Interpreting the LSND anomaly: sterile neutrinos or CPT-violation or...?*

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Abstract

We first study how sterile neutrinos can fit the $5\sigma \bar{\nu}_\mu \rightarrow \bar{\nu}_e$ LSND anomaly: 2+2 solutions are strongly disfavoured by solar and atmospheric data, while 3+1 solutions can still give a poor fit (for a specific range of oscillation parameters, to be tested by MiniBooNE). If MiniBooNE will see no $\nu_\mu \rightarrow \nu_e$ transitions, we will have a hint for CPT violation. Already now, unlike sterile neutrinos, CPT-violating neutrino masses can accomodate all safe and unsafe data. We study how much CPT must be conserved according to atmospheric and K2K data and list which CPT-violating signals could be discovered by forthcoming solar and long-baseline experiments.

Oscillations between the three Standard Model (SM) neutrinos are described by two independent squared neutrino mass differences, allowing to explain only two of the three neutrino anomalies (atmospheric [1], solar [2] and LSND [3]) as oscillations. A joint fit is not possible even if one trusts only the safest data from atmospheric, solar and reactor [4] neutrino experiments: the the up/down atmospheric asymmetries and a $\sim 50\%$ disappearance of solar $\nu_e$. Most global fits of neutrino data drop the LSND anomaly because the other ones are considered as more solid. In quantitative terms, we have a $3\sigma$ solar anomaly (although it can be reduced to $2\sigma$ by dropping solar model predictions), a $14\sigma$ atmospheric anomaly and a $5\sigma$ LSND anomaly. The ‘number of standard deviations’ is here naively computed as $\Delta \chi^2 \equiv (\chi^2_{\text{SM}} - \chi^2_{\text{best}})^{1/2}$, where $\chi^2_{\text{best}}$ is the $\chi^2$ value corresponding to the best-fit oscillation, and $\chi^2_{\text{SM}}$ corresponds to massless SM neutrinos.

*In the addendum at pages 9–12 we update our results including the first data from KamLAND and WMAP, which disfavour the CPT-violating and ‘3+1’ solutions.

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1 The $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ LSND anomaly is presented as an evidence for $\nu_\mu \rightarrow e$ oscillation probability of $(0.264 \pm 0.081)\%$ [3], that differs from zero only by slightly more than $3\sigma$. However, from a table of the likelihood $L_\chi^2$, obtained from the LSND collaboration and computed on an event-by-event basis, we read

$$\Delta \chi^2 = \chi^2_{\text{SM}} - \chi^2_{\text{best}} = -2 \ln \frac{L_{\text{best}}}{L_{\text{SM}}} = 29 \quad \text{rather than} \quad \sim 10.$$ 

A reanalysis of LSND data that chooses stronger cuts obtains $\Delta \chi^2 = 47$ (eq. (2.7) of [3]). These large $\Delta \chi^2$ mean that the LSND anomaly cannot be due to a statistical fluctuation. It is not clear which data really contain the LSND evidence. Apparently, some mark of oscillations that cannot be summarized by the number of $\bar{\nu}_e$ events is hidden in the full LSND data, maybe in the energy distribution.

In section 1 we discuss how and how well oscillations with extra sterile neutrinos can fit the LSND anomaly [4]. In particular we study which one of the two different kind of four-neutrino spectra (3+1 or 2+2) is favoured by the present data, and by an eventual future confirmation of the LSND data. Taking into account the recent SNO result [2] an extra sterile neutrino can improve the situation only in the 3+1 scheme, and even this case does not allow to fully reconcile all data.

This situation suggests to look for alternative interpretations of the LSND anomaly. One possibility is that either the atmospheric or solar or LSND anomaly is not due to oscillations. Various mechanisms (even unplausible ones) can fit the data as well as oscillations [4].

Using only oscillations, all data can be consistently fitted by the CPT-violating neutrino spectrum illustrated in fig. 1. This solution was proposed in [9] when the initial 2.6$\sigma$ LSND hint for $\nu_\mu \rightarrow \nu_e$ [1] decreased down to 0.6$\sigma$, leaving an anomaly only in $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ [3]. Unlike sterile neutrinos, this solution also satisfies (unsafe?) bounds from nucleosynthesis and SN1987A [9] [11]. Despite the lack of theoretical grounds, this speculation is interesting because it can be tested soon. If CPT violation were the right answer, MiniBooNE [12] (the experiment designed to test LSND, looking for $\nu_\mu \rightarrow \nu_e$) will not see the LSND oscillations; a $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ experiment is needed to directly test this possibility. If CPT is badly violated as in fig. 1, one generically expects detectable CPT-violating signals in atmospheric and solar oscillations. In any case it remains interesting to constrain CPT-violation in neutrino masses. In section 2 we compute the present bounds and list the possible CPT-violating signals and surprises that could ap-
pear in forthcoming solar and long-baseline experiments.

1 Sterile neutrinos

The sterile neutrino can be used to generate either the LSND or the solar or the atmospheric anomaly.

3+1 neutrinos

Within this scheme the sterile neutrino is employed to generate the LSND anomaly. In fact, in the jargon 3+1 indicates that the additional sterile neutrino is separated by the large LSND mass gap from the 3 active neutrinos, separated among them only by the small solar and atmospheric mass differences. A theoretical remark is in order. If the $4 \times 4$ neutrino mass matrix $m_{\nu_{i\ell}}$ ($i = (\ell, s)$ and $\ell = (e, \mu, \tau)$) has the naïve form

$$m_{\ell s} = m_{ss} = m_{LSND}, \quad m_{\ell\ell'} \ll m_{ss}$$

the sterile neutrino induces a contribution to the solar mass splitting of order†

$$\delta m_{\text{sun}}^2 = \Delta m_{\text{LSND}}^2 \sin^2 2\theta_{\text{LSND}} \approx 10^{-3(\pm 1)} \text{eV}^2$$

