Abstract

Lorentz covariance imposed upon a quantum logic of local propositions for which all observers can consistently maintain state collapse descriptions, implies a condition on space-like separated propositions that if imposed on generally commuting ones would lead to the covering law, and hence to a hilbert-space model for the logic. Such a generalization can be argued if state preparation can be conditioned to space-like separated events using EPR-type correlations. This suggests that the covering law is related to space-time structure, though a final understanding of it, through a self-consistency requirement, will probably require quantum space-time.

1 Introduction

The origin of hilbert-space quantum theory has been a nagging question ever since its creation. Axiomatic approaches, by which one attempts to derive the hilbert-space formalism from postulates whose content is supposed to be clear and whose truth is supposed to be compelling, have only had limited success. Even if progressive clarity has been achieved, the truth of the axioms never seems compelling. Something is missing, and the formalism continues to mystify. We shall here attempt to dispel part of this mystery by arguing

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that space-time considerations provide motivation for adopting some of the axioms that are hard to justify otherwise.

Though there are many axiomatizations of Hilbert-space quantum mechanics, we shall here focus on one, the well-known and much investigated Piron’s [1] “quantum logic”. The main object of consideration is a complete atomic orthomodular lattice of “physical propositions”. To have a generalized Hilbert-space model one has to assume, among others, an axiom called the “covering law.” It is this law that has received considerable attention, being the most controversial of the ingredients.

There are many attempts to reduce the covering law to clearer and more compelling physical statements, generally by introducing further structures into the quantum logic, such as measurements, transition probabilities, propensities, etc. We show here that some such structures provide us with means of deriving necessary conditions on the quantum logic if it is to describe a Lorentz-covariant theory. Generalizations of such conditions are then seen to be sufficient to derive the covering law and thereby a Piron-type Hilbert-space model.

There are three main ingredients in our argument. The first is the existence of Heisenberg-like physical states that suffer “collapse”-type transformations upon measurements. The second is Lorentz covariance, which, beyond the usual group-action type formulation, includes also what we call “covariance of objectivity” (Postulates [1] and [2] of section 6). These states roughly that if a state is prepared by a measuring apparatus with space-like separated parts then it has the usual covariance properties with respect to local observables in regions that are future time-like to all the parts of the measuring apparatus. An immediate consequence of this assumption is a condition on space-like separated propositions which, if applied to any commuting ones, would imply the covering law in existing axiomatic schemes. This suggests that the whole covering law may have a space-time origin. To reach such a conclusion however, one has to somehow relate time-like and space-like situations, which leads to the third major ingredient, that there are sufficiently many states with EPR-type correlations to be able to prepare arbitrary states conditioned to space-like separated events, as is the case for ordinary relativistic quantum mechanics.

Our approach is also of an axiomatic character, and so too suffers from the shortcomings we attribute to all such attempts. To its merit, it does clarify the nature of the final physical basis behind quantum mechanics. In particular, the third assumption suggests that a final justification could only
come through some form of generalized quantum gravity where the light cone is not a fundamental but an emergent object.

We have already argued for a space-time origin of the quantum formalism (Svetlichny [2]). There we use the hypothesis that it is impossible to communicate superluminally (ISC). Now the use of ISC to deduce constraints on physical theories must be considered at best heuristic, for ISC must be traceable to more basic considerations. In fact, in theories such as quantum gravity, where the light cone is an emergent object, ISC itself must be emergent. It is thus imperative that we try to re-establish the putative connection between hilbert space and lorentzian space-time in a way that makes no appeal to signals. To this end we must set up some of the machinery of what could be called relativistic quantum logic, quantum logic subject to the requirements of special relativity.

Relativistic quantum logic is relatively new, about two decades old. The paper of Mittelstaedt [3] could be said to be one of the first pioneering works published on the subject. The formalism was applied to the analysis of the Einstein-Podolsky-Rosen experiment by Mittelstaedt [4] and by Mittelstaedt and Stachow [5]. The approach is based on the dialogical (dialog logic) view of physical propositions upon which relativistic restrictions are applied in the form of spatio-temporal validity regions. The work of Neumann and Werner [6] is an elaboration of Ludwig’s [7] measurement axiomatics. A causality postulate is introduced for systems prepared in a space-time region and recorded in a space-like separated region. Examples are presented but no consequences are derived. The author’s own first ideas were also developing around the same time, but in contrast to the above mentioned works were inspired mainly by local algebraic quantum field theory introduced originally by Haag and Kastler (see Haag [8]). It is this viewpoint that we follow in this paper. One should also mention the work of Mugur-Schächter [9, 10] which though not explicitly “relativistic” is undeniably spatio-temporal and thus related.

Though the idea of relativistic quantum logic is not new, the application that we have in mind, to seek a space-time basis for the covering law, is new. It seems that to do so, we must incorporate structures that go beyond the usual ortho-algebraic ones. As a guide we try to adhere as much as possible to notions current within the usual “Copenhagen” interpretation and accepted by the greater part of the physics community. In doing so, we do not advocate this interpretation nor claim that it is in some sense correct, only that it provides a set of principles that are sufficiently characteristic of
quantum mechanics to be an interesting and familiar starting point.

We thus assume the usual notion of ensemble. Ensembles of physical systems give rise to representational elements in some abstract set of “physical states”, which for conventional quantum mechanics is the set of density matrices, positive hilbert space trace-class operators of trace one. Ensembles may consist of subensembles in which case these form well defined fractions given by a real number in [0, 1]. Ensembles and subensembles are to be considered as potential ontological entities capable of partial realization. Generally ensembles are partially realized by repetition of preparation procedures and subensembles identified by the occurrence of some physical results during preparation. The subensemble fraction is assumed to be approximated by the frequency of occurrence of the corresponding result. Besides ensembles of physical systems we can consider ensembles of measurements or experiments. These are partially realized by carrying out the corresponding acts a large number of times in such a way that they do not interfere with each other nor are interfered with by other acts and events in the universe. For this to make sense one must assume that one can individuate the necessary physical systems and the experimental apparatus in a way that warrants neglect of external influences, and posit some type of relativity theory by which act performed in different regions of space-time may be considered as performing the same experiment. Also for each one of these experiments it is usual to use a mathematical model in which the corresponding experiment is the only thing existing in all of space-time. This is a deliberate idealization of the isolation of the experiment from external influences. We shall tacitly subscribe to all such usual idealizations and conventions which underlie an “ensemble” interpretation of a physical theory.

In the recently introduced “consistent histories” approach to quantum mechanics [11, 12], many of the usual assumptions about “physical states” become considerably weakened, especially concerning the “collapse” of the state due to measurements. A relativistic quantum logic based on a consistent histories viewpoint would proceed in a radically different direction, and at first sight would not lead to the same conclusions, and so provide no justification for the covering law. We have suggested elsewhere [13] that quantum gravity would renormalize any such theory to one in which the covering law holds, as this would be a fixed point in a self-consistency requirement, however the argument used is still rather sketchy, and so will not be considered here, except for a few remarks toward the end. Based on this however, we feel that the present “collapse-biased” considerations are pertinent to a final
We must call attention to the distinction between ultimate physical facts and physical descriptions. Physical facts include at least such uncontroversial happenings as counter clicks, collisions, supernovas, etc., about which all observers agree. Descriptions are formal tools needed to deal with facts. The distinction is not at all clear-cut for one generally tries to include among the facts inferred objects such as the earth’s interior, and these may be argued by others to be just descriptive constructs that coordinate the true uncontroversial facts. One may maintain that elementary particles are just formal objects we have invented to provide a more visualizable description of the surprisingly complex and subtle antics of macroscopic bodies. In Feynman-Wheeler electrodynamics there are no electromagnetic fields, only charged bodies interacting along light-like intervals. If such a theory is taken as true, then the usual electromagnetic fields become just remote and formal descriptive paraphrases of the facts. A physical theory must make some declarations as to what is factual and what is descriptive, it must make some “ontological” commitment, though part of one category may slide over to the other as one changes the postulated relation of the descriptive elements to what are considered ultimate facts. Our concern in this paper is with classes of theories in which certain descriptions can be consistently maintained regardless of their relation to true ultimate facts. Descriptions belong to observers and are often frame-dependent. Ultimate facts are self-subsisting and have nothing to do with frames. All observers must agree upon them. Relativistic theories relate ultimate facts placing them in equivalence classes under the action of an appropriate relativity group. Frame-dependent descriptions and group action must coexist. This places constraints on the possible theories. It is some of these constraints that we try to explicit.

Our exposition, though roughly of an axiomatic nature, will gloss over mathematical details of purely technical type so as not to overburden the principal conceptual structure, whose presentation is the aim of this paper. We do not claim that we’ve found compelling reasons for quantum mechanics to be the way it is. We do claim that we’ve found a set of physical assumptions that can guide a more physically motivated axiomatics. That such a set of assumptions exists is worthy of note even if some of them can be seriously questioned in isolation. Our results must be considered in any attempt at unification of space-time with quantum mechanics.
2 Projection Rule and Objective Mixtures

Let a quantum state be represented by a density matrix $\rho$ and perform an ideal measurement represented by a self-adjoint operator $A$, which for simplicity’s sake we assume has a discrete spectrum. Thus $A = \sum \lambda P_\lambda$, where the sum is over distinct eigenvalues $\lambda$, and the $P_\lambda$ are spectral projectors. The projection rule states that outcome $\lambda$ occurs with frequency $\text{Tr}(\rho P_\lambda)$ and if this not zero, then the state after the measurement is given by $\rho_\lambda = P_\lambda \rho P_\lambda / \text{Tr}(\rho P_\lambda)$. We have here a “beam-splitter” interpretation of the measurement: the resulting states $\rho_\lambda$ for different values of $\lambda$ are maintained separate either formally (by conditioning further measurements or even data analysis to particular outcomes of the current one), or even physically by guiding the resultant states into different spatially separated regions.

One can however disregard which outcome occurs and consider each instance of any of the post-measurement states as being an instance of a single state which would now be represented by $\rho_A = \sum \lambda \text{Tr}(\rho P_\lambda) \rho_\lambda = \sum \lambda P_\lambda \rho P_\lambda$. This is usually referred to as an “incoherent mixture” of the resulting states $\rho_\lambda$.

