Cosmic acceleration sourced by modification of gravity without extra degrees of freedom

Abhineet Agarwal\textsuperscript{1}, R. Myrzakulov\textsuperscript{2}, S. K. J. Pacif\textsuperscript{3}, M. Sami\textsuperscript{4}, Anzhong Wang\textsuperscript{5}

\textsuperscript{1}International Institute of Information Technology, Gachibowli, Hyderabad, Telangana, India
\textsuperscript{2}Eurasian International Center for Theoretical Physics and Department of General & Theoretical Physics, Eurasian National University, Astana 010008, Kazakhstan
\textsuperscript{3}Department of Mathematics, School of Advanced Sciences, VIT University, Vellore, Tamil Nadu 632 014, India
\textsuperscript{4}Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi 110 025, India
\textsuperscript{4}Maulana Azad National Urdu University, Gachibowli, Hyderabad, Telangana 500 032, India
\textsuperscript{4,5}Institute for Advanced Physics & Mathematics, Zhejiang University of Technology, Hangzhou 310032, China
\textsuperscript{5}GCAP-CASPER, Department of Physics, Baylor University, Waco Texas 76798-7316, USA

agarwal.abhi93@gmail.com\textsuperscript{1}, rmyrzakulov@gmail.com\textsuperscript{2}, shibesh.math@gmail.com\textsuperscript{3}, samijamia@gmail.com\textsuperscript{4}, Anzhong_Wang@baylor.edu\textsuperscript{5}

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Abstract
In this paper, we investigate a scenario in which late time cosmic acceleration might arise due to coupling between dark matter and baryonic matter without resorting to dark energy or large scale modification of gravity associated with extra degrees of freedom. The scenario can give rise to late time acceleration in Jordan frame and no acceleration in Einstein frame - *generic modification of gravity* caused by disformal coupling. Using a simple parametrization of the coupling function, in maximally disformal case, we constrain the model parameters by using the age constraints due to globular cluster data. We also obtain observational constraints on the parameters using $H(z) + SNIa + BAO$ datasets. In this case, we distinguish between phantom and non-phantom acceleration and show that the model can give rise to phantom behavior in a narrow region of parameter space.

**keywords:** Jordan frame, Einstein frame, disformal coupling, late time acceleration

1 **Introduction**

It is common belief that late time cosmic acceleration [1, 2, 3, 4] is either sourced by dark energy [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] or by large scale modification of gravity associated with extra degrees of freedom [1]. As for dark energy, it could be represented by cosmological constant [7, 15, 16, 17] or by a slowly rolling scalar field [18, 19, 20, 21, 22], both of which are plagued with the theoretical problem of similar nature. On the other hand, observations at the background level are quite comfortable with dark energy scenario. Large scale modification of gravity amounting to Einstein general theory of relativity along with extra degrees of freedom generically includes their coupling to matter. The extra degree(s) of freedom, if massive, should be very light in order to be relevant to late time acceleration, but would in turn cause a havoc to local physics. Whether the degrees of freedom are massive or massless, one correspondingly invokes the chameleon [25, 26, 27] or Vainstein mechanism [28] to suppress them locally. In class of theories, with chameleon mechanism operative, imposition of local gravity constraints leaves no scope for self acceleration [29, 30]. On the other hand Vainstein screening sets in dynamically through non-linear derivative interaction and sounds superior to chameleon mechanism. For instance, it is at the heart of massive theories of gravity. Furthermore, in the case of massive gravity *a la* dRGT [31, 32], the scalar belongs to Galileon type [33] which gets screened via Vainstein screening. Unfortunately, FRW cosmology is absent in this theory. Promoting the latter to bi-gravity, might address the problem but then issues related to Higuchi bound pope in. It would be fare to say that, at present, a consistent model of large scale modification of gravity, associated with extra degrees of freedom, relevant to late time acceleration, is not known. In these scenarios, in the decoupling limit relevant to local physics, vector degrees of freedom decouple whereas the longitudinal (scalar) one couples to matter with the universal coupling.

In a recent review [34], a model-independent approach was considered to tackle the dark energy/modified gravity problem wherein $f(R)$ and $f(T)$ theories were explored in different formalisms and the role of conformal transformations in the Einstein and Jordan frames was clarified, see also Ref. [35] on related issues.

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*we did not mention here large scale modification caused by extra dimensions, non-local corrections to Einstein-Hilbert action and models based upon f(T) gravity.*
Clearly, it should be interesting to look for the third alternative leaving aside the exotic matter or extra degrees of freedom responsible for large scale modifications of gravity. Recently, a novel mechanism was proposed to do the needful, namely, interaction between baryonic and dark matter was shown to give rise to late time acceleration \[36\]. In this case, as demonstrated in Ref. \[36\], the purely conformal coupling is disfavored by the stability criteria. However, the maximally disformal coupling can give rise to late time cosmic acceleration in Jordan frame and no acceleration in the Einstein frame as the energy density of both matter components taken together follows standard conservation and redshifts as usual. On the other hand, in the Jordan frame, the matter components are not coupled but dynamics is modified such that late time acceleration might arise in this frame. In our opinion, this is a remarkable possibility of generic modification of gravity— acceleration in Jordan frame and no acceleration in Einstein frame.

In this paper, we further investigate the proposal of Ref. \[36\]. We use simple parametrization of metric function in a maximally disformal case and constrain the model parameters using the age constraints due to globular cluster data. We also find constraints on the parameters using \(H(z) + SNIa + BAO\) datasets for phantom and non phantom acceleration.

### 2 Interaction between dark matter and baryons

In this section, we briefly revisit the scenario introduced in Ref. \[36\]. As pointed out in the introduction, we shall consider a general coupling, to be specified later, between baryonic matter and dark matter. To this effect, we shall use the following action in the Einstein frame Ref. \[36\],

\[
\mathcal{L} = \frac{1}{16\pi G} \sqrt{-g} \mathcal{R} + \mathcal{L}_{DM}[g_{\mu\nu}] + \mathcal{L}_b[\tilde{g}_{\mu\nu}],
\]

(1)

where \(\mathcal{L}_{DM}\) describes the dark matter which is minimally coupled; \(\mathcal{L}_b\) the Lagrangian for baryonic matter that couples to dark matter through Jordan frame metric \(\tilde{g}_{\mu\nu}\) which is constructed from the Einstein frame metric \(g_{\mu\nu}\) and parameters that characterize the dark matter. From here onwards, quantities with a overhead tilde would be associated with Jordan frame.

In order to fix the Lagrangian for dark matter, for simplicity, we assume dark matter to be a perfect fluid which is legitimate on linear scales we are interested in. In this case, dark matter can be described by the Lagrangian of a single scalar field,

\[
\mathcal{L}_{DM} = \sqrt{-g} P(X),
\]

(2)

where \(X = -g^{\mu\nu} \partial_\mu \Theta \partial_\nu \Theta\), (\(\Theta\) being the dark matter field). For the action (2), the stress tensor is given by

\[
T_{\mu\nu} = 2P_X \partial_{\mu} \Theta \partial_{\nu} \Theta \mathcal{L}_b \tilde{g}_{\mu\nu},
\]

(3)

which has the form of energy momentum tensor of perfect fluid \(T_{\mu\nu} = (\rho_{DM} + P_{DM}) u_\mu u_\nu + P_{DM} g_{\mu\nu}\), provided that we make the following identification for the density and pressure of dark matter,

\[
\rho_{DM} = 2P_X (X) X - P(X) \mathcal{L}_b \tilde{g}_{\mu\nu},
\]

(4)

