A multi-objective programming model based on the relative deviation

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Abstract. Decision makers are often faced with the achievement of multiple goals in real life. The multi-objective programming model based on priority factors is an effective mathematical model to deal with multiple objectives, but it has some defects. In this paper, the concept of the relative deviation is introduced to deal with the defects of the multi-objective programming model based on priority factors, and the multi-objective programming model based on the relative deviation is proposed. A case study of portfolio decisions is given to illustrate the model. Results show that the model in this paper has more advantages than the traditional model based on priority factors.

1. Introduction
Decisions are made to achieve specific goals based on objective possibilities, information and experiences. With the help of certain tools, techniques and methods, we can make decisions about future actions after analyzing, calculating and judging the many factors that influence the achievement of the goal. Specifically, the decision-making is the process of selecting a satisfactory alternative from a number of possible alternatives. So the decision problem can be reduced to the programming problem under certain conditions. In practice, multiple goals are often considered in decision problems. According to the fuzzy set theory, stochastic theory and numerical theory, many scholars have presented different multi-objective programming methods in different environments.

The Multi-objective programming is an effective mathematical model to deal with multiple objectives. The multi-objective optimization problem is originated from the problem of coordinating multi-objective which proposed by Franklin in 1772. Colson and Bruyn (1989) proposed many models of multi-objective decision making and its computational methods. And then, the multi-objective decision model is widely used in various fields. For example, Kao et al (2014), constructed a multi-objective programming model to solve a network model by using Data Envelope Analysis. Dujardin et al (2015), established a multi-objective interactive system of adaptive traffic control based on traffic control theory. Mazicioglu and Merrick (2018) extended attacker/defender models to incorporate multiple objectives that a terrorist might consider in planning an attack. Ren and Tan (2019) proposed a new multi-attribute decision-making model of stope parameters based on set pair analysis and grey relational theory. Guu and Wu (2019) considered a multiple objective optimization problem which allows system manager to consider three objectives (system congestion, cost, and penalty) simultaneously. In this paper, a multi-objective programming model based on relative
deviation is established and it is applied to a class of portfolio decisions.

2. Multi-objective programming model based on relative deviation
The multi-objective method based on priority factors was proposed by Charnes and Cooper (1959). Its main idea is to minimize the deviation between the objective function and the target value. Although the priority-based model has been successfully applied in many decision problems, the model is still lack of generality. On the one hand, it is difficult to explain the meaning of the model’s goal clearly because of the different dimensions of the goals. On the other hand, the absolute deviation of the goal is often not concerned in the practical decision-making, but the decision maker considers the degree to which the goal value is actually met. It is difficult to achieve the goal absolutely of the decision maker, but we can make the results meet the needs of the decision maker as much as possible, and make the deviations between the results and the expectations as small as possible. Therefore, the concept of relative deviation is introduced based on the priority factor model.

Define 1. Relative Deviation (RD) is the ratio of the absolute deviation (the difference between the goal function \( h(x) \) and the goal value \( V \)) to the goal value, i.e., \( RD = \frac{h(x) - V}{V} \), \( V \neq 0 \).

The degree of deviation of the goal value can be reflected intuitively by the relative deviation. The smaller the relative deviation, the higher the degree to which the goal value is actually met. When the relative deviation is zero, there is no difference between the goal function and the goal value. This ideal value is difficult to realize. When the goal value \( V \) is equal to zero, the relative deviation of the target cannot be expressed correctly. To solve this problem, a number \( a \) (greater than 0) can be introduced to transform the goal constraint into an exponential function with the base \( a \) as follows.

\[
RD(h(x)) = \frac{a^{h(x)} - a^0}{a^0} = a^{h(x)} - 1 \quad (1)
\]

In reality, different decision makers attach different importance to different goal. Therefore, weights are introduced to indicate the importance of different goals. The minimum deviation degree of the goal is considered, and the priority-based model is improved to obtain the following multi-objective programming model based on relative deviation.

\[
\begin{align*}
\min & \sum_{i=1}^{l} R_i \sum_{k=1}^{s} (w^i_d d^i_k - w^i_u d^i_k) \\
\text{subject to:} & \quad \frac{h_k(x) - V_k}{V_k} + d^+_k - d^-_k = V_k, k = 1, 2, \ldots, s \\
& \quad g_i(x) \leq b_i, \quad i = 1, 2, \ldots, m \\
& \quad d^+_k, d^-_k, x \geq 0, \quad k = 1, 2, \ldots, s
\end{align*}
\]

Where, the relative deviation is expressed as \( (h_k(x) - V_k) / V_k \), it represents the degree to which the \( k \)-th objective function \( h_k(x) \) deviates from the goal value \( V_k \). \( d^+_k, d^-_k \) are called the \( k \)-th positive or negative relative deviation variables respectively, they are different from the positive or negative deviation variables of the formula. The values of \( d^+_k, d^-_k \) represent the degree of relative deviation between the \( k \)-th objective function \( h_k(x) \) and the goal value \( V_k \).

