Heretics of the False Vacuum: Gravitational Effects On and Of Vacuum Decay. 2.

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Abstract: This paper reexamines the question of vacuum decay in theories of quantum gravity. In particular it suggests that decay into stable flat or AdS vacua, never occurs. Instead, vacuum decay occurs, if at all, into a cosmological spacetime. If the latter has negative cosmological constant, it generically undergoes a Big Crunch, which suggests that the whole picture is inconsistent. The question of decay of de Sitter space must be very carefully defined.

Keywords: Vacuum Decay, Instantons.
1. Introduction

The possibility that we are living in a false vacuum has never been a cheering one to contemplate. Vacuum decay is the ultimate ecological catastrophe; in a new vacuum there are new constants of nature; after vacuum decay, not only is life as we know it impossible, so is chemistry as we know it. However, one could always draw stoic comfort from the possibility that perhaps in the course of time the new vacuum would sustain, if not life as we know it, at least some structures capable of knowing joy. This possibility has now been eliminated. S. Coleman, F. De Luccia, 1980

In a series of beautiful papers written in the 1970s and 80s, Coleman, with Callan and DeLuccia [17] applied the instanton methods invented for dealing with semiclassical decay in statistical mechanics[18] and multivariable quantum mechanics[19] to the decay of a metastable vacuum state in quantum field theory. The present work, which takes part of its name from the last paper in this series, is meant as a critical examination of what these results might mean in a full fledged theory of quantum gravity.

It is important to recognize the historical context in which the Coleman DeLuccia paper was written. Quantum and statistical mechanics, including quantum field theory,
primarily deal with isolated subsystems of the universe. The Poincare invariant vacuum state of quantum field theory, is a fiction, representing our conviction that spacetime is locally approximately flat and that the phenomena we are investigating are not terribly affected by the global structure of the universe. Coleman and DeLuccia realized that this could not be true of a truly cosmological application of the idea of false vacuum decay\(^1\). I will attempt to show that their results, and extensions of them constructed here, suggest that vacuum decay does not occur in many situations in quantum gravity. In many situations in which it may occur, I will argue that it cannot be thought of as the decay of one maximally symmetric spacetime into another.

Much of what I have to say is simply reemphasis and reinterpretation of the results of Coleman and DeLuccia. Those authors emphasized the use of the thin wall approximation to vacuum decay, because it gave very explicit and calculable expressions for decay amplitudes. This approximation is however misleading in one important respect. It neglects the variation in the scalar field (or fields) in most of spacetime and replaces it by a discontinuous jump between constant values. This is sometimes a good numerical approximation for the decay amplitude, but it gives the mistaken impression that the spacetime, both in and outside of the bubble, is well approximated by a maximally symmetric spacetime. Coleman and DeLuccia did not say this, and in fact, emphasize that in the case of decay into a spacetime with negative cosmological constant one rarely gets AdS space, but rather a Big Crunch spacetime. The actual result of this calculation has been ignored in most later discussions of the vacuum decay phenomenon\(^2\).

The first conceptual point that I want to make about vacuum decay is an elementary one. The idea of decay of a metastable state, presupposes the existence of a stable one, of which the metastable state is an excitation. In most (formal and informal) discussions of vacuum decay one supposes this to be the maximally symmetric spacetime that is found inside the bubble in the thin wall approximation. In the next section I will argue that the actual results of Coleman and DeLuccia, as well as elementary considerations about the formation of the metastable state starting from a maximally symmetric stable vacuum, give the lie to this supposition. Vacuum decay occurs, if at all, into

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\(^1\)I use the word cosmological to mean a discussion of the state of the entire universe. In much of the literature on inflationary cosmology, the part of the universe that we see is considered to be only a small subsystem of a much larger metauniverse. I will be cautious, and express reservations about whether or not the current discussion applies to the use of vacuum decay calculations in an inflationary context. The question cannot be answered without understanding the global structure of inflationary cosmology. It is inextricably intertwined with the discussion of whether there had to be a pre-inflationary state, or some kind of self-reproducing structure.

\(^2\)This is despite the fact that it was the occasion for one of the most amusing pieces of rhetoric in the Coleman canon.
open F(riedmann)-R(obertson)-W(alker) cosmologies, whose global geometry is very different from that of a maximally symmetric spacetime. The Big Crunch singularity of the CDL solution is, I believe, an argument that decays into spaces of negative cosmological constant do not occur in consistent theories of quantum gravity.

Section 3 is devoted to the decay of dS spacetimes. If dS spacetime can really be thought of as arising from a quantum theory with a fixed finite number of states[16] then the idea that it could decay into any sort of open universe violates the rules of quantum mechanics. What is really at issue is the question of whether some open universe model (with an infinite number of states) could contain a metastable subset of states which has observables similar to those in a dS universe. Similarly, we could ask whether the quantum theory of a dS space with one value of the cosmological constant, might contain a factor space of states which behaved for some time like a dS space of larger cosmological constant. There are thorny conceptual problems to be solved here, as well as technical problems with the definition of the instanton solutions, which might represent the decay. I will address these questions, and the reader must judge whether I have resolved them. The picture I have been led to suggests that dS to dS transitions may occur in a thermal manner. The interpretation of the relevant instanton is highly observer dependent, and various measurement theory issues must be dealt with. Decays of dS space into an open, matter dominated FRW universe with vanishing cosmological constant may also occur, if one can establish a consistent theory of gravity in the FRW background. This would require an understanding of the Big Bang singularity. For some values of parameters in the Lagrangian, even transitions to negative cosmological constant Big Crunch universes might make sense. For these parameter values, entropy bounds do not make the assumed transition paradoxical. I should emphasize that none of these considerations prove that dS decay really occurs. They simply show that there is nothing in the semiclassical arguments nor in general principles like holography and unitarity that would prevent them from occurring. To really demonstrate the existence of these transitions, one must construct the quantum theory of the putative stable ground state to which the dS space decays.

In section 4 I will argue that the results of this paper apply also to the membrane nucleation process discussed by Brown and Teitelboim[24], which has been used as a mechanism for relaxing the cosmological constant[25].

In the Conclusions I will discuss what these results mean for various concepts in string theory, in particular the notion of an effective potential. I wish however to pause here to make clear the spirit in which this paper was written. Despite years of work, we still do not have a clear and complete nonperturbative definition of a quantum theory of gravity. Matrix Theory[26] and AdS/CFT[21] give us definitions in certain restricted circumstances. The former, defined in light cone gauge, is not suitable for
the discussion of vacuum decay. Its very formulation presumes the existence of null Killing vectors that do not exist in the spacetimes into which flat space is hypothesized to decay. While this by itself may be suggestive, it is hard to argue that the problem is with vacuum decay rather than with the light cone formalism.

I will use some AdS/CFT wisdom to throw light on the problem of vacuum decay, but for the most part my arguments will be, in the spirit of CDL, reliant on properties of classical G(eneral) R(elativity). Fischler and I have argued elsewhere\cite{15} that this may be one of our most reliable guides to the structure of the quantum theory.

Finally, I would like to note that, as is the case for most Hollywood sequels, the director of the original has long since severed all connection with the project\textsuperscript{3}. As a consequence, although the present paper may contain some grains of truth that were missed by CDL, it is sure to be less entertaining. The present author can only apologize to his readers for this, and urge them to go back and really read the work of the Old Master.

2. The real Coleman and De Luccia

I will begin by summarizing the work of Coleman and collaborators. However, it is convenient to first place it in a modern context. In the 70s and early 80s, Higgs fields were the paradigm for the scalars which occur in vacuum decay, and one was careful to try to work in situations where gravitational corrections to local physics were irrelevant. Thus, it is assumed, and supposedly self consistently verified, that gravitational corrections to instanton amplitudes are small, and gravitational effects only become important for the large classical bubbles which result from the decay.

In the modern context, the most likely candidate scalars are moduli fields. Thus, we imagine a compactification of M-theory to $d \geq 4$ noncompact\textsuperscript{4} dimensions, and study the effective field equations of approximate moduli\cite{14}. Since it makes no difference to most of our considerations, we will take $d = 4$ for notational simplicity. This requires us to be in a region of moduli space where “SUSY is broken by a small amount ”. It is not at all clear to me in what situations it makes sense to use the phrase in quotes. To fix ideas, imagine a scenario in which some combination of asymmetric orbifolding, flux stabilization and/or timelike linear dilatons in supercritical string theory have convinced us that it makes sense to write down an effective potential for “approximate moduli”, with metastable minima. Generically, we will get a Lagrangian of the form:

\textsuperscript{3}He won’t even collect residuals.

\textsuperscript{4}For the moment, we will include dS space under this rubric, though its spatial sections and Euclidean continuation are compact. We will soon see that this is very important.
\[-M_P^2 \sqrt{-g} R - \frac{1}{2} G_{ij}(\phi/M_P) \nabla \phi^i \nabla \phi^j - M^4 V(\phi/M_P) \]  

(2.1)

Here $M_P$ is the four dimensional Planck mass, $M \ll M_P$, and is supposed to be so because of the large fluxes, large number of extra dimensions etc.. In principle, there could also be fields (e.g. boundary moduli in Horava-Witten compactifications) whose natural scale of variation was $M$ rather than $M_P$. This would introduce small dimensionless parameters into $V$, as would inclusion of standard model Higgs fields (which might be such boundary moduli) and the like.

