Edge state distribution in an Aharonov-Bohm electron interferometer in the integer quantum Hall regime

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Abstract. In this study we analyze the density distributions of the two dimensional electron system for an experimental geometry which is topologically equivalent of an Aharonov-Bohm interferometer in three dimensions in the quantum Hall regime and obtain the spatial distribution of the edge states. We employ the Thomas-Fermi approximation in our analysis and solve the Poisson equation in three dimensional using a multi grid technique. Also we obtain the distribution of incompressible strips for a wide range of magnetic fields strengths and comment on their relation with experimental results in literature.

1. Introduction
Aharonov-Bohm (AB) effect is a phenomenon which exhibits observable quantum mechanical effects with using a vector potential in a region where the magnetic field is zero [1]. The low dimensional experiments related to “AB effect” do not entirely display this effect because of the use of uniform magnetic field everywhere [2]. Similarities with the original AB effect are just geometry and a phase event in the presence of a magnetic field. One of these experimental studies is a quantum electron interferometer in the quantum Hall regime which is studied by Camino et al. [3]. They have used a ring shaped gate structure to experimentally observe resistivity oscillations. In this experimental setup, the incompressible strips (ISs) formed in two dimensional electron gas (2DEG) take place of the electron beams in the original AB effect. Therefore it is very important to have a knowledge of the distribution of the ISs in order to understand these effects.

In our study, we theoretically investigate the ISs formed in the structure used by Camino et al.[3] in their experiments. We obtain the distribution of the ISs for various magnetic field strengths using Thomas-Fermi-Poisson(TFP) approach. We consider a trenched structure which is illustrated in Fig. 1, where the gates are formed by applying a front gate potential to this trenched places. Under the gates there is a donor layer and 2DEG is placed under the donor layer. For such a geometry Poisson and Thomas-Fermi equations are solved self-consistently in three dimensions (3D) so the electron density and the screened potential are obtained then the position and structure of ISs are determined.

It was first proposed by Chklovskii et al.[4] that the electrostatics should be considered for determining the edge states. They also proposed the compressible and incompressible strips which identify the location of the current flowing in a 2DEG in a strong magnetic field. Self-consistent generalizations of model of Chklovskii et al.[4] have been done within the Thomas-Fermi [5-7] and Hartree [8] approximations during the recent two decades.
The electrostatics of a 2DEG within TFP approach consists of a simultaneous solution of Thomas-Fermi and Poisson equations. In a realistic approximation, Poisson equation must be solved in 3D. Solutions of Poisson equation in 3D has been obtained under open boundary conditions and applied to a quantum Hall bar [9] and an AB interferometer [10]. However, in these studies the Poisson solved only once without considering the distribution of the charges in the 2DEG and the gate potential obtained in this way is assumed to be constant throughout the calculations. This approach is called as frozen gate approximation. In the rest of the calculations the electrostatic potential is obtained by using a 2D Green’s function with periodic boundary conditions to solve the Poisson’s equation.

In order to obtain a completely self consistent solution in 3D, a solution of the Poisson equation should be obtained at every step throughout the calculations. Recently such an approach was developed for quantum Hall bar geometry[11] and in this study we have extended that work to AB geometry.

\[ \nabla^2 V(x, y, z) = -\frac{\rho(x, y, z)}{\varepsilon} \]

(1)

where \( V(x, y, z) \) is the screened potential, \( \rho(x, y, z) \) is the charge density and \( \varepsilon \) is the dielectric constant of the material.

The Poisson equation is solved numerically in 3D using a multigrid technique. Because we solve the Poisson equation in 3D with all the gates, donors and 2DEG we do not introduce a frozen gate approximation. We use two bar shaped top gates on each side of the interferometer for confining the system. These bar shaped gates are chosen to be wide enough so that the potential heals under the gates, i.e. \( \frac{\partial V}{\partial x} = 0 \) at the edges.

