Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
A recovery mathematical model for the impact of supply chain interruptions during the lockdown in COVID-19 using two warehouse perishable inventory policies

Dolagobinda Das \textsuperscript{a},* Gauranga Charan Samanta \textsuperscript{a}, Abhijit Barman \textsuperscript{b}, Pijus Kanti De \textsuperscript{b}, Kshitish Kumar Mohanta \textsuperscript{c}

\textsuperscript{a} P. G. Department of Mathematics, Fakir Mohan University, Vyasa Vihar, Balasore, 756019, Odisha, India
\textsuperscript{b} Department of Mathematics, National Institute of Technology, Silchar, 788010, Assam, India
\textsuperscript{c} Department of Mathematics, Indira Gandhi National Tribal University, Lalpur, Amarkantak, 484887, Madhya Pradesh, India

\textbf{A R T I C L E I N F O}

\textbf{MSC:}
90B05
90B06

\textbf{Keywords:}
Time dependent demand
COVID-19 pandemic
Two warehouse system
Lockdown
Perishable items
Deterioration

\textbf{A B S T R A C T}

This study develops an inventory model for things that deteriorate at a rate that depends on time. The research work is about minimizing the total cost of a two-warehouse system during the lockdown period. We consider two scenarios and the LIFO policy has been used in this model. In the first scenario, the stocks of the rented warehouse (RW) become empty after the lockdown eased, and in the second scenario, the stocks of the RW become empty during the lockdown. Here, two parametric Weibull distributions for the deterioration rate and a time-dependent demand are taken into consideration. Subsequently, sensitivity analysis is examined for both scenarios by using two different examples to make the research more realistic. In an emergency like the COVID-19 epidemic, the models may be used effectively when taking into consideration the actual circumstances.

1. Introduction

The novel corona virus SARS-n, CoV-2 spread the COVID-19 disease quickly across six continents, prompting many nations around the world to declare a state of a health emergency. COVID-19 is one of the most significant peacetime issues that industry and government have faced in the previous century [1]. The COVID-19 epidemic has slowed many industries. As a novel sort of disruption, the COVID-19 pandemic is the first long-term supply chain crisis in decades, causing great uncertainty in demand, supply, and supply chain planning. COVID-19 impacts the supply chain from both supply and demand perspectives. Getting resources from suppliers and ensuring the flow of goods from producers to customers is the most challenging problem in the supply chain during a pandemic. COVID-19 was described as a long-term supply chain crisis marked by ambiguity regarding the current position and future developments. The supply chain is confronted with numerous issues, including supply disruption and demand shifts [2,3].

To avoid the spread of the corona virus among the general population, all governments have decided that complete lockdowns are the best method to ensure that people have limited interaction with one another. No such inventory model has ever experienced a lockdown due to the COVID-19 pandemic. As a result, the supply chain company has been significantly impacted during COVID-19. A well-designed supply chain system provides increased efficiency, quality control, enhanced customer interactions and service, a shorter production cycle, cheaper production costs, and an overall improvement in a company’s financial performance. Due to global supply-chain disruptions, manufacturers and distributors have found it hard to repair or replenish their inventory, equipment,
or machinery. Due to restrictions and slowdowns in the industrial activities of trade partners, seaports (i.e., the main route for international trade) have been unable to transport the mass shipments of items across international borders. Players in the logistics, transportation, and supply chain sectors must put forward cutting-edge inventory management and distribution strategies and establish strategic alliances with participants and channel partners throughout the value chain to reduce the impact of COVID-19 on business operations. The global supply chain depends on the transportation sector, which is partially shut down due to the lockdown.

We know that deterioration is significant in an inventory model because, as the deterioration rate rises, a retailer's profit may falls; conversely, when the deterioration rate falls, a retailer's profit may rises; hence, profit changes inversely with the deterioration rate. Therefore, the rate of deterioration cannot be excluded in an inventory model [4]. Items subject to deterioration, damage, spoiling, evaporation, obsolescence, pilferage, loss of utility, or loss of marginal value are perishable or deteriorated items. Freshness is an essential quality factor for fresh produce. Perishable goods make up a significant portion of the stock carried in practice [5]. The Weibull distribution is commonly used to depict the time distribution until the item deteriorates. Because of their practical uses in agriculture, chemicals, medical products, photojournalism, and blood bank management, many articles have focused on perishable inventory control. Demand is either assumed to be constant or time-dependent in conventional deterministic inventory models. The rate of degradation may have an impact on demand for perishable goods like food and clothes. Researchers and practitioners in marketing have noted that improving product shelf space generally positively affects product sales.

It has been observed that business owners are compelled to acquire more than their storage capacity to take advantage of bulk purchasing discounts, avoid ordering costs, and so on. In these cases, businesses must rent a warehouse to store the extra quantity purchased. Due to the increased cost of protecting and maintaining material, per unit holding and deterioration costs in the rented warehouse (RW) are often higher than in owned warehouses, so things should be held first in the owned warehouse (OW) and only excess stock should be kept in RW. As a result, to lower total inventory costs, it is essential to finish the stock in the RW first, followed by the stock in the owned warehouse. The entire system cost is also reduced due to the greater capacity of the owned warehouse. As a result, studying a two-warehouse system is essential while designing inventory models. Already It is investigated that the concept of two warehouses, assuming that RW produces higher inventory holding costs than OW. As a result, goods in RW are moved to OW until RW stock reaches zero, at which point goods in OW are used. Because of the lack of suitable facilities in OW, many authors claim that the store was forced to hire RW to minimize waste caused by deterioration with enhanced storage facilities. The Weibull distribution is used in this model to represent deterioration (two parameters).

Following are the remaining sections of the manuscript: The literature review of this study is presented in Section 2. Section 3 contains a detailed comparison with past literature and our proposed study. The development of the mathematical model and analysis, notations, and hypotheses used in model formulation, the mathematical modelling, evaluation, and cost calculation for scenarios 1 and 2 are discussed in Section 4. Section 5 presents the solution technique and graphical insights. In Section 6, numerical examples are used to demonstrate the concepts. The sensitivity analysis is performed in Section 7 to validate the models. Finally, the current model's conclusion is drawn, and future scopes are recommended in Section 8.

