Radiative feedback from an early X-ray background

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ABSTRACT

The first generation of stars (commonly known as population III) are expected to form in low-mass protogalaxies in which molecular hydrogen is the dominant coolant. Radiation from these stars will rapidly build up an extragalactic ultraviolet (UV) background capable of photodissociating H2, and it is widely believed that this background will suppress further star formation in low-mass systems.

However, star formation will also produce an extragalactic X-ray background. This X-ray background, by increasing the fractional ionization of protogalactic gas, promotes H2 formation and reduces the effectiveness of ultraviolet feedback.

In this paper, we examine which of these backgrounds has the dominant effect. Using a simple model for the growth of the UV and X-ray backgrounds, together with a detailed one-dimensional model of protogalactic chemical evolution, we examine the effects of the X-ray backgrounds produced by a number of likely source models. We show that in several cases, the resulting X-ray background is strong enough to offset UV photodissociation in large H2-cooled protogalaxies. On the other hand, small protogalaxies (those with virial temperatures $T_{\text{vir}} < 2000$ K) remain dominated by the UV background in all of the models we examine.

We also briefly investigate the effects of the X-ray background upon the thermal and chemical evolution of the diffuse intergalactic medium.

Key words: molecular processes – radiative transfer – galaxies: formation – cosmology: theory.

1 INTRODUCTION

In cosmological models based on cold dark matter (CDM), the first stars are believed to form within small protogalaxies, with virial temperatures $T_{\text{vir}} < 10^4$ K (Couchman & Rees 1986; Haiman, Thoul & Loeb 1996a; Tegmark et al. 1997). Cooling within these protogalaxies is dominated by molecular hydrogen, H2, which forms via the gas-phase reactions

$$\text{H} + e^- \rightarrow \text{H}^- + \gamma \tag{1}$$

$$\text{H}^- + \text{H} \rightarrow \text{H}_2 + e^- \tag{2}$$

and

$$\text{H} + \text{H}^+ \rightarrow \text{H}_2^+ + \gamma \tag{3}$$

$$\text{H}_2^+ + \text{H} \rightarrow \text{H}_2 + \text{H}^+ \tag{4}$$

even in the absence of dust. Although the fractional abundance of H2 that forms in this way is small, it is sufficient to allow for effective cooling and the formation of stars (Haiman, Thoul & Loeb 1996a; Tegmark et al. 1997; Bromm, Coppi & Larson 1999; Abel, Bryan & Norman 2000).

As soon as massive stars form, however, they immediately begin to photoionize and photodissociate this H2. Photoionization requires photons with energies greater than 15.4 eV, which are strongly absorbed by neutral hydrogen, and is only of importance within HII regions. Photodissociation, in contrast, occurs through the absorption of photons in the Lyman–Werner band system (Stecher & Williams 1967), with energies in the range 11.18–13.6 eV. These photons are not strongly absorbed by neutral hydrogen and can readily escape into the intergalactic medium (IGM) (Omukai & Nishi 1999; Glover & Brand 2001).

Initially, many of these photons will be absorbed by intergalactic H2, but its abundance is small and it is rapidly photodissociated (see Section 4.4 below). Consequently, the onset of star formation is soon followed by the appearance of an ultraviolet background radiation field. This ultraviolet background acts to suppress further star formation by photodissociating H2 within newly forming protogalaxies. The effects of this background have been studied by a number of authors (Haiman, Rees & Loeb 1996b, 1997; Ciardi et al. 2000a; Ciardi, Ferrara & Abel 2000b; Haiman, Abel & Rees 2000; Machacek, Bryan & Abel 2001). In particular, Haiman et al. (2000) study the coupled problem of the evolution of the ultraviolet background and the formation of stars.
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2 THE UV AND X-RAY BACKGROUNDS

For an observer at redshift \( z_0 \), we can write the mean specific intensity of the radiation background at an observed frequency \( v_0 \) as (Madau, Haardt & Rees 1999)

\[
J(v_0, z_0) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \epsilon(v, z)e^{-\tau(v_0,\infty,z_0)} \frac{dl}{(1+z)^3} \frac{dz}{dz},
\]

where \( v = v_0(1+z)/(1+z_0) \), \( \epsilon(v, z) \) is the proper space-averaged volume emissivity, \( \tau(v_0, z_0, z) \) is the optical depth at frequency \( v_0 \) owing to material along the line of sight from redshift \( z_0 \) to \( z \) and \( dl/dz \) is the cosmological line element. To solve this equation, we need to know how the emissivity and opacity evolve with redshift.

2.1 Emissivity

For simplicity, we write the space-averaged emissivity in terms of the global star formation rate (SFR) as

\[
\epsilon(v, z) = L(v, z)M_* \text{ erg s}^{-1} \text{Mpc}^{-3} \text{Hz}^{-1},
\]

where \( L(v, z) \) is the luminosity density (i.e. the luminosity per unit frequency) per unit star formation rate (in solar masses per year), and \( M_* \) is the global star formation rate, with units of \( M_{\odot} \text{ yr}^{-1} \text{ Mpc}^{-3} \).

In principle, \( L(v, z) \) may be a complicated function of frequency and redshift. In practice, however, Lyman–Werner band emission is dominated by massive, short-lived OB stars and declines rapidly once star formation comes to an end (see, for example, the instantaneous burst models of Leitherer et al. 1999, in which the Lyman–Werner flux declines by an order of magnitude within 4–5 Myr).

Similarly, X-ray emission is dominated by short-lived sources such as massive X-ray binaries and supernova remnants that are end products of the same massive stars. Both kinds of emission are therefore strongly correlated with the star formation rate; to a first approximation, we can assume that they are directly proportional to it, and that any redshift dependence of \( L(v, z) \) can be neglected. With this simplification, determining the emissivity breaks down into two independent problems: determining the global star formation rate as a function of redshift, and determining the luminosity density as a function of the star formation rate.

2.1.1 The star formation rate

Although we have observational constraints on the star formation rate up to \( z \sim 5 \), we have no direct constraints (and few indirect ones) at higher redshift. Consequently, any model of high-redshift star formation must inevitably be highly theoretical. Moreover, this lack of constraints motivates us to choose as simple a model as possible; more complicated (and realistic) models can always be considered once our observational knowledge improves. A good example of this kind of simple model is the one used by Haiman et al. (2000); we adopt the same model here.

We assume that star formation proceeds primarily through starbursts, of duration \( t_{\text{on}} \text{ yr} \), that are triggered when galaxies form. During the starburst, the star formation rate is assumed to be constant.

The global star formation rate in this model is given by

\[
M_* = \frac{\epsilon_0 \Delta \rho_{\text{crit}}}{t_{\text{on}}} M_{\odot} \text{ yr}^{-1} \text{ Mpc}^{-3},
\]

where \( \epsilon_0 \Delta \rho_{\text{crit}} \) is the threshold for star formation.
where $\epsilon$ is the star formation efficiency, $f_b$ is the baryon fraction (i.e. the ratio of baryons to dark matter), and where $\Delta \rho_{gal}$ is the cosmological density of matter in newly formed galaxies (with units of $M_\odot$ Mpc$^{-3}$). We assume that the value of $f_b$ in the protogalaxies is the same as in the IGM, or, in other words, that $f_b = \Omega_b/\Omega_m$. For the cosmological model adopted in Section 4, this corresponds to $f_b = 0.124$.

We further assume that the rate of change of $\Delta \rho_{gal}$ is approximately the same as the rate of change of $F(z, T_{crit})$, the total fraction of matter in haloes with virial temperatures greater than a critical temperature $T_{crit}$, or in other words that

$$\frac{d\Delta \rho_{gal}}{dz} \simeq \rho_m(z) \frac{dF}{dz}(z, T_{crit}), \tag{8}$$

where $\rho_m = 2.8 \times 10^{11} \Omega_m (1 + z)^3 M_\odot$ Mpc$^{-3}$ is the cosmological matter density. Here, $T_{crit}$ represents the minimum virial temperature required for efficient cooling; to a first approximation, haloes with $T_{vir} < T_{crit}$ are unable to cool, while those with $T_{vir} > T_{crit}$ cool rapidly and can form stars. Various different definitions of $T_{crit}$ are in use in the literature (see, for example, Rees & Ostriker 1977; Tegmark et al. 1997; Haiman et al. 2000); we discuss our particular choice in a later section.

Making this approximation is equivalent to assuming that the growth in $F(z, T_{crit})$ is dominated by the formation of new haloes with $T_{vir} > T_{crit}$ (either by monolithic collapse or by the merger of smaller objects) rather than by the accretion of matter by existing haloes with $T_{vir} > T_{crit}$. This is justified at high redshift when such haloes are rare and $F(z, T_{crit})$ is dominated by objects near $T_{crit}$, but becomes less accurate at lower redshifts. For this reason (and others, to be discussed later) we do not attempt to simulate the evolution of the background below $z = 10$.

To solve equation (8) we need to know the value of $T_{crit}$. In general, this will depend both on redshift (see, e.g., Tegmark et al. 1997) and on the intensities of the UV and X-ray backgrounds. Indeed, understanding the evolution of $T_{crit}$ with redshift is one of the main goals of this paper. Our procedure for determining $T_{crit}$ is discussed at length in Section 3; for now, we assume that it is known. In this case, we can calculate $F(z, T_{crit})$ using the Press–Schechter formalism (Lacey & Cole 1993):

$$F(z, M_{crit}) = \text{erfc} \left( \frac{\delta_c(z)}{\sqrt{2} \sigma(M_{crit})} \right), \tag{9}$$

where $\delta_c(z)$ is the critical density threshold for collapse, $\sigma(M)$ is the square root of the variance of the cosmological density field, as smoothed on a mass scale $M$, and where $M_{crit}$ is the mass of a protogalaxy with virial temperature $T_{crit}$. Although both $\delta_c$ and $\sigma(M)$ depend upon the choice of cosmological model, their behaviour is well known, and the problem of determining $F(z, M_{crit})$ reduces to the relatively simple one of relating $M_{crit}$ to $T_{crit}$.

