A New Approach to Obtaining The Optical Indices From The Values Of The Spectrophotometric Measurement

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Abstract

Calculation of spectral transmittance and reflectance values of a multilayer coating need to be known the spectral refractive index and the extinction coefficient values. In order to calculate the spectral optical indices a software was developed. It uses the spectrophotometric transmittance and reflectance values of single layer coatings, each coated only with the material in question, and the thicknesses of these layers. Considering the effective functions it contains, the software was written in the Matlab programming environment.

Keywords: Spectral transmittance, spectral reflectance, refractive index, extinction coefficient, single layer coatings

1. INTRODUCTION

Light transmittance and reflectance averages, optical characters such as colour coordinates of the covered glass are computed from the transmittance and reflectance values which depend on the wavelength. To get a covered glass with some specific optical characters, a varied numbers of materials in a varied order and thickness to be covered on the glass, the transmittance of the obtained covered glass’s depending on the wavelength and the projection values to be measured, from these values optical characters to be computed, and therefore the obtained characters to be compared with the foresighted characters, the thickness of covering and therefore covering condition to be changed to eliminate the deviation between the obtained and the foresighted character, and the process to be repeated until the aforementioned deviations between the two values are negligible. To eliminate the deviations with this method, one needs to have many tests, so this method is lucubratory and in terms of material it is not a very cost effective way. However transmittance and reflectance values of the covered glass which depends on the wavelength can be computed if the ordering, thickness, optical indices depending on the wavelength (breaking indice, extinguishing parameters) of the materials constituting the covering are known. For this carrier glass must be covered only one type of glass in each turn in a specific thickness. And thus comprised glass with one layered covering whose transmittance and reflectance values to be measured in 300nm–2500nm wavelength interval. By using this

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measurements values and the thickness of covering, the changes of optical indices of covering materials depending on the wavelength to be computed [1].

This work was accomplished to enrich a faster and reliable software to satisfy this necessity. For this software, having had the efficient functions MATLAB is considered to be the right atmosphere [1]. After each covering materials, which is used to get the covered glass in the required standards, each of whose optical indices were obtained after a developed software as mentioned before, to arrive at the qualities that we talked at the beginning, the repeated experiments are carried by using a foresighted software instead of using laboratory conditions. After getting the desired results from the computers the production stage can be started. By this way we have saved both from time and also from the cost [1].

The main difficulty arising from obtaining the couple of optical indices by using the spectrophotometrical measurements lies in the possibility that for the two dimensional solution sought in the interval of breaking indices and extinguishing parameters we have more than one couple of breaking indices and extinguishing parameters which works for the same couples of transmittance and reflectance. In the interval in which the long wavelengths are expected we usually have one solution, however in the interval expected towards the short wavelengths the possibility of having more than one solution is getting higher. At this point the solution which has a physical meaning should be specified and the one which has no physical meaning should be eliminated. This procedure constituted an important element of developed system. To prove the accuracy of the system, the values of indices were entered in to the software, and the values of indices that software produce for these values were compared with the entered values of indices. At last it was observed that the values coincides [1].

2. THEORY

On the surface of the glass, by using different covering techniques, different materials, after obtaining homogeneous layers in a specified order and thickness, the optical characters of the glass can be changed to desired characters. These optical character changes because of the superposition of a great number of compounds of the light caused by the reflection in the intermediary surfaces. The light, which arrive in an intermediary surface between two layers, one of whose component arrives the other layer while the other component turns back from the same layer by reflection. The ratio between the arriving component and the component turning back from this surface by reflection depends on the breaking indices and the extinguishing parameters of the materials constituting the layers. While the light moves through the materials the conditions that help more or less to extinguish the light during this movement depend on the length of the way and the extinguishing coefficients of the materials. The phase difference of the compounds of the light that meet between the intermediary layers is the ratio of the way that the light moved between the layer and the wavelength of the light. Therefore this value depends on the breaking indices of the materials constituting the layer and the thickness of the material. Because of these, by using the suitable materials in a different order to constitute layers with different thickness we can obtain different superpositions, so to obtain some optical characters on the surface of the glass, this provides us with many alternative choices [1].

The transmittance (T) and the reflectance (R) of the covered glass system (Figure 1) can be computed by the help of complex breaking indices of the glass and the covering materials and also the thickness of the glass and the covering layers. Complex breaking indices (\( \bar{n} \)) is a complex number consists of two parts, one is the real part coming from the breaking indice (n), and the other one is the imaginary part coming from the extinguishing parameter (k) [1].

\[
\bar{n} = n - ik
\]

The surface between the two layers is called the intermediary surface. In the Figure 1, the amplitude of the electrical vectors of the light compounds moving through the intermediary surface which is between the surface with the breaking indice \( \bar{n}_1 \) and the surface breaking indice
\[ \vec{n}_{i+1} \text{ and the field that this intermediary surface separates is represented by } (E_i^+, E_i^-, E_{i+1}^+, E_{i+1}^-) \text{ [1].} \]

The ratios between the amplitudes are called Fresnel parameters. The ratio of the amplitude \( E_i^- \) of the reflected light from the intermediary surface to the amplitude \( E_i^+ \) which belongs to the light coming from the above according to the Figure 1 is the parameter \( r_i \) (the Fresnel reflection parameter of the intermediary surface) and the ratio of the amplitude \( E_{i+1}^+ \) which belongs to the light that has transmitted through the intermediary surface to the amplitude \( E_i^+ \) is \( t_i \) (the Fresnel transmittance parameter of the intermediary surface) is shown below [1].

\[ r_i = \frac{E_i^-}{E_i^+} = \frac{\vec{n}_i - \vec{n}_{i+1}}{\vec{n}_i + \vec{n}_{i+1}} \]  
\[ t_i = \frac{E_{i+1}^+}{E_i^+} = \frac{2 + \vec{n}_i}{\vec{n}_i + \vec{n}_{i+1}} = 1 + r_i \]  

Equations, are obtained according to the slanting coming on page 51 [2]. This equation is obtained from the Equations 4.19-4.20 and 4.21-4.22 by thinking that the coming is vertical.

