Convergence of Particle-Hole Expansions for the Description of Nuclear Correlations

N. Pillet\textsuperscript{(a,c)}, N. Sandulescu\textsuperscript{(b,c)}, Nguyen Van Giai\textsuperscript{(c)} and J.-F. Berger\textsuperscript{(a)}

\textsuperscript{(a)}Service de Physique Nucléaire, CEA/DAM-Ile de France, BP12, F-91680 Bruyères-le-Châtel, France
\textsuperscript{(b)}Institute of Physics and Nuclear Engineering, 76900 Bucharest, Romania
\textsuperscript{(c)}Institut de Physique Nucléaire, Université Paris-Sud, F-91406 Orsay Cedex, France

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The convergence properties of a multiparticle-multihole (mp-mh) configuration mixing approach whose purpose is to describe ground state correlations in nuclei without particle number and Pauli violations is investigated in the case of an exactly solvable pairing hamiltonian. Two different truncation schemes are tested by looking at quantities as correlation energies and single-particle occupation probabilities. Results show that pairing correlations present in usual superfluid nuclei can be accurately described using up to 6 particle–6 hole excitations, a convergence fast enough for envisaging extensions to fully microscopic calculations.

I. INTRODUCTION

One of the most powerful method of predicting nuclear ground state properties and excitations all over the nuclear chart is the microscopic approach based on the self-consistent mean-field theory \[1\]. Within this kind of approach, the nuclear mean-field is determined from a variational technique of the Hartree-Fock (HF) type applied to a many-body Hamiltonian. In order to be tractable up to the heaviest nuclei, the method is usually applied by employing phenomenological effective two-body interactions such as the Skyrme \[2\] or the Gogny \[3\] forces.

It is well known that an accurate description of nuclear structure almost always requires taking into account correlations beyond the simple HF approximation. One important class of these correlations, namely pairing correlations, are commonly treated using the BCS or Hartree-Fock-Bogoliubov (HFB) theories. A second class of correlations comes from the coupling of the nucleon motion to collective oscillations of the mean field. Depending on the amplitude of collective oscillations, these correlations are usually derived either from the Random Phase Approximation (RPA) theory or from the Generator Coordinate Method (GCM). Let us emphasize that, in addition to numerous standard nuclear properties, correlations are expected to play a key role in the interpretation of the unusual structure recently found in some nuclei such as the so-called parity inversions observed in the ground state and low-lying excitations of exotic light nuclei \[4,5\].

Most techniques used for including correlations suffer from defects that may be an obstacle to an accurate description of nuclear structure observables. For instance, proton and neutron numbers are not conserved in the BCS and HFB schemes. This deficiency is known to render these theories inadequate for describing superfluid to normal phase transitions and, more generally, situations where pairing correlations are small. As a consequence, the precision of the approximations consisting in treating simple excitations as multi-quasiparticle states in even-even and odd-even nuclei becomes questionable. Similarly, the quasi-boson approximation employed in RPA generates violations of the Pauli principle that require to introduce corrections to the mean-value of one-body operators and render delicate a microscopic treatment of particle-vibration coupling.

These difficulties can be obviated by having recourse to particle number projection techniques in the case of pairing or to extensions of RPA that do not make use of the quasi-boson approximation. However, these techniques complicate numerical algorithms considerably. For instance, in order to get a realistic description of weak pairing situations necessitates to perform the particle number projections before variation \[6\]. In the same way, extensions of the RPA avoiding the violation of Pauli correlations as e.g., the Self-Consistent RPA \[7\] require significant additional numerical effort.

