Curvature-driven AC-assisted creep dynamics of magnetic domain walls

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The dynamics of micrometer-sized magnetic domains in ultra-thin ferromagnetic films is so dramatically slowed down by quenched disorder that the spontaneous elastic tension collapse becomes unobservable at ambient temperature. By magneto-optical imaging we show that a weak zero-bias AC magnetic field can assist such curvature-driven collapse, making the area of a bubble to reduce at a measurable rate, in spite of the negligible effect that the same curvature has on the average creep motion driven by a comparable DC field. An analytical model explains this phenomenon quantitatively.

An arbitrarily weak quenched disorder has yet a notable qualitative effect in the driven motion of an extended elastic system such as an interface embedded in a random medium. A paradigmatic experimental example are pinned domain walls (DW) in thin film “Ising-like” ferromagnets with a uniform external magnetic field favouring the growth of a magnetic domain [1][3]. In these materials, due to the practically unavoidable presence of random heterogeneities, DW velocities can vary dramatically under relatively modest changes of a weak applied field. Strikingly, the quantitative way the velocity asymptotically vanishes in the small field limit is universal [4][6], and is thus sucessfully captured by minimal models that can be solved, in the limit of large systems, with powerful analytical [7] and numerical [8–11] techniques. These statistical-physics models yield, in particular, the celebrated creep-law ln(1/v) ∝ H−μ for the average velocity v of a DW in presence of weak uniform magnetic driving field H, with μ > 0 a universal exponent [4][5][12]. This law clearly signals the breakdown of linear-response in the collective transport. The success of this mathematical description unveils the basic physics fact that the glassy universal dynamics of DWs is mainly controlled by the interplay of pinning, elasticity and thermal fluctuations on the driven elastic interface. As such, creep theory is relevant for many other driven elastic systems with thermal or “thermal-like” fluctuations and quenched disorder, ranging from current driven vortices in superconductors [13][17], charge density waves [18] to tension driven cracks [19][20].

Many universal properties predicted by the creep theory, the velocity-force characteristics [4][5][2][12], the rough geometry of moving DWs [5][10], and even the event statistics behind the creep law [11][21], have been studied experimentally by applying external magnetic fields [1][2][6][22][28] or external currents [29][33] to drive DWs in ultra-thin ferromagnetic films with perpendicular anisotropy (PMA). Most of the studies focus in the DC-driven case while comparatively very few experimental [28][34] and theoretical [35][36] studies have focused on the universal properties that can emerge under a zero-bias AC-drive within the creep regime. Weak AC fields yield nevertheless a rich phenomenology which is worth studying. In particular, recent experiments have shown that roughly-circular magnetic bubbles evolve under a pure symmetric AC field in a very intriguing way [28]. The first interesting effect is that the (otherwise unstable) initial bubble monotonically shrinks with the number of alternated positive and negative magnetic field pulses of equal strength, apparently “rectifying” the AC drive. The second is the observation that the DW roughness increases at a much faster rate in the AC protocol compared to the DC for the same amplitude of the drive. An example of such evolution, captured by successive MO images, is shown in Fig. 1. These two intriguing effects have not been explained yet.

In this Letter we show that the elastic pressure arising from the domain mean curvature, even being orders of magnitude weaker than the driving field pressure, is the responsible for the shrinking of the domain area under AC fields. To show this, we first derive a model for the AC-driven DW dynamics and second, we quantitatively test two of its predictions experimentally: (i) the pulse asymmetry needed to stabilize the average size of the “beating domain” and, (ii) The area collapse rate in the initial dynamics for the case of symmetric positive-negative pulses. Finally, a qualitative argument is given to explain the AC enhancement of the DW dynamic roughening and its effect on the area collapse dynamics.

