Interval Parameter Estimation of Quantile Regression Using Bca-Bootstrap Approach with Application to Open Unemployment Rate Study

Solehatul Ummah¹, Vita Ratnasari†, Dedy Dwi Prasto¹
¹Institut Teknologi Sepuluh Nopember (ITS), Surabaya 60111, Indonesia
*Corresponding author’s email: vitaratna70@gmail.com

Abstract. Parameter estimation in the linear regression model using ordinary least square (OLS) method is less precise to analyze data containing outliers. It is because outliers can cause unstable parameter estimate. In addition, the existence of outliers causes residuals to be larger so that the residual’s variance is not constant (heteroscedasticity). One model that is able to overcome the effect of outliers is quantile regression because it can accommodate the non-homogeneous variances in modeling. In this study, the confidence interval of the parameter estimate in the quantile regression model was obtained, i.e., the Bias-Corrected and accelerated (BCa) bootstrap method. The proposed method was applied in modeling the open unemployment rate in Indonesia in 2017. The quantile value used in this study is quantile 0.05, 0.5, and 0.95 with 1500 resampling in BCa-bootstrap approach. The empirical result shows that the best quantile regression model is obtained at the value of quantile 0.95 which has a Pseudo R² value is 60.45 percent. The model at quantile 0.95 shows that the percentage of youth, economic growth rate, and labor force participation rate have a significant effect on the open unemployment rate in Indonesia.

1. Introduction

The method commonly used in estimating parameters in regression analysis is Ordinary Least Square (OLS). However, this method is sensitive to outliers which causes unstable parameter estimates [1]. The existence of outliers in the data causes the residuals to become large and the residual variance is not constant (heteroscedasticity). One model that is able to overcome the existence of outliers is quantile regression. Quantile regression method is one of the regression methods with the approach of dividing data into certain quantiles, by minimizing the asymmetric absolute error. Reference [2] showed that the interval estimation in quantile regression could be obtained by the resampling approach using the bootstrap method.

Researchers have developed several bootstrap techniques. Reference [3] have compared the confidence interval constructed from six bootstrap techniques. The results show that the recommended interval method is the Bias-Corrected Accelerated (BCa) approach because it has good performance as BCa consistently has shorter length intervals. The BCa method is a modification of percentile bootstrap, but in BCa method, there is a correction of bias and skewness. Another research that discussed BCa bootstrap is [4] that construct a confidence interval for the modeled response using a beta regression model. The confidence interval is constructed using the percentile and BCa bootstrap method.

From some of the illustrations above, it can be seen that the quantile regression method can be applied to various fields of life. One example is the problem in the development of the labor sector obtained
based on the report of the International Labor Organization (ILO) in 2017. Reference [5] shows that the unemployment rate in 2017 tends to decrease to 5.3 percent from 11.2 percent in 2015. In 2017, the distribution of unemployment rates among provinces shows quite high disparities. The high disparity in open unemployment rate in some of these provinces is a problem that indicates an outlier in the open unemployment rate data. Research related to open unemployment rate has been researched by [6], analyzing the influence of population indicators on open unemployment rate in Indonesia with a panel regression approach. The result is that the population growth rate, literacy rate, and gross enrollment rate have a significant effect on open unemployment rate.

Therefore, in this study, the researcher proposes to use quantile regression on open unemployment rate data in Indonesia in 2017 with the BCa bootstrap resampling method for estimating the interval. The independent variables that will be used are the percentage of the number of youth, the Labor Force Participation Rate, Minimum Wage, and the rate of economic growth to find out what factors cause unemployment in Indonesia.

2. Theoretical Review

In this section we will review some of the theories used.

