An $H_{-}/H_{\infty}$ optimization approach to fault detection for UAV

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Abstract: Unmanned aerial vehicles (UAVs) have good maneuverability and have been widely used in many fields. Therefore, it is of great significance to detect faults in the UAV flight control system. In this paper, a method based on $H_{-}/H_{\infty}$ optimization is proposed for the constant gain fault detection of UAV flight control system actuators, and the failure of UAV elevator and throttle lever parts are used as the example for simulation verification. The results show that the proposed method can realize the quick fault detection of UAV flight control system.

1. Introduction

The flight control system, as the core component of the UAV, plays a crucial role in the flight process of the UAV, and abnormality of the flight control system may lead to deviations in the controller, which may cause unexpected attitude movement of the UAV and accidents. Therefore, it is of great theoretical research significance and engineering application value to conduct fault detection of the UAV flight control system, see for example[1-4]. At present, domestic and foreign scholars have done a lot of research on the fault detection of UAV flight control system; In [5], the actuator fault for quadrotor UAV is modeled, and then carried out fault detection on the system according to the segmented system performance tolerance index; In [6], a mathematical model for the actuator fault of multi-rotor UAV is established, and then conducting fault detection by designing an adaptive observer-based fault reconstruction method; and [7] proposed a multi-model online fault diagnosis method for partial failure faults of UAV actuators; [8] proposed an EKF-based fault diagnosis method for UAV actuator faults.

Although numerous research results have been achieved for the fault detection of UAV flight control systems, many methods have more or less limitations, see for example [1], which attributed the modeling errors and atmospheric disturbances of UAVs to Gaussian white noise, however, when UAVs encounter more severe flight conditions such as discretegusts and atmospheric turbulence during flight, simply equating external disturbances and modeling errors to white noise will affect the performance of fault detection to a certain extent. In addition, robust filters based on $H_{-}$ and $H_{\infty}$ performance metrics are gaining more and more attention from scholars at home and abroad, and have been widely used in various fields. [9] discusses the linear discrete time-varying systems of observer-based fault detection problem and proposes a method based on $H_{-}/H_{\infty}$ and $H_{-}/H_{\infty}$ optimization for the fault detection filter design problem. In [10], an $H_{\infty}$ filter based on the T-S model is proposed for the estimation of the longitudinal attitude of hydrofoil catamarans; in [11], a $H_{\infty}$-optimized fault detection method is proposed for the fault detection problem of a class of linear continuous time-varying systems.

In summary, this study will investigate the fault detection problem of UAV longitudinal closed-loop flight control system based on [1]. For the model of UAV longitudinal closed-loop flight control system
established in [1], a fault detection filtering method based on $H_\infty$ optimization is proposed for fault detection of constant gain faults in the actuator.

2. Fault model of UAV flight control system and problem formulation
Consider the following state space model of longitudinal closed-loop flight control systems for UAVs [1]:

\[
\begin{align*}
\dot{x}(t) &= F(x(t)) + B(x(t))\beta u(t) + G(x(t))d(t) + w(t) \\
y(t) &= C(t)x(t) + v(t)
\end{align*}
\]  

(1)

Where the control inputs $u(t) = [\delta, \delta_p]^T$, the measurements $y(t) = [V, \alpha, \omega, \theta, H]^T$, the state vector $x(t) = [V, \alpha, \omega, \theta, H]^T$, the disturbance vector $d(t) = [\dot{\alpha}, \dot{\omega}, \dot{\theta}, \dot{H}]^T$. $w(t)$ is the system noise and $v(t)$ is the measurement noise. Here $w(t)$ and $v(t)$ are also energy bounded. In the above state space model,

\[
F(x(t)) = \begin{bmatrix}
-\frac{\rho V^2 S \alpha}{2m} (C_{\alpha 0} + C_{\alpha 0}^v \alpha + C_{\alpha 0}^v \alpha^2) - g \sin(\theta - \alpha) \\
\frac{1}{mV} (-\frac{\rho V^2 S \alpha}{2} (C_{\alpha 0} + \frac{C_{\alpha 0} \alpha}{2V} q) + mg \cos(\theta - \alpha)) + q \\
\frac{\rho V^2 S \alpha^2}{2I_\gamma} (m_{\alpha 0} + m_{\alpha 0}^v \alpha^2) \\
V \sin(\theta - \alpha)
\end{bmatrix}
\]

