One-pion exchange current corrections for nuclear magnetic moments in relativistic mean field theory

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Abstract

The one-pion exchange current corrections to isoscalar and isovector magnetic moments of double-closed shell nuclei plus and minus one nucleon with $A = 15, 17, 39$ and 41 have been studied in the relativistic mean field (RMF) theory and compared with previous relativistic and non-relativistic results. It has been found that the one-pion exchange current gives a negligible contribution to the isoscalar magnetic moments but a significant correction to the isovector ones. However, the one-pion exchange current doesn’t improve the description of nuclear isovector magnetic moments for the concerned nuclei.

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Nuclear magnetic moment is one of the most important physics observables. It provides a highly sensitive probe of the single-particle structure, serves as a stringent test of nuclear models, and has attracted the attention of nuclear physicists since the early days [1, 2].

The static magnetic dipole moments of ground states and excited states of lots of atomic nuclei have already been measured with several methods [3]. With the development of the radioactive ion beam (RIB) technique, it is now even possible to measure the nuclear magnetic moments of many short-lived nuclei near the proton and neutron drip lines with very high precision [4–6].

Theoretical description for nuclear magnetic moment is a long-standing problem. For the last decades, many successful nuclear structure models have been built up. However, the application of these models for nuclear magnetic moments is still not satisfactory.

The Schmidt values predicted by the extreme single-particle shell model qualitatively succeeded in explaining the magnetic moments of odd-$A$ nuclei near double-closed shells. Later on, the magnetic moment of nuclei farther away from closed shells were found to be sandwiched by the Schmidt lines. Therefore, lots of efforts have been made to explain the deviations of the nuclear magnetic moments from the Schmidt values. In shell model, the first-order configuration mixing (core polarization) [7], i.e., the single-particle state coupled to more complicated $2p-1h$ configurations, and the second-order core polarization as well as the meson exchange current (MEC) [8–10] are taken into account to explain the deviations.

The magnetic moments of $LS$ closed shell nuclei plus or minus one nucleon are of particular importance, in which, there are no spin-orbit partners on both sides of the Fermi surface and therefore all first-order core polarization corrections vanish. It has been shown in non-relativistic calculations that the second-order core polarization effect dominates the deviations of isoscalar magnetic moments and also gives large corrections to the isovector magnetic moments [11, 12]. The MEC effect, due to its isovector nature, gives rather small corrections to the isoscalar magnetic moments while gives important corrections to the isovector magnetic moments [10, 13]. As a result, the calculated corrections to the isoscalar magnetic moments are in reasonable agreement with the data, and the net effect of second order core polarization and MEC gives the right sign for the correction to the Schmidt isovector magnetic moments [11, 12].

In the past decades, the RMF theory, which can take into account the spin-orbit coupling naturally, has been successfully applied to the analysis of nuclear structure over the whole
periodic table, from light to superheavy nuclei with a few universal parameters \[14–16\]. However, a straightforward application of the single-particle relativistic model, where only sigma and the time-like component of the vector mesons were considered, cannot reproduce the experimental magnetic moments \[17, 18\]. It is because the reduced Dirac effective nucleon mass \( M^* \sim 0.6M \) enhances the relativistic effect on the electromagnetic current \[19\].

After the introduction of the vertex corrections to define effective single-particle currents in nuclei, e.g., the “back-flow” effect in the framework of the relativistic extension of Landau’s Fermi-liquid theory \[19\] or the random phase approximation (RPA) type summation of p-h and p-\( \bar{n} \) bubbles in relativistic Hartree approximation \[20, 21\], or the consideration of non-zero space-like components of vector meson in the self-consistent deformed RMF theory \[22–24\], the isoscalar magnetic moment can be reproduced quite well. Unfortunately, these effects cannot remove the discrepancy existing in isovector magnetic moments. To eliminate the discrepancy, the MEC corrections have been investigated in the linear RMF theory in Ref. \[25\], which was found to be significant but enlarge the disagreement with data.

