Polarimetric studies of comet Hale-Bopp

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ABSTRACT
In the present work, the non-spherical dust grain characteristics of comet Hale-Bopp are studied using the T-matrix method and the modified power law distribution function. It is found that the observed data fits very well with the power index (α) = −3. The best fit values of complex refractive index (n, k) and the aspect ratio (E) at α = −3 are calculated to be (1.382, 0.035, 0.936) and (1.379, 0.041, 0.936) at λ = 0.485μm and 0.684μm respectively. Kerola & Larson (K-L) analysed the same comet using the T-matrix method and the power law distribution function (α = −3), and found that the prolate grains can explain the observed polarization in a more satisfactory manner as compared to the other shapes. But their analysis could not reproduce the negative polarization branch beyond scattering angle 157°. However, the results obtained from the present work successfully generate the expected negative polarization curve beyond 157° and the fitting in this case is much better than K-L’s work. So it is concluded from the present study that the use of modified power law distribution function (with α = −3) can fit the observed data in a better way, as compared to the power law distribution function used by previous authors.

Key words: comets: general – dust, extinction – scattering – polarization

1 INTRODUCTION
Comets are believed to be the most primordial objects in our solar system. They are the least processed remnants from the early solar nebula era of dust grain formation. Thus, the study of comets gives information about the least processed and most pristine materials of early solar nebula, from which the present day solar system has been evolved. The knowledge of cometary grains come mainly from the light scattered by the grains, which are present in the coma. The linear polarization of the scattered light by dust particles depends upon (i) wavelength of incident light (λ), (ii) the shape and size of the particle, (iii) Scattering angle, θ (= 180° – Phase angle), and (iv) the composition of dust particles in terms of complex values of refractive index, m (= n – ik). For regularly shaped spheroidal grains one can characterise their shapes by an aspect ration E, where grains E > 1 for oblate, E < 1 for prolate and E = 1 for spheres. However, the grains are in general not solid but porous and irregularly shaped. In general, the study of the polarization of continuum radiation in comets is a powerful tool to determine the characteristics of the cometary dust grains (Kikuchi et al. 1987; Sen et al. 1991a, 1991b; Joshi et al. 1997; Chernova et al. 1993; Ganesh et al. 1998; Das et al. 2004 etc.).

Levasseur-Regourd et al. (1996), studied a polarimetric data base of 22 comets and from the nature of phase angle dependence, they concluded that there is a clear evidence for two class of comets. These two classes of comets are distinct only for the phase angles above 40°. However, Das et al.(2004) had shown that the comets on the basis of their dust properties need not be classified into such discrete classes, rather their dust properties are some smooth varying functions of cometary aging. Comet Hale-Bopp C/1995 O1 is an intrinsically bright comet, with positive polarization values much higher than those of other comets (although the highest phase angle observed was 47°). Hadamcik & Levasseur-Regourd (2003) compared the imaging polarimetry of seven different comets and suggested that Hale-Bopp itself represents a third class, marked by unusually high polarization.

It is now almost accepted that cometary grains are not spherical and may be fluffy aggregates or porous, with irregular or spheroidal shapes (Greenberg & Hage 1990). The measurement of circular polarization of comet Hale-Bopp (Rosenbush et al. 1997) revealed that cometary grains are mostly non-spherical in shape. In order to study the light scattering properties of these irregularly shaped and spheroidal grains, Discrete Dipole Approximation (DDA)
(Draine 1988) and T-matrix theory (Waterman 1965) are in general used respectively. Using the DDA method, Xing & Haner (1997) studied the fluffy nature of cometary grains of different shapes and sizes. Moreno et al. (2003) studied the composite grains using the DDA method for modelling the comet Hale-Bopp’s grains in the mid-infrared spectrum. But, the DDA method requires considerable computer time and memory. The T-matrix code (Mishchenko et al. 2002) on the other hand runs much faster and the results obtained can be tuned easily. Using the T-matrix code, Kerola & Larson (2001) analysed the polarization data of comet Hale-Bopp and found the grains to be mostly prolate in shape in that comet. Recently Das & Sen (2006) using the T-matrix code found that, the prolate grains can explain the observed polarization in a better way as compared to the other shapes in comet Levy 1990XX. However, the T-matrix code in its present form cannot be used for studying inhomogeneous (e.g. porous, fluffy, composite) particles (Mishchenko et al. 2002).

It is to be noted that the results obtained from the T-matrix code could not reproduce the negative polarization branch observed for comet Hale-Bopp, as seen in Kerola and Larson (2001), where the analysis has been restricted to \( \theta \leq 160^\circ \). Thus, the main objective of the present work is to see whether a better fit can be achieved by varying the dust size distributions and dust shapes.