2.1. Quantile Regression

Suppose that given data \( \{y_1, y_2, \ldots, y_n\} \) where \( i = 1, 2, \ldots, n \), and quantile-\( \tau \) is the cumulative distribution function of \( y \) such that \( P(Y \leq y) = \tau \). Then the quantile function is defined as the inverse \( Q_\tau(y) = Q^{-1}_\tau(y) = \inf \{y : F(y) > \tau\} \) for \( \tau \in (0, 1) \). The linear quantile regression equation is specific to conditional quintile \( Q(y_i | x_{i1}, x_{i2}, \ldots, x_{ik}) \) of the response variable \( y_i \), that is:

\[
y_i = \beta_0(\tau) + \beta_1(\tau)x_{i1} + \beta_2(\tau)x_{i2} + \cdots + \beta_k(\tau)x_{ik} + \varepsilon_i(\tau), \quad i = 1, 2, \ldots, n
\]  

Then the equation (1) can be written in the form of the following linear model:

\[
y = X\beta(\tau) + \varepsilon(\tau)
\]

where \( y \) is the vector of the response variable, \( X \) is the matrix of the predictor variable, \( \beta(\tau) \) is the parameter vector in the quintile- \( \tau \) where \( \tau \in (0, 1) \), \( \varepsilon(\tau) \) is the residual vector from the regression model in quintile \( \tau \).

Quantile regression is the expansion of median regression [7]. If the median regression is the parameter estimate by minimizing \( \sum_{i=1}^{n}|\varepsilon_i| \) by giving the same weight, quantile regression minimizes \( \sum_{i=1}^{n} \rho(\varepsilon_i) \) by giving different weight. The weight used is \( \tau \) for errors that are greater or equal to zero (underprediction), and \( 1 - \tau \) for errors that are less than zero (overprediction) with \( \tau \) is quantile. The multiplication between the error and the weight is then called the check function \( \rho(\varepsilon_i) \). The \( \beta \) estimation in quantile regression is done by minimizing the check function, then the \( \beta \) estimation in quantile regression is

\[
\hat{\beta}(\tau) = \arg\min_{\beta} \sum_{i=1}^{n} \rho(\varepsilon_i) = \arg\min_{\beta} \sum_{i=1}^{n} \rho_\tau(y_i - x_i^T\beta)
\]

where \( \tau \in (0, 1) \) and \( \rho(\varepsilon_i) \) in equation (3) are defined as

\[
\rho(\varepsilon)(\varepsilon_i) = \begin{cases} 
\tau\varepsilon_i & \text{if } \varepsilon_i \geq 0 \\
-(1-\tau)\varepsilon_i & \text{if } \varepsilon_i < 0
\end{cases}
\]

So the quantile regression estimator can also be written with

\[
\hat{\beta}(\tau) = \arg\min_{\beta} \sum_{i=1, \varepsilon_i \geq 0}^{n} \tau|y_i - x_i^T\beta| + \sum_{i=1, \varepsilon_i < 0}^{n} (1-\tau)|y_i - x_i^T\beta|
\]

The solution of equation (5) cannot be obtained analytically, but numerically one of them is by the simplex method.
2.2. Bootstrap Resampling
The Bootstrap method is a nonparametric approach to estimating various statistical quantities such as the mean, standard error, and bias of an estimate or to form a confidence interval by following a particular algorithm. The Bootstrap method is carried out by constructing a number of samples (B), where each sample is obtained by a random sampling procedure with returns from the original dataset. The resampling procedure is simultaneously applied to \( x \) and \( y \) vectors. Given bootstrap variance, as follows:

\[
\hat{\nu}_j = \frac{1}{B} \sum_{b=1}^{B} (\hat{\beta}_{b,j} - \bar{\beta}_j) (\hat{\beta}_{b,j} - \bar{\beta}_j)^T
\]

where \( j = 1, \ldots, p \) and the average of bootstrap parameters are:

\[
\bar{\beta}_j = \frac{1}{B} \sum_{b=1}^{B} \hat{\beta}_{b,j}
\]

where \( se(\hat{\beta}_j) \) is the square root of the variance in equation (6).

The percentile method is based on the percentile to \( \alpha \) \( (\hat{\beta}_{j,lo}) \) and to \( (1 - \alpha) \) \( (\hat{\beta}_{j,up}) \) of the cumulative distribution function of the bootstrap vector of parameter estimates

\[
[\hat{\beta}_{j,(lower)}^*, \hat{\beta}_{j,(upper)}^*] = [\hat{F}(\alpha), \hat{F}(1 - \alpha)]
\]

where \( lo \) and \( up \) each shows the lower and upper limits of the confidence interval [8].

