The density distribution and the physical origins of density intermittency in sub- to trans-Alfvénic supersonic turbulence

James R. Beattie 1*, Philip Mocz 2,3†, Christoph Federrath 1,4‡ and Ralf S. Klessen 4,5

1 Research School of Astronomy and Astrophysics, Australian National University, Canberra, ACT 2611, Australia
2 Department of Astrophysical Sciences, Princeton University, 4 Ivy Lane, Princeton, NJ 08544, USA
3 Lawrence Livermore National Laboratory, 7000 East Ave, Livermore, CA 94550, USA
4 Australian Research Council Centre of Excellence in All Sky Astrophysics (ASTRO3D), Canberra, ACT 2611, Australia
5 Universität Heidelberg, Zentrum für Astronomie, Institut für Theoretische Astrophysik, Albert-Ueberle-Str. 2, 69120 Heidelberg, Germany
6 Universität Heidelberg, Interdisziplinäres Zentrum für Wissenschaftliches Rechnen, Im Neuenheimer Feld 205, 69120 Heidelberg, Germany

† Einstein Fellow

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

The probability density function (PDF) of the logarithmic density contrast, \( s = \ln(\rho/\rho_0) \), with gas density \( \rho \) and mean density \( \rho_0 \), for hydrodynamical supersonic turbulence is well-known to have significant signatures of intermittency that monotonically increase with the turbulent Mach number, \( M \). By studying the mass- and volume-weighted \( s \)-PDF for an ensemble of 16 sub- to trans-Alfvénic mean-field, supersonic, isothermal turbulence simulations, relevant to molecular gas in the cool interstellar medium, we show that a more intricate picture emerges for the intermittency of \( s \). Using four independent measures of the intermittency we find hydrodynamical-like intermittency in the highly magnetised plasma for \( M \lesssim 4 \). However, for \( M \gtrsim 4 \), the signatures of intermittency disappear, leaving approximately lognormal \( s \)-statistics – exactly the opposite of hydrodynamical turbulence in the high-\( M \) limit. To understand the \( M \lesssim 4 \) intermittency we use one-dimensional (1D) pencil beams to explore the dynamics along and across the mean magnetic field, \( \mathbf{B}_0 \). We discuss kinetic, density and magnetic field fluctuations from the pencil beams, and identify physical sources of intermittency as single, strong shocks coupled to fast magnetosonic compressions that form along \( \mathbf{B}_0 \) and create large, volume-poor under-densities. These under-densities contribute significantly to the skewness of the \( s \)-PDF. We confirm this result independently using 1D fluid shock simulations. We discuss the Gaussianisation of the \( M \gtrsim 4 \) \( s \)-fields through the lens of two phenomenologies: the self-similarity of the \( s \)-field and homogenisation of the dynamical timescales between the over- and under-dense regions in the compressible gas.

Key words: MHD – turbulence – ISM: kinematics and dynamics – ISM: magnetic fields – ISM: structure

1 INTRODUCTION

The PDF of the gas density is a valuable tool for understanding the nature of compressible turbulence and star formation in the interstellar medium of galaxies (Vazquez-Semadeni 1994; Padoan et al. 1997; Passot & Vázquez-Semadeni 1998; Federrath et al. 2008b; Federrath et al. 2009; Brun et al. 2010a; Brun et al. 2010b; Krumholz & McKee 2005; Hennebelle et al. 2011; Federrath & Klessen 2012; Burkhardt & Lazarian 2012; Konstandin et al. 2012b; Molina et al. 2012; Hopkins 2013; Nolan et al. 2015; Federrath & Banerjee 2015; Squire & Hopkins 2017; Pan et al. 2019; Mocz & Burkhardt 2019; Menon et al. 2021, 2020; Khullar et al. 2021). Simple models of molecular clouds (MCs) in the interstellar medium (ISM), which are supersonic and magnetised, and have not yet started to collapse under their own self-gravity have an approximately normal volume-weighted \( s \)-PDF\(^1\), where \( s \equiv \ln(\rho/\rho_0) \), and \( \rho \) is the cloud density, with \( \rho_0 \) the value of the volume-weighted mean. It follows

\[
p_{N,V}(s;\sigma^2_{s,V}) = \frac{1}{\sqrt{2\pi \sigma^2_{s,V}}} \exp \left\{ -\frac{(s-s_{0,V})^2}{2\sigma^2_{s,V}} \right\},
\]

\[
s_{0,V} = -\frac{\sigma^2_{s,V}}{2},
\]

\[
\sigma^2_{s,V} = f(M, M_A, b, \gamma, \Gamma).
\]

The log-density variance, \( \sigma^2_s \), captures the density variations induced by different physical processes in a MC. It is a function of (i) the turbulent Mach number, \( M \equiv \sigma_V/c_s \),

\[
M = \sigma_V/c_s,
\]

\(^{1}\) Or equivalently, a lognormal \( \rho \)-PDF.
than that of the kinetic turbulent energy. We show that the nature of voids, rarefactions and over-dense structures in different from turbulence in the highly-magnetised mean-field (Padoan & Nordlund 2011; Molina et al. 2012; Beattie et al. 2021), (iii) the turbulent driving parameter, 2011; Molina et al. 2012; Beattie et al. 2021), (iii) the turbulent forcing (Federrath et al. 2008b; Federrath et al. 2010; Menon et al. 2021), (iv) the adiabatic index $\gamma$ (Nolan et al. 2015), and (v) the polytropic index $\Gamma$ (Federrath & Banerjee 2019).

This study is organised as follows. First, in §2 we revisit the lognormal density-fluctuation theory, the mass-weighted and volume-weighted distributions are intrinsically linked through the mean (Equation 7) and variance (Equation 8) of the volume-weighted $s$-PDF (Li et al. 2003). In hydrodynamical, supersonic turbulence it is well-known that there is a significant difference between $\sigma^2_{s,M}$ and $\sigma^2_{s,V}$ (an excess of $\sigma^2_{s,V}$), which in essence is from the emergence of non-Gaussian, intermittent events in the fluid (Passot & Vázquez-Semadeni 1998; Kritsuk et al. 2007; Federrath et al. 2010; Federrath 2013; Hopkins 2013; Squire & Hopkins 2017; Mocz & Burkhardt 2019).

The $s$-statistics and intermittency of strongly magnetised turbulence, where the flow is globally anisotropic about a coherent, strong mean-field has not been studied in great detail, even though it is indeed a relevant flow regime for either some regions inside of MCs (Li et al. 2013; Federrath et al. 2016; Hu et al. 2019; Heyer et al. 2020; Skalidis & Tassis 2020; Hwang et al. 2021; Hoang et al. 2021) or even perhaps most MCs (Li 2021). In this study we probe the physics of the $s$-PDFs in simulations with $2 \lesssim M \lesssim 20$ and $0.1 \lesssim M_{\Delta 0} \lesssim 2$, where $M_{\Delta 0} = c_s M \sqrt{\pi \rho_0} / |B_0|$, revealing the nature of voids, rarefactions and over-dense structures in the trans- to sub-Alfvénic mean-field turbulence, i.e., turbulence where the energy in the mean magnetic field is larger than that of the kinetic turbulent energy. We show that the $s$-intermittency for compressible, hydrodynamical turbulence is different from turbulence in the highly-magnetised mean-field regime. Specifically, we find that Equation 8 approximately holds for moderate to high-$M$ magnetised flows, which is not the case for purely hydrodynamical turbulence due to strong intermittency. We discuss a range of phenomenological models for why this may be the case and, for the first time, identify physical sources of $s$-intermittency in real-space, confirming that an important source of intermittency in these flows are strong shocks that produce intermittent, deep, volume-poor rarefactions in the gas density along the coherent magnetic field.

This study is organised as follows. First, in §2 we revisit the lognormal model for the density fluctuations. In §3 we extend the discussion to $s$-fluctuation models that include intermittency effects. In §4 we outline the supersonic MHD turbulence simulations that we use to explore the morphology of the full three-dimensional (3D) volume-weighted and mass-weighted $s$-PDFs from the simulation data, including fitting lognormal and non-lognormal models for the volume-weighted PDFs. Next, in §6 we compute four independent measures of $s$-intermittency, and show how the highly-magnetised regime has a more complicated intermittency structure than the hydrodynamical regime. In §7 we use one-dimensional (1D) probes of the low-$M$ turbulence to reveal the parallel and perpendicular to mean magnetic field $s$, velocity, and magnetic field dynamics, and use this 1D analysis to identify the rarest (intermittent) events in the turbulence, in real-space. In §8 we discuss the lack of intermittency in high-$M$, magnetised turbulence. Finally in §9 we summarise the key results of this study.

## 2 THE LOGNORMAL DENSITY-PDF

### 2.1 The lognormal model

Lognormal models for the PDF of turbulent $\delta = \rho / \rho_0$ fluctuations originate from Vazquez-Semadeni (1994). They consider $\delta$ in the self-similar, hierarchical structure of the cool, isothermal ISM, where global pressure is negligible ($M \gg 1$) and self-gravity is yet to dominate the dynamics of the region. The lognormal PDF is motivated by assuming that for time, $t_0$, the density can be expressed as a multiplicative interaction through independent jumps $\delta$ in the Eulerian frame of the gas,

$$\frac{\rho(t_0)}{\rho_0} = \delta_n \delta_{n-1} \ldots \delta_0 \frac{\rho(t_0)}{\rho_0} = \left( \prod_{i=0}^{n} \delta_i \right) \frac{\rho(t_0)}{\rho_0},$$

where $\rho(t_0)/\rho_0 = 1$ is the initial density in units of the mean. This means that under the log-transformation, fluctuations become additive,

$$s(t_n) = \ln \frac{\rho(t_n)}{\rho_0} = \sum_{i=0}^{n} \ln \delta_i = \sum_{i=0}^{n} c_i,$$

If the density fluctuations are

(i) generated by the same underlying probability distribution, and

(ii) are independent from one another (i.e., not temporally correlated),

then the central limit theorem states that the distribution of the logarithmic density fluctuations should tend towards a Gaussian distribution as the number of fluctuations increases. This framework lets us understand the nature of a single (Eulerian) density fluctuation changing in time. However, Vazquez-Semadeni (1994) further argued that since the hydrodynamical equations are self-similar in space (i.e., invariant to arbitrary length scaling) the fluctuations should be lognormal on all scales. With these assumptions, the lognormal distribution should aptly describe the density fluctuations on any length scale of the turbulence.

### 2.2 Key issues with the lognormal model

#### 2.2.1 Mass conservation

Hopkins (2013) articulates that having a lognormal distribu-
tion on all scales in the flow violates mass conservation. We show this by considering the $\rho/\rho_0$-PDF, $p_L(\delta)$, on the system scale, $L$. This distribution is the convolution of all PDFs from scales below the system scale, $L/\Gamma$, $L/\Gamma^n$, ..., $L/\Gamma$, ..., where $\Gamma \geq 1$, in the turbulence, which is simply an application of the Law of Total Probability \(^2\) (Castaing 1996). Writing this formerly,

$$p_L(\delta) = p_L/\Gamma(\delta) \otimes \cdots \otimes p_L/\Gamma^n(\delta) \otimes \cdots = \bigotimes_{n=1}^{\infty} p_L/\Gamma^n(\delta), \quad (11)$$

where $\otimes$ is the convolution operator, $f(x) \otimes g(x) = \int_{-\infty}^{\infty} dt f(x - t)g(t)$. Now we assume that all of the PDFs, $p_L/\Gamma(\delta) \ldots p_L/\Gamma^n(\delta) \ldots$, on the RHS of Equation 11 are lognormal distributions and inquire about the LHS of the equation. This is a well-known and historical problem in statistics, motivated by problems from a broad range of disciplines, from telecommunications to the biosciences (see for example, Wu et al. 2005; Lo 2012; Ben Hcine & Bouallegue 2015). The key conclusion one makes is that the LHS of Equation 11 cannot possibly be a lognormal distribution. In fact, because no scale is special in the turbulence, and instead of picking $p_L(\delta)$, we pick any arbitrary $p_L/\Gamma^n(\delta)$, then it is clear that only one of the infinite $p_L/\Gamma^n(\delta)$ can possibly be lognormal. This is directly related to mass conservation. It follows by assuming that all $p_L/\Gamma^n(\delta)$ are lognormal, then the total mass is,

$$M_1 = \int_{-\infty}^{\infty} d\delta \delta p_L(\delta), \quad (12)$$

which ought to equal

$$M_2 = \int_{-\infty}^{\infty} d\delta \delta \bigotimes_{n=1}^{\infty} p_L/\Gamma^n(\delta). \quad (13)$$

However, because $\bigotimes_{n=1}^{\infty} p_L/\Gamma^n(\delta)$ does not converge to a lognormal distribution, and because we also have assumed that all PDFs are, including $p_L(\delta)$, then $M_1 \neq M_2$, and hence a lognormal model on all scales violates mass conservation.