It is easy to see that a Weyl transformation, combined with passage to the dimensionless variables $\phi^i/M_P$ removes all trace of $M$ and $M_P$ from the equations of motion, and gives an action proportional to $(M_P/M)^4$. The dimensionful scale of variation of all fields is $M_P/M^2 \gg 1/M \gg 1/M_P$. Thus, the hypothesis that $M$ is small in Planck units, justifies the semiclassical approximation, as well as the neglect of higher derivatives in the effective action. There is, in general, no excuse for further approximation, though the small dimensionless parameters referred to above might provide one.

The solutions that we consider will all be four dimensional spacetimes with a three dimensional maximally symmetric subspace. Thus the metric will have the form

\[ ds^2 = \pm dz^2 + \rho(z)^2 d\Omega^2, \]  

(2.2)

where the second term is the metric of a manifold of a Lorentzian or Euclidean manifold with $SO(4)$ or $SO(3,1)$ isometry group. When $z$ is timelike, we will call it $t$. The scalar fields are functions of $z$ only. The solution thus defines a curve in the space of fields, $\phi^i$. For any such curve, define the path length $P$, by $P = \int dz \sqrt{G_{ij} d\phi^i/dz d\phi^j/dz}$. Then the scalar kinetic term in the Lagrangian will be proportional to $(dP/dz)^2$. The potential term, followed along the curve, will be some function $V(P)$ (a different function for each curve in field space). The variational problem can thus be split in two: find stationary points of the action for curves in field space with the constraint of fixed path length, and then solve the time dependent problem of how the path length varies with $z$. The latter is, up to notational changes, the one field problem studied by Coleman and De Luccia. The former is independent of gravity and depends only on the geometry of field space and the shape of the potential on it. The price that we pay for this simplification is that the typical path followed between two specified points in field space might be very long, and the $V(P)$ might be a very complicated function with many minima and maxima. For example, the typical metrics found on moduli space in the semiclassical approximation have chaotic geodesics. We must also worry about the fact that the transformation to path length becomes singular when the field space velocity vector, $d\phi^i/dz$ vanishes. For the standard CDL vacuum decay solutions this only happens at
the end of the trajectory, and causes no problem. We will see that in the analysis of dS decay, solutions with vanishing velocity are more common. For the most part, we will ignore this complication, and draw pictures of the simple double minimum potential used by CDL, but we will comment on the more complicated generic situation at various points.

Now let us recall the strategy of CDL. We will begin by making the assumption that the unstable vacuum has a negative cosmological constant, and then examine the zero cosmological constant limit. The more subtle case of a positive cosmological constant will be reserved for section 3. The instanton is a noncompact Euclidean geometry with metric

$$ds^2 = dz^2 + \rho(z)^2 d\Omega_3^2$$  \hspace{1cm} (2.3)

which is a three sphere bundle over a noncompact interval , $z \in [0, \infty]$. If we define, $U = -V$, the Euclidean equations are

$$(\rho')^2 = 1 + \frac{\rho^2}{3} E$$ \hspace{1cm} (2.4)

$$E = \frac{P'}{2} + U$$ \hspace{1cm} (2.5)

$$P'' + 3(\frac{\rho'}{\rho})P' + UP = 0.$$ \hspace{1cm} (2.6)

These are the Newtonian equations of motion for a particle moving in the potential $U$ with a dynamically determined friction (or anti-friction if $\rho'$ is negative)$^5$. The boundary conditions are that as $z \to \infty$ the geometry approaches that of Euclidean AdS and $P$ approaches $P_f$, the false minimum of the potential $V$. In fact, we should also require that near $z = \infty$, $P$ is a normalizable solution of the linearized AdS wave equation, since the instanton must represent an allowed fluctuation in AdS quantum gravity. The other boundary condition is that $\rho$ must vanish at the point we have conventionally chosen to call zero (the translation symmetry of the equations is a residual gauge symmetry, and we fix it by this choice). The equations are consistent and nonsingular only if $P'$ also vanishes at this point.

Geometrically, this is the requirement that the Euclidean manifold be smooth and have no other boundary than the one at infinity. From the point of view of instanton dynamics, the point $\rho = 0$ will be the tip of the light cone to which the bubble of true

$^5$They are also the equations for a scalar field in a negatively curved FRW universe with potential $U$. I am eschewing this interpretation to avoid confusion with the real FRW cosmologies, which arise after analytic continuation of the instanton.
vacuum asymptotes (using the language of the thin wall approximation). We obtain a Lorentzian manifold by analytically continuing the 3 sphere to a 2 + 1 dimensional dS space, obtaining an asymptotically (Lorentzian) AdS manifold, written as a dS fibration. The dS fibration becomes singular when \( \rho \) vanishes, but the manifold is smooth. We go to new coordinates by doing a double analytic continuation in which the dS space is continued to a Euclidean manifold with constant negative curvature and \( \rho(z) \to ia(it) \). \( t \) is then timelike and can be interpreted as the cosmic time of a negatively curved expanding\(^6\) FRW universe, with equations of motion:

\[
(\dot{\rho})^2 = 1 + \frac{\rho^2}{3} E
\]

\[
E = \frac{\dot{P}^2}{2} + V
\]

\[
\ddot{P} + 3\frac{\dot{P}}{\rho} \dot{P} + V_P = 0,
\]

where dots denote \( t \) derivatives. In order to keep the notation simple, we have used the same letter to denote the Lorentzian and Euclidean energies. They are not the same quantity, but we hope this will not cause confusion. This spacetime seems to have a big bang singularity at \( t = 0 \), but since \( \dot{P} = 0 \), this is no more dangerous than the apparent singularity in the FRW coordinates of AdS space. The space is not however AdS. We will see in a minute that \( P(0) \) cannot be sitting exactly at the true vacuum. It is determined instead by the conditions on the instanton at infinity. This accords with our usual ideas of tunneling: we tunnel not precisely to the true classical ground state, but to some point on the potential in its basin of attraction.

As the universe inside the bubble expands, the energy \( E \) decreases and becomes negative. Thus \( \dot{\rho}/\rho \) decreases and eventually goes to zero. If \( \dot{P} \) were exactly equal to zero at this point, the universe would stop expanding and \( P \) would oscillate forever around its minimum. This bizarre but possible state of affairs will not happen generically because the boundary conditions are all fixed at \( t = 0 \). Instead, the universe begins to recontract and eventually returns to \( \rho = 0 \). If \( \dot{\phi} \) happened to be exactly zero at this point, we would have indeed reached the true AdS vacuum. But again, this is a highly nongeneric situation which could only be achieved by fine tuning the potential. The actual solution has a Big Crunch Singularity for almost every potential. Thus, the conclusion of CDL is that if the true minimum has a negative cosmological constant (as it must if the false vacuum is AdS or flat) then the universe tunnels not to AdS

\(^{6}\)Since \( \rho \) starts from zero, it must increase.
space, but to disaster. We will reserve our comments about the meaning of this result until we have examined the instanton solution itself.

The potential $U$ is drawn in Fig. 1. Note that we have drawn the region near the minimum as a dashed curve, below zero. If the false vacuum energy were zero, this would be a necessity, but if it is negative it could be that the minimum of $U$ is positive. The dashes are supposed to indicate that this segment of the potential could be above zero. Our particle starts with zero velocity, near the higher maximum of $U$ at $z = 0$. It cannot start precisely at the higher maximum, because the solution with those boundary conditions remains at the higher maximum for all $z$ and does not asymptote to the Euclidean AdS space at the false maximum. Recall that starting precisely at the true maximum was the only way to save the system from the Big Crunch.
We now ask whether there are any solutions which have the right asymptotics. The parameter we have at our disposal to vary is the initial position of the particle. If there were no friction, we would be guaranteed a solution. Indeed, in that case energy is conserved and the particle would reach zero velocity at a value $P_f$ where the potential energy is the same as that at the initial point. If $P_f$ is a stationary point of the potential the transit time is infinite, as desired.

When we deal with a system with friction, such a conclusion is no longer guaranteed. We must surely start the particle out to the left of the point $P_1$ whose energy equals $U(P_f)$. In the case where gravity is neglected, Coleman argued that by starting the system very close to the higher maximum one could guarantee that the particle overshoots $P_f$ in finite time. Indeed, in that case the friction coefficient is just $\propto 1/z$ and is always positive. If we start with zero velocity, close to the maximum, then we stay there until very large $z$, friction is negligible, and when the particle finally moves it overshoots. When gravity is included, this strategy is ineffective, and in fact counterproductive. By going to large $z$, keeping $P$ near the maximum, we keep $P_2^2 + U(P)$ approximately constant and the friction coefficient stays approximately constant while $\rho$ gets large. This is like the slow roll regime of inflationary theories, where curvature is inflated away. But our goal is to get $P$ to its second maximum as $\rho$ goes to infinity and we have not progressed towards that goal. We still have motion in a potential with a substantial amount of friction and it is not clear if there will be enough energy to get to the second maximum.

If $U \geq 0$ everywhere, depending on the details of the potential, we may find that $P$ always oscillates down to the minimum of $U$ as $\rho \to \infty$. This would define a noncompact manifold, but would not be an instanton relevant to false vacuum decay. If $U$ is negative somewhere (as it must be if the false vacuum is at zero energy) an even worse disaster may occur. $\rho'$ may go to zero at a finite value of $z$, $\rho$ then turns around and begins to decrease, and we do not even get a noncompact instanton of the correct topology.

We see that, depending on the details of the potential $U$, there may or may not be an instanton that describes tunneling of a flat or AdS vacuum into a spacetime of lower cosmological constant. This is a generalization of a famous result of CDL. In the thin wall approximation, only one parameter characterizing the potential appears in the final answer - the vacuum energy difference. CDL show that when this parameter is too small, tunneling does not occur. Beyond the range of validity of the thin wall approximation, the answer to this question depends on the detailed structure of the potential.

