The stress-strain state simulation of slopes with dynamic loads

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Abstract. The article discusses the dynamic processes simulation in landslide slopes during dynamic effects. Dynamic effects can be caused by pulsed, including shock loads. Also, the movement of vehicles along the slope is considered. The solution is based on the general massive load movement dynamic problem formulation = on a massive basis. Discretization over the spatial domain is obtained in the finite element method (FEM) form. The equations solution is performed by the continuation method with respect to the time parameter. Direct methods of the integrating dynamics problems are used. Considering the high dimensionality of the problem in the spatial formulation, to construct a numerical solution, we use the previously obtained explicit absolutely stable direct integration schemes. The methods used can be considered as explicit, absolutely stable schemes for the direct integration of a dynamic problem with a variable mass in time. Recommendations for evaluating the numerical solution accuracy are given.

Introduction
Taking into account the true loading peculiarities leads to the need for dynamic calculations in determining the ultimate loads. The point is not only that the direct impact can be “fast”. But even with a sufficiently slow impact, considerable accelerations can occur in the system (as, for example, in the case of a structure fragment sudden destruction). Therefore, for a more accurate system response assessment (especially when taking into account unloading in certain zones that have reached the yield point), use the dynamic equilibrium equations and, therefore, calculation methods for dynamic effects [1-10].

In addition, impacts that are dynamic in nature can be transmitted to the slope. For example, shock and impulse effects resulting from construction and installation works; dynamic impact of mobile transport if there is a road on the slope; seismic effects, including induced seismic.

Often used in engineering practice, an approach in which dynamic effects are taken into account by multiplying the static load by a certain “dynamic factor” is often not justified. Since, firstly, the correct dynamic coefficient justification is possible only for an oscillator. Secondly, the dynamic structure response may differ from the static solution not only quantitatively but also qualitatively. A dynamic solution can have a completely different deformations distribution nature and efforts over the volume of a structure, rather than a static one, even in the case of a dynamic effect on an oscillator.

The technical methods main disadvantages analysis for calculating landslide slopes and the current state of computational mechanics made it possible to formulate the following approach to solving the problem stated. First, the model should not use a priori defined parameters and characteristics. For example, it is impossible to specify the slip areas shape in a landslide slope. Such characteristics and properties should be the model implementation results, and not the input data.
Secondly, the model needs to take into account the protective structures reinforcing effect (retaining walls, tongue-and-groove rows) and their compliance on the stress fields distribution and deformations in the landslide massif. The pressure distribution on reinforcement structures is determined by the joint calculation of the slope and reinforcement structures.

It is also necessary to take into account the dynamic effects arising in the construction or operation process of the structures near the landslide slopes. These include, for example: dynamic effects during construction (consolidation, pile driving, transport and installation strikes), traffic, seismic effects, dynamic effects when a sudden movement or soil mass partial collapse.

However, the spatial problem causes a sharp increase in the resolving equations system order. The increase consequence in the order is the impossibility of carrying out calculations on a finite-element grid with acceptable density on modern computers. Many authors use the rather emotional term “dimension curse”.

Traditional forms of FEM imply the system global stiffness matrix formation and its subsequent solution. The systems order of equations in real-life problems is measured in hundreds of thousands and millions of unknowns. Reducing the global matrix order with the help of the so-called “super-element” approach does not always allow to bypass the restrictions on the problem dimension. The fact is that the “super-elements” approach to the traditional FEM has two drawbacks. The first is the super-elements ensemble stiffness matrix conditionality deterioration, which leads to a decrease in the solution accuracy. The second is associated with a sharp increase in the super-elements ensemble stiffness matrix ribbon width. At the same time, the advantages of reducing the equations system order are lost, since often the storage of a matrix of a smaller order with an increased tape width requires more resources of the operational and external memory of a computer than for a standard stiffness matrix.

