Spin Alignment in the Production of Vector Mesons with Charm and/or Beauty via Heavy Quark Fragmentation

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Abstract

We calculate the process-independent fragmentation functions for a $\bar{b}$ antiquark to fragment into longitudinally and transversely polarized $B^*_c\ ({}^3S_1)$ mesons to leading order in the QCD strong coupling constant. In the special case of equal quark mass we recover previous results for the fragmentation of $c \rightarrow \psi$ and $b \rightarrow \Upsilon$. Various spin asymmetry parameters are defined as measures of the relative population of the longitudinally and transversely polarized vector meson states. In the heavy quark mass limit $m_b \rightarrow \infty$ our polarized fragmentation functions obey heavy quark spin symmetry, we therefore apply them as a model to describe the fragmentation of charm and bottom into heavy-light mesons like $D^*$ and $B^*$. The spin asymmetry parameter, $\alpha(z)$, is consistent with the existing CLEO data for $D^*$. The scaling behavior of $\langle z \rangle$ is studied in detail. We find excellent agreement between the predictions of $\langle z \rangle$ from our fragmentation functions and the experimental data for $D^*$ and $B^*$ from the LEP, CLEO, and ARGUS detectors. Finally, we also point out that the spin asymmetry depends significantly on the transverse momentum $p_\perp$ of the vector mesons relative to the fragmentation axis.
I. Introduction

The physics of hadrons containing a single heavy quark has been studied intensively in the past several years, mainly due to the development of the powerful technique of Heavy Quark Effective Theory (HQET) [1, 2, 3]. One crucial observation in HQET is that the heavy quark spin decouples from the strong interaction dynamics in the limit of infinite heavy quark mass. This happens because the leading operator, the chromo-magnetic dipole moment, that couples the heavy quark spin to the gluon field is inversely proportional to the mass of the heavy quark.

Whenever a heavy quark is produced with high degree of polarization (for example, bottom and charm quarks are produced with 94% and 67% left-handed polarization, respectively, at the $Z^0$ pole), it might be possible to extract the spin information of the heavy quark if the subsequent hadronization does not lead to substantial depolarization. The energy spectrum or the angular distributions of the lepton in the semi-leptonic decays of the top, bottom, or charm quarks are often used as the spin analyzer of the heavy quark spin. Several recent works (Refs. [4, 5, 6, 7, 8]) have been devoted to the polarization effects in the $D^*$ and $D^{**}$ mesons, $B^*$ and $B^{**}$ mesons, and $\Sigma_b$, $\Sigma_b^*$, and $\Lambda_b$ baryons at the LEP energy. In this article, we will investigate in detail the spin alignment of a heavy quark that undergoes fragmentation into the spin triplet S-wave meson state.

Recently, it has been pointed out [9] that the dominant production of $(\bar{b}c)$ mesons at the large transverse momentum region is due to fragmentation, in which a high energy $\bar{b}$ quark is produced from a hard process and subsequently fragments into various $(\bar{b}c)$ meson states. Furthermore, the corresponding process-independent fragmentation functions of $\bar{b} \to B_c(1S_0)$ and $\bar{b} \to B_c^*(3S_1)$ were calculated within perturbative QCD (PQCD) [3, 4] to leading order in both the strong coupling constant $\alpha_s$ and $v$, where $v$ is the typical velocity of the charm quark inside the meson. A lower bound of the inclusive branching fraction for the production of $B_c$ at $Z^0$ has been estimated to be about $2.3 \times 10^{-4}$ [4], including both 1S and 2S states. It implies that only about 230 $B_c^+$ or 230 $B_c^-$ are produced from $10^6 Z^0$. At the Tevatron
with a luminosity of 25 pb$^{-1}$, one expects \cite{11} about $2 \times 10^4 B_c$ mesons to be produced with $p_T(B_c) > 10$ GeV from the direct $\bar{b}$ quark and induced gluon fragmentation. The three-charged leptons from a secondary vertex observed in the decay $B_c^+ \rightarrow J/\psi + \ell' \nu_{\ell'}$ followed by $J/\psi \rightarrow \ell \bar{\ell}$, where $\ell, \ell' = e, \mu$, can provide a clean signature for $B_c$. The combined branching ratio of these decays is expected to be $\sim 0.2\%$, which implies 40 distinct events at the Tevatron. In addition, the $B_c$ meson can be fully reconstructed via hadronic decay modes, e.g., $B_c^+ \rightarrow J/\psi + \pi^+$, with $J/\psi \rightarrow \ell \bar{\ell}$. Thus, unless LEP can increase its luminosity by an order of magnitude or so in the near future, the best place to look for the $B_c$ meson will be at the Tevatron.

