A Novel Statistical Modeling Approach to Developing IDF Relations in the Context of Climate Change

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Abstract

Extreme rainfall intensity–duration–frequency (IDF) relations have been commonly used for estimating the design storm for the design of various urban water infrastructures. In recent years, climate change has been recognized as having a profound impact on the hydrologic cycle. Hence, the derivation of IDF relations in the context of a changing climate has been recognized as one of the most challenging tasks in current engineering practice. The main challenge is how to establish the linkages between the climate projections given by climate models at the global or regional scales and the observed extreme rainfall at a local site of interest. Therefore, our overall objective is to introduce a new statistical modeling approach to linking global or regional climate predictors to the observed daily and sub-daily rainfall extremes at a given location. Illustrative applications using climate simulations from 21 different global climate models and extreme rainfall data available from rain gauge networks located across Canada are presented to indicate the feasibility, accuracy, and robustness of the proposed modeling approach for assessing the climate change impact on IDF relations.

1 Introduction

Extreme rainfalls for short time scales (e.g. from 5 min to 1 h) are commonly used for estimating the design storm for the design of various urban infrastructures such as dams, culverts, and storm sewers. More specifically, the design storm is computed from the intensity–duration–frequency (IDF) relations, which provide extreme rainfall intensities for various durations and return periods at a given site of interest (WMO 2009; CSA 2019). In current engineering practice, the IDF relations are derived based on statistical frequency analyses of annual maximum (AM) rainfalls for different durations where available rainfall records of adequate lengths could be used to estimate the parameters of a selected probability distribution. However, these AM rainfall records for short durations (e.g. <1 d) are often limited or unavailable at the location of interest, while those for the daily scale are widely available. Hence, there exists an urgent need to develop new methods for modeling extreme rainfall processes over a wide range of time scales such that information related to sub-daily extreme rainfalls could be inferred from the daily extreme rainfall events available at a given site.

Furthermore, in recent years, climate change has been recognized as having a profound impact on the hydrologic cycle at different temporal and spatial scales. The temporal scales could vary from a very short time interval of 5 min (for an urban water cycle) to a yearly time scale (for annual water balance computation). The spatial resolutions could be from a few square kilometers (for urban and rural watersheds) to several thousand square kilometers (for large river basins). In particular, the intensity and frequency of extreme precipitation events in most urban regions are likely to be increased in the future (Zhang et al. 2017). Hence, there exists an urgent need to assess the possible impacts of climate change on extreme rainfall processes or on the IDF relations for improving urban infrastructure design in these areas. At present, the main challenge is how to establish the linkages between the daily climate change projections at coarse-grid global or regional scales given by global climate models (GCMs) and regional climate models (RCMs) and the observed extreme rainfalls at finer time scales for a given local site (Willems et al. 2012). If these linkages could be established, then the projected climate change conditions given by GCMs and RCMs could be used to predict the resulting changes of local extreme rainfalls and related runoff characteristics.

Consequently, different downscaling approaches have been proposed for establishing the linkages that are required for various impact studies in urban areas. In general, these downscaling methods can be grouped into two main categories (Nguyen and Nguyen 2008): dynamical downscaling (DD) techniques and statistical downscaling (SD) techniques. The DD methods could provide reasonable descriptions of the climate conditions for large regional scales, but they could not accurately capture observed characteristics of hydrologic processes such as precipitation at a local or station scale; while SD procedures have been found to be able to accurately describe the observed properties.
of various local processes. Hence, SD techniques are quite popular for various types of impact assessment studies, since they could be adapted to the climatic conditions for a specific site based on some established statistical relationships between large scale atmospheric variables (predictors) and local climate variables (predictands). However, since DD and SD methods require different associated skills, it is recommended that the best approach to climate change scenario development for impact and adaptation studies should be based on the combination of these two approaches (Willems et al. 2012).

