The relation between the model of a crystal with defects and Plebanski’s theory of gravity

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Abstract

In the present investigation we show that there exists a close analogy of geometry of spacetime in GR with a structure of defects in a crystal. We present the relation between the Kleinert’s model of a crystal with defects and Plebanski’s theory of gravity. We have considered the translational defects – dislocations, and the rotational defects – disclinations – in the 3- and 4-dimensional crystals. The 4-dimensional crystalline defects present the Riemann-Cartan spacetime which has an additional geometric property - “torsion” – connected with dislocations. The world crystal is a model for the gravitation which has a new type of gauge symmetry: the Einstein’s gravitation has a zero torsion as a special gauge, while a zero connection is another equivalent gauge with nonzero torsion which corresponds to the Einstein’s theory of ”teleparallelism”. Any intermediate choice of the gauge with nonzero connection $A^{iJ}_\mu$ is also allowed. In the present investigation we show that in the Plebanski formulation the phase of gravity with torsion is equivalent to the ordinary or topological gravity, and we can exclude a torsion as a separate dynamical variable.

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1 Introduction

There exists a close analogy of geometry of spacetime in General Relativity (GR) with a structure of defects in a crystal [1-3]. The crystal’s defects present a special version of the curved Riemannian spacetime - the Riemann-Cartan spacetime which has an additional geometric property called ”torsion” [4]. In the absence of matter, the world crystal is a model for Einstein’s gravitation with a new type of gauge symmetry in which a zero torsion is a special gauge, while a zero connection (and the absence of the Cartan curvature) is another equivalent gauge leading to the Einstein’s theory of ”teleparallelism” [5]. The relation of torsion to electromagnetism [6] makes photon massive. If torsion cannot be seen experimentally, then photon is massless. Limits on the range of magnetic fields emerging from planets and stars give the upper bounds on the photon mass which is extremely small: \( m_\gamma < 10^{-27} \text{ eV} \) (e.g. \( m_\gamma < 10^{-60} \text{ g} \)). Laboratory experiments yield much weaker bound: \( m_\gamma < 3 \times 10^{-16} \text{ eV} \) (e.g. \( m_\gamma < 10^{-49} \text{ g} \)). The recent critical discussion can be found in Ref. [7]. Later in Refs. [8-10] the relationship between torsion and the spin density of the gravitational field was considered.

2 The structure of defects in a crystal

2.1 The structure of defects in the 3-dimensional crystal

Crystalline defects may be generated via so called ”Volterra process”, when layers or sections of matter are cut from a crystal with a subsequent smooth rejoining of the cutting surfaces.

A crystal can have two different types of topological defects [1-3], which are line-like defects in the 3-dimensional space and form world surfaces in the 4-dimensional crystal (they may be objects of string theory).

A first type of topological defects are translational defects called dislocations: a single-atom layer is removed from the crystal and the remaining atoms relax to equilibrium under the elastic forces (see Fig. 1).

A second type of defects is of the rotation type and called disclinations. They arise by removing an entire wedge from the crystal and re-gluing the free surfaces (see Fig. 2).

Considering the displacement field \( u_i(\vec{x}) \) in the 3-dimensional crystal, we can calculate the dislocation density given by the following tensor:

\[
\alpha_{ij} = \epsilon_{kl} \nabla_k \nabla_l u_j(\vec{x}). \tag{1}
\]

The local rotation field

\[
\omega_i = \frac{1}{2} \epsilon_{ijk} [\nabla_j u_k(\vec{x}) - \nabla_k u_j(\vec{x})] \tag{2}
\]

determines the disclination density:

\[
\Theta_{ij} = \epsilon_{kl} \nabla_k \nabla_l \omega_j(\vec{x}). \tag{3}
\]

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The defect densities satisfy the conservation laws:

$$\nabla_i \Theta_{ij} = 0, \quad \nabla_i \alpha_{ij} = -\epsilon_{jkl} \Theta_{kl}. \quad (4)$$

The plastic displacement field $u^p_i(\vec{x})$ doesn’t obey the integrability conditions of Schwarz, i.e. near the plastic deformations the second derivatives don’t commute:

$$\left(\nabla_i \nabla_j - \nabla_j \nabla_i\right) u^p_k(\vec{x}) \neq 0. \quad (5)$$

