Hybrid simulations of fast electron propagation including magnetized transport and non-local effects in the background plasma

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Abstract

We present the first results from a 2D VFP-PIC hybrid code for fast electron transport which solves the Vlasov–Fokker–Planck (VFP) equation for the background electrons using the code IMPACT. This new type of hybrid code captures full Braginskii electron transport including magnetization, non-local corrections and electron inertial effects. We consider propagation of a relativistic electron beam, generated by a laser of intensity \( I = (1–5) \times 10^{19} \, \text{Wcm}^{-2} \) and focal radius of a few microns, inside a near solid-density carbon target. Electron thermal transport out of the resistively heated background plasma is strong enough to compete with ohmic heating after about a picosecond. The effect of heat flow on the plasma temperature is sufficient to alter resistive magnetic field generation over time scales beyond a few picoseconds. This includes removal of beam-hollowing field near the beam injection point and re-emergence of a collimating magnetic field. Background electrons become weakly magnetized after a few picoseconds; enough for magnetized transport effects to significantly alter the evolution of the background plasma temperature and the long term evolution of the fast electron filaments. A practical estimate for the evolution of electron magnetization is presented and shown to agree with the simulation results. Non-local modifications to transport of up to 20% have been found in this situation.

(Some figures may appear in colour only in the online journal)

1. Introduction

Successful propagation of an intense, laser-generated beam of relativistic (or ‘fast’) electrons from the critical surface to the ~1000 times solid density pre-compressed fuel core is crucial to the viability of ‘fast ignition’ inertial confinement fusion [1] using fast electrons. Requirements for the fast electron beam (FEB) and intense laser pulse needed to generate it—in the case of a pre-compressed core of density \( \rho \approx 500 \, \text{gcm}^{-3} \)—have been calculated [2] to be on the order of 5–7 PW of laser power delivered over 15 ps, focused to a radius and intensity of about 20 \( \mu \text{m} \) and \( I \approx 2 \times 10^{20} \, \text{Wcm}^{-2} \), respectively, in order to generate a beam of electrons with a kinetic energy spectrum of temperature \( T_e \approx 1.5 \, \text{MeV} \) and carrying energy \( W_f \approx 47 \, \text{kJ} \). This calculation involved a hybrid code and assumed a laser energy flux to fast electron energy flux conversion efficiency of \( \eta_L = 0.4 \) (i.e. the absorption efficiency). A typical FEB beam, suitable for ignition, possesses a current of several hundred MA and peak number density approaching \( n_f \approx 10^{22} \, \text{cm}^{-3} \) at the point of injection. Considering the laser is absorbed at the critical density which is \( n_{cr} \approx 10^{22} \, \text{cm}^{-3} \) at a laser wavelength of \( \lambda_L = 0.5 \, \mu \text{m} \) (and more towards \( 10^{23} \, \text{cm}^{-3} \) including relativistic corrections, appropriate at \( I \lambda_L^2 \approx 5 \times 10^{19} \, \text{Wcm}^{-2} \)), this beam will constitute a large fraction of the available background electrons at the point of absorption.
Accurate prediction of FEB propagation is challenging and has been an active area of research since fast ignition was proposed. The challenge stems from the large range of conditions (e.g. plasma densities ranging from critical density to 1000’s of times solid density) through which the FEB passes, and the different regimes of beam–plasma interaction that these conditions encompass. At solid density, resistively generated electromagnetic fields have been shown to be the dominant influence on fast electron propagation [3], whilst at higher densities collisions of the fast electrons on the background plasma dominates. Below solid density beam–plasma instabilities are expected to be the most important issue. For ‘solid’ density plasma and above, where \( n_f / n_e \ll 1 \) (with \( n_e \) the electron number density of the background plasma), current density neutralization is assumed to occur so that a ‘return current’ \( j_r \) is drawn in the background to locally cancel (to good approximation) the high current density of fast electrons \( j_f \), so one has \( j_f + j_r = \mu_e c \nabla \times B \approx 0 \) for wide enough beams. The return current is collisional hence an electric field \( E = \alpha j_f \) is required to draw it, where \( \alpha \) is the electrical resistivity. This electric field is responsible for induction of a magnetic field such that \( \partial \mathbf{B} / \partial t = \alpha \nabla \times j_f + \nabla \alpha \times j_f - \nabla \times [(\alpha / \mu_e) \nabla \times B] \). The terms on the right-hand side of the above induction equation (going left to right) are responsible for generation of (1) focusing, collimating field [3], (2) beam-hollowing, defocusing fields [4] and (3) resistive diffusion of magnetic field. Between critical and solid density, \( n_f \) is a large perturbation to the background plasma, with local current neutralization requiring a high drift speed for the background electrons. The nature of transport under these conditions is not well understood. Microscopic beam–plasma instabilities, such as resistive filamentation [5], the two-stream instability [6] and the Weibel instability [7] are thought to be prevalent here. For instance, the resistive filamentation instability—which is capable of splitting a FEB into multiple current filaments—typically has a peak growth rate of about the fast electron plasma frequency, \( \omega_{pe} = \sqrt{n_e^2 e^2 / \epsilon_0 m_e c^2} \), at a wave length of about the fast electron collisionless skin depth, \( (\delta_e)^c = c / \omega_{pe} \). These instabilities have been considered theoretically [8, 9] under simplifying assumptions about the distribution function of the background plasma and also numerically using hybrid FET codes [5] and particle-in-cell (PIC) codes. These works discuss more precisely how the instability growth rates and thresholds change under different conditions.

Augmenting the above induction equation with an electron fluid energy equation that describes ohmic heating and a prescription of the plasma resistivity is the standard way of describing resistive magnetic field generation. A simple yet suitable energy equation is \( c_v n_e \delta T_e / \partial t = \alpha j_f^2 \), where \( c_v n_e \) is the heat capacity at constant volume, \( T_e \) the electron temperature and \( \alpha \propto Z \ln A \propto T_e^{-3/2} \) the Spitzer resistivity. Standardly, ‘hybrid codes’ for fast electron transport [3, 5] use such a fluid description for the background, solving for \( T_e, E \) and \( B \) on a fixed mesh together with a macro-particle description of the fast particles which are relativistically ‘pushed’ using the \(-e(E + v \times B)\) force (and usually with an additional stochastic term to describe collisions). Many hybrid codes include phenomenological descriptions of resistivity and heat capacity, that fit cold material behaviour through to ideal plasma behaviour (Spitzer and \( \alpha = c_v^2 k_b \)). Fluid models can readily include hydrodynamic motion, solving for the evolution of the mass density \( \rho \) and the fluid velocity \( C \). Ionization physics has also been included in a few hybrid codes [10]. Recently, hybrid codes with full classical Ohm’s law and heat flow relations, including magnetization effects, have been reported [11]. (However the magnetized thermal conduction capability was not used in the results presented in [11].) Fully kinetic codes such as PIC codes have been applied to this problem [12, 13], but struggle to simultaneously consider realistic densities and volumes due to the need to resolve microscopic scales such as the Debye length and plasma frequency of the background. Although there are algorithms that can relax these temporal and spatial constraints somewhat, large scale simulation is still extremely challenging. ‘Hybrid-PIC’ algorithms that spatially interface a PIC code with a hybrid FET code are a new development [14, 15]. Full PIC calculations are used in the laser absorption region and can self-consistently resolve the generation of the FEB, whilst a conventional hybrid FET algorithm is applied in the high-density bulk of the system where high collisionality makes a PIC approach unnecessary for treating the background electrons. The hybrid components of these hybrid-PIC codes do not yet involve the full Ohm’s law, missing the thermo-electric term, and it is unclear what description of thermal transport in the background, if any, is currently included. A 2D fully kinetic VFP electron code has previously been applied to FET at solid density, being used to investigate the conditions needed to get whole beam self-collimation [16]. A 1D full-VFP electron code has found non-Spitzer return currents in the low-density regions of plasma that a fast ignition FEB would experience in propagating from absorption at critical density to the compressed core [17]. Non-Spitzer behaviour was found to be due to a combination of inertial effects and non-local effects due to long collisional mean-free-paths of plasma electrons in steep gradients. It neglected magnetic fields. Both of these fully kinetic VFP codes neglected hydrodynamic motion and material effects.

