnetic monopoles. We propose an axial magnetic current instead and show that such difficulties are solved. Charge quantization is shown to be intimately connected with results of theories of discrete space time.

PACS numbers: 14.80.Hv, 11.15.Ha

In 1931 Dirac proposes for the first time an electromagnetic theory with magnetic monopoles\textsuperscript{(1)}. Such polemic subject has fascinated many generations of physicists ever since. Its appeal is mainly connected to the (up to now) unique possibility of explaining the quantization of the electric charge.

Dirac's hypothesis, in spite of its undeniable theoretical appeal, brings out some important difficulties. Firstly, in Dirac's theory one is faced with a symmetry problem: the terms responsible for the monopole in the generalized Maxwell's equations violate their symmetry under space and time reversal. This new asymmetry remains up to now an open problem. Secondly, Dirac's quantization condition implies in half integer values for the electromagnetic field angular momentum. Finally, what seems to be the main problem: there has been, since Dirac's proposal, no conclusive experimental evidence of magnetic monopoles.

\textsuperscript{1}Work partially supported by CNPq and FAPESP
In this work we propose a new magnetic current, namely an *axial magnetic current* which presents the following differences as compared to previously proposed ones, always of vector structure:

a) the resulting theory preserves space and time inversion invariance without having to resort to a pseudo-scalar magnetic charge;

b) besides the usual conservation of the vector electromagnetic current, we have also the conservation of an axial current;

c) the charge quantization in the present theory leads to the conclusion that velocities are constrained to rational values only. This result already integrates theories of discrete space time;

d) as opposed to Dirac’s monopole theory, the quantum of the field’s angular momentum is not necessarily half integer;

e) we are able to reinterpret the current experimental results, explaining their “negative” conclusions and to suggest very special and (we hope) more favourable experimental conditions for the observation of the magnetic monopole;

We start with the generalized definition of the electromagnetic field tensor

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \epsilon_{\mu\nu\alpha\beta} \partial_\alpha B_\beta \]  

(1)

where \( B^\mu \) represents the new potential as defined in ref [2]. Maxwell’s equations for the fields \( A^\mu \) and \( B^\mu \) in Lorenz’s gauge (\( \partial_\mu A_\mu = \partial_\mu B_\mu = 0 \)) become

\[ \Box A_\mu = j_\mu \]  

(2)

\[ \Box B_\mu = g_\mu \]  

(3)
The quantity $F_{\mu\nu}$ in (1) is a tensor; $\epsilon_{\mu\nu\alpha\beta}$ is a pseudo-tensor and therefore the field $B_{\mu}$ must be a pseudo-vector or an axial field. From the point of view of quantum theory the field $B_{\mu}$ represents photon-like particles except for $P$, $T$ and $C$ parities. In other words, axial photons. From this it follows that (3) is not invariant under time and space reversal, unless $g^\mu$ is also a pseudo-vector. The simplest possibility is the assumption that the magnetic charge be a pseudo-scalar, if the current is of vectorial character.

We shall adopt here, however, a different hypothesis: magnetic monopoles are spin 1/2 fermions, the magnetic charge $g$ is a true scalar and the corresponding current is an axial vector current. Namely,

$$g_\mu = -g\bar{\psi}\gamma_\mu\gamma_5\psi$$  \hspace{1cm} (4)

Let us investigate the consequences of such current to the equations which govern the electromagnetic fields. The essential differences are given by the equation

$$\partial^\nu F^\dagger_{\nu\mu} = g_\mu = -g\bar{\psi}\gamma_\mu\gamma_5\psi$$  \hspace{1cm} (5)

where $F^\dagger_{\nu\mu}$ corresponds to $F_{\nu\mu}$'s dual tensor. Since $F^\dagger_{\nu\mu}$ is antisymmetric one gets

$$\partial^\mu g_\mu = 0$$  \hspace{1cm} (6)

which means axial current conservation.

In terms of electric and magnetic fields $\vec{E}$ and $\vec{H}$ one obtains

$$\nabla \cdot \vec{H} = -g\bar{\psi}\gamma_0\gamma_5\psi = -g\psi^\dagger\gamma_5\psi$$  \hspace{1cm} (7)

$$\nabla \times \vec{E} = -\frac{\partial \vec{H}}{\partial t} - g\bar{\psi}\gamma_5\psi = -\frac{\partial \vec{H}}{\partial t} - g\psi^\dagger \vec{\alpha}\gamma_5\psi$$  \hspace{1cm} (8)