We can also conclude from this analysis that in string compactifications with many moduli the chance that there is an instanton is likely to be smaller. We showed that the
effect of the higher dimensional moduli space and the chaotic behavior of its geodesics would be to make the distance in $P$ between the two maxima of $U$ large, and to introduce subsidiary maxima and minima into the effective potential. It seems clear that this will reduce the chance of the particle getting to the false vacuum maximum of $U(P)$.

Given the CDL conclusion about the nature of the spacetime that the instanton decays to, this is a welcome turn of events. The Big Crunch spacetime does not appear to be a stable ground state of anything. One might imagine a nongeometrical description of the quantum physics of this system, takes over at the Big Crunch. However, according to the FSB entropy bound [13], no observer in the spacetime can ever discover more than a finite (and fairly modest) number of quantum states. On the other hand, experiments done in the false vacuum, on times short compared to its decay time, can establish the existence of a much larger number of states. Thus, there seems to be a paradox in imagining one could decay to the other.

The nonexistence of the instanton for large classes of models provides us with a way of escaping this paradox. We are led to conjecture that in sensible theories of gravity in AdS or Minkowski vacua, there are no instanton solutions. This is of course well known to be the case for supersymmetric vacuum states [8]. In the thin wall approximation, the CDL bound on the vacuum energy difference necessary for vacuum decay, coincides precisely with the BPS bound on domain wall tensions. For two Susic vacua there is a static domain wall rather than a vacuum decay bubble. I suggest that no sensible theory of quantum gravity will contain a CDL vacuum bubble describing the decay of an asymptotically flat or AdS spacetime.

My conclusion from this analysis is that the semiclassical approximation gives us no evidence that decays of asymptotically flat or AdS vacua ever occur in sensible theories of gravity. As far as I can see, the only loophole in this argument is the possibility that supersymmetric theories might provide us with fine tuned potentials that guaranteed that the interior of the Lorentzian bubble was nonsingular. Numerical analysis of examples seems to show that this is not the case.

The AdS/CFT correspondence provides strong evidence that this interpretation of the semiclassical results is correct. According to the duality, the cosmological constant in Planck units is identified with a discrete parameter characterizing the CFT, for example a power of the rank of a gauge group. Smaller cosmological constants correspond to CFT’s whose high energy density of states grows more rapidly at infinity. The limit of flat space has a density of states which grows more rapidly than an exponential. From this point of view, it is difficult to understand how a system could tunnel into another with a smaller density of states. In non-gravitational contexts, tunneling is a low energy phenomenon. For energies above the barrier, the true and false vacua become
identical. AdS/CFT shows us without any doubt, that this is not true in quantum gravity.

If the low energy effective field equations gave unambiguous evidence for tunneling in a regime in which their validity was not in question, we would be faced with a paradox. Fortunately, they do not. Depending on the potential, we find either no tunneling, or tunneling to a state where effective field theory breaks down, and we seem to be faced with an entropy paradox. My conjecture would be that for any potential that we reliably compute from a genuine theory of quantum gravity, we will simply find that there are no instantons describing decays of flat or AdS vacua.

3. dS dK

3.1 The generic prescription

We now come to the question of the decay of dS space, about which there has been controversy both in the past[11] and more recently[20] . Indeed, it was the latter papers that stimulated my renewed interest in this issue. A very clear discussion of many of the issues can be found in a paper by Rubakov[9]. However, this author works in planar coordinates, which are ill suited for noticing many of the points that I will discuss below.

Before beginning I want to make a couple of points clear. First of all, if we admit that dS space is described by quantum mechanics with a fixed finite number of states, then it is obvious that it can decay neither into AdS space, flat space, a nonaccelerating FRW universe, nor a dS space with a smaller value of the cosmological constant. These systems all have larger Hilbert spaces than the original dS space. Whatever else quantum gravity does, if it obeys the most general principles of quantum mechanics, it cannot describe change of dimension of the Hilbert space. These remarks do not settle the issue, even if we believe that “real” dS space has a finite number of states. Attempts to find dS minima in controllable approximations to string theory will almost inevitably find them as metastable minima of an effective potential that goes to zero at infinity in moduli space. It is perfectly conceivable that the infinite system defined by a solution that stays near infinity in moduli space, could have a finite dimensional metastable subsystem that behaves like dS space for some period of time. One cannot use properties of finite systems to criticize this proposal (e.g. Poincare recurrences or a finite horizon) because the approximate dS space has to be presumed to be in contact with the other degrees of freedom of the system if one is not assuming from the start that decay does not occur. Thus, it makes sense to revisit the CDL proposal for dS space in the context of hypothetical metastable minima. Note that contrary to
the situation we encountered in discussing the decay of flat or AdS spaces, there is no paradoxical problem of trying to fit a large number of states into a box that is too small for them. In the decay of a noncompact vacuum, the false vacuum has a spectrum of high energy black hole excitations which is incompatible with any state of the system with more negative cosmological constant. In the decay of dS space, the true vacuum will have more states than the false vacuum as long as the true cosmological constant is nonnegative.

Another possible red herring in this discussion is the “decay” of dS space into a black hole[6]. Unlike the analogous decay of hot flat space[7] this is not really a decay. Viewed thru the lens of the static Hamiltonian, dS space is a thermal system. There is a finite probability for anything to be nucleated, including black holes. But unlike hot flat space, there is a maximal size black hole in dS space, and (for large dS radius in Planck units) it has much less entropy than the “dS vacuum”. It decays back into the vacuum state. Indeed, it is much more probable to nucleate a black hole that is not at rest with respect to the static observer, and falls back into the horizon much more rapidly than it decays. After this point it is interpreted as one of the states of the ”dS vacuum ensemble” by the static observer.

The first really troubling argument about the decay of dS space into flat space has to do with the inverse process. As mentioned in the introduction, decay into a stable state implies the possibility of producing the metastable state as an excitation of a stable one. This possibility was studied long ago by Guth and Farhi[27]. They showed that any attempt to create a local region of an asymptotically flat spacetime that was in a metastable dS vacuum led instead to the creation of a black hole. Furthermore, a singularity separates the external observer from the dS region, so even observers who jump into the black hole cannot see it. The mass of the black hole produced is determined not just by the value of the potential at the dS minimum, but also by the walls of the potential surrounding it. But as the dS minimum is made lower, we can also lower the walls (and keep metastability), so in this limit, the black hole mass is small. There is then a contradiction between the number of states of the system counted by the Bekenstein-Hawking entropy of the black hole, and the number counted by the Gibbons-Hawking entropy.

It is clear then, that one cannot create a metastable dS minimum as an excitation of flat space. Even if we wanted to resort to black hole complementarity [5] to invoke a complementary description of physics inside the black hole that might look like dS space, the clash of the number of states imputed to the system by the internal and external calculations, prevents us from doing so. Complementary observers may use non-commuting Hamiltonians to describe physics, but they agree on the Hilbert space dimension.
However, even this objection to metastable dS vacua may be overcome. Recall that CDL showed that the spacetime inside the bubble is not a symmetric space, but rather an open FRW cosmology. In the case of negative cosmological constant this leads to a disastrous Big Crunch, which we have chosen to regard as a signal that the decay will not occur in a sensible theory of quantum gravity\(^7\). However, a dS space can decay into a spacetime with non-negative cosmological constant, and in this case the FRW cosmology has a perfectly smooth future.

So the real question we must ask is whether this FRW cosmology defines a consistent set of asymptotic boundary conditions for quantum gravity. This is a thorny question, for any such cosmology has a Big Bang at a finite cosmic time in the past. It is conceivable that Big Bang cosmologies are acceptable. In [30], Fischler and the author argued that the Big Bang state should be thought of as one in which the universe was in a tensor product state of some number of two state systems. Each two state system describes the physics in a single, smallest, horizon volume. There is nothing singular about the quantum mechanics, though the geometrical description breaks down because areas are of order Planck scale.

The question of whether a metastable dS state can be formed in the throes of a hot Big Bang, was answered in the affirmative in Guth’s original inflationary models. One may question the validity\(^8\) of those models and the logic that led to them, but we do not really have a clear answer on this point. It is part of a general question of what the initial conditions are for inflationary cosmology, and whether the inflationary scenario is robust or fine tuned. It will be hard to answer these questions without a more complete quantum theory of cosmology than we have at present.

Thus, at the present time, there seems to be no paradox in assuming the existence of a metastable dS state that decays into an open FRW model with non-negative cosmological constant\(^9\).

Such dS decay would seem to be particularly natural for the models of [20]. These models have a potential, which goes to zero in the weak coupling region of moduli space. There are cosmological solutions, with negative spatial curvature in which the moduli remain within the weak coupling regime. It would seem natural for the metastable dS minimum constructed by these authors to decay into the state at infinity. However,

\(^7\)Actually, even this has to be rethought in the dS case, as we will see below.

\(^8\)I am speaking here of mathematical validity. These are certainly not good phenomenological models of cosmology.

\(^9\)The reader may wonder why I harp on the distinction between a late time open FRW model and asymptotically flat space. Although the local physics of these two spacetimes is similar in the asymptotic future, their spatial asymptotics are very different, and one has a Big Bang singularity which the other does not have. I believe the fundamental formulation of quantum theories of gravity depends crucially on the asymptotic boundary conditions.
they claim that within the range of validity of their approximations, no appropriate instanton exists. The current paper was begun in an attempt to further investigate this claim.