Hybrid codes have been extensively used to aid interpretation of ultra-high intensity laser–solid interaction experiments [18] and to undertake design studies of full-scale fast ignition, often involving coupling a hybrid FET code to a radiation-hydro code and/or a PIC code (e.g. to yield the fast electron momentum spectrum arising from laser absorption) [11, 19, 20]. They have also been used as theoretical tools, revealing potential fast electron transport phenomena based on evolution or control of the resistivity gradients, including beam-hollowing [4] and guiding via imposed resistivity structures [21, 22]. In a recent publication [23] we used a reduced kinetic description of the background electrons—the Vlasov–Fokker–Planck (VFP) equation in the \( f = f_0 + f_1 \) diffusion approximation—coupled to a rigid beam (RB) of fast electrons to undertake 1D calculations of the evolution of the background plasma \( (T_e, B) \) across the beam. This approach captured full, magnetized, non-local transport and showed the importance of thermal conduction including magnetization effects. In this paper we report on 2D calculations including a
self-consistent, evolving, fast-electron beam. This extends our previous findings to 2D and now addresses thermal conduction and magnetization effects on fast electron filamentations. The VFP-PIC hybrid code developed uses IMPACT [24] to treat the background electrons. IMPACT has previously been used on a range of problems relevant to non-local and magnetized electron transport and magnetic-field dynamics in the context of nanosecond laser-plasma interaction [25–27]. This represents the most detailed treatment of transport processes in the background plasma (in the regime where low-temperature materials effects are unimportant) to be attempted in 2D hybrid FET calculations, to the best of our knowledge. We note that full PIC codes and hybrid-PIC codes provide better treatment of transport in the relatively collisionless low-density plasma in the vicinity of the critical surface. Our hybrid VFP-PIC approach is more detailed than the fluid-PIC approach currently being used in hybrid modules of hybrid-PIC codes [14, 15], whilst being computationally less demanding and less prone to excessive noise compared to full PIC calculations of plasmas approaching solid density.

2. Physics of transport in the background

It is well known that electron transport in plasma is more complicated than \( E = \alpha j \) and for an ideal plasma (many, many particles in the Debye sphere) Braginskii’s transport relations [28] are appropriate

\[
e_n(E + C \times B) = -\nabla \cdot P + j \times B + \frac{m_e}{e \tau_{th}} \alpha \cdot j
\]

\[
q_e = -\frac{n_e \tau_{th}}{m_e} \alpha \cdot \nabla T_e
\]

\[\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V}_e \cdot \nabla \]

where \( \alpha \), \( \beta \) and \( \kappa \) are the dimensionless transport coefficients; electrical resistivity, thermodiffusive and thermal conductivity, respectively. In Ohm’s law, equation (1), \( C \) is the plasma fluid velocity (i.e. the centre of mass velocity, dominated by the ions) and \( \mathbf{V}_e \) is the electron fluid velocity. The electron pressure is often assumed to be isotropic (in which case \( \nabla \cdot P = \nabla \cdot P_e \) with \( P_e = n_e T_e \)) and electron inertia (the last term on the rhs) often neglected (unless fast, electron waves are being considered). These expressions use \( T_e \) in energy units. Other quantities: \( \tau_{th} = \frac{3(\sqrt{\pi}/4)v_{th}^3}{(YZ^2 n_i \ln \Lambda)} \) is the electron-ion collision time (i.e. electron angular scattering), \( v_{th} = \sqrt{2T_e/m_e} \) is the thermal speed, \( n_i \) the ion density, \( Z \) the ion charge, \( \Lambda \) is the Coulomb logarithm and \( Y = 4\pi (e^2/4\pi n_i m_e)^2 \). Equations (1) and (2) are valid for shallow gradients in macroscopic quantities so that \( \min[\lambda_{th}, r_{L}] \ll \min[L_T, \rho_n, L_H] \) holds, where \( \lambda_{th} = v_{th}/\kappa_{th} \) is the electron-ion mean-free-path, \( r_L = v_{th}/\omega_{pe} \) the Larmor radius (with \( \omega_{pe} = qB/m_e \) the electron Larmor frequency) and \( L_q = \sqrt{q/q-1} \) is the macroscopic scale length (with \( q \to [n, T, B] \)). Note that \( q_e \) is the intrinsic electron heat flux—transport of energy associated with the random motion of the electrons in a frame where there is no net electron drift—and appears in the following energy equation for electrons

\[
c_e \frac{d}{dt} T_e = -\nabla \cdot q_e - P_e \nabla \cdot V_e + E \cdot j + Q_{vis} + Q_{el}
\]

where \( d/dt = \partial/\partial t + \mathbf{V}_e \cdot \nabla \), \( Q_{vis} \) accounts for heating due to electron viscosity, and \( Q_{el} \) for thermal relaxation between electron and ion populations. Note that in the results section we will show ‘total’ electron heat flow in the local ion rest frame, \( (q_e)_{tot} = q_e - (j/n_e)2T_e/3 + \frac{1}{2}m_e(j/n_e)^2 \), which includes the enthalpy flux, \(-\frac{1}{2}j F_t/(n_e e)\), and a further correction [29].

The transport coefficients are more precisely tensors, describing the differencing transport fluxes generated by a thermodynamic driving force in three orthogonal directions—parallel (\( \parallel \)), perpendicular (\( \perp \)) and ‘cross’ (\( \times \)—with respect to the local magnetic field direction, e.g., \( \kappa \cdot \nabla T_e = \kappa_{\parallel} \nabla_{\parallel} T_e + \kappa_{\perp} \nabla_{\perp} T_e + \kappa_{\times} (\hat{b} \times \nabla T_e) \). (Notice that the ‘cross’ component of the flux is perpendicular to both the \( B \)-field and the driving force.) The same applies for the resistivity and thermodiffusive tensors, except that the sign of the cross component of \( \alpha \cdot j \) is negative by convention. The components of \( \alpha \), \( \beta \) and \( \kappa \) are functions of the Hall parameter \( \omega_{pe} = \tau_{th} \) and the ion charge \( Z \). (The Hall parameter will be written as \( \omega_{B} \) for brevity, henceforth.) See [29] for fits to the transport coefficients. Of particular relevance to the findings reported here are cross field thermal conduction \(-n_e T_e \tau_{th} / m_e \kappa_{\times} \nabla_{\times} T_e\) including its suppression as electrons magnetize \( \kappa_{\times} \approx \kappa_{\parallel}/(1 + (\omega_{B})^2) \). Ettingshausen heat flow \(-T_e/\omega_e \beta_{\parallel} (\hat{b} \times j) \) and cross field electrical resistivity \( (m_e/(e^2 n_e \tau_{th})) \alpha_{\perp} j \).