In Ref. \cite{12}, the asymmetry due to the relative probabilities for the production of the transversely versus longitudinally polarized $J/\psi$ states by $c$ and $\bar{c}$ quark fragmentation was discussed. As a result of this asymmetry, an anisotropic angular distribution of the leptons in the decay $J/\psi \rightarrow \ell \bar{\ell}$ was found to be of order 5\%, which might not be large enough to be observed due to the presence of another important source of highly polarized $J/\psi$ from $B$-meson decays. Since the heavy quark fragmentation probabilities, first calculated in \cite{13}, are known to have big cancellation in the equal mass quarkonium case, one expects to have larger asymmetry in the unequal mass case like the $B_c^*$ meson system. Such asymmetry can be measured in principle from the anisotropy of the photon angular distribution in the decay $B_c^* \rightarrow B_c + \gamma$. To predict the degree of anisotropy one needs to know the polarized fragmentation functions for the $\bar{b}$ quark splitting into various helicity states of the $B_c^*$ meson.

In this work, we will study the spin alignment in the $B_c^*$ production via the fragmentation of the $\bar{b}$ quark. First, we will calculate the fragmentation functions for $\bar{b} \rightarrow$ polarized $B_c^*$ states, as an extension to the previous calculation of $\bar{b} \rightarrow$ unpolarized $B_c^*$ \cite{4}, or as an unequal mass extension to the calculation of $c$ (or $\bar{c}$) $\rightarrow$ polarized $J/\psi$ \cite{12}. This extension is also useful beyond the $(\bar{b}c)$ system as the final results of the fragmentation functions can be applied phenomenologically to the case of heavy-light mesons such as $D^*$ and $B^*$. This allows us to get a better insight to the spin asymmetry in the $D^*$ and $B^*$ mesons, where
perturbative methods are not applicable.

Spin alignment of the $B_c^*, J/\psi, \Upsilon, D^*$, and $B^*$ systems will be studied. Two spin asymmetry parameters frequently quoted in the literature are

$$
\xi = \frac{T}{L+T} \quad \text{and} \quad \alpha = \frac{2L-T}{T},
$$

where $T(L)$ denotes the production probability of the transverse (longitudinal) state of the excited S-wave meson ($B_c^*, J/\psi, \Upsilon, D^*$, or $B^*$). Here we introduce another parameter

$$
\mathcal{W} = \frac{T}{T+2L}.
$$

Two useful relations of these spin asymmetry parameters are the following

$$
\mathcal{W} = \frac{\xi}{2-\xi}, \quad \alpha = \frac{2-3\xi}{\xi}.
$$

Note that we did not specify the production mechanism in the definitions of these spin asymmetry parameters.

To measure these spin asymmetry parameters, one can study the two body decay of the excited meson, e.g. $B^* \to B\gamma, D^* \to D\pi$, and $B_{c}^{*} \to B_{c}\gamma$ etc. The angular distribution of the emitted photon (or pion) depends on the helicity of the parent meson. For definiteness we will consider the $B_{c}^{*}$ meson in the following. Suppose the $B_{c}^{*}$ meson is polarized in such a way that a fraction $\xi$ is transverse while a fraction $(1-\xi)$ is longitudinal. The transversely polarized component of the $B_{c}^{*}$ gives rise to an angular distribution of $(1+\cos^2 \theta)/4$ weighted by the relative probability $\xi$, while the longitudinal polarized component has an angular distribution of $(1-\cos^2 \theta)/2$ weighted by the relative probability $(1-\xi)$, where $\theta$ is the angle between the outgoing photon 3-momentum and the polarization axis in the $B_{c}^{*}$ rest frame. We define the polarization axis to be the direction of the 3-momentum of the $B_{c}^{*}$ in the laboratory frame. Summing over all the helicity states of the $B_{c}^{*}$, the angular distribution of the emitted photon is given by

$$
\frac{d\Gamma}{d\cos \theta} \sim 1 + \left(\frac{3\xi - 2}{2-\xi}\right)\cos^2 \theta,
$$

5
\[
= 1 - \left( \frac{\alpha}{2 + \alpha} \right) \cos^2 \theta , \tag{5}
\]
\[
= 1 + (2W - 1) \cos^2 \theta . \tag{6}
\]

If the relative probability \(T/L\) equals 2 as suggested by the naive spin counting rule, \(\xi\) equals 2/3 and the resulting decay angular distribution will be flat. In other words, there is no spin alignment of the \(B^*_c\) meson. We will show that the above isotropic scenario is only true in the heavy quark mass limit and this limit is broken by the finite charm and bottom quark masses.

