Method of determining minimum area of residential development at different angles of rotation of buildings with allowance for insulation time length

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Abstract. The paper proposes an analytical method for constructing the shadows of buildings and structures with the aim of using them in residential development planning. This method based on generalized coordinates and small movement synthesis of residential facilities allows finding the minimum construction area that meets the requirements of insulation regime. With this method the position of shadows relative to buildings is investigated at all iterations.

1. Introduction
When planning the location of residential facilities in a definite area, it is necessary to take into account their mutual position of shadows. This is important for the assessment of the insolation regime of development areas and premises. Therefore, the estimated position of residential buildings must meet the specified standards for the insolation time length of the development area. The insolation index affects many areas of our live, ranging from comfortable accommodation to power engineering. Regulatory requirements in determining insolation, set out in SanR&S [1], are applied to objects at a design stage, reconstructed buildings, as well as erected residential accommodations and public buildings. In works [5, 6], geometric methods for determining the contours of shadows and buildings, and in general insolation of a given plot of the development area are represented. The use of these methods is based on the study of the specified area by grid nodes of a given discreteness. Insolation areas are investigated when the positions of buildings are predetermined.
In work [4] the technique of building positioning on the basis of generalized coordinates and their small movements synthesis is proposed. With this method, the position of shadows relative to buildings is investigated at each iteration. Calculation of insolation of the whole development area according to the method described in [5, 6] at each iteration in the movement synthesis of buildings in this case is not required. Therefore, the most effective way for these calculations is to assign the contour of the shadows by the set of nodal points for certain angles of building orientation relative to the East-West direction and the sunlight inclination angle. In this paper, an automated method for determining the contour shadows of an L-shaped or a rectangular-shaped buildings for arbitrary values of these angles is proposed.

2. Method of constructing the nodal points positions of shadow envelopes
As an example of hourly shadows contours imaging for different angles of sunlight rotation $\alpha$ for the L-shaped building is presented in figure 1. The insolation time length of a certain fragment of the $\Theta$ i-th building during a given time interval with a certain approximation will set the number of intersections of this area segment (these areas in figure 1 are indicated $\Delta_i$ and $\Sigma_j$, etc.) with a convex area $\Delta_i$ and $\Sigma_j$ hourly shadows during daylight hours, where the parameters $i$ and $j$ respectively specify the number of the building and the hourly shadow number. In figure 1, midday sunlight...
located in the plane $L$, inclined at an angle of $35^\circ$ to the horizontal plane, corresponds to the day of the autumnal and vernal equinox. Each individual region $\Delta_{ij}$ and $\Sigma_{ij}$ can be assigned by the intersection of regions $\lambda_{ij}$ (half-planes) defined by straight lines. Index $k$ indicates the time determining the position of the hourly shadow. The method of analytical setting of these convex areas of hourly shadows is described in [4].

The analysis of the shadow construction shows that with a change of a sunlight inclination angle, the coordinate $y$ of the nodal points $F_{ki}$ and $M_{ki}$ changes insignificantly (Figure 1). The $ki$ parameter specifies the number of the shadow node point. For the considered case $ki = 1 \div 5$. For automated calculation of shadow envelope positions, let us determine the coordinate change functions $x$ of the nodal points $F_1 \div F_5$ and $M_1 \div M_5$ of the regions $\Delta_{ij}$ and $\Sigma_{ij}$, from the angles $\alpha$ and $\beta$ (Figures 1, 2). The angle $\beta$ defines the building inclination in relation to the East-West direction (Figure 2). In figure 1, the points $F_3^{30}$, $F_3^{60}$, $F_3^{90}$ indicate the position of the point $F_3$ defining the shadow contour corresponding to eight hours and three different values of the angle $\beta$, namely, 30º, 60º, 90º. The specified points $F_i$ and $M_i$ determine in general the positions of two convex pentagons of shadows.

