Spin dynamics in Cuprates and its relation to superconductivity

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The relevance of magnetism for the mechanism responsible for high-temperature superconductivity remains an open and still interesting issue. The observation by inelastic neutron scattering of strong antiferromagnetic dynamical correlations in superconducting cuprates is discussed in relation to the unusual physical properties of the cuprates as well as in relation to the superconducting pairing.

1 Why neutron scattering in High-$T_c$ Cuprates?

Although conventional electron-phonon interaction alone within BCS theory can be dismissed, fourteen years after its discovery, the mechanism for high-temperature superconductivity in copper oxides is still debated. Among more exotic approach for superconductivity, spin fluctuations remain a serious candidate. Indeed, the systematic existence of strong antiferromagnetic (AF) dynamical correlations in the metallic and superconducting phases of all cuprates supports by principle that proposal. The observation of magnetic fluctuations around the AF wavevector $Q_{AF} = (\pi, \pi)$ was first emphasized by copper Nuclear Magnetic Resonance (NMR) and then widely reported by inelastic neutron scattering (INS). INS is actually playing an essential role on this matter as it is the only technique which directly measures the imaginary part of the spin susceptibility, $Im\chi(Q, \hbar\omega)$, over a wide energy range ($\hbar\omega \sim 1$ to 200 meV) and for any momentum transfer within the Brillouin zone. [In contrast, spin lattice relaxation rate in NMR experiments only probes a sum of $Im\chi(Q, \hbar\omega)$ in momentum space weighting by atomic hyperfine tensor and at frequencies $\omega \to 0$.] In principle, the full determination of the spin susceptibility by neutron experiments would ultimately answer whether the mechanism for high-temperature superconductivity is due to AF fluctuations or not.

The powerfulness of inelastic neutron scattering is unfortunately limited by the need of large single crystals (of cm$^3$ size) usually difficult to grow in complex systems such as high-$T_c$ cuprates. For that reason, only two cuprates families have been extensively studied by INS so far: La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) and YBa$_2$Cu$_3$O$_{6+x}$ (YBCO). Further, although they have common features (energy scale, absolute units), the spin susceptibility observed in the two systems also exhibits clear differences: namely the low energy spin fluctuations are peaked in LSCO at wavevectors $Q_{\delta} = (\pi(1 \pm \delta), \pi) \equiv (\pi, \pi(1 \pm \delta))$, incommensurate from the AF momentum. In YBCO, the spin
fluctuations are broader in momentum space but basically commensurate at $Q_{\text{AF}}$ (see section 4). More importantly, INS experiments in YBCO now reported for ten years (first observation in 1991) a sharp magnetic resonance peak at $Q_{\text{AF}}$ only in the superconducting (SC) state, which likely results from $d$-wave symmetry of the superconducting order parameter. Despite many efforts, this remarkable feature is absent in the spin spectrum of LSCO. Only recently, a third high-$T_c$ system Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCO) has been investigated and, up to now, only in the superconducting state. Results in BSCO are fully consistent with YBCO for equivalent doping level.

In conventional superconductors, electron-phonon interaction was most directly evidenced by tunneling experiments which was reproducing the phonon density of states (DOS). In copper oxides, a similar evidence of the mechanism responsible for the high-$T_c$ superconductivity is still missing. However, it has been recently argued that single particle spectrum (at finite wavevector measured by Angle Resolved Photoemission (ARPES) or integrated in momentum space in tunneling experiments) exhibits features in the superconducting state which map the resonance peak in the spin excitation spectrum. Independently, a similar connection has been inferred from infrared measurements showing that conducting carriers are strongly coupled to this magnetic mode. This is actually putting the resonance peak seen in INS experiments as a clearcut manifestation of a superconductivity mechanism based on magnetic fluctuations for the high-$T_c$ in these materials.