To do this, we need to know the protogalactic density profile. Hydrodynamical simulations suggest that it is approximately isothermal (Abel et al. 2000), but representing it as a singular isothermal sphere is physically unrealistic as a result of the infinite central density of the latter. Accordingly, we follow Haiman et al. and represent it as a truncated isothermal sphere (Iliev & Shapiro 2001). With this choice, we find that

$$M_{crit} = 1.5 \times 10^7 \left( \frac{T_{crit}}{10^4 K} \right)^{3/2} (1 + z)^{-3/2} \Omega_m^{-1/2} h^{-1} M_\odot. \tag{10}$$

To determine $\Delta \rho_{gal}(z)$, we must integrate equation (8) over a redshift interval $\Delta z(t_{star})$, corresponding to the duration of the starburst; hence,

$$\Delta \rho_{gal}(z) = \int_z^{z + \Delta z(t_{star})} \rho_m(z) \frac{dF}{dz}(z, T_{crit}) \, dz. \tag{11}$$

This simple model has a number of shortcomings. For instance, it assumes that the star formation efficiency $\epsilon$ and starburst duration $t_{star}$ are both constant, independent of redshift or galaxy mass. It also assumes that each galaxy forms its stars in a single starburst and thus ignores the effects of continuous star formation and of subsequent, merger-triggered starbursts. Nevertheless, it has the virtue of simplicity, and is a good point from which to start our examination of the effects of the X-ray background.

2.1.2 The UV luminosity density

The ultraviolet flux of a star-forming galaxy is dominated by emission from young, massive O- and B-type stars. These are short-lived, with the most massive having lifetimes of only a few Myr, and thus the ultraviolet luminosity density is closely correlated with the star formation rate. Its value depends upon the spectral properties of the newly formed stellar population, and thus on their initial mass function (IMF), metallicity and age.

In a recent paper, Schaerer (2002) presented values for the photon flux in the Lyman–Werner bands calculated for a number of different metal-free stellar populations. If we assume that the spectrum within the bands is flat (a reasonable approximation), then we can convert this photon flux into a luminosity density. For a Salpeter IMF with minimum mass $M_{min} = 1 M_\odot$ and maximum mass $M_{max} = 100 M_\odot$ (model A in Schaerer 2002), we find that

$$L_\nu = 1.1 \times 10^{28} \text{erg s}^{-1} \text{Hz}^{-1} \left( M_\odot \text{yr}^{-1} \right)^{-1}. \tag{12}$$

Reducing $M_{min}$ to the more conventional value of 0.1 $M_\odot$ reduces this luminosity to

$$L_\nu = 4.3 \times 10^{27} \text{erg s}^{-1} \text{Hz}^{-1} \left( M_\odot \text{yr}^{-1} \right)^{-1}, \tag{13}$$

as we form a greater number of low-mass stars that do not contribute significantly to the Lyman–Werner flux.

Both of these results assume that the dissipative flux has stabilized at its equilibrium value and is therefore proportional to the star formation rate. This equilibrium is typically established after only 2–3 Myr, so this is generally a good approximation, even for starbursts of relatively short duration.

The above figures are appropriate so long as we are dealing with stars formed out of entirely metal-free gas. Such stars are somewhat unusual, however, as the absence of carbon means that they are unable to generate energy via the CNO cycle, which otherwise would dominate energy production in stars of mass $M \gtrsim 1.1 M_\odot$. As a result, metal-free stars are hotter than their metal-enriched counterparts (Ezer & Cameron 1971; Cassisi & Castellani 1993; Tumlinson & Shull 2000; Cojazzi et al. 2000) and have harder spectra. A surprising consequence of this fact is that a low-metallicity stellar population will produce a larger dissipative flux than a metal-free population – the lower effective temperatures move the peak in the thermal emission from the most massive stars closer to the Lyman–Werner band, causing the ionizing flux to fall but the dissipative flux to rise.

We can use the data presented in Schaerer (2002) for a stellar population with $Z = 0.02 Z_\odot$ to examine the difference that this effect makes to the Lyman–Werner flux. Using the same IMF as in equation (12), we find that

$^1$ Although both of these can be mimicked to some extent by choosing a large value for $t_{star}$. 

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\[ L_x = 1.8 \times 10^{28} \text{erg s}^{-1} \text{Hz}^{-1} \left( \frac{M_\odot}{\text{yr}} \right)^{-1}. \] (14)

Thus, raising the metallicity increases the Lyman–Werner flux, but only by approximately 60–70 per cent; as we will see in Section 4.1, this has little effect on the evolution of \( T_{\text{ex}} \).

Clearly, there are many possible models other than those considered here. Indeed, there is growing evidence that the IMF of population III stars is strongly biased towards high masses (Larson 1998; Bromm et al. 1999; Bromm, Coppi & Larson 2002; Abel et al. 2000). However, this remains uncertain, and in this paper we have chosen to err on the side of caution and assume that the high-redshift IMF is similar to that at the present day.

2.1.3 The X-ray luminosity density

In a recent study, Helfand & Moran (2001) collate data on a number of local starburst galaxies and compare the 2–10 keV X-ray fluxes measured by ASCA with the 8–1000 \( \mu \)m infrared fluxes measured by IRAS. They find that a clear correlation exists, with the total X-ray and infrared fluxes related by

\[ F_X \simeq 10^{-4} F_{\text{IR}}. \] (15)

Similar correlations have previously been reported by David, Jones & Forman (1992) and Rephaeli, Gruber & Persic (1995) for X-rays in the 0.5–4.5 and 2–30 keV energy bands, respectively.

Theoretically, we would expect such a correlation, with both the X-ray and infrared emission tracing the underlying star formation rate. For the X-rays, this occurs because the emission is dominated by massive X-ray binaries (MXRBs): binary systems consisting of a massive OB star accreting on to a compact companion (a neutron star or black hole). X-ray emission from such systems generally switches on a few million years after the formation of the compact object, and the lifetime of the emitting phase is short, typically of the order of \( (2–5) \times 10^4 \) yr (Meurs & van den Heuvel 1989). These short time-scales tie the emission closely to the underlying star formation rate (Ghosh & White 2001). The far-infrared flux, on the other hand, tracks star formation far more directly, being dominated by emission from dust heated by short-lived, massive stars.

To use this observed correlation to determine the X-ray luminosity of a star-forming galaxy as a function of its star formation rate, we use the result from the starburst models of Leitherer & Heckman (1995) that

\[ L_{\text{IR}} \simeq 1.5 \times 10^{10} \left( \frac{\text{SFR}}{1 \, M_\odot \text{yr}^{-1}} \right) L_\odot. \] (16)

We then assume that this relationship between X-ray luminosity and star formation rate remains valid as we move to higher redshifts. Note that the same need not be true for the relationship between X-ray luminosity and infrared luminosity, or infrared luminosity and star formation rate: although they serve to establish the correlation between X-ray luminosity and star formation rate at \( z = 0 \), the assumption that this correlation remains valid at higher redshift does not imply that these other correlations also remain valid. Indeed, we would expect the infrared luminosity of dust-free protogalaxies to be very much lower than would be predicted by equation (16).

Evidence that the correlation between X-ray luminosity and star formation rate does indeed remain valid at high redshift is provided by the recent stacking analysis of individually undetected Lyman break galaxies in the \textit{Chandra} Deep Field North (Brandt et al. 2001). This analysis finds that the average rest frame luminosity of the Lyman break galaxies in the 2–8 keV energy band is

\[ L_X = 3.2 \times 10^{41} \text{erg s}^{-1}. \] (17)

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Consistent with the value derived above. Although far from conclusive, this result suggests that we can extrapolate the locally observed correlation to at least as far as \( z = 4 \).

Comparing our determination of \( L_X \) with a recent calculation by Oh (2001), we find a difference of a factor of 10 in our results. Some of this disagreement is caused by the difference in X-ray energy bands considered (0.2–10 keV in Oh 2001, compared with 2–10 keV here), but some must surely be a result of intrinsic scatter in the observational data, suggesting that equation (17) should properly be regarded as an order of magnitude estimate of the true X-ray luminosity.

Given equation (17) for the X-ray luminosity as a function of the star formation rate, we calculate the X-ray luminosity density by assuming a template spectrum of power-law form

\[ L_v = L_\odot \left( \frac{v}{v_0} \right)^\alpha, \] (19)

where \( hv_0 = 1 \text{ keV} \), and requiring that

\[ L_X = \int_{v_1}^{v_2} L_v \, dv, \] (20)

where \( hv_1 = 2 \) and \( hv_2 = 10 \text{ keV} \).

Rephaeli et al. (1995) find that a weighted average of the galaxies in their sample gives a value for the spectral index of \( \alpha = 1.5 \pm 0.3 \). Adopting this value and solving for \( L_\odot \), we find that

\[ L_\odot = 3.4 \times 10^{22} \text{erg s}^{-1} \text{Hz}^{-1} \left( \frac{M_\odot}{\text{yr}^{-1}} \right)^{-1}. \] (21)

Altering \( \alpha \) changes \( L_\odot \), but never by more than 50 per cent for values consistent with the Rephaeli et al. measurement. Clearly, individual galaxies may have spectra that differ markedly from this simple template, but it should be a reasonable approximation when averaging over a large number of galaxies.

2.1.4 Alternative X-ray models

The above model is simple, and empirically motivated, but does assume that the X-ray emission of high-redshift star-forming galaxies is very similar to that of starbursts observed locally. This is a reasonable assumption in the absence of evidence to the contrary, and is probably valid as long as massive X-ray binaries continue to dominate the galactic X-ray emission. However, it is quite possible that at high redshift some other type of source will come to dominate the emission, particularly if the number of binary systems is small, as is suggested by recent simulations (Abel et al. 2000). It is therefore prudent to consider the effects of other potential sources of X-rays.

The obvious candidates are supernova remnants (SNR); next to X-ray binaries, they are the most significant galactic sources.

\[ 2 \] Note that X-ray luminosity of equation (18) is measured in a slightly different energy band from that of equation (17), so the agreement between the two values is not quite as good as may at first appear. Nevertheless, the necessary correction is small, and the values agree to within 50 per cent.
(Helfand & Moran 2001). They can emit X-rays through a variety of different emission mechanisms, but at high redshift the most significant will be thermal bremsstrahlung emission and non-thermal inverse Compton emission.

