Entangled states in graphene- detection and use

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Abstract. Shot noise cross-correlations turning positive for electrons is a sign of spin singlet entanglement. We propose a setting where graphene with induced superconducting properties is employed to generate entangled electrons in spatially separated regions of graphene. The easy tunability of graphene in application of a gate voltage could make the detection of such entangled states possible.

1. Introduction
Graphene has captured the interest of the physics community because of its many versatile applications to microelectronics and its connection to relativistic quantum field theory[7]. Here, we create and detect entangled states in graphene sheets where superconductivity is induced via the proximity effect [1]. This idea involves extracting the Cooper pair, the most entangled state found in matter, into different sheets such that the electrons involved are not paired but remember their original correlations. In normal metals coupled to a superconductor it was found that such processes detected via the current-current correlations across the two normal metals could be positive [2,3]. This is in contrast to the expectation that since there are only fermionic excitations in normal metals these correlations should be negative. The possibility of obtaining positive value is a good indicator of spin singlet entanglement. Unfortunately, experiments in such a normal metal Y-junction coupled to a superconductor do not give the desired results. One reason being the difficulty in manipulating the Fermi energy of metals with the application of a gate voltage. This difficulty does not arise in graphene, where its Fermi energy can be very easily tuned. This is expected to lead to direct experimental verification of entanglement in superconducting materials.

The methodology used to calculate the current-current correlations, usually termed the noise, in mesoscopic parlance is well documented. The noise cross-correlations have been calculated in the context of normal metal-superconductor-normal metal structures[3]. The extension to graphene will follow similar steps. To be more precise, we consider a graphene sheet with superconductivity induced in a certain region(Gs), between the insulators(Gi) as in Fig. 1, while the rest is normal Graphene(Gn). This superconducting region is separated from the normal regions by two barriers, giving in effect, a region of effectively higher potential. The reason for this is that modulating the strength of the potentials can be quite easily done via gate voltages and this in turn makes the detection of entanglement, via the noise, possible.

2. Theory
One can demonstrate that there are additional processes occurring at the normal graphene-superconducting graphene junctions than those seen at normal metal-superconductor
Figure 1. An overview of the setting from the top. Two insulating layers of graphene (Gi’s) on either side of the superconducting graphene layer (Gs). Voltages $V_1$ and $V_2$ are applied to either ends of the normal graphene layers (Gn’s). Schematic of specular crossed Andreev reflection is also depicted. Incident electron at angle $\theta$ (IE). Reflected electron at angle $-\theta$ (RE). Andreev reflected hole at angle $\theta_A$ (AR). Specular Andreev reflected hole at angle $-\theta_A$ (SAR). Electron like quasiparticle (ELQ). Hole like quasiparticle (HLQ). Crossed Andreev reflection at angle $\theta_A$ (CAR). Specular crossed Andreev reflected hole at angle $-\theta_A$ (SCAR). Electron co-tunnelling at angle $\theta$ (EC).

Figure 2. Energy-Momentum diagram to explain specular crossed Andreev reflection where the regions of normal (N) graphene and superconducting (S) graphene are as indicated. (a) $E_F > \Delta$ regime where Andreev and crossed Andreev reflection occur in the same band, and (b) $E_F \ll \Delta$ regime where Andreev and crossed Andreev reflection occur in a specular fashion across the bands.

junctions [4]. These are local specular Andreev reflection and crossed (non-local) specular Andreev reflection. Importantly, Andreev reflection in graphene can switch the valleys. This process is known as, specular Andreev reflection [1]. In the process of normal Andreev reflection, an incident electron from the normal metal side is reflected as a hole which retraces the trajectory of the electron. In specular Andreev reflection, the reflected hole follows the trajectory which a normally reflected electron would have. In this work we see in addition to this the possibility of specular crossed Andreev reflection, wherein a hole is reflected at the other lead but in a specular fashion, see Fig. 1. Fig. 2 explains the processes involved in SCAR.

The system can be described by the Dirac Bogoliubov-de Gennes equation that assumes the form [7]

\[
\begin{pmatrix}
\hat{H} - E_F \hat{I} &=& \Delta \hat{I} \\
\Delta \hat{I} &=& E_F \hat{I} - \hat{T} \hat{H} \hat{T}^{-1}
\end{pmatrix}
\Psi = E \Psi,
\]

where $E$ is the excitation energy, $\Delta$ is the superconducting gap, $\Psi$ is the wavefunction and $\hat{T}$
represents $4 \times 4$ matrices. In the above equation

$$\hat{H} = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix}, \quad H_\pm = -i v_F (\sigma_x \partial_x \pm \sigma_y \partial_y)$$

(2)

Here $v_F$ is the energy independent Fermi velocity for graphene, while the $\sigma$’s denote Pauli matrices that operate on the sublattices $A$ or $B$. The subscripts of Hamiltonian $H$ refer to the valleys of $K_+$ and $K_-$ in the Brillouin zone. $T = -\tau_y \otimes \sigma_y C$, $C$ being complex conjugation, is the time reversal operator.

