The Nuts and Bolts of Brane Resolution

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ABSTRACT

We construct various non-singular $p$-branes on higher-dimensional generalizations of Taub-NUT and Taub-BOLT instantons. Among other solutions, these include $S^1$-wrapped D3-branes and M5-branes, as well as deformed M2-branes. The resulting geometries smoothly interpolate between product spaces which include Minkowski elements of different dimensionality. The new solutions do not preserve any supersymmetry.
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1 Introduction

Regular $p$-brane solutions can be constructed by a process which has been referred to as “resolution via transgression” \cite{1}. The first step is to replace the standard flat transverse space by a smooth space of special holonomy, which is Ricci-flat and has fewer covariantly constant spinors. Next, the $p$-brane solution is deformed by additional flux such that the Chern-Simons terms modify the equation of motion and/or Bianchi identity of the field strength. If the new transverse space has a non-collapsing $n$-cycle, then it can support a harmonic $n$-form which is square integrable at short distance. The result is a completely non-singular geometry \footnote{The latter portion of this procedure was first applied to the heterotic 5-brane, for which the singularity could be smoothed out by Yang-Mills fluxes \cite{2}. Curiously, the standard flat transverse space did not need to be replaced.}.

This procedure was applied to the D3-brane, for which the six-dimensional transverse space was replaced by the deformed conifold \cite{4,5,6}. Since the new transverse space has a non-collapsing 3-cycle, it supports a square integrable three-form flux, which serves to resolve the singularity. This singularity resolution procedure has been applied to many other $p$-branes, including D2-branes on spaces of $G_2$ holonomy \cite{1}, M2-branes on spaces of Spin(7) holonomy \cite{15,11,16,7,17,18,19,20,21,22,23,24,25,7,8}, and others \cite{7,8}, to give a small sample.

Since the standard flat transverse space is replaced by a space of special holonomy, the supersymmetry is reduced to a minimum. In fact, the main motivation for considering such solutions is because they may constitute viable gravity duals of strongly-coupled Yang-Mills field theories with reduced supersymmetry. This indicates that they may shed light on confinement and chiral-symmetry breaking. This was explored for the case of the D3-brane and the dual $\mathcal{N} = 1, D = 4$ superconformal Yang-Mills theory in \cite{3,4,5,6}.

Previous work on brane resolution has incorporated transverse spaces of special holonomy, in order to study dual gauge theories of minimal supersymmetry. However, resolved brane solutions are certainly of interest in their own right. Unlike typical brane solutions which require a source term that is beyond supergravity, resolved branes are complete purely within the framework of supergravity. An interesting point is that the procedure of resolving singularities does not depend on the presence of
supersymmetry. In this paper, we illustrate this point by considering $p$-brane solutions on higher-dimensional generalizations of Taub-NUT and Taub-BOLT instanton spaces [11, 12, 23] which do not preserve any supersymmetry.

## 2 $S^1$ wrapped D3-brane

The D3-brane of type IIB supergravity is supported by the self-dual 5-form field strength, with a six-dimensional Ricci-flat transverse space. Due to the Bianchi identity $dF_5 = F_{(3)}^{NS} \wedge F_{(3)}^{RR}$, one can construct a fractional D3-brane if the transverse space has a self-dual 3-cycle. If instead the transverse space has a 2-cycle $L_{(2)}$, we can construct an $S^1$-wrapped D3-brane with one of the world-volume coordinates fibred over the transverse space [13, 14]. The solution is given by

$$
\begin{align*}
\text{ds}_{10}^2 &= H^{-1/2} \left( -dt^2 + dx_1^2 + dx_2^2 + (dx_3 + A_{(1)})^2 \right) + H^{1/2} \text{ds}_6^2, \\
F_5 &= dt \wedge dx_1 \wedge dx_2 \wedge (dx_3 + A_{(1)}) \wedge dH^{-1} - *_6 dH \\
&\quad + m *_6 L_{(2)} \wedge (dx_3 + A_{(1)}) + dt \wedge dx_1 \wedge dx_2 \wedge L_{(2)}, \\
\text{d}A_{(1)} &= m L_{(2)},
\end{align*}
$$

where $L_{(2)}$ is a harmonic 2-form in the transverse space of the metric $\text{ds}_6^2$, and $*_6$ is the Hodge dual with respect to $\text{ds}_6^2$. The equations of motion are satisfied, provided that

$$
\Box H = -\frac{1}{2}m^2 L_{(2)}^2,
$$

where $\Box$ is the Laplacian in $\text{ds}_6^2$.

Note that one can also have a fibred time-like direction, in which case the D3-brane solution is given by

$$
\begin{align*}
\text{ds}_{10}^2 &= H^{-1/2} \left( - (dt + A_{(1)})^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) + H^{1/2} \text{ds}_6^2, \\
F_5 &= (dt + A_{(1)}) \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dH^{-1} + d^3 x \wedge L_{(2)} + \text{dual terms},
\end{align*}
$$

which can be interpreted as a rotating D3-brane.
2.1 On Taub-NUT/BOLT with $B = \mathbb{CP}^2$

The six-dimensional Taub-NUT/BOLT instanton metric with base space $\mathbb{CP}^2$ is given by

\[ ds_6^2 = F^{-1} \, dr^2 + 4N^2 \, (d\tau + A)^2 + R^2 \, d\Sigma_4^2, \]

(2.4)

where $R^2 = r^2 - N^2$. $d\Sigma_4^2$ is the metric over $\mathbb{CP}^2$, given by

\[ d\Sigma_2^2 = \frac{du^2}{V^2} + \frac{u^2}{4V^2} \,(d\psi + \cos \, \theta \, d\phi)^2 + \frac{u^2}{4V} \,(d\theta^2 + \sin^2 \, \theta \, d\phi^2). \]

(2.5)

\[ A = \frac{u^2}{2V} \,(d\psi + \cos \, \theta \, d\phi), \]

(2.6)

and

\[ V = 1 + \frac{u^2}{6}. \]

(2.7)