In view of this, an alternative approach has been proposed in Ref. \[8\], which consists of taking a ground state trial wave function in the form of a linear combination of multiparticle-multihole (mp-mh) operators acting upon the HF state. The relevance of taking as a starting point a set of HF single-particle states has been pointed out in Ref. \[9\]. The mp-mh expansion is thought to be truncated at a given order and the mixing coefficients are determined by minimizing the total energy of the system. The interest of such a trial wave function is of course to exactly conserve nucleon numbers and be fully consistent with the Pauli principle. In lowest order, in which only $1p-1h$ configurations are considered, one gets the well-known Tamm-Dancoff approximation (TDA) \[1\]. Adding $2p-2h$ and higher order configurations allows one to describe ground state correlations of the most general form, including pairing and RPA correlations. In the simplest application of this method, a HF calculation is performed and the secular equation involving the matrix elements of the residual interaction is diagonalized. The coefficients of the mp-mh superposition for the ground state are then taken from the eigenvector belonging to the lowest eigenvalue. The eigenvectors belonging to the next few higher eigenvalues can be interpreted as approximately representing the lowest excitations of the system. Let us point out that the advantage of building mp-mh configurations from HF single-particle states lies in the fact that an important part of two-body correlations is already included in the nuclear mean field. One there-
fore expects that the effect of the residual interaction is small enough to allow a fast convergence of the mp-mh expansion so that only the lowest orders will have to be retained.

A more elaborate version of this method is to allow the HF single particle themselves to be included as variational parameters in the energy minimization. This procedure yields additional equations from which renormalized single particle states can be derived. The method then is similar to the Multi-Configuration Hartree-Fock (MCHF) approach which is widely used in atomic and molecular physics. The interest of introducing such a consistency between the single-particle structure and the correlated ground state is that additional parts of two-body correlations are further included in the nuclear mean field. In this situation even lower orders in the mp-mh expansion should be necessary to describe ground state correlations compared to the case without single-particle state renormalization. An approximate but numerically far less demanding way of implementing such a self-consistent definition of single-particle states is to derive them from the mean-field calculated with the correlated ground state one-body density matrix. This latter kind of self-consistency has been tested in Ref. for correlations generated by a zero-range residual interaction in systems of 2N identical fermions distributed over a set of 2N equally spaced, twofold degenerate levels. The mp-mh method has been applied to this hamiltonian under the form of multiple $pp$-$hh$ excitations, where $p$ and $h$ are the time-reversed of $p$ and $h$. Particle numbers and pairing strengths have been explored within ranges of values corresponding to typical situations found in realistic nuclei. Ground state correlation energies, occupation probabilities and odd-even mass differences have been calculated and compared with the exact results given by Richardson for different numbers of $pp$-$hh$ excitations. Let us emphasize that the purpose of the present work is not to discuss the interpretation of the calculated quantities and their connection to pairing in real nuclei. Our goal is simply to check the ability of the mp-mh approach to describe pairing correlations within a model where exact solutions exist.

The paper is organized as follows. In Section II we present additional detail concerning the mp-mh configuration mixing method. We also give the Richardson expression of the exact ground state solution of the pairing hamiltonian. Results obtained with the mp-mh method are presented and discussed in Section III. A summary and conclusions are given in Section IV.

II. THE FORMALISM

In the mp-mh configuration mixing method the ground state wave function of the system is written as a superposition of a HF Slater determinant $|\phi_0> = \prod_h a^+_h |0>$ and particle-hole excitations built upon it:

$$ |\Psi> = A_{0ph}|\phi_0> + \sum_i A^i_{1ph} a^+_i a_h |\phi_0> + \frac{1}{2!} \sum_{i,j} A^{ij}_{2ph} a^+_i a^+_j a_h a_h |\phi_0> + \ldots $$

(1)

The creation operators associated with unoccupied and occupied single particle states in $|\phi_0>$ are denoted by $a^+_p$ and $a^+_h$, respectively and the $A_{\alpha p oh}$ are configuration mixing coefficients to be determined. In compact form, eq. (1) can be written:

$$ |\Psi> = \sum_{\alpha=0}^M \sum_{i=1}^{m_{\alpha}} A^i_{\alpha p oh} |\phi_{\alpha p oh_i}> , $$

