CONTRIBUTIONS OF VECTOR-LIKE QUARKS TO $B \to X_s \gamma$ AND $B_s^0 \overline{B}_s^0$ MIXING

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The radiative decay $B \to X_s \gamma$ and the mixing $B_s^0 \overline{B}_s^0$ are discussed in the vector-like quark model which contains extra SU(2)-singlet quarks with the same electric charges as the up-type and down-type quarks. Constraints on the extended Cabibbo-Kobayashi-Maskawa matrix are obtained from the experimental results for the branching ratio of $B \to X_s \gamma$. Within these experimental bounds, any value of the branching ratio can be accounted for by the contributions from the vector-like quarks. In sizable ranges of the model parameters, the mixing parameter for $B_s^0 \overline{B}_s^0$ is much different from the prediction of the standard model.

1 Introduction

A number of reasons suggest that some extension of the standard model (SM) be necessary for describing physics around or above the electroweak energy scale. Since several phenomena involved in the $B$-meson system are sensitive to the extension of the SM, new physics may be unveiled by detailed examinations of these phenomena in the present or near-future experiments, such as B factories, BTeV, and LHCb. For instance, the radiative $B$-meson decay and $B^0 \overline{B}^0$ mixing could receive non-trivial contributions from supersymmetry. Systematic studies of various extensions of the SM should be performed in order to pinpoint a plausible model.

In this report we discuss the decay $B \to X_s \gamma$ and the mixing $B_s^0 \overline{B}_s^0$ within the framework of the vector-like quark model (VQM) which is one of the minimal extensions of the SM. This model contains extra quarks with electric charges 2/3 and/or −1/3, whose left-handed components, as well as right-handed ones, are singlets under the SU(2) gauge transformation. Therefore, phenomena of flavor-changing neutral current (FCNC) are affected. It will be shown that the decay rate of $B \to X_s \gamma$ can deviate from the prediction of the SM. The experimental results for the decay rate available at present already impose certain constraints on the model. The amount of $B_s^0 \overline{B}_s^0$ mixing can also be much different from the SM prediction.

In sect. 2 a brief summary of the VQM is presented. We consider the branching ratio of $B \to X_s \gamma$ in sect. 3, and the mixing parameter $x_s$ for $B_s^0 \overline{B}_s^0$ in sect. 4. Discussions are given in sect. 5.

2 Generation Mixings

The VQM incorporates extra Dirac fermions which are transformed as $(3,1,2/3)$ and/or $(3,1,−1/3)$ under the SU(3)$\times$SU(2)$\times$U(1) gauge symmetry. The left-handed and right-handed components have the same properties. For definiteness, we assume that there exist one up-type and one down-type vector-like quarks. The masses of the quarks are generated through Yukawa couplings and bare mass terms. The whole mass terms are expressed by 4×4 matrices, which are denoted by $M^u$ and $M^d$ respectively for up-type and down-type vector-like quarks. The mass eigenstates are obtained by diagonalizing the mass matrices as

$$A^u_L M^u A^u_R = \text{diag}(m_{u1}, m_{u2}, m_{u3}, m_{u4}),$$

$$A^d_L M^d A^d_R = \text{diag}(m_{d1}, m_{d2}, m_{d3}, m_{d4}),$$

where $A^u_L$, $A^u_R$, $A^d_L$, and $A^d_R$ denote unitary matrices.

A large difference between the VQM and the SM resides in the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$ for the weak interactions. The CKM matrix of the VQM is enlarged to be a 4×4 matrix, which is given by

$$V_{ab} = \sum_{i=1}^{3} (A^u_L)_{ai} (A^d_L)_{ib}. \quad (3)$$

Since the left-handed components of the extra quarks are singlets under the SU(2) transformation, the matrix $V$ is...
The upper bounds on the branching ratio of the decay $B \to X_s \gamma$ are determined by the magnitudes of flavor-changing interactions at the tree level. Consequently, the interactions of the quarks with the $W$ boson become qualitatively different from those in the SM. The $Z$ boson couples directly to the quarks with different flavors. The neutral Higgs boson also mediates flavor-changing interactions at the tree level.