that is too large in most of the region allowed by solar and LSND data. One needs either a cancellation or a mass matrix of the special ‘approximatively rank one’ form

$$m_{\nu_{i\ell}} \approx \theta_{\ell s} \theta_{i s} m_{\text{LSND}}.$$  

Even ignoring this potential theoretical problem, 3+1 oscillations present a phenomenological problem, because predict that $\nu_{\mu} \rightarrow \nu_{\mu}$ oscillations at the LSND frequency proceed trough $\nu_{\mu} \rightarrow \nu_s \rightarrow \nu_e$ and $\nu_{e, \mu} \rightarrow \nu_s$ are strongly constrained by disappearance experiments. More precisely, keeping only oscillations at the dominant LSND frequency

$$S \equiv \sin^2 (\Delta m_{\text{LSND}}^2 L/4E_\nu)$$

one has

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta_{es}$$
$$P(\nu_{\mu} \rightarrow \nu_{\mu}) = 1 - \sin^2 2\theta_{\mu s}$$
$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta_{\text{LSND}}$$

with $\theta_{\text{LSND}} \approx \theta_{es}\theta_{\mu s}$, or more precisely §

$$\sin^2 2\theta_{\text{LSND}} = \frac{1}{4} \sin^2 2\theta_{es} \sin^2 2\theta_{\mu s}. \quad (1)$$

The $\theta_{es}$ mixing angle is constrained by Bugey, Chooz [1], SuperKamiokande (SK) atmospheric data [1] and the $\theta_{\mu s}$ mixing angle by SK, CDHS and CCFR [17]. Furthermore $\nu_{\mu} \rightarrow \nu_e$ oscillations are also directly constrained by Karmen [18]. Fig. 2 illustrates how accurately we reproduce such bounds³.

![Figure 1: The CPT-violating spectrum proposed in [4].](image)

The crucial question is if these bounds are too strong for allowing the oscillations suggested by LSND. At first sight the answer is that they are [19], but this negative conclusion was questioned in [17] and the first accurate statistical analysis of this issue was performed in [18] with Bayesian techniques. Our result, shown in fig. 5 basically agrees with [18]. Working in gaussian approximation we find that all 96% CL LSND confidence region is excluded at, at least, 96% CL level. Therefore 3+1 solutions have some goodness-of-fit problem. One needs to invoke a statistical fluctuation with around % probability to explain why only LSND sees the sterile oscillations.

Even if this conclusion is self-evident, we justify the adopted statistical strategy. As discussed in [19], due to the large number of d.o.f. (about 200) a naive Pearson global $\chi^2$ test is unable to notice this problem and would erroneously suggest that 3+1 oscillations give a good fit. While it is difficult to develop a general and efficient goodness-of-fit test, in this particular case the fit is bad for one specific reason: different sets of data are mutually exclusive (up to a 96% CL) within our theoretical assumptions. In such a situation the goodness-of-fit problem is efficiently

³We used the SK atmospheric results [1] after 79 kton-year (55 data), K2K [16] (at the moment K2K finds 44 events, versus an expected no-oscillation signal of 64 ± 6 events), the latest solar results from Homestake, Gallex, SAGE, GNO, SK, SNO (49 data), the final Bugey (60 data), Chooz (14 data), CDHS (15 data), CCFR (15 data), KARMEN and LSND results. We use the likelihoods computed by the KARMEN and LSND collaborations on an event-by-event basis. We have not included data from MACRO [1] (that confirms the atmospheric anomaly) and from earlier atmospheric experiments because are less statistically significant than SK. The data are combined by multiplying all likelihoods $L$ (i.e. by summing all $\chi^2 = -2\ln L$). At $\Delta m^2 \geq 10^{-2}\text{eV}^2$ CHOOZ and Bugey bounds could be considered as not fully trastable because limited by the theoretical error on the total $\nu_e$ fluxes generated by reactors.

⁴So that $\Delta \chi^2 = 7$ corresponds to 97% CL level for the two parameters $\theta_{\ell s}\theta_{\mu s}$ and $\Delta m_{\text{LSND}}^2$. The Gaussian approximation is not fully satisfied (e.g. our best fit regions are not ellipses). A Bayesian analysis can shift 97% to ~ 95% or ~ 98%, with ‘reasonable’ choices of the prior probability distribution. (the arbitrarily remains until there are ‘large’ allowed regions). As discussed in [19], a similar shift is typically obtained in a frequentist analysis, that cannot however be performed in a reasonable computing time. Therefore we stick to the Gaussian approximation.
recognized by fitting separately the two incompatible data. This is what is done in fig. 8.

Ignoring the poor quality of the fit, the best combined fit region for the LSND parameters is shown in fig. 1. It agrees reasonably well with the corresponding fig. in [20], taking into account that we show values of

$$\chi^2(\theta_{LSND}, \Delta m^2_{LSND}) = \min_p \chi^2(p, \theta_{LSND}, \Delta m^2_{LSND})$$

(where \(p\) are all other parameters in which we are not interested), so that we convert values of \(\chi^2 - \chi^2_{\text{best}}\) into confidence levels using the gaussian values appropriate for 2 d.o.f. (the 2 LSND parameters), while a statistically less efficient procedure with more d.o.f. is employed in [20].

2+2 neutrinos

In the jargon 2+2 indicates 2 couples of neutrinos (one generates the solar anomaly, and the other one the atmospheric anomaly), separated by the large LSND mass gap. Within this scheme, the sterile neutrino is employed to generate the solar or atmospheric anomaly, or one combination of the two. The fraction of sterile neutrino involved in solar oscillations, \(\eta_{s}^{\text{sun}}\), plus the fraction of sterile neutrino involved in atmospheric oscillations, \(\eta_{s}^{\text{atm}}\), is predicted to sum to unity [17]

$$\eta_{s}^{\text{tot}} = \eta_{s}^{\text{sun}} + \eta_{s}^{\text{atm}} = 1.$$  

Experiments now tell that both the solar and atmospheric anomalies are mostly generated by active neutrinos, and only a small sterile contribution is allowed. Consequently 2+2 oscillations give a global fit worse than 3+1 oscillations [22, 20]. Let us summarize the present experimental status of this issue.