In the present lecture, the INS experimental situation will not be presented in details: this has been already done in LSCO and in YBCO (a critical examination of the INS results in YBCO and BSCO has been recently given in Ref.). The objective is here rather to show how the magnetic INS measurements can give insight about the role of AF fluctuations for the high-$T_c$ mechanism. The energy dependence of the spin susceptibility will be discussed in both the normal and the SC states in section 3, and the momentum dependence in section 4. Before tackling these aspects, let first emphasize the anomalous phase diagram empirically inferred from many different measurements.

### 2 The High-$T_c$ Cuprates phase diagram

The high-$T_c$ cuprates are layered systems, systematically containing one or more CuO$_2$ planes, known to be responsible for the high-$T_c$ superconductivity. The physical properties of all high-$T_c$ Cuprates are strikingly controlled by the number of doped holes, $n_h$, within the CuO$_2$ planes. Part of the difficulty to describe the physical properties is related to that bidimensional
nature of cuprates. A generic phase diagram is shown in Fig. 1 versus doping level. The undoped system is an insulator with a long range ordered Néel state whose propagation wavevector is $Q_{AF}$. This AFM state disappears with small amount of doped holes, $n_h \sim 2-3\%$, and superconductivity occurs upon increasing further $n_h$. The SC transition temperature is going through a maximum for some doping, so-called optimal doping, separating an underdoped regime from an overdoped regime which exhibit quite different physical properties.

Superconductivity in cuprates is unconventional with a $k$-dependent SC order parameter of $d_{x^2-y^2}$-wave symmetry, $\Delta_k = \Delta_{\text{max}}/2 (\cos k_x - \cos k_y)$ (where $k_x$ and $k_y$ are in-plane wavevectors, $\Delta_{\text{max}}$ is the maximum of the SC $d$-wave gap). This has been very well documented by crossing ARPES experiments and Josephson effect experiments. A ratio $2\Delta_{\text{max}}/kT_c$ is found $\sim 7-9$ at optimal doping (see also electronic Raman scattering) far from the expected BCS value of $\sim 3.5$. Further, the single particle spectrum shows...
a doping dependent gap which increases for lower doping. However, other measurements of superconducting properties (Andreev reflection, penetration depth,...) do not follow that trend of the single particle gap and rather indicate a second lower gap, $\Delta_c$, which actually follows $T_c$ as $2\Delta_c/kT_c \sim 5-6$ for any doping. This second feature has been interpreted as a coherence gap, but, alternatively, it can also arise from different portions of the Fermi surface than the single particle gap.

In principle, an insulating-metal transition should occur at low doping before superconductivity sets up. However, unusual transport properties for a metal are observed in most of the phase diagram indicating a non-Fermi liquid behavior, for instance characterized by a linear thermal dependence of the resistivity. This has been subsequently confirmed by the absence above $T_c$ of well-defined quasiparticles in ARPES spectroscopy, questioning about the existence of a Fermi surface. The strangest part of this phase diagram is certainly the pseudogap state. Indeed, transport measurements as well as thermodynamics display anomalies (depression) below some temperature, referred as $T_{pg}$ in Fig. much higher than $T_c$. Further, various charge spectroscopies - ARPES experiments, optical conductivity, Raman scattering or tunelling experiments - have reported below $T_{pg}$ (but still above $T_c$) a loss of low energies electronic states suggesting an opening of a pseudogap. Magnetic properties also affected below $T_{pg}$. It is indeed known since 1989 that the uniform spin susceptibility as measured by NMR Knight shift does not behave as a Pauli susceptibility but systematically exhibits a pronounced decrease down to lower temperature. Copper NMR as well as INS experiments have also evidenced a depression of low energy fluctuations at $(\pi, \pi)$ already in the normal state, characteristic of a spin pseudogap behavior. Finally, it should be noticed that all anomalies in the physical properties occur, for a fixed doping level, over a certain temperature range rather than at a well-defined temperature. This is either related to the fact that each physical property might be differently sensitive to the growing of the pseudo gap, or that the chosen definition of the anomalous temperature for each experimental technique is quite ambiguous.

