Restrictions on B-L Symmetry Breaking
Implied by a Fourth Generation Neutrino †

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Abstract

We point out that if the fourth generation neutrino is a Majorana fermion,
then the astrophysical constraints coupled with the precision measurements
of the Z-width at LEP require that the corresponding B-L symmetry is un-
likely to be a spontaneously broken global symmetry. If B-L is chosen to be
a local symmetry, its breaking scale should be less than a few TeV.

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The recent precision measurements of Z-width[1] at LEP and SLC have shown that if there exists a fourth generation of leptonic doublet, the masses of both the charged lepton ($L^-$) and neutrino ($\nu_4$) must be heavier than about 45 GeV. They do not alter the standard big bang nucleosynthesis scenario[2] because due to their large mass, they annihilate away much earlier than the epoch of nucleosynthesis. It is therefore conceivable that a heavy fourth generation of quarks and leptons exists and remains to be discovered. It is then important to theoretically probe the constraints on the properties of these particles. In this brief note, we focus on the nature of the fourth generation neutrino, $\nu_4$.

First it is important to realize that unlike the case for the first three neutrino species $\nu_e$, $\nu_\mu$ and $\nu_\tau$, the above mentioned mass constraint implies that, if there is a fourth generation neutrino, it must be accompanied by a right-handed neutrino (to be denoted by $\nu_4^R$) to give it a mass. Note that, if a Higgs triplet with non-zero vev (vacuum expectation value) were to be introduced to give a Majorana mass to the $\nu_4^L$, (in which case $\nu_4^R$ would not be needed,) the vev of the triplet must be so large that it would contradict bounds on it implied by the $\rho$ parameter measurement.

The next question that arises is, whether $\nu_4$ is a Majorana or Dirac fermion. If it is a Dirac fermion, the only constraint on its mass is that which follows from the precision measurement of the $\rho$ parameter i.e.

$$|m_{\nu_4}^2 - m_{\nu_4^L}^2| \leq (152 \text{ GeV})^2$$  \hspace{1cm} (1)

On the other hand if it is of Majorana type, the constraints depend on how
its Majorana mass arises and are independent of $m_L$. In the standard model with only the fourth generation fermions and the $4^{th}$ generation right-handed neutrino $\nu_{4R}$, the $\nu_4$ has the following mass matrix (ignoring mixing with lower generation neutrinos)

$$
\begin{pmatrix}
0 & m_4 \\
-m_4 & M_4
\end{pmatrix},
$$

where $m_4$ denotes the Dirac mass induced by electroweak breaking and $M_4$ is the Majorana mass which is allowed in the standard model. The eigenvalues are

$$m_\pm = \frac{1}{2}(M_4 \pm \sqrt{M_4^2 + 4m_4^2}),$$

and the eigenstate are characterized by the mixing angle

$$\tan 2\theta = \frac{2m_4}{M_4}.$$  

Notice that the LEP Z-width constraints implies that $m_\pm$ should be heavier than about $M_Z/2$.

Let us now entertain the possibility that the Majorana mass of $\nu_{4R}$ arises from spontaneous breaking of either global or local B-L symmetry. In what follows, we will show that the first possibility is disfavored by astrophysical considerations. In the case of the second possibility, barring unnatural fine tuning of parameters, the scale of local B-L must have an upper-bound, if the Yukawa couplings are assumed to remain perturbative.

2. First we consider the case of spontaneously broken global B-L symmetry. In this case, there will emerge the massless particle, the Majoron[3]. We will show that for reasonable choice of parameters the coupling of the Majoron to
electron, up and down quarks in the presence of fourth generation neutrino is so large that it is in conflict with the bounds implied by the observed red giant abundances in the universe. This result is independent of whether the breaking of global B-L symmetry is dynamically induced or realized by an explicit scalar field. We will demonstrate our result using the latter example. The model is a simple extension of the CMP model[3] with the fourth generation fermions included and has been discussed in recent literature[4].

To remind the reader, the CMP model is the simplest extension of the standard model that adds only one complex, lepton-number carrying iso-singlet scalar field $\Delta$ and one right-handed neutrino per generation. The field $\Delta$ couples only to the right-handed neutrinos and when it acquires a non-zero vev i.e. $<\Delta> = \frac{1}{\sqrt{2}} v_{BL}$, it breaks the global B-L symmetry spontaneously and gives rise to the massless Majoron. In the polar decomposition of $\Delta$ i.e.

$$\Delta = \frac{1}{\sqrt{2}} (\rho + v_{BL}) e^{i \frac{\chi}{v_{BL}}} ,$$

(5)

