Baryogenesis and thermal history after inflation

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ABSTRACT

The basic idea of baryogenesis is lectured to introduce non-experts to this subject. Some recent topics, necessarily subjective in view of short time limitation, are also presented to show how the initial condition for baryogenesis is realized in the new framework of inflation.

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1 Introduction

Let us first recall the modern view of cosmology; big bang theory. According to general relativity the spacetime evolution is determined via the Einstein equation by the matter content of universe, which differs from epoch to epoch depending on what kind of energy dominates the energy density of the universe at that time. There are three important epochs characterized by different pressure-energy relation; (1) vacuum energy dominance with $p = -\rho$, (2) massless particle dominance with $p = \frac{1}{3}\rho$, (3) nonrelativistic particle dominance with $p \ll \rho$. With homogeneity and isotropy of 3-space the Einstein equation is much simplified with the Robertson-Walker metric: $ds^2 = dt^2 - a^2(t) \vec{x}^2$. The behavior of the scale factor $a(t)$ then follows:

\begin{align*}
(1) \quad a &\propto e^{Ht}, \quad H = \sqrt{8\pi G V/3}, \\
(2) \quad a &\propto t^{1/2}, \\
(3) \quad a &\propto t^{2/3},
\end{align*}

in their respective energy ranges. The first stage is the inflationary epoch where a constant vacuum energy $V$ gives the exponential growth of the scale factor, which is believed to solve the horizon and the flatness problems in the old big bang model.\[1\]

It would be of some interest to view the Hubble parameter defined by $H = \dot{a}/a$ as a function of $a$ or a typical energy scale $T \propto 1/a$ of dominant massless particles in the universe, instead of $a(t)$ as a function of time. In this picture the first inflationary stage is longest, lasting for the scale change $> 10^{30}$. Subsequent radiation and matter dominant epochs have energy changes of order, $10^{26}$ or $10^3$. The standard model of microphysics only probes the temperature range of $10^2$ GeV $- 10^{-13}$ GeV, of much smaller variation than in the inflationary epoch. Of great importance in subsequent cosmic evolution and of intensive current interest is the transient stage from inflation to radiation dominance; the epoch of reheating after inflation, which I shall touch upon later in this lecture.

For a long time the success of the big bang model rested with three cornerstones;

(1) Hubble expansion,
(2) Planck distribution of relic photons,
(3) light element abundance such as $^4$He, D, $^7$Li.
In the last decade there have been substantial improvements of this classic achievement both in observation and in theory. Perhaps the most significant is COBE measurement of the spectrum shape and large scale fluctuation of the relic microwave temperature: absolute motion of earth relative to the universal cosmic expansion has been observed with $\delta T/T \approx 10^{-3}$, and after subtraction of this dipole component there exists a large scale fluctuation of order $10^{-6}$, whose precise feature yet to be clarified is very important in the theory of structure formation. Nucleosynthesis has become a precise science developed to a fine detail, so much refined that one can even discuss dependence on the number of neutrino species and the neutron lifetime. But presumably the most important in nucleosynthesis is a consistent and reliable determination of the key cosmological parameter, the baryon to photon ratio $n_B/n_\gamma$, of order $10^{-10}$.

In order to probe microphysical processes that occur at each instant of cosmological time, it is necessary to compare the two time scales; physical time scale during which a relevant process takes place, and the Hubble time scale of order $1/H$. If the time scale of a physical process is much shorter than the Hubble time, then this physical process will occur frequently, and in the limit of large rate equilibrium is reached.

In a weak coupling theory such as supersymmetric grand unified theories beyond the TeV scale, the proper framework to discuss the interaction rate is the Boltzmann equation for one particle distribution function taking into account the cosmological expansion. When the Boltzmann equation is integrated with respect to the momentum phase space, an equation for the number density follows;

$$\left( \frac{d}{dt} + 3 \frac{\dot{a}}{a} \right) n_i = \int \frac{d^3 p_i}{(2\pi)^3} \frac{\Gamma_i[f]}{E_i},$$

where $\Gamma_i[f]$ is the invariant interaction kernel that depends on all distribution functions $f_j$ participating in the process. As an example let me give the form of the kernel when only two-body reactions are involved,

$$\Gamma_i[f] = \frac{1}{2} \sum_{j,k,l} \int dP_j dP_k dP_l \left[ (1 \pm f_i)(1 \pm f_j) f_k f_l \gamma_{kl \to ij} - (1 \pm f_k)(1 \pm f_l) f_i f_j \gamma_{ij \to kl} \right],$$

where $\gamma_{ij \to kl}$ is the invariant rate for the process $ij \to kl$ essentially given by the differential cross section times the initial relative velocity, and $\pm$ corresponds to
either stimulated emission for bosons or Pauli blocking for fermions. \( dP = \frac{d^3p}{E(2\pi)^3} \) is the invariant phase space element.

In general it is difficult to solve non-linear integro-differential equations of the Boltzmann type, but it is often possible to estimate reaction rate in the right hand side by some means. In certain situations the rate may roughly vary linearly with the number density itself \( n_i \);

\[
\left( \frac{d}{dt} + 3H \right) n_i = -\overline{\gamma_r} n_i ,
\]

with \( H = \dot{a}/a \). For instance, if the relevant particle freely decays having no interaction with medium particles and no process of particle production operates, \( \overline{\gamma_r} \) is simply the free decay rate.

A particular reaction is of cosmological relevance only if the reaction rate given by the right hand side of the Boltzmann equation is much larger than the Hubble rate \( 3H n_i \). What typically happens is that the process continues at early times and after some late time it is frozen. In cosmology it is thus important to estimate decoupling epoch of a particular reaction. The crucial quantity in determining relevance of the reaction is the ratio of the average rate to the Hubble rate \( \overline{\gamma_r}/H \). If this ratio is very large, the reaction occurs frequently.