One cannot reverse this, just from the density matrix $\rho_A$, there is no way of determining the constituent components $\rho_\lambda$ and the corresponding frequencies $\text{Tr}(\rho P_\lambda)$. Even if we seek pure components, a non-extreme density matrix $\rho$ can be decomposed in an infinite number of ways into a convex combination of extreme matrices, that is, there are infinitely many Borel probability measures $\mu$ with support in the subset $\mathcal{P}$ of extreme points such that $\rho = \int p \, d\mu(p)$. This of course raises a much ventilated controversy: given a non-extreme density matrix $\rho$, is any among its infinite integral representations as a convex combination of extreme points somehow better, or even objectively or ontologically “correct”? If some principle is assumed by which a unique representation is singled out, we shall call this representation an objective mixture, and to distinguish it from a purely mathematical integral representation, we shall use an indexed equality sign $=_{o}$ for the former. Thus $\rho =_{o} \int p \, d\mu(p)$ means that it is this representation that is singled out by the postulated principle. We use the term “objective” to give a deliberate bias to the notion, as we envisage that such unique representations have their roots in some objective reality. In particular, prior measurements may provide a basis for such representations.

One often sees another type of pure-to-mixed state transformation, the partial trace. For any density matrix $\rho$ defined in a tensor product hilbert space $H_1 \otimes H_2$ one can define the partial trace $\rho^{(1)} = \text{Tr}_{H_2} \rho$, a density ma-
trix in $H_1$, defined by requiring that for any bounded operator $A$ in $H_1$ one has $\text{Tr}(\rho^{(1)} A) = \text{Tr}(\rho (A \otimes I_{H_2}))$. In general even if $\rho$ is a pure state, the partial trace $\rho^{(1)}_\psi$ is not. What is usually said about this last situation is that $\rho^{(1)}_\psi$ represents an ensemble of first members, in an *undetermined* state, of a pure ensemble of a two-member composite system. Thus if we write $\psi = \sum_i \alpha_i \otimes \beta_i$ where $\alpha_i \in H_1$ and $\beta_i \in H_2$ then one can conceive $\psi$ as representing a composite system with two components, the states of one of which are represented in $H_1$ and of the other in $H_2$. If one now makes a measurement represented by the self adjoint operator $A$ only on the first component, the expected value is $\text{Tr}(\rho^{(1)}_\psi A)$ and so as far as the measurements on the first component are concerned, the system acts as though the first component is in a mixed stated given by $\rho^{(1)}_\psi$. The conventional wisdom concerning this situation is however that this is always a mathematical description and no objective mixture $\rho^{(1)}_\psi$ of any kind is present (except for the very particular case of the partial trace being extreme). The “partial trace” state is considered to be ontologically different from the other types of ensembles. The prototypical example of this situation is the singlet state of a two photon system. If no measurement is made on one of the photons, then the other one, in so far as it could be construed as a separate entity, is considered to be “unpolarized”, that is, in no definite state of polarization. In fact, if any polarizer is placed in front of it, the probability is always one half that it will pass through. One must point out that to be “unpolarized” is *not* a possible state that a photon may be in, as any one-photon state is always in some state of polarization. Thus to talk about an “unpolarized” photon is to employ a (useful) metaphor concerning the presence of an entangled state involving several photons.

A rather strong principle that leads to objective mixtures could be called “primacy of pure states”: given a mixed state, then any given physical instance of such a state is in fact a physical instance of a unique pure state which possibly varies from instance to instance. Primacy is given to pure states and mixed states arise through mere ensemble mixtures and do not represent new irreducible ontological entities. This is usually the attitude upheld in elementary textbooks on quantum mechanics especially for systems comprised of a small number of particles, as one can easily prepare mixtures that *prima facie* seem to obey it. One could seriously question it for mesoscopic and larger systems. We have argued (Svetlichny [14, 15]) that its negation can and does lead to interesting possibilities as there are combi-
natorial hidden-variable models in which mixed states violate this principle, opening up a new approach to the distinction between the classical and the quantum. A explicit revocation of the primacy of pure states can be found in Czachor’s proposals for non-linear quantum mechanics, in which density matrix evolution is not reducible to evolution of its component mixtures, the non-uniqueness of which is behind the causality problems of non-linear deformations of quantum theory. We shall see below that the principle cannot be universally upheld along with special relativity and usual notions of causality. Nevertheless, its simplicity makes it a useful heuristic device and it does bear examination on two grounds: 1) it becomes relevant once one contemplates alternative physical theories, and 2) it’s a useful starting point for seeking weaker criteria for objective mixtures.

Under the primacy of pure states, if \( \rho = \int p \, d\mu(p) \) and \( A \) an observable whose expected value in a pure state \( p \) is \( \langle A \rangle_p \) then the expected value \( \langle A \rangle_\rho \) in state \( \rho \) has to be \( \int \langle A \rangle_p \, d\mu(p) \). For a conventional quantum mechanical observable represented by a self adjoint operator \( A \) one has \( \langle A \rangle_\rho = \text{Tr}(pA) \) and then \( \langle A \rangle_\rho = \int \text{Tr}(pA) \, d\mu(p) = \text{Tr}(\int p \, d\mu(p) \, A) = \text{Tr}(\rho A) \) by the linearity of the trace and the operator \( A \). The representation as an objective mixture drops out and any other representation would lead to the same observable consequences. Operationally, there is no observable difference between two different representations, and some maintain that objective reality is related to the density matrix itself and representations as convex combinations of pure states is a purely mathematical affair. If however one wants to depart from ordinary linear quantum mechanics, the question of representation of mixtures becomes crucial as the integral formula for \( \langle A \rangle_\rho \) could very well depend on the representation used in which case some form of objective mixtures has to be maintained. Furthermore, maintaining some such version does lead to very interesting and important consequences as one is then able to influence “objective reality” at a location space-like to one’s own through EPR-type long-range quantum correlations. Consider the singlet state of a two-photon system when the two photons are space-like separated and, say, traveling along opposite arms of an EPR-type apparatus. If we now perform an observation represented by a non-degenerate quantum observable \( \hat{A} \) with normalized eigenvectors \( \psi_1 \) and \( \psi_2 \) on one arm of the apparatus, then by the projection postulate and strict correlations in the singlet state, the state on the other arm immediately after the measurement is an equal mixture of \( \psi_1 \) and \( \psi_2 \) represented by the density matrix

\[ \frac{1}{2} I = \frac{1}{2} (\psi_1, \cdot) \psi_1 + \frac{1}{2} (\psi_2, \cdot) \psi_2. \]

Now, if we believe in objective mixtures and
rewrite this with \(\omega\), we see that by changing the observable \(A\) to one with a different eigenbasis, we immediately change the objective mixture on the other arm. This “action at a distance” upon supposed objective mixtures has been extensively discussed ever since the original EPR paper (Einstein, Podolsky, and Rosen, [17]) and has recently been used to derive a series of strong constraints on possible alternatives conventional quantum mechanics (Gisin [18, 19, 20, 21]; Pearle [22, 23]; Svetlichny [2]). Such constraints stem from the fact that if one is not careful, such alternative theories will allow for humanly controlled superluminal signals and hence supposed difficulties with special relativity. We shall henceforth refer to these results as the ISC constraints. Theories in which objective mixtures as the result of measurement are allowed descriptive elements are strongly constrained and so it behooves us to try to give this notion some solid foundation.

Let us therefore assume that states collapse by measurements to objective mixtures and see what this may mean. Consider again a source of singlet two-photon states which then travel in opposing arms of an EPR-type apparatus. Assume we are in a reference frame, the rest frame, in which the apparatus and the source is at rest so that the two correlated photons are always at equal distances along the two arms. Put a vertically oriented linear polarizer at some distance along one arm, call it arm \(1\) and mark the other arm, call it arm \(2\), at the same distance without first placing anything in the way of the photon. By the usual arguments we must now conclude that just beyond the mark one has an objective mixture of vertically and horizontally polarized photons in equal proportions. We shall also consider a frame, the moving frame, in which a photon on arm \(2\) reaches the mark before its mate reaches the linear polarizer. In this frame, just beyond the mark, the photons are still unpolarized and an objective mixture of linearly polarized photons comes into being further down the arm. Thus the objective mixture description is frame dependent. In itself, frame dependence is not a defect and we face it all the time. A static magnetic field, viewed from a moving frame, becomes a magnetic and an electric field, so the presence or absence of an electric field is frame dependent. However, this type of behavior is easily explained by the notion of covariance under a group representation while the frame dependence of objective mixtures is of a completely different nature. Try now to give meaning to the statement that in the rest frame, just after the mark, there is an objective mixture of vertically and horizontally polarized photons. At first glance one might say that each photon would pass with certainty either through a vertically or horizontally oriented linear
polarizer and that this is not true for any other “filters” that can be placed in its path (uniqueness of objective mixtures). Let us call this the passage criterion. Note that in this situation the criterion is counterfactual for we have no way of knowing which polarization any individual photon has, but if we did know, it would pass through an appropriately oriented polarizer. Now call upon Maxwell’s demon’s cousin, the quantum demon. This being has knowledge of quantum mechanical systems that cannot be achieved by any humanly constructed apparatus, in our case the knowledge missing in the counterfactual criterion. Place now a linear polarizer just after the mark and have the quantum demon rotate it through a sequence of horizontal and vertical orientations in such a manner as to pass all the photons from the objective mixture that impinges upon it. How does this situation look from the moving frame? In this case the demon is twirling his polarizer in front of unpolarized photons and even so he is capable of letting all of them pass through. What’s responsible for this strange fact? We can of course try to blame the linear polarizer that sits at arm 1 but the event of a photon impinging there is to the future of its mate impinging upon the demon’s polarizer. This looks like inverted causal order, but it’s inverted order at space-like separation and so could be deemed innocuous (though there is the danger that concatenating two such could lead to time-like retrograde causality). Also it’s not surprising that such inverted causal order appears, as we have already posited something like “action at a distance” for manipulating distant objective mixtures and a Lorentz transformation can turn this into action into the past. What is often desired of theories that show such apparent causal anomalies (such as tachyon theories) is that the causal order of events can be reinterpreted as again to follow a strict temporal order. This would make the notion of cause and effect frame dependent, but in the end the usual notions of causality can be maintained in any frame. Let us call this desideratum upon physical theories “strict temporal causality”. If we assume this then we cannot blame the polarizer on arm 1 and we must assume that even a beam of unpolarized photons is an objective mixture of vertically and horizontally polarized photons. However we can restart the whole argument now with a circular polarizer on arm 1 and conclude that a beam of unpolarized photons is an objective mixture of left and right circularly polarized photons. Similarly for any other type of polarizer. This contradicts the whole idea of objective mixtures as uniqueness is important, and we must state that the conjunction of special relativity, strict temporal causality, and the counterfactual passage criterion for objective mixtures is
contradictory. Note however that primacy of pure states implies the passage criterion (counterfactual or not) and so the notion of primacy must be abandoned as a universal principle if we want to keep the other two. One way to weaken the passage criterion (or the primacy of pure states) is to abandon the uniqueness requirement. This is not a desirable step for us as this would undermine the ISC constraints. However such a step does bring its insights. We would then admit that an unpolarized photon already has a well defined value for any of its possible polarizations. We are now faced with a hidden-variable theory where each relevant quantum observable has a well defined value. We know that any such theory to be successful must be contextual and non-local (Redhead [24]). It is the appearance of non-locality in this context that is indicative. Primacy of pure states and the passage criterion seem at first to be local in nature, however let us examine them from an operational point of view. Confronted with a given physical instance of a mixed state we call upon our quantum demon to tell us which particular instance of a pure state, represented by a normalized vector $\psi$, we are dealing with. How can we be sure that the demon tells the truth? We test the state with the question represented by the hermitian projector $P_\psi$ upon the one-dimensional space spanned by $\psi$. If the test fails, the demon lied, if it passes we’re still not sure but this is the best we can do. If after a very long run all tests pass, we have strong statistical evidence to believe the demon is truthful. Now comes the crux of the matter: in any relativistic field theory, $P_\psi$ is not a local observable. We shall treat this some paragraphs below, but given this, we see that the notion of objective mixtures cannot be operationally a local notion, and it is the contradictory attempts to treat it as such that leads to major difficulties.