2.2.2 The sonic scale

This is the only reason why the lognormal distribution, which describes a scale-free random process, cannot be strictly "the right model" \(^3\) for the density fluctuations in supersonic turbulence. Recent numerical experiments by Federrath et al. (2021) reveal that supersonic turbulence is not scale-free, and has a characteristic length, the sonic scale, $\ell_s$, for which

$$\left(\left\langle|\mathbf{v}(\mathbf{r}) - \mathbf{v}(\mathbf{r} + \ell_s)|^2\right\rangle_r / 2\right)^{1/2} = c_s, \quad (14)$$

where $\mathbf{v}(\mathbf{r})$ is the velocity field of the turbulence. Furthermore, $c_s$ is the sound speed and $\langle \ldots \rangle_r$ is the ensemble average over positions, $\mathbf{r}$ where the turbulence above $\ell_s$ behaves like Burgers (1948) turbulence (the turbulence of interacting

\(^2\) This is simply because $(\rho/\rho_0)L/\Gamma^n$ on each scale is a random variable composed of the (volume-weighted) linear some of densities on smaller scales, $(\rho/\rho_0)L/\Gamma^n \propto (\rho/\rho_0)L/\Gamma^{n+1} + (\rho/\rho_0)L/\Gamma^{n+2} + \ldots$. This, by definition, leads to the PDF on each scale being an infinite convolution of PDFs from scales below it.

\(^3\) Note that it can, however, be very close to a lognormal model, and for practical purposes this might suffice, but theoretically the s-PDF can not be exactly a normal model.

sawtooth waves), and the scales below Kolmogorov (1941) turbulence (the turbulence of inertially interacting eddies). This means that one can not model the small-scale turbulence and scale the results to the system-scale, as in Vázquez-Semadeni (1994). In fact, through direct measurements of the s-PDF below and above $\ell_s$ (Federrath et al. 2021) show that on scales below $\ell_s$ the s-PDF becomes peaked and kurtotic, and on scales above, negatively skewed. It is clear that the s-PDF is non-Gaussian, even on the subsonic scales of the turbulence.

3 BEYOND LOGNORMALITY

In both ISM observations and in high-resolution numerical studies of supersonic turbulence, we find that the s-PDF deviates from perfect lognormality (Kritsuk et al. 2007; Federrath et al. 2008b; Federrath et al. 2010; Price et al. 2011; Konstandin et al. 2012a; Federrath 2013; Hopkins 2013; Pan et al. 2019; Menon et al. 2021). The reason for these deviations (ignoring gravity and power-law tails due to gravitationally bound, collapsing, dense structures; see Klissen 2000; Federrath et al. 2008a; Kritsuk et al. 2011; Federrath 2013; Girichidis et al. 2014; Mocz et al. 2017; Burkhardt 2018; Khullar et al. 2021, or due to non-isothermal effects; see Passot & Vázquez-Semadeni 1998; Nolan et al. 2015; Federrath & Bower 2015) are due to intermittency, in reference to intermittent, i.e., rare events that are non-Gaussian in nature. More quantitatively, Federrath et al. (2010) described three different realisations that intermittency manifests itself in turbulent flows:

(i) non-Gaussian wings / non-Gaussian higher-order moments of PDFs for turbulent quantities, e.g., $\rho$, $\mathbf{v}$, $B$, their derivatives, and combinations of the quantities (Burkhart et al. 2009; Hopkins 2013; Mocz & Burkhardt 2019; Seta et al. 2020; Mohapatra et al. 2020; Beattie et al. 2020);

(ii) anomalous scaling of the higher-order structure functions of the velocity field (She & Leveque 1994; Castaing 1996; Kowal et al. 2007; Schmidt et al. 2008; Konstandin et al. 2012a; Hopkins 2013);

(iii) structures with intense vorticity, $\nabla \times \mathbf{v}$, and energy dissipation, such as strong shocks and filamentary structures, $\nabla \cdot \mathbf{v} < 0$, and rarefied regions $\nabla \cdot \mathbf{v} > 0$ (Kritsuk et al. 2007; Federrath 2013; Squire & Hopkins 2017; Park & Ryu 2019a; Mocz & Burkhardt 2019; Beattie & Federrath 2020; Yoffe & McComb 2021).

In supersonic turbulence, let alone magnetised supersonic turbulence, the intermittent structures described above dominate the flow, for example through populations of interacting shocks. This has led to large efforts to understand intermittency, which is still one of the most fundamental problems in turbulence, and to characterise it in the ISM (Falgaron et al. 1995, 2009; Hily-Blant & Falgarone 2009; Falgarone et al. 2015). It means to properly model the density field one must include intermittency in the recipe. We now discuss two models that are based upon the life and happenings of shocked regions in the turbulence, and give some insight into the processes underpinning intermittency in the turbulent density field.
The $s$-PDF describes the continuous jump in density. The essence of the H2013 model is that an exponential PDF incorporates random multiplicative changes to the timescales than under-dense regions.

The value of $\langle s_{L/G^n} - s_{L/G^{n+1}} \rangle_{L/G^n}$ is significant, due to, for example, low-volume filling, high-mass over-dense regions. We show illustrations of the H2013 PDF in the left panel of Figure 1 for different $T$ values, showing how larger values of $T$ increase the negative skewness of the PDF. For small values of $T$, there are only negligible density jumps (on average) between any two nested scales. This means as $T \to 0$, the s-field becomes perfectly self-similar, $\langle s_{L/G^n} - s_{L/G^{n+1}} \rangle_{L/G^n} = 0$, and can be approximated by Gaussian-like statistics. Hence $T$ can also be thought of as the average deviation away from perfect self-similarity in the s-field, across all scales. Yet another way of interpreting $T$ is by rewriting Equation 18 in terms of the mass-weighted variance,

$$\frac{\sigma_{s,V}^2}{\sigma_{s,M}^2} = (1 + T)^{3/2},$$

$$T = \left( \frac{\sigma_{s,V}}{\sigma_{s,M}} \right)^{2/3} - 1. \quad (21)$$

where $I_1(\lambda)$ is the modified Bessel function of the first kind, $\lambda$ is a constant, and the $s$-field is a random variable. In particular, $T$ is the magnitude of the mean logarithmic density jump between these scales,

$$T = \langle s_{L/G^n} - s_{L/G^{n+1}} \rangle_{L/G^n} = \left\langle \ln \left( \frac{\rho}{\rho_0} \right)_{L/G^n} - \frac{1}{T} \right\rangle_{L/G^n}. \quad (19)$$

where $\langle \cdots \rangle$ is the ensemble average. Hence $T$ can be thought of as an average $s$-fluctuation between nested scales, averaged over all scales in the turbulence (Squire & Hopkins 2017). Large values (large in this context is not much greater than 0, noting that $T$ has scales similar to that of $s$, as shown in left panel of Figure 1) of $T$ implies that the $\langle s_{L/G^n} - s_{L/G^{n+1}} \rangle_{L/G^n}$ is significant, due to, for example, low-volume filling, high-mass over-dense regions. We show illustrations of the H2013 PDF in the left panel of Figure 1 for different $T$ values, showing how larger values of $T$ increase the negative skewness of the PDF. For small values of $T$, there are only negligible density jumps (on average) between any two nested scales. This means as $T \to 0$, the s-field becomes perfectly self-similar, $\langle s_{L/G^n} - s_{L/G^{n+1}} \rangle_{L/G^n} = 0$, and can be approximated by Gaussian-like statistics. Hence $T$ can also be thought of as the average deviation away from perfect self-similarity in the s-field, across all scales. Yet another way of interpreting $T$ is by rewriting Equation 18 in terms of the mass-weighted variance,

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As we mentioned in §1, for a lognormal density-fluctuation theory $\sigma_{s,V}^2 = \sigma_{s,M}^2$, corresponding to $T = 0$ in the above expression. Therefore non-zero values of $T$ lead a modified relationship between the $\sigma_{s,V}^2$ and $\sigma_{s,M}^2$, which defines a non-lognormal map between the mass-weighted and volume weighted distributions. We will discuss this in more detail in §3.

The H2013 and Castaing (1996) models were motivated for isotropic, homogeneous turbulence, and hence may not be phenomenologically appropriate for MHD. Still we find that the PDF, at least empirically, captures the morphology of the $s$ fluctuations well over a wide range of $M$ and for $|M_{A0}| < 2$. In this study we explore how well this PDF can be extrapolated to the highly-magnetised, highly-supersonic regime and
using the $T$ statistic, we explore how the intermittency of $s$ behaves in the presence of strong magnetic fields and high-$\mathcal{M}$, which was a parameter range not explored in the original H2013 analysis (large $s$ variance, strong $B_0$-field).

### 3.2 Langevin model

Recent work by Mocz & Burkhart (2019), herein MB2019, model the density-PDF of hydrodynamical density fluctuations using a Markov process framework. MB2019 construct a Langevin model,

$$s(t + dt) = s(t) + A(s) dt + \mathcal{N}(0,1)\sqrt{D(s)} dt,$$

$$A(s) = -\frac{s - s_0}{\tau_A} \left[ 1 + H(s - s_0) \frac{3f}{2} \right],$$

$$D(s) = \frac{2\sigma^2 s}{\tau_D},$$

where $A(s)$ is the deterministic or advective term in the model, $D(s)$ is the stochastic or diffusive term, and $\mathcal{N}(0,1)$ is a standard normal distribution. The stochastic term, $D(s)$, contains the turbulent fluctuations $\sigma^2_s$, which change on a dynamical timescale

$$\tau_D = \frac{\ell_0}{(c_s \mathcal{M})},$$

where $\ell_0$ is the turbulent driving scale. The deterministic term, $A(s)$, encodes how logarithmic density dynamically fluctuates about $s_0$ (as a mean-reverting random walk) on timescales

$$\tau_A = \frac{\tau_{A,0}}{1 + H(s - s_0) \frac{3f}{2}},$$

where $H(s - s_0)$ is the Heaviside function, $\tau_{A,0}$ is the characteristic timescale for the fluctuations and $f$ is a constant that encodes how high-density structures, such as the density contrast caused by a shock, live on shorter timescales than the rest of the density fluctuations in the fluid (Robertson & Goldreich 2018; Scannapieco & Safarzadeh 2018). For example, for $s > s_0$, $\tau_A$ is reduced by $1 + 3f/2$, and hence $f$ becomes the fitting parameter for how much shorter the dynamical timescales are for the shocked density structures compared to rarefied and mean-density regions in the turbulence. MB2019 found $f \approx 0.15$ using 1D simulations over a large range of $\mathcal{M}$, which means shocked regions operate on dynamical timescales $\approx 20\%$ shorter than the low-density regions. In this study, we test if $f$ is constant or rather varies over a large range of $\mathcal{M}$ and $\mathcal{M}_{A0}$, extending the analysis of MB2019.