$\chi$ corresponds to the Majoron. In order to study the properties of the Majoron, let us write down the leptonic part of the lagrangian involving only the 4th generation fermions:

$$\mathcal{L} = -\bar{L_4} \gamma_\mu D^\mu L_4 - \bar{\nu_{4R}} \gamma_\mu \partial^\mu \nu_{4R}$$

$$+ h_4 \bar{L_4} \phi \nu_{4R} + f_4 \nu_{4R}^T C^{-1} \nu_{4R} \Delta + h.c.$$ (6)

$$- \partial_\mu \Delta^* \partial^\mu \Delta - V(\phi, \Delta)$$

where $\phi$ is the Higgs doublet of the standard model with vev $v_{wk} \simeq 250 GeV$. (Note that terms such as $\Delta^3$, $\Delta^2 \Delta^*$, etc are forbidden from appearing in $V(\phi, \Delta)$ by lepton number conservation.) The massless $\chi$ can then be emitted
from the stars via processes like $\gamma + e \rightarrow \chi + e$, etc, providing the stars with an extra energy loss mechanism. Obviously, large values of the electron and quark couplings to $\chi$ would shorten the life of stars\cite{5}. In the CMP model, in the tree-level, there is no Majoron coupling to charged fermions. However, the neutrino exchange yields at one-loop level an effective operator:

$$\epsilon M_Z \partial_\mu \chi Z^\mu,$$

(7)

which lead to $\bar{e}\gamma_5 e\chi$, $\bar{u}\gamma_5 u\chi$, $\bar{d}\gamma_5 d\chi$ couplings. The upper limit on these couplings implied by the astrophysical considerations gives\cite{5}

$$\epsilon < 10^{-7}.\hspace{1cm}(8)$$

We have performed a detail calculation of the parameter $\epsilon$ at the one-loop level in the presence of the fourth generation neutrino with the mass matrix given in eq.(2). It is finite and expressed as follows

$$\epsilon = \frac{\sin^2 2\theta}{8\pi^2 v_{BL}} \frac{M_4}{v_{wk}} \left\{ \frac{m_-}{v_{wk}} \int_0^1 dx (1-x) \ln \frac{m_-^2 - x m_-^2 + x m_+^2}{m_-^2} \right\} + \frac{m_+}{v_{wk}} \int_0^1 dx x \ln \frac{m_-^2 - x m_-^2 + x m_+^2}{m_+^2},\hspace{1cm}(9)$$

where $m_\pm$ and $\theta$ are defined in eq.(3) and (4) respectively. In the limit $M_4 \gg m_4$ as usually assumed in the see-saw mechanism, $\epsilon$ is reduced to

$$\epsilon = \frac{1}{8\pi^2 v_{wk} v_{BL}} \frac{m_4^2}{v_{BL}},\hspace{1cm}(10)$$

This is the result obtained in ref.[3]. For the first three generations, the largest value of $m_4$ is likely to be of the order of 1-2 GeV, in which case
the astrophysical bound is satisfied for $v_{BL} \sim O(\text{TeV})$. Let us consider the constraints on the properties of the fourth generation neutrino resulting from LEP Z-width measurement and the constraint in eq.(8). Firstly we know that the scale of B-L symmetry breaking cannot be much bigger than the electroweak scale for $f_4 \sim 1$, otherwise the constraints from LEP Z-width measurement will not be satisfied. However, this is in contradiction with the requirement from eq.(8), which needs a large $v_{BL}$. So our arguments make highly unlikely the possibility that lepton number breaking associated with the fourth generation is realized via the Nambu-Goldstone mode.

If on the other hand, the B-L symmetry is local, then the astrophysical constraint does not apply. The Z-width constraint implies that the scale of local B-L symmetry breaking should be less than a few TeV, otherwise the see-saw mechanism would lead to a light neutrino with mass less than $M_Z/2$. It therefore appears that the existence of a fourth generation lepton with Majorana neutrino highly constrain the scale of B-L symmetry breaking.

The only way to avoid these constraints would be to have an unnaturally small value for the coupling $f_4$, in which case both constraints in the global symmetry case could be satisfied. For instance, if $f_4 \leq 10^{-5}$, then a $v_{BL} \geq 10^7$ GeV would satisfy both constraints.

Let us now see the constraint on $M_4$, when $m_4 \gg M_4$. In this case, $\epsilon$ is reduced to

$$\epsilon = \frac{1}{12\pi^2} \frac{M_4^2}{v_{BL} v_w k}. \quad (11)$$

Taking $v_{BL} \sim 1\text{TeV}$ as suggested above by the astrophysical constraint for
the first three generations, one has, from eq.(8)

\[
M_4 < O(1\text{GeV}).
\]

Thus the fourth generation neutrino, if it exists, will be a Dirac or a Pseudo-Dirac neutrino.

Our result has direct bearing on a recently proposed dynamical symmetry breaking model by Hill, Luty and Paschos[6]. In their model global B-L symmetry is dynamically broken near electroweak scale by neutrino (fourth generation’s) condensate. As we argued above, the astrophysical bounds are not satisfied in this model. We should also point out that these considerations can be extended to composite models and other models with the dynamical broken symmetries, where ultra-light Pseudo-Goldstone bosons will mix with Z directly and/or indirectly.

3. In summary, we have examined LEP Z-width measurement and astrophysical constraints on the properties of the heavy majorana neutrino, which get mass from spontaneously B-L breakdown. We have argued that if the 4\textsuperscript{th} generation exists, it cannot be naturally embedded into the singlet majoron model. In the case where B-L is a local symmetry, its breaking scale must be less than a few TeV.
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