Mechanical Bistability in Kerr-modified Cavity Magnomechanics

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Bistable mechanical vibration is observed in a cavity magnomechanical system, which consists of a microwave cavity mode, a magnon mode, and a mechanical vibration mode of a ferrimagnetic yttrium-iron-garnet sphere. The bistability manifests itself in both the mechanical frequency and linewidth under a strong microwave drive field, which simultaneously activates three different kinds of nonlinearities, namely, magnetostriction, magnon self-Kerr, and magnon-phonon cross-Kerr nonlinearities. The magnon-phonon cross-Kerr nonlinearity is first predicted and measured in magnomechanics. The system enters a regime where Kerr-type nonlinearities strongly modify the conventional cavity magnomechanics that possesses only a radiation-pressure-like magnomechanical coupling. Three different kinds of nonlinearities are identified and distinguished in the experiment. Our Letter demonstrates a new mechanism for achieving mechanical bistability by combining magnetostriction and Kerr-type nonlinearities, and indicates that such Kerr-modified cavity magnomechanics provides a unique platform for studying many distinct nonlinearities in a single experiment.

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Introduction.—Bistability, or multistability, discontinuous jumps, and hysteresis are characteristic features of nonlinear systems. Bistability is a widespread phenomenon that exists in a variety of physical systems, e.g., optics [1–3], electronic tunneling structures [4], magnetic nanorings [5], thermal radiation [6], a driven-dissipative superfluid [7], and cavity magnonics [8]. Its presence requires nonlinearity in the system. To date, bistability has been studied in various mechanical systems, including nano- or micro-mechanical resonators [9–11], piezoelectric beams [12], mechanical morphing structures [13], and levitated nanoparticles [14]. Bistable mechanical motion finds many important applications: it is the basis for mechanical switches [15,16], memory elements [17,18], logic gates [19], vibration energy harvesters [12,20], signal amplifiers [11,14], etc.

Different mechanisms can bring about nonlinearity in the system leading to bistable mechanical motion. Most commonly, a strong drive can induce bistability of a mechanical oscillator, of which the dynamics is described by the Duffing equation [11,21–23]. Mechanical bistability can also be caused by the Casimir force [9], nanomechanical effects on Coulomb blockade [10], magnetic repulsion [24], intrinsic nonlinearity in the optomechanical coupling [25], etc.

Here, we introduce a mechanism to induce mechanical bistability, distinguished from all the above mechanisms, by exploiting rich nonlinearities in the ferrimagnetic yttrium-iron-garnet (YIG) in cavity magnomechanics (CMM). In the CMM [26–29], magnons are the quanta of collective spin excitations in magnetically ordered materials, such as YIG. They can strongly couple to microwave cavity photons by the magnetic-dipole interaction, leading to cavity polaritons [30–35]. They can also couple to deformation vibration phonons of the ferrimagnet via the magnetostrictive force [26,28,36]. Such a radiation-pressure-like magnomechanical coupling provides necessary nonlinearity, enabling a number of theoretical proposals, including the preparation of entangled states [37–41], squeezed states [42–44], mechanical quantum-ground states [45–47], slow light [48,49], thermometry [50], quantum memory [51,52], exceptional points [53], and parity-time-related phenomena [54–57], etc. In contrast, the experimental studies on this system are, by now, very limited: Magnomechanically induced transparency and absorption [26] and mechanical cooling and lasing [28] have been demonstrated.

In this Letter, we report an experimental observation of bistable mechanical vibration of a YIG sphere in the CMM. We show that both the frequency and linewidth of the mechanical mode exhibit a bistable feature as a result of the combined effects of the radiation-pressure-like magnetostrictive interaction [26,28], the magnon self-Kerr [58,59], and the magnon-phonon cross-Kerr nonlinearities. Three different kinds of nonlinearities are simultaneously activated by applying a strong drive field on the YIG sphere. Their respective contributions to the mechanical frequency and linewidth are discussed.

