New Soliton Solutions for the Higher-Dimensional Non-Local Ito Equation

1 Introduction

Numerous complex phenomena that are encountered in mathematical physics, relativity, and economics are modeled via nonlinear differential equations [1]. There is no a single technique that can possibly solve different nonlinear evolution equations (NLEEs) kinds. Hence, several techniques have been introduced to construct exact solutions for NLEEs in the last few decades. Symbolic computational software programs can help interested researchers achieve these computational works. The obtained solutions allow scientists and engineers to understand the complex phenomena qualitative and measurable features in order to construct conclusions in an efficient way. As a result, various effective techniques have been proposed. (1/G′)-expansion algorithm [2], bifurcation method [3], Hirota bilinear approach [4], sine-cosine method [5], Adomian decomposition algorithm and its
extensions [6–9], Exp-function technique [10], technique of F-expansion [11], He’s variational iteration algorithm [12], inverse scattering approach [13], reduced differential transform method [14–17], new extended direct algebraic method [18], auxiliary equation method [19], first integral algorithm [20], residual power series method [21–25], simplest equation algorithm (SEM) [26, 27], modified simplest equation algorithm (MSEM) [28], and exp(−φ(ξ)) method [29], that will be presented in the coming context, are examples of some attempts to solve such equations. The fractional calculus (FC) has started to be incredibly known in a few fields of science and engineering (see [55–61]). For recent development with in this field in frame of the fractional calculus (FC) has started to be incredibly known in a few fields of science and engineering (see [55–61]).

For more information, readers can refer to some useful resources [50–54, 62–65] for deep understanding and learning about different aspects of this research area..

The SEM and its expansions [30–40] have constructed various NLEEs solutions. With the help of SEM and MSEM, this article constructs novel exact solutions of

\[ u_{tt} + u_{xxxx} + 3 (2 u_x u_t + u_{xxt}) + 3 u_{xx} \left( \int u_t dx \right) + \alpha u_{tt} + \beta u_{xt} = 0, \quad (1.1) \]

where the constants are denoted by α and β, and \( u(x, y, t) \) represents the waves amplitude. By making use of differential operator \( v = u_x \), Eq. (1.1) is going to be converted into the 5th-order PDE

\[ v_{xxt} + v_{xxxxx} + 3 (2 v_{xx} v_{xt} + v_x v_{xxt}) + 3 v_{xxxx} v_t + \alpha v_{xxt} + \beta v_{xxt} = 0. \quad (1.2) \]

The above Ito Eq. (1.2) (or equivalently Eq. (1.1)) was initially established by a generalized bilinear KdV equation [41]. For \( \alpha = \beta = 0 \), we have the 1-D Ito equation. Researchers have recently analyzed the (2+1)-dimensional Ito equation. Wazwaz [42] implemented tanh–coth algorithm to obtain single soliton and periodic solutions. In this article, we have investigated the (2+1)-dimensional Ito equation with two different methods involving the simplest equation and modified simplest equation for exact analytic solutions.

This article is prepared as follows: The algorithms of SEM and MSEM are presented in section 2. By applying the proposed techniques, the Logarithmic form of obtained exact solutions for Eq. (1.1) is studied in section 3. Results’ discussion including simulations of some obtained solutions is provided in section 4.

2 Mathematical Analysis

In this section, SEM and MSEM achieve the exact analytic solutions of constant-coefficients (2+1)-D PDE of the following form:

\[ P(v, v_x, v_y, v_t, v_{xy} \ldots) = 0, \quad (2.1) \]

where \( P \) is an assumed polynomial in \( v = v(x, y, t) \) and its derivatives involving the highest derivative and the higher power of linear terms. The solution process is also valid for the time-dependent coefficients and NPDEs’ systems.

