I. INTRODUCTION

The solution of the Schrödinger equation (SE) is essential to the energy spectrum of a particle since the early days of quantum mechanics. It is a major concern in many fields of physics and chemistry [1]. For instance, the solutions of the non-relativistic SE for a hydrogen atom and a harmonic oscillator represent two typical examples of exactly solvable problems in quantum mechanics. The solution of the SE with some potential is not known exactly in some quantum mechanical problems. The SE solution contains all the required information for the full depiction of a quantum state of the system. The study of the bound state processes is also essential to the understanding of the molecular spectrum of diatomic molecules. The Kratzer potential (KP) is one of the extensively used potential models in molecular physics and quantum chemistry [2]. The KP is made up of a repulsive part and long-range attraction [3]. Apart from that, when the inter-nuclear molecular distance approaches infinity, the potential decomposes to zero [4]. The potential is of the form:

$$V(\rho) = \eta_0 - \frac{\eta_1}{\rho} + \frac{\eta_2}{\rho^2}$$  \hspace{1cm} (1)
where \( \eta_0, \eta_1 \) and \( \eta_2 \) are potential strength and \( \rho \) is inter-nuclear distance. The KP has been used as a potential model to describe inter-nuclear vibration of diatomic molecules by different authors [5], [6]. The screened Coulomb potential (SCP) or Yukawa potential is used significantly in nuclear and condensed matter physics [7]. The SCP is a short-range potential [8]. In solid-state physics, it describes the charge particle effects of conduction electrons [9]. It has the form:

\[
V(\rho) = -\frac{\zeta e^{-\alpha \rho}}{\rho}
\]  
(2)

where \( \zeta \) is the potential parameter, \( \alpha \) is the screening parameter and \( \rho \) is the distance between two particles. The exact and approximate solutions to SE have been solved using different potentials such as Rosen-Morse, Coulomb, etc by many researchers [10]-[13]. Some of these potentials are solvable in SE using different methods. These methods include: The Variational principle [14], [15], the Nikiforov-Uvarov (NU) method [16]-[33], Series expansion method (SEM) [34] and others [35]. This potential has been used to describe diatomic molecular energy spectra by several authors [36]-[40]. For instance, Ikhdair et al. [41] obtain the exact solutions of the D-dimensional SE for the Kratzer and pseudoharmonic potentials. The energy states of some diatomic molecular systems were also obtained numerically by a proper transformation approach [42].

The Cornell potential is the essential potential used in studying quarkonium system. It contains two important terms of asymptotic freedom and quark confinement [43]. Several authors have carried out studies with Cornell potential to obtain the mass spectra of heavy quarkonia [44]-[46]. Recently, researchers are applying the exponential type of potential to study the mass spectra of heavy quarkonium system [47]-[50].

Many researchers are concerned about combining two or more potentials. The fundamental nature of combining potential models is to have a broader array of applications [51], [52]. Motivated by the above studies, we attempt to investigate the SE with a potential obtained from the combination of KP (1) and SCP (2) using the SEM to obtain the energy spectra of some selected diatomic molecules and mass spectra of heavy quarkonium system. The combine potential takes the form:

\[
V(\rho) = \eta_0 \cdot \eta_1 \cdot \frac{\eta_2}{\rho^2} - \frac{\zeta e^{-\alpha \rho}}{\rho}
\]  
(3)

The shape of the combined potential for different diatomic molecules is as shown in Fig. 1. Setting \( \zeta = 0 \), (3) reduces to KP. When \( \eta_0 = \eta_1 = \zeta = \alpha = 0 \), (3) reduces to standard Coulomb potential, when \( \eta_0 = \eta_1 = \eta_2 = 0 \), (3) reduces to SCP.

II. SOLUTION OF SCHRODINGER EQUATION WITH KRATZER-SCREENED COULOMB POTENTIAL (KSCP)

The SE of the form is considered [53]:

\[
\frac{d^2 U(\rho)}{d\rho^2} + \frac{2}{\rho} \frac{dU(\rho)}{d\rho} + \left[ \frac{2\mu}{\hbar^2} (E - V(\rho)) \right] \frac{l(l+1)}{\rho^2} U(\rho) = 0
\]  
(4)

where \( l \) is quantum number, \( \mu \) is the reduced mass, \( \rho \) is the inter-nuclear separation and, \( E \) denotes the energy eigenvalues of the system.