The PDF of MB2019’s Langevin model, $p_{MB}$, defines a solution to the steady-state (time-independent) Fokker-Planck equation,

$$\frac{\partial}{\partial s} \left[ A(s) p_{MB}(s) \right] = \frac{1}{2} \frac{\partial^2}{\partial s^2} \left[ D(s) p_{MB}(s) \right],$$

which has a solution of the form,

$$p_{MB}(s) \propto \exp \left\{ -\frac{(s - s_0)\pi^2}{2\sigma^2_{s,V}(\tau_A/\tau_D)} \right\},$$

which we show for a number of different $f$ parameters in the right panel of Figure 1, where $f = 0$ corresponds to no difference between the high- and low-density fluctuation timescales, i.e., a lognormal PDF, up to $f = 10$, which corresponds to high-density structures that operate on timescales 10 times shorter than the low-density regions. It is obvious that the value of $f$ encodes a $3^{\text{rd}}$ moment, skewness (discussed more in §6), into the PDF through the $O[(s - s_0)^3]$ term in the exponential. This provides a similar morphology to the H2013 model but, with a softer high-density tail (exponential rather than Bessel function).

We will use the MB2019 and H2013 non-lognormal models as building blocks to interpret results in §5. To summarise, MB2019’s Langevin model describes skewness in terms of the difference in dynamical timescales between over-dense and under-dense regions in the turbulence. In contrast, the H2013 model describes a continuous log-Poisson model process, which quantifies the intermittency with the parameter $T$, which is phenomenologically associated with the deviation from perfect self-similarity of the $s$-field, and can alternatively be seen as a modification to the mass-weighted and volume-weighted variance relation. Finally, we note that both of the models we discuss were constructed to address the hydrodynamical phenomenology of intermittency. We believe both models, which in essence rely upon the statistics and timescales of shocked density regions, are able to provide some insight into general supersonic turbulent flows, because large populations of shocked regions form in the density field, with or without a magnetic field present (Lehmann et al. 2016; Park & Ryu 2019b; Beattie & Federrath 2020; Beattie et al. 2020, 2021). Before discussing sub-Alfvénic s-PDF data, we turn to the details of the numerical experiments that we use in our study.

### 4 SUPERSONIC TURBULENCE SIMULATIONS

#### 4.1 Ideal (M)HD model and turbulent driving

In this study we analyse the $s$-PDFs using high-resolution, 3D turbulent, ideal magnetohydrodynamical (MHD) simulations. The corresponding equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \right) \mathbf{v} = \frac{(\mathbf{B} \cdot \nabla)\mathbf{B}}{4\pi} - \nabla \left( c_s^2 \rho + \frac{\mathbf{B} \cdot \mathbf{B}}{8\pi} \right) + \rho \mathbf{f},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \nabla \cdot \mathbf{B} = 0,$$

where $\mathbf{v}$ is the fluid velocity, $\rho$ the density, $\mathbf{B}$ the magnetic field, $c_s$ the sound speed, and $\mathbf{f}$ the turbulent acceleration field. The latter follows an Ornstein-Uhlenbeck process with finite auto-correlation time and is constructed such that we control the mixture of solenoidal and purely compressive modes in $f$ (see Federrath et al. 2010 for a detailed discussion of the turbulence driving). We choose to drive equal energy in both modes, i.e., a natural mixture of solenoidal and compressive driving modes. The driving is isotropic in wavenumber space, centred on $|kL/2\pi| = 2$, corresponding to real-space scales of $\ell_0 = L/2$, and falling off to zero with a parabolic spectrum within $1 \ll |kL/2\pi| \lesssim 3$. Thus, energy is injected only on large scales and the turbulence on smaller scales develops self-consistently through the turbulent energy cascade. The auto-correlation timescale of $f$ is equal to $\tau = \ell_0/\sigma_V = L/(2c_s \mathcal{M})$, hence, we use this timescale to
set the desired turbulent Mach number on $\ell_0 = L/2$ for each simulation. We vary the sonic Mach number between $M = 2$ and 20, encapsulating the range of observed $M$ values for turbulent MCs (e.g., Schneider et al. 2013; Federrath et al. 2016; Orkisz et al. 2017; Beattie et al. 2019).

We use a modified version of FLASH based on version 4.0.1 (Fryxell et al. 2000; Dubey et al. 2008) to solve the MHD equations in a periodic box with dimensions $L^3$, on a uniform grid with resolution $512^3 - 576^3$, using the multi-wave, approximate Riemann solver framework described in Bouchut.
et al. (2010) and implemented in FLASH in Waagan et al. (2011). We utilise up to approximately 6000 compute cores in parallel, for roughly one million compute hours for the most supersonic and magnetised simulations, running at almost 99% efficiency on the Gadi supercomputer hosted by the National Computing Infrastructure Australia. For details on the performance of the modified FLASH code we refer the readers to Federrath et al. (2021) and for more details about the current simulations Beattie & Federrath (2020), Beattie et al. (2020), and Beattie et al. (2021). With a grid resolution of \( \gtrsim 512^3 \) many previous studies have found that the 1-point density statistics are well converged (e.g. Kowal et al. 2007; Kitsionas et al. 2009; Federrath et al. 2010; Price & Federrath 2010; Kritsuk et al. 2011; Federrath 2013; Mohapatra et al. 2021), hence using a \( \gtrsim 512^3 \) computational grid is an appropriate resolution for discussing converged \( \sigma \) statistics.

### 4.2 Initial conditions and processing

The initial velocity field is set to \( v(x, y, z, t = 0) = (0, 0, 0) \), with units \( c_s = 1 \), and the density field \( \rho(x, y, z, t = 0) = \rho_0 \), with units \( \rho_0 = 1 \). The magnetic field is composed out of a fluctuating, \( \delta B(t) \), and a mean-field, \( B_0 \), as \( B(t) = B_0 \hat{z} + \delta B(t) \). The \( B \)-field is initialised with \( B_0 \) threaded through \( \hat{z} \) of the simulations. We consider systems where \( B_0 \) evolves on much larger timescales than \( \delta B(t) \). This could be, for example, a cool \( H_2 \) region that is pierced by a galactic scale mean-field. The mean-field would evolve on galactic timescales, \( \sim \mathcal{O}(\text{Gyr}) \), and the fluctuating field, akin to the turbulence in the region, on much shorter timescales, \( \sim \mathcal{O}(\text{Myr}) \). Hence, the mean, coherent field is approximately frozen in magnitude in the turbulence. We capture this by fixing \( B_0 \) to be constant throughout the entire simulation runtime, i.e. \( \partial_t B_0 = \partial_i B_0 = 0, x_i \in \{x, y, z\} \). Even though the mean-field gradients are zero, field lines, which are dominated by \( B_0 \), are able to walk stochastically (magnetic field line wander, Howes & Bourouaine 2017) in the plane \( \perp B_0 \). The turbulent field, \( \delta B \), evolves self-consistently from the MHD equations, with \( \delta B |_{t=0} = 0 \), and \( |\delta B| \propto \mathcal{M} \mathcal{M}_{\text{AO}} \) (Federrath 2016a; Beattie et al. 2020; Skalidis & Tassis 2020). \( B_0 \) is set by using the definition of the Alfvén velocity and \( \mathcal{M} \), \( B_0 = 2c_s \sqrt{\pi \rho_0} \mathcal{M} / \mathcal{M}_{\text{AO}} \), where \( \mathcal{M}_{\text{AO}} \) is the desired Alfvénic Mach number of \( B_0 \). For the magnetised simula-

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**Figure 3.** An example of the M2MA01 simulation (Table 1) shown at \( t/\tau = 6 \) with a section of the magnetic field (blue) and velocity streamlines (magma) surrounded by a box of \( \log(\rho/\rho_0) \) slices in each direction, and with volume-rendered over-dense regions (\( \rho/\rho_0 \gg 1 \)), shown in black. The velocity structure reveals a deformed vortex twisting around the magnetic field. Typical of sub-Alfvénic mean-field turbulence, the magnetic field is extremely ordered, showing no turbulent component at all via the streamline visualisation. This is because all of the energy is contained in the \( B_0 \) (direction indicated at the bottom-right of the plot) and in the sub-Alfvénic mean-field regime the turbulent velocity fluctuations are too weak to significantly bend the field. However, because \( B \) is not fixed at the boundary, the turbulent motions can advect the magnetic fields in the \( \tau \perp B_0 \) plane, establishing supersonic vertical motions with dynamical timescales \( \tau = t_0/(c_s \mathcal{M}) \), where \( t_0 \) is the energy injection or driving scale.

**Figure 4.** The volume-weighted (top) and mass-weighted (bottom) variances of the logarithmic density field for each of the simulations in Table 1, as a function of eddy turnover time, \( t/\tau \), coloured by \( \mathcal{M} \) and with different line styles for \( \mathcal{M}_{\text{AO}} \). We find that almost all simulations have become approximately stationary between \( 5 \leq t/\tau \leq 10 \), which is the interval where we average over to develop all of the statistics in our study. \( \mathcal{M} \) (changing colour) plays the largest role in setting the value of the variance, and the magnetic field (changing line style) only plays a significant role for the high-\( \mathcal{M} \) simulations, because it preferentially suppresses small-scale fluctuations that are introduced in high-\( \mathcal{M} \) flows (Beattie et al. 2021), coincident with when the turbulence is becoming more globally isotropic.
Table 1. Main simulation parameters.

| Simulation ID | $M (\pm 1\sigma)$ | $M_{\text{AO}} (\pm 1\sigma)$ | $N^3$ |
|---------------|-------------------|-------------------------------|------|
| M2Ma01        | 2.6 ± 0.2         | 0.131 ± 0.008                 | 512$^3$ |
| M4Ma01        | 5.2 ± 0.4         | 0.13 ± 0.01                   | 512$^3$ |
| M10Ma01       | 12 ± 1            | 0.125 ± 0.006                 | 512$^3$ |
| M20Ma01       | 24 ± 1            | 0.119 ± 0.003                 | 512$^3$ |
| M2Ma05        | 2.2 ± 0.2         | 0.54 ± 0.04                   | 512$^3$ |
| M4Ma05        | 4.4 ± 0.2         | 0.54 ± 0.03                   | 512$^3$ |
| M10Ma05       | 10.5 ± 0.5        | 0.52 ± 0.02                   | 512$^3$ |
| M20Ma05       | 21 ± 1            | 0.53 ± 0.02                   | 512$^3$ |
| M2Ma1         | 2.0 ± 0.1         | 0.98 ± 0.07                   | 512$^3$ |
| M4Ma1         | 3.8 ± 0.3         | 0.95 ± 0.08                   | 512$^3$ |
| M10Ma1        | 9.3 ± 0.5         | 0.93 ± 0.05                   | 512$^3$ |
| M20Ma1        | 18.8 ± 0.7        | 0.93 ± 0.03                   | 512$^3$ |
| M2Ma2         | 1.7 ± 0.1         | 1.7 ± 0.1                     | 512$^3$ |
| M4Ma2         | 3.5 ± 0.1         | 1.73 ± 0.07                   | 512$^3$ |
| M10Ma2        | 9.0 ± 0.4         | 1.8 ± 0.1                     | 512$^3$ |
| M20Ma2        | 18 ± 1            | 1.8 ± 0.1                     | 512$^3$ |
| M2Ma∞         | 2.15 ± 0.07       | ∞                             | 576$^3$ |
| M4Ma∞         | 4.1 ± 0.1         | ∞                             | 576$^3$ |
| M10Ma∞        | 10.1 ± 0.5        | ∞                             | 576$^3$ |
| M20Ma∞        | 20.2 ± 0.7        | ∞                             | 576$^3$ |

Notes: For each simulation we extract 51 realisations at 0.1 $\tau$ intervals, where $\tau = L/(2c_6 M)$ is the turbulent turnover time, between 5-10 $\tau$. All $\sigma$ fluctuations listed in the table and study are from time-averaging quantities over the 5 $\tau$. Column (1): the simulation ID. Column (2): the rms turbulent Mach number, $M = \sigma_v/c_s$. Column (3): the Alfvén Mach number for the mean-$B$ component, $B_0$, $M_{\text{AO}} = (2c_6 M \sqrt{\rho_0})/|B_0|$, where $\rho_0$ is the mean density and $c_s$ is the sound speed. Column (4): the number of grid cells in the discretisation of the spatial domain, $L^3$.

