CONSEQUENCES OF SPONTANEOUS LORENTZ VIOLATION IN GRAVITY

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Abstract

A brief summary of some of the main consequences of spontaneous Lorentz violation in gravity is presented, including evasion of a no-go theorem, concomitant spontaneous diffeomorphism breaking, the appearance of massless Nambu-Goldstone modes and massive Higgs modes, and the possibility of a Higgs mechanism in gravity.

1 Introduction

There has been a great deal of interest in the possibility of Lorentz violation in recent years stemming from ideas in quantum gravity, modified gravity theories, massive gravity, and cosmological models attempting to explain dark energy and/or dark matter. There have also been significant improvements in experimental tests of Lorentz violation, and sensitivities at levels involving suppression by the Planck scale have been attained. The theoretical framework known as the Standard-Model Extension[1] (SME) consists of the most general observer-independent effective field theory incorporating Lorentz violation, and it is routinely used by both theorists and experimentalists to study and obtain bounds on possible forms of Lorentz violation.[2, 3]

While the SME can accommodate both explicit forms of Lorentz breaking at the level of effective field theory as well as the process of spontaneous Lorentz violation, there are differences that arise between these types of Lorentz breaking in the context of gravity. This summary looks at distinguishing these differences and in particular what some of the consequences are of spontaneous Lorentz violation in the context of gravity.

2 Local Lorentz Symmetry and Diffeomorphisms

In the presence of gravity, Lorentz symmetry is a local symmetry that holds in local inertial frames at every spacetime point. For this reason, it is useful to use a vierbein formalism to investigate Lorentz violation in the context of gravity. The vierbein, \( e_\mu^a \), connects tensor components in local Lorentz frames (labeled using Latin indices) with tensor components in the space-time frame (labeled using Greek indices). The SME can be written using a vierbein formalism.

The SME Lagrangian consists of the most general scalar function under local Lorentz and diffeomorphism transformations that can be formed using gravitational fields, particle fields in the Standard Model, and Lorentz-violating coefficients known as SME coefficients. In SME models with explicit Lorentz breaking, the SME coefficients...
can be viewed as fixed background fields. However, in models arising from a process of spontaneous Lorentz violation, the SME coefficients are understood to be vacuum expectation values (vevs) of dynamical tensor fields. In either case, SME models with these forms of Lorentz breaking can be investigated experimentally, and bounds on the SME coefficients can be obtained. However, there are effects that arise that can further distinguish these types of symmetry breaking.

If the Lorentz violation is due to spontaneous breaking, then nonzero tensor-valued vacuum values, e.g., \( \langle T_{abc\ldots} \rangle \), occur in the local Lorentz frames. However, in a theory with spontaneous Lorentz breaking, the vierbein also has a vacuum value, \( \langle e_\mu^a \rangle \). When appropriate products of the vierbein vev act on the local tensor vevs, the result is that tensor vevs, e.g., \( \langle T_{\lambda\mu\nu\ldots} \rangle \), also appear in the spacetime frame. These tensor vevs in the spacetime frame spontaneously break local diffeomorphisms. Conversely, if a vev in the spacetime frame spontaneously breaks diffeomorphisms, then the inverse vierbein acting on it gives rise to vevs in the local frames that spontaneously break local Lorentz symmetry. Hence spontaneous local Lorentz breaking implies spontaneous diffeomorphism breaking and vice versa.[4]

Note that this concomitant symmetry breaking does not occur for the case of explicit symmetry breaking. For example, a Fierz-Pauli model describing massive metric excitations \( h_{\mu\nu} \) in a Minkowski background has explicit diffeomorphism breaking, but the theory does not break Lorentz symmetry. Similarly, one can write down models that explicitly break local Lorentz symmetry, e.g., using products of the spin connection, \( \omega_{\mu}^{ab} \), which are not Lorentz tensors, while maintaining diffeomorphism invariance.

In the gravity sector of the SME, a no-go theorem shows that when explicit Lorentz breaking occurs, an inconsistency can arise between conditions stemming from the field variations and symmetry considerations with geometrical constraints that must hold, such as the Bianchi identities.[5] However, it was also shown that the case of spontaneous Lorentz breaking evades the no-go theorem. The difference is that in a theory with explicit breaking, the SME coefficients are not associated with dynamical fields; while in the case of spontaneous Lorentz breaking they are. This creates a difference in the conditions that must hold with explicit symmetry breaking compared to spontaneous symmetry breaking. A consequence of the no-go theorem is that the gravity sector of the SME can only avoid incompatibility with conventional geometrical constraints if the symmetry breaking is spontaneous.

### 3 Spontaneous Lorentz Violation

Spontaneous breaking of Lorentz and diffeomorphism symmetry implies that massless Nambu-Goldstone (NG) modes should appear (in the absence of a Higgs mechanism). In general, there can be up to as many NG modes as there are broken symmetries. Since the maximal symmetry-breaking case would yield six broken Lorentz generators and four broken diffeomorphisms, there can be up to ten NG modes. A natural gauge choice is to put all the NG modes into the vierbein, and a simple counting argument shows that this is possible. With no spontaneous Lorentz violation, the six Lorentz and four diffeomorphism degrees of freedom can be used to reduce the vierbein from 16 down to six independent degrees of freedom. However, in a theory with spontaneous Lorentz breaking, alternative gauge-fixing choices can be made so that the NG modes
can naturally be incorporated in the vierbein. Of course, some of the NG modes might appear as ghosts, and it is for this reason that most models with spontaneous Lorentz breaking involve vevs that break fewer than ten spacetime symmetries.

In theories of spontaneous Lorentz breaking, the symmetry breaking is usually induced by a potential term in the Lagrangian that has a degenerate minimum space. The NG modes are excitations away from the vacuum that stay in the minimum space, while massive Higgs modes are excitations that go up the potential well away from the minimum. In conventional gauge theory, the fields in the potential are scalar fields, and the Higgs modes do not involve the gauge fields. However, with spontaneous Lorentz breaking, the metric typically appears in the potential, and for this reason massive Higgs modes can occur for the metric excitations in a process that has no analog in the case of conventional gauge theory.

In gravitational theories, since the symmetry breaking is local, the possibility of a Higgs mechanism occurs as well. In a Higgs mechanism, the would-be NG modes become additional degrees of freedom for massive gauge fields. In theories with spontaneous Lorentz breaking, since diffeomorphisms are also spontaneously broken, there are two potential Higgs mechanisms. The gauge fields associated with diffeomorphisms and Lorentz symmetry are the metric excitations and the spin connection. However, a Higgs mechanism involving the metric does not occur. This is because the mass term that is generated by covariant derivatives involves the connection, which consists of derivatives of the metric and not the metric itself. However, for the case of Lorentz symmetry, a conventional Higgs mechanism can occur. Here, the relevant gauge fields (for the Lorentz symmetry) are the spin connection. These appear directly in covariant derivatives acting on local tensor components, and when the local tensors acquire a vev, quadratic mass terms for the spin connection can be generated. Note, however, a viable Higgs mechanism involving the spin connection can only occur if the spin connection is a dynamical field. This then requires that there is nonzero torsion and that the geometry is Riemann-Cartan.

References

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