Eqs. (4) are Bianchi identities if densities $\Theta_{ij}$ and $\alpha_{ij}$ are expressed in terms of the plastic gauge fields $\beta^p_{ij}$ and $\phi^p_{ij}$:

$$\beta^p_{ij} \equiv \nabla_i u^p_j(\vec{x}) \quad (6)$$

- for dislocation, and

$$\phi^p_{ij} \equiv \nabla_i \omega^p_j(\vec{x}) \quad (7)$$

- for disclination. Then the defect densities

$$\Theta_{ij} = \epsilon_{ikl} \nabla_k \phi^p_{lj} \quad \text{and} \quad \alpha_{ij} = \epsilon_{ikl} \nabla_k \beta^p_{lj} + \delta_{ij} \phi^p_{kk} - \phi^p_{ji} \quad (8)$$
are invariant under the following gauge transformations:

\[ \beta^p_{ij} \rightarrow \beta^p_{ij} + \nabla_i u^p_j(\vec{x}) - \epsilon_{ijk} \omega^p_k(\vec{x}) \quad \text{and} \quad \phi^p_{ij} \rightarrow \phi^p_{ij} + \nabla_i \omega^p_j. \]  

(9)

Here we have chosen to use the same symbols for the two gauge functions \( u^p_k(\vec{x}) \) and \( \omega^p_k(\vec{x}) \) as we used for the fields describing the state of the (Kleinert) crystal, because these quantities have the same character. But it is of course a different meaning to a gauge function that can be chosen arbitrarily and the true field describing the crystal. Therefore, \( h_{ij} \equiv \beta^p_{ij} + \epsilon_{ijk} \omega^p_k \) and \( A_{ijk} \equiv \phi^p_{il} \epsilon_{ljk} \) are translational and rotational defect gauge fields in the crystal.

In the Riemann-Cartan geometry of the 3-dimensional crystal the connection is:

\[ \Gamma_{ijk} = \nabla_i \nabla_j u^p_k, \]  

(11)

the torsion is:

\[ S_{ijk} = \frac{1}{2}(\Gamma_{ijk} - \Gamma_{jik}), \]  

(12)

and the curvature is:

\[ R_{ijkl} = (\nabla_i \nabla_j - \nabla_j \nabla_i) u^p_k = \nabla_i \Gamma_{jkl} - \nabla_j \Gamma_{ikl}. \]  

(13)

In the 3-dimensional crystal the disclination density represents the Einstein tensor \( G_{ij} \) associated with the curvature (13):

\[ G_{ij} = \Theta_{ij}(\vec{x}), \]  

(14)

and the dislocation density represents the torsion:

\[ \alpha_{ij} = \epsilon_{ikl} S_{ikl}(\vec{x}). \]  

(15)

### 2.2 Topological defects in the 4-dimensional crystal

The geometry of the 4-dimensional Riemann-Cartan spacetime is described by the direct generalizations of the translational and rotational defect gauge fields \( h_{ij} \) and \( A_{ijk} \) on the vierbein field \( \theta^I_\mu \) and the spin connection \( A^I_{\mu J} \) with \( I, J = (0, 1, 2, 3) \) and \( I = (0, i) \) where \( i = 1, 2, 3 \) is the spatial index.

In four dimensions the indices \( I \) and \( J \) which function as the “flat” index on the vierbein may be chosen to be identified with the directions in the Kleinert-lattice (= world-chystal) meaning the index \( k \) on the \( u^p_k(\vec{x}) \), so that we may call it now - in four dimensions - \( I \) instead of \( k \), and write:

\[ \theta^I_\mu = \partial_\mu u^I_i(x). \]  

(16)
Similarly and analogously to the $A_{ijk}$ of equation (10) we can in four dimensions write:

$$A_I^\mu = \partial_\mu \omega_{IJ}. \quad (17)$$

Here we have analogously to (2) defined

$$\omega_{IJ} = \nabla_I u_J^\mu(x) - \nabla_J u_I^\mu(x). \quad (18)$$

In the traditional literature on gravity with spinning particles, a special role is played by single-valued vierbein fields $h_\alpha^\mu(x)$. Considering the coordinate transformation:

$$x^\alpha = x^\alpha(x^\mu), \quad (19)$$

we go from rectilinear coordinates $x^\alpha$ with $\alpha = 0,1,2,3$ to arbitrary curvilinear ones $x^\mu$ with $\mu = 0,1,2,3$, therefore, from the flat Minkowski metric $\eta_{\alpha\beta} = (1,-1,-1,-1)$ to the induced metric:

$$g_{\mu\nu} = \eta_{\alpha\beta} h_\alpha^\mu h_\beta^\nu, \quad (20)$$

where

$$h_\mu^\alpha \equiv \frac{\partial x^\alpha}{\partial x^\mu}, \quad (21)$$

are the vierbein fields (tetrads). They define local coordinate differentials $dx^\alpha$ by a transformation:

$$dx^\alpha = h_\mu^\alpha(x) dx^\mu. \quad (22)$$

Mathematically, the cutting and joining of the matter in the world crystal may be described by nonholonomic mappings [2] of the next-neighbor atomic distance vectors. This mapping is not integrable to a global coordinate transformation (19). It is described by a local transformation:

$$dx^I = \theta_I^\mu(x) dx^\mu, \quad (23)$$

whose coefficients $\theta_I^\mu(x)$ have a nonvanishing curl:

$$\partial_\mu \theta_I^\nu - \partial_\nu \theta_I^\mu \neq 0, \quad (24)$$

what means that any coordinate transformation $x^I = x^I(x^\mu)$ has to disobey the integrability conditions of Schwarz, and gives the noncommutativity:

$$[\partial_\mu \partial_\nu - \partial_\nu \partial_\mu] x^I(x^\mu) \neq 0. \quad (25)$$

Therefore, now the functions $x^I(x^\mu)$ are multivalued [2]. This is in contrast to the curl of the usual vierbein fields $h_\mu^\alpha(x)$ which is equal to zero and determines purely Riemannian spacetime, i.e. spacetime without torsion.
A set of new nonholonomic coordinates $dx^I$ are related to $dx^\alpha$ by a multivalued Lorentz transformation:

$$dx^I = \Lambda^I_\alpha(x)dx^\alpha,$$

and they are related to the physical $dx^\mu$ by the multivalued tetrad fields:

$$dx^I = \theta^I_\mu(x)dx^\mu \equiv \Lambda^I_\alpha(x)h^\alpha_\mu(x)dx^\mu.$$  

This procedure transforms the physical laws from the flat space to spaces with curvature and torsion.

The condition (25) is not enough to describe all topological defects in a crystal. Also the coefficients $\theta^I_\mu$ in Eq. (23) themselves must violate the Schwarz conditions leading to the noncommutative derivatives:

$$(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) \theta^I_\lambda \neq 0.$$  

They are called multivalued tetrads [2]. The multivaluedness distinguishes them from the well-known single-valued tetrads (or vierbeins) $e^I_\mu \equiv h^I_\mu$ considered in the standard literature on gravity.

The field strength of $A^{IJ}_\mu$ is:

$$F^{IJ}_\mu = \partial_\mu A^{IJ} - \partial_\nu A^{IJ}_\mu - [A^{IJ}_\mu, A^{IJ}_\nu],$$

and determines the Riemann-Cartan curvature [4]:

$$R_{\mu\nu\lambda\kappa} = \theta^I_\lambda \theta^J_\kappa F^{IJ}_{\mu\nu}.$$  

The field strength of $\theta^I_\mu$ is the torsion [4]:

$$S^{I}_\mu = D^{IJ}_\mu \theta^J_\nu - D^{IJ}_\nu \theta^J_\mu.$$  

where

$$D^{IJ}_\mu = \delta^{IJ}\partial_\mu - A^{IJ}_\mu$$

is a covariant derivative.

### 3 Plebanski formulation of gravity

The translational and rotational crystalline defect gauge fields – the tetrads $\theta^I_\mu$ (dislocations) and the spin connection $A^{IJ}_\mu$ with $I, J = (0, 1, 2, 3)$ (disclinations) – were used by J. Plebanski [11] for the construction of the gravitational action. The main idea of Plebanski’s formulation of the 4-dimensional theory of gravity was to present the gravitational action without metrics as a product of two 2-forms [11–17]. These 2-forms were
constructed using the connection $A^{IJ}$ and tetrads $\theta^I$ as independent dynamical variables. The tetrads $\theta^I$ were used instead of the metric $g_{\mu\nu}$.