Kerr-modified CMM.—The CMM system consists of a microwave cavity mode, a magnon mode, and a mechanical vibration mode; see Fig. 1. In the experiment, we use the...
where $a$, $m$, and $b$ ($a^\dagger$, $m^\dagger$, and $b^\dagger$) are the annihilation (creation) operators of the cavity mode, the magnon mode, and the mechanical mode, respectively. The last term is the driving Hamiltonian, where $\kappa_1$ is the cavity decay rate associated with the driving port (Port 1), and $\epsilon_d = P_d/(\hbar \omega_d)$, with $P_d (\omega_d)$ being the power (frequency) of the microwave drive field. The novel part of the Hamiltonian, with respect to the conventional CMM, is the Kerr nonlinear term $H_{\text{Kerr}}$ activated by the strong pump field [60],

$$
H_{\text{Kerr}}/\hbar = K_m m^\dagger m m + K_{\text{cross}} m^\dagger m b^\dagger b, \tag{2}
$$

where $K_m$ is the magnon self-Kerr coefficient, and $K_{\text{cross}}$ is the magnon-phonon cross-Kerr coefficient. The magnon self-Kerr effect is caused by the magnetocrystalline anisotropy [58,59], and the cross-Kerr nonlinearity originates from the magnetoelastic coupling by including the second-order terms in the strain tensor [61,62],

$$
e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial l_j} + \frac{\partial u_j}{\partial l_i} + \sum_k \frac{\partial u_k}{\partial l_i} \frac{\partial u_k}{\partial l_j} \right), \tag{3}
$$

where $u_i$ are the components of the displacement vector, and $l_i = i (i = x, y, z)$. The first-order terms lead to the conventional radiation-pressure-like interaction Hamiltonian [63], $\hbar g_{\text{hub}} m^\dagger m(b + b^\dagger)$. Under a moderate drive field, the second-order terms are negligible [26,28], but can no longer be neglected when the drive becomes sufficiently strong, as in our experiment, yielding an appreciable magnon-phonon cross-Kerr nonlinearity. As will be seen later, the cross-Kerr nonlinearity is indispensable in the model for fitting the mechanical frequency shift.

Both the magnon self-Kerr and magnon-phonon cross-Kerr terms, as well as the radiation-pressure-like term, cause a magnon frequency shift $\delta \omega_{m} = 2K_m |M|^2 + K_{\text{cross}} |B|^2 + 2g_{\text{hub}} \text{Re}[B]$, where $O = \langle \phi \rangle$ ($O = m, b, a$) denote the average of the modes. In our experiment, the dominant contribution is from the self-Kerr nonlinearity [60], which gives a bistable magnon frequency shift [8]. Also, the cross-Kerr nonlinearity causes a mechanical frequency shift $\delta \omega_b = K_{\text{cross}} |M|^2$. Using the Heisenberg-Langevin approach, we obtain the equation for the steady-state average $M$ [60],

$$
\eta_a \kappa_1 g_{\text{hub}} \epsilon_{d}^2 = |M|^2 \left[ (\Delta_m - \eta_a g_{\text{hub}} \Delta_d) + 2K_m |M|^2 \right]^2 + \left( \frac{K_m}{2} + \eta_a g_{\text{hub}} \kappa_1 \right)^2, \tag{4}
$$

where $\Delta_a^{(m)} = \omega_{a^{(m)}} - \omega_d$, $\eta_a = 1/|\Delta_a^{(m)} + (\kappa_a/2)|$. In deriving Eq. (4), we neglect contributions from the mechanical mode to the magnon frequency shift, because of a much smaller mechanical excitation number compared to the magnon excitations.
to the magnon excitation number for the drive powers used in this Letter. It is a cubic equation of the magnon excitation number $|M|^2$. In a suitable range of the drive power, there are two stable solutions, leading to the bistable magnon and phonon frequency shifts by varying the drive power.

The radiation-pressure-like coupling gives rise to an effective susceptibility of the mechanical mode [60],

$$\chi_{b,\text{eff}}(\omega) = \{\chi_b^{-1}(\omega) - 2i|G_{mb}|^2 [\chi_{ma}(\omega) - \chi_{ma}^{*}(-\omega)]\}^{-1},$$

where $\chi_b(\omega)$ is the natural susceptibility of the mechanical mode, but depends on the modified mechanical frequency $\tilde{\omega}_b = \omega_b + K_{\text{cross}} |M|^2$, which includes the cross-Kerr induced frequency shift. The effective coupling $G_{mb} = g_{mb} M$, and $\chi_{ma}(\omega) = [\chi_{ma}^{-1}(\omega) + g_{ma}^2 \chi_{ma}^{*}(\omega)]^{-1}$, where $\chi_{ma}(\omega)$ and $\chi_{ma}^{*}(\omega)$ are the natural susceptibilities of the magnon and cavity modes, with the magnon detuning in $\chi_{ma}^{*}(\omega)$ modified as $\tilde{\omega}_m = \Delta_m + 2K_m |M|^2$, which includes the dominant magnon self-Kerr induced frequency shift. See Ref. [60] for the explicit expressions of the susceptibilities.