5 MORPHOLOGY OF THE 3D DENSITY-PDFS

In §3 we discussed two non-lognormal volume-weighted s-PDF models that have been used to describe logarithmic density fluctuations in supersonic turbulence. In Figure 5 we plot the volume-weighted s-PDFs for each of the magnetised simulations from Table 1. We fit a $\text{H2013 model}$, $p_V(s;\sigma_v^2,T)$, Equations 15-18, $\text{MB2019 model}$, $p_{\text{MB}}(s;\sigma_v^2,T)$, Equations 28 and a lognormal model, $p_{\text{LN}}(s;\sigma_v^2)$, Equations 1-3 to the s-PDFs. The $\text{H2013}$ and $\text{MB2019}$ fits are qualitatively much better than the lognormal model for the low-$M$ simulations, which set the maximum limit of the spread of the PDF, along with or without a magnetic field. We draw purple dashed lines where $\ln \rho/\rho_0 = 0$. This shows that the low-$M$ intermittency is not necessarily from an excess of volume-weighted high-density structures, but from an excess of low-density rarefactions and density voids. As we discussed in §5, for $M \gtrsim 4$ the fits become, qualitatively, equally as good as the lognormal model. For the mass-weighted s-PDFs, shown in Figure 6, we fit a lognormal model, $p_{\text{LN}}(s;\sigma_v^2)$, Equations 6-8. In the next subsections we split the discussion of the volume- and mass-weighted PDFs and their fits into the low- and high-density tail.

5.1 High-density tail

The high-density tail of the PDFs in Figure 5 and 6 trace the shocks, filaments, sheets and other over-dense structures in the supersonic turbulence (Robertson & Goldreich 2018; Federrath et al. 2021). These structures make-up a small fraction of the total volume, and most of which is filled with rarefactions and voids that are found in the low-density tail between 1/$M \leq \rho/\rho_0 \leq 1$. However, most of the mass is found between 1 $\leq \rho/\rho_0 \leq M$, so the high-density tail contains most of the mass, but little of the volume (Robertson & Goldreich 2018).

Because the high-density tail is set by the shocked gas, it is no surprise that Pan et al. (2019) found the tail is amplified by the $\nabla \cdot \mathbf{v}$ term in the continuity equation and suppressed by the $\nabla P = c_s^2 \nabla \rho$ term in the momentum equation for hydrodynamical turbulence. Since $\nabla \cdot \mathbf{v} \sim v/L = c_s M/L$ for a fixed $L$, the high-density tail ought to increases systematically with $M$. However, we find that it is asymptotic at high-$M$, which Beattie et al. (2021) attributes to the total volume of the shocks forming along $B_0$ reducing until they no longer have a significant contribution to the spread of the PDF. Beattie et al. (2021) suggests that at this point the largest contribution is from weakly compressible MHD shocks that form from field line compressions (fast magnetosonic shocks), which set the maximum limit of the spread of the PDF, along with the type of turbulent driving.

We find that the lognormal model (shown with dotted markers) systematically overestimates the PDF in the high-density tail for the $M = 2-4$ simulations, regardless of $M_{\text{AO}}$. This is likely from a mixture of the strong magnetic pressure, $\nabla B^2/(8\pi)$, and the magnetic tension, $(\mathbf{B} \cdot \nabla) \mathbf{B}$. Both the pressure and tension terms scale with $M_{\text{AO}}^2$, and when $M_{\text{AO}}$ is sufficiently small, the pressure acts to homogenise the density, and the tension acts to smooth out any curvature in the magnetic field, which in turn reduces any large contrasts in the density through $B \propto \rho$ flux-freezing (Landau & Lifshitz...
In this $|B_\| \gg |\delta B|$ regime, the strongest shocks are only able to form along the mean magnetic field (Beattie & Federbath 2020; Beattie et al. 2020, 2021). This results in less available volume where strong shocks are able to form and therefore fewer over-densities. Because there are fewer shocked re-

1959; Mocz & Burkhart 2018; Yuen & Lazarian 2020). The overall result is that the magnetic field limits both high- and low-density fluctuations (Nordlund & Padoan 1999; Molina et al. 2012; Hennebelle 2013; Mocz & Burkhart 2018; Beattie et al. 2021).
regions there is less dense material that is able to contribute to the high-density tail of the PDFs. We explore this in more detail in §7 using one-dimensional pencil beams that probe the flow. However, as $\mathcal{M}$ increases to $\mathcal{M} \gtrsim 10$, the $s$-field on large scales becomes more isotropic, allowing for more space where shocks can form, filling up again the high-density tail of the PDF which is then fit well by the lognormal model. Shock frequencies have previously been found to increase with $\mathcal{M}$ and decrease with $|B_0|$ (Park & Ryu 2019b). Beattie & Federrath (2020) observed large-scale mixing and isotropisation in the density field by studying the 2D power spectra, which were isotropic on large scales, and then anisotropic on the scales of the individual shocks for $\mathcal{M} \gtrsim 10$.

In the mass-weighted PDFs, shown in Figure 6, we find that the lognormal model performs best for the high-density tails at low-$\mathcal{M}$, low-$\mathcal{M}_{Ao}$ and worst in the high-$\mathcal{M}$, trans-

Figure 6. The same as Figure 5 but for the mass-weighted $s$-PDFs. The lognormal fit, $p_{N,M}(s; \sigma_{s,M}^2)$, Equations 6-8, is shown with dots. Most of the lognormal PDFs overestimate the high-$s$ tail, and underestimate the low-$s$ tail. The high-$s$ tail has significant temporal fluctuations, revealing that the high-mass, high-density structures: filaments, over-dense sheets and shocked regions in the turbulence, fluctuate significantly in mass. Conversely, the lowest density structures, relative to the high-density tail, do not fluctuate significantly.
Alfvénic flows. Similar to the volume-weighted PDF, the lognormal model over-predicts the amount of high-density structures in the turbulence. Unlike the volume-weighted PDF, the 1σ temporal fluctuations (indicated by using transparent bands) from the mass-weighted PDF are strongest in the high-density tail. By tracking and analysing individual overdense regions Robertson & Goldreich (2018) showed that the temporal fluctuations come from the transient nature of the shock life-cycle, where shocks form, accumulate, lose mass and are torn apart within a fraction of $t_{\text{cross}} = L/c_s$. These are found to a much lesser degree in the volume-weighted PDFs, which demonstrates how the highest density structures in the turbulence must dynamically fluctuate through time, in mass, but be relatively stable in volume. The temporal fluctuations in the high-density bins increase with $M$, supporting the ideas of Padoan et al. (1997), Robertson & Goldreich (2018) and Mocz & Burkart (2018), which state that the internal structure and geometrical properties of in situ shocks is set by the properties of the turbulence.

5.2 Low-density tail

The low-density tail captures the density rarefaction waves and voids in the turbulence (Kritsuk et al. 2007; Federrath et al. 2010). These are coupled to the shocks because as the shocks compress the fluid they also evacuate large regions in the turbulence. These regions occupy the largest volumes in the flow, but have the smallest mass (Robertson & Goldreich 2018). For the volume-weighted PDF the low-density temporal fluctuations are significant, shown by the large 1σ values in the low-density tails in Figure 5. This can be interpreted to mean that the volumes of the low-density regions fluctuate throughout the life-cycle of the under-densities and hence the rarefactions are themselves strong sources of intermittency (Passot & Vázquez-Semadeni 1998; Kritsuk et al. 2007; Federrath et al. 2010).

For the mass-weighted PDFs in Figure 6, the temporal fluctuations are small in the low-density tail, hence, we find an interesting and understandable symmetry between the low-density and high-density regions. The voids and rarefactions in the turbulence fluctuate in volume, and much less so in mass, and the high-density regions fluctuate in mass, and much less so in volume. This is one of the key results from our analysis of the time-averaged PDFs, unrelated to the Gaussianity.

Similarly to what we found previously for the high-density tail in §5.1, as $M$ increases the lognormal model fits the $s$-PDF better. At low-$M$, and for most $M_{\text{AO}}$, the low-density tail under-predicts the amount of rarefactions and voids in the turbulence. We have until now restricted most of our attention to the lognormal fit. In the next section we consider the more general H2013 and MB2019 fits, which have associated intermittency parameters, $T$, that encodes the deviation away from perfect self-similarity through different scales in the $s$-field, and $f$, that captures the difference between the dynamical times of shocked regions and rarefactions, respectively.

Figure 7. Top: The volume-weighted skewness of the logarithmic density, $S_s$, as a function of turbulent eddy turnover time between $0 \leq t/\tau \leq 10$. The arrow describes the general trend we find in the skewness, which is as $M$ increases the skewness tends towards zero, corresponding to a more lognormal $s$ field. Bottom: $S_s$ as a function of $M$, averaged over $5 \leq t/\tau \leq 10$, with different line styles for $M_{\text{AO}}$, as indicated in the legend from the top panel.

6 INTERMITTENCY IN THE $s$-PDF

We qualitatively observed in the volume- and mass-weighted logarithmic density-PDFs that the moderate- to high-$M$ simulations are fit well by a lognormal distribution, and the low-$M$ simulations show significant deviation from the lognormal fit in both the low- and high-density tail. Now we aim to quantify these non-Gaussian features, focusing primarily on the volume-weighted PDF where the intermittency parameters computed from the H2013 and MB2019 models are valid.

6.1 Skewness

We quantify the intermittency of the volume-weighted $s$-PDFs in four ways. First, we calculate the 3rd central moment, the skewness of $s$. It is defined as

$$S_s = \frac{\langle (s - \langle s \rangle)^3 \rangle}{\sigma_{s,v}^3}.$$  \hspace{1cm} (32)

This gives $S_s = 0$ for the normal distribution, $S_s > 0$ for a distribution with an elongated tail towards the high-density
Figure 8. Left: The H2013 intermittency parameter, $T$, as a function of $M$, coloured by $M_{A0}$, split into three categories: sub-Alfvénic ($M_{A0} < 1.0$, orange), trans-Alfvénic ($1 \leq M_{A0} \leq 2$, aqua) and hydrodynamical ($M_{A0} = \infty$, black) with the H2013 hydrodynamical fit shown with the grey band for different driving parameters, $b = 0.3 - 1$. The Squire & Hopkins (2017) fit is shown with the black dot-dashed line, which is based on the shape and density jump of hydrodynamical shocks. A fit to the sub-Alfvénic models, $T(M) = 0.2^{+0.39}_{-0.19} M^{-0.81_{-0.60}^{+1.09}}$, is shown with the orange band and trans-Alfvénic, $T(M) = 1.66_{-1.0}^{+2.51} M^{-3.24_{-3.78}^{+2.70}}$, with the aqua band. The bands encapsulate the parameter uncertainties for each of the fits. Right: The mass-weighted $s$ dispersion as a function of volume-weighted $s$ dispersion, coloured the same as the left plot. The lognormal model defines the 1:1 line shown in black. The $T = 0.05$ and $T = 0.5$ lines show the cases for weak intermittency and strong intermittency, respectively, from the H2013 model. We show a band that encapsulated the H2013 fit with the grey band. In both plots, the $M = 2 - 4$ simulations show intermittency comparable to the H2013 fits. However, for $M \gtrsim 4$ the logarithmic density fields become significantly more Gaussian, with shrinking $T$ and dispersion relation closer to the 1:1 lognormal line, illustrating a key difference between magnetohydrodynamical and hydrodynamical $s$ statistics.

tail and $S_s < 0$ for an elongated low-density tail. Because we compute the central moment the magnitude of $S_s$ is weighted with respect to the volume-weighted variance, $\sigma_s^2$. Hence for $|S_s| \geq 1$ the skewness is just as, or more important for determining the morphology of the $s$-PDF than $\sigma_s$.

