Virtually Haken fillings and semi-bundles

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Suppose that $M$ is a fibered three-manifold whose fiber is a surface of positive genus with one boundary component. Assume that $M$ is not a semi-bundle. We show that infinitely many fillings of $M$ along $\partial M$ are virtually Haken. It follows that infinitely many Dehn-surgeries of any non-trivial knot in the three-sphere are virtually Haken.

57M10; 57M25

1 Introduction

In this paper manifold will always mean a compact, connected, orientable, possibly bounded, three-manifold. A bundle means a manifold which fibers over the circle. A semi-bundle is a manifold which is the union of two twisted $I$–bundles (over connected surfaces) whose intersection is the corresponding $\partial I$–bundle. An irreducible, $\partial$–irreducible manifold that contains a properly embedded incompressible surface is called Haken. A manifold is virtually Haken if has a finite cover that is Haken.

Waldhausen’s virtually Haken conjecture is that every irreducible closed manifold with infinite fundamental group is virtually Haken. It was shown by Cooper and Long [1] that most Dehn-fillings of an atoroidal Haken manifold with torus boundary are virtually Haken provided the manifold is not a bundle.

Theorem 1  Suppose that $M$ is a bundle with fiber a compact surface $F$ and that $F$ has exactly one boundary component. Also suppose that $M$ is not a semi-bundle and not $S^1 \times D^2$. Then infinitely many Dehn-fillings of $M$ along $\partial M$ are virtually Haken.

Corollary 1.2  Let $k$ be a knot in a homology three-sphere $N$. Suppose that $N – k$ is irreducible and that $k$ does not bound a disk in $N$. Then infinitely many Dehn-surgeries along $k$ are virtually Haken.
The main idea is to construct a surface of invariant slope (see Section 3) in a particular finite cover of $M$. Such surfaces are studied in arbitrary covers using representation theory in a sequel [2]. While writing this paper we noticed that Thurston’s theory of bundles extends to semi-bundles, and in particular there are manifolds which are semi-bundles in infinitely many ways. We discuss this in the next section.

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2 Bundles and semi-bundles

Various authors have studied semi-bundles, in particular Hempel and Jaco [6] and Zulli [10, 11]. Suppose a manifold has a regular cover which is a surface bundle. We wish to know when a particular fibration in the cover corresponds to a bundle or semi-bundle structure on the quotient. The following has the same flavor as some results of Hass [5].

Theorem 1 Let $M$ be a compact, connected, orientable, irreducible three-manifold, $p: \tilde{M} \to M$ a finite regular cover, and $G$ the group of covering automorphisms. Suppose that $\phi: \tilde{M} \to S^1$ is a fibration of $\tilde{M}$ over the circle. Suppose that the cyclic subgroup $V$ of $H^1(\tilde{M}; \mathbb{Z})$ generated by $[\phi]$ is invariant under the action of $G$. Then one of the following occurs:

1. The action of $G$ on $V$ is trivial. Then $M$ also fibers over the circle. Moreover there is a fibering of $M$ which is covered by a fibering of $\tilde{M}$ that is isotopic to the original fibering.

2. The action of $G$ on $V$ is non-trivial. Then $M$ is a semi-bundle. Moreover there is a semi-fibering of $M$ which is covered by a fibering of $\tilde{M}$ that is isotopic to the original fibering.

Proof Define $N = \ker[\phi_*: \pi_1\tilde{M} \to \pi_1S^1]$. Since $\phi$ is a fibration $N$ is finitely generated. If $N$ is cyclic then the fiber is a disc or annulus. In these cases the result is easy. Thus we may assume $N$ is not cyclic. Because $V$ is $G$-invariant, it follows that $N$ is a normal subgroup of $\pi_1M$ and $Q = \pi_1M/N$ is infinite. Using [6, Theorem 3] it follows that $M$ is a bundle or semi-bundle (depending on case 1 or 2) with fiber a compact surface $F$ and $N$ has finite index in $\pi_1F$. The pull-back of this (semi)fibration of $M$ gives a fibration of $\tilde{M}$ in the cohomology class of $\phi$ and is therefore isotopic to the given fibration. $\Box$