As mentioned before, the baryonic matter couples to dark matter via the Jordan frame metric \(\tilde{g}_{\mu\nu}\) which is constructed from the Einstein frame metric \(g_{\mu\nu}\) and components of dark matter. With the assumption that dark matter is perfect fluid, the most general form of \(\tilde{g}_{\mu\nu}\) is given by...
\[ \tilde{g}_{\mu\nu} = -Q^2(X)u_\mu u_\nu + R^2(X)(g_{\mu\nu} + u_\mu u_\nu), \] (5)

\[ \tilde{g}_{\mu\nu} = R^2(X)g_{\mu\nu} + S(X)\partial_\mu \Theta \partial_\nu \Theta; \quad S(X) \equiv \frac{R^2(X) - Q^2(X)}{X}, \] (6)

where \( R \) and \( Q \) are arbitrary functions to begin with and determinants of both the Einstein frame and Jordan frame metrics are related by \( \sqrt{-\tilde{g}} = QR^3 \sqrt{-g} \). With the above specialization of matter components, the equations of motion for DM can be obtained by varying the action with respect to the dark matter field \( \Theta \), Ref. [36],

\[ \partial_\nu \left( \left[ 2P,_{X}+QR^3\tilde{T}_{b}^{\alpha\beta}(2RR,_{X}g_{\alpha\beta} + S,_{X}\partial_\alpha \Theta \partial_\beta \Theta)g^{\mu\nu} - QR^3S\tilde{T}_{b}^{\mu\nu} \right] \sqrt{-g}\partial_\mu \Theta \right) = 0, \] (7)

where \( \tilde{T}_{b}^{\mu\nu} \) is the Jordan frame energy-momentum tensor for baryons given by,

\[ \tilde{T}_{b}^{\mu\nu} = \frac{2}{\sqrt{-\tilde{g}}} \frac{\delta \mathcal{L}_{b}}{\delta \tilde{g}_{\mu\nu}}. \] (8)

In Jordan frame, the matter components are not coupled though the dynamics might look complicated, see Appendix-1 for details. Thus both the components are separately conserved. In particular,

\[ \tilde{\nabla}_\mu \tilde{T}_{b}^{\mu\nu} = 0. \] (9)

However, in the Einstein frame, dark matter and baryonic matter do not conserve separately due to the presence of coupling. Nevertheless, the total energy density of both the components taken together should still follow standard conservation law. To this effect, following Ref. [36], let us first write down the Einstein equation, varying the action with respect to \( g_{\mu\nu} \), we obtain

\[ G_{\mu\nu} = 8\pi G \left[ T_{\mu\nu} + QR^3\tilde{T}_{b}^{\kappa\lambda} \left( R^2 g_{\kappa\mu}g_{\lambda\nu} + (2RR,_{X}g_{\kappa\lambda} + S,_{X}\partial_\kappa \Theta \partial_\lambda \Theta)\partial_\mu \Theta \partial_\nu \Theta \right) \right], \] (10)

where the DM stress-energy tensor \( T_{\mu\nu} \) is given in [3]. The second term on LHS in the above equation is the effective energy momentum tensor in the Einstein frame.

### 3 Cosmological dynamics in presence of coupling

The scenario under consideration is based upon the assumption of coupling between baryonic and dark matter components, introduced phenomenologically, in the Einstein frame. In order to investigate the observational consequences of (1), we shall, hereafter, specialize to homogeneous and isotropic Universe on a spatially-flat FRW (Friedmann-Robertson-Walker) background

\[ ds^2 = -dt^2 + a^2(t) \left( dx^2 + dy^2 + dz^2 \right). \] (11)

Since the metric functions \( R \) and \( Q \) are functions of the scale factor only on the FRW background, the Jordan frame metric tensor then takes the form,

\[ \tilde{g}_{\mu\nu} = \text{diag} \left( -Q^2(a), R^2(a)a^2, R^2(a)a^2, R^2(a)a^2 \right). \] (12)
The arbitrary functions $Q(a)$ and $R(a)$ need to be specified such that the thermal history, known to good accuracy, is left intact. Thus, in the early Universe, gravity should become standard so that in the limit of high matter density, $Q$, and $R$ become constant which without the loss of generality could be taken unity. This condition has to be imposed phenomenologically which also has implications for local physics. Indeed, the said choice would reduce the model to Einstein gravity in high density regime adhering to local gravity constraints. Secondly, at late times $Q$ and $R$ should be chosen such that the baryons experience accelerated expansion. Let us note that, since total energy density of matter in the Einstein frame follow standard conservation, such a scheme can not give rise to acceleration in this frame. We can expect acceleration in the Jordan frame only provided that we make appropriate choices. Indeed, the Jordan frame scale factor dubbed physical scale factor and its counterpart in the Einstein-frame are related as

$$\tilde{a} = Ra. \quad (13)$$

Since, $Q, R \to 1$ in the early Universe, $\tilde{a} \to a$ at early times. Although the Einstein frame scale factor is always decelerating, the expansion in Jordan frame governed by $\tilde{a}$ might exhibit acceleration if $R$ grows sufficiently fast at late times. In generic case, $R$ is concave up beginning from $R = 1$ at early times. In that case fast growth of $R$ might compensate the effect of deceleration of $a(t)$ making $\tilde{a}$ positive. Indeed,

$$\ddot{\tilde{a}} = \ddot{Ra} + 2\dot{R}\dot{a} + R\dot{a}. \quad (14)$$

The last term in Eq. (14) is negative and in case $R$ is concave up, the first term is positive. Hence, if $R$ grows fast at late times, the second term can compensate the effect of deceleration in Einstein frame giving rise to acceleration in the Jordan frame. In this case, acceleration is completely removed by disformal transformation which signifies that acceleration is generic feature of large scale modification of gravity in the Jordan frame.

Assuming the baryon component to be a perfect fluid,

$$\tilde{T}^\mu_\nu = (\tilde{\rho}_b + \tilde{P}_b)\tilde{u}^\mu_b \tilde{u}^\nu_b + \tilde{P}_b \tilde{g}^\mu_\nu \ ; \ \tilde{g}^\mu_\nu \tilde{u}^\mu_b \tilde{u}^\nu_b = -1. \quad (15)$$

Eq. (9) then gives the standard continuity equation

$$\frac{d\tilde{\rho}_b}{d\ln \tilde{a}} = -3(\tilde{\rho}_b + \tilde{P}_b). \quad (16)$$

The dark matter equation (7) reduces to [36],

$$\frac{d}{dt} \left( -P_{X} + Q R^2 \left( \frac{Q_{X}}{Q} \tilde{\rho}_b - 3 \frac{R_{X}}{R} \tilde{P}_b \right) \right) a^3 \dot{\Theta} = 0, \quad (17)$$

where we have used $g_{\lambda \kappa} \tilde{T}^{\lambda \kappa} = -Q^{-2} \tilde{\rho}_b + 3R^{-2} \tilde{P}_b$ and assumed, $\dot{\Theta} > 0$. Using equation (4) and integrating (17), we have

$$\rho_{DM} = \Lambda_{DM}^4 \sqrt{\frac{X}{X_{eq}}} \left( \frac{a_{eq}}{a} \right)^3 - P + 2XQR^3 \left( \frac{Q_{X}}{Q} \tilde{\rho}_b - 3 \frac{R_{X}}{R} \tilde{P}_b \right), \quad (18)$$
where the subscript ‘eq’ indicates matter-radiation equality. $\Lambda_{DM}^4$ is identified as the DM mass density at radiation-matter equality. The Friedmann equation can be derived from equation (10) as

$$3H^2 = 8\pi G \left( \rho_{DM} + \rho_b \right), \quad (19)$$

where an effective Einstein-frame baryon density is defined as

$$\rho_b = QR^3 \left( \bar{\rho}_b \left( 1 - 2X \frac{Q_X}{Q} \right) + 6X \frac{R_X}{R} \bar{P}_b \right). \quad (20)$$

Using Eqs. (18) and (20) in Eq. (19), the Friedmann equation becomes

$$3H^2 = 8\pi G \left( \Lambda_{DM}^4 \sqrt{\frac{X}{X_{eq}}} \left( \frac{a_{eq}}{a} \right)^3 - P + QR^3 \bar{P}_b \right). \quad (21)$$