Both $A^{IJ}$ and $\theta^I$ are 1-forms:

$$A^{IJ} = A_{\mu}^{IJ} dx^\mu \quad \text{and} \quad \theta^I = \theta^I_\mu dx^\mu. \quad (33)$$

In Eq. (1) the indices $I, J = 0, 1, 2, 3$ refer to the flat space-time with Minkowski metric $\eta^{IJ} = \text{diag}(1, -1, -1, -1)$, which is tangential to the curved space with the metric $g_{\mu\nu}$.

The world interval is represented as

$$ds^2 = \eta_{IJ} \theta^I \otimes \theta^J, \quad (34)$$

i.e.

$$g_{\mu\nu} = \eta_{IJ} \theta^I_\mu \otimes \theta^J_\nu. \quad (35)$$

In the Plebanski’s BF-theory [11], the gravitational action with nonzero cosmological constant $\Lambda$ is presented by the integral:

$$I_{GR} = \frac{1}{\kappa^2} \int \epsilon^{IJKL} \left( B^{IJ} \wedge F^{KL} + \frac{\Lambda}{4} B^{IJ} \wedge B^{KL} \right), \quad (36)$$

where $\kappa^2 = 8\pi G$ ($G$ is the gravitational constant).

Below we use units $\kappa = 1$.

The topological sector of gravity is described by the following integral [18]:

$$I_T = 2 \int \left( B^{IJ} \wedge F^{IJ} + \frac{\Lambda}{4} B^{IJ} \wedge B^{IJ} \right) = \int \epsilon^{IJKL} \left( B^{IJ} \wedge F^{*KL} + \frac{\Lambda}{4} B^{IJ} \wedge B^{*KL} \right), \quad (37)$$

where $F^{*IJ} = \frac{1}{2} \epsilon^{IJKL} F^{KL}$ is the dual tensor. Here we have the topological sector only in the flat space, and the dual tensor $F^{*IJ}$ corresponds only to the flat indexes $I, J$.

The antisymmetric tensor $F^{IJ}$ can be split into a self-dual component $F^+$ and an anti-self-dual component $F^-$, according to the relation:

$$F^\pm = \frac{1}{2} (F \pm F^*). \quad (38)$$

Here $B^{IJ}$ and $F^{IJ}$ are the following 2-forms:

$$B^{IJ} = \theta^I \wedge \theta^J = \frac{1}{2} \theta^I_\mu \theta^J_\nu dx^\mu \wedge dx^\nu, \quad (39)$$

$$F^{IJ} = \frac{1}{2} F^{IJ}_{\mu\nu} dx^\mu \wedge dx^\nu. \quad (40)$$

The tensor $F^{IJ}_{\mu\nu}$ is the field strength [29] of the spin connection $A^{IJ}_{\mu}$. 

As it was shown in Refs. [11–17], actions $I_{GR}$ and $I_{TG}$, given by Eqs. (36) and (37), respectively, can be presented in terms of the self-dual "left-handed" and anti-self-dual "right-handed" gravity [17, 19]:

$$I_{GR} = \int [\Sigma^i \wedge F^i - \bar{\Sigma}^i \wedge \bar{F}^i + \Lambda (\Sigma^i \wedge \Sigma^j - \bar{\Sigma}^i \wedge \bar{\Sigma}^j)],$$  \hspace{1cm} (41)

and

$$I_{TG} = \int [\Sigma^i \wedge F^i + \bar{\Sigma}^i \wedge \bar{F}^i + \Lambda (\Sigma^i \wedge \Sigma^j + \bar{\Sigma}^i \wedge \bar{\Sigma}^j)],$$  \hspace{1cm} (42)

where $F \equiv F^+$, $\bar{F} \equiv F^-$, $\Sigma \equiv \Sigma^+$, $\bar{\Sigma} \equiv \Sigma^-$, and the left-handed ("+" ) and right-handed ("−") $F_{\mu \nu}^{\pm i}$ and $\Sigma^{\pm i}$ are given by:

$$F_{\mu \nu}^{\pm i} = \partial_\mu A_\nu^{\pm i} - \partial_\nu A_\mu^{\pm i} + \epsilon^{ijk} A_\mu^{\pm j} A_k^{\pm k},$$  \hspace{1cm} (43)

$$- \Sigma^{\pm i} = -i\theta^0 \wedge \theta^i \pm \frac{1}{2} \epsilon^{ijk} \theta^j \wedge \theta^k.$$  \hspace{1cm} (44)

Choosing the correct gauge in his gravitational actions, Plebansky introduced the Lagrange multipliers $\psi_{ij}$, which are considered in theory as auxiliary fields, symmetric and traceless.

