Minimum Variability Time Scales of Long and Short GRBs

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ABSTRACT
We have investigated the time variations in the light curves from a sample of long and short Fermi/GBM Gamma ray bursts (GRBs) using an impartial wavelet analysis. The results indicate that in the source frame, that the variability time scales for long bursts differ from that for short bursts, that variabilities on the order of a few milliseconds are not uncommon, and that an intriguing relationship exists between the minimum variability time and the burst duration.

Key words: Gamma-ray bursts

1 INTRODUCTION
The prompt emission from Gamma-ray Bursts (GRBs) shows complex time profiles that have eluded a generally accepted explanation. Fenimore & Ramirez-Ruiz (2000) reported a correlation between variability of GRBs and the peak isotropic luminosity. The existence of the variability-luminosity correlation suggests that the prompt emission light curves have embedded temporal information related to the microphysics of GRBs. Several models have been proposed to explain the observed temporal variability of GRB light curves. Leading models such as the internal shock model (Kobayashi, Piran, and Sar 1997) and the photospheric model (Ryde 2004) link the rapid variability directly to the activity of the central engine. Others invoke relativistic outflow mechanisms to suggest that local turbulence amplified through Lorentz boosting leads to causally disconnected regions which in turn act as independent centers for the observed prompt emission. Within more recent models, both Morsony et al. (2010) and Zhang & Yan (2011) argue that the temporal variability may show two different scales depending on the physical mechanisms generating the prompt emission.

In order to further our understanding of the prompt emission phase of GRBs and to explicitly test some of the key ingredients in the various models, it is clearly important to extract the variability for both short and long gamma-ray bursts in a robust and unbiased manner. It is also clear that the chosen methodology should not only be mathematically rigorous but also be sufficiently flexible to apply to transient phenomena with multiple time scales and a wide dynamic range. A wide dynamic range is naturally provided by the bimodal separation of GRB duration occurring at $T_{90} = 2$ sec as observed by Kouveliotou et al. (1993) to distinguish between long and short duration GRBs.

In this paper, we extract variability time scales for GRBs using a method based on wavelets. The technique for such a temporal analysis is universal, and has the advantage over Fourier analysis that transients and frequency correlations can be more easily picked out in the data. Results presented herein were compared with Bhat et al. (2012) who gave pulse parameters for approximately 400 pulses obtained from 34 GRBs. It was shown by MacLachlan et al. (2012) that the minimum variability time scale tracks the rise times of pulses very well for over three orders of magnitude. The relation of minimum variability time scales to pulse parameters has been extended to four orders of magnitude by Sonbas et al. (2012) who applied the present technique to analyze X-ray flares.

The time scales being investigated here have power densities very near to that of the noise in the data which makes extracting these time scales nontrivial. A somewhat older but still interesting discussion of extracting signal in a noisy environment can be found in Scargle (1982) and a more recent discussion found in Kostelich & Schreiber (1993). The technique we offer is not necessarily new but is different from
previous published wavelet analyses [Fritz & Bruch 1998; Waller et al. 2000; Tamburini et al. 2009; Anzolin et al. 2010] in that we apply a number of modifications suggested by various authors [Addison 2002; Coifman 1995; Percival 2000; Strang and Nguyen 1997] as explained further in Sec 4.

The layout of the paper is as follows: the source of the data is described in section 2; the main aspects of the wavelet methodology are outlined in section 3; in section 4 we provide the details of the data analysis; in section 5 we present and discuss our main findings; finally, in section 6, we summarize our conclusions.

2 DATA
The Gamma-Ray Burst Monitor (GBM) on board Fermi observes GRBs in the energy range 8 keV to 40 MeV. The GBM is composed of 12 thallium-activated sodium iodide (NaI) scintillation detectors (12.7 cm in diameter by 1.27 cm thick) that are sensitive to energies in the range of 8 keV to 1 MeV, and two bismuth germanate (BGO) scintillation detectors (12.7 cm diameter by 12.7 cm thick) with energy coverage between 200 keV and 40 MeV. The GBM detectors are arranged in such a way that they provide a significant view of the sky (Meegan et al. 2009).

In this work, we have extracted light curves for the GBM NaI detectors over the entire energy range (8 keV - 1 MeV, also including the overflow beyond 1 MeV). Typically, the brightest three NaI detectors were chosen for the extraction. Lightcurves for both long and short GRBs were extracted at a time binning of 200 microseconds. The long GRBs were extracted over a duration starting from 20 seconds before the trigger and up to about 50 seconds after the $T_{90}$ for the burst without any background subtraction. For short GRBs, durations were chosen to be 20 seconds before the trigger and 10 seconds after the $T_{90}$. The $T_{90}$ durations were obtained from the Fermi GBM-Burst Catalog [Paciesas et al. 2012].

3 METHODOLOGY
We report on our model-independent statistical investigation of the variability of Fermi/GBM long and short GRBs. We extract this information by using a fast wavelet transformation to encode GRB light curves into a wavelet representation and then compute a statistical measure of the variance of wavelet coefficients over multiple time-scales.

