Rotating Black Hole sitting in Quintessence: Properties of Accretion

Ritabrata Biswas \textsuperscript{1} \hspace{1cm} Parthajit Roy \textsuperscript{2*}

\textsuperscript{1} Department of Mathematics, The University of Burdwan
Golapbag Academic Complex, City:Burdwan-713104,
Dist: Purba Bardhaman, State: West Bengal, India
\hspace{1cm} e-mail: biswas.ritabrata@gmail.com

\textsuperscript{2}Department of Computer Science,
The University of Burdwan
City:Burdwan-713104,
Dist: Purba Bardhaman, State: West Bengal, India
\hspace{1cm} e-mail: roy.parthajit@gmail.com

\textsuperscript{*}Corresponding Author:Parthajit Roy, roy.parthajit@gmail.com

Abstract

Viscous accretion flow around a rotating supermassive black hole sitting in a quintessence tub is studied in this article. To introduce such a dark energy contaminated black hole’s gravitation, pseudo-Newtonian potential is popularly used. Transonic, viscous, continuous and Keplerian flow is assumed to take place. Fluid speed, sonic speed profile and specific angular momentum to Keplerian angular momentum ratio are found out for different values of spinning parameter and quintessence parameter. Density variation is built and tallied with observations. Shear viscosity to entropy density ratio is constructed for our model and a comparison with theoretical lower limit is done.

Keywords: Accretion Disc, Viscosity, Quintessence, Supermassive Black Holes, Shear Viscosity to entropy ratio

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1 Introduction

Since 1998, despite the propositions of different conceptual models, we were not really able to construct a hundred percent correct theoretical support to the observations of late time cosmic acceleration. Stress energy tensor was redesigned to interpret the accelerated expansion via its abhorrent force. The names of candidates of such a repulsive energy/ hypothetical matter are familiar as dark energy (DE) \cite{1}. Molecular structures of DE were not revealed. This is the foremost cause we knew very less regarding the viscous nature of DE. Now a days, there are indirect ways to anticipate viscosity of the dark sector.
Table 1: Luminosity Distances of different GW events

| GW Event   | Luminosity Distance (MPc) | $\beta$ ($10^{-3}MPc^{-1}$) |
|------------|---------------------------|----------------------------|
| LVT 151012 | 1000$^{+500}_{-500}$     | 1.29                       |
| GW 170104  | 880$^{+150}_{-390}$       | 1.40                       |
| GW 170814  | 540$^{+150}_{-210}$       | 1.25                       |
| GW 151226  | 440$^{+180}_{-190}$       | 2.39                       |
| GW 150914  | 410$^{+190}_{-180}$       | 2.50                       |
| GW 170608  | 340$^{+140}_{-140}$       | 3.05                       |
| GW 170817  | 40$^{+8}_{-14}$           | 14.08                      |

But, before speaking about those indirect ways, we should focus on some references [2] [3] where dark matter(DM)-DE interactive models are proposed to support cosmic acceleration and viscosity of DE. These references have considered the phenomena of “delayed decay of cold DM” (from initial matter dominated flat cosmology with $\Lambda = 0$) into light undetectable relativistic species. This generates bulk viscosity and a $\Lambda = −1$ epoch near the neighborhood of present time. Deceleration for this model is found to be faster due to the presence of the matter content. Also the model is found to be consistent with the surveys of supernova magnitude-redshift relation [4] [5] and ages from the superannuated stars and globular clusters.

Now, we will focus on the “indirect ways” to realize and measure the viscous properties of DE. To do so, we will take the help of gravitational wave (GW) signals. It is quite possible to constrain the constituent present in our universe by observing the nature of propagation of GW through them. Till date, GW signals from several black hole(BH) binary mergers like GW150914, LVT151012, GW51226, GW170104, GW170608, GW170814 etc and GW signals from neutron star binary like GW170817 etc are announced by LIGO and Virgo collaborations [6] [7] [8] [9] [10] [11] [12]. Simultaneously, electromagnetic (EM) radiations coming out of the same sources have been detected. Tallying these, we are able to measure the arrival delay between the EM photons and GWs through the cosmological distances [13] [14] [15]. Different references like [16] [17] [18] predict that GWs should propagate freely without any absorption and dissipation if perfect fluid medium is considered to be embedded in Friedmann-Robertson-Walker universe is considered. Nevertheless, this scenario changes as soon as the fluid content is chosen to be non-ideal type [19], GW gets dissipated with a damping rate $\beta \equiv 16\pi G N \eta$ [20] [21] [22] when an amount of shear viscosity $\eta$ is incorporated into the fluid’s energy momentum tensor, $\tilde{G}_N$ being the Newtonian gravitational constant. So, changes in $\beta$ which can be noticed in GW attenuation indicate changes in the value of $\eta$ over time or over cosmic distances. This is nothing but the evolution of viscosity, especially the shear viscosity, of DE over time. The authors of the reference [23] present a statistics of different GW events, median value of source luminosity distances with 90% credible intervals in $MPc$ units and upper limit on the damping rate $\beta$ at 95% of confidence level in units of $10^{-3} M$ given by table 1.

It is clear from these data that as we look through longer distances, i.e., we look in past, the shear viscosity reduces. We are able to conclude that DE, as time grows, exerts more and more amount of shear viscosity.

Regarding bulk viscosity of DE, especially for generalized Chaplygin gas type model, we can find several works. The reference [24] uses the then-available cosmic observational data from SNLS3, BAO, HST and Planck and constrains the value of bulk viscosity coefficient as,

$$\zeta_0 = 0.0000138^{+0.00000614+0.0000145+0.0000212}_{-0.00000105-0.0000138-0.0000138}$$

However, shear and bulk viscosity are even related [25].
Viscous effects of a fluid are more realizable when it flows; particularly layer by layer. Most prominent examples are the narrow X-ray binaries where an accretion disc is likely to get formed. The most simple diagram of such a Roche lobe overflow incident can be imagined with some assumptions: Consider that the flow is axis-symmetric, i.e., causing a cylindrical structure around a compact star—preferably a black hole (BH)—where $r$, $\phi$, and $z$ are the coordinates. We will consider $\frac{\partial}{\partial \phi} \equiv 0$ and also a stationary disc, i.e., $\frac{\partial}{\partial t} \equiv 0$. We will further consider a thin disc, i.e., $h(r) \ll r$, where $h(r)$ is the disc height. The viscous effect of the disc is also taken to be small, i.e., the radial inward velocity $v_r \ll$ the local Keplerian rotational speed/azimuthal velocity $v_\phi = \Omega r = \frac{GM}{r}$, if the radial momentum is conserved. On the other hand, conservation of angular momentum requires effects of viscous forces to take into account. Keplerian balance always implies differential rotation. A transportation of angular momentum in the direction, perpendicular to the velocity, should be observed due to the differences in velocities at different locations. If this is absent, translational (shear) viscosity will turn down to zero. To simplify the nonzero viscosity, a simplistic description of the physics of accretion disc can be obtained by the $\alpha_{ss}[26]$ prescription. Though the accretion driven/driving viscosity is of magnetic origin, it is popular to use an effective hydrodynamic description of the related disc presented by the hydrodynamic stress tensor as $[27]$

$$\tau_{r\phi} = \rho \nu \frac{\partial v_\phi}{\partial r} = \rho \frac{d\Omega}{d\ln(r)} ,$$