In view of the above-mentioned issues, the overall objective of the present research is to focus on the development of a novel statistical modeling method for describing the linkage between GCM climate change projections to the observed daily and sub-daily extreme rainfall series at a given location over an urban area. Illustrative applications using climate simulations from different GCMs, and extreme rainfall data available from rain gauge networks located across Canada are presented to indicate the feasibility, accuracy, and robustness of the proposed statistical modeling approach for assessing the climate change impacts on the estimation of extreme design rainfalls and IDF relations.

2 Statistical modeling of extreme rainfall processes in climate change context

The proposed statistical modeling approach consists of two steps. The first step is to develop a temporal scale invariance (or scaling) probability model for describing the distribution of extreme rainfalls over a wide range of time scales (e.g., from several minutes to 1 day) for a given local site; and the second step is to develop the spatial downscaling procedure for linking the global or regional-scale daily climate projections given by GCMs or RCMs to the daily extreme rainfalls at the location of interest. Detailed descriptions of these two steps are provided in the following sections.

2.1 The temporal scaling generalized extreme value model

A mathematical framework for the temporal scaling generalized extreme value (GEV) model is proposed for estimating the probability distributions of sub-daily extreme rainfalls at a given location from the available daily rainfall measurements using the scaling concept. This scaling concept has increasingly become a promising method for the modeling of various hydrological processes across a wide range of time scales (Gupta and Waymire 1990). In particular, some pioneering works in the application of the scaling method for deriving short duration from longer duration AM rainfall series and for constructing IDF relations have been carried out over the last decade. For instance, Nguyen et al. (1998) proposed a mathematical framework for estimating the distribution of extreme rainfalls for different time scales based on the scaling properties of the empirical noncentral statistical moments (NCMs) and the three-parameter generalized extreme value (GEV) distribution (referred to as the GEV/NCM model).

Menabde et al. (1999) have used the NCM based scaling Gumbel distribution (GUM/NCM), a special case of GEV/NCM, for deriving the maximum rainfall IDF relations. The proposed GEV/NCM model has been also extended to include regional rainfall information for estimating the distribution of extreme rainfalls at locations without data (Nguyen et al. 2002).

Furthermore, it has been demonstrated that, compared to ordinary statistical moments, probability weighted moments (PWMs) and the linear combination forms (L-moments) were found to be more robust, especially against outliers when applied to small data samples; a common situation in the modeling of extreme hydrologic processes (Greenwood et al. 1979; Hosking and Wallis 1997). However, the use of PWMs for assessing the scaling properties of rainfall processes has been found in the scientific literature to be quite limited because of the difficulty in finding analytical solutions for describing the scaling behaviour of some theoretical probability distributions. Therefore, the present study proposes a new mathematical framework for modeling extreme rainfall processes over a wide range of time scales based on the GEV distribution and the scaling behaviour of the PWMs (hereafter referred to as the GEV/PWM model).

The GEV distribution has been widely used for describing the probability distributions of annual rainfall maxima and has been recommended in several technical guidelines for hydrological practices in the world (WMO 2009; ARR 2016). The cumulative distribution function (CDF) of the GEV distribution is given as follows:

\[ F(x) = \exp \left( -\left( 1 - \frac{(x - \xi)}{\alpha} \right)^{\frac{1}{\kappa}} \right); \quad (\kappa \neq 0) \]  

where:

- \( F(x) \) = CDF of \( x \),
- \( \xi \) = location parameter,
- \( \alpha \) = scale parameter, and
- \( \kappa \) = shape parameter.

The quantile \( X_T \) for a given return period \( T \) can be obtained using the following expression:

\[ X_T = \xi + \frac{\alpha}{\kappa} \left( 1 - \left( -\ln(F(x)) \right)^{\frac{1}{\kappa}} \right) \]  

In particular, when the shape parameter \( \kappa = 0 \), the GEV distribution becomes the two-parameter Gumbel distribution with the CDF and the quantile function as follows:

\[ F(x) = \exp \left( -\exp \left( -\frac{x - \xi}{\alpha} \right) \right); \quad (\kappa = 0) \]  

\[ X_T = \xi - \alpha \cdot \ln(-\ln F) \] 

The \( r \)th order NCMs \( \mu_r \) of the GEV distribution are given by (Nguyen et al. 2002):
where:

\[\Gamma()\] = the gamma function.