3.1 Minimum Variability Time Scales
It is often the case when multiple processes are present that one process will dominate the others at certain time scales but those same processes may exchange dominance at other time scales. A wavelet technique is useful in these situations because the variances of wavelet coefficients are sensitive to whichever processes dominate the light curve at a given time scale. Moreover, the technique can be used to classify those dominant processes as well as provide a means to determine the characteristic time scale, $\tau_\beta$, for which the processes exchange dominance. Determination of $\tau_\beta$ helps in the development of theoretical models and the understanding of observational data. Indeed, if there is a transition from a time-scaling region to that of white noise then there is a smallest variability time for the physical processes present.

3.2 Wavelet Transforms
Wavelet transformations have been shown to be a natural tool for multi-resolution analysis of non-stationary time-series [Flandrin 1989, 1992; Mallat 1989]. Wavelet analysis is similar to Fourier analysis in many respects but differs in that wavelet basis functions are well-localized, i.e. have compact support, while Fourier basis functions are global. Compact support means that outside some finite range the amplitude of wavelet basis functions goes to zero or is otherwise negligibly small (Percival 2000). In principle, a wavelet expansion forms a faithful representation of the original data, in that the basis set is orthonormal and complete.

3.2.1 Discrete Dyadic Wavelet Transforms
Given the discrete nature of the data, we employ a discrete wavelet analysis. The rescaled-translated nature of the wavelet basis functions make the wavelet transform well-localized in both frequency and time, which results in an insensitivity to background photon counts expressed by polynomial fits. The level of insensitivity, formally known as the vanishing moment condition, can be adjusted by the choice of wavelet basis function. By construction, the discrete wavelet transform is a multi-resolution operation (Mallat 1989). Such wavelets, denoted $\psi_{j,k}(t)$, form a dyadic basis set, i.e. wavelets in the set have variable widths and variable central time positions.

The wavelet analysis employed in this study, as with the fast Fourier transform, begins with a light curve with $N$ elements,

$$X_i = \{X_0 \ldots X_{N-1}\}.$$  

Figure 1. GBM GRB080925775. Preburst portion of the light curve, used for background removal, is shown in gray. The burst portion, from which a time scale is extracted, is shown in black.
where $N$ is an integer power of two. The light curve is convolved with a scaling function, $\phi_{j,k}(t)$, and wavelet function, $\psi_{j,k}(t)$ which are rescaled and translated versions of the original scaling and wavelet functions $\phi(t) = \phi_{0,0}$, and $\psi(t) = \psi_{0,0}$. Translation is indexed by $k$ and rescaling is indexed by $j$. The rescaling and translation relation is given by

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k).$$

The precise forms of the scaling and wavelet functions are not unique. The choices are made according to the features one wishes to exploit (Percival 2000; Addison 2002). The scaling function acts as a smoothing filter for the input time-series and the wavelet function probes the time-series for detail information at some time scale, $\Delta t$, which is twice that of the finest binning of the data, $T_{\text{bin}}$. In the analysis, the time scale is doubled

$$\Delta t \rightarrow 2\Delta t$$

and the transform is repeated until

$$\Delta t = NT_{\text{bin}}.$$

In this analysis we choose the Haar (Addison 2002) scaling/wavelet basis because it has the smallest possible support, has one vanishing moment, and is equivalent to the original scaling and wavelet functions indexed by $N$ where $N$ is an integer power of two. The light curve is convolved with Haar scaling and wavelet functions $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$.

### 3.2.2 The Haar Wavelet Basis

Convolving the light curve, $X$, with the scaling functions yields approximation coefficients,

$$a_{j,k} = \langle \phi_{j,k}, X \rangle.$$

Interrogating $X$ with the wavelet basis functions yields scale and position dependent detail coefficients,

$$d_{j,k} = \langle \psi_{j,k}, X \rangle.$$

It is interesting to point out that for the trivial $N = 2$ case the Haar wavelet transform and the Fourier transform are identical.

### 3.3 Logscale Diagrams and Scaling

Logscale diagrams are useful for identifying scaling and noise regions. Construction of a logscale diagram for each GRB proceeds from the variance of detail coefficients (Flandrin 1992),

$$\beta_j = \frac{1}{n_j} \sum_{k=0}^{n_j-1} |d_{j,k}|^2,$$

where the $n_j$ are the number of detail coefficients at a particular scale, $j$. A plot of $\log_2 \beta_j$ versus scale, $j$, takes the general form

$$\log_2 \beta_j = \alpha j + \text{constant},$$

and is known as a logscale diagram. A linear regression is made of each logscale diagram and the slope parameter, $\alpha$, (deicting a measure of scaling) is estimated. White-noise processes appear in logscale diagrams as flat regions while non-stationary processes appear as sloped regions with the following condition on the slope parameter, $\alpha > 1$ (Abry et al. 2003; Percival 2000; Flandrin 1992).

### 4 DATA ANALYSIS

#### 4.1 Background Subtraction

We now present a method for removing photometric background due to noise not intrinsic to the GRB so that physical variability arising from the GRB remains for further analysis. Background subtraction for a statistical analysis of variability via wavelet transforms should proceed in the space variances as opposed to a traditional flat or linear subtraction of counts. This owes to the fact that Haar detail coefficients are insensitive to polynomial trends in the signal up to first order. Subtraction of a flat or linear background from a light curve is an operation under which the wavelet transform is invariant (as are Fourier transforms) apart from the mean signal coefficient.