As an important practical example let us consider the four Fermi type interaction governed by a coupling \( \alpha/M^2 \). This could be the weak interaction involving neutrinos such as \( \nu \bar{\nu} \rightarrow e^+e^- \) or baryon number violating reaction, \( qq \rightarrow q\bar{q}l \), if one uses as the relevant mass \( M \) the electroweak or the GUT mass scale. Without any detailed discussion I would give you a suggestive form of the average rate,

\[
\overline{\gamma_r} \approx \frac{\alpha^2 T^2}{(T^2 + M^2)^2} \cdot T^3.
\]

Here \( T \) is a typical energy of participating particles and of order the temperature if all these particles are in thermal and chemical equilibrium, and the last factor \( T^3 \) represents the number density of colliding particles. The factor in front of this number density is the averaged reaction rate for the elementary process, which grows with energy \( T \) as \( \alpha^2 T^2/M^4 \) for \( T \ll M \), but is finally saturated and decreases as \( \alpha^2/T^2 \) in the high energy limit. On the other hand, the Hubble rate is of order \( \sqrt{N} T^2/m_{pl} \) with \( m_{pl} = 1/\sqrt{G} \sim 10^{19} \) GeV the Planck mass. \( N \) is the number of massless particle species in equilibrium. It is then easy to see that the ratio \( \overline{\gamma_r}/H \)
has a maximum at around $T = M$ and the maximum ratio is of order

$$\alpha^2 m_{\text{pl}}/(\sqrt{N} M).$$

(8)

The condition of the large rate for equilibrium is thus $M \ll \alpha^2 m_{\text{pl}}/\sqrt{N}$. Since the right hand side contains the Planck mass, this condition is easily obeyed for the weak processes. This is an important observation to understand why the element abundance cooked up in the early universe is insensitive to the initial condition: one can justify that the initial neutron-proton ratio is given by the value in thermal and chemical equilibrium.

As another example let us consider how much of baryons is left over if the universe is symmetrical with respect to baryons and anti-baryons. This is a problem relevant to baryogenesis, the main theme of this lecture. As the final element carrying the negative baryon number, anti-nucleons gradually pair-annihilate via many pion production; $NN \rightarrow$ many $\pi'$s, but after the decoupling of this annihilation process the leftover abundance $n_N/n_\gamma = n_{\overline{N}}/n_\gamma$ is frozen. The annihilation process has a cross section roughly of order $\langle \sigma v \rangle \approx 1/m_N^2$ at low energies, independent of the energy participating in the process. On the other hand, the thermal number density is suppressed by the Boltzmann factor when nucleons and anti-nucleons become nonrelativistic as actually is the case;

$$n_N = n_{\overline{N}} \sim 4 \left( \frac{m_NT}{2\pi} \right)^{3/2} e^{-m_N/T}.$$  

(9)

From the decoupling condition, $\langle \sigma v \rangle \cdot n_N = H$, one then finds for the decoupling temperature $T_d$ and the leftover abundance that

$$\frac{T_d}{m_N} \approx [\ln \frac{m_N m_{\text{pl}} \langle \sigma v \rangle}{60\sqrt{N}}]^{-1} \sim \frac{1}{45},$$

(10)

$$\frac{n_N}{n_\gamma} \approx \frac{40\sqrt{N}}{m_N m_{\text{pl}} \langle \sigma v \rangle} \sim 10^{-18}.$$  

(11)

The final abundance $n_N/n_\gamma$ is too small compared with the observed value $\approx 10^{-10}$, and this argument essentially rules out the symmetric cosmology.

In the asymmetric universe with respect to baryons and anti-baryons the present number ratio $n_B/n_\gamma$ is the measure of the imbalance between baryons and antibaryons prior to the annihilation process:

$$\left( \frac{n_B}{n_\gamma} \right)_{\text{present}} \approx \left( \frac{B - \overline{B}}{B + \overline{B}} \right)_{\text{before annihilation}},$$

(12)
since in thermal and chemical equilibrium baryons, anti-baryons, and photons are all in roughly equal abundance, $B \approx \overline{B} \approx \gamma$, ignoring the mass threshold and the statistical factor. Our task is thus to explain this quantity from microphysics.

## 2 Condition for baryogenesis

Although necessary ingredients for the baryogenesis were written a long time ago\[4\], intricacy of this condition has been spelled out much later \[5\],\[6\],\[7\],\[8\]. (For reviews, see \[9\],\[10\].) Their fascinating features still invite many interesting scenarios. The three obvious conditions are

1. baryon number non-conservation,
2. violation of discrete $C$ and $CP$ symmetry,
3. departure from thermal equilibrium.

The baryon number is not a sacred symmetry in modern gauge theories, and indeed violated explicitly by heavy $X$ boson mediated processes in grand unified theories \[11\]. Moreover, even in the standard $SU(3) \times SU(2) \times U(1)$ gauge theory the baryon number is violated by instanton effects at zero temperature\[12\], which is however unobservably suppressed. It has however been recognized that sphaleron-mediated processes \[13\] may enhance baryon non-conservation in the standard theory at high enough temperatures, higher than TeV scale, although $B - L$ is exactly conserved here. The sphaleron \[14\] is a finite energy field configuration which bridges between different nonperturbative vacuum configurations ordinarily suppressed by tunneling probability at zero temperature. The effect may be enhanced at a high temperature $T$ by the factor $e^{-F/T}$, with $F$ the free energy of this unstable sphaleron. The sphaleron effect can readily wash out the baryon asymmetry generated prior to this epoch if initially $B - L = 0$. It should however be noted that it is not a trivial matter to create from a symmetric state the asymmetry at the electroweak scale. This is related to that the last two conditions are much more subtle, as I shall explain shortly.

Is the standard model capable of explaining the baryon asymmetry? First of all, the first two conditions are met as a matter of principle even in the standard model, although strength of $B$ violation is to be detailed. The last condition of departure from equilibrium is in general difficult to meet. Most scenarios \[15\] use the first order phase transition of the electroweak gauge symmetry breaking as a means of setting
up non-equilibrium environment. This requires a Higgs boson mass \(< 45 \text{ GeV}\) in the standard model of one Higgs doublet, which already seems to be ruled out by LEP experiments. But this bound is based on a one-loop computation, so there might be some loophole in the argument not contemplated so far, and it may be worthwhile to explore the possibility of baryogenesis, ignoring the Higgs mass bound.

Farrar and Shaposhnikov \cite{2} recently proposed an interesting scenario that employs quark scattering off the electroweak bubble created at the first order phase transition. As an idealization one may assume that baryon number is strongly violated in the unbroken phase, while it is conserved in the broken phase. Quarks are scattered off within some length near the bubble wall and there may be many scattering amplitudes that interfere, giving non-trivial $\mathcal{CP}$ violating effect as required. Unfortunately the original calculation neglected the important effect of coherence in the cosmic plasma \cite{3}. Coherence is crucial to generate the baryon asymmetry by this mechanism and is only maintained by the coherence length which is severely limited in the cosmic plasma. The resultant asymmetry is too small to yield the one needed by observation, typically \(< 10^{-20}\) \cite{3}. Thus at the moment there is no viable model of baryogenesis that explains the observed ratio using only the source of $\mathcal{CP}$ violation identical to the CKM matrix.

