Axial anomaly as a collective effect of meson spectrum

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Abstract

We study the transition form factors of the light mesons in the kinematics, where one photon is real and other is virtual. Using the dispersive approach to axial anomaly we show that the axial anomaly in this case reveals itself as a collective effect of meson spectrum. This allows us to get the relation between possible corrections to continuum and to lower states within QCD method which does not rely on factorization hypothesis. We show, relying on the recent data of the BaBar Collaboration, that the relative correction to continuum is quite small, and small correction to continuum can dramatically change the pion form factor.

1 Introduction

The phenomenon of axial anomaly \cite{1,2} is known to be one of the most subtle effects of quantum field theory. Perhaps the most vivid manifestation of it in particle physics can be found in two-photon decays of pseudoscalars. Usually the differential form of the axial anomaly is utilized to study this kind of processes. However, less known dispersive approach to axial anomaly (\cite{3,4}, for a review, see \cite{5}) leads to anomaly sum rule (ASR) relation, which provides a very powerful tool for study of various characteristics of meson spectrum. The absence of corrections to ASR allows to get certain exact relations between characteristics of hadrons, such as decay width \cite{6} and relations between mixing parameters of pseudoscalars \cite{7,8}.

In the study of two-photon decays of pseudoscalars, usually the case of real photons is considered. However, ASR is valid for virtual photons also \cite{9,10} which leads to several interesting applications. As we will see, in the case of one real and one virtual photon \cite{9} the ASR takes into account the infinite number of meson states (which can contribute to it due to their quantum numbers), i.e. axial anomaly reveals itself as a collective effect of meson spectrum. This nontrivial fact together

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with the exactness of ASR gives us a tool to study different characteristics of meson spectrum (form factors of mesons, relation between possible corrections to lower meson states and continuum).

The experimental measurements of the photon-pion \( \gamma \gamma^* \rightarrow \pi^0 \) transition form factor \( F_{\pi\gamma}(Q^2) \) in the range of photon virtualities \( Q^2 < 9 \) GeV\(^2 \) were performed by CELLO\(^{[11]} \) and CLEO\(^{[12]} \) Collaborations. The measured values of \( F_{\pi\gamma}(Q^2) \) appeared to be consistent with predictions based on factorization approach to pQCD. Surprisingly, recent data of BaBar Collaboration on \( F_{\pi\gamma}(Q^2) \)\(^{[13]} \), which is available in the range \( 4 < Q^2 < 40 \) GeV\(^2 \), have shown a strong disagreement with the pQCD predicted behaviour of \( \gamma \gamma^* \rightarrow \pi^0 \) transition form factor.

Though the BaBar data in the range \( Q^2 < 10 \) GeV\(^2 \) fit well the CLEO data and is in a good agreement with theoretical predictions from the light-cone QCD sum rules (LCSR), offered in \(^{[14]} \), however, at larger virtualities strong disagreement takes place. Moreover, more precise recent LCSR analysis \(^{[15,16]} \) shows, that it is impossible to explain BaBar data on \( F_{\pi\gamma}(Q^2) \) at large \( Q^2 \) by use of usual (endpoint-suppressed) form of pion distribution amplitude. This have led (quite unexpectedly) to the question of pQCD factorization validity. Recently, there were proposed several approaches to explain such anomalous behaviour of \( F_{\pi\gamma}(Q^2) \)\(^{[17–21]} \), in particular, questioning pQCD factorization. At the same time, in \(^{[22]} \) the authors give some arguments against the related flat-type pion distribution amplitude, while in \(^{[23]} \) some doubts about BaBar results analysis were expressed.

In this paper we study what can be learnt about the meson-photon transition form factors from the anomaly sum rule for the case of one virtual photon. This generalizes the usual application of anomaly, providing the boundary condition in the real photon limit only. Our (non-perturbative) QCD method does not imply the QCD factorization and is valid even if the QCD factorization is broken. It is shown, that using axial anomaly in the dispersive approach we can get the exact relations between possible corrections to lower states and continuum providing a possibility of relatively large corrections to the lower states.