This paper is organized as follows. In Section II, we will derive the polarized fragmentation functions \(D^{L,T}_{b \to B^*_c}(z, s)\) that depend on both the usual fragmentation variable \(z\) and the variable \(s\) which measures the virtuality of the fragmenting \(\bar{b}\) quark. Covariant expressions for the transverse and longitudinal polarization sums for massive spin 1 objects will be derived so that covariant calculation can be performed. In Section III, we will discuss the heavy quark mass limit of \(m_b \gg m_c \gg \Lambda_{QCD}\). We show that our polarized fragmentation functions satisfy heavy quark spin symmetry in this limit, and that heavy quark symmetry breaking arises from the next-to-leading term in the heavy quark mass expansion. In Section IV, we will study the spin asymmetry parameters \(\xi, \alpha, \) and \(W\) in detail. We will point out that the anisotropic angular distribution in the two body decay of the excited meson (\(D^*, B^*,\) or \(B^*_c\)) first arises at the next-to-leading term in the heavy quark mass expansion. We will also introduce the \(z\)-dependence in the spin asymmetry parameters \(\xi(z), \alpha(z),\) and \(W(z),\) and study their variations with \(z\). In Section V, we will discuss the scaling behaviors of the mean longitudinal momentum fraction \(\langle z \rangle\) for the \(B^*_c\) meson, for quarkonia, and for the heavy-light excited mesons. In Section VI, we will present the results of the fragmentation functions that depend on both the variable \(z\) and the transverse momentum \(p_\perp\) of the \(B^*_c\) meson with respect to the fragmentation axis, which is defined as the direction of the 3-momentum of the fragmenting \(\bar{b}\) quark in the laboratory frame. Spin asymmetry parameters depending on \(p_\perp\) are also introduced and studied in detail. We conclude in Section VII.
II. Fragmentation Functions for Polarized $B^*_c$ Meson

The derivation of the polarized fragmentation functions of $\bar{b} \to B^*_c$ follows closely to the unpolarized case [9], but requires separate contributions from the longitudinal and transverse components of the $B^*_c$ meson. For the unpolarized case all the helicity states are summed by the following formula

$$\sum_\lambda \epsilon^\lambda_\mu(p) \epsilon^\lambda_\nu(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2},$$

(7)

where $p$ and $M$ are the momentum and mass of the meson, respectively. Since we are working entirely within the nonrelativistic approximation for the heavy quark bound state, we will take $M = m_b + m_c$. The two transverse helicity states are usually summed by using the following equation

$$\sum_T \epsilon_T^\mu(p) \epsilon_T^\nu(p) = \delta^{ij} - \frac{p^i p^j}{|\vec{p}|^2}.$$  

(8)

This noncovariant expression for summing the transverse polarizations often makes the manipulation unnecessarily cumbersome. Here we present covariant formulas for summing the longitudinal and transverse helicity states of massive spin 1 objects. Recall that the longitudinal polarization 4-vector $\epsilon_L(p)$ is usually written explicitly as

$$\epsilon^\mu_L(p) = \left( \frac{|\vec{p}|}{M}, \frac{E\vec{p}}{M|\vec{p}|} \right),$$

(9)

where $E^2 = M^2 + |\vec{p}|^2$. Let us define an auxiliary 4-vector $n^\mu = (1, -\vec{p}/|\vec{p}|)$ such that $n^2 = 0$ and $n \cdot p = E + |\vec{p}|$. With the help of this auxiliary vector one can rewrite $\epsilon^\mu_L(p)$ in the following covariant form,

$$\epsilon^\mu_L(p) = \frac{p^\mu}{M} - \frac{M n^\mu}{n \cdot p}.$$  

(10)

We can then obtain the following covariant expressions

$$\epsilon^\mu_L(p)\epsilon^\nu_L(p) = \frac{p^\mu p^\nu}{M^2} - \frac{1}{n \cdot p} (p^\mu n^\nu + p^\nu n^\mu) + \frac{M^2}{(n \cdot p)^2} n^\mu n^\nu,$$  

(11)

for the longitudinal polarization sum, and

$$\sum_T \epsilon_T^\mu(p)\epsilon_T^\nu(p) = -g^{\mu\nu} + \frac{1}{n \cdot p} (p^\mu n^\nu + p^\nu n^\mu) - \frac{M^2}{(n \cdot p)^2} n^\mu n^\nu,$$  

(12)
for the transverse polarization sum. In general, these covariant formulas are not only useful in our calculation but also in many other applications.

