Delay-line based adiabatic spin-dependent kicks on a hyperfine manifold

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(Dated: today)

High speed control of spinor matterwaves is instrumental to atom interferometry and ion-based quantum information processing. In this work, we extend the Raman adiabatic spin-dependent kick (SDK) technique into the nanosecond regime. Counter-propagating frequency-chirped laser pulses are programmed on an optical delay line to parallelly drive five $\Delta m = 0$ hyperfine Raman transition of $^{85}$Rb atoms within $\tau = 40$ nanoseconds. An average SDK fidelity of $f_{\text{SDK}} \approx 97.6\%$ is inferred from spin-dependent momentum transfer and Raman population measurements, combined with precise numerical modeling. At such a high speed, coherent spin leakage is driven by the weakly allowed $\Delta m = \pm 2$ transitions. We theoretically investigate the intricate dynamics to demonstrate that the leakage can be substantially suppressed by a chirp-alternating SDK sequence with precisely cancelled dynamic phases. The technique should thus enable high fidelity, parallel control of multi-Zeeman spinor matterwave within nanoseconds, for various quantum technological applications.

I. INTRODUCTION

Precise control of 2-level systems is instrumental to modern quantum technology. In atomic physics, such controllable two-level systems are naturally defined on a pair of long-lived atomic internal states and are often referred to as atomic spins. When the atomic spins are controlled by optical Raman transitions, quantized photon recoil momentum can be transferred to the center-of-mass motion during a spin-flip [1–3]. By laser cooling [4], the atomic motion can be sufficiently slowed that the spin-dependent momentum transfers driven by rapid optical pulses are effectively instantaneous “spin-dependent kicks” (SDK) [5, 6] with an accuracy insensitive to initial atomic velocity. SDKs are therefore a class of broadband spinor matterwave control techniques with an achievable accuracy similar to those for 2-level internal atomic spin controls [7–9]. Beyond traditional applications such as to enhance the enclosed area of light pulse atom interferometers [6, 10, 11], SDKs emerge as an important technique to control spin-motion entanglement and to improve the scalability of ion-based quantum information processing [12–14].

For coherent control of spinor matterwave, it is essential to operate SDK at high enough speed so as to suppress effects from low-frequency perturbations and to compose multiple operations within a limited duration. Multiple SDKs are important for enhancing the momentum transfer and to improve the resilience of operation against static control errors [15–17]. Practically, the operation speed of SDK is limited by intricate requirements for precise Raman control, including the following two categories. The first type is associated with the suppression of spin leakage: To avoid population of electric-dipole excited states and the associated spontaneous emission, the Raman transition bandwidth has to be much smaller than the single-photon detuning (Fig. 1a), which itself is constrained by limited laser power when taking into consideration the multi-level dynamics and diffraction phases [18]. In addition, when the 2-level spins are encoded on an atomic hyperfine manifold with Zeeman degeneracy (Fig. 1), coherent coupling of atomic population from one 2-level spin subsystem to another can be driven by the Raman pulses. Accordingly, the SDK speed is limited by the strength of the quantization field that lifts the Zeeman degeneracy [19]. The second type of SDK speed constraints are associated with directionality of SDK: As in Fig. 1, when counter-propagating $\mathbf{E}_{1,2}$ pulses are applied to drive a Raman transition, to ensure the preference for the atom to absorb a photon from the $\mathbf{E}_1$ field followed by a stimulated emission into the $\mathbf{E}_2$ field, certain mechanism is needed to prevent the time-reversed Raman process from occurring. For hyperfine spin control of alkaline-like atoms, the directionality is ultimately supported by the ground-state hyperfine splitting $\omega_{\text{hfs}}$ [5]. Practically, however, when the Raman interferometry is operated in the retro-reflection geometry with established advantages [20–25], the SDK speed is limited by the typically moderate 2-photon frequency differences introduced by additional frequency modulations [10, 20] or Doppler shifts of moving atoms [21].

The purpose of this work is to improve the speed and precision of spinor matterwave control in presence of the above mentioned technical barriers. In particular, we study Raman adiabatic SDK [6, 11] operating within nanoseconds in a retro-reflection optical setup [18, 24], for alkaline atoms with Zeeman degeneracy prone to coherent spin leakage. As schematically summarized in Fig. 1, counter-propagating frequency-chirped pulses are programmed on an optical delay line to adiabatically drive the $F_b = 2 \leftrightarrow F_a = 3$ Raman transitions of $^{85}$Rb through the D1 line while imparting $\pm \hbar k_{\text{eff}}$ momentum to atoms. Here $k_{\text{eff}} = k_1 - k_2$ is the difference of k-vector between the pulse pair. The adiabatic Ra-
man transfer technique [6, 11, 26] provides the necessary intensity-error resilience when the SDKs are driven by focused laser beams to achieve high speed operation with limited laser power. To enforce the directionality of the nanosecond SDK in the standard retro-reflection geometry [18, 24], the regular method of resolving the 2-photon frequency differences are abandoned. Instead, we exploit the repetitive technique [18, 24], the regular method of resolving the 2-photon limited laser power. To enforce the directionality of the focused laser beams to achieve high speed operation with nanosecond adiabatic SDK within tens of nanoseconds by drive adiabatic SDKs within tens of nanoseconds by while maintaining the established advantages of retro-reflection [20, 22–25].

Experimentally, equipped with a wideband optical arbitrary waveform generation system (OAWG) [27], we drive adiabatic SDKs within tens of nanoseconds by shaping milliwatt-level counter-propagating pulses with suitable single photon detuning and frequency chirp (Fig. 1d). The spin population and momentum transfer are measured and compared with full-level numerical simulations, with which we infer an SDK fidelity of $\approx 97.6(3)\%$. The $\delta f \sim 2.5\%$ infidelity, as unveiled by the full-level numerical simulation, is primarily limited by spontaneous emission and the $\Delta m = \pm 2$ leakage among Zeeman sublevels within the ground state hyperfine manifold.

Theoretically, we demonstrate that a tailored adiabatic SDK sequence with alternating up and down 2-photon chirps can substantially suppress the coherent spin leakage driven by cross-linearly polarized Raman excitations. Furthermore, the chirp-alternating SDK sequence can nullify laser intensity-dependent diffraction phase [6, 18] even in presence of significant optical losses by none-perfect retro-reflection [24]. The adiabatic SDK demonstrated in this work should thus enable highly accurate control of spin-motion entanglement with alkaline-like atoms, within nanoseconds, for various quantum technological applications [14, 28, 29].

In the following, this work is structured into four sections. First, in Sec. II, we set up the basic notation and formalism to describe SDK on a hyperfine manifold and to introduce the adiabatic SDK technique. In Sec. III we introduce the experimental implementation of nanosecond adiabatic SDK with $^{85}$Rb atoms and detail the data analysis to arrive at the $f_{SDK} \approx 97.6\%$ SDK fidelity. In Sec. IV we provide numerical results and theoretical analysis to demonstrate that simple repetitive applications of high quality SDK with $f_{SDK} \approx 1$ would lead to substantial $\Delta m$-leakage and unbalanced dynamic phase. However, the imperfections can be efficiently suppressed by a chirp-alternating adiabatic SDK sequence. We conclude this work in Sec. V with a discussion on the prospects of using the adiabatic SDK scheme that combines speed, precision and laser intensity error-resilience for power-efficient control of hyperfine spinor matterwave of alkaline-like atoms at the nanosecond time scale.

II. THEORETICAL MODEL

A. Spin-dependent kicks on a hyperfine manifold

As shown in Fig. 1 (a), a pair of counter-propagating laser pulses, $E_{1,2} = E_{1,2} \mathcal{E}_{1,2}(k_{1,2}, r - \nu_{1,2} t) + c.c.$ with cross-linear polarizations $\mathbf{e}_{1,2}$ and shaped slowly-varying amplitudes $\mathcal{E}_{1,2}(r, t)$, are applied to drive the Raman transitions of an alkaline-like atom that couple the hyperfine ground states $|a\rangle$ and $|b\rangle$ through the intermediate, optically excited states $|e\rangle$ within the D1 line. The difference of the optical carrier frequencies $\nu_1 - \nu_2$ is set to be equal to the ground state hyperfine splitting $\omega_{ab}$. We consider the $a, b$ and $e$, hyperfine states with total angular momentum $F_{a,b}$ (single-valued) and $F_{e}'$ (multi-valued for all hyperfine levels), with nuclear spin $F > 1/2$ so that all the $a, b, e$ manifolds possess Zeeman degeneracy. Specifically,
there are $2F_c + 1$ Zeeman sublevels $|c_m\rangle \equiv |c, F_c, m\rangle$ with magnetic quantum number $m \in [-F_c, F_c]$, for each manifold $c \in \{a, b\}$, and similarly for the $e$ manifold. Starting from the full semi-classical Hamiltonian (Appendix A) and for weak and smooth pulses, by the standard procedure to adiabatically eliminate the excited states we arrive at an effective Hamiltonian on the ground state manifold (Fig. 1c) as

$$
H(r, t) = 
$$

$$
\hbar \sum_e \sum_{j=1,2} \Omega_{a_m c_j}^j \Omega_{a_m e_j}^{j*} \sigma^{a_m a_n} + \frac{\Omega_{b_n a_m}^j \Omega_{b_n e_j}^{j*}}{4(\nu_j - \omega_{a_n})} \sigma b_n a_m
$$

$$
+ \hbar \sum \frac{\Omega_{a_m c_j}^j}{4\Delta_e} \Omega_{b_n e_j}^{j*} e^{i\mathbf{k}_{\text{eff}} \cdot \mathbf{r}} \sigma b_n a_m + \text{h.c.}
$$

$$
+ \hbar \sum \frac{\Omega_{a_m c_j}^j \Omega_{b_n e_j}^{j*}}{4(2\nu - \omega_{a_m})} e^{-i\mathbf{k}_{\text{eff}} \cdot \mathbf{r}} e^{-12\omega_{a_m} t} \sigma b_n a_m + \text{h.c.,}
$$

(1)

with the convention of summing over repeated $m, n, l$ indices [30]. The single photon detuning is defined as $\Delta_e = \nu_j - \omega_{a_m} = \nu_j - \omega_{a_b}$. The vector $\mathbf{k}_{\text{eff}} = \mathbf{k}_1 - \mathbf{k}_2$ is the k-vector associated with the Raman transition driven by the counter-propagating pulses. The $\sigma^{a_m e_j} = |a_m\rangle \langle e_j|$, $\sigma^{a_m a_n} = |e_j\rangle \langle a_m|$ are the raising and lowering operators between states $|a_m\rangle$ and $|e_j\rangle$. Similar $\sigma$ operators are defined for all the other $(|a_m\rangle, |b_n\rangle)$ and $|e_j\rangle$ state combinations. The associated optical Rabi frequencies are given by

$$
\Omega_{a_m(b_n)e_j}^j(r, t) \equiv \frac{-i\langle a_m(b_n)|d \cdot \mathbf{e}_j \mathbb{E}_j(r, t)|e_j\rangle}{\hbar},
$$

(2)

with $d$ to be the atomic electric dipole operator.

It is important to note that the atomic position $r$ parameter in Eq. (1) can be regarded as a quantum mechanical operator acting on the external atomic wavefunction. The Raman coupling associated with $\sigma^{b_n a_m}$ in line 2 of Eq. (1) is therefore accompanied by a $\pm \hbar \mathbf{k}_{\text{eff}}$ momentum transfer to the spinor matterwave. Similarly, the “counter-rotating” term in line 3 of Eq. (1) leads to an opposite, $\mp \hbar \mathbf{k}_{\text{eff}}$ momentum transfer. For similar laser pulses with bandwidth $\delta \nu \ll \omega_{ab}$ to be discussed, this “counter-rotating” Raman process is energetically suppressed. The regime of resonant Raman interaction with directional momentum transfer is the focus of this work.

