Thermal hadron production in relativistic nuclear collisions: the hadron mass spectrum, the horn, and the QCD phase transition

A. Andronic\textsuperscript{a}, P. Braun-Munzinger\textsuperscript{a,b}, J. Stachel\textsuperscript{c}

\textsuperscript{a}EMMI, GSI Helmholtzzentrum für Schwerionenforschung, D-64291 Darmstadt, Germany
\textsuperscript{b}Technical University Darmstadt, D-64289 Darmstadt, Germany
\textsuperscript{c}Physikalisches Institut der Universität Heidelberg, D-69120 Heidelberg, Germany

Abstract

We present, using the statistical model, a new analysis of hadron production in central collisions of heavy nuclei. This study is motivated by the availability of final measurements both for the SPS (beam energies 20-160 AGeV) and for the RHIC energies (\(\sqrt{s_{NN}}=130\) and 200 GeV) and by updates in the hadron mass spectrum, which is a crucial input for statistical models. Extending previous studies by inclusion of very high-mass resonances (\(m>2\) GeV), and the up-to-now neglected scalar \(\sigma\) meson leads to an improved description of the data. In particular, the hitherto poorly reproduced energy dependence of the \(K^+/\pi^+\) ratio at SPS energies ("the horn") is now well described through the connection to the hadronic mass spectrum and, implicitly, Hagedorn’s limiting temperature. We thereby demonstrate the intimate connection between the horn and the QCD phase transition.

1 Introduction

One of the major goals of ultrarelativistic nuclear collision studies is to obtain information on the QCD phase diagram \([1]\). A promising approach is the investigation of hadron production. Hadron yields measured in central heavy ion collisions from AGS up to RHIC energies can be described very well \([2,3,4,5,6,7,8,9,10,11,12,13,14]\) within a hadro-chemical equilibrium model. In our approach \([2,4,7,12]\) the only parameters are the chemical freeze-out temperature \(T\) and the baryo-chemical potential \(\mu_b\) (and the fireball volume \(V\), in case yields rather than ratios of yields are fitted); for a review see \([15]\).

The main result of these investigations was that the extracted temperature values rise rather sharply from low energies on towards \(\sqrt{s_{NN}}\approx 10\) GeV and reach afterwards constant values near \(T\approx 160\) MeV, while the baryochemical potential decreases smoothly as a function of energy. The limiting temperature \([16]\) behavior suggests a connection to the phase boundary and it was, indeed, argued \([17]\) that the quark-hadron phase transition drives the equilibration dynamically, at least for SPS energies and above. Considering
also the results obtained for elementary collisions, where similar analyses of hadron multiplicities, albeit with several additional, non-statistical parameters (see [18,19] and refs. therein), yield also temperature values in the range of 160 MeV, alternative interpretations were put forward. These include conjectures that the thermodynamical state is not reached by dynamical equilibration among constituents but rather is a generic fingerprint of hadronization [20,21], or is a feature of the excited QCD vacuum [22]. The analysis presented below lends further support to the interpretation that the phase boundary is reflected in features of the hadron yields.

While in general all hadron yields are described rather quantitatively, a notable exception was up-to-now the energy dependence of the $K^+ / \pi^+$ ratio which exhibits a rather marked maximum, “the horn”, near $\sqrt{S_{NN}} \simeq 10$ GeV [23]. The existence of such a maximum was, in fact, predicted [24] within the framework of the statistical model, but the observed rather sharp structure could not be reproduced [12]. Other attempts to describe the energy dependence of the $K^+ / \pi^+$ ratio within the thermal model [11,14] were also not successful, except when an energy-dependent light quark fugacity was used as an additional parameter [14]. Furthermore, all attempts to reproduce this structure within the framework of hadronic cascade models also failed, as is discussed in detail in [23]. As a consequence, the horn structure is taken in [23] as experimental evidence for the onset of deconfinement and quark-gluon plasma formation, and as support for the predictions of [25].