The initial fragmentation functions $D_{\bar{b} \rightarrow B_c^*(z, \mu_0)}^L$ and $D_{\bar{b} \rightarrow B_c^*(z, \mu_0)}^T$ are obtained by modifying the calculation in Ref. [9] using Eqns. (11) and (12) to separate the contributions from the longitudinal and transverse components of $B_c^*$. The starting point is the following expression [9]

$$D_{\bar{b} \rightarrow B_c^*(z)} = \int ds \theta \left( s - \frac{M^2}{z} - \frac{m_c^2}{1 - z} \right) D_{\bar{b} \rightarrow B_c^*(z, s)}, \quad (13)$$

with

$$D_{\bar{b} \rightarrow B_c^*(z, s)} = \frac{1}{16\pi^2} \lim_{q_0/m_\bar{b} \to \infty} \frac{|\mathcal{M}|^2}{|\mathcal{M}_0|^2}. \quad (14)$$

In Eqn.(14), $\mathcal{M}$ is the amplitude for producing a $B_c^*$ and a $\bar{c}$ quark from an off-shell $\bar{b}$ with virtuality $s = q^2$, where $q$ is the 4-momentum of the $\bar{b}$ quark (the leading contribution is an one-gluon exchange diagram given in Fig. [1]); and $\mathcal{M}_0$ is the amplitude for producing an on-shell $\bar{b}$ with the same 3-momentum $\vec{q}$. If the momentum of the $B_c^*$ is $p^\mu = (p_0, p_1, p_2, p_3)$, in a frame where $q^\mu = (q_0, 0, 0, q_3)$, the longitudinal momentum fraction $z$ is defined by $z = (p_0 + p_3)/(q_0 + q_3)$. The kinematical relation among the three variables $z, s$, and $p_\perp = |\vec{p}_\perp|$, where $\vec{p}_\perp = (p_1, p_2)$ is the transverse momentum of the $B_c^*$ meson with respect to the fragmentation axis, is

$$s = \frac{M^2 + p_\perp^2}{z} + \frac{m_c^2 + p_\perp^2}{1 - z}. \quad (15)$$

Note that the $\theta$ function constraint in Eqn. (13) arises from the positivity of $p_\perp$.

The amplitude $\mathcal{M}$ is evaluated in the axial gauge with an auxiliary 4-vector $n^\mu$ defined above in the covariant polarization sums. In this gauge, the dominant contribution arises from the Feynman diagram depicted in Fig. [1]. Other diagrams are suppressed by powers of $m_{b,c}/q_0$. In other words, factorization is manifest in this gauge [3]. The amplitude $\mathcal{M}$ is given by

$$i\mathcal{M} = \sqrt{\pi} \alpha_s C_F R(0) \frac{\delta_{ij}}{\sqrt{N_c}} \frac{1}{m_c} \frac{\sqrt{M}}{(s - m_b^2)^2} \times \Gamma \left[ 2M(\vec{q} + m_b)\xi^*_\lambda(p) + \frac{s - m_b^2}{n \cdot (q - \bar{r}p)}(\bar{p} + M)\xi^*_\lambda(p)\bar{\eta} \right] v(p'), \quad (16)$$
where \( p \) and \( p' \) are the momentum 4-vector of the outgoing \( B_c^* \) and \( \bar{c} \) respectively, \( \epsilon_\lambda(p) \) is the polarization 4-vector of the \( B_c^* \) meson with helicity \( \lambda \), \( R(0) \) is the radial wavefunction of the \((\bar{b}c)\) bound-state at the origin, \( M = m_b + m_c \) is the mass of the bound state, \( r = m_c/M \), \( \bar{r} = 1 - r \), and \( \Gamma \) is a symbolic Dirac structure representing the source to create the energetic \( \bar{b} \) quark in the hard subprocess. In Eqn. (16), \( C_F = (N_c^2 - 1)/2N_c \), where \( N_c \) is the number of color. \( R(0) \) can be determined by a potential model calculation [14] or extracted from the \( B_c \) decay constant \( f_{B_c} \). The latter can be calculated on a lattice or measured in the future experiments. The relation between \( R(0) \) and \( f_{B_c} \) is given by

\[
|R(0)|^2 = \frac{\pi M_{B_c} f_{B_c}^2}{3}.
\]