For Raman transitions driven by the cross-linearly polarized pulses as in Fig. 1 [22, 31] and with $\Delta_e \gg \omega_{\text{hfs}, e}$, the hyperfine dynamics is decomposed into those within $2F_b + 1$ copies of pseudo spin-1/2 sub-spaces $\{|a_m\rangle, |b_m\rangle\}$ at the large single-photon detuning limit. Specifically, we define the angular momentum quantization axis to be along the direction of light pulse propagation ($+z$ direction, Fig. 1b). The $\Delta m = 0$ Raman transitions are constructively enhanced by the cross-linearly polarized couplings, while the $\Delta m = \pm 2$ transitions are suppressed by a $\omega_{\text{hfs}, e}/\Delta_e$ factor [32]. The symmetry of the $c-g$ coefficients in addition guarantees that in this $\Delta_e \gg \omega_{\text{hfs}, e}$ limit, differential scalar light shifts in Eq. (1) are independent of the $m$-number [32]. Therefore, it is most convenient to rewrite the Hamiltonian $H$ in Eq. (1) as

$$
H(r, t) = \sum_{m=-F_b}^{F_b} H_0^{(m)}(r, t) + H'(r, t),
$$

(3)

with

$$
H_0^{(m)}(r, t) = \hbar \left( \frac{\delta_0}{2} \sigma_z^{(m)} + \frac{\Omega_{m}^{(m)}}{2} e^{i\mathbf{k}_{\text{eff}} \cdot \mathbf{r}} \sigma_+^{(m)} + \text{h.c.} \right)
$$

(4)

to be the Hamiltonian for the spinor defined in each $\{|a_m\rangle, |b_m\rangle\}$ subspace. The detuning $\delta_0$ describes the difference of scalar light shifts between the two hyperfine ground states (part of the first line of Eq. (1)). The Pauli matrices $\{\sigma_x^{(m)}, \sigma_y^{(m)}, \sigma_z^{(m)}\}$ are defined by $\sigma^{(m)} \equiv \sigma^{a_m b_m}$. For notation convenience in the following, we further define projection operators $1^{(m)} \equiv \sigma^{a_m a_m} + \sigma^{b_m b_m}$ and $\mathbf{1}^{(m)} = 1 - 1^{(m)}$ respectively. The complex, time-dependent Raman Rabi coupling driven by the cross-linearly polarized light is defined as $\Omega_{HR}^{(m)} = \chi^{(m)} \Omega^{1(2)} \Omega_{b}^{2(2)} / 2\Delta_e$ with $\chi^{(m)}$

$$
\Omega_{a(b)}^{1(2)} = \frac{\xi_{1(2)}}{\hbar} \frac{1}{\sqrt{3}} |J_g| \mathbf{d} |J_e|.
$$

(5)

as reduced Rabi frequencies. The close-to-unity factor $\chi^{(m)}$ is determined by $c-g$ coefficients, normalized at $\chi^{(0)} = 1$ and decreases slowly with $|m|$ [34].

In Eq. (3) the $H'(r, t)$ Hamiltonian include $\{a, b, m\}$-independent part of the first line of Eq. (1) as the “common shifts” . More importantly, it includes all the $m-$sensitive perturbations arising from the finite ratios $\Delta_e/\omega_{\text{hfs}, e}$ which induce $\Delta m = \pm 2$ couplings and $m-$dependent light shift ($m$-dependent part of the first line of Eq. (1), Fig. 1c). Finally, $H'$ includes the “counter-rotating” Raman couplings by the line 3 of Eq. (1).

A spin-dependent kick is a transfer of photon momentum to atoms accompanied by a spin-flip [6, 12]. Here, to define SDK on the ground state manifold, we ignore $H'$ so that the $\{|a_m\rangle, |b_m\rangle\}$ subsystems are decoupled. An ideal SDK referred to as $U_K(k)$ with $k = \mathbf{k}_{\text{eff}}$ by a $\sigma_+$ flip can be generated by the Eq. (4) Hamiltonian as

$$
U_K(k) = \prod_{m=-F_b}^{F_b} \left( e^{i\varphi^+} e^{i\mathbf{k}_{\text{eff}} \cdot \mathbf{r}} \sigma_+^{(m)} - \text{h.c.} + \mathbf{1}^{(m)} \right),
$$

(6)

where the $\varphi^+$ is a diffraction phase offset [35].

We now discuss tunneling of atomic wavefunction among the $2F_b + 1$ spin subsystems. The $\Delta m = \pm 2$ couplings in $H'$ in terms of $\sigma^{a_m b_m \pm 2}$ can be evaluated by summing all the intermediate $|e_{m \pm 1}\rangle$ states as in Eq. (1). A pictorial representation of such couplings is given in Fig. 1c. For the cross-linear polarized Raman beams and with the couplings between intermediate states of different
\( F_c \) nearly cancel each other, the reduced \( \Delta m = \pm 2 \) coupling strength of order

\[
\Omega^{\pm 2} = O(\frac{\omega_{\text{hfs}}\epsilon}{\Delta_\epsilon})\Omega_R.
\]  

(7)

Specifically, we consider Raman transition with balanced intensities and \( \Omega_\epsilon^2 \approx \Omega_R^2 \). In this case, the \( \Omega^{\pm 2} \) couplings share a similar strength regardless of whether they induce a hyperfine Raman transition.

**B. Numerical model**

We numerically simulate the full D1 light-atom interaction \([36, 37]\) driven by the counter-propagating Raman pulses as in Fig. 1. To account for radiation damping, we follow a stochastic wavefunction method \([38]\) to evaluate the wavefunction \( |\psi(r, t)\rangle \) for atom at location \( r \) under the non-Hermitian Hamiltonian \( H_{\text{eff}} = H_{\text{D1}} - i\Gamma_e/2 \sum c_m |e_m\rangle\langle e_m| \) (Appendix A). Here \( \Gamma_e \) is the natural linewidth of the D1 line. The simulations treat both the internal and external motion of the spinor matterwave quantum mechanically. For the purpose, we sample \( |\psi(r, t)\rangle \) densely over a uniform grid within \( 0 < z < \lambda/2 \) and sparsely in the \( x - y \) plane, and follow a split-operator method to evaluate internal/external atomic motion numerically with interleaved steps. Taking advantage of the short \( \tau_\epsilon \) for single SDK, the internal state dynamics is evaluated within a single step with frozen external motion under a local \( |\psi(r)\rangle \) basis, with atomic position \( r \) treated as a parameter of \( H_{\text{eff}} \). The evaluation of observables later is normalized by \( N = \sum_r \langle \psi(r, t = 0) | \psi(r, t = 0) \rangle \), with corrections from stochastic contributions to be discussed shortly. Between SDKs, a Fourier transform along \( e_z \) can be performed to evolve the free-flying spinor matterwave along \( z \) if necessary. To save computation resources, the relatively simple atomic dynamics in the \( x-y \) plane is ignored. To evaluate momentum distribution of spinor matterwave, we simply perform a Fourier transform to the spatial dependent \( \langle e_m | \psi(r, t) \rangle \) for any specific spin state \( |e_m\rangle \).

Beyond the coherent evolution, the simulation complexity is substantially reduced by skipping the evaluation of stochastic trajectories heralded by a single “quantum jump” \([39, 40]\). Specifically, after each pulsed interaction, the trajectories suffering a quantum jump are simply assumed to repopulate \( \{|a\}, |b\rangle \) in a uniform manner with properly shifted photon recoil momentum. Without further evolution, these trajectories contribute to the evaluation of incoherent, single-time observables such as hyperfine population and photon momentum transfer.

The overall probability of spontaneous emission is determined by the norm of the final wavefunction, \( \varepsilon_{\text{sp}} = 1 - \frac{1}{N} \sum_r \langle \psi(r, \tau_{\text{tot}}) | \psi(r, \tau_{\text{tot}}) \rangle \), after a total evolution time \( \tau_{\text{tot}} \). The simplification is generally justified for evaluating coherent observables of interest, since the expectation values shifted stochastically lead to zero coherent contributions. For the incoherent observables such as average photon momentum and hyperfine population, the simplification is supported by the simple D1 structure under consideration here \([36]\), where a single spontaneous emission effectively randomizes the following Raman interaction dynamics.

**C. \( f_{\text{SDK}} \) and \( \varepsilon_{\text{leak}} \) for non-ideal SDK**

The \( U_K(k_{\text{eff}}) \) in Eq. (6) is the target operation to be realized with the D1 atom-light interaction (Fig. 1). We refer the imperfect realization of SDK as \( \tilde{U}(k_{\text{eff}}; \eta) \) with \( \eta \) to generally represent relevant Hamiltonian parameters in Eq. (1) under discussions. We define an average fidelity of a single SDK acting on the \( \{|b_m\}, \{|a_m\} \) states as \([41]\)

\[
f_{\text{SDK}} = \left| \left\langle \left\langle c_m | \tilde{U}^n(k_{\text{eff}}) \tilde{U}(k_{\text{eff}}; \eta) c_m \right\rangle \right\rangle_z^2 \right|_{\eta, c_m} \right|^2.
\]  

(8)

Here the \( \tilde{U}(k_{\text{eff}}; \eta) c_m \) is compared with \( U_K(k_{\text{eff}}) c_m \) across the atomic sample with an \( \langle \ldots \rangle \) average which effectively performs a Fourier transform. An ensemble average of the mode-squared fidelity is then performed with \( \langle \ldots \rangle_{\eta} \) over Hamiltonian parameters of interest. Assuming light intensity hardly varies along \( z \) over the wavelength-scale distance of interest, the SDK fidelity defined this way becomes insensitive to the diffraction phase \( \varphi + \) and therefore provides a convenient measure for the quality of controlling incoherent observables, such as for the recoil momentum and hyperfine population transfer to be experimentally measured next. The \( \langle \ldots \rangle_{\text{SDK}} \) instead performs average over the \( 2(2F_b + 1) \) initial states of interest with \( m = -F_b, \ldots, F_b \) and \( c = a, b \).

