Probing the Higgs sector of $Y = 0$ Higgs Triplet Model at LHC

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Abstract

In this paper, we investigate the Higgs Triplet Model with hypercharge $Y_\Delta = 0$ (HTM0), an extension of the Standard model, characterized by a more involved scalar spectrum consisting of two CP even Higgs $h^0, H^0$ and two charged Higgs bosons $H^\pm$. We first show that the parameter space of HTM0, usually delimited by combined constraints originating from unitarity and BFB as well as experimental limits from LEP and LHC, is severely reduced when the modified Veltman conditions at one loop are also imposed. Then, we perform an rigorous analysis of Higgs decays either when $h^0$ is the SM-like or when the heaviest neutral Higgs $H^0$ is identified to the observed 125 GeV Higgs boson at LHC. In these scenarios, we perform an extensive parameter scan, in the lower part of the scalar mass spectrum, with a particular focus on the Higgs to Higgs decay modes $H^0 \rightarrow h^0 h^0, H^\pm H^\mp$ leading predominantly to invisible Higgs decays. Finally, we also study the scenario where $h^0, H^0$ are mass degenerate. We thus find that consistency with LHC signal strengths favours a light charged Higgs with a mass about $176 \sim 178$ GeV.

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Our analysis shows that the diphoton Higgs decay mode and $H \rightarrow Z\gamma$ are not always positively correlated as claimed in a previous study. Anti-correlation is rather seen in the scenario where $h$ is SM like, while correlation is sensitive to the sign of the potential parameter $\lambda$ when $H$ is identified to 125 GeV observed Higgs.

1 Introduction

Without no doubt, the neutral scalar boson discovered by ATLAS [1] and CMS [2] at the Large Hadron Collider (LHC) corresponds to the Higgs boson. All data collected at 7 and 8 TeV support the existence of Higgs signal with a mass around 125 GeV with Standard Model (SM) like properties. Moreover, the deviation in $\gamma\gamma$ channel for the gluon and vector boson fusion productions, the Higgs production and decays into $WW^*$ and $ZZ^*$ are all consistent with SM predictions, as can be seen from LHC run II measurements at 13 TeV [3, 4].

Similarly to our previous phenomenological analysis in the type II seesaw model [5–9] we focus in this work on the Higgs Triplet Model with hypercharge $Y_\Delta = 0$, hereafter referred to as HTM0. The main motivation of the HTM0 is related to the mysterious nature of dark matter (DM) and dark energy, which may signal new physics beyond the SM [10, 11]. Although a recent analysis of the HTM0 has been done in [12], we revisit this model in light of new data at LHC run II, with the aim to improve the previous analysis of the Higgs decays which suffered from some inconsistencies that produced inappropriate results for the correlation between Higgs to diphoton decay and Higgs to photon and a $Z$ boson. Furthermore, our work will investigate the naturalness problem in HTM0. We will show how the new degrees of freedom in the HTM0 spectrum can soften the quadratic divergencies and how the Veltman conditions are modified accordingly (VC) [13–16]. As a consequence, we will see that the parameter space of our model is severely constrained by the modified Veltman conditions.

This paper is organised as following. In section 2, we briefly review the main features of HTM0, and present the full set of constraints on the parameters of the Higgs potential. Section 3 is devoted to the derivation of the modified VC’s in HTM0. The Higgs sector is discussed in greater detail in section 4 where either $h^0$ or $H^0$ are identified to the SM-like Higgs, and at last we focus on the scenario of their mass degeneracy where both Higgses mimic the observed $\sim 125$ GeV. A full set of constraints were taken into account in the various analyses, including
theoretical (BFB, unitarity and Veltman conditions) as well as the experimental ones, and
scrutinised via HiggsBounds v4.2.1 [17] which we use to check agreement with all 2σ exclusion
limits from LEP, Tevatron and LHC Higgs searches. Our conclusion is drawn in section 5, while
some technical details are postponed into appendices.

2 Review of the HTM0 model

2.1 Lagrangian and Higgs masses

The Higgs triplet model with hypercharge $Y_\Delta = 0$ can be implemented in the Standard Model
by adding a colourless scalar field $\Delta$ transforming as a triplet under the $SU(2)_L$ gauge group
with hypercharge $Y_\Delta = 0$. The most general gauge invariant and renormalisable $SU(2)_L \times U(1)_Y$
Lagrangian of the scalar sector is given by,

$$
\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + Tr(D_\mu \Delta)^\dagger (D^\mu \Delta) - V(H, \Delta) + \mathcal{L}_{\text{Yukawa}}
$$

(2.1)

where the covariant derivatives are defined by,

$$
D_\mu H = \partial_\mu H + ig^W W^a_\mu H + ig'^B B^\mu H
$$

(2.2)

$$
D_\mu \Delta = \partial_\mu \Delta + ig[T^a W^a_\mu, \Delta].
$$

(2.3)

$(W^a, g)$, and $(B_\mu, g')$ are respectively the $SU(2)_L$ and $U(1)_Y$ gauge fields and couplings and
$T^a \equiv \sigma^a/2$, where $\sigma^a (a = 1, 2, 3)$ denote the Pauli matrices. The potential $V(H, \Delta)$ can be
expressed as [10],

$$
V(H, \Delta) = -m_H^2 H H^\dagger + \frac{\lambda}{4} (H H^\dagger)^2 - m_\Delta^2 Tr(\Delta \dagger \Delta) + \mu H^\dagger \Delta H
$$

$$
+ \lambda_1 (H^\dagger H) Tr(\Delta \dagger \Delta) + \lambda_2 (Tr(\Delta \dagger \Delta))^2 + \lambda_3 Tr(\Delta \dagger \Delta)^2
$$

$$
+ \lambda_4 H^\dagger \Delta \dagger \Delta H
$$

(2.4)

where $Tr$ is the trace over $2 \times 2$ matrices. Last, $\mathcal{L}_{\text{Yukawa}}$ contains all the Yukawa sector of the
SM plus an extra Yukawa term that leads after spontaneous symmetry breaking to (Majorana)
mass terms for the neutrinos, without requiring right-handed neutrino states.
Defining the electric charge as usual, $Q = I_3 + \frac{Y}{2}$ where $I$ denotes the isospin, we write the two Higgs multiplets in components as:

$$
\Delta = \frac{1}{2} \begin{pmatrix} \delta^0 & \sqrt{2}\delta^+ \\ \sqrt{2}\delta^- & -\delta^0 \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad (2.5)
$$

with

$$
\phi^0 = \frac{1}{\sqrt{2}}(v_d + h_1 + i z_1) \quad \text{and} \quad \delta^0 = v_t + h_2 \quad (2.6)
$$

For later convenience, the vacuum expectation values $v_d$ and $v_t$ are supposed positive values.

Assuming that spontaneous electroweak symmetry breaking (EWSB) is taking place at some electrically neutral point in the field space, and denoting the corresponding VEVs by

$$
\langle \Delta \rangle = \frac{1}{2} \begin{pmatrix} v_t & 0 \\ 0 & -v_t \end{pmatrix} \quad \text{and} \quad \langle H \rangle = \begin{pmatrix} 0 \\ v_d/\sqrt{2} \end{pmatrix} \quad (2.7)
$$

one finds, after minimisation of the potential Eq.(2.4), the following necessary conditions:

$$
M^2_\Delta = \frac{\lambda_a}{2} v_d^2 - \frac{\mu v_d^2}{4v_t} + \lambda_b v_t^2 \quad (2.8)
$$

$$
m^2_H = \frac{\lambda}{4} v_d^2 - \frac{\mu v_t^2}{2} + \frac{\lambda_a}{2} v_t^2 \quad (2.9)
$$

where $\lambda_a = \lambda_1 + \frac{\lambda_4}{2}$ and $\lambda_b = \lambda_2 + \frac{\lambda_3}{2}$.