The GRB light curves show power at various variability time scales. Most often, there is a region of the logscale diagram (log-power versus log-varibility time) with a single slope, indicating scaling in the power over those variability-times, and a flat region at the shortest variability times, indicating the presence of white-noise. Some of this white-noise may be intrinsic to the GRB. Some may be attributed to instrumental noise and to background emissions from sources not including the GRB in question. We therefore express the variability of the burst, $\beta_{\text{burst},j}$, at time scales $j$ as comprising of individual variances: a scaling component, $\beta_{\text{scaling},j}$; an intrinsic noise component, $\beta_{\text{noise},j}$; and a background component, $\beta_{\text{background},j}$. The variability of the burst can then be described as a linear combination of the component variances so long as the components have vanishing covariances. In this event we write,

$$\beta_{\text{burst},j} = \beta_{\text{scaling},j} + \beta_{\text{noise},j} + \beta_{\text{background},j}. \quad (7)$$

The minimum variability time scale, $\tau_{\beta}$, is identified from a logscale diagram by the octave, $j_{\text{intersection}}$, of the intersection of the flat intrinsic noise domain, $\beta_{\text{noise},j}$, with the sloped scaling domain, $\beta_{\text{scaling},j}$, as

$$\tau_{\beta} \equiv T_{\text{bin}} \times 2^{j_{\text{intersection}}}. \quad (8)$$

In practice, the octave at which the intersection occurs is determined by equating the polynomial fits to the flat intrinsic noise domain and the sloped scaling domain and solving for $j_{\text{intersection}}$. The uncertainty in $\tau_{\beta}$ is determined by propagating the uncertainty in the parameters from the fits to $\beta_j$ which in turn follow from a bootstrap procedure described in Sec. 4.1.2 and Sec. 4.1.3. It is at this time scale, $\tau_{\beta}$, that a structured physical process appears to give way to one that is stochastic and unstructured. Clearly one seeks to remove $\beta_{\text{background},j}$ from Eq. 4 to arrive at the cleanest possible signal,

$$\beta_{\text{burst},j} \rightarrow \beta_{\text{clean},j} \equiv \beta_{\text{burst},j} - \beta_{\text{background},j} = \beta_{\text{scaling},j} + \beta_{\text{noise},j}. \quad (9)$$

In order to estimate the variance of the background during the burst, we will assume that the variance obtained from a preburst portion of the light curve can serve as an acceptable surrogate for the background variance. That is,
and then the background is removed from the signal according to the relation,
\[ \log_2(\beta^\text{clean}_j) = \log_2(\beta^\text{burst}_j) - \beta^\text{preburst}_j. \]

A simple algebraic manipulation of Eq. (11) gives a form,
\[ \log_2(\beta^\text{clean}_j) = \log_2(\beta^\text{burst}_j) + \log_2\left(1 - \frac{\beta^\text{preburst}_j}{\beta^\text{burst}_j}\right). \]

For long GRBs, the preburst is defined relative to a 0 s trigger time as T-20 s to T-5 s and for short GRBs the preburst is defined to be from T-15 s to T-1 s. Here T is the trigger time of the burst.

4.1.1 Statistical Uncertainties

We have considered the statistical uncertainties in the light curve by a typical bootstrap approach in which the square root of the number of counts per bin is used to generate an additive poisson noise. A new poisson noise is considered for each iteration through the bootstrap process. More significant contributions to the uncertainties are discussed in Sec. 4.1.2 and Sec. 4.1.3.

4.1.2 Circular Permutation

Spurious artifacts due to incidental symmetries resulting from accidental misalignment (Percival 2000; Coifman 1992) of light curves with wavelet basis functions are minimized by circularly shifting the light curve against the basis functions. Circular shifting is a form of translation invariant denoising (Coifman 1992). It is possible a shift will introduce additional artifacts by moving a different symmetry into a susceptible location. Thus, our approach is to circulate the signal through all possible values, or at least a representative sampling, and then take an average over the cases which do not show spurious correlations.

4.1.3 Reverse-Tail Concatenation

Both discrete Fourier and discrete wavelet transformations imply an overall periodicity equal to the full time-range of the input data. This can be interpreted to mean that for a series of N elements, \( \{X_0, X_1 \ldots X_{N-1}\} \) then \( X_0 \) is made a surrogate for \( X_N \) and \( X_1 \) is made a surrogate for \( X_{N+1} \), and so forth. This assumption may lead to trouble if \( X_0 \) is much different from \( X_{N-1} \). In this case, artificially large variances may be computed. Reverse-tail concatenation minimizes this problem by making a copy of the series which is then reversed and concatenated onto the end of the original series resulting in a new series with a length twice that of the original. Instead of matching boundary conditions like,
\[ X_0, X_1, \ldots X_{N-1}, X_0, \]
we match boundaries as,
\[ X_0, X_1, \ldots X_{N-1}, X_{N-1}, \ldots X_1, X_0. \]

Note that the series length has thus artificially been increased to 2N by reversing and doubling of the original series. Consequently, the wavelet variances at the largest scale in a logscale diagram reflect this redundancy. This is the reason the wavelet variances at the largest scale are excluded from least-squares fits of the scaling region.