On the other hand, in the rest frame, the photons just beyond the mark seem to behave exactly like an objective mixture of linearly polarized photons because if we place a linear polarizer in a horizontal orientation just beyond the mark, we get exact coincidence with what happens at the other polarizer. In the moving frame the roles of the arms are reversed to maintain strict temporal causality but the same description applies. Another argument for objective mixtures is that we can reproduce the quantum demon’s exploit if we delay the photon on arm 2 sufficiently. At some point before the mark on arm 2 place a mirror that reflects the photon to a second distant mirror which then reflects it back to the arm at a position further down the arm from the first mirror but still before the mark, and at this position place a third mirror that redirects the photon again outward along the arm. Let the
distant mirror be so removed that information about what happened at the polarizer at arm 1 has time to reach an observer stationed at the mark before the detoured photon reaches him. The observer then uses this information to rotate a linear polarizer just beyond the mark so as to allow all the photons to pass through. Of course now the event of the photon passing through the polarizer on arm 2 is inside the future light cone of the event of its mate passing through the polarizer on arm 1 and this then remains true in all frames and we cannot invoke the argument presented above which knocked down the counterfactual passage criterion. So it seems we have all reason to believe in objective mixtures in this situation. In fact, the passage criterion is now factual and shows indeed that one deals with an objective mixture of linearly polarized photons. What happens then as the photon takes its detour. Does its ontological status changes from unpolarized to polarized somewhere along its path? If so, when does this happen? If we give a negative answer to the first question then the photons have been polarized all along including at points of space-like separation and by our previous argument we fall back into a particular hidden variable theory which we are trying to avoid, so the answer must be positive. A natural answer then to the second question would be that the change happens when the photon reaches a point along its path that is light-like to the event of its mate encountering the polarizer at arm 1. But now this means that a fundamental quantum mechanical feature changes merely due to a spatio-temporal arrangement. At space-like separation, given our bias for strict temporal causality, we can never maintain the (counterfactual) passage criterion of objective mixtures, but as soon as it becomes light-like, the passage criterion is applicable (and factual) and shows that one has an objective mixture of linearly polarized photons. This in itself demonstrates that quantum mechanics is linked to space-time structure for the photon becomes polarized just by penetrating a certain light cone. In other words, besides the measuring apparatus, space-time itself participates in state collapse.

This is in stark contrast with Galileian covariant theories. In such theories the primacy of pure states can be maintained universally and hence the passage criterion for objective mixtures can be used in all circumstances.

Now we must look again at the original space-like situation. We saw we cannot use the counterfactual passage criterion yet we would like to maintain some version of objective mixtures. The answer that offers itself is that whereas for time-like situations objective mixtures are ultimate physical facts, for space-like separations they are physical descriptions. By this
we do not mean they are arbitrary or “unreal” but that the criteria for their choice can depend on specifics of experimental arrangements and inertial frames. Thus we should not be surprised that if we place a circular polarizer placed just beyond the mark, in the rest frame there are (according to the description) linearly polarized photons impinging on it, whereas in the moving frame (again according to the description) there are no linearly polarized photons anywhere near it. We can base our criterion for objective mixtures on a previous measurement event, according to the time order in the given frame. Thus in the rest frame it is the linear polarizer on arm 1 that determines the objective mixtures in the time interval between one photon having reached its polarizer and its mate the other polarizer, and in the moving frame it is the circular polarizer on arm 2. It is also part of the conventional wisdom that this is a consistent way of proceeding and that observers in either frame, each one using his own description, will agree as to their predictions about ultimate physical facts.

Relativity theory forces us to abandon a naive picture of primacy of pure states and to adopt a sort of hybrid view in which objective mixtures are ultimate physical facts in some situations and physical descriptions in others. In a fixed frame one situation blends into the other without apparent discontinuity as soon as certain spatio-temporal relations are achieved.

The paradoxical nature of objectifying too much the state description after measurement in relativistic theories was also pointed out by Mielnik [25] who concludes that state-reduction and relativity are mutually inconsistent. This is a surprising conclusion as relativistic quantum field theory is highly successful. He is led to this by however by tacitly admitting results of counterfactual experiments. Applied to the singlet two-photon state considered above, his reasoning would lead to the same hidden-variable theory that the counterfactual passage criterion does. Disallowing such counterfactual definiteness blocks the contradiction. A different resolution of Mielnik’s paradox is given by Finkelstein [26].

Return again to the projection postulate and let $A$ be a self-adjoint operator with discrete spectrum and spectral decomposition $A = \sum \lambda P_\lambda$. Let the normalized vector $\psi$ represent the state upon which the observation is performed, and let $\psi_\lambda = P_\lambda \psi/||P_\lambda \psi||$ whenever $||P_\lambda \psi|| \neq 0$. We now write $\rho_A = \sum ||P_\lambda \psi||^2 (\psi_\lambda, \cdot) \psi_\lambda$ to indicate that $\rho_A$ occurred through a previous preparation (measurement) in which, had the “beam splitter” viewpoint been adopted, the $\psi_\lambda$ would describe the pure state in each “beam”. Objective mixtures are then tokens of preparations. Whether such a mixture is actual
and complies with the passage criterion or descriptive, depends now on some spatio-temporal situation. In all cases however one has a correlation criterion. If immediately after observing $A$ we perform a test of whether the resulting state is $\psi_\lambda$ (by using the orthogonal projector onto this vector as the observable), then the test is satisfied if and only if $\lambda$ occurs (exact correlation). This of course is due to the orthogonality of the $\psi_\lambda$ for different $\lambda$. This is not true for any other set of one-dimensional projectors associated to those outcomes $\lambda$ for which $P_\lambda \psi \neq 0$. Thus the correlation criterion picks out a unique convex combination of pure states and so is a legitimate basis for objective mixtures. Being an operational criterion (at least for ensembles) it is about as “objective” as one can wish. It works both in space-like and time-like situations. Its major disadvantage is that it doesn’t apply to the state in itself but involves the preparation that produced the state. Such knowledge of the preparation procedure can be used to separate the beams again if the original measurement was not of the beam splitter type. Just measure $A$ again (or any other observable for which the $\psi_\lambda$ are eigenstates) and adopt the beam splitter attitude.

For future reference, let us examine now what happens when we perform a second measurement represented by a self-adjoint operator $B$ with discrete spectrum and spectral decomposition $B = \sum \mu Q_\mu$. We now have $(\rho_\lambda)B = \sum \mu Q_\mu \rho_\lambda Q_\mu = \sum \mu \lambda (Q_\mu P_\lambda \psi, \cdot)Q_\mu P_\lambda \psi$. Now can this be interpreted as an objective mixture according to the correlation criterion? If by this we mean that each distinct (unnormalized) final state $Q_\mu P_\lambda \psi$ is uniquely correlated to a set of outcomes, then yes. If however we want that there be a test for each final state which passes if and only if that state is produced, or if we want to “separate the beams” as was done for a single measurement above, then the distinct final states must be orthogonal. We shall consider the objective mixture description legitimate in this case. This latter situation is always realized for any initial state $\psi$ whenever $A$ and $B$ commute.

How should one interpret then the ISC constraints? They must now be read in the following manner: in any theory for which the objective mixture criterion is the presence of an adequate previous measurement and for which such descriptions form a consistent logical system leading to frame independent predictions of ultimate physical facts, the results of the references hold. So reinterpreted, the validity of the works is maintained and they still provide strong criteria for selecting theories, but these theories are to be chosen among special types. This is an important insight, as one can now see under what conditions ISC may not constrain a theory or constrain it less. Thus if
“state collapse” as a descriptive element, along with ISC, argues strongly for a linear theory (as is expounded in [19, 2]), theories without this descriptive element, such as the consistent-histories approach to quantum mechanics, may possibly be made non-linear without violating ISC. We argue in this direction in [27, 28] and take up this point later in this paper.

3 Locality and Purity

Since the idea of objective mixtures involves decomposition into pure states, we must examine the nature of pure states in relativistic quantum mechanics. There are roughly two approaches to this, via fields (Streater and Wightman [29]) and via algebras of observables (Haag [8]).

