Conductance Characteristics between a Normal Metal and a Superconductor Carrying a Supercurrent

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The low-temperature conductance (G) characteristics between a normal metal and a clean superconductor (S) carrying a supercurrent \( I_s \) parallel to the interface is theoretically investigated. Increasing \( I_s \) causes lowering and broadening of (1) coherence peaks of s-wave S, and d-wave S at (100) contact, (2) midgap-states-induced zero-bias conductance peak for d-wave S at (110) contact, and (3) Andreev-reflection-induced enhancement of \( G \) within the gap near the metallic-contact limit. Novel features found include a current-induced central peak and a three-humped structure at intermediate barrier strength, etc.

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It is well-known that Andreev reflection plays a fundamental role in understanding the transport properties of a normal metal/superconductor junction (NSJ) [1]. From the current-voltage (I-V) characteristics of a clean NSJ with an intermediate barrier strength, etc. allows a dimensionless barrier-strength parameter \( z \> \frac{1}{\sqrt{\pi}} \) to range from metallic contact, \( z = 0 \), to the tunneling regime, \( z \gg 1 \). There only conventional s-wave symmetry for S is considered. Recently, much attention has been paid to the conductance characteristics of d-wave, cuprate S in both theory and experiment [3-13]. The low-temperature conductance (G) characteristics between a normal metal and a clean superconductor junction, one component excitation spectrum and its order-parameter symmetry, etc. Blonder et al. have developed a general theory [2] for studying I-V and G(V) of an NSJ that allows a dimensionless barrier-strength parameter \( z \) to range from metallic contact, \( z = 0 \), to the tunneling regime, \( z \gg 1 \). There only conventional s-wave symmetry for S is considered. Blonder et al. [2]. Contrary to Ref. [14], this work is not limited to large \( z \). We consider both s-wave and d-wave S with (100) and (110) contacts. Some novel results are obtained, especially for \( z \approx 1 \), when \( G(V) \) does not simply reflect the thermally-smeared quasi-particle density of states. Hopefully, these predictions can be confirmed experimentally. Unlike Ref. [14], the present work does not consider Coulomb blockade, which is presumably not so important in an extended system and in the clean limit. As in Ref. [14], we also assume a uniform \( I_s \), and neglect self-field.

When a uniform \( I_s \) passes through a conventional three-dimensional s-wave S, the phase of \( \Delta(k) \) has a spatial variation of \( 2q_x \cdot x \), where \( x \) is the center-of-mass position of a Cooper pair, \( q_x = (m_s/2)\nu_x \), with \( \nu_x \) the supercurrent velocity, and \( m_s \) the mass of a Cooper pair. \( \hbar = 1 \) is assumed throughout this work.) At temperature \( T = 0 \), the magnitude of the order parameter \( \Delta_q \) stays unchanged until the Landau criterion is satisfied (i.e., \( q \geq 0.5\Delta^0 \), where \( \Delta^0 = \Delta_0/E_F \). Here \( \Delta_0 \) is the superconducting gap when \( I_s = 0 \), \( k_F \) and \( E_F \) are the Fermi momentum and energy, respectively. When \( q \geq 0.5\Delta^0 \), S becomes gapless, and quasi-particles are generated in a portion of the Fermi surface [15]. We shall see that this can lead to a ZBCP in \( G(V) \) for an N/(s-wave S) junction with the barrier strength \( z \approx 1 \). This current-induced ZBCP is always quite broad and not very tall, and its height decreases for larger \( z \). It is therefore characteristically different from the ZBCP induced by the midgap surface states in a d-wave S with non-(n0m) contact that is narrower and taller for larger \( z \) [4]. The midgap-states-induced ZBCP has been ubiquitously observed in high-\( T_c \) cuprates and other unconventional Ss.)

As \( q \) is increased further, \( \Delta_q \) gradually decreases to zero at \( q = 0.67\Delta^0 \) [Fig. 1(a)]. The supercurrent density quickly reaches a peak (the thermodynamic critical current density) at \( q = q_c = 0.515\Delta^0 \) [Figs. 1(c)] [16].
FIG. 1: Dependence of the superconducting order parameter on the normalized supercurrent-velocity parameter \( q \) for (a) an s-wave and (b) a d-wave S. (\( \phi \) is the angle between the supercurrent and the antinodal direction in the latter case.) In (c) and (d), the corresponding dependences of supercurrent density on \( q \) are given.