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Suppose that \( G \cong (\mathbb{Z}_2)^n \) acts on a real vector space \( V \) and let \( X = \text{Hom}(G, \mathbb{C}) \) denote the set of characters on \( G \). Then \( X \cong \text{Hom}(G, \mathbb{Z}_2) \). For each \( \epsilon \in X \) there is a \( G \)-invariant generalized \( \epsilon \)-eigenspace

\[
V_\epsilon = \{ v \in V : \forall g \in G \ g \cdot v = \epsilon(g) v \}.
\]

Then \( V \) is the direct sum of these subspaces \( V_\epsilon \).

Suppose that \( M \) is an atoroidal irreducible manifold with boundary consisting of incompressible tori. According to Thurston there is a finite collection (possibly empty), \( \mathcal{C} = \{C_1, \cdots, C_k\} \), called fibered faces. Each fibered face is the interior of a certain top-dimensional face of the unit ball of the Thurston norm on \( H_2(M, \partial M; \mathbb{R}) \). It is an open convex set with the property that fibrations of \( M \) correspond to rational points in the projectivized space \( \mathbb{P}(\cup_i C_i) \subset \mathbb{P}(H_2(M, \partial M; \mathbb{R})) \).

Let \( G = H_1(M; \mathbb{Z}/2) \). The regular cover \( \tilde{M}_s \) of \( M \) with covering group \( G \) is called the \( \mathbb{Z}_2 \)-universal cover. Let \( \mathcal{D} = \{D_1, \cdots, D_l\} \) be the fibered faces for this cover. For each \( \epsilon \in H^1(M; \mathbb{Z}_2) \) there is an \( \epsilon \)-eigenspace \( H_{2,\epsilon} \) of \( H_2(\tilde{M}_s, \partial \tilde{M}_s; \mathbb{R}) \). For each \( 1 \leq i \leq l \) and \( \epsilon \in H^1(M; \mathbb{Z}_2) \) we call \( S_{i,\epsilon} = D_i \cap H_{2,\epsilon} \) a semi-fibered face if it is not empty. It is the interior of a compact convex polyhedron whose interior is in the interior of some fibered face for \( \tilde{M}_s \). Let \( S_i \) be the union of the \( S_{i,\epsilon} \) where \( \epsilon \) is non-trivial.

**Theorem 2** With the above notation there is a bijection between isotopy classes of semi-fibrations of \( M \) and rational points in \( \mathbb{P}(\cup_i S_i) \).

**Proof** A semi-fibration of \( M \) gives such a rational point by considering the induced fibration on \( \tilde{M}_s \). The converse follows from Theorem 1. We leave it as an exercise to check uniqueness up to isotopy.

We believe that all points in \( \mathbb{P}(\cup_i S_i) \) correspond to isotopy classes of non-transversally-orientable, transversally-measured, product-covered 2–dimensional foliations of \( M \). This is true for rational points and therefore holds on a dense open set (using the fact that the set of non-degenerate twisted 1–forms is open). However, since we have no use for this fact, we have not tried very hard to prove it.

**Definition** A manifold is a sesqui-bundle if it is both a bundle and a semi-bundle.

An example is the torus bundle \( M \) with monodromy \( -\text{Id} \). This is the quotient of Euclidean three-space by the group \( G_2 \) (Wolf [8, Theorem 3.5.5]). \( M \) has infinitely many semi-fibrations with generic fiber a torus and two Klein-bottle fibers. In addition, \( M \) is a bundle thus a sesqui-bundle.
A hyperbolic example may be obtained from $M$ as follows. Let $C$ be a 1–submanifold in $M$ which is a small $C^1$–perturbation of a finite set of disjoint, immersed, closed geodesics in $M$ chosen so that:

1. No two components of $C$ cobound an annulus and no component bounds a Möbius strip.
2. $C$ intersects every flat torus and flat Klein bottle.
3. Each component of $C$ is transverse to both a chosen fibration and semi-fibration.

