The baryon acoustic oscillation peak: a flexible standard ruler

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For about a decade, the baryon acoustic oscillation (BAO) peak at about $10^{5} h^{-1}$ Mpc has provided a standard ruler test of the $\Lambda$CDM cosmological model, a member of the Friedmann–Lemaître–Robertson–Walker (FLRW) family of cosmological models—according to which comoving space is rigid. However, general relativity does not require comoving space to be rigid. During the virialisation epoch, when the most massive structures form by gravitational collapse, it should be expected that comoving space evolves inhomogeneous curvature as structure grows. The BAO peak standard ruler should also follow this inhomogeneous evolution if the comoving rigidity assumption is false. This “standard” ruler has now been detected to be flexible, as expected under general relativity.

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1. Structure formation should curve space

For about a decade, the baryon acoustic oscillation (BAO) peak at about $10^{5} h^{-1}$ Mpc has provided a test of the Friedmann–Lemaître–Robertson–Walker (FLRW) family of cosmological models—according to which comoving space is rigid—strongly favouring a present–day matter density parameter and dark energy parameter of $\Omega_{m0} \approx 0.3$ and $\Omega_{\Lambda0} \approx 0.7$, respectively [1, 2]. However, general relativity does not require comoving space to be rigid. On the contrary, during the virialisation epoch during which the most massive structures form by gravitational collapse, it should be expected that comoving space evolves inhomogeneous curvature as structure grows inhomogeneously: overdensities contract while underdensities expand. By averaging over a spatial slice [3, 4], a generalised Friedmann equation (Hamiltonian constraint) is found to replace the homogeneous Friedmann equation [5]. Since voids dominate the recent volume, the effective (averaged) curvature at the present should be negative [6].

The coincidence argument—why does dark energy suddenly become non-negligible compared to the critical density during the epoch of galaxy formation?—has been quantified using the virialisation mass fraction, $f_{\text{vir}}(z)$, of massive dark matter haloes. This evolves with decreasing redshift $z$ similarly to the dark energy parameter $\Omega_{\Lambda}(z)$ interpreted under FLRW, from a tiny value to a big fraction of unity at the present [7]. The Virialisation Approximation, which gives one example of implementing virialisation in scalar averaging by using the observed Hubble constant and the peculiar expansion rate of voids as observational inputs, approximately agrees with the supernovae type Ia distance-modulus–redshift relation and the present–day effective matter density parameter [7]. Given initial results of other implementations of scalar averaging or Swiss cheese models: a power–law template...
metric [8], the Timescape model [9–11] and the Tardis [12] model, Occam’s razor favours dark energy as a phenomenological fit that physically represents the recent emergence of average negative curvature.

A key order-of-magnitude argument—how can the volume-weighted average curvature parameter (e.g. (123) in Ref. 5; (2.9) in Ref. 7) grow to a high enough amplitude in comparison to the matter density parameter?—follows from the similarity in order of magnitude between the Hubble constant and the void peculiar expansion rate (see (2.27), (2.22), (2.13a) in Ref. 7): both are indisputably several tens of km/s/Mpc; the former is well-studied, the latter is poorly studied. Another promising observational avenue for relativistic cosmology is measuring the emergence of average negative curvature, i.e. using the Clarkson, Bassett & Lu Ω_k(z) relation [15–17]. However, it should be possible to use a standard ruler to measure the inhomogeneity itself, rather than average curvature.

2. BAO peak location inhomogeneity

**Relativistic Zel’dovich approximation lower limit** The BAO peak standard ruler provides this: a method of detecting curvature inhomogeneity by measuring the environment dependence of the scale factor [13, 14]. Writing M (“Massive”) and E (“Empty”) to represent overdense and underdense spatial regions, respectively, the scalar averaged scale factors a_M and a_E should be low and high, respectively, i.e. a_M < a_E. Using equations (2), (13), (32), (50), and (54) of Ref. 18 to integrate the Raychaudhuri equation and 1σ initial overdensities/underdensities in a 105 h−1 Mpc diameter spherical domain gives a relativistic Zel’dovich approximation [18, 19] estimate of inhomogeneous scale factors a_M ≈ 0.91a_E [13].

**Method** The Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7) Luminous Red Galaxies (LRGs) provide a well-defined BAO peak. The two-point auto-correlation function ξ of pairs of SDSS DR7 LRGs that are preferentially tangential to the observer’s line-of-sight should have a peak that is less affected by redshift space distortions than that for radial pairs. Using Ref. 20’s catalogues of real and random galaxies, defining overlap ω between LRG pairs and Ref. 21’s supercluster catalogue (Sect. 2.3, Fig. 1, Ref. 14), assuming three-dimensional comoving separations s for a standard ΛCDM model (Ω_m0 = 0.32, Ω_Λ0 = 0.68) [22, 23], with the justification that this is a phenomenologically reasonable fit, using the Landy & Szalay correlation estimator [24], and subtracting a cubic fit to ξ from ranges of separation s away from the peak (s < 70h−1 Mpc and s > 140h−1 Mpc) yields BAO peaks such as those shown in Fig. 1.

