Disorder effects during the evolution from BCS to BEC superfluidity

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We describe the effects of disorder on the critical temperature of s-wave superfluids from the BCS to the BEC regime, with direct application to ultracold Fermi atoms. In the BCS regime the pairing breaking and phase coherence temperature scales are essentially the same allowing strong correlations between the amplitude and phase of the order parameter. As non-pair breaking disorder is introduced the rapidly overlapping Cooper pairs conspire to maintain phase coherence such that the critical temperature remains essentially unchanged. However, in the BEC regime the pairing breaking and phase coherence temperature scales are different such that non-pair breaking disorder can affect dramatically phase coherence, and thus the critical temperature, without the requirement of breaking tightly-bound fermion pairs simultaneously. Finally, we find that the superfluid is more robust against weak disorder in the intermediate region between the two regimes.

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Ultracold atoms are special systems for studying superfluid phases of fermions or bosons at very low temperatures, because of unprecedented tunability. In particular, ultracold fermions with tunable interactions were used to study experimentally the so-called BCS-to-BEC evolution, and population imbalanced systems. In addition, there are other interesting directions to be pursued, including studies of the BCS-to-BEC evolution in optical lattices [1, 2], and the effects of disorder during the BCS-to-BEC evolution, which would allow the very important study of the simultaneous effects of interactions and disorder at zero [3] and finite temperatures [1].

In ordinary condensed matter (CM) systems the control of interactions is not possible, and the control of disorder is very limited, because the disorder potential is not known and can not be changed at the turn of a knob. Thus, in standard CM the disorder is usually described in terms of defects or impurities, whose positions in the solid are assumed to be random. In ultracold atoms it is now possible to create controlled disorder using laser speckles or lasers with incommensurate wavelengths, which were used to study the phenomenon of Anderson localization in ultracold Bose atoms [3, 4], but that could also be used to study disorder effects in ultracold fermions.

Thus, here, we describe the finite temperature phase diagram of three dimensional (3D) s-wave Fermi superfluids from the BCS to the BEC limit as a function of disorder, which is independent of the hyperfine states of the atoms and is created by a Gaussian-correlated laser speckle potential. Our main results are as follows. First, in the BCS limit the amplitude and phase of the order parameter are strongly coupled, such that pair breaking and loss of phase coherence occur simultaneously. In this case, the critical temperature is essentially unaffected by weak disorder, since the disorder potential is not pair-breaking and phase coherence is not easily destroyed in accordance with Anderson’s theorem [2]. Second, in the BEC limit the breaking of local pairs and the loss of phase coherence occur at very different temperature scales. In this case, the critical temperature is strongly affected by weak disorder, since phase coherence is more easily destroyed without the need to break local pairs simultaneously, and Anderson’s theorem does not apply. Third, we find that superfluidity is more robust to disorder in the intermediate region between the BCS and BEC regimes.

To investigate the physics described above, we start with the real space Hamiltonian (\(\hbar = 1\)) density for three dimensional s-wave superfluids

\[
\mathcal{H}(\mathbf{x}) = \sum_{\sigma} \left( -\frac{\nabla^2}{2m} - \mu + V_{\text{dis}}(\mathbf{x}) \right) \psi_\sigma^\dagger(\mathbf{x}) \psi_\sigma(\mathbf{x}) + \hat{U}(\mathbf{x}), \tag{1}
\]

where \(\hat{U}(\mathbf{x}) = + \int d\mathbf{x}' V(\mathbf{x}, \mathbf{x}') \psi_\uparrow^\dagger(\mathbf{x}') \psi_\downarrow^\dagger(\mathbf{x}') \psi_\downarrow(\mathbf{x}) \psi_\uparrow(\mathbf{x})\) represents a term containing the interaction potential \(V(\mathbf{x}, \mathbf{x}') = -g \delta(\mathbf{x} - \mathbf{x}')\), and \(\psi_\sigma^\dagger(\mathbf{x})\) represents the creation of fermions with mass \(m\) and hyperfine state (spin) \(\sigma\). In addition, \(V_{\text{dis}}(\mathbf{x})\) represents the disorder potential, and \(\mu\), the chemical potential. We choose \(V_{\text{dis}}(\mathbf{x})\) to be independent of the hyperfine state, a choice that can be easily relaxed.

