Kinetic and chemical equilibrium of the Universe and gravitino production

Raghavan Rangarajan\textsuperscript{a}, Anjishnu Sarkar\textsuperscript{a,1}

\textsuperscript{a}Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India

Abstract

Flat directions in generic supersymmetric theories can change the thermal history of the Universe. A novel scenario was proposed earlier where the vacuum expectation value of the flat directions induces large masses for all the gauge bosons and gauginos. This delays the thermalization of the Universe after inflation and solves the gravitino problem. In this article we perform a detailed calculation of the above scenario. We include the appropriate initial state particle distribution functions, consider the conditions for the feasibility of the non-thermal scenario, and investigate phase space suppression of gravitino production in the context of heavy gauge bosons and gauginos in the final state. We find that the total gravitino abundance generated is consistent with cosmological constraints.

Keywords: physics of the early universe, supersymmetry and cosmology

1. Introduction

Large vacuum expectation values (vevs) for flat directions in supersymmetric (SUSY) theories, i.e., directions in the field space of scalars that have a flat potential, can affect the thermal history of the Universe in two ways as discussed in the literature. Firstly, they may delay the thermalization of the inflaton decay products \cite{1-3}. Secondly, the flat directions could potentially dominate over the inflaton decay products \cite{4, 5}, such that reheating is associated with the decay of flat directions as opposed to the decay of the inflaton. Below we shall consider gravitino production in the context of flat directions delaying thermalization of the decay products of the inflaton which are presumed to dominate the Universe.

\textsuperscript{1}Current address: The LNM Institute of Information Technology, Jaipur 302031, India.
Gauge symmetries are broken along flat directions and for certain flat directions all standard model (SM) gauge symmetries are broken in the early Universe. (Low energy phenomenology is not affected as the vevs of these flat directions ultimately disappear.) In refs. [1, 2] the authors have presented an interesting scenario in which flat directions give a large mass $\sim \sqrt{\alpha \varphi}$ to all gauge bosons and gauginos where $\varphi$ is the vev of the field $\phi$ that parametrizes the flat direction and $\alpha = g^2/(4\pi)$ for gauge coupling constant $g$. Since processes involving gauge bosons are important for thermalization, this results in a slow rate of thermalization after the end of inflation. Thus the Universe enters a period of pre-thermalization after inflation. Throughout this article we use the term thermalization to refer to chemical equilibration, and thermal equilibrium to refer to chemical equilibrium.

In the scenario of refs. [1, 2] the Universe reaches a state of kinetic equilibrium via $2 \rightarrow 2$ interactions after a certain time interval has elapsed following inflaton decay. Further thermal equilibrium is reached only when number violating interactions, i.e., $2 \rightarrow 3$ interactions, are also effective. Thus the rate of inflaton decay, of kinetic equilibration and of thermal equilibration follow the relation

$$\Gamma_d \gg \Gamma_{\text{kin}} > \Gamma_{\text{thr}},$$

(1)

where

$$\Gamma_d = \text{Inflaton decay rate}$$
$$\Gamma_{\text{kin}} = \text{Rate of kinetic equilibration}$$
$$\Gamma_{\text{thr}} = \text{Rate of thermal equilibration},$$

(2)

and we define $t_d, t_{\text{kin}}$ and $t_{\text{thr}}$ as the times of inflaton decay, kinetic equilibration and chemical equilibration respectively.

In refs. [1, 2] the authors argue that due to the delay in thermalization the generic problem of gravitino overproduction in SUSY models is also avoided. The lack of thermalization leads to a dilute plasma of high energy particles, and the low number density of particles effectively leads to low gravitino production. Moreover due to a low final reheat temperature after thermalization gravitino production after thermalization is also suppressed.

In this article we provide a detailed analysis of gravitino production in the non-thermal Universe scenario of refs. [1, 2]. To find the number density of gravitinos $n_{\tilde{G}}$, refs. [1, 2] use the integrated Boltzmann equation of the form

$$\dot{n}_{\tilde{G}} + 3Hn_{\tilde{G}} = \sum_{I \leq J} \langle \sigma v_{\text{rel}} \rangle n_In_J,$$

(3)

from inflaton decay till thermal equilibration for the process $I + J \rightarrow \tilde{G} + K$, where $I$ and $J$ are quarks or squarks and $K$ is a gauge boson or a gaugino.
We too obtain and use this equation, but only for $t_{\text{kin}} < t < t_{\text{thr}}$ where one can describe the particle distribution function by a thermal distribution with an effective chemical potential. For $t < t_{\text{kin}}$ we instead use a delta function distribution function for the incoming particles and then evaluate the collision integral for the integrated Boltzmann equation. As part of our analysis, we have also explicitly obtained the conditions for obtaining a non-thermal Universe after inflaton decay.

There is an important effect associated with large vevs of the flat directions as relevant for gravitino production that has not been considered earlier. Gauge bosons and gauginos are produced along with gravitinos in the relevant processes for gravitino production. As mentioned above these get large masses $m_{\tilde{g},\tilde{g}}$ due to the large vevs of the flat directions. Consequently gravitino production ceases in cases where the energy of quarks and squarks in the Universe falls below the mass of gauge bosons and gauginos due to phase space suppression.

As in ref. [2] we consider two cases, $m_0 < \Gamma_d$ and $m_0 > \Gamma_d$, where $m_0$ is the condensate mass. We find that for $m_0 < \Gamma_d$ phase space suppression shuts off gravitino production both before thermalization and even after thermalization, till the condensate decays. For $m_0 > \Gamma_d$, gravitino production is shut off after thermalization due to phase space suppression till the condensate decays.

It should be noted here that the longevity of these flat directions has been in dispute [3–12]. One of the key issues is whether or not non-perturbative effects lead to a fast decay of the condensate. In ref. [6, 7] it was pointed out that non-perturbative decay due to parametric resonance is suppressed for a complex scalar condensate if the real and imaginary parts of the scalar field oscillate out of phase. In that case the field does not pass through the minimum of the potential at $\varphi = 0$ and so the required non-adiabatic condition $|\dot{\omega}_k| \sim \omega_k^2$ is not satisfied, where $\omega_k$ is the the frequency of the $k$-th mode of the $\chi$ field produced by the decay of the condensate. (Backreaction effects can also be important in suppressing non-perturbative decay.) The condensate then decays only perturbatively and hence lasts for a long time. Ref. [4] however argued that in the case of multiple flat directions there is a mixing of excitations along different directions which makes non-perturbative effects important. Further investigations in ref. [8] implied that even with multiple flat directions resonant decay is most likely to be suppressed (in cases of physical interest) because the multiple flat directions effectively reduce to one flat direction or a collection of independent single flat directions. Raising certain issues related to gauge fixing, refs. [9, 10] worked in the unitary gauge to eliminate the Nambu-Goldstone boson that appears in the context of SUSY flat directions charged under the gauge group of the MSSM and
studied the circumstances under which the presence of multiple flat directions can lead to non-perturbative decay. Ref. [11] invoked charge conservation to argue that non-perturbative decay is suppressed. At the same time ref. [5] analysed the issue with proper gauge fixing and concluded that with one flat direction there is no resonant particle production, in agreement with ref. [11], while disagreeing with the conclusion of ref. [11] in the context of multiple flat directions. However refs. [3, 11] argue that even if non-perturbative decay occurs for multiple flat directions it leads to redistribution of energy of the condensate amongst the fields in the D flat superspace and hence to practically the same cosmological consequences (including large gauge boson and gaugino masses and delayed thermalization) as in the scenario with only perturbative decay.

Our analysis below presumes that perturbative decay is the relevant decay mechanism of the flat direction condensate and that the impact of the condensate lasts long enough to have cosmological consequences. This may be because either there is only a single flat direction that is excited, or because the multiple flat directions are excited such that parametric resonance is still suppressed, or because of the arguments of refs. [3, 11], or because of backreaction effects.

There are other effects that affect the evolution and decay of the flat direction condensate. Thermal contributions to the condensate potential can change the mass of the condensate leading to an earlier oscillation time for the condensate [13–15]. Below we take the condensate mass to be of order of the SUSY breaking scale $m_0 = 100 \text{ GeV}$. We present our results in terms of $m_0$ which can be replaced by larger values to include these effects. (Ref. [15] obtains masses of $\sim 10^{10} \text{ GeV}, 10^6 \text{ GeV}$ and $10^5 \text{ GeV}$ for $n = 1, 2, 3$ where terms that lift the flat directions have the form $\phi^{2n+4}/M^{*2n}$ where $M^* = 10^{18} \text{ GeV}$.) Another effect is that the condensate can decay due to scattering by inflaton decay products [13, 14, 16]. Both these effects have been discussed in Appendix A.4 of ref. [2] where it has been argued that these effects will not be relevant during the non-thermal phase.

The flat direction condensate can also decay to Q-balls due to inhomogeneities in the condensate field. This has been studied in the context of both gauge mediated and gravity mediated SUSY breaking [17–25]. In addition it has been seen that condensate lumps with different properties than Q-balls called Q-axitons may also be formed [26]. Note however that the time scale for the formation of Q-balls and Q-axitons can be large. For gauge mediated SUSY breaking scenarios the time scale for Q-ball formation $\tau \sim 5 \times 10^5 \text{ m}^{-1}$ [19], where $m$ is a characteristic mass scale for the flat direction condensate, and for gravity mediated SUSY breaking scenarios $\tau \sim 100 - 1000 \text{ m}^{-1}$ [18] or $5.5 \times 10^3 \text{ m}^{-1}$ [20]. Ref. [17] also obtains a Q-ball formation time which is
larger than the flat direction oscillation time by a factor of $1.5 \times 10^4$. The time scale for Q-axiton formation is $1600 \, m^{-1}$ [20]. Q-ball formation may be avoided for certain flat directions, including those with a large stop admixture, which allow for $K > 0$, where $K$ is a parameter in the potential in gauge mediated SUSY breaking scenarios [21]. Our analysis does not include Q-ball or Q-axiton formation.

We now present a summary of this article. In section 2 we provide the integrated Boltzmann equation and phase space distribution functions valid for a non-thermal Universe. Section 3 provides the scattering processes relevant for gravitino production in the scenario with heavy gauge bosons and gauginos. In section 4 we obtain the relevant time scales, namely, $t_{\text{kin}}$, $t_{\text{thr}}$, $t_f$ and $t_G$ where $t_f$ is the time of decay of the flat direction condensate and $t_G$ is the time in the radiation dominated era after thermalization when gravitino production could commence. In this section we also discuss the feasibility of gravitino production in the context of phase space suppression in different epochs. The cross section for gravitino production in the center-of-mass frame is obtained in section 5. Then in section 6 we obtain the gravitino abundance generated in different time intervals in the Universe. The parameters required to obtain the gravitino abundance are given in section 7. Our results are given in section 8 and our conclusions in section 9. The Appendices contain derivations of certain expressions used in the text.

2. The Boltzmann equation

To obtain the number density of a species $X_3$ participating in reactions $X_1X_2 \rightarrow X_3X_4$ the integrated Boltzmann equation is

$$\dot{n}_3 + 3Hn_3 = -\int d\pi_1 \, d\pi_2 \, d\pi_3 \, d\pi_4 \, (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \times [f_3f_4 - f_1f_2]|\mathcal{M}|^2,$$

(4)

where $f_i$ are phase space distribution functions and

$$d\pi_i \equiv \frac{g_i}{(2\pi)^3} \frac{d^3p_i}{2E_i}.$$

(5)

$g_i$ is the number of internal degrees of freedom of species $i$. The $|\mathcal{M}|^2$ includes an average over initial and final internal degrees of freedom [27].

Typically $X_1$ and $X_2$ are assumed to be in full thermal equilibrium with zero chemical potential and thus the phase space distribution function has the form

$$f_i = \exp(-E_i/T), \quad i = 1, 2.$$  

(6)
This is because $X_1$ and $X_2$ usually have other stronger interactions than their interaction with $X_3, X_4$. This assumption is not valid in the scenario considered in refs. [1,2] as the Universe is not in thermal equilibrium before $t_{thr}$. In fact $f_{1,2}$ have the following form

\begin{align}
 f_i &= C_i \delta \left( E_i - \frac{m_\phi a_d}{2} \right) \quad \text{for } t_d < t < t_{kin} \\
 &= \exp \left( -\frac{E - \xi_i}{T} \right) \quad \text{for } t_{kin} < t < t_{thr}
\end{align}

where,

\begin{equation}
 C_i = \frac{\rho_\phi(t_d)}{m_\phi^3} \left( \frac{a_d}{a} \right) \frac{(2\pi)^3}{\pi g_i}, \quad i = 1, 2
\end{equation}

as derived in Appendix A. In the above expressions $m_\phi$ is the mass of the inflaton, $\rho_\phi(t_d)$ is the energy density of inflaton $\phi$ at time $t_d$, $a$ is the scale factor and $\xi_i$ is the effective chemical potential. Eq. (7a) reflects the scenario that before $t_{kin}$ the relativistic energy of fermions and sfermions (whose interactions produce gravitinos) is their energy at the time of their production from (2 body) inflaton decay at $t_d$ scaled by $1/a$. Between $t_{kin}$ and $t_{thr}$ one does not have full thermalization as represented by $\xi_i$ in eq. (7b). (Also see footnote 24 of ref. [2].)

