Can universality of the QCD evolution be checked in W boson decays into hadrons?

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Abstract

Hadron multiplicity from W boson is calculated in pQCD. The agreement of our theoretical predictions with the LEP data says in favor of universality of the QCD evolution in hard processes.

1 Introduction

Experiments at LEP and SLAC colliders have shown that the multiple production of hadrons in $e^+e^-$ annihilation depend on the mass of the primary (anti)quarks which launch the process of the QCD evolution. Let us consider a heavy quark induced event,

$$e^+e^- \rightarrow Q \bar{Q} \rightarrow X,$$  \hspace{1cm} (1)

where $Q$ means a heavy ($c$ or $b$) quark, and an $e^+e^-$ event induced by the pair of light quarks:

$$e^+e^- \rightarrow l \bar{l} \rightarrow X.$$  \hspace{1cm} (2)

Here and in what follows $l$ denotes $u$, $d$ or $s$-quarks which are assumed to be massless. In both cases, $X$ means a system of final hadrons.

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Let $N_{QQ}(W^2, m_Q^2)$ and $N_{ll}(W^2)$ be the average multiplicities of charged hadrons in $e^+e^-$ events with heavy (1) and light primary quarks (2), respectively. $W$ is the invariant energy of colliding leptons, $m_Q$ is the mass of the (anti)quark $Q$. It appeared that differences between the light and heavy quark induced multiplicities,

$$\delta_{Ql} = N_{QQ} - N_{ll}, \quad (3)$$

become independent of the collision energy $W$, but depend only on the heavy quark mass $m_Q$. QCD calculations describe the phenomenon quite well [1]-[3] (see also [4]).

The QCD calculations of the hadron multiplicities in $e^+e^-$ events, in particular, hadron multiplicity from $Z$ boson, are in a good agreement with the data. Their energy dependence is defined by the QCD evolution of the parton showers. The aim of the present paper is to calculate the hadron multiplicity from the $W$ boson in pQCD, and thus to check once more the universality of the QCD evolution in hard processes.

## 2 Hadron multiplicity from the W boson

From now on we will consider the multiple hadron production in $e^+e^-$ annihilation mediated by production of a pair of $W$ bosons. We will refer to $e^+e^-$ event as the heavy quark event if one of the $W$ bosons produces a lepton pair, while the other decays into hadrons via charm production, for instance:

$$e^+e^- \rightarrow W^+W^-, \quad W^+ \rightarrow c \bar{s} \rightarrow X, \quad W^- \rightarrow \mu^-\bar{\nu}_\mu, . \quad (4)$$

In the light quark event final hadrons are fragments of the light quark-antiquark pair produced by one of the $W$ bosons:

$$e^+e^- \rightarrow W^+W^-, \quad W^+ \rightarrow l^+\bar{\nu} \rightarrow X, \quad W^- \rightarrow W^- \rightarrow \mu^-\bar{\nu}_\mu, . \quad (5)$$

Let us define the average multiplicities of charged hadrons in the above mentioned processes as $N_{Ql}(W^2, m^2)$ and $N_{ll}(W^2)$, where $m$ is the mass of the heavy (charm) quark\footnote{For generality, we will often use the notation $Q$ in our formulae, having in mind that $Q$ means charm quark in the most of the cases ($Q = c$).} Our main goal is to establish a relation between these two multiplicities. Namely, we will calculate the difference

$$\Delta_{Ql} = N_{Ql} - N_{ll}, \quad (6)$$
analogous to quantity (3). Let us note that the multiplicity \( N_l \) is equal to the multiplicity in the \( e^+e^- \) event taken at the energy \( W = m_W \), i.e. \( N_l = N_l(W^2 = m_W^2) \), where \( m_W \) is the \( W \) boson mass.

Hadron multiplicity in a heavy quark event is represented by the following equation:

\[
N_{Ql}(m_W^2, m^2) = (n_Q + n_l) + \tilde{N}_{Ql}(m_W^2, m^2),
\]

with

\[
\tilde{N}_{Ql}(m_W^2, m^2) = \frac{1}{N_0} \int \frac{d^4q}{(2\pi)^2} \delta(q^2 - m_W^2) \delta((p_1 + p_2 - q)^2 - m_W^2) \\
\times \Pi_{\mu\nu}(p_1, p_2, q) \int \frac{d^4k}{(2\pi)^4 k^2} \Phi^{\mu\nu}(q, k, m) n_g(k^2),
\]