For a simple scaling process, it can be shown that the NCMs of two different time scales \(t\) and \(\lambda t\) are related as follows (Nguyen et al. 2002):

\[
\mu_t(\lambda t) = \lambda^n \mu_t(t) = \lambda^n \mu_t(t)
\]

where:

\[\eta_r = r \eta,\]
\[\eta_1 = \eta\] is the scaling exponent of the NCM with order, \(r = 1\) (i.e. the mean), and
\[\lambda = \text{the scaling ratio.}\]

On the basis of the scaling properties of the NCMs of annual maximum rainfalls, it is possible to derive the GEV/NCM model parameters and the quantiles of extreme rainfalls for sub-daily scales from those for the daily scale based on the following mathematical relations:

\[\alpha(\lambda t) = \lambda^n \alpha(t)\]
\[\xi(\lambda t) = \lambda^n \xi(t)\]
\[\kappa(\lambda t) = \kappa(t)\]
\[X_r(\lambda t) = \lambda^n X_r(t)\]

where:

\[x_i = \text{observed values (} I = 1, 2, ..., n\text{) for probability } p_i,\]
\[y_i = \text{estimated values (} I = 1, 2, ..., n\text{) for probability } p_i\]
\[n = \text{sample length},\]
\[x = \text{average of observed quantiles},\]
\[y = \text{average of estimated quantiles},\]
\[m = \text{number of model parameters}.\]

The Gumbel distribution is a special case of the GEV distribution when the shape parameter \(\kappa = 0\). In this case, the scaling relationships between the scale and location parameters of the GEV distributions and the quantile estimate for the sub-daily and daily AM rainfalls as given by Equations 7, 8, and 10 continue to hold true. Thus these relationships represent the special features of the NCM based scaling Gumbel distribution (GUM/NCM) model as proposed by Menabde et al. (1999) and the PWM based scaling Gumbel (GUM/PWM) model as suggested by Yu et al. (2004). Hence, the GUM/NCM and GUM/PWM models are special cases of the more general scaling GEV modeling framework proposed in the present study.

In this study, 6 common goodness-of-fit (GOF) criteria were selected for assessing the feasibility and accuracy of the scaling models for estimating extreme rainfalls at a given site. These criteria are the following 6 numerical indices:

1. the root mean square error (RMSE),
2. the root mean square relative error (RMSEr),
3. the mean absolute deviation (MAD),
4. the mean absolute relative deviation (MADr),
5. the adjusted coefficient of determination \(R^2_{adj}\), and
6. the correlation coefficient (CC).

\[RMSE = \left[\frac{1}{n-m} \sum (x_i - y_i)^2 \right]^{1/2}\]
\[RMSEr = \left[\frac{1}{n-m} \sum \left(\frac{x_i - y_i}{x_i} \right)^2 \right]^{1/2}\]
\[MAD = \frac{1}{n-m} \sum |x_i - y_i|\]
\[MADr = \frac{1}{n-m} \sum \left|\frac{x_i - y_i}{x_i} \right|\]
\[R^2_{adj} = 1 - \frac{(n-1)}{(n-m-1)} \times (1-R^2)\]
\[CC = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\left[\sum (x_i - \bar{x})^2\right]^{1/2} \left[\sum (y_i - \bar{y})^2\right]^{1/2}} = R\]

where:

\(x_i = \text{observed values (} I = 1, 2, ..., n\text{) for probability } p_i,\)
\(y_i = \text{estimated values (} I = 1, 2, ..., n\text{) for probability } p_i,\)
\(n = \text{sample length},\)
\(\bar{x} = \text{average of observed quantiles},\)
\(\bar{y} = \text{average of estimated quantiles},\)
\(m = \text{number of model parameters}.\)

