Singularities and Closed String Tachyons

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Abstract

A basic problem in gravitational physics is the resolution of spacetime singularities where general relativity breaks down. The simplest such singularities are conical singularities arising from orbifold identifications of flat space, and the most challenging are spacelike singularities inside black holes (and in cosmology). Topology changing processes also require evolution through classically singular spacetimes. I briefly review how a phase of closed string tachyon condensate replaces, and helps to resolve, basic singularities of each of these types. Finally I discuss some interesting features of singularities arising in the small volume limit of compact negatively curved spaces and the emerging zoology of spacelike singularities.
1 Singularities and Winding Modes

In the framework of string theory, several types of general relativistic singularities are replaced by a phase of closed string tachyon condensate. The simplest class of examples involves spacetimes containing 1-cycles with antiperiodic Fermion boundary conditions. This class includes spacetimes which are globally stable, such as backgrounds with late-time long-distance supersymmetry and/or AdS boundary conditions.

In the presence of such a circle, the spectrum of strings includes winding modes around the circle. The Casimir energy on the worldsheet of the string contributes a negative contribution to the mass squared, which is of the form

\[ M^2 = -\frac{1}{l_s^2} + \frac{L^2}{l_s^4} \]  

where \( L \) is the circle radius and \( 1/l_s^2 \) is the string tension scale. For small \( L \), the winding state develops a negative mass squared and condenses, deforming the system away from the \( L < l_s \) extrapolation of general relativity. This statement is under control as long as \( L \) is static or shrinking very slowly as it crosses the string scale. This sort of system was studied first in string theory in [1].

Examples include the following. Generic orbifold singularities have twisted sector tachyons, i.e. tachyons from strings wound around the angular direction of the cone. The result of their condensation is that the cone smooths out [2], as seen in calculations of D-brane probes, worldsheet RG, and time dependent GR in their regimes of applicability (see [3] for reviews). Topology changing transitions in which a Riemann surface target space loses a handle or factorizes into separate surfaces are also mediated by winding tachyon condensation [4]. Tachyon condensation replaces certain spacelike singularities of a cosmological type in which some number of circles shrinks homogeneously in the far past (or future) [5].

Finally, tachyons condensing quasilocally over a spacelike surface appear in black hole problems and in a new set of examples sharing some of their features [6][7]. One interesting new example is an AdS/CFT dual pair in which an infrared mass gap (confinement) arises at late times in a system which starts out in an unconfined phase out on its approximate Coulomb branch. As an example, consider the \( \mathcal{N} = 4 \) SYM theory on a (time dependent) Scherk-Schwarz circle, with scalar VEVs turned on putting it out on its Coulomb branch. As the circle shrinks to a finite size and the scalars roll back toward the origin, the infrared physics of the gauge theory becomes dominated by a three dimensional confining theory. The gravity-side description of this is via a shell of D3-branes which enclose a finite region with a shrinking Scherk-Schwarz cylinder. When the cylinder’s radius shrinks below the string scale, a winding tachyon turns on. At the level of bulk spacetime gravity, a candidate dual for the confining theory exists [3]; it is a type of “bubble of nothing” in which the geometry smoothly caps off in the region corresponding to the infrared limit of the gauge theory. This arises in the time dependent problem via the tachyon condensate...
phase replacing the region of the geometry corresponding to the deep IR limit of the field theory.

For all these reasons, it is important to understand the physics of the tachyon condensate phase. The tachyon condensation process renders the background time-dependent; the linearized solution to the tachyon equation of motion yields an exponentially growing solution $T \propto \mu e^{\kappa X^0}$. As such there is no a priori preferred vacuum state. The simplest state to control is a state $|\text{out}\rangle$ obtained by a Euclidean continuation in the target space, and describes a state in which nothing is excited in the far future when the tachyon dominates. This is a perturbative analogue of the Hartle-Hawking choice of state. At the worldsheet level (whose self-consistency we must check in each background to which we apply it), the tachyon condensation shifts the semiclassical action appearing in the path integrand. String amplitudes are given by