The function $F$ is given by

\[ F = \frac{\frac{1}{3}r^4 - 2N^2 \, r^2 - 2M \, r - N^4}{R^4}. \]

(2.8)

For the appropriate periodicity of the fibre coordinate $\tau$, we can avoid a conical singularity. In fact, for the base space $\mathbb{CP}^2$, the six-dimensional Taub-NUT ($M = -\frac{4}{3}N^3$ and $r \geq N$) and Taub-BOLT ($M = \frac{4}{3}N^3$ and $r \geq 3N$) are free of singularities. At short distance, the Taub-NUT approaches $\mathbb{R}^8$ while the Taub-BOLT approaches $\mathbb{R}^2 \times \mathbb{CP}^2$. At large distance, they both are asymptotically cylindrical, having the form of an $S^1$ bundle over a cone with the base $\mathbb{CP}^2$, which we shall denote as $C(\mathbb{CP}^2) \times S^1$.

We are interested in finding a harmonic 2-form supported by the metric (2.4). The most general ansatz for a 2-form with respect to the isometry of (2.4) is given by

\[ L_{(2)} = u_1 \, e^0 \wedge e^5 + u_2 \, e^1 \wedge e^2 + u_3 \, e^3 \wedge e^4, \]

(2.9)

expressed in the vielbein basis

\[ e^0 = \frac{1}{\sqrt{F}} \, dr, \quad e^1 = R \, \frac{u}{2\sqrt{V}} \, d\theta, \quad e^2 = R \, \frac{u}{2\sqrt{V}} \, \sin \, \theta \, d\phi, \quad e^3 = \frac{R}{V} \, du, \]

\[ e^4 = R \, \frac{u}{2V} \,(d\psi + \cos \, \theta \, d\phi), \quad e^5 = \sqrt{F} \, [d\tau + \frac{u^2}{2V} \,(d\psi + \cos \, \theta \, d\phi)]. \]

(2.10)

The closure and co-closure of $L_{(2)}$ yield the following solutions:

\[ u_1 = \left( \frac{r + N}{r - N} \right)^{\frac{\sqrt{8 + N^2}}{2N}} \,(r^2 - N^2)^{-3/2}, \quad -u_2 = u_3 = \frac{1}{4}(r \mp \sqrt{8 + N^2}) \, u_1. \]

(2.11)
It has a non-trivial flux and is not normalizable at large distance. For the Taub-BOLT metric \( (M = \frac{4}{3}N^3 \text{ and } r \geq 3N) \), both two-forms are square integrable at short distance. On the other hand, for the Taub-NUT metric \( (M = -\frac{4}{3}N^3 \text{ and } r \geq N) \) only the two-form with the negative sign is square integrable at short distance. We solve for the corresponding \( H \) in the case \( N = 1 \).

First, we consider the Taub-NUT metric. For the top sign in (2.11), there are only singular solutions. This can be seen a priori, since the corresponding two-form is not square integrable. On the other hand, for the bottom sign in (2.11), there is a regular solution to \( H \). For this first example, we will show explicit details. In this case, \( (2.12) \) can be written as

\[
\frac{1}{(r-1)^2} \partial_r (r-1)^3 (r+3) \partial_r H = -\frac{3m^2 (r^2 + 6r + 17)}{8(r+1)^4}.
\]

(2.12)

The solution is given by

\[
H = 1 + \frac{3m^2}{16(r+1)^2} + \left( c - \frac{9}{512} m^2 \right) \left[ \frac{4}{r-1} - \frac{8}{(r-1)^2} + \log \left( \frac{r-1}{r+3} \right) \right].
\]

(2.13)

Choosing the integration constant \( c = \frac{9}{512} m^2 \) yields a regular solution given by

\[
H = 1 + \frac{3m^2}{16(r+1)^2}.
\]

(2.14)

For future examples given in this paper, we will only present the regular solution that results from the appropriate choice of the integration constant.

At both short and large distances, \( H \) asymptotes to a constant. Therefore, the D3-brane geometry smoothly interpolates between \( M_3 \times \mathbb{R}^6 \ltimes S^1 \) (a product space of three-dimensional Minkowski spacetime and a \( U(1) \) bundle over \( \mathbb{R}^6 \)) at short distance to \( M_4 \times C(\mathbb{CP}^2) \ltimes S^1 \) (a product space of four-dimensional Minkowski spacetime and a \( U(1) \) bundle over a cone with base \( \mathbb{CP}^2 \)) at large distance.

Next, we turn to the Taub-BOLT metric. In this case, since the two-form is square integrable for both signs in (2.11), both of the corresponding solutions are regular. These are given by

\[
H = 1 + \frac{3m^2 (\pm 6 + 5r \pm 25r^2 + 10r^3)}{160(r \pm 1)^5}.
\]

(2.15)

As before, \( H \) is asymptotically constant at short and large distances. The D3-brane geometry smoothly goes from \( M_3 \times (\mathbb{R}^2 \times \mathbb{CP}^2) \ltimes S^1 \) at short distance to \( M_4 \times C(\mathbb{CP}^2) \ltimes S^1 \) at large distance.
Since the Taub-BOLT instanton supports two independent harmonic two-forms, these can be superimposed to form a more general $S^1$-wrapped D3-brane. It is also possible to have a regular $T^2$-wrapped D3-brane, for which two of the worldvolume directions are fibred over the two two-forms respectively. The resulting short distance geometry of the D3-brane would then be $M_2 \times (\mathbb{R}^2 \times \mathbb{C}P^2) \times T^2$.