By default \(m = 0\) when comparing observed and estimated values. In particular, for performing a fair comparison of the performance of the four scaling models (GEV/PWM, GEV/NCM, GUM/PWM and GUM/NCM) in the estimation of the distributions of sub-daily extreme rainfalls from those of daily extreme
rainfalls, the number of model parameters \( m \) must be taken into account; the value \( m = 3 \) was used for the GEV/PWM and GEV/NCM models, and the value \( m = 2 \) was used for the GUM/PWM and GUM/NCM models. After computing the 6 GOF indices, a ranking scheme was used to rank all the selected distributions. Ranking scores are assigned to each model according to the value computed for each criterion. A distribution with the lowest RMSE, RMSEr, MAD, MADr, MAE, or highest \( R^2_{adj} \) is given the rank of 1 for the corresponding assessment category. In the case of a tie, equal ranks are given to those corresponding models. Furthermore, for each numerical criterion, the overall rank associated with each distribution is computed by summing the individual ranks.

### 2.2 The spatial downscaling of daily extreme rainfalls

The proposed spatial downscaling procedure consists of deriving the distribution of daily extreme rainfalls at a given location from the distribution of daily extreme rainfalls at the regional scale. In the present study, two approaches were employed for describing the relationship between the regional extreme rainfalls \( \hat{X} \) and the at-site extreme rainfalls \( X \). The first method is based on the use of the scaling factor to describe the linkage between the regional and at-site daily extreme rainfalls as shown by Equation 19. This scaling factor represents the correction of the difference between the mean values of regional and at-site daily extreme rainfalls:

\[
X_i(F) = \eta_i \cdot \hat{X}(F)
\]  

(19)

where:

- \( X_i(F) \) = adjusted daily extreme rainfall series at the local site of interest,
- \( \hat{X}(F) \) = daily regional extreme rainfall series at the grid containing that site,
- \( F \) = cumulative probability of interest,
- \( \eta_i \) = scaling factor at site \( i \), with \( \eta_i = \mu_i / \hat{\mu} \),
- \( \mu_i \) = computed sample mean of the daily extreme rainfalls at the local site of interest, and
- \( \hat{\mu} \) = computed sample mean of the daily extreme rainfalls at the grid containing the local site of interest.

The second method utilizes a bias correction function \( e(F) \) to correct the differences between the empirical cumulative distribution functions (ECDF) of regional and at-site daily extreme rainfalls as defined by Equation 20. This bias correction function could be described as a second order polynomial model as given by Equation 21 (Nguyen et al. 2010):

\[
X_i(F) = \hat{X}(F) + e(F)
\]  

(20)

\[
e(F) = c_0 + c_1 \cdot \hat{X}(F) + c_2 \cdot \left( \hat{X}(F) \right)^2 + \epsilon
\]  

(21)

where:

- \( e(F) \) = bias correction function associated with,
- \( c_0, c_1, c_2 \) = constant coefficients of this function, and
- \( \epsilon \) = resulting error term.

In summary, the proposed spatial downscaling procedure can be used for linking regional scale daily extreme rainfalls to the local daily extreme rainfalls at a given site. After this step, the computed at-site daily extreme rainfalls can be used to derive the statistical properties of sub-daily extreme rainfalls at the same location using the proposed temporal scaling GEV model.

### 3 Numerical application

To test the feasibility of the proposed statistical modeling method, an illustrative application is presented using the long records (\( \geq 40 \) y) of annual maximum (AM) rainfall data available from a network of 74 stations located in different regions that represent the diverse climate conditions of Canada. The selected AM rainfall series consist of nine different durations ranging from 5 min to 1 d (\( D = 5 \) min, 10 min, 15 min, 30 min, 60 min, 120 min, 360 min, 720 min, and 1440 min). In addition, the climate simulation outputs from 21 global climate models (GCMs) conducted under the Coupled Model Intercomparison Project Phase 5 (CMIP5) (Taylor et al. 2012) were used. The climate simulation outputs have been statistically downscaled by NASA from the global scale to the regional scale (approximately 25 km x 25 km) for two different IPCC scenarios, RCP 4.5 and RCP 8.5 (Thrasher et al. 2012). Each of the precipitation projections contains data for the periods 1950–2005 and 2006–2100. In this study, data for 1961–1990 were used for the calibration process, while those for 1991–2005 were used for validation.

