High-contrast Coherent Population Trapping based on Crossed Polarizers Method

Yuichiro Yano and Shigeyoshi Goka

Abstract—We developed a method based on crossed polarizers to observe high-contrast coherent population trapping (CPT) resonance. Since crossed polarizers have a simple optical system, our method is suitable for chip-scale atomic clocks (CSACs). We calculated the Faraday rotation in CPT in a linearly polarized light field (lin||lin) using two pairs of Λ system models based on the density matrix and estimated the spectrum of the Faraday rotation. After that, we measured the contrast and linewidth with the crossed polarizers method. A comparison of the theoretical model and experiment data showed they were in good agreement. Moreover, the experimental results showed that a high contrast (88.4%) and narrow linewidth (1.15 kHz) resonance could be observed using a Cs gas cell and D_{1}-line vertical-cavity surface-emitting laser (VCSEL).

Keywords—coherent population trapping, chip-scale atomic clocks, Faraday effect, polarization selective method

I. INTRODUCTION

Atomic clocks based on coherent population trapping (CPT) resonance with a vertical-cavity surface-emitting laser (VCSEL) have attracted attention as a means of fabricating very small atomic references [1]. CPT atomic clocks are in great demand for many applications, such as telecommunications, navigation systems, and synchronization of networks [2]. Such atomic clocks are in demand for their high frequency stability. In particular, short-term frequency stability is an important parameter for base stations of telecommunication systems. Short-term frequency stability, described as the Allan standard deviation σ_y(τ), is estimated as

\[ σ_y(τ) \propto \frac{Δf}{f_0 SNR} τ^{-1/2} \]  

where Δf is the resonance linewidth, f_0 is the resonance frequency, and SNR is the signal-to-noise ratio. Contrast, which is used as a measure of SNR, is defined as the amplitude of CPT resonance over the background signal level [3]. Therefore, short-term stability is determined from contrast and linewidth.

A high-contrast CPT resonance can easily be obtained in a high-intensity laser field; however, the resonance linewidth broadens as a result of the power broadening effect [4]. It is difficult to enhance contrast and make the linewidth narrower together. To resolve this issue, a number of methods have been developed (e.g., four-wave mixing [5], pulsed CPT [6], push-pull pumping [7]). However, since these methods occupy a large volume and consume a lot of power, they are not suitable for chip-scale atomic clocks (CSACs).

In this paper, we focus on the Faraday effect in CPT and propose a method based on crossed polarizers for observing high-contrast CPT. Since the crossed polarizers method has a simple optical system, it enables us to use CSACs to enhance frequency stability. Firstly, we calculate the Faraday rotation in CPT under a linearly polarized light field (lin||lin) using the two pairs of a Λ system model based on the density matrix. The CPT lineshape can be estimated by calculating the Faraday rotation angle. Both the resonance amplitude and linewidth behavior of the magnetic field are estimated. Secondly, we show the results of an experiment using a Cs gas cell and the D_{1}-line VCSEL. The contrast and linewidth were measured by varying the magnetic field and the relative transmission angle of the polarizers. A comparison of the experimental and calculated results showed that they had the same tendency, and the difference between experiment and calculation was no more than 20%. Finally, on the basis of the measurement and calculation data, we determine the optimal condition for enhancing the short-term stability of CPT atomic clocks.

II. THEORY

A. Faraday rotation in Coherent Population Trapping

Faraday rotation is a magneto-optical phenomenon, that is, an interaction between light and a magnetic field in a medium. The resonant Faraday rotation is known as the Macaluso-Corbino effect [8]. Though the Faraday rotation in CPT is
classified as such an effect, the way to calculate it in CPT has not been reported yet. In this section, we describe the Faraday rotation in CPT in a linear light field by using density matrix analysis.

Figure 1(a) shows the excitation scheme using a linear light field on the $^{133}$Rb $D_1$ line. In CPT phenomenon under linear light field, two schemes can be formed with two pairs of ground-state hyperfine sublevels simultaneously: $|F = 3, m_F = 1, |F = 4, m_F = -1 \rangle$ indicated by the label $\Lambda_a$, and $|F = 3, m_F = -1, |F = 4, m_F = 1 \rangle$ indicated label $\Lambda_b$ coupled with the common excited states $|F' = 3, m_F = 0 \rangle$ or $|F' = 4, m_F = 0 \rangle$. Since the dark state is a superposition between two ground states in a weak light field [10], these two resonances can be treated individually in the calculation.