(2)

where $i$ denotes an ordered set of indices specifying a given $\alpha p oh$ configuration, $|\phi_{\alpha p oh_i}>$ is the wave function obtained by acting with the corresponding $\alpha p oh$ operator on $|\phi_0> \equiv |\phi_{0ph}>$, $m_\alpha$ is the number of configurations of $\alpha p oh$ type ($m_0 = 1$) and $M$ is an integer parameter. The configuration mixing coefficients $A^i_{\alpha p oh}$
are obtained by minimizing the total energy of the system:

\[
\frac{\partial <\Psi|\hat{H} - \lambda|\Psi>}{\partial A_{\text{opah}}^{*}} = \frac{\partial <\Psi|\hat{H} - \lambda|\Psi>}{\partial A_{\text{opah}}} = 0, \tag{3}
\]

where \(\lambda\) is the Lagrange parameter related to the conservation of the norm of \(|\Psi>\).

When \(\hat{H}\) is independent of the nuclear density, the conditions (4) are equivalent to the secular equation expressing the diagonalization of \(\hat{H}\) in the mp-mh space:

\[
\sum_{\alpha'} \sum_{f} <\phi_{\alpha'f}|\hat{H}|\phi_{\alpha'f}, \alpha' \bar{f}> A_{\alpha'f, \alpha' \bar{f}}^{\dagger} = \lambda A_{\alpha'f, \alpha' \bar{f}} \tag{4}
\]

and the complex conjugate one.

In this work we choose for \(\hat{H}\) the pairing hamiltonian:

\[
\hat{H} = \sum_{f=1}^{2N} \epsilon_f (a_{f}^+ a_{f} + a_{f}^+ a_{f}^\dagger) - \gamma \sum_{f=1}^{2N} \sum_{f'=1}^{2N} a_{f}^+ a_{f'}^+ a_{f'} a_{f}, \tag{5}
\]

where \(\bar{f}\) denotes the time-reversed state of \(f\). The exact ground state of this Hamiltonian for the case of a system of 2N fermions has been derived by Richardson [15]:

\[
|\Psi^{\text{exact}}> = \prod_{i=1}^{N} B_{i}^+ |0>, \tag{6}
\]

where the operator \(B_{i}^+\) creates a collective pair:

\[
B_{i}^+ = \sum_{j=1}^{N} \frac{1}{2\epsilon_j - E_i} a_{j}^+ a_{j}^\dagger, \tag{7}
\]

and \(|0>\) is the fermion vacuum. The quantities \(E_i\) are the solutions of a set of N nonlinear equations [15]:

\[
1 - 2g \sum_{j\neq i=1}^{N} \frac{1}{E_j - E_i} + g \sum_{j=1}^{N} \frac{1}{2\epsilon_j - E_i} = 0. \tag{8}
\]

and the exact ground state energy of the system is given by:

\[
E = \sum_{i=1}^{N} E_i. \tag{9}
\]

The solutions of the pairing hamiltonian can be classified by the eigenvalues of the seniority operator:

\[
\nu = \sum_{f} (a_{f}^+ a_{f} - a_{f}^+ a_{f}^\dagger)^2, \tag{10}
\]

which counts the number of unpaired particles present in the system. In the mp-mh scheme the ground state of the pairing hamiltonian which corresponds to seniority zero can be generated by considering in (2) only configurations \(ap-\alpha h = \beta p\bar{p}-\beta h\) with even values \(\alpha=2\beta\). If one includes in expansion (2) all \(ap-\alpha h\) configurations up to \(M=2N\), the wave function \(|\Psi>\) becomes equivalent to the exact wave function \(|\Psi^{\text{exact}}>\).

