Observation of topological magnon insulator states in a superconducting circuit

W. Cai,1,∗ J. Han,1,∗ Feng Mei,2,3,† Y. Xu,1 Y. Ma,1 X. Li,1 H. Wang,1 Y. P. Song,1 Zheng-Yuan Xue,4 Zhang-qi Yin,1 Suotang Jia,2,3 and Luyan Sun1,‡

1Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing 100084, China
2State Key Laboratory of Quantum Optics and Quantum Devices, Institute of Laser Spectroscopy, Shanxi University, Taiyuan, Shanxi 030006, China
3Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan, Shanxi 030006, China
4Guangdong Provincial Key Laboratory of Quantum Engineering and Quantum Materials, and School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou 510006, China

Searching topological states of matter in tunable artificial systems has recently become a rapidly growing field of research. Meanwhile, significant experimental progresses on observing topological phenomena have been made in superconducting circuits. However, topological insulator states have not yet been reported in this system. Here, for the first time, we experimentally realize a spin version of the Su-Schrieffer-Heeger model and observe the topological magnon insulator states in a superconducting qubit chain, which manifest both topological invariants and topological edge states. Based on simply monitoring the time evolution of a single-qubit excitation in the chain, we demonstrate that the topological winding numbers and the topological magnon edge and soliton states can all be directly observed. Our work thus opens a new avenue to use controllable qubit chain system to explore novel topological states of matter and also offers exciting possibilities for topologically protected quantum information processing.

I. INTRODUCTION

Topological insulators, insulating in the bulk but conducting at their boundary, are a new state of matter and lie at the forefront of condensed matter physics [1, 2]. They are signified by topological invariants and topological edge states [1, 2]. Topological invariants are defined on the systematic bulk states and robust to smooth changes of systematic parameters. According to the bulk-edge correspondence, a nontrivial value of topological invariant guarantees the appearance of topological edge states, which are localized at the boundary between regions characterized by distinct topological invariants and are another hallmark of topological insulators. What makes these edge states particularly attractive is their topologically protected robustness to the presence of a wide class of disorders. Due to these topological protections, topological insulators hold tremendous promises for fundamental topological states of matter [1, 2] as well as for topological quantum computation [3].

The concept of topological insulators has recently been expanded to artificial systems. Topological invariants and edge states have been separately and extensively studied in ultracold atomic [4, 5], photonic [6, 7] and mechanical [8] systems. However, due to the difficulties in preparing photonic and phononic Bloch states in the momentum space and in engineering edges in an optical lattice, it is still challenging to experimentally explore the topological invariants in photonic and mechanical systems and the topological edge states in ultracold atomic systems. Therefore, to solidly confirm the emergence of topological insulator states in a separate artificial topological system by observing both the topological invariants and topological edge states is highly desirable.

Superconducting circuits now have become one of the leading quantum platforms for implementing scalable quantum computation [9–11] and large-scale quantum simulation [12–14]. In particular, topological effects recently have also been experimentally studied in superconducting circuits. Specifically, topological concepts have been investigated in the parameter space of superconducting qubits [15–18], including topological Chern numbers and topological transitions; Topological quantum walks have been implemented in the phase space of superconducting resonators [19, 20]; Synthetic gauge fields [21–23] and Hofstadter butterfly [24] have been realized and measured in superconducting circuits; Topological defects recently have been experimentally observed in a network of superconducting flux qubits [25]. However, topological insulator states have not been experimentally realized and explored before in this quantum computing platform.

In this paper, we experimentally demonstrate the first observation of topological insulator states in a superconducting qubit chain, which exhibits both the nontrivial topological invariants and topological edge states. Our experiment is based on a spin version of the Su-Schrieffer-Heeger (SSH) model [26], which supports topologically trivial or nontrivial magnon insulator states dependent on the qubit coupling configuration. In our experiment, different coupling configurations are realized through parametrical modulations of the transmon qubit frequencies in situ [27–31]. Based on exciting one of the qubits in the chain and then monitoring the dynamics of this excitation among all qubits, we obtain the trivial and nontrivial topological winding numbers and observe the topological magnon edge and soliton states all in one device. Distinct from the topological states of electrons [1, 2], atoms [4, 5], photons [6, 7] and phonons [8], the observed topological magnon insulator state here is a different topological state of matter, where magnons are bosonic quasiparticle excitations around the ground state of strongly correlated quantum magnets [32, 33]. Our experiment reveals the po-
topological magnon insulator states which are characterized with magnon energy bands in the momentum space and supports one-dimensional lattice with different intracell and intercell hopping amplitudes, as shown in Fig. 1(b). We implement such a model in a superconducting qubit chain, where each unit cell contains two qubits labelled by \( a \) and \( b \). The resulted qubit chain can be described by the spin version of the SSH model Hamiltonian after rotating wave approximation

\[
\hat{H} = \sum_{x=1}^{N} (J_1 \hat{\sigma}_{a_x}^+ \hat{\sigma}_{b_x}^- + J_2 \hat{\sigma}_{a_x}^+ \hat{\sigma}_{a_{x+1}}^- + \text{H.c.}),
\]

where \( x \) is the unit cell index, \( N \) is the number of the unit cells, \( J_1 \) and \( J_2 \) are the intracell and intercell qubit couplings, respectively, and \( \sigma_{a_x}^+ (\sigma_{a_x}^-) \) is the raising (lowering) operator associated with qubit \( a_x \). The single-qubit excitation in this spin chain is called a magnon in condensed matter physics [32, 33]. In the single-qubit excitation case, this qubit chain has two magnon energy bands in the momentum space and supports topological magnon insulator states which are characterized by topological winding number (see Supplementary Materials)

\[
\nu = \begin{cases} 
1, & J_1 < J_2; \\
0, & J_1 > J_2. 
\end{cases}
\]

One can find that the qubit chain exhibits two distinct topological magnon insulator states. When the qubit couplings have a dimerization \( J_1 < J_2 \) (\( J_1 > J_2 \)), the topological winding number value is nontrivial (trivial) and the system supports a topologically nontrivial (trivial) magnon insulator state. According to the bulk-edge correspondence, the non-zero topological winding number leads to the existence of topological magnon edge states in the topological case localized at the end of the qubit chain.

B. Experimental setup and parametrical modulations for tunable couplings

We implement the experiment in a superconducting circuit [9, 10, 34] consisting of five cross-shaped transmon qubits (Xmons, \( a_1, b_1, a_2, b_2, a_3 \)) [35, 36] arranged in a linear array with fixed capacitive nearest-neighbor couplings, as shown in Fig. 1(b). Each qubit has independent \( XY \) and \( Z \) controls. Separate \( \lambda/4 \) resonators with different frequencies couple to individual qubits for independent readouts. The average qubit \( T_1 \approx 18 \mu s \) and \( T_2 \approx 17 \mu s \) at the frequency sweet spots. We use a Josephson parametric amplifier [37, 38], a gain over 20 dB and a bandwidth about 260 MHz, for high-fidelity single-shot measurements of the qubits. To overcome the readout imperfections, we in addition use a calibration matrix to reconstruct the readout results based on Bayes’ rule.