This pseudo-gap phase shows the failure of conventional Fermi liquid to describe the high-$T_c$ cuprates. Further, the debate has been enriched by the observation of static spin and charge ordering in non superconducting cuprates $\La_{2-x-y}\Nd_x\Sr_y\CuO_4$. This has interpreted as an evidence of inhomogeneous distribution of charge carriers within the CuO$_2$ plane: the charge carriers would form lines, yielding the so-called stripes phase, separating antiferromagnetic regions free of charges. It has been then speculated that dynamical stripes occur in SC cuprates, so that the cuprates ground-state is a doped insulator with-
out formation of a Fermi surface. In such a case, the pseudo-gap phase is understood as a formation of preformed superconducting pairs without long range coherence. In approaches which still consider the Fermi surface, the pseudo-gap line is described either as simply a crossover line of physical properties, or as the trace of a quantum critical point (QCP) at optimal doping, with even a broken symmetry of an hypothetical hidden order parameter. Actually, the scaling of several physical properties at a common critical doping suggests the occurrence of a QCP in the lightly overdoped regime. The description of the physics behind the pseudo gap phase then would necessarily be important for the understanding of the high-$T_c$ superconductivity. In any case, the unusual phase diagram of Fig. 1 suggests an unconventional SC pairing.

3 Spin susceptibility

All INS experiments in the high-$T_c$ cuprates have unambiguously established the existence of strong antiferromagnetic correlations in both the normal and the superconducting states. This drastically differs from what we would expect in any conventional superconductor in both the normal state and the superconducting states. Typically, the spin susceptibility in standard paramagnetic metals (like Al, Cu,...) would be very weakly peaked in momentum space and extends up to high energy (limited by the electronic bandwidth, $t \sim 0.5$ eV), as a result of weak electronic correlations. The electronic spin fluctuations are then too weak to be detected in INS experiments. The amplitude of the magnetic fluctuations of several $100 \mu^2_B/eV$ observed in the metallic state of cuprates (at the AF wave vector, $(\pi, \pi)$, in YBCO or near $(\pi, \pi)$ in LSCO) is actually much larger than what expected in non-interacting metals of the order of $1/t \sim 2\mu^2_B/eV$. This, by itself, suggests the importance of antiferromagnetism in the microscopic description of the high-$T_c$ superconductors. The complete energy, wave vector and temperature dependences of the imaginary part of the spin susceptibility, $Im\chi(Q,\hbar\omega)$, have been reported in YBCO for several dopings. Let us first describe its behavior in the normal state.

3.1 Normal state

In a superconducting mechanism based on magnetism, the effective interaction would be directly proportional to the spin susceptibility measured in the normal state. The determination of the neutron spectrum above $T_c$ is then of
great importance. The spin susceptibility at $T=100$ K is reported in Fig. 2 at the AF wavevector for 4 different oxygen contents in YBCO corresponding to 4 doping levels in Fig. 1: $x=0.92$ is assumed to be slightly below the optimal doping, $x=0.5$ and $x=0.83$ are on the underdoped side and $x=0.97$ is slightly on the overdoped side (see refs. 10, 11 for details). Apart from $x=0.97$, $\text{Im}\chi(Q_{AF}, \hbar\omega)$ systematically exhibits a maximum around a characteristic energy, $\sim 25-30$ meV. Actually, such energy dependences remind that of paramagnons in metallic systems where strong electronic interactions enhance, at low energy, the bare spin susceptibility.