## 2 Anomaly sum rule

Following \(^{[9]} \), we briefly remind some results of the dispersive approach to axial anomaly which are relevant for this paper. The VVA triangle graph amplitude

\[
T_{\alpha\mu\nu}(k, q) = \int d^4x d^4y e^{i(kx+iyy)} \langle 0| T\{ J_5^\alpha(0), J_\mu(x), J_\nu(y)\} |0 \rangle
\]

contains axial current \( J_5^\alpha = (\bar{u}\gamma_5\gamma_\alpha u - \bar{d}\gamma_5\gamma_\alpha d) \) and two vector currents \( J_\mu = ((2/3)\bar{u}\gamma_\mu u - (1/3)\bar{d}\gamma_\mu d); k, q \) are momenta of photons. This amplitude can be presented as a tensor decomposition

\[
T_{\alpha\mu\nu}(k, q) = F_1 \varepsilon_{\alpha\mu\rho k^\rho} + F_2 \varepsilon_{\alpha\mu\rho q^\rho} + F_3 q_\nu \varepsilon_{\alpha\mu\rho} k^\rho q^\sigma + F_4 q_\nu \varepsilon_{\alpha\mu\rho} k^\rho q^\sigma + F_5 k_\mu \varepsilon_{\alpha\nu\rho} k^\rho q^\sigma + F_6 q_\mu \varepsilon_{\alpha\nu\rho} k^\rho q^\sigma,
\]

where the coefficients \( F_j = F_j(k^2, q^2, p^2, m^2), p = k + q, j = 1, \ldots, 6 \) are the corresponding Lorentz invariant amplitudes (form factors). Note that these form factors do not have kinematical singularities and are suitable for dispersive approach, which we use to derive anomaly sum rule.

Symmetries of the amplitude \( T_{\alpha\mu\nu}(k, q) \) impose the relations for the form factors \( F_j(k, q) \).
Bose symmetry, i.e. \( T_{\alpha\mu\nu}(k, q) = T_{\alpha\nu\mu}(q, k) \) leads to
\[
F_1(k, q) = -F_2(q, k), \\
F_3(k, q) = -F_6(q, k), \\
F_4(k, q) = -F_5(q, k).
\]

(3)

Vector Ward identities for the amplitude \( T_{\alpha\mu\nu}(k, q) \)
\[
k^\mu T_{\alpha\mu\nu} = 0, \quad q^\nu T_{\alpha\mu\nu} = 0
\]

(4)
in terms of form factors read:
\[
F_1 = k.q F_3 + q^2 F_4, \quad F_2 = k^2 F_5 + k.q F_6.
\]

(5)

Anomalous axial-vector Ward identity for \( T_{\alpha\mu\nu}(k, q) \) \[1, 2\]
\[
p^\alpha T_{\alpha\mu\nu}(k, q) = 2m T_{\mu\nu}(k, q) + \frac{1}{2\pi^2} \varepsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma
\]

(6)
in terms of form factors can be rewritten as follows:
\[
F_2 - F_1 = 2mG + \frac{1}{2\pi^2},
\]

(7)
where \( G \) is a form factor, related to the 2nd rank pseudotensor \( T_{\mu\nu} \) involved in the “normal term” on the r.h.s. of (6):
\[
T_{\mu\nu}(k, q) = G \varepsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma.
\]

(8)

Writing the unsubtracted dispersion relations for the form factors one gets the finite subtraction for axial current divergence resulting in the anomaly sum rule which for the kinematical configuration we are interested in \( (k^2 = 0, q^2 \neq 0) \) takes the form \[9\]:
\[
\int_{4m^2}^{\infty} A_{3a}(t; q^2, m^2) dt = \frac{1}{2\pi},
\]

(9)
where
\[
A_{3a} = \frac{1}{2} Im(F_3 - F_6).
\]

(10)
It holds for an arbitrary quark mass \( m \) and for any \( q^2 \) in the considered region. Another important property of the above relation is absence of any \( \alpha_s \) corrections to the integral \[24\]. Moreover, it is expected that it does not have any nonperturbative corrections too (’t Hooft’s principle).

