Microscopic study of neutron-rich Dysprosium isotopes

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Abstract

Microscopic studies in heavy nuclei are very scarce due to large valence spaces involved. This computational problem can be avoided by means of the use of symmetry based models. Ground-state, γ and β-bands, and their B(E2) transition strengths in 160−168Dy isotopes, are studied in the framework of the pseudo-SU(3) model which includes the preserving symmetry Q·Q term and the symmetry-breaking Nilsson and pairing terms, systematically parametrized. Additionally, three rotor-like terms are considered whose free parameters, fixed for all members of the chain are used to fine tune the moment of inertia of rotational bands and the band-head of γ and β-bands. The model successfully describes in a systematic way rotational features in these nuclei and allows to extrapolate toward the midshell nucleus 170Dy. The results presented show that it is possible to study full chain of isotopes or isotones in the region with the present model.

1 I. Introduction

Rare-earth neutron-rich nuclei offer an opportunity to study singular aspects of nuclear structure around the midshell region such as new modes of excitation and collectivity, the role of the single-particle levels [1] and even possible modifications of shell structure. Nevertheless, despite the recent progress in experimental techniques, the rare-earth neutron-rich nuclei remains a region of the nuclear chart which has been seldom studied. For example, in heavy-ion induced multinucleon transfer reaction, the combination of stable projectiles and stable targets has unfavorable kinematic matching conditions leading to heavy neutron-rich nuclei, resulting in small cross sections that limits its use for populating neutron-rich nuclei [2]. In this region, nuclei have large numbers of valence protons and neutrons, and many low-lying collective nuclear properties are expected to appear there. For example, Er, Yb and Hf neutron-rich nuclei exhibit a minimum in the 2+ energies and a maximum of deformation at N=104
Assuming standard spherical shell gaps, $^{170}$Dy has the larger number of valence particles of any nucleus lighter than the doubly magic lead, and hence it is expected to be the most collective nucleus in the region \cite{4, 5, 6, 7}. Experimental works in the lighter dysprosium isotopes have shown that there is an enhancement of the collectivity in $^{164}$Dy, a relative flatness of the moment of inertia in $^{166}$Dy, and $^{168}$Dy shows again an apparent enhancement of deformation \cite{8}. Nevertheless, to date the energies of the levels in $^{170}$Dy have not been measured, and hence its degree of collectivity is still unknown. In addition, the known saturation of B(E2) in the region \cite{5, 6, 7} and the decrease in moment of inertia between N=98-100 in the gadolinium isotopes \cite{9} open the following question of the collectivity in the region: is $^{170}$Dy indeed the most collective of the region or is a little of the collectivity lost when we reach N=104.

Among the different channels for producing the dysprosium neutron-rich nuclei, quasielastic two-neutron transfer reaction has been used to study the neutron-rich $^{166}$Dy \cite{2}, two-proton pick-up reaction allowed to populate $^{168}$Dy \cite{8, 10}, and projectile fragmentation of $^{208}$Pb was used \cite{11, 12} to synthetize the mid-shell $^{170}$Dy.

From the theoretical side, the large valence spaces associated with heavy-nuclei have implied a slow progress, limiting the number of models capable of dealing with $^{170}$Dy. The earlier studies of this nucleus have used the relativistic mean-field calculations \cite{13}, the Strutinsky shell correction \cite{14, 15}, the generator co-ordinate method \cite{16}, and the Monte Carlo shell-model \cite{17}. More recently, P. H. Regan \textit{et al.} \cite{18, 19} used the cranked shell-model with nonaxial deformed Woods-Saxon potential, and predicted a highly deformed $^{170}$Dy nucleus with a pure axial symmetric-shape deformation along the Yраст line and a $K^\pi = 6^+$ isomeric state at an approximated energy of 1.2 MeV.

A shell model description \cite{20} of heavy nuclei requires further assumptions that include a systematic and proper truncation of the Hilbert space \cite{21}. The symmetry based SU(3) shell model \cite{22, 23} has been successfully applied in light nuclei, where a harmonic oscillator mean-field and a residual quadrupole-quadrupole interaction can be used to describe dominant features of the nuclear spectra. However, the strong spin-orbit interaction renders the SU(3) model useless in heavier nuclei, while at the same time pseudo-spin emerges as a good symmetry \cite{24, 25}. The origin of this symmetry has been traced back to the relativistic mean field for heavy-nuclei \cite{26, 27}, and the success of the pseudo-SU(3) model \cite{28} lies on the consistency of this symmetry. On other hand, the backbending phenomenon in $^{154-164}$Dy chain has been studied with the projected shell model \cite{29}.