where we have assumed that both bosons are on-shell. Two first terms in the r.h.s. of Eq. (7), are the multiplicities from the leading (anti)quark \( Q \) and \( l \). They are known from the data. The tensor \( \Pi_{\mu\nu}(p_1, p_2, q) \) describes the process \( e^+(p_1) + e^-(p_2) \rightarrow W^+(q) + W^-(p_1 + p_2 - q) \), where \( \mu, \nu \) are the Lorentz indices of the \( W \) boson (see Fig. 1). Two blobs in Fig. 1 are tree diagrams with the \( \gamma/Z^0 \) exchange in the \( s \)-channel, and electronic neutrino exchange in the \( t \)-channel. The tensor \( \Phi^{\mu\nu}(q, k, m) \)

\[\text{Figure 1: The diagram describing the process } e^+(p_1) + e^-(p_2) \rightarrow W^+(q) + W^-(p_1 + p_2 - q), \text{ where } \mu, \nu \text{ are Lorentz indices of the } W \text{ boson (wavy lines). The cut line of the } W \text{ boson means that it is on-shell particle.} \]

\[\text{An account of the } W \text{ boson distribution in its invariant mass } q^2 \text{ will be discussed below (see Eqs. (50)-(53)).} \]

\[2\]
Figure 2: The inclusive distribution of the “massive” gluon jet with the 4-momentum $k$ (spiral line). The wavy line is the $W$ boson, whose 4-momentum is $q$, and Lorentz indices are $\mu$, $\nu$.

of the hadrons in the gluon jet with the invariant mass $k^2$. $N_0$ is the normalization factor.

It is convenient to use Lorentz gauge in which the tensor part of the $W$ boson propagator has the form:

$$d_{\mu\nu}(q) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2},$$

then

$$q^\mu \Pi_{\mu\nu}(p_1, p_2, q) = q^\nu \Pi_{\mu\nu}(p_1, p_2, q) = 0. \quad (10)$$

The quantity $\Phi_{\mu\nu}$ has the following tensor structure:

$$\Phi_{\mu\nu}(q, k, m) = (-g_{\mu\nu} q^2) C(q^2, k^2, qk, m^2) + k_\mu k_\nu D(q^2, k^2, qk, m^2)$$

$$+ \text{(terms } \sim q_\mu, q_\nu) + \text{(term } \sim \epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta). \quad (11)$$

Due to condition (10), terms proportional to $q_\mu$ or $q_\nu$ gives zero contribution to $N_{QI}$ after convolution in Lorentz indices with the tensor $\Pi_{\mu\nu}$.

In the first order in the strong coupling constant, $\Phi_{\mu\nu}(q, k, m)$ is represented by the sum of three QCD diagrams presented in Figs. 3, 4, and 5 (the crossed diagram is taken with the factor 2).

One can use the following useful relation

$$\int d^4k k_\mu k_\nu D(q^2, k^2, qk, m^2) = (-g_{\mu\nu} q^2) \int d^4k \frac{(qk)^2 - q^2k^2}{3q^4}$$

$$\times D(q^2, k^2, qk, m^2) + \text{(term } \sim q_\mu q_\nu). \quad (12)$$
Figure 3: The inclusive distribution of the massive gluon jet with the virtuality $k^2$. The wavy line is the $W$ boson, whose 4-momentum is $q$. The thick quark line is a heavy quark, while the thin line is a light quark. The cut quark lines mean that these quarks are on-shell quarks.

Figure 4: The same as in Fig. 10, but with the gluon jet emitted by the light quark.

Detailed analytical calculations of the diagrams in Figs. 3-5 show that

$$ D = -\frac{q^4}{(qk)^2 - q^2k^2} C, \tag{13} $$

up to small power-like corrections of the type $O(m^2/m_W^2)$ and $O(k^2/m_W^2)$. The corrections $O(k^2/m_W^2)$ can be neglected since it is the region $k^2 \sim m^2$ that makes the leading contribution to our main integral (25) (see Eq. (36) and a comment after it).

\footnote{Explicit analytic expressions for the functions $C(q^2, k^2, qk, m^2)$ and $D(q^2, k^2, qk, m^2)$ are quite complicated to be shown here. That is why, we will present only the final expression (27).}
Figure 5: The interference diagram which also contributes to the inclusive distribution of the gluon jets with the virtuality $k^2$ in the $W$ boson.