3 Transition form factors of mesons

The form factor \( F_{\pi\gamma} \) of the transition \( \pi^0 \rightarrow \gamma\gamma^* \) is defined from the matrix element:
\[
\int d^4 x e^{ikx} \langle \pi^0(p)|T\{J_{\mu}(x)J_{\nu}(0)\}|0\rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F_{\pi\gamma},
\]

(11)
where \( k, q \) are momenta of virtual photons, \( p = k + q \), and \( J_{\mu} = ((2/3)\bar{u}\gamma_{\mu}u - (1/3)\bar{d}\gamma_{\mu}d) \) is the electromagnetic current of light quarks.
Three-point correlation function $T_{\alpha\mu\nu}(k, q)$ has pion (pole at $p^2 = m^2_\pi$) and higher states contributions:

$$T_{\alpha\mu\nu}(k, q) = \frac{i\sqrt{2}f_\pi}{p^2 - m^2_\pi}p_\alpha k^\rho q^\sigma \epsilon_{\mu\nu\rho\sigma} F_{\pi\gamma} + \text{(higher states)} ,$$

where $f_\pi$ is a pion decay constant, which can be defined as a coefficient in the projection of axial current $J_5^\alpha$ onto one-pion state:

$$\langle 0| J_5^\alpha(0) |\pi^0(p) \rangle = i\sqrt{2}p_\alpha f_\pi .$$

The pion decay constant $f_\pi = 130.7$ MeV is experimentally well determined from the decay of charged pion $\pi^- \rightarrow \mu^- \nu$. Using the kinematical identities

$$\delta_{\alpha\beta}\epsilon_{\sigma\mu\nu\tau} - \delta_{\alpha\sigma}\epsilon_{\beta\mu\nu\tau} + \delta_{\alpha\mu}\epsilon_{\beta\sigma\nu\tau} - \delta_{\alpha\nu}\epsilon_{\beta\sigma\mu\tau} + \delta_{\alpha\tau}\epsilon_{\beta\sigma\mu\nu} = 0 ,$$

we can single out the pion contribution to $\frac{1}{2}(F_3 - F_6)/2$ (imaginary part is taken w.r.t. $p^2$) is:

$$\frac{1}{2} Im(F_3 - F_6) = \sqrt{2}f_\pi \pi F_{\pi\gamma}(Q^2)\delta(s - m^2_\pi) ,$$

where $Q^2 = -q^2$.

It is well known that at $Q^2 = 0$ the pion contribution saturates anomaly sum rule (9) and $F_{\pi\gamma}$ is known to be normalized by the $\pi^0 \rightarrow \gamma\gamma$ decay rate [2]:

$$F_{\pi\gamma}(0) = \frac{1}{2\sqrt{2}f_\pi^2} .$$

On the other hand at $Q^2 \neq 0$, factorization approach to perturbative quantum chromodynamics (pQCD) for exclusive process in the leading order in the strong coupling constant predicts [25, 26]:

$$F_{\pi\gamma}(Q^2) = \frac{\sqrt{2}f_\pi}{3Q^2} \int_0^1 dx \frac{\varphi_\pi(x, Q^2)}{x} + O(1/Q^4) ,$$

where $f_\pi = 130.7$ MeV and $\varphi_\pi(x)$ is a pion distribution amplitude (DA). The pion DA depends on the renormalization scale [25, 27] and at large $Q^2$ asymptotically acquires a simple form [28]:

$$\varphi_\pi^{\text{asymp}}(x) = 6x(1-x) .$$

This leads to asymptotic behaviour for the pion form factor:

$$F_{\pi\gamma}^{\text{asymp}}(Q^2) = \frac{\sqrt{2}f_\pi}{Q^2} + O(1/Q^4) .$$

From (17) and (15) we get the contribution of pion to anomaly sum rule (9):

$$2\pi f_\pi^2/Q^2 .$$

We see, that at $Q^2 \neq 0$ anomaly sum rule (9) cannot be saturated by pion contribution due to $1/Q^2$ behavior, so we need to consider higher states. The higher mass pseudoscalar states have the same behavior and suppressed by the factor $m^2_\pi/m^2_{\text{res}}$ as follows from the PCAC (since $\partial_\mu J_3^\mu$ should

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1. Note that in the case of the choice of the basis differing from (2) for AVV amplitude the results for transition formfactors do not change.
2. This may be compared with $Q^2$ dependence of pion-to-photons matrix element emerging in the analysis of sum rule [29, 30] for photon spin structure function.
vanish in the chiral limit). The other contributions are provided by axial mesons, the lightest of which is the $a_1(1260)$ meson. In fact, the contribution of longitudinally polarized $a_1$ is given by the similar equation to (18) at large $Q^2$. Indeed, the analysis of axial current bilocal matrix elements of $a_1$ is completely similar for that of vector current matrix elements of $\rho$. The contribution of transversally polarized $a_1$ decreases even faster. Actually, the same is true for all the higher axial mesons and mesons with higher spin. So we make an important observation: for the case $Q^2 \neq 0$ the anomaly relation (9) cannot be explained in terms of any finite number of mesons due to the fact that all transition form factors are decreasing functions. That is why we conclude that only infinite number of higher states can saturate anomaly sum rule and therefore at $Q^2 \neq 0$ the axial anomaly is a genuine collective effect of meson spectrum in contrast with the case of two real photons $Q^2 = 0$, where the anomaly sum rule is saturated by pion contribution only. Let us note that this conclusion does not depend on any choice of meson distribution amplitudes.

4 Quark-hadron duality

Now we proceed to particular analysis of anomaly sum rule (9) using quark-hadron duality. In some sense the discussion of the previous section is based on the quark-hadron duality. In this section we will apply local quark-hadron duality to anomaly sum rule (9). According to the quark-hadron duality, let us saturate the spectral density $A_{3a}$ by pion and continuum contributions:

$$A_{3a}(s, Q^2) = \sqrt{2} \pi f_\pi \delta(s - m_\pi^2) F_{\pi \gamma}(Q^2) + A_{3a}^{QCD} \theta(s - s_0),$$

where continuum contribution $A_{30}^{QCD} \theta(s - s_0)$ as usually supposed to be equal to QCD calculated spectral function $A_{3a}$, while $s_0$ is a continuum threshold.

Substituting (20) into (9), we obtain the anomaly sum rule in the following form:

$$\frac{1}{2\pi} = \sqrt{2} \pi f_\pi F_{\pi \gamma}(Q^2) + \int_{s_0}^{\infty} ds A_{3a}^{QCD}.$$

One-loop PT calculation [32, 33] leads to a simple result for the spectral density function:

$$A_{3a}^{QCD}(s, Q^2) = \frac{1}{2\pi} \frac{Q^2}{(s + Q^2)^2},$$

so we can rewrite the anomaly sum rule in the following form:

$$\frac{1}{2\pi} = \sqrt{2} \pi f_\pi F_{\pi \gamma}(Q^2) + \frac{1}{2\pi} \int_{s_0}^{\infty} ds \frac{Q^2}{(s + Q^2)^2},$$

and finally the pion form factor is

$$F_{\pi \gamma}(Q^2) = \frac{1}{2\sqrt{2} \pi f_\pi} \frac{s_0}{s_0 + Q^2},$$

where $s_0 = 0.7 \text{ GeV}^2$ is a continuum threshold.