The first applications of the pseudo-SU(3) model considered pseudo-spin as a dynamical symmetry \cite{30, 31, 32}. With the development of a computer code came a technical breakthrough that enabled mixed-representation calculations for calculating reduced matrix elements of physical operators (i.e. the symmetry-breaking Nilsson single-particle energies or pairing correlations) between different SU(3) irreps \cite{33}. This enabling technology allows fully microscopic studies of energy levels belonging to normal parity bands and transitions \cite{34, 35, 36, 37, 38, 39}.
The study of abnormal parity bands of rare earth nuclei, in addition of those with normal parity, requires the explicit consideration of intruder levels, which may be performed using the quasi-SU(3) scheme, as it was pointed out in Refs. [40, 41], coupling the intruder states of the quasi-SU(3) with those of normal parity described by the pseudo-SU(3). In Reference [20] the authors report some calculations of the quadrupole properties of $^{48}\text{Cr}$ and E2 transition probabilities of the Nd, Sm, Gd and Dy nuclei with $92 \leq N \leq 98$, showing the value of this theoretical framework, because their predictions are very close to values reported experimentally. In the sd-shell, the quasi-SU(3) scheme was applied to describe even-even, even-odd and odd-odd nuclei [42, 43].

However, a calculation of rare-earth nuclei which considers the coupling of quasi- and pseudo SU(3)-schemes is impossible at this time due to computational challenges unresolved yet. The first step in the development of these codes would be the building of the basis, which will emerge from the coupling of normal with intruder states, mixing the occupation numbers associated to each subspace. In this case, the use of pseudo + quasi scheme (P and Q, respectively) would involve the following couplings

$$
(\tilde{\lambda}_\pi, \tilde{\mu}_\pi)_P \otimes (\lambda_\pi, \mu_\pi)_Q = (\lambda_\pi, \mu_\pi)_{P+Q},
$$

$$
(\tilde{\lambda}_\nu, \tilde{\mu}_\nu)_P \otimes (\lambda_\nu, \mu_\nu)_Q = (\lambda_\nu, \mu_\nu)_{P+Q},
$$

$$
(\lambda_\pi, \mu_\pi)_{P+Q} \otimes (\lambda_\nu, \mu_\nu)_{P+Q} = (\lambda, \mu)_{\text{Total}}
$$

where the first row is for protons, the second is for neutrons and the third shows the final coupling that might be used in the description of the whole nucleus and where it has not been considered the changes in occupation numbers. The second step would be the extension of the Hamiltonian to include the coupling with the particles in the intruder sector. These two major extensions would give us the ability of realize an comprehensive description of heavy nuclei, as for example the study of abnormal parity bands, and the reduction of huge effective charges used up to now.

In order to illustrate this calculation, we can take as example the nucleus $^{160}\text{Dy}$. It has 66 protons and 94-104 neutrons, and of these, 16 protons and 12 neutrons are in the last unfilled (open) shells. Assuming a deformation $\beta \sim 0.25$, the deformed Nilsson single-particle levels of the active shells are filled from below [31, 32]. Ten protons are distributed in the $1g_{7/2}$ and $2d_{5/2}$ orbitals of the $\eta = 4$ shell, and the remaining six occupy the $1h_{11/2}$ intruder orbital. Eight neutrons occupy the $2f_{7/2}$ and $1h_{9/2}$ orbitals of the $\eta = 5$ shell and four are in $1i_{13/2}$ orbital. This calculation would consider a total of 28 nucleons in active shells, which increases the computational time by several orders of magnitude; so even considering the use of symmetries it would be necessary to modify the current codes to increase their efficiency. These studies are beyond the goals of this work and are left as future extensions of the model. Nevertheless, following the suggestions of the Refs. [20, 40, 41], it is possible to employ the quasi-SU(3) scheme to estimate the contributions of the intruder...
sector to the quadrupole moments \((Q_0)\) and relate them to the E2 transition probability, eliminating the use of the extremely large effective charges.