The mechanics real dynamic problems solution in the spatial formulation is almost impossible by the traditional methods. When solving the non-stationary problems, the methods of “continuation by time parameter” are used; otherwise, step-by-step approaches for solving the non-stationary problems are used. The time segment is divided into a series of segments. Using special formulas unique for each method, they make the transition from the VAT parameters at the initial moment to the time partition first point. Then a similar transition is made to the second point, using the initial values at the first point, etc. The number of steps in time can be quite large, and within each step a task is solved, in terms of complexity and appearance, corresponding to a certain quasi-static problem. As a result, the process overall complexity increases dramatically.

**Main text**

2.1 Explicit stable integration schemes for the motion equations with allowance for the moving mass

Constant increasing mass and speed of road transport cause a steady increase in the rolling stock effects on the road, and, accordingly, on the landslide slope.

Similarly, to the variable mass body motion equation derivation, one can apply to the medium differential element the theorem on the momentum change. In this case, the small volume element \( V_2 \) motion equation will obey the Meshchersky equation. Taking into account \( \rho_0=\text{const} \), the motion equation of an element with a variable mass is written as (1):

\[
A\sigma + p + (v_t - \dot{u})\frac{d\rho}{dt} = \rho \ddot{u}, \quad \epsilon \in V_2, t
\]  

(1)

Taking into account static boundary conditions on the surface \( S_1 \), kinematic boundary conditions on the surface \( S_2 \) and initial conditions, the elasticity dynamic equations theory system [8, 9] with a variable mass is written in the form (2):
\[
\begin{aligned}
A\sigma + p &= \rho u, \in V_1, t \\
A\sigma + p + (v_i - u) \frac{d\rho_t}{dt} &= \rho \dot{u}, \in V_2, t \\
\varepsilon &= A^T u, \in V, t \\
\sigma &= D\varepsilon, \in V, t \\
A_\sigma - p_s &= 0, \in S_1, t \\
u &= u_s, \in S_2, t \\
u &= u_0, \dot{u} = \dot{u}_0, \in V, t = 0
\end{aligned}
\]

The variable mass \( v_t \) movement speed as well as the mass change rate \( \frac{d\rho_t}{dt} \Delta V \) are the deterministic time functions. These functions are determined by the selected movement mode and the rolling stock type.

Using the well-known Gertin technique \([7–10]\), we construct a Lagrange-type variational equation in convolutions for a system with a variable mass. Performing the convolution \([122, 136, 164]\) of the equilibrium equation in the system volume and the static boundary equations conditions with \( \delta u \), we obtain the integral identity (3):

\[
\int_{(V)} \left( A\sigma + p + (v_i - u) \frac{d\rho_t}{dt} - (\rho_0 + \rho_t) \dot{u} \right) \delta u dV + \int_{(S_1)} (A_\sigma - p_s) \delta u dS = 0
\]

Transforming the integral over the volume according to the Ostrogradsky-Gauss formula, the integration formula in parts and taking into account the displacements variation isochronism \( \delta u(0) = \delta u(t) = 0 \), we obtain the Lagrange-type variational equation for the motion of a system with variable mass in convolutions (4):

\[
\int_{(V)} \delta \left( A^T u \right) \sigma dV - \int_{(V)} \delta u * p dV - \int_{(S_1)} \delta u * p_s dS - \int_{(V)} \delta u * (v_i - u) \rho_t + \\
\frac{1}{2} \int_{(V)} (\rho_0 + \rho_t) \dot{u} * \dot{u} dV = 0
\]

The motion equations in the finite element method form are constructed in a known manner. For this, the whole area is divided into a finite elements’ ensemble. Within each finite element, an approximation is made of a vector — functions of displacements by the finite element nodes displacements: \( u = \Phi q \), where \( \Phi \) is the finite element coordinate functions matrix, depending only on spatial coordinates. The deformations are determined by the nodal displacements according to the formula: \( \varepsilon = A^T u = A^T \Phi q = \Phi \Phi \), where \( \Phi = A^T \Phi \). After the stresses elimination by the formula \( \sigma = D\varepsilon = D\Phi q \), we have the one i-th finite element motion equation taking into account the moving mass (5):
\[
\begin{aligned}
&M_i + M_r \dot{q}_i + C_i \dot{q}_i + K_i q_i + M_i \left( \ddot{q}_i - \dot{q}_i \right) = P_{i0} + P_i(t), \\
&\text{if } t = 0 \quad q_i = q_{i0}, \quad \dot{q}_i = \dot{q}_{i0}
\end{aligned}
\]

