Analysis of scattering and breakup reactions of $^{12,14}\text{Be}$ within the microscopic model of optical potential

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Abstract. Analysis of the differential cross sections of scattering of neutron-rich nuclei $^{12,14}\text{Be}$ on $^{12}\text{C}$ at 56 MeV/nucleon and on protons at energy near 700 MeV/nucleon is carried out within the microscopic model of optical potential (OP). The real part of the $^{12,14}\text{Be}+^{12}\text{C}$ OP is calculated by using the double-folding procedure accounting for the anti-symmetrization effects, while the imaginary part of OP is obtained in the framework of the high-energy approximation (HEA). As to the $^{12,14}\text{Be}+p$ scattering at relativistic energies, calculations of the both real and imaginary OPs were made within the HEA approach. In this framework, the only free parameters of OP are the depths of its real and imaginary parts obtained by fitting to experimental data. The role of the $^{12,14}\text{Be}$ density models is considered when reproducing the experimental data. A contribution of the inelastic channels with excitations of $2^+$ and $3^-$ states in $^{12}\text{C}$ when calculating the quasielastic cross sections is analysed. Also, the $^{14}\text{Be}$ cluster model, in which this nucleus consists of a 2n-halo and the $^{12}\text{Be}$ core, is applied to calculate the cross sections of diffraction breakup and stripping reactions in the $^{14}\text{Be}+^{12}\text{C}$ collisions at the energy of 56 MeV/nucleon. A good agreement of the theoretical results with the available experimental data of both scattering and breakup processes is obtained.

1. Introduction

Interest in the study of light exotic nuclei with a neutron halo lasts more than 30 years, despite the large number of publications on this subject. A detailed review of the works devoted to the extra halo structure of the $^{12,14}\text{Be}$ nuclei is presented in [1]. First of all, the $^{12,14}\text{Be}$ nuclei are studied in [2] and [3] in a context of the $^{12,14}\text{Be}+^{12}\text{C}$ scattering. Theoretical analysis in [2] and [3] is based on a phenomenological approach using the Woods–Saxon optical potential (OP) and two very different sets of more than 10 phenomenological parameters have been obtained. Also, in [4], there were obtained the $^{12,14}\text{Be}+p$ scattering data at relativistic energy which were analysed within the Glauber theory suggesting the sets of phenomenological densities.

In our study, we employ the hybrid model of microscopic OP that was previously successfully used for explanation of experimental data on elastic cross sections and breakup reactions of light exotic nuclei of $^{6,8}\text{He}$, $^{8,11}\text{Li}$, $^{10,11}\text{Be}$ in [5, 6, 7, 8, 9, 10] Our calculations are based on the microscopic double-folding OP accounting for the antisymmetrization effects for the real part of OP and with the high energy approximation for its imaginary part.

We analyze, within the microscopic model of nucleus-nucleus optical potential (OP), the differential cross sections of scattering of exotic nuclei $^{12,14}\text{Be}$ on the $^{12}\text{C}$ target the 56.8
MeV/nucleon and on the proton target at 700 MeV. Also, we consider the breakup and stripping reactions of $^{14}$Be consisting of a 2n-halo and the $^{12}$Be core when interacting with the $^{12}$C target at energy of 56 MeV/nucleon.

2. Calculation of microscopic OP and cross sections

2.1. The double folding OP

The real double folding (DF) OP consists of the direct and exchange terms, $V^D$ and $V^{EX}$ [11, 12, 13]:

$$V^{DF}(r) = V^D(r) + V^{EX}(r).$$

Both potentials are composed from the isoscalar and isovector terms and the isoscalar part is as follows:

$$V^D(r) = \int d^3r_p d^3r_t \rho_p(r_p) \rho_t(r_t) \rho_{NN}(s),$$

$$V^{EX}(r) = \int d^3r_p d^3r_t \rho_p(r_p, r_t + s) \rho_t(r_t, r_t - s) \rho_{NN}(s) \exp \left[ \frac{iK(s)}{M} \right].$$