It is interesting to discuss the relationship between the mp-mh configuration mixing method presented above and the projected-BCS (PBCS) approach [6, 17], employed for restoring particle number conservation in finite Fermi systems. In the PBCS approach the ground state has a pair condensate structure obtained by replacing in eq. 10 the N pair operators \(B_{i}^+\) by a unique collective pair:

\[
\Gamma^+ = \sum_{j=1}^{N} x_j a_{j}^+ a_{j}^\dagger, \tag{11}
\]

where the amplitudes \(x_j\) are determined variationally. The PBCS wave function can be written also in terms of particle-hole operators acting on the HF state [18]:

\[
|\text{PBCS}>= \text{const.} \sum_{n=0}^{N} \frac{(\Gamma^+_h \Gamma_h)^n}{(n!)^2}|\text{HF}>, \tag{12}
\]

where the pair operators \(\Gamma_h (\Gamma_p)\) are formed by restricting the summation in (11) to hole (particle) states and replacing \(x_j\) by \(x_p (1/x_h)\). From (12) it can be seen that the PBCS wave function belongs to a particular subset of the mp-mh wave functions (2) in which the mixing coefficients have a separable form in the indices associated with particles and holes. It is known that, when the \(x_j\)’s are determined from projection before variation, PBCS wave-functions give a satisfactory description of weak pairing regimes and of the crossover from normal to superfluid phases in Fermi systems. One therefore expects that the more general mp-mh form (11) is able to provide an accurate description of pairing correlations in these situations. In addition, since the \(x_p\) and \(1/x_h\) are small, only the first few terms in (11) should be necessary.

In order to test the convergence of expansion (11) in all pairing regimes, the ground state solution of the Hamiltonian (6) has been calculated using eq. (11) for two systems, one with 2N=8 particles distributed among 8 equidistant levels, and the other one with 2N=16 particles distributed among 16 equidistant levels. The levels are twofold degenerate. These two systems simulate the level densities and pairing diffusivities found in weakly and moderately superfluid nuclei, respectively. The constant \(g\) has been varied in a large range of values around typical pairing strengths in nuclei.

III. NUMERICAL RESULTS

The number of seniority-zero states of the 2N particle system is equal to the binomial coefficient \(C_{2N}^N\). Thus, for the systems with 2N=8 and 2N=16 particles the total number of seniority-zero states which should eventually be used for exactly calculating the ground states of the two systems is equal to 70 and 12870, respectively. We
study here the convergence properties of the mp-mh expansion \( \text{corr} \) as a function of the maximum particle-hole order \( M \) and of an energy cut-off in the energy of the mp-mh configurations. The numbers of \( \text{corr} \) configurations for \( 2\alpha \) between 2 and 8 are shown in Table I for the two systems. The highest number of configurations is reached when \( \alpha \) is equal to half the number of particles. The numbers of configurations for various cut-off energies \( E_{\text{cut}} \) are given in Table II. Here and in the following, energies are in units of the equidistant level spacing \( d \).

The convergence of the mp-mh expansion towards the exact solution is studied below by looking at correlation energies, odd-even mass differences and level occupation probabilities. The same quantities calculated with the plain BCS (unprojected) approach are also given.

### A. Ground state correlation energy

The correlation energy \( E_{\text{corr}} \) in the ground state of the system is taken as:

\[
E_{\text{corr}} = E(g \neq 0) - E(g = 0),
\]

where \( E(g) \) is the total energy of the interacting system.

The amount of pairing correlations in the \( 2N \) particle ground state can be estimated from the odd-even mass difference defined by \( \text{corr} \) :

\[
P(2N) = 2E(2N - 1) - E(2N) - E(2N - 2),
\]

where \( E(N) \) is the ground state energy for \( N \) fermions. In the calculations we adopted the convention of Ref. [13], i.e., for a system of \( N \) particles (\( N \) odd or even), the pairing force is effective among \( N \) doubly-degenerated levels. Notice that \( P(2N) = 0 \) for non-interacting particles (\( g = 0 \)). Therefore, the mean-field effect which is contained in odd-even mass differences for real nuclei is not present in the equidistant model used here. Thus, \( P(2N) \) can be taken as a genuine measure of the intensity of pairing correlations.