**Results** In Fig. 1, the BAO peak location clearly shifts to smaller values for environments which, according to scalar averaging, should have lower values of the effective scale factor than that in the effective model, i.e. we see that a_M < a_E, and, as expected in a void-dominated model, a_E ≈ a_{ΛCDM}. The reality of this shift can be tested further by considering Eq. (9) of Ref. 18.
BAO peak flexible standard ruler

Fig. 1. BAO peak shift, as in Fig. 1, Ref. 13. A (above): BAO peak for LRG pairs whose paths’ overlap with a supercluster is \( \omega \geq \omega_{\text{min}} = 60h^{-1}\) Mpc or are completely contained in a supercluster, where the overlap \( \omega \) is defined in Sect. 2.3, Fig. 1 of Ref. 14. Individual curves show supercluster and “random” galaxy bootstrap resampling. The BAO peak mostly occurs at 95\(h^{-1}\) Mpc, shortward of the usual value. B (below): Complementary LRG pair subset. The peak occurs at the usual value of 105\(h^{-1}\) Mpc.

its dependence on the minimum overlap required for considering an LRG pair to overlap with a supercluster.

Figure 2 shows this dependence. The more that LRG pairs overlap with superclusters,
Fig. 2. Environment dependence of the BAO peak shift, as in Fig. 2, Ref. 13. The BAO peak shift $\Delta s := s_{\text{non-sc}} - s_{\text{sc}}$, where $s_{\text{sc}}$ and $s_{\text{non-sc}}$ are estimated with best-fit Gaussians for LRG pairs overlapping (sc) or not overlapping (non-sc) superclusters, with robust estimates of the standard errors. A best-fit line $\Delta s = 4.3 h^{-1} \text{Mpc} + 0.07 \omega_{\text{min}}$ is shown in green. A 9% shift (i.e. $a_M/a_E = 0.91$) would give $\Delta s = 0.09 \omega$, i.e. a lower limit from scalar-averaging, shown in blue.

the lower the averaged scale factor, in contrast with the rigid comoving space assumption of the FLRW model. The statistical significance of this relation can be estimated with the Pearson product–moment correlation coefficient for $\Delta s$ and $\omega_{\text{min}}$, estimated as 0.87. The null hypothesis that there is no correlation has a probability of $P \approx 0.0008$, i.e. the detection is highly significant. The linear least-squares best fit has slope and zero point $0.073 \pm 0.040$ and $4.3 \pm 2.0 h^{-1} \text{Mpc}$, respectively. The BAO peak standard ruler is flexible.

3. Conclusion

Work towards a relativistically more accurate cosmological model than $\Lambda$CDM is still in progress, but the early results are promising. This initial detection of inhomogeneity in the scale factor should, during the coming decades—with Euclid [25], eBOSS (extended Baryon Oscillation Spectroscopic Survey) [26], DESI (Dark Energy Spectroscopic Instrument) [27], 4MOST (4-metre Multi-Object Spectroscopic Telescope) [28, 29], and the LSST (Large Synoptic Survey Telescope) [30]—be able to help distinguish which implementations best match observations, as well as increase the statistical confidence in rejecting [31, 32] the Newtonian-structure-formation decoupled from relativistic-expansion hypothesis fundamental to $\Lambda$CDM. Although models that speculate beyond both the New-

Scalar averaging implies that these are coupled [4, 33].
tonian and general-relativistic models are presently very popular, the prospects for a dark-
ergy–free general-relativistic cosmological model look good.

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References

[1] D. J. Eisenstein, I. Zehavi, D. W. Hogg, R. Scocimarro, M. R. Blanton et al., *Astrophys. J.* **633**, 560 (November 2005), arXiv:astro-ph/0501171.

[2] S. Cole, W. J. Percival, J. A. Peacock, P. Norberg, C. M. Baugh et al., *Mon. Not. Roy. Astr. Soc.* **362**, 505 (September 2005), arXiv:astro-ph/0501174.

[3] T. Buchert, *General Relativity and Gravitation* **32**, 105 (January 2000), arXiv:gr-qc/9906015.

[4] T. Buchert, *General Relativity and Gravitation* **33**, 1381 (August 2001), arXiv:gr-qc/0102049.

[5] T. Buchert and M. Carfora, *Class. Quant. Gra.* **19**, 6109 (December 2002), arXiv:gr-qc/0210037.

[6] T. Buchert and M. Carfora, *Class. Quant. Gra.* **25**, 195001 (October 2008), arXiv:0803.1401.

[7] B. F. Roukema, J. J. Ostrowski and T. Buchert, *Journ. Cosm. Astr. Phys.* **10**, p. 043 (October 2013), arXiv:1303.4444.