This result coincides with the interpolation formula proposed by S. Brodsky and G.P. Lepage [26] ($s_0 = 4\pi^2 f_\pi^2$)

$$F_{\pi \gamma}^{BL}(Q^2) = \frac{1}{2\sqrt{2} \pi f_\pi} \frac{1}{1 + Q^2/(4\pi^2 f_\pi^2)}.$$
which was derived in the quark-hadron duality context by A.V. Radyushkin [34]. The contact with exact anomaly sum rule observed here allows to find the relations between contributions of $\pi^0$ with higher states which may be chosen as $a_1$ and continuum. With account of these contributions the anomaly sum rule (9) can be rewritten in the following form:

$$\frac{1}{2\pi} = \sqrt{2\pi f_{\pi} F_{\gamma\gamma}(Q^2)} + I_{a_1} + \frac{1}{2\pi} \int_{s_1}^{\infty} ds \frac{Q^2}{(s + Q^2)^2},$$

where $I_{a_1}$ is a contribution of $a_1$ meson to sum rule (which can be expressed in terms of $a_1$ form factors), $s_1 = 2.5 GeV^2$ is a continuum threshold for this case.

Using the asymptotic formula for pion form factor (24) we can estimate the behavior of $I_{a_1}$ at large $Q^2$ as

$$I_{a_1} = \frac{1}{2\pi} Q^2 \frac{s_1 - s_0}{(s_1 + Q^2)(s_0 + Q^2)}.$$

This equation can be treated as a good interpolation for $a_1$ contribution with correct asymptotic behavior (large and small $Q^2$).

The plot for contributions of pion, $a_1$ meson and continuum is shown in Fig.1. The figure illustrates the anomaly collective effect: indeed, the contribution of infinite number of higher resonances (continuum contribution) dominates starting from relatively small $Q^2 \simeq 1.5 GeV^2$.

5 Corrections’ interplay and experimental data

As we learned above, axial anomaly is a collective effect of meson spectrum. That is why it is natural study the relation between possible meson and continuum corrections from anomaly sum rule. Let us write out anomaly sum rule once more in the following form:
\[
\frac{1}{2\pi} = \int_0^\infty A_{3a}(s;Q^2)ds = I_\pi + I_{a_1} + I_{cont},
\]
where continuum contribution \( I_{cont} \) takes into account all other higher mass axial and higher spin states.

As we already mentioned, the anomaly sum rule (9) is an exact relation \( \int_0^\infty A_{3a}(s;Q^2)ds \) does not have any corrections). However, the continuum contribution

\[
I_{cont} = \int_{s_i}^\infty A_{3a}(s;Q^2)ds
\]

may have perturbative as well as power corrections.

Note that the two-loop corrections to the whole triangle graph were found to be zero \[35\] implying the zero corrections to all spectral densities\[3\]. To match this result with the earlier found non-zero corrections in the factorization approach (see \[16\] and references therein) one should be careful. When one is dealing with the corrections to the form factor itself, only the corrections to the coefficient function should be considered, while all other corrections are absorbed to the definition of the distribution amplitude. At the same time, when the factorization theorem is applied for calculations of the large \( Q^2 \) asymptotics of VVA diagram, all the corrections should be taken into account. To do so, one should add Eq. (3.11) of \[16\] with the projector to the local axial current (proportional to asymptotic pion distribution amplitude\[3\]) and Eq. (B1) (coinciding with Eq. (1) of the Erratum) of \[38\] and get a zero result, compatible with \[35\].