The topologically trivial and nontrivial phases require different qubit-qubit coupling configurations, necessitating full control of the effective couplings between neighboring qubits. Tunable couplings through parametrical modulations of the qubits can be realized in situ without increasing circuit complexity [27–31], therefore are ideal for topological simulations. We adopt this technique throughout our experiment to realize the required neighboring qubit coupling strengths as described in Eq. 1.

Explicitly, we apply

\[
\omega_{id} = \omega_{0, id} + \epsilon_{id} \sin(\mu_{id} t + \varphi_{id}),
\]

where \( \omega_{0, id} \) is the mean operating frequency, \( \epsilon_{id}, \mu_{id}, \) and \( \varphi_{id} \) are the modulation amplitude, frequency, and phase, respectively for the qubit \( id = a_x, b_x \) in the chain. By neglecting the higher order oscillating terms and under the resonant conditions \( \omega_{0, b_x} - \omega_{0, a_x} = \mu_{b_x} \) or \( \omega_{0, b_{x-1}} - \omega_{0, a_x} = \mu_{a_x} \), the effective coupling strengths are

\[
J_1 = g_{a_x, b_x} \mathcal{J}_1(\alpha_{a_x}) J_0(\alpha_{a_x}) e^{i(\varphi_{a_x} + \pi/2)}, \]

\[
J_2 = g_{b_{x-1}, a_x} \mathcal{J}_1(\alpha_{a_x}) J_0(\alpha_{a_x}) e^{-i(\varphi_{a_x} - \pi/2)},
\]

where \( \mathcal{J}_m(\alpha) \) is the \( m \)th Bessel function of the first kind and \( g_{a_x, b_x} \) is the static capacitive coupling strength between neighboring qubits. Both \( J_1 \) and \( J_2 \) can be conveniently tuned via changing \( \alpha_{id} = \epsilon_{id}/\mu_{id} \) of the external modulation. Note that the qubit at the edge (for example, \( a_1 \)) could be stationary without parametric modulation, while the middle qubit can be parametrically modulated with two independent sinusoidal
drives in order to tune the coupling strengths with its two neighboring qubits respectively. In our experiment, we first adjust \( \alpha_{ld} = \tilde{\varepsilon}_{ld}/\mu_{ld} \) to realize the wanted coupling strength roughly for each pair of the neighboring qubits without driving other qubits. Due to residual crosstalk, we then do optimization to achieve the exact wanted coupling configuration when all drives are on. The experimental setup, device parameters, and parametric modulation parameters are all presented in Supplementary Materials.

C. Experimental measurement of topological winding numbers

We firstly demonstrate that topological winding number can be measured by single-magnon quantum dynamics in a chain of four transmon qubits, provided the qubit chain is initially prepared in a single-magnon bulk state. This dynamic method for measuring topological winding number was originally proposed in a linear-optics system for studying discrete-time quantum walk [39]. We choose to excite one of the middle qubits to the excited state \( |e\rangle \) and leave the other qubits in the ground state \( |g\rangle \), leading to an initial state of the system \( |\psi(t = 0)\rangle = |gegg\rangle \). After an evolution time \( t \), the state of the system becomes \( |\psi(t)\rangle = e^{-i\hat{H}t}|\psi(t = 0)\rangle \). To reveal the relationship between this dynamics and the topological winding number, we introduce the chiral displacement (CD) operator \( \hat{P}_d = \sum_{x=1}^{2} x(P_{\mu x} - P_{\tilde{x} \mu}) \) with \( P_{\mu x} = |e\rangle_{id} \langle e| \) \((id = a_x, b_x)\). In the long-time limit, the topological winding number \( \nu \) can be extracted from the time-averaged CD, \( \nu = \lim_{T \to \infty} \frac{2}{T} \int_0^T dt \langle P_d(t) \rangle \), where \( T \) is the evolution time and \( \langle P_d(t) \rangle = \langle \psi(t)|\hat{P}_d|\psi(t)\rangle \) is the CD associated with the dynamics of the single-magnon state (see Supplementary Materials). As we can see, the topological winding number is two times the time-averaged CD, i.e., the oscillation center of the CD versus time. Experimentally, to measure the time-averaged CD, we only need to track the time evolution of the excitation for each qubit.

In the experiment, as shown in Figs. 2a and 2b, we tune the qubit chain into two configurations with the qubit coupling dimerization \( J_1 > J_2 \) and \( J_1 < J_2 \), corresponding to the topologically trivial and nontrivial magnon insulator states, respectively. After preparing the initial state \( |\psi(0)\rangle \), we measure the time evolution of the qubit excitation of the four qubits and show the experimental data in Figs. 2c and 2d. The measured excitation evolutions agree well with the theoretical predictions. Based on these time-resolved excitation data for each qubit, we directly derive the time evolution of CDs and plot them in Figs. 2e and 2f. Clearly the two curves oscillate around two different center values, qualitatively giving the signature of different topological winding numbers. The evolution time in our experiment is chosen as \( 1 \mu s \), during which the experimentally measured time-averaged CDs are 0.015 and 0.359 for the topologically trivial and nontrivial cases, respectively. Both experiments agree very well with the theoretically expected values 0 and 0.378, giving the experimentally measured topological winding numbers \( \nu = 0.030 \) and \( \nu = 0.718 \) for the two cases. The measured winding number for the topologically trivial case is quite close to the ideal value.

The reason for the difference in the topologically nontrivial case between the measured winding number \( \nu = 0.718 \)
and the ideal value $\nu = 1$ is that both the evolution time and the qubit chain we choose are not long enough and there is also inevitable system decoherence. Nevertheless, our experimental data within 1 μs agrees excellently with the theoretical expectation and demonstrates the validity of the method using single-magnon dynamics to measure topological winding number. The clear distinction between the measured nontrivial and trivial topological winding numbers thus can unambiguously distinguish the topologically nontrivial and trivial magnon insulator states.

### D. Experimental observation of topological magnon edge states

The second hallmark for topological magnon insulator states is the existence of topological magnon edge states at the boundary between regions with distinct topological states. When the qubit chain is in the topological magnon insulator state, topological magnon edge state will emerge at the edges of the qubit chain separating the topologically nontrivial magnon insulator and the topologically trivial vacuum state. The wavefunctions of the left and right magnon edge states can be analytically derived as $|\psi_L\rangle = \sum_x (-1)^x (J_1/J_2)^x \sigma_n^x |gg \cdots gg\rangle$ and $|\psi_R\rangle = \sum_x (-1)^{N-x} (J_1/J_2)^{N-x} \sigma_n^x |gg \cdots gg\rangle$, respectively (see Supplementary Materials). It turns out that the magnon in the left (right) edge state only occupies the $a$-type ($b$-type) qubit and is maximally distributed in the leftmost (rightmost) qubit. Such two features provide a mean to observe the topological magnon edge states. However, the coupling between the left and right magnon edge states is very large due to finite lattice size effect, we cannot unambiguously observe the left or right magnon edge state localization in a short qubit chain (see Supplementary Materials). This problem can be solved by considering a qubit chain with an odd number of qubits, where the right topological magnon edge state has been artificially removed.