Thus, with these preliminaries out of the way, we can turn to the technical problem of constructing the CDL instanton beyond the thin wall approximation. Here I find myself differing substantially from the CDL analysis. CDL’s calculational approach is based on the idea of small deviations from flat space results, at least for the computation of the probability for the bubble to materialize. However, there is a qualitative difference between the Euclidean AdS and flat spacetimes, on the one hand, and dS on the other. Euclidean dS space is $S^4$, a compact manifold without boundary. If we ask for a spacetime of the form

$$ds^2 = dz^2 + \rho(z)d\Omega^2,$$  \hspace{1cm} (3.1)

with the same topology, then it must have two values of $z$ where $\rho$ and $P'$ vanish. Set $z = 0$ at the vanishing point closest to the true vacuum maximum of $U$, and at a point $P_0$ to the right of it in Fig. 2. As before, we cannot take $P_0$ to be the maximum itself, because the solution would not move from there.

Let us consider the case where the true vacuum has non-negative cosmological constant. Then the potential $U$ is everywhere negative. In the Newtonian model system, the particle starts with zero velocity at $P_0$ and falls to the right, slowed by friction. $\rho$ is increasing and the energy is decreasing. If we choose $P_0$ to the right of $P_1$, the point where the energy is equal to that at $P_f$, then the particle gets stuck in the well and its energy goes negative. Inevitably, $\rho'$ goes to zero and changes sign. $\rho$ begins to decrease, anti-friction sets in and the energy increases.

It is clear that $\rho$ will get to zero at a finite value of $z$. The question is, can we tune $P_0$ so that $P'$ also vanishes at that point? The answer appears to be yes. There is a solution of the equations with constant $P = P_m$, the minimum of $U$. If $P$ sits at this point, there is an oscillatory solution for $\rho$, with a period $(-U_{\min})^{-1/2}$. If we set $P_0$ just to the left of this point $P$ will also oscillate, for a while with a frequency given by the curvature of $U$ at its minimum. By tuning the initial value of $P_0$ we can make a zero of $P'$ coincide with a zero of $\rho$. So we have found a compact Euclidean instanton for this system, but it is not clear what its relation is to vacuum tunneling. The Euclidean solution never visits either the true or the false vacuum. Furthermore, in the thin wall approximation $P$ never oscillates, so our oscillating solution looks nothing like the thin wall solution.

What does it describe? If we follow the CDL prescription, we must analytically continue some coordinate of the sphere, to Lorentzian signature in the region between the two zeros of $\rho$. This gives a Lorentzian four manifold foliated by $dS_3$ cross sections.
The scalar fields vary in a complicated manner in the spatial $z$ coordinate. It is this peculiar manifold that we should think of as the state to which (either the true or false) vacuum tunnels. There are then two zeroes of $\rho$ across which we must analytically continue. This means that this instanton always corresponds to the nucleation of two bubbles, which then grow at a rate approaching the speed of light. If one of the zeroes of $\rho$ occurs at a value of $P$ that is in the basin of attraction (for the true potential $V \equiv -U$) of the false vacuum, then the Lorentzian continuation of the instanton in this bubble, will relax back to the point $P_0$.

The geometry is initially spatially curved, but as the field settles in to $P_0$ it begins to inflate. Locally, the geometry then approaches dS space in planar coordinates, and has a finite area cosmological horizon. Thus, asymptotically, every local observer finds herself in empty dS space. Unlike the case of asymptotically infinite spaces, the exis-
ence of cosmological horizons implies that we cannot describe observations that would distinguish between this spacetime and dS space. Thus, I will interpret the asymptotic behavior as relaxation to the dS vacuum state.

Inside the true vacuum bubble, things are as they were for vacuum decay in the CDL analysis. One asymptotes to the "true vacuum" if it is dS space, to a negatively curved FRW if $\Lambda_{\text{true}} = 0$ and does not asymptote at all (there is a Big Crunch) if the true cosmological constant is negative.

In fact there will be many solutions of this sort\textsuperscript{10}. Consider the variation of the solution as we vary the value of $P_0$ between the true and false maxima. There might be some solutions starting near the true maximum, which overshoot the false maximum. These will not lead to compact manifolds (unless there are other minima of $U$). Since the derivative of $P$ does not vanish when $\rho$ goes to zero for the second time, such solutions are singular. However, there is certainly a point $P_s$ somewhere to the left of $P_1$, for which the solution does not overshoot. Such a solution will remain trapped in the basin of attraction of $P_m$ until $\rho' = 0$ and $\rho$ begins to decrease. Then it may oscillate a few more times and try to escape over one or the other of the maxima. By tuning $P_0$ we can make the last oscillation of $P$, a point where $P' = 0$, coincide with the point where $\rho$ vanishes. There will be a discrete set of such solutions. Each will correspond to a two bubble spacetime, after analytic continuation. The interiors of the two bubbles can be relaxing to either the same vacuum (true or false) or different ones, depending on the choice of $P_0$. There is in fact an accumulation point at $P_m$ of initial conditions which give a compact geometry. Thus there is a discrete infinite set of smooth compact instanton solutions. The situation is somewhat reminiscent of the discrete infinite set of Einstein metrics on higher dimensional spheres\textsuperscript{3}\textsuperscript{11}. The solution with $P_0 = P_m$ is the Hawking-Moss instanton\textsuperscript{10}. However, the Lorentzian continuation of this solution does not look like either one or two vacuum bubbles. It is simply a dS space with cosmological constant equal to the value of $V$ at its maximum and $P$ sitting at the maximum. It clearly has a classical instability. Nonetheless, the action of the infinite set of two bubble instantons approaches that of the Hawking-Moss instanton as the initial point approaches $P_m$.

\textsuperscript{10}This is probably the place to remind the reader that our parametrization of the multifield problem in terms of path length, will be singular for the oscillating solutions we are describing here. The reader may if he wishes, regard $P$ as a single scalar field from this point on, in which case the discussion needs no modification. As far as I can see, the multifield case does not introduce any new features except a much larger family of solutions.

\textsuperscript{11}I would like to thank G. Horowitz for bringing this work to my attention.
The Euclidean action of a general solution of the instanton equations is given by

\[ S_E = -4\pi^2 \int dz [\rho^3 U + 3\rho]. \] (3.2)

It takes on the largest possible negative value for solutions which stay for a long time in regions where the absolute value of \( U \) is small\(^{12}\). In particular the solutions which accumulate near the Hawking-Moss instanton give subleading contributions to the tunneling probability. There is no reliable way to take them into account. It is intuitively plausible, but I have not been able to prove in general, that the dominant contribution always comes from the instanton with the smallest number of oscillations. Like the flat space bounce, this solution makes one traverse from the vicinity of the true vacuum to that of the false one. The Lorentzian history of the decay consists of the nucleation of two bubbles in a compact ellipsoidal three space, with spatial metric

\[ ds_3^2 = dz^2 + \rho^2(z)d\Omega_2^2, \] (3.3)

where \( d\Omega_2^2 \) is the metric of a two sphere. The poles of the ellipsoid are the nucleation points of the bubbles. The bubble walls accelerate to the speed of light, but the space between them expands more rapidly. The two bubbles remain forever outside each other’s horizon. The interior of each bubble has spatial sections which are homogeneous spaces of constant negative curvature. If both the true and false vacuum energy are positive, then a local observer inside each bubble will rapidly find himself in an environment resembling empty dS space with the appropriate value of the cosmological constant. If the true vacuum energy is zero, the observer in one of the bubbles will experience (asymptotically) a negatively curved, matter dominated open FRW universe. If the true vacuum energy is negative, this observer will instead experience a Big Crunch. Note that we cannot make as strong a statement about the implausibility of Big Crunch solutions in the dS context, if we believe that there is any sense in which the metastable dS space has a finite number of states. For some potentials, the number of states accessible to observers in the Big Crunch universe may be larger than those available to the false vacuum dS observer.

An important technical point, which I have not been able to resolve is a calculation of the number of negative eigenvalues for this instanton solution. Coleman’s general argument\(^4\) for a single negative mode for instantons in quantum mechanics and field theory conspicuously omits discussion of the de Sitter case. The issue has to do with gauge invariance. The action

\(^{12}\)Tunneling probabilities are given by exponentials of differences between the action of two solutions, and are always less than one.
\[ S = \int dz [\rho^3 ((P')^2/2 - U) - 3(\rho(\rho')^2 - \rho)] \]  

(3.4)

for the equations of motion we have been studying, is a gauge fixed version of the reparametrization invariant action

\[ S = \int dz e(z) [\rho^3 (\frac{(P')^2}{2e^2} - U) - 3(\rho(\rho')^2/e^2 - \rho)] \]  

(3.5)

Indeed, the Friedmann equation is properly viewed as the equation obtained by varying the einbein \( e(z) \). One can gauge fix to constant \( e \), and then incorporate the constant value of \( e \) in the parameter length, \( L \), of the \( z \) interval. This is why all values of \( L \) are allowed. The question now is how to properly count the fluctuation modes, including \( L \). For example, for fixed \( L \), the fluctuation corresponding to an infinitesimal \( z \) translation of the instanton, which is formally a zero mode, does not satisfy the boundary conditions. So the usual argument for the existence of at least one negative mode, does not hold. A related problem is that the full system of equations is not a Sturm-Liouville system. One must do a complete gauge fixing and eliminate all but physical modes (which is awkward) to get a proper answer to these questions.

