Two-Dimensional Dilaton Gravity in a Unitary Gauge

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ABSTRACT

Reduced phase space formulation of CGHS model of 2d dilaton gravity is studied in an extrinsic time gauge. The corresponding Hamiltonian can be promoted into a Hermitian operator acting in the physical Hilbert space, implying a unitary evolution for the system. Consequences for the black hole physics are discussed. In particular, this manifestly unitary theory rules out the Hawking scenario for the endpoint of the black hole evaporation process.
1. Introduction

In a pioneering work [1], Callan, Giddings, Harvey and Strominger have proposed a theory of two-dimensional dilaton gravity coupled to matter as a toy model for studying the formation, evaporation and back-reaction of black holes. The attractive features of the model are that it is classically exactly solvable, it possesses black hole solutions and it is a renormalizable field theory. The last feature raises a possibility that the corresponding quantum theory may be tractable, and hence allow for the investigation of the elusive issues associated with the endpoint of the black hole evaporation [2]. As shown by a series of authors [3], the solutions of the one-loop matter corrected equations of motion are not free of singularities, in contrast to the initial expectation by CGHS. Hawking has even argued [4] that the solutions of any semi-classical approximation scheme will be singular, suggesting that the possible stabilization of the black hole by the quantum effects could be achieved only if the gravitational field is quantised together with the matter fields.

Non-perturbative quantisation of the gravitational field in four spacetime dimensions is still an unsolved problem. However, in 2d, significant simplifications occur, most notably the number of physical degrees of freedom of the gravitational field is finite. In addition, the CGHS model is a renormalizable field theory. However, the non-perturbative analysis is still a complicated problem. Instead of using the path-integral techniques, one could try using the canonical quantization methods, which were developed in the context of 4d quantum gravity (for a review and references see [6]). In [7] the canonical analysis of the model has been performed, and the Dirac type quantisation investigated. It was shown that a set of non-canonical variables can be found, forming an $SL(2,\mathbb{R})$ current algebra, such that the constraints become quadratic in the new variables. For a compact spatial manifold (i.e. circle) and piecewise continuous field configurations, Fourier modes can be defined, and the physical Hilbert space can be obtained from a cohomology of a Virasoro algebra. Although exactly solvable, the configuration space of this model does not contain singular solutions which can be associated with black holes. As suggested in [7], a Schrodinger type equation would be more appropriate for quantizing a more general configuration space, which naturally leads one to employ the extrinsic time variable approach [10].

In this paper we discuss the reduced phase space formulation of the CGHS theory in an extrinsic time gauge. Our gauge fixing conditions contain only the canonical variables, in contrast to the usual gauge fixings, where the Lagrange multipliers are involved, like the conformal gauge. Since we are dealing with a reparametrization invariant system, a consistent canonical gauge fixing must contain the definition of a
time variable \([4]\). We construct a time variable \(T(x,t)\), and in the gauge \(T(x,t) = t\) solve the constraints in terms of the independent canonical variables. We obtain an explicit expression for the Hamiltonian of the system in this gauge. That Hamiltonian can be promoted into a Hermitian operator, acting on the physical Fock space, implying a unitary evolution. Hence in this theory there are no anomalies associated with a non-unitary evolution, like transitions from pure into mixed states, a pathology expected at the endpoint of the black hole evaporation process \([2]\). However, it is still difficult to explicitly see what happens during the gravitational collapse in this theory. This problem together with some other caveats is discussed at the end of the paper.

2. Canonical Formulation

The CGHS action \([1]\) is given by

\[
S = -\frac{1}{8} \int_M d^2x \sqrt{-g} \left[ e^{-2\Phi} \left( R + 4(\nabla \Phi)^2 + \lambda^2 \right) + 4 \sum_{i=1}^N (\nabla \phi_i)^2 \right], \tag{2.1}
\]

where \(M\) is a 2d manifold, \(g_{\mu\nu}\) is a metric on \(M\), \(\Phi\) is a scalar field (dilaton), \(\lambda\) is a constant and \(R\) is the 2d curvature scalar. \(\phi_i\) are massless scalar fields, minimally coupled to gravity. We will label the time coordinate \(x^0 = t\) and the space coordinate \(x^1 = x\), while the corresponding derivatives will be denoted as \(\cdot\) and \(\dot{\cdot}\), respectively.

