Approximate Flavor Symmetries \textsuperscript{1,2}

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Abstract

We discuss the idea of approximate flavor symmetries. Relations between approximate flavor symmetries and natural flavor conservation and democracy models is explored. Implications for neutrino physics are also discussed.

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1 Introduction

In the Standard Model, Yukawa couplings $\lambda$ are defined as couplings between fermions and Higgs scalars:

$$\mathcal{L}_Y = \left( \lambda_{ij}^{Ua} \bar{Q}_i U_j \frac{\tilde{H}^a}{\sqrt{2}} + \lambda_{ij}^{Da} \bar{Q}_i D_j \frac{H^a}{\sqrt{2}} + \lambda_{ij}^{Ea} \bar{L}_i E_j \frac{H^a}{\sqrt{2}} + \text{h.c.} \right),$$

where $Q_i$ and $L_i$ are $SU(2)$ doublet quarks and leptons while $U_i$, $D_i$ and $E_i$ are $SU(2)$ singlets and $i = 1, 2, 3$ is a generation label. $H_a$ are Higgs $SU(2)$ doublets, and $a = 1, \ldots, n$, where $n$ is the number of Higgs doublets.

The Standard Model can easily be extended to accommodate neutrino masses. We can introduce higher dimensional operators $\frac{1}{M} H H L_i L_j$ which give Majorana neutrino masses. We can also add $SU(2)$ singlet neutrinos which can also have Majorana masses or Dirac masses similar to (1). Neutrino mass is then in general described by a $6 \times 6$ Yukawa matrix.

As opposed to, e.g. couplings of fermions and vector bosons, Yukawa couplings in the Standard Model are not constrained nor related to each other by any symmetry principle; i.e., they enter the Lagrangian as arbitrary complex numbers. These numbers are then only fixed by experiment, namely by measuring fermion masses and mixing angles.

Several classes of models/ansätze for Yukawa couplings exist today, and we list them below:

- Approximate flavor symmetries\cite{1}-\cite{3}, in which the entries in the Yukawa matrices are entered as small parameters by which the flavor symmetries are broken.
- Fritzsch and/or GUT inspired models\cite{4}-\cite{17}, in which some entries in the Yukawa mass matrices are assumed to be zero (e.g. by discrete flavor symmetry), and others may be related by some GUT relation.
- Flavor democracy models\cite{18}-\cite{19}, in which all the entries in the Yukawa matrices are
equal (i.e., no flavor symmetry), and hierarchy comes from diagonalization and RGE running.

- String inspired models, composite models, ...

This is in no way a complete list, of course. Also, most of the work done actually falls into several categories above, showing that there are common ideas, which will hopefully lead us to a certain trail beyond the Standard Model.

In this talk, we will concentrate on approximate flavor symmetries, since they are relatively simple and model independent.

## 2 Approximate flavor symmetries

In the Standard Model, the gauge interactions of the fermions

\[ \mathcal{L}_0 = i Q \bar{Q} Q + i U \bar{U} U + i D \bar{D} D + i L \bar{L} L + i E \bar{E} E \quad (2) \]

have global flavor symmetries and these are broken by Yukawa couplings \([1]\). We can understand the Yukawa couplings as being naturally small\([20]\), because in the limit that they become zero, the theory gets a larger (flavor) symmetry. This is certainly warranted by the smallness of fermion masses (except the top), which arise from Yukawa couplings.

One of the simplest assumptions one can make is that each of the chiral fermion fields carries a flavor symmetry which is broken by a small parameter, which we will call $\epsilon$. For example, Froggatt and Nielsen\([1]\) think of $\epsilon$ as $\epsilon \approx \frac{<\Phi_1>}{<\Phi_0>}$, where $<\Phi_1>$ is the vev of the scalar that breaks the flavor symmetry, and $<\Phi_0>$ is the vev of a superheavy field, therefore making $\epsilon$ small. Thus, for example

\[ \lambda^{Q}_{ij} \approx \epsilon_Q \epsilon_{U_j} \quad (3) \]

where $\epsilon_Q$ is the breaking parameter of the flavor carried by $Q_i$, and $\epsilon_{U_j}$ is the breaking parameter of the flavor carried by $U_j$. Froggatt and Nielsen\([1]\), as well as Leurer, Nir
and Seiberg[5] use one or two \( \epsilon \)s, which enter the Yukawa matrices with different powers, thus explaining the hierarchy of masses. We rather keep different \( \epsilon \)s for different flavors, as it keeps the discussion more model independent. Notice that in (3) we assumed that the Higgs field does not carry any flavor symmetry. If it did, then the Yukawa couplings would be multiplied by another \( \epsilon \) for the broken symmetry carried by the Higgs field.