In this letter, we present a new analysis of hadron production in central nucleus-nucleus collisions. The motivation for this is twofold: i) on the experimental side the data set has been recently extended (and/or finalized), in particular for the SPS energy regime by the NA49 collaboration [23,26,27] and for the RHIC energies by the STAR collaboration [28], and ii) we would like to explore the consequences of an improved hadronic mass spectrum in which the $\sigma$ meson and many higher-lying resonances are included. We note that, with the exception of the $\sigma$ meson, the full hadronic mass spectrum has already been used in our recent investigation [18] of hadron production in $e^+e^-$ collisions. In the following we first discuss the update in the hadronic mass spectrum and then explore its consequences for the description of all available data from SIS to RHIC energies.

For the hadronic mass spectrum we are using the complete mass states published recently by the Particle Data Group (PDG) [29]. Not all the states have known branching ratios and in such cases we have assigned values based on analogies to the nearest states with the same quantum numbers. The systematic uncertainty introduced by this is estimated to be well below the bias introduced if the high-mass states would not be considered. Another recent development in the field of hadron spectroscopy is the strengthening of the case for the existence of the $\sigma$ meson (labelled $f_0(600)$ in the PDG compilation [29]) [30,31,32]. The $\sigma$ meson is a broad structure, whose properties are extracted from fits of measurements in various channels (see [30] for a recent review) and decays into $\pi^+\pi^-$. We have adopted as “nominal” the values for the mass and (Breit-Wigner) width $\Gamma$ as extracted in ref. [30], $m_\sigma=484$ MeV, $\Gamma_\sigma=510$ MeV and will investigate the effect of different parameters, namely also the case $m_\sigma=600$ MeV, $\Gamma_\sigma=600$ MeV.

---

1 We ignore here that the width of the $\sigma$ meson is not small compared to its mass. Further investigations will have to deal with this issue but we note that for the $\rho$ meson the situation is not so different.
To explore the consequences of the improved mass spectrum we show as a function of energy, in Fig. 1, the increase of yields of pions after inclusion of high-mass resonances, relative to the case when hadrons up to a mass of 2 GeV were considered, as in our earlier study [12], and after the inclusion of the $\sigma$ meson. The observed energy dependence is driven mostly by the change in temperature, also shown in Fig. 1. The parametrizations for the energy dependence of $T$ and $\mu_b$ established in [12] were employed. The high-mass resonances lead to an increase of about 13% for the calculated pion yields. This increase levels off near the point where the temperature reaches its limiting value, thereby sharpening the structure in the $K^+/\pi^+$ ratio, as will be demonstrated below. A further 3.5% increase of the calculated pion yield is due to the presence of the $\sigma$ meson, with a rather small dependence on its mass and width. For $m_\sigma=600$ MeV, $\Gamma_\sigma=600$ MeV the calculations lead to about 1% fewer pions. For the $\Lambda$ hyperons, the new high mass resonances lead to an increase in the calculated production of about 22%, while the addition of the $\sigma$ meson has no effect. An increase of up to 6% is observed for protons, while for kaons this increase is about 7%.

We have shown earlier [12] that the thermal fits of hadron yields and of ratios of yields lead to very similar results. For the present analysis we focus on fits of yields. We mostly utilize mid-rapidity data, but, at lower SPS energies, fit also the hadron yields for the full phase space.

In Fig. 2 we present a comparison of measured and calculated hadron yields at the energies of $\sqrt{s_{NN}}=7.6$ GeV (beam energy of 30 AGeV at SPS) and $\sqrt{s_{NN}}=200$ GeV. The model is successful in reproducing the measurements and this applies to all energies, from 2 AGeV beam energy (fixed target) up to the top RHIC energy of $\sqrt{s_{NN}}=200$ GeV. The reduced $\chi^2$ values are reasonable. In most cases the fit quality is improved compared to our earlier analysis [12], even though the experimental errors are now smaller. Whenever
Figure 2. Experimental hadron yields and model calculations for the parameters of the best fit at the energies of 7.6 (left panel) and 200 GeV (right panel; the Ω yield includes both Ω− and Ω+).

several independent measurements are available, we have employed a weighted mean of the data following the recipe given in the introduction of [29]. A special case is that of the top SPS energy (\(\sqrt{s_{NN}}=17.3\) GeV), where the disagreement between the NA49 and the NA57 data persists. As in the case of our earlier analysis [12], we have moved the difference in the fit results into the respective systematic error. A disagreement between the experiments is seen at the top RHIC energy for pions and protons, see Fig. 2, which is the reason of the large reduced \(\chi^2\). A fit of ratios is in this case more suited, but we note that a fit of the STAR yields alone gives \(T=162\) MeV, \(\mu_b=32\) MeV, \(V=2400\) fm\(^3\), with a very good \(\chi^2/N_{df}=9.0/11\). The resonances were not included in the fits, but are quite well reproduced by the model.