2.3. Bias-Corrected and Accelerated
The Bias-Corrected and Accelerated (BCa) method is a modification of percentile bootstrap. In this method, there is a correction of bias and skewness. To get the confidence interval BCa in \( \beta \), initially by counting [9]:

\[
z = \Phi^{-1}\left(\frac{\#_{b=1..B}(\hat{\beta}_{b,j}^* < \hat{\beta}_j)}{B + 1}\right)
\]

where \( \Phi^{-1} \) is a normal-standard quantile function, and \( \#_{b=1..B}(\hat{\beta}_{b,j}^* < \hat{\beta}_j) / B + 1 \) is the proportion (adjusted) of bootstrap replication the \( \hat{\beta}_{b,j}^* \) value is smaller than \( \hat{\beta}_j \) as the estimator \( \hat{\beta}_j \).

For example, \( \hat{\beta}_{j(-i)} \) shows the value of \( \hat{\beta}_j \) which is generated when the \( i \)-observation is omitted from the sample and \( \hat{\beta}_j \) shows the average of \( \hat{\beta}_{j(-i)} \); then

\[
\hat{\beta}_j = \frac{\sum_{i=1}^{n} \hat{\beta}_{j(-i)}}{n}
\]

and

\[
a = \frac{\sum_{i=1}^{n} (\hat{\beta}_{j(-i)} - \hat{\beta}_j)^3}{6 \left[ \sum_{i=1}^{n} (\hat{\beta}_{j(-i)} - \hat{\beta}_j)^2 \right]^2}
\]

with \( z \) and \( a \) correction factors that have been obtained, it can be calculated

\[
a_1 = \Phi\left(z + \frac{z - z_{1 - a/2}}{1 - a(z - z_{1 - a/2})}\right)
\]

\[
a_2 = \Phi\left(z + \frac{z + z_{1 - a/2}}{1 - a(z + z_{1 - a/2})}\right)
\]

where \( \Phi(\cdot) \) is a normal-standard cumulative distribution function. Values \( a_1 \) and \( a_2 \) are used to find the end point of the corrected percentile confidence interval:

\[
\hat{\beta}_{j,(lower^*)} < \beta_j < \hat{\beta}_{j,(upper^*)}
\]
where lower$^*$ = $[B_{a_1}]$ and upper$^*$ = $[B_{a_2}]$. When the correction factors $a$ and $z$ are both 0, then $a_1 = \Phi(-z_{1-a/2}) = \Phi(z_{a/2}) = \alpha/2$ and $a_2 = \Phi(z_{1-a/2}) = \Phi(z_{a/2}) = 1 - \alpha/2$, which corresponds to the percentile interval (without correction).

To get a sufficient accurate confidence interval BCa, the number of bootstrap samples, $B$, must be in the order of 1000 or more. However, for the normal-theory bootstrap interval, we can use a smaller $R$, for example, in the order of 100 or more, because all we have to do is estimate the standard error of the statistics.

2.4. Quantile Regression Goodness of Fit
As OLS regression, the goodness of fit is based on R-squared values, so in quantile regression to determine the goodness of fit is $PseudoR^2(\tau)$ by comparing the absolute number of residuals from the weighting difference using the chosen model with the absolute residual number from the weighting difference use a model that is only intercepted. $PseudoR^2(\tau)$ can be calculated as follows [10].

$$PseudoR^2(\tau) = 1 - \frac{V^1(\tau)}{V^0(\tau)} ; \ 0 < R(\tau) < 1$$  \hspace{1cm} (14)

where,

$$V^1(\tau) = \sum_{i:y_i < \tau x_i} \tau |y_i - x_i^T \hat{\beta}| + \sum_{i:y_i \geq \tau x_i} (1 - \tau) |y_i - x_i^T \hat{\beta}|$$  \hspace{1cm} (15)

$$V^0(\tau) = \sum_{i:y_i < \tau x_i} \tau |y_i - \tau x_i| + \sum_{i:y_i \geq \tau x_i} (1 - \tau) |y_i - \tau x_i|$$  \hspace{1cm} (16)

3. Result and Discussion
The data used in this study is secondary data on the open unemployment rate and their indicators obtained by BPS Indonesia in 2017, with the object of its observations being 33 provinces in Indonesia. In this study, the response variables and predictor variables used in this study are summarized as of Table 1.

| Variable                           |
|------------------------------------|
| $Y$ Open Unemployment Rate by Province |
| $X_1$ Percentage of Youth by Province   |
| $X_2$ Economic Growth Rate by Province |
| $X_3$ Minimum Wage by Province       |
| $X_4$ Labor Force Participation Rate by Province |

Before going to the quantile regression modeling step, the data will be detected whether there are outliers or not and whether the classical assumptions of homoskedasticity are approved or not.