Interestingly, a drastic decreasing of the AF susceptibility amplitude is found as a function of doping. This effect is particularly pronounced near optimal doping where $\text{Im}\chi(Q_{AF}, \hbar\omega)$ is at least 4 times weaker for $x=0.97$ than $x=0.92$ corresponding to a small change of the doping level and for very similar $T_c$. This result is quite surprising as compared with the copper spin-lattice relaxation rate of NMR measurements, $T_1$, which remains roughly constant from YBCO$_{6.92}$ to YBCO$_7$ at $T=100$ K as, in principle, both probes measure spin fluctuations peaked around $(\pi, \pi)$. This might indicate either that NMR is sensitive to another source of magnetism too broad to be detectable in INS experiments or that the spin susceptibility has different momentum or energy dependences in the overdoped regime. [For instance, if the maximum of $\text{Im}\chi(Q_{AF}, \hbar\omega)$ is shifted down to $\sim 20$ meV, magnetic scattering can be partially occult by the large nuclear background in the INS spectra.] At present, it is premature to resolve this alternative: more accurate INS measurements in YBCO$_7$ are necessary to remove that difficulty.

However, to underline the observed doping dependence of Fig. 2, a partial energy integration of the spin susceptibility at $(\pi, \pi)$, $\int_0^{30\text{ meV}} \text{Im}\chi(Q_{AF}, \hbar\omega)d\omega$, is shown in Fig. 3 as a function of the oxygen content. It should be stressed that Fig. 3 does not represent any sum-rule, but just display the doping dependence of the spectral weight of the measured AF dynamical correlations within an arbitrary energy range, which is nevertheless the most interesting spectral region. At first glance, Fig. 3 suggests that AF fluctuations cannot be important for the high-$T_c$ mechanism as the magnetic fluctuations seem to vanish for samples where $T_c$ is almost as large as the maximum $T_c$. But actually, assuming the same momentum dependence for YBCO$_{6.97}$ as that for YBCO$_{6.92}$, the upper limit of the spin susceptibility reported in Fig. 2 for YBCO$_{6.97}$ is $\sim 80 \mu_B^2/eV$ at $(\pi, \pi)$ i.e. still $\sim 20$ times larger than the uniform susceptibility measured by macroscopic susceptibility or deduced from NMR knight shift. Therefore, Fig. 3 does not contradict the proposal that electronic interactions are responsible for the high-$T_c$ superconductivity. AF fluctuations can be still large enough to give rise to a sizeable $T_c$. Further, it is
instructive to mention that the doping dependence of AF fluctuations spectral weight reminds that of $T_{p9}$ in Fig. 1. The existence of strong AF fluctuations seems then directly related to the opening of the pseudo-gap, as for instance it has been suggested within QCP scenario.

3.2 Superconducting state

The spin dynamics in cuprates exhibit a sharp resonance peak in the SC state at some well-defined energy, which $\text{disappears}$ in the normal state for all doping. So far, the resonance feature has been found for only two systems, YBCO and BSCO, and not in LSCO. As the two former families contain bilayers of CuO$_2$ planes in each unit cell, it is not fully established that this phenomenon can be generalized to all cuprates. However, the absence of the resonance peak in the single layer LSCO system can be caused by few reasons, (i) too small $T_c$ ($T_c^{\text{max}} = 38$ K in LSCO) (ii) too much disorder, so, its non-observation in LSCO does not necessarily mean that the resonance peak phenomenon will not exist in other single CuO$_2$ layer cuprates with higher superconducting temperature.

In both underdoped and overdoped regimes, the resonance energy, $E_r$, is basically proportional to $T_c$ as $E_r/k_B T_c \sim 5-5.5$. Therefore, the resonance energy surprisingly does not follow the doping dependence of the single particle gap, but actually rather matches the doping dependence of the second gap behavior $\Delta_c$. This is certainly an important issue which needs further investigations. Similarly to the normal state, the overall spectral weight in the SC state decreases with increasing doping (Fig. 3). However, the absolute spectral weight related to the resonance peak itself remains approximately constant over the same doping range. The resonance peak is then a better defined feature for samples with high $T_c$ where the normal state peak becomes weaker.