The algebraic approach is closer in spirit to what we are contemplating here as it introduces algebras of local observables, that is, it associates to each bounded region \( O \) of space-time an algebra \( \mathfrak{A}(O) \) of observables that correspond to experiments that can be executed in \( O \). In relativistic quantum logic we would be interested in propositions that can be tested in \( O \) and so should in some natural way be related to the algebra \( \mathfrak{A}(O) \). It is however rather difficult to come across examples of local algebras except through quantum fields and so we present here one possible construction. Assume for simplicity that we have a real (uncharged) scalar relativistic Wightman field \( \Phi \). Such a field is an operator-valued distribution in the sense that there is a fixed dense domain \( D \) such for any \( f \in S(\mathbb{R}^4) \) there is an essentially self-adjoint operator \( \Phi(f) \) on the invariant domain \( D \) and such that for all \( \phi, \psi \in D \) the map \( f \mapsto (\phi, \Phi(f)\psi) \) defines a tempered distribution. Consider now the operators \( \Phi(f) \) for \( \text{supp} f \subset O \) for some bounded region of space-time \( O \). Let \( \mathfrak{A}^c(O) \) be the set of bounded operators \( A \) such that \( AD \subset D \) and which commute on \( D \) with all the operators \( \Phi(f) \) introduce above. Define the von-Neumann algebra \( \mathfrak{A}(O) \) as \( (\mathfrak{A}^c(O))' \), the commutant of \( \mathfrak{A}^c(O) \). The algebra \( \mathfrak{A}(O) \) is then taken to be the algebra of observables in \( O \). An alternative definition would be to take for \( \mathfrak{A}(O) \) the von-Neumann algebra generated by the bounded functions of the \( \Phi(f) \), that is by those operators of the form \( F(\Phi(f)) \) where \( F \) is a real Borel-measurable bounded function on \( \mathbb{R} \). The hermitian projectors in \( \mathfrak{A}(O) \) should then correspond to the testable proposition in \( O \). Now it is a know fact (Araki [30], Haag [8]) that the von Neumann algebra \( \mathfrak{A}(O) \) is of type III. This means that it contains no finite-dimensional projector and in particular no one-dimensional projector.
One-dimensional projectors are indicator propositions for pure states, that is if $P = (\psi, \cdot)\psi$ for a normalized vector $\psi$, and $\rho$ is a density matrix, then one has $\text{Tr}(\rho P) = 1$ if and only if $\rho = (\psi, \cdot)\psi$. It is these projectors that must be used to test for pure states, and therefore purity of states is not a local notion. This is the basic insight that relativistic quantum field theory provides for quantum logic. A consequence of this is that the notion of objective mixtures becomes a non-local notion, and in particular the correlation criterion, as it involves testing for pure states, is a non-local criterion.

Now if one cannot test for purity by projectors in $\mathcal{A}(\mathcal{O})$ for a bounded region $\mathcal{O}$, it is also highly plausible that one can test for pure states by projectors in the algebra associated to any time slice: $\mathcal{O} = \{(x, y, z, t) | t_1 < t < t_2\}$. Such an algebra is defined by an appropriate limiting procedure in term of the algebras associated to bounded regions contained in the time slice. For free fields any time-slice algebra is just $B(H)$, the full operator algebra of the physical Hilbert space. In general one expects the field to obey, in some appropriate sense, a hyperbolic differential equation and so the field values at any point can be determined from their values in a time slice. This would mean that the time-slice algebra coincides with the algebra associated to all of space-time, which again should be $B(H)$ for reasonable theories.

Suppose now that we perform a measurement in a bounded space-time region $\mathcal{O}$ upon a pure Heisenberg state represented by a normalized vector $\psi$. Consider at a space-like point to $\mathcal{O}$ two observers, one for whom, according to the time variable in his frame, the measurement is still to happen and another one for whom the measurement has already taken place. As they fly by each other they exchange notes, each one indicating what the quantum state is. One says it’s pure, the other one mixed. This apparent contradiction disappears when one realizes that an operational definition of purity is not local. Each one’s assessment depends on a time-slice which for one is prior to the experiment and posterior for the other. Thus in relativistic quantum mechanics, in the presence of measurements, whether a state is pure or not is frame dependent. If both observers are however in the future light cone of all points of $\mathcal{O}$, then their assessments of the state agree, both will say it is mixed and their respective descriptions should differ merely by the action of a representation of the Lorentz group, that is by usual Lorentz covariance. There is a point of consistency here: in the region where the observers do not agree, no local observable should be able to distinguish between the two conflicting descriptions. This in fact is the case, for let the experiment be represented by a self-adjoint operator $A$ with
discrete spectral decomposition \( A = \sum \lambda P_\lambda \) and let \( B \) be a self-adjoint operator pertaining to a space-like separated local algebra. Let us say the first observer’s description is that \( B \) is being observed on \( \psi \). The expected value then would be \( (\psi, B\psi) \). Assuming normal Lorentz covariance, the second observer describes the situation as observing \( U(g)^*BU(g) \) upon \( U(g)^*\rho_AU(g) \) where \( U(g) \) is a unitary operator representing the element \( g \) of the Lorentz group that connects the two observers. This last description gives a mean value of \( \text{Tr}(U(g)^*BU(g)U(g)^*\rho_AU(g)) = \text{Tr}(BP_\psi) = \text{Tr}(B\sum P_\lambda P_\psi P_\lambda) \) where \( P_\psi = (\psi, \cdot)\psi \). By the linearity and permutation symmetry of the trace this is \( \sum \text{Tr}(P_\lambda BP_\lambda P_\psi) \). Now in all relativistic field theories, local observables in space-like separated regions commute, so one has \( P_\lambda BP_\lambda = P_\lambda^2 B = P_\lambda B \) and since \( \sum P_\lambda = I \), the expected value is \( \text{Tr}(BP_\psi) = (\psi, B\psi) \) exactly as for the first observer. Hence the discrepancy in description of purity of states, due to the non-local nature of the correlation criterion, has no effect on locally observable quantities and this is precisely what accounts for the consistency of this criterion.

We are now in position to abstract from the above situation in conventional quantum mechanics and introduce a sketch of an axiom system for a measurement theory in relativistic quantum logic.

4 Propositions, Properties, States, and Ensembles

The basic ingredient of a “quantum logic” approach to a physical theory is a pair \((L,S)\) where \( L \) is an orthomodular poset (usually a lattice) of physical propositions, and \( S \) an abstract convex set whose elements correspond to physical states. The set \( P \) of extreme points of \( S \) correspond to pure states. The relation between \( L \) and \( S \) is usually that elements of \( S \) are \( \sigma \)-additive probability measures on \( L \), and given \( s \in S \) and \( a \in L \) the number \( s(a) \) corresponds to the probability that the proposition \( a \) be true in the state \( s \). We shall not necessarily adhere to such a view though much of the literature adopts it. We shall not here go into details about how one operationally prepares pure states nor determines their purity except in so far as is needed to consider the space-time relations involved.

One generally sees two concepts of physical state that. The most common one is that of an instantaneous state, and a physical system is supposed to
at each time instant be in some instantaneous state. The other notion is that of a state *sub specie aeternitatis* in which case the whole temporal history from \( t = -\infty \) to \( t = +\infty \) is subsumed in the notion. Of course this is an idealization as normally a state is prepared at some instant and destroyed at some future instant both by processes foreign to its “normal” isolated temporal evolution. Thus to maintain a view *sub specie aeternitatis* one has to rely on some deterministic evolution extendible to both temporal infinities. This also means that the states is considered as a self-subsisting entity. The ability to extend its evolution to a time prior to its creation, means that it could have been created at a different time and so it has no knowledge of its creation, and the ability to extend its evolution to a time beyond its destruction means that it has no presage of its demise. Such a physical state is an ontological entity all to itself. The two views coexist in ordinary quantum mechanics whereby the instantaneous state view is maintained in the Schrödinger picture and the other in the Heisenberg picture. The *sub specie aeternitatis* view is more convenient for space-time description as the notion of instantaneous state, even for normal time evolution, brings in frame-related considerations due to the frame-dependent nature of the notion of “instant”. Thus we maintain the *sub specie aeternitatis* viewpoint in this paper. Of course even this view cannot entirely avoid frame-related notions as this type of state is allowed to undergo change through the measurement process or other external interventions. Such changes are generally held to be instantaneous in some frame and so if the state is conceived as having a space-time extent, a frame dependent change of description is involved. We shall in fact be interested in states having sufficiently large space-time extents to be able to perform independent measurements at spatially separated distances.

Just as in quantum mechanics, we shall assume the notion of ensembles at least in so far as they can be partially realized by repetitions of preparation procedures. The notions of subensemble and subensemble fraction is also maintained. Ensembles are represented by elements of \( S \) and we admit, just as in quantum mechanics, that the same element of \( S \) could very well represent many ontologically distinct ensembles so one should not conflate the two.
5 Measurements and Objective Mixtures

We shall assume at least that \( L \) and \( S \) are related through measurement. By an instrument \( I \) we shall mean an exhaustive n-tuple \((a_1, \ldots, a_n) \in L^n\) of mutually exclusive propositions; that is, \( a_i \perp a_j \) for \( i \neq j \) and \( \bigvee_{i=1}^n a_i = 1 \).

We shall assume that such n-tuples correspond to physical measurements with \( n \) mutually exclusive and exhaustive outcomes; that is, when such a measurement procedure is executed, one and only one of the outcomes occurs.

We shall now make a series of assumptions concerning the act of measurement and discuss them later.

Postulate 1 (M1 – Frequency) Given an instrument \( I = (a_1, \ldots, a_n) \) and a state \( s \), then associated to an ensemble of measurements of \( I \) in \( s \), is a frequency function \( \omega^I_i(s) \) where \( \omega^I_i(s) \geq 0 \) and \( \sum_{i=1}^n \omega^I_i(s) = 1 \).

We’ve used the term “frequency function” as a neutral alternative to “probability” or “propensity” as there is no need to enter into interpretational questions at this moment. Of course, the subensembles of measurements corresponding to the occurrences of distinct \( a_i \) do not overlap.