Thermal bremsstrahlung is produced by the hot gas within the SNR. Detailed modelling properly requires a hydrodynamical treatment (see, for example, Chevalier 1999), but for our purposes a simple parametrization suffices. If we assume that the hot gas has a single characteristic temperature $T_x$, then we can write the luminosity density per unit star formation rate as

$$L_x = L_0 \left( \frac{h v_0}{k T_x} \right) \exp \left( -\frac{h v}{k T_x} \right) \text{erg s}^{-1} \text{Hz}^{-1} \left( \text{M}_\odot \text{yr}^{-1} \right)^{-1},$$

where $h v_0 = 1$ keV and where $L_0$ is constant. Moreover, we can write the total X-ray luminosity as

$$L_{\text{tot}} = 3.2 \times 10^{43} f_x E_{51} N_{\text{in}} \text{erg s}^{-1} \left( \text{M}_\odot \text{yr}^{-1} \right)^{-1},$$

where $E_{51}$ is the typical supernova explosion energy (in units of $10^{51}$ erg), $f_x$ is the fraction of this energy radiated as bremsstrahlung, and where $N_{\text{in}}$ is the number of supernovae that explode per solar mass of stars formed. The value of $N_{\text{in}}$ depends on the IMF; for the standard Salpeter IMF adopted previously, $N_{\text{in}} = 0.0075 \text{M}_\odot^{-1}$. Finally, since

$$L_{\text{tot}} = \int_0^\infty L_x \, dv,$$

we can fix the value of $L_0$; we find that

$$L_0 = 1.3 \times 10^{26} f_x E_{51} N_{\text{in}} \text{erg s}^{-1} \text{Hz}^{-1} \left( \text{M}_\odot \text{yr}^{-1} \right)^{-1}.$$

For typical supernova parameters ($E_{51} = 1$ and an ambient density of $n = 1 \text{ cm}^{-3}$), Helfand & Moran (2001) find that a fraction $f_x = 2 \times 10^{-4}$ of the explosion energy is radiated at a characteristic temperature $k T_x = 1$ keV. On the other hand, the higher mean density at high redshift, together with the comparative weakness of outflows from low-metallicity stars (Kudritzki 2000) suggest that the typical ambient density may be very much higher. In particular, if it is as high as $n \approx 10^7 \text{ cm}^{-3}$, then a supernova remnant will radiate its energy extremely rapidly, before the ejecta have time to thermalize (Terlevich et al. 1992). In this case, the fraction of energy radiated as bremsstrahlung is very much higher ($f_x \sim 0.01$), as is the characteristic temperature ($k T_x \sim 30$ keV). We examine both of these models in Section 4, with the understanding that the true picture lies somewhere in between.

In addition to this thermal emission, supernova remnants also produce non-thermal X-rays. These are generated as the relativistic electrons produced by the SNR gradually lose energy through synchrotron radiation, non-thermal bremsstrahlung and the inverse Compton scattering of photons from the cosmic microwave background (CMB). At high redshift, the latter is likely to dominate (Oh 2001). The spectrum of the resulting emission depends upon the energy spectrum of the relativistic electrons, but at the energies of interest is well represented by a power law: $L_x \propto v^{-1}$. The intensity of the emission depends upon the fraction of the supernova energy transferred to the electrons; this is not well constrained, with estimates ranging from 0.1 to 10 per cent. Accordingly, we model the emission as

$$L_x = 7.7 \times 10^{23} \left( \frac{v_0}{v} \right) f_e \text{erg s}^{-1} \text{Hz}^{-1} \left( \text{M}_\odot \text{yr}^{-1} \right)^{-1},$$

where $h v_0 = 1$ keV, $f_e$ is the fraction of energy deposited in the electrons, and where we have assumed that $E_{51} = 1$ and $N_{\text{in}} = 0.0075 \text{M}_\odot^{-1}$ as in the thermal bremsstrahlung case. This expression assumes a high-energy cut-off for the X-ray spectrum at 10 keV, but is only logarithmically dependent on the value of this cut-off. In Section 4, we examine results for models with $f_e = 10^{-3}$ and 0.1, which bracket the range of plausible values.

Much more detail on high-redshift inverse Compton emission, including a discussion of potential observational tests, is given in Oh (2001).

2.2 Opacity

The opacity $\tau(\nu_0, z_0, z)$ can be separated into two distinct components – absorption by dust and gas within the emitting protogalaxy, which we term intrinsic absorption, and absorption by gas along the line of sight through the IGM.

Intrinsic absorption is difficult to model with any degree of accuracy as it depends upon a number of variables – the size and shape of the galaxy, its ionization state, the position of the sources within it, the dust content, etc. Rather than attempt to model these in detail – a significant undertaking in itself – we instead adopt a highly approximate representation. We assume that the emitted X-rays are attenuated by a neutral hydrogen column density of $N_H = 10^{21} \text{ cm}^{-2}$ plus an associated neutral helium column density $N_{He} = 0.08 N_H$. This absorption is assumed to be the same for all sources. These values are chosen because they are representative of the column densities of the protogalaxies studied in this paper (which presumably contain the bulk of the X-ray sources). Reducing $N_H$ (as would be appropriate if much of the surrounding H I were photoionized and/or dispersed by the progenitors of the X-ray sources) has little or no effect on $T_{\text{em}}$, as gas in the IGM and within the protogalaxy itself quickly come to dominate the total absorption. Increasing $N_{He}$, on the other hand, has more significant effects: an order of magnitude increase in $N_{He}$ produces similar results to an order of magnitude decrease in the strength of the X-ray background, which, as we will see in Section 4, is generally sufficient to render X-ray feedback ineffective. Consequently, we will overestimate the effect of the background if the bulk of the X-ray sources reside within massive galaxies.

Turning to the Lyman–Werner bands, we note that intrinsic absorption will generally be negligible within small protogalaxies, as their H$_2$ content is rapidly photodissociated (Glover & Brand 2001). Moreover, we also assume that the effects of dust absorption are negligible. In galaxies of primordial composition, this is obviously true; in metal-poor galaxies, it should also be a good approximation, as very large column densities are required for significant dust obscuration (for instance, $N_H \gtrsim 10^{23} \text{ cm}^{-2}$ for $Z = 10^{-3} \text{ Z}_\odot$ gas, if the dust-to-gas ratio is similar to that in the Milky Way). Again, these assumptions break down if the majority of sources are to be found in massive, metal-rich galaxies, but we expect such galaxies to be extremely rare at the redshifts of interest in this paper.

Compared with intrinsic absorption, the effects of absorption arising from the IGM are much simpler to treat, particularly if we can assume that the bulk of the gas remains at an approximately uniform density. This assumption proves reasonable at high redshift for photons with mean free paths much greater than the typical clumping scale, as is the case for both Lyman–Werner band photons and X-rays. Our treatment of IGM absorption is discussed in the following sections.

2.2.1 Lyman–Werner band absorption

The continuum opacity of metal-free gas is very small (Lenzuni, Chernoff & Salpeter 1991) and for our purposes can be neglected.
Consequently, the only significant sources of opacity encountered by Lyman–Werner photons are absorption by the Lyman series lines of neutral hydrogen, and by the Lyman–Werner lines of molecular hydrogen. The Lyman series lines have the effect of absorbing any Lyman–Werner photons of the same frequency, and reprocessing them to Lyman–α photons plus associated softer photons. As Lyman–α lies outside of the Lyman–Werner band, the net effect is to block from view any sources at redshifts higher than some maximum, \(z_{\text{max}}\), given by

\[
\frac{1 + z_{\text{max}}}{1 + z_0} = \frac{v_0}{v_H},
\]

(27)

where \(v_0\) is the frequency of the Lyman series line and \(v_H\) and \(z_0\) are the observed frequency and redshift. Clearly, the size of \(z_{\text{max}}\) depends upon the distance between \(v_0\) and \(v_H\), and thus more sources are seen at frequencies that are a long way from a line. As a result, the spectrum develops a characteristic ‘sawtooth’ shape (see fig. 1 in Haiman et al. 2000), with the effect becoming more pronounced as one nears the Lyman limit.

Absorption by molecular hydrogen is rather more complicated, owing to the large number of Lyman–Werner lines that contribute to the opacity. If we approximate the lines as delta functions then an individual line produces an opacity

\[
\tau_i = \frac{\pi \sigma^2}{m_e c} f_{\text{osc},{\lambda}_i} \frac{n_{H_2}(z_i)}{H(z)},
\]

(28)

where \(f_{\text{osc}}\) and \(\lambda_i\) are the oscillator strength and wavelength of the transition, \(n_{H_2}(z_i)\) is the number density of \(H_2\) molecules in the level giving rise to the line at \(z_i\), the redshift of absorption, and \(H(z)\) is the Hubble constant at \(z_i\). The value of \(z_i\) is given by

\[
\frac{1 + z_i}{1 + z_0} = \frac{v_i}{v_0},
\]

(29)

where \(v_i = c / \lambda_i\).

We can see that all of the photons that are absorbed in the lines are permanently removed from the Lyman–Werner band, then the total opacity \(\tau(v_0, z_0, z)\) is simply given by the sum over all lines with \(z < z_i < z_{\text{max}}\):

\[
\tau(v_0, z_0, z) = \sum_i \frac{\pi \sigma^2}{m_e c} f_{\text{osc},{\lambda}_i} \frac{n_{H_2}(z_i)}{H(z)}. \]

(30)

This sum potentially involves a very large number of lines, but can be greatly simplified by assuming that all of the \(H_2\) is to be found in its ortho or para ground state; at the redshifts of interest, the population of excited states will be negligible.

In deriving this expression, we have assumed that every absorption permanently removes a Lyman–Werner band photon. This is not entirely correct. On average, only 15 per cent of absorptions are followed by photodissociation of the \(H_2\) molecule (Draine & Bertoldi 1996); the rest of the time, the molecule decays back to a bound state, emitting a photon. In their treatment of this problem, Haiman et al. (2000) assumed that the excited \(H_2\) molecule would always decay directly back into the original state, and would thus emit a photon of the same energy as the one initially absorbed. In fact, this is not correct (T. Abel, private communication); most decays occur initially to highly excited vibrational states, producing photons redwards of the Lyman–Werner bands. Only a small fraction of decays (approximately 5 per cent) take place directly into the original state, while a slightly larger fraction (approximately 15 per cent) produce photons that lie elsewhere in the Lyman–Werner band system (Glover & Brand 2001). We do not include the effect of these photons; however, an accurate treatment would be quite complicated and is almost certainly unnecessary – as we shall see in Section 4, \(H_2\) in the IGM is rapidly destroyed by the growing Lyman–Werner background and is completely negligible by the time that negative feedback begins.

### 2.2.2 X-ray absorption

At X-ray energies, the opacity of the intergalactic gas is dominated by the ionization of neutral hydrogen and helium; prior to recombination, the \(He^+\) abundance is small and can be neglected. The X-ray opacity can thus be written as

\[
\tau(v_0, z_0, z) = \int_{v_0}^{z} \left[ \sigma_H(v) n_{H} + \sigma_{He}(v) n_{He} \right] \frac{d\ell}{dz},
\]

(31)

where

\[
v = v_0 \left( \frac{1 + z}{1 + z_0} \right),
\]

(32)

and where \(\sigma_H(v)\) and \(\sigma_{He}(v)\) are the absorption cross-sections of neutral hydrogen and helium, respectively, with \(n_{He}(z)\) and \(n_{H}(z)\) being the corresponding number densities. As long as the fractional ionization of the IGM remains small (i.e. a few per cent or less), the ratio between \(n_{He}\) and \(n_{H}\) can be accurately approximated by its primordial value

\[
\frac{n_{He}}{n_H} = \frac{Y}{4 - 4Y},
\]

(33)

where \(Y = 0.247\) is the helium mass fraction, and we can write equation (31) purely in terms of \(n_{H}\) as

\[
\tau(v_0, z_0, z) = \int_{v_0}^{z} \left[ \sigma_H + \left( \frac{Y}{4 - 4Y} \right) \sigma_{He} \right] n_H \frac{d\ell}{dz} dz.
\]

(34)

This integral is readily computable by means of numerical integration.

