Features of application of bimoment theory of V.Z. Vlasov in the calculations of reinforced concrete bar structures

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Abstract. In this paper, an adaptation variant of the “deformation model” is proposed for estimating the parameters of the stress-strain state of a reinforced concrete bar element with flexible discrete reinforcement, taking into account the deplanation of concrete and the features of contact interaction between reinforcement and concrete. Basic principles of the bimoment theory of solid rods of V.Z. Vlasov are used to assess concrete deplanation.

Keywords: bond stiffness, plane section hypothesis, cross section deplanation, bimoment.

Introduction
The basis of the modern approach in the calculations of reinforced concrete bar structures is the deformation model, the main provisions of which are formulated in SP 63.13330.2012 [1]. Static, physical and geometric groups of equations are presented in this document to solve practical problems of assessing the parameters of the stress-strain state (SSS) of reinforced concrete elements with discrete flexible reinforcement.

The choice of static equations - equilibrium conditions is determined by the nature of the problem under consideration and depends on the type of SSS element.

Physical equations are considered in the form of accepting $\sigma - \varepsilon$ diagrams for concrete and reinforcing elements. The choice of state diagrams for the components of a reinforced concrete bar depends on the considered loading modes of the structure, loading history, and other factors. These issues were widely considered in the works of N.I. Karpenko and his school [2,5]

The geometric relationships in the deformation model are presented in the form of accepting the hypothesis about the distribution of relative deformations of concrete along the height of the section of the element, as well as the conditions of mutual deformation for concrete and the reinforcing element. Bernoulli’s hypothesis (plane section hypothesis) is accepted as the law of distribution of relative deformations along the height of the section in the current set of rules [4,7]

The mutual deformation of concrete and reinforcement is taken into account in the form of a condition for joint deformation of concrete and reinforcement. This means that at all stages of deformation, the final increments of concrete and reinforcement at one point are $\Delta \varepsilon_b = \Delta \varepsilon_s$. In this approach, the stiffness of the contact zone between reinforcement and concrete (bond stiffness) $G_{\text{link}} \to \infty$. In real reinforced concrete elements, even with “secure bond”, the $G_{\text{link}}$ parameter has a finite value. This fact has been experimentally confirmed in numerous works of M.M. Kholmyansky and his followers [10]. Since the bond is flexible, when a reinforced concrete element is deformed, in the zone of direct contact of concrete with reinforcement, a mutual concentrated shift $g_s(x)$ occurs. Since the parameter $g_s(x) \neq \text{const}$, along the length of the element, sections arise with different growth rates of relative strains and, especially, stretched concrete $\varepsilon_{bt}(x)$. Places, where these strains have a maximum growth rate, determine the position of discrete cracks.
During the force deformation of bent reinforced concrete elements in sections with normal discrete cracks, the plane section hypothesis is violated [3,8,9]. The nature of the distribution of the relative deformations of concrete becomes substantially non-linear, which is confirmed by the research of V.V. Belova [2], V.M. Bondarenko [3], V.M. Travusha [11] and other researchers. To take into account concrete deplanation by section height, the author previously used an approach that introduces into the calculation a variable gradient of relative deformations [1]. This work develops the problem of accounting for cross section deplanation of reinforced concrete beams. Resolving relations are obtained using the basic principles of the bimoment theory of solid rods of V.Z. Vlasov [6].

To describe the process of deformation of a flexible reinforced concrete element, a composite bar model is used (Fig. 1). The reinforced concrete element with double reinforcement consists of 3 branches. The middle part is a concrete branch. In the upper zone of the concrete branch, there is a reinforcing branch modeling the compressed reinforcement $A_{s1}$, in the lower part, there is a reinforcing branch characterizing the stretched reinforcement $A_{s2}$. All branches are interconnected by absolutely rigid transverse bonds and malleable shear bonds.

