Vertical Phase Mixing across the Galactic Disk

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Abstract

By combining the Large Sky Area Multi-object Fiber Spectroscopic Telescope and Gaia data, we investigate the vertical phase mixing across the Galactic disk. Our results confirm the existence of phase space snail shells (or phase spirals) from 6–12 kpc. We find that grouping stars by the guiding radius ($R_g$), instead of the present radius ($R$), further enhances the snail shell signal in the following aspects: (1) clarity of the snail shell shape is increased; (2) more wraps of the snail shell can be seen; (3) the phase spaces are less affected by the lack of stars closer to the disk midplane due to extinction; and (4) the phase space snail shell is amplified in greater radial ranges. Compared to the $R$-based snail shell, the quantitatively measured shapes are similar, except that the $R_g$-based snail shells show more wraps with a better contrast. These lines of evidence lead to the conclusion that the guiding radius (angular momentum) is a fundamental parameter tracing the phase space snail shell across the Galactic disk. Results of our test particle simulations with impulse approximation verify that particles grouped according to $R_g$ reveal well-defined and sharper snail shell features. By comparing the radial profiles of the snail shell pitch angle between the observation and simulation, the external perturbation can be constrained to $\sim$500–700 Myr ago. For future vertical phase mixing study, it is recommended to use the guiding radius with additional constraints on orbital hotness (ellipticity) to improve the clarity of the phase snail.

Unified Astronomy Thesaurus concepts: Milky Way Galaxy (1054); Milky Way dynamics (1051); Milky Way disk (1050); Milky Way evolution (1052)

1. Introduction

The Milky Way, as a massive pure disk galaxy, is not in dynamical equilibrium. The current status of the Galactic disk is mainly shaped via both the internal and external mechanisms together. Resonances from the bar and spiral arms have been shown to induce observed fine structures in the velocity phase space (Dehnen 2000; Fux 2001; Antoja et al. 2009, 2011; Quillen et al. 2011; Hunt & Bovy 2018) and the large scale bulk motions in the Galactic disk (Siebert et al. 2011, 2012; Carlin et al. 2013; Debattista 2014; Faure et al. 2014; Sun et al. 2015; Monari et al. 2015, 2016; Tian et al. 2017; Wang et al. 2018a, 2018b). The external perturbations from satellite galaxies or sub-halos can generate warps, flares, vertical density asymmetries, or high order velocity modes in the Galactic disk, such as the bending and breathing vertical motions (Hunter & Toomre 1969; Quinn et al. 1993; Kazantzidis et al. 2008; Purcell et al. 2011; Gómez et al. 2013; Williams et al. 2013; D’Onghia et al. 2016; Laporte et al. 2018a, 2018b).

External perturbations also excite vertical oscillation and phase mixing of stars in the Galactic disk. According to Widrow et al. (2012), the vertical density profile of the Galactic disk shows a clear asymmetry with wave-like patterns (also see Bennett & Bovy 2019 for a more recent update, and An 2019 from the chemical perspective). As first shown in Antoja et al. (2018) with Gaia DR2, stars near the solar radius exhibit a snail shell feature in the $Z - V_Z$ phase space, representing the ongoing vertical phase mixing that probably happened 300–900 Myr ago. This is a 2D representation of the signal discovered in Widrow et al. (2012). The snail shell forms as a result of anharmonic oscillation of stars in the vertical direction; the vertical oscillation frequency ($\Omega_z$) is anticorrelated with the vertical action ($J_z$) and the angular momentum ($L_z$), i.e., $V_\phi$ for stars in the solar neighborhood (Binney & Schönrich 2018). The phase space snail shell can be recognized for stars in a wide range of ages (Tian et al. 2018; Laporte et al. 2019) and different chemical properties (Bland-Hawthorn et al. 2019). By dissecting stars in the $V_R - V_\phi$ phase space into distinct arches, Li & Shen (2020) found clear snail shells only in the stars on dynamically colder orbits, i.e., stars with $|V_\phi - V_{LSR}| \lesssim 30$ km s$^{-1}$ (or $J_R < 0.04$ kpc$^2$/Myr). The hotter orbits, on the other hand, may have phase wrapped away already due to the much larger radial excitations to facilitate faster phase mixing. The absence of the clear snail shells on hotter orbits suggests that the vertical perturbation occurred at least 500 Myr ago. However, different opinions also exist for the origin of the vertical phase mixing process (see Khoperskov et al. 2019; Bennett & Bovy 2021).

The phase space snail shell has also been found beyond the solar neighborhood in the Milky Way disk. Based on the Gaia DR2 data around the solar neighborhood (within 1 kpc), slightly different phase space snail shells are found at different radii, suggesting the snail shell as a signature of corrugated waves propagating through the disk (Bland-Hawthorn et al. 2019). Laporte et al. (2019) later explored larger radial ranges from 6.6–10 kpc, and confirmed the existence of the snail shell at these locations. Similar results can also be seen in Wang et al. (2019) based on the Large Sky Area Multi-object Fiber Spectroscopic Telescope (LAMOST) and Gaia DR2 data from...
samples should help to enhance the phase space signal. The Astrophysical Journal, Schönrich 2019

For a volume limited sample, the angular momentum ($L_2$), i.e., the guiding radius ($R_g$), may help to better reveal the global kinematic patterns. For example, based on a local sample near the solar neighborhood, the Galactic warp was identified with a positive correlation with the wave-like pattern between the mean vertical velocity and the guiding radius (Schönrich & Dehnen 2018; Huang et al. 2018). Recently, Friske & Schönrich (2019) found that the dependence of the mean radial motion on the angular momentum (guiding radius) also shows wave-like behavior. The shape of the $V_R - R_g$ profile is similar to, but much stronger than the $V_R - R$ profile; patterns in the $V_R - R$ profile have probably been washed out by excursions of stars around their guiding centers (Friske & Schönrich 2019). Khanna et al. (2019) found that the snail shell shape remains the same at different radii (within 1 kpc from the Sun) for stars with the same $L_2$, but varies at different $L_2$. They suggested that the angular momentum could be a more robust quantity in characterizing the snail shell.

Instead of mapping the stellar kinematics and tracing the phase space maps in the traditional spatial volume, grouping stars according to the guiding radius may be another promising way to visualize the morphological or kinematic structures in the disk (e.g., Khoperskov et al. 2020; but see Hunt et al. 2020 for a different point of view). Aiming to probe the phase space snail shell in a large radial range in the Galactic disk with good number statistics, we combine the Gaia radial velocity sample (RVS) and the LAMOST DR6 sample together with all the stars in both samples included, i.e., LAMOST DR6 $∪$ Gaia RVS. Although the LAMOST survey does not sample the sky as uniformly as the Gaia survey, it has good spatial coverage in the Galactic anticenter direction with radial velocity information for fainter stars at greater distances, compensating the relatively brighter Gaia RVS sample. With the combined large sample, we can investigate the spatial variation of the snail shell with angular momentum in detail. 3

2. Sample

LAMOST DR6 provides reliable radial velocity values for 5,843,107 stars, focusing on the Galactic anticenter direction (Zhao et al. 2012; Deng et al. 2012; Liu et al. 2014). The Gaia RVS sample contains 7,225,631 stars with accurate position and velocity information (Gaia Collaboration et al. 2018b). To achieve good statistics, we utilize all the stars to form a main sample of 12,256,045 stars (LAMOST DR6 $∩$ Gaia RVS). Figure 1 summarizes the number of stars in the two samples.

