An explanation of the ABC enhancement in the dd $\rightarrow \alpha X$ reaction at intermediate energies

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Abstract

The dd $\rightarrow \alpha X$ reaction is studied in a model where each pair of nucleons in the projectile and target deuterons undergoes pion production through the NN $\rightarrow d\pi$ reaction. The condition that the two deuterons fuse to form an $\alpha$-particle then leads to peaks at small missing masses, the well-known ABC enhancement, but also a broad structure around the maximum missing mass. With a simplified input amplitude the model gives a quantitative description of both the $\alpha$-particle momentum and angular distributions for a deuteron beam energy of 1250 MeV.

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Measurements of the $\alpha$-particle momentum spectra in the $\text{dd} \rightarrow \alpha \text{X}$ reaction reveal rich structure for beam energies throughout the $0.8 - 1.4 \text{ GeV}$ region [1]. In particular there are very sharp peaks near the kinematic limits, \textit{i.e.} the maximum in the $\alpha$-particle c.m. momentum, and these correspond to missing masses of $M_X \sim 300 \text{ MeV}$. In addition, however, there is a broad central bump around the maximal missing mass, \textit{i.e.} in the region where the $\alpha$-particle is at rest in the c.m. system. Rather similar structures were observed many years ago in both the $\text{pd} \rightarrow \text{^3He} \text{X}$ [2] and $\text{np} \rightarrow \text{d} \text{X}$ [3, 4] reactions and this low missing-mass peak is commonly known as the ABC-effect. It is clear from the mass of the ABC that it must consist of two pions and the absence of an equivalent peak in the $\text{pd} \rightarrow \text{^3H} \text{X'}$ case [4] shows that it must have isospin $I = 0$. It is therefore likely that the ABC manifests itself so clearly in deuteron-deuteron collisions because it leads to a pure $I = 0$ channel for the pions.

The authors of ref. [2] originally suggested that the ABC might correspond to a large $s$-wave isoscalar $\pi-\pi$ scattering length but phase shifts obtained from high energy pion production experiments show this is in fact very small [5]. Since the mass and width of the ABC peak vary with kinematic conditions, it is natural to see if it can be understood as a kinematic effect associated with the presence of nucleons or nuclei. In this spirit Risser and Shuster [6] explored the $\text{np} \rightarrow \text{d} \text{X}$ reaction in a model where two different $\Delta$-resonances were excited through pion exchange. Though they could generate peaks in the right regions, the calculations had limited quantitative success, but it is not clear whether this was due to the simplifications in their formalism, and in particular their neglect of spin [6, 7]. Certainly double $p$-wave pion production should lead to kinematic enhancements in the spinless case because the simplest matrix element squared is proportional to $(\mathbf{k}_1 \cdot \mathbf{k}_2)^2$, \textit{i.e.} the square of the scalar product of the two pion momenta. This is maximal when the two momenta are parallel (the ABC peak) and antiparallel (the
central bump).

ABC production in deuteron-deuteron scattering is at its largest around 600 MeV per nucleon [1] and this is close to the maximum of single-pion production in pp → dπ^+ arising from ∆-excitation. It therefore seems natural to consider two-pion production as coming from two such independent reactions, involving two different pairs of nucleons from the projectile and target deuterons, where the α-particle is formed when the two final deuterons stick together. If we neglect the Fermi motion in the initial deuterons, the c.m. systems of the deuteron-deuteron and nucleon-nucleon channels coincide so that the produced deuterons (and pions) will have the same c.m. momenta. The relative momentum in the final d-d system will therefore be very small when the pions come out with low excitation energy in the ABC peak and there will then be little suppression coming from the α:dd sticking factor. This classical argument of favourable kinematics provides another reason why ABC production might be seen clearest in the dd → αX case.

The Feynman diagram corresponding to our model for dd → απ^+π^- is shown in fig. II, where the various momenta are defined in the c.m. system. In the non-relativistic limit of the Fermi momenta, the matrix element deduced from this diagram is

\[ M = \frac{-\sqrt{2m_\alpha} (\epsilon_1 \cdot \epsilon_2)}{\gamma^2 (2\pi)^{3/2}} \int \frac{d^3q_1 d^3q_2}{\sqrt{3}} \frac{\Delta E_1}{\Delta E_1 + i\epsilon} M_\pi^1 M_\pi^2 \psi_{\alpha}(q_\alpha) \phi_d(q'_1) \phi_d(q'_2) \]