Another difficulty in wavelet expansions is that the initialization procedure of the multi-resolution algorithm may pollute the detail coefficients at the finest scale (see Strang and Nguyen 1997; Abry et al. 2003). For this reason we follow the advice of Abry et al. (2003) and discard the detail coefficients at the finest scale.

4.2 Simulation

The efficacy of this background subtraction method and the sensitivity to signal to noise was tested using simulated data in the form of fractional Brownian motion (fBm) time series that were first discussed by Mandelbrot (1968). One advantage of using fBms for simulation of time series data is that short duration, statistically significant fluctuations which trigger the identification of a minimum variability time scale arise naturally as part of the random process which produces them. Another is that fBms have a scaling parameter, \( \alpha \), which is easily varied.

The outline of the simulation procedure begins with using the numerical computing environment MATLAB to produce 1000 realizations of fBms with scaling parameter \( \alpha \) randomly chosen from the range 1.0 \( \leq \alpha \leq 2.0 \) by using a uniform random number generator. The fBms were then acted upon by a Poisson operator which transformed each time series into a Poisson-distributed series but left other properties of the fBm intact, e.g., \( \alpha \).

The fBms were then combined with a Poisson noise with variance, \( \lambda_B \). These Poisson noises were regarded as intrinsic to the GRB. Another set of Poisson noises with variances,
In summary, 1000 simulated light curves were generated and background noise was added. The light curves with background noise were then denoised using the same algorithm applied to actual GRB data in which preburst data were used as a surrogate for background. The simulated background subtracted variances were then compared to the variances of the ideal light curves, i.e., light curves without external background noise. Signal to noise effects on the reliability of the method were also considered and found either to be small compared to our quoted errors or large enough that $\tau_\beta$ could not be determined. In the case of the latter the GRB was removed from the analysis. The results indicate that the background subtraction method is robust and gives confidence that external background noise can be subtracted from the GRB light curves with the assumption that preburst data can serve as a surrogate for background noise.

4.3 Selection Criteria

We analyzed 122 GRBs (61 long and 61 short) listed in the Fermi GBM-Burst Catalog (Paciesas et al. 2012) for the first two years of the GBM mission. As discussed in Sec. 4.2, the signal-to-background ratio is a factor to be considered in...
recovered the intrinsic light curve (see Eq. 12). We required the following condition on the ratio of variances,

$$\frac{\beta_j^{\text{preburst}}}{\beta_j^{\text{burst}}} < 0.75,$$

for one or more octaves, $j$. In addition, we also required that the first order polynomial fits to the noise region and to the scaling region each had a $\chi^2$/d.f. that was less than 2. This reduced the sample to 14 short GRBs (Tab. 1) and 46 long GRBs (Tab. 2) for a total of 60 and it is these GRBs which are used to create Figs. 4, 5, and 8. For boosting into the source frame (Figs. 6 and 7) a known $z$ is obviously required and this cut further reduced the data set to 2 short GRBs and 16 long GRBs for a total of 18 GRBs considered in the source frame (see Tab. 3).

5 RESULTS AND DISCUSSION

For a large sample of short and long GBM bursts, we have used a technique based on wavelets to determine the minimum time scale ($\tau_\beta$) at which scaling processes dominate.
over random noise processes. The $\tau_\beta$ is the intersection of the scaling region (red-noise) of the spectrum in the logscale diagram with that of the flat portion representing the (white-noise) random noise component. This transition time scale is the shortest resolvable variability time for physical processes intrinsic to the GRB. Histograms of the extracted $\tau_\beta$ values for long and short GRBs are shown in Fig. 1. We make two observations regarding these histograms: (1) There is a clear temporal offset in the extracted mean $\tau_\beta$ values for long and short GRBs. We believe this is the first clear demonstration of this temporal difference. [Walker et al. (2000)], who studied the temporal variability of long and short bursts using the BATSE data set did not report a systematic difference between the two types of bursts. (2) The two histograms are quite broad and very similar in dispersion. While the difference in the mean $\tau_\beta$ is understandable (a point we discuss further elsewhere) the similarity of the dispersion is somewhat surprising since the progenitors and the environment for the two types of bursts are presumably very different. The comparison is qualitative at best however because the $\tau_\beta$ scale has not been corrected for redshift ($z$), an effect that impacts the long bursts more than the short bursts. In passing we note that the dispersion of the $\tau_\beta$ histogram (for long bursts) is in agreement with the results of Ukwatta et al. (2011) who performed a power density spectral analysis of a large sample of Swift long GRBs. In that work the authors extracted threshold frequencies and related them to a variability scale.