### 3 METHOD

In the previous section, we showed that, given a simple star formation model, it is relatively easy to calculate the evolution of the Lyman–Werner and X-ray backgrounds. Two of the parameters of our star formation model – the star formation efficiency \(\epsilon\) and starburst lifetime \(t_{\text{on}}\) – we treat as free parameters (although they can be constrained to some extent – see, e.g. Haiman & Loeb 1997). The remaining parameter, \(T_{\text{cut}}\), the temperature at which efficient cooling becomes possible, is determined by the strength of the backgrounds themselves. This clearly presents us with a problem: the evolution of \(T_{\text{cut}}\) is coupled to that of the backgrounds, and to know one we must first know the other.

Fortunately, this difficulty is easily avoided. We know that at high redshift the number of protothalogalaxies, and hence the star formation rate, must be very small. Consequently, there must be some redshift above which the external radiation field will become too weak to affect galaxy formation. The precise redshift at which this occurs is model dependent, but for the models examined in this paper we typically find that radiative feedback is negligible above \(z = 40\). By choosing an initial redshift \(z_i = 50\), therefore, we can be sure that in our initial simulation the background radiation will have no effect.

Given this starting point, we next proceed incrementally to lower redshifts via the following procedure.

(i) Given \(T_{\text{cut}}(z_i)\), we calculate the background radiation field at \(z = z_i - \Delta z\), assuming that \(T_{\text{cut}}(z) = T_{\text{cut}}(z_i)\).
(ii) Using the computed background, we simulate the chemical and thermal evolution of a protogalaxy with $T_{\text{vir}} = T_{\text{crit}}(z)$; the details of this simulation are outlined in Sections 3.1–3.7 below. The main aim of this simulation is to determine whether the protogalactic gas can cool efficiently.

(iii) If the protogalactic gas cools, then our assumed value of $T_{\text{crit}}$ is correct; we store this result, and go to step (i) to proceed to the next redshift. If the gas fails to cool, we continue to step (iv).

(iv) We increment our assumed value of $T_{\text{crit}}(z)$ by a small amount $\Delta T$, and recalculate the background radiation field. We assume that $T_{\text{crit}}$ varies linearly over $\Delta z$. Given the new background, we return to step (ii).

Provided that $\Delta z$ and $\Delta T$ are both small, the error in $T_{\text{vir}}(z)$ will also be small; this is particularly the case once emission from larger protogalaxies (which cool via Lyman-$\alpha$ radiation) begins to dominate the background.

This approach reduces the coupled problem to the simpler one of determining whether a protogalaxy with virial temperature $T_{\text{vir}}$ and formation redshift $z_f$ will cool when exposed to a particular background radiation field. To answer this question, we need to be able to model the thermal and chemical evolution of the protogalaxy. Our approach to this problem is outlined in the following sections.

3.1 Computing the evolution of protogalactic gas

Ideally, we would like to use a high-resolution hydrodynamical simulation to follow the thermal and chemical evolution of the protogalactic gas (see, e.g., Machacek et al. 2001). Unfortunately, including the effects of radiative transfer, particularly of photons in the Lyman–Werner bands, into such a simulation is not currently feasible. We are thus forced to approximate. In choosing an appropriate approximation, we are also motivated by the desire to minimize the computational requirements of our simulations, so that we can explore the effects of a variety of different source models. We make three main approximations.

(i) We assume spherical symmetry. This is a reasonable approximation for the first generation of protogalaxies, but clearly is incorrect in detail (see, for example, fig. 2 of Abel et al. 2000).

(ii) We assume that the protogalactic gas is static, at least on the time-scale of the simulation. This allows us to ignore the hydrodynamical evolution of the gas, and also substantially simplifies the treatment of radiative transfer. This assumption clearly breaks down once the gas begins to cool strongly and loses its pressure support, but as we are only interested in the evolution up to this point, this is not a significant problem.

(iii) We assume that all of the $H_2$ molecules remain in the rotational and vibrational ground state, in either ortho or para form. This simplification allows us to evolve the chemistry and radiative transfer on the time-scale on which the total $H_2$ abundance changes (typically $10^{11}$–$10^{12}$ s) rather than that on which the individual level populations change ($10^5$–$10^{10}$ s). It also simplifies our treatment of the radiative transfer. We discuss this approximation in more detail in Section 3.4.2.

Together, these approximations allow us to solve for the chemical and thermal evolution of a model protogalaxy in a matter of minutes on a fast desktop computer. This allows us to study the redshift evolution of $T_{\text{vir}}$ at high resolution in both temperature and redshift for a number of different X-ray source models.

However, this approach has an obvious drawback – we cannot be sure that our approximations give a fair representation of the real protogalaxy. Of particular concern is the neglect of the hydrodynamic evolution of the gas, and the consequent error in the density profile. This is potentially significant because the $H_2$ cooling rate, along with many of the chemical reaction rates, scales as the square of the density. Small errors in the density can thus lead to larger ones in the computed temperature. These concerns are mitigated to some extent, however, by the close agreement between the results of detailed numerical simulations and previous semi-analytic treatments. For instance, the values of $T_{\text{vir}}$ obtained from the smoothed-particle hydrodynamics simulations of Fuller & Couchman (2000) agree well with the results of Tegmark et al. (1997), despite the highly approximate uniform density profile adopted by the latter group. Similarly, the results of Machacek et al. (2001), obtained with a three-dimensional adaptive mesh hydrodynamical code broadly agree with those of Haiman et al. (2000), who use a static model similar to that presented here. Together, these results suggest that $T_{\text{vir}}$ is insensitive to the precise details of the density profile, but clearly this remains an area of concern.

Our computational method can be broken down into three main stages: initialization of the density profile and the chemical abundances, computation of the thermal and chemical evolution of the gas, and termination of the simulation at a suitable point. These are described below.

3.2 The model protogalaxy

The protogalactic density profile is modelled as a truncated isothermal sphere, with central overdensity $\delta = 1.796 \times 10^4$ and truncation radius

$$r_t = 4.62 \times 10^3 \left(\frac{T_{\text{vir}}}{1000 \text{ K}}\right)^{1/2} (1 + z_i)^{-3/2} \Omega_m^{-1/2} h^{-1} \text{ pc.} \quad (35)$$

The virial temperature and redshift of formation of the protogalaxy completely specify its density profile.

We subdivide this profile into $N_g$ spherical shells of uniform thickness and compute the mean density within each shell. We have run a number of test simulations with different values of $N_g$, and find that setting $N_g = 100$ provides sufficient spatial resolution to accurately determine $T_{\text{vir}}$.

We assume that the initial chemical composition of the protogalactic gas is the same as that of the intergalactic medium. At our initial redshift ($z_f = 50$), we take this from Stancil, Lepp & Dalgarno (1998). At lower redshifts, the chemical evolution of the IGM is influenced by the Lyman–Werner and X-ray backgrounds. We therefore calculate the intergalactic abundances explicitly, using the chemical model outlined in Section 3.3, by solving the chemical rate equations:

$$\frac{d n_i}{dt} = C_i(n_j, T) - D_i(n_j, T)n_i, \quad (36)$$

where $C_i$ and $D_i$ are source and sink terms for $n_i$. At the same time we also solve for the temperature of the intergalactic gas (Galli & Palla 1998)

$$\frac{dT}{dt} = -2T H(z) + \frac{2}{3k n_{\text{tot}}}(\Gamma - \Lambda), \quad (37)$$

where $H(z)$ is the Hubble constant, $n_{\text{tot}}$ is the total particle number density and where $\Gamma$ and $\Lambda$ are the net heating and cooling rates (see Section 3.5).

We solve this set of ordinary differential equations with the STRIPB integrator of Press et al. (1986). As the size of the required time-step is generally much smaller than the redshift interval $\Delta z$ that separates
our individual determinations of $T_{\text{ex}}$, we compute intermediate values by interpolation and from these determine the strength of the radiation background and hence the photochemical rates. Although our main aim in following this chemistry is to determine the correct initial abundances for our simulations of protogalactic evolution, the results are of interest in their own right and are presented and discussed in Section 4.4.

Our treatment of the chemistry of the IGM does not include the effects of the ionizing photons from stars (and/or quasars) that are ultimately responsible for the reionization of the intergalactic gas. This is justified at early epochs, as these photons are confined within small H II regions surrounding the luminous sources, but this simplification restricts the validity of our results to the period prior to cosmological reionization. The post-reionization epoch, and the effect of reionization on galaxy formation, have received extensive study elsewhere (see Loeb & Barkana 2001 and references therein).

3.3 The chemical model

To simulate the chemical evolution of the protogalactic gas, we adopt a chemical model consisting of 30 reactions between nine species: H, H+, H2, He, He+, He++, H2, H2+ and free electrons. The reactions included in the model are summarized in Table 1, together with the source(s) of the data used. This model is based in large part on that of Abel et al. (1997), but has been modified to improve its accuracy when applied to optically thick gas. Aside from a number of updates to the reaction coefficients in the light of new data, the main differences are as follows.

(i) We include the contribution to the hydrogen ionization rate arising from the ionizing photons produced by He+ recombination, in line with the discussion in chapter 2 of Osterbrock (1989). Although commonly a small correction to the total rate, this can become significant when X-ray photoionization dominates.

(ii) To enable us to accurately determine the He+ abundance, we find that we need to include the effects of charge transfer between He+ and H (reaction 20), as this can be comparable to the recombination rate when the fractional ionization is small. For completeness we also include the inverse reaction (no 21), although this is unimportant at $T < 10^4$ K.

(iii) We include the contribution to the ionization rates of hydrogen and helium arising from secondary ionization by energetic photoelectrons, based on the recent calculations of Dalgarno, Yan & Liu (1999). The contribution of secondary ionization to the other photodissociation rates is small and can be neglected.

(iv) We do not include the photodissociation of H2 by photons above the Lyman limit (reaction 28 in Abel et al.,) as in optically thick gas this will be negligible compared with the effects of H2 photodissociation (reaction 28b). On the other hand, we do include the effects of dissociative photoionization (reaction 27), which becomes significant for photon energies greater than 30 eV.

More information concerning all of these points, and the chemical model generally, can be found in Glover (2001).

