Field-Theoretical Analysis of Singularities at Critical End Points

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Abstract

Continuum models with critical end points are considered whose Hamiltonian \( H[\phi, \psi] \) depends on two densities \( \phi \) and \( \psi \). Field-theoretic methods are used to show the equivalence of the critical behavior on the critical line and at the critical end point and to give a systematic derivation of critical-end-point singularities like the thermal singularity \( \sim |t|^{2-\alpha} \) of the spectator-phase boundary and the coexistence singularities \( \sim |t|^{1-\alpha} \) or \( \sim |t|^\beta \) of the secondary density \( \langle \psi \rangle \). The appearance of a discontinuity eigenexponent associated with the critical end point is confirmed, and the mechanism by which it arises in field theory is clarified.

Key words: critical end point, field theory, critical and coexistence singularities

1 Introduction

Critical end points are ubiquitous in nature. They occur when a line of critical temperatures \( T_c(g) \), depending on a nonordering field \( g \) such as chemical potential or pressure, terminates at a line \( g_\sigma(T) \) of discontinuous phase transitions [1–3]. Two familiar examples are: (i) the critical end point (CEP) of a binary fluid mixture where the critical line of demixing ends on the liquid-gas coexistence curve; (ii) the CEP of \( {}^4 \text{He} \), the terminus of the lambda line on the gas-phase boundary. On the critical (or lambda) line the disordered and ordered phases separated by it become identical critical phases; in the case of a binary fluid with components A and B, the disordered phase corresponds to a homogeneously mixed fluid \( \alpha\beta \), and the ordered ones to an A-rich phase \( \alpha \) and a B-rich phase \( \beta \); in the case of \( {}^4 \text{He} \), the disordered and ordered phases

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are normal fluid and superfluid, respectively. A crucial feature of a CEP is that a critical phase coexists with a noncritical (‘spectator’) phase $\gamma$ there.

Although CEPs were encountered in numerous studies of bulk and interfacial critical phenomena [1–11, and their references] in the past decades, they have rarely been investigated for their own sake. This may be due to the expectation that the critical phenomena at a CEP should not differ in any significant way from critical phenomena along the critical line $T_c(g)$ [1]. However, recently it has been pointed out [2–4] that even the bulk thermodynamics of a CEP should exhibit new critical singularities, not observable on the critical line. On the basis of the phenomenological theory of scaling it was predicted [2] that the first-order phase boundary $g_{\sigma}(T)$ should vary near the CEP ($T_e, g = g_e \equiv g_{\sigma}(T_e)$) as

$$g_{\sigma}(T) \approx g_{\sigma}^{\text{reg}}(T) - \frac{X^0_{\pm}}{(2 - \alpha)(1 - \alpha)} |t|^{2-\alpha}$$

in the limit $t \equiv (T - T_e)/T_e \to 0\pm$, where $g_{\sigma}^{\text{reg}}(T)$ is regular in $T$. Furthermore, the amplitude ratio $X^0_{\pm}/X^0_{0}$ should be equal to the usual universal (and hence $g$ independent) ratio $A_+/A_-$ of specific heat amplitudes $A_{\pm}$. These are defined by writing the specific heat singularity at constant $g \neq g_e$ on the critical line as $A_{\pm}(g) |T - T_c(g)|^{-\alpha}$. In other words, the singularities displayed by $g_{\sigma}(T)$ should be of the same form as those of the bulk free energy of the disordered ($\alpha\beta$) and ordered ($\alpha + \beta$) phases near $T_c(g)$.