The ratio of the amplitude \( E_{i+1}^- \) of the reflected light from the intermediary surface to the amplitude \( E_{i+1}^+ \) of the light coming from the bottom to the intermediary surface according to Figure 1 is \( r_i' \) and the ratio of the amplitude \( E_{i+1}^+ \) of the light passing through the intermediary surface to the amplitude \( E_{i+1}^+ \) is \( t_i' \) obtained from the previous Equations and the symmetry [1].

\[ r_i' = \frac{E_{i+1}^-}{E_{i+1}^+} = \frac{\vec{n}_{i+1} - \vec{n}_i}{\vec{n}_{i+1} + \vec{n}_i} = -r_i \]  
\[ t_i' = \frac{E_{i+1}^+}{E_{i+1}^+} = \frac{2 + \vec{n}_{i+1}}{\vec{n}_{i+1} + \vec{n}_i} = 1 + r_i' = 1 - r_i \]

The ratio of the energy that the reflected light carries to the coming light and the ratio of the energy of the passing light to the energy of the coming energy are represented by \( R \) reflectance and \( T \) transmittance respectively. The \( (T_i') \) transmittance and the \( (R_i') \) reflectance in the ith intermediary surface (Figure 2) are shown below.

\[ R_i = |r_i|^2 = \left| \frac{\vec{n}_i - \vec{n}_{i+1}}{\vec{n}_i + \vec{n}_{i+1}} \right|^2 \]  
\[ T_i = \left| \frac{\vec{n}_{i+1}}{\vec{n}_i} \right| * |t_i|^2 = \frac{4 + |\vec{n}_i| + \vec{n}_{i+1}|^2}{|\vec{n}_i| + |\vec{n}_{i+1}|^2} \]

The equations are transformed from the Equations 4.27 and 4.28 on page 53 [2]. The reflection \( R_i' \) in the ith surface (Figure 2) and the transmittance \( T_i' \) are written by the help of symmetry. As can be seen, there is an equality between \( R_i', R_i \) and \( T_i', T_i \) [1].
Figure 3 shows a construction consisting of m-1 covering layers and therefore showing a construction consisting of number of m intermediary surfaces. Figure 3 is taken from Figure 4.8 on page 70 [2].

While the intermediary surface in Figure -1 combining two half infinite fields the intermediary surfaces in Figure 3 restricts the finite atmospheres. Among the intermediary surfaces the relations between the amplitudes $E_i^+, E_i^-, E_{i+1}^+, E_{i+1}^-$ of the light compounds on the two sides of the ith intermediary surfaces are shown below by the matrice notations [1].

$$
\begin{align*}
\left( \begin{array}{c}
E_i^+ \\
E_i^-
\end{array} \right) &= \frac{1}{t_i} \left( \begin{array}{cc}
e^{i\delta_i} & r_i e^{i\delta_i} \\
1 & e^{-i\delta_i}
\end{array} \right) \left( \begin{array}{c}
E_{i+1}^+ \\
E_{i+1}^-
\end{array} \right)
\end{align*}
$$

(2.7)

The parameters $r_i$ and $t_i$ are the Fresnel parameters of the ith intermediary surface and are obtained from the complex breaking indices belonging to the fields with numbers i and i+1 restricted by the ith intermediary surface as shown in the Equations 2.1 and 2.2 [1].

$$
\begin{align*}
r_i &= \frac{\bar{n}_i - \bar{n}_{i+1}}{\bar{n}_i + \bar{n}_{i+1}} \\
t_i &= \frac{2 \bar{n}_i}{\bar{n}_i + \bar{n}_{i+1}} = 1 + r_i
\end{align*}
$$

(2.8) (2.9)

The phase in the ith intermediary surface is represented by $\delta_i$. This phase with the wavelength ($\lambda$) depends on the breaking indice ($\bar{n}_i$) of the layer with number i and the thickness ($d_i$). This is given as in the following [1].

$$
\delta_i = \frac{2\pi}{\lambda} * \bar{n}_i * d_i
$$

(2.10)

The Equation 2.7 was taken from the equation 4.100 on page 71 and the Equation 2.10 was taken from the Equation 4.41 on page 56 [2].

$$
C_i = \left( \begin{array}{cc}
e^{i\delta_i} & r_i e^{i\delta_i} \\
1 & e^{-i\delta_i}
\end{array} \right)
$$

(2.11)

By using the equation 2.11,

$$
\left( \begin{array}{c}
E_i^+ \\
E_i^-
\end{array} \right) = \frac{1}{t_i} \left( \begin{array}{cc}
c_i & r_i \frac{E_{i+1}^+}{E_{i+1}^-} \\
r_i e^{i\delta_i} & e^{-i\delta_i}
\end{array} \right)
$$

we can get the following [1].