Postulate 2 (M2 – State transformation) A state subject to a measurement undergoes a transformation to another state representable by an element of \( S \). Let \( \pi^I : S \to S \) be the map representing this transformation.

We are adopting here what in the quantum mechanical case we called the “incoherent mixture” view of measurement.

Postulate 3 (M3 – Subensemble) Given a state \( s \) the transformed state \( \pi^I s \) consists of subensembles corresponding to each particular outcome of the measurement. Each such subensemble is a fraction of the total ensemble given by the frequency function. Thus there are partial maps \( \pi^I_i \) such that \( \pi^I s = \sum_{i=1}^n \omega^I_i(s) \pi^I_i s \). The expression \( \pi^I_i s \) is considered to be defined only when \( \omega^I_i(s) \neq 0 \).

This assumption allows us now to also adopt the “beam splitter” view of measurements.

Postulate 4 (M4 – Ideality) Measurements are ideal, in the sense that for any pure state \( p \) and any instrument \( I \) one has that \( \pi^I_i p \) is also pure, when defined.
Postulate 5 (M5 – Objectivity) The ensemble produced by a measurement is an objective mixtures of the subensembles corresponding to the individual outcomes. That is, $\pi^I s =_o \sum_{i=1}^{n} \omega_i^I(s) \pi_i^I s$.

In particular, if one performs a measurement on a pure state, then by M4 and M5 one obtains an objective mixture of pure states.

We leave open as to what exactly is the criterion for objective mixtures. The precise nature of this criterion is not as relevant as some of its desirable properties to which we shall draw attention in due time. We mention, for the sake of concreteness, a possible correlation-type criterion just as in quantum mechanics:

(Correlation Criterion of Objectivity) The criterion for an observer in the coordinate future to the measurement by an instrument $I$ on a state $s$ to maintain $\pi^I s =_o \sum_{i=1}^{n} \alpha_i s_i$ with $\alpha_i \neq 0$ is the availability of dicotomic instruments $J^{(j)} = (b_1^{(j)}, b_2^{(j)})$ such that $\omega_i^{j^{(j)}}(\pi_i^I(s)) = \delta_{ij}$, and a strict correlation of the first outcome of $J^{(j)}$ with the $j$-th outcome of $I$, in which case $\alpha_i = \omega_i^I(s)$ and $s_i = \pi_i^I(s)$.

This criterion would suffice for what follows but others could probably do just as well.

One consequence of assuming the existence of objective mixtures is that if $s =_o \int p d\mu(p)$ then $\omega_i^I(s) = \int \omega_i^I(p) d\mu(p)$ and so it’s enough to know the frequency function only in pure states, and we can assume the functions $\omega_i^I$ are affine. The same goes for the map $\pi^I$.

How plausible are these assumptions? Taken together they express our ability to individuate physical system by appropriate “filtering” through measurements and then to perform statistical experiments on situations so created. This constitutes the basis of normal physical experimental practice, so to negate this is to radically change our view of the statistical nature of physical phenomena and our experimental access to them. Of course these assumptions are already idealized, and one could argue with the details of each one individually, but some such set must be postulated to even begin a formulation of a statistical science. Part of the above assumptions incorporate the notion of self-subsisting physical states, that is the sub specie aeternitatis view. This is not a logically necessary ingredient of a physical theory, ingrained as it may be. All a physical theory must be able to do is
predict the joint probabilities of events, and the mediation of these by physical states under evolution is not a necessity. As was mentioned before, the consistent histories approach does not make use of such a notion, and the preceding assumptions would have to be modified if one were to axiomatize such a viewpoint.

6 Space-time Structure of Measurements

The relation between a physical theory \((L, S)\) and space-time structure generally comes in through external considerations. Ordinary hilbert-space quantum mechanics admits both Galileian and Lorentz covariance; such considerations only enter through unitary representations of an appropriate group and have no expression in the fundamental formalism as such. In our context we cannot do much better while some basic mathematical questions have yet to be settled. We assume lorentzian space-time, and in relation to the physical theory we assume a series of postulates generalizing some of the usual external connections already seen in hilbert-space theory.

Postulate 6 (S1 – Localization) Given a bounded space-time region \(O\) then there is a sub-orthomodular-poset \(L(O) \subset L\) corresponding to propositions testable in \(O\).

This assumption reflects the notion that experiments are essentially bounded in space-time. Usually one also feels that propositions referring to unbounded regions should only be admitted as idealizations, that is, limits of local ones. Thus one could postulate that the union of the subsets \(L(O)\) is join-dense in \(L\) which is then viewed as a set of “quasi-local” propositions. Quantum field theory teaches us that one should not assume about \(L(O)\) properties that one usually postulates about \(L\); thus while the latter is often taken to be atomic and atomistic, this should not be the case for the local posets.

We say an instrument \(I = (a_1, \ldots, a_n)\) belongs to a region \(O\) if \(a_i \in L(O)\) for each \(i\). For two regions \(O_1, O_2\) we write \(O_1 \bowtie O_2\) in case they are space-like separated, that is, every point of one is space-like to every point of the other. For \(a_1, a_2 \in L\) we write \(a_1 \bowtie a_2\) in case \(a_1 \in L(O_1)\) and \(a_2 \in L(O_2)\) for some regions \(O_1 \bowtie O_2\). For two instruments \(I\) and \(J\) we write \(I \bowtie J\) in case they belong to space-like separated regions.

Postulate 7 (S2 – Locality) If \(O_1 \bowtie O_2\) then every element of \(L(O_1)\) commutes with every element of \(L(O_2)\).
It is customary in the lattice-theoretic approach to equate commutativity of propositions with their commensurability. It is likewise customary to assume that space-like separated regions are causally disjoint (locality or causality assumption). This then is a major assumption relating space-time structure and the poset of proposition. We shall write $a \leftrightarrow b$ whenever $a$ and $b$ commute. It is interesting to mention that in Mittelstaedt’s scheme, commutativity of space-like separated propositions is necessary for logical consistency.

If $I = (a_1, \ldots, a_n)$ and $J = (b_1, \ldots, b_m)$, are two instruments such that $a_i \leftrightarrow b_j$ for all $i$ and $j$, in particular if $I \cong J$, then we can form a new instrument $I \wedge J = (a_i \wedge b_j)_{i=1,\ldots,n; j=1,\ldots,m}$.

Now the execution of an experiment consists of physical acts leading to physical results. This in itself has nothing to do with our description of the experiment nor consequently with the adoption of any particular reference frame for space-time. The propositions being tested however do have something to do with reference frames. If an observer finds at time $t_0$ that a proposition $a$ is true about a state $s$, then the truth of $a$ is aspatial, hence to be considered as such at all points of space. If, as is customary, one considers that a proposition can become true at a time instant $t_0$ then it must become true instantly and simultaneously at all space points. An observer in a different frame has a different plane of simultaneity so his propositions become true in a different manner. This frame dependence of truth values and becoming-true is not necessarily in conflict with a frame-independent physics. The mutual consistency of the two however, given other requirements, can and does lead to constraints on physical theories.

**Postulate 8 (S3 – Measurement instant)** *If an experiment corresponding to an instrument belonging to a region $O$ has been executed, then every observer assigns a unique time instant at which the experiment is considered to be realized and at which instant one of the propositions related to the instrument becomes true and the others false. The plane of simultaneity of this instant intersects $O$.*

This assumption seems to be generally, even if grudgingly, accepted. The experimenter often doesn’t have control over the instant in question which is somehow decided by the physical processes that the experiment unleashes, yet such an instant is generally identified. Even more often the case is that the realization of the experiment is associated to some unique event (counter
click, for instance) and the instant is just the time coordinate of this event in the given frame. In this case different observers can agree on the same event, which lies in $\mathcal{O}$. There are of course important apparent exceptions to the single event viewpoint such as coincidence experiments, but even in this case some maintain that the experiment is only over when all the information is gathered in some single recording device, such as a brain, in which case one falls back on the single event hypothesis. We shall not however adhere to this viewpoint.

Consider now two experiments executed in space-like separated regions. Let $I = (a_1, \ldots, a_n)$ and $J = (b_1, \ldots, b_m)$ be the corresponding instruments. Suppose that an observer assigns the same time instant $t_0$ to the realization of both experiments. Then, by the commensurability of the two instruments, at $t_0$ the observer maintains not only that one of the $a_i$ becomes true and that one of the $b_j$ becomes true but also that one of the $a_i \land b_j$ becomes true.

As far as the observer is concerned, the two separate realizations of $I$ and $J$ is indistinguishable from a single realization of $I \land J$. This indistinguishability is our next assumption.

**Postulate 9 (S4 – Confluence of simultaneous measurements)** If an observer assigns the same time instant to the realization of two space-like separated experiments, these can be treated equivalently as the realization of a single experiment whose outcomes are the conjunctions of the outcomes of the separate experiments.

Another observer in a different frame would generally see a time interval between the two realizations, either $I$ first, followed by $J$, or vice-versa. In this case he must consider the two experiments as consecutive and for him the becoming-true of propositions related to the two experiments do not occur simultaneously. If physics is frame-independent then the two observers must agree about ultimate physical facts. For this to be the case, the pair $(\mathcal{L}, \mathcal{S})$ must satisfy certain constraints.

What we still lack are assumptions that express the supposed frame independence of ultimate physical facts. Just as in hilbert-space theory, we introduce this via group action.

**Postulate 10 (C – Lorentz covariance)** Let $\mathcal{G}$ be the Poincaré Group. There are actions $(g, a) \mapsto \lambda_g a$ of $\mathcal{G}$ on $\mathcal{L}$ and $(g, s) \mapsto \sigma_g s$ on $\mathcal{S}$ such that:

1. For all $g \in \mathcal{G}$, $\lambda_g$ is an orthomodular-poset isomorphism.
2. $\lambda_g \mathcal{L}(\mathcal{O}) \subset \mathcal{L}(g(\mathcal{O}))$

3. For all $g \in \mathcal{G}$, the action $s \mapsto \sigma_g s$ is affine and $\sigma_g(\mathcal{P}) \subset \mathcal{P}$.