In this Article, we present for the first time an application of the pseudo-SU(3) model to the \(^{160-168}\)Dy chain, where the most relevant quadrupole-quadrupole, Nilsson single-particle and pairing terms are parametrized systematically and at the same time, a best fit of the parameters \(a(K^2)\), \(b(J^2)\) and \(c(C_3)\) is done for this set of nuclei. The calculations for these known nuclei are extrapolated towards the neutron-rich nucleus \(^{170}\)Dy. Previous applications of the model made use of several nucleus by nucleus fitting procedure of several parameters, not allowing a systematic study of the evolution of properties (for example, the modifications in the moment of inertia of rotational bands) along chains of nuclei and making unreal the extrapolations predicting the spectroscopy of unmeasured nucleus. In the present approach, our goal is to study the evolution of collectivity along the dysprosium isotopic chain (\(Z=66\)) starting at \(N = 94\) and ending with predictions for ground-state, \(\gamma\) and \(\beta\) bands in the \(N = 104\) midshell \(^{170}\)Dy nucleus. To this end, the procedure we have followed is to employ a model with the Nilsson, Quadrupole-Quadrupole and Pairing terms of the Hamiltonian \([2]\) systematically parametrized \([14]\) in function of the mass \((A)\), whereas the parameters of the last three terms \([a(K^2), b(J^2)\) and \(c(C_3)\)] were determined by applying a best-fit procedure to the experimental data in all the chain.

In Section II a brief description of the pseudo-SU(3) classification scheme and the Hamiltonian of the model are discussed. Theoretical energies of levels, comparison with data (when available) for ground-state, \(\gamma\) and \(\beta\) bands in \(^{160}\)Dy, \(^{162}\)Dy, \(^{164}\)Dy, \(^{166}\)Dy and \(^{168}\)Dy, predictions in \(^{170}\)Dy and the analysis of collectivity implied are presented in the Section III. Intra and inter-band \(B(E2)\) transitions are presented in Section IV, and finally, a brief conclusion is given in Section V.

2 II. The model

The selection of the many-body basis is the starting point for any shell-model application. Many-particle states of \(n_\alpha\) active nucleons \((\alpha = \pi, \nu)\) in a given \((N)\) normal parity shell \(n_\alpha^N\) are classified by the following group chain \([30, 31, 32, 44]\):

\[
\begin{align*}
\{1^+\} \quad \{f_\alpha\} \quad \{f_\alpha\} \quad \gamma_\alpha \quad (\lambda_\alpha, \mu_\alpha) \quad \tilde{S}_\alpha \quad K_\alpha \\
U(\Omega_\alpha^N) \supset U(\Omega_\alpha^N \times U(2) \supset SU(3) \times SU(2) \supset \\
\tilde{L}_\alpha \quad J_\alpha \\
SO(3) \times SU(2) \supset SU_J(2),
\end{align*}
\]

where above each group the quantum numbers that characterize its irreducible representations (irreps) are given. \(\gamma_\alpha\) and \(K_\alpha\) are multiplicity labels of the indicated reductions. The occupation numbers for protons are constant along the chain. In shell model applications, the dysprosium isotopes are considered
Nucleus $\epsilon_2$ $n_\nu$ $n_\nu^N$ $n_\nu^A$

| Nucleus | $\epsilon_2$ | $n_\nu$ | $n_\nu^N$ | $n_\nu^A$ |
|---------|-----------|--------|--------|--------|
| $^{160}$Dy$_{94}$ | 0.250 | 12 | 8 | 4 |
| $^{162}$Dy$_{96}$ | 0.258 | 14 | 8 | 6 |
| $^{164}$Dy$_{98}$ | 0.267 | 16 | 10 | 6 |
| $^{166}$Dy$_{100}$ | 0.267 | 18 | 12 | 6 |
| $^{168}$Dy$_{102}$ | 0.275 | 20 | 12 | 8 |
| $^{170}$Dy$_{104}$ | 0.267 | 22 | 14 | 8 |

Table 1: Deformation ($\epsilon_2$) and occupation numbers for neutrons ($n$). The superscript $N$ and $A$ indicate normal and abnormal parity levels, respectively.

to have 16 protons out of the $Z = 50$ inert core, 10 of these in normal and 6 in abnormal $h_{11/2}$ parity levels. In Table 1 the occupation numbers for neutrons assigned to each nucleus are presented.