![Figure 1. Boxplot for each variable.](image-url)
Based on Figure 1, there are known outliers in the variables $X_1$, $X_2$, $X_3$ and $X_4$ in year 2017 of open unemployment rate data. In addition to the boxplot graph method, other methods can be used to detect statistical outliers with DfFITs, DfBETAS, or Cook Distance values. The decision criteria are outliers if the value:

- $DfFITs > 2 \sqrt{\frac{p}{n}} = 2 \sqrt{\frac{5}{33}} = 0.778499$
- $DfBETAS > 2 \sqrt{\frac{1}{n}} = 2 / \sqrt{33} = 0.348155$
- $Cook's distance > F(0.5, p, (n - p)) = 0.221988$

Based on the three criteria in the detection of outliers, it is known that several observations are indicated as outliers because the value is greater than the criteria. Outlier detection in each predictor variable shown from the $DfBETAS$ criteria, obtained several observations which are outliers, that are 16th and 33rd observations on the number of youth variable, 18th observation on the rate of economic growth variable, 18th observation and 33rd in the minimum wage variable, and 33rd observation in the TPAK variable. Where the 16th observation is Banten Province, the 18th observation is West Nusa Tenggara province, and the 33rd observation is Papua.

Furthermore, for heteroscedasticity detection, it will be obtained by the scatterplot graph method and Glejser test. Scatterplot between the response variable with each predictor variable to find out the distribution of data, which can be seen in Figure 2. From the results of the scatterplots, it can be seen that there is a pattern that spreads across the data. This empirical evidence indicates the existence of heteroscedasticity in the data.

![Scatterplot of Y vs X1, X2, X3, X4](Image)

**Figure 2.** Scatter plot between response variables with each predictor variable.

To test whether there is true heteroscedasticity in the data, the Glejser test is employed based the following hypothesis:

$H_0: \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_n^2 = \sigma^2$ against $H_1$: there is at least one $\sigma_i^2 \neq \sigma^2$; where $i = 1, 2, \ldots, n$, with $\alpha: 0.05$. The Glejser Test results on the data found that the Fhitung value is 2.77 and the $p$-value is 0.047. Because the $p$-value < $\alpha$ then Reject $H_0$, which means that data does not fulfill identical residual assumptions or heteroscedasticity occurs.

Furthermore, to detect heteroscedasticity by looking at the scatterplot between $e^2$ and variable $\hat{Y}$ can be seen in Figure 3, where $\hat{Y}$ is

$$\hat{Y} = 29.01 + 0.088X_1 - 0.227X_2 + 0.274X_3 - 0.381X_4$$
Figure 3. Scatter plot between $e^2$ and prediction of open unemployment rate 2017

Based on the scatter plot in Figure 3 above, it can be seen that the pattern is increasingly widening that indicates heteroscedasticity exist.

3.1. Quantile Regression Modeling

The previous data analysis informs that heteroscedasticity exist. Therefore, the quantile regression is employed at different level. In this work, there three level of quantiles is employed, that are $\tau = 0.05$; $\tau = 0.5$; and $\tau = 0.95$, such that the quantile regression at each level are as follow:

$$ Q_{\tau}(y|x) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \epsilon_i, i = 1, 2, \cdots n $$

Parameter estimation in quantile regression at level $\tau = 0.05$; $\tau = 0.5$; $\tau = 0.95$ applied on the open unemployment rate 2017 with resampling of 1500 times each is presented in Table 2, Table 3, and Table 4.

