Lorentz symmetry violation with higher-order operators and renormalization

J R Nascimento¹, A Yu Petrov¹ and C M Reyes²

¹ Departamento de Física, Universidade Federal da Paraíba Caixa Postal 5008, 58051-970, João Pessoa, Paraíba, Brazil
² Departamento de Ciencias Básicas, Universidad del Bío Bío, Casilla 447, Chillán, Chile
E-mail: creyes@ubiobio.cl

Abstract. Effective field theory has shown to be a powerful method in searching for quantum gravity effects and in particular for CPT and Lorentz symmetry violation. In this work we study an effective field theory with higher-order Lorentz violation, specifically we consider a modified model with scalars and modified fermions interacting via the Yukawa coupling. We study its renormalization properties, that is, its radiative corrections and renormalization conditions in the light of the requirements of having a finite and unitary $S$-matrix.

1. Introduction

Quantum gravity effects taking place at experiments at the energy scale $m$ are expected to be suppressed by the Planck mass $m_P = 10^{19}$ GeV with the factor $m/m_P$. Such violent suppression leads us to consider new strategies in order to be able to detect possible signals from the Planck scale. The strategy encoded in the framework of effective field theory is to consider minute effects in the conventional field theory Lagrangians. The approach has been widely used and has several advantages: 1) it provides a very general setup without any attachment to a particular quantum gravity model and 2) it allows us to experimentally put bounds on the new physics. In particular, the search for Lorentz and CPT violation at low energies has been intensively studied from both the experimental and theoretical points of view.

Originally the framework of the standard model extension [1] was proposed with mass-dimension operators $d$ lower than four and later generalized to include higher-order operators [2, 3]. Several higher-order effective models to describe departures from Lorentz symmetry have been proposed in the past years [4]. The effective field theory approach being a powerful tool, however, has to meet some constraints. In this work we emphasize the importance of having an $S$-matrix in the higher-order theory: (i) finite and (ii) unitary. For unitarity there have been some advances, see [5, 6, 7, 8].

Recently it has been shown that the Lorentz violation contained in quantum field theories leads to nontrivial modifications of the Källén–Lehmann representation and the LSZ reduction formalism [9]. This has led to modifications in the renormalization procedure, in the definition of the asymptotic Hilbert space and in general in the treatment for external-leg physics [10]; for other studies of the renormalization in Lorentz-breaking theories, see also [11]. A natural extension for these studies is to consider the nonminimal framework of Lorentz invariance violation, that is, when the Lorentz breaking is performed with higher-order operators [12].
2. The effective model

We focus on the higher-order Lorentz violating Lagrangian density

\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} M^2 \phi^2 + \bar{\psi} (i \partial - m) \psi + g_2 \bar{\psi} \eta (n \cdot \partial)^2 \psi + g (\bar{\psi} \phi \psi), \]  

(1)

where the first two terms involve a scalar field and the last part is the Myers and Pospelov fermion model [4] with a Yukawa coupling interaction. The Planck mass suppressed constant \( g_2 \) parametrizes the higher-order Lorentz invariance violation, \( g \) is the Yukawa coupling and \( n^\mu \) is a dimensionless four-vector defining a preferred reference frame.

The corresponding propagators in momentum space read

\[ \Delta(p) = \frac{i}{p^2 - M^2}, \quad S(p) = \frac{i}{\tilde{p} - m - g_2 \eta (n \cdot p)^2}. \]  

(2)

From now on we consider a purely time-like four-vector \( n^\mu = (1, 0, 0, 0) \). With this choice the scalar has the usual solutions but the number of fermion solutions increases being

\[ \omega_1 = \frac{1 - \sqrt{1 - 4 g_2 \mathcal{E}}}{2 g_2}, \quad \omega_2 = \frac{1 - \sqrt{1 + 4 g_2 \mathcal{E}}}{2 g_2}, \]

\[ W_1 = \frac{1 + \sqrt{1 - 4 g_2 \mathcal{E}}}{2 g_2}, \quad W_2 = \frac{1 + \sqrt{1 + 4 g_2 \mathcal{E}}}{2 g_2}, \]

(3)

(4)

with \( \mathcal{E} = \sqrt{\tilde{p}^2 + m^2} \). Note that the first two solutions are finite as \( g_2 \to 0 \) and the other two, the Lee-Wick modes, are singular [13, 14].