In view of these facts, it is essential to investigate the nuclear magnetic moments in the RMF theory with modern effective interactions. In this work, the isoscalar and isovector magnetic moments of light odd-mass nuclei near the double-closed shells will be studied in axially deformed RMF theory with the consideration of non-zero space-like components of vector meson. In particular, the one-pion exchange current corrections to nuclear magnetic moments will be examined.

The starting point of the RMF theory is the standard effective Lagrangian density constructed with the degrees of freedom associated with nucleon field \( \psi \), two isoscalar meson fields \( \sigma \) and \( \omega_\mu \), isovector meson field \( \vec{\rho}_\mu \) and photon field \( A_\mu \). The equation of motion for a single-nucleon orbit \( \psi_i(r) \) reads,

\[
\{ \alpha \cdot [p - V(r)] + \beta M^*(r) + V_0(r) \} \psi_i(r) = \epsilon_i \psi_i(r),
\]

where \( M^*(r) \) is defined as \( M^*(r) \equiv M + g_\sigma \sigma(r) \), with \( M \) referring to the mass of bare nucleon. The repulsive vector potential is \( V_0(r) = g_\omega \omega_0(r) + g_\rho \rho_0(r) + \frac{1}{2} A_0(r) \), where \( g_i (i = \sigma, \omega, \rho) \) are the coupling strengths of nucleon with mesons. The time-odd fields \( V(r) \) are naturally given by the space-like components of vector fields, \( V(r) = g_\omega \omega(r) \), where the space components of \( \rho \)-meson field \( \vec{\rho}(r) \) and Coulomb field \( \vec{A}(r) \) are neglected since they
turn out to be small compared with $\omega(r)$ field in light nuclei [22].

The non-vanishing time-odd fields in Eq. (1) give rise to splitting between pairwise time-reversal states $\psi_i$ and $\psi_T(\equiv \hat{T}\psi_i)$ and also the non-vanishing current in the core, where $\hat{T}$ is the time-reversal operator. Each Dirac spinor $\psi_i(r)$ and meson fields are expanded in terms of a set of isotropic harmonic oscillator basis in cylindrical coordinates with 16 major shells [26, 27]. The pairing correlations for these double-closed shell nuclei plus or minus one nucleon are neglected due to the quenching effect from unpaired valence nucleon. More details about solving the Dirac equation with time-odd fields can be found in Refs. [28, 29].

The effective electromagnetic current operator used to describe the nuclear magnetic moment is given by [23, 24]

$$\hat{J}^\mu(x) = \bar{\psi}(x)\gamma^\mu\frac{1 - \tau_3}{2}\psi(x) + \frac{\kappa}{2M}\partial_\nu[\bar{\psi}(x)\sigma^{\mu\nu}\psi(x)],$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, and $\kappa$ is the free anomalous gyromagnetic ratio of the nucleon, $\kappa_p = 1.793$ and $\kappa_n = -1.913$. The nuclear dipole magnetic moment is determined by

$$\mu = \frac{1}{2}\int d^3r r \times \langle \text{g.s.} | \hat{j}(r) | \text{g.s.} \rangle,$$

where $\hat{j}(r)$ is the operator of space-like components of the effective electromagnetic current.

In addition, for isovector magnetic moment, the one-pion exchange current correction should be taken into account. Although there is no explicit pion meson in the RMF theory, it is possible to study the MEC corrections due to the virtual pion exchange between two nucleons, which is given by the Feynman diagrams in Fig. 1.

![FIG. 1: Diagrams of the one-pion exchange current: seagull (left) and in-flight (right).](image)