Although there are many ways to introduce disorder, we will make a particular choice that the disorder potential is governed by a Gaussian distribution \(P[V_{\text{dis}}] = \exp[-\int d\mathbf{x} V_{\text{dis}}^2(\mathbf{x})/(2\kappa)]/A\), with normalization constant \(A = \int D[V_{\text{dis}}] \exp[-\int d\mathbf{x} V_{\text{dis}}^2(\mathbf{x})/(2\kappa)]\), which leads to the configurational average \(\langle V_{\text{dis}}(\mathbf{x}) V_{\text{dis}}(\mathbf{x}')\rangle = \kappa \delta(\mathbf{x} - \mathbf{x}')\).

To derive the effective action for a fixed configuration of disorder, we define the local chemical potential \(\mu(\mathbf{x}) = \mu - V_{\text{dis}}(\mathbf{x})\) and follow the functional integral formulation of the evolution from BCS to BEC superfluidity [3] to obtain \(S_0 + S_G\), where \(S_0\) is the action of unbound fermions in the presence of weak disorder and is given by

\[
S_0[V_{\text{dis}}] = -\frac{2}{V} \int d\mathbf{x} \sum_k \ln \left[ 1 + e^{-\xi(\mathbf{k}, \mathbf{x})/\ell} \right], \tag{2}
\]

with \(\xi(\mathbf{k}, \mathbf{x}) = \epsilon_k - \mu(\mathbf{x})\), and \(\epsilon_k = k^2/2m\). The second contribution to \(S_\text{eff}\) corresponds to Gaussian pairing
fluctuations

\[ S_{G}[V_{\text{dis}}] = \frac{1}{TV} \int dx \sum_{q} \Delta(q) \Gamma^{-1}(q, \text{dis}) \Delta(q), \tag{3} \]

where \( \Delta(q) \) is the pairing field, and

\[ \Gamma^{-1}(q, \text{dis}) = \sum_{k} \frac{X_{1} + X_{2}}{2[(iq_{\ell} - \xi_{1} - \xi_{2} - 2V_{\text{dis}}(x))] + C} \tag{4} \]

is the pair correlation function in the presence of disorder, where \( C = -(mV/4\pi a_{s}) + \sum_{k}(2\xi_{k})^{-1} \), \( q = (q, iq_{\ell}) \) represents the four-momentum, the function \( X_{1} = \tanh [(\xi_{1} + V_{\text{dis}}(x))/2T] \) describes the occupation of fermions with energy \( \xi_{1} = \xi(k - q/2) \), and the function \( X_{2} = \tanh [(\xi_{2} + V_{\text{dis}}(x))/2T] \) describes the occupation of fermions with energy \( \xi_{2} = \xi(k + q/2) \).

The description above corresponds to a semiclassical approximation which is valid for weak disorder potentials. However, quenched (as opposed to annealed) disorder is characterized by a static disorder potential and the necessity to average the thermodynamic potential \( \Omega(V_{\text{dis}}) \) rather than the partition function \( \mathcal{Z} \). The configurationally averaged thermodynamic potential \( \Omega_{\text{av}}(\kappa) = \langle \Omega(V_{\text{dis}}) \rangle \), and depends on disorder via the parameter \( \kappa \). The thermodynamic potential for a fixed configuration is \( \Omega(V_{\text{dis}}) = -T \ln Z(V_{\text{dis}}) \), where \( Z(V_{\text{dis}}) \) is defined in terms of the partition function \( Z(V_{\text{dis}}) = Z_{0}(V_{\text{dis}}) \times Z_{G}(V_{\text{dis}}) \), where \( Z_{0}(V_{\text{dis}}) = \exp[-S_{0}(V_{\text{dis}})] \) is the partition function for unbound fermions, and \( Z_{G}(V_{\text{dis}}) = \int D[\Psi, \bar{\Psi}] \exp[-S_{G}(\Psi, \bar{\Psi}, \text{dis})] \) is the partition function for the pairing field. Thus, \( \Omega(V_{\text{dis}}) = \Omega_{0}(V_{\text{dis}}) + \Omega_{G}(V_{\text{dis}}) \), where the unbound fermion thermodynamic potential is \( \Omega_{0}(V_{\text{dis}}) = -T \ln Z_{0}(V_{\text{dis}}) = TS_{0}(V_{\text{dis}}) \) while the pairing field contribution is \( \Omega_{G}(V_{\text{dis}}) = -T \ln Z_{G}(V_{\text{dis}}) \), which is approximated by

\[ \Omega_{G}(V_{\text{dis}}) = -\frac{T}{V} \int dx \sum_{q} \ln |T \Gamma(q, V_{\text{dis}}(x))|. \tag{5} \]