An important result of our analysis is that the resulting thermal parameters are close to those obtained earlier [12] and are in agreement with other recent studies [13,28]. This indeed confirms that the common practice of including in the thermal codes hadrons up to masses of 2 GeV (for instance in the publicly-available code THERMUS [33]) does not lead to significantly biased fit parameters. Nevertheless, there are small variations. In Fig. 3 we present the energy dependence of \(T\) and \(\mu_b\) in comparison to our earlier results [12]. We have parametrized \(T\) as a function of \(\sqrt{s_{NN}}\) with the following expression:

\[
T = T_{lim} \frac{1}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}}(\text{GeV}))/0.60)},
\]

with the "limiting" temperature \(T_{lim}=164\) MeV. This value is slightly higher compared to our earlier value of \(161\pm4\) MeV [12] due to the higher temperatures presently derived for the RHIC energies. The approach to \(T_{lim}\) is presently more gradual compared to our earlier parametrization.

The values of \(\mu_b\) extracted for the two lowest SPS energies deviate somewhat from the continuous trend suggested by all the other points. At these energies the fit to data in full

\[
\mu_b[\text{MeV}] = \frac{1303}{1+0.286\sqrt{s_{NN}}(\text{GeV})}
\]
Figure 3. The energy dependence of temperature and baryon chemical potential at chemical freeze-out. The results obtained here are compared to the values obtained in our earlier study [12]. The lines are parametrizations for $T$ and $\mu_b$ (see text).

phase space does lead, as expected, to larger values of $\mu_b$, which do fit in the systematics. At 40 AGeV ($\sqrt{s_{NN}}=8.8$ GeV) the resulting values of $T$ and $\mu_b$ from the fit of $4\pi$ data are very similar to those obtained from midrapidity data.

We employ the above parametrization to investigate the energy dependence of the relative production yields $K^+/\pi^+$ and $\Lambda/\pi^-$, shown in Fig. 4. The $K^+/\pi^+$ ratio shows a rather pronounced maximum at a beam energy of 30 AGeV [23], and the data are well reproduced by the model calculations. In the thermal model this maximum occurs naturally at $\sqrt{s_{NN}} \simeq 8$ GeV [24]. It is due to the counteracting effects of the steep rise and saturation of $T$ and the strong monotonous decrease in $\mu_b$. The competing effects are most prominently reflected in the energy dependence of the $\Lambda$ hyperon to pion ratio (lower panel of Fig. 4), which shows a pronounced maximum at $\sqrt{s_{NN}} \simeq 5$ GeV. This is reflected in the $K^+/\pi^+$ ratio somewhat less directly; it appears mainly as a consequence of strangeness
Figure 4. Energy dependence of the relative production ratios $K^+/\pi^+$ and $\Lambda/\pi^-$. With the dotted line we show for the $K^+/\pi^+$ ratio an estimate of the effect of higher mass resonances (see text). The dashed lines show the energy dependence of $T$ (upper panel) and $\mu_b$ (lower panel).

The model describes the $K^+/\pi^+$ data very well over the full energy range, as a consequence of the inclusion in the code of the high-mass resonances and of the $\sigma$ meson, while our earlier calculations [12] were overpredicting the SPS data. At RHIC energies, the quality of the present fits is essentially unchanged compared to [12], as also the data have changed somewhat. The model also describes accurately the $\Lambda/\pi^-$ data. We note that the maxima in the two production ratios are located at different energies [35]. The model calculations reproduce this feature in detail.

The calculated $K^+/\pi^+$ ratio (solid line in Fig. 4) is likely to decrease further at energies beyond the maximum and the peak is likely to sharpen somewhat if our presumably

---

3 Recent studies within the UrQMD model [34] suggest the possible presence of net strangeness at midrapidity; the present analysis lends no support to this conjecture.