2.2 On Taub-BOLT with $\mathcal{B} = S^2 \times S^2$

The six-dimensional Taub-NUT/BOLT instanton metric with base space $S^2 \times S^2$ is given by [11]

$$ds_6^2 = F^{-1} dr^2 + 4 N^2 F \sigma^2 + (r^2 - N^2)(d\Omega_2^2 + d\tilde{\Omega}_2^2), \quad (2.16)$$

where

$$d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2, \quad d\tilde{\Omega}_2^2 = d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\phi}^2,$$

$$\sigma = d\psi + \cos \theta d\phi + \cos \tilde{\theta} d\tilde{\phi}. \quad (2.17)$$

The function $F$ is given by [2.8]. For the base space $S^2 \times S^2$, the Taub-NUT has a curvature singularity. However, the Taub-BOLT ($M = \frac{2}{3} N^3$ and $r \geq 3N$) is completely smooth, and goes from $\mathbb{R}^2 \times S^2 \times S^2$ at short distance to $C(S^2 \times S^2) \times S^1$ at large distance.

The most general ansatz for a 2-form with respect to the isometry of (2.16) is given by [2.9] expressed in the vielbein basis $e^0 = 1/\sqrt{F} \, dr$, $e^1 = R \, d\theta$, $e^2 = R \sin \theta \, d\phi$, $e^3 = R \, d\tilde{\theta}$, $e^4 = R \sin \tilde{\theta} \, d\tilde{\phi}$ and $e^5 = 2N \sqrt{F} \, \sigma$. The closure and co-closure of $L_{(2)}$ yield the following solutions:

$$u_1 = -\frac{4N}{(r \pm N)^3}, \quad u_2 = u_3 = \frac{r \pm 3N}{(r \pm N)^3}. \quad (2.18)$$

It has a non-trivial flux. The square of this form is

$$L_{(2)}^2 = \frac{32N^2 + 4(r \pm 3N)^2}{(r \pm N)^6}. \quad (2.19)$$

Thus, it is square integrable for $r \to 0$ but not normalizable at large distance.

For the negative sign, for an appropriate choice of integration constant, there is a regular solution to $H$ in (2.2) given by

$$H = 1 + \frac{m^2}{(r - N)^2}. \quad (2.20)$$
For the positive sign, an appropriate choice for the integration constant leads to the regular solution

\[ H = 1 + \frac{(6N^3 + 5N^2 r + 25N r^2 + 10r^3) m^2}{10(N + r)^5}. \]  

(2.21)

All of these geometries smoothly interpolate from \( M_3 \times (\mathbb{R}^2 \times S^2 \times S^2) \times S^1 \) at short distance to \( M_4 \times C(S^2 \times S^2) \times S^1 \) at large distance. Again, we can superimpose the above solutions to get a regular \( T^2 \)-wrapped D3-brane.

However, Taub-BOLT instantons with \( B = S^2 \times S^2 \ldots \times S^2 \) do not admit a spin structure \([10, 11]\). This is because each \( S^2 \) factor generates an element of \( H_2 \) of odd self-intersection. This applies to the \( S^1 \)-wrapped D3-brane on a Taub-BOLT with \( B = S^2 \times S^2 \) of this section, as well as the deformed M2-brane on a Taub-BOLT with \( B = S^2 \times S^2 \times S^2 \) and the deformed or \( S^1 \)-wrapped 5-brane on a Taub-BOLT with \( B = S^2 \), which we will discuss shortly. Nevertheless, these spacetimes may still admit a \( \text{Spin}^C \) structure\(^2\).

### 2.3 On generalized Taub-NUT/BOLT

#### 2.3.1 Taub-BOLT

Recently a family of Taub-NUT and Taub-BOLT metrics were found which have an additional spherical element \([12]\). Included is a six-dimensional Taub-BOLT solution with the topology \( S^2 \) times a \( \mathbb{R}^2 \) bundle over the base space \( S^2 \). The metric is given by \([12]\)

\[ ds_6^2 = F^{-1} dr^2 + 4F \sigma^2 + (r^2 - 1)d\Omega_2^2 + r^2 d\tilde{\Omega}_2^2, \]  

(2.22)

where

\[ d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2, \quad d\tilde{\Omega}_2^2 = d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\phi}^2, \]

\[ \sigma = d\psi + \cos \theta d\phi, \]  

(2.23)

The function \( F \) is given by

\[ F = \frac{(r + 1)(r - 2)}{3r(r - 1)}. \]  

(2.24)

\(^2\)The author thanks Andrew Chamblin for clarifying this point.
Also, $r > 2$. This Taub-BOLT geometry goes from $\mathbb{R}^2 \times S^2 \times S^2$ at short distance to $C(S^2 \times S^2) \ltimes S^1$ at large distance.

We are interested in finding a harmonic 2-form supported by this metric. The most general ansatz for a 2-form with respect to the isometry of (2.22) is given by (2.9) expressed in the vielbein basis $e^0 = 1/\sqrt{F} dr$, $e^1 = \sqrt{r^2 - 1} d\theta$, $e^2 = \sqrt{r^2 - 1} \sin \theta d\phi$, $e^3 = r d\tilde{\theta}$, $e^4 = r \sin \tilde{\theta} d\tilde{\phi}$ and $e^5 = 2\sqrt{F} \sigma$. The closure and co-closure of $L_{(2)}$ yield three solutions. The first one is given by

$$u_1 = \frac{3r^2 - 1}{r^2 (r^2 - 1)^2}, \quad u_2 = \frac{2}{r(r^2 - 1)^2}, \quad u_3 = 0, \quad (2.25)$$

corresponding to the regular $H$ given by

$$H = \frac{m^2 (9 + 20r + 9r^2 - 12r^3 - 8r^4)}{6r(r - 1)(r + 1)^3} + \frac{4}{3} m^2 \log \left(\frac{r + 1}{r}\right). \quad (2.26)$$

The second solution is given by

$$u_1 = \frac{r}{(r^2 - 1)^2}, \quad u_2 = \frac{3 - r^2}{2(r^2 - 1)^2}, \quad u_3 = 0, \quad (2.27)$$

with

$$H = 1 + \frac{m^2 (11 + 18r - 3r^2 - 8r^3)}{24(r - 1)(r + 1)^3} + \frac{1}{3} m^2 \log \left(\frac{r + 1}{r}\right). \quad (2.28)$$

The third solution is

$$u_1 = u_2 = 0, \quad u_3 = \frac{1}{r^2}, \quad (2.29)$$

with

$$H = 1 - \frac{3m^2 (1 + 4r)}{2r(1 + r)} + 6m^2 \log \left(\frac{r + 1}{r}\right). \quad (2.30)$$

Notice that the second and third harmonic two-forms have non-trivial flux and are not normalizable at large distance. However, all three two-forms are square integrable for $r \to 2$. All of the corresponding D3-brane geometries asymptotically approach $M_4 \times C(S^2 \times S^2) \ltimes S^1$ at large distance. However, they have different bundle structures at short distance. The first two solutions are $M_3 \times S^2 \times (\mathbb{R}^2 \times S^2) \ltimes S^1$ at short distance, while the third is $M_5 \times S^2 \times S^2 \ltimes S^1$. These solutions can be superimposed to get, for example, a $T^3$-wrapped D3-brane.