There are 750,134 stars in common between LAMOST DR6 and Gaia RVS samples (LAMOST DR6 $∩$ Gaia RVS). Figure 2 compares the line-of-sight (LOS) velocities observed in Gaia ($V_{LOS}^{Gaia}$) and LAMOST ($V_{LOS}^{LAMOST}$) for these common stars. As shown in the left panel, the agreement between the velocities are quite good. However, there exists a small systematic offset, with the $V_{LOS}^{LAMOST}$ slightly smaller than $V_{LOS}^{Gaia}$. The distribution of the velocity difference ($\Delta V_{LOS} = V_{LOS}^{Gaia} - V_{LOS}^{LAMOST}$) is shown in the right panel. The peak position of the distribution is marked with the vertical dashed line. The LOS velocity observed in LAMOST is systematically lower than the Gaia radial velocity by 4.75 km s$^{-1}$. In the following analysis, for these common stars, only the Gaia measured velocities are adopted. For the rest of the stars in the LAMOST DR6 sample, we have added 4.75 km s$^{-1}$ to the LOS velocity to compensate this effect.

We adopt the Bayesian distance from Bailer-Jones et al. (2018) for the whole sample. There are 12,094,719 stars in the main sample with the Bayesian distance determined. Similar to previous works, we choose $(X_\odot, Y_\odot, Z_\odot) = (−8.34, 0, 0.027)$ kpc as the Sun position (Reid et al. 2014). The local standard of rest (LSR) circular velocity $V_{LSR}$ is set to 240 km s$^{-1}$ (Reid et al. 2014). Here the peculiar velocities of the Sun with respect to LSR are set to $(U_\odot, V_\odot, W_\odot) = (11.1, 12.24, 7.25)$ km s$^{-1}$ (Schönrich 2012). 5

The spatial distributions of the LAMOST and Gaia RVS samples are shown in Figure 3. The Gaia RVS sample distributions are quite symmetric in the $Y, Z, \theta$ directions. On the other hand, the LAMOST survey mainly covers the Galactic anticenter direction and $\theta > 0$ in the positive $Y$ axis. Since the limiting magnitude is deeper in the LAMOST spectroscopic survey, it also extends further in the vertical direction than the Gaia survey.

We further remove stars with relative distance uncertainties larger than 25%, velocity uncertainties larger than 50 km s$^{-1}$, and absolute velocities ($|V_R|$, $|V_Z - V_{LSR}|$, and $|V_Z|$) larger than 400 km s$^{-1}$. The final sample contains 11,350,423 stars in total. In the following analysis, we will focus on the radial range from 6--12 kpc for both the Galactocentric radius ($R$) and the guiding radius ($R_g$). In order to enhance the visualization of the phase space structures, at 6 and 7 kpc, the radial annulus width is set to 0.6 kpc (i.e., $±0.3$ kpc), while the annulus width is 0.4 kpc (i.e., $±0.2$ kpc) at the other radii. The typical velocity uncertainty is about 1 km s$^{-1}$ for the radial, azimuthal, and vertical velocities (Gaia Collaboration et al. 2018b; Antoja et al. 2018).

The guiding radius ($R_g$) of each star is calculated according to the rotation curve in Huang et al. (2016). As shown in Figure 12 in Huang et al. (2016), the rotation curve is roughly flat at $\sim240$ km s$^{-1}$ (within $\sim25$ kpc) with prominent wiggles at 11 and 20 kpc. Adopting the values of the rotation curve in Table 3 of Huang et al. (2016), the angular momentum corresponding to the circular motion at each radius (i.e., the guiding radius) can be derived to get the $L_2 - R_g$ profile. Then the guiding radius of each star in our sample is found by mapping the observed angular momentum to the derived $L_2 - R_g$ profile.

To highlight the snail shell feature, we adopt the number density contrast map used in Laporte et al. (2019) and Li & Shen (2020). To evaluate the influence of the parallax bias in the Gaia catalog, we also test the same analysis with the

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4 This systematic velocity offset in LAMOST data has been noted in previous works (Tian et al. 2015; Schönrich & Aumer 2017).

5 The main results are not affected if we choose other values of the solar peculiar motion, e.g., Tian et al. (2015) or Huang et al. (2015).

6 $\theta$ is the angle in the Galactocentric polar coordinate which increases clockwise ($\theta = 0^\circ$ for the Sun–Galactic center line).
Figure 1. Number of stars in the LAMOST DR6 and Gaia RVS samples used in this work. The combined main sample (LAMOST DR6 ∪ Gaia RVS) contains 12,256,045 stars in total. The crossmatched sample (LAMOST DR6 ∩ Gaia RVS) contains 750,134 stars (used in Figure 2). For the main sample (LAMOST DR6 ∪ Gaia RVS), after crossmatching with the Bayesian distance catalog from Bailer-Jones et al. (2018) and removing stars with large velocity uncertainties, the final sample contains 11,350,423 stars.

Figure 2. Comparison between the observed LOS velocities for the LAMOST DR6 and Gaia RVS common stars, with the correlation between $v_{\text{LOS}}^{\text{LAMOST}}$ and $v_{\text{LOS}}^{\text{Gaia}}$ in the left panel, and the distribution of their velocity difference ($\Delta v_{\text{LOS}} = v_{\text{LOS}}^{\text{Gaia}} - v_{\text{LOS}}^{\text{LAMOST}}$) in the right panel. The cyan dashed line in the left panel represents the one-to-one relationship. The LAMOST radial velocity is systematically lower than the Gaia radial velocity by 4.75 km s$^{-1}$ (vertical dashed line in the right panel).

Figure 3. Spatial distributions of the LAMOST DR6 (top) and Gaia RVS samples (bottom) in the X − Y plane (left column), the R − Z plane (middle column), and the $\theta$ − Z plane (right column). Compared to the relatively brighter Gaia RVS sample, the LAMOST sample has the advantage of more faint stars at relatively larger distances.

Figure 4. Number density distributions of our main sample in the $R − V_\phi$ (top) and $R − R_g(L_Z)$ phase spaces (bottom). The top panel shows several prominent diagonal ridge lines consistent with previous works. The dashed curve represents the constant angular momentum at $L_2 = 2000$ kpc km s$^{-1}$. In the bottom panel, there are several nearly horizontal stripes, suggesting a roughly constant angular momentum of stars at different radial ranges.

3. Phase Spaces across the Galactic Disk

In this section, we explore several different phase spaces of the sample, namely, the $R − V_\phi$, $R − R_g$, $V_R − V_\phi$, and the $Z − V_Z$ phase spaces. As we will show later, major structures in these phase spaces are consistent with previous studies, confirming the robustness of the sample to trace the kinematic status of the Milky Way disk.

3.1. $R − V_\phi$ and $R − R_g$ Phase Spaces

Figure 4 shows the number density maps of the main sample in the $R − V_\phi$ (top) and $R − R_g(L_Z)$ phase spaces (bottom). The top panel reveals several diagonal ridge lines, consistent with the previous observations (Antoja et al. 2018; Laporte et al. 2019; Fragkoudi et al. 2019). In the bottom panel, several nearly horizontal stripes can be seen across the phase space, suggesting a possible connection between the ridges in the $R − V_\phi$ phase space and the constant angular momentum curves. However, as shown in Khanna et al. (2019) and Fragkoudi et al. (2019), at higher $V_\phi$, the stripes may be more consistent with the constant energy lines. The observed diagonal ridges could have quite complicated origins, considering the presence of the ridge line structure in the $V_R$ or metallicity color-coded $R − V_\phi$ phase space (Fragkoudi et al. 2019; Laporte et al. 2019; Liang et al. 2019; Wang et al. 2020).

