Stress analysis of composite plate with cutout of various shape

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Abstract. Composite plate found numerous applications in various structures due to its superior properties like high stiffness to weight and strength to weight ratio. Generally, cutouts in these structural elements are unavoidable for many purposes like access to internal parts, ventilation and sometimes to prevent resonance. These geometrical discontinuities in the form of cutouts may be of different shapes and arise nonuniform stress in the plate. Therefore, it becomes necessary to analyse stress concentration factor near cutouts and effect of shape of cutouts on axial deformation and stress distribution for better safety and stability of structure. In the present investigation a Carbon/epoxy composite plate with various shape of cutout subjected to in-plane tensile loading is considered. Finite element based plane stress analysis is used for mathematical modeling. A program for the present analysis is developed in MATLAB. Variation of in-plane displacement and stress concentration factor (SCF) with various shape of cutout is determined. The effect of the geometry of hole on the stress distribution around the hole is also studied. The results of various analyses have been presented with the help of tables, graphs, and nodal plots.

1. Introduction
Composite plates as a structural member finds its application in various fields like automobile, space vehicle, submarines and many others where high strength and stiffness to weight ratio is desired. Sometimes holes and cutouts of various shapes are unavoidable in these type of plates for their proper functioning. These structural members usually subjected to in-plane tensile loading. With geometrical discontinuities these composite plates with holes are more vulnerable due to rises of stress near discontinuities (hole) which results in SCF near hole. Therefore, analysis of stress near holes for better safety and to evaluate load bearing capacity of these composite plates are essential. Many research works are reported for investigation of SCF of composite plates with holes of various shapes.

Investigation for SCF of an isotropic rectangular plate with circular hole subjected to uniform tensile load is done analytically and using Finite element method is done by Babulal et al. [1]. Influence of hole diameter to plate width ratio (D/A) on stress distribution is studied. Approximate solution for normal stress distribution in the form of polynomial is presented by Konish et al. [2] and found good agreement for SCF. Saddique and Mirzana [3] presented a review investigation of stress analysis of composite and isotropic plate with circular hole with finite element simulation using Ansys and reveal the effect of fiber orientation on deformation, Von Mises Stress and SCF. Pan et al. [4] studied stress distribution and their influencing parameters in a finite plate with rectangular hole using Muskhelishvili’s complex variable method and compared the results with those obtained from finite
element simulation. Effect of geometry of elliptical hole and its aspect ratio on stress distribution in a plate is studied by Gunwant and Singh [5] using Ansys and deflection of plate with Von-Mises stress are determined. Mekalke et al. [6] used finite element method to investigate stress and displacement distribution in a rectangular plate with cylindrical hole subjected to uniform stress. Variation of results are studied using various meshes in Ansys. Classical laminated plate theory is used to determine stress distribution around an elliptical hole in a finite composite plate and effect of various parameters like hole size, stacking sequence and loading conditions on SCF are studied by Xiwu et al. [7]. Ogonowski [8] investigated the influence of finite geometry of isotropic and anisotropic plate with hole on stress distributions and compared with the known solution. Alhazmi et al. [9] studied stress distribution in a carbon fiber reinforced composite plate with open and bolt-loaded holes subjected to tensile and fatigue loading experimentally and numerically. SCF of orthotropic composite plates with a circular hole subjected to tensile loading is measured experimentally and effect of various parameter on strain and SCF is revealed by Tan and Kim [10]. Isotropic analysis, orthotropic analysis and finite element analysis is done by Park and kim [11] for comparative study of SCF in fiber reinforced plastic composite plate with central circular hole. Wu and Mu [12] found application of Scale Factors (SFs) to determine SCF of isotropic plate under biaxial loading and cylinder under uniaxial loading for suitable circular hole and compared results with that obtained with finite element simulation.

From the above literature it is observed that displacement and stress distribution of a composite plate with hole of different shapes are studied separately. To the best of author’s knowledge there is no research work has been reported on comparative study of effect of different shapes of hole on displacement and SCF by keeping hole area constant. Therefore, in the present study stress analysis of carbon/epoxy composite plate with circular, square and rectangular hole by keeping cutout area constant subjected to uniaxial tensile loading is considered. Figure 1(a-c) shows the composite plate with hole of rectangular, square and circular shape under in-plane uniaxial tensile loading.

