Cluster-Enriched Yang-Baxter Equation from SUSY Gauge Theories

Masahito Yamazaki (山崎雅人)

Kavli IPMU (WPI), University of Tokyo, Kashiwa, Chiba 277-8583, Japan
Center for the Fundamental Laws of Nature, Harvard University, Cambridge, MA 02138, USA

Abstract

We propose a new generalization of the Yang-Baxter equation, where the R-matrix depends on cluster \( y \)-variables in addition to the spectral parameters. We point out that we can construct solutions to this new equation from the recently-found correspondence between Yang-Baxter equations and supersymmetric gauge theories. The \( S^2 \) partition function of a certain 2d \( \mathcal{N} = (2, 2) \) quiver gauge theory gives an R-matrix, whereas its FI parameters can be identified with the cluster \( y \)-variables.

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1 Introduction

The interplay between the physics of supersymmetric gauge theories and the integrable models (defined here as solutions to the Yang-Baxter equation (YBE) with spectral parameters [1][2]) has been a fascinating subject over the past several decades.

Recently, a new version of such an interplay (the Gauge/YBE correspondence) between supersymmetric gauge theories and integrable model has been found [3–5] (see also [6–14]). The correspondence states a rather surprising equivalence between the statistical partition function of a classical two-dimensional lattice model on the one hand, and a supersymmetric partition function of supersymmetric quiver gauge theories, on the other. The basic idea behind this correspondence is that the Yang-Baxter equation is promoted to a duality (Yang-Baxter duality) between supersymmetric gauge theories: since the two theories are dual, their partition functions are the same, and resulting mathematical equality turns out to have an interpretation as YBE. This is arguably one of the most direct correspondence between integrable models and supersymmetric gauge theories in the literature.

What is remarkable about this correspondence is that the insights from supersymmetric gauge theories helps us to find new integrable models hitherto unknown in the literature. Indeed, in [5] a new class of integrable models has been found from the lens space ($S^1 \times S^3/\mathbb{Z}_r$) index [15][16] of $SU(N)$ quiver gauge theories. This model is labeled by two integers $N > 1$ and $r > 0$, has spectral parameters, and depends on two elliptic parameters $p, q$.

Furthermore one can generalize the story by e.g. studying 2d $\mathcal{N} = (2, 2)$ quiver gauge theories [11]. We expect that there are many more solutions to the YBE yet to be found from this approach.

In this short note, we proceed further and point out that Gauge/YBE correspondence is useful to discover new equations generalizing the standard YBE. What is particularly nice about these equations is that the resulting equation naturally incorporates the mathematical machinery of cluster algebras [18][19]. For this reason we call our equations the cluster-enriched Yang-Baxter equation.

In the rest of this paper, we first quickly summarize the basic idea of the Gauge/YBE correspondence [2]. We then construct explicit solutions to the cluster-enriched YBE from 2d $\mathcal{N} = (2, 2)$ theories (section 3). The final section (section 4) contains concluding remarks.

[1] For $r = 1$, this newly-found solution reproduces the solutions found recently in [8] (see also [6][7][17]). For $N = 2$ and general $r$, the recent paper [13] mathematically proves integrability (star-triangle relation) of the model.
2 Gauge/YBE Correspondence

Let us quickly summarize the basic idea behind the Gauge/YBE correspondence. For full details, see \[3–5, 11\]. Readers familiar with these references can safely skip this section.

2.1 Star-Star Duality and R-matrix

One of the crucial ideas in the Gauge/YBE correspondence is to associate a quiver gauge theory (which we call $\mathcal{T}[R]$) to the R-matrix \[1\]. The quiver diagram for the theory $\mathcal{T}[R]$ is shown in Figure \[1\].

![Figure 1: The R-matrix is obtained from the partition function of the quiver on the right. Figure reproduced from \[11\].](image)

The quiver of Figure \[1\] has five nodes. The circle in the middle is the $G$ gauge group, while the four squares represent the $G$ flavor symmetries; $G$ in this paper is $U(N)$. This in particular means that the theory $\mathcal{T}[R]$ has $G$ flavor symmetries. This is the counterpart of the fact the R-matrix has four indices. We also note that some of the arrows are dotted—this is meant to be representing a “half chiral multiplet” \[11\], namely we take a square root when we discuss partition functions.