Here $s = r + r_t - r_p$ is the vector between two nucleons, one of which belongs to the projectile and another one to the target nucleus, $\rho_{p,t}$ are the projectile and target densities, and $\rho_i (r_{i+}, r_t \pm s)$ are the mixed one-particle density matrixes. $K(r)$ is the local momentum of the nucleus-nucleus relative motion, $v_{NN}^{DF}$, $v_{NN}^{EX}$ – effective Paris NN CDMS3Y potentials. The isovector potential is determined by the same formulae (2,3) but $\rho_i (i = t, p)$ should be exchanged by $\delta \rho_i$, the difference between proton and neutron densities for every $i$-nucleus.

2.2. The high energy approximation OP

At comparably high energies, the $NN$ potential is expressed through its explicit form [14]. In this framework, the HEA OP is determined as follows [11]:

$$U_{opt}^H(r) = V^H(r) + W^H(r) = -\frac{E}{k} \bar{\sigma}_N (i + \bar{\alpha}_N) \frac{1}{(2\pi)^3} \int e^{-iqr} \rho_p(q) \rho_i(q) f_N(q) d^3q.$$  

Here $\bar{\sigma}_N$ is the isospin averaged $NN$ total cross section, $\bar{\alpha}_N$ is the ratio of real to imaginary part of the forward nucleon-nucleon amplitude, and $f_N(q) = \exp(-\beta N q^2/2)$, where $\beta_N$ is the slope parameter.

2.3. Calculation of the cross sections

In case of the $^{12,14}$Be+$^{12}$C scattering, the microscopic nucleus-nucleus OP was taken in the hybrid form:

$$U(r) = N_R V^{DF}(r) + iN_I W^H(r),$$

where $N_R$ and $N_I$ are the renormalization factors of the real and imaginary OPs which are adjusted to the experimental data. In the case of the known densities of interacting nuclei, there are no more parameters to be fitted. The standard DWUCK4 [15] code is used for calculation of the cross sections. The Coulomb potential is taken in the standard form of the uniformly charged sphere of the radius $R_C$.

When considering the proton-nucleus scattering, the target density terms are removed from the formulae of OPs. In case of the $^{12,14}$Be+$^p$ scattering at relativistic energy, the cross sections are calculated by means of numerical solution of the respective relativistic equation [16] where the OP was taken in the form (4).
2.4. Density distributions

We use the following densities of $^{12,14}$Be.

The first one is the microscopic density calculated within the generator coordinate method (GCM) [17]. In this framework, the $^{14}$Be nucleus is considered as a three-cluster nucleus, involving several $^{12}$Be+$n+n$ configurations. The $^{12}$Be core nucleus is described in the harmonic oscillator model with all possible configurations in the $p$ shell.

The other microscopic density is the variational Monte Carlo model (VMC) [18], where the proton and neutron densities are computed with the so called AV18+UX Hamiltonian, in which the Argonne v18 two-nucleon and Urbana X three-nucleon potentials are used.

The third density is the phenomenological density in the form of the symmetrized Fermi function (SF):

$$\rho_{SF}(r) = \frac{A}{3\pi R^3} \left[ 1 + \left( \frac{a}{R} \right)^2 \right]^{-1} \times \frac{\sinh \left( \frac{R}{a} \right)}{\cosh \left( \frac{r}{a} \right) + \cosh \left( \frac{R}{a} \right)}.$$  \hspace{1cm} (6)

Here the parameters, radius $R$ and diffuseness $a$, were established in [4] by fitting (within the Glauber approach) to the data of $^{12,14}$Be+p elastic scattering at 700 MeV/nucleon: $R = 1.37$ fm, $a = 0.67$ fm for $^{12}$Be and $R = 0.99$ fm, $a = 0.84$ fm for $^{14}$Be. The $^{12}$C density is taken as the modified SF form:

$$\rho(r) = \rho_{SF}(r) + \rho_{SF}^{(1)}(r),$$  \hspace{1cm} (7)

where the $\rho_{SF}$ is determined by Eq. (6) with radius $R=2.275$ fm and diffuseness $a =0.393$ fm, while the surface term $\rho^{(1)}$ is calculated via the 1st derivative of $\rho_{SF}$. The parameters of this density were obtained in [19] by fitting to the $eA$ scattering data.