The first step is to perform a comparative study to assess the performance of the proposed temporal scaling GEV and Gumbel models as described in the previous section. A detailed analysis of the scaling properties of the selected AM rainfall series was carried out based on both PWSs and NCs. The scaling behaviour of these extreme rainfall processes for different time scales was investigated through examining the log–log plot of the relationships between the computed empirical statistical moments of the extreme rainfall amounts and the rainfall durations. If this log–log plot displays a straight line and does not indicate any breaking point, one can conclude that only one scaling regime exists for these extreme rainfall processes over all selected time scales. On the other hand, if the plot shows a breaking point at a specific duration that divides the rainfall durations into two distinct ranges, then one could identify two different scaling regimes of extreme rainfalls for these two ranges.

For purposes of illustration, Figure 1 shows the log–log plots of the relationships between the computed empirical rainfall NCs and PWSs and the rainfall durations for the Montreal Pierre Elliott Trudeau International Airport station. It can be seen that the AM rainfall series at this station shows a breaking point at 30 min duration indicating two distinct simple scaling behaviours for two different ranges of rainfall duration: the first scaling regime is for the range from 5 min to 30 min duration; and the second scaling regime for the range from 30 min duration to 1 d
duration. In general, the identification of the scaling behaviour of AM rainfalls based on both NCMs and PWMs for all 74 stations shows highly comparable results for a larger number (>96%) of stations, especially for those with long rainfall records. However, the scaling analysis results have indicated that the estimates of the empirical scaling exponents based on the PWMs (as shown in Figure 2) are more accurate and more robust than those given by the NCMs for different rainfall scaling regimes and for higher-order rainfall statistical moments. For instance, $R^2$ between the empirical and regression estimated scaling exponents of the second order PWM for two distinct scaling regimes was 0.982 and 0.874, while those values given by the third order NCM were only 0.955 and 0.812. In summary, it was found that the AM rainfalls for durations ranging from 5 min to 1d displayed two distinct scaling regimes with the breaking point duration that varies according to the diverse climate regions of Canada.

Figure 1 Log–log plot of (a) the first five NCMs and (b) PWMs over different rainfall durations (D = 5 min to 1440 min) for Montreal Pierre Elliott Trudeau International Airport station.

Figure 2 Plot of the empirical and regression-estimated scaling exponents based on NCMs and PWMs.

Figure 3 shows the computed values of the GOF numerical indices and the ranking of the performance of the four scaling models GEV/NCM, GUM/NCM, GEV/PWM, and GUM/PWM for all 74 stations considered in this study. For each station, the four scaling models are ranked from the best (rank = 1, darkest color with a diamond) to the worst (rank = 4, white). The total score for a model for each rank for all stations is computed by summing the individual ranks assigned to each station. Equal ranks are used for ties. The total number of first scores obtained by each model for all stations are displayed as bar graphs at the bottom. In general, it was found that all four scale-invariance models performed well. For instance, as can be seen from the boxplots to the whisker extents, the worst model produced values of <18% for RMSEr and <12% for MADr; errors of <4.5 mm for RMSE and <2.5 mm for MAD; and the high values of ≥0.95 for $R^2_{adj}$. In addition, as compared to the medians, the worst model produced values of <9% for RMSEr and <6% for MADr; errors <2.5 mm for RMSE and <1.5 mm for MAD; and the high values of ≥0.98 for $R^2_{adj}$.