Now we show that the left topological magnon edge state can be clearly observed in a chain of five qubits where there is no right topological magnon edge state. As shown in Fig. 3a, we can tune the qubit couplings in a chain of five qubits to make the system topologically trivial and nontrivial, respectively. Initially, the leftmost $a$-type qubit is excited and a single magnon has been put on the leftmost qubit with the initial systematic state $|\psi(t = 0)\rangle = |egggg\rangle$. Then, we let this magnon state evolve for certain time and measure the time evolution of the magnon density in the qubit chain. The results for the qubit chain being tuned into the topologically trivial state are shown in Fig. 3b. As expected, there is no magnon edge state localization and the wavepacket has a ballistic spread versus time, which is a typical feature of bulk Bloch state. The reason is that the initial magnon state in this case is a superposition of different bulk states, therefore, it evolves in the qubit chain via the bulk state wavepackets and does not support edge state localization.

In contrast, if the system is in the topologically nontrivial state that can support left magnon edge states, as shown in Fig. 3c, the measured magnon density is always maximal in the leftmost qubit. This is because the initial magnon state $|\psi(t = 0)\rangle$ has a large overlap with the left magnon edge state $|\psi_L\rangle$. The magnon state thus mainly evolves in the qubit chain based on the edge state wavepacket and always maximally localizes in the leftmost qubit. Moreover, the magnon only populates the $a$-type qubits, also satisfying the feature of left topological magnon edge state as shown before. These two features prove the existence of left topological magnon edge state and clearly indicate that the system is topologically nontrivial. In Figs. 3d and 3e, we also find that the measured qubit excitation evolutions agree excellently with the theoretical predictions.
FIG. 4: Observation of topological magnon soliton states. a Schematic of the experiment. Topological magnon soliton is an effect in the topological structure with the presence of defects. The couplings between neighboring qubits are tuned into $J_1-J_2-J_1-J_2 = 4-1-1-4$ (MHz). This special coupling configuration can simulate a topological structure with a defect and demonstrate the topological soliton effect. b Two-dimensional representation of time evolutions of all qubits’ excited state populations. The excitation is maximally localized on $a_2$ ($a$-type) and only has population in the $a$-type qubits as expected. c Time evolutions of $P_{id}^t$ ($id = a_1, a_2, a_3$). Dots are experimental data while solid lines are calculated from the ideal Hamiltonian (Eq. 1) with the measured system decoherence for an initial state $|ggeggg⟩$.

E. Experimental observation of topological magnon soliton states

The third important topological aspect for the SSH model is the emergence of topological soliton states. For our system, when the qubit chain is prepared with two different topological configurations, a topological magnon soliton state can be created at the boundary between the topologically trivial ($J_1 > J_2$) and nontrivial ($J_1 < J_2$) regions. In the experiment, as shown in Fig. 4a, we create such a boundary at qubit $a_2$ in a chain of five qubits. The magnon in the topological soliton state should only occupy $a$-type qubits and its density should be maximally distributed in qubit $a_2$ (see Supplementary Materials).

To experimentally observe the topological magnon soliton state, we initially excite qubit $a_2$ and prepare the system in $|ψ(t = 0)⟩ = |ggegg⟩$. Such an initial state has a large overlap with the wavefunction of the topological magnon soliton state. If the system has the topological soliton state, the magnon will prorogate in the qubit chain via the soliton state wavepacket. In the experiment, after evolving $|ψ(t = 0)⟩$ for certain time, we measure the final magnon density distribution in the qubit chain. The experimental results are shown in Fig. 4b and indeed indicate that the magnon is maximally localized in the center qubit $a_2$ and only has populations in the $a$-type qubits, unambiguously demonstrating the existence of a topological magnon soliton state. The time evolutions of qubit excitation for the five qubits are also shown in Fig. 4c, agreeing well with the theoretical expectations.

III. DISCUSSION

In conclusion, we have experimentally observed the topological magnon insulator states based on realizing a spin version of the SSH model in a superconducting qubit chain. Via parametrical modulations of the qubit frequencies to change the qubit coupling configurations, we have shown that such a qubit system can be tuned to support topologically trivial and nontrivial magnon insulator states. Moreover, through tracking the nonequilibrium dynamics of a single-qubit excitation in the qubit chain, we have measured the topological winding numbers and observed the topological magnon edge and soliton states, all in excellent agreement with the theoretical expectations.

Our experiment has demonstrated the possibility of exploring topological insulator states with a qubit chain system. It is quite natural to apply the technology presented here to a larger qubit network for studying high-dimensional topological states [1, 2]. Besides, through periodically driving the qubit frequencies or couplings, this experiment can also be generalized to study Floquet topological insulator states [40]. Via engineering the qubit nonlinearity, the qubit chain system in addition provides opportunities for searching fractional topological insulator states [41]. Thus our work opens a door for using controllable qubit systems to realize exotic topological states of matter towards topologically protected quantum information processing [3].

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These two authors contributed equally to this work.
† Electronic address: meifeng@sxu.edu.cn
‡ Electronic address: luyansun@tsinghua.edu.cn