3. Results

3.1. Quasielastic scattering $^{12,14}$Be+$^{12}$C

![Figure 1. Effect of accounting for states $2^+$ and $3^-$ in the $^{12}$Be+$^{12}$C (left) and $^{14}$Be+$^{12}$C (right) scattering. The calculations with the SF density of $^{12,14}$Be and $^{12}$C.](image)

It was suggested in [2] and [3] that the experimental data should be considered as quasielastic scattering, i.e., the contribution of inelastic channels related to excitations of the low-lying collective states of a nucleus, should be accounted for, too. When calculating inelastic cross sections, the transition potential is taken as $U_{inel} = -\tilde{R} \cdot dU/dr$, where $U$ is the microscopic OP in the form (5), $\tilde{R}$ is the potential radius (we put $\tilde{R}=4.25$ fm as in [2]). Contributions of inelastic channels ($E_{2^+} = 4.436$ MeV) and the $3^-$ state ($E_{3^-} = 9.641$ MeV) are shown in Fig.1. The calculations have been made with the SF density for both $^{12,14}$Be and $^{12}$C nuclei. It is seen that the account for the contribution of inelastic scattering to the first $2^+$ state improves the
Figure 2. Effect of the surface term of the $^{12}$C SF density on the differential cross sections of quasielastic scattering $^{12}$Be (left) and $^{14}$Be (right) on $^{12}$C. Solid, dashed, and dotted lines correspond, respectively, to the SF, GCM, and VMC densities of $^{12,14}$Be used in calculations of OP.

Table 1. Values of parameters of microscopic OP of the $^{12,14}$Be+$^{12}$C quasielastic scattering

| Nucleus | Density of $^{12,14}$Be | SF density of $^{12}$C | Modified SF density of $^{12}$C | $N_R$ | $N_I$ | $\beta_{2+}$ | $N_R$ | $N_I$ | $\beta_{2+}$ |
|---------|-------------------------|-----------------------|-------------------------------|------|------|-------------|------|------|-------------|
| $^{12}$Be | GCM                     | 0.496                 | 1.431                         | 0.437 | 0.592 | 1.133       | 0.526 |
|          | SF                      | 0.702                 | 1.294                         | 0.635 | 0.647 | 1.094       | 0.665 |      |
|          | VMC                     | 0.582                 | 1.156                         | 0.487 | 0.596 | 1.106       | 0.593 |      |
| $^{14}$Be | GCM                     | 0.638                 | 2.000                         | 0.375 | 0.708 | 1.920       | 0.369 |      |
|          | SF                      | 0.701                 | 1.252                         | 0.365 | 0.599 | 0.952       | 0.362 |      |

depth of the first minimum and leads to a left-shift correction of its place while the contribution of the excitation of the $3^-$ state is weak.

Fig.2 demonstrates the results of calculations with different densities of $^{12,14}$Be and the SF density of $^{12}$C with and without the accounting for the surface term. It is seen that inclusion of the surface part does not noticeably improve the agreement with experimental data at angles $\theta_{c.m.} < 5^\circ$. The respective best fit values of parameters $N_R$, $N_I$, and the deformation parameter $\beta_{2+}$ are given in Table 1.