For the sake of argument we now consider the nonrelativistic limit, in which the monopole's velocity is small $v \ll 1$, and the magnetic current in the classical limit. The
second component of $\psi$, $\chi$, is in this limit given by

$$\chi \simeq \frac{1}{2} \vec{\sigma} \cdot \vec{v}_\phi \ll \phi$$  \hfill (9)

$$\psi^\dagger \gamma_5 \psi = ( \phi^\dagger \chi^\dagger ) \left( \begin{array}{c} -\chi \\ -\phi \end{array} \right) = -\left( \phi^\dagger \chi + \chi^\dagger \phi \right) = -\phi^\dagger \lambda |\vec{v}| \phi$$  \hfill (10)

$$\psi^\dagger \vec{\alpha} \gamma_5 \psi = ( \phi^\dagger \chi^\dagger ) \left( \begin{array}{c} -\vec{\sigma} \phi \\ -\vec{\sigma} \chi \end{array} \right) \simeq -\phi^\dagger \vec{\sigma} \phi$$  \hfill (11)

where $\lambda/2$ is the expectation value of the monopole’s helicity. Substituting the above expressions in (7) and (8),

$$\nabla \cdot \vec{H} = \rho_m \lambda |\vec{v}|$$  \hfill (12)

$$\nabla \times \vec{E} = -\frac{\partial \vec{H}}{\partial t} + \rho_m \vec{\sigma}$$  \hfill (13)

$\rho_m = \phi^\dagger \phi g$ stands for the magnetic charge density, $\vec{\sigma}$ is the vector corresponding to the expectation value of the spin operator $\hat{\vec{\sigma}}$, with

$$\rho_m \vec{\sigma} \equiv g \phi^\dagger \hat{\vec{\sigma}} \phi$$  \hfill (14)

From equations (12) and (13) one sees that even if $g$ is large ($g = 2\pi/e$, $n = 1$), the contribution of the current $g_\mu$ to $\nabla \cdot \vec{H}$ and $\nabla \times \vec{E}$ may well be very small. This is basically due to three effects: $|\vec{v}|$, $\rho_m$ or $\lambda$ may independently be very small. On the light of such equations it is possible to reinterpret the experiments involving accelerators or cosmic rays.

In the present scheme the effects due to magnetic monopoles should become most conspicuous in experiments with polarized beams, at sufficiently high energies and densities. Experiments with non-polarized beams are therefore not conclusive. It is easy to check
that the same conclusions can be reached as to the magnitude of the effect of external fields on the monopoles. For the same reasons they will also be negligible, unless under the very special circumstances mentioned above: the Lorentz force density in this case is

\[ \vec{f} = \lambda |\vec{v}| \rho_m \vec{H} - \rho_m \vec{\sigma} \times \vec{E} \quad (15) \]

Let us now analyze the compatibility of the axial magnetic current with charge quantization. We shall approach the problem in two different ways.

We first consider the gauge invariant wave function of a charged spin 1/2 field in the presence of the electromagnetic field (see ref [2]),

\[ \Phi_e (x, P') = \Phi_e (x, P) \exp \left[ -\frac{ie}{2} \int_S F_{\mu\nu} d\sigma_{\mu\nu} \right] \quad (16) \]

\( S \) being any surface with contour \( P' - P \). Due to the arbitrariness of the surface \( S \) we can write

\[ \Phi_e (x, P) \exp \left[ -\frac{ie}{2} \int_S F_{\mu\nu} d\sigma_{\mu\nu} \right] = \Phi_e (x, P) \exp \left[ -\frac{ie}{2} \int_{S'} F_{\mu\nu} d\sigma_{\mu\nu} \right] \quad (17) \]

which leads to the condition

\[ \exp \left[ -\frac{ie}{2} \oint_{S-S'} F_{\mu\nu} d\sigma_{\mu\nu} \right] = 1 \quad (18) \]

or equivalently to

\[ \exp \left[ -ie \int_V \partial^\nu F^\dagger_{\nu\mu} dV^\mu \right] = 1 \quad (19) \]

where \( V \) is the volume corresponding to the arbitrary surface \( S - S' \). We have

\[ \partial^\nu F^\dagger_{\nu\mu} = g_\mu \neq 0 \quad (20) \]
and therefore

\[ \exp \left[-ie \int_V g_\mu dV^\mu \right] = 1 \]  

(21)

Using our definition of \( g_\mu \) (4), we get

\[ Q_V = \int_V (-g\bar{\psi}\gamma_\mu\gamma_5\psi) dV^\mu = \frac{2\pi n}{e} \]  

(22)

As \( Q_V \) is a Lorentz scalar, we can perform the calculation in a convenient reference frame. We choose a reference frame in which the monopole velocity is constant and \( v \ll 1 \).