Figure 1(b) shows a simple $\Lambda$-type three level system of the $\Lambda_a$ scheme. Here, the energy eigenstates $|1 \rangle$ and $|2 \rangle$ correspond to two ground states $|F = 3, m = -1 \rangle$ and $|F = 4, m = 1 \rangle$, and the excited state $|3 \rangle$ correspond to $|F' = 3, m = 0 \rangle$ or $|F' = 4, m = 0 \rangle$. In this system, the dynamical behavior of the density matrix $\rho$ is governed by the quantum Liouville equation,

$$\frac{\partial}{\partial t} \rho(t) = \frac{1}{i\hbar}[H, \rho] + i\mathbb{R} \rho.$$  \hspace{1cm} (2)

where $H$ is the Hamiltonian matrix for this three level system and $\mathbb{R}$ stands for the relaxation terms. Using a rotating wave approximation, Eq. (2) can be rewritten as

$$\dot{\rho}_{11} = i\frac{\Omega_p}{2}(\rho_{13} - \rho_{31}) + \Gamma_{31}\rho_{33} + \gamma_s(\rho_{22} - \rho_{11})$$
$$\dot{\rho}_{22} = i\frac{\Omega_c}{2}(\rho_{23} + \rho_{32}) + \Gamma_{32}\rho_{33} - \gamma_s(\rho_{22} - \rho_{11})$$
$$\dot{\rho}_{33} = i\frac{\Omega_p}{2}(\rho_{13} + \rho_{31}) + i\frac{\Omega_c}{2}(\rho_{23} + \rho_{32}) - \Gamma_{33}\rho_{33}$$
$$\dot{\rho}_{12} = i\rho_{12}(\delta_p - \delta_c) - i\frac{\Omega_p}{2}\rho_{13} + i\frac{\Omega_c}{2}\rho_{32} - \gamma_s\rho_{12}$$
$$\dot{\rho}_{13} = i\rho_{13}\delta_p - i\frac{\Omega_p}{2}\rho_{12} + i\frac{\Omega_c}{2}(\rho_{33} - \rho_{11}) - \gamma_f\rho_{13}$$
$$\dot{\rho}_{23} = i\rho_{23}\delta_c - i\frac{\Omega_p}{2}\rho_{21} + i\frac{\Omega_c}{2}(\rho_{33} - \rho_{22}) - \gamma_f\rho_{23}$$

and the trace of density matrix satisfies the closed system condition.

$$\text{Tr}(\rho) = \rho_{11} + \rho_{22} + \rho_{33} = 1.$$  \hspace{1cm} (4)

The density matrix elements $\rho_{13}$ and $\rho_{23}$ are responsible for the complex susceptibilities of left ($\sigma^+$) and right ($\sigma^-$) circularly polarized light, respectively. Regarding continuous wave excitations, the left terms in Eq. (3) equal zero for a time-independent solution ($\dot{\rho} = 0$). To simplify the equation of the Faraday rotation in CPT, we assume that $\Gamma_{31} = \Gamma_{32}$, $\Omega_c = \Omega_p$, $\Gamma_{33} = \Omega_p$, $\Gamma_{31} = \Gamma_{32}$.

In the case of bichromatic light, we have the relation: $\delta_p = -\delta_c = \delta/2$. Since $\delta_p$ and $\delta_c$ have opposite signs and the real parts of $\rho_{13}$ and $\rho_{23}$ are odd functions of the detuning $\delta$, we can obtain the relation between $\rho_{13}$ and $\rho_{23}$ from Eq. (3) and Eq. (4).

Re($\rho_{13}(\delta)$) = −Re($\rho_{23}(\delta)$) \hspace{1cm} (5)

Re($\rho_{13}$) and Re($\rho_{23}$) are responsible for the dielectric susceptibility of $\sigma^+$ and $\sigma^-$, respectively. The atomic susceptibility relation $\chi_{\pm}(\delta)$ is

$$\chi_+^\prime(\delta) = -\chi_-^\prime(\delta).$$  \hspace{1cm} (6)

Using Eq. (3), the refraction index $n_{\pm}$ for $\sigma^+$ and $\sigma^-$ can be expressed as

$$n_{\pm} = \sqrt{1 + \chi_\pm^\prime} \approx 1 \pm \frac{\chi_\pm^\prime}{2}.$$  \hspace{1cm} (7)

As light propagates through the atomic vapor, the two circular components acquire a relative phase shift:

$$\phi_a = \pi \frac{l}{\lambda} (n_+ - n_-)$$
$$\phi_b = \pi \frac{l'}{\lambda} \chi_+^\prime$$

where $l$ is the path length in the vapor, $\lambda$ is the wavelength of the light.