\[
B(X(t)) = \begin{bmatrix} 0 & B_1 \\
B_2 & 0 \\
B_3 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \end{bmatrix}
\]

\[
G(X(t)) = \begin{bmatrix} G_1 & G_2 & 0 \\
G_3 & G_4 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \end{bmatrix}
\]

\[
B_g = \frac{K}{m}, \quad B_2 = -\frac{\rho V S \alpha C_{\gamma}}{2m}, \quad B_3 = \frac{\rho V^2 S \alpha^2 m_{\gamma}}{2I_\gamma}, \quad G_1 = -\cos(\theta - \alpha), \quad G_2 = -\sin(\theta - \alpha), \\
G_3 = -\frac{\sin(\theta - \alpha)}{V}, \quad G_4 = \frac{\cos(\theta - \alpha)}{V}, \quad G_5 = I, \quad C(t) = I
\]

Here $\delta$ is the elevator deflection and $\delta_p$ is the throttle setting. $\rho$, $S$, $\overline{c}$ represent the air density, reference area and mean aerodynamic chord, respectively. $K$ is engine thrust coefficient and $C_{\alpha 0}$, $C_{\gamma 0}$, $m_{\alpha 0}$, $C_{\gamma 0}^v$, $m_{\alpha 0}^v$ are all aerodynamic coefficients obtained from a wind tunnel test. $\beta$ is gain coefficient.

For system (1), assume that the sampling time is $T$. Eulerian discretization is performed so as to obtain a nonlinear discretized failure model for the UAV longitudinal flight control system as follows:

\[
\begin{align*}
x(k+1) &= \Phi(x(k)) + B(x(k))u(k) + B_1(x(k))\beta I u(k) + G(x(k))\tilde{d}(k) \\
y(k) &= C(k)x(k) + D_1(k)\tilde{d}(k)
\end{align*}
\]

(2)

where:

\[
\Phi(x(k), u(k)) = x(k) + TF(x(k)), \quad B(x(k)) = TB(x(k)), \quad G(x(k)) = T[G(x(k)) \ I]
\]

\[
\tilde{d}(k) = [dt \ k \ w_{\gamma}(k) \ v_{\gamma}(k)]^T, \quad D_1(k) = \begin{bmatrix} 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0 \end{bmatrix}, \quad B_1(x(k)) = T(B(x(k)))
\]

The observer-based method of fault detection includes two parts: residual generation and residual evaluation, and residual generation is the first step of fault detection. The following residual generators can be designed.

\[
\begin{align*}
\hat{x}(k+1) &= \Phi(\hat{x}(k)) + B(\hat{x}(k))u(k) + L(k)\hat{y}(k) \\
\hat{y}(k) &= y(k) - C(k)\hat{x}(k) \\
r(k) &= W(k)\hat{y}(k), \quad \hat{x}(0) = 0
\end{align*}
\]

(3)

where $r(k)$ is the generated residual, $L(k)$ and $W(k)$ denote respectively the observer gain matrix and weighting matrix. Definition $e(k) = x(k) - \hat{x}(k)$. It follows from (3) and (4) that
\[
e(k+1) = \Phi(x(k)) - \Phi(\hat{x}(k)) + \tilde{B}(x(k))u(k) - \tilde{B}(\hat{x}(k))u(k) \\
+ \tilde{B}_f(x(k))(\beta - I)u(k) + \tilde{G}(x(k))\tilde{d}(k) - L(k)(y(k) - C(k)\hat{x}(k)) \\
r(k) = W(k)(C(k)e(k) + D_x(k)\tilde{d}(k)) \\
e(0) = x_0
\]

(4)

Assuming that the system noise and the measurement noise are attributed to the unknown input with bounded parametrization: \( w(k), v(k) \subset l_2[0, N] \). When designing the robust filter, the observer gain matrix \( L(k) \) and the posterior filter parameter matrix \( W(k) \) need to satisfy the following equation:

\[
\max_{L(k), W(k)} \left\| G_{\theta} \right\|_{[0, N]} = \frac{\sum_{k=0}^{N} r^T(k)r(k)}{\sum_{k=0}^{N} (x_0^T r_0 + d^T(k)d(k))}
\]

The second step of fault detection is to perform residual evaluation. Select the following residual evaluation function is:

\[
J_N(k) = \sum_{i=0}^{k} r^T(i)r(i)
\]

(5)

Where the positive integer \( N \) represents the length of the time window.