The phenomenological scaling arguments leading to (1) can be extended in a straightforward fashion to determine the singularities $\rho_g$, the thermodynamic density conjugate to the nonordering field $g$, should display as the CEP is approached along the coexistence boundary [12]. They yield the singular part

$$\rho_g^{\text{sing}} \equiv \rho_g - \rho_g^{\text{reg}}(T) \approx U^0_{\pm} |t|^\beta + V^0_{\pm} |t|^{1-\alpha}.$$

Having in mind binary fluid mixtures, we take the CEP to lie on the liquid (rather than the gas) side of the coexistence boundary. The quantity $\rho_g$ may be identified as the total density of the fluid. For a hypothetical symmetric binary fluid whose properties are invariant with regard to simultaneous interchange of its two constituents A and B and their respective chemical potentials $\mu_A$ and $\mu_B$, the amplitudes $U^0_{\pm}$ would vanish. More generally, this would be true for systems that are describable by a continuum Hamiltonian which is even in the order parameter field $\phi$. Just as $X^0_{\pm}/X^0_{0}$, the ratios $V^0_{\pm}/V^0_{0}$ as well as $U^0_{+}/U^0_{-}$ (if $U^0_{+} \neq 0$) are universal and can be expressed in terms of standard universal amplitude combinations [12].

The $|t|^{2-\alpha}$ singularity of (1) has been checked by Monte Carlo calculations [12] and verified for exactly solvable spherical models [8,9]; the $|t|^{1-\alpha}$ singularity
of (2) is consistent with the jump in the slope of $\rho_g(T)$ found in mean field and density functional calculations [10,11] and has also been seen in Monte Carlo simulations [12,13].

Here we will address the issue of CEP singularities via the field-theoretic renormalization group (RG) method. This approach is known to provide both a conceptually reliable basis of the modern theory of critical phenomena as well as powerful calculational tools (see, e.g., [14,15]). Surprisingly, it has not yet been applied with much success to the study of CEPs. We are aware of only one such work that goes beyond the Landau approximation, an $(\epsilon = 4 - d)$-expansion study of a scalar $\phi^8$ model with negative $\phi^6$ term [16]. Its one-loop result is that the critical line and the CEP are controlled by the same, standard $O(\epsilon)$ fixed point. Unfortunately, the model investigated has rather special features: its first-order line does not extend into the disordered phase; as its CEP is approached from the disordered phase, the order parameter $\langle \phi \rangle$ becomes critical and exhibits a jump to a nonvanishing value upon entering the ordered phase; and no critical fluctuations occur in its ordered phase. Hence it clearly does not reflect the typical CEP situation in which the two-phase coexistence surface bounded by the critical line $T_c(g)$ meets the spectator phase boundary in a triple line; its applicability appears to be quite limited.

One should also note that the above RG scenario differs from the one found in position-space RG calculations [5,6] of lattice models with conventional CEPs. In the latter scenario the critical line and the CEP are mapped onto separate fixed points, where the CEP fixed point has two relevant RG eigen-exponents that are identical to those of the former, plus the additional one $y = d$, characteristic of discontinuity fixed points [17], but absent in [16].

We conclude that systematic field-theoretic RG studies of appropriate models are urgently needed. A first obvious goal one would hope to achieve is a systematic derivation of the singularities in (1) and (2). This involves showing the equivalence of critical behavior at the CEP and on the critical line. Provided the above RG scenario with two separate fixed points prevails, one must prove that the associated critical spectra match, demonstrate the existence of the discontinuity eigenexponent $y = d$ and clarify its significance.

We have recently carried out such an investigation. In the sequel, we will briefly describe the main steps of our procedure and our findings. A more detailed exposition of our work will be given elsewhere [18].

\footnote{In their excellent survey [3] of the present state of the theory of CEP singularities, Fisher and Barbosa warn that this equivalence, with matching critical spectra of the corresponding two fixed points, need not be an invariable rule, even though their work confirms it, just as our own.}
2 Models