where the \((\epsilon, \epsilon')\) are the polarization vectors of the internal deuterons and \((\epsilon_1, \epsilon_2)\) those of the incident. The wave functions of the final α-particle and the initial deuterons \(\psi_{\alpha}(q_\alpha)\) and \(\phi_d(q'_i)\) depend upon the Fermi momenta in the rest systems of the corresponding nuclei. In the deuteron case the longitudinal component is boosted with respect to the c.m. system to give \(q'_i = (q^b_i, q^z_i / \gamma)\), where \(\gamma = E_d / m_d\) and \(b\) and \(z\) denote the transversal and
longitudinal components. The analogous relativistic effect is small for the $\alpha$-particle and so $q_\alpha = q_1 - q_2 - (k_1 - k_2)/2$. Since our model is not dependent upon exotic Fermi momentum components, it is reasonable to neglect the $D$-state components in the deuterons and the $\alpha$-particle.

The matrix element in eq. (1) is essentially that of second order perturbation theory, where the energy denominators corresponding to the two different time orderings are

$$\Delta E_i = \mp \left\{ v_d \cdot (q_1 + q_2) + (\omega_1 - \omega_2)/2 \right\},$$

with $v_d$ being the c.m. deuteron velocity and $\omega_i$ the pion total energies. This simple result only follows when retaining just the linear terms in the nuclear Fermi momenta. In such a case the two principal value integrals in eq.(1) cancel to leave only the $\delta$-function term.

The largest $pp \to d\pi^+$ amplitude in the $\Delta$ region corresponds to the $^1D_{2p}$ transition [8], for which

$$M_\pi = C \left[ 3(\hat{p} \cdot \hat{k})(\hat{p} \cdot \epsilon^\dagger) - k \cdot \epsilon^\dagger \right],$$

where $p$ and $k$ are the proton and pion c.m. momenta and $C$ is a function of energy. This form does not depend upon the proton spin variables and it is due to this that we have the simplification in eq.(1) which leaves only an $\epsilon_1 \cdot \epsilon_2$ dependence. Keeping only this term, the value of $C$ is fixed by experimental $pp \to d\pi^+$ data in the forward direction, as summarised in the SAID database [8].

Some care must be taken over the effect of the $\alpha$-particle binding energy in determining the energy in the proton-proton system. At the ABC-peak the pions emerge with zero relative momentum and we assume that the laboratory kinetic energy in the $\pi^+d$ system in the inverse reaction is one half of that for the $\pi^+\pi^-\alpha$ when the two pions have equal momentum vectors. With this prescription the interval of deuteron kinetic energies $T_d = 787 - 1412$ MeV is mapped onto a proton range of the $pp \to d\pi^+$ input $T_p = 410 - 720$ MeV.
Since in the linear Fermi momentum approximation only the $\delta$-function survives the integration of eq.(1), we define the form factor

$$W = \frac{1}{\gamma^2 \sqrt{m_\alpha \pi}} \int d^3q_1 d^3q_2 \varphi_d(q'_1) \varphi_d(q'_2) \psi_\alpha(q_\alpha) \delta(q'_1 + q'_2 + (\omega_1 - \omega_2)/2v_d).$$

(4)

This integral is most easily evaluated by transforming into configuration space:

$$W = \frac{2\pi}{\sqrt{m_\alpha \pi}} \int b db d\gamma z_1 d\gamma z_2 \Phi_d(b, \gamma z_1) \Phi_d(-b, \gamma z_2) \Psi_\alpha^\dagger(b, \frac{z_1 - z_2}{2}) \times$$

$$J_0\{|k_1^b - k_2^b|b/2\} \cos\{(k_1^b - k_2^b)(z_1 - z_2)/4\} \cos\{(\omega_1 - \omega_2)(z_1 + z_2)/4v_d\}.$$  (5)

After averaging over the spin directions of the initial deuterons, the matrix element squared becomes

$$\frac{1}{9} \sum |M|^2 = \frac{m_\alpha^2 N_\alpha^2}{9v_d^2} |W|^2 |C|^4 \left\{ 3(\hat{p} \cdot \hat{k}_1)(\hat{p} \cdot \hat{k}_2) + k_1 \cdot k_2 \right\}^2,$$

(6)

where $N_\alpha^2$ is the number of deuteron pairs in the $\alpha$-particle. In the ABC region where $k_1 \approx k_2$, this yields a characteristic $(3 \cos^2 \theta_\alpha + 1)^2$ angular distribution, which is just the square of that for the $pp \rightarrow d\pi^+$ differential cross section in the case of a pure $^1D_2p$ amplitude. This is precisely what one would expect in a classical picture. On the other hand in the central bump the $\alpha$-particle distribution is isotropic and it is the pion distribution with respect to the beam direction which inherits this shape.