3. Code details

Our VFP-PIC hybrid code simulates fast electron transport in a two-dimensional system in Cartesian \( x-y \) geometry. It includes electric field components \( E_x, E_y \) and magnetic field component \( B_z \). The parallelized VFP code IMPACT [24] is used to describe the background electrons; ions are assumed to be cold but respond hydrodynamically. IMPACT includes all the transport effects present in equations (1) and (2) including non-local changes to the transport coefficients and electron inertia. IMPACT makes the diffusive approximation by reducing the electron distribution function (EDF) to \( f_0(x, y, v, t) = f_0(x, y, v, t) + f_1(x, y, v, t) \cdot \hat{v} \) which is the first two terms in a Cartesian tensor expansion (or equivalent spherical harmonic expansion) of the EDF in velocity-space angle. Importantly, the isotropic part \( f_0 \) is allowed to be non-Maxwellian, allowing description of non-local effects. (Fixing \( f_0 \) to a Maxwellian would recover Braginskii’s Ohm’s law (but neglecting the anisotropic part of the electron pressure) and electron heat flow equation.) IMPACT solves the resulting VFP equations for the evolution of \( f_0 \) and \( f_1 \) together with Maxwell’s equations using implicit finite-differencing techniques for robustness and numerical stability. The time-discretization used for the resulting \( f_0 \) and \( f_1 \) equations is

\[
\frac{f_0^{n+1} - f_0^n}{\Delta t} + \frac{\nabla \cdot J_1^{n+1}}{3} = \frac{e}{3m_e v^2} \frac{\partial}{\partial v} (v^2 E_n^{n+1} \cdot J_1^{n+1}) + H_0^{n} = C_{en0}^{n+1}
\]
\[ \frac{f_i^{n+1} - f_i^n}{\Delta t} + v_n f_i^{n+1} - eE^{n+1} \frac{\partial f_i^n}{\partial v} = eB^{n+1} \times f_i^{n+1} + H_i^n \]
\[ = -YZ^2 n_i f_i^{n+1} \]

where \( n \) and \( n+1 \) denote time steps in the usual way. For non-linear terms we use a Picard iteration within the time step. The index \( n \) denotes quantities that get included in the implicit matrix and are updated at the latest \( n+1 \) solution values at the end of each Picard iteration. \( C_{\text{eh}}^{n+1} \) is the electron–electron collision operator involving Rosenbluth coefficients, described further in [24]. \( H_i^n \) and \( H_i^1 \) are explicit hydrodynamic correction terms involving the plasma fluid centre of mass velocity \( C \), as previously used in SPARK [30] (which ignores magnetic field). However, we retain all the hydro corrections including field transformation terms in \( C \times B \) and not just the PSD term \((-\frac{1}{2} \nabla \cdot C \partial f_0/\partial v) \) and the advective correction \((C \cdot \nabla f_0) \). The time-discretization used for Maxwell’s equations is

\[ \nabla \times \mathbf{B}^{n+\theta} = \mu_0 (j_t^{n+1} + j_t^{1/2}) + \frac{1}{c^2} \left( \frac{E^{n+1} - E^n}{\Delta t} \right) \]
\[ \nabla \times \mathbf{E}^{n+1} = -\left( \frac{B^{n+1} - B^n}{\Delta t} \right) \]

where \( j_t = -(4\pi e/3) \int_0^\infty f_i v_i \, dv \) is the return current in the background plasma that is computed kinetically via IMPACT. The implicitness parameter \( 0 \leq \theta \leq 1 \) allows for implicit \((\theta > 0)\) or explicit \((\theta = 0)\) treatment of the \( B \)-field. (For the results in this paper, \( \theta = 0 \) was used.) We retain the displacement current term which is novel for a hybrid code. This allows for quasi-neutrality. To achieve this, the background electron density \( n_e \) will electrostatically respond as a fast electron charge-pulse passes through. In the calculations presented here, the perturbation to the background electron number density almost exactly reflects the fast electron density (\( \delta n_e \approx -n_i \)) in any given cell, with the difference being a small amount of noise. The implicit treatment of the background electrons allows this to be followed without the need to resolve the plasma period of the background, which is prohibitively small at solid densities, so that \( \phi_{\text{pe}} \Delta t \gg 1 \) is possible (where \( \phi_{\text{pe}} = \sqrt{n_e e^2/(m_e c_0)} \) is the plasma frequency of the background.) However, the fast electron time scale \( \phi_{\text{pe}}^{-1} \) must still be resolved. The implicit matrix equation resulting from equations (4)–(7) is solved using a BICGSTAB solver and ILU preconditioner from PETSc [31, 32].

The fast electron current term, \( j_t^{1/2} \), in the Ampère-Maxwell equation is responsible for coupling the PIC macro-particle description to the VFP electron background and the electromagnetic field equations (solved by IMPACT). The fast electron current also appears in the MHD equation of motion as the Hall field contribution \(-j_t \times B \) which augments the plasma and magnetic pressure force densities. (Magnetic tension is absent in the geometry used here.) This MHD momentum equation is solved by IMPACT to evolve \( C \).

The fast electron macro-particles are dealt with by a standard PIC scheme. Each macro-particle is accelerated according to \( dp/dt = Q(E + v \times B) + F_{\text{coll}} \) where \( p \) and \( v \) represent a particle’s momentum and velocity, and \( Q \) its charge. \( F_{\text{coll}} \) is a stochastic force due to collisions with the background, but was not used in this investigation. The electromagnetic field at the position of a given macro-particle is found by bi-linear interpolation from the grid. A leap frog scheme is used for the particle pusher; the Boris algorithm [36] is employed to calculate the acceleration by EM fields. \( j_t \) is found by bi-linear interpolation of the macro-particles’ current to the spatial grid. Open boundary conditions are used for both the VFP and PIC components. The propensity for our VFP-PIC scheme to maintain quasi-neutrality means that the PIC macro-particles have to be introduced and removed in ghost-cells immediately outside the computational domain covered by IMPACT. A PIC particle drifts into (and out of) the computational domain whilst in these ghost-cells, gradually and smoothly contributing to \( j_t \) on the edge of the IMPACT’s grid. The fast electrons are injected with a phenomenological energy and momentum spectrum, as is usual in hybrid FET codes.

Finally we note that low-temperature materials effects—resistivity, heat capacity—which are present in many hybrid codes, are absent in this code. We use Fokker–Planck collision operators for the background electrons, which is valid when in \( \Lambda > 1 \). We currently neglect e–e collision terms from the \( f_i \) VFP equation (a very good approximation for high-Z; the Lorentz limit). The effects of viscous heating and electron–ion energy relaxation—i.e. \( Q_{\text{vis}} \) and \( Q_{\text{ei}} \) in equation (3)—are neglected. Full details of our VFP-PIC hybrid code, its algorithms and its testing will appear in a future publication.