The first application of the pseudo-SU(3) model, where symmetry-breaking terms were included in the Hamiltonian, considered a Hilbert space composed of those normal parity states ($|\beta, J M\rangle$) with the highest spatial symmetry, $\hat{S}_{\pi, \nu} = 0$ (for an even) and 1/2 (for an odd) number of protons or neutrons. That approach has allowed to describe in each nucleus typically three rotational bands in even-even [34, 35] and odd-mass [46] nuclei and intra- and inter-band [47] B(E2) transition strengths. As has been the case for almost all studies with the model to date, nucleons in abnormal parity orbital are considered to renormalize the dynamics that is described using only nucleons in normal parity states. This limitation is reflected, for example, by the use of very high effective charges to describe quadrupole electromagnetic transitions. While this is the most important limitation of the model and a very strong assumption, it has been shown to be a reasonable approach [43, 48].

In more recent studies [36, 37, 38, 39], the extension of the Hilbert space to those lesser spatially symmetric states ($\hat{S}_{\pi, \nu} = 1$ and 3/2, for even and odd number of protons or neutrons, respectively) has shown that the model can describe up to eight excited bands (in each nucleus), intra- and inter-band B(E2) transition strengths between them, and to discuss the interplay between the collective and single-particle nature of the M1 excitations, the so-called scissors mode [49]. A detailed analysis of the wave function content shows that the ground-state band is composed predominantly of $\hat{S}_{\pi, \nu} = 0$ and 1/2 states (for even and odd number of $\pi$ and $\nu$, respectively) with a very small mixing of $\hat{S}_{\pi, \nu} = 1$ and 3/2 irreps. Nevertheless, in excited bands (including the $\gamma$ and $\beta$ bands), the states with $\hat{S}_{\pi, \nu} = 1$ and 3/2 have a very important contribution, and the correct description of many spectroscopic properties requires a truncation scheme including the $\hat{S}_{\pi, \nu} = 1$ states (in even-even nuclei). As we are dealing with even-even dysprosium isotopes and interested in the ground-state, $\gamma$ and $\beta$-bands, in the present work those states with $\hat{S}_{\pi, \nu} = 0$ and 1 are considered.

The Hamiltonian contains spherical Nilsson single-particle terms for the protons and neutrons ($H_{sp, \pi[\nu]}$), the quadrupole-quadrupole ($Q \cdot Q$) and pairing
\begin{equation}
H = \sum_{\alpha=\pi,\nu} \{ H_{sp,\alpha} - G_\alpha H_{\text{pair},\alpha} \} - \frac{1}{2} \chi \hat{Q} \cdot \hat{Q} + a K^2 + b J^2 + c \tilde{C}_3.
\end{equation}

A detailed analysis of each term of this Hamiltonian and its systematic parametrization can be found in Ref. [44]. The first row contains the basic components of any realistic Hamiltonian: the single-particle levels, pairing correlations and the quadrupole-quadrupole interaction, essential in the description of deformed nuclei. They have been widely studied in nuclear physics literature, allowing to fix their respective strengths by systematics (A dependent) [50] [51], consequently they are not considered as free parameters of the model. The SU(3) mixing is due to the symmetry-breaking Nilsson single-particle and pairing terms.