The normalization factor $N_0$ in (8) is given by the expression

$$N_0 = \frac{1}{3\pi} \left(1 - \frac{m^2}{m_W^2}\right)^2 \left(1 + \frac{m^2}{2m_W^2}\right) \times \int \frac{d^4q}{(2\pi)^2} \delta(q^2 - m_W^2) \delta((p_1 + p_2 - q)^2 - m_W^2) \left(-g^{\mu\nu}q^2\right) \Pi_{\mu\nu}$$

$$\simeq \frac{1}{3\pi} \int \frac{d^4q}{(2\pi)^2} \delta(q^2 - m_W^2) \delta((p_1 + p_2 - q)^2 - m_W^2) \left(-g^{\mu\nu}q^2\right) \Pi_{\mu\nu} \cdot (14)$$

Let us define

$$\tilde{C} = C \sqrt{(qk)^2 - m_W^2 k^2} \left(\frac{m_W^2}{(2\pi)^2 q^2}\right). \quad (15)$$

Then we derive from (8), and (11)-(14):

$$\tilde{N}_{Ql}(m_W^2, m^2) = \int_{Q_0^2}^{(m_W^2-m^2)^2} \frac{dk^2}{k^2} n_g(k^2) \int \left(\frac{dq}{(qk)^2} \sqrt{(qk)^2 - m_W^2} \left(-g^{\mu\nu}q^2\right) \Pi_{\mu\nu}\right). \quad (16)$$

The integration limits for the variable $qk$ are:

$$(qk)_- = m_W\sqrt{k^2},$$

$$(qk)_+ = (m_W^2 + k^2 - m^2)/2. \quad (17)$$

Note that the antisymmetric part of $\Phi_{\mu\nu}$ (i.e. the last term in Eq. (11)) gives zero contribution to $N_{Ql}$.\"
The hadron multiplicity from the gluon jet with the fixed virtuality \( k^2 \) is related with the multiplicity from the gluon jet whose virtuality varies up to \( k^2 \) [3]:

\[
    n_g(k^2) = \left(k^2 \frac{\partial}{\partial k^2}\right) N_g(k^2) .
\] (18)

An explicit analytical expression for \( \tilde{C}(m_W^2, k^2, qk) \) shows that

\[
    \tilde{C}(m_W^2, k^2, qk, m^2) \bigg|_{qk=(qk)_-} = \tilde{C}(m_W^2, k^2, qk, m^2) \bigg|_{qk=(qk)_+} = 0 .
\] (19)

After introducing the notation

\[
    G(m_W^2, k^2, qk, m^2) = \left(- k^2 \frac{\partial}{\partial k^2}\right) \tilde{C}(q^2, k^2, qk, m^2) ,
\] (20)

we come to the following equation:

\[
    \tilde{N}_{Ql}(m_W^2, m^2) = \int_{Q_0^2}^{(m_W-m)^2} \frac{dk^2}{k^2} N_g(k^2) \int_{(qk)_-}^{(qk)_+} d(qk) G(m_W^2, k^2, qk, m^2) .
\] (21)

Let us define

\[
    \int_{(qk)_-}^{(qk)_+} d(qk) G(m_W^2, k^2, qk, m^2) = C_F \frac{\alpha_s(k^2)}{\pi} E_{Ql}(m_W^2, k^2, m^2) ,
\] (22)

where \( C_F = (N_c^2 - 1)/(2N_c) \), and \( N_c \) is a number of colors. \( E_{Ql}(m_W^2, k^2, m^2) \) is the inclusive spectrum of the gluon jet emitted by \( Q\bar{l} \)-quark pair (see Ref. [1, 2] for more details). Then the multiplicity in heavy quark event [21] can be represented in the form:

\[
    N_{Ql}(m_W^2, m^2) = (n_Q + n_l) + C_F \int_{Q_0^2}^{(m_W-m)^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{\pi} N_g(k^2) E_{Ql}(m_W^2, k^2, m^2) .
\] (23)

Correspondingly, the hadron multiplicity in the light quark event is given by

\[
    N_l(m_W^2) = 2n_l + C_F \int_{Q_0^2}^{m_W^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{\pi} N_g(k^2) E_l(m_W^2, k^2) ,
\] (24)
where \( E_l(m_W^2, k^2) \equiv E_Ql(m_W^2, k^2, 0) \). As a result, we obtain the multiplicity difference:

\[
\Delta_{Ql} = (n_Q - n_l) - \int_{Q_0^2}^{m_W^2} \frac{dk^2}{k^2} N_2(k^2) \Delta E_{Ql}(k^2, m^2),
\]