[1] M. Z. Hasan and C. L. Kane, “Colloquium: Topological insulators,” Rev. Mod. Phys. 82, 3045 (2010).
[2] X.-L. Qi and S.-C. Zhang, “Topological insulators and superconductors,” Rev. Mod. Phys. 83, 1057 (2011).
[3] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, “Non-abelian anyons and topological quantum computation,” Rev. Mod. Phys. 80, 1083 (2008).
[4] N. Goldman, J. C. Budich, and F. Zoller, “Topological quantum matter with ultracold gases in optical lattices,” Nat. Phys. 12, 639 (2016).
[5] D.-W. Zhang, Y.-Q. Zhu, Y. X. Zhao, H. Yan, and S.-L. Zhu, “Topological quantum matter with cold atoms,” arXiv:1810.09228 (2018).
[6] L. Lu, J. D. Joannopoulos, and M. Soljacic, “Topological phononics,” Nat. Photonics 8, 821 (2014).
[7] T. Ozawa, H. M. Price, A. Amo, N. Goldman, M. Hafezi, L. Lu, M. Rechtsman, D. Schuster, J. Simon, O. Zilberberg, and I. Carusotto, “Topological Phononics,” arXiv:1802.04173 (2018).
[8] S. D. Huber, “Topological mechanics,” Nat. Phys. 12, 621 (2016).
[9] J. Q. You and F. Nori, “Atomic physics and quantum optics using superconducting circuits,” Nature 474, 589 (2011).
[10] M. H. Devoret and R. J. Schoelkopf, “Superconducting circuits for quantum information: An outlook,” Science 339, 1169 (2013).
[11] C. Neill, P. Roushan, K. Kechedzhi, S. Boixo, S. V. Isakov, V. Smelyanskiy, A. Megrant, B. Chiaro, A. Dunsworth, K. Arya, R. Barends, B. Burkett, Y. Chen, Z. Chen, A. Fowler, B. Foxen, M. Giustina, R. Graff, E. Jeffrey, T. Huang, J. Kelly, P. Klimov, E. Lucero, J. Mutus, M. Neeley, C. Quintana, D. Sank, A. Vainsencher, J. Wenner, T. C. White, H. Neven, and J. M. Martinis, “A blueprint for demonstrating quantum supremacy with superconducting qubits,” Science 360, 195 (2018).
[12] I. Buluta and F. Nori, “Quantum simulators,” Science 326, 108 (2009).
[13] A. A. Houck, H. E. Türeci, and J. Koch, “On-chip quantum simulation with superconducting circuits,” Nat. Phys. 8, 292 (2012).
[14] I. M. Georgescu, S. Ashhab, and F. Nori, “Quantum simulation,” Rev. Mod. Phys. 86, 153 (2014).
[15] M. D. Schroer, M. H. Kolodrubetz, W. F. Kindel, M. Sandberg, J. Gao, M. R. Vissers, D. P. Pappas, A. Polkovnikov, and K. W. Lehnert, “Measuring a topological transition in an artificial spin-1/2 system,” Phys. Rev. Lett. 113, 050402 (2014).
[16] P. Roushan, C. Neill, Y. Chen, and M. K. et. al., “Observation of topological transitions in interacting quantum circuits,” Nature 515, 244 (2014).
[17] T. Wang, Z. Zhang, L. Xiang, Z. Gong, J. Wu, and Y. Yin, “Simulating a topological transition in a superconducting phase qubit by fast adiabatic trajectories,” Sci. China Phys. Mech. Astron. 61, 047411 (2018).
[18] X. Tan, D.-W. Zhang, Q. Liu, G. Xue, H.-F. Yu, Y.-Q. Zhu, H. Yan, S.-L. Zhu, and Y. Yu, “Topological Maxwell metal bands in a superconducting qutrit,” Phys. Rev. Lett. 120, 135003 (2016).
[19] V. V. Ramasesh, E. Flurin, M. Rudner, I. Siddiqi, and N. Y. Yao, “Direct probe of topological invariants using Bloch oscillating quantum walks,” Phys. Rev. Lett. 118, 130501 (2017).
[20] E. Flurin, V. V. Ramasesh, S. Hacohen-Gourgy, L. S. Martin, N. Y. Yao, and I. Siddiqi, “Observing topological invariants using quantum walks in superconducting circuits,” Phys. Rev. X 7, 031023 (2017).
[21] Y.-P. Wang, W. Wang, Z.-Y. Xue, W.-L. Yang, Y. Hu, and Y. Wu, “Realizing and characterizing chiral photon flow in a circuit quantum electrodynamics necklace,” Sci. Rep. 5, 8352 (2015).
[22] P. Roushan, C. Neill, A. Megrant, Y. Chen, R. Babbush, R. Barends, B. Campbell, Z. Chen, B. Chiaro, A. Dunsworth, A. Fowler, E. Jeffrey, J. Kelly, E. Lucero, J. Mutus, P. J. J. O’Malley, M. Neeley, C. Quintana, D. Sank, A. Vainsencher, J. Wenner, T. White, E. Kapit, H. Neven, and J. Martinis, “Chiral ground-state currents of interacting photons in a synthetic magnetic field,” Nat. Phys. 13, 146 (2017).
[23] C. Owens, A. LaChapelle, B. Saxberg, B. M. Anderson, R. Ma, J. Simon, and D. I. Schuster, “Quarter-flux hofstadter lattice in a qubit-compatible microwave cavity array,” Phys. Rev. A 97, 013818 (2018).
[24] P. Roushan, C. Neill, J. Tangpanitanon, V. M. Bastidas, A. Megrant, R. Barends, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, A. Fowler, B. Foxen, M. Giustina, E. Jeffrey, J. Kelly, E. Lucero, J. Mutus, M. Neeley, C. Quintana, D. Sank, A. Vainsencher, J. Wenner, T. White, H. Neven, D. G. Angelakis, and J. Martinis, “Spectroscopic signatures of localization with interacting photons in superconducting qubits,” Science 358, 1175 (2017).
[25] A. D. King, J. Carrasquilla, J. Raymond, I. Özfidan, E. Andriyash, A. Berkley, M. Reis, T. Lanting, R. Harris, F. Altomare, K. Boothby, P. I. Bunky, C. Enderud, A. Fräckhette, E. Hoskinson, N. Ladizinsky, T. Oh, G. Poulin-Lamarre, C. Rich, Y. Sato, A. Y. Smirnov, L. J. Swenson, M. H. Volkman, J. Whittaker, J. Yao, E. Ladizinsky, M. W. Johnson, J. Hilton, and M. H. Amin, “Observation of topological phenomena in a programmable lattice of 1,800 qubits,” Nature 560, 456 (2018).
[26] W. P. Su, J. R. Schrieffer, and A. J. Heeger, “Solitons in polyacetylene,” Phys. Rev. Lett. 42, 1698 (1979).
[27] L. Zhou, S. Yang, Y.-x. Liu, C. P. Sun, and F. Nori, “Quantum zero mode for single-photon coherent transport,” Phys. Rev. A 80, 062109 (2009).
[28] J. D. Strand, M. Ware, F. Beaudoin, T. A. Okhi, B. Johnson, A. Blais, and B. L. T. Plourde, “First-order sideband transitions with flux-driven asymmetric transmon qubits,” Phys. Rev. B 87, 220505 (2013).
[29] Y. Wu, L. Yang, M. Gong, Y. Zheng, H. Deng, Z. Yan, Y. Zhao, K. Huang, A. D. Castellano, W. J. Munro, K. Nemoto, D. Zheng, C. P. Sun, Y. X. Liu, X. Zhu, and L. Lu, “An efficient and compact switch for quantum circuits,” npj Quantum Inf. 4, 50 (2018).
[30] M. Reagor, C. B. Osborn, N. Tezak, A. Staley, G. Praweromadjo, M. Scheer, et al., “Demonstration of universal parametric entangling gates on a multi-qubit lattice,” Sci. Adv. 4, eaao3603 (2018).
[31] X. Li, Y. Ma, J. Han, T. Chen, Y. Xu, W. Cai, H. Wang, Y. P. Song, Z.-Y. Xue, Z.-Q. Yin, and L. Sun, “Perfect quantum state transfer in a superconducting qubit chain with parametrically tunable couplings,” Phys. Rev. Appl. 10, 054009 (2018).
[32] T. Fukuhara, P. Schaub, M. Endres, S. Hild, M. Cheneau, I. Bloch, and C. Gross, “Microscopic observation of magnon bound states and their dynamics,” Nature 502, 76 (2013).
[33] T. Fukuhara, A. Kantian, M. Endres, M. Cheneau, P. Schaub,
S. Hild, D. Bellem, U. Schollwöck, T. Giamarchi, C. Gross, I. Bloch, and S. Kuhr, “Quantum dynamics of a mobile spin impurity,” Nat. Phys. 9, 235 (2013).

