ASPECTS OF M THEORY AND PHENOMENOLOGY

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ABSTRACT

A brief review is presented of selected topics, including a world-sheet formulation of M theory, couplings and scales in M phenomenology, the perils of baryon decay and the possible elevation of free-fermion models to true M- or F-theory compactifications.

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1 Perspectives on $M$ Theory

The theories formerly known as strings have transmogrified into an incompletely understood higher-dimensional framework [1], initially with two formulations: $M$ theory that emerged originally in the strong-coupling limit of Type IIA string theory, and $F$ theory that appeared in the strong-coupling limit of Type IIB string theory [1]. There are by now several perspectives that offer clues to aspects of $M$ theory. In the low-energy limit, it becomes 11-dimensional supergravity. It may be formulated on the light-cone as Matrix theory [2]. From a world-sheet point of view, $M$ theory is a dual system of world-sheet vortices and monopoles (representing $D$ branes in target space) close to the Berezinskii-Kosterlitz-Thouless transition point [3]. At short distances $M$ theory may become a topological Chern-Simons theory with a non-compact gauge group [4]. It has recently appeared possible to relate many of these perspectives using the world-sheet formalism [5]. According to this formulation, string theory describes the dynamics of the Wilson loops that characterize matter in the topological gauge theory. An enticing aspect of this approach is the appearance of a twelfth dimension. This may be reminiscent of $F$ theory, which is related to $M$ theory by duality [1]: $M$ theory compactified on a space $\mathcal{M}$ is generally dual to $F$ theory compactified on $\mathcal{M} \times S^1$.

Among the interesting $M$ compactifications is that on $S_1/Z_2$, which yields the strong-coupling limit of the $E_8 \times E_8$ heterotic string [3]. This is the perspective on $M$ theory which is most often used in phenomenological papers, and provides the framework for most of the rest of this talk.

2 Overview of $M$ Phenomenology

In various different dual limits, $M$ theory manifests itself in 11-dimensional supergravity, the $E_8 \times E_8$ heterotic string, the $SO(32)$ heterotic string, Type I, IIA or IIB string. In the bad old days of weak-coupling string phenomenology, it was the $E_8 \times E_8$ heterotic string that attracted the most attention, and the first approaches to $M$ phenomenology were based on the strong-coupling version of this theory [3, 4]. It can be formulated by considering an 11-dimensional “bulk” space with 10-dimensional “walls” at each of its two ends. The low-energy theory in the bulk is 11-dimensional supergravity, whilst one $E_8$ gauge theory factor appears on each wall. This formulation appears in the strong-coupling limit $e^{2<\phi>} \gg 1$, where $\phi$ is the dilaton in 10 dimensions. The separation between the walls is $R_{11} \sim g_{\text{string}}^{2/3}$, which is large, whereas the old weak-coupling limit $g_{\text{string}} \gg 1$ corresponded to $R_{11} \to 0$ and hence a purely 10-dimensional theory.

Why did such a bizarre construction gain favour? The key phenomenological argument is the well-known conflict between the supersymmetric grand unification scale $m_\chi \sim 2 \times 10^{16}$ GeV inferred from low-energy data obtained at LEP and elsewhere [3], and the string unification scale calculated in weakly-coupled string theory: $m_U \sim 3 \times 10^{17}$ GeV [3]. There were many pre-$M$-theory attempts to resolve this discrepancy of more than an order of magnitude, as described below, but none of these was totally satisfactory.
The first option examined was to derive a GUT from string and break its symmetry at a scale \( m_{\text{GUT}} \ll m_U \). Unfortunately, this is not so easy, because the most amenable string model constructions based on level-one realizations of the world-sheet current algebra could not yield the adjoint or other large GUT Higgs representations required in (almost all) GUT models \([10]\). There have recently been heroic efforts to construct string GUTs using higher-level string constructions \([11]\), but these are very tightly constrained, and I am unaware of any completely convincing model.