2 SPHEROIDAL GRAIN MODEL

As already discussed the T-matrix method provides a powerful tool to study the spheroidal grains in comets. In this paper, calculation has been carried out for randomly oriented spheroids using Mishchenko’s (1998) single scattering T-matrix code, which is available in \url{http://www.giss.nasa.gov/~crmim}. The main feature of the T-matrix approach is that it reduces exactly to the Mie theory when the particle is a homogeneous or layered sphere composed of isotropic materials.

The \textit{in situ} dust measurement of comet Halley gave the first direct evidence of grain mass distribution (Mazets et al. 1986, Lamy et al. 1987). Mukai et al. (1987) and Sen et al. (1991a) based on Mie Theory analysed the polarization data of comet Halley using the power law dust distribution suggested by Mazets et al. (1986) and derived a set of refractive indices of cometary grains. Das et al. (2004) also analysed the polarization data of several comets including Halley using dust distribution function suggested by Lamy et al. (1987). For the analysis of polarimetric data of comet Hale-Bopp, a \textit{power law size distribution} was used by Kerola & Larson (K-L) (2001), where the minimum and maximum particle radius \( r_1 \) and \( r_2 \) respectively are automatically fixed for each and every run merely by specifying the particle’s effective radius \( r_{\text{eff}} \) and effective variance \( \nu_{\text{eff}} \).

This \textit{power law size distribution} (Hansen & Travis 1974) is given by

\[
   n(r) = \begin{cases} 
   \text{constant} \times r^{-3}, & 0 \leq r \leq r_1, \\
   \text{constant} \times (r/r_1)^{\alpha}, & r_1 \leq r \leq r_2, \\
   0, & r_2 < r, 
   \end{cases} \quad (1)
\]

Using equation (1) and the T-matrix code, K-L achieved reasonably good agreement with a set of \textit{spherical volume equivalent} values of effective radius \( r_{\text{eff}} \), effective variance \( \nu_{\text{eff}} \) and \( E = 0.216 \, \mu m \), 0.0105 and 0.415 respectively for prolate spheroids at \( \lambda = 0.485 \mu m \) and 0.684\( \mu m \). To analyse the data, the index of refraction for crystalline olivine (1.63, 0.00003) was taken. However, the analysis was restricted to \( \theta \leq 160^\circ \). The results obtained from the above calculations could not reproduce the negative polarization branch beyond 160°.

In the present study, the same comet is analysed using the \textit{modified power law distribution function} (Mishchenko et al. 1999), which is given by

\[
   n(r) = \begin{cases} 
   \text{constant}, & 0 \leq r \leq r_1, \\
   \text{constant} \times (r/r_1)^{\alpha}, & r_1 \leq r \leq r_2, \\
   0, & r_2 < r, 
   \end{cases} \quad (2)
\]

It is to be noted that the albedo of comet Hale-Bopp as derived from the comparison between infrared thermal emission and visible scattered light seems to be higher than those for other comets (Williams et al. 1997; Jones & Gehrz 2000). Williams et al. (1997) had interpreted this high albedo as an indication for the presence of a large number of small particles in Hale-Bopp, and Hanner et al. (1999) as an increase of the ratio between silicates and carbonaceous compounds. Thus the higher albedo and higher polarization could provide a clue to the presence of smaller grains in comet Hale-Bopp (Hadamzik & Levasseur-Regourd 2003).

In the present work, \( r_1 \) and \( r_2 \) are taken to be 0.01\( \mu m \) and 2\( \mu m \) respectively and a few early iterations using contrasting values of \( \alpha \) are tried. Initially, an index of refraction for crystalline olivine (1.63, 0.00003) is taken here as fixed for the analysis of data. Thus the shape parameter \( E \) was left as the only free parameter to vary. Taking \( \alpha = -3 \), the shape parameter \( E \) is varied to fit the observed data at \( \lambda = 0.485 \mu m \) by \( \chi^2 \) minimisation technique. But no good fit was observed at \( \alpha = -3 \). \( E \) is varied for other values of \( \alpha \) (say, -1.1, -1.2, -1.3, -2.9, -3.0, -3.1 etc.), but none of them could match the observed data well.

So a different approach is proposed here. The refractive index parameter \( (n, k) \) is now taken as another free parameter. Taking a particular value of \( \alpha \), the best fit values of \( (n, k) \) and \( E \) are determined at which the \( \chi^2 \) - value becomes minimum. This calculation is repeated for several other values of \( \alpha \). The results obtained from the present work are reproduced in Table 1. It can be seen from Table 1 that a value of \( \alpha = -3 \) fits the data well both at 0.485\( \mu m \) and 0.684\( \mu m \) and the best fit values of \( (n, k) \) and \( E \) are estimated to be \((1.382, 0.035, 0.936)\) and \((1.379, 0.041, 0.936)\) at \( \lambda = 0.485 \mu m \) and 0.684\( \mu m \) respectively. The \( \chi^2_{\text{min}} \) - values for spherical grains \( (E = 1) \) are also shown in Table 1. Thus one can see that the prolate grains \( (E = 0.936) \) represent a better fit to the observed data with lower \( \chi^2_{\text{min}} \) value, as compared to spherical grains. Moreover, the uniqueness of the estimated \( E \) and \( \alpha \) values at two different wavelengths, further strengthens our claim for a more suitable and realistic fit to the observed data.