Radiation do not couple to either baryon or DM. The acceleration equation can be derived as

$$2\frac{\ddot{a}}{a} + H^2 = -8\pi G (P + P_b), \quad (22)$$

where the effective baryon pressure is

$$P_b \equiv QR^3 \bar{P}_b. \quad (23)$$

Now, we have three field equations (17), (21) and (22) out of which two are independent. Hereafter, we shall assume matter to be pressureless, $\bar{P}_b \simeq 0$ and $P \ll 2XP_{X}$. Since both the matter components in Jordan frame conserve separately, they follow the standard conservation. Indeed, using Eqs. (23) in Eq. (16), one obtains, $\bar{\rho}_b \sim \bar{a}^{-3}$ as it should be in absence of pressure. Baryon matter density can conveniently be written as,

$$\bar{\rho}_b = \frac{\Lambda_b^4}{R^3} \left( \frac{a_{eq}}{a} \right)^3. \quad (24)$$

Since the function $R = 1$ in the early Universe, $\Lambda_b^4$ can be treated as the baryon mass density at equality. For zero pressure, Eq. (22) leads to $a(t) \sim t^2$ and the background is identical to matter dominated which is valid from matter-radiation equality up to the present time irrespective of the dark matter-baryon coupling. The total energy density can be obtained from Eq. (21) after substituting Eq. (24) as

$$\rho_{Total} \equiv \frac{3H^2}{8\pi G} \simeq \left[ \Lambda_{DM}^4 \sqrt{\frac{X}{X_{eq}}} + \Lambda_b^4 \right] \left( \frac{a_{eq}}{a} \right)^3, \quad (25)$$

The total energy density in absence of pressure in the Einstein frame should follow the standard conservation implying that $\rho_{Total} \sim a^{-3}$. It is important to note that the conservation holds for an arbitrary $Q$ which means that the term within square bracket in Eq. (25) is time independent for any $Q(X)$. Clearly, coupling does not give rise to acceleration in Einstein-frame. However, in
the Jordan frame, where the matter components are not directly coupled but gravity is modified, we might achieve acceleration at late times. We do not take this path, we shall rather directly work with the Jordan frame metric with a suitable parametrization\(^2\) of metric function \(R\).

Let us define the density parameters as

\[
\Omega_{DM} = \frac{\rho_{DM}}{3H^2}, \quad \Omega_b = \frac{\rho_b}{3H^2}.
\]  

(26)

In case of zero pressure, we have,

\[
\Omega_{DM} \equiv \Omega_{DM}^{(0)} \left( \frac{a_0}{a} \right)^3 \left( \frac{H_0}{H} \right)^2, \quad \Omega_b \equiv \Omega_{b}^{(0)} \left( \frac{a_0}{a} \right)^3 \left( \frac{H_0}{H} \right)^2.
\]  

(27)

The Friedmann equation,

\[
H^2 = H_0^2 \left[ \Omega_{DM}^{(0)} (1+z)^3 + \Omega_{b}^{(0)} (1+z)^3 \right],
\]  

(28)

has the standard equation of flat FRW cosmology with cold matter (dark matter + baryonic matter); subscript ‘0’ indicates the value of the quantity at present epoch as usual. We denote the present time mass densities as superscript ‘(0)’ for respective quantity.

As mentioned before, the two coupling functions \(Q\) and \(R\) can be chosen suitably such that \(Q, R \to 1\) in the early Universe and both \(a\) and \(\tilde{a}\) experience deceleration where as at late times the coupling function \(R\) grows sufficiently large such that the physical scale factor \(\tilde{a}\) experience acceleration. Let us distinguishes two choices of \(Q\) and \(R\). The conformal coupling that corresponds to \(Q(X) = R(X)\) and the disformal coupling \(Q(X) \neq R(X)\). The stability condition for \(0 < c_s^2 \leq 1\) requires fine tuning in case of conformal coupling and sounds unnatural. Indeed, the stability condition, \(\rho_{DM} >> Q^2 \rho_b/c_{DM}^2\) requires unnatural choice for \(Q\) corresponding to a given value of \(c_{DM}\), see Ref. [36] for details. The stability condition holds for the disformal coupling in case of \(Q = 1\) dubbed maximally disformal. In this case, one is left with one function \(R\) which can be conveniently parametrized and compared with data. We will focus on a simple parametrization of this function involving two parameters that can be constrained from data.

### 3.1 Simple parametrization and cosmological dynamics in Jordan frame

It is clear from Eq. (28) that the scale factor in Einstein frame does not experience acceleration. We should look for such a possibility in Jordan frame where matter components are not coupled but dynamics is modified. In order to progress further, we need to specify \(R(a)\). Equivalently, we can also parametrize \(a\) in terms of physical scale factor \(\tilde{a}\). In the discussion to follow, we consider the following simple form of \(a(\tilde{a})\),

\[
a(\tilde{a}) = \tilde{a} + \alpha \tilde{a}^2 + \beta \tilde{a}^3,
\]  

(29)

where \(\alpha\) and \(\beta\) are two parameters of the model to be determined from observational data. The functional form is chosen in such a way that in the early times \(a = \tilde{a}\) such that the distinction

\(^2\)In the generic case, \(Q\) can be fixed to unity.
between the two frames disappear at early times. We also note that this simple functional form, as series expansion in \(a = \tilde{a}\) is valid up to the present time, \(\tilde{a} \leq 1\). Extrapolating (29) to future evolution might bring in undesirable features. For simplicity, we shall restrict our discussion to the polynomial of third degree in \(\tilde{a}\) in (29) which is convenient for comparing results with observation. The first and second derivatives of the scale factor are given by

\[
\dot{a} = (1 + 2\alpha \tilde{a} + 3\beta \tilde{a}^2)\tilde{a}, \quad \ddot{a} = (1 + 2\alpha \tilde{a} + 3\beta \tilde{a}^2)\ddot{a} + (2\alpha + 6\beta \tilde{a})\dot{a}^2,
\]  

where \(\dot{a} = \frac{da}{dt}\). Using these, we can obtain the Hubble parameter and deceleration parameter in Jordan-frame as

\[
\tilde{H} = \frac{\dot{a}}{\tilde{a}} = \left(1 + \frac{\alpha \tilde{a} + \beta \tilde{a}^2}{1 + 2\alpha \tilde{a} + 3\beta \tilde{a}^2}\right)H,
\]

and

\[
\tilde{q} = -\frac{\ddot{a}}{\tilde{a}^2} = \frac{1}{2}\left(1 + \frac{2\alpha \tilde{a} + 3\beta \tilde{a}^2}{1 + \alpha \tilde{a} + \beta \tilde{a}^2}\right) + \left(\frac{2\dot{a}(\alpha + 3\beta \tilde{a})}{1 + 2\alpha \tilde{a} + 3\beta \tilde{a}^2}\right).
\]

(\because\) In Einstein frame, for zero pressure Eq. (22) gives \(q = \frac{1}{2}\). Also, using equation (30) along with the field equations, we obtain

\[
2\left(1 + \frac{2\alpha \tilde{a} + 3\beta \tilde{a}^2}{1 + \alpha \tilde{a} + \beta \tilde{a}^2}\right)\frac{\ddot{a}}{\tilde{a}} + \frac{2(\alpha + 3\beta \tilde{a})}{1 + \alpha \tilde{a} + \beta \tilde{a}^2}\frac{\dot{a}^2}{\tilde{a}^2} = -\frac{8\pi G}{3}\left[\Lambda_{DM}^4 \sqrt{\frac{X}{X_{eq}}} + \Lambda_b^4\right]\left(\frac{a_{eq}}{a}\right)^3,
\]

which gives an expression of deceleration parameter in Jordan frame as

\[
\tilde{q} = \frac{\Omega_{Total}}{2} \left(1 + \frac{2\alpha \tilde{a} + 3\beta \tilde{a}^2}{1 + \alpha \tilde{a} + \beta \tilde{a}^2}\right) + \left(\frac{2\dot{a}(\alpha + 3\beta \tilde{a})}{1 + 2\alpha \tilde{a} + 3\beta \tilde{a}^2}\right),
\]

where we have used the equation (25). Equations (32) and (34) are same because \(\Omega_{Total} = 1\).