Therefore, the model of the corrections to continuum discussed below should rather correspond to some non-perturbative corrections. Let us first consider the contributions of local condensates. Naively, they should strongly decrease with \( Q^2 \) compensating the mass dimension of gluon (as quark one is suppressed even more) condensate. However the ’t Hooft’s principle requires (see \[9\], Section 4) the rapid decrease of the corrections with Borel parameter \( M^2 \) (related to \( s \)) so that the power of \( Q^2 \) in the denominator may be not so large. In reality the actual calculations do not satisfy this property and the situation may be improved by the use of non-local condensates (see \[9\] and references therein). Another possibility is other non-perturbative contributions, like instanton-induced ones. So we assume the appearance of such corrections in what follows modelling the corrections to continuum.

In order to preserve the sum rule (28) (or in particular (23), (26)) the corrections to continuum contribution should be exactly compensated by corrections to lower states, in particular to pion. It turns out that this is a rather uncommon situation: corrections to continuum are compensated by the main terms of lower states which are of the same order in \( Q^2 \) as continuum corrections.

To be more specific, let us consider the model “\( \pi^0 + \)continuum”. From (23) the main contributions of pion and continuum read:

\[
I_\pi^0 = \sqrt{2\pi} f_\pi F_{\pi\gamma}^\pi(Q^2) = \frac{1}{2\pi} \frac{s_0}{s_0 + Q^2},
\]

\[
I_{cont}^0 = \frac{1}{2\pi} \frac{Q^2}{s_0 + Q^2}.
\]

\[^3\]Zero two loop corrections to spectral densities were also found in the massive case \[36\], although later the inconsistency of this result with asymptotic mass expansion was pointed out \[37\].

\[^4\]We are indebted to S.V. Mikhailov for clarification of this point.
If the corrections to pion and continuum to ASR are $\delta I_\pi$ and $\delta I_{\text{cont}}$ respectively

\[ I_\pi = I_\pi^0 + \delta I_\pi, \quad (32) \]

\[ I_{\text{cont}} = I_{\text{cont}}^0 + \delta I_{\text{cont}} \quad (33) \]

then, since $\delta I_\pi = -\delta I_{\text{cont}}$, the ratio of relative corrections to continuum and pion is

\[ \left| \frac{\delta I_{\text{cont}}}{\delta I_\pi} \right| = \frac{s_0}{Q^2}. \quad (34) \]

For instance, for $Q^2 = 20$ GeV$^2$, $s_0 = 0.7$ GeV$^2$ the ratio is

\[ \left| \frac{\delta I_{\text{cont}}}{\delta I_\pi} \right| \simeq 0.03. \quad (35) \]

We see, that the relative correction to continuum is suppressed by factor $1/Q^2$ as compared to the correction to pion. To illustrate our conclusion, we assume the correction to continuum at large $Q^2$ is $\delta I_{\text{cont}} = -c_0 \ln(Q^2/s_0) + b Q^2$. This correction preserves asymptotics of continuum contribution at large $Q^2$. Contributions of pion and continuum to ASR then have the following explicit forms:

\[ I_{\text{cont}} = \frac{1}{2\pi} \frac{Q^2}{s_0 + Q^2} - c_0 \ln(Q^2/s_0) + b Q^2, \quad (36) \]

\[ I_\pi = \frac{1}{2\pi} \frac{s_0}{s_0 + Q^2} + c_0 \ln(Q^2/s_0) + b Q^2. \quad (37) \]

If this correction corresponds, as it was discussed above, to (non-local) gluon condensate $\langle G^2 \rangle$ one may formally substitute the dimensional factor $c_0$ by $c_\langle G^2 \rangle/s_0$ stemming from $\langle G^2 \rangle/M^2$ for Borel transforms.

One can see, that the leading power correction to continuum preserving its asymptotics results in a substantial (of the order of the main term $I_0^\pi$) contribution to the pion state changing the pion form factor asymptotics at large $Q^2$.

The experimental data on pion form factor behavior at large $Q^2$ allows us to get estimation for the corrections to continuum. If the pion form factor expression (24) have matched the experimental data, the continuum leading correction might be only of order $1/Q^4$. However, the last BaBar data on pion transition form factor [13] manifests large discrepancy of $F_{\pi\gamma}$ values at large $Q^2$ with the expected asymptotic behaviour (24).