We plot the temporal evolution of the skewness for the volume-weighted distribution in the top panel of Figure 7, coloured by different $M$ and illustrated with different line styles for different $M_{A0}$, using the same style as Figure 4. $S_s$ becomes stationary for $t \gtrsim 5\tau$, which we demonstrate in the top panel. Before $t = 5\tau$, $S_s$ the skewness increases in magnitude with $M$, which is the opposite trend we found in averaged PDFs. We show the $S_s$ averaged over $5 \leq t/\tau \leq 10$ in the bottom panel of Figure 7. Tracing the different line styles through the plot, which correspond to different $M_{A0}$, we can immediately see that the absolute value of the skewness decreases and tends towards $S_s \approx 0$ as $M \gg 1$ for the sub-Alfvénic simulations, and $S_s \approx 0.2$ for the trans-Alfvénic simulations. For low-$M$ $S_s$ varies between $-0.3$ and $-0.5$, regardless of $M_{A0}$, meaning that the peak of the PDF is shifted towards the higher densities, with an elongated tail into the low-densities, consistent with our qualitative findings in §5, and the type of intermittency described in the H2013 and MB2019 models. The trans-Alfvénic simulations show some deviation to higher values than $S_s = 0$ but because $|S_s|$ decreases with $M$ all of the simulations share the same trend towards more Gaussian statistics. Since we find $|S_s| < 1$, $S_s$ is never more important than $\sigma_s^2$ for describing the morphology of the distribution in these strong mean-field, supersonic flows, i.e. a lognormal model is not strictly a bad empirical approximation for the $s$-PDF. However, because any non-zero $S_s$ is a result of asymmetry in the PDF; models that compute astrophysical properties, such as star-formation rate or efficiency from the high-density tail (Krumholz & McKee 2005; Hennebelle et al. 2011; Padoan & Nordlund 2011; Federrath & Klessen 2012; Federrath & Klessen 2013; Burkhardt 2018), which is significantly overestimated by the lognormal model (Figures 5 and 6) may overestimate the amount of shocked and over-dense gas, especially for lower-$M$, magnetised MCs.

Our $S_s$ results may seem to contradict previous findings of Kowal et al. (2007) and Burkhardt et al. (2009), but they analysed the higher-order statistics of not the logarithmic densities, but the linear densities, $\rho/\rho_0$. The higher-order statistics of $\rho/\rho_0$ and $s$ need not be alike; and in fact we have shown here they are significantly different. Our findings are consistent with those of Molina et al. (2012), who found that magnetised gas densities are much more lognormal than in hydrodynamical turbulence, but did not quantify it in detail.
6.2 H2013 intermittency parameter

To understand the transition from the low-\(M\) intermittency to the high-\(M\) Gaussianity we see both in the skewness statistics and directly from the PDFs we plot \(T\) as a function of \(M\). At low-\(M\) we find that the timescales of shocked gas evolve approximately twice as fast as low-density gas, but as \(M\) increases the timescales homogenise, resulting in more Gaussian \(s\)-PDFs.

\[
\frac{1 + 3f}{2} = \tau_A = \tau_{A,0}
\]

Figure 9. The MB2019 intermittency parameter, \(f\), written in terms of the reduction factor between the dynamical timescales for the rarefactions and the shocked gas density, \(1 + 3f/2\), as shown in Equation 26, as a function of \(M\). At low-\(M\) we find that the timescales of shocked gas evolve approximately twice as fast as low-density gas, but as \(M\) increases the timescales homogenise, resulting in more Gaussian \(s\)-PDFs.

6.3 MB2019 intermittency parameter

Using the MB2019 fits we extract the intermittency parameter, \(f\) (Equation 26), and plot \(1 + 3f/2\) as a function of \(M\), with different line styles for \(M_{A0}\) in Figure 9. We use \(1 + 3f/2\) because this is the reduction factor between the dynamical timescales for \(s > s_0\), \(\tau_A\) and \(s \leq s_0\), \(\tau_{A,0}\) structures in the MHD turbulence. The horizontal blue line shows the case where both the under-density and over-density dynamical timescale are the same, i.e. when \(f = 0\). At low-\(M\), \(\tau_A \approx \tau_{A,0}/2\), hence the shocked structures are operating on timescales twice as fast as the voids and rarefactions. This results in the most non-Gaussian \(s\)-PDFs at low-\(M\), similar to what we found using the skewness and \(T\) parameter. Consistent with our previous measurements, as \(M\) increases the field becomes more Gaussian, which can be interpreted as the two dynamical timescales becoming equal. The sub-Alfvénic simulations take the longest to reach Gaussian-like statistics, consistent with what we found in the MB2019 simulations. We discuss this in much more detail in §8.

6.4 Summary of §6

To summarise this section, we compute the logarithmic density intermittency with four independent measures: (i) the skewness, \(S_s\) (Equation 32), (ii) the H2013 \(T\) parameter (Equation 19), (iii) the \(\sigma_{s,M} - \sigma_{s,V}\) relation (Equation 20) and (iv) the MB2019 \(f\) parameter (Equation 26). All statistics point to the conclusion that \(s\)-intermittency in strong mean-field MHD turbulence is significantly different, and in fact to highly-supersonic, magnetised turbulent flows, assuming that the H2013 PDF can be used to interpret our MHD data. The sub-Alfvénic \(T\) values (orange) fit by a power law, \(T(M) = 0.27_{-0.19}^{+0.39} M^{-0.81_{-1.03}^{+0.60}}\) and the trans-Alfvénic (aqua) by \(T(M) = 1.66_{-0.24}^{+0.51} M^{-3.24_{-3.78}^{+3.0}}\). Hence, the sub-Alfvénic simulations seem to be more intermittent than the trans-sonic \(M_{A0}\) simulations, consistent with what we found in the skewness. We interpret this with the H2013 phenomenology as follows: the average jump in density between nested scales in the turbulence, \(T\) (Equation 19) scales with \(M^{-1}\) in the sub-Alfvénic simulations, and \(\sim M^{-3}\) in the trans-Alfvénic simulations. We discuss this in much more detail in §8.

One of the key motivations for H2013 was to address the discrepancy between the mass-weighted and volume-weighted variances (Equation 20), which, as we highlighted in §1, should be identically equal, \(\sigma_{s,V}^{2} = \sigma_{s,M}^{2}\), if a lognormal theory describes the density fluctuations. As another independent measure of the intermittency we therefore plot \(\sigma_{s,M}\) as a function of \(\sigma_{s,V}\) in the left panel of Figure 8. We plot a grey band that encapsulates the empirical relation that H2013 fit to the hydrodynamical data, the weak (\(T = 0.05\), blue, dotted) and strong (\(T = 0.5\), red, dotted) intermittency isocontours and the \(\sigma_{s,V} = \sigma_{s,M}\) line in black. We find that the low-\(M\) density structures follow more closely the H2013 intermittency relation, but as \(M\) increases the density field becomes lognormal, tending towards the one-to-one lognormal line with a small amount of scatter, consistent with our previous measurements.
opposite (decreases with $M$) compared to purely hydrodynamical turbulence (increases with $M$), which is a key result of this study. However, at low-$M$ ($M \lesssim 4$) there considerable H2013-like intermittency present in the s-field, i.e., $T$ values that are comparable to $T$ values computed for hydrodynamical turbulence. We now turn our attention to identifying what physics leads to this low-$M$ intermittency by identifying the intermittent structures in real-space, following the philosophy from Imara et al. (2021) – even theoretical astrophysics is, at heart, an observational endeavour that relies upon us being able to visualise the structures we are interested in.

### 7 THE PHYSICS OF LOW-$M$ INTERMITTENCY

Beattie & Federrath (2020) showed that the density structures in $M \approx 2 - 4$, sub- to trans-Alfvénic turbulence are anisotropic on all length scales. On average, equi-power surfaces in the power spectrum reveal that they are stretched along the mean magnetic field\(^4\). For this reason, this regime can naturally be decomposed into two domains: across ($\ell_\perp$) and along ($\ell_\parallel$) $B_0$, which on average defines a symmetrical axis ($\delta v_\parallel \sim \delta v_\perp$, $\delta B_\perp \sim \delta B_\parallel$) in the turbulence\(^5\). Since the M2MA01 simulation is the most anisotropic (Beattie & Federrath 2020; Beattie et al. 2020), this is where such a simple domain decomposition will work the best. Also, since the M2MA01 is significantly intermittent (of order the equivalent hydrodynamical simulation), we focus our analysis in this section primarily on the local dynamics and intermittency of this simulation.

To analyse the real-space intermittent structure we take pencil beams (1D slices) in the $\ell_\parallel$ and $\ell_\perp$ directions to examine examples of the frozen-in-time local dynamics along each direction. In Figure 10 we show pencil beams for the logarithmic density (blue), kinetic turbulent velocity, $\delta v$, (red, solid, dashed) and Alfvén turbulent velocity, $\delta v_A = |\delta B|/\sqrt{4\pi\rho}$ (red, dot-dashed) profiles. To make sure the beams we plot are representative of the average, typical dynamics we check the other 1024 possible pencil beams and find that these represent that average dynamics in the M2MA01 simulation faithfully (i.e., they are not outliers or peculiar in any way). The left panel illustrates a beam along $B_0$ (magnetic field direction is indicated in the top, left) and the right panel, across. To help the reader contextualise the pencil beams, in Figure 11 we indicate the positions of each of the pencil beams.

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\(4\) Note that on average for $M \gtrsim 4$ turbulence the density anisotropy is across and not along the mean magnetic field due to formation of perpendicular, high-density filaments; see Figures 2 and 3 in Beattie & Federrath (2020).

\(5\) From time-averaging experiments we find the symmetry is an average property of the turbulence and events, for example, strongly interacting vortices, can lead to velocity streams in preferential directions across the field, “breaking” the symmetry around $B_0$.  

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Figure 10. An example of profiles from two typical pencil beams along (left) and across (right) $B_0$ in the M2MA01 simulation. The plot is organised such that the logarithmic density is the blue solid line, corresponding to the left, blue axis, the kinetic velocities are either solid or dashed red lines, corresponding to the right red axis, in units of $c_s$ and Alfvén velocity $\delta v_A = |\delta B|/\sqrt{4\pi\rho}$, is a dot-dashed red line, also sharing the red axis with the kinetic velocity. Integer values of the global $\sigma_s$ for the $s$ field are shown as horizontal lines across the plots, indicating how extreme the $s$-fluctuations are. The $B_0$ direction is indicated in the top-left of each panel. Left: The parallel pencil beam reveals large-scale, over-densities with an exponential tail, caused by converging flows (in the grid frame) along $B_0$, consistent with type C filament formation (Abe et al. 2020). The structure of the over-density is qualitatively similar to atmospheric shocks studied in Robertson & Goldreich (2018) and Mocz & Burkhart (2018). A sawtooth shockwave travels down $B_0$ and is coupled to a volume-filling rarefaction wave, travelling in the opposite direction. At the shockwave interface fast magnetosonic (travelling at $\sim c_s$ with the mean-field subtracted) compression waves are excited that also travel down $B_0$. Right: The perpendicular pencil beam reveals high-frequency logarithmic density features that are correlated with velocity streams along the field, $\delta v_\parallel/c_s$. Large-scale vortical structures are seen in the $\delta v_\perp/c_s$ profile, with two vortices developing on $[-L_\perp/2, L_\perp/2]$. The shear Alfvén wave fluctuations, $\delta v_{A,\perp}/c_s$, are extremely small-scale compared to the kinetic fluctuations.
7.1 Along $B_0$-field pencil beam

In the left panel we see an in situ sawtooth shock coupled to an over-density forming perpendicular to $B_0$ (solid, blue) from converging kinetic velocities (solid, red line) along $B_0$, in the simulation frame. Converging flows are a popular hypothesis for filament formation in molecular clouds (e.g. Chen et al. 2020; Bonne et al. 2020), and in particular for sub-Alfvénic, compressible gases (Padoan & Nordlund 1999; Chen & Ostriker 2014; Abe et al. 2020). However, the interplay between the magnetic field and gravity orienting the filaments with respect to the magnetic field is still debated in the literature (Plack Collaboration et al. 2016; Soler et al. 2017; Tritsis et al. 2018; Mocz & Burkhart 2018; Heyer et al. 2020; Pillai et al. 2020; Barreto-Mota et al. 2021; Girichidis 2021). We show here that perpendicular filaments caused by converging flows along $B_0$ can be seeded without the presence of gravity, that such dense filaments are primarily created (or at least seeded) by turbulent compression (Federrath 2016b). The formation scenario is as follows: the strong $B_0$-field acts by constraining the turbulence along $B_0$ (suppressing diagonal velocity modes), compressible modes, which are injected isotropically from our turbulent driving source, are self-organised along $B_0$, and the gas density along the magnetic field becomes shocked, forming over-dense regions across the $B_0$. Beattie et al. (2020) found that the self-organisation of the compressible modes in the velocity was a natural reper-

\[ |\nabla \cdot v| = |\partial_0 v|, \]  

where $\partial_0$ is the derivative along $B_0$, and there is only negligible velocity divergence across the field, $\nabla \cdot \mathbf{v}_\perp \approx 0$, and the compressive motions are along the $\ell_0$ beam. It is also worth noting how similar this phenomenology is to the model for weakly-compressible MHD turbulence proposed by Bhattacharjee et al. (1998).