Based on these boxplots, especially boxplots of the dimensionless indices such as RMSEr, MADr, and $R^2_{adj}$ it can be seen that the GEV/PWM model performs best for all these criteria with the smallest box widths, the lowest medians, and the shortest upper whisker extents for RMSEr and MADr, and the highest median and the shortest lower whisker for $R^2_{adj}$. Furthermore, to identify the best overall model for all criteria and for all 74 selected stations, the overall rank (or the total score) for each model was computed by summing all the individual ranks computed for each numerical performance index and for each station. In general, as shown in Figure 3, GEV/PWM is the best model among the four considered models with the lowest computed total scores for all 6 numerical performance indices (122 for RMSE, 110 for RMSEr, 102 for MAD, 99 for MADr, 150 for MAE, and 110 for $R^2_{adj}$). In addition, GEV/PWM is the best model for the majority of considered stations as indicated by the highest numbers of first scores achieved for all 6 criteria (49 for RMSE, 51 for RMSEr, 55 for MAD, 56 for MADr, 38 for MAE, and 56 for $R^2_{adj}$). GEV/NCM is the second-best model that could provide comparable results, especially for the MAE criterion.
with a similar boxplot and a close total score of 155, as compared to the score of 150 for GEV/PWM.

The second step is to assess the feasibility and accuracy of the proposed spatial downscaling procedure for linking regional and at-site daily extreme rainfalls. As mentioned above, for calibration purposes, historical data for 1961–1990 were used to estimate the scaling factor for matching the mean values of daily regional and at-site daily extreme rainfalls as shown in Equation 19 (referred to hereafter as the MEAN method), and to derive the bias correction functions for matching the whole empirical cumulative distribution function (referred to hereafter as the ECDF method) as shown in Equation 20 and data for 1991–2005 were used to validate these estimations.

For purposes of illustration, Figure 4 shows the probability plots of results before and after bias corrections for the London CS station using these two bias correction methods. As expected, the ECDF method shows a much better fit than the MEAN method for the calibration period. However, for the validation period, these two methods provide a somewhat comparable performance based on a graphical assessment. As shown in Figure 5, a more objective comparison can be seen for an arbitrarily selected group of 7 stations in Ontario, using 5 numerical goodness-of-fit (GOF) criteria: root mean square error (RMSE), root mean square relative error (RMSER), mean absolute deviation (MAD), mean absolute relative deviation (MADr), and correlation coefficient (CC). Note that the median values of the results from 21 GCMs were used for this comparison. It can be seen that the MEAN method can provide comparable or slightly better results than the ECDF method for all these criteria.

Figure 4 Comparison of results before and after bias correction (BC) for calibration (1961–1990) and validation (1991–2005) periods using the mean correction (MEAN) and the bias correction function (ECDF) for London CS station.

After obtaining the daily extreme rainfalls for a given site from the regional values, the proposed spatial downscaling procedure can be used to derive the distributions of sub-daily extreme rainfalls. In this study, the data for 1961–1990 and 1991–2005 at each site were used for the calibration and validation of this scaling GEV/PWM model. For purposes of illustration, Figure 6 shows the probability plots of the computed extreme rainfalls \( X_t \) (mm) for different durations for London CS station for the 1961–1990 calibration period. The yellow markers show the empirical CDF using observed data from 1961–1990. The red discontinuous lines and cross markers show the theoretical CDF based on at-site frequency analysis. The gray lines and boxplots show the estimated CDF derived using the scaling GEV/PWM model and all 21 GCM outputs. Uncertainty associated with the estimation of the extreme rainfalls is displayed in the form of standard boxplots. It can be seen that the distributions of the estimated sub-daily extreme rainfalls derived from the distribution of daily extreme rainfalls using the scale-invariance GEV/PWM model did agree very well with the observed data.

Figure 6 Empirical and estimated cumulative distribution function plots of extreme rainfalls \( X_t \) (mm) for different durations for London CS station.
Figure 7 shows the Q–Q plots of the estimated extreme rainfalls given by the scale-invariance GEV/PWM model and the at-site frequency analysis using the GEV distribution for different rainfall durations and return periods for the 7 selected stations in Ontario. Note that the median values of the results from 21 GCMs were used for the computation. In addition, a numerical comparison was conducted to evaluate the results using the 3 selected GOF indices (RMSEr, MADr, and CC) for all sites as shown in Table 1 for both calibration and validation periods. The low values of RMSEr and MADr as well as the high values of CC indicate the feasibility and accuracy of the proposed temporal GEV/PWM statistical downscaling in the estimation of the extreme design rainfalls for a given ungauged location.

