Effect of the Temperature of Background Plasma and the Energy of Energetic Electrons on Z-mode Excitation

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Abstract

It has been suggested that the Z-mode instability driven by energetic electrons with a loss-cone type velocity distribution is one candidate process behind the continuum and zebra pattern of solar type-IV radio bursts. Both the temperature of background plasma (\(T_0\)) and the energy of energetic electrons (\(\nu_e\)) are considered to be important to the variation of the maximum growth rate (\(\gamma_{\text{max}}\)). Here we present a detailed parameter study on the effect of \(T_0\) and \(\nu_e\), within a regime of the frequency ratio (10 \(< \frac{\nu_e}{\Omega_{\text{ce}}} \leq 30\)). In addition to \(\gamma_{\text{max}}\), we also analyze the effect on the corresponding wave frequency (\(\omega_{\text{max}}\)) and propagation angle (\(\theta_{\text{max}}\)). We find that (1) \(\gamma_{\text{max}}\) generally decreases with increasing \(\nu_e\), while its variation with \(T_0\) is more complex depending on the exact value of \(\nu_e\). (2) With increasing \(T_0\) and \(\nu_e\), \(\omega_{\text{max}}\) presents stepwise profiles with jumps separated by gradual or very weak variations, and due to the warm plasma effect on the wave dispersion relation \(\omega_{\text{max}}\) can vary within the hybrid band (the harmonic band containing the upper hybrid frequency) and the higher band. (3) The propagation is either perpendicular or quasi-perpendicular, and \(\theta_{\text{max}}\) presents variations in line with those of \(\omega_{\text{max}}\), as constrained by the resonance condition. We also examine the profiles of \(\gamma_{\text{max}}\) with \(\frac{\nu_e}{\Omega_{\text{ce}}}\) for different combinations of \(T_0\) and \(\nu_e\) to clarify some earlier calculations which show inconsistent results.

Key words: instabilities – masers – plasmas – Sun: radio radiation – waves

1. Introduction

Recent studies on moving type-IV solar radio bursts (t-IVms, slowly drifting wide-band continua observed at metric-decimetric wavelengths) reveal that the t-IVm sources are associated with an eruptive high-temperature dense structure (Vasanth et al. 2016, 2019). This is possible because the events being studied are recorded at metric wavelengths by both the Nançay Radioheliograh (NRH: Kerdraon & Delouis 1997) and at Extreme Ultraviolet (EUV) by the Atmospheric Imaging Assembly on board the Solar Dynamics Observatory (AIA/ SDO: Lemen et al. 2012; Pesnell et al. 2012). Further differential emission analysis of AIA data shows that the source temperature is around several MK and the density is at the level of 10⁸ cm⁻³ at a heliocentric distance of \(\sim 1.2-1.5\) Solar Radii (\(R_\odot\)) at frequencies around 200–300 MHz (Vasanth et al. 2019). At this height of the solar atmosphere, the magnetic field strength is in general around or less than several Gauss (see, e.g., Dulk & McLean 1978; Cho et al. 2007; Ramesh et al. 2010; Chen et al. 2011; Feng et al. 2011). Thus, the metric t-IVms bursts are generated within a plasma regime that has plasma–electron–cyclotron frequency ratio \(\frac{\nu_e}{\Omega_{\text{ce}}}\) much larger than unity (mostly larger than 10). Based on observations and some related earlier theoretical studies (e.g., Winglee & Dulk 1986; Benáček et al. 2017), Vasanth et al. (2016, 2019) suggested that the t-IV continuum belongs to coherent plasma emission generated by energetic electrons trapped within the eruptive magnetic structure.

Energetic electrons trapped by a magnetic structure can develop a loss-cone type distribution with an inversion of population along the perpendicular direction in the velocity space, i.e., \(\frac{d\rho}{d\nu} > 0\), where \(f\) represents the velocity distribution function of energetic electrons. They can drive kinetic instabilities and excite plasma waves (e.g., Freund & Wu 1976, 1977; Wu & Lee 1979; Wu 1985; Winglee & Dulk 1986). In the parameter regime of \(\frac{\nu_e}{\Omega_{\text{ce}}} \gg 1\), such distribution can result in the Z-mode instability and excite enhanced Z-mode waves, which are the slow branch of the extraordinary (X) mode and correspond to obliquely (or perpendicularly) propagating Langmuir waves. Under certain conditions, such as propagation in inhomogeneous magnetic fields and nonuniform plasmas, Z-mode waves may transform into an escaping electromagnetic mode and be observed as radio bursts such as t-IV bursts (e.g., Winglee & Dulk 1986).