The $7 \times 7$ squared mass matrix,

$$
\mathcal{M}^2 = \frac{1}{2} \frac{\partial^2 V}{\partial \eta^2} \bigg|_{\Delta = \langle \Delta \rangle, H = \langle H \rangle} \quad (2.10)
$$

can be cast, thanks to Eqs.(2.8, 2.9), into a block diagonal form of three $2 \times 2$ matrices, denoted in the following by $\mathcal{M}^2_\pm, \mathcal{M}^2_{\text{CP-even}}$, and one odd eigenstate corresponding to the neutral Goldstone boson $G^0$. The mass-matrix for singly charged field given by,

$$
\mathcal{M}^2_\pm = \mu \begin{pmatrix} v_t & v_d/2 \\ v_d/2 & \frac{v_d^2}{4v_t} \end{pmatrix}
$$

is diagonalised by a $2 \times 2$ rotation matrix $\mathcal{R}_{\theta_\pm}$, where $\theta_\pm$ is a rotation angle. Among the two eigenvalues of $\mathcal{M}^2_\pm$, one is equal to zero indentifying the charged Goldstone boson $G^\pm$, while the other one corresponds to the mass of singly charged Higgs bosons $H^\pm$ given by,

$$
m^2_{H^\pm} = \left(\frac{v_d^2 + 4v_t^2}{4v_t}\right)\mu \quad (2.11)
$$
The mass-eigenstate $H^\pm$ and $G^\pm$ are rotated from the Lagrangian fields $\phi^\pm, \delta^\pm$ as follows:

\begin{align*}
G^\pm & = + \cos \theta^\pm \phi^\pm + \sin \theta^\pm \delta^\pm \\
H^\pm & = - \sin \theta^\pm \phi^\pm + \cos \theta^\pm \delta^\pm
\end{align*}

(2.12)

(2.13)

Diagonalization of $\mathcal{M}_\pm^2$ leads to the following relations involving the rotation angle $\theta^\pm$:

\begin{align*}
\frac{\mu v_d^2}{4v_t} & = \cos^2 \theta^\pm M_{H^\pm}^2 \\
\frac{\mu v_d}{2} & = - \frac{\sin 2\theta^\pm}{2} M_{H^\pm}^2 \\
\mu v_t & = \sin^2 \theta^\pm M_{H^\pm}^2
\end{align*}

(2.14)

(2.15)

(2.16)

since the Goldstone boson $G^\pm$ is massless. These three equations have a unique solution for $\sin \theta^\pm$ and $\cos \theta^\pm$ up to a global sign ambiguity. Indeed, Eq. (2.14) implies $\mu > 0$ in order to forbid tachyonic $H^\pm$ state, since our convention uses $v_t > 0$. Hence, from Eq. (2.15), $\sin \theta^\pm$ and $\cos \theta^\pm$ should have different signs; one gets:

\begin{align*}
\cos \theta^\pm & = \epsilon \frac{v_d}{\sqrt{v_d^2 + 4v_t^2}}, \quad \sin \theta^\pm = - \epsilon \frac{2v_t}{\sqrt{v_d^2 + 4v_t^2}}
\end{align*}

(2.17)

with a sign freedom $\epsilon = \pm 1$, which leads to negative $\tan \theta^\pm$.

As to the neutral scalar, its mass matrix reads:

\[
\mathcal{M}_{CP\ even}^2 = \begin{pmatrix} A & B \\ B & C \end{pmatrix}
\]

(2.18)

where

\[
A = \frac{\lambda}{2} v_d^2, \quad B = \frac{v_d}{2\sqrt{2}} \left[ -\mu + 2\lambda v_1 \right], \quad C = \frac{\mu v_d^2 + 8\lambda v_t v_3^3}{8v_t}
\]

(2.19)

This symmetric matrix is also diagonalised by a $2 \times 2$ rotation matrix $\mathcal{R}_\alpha$, where $\alpha$ denotes the rotation angle in the $CP_{even}$ sector.

After diagonalization of $\mathcal{M}_{CP\ even}^2$, one gets two massive even-parity physical states $h^0$ and $H^0$ defined by,

\begin{align*}
h^0 & = +c_\alpha h_1 + s_\alpha h_2 \\
H^0 & = -s_\alpha h_1 + c_\alpha h_2
\end{align*}

(2.20)

(2.21)
Their masses are given by the eigenvalues of $\mathcal{M}_{\text{CP}_{\text{even}}}^2$:

\[ m_{h^0}^2 = \frac{1}{2} [A + C - \sqrt{(A - C)^2 + 4B^2}] \quad (2.22) \]
\[ m_{H^0}^2 = \frac{1}{2} [A + C + \sqrt{(A - C)^2 + 4B^2}] \quad (2.23) \]

so that $m_{H^0} > m_{h^0}$. Note that the lighter state $h^0$ is not necessarily the lightest of the Higgs sector. Furthermore, the only odd eigenstate leads to one massless Goldstone boson $G^0$ defined by $G^0 = z_1$.

Once we know the above eigenmasses for the $\text{CP}_{\text{even}}$, one can determine the rotation angle $\alpha$ which controls the field content of the physical states. One has:

\[ C = s^2_\alpha m_{h^0}^2 + c^2_\alpha m_{H^0}^2 \quad (2.24) \]
\[ B = \frac{\sin 2\alpha}{2} (m_{h^0}^2 - m_{H^0}^2) \quad (2.25) \]
\[ A = c^2_\alpha m_{h^0}^2 + s^2_\alpha m_{H^0}^2 \quad (2.26) \]

Both Eq. (2.24) and Eq. (2.26) should be equivalent upon use of $s^2_\alpha + c^2_\alpha = 1$ and Eqs. (2.22, 2.23). Furthermore, $s_\alpha, c_\alpha$ also do not have definite signs, depending on the sign of $B$. The relative sign between $s_\alpha$ and $c_\alpha$ depends on the values of $\mu$ as can be seen from Eqs. (2.25, 2.19). While they will have the same sign and $\tan \alpha > 0$ for most of the allowed $\mu$ and $\lambda_1, \lambda_4$ ranges, there will be a small but interesting domain of small $\mu$ values and $\tan \alpha < 0$.

Finally, from Eqs. (2.24 - 2.26), it is easy to express $\alpha$ in terms of $A, B$ and $C$ (Eqs. (2.19)) via:

\[ \sin 2\alpha = \frac{2B}{\sqrt{(A - C)^2 + 4B^2}} \quad \text{and} \]
\[ \cos 2\alpha = \frac{A - C}{\sqrt{(A - C)^2 + 4B^2}} \quad (2.27) \]

### 2.2 Constraints in the HTM0

The full experimental validation of the HTM0 would require not only evidence for the neutral and charged Higgs states but also the experimental values for the various field couplings in the gauge and matter sectors of the model. Crucial tests would then be driven by the predicted correlations among these measurable quantities. For instance, the $\mu$ and $\lambda$’s parameters can be
easily expressed in terms of the physical Higgs masses and the mixing angle $\alpha$ as well as the VEV’s $v_d, v_t$, using equations (2.11), (2.24 - 2.26). One finds

$$\lambda_a = \frac{1}{v_t v_d} \left\{ \sqrt{2} s_\alpha c_\alpha (m_{h_0}^2 - m_{H^0}^2) + \frac{2 v_t v_d}{v_d^2 + 4 v_t^2} m_{H^\pm}^2 \right\} \quad (2.28)$$

$$\lambda_b = \frac{1}{v_t^2} \left\{ s_\alpha^2 m_{h_0}^2 + c_\alpha^2 m_{H^0}^2 - \frac{v_d^2}{2(v_d^2 + 4 v_t^2)} m_{H^\pm}^2 \right\} \quad (2.29)$$

$$\lambda = \frac{2}{v_d} \left\{ c_\alpha^2 m_{h_0}^2 + s_\alpha^2 m_{H^0}^2 \right\} \quad (2.30)$$

$$\mu = \frac{4 v_t}{v_d^2 + 4 v_t^2} m_{H^\pm}^2 \quad (2.31)$$

The remaining two Lagrangian parameters $m_{H^\pm}^2$ and $M_\Delta^2$ are then related to the physical parameters through the EWSB conditions Eqs. (2.8, 2.9).