Assimilating \( P(2N) \) with the pairing gap \( \Delta \), the weak (strong) pairing regime corresponds to values much smaller (larger) than 1 of the ratio \( \eta = \Delta/d \). For deformed nuclei in the rare earth region \( \Delta \) is about 1 MeV while \( d \) is of the order of 400-500 keV. Thus, for these nuclei \( \eta \) is of the order of 2-3. As we shall see below, for values of \( \eta \) in this range the correlation energies evaluated in the BCS approximation are largely underestimated while the mp-mh expansion converges quickly to the exact results. Let us add that, in units of \( d \), the constant \( g \) corresponding to typical nuclear pairing strengths is about 0.5.

### TABLE I: Number of \( \text{corr} \) configurations for given values of \( 2\alpha \) between 2 and 8 for the systems with \( 2N=8 \) and \( 2N=16 \) particles.

| \( E_{\text{cut}} \) | 2p2h | 4p4h | 6p6h | 8p8h | 2p2h | 4p4h | 6p6h | 8p8h |
|-------------------|------|------|------|------|------|------|------|------|
| 2                 | 1    | 0    | 0    | 0    | 2    | 1    | 0    | 0    |
| 8                 | 10   | 1    | 0    | 0    | 8    | 10   | 1    | 0    |
| 18                | 16   | 28   | 1    | 0    | 18   | 43   | 50   | 1    |
| 32                | 16   | 36   | 16   | 1    | 32   | 64   | 428  | 1    |
|                   | 50   | 64   | 776  | 1946 | 464  | 70   | 64   | 784  |
|                   | 128  | 64   | 784  | 3136 | 4862 |

### FIG. 1: Percentage error on correlation energies as a function of the pairing strength \( g \) for the \( 2N=8 \) and \( 2N=16 \) cases. The curves correspond to the results obtained by cutting the mp-mh expansion at 2p2h, 4p4h and 6p6h.

We first analyze the convergence of the mp-mh expansion for the two quantities \( E_{\text{corr}} \) and \( P(2N) \) when including successively all 2p2h, 4p4h, . . . , configurations in...
the ground state wave function \( \Psi \) (without any cut-off). The results of the calculations for the systems with 8 and 16 particles are shown in Tables III-IV and in Fig. The errors for correlation energies, \( \Delta E_{\text{corr}} \), and for the odd-even mass differences, \( \Delta P \), are calculated relative to the exact values. As mentioned above, for the system with 2N=8 particles an expansion up to \( 8P-8h \) corresponds to the exact solution.

In Tables III-IV the results are shown for several values of the pairing strength \( g \). In the BCS approximation there is a sharp transition between the normal and the superfluid phase which appears at \( g \approx 0.31 \) for 2N=8 and \( g \approx 0.24 \) for 2N=16. Below these critical values of the pairing strength the correlation energies are zero in the BCS approximation. This is not the case for the exact solution, which shows a rather smooth increase of the correlation energy with the pairing strength.

From Tables III-IV we also notice that, in the BCS approximation, the correlation energies are strongly underestimated. The error made is more than 20% for all the values of the pairing strength listed in Tables III-IV. The same large differences are obtained between the pairing gaps associated in BCS with the odd-even mass differences, and the exact values of \( P(2N) \). It is worth stressing that the errors in the BCS correlation energies remain rather large, more than 10%, even for large values of the pairing strength for which the ratio \( \eta \) between the

