Impact of hadronic interactions and conservation laws on cumulants of conserved charges in a dynamical model

Jan Hammelmann and Hannah Elfner

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Introduction

Explore the phase diagram of QCD by using heavy-ion collisions

Measure cumulants of conserved charges
Introduction

Signs of global charge conservation effects in measurements of heavy-ion collisions

Net protons are used as a proxy of the net baryon number fluctuations

Perform dynamical simulations to model the background signal
Model

Simulating Many Accelerated Strongly-interacting Hadrons (SMASH)
https://smash-transport.github.io/

Uses a geometric collision criterion

$$\pi d_{\perp}^2 < \sigma_{\text{tot}}$$

Incorporates particles with masses up to $\sim 2\text{ GeV}$

Types of processes are:

- $2 \leftrightarrow 1$ resonance formation / decay
- $2 \leftrightarrow 2$ elastic / inelastic interactions
- String excitation (not used in this work)
Model

Degrees of freedom

| N  | Δ   | Λ   | Σ   | Ξ   | Ω   | Unflavored | Strange |
|----|-----|-----|-----|-----|-----|------------|---------|
| N938 | Δ1232 | Λ1116 | Σ1189 | Ξ1321 | Ω1672 | π138 | f0 980 |
| N1440 | Δ1620 | Λ1405 | Σ1385 | Ξ1530 | Ω2250 | π1300 | f0 1370 |
| N1520 | Δ1700 | Λ1520 | Σ1660 | Ξ1690 |       | π1800 | f0 1500 |
| N1535 | Δ1905 | Λ1600 | Σ1670 | Ξ1820 |       | f0 1710 | f2 2010 |
| N1650 | Δ1910 | Λ1670 | Σ1750 | Ξ1950 |       | η548 | f2 2300 |
| N1675 | Δ1920 | Λ1690 | Σ1775 | Ξ2030 |       | η958 | a0 980 |
| N1680 | Δ1930 | Λ1800 | Σ1915 |       |       | η11295 | a0 1450 |
| N1700 | Δ1950 | Λ1810 | Σ1940 |       |       | η1405 | f1 1285 |
| N1710 | Δ1820 | Σ2030 |       |       |       | η1475 | f1 1420 |
| N1720 | Δ1830 | Σ2250 |       |       |       | η1880 | a1 1170 |
| N1875 | Δ1890 |       |       |       |       | σ800 | h1 1170 |
| N1900 | Δ2100 |       |       |       |       |      |  |
| N1990 | Δ2110 |       |       |       |       | ρ776 | η1 1400 |
| N2080 | Δ2350 |       |       |       |       | ρ1450 | b1 1235 |
| N2190 | Δ1220 |       |       |       |       | ρ1700 | a1 1260 |
| N2220 |       |       |       |       |       |      |  |
| N2250 |       |       |       |       |       | ω783 | η2 1645 |

From A. Schaefer, see [https://smash-transport.github.io/](https://smash-transport.github.io/)

https://theory.gsi.de/~smash/analysis_suite/SMASH-2.2/
Methodology

Initialization
Initialize a box event-by-event with the same number of particles with momenta according to the Boltzmann distribution.

Box
Time: 0 fm
Box Width: 10 fm
Temperature: 0.15 GeV

Calculate cumulants in this sub volume.

https://smash-transport.github.io/
Methodology

Infinite matter simulation

Calculate cumulants of particle distributions as a function of the size of the subvolume

\[ x = \frac{V}{V_0} \]

\[ \omega = \frac{C_2}{C_1} = \frac{\langle (\delta N)^2 \rangle}{\langle N \rangle} \]

\[ S\sigma = \frac{C_3}{C_2} = \frac{\langle (\delta N)^3 \rangle}{\langle (\delta N)^2 \rangle} \]

\[ \kappa\sigma^2 = \frac{C_4}{C_2} = \frac{\langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2}{\langle (\delta N)^2 \rangle} \]

