Proton Production in d+Au Collisions and the Cronin Effect

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Proton production in the intermediate \( p_T \) region in \( d+Au \) collisions is studied in the parton recombination model. The recombination of soft and shower partons is shown to be important in central collisions, but negligible in peripheral collisions. It is found that the large nuclear modification factor for proton production can be well reproduced by a calculation of the 3-quark recombination process.

In a previous paper\textsuperscript{1} we have shown that the Cronin effect\textsuperscript{2} on pion production in \( d+Au \) collisions can be understood in terms of the recombination of the soft and shower partons without \( p_T \) broadening by multiple scattering in the initial state. In this short note we extend the consideration to proton production and show that the same effect can similarly be interpreted.

The Cronin effect can best be displayed by the nuclear modification factor \( R_{CP}(p_T) \) that is the ratio of central-to-peripheral yields appropriately scaled by the average number of binary collisions \( \langle N_{\text{coll}} \rangle \). It is found in the PHENIX experiment that \( R_{CP}^p \), for proton reaches a value roughly 2 for \( 2 < p_T < 3 \text{ GeV}/c \), even higher than that for pion at \( \sim 1.4 \). Such a behavior of the enhancement effect is hard to interpret, if hadrons produced at intermediate \( p_T \) are the consequences of fragmentation of hard partons produced at higher \( p_T \). Indeed, since there is no energy loss in \( d+Au \) collision, one would expect \( R_{CP}^p \sim 1 \) for both pion and proton on the grounds that fragmentation outside the cold medium should be independent of the impact parameter. Thus the observed \( R_{CP}^p \) strongly suggests the dependence of the hadronization mechanism on the medium. In our view recombination is a mechanism, which, on the one hand, provides a way to describe fragmentation in terms of shower partons\textsuperscript{3}, and, on the other, can take into account the coalescence of soft and shower partons to form hadrons in the intermediate \( p_T \) range\textsuperscript{3}.

For the proton spectrum at low \( p_T \), one should be careful about the mass effect. The low-\( p_T \) region is, however, not the main part of our work where the model has any predictive power. As in\textsuperscript{1,3,4}, we fit the data in that region and use our model to predict the hadronic spectra at intermediate and high \( p_T \). We have found that for the purpose of data fitting at low \( p_T \) our 1D formulation of the recombination process is quite adequate when only the kinematical variables are suitably modified to account for the proton mass. Thus we start with the invariant inclusive distribution for proton formation at midrapidity in the recombination model\textsuperscript{3,4}

\[
\int_0^dN_p dp = \int \frac{dp_1}{p_1} \frac{dp_2}{p_2} F(p_1,p_2,p_3) R_p(p_1,p_2,p_3,p), \tag{1}
\]

where all momentum variables \( p_i \) and \( p \) are transverse momenta, and \( p^0 \) denotes the energy of the proton. Since the parton masses are set to zero, we continue to use \( p_i \) for their energies. \( F(p_1,p_2,p_3) \) is the joint distribution of \( u, u, \) and \( d \) quarks at \( p_1, p_2 \) and \( p_3 \), respectively. \( R_p(p_1,p_2,p_3,p) \) is the recombination function for a proton with momentum \( p \).

\[
R_p(p_1,p_2,p_3,p) = g(y_1 y_2)^{\alpha + \gamma} \frac{\delta \left( \sum_i y_i - 1 \right)}{}, \tag{2}
\]

where \( y_i = p_i/p \), \( \alpha = 1.75 \), \( \gamma = 1.05 \), and

\[
g = [6 \beta(\alpha + 1, \alpha + \gamma + 2)]^{-1}, \tag{3}
\]

\( \beta(a,b) \) being the beta function. As always in the recombination model, the main issue is about the distribution of the quarks that recombine. Here, it is \( F(p_1,p_2,p_3) \).

Following the same notation used in\textsuperscript{4} for \( Au+Au \) collisions, we write schematically

\[
F = TTT + TTS + TSS + SSS, \tag{4}
\]

where all the shower partons \( S \) are from one hard parton jet. Shower partons from different jets are ignored here for RHIC energies. \( T \) denotes thermal parton, even though in \( d+Au \) collisions the notion of thermal equilibrium may not be justified. To preserve the same notation as in\textsuperscript{4}, we continue to use \( T \) to signify the soft partons that are not associated with the shower components of a hard parton and are loosely referred to as thermal partons when convenient. The \( SSS \) term in Eq.\textsuperscript{4}, when convoluted with \( R_p \) in Eq.\textsuperscript{4}, gives rise to what is usually regarded as the fragmentation of a hard parton into a proton.\textsuperscript{4} The \( TTT \) term comes entirely from the soft partons, while \( TTS \) and \( TSS \) accounts for the interplay between the thermal (or soft) and shower partons.

