Propagation of broad meson resonances in a BUU type transport model: Application to di-electron production

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Abstract

We apply a BUU type transport model for the interpretation of the di-electron invariant mass spectrum measured by the HADES collaboration for the reaction C(2 AGeV) + C. Our model incorporates the propagation of broad meson resonances emerging from the decay of baryon resonances.

1 Introduction

Di-electrons serve as direct probes of dense nuclear matter stages during the course of heavy-ion collisions. The superposition of various sources, however, requires a deconvolution of the spectra by means of models. Of essential interest are the contributions of the light vector mesons $\rho$ and $\omega$. The spectral functions of both mesons are expected to be modified in a strongly interacting environment in accordance with chiral dynamics, QCD sum rules etc. After the first pioneering experiments with the DLS spectrometer now improved measurements with HADES start to explore systematically the baryon-dense region accessible in fixed-target heavy-ion experiments at beam energies in the few AGeV region.

We apply here our BRoBUU transport model to the di-electron data measured by HADES in the reaction C(2 AGeV) + C. Some features of the code are described in section 2, while section 3 is devoted to the presentation of simulation results and the comparison to data.
2 BRoBUU code

The BRoBUU computer code for heavy-ion collisions developed by a Budapest-Rossendorf cooperation solves the Boltzmann-Ühling-Uhlenbeck equation in the quasi-particle limit:

\[
\frac{\partial F}{\partial t} + \frac{\partial H}{\partial p} \frac{\partial F}{\partial x} - \frac{\partial H}{\partial x} \frac{\partial F}{\partial p} = C, \quad H = \sqrt{(m_0 + U(p, x))^2 + p^2}
\]  

for the one-body distribution function \(F(x, p, t)\) of a certain hadron species. This equation is applied to the motion of different hadron species, each with rest mass \(m_0\), in a momentum and density dependent mean field \(U\). The scalar mean field \(U\) is chosen in such a way that the Hamiltonian \(H\) equals:

\[
H = \sqrt{m_0^2 + p^2 + U_{nr}}
\]

where the parameters \(A, B, C, \tau, \Lambda\) define special types of potentials, while \(n, n_0\) and \(f_N\) stand for the baryon number density, saturation density and nucleon distribution function. The BRoBUU code propagates in the baryon sector the nucleons and 24 \(\Delta\) and \(N^*\) resonances and additionally \(\pi, \eta, \sigma, \omega, \rho\) mesons. Details will be reported elsewhere. Different particles species (each described by a corresponding distribution \(F\)) are coupled by the collision integral \(C\) which also contains the Ühling-Uhlenbeck terms responsible for Pauli blocking in the collision as well as particle creation and annihilation processes. The set of coupled Boltzmann-Ühling-Uhlenbeck equations is solved by using the parallel-ensemble test-particle method, where each distribution function is represented by:

\[
F = \frac{1}{N_{test}} \sum_{i=1}^{N_{test}} \delta^{(3)}(x - x_i(t)) \delta^{(4)}(p - p_i(t)).
\]

This method transforms the partial differential–integro equations into a set of ordinary differential equations (looking like equations of motion) for a large number of test particles simulating the ensemble averaging process for the respective function \(F\).

Recently theoretical progress has been made in describing the in-medium properties of particles starting from the Kadanoff-Baym equations for the Green functions of the particles. In the medium particles have a finite lifetime which is described by the width \(\Gamma\) in the spectral function \(A\) of the particles. The spectral function being the imaginary part of the retarded propagator in the Kadanoff-Baym equations, \(-2Im G^{\text{ret}}(x, p)\), is essentially defined by the self-energies \(\Sigma\) of the particle in the medium. For bosons it reads:

\[
A(p) = \frac{\hat{\Gamma}}{(E^2 - p^2 - m_0^2 - \text{Re}\Sigma^{\text{ret}})^2 + \frac{1}{4} \hat{\Gamma}^2},
\]

where \(\text{Im}\Sigma^{\text{ret}} \equiv \frac{1}{2} \hat{\Gamma} \) and \(\text{Re}\Sigma^{\text{ret}}\) depend on \(E, p\) and the local medium properties. The spectral function can significantly change during the heavy-ion collision process and can be simulated by an ensemble of test particles with different masses. The change of the
spectral function is now given by time variation of the test particle mass \( m_i \). For bosons this additional equation reads

\[
\frac{dm_i^2}{dt} \approx \left( \frac{\delta}{\delta t} \text{Re}\Sigma_{ret} + \frac{m_i^2 - m_0^2 - \text{Re}\Sigma_{ret}}{\Gamma} \frac{\delta}{\delta t} \Gamma \right),
\]

(4)

where the values of the selfenergy \( \Sigma_{ret} \) are taken at the positions of the test particles \( i \), and \( \delta / \delta t \) stands for the comoving time derivation. The real part of the selfenergy is related to the mean field \( U \). Eq. (4) is actually a corollary of three equations describing the propagation of energy, three-momentum and position of a test particle. This equation ensures that resonances are propagated towards their vacuum spectral functions at freeze-out [8]. This technique, allowing for a consistent propagation of broad resonances, is applied in the BRoBUU code for calculating the di-electron emission of \( \omega \) and \( \rho \) mesons.