When the number density of $X_3$ is small, as in our case where $X_3$ represents the gravitino, we can ignore the product $f_3 f_4$ in eq. (4). Thus, eq. (4) can be written as

\begin{equation}
 \dot{n}_3 + 3Hn_3 = \int d\Pi_1 d\Pi_2 f_1 f_2 W_{12}(s) \equiv A,
\end{equation}

where we have defined a variable $A$ to denote the collision integral on the right-hand-side of the above equation. Here, $W_{12}(s)$ is dimensionless and Lorentz invariant, and dependent only on the Mandelstam variable $s$. $W_{12}$ is given by

\begin{equation}
 W_{12}(s) = \int d\Pi_3 d\Pi_4 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |\mathcal{M}|^2,
\end{equation}

which can also be written as $4E_1 E_2 \sigma v_{rel}$ where $v_{rel}$ is $[(p_1, p_2)^2 - m_1^2 m_2^2]^{1/2}/(E_1 E_2)$.

2.1. The collision integral

For the completeness of the paper we provide below some details on how to obtain the collision integral. It is easier to calculate $W_{12}$ of eq. (9) in the centre-of-mass (CM) frame and it can then be expressed as

\begin{equation}
 W_{12}(s) = 4p_{12} \sqrt{s} \sigma_{CM}(s),
\end{equation}
where
\[ p_{12} = \frac{[s - (m_1 + m_2)^2]^{1/2} [s - (m_1 - m_2)^2]^{1/2}}{2\sqrt{s}} \] (12)
is the magnitude of the momentum of particle \(X_1\) (or \(X_2\)) in the center-of-mass frame of the particle pair \((X_1, X_2)\). The cross section in the CM frame \(\sigma_{CM}\) is
\[ \sigma_{CM}(s) = \int d\Omega \left( \frac{d\sigma}{d\Omega} \right)_{CM}, \] (13)
where
\[ \frac{d\sigma}{d\Omega}_{CM} = \frac{g_3 g_4}{64\pi^2 s} \left[ \frac{(s - (m_3 + m_4)^2)(s - (m_3 - m_4)^2)}{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)} \right]^{1/2} |\mathcal{M}|^2 \] (14)
is the differential scattering cross section in the CM frame \([29]\). Note that the averaging over the final state degrees of freedom in \(|\mathcal{M}|^2\) cancels the prefactor \(g_3 g_4\) in the above equation. \(|\mathcal{M}|^2(s, t, u)\) in eq. (14) can expressed in terms of \(s\) and \(\theta_{13}\), the angle between \(X_1\) and \(X_3\) in the CM frame, using \(u = -s-t+\sum_i m_i^2\) and \(t = m_1^2+m_3^2-2E_1E_3+2\sqrt{E_1^2 - m_1^2} \sqrt{E_3^2 - m_3^2} \cos \theta_{13}\), with \(E_1 = \sqrt{\vec{p}_{12}^2 + m_1^2}\) and \(E_3 = (s + m_3^2 - m_1^2)/(2\sqrt{s})\) \([29]\).

To perform the integration over \(d\Pi_1 d\Pi_2\) in (9) we note that
\[ \int d\Pi_1 d\Pi_2 \equiv \int \left( \frac{g_1}{(2\pi)^3} \frac{d^3p_1}{2E_1} \right) \left( \frac{g_2}{(2\pi)^3} \frac{d^3p_2}{2E_2} \right), \] (15)
and that the volume element \(d^3p_1 d^3p_2\) is given by \([28]\),
\[ d^3p_1 d^3p_2 = 4\pi |\vec{p}_1| \vec{E}_1 \, d\vec{E}_1 \, 4\pi |\vec{p}_2| \vec{E}_2 \, d\vec{E}_2 \, \frac{1}{2} d(\cos \theta_{21}), \] (16)
where \(\theta_{21}\) is the angle between \(X_1\) and \(X_2\). (Note that we now work in the rest frame of the Universe.) In order to simplify the integration we replace the variable \(\cos \theta_{21}\) by \(s\) via the relation \(s = m_1^2 + m_2^2 + 2E_1E_2 - 2|\vec{p}_1||\vec{p}_2| \cos \theta_{21}\). The lower limit of the integral over \(s\) should not be less than either \((m_1+m_2)^2\) or \((m_3+m_4)^2\) as dictated by the phase space factor in eq. (14). Therefore
\[ \int d\Pi_1 d\Pi_2 = \frac{g_1 g_2}{4(2\pi)^4} \int_{m_1}^{\infty} dE_1 \int_{m_2}^{\infty} dE_2 \int_{s_{\text{min}}}^{s_{2}} ds \, \int_{s_{1}}^{\infty} d\vec{E}_1 \, \int_{s_{1}}^{\infty} d\vec{E}_2 \, \int_{s_{\text{min}}}^{s_{2}} ds \, f_1 f_2 W_{12}, \] (17)
where \(s_{\text{min}} = \max \left[ s_1, (m_1+m_2)^2, (m_3+m_4)^2 \right]\) and \(s_{1} = m_1^2 + m_2^2 + 2E_1E_2 \mp 2|\vec{p}_1||\vec{p}_2|\). For \(m_1, m_2, m_3 \approx 0\), \(s_{\text{min}} \approx m_1^2\). Using eq. (17) in eq. (9) we get
\[ n_3 + 3H n_3 = \frac{g_1 g_2}{4(2\pi)^4} \int_{m_1}^{\infty} dE_1 \int_{m_2}^{\infty} dE_2 \int_{s_{\text{min}}}^{s_{2}} ds \, f_1 f_2 W_{12}. \] (18)
Thus, after calculating $|\mathcal{M}|^2$ for each process mentioned in the next section, we follow the procedure given above to find the parameter $W_{12}(s)$ and then with the $f_i$ as given in eq. (7) we use the above equation to calculate the number density of gravitinos produced between $t_d$ and $t_{kin}$, and $t_{kin}$ and $t_{thr}$.

3. Relevant processes for gravitino production

The production of gravitinos after inflation has been discussed in [30–49]. The processes involving strong interactions denoted by A to J are [34]

- A: $g^A + g^B \rightarrow \tilde{g}^C + \tilde{G}$
- B: $g^A + \tilde{g}^B \rightarrow g^C + \tilde{G}$
- C: $\tilde{q}_i + g^A \rightarrow q_j + \tilde{G}$, $\tilde{q}_i + g^A \rightarrow \tilde{q}_j + \tilde{G}$
- D: $q_i + g^A \rightarrow \tilde{q}_j + \tilde{G}$, $\bar{q}_i + g^A \rightarrow \tilde{q}_j + \tilde{G}$
- E: $q_i + \tilde{g}_j \rightarrow g^A + \tilde{G}$, $q_i + \tilde{g}_j \rightarrow g^A + \tilde{G}$
- F: $\tilde{g}^A + \tilde{g}^B \rightarrow \tilde{g}^C + \tilde{G}$
- G: $q_i + \tilde{g}^A \rightarrow q_j + \tilde{G}$, $\bar{q}_i + \tilde{g}^A \rightarrow \bar{q}_j + \tilde{G}$
- H: $\tilde{q}_i + \tilde{g}^A \rightarrow \tilde{q}_j + \tilde{G}$, $\bar{q}_i + \tilde{g}^A \rightarrow \tilde{q}_j + \tilde{G}$
- I: $q_i + \tilde{q}_j \rightarrow \tilde{g}^A + \tilde{G}$
- J: $\tilde{q}_i + \tilde{q}_j \rightarrow \tilde{g}^A + \tilde{G}$

The above can be generalized for all gauge groups. We shall consider $SU(3)_c$ only. In the non-thermal scenario we only have to consider processes involving the scattering of fermions and/or sfermions [2]. The reason is as follows: the gauge bosons and gauginos have large masses induced by the flat direction vev. Due to their large masses they decay quickly to lighter fermions and sfermions and therefore do not participate in the production of gravitinos. The processes not involving gauge bosons and gauginos as incoming particles are the E, I and J processes. These processes are shown in figures [1, 2] and [3]. (The second E process is not shown and its contribution will be the same as that of the first process.)

In ref. [41] it has been argued that the gravitino production cross section should be proportional to

$$\frac{g_s^2}{M^2} \left( 1 + \frac{m_3^2}{3m_{\tilde{G}}^2} \right),$$

(19)
Figure 1: Tree level diagrams for the $E$ process: $q_i + \bar{q}_j \to g^A + \tilde{G}$ scattering. The diagrams for $\bar{q}_i + q_j \to g^A + \tilde{G}$ are similar.

Figure 2: Tree level diagrams for the $I$ process: $q_i + q_j \to \bar{g}^A + \tilde{G}$ scattering.
where $M = M_{Pl}/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass (and $m_{\tilde{g}}$ in the expression in ref. [41] has been replaced by the supersymmetry breaking scale $m_0$). The factor 1 is related to the contribution of the $\pm 3/2$ helicity states of the gravitino while the term proportional to $m_0^2/(m_{\tilde{g}}^2)$ represents the contribution of the helicity $\pm 1/2$ states. To obtain the contribution of the $\pm 1/2$ helicity states, for $\sqrt{s} \gg m_{\tilde{g}}$ we may express the gravitino field as

$$\psi_\mu \rightarrow i \sqrt{\frac{2}{3}} \frac{1}{m_{\tilde{g}}} \partial_\mu \psi,$$

where $\psi$ represents a spin 1/2 fermionic field. Cross sections for spin 1/2 gravitino production can then be obtained from the corresponding effective lagrangian for $\psi$ [50–52]. The total cross section is then obtained in ref. [41] using eq. (19).

4. Obtaining $t_{\text{kin}}, t_{\text{thr}}, t_{G}$ and $t_f$ and checking the feasibility of gravitino production

In this section we obtain $t_{\text{kin}}$ and $t_{\text{thr}}$ for the cases $m_0 < \Gamma_d$ and $m_0 > \Gamma_d$ considered in ref. [2]. Since gravitino production is suppressed by lack of thermalization we impose the constraint that the $\Gamma_{\text{kin,thr}} < H$ at $t_d$ to study this effect. We also impose the constraint that the energy of the incoming quarks and squarks should be greater than $3m_{\tilde{g},\tilde{\tilde{g}}}$ to avoid phase space suppression of the gravitino production cross section by the $s - m_{\tilde{g},\tilde{\tilde{g}}}^2$ factor in eq. (14). These constraints give lower and upper bounds on $\varphi_0$ respectively, where $\varphi_0$ is the flat direction vev at $t_0 = m_0^{-1}$ when the flat direction starts oscillating. In the case of $m_0 < \Gamma_d$, these bounds are in fact incompatible indicating that lack of thermalization effectively shuts off gravitino production through phase space suppression. Surprisingly, we find
that even after thermalization, for both cases \( m_0 \lesssim \Gamma_d \), gravitino production is phase space suppressed till the flat direction condensate decays because of the large gluon and gluino mass.

We take the inflaton mass \( m_\phi \) to be \( 10^{13} \text{ GeV} \) and \( \alpha = \frac{g_2^2}{4\pi} \) to be \( 5 \times 10^{-2} \) as applicable for the relevant energy scales for \( t \leq t_{\text{thr}} \) and \( \alpha = 0.1 \) for \( t \gg t_{\text{thr}} \) as relevant below. We presume the inflaton decays at \( t_d = \Gamma_d^{-1} \) via \( \phi \to X_1 + X_2 \) so that the number density of the \( X_1 \) or \( X_2 \) particles equals that of the inflaton. The gluon mass is given by

\[
m_g^2 = \alpha \varphi^2 = \begin{cases} 
\alpha \varphi_0^2 & \text{for } t \leq t_0 \\
\alpha \varphi_0^2 \left( \frac{a_0}{a} \right)^3 & \text{for } t \geq t_0 
\end{cases}
\]  

(21)

4.1. \( m_0 < \Gamma_d \)

For \( m_0 < \Gamma_d \), the flat direction condensate starts oscillating after the inflaton decays. For this case we take \( m_0 = 100 \text{ GeV} \) and \( \Gamma_d = 10^4 \text{ GeV} \) as in section 3.1 of ref. [2]. One must separately consider the time intervals \( t_d - t_0, t_0 - t_{\text{kin}}, t_{\text{kin}} - t_{\text{thr}} \) and \( t > t_{\text{thr}} \).

The gluon mass is given by \( \sqrt{\alpha \varphi} \). The energy of the incoming particles is \( m_\phi(a_d/a) \) for \( t_d < t < t_{\text{kin}} \). Since kinetic equilibration does not change the number of particles while redistributing their energies, the average energy of the incoming particles for \( t_{\text{kin}} < t < t_{\text{thr}} \) is also given by \( m_\phi(a_d/a) \). Gravitino production for \( t > t_{\text{thr}} \) is considered in section 6.1.