(1)

where $\rho$ and $\nu$ are the density and kinematic viscosity coefficient respectively. Notifying total thermal pressure by $P$, isothermal sound speed $\sqrt{\frac{P}{\rho}}$ by $c_s$ and introducing a regulating parameter $\alpha_{ss}(\leq 1)$, Shakura and Sunyaev $[26]$ proposed the prescription

$$\tau_{r\Phi} = \alpha_{ss} P \Rightarrow \gamma = \alpha_{ss} c_s^2 \left[ \frac{dr}{dh} \right]^{-1} .$$

(2)

For Keplerian angular velocity $\Omega = \Omega_k = (\frac{GM}{r^2})^{-1}$, we have,

$$\gamma = \frac{2}{3} \alpha_{ss} c_s^2 / \Omega_k .$$

(3)

To keep the equilibrium, the gravitational force is counter acted by the force produced by the pressure gradient

$$\frac{\partial p}{\partial z} = \rho, \quad g_z = \rho \frac{G_N M}{R^2} \times \frac{z}{R}$$

(4)

and as $h(\lambda) \ll r$, we obtain

$$\frac{h}{r} \approx \frac{c_s}{v_k} \Rightarrow \gamma \approx \frac{2}{3} \alpha_{ss} c_s h .$$

This will be the way to replace the kinematic viscosity by $\alpha_{ss}$ parameter.

The value of $\alpha_{ss}$ is typically assumed to lie between 0.01 to 0.1 $[28]$. Fromang et al. $[29]$ have found a radially varying $\alpha_{ss}$, the overall size of which was over an order of magnitude lower, peaking at 0.013 and declining to below 0.002.

It is clear that, to measure viscosity, we will require to know the variation of density as well. Again accretion density and many other properties are dependent on the spin parameter of the central gravitating black hole (BH). Fink $[30]$ found the spin parameter $a$ for Arakelian 120 galaxy in the constellation of Orion at coordinates $\alpha_{J2000.0}0^h16^m11.395^s \delta_{J2000.0}00^\circ5'9.65''$ to be $a = 0.99^{+0.003}_{-0.004}$. But Turner et.al. $[2]$ shows the range to be $0.996 \leq a \leq 0.998$. Again for the same Seyfert I galaxy Fink measured the number density $n$ of the accretion disc to be $10^{15} cm^{-3}$. Reference $[2]$ has also measured it as $10^{15.95} cm^{-3}$. Mean
molecular weight of Sun is chosen as 0.62 (ionized gas) and hence the density of the taken accretion disc is found to be \( \propto 6.2 \times 10^{15} \text{gm cm}^{-3} \).

Primary physical model of spherically symmetric gas accretion falling onto an astrophysical object was studied by Bondi for the first time. If rotation of the accreting fluid is not taken into account, effective accretion begins from a characteristics radius (So called Bondi radius, given by \( R_B = \frac{GM}{c_s^2} \), \( c_s \) being the sound speed through the gas.) by dominating the thermal energy by negative gravitational energy. References like [10.1093/mnras/stn276] has considered the density distribution to follow \( \rho \propto \left(1 + \frac{R_B}{r}\right)^{3/2} \).

Bondi or Shakura-Sunyaev considered accretion dynamics by considering Newtonian potential. The essential general relativistic effects on the curvature, i.e. the gravity around a BH was not taken into account. The later work has considered the only general relativistic effect by truncating the innermost edge of the disc at the last stable orbit of the Schwarzschild geometry. Novikov and Thorne [31] have developed a complete general relativistic description of a thin Keplerian disc [32].

To reduce general relativistic non-linearity, it is helpful to consider stationary flow and to replace the general relativistic effect by the introduction of pseudo Newtonian potentials. Paczynsky & Witta [33] proposed such a force which exactly reproduces marginally stable orbit and marginally bound orbit of that infall GR. But this potential does consider only BH’s mass. As almost all the celestial objects are rotating, the BHs are also rotating and Mukhopadhyay (2002) [34] developed a PNP for a rotating BH for the first time. Sarkar and Biswas [35] has constructed a PNP for a rotating BH embedded in quintessence. This model proposes at most 4.95% error as compared to GR results. Roy and Biswas have modeled an accretion structure with Sarkar & Biswas’s potential. For this model, the first requisite cases should be known why accretion onto such a quintessence contaminated BH.

This is obvious to consider supermassive black holes (SMBHs) in the center of galaxies through the cases of such a presence is not justified. We are able to observe SMBHs at redshift \( z = 7.54 \) which must have formed within less than one billion years [36]. Alternative models to BHs, inclusion of extended objects in classical general relativity [37], consideration of existence of more exotic models, viz “naked singularity” [38] have been considered. So far the motivations of these works were to consider only the gravitational effects of alternatives to BHs and to find out their observational properties in order to distinguish a BH from a so called BH mimicker. Recently, in the references like [39], a possibility of DM, in the form of bosons, to form self gravitating bound structure in different galaxies are studied. Authors of [40] have compared the motion of test particles in the gravitational field of both SMBH and DM core. A significant discrepancy in the motion is noticeable 100 AU and this increases as we are approaching to the center.

Finer observations in-fature (Say VLBI, BH cam project etc) might be able to distinguish the shadows caused by BH and BH mimicker. As of now, we cannot exclude the idea of existence of SMBH candidates like gravastars or boson stars etc. These studies.realizations motivates us to consider quantum contaminated BHs. Besides DM clustering are chosen to be the cause of formation of different structures of universe, specially the galaxies. DM and DE interacts. As mentioned earlier DE and bulk viscosity even can be formed out of the delayed DM decay. As a result, we can expect the presence of DE at the vicinity of the core area of SMBHs. This motivates us to study the viscous accretion onto quantum contaminated SMBHs.