Finally, the resulting actions of the Plebanski gravity are [11–16]:

$$I_{GR} = \int [\Sigma^i \wedge F^i - \bar{\Sigma}^i \wedge \bar{F}^i + (\Psi^{-1})_{ij}(\Sigma^i \wedge \Sigma^j - \bar{\Sigma}^i \wedge \bar{\Sigma}^j)],$$  \hspace{1cm} (45)

and

$$I_{TG} = \int [\Sigma^i \wedge F^i + \bar{\Sigma}^i \wedge \bar{F}^i + (\Psi^{-1})_{ij}(\Sigma^i \wedge \Sigma^j + \bar{\Sigma}^i \wedge \bar{\Sigma}^j)],$$  \hspace{1cm} (46)

where

$$(\Psi^{-1})_{ij} = \Lambda \delta_{ij} + \psi_{ij}.$$  \hspace{1cm} (47)

The explanation of the introduction of the auxiliary field $\psi_{ij}$ is connected with the condition for metricity. The 'area metric' (see [19]) is:

$$m = \frac{1}{2} B^{IJ} \otimes B^{IJ},$$  \hspace{1cm} (48)

which can be written in terms of $\Sigma^{\pm i}$:

$$m^{\pm} = \Sigma^{\pm i} \otimes \Sigma^{\pm i} - \frac{1}{4} V^{\pm},$$  \hspace{1cm} (49)

with

$$V^{\pm} = \pm 4 \Sigma^{\pm i} \wedge \Sigma^{\pm i}.$$  \hspace{1cm} (50)
where $V^\pm$ both are equal to the 4-volume form $V = \theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^3$. Here $m^+$ and $V^+$ define a "left" geometry and $m^-$ and $V^-$ define a "right" geometry of spacetime. These geometries are defined, but they can be in disagreement. The stationarity with respect to $\psi_{ij}$ requires the equality $m^+ = m^-$ of the left and right area metrics $m^\pm$, which have no totally antisymmetric part, and leaves the totally antisymmetric parts of $\Sigma^\pm \otimes \Sigma^\pm$, which form $V$, unconstrained. We see that $m^+ - m^-$ has 20 linearly independent components. The equality $m^+ = m^-$ provides the correct number of constraints, reducing the 36 degrees of freedom of $(\Sigma^+, \Sigma^-)$ to the 16 degrees of freedom of tetrads $\theta^I_\mu$.

The main assumption of Plebanski was that our world, in which we live, is a self-dual left-handed gravitational world described by the action:

$$I_{(selfdual \ GR)}(\Sigma, A) = \int [\Sigma^i \wedge F^i + (\Psi^{-1})_{ij} \Sigma^i \wedge \Sigma^j],$$

and the anti-self-dual right-handed gravitational world is absent in Nature ($\bar{F} = 0$ and $\bar{\Sigma} = 0$), and we have the equality [17]:

$$I_{GR} = I_{TG},$$

i.e. the gravitational sector of gravity coincides with topological phase of gravity.

Postulating the existence of the Mirror (or Hidden) World [20–25], we assumed in Ref. [17], that the anti-self-dual right-handed gravity is the "mirror gravity" given by the equation:

$$I_{(antiselfdual \ GR)}(\bar{\Sigma}, \bar{A}) = \int [\bar{\Sigma}^i \wedge \bar{F}^i + (\Psi^{-1})_{ij} \bar{\Sigma}^i \wedge \bar{\Sigma}^j].$$

This "mirror gravity" describes the gravity in the Mirror World.

4 Torsion

The gravitational theory with torsion can be presented by the following integral:

$$I_S = 2 \int (2S^I \wedge S^I + \Lambda \frac{\Lambda}{4} B^{IJ} \wedge B^{IJ}),$$

where the 2-form

$$S^I = \frac{1}{2} S^I_{\mu\nu} dx^\mu \wedge dx^\nu$$

contains the torsion $S^I_{\mu\nu}$, which is the field strength (31) of the tetrads $\theta^I_\mu$.