### 2.3.2 Taub-NUT

Simply taking $r \to -r$ transforms the above six-dimensional Taub-BOLT metric into a Taub-NUT metric, where now $r \geq 1$. This Taub-NUT metric is given by (2.22).
Thus, the same six-dimensional local metric form extends smoothly onto two different manifolds. This Taub-NUT geometry runs from $\mathbb{R}^4 \times S^2$ to $C(S^2 \times S^2) \ltimes S^1$.

The three harmonic two-forms supported by these Taub-BOLT and Taub-NUT metrics are identical, since they do not depend on the function $F$. However, the resulting $H$ for each case are not related by taking $r \rightarrow -r$, and must be solved from scratch. In fact, only for the third harmonic two-form (2.30) does there exist a regular solution, given by

$$H = 1 + 3 \frac{m^2}{2r} - \frac{3}{4} m^2 \log \left( \frac{r + 2}{r} \right).$$

The D3-brane geometry smoothly runs from $M_3 \times (\mathbb{R}^4 \times S^2) \ltimes S^1$ to $M_4 \times C(S^2 \times S^2) \ltimes S^1$.

### 2.4 On Schwarzchild instanton

We will now consider the $S^1$-wrapped D3-brane solution given by (2.1) for which the six-dimensional transverse space is a Schwarzchild instanton, whose metric is given by

$$ds_6^2 = f \, dx^2 + f^{-1} \, dr^2 + r^2 \, d\Omega_4^2,$$

where

$$f = 1 - \frac{M}{r^3},$$

and $r^3 \geq M$. This geometry runs from $\mathbb{R}^2 \times S^4$ at short distance to $\mathbb{R}^6$ at large distance.

This metric supports a harmonic two-form

$$L_{(2)} = \frac{1}{r^2} \, dx \wedge dr,$$

which is normalizable at large distance and square integrable as $r \rightarrow M^{1/3}$. The corresponding regular solution to $H$ in (2.29) is given by

$$H = 1 + \frac{m^2}{9M \, r^3}.$$

This D3-brane geometry smoothly interpolates between $M_3 \times S^4 \times \mathbb{R}^2 \ltimes S^1$ and $M_{10}$. 

with

$$F = \frac{(r - 1)(r + 2)}{3r(r + 1)}. \quad (2.31)$$
3 Deformed M2-brane

The M2-brane of eleven-dimensional supergravity is supported by the 4-form field strength, with an eight-dimensional Ricci-flat transverse space. Due to the equation of motion $d\ast F(4) = \frac{1}{2} F(4) \wedge F(4)$, one can construct a resolved M2-brane if the transverse space has a (anti)-self-dual 4-cycle. This type of modification to the M2-brane, which makes use of the interaction in $d\ast F(4) = \frac{1}{2} F(4) \wedge F(4)$, has been greatly studied, for example in [15, 1, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. The deformed M2-brane is given by

$$\begin{align*}
  ds^2_{11} &= H^{-2/3} dx^\mu dx^\nu \eta_{\mu\nu} + H^{1/3} ds^2_8, \\
  F_{(4)} &= d^3x \wedge dH^{-1} + m G_{(4)}, \quad (3.1)
\end{align*}$$

where $G_{(4)}$ is a harmonic self-dual 4-form in the Ricci-flat transverse space $ds^2_8$. The equations of motion are satisfied, provided that

$$\Box H = -\frac{1}{48} m^2 G_{(4)}^2, \quad (3.2)$$

where $\Box$ is the Laplacian on $ds^2_8$.

Before discussing specific examples, we would like to mention that the deformed M2-brane can be dimensionally reduced along the worldvolume to give a deformed NS-NS string in type IIA theory, given by

$$\begin{align*}
  ds^2_{10} &= H^{-3/4} (-dt^2 + dx^2) + H^{1/4} ds^2_8, \\
  F_{(4)} &= m G_{(4)}, \quad F_{(3)} = dt \wedge dx \wedge dH^{-1}, \quad e^{2\phi} = H. \quad (3.3)
\end{align*}$$

This solution can be T-dualized to a regular type IIB pp-wave given by

$$\begin{align*}
  ds^2_{10} &= -H^{-1} dt^2 + H \left( dx + (H^{-1} - 1) dt \right)^2 + ds^2_8, \\
  F_{(5)} &= m_4 \left( dx + (H^{-1} - 1) dt \right) \wedge (G_{(4)} + *_8 G_{(4)}). \quad (3.4)
\end{align*}$$