![Figure 1. The Feynman contour \( C_F^{(\text{f})} \) for the fermion propagator.](image)

As can be seen from the solutions above, at the energy \( 1/2 g_2 \) the solutions labelled “1” pick up an imaginary part and jump to the complex plane. To define the Feynman contour \( C_F^{(\text{f})} \) we use a heuristic argument. First, we consider the contour which can be continuously deformed to the standard curve at low energies when all the solutions are real and the Lorentz violation is removed. Second, we consider the contour that allows us to preserve the unitarity of the \( S \)-matrix [15]. In this way, the integration contour \( C_F^{(\text{f})} \) is defined to pass below the negative pole and above the three positive ones, as shown in Fig. 1.
3. Two-point functions
As a first example of quantum corrections in our Yukawa-like model, we study the contribution with two external scalar legs depicted in Fig. 2 and represented by the integral
\[
\Pi(p) = -\frac{g^2}{2} \phi(-p) \phi(p) \int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}((Q_\mu \gamma^\mu + m)(R_\nu \gamma^\nu + m))}{(Q^2 - m^2)(R^2 - m^2)},
\]
where we define
\[
Q_\mu = k_\mu - g_2 n_\mu (n \cdot k)^2, \quad R_\mu = k_\mu + p_\mu - g_2 n_\mu (n \cdot (k + p))^2.
\]
We study the typical low-energy behavior of this contribution by expanding it into a Taylor series. After some algebra we find up to second order in \(p\)
\[
\Pi(p) = -2g^2 m^2 q_0 - 2g^2 p^2 q_1 - 2g^2 (n \cdot p)^2 q_n,
\]
where
\[
q_0 = -\frac{i}{48\pi^2 g^2 m^2} + \frac{i}{48\pi^2} \left(6\gamma_E - 0.46 + 12i\pi - 18 \ln \left(\frac{g_2 m}{2}\right)\right),
\]
\[
q_1 = -\frac{i}{2\pi^2} \left(i\pi - \ln \left(\frac{g_2 m}{2}\right) - \frac{1}{3}\right),
\]
\[
q_n = \frac{i}{\pi^2}.
\]
Now we focus on the contribution of the fermion self-energy graph depicted in Fig. 3. We take \(M = m\) and consider the fermion self-energy graph
\[
\Sigma_2(p) = g^2 \int \frac{d^4k}{(2\pi)^4} \frac{Q + m}{((k - p)^2 - m^2)(Q^2 - m^2)}.
\]
After considering its Taylor expansion along the lines of the previous contribution, we obtain
\[
\Sigma_2 = g^2 \gamma f_1^n + g^2 m f_0 + g^2 \gamma f_1 + g^2 m(n \cdot p) f_2^n + g^2 \gamma(n \cdot p) f_3^n + g^2 p^2 \gamma f_5^n
+ g^2 (n \cdot p) \gamma f_4^n + g^2 (n \cdot p)^2 \gamma f_5^n.
\]

4. Mass renormalization
Let us start with the renormalized scalar Lagrangian
\[
\mathcal{L} = \frac{1}{2} \partial_\mu \phi_R \partial^\mu \phi_R - \frac{1}{2} m^2 \phi_R^2.
\]
Consider the renormalized two-point function

\[(\Gamma_R^{(2)})^{-1} = p^2 - m_R^2 + \Pi_R(p),\]  

where \(\Pi_R(p) = \Pi(p) + \mathcal{O}(g^4)\). Let us write

\[\Pi_R(p) = p^2 A_\phi + m_R^2 B_\phi + (n \cdot p)^2 C_\phi,\]  

where \(A_\phi\), \(B_\phi\) and \(C_\phi\) are the radiative corrections already computed in the expressions (8), (9), (10). We define a quantity \(\bar{P}\) and \(\bar{P}_R^2 = p^2 - M_{\text{ph}}^2 + \bar{y}(n \cdot p)^2\) with the appropriate structure to make the two-point function vanish as seen from Eq. (15). The above quantities \(M_{\text{ph}}\) and \(\bar{y}\) are unknown constants we should find.

The new renormalization condition defined by \((\Gamma_R^{(2)})^{-1}(\bar{P}_R^2) = 0\) at \(\bar{P}_R^2 = 0\) gives the unknown constants

\[M_{\text{ph}}^2 = m_R^2 \frac{(1 - B_\phi)}{(1 + A_\phi)}, \quad \bar{y} = \frac{C_\phi}{1 + A_\phi}.\]  

which are determined in terms of the radiative corrections.