The adjoint-Higgs problem motivated interest in flipped \( SU(5) \times U(1) \) \([12]\), which only requires 10- and 5-dimensional Higgs representations that are freely available in a low-energy effective field theory derived from a level-one string theory. In the absence of any additional low-mass particles, the calculated value of \( \sin^2 \theta_W \) may be smaller than in a conventional \( SU(5) \) GUT, if the \( SU(5) \) subgroup is broken at a lower energy scale than the string scale where it is unified with the \( U(1) \) factor. However, the concordance between the measured and calculated values of \( \sin^2 \theta_W \) suggests that the string unification scale at which \( SU(5) \times U(1) \) emerges cannot be much higher than the conventional GUT scale at which \( SU(5) \) is broken, so the gap remains.

This problem could in principle be resolved by a suitable coalition of light particles, either in flipped \( SU(5) \times U(1) \) \([13]\) or some other string model. However, the retention of the successful prediction for \( \sin^2 \theta_W \) is not automatic in such models, and becomes a constraint rather than a glorious prediction.

Another suggestion was that the string threshold corrections might be large, invalidating the large estimate \( m_U \sim 3 \times 10^{17} \text{ GeV} \). However, this requires some moduli of the manifold of compactification to differ greatly from the Planck scale, which is difficult to arrange in an appealing string model. Moreover, the successful GUT value of \( \sin^2 \theta_W \) again becomes a constraint rather than a prediction \([14]\).

A structured approach to the possibility of additional light matter particles is offered by the idea that an extra space-time dimension appears at more than the Planck length, providing many additional Kaluza-Klein states that alter the energy-dependence of the gauge and/or gravitational couplings, and hence affect the calculation of \( m_U \). The strongly-coupled heterotic scenario for \( M \) phenomenology \([6]\) comes within this general category. In this case, the extra Kaluza-Klein states do not affect the running of the gauge couplings, which live on the walls at the end of the world, but they appear in the bulk and accelerate the running of the gravitational coupling, thereby reducing \( m_U \).

It is clear that, for this scenario to work, the eleventh dimension must be larger than \( m_{\text{GUT}}^{-1} \). This makes it larger than the compactification radius, so the sequence of events at increasing energies is \( 4 \to 5 \to 11 \) dimensions for gravity in the bulk. Algebraically, Newton’s constant is given by \([1, 13]\)

\[
G_N = \frac{\kappa^2}{16\pi^2 V_6 R_{11}}
\]  

(1)

where \( \kappa \) is the 11-dimensional supergravity coupling: \( \kappa = (m_{\text{p}}^{(11)})^{9/2}, V_6 \) is the six-dimensional
compactification volume, and the gauge coupling \[3, 15\]
\[
\alpha_{GUT} = \frac{(4\pi\kappa^2)^{2/3}}{2V_6}
\]
Putting in the numbers, one finds \[1, 13\]
\[
R_{11}^{-1} \sim m_{GUT} \left(\frac{m_{GUT}}{m_P}\right)^2 2\pi \sqrt{2} \alpha_{GUT}^{-3/2} m_{GUT} < m_{GUT}
\]
\[
m_P^{(11)} = \kappa^{-2/9} = m_{GUT} \left(\frac{4\pi}{2\alpha_{GUT}}\right)^{1/3} \lesssim m_{GUT}
\]
(3)

In this scenario, there is plenty of new physics at energies below the conventional 4-dimensional Planck scale. In particular, the spectre appears that 5-dimensional supergravity might be the appropriate effective field theory at energies between \(R_{11}^{-1}\) and \(m_{GUT}\) \[16, 17, 18\]. This could then provide the right framework for discussing the transmission of supersymmetry breaking from the hidden wall to the observable one \[16, 18\], as well as other issues \[17\].

The fact that \(M_P\) is now a derived composite scale, and that there is no fundamental scale much above \(m_{GUT}\), provides many phenomenological opportunities and some challenges, one of which we now discuss, in the hope that it may provide some inspiration for constructing interesting models derived from \(M\) theory.