Relying on the BaBar data, we can fit parameters $b, c$:

\[ b = -2.74, \quad c = 0.045. \quad (38) \]

The corresponding plot of combination $Q^2 F_{\pi\gamma}$ for the best-fit parameters (38) is shown in Fig. 2.

Basing on the model “pion+continuum” we can calculate the relative correction to continuum contribution to ASR $\delta I_{\text{cont}}/I_{\text{cont}}^0$ relying on different fits of $F_{\pi\gamma}^*$:

\[ \left| \frac{\delta I_{\text{cont}}}{I_{\text{cont}}^0} \right| = \sqrt{2} \pi f_\pi (F_{\pi\gamma}^* - F_{\pi\gamma}/I_{\text{cont}}^0) = \frac{2\sqrt{2} \pi^2 f_\pi (s_0 + Q^2)}{Q^2} F_{\pi\gamma}^* - \frac{s_0}{Q^2}. \quad (39) \]

In Fig. 3 the ratios $\delta I_{\text{cont}}/I_{\text{cont}}^0$ for our fit (36, 38) and fit, obtained in recent paper [18] are shown. We see, that the correction to continuum is indeed small, even though the BaBar data shows that the relative correction to pion contribution is large.
Figure 2: $Q^2F_{\gamma\gamma}(Q^2)$: BaBar data [13], pQCD calculated behaviour (24) (dashed curve) and logarithmically enhanced behaviour due to small correction to continuum (37, 38) (solid curve).

Figure 3: (Colour online). Relative correction to continuum $\delta I_{\text{cont}}/I_{\text{cont}}(Q^2)$: our fit (36) (solid curve) and fit, obtained in [18] (dashed blue curve).
Above we estimated the correction to continuum in the model \( \pi^0 + \text{continuum} \). However, one can consider more refined models like \( \pi^0 + a_1 + \text{continuum} \). This model may assume interplay between corrections to three terms in the ASR.

Moreover, in the case of the small correction to continuum which currently seems to be the most likely situation, ASR will lead to the relation between the transition form factors of pion and \( a_1 \) which may be further studied both theoretically and, most important, experimentally.

6 Discussion and Conclusions

Dispersive approach to axial anomaly proved to be a useful tool for studying the properties of meson spectrum. It is well known, that when both photons are real \( (Q^2 = 0) \) the ASR saturates by pion contribution only. However, when one of the photons is virtual \( (Q^2 \neq 0) \) we immediately get different situation: ASR can be saturated only with a full meson spectrum (any finite number of mesons cannot saturate the anomaly sum rule). So the axial anomaly is a collective effect of meson spectrum.

The anomaly sum rule and quark-hadron duality in case of model \( \pi^0 + \text{continuum} \) allows to reproduce the well-known Brodsky-Lepage interpolation formula for \( F_{\pi \gamma} \). We estimate the contribution of \( a_1 \) meson to anomaly sum rule in the model \( \pi^0 + a_1 + \text{continuum} \).

The exactness of the anomaly sum rule leads to the relation between corrections to continuum and lower mass states contributions. The last experimental data on pion transition form factor \( F_{\pi \gamma} \) at large \( Q^2 \) allows us to estimate the possible continuum correction.

One can also consider \( \gamma^* \gamma \rightarrow \eta \) transition form factor in the same way. Considering the similar ratio for the relative contributions of \( \eta \) meson and continuum as \( 34 \) (with the continuum threshold \( s_0^\eta \approx 2.5 \text{ GeV}^2 \)), we can estimate the relative correction to \( \eta \) to be several times smaller than the one for \( \pi^0 

\[
\frac{\delta I_{\eta}/I_{\eta}^0}{\delta I_{\pi}/I_{\pi}^0} \approx \frac{s_0^\pi}{s_0^\eta} \approx 0.3.
\]

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