Expanding in \( V_{\text{dis}} \) and taking the configurational average leads to \( \Omega_{\text{av}}(\kappa) = T \langle S_{0}[V_{\text{dis}}] \rangle = \Omega_{\text{av}}(0) + \Delta \Omega_{\text{av}}(\kappa) \), for the thermodynamic potential of unbound fermions, and \( \Omega_{G,\text{av}}(\kappa) = \Omega_{G,\text{av}}(0) + \Delta \Omega_{G,\text{av}}(\kappa) \) for the thermodynamic potential due to the pairing field. Here, \( \Omega_{\text{av}}(0) \) and \( \Omega_{G,\text{av}}(0) \) are just the thermodynamic potentials without disorder \( \mathcal{Z} \), while the disorder-dependent corrections to the thermodynamic potential are \( \Delta \Omega_{\text{av}}(\kappa) = -T \sum_{k} L(k) \kappa, \) with \( L(k) = 1/[2T^{2}(1 + \cosh(\xi_{k}/T))] \), and \( \Delta \Omega_{G,\text{av}}(\kappa) = -T \sum_{q} M(q) \kappa \) with \( M(q) = P(q) - |Q(q)|^{2}/2 \). The first term of \( M(q) \) can be expressed as

\[ \sum_{n=0}^{\infty} d_{n} [X_{1}^{n} + X_{2}^{n}] \Gamma_{0}(q)/V \] where the coefficients are \( d_{0} = -d_{2} = 1/(4T^{2}) \), \( d_{1} = 1/(8T^{2}z^{3}) \), and \( c_{3} = 1/(8T^{2}z^{3}) \), with \( z = iq_{\ell} - \xi_{1} - \xi_{2} \). The second term is \( Q(q) = -\sum_{n=0}^{\infty} d_{n} [X_{1}^{n} + X_{2}^{n}] \Gamma_{0}(q)/V \), where \( d_{0} = -d_{2} = 1/(4T^{2}) \), \( d_{1} = 1/(8T^{2}z^{3}) \), and \( c_{3} = 1/(8T^{2}z^{3}) \), with \( z = iq_{\ell} - \xi_{1} - \xi_{2} \). The second term is \( Q(q) = \frac{\pi}{2} \), where \( N_{\text{av}}(\kappa) = \partial \Omega_{\text{av}}(\kappa)/\partial \mu \) being due to unbound fermions, and \( N_{G}(\kappa) = \partial \Omega_{G,\text{av}}(\kappa)/\partial \mu \) being due to pairing. In the limit of \( \kappa \to 0 \), the standard results for the thermodynamic potential, number equation, and time-dependent Ginzburg-Landau theory are found \( \mathcal{Z} \). However, in the presence of disorder and in the vicinity of \( T_{c} \), the low-energy and long-wavelength Lagrangian density \( \mathcal{L}_{G} = \mathcal{L}_{ND} + \mathcal{L}_{D} \) derived from the action of Eqs. (4) and (5) has two contributions. The first is a non-dissipative part

\[ \mathcal{L}_{D}(x, \tau) = \Psi \left( \partial_{\tau} - \frac{\nabla^{2}}{2m_{*}} - \mu_{*} + \gamma V_{\text{dis}}(x) \right) \Psi + \frac{\gamma_{*}^2}{2} |\Psi|^{4}, \]

where \( \Psi = \Psi(x, \tau) \). Here, the term containing the effective mass \( m_{*} \) is the kinetic energy of the pairing field, \( \mu_{*} \) plays the role of the pairing field chemical potential, \( g_{*} \) is the effective interaction, and \( \gamma V_{\text{dis}}(x) \) is the effective disorder potential. The second term is the dissipative contribution reflecting the decay of fermion pairs into unbound fermions, which takes the Caldeira-Leggett form

\[ \mathcal{L}_{D}(x, \tau) = \frac{\lambda}{2 \pi} \int d\tau' \left| \frac{\Psi(x, \tau) - \Psi(x, \tau')}{(\tau - \tau')^{2}} \right|. \]