3. Application
In this section, a case study of portfolio decisions is given to apply the multi-objective programming model based on the relative deviation.

An enterprise manager plans to use 20 million yuan to carry out technical transformation on 4 subordinate enterprises, and the unit investment amount of each subordinate enterprise is known. We assume that the return is affected by the 5 factors, such as the change of three kinds of market demand, existing competitors and the threat of substitutes. The predicted return on the unit investment after technical transformation is shown in table 1. The manager have multiple goals to achieve, and they are ranked according to the importance as follows.
Table 1: The predicted return on the unit investment

| Subordinate enterprises | Enterprise 1 | Enterprise 2 | Enterprise 3 | Enterprise 4 |
|-------------------------|--------------|--------------|--------------|--------------|
| Unit investment (Ten thousand yuan) | 13 | 11 | 16 | 14 |
| The predicted return on the unit investment (Ten thousand yuan) | 5.24 | 6 | 5.87 | 5 |
| The market demand 1 | 4.17 | 5.22 | 4.74 | 4.45 |
| The market demand 2 | 3.25 | 4.23 | 3.23 | 3.27 |
| The market demand 3 | 2.36 | 3.34 | 3.25 | 2.65 |
| Existing competitors | 3.12 | 2.2 | 2.7 | 2.8 |
| Threat of substitutes | 4.22 | 3.22 | 5.52 | 4.02 |

(1) Wish to complete the total investment without exceeding the budget;
(2) The total expected return is 20% of the total investment;
(3) The risk in investment is as low as possible;
(4) The investment of the enterprise 3 accounts for about 30%.

Let \(x_j (j = 1, 2, 3, 4)\) be the manager’s investment shares in the \(j\)th enterprise. The constraint condition of the first goal is

\[
(13x_1 + 11x_2 + 16x_3 + 14x_4 - 2000)/2000 + d^-_1 - d^+_1 = 0 \quad (3)
\]

Where \(d^-_1, d^+_1\) represent the positive or negative relative deviation variables of amount of investment respectively.

The constraint condition of the second goal is

\[
(1.62x_1 + 1.02x_2 + 2.32x_3 + 1.22x_4)/0.2(13x_1 + 11x_2 + 16x_3 + 14x_4) + d^-_2 - d^+_2 = 0 \quad (4)
\]

Where \(d^-_2, d^+_2\) represent the positive or negative relative deviation variables of the total expected return respectively. Different from the general expression, the value of the risk in investment is expressed as deviation in our model. For example, the risk of the first case of market demand after investing 4 enterprises is

\[
(5.24 - 4.22)x_1 + (6 - 3.22)x_2 + (5.87 - 5.52)x_3 + (5 - 4.02)x_4 \quad (5)
\]

Since the goal value of the risk in investment appears to be zero, the number \(a (a > 0)\) is introduced to convert the relative deviation into an exponential form. According to the order of magnitude of the goal value, \(a=1.002\) is given in this model. The constraint conditions of the third goal are

\[
\begin{align*}
1.0002^{1.02x_1 + 2.78x_2 + 0.35x_3 + 0.98x_4} - 1 + d^-_3 - d^+_3 &= 0 \\
1.0002^{-0.05x_1 + 2.2x_2 - 0.78x_3 + 0.43x_4} - 1 + d^-_4 - d^+_4 &= 0 \\
1.0002^{-0.97x_1 + 1.01x_2 - 2.29x_3 - 0.75x_4} - 1 + d^-_5 - d^+_5 &= 0 \\
1.0002^{-1.86x_1 + 0.12x_2 - 2.27x_3 - 1.37x_4} - 1 + d^-_6 - d^+_6 &= 0 \\
1.0002^{-1.10x_1 - 1.02x_2 - 2.82x_3 - 1.22x_4} - 1 + d^-_7 - d^+_7 &= 0 
\end{align*} \quad (6)
\]

Where \(d^-_3, d^+_3, d^-_4, d^+_4, d^-_5, d^+_5, d^-_6, d^+_6, d^-_7, d^+_7\) represent the positive or negative relative deviation variables of the risk in five cases respectively. The constraint condition of the forth goal is

\[
-3.9x_1 - 3.3x_2 - 11.2x_3 - 4.2x_4)/0.3(13x_1 + 11x_2 + 16x_3 + 14x_4) + d^-_8 - d^+_8 = 0 \quad (7)
\]