4 On a statistical basis, with a relatively good $\chi^2/N_{\text{df}}=21/12$. 
still incomplete knowledge of the hadronic spectrum for masses larger than 2 GeV would improve. To get a qualitative idea about the size of the effect we have assumed that the true mass spectrum rises exponentially, with a Hagedorn temperature parameter \( T_H \approx 200 \text{ MeV} \) (as observed in the mass range up to 2 GeV). Based on this spectrum we have calculated the modification of the \( K^+/\pi^+ \) ratio by inclusion and exclusion of states above 3 GeV. The strongest contributions from these states to kaons comes from the decay of \( K^* \) mesons (where only 1 kaon is produced for each additional \( K^* \)) while all high mass resonances produce multiple pions, thereby reducing the \( K^+/\pi^+ \) ratio. Using these assumptions and Boltzmann suppression of high masses we arrive at the dotted line in Fig. [4] While the qualitative nature of the estimate should be underlined, it shows in any case the expected features, namely a sharpening of the peak and a further reduction in \( K^+/\pi^+ \) at high energies. The uncertainty of the calculations due to the mass and width of the \( \sigma \) meson are at the percent level only. Another few percent uncertainty, which is difficult to assess quantitatively, arises from the unknown branching ratios of the high-mass resonances.

Table 1
Predictions of the thermal model for selected hadron ratios in central Pb+Pb collisions at LHC.

| p/\pi^+ | K^+/\pi^+ | K^-/\pi^- | \Lambda/\pi^- | \Xi^-/\pi^- | \Omega^-/\pi^- |
|--------|-----------|-----------|--------------|------------|-------------|
| 0.072  | 0.164     | 0.163     | 0.042        | 0.0054     | 0.00093     |

In Table 1 we present our updated predictions for the LHC energy (\( \sqrt{s_{NN}} = 5.5 \text{ TeV} \)) [36].

In summary, we have demonstrated that by inclusion of the \( \sigma \) meson and many higher mass resonances into the resonance spectrum employed in the statistical model calculations an improved description is obtained of hadron production in central nucleus-nucleus collisions at ultra-relativistic energies. The most dramatic improvement is visible for the \( K^+/\pi^+ \) ratio, which is now well described at all energies. The “horn” finds herewith a natural explanation which is, however, deeply rooted in and connected to detailed features of the hadronic mass spectrum which leads to a limiting temperature and contains the QCD phase transition [16]. It is interesting to note that central questions in hadron spectroscopy such as the existence (and nature) of the \( \sigma \) meson apparently play an important role in quark-gluon plasma physics. Our results strongly imply that hadronic observables near and above the horn structure at a beam energy of 30 AGeV provide a link to the QCD phase transition. Open questions are whether the chemical freeze-out curve below the horn energy actually traces the QCD phase boundary at large values of chemical potential or whether chemical freeze-out in this energy range is influenced by exotic new phases such as have been predicted in [37]. In any case these are exciting prospects for new physics to be explored at RHIC low energy runs and, in particular, at the high luminosity FAIR facility. It remains a challenge, though, to identify clear signals for quark-gluon plasma properties in this energy range. At the high energy frontier, the measurements at LHC will be a crucial test of the present picture.

Acknowledgements: The authors would like to thank Jochen Wambach for pointing out the importance and special role of the \( \sigma \) meson in the hadronic mass spectrum. We acknowledge the support from the Alliance Program of the Helmholtz-Gemeinschaft.
References