(25)

where the following notation is introduced:

\[
\Delta E_{Ql}(k^2, m^2) = E_l(m_W^2, k^2) - E_Ql(m_W^2, k^2, m^2).
\]

(26)

Analytical calculations of the diagrams in Figs. 3, 4, and 5 result in the following expression:

\[
G(q^2, k^2, qk, 0) - G(q^2, k^2, qk, m^2) = C_F \frac{\alpha_s(k^2)}{\pi} \frac{8 m^2 qk [k^2(q^2 - 2qk) + 2m^2(q^4 - 2q^2 k + 2(kq)^2)(kq)^2]}{q^2[k^2 q^2 (q^2 - 2qk) + 4m^2(qk)^2]}. \]

(27)

As before, we have omitted small power-like corrections of the type \( O(m^2/m_W^2) \) and \( O(k^2/m_W^2) \).

One can conclude from Eqs. (22), (27) that \( \Delta E_{Ql}(k^2, m^2) \) is a function of the dimensional variable

\[
\rho = \frac{k^2}{m^2}.
\]

(28)

Indeed, let us define:

\[
qk = \frac{[q^2(1 - x) + k^2 - m^2]/2}.
\]

(29)

The new variable \( x \) varies within the limits:

\[
0 \leq x \leq x_{\text{max}} = \left(1 - \sqrt{k^2/m_W}\right)^2 - m^2/m_W^2.
\]

(30)

One can safely set \( x_{\text{max}} = 1 \), and obtain

\[
\Delta E_{Ql}(\rho) = \frac{1}{2} \int_0^1 dx (1 - x) \frac{(1 - x)^2 (1 + x^2) + 4 x^2 \rho}{[(1 - x)^2 + x \rho]^2}
\]

\[
= \frac{1}{4} [2 + \rho (3\rho - 2)] \ln \frac{1}{\rho} + \frac{1}{4} (5 + 6\rho)
\]

\[
+ \frac{1}{2} \rho (3\rho - 8) J(\rho) + 6 \frac{1 - J(\rho)}{\rho - 4},
\]

(31)

\footnote{Remember that we have to put \( q^2 = m_W^2 \) in (27).}
\[ J(\rho) = \begin{cases} \sqrt{\frac{\rho}{\rho-4}} \ln \left( \frac{\sqrt{\rho} + \sqrt{\rho-4}}{2} \right), & \rho > 4, \\ 1, & \rho = 4, \\ \sqrt{\frac{\rho}{4-\rho}} \arctan \left( \frac{\sqrt{4-\rho}}{\rho} \right), & \rho < 4. \end{cases} \]  

(32)

Remember that we considered the process \( e^- e^+ \to W^+ W^- \) and compared two possible subsequent hadronic decays of one of the bosons, \( W^+ \to c\bar{d} \ (c\bar{s}) + \) (gluon jets), and \( W^+ \to u\bar{d} \ (u\bar{s}) + \) (gluon jets). The quantity \( \Delta E_d(k^2/m_c^2) \) describes the difference of the distributions of the gluon jets in their invariant mass \( k^2 \) in these processes (\( m_c \) is the mass of the charm quark). Let us stress that Eq. (31) coincides with the expression for the difference of the gluon jet distributions derived in the case when the \( W^+ \) boson is a product of the top weak decay \( t \to b + W^+ \) (see Eq. (35) from Ref. [5]).

In terms of variables
\[ y = \ln \frac{m^2}{k^2} \equiv \ln \frac{1}{\rho}, \]  

(33)

and
\[ Y = \ln \frac{m^2}{Q_0^2}, \]  

(34)

the multiplicity difference looks like
\[ \Delta_Q = (n_Q - n_t) - \int_{-\infty}^{y} dy N_g(Y - y) \Delta E_Q(y), \]  

(35)

with \( E_Q \) defined by Eqs. (31), (32). The function \( \Delta E_Q(y) \) is shown in Fig. 6.