Squaring the amplitude \( \mathcal{M} \), summing over the color and spin of the \( \bar{c} \) quark, and using Eqn. (11) to project the longitudinal component of the \( B_c^* \) meson, we get

\[
\sum |\mathcal{M}|^2 = \frac{\pi^2 C_F^2 |R(0)|^2}{4} \frac{M^5}{m_c^2 (s - m_b^2)^4} \text{Tr}(\bar{\Gamma} \Gamma \hat{q}) \times \Delta^L(z, s),
\]

where \( \Delta^L(z, s) \) is given by

\[
\Delta^L(z, s) = \frac{16}{z^2} \left( 1 - 2\bar{r} z + \bar{r}(1 - 2r)z^2 \right) - \frac{4(s - m_b^2)}{z(1 - \bar{r}z)M^2} \left( -4 + 2(3 - 4r)z - (1 - 8r + 4r^2)z^2 - \bar{r}(1 - 2r)z^3 \right) + \frac{4(1 - z)(1 + rz)^2(s - m_b^2)^2}{(1 - \bar{r}z)^2M^4}.
\]

To arrive at this expression we have made the substitutions of \( p = zq \) and \( p' = (1 - z)q \) at the final step, which are accurate to leading order in \( m_{b,c}/q_0 \). In the fragmentation limit of \( q_0/m_b \to \infty \), the tree level amplitude \( \mathcal{M}_0 \) is simply

\[
\sum |\mathcal{M}_0|^2 = N_c \text{Tr}(\bar{\Gamma} \Gamma \hat{q}).
\]

Thus we obtain

\[
D_{b \to B_c^*}^L(z, s) = \frac{3}{2} r N M^6 \frac{\Delta^L(z, s)}{(s - m_b^2)^4},
\]

where we have defined

\[
N = \frac{\alpha_s^2(2m_c)C_F^2 |R(0)|^2}{24N_c \pi m_c^2}.
\]
The scale of the strong coupling constant has been set to be $2m_c$ – the minimal virtuality of the exchange gluon [9]. Doing the $s$ integration, we obtain the longitudinal fragmentation function

$$D_{b\to B_c^*}(z, \mu_0) = N \frac{rz(1-z)^2}{(1-\bar{r}z)^6} \left[ 2 + 2(2r - 3)z + (16r^2 - 10r + 9)z^2 \right.$$

$$- \left. 2\bar{r}(6r^2 - 5r + 4)z^3 + 3\bar{r}^2(2r^2 - 2r + 1)z^4 \right],$$

(22)

where the initial scale $\mu_0$ has been set to be $(m_b + 2m_c)$ – the minimal virtuality of the fragmenting $\bar{b}$ quark [9].

Similarly, one can use Eqn. (12) to project out the transversely polarized $B_c^*$ state and deduce

$$D_{\bar{b}\to B_c^*}(z, s) = \frac{3}{2} \frac{rN M^6 \Delta^T(z, s)}{(s - m_b^2)^4},$$

(23)

with

$$\Delta^T(z, s) = - \frac{16}{z^2} \left( 1 - 2\bar{r}z + \bar{r}(1 + r)z^2 \right)$$

$$+ \frac{8(s - m_b^2)}{z(1-\bar{r}z)M^2} \left( 2 + 2(r - 2)z + (2r + 1)z^2 + \bar{r}z^3 \right)$$

$$+ \frac{8z^2(1-z)(s - m_b^2)^2}{(1-\bar{r}z)^2M^4}. \quad (24)$$

Doing the $s$ integration, we obtain the transverse fragmentation function

$$D_{\bar{b}\to B_c^*}(z, \mu_0) = 2N \frac{rz(1-z)^2}{(1-\bar{r}z)^6} \left[ 2 + 2(2r - 3)z + (16r^2 - 4r + 9)z^2 \right.$$

$$- 2\bar{r}(r + 4)z^3 + 3\bar{r}^2z^4 \right].$$

(25)

At this point, a few cross checks can be made. Adding $D_{b\to B_c^*}(z)$ and $D_{\bar{b}\to B_c^*}(z)$, we reproduce the unpolarized result given in Refs. [9, 10]. By setting $r = 1/2$ in the Eqns. (22) and (25), we reproduce the results of $c \to \text{polarized } J/\psi$ given in Ref. [12]. The results given in Eqns. (22) and (25) also agree with a recent calculation of Ref. [15].