[34] X. Gu, A. F. Kockum, A. Miranowicz, Y.-X. Liu, and F. Nori, “Microwave photonics with superconducting quantum circuits,” Phys. Rep. 718-719, 1 (2017).

[35] R. Barends, J. Kelly, A. Megrant, D. Sank, E. Jeffrey, Y. Chen, Y. Yin, B. Chiaro, J. Mutus, C. Neill, P. O’Malley, P. Roushan, J. Wenner, T. C. White, A. N. Cleland, and J. M. Martinis, “Coherent josephson qubit suitable for scalable quantum integrated circuits,” Phys. Rev. Lett. 111, 080502 (2013).

[36] R. Barends, J. Kelly, A. Megrant, A. Veitia, D. Sank, E. Jeffrey, T. C. White, J. Mutus, A. G. Fowler, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, C. Neill, P. O’Malley, P. Roushan, A. Vainsencher, J. Wenner, a. N. Korotkov, a. N. Cleland, and J. M. Martinis, “Superconducting quantum circuits at the surface code threshold for fault tolerance,” Nature 508, 500 (2014).

[37] M. Hatridge, R. Vijay, D. H. Slichter, J. Clarke, and I. Siddiqi, “Dispersive magnetometry with a quantum limited SQUID parametric amplifier,” Phys. Rev. B 83, 134501 (2011).

[38] T. Roy, S. Kundu, M. Chand, A. M. Vadiraj, A. Ranadive, N. Nehra, M. P. Patankar, J. Aumentado, A. A. Clerk, and R. Vijay, “Broadband parametric amplification with impedance engineering: Beyond the gain-bandwidth product,” Appl. Phys. Lett. 107, 262601 (2015).

[39] F. Cardano, A. D’Errico, A. Dauphin, M. Maffei, B. Piccirillo, C. de Lisio, G. D. Filippis, V. Cataudella, E. Santamato, L. Marrucci, M. Lewenstein, and P. Massignan, “Detection of Zak phases and topological invariants in a chiral quantum walk of twisted photons,” Nat. Commun. 8, 15516 (2017).

[40] N. H. Lindner, G. Refael, and V. Galitski, “Floquet topological insulator in semiconductor quantum wells,” Nat. Phys. 7, 490 (2011).

[41] J. Maciejko and G. A. Fiete, “Fractionalized topological insulators,” Nat. Phys. 11, 385 (2015).
Supplementary Materials for “Observation of topological magnon insulator states in a superconducting circuit”

W. Cai,1,∗ J. Han,1,∗ Feng Mei,2,3,† Y. Xu,1 Y. Ma,1 X. Li,1 H. Wang,1 Y. P. Song,1 Zheng-Yuan Xue,2 Zhang-qi Yin,1 Suotang Jia,2,3 and Luyan Sun1,‡

1Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing 100084, China
2State Key Laboratory of Quantum Optics and Quantum Devices, Institute of Laser Spectroscopy, Shanxi University, Taiyuan, Shanxi 030006, China
3Collaborative Innovation Center of Extreme Optics, Shanxi University, Taiyuan, Shanxi 030006, China
4Guangdong Provincial Key Laboratory of Quantum Engineering and Quantum Materials, and School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou 510006, China

EXPERIMENTAL DEVICE, SETUP, AND SEQUENCES

Our experimental chip is anchored in an aluminum sample holder and measured in a dilution refrigerator with a base temperature of about 10 mK. An additional magnetic shield is used to cover the device for a clean electromagnetic environment. Figure S1 shows the measurement circuitry. For full frequency manipulation of the qubits, we use one four-channel arbitrary waveform generators AWG5014C to control the flux-biases of the qubits $a_1$, $b_1$, $a_2$, and $b_2$. This allows individual parametric modulation of each qubit frequency. The flux control line of $a_3$ is terminated with 50 ohm at room temperature and its frequency is at the sweet spot.

Due to the ground plane return currents, there are inevitable crosstalks (the maximum one in our device is about 10%) between flux-bias lines and qubits. This crosstalk can be corrected by orthogonalization of the flux-bias lines through an inversion of the qubit frequency response matrix, leading to independent control of only the desired qubit without changing the other qubit frequencies. Since $a_3$ is biased at its sweet spot and not sensitive to the crosstalk from other qubits’ flux control, we do the orthogonalization of the flux-bias lines only for $a_1$, $b_1$, $a_2$, and $b_2$, which appears sufficient for our experiment.

To achieve frequency modulations and fast switches between the idle and operating points, it is necessary to change the flux biases in fast time scale. However, the control circuit to generate the control pulses and wiring outside and inside the refrigerator cause finite rise time and ringing of the flux-control pulses, which need to be carefully calibrated out. We use the deconvolution method to correct the unwanted response in the control system based on the measured response function of the control circuit.

A two-channel AWG70002A, synchronized with AWG5014C, is used to realize all $XY$ controls and readouts of the qubits. Because of its large bandwidth and sampling rate, AWG70002A can directly generate the qubit control pulses without extra IQ modulations. In our experiment, the control of the five qubits do not need to be on simultaneously, therefore we use fast switches to manage the individual control of each qubit. The readout signals for individual qubits are also directly generated from AWG70002A without extra IQ modulations.

A Josephson parametric amplifier (JPA) [1, 2] at 10 mK is used as the first stage of the transmitted readout signal amplification. The JPA has a gain over 20 dB and a bandwidth about 260 MHz, therefore, it allows for high-fidelity single-shot measurements of all qubits individually and simultaneously. The readout frequencies of the five qubits are designed to span a range of about 80 MHz, well within the bandwidth of the JPA. Mainly due to the mismatch of the dispersive shifts and the readout resonator decay rates, the two Gaussians in each qubit’s readout histograms, corresponding to the ground state $|g⟩$ and the excited state $|e⟩$, are not perfectly separated.

To overcome this readout imperfection, we use a calibration matrix to reconstruct the readout results based on Bayes’ rule. Readout resonator frequencies, qubit frequencies, qubit coherence times, coupling strengths, and readout resonator decay rates are all presented in Table 1. The device fabrication, the orthogonalization and deconvolution of the flux-bias lines, the readout histograms, and the calibration matrix to reconstruct the readout results based on Bayes’ rule are all described in detail in Ref. 3.