First, we must choose an appropriate continuum model. Natural candidates are models whose Hamiltonian \( H[\phi, \psi] \) depends on two fluctuating densities: a (primary) order parameter field \( \phi(x) \) and a secondary (noncritical) density \( \psi(x) \). The form of \( H \) can be guessed on purely phenomenological grounds, but can also be derived by starting from an appropriate lattice model, such as the Blume-Emery-Griffiths model \( [19] \) on a \( d \)-dimensional simple cubic lattice. This is a classical spin \( S = 1 \) model with Hamiltonian

\[
H_{\text{BEG}}[S] = - \sum_{\langle i,j \rangle} \left[ J S_i S_j + K S_i^2 S_j^2 + L \left( S_i^2 S_j + S_i S_j^2 \right) \right] - \sum_i \left( H S_i + D S_i^2 \right), \quad S_i = 0, \pm 1 ,
\]

where \( \langle i,j \rangle \) indicates summation over nearest-neighbor pairs of sites. We presume the interaction constants \( K \) and \( J \) to be positive (‘ferromagnetic’), and \( L \geq 0 \). The quantities \( H \) and \( D \) correspond respectively to even and odd linear combinations of the chemical potentials \( \mu_A \) and \( \mu_B \) \( [20] \).

Performing a Gaussian (‘Kac-Hubbard-Stratonovich’) transformation with respect to both \( \{ S_i \} \) and \( \{ S_i^2 \} \), one can map the model (3) exactly on a lattice field theory with fields \( \phi_i \in \mathbb{R} \) and \( \psi_i \in \mathbb{R} \) \( [18] \). To make a continuum approximation, we replace these by smoothly interpolating fields \( \phi(x) \) and \( \psi(x) \), and Taylor expand nearby differences \( \phi_i - \phi_j \) about their midpoint \( (i + j)/2 \). We thus arrive at a continuum model with the Hamiltonian

\[
\mathcal{H}[\phi, \psi] = \mathcal{H}_1[\phi] + \mathcal{H}_2[\psi] + \mathcal{H}_{12}[\phi, \psi] ,
\]

\[
\mathcal{H}_1[\phi] = \int d^d x \left[ \frac{A}{2} (\nabla \phi)^2 + \frac{a_2}{2} \phi^2 + \frac{a_4}{4} \phi^4 - h \phi \right] ,
\]

\[
\mathcal{H}_2[\psi] = \int d^d x \left[ \frac{B}{2} (\nabla \psi)^2 + \frac{b_2}{2} \psi^2 + \frac{b_4}{4} \psi^4 - g \psi \right] ,
\]

\[
\mathcal{H}_{12}[\phi, \psi] = \int d^d x \left[ \psi \left( d_{11} \phi + \frac{d_{21}}{2} \phi^2 \right) + \Delta \psi \left( e_{11} \phi + \frac{e_{21}}{2} \phi^2 \right) \right] .
\]

The \( \phi^3 \) and \( \psi^3 \) terms have been eliminated by shifts \( \phi(x) \to \phi(x) + \phi_0 \) and \( \psi(x) \to \psi(x) + \psi_0 \). Monomials of higher order and higher-order gradient terms have been dropped.

The terms retained in \( \mathcal{H} \) require explanation. Consider, first, the case of a symmetric CEP, in which \( \mathcal{H}[\phi, \psi] = \mathcal{H}[\phi, \psi] \), i.e., \( h = d_{11} = e_{11} = 0 \). Owing
to this symmetry, $\phi$ and $\psi$ do not ‘mix’ and hence may be chosen as the fields that become critical or remain noncritical at the CEP, respectively. To assess the relevance of contributions to $\mathcal{H}$ via power counting, the coefficients $A$ and $b_2$ should be taken dimensionless. Thus $\phi$ has the usual momentum dimension $[\phi] = (d - 2)/2$, while $[\psi] = d/2$. In $\mathcal{H}_1[\phi]$, we have kept all monomials of the standard $\phi^4$ Hamiltonian, namely those (except $\phi^3$) having coefficients with nonnegative momentum dimensions for $\epsilon \geq 0$. For the remaining interaction constants, one finds $[g] = -[e_{21}] = 2 - \epsilon/2$, $[d_{21}] = \epsilon/2$, $[B] = -2$, and $[b_4] = \epsilon - 4$. This suggests that $B$, $e_{21}$, and $b_4$ may be expected to be irrelevant in the RG sense and hence can be set to zero. If we did this, $\mathcal{H}$ would reduce to the Hamiltonian of the dynamic model C [21]; it would be quadratic in $\psi$, so $\psi$ could be integrated out exactly. The resulting effective Hamiltonian would be identical to $\mathcal{H}_1[\phi]$, up to a change of its parameters $a_2$ and $a_4$, and an overall constant.