- **Solar data** give a 5.4\(\sigma\) evidence for pure active solar oscillations versus pure sterile oscillations: combining all solar data in a global fit we obtain [19]

$$\chi^2_{\text{sun}}(\text{best sterile}) - \chi^2_{\text{sun}}(\text{best active}) = 30$$

and \(\eta_{s}^{\text{sun}} = 0 \pm 0.18\). In particular, SNO/SK find a 5.1\(\sigma\) direct indication for \(\nu_{\mu,\tau}\) appearance.

- **Atmospheric data** give a 7\(\sigma\) indication for pure active atmospheric oscillations versus pure sterile oscillations. In fact, a global fit of atmospheric data gives [11, 21]**

$$\chi^2_{\text{atm}}(\text{best sterile}) - \chi^2_{\text{atm}}(\text{best active}) \approx 50$$

and \(\eta_{s}^{\text{atm}} = 0 \pm 0.16\). This strong evidence is obtained combining independent sets of data. SK claims [11] that pure sterile is disfavoured by the up/down ratio in a NC-enriched sample (3.4 standard deviations) and by matter effects in partially contained events.

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**Footnote:** Some words of caution. Arbitrary choices become more relevant when fitting disfavoured data (for example: the error is evaluated at the experimental point or at the theoretical point?). Furthermore, our bound on the sterile fraction allowed by solar data is obtained assuming the BP00 [24] prediction for the Boron \(n_{s}\) solar flux. It is proportional to the \(^{7}\text{Be} \rightarrow ^{8}\text{B} \gamma\) cross section: some authors think that systematic uncertainties in its measurement could be underestimated.

**Footnote:** A large amount of these atmospheric data is not included in theoretical reanalyses (because not yet accessible outside the SK collaboration in a form that allows to recompute them) that therefore obtain a much smaller \(\Delta \chi^2 \approx 15\) [20, 24] in place of 50 [11, 21]. This underestimation of the SK bound means that at the moment only SK can perform a sensible analysis of mixed sterile and active atmospheric oscillations and explains why the authors of [24] do not recognize that 2+2 oscillations are extremely disfavoured. One mixing angle is set to zero in the SK analysis; relaxing this unjustified simplification should not significantly weaken the bounds.
Figure 4: Best-fit regions at 90% and 99% CL (2 d.o.f.) for the LSND parameters assuming oscillations. Fig. 4a assumes that the LSND anomaly is generated trough a sterile neutrino (“3+1” scheme). Fig 4b assumes that the LSND anomaly is generated by active neutrinos, while something else (e.g. neutrino decay, sterile neutrinos, . . . ) generates either the atmospheric or the solar anomaly, without affecting LSND. The dotted lines show the regions suggested by only the LSND data. The dots show the best fit points.

\[ \theta^2_{\ell s} = \frac{\pi^2}{3} |V_{e3}|^2 \Delta m_{\text{atm}}^2 R^2 \]

\( \approx 2.9\sigma \) and upward through-going muons (\( \approx 2.9\sigma \)). In total 7\( \sigma \) [21]. Matter effects in MACRO [11] give another 3.1\( \sigma \) signal. Furthermore SK finds a direct 2\( \sigma \) hint for \( \tau \) appearance.

In summary, the two extreme cases (all the sterile in atmospheric oscillations and all the sterile in solar oscillations) have been excluded, as summarized in table 3. At the moment published results only allow an approximated analysis of intermediate cases. We find that \( \eta^\text{tot}_s = 0.5 \pm 0.25 \), with \( \eta^\text{tot}_a = 1 \) disfavoured at 4\( \sigma \). Intermediate cases are less disfavoured than the two extreme cases by only the amount expected, on a statistical basis, due to the presence of one more parameter: the ‘best’ fit is now obtained around the weighted average of the two incompatible solar and atmospheric determinations, \( \eta^\text{tot}_s = 1 - \eta^\text{tot}_a \approx 0.5 \). We do not present more precise results because fitting incompatible data makes little sense. Despite the approximation, the final conclusion is clear: 2+2 oscillations are too strongly disfavoured to be considered as a viable possibility.

In fig. 4b we show the best-fit region for the LSND parameters, assuming that the LSND anomaly is generated by oscillations of active neutrinos. This result applies to a general class of models where something different than oscillations between active neutrinos is the source of the solar or atmospheric anomalies. In particular it applies to 2+2 oscillations: despite they are strongly disfavoured the LSND best-fit regions are unaffected by the problems with solar and atmospheric data, and can therefore be reliably computed.

This region extends to values of the LSND parameters not accessible within 3+1 oscillations, see fig. 4. Therefore the value of \( P(\nu_\mu \rightarrow \nu_\tau) \) that will be measured at Mini-BooNE could discriminate between the two cases: roughly, 3+1 oscillations prefer a value of \( P(\nu_\mu \rightarrow \nu_\tau) \) somewhat smaller than the one suggested by LSND. Furthermore 3+1 spectra must be accompanied by a significant disappearance of \( \nu_\mu \) at the LSND frequency. For example, our 3+1 best-fit (marked with a dot in fig. 4b) has \( \sin^2 2\theta_{\mu s} = 0.2 \), around the sensitivity of MiniBooNE.

Both 2+2 and 3+1 oscillation patterns can be realized with different neutrino spectra. Since at the moment (and in the near future) no experiment can resolve the difference we do not consider all possibilities. For example, even knowing the oscillation parameters and the type of spectrum, we could not safely predict neutrinoless double \( \beta \) decay signals.