Following the analysis in \([7]\), we perform the field redefinitions \([5]\)

\[
\phi = \frac{1}{4} e^{-2\Phi}, \quad \tilde{g}_{\mu\nu} = 4\phi e^{-\phi} g_{\mu\nu}, \tag{2.2}
\]

so that the action becomes

\[
S = -\frac{1}{2} \int_M d^2x \sqrt{-\tilde{g}} \left( \left( \nabla \phi \right)^2 + \tilde{R}\phi + \frac{1}{4} \lambda^2 e^\phi + \sum_{i=1}^N (\nabla \phi_i)^2 \right). \tag{2.3}
\]

The canonical formulation requires that the 2d manifold \(M\) has a topology of \(\Sigma \times \mathbb{R}\), where \(\Sigma\) is the spatial manifold and \(\mathbb{R}\) is the real line corresponding to the time direction. \(\Sigma\) can be either a circle or a real line. The compact spatial topology is relevant for cosmological solutions and string theory, while the non-compact spatial topology is relevant for 2d black holes.

After introducing the canonical momenta, (2.3) takes the form \([7]\)

\[
S = \int dtdx \left( \dot{p}g + \pi^\phi \dot{\phi} + \pi^i \dot{\phi}_i - \frac{N}{\sqrt{g}} \left( G_0 - nG_1 \right) \right), \tag{2.4}
\]
where we have omitted the tildas, \( g = g_{11}, N \) and \( n \) are the laps and the shift vector and

\[
G_0(x) = -2g^2 p^2 - 2gp\pi + \frac{1}{2} (\phi')^2 + \frac{\lambda^2}{8} ge^{\phi} - \frac{1}{2} \frac{g'}{g} \phi' + \phi'' + \frac{1}{2} \sum_{i=1}^{N} (\pi_i^2 + (\phi_i')^2)
\]

\[
G_1(x) = \pi \phi' - 2p' g - pg' + \sum_{i=1}^{N} \pi_i \phi_i'.
\] (2.5)

The constraints \( G_0 \) and \( G_1 \) form a closed Poisson bracket algebra

\[
\{G_0(x), G_0(y)\} = -\delta'(x-y)(G_1(x) + G_1(y))
\]

\[
\{G_1(x), G_0(y)\} = -\delta'(x-y)(G_0(x) + G_0(y))
\]

\[
\{G_1(x), G_1(y)\} = -\delta'(x-y)(G_1(x) + G_1(y))
\] (2.6)

where the fundamental Poisson brackets are defined as

\[
\{p(x), g(y)\} = \delta(x-y) \quad \{\pi(x), \phi(y)\} = \delta(x-y)
\] (2.7)

\( G_1 \) generates the spatial diffeomorphisms, while \( G_0 \) generates the time translations of \( \Sigma \), in full analogy with the 4d gravity case. Note that the algebra (2.6) is isomorphic to two commuting copies of the one-dimensional diffeomorphism algebra, which can be seen by redefining the constraints as

\[
T_\pm = \frac{1}{2} (G_0 \pm G_1)
\] (2.8)

Since we are dealing with a reparametrization invariant system, the Hamiltonian vanishes on the constraint surface (i.e. it is proportional to the constraints). Therefore the dynamics is determined by the constraints only. Since \( G_0 \) and \( G_1 \) are independent, there will be \( (2+N)-2 = N \) local physical degrees of freedom. When \( N = 0 \), there is only a finite number of global physical degrees of freedom (zero modes of \( g \) and \( \phi \)), and one is dealing with some kind of a topological field theory. When \( N \neq 0 \), these global degrees of freedom will be present, together with the local ones.

The variables \((g, p, \phi, \pi)\) are not convenient for quantization, since \( G_0 \) is a non-polynomial function of these variables. First we perform a canonical transformation in order to get rid off the \( e^\phi \) term in \( G_0 \)

\[
g = e^{-\tilde{\phi}} \tilde{g} \quad , \quad p = e^{\tilde{\phi}} \tilde{p}
\]

\[
\phi = \tilde{\phi} \quad , \quad \pi = \tilde{\pi} + \tilde{p} \tilde{g}
\] (2.9)
The constraints now become

\[ G_0(x) = -4g^2 p^2 - 2gp\pi + (\phi')^2 + \Lambda g - \frac{1}{2} \frac{g'}{g} \phi' + \phi'' + \frac{1}{2} \sum_{i=1}^{N} (\pi_i^2 + (\phi_i')^2) \]

\[ G_1(x) = \pi \phi' - 2p' g - pg' + \sum_{i=1}^{N} \pi_i \phi_i' , \]