The one-pion exchange current contributions to magnetic moments can thus be obtained as [23],

$$\mu_{\text{MEC}} = \frac{1}{2}\int d^3r r \times \langle \text{g.s.} | \hat{j}_{\text{seagull}}(r) + \hat{j}_{\text{in-flight}}(r) | \text{g.s.} \rangle,$$
where the corresponding one-pion exchange currents $J^{\text{seagull}}(r)$ and $J^{\text{in-flight}}(r)$ are respectively,

\[
J^{\text{seagull}}(r) = -\frac{8e f_\pi^2 M}{m_\pi^2} \int d\vec{x} \bar{\psi}_p(\vec{r}) \gamma_5 \psi_n(\vec{r}) D_\pi(\vec{r}, \vec{x}) \bar{\psi}_n(\vec{y}) \frac{M^*}{M} \gamma_5 \psi_p(\vec{x}),
\]

\[
J^{\text{in-flight}}(r) = -\frac{16ie f_\pi^2 M^2}{m_\pi^2} \int d\vec{x} d\vec{y} \bar{\psi}_p(\vec{x}) \frac{M^*}{M} \gamma_5 \psi_n(\vec{x}) D_\pi(\vec{x}, \vec{r}) \nabla_{\vec{r}} D_\pi(\vec{r}, \vec{y}) \bar{\psi}_n(\vec{y}) \frac{M^*}{M} \gamma_5 \psi_p(\vec{y}),
\]

with the $\pi$-nucleon coupling constant $f_\pi = 1$ and the pion mass $m_\pi = 138$ MeV. The pion propagator in $r$-space is given by $D_\pi(\vec{x}, \vec{r}) = \frac{1}{4\pi} \frac{e^{-m_\pi|\vec{x} - \vec{r}|}}{|\vec{x} - \vec{r}|}$.

The magnetic moments of double-closed shell nuclei plus or minus one nucleon with $A = 15, 17, 39$ and $41$ are studied in the RMF theory using PK1 effective interaction [30], which includes the self-couplings of $\sigma$ and $\omega$ mesons.

The magnetic moments in Eq.(3) will be calculated using Dirac spinors $\psi_i$ from the axially deformed RMF calculations with space-like components of vector meson field. As small deformation in these nuclei, the one-pion exchange current contributions to magnetic moments in Eq.(4) are calculated using the spherical Dirac spinors of corresponding double-closed shell nucleus as done in Ref. [25].

**TABLE I**: The one-pion exchange current corrections to the isovector magnetic moments obtained from RMF calculations using PK1 effective interaction, in comparison with the Linear RMF [25] and non-relativistic results [8, 10, 31, 32] (see text for details).

| A    | Non-relativistic | Relativistic |
|------|-----------------|--------------|
|      | [8] [31] [10] [32] | [25] This work |
| 15   | 0.127 0.116 0.092 0.111 | 0.102 0.091 |
| 17   | 0.084 0.093 0.065 0.092 | 0.151 0.092 |
| 39   | 0.204 0.199 0.149 0.184 | 0.174 0.190 |
| 41   | 0.195 0.201 0.115 0.180 | 0.270 0.184 |

The one-pion exchange current corrections to the isovector magnetic moments obtained from RMF calculations using PK1 are compared in Table I with linear RMF calculations [25] using L3 [33] and non-relativistic calculations [8, 10, 31, 32]. It is shown that the obtained corrections to the isovector magnetic moments in this work are in reasonable agreement with
other calculations. As noted in Ref. [25], the differences between the various calculations presented in Table I are most likely due to relatively small changes in the balance of contributions from seagull and in-flight diagrams rather than any fundamental differences in the models used. Other nonlinear effective interactions are also used to calculate the one-pion exchange current corrections, and similar results are obtained as those given by PK1.

TABLE II: Isoscalar magnetic moments obtained from RMF calculations using PK1 effective interaction, in comparison with the corresponding data, Schmidt value, previous relativistic result [25] and non-relativistic results [11, 12] (see text for details).

| A   | Exp. | Non-relativistic | Relativistic |
|-----|------|------------------|--------------|
|     |      | Schmidt [11]     | [12]         | QHD+MEC [25] | RMF+MEC |
| 15  | 0.218| 0.187            | 0.228        | 0.233        | 0.200(0.199+0.001) | 0.216(0.216 + 0.000) |
| 17  | 1.414| 1.440            | 1.410        | 1.435        | 1.42 (1.43 −0.011) | 1.467(1.469 − 0.002) |
| 39  | 0.706| 0.636            | 0.706        | 0.735        | 0.659(0.660−0.001) | 0.707(0.707 + 0.000) |
| 41  | 1.918| 1.940            | 1.893        | 1.944        | 1.93 (1.94 −0.007) | 1.988(1.991 − 0.003) |