4. Given an instrument $I = (a_1, \ldots, a_n)$, let $\lambda_g I = (\lambda_g a_1, \ldots, \lambda_g a_n)$. One then has:

$$\omega^I_i(s) = \omega^{\lambda_g I}_i(\sigma_g s)$$

$$\pi^I_i \sigma_g p = \sigma_g \pi^I_i p$$

For simplicity’s sake we shall write $g \cdot a$ and $g \cdot s$ instead of $\lambda_g a$ and $\sigma_g s$.

Most of this assumption is a fairly straightforward rendition of rather standard covariance conditions on physical theories. There are two ways of understanding the group action. The first, or “passive” view, is that if an observer describes an experimental procedure as that of executing $I$ upon state $s$, then another observer whose frame is obtained from that of the first one by action of $g$ will describe the same procedure as that of executing $g^{-1} \cdot I$ upon $g^{-1} \cdot s$. The “active” view states that there is another procedure (whose execution has a clear operational relation to the execution of the first one) described by $g \cdot I$ and $g \cdot s$ by the first observer and whose description by the second observer is by $I$ and $s$. The equivalence of the two views is the essence of relativistic theories.

One possible strengthening of C3 would be to assert that objective mixtures map to objective mixtures, that is, if $s =_o \int p \, d\mu(p)$, then $g \cdot s =_o \int g \cdot p \, d\mu(p)$. This is prima facie a natural assumption, and we shall formulate a version of it. However we shall also need a more subtle manifestation of covariance in relation to objective mixtures. Since our only assumption concerning objective mixtures is that they come about through measurements and since the corresponding “collapse” is frame dependent, covariance of objective mixtures is a rather more involved concept since whatever criterion for objective mixtures one may adopt, one should not think of it as a local criterion as the quantum field theory case teaches us. One must imagine that such a criterion would utilize something like a time-slice region, idealized say to a space-like hyperplane. In some regions two observers would disagree even as to which (if any) of the relevant experiments have been carried out and in which order, leading to objective mixtures not related by the action of the Poincaré group. Objective mixtures as observer related descriptions would have varying relation to physical facts. Thus the simple relation for
objective mixtures given in the beginning of this paragraph can be main-
tained as referring to the state in question only by a pair of observers that are
both in the future light cone of all the relevant measurement processes that
enter into the objective mixture criterion.

**Postulate 11 (O I – Covariance of objectivity I)** If \( s =_o \int p \, d\mu(p) \), then
\[ g \cdot s =_o \int g \cdot p \, d\mu(p) \]. The right hand side of these equations refers to the state
in question as seen by two observers related by the element \( g \) of the Poincaré
group provided both are in the time-like future of all events involved in the
objective mixture criterion.

Let now \( I = (a_1, \ldots, a_n) \) and \( J = (b_1, \ldots, b_m) \) be two instruments and
assume \( I \bowtie J \). Consider now an observer, call him Observer 1, who assigns
the same time instant to the realization of both experiments upon a pure state
\( p \) (note well: we do not mean that each experiment acts on its own “copy” of
\( p \) but each on the same spatially extended state \( p \)). According to S4 we can
view this as a realization of \( I \land J \) and so by the measurement assumptions
the state is transformed into \( s =_o \sum_{i,j} \omega_{i,j}^{I \land J}(p)\pi_{i,j}^{I \land J} p \). Another observer, call
her Observer 2, will describe the situation as the realization of \( g \cdot I \) upon
\( g \cdot p \) followed by a realization of \( g \cdot J \) upon the resulting state of the first
measurement. The first experiment produces a state \( s_1 =_o \sum_i \omega_i^{g \cdot I}(g \cdot p)\pi_i^{g \cdot I} g \cdot p \)
and this is transformed by the second experiment into (note the lack of the
objective equality sign) \( s_2 = \sum_{i,j} \omega_{i,j}^{g \cdot J}(\pi_i^{g \cdot I} g \cdot p)\omega_{i,j}^{g \cdot I}(g \cdot p)\pi_j^{g \cdot J} \pi_i^{g \cdot I} g \cdot p \). Our
next assumption is that this is in fact an objective mixture:

**Postulate 12 (O II – Covariance of objectivity II)**

\[ a) \quad s_2 =_o \sum_{i,j} \omega_{i,j}^{g \cdot J}(\pi_i^{g \cdot I} g \cdot p)\omega_{i,j}^{g \cdot I}(g \cdot p)\pi_j^{g \cdot J} \pi_i^{g \cdot I} g \cdot p. \]

\[ b) \quad s_2 = g \cdot s. \]

This is a subtle point. One has no grounds on the basis of previous
assumptions or analyses to claim that the expression for \( s_2 \) is an objective
mixture and in fact one needs an additional argument to adopt this hypoth-
esis (it is true that M5 implies that \( s_2 \) is an objective mixture of certain
non-extreme states, but this is not what we are saying here, and M5 does
not imply O II). One saw in the quantum mechanical discussion that two
successive measurements with commuting instruments lead to an objective
mixture of the type expressed by \( s_2 \) according to the correlation criterion.
One could be tempted to use this as guideline and adopt a similar hypothesis in the quantum logic case, seeing that space-like separated instruments commute. This however has no more compelling reason to be than the covering law itself and we do not view O II as a statement about commutativity but about covariance. As has been pointed out, observers in different frames may even sustain objective mixture descriptions not related by the action of an element of the Poincaré group. Such disagreements as to the description of the state, objective as they may be, should of course not be detectable by local observations just as in the quantum field theory case, and this leads to definite constraints on the theory, but these as we shall see will be automatically satisfied. Let us however concentrate on the set of points that are future time-like to all events involved in the measurement process. As we found out in the quantum mechanical case, in this region, the passage criterion for objective mixtures can be maintained and actually carried out as we can have knowledge of the measurement results. This means that we can actually use the “beam-splitter” view of measurements and consider conditioning further measurements to the outcomes of the past measurements $I$ and $J$. Now if we condition to outcome $a_i$ and $b_j$, then these outcomes are ultimate facts whether they are simultaneous or not. In the future region we expect, by the very notion of Lorentz covariance, that the state defined by this “beam” should behave in a purely covariant fashion, that is, obeying $C$, when tested by local observables. As we move further and further into the future, local observables in this region can have increasingly greater space-like extent and asymptotically can approach time-slice observables. As these are presumed to be involved in the objective mixture criterion, these are then also subject to $C$. In particular, if Observer 1 finds the state to be $s_{ij}$ and if Observer 2 is related to Observer 1 by and element $g$ of the Poincaré group, she should find the state to be $g^{-1} \cdot s_{ij}$. Thus we can argue that the given representation for $s_2$ should be an objective mixture purely on the grounds of Lorentz covariance. We should also have $s_2 = g \cdot s$. This type of covariance is if course somewhat different from the one expressed in $C$ as that one makes no reference to measurements. Observer 1 has a different geometrical relation to the measurements than Observer 2 as in his situation there is only one plane of simultaneity related to the measurements, whereas for her there are two. The geometrical structures are not group transforms of each other. The frame-dependent description of the measurement process is in contrast with the conventional covariance behavior expressed by $C$. It is thus not surprising that a full expression of covariance should include statements concerning the
measurement process, and O II is such a statement. Another way to motivate this assumptions is to argue that any measurement process, occurring as it does in an interaction of a system with a macroscopic apparatus, is already likely to involve space-like separated events. Thus any physically reasonable system obeying C already must presuppose something like O II.

From \( s_2 = g \cdot s \) and Postulates [11] and [12], one has the following equality of objective mixtures:

\[
\sum_{i,j} \omega_{i,j}^I \pi_{i,j}^I \, p = \sum_j \sum_i \omega_j^I (\pi_i^I p) \omega_j^I (p) \pi_j^I \pi_i^I p,
\]

and thus, the final pure components and the corresponding fractions must co-incide. Assuming that the set of pure states is sufficiently large to distinguish the functions \( \omega \) and \( \pi \) one comes to the following consequence:

**Theorem 1** Given that \( I \bowtie J \) where \( I = (a_1, \ldots, a_n) \) and \( J = (b_1, \ldots, b_m) \) then:

\[
\forall p \in \mathcal{P}, \quad \omega_{i,j}^I \pi_{i,j}^I (p) = \omega_j^I (\pi_i^I p) \omega_j^I (p) \quad (1)
\]

\[
\pi_{i,j}^I = \pi_j^I \pi_i^I. \quad (2)
\]

These then are the constraints that any measurement theory of the type described here must satisfy if it is to be able to describe Lorentz-covariant physics.

One should note that in the two equations of the theorem, the left-hand side is the same if \( I \) and \( J \) are interchanged. This leads immediately therefore to the following relations:

\[
\forall p \in \mathcal{P}, \quad \omega_j^I (\pi_i^I p) \omega_j^I (p) = \omega_i^J (\pi_j^J p) \omega_j^J (p), \quad (3)
\]

\[
\pi_j^I \pi_i^I = \pi_i^I \pi_j^I. \quad (4)
\]

These relations of course could have been independently derived from an argument similar to ours if we considered two frames in which the temporal order of experiments \( I \) and \( J \) are opposite, without considering a frame in which they are simultaneous.

Now we come to the covering law. What is remarkable is that in various axiomatic schemes that have been proposed to replace the covering law with more “physical” assumptions, these coincide with or are readily deduced from one or both of equations (1–2) or even the weaker results (3–4), with
the difference that they are postulated for $I \leftrightarrow J$ and not only for $I \gg J$. We have been somewhat more general in this exposition than what is normally assumed, as usually the $\omega_{I}^{i}(p)$ and $\pi_{I}^{i}p$ are postulated not to depend on $I$ but only on the proposition $a_i$ in question. This is the absence of certain type of contextuality.

Bearing this in mind we thus see that the "Compatibility Postulate" in Guz [31] is a direct consequence of (2) stated for commuting pairs; axiom F4 in Guz [32] follows from (1), again stated for commuting pairs, as can be found in Svetlichny [2]. Pool's [33, 34] derivation of the semimodularity of quantum logic, necessary for a hilbert-space interpretation, follows a different chain of reasoning but is clearly related to our results. The axioms of the first paper lead to the equivalence of commutativity of propositions with a form of equation (4) while in the second paper semimodularity is derived from a further assumption which in our scheme is M4. Nicolas Gisin (private communications) has likewise derived the covering law from a form of equation (1) stated for propensities and has independently postulated a possible relation between the covering law and space-time structure.