Let $N$ be $M$ with a regular neighborhood of $C$ removed. Then the interior of $N$ admits a complete hyperbolic metric. By (3) it is a sesqui-bundle. This answers a question of Zulli who asked in [11] if there are non-Seifert 3–manifolds which are sesqui-bundles.

3 Virtually Haken fillings

The following is well-known, but we include it here for ease of reference.

**Lemma 1** Suppose $M$ is Seifert fibered and has one boundary component. Then one of the following holds:

1. $M$ is $D^2 \times S^1$ or a twisted $I$–bundle over the Klein bottle.
2. Infinitely many Dehn-fillings are virtually Haken.

**Proof** The base orbifold $Q$ has one boundary component and no corners. If $\chi_{\text{orb}} Q > 0$ then $Q$ is a disc with at most one cone point thus $M = D^2 \times S^1$. If $\chi_{\text{orb}} Q = 0$ then $Q$ is a Möbius band or a disc with two cone points labeled 2 and in either case $Q$ has a 2–fold orbifold-cover that is an annulus $A$. But then $M$ is 2–fold covered by a circle bundle over $A$. Since $M$ is orientable it follows that this bundle is $S^1 \times A$ and hence $M$ is a twisted $I$–bundle over the Klein bottle.

Finally, if $\chi_{\text{orb}} (Q) < 0$ then all but one filling of $M$ is Seifert fibered. There are infinitely many fillings of $M$ which give a Seifert fibered space, $P$, with base orbifold $Q'$ and $\chi_{\text{orb}} (Q') < 0$. There is an orbifold-covering of $Q'$ which is a closed surface of negative Euler characteristic. The induced covering of $P$ contains an essential vertical torus and is therefore virtually Haken.

**Definitions** A slope on a torus $T$ is the isotopy class of an essential simple closed curve on $T$. We say that a slope lifts to a covering of $T$ if it is represented by a loop which lifts. The following is immediate:
Lemma 2  Suppose $\tilde{T} \to T$ is a finite covering. Then the following are equivalent:

1. Some slope on $T$ lifts to $\tilde{T}$.
2. The covering is finite cyclic.
3. Infinitely many slopes on $T$ lift to $\tilde{T}$.

The distance, $\Delta(\alpha, \beta)$, between slopes $\alpha, \beta$ on $T$ is the minimum number of intersection points between representative loops. If $\alpha$ is a slope on a torus boundary component of $M$ then $M(\alpha)$ denotes the manifold obtained by Dehn-filling $M$ using $\alpha$. A surface $S$ in a manifold $M$ is essential if it is compact, connected, orientable, incompressible, properly-embedded, and not boundary-parallel. Let $M$ be a manifold with boundary a torus and $\alpha \subset \partial M$ a slope. Suppose that $N$ is a finite cover of $M$. An essential surface $S \subset N$ has invariant slope $\alpha$ if $\partial S \neq \phi$ and every component of $\partial S$ projects to a loop homotopic to a non-zero multiple of $\alpha$. We call a finite cover $p: N \to M$ a $\partial$–cover if there is an integer $d > 0$ and a homomorphism $\theta: \pi_1(\partial M) \to \mathbb{Z}_d$ such that for every boundary component $T$ of $N$ we have $p_*(\pi_1 T) = \ker \theta$. The existence of $\theta$ ensures each component of $\partial N$ is the same cyclic cover of $\partial M$.

The following lemma reduces the proof of the main theorem to constructing an essential non-fiber surface of invariant slope in a $\partial$-cover of $M$.

Lemma 3  Suppose that $M$ is a compact, connected, orientable irreducible 3–manifold with one torus boundary component. Suppose that there is a $\partial$–cover $N$ of $M$ and an essential non-separating surface $S \subset N$ of invariant slope. Assume that $S$ is not a fiber of a fibration of $N$. Then $M$ has infinitely many virtually-Haken Dehn-fillings.

Proof  We first remark that the particular case that concerns us in this paper is that $M$ is a bundle with boundary and thus $M$ is irreducible. Since $M$ is irreducible at most 3 fillings give reducible manifolds (Gordon and Luecke [4]). A cover of an irreducible manifold is irreducible (Meeks and Yau [7]). Therefore it suffices to show there are infinitely many fillings of $M$ which have a finite cover containing an essential surface.

