Optimization of the post-crisis recovery plans in scale-free networks

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Abstract  

General Motors or a local business, which one is better to be stimulated in post-crisis recessions, where government stimulation is meant to overcome recessions? Due to the budget constraints, it is quite relevant to ask how one can increase the chance of economic recovery. One of the key elements to answer this question is to understand metastable features of the economic networks. Ising model has been suggested for studying such features in the literature. In the homogenous networks one needs at least a minimum activation, forcing an Ising network to switch its local equilibria, where such minimum is independent of the nodes characteristics. In the scale free networks however, when one aims to push the network to switch its vacuum, she faces the question of which nodes are better to be stimulated to minimize the cost. In the paper it has been shown that stimulation of the high degree nodes costs less in general. Despite regular networks, in the scale free networks, the stimulation cost depends on the networks features such as assortativity. Though we have utilized the Ising model to tackle a problem in economics, our analysis shed lights on many other problems concerning stimulations of socio-economic systems.

I. Introduction  

In the aftermath of the 2007-2008 economic crisis, while the US government was going to stimulate the economy, some controversial issues had risen. For example, the debate about a recovery plan for the General Motors (GM) and Chrysler went up to the level of the US Senate. In that debate, some experts favored helping the big companies such as GM and Chrysler while the others favored small local businesses, see Stiglitz (2010) [1] and references therein.

To explain the problem more rigorously, we should recall that firms purchase products of their neighbors in the trade networks. Such trade then results in positive correlations between activities of firms. Similar to the ferromagnet systems, the positive correlation can result in dynamical hysteresis in the economic networks when in deep recessions we face a global reduction in the activities of firms. In deep recession if the partners of a firm reduce their production and the firm in a different manner work with maximum capacity, then there is a good chance that its products are not sold and depreciate resulting in a loss. So, managers have no choice but keeping steps with their partners. Such behavior can deepen economic harshness and may result in a long lasting depression.

In Keynesian economics, governments are suggested to intervene in the market and purchase from the firms, stimulating them to rise their activities in order to overcome recession. Due to the budget constraints, it will be critical to find the best strategy for stimulation of a heterogeneous network, at least in the simple agent based model of firms.

The questions of “whether one will better stimulate the recessed economy by helping the big companies or the small ones” is relevant due to the heterogeneous scale-free nature of the economic networks. In other words, heterogeneity rises the question of finding the best “strategy” to help the system. In a homogeneous network such as a regular lattice or a small-world Watts-Strogatz Network [2] all nodes are connected to almost similar numbers of neighbors, have the same practical roles in the structure and the similar priority to be stimulated after a crisis.

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The strategy question opens a much more general problem of dealing with heterogeneous networks, where one cannot easily use a mean-field solution. In such cases depending on what is going to be optimized, one needs to choose the best agents of the network to be stimulated. The low-degree nodes are easy to stimulate, while the high-degree nodes are more difficult but also more influential. So the answer to “what is the best choice?” will not in any way be trivial. To study this problem in a detailed way, we consider having several different scale-free networks with Ising spins on their nodes.

The Ising model has been a proper choice to model a wide range of phenomena such as opinion dynamics [3][4][5], neural function simulations [6] and many other real-world systems, see for example [7][8]. It could also be considered as a proper basis for our study. It has been suggested as a base to model the network of firms because the correlation between the activities of firms can be encapsulated in the interaction between some spins. Since firms trade with their neighbors in the network, when they increase/decrease productivity, they automatically force their neighbors to increase/decrease productivity. For the simplest model we can think of managers who can choose their level of activity from the binary choices of maximum or minimum capacity of production. To conclude, Ising model can be nominated as a simple model which addresses the correlation of activities in the economic network of firms [9][10][11].

Since the trade is the way that firms interact with and influence on each other, in recessions, government can stimulate the networks via purchasing the products. The budget constraint however limits the governments choices and therefore the question of which sectors or which classes of corporations are more appropriate choices for stimulation becomes a critical question.

