Basic idea of Corbino-type single-electron transistor

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Abstract. We have formulated the transmission probability of an electron in a Corbino quantum disk by taking into account charging effect. The geometrical potential of the Corbino disk has a singularity at the centre of the disk. In order to avoid this singularity problem, we have to reformulate the Schrödinger equation in the Riemannian manifold. The Schrödinger equation describing the motion of the electron in the Corbino disk must be expressed by introducing a momentum operator reformed by the metric tensor. In order to obtain a Hermitian momentum operator, we must deform the Hilbert space by introducing a new wave function. This deformation leads to the extra potential term in the Schrödinger equation, which depends on the metric, i.e., the geometry of the disk. It should be noted that the charging energy of confining electrons in the Corbino disk should depend on the geometry of the disk. We discuss the quantum tunneling of an electron confined in the Corbino disk in order to investigate the effect of both geometrical potential and charging energy of confining electrons in the Corbino disk by using the Wentzel-Kramers-Brillouin (WKB) method. It is expected that the charging energy, which depends on the effective confining potential, plays an important role in the transmission probability. This suggests that the formulated transmission probability is applicable to the analysis of the single-electron transistor.

1. Introduction
Recent technological advances enable us to fabricate a Corbino-type quantum disk in a two-dimensional (2D) electron system [1–5]. From the quantum mechanical viewpoint, it is interesting that a quantum centrifugal potential must be introduced in this system due to its geometry [6–8]. In general, it is necessary to reformulate the Schrödinger equation in terms of the Riemannian geometry to describe the motion of an electron constrained on the curved surface including the Corbino disk. The Schrödinger equation is generally given by [9–11]

\[-\frac{\hbar^2}{2m} \sum_{i,j=1}^{2} \frac{1}{\sqrt{g}} \frac{\partial}{\partial q_i} \left( \sqrt{g} g^{ij} \frac{\partial}{\partial q_j} \right) - \frac{\hbar^2}{8m} (\kappa_1 - \kappa_2)^2 \sigma = E \sigma,\]

where \( g = \det[g^{ij}] \), \( g^{ij} = g^{-1}_{ij} \)'s denote the metric tensors, and \( q_i, q_j \) the generalized coordinates. The most intriguing point is that a geometrical potential term appears through \(-\hbar^2(\kappa_1 - \kappa_2)^2/(8m)\), where \( \kappa_1 \) and \( \kappa_2 \) are the principal curvatures of the surface at \((q_1, q_2)\). This geometrical potential is uniquely determined if the configuration of the surface is given, in other words, the geometrical potential depends on the curvature of the surface. After we apply this equation to the Corbino disk, the geometrical potential diverges at the center of the disk since its functional form is proportional to \(1/r^2\), where \( r \) is a radial component in the polar...
coordinates system. Previous studies showed that this potential singularity can be safely avoided by introducing a new wave function so as to obtain a Hermitian momentum operator \(-i\hbar \partial / \partial r\) in the deformed Hilbert space [6–8]. These results have led us to the general formulation to describe the electronic states in the Corbino disk.

Corbino disk is a kind of the quantum dot whose transmission probability of an electron is tunable by changing the gate voltage as seen in a quantum point contact [12,13]. The amplitude of the charging energy is comparable with that of the centrifugal potential when the size of the system is to be of the order of micrometers. By applying the gate voltage, the transmission probability of an electron is also changed since the total charge in the Corbino disk associated with the Coulomb potential is tunable. This means that the charging energy should be taken into account to describe the quantum tunneling of the electron.

In this paper, we reformulate the Schrödinger equation by using the Riemannian geometrical method. The charging energy whose amplitude can be changed by the gate voltage is introduced in the potential term of the Schrödinger equation. In our model, the 2D Schrödinger equation can be reduced to the one dimensional one. Thus, the formula of the transmission probability of the electron confined in the Corbino disk is obtained by the Wentzel-Kramers-Brillouin (WKB) method.

2. The Schrödinger equation for an electron in a Corbino disk

Let us extend the general formula Eq. (1) to the case for an electron confined in the Corbino disk. Figure 1(a) shows a 2D electron system formed in the interface between e.g., GaAs and AlGaAs crystals in the Corbino disk [14]. Electron moves parallel to the interface of GaAs and AlGaAs layers. The Corbino disk is the concentric 2D circular disk (see Fig. 1(b)). Since the electron confined in the Corbino disk has a charge \(e\) (magnitude), a gate voltage \(V_{\text{gate}}\) can change the amount of charge \(Q = \sum_{i=1}^{N} e_i\), where \(N\) is the total number of electrons in the Corbino disk. This means that the static charging potential \(Q^2/(2C)\) induced by the presence of the electrons in the Corbino disk can be changed by applying \(V_{\text{gate}}\), where \(C\) is the static capacitance of the Corbino disk. This phenomenon could be useful for the application to the single-electron transistor by using the Corbino disk. When we apply the Corbino disk to a functional device such as the transistor, we need the expression for the transmission probability of an electron in order to investigate its performance.

