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LOW-ENERGY MAGNETIC RADIATION ENHANCEMENT WITHIN THE NUCLEAR SHELL MODEL

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Abstract.

The $\gamma$-ray strength function, the average reduced probability of absorbing or emitting a $\gamma$-ray of a given energy, is an indispensable quantity for calculations of astrophysical interest. Experimental studies of the $\gamma$SF have revealed an enhancement of this quantity in the low $E_\gamma$ energy region, which cannot be described by none of the known resonances or by semiclassical models. To understand the origin of the low-energy enhancement we have calculated the M1 transition probabilities, both in the emission and absorption regions, for the $^{49,50}$Cr and $^{48}$V nuclei in the $f_{7/2}$ shell-model basis. We find that the M1 strength distribution peaks at zero transition energy and falls off exponentially, independently of the excitation energy or spin range selected. The form of this exponential is the same across all three different nuclei studied within this model space. We also show that the slope of the exponential is proportional to the strength of the $T = 1$ pairing matrix elements.

1. Introduction

An important quantity that gives information on nuclear structure and is critical for calculations of nuclear reaction rates is the $\gamma$-ray strength function ($\gamma$SF) [1], namely the average reduced $\gamma$-decay probability of a particular multipole type.

The $\gamma$SF is dominated by the giant electric dipole (E1) resonance, which arises due to collective oscillations of protons against neutrons in the nucleus and is located at $E_\gamma \approx 78 \cdot A^{-1/3}$ MeV. For the description of this resonance, a standard Lorentzian shape [2, 3] can be used, however, the generalized Lorentzian [5] is usually preferred, by taking also into account the temperature dependence of the $\gamma$SF (Kadmensky-Markushev-Furman model) [4]. In the low $E_\gamma$ region, a damped electric dipole resonance is expected to be observed for neutron-rich nuclei, namely the pigmy dipole resonance [6, 7] (PDR), interpreted as the oscillation of the neutron skin against the proton plus neutron core.

The $\gamma$SF also has contributions of M1 character, which can be attributed to either spin-flip excitations of the nucleus [8, 9], typically at energy $E_\gamma \approx 8$ MeV, or to counter rotation of protons against neutrons in deformed nuclei (scissors mode) which generates a resonance around
The M1 contributions to the γ SF are also described by a Lorentzian line [5, 10] with parameters based on the recommended systematics [11].

Measurements of the γ SF dating back to 2004 [12] have revealed that it increases as the γ-ray energy decreases. This low-energy enhancement has been found in many nuclei extending from the pf shell, $^{56,57}$Fe [13, 14], $^{43,44,45}$Sc [16, 17], $^{60}$Ni [18], $^{44,45,46}$Ti [19, 20, 21], and to heavier nuclei, $^{90–98}$Mo [22, 23], $^{105,106}$Cd [24], $^{105}$Pd [25], $^{151,152}$Sm [27], $^{89}$Y [28], $^{73,74}$Ge [29]. Calculations of the radiative neutron capture integrating the low energy enhancement of the γ SF show that these cross sections can potentially increase considerably when approaching the neutron drip line [30].

The newly discovered behavior of the low-energy part of the γ SF needs a new physical interpretation. There are two questions that arise concerning the low-energy upbend: (i) the character of this resonance and its multipolarity and (ii) the mechanism which produces it. It has been found experimentally [14, 28] that E2 transitions are of minor importance, whereas dipole transitions dominate in the low-$E_\gamma$ enhancement region. Unfortunately, there are still no available experimental data distinguishing between the electric or magnetic character of the transition or providing an explanation of its origin.

The first theoretical attempt to explain the low energy enhancement suggested that this is of E1 character and is attributed to transitions from thermally unblocked states of the single-particle spectrum to the continuum [31]. At the same time, various groups performed shell-model calculations [28, 29, 32, 33] and found that the M1 contribution of the γ SF has an enhancement for low $E_\gamma$ energies and a maximum for $E_\gamma=0$ MeV, closely following the experimentally observed enhancement. The shell model studies suggest two interpretations of the low-energy upbend. According to the first interpretation [32], protons and neutrons, occupying high spin orbitals couple their spins forming a band, within which enhanced M1 transitions take place. The similarity of this mechanism to the “shears” mechanism previously found in nuclei [34], led to the suggestion that the the low-energy enhancement should take place mainly near closed shells, where the shears mechanism is strong. The second interpretation [33] proposes that strong M1 transitions originate from high spin diagonal single particle orbitals and that these M1 transitions will contribute to the low-energy M1 γ SF of all nuclei.