For \( \vec{\sigma} \) and \( \vec{\nu} \) in the same direction (e.g. \( z \) direction), using the previous results for \( \psi \) in this limit we obtain

\[ Q_v = g \left\{ -v \int \phi^\dagger \phi \, dx \, dy \, dz + \int \phi^\dagger \phi \, dx \, dy \, dt \right\} = \frac{2\pi n}{e} \]  

(23)

Since \( z = vt, \ dt = dz/v, \)

\[ Q_v = g \left\{ -v + \frac{1}{v} \right\} \int \phi^\dagger \phi \, dx \, dy \, dz = \frac{2\pi n}{e} \]  

(24)

which gives

\[ Q_v = g \left\{ \frac{1}{v} - v \right\} \simeq g \frac{2\pi n}{e} \]  

(25)

or

\[ \frac{eg}{2\pi v} = n \]  

(26)

The same result can alternatively be obtained semiclassically. As a consequence of the space-like nature of our magnetic current, we will not have radial fields for monopoles at rest (see (12)). This seems to suggest that an electric charge would not feel the action of such monopoles. In this case Goldhaber’s derivation of Dirac’s condition\(^{[3]}\) would not
be valid. However, this is not the case: a monopole at rest generates an electric field with nonvanishing curl, (see (13)) which is a sufficient condition for the validity of Goldhaber’s approach to Dirac’s charge quantization condition. This electric field is given by “Coulomb’s law”

\[ \vec{E} = \frac{g}{4\pi} \vec{\sigma} \times \frac{\vec{r}}{r^3} \]  

(27)

Consider the scattering of an electric charge by this field\(^4\). We assume that the initial charge’s velocity is along the direction of the vector \(\vec{\sigma}\). In such case, the variation of the charge’s angular momentum in this direction will be given by

\[ \Delta L_{\sigma} = \frac{eg}{2\pi v} \]  

(28)

independently of the impact parameter value. Using Bohr’s quantization rule

\[ \Delta L_{\sigma} = n \]  

(29)

one gets, as before,

\[ \frac{eg}{2\pi v} = n \]  

(30)

where, now, \(v\) is the charge’s velocity.

Let us now proceed to the analysis of the conditions (26) and (30). They will be fulfilled if one has simultaneously

\[ \frac{eg}{2\pi} = m \]  

(31)

\[ v = \frac{m}{n} \]  

(32)
The first equation is the celebrated Dirac’s condition for charge quantization. The second equation implies that the axial monopole’s velocity (or electric charge’s velocity) must be a rational number. This result is already integrated in discrete space time theories\cite{5,6}, since its initial proposition by Yukawa. It has recently been shown\cite{7} that demanding rational values for the velocity represents no contradiction at all with currently important theoretical results, such as Lorentz invariance, nor with experiments.

We last come to the question of the quantum of the field’s angular momentum. As discussed in ref \cite{4}, the angular momentum of the field produced by the pair charge-monopole, before and after scattering, is given by

\[ L_{em} = \frac{eg}{4\pi} \]  

(33)

In order to be compatible with Dirac’s quantization condition, \( L_{em} \) must be quantized by half integer values! No such problem arises in the present framework. It is simple to check that \( L_{em} \) is zero before and after scattering, since \( \vec{\sigma} \) is held fixed in z-direction. Therefore one is not confronted with the problem of having to impose a quantization condition in this situation.

In conclusion we see that the introduction of a conserved axial magnetic current, as proposed here, allows for a consistent parity conserving electromagnetic interaction, without having to resort to a pseudoscalar coupling constant. We are also able to solve the puzzle posed by the quantization of the angular momentum of the field. Finally, the condition of rational velocities obtained in the context of discrete space time theories is shown to arise naturally in connection to charge quantization. As to the observability of axial magnetic monopoles we show that it is intimately connected to polarization conditions.

References

\[ \text{[1] P.A.M.Dirac, Proc.Roy.Soc., A133(1931)60.} \]
[2] N.Cabibbo and E.Ferrari, Nuovo Cimento, 28(1962)1147.

[3] A.S.Goldhaber, Phys.Rev., 140B(1965)1407.

[4] J.D.Jackson, Classical Electrodynamics, second edition (1975), John Wiley (N.Y.), sections 6.12 and 6.13.

[5] T.D.Lee, Phys.Lett., 122B(1983)217.

[6] H.Yamamoto, Phys.Rev., D30(1984)1727.

[7] A.Horzela et al., Prog.Theor.Phys., 88(1992)1065.