The interactions mediated by the $W$, $Z$, and Higgs bosons give new contributions to processes of FCNC. However, taking into account the constraints on the QPM imposed from the presently available experiments, the effects by the $Z$ and Higgs bosons are found to be small. Our study is thus concentrated on the contributions coming from the $W$-mediated interactions. The interaction Lagrangian for the quarks with the $W$ and Goldstone bosons is given by

$$\mathcal{L} = \frac{g}{\sqrt{2}} \sum_{a,b=1}^{4} \bar{u}^a V_{ab} \gamma^\mu \frac{1-\gamma_5}{2} d^b W^{\mu}_\nu$$

$$+ \frac{g}{\sqrt{2}} \sum_{a,b=1}^{4} \bar{u}^a V_{ab} \left\{ \frac{m_{ua}}{M_W} \frac{1-\gamma_5}{2} \right\} d^b G^{\mu\nu}$$

$$+ \text{h.c.}.$$  \hspace{1cm} (5)

The mass eigenstates of the up-type and down-type quarks are denoted by $u^a$ and $d^b$, with $a$ and $b$ being the generation indices. These quarks are also called $(u, c, t, U)$ and $(d, s, b, D)$.

The decay width of $B \to X_s \gamma$ and the mixing parameter $x_s$ of $B^0 \to \bar{B}^0$ depend on the $U$-quark mass $m_U$ and the CKM matrix elements $V_{32}^*$, $V_{33}^*$, $V_{42}$, $(V^\dagger V)_{23}$. The mass should be larger than the $t$-quark mass. The matrix elements are related to $V_{23}$, $V_{13}$, $V_{22}$, and $V_{23}$, which have been directly measured in experiments. From their experimental values, a constraint

$$0.03 < |V_{32}^* V_{33} + V_{42}^* V_{43} - (V^\dagger V)_{23}| < 0.05 \hspace{1cm} (6)$$

is derived. The value of $(V^\dagger V)_{23}$ determines the magnitudes of flavor-changing interactions at the tree level. The upper bounds on the branching ratio of the decay $B \to K^{\pm} \mu^{\mp}$ lead to a constraint

$$|\langle V^\dagger V \rangle_{23}| < 2.0 \times 10^{-3}. \hspace{1cm} (7)$$

In principle, the $U$-quark mass and the CKM matrix elements are related to each other through the mass matrices of the up-type and down-type quarks. However, these relations depend on many unknown factors for the mass matrices, so that various possibilities for the relations are not forbidden. Therefore, in our analyses, we take the model parameters $m_U$, $V_{32}^*$, $V_{33}^*$, $V_{42}$ and $(V^\dagger V)_{23}$ as independent of each other. For simplicity, these matrix elements are assumed to be real.

### 3 Radiative Decay

The $B$-meson decay $B \to X_s \gamma$ is approximated by the $b$-quark decay $b \to s\gamma$. The relevant effective Hamiltonian is written as

$$\mathcal{H}_{eff} = \frac{-4G_F}{\sqrt{2}} \sum_{j=1}^{8} C_j(\mu) O_j(\mu), \hspace{1cm} (8)$$

where $O_j$ and $C_j$ stand for respectively an operator of the $\Delta B = 1$ transition and a Wilson coefficient, with $\mu$ being the evaluated energy scale. Four-quark operators are denoted by $O_1 - O_6$, while $O_7$ and $O_8$ represent the dipole operators for the photon and the gluon:

$$O_7 = \frac{\alpha}{16\pi^2} b_R G_{\mu\nu} F_{\mu\nu}, \hspace{1cm} (9)$$

$$O_8 = \frac{g_s}{16\pi^2} b_R G_{\mu\nu} T^a G^{a\mu\nu}. \hspace{1cm} (10)$$

In Table 1, the values of $V_{23}^*$ and $(V^\dagger V)_{23}$ in Figs. (i.a) and (i.b) are compared.

|            | (i.a) | (i.b) | (ii.a) | (ii.b) |
|------------|-------|-------|--------|--------|
| $V_{32}^*$ $V_{43}$ | -0.007 | -0.007 | 0.006  | 0.006  |
| $(V^\dagger V)_{23}$ | -0.002 | 0.002  | -0.002 | 0.002  |

Figure 1: The diagram which gives a contribution to $C_7$ or $C_8$. The photon or gluon line should be attached appropriately.
of $V_{23}^*V_{33}$ and $(V^1V)_{23}$ are not much dependent on $m_U$. For $|V_{23}^*V_{33}/V_{33}^*V_{33}| > 0.1$, the branching ratio is non-trivially different from that of the SM, and any value within the experimental bounds could be predicted.