In order to obtain the expression of the transmission probability, we have to solve the Schrödinger equation Eq. (1) in the Corbino geometry. In this geometry, it is convenient to
work in the polar coordinates, that is, \((q_i, q_j)\) is replaced by \((r, \theta)\). In this transformation, the solution of Eq. (1) specific to the Corbino geometry (see Fig. 1) is found by introducing \(\sigma(r, \theta) = R_{n,\ell}(r) \exp(i\ell\theta)\) in the separation of variables. The radial Schrödinger equation is then obtained in the following form:

\[
\left(-\frac{d^2}{dr^2} + V_{\text{eff}}(r)\right) R_{n,\ell}(r) = 0, \tag{2}
\]

where \(V_{\text{eff}}(r)\) is the effective confining potential given by

\[
V_{\text{eff}}(r) = \frac{1}{r^2} \left(\ell^2 - \frac{1}{4}\right) - (k_{n,\ell})^2 + \frac{2m}{R^2} \frac{Q^2}{2C} + \frac{2m}{R^2} V_{\text{conf}}(r). \tag{3}
\]

Here, \(k_{n,\ell} = \sqrt{2mE_{n,\ell}}/\hbar\) with the eigenenergy \(E_{n,\ell}\), where \(n = 1, 2, \ldots\) and \(\ell = 0, \pm 1, \pm 2, \ldots\) are the principal and angular quantum numbers, respectively. \(V_{\text{conf}}(r)\) is the potential barrier of the Corbino disk. The term \(-1/(4r^2)\) is arising from the geometrical potential given in Eq. (1) [9–11]. It should be noted that the effective confining potential \(V_{\text{eff}}(r)\) is formed by the confining potential barrier by the Corbino disk and the geometrical potential induced by the Corbino disk (see Fig. 2). Spatial profile of the effective potential is symmetric about the vertical axis at \(r = 0\) when the bias voltage \(V_{\text{bias}}\) is absent. It should be noted that the charging potential energy \(Q^2/2C\) is included in the effective confining potential in order to take account of the charging effect in the Corbino geometry.

3. The solution of the radial Schrödinger equation and transmission probability

Let us derive the formula for the transmission probability of the electron confined in the Corbino disk by applying the WKB method. Potential barrier \(V_{\text{conf}}(r)\) of the Corbino disk is set to be constant, i.e., \(V_{\text{conf}}(r) = V_0\) in the region of \(R - a/2 \leq r \leq R + a/2\), where \(R - a/2\) and \(R + a/2\) denote the radii of inner and outer disk, respectively, and \(a\) is the width of the disk. An electron cannot transmit through the Corbino disk when \(V_{\text{bias}} = 0\) while they can transmit when \(V_{\text{bias}} \neq 0\) since the effective potential becomes asymmetric by applying the bias voltage \(V_{\text{bias}}\) (see Fig. 3).

According to the WKB method, the eigenenergy of the electron \(E_{n,\ell}\) should be larger than \(V_0\) to obtain the transmission probability. The eigenfunction obtained by solving Eq. (2) is given in terms of the Bessel function \(J_n(k_{n,\ell}r)\) and the Neumann function \(N_n(k_{n,\ell}r)\) [15]:

\[
R_{n,\ell}(r) = \frac{a_{n,\ell}}{\sqrt{r}} J_n(k_{n,\ell}r) + \frac{b_{n,\ell}}{\sqrt{r}} N_n(k_{n,\ell}r), \tag{4}
\]
where the constants $a_{n,\ell}$ and $b_{n,\ell}$ are determined by the boundary conditions of the Corbino disk. The associated eigenenergy is

$$E_{n,\ell} = \frac{\hbar^2}{2m} (k_{n,\ell})^2,$$

where $k_{n,\ell}$ is numerically obtained. We finally obtain the formula of the transmission probability $T$ for an electron in the confining potential of the Corbino disk [16] as in the following form:

$$T = \exp \left[ -\frac{2}{\hbar} \int_{R-2/d}^{r_0(E_{n,\ell})} \sqrt{\frac{1}{r^2} \left( \frac{n^2}{4} - 1 \right) + 2m \frac{V_0 + Q^2/(2C) - E_{n,\ell}}{\hbar^2} \, dr} \right],$$

where

$$r_0(E_{n,\ell}) = \sqrt{\frac{\hbar^2 (4n^2 - 1)}{8m(E_{n,\ell} - V_0 - Q^2/(2C))}}.$$

It should be noted that the formula given in Eq. (6) contains the charging potential $Q^2/(2C)$. This potential is comparable to the energy of $E_{n,\ell} - V_0$ by tuning the gate voltage when the size of the system is less than micrometer order. Experimental observation of our theoretical result given in Eq. (6) is accomplished by measuring the conductance $G$ of the system. According to the Landauer formula, the conductance reads

$$G = \frac{2e^2}{h} T,$$

where $T$ is given by Eq. (6). We hope our theoretical prediction shall raise both the theoretical and experimental interests in the Corbino disk-based single-electron transistor.

4. Conclusion

We have formulated the expression of the transmission probability for an electron confined in the Corbino disk by applying the WKB method. The charging energy is introduced to the potential term in the expression of the WKB formula. This formula elucidates the electronic tunneling phenomenon of the Corbino disk and serves as a basic equation for the single-electron
transistor by using the Corbino disk. When the size of the system becomes the micrometer order, the charging potential is comparable with the geometrical potential. This means that it is important to incorporate the charging potential in the formulation of the transmission probability when one studies single-electron transistors.

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