3 Caveat Baryon Decay

The likelihood that quantum gravity might cause baryons to decay was discussed \[19\] before GUTS came on the scene. The basic reason is the no-hair theorem of quantum gravity, which indicates that the only exact symmetries are local (gauge) symmetries. Since baryon number is only a global quantum number with no associated massless gauge field, one would not expect it to be conserved. A dimension-six operator with coefficient \(1/m_G^2\) would yield a proton lifetime \(\tau_p \sim 10^{32} (m_G/10^{15} \text{ GeV})^4 y\). This would be unobservable if \(m_G \sim m_P \sim 10^{19} \text{ GeV} \sim 10^{48} y\) \[19\], and would be swamped by heavy-boson exchange in conventional GUTs, since \(1/m_P^2 \ll 1/m_{GUT}^2\).

In supersymmetric GUTs, there is a dimension-five mechanism for baryon decay, in which Higgsino exchange generates an effective superpotential term of the form \((\lambda^2/m_{H}) QQQL\) \[20\]. This yields \((\lambda^2/m_{H}) (qqq \tilde{q}, \tilde{q}qq \ell)\) interactions that become

\[
O \left(\frac{\alpha}{16\pi}\right) \left(\frac{\lambda^2}{m_{H} m}\right) (qqq\ell)
\]
interactions when dressed by sparticle loops. Thanks to the smallness of the Yukawa couplings \(\lambda\) for first-generation quarks and leptons and the loop factors, this mechanism is at the verge of observability for \(m_{H} \sim 10^{16} \text{ GeV}\) \[21, 22\].

However, what is to prevent a superpotential term \(\lambda_P QQQL\) from appearing in a quantum theory of gravity, with \(|\lambda_P| \sim 1/m_P\)? This would make baryon decay too observable: proton lifetime constraints impose \(|\lambda_P| \lesssim 10^{-6}/m_P\) \[23\].
We have studied this question in some specific models derived from string [24]. For example, no such dimension-five operators are generated by $\tilde{H}$ exchange in the effective field theory. Moreover, the coefficients of non-renormalizable superpotential terms are calculable in any given model, and may be absent in some specific models, as a result of $U(1)$ or other symmetries.

The problem becomes more acute in the new $M$-phenomenology framework [15, 25]. Consider a generic non-renormalizable interaction

$$0(1) \times \frac{g_{\text{string}}^{N+2}}{M^{N+1}} \ QQQL\Phi^N$$

(5)

The natural scale in the denominator is now $M \to m_{\text{GUT}} \sim 10^{16}$ GeV rather than $m_P \sim 10^{19}$ GeV, one can expect $<\Phi>/M \sim 0(1)$ in general, and the string coupling in the numerator is $0(1)$. We need some powerful symmetry or other principle to suppress such operators, perhaps to all orders in perturbation theory and at the non-perturbative level.

We have approached this problem [25] using one of the available technologies for string model-building, derived in the context of weakly-coupled heterotic string. In this way we may identify models with a chance of suppressing baryon decay, that one may be elevate to the strong-coupling limit of $M(F)$ theory.

4 Building Models with Free Fermions

This approach starts from free fermions on the world sheet [26], which are divided into sets $b_k$ with specified boundary conditions $f \to e^{i\alpha_k}f$, forming a finite additive group $\Xi$. The physical states in a given sector $\xi \in \Xi$ are then obtained by making generalized GSO projections. The choices of boundary conditions are subject to many consistency conditions imposed by modular invariance. There are $U(1)$ charges $Q(f) = \frac{1}{2}\alpha(f) + F(f)$, where $\alpha(f)$ is the fermionic boundary condition for $F$, and $F(f) = \pm 1$ for $f, f^*$. These may be enhanced to non-Abelian symmetries by appropriate choices of the boundary conditions.