According to our present wisdom the standard model must be extended in order to explain the baryon asymmetry. There are two attitudes in extending the established physics with this respect: one is to use the new physics of unification such as grand unified theories or its supersymmetric extension that have the natural source of all ingredients for baryogenesis, and the other \cite{1} is to minimize the extension utilizing the strong electroweak baryon violation maximally. The way I would like to characterize these two directions is that the first direction needs a big jump beyond the standard theory, while its physical mechanism of baryogenesis is straightforward and readily understandable. On the other hand, the second extension needs a small step beyond what we already know, such as a slight modification of the Higgs system, but physics involved is fairly complicated, and there seems no model everyone would like to consider seriously. With this situation in mind I would like to mainly discuss essential features of baryogenesis based on GUT. I shall also discuss obstacle against the GUT scenario and how to evade it.

Although not discussed below, there is another interesting scenario of baryogenesis that is becoming popular recently; the Affleck-Dine mechanism \cite{4}. This
mechanism uses a feature of supersymmetric models; existence of many flat field
directions in the scalar field potential. These directions may include field condensate
that carries the baryon number or the lepton number, both of which can be used
when combined with the strong electroweak baryon nonconservation at finite tem-
peratures. The resulting baryon to photon ratio tends to be large compared to the
observed one, and one usually has to consider some process of dilution at the same
time. A version of this mechanism is discussed in the lecture by L. Randall at this
school.

3 GUT baryogenesis

Baryon nonconserving processes in grand unified theories are mediated by gauge or
Higgs bosons generically called $X$ bosons. The simplest and workable scenario of
GUT baryogenesis employs $X$ boson decay, which has two types of decay modes,
the two quark decay mode $qq$, and the leptoquark mode $\bar{q}l$. Coexistence of the two
modes with different baryon numbers is a manifestation of baryon nonconservation.

Two simple examples of grand unified theories are $SU(5)$ and $SO(10)$ models
\[11\]. These models have unified multiplet structure of one family of quarks and
leptons of the form,

\[
SU(5); \quad 5 + 10^\ast, \quad \begin{pmatrix} d_R \\ d_G \\ d_B \\ e^+ \\ \bar{\nu} \end{pmatrix}_R + \begin{pmatrix} \overline{u}_B & -\overline{u}_G & u_R & d_R \\ 0 & \overline{u}_R & u_G & d_G \\ 0 & u_B & d_B & e^+ \\ 0 & 0 & 0 & 0 \end{pmatrix}_L, \tag{13}
\]

\[
SO(10); \quad 16, \quad \begin{pmatrix} u_R & u_G & u_B & \nu \\ d_R & d_G & d_B & e \end{pmatrix}_L + \begin{pmatrix} \overline{u}_R & \overline{u}_G & \overline{u}_B & \overline{N} \\ \overline{d}_R & \overline{d}_G & \overline{d}_B & \overline{\nu} \end{pmatrix}_L. \tag{14}
\]

For instance, in the $SU(5)$ model $X$ gauge bosons induce transitions from a quark
to a lepton in the representation $5$, while in $10$ they cause transitions both from a
quark to a lepton and from an anti-quark to a quark.

Recent precise measurements of coupling constants at LEP suggest \[19\] that
supersymmetric extention of the $SU(5)$ model gives a consistent picture of coupling
unification using the renormalization group. The $SO(10)$ model, on the other hand, is interesting if a finite neutrino mass hinted by the solar neutrino experiments is real. Despite of the lack of evidence for proton decay grand unified theories thus deserve serious consideration.

In discussing the GUT baryogenesis, I shall dismiss complication due to many decay channels and concentrate on the two decay modes for simplicity. Let us denote decay rates of the two modes by $\gamma_q$, $\gamma_l$ for $X$ and $\bar{\gamma}_q$, $\bar{\gamma}_l$ for its anti-particle $\bar{X}$. When a pair of $X$ and $\bar{X}$ decays, a finite baryon number may be created with a rate,

$$\Delta B = \frac{2}{3}\gamma_q - \frac{1}{3}\gamma_l - \frac{2}{3}\bar{\gamma}_q + \frac{1}{3}\bar{\gamma}_l = \gamma_q - \bar{\gamma}_q.$$  \hspace{1cm} (15)

This quantity was simplified by using the requirement of $\mathcal{CPT}$ theorem, which states the equality of particle and anti-particle total decay rate;

$$\gamma_q + \gamma_l = \bar{\gamma}_q + \bar{\gamma}_l.$$  \hspace{1cm} (16)

Clearly $\mathcal{CP}$ violation is called for $\gamma_q \neq \bar{\gamma}_q$.

Before discussing perturbative calculation of this asymmetry, I shall explain the non-equilibrium condition for the GUT baryogenesis. Departure from equilibrium demands in this case that the inverse decay process, $qq \rightarrow X$, $q\bar{q} \rightarrow X$ etc. is essentially frozen by threshold effect. This is realized if at the decay time the temperature is too low to create heavy $X$’s: $T < m_X$ when the Hubble $H(T) = \gamma_X$. This leads to $O[\sqrt{\frac{m_{pl}}{m_X}}] < m_X$. Since the total decay rate $\gamma_X \sim \alpha m_X$, it gives a constraint on the $X$ mass,

$$m_X > O[\alpha m_{pl}].$$  \hspace{1cm} (17)

A more precise estimate yields the $X$ mass bound of order $10^{15} - 10^{16}$ GeV, close to the GUT scale. This constraint may be obeyed without much difficulty by a Higgs $X$ boson, although with some difficulty by the gauge $X$ boson.

Calculation of the magnitude of the baryon asymmetry due to the $X$ boson decay involves interesting interference effect. Suppose that one computes the baryon production rate $\Delta B$ in perturbation theory;

$$\Delta B = \sum_{\text{phase space}} |g_1f_1 + g_2f_2 + \cdots|^2 - |g_1^*f_1 + g_2^*f_2 + \cdots|^2
\hspace{1cm} = -4 \Im(g_1g_2^*) \cdot \sum_{\text{phase space}} \Im(f_1f_2^*) + \cdots.$$  \hspace{1cm} (18)
Each decay amplitude $g_i f_i$ corresponds to a Feynman diagram and $g_i$ lumps together all coupling factors leaving the dynamical part of the amplitude to $f_i$. The rate for $X$ thus has the complex-conjugated coupling $g_i^*$, as required by $\mathcal{CPT}$ or hermiticity of the Hamiltonian. The final expression of $\Delta B$ clearly indicates need for a dynamical phase $\Im(f_1 f_2^*) \neq 0$ besides the $\mathcal{CP}$ violation, $\Im(g_1 g_2^*) \neq 0$. The dynamical phase may arise as a rescattering phase in ordinary two-body processes, and one may regard the phase above as a generalization of the rescattering phase. The constraint for a non-trivial dynamical phase is that intermediate states must have a threshold below the parent particle mass $m_X$. This observation also hints an efficient way to compute the asymmetry as a discontinuity according to the Landau-Cutkovsky rule: one can put intermediate particles on the mass shell which makes actual computation of the asymmetry much easier.