Since we are considering the case of weak disorder, these parameters can be easily related to original parameters of the theory without disorder \( \mathcal{Z} \). Using the notation \( X = \tanh(\xi_{k}/2T) \) and \( Y = \text{sech}^{2}(\xi_{k}/2T) \), the original coefficients are \( a(\mu, T) = -\frac{m_{*}^{2}}{4\pi a_{s}} + \sum_{k} \left[ \frac{\xi_{k}}{m_{*}^{2}} - \frac{\xi}{2\pi a_{s}} \right] \), corresponding to the order parameter equation in the absence of disorder when \( a(\mu, T) = 0 \),

\[ c(\mu, T) = \sum_{k} \left[ \frac{\chi_{k}^{4}}{8\pi^{2}} - \frac{Y}{16\pi k^{2}T^{2}} + \frac{XY}{16m_{*}^{2}k^{2}} \right], \]

d and \( d(\mu, T) = d_{R} + id_{I} \), with \( d_{R} = \sum_{k} X_{k}^{2}/4\pi^{2} \) and \( d_{I} = \sum_{k} X_{k}^{4}/4\pi k^{2} \), the dimensionless amplitude of the disorder potential is \( \gamma = -\frac{d_{R}d_{I}}{\partial \Omega_{\text{av}}/\partial \mu} \), and \( \lambda = d_{I}/d_{R} \). A natural choice of dimensionless parameters are \( 1/(k_{F} a_{s}) \) for interactions, and \( \eta = \kappa n_{F}/\epsilon_{F}^{2} \) for disorder, and \( T = T_{c}/T_{F} \) for temperature, where \( k_{F} \) is the Fermi momentum, \( n_{F} = k_{F}^{2}/3\pi^{2} \) is the fermion density and \( \epsilon_{F} = k_{F}^{2}/2m \) is the Fermi energy.
Notice that the condition
\[ \mu_s(T, 1/(k_F a_s)) = 0 \tag{7} \]
corresponds precisely to the order parameter equation in the absence of disorder \( a(\mu, T) = 0 \), indicating that this equation is not explicitly affected by weak disorder, as required by Anderson’s theorem \([7]\). However, the determination of the critical temperature \( T_c(\eta) \) and the chemical potential \( \mu_c(\eta) \) as a function of dimensionless disorder \( \eta \) for a fixed scattering parameter \( 1/(k_F a_s) \) requires the simultaneous solutions of the number Eq. \([8]\), and the order parameter Eq. \([7]\).

\[
\begin{align*}
\text{FIG. 1: The pairing temperature } T_p \text{ (dashed line) and the critical temperature } T_c \text{ (solid line) for uniform superfluidity in units of } \epsilon_F \text{ as a function of disorder parameter } \eta \text{ on a) the BCS side for } 1/k_F a_s = -1.8 \text{ and on b) the BEC side for } 1/k_F a_s = 1.2. \text{ The dotted line in b) corresponds to the temperature where the superfluid density vanishes.}
\end{align*}
\]

In the BCS limit, \( \gamma \rightarrow 2 \) indicating that the pairing field feels an effective disorder potential twice as large as that felt by individual unbound fermions, while \( \lambda = \pi/(2T_c) \gg 1 \) indicating that the dynamics of the pairing field is highly overdamped due to the decay of fermion pairs into unbound fermions. In the BCS limit without disorder the order parameter equation alone \( \mu_s = 0 \) (or \( a = 0 \)) determines the critical temperature \([8]\). As seen above this equation remains unchanged in the presence of weak disorder. Thus, the critical temperature is essentially unchanged, because the fermion chemical potential \( \mu \) is very large, positive, and remains pinned to \( \epsilon_F \), in the case of perfect particle-hole symmetry. This is a manifestation of Anderson’s theorem, and changes in the critical temperature can occur only via the disorder dependent shift in the chemical potential, which leads to a change in the single particle density of states.