The final expression for the differential cross section in the laboratory is

$$\left[ \frac{d^2\sigma}{d\Omega dp_\alpha} \right]_{\text{lab}} = \frac{1}{192(2\pi)^5 m_d M_X p_d E_\alpha} \int d\Omega^* \sum |M^*|^2,$$

(7)

where for quantities denoted by an asterix (*) the pion momenta and angles are evaluated in the $\pi\pi$ rest frame. Though we have only calculated $\pi^+\pi^-$ production, from isospin arguments the corresponding cross section for neutral pions should be half of this. However, because of the narrowness of the
ABC peak, it is important to evaluate the kinematics separately in the two cases.

In the numerical evaluation we use the $S$-state Paris wave function \cite{9} for the deuterons and the $\alpha$:dd cluster wave function of Forest et al. \cite{10}. According to the latter, the $I = 0$, $S = 1$ $S$-state nucleon-nucleon distributions look very similar to those of the deuteron when calculated with the corresponding nucleon-nucleon force provided that the separation $r_{np} < 2$ fm. The scale factor between the two intensities is 4.7. Since pion absorption in the deuteron occurs primarily when the nucleons are close together, we need only the number of such ‘small’ deuterons in the $\alpha$-particle. Thus, assuming that the ‘deuterons’ in the $\alpha$-particle have independent distributions, we normalise the $\alpha$:dd wave function to $N^2_\alpha = 2.3$.

The only beam energy for which the Saclay group measured an angular distribution is $T_d = 1250$ MeV \cite{11} and this corresponds to a region where the $1^D2p$ transition $pp \to d\pi^+$ amplitude is dominant \cite{8}. When comparing with such $\alpha$-particle momentum spectra, it is important to take into account the momentum resolution and this has been done by smearing the theoretical predictions over a resolution function with $\sigma = 10$ MeV/c. After doing this, our predictions are typically a factor of two too high and, for ease of comparison, they have been divided by that in fig. 2. Such a small reduction in overall normalisation could easily be due to initial distortion in the deuteron-deuteron system combined with uncertainty in the $\alpha$-particle normalisation.

The overall agreement between theory and experiment in fig. 3 is remarkably good and suggests strongly that, at least in this channel, the ABC is indeed a kinematic enhancement due to two independent $p$-wave pion productions. At a laboratory angle of $11^0$, the c.m. angle in the ABC peak is about $45^0$ and so the predictions are following the change of about a factor of 2.5 in c.m. differential cross section. The small deviations at the valleys and
the central bump at the larger angles may be due to having limited ourselves to just one amplitude. Away from 1250 MeV data the $^1D_2p$ dominance assumption is less good and, though we reproduce the other Saclay data [1] to better than a factor of two, the details of the momentum spectra must wait until we have included a more complete set of $pp \rightarrow d\pi^+$ amplitudes in the calculation. This is currently in progress.

A fuller set of input amplitudes is also required in order to predict the deuteron analysing powers which have been measured near the forward direction in our energy domain at the SPESIII spectrometer at Saclay [11]. In particular the $\epsilon_1 \cdot \epsilon_2$ form in eq.(1) predicts zero deuteron tensor analysing power whereas experiment yields a small value but with significant structure which might arise through interferences with other $pp \rightarrow d\pi^+$ amplitudes.

Extra confirmation of our approach might be provided by an exclusive measurement of the $dd \rightarrow \alpha\pi^+\pi^-$ reaction, since in the central region we predict that the angular distribution should behave like $(3\cos^2 \theta_{\pi} + 1)^2$.

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Figure 1: Feynman diagram for the $dd \rightarrow \alpha \pi \pi$ reaction showing the momenta in the c.m. system.
Figure 2: The $\alpha$-particle momentum spectra from the $d d \to \alpha X$ reaction for six different laboratory angles at $T_d = 1250$ MeV. The experimental data of ref. [1] are compared with the predictions of our double-pion-production model after division by an overall factor of two. The broken lines are the raw calculations and yield the solid lines after smearing over the experimental momentum resolution.