4.1.1. Non-equilibrium condition

At \( t = t_d \) we impose

\[ H(t_d) = \Gamma_d > \Gamma_{\text{kin}}(t_d) = n(\sigma v)_{\text{kin}} = \frac{\rho_\phi(t_d) \frac{4\pi\alpha^2}{m_\phi}}{m_g^2} \]  

(22)

where we have used \( \rho_\phi(t_d) = (3/(8\pi))M_{\text{Pl}}^2 \Gamma_d^2 \) and \( \langle \sigma v \rangle_{\text{kin}} \sim 4\pi\alpha^2/m_g^2 \). Then we get

\[
\Gamma_d > \frac{3}{2} \frac{M_{\text{Pl}}^2 \Gamma_d^2}{m_\phi} \frac{\alpha}{\varphi_0^2} \\
\text{or,} \quad \varphi_0 > \sqrt{\frac{3}{2} \left( \frac{\Gamma_d}{m_\phi} \right)^{1/2} M_{\text{Pl}} \sqrt{\alpha}} \\
\text{or,} \quad \varphi_0 > 10^{14} \text{ GeV} ,
\]  

(23)

where we have used \( m_g^2 = \alpha \varphi_0^2 \). \( H(t_d) > \Gamma_{\text{thr}}(t_d) \) gives \( \varphi_0 > 2 \times 10^{13} \text{ GeV} \).
Note that for \( t_d < t < t_0 \), \( \Gamma_{\text{kin}} \) is given by
\[
\Gamma_{\text{kin}} = n \langle \sigma v \rangle_{\text{kin}} \\
= \frac{\rho_\phi(t_d)}{m_\phi} \left( \frac{a_d}{a_0} \right)^3 4\pi \alpha^2 \frac{m_g^2}{m_\phi^2} \\
\sim \frac{1}{a^3} \sim \frac{1}{t^{3/2}},
\]
where we have used \( n = n(t_d)(a_d/a)^3 \). Since \( H \sim 1/t \), we see that \( \Gamma_{\text{kin}} \) is falling faster than \( H \). So, if \( H > \Gamma_{\text{kin}} \) holds at \( t_d \), then it also holds for \( t_d < t < t_0 \).

### 4.1.2. Equilibration times

For \( t > t_0 \), \( \Gamma_{\text{kin}} \) is given by eq. (24a) with \( m_g^2 = \alpha \varphi_0^2 (a_0/a)^3 \). Therefore
\[
\Gamma_{\text{kin}} = \frac{3}{2} \frac{M_H^2 \Gamma_d}{m_\phi} \left( \frac{a_d}{a_0} \right)^3 \alpha \frac{\varphi_0^2}{\varphi_d^2} \\
= \frac{3}{2} \frac{M_H^2}{m_\phi \varphi_d^2} \Gamma_d^{3/2} \frac{m_0^{3/2}}{\alpha} = \text{constant}.
\]

Since \( H \) is decreasing with time, after some time \( t_{\text{kin}} > t_0 \), \( H \) becomes less than \( \Gamma_{\text{kin}} \), and later it becomes less than \( \Gamma_{\text{thr}} \). Setting \( t_{\text{kin}} = \Gamma_{\text{kin}}^{-1} \), we get
\[
t_{\text{kin}} = \frac{2}{3} \frac{m_\phi}{M_H^2 \Gamma_d^{1/2}} \left( \frac{1}{m_0} \right)^{3/2} \frac{\varphi_0^2}{\alpha} \\
or, \quad t_{\text{kin}} = 9 \times 10^{-30} \text{GeV}^{-1} \left( \frac{\varphi_0^2}{\text{GeV}^2} \right).
\]

\( \Gamma_{\text{thr}}(t) = n \langle \sigma v \rangle_{\text{thr}} = \alpha \Gamma_{\text{kin}} \) as \( \langle \sigma v \rangle_{\text{thr}} \sim 4\pi \alpha^3/m_g^2 \) \[1, 2\]. For \( t > t_0 \), \( \Gamma_{\text{thr}}(t) \sim (1/a^3) \alpha^3/m_0^2 \sim \text{constant} \), as for \( \Gamma_{\text{kin}} \). Then \( t_{\text{thr}} = \alpha^{-1} t_{\text{kin}} \).

As we shall see below, there is also an upper bound on \( \varphi_0 \) which is highly restrictive. Taking \( \varphi_0 = 10^{14} \text{GeV} \), we get,
\[
t_{\text{kin}} = 9 \times 10^{-2} \text{GeV}^{-1}.
\]

Note that \( t_d = 10^{-4} \text{GeV}^{-1} \) and \( t_0 = 10^{-2} \text{GeV}^{-1} \). \( t_{\text{thr}} = 2 \text{GeV}^{-1} \). For larger \( \varphi_0 \), both \( t_{\text{kin}} \) and \( t_{\text{thr}} \) will be larger.

### 4.1.3. Avoiding phase space suppression

Gravitinos are produced along with gluons or gluinos in the processes discussed in section 3. To avoid phase space suppression by the \( s - m_g^2 \) factor in the differential cross section we impose
\[
3 m_{g,\tilde{g}} < E_1 + E_2.
\]
We apply this condition for gravitino production before and after thermalization (taking $m_{\tilde{g}} \approx m_g$ in this analysis).

**Before thermalization:** We first consider gravitino production in the time interval $t_d < t < t_0$. Eq. (28) implies

$$3\sqrt{\alpha} \varphi_0 < m_\phi \left(\frac{a_d}{a}\right),$$

where $E_1$ and $E_2$ are the incoming energies in the CM frame. We see that the RHS falls like $1/a$, whereas the LHS is constant. We wish to analyse whether delayed thermalization suppresses gravitino production. Therefore to consider maximum gravitino production, we impose the above inequality at $t_0$. If the above inequality holds at $t_0$, then it will hold at earlier times till $t_d$ too. Using the above inequality we find an upper bound on $\varphi_0$.

$$\varphi_0 < \frac{m_\phi}{3\sqrt{\alpha}} \left(\frac{t_d}{t_0}\right)^{1/2}$$

or,

$$\varphi_0 < \frac{m_\phi}{3\sqrt{\alpha}} \left(\frac{m_0}{\Gamma_d}\right)^{1/2}$$

or,

$$\varphi_0 < 1.5 \times 10^{12} \text{GeV}.$$  (30)

Thus we need $\varphi_0 < 1.5 \times 10^{12} \text{GeV}$ to get gravitino production from $t_d$ till time $t_0$. For the time interval $t_0 < t < t_{thr}$, $m_{g,\tilde{g}} \sim \varphi \sim 1/a^{3/2}$ and the condition in eq. (28) gives

$$3\sqrt{\alpha} \varphi_0 \left(\frac{a_0}{a}\right)^{3/2} < m_\phi \left(\frac{a_d}{a}\right).$$

We see that the RHS falls like $1/a$ and the LHS falls like $1/a^{3/2}$. So if the above inequality holds at $t_0$, which implies eq. (30) then it will hold for all time $t$ till $t_{thr}$.

We see that for the time interval $t_d < t < t_{thr}$ the upper bound on $\varphi_0$ (eq. (30)) is smaller than the lower bound on $\varphi_0$ (eq. (23)). Thus, gravitino production for the entire time interval $t_d < t < t_{thr}$ is not possible for the $m_0 < \Gamma_d$ case.

**After thermalization:** At $t = t_{thr}$, just after full equilibration, the average energy of particles in the Universe falls which further inhibits gravitino production. Gravitino production can start at $t_G$ when $3m_{g,\tilde{g}}$ becomes less than the incoming particle energy ($E_1 + E_2$), i.e.,

$$3m_{g,\tilde{g}} < E_1 + E_2$$

or,

$$3\sqrt{\alpha} \varphi_0 \left(\frac{a_0}{a}\right)^{3/2} < 2 \times \frac{3}{2} T_R \left(\frac{a_{thr}}{a}\right),$$

(32)
where the subscript $R$ denotes reheating or full equilibration at $t_{thr}$. This implies

$$\sqrt{\alpha \varphi_0} \left( \frac{a_0}{a_{thr}} \right)^{3/2} \left( \frac{a_{thr}}{a} \right)^{3/2} < T_R \left( \frac{a_{thr}}{a} \right)^{1/2}$$

or,

$$\sqrt{\alpha \varphi_0} \left( \frac{t_0}{t_{thr}} \right)^{3/4} < T_R \left( \frac{a}{a_{thr}} \right)^{1/2}$$

or,

$$\frac{\alpha \varphi_0^2}{T_R^2} (m_0 t_{thr})^{-3/2} < \left( \frac{a}{a_{thr}} \right). \quad (33)$$

Then,

$$t_G = \left[ \frac{\alpha \varphi_0^2}{T_R^2} \left( \frac{1}{m_0} \right)^{3/2} \left( \frac{1}{t_{thr}} \right) \right]^2$$

$$= 81 \left( \frac{g_*}{\mathcal{A}} \right) \frac{\alpha^2 \varphi_0^4 \Gamma_d^2}{m_\phi^4 m_0^3}. \quad (34)$$

where we have used section 4.1.2 for $t_{thr}$ and the expression for $T_R$ is derived later in section 7.5. The parameter $\mathcal{A}$ that enters via $T_R$ is obtained to be $4 \times 10^{-5}$ in section 7.3.

The perturbative flat direction decay rate is $\sim m_0^3/\varphi^2$ [4, 53] and the flat direction decays when the decay rate is of order $H$ at a time $t_f$. Then

$$t_f = \frac{\varphi_f^2}{m_0^3} \quad (35)$$

$$= \frac{\varphi_0^2}{m_0^3} \left( \frac{a_0}{a_f} \right)^3$$

$$= \frac{\varphi_0^2}{m_0^3} \left( \frac{t_0}{t_f} \right)^{3/2}$$

or,

$$t_f = \frac{\varphi_0^{4/5}}{m_0^{9/5}}. \quad (36)$$

We see that $t_G < t_f$ only for $\varphi_0 < 3 \times 10^{12}$ GeV, which conflicts with the lower bound on $\varphi_0$ in eq. (23). Thus the non-equilibrium condition implies that the condensate decays before gravitino production can commence in the radiation dominated era after thermalization. Subsequent to the decay of the condensate there will be gravitino production in the thermal Universe as discussed later in section 6.1.
4.1.4. Relaxing the bound from phase space suppression

If we relax the bound in eq. (30), we may get gravitino production for some part of the time interval \( t_d < t < t_{thr} \). We first consider \( t_d < t < t_0 \). Using \( a \sim t^{1/2} \) in eq. (29) and \( t_d = \Gamma_d^{-1} \), we find gravitino production is not phase space suppressed for

\[
\begin{align*}
t < \frac{m_\phi^2}{\Gamma_d} \frac{1}{9 \alpha \varphi_0^2} \\
or, \quad t < 2 \times 10^{-6} \text{ GeV}^{-1},
\end{align*}
\]

for \( \varphi_0 = 10^{14} \text{ GeV} \). Since this corresponds to times less than \( t_d = 10^{-4} \text{ GeV}^{-1} \), it implies that one cannot get gravitino production anywhere in this time interval.

We now consider \( t_0 < t < t_{thr} \). Gravitino production can start at some time after \( t_0 \), when the inequality in eq. (31) is satisfied. This is equivalent to

\[
3 \sqrt{\alpha} \varphi_0 \left( \frac{a_0}{a} \right)^{1/2} < m_\phi \left( \frac{a_d}{a_0} \right)
\]

or,

\[
81 \alpha^2 \frac{\varphi_0^4}{m_0 m_\phi^4} \frac{\Gamma_d}{m_0} < t.
\]

For \( \varphi_0 = 10^{14} \text{ GeV} \) this implies

\[
t > 2 \times 10^5 \text{ GeV}^{-1}.
\]

Since this is larger than \( t_{thr} = 2 \text{ GeV}^{-1} \), once again there is no gravitino production in this time interval.

Therefore the conditions required for lack of kinetic and chemical equilibration leads to a large outgoing gluon and gluino mass and thus to phase space suppression that shuts off gravitino production for the entire time interval from \( t_d \) to \( t_{thr} \) in the \( m_0 < \Gamma_d \) case. Increasing the value of \( \varphi_0 \) will not affect this conclusion as this will only lower or raise the limits in eqs. (37) and (39) respectively. Similarly, gravitino production is phase space suppressed in the radiation dominated era before the condensate decays at \( t_f \) and increasing the value of \( \varphi_0 \) will not alter this conclusion. (Decreasing the value of \( \varphi_0 \) to avoid phase space suppression will lead to the standard scenario of reheating and gravitino production in a thermal Universe.)

Thus, for the parameters chosen above for the scenario under consideration, there is no gravitino production for the case \( m_0 < \Gamma_d \) before the condensate decays.
4.2. \( m_0 > \Gamma_d \)

We now consider the case \( m_0 > \Gamma_d \), where the flat direction condensate starts oscillating before the inflaton decays. \( m_0 \sim 100 \text{ GeV} \) and we take \( \Gamma_d = 10 \text{ GeV} \) as in section 3.1 of ref. \[2\].