The rotorlike terms in Hamiltonian (2) are used to fine tune the spectra. Their three parameters $a$, $b$, and $c$ have been fixed following the prescriptions given in Ref. [44], where a detailed analysis of each term can be found. Only these three terms are taken as free parameters of the model, and once their magnitude are determined by best fit, their values are kept constant for the full chain of isotopes. To clarify the effect of these terms over the energies, we may take as example the $bJ^2$ term. Its effect is to give additional moment of inertia to the rotational bands, helping to diminish the energy of rotational states. When this term is added, the energy of the $J^\pi = 2^+_{g.s.b.}$ states in the neutron-rich dysprosium isotopes reduces its value between 18 and 20 keV. For example, in $^{162}\text{Dy}$ the energy of the $J^\pi = 2^+_{g.s.b.}$ state changes from 94 keV (when $b = 0$) to 74 keV ($b = -3.2$ keV). In $^{164}\text{Dy}$ it changes from 100 keV ($b = 0$) to 80 keV ($b = -3.2$ keV). Those states with higher angular moment are also affected by this rotor term with larger changes. Using $^{164}\text{Dy}$ as example, the state $J^\pi = 8^+_7$ changes from 1845 keV when $b = 0$ to 1606 keV when $b = -3.2$ keV. It is important to mention that the rotor terms have influence on the level spacings within the rotational bands, but the wave function holds practically constant [44] within 0.001 %.

### 3 III. Low-lying energy spectra

Using the basis and Hamiltonian presented in the previous section, we present the results for ground-state, $\gamma$ and $\beta$-bands in the $^{160-168}\text{Dy}$ isotopes. In addition, these results allow to extrapolate and predict energies and B(E2) transitions in $^{170}\text{Dy}$.

Since the value of the free parameters of the Hamiltonian (2) have a very strong influence on the energies, their parametrization is very important. They are therefore determined by applying best fit to the experimental data for the five even-even isotopes $^{160-168}\text{Dy}$. The states from $J^\pi = 0^+$ to $6^+$ belonging
Figure 1: Experimental and theoretical energies (in keV) of ground-state, \( \gamma \) and \( \beta \)-bands in \( ^{160-164}\text{Dy} \) nuclei. The labels indicate the total angular momentum and parity of each level. Experimental data are plotted on the left-hand side of each column and theoretical ones on the right-hand side. The correspondence between theoretical and experimental levels is indicated by dotted-lines.
to ground, $\gamma$ and $\beta$ bands were used in the best fit procedure. Higher angular moments presented in the Figures 1 and 2 are not used in the procedure. The values used in the present work are $a = 20$, $b = -3.2$ and $c = 0.033$ (all in [keV]). The corresponding root-mean-square to the experimental data (from $J^\pi = 0^+$ to $6^+$) is 111 keV. That value allows one to estimate how good the fit is. It should be pointed out that the absolute values of the present free parameters are significantly smaller than those considered in previous works [39, 36, 37] in the region, where they got variations from positive to negative values when going from one nucleus to the next. This fact comes from the strategy followed in the present work, where a single value of each parameter has been used along the chain, while previous applications of the model to the region used a different parametrization for each nucleus, finding variations in the parameter from one nucleus to another, some times of one order of magnitude, or even in the sign. This implies that a greater rms than in previous applications of the model is expected; this is the cost when we want to make the model more predictive. On the other hand, the benefits are that the more physically relevant terms of the Hamiltonian (first row of eq. 2) are determining completely the structure of spectra, since the contribution of the free parameters is very small, and that we can extrapolate and predict the energies of states in unknown nuclei.

In the present application of the model, we present results for the energies of levels and $B(E2)$ transitions in the dysprosium ($Z = 66$) isotopic chain, including the stable $^{160}$Dy$_{94}$, $^{162}$Dy$_{96}$ and $^{164}$Dy$_{98}$, and the unstable $^{166}$Dy$_{100}$ and $^{168}$Dy$_{102}$ isotopes. Finally, the extrapolation allows us to predict the energies of $^{170}$Dy$_{104}$.

Figures 1 and 2 show the experimental [8, 10, 52, 53, 54, 55] and theoretical energies for ground-state, $\gamma$ and $\beta$-bands in the $^{160-164}$Dy and $^{166-170}$Dy isotopic chains, respectively. For the ground-state band, the model predicts smaller energies than the experimental data, except in $^{164}$Dy (Figure 1) where the model overestimates almost all energies. The pattern of decreasing energy observed in experimental data in $^{164}$Dy is not described by the model. The case of the $\gamma$ band is different, where the model reproduces the decrease in energy of the band head in $^{164}$Dy and higher values in the other nuclei, corroborating the trend observed in experimental data (when available). As it happens with the ground-state band, theoretical $\gamma$ bands have an overestimated moment of inertia, except in $^{164}$Dy where the model overestimates almost all energies. For the theoretical predictions, a common feature in the isotopic chain studied here (except $^{164}$Dy) is that the model predicts a strong collectivity as is implied by the data. The lack of moment of inertia observed in theoretical values for $^{164}$Dy is probably due to the existence of some local effect that enhance nuclear deformation in this nucleus [8]. In $\beta$ bands the model agrees well with experimental values, with higher values of the $0^+$ band-head in $^{160}$Dy and $^{162}$Dy isotopes, and lower value in $^{166}$Dy. The nucleus $^{164}$Dy has peculiar rotational features, as it has been pointed out in Ref. [56]. It has been actively discussed the interpretation of the $\beta$-band in this nucleus, because from the experimental side there is not $\beta$-band reported up to now.