From the bottom panel, stars at a certain guiding radius are actually located within a large radial range. At the radius far from the solar neighborhood, the number of stars can be improved by selecting stars according to their guiding radii with the inclusion of stars close to the Sun. This could help enhance the phase space snail shell feature at a larger radial range.

3.2. $V_R − V_\phi$ Phase Space

Figure 5 shows the $V_R − V_\phi$ phase space of stars at different radii, with the number density and the $J_R$ color-coded maps in the top and bottom rows, respectively. From the main sample, we study the velocity phase space distributions at the 7 radial annuli centered at $R = 6, 7, 8, 9, 10, 11,$ and $12$ kpc with the same width as mentioned before. Gaia DR2 has revealed the
new arch-like structures in the $V_R - V_Z$ phase space for stars in the solar neighborhood, enclosing the classical moving groups (Gaia Collaboration et al. 2018a; Antoja et al. 2018; Li & Shen 2020). In fact, similar arch-like features can be seen at $R = 8, 9$, and $10$ kpc. As the radius increases, the arches seem to systematically shift toward lower $V_R$, consistent with previous observational studies (Antoja et al. 2014; Ramos et al. 2018).

According to the cycloidal theory, at a given radius, the azimuthal velocity difference between a star and the local circular velocity can be estimated as $V(0) - V_{\text{circular}}(R) = \kappa x / \gamma$, where $\kappa$ is the epicycle (radial oscillation) frequency, $x$ is the radial displacement from the guiding radius, and $\gamma = 2 \Omega_x / \kappa$. Under the flat rotation curve assumption, $\gamma$ equals $\sqrt{2}$. The ratio between the maximum radial velocity and the maximum azimuthal velocity difference can be estimated, which is simply $\gamma^{-1}$. Adopting the best-fit potential of Model I from Irrgang et al. (2013), we use the action-based galaxy modelling architecture package (Vasiliev 2019) to derive the epicycle frequencies and the action values. In Figure 5, from the left to right panels (6–12 kpc), the $\gamma$ values are 1.32, 1.35, 1.39, 1.40, 1.41, 1.43, and 1.44, respectively. In the bottom panels for the $V_R - V_Z$ phase spaces color coded with $J_K$, the black ellipses with this axis ratio are overplotted compared to the white curves of the constant $J_K$ contours. The white contours and the black ellipses agree well with each other.

Similarly, we choose seven different guiding radial annuli ranging from 6–12 kpc. The $V_R - V_Z$ phase space in different $R_g$ ranges are shown in Figure 6 with the same layout as that in Figure 5. Compared to Figure 5, the distribution is relatively smooth without clear arches. Similar to Figure 5, the azimuthal velocity difference between a star and the circular velocity at its guiding radius can be estimated as $V(0) - V_{\text{circular}}(R) = \gamma x / 2$. Therefore, the ratio between the maximum radial and azimuthal velocity difference is simply $\gamma / 2$. The $\gamma$ values from $R_g = 6$–12 kpc are 1.35, 1.37, 1.39, 1.41, 1.43, 1.45, and 1.46, respectively. The axis ratio is slightly smaller than that in Figure 5. The black ellipses with this axis ratio are shown in the bottom panels, with good agreement with the white contours.

### 3.3. Z – V2 Phase Space

In Figure 7, the $Z - V_Z$ phase spaces of stars at different Galactocentric radii ($R$) are shown in the left panel, with the...
first, second, and third columns representing the number density contrast ($\Delta N$), $V_R$, and $V_\phi$ color-coded maps, respectively. The number density contrast is derived as $\Delta N = N/\hat{N} - 1$, where $\hat{N}$ is the Gaussian kernel convolved number density distribution$^9$ (Laporte et al. 2019; Li & Shen 2020). The snail shell can be recognized in all the panels, confirming the snail shell feature as a global phenomenon across the Galactic disk. The shapes of the snail shells as revealed by the $\Delta N$ map vary at different radii. As the radius increases, the snail shell becomes more elongated along the Z axis, more squashed along the $V_Z$ axis, and less wound, due to the decreasing vertical restoring force and oscillation frequency at larger radius. These results are consistent with those of Laporte et al. (2019) and Wang et al. (2019).

The snail shells at 6, 7, and 12 kpc are relatively weak and hard to discern in the $\Delta N$ maps, although at $R = 7$ kpc, the $V_\phi$ color-coded phase space still reveals a snail shell pattern. At $R = 8$ kpc, the snail shell in the $\Delta N$ map looks roughly similar to those in the $V_\phi$ color-coded map, which is not true for $R = 9$.

$^9$ Before the convolution, we have taken the logarithm of the original number density map to enhance the fine structures in the phase space.
and 10 kpc. As already demonstrated by Li & Shen (2020), the snail shell that appeared in the $V_f$ color-coded phase space may not truly reflect the real phase mixing signal due to the variation of the snail shell shape at different $V_f$ (also see Laporte et al. 2019).

The $R_g$-based $Z - V_Z$ phase space maps are shown in the right panel of Figure 7. Clear snail shells can be seen in the $\Delta N$ maps, which look similar to the snail shells in the corresponding $R$-based phase spaces with improved clarity. On the other hand, there is no clear snail shell-like pattern in the $V_R$ or $V_\phi$ color-coded phase space shown in the second and third columns. The patterns in the $V_\phi$ color-coded phase spaces are discussed further in Appendix A. In Appendix B, we show $R_g$-based phase spaces for 30 sequential annuli, which are evenly sampled between 6 and 12 kpc with a 0.2 kpc width. A gradual variation of the snail shell shape toward larger radius is quite clear. In addition, in the $R_g$-based phase space, the gap in the $R$-based phase space around $Z = 0$ (lack of stars due to dust extinction) is also mitigated. In fact, the angular momentum space is not complete in terms of star counts. It is biased toward stars in the solar neighborhood where the stellar number density is the highest; at lower (higher) $R_g$, more stars in the

![Figure 8](image8.png)

**Figure 8.** Phase space snail shells shapes measured at different $R$ (top row) and $R_g$ (middle row) from 8–11 kpc. The overlaid green curves with error bars represent the extracted profiles of the snail shells. Apparently, compared to the top row, more wraps of the shell can be identified in the middle row at each $R_g$. The bottom row compares the phase space snail shell shapes between $R$ (blue) and $R_g$ (red) from 8–11 kpc. $\phi$ and $R_{norm}$ are the angle (increasing counterclockwise in the phase space) and normalized dimensionless radius of the measured snail shell profiles in the polar coordinates. The snail shell shapes at the same $R$ and $R_g$ are consistent, except that the profiles in $R_g$ extend further with larger $\phi$, which indicates more wraps of the snail shell in the phase space.

![Figure 9](image9.png)

**Figure 9.** The pitch angle $\alpha$ at different guiding radii $R_g$. The snail shells at the outer disk show larger pitch angle (i.e., loosely wound) compared to the snail shell in the inner disk. This is consistent with the theoretical expectation. The red point with a square denotes the value of the solar neighborhood.
lower (higher) $V_z$ tail in the velocity distributions are captured in the $R_g$ selected subsamples. Nonetheless, they still show coherent phase mixing signals to enhance the snail shell feature at each $R_g$. More details are given in Appendix C.