After the Equation 2.12 is rewritten for every intermediary surface from 1 to m and multiplying the left and right sides of the equations and doing the simplifications, we obtain the following equation [1].

$$
\left( \begin{array}{c}
E_1^+ \\
E_1^-
\end{array} \right) = \frac{c_1}{t_1} \left( \begin{array}{cc}
c_2 & c_3 \ldots c_m \frac{E_{m+1}^+}{E_{m+1}^-} \\
r_1 e^{i\delta_1} & e^{-i\delta_1}
\end{array} \right)
$$

(2.13)

Equation 2.13 is the same as the equation 4.101 on page 72 [2]. Since the phase of the intermediary surface with the number 1 is ($\delta_1$), the matrice $C_i$ could be written in the form. the equation 4.107 on page 72 [2].

$$
c_i = \left( \begin{array}{cc}
1 & r_i \\
r_i & 1
\end{array} \right)
$$

(2.14)

As seen from the Figure -4 the atmosphere (glass) with the number m+1 is being considered as half finite for the time being (at this step of the computation), there exist a light compound in the negative direction (coming from the bottom according to the Figure). For this reason we write 0 for $E_{m+1}^-$ in the equation 2.13, page 72 [2].

$$
P = \frac{c_1 \ldots c_m}{t_1 \ldots t_{m+1}}
$$

(2.15)

After some reductions in the Equations 2.13

$$
\left( \begin{array}{c}
E_1^+ \\
E_1^-
\end{array} \right) = P \left( \begin{array}{c}
E_{m+1}^+ \\
0
\end{array} \right)
$$

(2.16)

We get the above equation. Since $C_i$ are matrices of order 2x2, P is also a matrix of order 2x2.

$$
P = \left( \begin{array}{cc}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{array} \right)
$$

(2.17)

By using the Equations 2.16 and 2.17 the equivalent reflections and the transmittance Fresnel parameters of layers (the layer m-1) in the direction of $E_1^+$ (from up to bottom according to Figure 4) is obtained in the following way. The equations are equivalent to the Equations 4.104 on page 72 [2].

$$
\begin{align*}
r_f &= \frac{E_1^+}{E_1^-} = \frac{P_{21}}{P_{11}} \\
t_f &= \frac{E_{m+1}^-}{E_1^-} = \frac{1}{P_{11}}
\end{align*}
$$

(2.18) (2.19)
The equivalent reflections \( R_f \) and the transmittance \( T_f \) of the layers in the direction of air →layers →glass is described in the following equations [1].

\[
R_f = \left| r_f^2 \right| = \left| \frac{E_1^-}{E_1^+} \right|^2 = \left| \frac{P_{11}}{P_{12}} \right|^2 \tag{2.20}
\]

\[
T_f = \left| t_f^2 \right| = \left| \frac{\overline{P_{m+1} + 1}}{\overline{P_{m+1} - 1}} \right| * \left| t_f \right|^2 = \left| \frac{\overline{P_{m+1} + 1}}{\overline{P_{m+1} - 1}} \right| * \frac{1}{\left| P_{11} \right|^2} \tag{2.21}
\]

Here 2.20 is equivalent to the Equations 4.105 and to the 4.106a on page 72, the equation 2.21 is equivalent to 4.105 and to the 4.106b [1].

In Figure 4 it was assumed that the light arrives on the covered part of the glass from the air (according to the Figure from the above), reflected from the intermediary surface with the number 1 and transmitted to the atmosphere with the number m+1, so under these circumstances the value 0 was given to the compound \( E_{m+1}^+ \) in the Equation 2.13. In Figure 5 it was assumed that light came inside from the glass (according to the Figure from bottom) and therefore in this case the compound \( E_1^+ \) was given the value 0 [1].

\[
E_1^-(\text{from bottom to the up according to the Figure 5}) \text{ were obtained as follows.}
\]

\[
r_f' = \frac{E_{m+1}^-}{E_{m+1}^+} = -\frac{P_{12}}{P_{11}} \tag{2.23}
\]

\[
t_f' = \frac{E_1^-}{E_{m+1}^+} = P_{21} * \frac{E_{m+1}^+}{E_{m+1}^-} + P_{22}
\]

\[
t_f' = \frac{P_{11} + P_{22} - P_{21} + P_{12}}{P_{11}} \tag{2.24}
\]

\[
t_f' = \frac{\det(P)}{P_{11}}
\]

The equivalent reflection \( R_f' \) and the transmittance \( T_f' \) of the layers in the direction of glass →layers →air are obtained as in the following way similar to Equations 2.20 and 2.21 [1].

\[
R_f' = \left| r_f'^2 \right| = \left| \frac{E_{m+1}^-}{E_{m+1}^+} \right|^2 = \left| -\frac{P_{12}}{P_{11}} \right|^2 \tag{2.25}
\]

\[
T_f' = \left| t_f'^2 \right| = \left| \frac{\overline{P_{m+1} + 1}}{\overline{P_{m+1} - 1}} \right| * \left| t_f' \right|^2 = \left| \frac{\overline{P_{m+1} + 1}}{\overline{P_{m+1} - 1}} \right| * \left| E_1^- \right|^2
\]

\[
T_f' = \left| \frac{\overline{P_{m+1} + 1}}{\overline{P_{m+1} - 1}} \right| * \left| \frac{\det(P)}{P_{11}} \right|^2 \tag{2.26}
\]

By the help of the distributive property of the determinant function, the determinant of the matrice \( P \) given in the Equation 15

\[
P = \frac{C_1 \cdot C_2 \cdot C_3 \cdot \ldots \cdot C_m}{t_1 \cdot t_2 \cdot t_3 \cdot \ldots \cdot t_m}
\]

can be written in the following way (page 117 equation 22.3) [3].

\[
\det(P) = \det \left( \frac{C_1}{t_i} \right) \cdot \det \left( \frac{C_2}{t_i} \right) \cdot \ldots \cdot \det \left( \frac{C_m}{t_i} \right) \tag{2.27}
\]