The baryon to photon ratio, or more precisely the baryon to entropy ratio $n_B/s$ may be derived from the baryon production rate $\Delta B$ as follows. Define first the baryon number in the comoving volume, $\mathcal{N}_B = a^3 n_B$, together with a similar $X$ density, $\mathcal{N}_X = a^3 n_X$. These vary as

$$\dot{\mathcal{N}}_B = \Delta B \cdot \mathcal{N}_X, \quad \dot{\mathcal{N}}_X = -\gamma_{\text{tot}} \cdot \mathcal{N}_X,$$

with $\gamma_{\text{tot}} = \gamma_q + \gamma_l$ the total $X$ decay rate. This is integrated with the condition that initially no baryon number exists and finally no $X$ boson exists:

$$(\mathcal{N}_B)_f = \epsilon (\mathcal{N}_X)_i, \quad \epsilon \equiv \frac{\Delta B}{\gamma_{\text{tot}}}. \tag{20}$$

Since this reaction and subsequent evolution proceeds almost adiabatically, the entropy per comoving volume is approximately conserved; $a^3 s = \text{constant}$. This relation can be used to eliminate the volume factor $a^3$ in favor of the entropy density $s$;

$$\left(\frac{n_B}{s}\right)_f = \epsilon \left(\frac{n_X}{s}\right)_i. \tag{21}$$

There are many interesting details of how the magnitude of the baryon asymmetry may be correlated with some other physical quantity in a particular model. For instance, the $SU(5)$ model with the minimal Higgs structure yields too small a value of $n_B/n_\gamma$. On the other hand, the baryon asymmetry is correlated with the neutrino mass in $SO(10)$ models $[20]$: the smaller the neutrino mass is, the smaller the baryon asymmetry is. This can be utilized to constrain a finite, but a small neutrino mass. I shall however omit discussion of these subjects in this introductory lecture.
It is now appropriate to discuss difficulty associated with the GUT baryogenesis and how to overcome it. First, as I already mentioned above, there may be baryon number annihilation via electroweak processes at finite temperatures. One can always excuse for the GUT scenario that the electroweak baryon nonconservation keeps intact $B - L$, so if $B - L$ is created at the GUT epoch, later evolution only redistributes $B$ and $L$, but never annihilates the baryon number;

$$B_{\text{final}} = c \cdot (B - L)_{\text{initial}},$$

with $L_{\text{final}} = (1 - c) \cdot (B - L)_{\text{initial}}$. Here $c$ is a calculable number of order unity.

One has to be careful however to avoid a large lepton violation at intermediate temperature scales that may potentially dissipate away the baryon number with $\Delta B = \Delta L = 0$, when combined with electroweak $B$ nonconservation. Allowed range of this interaction, accordingly constraint on the neutrino mass via the seesaw type of neutrino mass generation may be estimated by introducing a generic type of $\Delta L = 2$ interaction;

$$\mathcal{L}_{\Delta L \neq 0} = \frac{m_\nu}{v^2} l_L l_L \varphi \varphi + (\text{h.c.}),$$

with $l_L$ and $\varphi$ the lepton and the Higgs doublet, respectively and $v \simeq 250$ GeV. Requirement of harmless lepton violation, $\Gamma_{\Delta L \neq 0} < H$, leads to the upper bound on the neutrino mass [21];

$$m_\nu < 4 \times 10^{-3} \text{eV} \left(\frac{T_{B-L}}{10^{16} \text{GeV}}\right)^{-1/2},$$

with $T_{B-L}$ the scale of $B - L$ generation at higher temperatures. This constraint is always applied to the lightest neutrino species, but when the neutrino mixing is large, it is also applied to the heaviest neutrino species.

The second problem with the GUT baryogenesis is more serious; a possible overproduction of gravitino and associated low reheating temperature after inflation. The gravitino is the superpartner of graviton in supergravity theories. It is a spin 3/2 particle and couples with ordinary matter field with the gravitational strength. Moreover, one usually associates supersymmetry breaking scale with the electroweak scale in order to ease the hierarchy problem, thus the mass of the gravitino $m_{3/2} = O[\text{TeV}]$. This has a consequence potentially very serious, because the decay rate of the gravitino is given by

$$\Gamma_{3/2} \approx \frac{m_{3/2}^3}{m_{\text{pl}}^2} \sim (10^5 \text{sec})^{-1} \cdot \left(\frac{m_{3/2}}{\text{TeV}}\right)^3,$$
the lifetime being close to the epoch of nucleosynthesis. If the gravitino abundance is larger than of order $10^{-10}$ relative to the entropy density, the successful nucleosynthesis would be destroyed.

For a short while inflation was considered to save this potential disaster by diluting away the gravitino abundance. But subsequently it has been recognized that regeneration of gravitinos after inflation severely constrains the maximally allowed reheating temperature. The point of this argument is that gravitino pairs may be produced from ordinary particles, whose cross section is calculable and of order $1/m^2_{\text{pl}}$, giving the abundance of order

$$\frac{n_{3/2}}{s} = O[10^{-2}] \frac{T}{m_{\text{pl}}},$$

in terms of the reheat temperature $T$ after inflation. For successful nucleosynthesis one gets the constraint on the reheat temperature

$$T < O[10^{10} - 10^{11}] \text{ GeV}$$

from $\frac{n_{3/2}}{s} < O[10^{-10} - 10^{-11}]$. This temperature is too low to create the baryon asymmetry by the $X$ boson decay. It thus appeared that supergravity models arising as the field theory limit of presumably the ultimate theory of superstring give too low a reheat temperature incompatible with the GUT baryogenesis.

I shall discuss in the following sections how a correct theory of reheating after inflation may provide a high temperature phase suitable to the baryon generation without overproduction of gravitinos.