4. 2D Simulations of near-solid-density carbon

We consider a laser pulse of peak intensity \( 10^{19} \text{W cm}^{-2} \), wavelength 1.054 \( \mu \text{m} \), Gaussian focal profile of full width half maximum \( (\text{FWHM}) \phi_{\text{FWHM}} = 5 \mu \text{m} \) and a fully ionized carbon plasma \((Z = 6)\) with an initial, uniform temperature \( T_0 = 100 \text{eV} \), density \( n_0 = 10^{29} \text{m}^{-3} \) (corresponding to about a fifth of the electron number density of solid amorphous carbon) and no magnetic or electric fields initially present. The Coulomb logarithm is taken to be \( \ln \Lambda = 2 \). The extent of the computational domain is 35 \( \mu \text{m} \) high (the \( y \) direction) by 80 \( \mu \text{m} \) deep (the \( x \) direction). Fast electrons are phenomenologically injected at the \( x = 0 \mu \text{m} \) boundary in a beam centred around \( y = 0 \). The energy and momentum distribution of the injected electrons is given by

\[ f_b = C p^2 \exp \left( -\frac{m_e c^2}{T_1} (\gamma - 1) \right) \cos^M \theta \]

where \( \gamma = \sqrt{1 + p^2/m_e c^2} \) is the relativistic gamma factor of an electron, \( T_i \) the fast electron temperature, \( m_e \) the electron rest mass, \( M \) controls the angular spread of the beam, \( \theta \) is the angle of the momentum vector with respect to the \( x \)-direction and \( C \) is a normalization constant. Beg’s \( T_i \propto (\Gamma)^{1/3} \) scaling law [37] was used giving \( T_i = 430 \text{keV} \) and average kinetic energy \( \langle K \rangle = (\gamma - 1)m_e c^2 = 990 \text{keV} \) at peak intensity. An angular parameter of \( M = 6 \) was used corresponding to half cone angle \( \theta_{1/2} = 27.0^\circ \) (defined as the half width at half maximum of the angular distribution). A constant laser energy to fast electron energy conversion efficiency value of
Figure 1. Fast electron number density \( \log_{10}(n_f) \) (m\(^{-3}\)) at 0.25 ps (a), 0.75 ps (b), and 2 ps (c) in a fully ionized carbon target of initial density \( n_{e0} = 10^{29} \text{ m}^{-3} \) and temperature \( T_{e0} = 100 \text{ eV} \), a laser intensity of \( I = 10^{19} \text{ W cm}^{-2} \), absorption efficiency \( \eta_L = 0.21 \), and width \( \phi_{\text{FWHM}} = 5 \mu\text{m} \) (Gaussian profile). The HWHM of the fast electron angular distribution is \( \theta_{1/2} = 27.0^\circ \). The laser intensity ramps up linearly over the first 0.5 ps.

\( n_{f} \) was obtained from energy flux balance arguments by inverting \( \eta_L I = n_f \langle v K_f \rangle; n_f \) and \( T_f \) were varied across the injection plane consistently with the intensity profile \( I(y) \). The laser intensity was linearly throttled from zero to the maximum over the first 0.5 ps and then kept constant. These conditions correspond to a peak beam to background electron density of \( n_f/n_e = 1/200 \).

A uniform spatial mesh was used with 192 and 250 cells in the \( x \) and \( y \) directions, respectively, yielding grid resolutions of \( \Delta x = 0.42 \mu\text{m} \) and \( \Delta y = 0.14 \mu\text{m} \). The fast electron number densities experienced peaked at about \( 5 \times 10^{20} \). At this density, the fast electron skin depth is \( 0.24 \mu\text{m} \), thus the cell size in the \( y \)-direction is just enough to resolve resistively generated filaments. For accurate calculation of the background transport effects, a time step of 0.2 times the initial collision time of the background plasma was used; \( \Delta t = 0.13 \text{ fs} \). This is smaller than the fast electron transit time across a grid cell (0.47 fs). A velocity grid of 20 \( v_{100} \), where \( v_{100} \) is the thermal speed at 100 eV, and 80 \( v \)-grid cells were used in IMPACT. Collisions of fast electrons with the background plasma were not used in the simulations to be shown. Up to 34000 macro-particles per spatial-cell are present in the injection region.

A run with twice the spatial resolution and duration of 1 ps was carried out, with no significant difference observed: We conclude that the results are grid resolved. Systems with twice the spatial extent, but the same resolution, were tested and good agreement was found between the results over 1.5 ps, indicating no serious influence from the spatial boundaries.

We now demonstrate that our VFP-PIC hybrid scheme works in 2D and can simulate FET over multi-picosecond timescales. Figure 1 shows profiles of the fast electron number density \( n_f \) at 3 successive times. At 0.25 ps the onset of magnetic focusing is visible, given the pinched shape of the density contours at \( x \sim 5 \mu\text{m} \). Deeper into the target the electron beam is still spreading laterally. Considering that the minimum fast electron transit time across the 80 \( \mu\text{m} \) deep domain is about 0.27 ps, the electrons deeper into the target would have passed through the injection region early on when the resistively generated magnetic field was relatively weak. By 0.75 ps the beam is beginning to experience global focusing deeper into the target and filaments have emerged. At 2 ps there is dramatic whole beam focusing, the main central filaments seen earlier on have strengthened while those exiting the right-hand boundary at \( |y| > 10 \mu\text{m} \) have vanished. Fast electrons at the periphery of the beam are forming a belt—much like for beam hollowing—yet a large proportion of injected electrons are still able to propagate close to \( y = 0 \) albeit redistributed in the strong filaments.

A collimating magnetic field next to the injection region and reaching 200 T is found at 0.5 ps as shown in figure 2. By 2 ps we see strong 2000 T fields cladding the fast electron filaments. At this time the electron temperature has reached 800 eV just beyond the injection region and about 450–500 eV next to the filaments. Interestingly the temperature profile in the filaments is hollow, which will be discussed further in section 4.2. This combination of \( T_e \) and \( B_z \) results in Hall
parameters $\omega\tau \propto B T_e^3/2$ of around 0.2 at the injection region and up to 0.7 for the filaments.

At 2 ps, the thermal pressure gradient $-\nabla P_e$ and the $j \times B$ force density has managed to accelerate the background to a speed of just over 0.02 $\mu$ms$^{-1}$ (see figure 3(a)) and perturbed the background density 5%, causing a broad cavitated profile at the 2% level in the injection region and narrower channels at the 5% level under the filaments (figure 3(b)). (Only the $y$ component of the plasma fluid velocity $C$ is shown; the $x$ component is much smaller.)

4.1. Heat flow and suppression of beam-hollowing field

In many hybrid codes, heat flow in the background plasma is ignored. However we previously found heat flow to be important in 1D VFP-RB (rigid beam) calculations [23] over multi-picosecond time scales, particularly to resistive magnetic field generation on these time scales. We now show that it is significant in 2D calculations with a non-RB. From the VFP-PIC code, we show the ‘total’ electron heat flow $(q_e)_\text{tot}$; the energy flux carried by the background electrons (as seen in the ion rest frame), defined just after equation (3), which includes the enthalpy flow $-\frac{5}{2} j P_e/(n_e e)$ and a further, small correction. (We drop the ‘e’ subscript and henceforth refer to it as $q_{\text{tot}}$.) Referring to figure 4, we observe the $y$-component of $q_{\text{tot}}$ approaching $10^{15}$ W cm$^{-2}$ at the sides of the ohmically heated hot spot at 2 ps. Strikingly, the $x$-component is up to 7 times larger in magnitude, the majority being enthalpy flux due to the strong return current. This enthalpy flux is able to carry thermal energy up the temperature gradient (i.e. into the hot plasma at the injection region). However, this enthalpy flux has a small divergence and thus a negligible effect on local plasma heating (or cooling). Although an energy flux of $\sim 10^{15}$ W cm$^{-2}$ is orders of magnitude smaller than the peak energy flux of the fast electrons, which is about $\sim 3 \times 10^{18}$ W cm$^{-2}$, it is still significant in how it affects the temperature evolution of the background plasma.