In our experiment, all qubits are initially at their idle points and only one of the qubit (dependent on the particular experiment) is prepared in the excited state with a π pulse. Then fast step pulses are used to bias the qubits from their idle points to the operating points quickly, followed by simultaneous modulations of the necessary qubits to achieve the required coupling configuration for various time $t$. In the end, fast step pulses immediately return all qubits back to their idle points for simultaneous final qubit state measurements to get $P_{\text{id}}(t)$ ($id = a_1, b_1, a_2, b_2,$ and $a_3$). Figure S2 shows an example of the experimental sequence for the topological edge state measurement. Table II shows the parameters of the parametric modulations to realize the required coupling configurations for the specified experiments. Note that for the experiment to measure the topological soliton state, qubit $a_2$ is modulated simultaneously with two sets of $\epsilon$ and $\mu$. When we measure the topologically trivial edge states (Fig. 3b of the main text), we use almost the identical coupling configuration (see Table II) as that for the topologically nontrivial edge states (Fig. 3e of the main text), but with the initial excitation on $a_3$ instead of $a_1$, i.e., the labellings of the five qubits are reversed and so is
FIG. S1. **Details of wiring and circuit components.** The experimental device consists of five cross-shaped transmon qubits (Xmons, \(a_1, b_1, a_2, b_2, \text{ and } a_3\)) \(^4, 5\) arranged in a linear array. Each qubit has independent \(XY\) and \(Z\) controls which are properly attenuated and low-pass filtered. Separate \(\lambda/4\) resonators with different frequencies couple to individual qubits for independent and simultaneous readouts. One four-channel AWG5014C is used to fully manipulate the flux biases of the first four qubits (\(a_1, b_1, a_2, \text{ and } b_2\)) while the flux-bias line of the fifth one \(a_3\) is terminated with 50 ohm at its sweet spot. One two-channel AWG70002A, synchronized with AWG5014C, is used to realize all \(XY\) controls and readouts of the qubits. Because of its large bandwidth and sampling rate, AWG70002A can directly generate the qubit control and readout pulses without extra IQ modulations. The \(XY\) manipulation signal is divided and managed through separate RF switches for selective control of individual qubits. A JPA at 10 mK with a gain over 20 dB and a bandwidth about 260 MHz is used for high-fidelity single-shot measurements of the qubits. A high-electron-mobility-transistor (HEMT) amplifier at 4 K and an amplifier at room temperature are also used before the down-conversion of the readout signal to different frequencies with a different generator as LO. To eliminate the readout signal phase fluctuation, part of the readout signal does not go through the refrigerator and is down-converted as a reference to lock the phase of the returning readout signal from the device.

The coupling configuration (seen from the perspective of the initial excitation). This is done only for reasons of simplicity without changing any underlying physics.

**TOPOLOGICAL EDGE STATE FOR FOUR QUBITS**

Besides the topological edge state measurement based on five qubits, as presented in Fig. 3 of the main text, we have also studied the case for four qubits. Figure S3a shows the schematic of the experiment. The couplings between neighboring qubits are first configured into \(J_1-J_2-J_1-J_2 = 1-5-1-\)
TABLE I. Device parameters.

| Parameters                        | $a_1$ | $b_1$ | $a_2$ | $b_2$ | $a_3$ |
|----------------------------------|-------|-------|-------|-------|-------|
| Readout frequency (GHz)          | 6.839 | 6.864 | 6.879 | 6.901 | 6.919 |
| Qubit frequency (GHz) (sweet spot)| 4.811 | 5.156 | 4.901 | 5.183 | 4.602 |
| $T_1$ (µs) (sweet spot)          | 20.0  | 17.0  | 14.8  | 17.9  | 20.0  |
| Ramsey $T_2^*$ (µs) (sweet spot) | 18.5  | 16.0  | 17.0  | 15.0  | 19.9  |
| Anharmonicity (MHz)              | 199.70| 181.53| 196.77| 212.05| 188.13|
| Adjacent qubit coupling strength $g_5/2\pi$ (MHz) | 16.70 | 17.50 | 17.50 | 16.85 |
| Qubit-readout dispersive shift $\chi_{qq}/2\pi$ (MHz) | 0.17  | 0.26  | 0.20  | 0.20  | 0.12  |
| Readout resonator decay rate $\kappa/2\pi$ (MHz) | 0.88  | 1.06  | 1.23  | 0.88  | 0.85  |

TABLE II. Parameters of the parametric modulations to realize the required coupling configurations for various experiments.

| Parameters                                                                 | $a_1$ | $b_1$ | $a_2$ | $b_2$ | $a_3$ |
|---------------------------------------------------------------------------|-------|-------|-------|-------|-------|
| Qubit operating frequency (GHz)                                           | 4.811 | 5.120 | 4.901 | 4.680 | 4.602 |
| $\varepsilon$ for topo. soliton state (MHz)                               | 4.760 | 4.940 | 4.830 | 4.680 | 4.602 |
| $\mu$ for topo. soliton state (MHz)                                       | 131.52| 14.73 | 38.38 | 38.17 |       |
| $\varepsilon$ for topo. trivial edge state (MHz)                          | 181.19|       | 107.16| 162.23| 70.71 |
| $\mu$ for topo. trivial edge state (MHz)                                  | 171.04| 100.62| 161.66| 71.10 |       |
| $\varepsilon$ for topo. nontrivial edge state (MHz)                       | 35.02 | 76.83 | 40.48 | 49.17 |       |
| $\mu$ for topo. nontrivial edge state (MHz)                               | 171.38| 100.92| 161.71| 71.75 |       |
| $\varepsilon$ for topo. nontrivial winding number (MHz)                   | 37.03 | 77.02 | 36.30 |       |       |
| $\mu$ for topo. nontrivial winding number (MHz)                           | 171.59| 102.57| 160.92|       |       |
| Operating frequency for topo. trivial winding number (GHz)                | 4.811 | 5.128 | 4.901 | 5.183 |       |
| Center frequency for topo. trivial winding number (MHz)                   | 4.780 | 4.930 | 4.830 | 4.990 |       |
| $\varepsilon$ for topo. trivial winding number (MHz)                      | 140.65| 17.72 | 129.12|       |       |
| $\mu$ for topo. trivial winding number (MHz)                              | 156.18| 108.74| 172.71|       |       |

FIG. S2. Experimental sequence for the topologically nontrivial edge state measurement. This is an example of the experimental sequence for Fig. 3e in the main text. All five qubits are initially at their idle points and $a_1$ is prepared in the excited state by a $\pi$ pulse. In the following, fast step pulses are used to bias the qubits (except for $a_3$) from their idle points to the operating points quickly and then the frequency modulations are on simultaneously to achieve the required coupling configuration for various time $t$. In the end, fast step pulses immediately return all qubits back to their idle points for simultaneous final qubit state measurements to get $P_{\text{id}}(t)$.