(2.10)

where we have dropped the tildas and \( \Lambda = \frac{N^2}{8} \). Now it is convenient to define the \( SL(2,\mathbb{R}) \) variables introduced in [7]

\[ J^+ = -\frac{\sqrt{2}}{2g} T_+ + \frac{\Lambda}{2\sqrt{2}} \]

\[ J^0 = gp + \frac{1}{4} \left( \pi - \frac{1}{2} \frac{g'}{g} \right) \]

\[ J^- = \frac{1}{\sqrt{2}} g , \]

(2.11)

and a \( U(1) \) current

\[ P_D = \frac{1}{2} \left( \pi - \frac{1}{2} \frac{g'}{g} + 2\phi' \right) . \]

The \( (J^a, P_D) \) variables satisfy an \( SL(2,\mathbb{R}) \otimes U(1) \) current algebra

\[ \{ J^a(x), J^b(y) \} = f^{ac} J^c(x) \delta(x - y) - \frac{1}{4} \eta^{ab} \delta'(x - y) \]

\[ \{ P_D(x), P_D(y) \} = -\delta'(x - y) , \]

(2.12)

where \( f^{ac} = 2\varepsilon^{abcd} \eta_{dc} \) with \( \eta^{+-} = \eta^{-+} = 2, \eta^{00} = -1 \), and \( \{ J, P_D \} = 0 \). Instead of using the canonical variables \( (\pi_i, \phi_i) \), we introduce the left/right moving currents

\[ P_i = \frac{1}{\sqrt{2}} (\pi_i + \phi_i') , \quad \tilde{P}_i = \frac{1}{\sqrt{2}} (\pi_i - \phi_i') \]

(2.13)

satisfying

\[ \{ P_i(x), P_j(y) \} = -\delta_{ij} \delta'(x - y) , \quad \{ \tilde{P}_i(x), \tilde{P}_j(y) \} = \delta_{ij} \delta'(x - y) \]

(2.14)

and \( \{ P, \tilde{P} \} = 0 \). Now one can show that the energy-momentum tensor associated to the algebra (2.12) via the Sugavara construction

\[ S = 2\eta_{ab} J^a J^b - (J^0)' + \frac{1}{2} P_D^2 + \frac{1}{2} P_D' + \frac{1}{2} \sum_{i=1}^{N} P_i^2 \]

(2.17)

satisfies \( S \approx T_+ \). Therefore the constraints become

\[ J^+(x) - \mu = 0 \quad , \quad S(x) = 0 \quad , \]

(2.18)
where \( \mu = \frac{\Lambda^2}{2\sqrt{2}} \).

Now it is convenient to introduce three new variables \( \beta(x), \gamma(x) \) and \( P_L(x) \) such that

\[
\begin{align*}
J^+ &= \beta \\
J^0 &= -\beta\gamma - \frac{1}{2}P_L \\
J^- &= \beta\gamma^2 + \gamma P_L - \frac{1}{2}\gamma',
\end{align*}
\]  

(2.18)

where

\[
\{ \beta(x), \gamma(y) \} = -\delta(x-y), \quad \{ P_L(x), P_L(y) \} = \delta'(x-y),
\]  

(2.19)

with the other Poissons brackets being zero. Then the expressions (2.18) satisfy the \( SL(2, \mathbb{R}) \) current algebra (2.12), and represent the classical analogue of the Wakimoto transformation \([8]\). The \( S \) constraint then becomes

\[
S = \beta'\gamma - \frac{1}{2}P_L^2 + \frac{1}{2}P_L' + \frac{1}{2}P_D^2 + \frac{1}{2}P_D' + \frac{1}{2}\sum_{i=1}^{N} P_i^2 = 0.
\]  

(2.20)

If we define \( B(x) = \beta(x) - \lambda \) and \( \Gamma(x) = \gamma(x) \), then the \( J^+ \) constraint implies that \( B = 0 \), and consequently we can omit the canonical pair \((B, \Gamma)\) from the theory. Therefore we are left with \( P_L, P_D \) and \( P_i \) variables, obeying only one constraint

\[
2S = -P_L^2 + P'_L + P_D^2 + P'_D + \sum_{i=1}^{N} P_i^2 = 0.
\]  

(2.21)