It is interesting to compare these tunneling arguments for dS space with finite temperature tunneling in quantum field theory. At finite temperature, Euclidean time is also compact. For low temperature there are an infinite set of periodic instantons, which must all be taken into account. The instanton with \( N \) oscillations, gives the term of order \( T^{-N} \) in the expansion of the exponential of the free energy. Half of these instantons have an even number of negative modes and half have an odd number. This corresponds to the expansion of \( e^{i\Gamma} \) where \( \Gamma \) is the width of the metastable state. For temperatures which are not low, we do not have to worry about large inverse powers of \( T \) and only the lowest action instanton contributes to the calculation. Our dS calculation for generic values of parameters in the potential is similar to the finite temperature case at temperatures of order one.

One might similarly guess that half of our dS instantons have an even and half an odd number of negative modes. In particular, my guess is that the lowest action instanton, which does not oscillate at all, has precisely one. I hope to report a proof of this conjecture, as well as the conjecture that the lowest action solution does not oscillate, in a future publication.

Let us return to the interpretation of the instanton solution. The Lorentzian continuation of the instanton is a spacetime with observer dependent horizons. To see this, introduce static coordinates in the dS fibers of that portion of the manifold between
the two zeros of $\rho(z)$. The metric is

$$ds^2 = dz^2 + \rho^2(z)((1 - r^2)d\tau^2 + \frac{dr^2}{1 - r^2} + r^2d\Omega^2),$$

where $-\infty \leq \tau \leq \infty$ and $d\Omega^2$ is the metric on a unit $d-3$ sphere. These are the natural coordinates for a timelike observer following a geodesic with fixed $z$ at the point where $\rho' = 0$, and fixed position on the $d-2$ sphere. She sees a static ellipsoidal geometry. The Penrose diagram of the Lorentzian bubble space-time (Fig. 3) shows that a finite volume of the phase space of timelike geodesics, correspond to observers for whom both bubble walls are outside the cosmological horizon.
At the present time, quantum measurement theory depends (in my opinion) on the description of the measuring apparatus by an approximate local field theory. The device should thus be thought of as a localized low energy observer, following a timelike trajectory in spacetime. Thus, many devices will not register the existence of the vacuum bubbles.

The correct state of the field theory describing a device using the static coordinate system, should be a local excitation of the Euclidean vacuum, whose Green’s functions are defined by analytic continuation from the Euclidean section. Thus, these static observers will see a thermal state, with $z$ dependent temperature. The temperature goes to infinity at the zeroes of $\rho$, as well as at $r = 1$, for all $z$. The temperature at the observer’s position is finite, and, as long as there are no large or small dimensionless parameters in $U(P)$, it is of order $M^2/M_P$. Again it is apparent that these observers see no vacuum decay. This observation is a restatement of the problem of the failure of the true vacuum bubbles to percolate through the universe in old inflation models[2]. The difference is that, in old inflation models the false vacuum was a temporary feature of the effective potential caused by finite temperature effects. Here we are speaking of a global dS vacuum.

There are of course local observers in this spacetime who experience the interior of the two expanding bubbles. However, they are causally disconnected from each other, as well as from the static observers of the previous paragraph. There are three kinds of freely falling timelike observers in this spacetime. Many timelike geodesics miss both vacuum bubble light cones. Observers following these trajectories can be viewed as moving observers in a static, thermally excited spacetime. Those observers on the submanifold with $\rho'(z) = 0$ are completely static. Observers who pass through the the vacuum bubble light cones, penetrate into the vacuum bubbles. They experience a dS vacuum (with either true or false values of the cosmological constant), which is excited by a tunneling event to a non-vacuum value of the homogeneous mode of the scalar field. The resulting Bose condensate is then diluted by inflation and the local observer sees the dS vacuum again.

The picture of vacuum tunneling event is thus much more complicated in the dS case than in asymptotically infinite spaces. The spacetime before the tunneling event is the false dS vacuum. For local observers, this means a thermal state of quantum fields in the false dS background. After the tunneling event, observers break up into three classes. Some indeed see an abrupt transition to a regime that locally asymptotes to the true vacuum (which in the case of vanishing cosmological constant is a negatively curved FRW universe rather than Minkowski space). Some see a static ellipsoidal spacetime with position dependent temperature. We will refer to these observers with the adjective inhomogeneous. Others find themselves in an excited state of the false
vacuum, which quickly reverts to the false vacuum, because of inflation.

Note also, that since the manifold is compact, there is an equally good interpretation of this instanton as an event occurring in the true vacuum. The difference is only in the manifold we choose to glue in to the past of the Lorentzian continuation of the instanton. Since we are interpreting this solution as a tunneling event and are consequently willing to tolerate discontinuities in the classical evolution, this choice is arbitrary.

These results have an obvious thermal interpretation[12]. All local observers in spacetimes with horizons, experience a temperature. In both the false and the true vacuum, there is some thermal plus quantum probability to nucleate the state of the fields described by the initial conditions of the Lorentzian continuation of the instanton. Although the instanton is the same for the jump from true to false as for false to true vacuum, the probability is different. The log of the probability is given by \((S_{F,T} - S_I)\), for the false and true vacuum decays respectively. Here \(S_I\) is the instanton action and \(S_{F,T}\) the actions of the true and false dS spaces. I would like to interpret this as a statement of detailed balance. That is, in a system with thermal excitation we can have transitions between the two states described by the stable and metastable minima of the potential. The total probability of transition is thus determined both by an intrinsic matrix element, and by the number of final states. The ratio of the two transition probabilities is the ratio of the number of initial and final states.

W. Fischler\textsuperscript{13} brought up the question of whether the entropy measured by the static inhomogeneous observer in the aftermath of the instanton event is always less than that of the true vacuum dS space. This would be required if we were to interpret the instanton in terms of the quantum mechanics of the true dS vacuum. Then the full instanton, as seen by any observer, could consistently be viewed as an excitation of the true vacuum state. The principle of Cosmological Complementarity[30][1] would imply that some of the information in this spacetime could be viewed in complementary ways by causally disconnected observers. Only the observer in the true vacuum would have (in principle) access to all of the information. The inhomogeneous, and false vacuum observers would be viewing very special factor spaces of the full Hilbert space. In fact, the appropriate bound on the entropy appears to follow from Bousso’s proof of the N-bound[29] for spherically symmetric spacetimes. The only thing that confuses me about this claim is that it is not clear how to identify the quantity that Bousso calls the cosmological constant in the present models. The bound proves what we want it to if I identify the cosmological constant with the vacuum energy at the true minimum.

\textsuperscript{13}Private Communication
cosmological constant instead.

I should note that in “practical” applications the number of true vacuum states is much much larger than the number available to either of the other observers. That is, although experiment and the anthropic principle force us to accept a fine tuning of parameters to obtain a small cosmological constant in the low energy effective Lagrangian which describes our observations, there is no reason to insist on any other fine tuning. Thus, the size of the horizons seen by the inhomogeneous and false vacuum observers is determined by microscopic parameters. The probability for forming a false vacuum bubble in the true vacuum is thus very small. Furthermore, the instanton which describes the false vacuum bubble formation, actually describes the creation of a spacetime which is, over most of its volume, a small excitation in the basin of attraction of the true vacuum. According to Cosmological Complementarity, there are many different observers in this spacetime, but most of them observe the instanton event as just a small excitation of the true vacuum. A small subset of rather short lived observers can actually see either the false vacuum, or the inhomogeneous state created. Furthermore, the waiting time for a true vacuum observer to see the instanton event, is exponentially larger than the time for quantum fluctuations to destroy the measurements that a true vacuum observer can make with any instrument whose workings can be described by cutoff quantum field theory[28]. It is also exponentially larger than the expected time for the observer to be killed by spontaneous nucleation of a black hole at his position. Nonetheless, there is a finite, though small, probability that the true vacuum observer will see the instanton event which corresponds to the creation of a region of false vacuum, even though he never sees the false vacuum itself. An interesting question, to which I do not know the answer, is whether this observer can create a non-vacuum state which will relax to the instanton configuration. Since the region of false vacuum is to be created outside the cosmological event horizon of the observer, a negative answer does not automatically follow from the negative answer provided by Guth and Farhi[27] to the corresponding question in flat spacetime.

There is another situation which some authors might think of as having practical interest. We can consider a situation in which our own world is identified with the false vacuum, and there is another true vacuum with vanishing or much smaller positive cosmological constant. Such a system apparently needs at least two fine tunings for its construction. Moreover, the second fine tuning has no anthropic justification. I consider it unlikely that any quantum theory of gravity will be found which would be described by such a low energy Lagrangian. Nonetheless, since such constructions have been considered in the literature, it is worth discussing them. I will consider only the case where the true vacuum has a nonzero cosmological constant.

In this case, two classes of observers have macroscopic lifetimes, namely those
observers who see the instanton as creating a fluctuation in the basin of attraction of either of the vacua. For a generic lagrangian with two fine tunings, the inhomogeneous observers will still have a lifetime of microscopic proportions. Again, although the waiting time for the false vacuum observer to see the instanton event is large compared to any practically realizable lifetime for his observations, there is a tiny probability that he will observe it. Most static observers in the original false vacuum dS space will make a quantum jump to a small excitation of the true vacuum, while a few observers will just see a small excitation of their own false vacuum. Thus, there will always be a few heretics who will remain forever justified in their belief that vacuum decay never occurs.