If $M$ contains an essential torus then this torus remains incompressible for infinitely many Dehn-fillings by Culler–Gordon–Luecke–Shalen [3, Theorem 2.4.2]. If $M$ is Seifert fibered then by Theorem 1 either the result holds or $M = S^1 \times D^2$ or is a twisted $I$–bundle over the Klein bottle. The latter two possibilities do not contain a surface $S$ as in the hypotheses. By Thurston’s hyperbolization theorem we are reduced to case that $M$ is hyperbolic.

Since $p: N \to M$ is a $\partial$–cover there is $d > 0$ such that every component of $\partial N$ is a $d$–fold cover of $\partial M$. Let $k$ be a positive integer coprime to $d$. Let $p_k: \tilde{N}_k \to N$ be the
We claim that there is an essential surface $S$. We claim that there is a homomorphism $\theta_k: \pi_1 M \to \mathbb{Z}_{kd}$ such that every slope in $\ker \theta_k$ lifts to every component of $\partial \tilde{N}_k$.

Assuming this, the filling $M(\gamma)$ of $M$ is covered by a filling, $\tilde{N}_k(\gamma)$, of $\tilde{N}_k$ if and only if the slope $\gamma \subset \partial M$ lifts to each component of $\partial \tilde{N}_k$. Since $S$ is non-separating, by Wu [9, Theorem 5.7], there is $K > 0$ such that if $k \geq K$ then there is an essential closed surface $F_k \subset \tilde{N}_k$ obtained by Freedman tubing two lifts of $S$. We choose such $k$ coprime to $d$. By [9, Theorem 5.3], there is a finite set of slopes $\beta_1, \ldots, \beta_n$ on $\partial M$ and $L > 0$ so that if $\gamma \subset \partial M$ is a slope and $\Delta(\gamma, \beta_i) \geq L$ for all $i$ then the projection of $F_k$ into $M(\gamma)$ is $\pi_1$–injective. Assuming the claim, there are infinitely many slopes $\gamma \in \ker \theta_k$ satisfying these inequalities. For such $\gamma$ the cover $\tilde{N}_k(\gamma) \to M(\gamma)$ contains the essential surface $F_k$.

It only remains to prove the claim. Let $T$ be a component of $\partial N$ and $\beta \subset T$ be the slope given by $S \cap T$. Let $\tilde{T}$ be a component of $\partial \tilde{N}_k$ which covers $T$. The cover $p_k|\tilde{T} \to T$ is cyclic of degree $k'$ some divisor of $k$ (depending only on $|S \cap T|$). Also $\beta$ lifts to this cover. Suppose that a slope $\gamma \subset \partial M$ lifts to a slope $\tilde{\gamma} \subset T$. It follows that $\tilde{\gamma}$ lifts to $\tilde{T}$ if $k'$ divides $\Delta(\tilde{\gamma}, \beta)$. If this condition is satisfied by some lift, $\tilde{\gamma}$, of $\gamma$ then, since $S$ has invariant slope and $N \to M$ is a $\partial$–cover, it is satisfied by every such lift.

Let $\tilde{T} \to T$ be the $k'$–fold cyclic cover dual to $\beta$. Since $k'$ and $d$ are coprime the composite of this cover and the cyclic $d$–fold cover $T \to \partial M$ is a cyclic cover of degree $dk'$. By Theorem 2 there are infinitely many slopes on $\partial M$ which lift to $\tilde{T}$. Every slope on $\partial M$ which lifts to $\tilde{T}$ also lifts to every component of $\partial \tilde{N}_k$. This proves the claim. $\square$

**Proof of Theorem 1** We attempt to construct $S$ and $N$ as in Theorem 3. The action of the monodromy on $H_1(F; \mathbb{Z}_2)$ has some finite order $m$. Therefore there is a finite cyclic $m$–fold cover $W \to M$ such that $W$ is a bundle with fiber $F$ and the action of the monodromy for $W$ on $H_1(F; \mathbb{Z}_2)$ is trivial. We then have

$$H^1(W; \mathbb{Z}_2) \cong H^1(F; \mathbb{Z}_2) \oplus H^1(S^1; \mathbb{Z}_2).$$