In this text we give arguments in favor of the second explanation. To test the suggestion that the low-energy enhancement comes from transitions between high spin orbitals, we calculate $B$(M1) values for $^{49}$Cr, $^{50}$Cr, and $^{48}$V in the small model space of $f_{7/2}$ using the OXBASH shell-model code [35] that allows us to calculate not only the emission, but also the absorption spectrum. We take into account only transitions between states with $T=T_\gamma$ and we find a characteristic peak of the γ SF at zero energy, falling off exponentially below and above that point. The characteristic upbend of the low-energy M1 γ SF is present independently of the nucleus, the range of excitation energies used, or the spin values of the initial states. We find that the low-energy enhancement is well approximated by an exponential function, similarly to previous studies [32], with the same parameters for all studied nuclei and that the slope of the exponential fall off is determined mainly by the $T=1$ (pairing) part of the Hamiltonian.

The slope of the exponent could be affected by the masses of the nuclei studied or by the presence of deformation. Details of the shell-model calculations presented in Section 2 could also play a noticeable role, for example mixing of different orbitals can quench the calculated low-energy enhancement. In Section 3 we suggest the interpretation of the results. We finish with the conclusions in Section 4.

2. Shell model calculations and results
As stated in the Introduction we calculate the transition rates $B$(M1) restricting ourselves to the $f_{7/2}$ orbital space. The nuclear states are obtained with the F742 Hamiltonian from [36] that reproduces the known low-lying energies in the nuclei of this region. Experimentally, the quantity
of interest is the $\gamma$-ray strength function $\gamma$SF defined by $[1]$ $f_{M1}^L(E_{\gamma}) = \rho_i \frac{\Gamma_{\gamma,M1}(E_{\gamma})}{E_{\gamma}},$ where $L$ is the multipolarity of the transition, $\rho_i$ the level density of the initial states and $\Gamma_{\gamma}$ the partial radiative M1 width given by $\Gamma_{\gamma,M1}(E_{\gamma}) = \frac{16\pi}{9} \left(\frac{E_{\gamma}}{\mu_N^2}\right)^{\frac{3}{2}} \langle B(1)|i\rangle,$ where the index $i$ specifies initial spin values and the initial energy region $E_i.$ Combining the two expressions, a new form of the $\gamma$SF is derived, $f_{M1}(E_{\gamma}) = a \langle B(1)|\gamma\rangle|\rho_i(E_i)$, where $a = 16\pi\mu_N^2/9 = 11.5473 \cdot 10^{-9} \cdot \mu_N^2$ MeV$^{-2}$. We are going to show our results in terms of the average $\langle B(1)|E_{\gamma}\rangle$. Actually the $\gamma$SF and $\langle B(1)|E_{\gamma}\rangle$ turn out to have very similar shapes. To determine $\langle B(1)|E_{\gamma}\rangle$, we first calculate the $B(1)$ values and then sort them according to increasing transition energy, $E_{\gamma}$. The results are grouped in energy bins of 0.2 MeV width. For each bin the average $B(1)$ value is found. This procedure guarantees that the average $B(1)$ value at given energy, $E_{\gamma}$, does not depend on the bin size.

For all ranges of initial energies and spins, strong low-energy M1 enhancement is observed as shown in Figs. 1 and 2 for $^{50}$Cr, in both the emission and the absorption spectrum. This result is very different from the Brink-Axel hypothesis in which the strength function for excited states is related to the absorption strength function in the ground state. In contrast, the low-energy distribution is a generic feature for excited states that cannot be obtained from information on the ground state since it peaks at zero energy. The red straight lines represent the low-energy distribution considerably steeper.

1. We find that the shape of the M1 distribution depends very little on the $T$ interaction, as shown in the upper panels of Fig. 4, but there is a strong dependence on the initial energy region $E_i$.

2. For all ranges of initial energies and spins, strong low-energy M1 enhancement is observed as shown in Figs. 1 and 2 for $^{50}$Cr, in both the emission and the absorption spectrum. This result is very different from the Brink-Axel hypothesis in which the strength function for excited states is related to the absorption strength function in the ground state. In contrast, the low-energy distribution is a generic feature for excited states that cannot be obtained from information on the ground state since it peaks at zero energy. The red straight lines represent the low-energy distribution considerably steeper.

3. Discussion

The approximation of the low-energy M1 strength by an exponential function, as was used here, has already been proposed in [19]. For the nuclei used in the context of this study, it is found that the slope, $T_B$, and the height, $B_0$, of the exponential functions, fitted on the $\langle B(1)|E_{\gamma}\rangle$, are almost constant for all three nuclei. Nuclei away from this mass region will have different parameters. For example, in [19] the $\langle B(1)|E_{\gamma}\rangle$ of $^{94,95,96}$Mo and $^{90}$Zr were calculated for a model space which allows for both positive and negative parity states. The slopes of the exponents were different for positive and negative parity and more steep than the ones found in the present study.

As shown previously, the pairing interaction strongly affects the M1 distribution in a way that weaker pairing gives a steeper slope to the $\langle B(1)|E_{\gamma}\rangle$. Since pairing in average depends on the mass number $A$ as $\alpha_p/A^{1/2}$ [38], in the $A=90-96$ region, pairing is 25% weaker than in
Figure 1. Average $B$(M1) values as a function of $\gamma$-ray energy $E_\gamma$ for $^{50}$Cr and various 2 MeV ranges of initial energies $E_i$. The lowest panel is for 0-2 MeV, the highest for 10-12 MeV. Each M1 distribution is compared to the same exponent, red line, with parameters $B_0 = 0.75 \mu_2^2 N$ and $T_B = 1.33$ MeV.