The predictions for the nucleus $^{170}$Dy are presented in the last three columns.
Figure 2: Experimental and theoretical energies (in keV) in $^{166-170}$Dy nuclei. The labels are the same as in Figure [1].
of Figure 2. It is surprising that the energies of levels predicted for this nucleus are higher than many presented in Figures 1 or 2 for other nuclei, for the respective states. This represents a first indication of the model that $^{170}$Dy is less deformed and collective than the lighter dysprosium isotopes studied here. The same tendency has been identified in $^{162-164}$Gd isotopes [9], where the moment of inertia between $N = 98 - 100$ decrease. It should be pointed out that the absence of nucleons in the intruder sector could help to explain the loss of collectivity in $^{164}$Dy and $^{170}$Dy, but at this stage of calculations there is no explanation for this loss of collectivity, presenting a challenge for future works. In addition, the model describes the $K^\pi = 6^+$ isomeric state which develops a rotational band. Its band-head is predicted at an energy of 1048 keV, very close to that predicted in Ref. [18] at 1200 keV. Here the band has its origin in the rotation of the intrinsic $K^\pi = 6^+$ state with pseudo-spin zero. Future calculations in other nuclei of the region with high collectivity and results coming from different models will contribute to confirm or reject the tendency discussed in the present work.

### 4 IV. Intra and inter-band B(E2)s

In Table 2 we show intra-band B(E2) transition strengths up to $J^\pi = 8^+$ in the ground-state, $\gamma$ and $\beta$ bands. The values are very collective between 200 and 700 $e^2b^2 \times 10^{-2}$. The experimental data are between parentheses. Unfortunately, there are no experimental measurements for almost all transitions in $\gamma$ and $\beta$ bands.

The effective charges used in the electric quadrupole operator $Q_{\mu}$ [44] are $e_\pi = 2.3$ and $e_\nu = 1.3$. These values are the same used in the pseudo-SU(3) studies up to now allowing to describe both intra- and inter-band B(E2)s. They are larger than those used in standard calculations of quadrupole transitions [50] due to the absence of nucleons in intruder levels, and they were not varied to fit any particular value.

The calculation of the electric quadrupole moment and B(E2) transition strengths with realistic effective charges ($e_\pi = 1.4$ and $e_\nu = 0.6$) can be done if the intruder sector is explicitly considered. As discussed in the introduction, even if a complete treatment of this sector is beyond the scope of this work, it can be obtained a simple evaluation of its effects by considering the quasi-SU(3) symmetry of the intruder sector, as proposed in Refs. [40, 41]. Having established for a given deformation the number of nucleons in the abnormal parity levels, their contribution to $Q_0$ is easily obtained by filling the orbits in the right panel of Fig. 34 in Ref. [20]. By summing this contribution and that of the normal parity sector, the B(E2) transitions are calculated with realistic values for effective charges $e_\pi$ and $e_\nu$. In Table 3 we show B(E2) intra-band transition strengths between the $0^+$ and $2^+$ states of the ground-state band.