Many sterile neutrinos

As shown in the last paper in [17], many sterile neutrinos cannot give a much better 3+1 fit than a single sterile neutrino. Of particular interest are minimal models where right-handed neutrinos live in a single extra dimension of radius \( R \) [25], that could be identified with the LSND scale. In such \( 3 + \infty \) models the problematic prediction [1] of 3+1 oscillations becomes slightly more problematic [2]. In fact, for small mixing angles and in the limit of averaged sterile oscillations, we now have \( \theta_{\text{LSND}} \approx \sqrt{7/10} \theta_{es} \theta_{\mu s} \) in place of \( \theta_{\text{LSND}} \approx \theta_{es} \theta_{\mu s} \). More importantly, the effective active/sterile mixing angles are now predicted to be
(for a hierarchical spectrum of active neutrinos, the other cases are more problematic). The Chooz bound on $V_{e3}$ (that will soon be tested and eventually strengthened by long-baseline experiments) now gives another constraint on $\theta_{es}$, making this minimal model more problematic than 3+1 oscillations. One can consider a large variety of less predictive non-minimal extra dimensional models.

In the case of sterile solar or atmospheric oscillations, many sterile neutrinos can be less disfavoured that a single sterile neutrino. As discussed above, pure atmospheric sterile oscillations are disfavoured mostly by matter effects (in the earth), that suppress $\nu_\mu \rightarrow \nu_\tau$ at large energy: SK data are better fitted by $\nu_\mu \rightarrow \nu_\tau$ oscillations, unsuppressed by matter effects. Even in the solar case, matter effects (in the sun) contribute to determine how much SMA sterile oscillations are disfavoured. In presence of a tower of many sterile neutrinos, matter effects do not suppress sterile oscillations at large energy or density, until there is a sufficiently heavy sterile resonance to cross. However, sterile oscillations must be strongly matter suppressed within a supernova. As discussed in supernovae strongly constrain sterile towers that continue up to masses of $10^{4-5}\text{eV}$. This is e.g. the case of an extra-dimensional Kaluza-Klein tower that continues up to the TeV scale. In conclusion, (2 + many) oscillations can be less disfavoured than 2 + 2 oscillations. However, even forgetting the lack of theoretical motivation, it does not seem possible to achieve a really satisfactory fit.

2 CPT violation

Theory

The only safe result is that CPT is conserved in Lorentz-invariant local quantum field theories (QFT). Therefore CPT-violating effects can be obtained by abandoning locality or Lorentz invariance:

1. In local QFT, CPT violation can be induced if the Lorentz symmetry is broken, e.g. spontaneously by vacuum expectation values of fields with spin 1 or higher, or cosmologically by interactions with some ‘æther’, or by a non-trivial extra-dimensional background, or...

This first possibility seems not promising for LSND: like anomalous matter effects and unlike oscillations, new effects are not enhanced at low neutrino energy. Therefore old experiments done at energies $2 \times 3$ orders of magnitude higher than LSND, disfavour the best fit Karmen/LSND region. Furthermore in this context it seems difficult to obtain $F_{ee} < 1/2$ (as suggested by the latest SNO data) in solar oscillations.

Therefore we focus on the second possibility, that could explain the LSND anomaly:

2. Strings, branes, quantum foams, wormholes, non commutative geometry (and other non local things like that) suggest CPT-violating effects, maybe suppressed by only one power of the quantum gravity scale $M$ (this case gives rise to interesting signals even for $M \sim 10^{19}\text{GeV}$).

If an effect at that level were an unavoidable phenomenon, quantum gravity at the TeV scale would be excluded by bounds on the $K_0 \bar{K}_0$ mass difference:

$$m_{K_0} - m_{\bar{K}_0} < 0.4 \times 10^{-9}\text{eV}.$$ 

The mass difference between neutrinos and anti-neutrinos that could explain LSND is larger by many orders of magnitude: we assume that CPT-violating effects are dominantly felt by neutrinos.

The generic Hamiltonian that describes non relativistic systems (e.g. Kaons) violates CPT, if the constraints from the underlying local relativistic QFT are not imposed. In the case of relativistic systems (e.g. neutrinos) one can mimic the standard Hamiltonian demanded by local relativistic QFT (particles together with anti-particles) but without imposing all the constraints demanded by QFT (particles degenerate with anti-particles), so that the generic Hamiltonian that describes free propagation of Dirac neutrinos has different mass terms for $\nu$ and $\bar{\nu}$. The social duty of studying how CPT-violating neutrino masses can arise in popular fundamental models has been exploited in obtaining the imprimatur from string brane-world orbifolds. Non commutative geometry was invoked in [3].

We do not consider other possible CPT violations in neutrino interactions, because experiments with (mainly) $\nu_\mu$, $\bar{\nu}_\mu$ beams and precision electroweak data find that neutrino NC couplings cannot differ from the SM prediction by more than few %. A global fit of electroweak precision data shows that the CC couplings of $e$ and $\mu$ neutrinos agree with the SM with few per-mille accuracy.

Fit of SK and K2K data

In absence of oscillations, the number of $\nu_\mu$-induced events at SK would be roughly double than the number of $\bar{\nu}_\mu$-induced events (the ratio is higher at sub-GeV energies. This is mainly due to the different $\nu_\mu$ and $\bar{\nu}_\mu$ cross-sections on matter, that we compute by summing the elastic and $\nu_\mu$-induced events). We use a (hopefully) self-explanatory notation for the $\nu$ and $\bar{\nu}$ parameters. An over-bar marks anti-neutrino parameters. For example, $\theta_{\text{atm}}$ and $\Delta m^2_{\text{atm}}$ parameterize the atmospheric $\nu_\mu \rightarrow \bar{\nu}_\tau$ oscillations.

Restricted analysis To begin, we assume that $\theta_{\text{sun}}$, $\theta_{\text{Chooz}}$, $\theta_{\text{Chooz}}$, $\theta_{\text{LSND}}$ have negligible effect on atmospheric oscillations, that are therefore described by $\Delta m^2_{\text{atm}}$, $\Delta m^2_{\text{atm}}$, $\theta_{\text{atm}}$ and $\theta_{\text{atm}}$.