4. Angular Momentum: A Fundamental Parameter to Trace the Snail Shell

In the epicycle approximation, a star revolves around the guiding center, which is on a circular orbit around the Galactic center. Grouping stars according to the guiding radius instead of the present Galactocentric radius can avoid the mixture of stars with different angular momentum. In this section, we perform quantitative comparisons of the phase space snail shells between different $R$ and $R_g$ ranges, and compare them to other previous works to emphasize the importance of the guiding radius in tracing the vertical phase mixing signal.

4.1. Snail Shell Shape Measurement

As shown in the $\Delta N$ maps of the $R$- and $R_g$-based phase spaces in Figure 7, the snail shell is reflected by a dark-brown color (local maximum) separated by a light-brown color (local minimum). Ideally, if we work in the polar coordinate of the phase space, and...
to choose a wedge centered at $(0, 0)$, then averaging the signal at each phase space radius in the wedge will result in a $\Delta N$ profile, with the peaks and troughs corresponding to the snail shell and inter shell regions, respectively. After identifying the shell positions in each wedge, the identified peak positions in all the wedges can be simply connected to recover the whole snail shell structure. The phase space is equally divided into 12 wedges with a $30^\circ$ azimuthal angle. The phase space angle $f = 0^\circ$ is defined in the direction of $Z = 0$ and $V_Z < 0$, which increases counterclockwise following the snail shell outwards from the central region of the phase space.

We have identified the snail shell structure in the $\Delta N$ maps at $R_g$. $R_g = 8, 9, 10$, and 11 kpc, which are shown in Figure 8 with the measured snail shell shape overlaid on the $\Delta N$ map. The outermost wrap of the snail shell is truncated depending on its relative amplitude. At 8 and 9 kpc, the snail shell is measured up to $\phi = 720^\circ$, then the measurement is terminated when the snail shell relative amplitude is below 0.025. For 10 and 11 kpc, the measurement is terminated when the snail shell relative amplitude is below 0.025.

10 In the phase space, the $Z$ and $V_Z$ values at each point are divided by 1.5 kpc and 70 km s$^{-1}$, respectively, to get a normalized dimensionless radius $R_{\text{norm}} = \sqrt{(Z/1.5 \text{ kpc})^2 + (V_Z/70 \text{ km s}^{-1})^2}$.

11 The snail shells at 6, 7, and 12 kpc are relatively weak and hard to measure quantitatively.

12 The relative amplitude of the snail shell is estimated by averaging the $\Delta N$ differences between the local peak position (the snail shell) and the adjacent local troughs (the inter shell regions on the two sides of each snail shell).
11 kpc, the snail shell is traced to $\phi = 540^\circ$. Then the profile is truncated when the snail shell relative amplitude is lower than 0.1. The error bar of the snail shell shape is determined by a $1/4$ separation between the two troughs on each side of the local peak (except for the outermost shell, which is determined by a $1/2$ separation between the peak and the inner trough, since no outer trough can be determined). As shown in Figure 8, the green curves with error bars generally follow the visually identified snail shell. Due to the large fluctuations in the signal, the outer shell is relatively difficult to map compared to the inner shell. Clearly, the green curve marking the $R_g$-based snail shell shows more wraps and a more well-defined shape than the $R$-based snail shell.

The bottom row of Figure 8 shows the shape of the $R$- and $R_g$-based snail shell in the $R_{\text{norm}} - \phi$ plane. At each radius, the two types of points follow the same track, suggesting that the snail shell shapes are similar. The red dot ($R_g$) extends to larger $\phi$ and $R_{\text{norm}}$ values to show more wraps in the phase space. In addition, the slope in the $R_{\text{norm}} - \phi$ plane is steeper at larger radius compared to the smaller ones. This is consistent with the
theoretical expectation that the snail shell at larger radius is more loosely wound than the snail shell at the smaller radius.

To quantify the degree of woundedness of the phase space snail shell, the concept of pitch angle $\alpha$ is adopted\textsuperscript{13}, which is often used in spiral arm measurement. By fitting a linear function between $R_{\text{norm}}$ and $\phi$ in the bottom row of Figure 8, the pitch angle at each guiding radius can be estimated. The result is shown in Figure 9. As expected, the pitch angle increases from $\sim 2.5-4^\circ$ with $R_g$ increasing from 8–11 kpc.

\textsuperscript{13} The pitch angle $\alpha$ can be calculated via $\cot(\alpha) = \left| \frac{\Delta R}{\Delta \phi} \right|$.

4.2. Comparison with Previous Works

We first compare our results with those of Antoja et al. (2018), the paper on the discovery of the phase space snail shell. As shown in the top row of Figure 10, we reproduce the phase space snail shell with stars near the solar radius in the Gaia RVS sample ($R = 8.34 \pm 0.1$ kpc), similar to Figure 1 in Antoja et al. (2018). For comparison, in the middle row, we also generate the $R_g$-based snail shell using Gaia RVS data. The bottom row shows the $R_g$-based snail shell using the LAMOST + Gaia sample. As shown in the $\Delta N$ maps (second column), it seems that the snail shell features are similar except that $R_g$-based snail shells are clear with one more wrap.
The snail shell shapes of the two phase spaces are measured with the method mentioned in the previous section. The $\Delta N$ maps with the snail shell curves overlaid are shown in the left three panels in Figure 11. The comparison between the three curves are shown in the fourth panel to the right. Clearly, the snail shell shapes are consistent within $\phi < 700^\circ$ (in the inner region of the phase space). The red and green dots ($R_g$) extend to larger $R_{\text{norm}}$, and $\phi$ values.

The snail shell clarity is also estimated with the $\Delta N$ profiles extracted along the $Z$ and $V_Z$ stripes; in the $\Delta N$ map, we choose the stripe with $|V_Z| < 15$ km s$^{-1}$ to get the $\Delta N - Z$ profile, and the stripe with $|Z| < 0.15$ kpc to get the $\Delta N - V_Z$ profile. The results are shown in Figure 12 with the left and right panels corresponding to the $\Delta N$ profiles along $Z$ and $V_Z$, respectively. The yellow vertical solid and dashed lines mark the snail shell turning-around points ($V_Z = 0$) and midplane points ($Z = 0$), respectively. The local peak position in the red and green curves ($R_g$) agree quite well with the yellow lines, with relatively larger differences between the peak and trough positions ($\sim 0.08$). The blue curve ($R$) also seems to agree with the position of the snail shells, but with a relatively smaller difference between the peaks and troughs ($\sim 0.05$). The comparison confirms that in the solar neighborhood, the snail shell in the $R_g$-based phase space is better revealed than the traditional $R$-based phase space.

Based on a small sample covering $\sim 1$ kpc distance from the Sun, Khanna et al. (2019) found that the phase spiral pattern for a given $L_Z$ is almost invariant with radius, suggesting the angular momentum as a more robust quantity to characterize the snail shell compared to $V_\phi$. As shown in Figure 16 of Khanna et al. (2019), the snail shell pattern revealed in the $L_Z$ color-coded phase space ($\Delta L_Z = 200$ kpc km s$^{-1}$) with the orientation of the snail shell changing with $L_Z$. Our result improves upon that of Khanna et al. (2019) in the aspect that we have explored a large radial range and performed comparisons of the $\Delta N$ phase spaces (representing the intrinsic shape of the snail shell rather than the $V_\phi$ or $L_Z$ color-coded phase space) of different $R$ and $R_g$ ranges. According to our results, $R_g$ (or $L_Z$) is not only robust, but also fundamental to trace the phase space snail shell across the Galactic disk.