The Hubble parameter (Eq. (31)) can be represented as

\[
\frac{\tilde{H}}{H_0} = \left(1 + \frac{2\alpha + 3\beta}{1 + \alpha + \beta}\right)\left(\frac{1 + \alpha \tilde{a} + \beta \tilde{a}^2}{1 + 2\alpha \tilde{a} + 3\beta \tilde{a}^2}\right)\left(\frac{H}{H_0}\right).
\]

Since, for pressureless matter, \(H(a) = H_0 \left(\frac{a_0}{a}\right)^3\) and \(a_0 = (1 + \alpha + \beta)\), using Eq. (29), Eq. (35) can be also be cast as

\[
\tilde{H}(\tilde{a}) = \tilde{H}_0 \left(\frac{1 + \alpha + \beta}{\tilde{a}}\right)^{\frac{3}{2}} \left(1 + 2\alpha + 3\beta\right) \left[1 + \alpha \tilde{a} + \beta \tilde{a}^2\right]^{\frac{1}{2}} \left[1 + 2\alpha \tilde{a} + 3\beta \tilde{a}^2\right]^{-\frac{1}{2}}.
\]

From Eq. (13), we have

\[
\frac{a}{\tilde{a}} = \frac{1}{R}.
\]

We set the physical scale factor at present to be 1 i.e. \(\tilde{a}_0 = 1\), but this implies that \(a_0 \neq 1\). The redshifts in both the Einstein frame and Jordan frame are then related as
\[ \ddot{a} = \frac{\ddot{a}_0}{1 + \ddot{z}}, \quad a = \frac{a_0}{1 + z}. \tag{38} \]

The present time corresponds to \( \ddot{z} = z = 0 \).

Now, using equations (38) and (29), the Hubble parameter can be expressed in terms of redshift \( \ddot{z} \) in Jordan frame as

\[ \ddot{H}(\ddot{z}) = \ddot{H}_0 \frac{(1 + \alpha + \beta)^{\frac{1}{2}} (1 + 2\alpha + 3\beta)(1 + \ddot{z})^{\frac{3}{2}}}{(1 + \ddot{z})^2 + \alpha (1 + \ddot{z}) + \beta \left[(1 + \ddot{z})^2 + 2\alpha (1 + \ddot{z}) + 3\beta\right]}. \tag{39} \]

Let us also quote the expression for the equation of state parameter versus the redshift (\( \ddot{z} \)) in Jordan frame as,

\[ \ddot{w}_{\text{eff}}(\ddot{z}) = \frac{\alpha (5 + 6\alpha + 5\ddot{z}) (1 + \ddot{z})^2 + \beta (14 + 23\alpha + 14\ddot{z})(1 + \ddot{z}) + 18\beta^2}{3\{(1 + \ddot{z})^2 + \alpha (1 + \ddot{z}) + \beta \}\{(1 + \ddot{z})^2 + 2\alpha (1 + \ddot{z}) + 3\beta\}}, \tag{40} \]

which clearly mimics cold matter in the limit of large redshift. In the scheme of two parameters, we have derived the effective equation of state induced by coupling between two known components of matter. Analytical expression (40) can directly be used for finding observational constraints on the model. Let us note that for generic negative values of parameters, the denominator in (40) vanishes for certain negative values of the redshift around which the numerator is always negative. Hence, if (29) is extrapolated to near future, the equation of state parameter after assuming the observed value at the present epoch, would attain larger and larger negative values. It is clear that the underlying system would then always evolve to phantom in future (even if we set the non-phantom behavior at present, see Fig. 1). This is, however, not the generic feature of the scenario but rather the artifact of parametrization (29).

![Phantom divide line](https://example.com/phantom_divide_line.png)

Figure 1: Figure shows the equation of state parameter versus the redshift in Jordan frame. Parameters are set such that we have non-phantom behavior at present; slight adjustment of parameters can give rise to phantom behavior consistent with the observation. Even if the present behavior is set to be non-phantom, equation of state evolves to large negative values in future as \( \ddot{z} \to \ddot{z}_s = -0.3902 \). Such a behavior is a clear manifestation of a future singularity which is specific to (29).
Expressions (39) and (40) are the important results of our analyses to be used for further manipulations. At the onset (39) (for non-vanishing couplings $\alpha$ and $\beta$ which imbibe new physics) does not look like the expression for Hubble parameter with standard matter plus an exotic (dark) fluid. Since, suitable, negative values of the couplings can give rise to late time acceleration, it should be possible to extract the required information from (39) by putting it in a convenient form. However, we shall first test it for the resolution of age problem in the hot big bang model.

3.2 Age of the Universe

It is well known that the FRW cosmology is plagued with age crises if Universe is inhabited by the standard form of matter alone which contributes to deceleration as gravity is attractive. Secondly, more than half the contribution to age of Universe comes from late stages, namely, evolution from $z = 0$ to $z = 1$. If hypothetically, we ignore gravity, then Hubble law gives rise to $t_0 = 1/H_0$, thereby the disturbing factor of $2/3$ in the formula for the age of Universe is contributed by gravity in presence of normal matter. Since we can not do away with normal matter (cold dark matter+baryonic matter to be called as cold matter or standard matter in the further discussion) the only way out in the standard frame work is provided by the assumption of existence of an exotic form of matter, repulsive in nature, that dominates the late Universe and can compensate deceleration caused by the standard matter for which gravity is attractive. Clearly, the latter would slow down the expansion rate at late times improving the age of Universe. Since in the present scenario, we have only standard matter, it is desirable to check for the age of Universe. Using expression (38), we can compute the age of Universe in the model under consideration in Jordan frame:

$$\tilde{t}_0 = \frac{1}{H_0} \int_0^\infty \frac{\left(1 + \tilde{z}\right)^2 + \alpha \left(1 + \tilde{z}\right) + \beta}{\left(1 + \alpha + \beta\right)\left(1 + 2\alpha + 3\beta\right)} d\tilde{z}. \quad (41)$$

In absence of coupling, $\alpha = \beta = 0$, we have $\tilde{t}_0 = t_0 = 2/3H_0$ as should be. The essence of acceleration lies in the non-vanishing values of coupling parameters $\alpha$ & $\beta$. In Fig. 2 we show parameter range consistent with globular cluster data on the age of Universe. The range is found using the expression (41) with $H_0 = 67.8 \text{ km/s/Mpc}$. We have plotted age of the Universe ranging from 12 Gyrs to 15 Gyrs in the $\alpha\beta$–plane. The shaded region shows the allowed values of $\alpha$ & $\beta$ corresponding to the range of age of the Universe as dictated by globular cluster data. The parameters $\alpha$ & $\beta$ can be set to get higher age of the Universe with large acceleration.

In order to understand the underlying physics, let us focus on the dot-dashed line corresponding to $t_0 = 14$ Gyrs. If we move down around this line towards smaller values of $\alpha$ (large values of $\beta$) (numerically), we observe an interesting pattern, namely, $\Omega^{(0)}_{\text{Meff}}$ increases whereas $\tilde{w}^{(0)}_{e\text{ff}}$ decreases towards larger negative values, see Table 1. Larger values of $\Omega^{(0)}_{\text{Meff}}$ amounts

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3 Radiation is also included in the standard matter if relevant, radiation, however, does not contribute to age of Universe.

4 We should emphasize that the age of Universe computed in the Einstein frame would still be, $t_0 = 2/3H_0$ as there is no acceleration in this case.
Figure 2: Different values of age of the Universe have been plotted in the $\alpha \beta-$plane. The solid line, dashed, dot-dashed and dotted lines correspond to $\tilde{t}_0(\tilde{H}_0/67.8) = 12$ Gyrs, 13 Gyrs, 14 Gyrs and 15 Gyrs, respectively. The shaded region shows the allowed values of $\alpha$ and $\beta$ for the aforementioned range of age of the Universe.

to more deceleration which decreases the age of the Universe. The effect is compensated by stronger repulsive effect caused by larger negative values of the effective equation of state such that the age of the Universe does not change. This pattern clearly shows the underlying importance of late time cosmic acceleration for the resolution of age crisis in the standard model of cosmology. We should, however, admit that age considerations give broad constraints on the parameters. One certainly requires other data sets for better constraints.