3.4 Radiative transfer

In order to determine rate coefficients for the photochemical reactions (nos 22–30), we need to calculate the evolution of the mean specific intensity $J_\nu$ within the protogalactic gas. For optically thin gas, this is simple, but the only photochemical reactions for which we can assume optically thin conditions are H+ photodetachment (reaction 25) and H2 photodissociation (reaction 28); for the rest, we must supplement our chemical model with a treatment of radiative transfer.

The high degree of symmetry of the problem makes a direct solution of the radiative transfer equation feasible, but the large spectral resolution required to properly treat the Lyman–Werner lines ($\Delta \nu / \nu \simeq 10^{-5}$) makes such an approach very time consuming, particularly since it must be repeated for a very large number of time-steps. This motivates our choice of a much simpler approximate treatment that nevertheless should produce results of an acceptable accuracy.

To begin with, we assume that the re-emission and/or scattering of photons within the protogalactic gas is negligible. Although highly accurate for the X-ray photons, this approximation is less so for the Lyman–Werner photons, since many Lyman–Werner absorptions are followed by re-emission. However, as noted previously in Section 2.2.1, the majority of these photons are emitted at frequencies that do not coincide with the ground-state Lyman–Werner lines, and

### Table 1

A list of the reactions included in our chemical model of protogalactic gas. Values for the rate coefficients (or radiative cross-sections where appropriate) are given in Glover (2001). References are to the primary source(s) of the data whenever possible; in many cases, we have also used analytical fits to this data from Abel et al. (1997), Galli & Palla (1998) or Stancil et al. (1998).

| No | Reaction | Reference |
|----|----------|-----------|
| 1  | H + e− → H+ + 2e− | Janev et al. (1987) |
| 2  | H+ + e− → H + γ | Ferland et al. (1992) |
| 3  | He + e− → He+ + 2e− | Janev et al. (1987) |
| 4  | He+ + e− → He++ + 2e− | Abel et al. (1997) |
| 5  | He++ + e− → He+ + γ | Hummer & Storey (1998), Aldrovandi & Pequignot (1973) |
| 6  | He++ + e− → He+ + γ | Ferland et al. (1992) |
| 7  | H+ + e− → H+ + γ | Wishart (1979) |
| 8  | H+ + H → H2 + e− | Launay, Le Dourneuf & Zeippen (1991) |
| 9  | H + H+ → H2+ + γ | Ramaker & Peck (1976) |
| 10 | H2+ + H → H2 + H+ | Karpas, Anicich & Huntress (1979) |
| 11 | H2 + H+ → H2+ + H | Holliday, Mackeran & Friedman (1971) |
| 12 | H+ + e− → 2H + e− | Stibbe & Tennyson (1999) |
| 13 | H+ + H → 3H | Martin, Schwarz & Mandy (1996) |
| 14 | H+ + e− → H + 2e− | Janev et al. (1987) |
| 15 | H+ + H → 2H + e− | Janev et al. (1987) |
| 16 | H+ + H2 → 2H | Moseley, Abers & Peterson (1970) |
| 17 | H− + H+ → H2+ + e− | Poulaert et al. (1978) |
| 18 | H2+ + e− → 2H | Schneider et al. (1994) |
| 19 | H+ + He → H+ + He− + e− | Hug et al. (1982) |
| 20 | H+ + H → H++ + H+ + γ | Zygelman et al. (1989) |
| 21 | He+ + H → He++ + H | Kimura et al. (1993) |
| 22 | H+ + γ → H++ + e− | Osterbrock (1989) |
| 23 | H+ + γ → He+ + e− | Yan et al. (1998) |
| 24 | He+ + γ → He++ + e− | Osterbrock (1989) |
| 25 | H+ + γ → H+ + e− | de Jong (1972) |
| 26 | H2 + γ → H2+ + e− | O’Neil & Reinhardt (1978), Wilms, Allen & McCray (2000), Yan et al. (1998) |
| 27 | H2 + γ → H+ + H+ + e− | Samson & Haddad (1994) |
| 28 | H2+ + γ → H + H+ + e− | Dunn (1968) |
| 29 | H2+ + γ → 2H+ + e− | Bates & Opik (1968) |
| 30 | H2 + γ → H2+ + e− | See text |
rapidly escape from the protogalaxy without being re-absorbed. By
ignoring the small fraction of re-emitted photons that do coincide
with ground-state lines, we underestimate the H2 photodissociation
rate slightly, but this effect is small compared with the uncertainty
in the incident spectrum.

In addition to the ionizations produced by the incident X-rays
(and secondary photoelectrons), we also must account for the ion-
izations caused by the emission of ionizing photons during H and He
recombination within the gas. This is done by use of the on-the-spot
approximation, as outlined in Osterbrock (1989). This approxima-
tion is valid as long as the mean free path of the ionizing photons is
much shorter than any length-scale of interest. For the protogalaxies
that we study, this condition is generally satisfied; for example, the
mean free path of a photon just above the Lyman limit in the core
region of a z = 10 protogalaxy is only 0.01 pc (and is even smaller in
higher-redshift objects). It breaks down somewhat in the outermost
regions of our protogalaxies, where the density is several orders
of magnitude lower, but H2 formation in these regions is negligible.

These approximations allow us to write the radiative transfer
equation for an arbitrary line of sight through the protogalaxy

\[ \frac{dI_\nu}{ds} = -\kappa_\nu(s)I_\nu + j_\nu(s) \]  

(38)

(where \(I_\nu\) is the specific intensity, \(\kappa_\nu\) and \(j_\nu\) are the opacity and emissivity, respectively, and where \(s\) is the position along the line of sight) in the simpler form

\[ \frac{dI_\nu}{d\nu} = -\kappa_\nu(s)I_\nu, \]  

(39)

which can be further simplified to

\[ \frac{dI_\nu}{d\nu} = -I_\nu. \]  

(40)

Solution of this equation is trivial; if the specific intensity at the edge
of the protogalaxy is \(I_\nu(0)\), then at a distance \(s\) into the protogalaxy, it
is

\[ I_\nu = I_\nu(0)e^{-\kappa_\nu(s)}. \]  

(41)

To obtain the mean specific intensity, we simply integrate this ex-
pression over a solid angle. Using the conventional variables \(r\) and \(\mu\) (see Fig. 1), we find that

\[ J_\nu(r) = \frac{1}{2} J_{\nu}(r_1) \int_{-1}^{+1} \exp \left[ -\tau(v, r, \mu) \right] d\mu, \]  

(42)

where \(J_{\nu}(r_1)\) is the incident intensity at the edge of the protogalaxy
(i.e. the background radiation) and where \(\tau(v, r, \mu)\) is the optical
depth between \(r\) and the edge of the protogalaxy in the direction
of \(\mu\).

3.4.1 Photoionization

For the photoionization reactions (numbers 22–24, 26–27 and 29),
the opacity is dominated by continuum absorption by neutral hy-
drogen and helium. For static gas, we can write this as

\[ \tau(v, r, \mu) = \sigma_H(v)N_H(r, \mu) + \sigma_{He}N_{He}(r, \mu) \]

(43)

where the second line follows if we assume primordial abundances
and a small fractional ionization (which also allows us to neglect
He\(^+\) absorption).

Substituting this expression into equation (42), we obtain

\[ J_\nu(r) = \frac{1}{2} J_{\nu}(r_1) \exp \left[ -\left( \sigma_H + \frac{Y}{4 - 4Y}\sigma_{He} \right) N_H(r, \mu) \right], \]  

(45)

where

\[ N_H(r) = \int_{-1}^{+1} \exp \left[ -N_H(r, \mu) \right] d\mu. \]

(46)

Aside from \(N_H(r)\), all of the terms in equation (45) are known
at the beginning of the simulation. Moreover, if we assume that
\(J_{\nu}(r_1)\) remains constant for the duration of the simulation, then it
becomes possible to calculate the photoionization rates in advance,
tabulating them as functions of \(N_H\). If we do this, then instead of
solving directly for the set of photoionization rates at each time-
step, we can simply calculate \(N_H(r)\) and then interpolate the rates
from the appropriate tables; this avoids a large number of frequency
integrations, and results in a significant speed-up.

3.4.2 Photodissociation

For Lyman–Werner photons, the situation is a little more complica-
ted, owing to the large number of spectral lines involved. Fortu-
nately, there is a simple approach that we can use, which signifi-
cantly reduces the amount of computation required to obtain the
photodissociation rate.

We begin by writing the total H\(_2\) photodissociation rate as a sum
of the contributions from each individual rotational and vibrational
level:

\[ k_{dis}N_{H_2} = \sum_j k_{j}n_{j}. \]

(47)

where \(n_{j}\) is the total number density of H\(_2\) molecules, and \(n_{j}\) is the
number density of molecules in the \(j\)th vibrational level. For each
level, we can write the photodissociation rate in the form (Draine &
Bertoldi 1996)

\[ k_{j} = \frac{2\pi J_{\nu}(r)}{h} \int_{-1}^{+1} \sum_i f_{dis,i}j_i dN_{H_2,j} d\mu, \]

(48)

where \(W_j\) is the dimensionless equivalent width of the \(j\)th Lyman–
Werner line originating from level \(j\), \(N_{H_2,j}\) is the column density of
H\(_2\) molecules in level \(j\), and \(f_{dis,i}\) is the dissociation fraction for level
i (i.e. the fraction of excitations that are followed by dissociation, rather than de-excitation to some other bound state).

The equivalent width can be written as

\[ W_{ij} = \int_0^{\infty} \left[ 1 - \exp \left( -\sigma_i(v)N_{H_2,j} \right) \right] \frac{dv}{v}, \]

where the radiative cross-section \( \sigma_i(v) \) can be written in terms of the line profile \( \phi(v, v_i) \) and oscillator strength \( f_{osc,i} \) as

\[ \sigma_i(v) = \frac{\pi e^2}{m_e c} f_{osc,i} \phi(v, v_i). \]

In general, the form of the line profile will depend upon the details of the temperature and velocity variations along the line of sight. For static, isothermal gas, however, it reduces to the familiar Voigt profile. Consequently, \( \sigma_i(v) \) is known a priori, allowing us to tabulate \( W_{ij} \), and hence \( dW_{ij}/dN_{H_2,j} \), as functions of \( N_{H_2,j} \) at the beginning of the simulation. Similarly, we can tabulate the photodissociation rates \( k_i \) as a function of \( N_{H_2,j}(r) = \int_{-\infty}^{\infty} \sum_i f_{abs,i} \frac{dW_{ij}}{dN_{H_2,j}} d\mu, \)

since the dissociation fractions \( f_{abs,i} \) are determined purely by atomic physics and are independent of the physical conditions in the gas. In this way, we reduce the problem of determining the photodissociation rates to the simpler one of determining the column density of \( H_2 \) molecules in each rovibrational level for a large number of lines of sight. Given a set of column densities for a particular level, we can obtain the corresponding values of \( dW_{ij}/dN_{H_2,j} \) by interpolation; from these, we can derive \( N_{H_2,j}(r) \) by numerical integration; and finally, the photodissociation rate \( k_i \) is obtained by another interpolation.