![Cross section of a reinforced concrete element](image)

**Figure 1.** Cross section of a reinforced concrete element;

schemes for determining the relative deformations of the concrete branch, taking into account the occurrence of the bimoment $M_{bi}$; parameter $\varphi(z^*)$ for efforts in reinforcing branches: $\varphi_1(z^*)$ for $N_{s1}$, $\varphi_2(z^*)$ for $N_{s2}$

Since the discrete flexible reinforcement $A_{s1}$ and $A_{s2}$ is localized along the peripheral zones of the cross section, after the formation of normal cracks, accompanied by the occurrence of mutual displacements between the concrete and the reinforcing branches: $g_{s1}(x)$, $g_{s2}(x)$, relative deformation of concrete deplanation $\varepsilon^d_b(x, z)$.

Considering V.Z. Vlasov [4] proposal, the displacement of the points of the bar in the direction of the $X(X^*)$ axis at a flat bend in the $OYZ$ plane is presented in the following form:

$$u(x, z) = u_1(x) + u_2(x)z + u_3(x)\varphi(z^*)$$  \hspace{1cm} (1)

$u_1(x)$ is the translational displacement of the cross section of the bar along the $X$ axis;

$u_2(x)$ characterizes the angle of rotation of a flat cross section relative to the axis $Y^*$, according to the Bernoulli hypothesis;

$u_3(x)$ is the measure of a cross section deplanation of the bar;

$\varphi(z^*)$ is the deplanation characteristics. This parameter is taken depending on the type of cross-section of the bar and the nature of the problem under consideration, taking into account the requirements of orthogonality to the functions $f_1(z^*) = 1, f_2(z^*) = z^*$. 

$$\varepsilon^l_b(x, z) = \rho^{-1}z; \quad \varepsilon^d_b(x, z) = u_3(x)\varphi(z^*)$$
$z^*$ is the distance from the center of gravity of the cross section to the fiber under consideration (from the $Y^*$ axis).

For the considered problem, only two components of the deformation of the concrete branch of the composite bar are taken into account: longitudinal relative deformations $\varepsilon_b(x, z)$ in the direction of the $X$ axis and relative shear deformations $\gamma_{zx}(x, z)$ in the $OXZ$ plane:

$$\varepsilon_b(x, z) = \frac{\partial u(x, z)}{\partial x} = u'_1(x) + u'_2(x)z^* + u'_3(x)\varphi(z^*).$$  \hspace{1cm} (2)

In another representation, the value of the longitudinal relative deformation of the concrete branch can be written as:

$$\varepsilon_b(x, z) = \varepsilon_0(x) + \theta'(x)z^* + u'_3(x)\varphi(z^*) = \rho^{-1}(x)z + u'_3(x)\varphi(z^*).$$  \hspace{1cm} (3)

Thus, the total longitudinal relative deformations of the concrete branch $\varepsilon_b(x, z)$ are defined as linear $\varepsilon^l_b(x, z)$ and deformations caused by the cross section deplanation of the concrete branch $\varepsilon^d_b(x, z)$ (Fig. 1)

$$\varepsilon_b(x, z) = \varepsilon^l_b(x, z) + \varepsilon^d_b(x, z).$$  \hspace{1cm} (4)

$\varepsilon^l_b(x, z) = \rho^{-1}(x)z$ is the linear relative deformations of concrete, determined using the hypothesis of flat sections;

$\theta'(x) = \rho^{-1}$ is the increment along the length of the rod of the angle of rotation of the cross section relative to the axis $Y^*$ is numerically equal to the curvature.

$\rho^{-1}$ is the curvature;

$z$ is the distance from the neutral axis ($X$) to the fiber in question;

$\varepsilon^d_b(x, z) = u'_3(x)\varphi(z)$ is the relative deformations caused by the cross section deplanation of a concrete branch.