3.3 Two fluid representation and connection to the standard lore

Since coupling between non components of matter in Einstein frame gives rise to acceleration in Jordan frame, it might be possible to define an effective hypothetical fluid that would mimic dark energy. Indeed, equation (39) can also be cast as

$$\frac{\tilde{H}^2}{\tilde{H}_0^2} = A(\alpha, \beta)(1 + \tilde{z})^3 + A(\alpha, \beta)f(\tilde{z}),$$  \hspace{1cm} (42) \hspace{1cm} \footnote{We have put the square of rational expression (39) in a convenient form isolating the term proportional to $(1 + \tilde{z})^3$ that can be identified with the effective fractional density of cold matter and the rest is pushed to a hypothetical matter expected to mimic dark energy.}
Table 1: Table lists points $A \to F$ on $\alpha \beta$–plane around $t_0 = 14$ Gyr line from top to bottom with the corresponding values of equation of state and density parameters. Table shows that weakening deceleration corresponds to weakening of acceleration.

\[
\begin{array}{|c|c|c|c|}
\hline
(\alpha, \beta) & w^{(0)}_{\text{eff}} & \Omega^{(0)}_{\text{Meff}} & \Omega^{(0)}_{X} \\
\hline
A(-0.1952, -0.0225) & -0.7484 & 0.2298 & 0.7701 \\
B(-0.1633, -0.0407) & -0.7927 & 0.2419 & 0.7581 \\
C'(-0.1075, -0.0720) & -0.8602 & 0.2656 & 0.7344 \\
D(-0.0814, -0.0864) & -0.8875 & 0.2780 & 0.7220 \\
E(-0.0340, -0.1140) & -0.9522 & 0.2965 & 0.7035 \\
F(-0.0024, -0.1343) & -1.0170 & 0.3028 & 0.6972 \\
\hline
\end{array}
\]

where $\left(\Omega^{(0)}_{DM} + \Omega^{(0)}_X\right) = 1$, in the case under consideration in Einstein frame and functions $A$ and $f$ are given by,

\[
A(\alpha, \beta) = (1 + \alpha + \beta)(1 + 2\alpha + 3\beta)^2, \quad f(\tilde{z}) = -5(1 + \tilde{z}^2)\alpha - \alpha \left(49\alpha^2 - 48\beta\right) + (1 + \tilde{z}) \left(17\alpha^2 - 7\beta\right) + \frac{(1 + \tilde{z})\alpha^6 - 5(1 + \tilde{z})\alpha^4\beta + \alpha^5\beta + 6(1 + \tilde{z})\alpha^2\beta^2 - 4\alpha^3\beta^2 - (1 + \tilde{z})\beta^3 + 3\alpha\beta^3}{(\alpha^2 - 4\beta)((1 + \tilde{z})^2 + (1 + \tilde{z})\alpha + \beta)}
\]

\[
\frac{128(1 + \tilde{z})\alpha^6 - 64\alpha^7 - 720(1 + \tilde{z})\alpha^4\beta + 576\alpha^5\beta + 864(1 + \tilde{z})\alpha^2\beta^2}{-1512\alpha^3\beta^2 - 135(1 + \tilde{z})\beta^3 + 918\alpha\beta^3} + \frac{128(1 + \tilde{z})\alpha^8 - 960(1 + \tilde{z})\alpha^6\beta + 192\alpha^7\beta + 2160(1 + \tilde{z})\alpha^4\beta^2 - 1296\alpha^5\beta^2}{-1512(1 + \tilde{z})\alpha^2\beta^3 + 2376\alpha^3\beta^3 + 162(1 + \tilde{z})\beta^4 - 1053\alpha\beta^4}{(\alpha^2 - 4\beta)((1 + \tilde{z})^2 + 2\alpha(1 + \tilde{z}) + 3\beta)^2}
\]

such that $[1 + f(0)] A(\alpha, \beta) \equiv 1$ where $f(\tilde{z} = 0) \equiv f(0)$. Let us now define the effective fractional density parameters, $\Omega^{(0)}_{\text{Meff}} \equiv A$ and $\Omega^{(0)}_x \equiv Af(0)$ which are functions of $\alpha, \beta$ only. Friedmann equation in Jordan-frame then takes the convenient form,

\[
\tilde{H}^2 = \tilde{H}_0^2 \left[ \Omega^{(0)}_{\text{Meff}}(1 + \tilde{z})^3 + \Omega^{(0)}_x F(\tilde{z}) \right],
\]

where $F(\tilde{z}) \equiv f(\tilde{z})/f(0)$. The first term in Eq. (45) is the effective fractional matter density for cold matter whereas the second term can be treated as the fractional energy density parameter of a hypothetical fluid ($x$-fluid). For a suitable choice of numerical values of $\alpha$ and $\beta$ (viz. $\alpha = -0.0655$ and $\beta = -0.0973$), we have the estimates, $\Omega^{(0)}_{\text{Meff}} \approx \{(1 + \alpha + \beta)(1 + 2\alpha + 3\beta)^2\} \approx 0.2789$ and $\Omega^{(0)}_x \approx \{(1 + \alpha + \beta)(1 + 2\alpha + 3\beta)^2\} f(0) \approx 0.7211$ as expected. Thus, we have succeeded to cast the Friedmann equation in the Jordan frame in the standard form with effective matter density, exotic fluid density. It will now be straightforward to analyze the observational constraints on model parameters using Eq. (45).

\textsuperscript{6}Let us note that both the (effective) dimensional density parameters $\Omega_{\text{Meff}}, \Omega_x$ are defined in the Jordan frame though we do not put tilde over them.
4 Simple parametrization and sudden future singularity in Jordan frame

In the preceding sections, we have elaborated on the details of the scenario based upon coupling between the known matter components in the Universe. In particular, the model gives rise to late time cosmic acceleration in Jordan frame. For generic values of $\alpha$ and $\beta$, Universe evolves to deep phantom region even if the behavior is non-phantom at present. To this effect, we quote the expressions for the effective density, pressure and equation of state parameter (rewriting it in a slightly different form) versus the redshift in Jordan frame in Appendix-1. Indeed, the denominator of $\tilde{w}_{\text{eff}}$ in Eq. (40) or Eq. (56) has four real roots which are negative for generic values of $\alpha$ & $\beta$. For the above choice of numerical values of the parameters ($\alpha = -0.0655$ and $\beta = -0.0973$), the highest root is $\tilde{z}_1 \simeq -0.3902$. The highest root is important as the evolution would terminate there. This expresses a future singularity which is reached for a finite value of the redshift $\tilde{z} = \tilde{z}_s \equiv \tilde{z}_1 \simeq -0.3902$ in future. The effective equation of state diverges to minus infinity at the quoted redshift. The effective matter density (Hubble parameter) in the Jordan frame also diverge there. Since the effective equation of state diverges, the effective pressure should also diverge. It is clear from the structure of Eqs. (54), (55) and (56) that they have the common roots. Secondly, $|\tilde{p}_{\text{eff}}|$ approaches infinity faster than $\tilde{\rho}_{\text{eff}}$ as $\tilde{z}$ approaches $\tilde{z}_s$, which is consistent with the behavior of equation of state parameter given by Eq. (40) or (56), see Fig. 3. Hence, the scenario under consideration with (29), is plagued with sudden future singularity of type III. In what follows, we show that the sudden future singularity is not the generic feature of the model but the artifact of simple parametrization.

![Figure 3](image_url)

Figure 3: Figure shows the behavior of effective energy density (solid line) and effective pressure (dashed line) versus the redshift in Jordan frame. Both the quantities diverge near $\tilde{z} = \tilde{z}_s = -0.3902$. This figure also shows that effective pressure diverges faster than the effective energy density near singularity which is consistent with the behavior of equation of state parameter given by Eq. (40) or (56), see Fig. 3. Hence, the scenario under consideration with (29), is plagued with sudden future singularity of type III. In what follows, we show that the sudden future singularity is not the generic feature of the model but the artifact of simple parametrization.