A simple approximation captures the main properties of the fit. The up/down asymmetry in the number of
multi-GeV muon events is $A = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = 0.327 \pm 0.045$

Assuming maximal mixings, in the CPT-conserving case one has

$$\Delta m^2_{\text{atm}} = \Delta m^2_{\text{sol}} \approx 3 \times 10^{-3} \text{eV}^2 : \quad A \approx 1/3 \quad (2a)$$

The asymmetry is smaller in CPT-violating cases, e.g.

$$\Delta m^2_{\text{atm}} \gg \Delta m^2_{\text{sol}} \approx 3 \times 10^{-3} \text{eV}^2 : \quad A \approx 1/4 \quad (2b)$$

$$\Delta m^2_{\text{atm}} \ll \Delta m^2_{\text{sol}} \approx 3 \times 10^{-3} \text{eV}^2 : \quad A \approx 1/7 \quad (2c)$$

$$\Delta m^2_{\text{atm}} \gg \Delta m^2_{\text{sol}} \approx 3 \times 10^{-3} \text{eV}^2 : \quad A \approx 1/11 \quad (2d)$$

$$\Delta m^2_{\text{atm}} \ll \Delta m^2_{\text{sol}} \approx 3 \times 10^{-3} \text{eV}^2 : \quad A \approx 1/11 \quad (2e)$$

and even smaller if mixings are non maximal. These considerations allow to understand the main features of our numerical result. In fig. 5 we show the $\chi^2$ minimized with respect to the mixing angles $\theta_{\text{atm}}$ and $\theta_{\text{sol}}$. While $\Delta m^2_{\text{atm}}$ is almost as strongly constrained as in a CPT-conserving fit, $\Delta m^2_{\text{atm}}$ can be about one order of magnitude larger or smaller that $\Delta m^2_{\text{sol}}$.‡‡ The global $\chi^2$ for SK data is here obtained by summing the $\chi^2$ corresponding to the individual zenith-angle distributions of sub-GeV and multi-GeV (10 $e$-like bins and 10 $\mu$-like bins each), stopping $\mu$ (5 bins) and upward-through-going $\mu$ (10 bins) events. The overall normalization in each kind of events has been considered as a free parameter.

Alternatively, one can try to take into account the theoretical predictions for the overall fluxes as in [28] employing a $55 \times 55$ correlation matrix. This second approach gives a slightly different bound on CPT-violation: larger values of $\Delta m^2_{\text{atm}}$ would not be significantly disfavoured up to the right border of fig. 4.

Since the best fit is obtained for almost CPT-conserving oscillations, the fit for the mixing angles is quite simple, and we do not need to show a dedicated figure. In the CPT-conserving case $\sin^2 2\theta_{\text{atm}}$ has to be close to one. We find that in the CPT-violating case the same bound applies replacing

$$\sin^2 2\theta_{\text{atm}} \rightarrow \frac{2}{3} \sin^2 2\theta_{\text{atm}} + \frac{1}{3} \sin^2 2\bar{\theta}_{\text{atm}}$$

so that both $\theta_{\text{atm}}$ and (to a lesser extent) $\bar{\theta}_{\text{atm}}$ have to be close to maximal.

‡‡ An analogous fit of sub- and multi-GeV SK data has been performed in [28], finding $\Delta m^2_{\text{atm}} \approx 3 \times 10^{-3} \text{eV}^2$, with a much larger $\Delta m^2_{\text{sol}} = 2.4 \times 10^{-3} \text{eV}^2$. While $\Delta m^2_{\text{atm}}$ is thus significantly larger than $\Delta m^2_{\text{sol}}$, we do not find any strong evidence for CPT-violation. As clearly discussed in [28], this large $\Delta m^2_{\text{sol}}$ could be an artifact due to neglecting the effect of the ratio between $\nu_\mu$ and $\nu_e$ fluxes. Our results also disagree with another CPT-violating fit presented in [23]: the difference is significant even in the CPT-conserving limit. A fit performed by the SK collaboration [11] agrees with our fig. 5.

In the case of K2K data (sensitive to neutrinos) we fitted the total number of events ignoring the information about their energy, finding a result in agreement with [37].

### General analysis

We now discuss the effects of the other mixing angles, $\theta_{\text{sun}}$, $\theta_{\text{CHOOZ}}$, $\theta_{\text{LMA}}$, $\theta_{\text{LSND}}$, that we have so far neglected. Some of them are allowed to be large, but cannot significantly affect our CPT-violating atmospheric fit shown in fig. 5.

In anti-neutrinos, disappearance experiments require small values of the two mixing angles that induce oscillations at the LSND frequency. These constraints allow for a novel possibility, somewhat disfavoured only by atmospheric data: the most splitted anti-neutrino eigenstate could be dominantly $\overline{\nu}_e$ (rather than $\overline{\nu}_\mu$ as in fig. 1). In this case, $\theta_{\text{CHOOZ}}$ (the remaining mixing angle that now gives oscillations at the atmospheric frequency) could be large, without conflicting with the CHOOZ bound, if $\Delta m^2_{\text{atm}}$ is below the CHOOZ sensitivity.

In neutrinos, solar experiments require $\theta_{\text{sun}} \sim 1$ as in the CPT-conserving case. Unlike in the CPT-conserving case CHOOZ does not force $\Delta m^2_{\text{sun}} \lesssim 0.7 \times 10^{-3} \text{eV}^2$, but a larger $\Delta m^2_{\text{sun}}$ has recently been disfavoured by the SNO NC data [23]. The angle $\theta_{\text{CHOOZ}}$ (that induces $\nu_\mu \rightarrow \nu_e$ oscillations at the atmospheric frequency; we improperly adopt the name used in CPT-conserving analyses) is not bounded by CHOOZ (i.e. by disappearance of $\overline{\nu}_e$), but only by global fits of solar and atmospheric data, that weakly disfavour a large $\theta_{\text{CHOOZ}}$ [39].

