Q-factor control of a microcantilever by mechanical sideband excitation

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We demonstrate the coupling between the fundamental and second flexural modes of a microcantilever. A mechanical analogue of cavity-optomechanics is then employed, where the mechanical cavity is formed by the second vibrational mode of the same cantilever, coupled to the fundamental mode via the geometric nonlinearity. By exciting the cantilever at the sum and difference frequencies between fundamental and second flexural modes, the motion of the fundamental mode of the cantilever is damped and amplified. This concept makes it possible to enhance or suppress the Q-factor over a wide range. © 2011 American Institute of Physics. [doi:10.1063/1.3650714]

Cantilevers have numerous scientific and technological applications and are used in various instruments. In sensing applications, the sensitivity is related to the Q-factor, and this has motivated researchers to increase the Q-factor of mechanical resonators, in particular, in dissipative environments. Among the techniques that have been employed are applying residual stress,1 parametric pumping,2 and self-oscillation by internal3 and external4 feedback mechanisms. When increasing the Q-factor in these ways, energy is pumped into the mechanical mode and the resonator heats up. The opposite effect leads to cooling of the resonator and attenuation of its motion.5 By pumping energy out of the mechanical resonator into a high quality-factor optical or microwave cavity, several groups have shown reduction of the effective temperature of the vibrational mode from room temperature to millikelvin temperatures.6–14 Such cooling schemes are now employed to bring down the mode temperature to below an average phonon occupation number of one, providing a promising route to study the quantum behavior of a mechanical resonator.15–17

In analogy to cavity optomechanics, where an optical or a microwave cavity is used to extract energy from the resonator, we employ a mechanical cavity to damp the mechanical mode. Here, the fundamental flexural mode of the cantilever is the mode of interest, and the mechanical cavity is formed by the second flexural mode of the same cantilever, which is geometrically coupled to the fundamental mode. In this paper, we demonstrate the presence of this coupling by strongly driving the cantilever on resonance, while monitoring its broadband frequency spectrum. Sidebands appear in the spectrum, which are located at the sum and difference frequencies of fundamental and second modes of the cantilever. Driving the cantilever at these sidebands results in positive or negative additional damping, which is demonstrated in this paper.

Cantilevers are fabricated from low pressure chemical vapor deposited silicon nitride by electron beam lithography and isotropic reactive ion etching in a O2/CHF3 plasma.18 The dimensions are length \times width \times height = 39 \, \mu m \times 8 \, \mu m \times 70 \, \mu m. An optical deflection technique, similar to the one employed in atomic force microscopy, is used to detect the cantilever motion. Figures 1(a) and 1(b) show the cantilever and the setup. The cantilever is mounted on a piezo crystal and placed in a vacuum chamber at a pressure of \sim 10^{-5} \, mbar. Two spectrum analyzers are used to simultaneously measure the thermal motion of the fundamental (i = 1) and second (i = 2) flexural modes. Figure 1(b) shows the power spectra without driving the piezo. The resonance frequencies and Q-factors are determined by fitting Lorentzian functions (solid lines), and we find \( f_1 = 63.2 \, \text{kHz} \) and \( f_2 = 385.4 \, \text{kHz} \) and \( f_3 = 1.068 \, \text{MHz} \) (not shown). The ratios \( f_2/f_1 = 6.1 \) and \( f_3/f_1 = 16.9 \) are close to the expected modal frequencies \( \omega_{21} = 6.3 \) and \( \omega_{31} = 17.5 \) representing the spectrum of a homogeneous cantilevered Euler-Bernoulli beam. For the fundamental and second resonance modes, the corresponding Q-factors are \( Q_1 = 5184 \) and \( Q_2 = 3922 \), respectively. The frequency difference, \( f_2 - f_1 = 322 \, \text{kHz} \), exceeds the bandwidth of the modes, \( f_1/Q_1 = 12 \, \text{Hz} \) and \( f_3/Q_2 = 98 \, \text{Hz} \), by four orders of magnitude.

![Diagram of the measurement circuit showing photodiode (D), laser (L), piezo (P), and the spectrum analyzers (SAs) to measure the thermal motion of the fundamental (SA 1) and the second (SA 2) flexural modes. The thermal noise spectra are shown at the fundamental (i = 1) and second (i = 2) flexural modes of the cantilever.](https://example.com/diagram.png)

**FIG. 1.** (Color online) (a) Scanning electron micrograph of the silicon nitride cantilever. (b) Diagram of the measurement circuit showing photodiode (D), laser (L), piezo (P), and the spectrum analyzers (SAs) to measure the fundamental (SA 1) and the second (SA 2) flexural modes. The thermal noise spectra are shown at the fundamental (i = 1) and second (i = 2) flexural modes of the cantilever.