$$< \prod \int V > \sim \int DX^0 D\vec{X} e^{-S_E} \prod \int V$$

where I work in conformal gauge and suppress the fermions and ghosts. Here $X^\mu$ are the embedding coordinates of the string in the target space and $\int V$ are the integrated vertex operators corresponding to the bulk asymptotic string states appearing in the amplitude. The semiclassical action in the Euclidean theory is

$$S_E = S_0 + \int d^2 \sigma \mu^2 e^{2\kappa X^0} \hat{T}(\vec{X})$$

with $S_0$ the action without tachyon condensation and $\hat{T}(\vec{X})$ a winding (sine-Gordon) operator on the worldsheet. These amplitudes compute the components of the state $|\text{out}\rangle$ in a basis of multiple free string states arising in the far past bulk spacetime when the tachyon is absent. The tachyon term behaves like a worldsheet potential energy term, suppressing contributions from otherwise singular regions of the path integration.

Before moving to summarize the full calculation of basic amplitudes, let me note two heuristic indications that the tachyon condensation effectively masses up degrees of freedom of the system. First, the tachyon term in (1.3) behaves like a spacetime dependent mass squared term in the analogue of this action arising in the case of a first quantized worldline action for a relativistic particle \[9\]. Second, the dependence of the tachyon term on the spatial variables $\vec{X}$ is via a relevant operator, dressed by worldsheet gravity (which in conformal gauge is encapsulated in the fluctuations of the timelike embedding coordinate $X^0$). The worldsheet renormalization group evolution with scale is different from the time dependent evolution, since fluctuations of $X^0$ contribute. However in some cases, such as localized tachyon condensates and highly supercritical systems, the two processes yield similar endpoints. In any case, as a heuristic indicator of the effect of tachyon condensation, the worldsheet RG suggests a massing up of degrees of freedom at the level of the worldsheet theory as time evolves forward.

Fortunately we do not need to rely too heavily on these heuristics, as the methods of Liouville field theory enable us to calculate basic physical quantities in the
problem. In the Euclidean state defined by the above path integral, regulating the bulk contribution by cutting off $X^0$ in the far past at $ln\mu^*$, one finds a partition function $Z$ with real part

$$Re(Z) = -\frac{ln(\mu/\mu^*)}{\kappa} \hat{Z}_{\text{free}}$$  \hspace{1cm} (1.4)$$

This is to be compared with the result from non-tachyonic flat space $Z_0 = \delta(0) \hat{Z}_{\text{free}}$ [5], where $\delta(0)$ is the infinite volume of time, and $\hat{Z}_{\text{free}}$ is the rest of the partition function. In the tachyonic background (1.4), the first factor is replaced by a truncated temporal volume which ends when the tachyon turns on. A similar calculation of the two point function yields the Bogoliubov coefficients corresponding to a pure state in the bulk with thermal occupation numbers of particles, with temperature proportional to $\kappa$. This technique was first suggested in [9], where it was applied to bulk tachyons for which $\kappa \approx 1/l_s$ and the resulting total energy density blows up. In the examples of interest for singularities, the tachyon arises from a winding mode for which $\kappa \ll 1/l_s$, and the method [9] yields a self-consistently small energy density [5]. In the case of an initial singularity, this gives a perturbative string mechanism for the Hartle/Hawking idea of starting time from nothing. This timelike Liouville theory provides a perturbative example of “emergent time”, in the same sense that spatial Liouville theory provides a worldsheet notion of “emergent space”.\(^1\)

So far this analysis applied to a particular vacuum. It is important to understand the status of other states of the system. In particular, the worldsheet path integral has a saddle point describing a single free string sitting in the tachyon phase. Do putative states such as this with extra excitations above the tachyon condensate constitute consistent states? This question is important for the problem of unitarity in black hole physics and in more general backgrounds where a tachyon condenses quasilocally, excising regions of ordinary spacetime. If nontrivial states persist in the tachyon phase in such systems, this would be tantamount to the existence of hidden remnants destroying bulk spacetime unitarity.