Many earlier studies have applied the Z-mode instability to explain the origin of the intriguing embedding zebra structure of t-IV bursts (e.g., Winglee & Dulk 1986; Yasnov & Karlický 2004; Zlotnik 2013; Benáček et al. 2017). Zebras refer to the numerous emission stripes that are almost parallel to each other that are superposed on the t-IV continuum, as manifested on the solar radio dynamic spectra (Kundu 1965; Slottje 1972; Krüger 1979; Chernov et al. 2001, 2012; Chernov 2010; Tan et al. 2014). Note that the Z-mode instability is also called the double-plasma resonance (DPR) in many references (Yasnov & Karlický 2004; Benáček et al. 2017; Benáček & Karlický 2018), because for cold plasmas the instability reaches the maximum growth rate when the upper hybrid frequency (\(\omega_{\text{UH}} = \sqrt{\nu_e^2 + \Omega_{\text{ce}}^2}\)) equals a harmonic of \(\Omega_{\text{ce}}\). It is found that the most important parameter relevant to solar radio bursts is the frequency ratio \(\frac{\nu_e}{\Omega_{\text{ce}}}\), which strongly modulates the values of the maximum of the Z-mode growth rate (\(\gamma_{\text{max}}\)). With increasing \(\frac{\nu_e}{\Omega_{\text{ce}}}\), the profile of \(\gamma_{\text{max}}\) manifests peaks at frequencies close to harmonics of \(\Omega_{\text{ce}}\), i.e., \(\gamma_{\text{max}}\) reaches maximum when Z-mode frequency is close to \(n\Omega_{\text{ce}}\) where \(n\) is an integer. Note that the exact mechanism(s)
out when the wave spectrum is not carefully analyzed. This, together with the observational indications from t-IVms (Vasanth et al. 2016, 2019), points to the significance of including the warm plasma effect when calculating the Z-mode growth rate.

Only a few studies considered this effect on Z-mode growth (Winglee & Dulk 1986; Yasnoff & Karlický 2004; Benáček et al. 2017), with various combinations of the background plasma temperature (represented by $T_0$ or the corresponding thermal speed $v_0 = \sqrt{\frac{k_{B}T_0}{m}}$) and energy of energetic electrons (represented by $v_e$) of the DGH distribution. Unfortunately, only a few discrete values of the two parameters have been taken into account. For example, Benáček et al. (2017) investigated the Z-mode instability for $v_0 = 0, 0.009$, and $0.018 c$, and $v_e = 0.1, 0.2$, and $0.3 c$, while Yasnov & Karlický (2004) only considered two different values of $T_0$ (2 and 20 MK). This means that present parameter studies on the effect of $T_0$ and $v_e$ are rather incomplete and more detailed parameter studies may be necessary. Indeed, according to the calculations presented below, such study leads to important novel results that have not been reported.

In summary, this section introduces a detailed parameter study on the effect of the temperature of background plasma, and the energy of energetic electrons is required to better understand the Z-mode instability driven by trapped electrons. This serves as the major motivation of the present study. The following section introduces basic assumptions, the wave dispersion relation (see also the Appendix), and parameters used in the calculations, and in Section 3 the results of the parameter study are presented. A summary and discussion are given in the last section.

2. Basic Assumptions, Dispersion Relation, and Parameters

The present study is based on the general kinetic dispersion relation for small-amplitude waves propagating in uniform magnetized warm plasmas (see, e.g., Baldwin et al. 1969; Wu 1985), which is a linear wave solution to the collisionless Vlasov–Maxwell system. The plasma consists of two components of electrons; one is the background warm plasma with the Maxwellian distribution ($f_0$), and the other is energetic electrons with the DGH distribution ($f_e$), given by

$$f(u_e, u_i) = \frac{n_e f_0}{n_0} + \left(1 - \frac{n_e}{n_0}\right)f_0,$$  \hspace{1cm} (1)

$$f_0 = \frac{1}{(2\pi)^{3/2}v_0} \exp \left(-\frac{u^2}{2v_0^2}\right),$$  \hspace{1cm} (2)

$$f_e = \frac{u_{\perp}^{2j}}{(2\pi)^{3/2}v_e^{2j+1}} \exp \left(-\frac{u_{\perp}^{2j}}{2v_e^{2j}}\right).$$  \hspace{1cm} (3)

where $f$ is the total electron distribution function, and $n_0$ and $n_e$ are number densities of thermal and energetic electrons, respectively. $j$ is the order of the DGH distribution function ($f_e$) and is set to be 1, $v_e$ is the mean velocity of energetic electrons, and $u_\perp (u_i)$ represents the averaged perpendicular (parallel) momentum per unit mass of electrons. The ions are assumed to be static because only modes with frequencies much higher than ion characteristic frequencies are considered, thus only electrons contribute to the general dispersion relation.
In Figure 1, we demonstrate the total electron distribution function (f) with white contours, superposed by maps of \( \log(\partial f / \partial u_e + 1) \), which represents the \( u_\perp \) gradient of f. The corresponding temperatures of panels (a), (b), and (c) are \( T_0 = 0.2, 2, \) and 4 MK, respectively, and the related electron velocity is fixed at 0.2c. With increasing \( T_0, f_0 \) occupies a larger area of the velocity space, and gets closer to the phase space occupied by energetic electrons, as expected. This of course affects the growth rate of Z-mode instability, which is determined by the integral (see Equation (10)) along the resonance curve as defined in the velocity space by the resonance condition

\[
\gamma_L \omega_r - n\Omega_{ce} - k_i u_i = 0, \tag{4}
\]

where \( \gamma_L \) is the Lorentz factor, \( k_i \) is the parallel wavenumber, and \( \Omega_{ce} \) is the electron cyclotron frequency.