The terms $\propto B$, $e_{11}$, and $e_{21}$ have been introduced because they play a role in the analysis of inhomogeneous states with a liquid-gas interface[18]. Since our main focus here is on bulk critical behavior, we can indeed set $B = e_{21} = 0$ in the sequel. However, $b_4$ must not be set to zero because, then, we would not be able to describe $\alpha\beta-\gamma$ coexistence, nor would the model have a CEP.

### 3 Landau theory and beyond

Application of the Landau approximation to the model (4)–(7) yields a phase diagram with a CEP and the correct topology, provided its parameter values are in the appropriate range [10,13,18]. With the choices $a_2 < 0$, $d_{21} > 0$ (aside from $a_4 > 0$ and $b_4 > 0$), one finds a critical line with

$$
\psi = \psi_c \equiv -a_2/d_{21}, \quad b_2 > b_{2c} \equiv \frac{d_{21}^2}{2 a_4} - \frac{b_4 a_2^2}{d_{21}^2}, \quad (8)
$$

and $g = g_c = b_2 \psi_c + b_4 \psi_c^3$ that is truncated by the liquid-gas coexistence boundary at the CEP $b_2 = b_{2c}$, $\psi = \psi_c$ (cf. case (a) in Figs. 8 and 9 of [10]). A detailed exposition of the Landau theory, with results for the phase boundaries and equilibrium values of $\phi$ and $\psi$ in the various phases will be given in [18].

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2 For $B > 0$, the mean-field correlation length of $\psi$ and hence the width of the interface region of classical kink solutions for $\psi$ are nonzero. The terms $\propto e_{11}$ and $e_{21}$ are significant for relating the problem of critical adsorption of the $\alpha\beta$-phase at the $\alpha\beta|\gamma$-interface to a wall problem.
To go beyond Landau theory, we use perturbation theory in combination with the RG. Writing \( \psi = \psi_{\text{ref}} + \tilde{\psi} \), we expand \( \mathcal{H} \) about a reference value \( \psi_{\text{ref}} \), which we take as the mean-field value of \( \psi \) at a reference point in the \( \alpha \beta \) phase away from the critical line. This gives

\[
\mathcal{H}[\phi, \tilde{\psi}] = \mathcal{H}[0, \psi_{\text{ref}}] + \mathcal{H}'[\phi, \tilde{\psi}; \psi_{\text{ref}}],
\]

with \( \mathcal{H}'[\phi, \tilde{\psi}] = \int d^d x \left[ \frac{A}{2} (\nabla \phi)^2 + \sum_{k=2,4} \frac{\tilde{a}_k}{k} \phi^k + \sum_{l=1}^4 \frac{\tilde{b}_l}{l} \tilde{\psi}^l + \frac{1}{2} d_{21} \tilde{\psi}^2 \right] \),

\[
\mathcal{H}'[\phi, \tilde{\psi}] = \int d^d x \left[ \frac{A}{2} (\nabla \phi)^2 + \sum_{k=2,4} \frac{\tilde{a}_k}{k} \phi^k + \sum_{l=1}^4 \frac{\tilde{b}_l}{l} \tilde{\psi}^l + \frac{1}{2} d_{21} \tilde{\psi}^2 \right],
\]

where

\[
\tilde{a}_2 = a_2 + d_{21} \psi_{\text{ref}}, \quad \tilde{a}_4 = a_4, \quad \tilde{b}_1 = -g + b_2 \psi_{\text{ref}} + b_4 (\psi_{\text{ref}})^3, \quad \tilde{b}_2 = b_2 + 3b_4 (\psi_{\text{ref}})^2, \quad \tilde{b}_3 = 3b_4 \psi_{\text{ref}}, \quad \text{and} \quad \tilde{b}_4 = b_4, \quad \text{where} \quad \tilde{b}_2 > 0.
\]