\[
M_i = \int_{(V)} \rho_i \phi_i^T \phi_i dV - \text{finite element mass matrix,}
\]

\[
K_i = \int_{(V)} \Phi_i^T D \Phi_i dV - \text{finite element stiffness matrix}
\]

For a finite elements ensemble that simulates a computational domain, the motion equations are constructed by the well-known elementwise summation technique. Finally, the finite element ensemble motion equation [5–9] with regard to the moving mass is written in (6):

\[
\begin{aligned}
&\left( M + M_r \right) \ddot{q} + C \dot{q} + K q + M \left( \ddot{q} - \dot{q} \right) = P_0 + P(t), \\
&\text{if } t = 0 \quad q = q_0, \quad \dot{q} = \dot{q}_0
\end{aligned}
\]

\[
q = q(t) - \text{nodal displacement vector function,} \quad P = P(t) - \text{external load vector function,}
\]

\[
M = \sum M_i - \text{global ensemble elements matrix,} \quad C = \sum C_i - \text{global system damping (viscosity) matrix,}
\]

\[
K = \sum K_i - \text{elements ensemble global stiffness matrix}
\]

We use the common hypothesis about the mass matrix diagonal nature. The stiffness and damping matrices are represented as the sum (4):

\[
\begin{aligned}
C &= C_1 + C_2, \\
K &= K_1 + K_2
\end{aligned}
\]

In (7) \( C_1, K_1 \) define the diagonal positive definite matrices in which the elements on the main diagonal do not exceed the damping and stiffness corresponding to those in the matrices. Let us transform (6) in view of (7) and perform the convolution operation of the obtained equation with some continuous function \( g(t) \). The system (6) is converted to (8):

\[
g(t) * \left[ \left( M + M_r \right) \ddot{q} + C \dot{q} + K q + M \left( \ddot{q} - \dot{q} \right) \right] = \\
g(t) * \left[ P_0 + P(t) - C_2 \ddot{q} + K_2 q \right]
\]

When approximating the desired nodal displacements vector function within a small time interval \( \Delta t \) according to [159, 180, 183, 184], we assume that the left side of the equation (8 contains a function with uncertain parameters: \( q \approx q_1(q_n, q_{n+1}, \dot{q}_n) \), and the parameters on the right are...
known from the previous steps: \( q \approx q_2(q_n, q_n') \). At the first stage, the vector functions \( q_1 \) and \( q_2 \) are represented as (9):

\[
\begin{align*}
q_1 &= q_n + q_n \tau + (q_{n+1} - q_n - s_n) \frac{\tau^2}{\Delta t^2}, \quad \tau \in [0, \Delta t], \quad s = \dot{q} \Delta t \\
q_2 &= q_n + q_n \tau 
\end{align*}
\]  

Having determined the stability parameter, an explicit unconditionally stable scheme of epy motion equations direct integration with variable mass is written in (10):

\[
\begin{align*}
\left[ M_n + M_{n+1} + \Delta t C_D + 0.5 \dot{\lambda} \Delta t^2 K_D \right] \ddot{q}_{n+1} &= \\
\left[ M_n + M_{n+1} + \Delta t C_D + 0.5 \dot{\lambda} \Delta t^2 K_D - \Delta t^2 K \right] \ddot{q}_n + \\
&+ \left[ M_n + M_{n+1} + \Delta t C_D - \Delta t C + 0.5 \dot{\lambda} \Delta t^2 K_D - 0.5 \Delta t^2 K \right] q_{n+1} + \\
0.5 \Delta t^2 (P_n + P_{n+1}), \\
s_{n+1} &= -s_n + 2(q_{n+1} - q_n), \quad s = \dot{q} \Delta t, \quad \dot{\lambda} = \max \{\text{eig}(K_D^{-1} K)\}
\end{align*}
\]  

The resolving equations system (10) has a diagonal structure that allows trivial inversion. The main thing is significantly reduced the need for computer memory resources. However, when developing an algorithm for solving a problem on a computer, one should correctly choose an algorithm for forming the system’s right-hand part.