$A = 48-50$, thus the slope of the M1 distribution is expected to be steeper. Another factor that affects the calculated slope is the mixing of orbitals. In [39] the $\gamma$SF for $^{48}$V was calculated using the GX1A interaction [40, 41] in the $pf$ model space. Nucleons were allowed to occupy either only the $f_{7/2}$ orbital or more orbitals of the $pf$ model space and compared with that obtained within the $f_{7/2}$ space. It turns out that the mixing of different orbitals with $f_{7/2}$ quenches the low-energy strengths.

In a recent study [42], where the average reduced M1 transition probability, $\langle B$(M1)$|E_\gamma\rangle$, was calculated using the shell model in a series of iron isotopes ($^{60,64,68}$Fe), it was found that the low-energy enhancement and the scissors mode are correlated. The sum of the strength of these two modes is constant, with the strength moving from the low-energy spike to the 3 MeV-resonance as deformation increases.

To address the role of the E1 transitions in the low-energy enhancement, a broad model space was employed, consisting of the $sd$-$pf$-$gds$ orbitals [43]. No low-energy enhancement was found for the E1 strengths when studying nuclei in the $A = 43-45$ region. On the contrary, the low-energy dipole strength function had a strong M1 contribution.

The exponential form, like that describing the low-energy enhancement, seems to be generic.
Figure 3. Average $B$(M1) values as a function of γ-ray energy $E_γ$ for $^{49}$Cr, $^{50}$Cr, and $^{48}$V for initial energies, $E_i$, in the interval 6-8 MeV. Each M1 distribution is compared to the same exponent, red line, with parameters used in Figures 1 and 2.

Figure 4. Average $B$(M1) values as a function of γ-ray energy $E_γ$ (black line) for $^{50}$Cr for initial energy, $E_i$, in the interval 6-8 MeV, compared with the average $B$(M1) values derived using (a) $k_0 = 0, k_1 = 1.0$, (b) $k_0 = 0.5, k_1 = 1.0$, (c) $k_0 = 1.0, k_1 = 0$, (d) $k_0 = 1.0, k_1 = 0.5$, red line. The green line is the exponential fit with $B_0 = 0.75 \, \mu^2_N$, $T_B = 1.33$ MeV.

for the problems which have a bilinear combination of random operators. An analog can be found in the statistical distribution of the off-diagonal matrix elements of a realistic many-body Hamiltonian used in the full shell-model calculations in a finite orbital space which show onset of quantum chaos. This was studied in detail for the $sd$ shell model space in [44] and examples can be found there in Figs. 8 and 9 and the Appendix. Contrary to the standard embedded ensembles of random matrices with Gaussian-like distribution of matrix elements [45], in these applications we typically have a distribution close to the exponential (sometimes prefactors are present, mostly important for the smallest matrix elements). This situation is supposed to emerge when the random quantities are matrix elements of multipole operators, while the main terms of the many-body Hamiltonian are their bilinear combinations, like multipole-multipole forces. For the components changing the seniority, the mean transition energy is of the order of the pairing gap $\Delta$, about 1.5 MeV, for this group of nuclei. This estimate agrees with the effective temperature $T_B$ found above.

This physics cannot support the Brink-Axel hypothesis which can be approximately valid for the excitations of general macroscopic nature. In the case of the giant dipole resonance, the main part is played by the local dipole polarization of the nuclear medium, which is essentially a universal property of nuclear matter. Such an excitation can be erected on top of any shell-model state, in the Brink-Axel spirit. In the case considered above, low-energy properties, such as isovector pairing and spin-orbit splitting of specific single-particle orbitals, are crucial.
4. Conclusion
We have performed shell-model calculations in the $f_{7/2}$ model space producing the full absorption and decay schemes of $^{48}$V, $^{49}$Cr, and $^{50}$Cr. The results indicate a strong low-$E_\gamma B(M1)$ component, in accordance with experimental and theoretical findings. The low-energy enhancement is essentially a one-partition phenomenon, which can be attributed to transitions stemming from diagonal high-spin orbitals. The low-energy upbend is independent of the initial spin and energy and it can be well fitted by an exponential with the same parameters for all energy and spin ranges. The $T=1$ matrix elements, with the pairing being the most important part, are responsible for the exponential shape of the $B(M1)$ distribution. We also discussed the possible factors which affect the effective temperature of the low-energy enhancement. The slope of the low-energy upbend could change depending on the mass of the nucleus involved, through pairing interaction, but also due to the mixing of different orbitals or truncations of the orbital space. A reference to the interesting correlation of the low-energy upbend with the scissors-like resonance was made which also affects the low-energy $B(M1)$ slope. We included more general arguments on relation of the phenomenon under consideration to the general problem of onset of chaotic dynamics.

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