Now, what is special about the theory $\mathcal{T}[R]$ (and its quiver diagram) is the fact that the theory often has dual (star-star dual), whose graphical representation of the Seiberg(-like) duality in Figure \[2\] coincides exactly with that for the star-star relation known from integrable models \[21, 22\].

Whether or not such a duality exists depends crucially on the details of the precise definition of the quiver gauge theory—we have the choice of the spacetime dimension, the gauge group at the vertex of the quiver, the number of supersymmetries, etc. In fact such flexibility is one reason which makes the Gauge/YBE correspondence so rich.

Let us denote the spacetime dimension by $D$. For $D = 4$, we can take $G = SU(N)$ or $U(N)$, and the start-star duality in question is the Seiberg duality \[23\]. For $D = 2$, we can also take $G = U(N)$, where the star-star duality coincides \[11\] with the 2d $\mathcal{N} = (2, 2)$ version of the Seiberg duality \[24, 25\].

\[2\]See also the forthcoming review \[20\].
Figure 2: Seiberg-like duality of a quiver gauge theory, which can also be read as the star-star relation of an integrable model. The dotted lines represent the “half chiral multiplet”, and the parameters on the edges (such as $u, 1 - u$ and 1) represent the R-charges of the bifundamental fields. For a closed loop (triangle) their R-charges sum up to two due to superpotential constraints.

### 2.2 Yang-Baxter Duality and YBE

In integrable model literature, the star-star equation, applied four times, is known to imply YBE. We can translate this into supersymmetric gauge theory. Since the Yang-Baxter equation is about the product of three R-matrices, we can glue together three copies of the quiver diagram for $\mathcal{T}[R]$, by gauging global symmetries of the theory. The Yang-Baxter duality is then the statement that the two resulting quiver gauge theories are dual, i.e. describe the same physics in the IR fixed point.

Once we obtain the duality, we can compute various supersymmetric partition functions and then obtain solutions to YBE. For $D = 4$ we can take $S^1 \times S^3 / \mathbb{Z}_r$ partition function, for example, and for $D = 2$ the $T^2$ partition function $[25, 26]$.

The partition function for the theory $\mathcal{T}[R]$ gives the R-matrix:

$$\mathcal{R}(u) \left[ \begin{array}{cc} d & c \\ a & b \end{array} \right] := \frac{1}{\sqrt{S^u S^d}} \sum_{g} S^g \mathcal{W}_u(a, g) \mathcal{W}_{1-u}(g, b) \mathcal{W}_u(c, g) \mathcal{W}_{1-u}(g, d).$$

Here we have chosen the R-charges of the four chiral multiplets to be $u, 1 - u, u, 1 - u$ in the cyclic order, and $u$ plays the role of the spectral parameter in integrable models.

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3Here we assumed conditions explicitly listed in [11]. These conditions are satisfied in the examples in the paper, except as we will see there are some issues in overall normalization.

4In general we have can associate a R-charges $\alpha, \beta, \gamma, \delta$ to the four bifundamental multiplets, with possibly further constraints in order to make sure that these parametrize global symmetries of the theory. This defines a more general R-matrix depending on multiple parameters. See [11] for more details.
In the expression (1), the factor $\mathcal{W}_{re}(t(e), h(e))$ is the 1-loop determinant for a matter supersymmetric multiplet associated with an edge $e$ with endpoints $t(e)$ and $h(e)$, and with R-charge $r_e$. Similarly, $S^v$ is the classical as well as the 1-loop contribution, for a gauge supersymmetric multiplet associated with a vertex $v$ of the quiver diagram.

The equivalence of the partition functions originating from the Yang-Baxter duality now is precisely the Yang-Baxter equation:

$$
\sum_{g} \mathcal{R}(u) \begin{bmatrix} f & g \\ a & b \end{bmatrix} \mathcal{R}(u + v) \begin{bmatrix} d & c \\ g & b \end{bmatrix} \mathcal{R}(v) \begin{bmatrix} e & d \\ f & g \end{bmatrix} = \sum_{g} \mathcal{R}(v) \begin{bmatrix} g & c \\ a & b \end{bmatrix} \mathcal{R}(u + v) \begin{bmatrix} e & g \\ f & a \end{bmatrix} \mathcal{R}(u) \begin{bmatrix} e & d \\ g & c \end{bmatrix} .
$$

(2)