4 Thermal history after inflation

How the hot big bang is started after inflation is a fascinating subject that can be discussed independently of baryogenesis. But since the subject directly addresses the origin of entropy in our universe at earliest times of evolution, one may very naturally entertain the possibility that the two basic quantities in cosmology, the baryon number and the entropy are both created roughly at the same time. Moreover, association of the origin of the cosmic entropy with inflation sets an ideal theoretical framework, because inflation dilutes away everything in our observable part of the universe: one must explain the origin of entropy starting from empty space, except the coherent inflaton oscillation around the minimum of inflaton potential.
For quite some time the theory of particle production due to coherent and homogeneous field oscillation was based on a naive picture of lowest order perturbation\cite{23}. The oscillating inflaton field, denoted here by $\xi(t)$, is regarded in this naive picture as an aggregate of condensed bosons at rest, and these bosons are assumed to decay stochastically according to the rate given by the Born approximation. Thus equating the decay rate with the Hubble rate $\gamma_\xi = H$, together with assumption of the instantaneous reheating, leads to

$$T_B \sim 0.1 \sqrt{\gamma_\xi m_{pl}} \sim 10^{14} \text{GeV} \cdot g \cdot \frac{m_\xi}{10^{13} \text{GeV}}.$$\hspace{1cm} (28)

The Born rate for massless particle decay, $\xi \to \varphi \varphi$,\hspace{1cm} (29)

$$\gamma_\xi = \frac{g^2}{32\pi} m_\xi$$

was used in this estimate.

This picture is valid when the amplitude of oscillation and the coupling $g$ to matter field is small enough, but is grossly wrong if this condition is violated, for instance when the amplitude of oscillation is large. As will be explained shortly, there exist an infinitely many bands of instability that may contribute to the inflaton decay, only one of which, the lowest band, when restricted to the small amplitude limit, is identified with the Born decay rate. This phenomenon of instability is known as the parametric resonance under periodic potential\cite{24,25}.

A systematic method to understand particle production and associated inflaton decay can be formulated\cite{26,27} in the Schrödinger picture of quantum field theory. (For other approaches and other aspects of this problem, see ref \cite{28,29,30,31}.) Quantum bose fields that couple to the inflaton field are treated as a quantum operator (in the Heisenberg picture), but the inflaton field is regarded as classical in this approach, although back reaction against the inflaton oscillation is also considered. In the Schrödinger picture the state vector describing behavior of quantum field coupled to the inflaton is given by a direct product of state vectors $|\psi(t)\rangle_k$ of independent spatial Fourier modes. During the time interval $\Delta t$ that obeys

$$1/m_\xi \ll \Delta t \ll 1/H,$$

one may assume exactly periodic inflaton oscillation with a periodically varying frequency, $\omega^2_k(t)$, containing the oscillating function $\xi(t)$. For longer time scales, $\Delta t \gg 1/H$, the amplitude of oscillation is taken to adiabatically change.
A salient feature of this approach is that one may solve the quantum state with a Gaussian ansatz;

\[ \langle \mathbf{k} | \psi(t) \rangle_k = \frac{1}{\sqrt{u_k}} \exp\left[ \frac{i \hat{u}_k}{2 u_k} q_k^2 \right], \quad (31) \]

where \( u_k(t) \) is shown to obey the classical oscillator equation;

\[ \left[ \frac{d^2}{dt^2} + \omega_k^2(t) \right] u_k = 0 . \quad (32) \]

The initial condition for this classical equation is determined by the choice of an initial quantum state. The simplest, and a reasonable choice is the ground state with respect to some reference frequency, most naturally the frequency at the onset of inflaton oscillation \( \omega_k(0) \). This is because after inflation the state is essentially devoid of matter. Under this circumstance the initial condition is

\[ u_k(0) = \left( \frac{\omega_0}{\pi} \right)^{-1/2}, \quad \frac{\dot{u}_k}{u_k}(0) = i \omega_0 , \quad (33) \]

with \( \omega_0 \) the reference frequency, and one may further simplify the wave function in terms of \( |u_k(t)| \) alone \[26\].

The most important consequence \[26,27\] of this formalism is that it gives a rationale to introduce a coarse grained density matrix \( \rho^{(D)} \) which has a classical probability distribution. The coarse graining here is defined by a short time average over the time scale of order a few oscillation periods. This seems a reasonable way to extract global behavior of the quantum system ignoring fine details of the quantum state. This makes it possible to replace the quantum density matrix by the time-averaged diagonal part \( \langle n | \rho^{(D)} | n \rangle \) of the density matrix in a convenient base such as the Fock base of frequency \( \omega_0 \):

\[ \langle n | \rho^{(D)} | n \rangle = \overline{\langle n | \psi(t) \rangle \langle \psi(t) | n \rangle} , \quad (34) \]

where \( |n\rangle \) is the \( n \)-th level of field oscillators and the overline represents the short time average. After the coarse graining a finite entropy may be assigned;

\[- \text{tr} \rho^{(D)} \ln \rho^{(D)} > 0 . \quad (35) \]

From this reduced density matrix one computes various physical quantities. For instance, the produced particle number in each mode is given by

\[ \langle N_k \rangle = \overline{\langle \frac{1}{\omega_0} \left( \frac{1}{2} p_k^2 + \frac{1}{2} \omega_k^2 q_k^2 \right) - \frac{1}{2} \rangle} = \text{tr} (\rho_k^{(D)} a_k^\dagger a_k) \]

\[ = 4 \left( \omega |u_k|^2 - \pi \right)^2 + \left( \frac{d|u_k|^2}{dt} \right)^2 \frac{16 \pi \omega |u_k|^2}{}, \quad (36) \]
with \( q_k, p_k \) oscillator coordinates. Suitable time average is understood here. As \( t \to \infty \),

\[
\langle N_k \rangle \to e^{\lambda_k m \xi t} \times \text{(polynomial in } t) \tag{37}
\]

for the momentum \( k \) in the instability band.

Two types of matter coupling have been considered;

\[
V_\xi = \frac{1}{2} g^2 \xi^2 \varphi^2 + \frac{1}{2} c g m \xi \xi \varphi^2, \tag{38}
\]

with \( c \) a constant of order unity and \( m_\xi \) the mass of the inflaton. As a model of inflation we consider the simplest chaotic type inflation \cite{32} with parameters; \( m_\xi \sim 10^{13} \) GeV consistent with COBE anisotropy, and the initial amplitude of order the Planck scale,

\[
\xi_0 \sim \sqrt{\frac{3}{4 \pi}} m_{pl}. \tag{39}
\]

The dimensionless coupling \( g \) is taken arbitrary at the moment.