The elastic relative deformations of shift:

$$\gamma_{zx}(x, z) = \frac{dw(x)}{dx} + \frac{\partial u(x, z^*)}{\partial z^*} = \frac{dw}{dx} + u_2(x) + u_3(x)\frac{d\varphi}{dz^*}.$$  \hspace{1cm} (5)

$w$ is the bar displacement in the direction of the $Z$ axis.

To determine the concrete deplanation for the parameter $\varphi(z^*)$, a parabolic dependence is taken. Given that the cross section of the reinforced concrete element has one axis of symmetry, the deplanation characteristic is defined as:

$$\varphi(z^*) = c \left\{ 1 - H(z^*, 0) \left( 1 + \frac{m_0}{z_0} \right) \left( \frac{z^*}{z_0} \right)^2 - H(-z^*, 0) \left( 1 + \frac{m_0}{z_0} \right) \left( -\frac{z^*}{z_0} \right)^2 \right\}$$  \hspace{1cm} (6)

where $c <> 0$ is a scale factor that determines the nature of the plot $\varepsilon^d_b(x, z)$ according to the section height depending on the type of concrete deplanation (for a section with a crack and for the section of the bar between adjacent cracks), as well as the sign of the longitudinal force in the concrete branch. To simplify the results, we assume that $\varphi(0) = 1$, then $c = \pm 1$;

$m_0, z_0$ are the coordinates of the center of gravity of the concrete branch;

$H(z^*, 0)$ is the Heaviside function.

Expression (6) for the deplanation characteristic $\varphi(z^*)$ defines a self-balanced plot of additional deformations during the concrete deplanation $\varepsilon^d_b(x, z)$ and the corresponding additional stresses in the concrete $\sigma^d_b(x, z) = E_b\varepsilon^d_b(x, z)$, caused by cross section deplanation. This means that the development of deplanation in concrete does not change the value of the moment of internal forces in the section of the element.

The system of equations of static equilibrium conditions in a section with a crack under the action of a bending moment in a concrete branch of a section $M_{sb}$ and longitudinal force $N_{sb}$ is supplemented by an equation that takes into account the occurrence of the bimoment $M_{bi}(x)$. 

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\[ \begin{aligned}
\int_F \sigma_b dF &= N_{sb} \\
\int_F \sigma_b z dF &= M_{sb} \\
\int_F \sigma_b \varphi dF &= M_{bi}
\end{aligned} \tag{7} \]

\( \sigma_b \) is the normal stress in the concrete branch;
\( z \) is the distance from the fiber in question to the neutral axis.

The third equation of system (7), taking into account the conditions of orthogonality, is converted into the expression:

\[ M_{bi}(x) = A_\varphi u'(x), \tag{8} \]

which is used to determine the deplanation component of the relative deformations of concrete:

\[ \varepsilon_d(x, z^*) = u'(x)\varphi(z^*) = \frac{M_{bi}(x)}{A_\varphi} \varphi(z^*), \tag{9} \]

\( A_\varphi \) is generalized longitudinal stiffness associated with longitudinal deformations during section deplanation, which is determined:

\[ A_\varphi = \int_{-m_0}^{m_0} E_b \varphi^2(z^*) b(z^*) d z^* \tag{10} \]

\( A_\phi \) - with elastic work of concrete and the absence of cracks. For a rectangular section with dimensions \(- b \times h = F\), \( A_e = FE_b e_d\), for a rectangular section \(- e_d = 1\), and \( E_b \) is the initial modulus of elasticity of concrete.

\[ \frac{A_\varphi}{A_e} = \left( \frac{5 m_0 z_0}{h} \right)^{-1} \tag{11} \]

To determine the bimoment \( M_{bi}(x) \), we use the equation obtained in [4] by the variational method, in which the work of all external and internal forces in the cross section of the concrete branch on the possible displacement \( \varphi(z^*) \) is considered:

\[ \int_F \frac{d\sigma_x}{dx} \varphi(z^*) dF - \int_F \tau_{xx} \frac{d\varphi}{dz^*} dF = 0. \tag{12} \]