3.2 Singularity of the flat space limit

In an attempt to gain some insight both into the form of the lowest action instanton, and the issue of negative modes, I have examined a limit in which the gravitational equations formally approach those of QFT in flat spacetime. It turns out that this limit is singular, and generates as much confusion as it solves. The requisite limit is one in which $U = U(aP)$ where $a$ is a large parameter. In terms of dimensionful scalar fields, this means that the potential has substantial variation on field scales much smaller than the Planck scale, as one would have expected for standard model Higgs potentials. Defining

$$p = aP, \quad (3.7)$$
$$x = az, \quad (3.8)$$

and

$$r = a\rho, \quad (3.9)$$

the equations become

$$p'' + 3\frac{r'}{r}p' + U_p = 0 \quad (3.10)$$

$$\left(r'\right)^2 = 1 + \frac{r^2}{3a^2}\left(\frac{P^2}{2} + U\right). \quad (3.11)$$

Primes now denote derivatives with respect to $x$. In the formal $a \to \infty$ limit, and taking the positive sign for the square root, these become the flat space equations and the interval for $x$ becomes half infinite.

Now consider following the flat space instanton, but keeping the “small” terms in the equations. For large $x$ one enters a region of large $r$ with very small $p'$. Eventually, the small term overwhelms the one in the equation for $r$. Furthermore it is negative. Thus, the perturbation is never negligible. This is not surprising. The topology of the
Euclidean manifold suffers a discontinuous change when $a$ goes to infinity. The finite $a$ manifolds are always compact.

In fact, we can see that boundary conditions for $P_0$ very close to that of the flat space instanton do not give rise to smooth compact solutions. When $r$ gets large, its derivative inevitably changes sign and it begins to decrease. But in this region, solutions close to the flat space instanton are very close to the false vacuum with small velocity, heading toward the false vacuum. When anti-friction sets in, they will be driven over the barrier and never come back. We will have to choose initial values at some finite distance (i.e. not proportional to an inverse power of $a$) from the flat space value in order to find a solution which does not fly over the barrier. Furthermore, in most nonsingular solutions in which $r$ grows to a value of order $a$ before beginning to decrease, there must be many oscillations. This is because the $p$ equation does not contain $a$, so its natural oscillation periods are of order one, while for $r$ of order $a$ or less, the decrease of $r$ by finite amounts also takes place over ranges of $x$ of order one.

For large $a$, it appears that the lowest action, nonsingular, instanton is quite close to the solution which sits at the true vacuum. As noted above, the increase and decrease of $r$ must take place over an $x$ range of order $a$, while the $p$ equation does not contain $a$. In order to obtain a nonsingular solution which does not have multiple oscillations, we must start near a maximum of $U$. If we start within $1/a$ of the maximum, with zero velocity, then $p$ will change by order one in an $x$ range of order $a$. By tuning the initial distance from the maximum we should be able to arrange that $p_x$ vanishes at the second zero of $r$. Note that this can only happen when $p$ is to the right of the minimum of $U$. Once $r$ begins to decrease, and anti-friction sets in the solution can decelerate only as it climbs the upward slope to the false maximum.

4. Other instantons

There are two other types of gravitational tunneling process, which have been widely discussed in the literature: the membrane creation of Brown and Teitelboim[24] and “decays to nothing”, as exemplified by the old work of Witten[23]. The Brown-Teitelboim work is closely related to that of Coleman-de Luccia. Decays to nothing are somewhat tangential to the work of this paper, but our general philosophy throws some light on them as well, so I will devote a small subsection to them below.

4.1 Membrane creation

The Brown-Teitelboim process seems to be a system in which the thin-wall approximation is nearly exact. It consists of the nucleation of a spherical shell of membrane in a background p-form gauge field, and its analysis is modelled on Schwinger’s calculation
of pair production in a constant electric field. More precisely, it is the higher dimensional analog of pair production in 1 + 1 dimensional electrodynamics - the massive Schwinger model.

The Schwinger model is amenable to systematic analysis using the technique of bosonization. It is exactly equivalent to a scalar field theory with Lagrangian

\[ \mathcal{L} = \frac{1}{2} \left[ (\nabla \phi)^2 - \frac{e^2}{\pi} \phi^2 + M^2 \cos(\phi) \right]. \] (4.1)

In this formulation of the theory, pair production is simply described as vacuum decay of the false vacua at minima of the cosine. These correspond to backgrounds with fixed quantized values of electric flux. The Schwinger approximation to tunneling, in which one considers a circular Euclidean electron path, corresponds to a step function approximation for \( \phi \), and is valid in only a limited region of the parameter space of the model. Even in that regime there are corrections to the step function approximation to \( \phi \).

In higher dimensions, although there is no exact bosonization formula, we can still introduce a scalar field through

\[ F_{\mu_1...\mu_d} = \epsilon_{\mu_1...\mu_d} \phi \] (4.2)

and write a derivative expansion for the effective action of \( \phi \), which includes effects of virtual membranes. General considerations show that it will have the above form, except that the cosine will be replaced by a more general periodic function. There are other, non periodic, terms coming from higher powers of the gauge field strength, but they are negligible at low energy.

Thus, I would claim that a more precise analysis of the Brown-Teitelboim process introduces corrections to the thin wall approximation, and that one must deal with the issues studied in this paper here as well.

In particular, in the discussions of relaxation of the cosmological constant by membrane creation[25], it is necessary to introduce a large negative cosmological constant in the low energy effective field theory in order to obtain vacua with very small cosmological constant. Tunneling transitions between a large number of vacua are then postulated. Many of these will be between two dS vacua. Proponents of this scenario would welcome a principle (such as the one we have proposed) which implied that there could never be transitions to the negative energy states. Nonetheless, it is disturbing to have the whole structure depend on the existence of the negative vacuum energy state, which can never be accessed by the system. My tentative conclusion is that such scenarios will not be meaningful in a real theory of quantum gravity.
4.2 Decay to nothing

Decay to nothing is an intrinsically gravitational effect, described in the simplest case by an instanton which is simply the Euclidean Schwarzchild solution. One treats the periodic coordinate as spatial, and analytically continues one of the coordinates of a sphere. The resulting manifold is asymptotically a circle times Minkowski space. In the interior it has nontrivial topology. This instanton is only allowed in nonsupersymmetric compactifications, because the fermion boundary conditions on the circle at infinity, violate supersymmetry.

The colorful phrase “decay to nothing” is not really accurate. In conventional vacuum decay in Minkowski space, any timelike observer is hit by a bubble wall in finite proper time, and its trajectory penetrates into a region of true vacuum. Assuming the measuring apparatus survives the passage through the bubble wall, it then measures the properties of the true vacuum forever after. In decays to nothing, the trajectories of timelike observers are bound to the bubble wall, but they cannot pass through it because there is no spacetime behind it. Spacetime has an asymptotic null infinity as well, that ends when it intersects the bubble wall.

Thus, there is no decay to a static vacuum state, but rather to a spacetime with no timelike Killing vector. What should one consider the candidate for the stable vacuum state into which the static, SUSY violating Minkowski space decays in this situation?

The answer to this question is intertwined with another serious problem with all known vacua in which instantons describing decays to nothing exist. They are perturbatively unstable. That is, although the tree level theory has solutions with a causal boundary similar to that of Minkowski spacetime\textsuperscript{14}, at one loop an effective potential is generated which completely changes the asymptotic behavior of solutions. In all examples I know of, the one loop corrected equations of motion have no solutions that remain under perturbative control.

The effects of loop corrections to the equations of motion are formally of lower order in the perturbation expansion than the instanton contribution. Thus, it seems mathematically inconsistent to consider the instanton as the dominant mode of “decay” of the tree level Minkowski vacuum state. There does not appear to be a metastable Minkowski vacuum state at all.

Equally disturbing is the fact that the instanton solution gives us no resolution of the problem posed by the perturbative effective potential. If the instanton indeed produced a bubble of a state which was not subject to the perturbative instability,\textsuperscript{14}

\textsuperscript{14}In the case of the Lorentzian continuation of the Witten bubble, future null infinity is cut off by its intersection with the “bubble of nothing”. By abuse of language, I call such a spacetime boundary “similar to that of Minkowski space”.
we might be willing to overlook the fact that the perturbative potential destroys the asymptotic behavior of the instanton. But no such bubble of true vacuum exists. All observers in the Lorentzian continuation of the instanton will feel the destabilizing effects of the perturbative corrections to the action.

It would appear that there are only two sensible conclusions about systems of this type. Either the semiclassical analysis is not an approximation to any quantum theory, or it is a bad approximation to a quantum theory defined with very different asymptotic conditions. In either case, the statement that a Minkowski vacuum state decays via a bubble of nothing would not be a valid description of the physics.

5. Conclusions

Since the seminal work of Coleman and De Luccia, instanton methods have been widely used in the study of quantum gravity. In this paper I have tried to argue that the idea of vacuum decay in quantum gravity must be re-examined with care. In fact, we have found no situations in which this notion makes unambiguous sense.

Implicit in the use of instanton methods in quantum mechanics and quantum field theory, were two propositions whose validity in theories of gravity is somewhat dubious. The first is that states with different ground state energies lie in the same Hilbert space. The second is that instantons demonstrate a decay of a metastable false vacuum state into a stable true vacuum. I have recently argued that the first proposition is in fact false[22]. Quantum theories are defined in terms of their high energy behavior. Among the high energy excitations of any theory of quantum gravity in asymptotically flat, or AdS spacetime, are stable or metastable black holes. Asymptotic darkness is the conjecture that these are in fact the generic high energy state of the theory. It is manifestly true in the AdS/CFT correspondence. Furthermore, the asymptotic spectrum of black holes in AdS/CFT is sensitive to the value of the cosmological constant, which is a discrete, tunable parameter. Thus, in this context, the idea that different values of the cosmological constant correspond to different states of the same theory, which can decay into a stable ground state, is manifestly false. The asymptotic spectrum of black hole states in asymptotically flat space, is completely different than that in any AdS space, so that instantons connecting such vacua do not seem to make any sense either.