In the Standard Model the custodial symmetry ensures that the $\rho$ parameter, $\rho \equiv \frac{M_{W^\pm}}{M_Z \cos^2 \theta_W}$, is 1 at tree level. In the HTM0, it is clear that $\delta^0$ don’t contribute to the Z boson mass, and one obtains the $Z$ and $W$ gauge boson masses readily from Eq.(2.7) and the kinetic terms in Eq.(2.1) as

$$M_Z^2 = \frac{(g^2 + g'^2)v_d^2}{4} = \frac{g^2 v_d^2}{4 c_w^2} \quad (2.32)$$

$$M_W^2 = \frac{g^2 (v_d^2 + 4 v_t^2)}{4} \quad (2.33)$$

Hence the modified form of the $\rho$ parameter is $\rho = \frac{v_t^2 + 4 v_t^2 v_d^2}{v_d^2}$.

Since we are interested in the limit $v_t \ll v_d$, we rewrite

$$\rho = 1 + 4 \frac{v_t^2}{v_d^2} = 1 + \delta \rho \quad (2.34)$$

with $\delta \rho = 4 \frac{v_t^2}{v_d^2} > 0$ and $\sqrt{v_d^2 + 4 v_t^2} = 246$ GeV.

From a global fit to EWPO one obtains the $1\sigma$ result [18],

$$\rho_0 = 1.0004^{+0.0003}_{-0.0004} \quad (2.35)$$

Consequently, in what follows, we adopt the bound

$$\left( \frac{2 v_t}{v_d} \right)^2 \lesssim 0.0006 \quad \text{or equivalently} \quad v_t \lesssim 3 \text{GeV} \quad (2.36)$$

The positivity requirement in the singly charged sector, Eq. (2.11), along with our phase convention $v_t > 0$, lead only to positive values of $\mu$. The tachyonless condition in the $C P_{\text{even}}$ sector,
Eqs. (2.22, 2.23), is somewhat more involved and reads:

\begin{align}
\mu v_d^2 + 4\lambda v_d^2 v_t + 8\lambda_b v_t^3 &> 0 \quad (2.37) \\
-2\mu^2 v_t + \mu(\lambda v_d^2 + 8\lambda_a v_t^2) + 8(\lambda\lambda_b - \lambda_a^2)v_t^3 &> 0 \quad (2.38)
\end{align}

The first equation is actually always satisfied thanks to the positivity of \(\mu\) and the boundedness from below conditions for the potential. The second equation, quadratic in \(\mu\), will lead to new constraints on \(\mu\) in the form of an allowed range

\(\mu_- < \mu < \mu_+\) \quad (2.39)

The full expressions of \(\mu_\pm\) are given by

\[\mu_\pm = \frac{8\lambda_a v_t^2 + \lambda v_d^2 \pm \sqrt{16\lambda\lambda_a v_d^2 v_t^2 + 64\lambda\lambda_b v_t^4 + \lambda^2 v_d^2}}{4v_t}\] \quad (2.40)

Let us discuss their behaviours in the favoured regime \(v_t \ll v_d\). In this case one finds a vanishingly small \(\mu_-\) given by

\[\mu_- = (\lambda_a^2 - \lambda\lambda_b) \frac{8 v_t^3}{\lambda v_d^2} + \mathcal{O}(v_t^4)\] \quad (2.41)

and a large \(\mu_+\) given by

\[\mu_+ = \frac{\lambda v_d^2}{2v_t} + 4\lambda_a v_t + \mathcal{O}(v_t^2)\]. \quad (2.42)

Depending on the signs and magnitudes of the \(\lambda\)'s, lower bound \(\mu > 0\) (positivity of Eq.(2.11)) or \(\mu_-\) will overwhelm the others. Moreover, these no-tachyon bounds will have eventually to be amended by taking into account the existing experimental exclusion limits. This is straightforward for the charged Higgs boson \(H^\pm\), thus we define for later reference:

\[\mu_{\text{min}} = \frac{4v_t}{v_d^2 + 4v_t^2} (m_{H^\pm})_{\text{exp}}\] \quad (2.43)

where \((m_{H^\pm})_{\text{exp}}\) denotes the experimental lower exclusion limit for the charged Higgs boson mass. So \(\mu\) must be larger than \(\mu_{\text{min}}\) in order for the mass to satisfy this exclusion limit.

Upon use of Eqs.(2.7, 2.8, 2.9) in Eq.(2.4) one readily finds that the value of the potential at the electroweak minimum, \(\langle V \rangle_{\text{EWSB}}\), is given by:

\[\langle V \rangle_{\text{EWSB}} = -\frac{1}{16}(\lambda v_d^4 + 4\lambda_b v_t^4 + 2v_t^2v_t(2\lambda_a v_t - \mu))\] \quad (2.44)
Since the potential vanishes at the gauge invariant origin of the field space, \( V_{H=0,\Delta=0} = 0 \), then spontaneous electroweak symmetry breaking would be energetically disfavoured if \( \langle V \rangle_{\text{EWSB}} > 0 \). One can thus require as a first approximation the naive bound on \( \mu \)

\[
\mu < \mu_{\text{max}} = \frac{\lambda v_d^2}{2 v_t} + 2 \lambda_a v_t + \mathcal{O}(v_t^2) \tag{2.45}
\]

Always with the theoretical considerations, these are the requirements for tree-level perturbative unitarity, namely that the eigenvalues of the 2 \( \rightarrow \) 2 scalar scattering matrix are below an absolute upper value given by \( 8\pi \), and boundedness form below (BFB), which means that the potential in Eq. (2.4) has to be bounded from below. Obviously, at large field values, this potential is generically dominated by its quartic part:

\[
V^{(4)}(H, \Delta) = \frac{\lambda (H\dagger H)^2}{4} + \lambda_1 (H\dagger H) Tr(\Delta\dagger \Delta)
+ \lambda_2 (Tr \Delta\dagger \Delta)^2 + \lambda_3 Tr(\Delta\dagger \Delta)^2 + \lambda_4 H\dagger \Delta\dagger \Delta H \tag{2.46}
\]

In Appendices A and B respectively, we demonstrate all the necessary and sufficient conditions for the BFB, and we give an introduction explaining the various sub-matrices for the unitarity in this model. These constraints are,

**BFB:**

\[
\lambda \geq 0 \ & \ \lambda_b \geq 0 \tag{2.47}
\]

\[
\lambda_a + \sqrt{\lambda \lambda_b} \geq 0 \tag{2.48}
\]

**Unitarity** [19]:

\[
|\lambda_a| \leq \kappa \pi \tag{2.49}
\]

\[
|\lambda| \leq 2\kappa \pi \tag{2.50}
\]

\[
|\lambda_b| \leq \frac{\kappa}{2} \pi \tag{2.51}
\]

\[
|3\lambda + 10\lambda_b \pm \sqrt{(3\lambda - 10\lambda_b)^2 + 48\lambda_a^2}| \leq 4\kappa \pi \tag{2.52}
\]

where we introduced the parameter \( \kappa \) that takes the values \( \kappa = 8 \), since we choose \( |Re(a_0)| \leq \frac{1}{2} \) as pointed out.
In order to make the space parameter more compact, and working out analytically these two sets of BFB and unitarity constraints, one can reduce them to a more compact system where the allowed ranges for the \( \lambda \)'s are easily identified. One can obtain a necessary domain for \( \lambda, \lambda_b \) that does not depend on \( \lambda_a \), by considering simultaneously Eqs. (2.50 - 2.52) together with Eq. (2.47), (the first two lines)

\[
0 \leq \lambda \leq \frac{2\kappa}{3}\pi \tag{2.53}
\]

\[
0 \leq \lambda_b \leq \frac{\kappa}{5}\pi \tag{2.54}
\]

\[
|\lambda_a| \leq \sqrt{\frac{5}{2}(\lambda - \frac{2}{3}\kappa\pi)(\lambda_b - \frac{\kappa}{5}\pi)} \tag{2.55}
\]

We stress here that the above constraints define the largest possible domain for \( \lambda, \lambda_b \) for any set of allowed values of \( \lambda_a \), although Eq. (2.52) has been used to determine this domain. Studying further Eq. (2.52), using Eqs. (2.53-2.54), one can rewrite it under simple form, Eq. (2.55), where the dependence on \( \lambda_a \) has been explicitly separated from that on \( \lambda, \lambda_b \).