Another motivation for the present work must be mentioned here. While studying the accretion and wind properties, we see for adiabatic fluid wind branches are almost parallel to \( x \) axis in \( u - x \) plane while we go far from the central BH. On the contrary, the wind branch turns to be parallel to \( u \) axis while modified Chaplygin gas is accreting (References [41] and [42]). These two extremely inclinedness are not smoothly changed at all. But no change in the physical constrain leads to such a drastically diversified solutions. So, there must exist some “missing links” between the two kinds of terminal cases(i.e. adiabatic and MCG flow). If even we are succeeded to find them, what should be the related nature of the density variations and the corresponding thermodynamics is more interesting.
point. We will try to find out the answers in the subsequent sections.

The rest of the paper is organized as follows. In section 2, first we reorganize the structure of the PNF for a rotating BH embedded in quintessence universe. Then we construct the mathematical problem for our model. In subsection 3.1, we find solutions for radially inward speed, speed of sound and specific angular momentum as function of radial distance from the BH. We thoroughly analyze these curves as well. In subsection 3.2, we find the variation of densities of accretion and wind for different parameters and have explained them. Subsection 3.3 deals with the study of the ratio of shear viscosity to entropy density for our model. In section 4, we concluded in brief.

2 Mathematical Modeling of Viscus Accretion onto Rotating SMBH

2.1 PNF Equations

Assuming
\[ \zeta(x) = aA_q x^{3\omega_q} \quad \text{and} \]
\[ \eta(x) = x^{3(\omega_q - 1)} \quad \text{along with} \]
\[ \alpha(x) = \zeta(x)\{3\omega_q(a^2 + x^2) + 3x^2 - 8x + a^2\}, \]
\[ \beta(x) = (2a^2 + 6x - 8)\zeta^2(x)(A_q^{-2}/a) - 2aA_q^2 x \quad \text{and} \]
\[ \gamma(x) = 2\eta^4(x)\{(x^2 - 2x + a^2)x^{3\omega_q} - A_qx\}^2(\eta^{-3}(x))\{A_q(3\omega_q + 1) + 2\zeta(x)/(aA_q)\} \]

and we construct the numerator the PNF as
\[ N(x) = \{\alpha(x) + \beta(x) - \sqrt{\gamma(x)}\}^2. \]

Next we again assume
\[ \phi(x) = a\zeta(x)\{3\omega_q + 1\} + 2a^2x^{6\omega_q} \quad \text{and} \]
\[ \psi(x) = \zeta(x)x^{-3\omega_q}a^{-1} + (2 - x)x^{3\omega_q} \]

and the denominator of the PNF os formed as
\[ D(x) = x^3\{\phi(x) + 2x\psi^2(x)\}^2. \]

Finally, we write our PNF as
\[ F_x = N(x)/D(x) \]

2.2 Sound and Fluid Speed Equations

In this section, we will construct the mathematical model from references [43] and [44]. First we will consider the continuity equation,
\[ \frac{\partial \rho}{\partial z} + \nabla \cdot (\rho \hat{V}) = 0 \]
which is simplified to:
\[ \frac{d}{dx} (x u \Sigma) = 0 \]  
for stationary and cylindrical structure, where \( \Sigma \) is vertically integrated density expressed as
\[ \Sigma = I_C \rho_e h(x), \]  
with \( I_C \) = constant (related to EoS of accreting fluid) = 1 (for simplicity), \( \rho_e \) = density of the accreting fluid at the equatorial plane, \( h(x) \) = halh thickness of the disc. \( u = u_x = \frac{v_x c}{x} \), \( v_x \) is the radially inward speed of accretion. Next, we will consider the radial component of stationary Navier Stoke's equation.

\[ \rho (\vec{V} \cdot \vec{\nabla}) \vec{V} = \vec{\nabla} \rho + \rho \gamma \nabla^2 u - F_{gx} \]  
as,
\[ \frac{du}{dx} + \frac{1}{\rho} \frac{dp}{dx} - \frac{\lambda^2}{x^3} + F_g (x) = 0, \]  
where \( F_g(x) \) is the radially inward gravitational force component. Assuming the vertical equilibrium from the vertical component, we get,
\[ h(x) = c_s \sqrt{\frac{x}{F_g}}. \]  
Assume \( \Psi = c_s^2 + (\alpha - c_s^2) n + \alpha \). Also Assume \( \mu(x) = \left(3 - \frac{x}{F_g}, \frac{dF_g}{dx}\right) \). Then,
\[ \frac{du}{dx} = \frac{u \{[\lambda^2 - x^3 F_g(x)] \Psi + x^3 \mu(x) c_s^4 \}}{x^3 \Psi u^2 - 2 c_s^4} \]  
\[ \frac{dc_s}{dx} = \left[ \frac{1}{2} \mu(x) + \frac{du}{u dx} \right] \Psi^{-1} [(n + 1) c_s (c^2 - \alpha)] \]  
\[ \frac{d\lambda}{dx} = \frac{x \alpha_s s}{u} \left[ \frac{1}{2} \left( \frac{5}{x} - \frac{1}{F_g} \frac{dF_g}{dx} \right) \left\{ \frac{(n_{MCG} + 1) \alpha_{MCG} - c_s^2}{n_{MCG}} + u^2 \right\} + 2u \frac{du}{dx} \right. \]  
\[ + \left\{ \frac{(n_{MCG} + 1) \alpha_{MCG} - c_s^2}{n_{MCG}} + u^2 \right\} \frac{1}{c_s} - \left( c_s^2 + u^2 \right) \left\{ \frac{1}{n_{MCG} + 1} - \frac{2 c_s}{c_s^2 - \alpha_{MCG}} \right\} \frac{dc_s}{dx} \right] . \]  
\[ A \left( \frac{du}{dx} \right)_{x=x_c}^2 + B \left( \frac{du}{dx} \right)_{x=x_c} + C = 0, \]  
where
\[ A = 2 \left[ 1 - \frac{2 (c_s^2 - \alpha) (n + 1) \{(1 - n) c_s^2 + 2 \alpha (n + 1)\}}{\{(1 - n) c_s^2 + \alpha (n + 1)\}^2} \right], \]  
\[ B = -\frac{2}{c_s^4} \left[ \frac{(c_s^2 - \alpha) (n + 1) \{(1 - n) c_s^2 + 2 \alpha (n + 1)\}}{\{(1 - n) c_s^2 + \alpha (n + 1)\}^2} \right] \left[ F_g(x_c) - \frac{\lambda^2}{x_c^3} \right], \]  
\[ C = \left\{ \frac{3 \lambda^2}{x_c} - \left( \frac{dF_g}{dx} \right)_{x=x_c} \right\} \left[ \left\{ \frac{1}{F_g} \left( \frac{dF_g}{dx} \right) \right\}^2 \right]_{x=x_c} - \frac{3}{x_c} \left\{ 1 - \left( \frac{1}{F_g} \frac{dF_g}{dx} \right) \right\}^2_{x=x_c} \frac{u_c^2}{2} \]  
here \( x_c \) indicates the critical point where both the denominator and the numerator of the equation vanishes and \( u_c \) is the value of radial velocity at \( x = x_c \) and \( c_s \) is the speed of sound at \( x = x_c \).
3 Solutions and Analysis

In this section, we will study different accretion properties, viz, fluid speed, sonic speed, \( \lambda/\lambda_k \) ratio, accretion/wind fluid density and \( \eta/s \) ratio. We have divided the whole study into three subsections: (i) Speeds, (ii) Density and (iii) \( \eta/s \) ratio.