4.2.1. Non-equilibrium condition and relevant time scales

The absence of kinetic equilibration of the inflaton decay products requires \( \Gamma_{\text{kin}} < H \). \( \Gamma_{\text{kin}} \) is given by eq. (25a). We require \( \Gamma_{\text{kin}} \) to be much less than \( H \) at \( t_d \) to obtain a significant phase of non-equilibrium since \( H \) decreases with time while \( \Gamma_{\text{kin}} \) remains constant. Then applying the non-equilibrium condition at \( t_d \), and setting \( \varphi_d^2 = \varphi_0^2 (a_0^3/\alpha_0^3) \) and \( H(t_d) = \Gamma_d \), we get

\[
\Gamma_d \gg \Gamma_{\text{kin}}(t_d)
\]

or,

\[
\varphi_0^2 \gg \frac{3}{2m_\phi} M_{\text{Pl}}^2 \Gamma_d \alpha \left( \frac{a_d}{a_0} \right)^3
\]

or,

\[
\varphi_0^2 \gg \frac{3}{2m_\phi} M_{\text{Pl}}^2 \Gamma_d \alpha \left( \frac{t_d}{t_0} \right)^2
\]

or,

\[
\varphi_0^2 \gg \frac{3}{2m_\phi} M_{\text{Pl}}^2 \Gamma_d \alpha \left( \frac{m_0}{\Gamma_d} \right)^2
\]

or,

\[
\varphi_0 > 3 \times 10^{13} \text{ GeV} ,
\]

where we have used \( a \sim t^{2/3} \) between \( t_0 \) and \( t_d \) as relevant for the inflaton oscillating in a quadratic potential before decay. Chemical non-equilibration at \( t_d \) requires \( \varphi_0 > 3 \times 10^{13} \text{ GeV} \sqrt{\alpha} \simeq 7 \times 10^{12} \text{ GeV} \).

Thus, for \( \varphi_0 < 7 \times 10^{12} \text{ GeV} \) the Universe is in kinetic and chemical equilibrium at \( t_d \). For \( 7 \times 10^{12} \text{ GeV} < \varphi_0 < 3 \times 10^{13} \text{ GeV} \) the Universe is in kinetic but not in chemical equilibrium at \( t_d \). Finally, for \( \varphi_0 > 3 \times 10^{13} \text{ GeV} \) the Universe is neither in kinetic nor chemical equilibrium at \( t_d \).

To obtain \( t_{\text{kin}} \) we use eq. (25a) and \( a \sim t^{2/3} \) for \( t_0 < t < t_d \). Then

\[
t_{\text{kin}}^{-1} = \Gamma_{\text{kin}} = \frac{3 m_\phi}{2} \frac{a_0^2}{\Gamma_d^2 \varphi_0^2} \left( \frac{m_0}{\Gamma_d} \right)^2
\]

\[
= \frac{3 \alpha}{2 \alpha m_\phi} \frac{M_{\text{Pl}}^2 \Gamma_d^2}{\Gamma_d^2 \varphi_0^2} \left( \frac{m_0}{\Gamma_d} \right)^2
\]

\[
= \frac{3 \alpha}{2 \alpha m_\phi} \frac{M_{\text{Pl}}^2 m_0^2}{\Gamma_d^2 \varphi_0^2} .
\]

As before, \( \Gamma_{\text{thr}} = \alpha \Gamma_{\text{kin}} \) and \( t_{\text{thr}} = t_{\text{kin}}^{-1} = \alpha^{-1} t_{\text{kin}} \).
4.2.2. Avoiding phase space suppression

For gravitino production not to be phase space suppressed by the $s - m_{\tilde{g}, \tilde{\tilde{g}}}^2$ factor for $t_d < t < t_{thr}$ we again impose the condition of eq. (28) and get eq. (31). To consider maximum gravitino production we assume that this condition is valid from $t_d$. Using eq. (31) at $t_d$ gives

$$3\sqrt{\alpha\varphi_d} < m_{\phi},$$

(42)

where $\varphi_d = \varphi_0(a_0/a_d)^{3/2} = \varphi_0(t_0/t_d) = \varphi_0(\Gamma_d/m_0)$. Therefore we get

$$\varphi_0 < 1.5 \times 10^{14} \text{ GeV}.$$ (43)

Since the LHS of eq. (51) falls faster than the RHS, this upper bound on $\varphi_0$ also ensures the viability of gravitino production at times after $t_d$.

We shall take $\varphi_0 = 10^{14} \text{ GeV}$ hereafter so that gravitino production commences at $t_d$ and there is neither kinetic nor chemical equilibration at $t_d$. For this value of $\varphi_0$, $t_{kin} = 0.9 \text{ GeV}^{-1}$ and $t_{thr} = 20 \text{ GeV}^{-1}$.

For gravitino production after $t_{thr}$, eq. (32) implies

$$3\sqrt{\alpha\varphi_0} \left(\frac{a_0}{a_d}\right)^{3/2} \left(\frac{a_d}{a_{thr}}\right)^{3/2} \left(\frac{a_{thr}}{a}\right)^{3/2} < 2 \times \frac{3}{2} T_R \left(\frac{a_{thr}}{a}\right)^{1/2},$$

or,

$$\sqrt{\alpha\varphi_0} \left(\frac{t_0}{t_d}\right) \left(\frac{t_d}{t_{thr}}\right)^{3/4} < T_R \left(\frac{a}{a_{thr}}\right)^{1/2},$$

or,

$$\frac{\alpha\varphi_0^2}{T_R^2} \left(\frac{\Gamma_d}{m_0}\right)^2 \left(\frac{1}{\Gamma_d t_{thr}}\right)^{3/2} < \left(\frac{a}{a_{thr}}\right).$$

(44)

Then

$$t_G = \left[\frac{\alpha\varphi_0^2}{T_R^2} \left(\frac{\Gamma_d^{1/2}}{m_0^2}\right) \frac{1}{t_{thr}}\right]^2 \left(\frac{g_*}{A}\right) \frac{\alpha^2 \varphi_0^4 \Gamma_d^3}{m_{\phi}^4 m_0^4},$$

(45)

where we have used the expression for $t_{thr}$ from section 4.2.1 and for $T_R$ as in eq. (82) or (84). $A = 4 \times 10^{-11}$ as obtained in section 7.3.
The time when the condensate decays, $t_f$, is given by eq. (35). Then

$$t_f = \frac{\varphi_0^2}{m_0^2} \left( \frac{a_0}{a_d} \right)^3 \left( \frac{a_d}{a_f} \right)^3$$

$$= \frac{\varphi_0^2}{m_0^2} \left( \frac{t_0}{t_d} \right)^2 \left( \frac{t_d}{t_f} \right)^{3/2}$$

$$= \frac{\varphi_0^2}{m_0^2} \left( \frac{\Gamma_d}{m_0} \right)^2 \left( \frac{1}{\Gamma_d t_f} \right)^{3/2}$$

or,

$$t_f = \frac{\varphi_0^{4/5} \Gamma_d^{1/5}}{m_0^2}. \quad (46)$$

For $\varphi_0 = 10^{14}$ GeV, $t_G$ is $5 \times 10^{11}$ GeV$^{-1}$ while $t_f$ is $3 \times 10^7$ GeV$^{-1}$. Thus, there is no gravitino production in the radiation dominated era after thermalization before the flat direction condensate decays.

4.2.3. Relaxing the bounds

The lower and upper bounds on $\varphi_0$ in eqs. (40) and (43) are restrictive. For later use we relax the upper bound from phase space suppression. If $\varphi_0 > 1.5 \times 10^{14}$ GeV it implies that gravitino production commences at a time $t_p > t_d$. To get an expression for $t_p$ we start with eq. (31) and solve for $a_p$, the scale factor at which gravitino production starts. We thus get

$$m_\phi \frac{a_d}{a_p} = 3\sqrt{\alpha} \varphi_0 \left( \frac{a_0}{a_d} \right)^{3/2}$$

or,

$$m_\phi \frac{a_d}{a_p} = 3\sqrt{\alpha} \varphi_0 \left( \frac{a_0}{a_d} \right)^{3/2} \left( \frac{a_d}{a_p} \right)^{3/2}$$

or,

$$t_p = \frac{1}{\Gamma_d} \left[ 3\sqrt{\alpha} \varphi_0 \left( \frac{\Gamma_d}{m_0} \right) \frac{1}{m_\phi} \right]^4, \quad (47)$$

where we have used the fact that for $t_0 < t < t_d$ the scale factor $a \sim t^{2/3}$ and for $t_d < t < t_p$, $a \sim t^{1/2}$. In this case one will get an extra factor of $1/(\Gamma_d t_p)^{7/2}$ along with $t_d$ replaced by $t_p$ in the expressions for gravitino number density generated before kinetic equilibration in eqs. (54a), (57a) and (59a).

We remark in passing that if $7 \times 10^{12}$ GeV $< \varphi_0 < 3 \times 10^{13}$ GeV and one has kinetic equilibration of the inflaton decay products at $t_d$ without chemical equilibration, then one would obtain gravitino production using only section 6.2.2 with $\Gamma_{kin}$ set equal to $\Gamma_d$. 

18
5. Calculations of $\sigma_{CM}$

Having discussed the feasibility of gravitino production in a non-thermal Universe in the context of phase space availability we now calculate the cross section of gravitino production. The expressions for $|M|^2$ in this section include an average over spins and colors of the initial and final state particles but are for a specific flavor for quarks and squarks. The cross sections obtained below are for the dominant $\pm \frac{1}{2}$ helicity states of the gravitino and will be used for obtaining the collision integral (the $A$ terms) for $t_d < t < t_{\text{kin}}$, as in Appendix B.

5.1. Differential scattering cross sections

The differential scattering cross sections in the CM frame is as given in eq. (14). We use $|M|^2$ as provided in Table 1 of ref. [41]. Then

\[
|M_E|^2 = \frac{g_s^2}{M^2} \frac{\sum_{A,i,j} |T_{j_1}^A|^2}{g_1 g_2 g_3 g_4} \left( 1 + \frac{m_0^2}{3m_G^2} \right) \frac{(-2t)}{g_1 g_2 g_3 g_4} \left( 1 + \frac{m_0^2}{3m_G^2} \right) t \tag{48a}
\]

\[
|M_I|^2 = \frac{g_s^2}{M^2} \frac{\sum_{A,i,j} |T_{j_1}^A|^2}{g_1 g_2 g_3 g_4} \left( 1 + \frac{m_0^2}{3m_G^2} \right) \frac{(-8t)(s + t)}{s} \tag{48b}
\]

\[
|M_J|^2 = \frac{g_s^2}{M^2} \frac{\sum_{A,i,j} |T_{j_1}^A|^2}{g_1 g_2 g_3 g_4} \left( 1 + \frac{m_0^2}{3m_G^2} \right) \frac{2(s^2 + 2ts + 2t^2)}{s} \tag{48c}
\]

where $T^A$ is the generator of the SU(3) group in the fundamental representation, and $i, j$ represent the color of the incoming particles. $|M_J|^2$ above differs from ref. [41] by a factor of 2 because we sum over both chiralities of the quarks. The expressions in Table 1 of ref. [41] do not include an average over incoming and final states and hence we have a sum over $A, i, j$ and an additional factor of $g_1 g_2 g_3 g_4$ in the denominators of $|M_{E,I,J}|^2$.

Using eq. (14) in the limit that the incoming energy $\sqrt{s}$ is much larger than all masses, the differential scattering cross sections for the three processes are

\[
\frac{d\sigma_E}{d\Omega} = -\frac{\sum_{A,i,j} g_s^2 |T_{j_1}^A|^2}{32 g_1 g_2 M^2 \pi^2 s} \left( 1 + \frac{m_0^2}{3m_G^2} \right) \frac{t}{g_1 g_2 g_3 g_4} \left( 1 + \frac{m_0^2}{3m_G^2} \right) t \tag{49a}
\]

\[
\frac{d\sigma_I}{d\Omega} = -\frac{\sum_{A,i,j} g_s^2 |T_{j_1}^A|^2}{8 g_1 g_2 M^2 \pi^2 s^2} \left( 1 + \frac{m_0^2}{3m_G^2} \right) \frac{t(s + t)}{g_1 g_2 g_3 g_4} \left( 1 + \frac{m_0^2}{3m_G^2} \right) t(s + t) \tag{49b}
\]

\[
\frac{d\sigma_J}{d\Omega} = \frac{\sum_{A,i,j} g_s^2 |T_{j_1}^A|^2}{32 g_1 g_2 M^2 \pi^2 s^2} \left( 1 + \frac{m_0^2}{3m_G^2} \right) \frac{(s^2 + 2ts + 2t^2)}{g_1 g_2 g_3 g_4} \left( 1 + \frac{m_0^2}{3m_G^2} \right) \frac{t(s^2 + 2ts + 2t^2)}{s} \tag{49c}
\]
5.2. Scattering cross sections

The cross section $\sigma_{CM}$ is obtained by integrating the differential cross section. The scattering cross sections for the E, I and J processes are

$$\sigma_E = \frac{\sum A_{i,j} g_s^2 |T_{ji}|^2}{16 g_1 g_2 M^2 \pi} \left( 1 + \frac{m_0^2}{3m_G^2} \right)$$  (50a)

$$\sigma_I = \frac{\sum A_{i,j} g_s^2 |T_{ji}|^2}{12 g_1 g_2 M^2 \pi} \left( 1 + \frac{m_0^2}{3m_G^2} \right)$$  (50b)

$$\sigma_J = \frac{\sum A_{i,j} g_s^2 |T_{ji}|^2}{12 g_1 g_2 M^2 \pi} \left( 1 + \frac{m_0^2}{3m_G^2} \right) .$$  (50c)

6. Gravitino abundances

Having obtained the cross section $\sigma_{CM}$ we use the procedure given in section 2 to calculate the number density of gravitinos via the E, I and J processes using the integrated Boltzmann equation. For $m_0 < \Gamma_d$ we only consider $t > t_{thr}$. For $m_0 > \Gamma_d$, we calculate the number density separately from $t_d$ to $t_{kin}$, $t_{kin}$ to $t_{thr}$ and for $t > t_{thr}$. Parameters needed to evaluate the gravitino abundance are given in section 7, Appendix C and Appendix D.