The classical mode equation is then

\[
\frac{d^2 u}{dz^2} + [h - 2 \theta \cos(4z) - 4c\sqrt{\theta} \sin(2z)]u = 0, \tag{40}
\]

\[
h = 4 \frac{k^2 + m^2}{m_\xi^2} + 2 \theta, \quad \theta = \frac{g^2 \xi_0^2}{m_\xi^2}, \quad z = \frac{m_\xi t}{2}, \tag{41}
\]

where \( \xi_0 \) is the amplitude of inflaton oscillation;

\[
\xi(t) = - \xi_0 \sin(m_\xi t). \tag{42}
\]

The criterion of large or small amplitude is thus given by the magnitude of \( \theta \). In the very small (\( \theta \ll 1 \)) or in the very large (\( \theta \gg 1 \)) amplitude limit the classical mode equation \cite{31} reduces to the Mathieu equation \cite{33} with a single oscillating term. In both these limits we developed analytic formulas suitable for detailed analysis of the reheating problem and related problems, too \cite{26},\cite{34}.

The structure of instability bands is as follows. Each band is labeled by an integer \( n = 1, 2, 3 \ldots \), and goes to \( h \to n^2 \) in the small amplitude limit, \( \xi_0 \to 0 \). The band width in the small amplitude region is

\[
\Delta h_n = \frac{\theta^{n/2} c_n}{2^{2n-1} [(n-1)!]^2}, \tag{43}
\]

at a fixed \( \theta \), with \( c_n \) some function of \( c \) \cite{26},\cite{34}. For instance, \( c_n = 2^n \) with the Yukawa coupling \( \frac{1}{2} g m_\xi \xi \varphi^2 \) alone. The instability bands are thus indeed very narrow.
in the small amplitude region. But as $\theta$ increases, the bands become broad, and for $h < 2\theta$ most of the $(\theta, h)$ parameter space is covered by instability bands except bounding narrow stability bands. For parameters $(h, \theta)$ or corresponding $(k, \xi_0)$ within an instability band the classical solution exhibits exponential growth:

$$u \to e^{\lambda \xi_0 t} \times P(t),$$

with $P(t)$ a periodic function. This has the important consequence of exponential particle production rate unless the back reaction stops it.

In the small amplitude limit exact results for the growth rate $\lambda$ and mode sum are available. After the coarse graining the initial state decays according to $e^{-\Gamma V t}$ where

$$\Gamma = \sum_n \Gamma_n, \quad \Gamma_n = \frac{m_\xi}{2V} \sum_{\bar{k} \in n-th \ band} \lambda_{\bar{k}}. \quad \text{(45)}$$

$\Gamma_n$ is the decay rate per unit volume of the $n$-th band, which is computed [26],[35] as

$$\Gamma_n = \frac{m_\xi^4}{64\pi} \sqrt{1 - \frac{4m^2}{n^2m_\xi^2} (\Delta h_n)^2}. \quad \text{(46)}$$

It was also shown recently [35] that the small amplitude result can be understood by familiar perturbation theory. The point is that the mode-summed rate grows with the amplitude as $\propto \xi_0^{2n} \propto (n \xi)^n$ where $n \xi = \frac{1}{2} m_\xi \xi_0^2$ is the number density of condensed inflatons. The decay rate precisely coincides with the zero-momentum limit of $n$ to 2 body process $n \xi \to \varphi \varphi$, worked out using the ordinary Feynman rule. In particular, the decay rate of the first band with $c = 1$ is given by

$$\Gamma_1 = \frac{g^2 m_\xi^2 \xi_0^2}{64\pi} \sqrt{1 - \frac{4m^2}{m_\xi^2}}. \quad \text{(47)}$$

This exactly coincides with the one-particle decay rate $\gamma_\xi$ for $\xi \to \varphi \varphi$, when divided by the inflaton number density $\frac{1}{2} m_\xi \xi_0^2$.

It is important for many interesting applications to work out analytic formulas in the large $\theta$ region. This seems a formidable task in view of that non-perturbative analysis is involved. A remarkable result [34] is that in the functional Schrödinger picture one can solve the fundamental quantity $u_k(t)$, which becomes rigorous in the region, $\theta \gg 1$ with $|h - 2\theta| \ll \theta$. Furthermore this is precisely the parameter region of prime importance when one considers cosmological evolution, as will be discussed.
shortly. In the rest of this lecture we shall neglect the mass $m$ of quantum bose field considering only those of $m \ll m_\xi$.

Cosmological evolution changes both the momentum and the inflaton amplitude according to

$$k \propto 1/a, \quad \xi \propto 1/a^{3/2}.\quad (48)$$

Taking in the large $\theta$ region the leading variation alone gives the approximate rule; $\Delta h \sim 2 \Delta \theta$. Thus parameters of the dominant contribution move parallel to the $h = 2\theta$ line with cosmological evolution. Moreover, the largest particle production with the largest rate $\lambda$ occurs in the deepest region within instability bands of the Mathieu equation (the original two terms of oscillations reduce to one term for $\theta \gg 1$). This implies that the most dominant region is along $h = 2\theta$ within some width $\delta = \delta(h - 2\theta)$.

The analytic result derived in ref [34] is summarized as

$$\lambda = \frac{1}{\pi} \ln\left(\frac{1}{\sqrt{2}}\right), \quad x = (1 + e^{-\pi \delta/(4\sqrt{\theta})}) \cos^2 \psi, \quad \psi = \frac{\pi^2}{4} \sqrt{\frac{\theta}{\pi}} + \frac{\delta}{4\sqrt{\theta}} \ln\left(\frac{\pi \theta^{1/4}}{\sqrt{2}}\right) + \frac{1}{2} \Im \ln\left[\frac{\Gamma\left(\frac{1}{2} - i\delta/(8\sqrt{\theta})\right)}{\Gamma\left(\frac{1}{2} + i\delta/(8\sqrt{\theta})\right)}\right].\quad (51)$$

The instability region is characterized by $x > 1$, while the stability region by $x < 1$. These two regions alternate roughly with equal band width of $\Delta \theta \approx \frac{1}{\pi} \sqrt{\theta}$ around along $h = 2\theta$. For $\delta \ll \sqrt{\theta}$, or $k \ll \sqrt{g m_\xi \xi_0}$, it approximately follows that

$$\lambda = \frac{1}{\pi} \ln\left(\sqrt{2} |\cos \psi| + \sqrt{\cos(2\psi)}\right), \quad \psi = \frac{\pi^2}{4} \sqrt{\theta}.\quad (52)$$

It can be readily shown that the maximal $\lambda$ along $h = 2\theta$ is

$$\lambda_{\text{max}} = \frac{1}{\pi} \ln(\sqrt{2} + 1) \approx 0.28\quad (53)$$

and the average in the instability band is $\bar{\lambda} \approx 0.22$. It is evident that in the large $\theta$ region the growth rate never diminishes, always with a sizable constant $\lambda$. Thus the exponent of particle production rate grows roughly in the time interval of $1/\bar{\lambda} (\approx 5) \times$ oscillation period $1/m_\xi$.