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7The effective matter density can be cast as effective matter density for cold matter plus matter density of an exotic fluid, the latter diverges as $\tilde{z} \to \tilde{z}_s$
4.1 Delaying the singularity to infinite future

In what follows, we show that by using an alternative parametrization, the sudden future singularity can be pushed to infinite future. To this effect, let us parametrize $a$ in terms of physical scale factor $\tilde{a}$ containing an exponential expression of the form

$$a(\tilde{a}) = \tilde{a} e^{\alpha \tilde{a}}, \quad (46)$$

with a single model parameter $\alpha$. Expression (46) mimics (29) for $\tilde{a} \lesssim 1$ with $\beta = \alpha^2 / 2$ but can crucially modify the future evolution.

In this case, the deceleration parameter in Jordan-frame is given by

$$\tilde{q} = -\frac{\ddot{\tilde{a}}}{\dot{\tilde{a}}^2} = \frac{(1 + \alpha \tilde{a})^2 + 2\tilde{a}(2\alpha + \alpha^2 \tilde{a})}{2(1 + \alpha \tilde{a})}. \quad (47)$$

The effective equation of state in Jordan frame is then obtained as

$$\tilde{w}_{\text{eff}}(\tilde{z}) = \frac{5\alpha(1 + \tilde{z}) + 3\alpha^2}{3(1 + \tilde{z})[(1 + \tilde{z}) + \alpha]}. \quad (48)$$

Unlike (29), we have only one model parameter in this expression of effective equation of state parameter induced by coupling between the known components of matter. In this case,

$$\tilde{w}_{\text{eff}}^{(0)} = \frac{5\alpha + 3\alpha^2}{3(1 + \alpha)}, \quad (49)$$

which can have a desired value for a chosen negative values of $\alpha$; for instance, $\tilde{w}_{\text{eff}}^{(0)} \simeq -0.98$ for $\alpha \simeq -0.44$. We have shown a plot (see Fig. 4) of the effective equation of state parameter (48) for different values of $\alpha$ corresponding to phantom (non-phantom) behavior at the present epoch. Let us note that unlike the previous case (40), the denominator in (48) does not vanish for any negative values of the redshift ($\tilde{z} > -1$). The underlying system would always evolve to phantom in future without hitting the singularity. In this case, the singularity would occur at $\tilde{a}_s = \infty$. Clearly, sudden future singularity is an artifact of extrapolation of (29) beyond the present epoch where, strictly speaking, the latter is not valid. It is really interesting that one parameter family (46) gives rise to late time acceleration without sudden future singularity.

5 Observational constrains

In the preceding sections, we have analyzed different cosmological aspects of the scenario based upon coupling between normal components of matter. We now proceed to constrain the model parameters $\alpha$ & $\beta$ using $H(z)$, SNIa and BAO data. The model parameters $\alpha$ & $\beta$ are constrained by employing the $\chi^2$ analysis. The maximum likelihood method is used and the total likelihood for $\alpha$ & $\beta$ as the product of individual likelihood for different datasets is obtained. For the joint data, the total likelihood function is written as

$$L_{\text{tot}}(\alpha, \beta) = e^{-\frac{\chi^2_{\text{tot}}(\alpha, \beta)}{2}}, \quad (50)$$
\[ \alpha = -0.50, \ w_{\text{eff}}(0) = -1.1666 \]
\[ \alpha = -0.4517, \ w_{\text{eff}}(0) = -1.009 \]
\[ \alpha = -0.41, \ w_{\text{eff}}(0) = -0.8732 \]

Figure 4: Plot of effective equation of state parameter (48) for different values of \( \alpha \) corresponding to phantom (non-phantom) behavior at the present epoch.

where we have
\[
\chi^2_{\text{tot}} = \chi^2_{\text{Hub}} + \chi^2_{\text{SN}} + \chi^2_{\text{BAO}},
\]
and is related to the Hubble \( (H(z)) \) dataset, the Supernovae of Type Ia (SNIa) and the Baryon Acoustic Oscillation (BAO) data. The best fit value of the model parameters \( \alpha \& \beta \) is obtained by minimizing the total chi-square \( \chi^2_{\text{tot}} \) with respect to \( \alpha \& \beta \). As usual, the likelihood contours at 1\( \sigma \) and 2\( \sigma \) confidence level are 2.3 and 6.17, respectively, in the two dimensional plane. The basic tools and the reference data used here for the data analysis are given in appendix-2.

In figure 5, we have shown the results based upon \( SNIa + BAO \) data and the combined data set, \( H(z) + SNIa + BAO \). For both the data sets, there is a region of parameter space where the model is close to \( \Lambda \text{CDM} \). The region with relatively larger values of \( \alpha \) and smaller values of \( \beta \) (numerically) is somewhat away from \( \Lambda \text{CDM} \) model. The best fit values of cosmological parameters are close to each other for the two data sets. For instance, \( \Omega_{M,\text{eff}}^{(0)} \simeq 0.26 \), thereby, the model favors relatively lower values of matter density parameter compared to \( \Lambda \text{CDM} \). The present values of other cosmological parameters are listed in the following table 2.

| Data sets | \( SNIa + BAO \) | \( H(z) + SNIa + BAO \) |
|-----------|-----------------|--------------------------|
| \( \alpha \) | \( -0.09295 \) | \( -0.10268 \) |
| \( \beta \) | \( -0.08305 \) | \( -0.07834 \) |
| \( \tilde{w}_{\text{eff}}^{(0)} \) | \( -0.91218 \) | \( -0.91015 \) |
| \( \Omega_{M,\text{eff}}^{(0)} \) | \( 0.26299 \) | \( 0.25648 \) |
| \( \Omega_X^{(0)} \) | \( 0.73701 \) | \( 0.74352 \) |
| \( t_0 \) (Gyr) | \( 14.054 \) | \( 14.102 \) |

Table 2: Table indicates the best fit values of \( \alpha, \beta \) obtained using \( SNIa + BAO \) and \( H(z) + SNIa + BAO \) data sets with corresponding values of other cosmological parameters.
Figure 5: The figure shows the 1σ (dark shaded) and 2σ (light shaded) likelihood contours in \( \alpha \beta \)-plane. The left panel corresponds to SN+BAO whereas right panel is for Hubble+SN+BAO. The black dots represent the best fit value of the model parameters which are found to be \( \alpha = -0.092954, \beta = -0.083058 \) (left panel) and \( \alpha = -0.102681, \beta = -0.078347 \) (right panel). The dashed line shown in both the plots is the \( \tilde{w}^{(0)} = -1 \) line which intersects the contours in 2σ region and separates the phantom and non-phantom region. We get a narrow strip from both the SN+BAO and Hubble+SN+BAO constraints for the allowed values of \( \alpha \) & \( \beta \) for phantom evolution.

6 Conclusions and outlook

In this paper we further investigated the proposal of Ref. [36] which is an alternative to dark energy and large scale modification of gravity associated with extra degrees of freedom coupled to matter. In this picture, the late time cosmic acceleration is sourced by a general type of coupling between dark matter and baryonic matter. In Einstein frame, both the components individually do not follow standard conservation due to coupling between them though the total energy density does and should. Assuming both the components to be pressure-less, their total energy density redshifts as usual in the Einstein frame. Clearly, there is no acceleration in this frame in the framework under consideration.

We should emphasize that coupling is defined in the Einstein frame only. As for the, Jordan frame, both the matter components adhere to standard conservation, however, the Einstein Hilbert action and matter action are modified, see Eq. (52) in Appendix-1. This modification of gravity is not endowed with extra degrees of freedom as there is non in the Einstein frame where the Lagrangian is diagonalized. The transformation of metric contains two functions \( R \) and \( Q \) to be chosen keeping in mind phenomenology, namely, in high density regime they should reduce to unity leaving the thermal history and local physics intact. Secondly, at late stages, the choice of metric functions should give rise to accelerating Universe. The choice, \( Q = 1 \) corresponds to maximally dis-conformal transformation. In this case, \( R \) being concave up and fast growing at late times can give rise to late time cosmic acceleration in the Jordan frame.