The effective mechanical susceptibility yields a frequency shift of the phonon mode (the so-called “magnon-mechanical” effect [28], in analogy to the “optical spring” in optomechanics [69]),

$$\delta \omega_b = -\text{Re} \{2i|G_{mb}|^2 [\chi_{ma}(\omega) - \chi_{ma}^{*}(-\omega)]\} + K_{\text{cross}} |M|^2,$$

where we write together the frequency shift induced by the cross-Kerr nonlinearity. Moreover, it leads to the following mechanical linewidth change:

$$\delta \Gamma_b = \text{Im} \{2i|G_{mb}|^2 [\chi_{ma}(\omega) - \chi_{ma}^{*}(-\omega)]\}.$$

Clearly, this linewidth change is only caused by the radiation-pressure-like coupling, distinguished from the frequency shift caused by the self-Kerr or cross-Kerr nonlinearity. By applying a red- or blue-detuned drive field, we can choose to operate the system in two different regimes, where either the magnomechanical anti-Stokes or Stokes scattering is dominant. This yields an increased ($\delta \Gamma_b > 0$) or a reduced ($\delta \Gamma_b < 0$) mechanical linewidth, corresponding to the cooling or amplification of the mechanical motion [28,69].

In our system, due to the strong coupling $g_{mb} > K_m$, the magnon and cavity modes form two cavity polariton (hybridized) modes (Fig. 2, left panels) [30–35]. Here, a red (blue)-detuned drive means that the drive frequency is lower (higher) than the frequency of the cavitylike polariton mode, i.e., the “deeper” polariton in the spectra close to the cavity resonance. For the red (blue)-detuned drive, we show the anti-Stokes (Stokes) sidebands associated with two mechanical modes for two drive powers in the enlarged plots of Fig. 2(a) [Fig. 2(b)]. When the detuning between the drive field and the deeper polariton matches the mechanical frequencies, the anti-Stokes (Stokes) sidebands are manifested as the magnomechanically induced transparency (absorption) [26].

Red-detuned drive.—To implement a red-detuned drive, we drive the cavity with a microwave field at frequency $\omega_d/2\pi = 7.645$ GHz. By increasing the drive power, the magnon frequency shift is negative; $\omega_{mb}^{\text{eff}}/2\pi = 7.658$ GHz (light green dashed line) at a lower power $P_d = 4.7$ dBm (upper panel) and $\omega_{mb}^{\text{eff}}/2\pi = 7.640$ GHz (dark green dashed line) at $P_d = 29.7$ dBm (lower panel). The green arrow indicates the direction in which the magnon frequency shifts by increasing the power. Right panel: enlargement of the red shaded areas in the left panel shows detailed spectra of the magnomechanically induced resonances, where $\Delta_{pd} = \omega_p - \omega_d$. The black lines are the fitting curves. (b) Left panel: measured reflection spectra under a blue-detuned drive. The drive frequency $\omega_d/2\pi = 7.660$ GHz. By adjusting the bias magnetic field, the magnon frequency is tuned close to the drive frequency $\omega_{mb}^{\text{eff}}/2\pi \approx \omega_d$ at the power $P_d = 4.7$ dBm. Increasing the power to 23.7 dBm, the magnon frequency $\omega_{mb}^{\text{eff}}/2\pi = 7.645$ GHz. Right panel: enlargement of the blue shaded areas in the left panel shows detailed spectra of the magnomechanically induced resonances. We observe two adjacent mechanical modes with the frequencies $\omega_{mb}^{\text{eff}}/2\pi = 11.0308$ MHz and $\omega_{mb}^{\text{eff}}/2\pi = 11.0377$ MHz. Because of their similar behaviors, we focus on the lower-frequency mode in the text.
magnonic spring effect yields a negative frequency shift \( \delta \omega_b = \Re[\chi_{b,\text{eff}}^{-1}(\omega) - \chi_{b}^{-1}(\omega)] < 0 \) [Fig. 3(a), the red line], and the maximum frequency shift is \(-200 \text{ Hz}\). Adding up these two frequency shifts gives the total mechanical frequency shift [Fig. 3(a), the black line], which fits well with the experimental data [Fig. 3(a), triangles] when the power is not too strong.