Table 2. Parameter Estimation in Quantile Regression with BCa Bootstrap Resampling on the $\tau = 0.05$

| $\tau$ |
|--------|
| 0.05 |

| Parameter | Estimate | CI 95% BR | CI 95% BCa |
|-----------|----------|-----------|------------|
| $\beta_0$ | 35.7041  | -0.75     | 58.59      |
| $\beta_1$ | -0.4347  | -0.8      | 0.56       |
| $\beta_2$ | -0.1352  | -0.71     | 0.05       |
| $\beta_3$ | 0.8203   | -2.19     | 3.07       |
| $\beta_4$ | -0.3336  | -0.67     | -0.04      |

Based on the results obtained in Table 2, $\hat{y}$ in open unemployment rate modeling with the $\tau = 0.05$ quantile regression approach is as follows.

$$ Q_{0.05}(y|x) = 3.7041 - 0.4347x_1 - 0.1352x_2 + 0.8203x_3 - 0.3336x_4 $$

Estimation results of quantile regression with quantile $\tau = 0.05$ can be explained that there are several indicators that have a significant effect on the Open Unemployment Rate in Indonesia in 2017. The constant value is 35.7041. That means if without the four independent variables, the percentage of open unemployment rate ($Y$) is 35.7041. Quantile regression coefficient -0.4347 in the percentage of youth ($X_1$) has a negative and significant effect on the model. It means that every addition of one unit percentage of the number of youth ($X_1$) will decrease the percentage of open unemployment rate ($Y$) by 0.4347 assuming other independent variables remain. Quantile regression coefficient -0.3336 on the
percentage of labor force participation rate \((X_4)\) has a negative and significant effect on the model. It means that every addition of one unit percentage of labor force participation rate \((X_4)\) will decrease the percentage of open unemployment rate \((Y)\) by 0.3336 assuming variables free others remain.

**Table 3.** Parameter Estimation in Quantile Regression with BCa Bootstrap Resampling on the \(\tau = 0.50\)

| \(\tau\) | Parameter | Estimate | CI 95% BR | CI 95% BCa |
|-----|----------|----------|-----------|------------|
| 0.5 | \(\beta_0\) | 30.040 | 16.28 | 52.39 | 17.58 | 54.5 |
|     | \(\beta_1\) | -0.550 | -0.39 | 0.45 | -0.42 | 0.39 |
|     | \(\beta_2\) | -0.3085 | -0.78 | 0.19 | -0.62 | 0.43 |
|     | \(\beta_3\) | 0.5977 | -1.79 | 1.98 | -1.84 | 1.89 |
|     | \(\beta_4\) | -0.3484 | -0.66 | -0.15 | -0.65 | -0.14 |

Based on the results obtained in Table 3, \(\hat{y}\) in open unemployment rate modeling in Indonesia in 2017 with the \(\tau = 0.5\) is as follows:

\[ Q_{0.5}(y|x) = 30.040 - 0.5550x_1 - 0.3085x_2 + 0.5977x_3 - 0.3484x_4 \]

Estimation results of quantile regression with quantile \(\tau = 0.5\) can be explained that there are indicators that have a significant effect on the Open Unemployment Rate in Indonesia in 2017. The constant value of 30.040 means that without the four independent variables the percentage of open unemployment rate \((Y)\) is 30.040. Quantile regression coefficient -0.3484 at the percentage of labor force participation rate \((X_4)\) has a negative and significant effect on the model. It means that every addition of one unit percentage of labor force participation rate \((X_4)\) will decrease the percentage of open unemployment rate \((Y)\) by 0.3484 with variable assumptions free others remain.

**Table 4.** Parameter Estimation in Quantile Regression with BCa Bootstrap Resampling on the \(\tau = 0.95\)

| \(\tau\) | Parameter | Estimate | CI 95% BR | CI 95% BCa |
|-----|----------|----------|-----------|------------|
| 0.95 | \(\beta_0\) | 24.810 | 6.84 | 45.99 | 4.51 | 43.08 |
|     | \(\beta_1\) | 0.6540 | -0.11 | 0.84 | 0.22 | 1.17 |
|     | \(\beta_2\) | -0.7285 | -1.28 | 0.63 | -1.49 | -0.0002 |
|     | \(\beta_3\) | -0.5429 | -1.71 | 1.23 | -2.12 | 0.91 |
|     | \(\beta_4\) | -0.4380 | -0.59 | -0.24 | -0.66 | -0.31 |