[1] P. Braun-Munzinger, J. Wambach, Rev. Mod. Phys., in print, arXiv:0801.4256.
[2] P. Braun-Munzinger, J. Stachel, J.P. Wessels and N. Xu, Phys. Lett. B 344 (1995) 43
   arXiv:nucl-th/9410026 and 365 (1996) 1 [arXiv:nucl-th/9508020].
[3] J. Cleymans, D. Elliott, H. Satz, R.L. Thews, Z. Phys. C 74 (1997) 319
   arXiv:nucl-th/9603004.
[4] P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. B 465 (1999) 15
   arXiv:nucl-th/9903010.
[5] J. Cleymans, K. Redlich, Phys. Rev. C 60 (1999) 054908 [arXiv:nucl-th/9903063].
[6] F. Becattini, J. Cleymans, A. Keranen, E. Suhonen, K. Redlich, Phys. Rev. C 64 (2001)
   024901 [arXiv:hep-ph/0002267].
[7] P. Braun-Munzinger, D. Magestro, K. Redlich, J. Stachel, Phys. Lett. B 518 (2001) 41
   arXiv:hep-ph/0105229.
[8] N. Xu, M. Kaneta, Nucl. Phys. A 698 (2002) 306c.
[9] F. Becattini, J. Phys. G 28 (2002) 1553.
[10] R. Rapp, E. Shuryak, Phys. Rev. Lett. 86 (2001) 2980 [arXiv:hep-ph/0008326].
[11] F. Becattini, M. Ga´dzicki, J. Manninen, Phys. Rev. C 73 (2006) 044905
   arXiv:hep-ph/0511092.
[12] A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A 772 (2006) 167
   arXiv:nucl-th/0511071.
[13] J. Manninen, F. Becattini, Phys. Rev. C 78 (2008) 054901 [arXiv:0806.4100].
[14] J. Letessier, J. Rafelski, Eur. Phys. J. A 35 (2008) 221 [nucl-th/0504028].
[15] P. Braun-Munzinger, K. Redlich, and J. Stachel, [nucl-th/0304013] invited review in Quark
   Gluon Plasma 3, eds. R.C. Hwa and X.N. Wang, (World Scientific Publishing, 2004).
[16] R. Hagedorn, CERN-TH-4100/85 (1985).
[17] P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B 596 (2004) 61
   arXiv:nucl-th/0311005.
[18] A. Andronic, F. Beutler, P. Braun-Munzinger, K. Redlich, J. Stachel, arXiv:0804.4132 [hep-
   ph].
[19] F. Becattini, P. Castorina, J. Manninen, H. Satz, Eur. Phys. J. C 56 (2008) 493
   arXiv:0805.0964.
[20] R. Stock, Phys. Lett. B 465 (1999) 277 [arXiv:hep-ph/9905247].
[21] U. Heinz, Nucl. Phys. A 685 (2001) 414 [arXiv:hep-ph/0009170].
[22] P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J. C 52 (2007) 187 [arXiv:0704.1426].
[23] C. Alt et al. (NA49), Phys. Rev. C 77 (2008) 024903 [arXiv:0710.0118].
[24] P. Braun-Munzinger, J. Cleymans, H. Oeschler, K. Redlich, Nucl. Phys. A 697 (2002) 902 [arXiv:hep-ph/0106066].

[25] M. Gaździecki, M.I. Gorenstein, Acta Phys. Polon. B 30 (1999) 2705 [arXiv:hep-ph/9803462].

[26] C. Alt et al. (NA49), Phys. Rev. C 78 (2008) 034918 [arXiv:0804.3770].

[27] C. Alt et al. (NA49), Phys. Rev. C 78 (2008) 044908 [arXiv:0806.1937].

[28] B.I. Abelev et al. (STAR), [arXiv:0808.2041]

[29] C. Amsler et al. (Particle Data Group), Phys. Lett. B 667 (2008) 1.

[30] R. García-Martín, J.R. Peláez, F.J. Ynduráin, Phys. Rev. D 76 (2007) 074034 [hep-ph/0701025].

[31] I. Caprini, G. Colangelo, H. Leutwyler, Phys. Rev. Lett 96 (2006) 132001 [hep-ph/0512364].

[32] G. Bonvicini et al. (CLEO Collab.), Phys. Rev. D 76 (2007) 012001 [arXiv:0704.3954].

[33] S. Wheaton, J. Cleymans, M. Hauer, Comput. Phys. Commun. 180 (2009) 84 [hep-ph/0407174].

[34] J. Steinheimer et al., [arXiv:0811.4077]

[35] J. Cleymans, H. Oeschler, K. Redlich, S. Wheaton, Phys. Lett. B 615 (2005) 50 [hep-ph/0411187].

[36] A. Andronic, P. Braun-Munzinger, J. Stachel, in N. Armesto et al., J. Phys. G 35 (2008) 054001 [arXiv:0711.0974].

[37] L. McLerran and R. Pisarski, Nucl. Phys. A 796 (2007) 83 [arXiv:0706.2191].