![Figure 1](image)

**2. Mathematical Formulation**

A MATLAB program is developed for plane stress analysis of carbon/epoxy composite plate with hole using four noded quadrilateral element with two degree of freedom. Material properties of composite plate considered for the present study is given in Table 1.
Table 1. Material properties of Carbon/epoxy

| $E_1$ (GPa) | $E_2$ (GPa) | $v_{12} = v_{21}$ | $v_{21}$ | $G_{12}$ (GPa) |
|-------------|-------------|-------------------|---------|---------------|
| 82.64       | 9.8034      | 0.3               | 0.0356  | 3.8545        |

Rectangular plate of 20 cm length, 10 cm width and 0.1 cm thickness with different shape of hole is considered unless otherwise specified for particular results. Uniformly distributed In-plane tensile load 5 KN is applied. The constant area of hole of different shape is 4 cm². Dimensions of holes are given in the Table 2.

Table 2. Dimensions of hole of different shape.

| Shape of hole | Dimensions (in cm) | Area of Hole (in cm²) |
|---------------|--------------------|-----------------------|
| Circular      | radius=1.128       | 4                     |
| Square        | 2x2                | 4                     |
| Rectangular   | 4x1                | 4                     |

2.1. Displacement Field
The displacement field for the present study is given as

$$
U_1 = u(x, y) \\
U_2 = v(x, y)
$$

where $u$ and $v$ are displacements in $x$ and $y$ direction respectively.

2.2. Strain-Displacement Relations
Strain in respective direction can be determined by differentiating displacement field and can be written as,

$$
\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
$$

(2)

2.3 Stress-Strain Relations
Stress in $x$, $y$- direction and in xy-plane can be evaluated using reduced constitutive matrix and Eq.1 as,

$$
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix} = \begin{bmatrix} Q \end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_{xy}
\end{bmatrix}
$$

(3)

Where $\begin{bmatrix} Q \end{bmatrix}$ is reduced constitutive matrix.

Boundary condition applied in the present study is given as,

$u = 0 \quad \text{at} \quad x = 0$

2.4 Finite Element Analysis
Displacement vector can be given as,

$$
U^e = \begin{bmatrix} u_1 \ v_1 \ u_2 \ v_2 \ldots \ u_n \ v_n \end{bmatrix}^T
$$

$$
U_i = \sum_{i=1}^{N} \phi_i^e u_i \quad \text{and} \quad U_i = \sum_{i=1}^{N} \phi_i^e v_i
$$

(4)

(5)
Where, \( N'_i \) is elemental interpolation function.

Strain vector can be given as
\[
\{ \varepsilon \} = [B]\{U'_i\} \tag{6}
\]

Where, \([B]\) is strain displacement matrix.

Elemental stiffness matrix using 2 points Gauss integration method can be written as
\[
[K'_e] = \int_{V} t[B]^T[Q][B]dV \tag{7}
\]

where, \( t \) = thickness of plate.

Nodal force vector in the absence of traction forces can be written as,
\[
\{ f \} = \int_{V} t[N]^TPdV \tag{8}
\]

Governing equation can be written as,
\[
\{ F \} = [K]\{U\} \tag{9}
\]

Stress Concentration Factor (SCF): It is the ratio of stress of a point on the circumference of hole to the stress at same point when there is no hole and can be given as,
\[
SCF = \frac{\text{Stress at a point on the circumference of hole}}{\text{stress at the same point when there is no hole}}
\]

### 3. Results and Discussions

Using developed MATLAB program for the present analysis distribution of displacement and stress along x-direction is investigated. Maximum displacement and SCF are also evaluated for different fiber orientations and thickness of plate.

Table 3. Shows validation study for maximum displacement \( U_x \) and SCF. Good agreement of present results with already published results are observed.

| Parameters          | Present Results | Published Results [1] |
|---------------------|-----------------|-----------------------|
| \( U_x \) (in mm)   | 2.43            | 2.71                  |
| SCF                 | 2.1435          | 1.9115                |

Figure 2. Shows distribution of (a) displacement and (b) Stress on deformed shape of plate with 90\(^0\) of fiber orientation for plate with circular hole.
It is well known fact that fibers have highest strength along its longitudinal direction which is observed from the Table 4 as orientation of fiber shifted from $0^0$ to $90^0$ maximum displacement increases. The variation of displacement with fiber orientation follows almost same trend in all shapes of hole due to equal cutout area. Maximum displacement is in case of plate with square hole followed by that of plate with circular hole. The maximum displacement in plate with square hole is approx. 1.64 % greater than that in case of plate with rectangular shape of hole.

![Figure 3. Shows distribution of (a) displacement and (b) Stress on deformed shape of plate with 90° of fiber orientation for plate with square hole.](image)

![Figure 4. Shows distribution of (a) displacement and (b) Stress on deformed shape of plate with 90° of fiber orientation for plate with rectangular hole.](image)

In case of SCF the plate with circular hole shows maximum value and with the increase in the fiber orientation SCF decreases as resisting force per unit area of the plate decrease and which results in increment in displacement. The plate with circular hole shows 63.6 % more SCF than that of plate with rectangular hole.