If the activity of our network of firms is modeled by the Ising spins, then their response to the external stimulation should be studied in the literature of the kinetic Ising. Kinetic behavior of the Ising model has been widely studied in the literature, see [12][13][14][15][16]. If you impose a magnetic field to an Ising system forcing it to switch its local equilibria, it takes time for the system to switch. Such metastable behavior has been widely investigated for the regular networks. In some works, the Ising network has been quenched by a magnetic field and then relaxed to settle to its local equilibrium, see for example the reference [17]. In such cases, the probability of switching local equilibria as a function of the magnitude of the external field has been studied. While the subject of such studies have been regular networks, we are interested in the same study on scale free networks.

Despite regular networks which are homogeneous, scale free networks have intrinsic heterogeneity and due to this heterogeneity have interesting features [18][19]. This heterogeneity raises one question as we aim to stimulate a scale-free network, should we discuss only the magnitude of the stimulating field, or it is relevant to ask: “Where in the network has the magnetic field to be imposed?”. The answer to this question is the subject of this paper. Though we have implemented the Ising model as a toy model of the network of firms, this study is interesting also from a statistical physics point of view. It may as well shed light on a wide range of phenomena concerning metastabilities which occur in scale free networks.

In this work, we stimulate an Ising system to transfer it from one of its minima with all nodes in downward direction to the other minimum with all nodes in upward direction. We will compare two general strategies of starting our stimulation from the high degree nodes (High-Degree-Stimulation strategy or HDS strategy) vs. starting it from the low degree nodes (Low-Degree-Stimulation strategy or LDS strategy). We are going to investigate the amount of resources needed for each strategy.

We observe that different strategies need different amount of hits, resources, or budget for success and show that the gap between different strategies depends on some of the network characteristics.

II. Results

In the Ising model of firms if firms work with maximum/minimum capacity, their state can be represented by upward/downward spins. At the beginning of each session, managers of firms and corporations look at their collaborating neighbors and subject to their level of activities decide to increase or decrease their production level in the coming session, resulting in hiring or firing some employees.

The chance for a manager’s decision upon working with maximum/minimum capacity is stochastic, following the probability rates

\[
P^\uparrow = \frac{\exp\left(-\frac{(N_1-N_d)J}{T}\right)}{\exp\left(-\frac{(N_1-N_u)J}{T}\right) + \exp\left(-\frac{(N_1-N_d)J}{T}\right)},
\]

\[
P^\downarrow = \frac{\exp\left(-\frac{(N_1-N_u)J}{T}\right)}{\exp\left(-\frac{(N_1-N_d)J}{T}\right) + \exp\left(-\frac{(N_1-N_u)J}{T}\right)},
\]

where \(P^\uparrow/\downarrow\) indicates the chance for working with maximum/minimum capacity and \(N_1/\downarrow\) indicates the number of neighbors which are working with maximum/minimum capacity. The value of \(J\) indicates the level of trade and the value of \(T\) shows how managers are tied to their neighbors. If the value of \(T\) is small then
the managers do not take risky actions in a sense that if the majority of their neighbors increase/decrease their production level, then they also increase/decrease theirs with a high probability. In the Ising model for reasonably small values of $T$ we observe symmetry breaking where if a big portion of firms decrease their production, then the system can stay there for a long time. In Keynesian economics government is suggested to compensate decline of neighbors to move economy from its metastable state. So, government is suggested to start purchasing from the private parties stimulating the system to move to its opposite local equilibrium. This action is similar to stimulation of the spins with an external magnetic field. The government pays for the compensation caused by the other firms’ declining trades. As a result, the probabilities are modified as

$$P_{\uparrow} = \frac{\exp \left( -\frac{(N_i - N_f)J - GP}{T} \right)}{\exp \left( -\frac{(N_i - N_f)J - GP}{T} \right) + \exp \left( -\frac{(N_i - N_f)J + GP}{T} \right)}$$

$$P_{\downarrow} = \frac{\exp \left( -\frac{(N_i - N_f)J + GP}{T} \right)}{\exp \left( -\frac{(N_i - N_f)J - GP}{T} \right) + \exp \left( -\frac{(N_i - N_f)J + GP}{T} \right)}$$

where $GP$ shows the level of purchase by government.