3.1 Profiles for Accreting Fluid Speed

We have plotted figures 1.1.a to 1.4.d to show the variations of fluid’s radially inward speed w.r.t radial distance. Rows entitled 1, 2, 3 and 4 are for \( \omega_q = 1/3, 0, -2/3 \) and \(-1\) respectively. Columns entitled (a) and (b) are for adiabatic accretions with polytropic index = 1.6 with viscosity given by \( \alpha_{ss} = 10^{-4} \) and \( 10^{-2} \) respectively. Columns entitled (c) and (d) are for adiabatic index 0.09 with viscosity parameter \( \alpha_{ss} = 10^{-4} & 10^{-2} \) respectively. In each figure, solid lines are (green) and wind(red) for \( a = 0 \), dotted lines are accretion(black) and wind(purple) for \( a = 0.5 \), dot-dashed lines are accretion(olive) and wind (orange) for \( a = 0.9 \) and dashed lines are accretion(blue) and wind(pink) for \( a = 0.998 \). Inset contains \( \log(u) \) vs. \( \log(x) \) curves whereas the offset shows \( u \) vs. \( \log(x) \) variations.

Now we are ready to analyze figure 1.1.a. The common features of the curves are same. Accretion speed is raising as we move towards the BH. Wind speed is low near the BH. As we go far, it increases and then becomes almost constant. Point to be noted that the accretion becomes fainter as we move far from the BH. This fainting rate increases as the value of the spin parameter increases. The more is the rotation, the nearer the sensible wind flow starts from the BH. As viscosity increases, in figure 1.1.b, we observe that the accretion to fall abruptly at the distant parts of the accretion disc. So, inclusion of viscosity reduces the angular momentum transport efficiency which ultimately causes the reduction of the physical radius of the disc. This is very much clear in 1.1.d, which is drawn for \( \omega_q = 1/3, 0, -2/3 \) respectively. In each figure, solid lines are (green) and wind(red) for \( a = 0 \), dotted lines are accretion(black) and wind(purple) for \( a = 0.5 \), dot-dashed lines are accretion(olive) and wind (orange) for \( a = 0.9 \) and dashed lines are accretion(blue) and wind(pink) for \( a = 0.998 \). Inset contains \( \log(u) \) vs. \( \log(x) \) curves whereas the offset shows \( u \) vs. \( \log(x) \) variations.

Before approaching to the other cases of \( \omega_q \), we try to find out the case for which the accretion stops or the wind reaches the speed of light at a finite distance. To do this, we will look at the sound speed and \( \lambda/\lambda_k \) curves which are plotted in figures 2.1.a to 2.4.d and 3.1.a to 3.4.d respectively.

For radiation dominated era, we find the sound speed of wind branch decreases slowly as we go far from the BH. As viscosity is considered, the sound speed for the wind branch is found to blow up at a finite distance \( x = x_{end}^{\omega_q, \alpha_{ss}} \). Exactly at \( x = x_{end}(\omega_q) \) the fluid speed for wind branch for wind branch is found to be zero. So, at this end regions of the corresponding disc, the fluid is acting mere stiff which causes the raising in the sound speed’s value but lowering in original fluid speed. As \( \alpha_{ss} \) increases, the value of \( x_{end} \) decreases. Similarly, as \( \omega_q \) is reduced, \( x_{end} \) decreases as well. So, both the effects of viscosity and quintessencial nature of the BH’s back ground shortens the effective disc length. Accretion branch’s accretion speed is found to have sound speed which is low at farthest point and decreases as the distance is reduced.

\( \lambda/\lambda_k \) curves are saying where the angular momentum is being greater than that possessed by a Keplerian orbit. If \( \lambda/\lambda_k > 1 \), the disc is rotating with a speed which is even greater than to fight the inward gravitational pull. This will break the structure of stable accretion disc after \( x = x_{end} \) and the part beyond it will be truncated off.

These three sets of graphs conclude that a viscous accretion onto a quintessence contaminated BH is weakened/shortened by four factors: the rotation of the BH, negativity of \( \omega_q \) in which the BH is embedded, value of \( \alpha_{ss} \), i.e. the viscosity imposed and the negativity inserted in EoS of accreting fluid. If all these factors increase, the accretion disc is fainted causing a weaker feeding process.
Figure 1.1: Images for $\lambda_c = 2.7, \omega_q = 1/3, A_q = 0.01$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blus dashed-dashed line shows accretion for $a = 0.998$.

Figure 1.2: Images for $\lambda_c = 2.7, \omega_q = 0, A_q = 0.01$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blus dashed-dashed line shows accretion for $a = 0.998$.

Figure 1.3: Images for $\lambda_c = 2.7, \omega_q = -2/3, A_q = 10^{-10}$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blus dashed-dashed line shows accretion for $a = 0.998$.

Figure 1: Curves for fluid speed vs radial distance from the BH.
To concretize these results, in the next subsection, we will study the variation of density of accretion and the wind for all the possible cases.

### 3.2 Profiles for Accreting Fluid Density

To sustain an accretion process to run the total system considered must not have luminosity greater than a maximum limit. Beyond this limit, the radiation pressure is so high, that any object will overcome gravitational pull. This will let no matter to fall inward and accretion will stop. This maximum limit is known as Eddington luminosity ($L_{\text{Edd}}$ say). To obtain this limit, we balance the gravitational force with radiation force as,

$$F_{\text{Grav}}(M, m, R) = \frac{G_N M m}{R^2} = P_{\text{rad}} km = \frac{L}{c} \cdot \frac{1}{4\pi R^2} km$$

where $k$, $\sigma_T$, $m_p$, and $L$ are opacity, Thompson Scattering crosssection, mass of proton and luminosity respectively. If this $L$ is taken to be equal to Eddington Luminosity,

$$L_{\text{Edd}} = \frac{4\pi G_N M c m_p}{\sigma_T} \quad (18)$$

Now, consider $\dot{M}_{\text{Edd}}$ is the Eddington mass accretion rate of the considered system. If the $\epsilon$ fraction of the mass is supposed to generate energy, then,

$$L_{\text{Edd}} = \epsilon \dot{M}_{\text{Edd}} c^2 \quad \Rightarrow \quad \dot{M}_{\text{Edd}} = \frac{4\pi G M m_p}{\epsilon c \sigma_T} \quad (19)$$

To choose the mass $M$ of the central BH, we will enlist a few of them in table 2.