The region \( q > q_c \), in which \( I_s \) is a decreasing function of \( q \), is unstable and cannot be observed experimentally. (For a two-dimensional s-wave S, superconductivity disappears immediately after the Landau criterion is met. Then \( q_c = 0.5\Delta^0 \).)

Different from that in an s-wave S, the \( \Delta^0 \)-vs-q relation in a d-wave S also depends on the direction of the supercurrent. (Here \( \Delta^0 \) denotes the maximum gap in the presence of \( I_s \).) For a two-dimensional d-wave S with a supercurrent, the gap-function order parameter at \( T = 0 \) is described by [17]

\[
\pi \ln \frac{\Delta^0}{\Delta^0} = \int_0^\infty d\theta \cos^2(2\theta) \ln(g + \sqrt{g^2-1}),
\]

where \( g = \frac{2q}{\Delta^0} \cos(\theta - \phi) \), \( \Delta^0 = \Delta^0/E_F \), \( \phi \) is the angle between the supercurrent and the antinodal direction, and the integral in Eq. (1) is from 0 to 2\( \pi \) with the constraint \( g^2 - 1 \geq 0 \).

Figure 1(b) shows the dependence of the d-wave \( \Delta^0 \) on \( q \) at \( \phi = 0 \) and \( \pi/4 \). We can see that when \( q \) is less than \( \sim 0.3\Delta^0 \), the changes of the order parameter with \( q \) in both the antinodal and nodal directions are almost the same. However, a great difference exists for larger \( q \). When \( I_s \) is applied along the antinodal direction, \( \Delta^0 \) has a sharp drop (from 0.88\( \Delta^0 \) to 0.58\( \Delta^0 \)) between \( q = 0.384\Delta^0 \) and 0.385\( \Delta^0 \). After that it drops continuously to zero at \( q = 0.53\Delta^0 \). When \( \phi = \pi/4 \), \( \Delta^0 \) gradually decreases to 0.689\( \Delta^0 \) at \( q = 0.469\Delta^0 \), and has no solution beyond. Fig. 1(d) gives the corresponding dependences of the supercurrent density on \( q \) [17]. It is seen that the thermodynamic critical current is reached at \( q = q_c = 0.35\Delta^0 \) (0.39\( \Delta^0 \)) for current in the antinodal (nodal) direction.

The elementary excitations in S are governed by the time-independent Bogoliubov-de Gennes equations [18]:

\[
Eu(x) = h_0 u(x) + \int dx' \Delta(s, r) v(x'), \quad (2a)
\]
\[
Ev(x) = -h_0 v(x) + \int dx' \Delta^*(s, r) u(x'), \quad (2b)
\]

where \( s = x - x' \), \( r = \frac{1}{2}(x + x') \), and \( h_0 = -\frac{q^2}{2m} + U\delta(x) - \mu \) with \( \mu \) the chemical potential. It is useful to express the superconducting order parameter in the form: \( \Delta(s, r) = \int dk e^{i k \cdot s} \Delta(k, r) e^{2i q k \cdot r} \) [3]. Neglecting the proximity effect near the N/S interface at \( x = 0 \), we have \( \Delta(k, r) = \Delta(k)\Theta(x) \), where \( \Theta(x) \) is a step function, and \( \Delta(k) \) is the order parameter of a bulk S in the presence of \( I_s \).

In the WKBJ approximation, Eqs. (2) have special solutions of the form

\[
\begin{pmatrix}
u & \bar{u} \\ \bar{v} & u
\end{pmatrix} = e^{ik_F \cdot x} \begin{pmatrix} e^{i q \cdot x} \bar{u} \\ e^{-i q \cdot x} \bar{v}
\end{pmatrix},
\]

where \( \bar{u}(x) \) and \( \bar{v}(x) \) obey the generalized Andreev equations [1]:

\[
(E - \frac{q^2}{2m} - \frac{q_s \cdot k_F}{m}) \bar{u} = -i\frac{(k_F + q_s)}{m} \nabla \bar{u} + \Delta(k) \Theta(x) \bar{v},
\]

\[
(E + \frac{q^2}{2m} - \frac{q_s \cdot k_F}{m}) \bar{v} = i\frac{(k_F - q_s)}{m} \nabla \bar{v} + \Delta^*(k) \Theta(x) \bar{u}.
\]