However, two potential problems remain with this method. First, equation (49) breaks down at high column densities as two effects that we have neglected – absorption by neutral hydrogen, and overlap of the individual Lyman–Werner lines – become significant. However, these effects are important only for \( H_2 \) column densities \( N_{H_2} > 10^{20} \) cm\(^{-2}\); since the largest \( H_2 \) column densities that we encounter in our simulations are typically no larger than \( N_{H_2} = 10^{18} \) cm\(^{-2}\), it is generally safe to ignore these complications.\(^3\)

The second and more significant problem is the fact that, in the method as outlined, we have to deal with all of the rotational and vibrational levels of \( H_2 \). This makes calculating the photodissociation rate much more costly than calculating the photoionization rates, as we need to compute several hundred different column densities for each line of sight. Similarly, the chemical network is also greatly enlarged and consequently takes longer to solve. Moreover, the excited vibrational levels typically have characteristic time-scales of the order of 10\(^6\) s, much smaller than the time-scale on which the total \( H_2 \) abundance changes, forcing us to solve the chemical network with a time-step that is orders of magnitude smaller than we could otherwise use.

To overcome these problems, we make another approximation. Since, at the temperatures and densities encountered in our simulations, most of the excited levels have very small populations, their net contribution to the photodissociation rate is also very small. We can therefore safely ignore them, and concentrate only on the few rotational levels that are responsible for the bulk of the photodissociation.

\(^3\) In any case, it is possible to reformulate equation (49) so as to avoid these difficulties, as in Draine & Bertoldi (1996).

The simplest approximation that we can make is to assume that all of the \( H_2 \) molecules are in the form of para-hydrogen and are to be found in the ground state. This approximation was suggested by Abel et al. (1997), and supposes that any ortho-hydrogen formed will be rapidly converted to para-hydrogen by collisions with protons (Gerlich 1990)

\[ H_2(ortho) + H^+ \rightarrow H_2(para) + H^+. \]

Unfortunately, this is only valid at very low temperatures, typically \( T \lesssim 50 \) K. At higher temperatures, conversion from para-hydrogen back to ortho-hydrogen via the inverse reaction becomes competitive. Moreover, at realistic temperatures and densities, ortho-para interconversion via proton collisions with excited states will also occur, and in general the ortho-to-para ratio will be close to its equilibrium value of 3:1.

We can therefore obtain a much better approximation to the true photodissociation rate by including ortho-hydrogen. For simplicity, we assume that the ortho-to-para ratio is fixed and is exactly 3:1. We continue to ignore the other excited rotational levels.

Comparing the results of simulations performed using this approximation with those of simulations in which the rotational and vibrational level populations are set to their local thermodynamic equilibrium (LTE) values, we find that it performs well: although differences in the final \( H_2 \) abundances are often observed, the effect on \( T_{crit} \) remains small – it is increased by at most 30–40 per cent at any given redshift, and by much less than this in the limits of low and high redshift. The uncertainty that this introduces is comparable to that owing to our poor knowledge of the high-redshift ultraviolet background (see Fig. 2 below). Moreover, since the true level populations will lie somewhere between our simple approximation and the LTE case, this suggests that our approximation is sufficient for our current purposes, and that a more detailed (and hence slower) treatment is not required.

Figure 2. The evolution of the critical temperature as a function of redshift in the absence of an X-ray background. Results are plotted for three different UV source models: two metal-free models, one with a minimum stellar mass \( M_{min} = 0.1 \, M_\odot \) and the other with \( M_{min} = 1 \, M_\odot \) (represented by the dotted and dashed lines, respectively) and a \( Z = 0.02 \, Z_\odot \) model with \( M_{min} = 1 \, M_\odot \) (solid line). In all three cases, the IMF is of Salpeter form, with a maximum mass \( M_{max} = 100 \, M_\odot \). The remaining parameters of the star formation model (star formation efficiency and starburst lifetime) are the same in all three simulations, as is the cosmological model.
3.4.3 Calculating the column densities

As demonstrated in the previous sections, we can determine the photochemical reaction rates for a given mass shell by computing the functions

\[ N_i(r, \mu) = \int_{-1}^{+1} \exp[-N_i(r, \mu)] \, d\mu, \]  

(53)

and

\[ N_{H_2,j}(r) = \int_{-1}^{+1} \sum_i f_{a_{ij}} \frac{dW_{ij}}{dN_{H_2,j}} \, d\mu, \]  

(54)

(where \( j = 0, 1 \) for the para- and ortho-hydrogen ground states, respectively) and then interpolating the rates from a pre-built look-up table. Moreover, knowledge of \( N_i(r, \mu) \) and \( N_{H_2,j}(r) \) is sufficient to specify \( N_i(r) \) and \( N_{H_2,j}(r) \), respectively, so the problem is essentially reduced to one of calculating these column densities.

To do this, we use a method very similar to that in Kepner, Babul & Spergel (1997). For each mass shell \( k \), we choose a point \( r_{ik} \) at the centre of the shell (see Fig. 1). For each of these points, we compute the column densities along \( N_i \) lines of sight with uniform angular separation. The column density of a chemical species \( i \) along one of these lines of sight is given by

\[ N_i(r, \mu) = \int_0^L n_i [r'(x)] \, dx, \]  

(55)

where \( L = r\mu + [r^2 - r^2(1 - \mu^2)]^{1/2} \) is the total distance along the ray to the edge of the protogalaxy, and where \( r'(x) = (r^2 + x^2 - 2rx\mu)^{1/2} \) is the distance from the point labelled by \( x \) to the centre of the protogalaxy. Solution of this equation by means of numerical integration is straightforward.

Given the set of column densities for a particular point \( r_{ik} \), we use equations (53) and (54) to compute \( N_i(r_{ik}) \) and \( N_{H_2,j}(r_{ik}) \), and then finally use these values to compute the photochemical rates at that point. Finally, the spherical symmetry of the problem allows us to generalize these rates to the whole of the mass shell containing \( r_{ik} \).

The accuracy of this procedure clearly depends both upon the spatial resolution of the grid \( N_{N} \) and upon the angular resolution with which we sample it \( (N_{\mu}) \). However, as the density, temperature and chemical abundances all vary smoothly within the protogalaxy, we find that very high resolution is not required: setting \( N_{N} = 100 \) and \( N_{\mu} = 20 \) proves to be sufficient for accurate calculation of the photochemical rates. Simulations run with substantially higher values produce very similar results.

3.5 Heating and cooling

The heating and cooling processes included in our model are listed in Table 2, together with references to the sources for the rates adopted. At low temperatures, cooling is dominated by rotational and vibrational line emission from H2; we include this according to the recent prescription of Le Bourlot, Pineau des Forêts & Flower (1999), which is probably the most accurate available to date. We do not include the effects of HD cooling: this becomes significant only at very low temperatures, and in any case does not appear to have a great effect on the evolution of the gas (Bromm et al. 2002).

At high temperatures \( (T \gtrsim 8000 \text{ K}) \), cooling through the atomic lines of hydrogen (often referred to simply as Lyman-\( \alpha \) cooling)

\(^4\)In the latter case, we make use of molecular data from Abgrall et al. (1993a,b) and Roueff (private communication), together with dissociation fractions from Abgrall, Roueff & Drira (2000).

(57)

using the STIFBS integrator of Press et al. (1986).

At the start of each time-step (hereafter time \( t \)), we compute the photochemical rates as outlined in Sections 3.4. We then use STIFBS to solve for the new chemical abundances and new temperature at the end of the time-step (time \( t + \Delta t \)), repeating this for each shell in turn. Next we store these values, return to time \( t \), and recalculate them using the same procedure, but with two time-steps of length \( \Delta t/2 \). We recalculate the photochemical rates at the intermediate point.

We next test for convergence by comparing our two sets of results. If any of the chemical abundances or temperatures of any of the shells rapidly becomes dominant. This sets an upper limit on \( T_{\text{crit}} \); for simplicity, we neglect any redshift dependence of this limit, and assume that protogalaxies with \( T_{\text{vir}} \gtrsim 8000 \text{ K} \) can always cool successfully.

Heating of the protogalactic gas is driven primarily by the photoionization of hydrogen and helium. H2 photodissociation also contributes to the heating rate, but is generally not significant, owing to the small H2 abundance.

3.6 Running the simulation

Given the initial temperature and chemical abundances, plus the set of chemical reaction rates, actually solving for the thermal and chemical evolution is relatively easy. As in the IGM case, we simply solve the coupled set of chemical rate equations

\[ \frac{dn_i}{d\tau} = C_i(n_j, T) - D_i(n_j, T)n_j, \]

(56)

\[ \frac{dT}{d\tau} = \frac{2}{3k n_{\text{tot}}} \left( \Gamma - \Lambda \right), \]

(57)

| Process | Reference |
|---------|-----------|
| Atomic line cooling: | |
| H\textsc{i} | Cen (1992) |
| He\textsc{i} | Bray et al. (2000) |
| He\textsc{ii} | Aggarwal et al. (1992) |
| Molecular line cooling: | |
| H2 | Le Bourlot et al. (1999) |
| Collisional ionization: | |
| H\textsc{i} | Janey et al. (1987) |
| He\textsc{i} | Janey et al. (1987) |
| He\textsc{ii} | Abel et al. (1997) |
| Recombination: | |
| H\textsc{ii} | Ferland et al. (1992) |
| He\textsc{ii} | Aldrovandi & Petquignon (1973) |
| He\textsc{iii} | Ferland et al. (1992) |
| Other chemistry: | |
| H\textsc{+} formation | Shapiro & Kang (1987) |
| H2\textsc{+} formation | Shapiro & Kang (1987) |
| Bremsstrahlung: | Spitzer (1978) |
| Compton scattering: | Peebles (1993) |
| Photoionization: | |
| H\textsc{i} | Osterbrock (1989), Yan et al. (1998) |
| He\textsc{i} | Yan et al. (1998) |
| He\textsc{ii} | Osterbrock (1989) |
| Photodissociation: | |
| H2 | Black & Dalgarno (1977) |
differ by more than 0.1 per cent, then we reject the results and begin again from time $t$ with a smaller time-step. Otherwise, we check to see whether we need to halt the simulation, using the criteria discussed below, and, if we do not, we begin the computations for a new time-step starting from $t + \Delta t$.

One final approximation that we find useful in practice is to fix the H$^-$ and H$_2$ abundances at their equilibrium values. This allows the integrator to take much larger time-steps than would otherwise be possible, but introduces very little error into the computed H$_2$ abundances.

