Bose-Einstein condensation and superfluidity of magnetobiexcitons in quantum wells' and graphene superlattices

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We propose the Bose-Einstein condensation (BEC) and superfluidity of quasi-two-dimensional (2D) spatially indirect magnetobiexcitons in a slab of superlattice with alternating electron and hole layers consisting from the semiconducting quantum wells (QWs) and graphene superlattice in high magnetic field. The two different Hamiltonians of a dilute gas of magnetoexcitons with a dipole-dipole repulsion in superlattices consisting of both QWs and graphene layers in the limit of high magnetic field have been reduced to one effective Hamiltonian a dilute gas of two-dimensional excitons without magnetic field. Moreover, for \(N\) excitons we have reduced the problem of \(2N \times 2\) dimensional space onto the problem of \(N \times 2\) dimensional space by integrating over the coordinates of the relative motion of an electron (e) and a hole (h). The instability of the ground state of the system of interacting two-dimensional indirect magnetoexcitons in a slab of superlattice with alternating electron and hole layers in high magnetic field is established. The stable system of indirect quasi-two-dimensional magnetobiexcitons, consisting of pair of indirect excitons with opposite dipole moments is considered. The density of superfluid component \(n_s(T)\) and the temperature of the Kosterlitz-Thouless phase transition to the superfluid state in the system of two-dimensional indirect magnetobiexcitons, interacting as electrical quadrupoles, are obtained for both QW and graphene realizations.

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The many-particle systems of the spatially-indirect excitons in coupled quantum wells (CQWs) in the presence or absence of a magnetic field \(B\) have been the subject of recent experimental studies\(^1,2,3\). These systems are of interest, in particular, in connection with the possibility of BEC and superfluidity of indirect excitons or electron-hole pairs, which would manifest itself in the CQWs as persistent electrical currents in each well and also through coherent optical properties and Josephson phenomena\(^4,5,6,7,8\). In strong magnetic fields \((B > 7\text{T})\) 2D excitons survive in a substantially wider temperature region, as the exciton binding energies increase with magnetic field\(^9,10,11,12,13,14,15\). The problem of essential interest is also collective properties of magnetoexcitons in high magnetic fields in superlattices and layered systems\(^16\).

Recent technological advances have allowed the production of graphene, which is a 2D honeycomb lattice of carbon atoms that form the basic planar structure in graphite\(^17,18\). Graphene has been attracting a great deal of experimental and theoretical attention because of its unique properties in its bandstructure\(^17,18,19,20\). It is a gapless semiconductor with massless electrons and holes which have been described as Dirac-fermions\(^21\). Since there is no gap between the conduction and valence bands in graphene without magnetic field, the screening effects result in the absence of excitons in graphene in the absence of a magnetic field. A strong magnetic field produces a gap since Landau levels form the discrete energy spectrum. The gap reduces screening and leads to the formation of magnetoexcitons. We consider magnetoexcitons in the superlattices with alternating electronic (e) and holes (h) QWs as well as graphene layers (GLs) and suppose that recombination times may be much greater than relaxation times \(\tau\), due to small overlapping of the spatially separation of e- and h- wave functions in QWs or GLs. In this case electrons and holes are characterized by different quasiequilibrium chemical potentials. Therefore, in the system of indirect excitons in superlattices the quasiequilibrium phases appear, as, for example, in CQWs\(^12\). While coupled-well structures with spatially separated electrons and holes are typically considered to be under applied electric field, which separates electrons and holes in different QWs\(^2\), we assume there are no external fields applied to a slab of superlattice. If electron and hole QWs alternate, there are excitons with parallel dipole moments in one pair of wells, but dipole moments of excitons in other neighboring pairs of neighboring wells have opposite direction. This fact leads to essential distinction of properties of \(e - h\) system in superlattices from one for CQWs with spatially separated electrons and holes, where indirect exciton system is stable due to a dipole-dipole repulsion of all excitons. This difference manifests itself already beginning from threelayer \(e - h - e\) or \(h - e - h\) system. The superlattice of the GLs with the electrons and holes separated in alternating layers can be created by controlling the chemical potential of the charge carriers due to the doping of the GLs by the charged impurities\(^22\).