As it is clear from the probabilities, government purchase has effect if it is comparable with trade between firms or strictly the value of $N_f / J$. Now, to address the response of the network to our stimulations, we should analyze the kinetic behavior of the system under the change in size and strategy of stimulations. To this aim, we first generate a preferential attachment scale-free network (B-A Model [20]). We then set all nodes to downward direction indicating a situation where all firms work with minimum capacity.

In the HDS strategy we start the action by stimulating the high degree nodes. We consider a value for the number of nodes, $n$, and impose a magnetic field on the $n$ high degree nodes updating them with the probabilities in Eq 2. Stimulation imposed on each node is proportional to its degree

$$GP_i = k_i J.$$  \hfill (3)

After a number of nodes are stimulated, we let the system relax in some Monte Carlo steps to see if it changes its local equilibrium in the way that the majority of firms start working with their maximum capacity.

$$R_{\text{max}} = \sum_{i=1}^{n} GP_i.$$  \hfill (4)

After a number of nodes are stimulated, we let the system relax in some Monte Carlo steps to see if it changes its local equilibrium in the way that the majority of firms start working with their maximum capacity.
The resource needed for each strategy denoted by $R_n$ is the cumulative magnetic field imposed on the nodes for the stimulation

$$R_n = \sum_{i=1}^{n} GP_i.$$  \hspace{1cm} (5)

We repeat this for an ensemble of 1,000 experiments for all given values of $n$ and obtain the success rate for such stimulation and its related resource. The result of the simulation has been depicted in Fig 1. In this figure, the blue curve shows the success rate for each value of $n$. For any given value of $n$ we then measure the cumulative field, i.e. $R_n$, as the resource needed for stimulation and translate it to the economic language as the cost of stimulation.

In our simulation we looked for stimulations which by a rate of 80% could change global state of the spins. For this success rate in HDS strategy, we need to stimulate about 280 high degree nodes which is equal to spending $0.552 \Delta GDP$ in economic language or $R_n = 1.104 mN J$ in the Ising language where $m$ is the average degree.

In LDS strategy everything is similar to HDS. We first set all spins downward and stimulate them. In this strategy however we stimulate low degree nodes. As shown in Fig 1 to obtain a 80% success rate, we need a stimulation equal to $0.663 \Delta GDP$. This means that the cost of successful stimulation through the HDS strategy is about 20% less than the LDS strategy.

The observation of difference in the outcomes of different strategies is significant. We however need to solidify the results. The first analysis to be done is the investigation of the size dependency. We repeated our simulations in three different network sizes and observed that our normalized results seem to be independent of the network size, see Fig 2.

Our first investigation was on Barabasi-Albert (B-A) network. A known feature of the B-A networks is that they have lower clustering coefficients than many real-world scale-free networks. To test the effect of clustering coefficient, we changed the clustering of our networks with the Holme-Kim method mentioned in the Methods section. As you can see in Fig 2 changing the clustering coefficient had insignificant effect on the size of the gaps.

Another feature of the real networks is their different assortativity structure. So, we did another analysis to check the effect of assortativity on the stimulation strategies. Our simulation showed that despite clustering, assortativity can significantly influence the gap between HDS and LDS strategies. When we increase the assortativity, we observe that the cost for LDS strategy remains unchanged. This is while HDS strategy becomes relatively cheaper and as a result, the gap between two strategies grows in size. For the big values of assortativity, the cost for LDS stimulation becomes more than twice the cost for

Figure 2: Evaluation of the impact of size and clustering on our results.
(A) Stimulation of different sizes; The difference between HDS and LDS strategies as we change the size of the network from 1000 nodes to 4000 nodes, normalizing the $\Delta GDP$ to the number of nodes, there is no significant difference between different sizes. (B) Stimulation of different clusterings; Changing the clustering of our Scale-free networks, the difference between two strategies remains the same.
HDS stimulation. Another observation is that the gap is saturated for large and small values of assortativity, see Fig 3.

Figure 3: The impact of assortativity on the gap between different strategies. The gaps between the cost for HDS and LDS strategies grow as we increase the assortativity of our networks.

III. Methods

The Ising model is identified by the Hamiltonian

\[ H = -J \sum_{\langle i,j \rangle} s_i s_j - \sum_j h_j s_j, \]

where \( s_i \) is the spin of the \( i \)th site, \( J \) is the coupling constant, the notation \( \langle i,j \rangle \) means the sum is over the nearest neighbor sites and \( h_j \) indicates the stimulating external field applied on \( j \)th node.