We generalize this as $M = 10^{6+\sigma} \cdot M_\odot$.

To determine $\epsilon$, we point out the value as $0.01 - 0.1 L_{\text{Edd}}$ for quasars and $0.001 - 0.3$ for Seyfert galaxies. So, we choose

$$\epsilon = 3 \times 10^{-1-\psi}$$

**Figure 1.4:** Images for $\lambda_c = 2.7, \omega_q = -1, A_q = 10^{-10}$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blus dashed-dashed line shows accretion for $a = 0.998$.

Figure 1: Curves for fluid speed vs radial distance from the BH.
Figure 2.1: Images for $\lambda_c = 2.7, \omega_q = 1/3, A_q = 0.01$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blue dashed-dashed line shows accretion for $a = 0.998$

Figure 2.2: Images for $\lambda_c = 2.7, \omega_q = 0, A_q = 0.01$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blue dashed-dashed line shows accretion for $a = 0.998$

Figure 2.3: Images for $\lambda_c = 2.7, \omega_q = -2/3, A_q = 10^{-10}$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blue dashed-dashed line shows accretion for $a = 0.998$

Figure 2: Curves for sound speed vs radial distance from the BH.
combining we have

$$\dot{M}_{Edd} = \frac{4\pi G M_\odot m_p 10^7}{3c\sigma_T} 10^{\sigma+\psi}$$

Now, let us assume the accretion disc concerned in this work is consuming mass at Eddington mass accretion limit. Then,

$$\rho = \frac{\dot{M}_{Edd} \sqrt{\frac{E_g}{x^3}}}{uc_s} \Rightarrow \rho = \frac{4\pi m_p 10^{1+\psi}}{3c\sigma_T} \times \sqrt{\frac{E_g}{x^3}} uc_s$$

We plot density vs radial distances in figures 4.1.a to 4.4.d. Among them, figures 4.1.a to 4.1.d are for the EoS value $\omega_q = \frac{1}{3}$. In figure 4.1.a, where viscosity is low ($\alpha_{SS} = 10^{-4}$), the wind density varies from $10^{-21}$ to $10^{14} gmcm^{-3}$. In figure 4.1.b, viscosity is high ($\alpha_{SS} = 10^{-2}$) and as a result we observe the wind density to raise from the order of $10^{-24} gmcm^{-3}$ at $10^3$ Schwarzschild radius to $10^{12} gmcm^{-3}$ near the event horizon. For low value of $\Gamma$, we observe that the wind varies for a larger range.

Accretion density, however, varies for a comparatively smaller range.

The same wide range of variations are observed for all the $\omega_q$ cases.

We can explain this issue like: the accretion density varies through an order of $10^4$ centering commonly around the value $10^{-18} gmcm^{-3}$. The negative pressure of quintessence may oppose the accreting matter to fall in and hence near the BH, the wind speed is tremendously high. Wind density profiles which does match with the density profile predictions of the article like [50].
Figure 3.1: Images for \( \lambda_c = 2.7, \omega_q = 1/3, A_q = 0.01 \). Red solid line shows wind for \( a = 0 \), Green solid line shows accretion for \( a = 0 \), Purple dotted line shows wind for \( a = 0.5 \), Black dotted line shows accretion for \( a = 0.5 \), Orange dash-dotted line shows wind for \( a = 0.9 \), Olive dash-dotted line shows accretion for \( a = 0.9 \) and Pink dashed-dashed line shows wind for \( a = 0.998 \), Blus dashed-dashed line shows accretion for \( a = 0.998 \).

Figure 3.2: Images for \( \lambda_c = 2.7, \omega_q = 0, A_q = 10.01 \). Red solid line shows wind for \( a = 0 \), Green solid line shows accretion for \( a = 0 \), Purple dotted line shows wind for \( a = 0.5 \), Black dotted line shows accretion for \( a = 0.5 \), Orange dash-dotted line shows wind for \( a = 0.9 \), Olive dash-dotted line shows accretion for \( a = 0.9 \) and Pink dashed-dashed line shows wind for \( a = 0.998 \), Blus dashed-dashed line shows accretion for \( a = 0.998 \).

Figure 3.3: Images for \( \lambda_c = 2.7, \omega_q = -2/3, A_q = 10^{-10} \). Red solid line shows wind for \( a = 0 \), Green solid line shows accretion for \( a = 0 \), Purple dotted line shows wind for \( a = 0.5 \), Black dotted line shows accretion for \( a = 0.5 \), Orange dash-dotted line shows wind for \( a = 0.9 \), Olive dash-dotted line shows accretion for \( a = 0.9 \) and Pink dashed-dashed line shows wind for \( a = 0.998 \), Blus dashed-dashed line shows accretion for \( a = 0.998 \).

Figure 3: \( \lambda/\lambda_k \) vs radial distance plotting for accretion and wind branches for different parameters.
3.3 Shear Viscosity Coefficient to Entropy Density Ratio

Dual holographic nature of states is predicted by strongly interacting quantum field theories. For example, we can choose the systems where BHs are embedded in AdS space. For such a system, a universal lower bound of the shear viscosity coefficient ($\eta$) to entropy density ($s$) ratio is prescribed as $\frac{\eta}{s} \geq \frac{1}{4\pi \kappa_B}$.

This lower bound is popularly known as the Kovtun-Starinets-Son (KSS) bound.

However, Jakovac [56] has calculated the $\frac{\eta}{s}$ ratio mathematically. To do this, he has assumed some physical conditions for the spectral functions and kept the entropy density constant. He observed that the lower bound may not be universal for some systems which carry quasi-particle constituents with small wave function re-normalization constant, high temperature strongly interacting systems or systems with low temperatures and zero mass excitation. As we have discussed in the introductory section of this article, DE may possess a very small amount of shear viscosity. Besides, the entropy density for DE should be high due to its repulsive nature. Entropy for different components of universe shows [57]. That for cosmic cosmic event horizon it may raise upto $2.6 \pm 0.3 \times 10^{122}$ and for SMBHs it may go upto $1.2^{+1.1}_{-0.7} \times 10^{103}$. So for a phenomenon which involves both SMBHs and DE, the $\frac{\eta}{s}$ ratio may fall and can become lower than the theoretical prediction as well.