We assume that \( n_e \ll n_0 \). Then, the wave modes are determined by thermal electrons and the instability is energized by energetic electrons. This allows us to utilize fluid equations of warm plasmas to derive the dispersion relation of the Z-mode from which the real part of the wave frequency \( \omega_r = \omega_r(k) \) is deduced, where \( k = k \cos \theta_x + k \sin \theta_x \) represents the wave vector, \( \theta \) is the angle between the background magnetic field \( B_0 = B_0 \hat{\mu}_z \) and \( k \). The solution of the growth rate (i.e., the imaginary part of the wave frequency, \( \gamma \)) can be greatly simplified with this assumption (see Equation (9)). Please check the Appendix for details on \( \gamma \) and the general kinetic and fluid dispersion relations.

Both dispersion relations are solved numerically. Major parameters are \( T_0, v_e, \) and \( \Omega_{ce} \). The density ratio \( \frac{n_0}{n_e} \) is included in the growth rate, therefore its exact value is not important as long as it remains small enough. For \( T_0 \), considering the t-IVm observations introduced earlier and the usual warm plasma parameters of the solar corona, we vary \( T_0 \) in a range of \([0, 8]\) MK or \([0, 1]\) keV; for \( v_e \), we vary it in a range of \([0.15, 0.4]\) c or \([5, 50]\) keV, where c is the speed of light. Thus, the weakly relativistic approximation can be applied.

Regarding the range of \( \frac{\omega_r}{\Omega_{ce}} \), as mentioned in the introduction this parameter can be much larger than unity for plasmas within the t-IVm sources; we therefore set its range to be \([10, 30]\). According to observations on t-IV bursts with zebra patterns, the number of stripes and the corresponding estimated harmonics are often larger than 10, sometimes reaching 30 (e.g., Kuijpers 1975; Aurass et al. 2003; Zlotnik et al. 2003; Chernov et al. 2005; Kuznetsov & Tsap 2007). This means the adopted range of \( \frac{\omega_r}{\Omega_{ce}} \) is relevant to the study of t-IV radio bursts.

Another reason for using this range of \( \frac{\omega_r}{\Omega_{ce}} \) stems from the limitation of the fluid dispersion relation of Z-mode. For cold plasmas, the frequency of Z-mode is determined by the upper hybrid frequency \( \omega_{UH} \). For warm plasmas, its kinetic dispersion relation is greatly affected by the cyclotron resonance effect, and is split into many branches (usually called electron Bernstein modes). The branch within the hybrid band (i.e., the band containing \( \omega_{UH} \), given by \((s-1)\Omega_{ce} \leq \omega_r \leq s\Omega_{ce} \), \( s \) is a positive integer) with normal dispersion corresponds to the Z-mode, and the part with abnormal dispersion corresponds to the electron cyclotron mode. Only the normal dispersion part is of interest here. The dispersion relations given by the fluid equations and the plasma kinetic theory for the Z-mode in warm plasmas of Maxwellian distribution are plotted in Figure 2, for \( \frac{\omega_r}{\Omega_{ce}} = 5, 10, 15, \) and 20.

It can be seen that for \( \frac{\omega_r}{\Omega_{ce}} = 5 \), the fluid dispersion relation deviates away from the kinetic one at frequencies higher than \( 5.7\Omega_{ce} \) while for larger values (\( \geq 10 \)) the fluid dispersion relation represents a good approximation to the kinetic one, at least for the normal dispersion parts within the hybrid band and one band higher (i.e., the band of \( s\Omega_{ce} \leq \omega_r \leq (s + 1)\Omega_{ce} \), see also Zlotnik et al. 2003). We therefore use the fluid dispersion relation of the Z-mode and limit our discussion to the regime of \( 10 \leq \frac{\omega_r}{\Omega_{ce}} \leq 30 \). As seen from our results, all the obtained values of \( \omega_r \) are within these two bands, justifying the usage of the fluid dispersion relation.

The general resonance condition (Equation (4)) can be simplified under weak relativistic approximation as

\[
u_e^2/c^2 + (u_i/c - u_0/c)^2 = r^2, \quad r^2 = N^2 \cos^2 \theta + 2\left(n\Omega_{ce}/\omega - 1\right), \tag{5}\]

where \( u_0/c = N \cos \theta \) and \( N = kc/\omega_r \). Thus, the resonance curve in the \( \nu_e \) space is a circle with the radius given by \( r \) and the location of the center given by \( u_0 \). The growth rate is determined by both the details of the distribution function and the resonance curve. The instability grows when the resonance curve passes through regions with large and positive \( \partial f / \partial u_\perp \). Such examples are shown in Figure 1 as black arcs or half circles. On the other hand, if the resonance curve passes through regions of small and/or negative values of \( \partial f / \partial u_\perp \), the growth rate is either small or negative, corresponding to wave damping. Such examples are plotted as yellow and green half circles in Figure 1. In our calculations, the absorption or damping effect of thermal electrons is taken into account.
3. Parameter Study on the Z-mode Instability

In this section, we present the parameter study on Z-mode excitation, focusing on the effect of $T_0$ and $v_e$. We numerically solve the dispersion relation using the fluid approximation of warm plasmas to determine the mode frequency ($\omega_r$, normalized by $\Omega_{ce}$, Equation (6)), and integrating the kinetic dispersion relation to get the growth rate ($\gamma$, normalized by $\Omega_{ce} n_e/n_0$, Equations (9)). As mentioned, we limit our study in the parameter regime of $10 \leq \omega_{pe}/\Omega_{ce} \leq 30$, $0 \leq T_0 \leq 8$ MK, and $0.15 c \leq v_e \leq 0.4 c$, with the weakly relativistic approximation. For each set of parameters, we calculate $\gamma$ within an appropriate range of $(\omega_r, \theta)$. This yields a map of $\gamma$ over $(\omega_r, \theta)$, through which we find the maximum growth rate $\gamma_{\text{max}}$ and the corresponding wave frequencies $\omega_{r,\text{max}}$ and $\theta_{\text{max}}$ for further analysis.