The basic argument is thus that maximally symmetric spacetimes with nonpositive values of the cosmological constant cannot decay into AdS space with a more negative value of the cosmological constant because the former have, by virtue of their existence as metastable states, black hole excitations which simply cannot exist in the putative true vacuum. Note that in conventional quantum theories, generic high energy excitations of the true and false vacuum states are identical, and there is no tunneling barrier
for decay of an excitation of one into an excitation of the other. This is manifestly untrue for black holes.

Here I have argued that the CDL analysis of decay into a state with negative cosmological constant does not really give us evidence for transitions between flat or AdS vacua, and AdS vacua with lower cosmological constant. CDL pointed out that the spacetime inside the “bubble of true vacuum” is in fact a negatively curved FRW universe with a Big Crunch singularity. The Lorentzian continuation of the CDL instanton thus resembles a black hole. If the false vacuum is asymptotically flat, then the main difference is that the bubble wall intersects null infinity at finite affine parameter, and there is no timelike infinity. One might argue that such configurations should simply not be allowed, because the quantum theory is defined only in terms of configurations that are truly asymptotically flat. For example, if the quantum theory is formulated as a holographic theory on all of null infinity, it would be hard to understand how it could contain configurations like the Lorentzian CDL instanton.

We are familiar in quantum field theory with the statement that field configurations should satisfy the boundary conditions at infinity. In field theory the Euclidean and Lorentzian versions of this statement are equivalent to each other. Fundamentally, I believe that this is a consequence of the Cauchy-Kowalevska (CK) parametrization of solutions of the Lorentzian equations. Requiring spatial fall off of the initial data is equivalent, upon analytic continuation, to falloff in all directions in Euclidean space.

It is becoming more and more clear, that in theories of gravity, because of the ubiquitous occurrence of spacelike singularities, the CK parametrization of phase space is inadequate. We do not know a general rule for describing the global phase space of general relativity, but there are special cases where we know the answer\textsuperscript{15}. That is, we know how to parametrize the space of asymptotically flat or AdS solutions, if we are willing to accept the conjecture that smooth (perhaps locally bounded) data on the conformal boundary define solutions all of whose singularities are shrouded behind black hole horizons. Technically, asymptotically flat or AdS asymptotics, includes the provision that the spacetime has a conformal boundary identical to that of the indicated maximally symmetric spacetime.

The Lorentzian continuations of CDL instantons for decay of flat or AdS space, do not obey this boundary condition, despite the fact that the Euclidean solutions fall off at infinity, and have finite action. The interior of the bubble contains a spacelike singularity and the singularity intersects null infinity at a finite affine parameter. Future null infinity is not geodesically complete in these spacetimes. Timelike infinity is a $d-2$ sphere instead of a point.

\textsuperscript{15}If we accept a version of the famous Cosmic Censorship conjecture.
I believe that the simplest conclusion from all of these data is that decay of a maximally symmetric space of nonpositive cosmological constant into another one is not a valid concept in properly formulated theories of quantum gravity. The asymptotics at large spacelike distance, and the value of the cosmological constant are built into the structure of the quantum theory.

One may object that by appropriate choice of discrete parameters (e.g. fluxes) one can construct effective potentials in string theory that contain metastable, as well as stable, AdS vacua, in a regime where perturbative calculations are sensible. I would conjecture that in all such cases the CDL instanton will not exist. We have seen that this is a generic possibility, which depends on the details of the potential. Alternatively, the existence of the instanton might be taken to imply that the metastable state described by the false vacuum did not exist. I have argued elsewhere that the concept of an effective potential is suspect in theories of quantum gravity[22]. What is certain is that the hypothetical instanton is not evidence for decay into the stable AdS vacuum.

Our assessment of the validity of the CDL analysis for the case of a false vacuum with positive cosmological constant was much less conclusive because there does not yet exist a rigorous formulation of the quantum theory of dS spacetime. If it is a theory with a finite number of states, then it is obvious that vacuum decays cannot occur. This is not the end of the story. We can imagine, in a quantum theory of dS space with a small cosmological constant, a metastable factor of the Hilbert space, which behaves in some way like a dS space of larger cosmological constant. Similarly, in the infinite dimensional Hilbert space of the quantum theory of an open Big Bang FRW cosmology (the apparent geometry of the interior of the CDL bubble describing decay to a space of vanishing cosmological constant), we can imagine a finite tensor factor which describes a metastable dS space. It is to these hypothetical situations that the CDL instanton analysis might apply.

I found that the CDL instanton equations generically have an infinite discrete set of solutions, with an accumulation point at the Hawking-Moss instanton. This is true independent of the parameters in the Lagrangian. Thus, I disagree with the claims of [20] that metastable dS minima of an effective potential can sometimes be exactly stable. The Lorentzian continuation of the generic solution describes two bubbles nucleated in an inhomogeneous spacetime, which is a dS fibration. The two bubbles are out of causal contact. Furthermore, many timelike observers in the spacetime are out of causal contact with either of the bubbles. Each bubble evolves to either the false or the true vacuum, depending on our choice of instanton. I argued that the lowest action instanton was probably the one with the smallest number of oscillations, in which case, one bubble evolves to the true and one to the false vacuum. The interpretation I proposed of this instanton was thermal. The system described by the effective field theory is a quantum
system with a number of states given in terms of the true cosmological constant. It has a subsystem with a much smaller number of states (within the range of validity of the semiclassical approximation, and in the absence of extreme fine tuning) which local observers view as a dS space with the false value of the cosmological constant. Transitions between these two states can occur, because local observers in either state are in thermal equilibrium with a random system on their horizon. The CDL instanton describes both of these transitions with probabilities that satisfy a law of detailed balance.

The actual effect of instanton transitions on local observers is somewhat peculiar. There are, in the aftermath of the instanton event, three classes of local observers, whom I have characterized as true, false and inhomogeneous. In the generic case where only one fine tuning has been made (to make the true cosmological constant much smaller than its natural value $M^4$), only the true observers live in a spacetime with a macroscopic cosmological horizon. When the instanton occurs in the true vacuum, the true observers see it as a small excitation of the vacuum, which quickly inflates away. Observers outside the cosmological horizon of any true observer see a quantum jump to an inhomogeneous static spacetime, or to a small excitation of the false vacuum. Note that in this case, both inhomogeneous and false observers must be microscopically small to fit inside the cosmological horizons of their respectively regions. If we do two fine tunings, then the false vacuum observers can also be macroscopically large.

This case is the one in which it is most interesting to think about the effect of the instanton transition on an observer initially in the false vacuum. The majority of such observers will, shortly after the nucleation of the bubble, be hit by the bubble wall. If these observers survive the collision, they find themselves in the true vacuum, able to perform a much larger number of observations than they had previously thought possible. But there will always be some heretics, who see the instanton event as merely a thermal/quantum jump to an excited state of the false vacuum. These infidels will live out the rest of their lives in their narrow world, and can continue to refuse to believe in the existence of vacuum decay. Fortunately, these fanatics remain forever causally disconnected from the true believers, and bother their existence no more than does a quantum fluctuation on the faraway horizon.

The instanton events always have a very small probability, much smaller than the probability that quantum fluctuations will render an observer’s measuring device inoperable, or that a black hole will be spontaneously nucleated on top of him. Nonetheless, quantum mechanics remains quantum mechanics, and the theory predicts that there is a finite but small probability that a given observer will actually get to view the decay of the vacuum. He should live so long!

The above interpretation of CDL instantons is appropriate for the case where both
true and false vacua have positive cosmological constant. In the case where the true cosmological constant is zero, other issues must be addressed. One must first establish that the negatively curved FRW universe which plays the role of the true vacuum, is actually a sensible solution to quantum gravity. Then one must understand whether processes in such a spacetime could indeed excite a portion of it into the false dS minimum of the potential. The case of a negative value of the true cosmological constant might also make sense for a false dS vacuum. At a minimum, one would require the number of states observable in the Big Crunch universe that results from vacuum decay, to be larger than the number in the false dS vacuum. This will be true for a range of parameters in the effective potential.

To summarize, we have not found any gravitational context in which the concept of false vacuum decay makes unambiguous sense. The most promising case is that of dS to dS transitions. A model in which this case is of phenomenological interest would require two fine tunings, one of which could not be justified even by the anthropic principle. The case of dS space tunneling to a space with vanishing cosmological constant might provide a model of quintessence with the dubious virtue of never being testable. Both this case, and the case of dS space decaying into a spacetime with negative cosmological constant, might be realizable in perturbative stringy constructions. The analysis of this paper suggests several constraints that must be satisfied by such constructions, and one could imagine checking whether these constraints were satisfied. The most important message of this paper is that the use of effective field theory to study these issues must proceed with caution. Vacuum decay in the presence of gravity is a much more subtle issue than even Coleman and DeLuccia realized. Later, discussions, which have largely ignored the subtleties, have given the impression that establishing the existence (or not) of an instanton solution in the thin wall approximation establishes (or negates) the validity of the concept of vacuum decay in particular examples. I would submit that no such claim has been established. In fact, evidence from rigorous formulations of quantum gravity in asymptotically AdS space, and from the principle of Asymptotic Darkness, suggest that the effective potential formalism which is at the basis of instanton calculations has only limited applicability in theories of quantum gravity.