The occurrence of the resonance peak has motivated a large theoretical activity (see e.g. and other references in these papers). The simplest approach is to consider the spin response in a BCS superconductor within a Fermi-liquid approach. In such a case, the resonance peak $\text{primarily}$ results from electron-hole pair production across the SC energy gap. In that sense, the resonance peak occurs due to coherence effects in a $d$-wave superconductor in a similar way as, in conventional superconductors with isotropic $s$-wave gap, the Hebel-Slichter peak is observed in NMR experiments. In such a case, the resonance peak is described as an excitation which creates two quasi-particles at the Fermi level whose momentums differ by exactly $(\pi, \pi)$. This requires a $d_{x^2-y^2}$-wave symmetry of the SC gap function as $\Delta_k$ should change sign from
any $k$ wavevectors to $k + Q_{AF}$. The spectrum of this excitation exhibits a gap, corresponding to the threshold of the electron-hole continuum, $\omega_c$, defined by the minimum of $\sum_k (E_k + E_{k+Q_{AF}})$ (where $E_k = \sqrt{\epsilon_k^2 + \Delta_k^2}$ is the quasiparticle energy in the SC state, $\epsilon_k$ is the electronic dispersion in the normal state). This threshold can actually be experimentally determined from ARPES measurements. For instance, let us consider the case of BSCO at optimal doping: the SC gap is measured to be $\Delta_{max} = 35$ meV. A close inspection of the Fermi surface shows that the quasi-particles which are connected by the $(\pi, \pi)$ momentum have the energy $E_k \approx 0.9\Delta_{max}$ at the Fermi level. The electron-hole continuum is then, $\omega_c \approx 1.8\Delta_{max} \approx 63$ meV. The resonance energy for that composition has been reported at $E_r = 43$ meV, clearly lower than $\omega_c$. The above scenario of non-interacting spin susceptibility, $\chi_0$, is then not enough to account for the observed sharp peak. A ratio $E_r \approx 1.2\Delta_{max}$ is found indicating that the resonance peak occurs well below the electron-hole continuum. Within a Fermi-liquid-like approach, this experimentally shows that the resonance peak is a true collective mode of $d$-wave superconductivity, corresponding to the strong coupling limit. Interactions, like $J(Q)$, in a RPA scheme actually produces this collective excitation under the condition that, $1 - J(Q_{AF})Re\chi_0 = 0$. A momentum dispersion of this collective mode has been also theoretically predicted in agreement with recent INS reports.

3.3 Spin dynamics and single particle excitation

As mentioned in the introduction, the unusual spectral lineshape, known as the peak-dip-hump structure, of the quasi-particles measured by ARPES in the superconducting state of BSCO has been interpreted as a result of a coupling with collective excitations centered at the AF momentum and more specifically with the magnetic resonance peak. Namely, quantitative correspondence of the resonance energy with the energy separated the peak and the dip have been proposed through electron-electron contribution to the electronic self energy. This proposal, discussed in details in this book by J. Mesot, seems to properly agree with the neutron data.

In many aspects, this proposal is similar to the recent claim of superconductivity mediated by spin fluctuations in the heavy-fermion compound, UPd$_2$Al$_3$. This system is remarkable as it simultaneously exhibits an AF Néel state below $T_N = 14.3K$ and a superconducting state below $T_c \approx 2K$ with also an unconventional symmetry of the SC gap. Tunneling spectroscopy in the SC state displays, above the SC gap, oscillations at energies comparable to the spin-wave energy $\sim 1.4$ meV, which has been directly measured by INS at the propagation wavevector of the AF structure, $Q_0 = (0, 0, \frac{1}{2})$. 
The similarity with tunneling data showing the phonon DOS in conventional superconductors is striking and suggests a magnetic origin of the superconductivity in that system. Further, a new magnetic excitation appears in the neutron spectrum only in the SC state at an energy, \( \hbar \omega \approx 0.36 \text{ meV} \), and at \( Q_0 \) \text{57, 58}. This peak has also other common features with the resonance peak in the cuprates as it occurs at an energy lower than twice the SC gap at the wavevector characteristic of the magnetic correlations. Therefore, the similarity with cuprates is quite striking, and the main difference with the proposal made in the high-\( T_c \) cuprates is that, in UPd\(_2\)Al\(_3\), the tunneling spectrum shows the pre-existing normal state spin excitation spectrum (not the additional "resonance" peak of the SC state).