Figure 5(a) shows that by 2 ps plasma cooling due to $\nabla \cdot q_{\text{tot}}$ under the beam centre is approximately 50% of the peak ohmic heating rate there, just beyond beam injection (at $x = 5 \mu$m). Each component of $q_{\text{tot}}$ contributes similarly there which is consistent with the similar spatial extents of the hot spot in the $x$ and $y$ directions. Later on, at 3 ps, the cooling rate in the vicinity of $y = 0$ (and the heating rate at the sides of the beam; $|y| \sim 5 \mu$m) due to the diffusive heat flow is even larger. Deeper into the target, at 60 $\mu$m, we see that the lateral heat flow is also able to counteract ohmic heating of plasma traversed by the fast electron filaments, as shown in figure 5(b). It removes thermal energy where ohmic heating peaks and deposits it between the filaments, leading to a much less strongly peaked overall heating rate. The $x$-component has a smaller effect.

In a previous paper we showed that heat flow could remove beam-hollowing magnetic fields near the beam centre and permit the re-emergence of field with the correct polarity for collimating the fast electrons [23]. This finding was in a 1D calculation with a RB. Significantly, we find that this effect persists in 2D and when dynamical evolution of the
fast electrons is included. This is demonstrated in figure 6(a) at a depth of $x = 5 \mu m$ in the core region (approximately $|y|/\mu m < 5$) where the slow creation of beam-hollowing field at 1 ps reverts to creation of collimating field at 2 ps and beyond. It would be instructive to repeat the 2D VFP-PIC calculations with heat flow in the background turned off, but this is not readily possible without interfering with the return current (which would have unintended side effects) since both are moments of $f_1$ and IMPACT solves for $f_0$ and $f_1$ implicitly.

Finally we consider the composition of the lateral total heat flow $q_y |_{\text{tot}}$. At $x = 5 \mu m$, 2 ps it is dominated by similar amounts of diffusive heat flow $-\kappa_\perp \partial T_e / \partial y$ and Ettingshausen heat flow $-(T_e/e) \beta_\perp [\hat{b} \times \vec{j}]_y$, as shown in figure 6(b). The latter effect is due to the magnetic field deflecting the hot tail of the return current carrying electrons away from the centre of the beam. These particles are more magnetized and less collisional than the colder thermal ‘body’ of electrons. At $x = 60 \mu m$, 1 ps, the heat flux out of the filaments is very

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**Figure 3.** (a) Plasma fluid velocity $C_v (\mu m \text{ps}^{-1})$ and (b) perturbation to background electron density $\delta n_e = n_e - n_{e0} (m^{-3})$, both at 2 ps. Conditions are as described in figure 1.

**Figure 4.** Electron heat flow components at 2 ps; $q_y$ (a) and $q_x$ (b), in units of W cm$^{-2}$. (Note that the total heat flow $(q_{\text{tot}})$, rather than the intrinsic heat flow, is shown here.) Conditions are as described in figure 1.

**Figure 5.** Comparison of main contributions to heating rates at 2 ps. (a) at $x = 5 \mu m$ and (b) at $x = 60 \mu m$. Contributions to $\nabla \cdot q_{\text{tot}}$, the ohmic heating rate and the total rate are depicted. Conditions are as described in figure 1.
roughly $\sim 60\%$ Ettingshausen and $\sim 40\%$ diffusive heat flow, with some contributions from other terms in the outer fast electron 'belt' at $4 < |(y)/\mu m| < 8$. (See figure 6(c).) The situation here looks similar later on.

4.2. Influence of magnetized background plasma

For this system comprising of a $10^{19}$ W cm$^{-2}$ laser, $\eta_L = 0.21$, $\phi_{\text{FWHM}} = 5 \mu m$, $n_{e0} = 10^{23}$ cm$^{-3}$, $T_{e0} = 100$ eV and $Z = 6$ we see that magnetization of the transport in the background plasma makes a difference to evolution of the system. We compare two calculations which are identical except that the magnetic field in the $f_i$ equation for the background is retained in one (the magnetized transport case, with $\omega \tau = 0$). Figure 7(a) shows the $T_e$ profile at 3 ps with magnetized transport included and has three features not seen in the $\omega \tau = 0$ case (figure 7(b)). (1) The hot spot just beyond the injection point is wider in the $y$-direction. (2) There are slight double temperature peaks either side of $y = 0$ (see figure 7(c)). (3) The temperature profile around each filament is also hollow; the temperature does not peak at the centre of the filament where $j_i$, and therefore $\eta j_i^2$ is maximum, but either side. The amplitude of these hollow profiles, both near the injection region and in the filaments, grow with time.

The explanation for these differences is likely the same as proposed in 1D VFP-RB (RB) calculations [23]. The 'off-axis' peaks in the lateral $T_e$ profiles in the beam injection region, and also around each filament, occur near where the Hall parameter $\omega \tau \propto B T_e^{3/2}$ peaks (see figure 2). This has three consequences. (1) The resistivity $\alpha_\perp$ depends on $\omega \tau$ (increasing by a factor of $\sim 3$ as the Hall parameter varies from 0 to $\infty$, in the Lorentz $Z \rightarrow \infty$ case considered here [29]) and peaks where the Hall parameter does, thus enhancing the ohmic heating rate off axis compared to on axis, where the magnetic field and thus $\omega \tau$ vanish. (2) The diffusive heat flow depends on $\kappa_\perp$ which approximately varies with magnetization as $\kappa_\perp \propto [1 + (\omega \tau)^2]^{-1}$. This enhances $\partial q_y/\partial y$ where the Hall parameter rises, compared to when magnetization effects are ignored. (3) The Ettingshausen heat flow is strongly divergent on the beam axis, which sucks thermal energy from the beam centre and deposits it further out. With $\omega \tau = 0$, Ettingshausen is absent. Figure 8 shows that the total heat flow that would occur with $\omega \tau = 0$ is significantly different to that actually seen. These unmagnetized results are reconstructions of the instantaneous $(q_e)_\text{tot}$ obtained using Braginskii heat flow (but calculated with $\omega \tau = 0$) using the temperature, density and current profiles from the VFP-PIC simulation, and then adding the enthalpy flux contribution and the small correction in $j_3$.

Finally, we demonstrate that magnetized transport can significantly affect the gross evolution of the FEB. We now consider a more extreme case of $I = 5 \times 10^{19}$ W cm$^{-2}$ and $\phi_{\text{FWHM}} = 4.2 \mu m$. The target is still carbon with $n_{e0} = 10^{23}$ cm$^{-3}$ and has an initial temperature of $T_{e0} = 200$ eV. $\Lambda_{\text{in}}$ was set to 3.8 in these calculations. These conditions correspond to a peak beam to background electron
Figure 7. The effect of magnetized transport in the background plasma on $T_e$ at 3 ps: with (a) and without (b) magnetization effects. (c) Slices of $T_e(y)$ with magnetized and unmagnetized transport at $x = 5 \mu m$ at 2 ps (lowest curve), 3 ps and 4 ps (highest curve). Conditions are as described in figure 1.