3.7 Halting the simulation

Using the method outlined in the preceding sections, we compute the chemical and thermal evolution of the protogalaxy until one of two conditions is met: either the protogalactic gas begins to cool strongly, or we exceed a preset time limit, $t_{\text{lim}}$.

To assess whether gas cooling is ‘strong’ enough requires an objective cooling criterion. A number of different possibilities have been suggested in the literature (Rees & Ostriker 1977; Tegmark et al. 1997; Haiman et al. 2000). In our simulations, we adopt the criterion used by Haiman et al. (2000): we require that the elapsed time exceeds the cooling time, as computed at the edge of the protogalactic core, at a distance $r_0 = 0.034 r_h$ from the centre of the protogalaxy. The advantage of this choice is that it avoids giving us a false positive result in cases where $t_{\text{cool}}$ drops briefly below $t_{\text{dyn}}$ at early times, but remains so for a time $t < t_{\text{cool}}$. As an additional sanity check, we also require that the final temperature be smaller than the initial temperature.

If the protogalactic gas does not cool strongly, then the simulation will terminate when it reaches $t_{\text{lim}}$. This pre-set time limit is required on purely practical grounds, to prevent simulations in which the gas does not cool from running for excessive amounts of time, but also has a physical justification. In our simulations, we treat protogalaxies as isolated objects, uninfluenced by external events. In reality, they are part of a dynamically evolving mass distribution, and the majority will only survive for a limited time before merging with other protogalaxies of a similar or larger size. It is possible to use the Press–Schechter formalism to calculate the distribution of survival times as a function of mass (Lacey & Cole 1993), but for rare objects the mean survival time is typically of the order of the Hubble time and thus for simplicity we set $t_{\text{lim}} = t_H$.

4 RESULTS

In the following sections, we present results from a number of simulations that examine the effects of the X-ray backgrounds produced by the various source models discussed in Sections 2.1.3 and 2.1.4. Unless otherwise noted, all of these simulations assume the same parameters for the star formation model: a standard Salpeter IMF, with $M_{\text{min}} = 0.1 M_\odot$, and $M_{\text{max}} = 100 M_\odot$; a star formation efficiency $\epsilon = 0.1$; and a starburst lifetime $t_{\text{burst}} = 10^7$ yr. Additionally, all of the simulations use the same cosmological model, the $\Lambda$CDM concordance model of Wang et al. (2000), which has parameters $(\Omega_\Lambda, \Omega_m, \Omega_k, h, n, \sigma_8) = (0.67, 0.33, 0.041, 0.65, 1.0, 0.9)$.

4.1 Simulations without an X-ray background

Before examining the effect of an X-ray background on protogalactic evolution, we first briefly study the evolution of $T_{\text{crit}}$ in its absence. As well as allowing us to determine the sensitivity of our results to variations in the UV source model, this also provides us with a necessary baseline against which to compare our other results.

In Fig. 2, we plot the evolution of $T_{\text{crit}}$ with redshift for the three different star formation models discussed in Section 2.1.2. All three models assume a Salpeter IMF, with maximum stellar mass $M_{\text{max}} = 100 M_\odot$, and our standard star formation efficiency, starburst lifetime and cosmological model, described previously. Our basic model assumes a metal-free stellar population, with a minimum stellar mass of $M_{\text{min}} = 0.1 M_\odot$; the corresponding results are given by the dotted line in Fig. 2. The dashed line illustrates the effect of increasing the minimum mass to $M_{\text{min}} = 1 M_\odot$; the solid line assumes the same $M_{\text{min}}$, together with a metallicity $Z = 0.02 Z_\odot$.

Fig. 2 demonstrates that although the strength of the Lyman–Werner background increases by almost a factor of 5 as we move from our basic model to the metal-enriched model, this has less effect on $T_{\text{crit}}$ than we might expect: the difference in $T_{\text{crit}}$ between the three models is never more than 50 per cent, and the qualitative details of its evolution are very similar in all three models. This suggests that the uncertainty introduced by our lack of knowledge of the properties of the primordial stellar population need not be unduly limiting. However, it is also clear that some uncertainty remains, and this will place a lower limit on the magnitude of any effect that we can reliably claim to detect, as small variations in $T_{\text{crit}}$ owing to the X-ray background will be swamped by the error resulting from the uncertainty in the Lyman–Werner background.

In the work that follows, we take as our baseline the results of our basic, metal-free, low-$M_{\text{min}}$ model; as Fig. 2 demonstrates, this minimizes the strength of the Lyman–Werner background, and thus will tend to maximize the effectiveness of the X-ray background.

4.2 Massive X-ray binaries

In Fig. 3, we plot the evolution of $T_{\text{crit}}$ in the presence of the X-ray background generated by the massive X-ray binary model described in Section 2.1.3. For the purposes of comparison, we also plot the results of our basic X-ray-free model. Initially, the evolution of $T_{\text{crit}}$ is the same in both models, implying that the X-ray background has little or no effect on the gas. At a redshift $z \simeq 20$ and critical temperature $T_{\text{crit}} \simeq 2000$ K, however, the models begin to diverge.

![Figure 3](https://academic.oup.com/mnras/article-abstract/340/1/210/1125960)
significant. In the X-ray-free model, the critical temperature continues to increase rapidly until it reaches its maximum value of \( T_{\text{crit}} = 8000 \) K. In the X-ray binary model, in contrast, the rate of increase of \( T_{\text{crit}} \) is significantly slower, and it fails to reach its maximum value by the end of the simulation at \( z = 10 \).\(^5\)

It is clear from Fig. 3 that the presence of the X-ray background significantly affects the evolution of gas in the larger of the H\(_2\)-cooled protogalaxies. In small protogalaxies, on the other hand, the negative feedback caused by the Lyman–Werner background remains as strong as ever.

A simple way to judge the importance of this effect is to examine the difference it makes to the fraction of gas in the Universe that can collapse and cool. As we saw in Section 2.1.1, we can use the Press–Schechter formalism to express the cool gas fraction as

\[
F(z, M_{\text{crit}}) = \text{erfc} \left[ \frac{\delta_c(z)}{\sqrt{2} \sigma(M_{\text{crit}})} \right].
\]

Using the relationship between \( M_{\text{crit}} \) and \( T_{\text{crit}} \) derived in that section, it is straightforward to calculate the evolution of \( F(z, M_{\text{crit}}) \) in both the X-ray binary and X-ray-free models.

To better highlight the difference between the two models, in Fig. 4 we plot the ratio of the cooled gas fraction in the X-ray binary model to that in the X-ray-free model. At high redshift, the evolution of \( T_{\text{crit}} \) is the same in both models, and thus the ratio is unity. Below \( z = 30 \), the behaviour of the models begins to diverge, but this has little effect on the ratio until the models begin to diverge sharply at \( z \sim 20 \). Thereafter, the ratio rises sharply to a peak at \( z \sim 15 \), and subsequently declines as the growth in the cooled mass fraction becomes dominated by the formation of larger protogalaxies that cool via Lyman–\( \alpha \) emission.

Fig. 4 shows us that by ignoring the effect of the X-ray background we underestimate \( F(z, M_{\text{crit}}) \) by at most a factor of 2. In fact, the difference is likely to be even smaller: we have assumed that all of the gas in a protogalaxy with \( M > M_{\text{crit}} \) will cool, but this need not be the case if an X-ray background is present, as X-ray heating will prevent gas from cooling in the low-density outer layers of the protogalaxy. Whether this will affect the star formation rate within individual protogalaxies is not clear, but in any case the difference in the globally averaged star formation rate is unlikely to be greater than a factor of 2. Note, however, that this extra star formation occurs entirely in low-mass systems, from which ionizing photons (Ricotti & Shull 2000) and supernova ejecta (Ferrara & Tolstoy 2000) can more readily escape. Doubling the star formation rate may thus significantly increase the feedback of primordial star formation on the IGM.

Finally, given the uncertainties in the data underlying our simple massive X-ray binary model (see Section 2.1.3), it is of interest to investigate the sensitivity of our results to changes in the strength of the X-ray background. Accordingly, we have run additional simulations in which the strength of the X-ray sources was increased or decreased by a factor of 10. The results are plotted in Fig. 5, together with the results of our basic X-ray binary model and of the X-ray-free model.

Unsurprisingly, increasing or decreasing the strength of the X-ray background alters the evolution of \( T_{\text{crit}} \). Decreasing it by an order of magnitude increases \( T_{\text{crit}} \) to the point where its evolution is little different from that in the X-ray-free model. Increasing it by an order of magnitude, on the other hand, systematically lowers \( T_{\text{crit}} \), although never by more than a factor of 2.

### 4.3 Supernova remnants

As we discussed in Section 2.1.4, it is possible that massive X-ray binaries are much less abundant at high redshift than at the present day. If so, then the X-ray emission of star-forming galaxies will be dominated by supernova remnants. These will generate X-rays through two main emission mechanisms: thermal bremsstrahlung from hot gas and inverse Compton scattering of the CMB by relativistic electrons. We consider the effects of these mechanisms separately.

We examined two possible variants of the thermal bremsstrahlung model. In one, we assumed that the characteristics of the emission are broadly the same as those observed locally, with a fraction \( f_s = 2 \times 10^{-4} \) of the supernova energy being radiated as X-rays with

\[\text{(58)}\]

---

\(^5\) We choose to end our simulations at \( z = 10 \) because at a lower redshift we expect the effects of ionizing radiation from both stellar sources and quasars to become increasingly important and thus our results to become unreliable. Clearly, if reionization occurs at \( z > 10 \), the same is true for some portion of the results plotted here.

---

**Figure 4.** The ratio of the cooled gas fraction in the massive X-ray binary model to that in the X-ray-free model. The mass fractions are calculated using the Press–Schechter formalism, as outlined in the text.

**Figure 5.** As in Fig. 3, but including variants of the X-ray binary model with 10 per cent (dash-dotted line) and 1000 per cent (dotted line) of the flux of the basic model (dashed line). As before, we also plot the evolution of \( T_{\text{crit}} \) for the X-ray-free model (solid line).
Figure 6. As in Fig. 3, but for an X-ray background produced by inverse Compton emission from supernova remnants. We assume that 10 per cent of the total supernova energy is transferred to the relativistic electrons powering the emission, and again plot the results of the X-ray-free model for comparison.

Figure 7. The evolution with redshift of the fractional abundance of H$_2$ in the intergalactic medium, plotted for the X-ray-free model (solid line) and the massive X-ray binary model (dashed line).