Using Mie scattering theory and grain model of Mazets et al. (1986), Mukai et al. (1987) analysed comet Halley and found a set of three complex refractive indices \( (n, k) \) at three HIW filters which best matched their observations. Sen et al. (1991a) combined their polarimetric observations with those of other investigators and estimated \( (n, k) \) values which are slightly different from those of Mukai et al. (1987). Based
on the dust size distribution function derived by Das et al. (2004) for comet Halley (on the basis of the work reported by Lamy et al. (1987)) and Mie theory, Das et al. (2004) also analysed polarization data and found a set of refractive indices \( (n, k) \) for comet Halley. The best fit values of \((n, k)\) derived by them for comet Halley are reproduced in Table 2. Lamy et al. (1987) denoted these hypothetical refractive indices \((n, k)\) emerging out from the Mie code as ‘Silicate B’. The present analysis also suggests the ‘Silicate B’ nature of comet Hale-Bopp’s dust grains.

The \( \chi^2_{\text{min}} \) - values obtained from present work are 20.8 and 39.1 at \( \lambda = 0.485 \mu m \) and 0.684 \( \mu m \) respectively, whereas the values obtained from K-L’s work are 212.9 and 174.4 respectively. Thus the present analysis is clearly giving better fit to the observed polarization data of comet Hale-Bopp, as compared to K-L. In Fig 1 and Fig 2, the best fitted polarization curves obtained from the T-matrix code are drawn on the observed polarization data (Ganesh et al. 1998, Manset & Bastien 2000) at \( \lambda = 0.485 \mu m \) and 0.684 \( \mu m \) respectively. It is interesting to note that the present analysis can reproduce the negative polarization curves beyond 157\(^0 \) at \( \lambda = 0.485 \mu m \) and 0.684 \( \mu m \) respectively, which were not possible in K-L’s work. The simulated polarization curves from K-L’s work are also shown in Fig 1 and Fig 2.

3 DISCUSSIONS

It can be seen from the above analysis that the simulated polarization values in the present case fit much better to the observed data as compared to K-L’s work. The main reasons for getting better fit are: (i) use of modified power law distribution (though the power index is same in both the distributions), (ii) the size range of the grains, which is slightly broader in the present work as compared to K-L’s work, (iii) the refractive index \( (n, k) \) in the present analysis, which is also different from K-L’s work where the index of refraction for crystalline olivine was considered and finally (iv) the shape parameter which is \( E = 0.936 \) in the present work, as against the value \( E = 0.415 \) used by K-L.

In the present study, one can generate the expected polarization curves beyond 157\(^0 \) at \( \lambda = 0.485 \mu m \) and 0.684 \( \mu m \). But, the results obtained by K-L could not reproduce the negative polarization branch for comet Hale-Bopp beyond 157\(^0 \). Their analysis was restricted to \( \theta \leq 160^0 \). K-L concluded that combination of viewing geometry effects and enhanced multiple scattering might provide a quantitative explanation of the negative polarization for scattering angle beyond 160\(^0 \).

Greenberg & Hage (1990) originally proposed the presence of large numbers of porous grains in the coma of comets to explain the spectral emission at 3.4 \( \mu m \) and 0.7 \( \mu m \). Dollfus (1989) discussed the results of laboratory experiments by microwave simulation and laser scattering on various complex shapes with different porosities. It has been observed that the measurement of 10 \( \mu m \) flux in comet Hale-Bopp is dominated by small, porous, amorphous silicate grains (Harker et al. 2002). The higher polarization observed for the dust coma of comet Hale-Bopp also suggests the existence of smaller grains included in highly porous large aggregates, and possibly an increase in the number of high albedo grains in these aggregates (Hadamcik & Levasseur-Regourd 2003).