A comment about the conformal coupling which corresponds to \( Q = R \) is in order. The
conformal choice though disfavored from stability criteria but not prohibited [36]. It simply requires the fine tuning of the metric functions, thereby, in principle, late time cosmic acceleration is possible in both the conformal and disformal cases.

Using the two parameter functional form for \(a(\tilde{a})\), we derived expression for the ratio, \(\tilde{H}(\tilde{z})/\tilde{H}(0)\), which is a rational expression in \(\alpha, \beta \& \tilde{z}\). We show that the age computed using the rational expression in the Jordan frame is consistent with observation provided we choose the couplings in accordance with the observed value of deceleration parameter. In the Einstein frame, we have standard FRW Universe inhabited with cold matter giving rise to \(t_0 = 2/3H_0\) as expected. In order to reconcile the scenario with the standard lore, we transformed the ratio, \(\tilde{H}(\tilde{z})/\tilde{H}(0)\), to a suggestive form isolating the cold matter like term proportional to \((1 + \tilde{z})^3\) and attributing the rest to a hypothetical \("x"-fluid. The so defined effective fractional energy densities \(\Omega_{Meff}^{(0)} \& \Omega_x^{(0)}\) are functions of \(\alpha \& \beta\) alone. The best fit values for the couplings allows us to reconcile with the standard lore with cold matter and an exotic fluid with large negative pressure giving rise to \(\Omega_{Meff}^{(0)} \simeq 0.26299 \& \Omega_x^{(0)} \simeq 0.73701\) (for \(SNIa + BAO\) data) and \(\Omega_{Meff}^{(0)} \simeq 0.25648 \& \Omega_x^{(0)} \simeq 0.74352\) (for \(H(z) + SNIa + BAO\) data).

The underlying picture should be contrasted with large scale modification of gravity due to extra degree(s) of freedom where both the frames are related to each other by conformal transformation. In that case, proper screening of the extra degrees does not leave any scope for late time acceleration—acceleration in Jordan frame and no acceleration in Einstein frame. Indeed, if one screens out the local effect of the extra degrees of freedom, conformal transformation from Jordan to Einstein frame fails to remove acceleration completely, thereby, acceleration is not due to modification of gravity. This result applies to any scheme of large scale modification caused by massive extra degrees of freedom coupled to matter such as \(f(R)\) theories [34, 35]. It is remarkable that in the framework, under consideration, acceleration is completely removed by disformal (conformal) transformation, though one adheres to disformal transformations only in order to avoid fine tuning related to instability. Hence, the scenario based upon disformal coupling between dark matter and baryonic matter, represents the true large scale modification of gravity as the underlying cause of late time cosmic acceleration.

Let us also mention that the scenario admits phantom as well as non-phantom behavior provided that we make suitable choice of parameters \(\alpha \& \beta\). Indeed, as shown, in Fig. 5, there is narrow region below the phantom divide line in the 2\(\sigma\) contour allowed by the combined data. In this framework, if simple parametrization used, Universe inevitable evolves to phantom even if the present behavior is set to be non-phantom. In case of the parametrization (29), the Hubble parameter, effective matter density and effective pressure diverge at a finite value of the red-shift, \(\tilde{z}_s \simeq -0.39932\) (for \(SNIa + BAO\) data) and \(\tilde{z}_s \simeq -0.40\) (for \(H(z) + SNIa + BAO\) data) a la a type III sudden future singularity. This is, however, not the generic feature of the model but rather the non-judicial use of (29) beyond present epoch in future. Indeed, we have shown that replacing (29) by an alternative parametrization (46), the sudden future singularity can be delayed to infinite future leaving intact the dynamics from past to the present epoch.

In our opinion, the scenario is of great interest and deserves further investigations both at the background as well as the level of perturbations.
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8 Appendix-1: Ricci scalar in Jordan frame

In the present scenario, dark matter and baryonic matter are disformally coupled in Einstein frame such that the individual components do not conserve on their own but standard conservation applies to both of them take together. As a result, Universe is matter dominated after radiation matter equality, thereby decelerating, in Einstein frame. Let us emphasize again that coupling is defined in Einstein frame only. We can transform to Jordan frame where the matter components adhere to standard conservation separately but dynamics becomes complicated. In particular, the Einstein-Hilbert action transforms. In the flat FRW background, the Jordan frame metric is $\tilde{g}_{\mu\nu}$ with the coupling functions $Q = Q(a)$ and $R = R(a)$ is

$$\tilde{g}_{\mu\nu} = \text{diag}(-Q^2, R^2 a^2, R^2 a^2, R^2 a^2).$$

In this background, the Ricci scalar in Jordan frame ($\tilde{R}_J$), is given by

$$\tilde{R}_J = Q^{-2}R_E + 6Q^{-2} \left[ \frac{\dot{a} R'}{R} + 4 \frac{\dot{a}^2 R'}{a R} + \dot{a}^2 \frac{R'^2}{R^2} + \dot{a}^2 \frac{R''}{R} - \dot{a}^2 \frac{Q'}{Q} \frac{R'}{R} - \frac{\dot{a}^2 Q'}{a Q} \right],$$  \hspace{1cm} (52)

where dot and dash designate derivatives with respect to time cosmic time “$t$” and scale factor “$a$” respectively; the Einstein frame Ricci scalar is given by, $R_E = \frac{6}{a^2} (a\ddot{a} + \dot{a}^2)$. For maximally disformal case, $Q = 1$, $R = R(a)$, we have,

$$\tilde{R}_J = R_E + 6 \left[ \frac{\dot{a} R'}{R} + 4 \frac{\dot{a}^2 R'}{a R} + \dot{a}^2 \frac{R'^2}{R^2} + \dot{a}^2 \frac{R''}{R} \right].$$  \hspace{1cm} (53)

which reduces to Ricci scalar in Einstein frame at early times as $R$ turns constant there. For the parametrization $a(\tilde{a}) = \tilde{a} + \alpha \tilde{a}^2 + \beta \tilde{a}^3$, $\tilde{R}_J$ can be recast in terms of redshift ($\tilde{z} = \frac{1}{\tilde{a}} - 1$) as

$$\tilde{R}_J = \left[ \begin{array} \{3H_0^2(1 + \alpha + \beta)(1 + 2\alpha + 3\beta)^3(1 + \tilde{z})^9 \\
(1 + \tilde{z})^3[(1 + \tilde{z})^3 - \alpha(1 + \tilde{z})^2 + 2\alpha^2(1 + \tilde{z})(-3 + 4\tilde{z}) + 4\alpha^3(-1 + 3\tilde{z})] \\
+ \beta(1 + \tilde{z})^2[1 + (1 + \tilde{z})^2(-3 + 4\tilde{z}) + 6a(1 + \tilde{z})(-1 + 8\tilde{z}) + 2\alpha^2(-3 + 8\tilde{z})] \\
+ \beta^2(1 + \tilde{z})[7(1 + \alpha) + \tilde{z}(63 + 144\alpha + 56\tilde{z})] \\
+ \beta^3(3 + 28\tilde{z}) \end{array} \right] \times \left[ \begin{array} \{(1 + \tilde{z})(1 + \tilde{z} + \alpha) + \beta \}^3 \{1 + (1 + \tilde{z} + \alpha) + \beta \}^3 \right]^{1/3}.$$
The denominator of $\tilde{R}_J$ has the same structure present in the equation of state parameter, effective pressure and effective energy density. Clearly $\tilde{R}_J$ diverges as $\tilde{z} \to \tilde{z}_s$. Indeed, there are four roots,

$$\tilde{z} = \frac{1}{2} \left\{ -2 - \alpha - \sqrt{\alpha^2 - 4\beta} \right\}, \frac{1}{2} \left\{ -2 - \alpha + \sqrt{\alpha^2 - 4\beta} \right\}, \left\{ -1 - \alpha - \sqrt{\alpha^2 - 3\beta} \right\}, \left\{ -1 - \alpha + \sqrt{\alpha^2 - 3\beta} \right\}. $$

For $\alpha = -0.0655, \beta = -0.0973$, we have $\tilde{z} \simeq -0.3902, -0.6536, -1.2806, -1.4781$. At $\tilde{z} = \tilde{z}_s \simeq -0.3902$ around which $\tilde{R}_J$ diverges.