After transformations, taking into account that \( \tau_{xx} = G_b y_{xx} \) and conditions (8), we obtain the equation:

\[ M_{bi}''(x) - k^2 M_{bi}(x) = 0, \tag{13} \]

\( k^2 = C_\varphi/A_\varphi \).

where \( C_\varphi \) is the transverse generalized stiffness associated with longitudinal shear deformations in a concrete branch:

\[ C_\varphi = \int_{-m_0}^{m_0} G_b \left( \frac{d\varphi}{dz^*} \right)^2 b(z^*) d z^*. \tag{14} \]

For an element of rectangular cross section during elastic work of concrete for the section between cracks \( C_\varphi = C_{\varphi e} \):

\[ \frac{C_{\varphi e}}{G_b} = \frac{4 b}{3 h} \left( 1 + \frac{h}{z_o} \right)^2 \frac{h}{z_o} + \left( 1 + \frac{h}{m_0} \right)^2 \frac{h}{m_0} \tag{15} \]

\( G_b \) is the concrete shear modulus.

The generalized stiffnesses \( A_\varphi \) and \( C_\varphi \) in the cross section with a crack will have different values taking into account the normal crack depth \( \lambda \), and also taking into account the development of inelastic deformations of concrete.
The parameter $\lambda$ can be identified as the height of the cross section of the concrete branch, where the concrete is inelastically deformed or completely turned off from the resistance mode due to the separation of the crack faces. If we additionally introduce the parameters: $E_{bp}$, $G_{pp}$ – averaged value of the deformation modulus and concrete shear modulus in the range $-m_0 < z^* < h - \lambda$, then we can obtain expressions for the generalized stiffnesses $A_\varphi$ and $C_\varphi$ for the cross section with a crack:

$$A_\varphi = A_{\varphi e} - \Delta A_\varphi.$$  

If $\lambda \leq m_0$

$$\frac{\Delta A_\varphi}{A_e} = \left(1 - \frac{E_{bp}}{E_b}\right) \left(1 - \frac{2m_0}{3h} \left(1 - \frac{\lambda}{m_0}\right)^3\right) + \frac{m_0}{5h} \left(1 + \frac{h}{m_0}\right)^2 \left(1 - \left(1 - \frac{\lambda}{m_0}\right)^5\right). \quad (16)$$

If $\lambda > m_0$

$$\frac{\Delta A_\varphi}{A_e} = \left(1 - \frac{E_{bp}}{E_b}\right) \frac{m_0}{h} \left(1 - \frac{2}{3} \left(1 + \frac{h}{m_0}\right) + \frac{1}{5} \left(1 + \frac{h}{m_0}\right)^2\right) + \left(1 - \frac{E_{bp}}{E_b}\right) \frac{z_0}{h} \left(1 - \frac{h - \lambda}{z_0}\right)^2 \left(1 + \frac{h}{z_0}\right)^2 \left(1 - \left(1 - \frac{h - \lambda}{z_0}\right)^3\right). \quad (17)$$

Similarly for $C_\varphi$

$$C_\varphi = C_{\varphi e} - \Delta C_\varphi,$$

if $\lambda \leq m_0$

$$\frac{\Delta C_\varphi}{C_b} = \frac{4b}{3h} \frac{G_{bp}}{G_b} \left(1 + \frac{h}{m_0}\right)^2 \frac{h}{m_0} \left(1 - \left(1 - \frac{\lambda}{m_0}\right)^3\right), \quad (18)$$

if $\lambda > m_0$

$$\frac{\Delta C_\varphi}{G_b} = \frac{4b}{3h} \frac{G_{bp}}{G_b} \left(1 + \frac{h}{m_0}\right)^2 \frac{h}{m_0} + \left(1 + \frac{h}{z_0}\right)^2 \frac{h}{z_0} \left(1 - \frac{h - \lambda}{z_0}\right)^3. \quad (19)$$

