Radiation-induced magnetoresistance oscillations in monolayer and bilayer graphene

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We examine the characteristics of the microwave/mm-wave/terahertz radiation-induced magnetoresistance oscillations in monolayer and bilayer graphene and report that the oscillation frequency of the radiation-induced magnetoresistance oscillations in the massless, linearly dispersed monolayer graphene system should depend strongly both on the Fermi energy, and the radiation frequency, unlike in the case of the massive, parabolic, GaAs/AlGaAs 2D electron system, where the radiation-induced magnetoresistance oscillation frequency depends mainly on the radiation frequency. This possible dependence of the magnetoresistance oscillation frequency on the Fermi level at a fixed radiation frequency also suggests a sensitivity to the gate voltage in gated graphene, which suggests an in-situ tunable photo-excitation response in monolayer graphene that could be useful for sensing applications. In sharp contrast to monolayer graphene, bilayer graphene is expected to show radiation-induced magnetoresistance oscillations more similar to the results observed in the GaAs/AlGaAs 2D system. Such expectations for the radiation-induced magnetoresistance oscillations are presented here to guide future experimental studies in both of these modern atomic layer material systems.

Carrier scattering gives rise to electrical resistance – a measure of frictional losses within semiconductor specimens. Under microwave/mm-wave/terahertz photoexcitation, high mobility 2D electron systems confined, for example, within GaAs/AlGaAs heterostructures exhibit large amplitude “1/4 cycle shifted” magnetoresistance oscillations with resistance maxima in the vicinity of \( E = (j + 3/4) \hbar \omega_c \), and nodes in the resistance oscillations in the vicinity of the cyclotron resonance, and integral and half-integral cyclotron resonance harmonics, i.e., \( E = j \hbar \omega_c \) and \( E = (j + 1/2) \hbar \omega_c \). Here, \( E \) = energy, \( \omega_c = eB/m^* \), \( e \) = electron charge, \( m^* \) = electron effective mass, \( \hbar \) = the reduced Planck constant, and \( B \) = the magnetic field. Most remarkably, at the lowest temperatures under modest photo-excitation, the deepest resistance minima saturate into zero-resistance states, about \( m^* \) = electron effective mass, \( \hbar \) = the reduced Planck constant, and \( B \) = the magnetic field. Such expectations for the radiation-induced magnetoresistance oscillations are presented here to guide future experimental studies in both of these modern atomic layer material systems.

Radiation-induced magnetoresistance oscillations have already served to characterize material systems such as GaAs/AlGaAs heterostructures, strained Si/SiGe, and the oxide MgZnO/ZnO 2DES system for scattering lifetimes, and effective masses. Thus, one expects that photo-excited magnetotransport studies of modern atomic layer 2D systems such as monolayer and bilayer graphene could potentially unveil new science and applications. However, the expectations for device response of these materials is not known and such phenomena have not been observed thus far in the graphene system. As a consequence, a question of interest is: what will be the oscillatory resistance response of modern atomic layered 2D materials such as, for example, graphene under electromagnetic wave excitation in a magnetic field? Below, we examine the expected characteristics for both the monolayer and bilayer graphene systems.

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systems. As mentioned above, our results suggest a strong sensitivity of the response in monolayer graphene to both the Fermi energy and the radiation frequency. However, bilayer graphene is expected to show a more GaAs/AlGaAs like behavior, characteristic of parabolically dispersed systems. The results are summarized below.

It is well known that the Landau level dispersion for Dirac fermions in monolayer graphene is given by

$$E_N = \text{sgn}(N)(2\hbar e B N)^{1/2} v_F$$

Here, $N$ is the Landau level index, and $v_F$ is the carrier velocity. Suppose that the Fermi level lies in the $N$th Landau level, $E_F = E_N$, and the electromagnetic radiation induces transitions from the $N$th Landau level to the $N+q$th Landau level, such that $E_{N+q} = E_F + hf$, where $h$ is Planck’s constant, and $f$ is the radiation frequency. From studies of the GaAs/AlGaAs system, it is known that nodes in the radiation-induced magnetoresistance oscillations appear when the radiation energy spans such an integral number of Landau levels, i.e., $E_{N+q} - E_N = hf$, where $q$ denotes the order of the node. Consider $E_{N+q}^2 = 2\hbar e B q v_F^2$, and note that