What do we know about \( \epsilon \)s? Although their approximate value can be fixed by the known masses and mixings (at least in the quark sector[3]; lepton sector is much more speculative as the neutrino masses and mixings are not known[4]; we will talk more on this later), their exact values depend on the underlying theory, which we do not know. Therefore, relation like (3) is not meant to be an exact relation but rather an order of magnitude estimate, and we are interested here mainly in general features, rather than specific predictions. It was shown that flavor changing interactions, with couplings determined by approximate flavor symmetries, can involve new scalars at the scale as low as the weak scale, and still satisfy stringent experimental limits([2],[3]). This is opposed to the common view that, for example, \( K_L - K_S \) mass difference, implies high bounds on the scale of new interactions, typically \( \sim 1000 \) TeV. However, it is precisely because of the approximate flavor symmetries that the couplings of the new scalars are small, lowering the naive bound considerably.

3 Many Higgs doublet model

In this section we discuss the case of the minimal Standard Model extended only by the addition of an arbitrary number of Higgs doublets. In this case it is already known that, for the special case of Fritzsch-like Yukawa matrices, the additional scalars need not be heavier than a TeV [8]. However, our results are independent of the particular texture and depend only on the approximate flavor symmetry.
Let us look at a two Higgs doublet model (the generalization to many Higgs doublets is trivial). For example, the up quark Yukawa couplings are

\[
\begin{pmatrix}
\bar{Q}_1 & \bar{Q}_2 & \bar{Q}_3 \\
\end{pmatrix}
\begin{pmatrix}
\sim \epsilon_{Q_1} \epsilon_{U_1} & \sim \epsilon_{Q_1} \epsilon_{U_2} & \sim \epsilon_{Q_1} \epsilon_{U_3} \\
\sim \epsilon_{Q_2} \epsilon_{U_1} & \sim \epsilon_{Q_2} \epsilon_{U_2} & \sim \epsilon_{Q_2} \epsilon_{U_3} \\
\sim \epsilon_{Q_3} \epsilon_{U_1} & \sim \epsilon_{Q_3} \epsilon_{U_2} & \sim \epsilon_{Q_3} \epsilon_{U_3} \\
\end{pmatrix}
\begin{pmatrix}
U_1 \\
U_2 \\
U_3 \\
\end{pmatrix}
\frac{\mu_1}{\sqrt{2}}
\]

and similarly for down type quark matrices, keeping in mind that each entry in the matrices is uncertain by a factor of 2 or 3 (denoted by \(\sim\)).

Notice that because of the numerical factors in front of the \(\epsilon\)s, the matrices for \(H_1\) and \(H_2\) are not equal in general. That means that if we diagonalize the matrix of \(H_1\), the matrix of \(H_2\) will not be diagonalized and will keep the same general form as above. In particular, we can always choose \(H_1\) to be the only doublet that acquires a vev (rotating the doublets will not change the above form of matrices). Therefore we see that in the quark mass eigenstate basis, the new Higgs \(H_2\) couplings are not diagonal: we have flavor changing couplings. The nice thing now is that since the flavor changing couplings are small the stringent experimental limits on flavor changing neutral currents (FCNC) actually translate only into lower limits on the mass of new scalars, about 1 TeV, as discussed above.

To avoid problems with large flavor-changing neutral currents, Glashow and Weinberg \[21\] argued that only one Higgs doublet could couple to up-type quarks and only one Higgs to down-type quarks. However, this naturality constraint, known as the Glashow-Weinberg criterion (or natural flavor conservation (NFC)), was based on an unusual definition of what is “natural.” For them the avoidance of flavor-changing neutral currents was natural in a model only if it occurred for all values of the coupling constants of that model. For us a model will be natural provided the smallness of any coupling is guaranteed by approximate symmetries \[20\], and we find that this implies the Glashow-
The Weinberg criterion is not necessary (however, see caveat below).

One potential problem arises from the smallness of the observed CP violation, as noted by Hall and Weinberg\cite{HallWeinberg}. The CP violating parameter \( \epsilon_{CP} = \frac{\text{Im}(\Delta M_K)}{\sqrt{2}|\Delta M_K|} \) would naively be expected to be of order unity in our case (we have no reason to assume \textit{a priori} that the Yukawa couplings a real), contrary to the observed value of \( 10^{-3} \). To avoid this problem one might go back to NFC, or, more in the philosophy of naturalness of small couplings, just say that CP is another approximately conserved quantity broken by \( \epsilon_{CP} \).

Here we would like to mention two different limits of the Yukawa matrices which obey approximate flavor symmetries (as in (3)), one of which gives NFC, and the other one which gives democratic matrices.

Notice that if the matrix for \( H_2 \) is nearly equal to the matrix for \( H_1 \), then diagonalization of the \( H_1 \) matrix will almost diagonalize the second matrix. In this case the flavor changing couplings become even smaller. This is of course no surprise, because if the two matrices were exactly equal then only one linear combination of Higgses \( (H_1 + H_2) \) couples to the quarks: we have NFC! This is actually the starting point of Leurer, Nir and Seiberg \cite{LeurerNirSeiberg}; i.e., use broken flavor symmetries in combination with weakly broken NFC.