6.1. $m_0 < \Gamma_d$

As shown in section 4.1 there is no gravitino production until the flat direction condensate decays at $t_f$. The temperature $T_f$ at $t_f$ is derived in section 7.7 and equals $1 \times 10^5$ GeV. The gravitino abundance generated after $t_f$ is given by the standard expression for gravitino production in a thermal Universe after inflation with the reheat temperature replaced by $T_f$. For this low value of $T_f$, the gravitino abundance will be small enough to satisfy cosmological constraints.

6.2. $m_0 > \Gamma_d$

6.2.1. $t_d$ to $t_{kin}$

The $A$ terms for the processes E, I, J that enter the RHS of the integrated Boltzmann eq. (9) for the phase space distribution in eq. (7a) are given in Appendix B.
For the process $E$,

\[
\frac{dn_G^{(E)}}{dt} + 3Hn_G = A_E = 144 \ B \ m_\phi^4 \left( \frac{a_d}{a} \right)^6 \\
= 144 \ B \ m_\phi^4 \left( \frac{t_d}{t} \right)^3 \\
= 144 \ Q \ t^{-3},
\]

(51)

where

\[
B = \frac{3 \sum_{A,i,j} g_s^2 M_P^2 |T_{ji}|^2 \Gamma_d^4}{256 \pi^2 g_1 g_2 m_\phi^6} \left( 1 + \frac{m_0^2}{3m_G^2} \right) \\
Q = \frac{B m_\phi^4}{\Gamma_d^3}
\]

(52)

(53)

The parameter $B$ is derived in Appendix C. Solving eq. (51) we then get

\[
n_G^{(E)}(t) = 144K \left[ 1 - \left( \frac{t_d}{t} \right)^{1/2} \right] \left( \frac{t_d}{t} \right)^{3/2} \\
= 144K \left[ 1 - \left( \frac{a_d}{a} \right) \right] \left( \frac{a_d}{a} \right)^3,
\]

(54a)

(54b)

where $K$ is defined as

\[
K = \frac{2Bm_\phi^4}{\Gamma_d^3}
\]

(55)

Similarly for the process $I$,

\[
\frac{dn_G^{(I)}}{dt} + 3Hn_G = A_I = 48 \ B \ m_\phi^4 \left( \frac{a_d}{a} \right)^6 \\
= 48 \ B \ m_\phi^4 \left( \frac{t_d}{t} \right)^3 \\
= 48 \ Q \ t^{-3}.
\]

(56)

Eq. (56) implies that

\[
n_G^{(I)}(t) = 48K \left[ 1 - \left( \frac{t_d}{t} \right)^{1/2} \right] \left( \frac{t_d}{t} \right)^{3/2} \\
= 48K \left[ 1 - \left( \frac{a_d}{a} \right) \right] \left( \frac{a_d}{a} \right)^3.
\]

(57a)

(57b)
For the process $J$,
\[
\frac{dn^{(J)}_{G}}{dt} + 3Hn_{G} = A_{J} = 96\ B\ m_{\phi}^{4}\ \left(\frac{a_{d}}{a}\right)^{3} = 96\ Q\ t^{-3}.
\]
Eq. (58) implies that
\[
n^{(J)}_{G}(t) = 96K\left[1 - \left(\frac{a_{d}}{a}\right)^{3}\right].
\]

One observes that the factor in the square brackets in the above expressions is related to gravitino production while the factor $\sim 1/a^3$ reflects dilution due to Hubble expansion. Because of the fast decline with time of the source term on the RHS of the integrated Boltzmann equation, the production of gravitinos shuts off quickly and then the number density falls as $1/a^3$.

6.2.2. $t_{kin}$ to $t_{thr}$

For $t_{kin}$ to $t_{thr}$ we evaluate $A$ defined in eq. (9) with the phase space distribution function given in eq. (7b).

\[
A = \exp(\xi_{1}/T)\exp(\xi_{2}/T)\int d\Pi_{1}\ d\Pi_{2}\exp(-E_{1}/T)\exp(-E_{2}/T)W_{12}
\]
\[
= \exp(\xi_{1}/T)\exp(\xi_{2}/T)\ n_{1}^{eq}\ n_{2}^{eq}\left[\frac{1}{n_{1}^{eq}\ n_{2}^{eq}}\int d\Pi_{1}\ d\Pi_{2}\exp(-E_{1}/T)\exp(-E_{2}/T)W_{12}\right]
\]
\[
\equiv n_{1}(t)n_{2}(t)\langle\sigma v\rangle,
\]
where $\langle\sigma v\rangle$ is the expression in square brackets and
\[
n_{i} = \frac{g_{i}}{(2\pi)^{3}}\int\exp[-(E_{i} - \xi_{i})/T]\ d^{3}p_{i} \quad \text{for } i = 1, 2
\]
\[
= A_{i} \ n_{i}^{eq}
\]
\[
= A_{i} \ \frac{g_{i}}{\pi^{2}}T^{3}.
\]
Here,
\[
A_{i} \equiv \exp(\xi_{i}/T) < 1 \quad \text{for } t_{kin} < t < t_{thr},
\]
\[
= 1 \quad \text{for } t = t_{thr},
\]
and
\[ n_i^{eq} = \frac{g_i}{(2\pi)^3} \int \exp(-E_i/T) \, d^3p_i, \] (63)
and we have ignored factors of \( O(1) \). Our expression for \( n_i \) includes \( g_i \), unlike in ref. [2]. The number density of all particles in the Universe is
\[ n = \sum_j n_j = \sum_j A_j g_j T^3 \equiv \mathcal{A} \frac{T^3}{\pi^2}. \] (64)
\( \mathcal{A} \rightarrow \mathcal{g}_* \) in thermal equilibrium, where \( \mathcal{g}_* \) is the total number of relativistic degrees of freedom for a fully thermalized plasma. For the minimal supersymmetric Standard Model (MSSM) \( \mathcal{g}_* = 228.75 \). Since we can express \( \mathcal{A} \) for \( t_{\text{kin}} \) to \( t_{\text{thr}} \) in terms of the standard thermally averaged cross section we can therefore use expressions for \( \langle \sigma v \rangle \) from the literature.

In kinetic equilibrium, temperature is well defined and we choose to re-express the integrated Boltzmann eq. (9)
\[ \dot{n}_\tilde{G} + 3H n_\tilde{G} = \langle \sigma v \rangle n_1 n_2, \] (65)
as
\[ T \frac{dn_{\tilde{G}}}{dT} + 3H n_{\tilde{G}} = \langle \sigma v \rangle n_1 n_2, \] (66)
where a sum over all processes E, I, J is implicit. Defining \( Y' = n/s' \), where \( s' = \frac{2\pi^2}{45} \mathcal{A} T^3 \), we can rewrite the Boltzmann equation as
\[ T \frac{dY'}{dT} = \langle \sigma v \rangle Y'_1 n_2. \] (67)
Since \( T \sim 1/a, \dot{T}/T = -H \) and so
\[ \dot{T} = - \left( \frac{\mathcal{A} \pi^2}{90} \right)^{1/2} \frac{T^3}{M}, \] (68)
where we have used \( H^2 = \rho/(3M^2) \) and \( \rho = (\pi^2/30)\mathcal{A} T^4 \). Expressions for \( \langle \sigma v \rangle \) from ref. [34], were used in ref. [2]. These results were for the \( \pm 3/2 \) helicity states of the gravitino. Including the \( \pm 1/2 \) helicity states we get
\[ \langle \sigma v \rangle_E = \frac{1}{32M^2} \left( 1 + \frac{m_0^2}{3m_{\tilde{G}}^2} \right) 48\alpha \] (69a)
\[ \langle \sigma v \rangle_I = \frac{1}{32M^2} \left( 1 + \frac{m_0^2}{3m_{\tilde{G}}^2} \right) 16\alpha \] (69b)
\[ \langle \sigma v \rangle_J = \frac{1}{32M^2} \left( 1 + \frac{m_0^2}{3m_{\tilde{G}}^2} \right) 8\alpha, \] (69c)
where we have considered only the SU(3)_c gauge group. The gravitino abundance just before full thermalization at \( t_{\text{thr}} \) is

\[
Y_G'(t_{\text{thr}}) = \frac{n_{\tilde{G}}(t_{\text{thr}})}{s'(t_{\text{thr}})} = Y_G'(t_{\text{kin}}) + \frac{45}{2\pi^6} \left( \frac{90}{A^2} \right)^{1/2} \frac{1}{A} \int_{T_{\text{min}}}^{T_{\text{kin}}} \langle \sigma v \rangle A_1 g_1 A_2 g_2 M dT
\]

(70)

where \( s'(t_{\text{thr}}) \) and \( T_{\text{min}} \) are the entropy density and temperature at \( t_{\text{thr}} \) just before thermal equilibration. \( s'(t_{\text{thr}}) = \left( \frac{2\pi^2}{45} \right) A T_{\text{min}}^3 \) and \( \langle \sigma v \rangle \) includes a sum over all the processes in eq. (69). \( Y_G'(t_{\text{kin}}) \) is obtained using eqs. (54a), (57a) and (59a) and the temperature at \( t_{\text{kin}} \) needed for the entropy is given by eq. (76). Then

\[
Y_G'(t_{\text{kin}}) = \frac{n_{\tilde{G}}(t_{\text{kin}})}{s(t_{\text{kin}})} \sum_l q_l K \left[ 1 - \left( \frac{t_d}{t_{\text{kin}}} \right)^{1/2} \right] \left( \frac{t_d}{t_{\text{kin}}} \right)^{3/2}
\]

\[
= \frac{\sum_l q_l K \left[ 1 - \left( \frac{T_{\text{kin}}}{T_d} \right)^{1/2} \right] \left( \frac{T_{\text{kin}}}{T_d} \right)^{3/2}}{(2\pi^2/45) A (m_\phi/3)^3 (T_{\text{kin}}/T_d)^{3/2}}
\]

\[
\equiv \sum_l q_l K \left[ 1 - \left( \frac{T_{\text{kin}}}{T_d} \right)^{1/2} \right] \left( \frac{2\pi^2}{45} \right) A (m_\phi/3)^3 \int_{T_{\text{min}}}^{T_{\text{kin}}} \langle \sigma v \rangle A_1 g_1 A_2 g_2 M dT
\]

(71)

where \( q_l = 144, 48, 96 \) for the \( l = E, I, J \) processes respectively. Note that \( K \propto B \) is also process dependent, as mentioned in Appendix C.

After equilibration at \( t_{\text{thr}} \) the temperature falls to

\[
T_R = \left( \frac{A}{g_*} \right)^{1/4} T_{\text{min}},
\]

(72)

Then,

\[
Y_G(t_{\text{thr}}) = Y_G'(t_{\text{thr}}) \frac{s'(t_{\text{thr}})}{s(t_{\text{thr}})} \equiv \left( \frac{A}{g_*} \right)^{1/4} Y_G'(t_{\text{kin}}) + \frac{0.07}{g_*^{5/2}} \left( \frac{g_*}{A} \right)^{5/4} \int_{T_{\text{min}}}^{T_{\text{kin}}} \langle \sigma v \rangle A_1 g_1 A_2 g_2 M dT
\]

(73)

\[
\equiv Y_G^{(1)}(t_{\text{thr}}) + Y_G^{(2)}(t_{\text{thr}}),
\]

24
where \( Y_G(t_{thr}) = n_G(t_{thr})/s(t_{thr}) \) and \( s(t_{thr}) = (2\pi^2/45)g_*T^3_{thr} \). \( Y_G^{(1)} \) and \( Y_G^{(2)} \) are associated with gravitino production between \( t_d \) and \( t_{kin} \), and between \( t_{kin} \) and \( t_{thr} \), respectively.

For \( Y_G^{(2)}(t_{thr}) \), substituting eqs. (69) in eq. (73) we have

\[
\begin{align*}
Y_G^{(2)}(t_{thr}) &= \left[ \frac{0.07}{g_*^{3/2}A} \right] \sum_l f_l \frac{\alpha}{32M^2} \left( 1 + \frac{m_0^2}{3m_Z^2} \right) A_1 g_1 A_2 g_2 M \int_{T_{kin}}^{T_{min}} dT \\
&= \left[ \frac{0.07}{g_*^{3/2}A} \right] \sum_l f_l \frac{\alpha}{32M^2} \left( 1 + \frac{m_0^2}{3m_Z^2} \right) A_1 g_1 A_2 g_2 M (T_{kin} - T_{min})
\end{align*}
\]

where \( f_l = 48, 16, 8 \) from eq. (69) for \( l = E, I, J \) processes respectively and \( g_{1,2} \) are as given in Appendix C. We have ignored the variation of \( \alpha \) with temperature and shall use the value of \( \alpha = 5 \times 10^{-2} \).