Particle production is halted by the back reaction. This problem is studied by solving a coupled system of the inflaton and the radiation energy densities in the
expanding universe \[36\],

\[
\frac{d\rho_\xi}{dt} + 3H\rho_\xi = -N\frac{d}{dt}\langle\rho_\varphi\rangle, \tag{54}
\]

\[
\frac{d\rho_r}{dt} + 4H\rho_r = N\frac{d}{dt}\langle\rho_\varphi\rangle, \tag{55}
\]

\[
H^2 = \frac{8\pi}{3}G(\rho_\xi + \rho_r), \tag{56}
\]

where \(N\) is the number of boson species contributing to parametric resonance effects, roughly with the coupling \(g\).

The idea behind this evolution equation is that irrespective of interaction among created particles towards thermalization, the energy balance between the inflaton and radiation should hold at any instant of time because of energy conservation. If one separately checks that thermalization is realized, the radiation energy density \(\rho_r\) can be used to estimate the temperature in equilibrium;

\[
T = \left(\frac{30}{\pi^2 N'} \rho_r\right)^{1/4}. \tag{57}
\]

We allow the possibility that there might be more particle species participating in equilibration than the number of created boson species \(N\): for instance, in supersymmetric theories of \(N\) boson species there are roughly equal number of fermion species that may be produced from the bosons by secondary processes, giving \(N' \approx 2N\). The system of evolution equation can be extended to include the mass variation by

\[
\frac{dm^2_{\xi}}{dt} = N \cdot g^2 \frac{d}{dt}\langle\varphi^2\rangle. \tag{58}
\]

It might be instructive to give derivation of this evolution equation in view of that the meaning of energy densities in the presence of interaction terms is not clear. We define the inflaton energy density including the mass variation;

\[
\rho_\xi = \frac{1}{2} \xi^2 + \frac{1}{2} m^2_{\xi}(t) \xi^2, \tag{59}
\]

with the time dependent part of the mass given by \(g^2 \sum_\varphi \langle\varphi^2\rangle\). From the \(\xi\) equation of motion,

\[
\ddot{\xi} + 3H\dot{\xi} + m^2_{\xi}(t) \xi = -\langle \frac{\partial V_\gamma}{\partial \xi} \rangle, \tag{60}
\]

where \(\langle A \rangle \equiv \text{tr} (A\rho) \ (\rho = \text{density matrix})\) and \(V_\gamma = \frac{1}{2} gm_\xi \sum_\varphi \varphi^2\), one derives for evolution of the \(\xi\) energy density;

\[
\dot{\rho}_\xi + 3H\dot{\xi}^2 = -\langle \xi \frac{\partial V_\gamma}{\partial \xi} \rangle + \frac{g^2}{2} \xi^2 \frac{d}{dt} \sum_\varphi \langle\varphi^2\rangle. \tag{61}
\]
Note that we included the quartic interaction term \( \frac{1}{2} g^2 \xi^2 \sum \varphi \langle \varphi^2 \rangle \) in the time variant mass term \( m_\xi^2(t) \). The left hand side becomes \( \dot{\rho}_\xi + 3H \rho_\xi \) after time average over the oscillation period, since
\[
\overline{\dot{\xi}^2} = m_\xi^2(t) \overline{\xi^2} = \overline{\rho}_\xi ,
\] (62)
under the assumption of slow \( \xi \) variation.

On the other hand, it can be shown using the time evolution equation for the density matrix,
\[
\dot{\rho} = -i[H, \rho] = -i \left[ \sum_\varphi H_\varphi + V_Y + V_4, \rho \right] ,
\] (63)
that
\[
\langle \xi \frac{\partial V_Y}{\partial \xi} \rangle = \operatorname{tr} \left( \frac{\partial V_Y}{\partial t} \rho \right) = \frac{d}{dt} \langle V_Y \rangle - \operatorname{tr} (V_Y \dot{\rho})
\]
\[
= \frac{d}{dt} \langle V_Y \rangle + \frac{d}{dt} \sum_\varphi \langle H_\varphi \rangle + \frac{d}{dt} \langle V_4 \rangle - \langle \frac{\partial V_4}{\partial t} \rangle ,
\] (64)
where
\[
H_\varphi = \frac{1}{2} \varphi^2 + \frac{1}{2} (\nabla \varphi)^2
\] (65)
is the free Hamiltonian density of created boson \( \varphi \) and \( V_4 = \frac{1}{2} g^2 \xi^2 \sum \varphi \varphi^2 \). The short time average over one oscillation period of \( \langle V_Y \rangle \) vanishes, since this quantity has a linear dependence on \( \xi \). Thus one has
\[
- \langle \xi \frac{\partial V_Y}{\partial \xi} \rangle + \frac{g^2}{2} \xi^2 \frac{d}{dt} \sum_\varphi \langle \varphi^2 \rangle
\]
\[
= - \frac{d}{dt} \sum_\varphi \langle H_\varphi \rangle - \frac{g^2}{2} \sum_\varphi \langle \varphi^2 \rangle \frac{d \xi^2}{dt} + \langle \frac{\partial V_4}{\partial t} \rangle ,
\] (66)
leading to
\[
\dot{\rho}_\xi + 3H \rho_\xi = - \frac{d}{dt} \sum_\varphi \langle H_\varphi \rangle ,
\] (67)
\[
\dot{\rho}_r + 4H \rho_r = \frac{d}{dt} \sum_\varphi \langle H_\varphi \rangle .
\] (68)

The second equation simply follows from energy conservation of the combined system of \( \xi \) and radiation.