2. Literature review

There are many factors which are consider to create a appropriate EOQ model, and two of them are deterioration and demand rate. Few researches considered rate of deterioration as a constant rather than variable. Jaggi et al. [6] consider a constant deterioration rate where demand is vary exponentially with time for a single warehouse inventory system. Subsequently, Dey et al. [7] consider constant deterioration rate but taking linear time dependent demand for two warehouse inventory system. However, considering a constant deterioration is not enough to describe the real issues of an inventory system. So to make more realistic, many researchers consider linear as well as non linear time dependent deterioration rate. Wu and Ouyang [8] and Skouri et al. [9] used a two-parameter Weibull distribution and a ramp type demand rate to create an appropriate EOQ model for a perishable inventory system. Some researcher add some extra factor like shortage and backlog to create a better EOQ model. Then [10] builds an inventory model by using a general type demand rate and deterioration where shortage is partially backlogged. Now demand is an important factor which affect the EOQ inventory model. Apart from time dependent demand some researchers studied that demand may dependent on price, stocks, price and time, price and stocks, credit, etc. [11] developed an EOQ model for degrading goods where demand is dependent on credit period with a maximum lifetime to maximize total profit. Zhang et al. [12] make use of restoration technology infrastructure to optimize price policies for degrading items, where demand is based on selling price, and deterioration is based on natural conditions. Prasad and Mukherjee [13] developed a model that deals with stock and time-dependent demand and deterioration as two parameters of the Weibull distribution to maximize the profit function. Rastogi et al. [14] created a study on a two-warehouse inventory policy with price-dependent demand and constant deterioration under potential backlogged to minimize the overall costs. Tiwari et al. [15] developed a model for the inventory of deteriorating items that incorporates expiry dates and partial backlogs with price-dependent demand for a two-econom trade-credit strategy to reflect actual situations. Panda et al. [16] developed a model that minimizes the total cost of a two-warehouse system where they take price and stock-dependent demand and a constant deterioration rate. A model of the two-warehouse inventory system for non-instantaneous deteriorating items with an exponential demand rate was studied by Bishi et al. [17] to reduce overall costs. In order to maximize overall profit, Shaikh et al. [18] constructed a model for a two-warehouse inventory system for non-instantaneously decaying items with interval-valued product costs and stock-dependent demand. Using a partial backlog and a demand model based on stocks, Ahmad and Benkherouf [19] suggested an inventory model reduce the total cost for slowly deteriorating goods. In order to maximize overall profit, Garg et al. [20] created a model that addressed a two-warehouse inventory model for perishable items with ramp type demand and partial backlog. An inventory model was investigated by Rout et al. [21] to reduce the overall cost of the system for deteriorating goods.
when demand is dependent on the backlog. Due to offering concessions, some buy more stocks which is greater than the capacity of their warehouse. So they hired an RW to keep the extra stocks. To reduce costs and maximize efficiency, Kumar et al. [22] studied a model for a two-warehouse inventory system with stock-dependent demand for depreciating goods. Rana et al. [23] developed a model for two warehouse systems; demand is based on time-dependent and Weibull distribution is used for deterioration rate where some finite demand disruption used due COVID-19 lockdown. Some researchers are studied that holding cost may depend on time. So [24] studied a EOQ inventory model by taking holding cost as time dependent as well as demand is also time dependent for a non-instantaneous deterioration rate under back-ordering and non-terminating circumstances.

3. Comparison study with our contribution

The essential features of our contribution are compared to existing publications in this study. According to Table 1, numerous researchers created models for deteriorating items without using the demand disruption idea. However, in this model, we concentrate mainly on demand disruption that occurs due to lockdown restrictions. We are thinking of two scenarios: In scenario 1, the RW is completely evacuated after the lockdown period, and in scenario 2, the RW is completely evacuated during the lockdown period. Here, we apply a small-scale fall in demand rate brought on by lockdown restriction, and a small-scale rise brought on by the customer’s panic-buying tendencies after lockdown relaxation. Because the holding cost of RW is greater than that of OW, we apply the LIFO policy in this article to reduce the system’s overall holding cost. The present paper is about minimizing the total holding cost of the two warehouse systems by using the LIFO principle, i.e., first use stocks of RW, after the complete evacuation of RW, use stocks of the owned warehouse because holding costs in RW is more than the cost in OW. This paper also uses the demand based on time and the two-parametric Weibull distribution to explain the deterioration rate.

4. Development of mathematical model and analysis

4.1. Motivation of the work

According to the extensive literature studies, almost all of the study recommended in this field is theoretical. But a theoretical idea is insufficient without supporting evidence from a real-world scenario. As a result, it is thought necessary to provide a mathematical model that aids in the analysis of supply chain disruption caused by COVID-19. In this technique, linear time dependent demand is consider and made more realistic by using a two-parameter Weibull distribution deterioration. A finite increase and decrease in demand have been considered, with this finite value of demand disruption taken from [25] production inventory model for single storage, i.e., decreased demand rate due to the government’s lockdown and an increase in demand due to overstocking made by the people when the government eased the rules (panic buying behaviour of the people has been observed). Their consequences have been visually displayed to understand better how various parameters affect the results. To determine the critical parameters, a sensitivity analysis was performed.

4.2. Notation

For simplicity, the additional notation have been added to the paper.
4.3. Assumptions

These fundamental assumptions were used to construct the mathematical inventory model.

1. The Weibull distribution with two parameter deterioration rate is taken into account.
2. The pace of replenishing is infinite.
3. The storage space of the OW to store $W$ units is limited, whereas the RW’s storage space is limitless.
4. When an OW is totally full, we can begin storing products in a RW.
5. The deteriorated item cannot be repaired or replaced.
6. The inventory systems only deal with one type of item.
7. The terms $S_T$ and $E_T$ refer to the times when the government imposes and then lifts the total lock-down.
8. The initial demand rate ($\gamma$) is supposed to be a proportion of $\Delta T$ and $\Delta T_1$.
9. The LIFO dispatching strategy is applied when the RW runs out of storage, which implies that the first item in the RW is consumed before the last item in the OW.

4.4. Mathematical model

4.4.1. Scenario 1

Fig. 1 depicts the graphs of inventory reduction in Scenario 1. At first, the retailer orders $Q$ units, which are greater than the OW’s storage space ($W$ units). As a result, the shop must rent a warehouse to store the excess inventory, referred to as a RW. The excess units, $X = (Q - W)$ are kept in the $RW$ once the OW is filled to its total capacity of $W$ units. According to the LIFO dispatching policy, RW’s items are predominantly supplied until $RW$ is entirely empty; the OW is vacated to lower overall holding costs. The items in $RW$ decreases throughout the time period $0$ to $T$ as a results of the combined impact of demand and deterioration, and $RW$ is empty at $T$. But the $OW$ sees only deterioration, and the amount decreases from $W$ to $W^0$ at time $T$. Furthermore, items in $OW$ diminish owing to demand and deterioration concurrently during the time $T$ to $T$, eventually reducing to zero at $T$.