### Signals

At the light of these results, we can now list the CPT-violating signals that could appear in forthcoming experiments (some signals were discussed in [11] [32] [28]).

- MiniBooNE will not see the LSND oscillations, if we only search them as $\nu_\mu \rightarrow \nu_e$ rather than as $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$.

While this signal is mandatory if the CPT-violating interpretation of the LSND anomaly is correct, the following signals can but need not to appear, depending on the values of the unknown parameters:

- We would have a signal for CPT violation if KamLAND will find no solar oscillations in its reactor data, and Borexino will indirectly favour LMA by finding a $\sim 1/2$ suppression and no matter nor seasonal effects.

- If $\theta_{\text{CHOOZ}}$ were large, KamLAND would discover its effects and misinterpret them as LMA oscillations. In particular this means that if KamLAND will confirm LMA, a CPT-violating interpretation of the LSND anomaly would not be immediately excluded, but only disfavoured. We do not list other possible situations that could happen depending on future Borexino and KamLAND results.

- According to our fit in fig. 5, long-baseline experiments that plan to employ a $\nu_\mu$ beam (like K2K, ATP, Future LSND) will definitely see a signal for CPT violation. As a result, we will update the hep-ph version of this paper, adding a precise discussion.

†† When these results will be announced, we will update the hep-ph version of this paper, adding a precise discussion.
Minos and CNGS) have almost the same capabilities of confirming atmospheric oscillations as in the CPT-conserving case. Using a a $\bar{\nu}_\mu$ beam they can also test if $\Delta m_{\text{atm}}^2$ is higher than $\Delta m_{\text{atm}}^2$ (if $\Delta m_{\text{atm}}^2$ is as large as possible, a 5% $\bar{\nu}_\mu$ contamination in the $\nu_\mu$ beam could also give detectable $\tau$-appearance effects).

- These long-baseline experiments can test if $\theta_{\text{Chooz}}$ is larger than what allowed in the CPT-conserving case by looking at $\nu_\mu \rightarrow \nu_e$.

In longer terms, an atmospheric experiment that separately measures $\Delta m_{\text{atm}}^2$ and $\Delta m_{\text{atm}}^2$ (and sees the first oscillation dip) seems feasible \[^2\] , although KEK, CERN and Fermilab preferred to pursue 3 long-baseline experiments.

With a hierarchical $\bar{\nu}$ spectrum (rather than with the inverted spectrum motivated in \[^3\]) planned $\beta$-decay experiments like KATRIN \[^11\] can test the upper part of the $\Delta m^2$ range suggested by LSND \[^2\]. Planned neutrinoless double $\beta$-decay experiments \[^12\] have brighter perspectives of improvement than $\beta$-decay experiments, but CPT-violating neutrino masses seem to require Dirac (rather than Majorana) neutrinos, if the Lorentz symmetry is unbroken (because there is no Lorentz-invariant distinction between massive Majorana $\nu$ into a $\bar{\nu}$: a sufficiently 'fast' Lorentz transformation transforms $\nu$ in $\bar{\nu}$).

In the far future, with a neutrino factory it should be possible to test CPT conservation in atmospheric oscillations at the $\%$ level \[^13\].

### 3 Conclusions

A possible global explanation of the three neutrino anomalies (atmospheric, solar and LSND) is that an extra sterile neutrino generates one of them. Each anomaly, when fitted independently from the other ones, prefers active oscillations refusing the sterile neutrino. The relatively better global fit is obtained with a 3+1 spectrum (sterile LSND oscillations) rather than with a 2+2 spectrum (sterile solar or atmospheric oscillations: this case is disfavoured at 4$\sigma$, after the recent SNO NC results \[^2\]). However the fit is not good: within the 3+1 scheme the LSND anomaly conflicts with $\nu_e$ or $\nu_\mu$ disappearance experiments. One needs to invoke a statistical fluctuation with around $\%$ probability to understand why Bugey, Chooz, CDHS or SK have not seen sterile effects. Our main results are summarized in table 1.

The best-fit LSND regions are shown in fig. 4 assuming that the LSND anomaly is generated through a sterile neutrino (3+1 case, fig. 4a) or by oscillations of active neutrinos (fig. 4b), assuming that a sterile neutrino or something else (e.g. neutrino decay) generates the solar or atmospheric anomaly. The best fit LSND regions are somewhat different: MiniBooNE could discriminate the two cases.

Many sterile neutrinos (motivated e.g. in extra dimensional models) can somewhat improve the fit, but it does not seem possible to obtain a good sterile solution.

In view of these unsatisfactory sterile fits, and of the latest LSND results \[^2\]

\[ P(\nu_\mu \rightarrow \nu_e) = (1.0 \pm 1.6) \times 10^{-3} \]

\[ P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = (2.6 \pm 0.8) \times 10^{-3} \]

one might want to speculate on CPT-violation. A satisfactory global fit of all neutrino data (see table 1) can be obtained with the CPT-violating neutrino masses proposed in \[^2\]. Theory gives no useful restriction, and in particular does not tell if CPT should be violated also in atmospheric oscillations, although it looks plausible. Fig. 4 shows how present SK and K2K data restrict the atmospheric oscillation parameters $\Delta m_{\text{atm}}^2$ and $\Delta m_{\text{atm}}^2$. They can differ by about one order of magnitude. In section 2 we studied which CPT-violating oscillations are compatible with present data, and listed the unusual signals that could be seen at forthcoming solar (KumLAND, Borexino) and long-baseline experiments (K2K, MINOS, CNGS) — and of course at MiniBooNE.