After determining the residual evaluation method, the fault detection threshold should be selected. This paper is based on the traditional fault detection threshold design method. Let the residual evaluation function in the fault-free case be \( J_{N0}(k) \), and take its upper exact boundary as the fault detection threshold. In this paper, we obtain the set of sampling points by sampling the residual evaluation function \( J_{N0}(k) \) in a large number of random samples in the fault-free case of the system, and select the maximum value among all the sampling points as the threshold \( J_{th} \). According to the relationship between the residual evaluation function and the fault detection threshold, we can determine whether a fault has occurred:

\[
\begin{align*}
J_N(k) & \leq J_{th}, \quad \text{no fault alarm} \\
J_N(k) & > J_{th}, \quad \text{fault alarm}
\end{align*}
\]

3. Design of fault detection system for UAV

The design of \( H_\infty / H_\infty \) -based fault detection filter for UAV control system is carried out. First, the following citation is given. For the following linear discrete time varying system[9]:

\[
\begin{align*}
x(k+1) &= A(k)x(k) + B(k)u(k) + B_d(k)d(k) + B_f(k)f(k) \\
y(k) &= C(k)x(k) + D_d(k)d(k) + D_f(k)f(k)
\end{align*}
\]

(6)

where the state vector \( x(k) \in \mathbb{R}^n \), the measurements \( y(k) \in \mathbb{R}^q \), the control inputs \( u(k) \in \mathbb{R}^p \), the disturbance vector \( d(k) \in \mathbb{R}^m \), the fault vector \( f(k) \in \mathbb{R}^l \) and \( d(k) \in l_2[0, N] \), \( f(k) \in l_2[0, N] \), \( A(k) \), \( B(k) \), \( B_d(k) \), \( B_f(k) \), \( C(k) \), \( D_d(k) \), \( D_f(k) \) is a known matrix of appropriate dimensions. The following filter can be designed:

\[
\begin{align*}
\dot{x}(k+1) &= A(k)\dot{x}(k) + B(k)u(k) + L_0(k)(y(k) - C(k)\dot{x}(k)) \\
\dot{y}(k) &= y(k) - C(k)\dot{x}(k) \\
r(k) &= W_0(k)\hat{y}(k), \quad \hat{x}(0) = \hat{x}_0
\end{align*}
\]

(7)
Where \( r(k) \) is the generated residual, \( L_0(k) \) and \( W_0(k) \) denote respectively the observer gain matrix and weighting matrix. Definition \( e(k) = x(k) - \hat{x}(k) \). It follows from (5) and (6) that
\[
\begin{cases}
    e(k+1) = (A(k) - L_0(k)C(k))e(k) + (B_d(k) - L_0(k)D_d(k))d(k) + (B_f(k) - L_0(k)D_f(k))f(k) \\
    r(k) = W_0(k)(C(k)e(k) + D_d(k)d(k) + D_f(k)f(k))
\end{cases}
\]
(8)

The \( L_0(k) \) and \( W_0(k) \) given by:
\[
\begin{align*}
    L_0(k) &= (A(k)P_0(k)C^T(k) + B_d(k)D_d^T(k))W_0^2(k) \\
    W_0(k) &= R_d^{-1/2}(k)
\end{align*}
\]
(9)

where \( P_0(k) \) is computed recursively by
\[
\begin{align*}
    R_d(k) &= C(k)P_0(k)C^T(k) + D_d(k)D_d^T(k) \\
    P_0(k+1) &= A(k)P_0(k)A^T(k) + B_d(k)B_d^T(k) - L_0(k)W_0^2(k)L_0^T(k) \\
    P_0(0) &= I
\end{align*}
\]

Linearizing the UAV nonlinear system of equation (3), here the nonlinear function of the system can be Taylor expanded at the filter point as follows.

\[
\begin{align*}
    \Phi(x(k)) + \bar{B}(x(k))u(k) &= \Phi(\hat{x}(k)) + \bar{B}(\hat{x}(k))u(k) + \left[ \frac{\partial \Phi(x(k))}{\partial x(k)} + \frac{\partial B(x(k))}{\partial x(k)} - u(k) \right] \times (x(k) - \hat{x}(k)) + \cdots \\
    \bar{G}(x(k)) &= \bar{G}(\hat{x}(k)) + \cdots, \quad \bar{B}_f(x(k)) = \bar{B}_f(\hat{x}(k)) + \cdots
\end{align*}
\]