The form of the Poisson brackets of \( P_L \) and \( P_D \) allow us to introduce a canonical pair \((P(x), T(x))\) such that

\[
P_L = \frac{1}{\sqrt{2}}(P - T') \quad , \quad P_D = \frac{1}{\sqrt{2}}(P + T')
\]  

(2.22)

Note that the definition (2.22) implies that the zero-mode parts of \( P_L \) and \( P_D \) are equal. When \( N = 0 \), this is true on the constraint surface, but away from the constraint surface these zero modes are independent. Therefore we are going to modify the eq. (2.22) by introducing an independent zero-mode momentum \( p \) such that

\[
P_L = \frac{1}{\sqrt{2}}(P - T') \quad , \quad P_D = p + \frac{1}{\sqrt{2}}(P + T')
\]  

(2.23)

Then the \( S \) constraint becomes

\[
S = (p + \sqrt{2}T')(p + \sqrt{2}P) + \sqrt{2}P' + \sum_{i=1}^{N} P_i^2 = 0.
\]  

(2.24)
Now one can easily solve the eq. (2.24) for $T$ or $P$, and therefore put $S$ into form which is linear in one of the momenta, a step which is crucial for formulating a Schrodinger type equation [11]. Although in this way one preserves the manifest diffeomorphism covariance, the corresponding multifingered time Schrodinger equation is difficult to solve. Instead, we fix the time reparametrization invariance by choosing the gauge $T(x,t) = t$. \\

(2.25)

Then from the eq. (2.24) we get

\[ P(x) = -\frac{p}{\sqrt{2}} - \frac{1}{\sqrt{2}} e^{-px} \int_{-\infty}^{x} dy e^{py} \sum_{i=1}^{N} P_{i}^{2}(y) \ . \]

(2.26)

Hence the independent canonical variables are $(\pi_{i}(x), \phi_{i}(x))$ together with the $x$-independent variables $(p, q)$. The $(p, q)$ variables are the global remnants of the graviton-dilaton sector, and they represent the physical degrees of freedom of that sector. The Hamiltonian for the independent canonical variables can be deduced from the $\int dx P \dot{T}$ part of the action to be

\[ H = \frac{cp}{\sqrt{2}} + \frac{1}{\sqrt{2}} \int_{-\infty}^{\infty} dx e^{-px} \int_{-\infty}^{x} dy e^{py} \sum_{i=1}^{N} P_{i}^{2}(y) \ , \]

(2.27)

where $c$ is a constant. In the compact case $c$ is proportional to the volume of $\Sigma$, and can be set to 1. In the non-compact case, the value of $c$ can be determined from the requirement of the asymptotic flatness of the black-hole solution, whose ADM mass is asymptotically conserved energy [12], and therefore $M = H = cp$.

The formulas (2.26) and (2.27) simplify if we use the Fourier modes of $P_{i}$

\[ P_{i}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk e^{ikx} \alpha_{k}^{i} \ , \]

(2.28)

and analogously for $\tilde{P}_{i}$. In particular one gets for the Hamiltonian (2.27)

\[ H = \frac{cp}{\sqrt{2}} + \frac{1}{2\sqrt{2p}} \int_{-\infty}^{\infty} dk \alpha_{-k}^{i} \alpha_{k}^{i} \ , \]

(2.29)

which is almost like a free-field Hamiltonian, except for the non-polynomial dependence on the momentum $p$.

Note that $H$ would be positive definite if $p$ was restricted to $\mathbb{R}_{+}$. It is a non-trivial task to deduce directly from our approach what is the range of $p$, but when compared to the results of the Dirac analysis [10], $p$ can be identified with the energy of a free relativistic 2d particle, whose range is positive. There is also a disconnected piece, corresponding to the negative energies. One obtains a similar result for the reduced
configuration space of the gravity plus dilaton sector in the $V(\phi) = \text{const.}$ model, i.e. two disconnected $\mathbb{R}_+$ spaces [5]. Both models have the same reduced phase space since the constraints can be brought into an identical form [7].