To investigate Eq. (2.1), we transform it into an ODE under the transformation:

\[ v(x, y, t) = v(\xi), \quad (2.2) \]

where \( \xi = x + y - \eta t \) is the wave variable and \( \eta \) is the wave frequency. In case of time-dependent, one can use \( \eta = \eta(t) \). Under this consideration, Eq. (2.1) is reduced to be

\[ F(v, v', v'', \ldots) = 0, \quad (2.3) \]

where \( v' = \frac{dv}{d\xi}, v'' = \frac{d^2v}{d\xi^2}, v''' = \frac{d^3v}{d\xi^3}, \ldots \). By integrating Eq. (2.3) several times as possible as we can and setting the integration’s constants to 0. The transformed equation is reduced and kept the solution process very simple.

From the above schemes, positive integer \( m \) is determined by balancing the highest order derivative and the nonlinear term of the highest order in the completely integrated form of Eq. (2.3). As a result, each method’s solution process is discussed.

2.1 The Method of Simplest Equation

The Eq. (2.3) solution via the SEM [26, 27] is expressed as:

\[ v(\xi) = \sum_{i=0}^{m} A_i \phi^i(\xi), A_m \neq 0, \quad (2.4) \]

where \( A_i (i = 0, 1, \ldots, m) \) are parameters to be found later. \( \phi(\xi) \) is the function that with the help of by the simplest equation can satisfy an ODE. The simplest equations in this article are the 1st order ODEs including the Bernoulli equation and Riccati equation. In the Bernoulli equation case, we have:

\[ \phi'(\xi) = \lambda \phi(\xi) + \mu \phi^2(\xi). \quad (2.5) \]

The solution function \( \phi(\xi) \) is expressed as:
If $\lambda = 0$, we have the rational form:

$$
\phi (\xi) = \frac{1}{\mu (\xi_0 - \xi)}.
$$

(6.6)

**II.** If $\lambda > 0$ and $\mu < 0$, we have the rational-exponential form:

$$
\phi (\xi) = \frac{A e^{(\lambda (\xi + \xi_0))}}{1 - \mu e^{(\lambda (\xi + \xi_0))}}.
$$

(6.7)

**III.** If $\lambda < 0$ and $\mu > 0$, we have the rational-exponential form:

$$
\phi (\xi) = \frac{A e^{(\lambda (\xi + \xi_0))}}{1 + \mu e^{(\lambda (\xi + \xi_0))}}.
$$

(6.8)

Whereas in the case Riccati equation

$$
\phi' (\xi) = \lambda \phi^2 (\xi) + \mu,
$$

(6.9)

and the solutions of Eq. (6.9) can be expressed in the following forms:

**I.** If $\lambda \mu < 0$, we have the hyperbolic form:

$$
\phi (\xi) = -\frac{\sqrt{-\lambda \mu}}{\lambda} \tanh \left( \sqrt{-\lambda \mu} \xi - \frac{\varepsilon \ln \xi_0}{2} \right), \quad \varepsilon = \pm 1,(6.10)
$$

or,

$$
\phi (\xi) = -\frac{\sqrt{-\lambda \mu}}{\lambda} \coth \left( \sqrt{-\lambda \mu} \xi - \frac{\varepsilon \ln \xi_0}{2} \right). \quad (6.11)
$$

**II.** If $\lambda \mu > 0$, we have the periodic form:

$$
\phi (\xi) = \frac{\sqrt{\lambda \mu}}{\lambda} \tan \left( \sqrt{\lambda \mu} \xi + \xi_0 \right),
$$

(6.12)

or,

$$
\phi (\xi) = -\frac{\sqrt{\lambda \mu}}{\lambda} \cot \left( \sqrt{\lambda \mu} \xi + \xi_0 \right), \quad (6.13)
$$

where $\xi_0$ is an integration constant.