In Fig. 5 we show a log-log plot of $\tau_\beta$ versus $T_{90}$ (the duration of the bursts); long GRBs are indicated by circles, the short ones by squares and both time scales are with respect to the observer frame. As in the histograms above, the fact that short GRBs, in general, tend to have smaller $\tau_\beta$ values compared to long GRBs, is evident in this figure. Also shown in the figure (as a dash line) is the trajectory of $\tau_\beta$ equal to $T_{90}$. As we expect, no long GRBs exhibit a $\tau_\beta$ longer than $T_{90}$ although interestingly a few short GRBs of extremely short duration appear to be approaching the limit of equality. In addition to establishing a characteristic time scale for short and long bursts, this figure also hints at a positive correlation between this time and the duration of bursts. We note that the $\tau_\beta$ scale spans approximately two decades for both sets of GRBs and that the two groups are fairly well clustered in the $\tau_\beta$-$T_{90}$ plane. A closer examination of the two groups, however, indicates that a correlation between $\tau_\beta$ and $T_{90}$, if present, is marginal at best. This is certainly true for the short-GRB group, especially given the large uncertainties in the $T_{90}$ for these bursts. The situation for the long-burst group on the other hand is not immediately clear. In order to explore this further we cast the $\tau_\beta$ and the $T_{90}$ time scales into the source frame by applying the appropriate $(1+z)$ factor to the GRBs for which the $z$ is known. Unfortunately the $z$ is not available for the majority of the short GRBs but we note that the correction is the same for both axes and is, to first order, small for the short GRBs since the mean $z$ for this group is $<0.8$. The corrected results for long-GRBs are shown as a log-log plot in Fig. 6. We see from this figure (and Fig. 5) the appearance of a very intriguing feature: A plateau region in which the $\tau_\beta$ is essentially independent of $T_{90}$ and a scaling region in which the $\tau_\beta$ appears to increase with $T_{90}$, with the transition occurring around $T_{90}$ on the order of a few seconds.

If one assumes a positive correlation between luminosity and variability as suggested by a number of authors, then one might expect smaller $\tau_\beta$ values for higher luminosity bursts compared to those of lower luminosity. To investigate this, the data (in Fig. 6) are re-plotted in Fig. 7 in which the size of each datum symbol has been modulated by the gamma-ray luminosity of the burst, i.e., a large symbol implies a high luminosity and a small symbol a low luminosity. We see from Fig. 6 that no obvious connection between $\tau_\beta$ and luminosity is evident.

Under the assumption that the $\tau_\beta$ is a measure proportional to the smallest causally-connected structure associated with a GRB light curve, it is then possible to interpret the scaling trend in terms of the internal shock model in which the basic units of emission are assumed to be pulses that are produced via the collision of relativistic shells emitted by the central engine. Indeed, we note that Quilligan et al. (2002) in their study of the brightest BATSE bursts with $T_{90} > 2$ sec explicitly identified and fitted distinct pulses and demonstrated a strong positive correlation between the number of pulses and the duration of the burst. More recent studies (Bhat & Guiriec 2011, Bhat et al. 2012, Hakkila & Cumbe 2008, Hakkila & Precock 2011) provide further evidence for the pulse paradigm view of the prompt emission in GRBs. In our work we have not relied on identifying distinct pulses but instead have used the multi-resolution capacity of the wavelet technique to resolve the smallest temporal scale present in the prompt emission. If the smallest temporal scale is made from pulse emissions from the smallest structures, then we can get a measure of the number of pulses in a given burst through the ratio $T_{90}/\tau_\beta$. In the simple model in which a pulse is produced every time two shells collide then the ratio, $T_{90}/\tau_\beta$, should show a correlation with the duration of the burst. A plot of this ratio versus $T_{90}$ is shown for a sample of short and long bursts in Fig. 6. The correlation is apparent.

It is now widely accepted that the progenitors for the two classes of GRBs are quite distinct i.e., the merger of compact objects in the case of short GRBs and the collapse of rapidly rotating massive stars in the case of long GRBs. Formation of an accretion disk in the two cases is posed in a number of models but important factors such as the size of the disk, the mass of the disk, the strength of the magnetic field, in addition to the magnitude of the accretion rate during the prompt phase, remain largely uncertain. With contributions from intrinsic variability of the central engine or nearby shock-wave interactions within a jet, we should not be surprised to observe a systematic difference in the extracted variability time scales for long and short bursts, since the progenitors have different spatial scales. Knowing the variability timescales, we can estimate the size of an assumed emission region. From Fig. 6 we note that the smallest temporal-variability scale for the short bursts is approximately 3 ms and that for the long bursts is approximately 30 ms: These times translate to emission scales of approximately $10^8$ and $10^9$ cm respectively. Our variability times and size scales are generally consistent with the findings of Walker et al. (2000) although these authors also reported observing time scales as small as few microseconds.
We find no evidence for variability times as low as a few microseconds.

Morseny et al. (2010) modeled the behavior of a jet propagating through the progenitor and the surrounding circumburst material and showed that the resulting light curves exhibited both short-term and long-term variability. They attribute the long-term variability, at the scale of few seconds, to the interaction of the jet with the progenitor. The short-term scale, at the level of milliseconds, they attribute to the variation in the activity of the central engine itself. Alternatively, Zhang & Yan (2011) consider a model in which the prompt emission is the result of a magnetically powered outflow which is self-interacting and triggers rapid turbulent reconnections to power the observed GRBs. This model also predicts two variability components but interestingly and in contrast to the findings of Morseny et al. (2010), it is the slow component that is associated with the activity of the central engine, and the fast component is linked to relativistic magnetic turbulence. While we are not in a position to distinguish between these two models it is intriguing nonetheless to note (see Fig. 5) that indeed there do appear to be two distinct time domains for the \( \tau_\beta \): a plateau region dominated primarily by short bursts although it includes some long bursts too, and a scaling region (i.e., a hint of a correlation between \( \tau_\beta \) and \( T_{90} \)) that is comprised solely of long bursts. In addition, we observe that the time scale in the plateau region is the order of milliseconds whereas that for the scaling region is approaching seconds.