One can easily extend these results to the case where the initial fragmenting $\bar{b}$ quark is also polarized. Let us denote $D_{(b;\lambda)}(z)$ to be the fragmentation function for a heavy quark
Q with helicity $h = \pm 1/2$ to split into a vector meson $V^*$ with helicity $\lambda = 0, \pm$. Thus, by definition, we have

$$D^L(z) = \frac{1}{2} \left( D_{(\frac{1}{2},0)}(z) + D_{(-\frac{1}{2},0)}(z) \right) ,$$  

and

$$D^T(z) = \frac{1}{2} \left( D_{(\frac{1}{2},+)}(z) + D_{(\frac{1}{2},-)}(z) + D_{(-\frac{1}{2},+)}(z) + D_{(-\frac{1}{2},-)}(z) \right) .$$

Parity invariance implies

$$D_{(h;\lambda)}(z) = D_{(-h;-\lambda)}(z) .$$

Therefore, we deduce the following relations

$$D_{(\frac{1}{2},0)}(z) = D_{(-\frac{1}{2},0)}(z) = D^L(z) ,$$  

and

$$D_{(\frac{1}{2},+)}(z) + D_{(\frac{1}{2},-)}(z) = D_{(-\frac{1}{2},+)}(z) + D_{(-\frac{1}{2},-)}(z) = D^T(z) .$$

These relations immediately imply that the polarized heavy quark fragmentation functions $D^L(z)$ and $D^T(z)$ are the same whether the initial heavy quark is polarized or unpolarized.

### III. Heavy Quark Spin Symmetry

Using the technique of HQET $\text{[1, 2, 3]}$, Jaffe and Randall $\text{[16]}$ have recently shown that the fragmentation functions $D_{Q \to H}(z)$ for a heavy quark $Q$ to split into a hadron $H$ with one heavy constituent quark can be expanded as a power series in $r$,

$$D_{Q \to H}(z) = \frac{a(y)}{r} + b(y) + \mathcal{O}(r) ,$$  

where $a(y)$ and $b(y)$ are functions of $y = (1 - \bar{r}z)/rz$ and $\mathcal{O}(r)$ denotes all other terms higher order in $r$. The leading term $a(y)$ is independent of the heavy quark spin and flavor; while the next-to-leading term $b(y)$ and all higher order terms contain heavy quark spin-flavor symmetry breaking effects. One can verify easily that our polarized $B_c^*$ fragmentation
functions can be expressed in this form by carefully expanding the powers of \( r \) and \( (1 - \bar{r}z) \). The results are

\[
D^L_{b \to B^*_c}(z) = \frac{N(y - 1)^2}{y^6} \left[ \frac{1}{r} (8 + 4y + 3y^2) - (8 - 8y + 5y^2 + y^3) + ... \right], \tag{32}
\]

\[
D^T_{b \to B^*_c}(z) = \frac{2N(y - 1)^2}{y^6} \left[ \frac{1}{r} (8 + 4y + 3y^2) - (8 - 8y - y^2 + y^3) + ... \right]. \tag{33}
\]

Obviously, the leading terms of \( D^L(z) \) and \( D^T(z) \) in Eqns. (32) and (33) differ only by a factor of 2 and thus obey heavy quark spin symmetry. The \( \mathcal{O}(r^0) \) terms and beyond in Eqns. (32) and (33) are different due to heavy quark spin-flavor symmetry breaking effects. In fact, one can show that \[17\] the leading order terms in Eqns. (32) and (33) can be derived by using the Feynman rules of the leading operator in the HQET Lagrangian, while the \( \mathcal{O}(r^0) \) pieces arise not only from the next-to-leading \( (1/M) \) operators but also from the small component of the heavy quark spinor in the HQET.