We highlight and emphasise here that in situ turbulent over-densities are not morphologically the same as simple shock tube models, which is shown both by the over-density (solid, blue line) and the shock (solid, red line) in the left panel of Figure 10, which have exponential tails, and sawtooth structure, respectively. This demonstrates that in situ over-densities have internal structures (e.g., scale heights), i.e., are not just discontinuities, and that in the strong $B_0$ regime Burgers (1948)-like turbulence develops along $B_0$. The over-density we highlight is qualitatively similar to the hydrodynamical atmospheric shocks studied in Robertson & Goldreich (2018) and extended to MHD in Mocz & Burkhart (2018). Robertson & Goldreich (2018) explains the exponential tail by balancing the ram pressure, which is $\propto c^2 v^2$, with the ambient pressure gradient $\nabla P$. We also note how the over-density is approximately consistent with the simple hydrodynamical model proposed by Padoan & Nordlund (2011) for the thickness of the post-shock layers. Like the Robertson & Goldreich (2018) model, Padoan & Nordlund (2011) uses a pressure balance to derive a characteristic width of a shock. For the sawtooth shock we find, $\ln \rho_0/\rho_0 = \ln M^2 \sim 1.4$, and shock width $L_0/M^2 = L_0/4$ ($M = 2$ for the M2RA01 simulation), which defines a rectangle that captures the basic geometry of the over-density captured in the pencil beam. Many models for the logarithmic density variance (e.g. Molina et al. 2012; Federrath 2015; Nolan et al. 2015; Beattie et al. 2021) rely upon this model for relating the shock jump relations to the variance. Beattie et al. (2021) in particular modelled the variance along $B_0$ with hydrodynamical shock jump and width conditions, which is supported here.

At the sawtooth shock front (at roughly $-L_0/8$ in the left panel of Figure 10) some fast magnetosonic ($\delta B_0^2$) compression ($\delta B_0^2$) waves are excited (shown with dot-dashed, red line), which propagate down $B_0$. These magnetic compression waves become amplified in the sub-Alfvénic regime when shock formation becomes highly oriented along $B_0$ (Beattie et al. 2020), favouring the excitation of high-amplitude, fast magnetosonic compression waves compared to perpendicular shear Alfvén waves. Beattie et al. (2020) found that the compression waves preferentially oppose the direction of $B_0$, reducing the total $B$, and allowing pressure-supported vortices to be maintained in the turbulence through $\sim \nabla B^2$.

\[ v = \sqrt{c_s^2 + v_{A0}^2}, \quad v \propto \sqrt{c_s^2 + v_{A0}^2}, \]

Note here that the MHD waves speeds in the parallel direction are $v = \delta v_A + v_{A0}$, but we show the mean-subtracted speeds, $\delta v_A$, consistent with fast magnetosonic waves, which travel at these speeds isotropically in the fluid, at least for trans-sonic turbulence (Makwana & Yan 2020).

Figure 11. Positions of the 1D pencil beams shown in a 2D slice of the logarithmic density, which are randomly chosen and represent a typical, average pencil beam through the turbulence. The along $B_0$-field pencil beam (left panel of Figure 10) is shown with the vertical line and the across $B_0$-field pencil beam (right panel of Figure 10) is shown with the horizontal line. The along $B_0$-field pencil beam has been shifted into the centre of the 2D map for ease of viewing.

\[ F_{\text{MHD log-density intermittency}} 15, MNRAS 000, 1–24 (2021) \]
Away from the shock front the $\delta B_\parallel$ gradient becomes very small, $|\partial B/\partial t_\parallel| \approx 0$. This has repercussions for the magnetic tension. The magnetic tension written in normal $\hat{e}_n$ and tangential $\hat{e}_t$ $B$-field coordinates is,
\begin{equation}
(B \cdot \nabla) B = B \frac{\partial B}{\partial \lambda} \hat{e}_t - \kappa B^2 \hat{e}_n, \tag{34}
\end{equation}
where $\lambda$ is the arc length parameterisation variable along $B$, and $\kappa$ is the $B$-field curvature. If gradients along the field are small, as they are everywhere except at the shock interface in the left panel of Figure 10, then clearly
\begin{equation}
(B \cdot \nabla) B \approx -\kappa B^2 \hat{e}_n \implies (B \cdot \nabla) B \perp B_0, \tag{35}
\end{equation}
and the tension becomes purely a restoring force for the Alfvénic fluctuations. At the shock interface the parallel $B$-field gradient is very large compared to the amplitudes of the shear Alfvén waves as we show in the right panel of Figure 10, and
\begin{equation}
(B \cdot \nabla) B \approx B \frac{\partial B}{\partial \lambda} \hat{e}_t \implies (B \cdot \nabla) B \parallel B_0. \tag{36}
\end{equation}
This demonstrates that in the sub-Alfvénic mean-field turbulence regime the $(B \cdot \nabla) B$ term becomes anisotropic. This has repercussions for both the energy cascade in MHD turbulence, where $(B \cdot \nabla) B$ is found to play a role in suppressing the kinetic energy transfer (Grete et al. 2020), and the nature of solenoidal modes in the velocity field, where $(B \cdot \nabla) B$ is found to be a strong source of vorticity generation when the $B_0$ is strong (Lim et al. 2020).

### 7.2 Across $B_0$-field pencil beam

Turning our attention to the pencil beam perpendicular to $B_0$ in the right panel of Figure 10, we find significant amounts of high-frequency density structure, which do not correlate with the perpendicular kinetic velocity (dashed, red line). The perpendicular velocities trace the structure of two large, supersonic, rigid body ($u_\perp/c_s \propto \ell/L_\parallel$) counter-rotating vortices in the plane $\ell_\perp$ around $B_0$. These vortices are visualised in 3D, through the velocity streamlines in Figure 3, and previously in the 3D sub-Alfvénic turbulence renderings in Figure 2 in Beattie et al. (2020). The large density contrasts, up to roughly $M^2$, are more strongly correlated with $v_{||}/c_s$ (red, solid line), which we observed were caused from converging flows along $B_0$. We conclude that the largest over-densities in the strong-$B_0$ MHD regime are caused by convergent flows along $B_0$, consistent with the anisotropic $p/p_0$-variance model proposed by Beattie et al. (2021). The shear Alfvén wave fluctuations (dot-dashed, red line) are extremely weak and on small scales compared to both the kinetic velocities and the magnetic compression waves in the left panel, consistent with the ratios between $\delta B_\parallel/\delta B_\perp > 2$ found for the sub-Alfvénic regime in Beattie et al. (2020). This suggests that models like Goldreich & Sridhar (1995), which rely on a critical balance between the turbulent eddies and shear Alfvén wave packets may only be appropriate on very small scales in supersonic turbulence, where the flow becomes strongly Alfvénic (with respect to the Alfvénic fluctuations and not the mean field). This illustrates what is a commonly not considered by some of the astrophysical turbulence community – sub-Alfvénic mean-field turbulence does not mean that the shear Alfvén waves are dominating the fluid dynamics. In fact, the perpendicular fluctuating field is clearly super-Alfvénic with respect to the kinetic turbulence, and all of the magnetic energy is stored in $B_0$.  

### 7.3 Intermittent events and 1D shock experiments

Now that we understand how to interpret the pencil beams, we turn our attention to using them to seek the events that contribute to the s-PDF intermittency. The H2013-like intermittency manifests itself in the volume-weighted s-PDF by elongating the low-$s$ tail, and truncating the high-$s$ tail. As a proxy for understanding what kind of flow events give rise to the elongated low-$s$ tail we create pencil beams that intersect through the grid element in the simulation that has the minimum value of $s$, $s_{\text{min}}$. We construct parallel pencil beams, because as we noted in the previous section, the largest density contrasts are formed by compressive motions along $B_0$. We show these pencil beams in Figure 12 for the density, and kinetic and fast magnetosonic field fluctuations (with the same colouring and line style scheme as Figure 10), for time realisations between $t/\tau = 5 - 9.8$, where $t/\tau = 5$ is shown in the top-left corner, and $t/\tau = 9.8$ in the bottom-right corner of the plot. We annotate the skewness, $S_s$, in the top-left corner of the plot.

Our first conclusion is that many of the 49 visualised extreme low-$s$ events are formed by single, strong shocks, which is especially apparent in the time series from $t/\tau = 6.2 - 7.5$. In this time-correlated⁷ set of pencil beams we see a $M \approx 3$ shock (a $\sim 1.5\sigma$ event in the velocity field for the M2MA01 simulation) moving down the field, evacuating a large void in the turbulence, where $s_{\text{min}}$ resides. The strong shock also facilitates an equally large fast magnetosonic modes. Clearly this hints at density, magnetic and velocity intermittency being linked through these strong-shock events, and explains the physical origins of the fast magnetosonic wave intermittency studied in Ho & Lazarian (2021), i.e., as a side effect of shock generation in the supersonic plasma. The median, 16th and 84th percentiles for the skewness of all of the intermittent beams is $S_s = -0.70_{-1.33}^{+0.33}$, almost a factor of 2 more skewed than the average $S_s$ calculated for the global s-field in M2MA01 in §6. This demonstrates that these events contribute significantly to the non-Gaussian features of the s-PDF. The most skewed beam is at $t/\tau = 5.5$, with $S_s = -1.9$, where a strong, almost $M \approx 4$ ($2\sigma$ event in the velocity) shock compresses an under-dense region into a thin, low-volume rarefaction.

A few of the panels show multi-shock interactions along $B_0$ give rise to $s_{\text{min}}$, like, for example, at $t/\tau = 5.0$ and $t/\tau = 8.4$, where two sawtooth shocks travel along the field. To confirm that strong, single shocks, give rise to the most non-Gaussian features we perform four simple numerical experiments. We give details about the numerical experiments in §5A. To summarise the experiments, we follow the 1D setup in Mocz & Burkhardt (2018), solving the compressible Euler equations on a periodic domain $x \in [0,2L]$ and creating shocks by perturbing the velocity field with a Gaussian pulse that has an amplitude which sets the root-mean-squared $M$. We choose $M \approx 2$, mimicking the characteristic shock velocities from the M2MA01 simulation. The four experiments are:

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⁷ Note that there need not be any time correlation, by construction, but clearly some of the most under-dense events are so extreme that they define $s_{\text{min}}$ for a full $\sim \tau = L/(2c_s M)$. 