From the Bernoulli equation given in Eq. (2.5), by the substitution of Eq. (2.4) into Eq. (2.3) and equating each coefficient with the same power in the constructed polynomial of $\phi (\xi)$ to $0$, an algebraic equations' system in the variables $\mu$, $\lambda$ and the $A_i$'s is constructed. This system is solved and substituted the determined values of $\mu$, $\lambda$ and the $A_i$'s, associated with the Eq. (2.5) general solutions into Eq. (2.4). Then, the exact solution is obtained in a traveling-wave form for Eq. (2.1). By similarly doing this process again by the replacement of Eq. (2.5) by Eq. (2.9), new solutions' classes can be obtained. The scheme of the simplest equation is applicable at the same time that the obtained system is solved in the undetermined parameters.

### 2.2 Modified Simple Equation Method

The MSEM [28] considers the Eq. (2.1) solution as follows:

$$
v (\xi) = \sum_{i=0}^{m} A_i \left( \frac{\phi' (\xi)}{\phi (\xi)} \right)^i, \quad A_m \neq 0,
$$

(6.14)

where $A_i$ ($i = 0, 1, \ldots, m$) are parameters to be determined later. Positive integer $m$ is found by the homogeneous balance principle. $\phi (\xi)$ is an unspecified function to be determined later. Once Eq. (6.14) is substituted into Eq. (2.3), an algebraic equations' system, which can be an algebraic-differential system, is resulted. When the constructed system's numerator is forced to be vanished and substituted our results into Eq. (6.14), the exact solution is determined for the studied problem.

### 3 The Ito Equations' Applications

The (2+1)-D non-local Ito equation is investigated via the Kudryashov SEM, which was discussed in the previous section. With the application of the transformation given in Eq. (2.2), Eq. (1.2) will be carried into the following ODE form:

$$
(\eta - \alpha - \beta) v^{(3)} - v^{(5)} - 6 (v')^2 = 0.
$$

(3.1)

In further compact form, Eq. (3.1) can be expressed as:

$$
(\eta - \alpha - \beta) v^{(3)} - v^{(5)} - 3 (v')^2 = 0.
$$

(3.2)

By the twice integration Eq. (3.2) w.r.t $\xi$ and let the integration's constants to be 0, we have:

$$
(\eta - \alpha - \beta) v' - v^{(3)} - 3 (v')^2 = 0.
$$

(3.3)

Let $w (\xi) = v' (\xi)$ to get:

$$
(\eta - \alpha - \beta) w - w'' - 3 w^2 = 0.
$$

(3.4)

The balance is made between $w''$ and $w^2$ in Eq. (3.4) which $m = 2$ is obtained.
3.1 Application of SEM

As a result, Eq. (3.5) has a solution as:

\[
    w(\xi) = A_0 + A_1 \phi(\xi) + A_2 \phi(\xi)^2. \tag{3.5}
\]

By the substitution of Eq. (3.5) into Eq. (3.4), and from the Bernoulli Eq. (2.5), as well as, let us set up the coefficients of \( \phi^i \), \( i = 0, 1, \ldots, 4 \), to be zero, the following system is constructed in terms of \( A_0, A_1, A_2, \lambda, \mu \) and \( \eta \):

\[
    A_0 (3A_0 + \alpha + \beta - \eta) = 0, \tag{3.6}
\]
\[
    A_1 \left( 6A_0 + \alpha + \beta - \eta + \lambda^2 \right) = 0, \tag{3.7}
\]
\[
    3A_1 (A_1 + \lambda \mu) - A_2 \left( 6A_0 + \alpha + \beta - \eta + 4\lambda^2 \right) = 0, \tag{3.8}
\]
\[
    2 \left( 3A_1 A_2 + 5\lambda \mu A_2 + \mu^2 A_1 \right) = 0, \tag{3.9}
\]
\[
    3A_2 \left( A_2 + 2\mu^2 \right) = 0. \tag{3.10}
\]
Solving Eq. (3.6) and Eq. (3.10) implies
\[ A_0 = 0, \quad \frac{1}{3} (\eta - \alpha - \beta) \quad \text{and} \quad A_2 = -2\mu^2, \]
where \( \mu \) is a nonzero constant. As a result, the exact traveling wave solutions of Ito Eq. (1.1) is constructed as:

**Case 1.** If \( A_0 = 0, \eta = \alpha + \beta, \text{and} \lambda = 0 \), we get \( A_1 = 0 \) and
\[ u_{01}(x, y, t) = 2 \ln (\xi - \zeta). \] (3.11)

**Case 2.** If \( A_0 = 0, \eta = \alpha + \beta + \lambda^2, \text{and} \lambda \neq 0 \), we get \( A_1 = -2\lambda\mu \) and
\[ u_{02}(x, y, t) = -2 \left( \lambda x - \ln \left( 1 - \mu e^{\lambda \xi + \xi_0} \right) \right), \quad \text{for} \lambda > 0 \text{ and} \mu < 0. \] (3.12)
\[ u_{03}(x, y, t) = -2 \left( \lambda x - \ln \left( 1 + \mu e^{\lambda \xi + \xi_0} \right) \right), \quad \text{for} \lambda < 0 \text{ and} \mu > 0. \] (3.13)

**Case 3.** If \( A_0 = \frac{1}{2} (\eta - \alpha - \beta), \eta = \alpha + \beta - \lambda^2, \lambda > 0, \mu < 0 \text{ and} \lambda = 0 \), we get
\[ u_{04}(x, y, t) = \frac{\lambda^2}{3} x \left( \frac{x}{2} - \frac{6}{\lambda} - \xi \right) + 2 \ln \left( 1 - \mu e^{\lambda \xi + \xi_0} \right), \quad \text{for} \lambda > 0 \text{ and} \mu < 0. \] (3.14)
\[ u_{05}(x, y, t) = \frac{\lambda^2}{3} x \left( \frac{x}{2} - \frac{6}{\lambda} - \xi \right) + 2 \ln \left( 1 + \mu e^{\lambda \xi + \xi_0} \right), \quad \text{for} \lambda < 0 \text{ and} \mu > 0. \] (3.15)

As in the Bernoulli equation case, by using the Riccati equation provided in Eq. (2.9), we obtain:
\[ 3A_0^2 + A_0 (\alpha + \beta - \eta) + 2\mu^2 A_2 = 0, \] (3.16)
\[ A_1 (6A_0 + \alpha + \beta - \eta + 2\lambda\mu) = 0, \] (3.17)
\[ 3A_1^2 + A_2 (6A_0 + \alpha + \beta - \eta + 8\lambda\mu) = 0, \] (3.18)
\[ 2A_1 \left( 3A_2 + \lambda^2 \right) = 0, \] (3.19)
\[ 3A_2 \left( A_2 + 2\lambda^2 \right) = 0. \] (3.20)

Eliminating the trivial solution, Eq. (3.16) and Eqs. (2.2)-(3.20) imply that \( A_2 = -2\lambda^2, A_1 = 0, A_0 = -\frac{1}{6} (\alpha + \beta - \eta + 8\lambda\mu) \) and the following cases:

**Case 4.** If \( \eta = \alpha + \beta - 4\lambda\mu, \text{and} \lambda \mu < 0 \), we get:
\[ u_{06}(x, y, t) = 2 \ln \left( \cosh \left( \sqrt{-\lambda \mu} \xi \right) - \frac{\epsilon \ln (\xi_0)}{2} \right). \] (3.21)

Or,
\[ u_{07}(x, y, t) = 2 \ln \left( \sinh \left( \sqrt{-\lambda \mu} \xi \right) - \frac{\epsilon \ln (\xi_0)}{2} \right). \] (3.22)

**Case 5.** If \( \eta = \alpha + \beta + 4\lambda\mu, \text{and} \lambda \mu < 0 \), we get:
\[ u_{08}(x, y, t) = \mp \sqrt{-\lambda \mu} \epsilon \ln (\xi_0) x + \frac{2}{3} \lambda \mu x (2\xi - x) + 2 \ln \left( \cosh \left( \sqrt{-\lambda \mu} \xi \right) \pm \frac{\epsilon \ln (\xi_0)}{2} \right). \] (3.23)