3. Quantization

Given the reduced phase space and the corresponding Hamiltonian, one can define the quantum theory by the Schrodinger equation

$$i\partial_t \Psi = \hat{H} \Psi,$$

where $\hat{H}$ is an operator corresponding to the classical expression (2.29). $\Psi$ belongs to a Hilbert space constructed from the canonical algebra of the basic variables $(p, q, \pi_i(x), \phi_i(x))$, which are now promoted into hermitian operators. As in the classical case, it is convenient to use the $P_i$ and $\tilde{P}_i$ operators, satisfying

$$[P_i(x), P_j(y)] = -i\delta'(x-y)\delta_{ij}, \quad [\tilde{P}_i(x), \tilde{P}_j(y)] = i\delta'(x-y)\delta_{ij},$$

and $[P_i, \tilde{P}_j] = 0$, while for $p$ and $q$ we will take

$$[p, q] = ip,$$

since $p \in \mathbb{R}_+$. Given the relations (3.2) there is an immediate problem of how to order the $P$’s in the expression (2.29). However, given the simple form of $H$ in terms of the $\alpha$ modes, and the fact that they resemble particle creation and annihilation operators, we can define a quantum theory based on the Hilbert space

$$\mathcal{H}^* = \mathcal{H}(p) \otimes \mathcal{F}(\alpha) \otimes \mathcal{F}(\tilde{\alpha})$$

where $\mathcal{H}(p)$ is the Hilbert space associated with the $(p, q)$ algebra, while $\mathcal{F}(\alpha)$ and $\mathcal{F}(\tilde{\alpha})$ are the Fock spaces built on the vacuum

$$\alpha_k^i |0\rangle = \tilde{\alpha}_{-k}^i |0\rangle = 0, \quad k \geq 0.$$

One can now introduce the standard field-theory creation and annihilation operators as

$$a_i(k) = \frac{1}{\sqrt{k}} \alpha_k^i, \quad k > 0$$

$$a_i(k) = \frac{1}{\sqrt{|k|}} \tilde{\alpha}_k^i, \quad k < 0.$$
Therefore $\alpha_k$ corresponds to the right-moving ($k > 0$) quant, while $\tilde{\alpha}_k$ corresponds to the left-moving ($k < 0$) quant.

Given the Hilbert space $H^*$, the hamiltonian $H$ can be promoted into a hermitian operator
\[ \hat{H} = \frac{cp}{\sqrt{2}} + \frac{1}{\sqrt{2}p} \int_0^\infty dk |k| a_i^\dagger(k) a_i(k) \quad . \] (3.7)
Note the absence of the left-moving modes in the expression (3.7). This is the consequence of the fact that the $S$ constraint does not depend on the $\tilde{P}_i$ variables. This asymmetry arises from our choice of the variables and the gauge-fixing procedure. In (2.11) we set $J^+ \approx T_-$ and subsequently $S \approx T_+$. Then we solve the $J^+$ constraint by setting $\beta = \mu$ while the $S$ constraint is solved by choosing the gauge (2.25), which is a choice of the time variable and therefore the $S$ constraint is transformed into a Schrodinger equation. Hence in the gauge (2.25) the $T_+$ constraint generates the time translations, while $T_-$ generates the spatial diffeomorphisms and consequently $\tilde{P}_i$ are frozen (integrals of motion). Clearly our choice of the variables and the gauge is convenient for describing a one-sided collapse, i.e. when initially one has only a right-moving matter.

4. Concluding Remarks

The Hamiltonian of our theory is a Hermitian operator in the physical Hilbert space, and therefore any time evolution will be unitary. This implies in particular that there will not be any transitions from pure into mixed states, which rules out the Hawking scenario [2] for the endpoint of the black hole evaporation process. However, in order to see what really happens during the gravitational collapse one has to carefully study the black-hole solutions in this theory. The spatial metric $g(x)$ can be written as
\[ g(x) = \frac{\lambda^2}{16} \gamma^2(x) - \frac{\gamma(x)}{\sqrt{2}} \left( p + \frac{1}{2\pi} \int_{-\infty}^{+\infty} dk \ e^{ikx} \frac{e^{ikx}}{p + ik} \right) - \sqrt{2}\gamma'(x) \quad , \] (4.1)
where
\[ L_k = \int_{-\infty}^{+\infty} dq \ a_{k-q} i a_q i \quad . \] (4.2)
Classical equations of motion imply
\[ \dot{p} = \{H,p\} = 0 \quad , \quad \dot{\alpha}_k = \{H,\alpha_k\} = \frac{c}{2\sqrt{2}p} \alpha_k \quad , \] (4.3)
so that from the eq. (4.1) one can find the spatial metric at any time. Note the similar structure of the expression (4.1) and the corresponding expression of CGHS
where our $p$ is analogous to their $M$ (ADM mass of the black hole), our arbitrary
gauge function $\gamma(x)$ is analogous to their $w_\pm(x)$ gauge functions, and the dependence
on the scalar fields is similar. The differences come from the fact that we are working
in some type of the Polyakov light-cone gauge \cite{13}, while CGHS are in the conformal
gauge.