![Q–Q plots](image)

Figure 7  Q–Q plots of the extreme rainfalls computed using the proposed statistical downscaling procedure ($X_{\text{STSD}}$, mm) and the at-site frequency analysis ($X_{\text{at-site}}$, mm) for different rainfall durations (from $D = 5$ min to $D = 1440$ min) and for different return periods ($T = 2$ y to $T = 100$ y) for the 1961–1990 calibration period.

Table 1  Goodness-of-fit test results for both calibration and validation periods.

|          | Calibration period 1961–1990 | Validation period 1991–2005 |
|----------|-------------------------------|-------------------------------|
| T (y)    | 2    | 5    | 10   | 25   | 50   | 2    | 5    | 10   | 25   |
| RMSE (%) | 10.3 | 10.3 | 11.4 | 13.9 | 16.3 | 15.9 | 15.2 | 16.5 | 20.2 |
| MAD (%)  | 8.0  | 8.1  | 9.3  | 11.8 | 13.8 | 13.4 | 12.5 | 12.8 | 15.9 |
| CC (dmn) | 0.965 | 0.958 | 0.950 | 0.931 | 0.910 | 0.929 | 0.903 | 0.885 | 0.866 |

In summary, it can be seen that the proposed statistical modeling approach based on the temporal scaling of GEV/PWM and the spatial downscaling procedure can provide an accurate estimation of extreme design rainfalls for a given location of interest in the context of climate change.

4 Conclusions

The present study has introduced a novel statistical modeling approach to the estimation of extreme design rainfalls for a location of interest in the climate change context. The proposed approach consists of two steps. The first step is based on the temporal scale-invariance GEV/PWM probability model to represent the distributions of extreme rainfalls over a wide range of time scales (from several minutes to 1 d) for a given site, and the second step relies on a spatial downscaling procedure to describe the linkages between regional daily extreme rainfall projections given by climate models and the observed at-site daily extreme rainfalls. Results from an illustrative case study using observed annual maximum (AM) rainfall data available at different locations across Canada and climate simulation outputs from 21 different GCMs have indicated the feasibility, accuracy, and robustness of the statistical modeling approach proposed in this study.

More specifically, in the first step, a detailed analysis of the temporal scaling properties based on both PWMs and NCMs of the long records (≥40 y) of AM rainfalls available from a network of 74 stations across Canada was carried out. In general, it was found that the AM rainfalls for durations ranging from 5 min to 1 d displayed two distinct scaling regimes with the breaking point duration that varies according to the diverse climate regions of Canada. In addition, the scaling analysis results have indicated that the estimates of the empirical scaling exponents based on the PWMs are more accurate and more robust than those given by the NCMs for different rainfall scaling regimes and for higher order rainfall statistical moments. Finally, on the basis of different graphical and numerical performance criteria, the proposed GEV/PWM model was found to be the best overall model as compared to the existing GEV/NCM, GUM/NCM and GUM/PWM models.

Furthermore, in the spatial downscaling step, two methods were proposed for describing the linkages between the daily extreme rainfalls at a given local site and the downscaled regional values at the 25 km scale. The MEAN method used a scaling factor to correct the mean values of the regional and at-site daily extreme rainfall amounts, and the ECDF method utilized a bias correction function to correct the differences between the empirical cumulative distribution functions of these two daily rainfall amounts. In general, based on the results of an illustrative application using daily AM rainfall data available at different locations in Canada and the daily downscaled 25 km regional values from the outputs of 21 different GCMs, it was found that the MEAN method can provide a better performance than the ECDF method.

In summary, the proposed statistical modeling approach can be used to estimate extreme design rainfalls for different time scales and to construct IDF relations for the current climate, as well as for future climate, under different climate change scenarios.

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