Many interesting models are derived by starting with a particular set (called NAHE) of five boundary-condition vectors $\{1, S, b_1, b_2, b_3\}$, which yield after the generalized GSO projections an $N = 1$ supersymmetric $SO(1) \times SO(6)^3 \times E_8$ gauge group [27]. The boundary-condition vectors $\{b_1, b_2, b_3\}$ provide three twisted sectors that each yield 16 $16$ representations of $SO(10)$. The models are differentiated by their choices of additional basis vectors, which reduce the spectrum to three generations with an observable-sector gauge group that may be $SU(5) \times U(1)$ [12], $SO(6) \times SO(4)$ [28] or $SU(3) \times SU(2) \times U(1)^2$ [29], with extra observable-sector Higgs representations in a $16 + \bar{16}$ of $SO(10)$, and a hidden sector gauge group that is a subgroup of $E_8$, and has matter representations in general.

We have studied [25] two specific models to see whether they avoid the baryon-stability problem. The first of these has [30] two dangerous sixth-order terms in the superpotential of the forms $\frac{1}{M}QQQL\Phi\Phi$, that would yield dangerous dimension-five proton decay operators if $<\Phi> <\bar{\Phi}> \neq 0$. Some such vacuum expectation values are necessarily generated by an
anomalous $U(1)$, and we can expect the flatness conditions on the potential to generate

$$\langle \Phi \rangle \neq 0$$

In the $M$-theory context, we do not expect these vacuum expectation values to be much smaller than the mass scale $M \sim 10^{16}$ GeV. Moreover, many analogous operators appear at higher orders. Therefore, this model exemplifies the generic problems we expect with baryon stability in $M$ theory.

A more promising model \[31\] is one with an enhanced gauge symmetry. It has a Neveu-Schwarz sector that yields an $SU(3) \times SU(2) \times U(1)_C \times U(1)_L \times U(1)^6$ gauge group. Suitable further choices of boundary conditions elevate one particular combination of $U(1)_C, U(1)_L, \cdots$ to become an $SU(2)$ gauge group. The conventional electric charge $Q_{em} = T^3_L + Y + \frac{1}{2} T^3_{\text{cust}}$, with a component in this custodial $SU(2)$ symmetry group. Among the conventional three light generations, the leptons $L$ and $e^C_L$ are $SU(2)_{\text{cust}}$ doublets, whilst the quarks $Q, u^C_L, d^C_L$ are $SU(2)_{\text{cust}}$ singlets. This immediately implies that there are no $QQQL$ terms, and the quantum numbers of the candidate $\Phi, \bar{\Phi}$ fields ensure that no such terms are generated in any order of perturbation theory \[25\].

This is certainly a promising start, though there is no guaranteed what non-perturbative effects may appear in $M$ theory. On the other hand, a generic $M$-theory model may even possess additional non-perturbative gauge symmetries. If one stays within the weak-coupling free-fermion approach, the strategy is to add to the NAHE set of basis vectors some new vector $\gamma$ which, in combination with others, yields a sector containing additional massless space-time gauge bosons. However, the quarks and leptons must transform non-trivially under this enhanced symmetry. It is no good if it only acts on hidden-sector states, for example, and this depends on details of the GSO projection.

This motivates a search for more powerful analysis tools that may be elevated to the full $M(F)$-theory context. At the moment, we are unaware of a general strong-coupling equivalent of the free-fermion model. On the other hand, some geometric tools for compactifying $M(F)$ theory have been developed. Hence it is desirable, as a first step, to understand the geometry underlying the NAHE set of boundary conditions \[32\]. We have first identified how the NAHE set may be found within the general class of $Z_2 \times Z_2$ orbifolds. The NAHE free-fermion point corresponds to a compactified lattice with enhanced $SO(12)$ symmetry, rather than the more familiar $Z_2 \times Z_2$ orbifold that is based on a $(T_2)^3$ Narain lattice with $SO(4)^3$ symmetry. The NAHE model has Euler characteristic $\chi = 48$, with $h_{11} = 27, h_{21} = 3$, whereas the more familiar model has $h_{11} = 51, h_{21} = 3$.