Since the determinant of the matrice,

\[
C_i = \begin{pmatrix}
e^{i\delta_i} & r_i e^{i\delta_i} \\
e^{-i\delta_i} & r_i e^{-i\delta_i}
\end{pmatrix}
\]

in equation 2.11 is, \( 1 - r_i^2 \) if we take \( t_i = 1 + r_i \) in equation 2.9, we can write the determinant of the matrice \( P \) in equation 2.15 as,

\[
\det(P) = \prod_{i=1}^{m} \det \left( \frac{C_i}{t_i} \right) = \prod_{i=1}^{m} \frac{1 - r_i^2}{t_i^2}
\]
\[ \prod_{i=1}^{m} \frac{(1-r_i^2)(1+r_i^2)}{t_i^2} \]  
\[ \prod_{i=1}^{m} \text{det}(C_{i}) = \prod_{i=1}^{m} \frac{1-r_i^2}{1+r_i^2} = \prod_{i=1}^{m} \frac{1}{1+r_i} \]  
(2.28)

The expression
\[ r_i = \frac{n_i - n_{i+1}}{n_i + n_{i+1}} \]
showing the dependence of \( r_i \) to the indices, by using the equation 2.1, the equation 2.28 can be written as,
\[ \text{det}(P) = \prod_{i=1}^{m} \frac{1-r_i^2}{1+r_i^2} = \prod_{i=1}^{m} \frac{n_{i+1}}{n_i} \]  
(2.29)

By writing equation 2.29 clearly we can write for the \( \text{det}(P) \)
\[ \text{det}(P) = \frac{\pi_2}{\pi_1} \ast \frac{\pi_3}{\pi_2} \ast \frac{\pi_4}{\pi_3} \ast \ldots \ast \frac{\pi_{m+1}}{\pi_m} = \frac{\pi_{m+1}}{\pi_1} \]  
(2.30)

And for \( T'_f \)
\[ T'_f = \left| \frac{\pi_1}{\pi_{m+1}} \right|^2 \ast \left| \frac{\pi_{m+1}}{\pi_{m+1}} \right|^2 \ast \left| \frac{1}{\rho_{11}} \right|^2 = T_f \]  
(2.31)

As can be seen the transmittance doesn’t depend on the direction from which the light comes. Different from the reflections \( (R_f, R'_f) \), the transmittance \( (T_f) \) has the same value from both directions [1].

Contrary to the one done in the thin films, the superposition of the light compounds in the thick layers is calculated by assuming there is no coherency. In other words, instead of summing the complex amplitude ratios in the thick layers, the energy ratios (the reflections and the transmittance) are summed. If this way was ignored and not followed, in an interval of small wavelength, there would be a number of foresighted oscillation that couldn’t be responded by the resolution of the spectrometer. As a result of this, there would be both unnecessary calculations and measurements as well as the foresight that we get after the calculations that don’t coincide with the measurement of the spectrometer. On the other hand, the calculation which is done by incoherent foresight for thick layers will be equal to the average values chosen in the interval of wavelength for the spectrometric measurements, in other words to the measurement values of the spectrometer. Since the calculations are done on the basis of existence of coherency in the thin layers (covering layers) or the nonexistence of coherency in the thick layers (glass), the total transmittance and the reflections (Figure 6) of the covered system of glass are computed by a procedure having two steps. The first step of this procedure involves obtaining restricted equivalent transmittance and reflections \( (T'_f, R_f, R'_f) \) with the half finite air and half finite glass atmospheres [1].

The values \( T_f, R_f \) ve \( R'_f \) obtained at this solution stage are the same as in the Equations 2.20, 2.21, 2.25. In the second step of this procedure having two stages, for one of the surfaces the total transmittance and the reflections \( (T', R, T', R') \) are computed from the equivalent intermediary surface in which the equivalent transmittance and the reflections are satisfied while for the other intermediary surface the total transmittance and the reflections \( (T, R, T', R') \) are computed (Figure 7b) from the equivalent thick layer (Figure -7a) corresponding to the half -finite - glass-the-half finite - air, constituting the intermediary surface satisfying the transmittance and the reflections \( T'_f, R'_f, T'_S, R'_S \) and the extinction of the amplitudes between the intermediary surfaces is a [1].

![Figure 4 The reduction of covering layers to the equivalent intermediary surface](image)

Rewriting the Equations 2.5 and 2.6 for the atmosphere of air and glass for the values \( T_s \) and \( R_s \)
\[ R_s = \left| \frac{\pi_{m+1} - \pi_s}{\pi_{m+1} + \pi_s} \right|^2 \]  
(2.32)
The value $E_f$ of the vectoral amplitudes of electric on the equivalent intermediary surface will reduce along the way $E_f$ and finally will come down to the value $aE_f$ on the glass-air intermediary surface. Therefore, the value $|E_f|^2$ of the energy of light in the equivalent intermediary surface will be reduced to the value $a^2|E_f|^2$ on the glass-air intermediary surface and the ratio $T_f$ on the equivalent intermediary surface will be reduced to the value $a^2 T_f$ on the glass-air intermediary surface [1].

\[
T_s = \frac{4\pi |\overline{m}_{m+1}\overline{n}_1|}{|\overline{m}_{m+1}\overline{n}_1|^2} \quad (2.33)
\]

is obtained. By using the symmetry,

\[
R'_s = R_s \quad (2.34)
\]

\[
T'_s = T_s \quad (2.35)
\]

can be easily seen. According to the Equation 4.41 on page 50 the extinction of amplitudes between the intermediary surface is described as follows [2].

\[
a = e^{-2\pi \frac{k m+1 \cdot d m+1}{\lambda}} \quad (2.36)
\]

Figure 5 The transmitty compounds constituted by the reflections of light arriving on the covered surface of the glass

The values $T$ and $R$ in the Figure 7b as shown in the Figure 8. The infinite term can be computed by the help of two sums. In this work, all the equations are obtained by assuming that the light arrives to the intermediary surfaces vertically. The sloping lines in Figure 8 are drawn to acquire an easy drawing. The sum to be obtained for $R$ is given as follows, [1]

\[
R = R_f + a^4 T_f R_s (1 + a^4 R_s R'_f + a^8 R^2_s R^2_f + \cdots) \quad (2.37)
\]