For the Doppler-free case and narrow-band light, the susceptibility $\chi_\pm^\prime(\delta)$ can be described by a Lorentzian line-shape function. The relative phase shift is as follows:

$$\phi_a(\delta) \approx \frac{\pi l\chi_0}{\lambda} \frac{\gamma \delta}{\delta^2 + \gamma^2}.$$  \hspace{1cm} (9)

where $\chi_0$ is the amplitude of the linear susceptibility, and $\gamma$ is the linewidth (half width at half maximum: HWHM) of the CPT resonance.

The relative phase shift $\phi_b$ of scheme $\Lambda_b$ can be calculated in the same way:

$$\phi_b(\delta) \approx -\pi \frac{l\chi_0}{\lambda} \frac{\gamma \delta}{\delta^2 + \gamma^2}.$$  \hspace{1cm} (10)

The total relative phase shift $\phi$ is obtained from Eq. (9) and Eq. (10):

$$\phi = \phi_a(\delta) + \phi_b(\delta).$$  \hspace{1cm} (11)

Next, we calculate the Zeeman shift in the linear light field. The Zeeman shift of two pairs in a weak magnetic field ($< 50$ mT) is as follows [11]:

$$f_{a,b} = f_0 \pm \frac{2g_I\mu_B h}{\lambda} B + \frac{15g_I^2\mu_B^2 h}{32f_{a,b}^2} B^2.$$  \hspace{1cm} (12)

where $B$ is the magnetic field, and $f_0$ is the hyperfine splitting frequency of the ground states in the absence of the magnetic field, $g_I$ is the nuclear $g$-factor, $g_J$ is the Landé $g$-factor, $\mu_B$ is the Bohr magneton, and $h$ is Planck’s constant. From Eq. (12), the two resonances shift in opposite frequency directions in the magnetic field. The frequency difference $f_B$ between $f_a$ and $f_b$ can be expressed as

$$f_B = f_a - f_b = \frac{4g_I\mu_B h}{f_{a,b}} B.$$  \hspace{1cm} (13)
The AM conversion noise on the atomic absorption [12]. The AM limited by a combination of light source AM noise and FM-AM conversion noise is caused by the effect that laser frequency fluctuations have on the absorption [12]. It is known that these noises are proportional to the background signal level (DC level) of the photo detector [3]. Therefore, to enhance the short-term stability of CPT atomic clocks, it is necessary to reduce the DC level.

The crossed polarizers method is a way of measuring the birefringence of the optical medium. It has high sensitivity for birefringence detection because it suppresses the background signal level [3]. The schematic optical layout is shown in Fig. 2. Two linear polarizers are placed on both sides of the gas cell, and the transmission axes are set nearly orthogonal to each other. The first polarizer and the second polarizer are called the polarizer and analyzer, respectively. The transmission axes between the polarizer and analyzer are defined by the relative angle $\theta$. In this paper, $\theta$ is defined as zero when the polarizers are orthogonal to each other. Here, let the Faraday rotation angle caused by the gas cell be $\phi$; the transmitted light $I$ can be expressed as

$$I = I_c \cos^2(\phi + \theta - \pi/2) + I_{nc} \cos^2(\theta - \pi/2)$$

$$= I_c \sin^2(\phi + \theta) + I_{nc} \sin^2(\theta)$$

where $I_c$ is the sum of light intensities contributing to CPT, and $I_{nc}$ is the sum of light intensities not contributing to CPT.