[8] J. Larena, J.-M. Alimi, T. Buchert, M. Kunz and P.-S. Corasaniti, *Phys. Rev. D* **79**, p. 083011 (April 2009), arXiv:0808.1161.

[9] D. L. Wiltshire, *Phys. Rev. D* **80**, p. 123512 (December 2009), arXiv:0909.0749.

[10] J. A. G. Duley, M. A. Nazer and D. L. Wiltshire, *Class. Quant. Gra.* **30**, p. 175006 (September 2013), arXiv:1306.3208.

[11] M. A. Nazer and D. L. Wiltshire, *Phys. Rev. D* **91**, p. 063519 (March 2015), arXiv:1410.3470.

[12] M. Lavinto, S. Räsänen and S. J. Szybka, *Journ. Cosm. Astr. Phys.* **12**, p. 51 (December 2013), arXiv:1308.6731.

[13] B. F. Roukema, T. Buchert, H. Fujii and J. J. Ostrowski, *Mon. Not. Roy. Astr. Soc.* **456**, L45 (February 2016), arXiv:1506.05478.

[14] B. F. Roukema, T. Buchert, J. J. Ostrowski and M. J. France, *Mon. Not. Roy. Astr. Soc.* **448**, 1660 (April 2015), arXiv:1410.1687.

[15] C. Clarkson, B. Bassett and T. H.-C. Lu, *Physical Review Letters* **101**, p. 011301 (July 2008), arXiv:0712.3457.
[16] C. Clarkson, *Comptes Rendus Physique* **13**, 682 (July 2012), arXiv:1204.5505.

[17] D. Sapone, E. Majerotto and S. Nesseris, *Phys. Rev. D* **90**, p. 023012 (July 2014), arXiv:1402.2236.

[18] T. Buchert, C. Nayet and A. Wiegand, *Phys. Rev. D* **87**, p. 123503 (June 2013), arXiv:1303.6193.

[19] T. Buchert and M. Ostermann, *Phys. Rev. D* **86**, p. 023520 (July 2012), arXiv:1203.6263.

[20] E. A. Kazin, M. R. Blanton, R. Scoccimarro, C. K. McBride, A. A. Berlind et al., *Astrophys. J.* **710**, 1444 (February 2010), arXiv:0908.2598.

[21] S. Nadathur and S. Hotchkiss, *Mon. Not. Roy. Astr. Soc.* **440**, 1248 (May 2014), arXiv:1310.2791.

[22] D. N. Spergel, L. Verde, H. V. Peiris, E. Komatsu, M. R. Nolta et al., *Astrophys. J. Supp.* **148**, p. 175 (September 2003), arXiv:astro-ph/0302209.

[23] P. A. R. Ade, N. Aghanim, C. Armitage-Caplan, M. Arnaud, M. Ashdown et al., *Astron. Astroph.* **571**, p. A16 (November 2014), arXiv:1303.5076.

[24] S. D. Landy and A. S. Szalay, *Astrophys. J.* **412**, 64 (July 1993).

[25] A. Refregier, A. Amara, T. D. Kitching, A. Rassat, R. Scaramella et al., *ArXiv e-prints* (January 2010), arXiv:1001.0061.

[26] G.-B. Zhao, Y. Wang, A. J. Ross, S. Shandera, W. J. Percival et al., *ArXiv e-prints* (October 2015), arXiv:1510.08216.

[27] M. Levi, C. Bebek, T. Beers, R. Blum, R. Cahn et al., *ArXiv e-prints* (August 2013), arXiv:1308.0847.

[28] R. S. de Jong, O. Bellido-Tirado, C. Chiappini, E. Depagne, R. Haynes et al., 4MOST: 4-metre multi-object spectroscopic telescope, in *Ground-based and Airborne Instrumentation for Astronomy IV*, eds. I. S. McLean, S. K. Ramsay and H. Takami, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 8446 (September 2012), arXiv:1206.6885.

[29] R. S. de Jong, S. Barden, O. Bellido-Tirado, J. Brynnel, C. Chiappini et al., 4MOST: 4-metre Multi-Object Spectroscopic Telescope, in *Ground-based and Airborne Instrumentation for Astronomy V*, eds. S. K. Ramsay, I. S. McLean and H. Takami, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series, Vol. 9147 (July 2014).

[30] J. A. Tyson, D. M. Wittman, J. F. Hennawi and D. N. Spergel, *Nuclear Physics B Proceedings Supplements* **124**, 21 (July 2003), arXiv:astro-ph/0209632.

[31] T. Buchert, A. A. Coley, H. Kleinert, B. F. Roukema and D. L. Wiltshire, *Int. J. Mod. Phys. D* submitted (2016), arXiv:1512.03313.

[32] P. Bull, Y. Akrami, J. Adamek, T. Baker, E. Bellini et al., *ArXiv e-prints* (December 2015), arXiv:1512.05356.

[33] X. Roy and T. Buchert, *Class. Quant. Gra.* **29**, p. 115004 (June 2012), arXiv:1202.5766.