Figure 8. Comparison of $q_y|_{\text{tot}}$ from the VFP-PIC simulation (red curves) and reconstructed total heat flow (using Braginskii’s transport relations) with all magnetization effects retained (dashed black curves) and for unmagnetized transport ($\omega \tau = 0$) (dashed blue curve). (a) At $x = 5 \mu m$ and 2 ps. (b) At $x = 60 \mu m$ and 1 ps. Conditions are as described in figure 1.

density of $n_f/n_e = 1/70$. The above intensity corresponds to a fast electron temperature of $T_f = 0.8 \text{MeV}$. Other important FEB injection parameters are $\eta_L = 0.071$ and $\theta_{1/2} = 32.8^\circ$ ($M = 4$). A higher spatial resolution of $\Delta x = 0.2 \mu m$ and $\Delta y = 0.05 \mu m$ was used, corresponding to 280 and 400 grid cells in the $x$ and $y$ directions, respectively. IMPACT used 40 $v$-grid cells and $v_{\text{max}} = 10v_0$ where $v_0$ is the thermal velocity at the initial temperature.

At 0.66 ps, the fast electron current-density profiles with magnetized and unmagnetized transport are almost identical; figure 9(a) shows the magnetized case. By this time the Hall parameter has reached about 0.1 around each filament and also around the injection region. However by 2.9 ps, we see from comparing figures 9(b) and (c) that the fast electrons are choosing different filament paths in the magnetized and unmagnetized cases. In both cases, there appears to be more hosing of the FEB at 2.9 ps compared to 0.66 ps. At this time $B_z$ reaches a significantly higher peak value of about 3200 T around the filaments in the magnetized transport case, compared to 2500 T with unmagnetized transport. The peak Hall parameters observed are 3.4 and 1.8 for the magnetized and unmagnetized transport cases, respectively. This difference is a consequence of how magnetized transport affects the evolution of $T_e$ and $B_z$, which in turn influences the
Figure 9. Fast electron current density $|j_f|$ (Am$^{-2}$) at 0.66 ps (a) and 2.9 ps with (b) and without (c) magnetized electron transport in the background plasma. Conditions: $I = 5 \times 10^{19}$ W cm$^{-2}$, $\phi_{FWHM} = 4.2$ $\mu$m (Gaussian), $\eta_L = 0.071$, $\theta_{1/2} = 32.8^\circ$, $T_0 = 200$ eV, $n_0 = 10^{23}$ cm$^{-3}$ and $Z = 6$. $|j_f|$ at 0.66 ps is almost identical with and without magnetized electron transport. FEB propagation. Noticeable in the unmagnetized case is that two filaments have merged forming one dominant filament. Conversely, the upper filament splits into two in the $\omega\tau = 0$ transport case. Notice also that the roots of the filaments occur at a shallower depth of approximately $x = 9$ $\mu$m in the magnetized case, compared to $x = 12$ $\mu$m in the unmagnetized case. We also see hollow $T_e$ profiles in the filaments and in the hot spot just beyond beam injection for this more extreme system; it is qualitatively similar to that seen in figure 7, but more pronounced. The gross effect of magnetized transport on the FEB path is also more pronounced for the conditions here in comparison to those used in figure 7.

Two possible reasons for the different filament paths could be: (1) The way that the filaments ‘collect’ the fast electrons from the wide injected beam is affected by the depth at which the filaments start. (2) The different evolution of the background plasma under the current filaments in the magnetized and unmagnetized cases is feeding back onto the propagation of the filaments.

4.3. Non-local effects

We observe that lateral electron thermal transport in the background plasma is marginally non-local in the region just beyond injection. For the $I = 10^{19}$ W cm$^{-2}$, $\theta_{1/2} = 27^\circ$, $\eta_L = 0.21$ calculation presented in figures 1 to 8, the peak value of $q_y|_{\text{tot}}$ obtained at $x = 5$ $\mu$m, at 1 ps is about 12% lower than $q_B^y$, the peak reconstructed value (found using Braginskii’s heat flow relation, plus the enthalpy/current contributions). At 2 ps the VFP-PIC result is about 20% lower as shown in figure 8(a). Note that no flux limiter was used when obtaining $q_B^y$. At $y = 2.5$ $\mu$m, approximately where the peak heat flux occurs, and at 2 ps the temperature is $T_e = 710$ eV, the lateral temperature scale length is $L_T = T_e/|dT_e/dy| = 9.1$ $\mu$m and the collisional mean-free-path is $\lambda_{\text{th}} = 0.086$ $\mu$m. This yields a non-locality parameter of $\lambda_{\text{th}}/L_T = 9.5 \times 10^{-3}$. Previously, in the context of nanosecond laser-plasma interactions, VFP calculations by others (neglecting B-fields) have demonstrated non-local reductions to heat flow for approximately $\lambda_{\text{th}}/L_T \geq 0.01$ [38, 39], with almost an order of magnitude suppression in very steep gradients ($\lambda_{\text{th}}/L_T \sim 1$). The reduction observed here is consistent with such previous findings. Deeper into the target at $x = 60$ $\mu$m, where there are filaments, lateral heat flow is described very well by Braginskii’s prescription at 1 ps (see figure 8(b)), but only with magnetized transport included. The reason why $q_y$ at 1 ps manifests greater non-local modification in the injection region than in the filaments could be down to differences in the magnetization levels. Where peaks in $q_y$ occur, $\lambda_{\text{th}}/L_T \sim 5 \times 10^{-3}$ at both 5 and 60 $\mu$m depths into the target. However, $\omega\tau$ is approximately 0.05 at $x = 5$ $\mu$m compared to approximately 0.1 at 60 $\mu$m.
depth. Quenching of non-locality by magnetic fields has been previously inferred in nanosecond laser–plasma interaction experiments [40]. Indications are that this is happening in the FET system here.

The electrical resistivity is crucially important to fast electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D VFP-PIC simulation, we observe a 1% reduction of field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping. At 1 ps in the 2D electron transport, through its role on resistive magnetic field generation and resistive stopping.

5. Discussion

5.1. Approximate model for magnetization growth

In the previous section we showed that significant levels of magnetization of the background electrons developed in the particular cases considered. This magnetization was shown to be sufficient to influence transport in the background and, in turn, the propagation of the fast electrons. We now develop an approximate and easy to use analytical model to estimate how the magnetization develops in time. We then show that this agrees well with the hybrid code results. The model serves to highlight the important parameters for achieving (or mitigating) magnetization, and allows approximate predictions of the peak $\sigma$ to be expected around a FEB or filament.