The solution of equation (13) is taken in the form:

$$M_{bl}(x) = C_1 ch kx + C_2 sh kx. \quad (20)$$

A fragment of the bar is considered with a length equal to the distance between cracks $l_{cr,c}$. The origin of the $x$ coordinate is taken in the middle of the concrete block bounded by adjacent cracks. Boundary conditions are accepted:

when $x = -l_{cr,c}/2, M_{bl}(-l_{cr,c}/2) = N_{s2,1} \varphi_2(-(m_0 - a_2)) + N_{s1,1} \varphi_1(z_0 - a_1)$,

when $x = l_{cr,c}/2, M_{bl}(l_{cr,c}/2) = N_{s2,1} \varphi_2(-(m_0 - a_2)) + N_{s1,1} \varphi_1(z_0 - a_1)$,

where $N_{s1,1}(x,\varphi)$ are forces in reinforcing branches $A_{s1}(A_{s2})$ at the beginning and at the end of the block section between adjacent cracks. Since the force in the reinforcing branch is $A_{s1}$ compressive, and in $A_{s2}$ is tensile, then $\varphi_2(z^*) = -\varphi_1(z^*)$.

The expression for the bimoment is defined as:

$$M_{bl}(x) = \left(\frac{N_{s2,1} + N_{s2,1}}{2}\varphi(-(m_0 - a_2)) + \frac{N_{s1,1} + N_{s1,1}}{2}\varphi(z_0 - a_1)\right) \frac{ch kx}{ch l_{cr,c}} + \left(\frac{N_{s2,1} - N_{s2,1}}{2}\varphi(-(m_0 - a_2)) + \frac{N_{s1,1} - N_{s1,1}}{2}\varphi(z_0 - a_1)\right) \frac{sh kx}{sh l_{cr,c}}. \quad (21)$$
Figure 2. The stress-strain state in a cross section with a discrete crack

For the portion of the bar, where the bending moment from the external load varies slightly: $M(x) \cong \text{const}$

$$M_{bl}(x) = \{N_{s2} \varphi(-(m_0 - a_2)) + N_{s1} \varphi(z_0 - a_1)\} \frac{ch\;kx}{ch\;k \frac{l_{crc}}{2}}$$

The solution of the homogeneous equation (13) in the form of (21) can be used to estimate the bimoment in the sections along the length of the bar when $G_{link} \to 0$. These are areas directly adjacent to the crack, reinforced concrete elements with a discrete method of contact of reinforcement with concrete.

To evaluate the bimoment when $0 < G_{link} < \infty$, considering the section between cracks of length $l_{crc}$, we examine an inhomogeneous differential equation obtained from expression (12) taking into account the action of the distributed surface longitudinal load acting on the contact of the concrete branch from the influence of reinforcing branches $A_{s1}$ and $A_{s2}$

$$\int_{F} F \frac{d\varphi}{dz} \;dF - \int_{F} \tau_{xz} \frac{d\varphi}{dz} \;dF + \int X_n \varphi(z^*) \;ds = 0.$$ (23)

The third term of equation (23) represents the sum of the work of all external surface forces $X_n$ falling on the “elementary disk” of thickness $dx$. The integral is calculated by the coordinate $s$ for the entire contour bounding the cross section of this rod in the OYZ plane (Fig. 1.). The surface forces $X_n$, in this case, are represented by the tangential stresses acting on the contact of the reinforcing and concrete branches: for $A_{s1}$ and $A_{s2}$ the shear forces acting along the length of the corresponding contact zones. In the absence of initial prestressing in reinforcing bars $A_{s1}$ and $A_{s2}$ the shear forces $T_{s1}(x), T_{s2}(x)$ are identically equal to the forces in these reinforcing branches $N_{s1}(x), N_{s2}(x)$.