Since all the values $a, R_s, R'_f$ are smaller than 1, taking the value $x$ as

\[
x = a^4 R_s R'_f
\]

$x$ is smaller than $1$. The relation

\[
1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}
\]

which is true for $x$ smaller than 1, page 555 according to the Equation 9, the equation 2.37 is reduced to the following equation [4].

\[
R = R_f + \frac{a^4 T_f R_s}{1-a^4 R_s R'_f} \quad (2.38)
\]

Similar to this one, an equation is obtained for $T$ as in the following way,

\[
T = a^2 T_f T_s + a^6 T_f R_s R'_f T_s + a^{10} T_f R^2_s R^2_f T_s \cdots
\]

\[
T = a^2 T_f T_s \left(1 + a^4 R_s R'_f + a^8 R^2_s R^2_f + \cdots\right) \quad (2.39)
\]

\[
T = \frac{a^2 T_f T_s}{1-a^4 R_s R'_f} \quad (2.40)
\]

The sum that can be obtained $R'$ is as follows

\[
R' = R_s + a^4 T^2_f R'_f + a^8 T^2_f R^2_f R_s + a^{12} T^2_f R^2_f R^2_f R_s^2 + \cdots
\]

\[
R' = R_s + a^4 T^2_f R'_f \left(1 + a^4 R_s R'_f + a^8 R^2_s R^2_f + \cdots\right) \quad (2.41)
\]

By performing a similar work to the one in the above the Equation 2.41 is reduced to the following equation

\[
R' = R_s + \frac{a^4 T^2_f R'_f}{1-a^4 R_s R'_f} \quad (2.42)
\]

And by a similar method we get the followings for $T'$.
As could be seen the transmittance of the system doesn’t depend on the direction the light comes from [1].

In the calculation of the total transmittance ($T$) and the reflections ($R, R'$) of covered glass instead of a method consisting of computation in two stages, the first one over the complex amplitudes and the second one over the real energy values, the method, which is developed by B. Harbecke, both of whose stages are carried out over the complex amplitudes, can be used as a computational method [5]. The formulas in the related article of B. Harbeckee are written assuming that the covering layers can be on both sides of carrying glass. The covered system discussed in that paper is named as a-layers-s-layers-b. a and b represent the air and s represents the transporter glass (substrate) [1].

The considered equations are obtained by rewriting the Equations 2.3 and 2.4 for the the air and the glass atmospheres

The Equations 2.53 and 2.55 [5], which provides us with the reflections ($R, R'$) and the transmittance ($T$) of the system are obtained from the Equation 16 and the Equation 17. The Equation 2.54 is written using the symmetry. The Equations 2.53, 2.54, 2.55 are equivalent to the Equations 2.38, 2.42, 2.40, respectively [1].

$$\begin{align*}
T' &= a^2T_2T_f + a^6T_s R_s' R_s T_f + a^{10} T_s R_s'^2 R_s^2 T_f + \ldots \\
T' &= a^2T_2T_f(1 + a^6R_s' R_s + a^{10} R_s'^2 R_s^2 + \ldots) \quad (2.43) \\
T' &= a^2r_f T_s \left( \frac{1}{1 - a^4 R_s R_s'} \right) = T \quad (2.44)
\end{align*}$$

The Equation 2.45 and 2.46 are taken from Equation 9, the Equations 2.47 and 2.48 are taken from Equation 11, [5]. The terms $r_{as}, t_{as}$, $r_{sa}$, $t_{sa}$ which are defined by the equations are equal to the terms $r_f, t_f, r_f', t_f'$ in the Equations 2.18, 2.19, 2.23, 2.24 respectively [1].

$$\begin{align*}
r_{as} &= \frac{p_{21}}{p_{11}} \quad (2.45) \\
t_{as} &= \frac{1}{p_{11}} \quad (2.46) \\
r_{sa} &= -\frac{p_{12}}{p_{11}} \quad (2.47) \\
t_{sa} &= \frac{p_{11}p_{22} - p_{12}p_{21}}{p_{11}} \quad (2.48)
\end{align*}$$

The transmittances of covered glass instead of layer $s$ are taken from Equation 2.47 and 2.48 are taken.
\[ T = \frac{a^2 + t_{as} + t_{sb}^2}{1 - a^4 + r_{sa} + r_{sb}^2} \]  
\[ \alpha = e^{-2\pi km_1 + r_{dm_1}} \]  

(2.55)

(2.56)

The parameter \( a \) in the equations is equivalent to the extinction factor in the Equation 2.36 [1]

3. DEVELOPED METHOD

3.1. The essence of the procedure

To obtain the breaking indice and the extinction parameters of the covered materials, the material is covered on the glass in specified amount of thickness, after this the transmittance and the reflections values of the obtained covered glass are measured depending on the wavelength. After this, the breaking indice and the extinction parameters depending on the wavelength which is needed to be known for the foresight for the above considerations about the material, are calculated from the values of the measured transmittance and the reflections [1].

This work is a study of developing method for the considered computation. The essence of the developed method consists of the following:

- For every wavelength in the n,k (breaking indice, extinction parameters) plane having a physical meaning, the computations of T,R (transmittance, reflection) couples corresponding to many couples of n, k chosen in appropriate intervals [1].
- The comparison of the couples T,R with the couples Tm, Rm which are measured for the considered wavelength, and obtaining the matching couples T,R and therefore the n,k couples [1].
- The system involves the case if there are more than one n,k couples matching for the couple T,R we can choose the one having the physical meaning among those [1]

3.2. The Stages of The Procedure

T and R Contours

The values T,R (transmittance, and reflection) which are computed by the probable couples n,k whose bounds are previously specified constitute a surface as shown in the Figure 7 and Figure 8. The n,k couples which satisfy the values of Tm and Rm obtained after the measurement, are found on the intersection curves between the surfaces T and R and the surfaces Tm and Rm. The projection (The contours T= Tm and R = Rm) of these intersection curves on the plane k are shown in the Figures (Figure 7,8) [1].