In this Letter we propose a new physical realization of magnetoexcitonic BEC and superfluidity in superlattices with alternating electronic and hole layers, that is in a sense representing an array of QWs or GLs with spatially
separated electrons and holes in high magnetic field. We reduce the problem of magnetoelectrons to the problem of excitons at $B = 0$. The instability of the ground state of the system of interacting indirect excitons in slab of superlattice with alternating $e-$ and $h-$ layers is established in a strong magnetic field. Two-dimensional indirect magnetobielectrons, consisting of the indirect magnetoelectrons with opposite dipole moments, are considered in a high magnetic field. These magnetobielectrons repel as electrical quadrupoles at long distances. As a result, the system of indirect magnetobielectrons becomes stable. Below the radius and the binding energy of indirect magnetobielectrons are calculated. By applying the ladder approximation we consider a collective spectrum of the weakly interacting by the quadrupole law two-dimensional indirect magnetobielectrons and calculate their superfluid density $n_s(T)$ in superlattices at low temperatures $T$. We analyze the dependence of the Kosterlitz-Thouless transition temperature and superfluid density on magnetic field.

The Hamiltonian of a single 2D magnetoelectron is given in Refs. [3][11] for CQWs and in Ref. [24] for GLs. A conserved quantity for an isolated electron-hole pair in magnetic field $B$ is the exciton generalized momentum $\mathbf{P}$ defined as $\mathbf{P} = -i\hbar \nabla_e - i\hbar \nabla_h + e/c(A_e - A_h) - e/c(\mathbf{B} \times (\mathbf{r}_e - \mathbf{r}_h))$, for the Dirac equation in GLs [22] as well as for the Schrödinger equation in CQWs [11][23]. Here $\mathbf{r}_e$ and $\mathbf{r}_h$ are electron and hole locations along QWs, respectively. $D$ is the distance between electron and hole QWs, $e$ is the charge of an electron, $c$ is the speed of light and $\hbar$ is a dielectric constant. We find it is convenient to work with the symmetric gauge for a vector potential $\mathbf{A}_e(h) = 1/2[\mathbf{B} \times \mathbf{r}_e(h)]$, where $\mathbf{r}_e(h)$ is the location of an electron or a hole, respectively.

The Hamiltonian of a single isolated magnetoelectron without any random field ($V_e(\mathbf{r}_e) = V_h(\mathbf{r}_h) = 0$) is commutated with $\mathbf{P}$, and hence they have the same eigenvalues, which have the following form (see Refs. [3][25]):

$$\Psi_{k\mathbf{P}}(\mathbf{R}, \mathbf{r}) = e^{i\mathbf{R}(\mathbf{P} + \hbar^2 \mathbf{B} \times \mathbf{R}) + \gamma \mathbf{r} \cdot \mathbf{P}} \Phi_{k}(\mathbf{P}, \mathbf{r}),$$

where $\gamma = (m_h - m_e)/(m_h + m_e)$ ($m_e$ and $m_h$ are the masses of an electron and hole in QWs, and $\gamma = 0$ for GLs), $\Phi_{k}(\mathbf{P}, \mathbf{r})$ is a function of internal coordinates $\mathbf{r}$ and the eigenvalue of the general magnetic momentum $\mathbf{P}$, and $k$ represents the quantum numbers of exciton internal motion. The expression for $\Phi_{k}(\mathbf{P}, \mathbf{r})$ for CQWs is given in Ref. [13], and in Ref. [24] for graphene layers. In high magnetic fields the magnetoelectron quantum numbers $k = \{n_+, n_-\}$ for an electron in Landau level $n_+$ and a hole in level $n_-.$