6.2.3. \( t > t_{thr} \)

As discussed in section 4.2 gravitino production is suppressed until the flat direction condensate decays at \( t_f \). The temperature \( T_f \) at \( t_f \) is given in section 7.7 and equals \( 1 \times 10^5 \) GeV. Such a low temperature will lead to a gravitino abundance consistent with cosmological constraints.

7. Obtaining parameters

The values of parameters which we require to obtain the gravitino abundances are evaluated in this section for both \( m_0 < \Gamma_d \) and \( m_0 > \Gamma_d \).

7.1. \( T_{kin} \) and \( \Gamma_{kin} \)

\( T_{kin} \) is the temperature at \( t_{kin} \) just after kinetic equilibration. Since kinetic equilibration only re-distributes energy among particles without changing their number, the average energy per particle is the same before and after kinetic equilibration. Therefore

\[
\frac{3}{2} k_B T_{kin} = \frac{m_\phi}{2} \frac{a_d}{a_{kin}} = \frac{m_\phi}{2} \left( \frac{t_d}{t_{kin}} \right)^{1/2} = \frac{m_\phi}{2} \left( \frac{\Gamma_{kin}}{\Gamma_d} \right)^{1/2},
\]

(75)

25
where the RHS represents the energy of the particles just before kinetic equilibration and we presume 2 body decay of the inflaton. The Boltzmann constant $k_B$ is hereafter set to 1. Therefore

$$T_{\text{kin}} = \frac{m_\phi}{3} \left( \frac{\Gamma_{\text{kin}}}{\Gamma_d} \right)^{1/2},$$

(76)

where $\Gamma_{\text{kin}}$ is given in eqs. (25b) and (41).

For $m_0 < \Gamma_d$,

$$\Gamma_{\text{kin}} = \frac{3 \alpha M_P^2 \Gamma_d^{1/2} m_0^{3/2}}{m_\phi \varphi_0^2} = 11 \text{ GeV}.$$  

(77)

For $m_0 > \Gamma_d$,

$$\Gamma_{\text{kin}} = \frac{3 \alpha M_P^2 m_0^2}{2 m_\phi \varphi_0^2} = 1 \text{ GeV}.$$  

(78)

Thus, the value of $T_{\text{kin}}$ for $m_0 < \Gamma_d$ is $1 \times 10^{11}$ GeV and for $m_0 > \Gamma_d$ is $1 \times 10^{12}$ GeV.

7.2. $T_{\text{min}}$

The temperature $T_{\text{min}}$ just before thermalization at $t_{\text{thr}}$ can be obtained by

$$T_{\text{min}} = T_{\text{kin}} \left( \frac{a_{\text{kin}}}{a_{\text{thr}}} \right) = T_{\text{kin}} \left( \frac{t_{\text{kin}}}{t_{\text{thr}}} \right)^{\frac{1}{2}} = T_{\text{kin}} \left( \frac{\Gamma_{\text{thr}}}{\Gamma_{\text{kin}}} \right)^{\frac{1}{2}}$$

(79)

$$T_{\text{min}} = \begin{cases} 2 \times 10^{10} \text{ GeV} & \text{for } m_0 < \Gamma_d \\ 2 \times 10^{11} \text{ GeV} & \text{for } m_0 > \Gamma_d \end{cases}$$

(80)

where we have used the relation $\Gamma_{\text{thr}} = \alpha \Gamma_{\text{kin}}$ as discussed in section [3].

7.3. $\mathcal{A}$

To obtain $\mathcal{A}$ we note that the energy density of the Universe $\rho_{\text{kin}+}$ just after kinetic equilibration at $t_{\text{kin}}$ is equal to the energy density $\rho_{\text{kin}-}$ just
before kinetic equilibration. Then

$$\rho_{\text{kin}}^+ = \rho_{\text{kin}}^-$$

or,

$$\frac{\pi^2}{30} A T_{\text{kin}}^4 = \rho_\phi(t_d) \left( \frac{a_d}{a_{\text{kin}}} \right)^4$$

or,

$$\frac{\pi^2}{30} A \left( \frac{m_\phi}{3} \left( \frac{\Gamma_{\text{kin}}}{\Gamma_d} \right)^\frac{1}{2} \right)^4 = \rho_\phi(t_d) \left( \frac{\Gamma_{\text{kin}}}{\Gamma_d} \right)^2$$

or, \( A \approx \frac{2430}{\pi^2} \left( \frac{\rho_\phi(t_d)}{m_\phi^4} \right) \)

or,

$$A \approx \frac{2430}{\pi^2} \left( \frac{3}{8\pi} \right) \left( \frac{M_P^2 \Gamma_d^2}{m_\phi^4} \right)^{1/2}$$

or,

$$A \sim \begin{cases} 4 \times 10^{-5}, & \text{for } m_0 < \Gamma_d = 10^4 \text{ GeV} \\ 4 \times 10^{-11}, & \text{for } m_0 > \Gamma_d = 10 \text{ GeV} \end{cases}$$

(81)

Thus \( A \ll 1 \).

7.4. \( A_i \)

The number density of a species \( i \) just after kinetic equilibration is \( A_i g_i T_{\text{kin}}^3 / \pi^2 \). This equals the number density of the species just before kinetic equilibration which, in turn, is the number density of the species at \( t_d \) scaled by \( 1/a^3 \). The decay rate of the inflaton to a species \( i \) will be proportional to \( g_i \) and so \( n_i(t_d) \) is also proportional to \( g_i \). Then \( A_i \) is the same for all species. Since \( A = \sum_j A_j g_j = A_i \sum_j g_j \approx 200 A_i, A_i \approx A/200 \).

7.5. \( T_R \)

As stated earlier in eq. (72) the temperature at \( t_{\text{thr}} \) is given by

$$T_R = \left( \frac{A}{g_s} \right)^{1/4} T_{\text{min}}$$

$$= \left( \frac{A}{g_s} \right)^{1/4} \sqrt{\alpha} T_{\text{kin}}$$

$$= \left( \frac{A}{g_s} \right)^{1/4} \sqrt{\alpha} \frac{m_\phi}{3} \left( \frac{\Gamma_{\text{kin}}}{\Gamma_d} \right)^{1/2}.$$  

(82)
For $m_0 < \Gamma_d$, 
\[
T_R = \frac{1}{\sqrt{6}} \left( \frac{A}{g_*} \right)^{1/4} \frac{\alpha m_\phi^{1/2} M_{Pl} m_0^{3/4}}{\varphi_0 \Gamma_d^{1/4}} 
\]
\[
= 5 \times 10^8 \text{ GeV} . 
\] (83)

For $m_0 > \Gamma_d$, 
\[
T_R = \frac{1}{\sqrt{6}} \left( \frac{A}{g_*} \right)^{1/4} \frac{\alpha m_\phi^{1/2} M_{Pl} m_0}{\varphi_0 \Gamma_d^{1/2}} 
\]
\[
= 2 \times 10^8 \text{ GeV} . 
\] (84)

We have used eqs. (77) and (78) for the expression of $\Gamma_{\text{kin}}$ for $m_0 < \Gamma_d$ and $m_0 > \Gamma_d$ respectively, and $\alpha = 5 \times 10^{-2}$.

7.6. $T_G$

The temperature $T_G$ at $t_g$ is given by 
\[
T_G = T_R \left( \frac{a_{\text{thr}}}{a_G} \right) . 
\] (85)

For $m_0 < \Gamma_d$, using eq. (33) to define $a_G$, we get 
\[
T_G = \frac{T_R^3}{\alpha \varphi_0^2} (m_0 t_{\text{thr}})^{3/2} 
\]
\[
= \left( \frac{1}{27} \right) \left( \frac{A}{g_*} \right)^{3/4} \frac{m_\phi^3 m_0^{3/2}}{\alpha \varphi_0^2 \Gamma_d^{3/2}} 
\]
\[
= 300 \text{ GeV} , 
\] (86)

where we have used section 4.1.2 for $t_{\text{thr}}$ and eq. (82) or (83) for $T_R$, and $\alpha = 0.1$.

For $m_0 > \Gamma_d$, using eq. (44), the temperature at $t_G$ is 
\[
T_G = \frac{T_R^3}{\alpha \varphi_0^2} \left( \frac{m_0}{\Gamma_d} \right)^2 (\Gamma_d t_{\text{thr}})^{3/2} 
\]
\[
= \frac{1}{27} \left( \frac{A}{g_*} \right)^{3/4} \frac{m_\phi^3 m_0^2}{\alpha \varphi_0^2 \Gamma_d^{2}} 
\]
\[
= 1000 \text{ GeV} , 
\] (87)

where we have used section 4.2.1 for $t_{\text{thr}}$ and eq. (82) or (84) for $T_R$, and $\alpha = 0.1$.  

28
7.7. \( T_f \)

The temperature at \( t_f \) is given by

\[
T_f = T_R(t_{\text{thr}}/t_f)^{1/2}.
\]

(88)

For \( m_0 < \Gamma_d \),

\[
T_f = \frac{1}{3} \left( \frac{A}{g_*} \right)^{1/4} \frac{m_\phi m_0^{9/10}}{\varphi_0^{2/5} \Gamma_d^{1/2}}
= 1 \times 10^5 \text{ GeV},
\]

(89)

where the expressions for \( t_{\text{thr}}, t_f \) and \( T_R \) are obtained from section 4.1.2, eqs. (36) and (82) or (83) respectively.

For \( m_0 > \Gamma_d \),

\[
T_f = \frac{1}{3} \left( \frac{A}{g_*} \right)^{1/4} \frac{m_\phi m_0}{\varphi_0^{2/5} \Gamma_d^{3/5}}
= 1 \times 10^5 \text{ GeV},
\]

(90)

where the expressions for \( t_{\text{thr}}, t_f \) and \( T_R \) are obtained from section 4.2.1, eqs. (46) and (82) or (84) respectively.

8. Results

For the case \( m_0 < \Gamma_d \), with \( m_0 = 100 \text{ GeV} \), \( \Gamma_d = 10^4 \text{ GeV} \) and \( \varphi_0 = 10^{14} \text{ GeV} \), \( t_d, t_0, t_{\text{kin}}, t_{\text{thr}} \) and \( t_f \) are \( 10^{-4} \text{ GeV}^{-1}, 10^{-2} \text{ GeV}^{-1}, 9 \times 10^{-2} \text{ GeV}^{-1}, 2 \text{ GeV}^{-1} \) and \( 4 \times 10^7 \text{ GeV}^{-1} \) respectively. \( \varphi \) is constant at \( 10^{14} \text{ GeV} \) from \( t_d \) to \( t_0 \) and falls to \( 2 \times 10^{13} \text{ GeV} \) and \( 2 \times 10^{12} \text{ GeV} \) at \( t_{\text{kin}} \) and \( t_{\text{thr}} \) respectively.

From Appendix D the gluon mass is constant from \( t_d \) to \( t_0 = 2 \times 10^{13} \text{ GeV} \) and falls to \( 4 \times 10^{12} \text{ GeV} \) and \( 2 \times 10^{11} \text{ GeV} \) at \( t_{\text{kin}} \) and \( t_{\text{thr}} \) respectively. The final reheat temperature \( T_R \) after full equilibration is \( 5 \times 10^8 \text{ GeV} \), which is less than the gluon and gluino mass at that time. As we have argued in section 4.1 there is no gravitino production in the non-thermal phase, nor after thermalization till the flat direction condensate decays at \( t_f \), because of phase space suppression associated with the large mass of the outgoing gluon or gluino. The temperature \( T_f \) is \( 1 \times 10^5 \text{ GeV} \) and gravitino production after \( t_f \) is hence small and consistent with cosmological constraints.

For \( m_0 > \Gamma_d \), with \( m_0 = 100 \text{ GeV} \), \( \Gamma_d = 10 \text{ GeV} \) and \( \varphi_0 = 10^{14} \text{ GeV} \), \( t_0 \), \( t_d \), \( t_{\text{kin}} \) and \( t_{\text{thr}} \) are \( 0.01 \text{ GeV}^{-1}, 0.1 \text{ GeV}^{-1}, 0.9 \text{ GeV}^{-1} \) and \( 20 \text{ GeV}^{-1} \). \( \varphi \) falls to \( 10^{13} \text{ GeV} \) at \( t_d \), \( 2 \times 10^{12} \text{ GeV} \) at \( t_{\text{kin}} \) and \( 2 \times 10^{11} \text{ GeV} \) at \( t_{\text{thr}} \). The corresponding gluon mass is \( 2 \times 10^{13} \text{ GeV} \), \( 2 \times 10^{12} \text{ GeV} \), \( 4 \times 10^{11} \text{ GeV} \) and
The final reheat temperature $T_R$ is $2 \times 10^8$ GeV. The gravitino abundances generated in the different epochs are

$$Y_{\tilde{G}}^{(1)} = 4 \times 10^{-18}$$
$$Y_{\tilde{G}}^{(2)} = 1 \times 10^{-20}$$

where $Y_{\tilde{G}}^{(1)}$ is the abundance due to gravitino production between $t_d$ and $t_{kin}$ and $Y_{\tilde{G}}^{(2)}$ is the abundance due to production between $t_{kin}$ and $t_{thr}$. The gravitino abundance in both epochs is less than the upper bound of $10^{-14}$ on the gravitino abundance from various cosmological constraints [54]. Gravitino production after thermalization at $t_{thr}$ is suppressed until the condensate decays at $t_f$. Since the temperature at $t_f$ is $1 \times 10^5$ GeV the gravitino abundance generated after $t_f$ will be small and within cosmological bounds.

