Temporal Modulation of Bianisotropic Metasurfaces for Unidirectional Wave Amplification

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Abstract—Nonreciprocity in time-modulated metasurfaces is normally achieved only when it is combined with space modulation, using at least two time-varying elements located at different positions in space. In this talk, we present the idea of time-modulated bianisotropic metasurfaces where nonreciprocal wave propagation is realized with time modulation of only a single element. The results show that by uniformly modulating a capacitive sheet mounted on a sub-wavelength dielectric layer, one can obtain strong nonreciprocity and achieve unidirectional wave amplification.

Index Terms—nonreciprocity, time modulation, wave amplification, bianisotropic metasurfaces.

I. INTRODUCTION

Temporal modulation of macroscopic parameters such as reactive lumped elements (inductance and capacitance), permittivity and surface impedance, gives us an exceptional opportunity for extreme manipulation of electromagnetic radiation. Consequently, using this technique, extraordinary functionalities such as magnet-free nonreciprocity \cite{ref1, ref2}, virtual absorption \cite{ref3} and control of scattered fields \cite{ref4} have been realized in recent years. Regarding metasurfaces as 2D arrays of periodically positioned subwavelength particles, temporal modulation provides an additional degree of freedom which enables nonreciprocal wave propagation, frequency conversion (generation of harmonics), etc. However, to realize nonreciprocal response, most of the current works apply modulations both in space and time, usually in form of a travelling wave (such spatiotemporally modulated metasurfaces have been demonstrated for wave isolation \cite{ref5}, circulation \cite{ref6} and nonreciprocal leaky-wave antennas \cite{ref7}). This approach has a significant drawback in practical implementations since different meta-atoms should be modulated with different phases. This dramatically increases complexity of the feed network. If there would be no need for spacial modulation, all the meta-atoms could be fed by the same pumping signal, immensely simplifying the pumping network.

In this paper, we propose the concept of time-modulated bianisotropic structures which provide strong nonreciprocity with only uniform temporal modulation of meta-atoms. The idea is based on an asymmetrical time-modulated $T$-circuit. It is shown that, with proper modulation function of the lumped components in such $T$-circuit, the transmission of the input signal (frequency $f_0$) can be engineered so that it is dramatically different for two opposite directions of the incidence, resulting in strong nonreciprocity. This conceptual idea is implemented with time-modulated bianisotropic metasurfaces, where a time-varying capacitive sheet is mounted on a substrate of subwavelength thickness. The results indicate that this structure can realize one-way amplification, i.e., for illuminations from one side the transmitted wave is amplified, while for the opposite incidence direction it is nearly fully attenuated.

II. TRANSMISSION-LINE MODEL

The main idea of this work is based on the use of an asymmetric time-modulated $T$-circuit, introduced in \cite{ref7}. The circuit is shown in Fig. 1(a). Here, the impedances $Z_1(t)$ and $Z_2(t)$ are not identical in order to break the circuit symmetry. We also assume that the impedances are purely reactive, i.e, we neglect dissipation. We consider a special case when the elements are inductors or capacitors. Normal incidence is assumed throughout the text. As an example, in Fig. 1(b) we replace $Z_1(t)$ with a constant inductance $L$, $Z_2(t)$ is shorted, and finally $Z_4(t)$ is substituted by a time-dependent capacitance $C(t)$ which is periodically modulated at $\omega_M = 2\omega_0$ ($\omega_0$ is the angular frequency of the incident signal). Due to the periodic modulation, the reflected and transmitted signals contain infinite numbers of harmonics $\omega_n = \omega_0 \pm n\omega_M$. Here, $n$ is an integer and refers to the harmonic order.

For a specific excitation $v_i(t) = e^{j\omega t}$ and modulation waveform $C(t)$, the harmonics of the transmitted signal can be analytically determined by employing the known mode-matching method \cite{ref8}. Performing mathematical optimizations, we can find the optimal values for the time-modulated capacitance and the static inductance – $C(t) = 0.38[1 - \sin(\omega_M t)]$ [pF] and $L = 9.15$ [nH] – such that strong nonreciprocity at the fundamental frequency $\pm \omega_0$ is obtained. This property is explicitly shown in Fig. 1(c) Intriguingly, the amplitude of the transmitted signal at the fundamental frequency (the sum of $+\omega_0$ and $-\omega_0$ contributions corresponding to harmonics $n = 0$ and $n = -1$) is doubled, which means that the pumping system (needed for changing the capacitance in time) supplies energy to the circuit.

III. REALIZATION WITH TIME-MODULATED BIANISOTROPIC METASURFACES

The asymmetric $T$-circuit described in the previous Section can be considered as a circuit model of bianisotropic...
metasurfaces. Figure 2 illustrates a time-modulated capacitive sheet which is positioned on a thin dielectric slab. The presence of the substrate breaks the structure symmetry with respect to inversion of the $z$-axis and introduces bianisotropic effects. This structure can be modeled as a shunt capacitance connected with a section of a transmission line. The corresponding amplitudes and phases of scattered waves are analytically solved using the generalized transmission-line method. Similarly as the previous section, we optimize the modulation waveform $C(t)$ and the substrate properties in order to obtain strong nonreciprocity. When $\omega_M = 2\omega_0$, $C(t) = 9.1 + 8.8\sin(\omega_M t)$ [pF], $\epsilon_d = 65$ and $d = 0.034\lambda_0$, the amplitudes of the transmitted harmonics corresponding to the opposite propagation directions of the input wave become dramatically different, as seen in Fig. 3(a). Here, $\epsilon_d$ denotes the relative permittivity of the substrate material, $d$ represents the substrate thickness, and $\lambda_0$ is the free-space wavelength related to the input wave.

![Figure 1](image1.png)

**Fig. 1.** (a) Asymmetric time-modulated $T$-circuit. (b) Realization with lumped components. (c) Amplitudes of transmitted harmonics for circuit in Fig. 1(b). The operating frequency is $f_0 = 10$ GHz and the source impedance is $\eta_0 = 377$ $\Omega$.

IV. Conclusions

This paper develops the concept of achieving nonreciprocity with only temporal (spatially local) modulation. We have presented example structures with realistic parameters that realize nonreciprocal transmission in a time-modulated $T$-circuit. Furthermore, we have adapted this circuit analogy to transmission lines and presented a design of a nonreciprocal metasurface with realistic parameters. We have shown that using only temporal modulation it is possible to realize amplification of signals at the carrier frequency in one direction and completely suppress the carrier wave propagation in the opposite direction. In the talk, we will also present results regarding the polarizabilities of the unit cells that provide this nonreciprocal behavior and discuss the physical properties of these time-modulated unit cells.

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