4 Brief Discussions and Conclusions

This present article can be treated as a detailed study of the viscous accretion onto a rotating black hole embedded in a quintessence universe and the consequent thermodynamic phenomena. To construct the mathematical model we have chosen a particular type of black hole which has mass and rotation as signature properties along with a special type of back ground. Quintessence is a hypothetical fluid which is theorized to create repulsive force responsible for late time cosmic acceleration. We choose a rotating black hole solution which carries effects of quintessence universe in it. The gravitational effect of such a black hole is implied through a pseudo Newtonian potential. This is done as direct general relativistic nonlinear differential equations are difficult to solve. Viscous effect is adopted through the Shakura and Sunyaev $\alpha_{SS}$ effects.
\( \Gamma = 1.6, \alpha_{ss} = 10^{-4} \)

\( \Gamma = 1.6, \alpha_{ss} = 10^{-2} \)

\( \Gamma = 0.09, \alpha_{ss} = 10^{-4} \)

\( \Gamma = 0.09, \alpha_{ss} = 10^{-2} \)

**Figure 4.1:** Images for \( \lambda_c = 2.7, \omega_q = 1/3, A_q = 0.01 \). Red solid line shows wind for \( a = 0 \), Green solid line shows accretion for \( a = 0 \), Purple dotted line shows wind for \( a = 0.5 \), Black dotted line shows accretion for \( a = 0.5 \), Orange dash-dotted line shows wind for \( a = 0.9 \), Olive dash-dotted line shows accretion for \( a = 0.9 \) and Pink dashed-dashed line shows wind for \( a = 0.998 \), Blus dashed-dashed line shows accretion for \( a = 0.998 \)

\( \Gamma = 1.6, \alpha_{ss} = 10^{-4} \)

\( \Gamma = 1.6, \alpha_{ss} = 10^{-2} \)

\( \Gamma = 0.09, \alpha_{ss} = 10^{-4} \)

\( \Gamma = 0.09, \alpha_{ss} = 10^{-2} \)

**Figure 4.2:** Images for \( \lambda_c = 2.7, \omega_q = 0, A_q = 0.01 \). Red solid line shows wind for \( a = 0 \), Green solid line shows accretion for \( a = 0 \), Purple dotted line shows wind for \( a = 0.5 \), Black dotted line shows accretion for \( a = 0.5 \), Orange dash-dotted line shows wind for \( a = 0.9 \), Olive dash-dotted line shows accretion for \( a = 0.9 \) and Pink dashed-dashed line shows wind for \( a = 0.998 \), Blus dashed-dashed line shows accretion for \( a = 0.998 \)

\( \Gamma = 1.6, \alpha_{ss} = 10^{-4} \)

\( \Gamma = 1.6, \alpha_{ss} = 10^{-2} \)

\( \Gamma = 0.09, \alpha_{ss} = 10^{-4} \)

\( \Gamma = 0.09, \alpha_{ss} = 10^{-2} \)

**Figure 4.3:** Images for \( \lambda_c = 2.7, \omega_q = -2/3, A_q = 10^{-10} \). Red solid line shows wind for \( a = 0 \), Green solid line shows accretion for \( a = 0 \), Purple dotted line shows wind for \( a = 0.5 \), Black dotted line shows accretion for \( a = 0.5 \), Orange dash-dotted line shows wind for \( a = 0.9 \), Olive dash-dotted line shows accretion for \( a = 0.9 \) and Pink dashed-dashed line shows wind for \( a = 0.998 \), Blus dashed-dashed line shows accretion for \( a = 0.998 \)

Figure 4: Curves for \( \eta/s \) vs radial distance from BH.
We follow that if the viscosity is high the accretion branch’s fluid speed steeply falls down as we go far from the central black hole. Wind speed increases as we increase viscosity. But the radial distance wise shift is small. At a finite distance fluid speed becomes equal to that of light. As we increase the quintessential effect, wind speed increases.

Truncation in the accretion length is supported by the sonic speed curves and specific angular momentum to Keplerian angular momentum ratio curves. Either the sonic speed reaches the speed of light or the $\frac{\Lambda}{\Lambda_k}$ ratio reaches the value 1 where the accretion turns zero. Steep fall in accretion due to the increase in viscosity signifies the weakening of accretion procedure.

Density profiles are found to be very interesting. At the edges of the disc, approximately at the order of thousand Schwarzschild radius distance the density is found to be very low. But of course this was higher than the density of universe. At the nearer vicinity of the SMBH, we see the wind density to rise up to the order of $10^{12} \text{gmcm}^{-3}$. This quite matches with the observational results.

Finally, we study the $\frac{\eta}{s}$ ratio and follow that this ratio turns to be less than the theoretical predictions. Present day speculation about the shear viscosity of DE supports this result. Interestingly, we achieve the result where $\frac{\eta}{s}$ is lower than the pre-predicted value for adiabatic accretion. Only the dark energy contamination is considered for the BH metric itself, not onto the accreting fluid’s property. Our result strongly states that far late time BHs, accretion of adiabatic fluid can even reduce the $\frac{\eta}{s}$ ratio.

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**References**

[1] Peter M. Garnavich et al. Supernova limits on the cosmic equation of state. *The Astrophysical Journal*, 509(1):74–79, dec 1998.
(a) $\Gamma = 1.6, \alpha_{ss} = 10^{-4}$  
(b) $\Gamma = 1.6, \alpha_{ss} = 10^{-2}$  
(c) $\Gamma = 0.09, \alpha_{ss} = 10^{-4}$  
(d) $\Gamma = 0.09, \alpha_{ss} = 10^{-2}$

**Figure 5.1:** Images for $\lambda_c = 2.7, \omega_q = 1/3, A_q = 0.01$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blus dashed-dashed line shows accretion for $a = 0.998$

(a) $\Gamma = 1.6, \alpha_{ss} = 10^{-4}$  
(b) $\Gamma = 1.6, \alpha_{ss} = 10^{-2}$  
(c) $\Gamma = 0.09, \alpha_{ss} = 10^{-4}$  
(d) $\Gamma = 0.09, \alpha_{ss} = 10^{-2}$

**Figure 5.2:** Images for $\lambda_c = 2.7, \omega_q = 0, A_q = 0.01$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blus dashed-dashed line shows accretion for $a = 0.998$