If \( \mathcal{H}' \) is taken into account by perturbation theory, the critical and first-order lines, and hence the CEP, get shifted. When studying the behavior near these lines, one must use their corresponding new locations that are compatible with the level of approximation. Suppose that a point \((T_c(g_c), g_c)\) on the critical line is approached, which may be the CEP \((g_c = g_c)\). Let us ignore the \( \tilde{\psi}^3 \) and \( \tilde{\psi}^4 \) terms for the present. Then \( \mathcal{H} \) becomes quadratic in \( \tilde{\psi} \), so that \( \tilde{\psi} \) can be integrated out exactly. The resulting effective Hamiltonian \( \mathcal{H}_{\text{eff}}[\phi] \) is given by \( \mathcal{H}_1[\phi] \), with the replacements \( a_2 \rightarrow \tilde{a}_2 = (b_1 d_{21}/\tilde{b}_2) \) and \( a_4 \rightarrow \tilde{a}_4 = (d_{21}^2/\tilde{b}_2) \), plus a \( \phi \)-independent term \( \int d^d x f_G(\tilde{b}_2, \tilde{b}_1) \) corresponding to the free energy of the Hamiltonian \( \mathcal{H}_G[\tilde{\psi}] = \int d^d x [ (\tilde{b}_2/2) \tilde{\psi}^2 + \tilde{b}_1 \tilde{\psi} ] \). As usual, we may presume that parameters such as \( \tilde{a}_2 \), \( \tilde{a}_4 \) have a Taylor expansion in \( t = (T - T_c(g_c))/T_c(g_c) \) and \( \delta g = (g - g_c)/g_c \) near \((T_c, g_c)\). Both \( \mathcal{H}[0, \psi_{\text{ref}}] \) as well as \( f_G \) are regular at \((T_c, g_c)\). Hence the singular part of the total free energy results solely from \( \mathcal{H}_{\text{eff}} \).

It is instructive to consider these nonlinearities first in the case \( d_{21} = 0 \) of a massive field \( \tilde{\psi} \) decoupled from \( \phi \). Let \( \mathcal{G}_{\text{tra}}[\Psi] \) be the generator of transformations of Hamiltonians \( \mathcal{H}[\tilde{\Psi}] \) induced by the change of variable \( \psi \rightarrow \psi + \Psi \):

\[
\mathcal{G}_{\text{tra}}[\Psi] \mathcal{H}[\tilde{\Psi}] = \int d^d x \left[ \Psi(x) \frac{\delta \mathcal{H}[\tilde{\Psi}]}{\delta \tilde{\Psi}(x)} - \frac{\delta \Psi(x)}{\delta \tilde{\Psi}(x)} \right].
\]

\(^3\) Implicit in our analysis is the well-founded assumption that the CEP, the critical line, and the first-order line \( g_\sigma \) will survive the inclusion of fluctuation corrections.
Choosing
\[ \Psi_k = \frac{\tilde{\nu}_k}{k \tilde{\nu}_2} \left[ \psi^{k-1} + \delta(0) \frac{k-1}{\tilde{\nu}_2} \psi^{k-3} \right], \quad k = 3, 4, \quad (12) \]
gives
\[ \frac{\tilde{\nu}_k}{k} \int d^d x \psi^k = \mathcal{G}_{\text{tra}}[\Psi_k] \mathcal{H}_G[\psi] + \delta_{k,4} c_4 \int d^d x, \quad k = 3, 4, \quad (13) \]
with \( c_4 = -3\tilde{b}_4 [\delta(0)/2\tilde{b}_2]^2 \), where \( \delta(0) = \int d^d q/(2\pi)^d \) is a cutoff-dependent constant. Hence, at the Gaussian fixed point \( \mathcal{H}_G[\psi] \), the \( \psi^3 \) and \( \psi^4 \) terms correspond to a redundant operator, and a redundant operator plus a constant, respectively. More generally, Wegner [22, Sec. III.G.2] has shown that any translationally invariant local operator can be represented as a redundant operator plus a constant at such a noncritical Gaussian fixed point.