The reduced couplings \( g_{Hff} \) and \( g_{HVV} \) of the Higgs bosons to fermions and \( W \) bosons are given in Tab. 1, while the trilinear couplings to charged Higgs bosons can be extracted from the Lagrangian as \( \mathcal{L} = g_{H \pm H \mp} \mathcal{H} H^+ H^- + g_{Z \pm H \mp} Z(\partial_\mu H^+) H^- + \ldots \). We will use the reduced HTM0 trilinear coupling of \( \mathcal{H} \) and \( Z \) to \( H^\pm \) given by:

\[
g_{ZH^+H^-} = \frac{e}{2 s_w c_w} (1 - 2 c_w^2) s_\theta^2 \]

\[
\tilde{g}_{H^+H^-} = -s_w \frac{m_W}{e m_{H^+}^2} g_{HH^+H^-} \tag{2.56}
\]

| \( \mathcal{H} \)  | \( g_{Hff} \)     | \( g_{HWW} \)   | \( g_{HZZ} \)   |
|----------------|------------------|----------------|----------------|
| \( h^0 \)     | \( c_\alpha/c_{\theta_\pm} \) | \( c_{\theta_\pm} c_\alpha - 2 s_{\theta_\pm} s_\alpha \) | \( c_{\theta_\pm} c_\alpha \) |
| \( H^0 \)     | \( -s_\alpha/c_{\theta_\pm} \) | \( -c_{\theta_\pm} s_\alpha - 2 s_{\theta_\pm} c_\alpha \) | \( -c_{\theta_\pm} s_\alpha \) |

Table 1: The CP-even neutral Higgs couplings to fermions and gauge bosons in the HTM0 relative to the SM Higgs couplings. \( \alpha \) and \( \theta_\pm \) are the mixing angles respectively in the CP-even and charged Higgs sectors, \( e \) is the electron charge, \( m_W \) the \( W \) gauge boson mass and \( s_W \) the sinus of the weak mixing angle.
The trilinear coupling \( g_{h^0H^+H^-} \) for the light CP-even Higgs boson is given by:

\[
g_{h^0H^+H^-} = -\frac{1}{2} \left\{ c_\alpha (-2c_{\theta_\pm} s_{\theta_\pm} \mu + 2\lambda_a c_{\theta_\pm}^2 v_d + \lambda s_{\theta_\pm}^2 v_d) \\
+ s_\alpha (4\lambda_b c_{\theta_\pm}^2 v_t + s_{\theta_\pm}^2 (\mu + 2\lambda_a v_t)) \right\}
\]

The couplings for the heavy Higgs boson are obtained from the previous ones by simple substitutions \( g_{H^0H^+H^-} = g_{h^0H^+H^-} [c_\alpha \rightarrow -s_\alpha, s_\alpha \rightarrow c_\alpha] \).

### 3 Veltman conditions

To derive the Veltman conditions (VC), one just has to collect the quadratic divergencies [20]. There are various ways to do that, and to be on a safer side, we use the dimensional regularisation because this procedure ensures gauge as well as Lorentz invariances. To work out these quadratic divergencies, we follow exactly the procedure of calculations used in our previous work on the Higgs Triplet Model with hypercharge \( Y = 2 \) [9]. Moreover, it is worth to note that the main difference with [9] is the absence of the CP odd neutral Higgs \( A^0 \) and the doubly charged Higgs \( H^{\pm\pm} \), from HTM0 spectrum. Also we have calculated the quadratic divergencies of the CP-neutral Higgs \( H^0 \) and \( h^0 \) tadpoles in a general linear \( R_\xi \) gauge respectively, leading to results which are independent of the \( \xi \) parameters but depending on the model mixing angles. As noted in [9], it is more convenient to combine these two results to get the tadpoles quadratic divergencies of the real neutral components of the doublet \( (h_1) \) and triplet \( (h_2) \) which are free of any mixing angles. After their VEV shifts, one finds, for the doublet:

\[
T_d = v_d \left(-2Tr(I_n)\Sigma f_{\bar{f}} \frac{m_f^2}{v_d^2} + 3(\lambda + \lambda_a) + 2\frac{m_W^2}{v_{sm}^2} \left(\frac{1}{c_w^2} + 2\right)\right)
\]

and for the triplet:

\[
T_t = v_t \left(8\frac{m_W^2}{v_{sm}^2} + 2\lambda_a + 5\lambda_b\right)
\]

where the simplified notations \( c_w = \cos \theta_{W\text{Einst}} \) and \( v = \sqrt{v_d^2 + v_t^2} \) have been used.

Notice that the quadratic divergencies of the Standard Model are easily recovered in \( T_d \) when the \( \lambda_1 \) and \( \lambda_4 \) couplings vanish, implying \( \lambda_a = 0 \).

Now to proceed with the implementation of the two VC’s in the parameter space and the subsequent scans, we usually assume that the deviations \( \delta T_t \) and \( \delta T_d \) should not exceed the
Higgs mass scale. In our analysis, we will allow them to vary within the reduced conservative range from 0.1 to 10 GeV.

Figure 1: The allowed regions in \((\lambda_a, \lambda_b)\) for two values of \(\delta T = 5, 10\). Color codes are as follows, Orange: Excluded by Unitarity constraints. Red: Excluded by Unitarity & BFB constraints. Blue: Excluded by Unitarity & BFB & VC constraints Only the Green area obeys ALL constraints. Our inputs are \(\lambda = 0.52, -5 \leq \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \leq 5, v_t = 1\) GeV and \(2 \leq \mu \leq 5\) (GeV).

In addition to the theoretical constraints shown in Eqs\[2.48, 2.52\] namely the unitarity, BFB and \(R_{\gamma\gamma}\) from LHC measurements, if the supplementary VC constraints are imposed as well, we see that the allowed region of the parameter space dramatically reduces and its extent depends on the value given to the deviation \(\delta T\). This salient feature is illustrated in Fig.1 which exhibits the allowed domains in the \((\lambda_a, \lambda_b)\) plan. Our analysis shows that naturalness constraint is stronger than the other theoretical conditions and that deviations \(\delta T\) should be larger than 3 GeV in order to keep a viable model. Moreover, taken those constraints together, one might see that \(\lambda_a\) will be restricted around \(\sim 1.2\).