By comparing the B(E2) values in Table 3 with those of Table 2, it can be clearly seen that the values in both tables are very similar. Therefore, the closeness of both results implies that the explicit consideration of the intruder sector.
Table 2: Theoretical $B(E2; J_i^\pi \rightarrow J_f^\pi) \ [\text{given in } e^2 b^2 \times 10^{-2}]$ intra-band transitions in $^{160-170}$Dy nuclei. Known experimental data [52, 53, 54, 55] are shown between parentheses. Effective charges are $e_\pi=2.3$ and $e_\nu=1.3$.

| $J_i^\pi_{, \text{band}} \rightarrow J_f^\pi_{, \text{band}}$ | $^{160}$Dy | $^{162}$Dy | $^{164}$Dy | $^{166}$Dy | $^{168}$Dy | $^{170}$Dy |
|------------------------------------------------------|----------|----------|----------|----------|----------|----------|
| $0^+_{g.s.} \rightarrow 2^+_{g.s.}$                  | 591 (500)| 596 (535)| 664 (557)| 717      | 723      | 712      |
| $2^+_{g.s.} \rightarrow 4^+_{g.s.}$                  | 302 (146)| 305 (151)| 342 (145)| 368      | 371      | 365      |
| $4^+_{g.s.} \rightarrow 6^+_{g.s.}$                  | 265 (122)| 267 (157)| 302 (173)| 323      | 325      | 322      |
| $6^+_{g.s.} \rightarrow 8^+_{g.s.}$                  | 249 (170)| 251 (181)| 287 (165)| 302      | 304      | 304      |
| $8^+_{g.s.} \rightarrow 10^+_{g.s.}$                 | 239 (169)| 241 (183)| 279 (188)| 288      | 290      | 292      |
| $10^+_{g.s.} \rightarrow 12^+_{g.s.}$                 | 232 (160)| 234 (173)| 274 (189)| 275      | 277      | 278      |
| $0^+_\beta \rightarrow 2^+_\beta$                   | 298      | 269      | 638      | 424      | 616      | 600      |
| $2^+_\beta \rightarrow 4^+_\beta$                   | 127      | 160      | 294      | 141      | 140      | 282      |
| $4^+_\beta \rightarrow 6^+_\beta$                   | 138      | 173      | 240      | 68       | 71       | 275      |
| $6^+_\beta \rightarrow 8^+_\beta$                   | 148      | 165      | 236      | 69       | 73       | 173      |
| $8^+_\beta \rightarrow 10^+_\beta$                  | 205      | 200      | 160      | 144      | 150      | 233      |
| $10^+_\beta \rightarrow 12^+_\beta$                  | 198      | 151      | 201      | 144      | 144      | 227      |
| $2^+_\gamma \rightarrow 3^+_\gamma$                  | 289      | 292      | 329      | 194      | 181      | 237      |
| $3^+_\gamma \rightarrow 4^+_\gamma$                  | 192      | 193      | 217      | 57       | 61       | 151      |
| $4^+_\gamma \rightarrow 5^+_\gamma$                  | 133      | 134      | 153      | 99       | 94       | 77       |
| $5^+_\gamma \rightarrow 6^+_\gamma$                  | 88       | 88       | 95       | 81       | 82       | 67       |
| $6^+_\gamma \rightarrow 7^+_\gamma$                  | 75       | 77       | 82       | 48       | 49       | 32       |
| $7^+_\gamma \rightarrow 8^+_\gamma$                  | 33       | 19       | 45       | 67       | 16       | 14       |
| $2^+_\gamma \rightarrow 3^+_\gamma$                  | 122      | 123      | 139      | 102      | 99       | 84       |
| $3^+_\gamma \rightarrow 5^+_\gamma$                  | 172      | 173      | 117      | 50       | 54       | 205      |
| $4^+_\gamma \rightarrow 6^+_\gamma$                  | 186      | 184      | 214      | 135      | 134      | 236      |
| $5^+_\gamma \rightarrow 7^+_\gamma$                  | 195      | 197      | 234      | 156      | 159      | 245      |
| $6^+_\gamma \rightarrow 8^+_\gamma$                  | 130      | 96       | 223      | 134      | 123      | 173      |
| $7^+_\gamma \rightarrow 9^+_\gamma$                  | 179      | 181      | 242      | 152      | 152      | 256      |

Table 3: Pseudo + quasi-SU(3) $B(E2; J_i^\pi \rightarrow J_f^\pi) \ [\text{given in } e^2 b^2 \times 10^{-2}]$ intra-band transitions in $^{160-170}$Dy nuclei. Effective charges are $e_\pi=1.4$ and $e_\nu=0.6$.