To compare our results with those of Khanna et al. (2019), we explore the phase space distributions at each $L_Z$. We first derive the 6D information for stars in the Gaia RVS sample using the same solar position and kinematic configuration as in Khanna et al. (2019). Then we are able to reproduce Figure 16 in Khanna et al. (2019) by selecting stars in different angular momentum bins with $\Delta L_Z = 200$ kpc km s$^{-1}$ ($\sim 0.4$ kpc in $R_g$).

Figure 13 shows the number density, $\Delta N$, and $L_Z$ color-coded phase spaces from the top to bottom rows, respectively. Apparently, the snail shell shapes revealed in the top and middle rows ($N$ and $\Delta N$ maps) are quite different from the corresponding $L_Z$ color-coded maps in the bottom row. This confirms our previous conclusion that it is the number density map representing the intrinsic phase space structures. The $L_Z$ color-coded phase space may not accurately reflect the phase space snail shell structure because it is the number weighted average of the angular momentum of stars. In an imaginary extreme case, where all the stars in each panel of the top row in Figure 13 have the same angular momentum, the $L_Z$ color-coded $Z - V_Z$ phase space will not reveal any feature at all (see Appendix B for a further discussion). We can conclude that $R_g$-based snail shells can reveal more clear and intrinsic information on the phase mixing process than the $L_Z$ color-coded ones.

### 4.3. Colder and Hotter Orbits Dichotomy

Li & Shen (2020) found that, for stars near the solar neighborhood, a clear phase space snail shell can only be seen in the dynamically colder orbits ($|V_\phi - V_{\text{LSR}}| < 30$ km s$^{-1}$ or $J_R < 0.04$), while the hotter orbits have probably phase wrapped away already. The perturbation should have happened at least $\sim 500$ Myr ago to facilitate the phase mixing of the hotter orbits. Here with a larger sample we also investigate the cold/hot dichotomy across the Galactic disk. Similar to Li & Shen (2020), at different radii, we use the same criteria to separate the colder and hotter orbits. The phase space distributions for the colder and hotter orbits at different radii are shown in the left and right panels of Figure 14, respectively. Consistent with Li & Shen (2020), a prominent snail shell can only be seen on the colder orbits at different radii.

We also test dividing stars at each guiding radius into colder and hotter orbits. The $Z - V_Z$ phase space distributions are shown in Figure 15 with the colder and hotter orbits shown in the left and right panels, respectively. In fact, we have raised the orbital hotness criteria to $J_R > 0.06$, since snail shell features are still visible in the warmer orbit at intermediate $J_R$. The profiles are truncated with $\Delta N$ uncertainty larger than 0.14.
values (0.04 < \( J_R \) < 0.06). This test again confirms the importance of the guiding radius in revealing the phase space snail shell features, with the radial orbit hotness (ellipticity) playing a less important role.

As shown in Figure 15, the phase space number density maps of the hotter orbits (right panel) seem to show less vertical excursion than the colder orbits (left panel), inconsistent with the expectation that hotter orbits are more likely to drift away from the disk and reach a higher vertical distance. Using \( R_g = 9 \) kpc as an example, we compare the distribution of parameters representing the vertical excursion (i.e., \( J_Z \), \( Z \), and \( V_Z \)) between the colder orbits and hotter orbits\(^{15}\). As shown in the normalized histograms in Figure 16, both the hotter and colder orbits peak at 0, except that the hotter orbits show larger dispersions in \( J_Z \), \( Z \), and \( V_Z \) than the colder ones, consistent with the expectation. The reason that we found smaller phase space coverage as shown in Figure 15 for the hotter orbits is mainly due to the much lower numbers of stars; at \( R_g = 9 \) kpc, there are 944,231 stars in the colder orbits (\( J_R < 0.06 \)), and only 108,095 stars in the hotter ones (\( J_R > 0.06 \)). Another possibility for this inconsistency originates from our method to calculate \( R_g \), where we ignored the fact that the circular velocity decreases as a function of \( Z \) at a given radius. However, this effect should be minor. According to Bovy & Tremaine (2012), at the solar radius with \( |Z| < 1.5 \) kpc, the vertical gradient of the circular velocity can be estimated as \( \frac{\partial V_c}{\partial Z} \approx -2.8 \text{ km s}^{-1} \text{ kpc}^{-1} \left( \frac{Z}{100 \text{ pc}} \right) \). At \( Z = 1 \) kpc, the circular velocity is expected to be 14 km s\(^{-1}\) lower than the circular velocity in the midplane. The angular momentum

\(^{15}\) The colder and hotter orbits in Figure 16 are selected with \( J_R < 0.02 \) and \( J_R > 0.07 \), respectively, to enhance the difference in the parameter distributions.
difference is $\sim 110 \text{kpc km s}^{-1}$, 5% of the angular momentum at $R_e$, which should play a minor role in the guiding radius estimation.

4.4. Vertical Action and Oscillation Frequency Analysis

Aiming to better understand the vertical phase mixing process, we explore the action space of the sample. Figure 17 shows the $\Omega_Z - \sqrt{J_Z}$ correlations for stars at $R, R_g = 8 \text{kpc}$. At each radius, the anticorrelation between $\Omega_Z$ and $\sqrt{J_Z}$ is the main reason behind the phase space snail shell, representing the anharmonic vertical oscillation. Comparing the top and middle panels, the $\Omega_Z - \sqrt{J_Z}$ distribution at each $R_g$ is much tighter than that at different $R$, resulting in a clear snail shell in the number density map of the $R_g$ selected sample. This also implies that regardless of the current position in the disk, stars with the same angular momentum tend to follow a similar $\Omega_Z - \sqrt{J_Z}$ correlation (and similar snail shell shape in the $Z - V_Z$ phase space).

The top right panel in Figure 17 shows the same $\Omega_Z - \sqrt{J_Z}$ correlation at $R = 8 \text{kpc}$ but color coded with $V_f$. A clear trend can be seen in that $\Omega_Z$ decreases for larger $V_f$ at a given $J_Z$, consistent with Binney & Schönrich (2018). On the other hand, the middle right panel shows the $\Omega_Z - \sqrt{J_Z}$ correlation of stars

Figure 18. The $Z - V_Z$ phase space distributions of stars in different radial ranges (left panel) and guiding radius ranges (right panel) of the test particle simulation in the first approach. In the left (right) panel, the first, second, and third columns show the $\Delta N$, $V_R$, and $V_f$ color-coded phase spaces, respectively.
Figure 19. The evolution of the test particle simulation with the impulse approximation implemented. The intruder is assumed to hit the disk at 18 kpc along the X axis from the center, with a vertical downward velocity of 300 km s$^{-1}$. From left to right, the four columns show the evolution of the model at $T = 50, 250, 500,$ and 800 Myr after the impact. We adopt the snapshot at $T = 500$ Myr in the phase space analysis.

at $R_g = 8$ kpc color coded with the guiding radius deviation ($\Delta R_g = R_g - 8$ kpc). The relatively smaller guiding radius corresponds to higher vertical oscillation frequency $\Omega_Z$.