The Taylor expansion of the exponential term in (3) up to third order correction is carried out, in other for the potential to be solvable using the SEM, this yields:

\[
\frac{e^{-\alpha \rho}}{\rho} = \frac{1}{\rho} - \frac{\alpha \rho}{2} - \frac{\alpha^2 \rho^2}{6} + ...
\]  
(5)

Upon substituting (5) into (3) we obtain:

\[
V(\rho) = \frac{\zeta a}{\rho^2} + \frac{\zeta a^2}{2} + \frac{\zeta a^3}{6}
\]  
(6)

where

\[
\begin{align*}
\zeta_0 &= \eta_2, \\
\zeta_1 &= \eta_1 - \zeta, \\
\zeta_2 &= \frac{\zeta a^2}{2}, \\
\zeta_3 &= \frac{\zeta a^3}{6}, \\
\zeta_4 &= \eta_0 + \zeta a
\end{align*}
\]  
(7)

Substituting (6) into (4) gives,

\[
\frac{d^2 U(\rho)}{d\rho^2} + \frac{2}{\rho} \frac{dU(\rho)}{d\rho} + \left[ \frac{e^{-\alpha \rho}}{\rho} \frac{L(L+1)}{\rho^2} \right] U(\rho) = 0,
\]  
(8)

where

\[
\begin{align*}
\varepsilon &= \frac{2\mu}{\hbar^2} (E - \zeta_4), \\
-P &= \frac{2\mu \zeta_0}{\hbar^2}, \\
Q &= \frac{2\mu \zeta_2}{\hbar^2}, \\
S &= \frac{2\mu \zeta_3}{\hbar^2}
\end{align*}
\]  
(9)

\[
L(L+1) = \frac{2\mu \zeta_0}{\hbar^2} + l(l+1)
\]  
(10)

Equation (10) yields,
Anzats wave function is considered as [54]:

\[ U(\rho) = e^{-\alpha^2 - \beta^2} F(\rho) \]  

(12)

where \( \alpha \) and \( \beta \) are positive constants. From (12) we achieve the following:

\[ U'(\rho) = F'(\rho)e^{-\alpha^2 - \beta^2} + F(\rho)(-2\alpha \rho - \beta)e^{-\alpha^2 - \beta^2} \]  

(13)

\[ U''(\rho) = F''(\rho)e^{-\alpha^2 - \beta^2} + F(\rho)(-2\alpha \rho - \beta)e^{-\alpha^2 - \beta^2} \]  

(14)

By putting (12), (13) and (14) into (8) and divide through by \( e^{-\alpha^2 - \beta^2} \) we obtain:

\[ F'(\rho) + \left[ -4\alpha \rho - 2\beta + \frac{2}{\rho} \right] F'(\rho) + \left[ 4\alpha^2 - S \right] \rho^2 + (4\alpha \beta - Q) \rho + \left[ P - 2\beta \right] - \frac{L(L+1)}{\rho^2} + \left( \varepsilon + \beta^2 - 6\alpha \right) \right] F(\rho) = 0. \]  

(15)

We consider the function \( F(\rho) \) as a series of the form,

\[ F(\rho) = \sum_{n=0}^{\infty} a_n \rho^{2n+L}. \]  

(16)

Differentiating (16), we achieve the following:

\[ F'(\rho) = \sum_{n=0}^{\infty} (2n + L) a_n \rho^{2n+L-1} \]  

(17)

\[ F''(\rho) = \sum_{n=0}^{\infty} (2n + L)(2n + L - 1) a_n \rho^{2n+L-2} \]  

(18)

By substituting (16), (17) and (18) into (15) we have:

\[ \sum_{n=0}^{\infty} \left[ (2n + L)(2n + L - 1) a_n \rho^{2n+L-1} \right] + \left[ -4\alpha \rho - 2\beta + \frac{2}{\rho} \right] \sum_{n=0}^{\infty} (2n + L) a_n \rho^{2n+L-1} + \left[ 4\alpha^2 - S \right] \rho^2 + (4\alpha \beta - Q) \rho + \left[ P - 2\beta \right] - \frac{L(L+1)}{\rho^2} + \left( \varepsilon + \beta^2 - 6\alpha \right) \sum_{n=0}^{\infty} a_n \rho^{2n+1} = 0 \]  

(19)