Since $F$ has boundary and $F \neq D^2$ we may choose a non-zero element $\phi = (b, 0) \in H^1(F; \mathbb{Z}_2) \oplus H^1(S^1; \mathbb{Z}_2)$. This determines a two-fold cover $\tilde{W}$ of $W$. Since $F$ has one boundary component, $\phi$ vanishes on $H_1(\partial W; \mathbb{Z}_2)$, and since $W$ has one boundary component, $\tilde{W}$ has exactly two boundary components $T_1$ and $T_2$. The action of the covering involution, $\tau$, swaps these tori. In particular $\tilde{W} \to M$ is a $\partial$–cover.

We claim that there is an essential surface $S$ in $\tilde{W}$ such that

$$\tau_*[S] = -[S] \neq 0 \in H_2(\tilde{W}, \partial \tilde{W}; \mathbb{Z}).$$
Using real coefficients, all cohomology groups have direct-sum decomposition into $\pm 1$ eigenspaces for $\tau^*$; thus $H^1(\partial \tilde{W}; \mathbb{R}) = V_+ \oplus V_-$. Since $\tau$ swaps $T_1$ and $T_2$ then, with obvious notation, it swaps $\mu_1$ with $\mu_2$ and $\lambda_1$ with $\lambda_2$. If $\epsilon = \pm 1$ then $V_\epsilon$ has basis $\{\mu_1 + \epsilon \mu_2, \lambda_1 + \epsilon \lambda_2\}$ and thus has dimension 2. Let

$$K = \text{Im} \left[ \text{incl}^*: H^1(\tilde{W}; \mathbb{R}) \to H^1(\partial \tilde{W}; \mathbb{R}) \right].$$

Decompose $K = K_+ \oplus K_-$. We claim that $\dim(K_+) = \dim(K_-) = 1$. Since $\dim(K) = 2$ the only other possibilities are that $K_+ = V_+$ or $K_- = V_-$. The intersection pairing on $\partial \tilde{W}$ is dual to the pairing on $H^1(\partial \tilde{W}, \mathbb{R})$ given by $\langle \phi, \psi \rangle = (\phi \cup \psi) \cap (\partial \tilde{W})$. This pairing vanishes on $K$. Since $\langle \mu_1 + \epsilon \mu_2, \lambda_1 + \epsilon \lambda_2 \rangle = 2 < \mu_1, \lambda_1 \rangle = \pm 2$, the restriction of $\langle , \rangle$ to each of $V_\pm$ is non-degenerate. This contradicts $K = V_\pm$.

Choose a primitive class $\phi \in H^1(\tilde{W}; \mathbb{Z})$ with $\text{incl}^* \phi \in K_-$. Let $S$ be an essential oriented surface in $\tilde{W}$ representing the class Poincaré dual to $\phi$. Then $\tau_*[S] = -[S]$ as required.

The 1–manifold $\alpha_i = T_i \cap \partial S$ with the induced orientation is a 1–cycle in $\partial \tilde{W}$. Then $[\partial S] = [\alpha_1] + [\alpha_2] \in H_1(\partial \tilde{W})$. Since $T_i$ is a torus all the components of $\alpha_i$ are parallel. Since $\tau(T_1) = T_2$ all components of $\partial S$ project to isotopic loops in $\partial W$ thus $S$ has invariant slope for the cover $\tilde{W} \to M$. This gives:

**Case (i)** If $S$ is not the fiber of a fibration of $\tilde{W}$ then the result follows from Theorem 3.

Thus we are left with the case that $S$ is the fiber of a fibration of $\tilde{W}$. Let $N$ be the $\mathbb{Z}_2$–universal covering of $W$. This is a regular covering and each component of $\partial N$ is a two-fold cover of $\partial W$. We claim that the composition of coverings $N \to W \to M$ is regular.