Let us consider an incident electron from the normal side of the junction ($x < -d$) with energy $E$. For a right moving electron with an incident angle $\theta$ the eigenvector and corresponding momentum reads $\psi^{(e)}_{\pm} = [1, e^{i\theta}, 0, 0]^T e^{i p^e_{\pm} x}$, $p^e = (E + E_F)/v_F$. A left moving electron is described by the substitution $\theta \rightarrow \pi - \theta$. If Andreev-reflection takes place, a left moving hole is generated with energy $E$, angle of reflection $\theta_A$ and its corresponding wavefunction is given by $\psi^{(h)} = [0, 0, 1, e^{-i\theta_A}]^T e^{-i p^h x}$, $p^h = (E - E_F)/v_F$. The superscript $e(h)$ denotes an electron-like (hole-like) excitation. Since translational invariance in the $y$-direction holds the corresponding component of momentum is conserved. This condition allows for the determination of the Andreev reflection angle $\theta_A$ through $p^h \sin(\theta_A) = p^s \sin(\theta)$. There is no Andreev reflection and consequently no sub-gap conductance for angles of incidence above the critical angle $\theta_c = \sin^{-1} ((|E - E_F|)/E + E_F)$.

In the insulators, $-d < x < 0$ and $L < x < L + d$, the eigenvector and momentum of a right moving electron are given by, with $i = 1, 2$: 

$$\psi^{(i)}_{\pm} = [1, e^{i\theta_i}, 0, 0]^T e^{i p_{\pm}^{(i)} x}, p_{\pm}^{(i)} = (E + E_F - V_i)/v_F, \quad \text{while a left moving hole is described by} \quad \psi^{(i)}_{\pm} = [0, 0, 1, e^{-i\theta_i}]^T e^{-i p^{(i)}_{\pm} x}, p^{(i)}_{\pm} = (E - E_F + V_i)/v_F.$$ 

On the superconducting side of the system, $(0 < x < L)$, the possible wavefunctions for transmission of a right-moving quasiparticle with excitation energy $E > 0$ read

$$\Psi^+_{\pm} = (u(\theta^+), u(\theta^+), e^{i\theta^+}, e^{i\theta^+})^T e^{i p_{\pm}^+ x}, \Psi^-_{\pm} = (v(\theta^-), v(\theta^-), e^{-i\theta^-}, e^{-i\theta^-})^T e^{i p_{\pm}^- x}, \theta^+ = (E + E_F + U_0\sqrt{E^2 - |\Delta(\theta^+)|^2})/v_F \quad \text{and} \quad q^\pm = (E + E_F + U_0 - \sqrt{E^2 - |\Delta(\theta^-)|^2})/v_F.$$ 

In the sub-gap regime the quasi-particle wave-vectors have a small imaginary component as $q^{\pm \theta} = (1/v_F)(E_F + U_0 \pm 1/\xi)$, where $\xi = hv_F/\Delta$, is the coherence length. The coherence factors are given by $u(\theta) = \sqrt{1 + \sqrt{1 - |\Delta(\theta)|^2/E^2}/2, \quad v(\theta) = \sqrt{1 - 1 - |\Delta(\theta)|^2/E^2}/2}$.