Obviously, the eigenenergy \( E \) is symmetric about \( E = q_s \cdot k_F / m \). When \( q_s \) is applied parallel to the interface of the NSJ, i.e. \( q_s = -q_s \epsilon_y \), we have

\[
\begin{pmatrix}
u \bar{u} \\ \bar{v}
\end{pmatrix} = e^{i\alpha x} \begin{pmatrix} u_{\nu} \\ \bar{u}_{\nu}
\end{pmatrix} \quad (\text{for } x > 0),
\]

\[
\begin{pmatrix}
u \bar{u} \\ \bar{v}
\end{pmatrix} = \begin{pmatrix} e^{i\beta x} u_{\nu} \\ e^{i\gamma x} \bar{u}_{\nu}
\end{pmatrix} \quad (\text{for } x < 0),
\]

where \( \nu = \text{sign}(k_{Fx}) \), \( \alpha_{\nu} = -\nu q_s^2/2 + m A_{\nu} \) (\( |k_{Fx}| \), with \( A_{\nu} = \sqrt{(E + q_s k_{Fy}/m)^2 - \Delta^0(k_F) \Delta^*(k_F)} \), \( \beta_{\nu} = \nu q_s k_{Fy}/|k_{Fx}| \), \( \gamma_{\nu} = -m(v_{\nu} q_s^2/2m + E + q_s k_{Fy}/|k_{Fx}|) \), \( u_{\nu}^{(+)(<)} \) and \( \bar{u}_{\nu}^{(+)(<)} \) are constants. For example, in S, we have \( B_{\nu} = u_{\nu}^{(+)} / \bar{u}_{\nu}^{(-)} = \Delta(k_{Fx})/(E + q_s k_{Fy}/m - \nu A_{\nu}) \).

Following Ref. [2], after a tedious but straightforward calculation, we obtain the Andreev and normal reflection coefficients, \( a(E) \) and \( b(E) \):

\[
a(E) = \frac{2q_s (k_+ + k_-)}{B_-(k_- + q_+ + 2i m U)(k_+ - q_- + 2i m U) - B_+(k_+ + q_+ + 2i m U)(-k_- - q_- + 2i m U)},
\]
Here \( q_+ = |k_{Fx}| + \beta_+, q_- = |k_{Fx}| + \gamma_+ \), and \( k_0 = |k_{Fx}| + \nu_0 \). The critical supercurrent velocity is much less than the Fermi velocity. So the Andreev approxima-

tion, \( q_+ \approx k_\pm \approx |k_{Fx}| \), also holds in the presence of a supercurrent. The normalized conductance can then be calculated according to a formula given in Ref. [2]:

\[
G = \frac{G_s}{G_n}, \quad G_n = -\frac{e^2}{\pi} \int_{-\infty}^{+\infty} dE \int_{-\pi/2}^{\pi/2} d\theta \frac{\partial f(E-eV)}{\partial E} [1 - |b(+\infty)|^2],
\]

\[
G_s = -\frac{e^2}{\pi} \int_{-\infty}^{+\infty} dE \int_{-\pi/2}^{\pi/2} d\theta \frac{\partial f(E-eV)}{\partial E} [1 + |a(-E)|^2 - |b(E)|^2],
\]

where \( |k_{Fx}| = k_F \cos \theta \), \( f(E) \) is the Fermi distribution function, \( G_n \) and \( G_s \) are the differential conductance for \( S \) in the normal and superconducting states, respectively.

**S-wave superconductor.** In this case, the superconducting order parameter \( \Delta_0(k_F) = \Delta_q \) is independent of \( \nu \).

In Fig. 2, \( G(V) \) at various \( q = 2mU/k_F \) is plotted. [We have used \( k_BT = 0.01E_F \) and \( \Delta_0 = 0.1E_F \).] When \( z = 0 \) and \( q = 0 \), electrons entering with all momenta \( k_F \) with \( k_{Fx} > 0 \) can enter \( S \) and equal number of holes at opposite momenta are retro-reflected into \( N \) if \( |eV| < \Delta_0 \). So the normalized conductance \( G = 2.0 \) within the superconducting gap if \( T = 0 \). With increasing \( q \), the range of \( G = 2.0 \) diminishes and the \( G(V) \) curve turns into a nearly triangular peak centered at zero bias [Fig. 2(a)]. At large \( z \) [Fig. 2(d)], the coherence peaks are suppressed and broadened with increasing \( q \), but contrary to the case of a diffusive superconducting wire [14], here the peaks of \( G(V) \) move outward while the

gap shrinks. The intermediate- \( z \) results are even richer in behavior [Figs. 2(b) and (c)]: A fairly broad and not very tall peak appears at zero bias and a three-humped struc-
ture can also appear for nearly critical \( q \). Note that the larger is \( z \), the lower is this current-induced ZBCP. The area under this peak is also not conserved as \( z \) changes.