Recall that a subgroup $H < G$ is characteristic if it is preserved by $\text{Aut}(G)$. The $\mathbb{Z}_2$–universal covering $N \to W$ corresponds to the characteristic subgroup $\pi_1 N < \pi_1 W$. The cover $W \to M$ is cyclic and so $\pi_1 W$ is normal in $\pi_1 M$. A characteristic subgroup of a normal subgroup is normal. Hence $\pi_1 N$ is also normal in $\pi_1 M$. This proves the claim. It follows that $N \to M$ is a $\partial$–cover. A pre-image, $\tilde{S}$, of $S$ in $N$ is a fiber of a fibration.

**Case (ii)** Suppose the one-dimensional vector space of $H_2(N, \partial N; \mathbb{R})$ spanned by $[\tilde{S}]$ is invariant under the group of covering transformations of $N \to M$.

Then, by Theorem 1, $M$ is semi-fibered which contradicts our hypothesis. This completes case (ii). Therefore there is some covering transformation, $\sigma$, such that $\sigma_*[\tilde{S}] \neq \pm [\tilde{S}]$.  

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Because $\tilde{S}$ and $\sigma \tilde{S}$ are fibers, they both meet every boundary component of $N$. Since $S$ has invariant slope for the cover $N \to M$ it follows that $\tilde{S}$ and $\sigma \tilde{S}$ have the same invariant slope for this cover.

**Case (iii)** Suppose $S$ is a fiber and $[\partial \tilde{S}] \neq \pm \sigma_* [\partial \tilde{S}] \in H_1(\partial N)$.

Given a boundary component of $N$, there are integers $a$ and $b$ such that the class $a \tilde{S} + b \cdot \sigma_* S \in H_2(N, \partial N)$ is non-zero and represented by an essential surface $G$ that misses this boundary component. Thus $G$ is not a fiber of a fibration. Clearly $G$ has invariant slope. The result now follows from Theorem 3 applied to the surface $G$ in the $\partial$–cover $N$. This completes case (iii). The remaining case is:

**Case (iv)** $S$ is a fiber and there is $\epsilon \in \{ \pm 1 \}$ with $\sigma_* [\partial \tilde{S}] = \epsilon \cdot [\partial \tilde{S}] \in H_1(\partial N)$.

Consideration of the homology exact sequence for the pair $(N, \partial N)$ shows $x = \sigma_* [\tilde{S}] - \epsilon \cdot [\tilde{S}] \in H_2(N, \partial N)$ is the image of some $y \in H_2(N)$. Using exactness of the sequence again it follows that $y + i_* H_2(\partial N)$ is not zero in $H_2(N) / i_* H_2(\partial N)$. Hence every filling of $N$ produces a closed manifold with $\beta_2 > 0$. Infinitely many slopes on $\partial M$ lift to slopes on $\partial N$. The result follows. This completes the proof of case (iv) and thus of the Theorem 1.

**Proof of Corollary 1.2** Let $\eta(K)$ be an open tubular neighborhood of $k$. By hypothesis the knot exterior $M = N \setminus \eta(K)$ is irreducible. Every semibundle contains two disjoint compact surfaces whose union is non-separating, thus the first Betti number with mod-2 coefficients of a semi-bundle is at least 2. Because $N$ is a homology sphere $H_1(M; \mathbb{Z}_2) \cong \mathbb{Z}_2$, therefore $M$ is not a semi-bundle. Since $N$ is a homology sphere it, and therefore $M$, are orientable.

If $M$ is a bundle with fiber $F$ then, since $N$ is a homology sphere, $F$ has exactly one boundary component. Since $k$ does not bound a disk in $N$ it follows that $M \neq D^2 \times S^1$. The result now follows from Theorem 1. If $M$ contains a closed essential surface then infinitely many fillings are Haken, [3, Theorem 2.4.2]. The remaining possibilities are that $M$ is hyperbolic and not a bundle, or else Seifert fibered. The hyperbolic non-bundle case follows from [1].

This leaves the case that $M$ is Seifert fibered. The manifold $M$ is not a twisted $I$–bundle over the Klein bottle because the latter has mod-2 Betti number 2. The result now follows from Theorem 1.

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