The next question is then: in the cuprates, what might be the link between the resonance peak below \( T_c \) and the normal state maximum of the spin susceptibility? For instance, it has been proposed that the normal state peak observed in neutron scattering is a precursor peak of the resonance peak \text{59}. However, the normal state maximum has not the same doping dependence as the resonance peak \text{10}. Especially, spin susceptibilities in underdoped YBCO\(_{6.83}\) and in optimally doped YBCO\(_{6.92}\) exhibit a maximum at \( \hbar \omega \sim 30 \text{ meV} \) in the normal state (see Fig. 2) whereas they are characterized by two distinct resonance energies of 35 and 41 meV, respectively \text{11}. Independent physical process are likely needed to account for the two characteristic energies (a possible scenario is given in \text{60}). Nevertheless, both features are also necessarily inter-related as, for instance, the resonance intensity is formed from the broad normal state feature. Further, they systematically occur in a very similar energy range which makes a clear difference with the heavy-fermion compound, UPd\(_2\)Al\(_3\), where in the SC state the two excitations - spin-waves and the additional "resonance" peak - occur in two distinct energies ranges. Therefore, the fact that, in the cuprates, the resonance peak in the SC state replaces the broad normal state peak - affecting both the energy \text{12, 13} and the momentum \text{14, 15} line-shapes of the spin susceptibility - might explain why the ARPES anomalous lineshape is sensitive to the resonance energy. Actually, the energy between the peak and the dip is also within errors fully consistent with the normal state maximum of the spin susceptibility. Further, to complete the comparison, it should be stressed that the magnetic fluctuations in UPd\(_2\)Al\(_3\) are typically thought to arise from two different subsystems characterized, above \( T_c \) by two coupled modes \text{12} whereas, in cuprates, a single broad response is observed (see Fig. 2).
4 Momentum dependence

The momentum dependence the peak in the spin susceptibility in the normal state exhibits also a very interesting feature. Indeed, in the underdoped regime of both LSCO and YBCO, the superconducting $T_c$ is linearly proportional to the typical momentum extension of the AF fluctuations.

In the case of LSCO, the low energy spin fluctuations are peaked at some wavevectors displaced by a doping dependent amount, $\delta$. $\delta$ is temperature independent and does not depend on energy, at least up to $\sim 20$-25 meV (the question whether the fluctuations remain incommensurate up to higher energy or becomes commensurate is actually controversial). This discommensuration is then characteristic of only the doping level. However, both features are not simply proportional each other as $\delta$ saturates at high doping, rather following the doping behavior of $T_c$.

In YBCO, the spin susceptibility in the normal state is basically commensurate. [Incommensurability in YBCO have been also reported but they predominantly occur in the SC state, and actually, belong to the same excitation as the resonance peak. The different temperature, energy and doping dependences of the discommensurations in both YBCO and LSCO basically call for a different interpretation.] Interestingly, the typical momentum extension of commensurate AF fluctuations in YBCO, the half width at half maximum of the peak in the spin susceptibility, $\Delta_q$, behaves very similarly to the discommensuration parameter in LSCO. $\Delta_q$ is also found temperature independent and very weakly energy dependent. As $\delta$ in LSCO, $\Delta_q$ is then related to the doping level in YBCO and a linear relationship, $T_c = \hbar v^* \Delta_q$ for a large number of INS experiments is also found (Fig. 4), similar to the linear relation between incommensurate peak splitting in LSCO.