9. Conclusions

In this article we have studied the novel scenario proposed by refs. [1, 2] where the vacuum expectation values of SUSY flat directions generate a large mass for gauge bosons and gauginos. Due to the large gauge boson masses thermalization is delayed after the end of inflation. In the non-thermal Universe gravitino production in refs. [1, 2] is suppressed because of the dilute nature of the plasma. The delay in thermalization also leads to a low reheat temperature and so gravitino production after thermalization is also suppressed. Thus it is argued that the gravitino problem of SUSY models is avoided in this scenario.

In this article we have carried out a more careful analysis of gravitino production in the non-thermal Universe scenario of refs. [1, 2]. We have identified the initial state particle distribution functions in the gravitino production process for the eras before kinetic equilibration and after kinetic equilibration in a non-thermal Universe (the appropriate particle distribution function for the pre-kinetic equilibration era was not used in refs. [1, 2]). These have then been included in the calculation of the collision integral in the integrated Boltzmann equation. We have also explicitly obtained the conditions for obtaining a non-thermal Universe after inflaton decay. We have further investigated final state phase space suppression in the context of heavy final state gluons and gluinos in both the non-thermal and thermal eras.

We have investigated two cases (i) where the flat direction starts oscillating after inflaton decay ($m_0 < \Gamma_d$) and (ii) where the flat direction starts
oscillating before inflaton decay \((m_0 > \Gamma_d)\). In the first case \((m_0 < \Gamma_d)\) we find that there is no gravitino production during the non-thermal phase because of phase space suppression due to the large gluon and gluino mass. Gravitino production is suppressed even after thermalization till the flat direction condensate decays at \(t_f\). However the temperature \(T_f\) is low and as such any gravitino abundance produced after \(t_f\) is small and is within the standard cosmological bounds.

In the second case \((m_0 > \Gamma_d)\) there is gravitino production before thermalization. The gravitino abundance generated is consistent with cosmological constraints. After thermalization, as in the case \(m_0 < \Gamma_d\), there is no gravitino production before the flat direction condensate decays. After the decay of the condensate the temperature \(T_f\) is low and does not lead to conflict with cosmological bounds.

Acknowledgments

We would like to thank H. Baer, A. Joshipura, P. Konar, N. Mahajan, H. Mishra, K. Patel and P. Roy for very useful discussions. We would also like to thank R. Allahverdi for clarifying aspects of ref. [2].

Appendix A. Normalization of the phase space distribution functions for \(t_d < t < t_{\text{kin}}\)

The number of incoming particles \(N_i\) for gravitino production is given by

\[
\frac{g_i}{(2\pi)^3} \int f_i \, d^3 p_i \, dV = N_i. \quad (A.1)
\]

For the time interval \(t_d < t < t_{\text{kin}}\) the form of \(f\) is given by eq. (7a). Substituting that in the above equation we get,

\[
N_i/V \equiv n_i(t) = \frac{g_i}{(2\pi)^3} \int C_i \delta \left( E_i - \frac{m_\phi a_d}{2} \right) 4\pi p_i^2 \, dp_i
\]

\[
= \frac{g_i}{(2\pi)^3} \int C_i \delta \left( E_i - \frac{m_\phi a_d}{2} \right) 4\pi \sqrt{E_i^2 - m_i^2} \, E_i \, dE_i
\]

or,

\[
C_i = \frac{n_i(t)}{m_\phi^2} \left( \frac{a}{a_d} \right)^2 \frac{(2\pi)^3}{\pi g_i}. \quad (A.2)
\]

where we have considered \(E_i \gg m_i\). Now at \(t = t_d\), the number density of the particles is equal to the number density of the inflaton, as \(\phi \to X_1 + X_2\). So, \(n_i(t_d) = n_\phi(t_d) = \rho_\phi(t_d)/m_\phi\). Then for \(t_d < t < t_{\text{kin}}\), the number density \(n_i(t)\)
is given by \( n_i(t) = (\rho_\phi(t_d)/m_\phi)(a_d/a)^3 \). Substituting the above expression for \( n_i(t) \) in eq. (A.2) we get
\[
C_i(t) = \frac{\rho_\phi(t_d)}{m_\phi^3} \left( \frac{a_d}{a} \right) \left( \frac{2\pi}{g_i} \right)^3.
\] (A.3)

**Appendix B. A terms for \( t_d < t < t_{kin} \)**

Below we present the A terms for the processes E, I and J during the interval \( t_d < t < t_{kin} \) as needed for the \( m_0 > \Gamma_d \) case using eqs. (9), (11), (12) and (17), and the phase space distribution in eq. (7a), with the expressions for \( \sigma_{CM} \) for the ±1/2 helicity gravitino states as given in eqs. (50a), (50b) and (50c).

The A term for the E process is
\[
A_E(t) = \frac{6 \times 2 \times 2 \times \sum_{A,i,j} C_1 C_2 g_s^2 |T^A_{ji}|^2 a_d^4 m_\phi^4}{1024 M^2 \pi^5 a^4} \left( 1 + \frac{m_0^2}{3 m_G^2} \right),
\] (B.1)

where \( T^A \) is the generator of the SU(3) group in the fundamental representation and \( C_i \propto (a_d/a) \) are given in eq. (A.3). \( A_E \) includes a factor of 6 for 6 quark/squark flavors, a factor of 2 for the 2 squark ‘chiralities’ and another factor of 2 for the 2 hermitian conjugate processes. (One should actually exclude the heavy quarks and squarks associated with the flat direction. That would decrease the final abundance by a factor of \( O(1) \).) Then we get
\[
A_E(t) = 144 B m_\phi^4 \left( \frac{a_d^4}{a^6} \right),
\] (B.2)

where the parameter \( B \) is as defined in Appendix C.

The A term for the I process is
\[
A_I(t) = \frac{6 \times \sum_{A,i,j} C_1 C_2 g_s^2 |T^A_{ji}|^2 a_d^4 m_\phi^4}{768 M^2 \pi^5 a^4} \left( 1 + \frac{m_0^2}{3 m_G^2} \right).
\] (B.3)

\( A_I \) includes a factor of 6 for the quark flavors. Then we get
\[
A_I(t) = 48 B m_\phi^4 \left( \frac{a_d^6}{a^6} \right).
\] (B.4)

The A term for the J process is
\[
A_J(t) = \frac{6 \times 2 \times \sum_{A,i,j} C_1 C_2 g_s^2 |T^A_{ji}|^2 a_d^4 m_\phi^4}{768 M^2 \pi^5 a^4} \left( 1 + \frac{m_0^2}{3 m_G^2} \right).
\] (B.5)
\( A_J \) gets a factor of 6 for squark flavors and a factor of 2 for the squark ‘chiralities’. Then we get

\[
A_J(t) = 96B m_\phi^4 \left( \frac{a_d^6}{a_b^6} \right).
\] (B.6)

Appendix C. B parameter

To simplify our equations we define new parameters called \( B \) and \( B' \) for the pre-factor present in \( A \) terms, i.e.,

\[
B = \left( \sum_{A,i,j} \frac{8\pi C_1 C_2 g_s^2 |T^{A}_{ji}|^2}{6144 M_{Pl}^2 \pi^5} \right) \left( 1 + \frac{m_0^2}{3m_G^2} \right),
\] (C.1)

where \( M_{Pl}^2 = 8\pi M^2 \). Substituting the values of \( C_1 \) and \( C_2 \) from eq. (A.3) in the above equation gives us

\[
B = \left( \frac{\rho_\phi(t_d) (2\pi)^3}{m_\phi^3} \frac{1}{\pi} \right)^2 \left( \frac{1}{g_1 g_2} \right) \left( \frac{\sum_{A,i,j} 8\pi g_s^2 |T^{A}_{ji}|^2}{6144 M_{Pl}^2 \pi^5} \right) \left( 1 + \frac{m_0^2}{3m_G^2} \right) \left( \frac{a_d}{a} \right)^2 \equiv B \left( \frac{a_d}{a} \right)^2,
\] (C.2)

where

\[
B = \left( \frac{\rho_\phi(t_d) (2\pi)^3}{m_\phi^3} \frac{1}{\pi} \right)^2 \left( \frac{1}{g_1 g_2} \right) \left( \frac{\sum_{A,i,j} 8\pi g_s^2 |T^{A}_{ji}|^2}{6144 M_{Pl}^2 \pi^5} \right) \left( 1 + \frac{m_0^2}{3m_G^2} \right)
\]

\[
= \left( \frac{3 \Gamma_d^2 (2\pi)^3}{m_\phi^3} \frac{1}{\pi} \right)^2 \left( \frac{M_{Pl}^2}{8\pi} \right) \left( \frac{\sum_{A,i,j} g_s^2 |T^{A}_{ji}|^2}{6144 g_1 g_2 \pi^5} \right) \left( 1 + \frac{m_0^2}{3m_G^2} \right)
\]

\[
= \frac{3 \sum_{A,i,j} g_s^2 M_{Pl}^2 |T^{A}_{ji}|^2 \Gamma_d^4}{256 \pi^2 g_1 g_2 m_\phi^6} \left( 1 + \frac{m_0^2}{3m_G^2} \right)
\] (C.3)

and we have used \( \rho_\phi(t_d) = (3/(8\pi)) M_{Pl}^2 \Gamma_d^2 \). We suppress the index \( E,I,J \) for \( B \) associated with the different \( g_{1,2} \) for each process. Considering the spin and color of the incoming particles, \((g_1, g_2) = (6, 3), (6, 6) \) and \((3, 3) \) for the \( E, I, J \) processes respectively.

Appendix D. Mass of the gluon

The mass of the gluon \( m_g = \sqrt{\alpha \varphi} \) which varies as \( 1/a^{3/2} \) after the condensate starts oscillating at \( t_0 \). Below we derive the mass of the gluon for various epochs in time for the two cases, \( m_0 < \Gamma_d \) and \( m_0 > \Gamma_d \).
Appendix D.1. $m_0 < \Gamma_d$ or $t_0 > t_d$

1. For $t_d < t < t_0$, $\varphi = \varphi_0$. So

$$m_g(t_d) = \sqrt{\alpha \varphi_0}.$$  \hfill (D.1)

2. For $t_0 < t < t_{thr}$,

$$m_g = m_g(t_0) \left( \frac{a_0}{a} \right)^{3/2}$$
$$= \sqrt{\alpha \varphi_0} \left( \frac{t_0}{t} \right)^{3/4}$$
$$= \sqrt{\alpha \varphi_0} \left( \frac{1}{m_0 t} \right)^{3/4}. \hfill (D.2)$$

At $t = t_{kin}$,

$$m_g(t_{kin}) = \sqrt{\alpha \varphi_0} \left( \frac{\Gamma_{kin} m_0}{m_0} \right)^{3/4}, \hfill (D.3)$$

where $\Gamma_{kin}$ is given by eq. (77).

At $t = t_{thr}$,

$$m_g(t_{thr}) = \sqrt{\alpha \varphi_0} \left( \frac{\Gamma_{thr} m_0}{m_0} \right)^{3/4}. \hfill (D.4)$$

3. For $t > t_{thr}$,

$$m_g = m_g(t_{thr}) \left( \frac{a_{thr}}{a} \right)^{3/2}$$
$$= \sqrt{\alpha \varphi_0} \left( \frac{\Gamma_{thr} m_0}{m_0} \right)^{3/4} \left( \frac{T}{T_R} \right)^{3/2}. \hfill (D.5)$$

Appendix D.2. $m_0 > \Gamma_d$ or $t_0 < t_d$

1. At $t = t_0$, 

$$m_g(t_0) = \sqrt{\alpha \varphi_0}. \hfill (D.6)$$

2. For $t_0 < t < t_d$,

$$m_g = \sqrt{\alpha \varphi_0} \left( \frac{a_0}{a} \right)^{3/2}$$
$$= \sqrt{\alpha \varphi_0} \left( \frac{t_0}{t} \right)$$
$$= \sqrt{\alpha \varphi_0} \left( \frac{1}{m_0 t} \right), \hfill (D.7)$$

34
where we have used the fact that the Universe was matter dominated for the period \( t_0 < t < t_d \) and \( a \sim t^{2/3} \), as relevant for an inflaton oscillating in a quadratic potential after inflation.