To quantify the leakage of atomic state out of the spin sub-space \( \{|a_m\}, |b_m\} \), we define an average spin leakage probability during the non-ideal SDK as

\[
\varepsilon_{\text{leak}} = 1 - \left| \left\langle c_m | \tilde{U}^n(k_{\text{eff}}; \eta) 1^{(m)}(k_{\text{eff}}; \eta) c_m \right\rangle \right|_{\eta, c_m} \right|^2.
\]  

(9)

Since any spin leakage leads to inefficient control, we generally expect \( f_{\text{SDK}} \leq 1 - \varepsilon_{\text{leak}} \). The spin leakage probability \( \varepsilon_{\text{leak}} \) defined by Eq. (9) includes contributions from \( \Delta m = \pm 2 \) transition as well as those due to spontaneous emission, as \( \varepsilon_{\text{leak}} = \varepsilon_{\Delta m} + \varepsilon_{\text{sp}} \). Here the spontaneous emission probability \( \varepsilon_{\text{sp}} \) during the SDK control is obtained by evaluating \( \tilde{U}(k_{\text{eff}}; \eta) \) with the stochastic wavefunction method in Sec. II.B. For the \( \Delta m = \pm 2 \) leakage, we find \( \varepsilon_{\Delta m} \propto \omega_{\text{hfs}}/\Delta_\epsilon^2 \), as by Eq. (7), and is thus suppressible at large \( \Delta_\epsilon \), similar to the suppression of spontaneous emission. However, one should note that unlike spontaneous emission which simply leads to decoherence, the \( \Delta m = \pm 2 \) leakage is a coherent process where a sequence of leakage driven by multiple SDKs may interfere constructively to amplify the effect, even if \( \varepsilon_{\Delta m} \) for a single SDK is negligibly small.
D. Adiabatic SDK

The operation speed of SDK determined by the Raman Rabi frequency $\Omega_R$ (Eq. (4)) is proportional to the local laser intensities $I_{1,2}(r) = |E_{1,2}(r)|^2$. Increasing $\Omega_R$ necessarily leads to an increased spontaneous scattering rate $\gamma \sim \Gamma_c((\Omega_R^2)^2 + (\Omega_R^2)^2)/4\Delta^2_z$ associated with off-resonant excitation to the $|e\rangle$-manifold. To shorten the SDK time $\tau_c$ at a fixed spontaneous emission rate, the laser intensity $I_{1,2}$ should be increased with $\Delta_z^2$ in proportion, so as to achieve an increased $\Omega_R \propto \Delta_z$ slowly. With a limited laser power and to locally obtain strong enough $I_{1,2}(r)$, the laser beam sizes associated with the spatial envelopes $\mathcal{E}_{1,2}$ (Fig. 1) can be reduced. However, with the more focused laser beams, one needs to overcome control errors associated with position-dependent laser intensities to ensure $f_{\text{SDK}} \approx 1$ by Eq. (8) insensitive to the magnitudes of $\Omega_R(r)$ and $\delta_0(r)$ parameters in Eq. (4).

To achieve a uniformly high SDK fidelity across an intensity-changing sample volume, a standard technique is to exploit the geometric robustness of 2-level system by inducing an adiabatic rapid passage (ARP) [6, 11, 26, 42, 43]. We parametrize the Rabi frequency of the two pulses as $\Omega_{a,b}^{(2)} = C_{a,b}^{(0)} e^{i\varphi_{a,b}(t)}$, and specifically consider the time-dependent phase difference and the amplitude profile as $\Delta\varphi(t) = \phi_0 \sin(\pi t/\tau_c)$, $C_{a,b}(t) = C_{a,b}^{(0)} \sin(\pi t/\tau_c)$ respectively for $t \in [0, \tau_c]$ (Fig. 2b inset). With a large enough 2-photon sweep frequency $\delta_{\text{swp}} = \pi \phi_0 / \tau_c$, $|\delta_{\text{swp}}| > \delta_0$ to let the 2-photon detuning $\delta_R = \Delta \hat{\phi} + \delta_0$ cover the light-induced 2-photon shift, a strong enough Raman Rabi amplitude $C_{a,b}^{(0)} = C_a^{(0)} C_b^{(0)}/2\Delta_z = 2A_R/\tau_c$ with Raman pulse area $A_R \gg 1$, and matched magnitudes between $C_{a,b}^{(0)}$ and $\delta_{\text{swp}}$, SDK can be generated by the Eq. (4) Hamiltonian in a quasi-adiabatic manner [44], i.e., with atomic state $\psi(t) \approx c_a(t) e^{i\varphi_a(t)} |a(t)\rangle + c_b(t) e^{i\varphi_b(t)} |b(t)\rangle$ following the adiabatic basis $\{|a(t)\rangle, |b(t)\rangle\}$ which are simply the eigenstates of the instantaneous Hamiltonian $H_0(t)$. Population inversion are thus driven quasi-adiabatically during $0 < t < \tau_c$ with the efficiency insensitive to the laser intensity, detuning, and their slow deviations from the specific time-dependent forms. Putting back the r-dependence, the diffraction phase accompanying the population inversion, $\varphi^+(r) \equiv \varphi^+ + k_{\text{eff}} \cdot r$, is evaluated as $\varphi^+(r) = \varphi_a(r_c) - \varphi_b(r_c)$ which includes not only a geometric phase $\varphi_G = \pi/2 + k_{\text{eff}} \cdot r$ [45, 46], but also a dynamic phase $\varphi_D \propto \int_0^{\tau_c} (\hat{a} |H_0| \hat{a}) - (\hat{b} |H_0| \hat{b}) \, d\tau$ sensitive to the control parameters $\Omega_{a,b}^{(2)}$ characterized by the laser intensity profiles.

E. Double SDKs with (un)balanced dynamic phase

Clearly, a high $f_{\text{SDK}} \approx 1$ by Eq. (8) does not guarantee faithful spinor matterwave control, since the diffraction phase $\varphi^+$ (Eq. (6)) to print into the matterwave may vary across the sample as a function of laser intensity. For perfect $U_K$ by Eq. (6), the dynamic part of the diffraction phase can in principle be cancelled by applying pairs of SDKs with alternating $\pm k_{\text{eff}}$ [6, 11], leading to

$$U_K^{(2N)}(k) = U_K(-k)U_K(k)\cdots U_K(-k)U_K(k) = \prod_{m=-F_k}^{2N} (-1)^{N_m} e^{i(\varphi_{m} + \tilde{\varphi}(m))} . \tag{10}$$

Therefore, within each $\{a_m, b_m\}$ subspace, the $U_K^{(2N)}(k_{\text{eff}})$ acts as a position-dependent phase gate to pattern the two components of spinor matterwave with opposite $\pm(2Nk_{\text{eff}} \cdot r)$ phases.

To swap the k-vectors of $E_{1,2}$ without affecting other pulse parameters [20, 21, 24, 47] can be difficult practically. For this work in particular, the k-vector swapping is achieved in a retro-reflection setup (Fig. 1c) by programming the carrier frequencies $\nu_1 \leftrightarrow \nu_2$ for the delayed pulses meeting at the atomic sample. We consider a fixed peak amplitude for the incident pulses, so that the reflected pulse is reduced in amplitude by a reflective coefficient $\kappa$, i.e., $\mathcal{E}_{2,r} = \kappa \mathcal{E}_1, \mathcal{E}_{1,r} = \kappa \mathcal{E}_2$ for the imperfect $U_k(\mathcal{U})$ and $U(-k_{\text{eff}})$ operations respectively. The imperfect mirror reflection with $\kappa < 1$ generally leads to unbalanced diffraction phases $\varphi^+ \mp \varphi_\pm$ associated with $U(\pm k_{\text{eff}})$.

For the convenience of related discussions, we generally refer the dynamic part of diffraction phase introduced in Sec. II D as dynamic phase $\varphi_D$, which can be conveniently evaluated by comparing the phase of imperfectly controlled $\mathcal{U}^{(2N)}(k_{\text{eff}})$ with the ideal $U_K^{(2N)}$ to have $\varphi_{D,m} = \arg\{a_m|U_K^{(2N)}(k_{\text{eff}})U^{(2N)}(k_{\text{eff}}; \eta)a_m\rangle\} - \arg\{b_m|U_K^{(2N)}(k_{\text{eff}})U^{(2N)}(k_{\text{eff}}; \eta)b_m\rangle\}$. \tag{11}

To construct perfect $U_K^{(2N)}(k_{\text{eff}})$ quantum gates from $2N$ imperfect $\mathcal{U}(\pm k_{\text{eff}})$, in presence of the unbalanced dynamic phase and the $\Delta m = \pm 2$ leakage, is the topic for Sec. IV. To facilitate related discussions in the following, we refer the experimentally realized non-ideal adiabatic SDKs with equal amount of positive ($\delta_{\text{swp}} > 0$) and negative ($\delta_{\text{swp}} < 0$) 2-photon sweeps as $\mathcal{U}_u(\pm k_{\text{eff}})$ and $\mathcal{U}_d(\pm k_{\text{eff}})$ respectively, with the associated control Hamiltonians as $H_u(t)$ and $H_d(t)$.

III. EXPERIMENTAL IMPLEMENTATION

A. Nanosecond SDK on a delay line

The adiabatic SDK is implemented on the $^{85}\text{Rb} 5S_{1/2} - 5P_{3/2}$ D1 line as depicted in Fig. 1, with counterpropagating chirp pulses programmed by a wideband optical waveform generator (OAWG) [27] on an optical delay line [48]. The cross-linear polarization is realized by double-passing the light beam with a quarter waveplate before the end mirror (Fig. 1b) which
converts the incident $\mathbf{e}_x$ polarization to $\mathbf{e}_y$. With the OAWG output peak power limited to $P_{\text{max}} \approx 20$ mW, the incident control pulse $\mathcal{E}_{1,2}$ is focused to a waist radius of $w \approx 13 \, \mu\text{m}$ to reach a peak Rabi frequency of $\Omega_{\text{R}}(b) \approx 2\pi \times 2$ GHz. The imperfect retro-reflection with $\kappa \approx 0.7 \left( r = |\kappa|^2 \approx 50\% \right)$ reflectivity, primarily limited by increased focal beam size due to wavefront distortion, leads to decreased $\Omega_{\text{R}}(a) = \kappa \Omega_{\text{R}}(a)$ for the reflected pulses seen by the atomic sample. We set the single-photon detuning to be $\Delta_c = 2\pi \times 10$ GHz to achieve a peak Raman Rabi frequency of $\Omega_{\text{R}}(0) \approx 2\pi \times \kappa \times 300$ MHz estimated at the center of the Gaussian $\mathcal{E}_{1,2}$ beams. An $\tau_d = 140.37$ ns optical delay is introduced by the $L \approx 20$ m folded delay line, which is long enough to spatially resolve the counter-propagating nanosecond pulses. To form the counter-propagating $\mathcal{E}_{1,2}$ pulse pair, we pre-program $\mathcal{E}_{1,2}(t)$ and $\mathcal{E}_{2,1}(t - \tau_d)$, with a relative delay matching the optical delay line, to ensure the pulse pair with proper carrier frequency $\nu_{1,2}$ meeting head-on-head in the atomic cloud. To continue multiple SDKs, additional, individually shaped pulses with alternating $\nu_{1,2}$ can be applied. As in Fig. 1e, it is worth pointing out that to generate multiple SDK pulses this way, we always introduce an extra “pre-pulse” before and a “post-pulse” after the SDKs. However, without counter-propagating pulses to help driving the resonant Raman transition, these two extra off-resonant pulses impact negligibly the atomic hyperfine population and momentum transfer. Their impact to interferometric applications is discussed in Sec. V.

### B. Optimizing Adiabatic SDK

We prepare $N_A \sim 10^6 \ ^{85}\text{Rb}$ atoms in a compressed optical dipole trap at a temperature of $T \sim 200 \, \mu\text{K}$ [27]. The atomic sample is optically pumped into the $F = 2$, $|b_m\rangle$ hyperfine states, elongated along $z$, with a characteristic radius of $\approx 7 \, \mu\text{m}$ in the x-y plane (Fig. 2a). Immediately after the atoms released from the trap, multiple SDKs programmed on the optical delay line with alternating $\pm \mathbf{k}_{\text{eff}}$, $\mathbf{k}_{\text{eff}} = 2k_0\mathbf{e}_z$ are applied to transfer photon momentum by repetitively inverting the atomic population between $F = 2$, $|b_m\rangle$ and the $F = 3$, $|a_m\rangle$ hyperfine states. Here $k_0 = 2\pi/\lambda$ is the wavenumber of the D1 line SDK pulses at $\lambda = 795$ nm.