5 (MHz) (topologically nontrivial) as in Fig. 3 of the main text. Then the modulation on the fourth qubit ($b_2$) is turned off. Due to the vanish of this modulation, $a_3$ is then decoupled from the first four qubits and the coupling between $a_2$ and $b_2$ is also expected to become larger. However, due to the topological protection, this small imperfection does not affect the appearances of the topological magnon edge states.

Figures S3b and S3c show the time evolutions of all four qubits’ excited state populations [$P_{\text{id}}(t)$]. In sharp contrast to the behavior in Fig. 3 of the main text, in this case the population is not localized on $a_1$ only and can be transferred to $b_2$, in good agreement with theoretical calculations from the ideal Hamiltonian (Eq. 1 in the main text) for an initial state ($eggg$) with the coupling configuration $J_1,J_2,J_1 = 1.5-1.1$ (MHz) and the system decoherence.

A SPIN VERSION OF THE SSH MODEL

In the experiment, we realize a spin version of the topological Su-Schrieffer-Heeger (SSH) model in a superconducting qubit chain, where each unit cell contains two qubits labelled by $a$ and $b$. The resulted qubit chain can be described by the spin version of the SSH model Hamiltonian, Eq. 1 in the main text. We omit the constant qubit frequencies and only consider a singe-qubit excitation. Because the number of excitations is conserved in our model, the SSH model Hamiltonian can be reduced to the single-excitation subspace. In condensed matter physics, this single excitation is called a single magnon [6, 7]. Based on the Matsubara-Matsuda transformation [8], the qubit chain can be described with the following...
with $d_x = J_1 + J_2 \cos(k_z)$, $d_y = J_2 \sin(k_z)$, and $\hat{\tau}_x$ and $\hat{\tau}_y$ being the Pauli spin operators defined in the momentum space. The energy bands of the Hamiltonian (S1) are characterized by the topological winding number

$$\nu = \frac{1}{2\pi} \int dk_z n \times \partial k_x,$$

(S3)

where $n = (n_x, n_y) = (d_x, d_y)/\sqrt{d_x^2 + d_y^2}$. Through a straightforward calculation, we find that

$$\nu = \begin{cases} 1, & J_1 < J_2; \\ 0, & J_1 > J_2. \end{cases}$$

(S4)

The winding number $\nu = 1 (0)$ shows that the above SSH-type qubit chain is in the topologically nontrivial (trivial) magnon insulator state.

**THE RELATIONSHIP BETWEEN THE SINGLE-MAGNON DYNAMICS AND THE TOPOLOGICAL WINDING NUMBER**

It has been previously demonstrated that topological winding number can be dynamically detected via single-particle discrete- and continuous-time quantum dynamics [9, 10]. In the following, we will show that such a method also can be used in a chain of superconducting qubits [11]. We choose to excite one of the middle qubits to the excited state $|e\rangle$ and the other qubits are all in the ground state $|g\rangle$. Thus, the initial state of the system is

$$|\psi(0)\rangle = |gg\cdots e\cdots gg\rangle.$$  

(S5)

The quantum dynamics of this single-excitation state is governed by the Hamiltonian in Eq. S1. After an evolution time $t$, the state of the system becomes

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle.$$  

(S6)

The relation between the above quantum dynamics and the topological feature of the SSH-type qubit chain can be revealed through the chiral displacement (CD) in the qubit chain. The CD operator is defined as

$$\hat{P}_d = \sum_{\alpha=a,b}^N (\hat{P}_{e\alpha}^\alpha - \hat{P}_{e\alpha}^{\alpha})$$

(S7)

with $\hat{P}_{e\alpha}^\alpha = |e\rangle_{id}\langle e|$ (id = $a_x$, $b_x$). Then the time-dependent average of the CD associated with the above single-excitation quantum dynamics is given by

$$\bar{P}_d(t) = \langle \psi(t)|\hat{P}_d|\psi(t)\rangle.$$  

(S8)

Furthermore, we transfer Eq. S8 into the momentum space and get

$$\bar{P}_d(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk_z \langle \psi(0)|e^{i\hat{h}(k_z)t}\hat{\tau}_z e^{-i\hat{h}(k_z)t}|\psi(0)\rangle.$$  

(S9)
By substituting Eq. S2 into Eq. S9, we find $\bar{P}_d(t)$ is tied closely with the topological winding number $\nu$ defined in Eq. S3, i.e.,
\[ \bar{P}_d(t) = \frac{\nu}{2} - \frac{1}{4\pi} \int dk_x \cos(2d_1 t) n \times \partial_x n, \quad (S10) \]
where $d_1 = \sqrt{J_1^2 + J_2^2 + 2J_1J_2 \cos(k_x)}$. In the long time limit, we can obtain the relationship between the winding number and the time-averaged CD [9–11], i.e.,
\[ \nu = \lim_{T \to \infty} \frac{2}{T} \int_0^T dt \bar{P}_d(t). \quad (S11) \]
Therefore, the time-averaged CD value depends on the topology of the band structure of the qubit chain.

**WAVEFUNCTION OF ZERO-ENERGY TOPOLOGICAL EDGE STATES**

According to the bulk-edge correspondence for topological states, the existence of edge states is a seminal feature associated with topological insulator states. In the following, we will show that the topological SSH qubit chain supports zero-energy topological edge states. Firstly, we show that the wavefunction of the zero-energy state in an SSH model can be exactly derived, even in the absence of translational invariance. For this purpose, we consider a generalized SSH model with its Hamiltonian written as
\[ \hat{H}' = \sum_{x=1}^N \left( u_x \hat{\sigma}^\dagger_{ax} \hat{\sigma}_{bx} + w_x \hat{\sigma}^\dagger_{bx} \hat{\sigma}_{ax+1} + \text{H.c.} \right), \quad (S12) \]
which breaks the translational invariance. Suppose the wavefunction of the zero-energy state is
\[ |\psi_E\rangle = \sum_{x} [\lambda(a_x)\hat{\sigma}^\dagger_{ax} + \lambda(b_x)\hat{\sigma}^\dagger_{bx}] |G\rangle, \quad (S13) \]
where $|G\rangle = |gg \cdots gg\rangle$ is the vacuum magnon state. By substituting this wavefunction into the Schrödinger equation $H' |\psi_E\rangle = 0$, when $u_x < w_x$ and in the thermodynamic limit, we can get two solutions
\[ \lambda(a_x) = -\frac{u_x}{w_x} \prod_{j=1}^{x-1} \lambda(a_j), \lambda(b_x) = 0; \]
\[ \lambda(b_x) = -\frac{u_N}{w_x} \prod_{j=x+1}^{N-1} \lambda(b_j), \lambda(a_x) = 0. \quad (S14) \]
For the standard translational invariant SSH model, $u_x = J_1$ and $w_x = J_2$, and we can derive the wavefunctions of the left and right zero-energy edge states as
\[ |\psi_L\rangle = \sum_x (-\frac{J_1}{J_2})^x \hat{\sigma}^\dagger_{ax} |G\rangle; \]
\[ |\psi_R\rangle = \sum_x (-\frac{J_1}{J_2})^{N-x} \hat{\sigma}^\dagger_{bx} |G\rangle. \quad (S15) \]
It is found that the magnon in the left (right) edge state only occupies the $a$-type ($b$-type) qubit and its density is maximally distributed in the leftmost (rightmost) qubit. This feature has been clearly demonstrated in Fig. 3c of the main text.