We assume a RB described by $j_L(y) = -j_0 \exp(-ay^2) \hat{x} = -j_0(y) \hat{x}$ which has a diameter of $\phi_{\text{FWHM}} = 2\sqrt{\ln 2}/a$, a stationary plasma that is initially uniform (starting at temperature $T_{\text{e0}}$) and with static $Z$ and $n_e$; induction of magnetic field ignoring resistivity gradients ($\partial_t \mathbf{B} = \alpha \nabla \times \mathbf{j}$), and that the temperature only changes due to ohmic heating. Given this, one finds that the Hall parameter $\omega \tau = (eB_z/m_e)\tau_{\text{OH}} \propto B, T_e^{3/2}$ develops according to

$$\omega \tau(y, t) = \frac{2\alpha_e a |y| j_L(y) \tau_{\text{OH}}}{n_e e} \left[ \frac{T_e}{T_{\text{e0}}} - 1 \right] \left( \frac{T_e}{T_{\text{e0}}} \right)^{3/2},$$

where $\tau_{\text{OH}} = \frac{5t}{2\tau_{\text{WHL}}} + 1$ (9)

where $\tau_{\text{WHL}}(y) = c_t n_e T_{\text{e0}}/\alpha_0 |j_L(y)|^2$ is a characteristic ohmic heating time constant. The above expressions assume that initially, $B = 0$ everywhere. Spitzer resistivity (unmagnetized) is used: $\alpha = \alpha_0 m_e/n_e^2 \tau_{\text{WHL}} = \alpha_0 (T_e/T_{\text{e0}})^{3/2}$, where $\alpha_0$ is the dimensionless resistivity coefficient [29] and $\alpha_t$ is the initial resistivity. Similar expressions for $T_e/T_{\text{e0}}$ and expressions for $B(y, t)$ can be found in [16] (also with $\nabla \alpha$ ignored in obtaining $B$) and [4, 23] (with $\nabla \alpha$ included). In the weak ohmic heating limit (WHL), defined by $t \ll \tau_{\text{WHL}}$, the Hall parameter becomes $(\sigma \tau)_{\text{WHL}} = 2\alpha_e a |y| j_L(y) t/(n_e e)$ which peaks at $|y| = \phi_{\text{FWHM}}/\sqrt{8 \ln 2}$ and yields

$$(\sigma \tau)_{\text{max-WHL}} = 0.421 f(Z) \frac{j_0}{n_e e} \phi_{\text{FWHM}} \propto \frac{(n_t/n_e) (\beta_e) t}{\phi_{\text{FWHM}}},$$

(10)

$$(\sigma \tau)_{\text{max-WHL}} = 0.514 f(Z) \frac{n_t \beta_e t}{(K_i/m_e c^2)^{1/2} n_{23}},$$

(11)

The form in (11) is a practical and convenient to use version of (10), made possible by using energy-flux balance arguments; $n_t I = n_t (v K_i) = j_L (v_{\text{th}})/e$. Here, $I_{\text{th}}$ is the laser intensity in units of $10^{18}$ W cm$^{-2}$, $t_{\text{ps}}$ the time in picoseconds, $(K_i/m_e c^2)$ the normalized distribution averaged fast electron kinetic energy, $n_{23}$ the background electron number density in units of $10^{23}$ cm$^{-3}$. The above expressions for $T_e/T_{\text{e0}}$ and expressions for $B(y, t)$ can be found in [16] (also with $\nabla \alpha$ ignored in obtaining $B$) and [4, 23] (with $\nabla \alpha$ included). In the weak ohmic heating limit (WHL), defined by $t \ll \tau_{\text{WHL}}$, the Hall parameter becomes $(\sigma \tau)_{\text{WHL}} = 2\alpha_e a |y| j_L(y) t/(n_e e)$ which peaks at $|y| = \phi_{\text{FWHM}}/\sqrt{8 \ln 2}$ and yields

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of $10^{23}$ cm$^{-3}$, $\phi_{\mu m}$ the beam FWHM diameter in microns and $(\beta_i)$ the distribution averaged fast electron speed normalized to $c$. The quantity $f(Z)$ is a weakly varying factor such that $f(1) = 1.719$ and $\lim_{Z \to \infty} f(Z) = 1$ (the Lorentz limit). This accounts for the variation of $\alpha_0$ with $Z$ [29]. The numerical constants in the above equations are given to three significant figures. In the strong heating limit (SHL), $t \gg \tau_{OH}$, the Hall parameter becomes $(\omega t)_{SHEL} = \frac{1}{2} (\omega t)_{WHL}$ so that equations (10) and (11) can be applied if multiplied by a factor of 5/2. The weak and strong heating limits are accurate for $t < \tau_{OH}/10$ and $t > 100 \tau_{OH}$, respectively, with $(\omega t)_{\text{max}}$ transitioning smoothly with $\log(t)$ for intermediate times.

Figure 11 shows a comparison between the magnetization growth rate—$d(\omega t)_{\text{max}}/dt$—predicted by the approximate model and that measured in a VFP-PIC simulation. These results are for the $I = 5 \times 10^{19}$ W cm$^{-2}$ system considered at the end of section 4.2 and shown in figure 9. There is good agreement in the injection region, with the observed value of $d(\omega t)_{\text{max}}/dt$ (measured at $x = 2 \mu m$) initially close to the weak heating limit (i.e. $d/dt$ of equation (11)) and then gradually rising towards the strong heating limit prediction. For this system, the characteristic ohmic heating time constant is about $\tau_{OH} = 50$ fs at the centre of the beam in the injection region where $j(y) = j_0 \approx 2 \times 10^{16}$ A m$^{-2}$ and using the initial resistivity value of $\alpha_0 = 2.5 \times 10^{-7}$ $\Omega$ m. As stated above, the SHL is only accurate for $t > 100 \tau_{OH}$, i.e., $t > 50$ ps in our case. This explains why the SHL is not reached in the hybrid simulation. Deeper into the target ($x = 50 \mu m$) where the FEB is filamented, the model overestimates initially but later on the simulation values lie between the WHL and SHL. Equation (10) is more practical for use in filaments, which carry only a portion of the injected FEB, and was used to obtain the theory estimate for the filaments in figure 11.

Note that these ‘filament theory’ estimates are relatively crude, simply using a single, representative value of filament current density ($j_0 = 1.3 \times 10^{12}$ A cm$^{-2}$) and filament width ($\phi_{\text{WHL}} = 0.8 \mu m$) at all times. These values are typical for the strongest, fully developed filaments and are over-optimistic when applied to the filament emergence phase. This likely accounts for the poor agreement for $t < 1$ ps.

The approximate model also agrees well for the main system under investigation (described in figure 1). In the beam injection region at 2 ps, we estimate $(\omega t)_{\text{max-Whl}} = 0.22$ and observe $\omega t \approx 0.2$ from the VFP-PIC simulation. The filaments here have a diameter of about $\phi_{\text{WHL}} \approx 1 \mu m$. Assuming they emerge at 0.5 ps, equation (11) estimates that $(\omega t)_{\text{max-WHL}} = 0.83$ after 1.5 ps of heating, whereas we observe peak $\omega t$ values in the range 0.6–0.8 in the filaments.

### 5.2. Implications of magnetization model

The good agreement between the approximate model of magnetization growth and the detailed VFP-PIC calculations occur despite the latter including dynamic FEBS, resistivity gradient effects and thermal conduction. This suggests that the estimates are quite robust indicators. It raises the prospect that the estimates are likely to be useful in other cases than considered here. The crucial prediction is then that the magnetization grows approximately linearly with time (and exactly linearly in the weak and strong heating limits) so that reaching values of $\omega t = 0.1–1$ is inevitable, given enough time. Such Hall parameters have been shown here to affect FET. Narrower current beams will speed up the time until magnetization sets in. A sensible lower limit for $\phi_{\text{WHL}}$ is the collisionless skin depth. Stronger beam currents, tantamount to larger $n_i/n_e$ ratios, will also accelerate the onset of magnetization. A closer look at equation (10) implies that magnetization is controlled by the transit time across the current beam at a speed equal to the drift velocity of the return current electrons.