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4 Addendum about the first KamLAND and WMAP data

CPT-violating solution The CPT-violating neutrino spectrum suggested in [9, 32] allowed to reconcile the solar, atmospheric and LSND neutrino anomalies but predicted no effect in KamLAND. The evidence seen by KamLAND [1A] can be fitted by the alternative CPT-violating spectrum proposed at page 6 of the original version of this paper, at the price of a non standard fit of atmospheric data. We now show that, as anticipated in [2A], this solution is disfavoured by atmospheric data (we disagree with the claim in [3A] that it gives an atmospheric fit “clearly favored over the CPT conserving one”).

We denote anti-neutrino parameters with an over-bar. Fitting SK and K2K data in the usual two neutrino approximation, in fig. 5 we plotted the best-fit regions for the atmospheric mass splittings $(\Delta \bar{m}_{\text{atm}}^2, \Delta m_{\text{atm}}^2)$, marginalizing the global $\chi^2$ with respect to the atmospheric mixing angles $\theta_{\text{atm}}$ and $\bar{\theta}_{\text{atm}}$. We can now do the same test on solar $\nu$ data and reactor $\bar{\nu}$ data (from the Homestake, SAGE, Gallex, GNO, SK, SNO, KamLAND, CHOOZ experiments). The result is shown in fig. 6. The plot is restricted to the LMA solution: other solutions with smaller $\Delta m_{\text{sun}}^2$ (LOW and QVO solutions) are disfavoured but not excluded by solar $\nu$ data. The best fit is close to CPT-conservation in both the atmospheric and in the solar cases. We do not show the corresponding fits for the mixing angles $\theta_{\text{atm}}, \bar{\theta}_{\text{atm}}, \theta_{\text{sun}}$ and $\bar{\theta}_{\text{sun}}$. All these mixing angles must be large.

After including KamLAND data, CPT-violation can no longer perfectly fit all data. We now have some evidence of ‘solar’ and ‘atmospheric’ oscillations not only in $\nu$ but also in $\bar{\nu}$, leaving no room for the larger $\Delta m^2$ that should give rise to the LSND anomaly.

In both ‘solar’ and ‘atmospheric’ cases $\bar{\nu}$ data do not yet provide a conclusive evidence, so that we may attempt to fit all neutrino anomalies by sacrificing either i) solar $\bar{\nu}$ data or ii) atmospheric $\bar{\nu}$ data.

Concerning case i), we just mention that the CPT-violating spectrum proposed in [31, 32] (that predicted no anomaly in KamLAND) still gives a reasonably good global fit. We do not consider this possibility, that KamLAND should exclude with more statistics.

Rather, we explore case ii) and sacrifice atmospheric data, taking the smaller anti-neutrino $\Delta \bar{m}^2$ in the KamLAND (KL) range, rather than in the atmospheric range,
and the larger $\Delta m^2$ in the LSND range. There are three anti-neutrino mixing angles. Two of them (that from now on we name $\tilde{\theta}_{\text{LSND}}$ and $\tilde{\theta}_{\text{atm}}$) give oscillations at the larger LSND frequency, $\Delta m_{\text{LSND}}^2$. The third mixing angle (that from now on we name $\tilde{\theta}_{\text{KL}}$) gives oscillations at the smaller $\Delta m_{\text{KL}}^2$. KamLAND and LSND data want a large $\Delta m_{\text{atm}}^2$, induces $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_\tau$ oscillations at the LSND frequency. As usual, the relatively better atmospheric fit is obtained for maximal $\tilde{\theta}_{\text{atm}}$.

Beyond performing a global fit, it is useful to present a semi-quantitative understanding of SK data. For our purposes the main observables are the number of up-ward going and down-ward going $\mu$-like events in the multi-GeV sample. We recall that SK cannot distinguish $\nu$ from $\bar{\nu}$. Neutrinos have roughly the same flux and a two times larger cross section than anti-neutrinos: Analytical estimates are performed by just using this factor two. The total flux has a large overall uncertainty.

We first give an argument similar to the one used in [3A] to state that atmospheric data favour CPT-violation, so that a comparison shows the reason of the disagreement.

1. The maximal up/down asymmetry that CPT-conserving $\nu_\mu \rightarrow \nu_\tau$ oscillations can produce in the ‘multi-GeV $\mu$-like + PC’ sample of SK atmospheric data is (eq. (2b))

\[
A_{\text{ideal}} = \frac{N_\mu(\cos \vartheta = 1) - N_\mu(\cos \vartheta = -1)}{N_\mu(\cos \vartheta = 1) + N_\mu(\cos \vartheta = -1)} = \frac{1}{3}
\]

where $\vartheta$ is the zenith angle and $\cos \vartheta = 1$ corresponds to vertical down-going events. The corresponding upward/downward asymmetry is

\[
A_{\text{real}} = \frac{N_\mu(\cos \vartheta > 0.2) - N_\mu(\cos \vartheta < -0.2)}{N_\mu(\cos \vartheta > 0.2) + N_\mu(\cos \vartheta < -0.2)} = 0.28,
\]

defined ignoring ‘horizontal’ events with $|\cos \theta| < 0.2$. The most recent SK data [4A] give the value

\[
A_{\text{real}} = 0.288 \pm 0.030.
\]

The proposed CPT-violating scenario can give at most an up/down asymmetry $A_{\text{ideal}} = 1/4$, which corresponds to $A_{\text{real}} = 0.21$, $2.5\sigma$ below the experimental value.