Figure 8. The evolution with redshift of the fractional ionization of the intergalactic medium, plotted for the X-ray-free model (solid line) and the massive X-ray binary model (dashed line).

a characteristic temperature $T_x = 1$ keV (Helfand & Moran 2001). In the other model, we assumed that all high-redshift supernovae explode in extremely dense surroundings, producing X-ray bright, ultracompact remnants with $f_e = 0.01$ and $T_x = 30$ keV (Terlevich et al. 1992). Realistic models should lie somewhere between these two extremes.

However, we found that in neither of these cases does the X-ray background affect the evolution of $T_{\text{crit}}$: at our level of temperature resolution, the results are identical to those obtained for the X-ray-free model. The obvious conclusion is that the background produced by bremsstrahlung is simply too weak to be effective.

Our inverse Compton model fares somewhat better. In this model, the strength of the X-ray background is proportional to the mean fraction $f_e$ of the supernova explosion energy that is transferred to relativistic electrons within the remnant. This value is not known accurately and so we considered two possible cases, one with $f_e = 10^{-3}$ and another with $f_e = 0.1$, these being conservative lower and upper bounds on the true value. In the former case, we again saw no significant effect on the evolution of $T_{\text{crit}}$. In the latter case, on the other hand, we saw results very similar to those obtained for the X-ray binary model, as illustrated in Fig. 6.

In principle, therefore, inverse Compton emission from supernova remnants could be as important an X-ray source as emission from X-ray binaries. Ultimately, however, its importance depends upon the value of $f_e$, and no firm conclusions are possible until this value is better constrained. In this context, the possible observational tests suggested by Oh (2001) could prove extremely valuable.

4.4 The thermal and chemical evolution of the IGM

As well as determining the evolution of $T_{\text{crit}}$, our simulations also allow us to study the thermal and chemical evolution of the diffuse intergalactic medium, as outlined in Section 3.2. As a simple example, we plot in Fig. 7 the evolution of the fractional H$_2$ abundance in the IGM for the X-ray-free model (solid line) and the massive X-ray binary model (dashed line).

In both cases, the fractional abundance rapidly decreases from its primordial value owing to photodissociation by the ultraviolet background, reaching $f_{H_2} = 10^{-9}$ by $z \simeq 35$. Subsequently, its decline slows, in part because the rate of increase in the strength of the ultraviolet background also slows. Below $z = 20$, the behaviour of the two models diverges. In the X-ray-free model, $f_{H_2}$ continues to decline until the end of the simulation. In the X-ray binary model, on the other hand, the increasing ionization of the IGM boosts the H$_2$ formation rate to the point where it overtakes the photodissociation rate and the H$_2$ abundance, after reaching a minimum at $z \simeq 17$, begins to climb. Nevertheless, it remains extremely small at the end of the simulation, readily justifying our assertion in Section 2.2.1 that absorption by intergalactic H$_2$ does not play a significant role in determining the strength of the Lyman–Werner background.

In Fig. 8, we plot the evolution of the fractional ionization of the IGM. In the X-ray-free model, this remains approximately constant over the lifetime of the simulation, as the recombination time-scale is significantly longer than the Hubble time. In the X-ray binary model, on the other hand, photoionization by the growing X-ray background eventually overcomes the very small recombination rate and drives the fractional ionization upwards, increasing it by just
The evolution with redshift of the temperature of the intergalactic medium, plotted for the massive X-ray binary model. In the X-ray-free model, the He\(^+\) abundance remains negligible throughout the simulation.

The effect on the fractional ionization of helium (the ratio of He\(^+\) to He) is rather more striking. The post-recombination He\(^+\) abundance is extremely small (Stancil et al. 1998), and in the X-ray-free model remains at this low level throughout the simulation. In the X-ray binary model, on the other hand, it increases dramatically over the course of the simulation, reaching \(f_{\text{He}^+} = 4 \times 10^{-3}\) by \(z = 10\). This is illustrated in Fig. 9.

Finally, in Fig. 10 we plot the evolution with redshift of the temperature of the IGM. In the X-ray-free model, adiabatic cooling dominates the thermal evolution, and the temperature falls off approximately as \((1 + z)^2\). In the X-ray binary model, on the other hand, photoelectric heating begins to heat the IGM strongly at \(z = 20\), driving the temperature up to \(T = 100\) K by the end of the simulation.

Thus, although the X-ray background does not contribute significantly to the reionization of the IGM, it does produce substantial reheating prior to reionization. Moreover, given the large mean free path of the X-ray photons, this reheating occurs almost uniformly throughout the IGM, rather than being localized to the vicinity of star-forming galaxies.

One consequence of this reheating is that the formation of very small-scale structure will be suppressed, as the increased temperature of the IGM leads to an increased Jeans mass. This is unlikely to affect the global star formation rate, however, as star formation within these small structures would in any case be strongly suppressed by the ultraviolet background. Nevertheless, it will reduce the mean clumping factor of the IGM below the level that we would otherwise predict, which may in turn speed up reionization (Haïman, Abel & Madau 2001; Barkana & Loeb 2002).

Reheating also affects the visibility of the IGM in the redshifted 21-cm line of neutral hydrogen. Madau, Meiksin & Rees (1997) show that scattered Lyman-\(\alpha\) emission from high-redshift galaxies efficiently couples the spin temperature of the H\(_{\text{i}}\) hyperfine levels to the kinetic temperature of the gas. If the gas temperature is lower than the CMB temperature, this results in 21-cm line absorption; if it is higher, then it results in emission. Absorption is easier to detect than emission (Scott & Rees 1990), but the heating produced by the X-ray background implies that absorption occurs only at \(z \gtrsim 15\), and that concentrating on detecting 21-cm emission may be the more viable strategy.

5 CONCLUSIONS

The results of the previous section allow us to assess the impact of the high-redshift X-ray background that is produced by star-forming galaxies. If we assume that the X-ray emission of these galaxies is similar to that observed locally, and that the same correlation between X-ray luminosity and star formation rate applies, then we find that the background produced is strong enough to partially offset the effects of UV photodissociation in large (\(T_{\text{vir}} > 1000\) K), H\(_2\)-cooled protogalaxies.

However, local emission is dominated by massive X-ray binaries, which may not form in large numbers at high redshift. Therefore, we have also explored the effect of an X-ray background produced by emission from supernova remnants. If this emission is dominated by inverse Compton scattering and if the fraction of the supernova energy transferred to the relativistic electrons powering this emission is large, then the resulting background has very similar effects to that produced by X-ray binaries. On the other hand, if the fraction of energy transferred is small, then the background has little or no effect.

In addition to inverse Compton emission, we have also examined the effect of thermal bremsstrahlung emission from hot gas in the remnants, and find that even if all supernovae were to form X-ray bright, ultracompact remnants, the resulting X-ray background would still be too weak to significantly affect protogalactic evolution.

Finally, none of these models produces an X-ray background that is strong enough to balance UV photodissociation in small protogalaxies, with virial temperatures \(T_{\text{vir}} < 1000\) K. In these protogalaxies, negative feedback always dominates.

How significant are these results? One simple way to assess this is to study the evolution of the mass fraction of cooled gas, which represents the total amount of matter available to form stars. Comparing its evolution in the X-ray binary model with that in the absence of an X-ray background, we find that it is increased by approximately a factor of 2. Given our star formation model, this corresponds to an increase in the global star formation rate by the same amount. However, this is small compared with the order of magnitude increase.
that would result if we were simply to ignore the effect of the UV background (see fig. 7 of Haiman et al. 2000).

In reality, the difference between the two models may be greater than this because the additional star formation takes place entirely in low-mass systems, from which ionizing photons and supernova-produced metals can readily escape (Ricotti & Shull 2000; Ferrara & Tolstoy 2000). Doubling the star formation rate may therefore have more than double the impact on the intergalactic medium. However, to properly assess the ultimate importance of this effect requires more detailed modelling, which is beyond the scope of this paper.

Ultimately, understanding the history of star formation in the small protogalaxies studied in this paper remains important even if they do not contribute to the reionization or enrichment of the IGM to any great degree. This is simply because, in a hierarchical universe, these protogalaxies are the building blocks from which larger galaxies form and therefore set the initial conditions for later stages of galaxy formation. In particular, very little metal enrichment of the primordial gas is required in order to allow the CNO cycle to operate and population II (rather than population III) stars to form, and yet this can have a profound effect on the predicted spectral energy distribution of an early stellar population.

Although the main purpose of our study was to examine the effects of the X-ray background on the thermal and chemical evolution of gas within protogalaxies, our approach also allows us to examine the effects of the background on the diffuse IGM. Our main results are threefold.

(i) We confirm the rapid destruction of H₂ in the intergalactic medium as noted by Haiman et al. (2000), but also show that when an X-ray background is present the H₂ abundance does not continue to decline indefinitely, but eventually stabilizes and may even begin to increase. However, it never becomes large enough to significantly affect the Lyman–Werner background.

(ii) We show that although photoionization by the X-ray background significantly increases the fractional ionization of the IGM (and, in particular, the fractional ionization of helium), the bulk of the gas remains mostly neutral, demonstrating that the contribution of the X-ray background to cosmological reionization is small.

(iii) We find that the X-ray background will also heat the intergalactic gas, raising its temperature to T ≃ 100 K by z = 10 (compared with T = 3.5 K in the X-ray-free model). This will suppress the formation of structure on the smallest scales by increasing the Jeans mass. It is unlikely to affect the global star formation rate, but that the latter still dominates. They do not find evidence for the positive feedback predicted by Haiman et al. (2000). However, their simulations do not include the effects of H₂ self-shielding, and thus potentially underestimate the amount of H₂ that forms (although see Machacek et al. 2001 for a different view).

There are several significant differences between our work and these previous investigations. First, we do not assume a fixed spectrum or intensity for the X-ray background; rather, we specify the properties of the X-ray sources and subsequently compute the build-up of the background in a self-consistent fashion. Moreover, we consider source models where the X-ray emission is proportional to the star formation rate, as is observed to be the case for star-forming galaxies at low redshift; the relationship between star formation rate and X-ray emission in quasar-based models is far less clear. As a result, we generally consider X-ray backgrounds significantly fainter than those studied in the papers cited above. This makes a direct comparison of our results difficult. However, we note that Machacek et al. (2003) find X-ray feedback to be ineffective below M₂vir = 10⁶ M☉, regardless of the strength of the X-ray background; at the redshifts they consider this corresponds to a virial temperature T₂vir ≃ 1000 K, and thus agrees well with the similar result obtained in this paper.

In closing, we note that a number of uncertainties still remain in our treatment of this problem. Some of these – the high-redshift star formation rate or the appropriate population III initial mass function, for instance – we simply do not know at the present time. Fortunately, changes to our assumed values can be readily incorporated within the framework laid out in this paper. Other uncertainties arise from our method of simulation; in particular, from our assumption of a static density profile for the protogalactic gas. We hope to address these issues in future work.

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