(a) $\Gamma = 1.6, \alpha_{ss} = 10^{-4}$  
(b) $\Gamma = 1.6, \alpha_{ss} = 10^{-2}$  
(c) $\Gamma = 0.09, \alpha_{ss} = 10^{-4}$  
(d) $\Gamma = 0.09, \alpha_{ss} = 10^{-2}$

**Figure 5.3:** Images for $\lambda_c = 2.7, \omega_q = -2/3, A_q = 10^{-10}$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blus dashed-dashed line shows accretion for $a = 0.998$

Figure 5: Curves for $\eta/s$ vs radial distance from BH.
Figure 5.4: Images for $\lambda_c = 2.7, \omega_q = -1, A_q = 10^{-10}$. Red solid line shows wind for $a = 0$, Green solid line shows accretion for $a = 0$, Purple dotted line shows wind for $a = 0.5$, Black dotted line shows accretion for $a = 0.5$, Orange dash-dotted line shows wind for $a = 0.9$, Olive dash-dotted line shows accretion for $a = 0.9$ and Pink dashed-dashed line shows wind for $a = 0.998$, Blus dashed-dashed line shows accretion for $a = 0.998$

Figure 5: Curves for $\eta/s$ vs radial distance from BH.

[2] Michael S. Turner, Gary Steigman, and Lawrence M. Krauss. Flatness of the universe: Reconciling theoretical prejudices with observational data. *Phys. Rev. Lett.*, 52, 2090, 1984.

[3] G. J. Mathews, N. Q. Lan, and C. Kolda. Late decaying dark matter, bulk viscosity, and the cosmic acceleration. *Phys. Rev. D*, 78:043525, Aug 2008.

[4] Adam G. Riess et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*, 116(3), 1998, [arXiv:astro-ph/9805201](https://arxiv.org/abs/astro-ph/9805201).

[5] S. Perlmutter, G. Aldering, and M. et al. Valle. Discovery of a supernova explosion at half the age of the universe. *Nature*, 391:51–54, 1998, [arXiv:astro-ph/9712212](https://arxiv.org/abs/astro-ph/9712212).

[6] LIGO Scientific Collaboration and Virgo Collaboration, Abbott, B. P. et al. Observation of gravitational waves from a binary black hole merger. *Phys. Rev. Lett.*, 116:061102, Feb 2016.

[7] LIGO Scientific Collaboration and Virgo Collaboration, Abbott, B. P. et al. Binary black hole mergers in the first advanced ligo observing run. *Phys. Rev. X*, 6:041015, Oct 2016.

[8] LIGO Scientific Collaboration and Virgo Collaboration, Abbott, B. P. et al. Gw151226: Observation of gravitational waves from a 22-solar-mass binary black hole coalescence. *Phys. Rev. Lett.*, 116:241103, Jun 2016.

[9] LIGO Scientific Collaboration and Virgo Collaboration, Abbott, B. P. et al. Gw170104: Observation of a 50-solar-mass binary black hole coalescence at redshift 0.2. *Phys. Rev. Lett.*, 118:221101, Jun 2017.

[10] B. P. Abbott et al. GW170608: Observation of a 19 solar-mass binary black hole coalescence. *The Astrophysical Journal*, 851(2):L35, dec 2017.

[11] LIGO Scientific Collaboration and Virgo Collaboration, Abbott, B. P. et al. Gw170814: A three-detector observation of gravitational waves from a binary black hole coalescence. *Phys. Rev. Lett.*, 119:141101, Oct 2017.

[12] LIGO Scientific Collaboration and Virgo Collaboration, Abbott, B. P. et al. Gw170817: Observation of gravitational waves from a binary neutron star inspiral. *Phys. Rev. Lett.*, 119:161101, Oct 2017.
13] Clifford M. Will. Bounding the mass of the graviton using gravitational-wave observations of inspiralling compact binaries. *Phys. Rev. D*, 57:2061–2068, Feb 1998.

14] Atsushi Nishizawa. Constraining the propagation speed of gravitational waves with compact binaries at cosmological distances. *Phys. Rev. D*, 93:124036, Jun 2016.

15] Xiang Li, Yi-Ming Hu, Yi-Zhong Fan, and Da-Ming Wei. GRB/GW ASSOCIATION: LONG–SHORT GRB CANDIDATES, TIME LAG, MEASURING GRAVITATIONAL WAVE VELOCITY, AND TESTING EINSTEIN’s EQUIVALENCE PRINCIPLE. *The Astrophysical Journal*, 827(1):75, Aug 2016.

16] Clifford M. Will. The confrontation between general relativity and experiment. *Living Reviews in Relativity*, 17(1):4, Jun 2014.

17] Emre O. Kahya and Shantanu Desai. Constraints on frequency-dependent violations of shapiro delay from gw150914. *Physics Letters B*, 756:265 – 267, 2016.

18] Yue-Liang Wu. Quantum field theory of gravity with spin and scaling gauge invariance and spacetime dynamics with quantum inflation. *Phys. Rev. D*, 93:024012, Jan 2016.

19] S. W. Hawking. Perturbations of an expanding universe. *Astrophysical Journal*, 145:544, 1966.

20] F. P. Esposito. Interaction of gravitational radiation with an inviscid fluid in simple motion. *Astrophysical Journal*, 168:495, 1971.

21] J. Madore. The absorption of gravitational radiation by a dissipative fluid. *Comm. Math. Phys.*, 30(4):335–340, 1973.

22] A.R. Prasanna. Propagation of gravitational waves through a dispersive medium. *Physics Letters A*, 257(3):120 – 122, 1999.

23] Bo-Qiang Lu, Da Huang, Yue-Liang Wu, and Yu-Feng Zhou. Damping of gravitational waves in a viscous universe and its implication for dark matter self-interactions, 2018.

24] Wei Li and Lixin Xu. Viscous generalized chaplygin gas as a unified dark fluid: including perturbation of bulk viscosity. *The European Physical Journal C*, 74(2):2765, Feb 2014.

25] Karl F. Herzfeld. Bulk viscosity and shear viscosity in fluids according to the theory of irreversible processes. *J. Chem. Phys.*, 28:595, 2004.

26] N. I. Shakura and R. A. Sunyaev. Black holes in binary systems. observational appearance. *Astronomy and Astrophysics*, 24:337–355, 1973.

27] L. D. Landau and E. M. Lifshitz. *Fluid Mechanics*, volume 6 of Course of Theoretical Physics. Pergamon Press, Moscow, 2nd edition, 1987.