Since
\[ \Delta E_Q(y) \left|_{y\to-\infty} \right. \approx \frac{11}{3} e^{-|y|}, \]  

(36)

the integral in (35) converges rapidly at the lower limit. Asymptotics of \( \Delta E_Q(y) \) at large \( y \) is the following:
\[ \Delta E_Q(y) \left|_{y\to\infty} \right. \approx \frac{1}{2} \left( y - \frac{1}{2} \right). \]  

(37)
In Ref. [1] the analogous formula for the multiplicity difference in \( e^+e^- \) events not mediated by the \( W \) bosons (3) was derived:

\[
\delta_{Ql} = 2(n_Q - n_l) - \int_{-\infty}^{Y} dy N_g(Y - y) \Delta E_Q(y) ,
\]

(38)

where

\[
\Delta E_Q(\rho) = 1 + \rho \left( \frac{7}{2} \rho - 3 \right) \ln \frac{1}{\rho} + \left( \frac{9}{2} + 7\rho \right)
\]

\[
+ \rho \left( 7\rho - 20 \right) J(\rho) + 20 \frac{1 - J(\rho)}{\rho - 4} ,
\]

(39)

with \( J(\rho) \) defined above (32).

The function \( \Delta E_Q(y) \) has the following asymptotics at \( y \to \infty \):

\[
\Delta E_Q(y) \bigg|_{y \to \infty} \simeq y - \frac{1}{2} .
\]

(40)

As one can see from Eqs. (40), \( \Delta E_Q(y) = \Delta E_{Ql}(y)/2 \) at large \( y \). Moreover, numerical calculations show that \( \Delta E_{Ql} \) is very close to \( \Delta E_Q/2 \) at all \( y \) (see Fig. 7). Thus, we get the prediction:
Figure 7: The function $\Delta E_Q(y)$ (solid curve) vs. function $2\Delta E_{Ql}(y)$ (dashed curve).

$$\Delta_Q = \frac{1}{2} \delta_Q .$$

Now we can calculate the total hadron multiplicity from the $W$ boson:

$$N_W = N_l(m_W) + \frac{|V_{cd}|^2 + |V_{cs}|^2}{|V_{ud}|^2 + |V_{us}|^2 + |V_{cd}|^2 + |V_{cs}|^2} (N_{cl} - N_l)$$

$$= N_l(m_W) + \frac{1}{4} \delta_{cl} .$$

For our numerical estimates, we will use the corrected experimental value of $\delta_{cl}$ from Ref. [4]:

$$\delta_{cl} = 1.03 \pm 0.34$$

The light quark multiplicity at the energy $W = m_W$ was recently estimated to be [5]:

$$N_l(m_W) = 19.09 \pm 0.18 ,$$

that results in

$$N_W = 19.34 \pm 0.21 .$$

Let us estimate effects associated with $W$ boson decays into $u\bar{b}$ and $c\bar{b}$-pairs. The account of the $b$-quark production increase the multiplicity from the $W$ boson [12] by the quantity $\Delta N_W^b$, i.e. $N_W \rightarrow N_W + \Delta N_W^b$, where

$$\Delta N_W^b = |V_{ub}|^2 (N_{ul} - N_l) + |V_{cb}|^2 (N_{bc} - N_l) .$$
The first term in (46) is equal to

$$\frac{1}{2} |V_{ub}|^2 \delta_{bl} .$$

(47)

In order to estimate the second term, we will use the fact that the emission of the massive gluon jets by heavier quark is suppressed, that results in $E_{bc} < E_{bl}$. Thus, we get the inequality:

$$N_{bc} - N_l < N_{bl} - N_l = (n_c - n_l) + \frac{1}{2} \delta_{bl} .$$

(48)

Using the average values of $n_c = 2.6$, $n_l = 1.2$, and $\delta_{bl} = 3.12$, as well as CKM matrix elements $|V_{ub}| = 3.95 \cdot 10^{-3}$, $|V_{cb}| = 38.6 \cdot 10^{-3}$ we obtain from (46)-(48) that

$$\Delta N_b^W < 1.2 \cdot 10^{-2} .$$

(49)

It means that one can ignore the contribution to $N_W$ from the multiplicity difference between $ub(cb)$-event and light quark event.

Now let us take into account that the $W$ boson with the 4-momenta $q$ in Fig. 1 is not on-shell, but has a distribution in its invariant mass $q^2$. The denominator of the $W$ boson propagator is equal to

$$q^2 - m_W^2 + i m_W \Gamma_W ,$$

(50)

where $\Gamma_W$ is the full $W$ width. As a result, the hadron multiplicity from the $W$ boson is given by the formula:

$$N_W - \frac{1}{4} \delta_{el} = \frac{1}{H} \int_{Q_0^2}^{(W-m_W)^2} dq^2 \frac{1}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} F(W^2, q^2) N_l(q^2) ,$$

(51)

where

$$H = \int_{Q_0^2}^{(W-m_W)^2} dq^2 \frac{1}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} F(W^2, q^2) .$$

(52)

The function $F(W^2, q^2)$ is defined by the invariant part of the diagram in Fig. 1 integrated in $d^3q$ at fixed $q^2$. It can be chosen to be dimensionless i.e. dependent on the ratio $q^2/W^2$.