To better understand the tight correlation between $\Omega_Z$ and $\sqrt{J_Z}$ in the $R_g$-based sample, we first choose stars in a very narrow guiding radius range, i.e., $R_g = 8 \pm 0.01$ kpc, which are further separated into a dynamically colder subsample with $J_R < 0.01$ and a dynamically hotter subsample with $J_R > 0.06$. The $\Omega_Z$ and $\sqrt{J_Z}$ distributions of the two subsamples are shown in the bottom left panel of Figure 17. The correlation is very tight for the dynamically colder orbits at very small $J_R$, with the dispersion increasing considerably at larger $J_R$ for the dynamically hotter orbits. This is expected, since the dynamically colder stars are close to circular orbits; these stars with different vertical actions move in the vertical gravitational potential well at the same radius, thus resulting in a tight correlation between $\Omega_Z$ and $J_Z$. On the other hand, the dynamically hotter stars have large radial excursion, with the corresponding vertical profile of the gravitational potential varying to result in the large dispersion. Notice that the dispersion is mainly toward a lower vertical frequency. This is probably due to the fact that the dynamically hotter stars spend more time around the apocenter in the outer disk, with the vertical oscillation frequency reduced in the shallower potential well.

The bottom right panel in Figure 17 shows the tight $\Omega_Z - \sqrt{J_Z}$ correlation for dynamically colder stars at different guiding radii from 6–11 kpc, each with 0.02 kpc width annulus and $J_R < 0.01$. At a given vertical action value, the vertical oscillation frequency decreases progressively at larger guiding radius. The slope at larger $R_g$ is also shallower, consistent with the loosely wound snail shell shown in Figure 7.

5. Test Particle Simulations

To better understand the spatial variation and the importance of the angular momentum of the phase space snail shell across the Galactic disk, we perform test particle simulations in a realistic Milky Way potential, i.e., Model I in Irgang et al. (2013). We use AGAMA to construct the stellar distribution function (DF) for the initial condition and to perform the orbit integration of the test particle simulation (Vasiliev 2019). We adopt the quasi-isothermal DF for the thin disk (Binney 2010; Binney & McMillan 2011), with the radial scale length as 3.7 kpc, and vertical scale height as 0.3 kpc (Binney & Piffl 2015; Bland-Hawthorn & Gerhard 2016).

The vertical perturbation is imposed on the test particles using two different approaches. The first approach is similar to Li & Shen (2020), where the particles receive vertical downward velocities with the vertical positions barely changed; each particle receives a random vertical velocity perturbation with the median value of $-30$ km s$^{-1}$ and dispersion of 5 km s$^{-1}$. For the second approach, we consider a more realistic impulse approximation on the test particle velocities following Binney & Schöning (2018) to mimic the effect of a fly-by external perturber. Although the test particle simulation is relatively simple, it has the advantage of numerical efficiency and could also reflect the important physical processes of vertical oscillation and phase mixing (see Bland-Hawthorn & Tepper-Garcia 2021 for a more recent comprehensive numerical modeling with N-body simulation).

In the first approach, 1 million test particles have been sampled from the DF. Regardless of the different azimuthal angles, the test particles at the same radius receive the same vertical perturbation, and thus the same vertical velocity distributions. In the following analysis, all the particles in each radial (or guiding radial) annulus are used. After 500 Myr of evolution, the $Z - V_Z$ phase space distributions are shown in Figure 18 for different radial annuli (left panel) and different guiding radial annuli (right panel) similar to Figure 7. The $R$-based snail shell shapes are similar to $R_g$-based snail shell, with the $R_g$-based snail shells narrower and more well defined than the $R$-based ones. These results agree with the observational results in Figure 7. Stars with the same angular momentum tend to follow the same vertical oscillation pattern (although they were located at different radii). On the other hand, in a given radial annulus, stars usually have a wide distribution of

$\Delta V_z = V_{z,\text{max}} - V_{z,\text{min}}$.

To enhance the prominence of the snail shell, we only keep the colder orbits at each radius ($V_{z} - V_{z,\text{circ}} < 20$ km s$^{-1}$).
angular momentum. The mixture of stars with different angular momenta (and the related phase space pattern) results in the blurring of the snail shell.

In the left panel of Figure 18, the third column shows the \( V_f \) color-coded phase space. Different colors show different snail shell patterns. For particles at a given radius, the azimuthal velocity is proportional to the angular momentum. Since stars with different angular momenta have different snail shell patterns, when color coded with \( V_f \), different snail shells naturally emerge in the phase space. Comparing the results with different \( R_g \) ranges in the right panel, there is no systematic variation of the snail shell shape with azimuthal velocity, since the median \( V_f \) at different parts of the phase space along the snail shell is close to 0. In Figure 18 for both the \( R \)- and \( R_g \)-based phase spaces, no variation of the snail shell pattern with \( V_R \) can be seen. The median radial velocity in the phase space is close to 0. This is mainly due to the initially imposed symmetric distribution of \( V_R \) in our test particle simulation.

For the second approach, we sampled 3 million test particles based on the DF to account for the azimuthal variation of the phase space snail shell. The perturbation to the vertical velocities of the particles are estimated following the impulse approximation in Binney & Schönrich (2018). The impact parameter of the external perturber is 18 kpc along the X-axis from the Galactic center at a speed of \( \sim 300 \text{ km s}^{-1} \). The mass of the perturber is \( 2 \times 10^{10} M_\odot \) and \( T = 66 \text{ Myr} \) as the passage timescale. To work in the non-inertial frame with the Galactic center stationary, the gravitational pull from the intruder to a point mass at the Galactic center is subtracted from the acceleration of the intruder on the other test particles. The in-plane velocity change \( \delta v_p \) is computed by multiplying the characteristic timescale of the passage with the acceleration. A downward component \( \delta v_\perp \) is added as \( \delta v_\perp = \alpha \delta v \left[ 1 - \beta \sin(\psi - \psi_{\text{intruder}}) \right] \) with \( \alpha = 1.5 \) and \( \beta = 0.5 \) (see Equation 3 in Binney & Schönrich 2018).\(^{17}\) The vertical perturbation is composed of two components: a constant term

\[^{17}\text{We have increased } \alpha \text{ from 0.4–1.5 to enhance the vertical perturbation effect.}\]
associated with the standard dynamical friction and a term proportional to \( \sin(\psi - \psi_{\text{intruder}}) \). The second term arises from the consideration that stars initially moving toward the intruder \((\psi - \psi_{\text{intruder}} < 0)\) receive a net downward kick, while those stars initially moving away from the intruder \((\psi - \psi_{\text{intruder}} > 0)\) receive a net upward kick. We define \( \psi_{\text{intruder}} = 0^\circ \) along the \( X \) axis, and the azimuthal angle increases counterclockwise.

After the impact, the time evolution of the test particles is shown in Figure 19. As shown in the first column, 50 Myr after the perturbation, the disk becomes lopsided and is stretched toward the intruder with a prominent single arm. The amplitude of the vertical perturbation of each particle depends on the azimuthal angle, with larger amplitudes for particles closer to the intruder. This is the main difference compared to the previous test particle simulation where the imposed vertical perturbation is azimuthally symmetric. In the second column, 250 Myr after the impact, the initial perturbation gradually evolves to form the \( m = 2 \) mode due to the differential rotation, with the vertical oscillation in different parts of the disk. After 500 Myr of evolution (third column), the \( m = 2 \) mode is still present, with continuous phase mixing in the vertical direction of the disk. In fact, this is the snapshot we adopt in the following analysis, since the phase space snail shell is most prominent and roughly similar to the observations. At later times, as shown in the fourth column (800 Myr), the face-on image still exhibits a prominent \( m = 2 \) mode feature, with the roughly symmetric distribution of the particles with respect to the disk midplane in the edge-on view of the model.