Figure 7 The surfaces T and Tm and the contour T obtained by intersecting the surfaces

Figure 8 The surfaces R and Rm and the contour R obtained by intersecting the surfaces.

The couple n,k which satisfy both of the values of the measurements Tm and Rm (from now on we call T and R shortly) is the couple n,k corresponding to the point in which the contours T=Tm and the contour R=Rm coincide (Figure 9) [1].
The intersections of the contours T and R

The contours as exemplified in the table 1 consisting of a table containing the couples of n,k. To obtain the intersections of the contours the following method is developed [1].

Table 1 The tables CT and CR for the wavelength 810nm.

| Point | n   | k   | | Point | n   | k   |
|-------|-----|-----| |-------|-----|-----|
| 2     | 1.5000 | 0.5621  | 2 | 2.3038 | 0.0000 |
| 3     | 1.6000 | 0.5256  | 3 | 2.3000 | 0.0149 |
| 4     | 1.6727 | 0.5000  | 4 | 2.2740 | 0.1000 |
| 5     | 1.7000 | 0.4912  | 5 | 2.2342 | 0.2000 |
| 6     | 1.8000 | 0.4591  | 6 | 2.2000 | 0.2711 |
| 7     | 1.9000 | 0.4274  | 7 | 2.1836 | 0.3000 |
| 8     | 1.9861 | 0.4000  | 8 | 2.1197 | 0.4000 |
| 9     | 2.0000 | 0.3960  | 9 | 2.1000 | 0.4281 |
| 10    | 2.1000 | 0.3659  | 10| 2.0414 | 0.5000 |
| 11    | 2.2000 | 0.3350  | 11| 2.0000 | 0.5476 |
| 12    | 2.3000 | 0.3030  | 12| 1.9474 | 0.6000 |
| 13    | 2.3088 | 0.3000  | 13| 1.9000 | 0.6453 |
| 14    | 2.4000 | 0.2717  | 14| 1.8344 | 0.7000 |
| 15    | 2.5000 | 0.2388  | 15| 1.8000 | 0.7278 |
|       |       |       | | 16    | 1.7000 | 0.7992 |
|       |       |       | | 17    | 1.6987 | 0.8000 |
|       |       |       | | 18    | 1.6000 | 0.8605 |
|       |       |       | | 19    | 1.5284 | 0.9000 |
|       |       |       | | 20    | 1.5000 | 0.9153 |

The intersection of the contours T and R

For the beginning step of the developed method, for each point on the contour T, the distances of this point to the points on the contour R is compared and then the point with the shortest distance is connected to the point on the contour T (Figure 10). This connection is illustrated in Figure -11 and Table 2, as p showing the row numbers of the points on the contour T in the table CT and q showing the row numbers of the points on the contour R in the table CR [1].

Table 2 The table p-q for 810nm

| PQ | i | p | q | Distance |
|----|---|---|---|----------|
| 1  | 2 | 17| 0.0961 |
| 2  | 3 | 15| 0.0809 |
| 3  | 4 | 14| 0.0661 |
| 4  | 5 | 14| 0.0617 |
| 5  | 6 | 12| 0.0416 |
| 6  | 7 | 11| 0.0245 |
| 7  | 8 | 10| 0.0131 |
| 8  | 9 | 9 | 0.0110 |
| 9  | 10| 8 | 0.0016 |
| 10 | 11| 7 | 0.0015 |
| 11 | 12| 6 | 0.010 |
| 12 | 13| 6 | 0.0127 |
| 13 | 14| 5 | 0.0326 |
| 14 | 15| 4 | 0.0703 |
In the second step of this method, Under the condition of applying all the points, the consecutive points of p are pA and pB and relating points of q which are connected to these points (pA, pB) are qA, qB. The points qA, qB are checked whether or not they stay at the same sides of the line passing through the points pA, pB. After the checking, the point n,k obtained by intersecting the lines passing through the two p points and the line passing through the two q points is taken as the solution [1].

Figure 12 The contours T and R and their intersection point

If the points pA, pB depend on the same point q, the control is performed by choosing the points qA, qB in the rows preceding and succeeding, respectively the row containing q [1]

Once the control is ensured, the lines passing through pA, pB and the points qA, q, qB are intersected [1].

The elimination of multiple intersections

The procedure is carried even if an intersection point is obtained, the reason for this as seen from the graph of f in the Figure 14, we can find a second, third even a fourth intersection points and therefore different n,k couples of candidate solution for the same couple Tm, Rm. A method is developed to determine which of these has a physical meaning [1].

Figure 14 The projections for a variety of wavelengths from 2500nm to 300nm

In the Figure 14 the contours are shown obtained for the different wavelengths. The contours intersect in regions shown in the Figure by small circles. This method can be explained by the Figures 15, 16, 17. Figure 15 contains the points corresponding to the couples n,k obtained for all the wavelengths. Since only one solution is obtained on the n,k plane from 2500 to 350nm, the points are found on a smooth curve (route). The curve here under consideration continues its path by zigzagging because of the existence of the couples of solutions after 350 nm. As seen obviously from the Figure 19, 20 and explained by diagrams the curve is constituted by a different path. The path (the path 2) after the apparition 350 nm is viewed to move towards the region in which the couples n,k have no meaning for the wavelengths bigger than 350 nm [1].
The principle of the nearest point computation of the distance

The developed method depends on the elimination of the path whose remaining part doesn’t have a physical meaning. The method which enables to eliminate the path 2 works with regard to the principle of the nearest point. By using the fact that only the single solutions appear for the big wavelengths, a loop is constituted starting from the big wavelengths. In case there are more solutions for the next wavelength, the points n,k corresponding to the solutions are checked if their previous couples n,k are near or not. If they are not near, they are eliminated.[1]