During time $0$ to $S_T$, the RW stocks deplete due to demand and deterioration; we use the Weibull distribution to express the deterioration. The inventory of RW ($R_{ij}$) represented by differential equation (1).

\[
\frac{dR_{ij}(t)}{dt} + q^{\gamma-1}R_{ij}(t) = -\gamma - \delta t
\]

Using initial condition $R_{ij}(0) = X = Q - W$, the solution of the above differential equation is

\[
R_{ij}(t) = e^{-q^{\gamma}} \left[ X - t \left( \gamma + \frac{\delta t}{2} \right) - q^{\gamma+1} \left( \frac{\gamma}{p+1} + \frac{\delta t}{p+2} \right) \right]
\]

Inventory in the RW at time $t = S_T$

\[
R_{ij}(S_T) = e^{-q^{\gamma}S_T} \left[ X - S_T \left( \gamma + \frac{\delta S_T}{2} \right) - q^{\gamma+1} \left( \frac{\gamma}{p+1} + \frac{\delta S_T}{p+2} \right) \right]
\]
The government declared a lockdown at time \( S_T \), causing a sharp drop in demand of amount \( \Delta d \). As a result, demand is slowing down until the situation is eased at time \( E_T \). Consequently, the decrease in stock during \( (S_T \leq t \leq E_T) \) is demonstrated by Eq. (4).

\[
\frac{dR_{I2}(t)}{dt} + q \tau^{p-1} R_{I2}(t) = -\gamma - \delta t + \Delta d 
\]  

Using the given condition \( R_{I1}(S_T) = R_{I2}(S_T) \), the solution of the above differential equation is

\[
R_{I2}(t) = e^{-\tau^p} \left[ t(\Delta d - \gamma) \left( 1 + \frac{q \tau^p}{p+1} \right) - \delta t^2 \left( \frac{1}{2} + \frac{q \tau^p}{p+2} \right) + X - S_T \Delta d \left( 1 + \frac{q S_T^p}{p+1} \right) \right] 
\]  

since \( E_T \leq \Delta_T \), RW completely evacuated after lockdown period, so \( R_{I2}(E_T) > 0 \). The demand rate increases by an amount \( \Delta d_1 \) as soon as the lockdown is relaxed at \( t = E_T \) because public start creating overstock out of fear of another lockdown, as a result of panic buying by the public [26]. During \( (E_T \leq t \leq \Delta_T) \), the rest inventory loss in the RW throughout this period is illustrated by Eq. (7).

\[
\frac{dR_{I3}(t)}{dt} + q \tau^{p-1} R_{I3}(t) = -\gamma - \delta t - \Delta d_1 
\]  

The solution of the above differential equation with given condition \( R_{I2}(E_T) = R_{I3}(E_T) \), is

\[
R_{I3}(t) = e^{-\tau^p} \left[ -\gamma t \left( 1 + \frac{q \tau^p}{p+1} \right) - \delta t^2 \left( \frac{1}{2} + \frac{q \tau^p}{p+2} \right) + \Delta d \left( E_T - S_T \right) + \frac{q S_T^p}{p+1} (E_T^{p+1} - S_T^{p+1}) \right] 
\]  

Calculate time at which RW becomes empty(\( \Delta_T \)):

Suppose that the RW fully empty at time \( \Delta_T \), i.e., \( R_{I3}(\Delta_T) = 0 \), where \( \Delta_T \leq T \).

\[
R_{I3}(\Delta_T) = e^{-\tau^p} \left[ -\gamma \Delta_T \left( 1 + \frac{q \Delta_T^p}{p+1} \right) - \delta \Delta_T^2 \left( \frac{1}{2} + \frac{q \Delta_T^p}{p+2} \right) + \Delta d \left( E_T - S_T \right) \right] 
\]
Using the given condition, 

\[ O(t) = W \]

or 

\[ (\gamma + d_1)(\gamma + d_1) \delta \left( \frac{\gamma + d_1}{2} + \frac{q\gamma + d_1}{p + 1} \right) = X + \Delta d_1 \left( E_T + \frac{q\gamma + d_1}{p + 1} \right) + \Delta d \left( E_T - S_T \right) \]

Neglecting \( \Delta_T^{p+1} \) and \( \Delta_T^{p+2} \) from Eq. (10).

\[ \Delta_T = \frac{\Delta_T^2}{2} = X + \Delta d_1 \left( E_T + \frac{q\gamma + d_1}{p + 1} \right) + \Delta d \left( E_T - S_T \right) \]

Throughout the time \( 0 \leq t \leq \Delta_T \), the stoke in OW. \( O_1 \) decreases due to only deterioration, so it is represented by the differential equation (13)

\[ \frac{dO_1(t)}{dt} + q\gamma t^{p-1} = 0 \]

Using given condition \( O_1(0) = W \), and \( O_1(\Delta_T) = W^0 \), the solution of the above Eq. (13) is

\[ O_1(t) = W e^{-\gamma t} \]

\[ W^0 = W e^{-\gamma \Delta_T} \]

Due to the combined effects of demand and deterioration, the stock in OW, \( O_1 \) decreases with time \( (\Delta_T \leq t \leq T) \), so it is represented by differential equation (16)

\[ \frac{dO_2(t)}{dt} + q\gamma t^{p-1} O_2(t) = -\gamma - \delta t - \Delta d_1 \]

Using the given condition, \( O_2(T) = 0 \), the solution of the differential equation (16) is

\[ O_2(t) = e^{-\gamma t} \left[ (\gamma + d_1)(T - t) + \frac{q}{p + 1} (T^{p+1} - t^{p+1}) \right] + \delta \left[ \frac{1}{2} (T^2 - t^2) + \frac{q}{p + 2} (T^{p+2} - t^{p+2}) \right] \]

### 4.4.2. Cost calculation:

The ordering cost is B.

(i) Inventory holding cost in RW \( (C_{iRW}) \) is

\[ C_{iRW} = E \left[ \int_0^{S_T} R_1(t) \ dt + \int_{S_T}^{E_T} R_2(t) \ dt + \int_{E_T}^{\Delta_T} R_3(t) \ dt \right] \]

or \( C_{iRW} = E[K_1 + K_2 + K_3] \)

where \( K_1 = \int_0^{S_T} R_1(t) \ dt, K_2 = \int_{S_T}^{E_T} R_2(t) \ dt, \) and \( K_3 = \int_{E_T}^{\Delta_T} R_3(t) \ dt \)