Collecting powers of \( \rho \) in (19), yields:

\[ \sum_{n=0}^{\infty} \left[ \left[ (2n + L)(2n + L - 1) + 2(2n + L - L(L+1)) \rho^{2n} + (P - 2\beta) \right] a_n \rho^{2n+L-1} \right] + \left[ 4\alpha^2 - S \right] \rho^{2n+1} + \left[ 4\alpha \beta - Q \right] \rho^{2n+1} + \left( \varepsilon + \beta^2 - 6\alpha \right) \sum_{n=0}^{\infty} a_n \rho^{2n+1} = 0 \]  

(20)

We note that (20) is linearly independent, meaning that each of the terms is separately equal to zero. Note that \( \rho \) is a non-zero function; therefore, it is the coefficient of \( \rho \) that is zero. With this in mind, we obtain the relation for each of the terms.

\[ (2n + L)(2n + L - 1) + 2(2n + L - L(L+1)) = 0 \]  

(21)

\[ -2\beta(2n + L) + P - 2\beta = 0 \]  

(22)

\[ -4\alpha(2n + L) + \varepsilon + \beta^2 - 6\alpha = 0 \]  

(23)

\[ 4\alpha \beta - Q = 0 \]  

(24)

\[ 4\alpha^2 = S = 0 \]  

(25)

From (22) we have,

\[ \beta = \frac{P}{4n + 2L + 2} \]  

(26)

From (25) we have,

\[ \alpha = \sqrt{S} \]  

(27)

The energy eigenvalue equation using (23), yields:

\[ \varepsilon = 2\alpha(4n + 2L + 3) - \beta^2 \]  

(28)

Upon substituting (9), (11), (26) and (27) into (28), we obtain:

\[ E_n = \frac{\beta^2 \epsilon_n}{2\mu} \left( \frac{4n + 2 + \sqrt{(2n + 1)^2 + 8\mu \epsilon_n / \hbar^2}}{4n + 1 + \sqrt{(2n + 1)^2 + 8\mu \epsilon_n / \hbar^2}} \right)^2 + \gamma \]  

(29)

By putting (7) into (29) the energy eigenvalue for the combined potential of (3) is obtain as:

\[ E_n = \frac{\beta^2 \epsilon_n}{12\mu} \left( 4n + 2 + \sqrt{(2n + 1)^2 + 8\mu \epsilon_n / \hbar^2} \right)^2 - \frac{4\mu \epsilon_n}{\hbar^2} \left( 4n + 1 + \sqrt{(2n + 1)^2 + 8\mu \epsilon_n / \hbar^2} \right)^2 + \gamma \]  

(30)

III. SPecial CASES

1. Setting \( \zeta = 0 \) in (30), the energy eigenvalues for KP is in the form:

\[ E_n = \left( 4n + 2 + \sqrt{(2n + 1)^2 + 8\mu \epsilon_n / \hbar^2} \right)^2 - \frac{4\mu \epsilon_n}{\hbar^2} \left( 4n + 1 + \sqrt{(2n + 1)^2 + 8\mu \epsilon_n / \hbar^2} \right)^2 + \gamma \]  

(31)
2. Setting $\zeta = \eta_0 = \eta_2 = \alpha = 0$ in (30) we obtain the energy eigenvalue for standard Coulomb potential in the form:

$$E_{sd} = -\frac{2\mu\eta^2}{\hbar^2 (2n + l + 1)^2}$$  \hspace{1cm} (32)$$

Equation (32) is in good agreement with (30) of [49].

3. Setting $\eta_0 = \eta_1 = \eta_2 = 0$ in (30) we obtain the energy eigenvalues for SCP in the form:

$$E_{sd} = \sqrt{\frac{-\hbar^2 \zeta^2}{12\mu} \left(4n + 2 + \sqrt{(2l + 1)^2 + 16\mu Dr_r^{-1}}\right) - \frac{2\mu}{\hbar^2} \left(4n + 1 + \sqrt{(2l + 1)^2 + 16\mu Dr_r^{-1}}\right)^2} + \zeta \alpha$$  \hspace{1cm} (33)$$