Or,
\[ u_{09}(x, y, t) = \pm \sqrt{-\lambda \mu} \epsilon \ln (\xi_0) x + \frac{2}{3} \lambda \mu x (2\xi - x) + 2 \ln \left( \sinh \left( \sqrt{-\lambda \mu} \xi \right) \pm \frac{\epsilon \ln (\xi_0)}{2} \right). \] (3.24)
Case 6. If \( \eta = \alpha + \beta - 4 \lambda \mu \), and \( \lambda \mu > 0 \), we get
\[
    u_{10}(x, y, t) = 2 \ln \left( \cos \left( \sqrt{\lambda \mu} \xi \right) + \xi_0 \right). \tag{3.25}
\]
Or,
\[
    u_{11}(x, y, t) = 2 \ln \left( \sin \left( \sqrt{\lambda \mu} \xi \right) + \xi_0 \right). \tag{3.26}
\]
Case 7. If \( \eta = \alpha + \beta + 4 \lambda \mu \), and \( \lambda \mu > 0 \), we get:
\[
    u_{12}(x, y, t) = 2 \sqrt{\lambda \mu} \xi_0 x + \frac{2}{3} \lambda \mu x (2 \xi - x) + 2 \ln \left( \cos \left( \sqrt{\lambda \mu} \xi \right) + \xi_0 \right). \tag{3.27}
\]
Or,
\[
    u_{13}(x, y, t) = 2 \sqrt{\lambda \mu} \xi_0 x + \frac{2}{3} \lambda \mu x (2 \xi - x) + 2 \ln \left( \sin \left( \sqrt{\lambda \mu} \xi \right) + \xi_0 \right). \tag{3.28}
\]

3.2 Application of MSEM

The MSEM is applied for Eq. (1.2) with \( m = 2 \), Eq. (3.4) constructs a solution in the form:
\[
    w(\xi) = A_0 + A_1 \frac{\phi'(\xi)}{\phi(\xi)} + A_2 \left( \frac{\phi'(\xi)}{\phi(\xi)} \right)^2, A_2 \neq 0. \tag{3.29}
\]

It is simple to find that
\[
    w'(\xi) = A_1 \left( \frac{\phi''}{\phi} - \left( \frac{\phi'}{\phi} \right)^2 \right) + 2A_2 \left( \frac{\phi'''}{\phi^2} - \left( \frac{\phi'}{\phi} \right)^3 \right), \tag{3.30}
\]
\[
    w''(\xi) = A_1 \left( \frac{\phi'''}{\phi} - \frac{3}{2} \left( \frac{\phi''}{\phi^2} + 2 \left( \frac{\phi'}{\phi} \right)^3 \right) + 2A_2 \left( \frac{2\phi''^2 + \phi' \phi'''}{\phi^2} - 5 \frac{\phi''^2}{\phi^3} + 3 \left( \frac{\phi'}{\phi} \right)^4 \right) \right). \tag{3.31}
\]

Substituting Eqs. (3.30)-(3.31) into Eq. (3.4) and the coefficients of \( \phi^{-i} (i = 0, \ldots, 4) \) are equated to be vanished which imply the following:
\[
    A_0 \left( 3A_0 + \alpha + \beta - \eta \right) = 0, \tag{3.32}
\]
\[
    3A_2 \left( 2 + A_2 \right) \phi'(\xi)^4 = 0, \tag{3.33}
\]
\[
    2 \left( A_1 \phi'(\xi) \left( 1 + 3A_2 \right) - 5 \phi''(\xi) \right) \phi'(\xi)^2 = 0, \tag{3.34}
\]
\[
    A_1 \left( \phi'(\xi) \left( 6A_0 + \alpha + \beta - \eta \right) + \phi''(\xi) \right) = 0, \tag{3.35}
\]
\[
    2 A_2 \phi'' - \left( 3A_1 \phi'' - 2A_2 \phi''' \right) \phi' + \left( 3A_1^2 + A_2 \left( 6A_0 + \alpha + \beta - \eta \right) \right) \phi'^2 = 0. \tag{3.36}
\]