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To demonstrate the coupling between the fundamental and second flexural modes of the cantilever, we drive the cantilever on resonance, while measuring its broadband spectrum. Figure 2(a) shows this spectrum as a function of the drive strength. When the amplitude of the second mode increases, mechanical sidebands become visible in the spectrum. These sidebands occur at \( f_2 \pm f_1 \) and clearly indicate the presence of mechanical coupling between the two modes. Traces for weak and strong driving are extracted from (a) in Fig. 2(b), to show the shape and relative amplitudes of the sidebands. As the spacing between the sidebands is much larger than the linewidth of the mode, we operate in the resolved sideband regime.\(^6\)

The mechanism that couples the vibrational modes in a cantilever can be qualitatively understood as follows. A non-zero amplitude of one flexural mode of the cantilever changes the shape of the cantilever.\(^19\) This geometric change has a small but measurable effect on the resonance frequency of all the other vibrational modes. The effect of the cantilever amplitude on its own resonance frequency was recently analyzed in detail.\(^20\) For the first few modes, any nonzero amplitude stiffens the frequency response, and this gives rise to frequency pulling. Recently, we also presented a detailed study on the coupling mechanism between the vibrational modes in clamped-clamped resonators.\(^21\) Here, the coupling between the modes is fully described by the displacement-induced tension. A similar analysis can be carried out for the coupling between vibration modes of a cantilever beam. The only difference is that in the inextensional cantilever, the modes are coupled by the geometric nonlinearity, whereas for the (extensional) clamped-clamped resonator, the modes are coupled by the displacement-induced tension. For a cantilever, the modal amplitudes \( u_i \) are calculated by solving the (dimensionless) coupled equations: \(^22\)

\[
\ddot{u}_i + \eta_i \dot{u}_i + \omega_i^2 u_i + \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} \left( z_{ijkl} \dot{u}_j \dot{u}_l + \frac{1}{2} \beta_{ijkl} \dot{u}_j \dot{u}_l \right) = f_i \cos(\Omega t),
\]

where \( \eta_i \) represents a damping constant, \( \omega_i \) the resonance frequency, \( \Omega \) the drive frequency, and \( f_i \) the excitation strength of mode \( i \). The dots and primes denote derivative to time and coordinate, \( s \), respectively. The coupling coefficients \( z_{ijkl} \) and \( \beta_{ijkl} \) are calculated by integrating the cantilever modes shapes \( \xi \), as follows:

\[
z_{ijkl} = \int_0^1 \xi_i (\xi_j (\xi_k \xi_l'))' \, ds,
\]

\[
\beta_{ijkl} = \int_0^1 \xi_i (\xi_j (\int_0^1 \int_0^1 \xi_k \xi_l ds_1 ds_2)') \, ds.
\]

Taking only the fundamental and the second modes into consideration, Eq. (1) yields two coupled nonlinear differential equations with constant coefficients, which can be solved numerically.

The coupling between the vibrational modes can be used to transfer energy by employing a process similar to sideband cooling in cavity-optomechanics, where the cavity is used to extract energy from the mechanical mode. The mechanical resonator is embedded in an optical\(^6–10\) or microwave cavity.\(^11–13\) In analogy to those experiments and given the presence of the mechanical mode-coupling, the damping of one mechanical mode by another mode of the same resonator can be envisioned. Using the coupling mechanism described in the previous section, any change in the position of the mode under consideration (the fundamental flexural mode in the experiments that follow) changes the stiffness of the mode that acts as the cavity (the second flexural mode). The energy change in the cavity mode is retarded by the cavity relaxation time, equal to \( \sim Q/\omega f_2 \) for our mechanical cavity. Due to the delayed response of the cavity mode, a force is exerted by the cavity mode on the fundamental mode. This velocity-proportional force can either amplify or attenuate the motion of the fundamental mode.\(^23\) In case of red-detuned driving, the damping force on the fundamental mode is increased. When the driving is blue-detuned, the motion of both the cavity mode and the fundamental mode is amplified. The schemes are illustrated in Fig. 3(a), where the two Lorentzian shaped curves represent the two flexural modes of the cantilever, and the driving frequencies corresponding to blue and red detunings are indicated by the arrows. The damping rate is maximized by driving at the sum and difference frequencies and is increased by decreasing the linewidth of the cavity mode.

The effect of sideband excitation on the damping of the cantilever is demonstrated by measuring the thermal noise spectra of the fundamental and second flexural resonance modes, while driving the piezo sinusoidally at their sum and difference frequencies. Figure 3(b) shows the spectrum without driving (indicated by the black open circles). When the cantilever is driven at the blue-detuned sideband, its amplitude increases as shown by the blue curve. The blue and red curves in the power spectral density plots of Fig. 3(b) correspond to driving at the blue and red-detuned sidebands of the cavity mode shown in Fig. 3(a). By fitting Lorentzian functions to the data, we obtain the temperature and the \( Q \)-factors of the fundamental mode while driving the sidebands. When the cantilever is driven at the red sideband, the \( Q \)-factor of the fundamental mode decreases from 4599 to 1421. No changes in the temperature of the mode are observed, which indicates that the energy extracted via the modal interactions.

![Figure 2](image-url)
by mechanical sidebands in the frequency spectrum, which
flexural modes of a microcantilever. This coupling is marked
to obtain a significant change in the damping.

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