$fb$ is proportional to the magnetic field. The coefficient of the difference frequency is -22.3 Hz/µT of $^{133}$Cs. The second-order Zeeman shift terms do not influence the spectrum of the Faraday rotation. Therefore, we only need treat the first-order Zeeman shift terms. The Faraday rotation in CPT with the Zeeman shift can be expressed by adding Zeeman shift terms. The Faraday rotation in CPT with the Zeeman shift is defined as zero when the polarizers and analyzer are orthogonal. Here, let the Faraday rotation caused by the gas cell be $\phi$; the transmitted light $I$ can be simplified into

$$\phi = \frac{\phi_0 (\delta - f_B/2) + \phi_b (\delta + f_B/2)}{\lambda} \gamma (\delta - f_B/2) + \frac{\gamma (\delta + f_B/2)}{\lambda} (15)$$

and by using a normalized detuning $d \equiv \delta/\gamma$ and normalized Zeeman shift $\beta \equiv f_B/2\gamma$, Eq. (14) can be simplified into

$$\phi = -\frac{\pi l_{x0}}{(1 + b^2 + d^2)^2 - 4b^2 d^2}.$$

### TABLE I. $\alpha_n$ VALUES

| $\alpha_n$ | Value    |
|-----------|----------|
| $\alpha_0$ | 3.681 $\cdot 10^{-1}$ |
| $\alpha_1$ | 3.988 $\cdot 10^{-1}$ |
| $\alpha_2$ | 1.644 $\cdot 10^{-2}$ |
| $\alpha_3$ | 3.706 $\cdot 10^{-4}$ |
Taking into account that the conventional resonance contrast is no more than 10% [14], the relation between \( I_c \) and \( I_{DC} \) is \( I_c < I_{DC} \). Therefore, the DC level is dominated by the relative angle \( \theta \).

When the relative angle \( \theta \) is set larger than the phase shift \( \phi \), which enables us to ignore the phase shift \( \phi \), the DC level \( I_{DC} \) is given as

\[
I_{DC} = I_c \sin^2(\theta) + I_{Ne} \sin^2(\theta).
\]

(17)

And the resonance amplitude \( I_S \) is

\[
I_S \approx \frac{\partial I_c}{\partial \theta} \phi = I_c \sin(2\theta) \phi
\]

(18)

From Eq. (18), as the resonance amplitude is maximized by setting the relative angle \( \theta \) to \( \pi/4 \). However, since the DC level increases with \( \theta \) (from Eq. (17)), the highest contrast resonance can be obtained around \( \theta \approx 0 \).

When the relative angle \( \theta \) is close to 0, Eq. (18) can not be used to describe the resonance amplitude behavior because the amplitude of the phase shift \( \phi \) can not be ignored. Instead, from Eq. (16) and using a small rotation approximation, the resonance amplitude \( I_S \) is given as

\[
I_S = I_c \cos^2(\phi - \pi/2) = I_c \sin^2 \phi
\]

\[
\simeq I_c \sin^2 \phi.
\]

(19)

Next, we calculate the spectrum characteristics of the transmitted light. Figure 4 shows the resonance lineshape calculated from Eq. (15) and Eq. (19) when \( \theta = 0 \). It is clear that the both the amplitude and linewidth of the resonance increase with increasing normalized magnetic field \( b \).

The resonance amplitude \( I_{pp} \) can be gotten from Eq. (15) and Eq. (19):

\[
I_{pp} = \frac{\pi^2 I_c^2 \lambda^4}{2} \left( \frac{b^2 + \sqrt{b^2 + 1} - 1}{b(b^2 + 1)} \right)^2
\]

(20)

Figure 4 plots the resonance amplitude as a function of the magnetic field using Eq. (20). In the weak magnetic field \( b \leq 0.2 \), the resonance amplitude is small because the Faraday rotation is small. In the range of \( 0.2 < b \leq 1.16 \), the resonance amplitude significantly increases with increasing magnetic field. The maximum resonance amplitude is obtained at \( b = 1.16 \). For magnetic fields \( b \) over 1.16, the resonance amplitude tends to decrease because the two resonances separate from each other.

Figure 5 shows the linewidth with the crossed polarizers method \( \gamma_{cp} \) normalized by the conventional linewidth \( \gamma \) as a function of normalized magnetic field strength. The polynomial approximation of the normalized linewidth \( \gamma_{cp}/\gamma \) is

\[
\gamma_{cp}/\gamma \approx \sum_{n=0} a_n b^{2n}
\]

(21)

where \( a_n \) are constant values shown in Table 2 in the \( n \) range from 0 to 3. The linewidth with the proposed method increases with increasing magnetic field. Moreover, since the linewidth is the sum of even functions and has a positive second derivative, the linewidth has a minimum value in the absence of the magnetic field. The minimum linewidth is the conventional one of 36.8%. The linewidth of the proposed method is equal to that of the conventional one at a \( b = 1.27 \).