The number equation in the BCS limit is approximated by \( N \approx N_0(0) + \Delta N_0(\eta) \). The first coefficient represents the contribution in the absence of disorder \( N_0(0) = \sum \delta(1 - X) \), while the second contribution involves the effects of disorder \( \Delta N_0(\eta) = \eta \sum \delta(1 - X) \). In the case of perfect particle-hole symmetry \( \Delta N_0(\eta) = 0 \), and indeed \( \mu = \epsilon_F \). Thus, strictly in the BCS limit \( 1/(k_F a_s) \rightarrow -\infty \), where the assumption of perfect particle-hole symmetry is made, the chemical potential is pinned to the Fermi energy \( \mu = \epsilon_F \) and the critical temperature is unchanged \( T_c = (\epsilon_F/\pi)e^{-2}\exp(-\pi/2k_F |a_s|) \). Since the presence of disorder does not alter the total density of fermions \( n_F = k_F^2/3\pi^2 \), relaxing the condition of perfect particle-hole symmetry makes the chemical potential adjust itself to \( \mu = \epsilon_F - \Delta \mu(\eta) \), with \( \Delta \mu(\eta) > 0 \) leading to a corresponding reduction in the critical temperature \( T_c(\eta) = T_c(0) - \Delta T_c(\eta) \), with \( \Delta T_c(\eta) > 0 \). In Fig. 1, the behavior of the pairing temperature \( T_p(\eta) \) and the critical temperature \( T_c(\eta) \) are shown in the BCS regime as a function of \( \eta \) for \( 1/k_F a_s = -1.8 \). These curves are obtained by solving the corresponding number and order parameter equations for varying \( \eta \).

As the attraction is increased towards unitarity and \( \mu = 0 \), the relative change in \( T_c \) for fixed disorder parameter \( \eta \), expressed as \( \Delta T_c/T_c(0) = [1 - T_c(\eta)/T_c(0)] \), decreases as the attraction strength is increased, and reaches a minimum as indicated in Fig. 2. Beyond this minimum \( \Delta T_c/T_c(0) \) increases again, being the largest in the BEC regime. These results indicate that superfluidity is more robust to disorder in the intermediate regime of \( -1 < 1/(k_F a_s) < 1 \), where the coherence length reaches a minimum \([8]\) (or the critical current reaches a maximum). However, the relative change \( \Delta T_p/T_p(0) = [1 - T_p(\eta)/T_p(0)] \) for fixed disorder always decreases with \( 1/(k_F a_s) \) indicating that it becomes increasingly more difficult to break pairs with larger binding energy as the BEC regime is approached. As discussed next, the difference in behavior between \( \Delta T_c/T_c(0) \) and \( \Delta T_p/T_p(0) \) in the BEC regime is attributed to temporal phase fluctuations, which become increasingly more important as \( 1/(k_F a_s) \) increases.

\[
\begin{align*}
\text{FIG. 2: The relative change } \Delta T_c/T_c(0) \text{ (solid line) and } \Delta T_p/T_p(0) \text{ (dashed line) throughout the BCS to BEC evolution for } \eta = 0.1. \text{ Notice the minimum in } \Delta T_c/T_c(0) \text{ occurring in the intermediate region } -1 < 1/k_F a_s < 1 \text{ indicating that the superfluid is more robust to disorder in this region, while the } \Delta T_p/T_p(0) \text{ always decreases indicating that the pairing temperature becomes less sensitive to disorder in the BEC regime. The dotted line corresponds to the analytical result in the BEC limit.}
\end{align*}
\]