To generalize these considerations to the case \( d_{21} \neq 0 \) with \( \tilde{\nu}_1 \neq 0 \), we insert
\[ \mathcal{G}_{\text{tra}}[\Psi_k] \mathcal{H}_G[\psi] = \mathcal{G}_{\text{tra}}[\Psi_k] \mathcal{H}^t[\phi, \psi] - \int d^d x \left( \frac{d_{21}}{2} \phi^2 + \tilde{\nu}_1 \right) \psi_k \quad (14) \]
into (13). The result tells us that, to first order in \( \tilde{\nu}_3 \), the \( \psi^3 \) term is equivalent to shifts of \( \tilde{a}_2, \tilde{b}_2 \), and the constant part of \( \mathcal{H} \), plus a generated \( \phi^2 \tilde{\psi}^2 \) contribution. Likewise, the \( \psi^4 \) term corresponds (to order \( \tilde{b}_4 \)) to the generation of \( \phi^2 \tilde{\psi}^3 \) and \( \psi^3 \) contributions, and shifts of \( d_{21} \) and \( \tilde{b}_1 \). Owing to the high naive dimensions \( 2(d-1) \) and \( (5d-4)/2 \) of the operators \( \phi^2 \tilde{\psi}^2 \) and \( \phi^2 \tilde{\psi}^3 \), we may trust that both produce only irrelevant corrections and hence may be dropped. Consequently, the effects of the \( \psi^3 \) and \( \psi^4 \) terms can be absorbed through shifts of the parameters \( \tilde{a}_2, \ldots, \tilde{b}_1 \) of the Hamiltonian with \( \tilde{\nu}_3 = \tilde{\nu}_4 = 0 \), apart from irrelevant corrections. This means a change of the locations of the critical and first-order lines, and a corresponding adjustment of, e.g., the temperature scaling field. In summary, we arrive at an effective \( \phi^4 \) Hamiltonian \( \mathcal{H}_{\text{eff}}[\phi] \), irrespective of whether the critical line or the CEP is approached.

Next, we turn to the issue of the discontinuity eigenexponent \( y = d \). Consider changes \( g_e \rightarrow g_c = g_e + \delta g \), \( \tilde{a}_{2c} \rightarrow \tilde{a}_{2c} = \tilde{a}_{2c} + \delta \tilde{a}_2 \), away from the CEP such that the theory remains critical. As we have seen above, varying \( g \) (i.e., \( \tilde{b}_1 \)) alters the configuration-independent part of \( \mathcal{H} \), i.e., the coefficient, \( \mu_0 \), of the ‘volume operator’ \( \int d^d x \). Since \( \mu_0 \) trivially scales with the exponent \( d \) under RG transformations, it is formally relevant; but it does not contribute to the critical behavior at continuous phase transitions. Wegner [22] therefore proposed to call it ‘special’ scaling field. The above variation within the critical
manifold implies a change $\delta \mu_0 \sim (T_c - T_e)$ of $\mu_0$. This is the analog of the
eigenperturbation with eigenexponent $y = d$ found at the CEP fixed point in
position-space RG calculations $[5,6]$.

The above analysis of the symmetric CEP can be extended to the 
\textit{nonsymmetric} case, in which the $\phi \rightarrow -\phi$ symmetry of $\mathcal{H}[\phi, \psi]$ is broken $[18]$. As expected,
the principal modifications are of a geometrical nature: the first-order surface
bounded by the critical line is no longer confined to the $h = 0$ plane, and the
ordering field $h$ and the temperature variable $t$ mix in the scaling fields.