\[ K_1 = \int_0^{S_T} R_1(t) \ dt = \int_0^{S_T} e^{-\gamma t} \left[ X - t \left( \gamma + \frac{\delta t}{2} \right) - q(t^{p+1}) \left( \frac{\gamma}{p + 1} + \frac{\delta t}{p + 2} \right) \right] dt \]
\begin{align}
K_1 &= X \left( S_T - \frac{qS_T^{p+1}}{p+1} \right) - \gamma S_T^2 \left( \frac{1}{2} - \frac{qS_T}{p+2} \right) - \gamma q S_T^2 \left( \frac{S_T^p}{p+2} - \frac{qS_T^{2p}}{2(p+2)} \right) \\
&\quad - \frac{\delta p S_T^2}{2} \left( \frac{1}{3} - \frac{qS_T}{p+3} \right) - \delta q S_T^2 \left( \frac{S_T^p}{p+2} - \frac{qS_T^{2p}}{2(p+3)} \right) \\
&\quad + \int_{S_T}^{E_T} R_{12}(t) \, dt = \int_{S_T}^{E_T} e^{-qt} \left[ \alpha (\Delta d) - \gamma \right] \left( 1 + \frac{qt}{p+1} \right) - \delta t \left( \frac{1}{2} + \frac{qt}{p+2} \right) dt \\
K_2 &= (\Delta d - \gamma) \left\{ \frac{E_T^2 - S_T^2}{2} - \frac{q(E_T^{p+2} - S_T^{p+2})}{p+2} \left( 1 - \frac{1}{p+1} \right) - \gamma q(E_T^{2p+2} - S_T^{2p+2}) \right\} \\
&\quad - \delta \left\{ \frac{E_T^3 - S_T^3}{6} - \frac{q(E_T^{p+3} - S_T^{p+3})}{p+3} \left( 1 - \frac{1}{p+2} \right) - \gamma q(E_T^{3p+3} - S_T^{3p+3}) \right\} \\
&\quad + \left\{ X - S_T \Delta d \left( 1 + \frac{qS_T}{p+1} \right) \right\} \left\{ E_T - S_T - \frac{q(E_T^{p+1} - S_T^{p+1})}{p+1} \right\} \\
K_3 &= (\Delta T - \Delta d) \left\{ \frac{q}{p+1} \right\} - \delta \left\{ (\Delta T - E_T) + \frac{q}{p+1} (E_T^{p+1} - E_T^{p+3}) \right\} + \Delta d \left\{ (E_T - S_T) + \frac{q}{p+1} (E_T^{p+1} - S_T^{p+1}) \right\} dt \\
K_4 &= \int_{0}^{T} O_{11}(t) \, dt + \int_{0}^{T} O_{12}(t) \, dt \\
K_5 &= \int_{0}^{T} O_{12}(t) \, dt = \int_{0}^{T} e^{-qt} \left[ (T + \Delta d) \left\{ (T - t) + \frac{q}{p+1} (T^{p+1} - t^{p+1}) \right\} \\
&\quad + \delta \left\{ \frac{1}{2} (T^2 - t^2) + \frac{q}{p+2} (T^{p+2} - t^{p+2}) \right\} \right] dt \\
K_5 &= (\Delta d + \gamma) \left\{ (T - \Delta T) \left( T - \frac{q(T^{p+1} - \Delta T^{p+1})}{p+1} \right) - \left( \frac{q(T^{p+1} - \Delta T^{p+1})}{p+1} \right) \left( T - \frac{q(T^{p+1} - \Delta T^{p+1})}{p+1} \right) - \frac{T^2 - \Delta T^2}{2} \right\}
\end{align}

(ii) The inventory holding cost in OW \((C_{HOW})\) is
\begin{equation}
C_{HOW} = G \int_{0}^{T} O_{11}(t) \, dt + \int_{0}^{T} O_{12}(t) \, dt
\end{equation}

or \(C_{HOW} = G \left[ K_4 + K_5 \right] \)

where \(K_4 = \int_{0}^{T} O_{11}(t) \, dt\), and \(K_5 = \int_{0}^{T} O_{12}(t) \, dt\)

\begin{align}
K_4 &= \int_{0}^{T} O_{11}(t) \, dt = \int_{0}^{T} W e^{-qt} \, dt \\
K_4 &= W \left[ \Delta T - \frac{\Delta T^{p+1}}{p+1} \right] \\
K_5 &= \int_{0}^{T} O_{12}(t) \, dt = \int_{0}^{T} e^{-qt} \left[ (T + \Delta d) \left\{ (T - t) + \frac{q}{p+1} (T^{p+1} - t^{p+1}) \right\} \\
&\quad + \delta \left\{ \frac{1}{2} (T^2 - t^2) + \frac{q}{p+2} (T^{p+2} - t^{p+2}) \right\} \right] dt \\
K_5 &= (\Delta d + \gamma) \left\{ (T - \Delta T) \left( T - \frac{q(T^{p+1} - \Delta T^{p+1})}{p+1} \right) - \left( \frac{q(T^{p+1} - \Delta T^{p+1})}{p+1} \right) \left( T - \frac{q(T^{p+1} - \Delta T^{p+1})}{p+1} \right) - \frac{T^2 - \Delta T^2}{2} \right\}
\end{align}
\[ R = \frac{q_p(T^{p+2} - \Delta_i^{p+2})}{(p+1)(p+2)} + \frac{q^2(T^{2p+2} - \Delta_i^{2p+2})}{2(p+1)^2} + \delta \left[ (T - \Delta_i) \left( \frac{T^2}{2} - qT^{p+2} + \frac{T^3 - \Delta_i^3}{6} \right) \right. \]

Total holding cost in RW and OW (\(C_{hi}\)) is

\[ C_{hi} = C_{H RW1} + C_{HOW1} = E \sum_{n=1}^{3} K_n + G \sum_{n=1}^{5} K_n \]

(iii) Purchase cost per cycle = \(CQ\).

(iv) Deterioration cost in RW (\(C_{DRW1}\)) is

\[ C_{DRW1} = C \left[ \int_0^{S_T} q_{p+1} R_{11}(t) \, dt + \int_{S_T}^{E_T} q_{p+1} R_{12}(t) \, dt + \int_{E_T}^{d_T} q_{p+1} R_{13}(t) \, dt \right] \]

or \(C_{DRW1} = C[K_6 + K_7 + K_8]\)

where \(K_6 = \int_0^{S_T} q_{p+1} R_{11}(t) \, dt, K_7 = \int_{S_T}^{E_T} q_{p+1} R_{12}(t) \, dt, \) and \(K_8 = \int_{E_T}^{d_T} q_{p+1} R_{13}(t) \, dt\)

\[ K_6 = \int_0^{S_T} q_{p+1} e^{-q_t} \left[ X - t \left( \frac{\gamma + \delta t}{p} \right) - \frac{qT^{p+1}}{p+1} \right] \, dt \]

\[ K_7 = \int_{S_T}^{E_T} q_p(\Delta d - \gamma) \left[ \frac{E_T^{p+1} - S_T^{p+1}}{p+1} - q(E_T^{p+2} - S_T^{p+2}) \frac{(p+1)(2p+1)}{(p+1)(3p+2)} + q^2(E_T^{p+3} - S_T^{p+3}) \right] \, dt \]

\[ K_8 = \int_{E_T}^{d_T} q_p(\Delta d - \gamma) \left[ \frac{E_T^{p+1} - S_T^{p+1}}{p+1} - q(E_T^{p+2} - S_T^{p+2}) \frac{(p+1)(2p+1)}{(p+1)(3p+2)} + q^2(E_T^{p+3} - S_T^{p+3}) \right] \, dt \]
Because a total lockdown when things are despatched from the RW, and the lockdown is lifted when the RW is vacated.