This argument takes into account only a part of SK data; furthermore the precise value of $A_{\text{real}}/A_{\text{ideal}}$ depends on $\Delta m_{\text{atm}}^2$. We present another argument which avoids these drawbacks:

2. Since $\Delta m_{\text{LSND}}^2 \gg \Delta m_{\text{atm}}^2$, the proposed CPT-violating oscillations give a reduction in the muon rate with an energy and zenith-angle dependence which (up to an overall factor that does not play an important rôles in the SK analysis) can be mimicked by normal CPT-conserving oscillations with an appropriate effective value of the mixing angle

\[
\sin^2 2\theta_{\text{atm}} \leftrightarrow \begin{cases} \frac{4}{9} & \text{CPT-conserving} \\ \frac{6}{9} - \sin^2 2\theta_{\text{atm}} & \Delta m_{\text{atm}}^2 \gg \Delta m_{\text{LSND}}^2 \end{cases}
\]

The maximal value of the effective CPT-conserving $\sin^2 2\theta_{\text{atm}}$ allowed by the proposed CPT-violating oscillations is $4/5$, which is $5\sigma$ below the experimental value

\[
\sin^2 2\theta_{\text{atm}} = 1.00 \pm 0.04 [4A].
\]

Our global fit of SK data, shown in fig. 5 gives

\[
\chi^2(\Delta m_{\text{atm}}^2 = \Delta m_{\text{LSND}}^2) - \chi^2(\Delta m_{\text{atm}}^2 = \Delta m_{\text{atm}}^2) \approx 5^2
\]

confirming the second argument. We remark that we fit all SK data and not only the up/down asymmetries. We computed the $\Delta \chi^2$ using the latest SK data (1489 days of data taking[1 4A]). Our numerical code contains precise neutrino and anti-neutrino fluxes and cross-sections.

L-violating muon decay We extend our analysis considering one more tentative interpretation of the LSND anomaly, in terms of a speculative $\Delta L = 2$ muon decay channel with branching ratio roughly equal to the oscillation probability suggested by LSND:

\[
\text{BR}(\mu \rightarrow e\bar{\nu}_e\nu) \approx P(\nu_\mu \rightarrow \nu_e) = (2.6 \pm 0.8) \times 10^{-3}.
\]

Since the fact that LSND has a longer path-length than KARMEN plays no rôles according to this interpretation, one naively expects that it is disfavoured by KARMEN as much as oscillations with large $\Delta m^2$.

This expectation was questioned in [5A], that presented one explicit model that produces the $\mu \rightarrow e\bar{\nu}_e\nu$ decay with

[1] We thank M. Shiozawa for providing us the data presented at the Neutrino 2002 conference. Using the slightly older SK data set employed in the original version of this paper would make no significant difference.
Figure 7: Status of 3+1 oscillations including WMAP data. Fig. 7a: LSND favours the shaded region. Values of $\Delta m_{\text{LSND}}^2$ above the horizontal dashed line are disfavoured by WMAP. The other dashed line shows the upper bound on $\theta$ from all other neutrino experiments. The continuous line shows the combination of the two previous constraints. All bounds are at 99% CL for 2 dof. In fig. 7b we show the best fit 3+1 solution, including all data.

Michel parameter $\rho = 0$ (while $\rho = 3/4$ in ordinary muon decay) and consequently a $\bar{\nu}_e$ spectrum softer than the $\nu_e$ spectrum produced by oscillations with large $\Delta m^2$. As a consequence the KARMEN bound on $\bar{\nu}_e$ appearance gets relaxed $\text{[5A]}$ by a factor $\lambda = 1.9$  \text{[6A]}^3 with respect to the bound obtained from the analysis in terms of oscillations with large $\Delta m^2$. In fact, KARMEN detects $\bar{\nu}_e$ using the $\bar{\nu}_e p \rightarrow \bar{\nu} n$ reaction, which cross-section is roughly proportional to $E_{\nu_e}^2$.

However also the LSND experiment detects $\bar{\nu}_e$ using the $\bar{\nu}_e p \rightarrow \bar{\nu} n$ reaction. Therefore an interpretation of the LSND anomaly needs a BR($\bar{\mu} \rightarrow \bar{\nu}_e \bar{\nu}_e \bar{\nu}_e$) larger than $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ by the same factor $\lambda$. The two correction factors compensate each other when comparing LSND with KARMEN, indicating that the naive expectation is right. Table 2 quantifies how much the $\bar{\mu} \rightarrow \bar{\nu}_e \bar{\nu}_e \bar{\nu}_e$ solution is disfavoured. A fully precise result would need a dedicated analysis, that only the LSND collaboration can perform.

2+2 sterile neutrinos Refs 7A, 8A questioned the conclusion that an interpretation of the LSND anomaly in terms of an extra sterile neutrino with ‘2+2’ spectrum have been excluded by solar and atmospheric experiments.

3+1 sterile neutrinos After the first WMAP data cosmology gives the dominant bound on neutrino masses 9A

$$\sum m_\nu < 0.69 \text{ eV} \quad (95\% \text{ CL, 1 dof}).$$

We assume that the extra sterile neutrino has a thermal abundance (a possibility still compatible with primordial nucleosynthesis, unless uncertainties are aggressively esti-
Table 2: Interpretations of all oscillation data, ordered according to the quality of their global fit. A $\Delta \chi^2 = n^2$ roughly signals an incompatibility at $n$ standard deviations.

| model and number of free parameters | $\Delta \chi^2$ | mainly incompatible with |
|------------------------------------|----------------|------------------------|
| ideal fit (no known model)         | 0              | Karmen                 |
| $\Delta L = 2$ decay $\mu \rightarrow e\bar{\nu}_e\bar{\nu}_e$ | 12             | Bugey, WMAP            |
| $3 + 1$: $\Delta m^2_{\text{sterile}} = \Delta m^2_{\text{LSND}}$ | 15             | KamLAND                |
| 3 neutrinos and $\mathcal{O}P\mathcal{T}$ (no $\Delta \tilde{m}^2_{\text{sun}}$) | 25             | SK atmospheric         |
| normal 3 neutrinos                 | 25             | LSND                   |
| $2 + 2$: $\Delta m^2_{\text{sterile}} = \Delta m^2_{\text{sun}}$ | 30             | SNO                    |
| $2 + 2$: $\Delta m^2_{\text{sterile}} = \Delta m^2_{\text{atm}}$ | 50             | SK atmospheric         |

Conclusion We collect in table 2 the present status of various global interpretations of the solar, atmospheric and LSND neutrino anomalies. None of them allows to reconcile all neutrino data in a clean way. It will be interesting to see if MiniBoone will confirm LSND.

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