We use a double-imaging technique to characterize the performance of the adiabatic SDKs, by simultaneously measuring the resulting spin-dependent momentum transfer and population inversion (Appendix B). Specifically, immediately after the last of $n$ SDK pulses, a probe pulse resonant to the D2 line $F = 3 - F' = 4$ hyperfine transition is applied for $\tau_p = 20 \, \mu$s to record the spatial distribution of atoms in state $|a\rangle$, in the $x - z$ plane, with calibrated absorption imaging [48]. Next, after a $\tau_{\text{tof}} = 160 \, \mu$s free-flight time, the 2nd $\tau_p = 20 \, \mu$s resonant probe pulse is applied to image all the atoms. For the purpose, during the time of flight an additional $50 \, \mu$s pulse along $\mathbf{e}_z$, resonant to $F = 2 - F' = 3$ transition, repumps the $|b\rangle$ atoms to $|a\rangle$ for the 2nd imaging. By comparing atom number $N_a$ in state $|a\rangle$ and the total atom number $N_A = N_a + N_b$, inferred from the first and second images respectively, the probability of atoms ending up in $|a\rangle$ can be measured as a function of the number of SDKs $n$ as $\rho_{aa,n} = N_a/(N_a + N_b)$. In addition, by fitting both images to locate the center-of-mass vertical positions $z_1, z_2$, the atomic velocity $v_a = \delta z/\tau_{\text{tof}}$ can be retrieved to estimate the photon momentum transfer $p_n = M v_a$ in unit of $h\mathbf{k}_{\text{eff}}$.

Typical $v_a$ measurement results are given in Fig. 2b. Here, for atoms prepared in $|b\rangle$ states subjected to an $n = 25$ SDK sequence starting with $\hat{U}_d(\mathbf{k}_{\text{eff}})\hat{U}_a(-\mathbf{k}_{\text{eff}})$ (a $2N = 24$ double-SDK followed by an additional kick to drive the final Raman transition), the atomic population is largely in $|a\rangle$ while $v_a$ is unidirectional along $\mathbf{e}_z$. We optimize $v_a$ by varying the peak Raman coupling amplitude $C_{\text{R}}^{(0)}$ and sweep frequency $\delta_{\text{wp}}$ of the adiabatic SDK pulses at fixed $\tau_c$. As in Fig. 2b, for a fixed peak Raman pulse area $\mathcal{A}_R$, we generally find $\delta_{\text{wp}}$ to be optimized for efficient photon momentum transfer when it matches $C_{\text{R}}^{(0)}$ (See the arrow markers in Fig. 2b). However, unlike 2-level ARP [44] where an increased pulse area always leads to improved adiabaticity and population inversion robustness, here we find the peak $\mathcal{A}_R \approx 9 \pi$ reaches optimal to ensure the resilience of adiabatic SDK against the up to 50% intensity variation in the setup. Larger $\mathcal{A}_R$
is accompanied by slow decrease of $v_n$, due to increased probability of spontaneous emission. Here, for $\tau_c = 60$ ns, we need to attenuate $C_R^{(0)} \approx 150$ MHz to keep the peak $A_R \approx 9\pi$. By using the full $C_R^{(0)} \approx 200$ MHz available in this work, we are able to reduce $\tau_c$ to 40 nanoseconds while maintaining nearly identical momentum transfer efficiency at $\delta_{swp} = 2\pi \times 150$ MHz.

C. Inference of $f_{SDK}$

![FIG. 3. Measurements of recoil momentum and hyperfine population transfer by multiple adiabatic SDKs optimized at $\tau_c = 40$ ns. Normalized momentum transfer $p_n$ and hyperfine population $\rho_{aa,n}$ vs kick number $n$ are plotted in (a)(b) respectively. With atoms starting in $F = 2, |b_n\rangle$ states, the SDK sequence starting with $U_u(k_{eff})U_a(-k_{eff})$ and $U_a(-k_{eff})U_u(k_{eff})$ are plotted in top (i) and bottom (ii) panels with solid symbols. Corresponding chirp alternating $n = 4N$ sequence (Eq. (15)) with $N = 2 - 6$ are plotted via open squares. For comparison, spontaneous-emission-limited $p_n$ and $\rho_{aa,n}$ according to numerical simulation are plotted with black dashed lines. A phenomenological fit (solid red and blue curves) suggests hyperfine Raman transfer efficiency $f_R = 98.8(3)\%$ for the experimental data. See discussions in the main text for inference of $f_{SDK} = 97.6(3)\%$ from the data. (c) Top: State-selective absorption images of atomic sample in $F = 3, \{|a_{n}\rangle\}$ states, probed immediately after $n = 1 - 25$ SDK pulses starting with $U_u(k_{eff})U_a(-k_{eff})$. Bottom: Corresponding absorption images of optically repumped atomic samples with $\tau_{opt} = 160$ µs.

With the optimal $\delta_{swp} = 2\pi \times 150$ MHz and peak $A_R \approx 9\pi$ at $\tau_c = 40$ ns, we now apply $n = 1 - 25$ SDKs to characterize the momentum transfer $p_n = M\epsilon_n$ and normalized population $\rho_{aa,n}$ as a function of kicking number $n$. Typical results are given in Fig. 3. Here the momentum change $p_n$ along $e_z$ is again unidirectional along $z$ as in Figs. 3(a,i). The direction is conveniently reversed by programming $\hat{U}_u(-k_{eff})$ first in the $\pm k_{eff}$ alternating sequence, resulting in acceleration of atoms along $-e_z$ instead as in Figs. 3(a,ii). In contrast, the hyperfine population $\rho_{aa,n}$ is suppressed and revived after an even and odd number of SDKs respectively, as demonstrated in Fig. 3b. In addition, we program a chirp-alternating sequence $n = 4N$ that combines $\hat{U}_u(\pm k_{eff})$ with $\hat{U}_a(\mp k_{eff})$ according to Eq. (15), a sequence which will be detailed in Sec. IV for interferometric applications, with $p_n$ and $\rho_{aa,n}$ measurement results also given in Fig. 3 to demonstrate similar momentum and population transfer efficiency.

We estimate $f_{SDK}$ by comparing the $p_n$ and $\rho_{aa,n}$ measurements as in Fig. 3 with precise numerical modeling outlined in Sec. II, taking into account the finite laser beam sizes and imperfect reflection with reflectivity $r = |\epsilon|^2$. The comparison is facilitated by fitting both the measurement and simulating data according to a phenomenological model (Appendix C), which assumes that errors between successive SDKS are uncorrelated and are solely parametrized by $f_R$, a hyperfine Raman population transfer efficiency. The model predicts exponentially reduced increments $|p_{n+1} - p_n|/\hbar k_{eff} = |\rho_{aa,n+1} - \rho_{aa,n}| = f_R(2f_R - 1)^n$ for $f_R \approx 1$ by each SDK. From measurement data in Fig. 3a,b, $f_R \approx 98.8\%$ can be estimated in both $U_u(k_{eff})$ and $U_a(-k_{eff})$ kicks (Fig. 3a), slightly less than spontaneous-emission-limited $f_R \approx 99.2\%$ predicted by numerical simulation of the experiments, assuming perfect reflection with $n = 1$.

We simply attribute the slightly reduced $f_R$ from the theoretical value to imperfect retro-reflection (Sec. D). In particular, we numerically find $f_R$ reduces with $f_{SDK}$ when $\epsilon_{1,2}$ are unbalanced in amplitudes, so that both spin leakage and spontaneous emission are increasingly likely to occur. Taking into account the independently measured $R \approx 0.5$ in this work and with numerically matched $f_R$, $\epsilon_{\text{leak}} \approx 2.5\%$ with $\epsilon_{\Delta m} \approx 0.5\%$ and $\epsilon_{\text{sp}} \approx 2\%$ can be estimated respectively. We therefore infer from the combined analysis an $f_{SDK} = 97.6(3)\%$ for the adiabatic SDK in this experiment, slightly less than the spontaneous-emission-limited $f_{SDK} \approx 98\%$ for the nearly perfect adiabatic SDK. The numbers can be improved further by increasing $\Delta_c$ to suppress spontaneous emission as well as the $\Delta m$-leakage. It is important to note that while the imperfect reflection only moderately reduce $f_{SDK}$ here, the resulting imperfection of $k_{eff} \leftrightarrow -k_{eff}$ swapping in successive $U_u(\pm k_{eff})$ control can greatly compromise the cancellation of dynamic phase $\varphi_D$ in double SDK (Eq. (10)), a topic to be detailed in Sec. IV.D.

The adiabatic SDK demonstrated in this experimental section is among the fastest realization to neutral atoms [6, 11]. By equipping a more powerful laser, the SDK time can be further reduced to sub-nanosecond level [5], only limited by the ground state hyperfine splitting $\omega_{hfs,g}$. At such high speed an optical delay line of less than one meter is long enough to resolve counter-propagating pulses for conveniently performing $\pm k_{eff}$ kicks with suppressed systematics.
IV. ADIABATIC SDK FOR SPINOR MATTERWAVE CONTROL

FIG. 4. Management of $\Delta m = \pm 2$ leakage dynamics for high-fidelity realization of $U_{k}^{(4N)}$ with a $\tilde{U}_{ud}^{(4N)}$ sequence. The Raman couplings in Fig. 1(c) is decomposed to Fig.(a) here for $^{85}$Rb atom initialized in $m = 0, \pm 2$ (top) and $m = \pm 1$ (bottom) sub-space respectively. (b) Bloch sphere representation of $\{|a_m\rangle, |b_m\rangle\}$ spin-dynamics for $A_R = 9\pi$, $\delta_{\text{swp}}/2\pi = 150$ MHz (left) and -150 MHz (right). multiple trajectories with $\eta = \lambda^R/\lambda^K$ are displayed to demonstrate the robustness of ARP. The $\tilde{\xi}_{1,2}$ profile for the $\tilde{U}_{ud}^{(4N)}$ and $\tilde{G}_{ud}^{(4N)}$ sequence with $N = 1$ are sketched in the bottom. (c-f): Numerical simulations of $\Delta m$-leakage dynamics. The parameters for the simulations are chosen according to the experimental section III, as described in the text. (c) $\epsilon^{(2N)}_{m,\Delta m}$ as a function of pulse number $2N$ for the $\tilde{U}_{ud}^{(2N)}(k)$ (left) and $\tilde{G}_{ud}^{(2N)}(k)$ (right) controls, with $A_R = 9\pi$. (d-f): $\Delta m$-leakage for $m = 0, 1, 2$ sub-spin as a function of kicking number $2N$ and $A_R$ for the $\tilde{U}_{ud}^{(2N)}(k)$ (left) and $\tilde{U}_{ud}^{(2N)}(k)$ (right) controls. The corresponding $f_{\text{SDK}}$ is displayed on the top.

FIG. 5. Numerical results of gate infidelity $1-F$ for $\tilde{U}_{udd}(4N)$ (left) and $\tilde{U}_{udd}(4N)$ (right), to be compared with the spin-leakage in Fig. 4(d-f). Here Figs. (a,b,c) are for the composite SDKs within $m = 0, 1, 2$ spin sub-spaces respectively. The corresponding $f_{\text{SDK}}$ is displayed on the top.