Besides the propagation of broad resonances the spectral function also controls the production of mesons. Vector mesons (V) are essentially created by the decays of baryon resonances (R). The mass distribution reads

\[
\frac{dN^{R \rightarrow NV}}{dm_V} \propto A(m_V) m_V (m_R^2 + m_N^2 - m_V^2) \lambda^{1/2}(m_R^2, m_N^2, m_V^2),
\]

(5)

where \( \lambda(a^2, b^2, c^2) = (a^2 + b^2 - c^2)^2 - 4a^2b^2 \) denotes the triangular function. The resonances \( R \) is created in nucleon-nucleon collisions \( NN \rightarrow NR \) and meson-nucleon collisions, \( MN \rightarrow R \). \( R \) may more generically decay as \( R \rightarrow R'M \) into other channels (\( R' = \Delta(1232), N(1440), N(1520), N(1535) \)). Resonance parameters for \( NN \leftrightarrow NR \) and \( MN \leftrightarrow R \) are fitted to available data for meson production in nucleon-nucleon and pion-nucleon reactions [9].

In our calculations we employ a simple form of the selfenergy:

\[
\text{Re}\Sigma_{V, ret} = 2m_V \Delta m_V \frac{n}{n_0},
\]

(6)

\[
\text{Im}\Sigma_{V, ret} = m_V (\Gamma_{V, ret}^{vac} + \frac{nn_V\sigma_V}{\sqrt{1-v^2}}).
\]

(7)

Eq. (6) describes schematically a mass shift proportional to the density \( n \) of the surrounding matter related to the normal matter density \( n_0 \). The effect of the collision broadening is given in Eq. (7) which depends on density, relative velocity \( v \) and the cross section \( \sigma_V \) of the vector meson in matter. This cross section is calculated via the Breit-Wigner formula

\[
\sigma_V = \frac{4\pi}{q_{in}^2} \sum_R \frac{2J_R + 1}{3(2J_i + 1)} \frac{s\Gamma_{V,R}^{tot}}{(s - m_R^2)^2 + s(\Gamma_{R}^{tot})^2}
\]

(8)

for forming resonances with masses \( m_R \), angular momenta \( J_R \), partial widths \( \Gamma_{V,R} \), total widths \( \Gamma_{R}^{tot} \) with energy \( \sqrt{s} \) and relative momentum \( q_{in} \) in the entrance channel. In vacuum the baryon density \( n \) vanishes and the resulting spectral function \( A_{vac} \) is solely determined by the energy dependent width \( \Gamma_{V}^{vac} \).
The di-electron production from vector meson decays $V \rightarrow e^+e^-$ is calculated by integrating the local decay probabilities along their paths in the collision. The subleading so-called direct channel $\pi\pi \rightarrow \rho \rightarrow e^+e^-$ is treated with the cross section

$$\sigma(M) = \frac{\sigma_{vac}}{m_{\rho,0} A_{vac}}, \quad \sigma_{vac} = \frac{4\pi}{3} \left( \frac{\alpha}{M} \right)^2 \sqrt{1 - \frac{4m_{\rho}^2}{M^2}} \left( \frac{\tilde{m}_\rho^4}{(M^2 - \tilde{m}_\rho^2)^2 - (\tilde{m}_\rho \tilde{\Gamma})^2} \right)$$

with $m_\rho = m_{\rho,0} + \Delta m_\rho n/n_0$, $\tilde{m}_\rho = 775$ MeV, $\tilde{m}_\rho = 761$ MeV, $\tilde{\Gamma}_\rho = 118$ MeV. Analog considerations apply to the channel $\pi\rho \rightarrow \omega$.

