BLADDER EMPTYING FLOW RATE AS A FUNCTION OF BLADDER VOLUME†

The recent introduction of radioisotopic technique to measure bladder emptying flow rate has prompted a resurgence of interest in the topic. Since no comprehensive description of bladder emptying flow rate (as a function of bladder volume) has been discussed in the literature, we present such an approach in this communication. The topic is of general interest since radioisotopic methods provide a means for estimating bladder function without the dangers inherent in catheterization. Further, an empirical understanding of the normal dynamics of micturation is helpful in understanding the altered dynamics occurring in bladder diseases. The present discussion, limited to the normal bladder, will hopefully be followed by studies on patients with specific urologic problems.

DERIVATION

There are two constraints on a curve of voiding flow rate versus bladder volume. First, the curve must pass through the origin. That is, at zero volume there must be a zero flow rate. Second, the curve (although initially steep) begins to approach a limiting value. The rationale for the behavior of this portion of the curve is presumably that the normal bladder musculature can only distend to a fixed maximum length. Each of the complete curves of voiding flow rate versus bladder volume that we have been able to find in the literature (Aberle and Krepler,* Von Garrelts,*4 Drake7) exhibit these characteristics.

We can begin to describe these curves by assuming that the bladder emptying rate depends upon the volume at the start of the voiding (V). The description of the rate of change of the volume, between various initial bladder volumes, is one of a decreasing exponential. Letting F be the flow, V the initial volume, and K and λ constants:

\[
\frac{dF}{dV} = -Ke^{-\lambda V}
\]

This integrates to (k = mL):

\[
F = m(1-e^{-\lambda V})
\]

* Professor of Nuclear Medicine, Yale University School of Medicine.
** Head of Division of Nuclear Medicine, Children's Hospital Medical Center, Boston, Massachusetts.
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The term $m$ can be considered the theoretical maximal flow rate (the asymptote the curve is approaching). The term $m$ is to be distinguished from the actually achieved flow values which are always less than $m$. While $m$ has the units of ml/sec, the voiding rate constant has the units of reciprocal volume ($1$/ml.). These concepts are illustrated in Figure 1. Figure 1A shows a typical curve of voiding flow rate as a function of the bladder volume. The curve corresponds to the equation:

$$F = 25(1-e^{-0.004V})$$  \[3\]

Some studies reported in the literature have neglected flow rates at low bladder volumes. The resulting plots resemble straight lines (Fig. 1B). If only data points for flow corresponding to bladder volumes of 100 ml and greater are considered, the values can be approximated by a straight line. In this case, the straight line passing through the 100 ml and 300 ml points is:

$$F = 3.63 + 0.0462V$$  \[4\]

![Figure 1](image-url)  
Fig. 1. A plot of voiding rate (see text for details).
Such attempts to linearize bladder voiding data can be recognized by two pieces of information.

1. There is a paucity or lack of points corresponding to bladder volumes under 100 ml.

2. The line does not pass through the origin, but intersects at a flow rate greater than zero (this is physiologically unsound, but a consequence of the neglect of the data at the lower portion of the voiding curve).

Such linear plots mask the true nature of the voiding event, but do provide a plausible clinical approach for the comparison of normal and abnormal populations as exemplified by Strauss and Blaufox.

In addition, Figure 1B shows that the tangent to the curve, when it parallels the artificially produced straight line, has the same slope as the line. To find the tangent to equation [1] at \( V = 200 \), we substitute the numerical values (recalling that \( K = m \lambda \)). This yields:

\[
\frac{dF}{dV} = -\lambda m e^{-\lambda V} = 0.1(e^{-0.004(200)}) = 0.0449
\]

This value is rather close to the slope (0.0462) of the line drawn to approximate the fairly straight portion of the exponential curve. It can readily be shown that the full voiding curve described by equation [3] and the linear approach of equation [4] have two intersecting points at 100 and 300 ml bladder volumes.

At zero bladder volume there is zero flow. The tangent to the curve coming in at this point is found by solving equation [1] at \( V = 0 \). This leads to a slope of \( K \) or \( m \lambda \) (the maximal voiding rate times the rate constant). The line of \( m \lambda V \) is shown in Figure 1C. Initially this line can be approximated by connecting the lower points for bladder flow to the origin. The line \( m \lambda V \) represents the maximal possible voiding rate. The exponential term in equation [2] can be expanded in a Taylor's series. Doing this and truncating after three terms yields:

\[
F = m \left( \lambda V - \frac{(\lambda V)^2}{2} + \frac{(\lambda V)^3}{6} \ldots \right)
\]

This expression gives a close approximation of the full equation.