There is considerable dispersion in the extracted \( \tau_\beta \). The variation is evident for both short and long-duration GRBs. The main cause of this dispersion is not fully understood but one factor that may play a significant role is angular momentum. As Lindner et al. (2010) note, the basic features of the prompt emission can be understood in terms of accretion that results via a simple ballistic infall of material from a rapidly rotating progenitor. Material with low angular momentum will radially accrete across the event horizon whereas the material with sufficient angular momentum will tend to circularize outside the innermost stable circular orbit and form an accretion disk. Simulations that go beyond the simple radial infall model (Lindner et al. 2011) suggest that the formation of the disk leads to an accretion shock that traverses outwards through the infalling material. If one assumes that the initiation of such an accretion shock and the subsequent emission of the prompt gamma-rays are associated with a particular time scale, the variability of this scale then (as given by the dispersion in \( \tau_\beta \) for example) may reflect the different dynamics (initial angular momentum and the mass of the black hole) of each GRB in our sample. In the case of long GRBs, the mass of the central black hole can vary by an order of magnitude thus potentially explaining a large part of the dispersion seen in the \( \tau_\beta \). However a similar dispersion for short bursts is difficult to reconcile using the same arguments since the mass range for the central black hole in standard merger models (at least for NS-NS mergers) is expected to be significantly smaller.

6 CONCLUSIONS

We have studied the temporal properties of a sample of prompt-emission light curves for short and long-duration GRBs detected by the Fermi/GBM mission. By using a technique based on wavelets we have extracted the variability timescales for these bursts. Our main results are summarized as follows:

a) Both short and long-duration bursts indicate a temporal variability at the level of a few milliseconds. Variability of this order appears to be a common feature of GRBs. This finding is consistent with the work of Walker et al. (2000). However, unlike these authors we do not find evidence of variability at a time scale of few microseconds.

b) In general the short-duration bursts have a variability time scale that is significantly shorter than long-duration bursts. In addition, the \( \tau_\beta \) values seem not to depend in any obvious way on the luminosity of the bursts. The dispersion over different GRBs in the extracted time scale for short-duration bursts is an order of magnitude within the smallest variability time, that time being approximately 3 milliseconds. The dispersion for the long-duration bursts is somewhat larger. The origin of the dispersion in either case is not known, although we should expect that the size of the initial angular momentum and the mass of the system play significant roles. We note in passing that the 3 millisecond time scale may not be a physical lower limit and may be a result of signal to noise and the set of GRBs used in this analysis. We remind the reader that our light curve resolution was 200 \( \mu s \) and if a strong enough signal within a range of time scales between 0.5 - 3 milliseconds were present we would expect our technique to be sensitive to it.

c) The ratio of \( T_{90}/\tau_\beta \) appears to be positively correlated with the minimum variability time scale. This suggests further support for the pulse paradigm view of the prompt emission as being the result of shell collisions. In this respect, the minimum variability time scale is likely related to key pulse parameters such as risetimes and/or widths.

d) For short-duration bursts, the variability parameter \( \tau_\beta \) shows negligible dependence on the duration of the bursts (characterized by \( T_{90} \)). In contrast, the long-duration bursts indicate evidence for two variability time scales: a plateau region (at the shortest time scale) which is essentially independent of burst duration and a scaling region (at the higher time scale) that shows a positive correlation with burst duration. The transition between the two regions occurs around \( T_{90} \) on the order of a few seconds in the source frame.

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Table 1. Short GRBs (Observer Frame).

| GRB          | $T_{90}$ [sec] | $\delta T_{90}$ [sec] | $\tau_\beta$ [sec] | $\delta \tau^-_\beta$ [sec] | $\delta \tau^+_\beta$ [sec] |
|--------------|----------------|-----------------------|---------------------|-----------------------------|-----------------------------|
| 080723913    | 0.192          | 0.345                 | 0.0307              | 0.0192                      | 0.0510                      |
| 081012045    | 1.216          | 1.748                 | 0.0052              | 0.0024                      | 0.0044                      |
| 081102365    | 1.728          | 0.231                 | 0.0258              | 0.0100                      | 0.0165                      |
| 081105614    | 1.280          | 1.368                 | 0.0306              | 0.0147                      | 0.0282                      |
| 090108020    | 0.704          | 0.143                 | 0.0241              | 0.0064                      | 0.0088                      |
| 090206620    | 0.320          | 0.143                 | 0.0143              | 0.0063                      | 0.0112                      |
| 090227772    | 1.280          | 1.026                 | 0.0053              | 0.0009                      | 0.0011                      |
| 090228204    | 0.448          | 0.143                 | 0.0028              | 0.0005                      | 0.0005                      |
| 090308734    | 1.664          | 0.286                 | 0.0120              | 0.0040                      | 0.0059                      |
| 090429753    | 0.640          | 0.466                 | 0.0285              | 0.0115                      | 0.0193                      |
| 090510016    | 0.960          | 0.138                 | 0.0049              | 0.0009                      | 0.0011                      |
| 100117879    | 0.256          | 0.834                 | 0.0331              | 0.0122                      | 0.0192                      |
Table 2. Long GRBs (Observer Frame).