These features are characteristically different from the ZBCP induced by the midgap surface states in d-wave \( S \) with non-(\( n \)0\( m \)) contacts. [4]

For electrons entering an NSJ at a fixed incident angle \( \theta \), a ZBCP would result from their contributions to the normalized conductance if \( 2q|\sin \theta| > \Delta^0 \) is satisfied. Thus, one can see this peak only if \( q > 0.5\Delta^0 \) is satisfied. For \( 0.5\Delta^0 < q < 0.67\Delta^0 \), there is a critical angle \( |\theta_c| = \arcsin(\Delta^0/2q) \), which decreases from 90° to 48.3° in this range. No ZBCP is induced by electrons with incident angle \( |\theta| < |\theta_c| \). However, only a small portion of this regime can be observed, because only the region \( q \leq 0.515\Delta^0 \) is stable.

**D-wave superconductor.** In this case, the pair potential has the form \( \Delta_0(k_F) = \Delta_q \cos(2\theta_\nu) \). Here, \( \theta_\nu = \theta + \nu \alpha \), \( \alpha \) is the angle between the antinodal direction and the positive \( x \) axis.

Figure 3 presents the normalized conductance at different \( z \) and \( q \) for a d-wave \( S \) with (100) contact (i.e. \( \alpha = 0^0 \)). For \( z = 0 \) [Fig. 3(a)], the central peak due to Andreev reflection is gradually suppressed and slightly broadened. For large \( z \) [Fig. 3(d)] one sees mainly the filling up of the central dip with only a slight suppression of the coherence peaks as \( q \) increases. For intermediate \( z \) [Figs. 3(b) and (c)], one sees intricate behavior with some similarity to the corresponding figures in Fig. 2.

Figure 4 shows the normalized conductance at different \( z \) and \( q \) for a d-wave \( S \) with (110) contact (i.e. \( \alpha = 45^0 \)). It is seen that the ZBCP induced by the midgap surface
at which the thermodynamic critical current is reached.

\[ q \approx 3. \] The age for a normal metal/d-wave superconductor junction with (100) contact: (a) \( z = 0 \), (b) \( z = 0.5 \), (c) \( z = 1.0 \), and (d) \( z = 5.0 \). Red: \( q = 0 \), green: \( q = 0.2\Delta_0 \), and blue: \( q = 0.35\Delta_0 \), at which the thermodynamic critical current is reached.

FIG. 4: The normalized differential conductance vs voltage for a normal metal/d-wave superconductor junction with (100) contact. The \( z \) values considered are the same as in Fig. 3. The \( q \) values considered are: Red: \( q = 0 \), green: \( q = 0.2\Delta_0 \), and blue: \( q = 0.39\Delta_0 \), at which the critical current is reached.

states is suppressed, broadened, and eventually split at sufficiently large \( z \) when \( q \) is increased.

In conclusion, we have studied the differential conductance of a clean normal metal/superconductor junction carrying a supercurrent parallel to the junction interface, for barrier strength ranging from metallic-contact to the tunneling regime. In the tunneling regime, we obtain results similar to the case of a diffusive s-wave superconducting wire studied recently, viz., suppression and broadening of the coherence peaks for both an s-wave superconductor and a d-wave superconductor with (100) contact, except that the coherence peaks are found to move outward in the s-wave case. For d-wave superconductor with (110) contact we also find the midgap-surface-states-induced ZBCP to be suppressed and broadened and eventually split with increasing supercurrent. In the metallic-contact limit, supercurrent causes the Andreev-reflection-induced conductance enhancement within the (maximum) gap to become weakened and broadened. For intermediate barrier strengths some novel features are revealed including a current-induced zero-bias peak and a three-humped structure near the thermodynamical critical current density. It is hoped that these predictions can be observed experimentally. We conclude with the remark that this formulation can also be applied to the case of an d+s superconductor. Because the critical current for an s-wave superconductor is larger than that for a d-wave one, the existence of an s component can be verified by a supercurrent reaching a magnitude between the critical values of the two waves.

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