To demonstrate the method, we first present the study with a fixed $\omega_{pe}/\Omega_{ce} (=15)$. This allows us to look into the individual contribution from various harmonics ($n$) and reach some general conclusions regarding the effect of $T_0$ and $v_e$. Then, we examine more details of their effect on the Z-mode instability over the given regime of $\omega_{pe}/\Omega_{ce}$.

3.1. Effect of $T_0$ and $v_e$ on Z-mode Growth with $\omega_{pe}/\Omega_{ce} = 15$

In Figures 3–5, we plot the map of the growth rate at various individual harmonic ($n$) (panels (a)–(e)) as well as their sum (panel (f)), for $\omega_{pe}/\Omega_{ce} = 15$, $v_e = 0.15 c$, and $T_0 = 0, 2, 4$ MK. From these maps, it is easy to tell the maximum growth rate for each harmonic. As expected, for different combinations of parameters there always exists a specific harmonic $n_d$ at which the growth rate dominates over other harmonics, and only the two nearby harmonics with $n = n_d \pm 1$ contribute significantly to the wave growth rate (given by the sum over harmonics). In addition, for all cases considered in this study, the Z-mode always achieves the maximum growth rate at the perpendicular or quasi-perpendicular direction. These features are consistent with earlier studies (e.g., Winglee & Dulk 1986) that used the DGH distribution functions for energetic electrons.

For $T_0 = 0$, i.e., the cold plasma case, we have $n_d = 15$ with $\gamma_{\text{max}}^n = 4.372$, $\omega_{\text{pe}}^n = 15.032$, and $\theta^n = 88.8^\circ$, and similar values are obtained for the summed growth rate, indicating the dominance of this harmonic. For $T_0 = 2$ MK, we have $n_d = 16$ with $\gamma_{\text{max}}^n = 4.303$, $\omega_{\text{pe}}^n = 15.892$, and $\theta^n = 90^\circ$, and the summed growth rate also has similar or identical values. For $T_0 = 4$ MK, we have $n_d = 16$ with $\gamma_{\text{max}}^n = 5.527$, $\omega_{\text{pe}}^n = 15.761$, and $\theta^n = 90^\circ$: again the summed grow rate has similar or identical values. These results indicate that for warm plasmas the maximum of the dominant harmonic can deviate away from the value given by $\omega_{pe}/\Omega_{ce} (=15)$.

This demonstrates one important difference between the cold- and warm- plasma situations. For cold plasmas the frequency is fixed to the upper hybrid frequency $\omega_{\text{UH}}$, while for warm plasmas the Z-mode can grow in a much broader range covering the whole hybrid band and the bands higher (see Figure 2).

As seen from Figure 4, we see that the wave growth rates at $n = 17$ and $n = 15$ have smaller yet comparable maximum growth rates with that at $n_d = 16$. The contributions from these three harmonics have been labeled in the map of the total wave growth rate (Figure 4(f)). From Figure 5, a very similar situation is observed for the three harmonics (15, 16 (nd), and 17), yet the contribution of $n = 15$ cannot be recognized from the summed map (Figure 5(f)). This is simply due to the strong damping effect at $n_d = 16$ canceling the wave growth at $n = 15$ within the corresponding regime of $(\omega_r, \theta)$.

Another interesting observation is that, for both $T_0 = 2$ and 4 MK, the wave growth pattern with $\gamma > 0$ at the dominant harmonic splits into two parts through a strong absorption (or
damping) regime, due to the presence of warm plasmas. In general, the thermal damping extends to a larger parameter space of \((\omega_r, \theta)\) according to Figures 4 and 5.

The above results demonstrate the significance of including the warm plasma effect in the calculation of the Z-mode dispersion relation and their growth rates.

To further understand the resonance condition that decides the instability, we select three points in \((\omega_r, \theta)\) space, including the maximum growth rate at the dominant harmonic \(n_f\), a weaker growth rate located nearby (at \(\theta = 88^\circ\)), and a point in the strong wave damping region (see vertical arrows in panels (c) of Figures 3–5). The resonance curves given by the corresponding parameters have been plotted onto the maps of relevant velocity distribution functions, as shown in Figure 1.

Note that for cold plasmas, the absorption effect is due to the negative gradient of the distribution of energetic electrons \(f_e\), and only two points, corresponding to the maximum and a weaker growth rate have been selected. As seen from Figure 1(a), both curves pass through a significant part of the region with a positive gradient of \(f_e\).