In the previous section, we experimentally characterize nanosecond adiabatic SDK by measuring the transfer of photon momentum and hyperfine population by a SDK sequence. A natural question to ask is whether it is possible to exploit the control technique for coherent spinor matterwave control. Unlike control of photon recoil or hyperfine population where a high $f_{\text{SDK}} \approx 1$ by Eq. (8) guarantees nearly perfect outcomes, to apply SDKs for spinor matterwave control one needs to ensure that for the general spinor state $|\psi_m\rangle = |a_m\rangle + |b_m\rangle$ within each $\{|a_m\rangle, |b_m\rangle\}$ sub-space, $U(k_{\text{eff}}; \eta) |\psi_m\rangle$ overlaps well with $U(k_{\text{eff}})|\psi_m\rangle$ in a $\eta$-insensitive manner against the spreading of control parameters. Unfortunately, as discussed in Sec. II D, for the adiabatic SDK the diffraction phase $\varphi^+_{\text{D}}$ includes a laser intensity-dependent dynamic phase $\varphi^+_{\text{D}}$, precluding coherent control in presence of laser intensity inhomogeneities. Within the 2-level model, in Sec. II E we introduced the double SDK sequence (Eq. (10)) by pairing $U_k(k_{\text{eff}})$ with $U_k(-k_{\text{eff}})$ to cancel the dynamic phase [11, 49]. However, as to be demonstrated in this section, the double SDK technique becomes insufficient in presence of coherent spin leakage among the $2F_g + 1 \{|a_m\rangle, |b_m\rangle\}$ sub-spaces, or when the none-perfect $k_{\text{eff}} \leftrightarrow -k_{\text{eff}}$ swapping introduces additional dynamic phases.

Here, in presence of the seemingly difficult challenges, we demonstrate that the $\Delta m$-leakage can be quite efficiently suppressed with a chirp-alternating $n = 4N$ SDK sequence, designated as $\tilde{U}_{udd}^{(4N)}(k_{\text{eff}})$ (Eq. (15)), which can also efficiently suppress the additional dynamic phases for faithful implementation of the $U_k^{(4N)}(k_{\text{eff}})$ phase gate...
by Eq. (10). We further numerically demonstrate the utility of the composite adiabatic SDK sequence by simulating an area-enhanced atom interferometry sequence, using the experimental parameters both within and beyond this experimental work.

We notice control of spin leakage in quasi-two-level systems is an important topic in quantum control theory [50–52]. Previous studies on the topic typically involve a Morris–Shore transformation of the interaction matrix to decompose the multi-level dynamics [53]. However, as being schematically summarized in Fig. 1(c) and Fig. 4(a)(b), here the spin leakages are coherently driven through multiple paths with multiple Raman couplings to preclude a straightforward transformation, nor a direct application of the associated leakage-suppression techniques [50–52].

### A. Gate fidelity of double-SDK

Generally, to quantify imperfect $\tilde{U}$ for realizing quantum control $U$, we need to investigate the control fidelity $\mathcal{F}$ [54] and further averaging over parameter $\eta$ of interest to ensure robustness. Here, to conveniently evaluate an imperfect double-SDK sequence $\tilde{U}^{(2N)}(k_{\text{eff}})$, we define an average gate fidelity for spinor matterwave control as [41]

$$\mathcal{F}^{(2N)}_m = \left| \langle \psi_{m,j} | U^{(2N)}_R(k_{\text{eff}}) \tilde{U}^{(2N)}(k_{\text{eff}}; \eta) \psi_{m,j} \rangle \right|^2.$$  

(12)

The evaluation samples the initial state of $|\psi_{m,j}\rangle$ within the $m$-spin subspace of interest, with six eigenstates of $\sigma_{x,z}^{(m)}$, listing as $\{|\psi_{m,j}\rangle\} = \{|b_m\rangle, |a_m\rangle, (|b_m\rangle \pm |a_m\rangle)/\sqrt{2}, (|b_m\rangle \pm i|a_m\rangle)/\sqrt{2} \}$ respectively.

For the convenience of related discussions, we define an average $\Delta m$-leakage probability, similar to Eq. (9), as

$$\varepsilon^{(n)}_{m,\Delta m} = 1 - \langle \psi^{(n)}_{m,j} | 1^{(m)} | \psi^{(n)}_{m,j} \rangle_{\eta,j}.$$  

(13)

where $|\psi^{(n)}_{m,j}\rangle = \tilde{U}^{(n)}(k_{\text{eff}}; \eta) |\psi_{m,j}\rangle$ is defined as the final atomic state after the imperfect control. To exclude spontaneous emission, we simply renormalize to have $\langle \psi^{(n)}_{m,j} | \psi^{(n)}_{m,j} \rangle = 1$ before the evaluation of the $\Delta m$-leakage.

Similar to Eq. (13), here the $\Delta m$-leakage probability is also evaluated for a specific $m$-subsystem of interest.

Similar to Eqs. (8)(9), here we expect $\mathcal{F}^{(n)}_m \leq 1 - \varepsilon^{(n)}_{m,\Delta m}$ since any spin leakage results in gate infidelity.

With $\omega_{\text{fs}} \gg 1/\tau_c, \delta_{\text{swp}}$ so that the “counter-rotating terms” in Eq. (4) hardly affects the spin dynamics of each adiabatic SDK, we find accordingly that both $\mathcal{F}^{(2N)}_m$ and $\varepsilon^{(n)}_{m,\Delta m}$ hardly depend on the atomic position $z$ along $k_{\text{eff}}$. Nevertheless, to quantify the gate fidelity and spin leakage for the spinor matterwave control, we simply include $z$ into the set of Hamiltonian parameters $\eta$ for the numerical average.

### B. Spin leakage for non-ideal double-SDK

We define the non-ideal realization of the double-SDK sequence with $\tilde{U}_u(\pm k_{\text{eff}})$ or $\tilde{U}_d(\pm k_{\text{eff}})$ the same way as in Eq. (10) for the ideal case, which are referred to as $\tilde{U}_u^{(2N)}(k_{\text{eff}})$ and $\tilde{U}_d^{(2N)}(k_{\text{eff}})$ respectively in the following. We numerically investigate the $\Delta m$-leakage probability $\varepsilon^{(n)}_{m,\Delta m}$ and the gate fidelity $\mathcal{F}^{(n)}_m$, for an atom initialized in an $\{|a_m\rangle, |b_m\rangle \}$ subsystem being subjected to non-ideal double-SDK sequence $\tilde{U}_u^{(2N)}(k_{\text{eff}})$. The Hamiltonian parameters in the simulation follow the experimental setup described in Sec. III for the $^{85}$Rb D1 line scheme (in particular, $\delta_{\text{swp}} = 2\pi \times 150$ MHz), except here the reflective coefficient is set as $\kappa = 1$ for simplicity, and $\Gamma_z = 0$ to focus on the coherent control dynamics. To elucidate the role of laser intensity for the coherent control, we repeat the simulation while scanning the laser intensities in proportion, parametrized by the Raman pulse area $A_R$ in the following. Typical results for $^{85}$Rb are shown in the left panel of Fig. 4(c-f,i) and Fig. 5(a-c,i).

We first discuss the average $\Delta m$-leakage probability $\varepsilon^{(n)}_{m,\Delta m}$ as a function of kick number $n$ for typical $A_R = 9\pi$ presented in Fig. 4(c,i). Here, for $n = 1$, the tiny $\varepsilon^{(n)}_{m,\Delta m}$ for both $m = 0, 1$ are close to the single kick leakage $\varepsilon_{\Delta m} \approx 0.5\%$ as being inferred experimentally. However, with increased $n$, $\varepsilon^{(n)}_{m,\Delta m}$ increases rapidly to approach unity for merely $n \approx 20$, i.e., atoms starting from any of the subsystems $\{|a_m\rangle, |b_m\rangle\}$ has a substantial probability of ending up in a different $m$-subspace. We also note that with increased $n$ the leakage within the hyperfine manifold is accompanied by partial or full returns in a laser-intensity dependent manner.

To investigate the laser intensity dependence, the $\Delta m$-leakage $\varepsilon^{(2N)}_{\Delta m}$ is further plotted in Figs. 4(d-f,i) as a function of both kicking number $n = 4N$ and pulse area $A_R$, for $m = 0, 1, 2$ subsystems respectively (The choice of $n = 4N$ is for comparison with a chirp-alternating sequence to be discussed shortly). We see that over a broad range of pulse area $A_R$, even with the near unity $\mathcal{F}^{(2N)}_m$ (Fig. 4(d,i), top), the spin-leakage probability $\varepsilon^{(2N)}_{\Delta m}$ increases rapidly with $n$ in oscillatory fashions. There is hardly any continuous region of laser intensity with $\varepsilon^{(2N)}_{m,\Delta m} < 0.1$.

It is important to note that although the strong $\Delta m$-leakages hardly affect the photon momentum and hyperfine population transfer (Sec. III), they do limit the gate fidelity for faithful spinor matterwave control with the multiple SDKs. The impact of spin leakage to average gate fidelity $\mathcal{F}^{(2N)}_m$ is demonstrated by comparing Figs. 4(d-f,i) with Figs. 5(a-c,i), where we see the gate infidelity $1 - \mathcal{F}^{(2N)}_m$ closely follows $\varepsilon^{(2N)}_{m,\Delta m}$ to hardly reach 0.1 over most laser intensities, except when the laser intensity is too low to adiabatically drive the Raman transition at all (with $A_R < 3\pi$ here) where we instead find $\mathcal{F}^{(2N)}_m \approx 0.5$, as expected.
C. A chirp-alternating \( n = 4N \) SDK sequence

Clearly, the coherent accumulation of \( \Delta m \)-leakage error as in Figs. 4(c-f,i) needs to be efficiently suppressed before the adiabatic SDK technique can be exploited for coherent control of spinor matterwave. Traditional methods for such suppression include applying a moderate but sufficient quantization field to lift the Zeeman degeneracy [6, 11, 19]. In addition, the \( \Delta m \) spin leakage are naturally suppressed if the Raman transitions are driven by beams with a same circular polarization [47]. These traditional techniques, while showing great success in precision measurements, can be challenging to implement into future compact, flexible devices with SDK control in the \( \omega_{\text{bd},g} \)-limited nanosecond regime.

Instead of resorting to strong quantization fields, we find out a simple way to suppress the \( \Delta m \)-leakage in the cross-linear polarization configuration when transferring \( 2N\hbar k \) momentum to the atom. The idea is to pair \( \tilde{U}_{ud}(-k) \) with \( \tilde{U}_{d}(k) \) which share a same \( \delta_{\text{wp}} \) with opposite sign as

\[
\tilde{U}_{ud}^{(2N)}(k) = \tilde{U}_{ud}(-k)\tilde{U}_{d}(k)\cdots\tilde{U}_{ud}(-k)\tilde{U}_{d}(k). \tag{14}
\]

Similar leakage-suppression sequence can be defined for \( \tilde{U}_{du}^{(2N)}(k) \) by swapping \( u \leftrightarrow d \).

To understand why the accumulation of \( \Delta m \)-leakage error can be partly suppressed by the \( \tilde{U}_{ud}^{(2N)}(k) \) or \( \tilde{U}_{du}^{(2N)}(k) \) sequence, we come back to Eq. (3) to better understand the \( \Delta m = \pm 2 \) leakage itself. In particular, for atom starting in \( |a_m \rangle \) or \( |b_m \rangle \) and subjected to a close-to-ideal adiabatic SDK control, with a \( H_0(r, t) \) Hamiltonian during \( 0 < t < \tau_c \) as prescribed in Sec. II E, the \( \Delta m = \pm 2 \) transitions driven by \( H' \) are often a result of non-adiabatic couplings among systems with nearly equal energy (for example, the \( m = \pm 1 \) subsystems here). By reversing the time-dependence of the Hamiltonian, here with \( H_0(r, t) = H_0(r, \tau_c - t) \) with \( \tau_c < t < \tau_c + \tau_c \), the sign of the non-adiabatic couplings are reversed. Such a sign reversal would lead to complete cancellation of the non-adiabatic transitions if the adiabatic states involved in the couplings are truly degenerate. Here, for atoms being addressed by the cross-linear polarized light, the fact that the \( m \)-spin subsystems are nearly degenerate makes the sign reversal efficient for the suppression of the unwanted leakages.