$$E_{N+q}^2 - E_N^2 = 2\hbar e B q v_F^2$$

Apply the identity $E_{N+q}^2 - E_N^2 = (E_{N+q} - E_N)(E_{N+q} + E_N)$. Let $E_{N+q} - E_N = hf$ and $E_{N+q} + E_N = 2E_F + hf$, then, by substituting into Eq. 1, we obtain

$$(2E_F + hf)hf = 2\hbar e B q v_F^2$$

or

Figure 1. (a) For monolayer graphene, the magnetic field values, $B$, for nodes in the microwave induced magnetoresistance oscillations are plotted vs. the inverse node index, $1/q$, with $q = 1, 2, 3, \ldots$, for different values of the Fermi energy, $E_F$, at a microwave frequency of $f = 100$ GHz. The inset shows the characteristic field or oscillation frequency $B_f$ of the microwave induced magnetoresistance oscillations as a function of the Fermi energy, $E_F$, for radiation frequencies $f = 25, 50, 100, 200,$ and $400$ GHz. (b) This figure illustrates the expected oscillatory magnetoresistance, $\Delta R$, vs. $B$, in monolayer graphene under microwave photo-excitation at $f = 100$ GHz for three values of the Fermi energy, $E_F$. Note that the characteristic magnetic field, $B_f$, of the oscillatory magneto-resistance increases with $E_F$.
Thus, Eq. 3 describes the magnetic field values for the $q$’th node of radiation-induced magnetoresistance oscillations in monolayer graphene and it suggests that such oscillations in monolayer graphene are also dependent upon value of the Fermi energy, $E_F$, unlike in the GaAs/AlGaAs system. In Fig. 1(a), we plot the nodal positions in magnetic field, $B$, vs. the inverse of integers, $1/q$, for several typical values of the Fermi energy in monolayer graphene. The figure shows a set of straight lines, which indicates that the expected magnetoresistance oscillations are periodic in $B^{-1}$, just as in the GaAs/AlGaAs system. The characteristic frequency or field, $B_f$, of the radiation-induced magnetoresistance oscillations that appears in the empirical formula for the oscillatory magnetoresistance lineshape $\Delta R \approx - \exp(-\lambda B) \sin(2\pi B_f/B)$, is found from the slopes in Fig. 1(a) of $B$ vs. $1/q$, which indicates that $B_f$ increases with $E_F$, at a fixed radiation frequency, $f = 100$ GHz. The inset of Fig. 1(a) shows the variation of this characteristic frequency $B_f$ of the radiation-induced magnetoresistance oscillations with $E_F$ at several radiation frequencies, $f = 25, 50, 100, 200$ and $400$ GHz. The inset conveys that $B_f$ increases faster with $E_F$ at larger radiation frequencies $f$. Note that $B_f$ increases linearly with $f$ over this range of $f$ since $E_F \gg hf$, see eqn. 3. For $f = 100$ GHz, $hf = 4.125 \times 10^{-4}$ eV is much smaller than a small practical value for $E_F$ such as $E_F = 5 \times 10^{-3}$ eV.

The expectations for the oscillatory resistance, $\Delta R \approx - \exp(-\lambda B) \sin(2\pi B_f/B)$, in the monolayer graphene system are illustrated in Fig. 1(b), which exhibits $\Delta R$ vs. $B$ at $E_F = 0.5, 0.25$, and $0.166$ eV for photoexcitation at $f = 100$ GHz. Since the characteristic field $B_f$ increases with $E_F$, and the parameters $B, f, e$, and $v_F$ in Eq. 3 can be

$$B = \frac{1}{n} (\pi f/e)(2E_F/v_F^2 + hf/v_F^2)$$

(3)
determined or are well known, the radiation-induced magnetoresistance oscillations in monolayer graphene can be utilized to accurately determine $E_F$.