**TABLE III**

| \( g \) | npnh | \( E_{\text{corr}} \) | \( \Delta E_{\text{corr}}(\%) \) | \( P(2N) \) | \( \Delta P(2N)(\%) \) |
|---|---|---|---|---|---|
| 0.2 | 2 | -0.924 | 0.36 | 0.299 | 1.65 |
|    | 4 | -0.927 | 0.00 | 0.303 | 0.33 |
|    | 6 | -0.927 | 0.00 | 0.304 | 0.00 |
|    | 8 | -0.927 | 0.00 | 0.304 | 0.00 |
|    | BCS | 0 | 100.00 | 0 | 100.00 |
| Exact | -0.927 | 0.304 |
| 0.6 | 2 | -3.757 | 9.18 | 1.381 | 20.59 |
|    | 4 | -4.114 | 0.55 | 1.718 | 1.27 |
|    | 6 | -4.136 | 0.01 | 1.739 | 0.00 |
|    | 8 | -4.136 | 0.00 | 1.739 | 0.00 |
|    | BCS | -2.942 | 28.92 | 1.570 | 9.72 |
| Exact | -4.136 | 1.739 |
| 0.8 | 2 | -5.602 | 15.25 | 2.618 | 29.21 |
|    | 4 | -6.511 | 1.49 | 3.594 | 2.81 |
|    | 6 | -6.608 | 0.02 | 3.696 | 0.05 |
|    | 8 | -6.610 | 0.00 | 3.701 | 0.00 |
|    | BCS | -5.028 | 23.93 | 2.502 | 32.40 |
| Exact | -6.610 | 3.701 |

**TABLE IV**

| \( g \) | npnh | \( E_{\text{corr}} \) | \( \Delta E_{\text{corr}}(\%) \) | \( P(2N) \) | \( \Delta P(2N)(\%) \) |
|---|---|---|---|---|---|
| 0.18 | 2 | -1.659 | 0.64 | 0.305 | 4.09 |
|    | 4 | -1.669 | 0.01 | 0.317 | 0.31 |
|    | 6 | -1.669 | 0.00 | 0.318 | 0.00 |
|    | 8 | -1.669 | 0.00 | 0.318 | 0.00 |
|    | BCS | 0.00 | 100.00 | 0.00 | 100.00 |
| Exact | -1.669 | 0.318 |
| 0.54 | 2 | -6.883 | 20.92 | 1.941 | 49.51 |
|    | 4 | -8.250 | 5.22 | 3.301 | 14.13 |
|    | 6 | -8.652 | 0.60 | 3.781 | 1.64 |
|    | 8 | -8.702 | 0.03 | 3.842 | 0.05 |
|    | BCS | -6.636 | 23.76 | 2.580 | 32.88 |
| Exact | -8.704 | 3.844 |
| 0.66 | 2 | -9.149 | 29.37 | 2.598 | 57.16 |
|    | 4 | -11.711 | 9.59 | 4.746 | 21.74 |
|    | 6 | -12.738 | 1.66 | 5.819 | 4.04 |
|    | 8 | -12.938 | 0.12 | 6.047 | 0.28 |
|    | BCS | -10.357 | 20.05 | 3.701 | 43.96 |
| Exact | -12.954 | 6.064 |

**BCS pairing gap and the level spacing (which in our case is the energy unit) is of the order of 20.**

Let us emphasize however that in BCS or HFB calculations applied to real nuclei, the pairing strengths for neutrons and protons – or the matrix elements of the pairing residual interaction – are adjusted in order to correctly describe pairing correlations in strongly superfluid nuclei. Consequently, BCS or HFB results should be taken with caution only in weak pairing regimes.

From Tables III-IV one can see that the mp-mh expansion is converging rapidly to the exact results. Thus, for the system with 16 particles a truncation to 6p-6h provides rather accurate values for the correlation energies (\( \Delta E_{\text{corr}} \sim 1\% \)). This truncation order should be compared to the maximum possible order of the mp-mh expansion, which is 16p-16h in this case. As seen in Table II for a truncation at 6p-6h the number of configurations is equal to 3136, while the total number of seniority-zero configurations is 12870.

Next, we examine the convergence of the results as a function of the energy cut-off \( E_{\text{cut}} \). In this truncation scheme we consider only mp-mh configurations whose excitation energies are smaller than \( E_{\text{cut}} \). The number of configurations for various \( E_{\text{cut}} \) values are shown in Table III. In Fig.2 the evolution of \( \Delta E_{\text{corr}} \) with \( E_{\text{cut}} \) is shown for several values of the pairing strength \( g \).