For the study of the Ising dynamics on different networks, we used the Glauber weight [21]

\[ W(s_i \rightarrow -s_i) = \frac{\exp^{-\beta \Delta E}}{1 + \exp^{-\beta \Delta E}}, \]

where \( \Delta E \) is the energy difference for the system changing the sign of \( i \)th unit and \( \beta = \frac{1}{k_B T} \). The main targets of our study are scale-free networks (networks with power-law degree distribution). To have our desired meta-stable states, we considered the systems below the critical temperature (in our case 0.1\( T_c \)). There are some analytical ways to find the critical temperature of Barabasi-Albert networks [22, 23, 20]. For the other networks introduced here, we used numerical methods and simulations to find the critical temperature.

There are several known ways to produce scale-free networks [20, 24, 19, 25, 26].

The methods we used to produce or change them are listed below:

- Barabasi-Albert Network was reconstructed using the algorithm mention in [20], we generated the ensembles of B-A networks with total sizes of 1000, 2000 and 4000 nodes. The number of edges of new coming nodes ranged from 2 to 8 in different ensembles.
- To change the clustering of our scale-free networks, we used the triad formation step by Holme and Kim [27] keeping fixed the number of links, to be able to compare the results.
- We changed the assortativity [28] of our scale-free networks using the Brunet and Sokolov reshuffling procedure [29] and finding the largest connected component in case the network was not a single component anymore, then checking the degree distribution to be sure of power-law existence.

IV. Discussion

Our investigations showed that in general, it is more efficient to start the stimulation from high degree nodes. The resource gaps between starting from high or low degrees are independent of the network size and clustering coefficient after normalizations, but depend on the assortativity of network. Networks with higher assortativity show larger gaps. Such results indicate that in order to better estimate the gap between HDS and LDS strategy, beside the degree distribution, we need to know other features of the studied network. In economics, the role of networks has been studied widely for the fragility of the markets and the diffusion of the cascades.

It has been shown that economies having more connections within the production networks are more fragile. In the homogeneous networks the central limit theorem rules out the chance for the systematic failure of the market due to the local fluctuations. In an interesting work by Acemoglu et al. [30] it has been shown that unlike the homogeneous networks, in the scale free networks, local fluctuations in the network may blow up and be the triggers for a crisis.

Despite the mentioned work and many other works which discuss the occurrence and diffusion of the crisis [31, 32, 33] in complexity economics [34, 35, 36, 37], the current work addresses the recovery plans and its possible concerns. In the world of economy, budget constraint is usually a serious problem during the recessions. This work emphasizes on the role of network features for addressing the response of the market to different recovery plans and suggests strategies that target big firms or corporations.

If we assume the real world is as simple as our model, then the US government should have focused on big companies such as Chevrolet instead of small businesses. Due
to the simplicity of the model, our findings on the role of strategy and as well the role of network characteristics in the recovery chance might not be reliable for the real cases, but they strongly suggest further studies and simulations in this area. Despite the fact that we had an economic problem in mind; this approach may find applications in many other socio-economics areas.

There are a lot of situations in the socio-economic systems where the system is trapped in an unfavored local equilibrium and the related organizations prefer to lead the system to another equilibrium [38]. All phenomena which are categorized as social dilemma, social trap, or the tragedy of the commons could be considered as our examples [39, 40, 41]. In social traps which are an extension of the prisoner dilemma to the society, the society can choose to live in different local Nash’s equilibria. The positive interaction between utilities of agents however results in the system showing hysteresis even in its unfavored equilibrium where the utility is not a global maximum. In such cases the government may try to drive the system to the better equilibrium. If agents are connected in heterogenous network, then our study reveals that different strategies have different costs for the government.

In a voting model as well, if we think that the agents of a network can influence each other then our simulation suggest that in heterogenous networks finding the best strategy to influence society finds application. One should ask if trying to influence big names has less cost or influencing individuals. Trivially the real world needs a much more complicated model which considers the behavior of the agents in a more precise manner. Our study however reveals that one should considers the features of the heterogenous social network to initiate a more successful stimulation with the same budget.

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