From Fig. 4 and Fig. 5, we can obtain the relation between \( b|_{I_{pp}=\text{max}} \) and \( b|_{\gamma_{cp}/\gamma=1} \):

\[
b|_{I_{pp}=\text{max}} < b|_{\gamma_{cp}/\gamma=1}
\]

(22)

Thus, the linewidth of the proposed method is narrower than that with the conventional method in the magnetic field range from 0 to \( b|_{I_{pp}=\text{max}} \).

III. EXPERIMENTAL SETUP

Figure 6 shows the experimental setup of the proposed observation method. The two polarizers were near-infrared sheet polarizers. A parallel linear beam (lin||lin light field) was incident on the gas cell. The analyzer selected the optical polarization of wavelength components incident on the photodiode. The photodiode signal was connected to the oscilloscope.

A single-mode VCSEL fabricated by Ricoh Company, Ltd was used as the laser source. The 894.6 nm wavelength of the VCSEL excites \(^{133}\)Cs at the D\(_1\)-line. The VCSEL was driven by a DC injection current using a bias \( T \) and was modulated at 4.6 GHz using an analog signal generator to generate the first-order sidebands around the laser carrier. The absorption profile of the Cs-D\(_1\) line using the VCSEL modulated at 4.6 GHz is
shown in Fig. 7 Since the incident light contains first-order sidebands and high-order sidebands, the plot shows some of the absorption lines. The two center absorption lines are excited by the first-order sidebands. Moreover, the minimum and second absorption lines. The two center absorption lines are excited by using a neutral density filter in place of the analyzer and added to the conventional method's value of 2.15 kHz. This result means that the resonance with the crossed polarizers method has not only higher contrast but also a narrower linewidth.

IV. EXPERIMENTAL RESULTS AND DISCUSSION
A. Line shape of CPT resonance

Figure 8 shows the observed CPT resonance with the crossed polarizers method. A good reduction in DC level was achieved because the transmission axis of the analyzer was optimized. Considering the transmitted light intensity, we estimate that the peak rotation angle is few tens of milli-radians. Since the signal was greater than the DC level, the conventional contrast, which was simply defined as the signal over the DC level, exceeded 100%. In this paper, contrast is defined as not to exceed 100% as follows.

\[
\text{Contrast} := \frac{\text{Signal}}{\text{Signal} + \text{DC level}} \tag{23}
\]

Although the DC level was suppressed with the crossed polarizers method, weak light leakage was picked up by the photo detector. Since the leakage could not be reduced below the DC level by varying the relative angle \( \theta \), the leakage was dependent on the extinction ratios of the polarizers. Owing to the DC level reduction, the proposed method yielded a contrast of 88.4% with under a static magnetic field of 93 \( \mu \)T. Since the conventional contrast was 3.3 % under a static magnetic field of 5.0 \( \mu \)T when using the neutral density filter instead of the analyzer, the crossed polarizers method obtained about 25 times better contrast than the conventional method did. In addition, the linewidth obtained with the crossed polarizers method was 1.15 kHz, which was about twice as narrow as the conventional method's value of 2.15 kHz. This result means that the resonance with the crossed polarizers method has not only higher contrast but also a narrower linewidth.

B. Contrast as a function of the relative angle \( \theta \)

Figure 9 shows the DC level as a function of the relative angle \( \theta \). The square dot is the measured data, and the solid line is the fitting curve calculated from Eq. (17). The relative angle \( \theta \) giving the minimum DC level is shifted from 0°. This shift is caused by misalignment between the scale of the polarizer's mount and the transmission axis of polarizers. The extinction ratio of the polarizers was estimated to be about 40 dB.

Figure 10 shows the signal of the resonance as a function of the relative angle \( \theta \). The square dot is the measured data, and...
the solid line is the fitting curve calculated from Eq. (18). The calculated curve is in good agreement with the experimental data. By comparing the signal and DC level, it can be seen that the behavior of the signal was different from that of the DC level. Therefore, this is proof that the polarization of the wavelength components contributing to CPT is rotated.

Figure 11 shows the contrast as a function of the relative angle $\theta$ from the measurements. Since the signal has a high value despite the DC level reduction, the contrast significantly increased near 0°. A resonance contrast over 10% was obtained in the range from -15 to 5°.