From these figures and from Table III one can see that, for the values of \( g \) of physical interest (\( g \sim 0.5 \)) the number of configurations needed to achieve the same accuracy as previously is smaller. Thus, in the case of the system with 16 particles and for \( g=0.54 \) one needs a cut-off energy \( E_{\text{cut}}=50 \) in order to get an accuracy of
\[ \Delta E_{\text{corr}} \sim 0.5\%. \] As seen in Table III for this cut-off energy one selects 3250 configurations up to 8p-8h. On the other hand, in the truncation scheme based on the order of the particle-hole expansion, to achieve the same accuracy one needed to consider all configurations up to 6p-6h that is, from Table I 3984 configurations. This is about 20\% more than in the truncation based on cut-off energy. In particular a large number – about one third – of the 6p-6h configurations of Table I are eliminated by the cut-off. Since the same precision is achieved, this means that these eliminated configurations do not contribute to the correlation energy. Let us mention that a 50 MeV cut-off in the excitation energy of scattered pairs is consistent with the results of BCS or HFB calculations in actual nuclei, even in strongly superfluid ones.

FIG. 2: Percentage errors on correlation energies as a function of cut-off energy \( E_{\text{cut}} \). The curves corresponds to various pairing strength values listed in the inset. The results are for the systems with 2\( N=8 \) and 2\( N=16 \) particles.

FIG. 3: Occupation probabilities \( v_n^2 \) for the single-particle levels \( n \). The calculated points are joined by straight lines. The curves correspond to different truncation orders specified in the insets, the full line showing the exact values. The results are for the system with 2\( N=8 \) particles.

B. Occupation probabilities

The occupation probability \( v_n^2 \) of the single-particle level \( n \) is given by:

\[
v_n^2 = \frac{1}{2} \langle \Psi | a_n^+ a_n + a_n^{\dagger} a_n | \Psi \rangle .
\]  

The occupation probabilities of all single-particle levels in the cases 2\( N=8 \) and 2\( N=16 \) are plotted in Figs. III for three values of the pairing strength \( g \). It can be seen
that for small values of $g$ the exact values of occupation probabilities are rather well reproduced by a truncation to $2p2h$ configurations.

However, for the highest values of $g$ listed in Tables III-V, i.e., $g=1.0$ for $2N=8$ and $g=0.66$ for $2N=16$, one needs to introduce up to $6p6h$ configurations in order to get an accurate description of occupation probabilities in the $mp-mh$ scheme. This truncation order appears consistent with the behaviour of correlation energies discussed in the previous subsection.

IV. SUMMARY AND CONCLUSIONS

In this paper we have analyzed the convergence properties of the multiparticle-multihole ($mp-mh$) configuration mixing approach. The study was done for a pairing Hamiltonian with a constant pairing force, which can be solved exactly. For the single-particle model we have chosen a sequence of doubly-degenerate and equidistant levels, half-filled with one kind of fermions. The parameters of the model and the number of particles have been chosen so as to simulate the physical situation met in light and medium nuclei. The pairing strength $g$ has been varied from zero up to about twice the value corresponding to typical conditions in atomic nuclei.

We have shown that a truncation based on the $mp-mh$ expansion is converging rather rapidly to the exact results for the ground state correlation energies and occupation probabilities of single-particle levels. For instance, with a value of $g$ corresponding to standard nuclear pairing, one can get an accuracy of about 1% for the correlation energy in a system of 16 particles if one considers configurations up to $6p6h$. The number of these configurations is about four times smaller than the total number of possible $mp-mh$ configurations with seniority zero that the system can form. An even faster convergence without precision loss is found by selecting configurations whose excitation energies are limited by a cut-off corresponding to the maximum energy of scattered pairs. These encouraging results indicate that applications of the $mp-mh$ approach to fully microscopic self-consistent calculations in nuclei should be possible with present-day numerical capabilities.

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