\textsuperscript{6}Since $q^2$-independent dimensional factors may be safely omitted in $F$. 

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The main contributions to the integrals in (51), (52) come from the region $q^2 - m_W^2 \sim m_W \Gamma_W$. The argument of the function $F(q^2/W^2)$ varies rather slowly in this region, contrary to the factor $[(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2]^{-1}$ which has a sharp peak around the point $q^2 = m_W^2$. Thus, we can assume that an explicit form of $F(q^2/W^2)$ is not important and put $F = 1$ in Eqs. (51), (52).

By using a fit of the data on light quark multiplicity $N_l(E_2)$, we can estimate a possible variation of the multiplicity (51). The numerical calculations show that at $W = 183$ GeV (collision energy of LEPII experiments on $W^+W^-$ production [7]-[8]) it differs from $N_l(m_W)$ by the quantity $0.05$. Then we sum the uncertainty $2 \times 0.05 = 0.10$ with the errors of $N_W$ (45) in quadrature and obtain:

$$N_l(m_W) = 19.34 \pm 0.23.$$  \hspace{1cm} (53)

Our result (53) is in a good agreement with the experimental data from OPAL [7],

$$N_W = 19.3 \pm 0.3 \pm 0.3,$$  \hspace{1cm} (54)

and DELPHI [8],

$$N_W = 19.44 \pm 0.13 \pm 0.12.$$  \hspace{1cm} (55)

Formulae (41) and (42) is our main theoretical result. Let us underline that these equations relate measurable quantities, and they do not depend on the explicit form of the hadron multiplicity from the gluon jet $N_g(k^2)$.

Our main prediction is

$$\Delta cl = N_{cl} - N_l = 0.52 \pm 0.17.$$  \hspace{1cm} (56)

This prediction can be checked by using available LEPII data on hadron multiplicities in $W^+W^- \rightarrow q\bar{q}' l\bar{\nu}_l$ and $W^+W^- \rightarrow q\bar{q}' q\bar{q}'$ events [7,8] as well as future data from the ILC. It will be one more experimental test of the universality (i.e. the process-independent character) of the QCD evolution in the multiple hadron production.

Both the multiplicity (53), and multiplicity difference (56) can be also measured at the LHC. One of the processes to look for mass-dependent effects in the QCD evolution is a single $W$ production with its subsequent decay into hadrons. Another possibility is a production of two $W$’s, one of which decays in a lepton mode, while the other goes into hadrons with (without) charm production.

\footnote{This difference becomes smaller with the increase of $W$.}
3 Conclusions

In the present paper we have calculated the hadron multiplicity from $W$ boson in pQCD. The difference of the hadron multiplicities in the $W$-boson hadronic decays with and without charm production is also estimated. Previously [1]-[3] the multiplicity difference in $b\bar{b}$ ($c\bar{c}$)-event and light quark event was calculated on the same ground. The nice agreement of these theoretical predictions with the LEP data confirms the universal character of the mechanism of multiple hadron production in QCD via evolution of quark-gluon showers.

References

[1] V.A. Petrov and A.V. Kisselev, Z. Phys. C 66, 453 (1995); Nucl. Phys. (Proc. Suppl.) B, 39, C, 364 (1995).

[2] A.V. Kisselev and V.A. Petrov, Eur. Phys. J. C, 50, 21 (2007).

[3] A.V. Kisselev and V.A. Petrov, Phys. Part. Nucl. 39, 798 (2008).

[4] Yu.L. Dokshitzer, F. Fabbri, V.A. Khoze, and W. Ochs, Eur. Phys. J. C 45, 387 (2006).

[5] A.V. Kisselev and V.A. Petrov, PMC Physics A 2, 3 (2008).

[6] Review of Particle Physics, Phys. Lett. B 667, 1 (2008).

[7] G. Abbiendi et al. (OPAL Collaboration), Phys. Lett. B 453, 153 (1999).

[8] P. Abreu et al. (DELPHI Collaboration), Eur. Phys. J. C 18, 203 (2000).