Let us now consider the constraints that express the fact that local observables must not distinguish between two objective mixtures not related by an element of the group action but that are legitimately maintained by two observers who disagree as to which experiments have already been carried out. Suppose that in some region $O$ an experiment is performed which the first observer, space-like to $O$, says has already happened and consisted of an observation by an instrument $I$ upon a pure state $p$. He would say the new state is $\sum_{i} \omega_{I}^{i}(p)\pi_{I}^{i}p$. Another observer flying past him would claim that since the experiment has not been performed in his frame, the state is $g \cdot p$ where $g$ is the element of the Lorentz group that relates the two observers. Suppose now that a local experiment is performed in a region $O'$ space-like to $O$. The first observer describes this by an instrument $J$ and the frequency of the $j$-th result is thus given by $\sum_{i} \omega_{I}^{i}(p)\omega_{J}^{j}(p)$. This by (3) $\sum_{i} \omega_{I}^{i}(\pi_{J}^{j}p)\omega_{J}^{j}(p) = \omega_{J}^{j}(p)$. The second observer will assign frequency $\omega_{J}^{j}(g \cdot p)$ to the same result but this by C is also $\omega_{J}^{j}(p)$ and so the two observers will agree as to the frequencies of outcomes of local experiments and the two descriptions are consistent just as in the quantum mechanical case. Finally we mention that (4) also implies that no superluminal signal can be sent through long-range correlation by mere change of a local measuring device. The frequency assigned to the $i$-th result of instrument $I$, under lack of
knowledge of the outcome of instrument $J$, is by the right-hand side of (1)
just $\sum_j \omega^J_j (\pi^J_i p) \omega^I_i (p) = \omega^I_i (p)$ which is independent of instrument $J$ just as in the quantum-mechanical case and so no signal of this type is possible.

7 Bridging the Light Cone

One sees that in some of the existing schemes the crucial axiom that leads to the covering law can be substituted by Lorentz covariance and an appeal to a principle generalizing necessary conditions on space-like separated propositions to merely commuting ones. Let us first examine the purely logical nature of such a principle which can be stated as follows:

(E – Equivalence of commutativities) Let $Q(a,b)$ be two-place predicate concerning $L$ then,

$$(a \triangleright \triangleright b \Rightarrow Q(a,b)) \Rightarrow (a \leftrightarrow b \Rightarrow Q(a,b))$$

Assuming the metatheoretic principle E, the covering law can be deduced from Lorentz covariance and axioms of generally less controversial nature. The covering law would thus be compelling if one can turn E compelling. One has to be a bit careful though about this principle, one should probably not maintain it for all possible $Q$. If for $Q(a,b)$ we take $a \triangleright \triangleright b$ then we deduce $(a \leftrightarrow b) \Leftrightarrow (a \triangleright \triangleright b)$ which is likely too strong. It may be that all commutativity in physics can be reduced to space-like commutativity, but one should see this in detail and not through a metatheoretic principle. Is there reason to believe E? One way that E could be true is if there is a symmetry group on $L$ by which if $a$ and $b$ are local and $a \leftrightarrow b$ then there is an element $\phi$ of this group such that $\phi(a) \triangleright \triangleright \phi(b)$. This would establish E for group invariant predicates and a join-dense set of propositions, which would probably be sufficient. In ordinary quantum mechanics the full unitary group of hilbert space is apparently such a symmetry group. There seems to be no compelling reason however to believe in such a group. Given a complete lack of any mathematical development of lattice-theoretic approaches “localized” into space-time regions and with Poincaré group action, “local relativistic quantum logic” in other words, it’s even hard to say how restrictive E is. From the physical side one lacks any real understanding of commutativity of propositions except for space-like separated ones where causality arguments seem compelling. Thus the spin and orbital angular momenta of a particle
commute, but we see this in a purely formal way: the corresponding operators act on different tensor factors. We cannot say we have a true physical understanding of this. Without this understanding, the covering law is still not compelled.

To better understand what can be physically involved in assumptions such as E, it is necessary to examine more closely the meaning of commutativity, which formalizes the notion of compatibility. For the rest of this paragraph we assume a more primitive notion of an instrument than that used in the previous section. An instrument \( I \) corresponds to some procedure or physical construct which leads to a finite set of outcomes \( x_1, \ldots, x_n \) but we do not assume that these are necessarily represented by some algebraic elements. We can still talk about states \( p \), frequencies \( \omega^I_i(p) \) and projections \( \pi^I_i p \) as before.

Operationally, we say that an instruments \( I \) with outcome set \( x_1, \ldots, x_n \) and another one \( J \) with outcome set \( y_1, \ldots, y_m \) are compatible if there is a compound instrument \( K \) with outcome set \( (x_i, y_j), i = 1, \ldots, n, j = 1, \ldots, m \) whose outcome frequencies in any state has as marginals the outcome frequencies of \( I \) and \( J \) in the same state. The actual physical construction of \( K \) may have in general no clear relation to the physical constructions of \( I \) and \( J \), but in certain cases it is customary to consider \( K \) as just \( I \) and \( J \) physically coexisting and consider the outcomes of \( K \) as coincidences of outcomes of \( I \) and \( J \). This is the case when \( I \bowtie J \) and also when \( I \) and \( J \) belong to limited space-time regions in which all points of one are future time-like to all points of the other. In ordinary quantum mechanics, the compound observation of compatible observations that succeed temporally are taken to be just these successive individual measurements. Two instruments that satisfy (1) for both orders of \( I \) and \( J \) on the right hand side are compatible for then the compound instrument \( K \) is \( I \land J \). Indeed one has for the two marginals: \( \sum_j \omega^{I \land J}_{i,j}(p) = \sum_j \omega^I_j(\pi^I_i p)\omega^I_i(p) = \omega^I_i(p) \) and \( \sum_i \omega^{I \land J}_{i,j}(p) = \sum_i \omega^I_i(\pi^J_j p)\omega^J_j(p) = \omega^J_j(p) \). For space-like separated instruments one can argue for both orders on the right-hand side of (1) from covariance arguments as there are frames in which the measurements occur in either temporal order. In the time-like case one cannot use this argument. Let us consider the case in which \( J \) belongs to a space-time region all points of which are future time-like in relation to each point of the region to which \( I \) belongs. One can then consider the instrument \( J \circ I \) consisting of the successive measurements by \( I \) and then by \( J \). By our postulates one has immediately \( \omega^{J \circ I}_{i,j}(p) = \omega^J_j(\pi^I_i p)\omega^I_i(p) \) and so (1) holds for one order by definition even if \( J \) and \( I \) are not compatible. Now if \( I \) and \( J \) are compati-
ble, then conventional wisdom deems $J \circ I$ to be the compound instrument $K$, and so half of (1) is trivially true. The difficulty then is in arguing for the other half, that is, $\omega_{i,j}^J(p) = \omega_i^I(\pi_j^J p) \omega_j^J(p)$. The operational verification of this cannot now be simply done by successive measurements as the right hand side now has an $I$ measurement on a state conditioned by a future time-like $J$ measurement. The frequencies $\omega_{i}^J(p)$ can be established just from $J$ measurements, but then to verify the frequencies $\omega_i^I(\pi_j^J p)$ one needs to be able to create states $\pi_j^J p$ prior to the execution of $I$, that is one needs an instrument $L$ with $m$ outcomes, belongs to a region in the temporal past of $I$, whether time-like or space-like, and a state $p'$ such that $\pi_j^L p' = \pi_j^J p$. One can then verify (1). Thus the operational verification that this equation holds for generally commutative instruments involves an assumption concerning the producibility of given states by actions in distinct and possibly widely separated regions and so transcends purely covariant considerations. It seems that to be able to perform the generalization mentioned in the beginning of this section one needs a physical assumption going beyond covariance.

Standard relativistic quantum theory provides a clue by the presence of long-range correlated states, that is, states of the EPR type. As a simplified version of this situation suppose you want to study right-hand circularly polarized photons. One way is to simply put an appropriate filter in front of a light source and those photons that get through are of the right kind and so can be observed at will. Another equivalent way is to set up an EPR-type arrangement that creates singlet two-photon states with the individual photons flying off in opposite directions. Put now the same filter on the distant arm of the EPR apparatus and nothing on the near arm. Observe at will. Half of the photons observed are right-hand circularly polarized and half are in the orthogonal left-hand circularly polarized state, and as the measurements are done, there is no way of knowing which is which. If all one wants however is analysis of experimental outcomes, this is no problem, just wait enough time that the results (passage through the filter or not) at the distant arm of each photon pair are available (typical correlation experiment situation) and simply throw out all the experimental data for the instances where the distant photon did not pass through the filter. This provides you with data now of just the right-hand circularly polarized photons at the near arm. The fact that these two experimental procedures are equivalent is a feature of ordinary quantum mechanics and depends on the existence of a particular entangled state, the two-photon singlet. We now show that this simplified situation is quite general. Assume we are working with a
local relativistic quantum field theory of the Haag type in a Hilbert space $\mathcal{H}$. Let $I = (P_1, \ldots, P_n)$ and $J = (Q_1, \ldots, Q_m)$ be two compatible instruments belonging to two limited space-time regions, $\mathcal{O}_I$ and $\mathcal{O}_J$ respectively, which need not be space-like separated. Let $\Psi$ be a pure state. Consider now an element $g$ of the Poincaré group such that $\mathcal{O}_J' = g(\mathcal{O}_J)$ is space-like separated from both the original regions. Let $U(g)$ be the unitary symmetry operator associated to $g$, and let $J' = (Q_1', \ldots, Q_m')$, where $Q_j' = U(g)Q_jU(g)^\dagger$, be the transformed instrument. One can choose $\mathcal{O}_J'$ in such a manner [3, 35] that $\mathcal{H}$ decomposes into a tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ with $P_i = P_i \otimes I$, $Q_j = Q_j \otimes I$, and $Q_j' = I \otimes Q_j'$. The projectors of all three instruments commute among themselves. Let $\Lambda$ be the set of triples $ijk$ such that $P_i Q_j Q_k' \mathcal{H} \neq \{0\}$, and for $ijk \in \Lambda$ let $e_{ijk\alpha}$, $\alpha \in A(ijk)$ be an orthonormal basis for $P_i Q_j Q_k' \mathcal{H}$. One has $\Psi = \sum_{ijk \in \Lambda} \sum_{\alpha \in A(ijk)} \psi_{ijk\alpha} e_{ijk\alpha}$. Now for fixed $ij$ one can find an index $p$ such that $ipj \in \Lambda$, for otherwise one would have $\sum_{p} P_i Q_p Q_j' = P_i Q_j' = 0$ which is impossible given the tensor product decomposition. Choose $\beta \in A(ipj)$, set