3.1 On Taub-NUT/BOLT with $\mathcal{B} = \mathbb{C}P^3$

For the transverse space, we will consider an eight-dimensional Taub-NUT/BOLT instanton, whose metric is given by [23]

$$\begin{align*}
  ds^2_8 &= F^{-1} dt^2 + N^2 F (d\tau + A)^2 + R^2 d\Sigma^2_6, \quad (3.5)
\end{align*}$$
where
\[ F = \frac{8(r^6 - 5N^2 r^4 + 15N^4 r^2 - 10M r + 5N^6)}{5(r^2 - N^2)^3}, \]  
and \( R^2 = r^2 - N^2 \). \( d\Sigma_6^2 \) is the metric for \( \mathbb{CP}^3 \) given by
\[ d\Sigma_6^2 = d\xi^2 + \frac{1}{4} c^2 d\Omega_2^2 + \frac{1}{4} s^2 \xi d\tilde{\Omega}_2^2 + \frac{1}{4} s^2 \xi c^2 \sigma^2, \]  
where
\[ d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2, \quad d\tilde{\Omega}_2^2 = d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\phi}^2, \]
\[ \sigma = d\psi - \cos \theta d\phi + \cos \tilde{\theta} d\tilde{\phi}, \]  
and \( c = \cos \xi \) and \( s = \sin \xi \). Also,
\[ A = \frac{1}{4} (c^2 - s^2) d\psi - \frac{1}{2} c^2 \cos \theta d\phi - \frac{1}{2} s^2 \cos \tilde{\theta} d\tilde{\phi}. \]

For the appropriate periodicity in the fibre coordinate \( \tau \), there is no conical singularity.

At short distance, the Taub-NUT geometry is \( \mathbb{R}^8 \) while the Taub-BOLT geometry is \( \mathbb{R}^2 \times \mathbb{CP}^3 \); both geometries asymptotically approach \( C(\mathbb{CP}^3) \rtimes S^1 \) at large distance.

The veilbein for the 8-space described by (3.5) are given by
\[ e^0 = \sqrt{F} dr, \quad e^1 = \frac{R}{2} c d\theta, \quad e^2 = \frac{R}{2} \sin \theta d\phi, \]
\[ e^3 = \frac{R}{2} s d\tilde{\theta}, \quad e^4 = \frac{R}{2} \sin \tilde{\theta} d\tilde{\phi}, \quad e^5 = R d\xi, \]
\[ e^6 = \frac{R}{2} sc (d\psi - \cos \theta d\phi + \cos \tilde{\theta} d\tilde{\phi}), \quad e^7 = N \sqrt{F} (d\tau + A). \]  

An (anti)self-dual 4-form on this 8-space is given by
\[ G_{(4)} = u^+_1 (e^0 \wedge e^1 \wedge e^2 \pm e^3 \wedge e^4 \wedge e^5 \wedge e^6) \]
\[ + u^+_2 (e^0 \wedge e^1 \wedge e^3 \wedge e^4 \pm e^1 \wedge e^2 \wedge e^5 \wedge e^6) \]
\[ + u^+_3 (e^0 \wedge e^1 \wedge e^5 \wedge e^6 \pm e^1 \wedge e^2 \wedge e^3 \wedge e^4). \]  

The closure of \( G_{(4)} \) yields the (anti)self-dual solution
\[ u^+_1 = u^+_2 = - u^+_3 = \frac{1}{(r \mp N)^4}. \]  

Both of these two-forms are normalizable at large distance. In the case of the Taub-NUT \( (M = \frac{8}{5} N^5 \text{ and } r \geq N) \), only the anti self-dual two-form is integrable at short distance. This has the corresponding regular solution for \( H \) given by
\[ H = 1 + \frac{5m^2 (2N^2 + 3N r + 3r^2)}{256 N^5 (r + N)^3} + \frac{15m^2}{512 N^6} \arctan \left( \frac{(r + N)(r + 3N)}{2N (r + 2N)} \right). \]
The M2-brane geometry smoothly interpolates from $M_1$ at short distance to $M_3 \times C(\mathbb{CP}^3) \ltimes S^1$ at large distance.

Both the anti self-dual and the self-dual two-forms are square integrable at short distance for the Taub-BOLT, which indicates that there exists corresponding regular solutions for $H$. However, we have been unable to express these in a closed analytical form. We expect that such M2-brane geometries would smoothly run from $M_5 \times \mathbb{CP}^3$ at short distance to $M_3 \times C(\mathbb{CP}^3) \ltimes S^1$ at large distance.

### 3.2 On Taub-BOLT with $\mathcal{B} = S^2 \times S^2 \times S^2$ or $\mathbb{CP}^2 \times S^2$

We can replace the base space $\mathbb{CP}^3$ of the Taub-NUT/BOLT metric (3.5) by any other six-dimensional Einstein-Kähler space, such as $S^2 \times S^2 \times S^2$ or $\mathbb{CP}^2 \times S^2$ [11]. Since, in these cases, only the Taub-BOLT is free of singularities, we will not consider the Taub-NUT in this section.

In the case of $\mathcal{B} = S^2 \times S^2 \times S^2$, the corresponding $G^2_{(4)}$ is the same as in the previous section. This indicates that there is a corresponding regular solution for $H$ though, again, there does not seem to be a closed form analytical expression for it. Presumably this M2-brane geometry would run from $M_5 \times S^2 \times S^2 \times S^2$ to $M_3 \times C(S^2 \times S^2 \times S^2) \ltimes S^1$. However, as previously mentioned, Taub-BOLT instantons of this topology do not admit a spin structure, though they may still admit a $Spin^C$ structure [11].

We will now consider $\mathcal{B} = \mathbb{CP}^2 \times S^2$ explicitly, though we restrict ourselves to $N = 1$ for simplicity. In this case, the Taub-NUT/BOLT metric can be written as

$$ds_8^2 = F^{-1} dr^2 + 4F (d\tau + A)^2 + R^2 (d\Sigma_4^2 + d\Omega_2^2), \quad (3.14)$$

where $F$ is given in (3.6) divided by a factor of 8, and $R^2 = r^2 - 1$. $d\Sigma_4^2$ is the metric for $\mathbb{CP}^2$ given by (2.3) and (2.7). Also, $A$ is given by (2.6), and $d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2$. This Taub-BOLT geometry goes from $\mathbb{R}^2 \times S^2 \times \mathbb{CP}^2$ to $C(S^2 \times \mathbb{CP}^2) \ltimes S^1$.