A. Application of Kratzer Potential (KP)

We compute the energy eigenvalues of some selected diatomic molecules from the special case 1 of (32), and the general case of (30). Keeping this in mind, we apply the result to obtain the energy eigenvalues spectra for four diatomic molecules such as N$_2$, CO, NO, and CH, noting that $\eta_0 = D_0$, $\eta_1 = D_r r^{-1}$ and $\eta_2 = 2Dr_r$, [55], where $D_0$ is the dissociation energy and $r_r$ is the equilibrium inter-nuclear separation, which gives the energy eigenvalue in the form:

$$E_{sd} = \left\{4n + 2 + \sqrt{(2l + 1)^2 + 16\mu D_r r^{-1}}\right\} - \frac{2\mu (D_0 r^{-1})^2}{\hbar^2} \left\{4n + 1 + \sqrt{(2l + 1)^2 + 16\mu D_r r^{-1}}\right\}^2 + D_0$$  \hspace{1cm} (34)$$

and

$$E_{sd} = \sqrt{\frac{-\hbar^2 \zeta^2}{12\mu} \left(4n + 2 + \sqrt{(2l + 1)^2 + 16\mu D_r r^{-1}}\right) - \frac{2\mu}{\hbar^2} \left(-D_0 r^{-1} - \zeta\right) \left\{4n + 1 + \sqrt{(2l + 1)^2 + 16\mu D_r r^{-1}}\right\}^2 + D_0 + \zeta \alpha}$$  \hspace{1cm} (35)$$

respectively.

B. Mass Spectra

In this subsection, the mass spectra of the heavy quarkonium system that have the quark and antiquark flavor are calculated by applying the following relation [56], [57].

$$M = 2m + E_{sd}$$  \hspace{1cm} (36)$$

where $m$ is quarkonium mass, and $E_{sd}$ is energy eigenvalues. By substituting (30) into (36) we obtain the mass spectra for KSCP as:

$$M = 2m + \frac{-\hbar^2 p^2 \alpha^2}{12\mu} \left\{4n + 2 + \sqrt{(2l + 1)^2 + 8\mu \alpha} \right\} - \frac{2\mu}{\hbar^2} (-b - p) \times \left\{4n + 1 + \sqrt{(2l + 1)^2 + 8\mu \alpha} \right\}^2 + \alpha \mu$$  \hspace{1cm} (37)$$

IV. RESULTS AND DISCUSSION

Table I shows the standard results of the spectroscopic parameters for selected diatomic molecules of N$_2$, CO, NO, and CH.

| Molecules | $D_i$ (eV) | $r_i$ (Å) | $\mu$ (amu) | $\alpha$ (Å$^{-1}$) |
|-----------|------------|-----------|-------------|------------------|
| N$_2$     | 11.93819382 | 1.0940    | 7.003350    | 2.63960          |
| CO        | 8.10547534 | 1.1283    | 6.860586    | 2.29940          |
| NO        | 8.043729855 | 1.1508    | 7.468441    | 2.75340          |
| CH        | 3.9479221048 | 1.1198    | 0.929931    | 1.52179          |

The spectroscopic parameters are taken from [38], [39], also the following conversion $1 \text{amu} = 931.494028 \text{MeV} / c^2$ [38], [39] and $\hbar c = 1.97329 eV \cdot Å$ [38], [39] are used throughout the computations.

TABLE II: COMPARISON OF BOUND STATE ENERGY EIGENVALUES (eV) OF N$_2$ MOLECULES

| $n$ | $l$ | $E_{sd}$ (eV) | [36] | [19] |
|-----|-----|---------------|------|------|
| 0   | 0   | 0.10918452    | 0.109180 |      |
| 1   | 0   | 0.32742891    | 0.327414 |      |
| 2   | 0   | 0.54567311    | 0.545648 |      |
| 0   | 0   | 0.54671203    | 0.546147 |      |
| 0   | 0   | 0.54670928    | 0.546137 |      |
| 0   | 0   | 0.54716942    | 0.547117 |      |
| 0   | 0   | 0.76391770    | 0.763883 |      |
| 1   | 0   | 0.76416411    | 0.764169 |      |
| 2   | 0   | 0.76541381    | 0.765417 |      |
| 0   | 0   | 0.76690882    | 0.766913 |      |
| 0   | 0   | 0.98216208    | 0.982166 |      |
| 0   | 0   | 0.98266080    | 0.982665 |      |
| 0   | 0   | 0.98365820    | 0.983662 |      |
| 0   | 0   | 0.98515421    | 0.985154 |      |
| 0   | 0   | 0.98714876    | 0.987153 |      |