Table 3 Z (Probable solutions) and Zt tables (The solutions having the physical meaning)

| λ(nm) | n     | k     | λ(nm) | n     | k     |
|-------|-------|-------|-------|-------|-------|
| 350   | 2.3269| 1.3305| 350   | 2.3263| 1.3296|
| 350   | 2.3263| 1.3296| 340   | 1.5626| 1.7785|
| 340   | 2.3048| 1.7782| 340   | 1.6496| 1.4013|
| 340   | 1.6503| 1.7875| 330   | 2.2689| 1.4831|
| 330   | 1.7520| 1.7968| 320   | 2.2246| 1.5721|
| 320   | 1.8741| 1.8049| 310   | 2.1691| 1.6701|
| 310   | 2.0401| 1.8045| 300   | 2.1014| 1.7779|
| 300   |       |       | 300   |       |       |

The result is in the form shown by Figure 17. By drawing the couples n,k using the corresponding values (wavelength) of λ ,the optical indices λ, n looked for the material in the Figure 18 and λ, k values are obtained [1].

Figure 17 The curve whose nearest point principle is obtained

Figure 18 The graph of n,k depending on the wavelength

Figure 19 The graph of n,k depending on the wavelength obtained by a method with the theoretical values
4. THE DEVELOPED SOFTWARE

4.1. The general scheme of the software

The general scheme of the developed software, is shown in the Figure 20. The main programme of the software is constituted by the m.m and fnk.m, Ftr.m working connected with the m.m. The programme m.m operates the multiple datas as the entry in the files subdat.mat, measdat.mat, NK.mat, and keep the results obtained by registering in the excel file [1].

The software programmes m, addmeasdat.m and setNK.m are the data for the programme m.m and constitute to procure the programmes "mat". The first two of them constitute the data with the related Excel files [1].

As seen, the software takes the operated data from the Excel atmosphere in which the data are kept safely and writes the results into this atmosphere [1].

4.2. The explanations of the software

4.2.1. addsubdat.m

Addsubdat.m consists of five columns, order number, the name of the transformer glass, wavelengths, breaking indices, the data of extinguishing parameters, respectively and provide the cell-type matrice kept in the file subdat.m with the name subdat a new row for a new transformer glass. The programme reads datas added to the matrice (variable) subdat from the page the user foresights from the Excel file subdat.xls [1].

4.2.2. addmeasdat.m

Addmeasdet.m consists of nine columns, the number of order, the name of the covering material, the thickness of the covering material, the code of the transformer glass, the name of the transformer glass, the thickness of the transformer glass, wavelengths, reflections, the matrices of transmittance, respectively and provides a new row for a new covering for the cell-type matrice kept in the file measdet.mat. The programme reads the datas, which is added to the matrice meadset, from the page user foresights from the file measdet.xls [1].

4.2.3. setNK.m

The method is based on comparing the measured values Tm, Rm (transmittance, reflection) with the T,R values(transmittance, reflection) which are computed for various couples of n,k [1].

The programme setNK.m is determined by the intermediary surface whose upper and lower boundary values is satisfied by n,k indices under consideration and whose step intervals are satisfied by the indices. The values entered with this intermediary surface is placed into the variable NK consisting of seven columns, wavelength, the lower bound of breaking indice, the upper bound of breaking indice, width of scanning step, the lower bound of extinguishing parameter, the upper bound of extinguishing parameter and width of scanning step, respectively and they also help to save this variable in to the file NK.mat [1].
4.2.4. m.m
This is the main module of the programme for the glass in a specified thickness in a specified material whose T and R (transmittance, reflections) values measured in the interval 300 nm-2500 nm helping to obtain the values n,k (breaking indice, extinguishing parameter) in the considered interval in the considered material [1].

4.2.5. fnk.m
The function in the loop of wavelength contained by the mm (mt.m) modules getting the values for the material’s n, k couples for each wavelength. The thickness value of the glass and the covering, the considered wavelength and the corresponding breaking indice, extinguishing parameter, the foresighted breaking indices and the intervals of extinguishing parameters of the covering for this wavelength, the measurement values of transmittance and the reflection of the covered glass are the inputs of the function, the z matrice, containing the values of one or more than one candidate couples’ n,k wavelength, breaking indices and the extinguishing parameters, only one of which is chosen taking the values from the obtained module m.m (mt.m) for the considered wavelength is the output of the function. By using the previously determined starting and the final values n,k intervals contained by the data matrices N,K for the function and step intervals to find the values Tm, Rm for the considered wavelength, by sending every couple of the considered n,k intervals into fTR function, it provides producing many couples T,R to be compared with the Tm,Rm [1].

4.2.6. fTR.m
The function Ftr is a function written aiming to compute the transmittance and the reflection values of the system of the covered glass with many layers. It is the module of the packet which is written to foresight the performance values (visible region, the averages of solar energy region, colour) of the systems of the covered glass containing layers in a different numbers, different order and different thickness and constituted by the materials whose optical values are known [6].

This work as mentioned in the previous chapters, by using a clearer language for the knowledge under consideration and needed for the similar foreseeing programmes aiming to determine the optical indices of the covered materials. Therefore, the only type of the system under consideration is the single layered systems constituted by covering the transformer glass with only one material in each turn. In other words, there exist two intermediary surfaces between the air and the transformer glass, and therefore, two C matrices are enough [1].

The material whose indices are going to be determined must be isolated from the air and therefore another material might need to be covered on it. In this work, this condition isn’t foresighted. But since both the indices of the second material is known and in addition to this if the the needed changes are performed in the m.m, and fnk.m, the number of layers can be raised to two (and the the number of intermediary surfaces to three). Since fTR is written for multi purpose constructions, it doesn’t need any change [1].