4 Results and Discussions

Since HTM0 spectrum contains two CP even Higgs boson \(h^0\) and \(H^0\), either \(h^0\) or \(H^0\) can be identified as the observed SM-like boson with mass \(\approx 125\)GeV. Therefore, we are facing two
Figure 2: Higgs bosons masses as a function of $\mu$ parameter in the HTM0. We take as inputs $\lambda = 0.52$, $-1 \leq \lambda_a \leq 1$, $\lambda_b = 1$, $v_t = 1$ GeV and $0.1 \leq \mu \leq 25$ (GeV).

choices: $M^0_h \approx 125$ and $M^0_h \leq M^0_H$, or $M^0_h \approx 125$ and $M^0_h \leq M^0_H$. The third scenario is when both Higgs bosons are mass degenerate, $M^0_h \approx M^0_H$. For the three cases, and in order to infer limits on the parameters of our model from the experimental searches, we present the Higgs to diphoton and photon+Z gauge boson signal strengths, $R_{\gamma\gamma}$ and $R_{Z\gamma}$, defined by

$$R_{\gamma\gamma(Z\gamma)}(\phi) = \frac{\Gamma_{\phi \rightarrow gg}^{HTM} \times BR_{\phi \rightarrow \gamma\gamma(Z\gamma)}^{HTM}}{\Gamma_{\phi \rightarrow gg}^{SM} \times BR_{\phi \rightarrow \gamma\gamma(Z\gamma)}^{SM}} \quad (4.1)$$

For each benchmark scenario, we investigate the allowed parameters space by the 1$\sigma$ limit of the current Higgs data after run-II in the $gg \rightarrow \mathcal{H} \rightarrow \gamma\gamma$ channel, reported by ATLAS $\mu_{\gamma\gamma} = 0.85^{+0.22}_{-0.20}$ [21,23] and CMS $\mu_{\gamma\gamma} = 1.11^{+0.19}_{-0.18}$ [24], which are consistent with the Standard Model expectation either for ATLAS or for CMS at 1$\sigma$. Furthermore, we can see that the errors reported are smaller from those reported at 7$\oplus$8 TeV.

4.1 $h^0$ SM-like

Fig.3 displays the allowed region in the $(v_t, \mu)$, $(\mu, m_{H^0})$ and $(\mu, m_{H^\pm})$ planes, where $h^0$ is chosen to be SM-like. It is interesting to note that significant amount of parameter space is allowed once we impose either theoretical or experimental constraints, even for small nonzero value of $v_t$ and $\mu$. 

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Figure 3: \( R_{\gamma\gamma}(h^0) \) variation in the \((\mu, v_t)\) plane (left), \(H^0\) and \(H^\pm\) Higgs bosons masses as a function of \(\mu\) (middle) and \(v_t\) (right). Inputs: \(\lambda \approx 0.52\) (\(m_{h^0} \approx 125\) GeV), \(|\lambda_a| \leq 1.5\), \(|\lambda_b| \leq 1\), \(10^{-2} \leq \mu \leq 25\) (GeV) and \(10^{-2} \leq v_t \leq 3\) (GeV)

Figure 4: \(\text{BR}(H^0 \to h^0 h^0)\) variation in the \((R_{\gamma\gamma}(h^0), \lambda_a)\) plane taking into account ATLAS result at \(1\sigma\), and as inputs: \(\lambda \approx 0.52\), \(|\lambda_a| \leq 1.5\), \(\lambda_b = 1\), \(5 \leq \mu \leq 25\) (GeV) and \(v_t = 1\) GeV (left). The \(\text{BR}(H^0)\) as a function of \(m_{h^0}\) for a benchmark point where \(\lambda_a = -1\) (right)

In order to establish in this case the branching ratios of the heaviest CP even neutral Higgs boson, we present in Fig. 4 (right) the decay branching fractions of the heavier Higgs boson \(H^0\) in the HTM0, for a benchmark point where \(\lambda_a = -1\). We see that for \(200\) GeV < \(m_{H^0}\) < 250 GeV, the dominant decay channels are the \(H^0 \to ZZ\) and \(W^+W^-\) decay modes, whereas \(h^0h^0\) is off-shell and consequently its corresponding ratio gets a tiny values of order of 1%, regardless of what \(\lambda_a\) can be. Once \(h^0h^0\) threshold takes place, this channel becomes predominant for
Figure 5: $R_{\gamma\gamma}(h^0)$ as a function of $m_{H^\pm}$ for various values of $\lambda_a$ (left). Correlation between $R_{\gamma\gamma}(h^0)$ and $R_{\gamma Z}(h^0)$ for various of $\lambda_a$ (right). We take as inputs: $\lambda \approx 0.52$, $2.5 \leq \mu \leq 15$ (GeV) ($m_{h^0} \approx 125$ GeV), $\lambda_b = 1$ and $v_t = 1$ GeV.

negative $\lambda_a$, with the ratio $R_{\gamma\gamma}(h^0)$ almost equal to its standard value, and $250 \leq m_{H^0} < 450$ (GeV). This feature persists even when $t\bar{t}$ threshold is reached at $m_{H^0} > 350$ GeV.

According to the ratio definition adopted in [25], we display the deficit of $R_{\gamma\gamma}(h^0)$ in the left panel of Fig.5 as a function of $H^\pm$ mass for various values of $\lambda_a$ and with $m_{H^0} \geq 140$ GeV. As it can be seen, a mass about 255 GeV and above is allowed for $H^\pm$ within $+1\sigma$ of ATLAS value for $\lambda_a = -1.4$. Once $\lambda_a$ increases, this lower bound decreases consistently to reach its lowest value around $\sim 197$ GeV, given $\lambda_a > -0.5$. This situation is exactly the opposite for CMS, where only the range $200 \leq m_{H^\pm} \leq 250$ (GeV) is excluded for $\lambda_a = 1.4$. Besides, $R_{\gamma\gamma}(h^0)$ tends towards its standard value for $\lambda_a \neq 0$, and to 1 for large $m_{H^\pm}$ whatever the variation of $\lambda_a$.

In this scenario, the anti-correlation between $R_{\gamma\gamma}(h^0)$ and $R_{\gamma Z}(h^0)$ is displayed in the left panel of Fig.5, taking into account the experimental tests at $1\sigma$. At first sight, the $R_{\gamma Z}(h^0)$ deviation is almost nul relatively to its standard value, and contrary to what has been claimed in [12], $R_{\gamma\gamma}(h^0)$ and $R_{\gamma Z}(h^0)$ are always anti-correlated, independently of $\lambda_a$ sign.

4.2 $H^0$ SM-like invisible decays

This section investigates the possible existence of a scalar state $h^0$ lighter than $H^0$, with $M^0_{H} \approx 125$. Such a scenario has attracted attention within a plethora of theoretical frameworks dealing with new physics beyond standard model [26, 27], particularly those considering enlargement
of the Higgs sector of the SM via doublet or triplet fields. However, to our knowledge, it has not yet been addressed in the HTM0.

Figure 6: $m_{h^0}$ (left) and $m_{H^\pm}$ (right) dependences on $v_t$ (upper panel) and $\mu$ (lower panel).

Inputs : $\lambda \approx 0.52$ ($m_{H^0} \approx 125$ GeV), $|\lambda_a| \leq 1.5$, $0 \leq \lambda_b \leq 1$, $10^{-2} \leq \mu \leq 10$ (GeV) and $10^{-2} \leq v_t \leq 3$ (GeV)

Figure 6 displays the dependence of light and charge Higgs bosons masses on $\mu$ and $v_t$ parameters when the heavier CP-even state $H^0$ is identified to the SM-like Higgs boson. At first glance, the default values of these parameters for a given region where $m_{h^0} \leq \frac{m_t^2}{2}$ should not be of the same order of magnitude, indeed, to fulfil such situation, we request $v_t$ to be equal or slightly higher than 1 GeV for a given $\mu$ below 1 GeV. As a results, the parameter space is quite restricted offering many new interesting features. Indeed, the Higgs charged is very light with an upper bound on its mass about 180 GeV, as can been seen from Eq. (2.11). Also, for such small values of $\mu$, the lighter CP-even state $h^0$ is mostly dominated by a triplet component and is typically very light mass as shown in Eq. (2.22). It is worth to notice that, according to
Figure 7: Branching ratio of $H^0 \to b\bar{b}$, $c\bar{c}$, $\tau^+\tau^-$, $W^+W^-$, $ZZ$, $h^0h^0$ and $H^\pm H^\mp$ as a function of $\lambda_a$. Our inputs are $\lambda \approx 0.52$, $\lambda_b = 1$, $v_t = 1$ GeV and $0.1 \leq \mu \leq 0.52$ (GeV) ($m_{H^0} \approx 125$ GeV).