In Table II the isoscalar magnetic moments and corresponding pion exchange current corrections obtained from RMF calculations using PK1 are presented in comparison with the corresponding data, Schmidt value, previous relativistic result [25] and non-relativistic results [11, 12]. The isoscalar magnetic moments obtained from deformed RMF theory with space-like components of vector meson are labeled as RMF and corresponding one-pion exchange current corrections calculated similarly as in Ref. [25] are labeled as MEC.

The isoscalar magnetic moment in Ref. [25] consists of two parts, i.e., the QHD calculations taken from Ref. [23] and the additional one-pion exchange current corrections calculated with L3 effective interaction.

For the non-relativistic calculations in Refs. [11, 12], the harmonic oscillator wave functions are used for single-particle states and one-boson-exchange potential [11] and Hamada-Johnston potential [12] were respectively employed for the residual interaction. For the corrections to magnetic moments, the second-order core polarization, MEC, and the crossing term between MEC and core polarization have been included. For the MEC corrections, the Δ isobar current as well as the exchange current of the mesons π, σ, ω, and ρ have been taken into account.
It is shown that all calculated results in Table II are in good agreement with data, and same as the previous relativistic \cite{25} and non-relativistic calculations \cite{11, 12}, the MEC corrections to isoscalar moments in present calculations are negligible. For the mirror nuclei with double-closed shell plus or minus one nucleon, the MEC corrections to isoscalar moments reflect the violation of isospin symmetry in wave functions. With the small MEC corrections to isoscalar moments here, it is easy to understand the excellent description of the isoscalar magnetic moments in deformed RMF theory with space-like components of vector meson in Refs. \cite{22, 24}.

| A  | Exp. | Non-relativistic | Relativistic |
|----|------|-----------------|--------------|
|    |      | Schmidt         |              |
|    |      | \[11\]          | \[12\]      |              |
| 15 | −0.501 | −0.451          | −0.456       |
| 17 | 3.308  | 3.353           | 3.281        |
| 39 | −0.316 | −0.512          | −0.286       |
| 41 | 3.512  | 3.853           | 3.803        |

In Table III, the isovector magnetic moments and corresponding pion exchange current corrections in RMF calculations using PK1 are compared with the data, Schmidt value, previous relativistic \cite{25} and non-relativistic results \cite{11, 12}.

It is shown that the pion exchange current gives a significant positive correction to isovector magnetic moments, which is consistent with the calculations in Ref. \cite{25} as well as most non-relativistic calculations \cite{11, 12}. However, compared with the case for the isoscalar magnetic moments, the results of relativistic calculations deviate much more from data explicitly, namely, this positive contribution is not welcome to improve the agreement with data. Such a phenomenon is also found from RMF calculations with other effective interactions. Therefore, the RMF theory with one-pion exchange current corrections could not improve the description of isovector magnetic moment for the concerned nuclei.

In the future relativistic investigation, the other effects due to the second-order core polarization, the Δ isobar current, exchange current corrections due to other mesons, and the crossing term between MEC and core polarization should be taken into account, as noted already in the non-relativistic calculations \cite{11, 12}.
In summary, the one-pion exchange current corrections to the isoscalar and isovector magnetic moments have been studied in the RMF theory with PK1 effective interaction and compared with previous relativistic and non-relativistic results. It has been found that the one-pion exchange current gives a negligible contribution to the isoscalar magnetic moments but a significant correction to the isovector ones. However, the one-pion exchange current doesn’t improve the description of nuclear isovector magnetic moments for the concerned nuclei. In the future investigation, similar as the non-relativistic cases [11, 12], the second-order core polarization effects, the Δ isobar current, crossing term between MEC and core polarization, and exchange current corrections due to other mesons should be taken into account. In addition, the correction due to the restoration of the rotational symmetry [34] may play a role as well. The investigation towards these directions is in progress.

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