5 Connection to $M(F)$ Compactifications?

There is rather little literature on these. Much of it concerns $Z_2 \times Z_2$ orientifolds related to the orbifold model with $h_{11} = 51, h_{21} = 3$ \[33\]. Known $M(F)$ theory compactifications on Calabi-Yau threefolds may be classified in terms of three invariants $(r, a, \delta) : h_{11} = 5 + 3r - 2a,
\( h_{21} = 65 - 3r - 2a \). There is a limited catalogue of such models due to Voisin, Borcea and Nikulin \[34\]. The “standard” \( Z_2 \times Z_2 \) orbifold model has \((r, a, \delta) = (18, 4, 0)\), whilst the NAHE orbifold “should” have \((r, a, \delta) = (14, 10, 0)\), which is not in the catalogue!

We are currently looking directly for a Calabi-Yau threefold compactification that corresponds to the NAHE set. A convenient way to tackle this problem is to use the Landau-Ginzburg formalism. We have found \[32\] an interesting class of such models based on the superpotential

\[
W = X_1^4 + X_2^4 + X_3^2 + X_4^4 + X_5^4 + X_6^2 + X_7^4 + X_8^4 + X_9^2
\]  

with spectra modded out by discrete symmetries. We have identified one such model that has \((h_{11}, h_{21}) = (51, 3)\) and has the right symmetries to correspond to the known Voisin-Borcea model \[31, 33\], and another that has \((h_{11}, h_{21}) = (27, 3)\) and apparently corresponds to the NAHE set. If so, the next step will be to see whether it yields a consistent extension of the known Voisin-Borcea models, and in particular whether it an elliptic fibration as sought for \(M\)-\text{and} \(F\)-theory compactifications \[33\]. If so, we would have a consistent \(M(F)\) elevation of the NAHE free-fermion models, that may provide new phenomenological insights into issues such as proton decay.

\section{Conclusions}

\(M\) phenomenology is very much in its infancy. Although impressive technical progress in \(M\) and \(F\) theory has been achieved, little progress has yet been made on the construction of interesting models. Some purists would consider any such effort premature before all the theoretical problems have been resolved. Perhaps theoretical consistency will even determine uniquely the choice of vacuum. I disagree. I believe that experiment surely has a rôle to play, and think that it is useful to pursue a complementary bottom-up approach that uses our empirical knowledge in an attempt to figure out aspects of the Big Picture even before all the pieces of the theoretical jigsaw are in place.

Baryon stability may be one of the important clues. It was already a headache for compactifications of the weakly-coupled heterotic string. It may be a more serious problem in \(M\) theory, with its agglomeration of physics scales around \(m_{GUT} \approx 10^{16} \) GeV \(\ll m_P \[25\]\). Free-fermion models may provide a useful tool for analyzing this problem, and models based on the NAHE structure with an enhanced gauge symmetry may be particularly promising. Elevating these models to true \(M(F)\) theory compactifications requires more geometric intuition. The NAHE set would correspond to some generalization of the known Voisin-Borcea models, and some progress towards identifying this seems to be emerging \[32\].

Looking beyond this horizon, non-perturbative string theory offers many further prospects for model building that had not previously been considered \[35\]. These include possible enlargements of the gauge group to a rank (considerably) larger than 22 and the treatment of transitions that change the number of chiral fields, as well as duality relations between strong-

\footnote{Other members of this family of models include the mirrors of these examples.}
and weakly-coupled models. It will take time to learn how to apply all these tricks to the construction of realistic phenomenological models, but they offer exciting prospects, such as the hope of determining dynamically the number of generations. It will be interesting to see whether the ultimate string- or $M$-theory model bears a close relation to the specific models that have been studied up to now. Very likely not, but I believe that at least some of the phenomenological lessons we have learnt may stand us in good stead as we search for this ultimate model.

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