As in\textsuperscript{1,3,4}, we can calculate the distributions of the shower partons \( S \) from the QCD processes of producing hard partons and their induced shower partons. We cannot calculate the thermal component \( T \), which is deduced from fitting the low-\( p_T \) data. Thus what we can calculate that is new is only the effect of the \( TTS + TSS \) terms at intermediate \( p_T \), knowing that the \( SSS \) term dominates at very high \( p_T \).

For \( T \) we use the same parametrization as before\textsuperscript{1,3,4}, and write

\[
T(p_1) = p_1 \frac{dN_T}{dp_1} = C p_1 \exp(-p_1/T), \tag{5}
\]
where $T$ should be regarded as just an inverse slope. The thermal contribution to the proton spectrum arising from $TTT$ recombination is then

$$\frac{dN^\text{thermal}_p}{dp} = \frac{C^3 p^2}{6} \frac{e^{-p/T}}{p^6} B(\alpha + 2, \gamma + 2) B(\alpha + 2, \alpha + \gamma + 4) \times B(\alpha + 1, \gamma + 1) B(\alpha + 1, \alpha + \gamma + 2),$$

which differs from a similar formula in Ref. [3] by only the presence of $p^0$, instead of $p$. For the other three terms in Eq. (4) that involve $S$, the contributions to the proton spectrum are the same as those in Ref. [3] except that each equation should be multiplied by the factor $p^0/p^6$ on the RHS and the factor $\xi$ should be omitted. The latter refers to the mean energy loss in Au+Au collisions, and should be absent in $d+Au$ collisions. Also the hard parton distributions $f_i(k)$ in Ref. [3] should be changed to the corresponding ones for $d+Au$ collisions, as given in Ref. [1]. What is to be emphasized is that there are no free parameters to adjust in those terms. The shower parton distributions are the main input, and they have previously been determined in Ref. [3].

We must now determine $C$ and $T$ by fitting the low-$p_T$ data using Eq. (6) for both central and peripheral collisions. The data available are from PHENIX, given as figures online [5]. We fit them in the region $0.5 < p_T < 1.5$ GeV/c and the results are shown by the light solid lines in Fig. 1 (a) and (b) for 0-20% and 60-90% centralities, respectively. The values determined are $C = 11.5$ (8.0) (GeV/c)$^{-1}$ and $T = 0.24$ (0.21) GeV/c for 0-20% (60-90%) centrality. The contributions from $TTS+TSS$ and $SSS$ components are determined without free parameters, and are shown by the dashed and dash-dot lines in the same figures. The sum of all four components are shown by the thick solid lines. Evidently, they agree with the data very well.

We note that at 0-20% centrality the thermal-shower ($TTS+TSS$) contribution crosses over the fragmentation ($SSS$) component at $p_T \approx 2.5$ GeV/c, roughly the same as in the case of pion production in $d+Au$ collisions. However, the thermal $TTT$ contribution is roughly the same as each of the ($TTS+TSS$) and $SSS$ contributions at the cross-over point, whereas for pion production the thermal $TTT$ contribution is much lower than $TTS$ and $SSS$ at the same point. Thus the thermal contribution to proton formation dominates over a wider range of $p_T$ than that for pion. That is because of the $C^3$ dependence in Eq. (6), and is consistent with the findings in Refs. [4, 10], where the recombination of thermal partons can account for the large $p_T$ ratio up to $p_T \approx 3-4$ GeV/c in $Au+Au$ collisions. The same cannot be said about 60-90% centrality in $d+Au$ collisions. The cross-over between $TTS+TSS$ and $SSS$ occurs at $p_T \approx 1$ GeV/c, where the distributions are far lower than the thermal contribution. Throughout all $p_T$ in Fig. 1 (b) the ($TTS+TSS$) component is negligible, a situation that is similar to what we have found separately for either $p$ or $\pi$ production in $pp$ collisions. The reason is the substantial reduction of the soft parton environment in peripheral $d+Au$ or $pp$ collisions so that the ($TTS+TSS$) component does not develop any strength relative to the $SSS$ component. We should, however, note parenthetically that the low density of soft partons that are produced in $pp$ collisions is sufficient to make the jet structure different from that in $e^+e^-$ collisions, which have no soft partons of the same origin at all.

The fact that our calculated results agree well with the data for both 0-20% and 60-90% centralities implies that the ratio $R_{CP}^p(p_T)$, defined by

$$R_{CP}^p(p_T) = \frac{\langle N_{Coll} \rangle_{60-90\%} \frac{dN_p}{dp_T}(0-20\%)}{\langle N_{Coll} \rangle_{0-20\%} \frac{dN_p}{dp_T}(60-90\%)}$$