3 Cluster-Enriched YBE from 2d $\mathcal{N} = (2, 2)$ Theories

3.1 FI Parameters as Cluster Variables

In this section we consider 2d $\mathcal{N} = (2, 2)$ quiver gauge theory, where the gauge group at the vertex of the quiver diagram is $U(N)$, with the same value of $N$ for all vertices.\(^5\)

The star-star duality in this case will then be the 2d version of the Seiberg duality, as proposed in \cite{24, 25} (see also \cite{26, 28}), and the integrable model associated with the $T^2$ partition function \cite{25, 26, 29} was studied in \cite{11}. Here we instead study their $S^2$ partition functions \cite{24, 30}.

Compared with the $T^2$ partition function, the $S^2$ partition function depends on a set of extra parameters, the complexified FI parameters, which transform non-trivially under the Seiberg-duality, and hence under the Yang-Baxter duality.

Recall that in 2d $\mathcal{N} = (2, 2)$ theories the FI (Fayet-Iliopoulos) parameter $r$ is naturally

\(^5\)One advantage of this choice is that the rank of the gauge group, and hence number of the components of the spin of the integrable model at a lattice site, is preserved by the Seiberg-like duality. It is straightforward to allow different gauge groups to different nodes. Such a generalization will further refine our YBE. Note that in \cite{27} the ranks of the gauge groups transform as cluster $x$-variables.
complexified by the theta angle $\theta$ into an exponentiated variable $y^\theta$

$$y := (-1)^N e^{r+i\theta} .$$  \hspace{1cm} (4)

The factor of $(-1)^N$ is inserted for a better match with cluster algebra literature.

Now the highly nontrivial statement found in [27] was that the complexified FI parameters for the Seiberg-like dual pair theories (Figure 2) are given by

$$y'_g = z_g^{-1} , \\ y'_a = z_a(1 + z_g) , \\ y'_b = z_b(1 + z_g)^{-1} , \\ y'_c = z_c(1 + z_g) , \\ y'_d = z_d(1 + z_g)^{-1} ,$$  \hspace{1cm} (5)

with primed (unprimed) variables representing the parameters for the after (before) the Seiberg-like duality. Interestingly, that this transformation rule is the same as the transformation rules of the “cluster $y$-variable” under a “mutation” of the quiver.

By using this transformation property we can easily compute the transformation properties of the complexified $y$-variables. Let us parametrize the exponentiated complexified FI parameters as in Figure 3. Then the $y$-variables at the 10 vertices are given by

$$y_1 \rightarrow a_3 b_4 c_2 , \\ y_2 \rightarrow a_5 , \\ y_3 \rightarrow b_5 , \\ y_4 \rightarrow c_5 , \\ y_5 \rightarrow a_1 , \\ y_6 \rightarrow a_2 b_1 , \\ y_7 \rightarrow b_2 , \\ y_8 \rightarrow b_3 c_3 , \\ y_9 \rightarrow c_4 , \\ y_{10} \rightarrow c_1 a_4 ,$$  \hspace{1cm} (6)

and

$$y'_1 \rightarrow a'_5 , \\ y'_2 \rightarrow a'_1 b'_2 c'_4 , \\ y'_3 \rightarrow c'_5 , \\ y'_4 \rightarrow b'_5 , \\ y'_5 \rightarrow b'_1 c'_1 , \\ y'_6 \rightarrow c'_2 , \\ y'_7 \rightarrow a'_2 c'_3 , \\ y'_8 \rightarrow a'_3 , \\ y'_9 \rightarrow a'_4 b'_3 , \\ y'_{10} \rightarrow b'_4 ,$$  \hspace{1cm} (7)

where we used that fact that we need to combine the FI parameters when we glue quivers (we simply add terms in the Lagrangian).

After 4 mutations at vertices 1, 3, 4, 2 (in this order; see Figure 4), the $y$-variables

\[ L_{\text{FI}} = -rD + \theta F_{01} , \]  \hspace{1cm} (3)

where $D$ is an auxiliary field in the $\mathcal{N} = (2, 2)$ vector multiplet.

\footnote{The relevant term in the Lagrangian is given by $L_{\text{FI}} = -rD + \theta F_{01}$,}

\footnote{This was denoted as $z$ in [11][27].}
Figure 3: Labeling of the complexified FI parameters. Each R-matrix has five vertices, and hence five FI parameters, which we label as $\vec{a} = (a_1, \ldots, a_5)$, $\vec{b}$ and $\vec{c}$. On the other side of the YBE we use the similar labeling, with the primed variables.