Before concluding, let me caution again that these negative remarks do not necessarily apply to the discussions of vacuum decay in inflationary cosmology. One common view of inflation is that we should describe the entire inflationary patch as a subsystem of a much larger universe. Applications of the CDL analysis in such a situation would conform more closely to the original condensed matter context, in which vacuum decay was merely an idealization of extensive behavior of finite systems. Deep questions about the total number of states in the universe could be reserved for the discussion of what
the quantum theory of gravity, into which the inflationary subsystem fits, consisted of. In this context it might make sense to truncate the dS instanton and consider only the true vacuum bubble. Inflationary cosmology is only locally the same as dS space.

References

[1] L. Dyson, J. Lindsey, L. Susskind, Is There Really a deSitter/CFT Duality?, JHEP 0208, 045, (2002), hep-th/0202163.

[2] A. Guth, E. Weinberg, Could the Universe Have Recovered From a Slow First Order Phase Transition, Nucl. Phys. B212, 321, (1983).

[3] C. Bohm, Inhomogeneous Einstein Metrics on Low Dimensional Spheres and Other Low Dimensional Spaces, Invent. Math. 134, 145, (1998).

[4] S. Coleman, Quantum Tunneling and Negative Eigenvalues, Nucl. Phys. B307, 867, (1988).

[5] C.R. Stephens, G. ’t Hooft, B.F. Whiting, Black Hole Evaporation Without Information Loss, Class. Quant. Grav. 11, 621, (1994), gr-qc/9310006; G. ’t Hooft, Quantum Information and Information Loss in General Relativity, Presented at Int. Symp. on Quantum Mechanics, Tokyo, Japan, August 1995, gr-qc/9509050; The Scattering Matrix Approach for the Quantum Black Hole: An Overview, Int. J. Mod. Phys. A11, 4623, (1996), gr-qc/9607022. L. Susskind, L. Thorlacius, J. Uglum, The stretched horizon and black hole complementarity, Phys. Rev. D48, (1993), 3743, hep-th/9306069.

[6] P. Ginsparg, M. Perry, Semiclassical Perdurance of de Sitter Space, Nucl. Phys. B222, 245, (1983); R. Bousso, S.W. Hawking, (Anti) Evaporation of Schwarzchild de Sitter Black Holes, Phys. Rev. D59, 103501, (1998), hep-th/9807148.

[7] D. Gross, M. Perry, L. Yaffe, Instability of Flat Space at Finite Temperature, Phys. Rev. D25, 330, (1982).

[8] M. Cvetic, S. Griffies, S-J. Rey, Static Domain Walls in N=1 Supergravity, Nucl. Phys. B389 3, (1993), hep-th/9206004 ; S. Weinberg, Does Gravitation Resolve the Ambiguity Among Supersymmetry Vacua?, Phys. Rev. Lett. 48, 1776, (1982).

[9] V.A. Rubakov, Quantum Mechanics in the Tunneling Universe,Phys. Lett B148, 280, (1984).

[10] S.W. Hawking, I.G. Moss, Supercooled Phase Transitions in the Very Early Universe, Phys. Lett. B110, 35, (1982).

[11] S.W. Hawking, I. Moss; A. Linde, Quantum Creation of An Open Inflationary Universe, Phys. Rev. D58, 083514, (1998), gr-qc/9802038.
[12] V.A. Rubakov, *Quantum Mechanics in the Tunneling Universe*, Phys. Lett B148, 280, (1984); A.S. Goncharov, A.D. Linde *Tunneling in Expanding Universe: Euclidean and Hamiltonian Approaches*, Sov. J. Part. Nucl. 17, 369, (1986); A.D. Linde, *Hard Art of the Universe Creation (stochastic approach to tunneling and baby universe formation)*, Nucl. Phys. B372, 421, (1992); A.D. Linde, D.A. Linde, A. Mezhulman, *From the Big Bang Theory to the Theory of a Stationary Universe*, Phys. Rev. D49, 1783, (1994), gr-qc/9306035, and Chapter 7 in A.D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Academic Publishers, Chur, Switzerland, 1990.  

[13] W. Fischler, L. Susskind *Holography and Cosmology*, hep-th/9806039. R. Bousso *A Covariant Entropy Conjecture*, JHEP 9907 (1999) 004, hep-th/9905177; *Holography in General Space Times*, JHEP 9906 (1999) 028, hep-th/9906022; *The Holographic Principle for General Backgrounds*, Class. Quant. Grav. 17 (2000) 997, hep-th/9911002  

[14] T. Banks, *M-theory and Cosmology*, Lectures at the 71st Les Houches Summer School, *The Primordial Universe*, Les Houches, France, 28 Jun - 23 Jul, 1999, p. 495.  

[15] T. Banks, W. Fischler, *A Model for High Energy Scattering in Quantum Gravity*, hep-th/9906038.  

[16] T. Banks, *Quantum Mechanics and Cosmology*, Talk given at the festschrift for L. Susskind, Stanford University, May 2000; *Cosmological Breaking of Supersymmetry?*, Talk Given at Strings 2000, Ann Arbor, MI, Int. J. Mod. Phys. A16, 910, (2001), hep-th/0007146; W. Fischler, *Taking de Sitter Seriously*, Talk given at *The Role of Scaling Laws in Physics and Biology (Celebrating the 60th Birthday of Geoffrey West)*, Santa Fe Dec. 2000, and unpublished.  

[17] S. Coleman, *The Fate of the False Vacuum. 1. Semiclassical Theory*, Phys. Rev. D15, 2929, (1977), Erratum-ibid. D16, 1248, (1977); S. Coleman, C.G. Callan, *The Fate of the False Vacuum. 2. First Quantum Corrections*, Phys. Rev. D16, 1762, (1977); S. Coleman, F. De Luccia, *Gravitational Effects on and of Vacuum Decay*, Phys. Rev. D21, 3305, (1980).  

[18] J.S. Langer, Ann. Phys. 41 (NY), 108, (1967).  

[19] T. Banks, C.M. Bender, T.T. Wu, Phys. Rev. D8, 3346, (1973), D8, 3366, (1973) ; L.N. Lipatov, *Divergence of the Perturbation Theory Series and the Quasiclassical Theory*, Sov. Phys. JETP, 45, 216, (1977); E. Brezin, J.C. Le Guillou, J. Zinn-Justin, *Perturbation Theory at Large Order. 1. The φ^2N Interaction*, Phys. Rev. D15, 1544, (1977); *Perturbation Theory at Large Order. 2. The Role of Vacuum Instability*, Phys. Rev. D15, 1558, (1977).
[20] A. Maloney, E. Silverstein, A. Strominger, *de Sitter Space in Noncritical String Theory*, hep-th/0205316; A. Maloney, A. Strominger, *Talk Given by A. Strominger at the Festschrift for Steven Hawking*.

[21] J. Maldacena, *The Large N Limit of Superconformal Field Theories and Supergravity*, Adv. Theor. Math. Phys. 2, 231, (1998), Int J. Theor. Phys. 38, (1999); S. Gubser, I. Klebanov, A. Polyakov, *Gauge Theory Correlators From Non Critical String Theory*, Phys.Lett.B428:105-114,1998, hep-th/9802109 ;E. Witten, *Anti-De Sitter Space and Holography*, Adv.Theor.Math.Phys.2:253-291,1998, hep-th/9802150.

[22] T. Banks, *On Isolated Vacua and Background Independence*, hep-th/0011255; *A Critique of Pure String Theory*, Talk Given at Strings 2002, Cambridge, UK, July 2002; *A Critique of Pure String Theory: Heterodox Opinions of Diverse Dimensions*, manuscript in preparation.

[23] E. Witten, *Instability of the Kaluza-Klein Vacuum*, Nucl. Phys. B195, 481, (1982).

[24] J.D. Brown, C. Teitelboim, *Neutralization of the Cosmological Constant by Membrane Creation*, Nucl. Phys. B297, 787, (1988), *Dynamical Neutralization of the Cosmological Constant*, Phys. Lett. B195, 177, (1987).

[25] T. Banks, *Relaxation of the Cosmological Constant*, Phys. Rev. Lett. 52, 1461, (1984); *TCP, Quantum Gravity, The Cosmological Constant and All That*, Nucl.Phys. B249, 332, (1985); L.F. Abbott, *Scalar Fields and the Cosmological Constant*, Lectures given at 5th Latin American Symp. on Relativity and Gravitation, Bariloche Argentina, 1985, Bariloche SILARG Symp. 1985 ; R. Bousso, J. Polchinski, *Quantization of Four Form Fluxes and Dynamical Neutralization of the Cosmological Constant*, JHEP 0006, 006, (2000), hep-th/0004134 ; J.L. Feng, J. March-Russell,S. Sethi, F. Wilczek, *Saltatory Relaxation of the Cosmological Constant*, Nucl. Phys. B602, 307, (2001), hep-th/0005276. .

[26] T. Banks, W. Fischler, S. Shenker, L. Susskind *M-theory as a Matrix Model: A Conjecture*, Phys.Rev.D55:5112-5128,1997, hep-th/9610043

[27] A. Guth, E. Farhi, *An Obstacle to Creating a Universe in the Laboratory*, Nucl. Phys. B339, 417, (1990).

[28] T. Banks, W. Fischler, S. Paban *Recurrent Nightmares?: Measurement Theory in de Sitter Space*, hep-th/020160.

[29] R. Bousso, *Positive Vacuum Energy and the N Bound*, hep-th/0010252.

[30] T. Banks, W. Fischler, *M-theory Observables for Cosmological Spacetimes*, hep-th/0102077.