A rapid way of plotting the voiding rate data is to rewrite equation [2] in logarithmic form which yields:

\[
1 - \frac{F}{m} = e^{-\lambda V}
\]

\[
\ln \left( 1 - \frac{F}{m} \right) = -\lambda V
\]
The term \(1 - \frac{F}{m}\) is plotted logarithmically as a function of the volume \(V\).

Figure 1D shows application of this plot to the data depicted in the other parts of Figure 1.

**REVIEW OF DATA**

The uroflowmeter was introduced by Drake,\(^7\) for the study of the rate of voiding as a function of the bladder volume. Drake’s values are plotted in Figure 2, using the format of equation [7]. There is excellent agreement with the reported points.

Von Garrelts refined the use of the uroflowmeter and presented several series of data. Shown in Figure 3 are values taken from Table 3 of Von Garrelts\(^8\) on many individual points of voiding data obtained on the same person. Although there is some scatter around the line, the overall fit is good (correlation coefficient of 0.94). In another study,\(^9\) Von Garrelts presented values on 75 voidings by one person. Depicted in Fig. 4 are points taken from Von Garrelts drawing of the best fitting line; it can be seen that the description of equation [7] is quite accurate.

![Graph](image-url)

**Fig. 2.** Bladder emptying rate as a function of bladder volume, plotted according to equation [7]. Data were obtained from the Table given by Drake\(^7\) (The group with bladder volumes between 100 and 199 ml were assumed to have a mean value of 150 ml, and so on for each of the other groupings).
Bladder emptying rate

Fig. 3. Bladder emptying rate as a function of bladder volume, plotted according to equation [7]. Data are from Table 3 of Von Garrelts, representing voidings by the same catheterized individual.

Fig. 4. Bladder emptying rate as a function of bladder volume, plotted according to equation [7]. Data were obtained from the smooth curve drawn by Von Garrelts on a study of 75 voidings by the same patient.
Aberle and Krepler\textsuperscript{4} studied voiding in 6-12 year old children and constructed a table showing the average values for these youngsters. These values are plotted in Fig. 5, according to equation [7].

The four series of data are summarized in Table 1 for purposes of comparison. The Table also presents the limiting value of the slope to the line at low flows (m.l, with the units 1/sec.). The values only vary between a narrow range (0.097 per second to 0.231 per second). The physiologic significance of this narrow range remains to be determined. Also to be determined are the values of m and \( \lambda \) for each population so as to separate those with voiding problems from those without.

**SUMMARY**

An expression for describing the bladder emptying rate as a function of bladder volume was derived, so that the results fitted known clinical data. A necessary assumption was that the rate of change of flow, with volume, depended upon the initial volume, and was a decreasing exponential. The resulting expression contained two descriptive parameters: the maximum voiding flow rate (ml/sec.) and the emptying rate constant (1/volume).
Table 1. A Summary of Literature Data Plotted According to Equation [7]. The Data Are Consistent in Having a Small Range for the Product of m.λ. The One Patient Repeatedly Studied by Von Garrelts' Appears to Have a Flow Rate Maximum Considerably Above the Others

| Author       | Subject                           | Points | Equation                  | Product of m.λ. |
|--------------|-----------------------------------|--------|---------------------------|-----------------|
| Drake        | Adults (mean of series)           | 4      | F = 30(1−e⁻₀.₀₀₀₆₇₇₆T)    | 0.195           |
| Von Garrelts | Adult, catheterized               | 16     | F = 20(1−e⁻₀.₀₀₀₉₇₉₆T)    | 0.098           |
| Von Garrelts | Adult, best line of 75 voidings   | 9      | F = 50(1−e⁻₀.₀₀₀₉₇₉₆T)    | 0.097           |
| Aberle and   | Children, aged 6-12 years (mean   | 13     | F = 28(1−e⁻₀.₀₀₀₇₆T)      | 0.213           |
| Kepler       | values                            |        |                            |                 |

The latter part of the predicting equation was nearly straight, and agreed with previous attempts to describe voiding (limited to this section of the curve) which showed good fit with a straight line. The predicting equation was applied to four reported series of data and corresponded well in each case. The product of the maximum voiding flow rate and the emptying rate constant (m.λ) in each case gave a value between 0.097 per second and 0.213 per second; the physiological significance of this narrow range remains to be determined.

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