3.2. Elastic scattering $^{12,14}$Be+$p$

The results of calculation of differential cross sections of the $^{12,14}$Be+$p$ elastic scattering are presented in Fig. 3. The cross sections are calculated by means of numerical solution of the respective relativistic equation, see [16] for details. In this framework, the Klein – Gordon – Fock equation at $E \gg |U|$ is reduced to the form of Schrödinger equation with relativistic momentum.
\( p \) and effective potential:
\[
(\Delta + p^2)\psi(r) = 2\mu U_{\text{eff}}(r)\psi(r).
\]
(8)

Here \( \mu = m_pm_p/(m_p + M_A) \) is non-relativistic reduced mass and the relativistic momentum \( p \) in c.m. system depends on total energy \( E = T_{\text{lab}} + m_1 \), where \( T_{\text{lab}} \) is kinetic energy in laboratory system, \( m_1 \) – the proton mass, and \( m_2 \) – the \(^{12,14}\text{Be} \) mass:
\[
p = \frac{p_{\text{lab}}m_2}{\sqrt{2Em_2 + m_1^2 + m_2^2}} \quad p_{\text{lab}} = \sqrt{T_{\text{lab}}(T_{\text{lab}} + 2m_1)}.
\]
(9)

Effective OP \( U_{\text{eff}}(r) \) consists of Coulomb potential \( U_C \) and the microscopic OP \( U \):
\[
U_{\text{eff}}(r) = \gamma(r) [U(r) + U_C(r)].
\]
(10)

Relativization factor is calculated as follows:
\[
\gamma(r) = \frac{\bar{\mu}}{\mu} = \frac{m_1 + m_2}{\bar{m}_1 + m_2}, \quad \bar{m}_1 = \sqrt{p^2 + m_1^2} = E.
\]
(11)

In the case of the \(^{12,14}\text{Be}+p \) scattering at \( T_{\text{lab}} = 0.7 \text{ GeV} \) we obtain \( \gamma = 1.6 \) for \(^{12}\text{Be} \) and \( \gamma = 1.63 \) for \(^{14}\text{Be} \). As shown in [20], the calculations can reasonably reproduce the data of \(^{12,14}\text{Be}+p \) elastic scattering even without fitting of the OP parameters. Having in mind that the cross sections in our study and in [4] are calculated on different theoretical framework, we have fitted parameters \( \bar{\sigma}_N \) (in \( \text{fm}^2 \)) and \( \bar{\alpha} \) of the microscopic OP (4) improved the agreement of calculations with experimental data. The calculated cross sections are shown in Fig. 3. The best fit values of parameters are given in Table 2. In calculations, the values of slope parameter \( \beta_N \) were taken as in [4]. One sees that the calculations with SF and VMC densities well reproduce the experimental data, while the GCM model does not provide so good agreement at \( \Theta > 5^\circ \).

**Figure 3.** The differential cross sections of the \(^{12,14}\text{Be}+p \) elastic scattering at 700 MeV/nucleon. Calculations have been performed with the modified SF density of \(^{12}\text{C} \) (7) and the different densities of \(^{12,14}\text{Be} \). Solid, dashed and dotted lines correspond, respectively, the SF, GCM, and VMC densities of \(^{12,14}\text{Be} \).

3.3. The breakup cross section of \(^{14}\text{Be}+^{12}\text{C} \rightarrow (^{12}\text{Be}+2n)+^{12}\text{C} \)

The cross section of the breakup of the incident nucleus \( a \) into two clusters \((a + A \rightarrow c + v + A) \) is determined as follows:
\[
\frac{d\sigma}{dk_{||}dk_{\perp}} = \frac{1}{2l + 1} \frac{4k_{\perp}}{k_2^2} \int d^2b \sum_{M,m} \int dr \int d(\cos \theta) \sum_L (-i)^L u_{k,L}(r)g_L(r)
\]
(12)
Table 2. Parameters $\bar{\sigma}_N$ (in fm$^2$) and $\bar{\alpha}$ of the microscopic OP for the $^{12,14}$Be+$p$ elastic scattering.