As a result, Eqs. (4) and (5) can be used to explain inventory depletion during scenario 1. Suppose the RW get completely empty at the time $τ = τ(0)$.

\[
\text{Total deterioration cost in RW and OW (C_D)} \]

\[
\text{C_D(1)} = \text{C_DRW} + \text{C_DOW} = C \left[ \sum_{n=0}^{10} K_n \right]
\]

Total cost per cycle (TC) for scenario 1 is

\[
\text{TC} = B + \text{C_H} + \text{CQ} + \text{C_D(1)}
\]

4.4.3. Scenario 2

During (0 to $S_T$), and the products in the RW are decreased by both demand and deterioration rate, as shown in Eq. (1). Because $R_{11}(0) = X$, the equation’s solution is equivalent to Eq. (2). When lockdown imposed by government, the inventory of RW represented as Eq. (3). Due to lockdown restriction the demand decreased by an amount $Δd$, causing inventory to decrease at a slower rate. As a result, Eqs. (4) and (5) can be used to explain inventory depletion during ($S_T \leq Δ d \leq E_T$).

\[
\text{Calculate time at which RW becomes empty (ΔF):}
\]

Suppose the RW get completely empty at the time $ΔF$, i.e., $R_{12}(ΔF) = 0$.

\[
\Delta F(Δd - γ) \left( 1 + \frac{q Δ F}{p + 1} \right) - q Δ F \left( \frac{1}{2} + \frac{q Δ F}{p + 2} \right) + X - S_T Δd \left( 1 + \frac{q S_T}{p + 1} \right) = 0
\]
Neglecting $\Delta \rho_1^+ \Delta \rho_2^+$ in Eq. (36)

$$\Delta \Gamma (\Delta d - \gamma) - \frac{\delta \Delta \rho_1^+}{2} + X - S_T \Delta d \left(1 + \frac{q S_T^p}{p + 1}\right) = 0$$

(37)

$$\Delta \Gamma = \frac{(\gamma - \Delta d) \pm \left[(\Delta d - \gamma)^2 + 2 \delta \left(X - S_T \Delta d \left(1 + \frac{q S_T^p}{p + 1}\right)\right)\right]^{\frac{1}{2}}}{\delta}$$

(38)

Eq. (14) can be used to define the inventory in OW, $O_{I1}(t)$, which depletes exclusively due to deterioration during $(0 \ to \ \Delta \Gamma)$, and Eq. (15) can be used to express the inventory at the time $\Delta \Gamma$. Furthermore, the inventory in OW, $O_{I2}(t)$, depletes during $(\Delta \Gamma \ to \ E_T)$ as a result of demand and deterioration and it is represented by differential equation (39)

$$\frac{dO_{I2}(t)}{dt} + q \rho p^{-1} O_{I2}(t) = -\gamma - \delta t + \Delta d$$

(39)

Using given condition $O_{I2}(\Delta \Gamma) = O_{I3}(\Delta \Gamma)$ such that $\Delta \Gamma \leq E_T$, the solution of the above differential Eq. (39) is

$$O_{I2}(t) = e^{-q\rho} \left[W + (\Delta d - \gamma) \left\{(t - \Delta T) + \frac{q}{p + 1} \left(T^{\rho+1} - T^\rho\right)\right\} - \delta \left\{\frac{1}{2} (T^2 - t^2) + \frac{q}{p + 2} \left(T^{\rho+2} - T^{\rho+1}\right)\right\}\right]$$

(40)

When the lockdown is lifted, the demand is expected to increase by $\Delta d_1$. As a result, during $E_T \leq t \leq T$, the change in inventory in OW is provided by Eq. (41)

$$\frac{dO_{I3}(t)}{dt} + q \rho p^{-1} O_{I3}(t) = -\gamma - \delta t - \Delta d_1$$

(41)

Using the given condition $O_{I3}(T) = 0$, the solution of the differential equation (41) is

$$O_{I3}(t) = e^{-q\rho} \left[(\gamma + \Delta d_1) \left\{(T - t) + \frac{q}{p + 1} \left(T^{\rho+1} - T^\rho\right)\right\} + \delta \left\{\frac{1}{2} (T^2 - t^2) + \frac{q}{p + 2} \left(T^{\rho+2} - T^{\rho+1}\right)\right\}\right]$$

(42)

4.4.4. Cost calculation:

The ordering cost is $B$.

(i) Inventory holding cost in RW ($C_{HRW_2}$) is

$$C_{HRW_2} = E \left[\int_0^{S_T} R_{I1}(t) \ dt + \int_{S_T}^{\Delta \Gamma} R_{I2}(t) \ dt\right]$$

or $C_{HRW_2} = E[N_1 + N_2]$
where \( N_1 = \int_0^{S_T} R_{11}(t) \, dt \) and \( N_2 = \int_0^{S_T} R_{12}(t) \, dt \).

\[
N_1 = \int_0^{S_T} R_{11}(t) \, dt = \int_0^{S_T} e^{-\varrho t} \left[ X - \gamma \left( \frac{\varrho t}{p+1} + \frac{\varrho t}{p+2} \right) \right] \, dt
\]

\[
N_1 = \frac{X}{\delta} \left( S_T - \frac{q S_T^p}{p+1} - \frac{q S_T^p}{p+2} \right) - \gamma S_T^2 \left( \frac{1}{2} - \frac{q S_T^p}{p+2} \right) - \gamma q S_T^p \left( \frac{S_T^p}{p+1} - \frac{q S_T^p}{p+2} \right)
\]

\[
- \frac{\delta S_T^3}{2} \left( \frac{1}{3} - \frac{q S_T^p}{p+3} \right) - \frac{\delta q S_T^p}{p+2} \left( \frac{S_T^p}{p+3} - \frac{S_T^p}{2 p+3} \right)
\]

\[
(44)
\]

\[
N_2 = \int_0^{S_T} R_{12}(t) \, dt = \int_0^{S_T} e^{-\varrho t} \left[ t(\Delta d - \gamma) \left( 1 + \frac{q p}{p+1} \right) - \delta t \left( \frac{1}{2} + \frac{q p}{p+2} \right) \right] \, dt
\]

\[
+ X - \frac{S_T}{\Delta d} \left( 1 + \frac{q S_T^p}{p+1} \right) \left( \Delta d - \gamma \right) \left( \frac{1}{2} + \frac{q S_T^p}{p+1} \right) \left( \Delta d - \gamma \right)
\]

\[
- \frac{\delta}{2} \left( \frac{2 S_T^2}{6} + \frac{q(\Delta d - \gamma)(S_T^p - S_T^{p+1})}{2(p+3)} + q(\Delta d - \gamma)(S_T^p - S_T^{p+1}) \right)
\]

\[
\left( \frac{1}{2} - S_T^2 \right) \left( \frac{1}{2} + \frac{q S_T^p}{p+1} \right) - \frac{q p(\Delta d - \gamma)(S_T^p - S_T^{p+1})}{(p+1)(p+2)}
\]

\[
(45)
\]