Solving Eq.(3.33), with \( A_2 \neq 0 \) and \( \phi'(\xi) \neq 0 \) to avoid trivial solution, gives that \( A_2 = -2 \). Accordingly, Eq.(1.1) exact travelling wave solutions are constructed as:

Case 1. If \( A_0 = 0 \), \( \phi(\xi) = \frac{2 e^{\frac{\alpha}{A_1}}}{A_1} - C_1 + C_2 \), and \( \eta = \frac{1}{\lambda} (A_1^2 + 4\alpha + 4\beta) \), we get
\[
    u_{14}(x, y, t) = -A_1 x + 2 \ln \left( 2C_1 e^{\frac{\alpha}{A_1}} \xi + C_2 A_1 \right). \tag{3.37}
\]
Case 2. If $A_0 = \frac{1}{3} (\eta - \alpha - \beta)$, $\phi (\xi) = \frac{2 \alpha}{A_1} C_1 + C_2$, and $\eta = \frac{1}{3} (-A_1^2 + 4 \alpha + 4 \beta)$, we get

$$u_{15} (x, y, t) = A_1 x \left( \frac{A_1}{12} \left( \frac{x}{2} - \xi \right) - 1 \right) + 2 \ln \left( 2 C_1 e^{\frac{1}{2} A_1 \xi} + C_2 A_1 \right),$$

(3.38)

where $C_1$ and $C_2$ are arbitrary constants. $A_1$ is nonzero arbitrary constant.

Figure 1: (a) 2D plot for $u_2$. (b) 3D plot for $u_2(x, y)$, when $t = 0$.

Figure 2: (a) 2D plot for $u_3$. (b) 3D plot for $u_3(x, y)$, when $t = 0$. 
Figure 3: (a) 2D plot for $u_4$, (b) 3D plot for $u_4(x, y)$, when $t = 0$.

Figure 4: (a) 2D plot for $u_6$, (b) 3D plot for $u_6(x, y)$, when $t = 0$.

Figure 5: (a) 2D plot for $u_{10}$, (b) 3D plot for $u_{10}(x, y)$, when $t = 0$. 
Three classes of traveling-wave solutions for the nonlocal (2+1) Ito equation have been explored via the SEM and MSEM. Using SEM along using of Bernoulli equation produced bright solitons provided in Eqs. (3.11)-(3.15). Similarly, with the Riccati equation, bright solitons are obtained in Eqs. (3.21)-(3.24) which include these ones in Bernoulli equation case, singular periodic solitons in Eq. (3.25) and Eq. (3.26), and singular bright solitons in Eqs. (3.27)-(3.28) are obtained. The output of MSEM in the form of bright soliton shapes coincides with SEM associated with the Bernoulli equation via a special choice of parameters. In Figure 1, the 3-D profiles of solutions in Eq. (3.14) are shown. The singular periodic solitons given in Eq. (3.25) are plotted in Figure 2. Also, some of the solutions obtained are shown in Figures 1-7. The constructed solutions are new and have not been obtained before in any research work. The wave solutions are tested by substituting them into the fundamental equations. According to the free parameters’ choice in the constructed solutions, various physical structures can result. Since solving NODEs is complicated, particularly the ones that are appeared from the MSEM, Eqs. (3.35)-(3.36), the SEM is more effective in processing the considered Ito equation reported in Eq. (1.1). The proposed techniques can be applied to various related NPDEs via Mathematica symbolic computation package 11. The considered can be investigated by using different definitions of fractional derivatives. Also, solitonic, super nonlinear, periodic, quasi-periodic, and chaotic waves can be done in future works.
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