A toy model for the coincidence problem *

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Abstract

The measured values of the matter energy density and the vacuum energy density are obtained using an adiabatic black hole percolating model at the critical point. The percolation of black holes is related with an expanding isotropic universe filled with the most entropic fluid saturating the holographic bound.

KEYWORDS: cosmological constant, black holes, holography, string/M theory.

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It has been a surprise that the recent results reported by the WMAP and others teams ([1]) converge to a universe with an important contribution to the energy content in the form of vacuum energy. This observation acute the cosmological constant problem; now not only the value of the cosmological constant is very small with respect to the theoretical predictions but it is

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actually non-zero and of the order of the energy density of ordinary matter (dark and visible). Usually one believes that there is some unknown mechanism originating on unknown symmetries of the full quantum gravity theory which can give a cosmological constant equal to zero. But the fact that the cosmological constant is very small and also of the same order of the rest of the energy, that evolves with the cosmological evolution, is a paradox for which we have no mechanism that can explain this phenomenon.

In this research we address this so-called coincidence problem ([2]) by envisaging a toy model with the desired properties, and related to the vacuum structure of the effective theory. In fact the main problem of the string physics is the nature of the vacuum of the theory ([3]); in spite that different string theories are related by dualities pointing to one single underlying structure, the vacuum is not fixed and on the contrary, a great number of vacua are in principle possible; there is not a preferred vacuum and due to such a high diversity actually it begins to increase the suspicion that arguments beyond the theory such as the anthropic principle must be advocated to understand the observations([4]).

In fact the observations must be the clue to gain insight on the nature of the vacuum of the string/M theory, and observation, mainly cosmological measurements offers us with the surprise of a flat, accelerated universe with a contribution of vacuum energy, and ordinary energy of the same order. It is not easy to relate these observations to the fundamental theory of quantum gravity but they must be a consequence of the classical effective theory. That is, the string/M theory has to have a vacuum structure that in the classical regime must explain the observed cosmological data.

We propose a mechanical model to account for the cosmological observations of the composition of the universe; we consider a fluid of black holes evolving adiabatically as the substratum behind the actual universe. A fluid of black holes has been considered before by T. Banks and W. Fischler as the initial state for the Universe ([5]) because such a fluid saturates the holographic bound([6]) underlying a holographic cosmology ([7]). We insist, with variants, on this mechanical model to account for the actual observations.

We consider a system of black holes in a process of coalescence. We neglect the gravitational interaction between them so that the process is purely random; as the density of black holes increase the system approach the critical point at which it percolates. We consider the vicinity of this critical point when the infinite cluster develops; then we identify a cosmological
model with the infinite cluster of a network of percolating black holes ([8]).

This model at the critical point shares many properties with the dense black hole fluid of ([5]); in fact the infinite cluster or infinite droplet will be a black hole covering all the disposable space. We envisage a dual description for the system. For an observer \(^1\) surfing the expanding event horizon the geometry is that of a Friedman-Lemaître Robertson Walker (FLRW) universe filled with a fluid with equation of state \(p = \omega \rho\) with \(\omega = 1\) ([5]). For an observer that traverses the event horizon the geometry is the interior of a growing black hole; in its expansion the density of matter is diluted and asymptotically tends to zero as the radius of the black hole goes to infinity; the geometry is described by a flat expanding FLRW model.

At the percolating critical point there is a duality between the regions covered by black holes, that are inside an event horizon (black regions), and regions outside the event horizon (white regions) (Figure 1). In ([5]) the white regions are used to contact with the observational cosmology. We need to use the Israel Junction conditions ([9]) to match the geometry at both sides of the event horizon for this interstitial regions. What it is evident is that this white regions are covered by an event horizon and are candidates to develop accelerated cosmologies.

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Figure 1: The two dual possibilities are sketched. In the right the horizon of a black hole is surrounded by matter. In the left ordinary matter is covered by an horizon of the percolating black hole system.

\(^1\)We use the term observer without any anthropomorphic intention; the geometry is independent of the presence of any observer.
Our toy model of a percolating network of black holes at the critical point can describe two different scenarios. In the black regions the expansion is decelerated, the matter is diluted as the infinite cluster grows, and asymptotically the density is zero (the relation between the density and size for a black hole is $\rho \sim R^{-2}$). This realization of the model is a candidate for the initial expansion of the universe; we have a FLRW expanding model isotropic and flat without an initial singularity\(^2\). The isotropy and flatness are consequences of the random distribution of black holes and the fact that we are at the critical percolating point; this two properties appear in the model of Banks and Fischler and can be related with the saturation of the entropy bounds proposed in ([5]) to construct a holographic cosmology; moreover, the percolating critical point is scale invariant and the distribution of clusters is described by a power law; we have not been able to determine the critical exponents of our model but if the relation with the cosmic fluid $p = \rho$ can be made we can use the results of ([5]) to assert that the distribution is a Gaussian random scale invariant one.

In the white regions we have ordinary radiated matter surrounded by an event horizon; this geometry has the causal structure of accelerated cosmologies; if a cosmological constant is the responsible of the dark energy, the event horizon is at a fixed proper distance $R \sim \Lambda^{-2}$, whereas if the dark energy is some sort of quintessence with $-1/3 > \omega > -1$, the event horizon grows with the cosmic time at a rate $R \sim t$, $t$ being the cosmic time; so in this regions covered by an event horizon the expansion can be accelerated. Is in these last regions that we make contact with the observed universe. With this end we analyze the merging of black holes.