5. ACHIEVING THE SOFTWARE
n,k are obtained for the measured T,R values, thereafter the T and R are recalculated from the these n,k values and compared to the original T and R values. Since the measured T,R couples contain measurement errors the corresponding n,k values for these T and R contain the measurement errors, therefore the curves having folds, not having rather smooth lines emerge. Instead of ensuring such a thing to actualize the system, the converse is being performed [1].

To acquire the software, two theoretical breaking indices and the couples of extinguishing parameters (n,k) are produced, by taking the one from the relatively high standards materials as sample and the other from the sample of material showing a very low absorption, the computed transmittance and the reflection values of (T,R) couples are entered as the measured couples of T,R into the software and the couples n,k are obtained and it is seen that the obtained couples of n,k coincides with the theoretical couples of n,k [1].
5.1. addtestdat.m

Addtetsdat.m consists of eleven columns, number of order, the name of the covering material, the thickness of the covering material, the code of the transformer glass, the name of the glass, the thickness of the glass, wavelengths and the breaking indices corresponding to this wavelength, extinguishing parameters, reflections, the matrices of transmittance and provides the cell type matrice testdat, kept in the file testdat.mat, with a new row for a new covering. The programme reads the datas which are going to be added to the matrice testdat (variable) from the Excel file testdat.xls from the page the user foresees [1].

5.2. mt.m

This is the module of the programme doing the same job with the m.m module obtained to make the the developed programme work, different from the previous one in that applied to the data T,R obtained from the theoretical values n,k by computations instead of measurement data and comparing the values n, k obtained as a result of these data with the considered theoretical n,k values [1].

5.3. Ensuring

While the indice plane for the testdata-1 is prepared for the values between 3000 nm and 2500 nm, for all the wavelengths between the the intervals 300 nm and 2500 nm the following values are taken; as for the lower bound of the breaking indice 1.5, for upper bound 2, for the scanning step width of the breaking indice 0.1, in the same way for the lower bound of the extinguishing parameter 0, for upper bound 2 and for the scanning step width of extinguishing parameter 0.1 [1].

Table 4 The values of the n,k indice plane for the testdata-1

| Wavelength | Breaking indices (n) Lower | Upper | Scan Step |
|------------|----------------------------|-------|-----------|
| 300        | 1.5                        | 2.5   | 0.1       |
|            | Extinguishing parameters Lower | Upper | Scan Step |
|            | 0                          | 2     | 0.1       |

Figure 21 The test scheme of the developed software

Figure 22 Intermediary surface setting (n,k) indice plane for the testdata-1

After the indice plane is obtained, to check out the validity of the method, the testdata-1 is chosen.

Figure 23 The intermediary surface for choosing test materials
In the above Figures the graph $Z$ obtained by the method for all the $n,k$ values between the wavelengths 300 nm and 2500 nm and the graph $Z_t$ obtained by the technique the nearest point and constituted by the right couples of $n,k$ are seen [1].

**6. THE APPLICATION OF THE SOFTWARE**

| Wavelength | Breaking indices $(n)$ Lower | Upper | Scan Step | Extinguishing parameters Lower | Upper | Scan Step |
|------------|-----------------------------|-------|-----------|--------------------------------|-------|-----------|
| 300-490    | 1.5                         | 2.5   | 0.1       | 0                             | 2     | 0.1       |
| 500-2500   | 2.4                         | 2.8   | 0.02      | 0                             | 0.2   | 0.01      |

Figure 24 The graphs of testdata-1 $Z$ and $Z_t$

Figure 25 testdata-1 the graph of wavelength $n,k$

Figure 26 The results of the method and the graph of the wavelength $n,k$ of the original values

Obtained by the procedure as in the above diagrams for the values of the wavelengths between 300 nm and 2500 nm, the breaking indices, the breaking indices of the original wavelength values with the graph of the extinguishing parameters, the graph of extinguishing parameters are given together. As understood from the diagrams, the results of the method coincide with the original values on the determined indice plane [1].

Table 5 The values of $n,k$ indice plane for TiOx 40

Figure 27 The intermediary $(n,k)$ indice plane for TiOx 40

Figure 28 The selection of the covering mateiral TiOx 40
7. CONCLUSIONS

To compute the spectral transmittance and the reflections values of the multi layers covering, the breaking indice and the extinguishing parameters of the covering materials are needed to be known. For this, taking the datas for the thickness of the covering layer, spectrometric transmittance and reflection values of single layered covered glass constituted in each turn by only one material covered in a specified thickness, a software is developed to compute the spectral optical values of the material under consideration [1].

To acquire the software, two theoretical breaking indice one is modelled from the relatively high standards materials, the other from the materials showing a very low absorption and the couples of extinguishing parameters (n,k) are produced, the couples of the values of the transmittance and the reflectance (T,R) computed from these couples are entered to the software as if they are the measured couples of T,R to obtain the couples n, k, after this it is seen that the couples of n,k coincides with the theoretical indices n,k [1].

In the interval, n-k for the long wavelengths usually exists in a single solution. However in the interval encountered approaching towards the short wavelengths, the probability of having more than one solution rises. Among these, we need to determine the solution having a physical meaning and eliminate the one not having a physical meaning. The developed method contains a procedure to function for this. In the proofs under consideration it is seen that the elimination process is accomplished successfully [1].

The software is worked for the single layered coverings produced in the AR-GE department of the Şişecam fabrics whose thickness of covering and transmittance and reflectance values between the wavelengths 300 nm and 2500 nm are measured to determine the optical indices of the covering materials in the same interval [1].

The software has the ability to determine the optical indices under the considered interval in a time interval of 10-20 seconds in a standard computer [1].

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