In fact, we find significant indications that the state where a string sits by itself in the tachyon phase does not survive as a consistent state in the interacting theory [10/7]. The saddle point solution has the property that the embedding coordinate $X^0$ goes to infinity in finite worldsheet time $\tau$. This corresponds to a hole in the worldsheet, which is generically not BRST invariant by itself. If mapped unitarily to another hole in the worldsheet obtained from a correlated negative frequency particle impinging on the singularity, worldsheet unitarity may be restored. This prescription is a version of the Horowitz/Maldacena proposal of a black hole final

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\(^1\)This was also noted by M. Douglas in the discussion period in the session on emergent spacetime, in which G. Horowitz also noted existing examples. As explained by the speakers in that session, no complete non-perturbative formulation involving emergent time exists, in contrast to the situation with spatial dimensions where matrix models and AdS/CFT provide examples (but see [14] for an interesting example of a null singularity with a proposed non-perturbative description in terms of matrix theory).
state [11]; the tachyon condensate seems to provide a microphysical basis for this suggestion. Note that this proposal is consistent with the idea of complimentarity: unitary evolution along the spacelike slice of the singularity is mapped to unitary evolution as seen by the asymptotic observer.

A more dynamical effect which evacuates the tachyon region also arises in this system. A particle in danger of getting stuck in the tachyon phase drags fields (for example the dilaton and graviton) along with it. The heuristic model of the tachyon condensate as an effective mass for these modes [9] suggests that the fields themselves are getting heavy. The resulting total energy of the configuration, computed in [7] for a particle of initial mass $m_0$ coupled with strength $\lambda$ to a field whose mass also grows at late times like $M(x^0)$, is

$$E = m_0^2 \lambda^2 M(x^0) \cos^2 \left( \int x^0 M(t') dt' \right) F(R)$$  \hspace{1cm} (1.5)

This is proportional to a function $F(R)$ which increases with greater penetration distance $R$ of the particle into the tachyon phase. Hence we expect a force on any configuration left in the tachyon phase which sources fields (including higher components of the string field). This does not mean every particle classically gets forced out of the tachyonic sector: for example in black hole physics, the partners of Hawking particles which fall inside the black hole provide negative frequency modes that correlate with the matter forming the black hole.

The analysis of this dynamical effect in generic states relies on the field-theoretic (worldsheet minisuperspace) model for tachyon dynamics. It is of interest to develop complete worldsheet techniques to analyze other putative vacua beyond the Euclidean vacuum. In the case of the Euclidean vacuum, the worldsheet analyses [9, 5] reproduce the behavior expected from the heuristic model, so we have tentatively taken it as a reasonable guide to the physics in more general states as well.

The string-theoretic tachyon mode which drives the system away from the GR singularities is clearly accessible perturbatively. But it is important to understand whether the whole background has a self-consistent perturbative string description. In the Euclidean vacuum, this seems to be the case: the worldsheet amplitudes are shut off in the tachyon phase in a way similar to that obtained in spatial Liouville theory. In other states, it is not a priori clear how far the perturbative treatment extends. One indication for continued perturbativity is that according to the simple field theory model, every state gets heavy in the tachyon phase, including fluctuations of the dilaton, which may therefore be stuck at its bulk weak coupling value. It could be useful to employ AdS/CFT methods [12] to help decide this point.

## 2 Discussion and Zoology

Many timelike singularities are resolved in a way that involves new light degrees of freedom appearing at the singularity. In the examples reviewed in section 1, ordinary
spacetime ends where the tachyon background becomes important. The tachyon at first constitutes a new light mode in the system, but its condensation replaces the would-be short-distance singularity with a phase where degrees of freedom ultimately become heavy. However, there are strong indications that there is a whole zoo of possible behaviors at cosmological spacelike singularities, including examples in which the GR singularity is replaced by a phase with more light degrees of freedom (see [14] for an interesting null singularity where a similar behavior obtains).