| $J_i^\pi_{, \text{band}} \rightarrow J_f^\pi_{, \text{band}}$ | $^{160}$Dy | $^{162}$Dy | $^{164}$Dy | $^{166}$Dy | $^{168}$Dy | $^{170}$Dy |
|------------------------------------------------------|----------|----------|----------|----------|----------|----------|
| $0^+_{g.s.b.} \rightarrow 2^+_{g.s.b.}$              | 565      | 626      | 653      | 673      | 739      | 741      |
allows using realistic values for the effective charges $e_\pi$ and $e_\nu$. In other words, it means that the huge effective charges used in Table 2 are adequately compensating the absence of the intruder sector, which was not explicitly considered in the pseudo-SU(3) scheme.

Table 4 reports the inter-band B(E2) strengths between states of ground-state and $\gamma$ bands. These values are significantly smaller than those shown in Table 2, because the wave functions of states belong to different bands have very different components. Nevertheless, there are some B(E2)s with large values, which is the result of a strong overlap between the wave functions of the states.

The total pseudo-spin content of the nuclear wave function is built through the coupling of the $\tilde{S}_\pi$ and $\tilde{S}_\nu$ components. As it has been the case in previous works with the model, the ground-state bands in the chain of isotopes $^{160-170}$Dy are composed predominantly by $\tilde{S} = 0$, with very small mixing of the $\tilde{S} = 1$, varying from 0% in $^{164}$Dy to 19% in $^{160}$Dy. The $\gamma$-bands have larger components of $\tilde{S} = 1$, varying from 7% in $^{164}$Dy to 90% in $^{166}$Dy. Large components of $\tilde{S} = 1$ are also observed in $\beta$-bands. The pseudo-spin contents for each band are practically constant along all states of the band. These results show the importance of the $\tilde{S} = 1$ contribution in the description of excited $\gamma$ and $\beta$-bands.

5 V. Conclusions

The pseudo-SU(3) shell model offers a quantitative microscopic description of heavy deformed nuclei. For the first time, employing a systematically parametrized Hamiltonian and the best fit of three parameters for a set of nuclei, the model has been used to study energies and B(E2) transitions of the chain of dysprosium isotopes and to predict the excitation energies in the ground-state, $\gamma$ and $\beta$-bands in $^{170}$Dy. The model describes a maximum of collectivity in $^{168}$Dy, but it fails in $^{164}$Dy where no enhancement of collectivity is found. In $^{170}$Dy a lower degree of collectivity is found, corroborating the tendency found in $^{162-164}$Gd isotopes [3], where the moment of inertia between $N = 98 - 100$ decrease. At the present, the explanation for this loss of collectivity in $^{164}$Dy and $^{170}$Dy is the absence of a dynamical treatment of the intruder states in the model, presenting a challenge for future works.

Intra- and inter-band B(E2) transition strengths were presented, showing
very collective bands and some degree of mixing between ground-state and $\gamma$ and $\beta$ bands. Likewise, it was shown that the explicit consideration of the intruder sector using the quasi-SU(3) symmetry allows to employ realistic effective charges. Nevertheless, the large effective charges used in our calculations work adequately to compensate the absence of the intruder sector in our model, as it was checked for the $B(E2;0^+_{g.s.b.} \rightarrow 2^+_{g.s.b.})$ strength, where the values obtained with the large effective charges are similar to those obtained with realistic effective charges when is considered the effect of the abnormal parity states within a quasi-SU(3) scheme. A more complete study of the rare earth neutron-rich region, exploiting the quasi-SU(3) symmetry to include dynamically the intruder states, would be a natural extension of the present work.

In addition, we have presented the prediction of the model for the $K^\pi = 6^+$ isomeric state in $^{170}$Dy, which is at an energy of 1048 KeV, lower that the value of Ref. [18]. Finally, the results presented confirm the adequacy of the model to predict properties of rare-earth exotic nuclei. Experimental information for the neutron-rich nuclei is highly desired to understand nuclear structure around the midshell region.

6 Acknowledgments

The authors are grateful to ICN-UNAM for the valuable bibliographical support. This work was supported in part by CONACyT (México). CV would like to thank to Dr. Fred Mason Lambert the style corrections and to Dr. A. E. Stuchbery and Dr. O. Castaños for their valuable suggestions to the present work.

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