To see how the branching ratio of $B \to X_s \gamma$ constrains the CKM matrix elements, we show allowed regions for $V_{32}^*V_{33}$ and $V_{42}^*V_{43}$ in Fig. 3. The shaded regions are compatible with the experimental results for $B \to X_s \gamma$ by CLEO and for other CKM matrix elements translated as Eq. \( \text{(8)} \). The regions between the solid lines satisfy the latter. We have taken the $U$-quark mass for $200 \text{ GeV} < m_U < 800 \text{ GeV}$ and $(V^1V)_{23}$ for $1 \times 10^{-3}$. The branching ratio of $B \to X_s \gamma$ imposes stringent constraints on the CKM matrix elements in the VQM.

### 4 $B_s^0 \bar{B}_s^0$ Mixing

The effective Hamiltonian for $B_s^0 \bar{B}_s^0$ mixing is given by

$$H_{eff} = \frac{G_F}{4\pi^2} M_W C(\mu) O(\mu),$$

where $O$ denotes the four-quark operator of the $\Delta B = 2$ transition

$$O = \bar{s}_L \gamma_\mu b_L \bar{s}_L \gamma^\mu b_L,$$

and $C$ stands for a coefficient. At $\mu = M_W$, the coefficient $C$ receives contributions from one-loop diagrams mediated by the $W$ boson, as shown in Fig. 4. The mixing $B_s^0 \bar{B}_s^0$ is also generated at the tree level by the $Z$ and Higgs bosons. However, taking into account the experimental constraints on the flavor-changing interactions, the tree-level contributions turn out to be smaller than the one-loop contribution. Accordingly, the one-loop diagrams with the $Z$ or Higgs boson are negligible. The coefficient $C$ at $\mu = m_b$ receives QCD corrections.

One observable for $B_s^0 \bar{B}_s^0$ mixing is the parameter $x_s$, which is defined by

$$x_s = \frac{\Delta M_{B_s}}{\Gamma_{B_s}}.$$  

Here $\Delta M_{B_s}$ and $\Gamma_{B_s}$ represent the mass difference and the average width for the $B_s^0$-meson mass eigenstates. The mass difference is induced dominantly by the short distance contributions. The decay constant and the bag factor of the $B_s^0$ meson for the matrix element are evaluated by lattice calculations.

In Fig. 4 the mixing parameter $x_s$ is shown for the same parameter sets as those in Fig. 2. The solid line represents the SM prediction. The value of $x_s$ varies in the region between the curves (i) and (ii) for the ranges of $V_{32}^*V_{33}$ consistent with the branching ratio of $B \to X_s \gamma$. The mixing parameter can be larger than the SM value by as much as a factor of two. As the value of $V_{32}^*V_{33}$ increases, the difference between the VQM and the SM
the branching ratio, the values of yield sizable effects. From the experimental results for other new contributions, the W boson can be examined by summing the SM value, which does not happen in the supersymmetric standard model. For some ranges of the model parameters, the mixing parameter becomes smaller than the SM value, which does not happen in the supersymmetric standard model.

Within the framework of the SM, the measurement of $x_s$ determines the value of $V_{32}^* V_{33}$. Then, the unitarity of the CKM matrix can be examined by summing $V_{12}^* V_{13}$, $V_{22}^* V_{23}$, and $V_{32}^* V_{33}$. Another possible examination of the SM is to compare the value of $V_{32}^* V_{33}$ thus determined with that measured by the $t$-quark decays. If a contradiction is found in these studies, the CKM matrix should be reanalyzed from the viewpoint of the VQM.

5 Discussions

We have discussed the effects of the VQM on the branching ratio for the radiative $B$-meson decay and the parameter $x_s$ for $B_s^0 \to B_d^0$ mixing. Although there are several new contributions, the $W$-mediated diagrams only yield sizable effects. From the experimental results for the branching ratio, the values of $V_{32}^* V_{33}$ and $V_{42}^* V_{43}$ are severely constrained. These constraints do not strongly depend on the mass of the extra quark $U$. The VQM could make the branching ratio non-trivially different from the SM prediction. The mixing parameter may be either larger or smaller than the SM prediction by a factor of two or more.

In addition to causing effects on FCNC processes, the VQM induces $CP$ violation differently from the SM. The extended CKM matrix contains more than one physical complex phases, so that new $CP$-violating phenomena could occur. For instance, a $CP$-odd coupling is generated at the one-loop level in the gauge-boson self-interactions for $WWZ$, which could be examined in future experiments at linear $e^+e^-$ colliders.

The contributions of the VQM are also found in the semi-leptonic decay $B \to X_s l^+ l^-$. This decay is induced both at the tree level and at the one-loop level. Its branching ratio and the forward-backward asymmetry for the final lepton could be different between the VQM and the SM. The decay rate asymmetry for $B$ and $\bar{B}$ is affected by the new source of $CP$ violation.

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