The differential equation for determining the bimoment for the sections between the cracks, taking into account the previously mentioned transformations, has the following form:

$$M_{bl}'(x) - k^2M_{bl}(x) = -T_{s1}''(x)\varphi(z_0 - a_{s1}) - T_{s2}''(x)\varphi(-(m_0 - a_{s2})).$$ (24)
In the process of the force deformation of a reinforced concrete bending element in the zones of contact interaction of the concrete branch with reinforcing branches $A_{s1}$ and $A_{s2}$ encountered shear stresses $T'_{s1}(x), T'_{s2}(x)$, define the mutual shifts $g_{s1}(x), g_{s2}(x)$. In expanded form, the system of differential equations for determining these parameters, taking into account discrete normal cracks (Fig. 2, 3), is as follows:

$$\begin{align*}
\left( T'_{s1}(x)G_{\text{link}1}^{-1}(x) \right)' &= \Delta_{11}(x)T_{s1}(x) + \Delta_{12}(x)T_{s2}(x) + \Delta_{10}(x) \\
\left( T'_{s2}(x)G_{\text{link}2}^{-1}(x) \right)' &= \Delta_{21}(x)T_{s1}(x) + \Delta_{22}(x)T_{s2}(x) + \Delta_{20}(x)
\end{align*}$$

Most often, the boundary conditions are written for the shear forces $T_s$:

$$T_{sj}(0) = T_{sj0}, \quad T_{sj}(L) = T_{sjL}, \quad j = 1, 2, L$$

is the bar length.

![Adapted deformation model for reinforced concrete bar element](image)

**Figure 3.** Adapted deformation model for reinforced concrete bar element taking into account the stiffness of adhesion and concrete deplanation

$G_{\text{link}1}, G_{\text{link}2}$ are stiffnesses of shear bonds between concrete and reinforcing branches $A_{s1}, A_{s2}$. With the elastic operation of all components of the reinforced concrete element, the bond stiffness for reinforcement located in the stretched zone is estimated as:

$$G_{\text{link}2}(m_0 - a_{s2})(bG_b)^{-1} + t_{\text{con}2}(u_{s2}G_{\text{con}2})^{-1};$$

(26)

(the formula for $G_{\text{link}1}$ has a similar form);

$G_{\text{con}2}$ is the deformation modulus in the zone of direct contact of reinforcement and concrete, determined on the basis of experimental data;

$u_{s2}$ is the total perimeter of the lateral surface of the reinforcing elements $A_{s2}$ in the stretched zone;

$t_{\text{con}2} = 2d_{s2}$ is the thickness of the conditional contact layer between reinforcement and concrete;
\( N = N_{s1} + N_{s2} \) is the longitudinal force arising in the concrete branch during deformation in the composition of the composite bar; in this case, in the absence of an external longitudinal force \( N_{s2} = T_{s2} \), \( N_{s1} = -T_{s1} \).

The functions \( F_1(N, \rho^{-1}) \), \( F_2(N, \rho^{-1}) \) take into account the nonlinearity of deformation of the concrete branch: the development of inelastic deformations in concrete, the presence of cracks, depend on the type of the initial diagram \( \sigma_b - \varepsilon_b \). These parameters are taken into account when considering the initial problem for obtaining a reinforced concrete bar, with a system of discrete cracks located with steps \( \{ l_{crc,i} \}, i = 1, 2 ... n_{crc} \), here \( n_{crc} \) is the number of cracks. To evaluate bimoment, it is assumed that \( F_1(N, \rho^{-1}) = 1, F_2(N, \rho^{-1}) = 1 \).

When using the linear transformation of free terms \( \Delta_{10}, \Delta_{20} \) in the form of a method of decomposing an external load into eigenfunctions of the main system of equations of a compound bar, presented in A.R. Rzhanitsyna [6], the original system of differential equations (25) splits into two independent linear inhomogeneous second-order differential equations. Together with the boundary conditions for the reinforcing branches \( A_{s1}, A_{s2} \), which are also written concerning the shear forces \( T_{si}, i = 1, 2 \), two boundary value problems are solved.