It is now appropriate to discuss the thermal history after inflation, namely after the inflaton commences oscillation at \( t \approx \frac{2}{3} \frac{\xi}{m_\xi} \). The result should be compared with the naive estimate of the reheat temperature, \( T_B \sim 0.1 \sqrt{\gamma_\xi m_{\text{pl}}} \sim 10^{-2} g \sqrt{m_\xi m_{\text{pl}}} \).
Particle production is significant only when the exponent in the rate is appreci-
able, but once it becomes of order unity, the production rate becomes accelerated. Indeed, when the exponent becomes of order 100, a catastrophic particle production occurs and back reaction immediately stops particle production. The time when this happens can be estimated by

\[ t = t_d \approx O[100] / (\bar{\lambda} m_\xi) . \]  

(69)

This is an abrupt change, as numerically checked by solving the time evolution equation [36]. Prior to this time the inflaton density varies according to the $\xi$–matter dominance, $\rho_\xi = \frac{1}{2} m_\xi^2 \xi^2 \propto 1/t^2$. Combined with the time of catastrophic decay $t_d$ above, the abrupt change occurs at

\[ \frac{\xi}{\xi_0} \approx O[10^{-2}] \frac{2}{3} \bar{\lambda} . \]  

(70)

A typical energy of produced particles at this time, prior to thermalization, is of order

\[ \bar{E} = O[\sqrt{gm_\xi \xi}] = O[0.3] \sqrt{g\bar{\lambda} m_\xi m_{\text{pl}}} , \]  

(71)

with $\xi$ given as above. Using this energy, one estimates the rate of two body reactions among created particles, and the corresponding ratio of this to the Hubble rate,

\[ \frac{\Gamma}{H} \approx \frac{\alpha_s/E^2 \cdot N n_\varphi}{\sqrt{\rho_\xi + \rho_r/m_{\text{pl}}}} , \]

\[ \approx \frac{\alpha_s m_{\text{pl}}}{\bar{E}} \left( \frac{\rho_r}{\rho_\xi} \right)^{1/2} \sqrt{\frac{\rho_r/\rho_\xi}{1 + \rho_r/\rho_\xi}} , \]  

(72)

with $\alpha_s$ a typical coupling involving ordinary particles. What usually happens [36] is that

\[ \rho_r \approx \rho_\xi , \quad \frac{\rho_r}{\bar{E}^4} \gg 1 , \]  

(73)

hence $\Gamma \gg H$. Thus two-body reactions frequently take place. Even multi-particle reactions occur and one may conclude that thermalization takes place immediately after the catastrophic particle production. Under this circumstance the reheat temperature right after the explosive particle production is

\[ T_R \approx O[10^{-2}] \sqrt{\bar{\lambda} m_\xi m_{\text{pl}}} . \]  

(74)

With $m_\xi \sim 10^{13} \text{ GeV}$, $\bar{\lambda} \approx 0.2$, this reheat temperature is close to the GUT scale. It is possible that at least the Higgs $X$–boson of mass $\sim 10^{14} \text{ GeV}$ can copiously be
produced. In any case $T_R \gg T_B$ for a small coupling $g$. More precise computation has been done by numerically integrating time evolution equation \cite{36}.

The entire thermal history may in general be fairly complicated. Even if the radiation dominance is realized at the early epoch simultaneous to the catastrophic production, there exists a residual inflaton field after the catastrophic stage:

$$\theta \approx O\left[10^{-4}\right] \frac{g \lambda m_{\text{pl}}}{m_\xi} \ll 1.$$  \hspace{1cm} (75)

If this value is still large, $\gg 1$, there may be a second explosive decay. On the other hand, if this value is moderately small, but not small enough such that the naive perturbative analysis is no longer valid, then there may be gradual particle production continuously down to $\theta \ll 1$. This region is difficult to analyze, but under study currently.

The ultimate end point of the inflaton decay is the decay in the first band, namely the Born decay when the inflaton amplitude becomes very small. This is because only the first band appreciably contributes with very small $\theta$, and the Born decay rate always satisfies $\gamma_\xi > H$ at very late times. Under the assumption that intermediate amplitude decay is not significant, one can estimate entropy creation at the Born decay. According to our numerical integration \cite{36} the entropy production by the Born decay is significant for the coupling range $g < O[10^{-3}]$, but negligible for $g > O[10^{-2}]$. In all cases numerically checked \cite{36}, the final temperature at the end of the complete inflaton decay is given approximately by the estimate due to the Born formula $T_B$. This does not mean that effects of parametric resonance can be forgotten. High energy processes that may occur in the temperature range, $T_R > T > T_B$, have to be reconsidered. As such, the baryogenesis and the gravitino production are of prime importance.

5 Gravitino abundance

The old formula of the gravitino abundance $n_{3/2}/s \sim 10^{-2} \cdot T_B/m_{\text{pl}}$ with $s$ the entropy density is not valid since the thermal history right after the catastrophic particle production is complicated in some region of the coupling $g$, and $T_B$ is not a good measure to characterize the thermal history. The truth is that no single temperature represents the state after inflation. In order to estimate the gravitino abundance with non-trivial thermal history after inflation, it is necessary to follow
time evolution of the gravitino number density $n_{3/2}$

$$\frac{dn_{3/2}}{dt} + 3H n_{3/2} = \langle \Sigma v \rangle n_{\varphi}^2,$$

where $n_{\varphi}$ is the thermal number density of one species of created bosons. The cross section $\langle \Sigma v \rangle$ of gravitino production $\varphi \varphi \rightarrow g_{3/2} g_{3/2}$ has been computed, and roughly $\langle \Sigma v \rangle \sim 250/m^2_{\text{pl}}$ unless the gravitino is lightest supersymmetric particle (LSP). Destructive term has been neglected in the evolution equation above, which is justified for the gravitino mass larger than $O[1]\text{ keV}$.

Both possibilities of stable and unstable gravitino remain viable. Let us only mention a possibility of the gravitino dominated universe at the present epoch. With the initial temperature $T_R > 2 \times 10^{14}$ GeV imposed to give a favorable situation for GUT baryogenesis, there is a region of parameters for the closure density of gravitino dominated universe if $m_{3/2} = 0.1 - 10$ GeV. The basic reason this becomes possible is that the initial large gravitino yield created right after the catastrophic particle production is much diluted via the late phase of Born decay. This is again a reflection of the large disparity of the two temperatures; $T_R \gg T_B$. Of course, it remains to demonstrate a sizable baryon to photon ratio. But things are not bad: there is an epoch immediately before the catastrophic particle production in which non-equilibrium environment necessary for baryon generation exists, and moreover the observed baryon to photon ratio is of order $10^{-10}$, allowing some amount of dilution in later epochs.

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