The fluffy aggregate model originally proposed by Greenberg and Hage (1990) and later adopted by Xing and Hanner (1997) is also used for the study of negative polarization in comets. Tanga et al. (1997) and Levasseur-Regourd et al. (1998) suggested that multiple scattering may well explain the negative polarization, because lower polarization is found in the near-nucleus region of comets where dusty jets are most pronounced. Xing & Hanner (1997) have done calculations with porous grains of various shapes and sizes using the DDA method. Petrova et al. (2000) have shown that aggregates composed of touching spheres (< 50 \( \mu m \)) with size parameters ranging from 1.3 to 1.65, display properties typical of cometary dust particles, namely, a weak increase of the backscattering intensity, negative linear polarization at small phase angles (\( \leq 20^0 \)), and a positive wavelength gradient of polarization. Their results on the aggregates indicate that more compact particles have a more pronounced negative branch of polarization. They also commented that the increase of polarization with wavelength is reduced, if the imaginary part of the refractive index decreases with wavelength. Moreno et al. (2003) also studied the irregular and composite grain characteristics of comet Hale-Bopp in the mid infrared spectrum using the DDA method. The favoured grain model is now that of porous fluffy grains with irregular shapes. So, it is important to study the fluffy grains with irregular shapes and enhanced multiple scattering, which may better explain the observed polarization data in comets. However, a systematic approach in this direction is beyond the scope of the present work.

4 CONCLUSIONS

Based on the T-matrix method and the modified power law distribution function, the following conclusions can be drawn from the present work:

(i) The complex refractive indices and shape parameters of Hale-Bopp’s grains as derived from present work are: (1.382, 0.035, 0.936), (1.379, 0.041, 0.936) at \( \lambda = 0.485 \mu m \) and 0.684 \( \mu m \) respectively.

(ii) It has been found from the present analysis that the power index \( (\alpha) \) in the modified power law distribution is same as that of the power law distribution function used by K-L, which is -3.

(iii) The \( \chi^2_{\text{min}} \) - values obtained from present work are 20.84 and 39.08 at \( \lambda = 0.485 \mu m \) and 0.684 \( \mu m \) respectively, whereas the corresponding values from K-L’s work are 212.9 and 174.4 respectively. Thus the present analysis is giving better fit to the observed polarization data of comet Hale-Bopp as compared to K-L.

(iv) The expected negative polarization values have been successfully generated for comet Hale-Bopp using the T-matrix method, which was not possible earlier by K-L.

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Table 1. The best fit values of \((n, k)\) and \(E\) obtained in the present work for comet Hale-Bopp at \(\alpha = -2.9, -3.0\) and \(-3.1\).

| \(\lambda\) (in \(\mu\)m) | Scattering angle range (in degrees) | No. of data points | \(\alpha\) | \(n\) | \(k\) | \(E\) | \(\chi^2_{\text{min}}\) |
|-------------------------|------------------------------------|-------------------|---------|----|----|----|----------------|
|                         |                                    |                   |         |   |   |    |                |
| 0.485                   | 133 - 163                          | 29                | -2.9    | 1.381 | 0.039 | 0.936 | 22.13          |
|                         |                                    |                   |         |     |    |    |                |
|                         |                                    |                   |         |     |    |    |                |
|                         |                                    |                   |         |     |    |    |                |
|                         |                                    |                   |         |     |    |    |                |
|                         |                                    |                   |         |     |    |    |                |
| 0.684                   | 133 - 177                          | 57                | -3.0    | 1.379 | 0.046 | 0.936 | 40.18          |
|                         |                                    |                   |         |     |    |    |                |
|                         |                                    |                   |         |     |    |    |                |
|                         |                                    |                   |         |     |    |    |                |
|                         |                                    |                   |         |     |    |    |                |
|                         |                                    |                   |         |     |    |    |                |

Table 2. The \((n, k)\) values as obtained by different authors for comet Halley using the Mie code at different wavelengths.

| \(\lambda\) (in \(\mu\)m) | \(n\) | \(k\) | Authors |
|-------------------------|------|------|--------|
| 0.365 \(\mu\)m         | 1.392 | 0.024 | Mukai et al. (1987) |
|                         | 1.387 | 0.032 | Sen et al. (1991a)  |
|                         | 1.403 | 0.024 | Das et al. (2004)   |
| 0.485 \(\mu\)m         | 1.387 | 0.031 | Mukai et al. (1987) |
|                         | 1.375 | 0.040 | Sen et al. (1991a)  |
|                         | 1.390 | 0.026 | Das et al. (2004)   |
| 0.620 \(\mu\)m         | 1.385 | 0.035 | Mukai et al. (1987) |
| 0.684 \(\mu\)m         | 1.374 | 0.052 | Sen et al. (1991a)  |
|                         | 1.386 | 0.038 | Das et al. (2004)   |

Figure 1. The observed polarization values of comet Hale-Bopp at \(\lambda = 0.485\mu\)m. The solid line represents the best fitted polarization curve obtained from the present work. The simulated polarization curve from Kerola & Larson (2001) is also shown by dotted line.
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Figure 2. The observed polarization values of comet Hale-Bopp at $\lambda = 0.684\mu m$. The solid line represents the best fitted polarization curve obtained from the present work. The simulated polarization curve from Kerola & Larson (2001) is also shown by dotted line.

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