This simple picture of leakage suppression is verified with numerical simulation in Fig. 4(c,ii) for the chirp-alternating \( \tilde{U}_{ud}^{(2N)}(k_{\text{eff}}) \) sequence, by plotting \( \varepsilon^{(n)}_{m,\Delta m} \) for \( m = 0, 1 \) after each adiabatic SDK. Here, in contrast to the \( \tilde{U}_{ud}^{(2N)}(k_{\text{eff}}) \) case in Fig. 4(c,i), the spin leakage \( \varepsilon^{(n)}_{m,\Delta m} \) oscillates and is overall efficiently suppressed. Comparing with \( m = 1 \) where the leakage is strongly suppressed for even \( n \), the leakage suppression from the \( m = 0 \) subsystem follows a more complicated pattern with an approximate periodicity of 4 to 5, suggesting more complex multi-level dynamics (Fig. 4a).

We further investigate the \( m \)-dependent \( \varepsilon^{(n)} \) dynamics as a function of \( \mathcal{A}_R \) in general. The results for \( n = 4N \) are presented in Fig. 4(d-f,i) to be compared with those in Fig. 4(d-f,i). Here we find the \( \Delta m \)-leakage suppression works nearly perfectly for \( m = 1 \) subsystem. For \( m = 0, 2 \) subsystems, the suppression works fairly well for most \( \mathcal{A}_R \) while there are stripes of \( \mathcal{A}_R \)-region (around \( \mathcal{A}_R = 12\pi \) and \( \mathcal{A}_R = 22\pi \) here for example) where the leakage still accumulate with increased \( n = 4N \). The intricate \( \mathcal{A}_R \)-dependent \( \varepsilon^{(2N)}_{nm} \) as in Figs. 4(d,ii)(f,ii) suggests that the difference of dynamical phases among \( m = 0, \pm 2 \) subsystems by each SDK is large enough to affect the coherent leakage cancellation.

With a detailed investigation of the intricate coupling dynamics left for future work, here we proceed with the numerical evidence demonstrated in Fig. 4(d-f, ii) to construct faithful spinor matterwave control with the \( \tilde{U}_{ud}^{(2N)} \) and \( \tilde{U}_{du}^{(2N)} \) sequences. From the analysis above it becomes evident that while the non-adiabatic couplings by \( \tilde{U}_{ud} \) can be partially canceled by a following \( \tilde{U}_{d} \), the dynamic phase for the \( \text{adiabatically followed} \) amplitudes cannot be cancelled at the same time. To achieve the dynamic phase cancellation as those in Eq. (10), we simply combine the \( \tilde{U}_{ud}^{(2N)} \) and \( \tilde{U}_{du}^{(2N)} \) sequence, leading to

\[
\tilde{U}_{uddu}^{(4N)}(k) = \tilde{U}_{ud}^{(2N)}(k)\tilde{U}_{du}^{(2N)}(k). \tag{15}
\]

The idea is to let the dynamic phases by \( \tilde{U}_{ud}^{(2N)}(k_{\text{eff}}) \) and \( \tilde{U}_{du}^{(2N)}(k_{\text{eff}}) \) cancel each other.

We evaluate the average gate fidelity for realizing the quantum gate for \( \tilde{U}_{K}^{(4N)}(k_{\text{eff}}) \) prescribed by Eq. (10) with the composite \( \tilde{U}_{uddu}^{(4N)}(k_{\text{eff}}) \) sequence. The results of \( F^{(4N)} \), with otherwise identical Hamiltonian parameters as those in Fig. 5(a-c,i), are shown in Fig. 5(a-c,ii). Similar to Figs. 5(a-c,i), here the infidelity \( 1 - F^{(4N)} \) for the \( \tilde{U}_{uddu}^{(4N)}(k_{\text{eff}}) \) sequence closely follow \( \varepsilon^{(4N)} \) for \( \tilde{U}_{ud}^{(4N)} \). Therefore, the improvment is most significant for the \( m = \pm 1 \) spin subsystems. For \( m = 0, \pm 2 \) subsystems, we also see improved gate fidelity \( F^{(4N)} > 95\% \) span a substantial range of intensity for 4N up to 80.

Finally, it is interesting to note that for the nearly perfect SDK with \( f_{\text{SDK}} \approx 1 \), the gate infidelity \( 1 - F^{(4N)} \) is dominantly due to the spin leakage \( \varepsilon^{(4N)} \) as demonstrated by the remarkably similar \( 1 - F^{(4N)} \) and \( \varepsilon^{(4N)} \) data in Fig. 4 and Fig. 5. This is a result of dynamic phase cancellation, which is guaranteed by both the \( \tilde{U}_{ud}^{(4N)} \) and \( \tilde{U}_{du}^{(4N)} \) sequences in the adiabatic limit, if the intra-sequence \( \pm k_{\text{eff}} \) swapping as detailed next is perfect.

D. Robust cancellation of dynamic phase

The numerical results in Fig. 5 demonstrate that precise dynamic phase cancellation can be achieved by pairing \( \tilde{U}_{ud}^{(2N)}(k_{\text{eff}}) \) with \( \tilde{U}_{du}^{(2N)}(k_{\text{eff}}) \) into \( \tilde{U}_{uddu}^{(4N)}(k_{\text{eff}}) \) as by
FIG. 6. Unbalanced dynamic phase $\varphi_D = \varphi_D^{(4)}$ according to Eq. (11) as a function of pulse area $A_R$ with $m = 0$ (Fig. (a)) and $m = 1$ (Fig. (b)) subjected to $U_{udd}(k_{eff})$ (left panel) and $U_{udd}(k_{eff})^*(right\ panel)$ controls. The data are simulated for $\tau = 40$ ns adiabatic SDK sequence as those in Fig. 5, except here we allow the mirror reflectivity $r$ to vary. In all the graphs the red dashed lines give $f_{SDK}$ for single kicks where different $r$-curves closely overlap.

Eq. (15). In fact, we find that the dynamic phase cancellation in the $U_{udd}(k_{eff})$ chirp-alternating scheme is substantially more robust than the standard double-SDK by Eq. (10), as following.

As in Sec. II E, the standard method of dynamic phase cancellation [6] requires perfect $k_{eff}$ reversal for the successive $U_{K}(k_{eff}) U_{K}^*(-k_{eff})$ controls. Practically the $k$-vector swapping is typically accompanied by a modification of $E_{1,2}$ intensity ratio. For example, in the retro-reflection setup (Fig. 1c), the amplitude of the reflected beam is reduced by a $\kappa < 1$ factor due to the imperfect reflection, leading to unbalanced dynamic phases $\varphi_D$ associated with $U(\pm k_{eff})$ to compromise their cancellation in the standard double-SDK (Eq. (10)). This systematic exists quite generally in retro-reflection setups since the 2-photon shift $\delta_0$ is sensitive to the laser intensities ratios [18, 47].

In contrast, here we notice that in the $U_{udd}(k_{eff})^*$ sequence the dynamic phase by any $U_{du}(2) (k_{eff})$ pair is expected to be cancelled by a $U_{ud}(2) (k_{eff})$ pair later. In the adiabatic limit the cancellation is guaranteed, since for free atom starting from any specific 2-level spin state, the sign of the chirp frequency $\delta_{dep}$ dictates the adiabatic quantum number [46] and thus the sign of the dynamic phase in the adiabatic limit.

To demonstrate the robust dynamic phase cancellation, in Fig. 6 we compare $\varphi_D$ according to Eq. (11) for the $U_{uu}(4) (k_{eff})^*$ and $U_{udd}(4) (k_{eff})$ controls, for atom starting from $m = 0, 1$ sub-spaces as examples. The $f_{SDK}$ values are given in the same plots, with which we see that the high $f_{SDK}$ is hardly affected by a poor reflectivity $r = |k|^2 = 0.5$. On the other hand, we see that in contrast to Fig. 6(a-b,i) where $r \approx 1$ is required for precise suppression of $\varphi_D$ (blue line), in Fig. 6(a-b,ii) the $\varphi_D$ is largely suppressed so long as $f_{SDK} \approx 1$, even for a poor $r = 0.5$ as in this experiment. It is worth noting that the residual $\varphi_D$ variation in Fig. 6(a,ii) around $A_R \approx 12\pi$ is related to coherent spin leakage in the $m = 0, \pm 2$ manifold (Fig. 4(d,f)). For atom starting with $m = 1$ (Fig. 6(b,ii)), the dynamic phase cancellation is essentially perfect even for $r = 0.5$.

E. Adiabatic SDK for atom interferometry

So far in this section, we have shown that the double-SDK sequence of $U_{udd}(k_{eff})^*$ (Eq. (10)) as a local phase gate can be faithfully implemented by a chirp-alternating $U_{udd}(k_{eff})^*$ sequence, to coherently shift any $|a_m\rangle, |b_m\rangle$ components of hyperfine spinor matterwave with opposite $\pm 4\hbar k_{eff}$ momentum within nanoseconds. For control parameters in this experimental demonstration, our numerical results already suggest high gate fidelity with efficient suppression of the coherent spin leakage and inhomogenous dynamic phase. In future work, by increasing $\Delta_\gamma/T$ and $\Delta_e/\hbar\Delta_k$ ratios and the laser intensities $I_{1,2}$ in proportion, the residual imperfections can be further suppressed to meet the requisite requirements in the applications of quantum information processing [14, 28, 29].

Here, to demonstrate the utility of the adiabatic SDK sequence for precision measurements, we numerically investigate a simple atom interferometry scheme [6, 10, 11] where an “enclosed area” $A$ is enhanced by the $U_{udd}(4N) (\pm k_{eff})$ sequences. As in the Mach-Zehnder configuration in Fig. 7a, we consider the two spinor matterwave components forming a loop to interfere at $t = 2T$ (the dashed lines): when the duration of the pulsed rotations are negligibly short compared to the “interrogation time” $T$, then the spatial-temporal “area” enclosed by the loop is easily evaluated as $A = v_R T^2$ with $v_R = \hbar k_{eff}/M$. Importantly, the “enclosed area” $A$ is an integrated phase-space separation between the two interfering paths of matterwave, which is often proportional to the differential action experienced by the atom along the two paths, such as by a gravitational force or a Coriolis force to be read out interferometrically. To achieve as large “area” $A$ as possible within a measurement time $T$ is thus of essential importance to precision measurements with light pulse atom interferometry [10, 18, 55].