Also notice that if the Yukawa matrix elements are \textit{exactly} a product of \( \epsilon \)s by which the symmetries are broken, then the matrices have one large eigenvalue and two eigenvalues equal to zero: we have flavor democracy(\cite{LeurerNirSeiberg,LeurerNirSeiberg1})! This is easily understood, since this limit means that only one linear combination of the left handed fields couples to one linear combination of right handed fields (while the other two combinations remain massless).
None of these limits follow from the idea of approximate flavor symmetries alone. Unless they are motivated by a specific model, we must stay with the general relation of type (3).

4 Lepton sector

By adding the right-handed neutrinos $N_i$, $i = 1, 2, 3$, to the particle content of the Standard Model we can allow for Dirac type masses. Under the action of approximate flavor symmetries, whenever an $N_i$ enters a Yukawa interaction, the corresponding coupling must contain the symmetry breaking parameter $\epsilon_{N_i}$.

A natural way to justify the smallness of neutrino masses is to use the see-saw mechanism[22], in which the smallness of the left-handed neutrino masses is explained by the new scale of heavy right-handed neutrinos. The mass matrices will have the structure[4]

$$m_{ND_{ij}} \approx \epsilon_{L_i} \epsilon_{N_j} v_{SM},$$

$$m_{NM_{ij}} \approx \epsilon_{N_i} \epsilon_{N_j} v_{Big},$$

$$m_{E_{ij}} \approx \epsilon_{L_i} \epsilon_{E_j} v_{SM},$$

where $m_{ND}$ and $m_E$ are the neutrino and charged lepton Dirac mass matrices, $m_{NM}$ is the right-handed neutrino Majorana mass matrix, $v_{SM} = 174$ GeV and $v_{Big}$ is the new large mass scale. The generation indices $i$ and $j$ run from 1 to 3. In the following we assume a hierarchy in the $\epsilon$s (i.e. $\epsilon_{L_1} << \epsilon_{L_2} << \epsilon_{L_3}$, etc.) as suggested by the hierarchy of quark and charged lepton masses. Then the diagonalization of the neutrino mass matrix will give a heavy sector with masses $m_{NH_i} \approx \epsilon_{N_i}^2 v_{Big}$ and a very light sector with mass matrix

$$m_{NL_{ij}} \approx (m_{ND} m_{NM}^{-1} m_{ND}^T)_{ij} \approx \epsilon_{L_i} \epsilon_{L_j} \frac{v_{SM}^2}{v_{Big}},$$

where the number $Tr(\epsilon_N \epsilon_N^{-1} \epsilon_N)$ is assumed to be of order unity. We have the expected result: the heavy right-handed neutrino decouples from the theory leaving
behind a very light left-handed neutrino. The masses and mixing angles are independent of the right-handed symmetry breaking parameters $\epsilon_{N_i}$:

$$m_i^N \approx \epsilon_{L_i}^2 \frac{v_{SM}^2}{v_{\text{Big}}},$$

$$m_i^E \approx \epsilon_{L_i} \epsilon_{E_i} v_{SM} \quad \text{(no sum on $i$)},$$

$$V_{ij} \approx \frac{\epsilon_{L_i}}{\epsilon_{L_j}} \quad (i < j). \quad (8)$$

Therefore, besides the unknown scale $v_{\text{Big}}$, only two sets of $\epsilon$s are needed: $\epsilon_{L_i}$ and $\epsilon_{E_i}$. In fact, the neutrino masses and mixings depend only on $\epsilon_{L_i}$ and they are approximately related through

$$V_{ij} \approx \sqrt{\frac{m_i^N}{m_j^N}}. \quad (9)$$

Equation (9) reduces the number of parameters needed to describe neutrino masses and mixings by three; for example, given two mixing angles and one neutrino mass, we can predict the third mixing angle and the other two neutrino masses. These results are extremely general. They follow simply from the approximate factorization of the Dirac masses, regardless of the specific form of $m_{N_M}^{-1}$, which only contributes to set the scale.

To get further relations one needs some additional information about the $\epsilon_{LS}$ and $\epsilon_{ES}$. We tried several plausible ansätze and found that in all of them the solar neutrino problem (SNP) can easily be accommodated with MSW $\nu_e - \nu_\mu$ mixing solution. However, if we now fix the mass scale from the SNP solution (requiring $m_{\nu_\mu}$ to be about $10^{-3}\text{eV}$), then all neutrino masses are too small to close the Universe. This comes about because the approximate flavor symmetries tell us that the ratios of neutrino masses are likely to be of the order of ratios of charged lepton masses (and therefore, the heaviest neutrino, $\nu_\tau$, is not likely to be heavier than $1\text{eV}$), as opposed to some proposed quadratic relations.
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