| GRB       | $T_{90}$ [sec] | $\delta T_{90}$ [sec] | $\tau_0$ [sec] | $\delta \tau_0$ [sec] | $\delta \tau_+^+$ [sec] |
|------------|---------------|-----------------------|----------------|-----------------------|--------------------------|
| 080723557  | 58.369        | 1.985                 | 0.0440         | 0.0113                | 0.0151                   |
| 080723985  | 42.817        | 0.659                 | 0.1894         | 0.0557                | 0.0789                   |
| 080724401  | 379.307       | 2.202                 | 0.0741         | 0.0208                | 0.0290                   |
| 080804972  | 24.704        | 1.460                 | 0.4306         | 0.1336                | 0.1937                   |
| 080806896  | 75.777        | 4.185                 | 0.4189         | 0.1471                | 0.2268                   |
| 080807993  | 19.072        | 0.181                 | 0.0232         | 0.0096                | 0.0164                   |
| 080810549  | 107.457       | 15.413                | 0.1353         | 0.0648                | 0.1243                   |
| 080814503  | 64.769        | 1.810                 | 0.1067         | 0.0428                | 0.0715                   |
| 080817161  | 60.289        | 0.466                 | 0.1919         | 0.0402                | 0.0509                   |
| 080825593  | 20.992        | 0.231                 | 0.0775         | 0.0138                | 0.0168                   |
| 080906212  | 2.875         | 0.767                 | 0.1011         | 0.0182                | 0.0222                   |
| 081009009  | 62.977        | 0.810                 | 0.2266         | 0.0630                | 0.0872                   |
| 081025775  | 31.744        | 3.167                 | 0.1748         | 0.0425                | 0.0562                   |
| 081009140  | 41.345        | 0.264                 | 0.1095         | 0.0170                | 0.0201                   |
| 081150532  | 8.256         | 0.889                 | 0.0948         | 0.0302                | 0.0444                   |
| 081125496  | 9.280         | 0.607                 | 0.2182         | 0.0504                | 0.0656                   |
| 081129161  | 62.657        | 7.318                 | 0.0912         | 0.0292                | 0.0429                   |
| 081215784  | 5.568         | 0.143                 | 0.0319         | 0.0043                | 0.0050                   |
| 081222290  | 18.800        | 2.318                 | 0.1956         | 0.0533                | 0.0732                   |
| 08124887   | 16.448        | 1.159                 | 0.2055         | 0.0356                | 0.0431                   |
| 090102122  | 26.624        | 0.810                 | 0.0347         | 0.0111                | 0.0164                   |
| 090130900  | 35.073        | 1.056                 | 0.0733         | 0.0169                | 0.0220                   |
| 090223471  | 12.608        | 0.345                 | 0.1444         | 0.0575                | 0.0954                   |
| 090323002  | 135.170       | 1.448                 | 0.1598         | 0.0436                | 0.0599                   |
| 090328401  | 61.697        | 1.810                 | 0.0682         | 0.0139                | 0.0175                   |
| 090411991  | 14.336        | 1.086                 | 0.0673         | 0.0391                | 0.0935                   |
| 090424592  | 14.144        | 0.264                 | 0.0249         | 0.0031                | 0.0036                   |
| 090425377  | 75.393        | 2.450                 | 0.1346         | 0.0309                | 0.0508                   |
| 090516353  | 118.018       | 4.028                 | 0.4938         | 0.2063                | 0.3544                   |
| 090516353  | 123.074       | 2.896                 | 0.7992         | 0.5686                | 1.9711                   |
| 090628516  | 79.041        | 1.088                 | 0.1314         | 0.0320                | 0.0423                   |
| 090628516  | 112.386       | 1.086                 | 0.2631         | 0.0536                | 0.0673                   |
| 090620400  | 13.568        | 0.724                 | 0.1667         | 0.0422                | 0.0564                   |
| 090628189  | 48.897        | 2.828                 | 0.0498         | 0.0078                | 0.0093                   |
| 090718762  | 23.744        | 0.802                 | 0.1621         | 0.0482                | 0.0686                   |
| 090809978  | 11.008        | 0.320                 | 0.2436         | 0.0515                | 0.0652                   |
| 090810659  | 123.458       | 1.747                 | 0.7319         | 0.3027                | 0.5161                   |
| 090829672  | 67.585        | 2.896                 | 0.0678         | 0.0141                | 0.0177                   |
| 090931317  | 39.424        | 0.572                 | 0.0266         | 0.0103                | 0.0169                   |
| 090902462  | 19.328        | 0.286                 | 0.0223         | 0.0026                | 0.0029                   |
| 090926181  | 13.760        | 0.286                 | 0.0435         | 0.0061                | 0.0070                   |
| 091003191  | 20.224        | 0.362                 | 0.0300         | 0.0051                | 0.0062                   |
| 091125976  | 8.701         | 0.571                 | 0.0395         | 0.0059                | 0.0069                   |
| 091208410  | 12.480        | 5.018                 | 0.0621         | 0.0180                | 0.0254                   |
| 100414097  | 26.497        | 2.073                 | 0.0418         | 0.0074                | 0.0090                   |
Table 3. Long and Short GRBs ($T_{90}$ and $\tau_{\beta}$ in Observer Frame). Luminosities are taken from references given in footnotes.