We have also defined $\theta^+ = \theta^\xi_A$, $\theta^- = \pi - \theta^\xi_A$, and $e^{i\theta^x} \triangleq e^{i\theta^x} / |\Delta(\theta^x)|/|\Delta(\theta^x)|$. The transmission angles $\theta^\xi_A$ for the electron-like and hole-like quasi-particles are given by $q_i \sin \theta^\xi_A = p^e \sin \theta, i = e, h$. In the following we limit ourselves to the regime where $U_0 \gg \Delta$, such that the mean field conditions for superconductivity are satisfied. The trajectory of the quasi-particles in the insulating region are defined by the angles $\theta_0$ and $\theta^A_0$. These angles are related to injection angles by $\sin \theta_0 = (E + E_F)/(E + E_F - V_i), \sin \theta^A_0 = (E + E_F)/(E - E_F + V_i)$, wherein $V_i, i = 1, 2$ denotes the strengths of the potential barrier which model the insulating regions Gi1 and Gi2. Here, we adopt the thin barrier limit defined as, $\theta_0, \theta^A_0$ and $d \rightarrow 0$, while $V \rightarrow \infty$, such that $p^e_{\pm} d, p^h_{\pm} d \rightarrow \chi_0$. From the above equation, when $V_i \rightarrow \infty, \theta_0, \theta^A_0 \rightarrow 0$. To solve the scattering problem, we match the wavefunctions at four interfaces: $\psi_{|x = -d} = \psi_{|x = 0} = \Psi_{|x = L} = \Psi_{|x = L + d} = \psi_{|x = L + d}$. Solving the above equations leads to the amplitude of Andreev reflection, $s_{eh}^{11}$, normal reflection, $s_{eh}^{11}$, and of crossed Andreev reflection, $s_{eh}^{11}$. To derive an expression for the shot noise in multi terminal settings we proceed similarly to [2]. The fluctuations of the current away from the average is termed noise. A general expression for current fluctuations between any two arbitrary leads is given by $N_{ij}(\tau) = \langle \Delta I_j(t) \Delta I_i(t + \tau) + \Delta I_i^+(t) \Delta I_i^-(t + \tau) \rangle$, where $\Delta I_i(t) = I_i(t) - \langle I_i(t) \rangle$. The Fourier transform of this gives $N_{ij}(w) \delta(w + w^*) = \langle \Delta I_j(w^*) \Delta I_i(w) + \Delta I_i^+(w) \Delta I_i^-(w) \rangle$. For simplicity we consider the experimentally feasible zero frequency noise limit, since displacement currents are absent in this
The current operator is given by
\[
\hat{I}_i (w = 0) = \sum_{k, l \in G_{n_1}, G_{n_2}, G_s} q_\alpha \int dE A_{k\gamma, l\delta}^{(i\alpha)} \hat{a}^\dagger_{k\gamma} \hat{a}_{l\delta},
\]
with \( A_{k\gamma, l\delta}^{(i\alpha)} = \delta_{ik}\delta_{l} - s_{ik\gamma}^{\alpha\delta} \), where Greek indices denotes the nature (\( e \) for electrons, \( h \) for holes) of the incoming/outgoing particles with their associated charges \( q_\alpha \), while Latin indices \( l, k \) identify the graphene sheets. From the expressions of the noise and Eq.3 the zero frequency noise cross-correlations between the currents at left and right normal graphene sheets \((G_{n_1}, G_{n_2})\) become [2]
\[
N_{12} = \sum_{k, l \in G_{n_1}, G_{n_2}, G_s} q_\alpha q_\beta \int \frac{\pi}{2} d\theta \cos \theta \int dE A_{k\gamma, l\delta}^{(1\alpha)} A_{l\delta, k\gamma}^{(2\beta)} f_{k\gamma}(1 - f_{l\delta})
\]
\( f_{k\gamma} \) is a Fermi function for particles of type \( \gamma \) in reservoir \( k \).

3. Results

![Figure 3](image)

**Figure 3.** Noise cross-correlations as function of the gate voltage \((\chi = \chi_1)\) applied to the left insulator. The right insulator is fixed at gate voltage \(\chi_2 = 0\), while \(U_0 = 1000\Delta\) and \(V_2 = 0.2\Delta\).

We detect shot noise cross-correlations across two graphene sheets (left and right) separated by a graphene-superconducting region. Shot noise cross-correlations are current-current correlations. The currents generated are because of two processes the crossed Andreev(CAR) and electron co-tunneling(EC). EC gives rise to negative correlations while CAR leads to positive correlations. CAR processes are those in which either a cooper pair breaks apart across two strips or two electrons travelling from either strip come together in the superconducting region, this naturally leads to positive correlations. EC processes on the other hand involve electron tunnelling across the superconducting region leading to negative correlations.

In Fig. 3 we plot the shot noise cross-correlations as function of the gate voltage which tunes the strength of the insulators in the system, the bias voltage applied is as stated in the figure caption with \( L < \xi \) and \( L = \xi \). As the effective barrier strength changes, one sees negative cross-correlations turning positive for \( L < \xi \). This indicates that a gate voltage can tune the entanglement properties. More interesting is the case \( L = \xi \), where noise cross-correlations turn completely positive enabled by the strong CAR signal. In the specular regime the noise is enhanced. The behavior depicted in Fig. 3 is of significance for the experimental detection of entanglement in solid state systems. It shows for the first time that a gate voltage can change the sign of noise cross-correlations unlike that seen in normal metal counterparts. The magnitude of the noise cross-correlations for \( L > \xi \) are very much reduced (not plotted). However herein also one also sees completely positive noise cross-correlations.
4. Conclusions

Finally, to conclude, these generated entangled states could be used to teleport and swap\textsuperscript{5} the entanglement between pairs of electrons. In theory, it is possible to teleport the information encoded in a quantum object to another place arbitrarily far via entangled states. In practice, though, only information about a photon has been teleported. It is advantageous to perform quantum teleportation via electrons. The aim being to speed up processing since teleportation would obviate any need for the physical transport of electrons.

5. References

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