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**Figure 12.** The same as the left panel of Figure 10 but for the events in the turbulence that give rise to the largest under-densities across $5 \leq t/\tau \leq 10$, i.e. a proxy for the events in the turbulence that contribute most to the H2013-like intermittency in the $s$-PDF. The panels are organised such that top-left panel corresponds to the 1D pencil beam at $t/\tau = 5$, and the bottom-right panel is $t/\tau = 9.8$. We find that most of the extreme under-densities ($s_\beta = \ln(\rho_\beta/\rho_0)$, blue, solid lines) correspond to single, strong sawtooth shocks (red, solid lines) that evacuate mass from large-in-volume regions along the direction of $B_0$. Sometimes these intermittent events are also coupled with large parallel magnetic field fluctuations caused by fast magnetosonic waves (red, dashed lines).
increasing $t/t_{\text{cross}}$ for the four 1D shock experiments: (i) single shock, (ii) two shocks, (iii) two shocks, constant driving, (iv) many shocks, constant driving. We use these experiments to explore what kinds of 1D shock dynamics lead to the largest non-Gaussian components in the $s$-field, analogous to the dynamics that are constrained to along $B_0$ in the 3D sub-Alfvénic mean-field turbulence. $t/t_{\text{cross}} = 0$ is at the bottom of the space-times, and $t/t_{\text{cross}} = 2$ is at the top.

We show the space-time diagrams for $s$, in each of the experiments in Figure 13, from left-to-right for experiments (i)-(iv), and the $s$-profiles as a function of $t/t_{\text{cross}}$ in Figure A1. In Figure 13, the over-densities are coloured in red, which trace the shock fronts and the under-densities in blue, which are almost stationary in the first three experiments. We evolve the experiments for $2t_{\text{cross}}$, which is shown by going up the vertical axis as indicated with the $t/t_{\text{cross}}$ annotation. In the experiments (iii) and (iv) one can see the that repeated pulses in the $s$-field give rise to intricate, interacting shock networks, analogous to Burgers (1948) turbulence (Mocz & Burkhart 2018).

We compute the $S_s$ for each of the experiments as a function of $t/t_{\text{cross}}$, shown in Figure 14. We only take statistics for $0 \leq t/t_{\text{cross}} \leq 0.8$, the time for which the periodic boundaries in the first three experiments do not play a role in the flow. We find that the largest $|S_s|$ values, $S_s \approx -5$, are associated with the single shock experiment, consistent with our previous intermittent pencil beam analysis. For both the one-shock (i) and two-shock (ii), (iii) experiments, the initial generation of the over-dense regions, where both the under-density and over-density occupy the smallest volumes, have the most negative $S_s$, similar to the features we saw in the most non-Gaussian event in Figure 12 at $t/\tau = 5.5$. The interaction between the two shocks in experiment (ii), which happens at $t/t_{\text{cross}} \approx 0.4$, results in $S_s > 0$ as the shocked gas fills the volume of the domain after the interaction. The two-shock experiment with repeated injections almost matches experiment (ii), but the $S_s$ reduces after each pulse in the...
driving, before slowly relaxing back to a more Gaussian field. The many-shock experiment settles into a stationary state quickly, fluctuating around $S_s = 0$, slightly tending towards a negative value of $S_s$, similar to our global $S_s$ statistics we computed in §6. Even in this experiment it is probably the rare, single, strong shocks that we can see traced by the red, over-dense gas, that contribute to the non-Gaussian components by creating the strongest under-densities in the $s$-field.

The key conclusion from this analysis is that we find strong, single shocks in the 3D, low-$M$, sub-Alfvénic mean-field turbulence that give rise to some of the lowest values in $s_{\text{min}}$. We show that it is these strong, single shocks that give rise to the greatest non-Gaussian contributions to the $s$-PDF. We have now solidified our understanding of the $s$-intermittency for the $M \lesssim 4$ sub-Alfvénic mean-field turbulence, and now turn focus on the $M \gtrsim 4$ regime.

8 PHENOMENOLOGY OF HIGH-$M$ GAUSSIANISATION

In this section we outline a number of reasonable explanations for the high-$M$ lognormality that we observed in the trans- and sub-Alfvénic mean-field regime in Figures 7, 8 and 9. The arguments for this are based upon the self-similarity of $s$ and timescale phenomenologies developed by H2013 and MB2019, respectively.

8.1 Self-similarity interpretation

Because the H2013 $s$-PDF model is characterised by the $T$ parameter (Equations 19 & 20), discussed in §3, which is an average measure of how much the $s$-field in the turbulence deviates from perfect self-similarity at each neighbouring spatial scale, $\langle s_L/T^n - s_L/T^{n+1} \rangle_{L/T^n}$, we call this the self-similarity (SS) phenomenology. The H2013 SS phenomenology may be the most robust of the phenomenologies we discuss, because the formulation relies upon the Castaing (1996) method of probing probabilities for very general steady-state classes of multiplicative-random-relaxation processes within the turbulence cascade. Under this interpretation, the decreasing $T$ with high-$M$ in the magnetised flows is a simple statement about the $s$-field becoming closer to perfect self-similarity. As Squire & Hopkins (2017) points out, this does not mean that the underlying statistics are Gaussian (we have shown that this cannot be the case if one considers mass conservation or supersonic turbulence, which is not scale-free in §2.2), but rather, it means that the flow is full of small-in-volume, small-amplitude density contrasts, making $\langle s_L/T^n - s_L/T^{n+1} \rangle_{L/T^n}$ approach zero, and hence decreasing $T$.

We believe the $T$ trends we find in Figure 8 can be at least conceptually understood through (1) the typical shock-jump conditions in MHD turbulence and (2) the isotropic turbulent mixing of the flow as $M_{\text{AD}}$ increases. Beattie et al. (2021) found that the magnetic field preferentially suppresses the $\rho/\rho_0$-variance at high-$M$ and attributed this to the magnetic field being able to more easily smooth out small-scale fluctuations, which are flux-frozen to the field (Landau & Lifshitz 1959), and are introduced into the flow as $M$ increases (Kim & Ryu 2005). One can see this immediately by considering a slab of isothermal gas, threaded by magnetic fields. Perpendicular to the magnetic field $B \propto \rho$ (Hennebelle & Inutsuka 2019), and by using the regular Rankine-Hugoniot jump conditions (Landau & Lifshitz 1959) one can show that the density jump for these magnetised fluctuations is

$$\rho/\rho_0 = \frac{1}{2} \left[ \sqrt{\left(1 + 2 M_{\text{AD}}^2 \frac{b}{(bM)^2}\right)^2 + 8 M_{\text{AD}}^2 - \left(1 + 2 M_{\text{AD}}^2 \frac{b}{(bM)^2}\right)} \right]$$

(Molina et al. 2012; Mocz & Burkhart 2019; Beattie et al. 2021). We plot $\rho/\rho_0$ as a function of $M$ in Figure 15 (black), as well as the regular hydrodynamical shock jump conditions $\rho/\rho_0 = (bM)^2$ (red) for comparison. The magnetised gas density jump diverges from the hydrodynamical value at high-$M$ because the small-scale density fluctuations are suppressed by the magnetic field. This significantly reduces the amplitude of the fluctuations (both for over- and under-densities; see e.g., Figure 3 in Mocz & Burkhart 2018), which means that $\langle s_L/T^n - s_L/T^{n+1} \rangle_{L/T^n}$ is also reduced, giving rise to a more self-similar (small $T$), Gaussian-like set of global $s$-statistics for the high-$M$ simulations (Squire & Hopkins 2017), regardless of $M_{\text{AD}}$.

In Figure 8 we found that the $T$ parameter depends upon $M_{\text{AD}}$: high-$M_{\text{AD}}$ simulations become lognormal, $T \sim M^{-3}$ faster than low-$M_{\text{AD}}$, $T \sim M^{-2}$. We believe this could be due to the global transition from an weakly-isotropic to strongly-anisotropic $s$-field. For the sub-Alfvénic mean-field turbulence, the high-energy $B_0$ provides a domain, $\ell_\parallel$, in the turbulence along $B_0$, where hydrodynamical-like shocks, as we showed in §7, increasing $S_s$ and hence $T$. But for the trans-Alfvénic simulations, $M_{\text{AD}} = 1 - 2$, $B_0$ and $\delta B$ are at almost energy equipartition (Beattie et al. 2020, see Figure 7). The transition is qualitatively illustrated in Figure 2 through the difference between the $B$-field lines in the top row, where the field is highly-coherent across the whole box, dominated by the $B_0$, and bottom row, where the field is becoming tangled as the $\delta B$ grows. Once there is significant magnetic energy in $\delta B$ there is no longer a region in the turbulence, $\ell_\parallel$, driving, before slowly relaxing back to a more Gaussian field. The many-shock experiment settles into a stationary state quickly, fluctuating around $S_s = 0$, slightly tending towards a negative value of $S_s$, similar to our global $S_s$ statistics we computed in §6. Even in this experiment it is probably the rare, single, strong shocks that we can see traced by the red, over-dense gas, that contribute to the non-Gaussian components by creating the strongest under-densities in the $s$-field.

The key conclusion from this analysis is that we find strong, single shocks in the 3D, low-$M$, sub-Alfvénic mean-field turbulence that give rise to some of the lowest values in $s_{\text{min}}$. We show that it is these strong, single shocks that give rise to the greatest non-Gaussian contributions to the $s$-PDF. We have now solidified our understanding of the $s$-intermittency for the $M \lesssim 4$ sub-Alfvénic mean-field turbulence, and now turn focus on the $M \gtrsim 4$ regime.
where hydrodynamical-like shocks can form (left panel of Figure 10), since the $B$-fields are tangled into random orientations all through space. This means magnetised $s$-fluctuations are more uniformly distributed through the fluid, and hence, there are many more small amplitude fluctuations in the fluid than in the highly-sub-Alfvénic flows. As we discussed above, for the high-$M$ simulations, having many small amplitude fluctuations leads to lower values of $T$, and more, self-similar, $\langle s_{L/T^n} - s_{L/T^{n+1}} \rangle_{L/T^n} \approx 0$, Gaussian-like $s$-statistics.

Finally, we note that our interpretation relies upon measuring $T$ via the $s$-PDF fits, and interpreting this parameter in the context of SS. One way to directly measure the SS would be to compute the scale-dependent $\langle s_{L/T^n} - s_{L/T^{n+1}} \rangle_{L/T^n}$-PDF, which we do not do in this study, but we think is feasible to do with high-resolution turbulence data that was recently produced in Federrath et al. (2021). One can compare this with the $T$ computed from the $s$-PDF with the direct measurement, $\langle s_{L/T^n} - s_{L/T^{n+1}} \rangle_{L/T^n}$. This would allow us to understand better the exact physics of $T$, and how we can use it to interpret the $s$-PDF intermittency.

### 8.2 Timescale interpretation

The central tenant of the MB2019 $s$-PDF phenomenology is timescales that determine the lifetimes of high-density and low-density structures in the turbulence are different (see §3). High-density objects, $s > s_0$, live on shorter timescales compared to lower-density objects, $s < s_0$, reduced by a factor of $1/1-3f/2$ (see Equation 26) which amounts to skewness in the $s$-PDF. It is true that the magnetic field can act to shield density fluctuations, prolonging the life of the fluctuations. Hennebelle (2013) found this was the case for $\approx 55 - 65\%$ of the dense, magnetised filamentary structures in a suite of MHD box simulations. The radial Lorentz force was found to mostly point towards the filaments, opposing thermal pressure and preventing the fluctuation from expanding, prolonging the life of the over-density. This could be in part responsible for the Gaussianisation of the statistics, coupled with $M$ increasing, increasing the number of shocks and getting larger and more complete samples of shock lifetimes. The $M_{A0}$ dependency could be explained in the same fashion as the previous SS phenomenology, i.e., sub-Alfvénic flows allow for hydrodynamical-like shocks to form along $B_0$, which live on shorter timescales than magnetised shocks, whereas shocks in more trans-Alfvénic, isotropic flows, always feel the magnetic field, prolonging the lifetimes for all of the density fluctuations. This is consistent with what we find in Figure 9, where we directly compute $1/1-3f/2$ for our simulations. It shows a spread in $1/1-3f/2$ for different $B$-field strengths, where $M_{A0} = 2$ is about 25% slower than $M_{A0} = 0.1$, at $M \approx 4$, before becoming completely homogenised at $M \geq 4$.