Another interesting finding is the bistable feature of the mechanical linewidth [Fig. 3(b)]. The magnomechanical backaction leads to the variation of the mechanical linewidth \( \delta \Gamma_b = -\Im[\chi_{b,\text{eff}}^{-1}(\omega) - \chi_{b}^{-1}(\omega)] \). For a red-detuned drive, the anti-Stokes process is dominant, resulting in an increased mechanical linewidth \( \delta \Gamma_b > 0 \) and the cooling of the motion. The bistable mechanical linewidth is also induced by the bistable \( |M|^2 \) [see Eq. (7)], similar to the mechanical frequency. The theory fits well with the experimental results, and the discrepancy appears only in the high-power regime. This is because the considerable heating effect at strong pump powers can broaden the mechanical linewidth \([70,71]\), which is not included in our model.

**Blue-detuned drive.**—When a blue-detuned drive is applied, the system enters a regime where the Stokes scattering is dominant. The magnomechanical parametric down-conversion amplifies the mechanical motion with the characteristic of a reduced linewidth. Furthermore, the mechanical frequency shift induced by the spring effect will move in the opposite direction compared with the red-detuned drive.

To implement a blue-detuned drive, we drive the cavity with a microwave field at frequency \( \omega_d/2\pi = 7.66 \text{ GHz} \), and tune the magnon frequency close to the drive frequency [see Fig. 2(b)]. We attempted to make the Stokes sideband and the magnon resonate. Therefore, we only achieve this at lower drive powers, giving a faint magnomechanically induced absorption [Fig. 2(b), upper panels].

From the red to the blue detuning, we only adjust the magnon and the drive frequencies. Because the direction of the crystal axis is unchanged, the magnon self-Kerr coefficient \( K_m \) is still negative, so again a negative frequency shift occurs by increasing the power [green arrow in Fig. 2(b)]. For the power up to 23.7 dBm, which yields \( G_{mb}/2\pi = 42.7 \text{ kHz} \), the frequency of the cavitylike polariton is always lower than the drive frequency, so the system is operated under a blue-detuned drive.
plays a dominant role [Fig. 4(a), the green line], as in the case of the red-detuned drive, yielding a frequency shift of $-6.5 \text{ kHz}$ at the power of $23.7 \text{ dBm}$. Differently, the spring effect induced frequency shift ($370 \text{ Hz}$ at $23.7 \text{ dBm}$) is positive [Fig. 4(a), the red line]. The opposite frequency shifts by the spring effect in the blue- and red-detuned drives agree with the finding of Ref. [28], but no bistability was observed in their work.

The reduced mechanical linewidth $\delta \Gamma_b < 0$ under a blue-detuned drive is confirmed by Fig. 4(b). However, unlike the bistable curve in the red-detuned drive case [Fig. 3(b)], $\delta \Gamma_b$ manifests the bistability in an $\alpha$-shaped curve by sweeping the drive power. This is the result of the trade-off between the growing coupling strength $G_{\text{mb}}$ (which enhances the Stokes scattering rate, yielding an increasing $|\delta \Gamma_b|$) and the larger detuning between the Stokes sideband and the “deeper” polariton [Fig. 2(b)] (which reduces the Stokes scattering rate, resulting in a decreasing $|\delta \Gamma_b|$) by raising the drive power. These two effects are balanced when the power is in the range of 12 to 15 dBm.

Conclusions.—We have observed bistable mechanical frequency and linewidth and the magnon-phonon cross-Kerr nonlinearity in the CMM system. The mechanical bistability results from the magnomechanical backaction on the mechanical mode and the strong modifications on the backaction due to the magnon self-Kerr and magnon-phonon cross-Kerr nonlinearities. The effects of the magnon self-Kerr, the magnon-phonon cross-Kerr, and the radiation-pressure-like interactions can be identified by measuring primarily the magnon frequency shift, the mechanical frequency shift, and the mechanical linewidth, respectively. The new mechanism for achieving bistable mechanical motion revealed by this Letter promises a wide range of applications, such as in mechanical switches, memories, logic gates, and signal amplifiers.

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[14] F. Ricci, R. A. Rica, M. Spasenović, J. Gieseler, L. Rondin, L. Novotny, and R. Quindt, Optically levitated nanoparticle as a model system for stochastic bistable dynamics, Nat. Commun. 8, 15141 (2017).

[15] Q. P. Unterreithmeier, T. Faust, and J. P. Kotthaus, Nonlinear switching dynamics in a nanomechanical resonator, Phys. Rev. B 81, 241405(R) (2010).

[16] R. J. Dolleman, P. Belardinelli, S. Houri, H. S. J. van der Zant, F. Alijani, and P. G. Steeneken, High-frequency stochastic switching of graphene resonators near room temperature, Nano Lett. 19, 1282 (2019).