must also agree with the data. That agreement is shown explicitly in Fig. 2 by the solid line, where the data are from Ref. [5]. The calculated curve approaches 1 at high $p_T$, where the yields are dominated by the fragmentation of hard partons ($SSS$), which is independent of the soft partons. The dashed line in Fig. 2 shows the contribution to $R_{CP}^p$ from the thermal components only. The small difference in $T$ for the two centralities results in an exponential growth in $p_T$ as can be seen directly from Eq. (6). Thus the effect of thermal-shower recombination is the damping of that exponential increase in $R_{CP}^p(p_T)$, as shown by the solid line. Because of the ineffectiveness of the thermal-shower contribution at 60-90% centrality, that damping does not take place until $p_T$ reaches near 2 GeV/c. By then $R_{CP}^p$ is already greater than 1.7, which is higher than $R_{CP}^\pi$. Thus the origin of $R_{CP}^p$ being greater than $R_{CP}^\pi$ is mainly in the $TTT$ recombination for proton being more sensitive to centrality than $TT$ for pion. The role of shower partons is limited in that comparison.

![FIG. 1: (a) Proton transverse momentum distribution in $d+Au$ collisions at 0-20% centrality. The preliminary data are from [3]. (b) Same as in (a), but for 60-90% centrality.](image-url)
It should be pointed out that our fitting procedure in the determination of $C$ and $T$ has not ignored $R_{CP}$, as an outcome. Since Fig. 1 has logarithmic vertical scale, those data points can determine $C$ and $T$ only within narrow ranges. The data in Fig. 2 involve their ratios and are in linear scale. Thus the parameters can be more accurately determined by including Fig. 2 in the fit. Since the distributions of the soft partons at low $p_T$ are not generated from first-principle calculations, we have taken the liberty to use all the data available to make the best determination of them. Our predictions are only for the behavior at $p_T$ above the region that is dominated by the soft partons.

The values of $C$ and $T$ obtained here should be compared with those determined from the pion spectrum [1]. They are $C_\pi = 12 (5.65)$ (GeV/c)$^{-1}$ and $T_\pi = 0.21 (0.21)$ GeV/c for 0-20% (60-90%) centrality, where the subscripts $\pi$ are added for distinction. Whereas there is no essential dependence of some of those parameters on whether the formed hadrons are pions or protons, i.e., $C_p \simeq C_\pi$ at 0-20% and $T_p = T_\pi$ at 60-90%, there exist some significant differences in others: $C_p = 8.0$, $C_\pi = 5.65$ at 60-90% and $T_p = 0.24$, $T_\pi = 0.21$ at 20-60%.

The species dependence of those parameters reflects the general properties of the spectra, especially at $p_T < 1$ GeV/c. In [1] it is shown in Au+Au collisions that the low-$p_T$ spectra can be fitted by exponential behavior in $m_T$ with the inverse slope increasing linearly with hadronic mass at a rate that increases with centrality. It strongly suggests hydrodynamical expansion radially, which is well known to exist. It means that a significant portion of the hydrodynamical fluid is hadronic at very low $p_T$. Only the portion that remains partonic at $p_T$ around 1 GeV/c and above are available for recombination with the shower partons. How much of this scenario is valid (and in what quantitative way) for $d+Au$ collisions is not known. Since our hadronization model does not treat the hadronizmation phase, we can only take the thermal hadrons in the 0.5 < $p_T$ < 1.5 GeV/c range as observed, but not lower, to determine our parameters for the thermal partons $T$. The species dependence of some of the $C$ and $T$ parameters is clearly a result of our inadequacy in subtracting out the already-formed hadrons from the medium at low $p_T$, since the thermal partons $T$ should have no knowledge of what hadrons they are to form. Given our inability to treat very low $p_T$ physics, we can only regard what we have done as a determination of the intermediate $p_T$ behavior in the separate cases of specific hadrons without a way to enlighten the problem of overall species dependence at very low $p_T$ in a broader scheme.

The important properties of hadron production in $d+Au$ collisions that we have learned from this study is that the protons are formed by recombination at all $p_T$ and that the underlying partons that give rise to their formation change smoothly from the soft component to the semi-hard shower partons that are created by the hard partons. The recombination formalism allows us to calculate the $p_T$ distribution in the intermediate and higher $p_T$ regions with good agreement with the data. The contribution from the recombination of soft and shower partons cannot be interpreted as a modification of the fragmentation function, since the hard partons in $d+Au$ collisions are not significantly affected by the cold medium that they traverse. In this treatment the Cronin effect of the proton spectrum is not induced by transverse broadening due to initial scatterings, but is caused mainly by the centrality dependence of the soft partons that recombine. $R_{CP}$ is higher than $R_{CP}^T$ because the number of such recombining quarks is 3 instead of 2. Our approach cannot be viewed as being totally successful until the data for $p_T > 3$ GeV/c turn out to support our predictions in the higher $p_T$ region.

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![FIG. 2: Nuclear modification factor $R_{CP}$ for proton production in $d+Au$ collisions. The data are for 0-20% to 60-90% centralities [2]. The solid line is the result of our calculation when all contributions are taken into account, while the dashed line gives the ratio when only the thermal contributions are included.](image)

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