Let us also quote the expressions for the effective pressure, matter density and equation of state parameter in Jordan frame required for the classification of singularity

$$\tilde{p}_{\text{eff}}(\tilde{z}) = \frac{3H_0^2}{8\pi G} \frac{(1 + \alpha + \beta)(1 + 2\alpha + 3\beta)^2 (1 + \tilde{z})^9}{[(1 + \tilde{z})^2 + \alpha (1 + \tilde{z}) + \beta] [(1 + \tilde{z})^2 + 2\alpha (1 + \tilde{z}) + 3\beta]^2}, \quad (54)$$

$$\tilde{\rho}_{\text{eff}}(\tilde{z}) = \frac{\tilde{H}_0^2}{8\pi G} \left[ \frac{(1 + \alpha + \beta)(1 + 2\alpha + 3\beta)^2 (1 + \tilde{z})^9}{[(1 + \tilde{z})^2 + \alpha (1 + \tilde{z}) + \beta] [(1 + \tilde{z})^2 + 2\alpha (1 + \tilde{z}) + 3\beta]^2} \right]$$

$$\times \left[ -1 + \frac{(1 + \tilde{z})^2 + 2\alpha (1 + \tilde{z}) + 3\beta}{(1 + \tilde{z})^2 + \alpha (1 + \tilde{z}) + \beta} + \frac{4((1 + \tilde{z}) + 3\beta)}{(1 + \tilde{z})^2 + 2\alpha (1 + \tilde{z}) + 3\beta} \right], \quad (55)$$

$$\tilde{w}_{\text{eff}}(\tilde{z}) = \frac{1}{3} \left[ -1 + \frac{(1 + \tilde{z})^2 + 2\alpha (1 + \tilde{z}) + 3\beta}{(1 + \tilde{z})^2 + \alpha (1 + \tilde{z}) + \beta} + \frac{4((1 + \tilde{z}) + 3\beta)}{(1 + \tilde{z})^2 + 2\alpha (1 + \tilde{z}) + 3\beta} \right], \quad (56)$$

### 9 Appendix-2: Tools for Data Analysis

For analysis with H(z) data, We have used the compiled dataset used by Farooq and Ratra [37] of 28 H(z) data points in the redshift range $0.07 \leq z \leq 2.3$. We use $H_0 = 67.3 \pm 1.2 \text{ Km}/\text{S}/\text{Mpc}$ to complete the dataset [38]. We apply the data to the model with normalized Hubble parameter, $h = H/H_0$. In this case, the $\chi^2$ is defined as

$$\chi^2_{\text{Hub}}(\theta) = \sum_{i=1}^{29} \frac{[h_{\text{th}}(z_i, \theta) - h_{\text{obs}}(z_i)]^2}{\sigma_h(z_i)^2},$$

where $h_{\text{obs}}$ and $h_{\text{th}}$ are the observed and theoretical values of the normalized Hubble parameter, respectively, and $\sigma_h = \left( \frac{\sigma_H}{H} + \frac{\sigma_{H_0}}{H_0} \right) h$, where $\sigma_H$ and $\sigma_{H_0}$ are the errors associated with $H$ and $H_0$, respectively.

As we know that Type Ia supernova is considered as an ideal astronomical object and observed as very good standard candles. It is one of the direct probe for the cosmological expansion. For our analysis, we take 580 data points from Union2.1 compilation data [39]. The appropriate quantity luminosity distance $D_L(z)$ can be defined as

$$D_L(z) = (1 + z) \int_0^z \frac{H_0 dz'}{H(z')}, $$

(58)
The distance modulus $\mu(z)$ is the observed quantity and is related to $D_L(z)$ as $\mu(z) = m - M = 5 \log D_L(z) + \mu_0$, where $m$ and $M$ are the apparent and absolute magnitudes of the Supernovae and $\mu_0 = 5 \log \left( \frac{H_0}{\text{Mpc}} \right) + 25$ is the nuisance parameter that should be marginalized. The corresponding $\chi^2$ is given by

$$
\chi^2_{\text{SN}}(\mu_0, \theta) = \sum_{i=1}^{580} \frac{[\mu_{\text{th}}(z_i, \mu_0, \theta) - \mu_{\text{obs}}(z_i)]^2}{\sigma_\mu(z_i)^2},
$$

(59)

where $\mu_{\text{th}}$, $\mu_{\text{obs}}$ and $\sigma_\mu$ represent the theoretical, observed distance modulus and uncertainty in the distance modulus, respectively. The $\theta$ is an arbitrary parameter of the corresponding model. After marginalization of $\mu_0$, one obtains

$$
\chi^2_{\text{SN}}(\theta) = A(\theta) - \frac{B(\theta)^2}{C(\theta)},
$$

(60)

where,

$$
A(\theta) = \sum_{i=1}^{580} \frac{[\mu_{\text{th}}(z_i, \mu_0 = 0, \theta) - \mu_{\text{obs}}(z_i)]^2}{\sigma_\mu(z_i)^2},
$$

(61)

$$
B(\theta) = \sum_{i=1}^{580} \frac{\mu_{\text{th}}(z_i, \mu_0 = 0, \theta) - \mu_{\text{obs}}(z_i)}{\sigma_\mu(z_i)^2},
$$

(62)

$$
C(\theta) = \sum_{i=1}^{580} \frac{1}{\sigma_\mu(z_i)^2}.
$$

(63)

Finally, we worked out with BAO data of $d_A(z_\star)/D_V(Z_{BAO})$ [41, 42, 43, 44, 45, 46], where $z_\star \approx 1091$ is the decoupling time, $d_A(z)$ is the co-moving angular-diameter distance and $D_V(z) = (d_A(z)^2 z/H(z))^{1/3}$ is the dilation scale. The required data is presented in Table ???. The corresponding $\chi^2_{\text{BAO}}$ is given by [46]:

$$
\chi^2_{\text{BAO}} = Y^T C^{-1} Y,
$$

(64)

where,

$$
Y = \begin{pmatrix}
\frac{d_A(z_{0.106})}{D_V(0.106)} & -30.95 \\
\frac{d_A(z_{0.2})}{D_V(0.2)} & -17.55 \\
\frac{d_A(z_{0.35})}{D_V(0.35)} & -10.11 \\
\frac{d_A(z_{0.44})}{D_V(0.44)} & -8.44 \\
\frac{d_A(z_{0.6})}{D_V(0.6)} & -6.69 \\
\frac{d_A(z_{0.73})}{D_V(0.73)} & -5.45 \\
\end{pmatrix},
$$

(65)

and $C^{-1}$ is the inverse covariance matrix given by [46].
### Values of $\frac{d_A(z)}{D_V(Z_{BAO})}$ for distinct values of $z_{BAO}$.

| $z_{BAO}$ | $\frac{d_A(z)}{D_V(Z_{BAO})}$ |
|----------|--------------------------------|
| 0.106    | 30.95 ± 1.46                 |
| 0.2      | 17.55 ± 0.60                 |
| 0.35     | 10.11 ± 0.37                 |
| 0.44     | 8.44 ± 0.67                  |
| 0.6      | 6.69 ± 0.33                  |
| 0.73     | 5.45 ± 0.31                  |

\( C^{-1} = \begin{pmatrix} 
0.48435 & -0.101383 & -0.164945 & -0.0305703 & -0.097874 & -0.106738 \\
-0.101383 & 3.2882 & -2.45497 & -0.078798 & -0.252254 & -0.2751 \\
-0.164945 & -2.45499 & 9.55916 & -0.128187 & -0.410404 & -0.447574 \\
-0.0305703 & -0.078798 & -0.128187 & 2.78728 & -2.75632 & 1.16437 \\
-0.097874 & -0.252254 & -0.410404 & -2.75632 & 14.9245 & -7.32441 \\
-0.106738 & -0.2751 & -0.447574 & 1.16437 & -7.32441 & 14.5022 
\end{pmatrix} \)

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