28] Lijun Gou, Jeffrey E. McClintock, and Mark J. Reid et al. THE EXTREME SPIN OF THE BLACK HOLE IN CYGNUS x-1. *The Astrophysical Journal*, 742(2):85, Nov 2011.

29] S. Fromang, W. Lyra, and F. Masset. Meridional circulation in turbulent protoplanetary disks. *A&A*, 534(A107):460, 2011.

30] Marco Fink. Lamppost source height measurements: In unobscured active galactic nuclei. *Master’s Thesis*, September, 30, 2016.
[31] I. D. Novikov and K. S. Thorne. Astrophysics of black holes. *Black Holes (Les Astres Occlus), ed. C. DeWitt & B. S. DeWitt (New York: Gordon and Breach)*, pages 343–450, 1973.

[32] Shubhrangshu Ghosh and Banibrata Mukhopadhyay. Generalized pseudo-newtonian potential for studying accretion disk dynamics in off-equatorial planes around rotating black holes: Description of a vector potential. *The Astrophysical Journal*, 667(1):367–374, sep 2007, [arXiv:0706.2221](http://arxiv.org/abs/0706.2221).

[33] B. Paczynsky and P. J. Wiita. Thick accretion disks and supercritical luminosities. *Astronomy and Astrophysics*, 88(1-2):23–31, Aug 1980.

[34] Banibrata Mukhopadhyay. Description of pseudo-newtonian potential for the relativistic accretion disks around kerr black holes. *The Astrophysical Journal*, 581(1):427–430, dec 2002.

[35] Siddhartha Sankar Sarkar and Ritabrata Biswas. Pseudo newtonian potential for a rotating kerr black hole embedded in quintessence. *The European Physical Journal C*, 79(5):380, May 2019.

[36] Eduardo Banados et al. An 800-million-solar-mass black hole in a significantly neutral universe at a redshift of 7.5. *Nature*, 553:473–476, January 2018.

[37] Cecilia B M H Chirenti and Luciano Rezzolla. How to tell a gravastar from a black hole. *Classical and Quantum Gravity*, 24(16):4191–4206, Jul 2007.

[38] Pankaj S Joshi, Daniele Malafarina, and Ramesh Narayan. Equilibrium configurations from gravitational collapse. *Classical and Quantum Gravity*, 28(23):235018, nov 2011.

[39] D. G. Levkov, A. G. Panin, and I. I. Tkachev. Gravitational bose-einstein condensation in the kinetic regime. *Phys. Rev. Lett.*, 121:151301, Oct 2018.

[40] K Boshkayev and D Malafarina. A model for a dark matter core at the Galactic Centre. *Monthly Notices of the Royal Astronomical Society*, 484(3):3325–3333, 01 2019.

[41] Ritabrata Biswas, Subenoy Chakraborty, Tarun Deep Saini, and Banibrata Mukhopadhyay. Accretion of chaplygin gas upon black holes: formation of faster outflowing winds. *Classical and Quantum Gravity*, 28(3):035005, jan 2011.

[42] Ritabrata Biswas and Sandip Dutta. Threshold drop in accretion density if dark energy is accreting onto a supermassive black hole. *The European Physical Journal C*, 79(9):742, Sep 2019.

[43] Banibrata Mukhopadhyay. Stability of accretion disks around rotating black holes: A pseudoâĂŞgeneral-relativistic fluid dynamical study. *The Astrophysical Journal*, 586:1268âĂŞ1279, Apr 2003.

[44] Ritabrata Biswas. Density profiles for chaplygin gas accretion upon black holes: Moderately differentiated minima in wind branch. *EPL (Europhysics Letters)*, 96(4):49001, Nov 2011.

[45] Wenwen Zuo, Xue-Bing Wu, Xiaohui Fan, Richard Green, Ran Wang, and Fuyan Bian. BLACK HOLE MASS ESTIMATES AND RAPID GROWTH OF SUPERMASSIVE BLACK HOLES IN LUMINOUSz~ 3.5 QUASARS. *The Astrophysical Journal*, 799(2):189, jan 2015.

[46] L. J. Oldham and M. W. Auger. Galaxy structure from multiple tracers âĂŞ II. M87 from parsec to megaparsec scales. *Monthly Notices of the Royal Astronomical Society*, 457(1):421–439, 01 2016.

[47] B.M. Peterson. Measuring the masses of supermassive black holes. *Space Sci. Rev.*, 183:253, 2014.
[48] Nick Devereux, Holland Ford, Zlatan Tsvetanov, and George Jacoby. STIS spectroscopy of the central 10 parsecs of m81: Evidence for a massive black hole. *The Astronomical Journal*, 125(3):1226–1235, mar 2003.

[49] A. M. Ghez, S. Salim, N. N. Weinberg, J. R. Lu, T. Do, J. K. Dunn, K. Matthews, M. R. Morris, S. Yelda, E. E. Becklin, T. Kremenek, M. Milosavljevic, and J. Naiman. Measuring distance and properties of the milky way’s central supermassive black hole with stellar orbits. *The Astrophysical Journal*, 689(2):1044–1062, dec 2008.

[50] Tiziana Di Matteo, Steven W. Allen, Andrew C. Fabian, Andrew S. Wilson, and Andrew J. Young. Accretion onto the supermassive black hole in m87. *The Astrophysical Journal*, 582(1):133–140, jan 2003.

[51] P. K. Kovtun, D. T. Son, and A. O. Starinets. Viscosity in strongly interacting quantum field theories from black hole physics. *Phys. Rev. Lett.*, 94:111601, Mar 2005.

[52] G. Policastro, D. T. Son, and A. O. Starinets. Shear viscosity of strongly coupled $n = 4$ supersymmetric yang-mills plasma. *Phys. Rev. Lett.*, 87:081601, Aug 2001.

[53] S. Tamaryan, H. J. W. Mueller-Kirsten, and D. K. Park. D3-brane intersecting with dyonic bion. *arXiv: hep-th/0309231* 2003.

[54] Alex Buchel and James T. Liu. Universality of the shear viscosity from supergravity duals. *Phys. Rev. Lett.*, 93:090602, Aug 2004.

[55] Dam T. Son and Andrei O. Starinets. Viscosity, black holes, and quantum field theory. *Annual Review of Nuclear and Particle Science*, 57:95–118, 2007.

[56] Antal Jakovac. Nonuniversal lower bound for the shear viscosity to entropy density ratio. *Phys. Rev. D*, 81:045020, Feb 2010.

[57] Chas A. Egan and Charles H. Lineweaver. A LARGER ESTIMATE OF THE ENTROPY OF THE UNIVERSE. *The Astrophysical Journal*, 710(2):1825–1834, feb 2010.