Figure 4: The labeling of the vertices, on both sides of the YBE. We associate a complexified FI parameter $y_i$ ($y'_i$) to the $i$-th vertex of the quiver on the left (right).
\[
y_1' = \frac{y_1 y_2 y_3 y_4}{y_1^2 y_2 (y_3 + 1)(y_4 + 1) + y_1 (y_2 (y_3 + y_4 + 2) + 1) + y_2 + 1},
\]
\[
y_2' = \frac{y_1 + 1}{y_2(y_1 y_3 + y_1 + 1)(y_1 y_4 + y_1 + 1)},
\]
\[
y_3' = \frac{y_1 y_3}{y_1 y_2 (y_3 + 1)(y_4 + 1) + y_1 (y_2 (y_3 + y_4 + 2) + 1) + y_2 + 1},
\]
\[
y_4' = \frac{y_1 y_4}{y_1 y_2 (y_3 + 1)(y_4 + 1) + y_1 (y_2 (y_3 + y_4 + 2) + 1) + y_2 + 1},
\]
\[
y_5' = \frac{y_1 y_3 y_6}{y_1 y_3 (y_3 + y_1 + 1)} ,
\]
\[
y_6' = \frac{y_1 y_3 (y_3 + y_1 + 1)}{y_1 y_3 y_7 + y_7},
\]
\[
y_7' = \frac{y_1 y_4 y_9}{y_1 + 1} + y_9 ,
\]
\[
y_8' = (y_1 + 1) y_8,
\]
\[
y_9' = \frac{y_1 y_4 y_9}{y_1 + 1} + y_9 ,
\]
\[
y_{10}' = \frac{y_1 y_{10} y_4}{y_1 y_4 + y_1 + 1}.
\]

The R-matrix now depends explicitly on the FI parameters,
\[
\mathcal{R}(u; \vec{a}) \begin{bmatrix} d & c \\ a & b \end{bmatrix},
\]
where \(\vec{a} = (a_1, a_2, \ldots, a_6)\) is a set of the FI parameters for the five vertices of the theory \(\mathcal{T}[R]\), and primed/unprimed variables should satisfy the constraint (8).

The identity representing the Yang-Baxter duality reads
\[
\sum_g \mathcal{R}(u; \vec{a}) \begin{bmatrix} f & g \\ a & b \end{bmatrix} \mathcal{R}(u + v; \vec{b}) \begin{bmatrix} d & c \\ g & b \end{bmatrix} \mathcal{R}(v; \vec{c}) \begin{bmatrix} e & d \\ f & g \end{bmatrix} = \sum_g \mathcal{R}(v; \vec{c}) \begin{bmatrix} g & c \\ a & b \end{bmatrix} \mathcal{R}(u + v; \vec{b}) \begin{bmatrix} e & g \\ f & a \end{bmatrix} \mathcal{R}(u; \vec{a}) \begin{bmatrix} e & d \\ g & c \end{bmatrix}.
\]

There is one subtlety in (10). The \(S^2\) partition function has an ambiguity of the Kähler
transformation \[31\]:
\[
\log Z_{S^2} \rightarrow \log Z_{S^2} + f(y) + \bar{f}(\bar{y}) ,
\]
(11)

where \(f(y) (\bar{f}(\bar{y}))\) is a holomorphic (anti-holomorphic) function of \(y\). This means that more naturally the identity \([10]\) should be interpreted as an identity up to this ambiguity.

If we wish we can eliminate this ambiguity by using the explicit numerical factor derived in \([27]\). The result is that \([10]\) holds including the overall factor if we further impose the condition
\[
f(y_1) f\left(\frac{y_1 y_3}{1 + y_1}\right) f\left(\frac{y_1 y_4}{1 + y_1}\right) f\left(\frac{y_2 (1 + y_1 + y_1 y_3)(1 + y_1 + y_1 y_4)}{1 + y_1}\right) = 1 ,
\]
(12)

where \(f(y)\) is a function denoted by \(f^{(r)}_{\text{ctc}}\) in \([27]\). It would be nice to better understand the cluster-algebraic significance of this constraint.