More specifically, the enclosed area is defined as $A = \int_0^T \Delta z(t) dt$ during a 3-pulse Raman interferometry sequence by integrating the relative displacement $\Delta z(t)$ between the two matterwave diffraction paths under the three operations as splitter ($t = 0$), mirror ($t = T$) and
FIG. 7. Enhancing the enclosed area of an atom interferometer with four $\tilde{U}_{\text{uddu}}(\pm\mathbf{k}_\text{eff})$ adiabatic SDK sequences. (a): Schematic of the interferometry scheme. The orange dashed lines at $t = 0, T, 2T$ represent regular Raman interferometry controls for $R_1 = R_\varphi(\pi/2)$ splitter, $R_2 = R_\varphi(\pi)$ mirror and $R_3 = R_\varphi(\pi/2)$ combiner of the spinor matterwave respectively with $\varphi = \mathbf{k}_\text{eff} \cdot \mathbf{r}$. The atom in $|b\rangle$ and $|a\rangle$ states is represented by the blue and red lines. The thick purple-colored vertical lines at $t = t_1, T - t_1, T + t_1, 2T - t_1$ with red curved arrows represent $\tilde{U}_1 = \tilde{U}_{\text{uddu}}^N(\mathbf{k}_\text{eff})$, $\tilde{U}_2 = \tilde{U}_{\text{uddu}}^N(-\mathbf{k}_\text{eff})$, $\tilde{U}_3 = \tilde{U}_{\text{uddu}}^N(-\mathbf{k}_\text{eff})$ and $\tilde{U}_4 = \tilde{U}_{\text{uddu}}^N(\mathbf{k}_\text{eff})$ sequences respectively. We consider $\tau_1 < T$. By properly choosing $\tau_1/T$ ratio, spurious interference by multiple imperfect controls can be suppressed, and are not included in the simulations. (b-d): The interferometry phase offset $\delta \Phi$ and contrast $C$ as a function of SDK density number $n = 4N$ and pulse area $A_{R\varphi}$. Here the single-photon detuning is chosen as $\Delta_c = 3.3 \omega_{\text{phys}}$. The simulations average over unpolarized $m = -2, -1, 0, 1, 2$ states, and include $\Gamma_c = 0.017 \omega_{\text{phys}}$ as for the case of $^{85}\text{Rb}$. The phase offsets and interferometry contrasts locally averaged over a $50\%$ intensity are given in (c,d). The data in Fig. (e-g) are similar to Fig. (b-d), but with an increased single-photon detuning of $\Delta_c = 6.6 \omega_{\text{phys}}$. combiner ($t = 2T$). We generally refer the idealized local spin rotations as $R_\varphi(\theta) = \cos(\theta/2)I + i \sin(\theta/2)(e^{i\varphi} \sigma_+ + e^{-i\varphi} \sigma_-)$ for the Raman interferometer, for any spin state within $\{|a_m\rangle, |b_m\rangle\}$. Here $\varphi = \mathbf{k}_\text{eff} \cdot \mathbf{r}$ is the local Raman optical phase. Notice the spatial-dependent $R_\varphi(\theta)$ rotation can in principle be generated by the Eq. (3) Hamiltonian [56] as phase-coherent “half” and “full” kicks. The splitter and mirror operations in the standard light Raman interferometer can then be conveniently expressed as $R_1 = R_\varphi(\pi/2)$, $R_2 = R_\varphi(\pi)$ and $R_3 = R_\varphi(\pi/2)$ respectively to manipulate the spin states while imparting the $\pm \hbar \mathbf{k}_\text{eff}$ photon recoil momentum.

We now consider enhancing the enclosed area $A$ of the standard 3-pulse Raman interferometer with the chirp-alternating SDK sequence. In particular, we consider the Fig. 7a scheme with the spinor matterwave diffraction paths marked with solid lines: a $\tilde{U}_1 = \tilde{U}_{\text{uddu}}^N(\mathbf{k}_\text{eff})$ is first applied at $t = \tau_1$ to increase the momentum displacement between the two interfering paths from $\Delta \mathbf{p} = \hbar \mathbf{k}_\text{eff}$ by $R_\varphi(\pi/2)$ to $\Delta \mathbf{p} = (2 \times 4N + 1) \hbar \mathbf{k}_\text{eff}$ with the spin-dependent kicks. This $\Delta \mathbf{p}$-enhancement is followed by an opposite $\tilde{U}_2 = \tilde{U}_{\text{uddu}}^N(-\mathbf{k}_\text{eff})$ at $t = T - \tau_1$ before the $R_2$-operation to recover the initial $\Delta \mathbf{p}$. To ensure that the interfering paths spatially overlap at $t = 2T$, an additional pair of opposite momentum boosts, $\tilde{U}_{3,4}$ are applied at $t = T + \tau_1$ after the $R_2$ and $t = 2T - \tau_1$ before the $R_3$ operation respectively. By properly choosing the $\tau_1/T$ ratio, spurious interference by imperfect $R_{1,2,3}$ and $\tilde{U}_{1,2,3,4}$ controls can be suppressed [57, 58]. With $T \gg \tau_1$, the enclosed area of the resulting interfering loop is enhanced to $A' = (2 \times 4N + 1) A$.

For the numerical simulation, we consider at $t = 0$ the atomic state to be initialized at certain $|b_m\rangle$ and subjected to the $\tilde{U}_{\text{uddu}}^N(\mathbf{k}_\text{eff})$-enhanced 3-pulse interferometry sequence. The output atomic state, right before the final matterwave combiner $R_3$, can then be written as $|\psi_m(2T^-)\rangle = U_{AI}|\psi_m(\tau_1)\rangle$, with $U_{AI} = U_f(\tau_1)\tilde{U}_3 R_2 \tilde{U}_2 U_f(T - 2\tau_1)\tilde{U}_3 R_2 \tilde{U}_2 U_f(T - \tau_1)\tilde{U}_3 R_2$. $U_f(\tau_1)$ designates free propagation of matterwave for time $t$. The $R'_1 = U_f(\tau_1) R_1$, $R'_2 = U_f(\tau_1) R_2 U_f(\tau_1)$ take into account the free propagation of matterwave between the standard $R_3$ and area-enhancing $\tilde{U}_3$ sequences. We numerically evaluate $|\psi_m(z, 2T^-)\rangle$ for 1D spinor matterwave between $0 < z < \lambda/2$, as described in Sec. II B. To focus on the performance of SDK, we set $R_{1,3}$ as perfect $\pi/2$ pulses and $R_2$ as perfect $\pi$ mirror pulse respectively. A further simplification sets $\mathbf{k}_\text{eff} = 0$ for the idealized $R_{1,2,3}$ controls, with which we numerically evaluate $|\Sigma_j\rangle = \langle \psi_m(z, 2T^-)|\psi_m(z, 2T^-)\rangle|_{z,m}$ for an initially unpolarized atomic sample right before the $R_3$ operation. Here $\Sigma_j = \sum_{m'} (\mathbf{R}_j^{m'} - \mathbf{j} \sigma_j^{m'})$ are summed over all $m$ subspins for Pauli matrices with $\mathbf{j} = x, y, z$. $\Sigma_j$ corresponds to observables of experimental measurements in which Zeeman sublevels are not resolved, as in most atom interferometry experiments with hyperfine state-dependent fluorescence readouts [18].

With $U_f$ chosen as free 1D propagation, the values of
\( \tau_1 \) and \( T \) only affects contributions of spurious interfering paths into the final readouts \([57, 58]\) in the simulation. With \( R_{1,2,3} \) set as ideal, the spurious interfering paths are from imperfect \( U_{1,2,3,4} \) diffractions only. Notably, since successive adiabatic SDKs here within each \( U_j \) last merely tens of nanoseconds, the spatial displacements among the spurious interfering paths are negligibly small comparing with the typical coherence length of cold atom samples, and therefore do not alter the matterwave dynamics \([59, 60]\). We have randomly sampled the atomic initial position and velocity to numerically verify that the residual spurious interference are indeed suppressed. Practically, to generate the results in Fig. 7(b-d) with all spurious interference removed in an efficient manner, we simply apply a digital filter to remove unwanted diffraction orders after each \( U_j \) sequence.

We are particularly interested in the interferometry contrast \( C \) and diffraction phase offset \( \delta \Phi \). The contrast \( C \) decides the quality of the final matterwave interference fringes. The phase offset \( \delta \Phi \), stemming from the unbalanced dynamic phase by the four \( U_{uddu} \) sequences, enters the interferometry readout as systematic bias against any precision measurements or controls. Numerically, we conveniently evaluate the interferometry contrast as

\[
C = \sqrt{\langle \Sigma_x \rangle^2 + \langle \Sigma_y \rangle^2}.
\]

The diffraction phase offset is instead evaluated as

\[
\delta \Phi = \arg [\langle \Sigma_x \rangle + i \langle \Sigma_y \rangle] - \Phi_0 \] with \( \Phi_0 \) to be the relative phase between \( |a_m\rangle \) and \( |b_m\rangle \) right after the ideal \( R_1 \) splitter. Typical numerical results are presented in Figs. 7(b-d). The simulation is again performed on the \( ^{85} \text{Rb} \) D1 line, here with spontaneous emission included. For Figs. 7(b-d) on the left panel, the control laser parameters are chosen close to this experimental work, with \( \Delta_c = 3.3 \, \omega_{\text{hfs},g} \) so both \( \varepsilon_{\text{sp}} \) and \( \varepsilon_{\text{ramp}} \) are quite substantial. Nevertheless, we find \( \varepsilon_{\text{sp}} > 0.5 \) with \( \delta \Phi < 0.01 \) after four \( n = 4N = 12 \) chirp-alternating SDKs are applied for a 25-fold enhancement of interferometry enclosed area. Here, to avoid excessive spontaneous emission and coherent leakage (Fig. 4(d-f,ii)), The peak \( A_R \approx 6 - 8N \) needs to be chosen (Fig. 7(b-d)). On the other hand, by doubling the single photon detuning to \( \Delta_c = 6.6 \, \omega_{\text{hfs},g} \) (with laser intensity increased in proportion to maintain the Raman Rabi frequency), \( \varepsilon_{\text{sp}} \) are halved, while the impact of \( \varepsilon_{\text{ramp}} \) leakage are dramatically suppressed in the \( U_{uddu} \) sequence (Figs. 7(e-g)). The further detuned \( U_{uddu} \) sequence should thus support up to 50-fold enhancement of interferometry enclosed area, with spontaneous-emission-limited \( C > 0.5 \) contrast and negligible \( \delta \Phi \) offset.

V. DISCUSSIONS

Significant aspects of advanced quantum technology today are based on controlling alkaline atoms through their center-of-mass motion and ground-state hyperfine interaction. The two long-lived degrees of freedom are naturally coupled to each other in resonant laser fields. Precise control of the spinor matterwave with light requires carefully tailored light-atom interactions. Unlike microwave control of magnetic spins, quantum control of macroscopic matterwave with lasers is substantially more demanding on the intensity-error resilience \([48, 61]\). To meet the high fidelity requirements by the next generation quantum technology, particularly on macroscopic samples, implementation of error-resilient quantum techniques \([15–17]\) are likely required. To this end, high speed quantum control into the nanosecond regime allows efficient suppression of low-frequency noises, including those arising from uncontrolled atomic motion, for nearly ideal implementation of the error-resilience techniques. In addition, high-speed, repetitive application of optical Raman control enhances the spin-dependent force, in spite of the tiny atomic recoils, for improving the scalability of precision measurements \([6, 10, 11]\) and quantum information processing \([12–14]\).