An (anti)self-dual 4-form on this 8-space can be written as (3.11), in the veilbein
We will restrict ourselves to the case $N = 1$. The closure of $G_{(4)}$ yields the following self-dual solutions:

$$u_1 = -u_2 = \frac{1}{2(r+1)^4}, \quad u_3 = \frac{1}{(r+1)^4},$$

(3.16)

and

$$u_1 = -u_2 = \frac{1 - 9r + 3r^2 - 3r^3}{24(r+1)^4(r-1)^3}, \quad u_3 = \frac{3r^2 + 1}{3(r+1)^4(r-1)^3}.$$

(3.17)

There are also two anti self-dual solutions given by

$$u_1 = -u_2 = \frac{1}{4(r+1)^3(r-1)}, \quad u_3 = \frac{1}{(r+1)^3(r-1)},$$

(3.18)

and

$$u_1 = -u_2 = \frac{1 + 9r + 3r^2 + 3r^3}{24(r+1)^3(r-1)^4}, \quad u_3 = \frac{3r^2 + 1}{3(r+1)^3(r-1)^4}.$$

(3.19)

All of these two-forms are normalizable at large distance. For the Taub-BOLT, all of them are also square integrable at short distance. This indicates that each two-form has a corresponding regular solution for $H$, even though we cannot express it in closed analytical form. These M2-brane geometries would smoothly go from $M_5 \times S^2 \times \mathbb{CP}^2$ to $M_3 \times C(S^2 \times \mathbb{CP}^2) \ltimes S^1$.

### 3.3 On generalized Taub-NUT

For the transverse space, we will now consider a generalized Taub-NUT instanton, whose metric is given by

$$ds_8^2 = \frac{5}{U} dr^2 + \frac{4U}{5} (\tau + A)^2 + (r^2 - 1) d\Sigma_4^2 + r^2 d\Omega_2^2,$$

(3.20)

where

$$U = 1 - \frac{4(r^3 - 3r + 2)}{3r (r^2 - 1)^2}.$$

(3.21)
\(d\Sigma^2_2\) is the metric for \(\mathbb{C}\mathbb{P}^2\) given by (2.5) and (2.7). Also, \(A\) is given by (2.6). This Taub-NUT geometry goes from \(\mathbb{R}^6 \times S^2\) to \(C(S^2 \times \mathbb{C}\mathbb{P}^2) \ltimes S^1\).

The veilbein for the 8-space described by (3.20) are given by
\[
\begin{align*}
e^0 &= \sqrt{\frac{5}{U}} \, dr, \\
e^1 &= \frac{u R}{2\sqrt{V}} \, d\theta, \\
e^2 &= \frac{u R}{2\sqrt{V}} \sin \theta \, d\phi, \\
e^3 &= \frac{R}{V} \, du, \\
e^4 &= \frac{u R}{2V} \, (d\psi + \cos \theta \, d\phi), \\
e^5 &= r \, d\tilde{\theta}, \\
e^6 &= r \sin \tilde{\theta} \, d\tilde{\phi}, \\
e^7 &= 2\sqrt{\frac{U}{5}} \, (d\tau + A).
\end{align*}
\]

(3.22)

An (anti)self-dual 4-form on this 8-space is given by
\[
G^{(4)} = u_1 (e^0 \wedge e^7 \wedge e^1 \wedge e^2 \pm e^3 \wedge e^4 \wedge e^5 \wedge e^6) + u_2 (e^0 \wedge e^7 \wedge e^3 \wedge e^4 \pm e^1 \wedge e^2 \wedge e^5 \wedge e^6) + u_3 (e^0 \wedge e^7 \wedge e^5 \wedge e^6 \pm e^1 \wedge e^2 \wedge e^3 \wedge e^4).
\]

(3.23)

The closure of \(G^{(4)}\) yields the following solutions:
\[
u_1 = \pm \frac{3r + 1}{2r^2 (r + 1)^3} = -u_2, \quad u_3 = \frac{1}{r (r + 1)^3},
\]

(3.24)

and
\[
u_1 = \pm \frac{1 - 10r^2 - 15r^4}{2r^2 (r^2 - 1)^5} = -u_2, \quad u_3 = \frac{3r^2 + 1}{r (r^2 - 1)^3}.
\]

(3.25)

All of these four-forms are normalizable at large distance. However, only the (anti)self-dual pair given in (3.24) are square integrable as \(r \to 1\) and, thus, have a corresponding regular \(H\) given by
\[
H = 1 + \frac{15m^2}{16r} + \frac{45m^2}{8(r+1)} - \frac{15m^2}{8(r+1)^2} + \frac{207}{128}\sqrt{15m^2} \arctan\left[\sqrt{\frac{3}{5}}(3 + 2r)\right] + \frac{15}{256}m^2 \log\left[\frac{r^3}{(r+1)^6 (r^2 + 3r + 8/3)^9}\right].
\]

(3.26)

The geometry of this M2-brane interpolates from \(M_9 \times S^2\) at short distance to \(M_3 \times C(S^2 \times \mathbb{C}\mathbb{P}^2) \ltimes S^1\) at large distance.
4 Deformed and \( S^1 \)-wrapped 5-brane

4.1 Deformed heterotic 5-brane

The deformed heterotic 5-brane is given by \[ \text{(4.1)} \]
\[
\begin{align*}
    ds_{10}^2 &= H^{-1/4} dx_\mu^2 + H^{3/4} ds_4^2, \\
    e^{-\phi} * F_{(3)} &= d^6 x \wedge dH^{-1}, \quad \phi = \frac{1}{2} \log H, \quad F_{(2)} = m L_{(2)},
\end{align*}
\]
where * is the Hodge dual with respect to \( ds_{10}^2 \) and \( L_{(2)} \) is a harmonic two-form on \( ds_4^2 \). The equations of motion are satisfied, provided that \( L_{(2)} \) is a self-dual two-form and
\[
\Box H = -\frac{1}{4} m^2 L_{(2)}^2, \tag{4.2}
\]
where \( \Box \) is the Laplacian on \( ds_4^2 \). Note that an overlapping 5-brane configuration can also be resolved [14].