**THE INFLUENCE OF QUBIT LATTICE SIZE ON OBSERVING THE TOPOLOGICAL MAGNON EDGE STATES**

Based on Eq. S15, one can find that the overlap between the left and right edge states decreases exponentially with the increase of the number $N$ of the unit cells in the qubit chain. This overlap determines the coupling between the two edge states. When the qubit chain is very long, the coupling between the left and right edge states is almost zero and can be ignored.

However, for a practical short qubit chain, the coupling cannot be ignored. We have experimentally observed this feature in a chain of four superconducting qubits. Initially, we prepare a single-magnon state by exciting the leftmost qubit to an excited state, i.e., $|\psi(t = 0) = |\text{egg} \cdots \text{gg}\rangle$. After that, we measure the time evolution of this single-magnon state in the SSH qubit chain. Note that the initial single-magnon state $|\psi(t = 0)\rangle$ is very close to the left magnon edge state $|\psi_L\rangle$. The experimental result is shown in Fig. S3, which indicates that the magnon finally goes to the rightmost qubit. This is because the wavepacket of the left edge state has a large overlap with that of the right edge state, and the leftmost magnon evolves to the rightmost qubit. Although the topological edge state localization has not been observed, our measured result in a chain of four qubits still provides indirect evidence for the appearances of the left and right topological magnon edge states.

A direct method to eliminate the coupling between the left and right edge states and to only observe a single edge state is to perform the experiment in a long qubit chain, which however is not the case for our current device. We solve this problem through an alternative way by using a qubit chain with an odd number of qubits. In this case, the system only supports the left edge state while the right edge state has been removed. We have successfully observed the left magnon edge state in a chain of five qubits. The experimental results are shown in Fig. 3 of the main text.

**WAVEFUNCTION OF ZERO-ENERGY TOPOLOGICAL MAGNON SOLITON STATES**

The SSH qubit chain also supports the zero-energy topological magnon soliton state [12]. Such a state is generated at the boundary between the topologically trivial ($J_1 > J_2$) and nontrivial ($J_1 < J_2$) regions. In this case, the qubit lattice breaks the translational invariance. Suppose the boundary is located at qubit $a_x$, and based on Eqs. S13 and S14, we can derive the wavefunction of the zero-energy topological magnon soliton...
\[ |\psi_S\rangle = \left( \sum_{x < x_e} (-J_1 / J_2)^{x_e - x} + \sum_{x \geq x_e} (-J_1 / J_2)^{x - x_e} \right) \hat{\sigma}^\dagger_{ax} |G\rangle. \]

(S16)

This shows that the magnon in the topological soliton state only occupies a-type qubit and its density is maximally distributed in qubit \( a_{x_e} \) at the boundary. Our experimental data, plotted in Fig. 4 of the main text, show exactly this expected behavior.

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* These two authors contributed equally to this work.

† meifeng@sxu.edu.cn

‡ luyansun@tsinghua.edu.cn

[1] M. Hatridge, R. Vijay, D. H. Slichter, J. Clarke, and I. Siddiqi, “Dispersive magnetometry with a quantum limited SQUID parametric amplifier,” Phys. Rev. B 83, 134501 (2011).

[2] T. Roy, S. Kundu, M. Chand, A. M. Vadiraj, A. Ranadive, N. Nehra, M. P. Patankar, J. Aumentado, A. A. Clerk, and R. Vijay, “Broadband parametric amplification with impedance engineering: Beyond the gain-bandwidth product,” Appl. Phys. Lett. 107, 262601 (2015).

[3] X. Li, Y. Ma, J. Han, T. Chen, Y. Xu, W. Cai, H. Wang, Y. P. Song, Z.-Y. Xue, Z.-Q. Yin, and L. Sun, “Perfect quantum state transfer in a superconducting qubit chain with parametrically tunable couplings,” Phys. Rev. Appl. 10, 054009 (2018).

[4] R. Barends, J. Kelly, A. Megrant, D. Sank, E. Jeffrey, Y. Chen, Y. Yin, B. Chiaro, J. Mutus, C. Neill, P. O’Malley, P. Roushan, J. Wenner, T. C. White, A. N. Cleland, and J. M. Martinis, “Coherent josephson qubit suitable for scalable quantum integrated circuits,” Phys. Rev. Lett. 111, 080502 (2013).

[5] R. Barends, J. Kelly, A. Megrant, A. Veitia, D. Sank, E. Jeffrey, T. C. White, J. Mutus, A. G. Fowler, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, C. Neill, P. O’Malley, P. Roushan, A. Vainsencher, J. Wenner, a. N. Korotkov, a. N. Cleland, and J. M. Martinis, “Superconducting quantum circuits at the surface code threshold for fault tolerance,” Nature 508, 500 (2014).

[6] T. Fukuhara, P. Schaub, M. Endres, S. Hild, M. Cheneau, I. Bloch, and C. Gross, “Microscopic observation of magnon bound states and their dynamics,” Nature 502, 76 (2013).

[7] T. Fukuhara, A. Kantian, M. Endres, M. Cheneau, P. Schaub, S. Hild, D. Bellem, U. Schollwöck, T. Giamarchi, C. Gross, I. Bloch, and S. Kuhr, “Quantum dynamics of a mobile spin impurity,” Nat. Phys. 9, 235 (2013).

[8] T. Matsubara and H. Matsuda, “A lattice model of liquid helium, I,” Prog. Theor. Phys. 16, 569 (1956).

[9] F. Cardano, A. D’Errico, A. Dauphin, M. Maffei, B. Piccirillo, C. de Lisio, G. D. Filippis, V. Cataudella, E. Santamato, L. Marrucci, M. Lewenstein, and P. Massignan, “Detection of Zak phases and topological invariants in a chiral quantum walk of twisted photons,” Nat. Commun. 8, 15516 (2017).

[10] M. Maffei, A. Dauphin, F. Cardano, M. Lewenstein, and P. Massignan, “Topological characterization of chiral models through their long time dynamics,” New J. Phys. 20, 013023 (2018).

[11] F. Mei, G. Chen, L. Tian, S.-L. Zhu, and S. Jia, “Topology-dependent quantum dynamics and entanglement-dependent topological pumping in superconducting qubit chains,” Phys. Rev. A 98, 032323 (2018).

[12] W. P. Su, J. R. Schrieffer, and A. J. Heeger, “Solitons in polyacetylene,” Phys. Rev. Lett. 42, 1698 (1979).