TABLE III: COMPARISON OF BOUND STATE ENERGY EIGENVALUES (eV) OF CO MOLECULES

| $n$ | $l$ | $E_{sd}$ (eV) | [36] | [19] |
|-----|-----|---------------|------|------|
| 0   | 0   | 0.10194829    | 0.101980 |      |
| 1   | 0   | 0.32742891    | 0.327414 |      |
| 2   | 0   | 0.54567311    | 0.545648 |      |
| 1   | 0   | 0.54671203    | 0.546147 |      |
| 2   | 0   | 0.54716942    | 0.547117 |      |
| 0   | 0   | 0.76391770    | 0.763883 |      |
| 1   | 0   | 0.76416411    | 0.764169 |      |
| 2   | 0   | 0.76541381    | 0.765417 |      |
| 0   | 0   | 0.76690882    | 0.766913 |      |
| 0   | 0   | 0.98216208    | 0.982166 |      |
| 0   | 0   | 0.98266080    | 0.982665 |      |
| 0   | 0   | 0.98365820    | 0.983662 |      |
| 0   | 0   | 0.98515421    | 0.985154 |      |
| 0   | 0   | 0.98714876    | 0.987153 |      |
TABLE IV: Comparison of Bound State Energy Eigenvalues (eV) of NO Molecules

| n  | l  | E_D [36]     | [42]     |
|----|----|--------------|----------|
| 0  | 0  | 0.08250989   | 0.0824883 |
| 1  | 0  | 0.24742405   | 0.2473352 |
|    | 1  | 0.2478667    | 0.2477817 |
| 2  | 0  | 0.41233921   | 0.4122301 |
|    | 1  | 0.41276083   | 0.4126526 |
|    | 2  | 0.41360605   | 0.4134977 |
| 3  | 0  | 0.5772524    | -         |
|    | 1  | 0.5776349    | -         |
|    | 2  | 0.5782520    | -         |
|    | 3  | 0.5797879    | -         |
| 4  | 0  | 0.74216650   | 0.7419718 |
|    | 1  | 0.74258914   | 0.7423944 |
|    | 2  | 0.7434347    | -         |
|    | 3  | 0.7440213    | -         |
|    | 4  | 0.74639231   | 0.7461969 |

TABLE V: Comparison of Bound State Energy Eigenvalues (eV) of CH Molecules

| n  | l  | E_D [36]     | [42]     |
|----|----|--------------|----------|
| 0  | 0  | 0.16867600   | 0.1686344 |
| 1  | 0  | 0.50513964   | 0.5050072 |
|    | 1  | 0.50872344   | 0.5087593 |
| 2  | 0  | 0.84160069   | 0.8413804 |
|    | 1  | 0.84518448   | 0.8449631 |
|    | 2  | 0.85234720   | 0.8521246 |
| 3  | 0  | 1.18164552   | -         |
|    | 1  | 1.18165060   | -         |
|    | 2  | 1.18880823   | -         |
|    | 3  | 1.19554015   | 1.1955459 |
| 4  | 0  | 1.51452277   | 1.5141255 |
|    | 1  | 1.51810856   | 1.5181130 |
|    | 2  | 1.52526928   | 1.5248701 |
|    | 3  | 1.53600120   | 1.5356002 |
|    | 4  | 1.55028784   | 1.5498843 |

TABLE VI: Energy Eigenvalues (eV) of KSCP for Different Values of n and l for Different Diatomic Molecules

| n  | l  | N_2  | CO   | NO   | CH   |
|----|----|------|------|------|------|
| 0  | 0  | 9.4749839 | 8.5418202 | 6.3038646 | 3.1242806 |
| 0  | 1  | 9.4749538 | 8.5417902 | 6.3038367 | 3.1240224 |
| 1  | 0  | 9.4749596 | 8.5417961 | 6.3038421 | 3.1240615 |
| 1  | 1  | 9.4749293 | 8.5417604 | 6.3038089 | 3.1237742 |
| 1  | 2  | 9.4749297 | 8.5417309 | 6.3038144 | 3.1238135 |
| 2  | 0  | 9.4749413 | 8.5417487 | 6.3038253 | 3.1238921 |
| 2  | 1  | 9.4749842 | 8.5417665 | 6.3037815 | 3.1235335 |
| 2  | 2  | 9.4749117 | 8.5417075 | 6.3033859 | 3.1236539 |
| 3  | 0  | 9.4749292 | 8.5417194 | 6.3037543 | 3.1237726 |
| 3  | 1  | 9.4748649 | 8.5417372 | 6.3037597 | 3.1233054 |
| 3  | 2  | 9.4748707 | 8.5417610 | 6.3037706 | 3.1233058 |
| 3  | 3  | 9.4748823 | 8.5417672 | 6.3037870 | 3.1233451 |
| 3  | 4  | 9.4748998 | 8.5417194 | 6.3038087 | 3.1234246 |
| 4  | 0  | 9.4749231 | 8.5417610 | 6.3037274 | 3.1234246 |
| 4  | 1  | 9.4748357 | 8.5416726 | 6.3037328 | 3.1235438 |
| 4  | 2  | 9.4748415 | 8.5416785 | 6.3037328 | 3.1231237 |
| 4  | 3  | 9.4748532 | 8.5416904 | 6.3037437 | 3.1232035 |
| 4  | 4  | 9.4748707 | 8.5417082 | 6.3037600 | 3.1233232 |
| 4  | 5  | 9.4748940 | 8.5417320 | 6.3037818 | 3.1234830 |
| 5  | 0  | 9.4749231 | 8.5417617 | 6.3038090 | 3.1236828 |