The exact meaning of this relation remains unclear and, so far, has been discussed within the stripes picture as an evidence of charged stripes formation in all cuprates. Other interpretations have to be considered as, on general grounds, this linear relation indicates that $T_c$ in the underdoped cuprates is controlled by the momentum extension of the AF fluctuations in the normal state. Superconductivity is indeed limited by the strength of the magnetic correlations, their amplitude at $(\pi, \pi)$ as shown by Fig. 3, but also by their location in momentum space: broader AF correlations, better $T_c$. Actually, a relation between $T_c$ and the inverse magnetic correlation length, $\xi^{-1}$ (which basically corresponds to $\Delta_q$), has been recently discussed in a spin-fermion model where superconductivity is magnetically induced. More works are certainly needed in that direction.
5 Conclusion

Finally, the occurrence of strong and doping dependent antiferromagnetic fluctuations as well as their close link with the unusual physical properties of high-$T_c$ superconductors and with the SC temperature itself naturally militates in favor of an electron-electron origin for the SC pairing. Further, the magnetic resonance peak is proposed to be responsible for the anomalous shape of the microscopic electronic properties (single particle spectrum and optical conductivity). If this idea is correct, it gives a clearcut signature of the high-$T_c$ pairing based on antiferromagnetism (by analogy with the tunneling experiments in conventional superconductors which were reproducing the phonon DOS).