Eq. (2.30), the mass of the heavier CP-even state $H^0$ matches the observed value $m_{H^0} \approx 125$ GeV, if the coupling $\lambda$ is approximately set to the value $\lambda \approx 0.52$. Such scenario offers a particularly rich phenomenology. Our analysis will focus on two interesting Higgs to Higgs decays, namely: $H^0 \to h^0h^0$, $H^\pm H^\mp$. These invisible Higgs decay channels might become kinematically favoured with significant branching ratios for certain regions of the HTM0 parameter space. Indeed, as $|s_\alpha| \approx 1$, $c_\alpha \approx 0$ in these regions, the $h^0h^0H^0$ and $H^\pm H^\mp H^0$ couplings reduce to,

$$g_{h^0h^0H^0} = g_{H^\pm H^\mp H^0} \approx \lambda_a v_d + \mathcal{O}(v_t) \quad (4.2)$$

Then, we plot in Fig.7 the branching ratios of the $H^0$ decays into $b\bar{b}$, $c\bar{c}$, $W^+W^-$, $ZZ$, and into the invisible decay modes $h^0h^0$ and $H^\pm H^\mp$. We clearly see that the branching ratios into $h^0h^0$ and $H^\pm H^\mp$ become dominant for non-vanishing values of $|\lambda_a|$, as can be seen from Eq. (4.2) where the corresponding couplings get substantially large values. However, once $\lambda_a$ approaches zero, these decay channels fade away.

By the following, we fix $v_t = 1$ GeV and $\lambda_b = 1$, we present in Fig.8 the branching ratios for $H^0 \to h^0h^0$ and $H^0 \to H^\pm H^\mp$. From the left panel, we can see that decay into $h^0h^0$ gets sizeable values important for values of the $\mu$ parameter larger than 0.15 GeV ($m_{h^0} \approx 35$ GeV), reaching up to 7% when $m_{h^0}$ is around 45 $\sim$ 50 GeV. When $\mu$ becomes larger than 0.26 GeV...
(m_{h^0} \approx 45 \text{ GeV}), this ratio decreases slightly but still remains relatively important, and never falls below 6\%. Furthermore, it shall get greater for m_{h^0} \approx 60 \sim 65 \text{ GeV} and drop 7\% again.

Figure 8: m_{h^0} (left) and m_{H^\pm} (right) dependences on v_t (upper panel) and \mu (lower panel). Inputs : \lambda \approx 0.52 (m_{h^0} \approx 125 \text{ GeV}), |\lambda_a| \leq 1.5, 0 \leq \lambda_b \leq 1, 10^{-2} \leq \mu \leq 10 \text{ (GeV)} and 10^{-2} \leq v_t \leq 3 \text{ (GeV)}

Figure 9: (Left) : R_{\gamma\gamma}(H^0) as a function of m_{H^\pm} for various values of \lambda_a. (Right) : correlation between R_{\gamma\gamma}(H^0) and R_{\gamma Z}(H^0) for various of \lambda_a. Our inputs are : \lambda \approx 0.52, 0.5 \leq \mu \leq 1.6 \text{ (GeV)} (m_{h^0} \approx 125 \text{ GeV}), \lambda_b = 0.1 and v_t = 1 \text{ GeV}.

The situation is quite different for the BR(H^0 \rightarrow H^\pm H^\pm) as illustrated in the right panel of Fig.8. This ratio tends to its maximal value, \approx 2.7\%, for very tiny \mu about \approx 0.1 \text{ GeV},
corresponding to small values of \( m_{H^\pm} \approx 39 \) GeV, and decreases inversely when \( \mu \) increases up to the value \( \mu \approx 0.26 \) GeV. In contrast to the decay into \( h^0 h^0 \), beyond this value, the branching ratio is almost vanishing.

From the left side of Fig. 9, the ratio \( R_{\gamma\gamma}(H^0) \) reaches its SM-like value for \( \lambda_a \approx 0 \) and for the charged Higgs mass in the range 40 ∼ 160 GeV, while an excess up to 20% can be achieved for negative values of \( \lambda_a \). If ATLAS/CMS exclusions data at 1σ, is taken into account, then this excess is largely reduced to less than 10%. As a byproduct, this analysis sets up a lower limit on the \( m_{H^\pm} \) of order ∼ 115 GeV (for \( \lambda_a = -0.2 \)). In addition, \( R_{\gamma\gamma}(H^0) \) remains below its SM value when \( \lambda_a > 0 \), even for \( m_{H^\pm} \) above this lower value. At last, we study correlation of \( R_{\gamma\gamma}(H^0) \) with \( R_{\gamma Z}(H^0) \) in this scenario. Unlike the \( h^0 \) SM-like case, one can see from the right panel of Fig. 9 that these observables are correlated for \( \lambda_a < 0 \) or anti-correlated for \( \lambda_a > 0 \) with a predicted charged Higgs mass in the range [130 ∼ 160] or [110 ∼ 160] GeV respectively.

### 4.3 Degenerate case : \( m_{H^0} \approx m_{h^0} \approx 125 \) GeV

In this subsection, we consider the CP-even neutral Higgs bosons \( h^0 \) and \( H^0 \) with nearly degenerate mass. This scenario has recently attracted attention and been taken seriously in many SM extensions [8, 28–30]. Here we would like to ask to what extent this survives in HTM0 in light of LHC data at 13 TeV. In other words, we probe the region of the parameter space where the twin Higgs decays into diphoton Higgs with branching ratio (or signal strength \( R_{\gamma\gamma} \)) consistent with ATLAS and CMS data. A first analysis has been performed in [12]. This analysis used an intriguing and unjustified hypothesis considering the charged Higgs mass equals to the neutral ones. In this model, this possibility is excluded by theoretical constraint as we will show shortly. But first, we will demonstrate that the parameter space is restricted further by an additional constraint, induced by the Higgs mass degeneracy, and leading to a severe control of the potential parameters.

The two eigenvalues \( m_{\pm} \) (with \( m_- = m_{h^0}^2 < m_+ = m_{H^0}^2 \)), representing the squared masses of \( h^0 \) and \( H^0 \), are:

\[
m_{\pm} = \frac{A + C \pm \sqrt{(A - C)^2 + 4B^2}}{2}.
\] (4.3)
Then
\[ m_+ - m_- = (m_{H^0} - m_{h^0})(m_{H^0} + m_{h^0}) \]
\[ \approx (m_{H^0} - m_{h^0})2M_{ex} = 2M_{ex}\Delta M. \]

where \( \Delta M \), the difference of masses between the two neutral Higgs \( H^0 \) and \( h^0 \) is set to about 1 GeV, corresponding to the detector inability to resolve two nearly Higgs signals, and \( M_{ex} \) is the experimental Higgs boson mass \( \approx 125 \) GeV. Taking into account these considerations one gets \( \sqrt{(A - C)^2 + 4B^2} \leq 2M_{ex}\Delta M \), that obviously leads to two constraints: \( |B| \leq M_{ex}\Delta M \) and \( |A - C| \leq 2M_{ex}\Delta M \).