The determination of bimoment in the presence of double reinforcement is the most common case for real reinforced concrete elements. When the adhesion and concrete work in elastic between the cracks, the solution of the system of equations (25) can be obtained in a closed form. This is of interest for analyzing the obtained result and establishing the degree of influence of various factors on the level of cross section deplanation:

\[
M_{bi}(x) = M_{bic} \frac{ch \frac{kx}{l_{crc}}}{ch \frac{l_{crc}}{2}} - D_1 \left( \frac{ch \frac{kx}{l_{crc}}}{ch \frac{l_{crc}}{2}} - \frac{ch \frac{kx}{l_{crc}}}{ch \frac{l_{crc}}{2}} \right) - D_2 \left( \frac{ch \frac{kx}{l_{crc}}}{ch \frac{l_{crc}}{2}} - \frac{ch \frac{kx}{l_{crc}}}{ch \frac{l_{crc}}{2}} \right),
\]

(27)

Where \( M_{bic} = T_{s1,c} \varphi(z_0 - a_{s1}) + T_{s2,c} \varphi(-(m_0 - a_{s2})) \) is the value of the bimoment in the section with a crack.

The shear forces in the section with a crack \( T_{s1,c} = N_{s1,c} \), \( T_{s2,c} = N_{s2,c} \) are defined as the forces in the reinforcing branches in the section with a crack

\[
\lambda_2^2 = \frac{G_{link1,1} \Delta_{11} + G_{link2,2} \Delta_{22}}{2} \pm \sqrt{\left( \frac{G_{link1,1} \Delta_{11} - G_{link2,2} \Delta_{22}}{2} \right)^2 + G_{link1,1} G_{link2,2} \Delta_{12}^2},
\]

(28)

\[
D_1 = \left( \frac{G_{link1,1} \cos \alpha \varphi(x_0 - a_{s1}) + G_{link2,2} \sin \alpha \varphi(-(m_0 - a_{s2}))(T_{1c} \lambda_2^2 + R_1)}{k^2 - \lambda_1^2} \right),
\]

(29)

\[
D_2 = \left( \frac{G_{link2,2} \cos \alpha \varphi(-(m_0 - a_{s2})) - G_{link1,1} \sin \alpha \varphi(x_0 - a_{s1}))(T_{2c} \lambda_2^2 + R_2)}{k^2 - \lambda_1^2} \right),
\]

(30)

\[
T_{1c} = \frac{T_{1c} \sin \alpha}{\sqrt{G_{link2}}}, \quad T_{2c} = \frac{T_{2c} \cos \alpha}{\sqrt{G_{link1}}} - \frac{T_{1c} \sin \alpha}{\sqrt{G_{link1}}},
\]

(31)

\[
R_1 = \sqrt{G_{link1}} (\Delta_{10} \cos \alpha - \Delta_{20} \sin \alpha), \quad R_2 = \sqrt{G_{link2}} (\Delta_{10} \sin \alpha + \Delta_{20} \cos \alpha).
\]

(32)

\[
tg \alpha = \frac{G_{link1} G_{link2} \Delta_{12}}{\lambda_1^2 - G_{link2,2} \Delta_{22}}, \quad \cos \alpha = (1 + t g^2 \alpha)^{-1/2},
\]

(33)

The results (28), (31), (32), (33) are presented in [6].

The increment of mutual shifts along the length of the bar for a stretched reinforcing branch \( A_{s2} \) with elastic coupling work is written as:

\[
g_{s2}(x) = \varepsilon_{s2}(x) - \varepsilon_{bts}(x) = (T_{s2} \varphi) G_{link1,2}^{-1} T'_{link2} G_{link2,2}^{-1} < 0.
\]

(34)

In the section with a crack: \( g_{s2}'(x) = \varepsilon_{s2}(x) - \varepsilon_{bts}(x) < 0 \) : \( T''_{s2} < 0 \). In the interval between the cracks: \( g_{s2}(x) > 0 \) : \( T''_{s2} > 0 \). In addition, for real reinforced concrete bending elements: \( g_{s2}(x) > 0 \)
This means that, taking into account the change in the sign of $g'_{s2}(x)$, the deplanation deformation diagram $\varepsilon_{b}^{d}(x,z)$ and, accordingly, the diagram of the relative concrete deformation $\varepsilon_{b}(x,z)$ also change depending on the location of the section along relative to the nearest crack.