Now we pass to the renormalization in the fermion sector and consider the renormalized Lagrangian

\[\mathcal{L} = i \bar{\psi} R \psi R - m_R \bar{\psi} R \psi R + g_2 \bar{\psi} R \slashed{\partial}(n \cdot \partial) \psi R + \delta_{g_2} \bar{\psi} R \slashed{\partial} \psi R,\]  

with \(\delta_{g_2}\) a counterterm. The renormalized two-point function is \((\Gamma_R^{(2)})^{-1} = \bar{\psi} - m_R + \Sigma_R\), with

\[\Sigma_R = \Sigma_2 + \delta_{g_2} \bar{\psi} + \mathcal{O}(g^4).\]  

We can write the finite contribution to the renormalized two-point function up to second order in \(p\) with \(\Sigma_R = \bar{p} A_\psi + m_R B_\psi + \bar{\pi} C_\psi\) where again the constants \(A_\psi\), \(B_\psi\) and \(C_\psi\) have been determined in (12) through the expressions

\[A_\psi = A^{(0)} + g_2 A^{(1)}(n \cdot p), \quad B_\psi = B^{(0)} + g_2 B^{(1)}(n \cdot p),\]  

\[C_\psi = \frac{C^{(0)}}{g_2} + C^{(1)}(n \cdot p) + g_2 C^{(2)}(n \cdot p)^2 + g_2 C^{(3)} p^2.\]  

We work in the minimal subtraction scheme and hence we fix the divergent term to be \(\delta_{g_2} = \frac{-i g^2}{4 g_2^2 m^2}.\) To find the pole we consider the ansatz \(\bar{P}_\psi = \bar{p} - \bar{m} + \bar{\pi} \bar{p}\) where \(\bar{m} = m_{\text{ph}} + g_2 m_n(n \cdot p)\) and \(\bar{\pi} = \bar{x}^{(0)} + \bar{x}^{(1)}(n \cdot p) + g_2 \bar{x}^{(2)}(n \cdot p)^2 + g_2 \bar{x}^{(3)} p^2.\) We include the large Lorentz violating terms in the pole extraction process which has been included explicitly in \(\bar{\pi}\).

Considering the condition for \(\bar{P}_\psi\) to be a pole, that is to say,

\[(\Gamma_R^{(2)})^{-1}(\bar{P}_\psi = 0) = \bar{p} - m_R + \Sigma_R = 0,\]  

gives the six terms \(m_{\text{ph}}, m_n, \bar{x}^{(0)}, \bar{x}^{(1)}, \bar{x}^{(2)}, \bar{x}^{(3)},\)

\[m_{\text{ph}} = m_R \frac{(1 - B^{(0)})}{(1 + A^{(0)})}, \quad m_n = -m_R \left( \frac{(A^{(1)} - B^{(0)})}{(1 + A^{(0)})^2} + \frac{B^{(1)}}{1 + A^{(0)}} \right),\]  

and

\[\bar{x}^{(0)} = \frac{C^{(0)}}{1 + A^{(0)}}, \quad \bar{x}^{(1)} = -\frac{A^{(1)} C^{(0)}}{(1 + A^{(0)})^2} + \frac{C^{(1)}}{1 + A^{(0)}},\]  

\[\bar{x}^{(2)} = \frac{(A^{(1)})^2 C^{(0)}}{(1 + A^{(0)})^3} - \frac{A^{(1)} C^{(1)}}{(1 + A^{(0)})^2} + \frac{C^{(2)}}{1 + A^{(0)}}, \quad \bar{x}^{(3)} = \frac{C^{(3)}}{1 + A^{(0)}}.\]  

This finishes the pole extraction in both sectors.
5. Summary
We considered the Myers-Pospelov-like higher-derivative extension of the Yukawa model which incorporates possible new physics from the Planck scale through dimension five operators coupled to a preferred four vector $n^\mu$ which breaks Lorentz symmetry. We have selected a particular configuration of Lorentz symmetry violation in which the preferred four-vector $n^\mu$ is purely timelike. This choice produces higher-order time-derivative terms and leads to extra solutions and new poles in the model. We have found and identified the poles associated to standard solutions and those ones corresponding to negative-metric states or Lee-Wick solutions. Some of these poles move on the real axis, as in the usual Yukawa model. However, above a specific energy called the critical energy some solutions become complex introducing an extra difficulty to define a consistent prescription for the contour of integration in the complex plane. To solve this problem we have analyzed the motion of the poles in the complex plane.

A central part of this work has been the study of mass renormalization in the presence of both Lorentz invariance violation and higher-order operators. We have computed the one-loop corrections in the model and shown how they lead to modifications in the renormalization procedure by pushing the pole mass to a sector involving other gamma matrices besides of $\not{p}$. This means basically that tree level physics is modified due to quantum corrections so that at asymptotic times one has to include corrections in the propagation of particles with the operators $\not{P}_\phi$ and $\not{P}_\psi$.

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