For large electron-hole separation $D \gg r_B$, where $r_B = \sqrt{\hbar c/(eB)}$, transitions between Landau levels due to the Coulomb electron-hole attraction can be neglected, if the following condition is valid for the magnetoelectron binding energy $E_b$: $E_b = e^2/(\epsilon D) \ll \hbar \omega_c = \hbar eB(m_e + m_h)/(2m_em_h\epsilon)$ for QWs; $E_b = 4e^2/(\epsilon D) \ll h v_F/r_B$ for GLs, where $v_F = \sqrt{3at/(2\hbar)}$ is the Fermi velocity of electrons for a lattice constant $a = 2.566\AA$ and the overlap integral between nearest carbon atoms $t \approx 2.71 eV$ [25]. This corresponds to high magnetic field $B$, large interlayer separation $D$ and large dielectric constant of the insulator layer between the GLs. At small densities $n^2 \ll 1$ the system of indirect excitons at low temperatures is two-dimensional dilute weakly nonideal Bose gas with normal to wells dipole moments $\mathbf{d}$ in the ground state ($d \sim eD$, $D$ is the interwell separation), increasing with the distance between wells $D$. In contrast to ordinary excitons, for a low-density spatially indirect magnetoelectron system the main contribution to the energy is originated from dipole-dipole interactions $U_- + U_+$ of magnetoelectrons with opposite and parallel dipoles, respectively. Two parallel (+) and opposite (−) dipoles in low-density system interact as $U_- = -U_+ = e^2 D^2/\epsilon R^3$, where $R$ is the distance between dipoles along wells planes. We suppose that $D/R \ll 1$ and $L/R \ll 1$ ($L$ is the mean distance between dipoles normal to the wells). We consider the case, when the number of QWs or GLs $k$ in superlattice is restricted $k \ll (D/\sqrt{\pi n})^{-1}$, where $n$ is exciton surface density, and this is valid for small $k$ or for sufficiently low exciton density.

Due to the orthonormality of the single magnetoelectron wave functions $\Phi_{n_+, n_-}(\mathbf{r}, \mathbf{r})$ ($\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$) the projection of the the many-particle Hamiltonian onto the lowest Landau level results in the effective Hamiltonian, which does not reflect the spinor nature of the four-component magnetoelectron wave functions in graphene. Typically, the value of $r$ is $r_B$, and $P \ll \hbar/2B$. In this approximation, the effective Hamiltonian $H_{\text{eff}}$ in the momentum representation $P$ in the subspace the lowest Landau level (for QWs $n_+ = 0$; for GLs $n_+ = n_- = 1$) has the same form (compare with Ref.[3]) as for two-dimensional boson system without a magnetic field. Only differences are that, instead of the exciton mass $M = m_e + m_h$ and ordinary momenta, we have the magnetoelectron magnetic mass, which depends on $B$ and $D$ and magnetic momenta, respectively. For the lowest Landau level we denote the spectrum of the single exciton $\varepsilon_0(P) = \varepsilon_0(\mathbf{P})$. The Hamiltonian can be represented in the form: $\hat{H}_{\text{tot}} = \hat{H}_0 + \hat{H}_{\text{int}}$, where $\hat{H}_0$ is the effective Hamiltonian of the system of noninteracting magnetoelectrons:

$$\hat{H}_0 = \sum_{\mathbf{p}} \varepsilon_0(p)(a_{\mathbf{p}}^+ a_{\mathbf{p}} + b_{\mathbf{p}}^+ b_{\mathbf{p}} + a_{-\mathbf{p}}^+ a_{-\mathbf{p}} + b_{-\mathbf{p}}^+ b_{-\mathbf{p}}).$$

In Eq. (2) $\varepsilon_0(p) = p^2/(2m_B)$ is the spectrum of isolated two-dimensional indirect magnetoelectron; $\mathbf{p}$ represents the excitonic magnetic momentum and $a_{\mathbf{p}}^+, a_{\mathbf{p}}, b_{\mathbf{p}}^+, b_{\mathbf{p}}$ are creation and annihilation operators of magnetoelectrons with up and down dipoles. For an isolated magnetoelectron on the lowest Landau level at the small magnetic momenta under consideration, $\varepsilon_0(\mathbf{P}) \approx P^2/(2m_B)$, where $m_B$ is the effective magnetic mass of a magnetoelectron in the lowest Landau level and is a function of the
distance $D$ between $e$ - and $h$ - layers and magnetic field $B$. In a strong magnetic field at $D \gg r_B$ the exciton magnetic mass is $m_B(D) = eD^3/(\epsilon^2 r_B^4)$ for QWs and $m_p(D) = eD^3/(4\epsilon^2 r_B^4)$ for GLs. The effective Hamiltonian of the interaction between magnetoexcitons $\hat{H}_\text{int}$ is:

$$\hat{H}_\text{int} = \frac{\nu}{2S} \sum_{p_1+p_2=p_3+p_4}(a_{p_1}^+ a_{p_2}^+ b_{p_3} b_{p_4} + b_{p_1}^+ b_{p_2}^+ a_{p_3} a_{p_4} - a_{p_1}^+ b_{p_2}^+ b_{p_3} a_{p_4} - b_{p_1}^+ a_{p_2}^+ a_{p_3} b_{p_4}),$$

where $S$ is the surface of the system. Let us consider the temperature $T = 0$. We apply Bogolubov approximation and assume $(N - N_0)/N_0 \ll 1$, where $N$ and $N_0$ are the total number of particles and the number of particles in condensate, respectively. In Bogolubov approximation the interaction between non-condensate particles is neglected, and only the interactions between condensate particles and excited particles with condensate particles are considered. Therefore, in the total Hamiltonian the terms, arising from first and second terms of the Hamiltonian (3), which describe the repulsion of the indirect magnetoexcitons with parallel dipole moments, are compensating by other terms of this Hamiltonian, describing the attraction of indirect magnetoexcitons with opposite dipoles. As the result only terms describing the attraction survive. Let us diagonalize the total Hamiltonian by using of the Bogolubov type unitary transformation

$$a_p = \frac{1}{\sqrt{1 - A_p^2}} b_p (\alpha_p + A_p \alpha_p^+ + B_p \beta_p^+),$$

$$b_p = \frac{1}{\sqrt{1 - B_p^2}} c_p (\beta_p + A_p \beta_p^+ + B_p \alpha_p^+),$$

where the coefficients $A_p$ and $B_p$ are found from the condition of vanishing of coefficients at nondiagonal terms in the Hamiltonian. As the result we obtain $\hat{H}_\text{tot} = \sum_{p \neq 0} \varepsilon(p) (\alpha_p^+ \alpha_p + \beta_p^+ \beta_p)$ with the spectrum of quasiparticles $\varepsilon(p) = |\varepsilon_0(p)|^2 - (nU)^2|^{1/2}$. At small momenta $p < \sqrt{2m_B nU}$ the spectrum of excitations becomes imaginary. Hence, the system of weakly interacting indirect magnetoexcitons in a slab of superlattice is unstable. The instability of magnetoexcitons becomes stronger as magnetic field higher, because $m_B$ increases with the increment of magnetic field, and, therefore, the region of $p$ resulting in the imaginary collective spectrum increases as $B$ increases.

This, on first view, strange result can be illustrated by the following example. There are equal number of dipoles oriented up and down. Let us consider four dipoles, two of them being oriented up and two — down. It is easy to count that number of repelling pairs is smaller than that of attracting ones. The prevailing of attraction leads to instability.

We assume the energy degeneracy respect to two possible spin projections in QWs and graphene and two graphene valleys (two pseudospins). Since electrons on a graphene lattice can be in two valleys, there are four types of excitons in bilayer graphene. Due to the fact that all these types of excitons have identical envelope wave functions and energies, we consider below only excitons in one valley. Also, we use $n_0 = n/(4s)$ as the density of excitons in graphene superlattice, with $n$ denoting the total density of excitons, and $s$ the spin degeneracy, which equals to 4 for magnetoexcitons in bilayer graphene. Besides, we use $n_0 = n/s$ as the density of excitons in QWs, with the spin degeneracy $s$, which equals to 4 for magnetoexcitons in QWs.