(ii) Inventory holding cost in OW (\( C_{HOW2} \)) is

\[
C_{HOW2} = G \left[ \int_0^{T_2} O_{11}(t) \, dt + \int_0^{E_2} O_{12}(t) \, dt + \int_0^{E_3} O_{13}(t) \, dt \right]
\]

or \( C_{HOW2} = G[N_3 + N_4 + N_5] \)

where \( N_3 = \int_0^{T_2} O_{11}(t) \, dt \), \( N_4 = \int_0^{E_2} O_{12}(t) \, dt \), and \( N_5 = \int_0^{E_3} O_{13}(t) \, dt \)

\[
N_3 = \int_0^{T_2} O_{11}(t) \, dt = \int_0^{T_2} W e^{-\varrho t} \, dt
\]

\[
N_3 = W \left[ \Delta d - \frac{q \Delta d^{p+1}}{p+1} \right]
\]

\[
N_4 = \int_0^{E_2} O_{12}(t) \, dt = \int_0^{E_2} e^{-\varrho t} \left[ W + (\Delta d - \gamma) \left( t - \Delta d \right) + \frac{q}{p+1} \left( T^{p+1} - \Delta d^{p+1} \right) \right] \, dt
\]

\[
- \frac{\delta}{2} \left( \frac{1}{2} (T^2 - \Delta d^2) + \frac{q}{p+1} \left( T^{p+1} - \Delta d^{p+1} \right) \right) \left( T^2 - \Delta d^2 \right)
\]

\[
N_4 = W \left[ \left( E_T - \frac{q E_T^{p+1}}{p+1} \right) - \left( \Delta d - \frac{q \Delta d^{p+1}}{p+1} \right) \right] + (\Delta d - \gamma) \left[ E_T^2 - \frac{E_T^2}{2} - \frac{q p (E_T^{p+2} - \Delta d^{p+2})}{(p+1)(p+2)} \right]
\]

\[
- \left( \frac{q p+4}{2(p+3)(p+2)} \right) \left( E_T - \frac{q E_T^{p+1}}{p+1} \right) - \left( \Delta d - \frac{q \Delta d^{p+1}}{p+1} \right)
\]

\[
- \frac{\delta}{2} \left( \frac{2 E_T^2}{2} + \frac{q^2 (E_T^{p+2} - \Delta d^{p+2})}{2(p+3)(p+2)} \right)
\]

\[
(48)
\]

\[
N_5 = \int_0^{T_2} O_{13}(t) \, dt = \int_0^{T_2} e^{-\varrho t} \left[ (\gamma + \Delta d) \left( T - t \right) + \frac{q}{p+1} \left( T^{p+1} - \Delta d^{p+1} \right) \right] \, dt
\]

\[
+ \frac{\delta}{2} \left( \frac{1}{2} (T^2 - t^2) + \frac{q}{p+1} \left( T^{p+2} - \Delta d^{p+2} \right) \right) \left( T^2 - t^2 \right)
\]
\[ N_5 = (\gamma + \Delta d) \left[ \left( T + \frac{qT^{p+1}}{p+1} \right) - \left( T - \frac{qT^{p+1}}{p+1} \right) \right] \]
\[ + \frac{q(T^{p+2} - E_T^{p+2})}{(p+1)(p+2)} - \frac{T^2 - E_T^2}{2} + \frac{q(T^{p+2} - E_T^{p+2})}{2(p+1)^2} \]
\[ + \delta \left( \frac{T^2}{2} - \frac{qT^{p+1}}{p+2} \right) \left( T - \frac{qT^{p+1}}{p+1} \right) - \left( T - \frac{qT^{p+1}}{p+1} \right) \left( T - \frac{qT^{p+1}}{p+1} \right) \]
\[ - \frac{T^2 - E_T^2}{2} + \frac{q(p+4)(T^{p+3} - E_T^{p+3})}{2(p+1)(p+3)} - \frac{q^2(T^{p+3} - E_T^{p+3})}{(p+2)(2p+3)} \]  

Total holding cost in RW and OW (\(C_{H2}\)) is

\[ C_{H2} = C_{HRW2} + C_{HOW2} = E \sum_{n=1}^{N} N_n + G \sum_{n=3}^{5} N_n \]  

(iii) Purchasing cost per cycle \(C_Q\).

(iv) Deterioration cost in RW (\(C_{DRW2}\))

\[ C_{DRW2} = C \left[ \int_{0}^{S_T} qpte^{-1} R_{I1}(t) \, dt + \int_{0}^{\Delta t} qpte^{-1} R_{I2}(t) \, dt \right] \]  

or \(C_{DRW2} = C[N_6 + N_7]\)

where \(N_6 = \int_{0}^{S_T} qpte^{-1} R_{I1}(t) \, dt\), and \(N_7 = \int_{0}^{\Delta t} qpte^{-1} R_{I2}(t) \, dt\)

\[ N_6 = XqS_T^t \left( 1 - \frac{qS_T^{p+1}}{2} \right) - \gamma qS_T^{p+1} \left( \frac{qS_T^p}{p+1} + \frac{1}{2(p+1)} \right) + \frac{qS_T^p}{p+1} \]
\[ + \frac{1}{2(p+1)(3p+1)} \]
\[ + \frac{qS_T^p}{p+1} \left( \frac{1}{4(p+1)} + \frac{qS_T^p}{p+1} \right) \]
\[ + \gamma q^2 pS_T^{2p+1} \left( \frac{1}{2p+1} + \frac{qS_T^p}{p+1} \right) \]
\[ + \frac{qS_T^p}{p+1} \left( \frac{1}{4(p+1)} + \frac{qS_T^p}{p+1} \right) \]

\[ N_7 = \frac{q^2 T^{p+2} - E_T^{p+2}}{2(p+2)} + \frac{q^2 T^{p+2} - E_T^{p+2}}{4(p+1)(p+2)} + \frac{q^2 T^{p+2} - E_T^{p+2}}{(p+2)(3p+2)} \]
\[ + \frac{q^2 T^{p+2} - E_T^{p+2}}{2} \]

(v) Deterioration cost in OW (\(C_{DOW2}\))

\[ C_{DOW2} = C \left[ \int_{0}^{\Delta t} qpte^{-1} O_{I1}(t) \, dt + \int_{0}^{E_T} qpte^{-1} O_{I2}(t) \, dt + \int_{0}^{T} qpte^{-1} O_{I3}(t) \, dt \right] \]

or \(C_{DOW2} = C[N_8 + N_9 + N_{10}]\)

where \(N_8 = \int_{0}^{\Delta t} qpte^{-1} O_{I1}(t) \, dt\), \(N_9 = \int_{0}^{E_T} qpte^{-1} O_{I2}(t) \, dt\), and \(N_{10} = \int_{0}^{T} qpte^{-1} O_{I3}(t) \, dt\)

\[ N_8 = \int_{0}^{\Delta t} qpte^{-1} O_{I1}(t) \, dt = \int_{0}^{\Delta t} qpte^{-1} W e^{-qT} \, dt \]
\[ N_8 = W q\Delta_T^t \left( 1 - \frac{q\Delta_T^t}{2} \right) \]

\[ N_9 = \int_{0}^{E_T} qpte^{-1} O_{I2}(t) \, dt = \int_{0}^{E_T} qpte^{-1} e^{-qT} \left[ W + (\Delta d - \gamma) \left( t + \Delta_T^t \right) + \frac{q}{p+1} (p^{+1} - \Delta_T^{p+1}) \right] \]
We can see from the Figs. 3 and 4 curves for scenarios 1 and scenario 2. We have a positive impact on total cost. So the minimal total cost exists when \( \Delta d \) and \( Z \) (lockdown period) have a negative impact on the total cost, and \( \Delta d_1 \) have a positive impact on total cost. So the minimal total cost exists when \( \Delta d \) and \( Z \) decreases and \( \Delta d_1 \) increases.