Additionally study the effects of cuts on transverse momentum \(0.4 < p_T < 2 \text{ GeV}\)
Methodology

Analytic Comparison
When the net charge number is conserved and the total charge number can fluctuate

\[ P(n_+, n_-; x) = \sum_{N_+, N_-} P(N_+, N_-) B(n_+, n_-; x) B(N_-, n_-; x) \]

Binomial distribution

\[ P(N_+, N_-) = \delta(N_+ - N_- - Q) P(N_{ch}) \]

Net charge conservation  Fluctuation of the total charge number

Analytic expressions for cumulants as function of \( x \)

\[ \omega = 1 - x \]
\[ S\sigma = \frac{Q}{\langle N_{ch} \rangle} (1 - 2x) \]
\[ \kappa\sigma^2 = 1 + 3x(1 - x)(\omega[N_{ch}] - 2) \]

For more details see *Phys.Rev.C* 101 (2020) 2, 024917
Thermal and chemical equilibrium as well as isotropic densities are a pre-requisite to perform these kind of calculations.

Temperature and baryon chemical potential are calculated assuming

\[ \frac{dN}{dp} \sim \exp\left( - \left( \sqrt{p^2 + m^2} - \mu_B \right) / T \right) \]

The density is distributed isotropically in the system.
Simple $\pi \rho$ hadron gas interacting via an energy dependent cross section $\pi^\pm \pi^\mp \leftrightarrow \rho^0$

Cumulants follow lines of perfect conservation

Correlations are present within a cut in momentum space

$\kappa \sigma^2$ dependency on the charge density $\rightarrow$ total charge number fluctuation

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Test System

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**Full Hadron Gas**

Full set of hadrons from SMASH with all possible interactions

Perform final decays after the dynamical evolution

Possibility to extract $C_2[N_{ch}]$ for a realistic interacting hadron gas

Effects of conservation still present within the net-proton numbers
In the limit $x \to 0$, $\omega / k \sigma^2 \to 1$ (Poisson Limit)

For small volumes and large baryon chemical potentials, the cumulants don’t strictly follow the analytic expressions
Mapping between protons and baryons

$\delta B$ fluctuations are of interest, however only $\delta P$ fluctuations are accessible

Is it possible to map fluctuations $\delta N_p \rightarrow \delta N_B$?

Mapping between proton and baryon number fluctuations $\delta P \leftrightarrow \delta B$

Kitazawa and Asakawa Phys.Rev.C 85 (2012) 021901

Based on binomial unfolding procedure
input: Probability $p = \langle N_p \rangle / \langle N_B \rangle$
Mapping between protons and baryons

Dynamical correlations within the model limits the applicability in large acceptance regions

Dependency on the complexity of the system
Impact of deuteron formation

Build a relatively easy hadron gas such that the impact of deuterons can be extracted

$\pi \rho h_1(1170) \Delta N d d'$

Deuteron formation on the basis of a fictional resonance $d'$

$pn \leftrightarrow d' \quad \pi d' \leftrightarrow \pi d \quad Nd' \leftrightarrow Nd$

Run system with and without deuterons and compare the fluctuations

Deuterons are almost not affected by conservation effects
Conclusion

Summary

Charge annihilation processes affect the kurtosis at large densities

At large baryon chemical potentials the scaled variance is modified in small volumes

Proton number fluctuations cannot be recovered at large acceptance regions from baryon number fluctuations due to dynamical correlations

Deuteron number has no large impact on conservation effect

Outlook

Critical initial conditions in an expanding sphere

Impact of baryon annihilation and rescattering in more realistic scenarios
Backup
Mapping between protons and baryons

Dependence on the complexity of the system
Fluctuations of the total charge number $C_2[N_{ch}]$ becomes important at large densities

More charge annihilation processes ($C_4$ depends on $C_2[N_{ch}]$)

Fluctuations can be described by analytic formulas with the input of $\langle N_\pm \rangle$ and $C_2[N_{ch}]$