3.2 Expression for R-matrix

Let us also write down explicit expression for the R-matrix.

The \(S^2\) partition function is represented as an integral over the Cartan of the gauge group, which for a \(U(N)\) gauge group is parametrized by \(N\) parameters. We in addition have a monopole flux, a set of \(N\) integers, and we take a sum over them. Correspondingly, the integrable model has spins \(s_v\) taking values in \(\mathbb{R}^N \times \mathbb{Z}^N\). First, we have \(N\) continuous variables \(\sigma_{v,i}\), corresponding to the values of the Coulomb branch scalar inside the \(\mathcal{N} = (2,2)\) vector multiplet. We also have \(N\) discrete variables \(m_{v,i}\), corresponding to magnetic fluxes on \(S^2\). Correspondingly, the sum over \(s_v\) reads
\[
\sum_v \rightarrow \sum_{m_{v,i}} \int \prod_i d\sigma_{v,i} .
\]
(13)

For an edge \(e\) corresponding to a 2d \(\mathcal{N} = (2,2)\) chiral multiplet, the Boltzmann weights is given by
\[
\mathbb{W}_e(s_{t(e)}, s_{h(e)}) = \prod_{i,j} \frac{\Gamma \left( \frac{1}{2} - i \sigma_{t(e),i,j} - m_{t(e),i,j} \right)}{\Gamma \left( 1 - \frac{1}{2} + i \sigma_{t(e),i,j} - m_{t(e),i,j} \right)} ,
\]
(14)

where \(\sigma_{v,i,j} =: \sigma_{v,i} - \sigma_{v,j}\) and \(m_{v,i,j} =: m_{v,i} - m_{v,j}\).
For a vertex $v$ corresponding to a 2d $\mathcal{N} = (2, 2)$ vector multiplet, we have

$$S^u_v (s_v) = S^u_{\text{gauge}} (s_v) S^u_{\text{FI}} (s_v)$$

(15)

with

$$S^u_{\text{gauge}} (s_v) := \frac{1}{N!} \prod_{i \neq j} \left[ (\sigma_{v,i} - \sigma_{v,j})^2 + \left( \frac{m_i - m_j}{2} \right)^2 \right],$$

(16)

$$S^u_{\text{FI}} (s_v) := (-1)^{(N-1) \sum_j m_{v,j}} e^{2i \left( \sum_j \sigma_{v,j} \right) t + i \theta \left( \sum_j m_{v,j} \right)}.$$

(17)

4 Discussion

In this paper, we provided solutions to a version the Yang-Baxter equation where the R-matrix also depends on a cluster variable (or its tropical counterpart).

There could be other examples of supersymmetric gauge theories leading to novel cluster-algebra-enrichment of YBE. For example, we can appeal to the Giveon-Kutasov duality [33] for 3d $\mathcal{N} = 2$ Chern-Simons-matter theories. In this case, the ranks of the gauge groups (the number of components of spins) and the Chern-Simons levels (extra parameter at a vertex of the quiver) transform as tropical $x$- and $y$-variables, respectively [34], and for example the $S^1 \times S^2$ partition function [35, 36] of the theory gives some refinement of the YBE. It is also a natural question if there is any connection with another cluster algebra structure found in the literature, namely “3d cluster $\mathcal{N} = 2$ theories” of [37, 38], where a 3d $\mathcal{N} = 2$ theories was associated with a mutation sequence of a quiver.

As we have seen, the cluster-enriched YBE is a natural equation from the viewpoint of supersymmetric gauge theory. The real significance of the equation, however, is not clear as of this writing, since for example the equation does not ensure existence of an infinite number of conserved charges. In this respect one useful analogy is another generalization of the YBE, the so-called dynamical YBE. Historically, the dynamical YBE first appeared in 1983 in the study of Liouville theory [39]. However, it was only 10 years later when people begin to appreciate the underlying mathematical structure of the dynamical YBEs [40, 41].

While or not whether the history repeats itself only time will tell, it is fair to say that one should take this lesson seriously.

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8 In $S^u_{\text{FI}} (s_v)$ we included a sign factor $(-1)^{(N-1) \sum_j m_{v,j}}$ as suggested by [32], for better comparison with [27]. For our considerations only the identity of the partition functions for the star-star (and Yang-Baxter) duals is of importance, and this is not affected by such a sign.
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