In this work, we have demonstrated a novel configuration of adiabatic SDK implemented on an optical delay line, which is able to reach the speed limit of Raman SDK control \([5, 62]\), featuring robust intensity-error resilience, while maintaining various advantages of optical retro-reflection established for precision atom interferometry. Constrained by experimental resources during the completion of this work, the characterization of the technique is limited to inference of single kick \( f_{\text{SDK}} \) with atomic velocity and population as observables. We clarify in Sec. IV that high precision phase gates can be realized by the adiabatic SDK scheme. In particular, we numerically demonstrate that by properly alternating the sign of frequency-chirps in successive adiabatic SDKs, major error terms associated with coherent spin-leakage and unbalanced dynamic phases can be mitigated or even completely suppressed. The robust suppression of dynamic phase in particular is due to a time-reversal symmetry for the driven spin dynamics in the chirp-alternating scheme, much like those in the traditional spin-echo schemes, but is achieved here in the adiabatic limit to support an even stronger intensity-error resilience. With atom interferometry as an example, we have provided numerical evidence that the delay-line based adiabatic SDK scheme support faithful, parallel \( \Delta m = 0 \) control of multi-Zeeman spinor matterwave, with giant spin-dependent optical forces applied within nanoseconds to rapidly shift the phase-space spin separation, even with a moderate laser power in the 10 mW range as in this work.

On the other hand, the performance of the adiabatic SDK demonstrated in this experimental work is limited by the moderate laser power \([27]\). New laser technology to increase the peak laser power to Watt level would support a ten-fold increase of \( \Delta_c \) to substantially suppress \( \varepsilon_{\text{sp}} \) and \( \varepsilon_{\text{ramp}} \) errors, and to allow a ten-fold reduction of SDK pulse duration at the same time. For SDK within a few nanoseconds, a meter-long delay line is able to temporally resolve counter-propagating pulses to flexibly drive bidirectional SDKs as in this work. It would also be tech-
nically easier to achieve nearly perfect retro-reflection in the compact device. The improved compact device combined with the chirp-alternating technique should support nearly ideal spinor matterwave control with unprecedented speed and precision.

Finally, we remark that for the spinor matterwave control with the delay-line based SDK scheme, as in Fig. 1, the extra dynamic phases (Stark shifts) by the pre- and post-pulses need to be precisely compensated. The compensation can be effectively achieved by pairing SDK sequence to enforce that their additional pulses lead to opposite Stark shifts to the spinor components under control. Similar to the frequency domain Stark shift compensation techniques [55], for the nanosecond SDKs here a more straightforward method is to fire additional pulses with suitable single-photon detunings to trim the overall dynamic phase. Within nanoseconds, atoms hardly move to change the local laser intensity. We therefore expect the dynamic phase compensation to function well in the time domain.

ACKNOWLEDGEMENTS

We are grateful to Prof. Yiqiu Ma for insightful comments to the manuscript, and to Prof. Xiaopeng Li and Prof. Haidong Yuan for helpful discussions. We acknowledge support from National Key Research Program of China under Grant No. 2016YFA0302000 and No. 2017YFA0304204, from NSFC under Grant No. 12074083.

Appendix A: Full Hamiltonian

The numerical simulation in this work is according to the full light-atom interaction Hamiltonian on the D1 line. Following the notation in the main text, the effective, non-Hermitian Hamiltonian is written as

$$H_{\text{D1,eff}}(r, t) = \hbar \sum_e \{\omega_e - \omega_{e0} - i \Gamma_e / 2\} \sigma^{e*e} + $$

$$\hbar \sum_{e=a,b} \{\omega_e - \omega_{g0}\} \sigma^{e*e} + $$

$$\frac{\hbar}{2} \sum_{e=a,b} \sum_{e1} \Omega_{e=e1}(r, t) \sigma^{e*e1} + \text{h.c.}$$

(A1)

Here $\omega_{e0}, \omega_{g0}$ are decided by the energy of reference level in the excited and ground state manifolds respectively, chosen as the top hyperfine levels in this work. The laser Rabi frequency is accordingly written in the $\omega_{e0, g0}$ frame under the rotating wave approximation.

Appendix B: Absorption imaging analysis

In Sec. III we have introduced the double imaging technique. This section provides details on deriving recoil momentum $p_n$ and population transfer $\rho_{aa}$ from the imaging data.

Our atomic sample is prepared in a cross-dipole trap with slight asymmetry. When deriving the momentum transfer from the double images as in Fig. 2, we found that neither before nor after the time-of-flight, the absorption profile can fit perfectly to a 2D Gaussian. In addition, due to the relatively short exposure time of $20 \mu s$ to the weak probe ($s = 1$), there are substantial photon shot noise in single-shot images. To faithfully retrieve central position and atom number from each pair of double-image, we take the following procedure. First, we repeat a same type of measurements for $N = 80$ shots, and do a principle components analysis to all pairs of double-images after background subtractions. The first three components are kept for the following analysis. We then do a 2D Gaussian fit, expanding the fitted Gaussian profiles to 1.5 times the waist into a wide enough step-wise mask. The population ratios $\rho_{aa/\bar{b}b}$ are evaluated by the ratio of total counts between the two images within the mask, where the center-of-mass (COM) positions $z_1$ and $z_2$ are also directly evaluated.

We note that the first image is contributed by atoms at $F = 3$ “$|a_m\rangle$” states only. In other words, the atoms kicked to the “visible” (or “invisible”, depends on odevity of kicking number $n$) hyperfine levels are post-selected. Since there is a small interval $\tau_{p,1}$ between SDK and the first image, there is a bias to the COM position $z_1$ of the whole cloud due to this post-selection. To correct for the bias, we rewrite the position difference as $z_2 - z_1 = \bar{v}_n,\bar{v}_{b,n}$. Here $\bar{v}_n$ and $\bar{v}_{b,n}$ are the mean velocity for the whole atomic ensemble and for the atoms in $F = 2$ after $n$ SDKs, respectively. The correction to the post-selection induced velocity bias is then given by

$$\bar{v}_n = v_n \left(1 - \rho_{b,n} v_{b,n} \frac{\tau_{p,1}}{\tau_{\text{tof}}}\right) \equiv v_n \left(1 - \xi(n, v_n) \frac{\tau_{p,1}}{\tau_{\text{tof}}}\right).$$

(B1)

Here, $\xi(n, v_n)$ is a function on the relation between $v_n$ and $\rho_{b,n} v_{b,n}$. The relation can be approximated with the model in Appendix. C. With $\tau_{p,1} \sim 15 \mu s$ in our experiment, this correction is typically $1 - \bar{v}_n / v_n \sim \pm 5\%$ (The $\pm$ signs depend on $n$), which impacts $f_R$ at $3\%$ level. The correction is model-dependent. We correct for the bias in our final estimation of $f_{\text{SDK}}$ and leave $3\%$ as the dominant uncertainty in our $f_{\text{SDK}}$ estimation.

Appendix C: A Markovian model for $f_{\text{SDK}}$ estimation

For atoms subjected to multiple SDKs, the dynamics of spinor matterwave that deviates from the ideal control can be depicted as diffusing in a “momentum-lattice” [48]. Our numerical simulation suggests that with fair efficiency of single ARP pulse to achieve hyperfine transfer efficiency of $f_R > 95\%$, the resulting average momentum $p_n$ and population $\rho_{aa/\bar{b}b,n}$ roughly follow a simple Markovian model. The model assumes that both
the momentum and population transfer by the next kick are decided by the present population difference ρ_{aa} − ρ_{bb} only. The details of the Markovian model is describe as following.

Suppose that after n kicks, the atomic ensemble is with momentum \( p_n \) (in unit of \( \hbar k_{\text{eff}} \)) and population contrast \( C_n \equiv |\rho_{aa,n} − \rho_{bb,n}| \); then the next kick will impart momentum as

\[
\Delta p_{n+1} = p_{n+1} - p_n = f_0 (1 - \varepsilon_{\text{sp}}/2) C_n, \tag{C1}
\]

where \( f_0 \) is the hyperfine population transfer efficiency in absence of the spontaneous emission. Here we have assumed that during the single pulse process, the spontaneous emission occurs with a uniform distribution of probability, thus the associated population recycled to the ground states acquires half of \( \hbar k_{\text{eff}} \) momentum on average.

Similarly, the population distribution can be written as

\[
\rho_{aa,n+1} = (1 - \varepsilon_{\text{sp}}) \left[ (1 - f_0) \frac{1 + C_n}{2} + f_0 \frac{1 - C_n}{2} \right] + \varepsilon_{\text{sp}}/2, \tag{C2}
\]

\[
\rho_{bb,n+1} = (1 - \varepsilon_{\text{sp}}) \left[ f_0 \frac{1 + C_n}{2} + (1 - f_0) \frac{1 - C_n}{2} \right] + \varepsilon_{\text{sp}}/2,
\]

so that

\[
C_{n+1} = (1 - \varepsilon_{\text{sp}})(2f_0 - 1)C_n. \tag{C3}
\]

We define Raman transfer efficiency as \( f_R = f_0 (1 - \varepsilon_{\text{sp}}/2) \). When both \( f_0 \) and \( 1 - \varepsilon_{\text{sp}} \) are close to unity, Eqs. (C1)/(C3) can be approximated as

\[
p_{n+1} - p_n = f_R C_n, \tag{C4}
\]

\[
C_{n+1} = (2f_R - 1)C_n.
\]

With the recursion relations by Eq. (C4), we arrive at

\[
p_n = f_R^{n-1} \frac{1 - (2f_R - 1)^n}{1 - (2f_R - 1)}, \tag{C5}
\]

\[
C_n = (2f_R - 1)^n.
\]

Finally, we remark that for the Raman SDK, there is a slight difference of spontaneous emission loss for single kicks between the \( a \to b \) and \( b \to a \) process. As we consider repetitive SDK with \( n \) up to a quite large number (e.g. \( n_{\text{max}} = 25 \) in our experiment), we effectively set a same \( \varepsilon_{\text{sp}} \) parameter for the opposite population transfer processes.

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**Appendix D: Impact of mirror optical loss to \( f_{\text{SDK}} \)**

Numerical simulations suggest that the reflectivity \( r = |\kappa|^2 \) of the retro-reflection mirror (Fig. 1) affect \( f_{\text{SDK}} \)

![Figure 8](image_url)

FIG. 8. (a) Numerical simulations for \( f_R \) and \( f_{\text{SDK}} \) vs Raman pulse area \( A_R \) for different reflectivity \( r = |\kappa|^2 \) of the retro-reflection mirrors. The red bars show estimated distribution of atoms subject to different pulse areas under the Gaussian beam illumination in this experiment. (b) \( n(A_R) \)-weighted average \( f_R \) and \( f_{\text{SDK}} \) vs reflectivity \( r \). The experimentally measured \( f_R \) and inferred \( f_{\text{SDK}} \) are marked with error bars.

slightly. To investigate the effect, we sample \( 0 < r < 1 \) during the simulation, and calculate Raman transfer efficiency \( f_R \) and SDK fidelity \( f_{\text{SDK}} \) with peak Raman pulse area \( A_R \) as in Fig. 8(a). In light of the mixed state nature of the experimental measurements, here the results are again averaged over the initial states \( |b_n\rangle \). For all the simulation, we evaluate peak Rabi frequency \( \Omega_{a,b} \) as in previous work of electric dipole transition control [48] where the laser beam waist and the atomic ensemble size are also carefully characterized. Based on the geometry parameters, we histogram the fractions of atoms with peak Raman pulse area in Fig. 8(a). The fractions are applied to weight the average over all pulse areas for the evaluation of \( \langle f_R \rangle \) and \( \langle f_{\text{SDK}} \rangle \) at various reflectivity \( r \) in Fig. 8(b). On the plot, the measured Raman transfer efficiency \( f_R \) suggest \( r \sim 55(5)\% \). This reflectivity is consistent with experimental measurements on the ratio of Stark shifts by the incident and reflected beams.

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