In particular, consider a spacetime with compact negative curvature spatial slices, for example a Riemann surface. The corresponding nonlinear sigma model is strongly coupled in the UV, and requires a completion containing more degrees of freedom. In supercritical string theory, the dilaton beta function has a term proportional to \( D - D_{\text{crit}} \). The corresponding contribution in a Riemann surface compactification is \( (2h - 2)/V \sim 1/R^2 \) where \( V \) is the volume of the surface in string units, \( h \) the genus and \( R \) the curvature radius in string units. This suggests that there are effectively \( (2h - 2)/V \) extra (supercritical) degrees of freedom in the Riemann surface case. Interestingly, this count of extra degrees of freedom arises from the states supported by the fundamental group of the Riemann surface. For simplicity one can work at constant curvature and obtain the Riemann surface as an orbifold of Euclidean AdS\(_2\), and apply the Selberg trace formula to obtain the asymptotic number density of periodic geodesics (as reviewed for example in [15]). This yields a density of states from a sum over the ground states in the winding sectors proportional to \( e^{m \sqrt{2h/V}} \) where \( m \) is of order the mass of the string state. Another check arises by modular invariance which relates the high energy behavior of the partition function to the lowest lying state: the system contains a light volume mode, which is normalizable on the compact surface and propagates in loops, whose mass scales the right way to account for the modular transform of this density of states.

At large radius, the Riemann surface component of the target space is clearly two dimensional to a good approximation, and the 2d oscillator modes are entropically favored at high energy. It is interesting to contemplate possibility of cases where the winding states persist to the limit \( V \rightarrow 1 \), in which case the density of states from this sector becomes that of a 2h dimensional theory and the system crosses over to a very supercritical theory in which these states become part of the oscillator spectrum. In particular, states formed from the string wrapping generating cycles of the fundamental group in arbitrary orders (up to a small number of relations) constitute a 2h-dimensional lattice random walk. At large volume, the lattice spacing is much greater than the string scale and the system is far from its continuum limit, so these states are a small effect. But at small Riemann surface volume it is an interesting possibility that these states cross over to the high energy spectrum of oscillator modes in 2h dimensions [13].

Of course as emphasized in [13], there are many possible behaviors at early

\(^2\)I thank O. Aharony, A. Maloney, J. McGreevy, and others for discussions on these points.
times, including ones where the above states do not persist to small radius \[^{[2]}\] and ones where they do persist but are part of a still larger system. One simple way to complete the sigma model is to extend it to a linear sigma model (containing more degrees of freedom) which flows to the Riemann surface model in the IR. Coupling this system to worldsheet gravity yields in general a complicated time dependent evolution, whose late time behavior is well described by the nonlinear sigma model on an expanding Riemann surface. If one couples this system to a large supercritical spectator sector, then during the epoch when the coupling is weak (obtainable for any semi-infinite time span by tuning the bare string coupling) the time dependent evolution approaches the RG flow of the linear sigma model, which yields a controlled regime in which it is clear that at earlier times the system had more degrees of freedom.

Clearly a priori this can happen in many ways. In addition to the landscape of metastable vacua of string theory I believe the conservative expectation is that there will be a zoo of possible cosmological histories with similar late time behavior; indeed inflationary cosmology already has this feature. While it may be tempting to reject this situation out of hand in hopes of a universal prediction for all cosmological singularities, that would be much more speculative. Given the evidence for a plethora of solutions, there are various indications that gravity may simplify in large dimensions (see e.g. \[^{[10]}\]) and it would be more constructive to try to obtain from this an organizing principle or measure applying to the multitude of cosmological singularities of this type.\[^{3}\]

In any case, the singularities discussed in section 1, which are replaced by a phase of tachyon condensate, are simpler, appear more constrained \[^{[11][7]}\], and apply more directly to black hole physics. It would be interesting to understand if there is any principle relating black hole singularities and cosmological singularities, and to try to apply these techniques to broad classes of GR singularities \[^{[17]}\].

The relative simplicity with which the tachyon condensate removes the GR singularity provides a positive role for tachyons; the instability which often motivates discarding systems with tachyons appears here in such a way as to save the system from a worse (singular) fate. In fact even in low energy supersymmetric phenomenology it is also not necessary to discard the whole space of models with bare negative scalar mass squareds \[^{[18]}\]; it will be interesting to see if string compactifications can realize this predictive swath of parameter space in a natural way \[^{[19]}\].

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\[^{3}\]as mentioned for example in J. Polchinski’s talk in the cosmology session.
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