6. Numerical example

For each scenario, a numerical example demonstrate the impact of demand disruption on overall cost as a result of the enforced lockdown. The MATHEMATICA software is used to solve and plot the graphs using the provided data.
Example 6.1 (Scenario 1). Let $q = 0.5$, $p = 2$, $Z = 0.3$, $Q = 400$, $X = 250$, $W = 150$, $\Delta d = 130$, $\Delta d_1 = 70$, $\gamma = 230$, $\delta = 20$, $E = 3$, $G = 2$, $S_T = 0.3$, $E_T = S_T + Z = 0.6$, $\Delta_T = 0.8$, $B = 300$, $C = 20$, $T = 1$, then $TC$ (Total Cost) = $10643.4$.

Example 6.2 (Scenario 2). Let $q = 0.5$, $p = 2$, $Z = 0.8$, $Q = 400$, $X = 250$, $W = 150$, $\Delta d = 130$, $\Delta d_1 = 70$, $\gamma = 230$, $\delta = 20$, $E = 3$, $G = 2$, $S_T = 0.4$, $E_T = S_T + Z = 1.2$, $\Delta_T = 1$, $B = 300$, $C = 20$, $T = 1.5$, then $TC$ (Total Cost) = $13529$.

We can observe from the preceding Examples 6.1 and 6.2, the total cost is more in scenario 2, because both holding costs and deterioration costs increases due to longer lockdown period in scenario 2 as compare to scenario 1.

7. Sensitivity study

Some essential parameters are selected to study the sensitivity analysis to determine the impact of changing specific parameters on the result i.e., total cost. These parameters are (i.e., $\gamma$(the initial rate of demand), $\delta$(the rate at which demand grows over time),}
Fig. 4. In Scenario 2, Total cost(TC) change with respects to $\Delta d_1$, $\Delta d$, and $Z$ shown in Table 2. The Table 2 below gives a percentage change in overall cost for each cycle and analyses parameter fluctuations using 20% and 10%, respectively.

(i) Table 2 shows that any change in these parameters has a considerable impact on the output in Scenario 2 as compare to scenario 1. The reason for this is that Scenario 2 has a longer lockdown duration than Scenario 1, so Scenario 2 has higher deterioration and holding costs.

(ii) Variation to the demand factors ($\gamma$ and $\delta$) have a major impact on the total cost of each cycle because when demand factor decreases, the demand rate also decreases, so it rises the holding cost. Additionally, as items are stored in the warehouse for extended periods of time, the cost of deterioration rises as a result of the rise in deterioration rate.
Table 2
Sensitivity study for both Scenario 1 and 2.

| Parameter | Scenario 1 | Scenario 2 |
|-----------|------------|------------|
| | Value | Total cost | Total cost variation (%) | Value | Total cost | Total cost variation (%) |
| $q$ | 0.6 | 11,048.7 | 3.81 | 0.6 | 14,484 | 7.06 |
| 0.55 | 10,844.5 | 1.89 | 0.55 | 14,002.3 | 3.54 |
| 0.45 | 10,449.4 | –1.82 | 0.45 | 13,524 | –3.52 |
| 0.4 | 10,261.2 | –3.59 | 0.4 | 12,579.3 | –7.02 |
| $p$ | 2.4 | 10,472.5 | –1.61 | 2.4 | 14,230.2 | 5.18 |
| 2.2 | 10,555.6 | –0.82 | 2.2 | 13,863.1 | 2.47 |
| 1.8 | 10,735.3 | 0.86 | 1.8 | 13,226 | –2.23 |
| 1.6 | 10,830.2 | 1.76 | 1.6 | 13,952.3 | –4.26 |
| $\Delta d$ | 156 | 10,679.6 | 0.34 | 156 | 13,577.4 | 0.36 |
| 143 | 10,661.5 | 0.17 | 143 | 13,554.4 | 0.19 |
| 117 | 10,625.3 | –0.17 | 117 | 13,508.3 | –0.15 |
| 104 | 10,607.2 | –0.34 | 104 | 13,485.3 | –0.32 |
| $\Delta d_1$ | 84 | 10,581.1 | –0.59 | 84 | 13,483 | –0.34 |
| 77 | 10,612.2 | –0.29 | 77 | 13,503.3 | –0.18 |
| 63 | 10,674.5 | 0.29 | 63 | 13,633.6 | 0.77 |
| 56 | 10,705.7 | 0.59 | 56 | 13,683.8 | 1.14 |
| $\gamma$ | 276 | 10,521.2 | –1.15 | 276 | 13,609.8 | 0.6 |
| 253 | 10,578.8 | –0.61 | 253 | 13,569.4 | 0.3 |
| 207 | 10,706.1 | 0.59 | 207 | 13,488.6 | –0.3 |
| 184 | 10,768.8 | 1.18 | 184 | 13,448.2 | –0.6 |
| $\delta$ | 24 | 10,643.4 | –0.06 | 24 | 13,516.3 | –0.09 |
| 22 | 10,640.4 | –0.03 | 22 | 13,522.7 | –0.05 |
| 18 | 10,646.4 | 0.03 | 18 | 13,535.3 | 0.05 |
| 16 | 10,649.4 | 0.06 | 16 | 13,541.7 | 0.09 |
| $Z$ | 0.36 | 10,759.4 | 1.09 | 0.36 | 14,047.6 | –4.99 |
| 0.33 | 10,704.4 | 0.57 | 0.33 | 13,799.8 | –2.27 |
| 0.27 | 10,582.1 | –0.58 | 0.27 | 13,222.4 | 2.00 |
| 0.24 | 10,516 | –1.2 | 0.24 | 12,853.8 | 3.83 |
| $E$ | 3.6 | 10,721.4 | 0.73 | 3.6 | 13,605.4 | 0.56 |
| 3.3 | 10,682.4 | 0.37 | 3.3 | 13,567.2 | 0.28 |
| 2.7 | 10,604.4 | –0.37 | 2.7 | 13,490.8 | –0.28 |
| 2.4 | 10,565.4 | 0.73 | 2.4 | 13,452.6 | –0.56 |
| $G$ | 2.4 | 10,683 | 0.37 | 2.4 | 13,610.5 | 0.60 |
| 2.2 | 10,663.2 | 0.19 | 2.2 | 13,569.8 | 0.30 |
| 1.8 | 10,623.6 | –0.19 | 1.8 | 13,488.2 | –0.30 |
| 1.6 | 10,603.8 | –0.37 | 1.6 | 13,447.5 | –0.60 |

(iii) The total holding cost rises when the per-item holding costs of RW ($E$) and OW ($G$) rises, but other costs remain constant, resulting in an increase in total cost each cycle.

(iv) Both holding costs and degradation costs rise as a result of the items being stored in warehouses for a longer period of time if the demand rate declines by $\Delta d$. The demand instability caused by the imposed lockdown has a major impact on total cost. Both holding costs and deterioration costs rise as a result of the items being stored in warehouses for a longer period of time if the demand rate declines by $\Delta d$.