Using the partial integration and putting

$$\int \partial_\mu (T_{\nu\lambda}) dx^\mu \wedge dx^\nu \wedge dx^\sigma \wedge dx^\lambda = 0,$$

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it is not difficult to show that
\[
\int B^{IJ} \wedge F^{IJ} = 2 \int S^I \wedge S^I. \quad (57)
\]

According to Eqs. (37) and (54), we have:
\[
I_{TG} = 2 \int \left( B^{IJ} \wedge F^{IJ} + \frac{\Lambda}{4} B^{IJ} \wedge B^{IJ} \right) = 2 \int \left( 2S^I \wedge S^I + \frac{\Lambda}{4} B^{IJ} \wedge B^{IJ} \right) = I_S. \quad (58)
\]

This means that in the self-dual Plebanski formulation of gravitational theory (i.e. in our world) all sectors of gravity coincide [17]:
\[
I_{GR} = I_{TG} = I_S. \quad (59)
\]

Here we see a close analogy of the geometry of spacetime in GR with a structure of defects in a crystal [1–3]. Torsion is presented by the translational defects – dislocations, and curvature is given by the rotational defects – disclinations. But these defects are not independent of each other: a dislocation is equivalent to a disclination-antidisclination pair, and a disclination gives a string of dislocations.

The crystalline defects present a special version of the curved spacetime - the Riemann-Cartan spacetime which has an additional geometric property - ”torsion” [4], described by dislocations. The world crystal is a model for the Einstein’s gravitation which has a new type of gauge symmetry with a zero torsion as a special gauge, while a zero connection (with zero Cartan curvature) is another equivalent gauge with nonzero torsion which corresponds to the Einstein’s theory of ”teleparallelism” [5]. But any intermediate choice of the gauge for the field \( A^{IJ}_\mu \) is also allowed [3]. In the present investigation we have shown that in the Plebanski formulation the phase of gravity with torsion is equivalent to the ordinary or topological gravity, and we can exclude a torsion as a separate dynamical variable. This explains why the Einstein’s theory of gravity described only by curvature corresponding to the disclination defects of a crystal can be rewritten as a teleparallel theory of gravity described only by torsion which corresponds to the dislocation defects.

5 Summary and Conclusions

In this paper we have shown the close analogy of geometry of space-time in GR with a structure of defects in a crystal, given by H. Kleinert in his model [1–3]. This model was related with the Plebanski formulation of gravity. In the Section 1 we have discussed a role of torsion. We have reviewed the Kleinert’s model of a crystal with defects in the Section 2 considering the translational defects – dislocations, and the rotational defects – disclinations – in the 3- and 4-dimensional crystals. In the Section 3 we have explained the main idea of Plebanski [11] to construct the 4-dimensional gravitational action without
metric. The integrand of this action contains a product of two 2-forms, which are constructed from the tetrads $\theta^I$ and the connection $A^{IJ}$ considered as independent dynamical variables. Here we have presented the relation between the Kleinert’s model of a crystal with defects and Plebanski’s formulation of gravity. It was shown that the tetrads $\theta^I$ are dislocation gauge fields, and the connections $A^{IJ}$ are the disclination gauge fields, the curvature is the field strength of connections $A^{IJ}_{\mu}$, and the torsion is the field strength of tetrads $\theta^I_{\mu}$. Both $A^{IJ}$ and $\theta^I$ are 1-forms. The tetrads $\theta^I_{\mu}$ were used instead of the metric $g_{\mu\nu}$.

Section 4 is devoted to the torsion. The crystalline defects represent a special version of the curved space-time – the Riemann–Cartan space-time with torsion [4]. The world crystal gives a model with a new type of gauge symmetry for GR [3]. The Einstein’s gravitation, which has a zero torsion, corresponds to a special gauge, while the theory with a zero connection (i.e. zero Cartan curvature) is another equivalent gauge with nonzero torsion which corresponds to the Einstein’s theory of “teleparallelism” [5]. In this paper we have showed that in the Plebanski formulation of gravity the torsional gravitational phase (phase described only by nonzero torsion) is equivalent to the ordinary or topological gravity, and we can exclude torsion as a separate dynamical variable of gravity. In the Plebanski formulation of 4-dimensional gravity the Einstein’s gravity with a space-time corresponding to the structure of a crystal with disclinations, i.e. rotational defects, coincides with the Einstein’s “teleparallel” gravitational theory [5] corresponding to a crystal having only the translational defects – dislocations.

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