We will consider the case in which \( ds_4^2 \) is the metric for the Taub-NUT instanton, given by
\[
\begin{align*}
    ds_4^2 &= F^{-1} dr^2 + F (d\psi - 2N \cos \theta d\phi)^2 + (r^2 - N^2)(d\theta^2 + \sin^2 \theta d\phi^2), \tag{4.3}
\end{align*}
\]
where
\[
F = \frac{r^2 - 2Mr + N^2}{r^2 - N^2}. \tag{4.4}
\]

Harmonic 2-forms supported by this metric are given by
\[
L_{(2)}^\pm = \frac{2}{(r \pm N)^2} (e^0 \wedge e^3 \pm e^1 \wedge e^2), \tag{4.5}
\]
expressed in the vielbein basis \( e^0 = \frac{1}{\sqrt{F}} dr, e^1 = \sqrt{r^2 - N^2} d\theta, e^2 = \sqrt{r^2 - N^2} \sin \theta d\phi, \) and \( e^3 = \sqrt{F} (d\psi - 2N \cos \theta d\phi) \). It has a non-trivial flux, and the square of this form is
\[
L_{(2)}^{\pm 2} = \frac{16}{(r \pm N)^4}. \tag{4.6}
\]

We will now break up further analysis for the cases of the Taub-NUT and Taub-BOLT instantons.
In the BPS limit, \( M = N \), for which the four-dimensional metric \((4.3)\) is that of the Taub-NUT instanton. In this case, \( r \geq N \). The geometry goes from \( \mathbb{R}^4 \) to \( C(S^2) \times S^1 \).

Only \( L^+_{(2)} \) is square integrable for \( r \rightarrow N \). Neither two-form is normalizable at large distance for any of the three instantons. For \( L^+_{(2)} \), a regular solution is given by

\[
H = 1 + \frac{m^2}{N(N + r)},
\]

\textit{(4.7)}

For \( L^-_{(2)} \), the unavoidably singular solution is given by

\[
H = 1 + \frac{c}{r - N} - \frac{2m^2 (r - 3N)}{3(r - N)^3}.
\]

\textit{(4.8)}

The geometry of this 5-brane interpolates from \( M_{10} \) at short distance to \( M_6 \times C(S^2) \times S^1 \) at large distance. This solution preserves minimal supersymmetry \cite{[1]}.

\textbf{Taub-BOLT}

In this case, \( M = \frac{5}{4}N \) and \( r \geq 2N \). The geometry goes from \( \mathbb{R}^2 \times S^2 \) to \( C(S^2) \times S^1 \).

Both \( L^\pm_{(2)} \) are square integrable for \( r \rightarrow 2N \). Regular solutions are given by

\[
H = 1 + \frac{8m^2}{9N(N + r)},
\]

\textit{(4.9)}

and

\[
H = 1 + \frac{8m^2}{N(r - N)},
\]

\textit{(4.10)}

for \( L^+_{(2)} \) and \( L^-_{(2)} \), respectively. Both of these 5-brane geometries run from \( M_8 \times S^2 \) to \( M_6 \times C(S^2) \times S^1 \). \( L^+_{(2)} \) and \( L^-_{(2)} \) can be linearly superimposed to yield a more general deformed 5-brane solution. However, as we previously mentioned, the four-dimensional Taub-NUT instanton does not admit a spin structure, though it may still admit a \( Spin^C \) structure \cite{[10]}.

\textbf{Schwarzschild instanton}

For \( N = 0 \), \( r \geq 2M \). \( L^\pm_{(2)} \) are square integrable as \( r \rightarrow 2M \). This geometry goes from \( \mathbb{R}^2 \times S^2 \) to \( \mathbb{R}^4 \).

Both \( L^\pm_{(2)} \) have a regular solution given by

\[
H = 1 + \frac{2m^2}{Mr}.
\]

\textit{(4.11)}

The corresponding 5-brane geometry go from \( M_8 \times S^2 \) to \( M_{10} \).
4.2 $S^1$-wrapped 5-brane

The above resolution of the heterotic 5-brane requires the presence of matter Yang-Mills fields which are absent in the type II theories. However, a resolved type II 5-brane can be constructed by wrapping worldvolume directions around the transverse space. For example, a regular $S^2$-wrapped 5-brane was obtained in [27] by lifting the four-dimensional $SU(2)$ gauged black hole [28]. This solution can apply for both type II and heterotic 5-branes.

Another example of a regular 5-brane solution of both type II and heterotic theories is the $S^1$-wrapped 5-brane given by [13]

$$ds_{10}^2 = H^{-1/4} \left( -dt^2 + dx_1^2 + \cdots + dx_4^2 + (dx_5 + A_{(1)})^2 \right) + H^{3/4} ds_4^2,$$

$$F_{(3)}^{RR} = \ast_4 dH - m L_{(2)} \wedge (dx_5 + A_{(1)}), \quad \phi = -\frac{1}{2} \log H,$$  \hfill (4.12)

where $dA_{(1)} = mL_{(2)}$. $L_{(2)}$ is a harmonic 2-form on $ds_4^2$ and $\ast_4$ is the Hodge dual with respect to $ds_4^2$. The equations of motion are satisfied, provided that (4.12) and $L_{(2)}$ is a self-dual two-form. Note that overlapping $S^1$-wrapped 5-branes can also be resolved [14].