[17] R. L. Badzey, G. Zolfagharkhani, A. Gaidarzhy, and P. Mohanty, A controllable nanomechanical memory element, Appl. Phys. Lett. 85, 3587 (2004).

[18] D. Roodenburg, J. W. Spronck, H. S. J. van der Zant, and D. N. Guerra, A. Bulsara, W. Ditto, S. Sinha, K. Murali, and R. L. Badzey, G. Zolfagharkhani, A. Gaidarzhy, and P. Mohanty, A noise-assisted reprogrammable nanomechanical logic gate, Nano Lett. 10, 1168 (2010).

[19] R. L. Harne and K. W. Wang, A review of the recent research on vibration energy harvesting via bistable systems, Smart Mater. Struct. 22, 023001 (2013).

[20] J. S. Aldridge and A. N. Cleland, Noise-Enabled Precision Measurements of a Duffing Nanomechanical Resonator, Phys. Rev. Lett. 94, 183501 (2005).

[21] R. Almog, S. Zaitsev, O. Shtempluck, and E. Buks, Noise Squeezing in a Nanomechanical Duffing Resonator, Phys. Rev. Lett. 98, 078103 (2007).

[22] I. Katz, A. Retzker, R. Straub, and R. Lifshitz, Signatures for a Classical to Quantum Transition of a Driven Nonlinear Nanomechanical Resonator, Phys. Rev. Lett. 99, 040404 (2007).

[23] M. López-Suárez and I. Neri, Micro-electromechanical memory bit based on magnetic repulsion, Appl. Phys. Lett. 109, 133505 (2016).

[24] H. Seok, L. F. Buchmann, E. M. Wright, and P. Meystre, Multimode strong-coupling quantum opticsmechanics, Phys. Rev. A 88, 063850 (2013).

[25] X. Zhang, C.-L. Zou, L. Jiang, and H. X. Tang, Cavity magnammechancics, Sci. Adv. 2, e1501286 (2016).

[26] D. Lachance-Quirion, Y. Tabuchi, A. Gloppe, K. Usami, and Y. Nakamura, Hybrid quantum systems based on magnons, Appl. Phys. Express 12, 070101 (2019).

[27] C. A. Potts, E. Varga, V. A. S. V. Bittencourt, S. V. Kusmienskiy, and J. P. Davis, Dynamical Backaction Magnomechanics, Phys. Rev. X 11, 031053 (2021).

[28] H. Y. Yuan, Y. Cao, A. Kamra, R. A. Duine, and P. Yan, Quantum magnons: When magnon spintronics meets quantum information science, Phys. Rep. 965, 1 (2022).

[29] H. Huebl, C. W. Zollitsch, J. Lotze, F. Hocke, M. Greifenstein, A. Marx, R. Gross, and S. T. B. Goennenwein, High Cooperativity in Coupled Microwave Resonator Ferrimagnetic Insulator Hybrids, Phys. Rev. Lett. 111, 127003 (2013).

[30] Y. Tabuchi, S. Ishino, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Hybridizing Ferromagnetic Magnons and Microwave Photons in the Quantum Limit, Phys. Rev. Lett. 113, 083603 (2014).

[31] X. Zhang, C. L. Zou, L. Jiang, and H. X. Tang, Strongly Coupled Magnons and Cavity Microwave Photons, Phys. Rev. Lett. 113, 156401 (2014).

[32] M. Goryachev, W. G. Farr, D. L. Creedon, Y. Fan, M. Kostylev, and M. E. Tobar, High-Cooperativity Cavity QED with Magnons at Microwave Frequencies, Phys. Rev. Applied 2, 054002 (2014).

[33] L. Bai, M. Harder, Y. P. Chen, F. X. Fan, J. Q. Xiao, and C. M. Hu, Spin Pumping in Electrodynamically Coupled Magnon-Photon Systems, Phys. Rev. Lett. 114, 227201 (2015).

[34] D. Zhang, X.-M. Wang, T.-F. Li, X.-Q. Luo, W. Wu, H. Nori, and J. Q. You, Cavity quantum electrodynamics with ferromagnetic magnons in a small yttrium-iron-garnet sphere, npj Quantum Inf. 1, 15014 (2015).

[35] C. Kittel, Interaction of spin waves and ultrasonic waves in ferromagnetic crystals, Phys. Rev. 110, 836 (1958).

[36] J. Li, S.-Y. Zhu, and G. S. Agarwal, Magnon-Photon Entanglement in Cavity Magnomechanics, Phys. Rev. Lett. 121, 203601 (2018).