When two black holes meet due to the contact of its event horizons they form a new black hole\(^3\). We consider two magnitudes in this process, the energy and the entropy. For the energy conservation law we have\(^4\)

$$2M = M_f + E_r,$$

\(^2\)We have not the big bang singularity but because we are in the interior of black holes we confront the black holes singularities; however if the time of merging is smaller than the time of collapse the singularities are not formed and we have a singularity free model.

\(^3\)When we say black hole we refer to a closed event horizon; it is clear that the geometry is strongly distorted with respect to the Schwarzschild one.

\(^4\)We use units in which $G = c = \hbar = k_B = 1$. To restore the correct dimensions on a formula multiply by the appropriate powers of the previous constants.
where $M$ is the mass of each of the initial black holes, that we suppose the same, $M_f$ is the mass of the final black hole and $E_r$ is the energy emitted in the process of merging; using the relation $R = 2M$ between radius and mass for Schwarzschild black holes we have

$$R = \frac{R_f}{2} + E_r.$$  \hfill (2)

On the other hand, the second law of thermodynamics establishes that the entropy of the final state is equal or higher than the initial one; for black holes the entropy is $\frac{1}{4}$ the area of the event horizon in Planck units, so we have

$$2R^2 \leq R_f^2 + \frac{S_r}{\pi},$$  \hfill (3)

where $S_r$ is the entropy of the radiated energy $E_r$. A good approximation is to consider $S_r \ll R_f^2 \sim R^2$ so that for an adiabatic evolution we have

$$2R^2 = R_f^2.$$  \hfill (4)

In this adiabatic coalescence of two equal mass black holes we finish with two forms of energy, one constituting a final black hole and other in form of radiated energy. The amount of energy in the final black hole with respect of the total energy $2M$ can be obtained from (4),

$$\Omega_{bh} = \frac{M_f}{2M} = \frac{R_f/2}{R} = \frac{1}{\sqrt{2}} = .707;$$  \hfill (5)

consequently, the percentage of energy radiated away during this process is $\Omega_r = E_r/2M = 1 - 1/\sqrt{2} = .293$ ([10]). This two forms of energy, one associated to the event horizon of a final black hole and the other to ordinary energy, coincide with the two contributions to the energy of the universe reported by observations ([1]) in terms of an accelerating amount of energy $\Omega_\Lambda \sim 0.7$, and a decelerating one $\Omega_m \sim 0.3$.

We can relax the equality of the initial black hole masses; if we consider the union of two black holes of mass $M$ and $\alpha M$ with the same premises as before we can show that the amount of radiated energy is given by

$$\Omega_r = 1 - \frac{\sqrt{1 + \alpha^2}}{1 + \alpha},$$  \hfill (6)
that takes the maximum value for \( \alpha = 1 \). Also it is clear that without radiated energy \( (E_r = 0) \), the conservation of energy (1) and the conservation of entropy (4) are not compatible.

If we consider the entropy of the radiated energy \( S_r \) in the process of adiabatic coalescence between two equal mass black holes, our model allows us to relate the properties of the matter radiated (which according to the observation is made mainly by dark matter) with the value of \( \Omega_\Lambda \). Manipulating (1) and (3), this last with an equal sign, we obtain,

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\frac{E^2_r}{S_r} = \frac{1}{2\pi} \sqrt{1 - 2\Omega^2_\Lambda},
\]

Let us now elucidate the loopholes that the previous model has. In the case in which the formation of a single black hole is given by the coalescence of an arbitrary number \( n \) of initial ones we obtain \( \Omega_{bh} = 1/\sqrt{n} \); so the model accords with observation only if the dominant channel is the approximately equal mass pairing. A sort of bootstrap model can be envisaged in which the merging process is due to the contact of closed event horizons favoring the single pairing as the elementary mode. If the adiabatic merging of pairs of equal mass black holes is the elementary process of the percolation of black holes the percolation process will share the same partitioning for the total energy as the elementary one.

Our model predicts a constant relation between \( \Omega_\Lambda \) and \( \Omega_m \) during the process of percolation. The pairing process extends to all spatial scales and consequently has in principle an infinite duration in time. In order to respect primordial nucleosynthesis we must put an initial cosmic time for the beginning of the percolation process after primordial nucleosynthesis be developed in the standard way. There is an scale in this approach given by the size of the initial black hole that we can relate with the observational indications of a possible cutoff in the spectrum of multipoles of the cosmic microwave background radiation at low momentum ([11]).

Despite the previous problems the black hole fluid is a very interesting sort of vacuum of the underlying theory of quantum gravity. It saturates the entropy bounds, so it is the more entropic (and consequently anthropic) fluid we can construct. Also it is a good candidate for the initial state of the universe, and we have shown that using its properties we can explain the observed values for the two sort of energies that we observe, with simple
assumptions. It is a vacuum that seems to have in its nature different realization for accelerated and decelerated cosmologies. We think it merits a deeper study.

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