(v) Demand rise by $\Delta d_1$ after the lockdown limitation was released as a result of the nature of consumers' panic buying behavior. Deterioration and holding costs drop as a result of the warehouse being vacant in less time, which lowers the overall cost each cycle.

(iv) The effect of various parameter such as $\Delta d$, $\Delta d_1$, and $Z$ on Total cost (TC) in both scenario 1 and scenario 2 are depicted in Figs. 3 and 4, to help the reader understand.

8. Conclusion

This study used a single-item inventory with two warehouses, a linearly time-dependent demand, and a Weibull distribution to more precisely depict the deterioration rate. Action is determined to control the degradation since it substantially impacts the perishable inventory issue. To demonstrate the versatility and applicability of the Weibull distribution for studying deterioration processes, we examine the Weibull parameter changes on the properties of perishability. The total cost of each cycle is provided for two scenarios using the suggested mathematical analysis. Because the lockdown duration in scenario 2 is long, it has a higher total cost than scenario 1 in this model. The vendor purchases more than the warehouse can hold because of discounts offered for bulk orders and other circumstances. Therefore, under these circumstances, the vendor must store the extra stocks in any RW. This study has offered a meaningful conclusion that the additional cost of preserving and holding material, etc., might be decreased utilizing
the proposed technique, keeping in mind that the inventory cost, such as holding cost in RW, is more than OW. However, it will be more practical and financially advantageous for businesses to keep goods in OW before RW and use the stocks in RW before OW. This research leads to the conclusion that raising OW capacity reduces the total cost of the system. Additionally, a sensitivity analysis is done concerning different system parameters, and an analytic example is given.

This model can be applied to studying the inventory management for continuously degrading products, such as those sold primarily in supermarkets and grocery stores. By simulating various demand scenarios, the proposed model can assist managers in making predictions and evaluating several potential courses of action. In the future, the model can be expanded to include variables such as the demand based on price, stock, freshness, price and freshness, and many others. We can also think of other costs as being time-dependent, such as the cost of unit purchases and inventory keeping.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

The author D Das is thankful to University Grant Commission (UGC), Govt. of India for providing JRF (Ref No. 201610142889) to carry out his research.

References

[1] Mohanta KK, Sharanappa DS, Aggarwal A. Efficiency analysis in the management of COVID-19 pandemic in India based on data envelopment analysis. Curr Res Behav Sci 2021;100063.
[2] Basu R. COVID control in India: A look back. J Compr Health 2020;8(2):129–31.
[3] Barman A, Das R, De PK. Impact of COVID-19 in food supply chain: Disruptions and recovery strategy. Curr Res Behav Sci 2021;1:100017.
[4] Wei H-M. Economic production lot size model for deteriorating items with partial back-ordering. Comput Ind Eng 1993;24(3):449–58.
[5] Bakhshi ZT. On the global optimal solution to an integrated inventory system with general time varying demand, production and deterioration rates. European J Oper Res 1999;114(1):29–37.
[6] Jaggi CK, Aggarwal K, Goel S. Optimal order policy for deteriorating items with inflation induced demand. Int J Prod Econ 2006;103(2):707–14.
[7] Dey JK, Mondal SK, Maiti M. Two storage inventory problem with dynamic demand and interval valued lead-time over finite time horizon under inflation and time-value of money. European J Oper Res 2008;185(1):170–94.
[8] Wu K-S, Guyang L-Y. A replenishment policy for deteriorating items with ramp type demand rate. Proc-Natl Sci Counc Rep China A 2000;24(4):279–86.
[9] Skouri K, Konstantaras I, Papachristos S, Ganas I. Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. European J Oper Res 2009;192(1):79–92.
[10] Hung K-C. An inventory model with generalized type demand, deterioration and backorder rates. European J Oper Res 2011;208(3):239–42.
[11] Wang W-C, Teng J-T, Lou K-R. Seller's optimal credit period and cycle time in a supply chain for deteriorating items with maximum lifetime. European J Oper Res 2014;232(2):315–21.
[12] Zhang J, Bai Z, Tang W. Optimal pricing policy for deteriorating items with preservation technology investment. J Ind Manag Optim 2014;10(4):1261.
[13] Prasad K, Mukherjee B. Optimal inventory level under stock and time dependent demand for time varying deterioration rate with shortages. Ann Oper Res 2016;243(1):323–34.
[14] Rastogi M, Singh S, Kushwah P, Tayal S. Two warehouse inventory policy with price dependent demand and deterioration under partial backlogging. Decis Sci Lett 2017;6(1):11–22.
[15] Tiwari S, Cárdenas-Barrón LE, Goh M, Shaikh AA. Joint pricing and inventory model for deteriorating items with expiration dates and partial backlogging under two-level partial trade credits in supply chain. Int J Prod Econ 2018;200:16–36.
[16] Panda GC, Khan M, Shaikh AA, et al. A credit policy approach in a two-warehouse inventory model for deteriorating items with price and stock-dependent demand under partial backlogging. J Ind Eng Int 2019;15(1):147–70.
[17] Bishi B, Behera J, Sahu SK. Two-warehouse inventory model for non-instantaneous deteriorating items with exponential demand rate. Int J Appl Eng Res 2019;14:495–515.
[18] Shaikh AA, Cárdenas-Barrón LE, Tiwari S. A two-warehouse inventory model for non-instantaneous deteriorating items with interval-valued inventory costs and stock-dependent demand under inflationary conditions. Neural Comput Appl 2019;31(6):1931–48.
[19] Ahmad B, Benkherouf L. On an optimal replenishment policy for inventory models for non-instantaneous deteriorating items with stock dependent demand and partial backlogging. RAIRO-Oper Res 2020;54(1):69–79.
[20] Garg G, Singh S, Singh V, et al. A two warehouse inventory model for perishable items with ramp type demand and partial backlogging. Int J Eng Res Technol (IJERT) 2020;9(06).
[21] Rout C, Chakraborty D, Goswami A. A production inventory model for deteriorating items with backlog-dependent demand. RAIRO-Oper Res 2021;55:S549–70.
[22] Kumar N, Dahysi S, Kumar S. Two warehouse inventory model for deteriorating items with fixed shelf-life stock-dependent demand and partial backlogging. J Math Comput Sci 2022;12:Article-ID.
[23] Rana RS, Kumar D, Prasad K. Two warehouse dispatching policies for perishable items with freshness efforts, inflationary conditions and partial backlogging. Oper Manag Res 2021;1–18.
[24] Khan MA-A, Halim MA, AlArjani A, Shaikh AA, Uddin MS. Inventory management with hybrid cash-advance payment for time-dependent demand, time-varying holding cost and non-instantaneous deterioration under backordering and non-terminating situations. Alexandria Eng J 2022;61(11):8469–86.
[25] He Y, Wang S. Analysis of production-inventory system for deteriorating items with demand disruption. Int J Prod Res 2012;50(16):4580–92.
[26] Hobbs JE. Food supply chains during the COVID-19 pandemic. Can J Agric Econ/Rev Can d'Agroecon 2020;68(2):171–6.