| Model of $^{12,14}$Be | $^{12}$Be, $E/A=703.5$ MeV | $^{14}$Be, $E/A=702.9$ MeV |
|-----------------------|-------------------------------|-------------------------------|
| SF                    | $\bar{\sigma}_N$ = 4.4       | $\bar{\sigma}_N$ = 4.136     |
|                       | $\bar{\alpha}$ = -0.237       | $\bar{\alpha}$ = -0.209       |
| GCM                   | $\bar{\sigma}_N$ = 3.5        | $\bar{\alpha}$ = -0.483       |
|                       | $\bar{\alpha}$ = 3.460        | $\bar{\alpha}$ = -0.350       |
| VMC                   | $\bar{\sigma}_N$ = 3.8        | $\bar{\alpha}$ = -0.416       |

\[
\times \tilde{Y}_{L,M}(\theta_k)\tilde{Y}_{L,M}^*(\theta)\tilde{Y}_{L,M}(\theta) Y_{L,M}(\hat{k}) \int d\varphi \exp (i(m - M)\varphi) S_c(b_c)S_v(b_v) \Bigg| ^2 ,
\]

where $\mathbf{k}$ – relative momentum of both clusters in their c.m. frame, $k_\parallel$ and $k_\perp$ – its parallel and transversal components. The relative motion wave function of the fragments of $a = c + v$ in the continuum final state has the form:

\[
\phi_k(r) = 4\pi \sum_{L,M} u_{k,L}(r)Y_{LM}(\hat{r})Y^*_{LM}(\hat{k}), \quad Y_{LM}(\hat{k}) = \tilde{Y}_{L,M}(\theta_k) \exp(iM\varphi_k)
\]

where we neglect the distortion effect and thus use $u_{k,L}(r) = kr j_L(kr)$, where $g_l(r)$ – radial part of the initial bound state wave function of clusters $c$ and $v$.

In the case of the $s$-state for the mutual motion of the clusters in the incident nucleus $a = c + v$, the cross sections of the stripping reaction when the valance cluster $v = 2n$ leaves the elastic channel are determined as follows [10]:

\[
\left( \frac{d\sigma}{dk_\parallel} \right)_{\text{str}} = \frac{1}{2\pi^2} \int d^2b_v \left[ 1 - |S_v(b_v)|^2 \right] \int d^2\rho |S_c(b_c)|^2 \times \left[ \int dz \cos(k_\parallel z)\phi_0 \left( \sqrt{\rho^2 + z^2} \right)^2 \right],
\]

with $\mathbf{r} = \rho + z$ and $\rho = b_v - b_c$. The $^{12}$Be longitudinal momentum distribution from $^{14}$Be

**Figure 4.** The $^{12}$Be longitudinal momentum distribution from $^{14}$Be fragmentation on $^{12}$C at incident energy of 56.8 MeV/nucleon for stripping (left) and diffraction (right) processes.
fragmentation on $^{12}$C at incident energy of 56.8 MeV/nucleon for the stripping and diffraction processes are given in Fig.4. The respective values of the width 80 MeV/c and 115.7 MeV/c adequately reproduce the experimental estimation $92.2 \pm 2.7$ MeV/c [2].

4. Summary
Differential cross sections of the $^{12,14}$Be+$^{12}$C scattering at energy 56 MeV/nucleon and of the $^{12,14}$Be+$p$ scattering at 700 MeV have been analyzed within the microscopic model of OP using three models of the $^{12,14}$Be density distribution to be tested. Shown that the inelastic channel should be added to the elastic one to reasonably explain the experimental data on the $^{12,14}$Be+$^{12}$C elastic scattering with the given resolution. Our approach is shown to provide a good agreement with experimental data on the $^{12,14}$Be+$p$ scattering at 700 MeV when used VMC and SF densities. The calculated longitudinal momentum distributions of $^{12}$Be fragments produced in the breakup of $^{14}$Be+$^{12}$C at 56.8 MeV/nucleon are in a reasonable agreement with experimental data. More details on the microscopic analysis of scattering and breakup reactions with participation of $^{12,14}$Be nuclei are presented in [1].

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