Where \(d^-_8, d^+_8\) represent the positive or negative relative deviation variables of the amount of
investment respectively. Let $w_1, w_2, w_3, w_4$ represent the weights of the four goals respectively. The multi-objective programming model based on the relative deviation is established as follows.

$$\begin{align*}
\min z &= w_1 (d_1^- + d_1^+) + w_2 d_2^- + w_3 \left[ \sum_{i=3}^7 (d_i^- + d_i^+) \right] + w_4 (d_8^- + d_8^+) \\
&= \left( 13x_1 + 11x_2 + 16x_3 + 14x_4 - 2000 \right) / 2000 + d_1^- - d_1^+ = 0 \\
&= \left( 1.62x_1 + 1.02x_2 + 2.32x_3 + 1.22x_4 \right) / 0.2 \left( 13x_1 + 11x_2 + 16x_3 + 14x_4 \right) + d_2^- - d_2^+ = 0 \\
&= 1.0002^{1.02x_1 + 2.78x_2 + 0.35x_3 + 0.98x_4} - 1 + d_3^- - d_3^+ = 0 \\
&= 1.0002^{-0.05x_1 + 2x_2 - 0.78x_3 + 0.43x_4} - 1 + d_4^- - d_4^+ = 0 \\
&= 1.0002^{-0.97x_1 + 0.1x_2 - 2.29x_3 - 0.75x_4} - 1 + d_5^- - d_5^+ = 0 \\
&= 1.0002^{-1.86x_1 + 0.12x_2 - 2.27x_3 - 1.37x_4} - 1 + d_6^- - d_6^+ = 0 \\
&= 1.0002^{-1.01x_1 - 1.02x_2 - 2.82x_3 - 1.22x_4} - 1 + d_7^- - d_7^+ = 0 \\
&= \left( -3.9x_1 - 3.3x_2 + 11.2x_3 - 4.2x_4 \right) / 0.3 \left( 13x_1 + 11x_2 + 16x_3 + 14x_4 \right) + d_8^- - d_8^+ = 0 \\
&\ |x_j, d_j^+, d_j^-| \geq 0; i = 1, 2, \ldots, 7; j = 1, 2, 3, 4.
\end{align*}$$

According to the ranking of the four goals above, we take $w_1 = 0.4, w_2 = 0.3, w_3 = 0.2, w_4 = 0.1$

The solution of the model by LINGO software is:

$$\begin{align*}
x_1 &= x_2 = 0, x_3 = 37.50, x_4 = 100, z = 0.029 \\
d_1^+ &= 0.52, d_1^- = 0.022, d_2^+ = 0.0027, d_2^- = 0.032, d_3^- = 0.043, d_3^+ = 0.045 \\
d_4^+ &= d_4^- = d_5^+ = d_5^- = d_6^+ = d_6^- = 0
\end{align*}$$

4. Conclusions

In this paper, the relative deviation is introduced to reflect the deviation degree of the goal value, and the multi-objective programming model based on the relative deviation is proposed and analyzed. Results show that the multi-objective programming model based on the relative deviation not only guarantees the existence of the solution, but also reflects the degree of deviation of each objective. Moreover, it can be applied not only to linear programming problems, but also to nonlinear programming problems.

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References

[1] Colson, G., Bruyn, C.D. (1989) Models and methods in multiple objectives decision making. Math.Comput. Model., 12: 1201-1211.
[2] Kao, H. Y., Chen, C. Y and Wu, D. J. (1989) A multi-objective programming method for solving network DEA. Appl. Soft. Comput., 24: 406-413.
[3] Dujardin, Y., Vanderpooten, D and Boillot, F. (2015) A multi-objective interactive system for adaptive traffic control. Eur. J. Oper. Res., 244: 601-610.
[4] Mazicioglu D, Merrick J R W. (2018) Behavioral modeling of adversaries with multiple objectives in counterterrorism. Risk Anal.,38: 962-977.
[5] Hong-gang Ren, Zhuo-ying Tan. (2019) Multiple Attribute Decision Making of Stope Parameters Based on SPA-GRA Model. Geot. Geol .Eng., 37: 5341–5348.
[6] Guo S M, Wu Y K. (2019) Multiple objective optimization for systems with addition-min fuzzy relational inequalities. Fuzzy Optim. Decis. Mak., 18: 529-544.
[7] Charnes, A, Cooper, W. W. (1959) Chance-Constrained Programming. Manage. Sci., 6: 73-79.