C. Characteristics as a function of magnetic field

Figure 12 shows the signal and DC level as a function of magnetic field. The relative angle $\theta$ was optimized in order to maximize contrast. In weak magnetic fields ($< 15 \mu T$), the signal was so small that we could not observe CPT resonance, and the maximum magnetic field (93 $\mu T$) was limited by the current source output supplying the Helmholz coil. The signal linearly increased with increasing magnetic field. On the other hand, the DC level was constant regardless of the change in the magnetic field. The results are evidence that the Faraday rotation affected only the wavelength components contributing to CPT. The solid line is a fitting curve of Eq. (20). The experimental signal has the same tendency as the theoretical curve.

Figure 13 shows the contrast estimated from the measurement results in Fig. 12. The contrast increased with increasing magnetic field. We assume that nearly 100% contrast can be obtained in a larger magnetic field. The DC level is independent of the magnetic field, and this indicates that the resonance contrast has reached a peak value. From Eq. (20), it is estimated that the maximum contrast of 94.0% can be obtained at 224 $\mu T$.

The linewidth as a function of the magnetic field is shown in Fig. 14. The linewidth broadened with increasing magnetic field and was approximately proportional to it. The linewidth obtained with the crossed polarizers method is less than 2.15 kHz of the linewidth with the conventional excitation method.

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![Fig. 11. Contrast as a function of the relative angle $\theta$.](image1)

![Fig. 12. Signal and DC level as a function of magnetic field: The square and triangle are the measured signal and DC level, respectively. The solid line is a fit of Eq. (20); resonance linewidth $\gamma = 1.08$ kHz.](image2)

![Fig. 13. Contrast as a function of magnetic field estimated from Fig. 12.](image3)

![Fig. 14. Linewidth as a function of magnetic field: The solid line is data calculated from Eq. (21); resonance linewidth $\gamma = 1.08$ kHz.](image4)

![Fig. 15. Figure of merit as a function of magnetic field. The figure of merit is normalized by that of the conventional resonance.](image5)
in this range of magnetic field. At a magnetic field of $15 \, \mu T$, the narrowest linewidth obtained was 760 Hz, which is about three times narrower than the conventional one. The solid line in Fig. 13 is calculated data based on Eq. (21). The difference between the measurement and calculation is no more than 20%.

Figure 15 shows the figure of merit (FoM) as a function of magnetic field. Since short-term stability is determined by the contrast and linewidth from Eq. (21), the FoM is defined as follows.

$$\text{FoM} := \frac{f_0}{\Delta f} \cdot \text{Contrast} \quad (24)$$

In small magnetic fields ($< 40 \, \mu T$), the FoM increased because the increase in contrast was dominant. However, in large magnetic fields ($> 60 \, \mu T$), the FoM decreased with broadening linewidth. This shows that the FoM of the CPT resonance has a peak value with respect to the magnetic field. In this experiment, the maximum value of FoM was obtained at 55 $\mu T$, and this value is 58 times better than the conventional one.

V. Conclusion

We developed a new method based on crossed polarizers for observing high-contrast CPT resonance. Firstly, we calculated the Faraday rotation in CPT under a linear light field by using two pairs of $\Lambda$ system models based on the density matrix. The calculated results indicated that the resonance amplitude has a peak value with respect to the magnetic field, and the resonance linewidth increases with increasing magnetic field. The minimum linewidth obtained with the crossed polarizers method is 36.8% that of the conventional method in the absence of the magnetic field. Secondly, we measured the resonance characteristics with crossed polarizers method using a $^{133}$Cs gas cell and the $D_1$-line VCSEL. It was shown that the background signal level of the photodetector is suppressed by the crossed polarizers method and a high contrast resonance can be obtained. As the relative angle $\theta$ and magnetic field were optimized, the observed resonance had a contrast of 88.4% and linewidth of 1.15 kHz. The measurement data was in good agreement with the theoretical data and the difference between experiment and theory was no more than 20%. Finally, we investigated the optimal conditions for enhancing short-term stability. The figure of merit has a peak value with respect to magnetic field. By optimizing the relative angle $\theta$ of the analyzer and magnetic field, the figure of merit was 58 times better than the conventional one. Since a high contrast and narrow linewidth resonance can be obtained with such a simple optical system, the crossed polarizers method is an attractive means of enhancing the frequency stability of CPT atomic clocks.

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