For warm plasmas, the resonance circle at the maximum growth rate is given by the zero of a small value of \(u_0\) \((\approx 0)\), corresponding to a nearly perpendicular propagation \((\theta \approx 90^\circ)\). This makes the curve sample the positive gradient region of \(f_e\) most efficiently and thus leads to the maximum growth rate at the perpendicular propagation. The resonance curve corresponding to the weaker growth rate has a much larger radius and samples a part of the Maxwellian region with a significant negative gradient due to large number of electrons there, which makes the growth rate less efficient. On the other hand, the curve corresponding to strong damping passes through a significant section, including the central part, of the Maxwellian. This makes the growth rate negative and the Z-mode wave cannot grow.

In Figure 6, we show the maps of the summed growth rate for \(v_e = 0.15 c\) (Upper), \(0.2 c\) (Middle), and \(0.3 c\) (Lower) and \(T_0 = 0\) (Left), \(2\) (Middle), \(4\) (Right) MK. For ease of comparison, we reproduce the results for \(v_e = 0.15 c\). The variation trend is very clear from cold to warm plasmas with increasing \(T_0\) and \(v_e\), which can be summarized as: (1) the frequency range of wave growth increases significantly due to the warm plasma effect on the wave dispersion relation, as already mentioned; (2) the appearance of strong wave absorption or damping due to the inclusion of thermal electrons, in particular, the growth region at the dominant harmonic \((n_f)\) splits into two parts by the damping of Maxwellian; (3) the maximum wave growth always appears at perpendicular or quasi-perpendicular directions, as required by the resonance curve to sample the most positive gradient of \(f_e\), as elaborated above; and (4) with increasing \(v_e\), a clear trend with non-negligible contributions from more harmonics and a slight decrease of \(\gamma_{max}\) are observed.

The above studies are based on a few discrete values of \(T_0\) and \(v_e\) as done in most earlier studies. In Figure 7 we show variation profiles of the three parameters of Z-mode instability \((\gamma_{max}, \omega_{max}, \theta_{max})\) with \(T_0\) and \(v_e\). This allows us to explore more details of the parameter dependence.

As seen from the left panels of Figure 7, for \(v_e \leq 0.25 c\) the maximum growth rate manifests as an obvious oscillation with increasing \(T_0\). For instance, for \(v_e = 0.15 c\), \(\gamma_{max}\) reaches its peak of 5.349 at 3 MK; for \(v_e = 0.2 c\), \(\gamma_{max}\) reaches its peak of 2.926 at 4.5 MK. On the other hand, for \(v_e > 0.25 c\), the oscillation pattern is not significant; in other words, \(\gamma_{max}\) is almost independent of \(T_0\). The wave frequencies \(\omega_{max}\) vary within the hybrid band for lower \(T_0\) and may jump into the band higher for larger \(T_0\). The jumping point is different for different \(v_e\). For \(v_e = 0.15 c\), \(\omega_{max}\) jumps from 15.270 to 15.932 around 1.125 MK, and for \(v_e = 0.25 c\), \(\omega_{max}\) jumps from 15.383 to 15.776 around 3.375 MK. Before the jumps, \(\omega_{max}\) increases gradually, while after the jumps \(\omega_{max}\) presents a slow declining trend. This gives the interesting behavior of the stepwise variation of \(\omega_{max}\). The stepwise jumping point happens at different values of \(T_0\) for different \(v_e\). This behavior is mainly due to the increase of the number of dominant harmonic \((n_f)\) by unity, in response to the continuous increase of \(T_0\). Note that the values of \(n_f\) have been written onto the upper panel. A further discussion will be presented in the last section.

![Image](image_url)
For large $T_0$ and low $v_c$, it can be seen that $\omega'_\text{max}$ is in the band higher than the hybrid band. We emphasize that $\omega'_\text{max}$ can appear in both bands, but its exact values depend on the values of $T_0$ and $v_c$. These results are significant to studies on t-IV solar radio bursts with or without zebra patterns because this basically determines the frequencies of emission. It also affects any further studies to infer the magnitude of the magnetic field strength and plasma density on the basis of radio spectral data. More discussion is presented in the last section.

The propagation angle at the maximum growth rate ($\theta_{\text{max}}$) is or very close to $90^\circ$, consistent with earlier studies on Z-mode instability using the DGH distribution of energetic electrons. Here, we show that $\theta_{\text{max}}$ varies in accordance with $\omega'_\text{max}$. This is because the two parameters must vary coherently to meet the resonance condition (Equation (4)).

From the right panels of Figure 7, we see that both $\gamma_{\text{max}}$ and $\omega'_{\text{max}}$ generally decline with increasing $v_c$. This trend becomes not significant when $v_c$ is large enough, say, $v_c > 0.25 c$. In other words, when $v_c$ is large enough, both wave parameters only weakly depend on $v_c$. Again, a stepwise variation of $\omega_{\text{max}}$ exists when $v_c$ increases continuously. This happens at $v_c = 0.189 c$ for $T_0 = 2$ MK, and at $v_c = 0.255 c$ for $T_0 = 4$ MK, also due to the jump of the dominant harmonic number by unity. Note that the number $n_0$ has been written onto the upper panel. The wave grows and propagates mainly along the perpendicular or quasi-perpendicular direction, and the angle $\theta_{\text{max}}$ presents a variation pattern in accordance with that of the $\omega'_\text{max}$.