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References

1. J. Bardeen, L.N. Cooper and J.R. Schrieffer, Phys. Rev. 108, 1175 (1957).
2. J.G. Bednorz, and K. A. Müller, Z. Phys B 64, 189 (1986).
3. D. Pines, in The gap Symmetry and Fluctuations in High Temperature Superconductors, ed. by J. Bok et al., (Plenum Press, New York, 1998), p. 111.
4. See e.g. D. Scalapino, Phys. Rep. 250, 329 (1995).
5. See e.g. C. Berthier, M.H. Julien, M. Horvatic and Y. Berthier, Journal de Physique I (France) 6, 2205 (1997).
6. M.A. Kastner et al., Rev. Mod. Phys. 70, 897 (1998).
7. G. Aeppli et al., Science 278, 1432 (1997).
8. J. Rossat-Mignod et al., Physica C 185-189, 86 (1991).
9. H. F. Fong et al., Phys. Rev. Lett. 75, 316 (1995).
10. P. Bourges, in p. 349 (cond-mat/9901333).
11. L.P. Regnault et al., in Neutron Scattering in Layered Copper-Oxide Superconductors, ed. by A. Furrer, (Kluwer, Amsterdam, 1998), p. 85.
12. H.A. Mook et al., Nature 395, 580 (1998).
13. P. Dai et al., Science 284, 1344 (1999).
14. H.F. Fong et al., Phys. Rev B. 61, 14773 (2000) (cond-mat/9910041).
15. P. Bourges et al., Science 288, 1234 (2000).
16. P. Bourges, B. Keimer, L.P. Regnault and Y. Sidis, J. of Superconductivity 13, 735 (2000) (cond-mat/0006084).
17. H.F. Fong et al., Nature 398, 588 (1999).
18. H.F. He et al., Phys. Rev. Lett. (submitted for publication) (cond-mat/0002013).
19. J.R. Schrieffer, Theory of Superconductivity, Frontiers in Physics Vol. 20, (Addison Wesley, Reading MA, 1988).
20. J.C. Campuzano et al., Phys. Rev. Lett. 83, 3709 (1999).
21. A. Abanov and A.V. Chubukov, Phys. Rev. Lett. 83, 1652 (1999).
22. J.P. Carbotte et al., Nature 401, 304 (1999).
23. D. Munzar et al., Physica C 312, 121 (1999).
24. B. Batlogg and C.M. Varma, Physics World, Feb. 2000, p 33.
25. J. Orenstein and A.J. Millis, Science 288, 468 (2000).
26. Z.X. Shen et al., Phys. Rev. Lett. 70, 1553 (1993).
27. C.C. Tsuei et al., Phys. Rev. Lett. 73, 593 (1994).
28. R. Hackl, in [ ], p. 249.
29. J. Mesot et al., Phys. Rev. Lett. 83, 840 (1999).
30. G. Deutscher, Nature 397, 410 (1999).
31. T. Ito et al., Phys. Rev. Lett. 70, 3995 (1993).
32. V.V. Moschalkov et al., in [ ], p. 91.
33. M.R. Norman et al., Nature 392, 157 (1998).
34. J.W. Loram et al., Physica C 282-287, 1405 (1997).
35. H. Ding et al., Nature 382, 51 (1996).
36. C.C. Hones et al., Phys. Rev. Lett. 71, 1645 (1993).
37. R. Nemetschek et al., Phys. Rev. Lett. 78, 4837 (1997).
38. C. Renner et al., Phys. Rev. Lett. 80, 149 (1998).
39. H. Alloul et al., Phys. Rev. Lett. 63, 1700 (1989).
40. J. Rossat-Mignod et al., Physica B 169, 58 (1991).
41. J.M. Tranquada et al., Nature 375, 561 (1995).
42. J.M. Tranquada, in [ ], p. 225.
43. J. Zaanen, Science 286, 251 (1999).
44. V.J. Emery et al., Phys. Rev B. 56, 7120 (1997).
45. N. Furukawa, T.M. Rice and M. Salmhofer, Phys. Rev. Lett. 81, 3195 (1998). See also, C. Honerkamp et al., preprint, (cond-mat/9912358).
46. S. Chakravarty, R.B. Laughlin, D.K. Morr and C. Nayak, preprint, (cond-mat/0005443).
47. J.L. Tallon et al., Phys. Stat. Sol. 215, 531 (1999) (cond-mat/9911157). preprint, (cond-mat/0005063).
48. R.M. White, Quantum Theory of Magnetism, Springer series to Solid-State Science 32, (Springer Verlag, Berlin, 1983).
49. C.M. Varma, Phys. Rev. Lett. 77, 3431 (1996).
50. A.J. Millis, H. Monien and D. Pines, Phys. Rev. B 42, 67 (1990).
51. H. Ding et al., Phys. Rev. Lett. 78, 2628 (1997).
52. See e.g. A.J. Millis and H. Monien, Phys. Rev. B 54, 16172 (1996).
53. F. Onufrieva and P. Pfeuty, preprint (cond-mat/9903097).
54. Z.X. Shen and J.R. Schrieffer, Phys. Rev. Lett. 78, 1771 (1997).
55. M.R. Norman and H. Ding, Phys. Rev. B 57, R11089 (1998).
56. M. Jourdan et al., Nature 398, 47 (1999).
57. N. Metoki et al., Phys. Rev. Lett., 80, 5417 (1998).
58. N. Bernhoeft et al., Phys. Rev. Lett. 81, 4244 (1998).
59. D.J. Morr and D. Pines, Phys. Rev. Lett. 81, 1086 (1998).
60. D. Manske, I. Eremin and K.H. Bennemann, preprint, (cond-mat/0007083).
61. K. Yamada et al., Phys. Rev. B 57, 6165, (1998).
62. A.V. Balatsky and P. Bourges, Phys. Rev. Lett. 82, 5337 (1999).
63. A. Abanov, A.V. Chubukov and J. Schmalian, preprint, (cond-mat/0005163).
Figure 2: Normalized imaginary part of the spin susceptibility at the AF wavevector in the normal state, at $T = 100$ K, for four oxygen contents in YBCO ($T_c = 45, 85, 91, 92.5$ K for $x = 0.5, 0.83, 0.92, 0.97$ respectively). These curves have been normalized to the same units using standard phonon calibration. (100 counts in the vertical scale roughly correspond to $\sim 350 \mu_B^2/\text{eV i absolute units}$) (from [10]).
Figure 3: Doping dependence of $\int_0^{50\text{meV}} \text{Im} \chi(Q_{AF}, \hbar \omega) d\omega$ in both the normal state ($T=100$ K from the energy dependences of Fig. 2) as well as in the SC state (the energy dependences of $\text{Im} \chi(Q_{AF}, \hbar \omega)$ at $T=5$ K are from [1]). Lines are only guides to the eye.
Figure 4: Superconducting transition versus the half width at half maximum of the peak in the spin susceptibility, $\Delta_q$ (from [4]).