Our analysis confirms that the singular part of the free energy of the $\alpha\beta$ and
$\alpha + \beta$ phases has the usual scaling form anticipated in the phenomenologi-
cal scaling theory, both on the critical line and at the CEP. Hence it is clear
that the limiting forms (1) and (2) must hold and can be derived in a similar
fashion as in phenomenological investigations $[2,12]$. We will therefore restrict
ourselves here to a few remarks. The singularity (1) in $g_{\sigma}(T)$ follows by ex-
ploring the equality of the free energies (grand potential) of the liquid ($\alpha\beta$, 
$\alpha$, $\beta$) and the spectator ($\gamma$) phases at coexistence. To derive the behavior of
the nonordering density $\rho_g = \langle \psi \rangle$ near the CEP, we have generalized the field-
theoretic RG analysis $[23]$ to the $g$-dependent model-C-type Hamiltonian that
result in the symmetric and nonsymmetric cases. In the nonsymmetric case $\psi$
do not only couple to the energy density ($\sim |t|^{1-\alpha}$), but also to the order
parameter, which produces the additional $|t|^{\beta}$ singularity in (2).

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\textbf{References}

[1] R. B. Griffiths, J. Chem. Phys. 60 (1973) 195.

[2] M. E. Fisher, In D. Caldi and G. D. Mostow, editors, \textit{Proceedings of the
Gibbs Symposium, Yale University, 1989} 39–72 Rhode Island 1990. American
Mathematical Society.

[3] M. E. Fisher and M. C. Barbosa, Phys. Rev. B 43 (1991) 11177.

[4] M. E. Fisher and P. J. Upton, Phys. Rev. Lett. 65 (1990) 2402; Phys. Rev.
Lett. 65 (1990) 3405.
[5] A. N. Berker and M. Wortis, Phys. Rev. B 14 (1976) 4946.
[6] M. Kaufman, R. B. Griffiths, J. M. Yeomans and M. E. Fisher, Phys. Rev. B 23 (1981) 3448.
[7] B. Widom, J. Chem. Phys. 67 (1977) 872; Chem. Soc. Rev. 14 (1985) 121.
[8] M. C. Barbosa and M. E. Fisher, Phys. Rev. B 43 (1991) 10635.
[9] M. C. Barbosa, Physica A 177 (1991) 153.
[10] D. Roux, C. Coulon and M. E. Cates, J. Phys. Chem. 96 (1992) 4174.
[11] B. Groh and S. Dietrich, In C. Caccamo, J. P. Hansen and G. Stell, editors, New Approaches to Old and New Problems in Liquid State Theory: Inhomogeneities and Phase Separation in Simple, Complex and Quantum Fluids 147 Dordrecht 1998. Kluwer.
[12] N. B. Wilding, Phys. Rev. Lett. 78 (1997) 1488; Phys. Rev. E 55 (1997) 6624; Phys. Rev. E 58 (1998) 2201.
[13] C. Domb and M. S. Green, editors, Phase Transitions and Critical Phenomena volume 6, (Academic Press, London, 1976).
[14] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena, International series of monographs on physics. (Clarendon Press, Oxford, 1996) 3rd edition.
[15] T. A. L. Ziman, D. J. Amit, G. Grinstein and C. Jayaprakas, Phys. Rev. B 25 (1982) 319.
[16] B. Nienhuis and M. Nauenberg, Phys. Rev. Lett. 35 (1975) 477.
[17] M. Smock and H. W. Diehl, to be published; M. Smock, Doktorarbeit, U. Essen, Essen, May 1999.
[18] M. Blume, V. J. Emery and R. B. Griffiths, Phys. Rev. A 4 (1971) 1071.
[19] J. Sivardière and J. Lajzerowicz, Phys. Rev. A 11 (1975) 2090.
[20] B. I. Halperin and P. C. Hohenberg, Rev. Mod. Phys. 49 (1977) 435.
[21] F. J. Wegner, In C. Domb and M. S. Green, editors, Phase Transitions and Critical Phenomena volume 6 chapter 2, 7–124. Academic Press London 1976.
[22] E. Brézin and C. de Dominicis, Phys. Rev. B 12 (1975) 4954.