For the electron density which is the other stand of the self-consistent calculation, we use Thomas-Fermi approximation. Within this approximation, in the presence of a strong magnetic field \( B \) we can write the electron density as

\[ n_e(\vec{r}) = \frac{g_s}{2\pi^2} \sum_{n=0}^{\infty} \int \left( \frac{E_n + V(r) - \mu}{k_f T} \right) \]

(2)

Figure 1. A schematic layout of an AB interferometer in 3D

2. Electrostatics of the system and numerical procedure

We can write 3D Poisson equation for the structure in Fig. 1 as

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(2)
where $l = \sqrt{\frac{\hbar}{eB}}$ is the magnetic length, $\mu$ is the chemical potential, $E_n$ is the energy of the $n$th Landau level, and $T$ is the temperature. The degeneracy of Landau levels are given by $\frac{g_s}{2\pi l^2}$, where $g_s = 2$ is the spin degeneracy.

3. Results and Discussion
The structure considered in this study is based on a recent experimental study of an AB interferometer fabricated on AlGaAs/GaAs heterojunction with etching techniques [3]. Here the gates are deposited in trenches. In our study for the similarity with experimental study we consider the trenched structure in Fig 1. The gates are obtained by filling these trenches down to 80 nm from the surface. The applied potential on all the gates is -0.7 V. We placed two gates to the edge of the structure in x direction. The widths of these edge gates are 50 nm, the length of the structure is 2000 nm in y direction. The 2DEG is at 210 nm below the surface. The average electron density $n_e$ and the ionized donor density $n_d$ are taken as equal $n_d = n_e = 0.97 \times 10^{11} \text{cm}^{-2}$ so that overall charge neutrality is maintained. In x, y and z directions the dimensions of the structure are taken 2600 nm, 2000 nm and 1000 nm, respectively. For numerical calculations a mesh is formed with 10 nm steps in all three directions.

Combining the Poisson and Thomas-Fermi equations one obtains a nonlinear differential equation for the electron density distribution. We have obtained a self consistent solution of this nonlinear equation using an iterative approach. The numerical procedure used to obtain the density distribution is as follows. First, we solve the equation to obtain electron density distribution for $T = 0$ and $B = 0$. Using this solution as the initial distribution, the electron density distribution is calculated iteratively for some sufficiently high temperature and zero magnetic field. Then we set the magnetic field and repeat iterative procedure for the self consistent solution. After this, the temperature is reduced slowly obtaining a self consistent density distribution at each temperature step until the system reaches to aimed temperature ($T=1.4 \text{ K}$).

The ISs which are formed by completely filled Landau levels are located where the filling factor $\nu = n_e h / eB$ is equal to an even integer number. In order to obtain an AB like structure the ISs should form a circular path enclosing a magnetic flux. The results are illustrated Fig 2, where filling factors are shown as a function of the spatial coordinates for different magnetic field values. As shown in Fig. 2, for different values of magnetic field the ISs are easily seen in the electron distribution, the ISs are indicated with yellow colour and correspond to filling factor two. In Fig. 2(a) the magnetic field at 2.8 T, the ISs are observed but do not contact with each other. We do not expect observation of any interference pattern at this value of magnetic field. In Fig 2(b) the magnetic field strength is 2.9 T, here the ISs become closer, however there is still no overlap and again no interference is expected. Figs. 2(c) and 2(d) correspond to 3.0 T and 3.1 T respectively, at these values of magnetic field the ISs touch to each other and we expect observation of an interference pattern. If we increase the magnetic field still further then the ISs will split again (this time in the vertical direction) and again no interference is expected. The general behaviour of the ISs obtained in this study is consistent with the experimental findings of Camino et al. [3].

4. Conclusion
We have calculated the spatial electron distributions for a quantum Hall effect based electronic AB interferometer with using a self-consistent TFP approach. We determined the spatial distribution of ISs and discussed their relation with the interference phenomena.

Acknowledgments
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Figure 2. Contour graphs of the filling factors for different magnetic field values. The yellow colour indicates the ISs for filling factor two.

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