Let us consider as the ground state of the system the low-density weakly nonideal gas of two-dimensional indirect magnetoexcitons, created by indirect magnetoexcitons with opposite dipoles in neighboring pairs of wells (Fig. 1). The small parameter for the adiabatic approximation is the numerical small parameter which is equal to the ratio of magnetoexciton and magnetoexciton energies or the ratio between radii of magnetoexciton and magnetoexciton along QWs or GLs. These parameters are small, and they are even smaller than analogous parameters for atoms and molecules. The smallness of these parameters will be verified below by the results of the calculation of indirect magnetoexciton. Here it was assumed, that the distance between wells (GLs) $D$ is greater than the radius of indirect magnetoexciton. Here it was assumed, that the distance between wells (GLs) $D$ is greater than the radius of indirect magnetoexciton. Hence, the system of weakly interacting indirect magnetoexcitons with opposite dipoles $U(r)$ has the form

$$U(r) = \frac{e^2}{cr} - \frac{2\epsilon^2}{\epsilon \sqrt{r^2 + D^2}} + \frac{e^2}{\epsilon \sqrt{r^2 + 4D^2}}.$$  

At $r > 1.11D$ indirect magnetoexcitons attract, and at $r < 1.11D$ they repel. The minimum of potential energy $U(r)$ locates at $r = r_0 = 1.67D$ between indirect excitons. So at large $D$ magnetoexciton levels correspond to the two-dimensional harmonic oscillator with the frequency $\omega = 0.88\epsilon^2/(m_B D^3): E_n = -0.04\epsilon^2/(\epsilon D) + 2\sqrt{2}E_0 (r^*/D)^{3/2} (n + 1)$ where $E_0 = m_B \epsilon^4/(\epsilon^2 D^3)$, $r^* = \hbar^2/(2m_B \epsilon^2)$, $e^2 = 0.88\epsilon^2$. In the ground state the characteristic spread of magnetoexciton $a_0$ along QWs/GLs (near the mean radius of magnetoexciton $r_0$ along wells/GLs) is: $a_0 = \sqrt{2h/(n_0 m_B \omega)} |^{1/2} = (8\epsilon^4)^{1/4} D^{3/4} = 1.03a_{ex}$, where $a_{ex} = (8\epsilon^4)^{1/4} D^{3/4}$, $r^{ex} = \hbar^2/(2m_B \epsilon^2)$ is the two-dimensional effective Bohr radius with the effective magnetic mass $m_B$. Hence, the ratio of the binding energies of the magnetoexciton and magnetoexcit-
FIG. 2: Dependence of temperature of the Kosterlitz-Thouless transition $T_c = T_c(B)$ for the superlattice consisting of QWs for GaAs/AlGaAs: $\epsilon = 13$; and for GLs separated by the layer of SiO$_2$ with $\epsilon = 4.5$ on the magnetoexciton density $n$ at $D = 10\,\text{nm}$ at different magnetic fields. The solid, dashed and thin solid curves for QWs, dotted, dashed-dotted and thin dotted curves for GL at $B = 20T, B = 15T$ and $B = 10T$ respectively.

Thus, it is shown that the low-density system of indirect magnetoexcitons in a slab of superlattice of second type or consisting of QWs or GLs in high magnetic field occur to be *instable* due to the attraction of magnetoexcitons with opposite dipoles at large distances. Note that in spite of both QW and graphene realizations represented by completely different Hamiltonians, the effective Hamiltonian in a strong magnetic field was obtained to be the same. Moreover, for $N$ excitons we have reduced the number of the degrees of freedom from $2N \times 2$ to $N \times 2$ by integrating over the coordinates of the relative motion of $e$ and $h$. The instability of the ground state of the system of interacting two-dimensional indirect magnetoexcitons is considered. The low-density system of indirect quasi-two-dimensional magnetoexcitons, consisting from indirect excitons with opposite directed dipole moments. The stable system of indirect magnetoexcitons in superlattices is *stable* due to the quadrupole-quadrupole repulsion.

In a 2D system, superfluidity of magnetoexcitons appears below the Kosterlitz-Thouless transition temperature $T_c = \pi n_S(T)/(2m_B^{1/2})$ ($n_S(T)$ is the density of the superfluid component), where only coupled vortices are present. The dependence of $T_c$ on the density of magnetoexcitons at different magnetic field $B$ for superlattice consisting of QWs and GLs was calculated in the ladder approximation for the dilute magnetoexciton gas, and the results are represented on Fig.

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