At \( t = t_d \),
\[
m_g(t_d) = \sqrt{\alpha} \varphi_0 \left( \frac{\Gamma_d}{m_0} \right),
\]

(D.8)

3. For \( t_d < t < t_{\text{kin}} \),
\[
m_g = m_g(t_d) \left( \frac{a_d}{a} \right)^{3/2}
= m_g(t_d) \left( \frac{t_d}{t} \right)^{3/4}
= \sqrt{\alpha} \varphi_0 \left( \frac{\Gamma_d}{m_0} \right) \left( \frac{1}{\Gamma_d t} \right)^{3/4},
\]

(D.9)

where we have used the fact that the Universe is radiation dominated for the period \( t_d < t < t_{\text{kin}} \) and \( a \sim t^{1/2} \).

4. For \( t_{\text{kin}} < t < t_{\text{thr}} \)
\[
m_g = m_g(t_{\text{kin}}) \left( \frac{a_{\text{kin}}}{a} \right)^{3/2}
= \sqrt{\alpha} \varphi_0 \left( \frac{\Gamma_d}{m_0} \right) \left( \frac{\Gamma_{\text{kin}}}{\Gamma_d} \right)^{3/4} \left( \frac{T}{T_{\text{kin}}} \right)^{3/2},
\]

(D.10)

where we have used the relation \( a \sim 1/T \) for \( t_{\text{kin}} < t < t_{\text{thr}} \).

5. For \( t > t_{\text{thr}} \)
\[
m_g(t) = m_g(t_{\text{thr}}) \left( \frac{a_{\text{thr}}}{a} \right)^{3/2}
= \sqrt{\alpha} \varphi_0 \left( \frac{\Gamma_d}{m_0} \right) \left( \frac{\Gamma_{\text{thr}}}{\Gamma_d} \right)^{3/4} \left( \frac{T}{T_{\text{R}}} \right)^{3/2},
\]

(D.11)

where we have used eqs. (D.10) and (19), and \( a \sim 1/T \) for \( t > t_{\text{thr}} \).

References

[1] R. Allahverdi and A. Mazumdar, Quasi-thermal universe and its implication for gravitino production, baryogenesis and dark matter, [hep-ph/0505050].

[2] R. Allahverdi and A. Mazumdar, Supersymmetric thermalization and quasi-thermal universe: Consequences for gravitinos and leptogenesis, JCAP 0610 (2006) 008, [hep-ph/0512227].
[3] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine, and A. Mazumdar, *Reheating in Inflationary Cosmology: Theory and Applications*, Ann. Rev. Nucl. Part. Sci. **60** (2010) 27–51, [arXiv:1001.2600](http://arxiv.org/abs/1001.2600).

[4] K. A. Olive and M. Peloso, *The fate of SUSY flat directions and their role in reheating*, Phys. Rev. **D74** (2006) 103514, [hep-ph/0608096](http://arxiv.org/abs/hep-ph/0608096).

[5] A. E. Gumrukcuoglu, K. A. Olive, M. Peloso, and M. Sexton, *The nonperturbative decay of SUSY flat directions*, Phys. Rev. **D78** (2008) 063512, [arXiv:0805.0273](http://arxiv.org/abs/0805.0273).

[6] R. Allahverdi, R. H. A. D. Shaw, and B. A. Campbell, *Parametric resonance for complex fields*, Phys. Lett. **B473** (2000) 246–257, [hep-ph/9909256](http://arxiv.org/abs/hep-ph/9909256).

[7] M. Postma and A. Mazumdar, *Resonant decay of flat directions: Applications to curvaton scenarios, Affleck-Dine baryogenesis, and leptogenesis from a sneutrino condensate*, JCAP **0401** (2004) 005, [hep-ph/0304246](http://arxiv.org/abs/hep-ph/0304246).

[8] R. Allahverdi and A. Mazumdar, *Longevity of supersymmetric flat directions*, JCAP **0708** (2007) 023, [hep-ph/0608296](http://arxiv.org/abs/hep-ph/0608296).

[9] A. Basboll, D. Maybury, F. Riva, and S. M. West, *Non-Perturbative Flat Direction Decay*, Phys. Rev. **D76** (2007) 065005, [hep-ph/0703015](http://arxiv.org/abs/hep-ph/0703015).

[10] A. Basboll, *SUSY Flat Direction Decay - the prospect of particle production and preheating investigated in the unitary gauge*, Phys. Rev. **D78** (2008) 023528, [arXiv:0801.0745](http://arxiv.org/abs/0801.0745).

[11] R. Allahverdi and A. Mazumdar, *Affleck-Dine condensate, late thermalization and the gravitino problem*, Phys. Rev. **D78** (2008) 043511, [arXiv:0802.4430](http://arxiv.org/abs/0802.4430).

[12] A. E. Gumrukcuoglu, *Non-perturbative decay of udd and QLd flat directions*, Phys. Rev. **D80** (2009) 123520, [arXiv:0910.0854](http://arxiv.org/abs/0910.0854).

[13] R. Allahverdi, B. A. Campbell, and J. R. Ellis, *Reheating and supersymmetric flat direction baryogenesis*, Nucl. Phys. **B579** (2000) 355–375, [hep-ph/0001122](http://arxiv.org/abs/hep-ph/0001122).

[14] A. Anisimov and M. Dine, *Some issues in flat direction baryogenesis*, Nucl. Phys. **B619** (2001) 729–740, [hep-ph/0008058](http://arxiv.org/abs/hep-ph/0008058).
[15] A. Anisimov, *Thermal effects and flat direction baryogenesis*, Phys. Atom. Nucl. 67 (2004) 640–647, [hep-ph/0111233].

[16] M. Dine, L. Randall, and S. D. Thomas, *Baryogenesis from flat directions of the supersymmetric standard model*, Nucl. Phys. B458 (1996) 291–326, [hep-ph/9507453].

[17] A. Kusenko and M. E. Shaposhnikov, *Supersymmetric Q-balls as dark matter*, Phys. Lett. B418 (1998) 46–54, [hep-ph/9709492].

[18] K. Enqvist and J. McDonald, *Q balls and baryogenesis in the MSSM*, Phys. Lett. B425 (1998) 309–321, [hep-ph/9711514].

[19] S. Kasuya and M. Kawasaki, *Q-ball formation through Affleck-Dine mechanism*, Phys. Rev. D61 (2000) 041301, [hep-ph/9909509].

[20] S. Kasuya and M. Kawasaki, *Q-ball formation in the gravity-mediated SUSY breaking scenario*, Phys. Rev. D62 (2000) 023512, [hep-ph/0002285].

[21] K. Enqvist, A. Jokinen, and J. McDonald, *Flat direction condensate instabilities in the MSSM*, Phys. Lett. B483 (2000) 191–195, [hep-ph/0004050].

[22] S. Kasuya and M. Kawasaki, *A new type of stable Q balls in the gauge-mediated SUSY breaking*, Phys. Rev. Lett. 85 (2000) 2677–2680, [hep-ph/0006128].

[23] K. Enqvist, A. Jokinen, T. Multamaki, and I. Vilja, *Numerical simulations of fragmentation of the Affleck-Dine condensate*, Phys. Rev. D63 (2001) 083501, [hep-ph/0011134].

[24] S. Kasuya and M. Kawasaki, *Q-ball formation: Obstacle to Affleck-Dine baryogenesis in the gauge-mediated SUSY breaking?*, Phys. Rev. D64 (2001) 123515, [hep-ph/0106119].

[25] T. Multamaki and I. Vilja, *Simulations of Q-ball formation*, Phys. Lett. B535 (2002) 170–176, [hep-ph/0203195].

[26] K. Enqvist and J. McDonald, *The Dynamics of Affleck-Dine condensate collapse*, Nucl. Phys. B570 (2000) 407–422, [hep-ph/9908316].

[27] E. W. Kolb and M. S. Turner, *The Early Universe*. Addison-Wesley, Redwood City, 1990.
[28] J. Edsjo and P. Gondolo, *Neutralino relic density including coannihilations*, Phys. Rev. **D56** (1997) 1879–1894, [hep-ph/9704361](http://arxiv.org/abs/hep-ph/9704361).

[29] A. Lahiri and P. B. Pal, *A First Book of Quantum Field Theory, 2nd ed.* Harrow, Alpha Sci. Int., UK, 2005.

[30] D. V. Nanopoulos, K. A. Olive, and M. Srednicki, *After Primordial Inflation*, Phys. Lett. **B127** (1983) 30.

[31] L. M. Krauss, *New Constraints on Ino Masses from Cosmology. 1. Supersymmetric Inos*, Nucl. Phys. **B227** (1983) 556.

[32] I. V. Falomkin, G. B. Pontecorvo, M. G. Sapozhnikov, M. Y. Khlopov, F. Balestra, and G. Piragino, *Low-energy anti-p He-4 annihilation and problems of the modern cosmology, GUT and SUSY models*, Yad. Fiz **39** (1984) 990, [Nuovo Cim. A **79**, 193 (1984)].

[33] M. Y. Khlopov and A. D. Linde, *Is it easy to save the gravitino?*, Phys. Lett. **B138** (1984) 265–268.

[34] J. R. Ellis, J. E. Kim, and D. V. Nanopoulos, *Cosmological Gravitino Regeneration and Decay*, Phys. Lett. **B145** (1984) 181.

[35] R. Juszkiewicz, J. Silk, and A. Stebbins, *Constraints on cosmologically regenerated Gravitinos*, Phys. Lett. **B158** (1985) 463–467.

[36] J. R. Ellis, D. V. Nanopoulos, and S. Sarkar, *The cosmology of decaying gravitinos*, Nucl. Phys. **B259** (1985) 175.

[37] M. Kawasaki and K. Sato, *Decay of Gravitinos and photodestruction of light elements*, Phys. Lett. **B189** (1987) 23.

[38] M. Y. Khlopov, Y. L. Levitan, E. V. Sedelnikov, and I. M. Sobol, *Nonequilibrium cosmological nucleosynthesis of light elements: Calculations by the Monte Carlo method*, Yad. Fiz. **57** (1994) 1466, [Phys. At. Nucl., **57**, 1393 (1994)].

[39] T. Moroi, H. Murayama, and M. Yamaguchi, *Cosmological constraints on the light stable gravitino*, Phys. Lett. **B303** (1993) 289–294.

[40] M. Kawasaki and T. Moroi, *Gravitino production in the inflationary universe and the effects on big bang nucleosynthesis*, Prog. Theor. Phys. **93** (1995) 879–900, [hep-ph/9403364](http://arxiv.org/abs/hep-ph/9403364).
[41] M. Bolz, A. Brandenburg, and W. Buchmuller, *Thermal Production of Gravitinos*, Nucl. Phys. B606 (2001) 518–544, [hep-ph/0012052].
Erratum-ibid. B790 (2008) 336.

[42] R. H. Cyburt, J. R. Ellis, B. D. Fields, and K. A. Olive, *Updated nucleosynthesis constraints on unstable relic particles*, Phys. Rev. D67 (2003) 103521, [astro-ph/0211258].

[43] G. F. Giudice, A. Riotto, and I. Tkachev, *Thermal and nonthermal production of gravitinos in the early universe*, JHEP 11 (1999) 036, [hep-ph/9911302].

[44] M. Kawasaki, K. Kohri, and T. Moroi, *Big-Bang nucleosynthesis and hadronic decay of long-lived massive particles*, Phys. Rev. D71 (2005) 083502, [astro-ph/0408426].

[45] J. Pradler and F. D. Steffen, *Thermal Gravitino Production and Collider Tests of Leptogenesis*, Phys. Rev. D75 (2007) 023509, [hep-ph/0608344].

[46] J. Pradler and F. D. Steffen, *Constraints on the reheating temperature in gravitino dark matter scenarios*, Phys. Lett. B648 (2007) 224–235, [hep-ph/0612291].

[47] R. Rangarajan and N. Sahu, *Gravitino production in an inflationary universe: A fresh look*, Mod. Phys. Lett. A23 (2008) 427–436, [hep-ph/0606228].

[48] R. Rangarajan and N. Sahu, *Perturbative Reheating and Gravitino Production in Inflationary Models*, Phys. Rev. D79 (2009) 103534, [arXiv:0811.1866].

[49] V. S. Rychkov and A. Strumia, *Thermal production of gravitinos*, Phys. Rev. D75 (2007) 075011, [hep-ph/0701104].

[50] T. Moroi, *Effects of the gravitino on the inflationary universe*, [hep-ph/9503210] (Ph.D. Thesis).

[51] M. Drees, R. Godbole, and P. Roy, *Theory and Phenomenology of Sparticles: An account of four-dimensional N=1 supersymmetry in high energy physics*. Hackensack, USA: World Scientific, 2004.

[52] H. Baer and X. Tata, *Weak Scale Supersymmetry: From superfields to scattering events*. Cambridge University Press, 2006.
[53] I. Affleck and M. Dine, *A New Mechanism for Baryogenesis*, *Nucl. Phys. B* **249** (1985) 361.

[54] R. H. Cyburt, J. Ellis, B. D. Fields, F. Luo, K. A. Olive, et al., *Nucleosynthesis Constraints on a Massive Gravitino in Neutralino Dark Matter Scenarios*, *JCAP* **0910** (2009) 021, [arXiv:0907.5003](http://arxiv.org/abs/0907.5003).