**Figure 4.** A characteristic view of the diagram of the relative deformations of concrete (above - for a section with a crack, below - for a section in the middle, between - normal cracks)
Using the described approach, the nature of the distribution of relative concrete deformations for a reinforced concrete element of rectangular cross section with double reinforcement is estimated. Cross-sectional dimensions: h\times b=200\times150, symmetrical reinforcement: A_{s1} = A_{s2} = 2012 \text{A400} \ a_{s1} = a_{s2} =30. Concrete characteristics: R_b = 20 \text{MPa}, R_{bt} = 2 \text{MPa}, E_b = 2.7\cdot104 \text{MPa}, v_{bt} = 1. The calculation results are shown in Fig. 4.

An analysis of preliminary verification calculations shows that the phenomenon of concrete deplanation is manifested as a factor in the adaptation of a mechanical system — a reinforced concrete element under stress deformation. For a cross section with a crack, deplanation in the compressed zone reduces the gradient of relative deformation. A similar effect - a decrease in relative deformations is manifested for the cross section between cracks, but the concrete of the stretched zone. This fact has experimental evidence - as the effect of stabilization of the cracking process at a certain level of loading of a reinforced concrete beam, when new cracks are no longer formed, and only the already formed cracks open.

The calculation of the reinforced concrete bar element is carried out in the process of virtual deformation during step loading. Concrete deplanation is accounted for after cracks are formed. When solving the system of equations (25), the bimoment value is taken from the previous calculation step.

The development of inelastic deformations in stretched concrete in the areas between the cracks can be taken into account by introducing a different modulus of concrete. For this, the tensile modulus of concrete under tension $E_{bt} < E_{bc}$ is assumed to be averaged over the height of the stretched zone, and in the initial dependencies, it is necessary to adjust the parameters: $B_e, A_e, m_0, z_0$.

Bimoment $M_{bi}(x)$ is the edge load applied in cross sections with a crack; therefore, its maximum effect is manifested in areas adjacent to the crack. For a bent element with a rectangular cross section having a cross section dimension = (0.25 ... 0.3)h, taking into account that $G_p = 0.42E_b$, we can obtain an estimate when the value of the bimoment decreases by an order of magnitude $M_{bi}(x) = 0.1M_{bi}(x_{crc})$. Taking into account the bimoment and, correspondingly, concrete deplanation will be relevant if the crack spacing $l_{crc} \leq (0.7 ... 0.92)h$, that is, when $l_{crc} < h$, which occurs when deforming beams with secured adhesion under operating loads. If the crack spacing $l_{crc} > h$, which occurs at the initial stage of crack formation for beams with provided adhesion, as well as for elements with “weakened adhesion”, account for cross section deplanation should be performed for sections with cracks and for sections between cracks located at a distance not further 0.35h from the crack section.

Conclusions
1. Concrete deplanation causes a change like the distribution of relative deformations along the height of the section, without changing the magnitude of the moment of internal forces.
2. The most important consideration is the possible concrete deplanation in a section with a crack for an adequate estimate of the maximum relative deformations of compressed concrete when predicting the scenario of a subsequent failure of a reinforced concrete structure.
3. Changes in the values of the relative deformations of concrete $\varepsilon_b(z = x - a_{s1})$ and $\varepsilon_{bt}(z = -(m - a_{s2}))$ affect the relative deformations in the reinforcing branches: $\varepsilon_{s1}$ and $\varepsilon_{s2}$, i.e., the forces $N_{s1}$ and $N_{s2}$.

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