let: \( \alpha(k) = \left[ \frac{\partial \Phi(x(k))}{\partial x(k)} + \frac{\partial B(x(k))}{\partial x(k)} - u(k) \right] \), \( E_d(k) = \bar{G}(\hat{x}(k)) \), \( E_f(k) = \bar{B}_f(\hat{x}(k)) \). Brining this equation into equation (5) yields:
\[
\begin{cases}
    e(k+1) = (\alpha(k) - L(k)C(k))e(k) + E_f(k)(\beta - I)u(k) + (E_d(k) - L(k)D_d(k))\bar{d}(k) \\
    r(k) = W(k)(C(k)e(k) + D_d(k)\bar{d}(k))
\end{cases}
\]
(10)

the \( L(k) \) and \( W(k) \) given by:
\[
\begin{align*}
    L(k) &= (\alpha(k)P(k)C^T(k) + E_d(k)E_d^T(k))R_d^{-1/2}(k) \\
    W(k) &= R_d^{-1/2}(k)
\end{align*}
\]
(11)

\[
\begin{align*}
    P(k+1) &= \alpha(k)P(k)\alpha^T(k) + E_d(k)E_d^T(k) - L(k)R_d(k)L^T(k) \\
    R_d^{-1/2}(k) &= C(k)P(k)C^T(k) + D_d(k)D_d^T(k)
\end{align*}
\]
(12)

4. Simulations

Taking a certain type of UAV as an example, considering its longitudinal closed-loop flight control system, we build a UAV flight control system fault detection platform based on Simulink simulation platform, and simulate the failure fault of elevator part and throttle stick part. The aerodynamic coefficients and aerodynamic moment coefficients in the simulation are obtained by interpolation on the basis of the experimental data of the UAV wind tunnel [1].

Set the simulation time as \( 100s \), and the discrete period as \( T = 0.01s \); the length of the moving time window is \( x_0 = [24 \ 0 \ 0 \ 0 \ 200]^T \); \( w(k) \) and \( v(k) \) are Gaussian white noise with a mean of zero and a variance of 0.01. \( d(t) = [\alpha_{z} \ \omega_{z} \ \omega_{h}]^T \), ignore the \( \omega_{h} \), and assuming that the turbulence disturbance at 200 meters is isotropic, the Dryden model in Reference [12] is used to generate wind disturbance:

\[
\begin{align*}
    \omega_{z} &= F_{z}(s)n_{z}, \quad F_{z}(s) = \sqrt{3V_{0}\sigma_{z}^{2}/L_{z}}/((3L_{z})^{2} + s), \quad \omega_{h} = F_{h}(s)n_h, \quad F_{h}(s) = \sqrt{2V_{0}\sigma_{h}^{2}/L_{h}}/((V_{0}/L_{h})^{2} + s)
\end{align*}
\]
Where $L_x = L_h = 580 m$, $n_x$ and $n_h$ is zero mean Gaussian white noise with 0.01 variance, turbulence intensity $\sigma_x = \sigma_h = 7 m/s$. Wind disturbance simulation is shown in Figure 1.

Let $(\beta I)u(k) = [\beta_1 u_1(k) \beta_2 u_2(k)]^T$. As the system operation up to 50s and 140s, the faults are injected, where $\beta_1 = 0.2$ and $\beta_2 = 0.3$, respectively. The filter residual and residual evaluation function can be obtained as shown in Figure 2 and Figure 3.

At the system operation up to 50s, the fault is injected, so that $\beta_2 = 0.25$, the filter residual and residual evaluation function can be obtained as shown in Figure 4 and Figure 5.

As shown in Figure 2 and Figure 4, the airspeed residuals and altitude residuals showed a sudden change when the fault was injected and then disappeared; while the residual evaluation functions in
Figure 4. Residuals in the case of throttle rod fault

Figure 5. Evaluation function in the case of elevator throttle rod fault

Figure 3 and Figure 5 rapidly exceeded the threshold value after the fault occurred and remained above the threshold value until the fault disappeared, thus generating an alarm signal and thus achieving the detection of the partial failure of the UAV elevator and throttle lever.

5. Conclusions

In this paper, a robust fault detection filter method based on $H_\infty$ performance index is used for fault detection of UAV closed-loop nonlinear flight control system. The experimental results show that the robust fault detection filter method based on $H_\infty$ performance index can quickly and effectively implement the UAV flight control system fault detection, which can provide a more reliable guarantee for the safe flight of UAV.

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