[37] J. Li and S.-Y. Zhu, Entangling two magnon modes via magnetostrictive interaction, New J. Phys. 21, 085001 (2019).

[38] H. Tan, Genuine photon-magnon-phonon Einstein-Podolsky-Rosen steerable nonlocality in a continuously-monitored cavity magnomechanical system, Phys. Rev. Research 1, 033161 (2019).

[39] Y. Li, Z.-O. Wang, C.-H. Bai, T. Wang, S. Zhang, and H.-F. Wang, Generation and transfer of squeezed states in a cavity magnomechanical system by two-tone microwave fields, Opt. Express 29, 11773 (2021).

[40] J. Li, Y.-P. Wang, J. Q. You, and S.-Y. Zhu, Squeezing microwaves by magnetostriction, arXiv:2101.02796.

[41] M.-S. Ding, L. Zheng, and C. Li, Ground-state cooling of a magnomechanical resonator induced by magnetic damping, J. Opt. Soc. Am. B 37, 627 (2020).

[42] Z.-X. Yang, L. Wang, Y.-M. Liu, D.-Y. Wang, C.-H. Bai, S. Zhang, and H.-F. Wang, Ground state cooling of magnomechanical resonator in $PT$-symmetric cavity magnomechanical system at room temperature, Front. Phys. 15, 52504 (2020).

[43] M. Asjad, J. Li, S.-Y. Zhu, and J. Q. You, Magnon squeezing enhanced ground-state cooling in cavity magnomechanics, arXiv:2203.10767.

[44] C. Kong, B. Wang, Z.-X. Liu, H. Xiong, and Y. Wu, Magnetically controllable slow light based on magnetostrictive forces, Opt. Exp. 27, 5544 (2019).

[45] K. Ullah, M. Tahir Naseem, and Ö. Müstecaplıoğlu, Tunable multiwindow magnomechanically induced transparency, Fano resonances, and slow-to-fast light conversion, Phys. Rev. A 102, 033721 (2020).
See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.129.123601 for additional proofs, which includes Refs. [26,28,61–68].

[50] C. A. Potts, V. A. S. V. Bittencourt, S. V. Kusminskiy, and J. P. Davis, Magnon-Phonon Quantum Correlation Thermometry, Phys. Rev. Applied 13, 064001 (2020).

[51] S.-F. Qi and J. Jing, Magnon-assisted photon-phonon conversion in the presence of structured environments, Phys. Rev. A 103, 043704 (2021).

[52] B. Sarma, T. Busch, and J. Twamley, Cavity magnomechanical storage and retrieval of quantum states, New J. Phys. 23, 043041 (2021).

[53] T.-X. Lu, H. Zhang, Q. Zhang, and H. Jing, Exceptional-point-engineered cavity magnomechanics, Phys. Rev. A 103, 063708 (2021).

[54] S.-N. Huai, Y.-L. Liu, J. Zhang, L. Yang, and Y.-X. Liu, Enhanced sideband responses in a $\mathcal{PT}$-symmetric-like cavity magnomechanical system, Phys. Rev. A 99, 043803 (2019).

[55] M. Wang, D. Zhang, X.-H. Li, Y.-Y. Wu, and Z.-Y. Sun, Magnon chaos in $\mathcal{PT}$-symmetric cavity magnomechanics, IEEE Photonics J. 11, 5300108 (2019).

[56] Y.-T. Chen, L. Du, Y. Zhang, and J.-H. Wu, Perfect transfer of enhanced entanglement and asymmetric steering in a cavity-magnomechanical system, Phys. Rev. A 103, 053712 (2021).

[57] M.-S. Ding, X.-X. Xin, S.-Y. Qin, and C. Li, Enhanced entanglement and steering in $\mathcal{PT}$-symmetric cavity magnomechanics, Opt. Commun. 490, 126903 (2021).

[58] Y.-P. Wang, G. Q. Zhang, D. Zhang, X. Q. Luo, W. Xiong, S. P. Wang, T. F. Li, C. M. Hu, and J. Q. You, Magnon Kerr effect in a strongly coupled cavity-magnon system, Phys. Rev. B 94, 224410 (2016).

[59] R.-C. Shen, Y.-P. Wang, J. Li, S.-Y. Zhu, G. S. Agarwal, and J. Q. You, Long-Time Memory and Ternary Logic Gate Using a Multistable Cavity Magnonic System, Phys. Rev. Lett. 127, 183202 (2021).