We can also consider a rotating 5-brane, given by

$$ds_{10}^2 = H^{-1/4} \left( -(dt + A_{(1)})^2 + dx_1^2 + \cdots + dx_5^2 \right) + H^{3/4} ds_4^2,$$

$$F_{(3)}^{RR} = \ast_4 dH - m L_{(2)} \wedge (dt + A_{(1)}), \quad \phi = -\frac{1}{2} \log H.$$  \hfill (4.13)

For the Taub-NUT/BOLT metric given by (4.3), the computation of $L_{(2)}$ and $H$ carry over from the deformed 5-brane of the previous section. Since the Taub-BOLT and Schwarzchild instantons both support two independent harmonic two-forms, these can be superimposed to form a regular $T^2$-wrapped 5-brane. In the case of the Schwarzchild instanton, for example, the resulting $T^2$-wrapped 5-brane geometry goes from $M_4 \times (\mathbb{R}^2 \times S^2) \times T^2$ to $M_{10}$.
5 Resolved D4/M5/NS5-branes

The deformed D4-brane solution is given by

\[ ds_{10}^2 = H^{-3/8} dx_\mu^2 + H^{5/8} ds_5^2 , \]
\[ F_{(4)} = * dH , \quad \phi = -\frac{1}{4} \log H , \]
\[ F_{(2)} = m L_{(2)} , \quad F_{(3)} = m * L_{(2)} , \] (5.1)

where * is the Hodge dual with respect to \( ds_5^2 \). The equations of motion are satisfied provided that \( H \) is given by (2.2), where \( L_{(2)} \) is a harmonic two-form on the five-dimensional transverse space.

5.1 On Schwarzchild instanton

The \( D = 5 \) Schwarzchild instanton metric is given by

\[ ds_5^2 = f dz^2 + f^{-1} dr^2 + r^2 d\Omega_3^2 , \] (5.2)

where

\[ f = 1 - \frac{M}{r^2} , \] (5.3)

and \( r \geq \sqrt{M} \). This geometry goes from \( \mathbb{R}^2 \times S^3 \) to \( \mathbb{R}^5 \).

This metric supports a harmonic two-form

\[ L_{(2)} = \frac{1}{r^3} dz \wedge dr , \] (5.4)

which is normalizable at large distance and square integrable at short distance. The corresponding regular solution to \( H \) is given by

\[ H = 1 + \frac{m^2}{4Mr^2} . \] (5.5)

The geometry of this D4-brane smoothly interpolates from \( M_7 \times S^3 \) at short distance to \( M_{10} \) at large distance.

The above deformed D4-brane can be lifted to eleven dimensions as an \( S^1 \)-wrapped M5-brane solution given by

\[ ds_{11}^2 = H^{-1/3} \left( dx_\mu^2 + (dx_5 + \frac{m}{2r^2} dz)^2 \right) + H^{2/3} ds_5^2 , \]
\[ F_{(4)} = * dH + m \left( dx_5 + \frac{m}{2r^2} dz \right) \wedge \Omega_{(3)} , \] (5.6)
where $\Omega_3$ is the volume-form of the transverse $S^3$ corresponding to the metric $d\Omega_3^2$. The M5-brane geometry runs from $M_5 \times S^3 \times \mathbb{R}^2 \times S^1$ to $M_{11}$.

We can now reduce back to ten dimensions along one of the non-fibred spatial directions of the M5-brane. The result is the $S^1$-wrapped D4-brane solution

$$
 ds_{10}^2 = H^{-3/8} \left( dx_\mu^2 + (dx_4 + \frac{m}{2r^2} dz)^2 \right) + H^{2/3} d\Omega_3^2, \\
 F_4 = *dH + m (dx_4 + \frac{m}{2r^2} dz) \wedge \Omega_3.
$$

(5.7)

Alternatively, we can express the transverse $S^3$ metric as a fibre bundle over $S^2$:

$$
 d\Omega_3^2 = \frac{1}{4} (d\psi + \cos \theta d\phi)^2 + \frac{1}{4} d\Omega_2^2, \\
 d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2.
$$

(5.8)

where $d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2$. reducing over the fibre bundle direction yields the type IIA $S^1$-wrapped NS5-brane solution

$$
 ds_{10}^2 = \left( \frac{r}{2} \right)^{1/4} \left[ H^{-1/4} \left( dx_\mu^2 + (dx_5 + \frac{m}{2r^2} dz)^2 \right) + H^{3/4} (f dz^2 + f^{-1} dr^2 + \frac{r^2}{4} d\Omega_2^2) \right], \\
 F_3 = \frac{r}{2} *dH + \frac{m}{8} (dx_5 + \frac{m}{2r^2} dz) \wedge \Omega_{(2)}, \quad F_2 = \Omega_{(2)},
$$

(5.9)

where $\Omega_{(2)}$ is the volume-form corresponding to the metric $d\Omega_2^2$.

All of the above solutions are regular, since $r \geq \sqrt{M}$.

### 5.2 On Schwarzchild on Taub-NUT

A metric that resembles a $D = 5$ Schwarzchild instanton superimposed with a $D = 4$ Taub-NUT instanton can be written as

$$
 ds_5^2 = f dz^2 + f^{-1} dr^2 + 2N(2N + M) W (d\psi + \cos \theta d\phi)^2 + (r^2 - N^2) d\Omega_2^2,
$$

(5.10)

where

$$
 f = 1 - \frac{M}{r - N}, \quad W = \frac{r - N}{r + N},
$$

(5.11)

and $d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2$. Also, $r \geq M + N$.

A harmonic two-form supported by this metric is given by

$$
 L_{(2)} = \frac{1}{\sqrt{W} (r^2 - N^2)} dz \wedge dr.
$$

(5.12)
which is normalizable at large distance and square integrable at short distance. The corresponding regular solution to $H$ is given by

$$H = 1 + \frac{m^2}{MN} \log \left[ \frac{\sqrt{M(M+2N)(r^2-N^2)} + [(M+N)r-N^2]}{(\sqrt{M(M+2N)}+M+N)(r-N)} \right].$$

(5.13)

As in the previous section, we can obtain regular $S^1$-wrapped M5, D4 and NS5-branes from the above solution.

6 Conclusions

We have constructed many regular $p$-brane solutions on higher-dimensional generalizations of Taub-NUT and Taub-BOLT instantons. These new solutions do not preserve any supersymmetry, which serves to demonstrate that the resolution of brane singularities works without the presence of supersymmetry. The resulting geometries smoothly interpolate between two phases of Minkowski spacetime of differing dimensionality. Since these new solutions do not preserve any supersymmetry, the stability is not assured. We leave this issue for future work.

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