### 3.2. Effect of $T_0$ and $v_c$ on Z-mode Growth with $10 \leq \omega_{\text{pe}}/\Omega_{\text{ce}} \leq 30$

In this subsection, we investigate the effect of $\omega_{\text{pe}}/\Omega_{\text{ce}}$ on the $\gamma_{\text{max}}$ of the instability, as done in many earlier studies. Varying $\omega_{\text{pe}}/\Omega_{\text{ce}}$ can be understood as varying the magnitude of the background magnetic field and the plasma density. Thus, this allows a preliminary study of wave excitation in inhomogeneous media if assuming the spatial scale of the inhomogeneity is much larger than the wavelength.

The ratio $\omega_{\text{pe}}/\Omega_{\text{ce}}$ varies in a range of [10, 30]. For any specific value of $\omega_{\text{pe}}/\Omega_{\text{ce}}$, we conduct the parameter study like those described above and find the corresponding $\gamma_{\text{max}}$, then plot the profiles of $\gamma_{\text{max}}$ versus $\omega_{\text{pe}}/\Omega_{\text{ce}}$. A major purpose of this subsection is to clarify some inconsistent results given by earlier publications, as stated earlier.

From Figures 8(a) and (b), we plot profiles for different values of $T_0$ while fixing $v_c$ at $0.2 c$. The most prominent feature of the profiles is their oscillations, with quasi-periodic peaks and valleys. The distance between neighboring peaks is about one $\Omega_{\text{ce}}$. This pattern is well known and has been applied to explain the presence of zebra patterns of solar t-IV radio bursts (e.g., Winglee & Dulk 1986; Yasnov & Karlický 2004; Kuznetsov & Tsap 2007; Benáček et al. 2017). Another important feature is the overall increasing trend of $\gamma_{\text{max}}$ with increasing $\omega_{\text{pe}}/\Omega_{\text{ce}}$ and the very weak-yet-discernible decreasing trend with increasing $T_0$. This result is different from that presented by Benáček et al. (2017), who showed that $\gamma_{\text{max}}$ decreases with increasing $\omega_{\text{pe}}/\Omega_{\text{ce}}$. The difference might be due to the specific simplifications used in their model. For example, they have neglected the term containing the parallel gradient of $f_e$ (i.e., $\partial f_e/\partial u_\parallel$) when calculating the growth rate. In addition, the specific peaks shift toward lower values of $\omega_{\text{pe}}/\Omega_{\text{ce}}$ with increasing $T_0$. This is basically consistent with those given by Benáček et al. (2017) (see Figure 6 of this reference).

The ratio between values of $\gamma_{\text{max}}$ at neighboring peaks and bottoms can be used to characterize the flatness of the profile, as plotted in Figure 8(c). It can be seen that the ratio declines with increasing $\omega_{\text{pe}}/\Omega_{\text{ce}}$. For instance, for $\omega_{\text{pe}}/\Omega_{\text{ce}} = 15$, the ratio is about 2.1 at $T_0 = 1$ MK and $\sim 1.9$ at $T_0 = 3$ MK, while for $\omega_{\text{pe}}/\Omega_{\text{ce}} = 25$, the ratio is $\sim 1.6$ at $T_0 = 1$ MK and $\sim 1.3$ at $T_0 = 3$ MK. The flatness variation trend of $\gamma_{\text{max}}$ versus $\omega_{\text{pe}}/\Omega_{\text{ce}}$ is basically consistent with the result of Benáček et al. (2017). For $\omega_{\text{pe}}/\Omega_{\text{ce}} < 15$, the peak-bottom ratio decreases with increasing $T_0$, while the variation of flatness of $\gamma_{\text{max}}$ with $T_0$ is not very regular for larger values of $\omega_{\text{pe}}/\Omega_{\text{ce}} (> 15)$. The latter might be caused by irregular oscillations and steep changes of $\gamma_{\text{max}}$ at the bottom of the profiles.
In Figure 9, we show similar profiles of $\gamma_{\text{max}}$ with $\omega_{pe}/\Omega_{ce}$ for different $v_e$ while fixing $T_0$ to be 2 MK. A very weak overall increasing trend of $\gamma_{\text{max}}$ with increasing $\omega_{pe}/\Omega_{ce}$ can be identified from the upper panel. From the profiles of peak-bottom ratio (Figure 9(b)), it can be seen that the ratio declines with increasing $\omega_{pe}/\Omega_{ce}$ and also with increasing $v_e$. For $v_e \geq 0.3c$ and $\omega_{pe}/\Omega_{ce} \geq 15$, the ratio is around or less than 1.2, indicating that the zebra pattern may not be recognizable under these conditions. The result presented in Figure 9 is basically consistent with those presented by Winglee & Dulk (1986) and Benáček et al. (2017). Here we extend the calculations to a larger parameter regime of $\omega_{pe}/\Omega_{ce}$.

4. Summary and Discussion

Many earlier studies studied the Z-mode instability to understand the origin of zebras of t-IV bursts. Among various parameters, the ratio of $\omega_{pe}$ and $\Omega_{ce}$ (decided by the magnetic field strength and plasma density) plays a major role on the...
maximum of the instability growth rate ($\gamma_{\text{max}}$). In addition to this ratio, the temperature of background plasma ($T_0$) and the energy of energetic electrons ($v_e$) are also important. Earlier studies only considered a few discrete values of the two parameters and revealed inconsistent results. Here we revisit the problem with a more complete parameter study on the effect of these parameters.