| GRB   | $z$ | $T_{90}$ [sec] | $\delta T_{90}$ [sec] | $\tau_{\beta}$ [sec] | $\delta \tau_{\beta}$ [sec] | $L_{\text{iso}}$ [ergs/s] | $\delta L_{\text{iso}}$ [ergs/s] | $\log \delta L_{\text{iso}}$ [ergs/s] |
|-------|-----|----------------|-----------------------|-----------------------|-----------------------------|-------------------------|-------------------------------|----------------------------------|
| 080804972 | 2.204 | 24.704 | 1.460 | 0.4306 | 0.1336 | 0.1937 | $^{1}3.58 \cdot 10^{52}$ | 5.82 $\cdot 10^{51}$ | 7.85 $\cdot 10^{51}$ |
| 080810549 | 3.350 | 107.457 | 15.413 | 0.1353 | 0.0648 | 0.1243 | $^{2}9.59 \cdot 10^{52}$ | 1.28 $\cdot 10^{52}$ | 1.28 $\cdot 10^{52}$ |
| 080916009 | 4.350 | 62.977 | 0.810 | 0.2266 | 0.0630 | 0.0872 | $^{3}1.04 \cdot 10^{54}$ | 8.79 $\cdot 10^{52}$ | 8.79 $\cdot 10^{52}$ |
| 081222204 | 2.770 | 18.880 | 2.318 | 0.1956 | 0.0533 | 0.0732 | $^{1}1.26 \cdot 10^{53}$ | 7.10 $\cdot 10^{51}$ | 6.10 $\cdot 10^{51}$ |
| 090102122 | 1.547 | 26.624 | 0.810 | 0.0347 | 0.0111 | 0.0164 | $^{3}8.71 \cdot 10^{52}$ | 5.6 $\cdot 10^{51}$ | 5.6 $\cdot 10^{51}$ |
| 090323002 | 3.570 | 135.170 | 1.448 | 0.1598 | 0.0436 | 0.0599 | $^{6}8.79 \cdot 10^{53}$ | 6.55 $\cdot 10^{53}$ | 4.45 $\cdot 10^{52}$ |
| 090328401 | 0.736 | 61.697 | 1.810 | 0.0249 | 0.0031 | 0.0036 | $^{1}1.62 \cdot 10^{52}$ | 4.10 $\cdot 10^{50}$ | 5.10 $\cdot 10^{50}$ |
| 090510016 | 0.903 | 0.960 | 0.138 | 0.0049 | 0.0009 | 0.0011 | $^{3}1.78 \cdot 10^{53}$ | 1.2 $\cdot 10^{51}$ | 1.2 $\cdot 10^{51}$ |
| 090516353 | 4.100 | 123.074 | 2.896 | 0.7992 | 0.5686 | 1.9711 | $^{6}8.17 \cdot 10^{52}$ | 2.85 $\cdot 10^{52}$ | 6.1 $\cdot 10^{51}$ |
| 090618353 | 0.540 | 112.386 | 1.086 | 0.2631 | 0.0536 | 0.0673 | $^{5}8.47 \cdot 10^{51}$ | 1.17 $\cdot 10^{51}$ | 3.4 $\cdot 10^{50}$ |
| 090902462 | 1.822 | 19.328 | 0.286 | 0.0223 | 0.0026 | 0.0029 | $^{5}5.89 \cdot 10^{53}$ | 9.71 $\cdot 10^{51}$ | 9.71 $\cdot 10^{51}$ |
| 090926181 | 2.106 | 13.760 | 0.286 | 0.0435 | 0.0061 | 0.0070 | $^{3}7.40 \cdot 10^{53}$ | 1.45 $\cdot 10^{52}$ | 1.45 $\cdot 10^{52}$ |
| 091003191 | 0.897 | 20.224 | 0.362 | 0.0300 | 0.0051 | 0.0062 | $^{1}4.53 \cdot 10^{52}$ | 3.71 $\cdot 10^{51}$ | 6.55 $\cdot 10^{51}$ |
| 091127976 | 0.490 | 8.701 | 0.571 | 0.0395 | 0.0059 | 0.0069 | $^{7}3.70 \cdot 10^{51}$ | 1.38 $\cdot 10^{50}$ | 1.06 $\cdot 10^{50}$ |
| 091208410 | 1.063 | 12.480 | 5.018 | 0.0621 | 0.0180 | 0.0254 | $^{1}1.45 \cdot 10^{52}$ | 1.48 $\cdot 10^{51}$ | 3.45 $\cdot 10^{51}$ |
| 100117879 | 0.920 | 0.256 | 0.834 | 0.0331 | 0.0122 | 0.0192 | $^{2}6.3 \cdot 10^{52}$ | 5.01 $\cdot 10^{51}$ | 1.08 $\cdot 10^{52}$ |
| 100414097 | 1.368 | 26.497 | 2.073 | 0.0418 | 0.0074 | 0.0090 | $^{2}1.00 \cdot 10^{53}$ | 1.58 $\cdot 10^{52}$ | 7.6 $\cdot 10^{51}$ |

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