**Figure 1.** Images of the L-shaped building shadows envelope corresponding to 8, 10, 12, 14 and 16 hours

Figure 2 shows the shadow envelopes of the rectangular building with its different rotation relative to the East-West direction. As can be seen from the figure, the angle $\beta_i$ affects the position of the nodal points $F_{ki}$ of the shadow contours.

Let us solve the problem with automated computation of shadow envelope nodal points coordinates with arbitrary values of the angles $\alpha$ and $\beta_i$. 
3. Automation of calculation for shadow envelope nodal points coordinates

Table 1 shows the nodal points $F_{ki}$ and $M_{ki}$ of the shadow contours $\Delta_{ki}$ and $\Sigma_{ki}$ of the pentagons for different time points, namely 8, 10, 12, 14, and 16 hours. These points can be used in the automated calculation of the areas $\Delta_{ki}$ and $\Sigma_{ki}$ on the development plan in movement synthesis of buildings to determine the minimum construction area and ensure the prescribed requirements for insolation [8].

As is clear from figure 1 and table 1, at different times the current points $F_{ki}$ and $M_{ki}$ are defined by different points $A, B, \ldots, G$ specifying the horizontal contour of the building. The subscripts 8, 10, etc. of the points in the table and figures 1 and 2 specify the time corresponding to the hourly shadow.

The graphs of figure 3 show the dependencies of the values of the time varying coordinates $x$ (in meters) of points $F_2, F_3$ and $F_4$ for different values of the angles $\alpha$ and $\beta_i$. To determine the coordinate $x$ of the intermediate node points $F_{ki}$ and $M_{ki}$, the most convenient is to use polynomials of the third degree, reflecting the dependencies presented in figure 3. The equations of polynomials of the third degree have the following form [5]:

$$
\begin{align*}
xF_2 &= a_1^{F_2} \alpha^3 + a_2^{F_2} \alpha^2 + a_3^{F_2} \alpha + a_4^{F_2}, \\
xF_3 &= a_1^{F_3} \alpha^3 + a_2^{F_3} \alpha^2 + a_3^{F_3} \alpha + a_4^{F_3}.
\end{align*}
$$

The coefficients $a_1^{F_2}, a_2^{F_2}, \ldots, a_4^{F_4}$ for the definitely specified value of the angle $\beta_i$ are obtained by substitution of the coordinates of the four points $F_{ki}$.

Table 1. Assignment of current nodal points $f_{ki}$ and $m_{ki}$ of the shadow envelope

| Region. $\Delta$ | Time interval (hour) |
|------------------|----------------------|
| $F_1$ $F_2$ $F_3$ $F_4$ $F_5$ | 8 10 12 14 16 |
| $A_1$ $A_8$ $B_8$ $C_8$ $C_1$ | $B_1$ $B_{10}$ $C_1$ $C_{10}$ $C_1$ |
| $B_1$ $B_{12}$ $D_{12}$ $E_{12}$ $E_1$ | $B_{14}$ $D_{14}$ $E_{14}$ $E_{14}$ $E_1$ |
| $D_1$ $D_{16}$ $E_{16}$ $G_{16}$ $G_1$ | $E_{16}$ $G_{16}$ $G_1$ $G_1$ $G_1$ |

| Region. $\Sigma$ | |
|------------------|----------------------|
| $M_1$ $M_2$ $M_3$ $M_4$ $M_5$ | |
| $C_1$ $C_{10}$ $D_{10}$ $E_{10}$ $E_1$ | $C_1$ $C_{12}$ $E_{12}$ $E_{14}$ $E_1$ |
| $C_1$ $C_{12}$ $E_{12}$ $E_{14}$ $E_1$ | $C_1$ $C_{12}$ $E_{14}$ $E_{16}$ $E_1$ |
| $E_1$ $E_{14}$ $G_{14}$ $G_{16}$ $G_1$ | $E_{14}$ $G_{16}$ $G_1$ $G_1$ $G_1$ |
For example, to determine the second equation of the system (1), it is necessary to substitute the values of the coordinate \( x \) of points \( B_8, B_{10}, D_{14} \) and \( E_{16} \) into this equation and solve a system of four equations with four unknowns. Determining the coefficients \( a_1^{F2} + a_4^{F2} \) for four different angles \( \beta_i \), the dependences \( a_1^{F3} = f_{1-4}(\beta_i) \) of these values on the angle \( \beta_i \) can be determined. To find these dependencies, we use other polynomials of the third degree:

\[
\begin{align*}
    a_1^{F3} &= b_{11}^{F3} \beta_i^3 + b_{12}^{F3} \beta_i^2 + b_{13}^{F3} \beta_i + b_{14}^{F3}, \\
    a_2^{F3} &= b_{21}^{F3} \beta_i^3 + b_{22}^{F3} \beta_i^2 + b_{23}^{F3} \beta_i + b_{24}^{F3}, \\
    a_3^{F3} &= b_{31}^{F3} \beta_i^3 + b_{32}^{F3} \beta_i^2 + b_{33}^{F3} \beta_i + b_{34}^{F3}, \\
    a_4^{F3} &= b_{41}^{F3} \beta_i^3 + b_{42}^{F3} \beta_i^2 + b_{43}^{F3} \beta_i + b_{44}^{F3}.
\end{align*}
\]

(2)

Using the polynomials (1) and (2) makes it possible to calculate the coordinate \( x \) of the nodal points \( F_{ki} \) for arbitrarily given angles \( \alpha \) and \( \beta_i \).
Figure 3. Functions graphs: a) $x_{F2}=f_1(\alpha, \beta)$; b) $x_{F3}=f_2(\alpha, \beta)$; c) $x_{F4}=f_3(\alpha, \beta)$

The example of the obtained values of the polynomial coefficients (2) for the nodal point $F_3$ is represented in table 2.

| Points notation | Polynomials coefficients |
|-----------------|--------------------------|
| $F_3$           |                          |
|                 | $b_{11}^{Fki} + b_{14}^{Fki}$ | $b_{21}^{Fki} + b_{24}^{Fki}$ | $b_{31}^{Fki} + b_{34}^{Fki}$ | $b_{41}^{Fki} + b_{44}^{Fki}$ |
| $-0.0026$       | $-0.0022$                | $-0.0019$                   | $0.0003$                      |
| $0.2453$        | $0.2068$                 | $0.1877$                    | $-0.0323$                     |
| $-5.3489$       | $-4.4998$                | $-4.0901$                   | $0.6900$                      |
| $9.7373$        | $8.1898$                 | $7.4453$                    | $-1.2576$                     |

The algorithm for determining the relative position of buildings and shadow envelopes is shown in figure 4. The following notations are taken in figure 4:

1 is data input $i = 1$, $i_{\text{max}}$ is maximum number of placed buildings, $\beta_1 = 0$, $\beta_2 = 0$, ... $\beta_i = 0$; ($\beta_i$ is the angle of rotation of the $i$-th building relative to the direction East–West), $k_1 = 0$, $k_2 = 0$ ... $k_i = 0$, ($k_1$, $k_2$, ..., respectively is the number of intersections of the first, second, etc. fragments of the $i$-th building with envelopes of shadows), $k_{\text{max}}$ (for test case $=2$, $2$, $4$) ($k_{\text{max}}$ is the maximum value of the number of fragments of the $i$-th building), $\alpha = 0$, $k_{\text{max}}$ is the maximum number of the region intersection of the building fragment and shadows prescribed in accordance with the standards of insolation; 2 is the displacement of the buildings on movement minimization criterion in the vector of velocities [4]; 3 is the definition of polynomials coefficients $a_{11}^{F}, a_{21}^{F}, a_{31}^{F}, a_{41}^{F}$, $b_{11}^{F}$, $b_{12}^{F}$, $b_{21}^{F}$, $b_{22}^{F}$ in accordance with the database for each residential structure for the angle values $\alpha$ and $\beta_i$ (database of coefficient values is determined in advance); 4 is the calculation of the coordinates of the current points $F_i$ and $M_i$ in moving and fixed coordinate systems in accordance with the angles $\alpha$ and $\beta_i$ (1) and (2); 5 is the calculation of coefficients for equations of straight lines passing through the points $F_i$ and $M_i$ specifying the half-planes $\lambda$ prescribed by the points $F_k$, $F_{k+1}$, $M_k$ and $M_{k+1}$ of the $i$-th building [4]. Determination of regions $A_{ij}^k$ and $\Sigma_{ij}^k$ based on Boolean operations of the set theory [10]; 6 is the determination of intersection of the $kk$-th fragment for the horizontal projection of the $i$-th building with the regions $A_{ij}^k$ and $\Sigma_{ij}^k$; 7 – there is an intersection; 8 – $k_i = k_i + 1$; 9 – $kk = kk + 1$; 10 – $kk > k_{\text{max}}$; 11 – $i = i + 1$, $kk = 0$; 12 – $i > i_{\text{max}}$, $13 – k_i > k_{\text{max}}$; 14 – $\alpha > 150^\circ$.
Figure 4. Algorithm scheme for determining the insolation time length based on the position analysis of the buildings fragments and contours of the shadows

15 – the position of the \( i \)-th building at the current angle \( \beta_i \) does not meet the standards of insolation time length; \( \beta_i = \beta_i + 5^\circ \); 16 – \( \beta_i > \beta_i^{\max} \); 17 - the position of all buildings satisfies the insolation time length; 18–\( q_{it} = q_{it} + \Delta q_{it} \) (\( q_{it} \) are the values of generalized coordinates); \( \Delta q_{it} \) is the increment of generalized coordinates [4]. The determination of a new position of buildings in which the condition \( \sum_{i=1}^{n} q_{it} < \sum_{i=1}^{n} q_{it-1} \) is investigated (\( n \) is the number of the generalized coordinates) [4]; 22 is the output final values of generalized coordinates \( q_{ii} \) specifying the position of the buildings under which insolation standards are met.

4. Results of calculation of test case with the location of buildings in compliance with insolation requirements

Figure 5 shows the simulation results of the movement of buildings with the fulfillment of the conditions \( \sum q_{it} < \sum q_{it-1} \) and \( \Delta^i \Theta \), \( \Sigma^i \Theta \) at each calculation step. The sum of generalized coordinate values \( q_{it} \) decreases in modeling at each iteration, and hence, the construction area reduces.
In the figure, the symbols $q_1, q_2, ..., q_6$ define the segments that determine the initial values of the generalized coordinates. Rectangles $P_1$, $P_2$ and $P_3$ define the original position of buildings and their fragments. Points $O_1$, $O_2$ and $O_3$ determine the beginning of the moving coordinates associated with buildings. The contour specified by points $D_1$, $D_2$, ..., etc. is determined by linear objects such as roads, power lines, industrial sites, etc. On modeling small displacements of buildings, the generalized coordinates $q_i$ were calculated by the increment vector $V_3 (\Delta x, \Delta y)$ for the third building [4]. The technique of determining the direction of vector $V_3$ is described in [4]. To reduce the values of the generalized coordinates $q_i$, negative values of the weight coefficients of the generalized coordinates increments $a_3$ and $a_5$ were used [4]. The height of the buildings in the test case was taken $h = 50$ m.

5. Summary and Conclusion
Functions specified by polynomials can be used in the automated determination of the building location [4] with the implementation of insolation requirements. The simulation results of small movements synthesis for buildings under construction to ensure the maximum level of illumination using the calculation of the nodal points of the shadow envelopes according to the developed technique allow reducing the calculation time. The method of determining the shadows and location of buildings with the minimum occupation area can be used in planning new build development of various residential buildings.

6. References
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