$$
\psi'_{ipj\beta} = \sqrt{\sum_{\{ijkl \in \Lambda\}} \sum_{\alpha \in A(ijk)} |\psi_{ijk\alpha}|^2}
$$

and set all other components $\psi'_{ik\alpha} = 0$ for $(k, \alpha) \neq (p, \beta)$. Obviously $\Psi' = \sum_{ijk \in \Lambda} \sum_{\alpha \in A(ijk)} \psi_{ijk\alpha} e_{ijk\alpha}$ is another normalized state vector. One easily verifies $\|P_i Q_k' \psi'\|^2 = \|P_i Q_k \psi\|^2$. In standard quantum mechanics one has $\omega_i'(\Psi) = \|P_i \Psi\|^2$ and $\pi_i' = P_i \psi / \|P_i \psi\|$, whenever $\omega_i'(\Psi) \neq 0$. We have from our construction that $\omega_i'^{\land J'}(\Psi') = \omega_i'^{\land J}(\Psi)$, $\omega_j'(\pi_i' \Psi) = \omega_j'(\pi_i \Psi)$ and $\omega_i'(\Psi') = \omega_i'(\Psi)$. So the validity of (1) for the pair of instruments $(I, J)$ can be deduced from the validity for the pair $(I, J')$ of space-like separated instruments due to the existence of the state $\Psi'$. Note that no distinction is made between the two instruments so that the right-hand side of (1) holds for any order. The validity of (1) for space-like separated instruments follows basically from Lorentz covariance and other assumptions of this paper and so is not peculiar to standard quantum theory. The existence of $\Psi'$ has to be viewed however as an additional physical assumption, that happens to be true in standard quantum theory and is in some sense also characteristic of it. This state is in general one that has long-range correlations of the EPR-type. We can now envisage a more physically plausible version of principle E:

**Postulate 13 (EP - Equivalence of conditioning)** For any pair of instruments $(I, J)$ with $I \leftrightarrow J$ and pure state $p$ there is an instrument $J'$
such that $J' \cong I$, and a pure state $p'$ such that the joint experiment $(I, J)$ on the state $p$ is equivalent to the joint experiment $(I, J')$ on $p'$. That is, 

$$\omega_{i,j}(p') = \omega_{i,j}(p), \quad \omega_{j}(\pi_i^I p') = \omega_{j}(\pi_i^I p) \quad \text{and} \quad \omega_{i}(p') = \omega_{i}(p).$$

We call this equivalence of conditioning since it allows one to prepare states conditioned to outcomes at space-like separations as was the case for the simplified photon polarization experiment. With this additional postulate, as was pointed out before, one can now deduce the covering law in some of the existing axiomatization schemes.

8 Conclusions

One of the most intriguing features of quantum mechanics is its universality. All phenomena, to the extent that their quantum behavior can be exhibited experimentally, are subject to the same general formalism. The postulated connection of quantum mechanics to space-time structure makes this understandable. The measurements to which the above discussion refer could be any measurements. To the extend that any measurement takes place in space-time, it must exhibit universal quantum behavior. The universality of quantum mechanics is a reflection of the universality of space-time as the arena for our experiments.

To be able to deduce such universality one however has to be able to make some assumption such as EP compelling. Now why should EP be compelling? It is unlikely that one can find an argument on purely formal grounds or by appeal to “reasonableness” of any kind just as such appeals are ultimately unconvincing in all the axiomatic approaches to quantum mechanics. Space-like and time-like situations are in logically distinct domains and any relation between them must come from some realm in which this distinction is weakened. The only existing considerations of this sort come from what is loosely known as “quantum gravity”. In fact, at this point one perceives a fundamental difficulty with the whole argument of this paper. One has started with a definitely classical view of space-time and traced out a route which leads to a universal mechanics. But now space-time phenomena themselves must, by universality, be subject to the same mechanics, which distorts the original starting point. Space-time itself must be quantum mechanical. There is a self-consistency question. The constraints that space-time structure places on mechanics must govern the structure of space-time.
itself. Unfortunately “quantum-space-time quantum logic” is just a glimmer of an idea at this moment, more remote than the “relativistic quantum logic” that we’ve embarked upon. In any case, in all present-day approaches to quantum gravity, the rigid structure of the light cone disappears and the usual notions of space-like and time-like are emergent and not fundamental. In such a context it is perfectly understandable that relations between space-like and time-like situations arise out of a more basic theory in which such a distinction is not fundamental.

On a more technical side, we note that the above argument utilizes more the lorentzian causal structure of space-time and the relativity of simultaneity than the exact details of Lorentz covariance. Thus one can expect that a similar result can be obtained for curved space-time as well. Strict Lorentz covariance should therefore be viewed as a simplifying assumption for these preliminary studies. One also has the awkwardness of deriving a global feature (covering law) through local considerations. One knows from algebraic quantum field theory that local von Neumann algebras of observables have a unique form (Haag [8]), they are all type III$_1$ hyperfinite factors for a causal diamond $O$ (intersection of a forward and a backward light cone). It would be more in keeping with the local approach to try to deduce that $L(O)$ is isomorphic to the projection lattice of a type III$_1$ hyperfinite factor and then only secondarily argue for the covering law through global considerations. This of course requires a lattice-theoretic characterization of such factors, which to our knowledge is not available.

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References
[1] Piron, C. (1976) “Foundations of Quantum Physics”, W. A. Benjamin, Inc., London.

[2] Svetlichny, G. (1998) Foundations of Physics, 28, 131; quant-ph/9511002.

[3] Mittelstaedt, P. (1983a) International Journal of Theoretical Physics 22, 293.

[4] Mittelstaedt, P. (1983b) Proceedings of the International Symposium on the Foundations of Quantum Mechanics Tokyo, pp. 251-255.

[5] Mittelstaedt, P. and Stachow E. W. (1983) International Journal of Theoretical Physics 22, 517.

[6] Neumann, N. and Werner, R. (1983) International Journal of Theoretical Physics 22, 781.

[7] Ludwig, G. (1983) “Foundations of Quantum Mechanics”, Springer, New York.

[8] Haag, R. (1992) “Local Quantum Physics”, Springer Verlag, Berlin.

[9] Mugur-Schäcter, M. (1991) Foundations of Physics 21, 1387.

[10] Mugur-Schäcter, M. (1992) Foundations of Physics 22, 235.

[11] Hartle, J. B., “Spacetime Quantum Mechanics and the Quantum Mechanics of Spacetime”, in 1992 Les Houches Ecole d’été, Gravitation et Quantifications.

[12] Omnès, R., The Interpretation of Quantum Mechanics, Princeton University Press, (1994)

[13] Svetlichny, G. (1999) “Space-time Structure and Quantum Mechanics” to appear in the proceedings of the XXXth Workshop on Geometric Methods in Physics, Coherent States, Quantization and Gravity, Białowieża, Poland, 1998.

[14] Svetlichny, G. (1990) Foundations of Physics 20, 635 - 650.

[15] Svetlichny, G. (1992) International Journal of Theoretical Physics 31, 1797.
[16] Czachor, M., quant-ph/9501008

[17] Einstein, A., Podolsky, B. and Rosen, N. (1935) Physical Review 47, 777.

[18] Gisin, N. (1984a) Physical Review Letters 52, 1657.

[19] Gisin, N. (1984b) Physical Review Letters 53, 1776.

[20] Gisin, N. (1989) Helvetica Physica Acta 62, 363.

[21] Gisin, N. (1990) Physics Letters A 143, 1.

[22] Pearle, P. (1984) Physical Review Letters 53, 1775.

[23] Pearle, P. (1986) Physical Review D 33, 2240.

[24] Redhead, M. (1987) “Incompleteness, Nonlocality, and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics”, Claredon Press, Oxford.

[25] Mielnik, B. (1990) Foundation of Physics 20, 745.

[26] Finkelstein, J. (1992) Foundation of Physics Letters 5, 383.

[27] Svetlichny, G., “Quantum Evolution and Space-Time Structure” in H.-D. Doebner, V. K. Dobrev, and P. Nattermann, eds., Nonlinear, Deformed and Irreversible Quantum Systems. Proceedings of the International Symposium on Mathematical Physics, Arnold Sommerfeld Institute, 15-19 August 1994, Clausthal, Germany, p. 246, World Scientific, Singapore, 1995, or in a slightly expanded form in quant-ph/9512004.

[28] Svetlichny, G., “On Relativistic Non-linear Quantum Mechanics” in M. Shkil, A. Nikitin, V. Boyko, editors, Proceedings of the Second International Conference “Symmetry in Nonlinear Mathematical Physics. Memorial Prof. W. Fushchych Conference”, Institute of Mathematic of the National Academy of Sciences of Ukraine, Kiev, Ukraine, 1997, Vol. 2, pp. 262–269.

[29] Streater, R. R. and Wightman, A. S. (1964) “PCT, Spin and Statistics and all that”, Benjamin, New York.
[30] Araki, H. (1964) *Progress in Theoretical Physics* 32, 956.

[31] Guz, W. (1979) *Reports on Mathematical Physics* 16, 125.

[32] Guz, W. (1980) *Reports on Mathematical Physics* 17, 385.

[33] Pool, J. C. T., (1968a) *Communications in Mathematical Physics* 9, 118.

[34] Pool, J. C. T., (1968b) *Communications in Mathematical Physics* 9, 212.

[35] Schroer, B., “New Concepts in Particle Physics from Solution of an Old Problem” hep-th/9908021