When $N = 0$, then the black-hole solution is equivalent to a choice of $\gamma(x)$ such
that
\begin{equation}
\frac{\lambda^2}{16} \gamma^2(x) - \frac{\gamma(x)}{\sqrt{2}} p - \sqrt{2} \gamma'(x) = \frac{e^{\lambda x}}{1 - \frac{M}{\lambda} e^{-\lambda x}}.
\end{equation}

Solutions of this equation exist; however, we could not find an explicit expression.
Such an expression will give the relation between the parameters $p$ and $M$, and it will
constitute an independent check of $M = cp$.

Since $H \approx p$ in the $N = 0$ case, one has an eternal black hole. Clearly in order to
get some interesting effects, $N$ must be different from zero. Then $g(x)$ is given by the
expression (4.1), which becomes an operatorial expression in the quantum theory. A
normal ordering ambiguity in the $L_k$ operators then appears. The standard normal
ordering prescription causes a $c$-number anomaly in the commutator $[\hat{g}(x), \hat{g}(y)]$, and
we believe that this is a technical problem which could be overcomed by appropriate
modifications. Note that in the Dirac approach a $c$-number anomaly appears in the
diffeomorphism algebra \cite{11}. It would be interesting to see whether this anomaly is in
any way equivalent to the metric anomaly in our approach. A more difficult problem
is the construction of a hermitian operator associated with the scalar curvature $R$.
This operator is important since it will give a measure of a singularity. $R$ is certain to
be a non-polynomial function of the $p$ and the $a(k)$’s, which will be the main source
of difficulties in constructing the $\hat{R}$ operator.

An important issue which has to be analyzed is the Hawking effect. A natural
way to do this in our model is to construct a state $|\psi_0\rangle$ such that
\begin{equation}
\hat{g}(x) |\psi_0\rangle = g_{\text{reg}}(x) |\psi_0\rangle,
\end{equation}
where $g_{\text{reg}}(x)$ is a non-singular metric. Then evolve $|\psi_0\rangle$ in time by the evolution
operator $e^{-i\hat{H}t}$. At some time $t_A$ an apparent horizon will form in the effective
metric $\langle \psi_0 | e^{i\hat{H} t} \hat{g} e^{-i\hat{H} t} | \psi_0 \rangle$. Then a reduced density matrix $\hat{\rho}$ can be introduced, by tracing
out the states which are beyond the horizon \cite{2, 12}. How to define these states is not
clear at the moment, but when this problem is resolved then one could in principle
answer the questions about the thermal nature of $\hat{\rho}$, i.e. when
\begin{equation}
\hat{\rho} \approx \frac{1}{Z} e^{-\beta \hat{H}},
\end{equation}
\]
and what are the non-perturbative corrections to the Hawking temperature

$$\beta = \frac{4\pi}{\lambda} + .... \quad .$$  \hspace{1cm} (4.7)

Moreover, by analyzing the effective scalar curvature

$$R_{\text{eff}}(x,t) = \langle \psi_0 | e^{i\hat{H}t}\hat{R}(x)e^{-i\hat{H}t} | \psi_0 \rangle$$

one should be able to say what happens with the singularity. Ideally, $R_{\text{eff}}(x,t)$ should stay a regular function for any $t$.

We should emphasize that in our quantization scheme the topology of the space-time stays fixed. One could argue that this is the main reason why the theory is unitary, and no violations of quantum mechanics occur. This may well be the case, and we should point out that in the context of 2d gravity introduction of the topology change is equivalent to introducing the interactions in the corresponding string field theory. As a result, the string field $\Psi$ will not satisfy the linear equation (3.1), but instead the eq. (3.1) will be modified by $\Psi^2$ and higher order terms. This will directly violate the quantum mechanics. Formally, one can invoke the third quantization in order to get around this problem; however, it is not clear how to define such a theory. Matrix models approach offers a definition [15], and there are indications that the Wheeler-DeWitt equation is not satisfied [16].

Clearly a lot of work remains to be done in order to answer all these questions. The main difficulty at the moment are the explicit calculations. However, we believe that the reduced phase-space quantization approach can answer, at least qualitatively, some of the issues raised in our discussion. Study of the theory in other gauges should also be beneficial, since then the questions of the diffeomorphism invariance could be analyzed and the results in different gauges compared.

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