For a specific value of $\omega_{pe}/\Omega_{ce}$ ($=15$), it was found that $\gamma_{\text{max}}$ presents a general decreasing trend with increasing $v_e$, and an obvious oscillation with increasing $T_0$ for $v_e = 0.15$–0.25 $c$, while it is almost independent of $T_0$ for larger $v_e$. In addition, with increasing $T_0$ and $v_e$, the frequency at $\gamma_{\text{max}}$ presents stepwise profiles with jumps separated by gradual or weak variations in the

![Figure 7](image1.png)

**Figure 7.** Variation profiles of $\omega_{r\text{max}}$, $\theta_{\text{max}}$, $\gamma_{\text{max}}$ with $T_0$ (left) and $v_e$ (right).

![Figure 8](image2.png)

**Figure 8.** (a) and (b) $\gamma_{\text{max}}$ as a function of $\omega_{pe}/\Omega_{ce}$ with $T_0 = 0$–8 MK and $v_e = 0.2$ $c$. Different colors represent different values of $T_0$. (c) Ratios of maxima and neighboring minima of $\gamma_{\text{max}}$ vs. $\omega_{pe}/\Omega_{ce}$, to indicate the flatness of the profiles shown in the upper panel.

![Figure 9](image3.png)

**Figure 9.** (a) $\gamma_{\text{max}}$ as a function of $\omega_{pe}/\Omega_{ce}$ with $v_e = 0.15$–0.4 $c$ and $T_0 = 2$ MK. Different colors represent different values of $v_e$. (b) The peak—bottom contrast vs. $\omega_{pe}/\Omega_{ce}$, to indicate the flatness of the profiles shown in the upper panel. The black dashed line is at 1.2.
range covering the hybrid band and the band higher. The
propagation angle \( \theta_{\text{max}} \) varies accordingly as constrained by the
resonance condition. The cause of these jumps, as mentioned in
Section 3, is due to the change of the dominant harmonic number
\( (n, \ell) \) by unity. Further explanation is given as follows. To achieve
the maximum growth rate, the resonance curve must sample the
appropriate region of the velocity distribution \( f \), i.e., with a large
positive velocity gradient of \( f \) and enough number of particles
along the curve. This puts strong constraints on \( \omega_r \), \( \kappa \), and \( \theta \), at
which the maximum growth rate is obtained. These parameters,
along with \( n \), decide the resonance curve. In Figure 7, we increase
\( T_0 \) and \( \nu_e \) gradually. The two parameters change the velocity
gradient of \( f \) and thus the resonance condition at \( \gamma_{\text{max}} \). Generally
speaking, a larger \( T_0 \), i.e., a more-expanded Maxwellian
distribution, corresponds to a more-expanded resonance curve at
\( \gamma_{\text{max}} \), while \( f \) with a larger \( \nu_e \), will result in the opposite trend of the
resonance curve at \( \gamma_{\text{max}} \). In addition, the dispersion relation of
Z-mode changes with \( T_0 \) and \( \nu_e \). This further affects the growth rate.

As seen from our results, below certain thresholds, the
maximum rate can still be obtained for fixed harmonic number \( n \)
with gradually changing values of \( \omega_r \), \( \gamma \), and \( \theta \). Yet, above the
thresholds, the maximum growth rate moves to the nearby
harmonic, \( n + 1 \) for increasing \( T_0 \) and \( n - 1 \) for increasing \( \nu_e \),
due to the abovementioned opposite trend of \( T_0 \) and \( \nu_e \) on the
velocity distribution function \( f \). This change of harmonic number
leads to the jumps of various parameters observed in Figure 7.

One explanation of zebras is that each stripe is given by a
peak of growth rate that appears for continuous variation of
\( \omega_{pe}/\Omega_{ce} \). For cold plasmas the peak is reached when
\( \omega_{pe}/\Omega_{ce} = s \), i.e., when the upper hybrid frequency equals
a harmonic of electron cyclotron frequency. This is not correct for
warm plasmas. Taking the warm plasma effect into account, within a range of \( 10 < \omega_{pe}/\Omega_{ce} < 30 \), we studied the influences of
\( T_0 \) and \( \nu_e \) on the variation of the peaks and relevant
maximum–minimum ratios of growth rate. It was found that the
ratios always decline and the locations of peaks shift toward
lower values of \( \omega_{pe}/\Omega_{ce} \), with increasing \( \omega_{pe}/\Omega_{ce} \). The ratios do
not present a simple variation trend with increasing \( T_0 \), and it
generally declines with increasing \( \nu_e \) for \( \nu_e < 0.3 \), c.l. For larger \( \nu_e \), the ratio remains around or less than 1.2. Such small ratio
values ratio may lead to continua without recognizable zebras.