From the experimental perspective, measurements can be carried out as a function of the magnetic field, $B$, or, as is more typical for graphene, vs. the gate voltage $V_G$. In monolayer graphene, the carrier density, $n_q$, varies as the square of the Fermi energy, i.e., $n_q = \frac{\pi}{h^2} \left( \frac{E_F}{v_F} \right)^2$. Then,

$$
B_f = \left( \frac{\pi f}{e} \right) \left( \frac{1}{v_F^2} \right) \left( 2 \hbar v_F (n_q^2/4\pi)^{3/2} + hf \right)
$$

Further, for graphene on top of 300 nm SiO$_2$ on doped Si the relation between the gate voltage and the carrier density is $n_q = \text{sgn}(\Delta V_G)\alpha|\Delta V_G|$ with $\alpha = 7.2 \times 10^{10}$ cm$^2$/V$^2$ \cite{Gusynin2000}. Upon inserting these relations, we obtain from eqn. 4:

$$
B_f = \left( \frac{\pi f}{e} \right) \left( \frac{1}{v_F^2} \right) \left( 2 \hbar v_F (\alpha|\Delta V_G|/4\pi)^{3/2} + hf \right)
$$

Figure 2(a) exhibits the dependence of the frequency or characteristic field $B_f$ of the magnetoresistance oscillations vs. the electron density, i.e., $n_q = n$, and vs. the difference in the gate voltage with respect to the neutrality voltage, $V_N$, i.e., $\Delta V_G = V_G - V_N$. The square root dependence observed in Eqs 4 and 5 is manifested as a sub-linear variation of $B_f$ with respect to these parameters at a fixed radiation frequency, $f$, i.e., $B_f \approx n^{1/2}$ and $B_f \approx \Delta V_G^{1/2}$. 

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**Figure 3.** (a) For radiation-induced magneto-resistance oscillations in bilayer graphene, the inverse node index, $1/q$, for $q = 1, 2, 3...$ is plotted vs the magnetic field, $B$, for various Fermi energies, $E_F$, at a radiation frequency, $f = 100$ GHz. (b) For radiation-induced magnetoresistance oscillations in bilayer graphene, the inverse node index, $1/q$, for $q = 1, 2, 3...$ is plotted vs. the magnetic field, $B$, for radiation frequencies $f = 25, 50, 100, 200, \text{and} 400$ GHz. The plot implies that increasing the radiation frequency shifts the magnetoresistance oscillations to higher magnetic fields. The inset shows the magnetoresistance oscillation frequency, $B_f$, vs the radiation frequency, $f$. The plot shows that the slope of the line is 1.32 mT/GHz which corresponds to an effective mass ratio $m^*/m = 0.037$. 

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Expectations for gate voltage dependence of the radiation-induced magnetoresistance oscillations in monolayer graphene following from Eq. 5 are exhibited in Fig. 2(b) for $f = 200 \text{GHz}$. The Fig. 2(b) shows that oscillations grow in amplitude with increasing magnetic field. Further, it is evident that the number of oscillations over a given span of gate voltage decreases with increasing $B$. Finally, the spacing between, say, successive oscillatory minima, increases with increasing $\Delta V_B$ because the magnetoresistance oscillations are actually periodic in $\Delta V_B^{1/2}$. From these numerical studies, it is clear that radiation-induced oscillations in monolayer graphene should be substantially different than the oscillations observed in the GaAs/AlGaAs system. The key differences are the dependence of the characteristic field or frequency $B_0$ of the magnetoresistance oscillations upon the Fermi energy when $E_F \gg h\nu$ even at a fixed radiation frequency in monolayer graphene (Fig. 1(a)), the expected dependence of $B_0$ on the gate voltage (Fig. 2), and the possibility of using such magnetoresistance oscillations to measure a momentary Fermi energy in monolayer graphene.

Unlike in monolayer graphene, charge carriers in bilayer graphene are massive fermions as a consequence of the parabolic band structure. Bilayer graphene is zero-gap system in the absence of a transverse electric field and it develops a bandgap under a transverse electric field. Consider the zero-gap case: The dispersion of carriers in zero-gap bilayer graphene under the influence of a magnetic field is given by $E_{\text{nk}} = (N(N+1))^{1/2}\omega_0 B$, where $\omega_0 = eh/m^*$, and $m^* = 0.037 m$ is the nominal effective mass of the carriers. The key difference is the relative insensitivity of the results to the value of the Fermi energy for bilayer graphene, unlike in the case of the parabolic band structure. Bilayer graphene is zero-gap system in the absence of a transverse electric field and it develops a bandgap under a transverse electric field. Consider the zerogap case: The dispersion of carriers in zero-gap bilayer graphene under the influence of a magnetic field is given by $E_{\text{nk}} = (N(N+1))^{1/2}\omega_0 B$, where $\omega_0 = eh/m^*$, and $m^* = 0.037 m$ is the nominal effective mass of the carriers. Here, the + and $-$ describe the response of the electron and the hole systems, and $N$ can take on positive and negative integers for electrons and holes, respectively. To determine the expected response for the radiation-induced magnetoresistance oscillations in bilayer graphene with electrons, we suppose that at a magnetic field, $B$, the Fermi level lies in the $N$th Landau level, $E_{\text{F}} = E_{\text{nk}}$, and the electromagnetic radiation induces transition from Landau level $N$ to the $N + q$ Landau level so that $E_{\text{nk}+q} = E_{\text{nk}} + h\nu$. Further, assume that nodes in the radiation-induced magnetoresistance oscillations will appear when the radiation energy spans an integral number of Landau levels, i.e., $E_{\text{nk}+q} = E_{\text{nk}} + h\nu$, where $q$ is an integer that denotes the order of the node. To extract the characteristics for radiation-induced magnetoresistance oscillations in such a system, we examine:

$$E_{\text{nk}+q}^2 - E_{\text{nk}}^2 = (h\nu)^2(2E_{\text{nk}} + h\nu) = \omega_0^2 B^2 q(q + 2N + 1)$$

(6)

By setting $E_{\text{nk}+q}^2 = E_{\text{nk}}^2 = N(N+1)\omega_0^2 B^2$, we find $(N+1/2) = ((E_{\text{nk}}^2/\omega_0^2 B^2) + 1/4)^{1/2}$, to obtain

$$q = \sqrt{[(h\nu)^2(2E_{\text{nk}} + h\nu)]/(\omega_0^2 B^2) + 1/4} - \sqrt{E_{\text{nk}}^2/(\omega_0^2 B^2) + 1/4}^{1/2}$$

(7)

Equation 7 serves to determine the $B$-position of the nodes in the radiation-induced magnetoresistance oscillations as a function of the magnetic field for different values of the Fermi energy, $E_{\text{F}}$. Figure 3(a) presents $1/q$ vs. $B$ for different values of the $E_{\text{F}}$ at $f = 100 \text{GHz}$ for bilayer graphene. The remarkable feature in this figure is the relative insensitivity of the results to the value of the Fermi energy for bilayer graphene, unlike in the case of monolayer graphene (see Fig. 1(a)). Thus, in this sense, bilayer graphene looks more similar to the GaAs/AlGaAs system.

The expected nodal $B$-positions, which are insensitive to $E_{\text{F}}$ in bilayer graphene, are examined for different $f$ in Fig. 3(b). Figure 3(b) shows that a given node, $q = 1, 2, 3, \ldots$, shifts to higher $B$ as $f$ increases. Further, the frequency or characteristic field $B_0$ of the radiation-induced magneto-resistance oscillations increases linearly with $f$ in the limit where $E_{\text{F}} \gg h\nu$, as indicated in the inset of Fig. 3(b). Indeed, the characteristic field of or frequency $B_0$-field should shift at the rate of $1.37 \emph{mT}/\emph{GHz}$ in bilayer graphene vis-à-vis the $\approx 2.35 \emph{mT}/\emph{GHz}$ shift observed in the GaAs/AlGaAs system. These calculations have assumed a fixed effective mass of $m^* = 0.037 m$ for bilayer graphene, while it is known that non-parabolicity could provide for variation in $m^*$ with $E_{\text{F}}$. Thus, studies of radiation induced magnetoresistance oscillations in bilayer graphene as a function of the carrier density could serve to characterize the non-parabolicity and determine the effective mass as a function of the energy.

In comparing the expectations for monolayer and bilayer graphene, the striking feature is the great dissimilarity in the $E_{\text{F}}$ dependence of the radiation-induced oscillatory magnetoresistance characteristics. For monolayer graphene, the oscillations depend strongly on $E_{\text{F}}$ and therefore also on $\Delta V_B$. On the other hand, for bilayer graphene, there is a lack of sensitivity to $E_{\text{F}}$ and therefore also on $\Delta V_B$. This suggests that the characteristics of the observed magnetoresistance oscillations could also serve to differentiate between these two types of graphene.

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**Author Contributions**

Modeling by R.G.M., A.K. and R.M. Manuscript by R.G.M. and A.K.

**Additional Information**

**Competing Interests:** The authors declare no competing interests.

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