The proposed model is very simple. Let us consider a segment of a DW in a film of thickness d. In the absence of disorder and thermal fluctuations, the total force F per unit-length acting on the effectively one-dimensional DW is the sum of the magnetic field force...
Above results are not valid as the normal velocity \( v_\perp \) per unit length. This generalizes the sequence of PMOKE images in the bottom corresponds to the point \( t_1 \), the topological index of the curve \( s \) scales as \( 2\pi \). If we assume a linear and instantaneous response we can write Eq. (1) as

\[
\frac{dA_t}{dt} = \int_{\Gamma_t} v_i(r_s)ds
\]

with \( v_i(r_s) \) the local instantaneous normal velocity at point \( r_s \) in \( \Gamma_t \). If we assume a linear and instantaneous response \( v_i(r_s) = m(H_t + C\kappa_t(r_s)) \), with \( \kappa_t(r_s) \) the instantaneous signed local curvature we easily obtain, using the topological index of the curve \( \int_{\Gamma_t} \kappa_t(r_s)ds = -2\pi \), the rate \( \frac{dA_t}{dt} = mH_tP_t - 2\pi Cm \), with \( P_t \equiv \int_{\Gamma_t} ds \) the perimeter. This generalizes the \( H_t \) is constant decay rate \( \frac{dA_t}{dt} = -2\pi Cm \) to any initial simple closed curve \( \Gamma_t \).

In the presence of quenched and thermal disorder the above results are not valid as the normal velocity \( v_i(r_s) \) is in general expected to be an inhomogeneous non-linear function of \( H_t + C\kappa_t \). Assuming again instantaneous response we can write Eq. (1) as

\[
\frac{dA_t}{dt} \approx \int_{\Gamma_t} V_T(H_t + C\kappa_t(r_s), r_s)ds
\]

with \( V_T(h, r) \) a temperature and position dependent velocity response to a local field \( h_t = H_t + C\kappa_t \). We will argue that for weak enough fields, \( V_T(h, r) \) in Eq. (2) can be approximated by the well known creep law for DC-driven DWs. This hydrodynamic approach can be formally justified: in the creep regime, DW velocity is mainly controlled by creep events with a cut-off radius estimated to be, for ultra-thin ferromagnet, less than 0.1 \( \mu \)m, clearly well below the \( \sim 1 \mu \)m PMOKE resolution. Therefore, larger size fluctuations are expected to introduce only negligible logarithmic corrections into the creep law \( V_T(h) \sim \exp[-(T_d/T)(H_d/h)^{\frac{n}{2}}] \) that describes the DW velocity in terms of the effective field \( h \), temperature \( T \) and also the disorder and elasticity through \( T_d \) and \( H_d \). Similarly, the characteristic time associated to individual creep events is much smaller than the experimental time-scale used for resolving DW displacements so the velocity response can be considered local and instantaneous.

Replacing the creep velocity \( V_T(h, r) \) in Eq. (2) is a step forward but still yields a non-closed equation for \( dA_t/dt \) as it requires the knowledge of the time dependent curvature field \( \kappa_t(r) \), together with a model for the spatially fluctuating pinning parameters of the creep law. Nevertheless, to extract the basic physics some progress can be made by first making the well justified approximation that \( H_t \gg C\kappa_t \). Second, the complexity of Eq (2) is greatly reduced if we neglect the heterogeneity of the creep-law and replace it by its average \( V_T(h, r) \approx V_T(h) \) or velocity-field characteristics. This approximation is not equivalent to neglect disorder completely, as \( V_T(h) \) is in general quite different from the \( V_T(h) \equiv 0 \) expected for an homogeneous sample, particularly in the strongly nonlinear creep-regime. Developing then at first order in \( C\kappa_t \) from Eq. (2) we obtain

\[
\frac{dA_t}{dt} \approx V_T(H_t)P_t - 2\pi CV_T(H_t),
\]

only relating the geometric variables \( A_t \) and \( P_t \). The position and time dependent curvature \( \kappa_t(r) \) disappears thanks to the topological invariant \( \int_{\Gamma_t} \kappa_t = -2\pi \). Let us now focus on the experiments and make some concrete predictions with Eq. (3).