During solar flares, both heating and particle acceleration
take place. This leads to change of the plasma temperature \( T_0 \)
and energy of energetic electrons \( (\nu_e) \). Thus, it is natural to
suggest that the frequency variations of zebra stripes may be
partially due to the ongoing heating and acceleration processes
(e.g., Yasnov & Karlický 2004). For example, according to our
calculations, with continuous plasma heating and particle
acceleration, the wave frequency may change either gradually
or suddenly. And the maximum growth rate also changes
accordingly. This may affect the morphology of the stripes and
their emission intensity. Thus, the calculations presented here
are significant for explanations of zebras and further studies of
coronal parameters, such as the magnetic field strength in the
source region (e.g., Tan et al. 2012). In addition, it should be
highlighted that both magnetic field and plasma density change
rapidly during solar flares, and this may also have an important
effect on the wave growth rate as well as the presence of zebra
stripes and their spectral morphology. Future studies that
explore the origins of various types of zebras should take these
factors into account.

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Appendix

The Growth Rate and Dispersion Tensor of Z-mode
Instability for Warm Plasmas

The wave frequency in a collisionless Vlasov–Maxwell
system is written as \( \omega = \omega_r + i \gamma \), where \( \omega_r \) is determined by
the dispersion relation using the following fluid approximation
of warm plasmas for X (Z) mode

\[
\Re \Lambda(k, \omega_r) = \left( \begin{array}{cc}
-N^2 \cos^2 \theta & N^2 \sin \theta \cos \theta \\
0 & -N^2 \sin \theta \cos \theta \\
N^2 \sin \theta \cos \theta & 0 & -N^2 \sin^2 \theta \\
\end{array} \right) + \epsilon = 0,
\]

\[
\epsilon = I - \frac{\omega_{pe}^2}{\omega_r^2} \frac{\Omega_{ce}}{\Delta_e} \Delta_e = (1 - \frac{\Omega_{ce}^2}{\omega_r^2})(1 - 3N^2v_0^2\cos^2 \theta) - 3N^2v_0^2 \sin^2 \theta,
\]

where \( N = kc/\omega_r \) and \( k \) are the refractive index and the
wavenumber, respectively, and \( \theta \) is the angle of propagation
(i.e., the angle between \( k \) and \( \mathbf{B} \)), and \( v_0 = \sqrt{k_B T_0/m_e} \) is the
thermal velocity of background electrons.

Under the assumption of \( \omega_r \gg |\gamma| \), the growth rate \( \gamma \) is
given by

\[
\gamma = -\frac{\Im \Lambda(k, \omega_r)}{\frac{2}{\Omega_{ce}} \Re \Lambda(k, \omega_r)}.
\]
Im\(\Lambda(k, \omega_r)\) is the imaginary part of the kinetic dispersion relation given by, e.g., Baldwin et al. (1996) as

\[
\text{Im} \Lambda(k, \omega_r) = \frac{2\pi^2 \omega_{pe}^2}{\omega_r^2} \int_{-\infty}^{+\infty} d\mu \int_{0}^{+\infty} d\nu \left\{ \frac{u_{\parallel}}{\gamma_L} \left( \frac{\partial}{\partial \mu} - \frac{\partial}{\partial \nu} \right) + \frac{\partial}{\partial \nu} + \frac{k_{\parallel}}{\gamma_L \omega_r} \left( \frac{\partial}{\partial \mu} - \frac{\partial}{\partial \nu} \right) \right\} \times f(u_{\perp}, u_{\parallel}) \hat{e}_r \hat{e}_\perp + \omega_r \left[ \frac{\partial}{\partial \mu} + \frac{k_{\parallel}}{\gamma_L \omega_r} \left( \frac{\partial}{\partial \mu} - \frac{\partial}{\partial \nu} \right) \right],
\]

\[
\times f(u_{\perp}, u_{\parallel}) \sum_{n=-\infty}^{+\infty} \frac{T_n(b)}{\gamma_L \omega_r - n\Omega_{ee} - k_{\parallel} u_{\parallel}}
\]

(10)

\[
T_n(b) = \begin{cases} 
\frac{n^2 \Omega_{ee}^2 f_n^2(b)}{k_L^2} & -i \frac{n \Omega_{ee}}{k_L} u_{\parallel} J_n(b) J'_n(b) + \frac{n \Omega_{ee}}{k_L} u_{\parallel} J'_n(b) f_n(b) \\
\frac{i n \Omega_{ee}}{k_L} u_{\parallel} J_n(b) J'_n(b) + \frac{u_{\parallel}^2}{k_L} J^2_n(b) & i u_{\parallel} u_{\perp} J_n(b) J'_n(b) + \frac{u_{\parallel}^2}{k_L} J^2_n(b) \\
\frac{n \Omega_{ee}}{k_L} u_{\perp} J_n^2(b) & -i u_{\parallel} u_{\perp} J_n(b) J'_n(b) + \frac{u_{\parallel}^2}{k_L} J^2_n(b) 
\end{cases}
\]

(11)

where \(u_{\parallel} = p_{\parallel}/m_e = \gamma_L v_{\parallel}\), \(u_{\perp} = p_{\perp}/m_e = \gamma_L v_{\perp}\), \(\omega_{pe} = \sqrt{n_e e^2/m_e c_0}\) is the plasma frequency, \(\gamma_L = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\) is the Lorentz factor, \(f(u_{\perp}, u_{\parallel})\) is the total distribution function of electrons, and \(J_n(b)\) is the first-kind Bessel function of the \(n\)th order, where \(J'_n(b)\) is its partial derivative with respect to \(b\) (\(= k_{\parallel} u_{\perp}/\Omega_{ee}\)).

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