In the BEC regime, \( \gamma \rightarrow 2 \) indicating that the pairing field feels an effective disorder potential twice as large as that felt by individual unbound fermions, but \( \lambda \rightarrow 0 \) indi-
cating that the dynamics of the pairing field is undamped, reflecting that tightly-bound fermion pairs have very long lifetimes. In the BEC regime \((1/k_F a_s \gg 1)\) the number equation is dominated by the contributions coming from \(N_G \gg N_0\) given that all fermions are paired into molecular bosons. Here, \(N_G\) is well approximated by the number of bound fermions (molecular bosons) \(N_B\), since the number of scattering states \(N_{SC}\) is very small. The configurational average can be performed using the replica technique \([9, 10]\) on the Lagrangian density \(\mathcal{L}_G\), leading to the action \(S_G = \langle 1/T \rangle \sum_q \bar{b}(q) G^{-1}(q) b(q)\), with \(G^{-1}(q) = \frac{\gamma^2}{2m^*} - \mu_s - i\eta c - \Sigma(q)\), where the self-energy is

\[
\Sigma(q) = g_s n_B + \gamma^2 \kappa \sum_{|k|<|k_c|} \frac{m_s}{k^2 V} - \frac{\gamma^2 \kappa}{4\pi} \frac{2m^* \sqrt{\vert \mu_s \vert} - i\eta c}{|\mu_s| - i\eta c}.
\]

Here, \(G(q)\) plays the role of the molecular boson propagator and leads to the thermodynamic potential \(\Omega_{c,s} = -2T \sum_q \ln [G^{-1}(q)/T]\) with total number of fermions \(N \approx 2N_B\), where \(N_B = T \sum_q G_{av}(q)\), and renormalized chemical potential \(\mu_{s*} = \mu_s + g_s n_B + \gamma^2 \kappa \sum_{|k|<|k_c|} m_s/k^2 V\). Upon summation over Matsubara frequencies \(i\eta c\), we arrive to the boson density \(n_B = N_B/V\) corrected by disorder

\[
n_B = \zeta (3/2) (m_s T_c / 2\pi)^{3/2} + \gamma^2 \kappa T_c m_s^2 / 4\pi^2, \quad (8)
\]

when \(\mu_{s*} = 0\). The solution for \(T_c\) can be obtained numerically, but an analytical solution is possible when \(\eta = \kappa T_c / \gamma^2 \ll 1\). This leads to \(T_c = T_c(0) \left[ 1 - \gamma^2 \kappa T_c(0) m_s^3 / 6\pi^2 n_B \right]\), where \(T_c(0) = 2\pi n_B / \zeta (3/2)^{2/3} / m_s\) is the Bose-Einstein condensation temperature for a gas of molecular bosons, which becomes \(T_c(0) = 0.218\kappa T_c\), when \(m_s / 2m\) and \(n_B / n_F / 2\).

Unlike in the BCS regime, the critical temperature \(T_c\) is directly affected by the presence of disorder in the BEC regime, and is determined essentially by the number equation. A small amount of disorder affects the phase coherence of the molecular bosons through the emergence of an incoherent part in the molecular boson Green’s function \(G^{-1}(q)\) manifested by the branch cut \(\sqrt{\vert \mu_s \vert} - i\eta c\) present in the self-energy \(\Sigma(q)\). If only the zero frequency \(i\eta c = 0\) contribution were considered, one would be led to the conclusion that disorder would not affect the critical temperature in the BEC regime. However, disorder introduces quantum (temporal) phase fluctuations in the BEC regime, where fermions are largely non-degenerate, particle-hole symmetry is absent, and Anderson’s theorem does not apply, thus leading to a strong suppression of \(T_c\).

The critical temperature calculated corresponds to the transition between the superfluid and a non-uniform phase containing local superfluid islands separated from each other by peaks of the disorder potential. The transition from the local superfluid regime to the normal phase may be determined by imposing the condition that the superfluid density \(\rho(\bar{T}, \eta) = 0\), which leads to the schematic phase diagram in Fig. 3.

We analyzed the effects of disorder on the critical temperature for superfluidity of ultracold fermions during the evolution from the BCS to the BEC regime. For s-wave superfluids, we showed that weak disorder does not affect the critical temperature of a BCS superfluid with perfect particle-hole symmetry in accordance with Anderson’s theorem, as the breaking of fermion pairs and the loss of phase coherence must occur at the same temperature. However in the BEC regime, phase coherence is more easily destroyed by disorder without the need of simultaneously breaking fermion pairs. Finally, we also showed that the superfluid is more robust to the presence of weak disorder in the intermediate (crossover) region.

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