The first constraint reads as:
\[ |2\lambda_\alpha v_t - \mu| \leq 2\sqrt{2} \frac{M_{ex}\Delta M}{v_d}, \]

while, for small ratio of the two vevs \( \frac{v_t}{v_d} \), the second constraint reduces to,
\[ |4\lambda v_t - \mu| \leq \frac{8v_t}{v_d} 2\frac{M_{ex}\Delta M}{v_d}, \]

Since the ratio \( \frac{2M_{ex}\Delta M}{v_d} \) is about 1 GeV, these two relations simplify to \( |2\lambda_\alpha v_t - \mu| \leq \sqrt{2} \) GeV and \( \frac{\mu}{\lambda} \approx 4v_t \), providing strict bounds to the three potential parameters \( \mu, \lambda \) and \( \lambda_\alpha \), hence severely reducing the allowed regions in the parameter space, as it is illustrated in Fig.10.

This feature has a dramatic effect on the discrepancy between the neutral and charged Higgs masses as can be seen from Fig.2. In such figure, the Higgs bosons masse behaviours are plotted as a function of the \( \mu \) parameter; these values satisfy the above resulting relation in the degenerate case. The seemingly constant \( m_{h^0}^2 \) for \( \mu > \mu_c \) and constant \( m_{H^0}^2 \) for \( \mu < \mu_c \) are clearly achieved around the critical value \( \mu_c \approx 2.1 \) GeV. Contrary to what one might think, if we take the Higgs bosons masses as inputs [12], such a situation matches a splitting between the charged Higgs boson mass and the \( H (= h^0 = H^0) \) degenerate state mass in the range of \( \Delta m = m_{H^\pm} - m_{H} \approx 51 \) GeV. Hereafter we define the diphoton signal strength \( R_{\gamma\gamma} \) by the following quantity,
\[ R_{\gamma\gamma} = R_{\gamma\gamma}(h^0) + R_{\gamma\gamma}(H^0) \] (4.6)

and by the same way \( R_{\gamma Z} \) is introduced. In this scenario, the charged Higgs boson loops are included with the \( g_{H^\pm vv}, g_{H^\pm ff} \) couplings given by Table.1.
Figure 10: The allowed regions in $(\lambda_a, \lambda_b)$ for $\delta T = 10$ in the degenerate case. Color codes are as follows, **Orange**: Excluded by Unitarity constraints. **Red**: Excluded by Unitarity+BFB constraints. **Blue**: Excluded by Unitarity+BFB & $\mu/\lambda \approx 4 v_t$ constraints. **Yellow**: Excluded by Unitarity+BFB & $T_d \approx \delta T \wedge T_l \approx \delta T$ & $\mu/\lambda \approx 4 v_t$ constraints. Only the **Green** area obeys ALL constraints. Our inputs are $\lambda = 0.52$, $-5 \leq \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \leq 5$, $10^{-3} \leq v_t \leq 3$ (GeV) and $10^{-3} \leq \mu \leq 5$ (GeV).

Fig.11 illustrates the HTM0 degenerate case effect on $R_{\gamma\gamma}$. Similarly to the previous scenarios, we fix $\lambda \sim 0.518$ and scan over $\lambda_a, \lambda_b, \mu$ and $v_t$, with the Higgs masses given by Eqs.4.3, 4.4 and 4.5. In the left panel, we show the scatter plot for the mixing angle $\alpha$ in the $(R_{\gamma\gamma}, v_t)$ plane. Again we see that small values below 0.5 are favoured for the triplet vev $v_t$ to achieve the standard limit, corresponding to $\sin \alpha \sim 0.55 - 0.65$. In the right panel, we show the variation of $R_{\gamma\gamma}$ a function of $\mu$ and $v_t$ within 1$\sigma$ of ATLAS/CMS measurements.

Finally, we display in Fig.12 we have plotted $R_{\gamma\gamma}$ versus $R_{\gamma Z}$ in mass degenerate scenario for various values of $\lambda_a$. From this plot one can see that the correlation is always positive whatever the value of $\lambda_a$. We also note that no noticeable enhancement can be achieved, since most part of the parameter space is drastically constrained by a constant charged Higgs mass at about $m_{H^\pm} \sim 176$ GeV; as shown form Fig.2.
5 Conclusion

In this paper, we have discussed some features of the Higgs triplet model with null hypercharge (HTM0), an extension of the SM with a larger scalar sector. First, we have shown that the space parameter of HTM0 generally constrained by unitarity and boundedness from below, is severely reduced when the modified Veltman conditions are imposed. Then, we have investigated some Higgs decays, including Higgs to Higgs decays, in light of LHC data, either when \( h^0 \) is the SM-like Higgs or when the heaviest neutral Higgs \( H^0 \) is identified to the 125 observed GeV Higgs. In addition, we have analysed the degenerate scenario and shown that LHC signal strengths favours a light charged Higgs mass about 176 \( \sim \) 178 GeV. Finally, we have pointed out some discrepancies with previous analysis, regarding the correlations between the diphoton Higgs decay mode and \( H \rightarrow Z\gamma \) mode.

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Figure 11: Left: Scatter plot for \( \sin \alpha \) in the plan \((R_{\gamma\gamma}, v_t)\) with \(0.1 \leq \mu \leq 5\) (GeV). Right: \( R_{\gamma\gamma} \) as function of \( v_t \), where the palette shows the size of \( \mu \).
Figure 12: $R_{\gamma\gamma}$ and $R_{\gamma Z}$ correlation in the degenerate case for various $\lambda_a$. Inputs are the same as in Fig.11 except for $\lambda_a$.

**Appendix A : Boundedness from below of the potential**

Obviously, at large field values the potential Eq. (2.4) is generically dominated by the part containing the terms that are quartic in the fields,

$$
V^{(4)}(H, \Delta) = \left(\lambda/4\right)(H^\dagger H)^2 + \lambda_1(H^\dagger H)Tr(\Delta^\dagger \Delta) + \lambda_2(Tr\Delta^\dagger \Delta)^2 + \lambda_3Tr(\Delta^\dagger \Delta)^2 + \lambda_4H^\dagger \Delta^\dagger \Delta H
$$

(5.1)

The study of $V^{(4)}(H, \Delta)$ will thus be sufficient to obtain the main constraints. To obtain BFB conditions it is common in the literature to pick up specific field directions or to put some of the couplings to zero. To proceed to the most general case, we adopt the same parameterisation as in [7]. The $\xi$ and $\zeta$ parameters defined in [7] become in our model,

$$
\xi = \frac{1}{2} \quad \text{and} \quad \zeta = \frac{1}{2}
$$

(5.2)

With this parameterisation, boundedness from below is then equivalent to requiring $V^{(4)} > 0$ for all directions, and one gets

$$
\lambda > 0 \quad \& \quad \lambda_b > 0 \quad \& \quad \lambda_a + \sqrt{\lambda \lambda_b} > 0.
$$

(5.3)
Appendix B : Unitarity constraints

The quartic scalar vertices in the basis of unrotated states have a much simpler form than the complicated functions of $\lambda_i, \alpha$ and $\theta_\pm$ obtained in the physical basis ($H^\pm, G^\pm, h^0, H^0$, and $G^0$) of mass eigenstate fields. The $S$-matrix for the physical fields is related by a unitary transformation to the $S$-matrix for the unrotated fields. Close inspection shows that the full set of 2-body scalar scattering processes leads to a $19 \times 19$ $S$-matrix which can be decomposed into 5 block submatrices corresponding to mutually unmixed sets of channels with definite charge and CP states. One has the following submatrix dimensions, structured in terms of net electric charge in the initial/final states: $S^{(1)}(4 \times 4)$, $S^{(2)}(5 \times 5)$ and $S^{(3)}(1 \times 1)$ corresponding to 0-charge channels, $S^{(4)}(6 \times 6)$ corresponding to the 1-charge channels and $S^{(5)}(3 \times 3)$ corresponding to the 2-charge channels.

Appendix C : Feynman Rules for tadpoles

In this appendix, we list the couplings used to calculate the tadpoles of the two neutral CP-even Higgs $h^0$ and $H^0$ as explained in [9].