Based on the results obtained in Table 4, \(\hat{y}\) in the 2017 open unemployment rate modeling in Indonesia with the \(\tau = 0.95\) quantile regression approach \(= 0.95\) are as follows:

\[ Q_{0.95}(y|x) = 24.810 + 0.6540x_1 - 0.7285x_2 - 0.5429x_3 - 0.4384x_4 \]

Estimation results of quantile regression with quantile \(\tau = 0.95\) can be explained that several indicators have a significant effect on the Open Unemployment Rate in Indonesia in 2017. The constant value of 24.810 means that without the four independent variables the percentage of open unemployment rate \((Y)\) is 24.810. The 0.6540 quantile regression coefficient on the percentage of the number of youth \((X_1)\) has a positive and significant effect on the model. It means that every addition of one unit percentage of youth \((X_1)\) will increase the percentage of open unemployment rate \((Y)\) by 0.6540 assuming other independent variables remain. The quantile regression coefficient -0.7285 at the rate of economic growth \((X_2)\) has a negative and significant effect on the model. It means that every addition of one unit percentage of economic growth rate \((X_2)\) will decrease the percentage of open unemployment rate \((Y)\) by 0.7285 assuming other independent variables remain. Quantile regression coefficient -0.4380 at the percentage of labor force participation rate \((X_4)\) has a negative and significant effect on the model. It
means that every addition of one unit percentage of labor force participation rate \((X_4)\) will decrease the percentage of open unemployment rate \((Y)\) by 0.4380 assuming variables free others remain.

Once the model with several quantile values \((\tau = 0.05; \tau = 0.50; \tau = 0.95)\) was obtained, the next step is to choose the best model by looking at the greatest Pseudo \(R^2(\tau)\) value of each quantile value.

**Table 5. Pseudo \(R^2\) at Each Level of Quantile**

| Model     | Pseudo \(R^2\) |
|-----------|-----------------|
| \(Q_{0.05}\) | 0.5775          |
| \(Q_{0.50}\) | 0.4445          |
| \(Q_{0.95}\) | 0.6045          |

From Table 5 it can be seen that the open unemployment rate 2017 model in quantile 0.95 \((Q_{0.95})\) has the greatest Pseudo \(R^2\) value among the other quintile values, that is, the Pseudo \(R^2(0.95)\) = 0.6045. The Pseudo \(R^2\) value (0.95) shows that the model in quantile 0.95 \((Q_{0.95})\) can explain 60.45% of the variability of open unemployment rate in Indonesia in 2017. In addition, based on Table 4, it is in quantile \(\tau = 0.95\) also shows that by using the value of \(\alpha = 5\%\), the percentage of the number of youth \((X_1)\), the percentage of economic growth rate \((X_2)\), and the percentage of labor force participation rate \((X_4)\) which has a significant effect on open unemployment rate \((Y)\). So it can be concluded that quantile \(\tau = 0.95\) is the best model.

Furthermore, the data distribution for each predictor variable is the percentage of youth \((X_1)\), the percentage of economic growth rate \((X_2)\), and the percentage of labor force participation rate \((X_4)\) on the open unemployment rate \((Y)\) from the best model quantile regression \(\tau = 0.95\) is robust and stable. Based on Figure 4, in general, the distribution of data on the percentage of youth \((X_1)\), the percentage of economic growth rate \((X_2)\), and the percentage of labor force participation rate \((X_4)\) on the open unemployment rate \((Y)\) can follow the actual data pattern. As a result, quantile regression has parameter estimates that are robust, efficient, and consistent.

**Figure 4.** Quantile Regression of Open Unemployment Rate Value \((Y)\) with Each Variable Using 95% Confidence Interval.

### 4. Conclusion

In the analysis process, in general, quantile regression whose interval estimation uses BCa bootstrap resampling provides information that the predictor variable has a more significant effect than the ordinary bootstrap resampling in this case study. Interval estimation using BCa bootstrap resampling has three predictor variables that have a significant effect, while interval estimation uses ordinary
bootstrap resampling, only one predictor variable has a significant effect. The empirical result shows that the best quantile regression model is obtained at the value of quantile 0.95 which has a Pseudo R² value is 60.45 percent. The model at quantile 0.95 shows that the percentage of youth, economic growth rate, and labor force participation rate have a significant effect on the open unemployment rate in Indonesia in 2017.

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