Recently, Bland-Hawthorn & Tepper-Garcia (2021) used a high-resolution \( N \)-body model to simulate the impact of an external intruder on the main disk. They found good agreement with the model in Binney & Schönrich (2018), indicating that the impulse approximation is valid to describe such process. After the impact, Bland-Hawthorn & Tepper-Garcia (2021) noticed a strong, \( m = 1 \) bending mode across the disk is set up, which gradually wraps up due to the differential rotation of the disk to result in a superposition of two distinct bisymmetric \( (m = 2) \) modes, namely, a spiral pattern and a bending wave.

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**Figure 21.** The \( Z - V_Z \) phase space distributions of stars in different guiding radial ranges (from \( R_g = 6 - 12 \) kpc) of the test particle simulation in the second approach. The left three columns show the number density maps in the \( Z - V_Z \) phase space at each guiding radius (different rows) and each azimuthal wedge (different columns). The middle three columns show the \( V_Z \) color-coded \( Z - V_Z \) phase spaces, and the right three columns show the \( V_R \) color-coded phase spaces corresponding to these number density maps.
Their self-consistent N-body simulation results are quite similar to our simple test particle simulation under the impulse approximation.

Since the vertical perturbation is not azimuthally symmetric, different phase space snail shells are expected to emerge at different azimuthal angles. The disk is separated into three azimuthal wedges with 120° each, in order to trace the azimuthal variation of the phase space snail shell with a sufficient number of particles. The $Z-V_Z$ phase space distributions at different radii and azimuthal wedges are shown in Figure 20. At each radius, as expected, the shape of the phase space snail shell is different in different azimuthal wedges. For example, at $R=8$ and 9 kpc, the snail shell is barely seen in the azimuthal wedge with $\phi = (4\pi/3, 2\pi)$ (first column), while in the other azimuthal wedges at $(2\pi/3, 4\pi/3)$ (second column) and $(0, 2\pi/3)$ (third column), the snail shells are clear. In the $V_R$ (middle three columns) and $V_\phi$ (right three columns) color-coded phase spaces, evidence of snail shell shape changing at different velocities can be seen, roughly consistent with observations.

The phase space distributions at different guiding radii are shown in Figure 21. With azimuthal variations still present, the $R_g$-based number density phase space (left three columns) show prominent snail shells compared to the $R$-based phase spaces in Figure 20. In the $V_R$ and $V_\phi$ color-coded phase spaces, there is no systematic variation of the snail shell shape with the velocities. Bland-Hawthorn & Tepper-Garcia (2021) commented that they could not find the phase space snail shell feature as a function of the angular momentum. Still, their model is one of the best available in the literature that fits the global Milky Way properties well.

Based on the first test particle simulation, the pitch angles of the phase space snail shells in different guiding radii ($8 \sim 11$ kpc) at different times (300 $\sim$ 800 Myr) are measured following the method in Section 4.1. By comparing to observational pitch angles, this could help date the perturbation event: the pitch angle decreases monotonically as the phase space snail shell winds up with time. The radial profiles of the measured pitch angle at different times are shown in Figure 22.

The pitch angle decreases from $\sim 6^\circ$ at 300 Myr to $\sim 2.2^\circ$ at 800 Myr, consistent with our expectation for tighter snail shells at earlier perturbation events. The observational result seems to agree well with the pitch angle profiles from 500–700 Myr, with the best agreement at $T = 600$ Myr. This constraint of the perturbation event is also consistent with previous results using other different methods, such as the consecutive turns of the snail shell (Antoja et al. 2018), the cold/hot orbit dichotomy of the snail shell (Li & Shen 2020), and vertical potential reconstruction with oscillation frequency estimation (Li & Widrow 2021).

6. Summary

We utilize $\sim$11 million stars from both the Gaia RVS sample and the LAMOST DR6 sample to study the vertical phase mixing across the Galactic disk. We confirm the existence of the snail shells in the $Z-V_Z$ phase space in the Galactic disk from 6–12 kpc, and quantitatively measure the shape of the snail shells, with the corresponding pitch angles also derived. We find that the guiding radius (angular momentum) is fundamental to reveal the vertical phase mixing signals. Compared to the $R$-based phase spaces, the clarity of the snail shell in the $R_g$-based phase space is increased; more wraps of the snail shell can be seen; phase space is less affected by the lack of stars close to the disk midplane due to extinction; snail shell signal is also amplified in a greater radial range.

In the epicycle theory, a star revolves around the guiding radius that is on circular orbits in the disk plane. When perturbed in the vertical direction, the amplitudes and the frequency of the stellar vertical oscillation depend on the shape of the vertical potential at different radii. Along its orbit, as the star moves inside $R_g$, the vertical oscillation frequency generally increases with lower $Z_{\text{max}}$ and higher $V_{Z_{\text{max}}}$, and vice versa for the star moving outside $R_g$. Averaging along the orbits, the $Z-V_Z$ phase space trajectories of the stars with the same angular momentum generally showed a coherent pattern with small deviations. Therefore, grouping stars into different $R_g$ helps enhance such phase space signal with a more prominent snail shell.

Additional evidence comes from the cold/hot dichotomy discovered in Li & Shen (2020) in the solar neighborhood. For the $R$-based phase space, the cold/hot dichotomy still holds, with a prominent snail shell only in the colder orbits. However, in each $R_g$ range, a snail shell can still be seen in warmer orbits. The snail shell eventually disappears for very hot orbits with $J_R > 0.06$ (compared to the original criteria $J_R > 0.04$ for the hotter orbits in Li & Shen 2020). Compared to the colder orbits, the warmer orbits (0.04 $< J_R < 0.06$) show larger radial excursion. Within each $R_g$ range, they still show a similar snail shell feature as the colder orbits, indicating that the vertical perturbation is probably global and of external origin. If the perturbation happens in the inner disk, the orbits with the same $L_{\phi}$ but different radial excursion will likely encounter the outward propagating wave at different times, to show differences in the initial phase angle of the snail shell, which is likely to blur the phase space signals. Therefore, to study the vertical phase mixing process, it is recommended to use $R_g$ (with additional constraints on $J_\phi$) to better reveal the intrinsic shape of the snail shell in the Galactic disk. In another ongoing work, we are trying to model the snail shell shapes at different radii to constrain the shape of the Galactic potential.
The difference between the phase space distributions in the Galactocentric radial range and the guiding radius range can also be understood in the $\Omega_z - \sqrt{I_z}$ plane. At each radius, the distribution of stars in the $\Omega_z - \sqrt{I_z}$ plane shows anticorrelation with a broad distribution, while stars in the corresponding $R_g$ range show a much tighter correlation, corresponding to more prominent snail shells in the phase space.

Test particle simulations are also performed using two approaches to understand this phenomenon. In the first approach, a simple azimuthally symmetric vertical perturbation is imposed on all the test particles. Snail shell features can be seen across the disk, which are more prominent when the particles are grouped into different guiding radial ranges. In the second approach, the perturbation with the impulse approximation is applied on all the test particles. Again, the phase space snail shell is more prominent in different guiding radial ranges, consistent with the observations. By comparing the pitch angle profiles between the observation and the simulation, the perturbation event is constrained to happen at $\sim 500$–$700 \text{ Myr}$ ago, consistent with other studies using different methods.