We also include into our simulations a bremsstrahlung contribution which is guided by a one-boson exchange model adjusted to $pp$ virtual bremsstrahlung and transferred to $pn$ virtual bremsstrahlung [13]. Actually we use $\frac{d\sigma_{pp}}{dM} = \frac{4\pi}{3} \sigma_\perp \frac{a^2}{6\pi} \int dq^2/q^3 R_2(s)/R_2(s)$ and $\sigma_{pp} = 0$ with $\sigma_\perp = \sigma_{pn,tot}(s) \frac{s - 4m_N^2}{2m_N^2}$, where $M$ is again the $e^+e^-$ invariant mass, $R_2$ denotes the two-particle phase space volume, $\sqrt{s}$ stands for the c.m.s. energy in a nucleon-nucleon collision, $\tilde{s}$ is the reduced energy squared after the dilepton emission, and $\sigma_{pn,tot}(s)$ is the corresponding total cross section.

3 Results and comparison to data

We employ the above described code for the reaction $C(2$ AGeV$) + C$, where first data from HADES [10] are at our disposal. In the present explorative study we are going to contrast simulations with and without medium modifications of $\rho$ and $\omega$ mesons to elucidate to which degree medium effects can become visible in the light collision system under consideration. In doing so we use fairly schematic medium effects condensed in a ”mass shift” of $\Delta m_\omega = -50$ MeV for the $\omega$ meson. Such a shift is suggested by recent CB-TAPS data [11]. The use of QCD sum rules [12] then can be used to translate this shift into a significantly larger shift for the $\rho$ meson (dictated essentially by the Landau damping term); we use here $\Delta m_\rho = -200$ MeV.

To illustrate the time evolution of the test particles we have shown in the lower part of Fig. 1 the masses of four test particles randomly selected out of the ensemble representing the spectral function of an $\omega$ meson as a function of the collision time. (These curves were obtained by artificially switching off the decay of the $\omega$ mesons. Otherwise they would decay in a very short time in the high density region. Thus, particles with masses strongly deviating from their peak mass have little chance to radiate di-electrons off.) The upper part shows the respective local densities of the matter surrounding these particles. The tendency to acquire their vacuum mass starting from very different initial masses is clearly seen. For the test particle depicted by the dash-dotted curve which moves with a velocity of 0.6 c we have also shown the spectral function of the $\omega$ meson at the collision times of 6 fm/c and 10 fm/c together with the vacuum function in Fig. 2.

The obtained di-electron spectra are represented in Figs. 3 and 4. In Fig. 3 the vacuum parameters are employed while Fig. 4 shows the results obtained by including the above described medium modifications of $\rho$ and $\omega$ mesons. For comparison with the data the
Figure 1: Time evolution of the masses (lower panel) and the surrounding baryon densities (upper panel) of four randomly selected test particles.

HADES filter has been applied accounting for the geometrical acceptance, momentum cuts and pair kinematics. The filter causes a reduction of the strength and a smearing of the invariant masses of the di-electrons. The result of this filtering is always shown on the right hand panel of the figures.

In these figures we show different contribution to the di-electron rate. Important di-electron sources are $\pi^0$ and $\eta$ Dalitz decays which are proportional to the multiplicities of their parents. The TAPS collaboration has measured \cite{ref} the $\pi^0$ and $\eta$ production cross sections of $707\pm72$ mb and $25\pm4$ mb for the same system and same beam energy. These values have to be compared to our calculations yielding 700 mb and 20 mb. While the value for pion production is in good agreement, there is a slight underestimation of the $\eta$ production. (Note that the presently employed cross sections rely on a global fit of many elementary reactions which is not optimized for the $\eta$ channel.) The Dalitz decays of $\rho$ and $\omega$ mesons and nucleon resonances do not contribute essentially. The employed elementary cross sections for $pp \rightarrow pp\omega$ and $pn \rightarrow pn\omega$ agree with the ones in \cite{ref2} which are adjusted to near-threshold data for $pp \rightarrow pp\omega$. One recognizes comparing Fig. 3 and 4 that the medium modification due to collision broadening diminishes the yield of $\rho$ di-electrons. Still an indication of a shoulder is expected from the calculations. Our calculations do not
show a considerable effect of a $\rho$ mass shift, however this finding depends sensitively on our elementary $\rho$ production rates employed in the calculations. Since the fine structure is not yet resolved in the data a conclusive decision cannot be made at present.

4 Summary

In summary we have compared results of a transport code to recent measurements of the di-electron invariant mass spectrum for the reaction $C(2\ AGeV) + C$. The code incorporates the production and propagation of broad resonances, in particular $\rho$ and $\omega$ mesons. Including even fairly schematically parameterized in-medium modifications of these vector mesons the emerging differences to results which employ vacuum parameters show up in a better agreement with the data.

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Figure 3: Various sources of the di-electron invariant mass spectrum for C(2 AGeV) + C. Left panel: without filter. Right panel: with experimental filter and compared to HADES data [10]. Vacuum selfenergies are used.

Figure 4: The same as Fig. 3 but with modified selfenergies of ρ and ω mesons as described in text.
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