Our measurements were carried out in ultrathin ferromagnetic films with PMA, by Magneto-optical imaging, using a homemade polar Magneto-optical Kerr effect (PMOKE) microscope. Two kinds of samples from different sources were used: a Pt/Co/Pt magnetic monolayer (S1) and a Pt/[Co/Ni]/Al multilayer (S2), both grown by DC magnetron sputtering. Helmholtz coils allow to apply well conformed square magnetic field pulses with amplitude \( H \) up to 700 Oe and duration \( \tau_1 > 1 \) ms. DW dynamics is characterized with the usual quasi-static technique (see experimental details). The AC field is applied to an already grown domain (see Fig. 1) and consists in alternated square pulses of identical duration \( \tau_1 \) and amplitude \( H = H_{\tau1} > 0 \) (expanding the domain), and \( H = -H_{\tau1} < 0 \) (compressing the do-
main). The two pulses are periodically repeated with period $\tau \geq 2 \tau_1$, as schematized in Fig. 1. The magnitudes of all applied fields are such that the creep-law with $\mu = 1/4$ is well observed in the DC protocol (see [42]).

Since we are only interested in the smooth evolution of $A_t$ and $P_t$ as a function of the number $N$ of AC cycles we define $A$ and $P$ such that $dA/dN = A_{n+1} - A_n$ and $dP/dN = P_{n+1} - P_n$, where $n = t/\tau$. Integrating Eq. (3) from $t = n\tau$ to $t = (n+1)\tau$ we thus obtain

$$\frac{dA}{dN} \approx -2\pi CV^f_t(H_t)\tau - \frac{\tau}{4} V^f_t(H_t) \frac{dP}{dN} + \frac{\tau}{2} \Delta HV^f_t(H_t)P,$$

(4)

where $\Delta H = H^\uparrow - H^\downarrow \ll H^\uparrow$ quantifies a possible pulse asymmetry, and we have used the expected symmetry $V^f_t(h) = -V^f_t(-h)$.

We test Eq. (4) in two different ways. On one hand, we can choose $\Delta H = \Delta H^*$ such that $dA/dN = dP/dN = 0$,

$$\Delta H^* = \frac{2C}{\mathcal{R}}$$

(5)

where we have defined $\mathcal{R} \equiv P/2\pi$, approximately the observed average domain radius. This is a simple but rather general prediction: $\Delta H^*$ is independent of $V^f_t$ only provided that $V^f_t(h) = -V^f_t(-h)$, and of the AC parameters $\tau$ and $H^\uparrow$. In physical terms, Eq. (5) states that even weak compressing forces arising from mean curvature are relevant because they break the forward-backward symmetry of the DW velocity in the AC field. Importantly, Eq. (5) connects with micro-magnetism through $C = \sigma/2M_s$. Using that $A_N \approx A_N$ in Fig. 2 we test Eq. (5) experimentally. The main panel shows the field asymmetry $\Delta H$ stabilizing the average area $A$ of initially nucleated domains with different initial radius $R$. An example of such compensation is shown in the inset, where the evolution of $A(N)$ under symmetric field pulses and asymmetric compensating pulses are compared for sample S1, with an initial $R = 25 \mu$m. The corresponding videos are available in the Supplemental Material. For both samples, there is a good agreement with the linear relation between $\Delta H^*$ and $R/\mathcal{R}$ predicted in Eq. (5), for four $R$ ranging from 15$\mu$m to 35$\mu$m. The ordinates predicted to be zero in Eq. (5) are small for the two samples, compatible with a small hazard DC field present in the Lab. The fitted value of $C$ is in both cases of order $10^{-3}$Oe cm, fairly agreeing with $C_{S1} = 2.1 \times 10^{-3}$ Oe cm and $C_{S2} = 1.2 \times 10^{-3}$ Oe cm estimated as $C = \sigma/2M_s$ from the micromagnetic parameters respectively [42].