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Guoshoujing Telescope (LAMOST) is a National Major Scientific Project built by the Chinese Academy of Sciences. This work has made use of data from the European Space Agency (ESA) mission Gaia (http://www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC; http://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement.

Appendix A

$V_\phi$ Color-coded Phase Space Properties

As shown in the right panel in Figure 7, in the $V_\phi$ color-coded phase space at $R_g = 8, 9, 10, 11$, and (possibly) 12 kpc, the average azimuthal velocity seems smaller at larger $|Z|$ and higher at larger $|V_Z|$. This feature can be understood from the statistical distribution of stellar orbital properties. Focusing on $R_g = 8, 9, 10, 11$, we select three subsamples at each guiding radius bin, with subsample 1 at $|Z| \leq 0.5 \text{ kpc}$ and $|V_Z| \leq 15 \text{ km s}^{-1}$, subsample 2 at $|Z| \leq 0.5 \text{ kpc}$ and $|V_Z| \geq 20 \text{ km s}^{-1}$, and subsample 3 at $|Z| > 0.7 \text{ kpc}$. Figure 23 shows the distributions of $I_Z, Z, R$, and $V_\phi$ at each guiding radius, with the black solid line, the red dotted line, and the blue dashed line representing subsamples 1, 2, and 3, respectively. As expected from the location in the $Z - V_Z$ diagram, and also shown in the first column, the vertical action is smallest in subsample 1, and largest in subsample 3. The vertical height is largest in subsample 3 (second column), corresponding to a larger radial range shown in the third column; stars with higher vertical heights tend to be located outwards at larger radius where the disk potential well is shallower. At each guiding radius, these three subsamples have similar angular momentum. Therefore, the larger radii naturally result in smaller azimuthal velocities in subsample 3 (fourth column).

For subsample 2, although the vertical action is higher than that of subsample 1 as shown in the first column, they show similar vertical height distributions (second column), with subsample 1 slightly narrower. The much larger vertical velocities of subsample 2 hint at a smaller radius for this subsample; stars are more likely to acquire larger vertical velocities when traveling at smaller radii with a deeper potential well. This picture is consistent with the third column. Therefore, the azimuthal velocity of subsample 2 is relatively higher than that of subsample 1.

Besides the $V_\phi$ pattern of the phase space at each $R_g$, careful inspection of the left columns in Figure 7 for the snail shell at different radial annuli indicates differences between the snail shell shapes of the $\Delta N$ map (first column) and $V_\phi$ color-coded phase spaces (third column), especially for the annulus at 10 kpc. This difference is likely caused by the snail shell shape variation at different $V_\phi$. According to Li & Shen (2020), from a mathematical point of view, the $V_\phi$ color-coded phase space can be considered as the number weighted average of the azimuthal velocity of stars in each radius. With $V_\phi$ color coding, problems with the incomplete sampling close to the midplane can be alleviated. However, the $V_\phi$ color-coded phase space may not truly reflect the real phase space snail shell shape. Li & Shen (2020) have shown in their Figure 14 that by combining stars following different snail shell shapes at slightly different $V_\phi$, the phase space color coded with $V_\phi$ would look quite different from the $\Delta N$ map. As shown in the second and third columns of the right panel for different guiding radius ranges, there is no well-recognized snail shells at all for the $V_R$ and $V_\phi$ color-coded phase spaces.
Appendix B

Z − VZ Phase Space at Different Rg

In order to better visualize the gradual transition of the phase space snail shell across the Galactic plane, we separate the sample into 30 guiding radius bins from 6–12 kpc with the bin width of 0.2 kpc. The number density map and the contrast maps are shown in Figures 24 and 25, respectively. As the guiding radius increases, the snail shell becomes less wound and more extended along the Z direction. The gradual transition is very clear. The \( V_R \), \( V_{\phi} \), and \( L_Z \) color-coded phase spaces are shown in Figures 26, 27, and 28, respectively. No clear pattern can be seen, consistent with our previous results.
Figure 24. The number density map $N$ of the $Z - V_Z$ phase space distributions for stars with different guiding radii $R_g$. 
Figure 25. The number density contrast map $\Delta N$ of the $Z - V_Z$ phase space distributions for stars with different guiding radii $R_g$. 
Figure 26. The radial velocity ($V_R$) color-coded $Z - V_Z$ phase space distributions for stars with different guiding radii $R_g$. 

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Figure 27. The azimuthal velocity ($V_\phi$) color-coded $Z - V_Z$ phase space for stars with different guiding radii $R_g$. 
Appendix C

The Gap in the Phase Space around $Z = 0$

In Figure 7, a clear gap close to $Z = 0$ in the $R$-based phase space number density map can be seen, especially at radii smaller than 8 kpc. This feature is mainly caused by the stronger dust attenuation in the disk midplane. However, this gap becomes much weaker or even disappears in the corresponding phase space at $R_g$. Here we perform a detailed comparison between the LAMOST and Gaia RVS sample at each $R$ and $R_g$ to better understand this difference. Figure 29 shows the comparison at 8 kpc. The gap is very clear in the LAMOST sample at $R = 8$ kpc (top row), and much weaker at $R_g = 8$ kpc (middle row). For the Gaia RVS sample, the gap is weaker, but the improvement in $R_g$ is apparent. The bottom panel shows the same $R_g$ phase space as the middle row but color coded with $\Delta R = R - 8$ kpc. It seems that at $R_g = 8$ kpc, most stars come from $R = 8 \sim 9$ kpc. In this case, stars in the disk are reshuffled with a large fraction of stars near the solar radius (where the observed stellar number density is highest close to the midplane) redistributed to other $R_g$ values. Figure 30 shows the $R - Z$ distribution of stars with guiding radii $R_g = 7-10$ kpc in the LAMOST (top row) and Gaia samples (bottom row). The contribution from stars near the solar radius is significant at each guiding radius. These stars are also well sampled across the vertical direction, especially close to the midplane around $Z = 0$. In fact, stars at each guiding radius originate from a large radial range, with a high contribution especially from 8–9 kpc.

Figure 28. The angular momentum $L_z$ color-coded $Z - V_z$ phase space for stars with different guiding radii $R_g$. 

Figure 29. The angular momentum $L_z$ color-coded $Z - V_z$ phase space for stars with different guiding radii $R_g$. 

Figure 30. The angular momentum $L_z$ color-coded $Z - V_z$ phase space for stars with different guiding radii $R_g$. 

Figure 29. The number density contrast map $\Delta N$ of the $Z - V_z$ phase space distributions for stars in the LAMOST and Gaia RVS samples at 8 kpc. The left and right panels show the LAMOST and Gaia RVS results, respectively. The top and middle rows show the phase space map at the radius $R$ and guiding radius $R_g$, respectively. The bottom row is the phase space at $R_g$ and color coded with $\Delta R = R - 8$ kpc.
Figure 30. The $R - Z$ distributions for stars at different guiding radii ($R_g$) in the LAMOST (top row) and Gaia samples (bottom row). The cyan dot marks the position of the Sun.

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**Figure 30.** The $R - Z$ distributions for stars at different guiding radii ($R_g$) in the LAMOST (top row) and Gaia samples (bottom row). The cyan dot marks the position of the Sun.
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