Robertson & Goldreich (2018) has probably done the most robust measurements of lifetimes for over-dense regions using clustering and filtering techniques on passive tracer particles. The lifetimes of dense shocked regions in the supersonic isothermal turbulence simulation are short, $O(10^{-3}L/c_s)$. In the frame of turbulent turnover times on the driving scale that is $\sim O(10^{-2}L/[2M_c])$ for $M \geq 4$. Based on our observations of the shocks in the magnetised flows it is very reasonable to suggest that the shock lifetimes are well beyond that of $\sim O(10^{-2}L/[2M_c])$. For example, we find some of the extreme shock events that we identified in Figure 12 persist for $O(L/[2M_c])$. Regardless, a more detailed study of the lifetimes of over-dense objects, such as the one performed for hydrodynamical simulations in Robertson & Goldreich (2018) would be needed to test this directly, not relying on the indirect $f$ parameter to determine how large of an affect the magnetic field has on the lifetimes of the dense regions with different $M$ and $M_{A0}$.

### 9 SUMMARY AND KEY FINDINGS

We provide a detailed analysis of the 1-point logarithmic density ($s \equiv \ln \rho/\rho_0$) statistics for magnetised supersonic turbulence in the sub- to trans-Alfvénic mean-field ($M_{A0} \lesssim 2$) and supersonic ($M > 1$) regime, relevant to the cool interstellar medium. In §§1-2 we motivate and discuss lognormal models for the $s$ probability density function (PDF) and the fundamental assumptions and problems associated with lognormal fluctuation models. Next in §3 we focus on two models that go beyond the lognormal fluctuation picture, motivated by relaxing the self-similarity assumption (H2013) and formulating the $s$-fluctuations as a Langevin model with non-symmetric timescales for high- and low-$s$ fluctuations (MB2019). In §4 we describe the numerical methods and setup for the 16 MHD simulations we use to compute the density statistics. Next in §5 we discuss in detail the morphology of the volume-weighted and mass-weighted $s$-PDFs, focusing on the high- and low-$s$ tails and the temporal fluctuations from the time-averaging. In §6 we provide a detailed analysis of the $s$-intermittency based on four independent statistical measures and identify, unlike hydrodynamical turbulence, that the sub- to trans-Alfvénic mean-field turbulence becomes more lognormal with increasing $M$. In §7 we use 1D pencil beams to probe the 3D turbulence in the low-$M$ regime and analyse the across and along $B_0$ dynamics, and show that the low-$M$ intermittency is caused by strong, single shocks that form along $B_0$. We run 1D shock simulations that support our conclusions. Finally in §8 we discuss two different phenomenological explanations for the lack of $s$ intermittency in the high-$M$ regime: self-similarity and inhomogenous timescales between high- and low-density structures. We list the key results below:

- We show that the mass and volume-weighted statistics (Figures 4 & 7) only become stationary in trans-to-sub-Alfvénic mean-field turbulence after about five turbulent turnover times, $t = \ell_0/(c_s M)$, where $\ell_0$ is the driving scale, more than double the time required for hydrodynamical turbulence to reach a stationary state.

- The temporal fluctuations in the volume-weighted (Figure 5) and mass-weighted (Figure 6) $s$-PDFs show how the low-density, high volume-filling structures, such as rarefactions and voids, fluctuate significantly in volume, and much less in mass. In contrast, the highest-density and lowest volume-filling structures, such as shocked regions, filaments, and over-dense sheets, fluctuate significantly in mass, but not in volume. This paints a picture of magnetised turbulence where the densest regions are gaining and losing mass over short timescales, using neighbouring voids as reservoirs, which are expanding and contracting in volume as mass is exchanged between them.
• We compare the non-lognormal models, H2013 (Equation 15-18) and MB2019 (Equation 28), and the lognormal model (Equation 1) for the volume-weighted $s$-PDF. We find that unlike hydrodynamical turbulence, which monotonically increases the non-Gaussian features of the $s$-PDF with $M$, the H2013 or MB2019 are required to capture the PDF morphology at $M \lesssim 4$, but for $M \gtrsim 4$ the lognormal model is empirically (but not theoretically) sufficient.

• Motivated by the complex non-Gaussian behaviour of the $s$-PDF we calculate four independent measures of the global, volume-weighted intermittency (1) the skewness (Equation 32, Figure 7), (2) the mass-weighted versus volume-weighted $s$ standard deviation relation (right panel of Figure 8), (3) the H2013 $T$ parameter (Equation 19, left panel of Figure 8) and (4) the MB2019 $f$ parameter (Equation 26). All four independent statistics confirm that hydrodynamical-like intermittency is found for low-$M$ highly-magnetised simulations, but the intermittency monotonically reduces as a function of $M$, with some dependence upon $M_{\text{Ao}}$, completely opposite to the hydrodynamical turbulence studied in H2013 and Squire & Hopkins (2017).

• Using 1D pencil beams (Figure 10, §7) we explore the low-$M$ local parallel and perpendicular dynamics along $B_0$. Along $B_0$ large-scale exponential shocks form through convergent flows and sawtooth shocks. We stress, that unlike simple shock-tube experiments, the over-densities have an internal structure, similar to over-densities described in Robertson & Goldreich (2018). The shocks excite magnetic compression waves at the shockfront and give rise to the largest over-densities in the flow. This also leads to anisotropies in the magnetic tension. Across $B_0$ we find large-scale, rigid body vortices and very small-amplitude shear Alfvén waves (compared to the kinetic and fast magnetosonic compression waves). We comment that in sub-Alfvénic mean-field turbulence shear Alfvén waves can only possibly play a dominant role in the dynamics on very small scales and the large scales are dominated by sawtooth shocks in the gas velocity (i.e., akin to large-scale Burgers (1948) turbulence).

• Using our pencil beams we search for the low-$s$ intermittent events that are captured by the H2013-type intermittency. We look for the most extreme under-dense events through 49 time realisations (Figure 12), and find that the largest contributors to the non-Gaussian $s$-statistics are strong, single shocks ($1.5 - 2\sigma$ events in the velocity) that form along $B_0$ and create volume-poor $s$ rarefactions, which are seemingly also coupled to rare velocity and magnetosonic field fluctuation events. We show explicitly that it is single shocks and not multi-shock interactions along $B_0$ that are the largest contributors to the non-Gaussian components of $s$ using 1D numerical shock experiments that we show in Figures 13, 14 and A1.

• For the high-$M$ flows, we discuss the two phenomenologies that we use to understand why the $s$-statistics become more Gaussian with increasing $M$ (§8). These are the self-similarity (H2013, Squire & Hopkins 2017) and inhomogeneous timescale (Mocz & Burkhart 2019) interpretations of the $s$-statistics. We conclude it is most likely that the self-similarity of the flow is maintained through suppression of over- and under-dense $s$-fluctuations by $B$. This effect is stronger when the flows are trans-Alfvénic and are becoming dominated by isotropic magnetised $s$-fluctuations. We also conjecture that the magnetic field plays role in facilitating more uniform dynamical timescales between the low- and high-density structures in the gas.

ACKNOWLEDGEMENTS

J. R. B. thanks Christoph Federrath’s and Mark Krumholz’s research groups for the many productive discussions, in particular, Shyam H. Menon, and acknowledges financial support from the Australian National University, via the Deakin PhD and Dean’s Higher Degree Research (theoretical physics) Scholarships, the Research School of Astronomy and Astrophysics, via the Joan Duffield Research Scholarship and the Australian Government via the Australian Government Research Training Program Fee-Offset Scholarship. P. M. acknowledges support for this work provided by NASA through Einstein Postdoctoral Fellowship grant number PF7-180164 awarded by the Chandra X-ray Centre, which is operated by the Smithsonian Astrophysical Observatory for NASA under contract NAS8-03060. C. F. acknowledges funding provided by the Australian Research Council (Future Fellowship FT180100495), and the Australia-Germany Joint Research Cooperation Scheme (UA-DAAD). R. S. K. acknowledges financial support from the German Research Foundation (DFG) via the Collaborative Research Center (SFB 881, Project-ID 138713538) ‘The Milky Way System’ (subprojects A1, B1, B2, and B8). He also thanks for funding from the Heidelberg Cluster of Excellence STRUCTURES in the framework of Germany’s Excellence Strategy (grant EXC-2181/1 - 390900948) and for funding from the European Research Council via the ERC Synergy Grant ECOGAL (grant 855130). We further acknowledge high-performance computing resources provided by the Australian National Computational Infrastructure (grant ek9) in the framework of the National Computational Merit Allocation Scheme and the ANU Merit Allocation Scheme, and by the Leibniz Rechenzentrum and the Gauss Centre for Supercomputing (grants pr32lo, pr48pi, pr74nu, pn73li).

The simulation software, FLASH, was in part developed by the DOE-supported Flash Centre for Computational Science at the University of Chicago. Data analysis and visualisation software used in this study: C++ (Stroustrup 2013), CYTHON (Behnel et al. 2011), VISIT (Childs et al. 2012), NUMPY (Oliphant 2006; Harris et al. 2020), MATPLOTLIB (Hunter 2007), SCIPY (Virtanen et al. 2020), SCIKIT-IMAGE (van der Walt et al. 2014), PANDAS (pandas development team 2020).

DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author, James R. Beattie.
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We perform 1D shock experiments with a similar setup to the one described in Mocz & Burkhart (2018). We solve the compressible Euler equations in 1D,

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \rho v_x = 0, \tag{A1}
\]

\[
\rho \left( \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} \right) v_x = c_s^2 \frac{\partial}{\partial x} \rho, \tag{A2}
\]

based on the multi-state Harten-Lax-van Leer approximate Riemann method described in Miyoshi & Kusano (2005). We solve the equations on a periodic domain \( x \in [0, 2L] \), and with \( \rho(x, t = 0) = \rho_0 = 1 \). Following
Mocz & Burkhart (2018), to create an over-dense region we perturb the velocity field with a Gaussian pulse,

\[ v(x, t = 0)/c_s = M \exp \left\{ -\frac{(x - x_0)^2}{2\ell_0^2} \right\} , \tag{A3} \]

hence we are able to tune \( M \) and driving scale, \( \ell_0 \) directly through the pulse. For all of the experiments we set \( M \approx 2.0 \), capturing along-\( B_0 \)-field dynamics of the \textsc{M2Ma01} simulation studied in detail, in §7. We run all experiments from \( t/t_{\text{cross}} = 0 - 2 \), where \( t_{\text{cross}} = L/c_s \). As outlined in §7.3, we perform four experiments,

(i) a single shock travelling across the domain,  
(ii) two shocks colliding at the centre of the domain, 
(iii) two shocks repeatedly injected, 
(iv) multiple shocks repeatedly injected.

which we show in terms of the space-time diagrams, Figure 13, and the time-evolving \( s \) profiles, Figure A. For experiments (i) and (ii) we simply initialise the velocity perturbations and track the over-densities. For (iii) and (iv) we repeatedly add pulses to the velocity. For (iii) we pulse every \((1/3)t_{\text{cross}}\) and for (iv) \((1/5)t_{\text{cross}}\), for the latter, ensuring that the 1D “turbulence” becomes stationary. For (i)-(iii) \( \ell_0 = 2.5 \times 10^{-3} L \), providing ample amounts of space for the shocks to develop and evolve, and for (iv) \( \ell_0 = 1.25 \times 10^{-3} L \), which allows us to create many small-scale shocks that fill the domain completely.

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