Maximum stress along x-direction for plate with various shape of hole by keeping equal cutout area are shown in Table 5. It is observed that with the increase of fiber orientation stress decrease up to 45° of fiber angle while after that 90° it shows minimum stress which is due to weaker of fiber along
transverse direction. Plate with circular hole shows maximum axial stress followed by plate with square hole. Plate with circular hole shows 63.73 % higher axial stress than that of plate with rectangular hole.

Table 4. Variation of deformation and stress concentration factor with fiber angle.

| Types of cutout            | SCF 0° | SCF 30° | SCF 45° | SCF 60° | SCF 90° |
|----------------------------|--------|---------|---------|---------|---------|
| Plate with circular hole   | 3.6487 | 2.7208  | 2.7828  | 3.8137  | 2.7392  |
| Plate with square hole     | 2.8601 | 2.5716  | 2.4888  | 3.3232  | 2.4642  |
| Plate with rectangular hole| 2.2291 | 1.9186  | 1.8646  | 2.4329  | 1.8749  |

Table 5. Variation of maximum axial stress with fiber angle

| Types of cutout          | Maximum axial stress ($\sigma_{\text{f}_{\text{max}}}$ in N/m²) |
|--------------------------|---------------------------------------------------------------|
|                          | 0°                | 30°                | 45°                | 60°                | 90°                |
| Plate with circular hole | 1.824e+04         | 1.360e+04          | 1.391e+04          | 1.906e+04          | 1.369e+04          |
| Plate with square hole   | 1.430e+04         | 1.285e+04          | 1.244e+04          | 1.661e+04          | 1.232e+04          |
| Plate with rectangular hole | 1.114e+04       | 9.593e+03          | 9.323e+03          | 1.216e+04          | 9.374e+03          |

Table 6. Variation of maximum displacement, maximum axial stress and SCF with thickness of plate having various shapes of hole.

| Thickness of plate (in cm) | Plate with circular hole | Plate with square hole | Plate with rectangular hole |
|---------------------------|--------------------------|------------------------|-----------------------------|
| U_{\text{f}_{\text{max}}} (in m) | $\sigma_{\text{f}_{\text{max}}}$ (N/m²) | SCF | U_{\text{f}_{\text{max}}} (in m) | $\sigma_{\text{f}_{\text{max}}}$ (N/m²) | SCF | U_{\text{f}_{\text{max}}} (in m) | $\sigma_{\text{f}_{\text{max}}}$ (N/m²) | SCF |
| 0.1 | 0.0184 | 1.82e+04 | 3.6487 | 0.0185 | 1.391e+04 | 3.6487 | 0.0182 | 1.360e+04 | 3.6487 |
| 0.2 | 0.0092 | 9.12e+03 | 3.6487 | 0.0092 | 7.150e+03 | 3.6487 | 0.0091 | 5.572e+03 | 3.6487 |
| 0.4 | 0.0046 | 4.56e+03 | 3.6487 | 0.0046 | 3.575e+03 | 3.6487 | 0.0045 | 2.786e+03 | 3.6487 |
| 0.6 | 0.0031 | 3.04e+03 | 3.6487 | 0.0031 | 2.383e+03 | 3.6487 | 0.0030 | 1.857e+03 | 3.6487 |
| 0.8 | 0.0023 | 2.28e+03 | 3.6487 | 0.0023 | 1.787e+03 | 3.6487 | 0.0023 | 1.393e+03 | 3.6487 |
| 1.0 | 0.0018 | 1.82e+03 | 3.6487 | 0.0018 | 1.430e+03 | 3.6487 | 0.0018 | 1.114e+03 | 3.6487 |

Variation of maximum axial displacement, stress and SCF with thickness of plate having various shapes of hole are shown in table 6. It is observed that with the increase in thickness of the plate, axial displacement increases, maximum axial stress increases in the same proportion and there is no significant variation in SCF due to increment of corresponding nominal stresses.

4. Conclusions

In the present work analysis distribution of axial displacement and stress of composite plate with hole of various shape has been studied. The result of this analysis reveals the influence of fiber orientation and plate thickness on displacement, stress, and SCF when the plate is subjected to in-plane tensile loadings. Based on present analysis and results the following conclusion can be drafted,

1. Plate with square hole is more vulnerable and shows maximum displacement while cutout area of all shape of hole is kept equal.
2. Composite plate having any shape of hole shows minimum displacement with 0° fiber orientation when subjected to in-plane tensile loadings.
3. Plate with circular hole shows maximum SCF and plate with rectangular hole gives minimum SCF.
4. Composite plate with any shape of hole shows maximum value of SCF for 0\(^0\) fiber orientation when subjected to in-plane tensile loadings.

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