### 5.3. Influences on the degree of non-locality

In this paper, we have only addressed fully ionized carbon plasma at near-solid density. In this case we see a 20% non-local reduction to heat flow and a ~1% non-local reduction to the electrical resistivity, in the FEB injection region. Non-locality should be more prominent (for the same FEB parameters, i.e., current and radius) at lower densities and/or in lower $Z$ plasmas since the collisional mean-free-path of the background electrons—which scales as $\lambda_{th} \propto T_{e}^{2}/(Zn_{e})$—will be longer. Similarly the collision time, $\tau_{th} = \propto T_{e}^{2}/(Zn_{e})$, will be longer. The non-locality parameter for thermal conduction is $\lambda_{th}/L_{T}$. Lowering $n_e$ also provides an additional indirect boost to non-locality by reducing $L_{T}$ (i.e. steeping temperature gradients). Referring to equation (9), reducing $n_e$ accelerates ohmic heating of the background by reducing the heating constant $\tau_{OH} \propto n_e$. The overall scaling of $\lambda_{th}/L_{T}$ on $n_e$ can be estimated as follows. Choosing the strong heating limit, equation (9) implies that $T_{e}/T_{0} \propto (t/t_{OH})^{5/2} \propto (t/n_{e})^{2/5}$ and it can readily be shown that $L_{T} \propto (t/n_{e})^{-9/25}$. Together this
implies the following dependence on \( n_e, t \) and \( Z \) (at fixed FEB parameters) in the SHL:

\[
\frac{\lambda_{th}}{L_T} \propto \frac{1}{Zn_e} \left( \frac{t}{n_e} \right)^{2\frac{2}{3}}.
\]  

(12)

Notice that the degree of non-locality is sensitive to lowering the density, depending on it slightly more strongly that \( n_e^{\frac{2}{3}} \), and also lowering \( Z \). It should also grow in time, slightly faster than linearly.

However, the VFP-PIC simulations here point to magnetization suppressing non-local modifications to heat flow. Quenching of non-locality by magnetizing the electrons has previously been inferred in nanosecond laser–plasma interaction \[40\]. Comparing equations (10) and (12) suggests that lowering \( n_e \) should result in the development of a higher degree of non-locality before magnetization builds up to a sufficient level to start quenching it. Taking non-locality suppression to occur at \( \sigma \tau \sim 1 \), suggests that the peak non-locality attained scales simply as \( (\lambda_{th}/L_T)_{\text{max}} \propto (Zn_e)^{-2} \). This scaling estimate glosses over the precise variation of \( L_T \) and \( \lambda_{th} \) across the FEB. Like the magnetization model, it omits processes such as thermal conduction.

Precisely how non-locality affects the evolution of the background plasma and the propagation of fast electrons is an open question. Because non-locality cannot easily be ‘turned off’ in our VFP-PIC hybrid code, we have only been able to make instantaneous comparisons between calculated fluxes (or transport coefficients) and values reconstructed from classical transport theory. An assessment of how non-local effects influence the long term evolution of FEBs should be possible by replacing the VFP description of the background electrons with a Braginskii-transport description and simulating the same system. The applicability of using a simple flux limiter to treat non-local suppression of heat flow (as widely used in ICF rad-hydro design codes) could readily be explored this way too.

5.4. Overall implications

The findings here have interesting implications for fast ignition and other situations where lower densities occur than considered here. The model developed in section 5.1 suggests that magnetization will occur more rapidly in lower density background plasmas than the several picoseconds needed for the near-solid densities considered here. Magnetization has already been shown to make a difference to FEB propagation under the conditions here. Greater magnetization at lower densities could see such effects enhanced. The approximate scaling analysis for the degree of non-locality presented in section 5.3, suggests that lowering the density will also enhance the level of non-local modification to thermal transport that develops before magnetization becomes sufficient to quench non-locality. In fast ignition, low-density regions that may be susceptible to high magnetization and high non-locality include the ablated plasma inside the gold cone (due to any prepulse or pedestal on the PW ignitor laser pulse) and plasma between the outer tip of the cone and the compressed fuel. In addition, ohmic heating is more rapid in such regions and heat flow will play a role sooner.

Thermal conduction and magnetization may also be important for FEB-divergence mitigation schemes such as guiding by resistivity structures \[22\] and using pre-applied guiding magnetic field \[11\]. We already showed the importance of thermal conduction in 1D VFP-RB calculations \[23\], including engineered resistivity structures. Though the work reported here only considers uniform \( Z \) plasmas, the persistence of the \( \nabla Z = 0 \) phenomena seen in 1D VFP-RB to 2D VFP-PIC calculations suggests that \( \nabla Z \neq 0 \) findings will still be relevant going to 2D and with dynamic FEBs. Considering the proposed pre-applied guiding magnetic-field scheme; we have seen the importance of Nernst advection of magnetic-field and magnetized heat flow in a different context of nanosecond LPI \[25, 27\]. The effect of these processes on the dynamics of the guide field need to be considered in the context of fast ignition.

6. Conclusions

We have demonstrated the first results from a novel 2D VFP-PIC hybrid code for fast electron transport, at fast electron beam density to background electron density ratios of up to 1/70. This approach solves the Vlasov–Fokker–Planck equation for the background plasma, rather than employing a fluid model used in other hybrid FET codes. The results shown here extend the 1D VFP–rigid-beam calculations reported recently \[23\] into 2D including an evolving fast electron beam. They show that electron thermal transport in the background plasma can significantly affect the temperature and \( B \)-field evolution for a near-solid-density carbon target, when the background is ohmically heated by a relativistic electron beam generated by a laser of intensity \( I = (1–5) \times 10^{19} \text{W cm}^{-2} \) and focal radius of a few micrometres. We see strong heat fluxes capable of removing \( \sim 50\% \) of the ohmic power density deposited at the peak of the injected FEB and its filaments (that emerge a few tens of microns into the target). Heat flow alters resistivity gradients leading to the removal of beam-hollowing and defocusing resistively generated magnetic fields near beam injection point and the re-establishment of collimating magnetic field near the centre of the beam. In the cases considered, the background electrons become weakly magnetized after a few picoseconds, with a peak Hall parameter of \( \sim 4 \) observed. Even these modest levels of magnetization have been shown to have a significant effect on the evolution of the temperature of the background plasma and the long term evolution of the fast electron filaments beyond a few picoseconds. Magnetized suppression of diffusive heat flow across field lines and Ettingshausen heat flow (associated with the return current) are found to be primarily responsible for a hollow temperature profile across the FEB and filaments. The presented analytical model for the evolution of magnetization—which agrees well with the VFP-PIC results—indicates that peak magnetization develops linearly with time and is inversely proportional to the electron beam width. This estimate accounts for thermal changes to the resistivity. Non-local suppression to thermal heat flow...
of up to 20% has been observed near the beam injection region. In the vicinity of the centre of the injected beam, where the plasma is unmagnetized, a non-local modification to the electrical resistivity of around 1% occurs. Where the plasma becomes magnetized, this magnetization dominates the change of the resistivity. Estimates indicate that non-local effects are expected to become more prominent at lower plasma densities than addressed here, but that even then magnetization will eventually reach levels capable of suppressing non-locality.

While these calculations concern an idealized case in comparison to a full-scale Fast Ignition scenario, they serve to highlight the potential importance of heat flow including magnetization effects in full-scale calculations. This work also suggests that thermal transport out of plasma ohmically heated by the fast electron beam will be non-local in regions well below solid density.

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