Let us now go further and focus in the interesting case of symmetric field pulses $H^\uparrow = H^\downarrow = H$, i.e. $\Delta H = 0$. From Eq. (4) we simply predict

$$- \frac{d}{dN} \left[ \frac{A + \frac{\tau}{4} V^f_t(H)P}{2\pi V^f_t(H)\tau} \right] = \frac{d\Lambda}{dN} \approx C.$$

(6)

For circular DWs with radius $R_\ell$, this equation can be readily obtained from $dR_\ell/dt \approx V(H_\ell + C\kappa_\ell)$ with $\kappa_\ell = -1/R_\ell$. Remarkably however, Eq. (6) is valid regardless of the circular shape assumption (see [42] for further details) and contains the spontaneous ($H = 0$) collapse as a special case. Fig. 3(a) shows the evolution of the function $\Lambda(N)$ defined in Eq. (6), for 4 different field amplitudes, measured in sample S2. The initial slope for the highest amplitudes, using the creep-regime velocity-field characteristics measured in S2, gives $C \approx 10^{-3}$Oe cm, again in fair agreement with the micromagnetic estimate for $C$, hence reinforcing the curvature argument.

Results shown in Figs. 2 and 3(a) confirm that surface tension forces arising from curvature are responsible for the domain collapse, in spite of being two orders of magnitude smaller than the AC forces. They also validate the proposed model but, as can be appreciated in Fig. 3(a), only for the very first few AC cycles (small $N$); the lowest the field the sooner the deviation. We argue that these deviations are due to large-scale dynamic roughening, neglected in our simple model. To show it we exploit that creep dynamics displays a “depinning-like” regime upon coarse-graining many creep events [7,9] and thus numerically emulate the experimental protocol using the time-dependent Ginzburg-Landau model [45,48].

$$\eta \partial_t \phi = c \nabla^2 \phi + \epsilon_0 [(1 + r(x,y))\phi - \phi^3] + \tilde{h}_t,$$

(7)

near the depinning transition, where $\phi \equiv \phi(x,y,t)$ models the local magnetization, and $r(x,y)$ an uncorrelated random-bond type of disorder of strength $r_0$ [49]. The effective AC field $\tilde{h}_t = \pm h_0$ of period $\tilde{\tau}$ is chosen so to impose a given average displacement of the DW in half a period comparable to that observed in the experiments. After nucleating a circular domain with saturation magnetization $\phi_s \sim 1$, this model generates a closed curve $\Gamma_t$ (i.e. $\phi(r_s) = 0$ for $r_s \in \Gamma_t$) describing a DW with width $\delta \sim \sqrt{c/\epsilon_0}$, and surface tension
Figure 3. Open symbols show the evolution of $\Lambda(N)$ (see text) (a) and of the relative domain area change $\Delta A/A_0$ (b), for different amplitudes of symmetric pulses in sample S2. The black-line in (a) is a linear fit of the initial evolution of $\Lambda(N)$, yielding the micromagnetic constant $C$. In (b) numerical simulation results are shown with full symbols. Inset (c): DW mean squared displacements for $H = 160$ Oe pulses.

Summarizing, we have proposed and experimentally tested a model for the DW creep dynamics of an isolated magnetic domain in an ultra-thin ferromagnet under AC fields at ambient temperature. We showed that curvature effects, with a negligible effect on the average DC-driven motion, play nevertheless a central role in the AC-driven case. The intriguing “rectification effect” in the zero-bias AC case of Ref[28] is then explained by the curvature-induced symmetry breaking of forward-backward DW motions. Rather strikingly, the same curvature effects are unable to produce, without AC-assistance, any experimentally observable DW displacement [54]. We have also explained, qualitatively, the AC enhancement of large-scale dynamic roughening. Although we have focused in their important role in the AC-assisted motion, curvature effects can be relevant in some DC-driven systems as well: a non-steady velocity in DC-driven circular domains was experimentally reported [53]; on the other hand, in thin and narrow ferromagnetic wires the universal creep-law is satisfied by the DC-driven steady DW velocity only if an effective “counterfield” $\Delta H$ [50, 57], proportional to the observed average curvature of the narrow DW, is added, in agreement with our arguments. In the latter case, however, average curvature is not inherited from the initial conditions but steadily maintained by the strong localized “dynamic friction” at the wire edges. Due to the simplicity and generality of our arguments, we hope that the present work will open new perspectives for modelling and controlling DW creep motion in a variety of elastic systems, far beyond ferromagnetic films.
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