TABLE VII: Mass Spectra of Charmonium in (GeV) (m_c = 1.488 GeV , μ = 0.744 GeV, α = 3.1674, h = 1, η = -0.2860 GeV, η = 0.001 GeV, η = 0.1306 GeV and ζ = 0.0022 GeV )

| State | Present work | [49] | [47] | Experiment | [46] |
|-------|--------------|------|------|------------|------|
| 1S    | 3.096        | 3.096| 3.096| 3.096      |      |
| 2S    | 3.686        | 3.686| 3.686| 3.686      |      |
| 1P    | 3.295        | 3.527| 3.527| 3.527      |      |
| 2P    | 3.802        | 3.871| 3.687| 3.773      |      |
| 3S    | 4.040        | 4.040| 4.040| 4.040      |      |
| 4S    | 4.269        | 4.341| 4.360| 4.322      |      |
| 1D    | 3.583        | 3.098| 3.770| 3.770      |      |
| 2D    | 3.976        | 3.976| 4.159| 4.159      |      |
| 1F    | 3.862        | 4.162|      |            |      |

Fig. 1. Shape of Kratzer potential plus screened Coulomb potential for different diatomic molecules.

Fig. 2. Energy eigenvalues variation with dissociation energy for different diatomic molecules.

Fig. 3. Energy eigenvalues variation with equilibrium bond length for different diatomic molecules.
cases. The mass spectra energy of charmonium and bottomonium for states 1S to 1F were obtained and compared with experimental data and other recent theoretical works, which are in good agreement. Our results are consistent with the results in the available literature. The present study can be extended to scrutinize the thermodynamic properties of energy dependent systems [60]-[72].

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A. Discussion
The energy eigenvalues of the KSCP and KP using (35) and (34) for \( \text{N}_2 \), CO, NO, and CH diatomic molecules are computed, respectively as presented in Tables II to VI, explicitly, for different quantum numbers. Our results are in good agreement with the works of others, like [36] and [19] as shown in Tables II and III. Also, Tables IV and V of [36] and [42], Table VI with the results of [37] are in agreement. We have plotted the shape of KSCP for selected diatomic molecules, as shown in Fig. 1. It shows a close look into the behavior of the potential under study. Also, we plotted the energy eigenvalues with different parameters such as \( D_\nu \), \( r_c \) and \( \alpha \) as shown in Fig. 2-4, respectively, for various values of vibrational and rotational quantum numbers. There is an increase in energy eigenvalues as the \( D_\nu \), \( r_c \) and \( \alpha \) increases in these Figures. In Fig. 2 and 3, it was noticed that there is an asymptotic convergence towards zero energy while in Fig. 4, the increase tends to spread out from zero position for different vibrational quantum numbers.

We calculate mass spectra of heavy quark-antiquark system such as charmonium and bottomonium for some states using (37). The numerical values of bottomonium (\( bb \)) and charmonium (\( cc \)) masses as 4.680 GeV and 1.488 GeV , respectively were adopted [58]. Then, the reduced masses are \( \mu_b = 2.340 \text{ GeV} \) and \( \mu_c = 0.744 \text{ GeV} \). The parameters of (37) were then obtained by fitting with experimental data of mass spectra for 2S, 2P in the case of charmonium. In the case of bottomonium the values of the parameters in (37) are calculated following the same procedure, with experimental data of mass spectra for 1S, 2S . Experimental data is taken from [59].

We note that prediction of mass spectra of charmonium and bottomonium are in acceptable concurrence with experimental data and other recent theoretical predictions, like in [47] and [49], as shown in Tables VII and VIII.

V. Conclusion
In this study, we obtain the bound state solutions of the Schrödinger equation with KSCP via the SEM. Explicitly, we compute the energy eigenvalues of the selected diatomic molecules: \( \text{N}_2 \), CO, NO, and CH, and also consider special
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