We note $c_{F_i}$ ($C_{F_i}$) the couplings to the Higgs $h^0$ ($H^0$) where $F_i$ stands for any quantum field of the HTM0: scalar and vectorial bosons, fermions, Goldstone fields $G_i$ and Faddeev-Popov ghost fields $\eta_i$. Because the field $F_i$ fixes the propagator, we also list the values $t_i$ ($T_i$) of the loop due to the propagator of the $F_i$ particle which gain a factor 2 in case of charged fields, and the symmetry factor $s_i$.

\[
\begin{align*}
  c_1 &\equiv c_{h_0 h_0} = -\frac{3i}{2}(\lambda v_d c_\alpha^3 + 2\lambda_a v_d c_\alpha s_\alpha^2 + 4\lambda_b v_t s_\alpha^3 + (-\mu + 2\lambda_a v_t)c_\alpha^2 s_\alpha), \\
  C_1 &\equiv C_{H_0 H_0} = \frac{3i}{2}(\lambda v_d s_\alpha^3 + 2\lambda_a v_d s_\alpha c_\alpha^2 - 4\lambda_b v_t c_\alpha^3 - (-\mu + 2\lambda_a v_t)s_\alpha^2 c_\alpha), \\
  t_1 & = iA_0(m_{h_0}^2), \\
  T_1 & = iA_0(m_{H_0}^2), \\
  s_1 & = \frac{1}{2}.
\end{align*}
\] (5.4)
\[ c_2 \equiv C_{e_0e_0} = -\frac{i}{2}(-\mu s_\alpha + \lambda v_d c_\alpha + 2\lambda_a s_\alpha v_t), \]
\[ C_2 \equiv C_{G_0G_0} = +\frac{i}{2}(\mu c_\alpha + \lambda v_d s_\alpha - 2\lambda_a c_\alpha v_t), \]
\[ t_2 = T_2 = iA_0(\xi_Z m_Z^2), \]
\[ s_2 = \frac{1}{2}, \quad (5.5) \]
\[ c_3 \equiv C_{G_+G_-} = -\frac{i}{2}(2\mu c_\alpha c_{\theta \pm} s_{\theta \pm} + (\mu s_\alpha + \lambda v_d c_\alpha + 2\lambda_a v_t s_\alpha) c_{\theta \pm}^2 + 2(\lambda v_d c_\alpha + 2\lambda_b v_t s_\alpha) s_{\theta \pm}^2), \]
\[ C_3 \equiv C_{G_+G_-} = -\frac{i}{2}(-2\mu c_{\theta \pm} s_{\theta \pm} + \lambda v_d c_{\theta \pm}^2 + 2\lambda_a v_t s_{\theta \pm}^2) s_\alpha + (4\lambda_b s_{\theta \pm}^2 v_t + 2\lambda_a v_t c_{\theta \pm}^2 + \mu c_{\theta \pm}^2) c_\alpha), \]
\[ t_3 = T_3 = 2 \times iA_0(\xi_W m_W^2), \]
\[ s_3 = \frac{1}{2}, \quad (5.6) \]
\[ c_4 \equiv C_{H_0H_0} = -\frac{i}{2}(2\lambda_d c_\alpha^2 v_d + (3\lambda - 4\lambda_a) c_\alpha s_\alpha^2 v_d - (\mu - 2\lambda_a v_t) s_\alpha^3 + 2c_\alpha^2 s_\alpha (\mu - 2(\lambda_a - 3\lambda_b)v_t)) \]
\[ C_4 \equiv C_{H_0H_0} = -\frac{i}{2}(-2\lambda_a s_\alpha^3 v_d - (3\lambda - 4\lambda_a) s_\alpha c_\alpha^2 v_d - (\mu - 2\lambda_a v_t) c_\alpha^3 + 2s_\alpha^2 c_\alpha (\mu - 2(\lambda_a - 3\lambda_b)v_t)) \]
\[ t_4 = iA_0(m_{H_0}^2), \]
\[ T_4 = iA_0(m_{H_0}^2), \]
\[ s_4 = \frac{1}{2}, \quad (5.7) \]
\[ c_5 \equiv C_{H_+H_-} = -\frac{i}{2}((-2\mu c_{\theta \pm} s_{\theta \pm} + 2\lambda_a c_{\theta \pm}^2 v_d + \lambda s_{\theta \pm}^2 v_d) c_\alpha + (4\lambda_b c_{\theta \pm}^2 v_t + (\mu + 2\lambda_a v_t) s_{\theta \pm}^2) s_\alpha) \]
\[ C_5 \equiv C_{H_+H_-} = -\frac{i}{2}(2\mu c_{\theta \pm} s_{\theta \pm} s_\alpha + (\mu c_\alpha - \lambda v_d s_\alpha + 2\lambda_a v_t c_\alpha) c_{\theta \pm}^2 + 2(-\lambda_a v_d s_\alpha + 2\lambda_b v_t c_\alpha) c_{\theta \pm}^2) \]
\[ t_5 = T_5 = 2 \times iA_0(m_{H_\pm}^2), \]
\[ s_5 = \frac{1}{2}, \quad (5.8) \]
\[ c_6 \equiv C_{ZZ} = iem_{W} c_\alpha c_{\theta \pm}/(c_{\theta \pm}^2 s_{\theta \pm}), \]
\[ C_6 \equiv C_{ZZ} = -i em_{W} c_{\theta \pm} s_\alpha/(c_{\theta \pm}^2 s_{\theta \pm}), \]
\[ t_6 = T_6 = -i(n - 1)A_0(m_Z^2) + \xi_Z A_0(\xi_Z m_Z^2), \]
\[ s_6 = \frac{1}{2}, \quad (5.9) \]
\begin{align}
  c_7 & \equiv c_{W_+W_-} = i e m_W (c_\alpha c_{\theta_{\pm}} - 2 s_\alpha s_{\theta_{\pm}}) / s_w, \\
  C_7 & \equiv C_{W_+W_-} = -i e m_W (c_{\theta_{\pm}} s_\alpha - 2 c_\alpha s_{\theta_{\pm}}) / s_w, \\
  t_7 = T_7 & = 2 \times (-i((n-1) A_0(m_W^2) + \xi_W A_0(\xi_W m_W^2))), \\
  s_7 & = \frac{1}{2},
\end{align}

\begin{align}
  c_8 & \equiv c_{ff} = -\frac{i}{2} e (c_\alpha / c_{\theta_{\pm}}) m_f / (m_W s_w), \\
  C_8 & \equiv C_{ff} = \frac{i}{2} e (s_\alpha / c_{\theta_{\pm}}) m_f / (m_W s_w), \\
  t_8 = T_8 & = i m_f A_0(m_f^2) Tr(I_\alpha), \\
  s_8 & = 1,
\end{align}

\begin{align}
  c_9 & \equiv c_\eta\bar{\eta}_Z = -\frac{i}{2} e m_W (c_\alpha c_{\theta_\pm}) \xi_Z / (c_w^2 s_w), \\
  C_9 & \equiv C_\eta\bar{\eta}_Z = \frac{i}{2} e m_W (c_{\theta_\pm} s_\alpha) \xi_Z / (c_w^2 s_w), \\
  t_9 = T_9 & = i A_0(\xi_Z m_Z^2), \\
  s_9 & = 1,
\end{align}

\begin{align}
  c_{10} & \equiv c_{\eta_\pm\bar{\eta}_\pm} = -\frac{i}{2} e m_W (c_\alpha c_{\theta_\pm} - 2 s_\alpha s_{\theta_\pm}) \xi_W / s_w, \\
  C_{10} & \equiv C_{\eta_\pm\bar{\eta}_\pm} = \frac{i}{2} e m_W (c_{\theta_\pm} s_\alpha - 2 c_\alpha s_{\theta_\pm}) \xi_W / s_w, \\
  t_{10} = T_{10} & = 2 \times i A_0(\xi_W m_W^2), \\
  s_{10} & = 1.
\end{align}

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