If we consider the matrices multiplication by vectors (the result is a vector) directly, then it will be necessary to form these matrices. And, thereby, all the explicit scheme advantages will be lost. Therefore, the basic formulas should be used quite carefully. When forming the right-hand side, it should be taken into account that the resulting vectors can be built and accumulated elementwise, and the need to build high order global matrices is eliminated.

2.2 Reduced quasi-static load when taking into account long-term dynamic effects
To assess the structure stress - strain state in the long - term forecast, taking into account the dynamic effects, the proposed by Matua V.P. [103], the converting dynamic load method to a time variable is “quasi-static”, constant within certain “basic” time intervals. As such, the time intervals during which the intensity should be selected and the movement composition can be taken quite constant (1 hour is accepted in the work). Accounting for seasonal and daily fluctuations of temperature and humidity factors in the road construction will be provided automatically, since their sensitive change takes much longer.

The method of constructing a "quasi-static load" is as follows:

A calculations series are made of the various brands impact of motor vehicles moving on the road structure in a spatial formulation (taking into account the actual and future intensity and composition of traffic, the vehicles movement speed and their location on the roadway).

For each possible impact option, the maximum “quasi-static load” is first calculated from the maximum displacements equality condition at the cross-sectional road points (from a moving vehicle and the maximum reduced load:

\[
\overline{P}_{\text{max.stat}} = A_1 \times \overline{q}_{\text{max.dyn}}
\]

\( A_1 \) - is a compliance matrix at the section characteristic points.
Then, the “reduced load” is determined for this vehicle impact variant and the estimated time period T, using the long-term integral effects coincidence condition. The decisions coincidence of the operator’s analysis according to the dynamic and quasi-static statements:

\[ M\ddot{q} + K_0(1 + K) * q = P(t), \quad K_0(1 + K) * q = \tilde{P}(t) \]

shows that the displacements q equality is possible under the corresponding impulses equivalence condition for the billing period:

\[ \int_0^T \sigma_0 dt = \int_0^T \sigma(t) d(t). \]

Then the “reduced static load” for this impact type (this type of vehicle, its speed of movement and location on the roadway) for the billing period

\[ \Pr, \text{ sing} = \frac{\int_0^T \sigma(t) dt}{T} \cdot \text{Pmax, stat.} \]

It should be noted that a similar operation is carried out for each vehicle intended for inclusion in the movement.

Further, all the “reduced loads” from those vehicles included in the movement package for period T are summed up:

\[ \Pr = \sum_{i=1}^N (P_{T, \text{Sing}})i. \]

2.3 Experimental research

In the experimental part, a series of numerical experiments was performed on the dynamic effect of transport on a landslide slope. The real problem - the landslide slope in the Taganrog city with new buildings, and the passage of the road on the slope was considered. The slope simulation was performed for different variants of its strengthening.

Varied speed and vehicles type. For illustration, some graphs of maximum deflections and bulging points of the road structure are shown, depending on the vehicle speed.
Summary

The main spatial domain discretization method is the finite element method (FEM), which allows to take into account the system topology complex form and arbitrary boundary conditions. We describe the non-linear soil work using the elementary volumes limiting equilibrium hypotheses [2-4]. The load parameter problem evolution will, within the limit state hypothesis, determine the maximum load limit. Rapid dynamic effects are taken into account using absolutely stable direct integration schemes for the motion equations are taken into account, including the limit state hypothesis. Dynamic effects associated with a change in the qualitative picture of the VAT distribution, take into account the reduced quasi-static load, which gives the same quantitative and qualitative integral characteristics of the impact on the slopes.

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