In Fig. 2, we plot the full PQCD fragmentation functions \( D^L_{b \to B^*_c}(z) \) and \( D^T_{b \to B^*_c}(z) \) from Eqns. (22) and (25), and the corresponding heavy quark mass expansion from Eqns. (32) and (33). We take \( m_b = 4.9 \) GeV and \( m_c = 1.5 \) GeV throughout the paper. We employ a simple form of \( \alpha_s \) by evolving from its well measured experimental value at the \( Z^0 \)-mass, namely

\[
\alpha_s(\mu) = \frac{\alpha_s(m_Z)}{1 + (b/2\pi)\alpha_s(m_Z) \log(\mu/m_Z)}, \tag{34}
\]

where \( b = (11N_c - 2n_f)/3 \), \( n_f \) is the number of active flavors at the scale \( \mu \), and \( \alpha_s(m_Z) = 0.12 \). It is clear from Fig. 2 that the sum of the leading and the next-to-leading terms in the heavy quark mass expansion is a very good approximation to the full PQCD result, and the difference is of order \( \mathcal{O}(r) \). We note that the width between the peak and the endpoint of the fragmentation functions scales as \( r \).

Since our polarized fragmentation functions given in Eqns. (22) and (25) are consistent with the general analysis of Jaffe and Randall \[16\], we can apply them to describe the heavy-light mesons as well. The formulas given in Eqns. (22) and (25) can be regarded as phenomenological fragmentation functions with two free parameters \( N \) and \( r \), to describe
the nonperturbative process of a heavy quark splitting into a polarized heavy-light excited meson, such as \( c \rightarrow \text{polarized } D^* \) and \( b \rightarrow \text{polarized } B^* \). The parameter \( N \) can be adjusted to describe the overall normalization and \( r \) is the mass ratio of the light constituent quark mass to the meson mass. Although in the \( D^* \) and \( B^* \) systems there are probably large nonperturbative and relativistic effects that we have not taken into account, our perturbative QCD fragmentation functions with the free parameters \( N \) and \( r \) can at least provide some insights to these systems while precise nonperturbative fragmentation functions for \( c \rightarrow D^* \) and \( b \rightarrow B^* \) are not available yet. Our PQCD fragmentation functions only depend on two free parameters as does the phenomenological Peterson fragmentation function [18], which is widely used in the literature to describe \( c \rightarrow D^* \) and \( b \rightarrow B^* \). However, our fragmentation functions also carry spin informations. We expect our functions should be equally successful as phenomenological description of fragmentation, but they are more predictive since they also contain spin informations. They also have an additional virtue of being rigorously correct in some limit, namely, the limit in which the lighter quark mass is much greater than \( \Lambda_{\text{QCD}} \).

Finally, we note that the 2-to-1 spin counting ratio for the transversely and longitudinally polarized states is only true for the S-wave excited states in the heavy quark limit. For the P-wave excited states, this statement is incorrect [7].

IV. Spin Alignment in Heavy Quark Fragmentation

In this Section, we will study the spin alignment of the excited S-wave heavy meson (\( J/\psi \), \( \Upsilon \), \( B_c^* \), \( D^* \), or \( B^* \)) produced by heavy quark fragmentation. To begin with, we integrate \( D^{L,T}(z, \mu_0) \) over \( z \) to get the total fragmentation probabilities, or equivalently the first moments \( D^{L,T}(1) \) of the corresponding fragmentation functions:

\[
D^L(1) = \int_0^1 dz D^L(z, \mu_0) = N \left[ \frac{24 + 89r - 486r^2 + 354r^3 + 289r^4}{15r^5} \right]
\]
\begin{align*}
&+ r(7 - 16r - 9r^2 + 30r^3 + 6r^4) \log(r) \right], \\
D^T(1) &= \int_0^1 dz D^T(z, \mu_0) \\
&= 2N \left[ \frac{24 + 119r + 54r^2 + 84r^3 - 11r^4}{15r^5} + \frac{7 + 2r + 9r^2}{r^6} \log(r) \right],
\end{align*}

(35)

where $r$ is defined as the mass ratio $m_{\text{light quark}}/m_{\text{meson}},$ just the same way as we defined for the $B^*_c$ meson. From now on we will omit the subscript $\bar{b} \to B^*_c$ in the fragmentation function and understood that it can refer to either $B^*_c,$ $J/\psi,$ $\Upsilon,$ or the heavy-light mesons $D^*$ and $B^*.$ It is well known that the first moment of fragmentation function has zero anomalous dimension and hence it does not evolve with the scale. We therefore drop the $\mu$ dependence in the first moments $D^{L,T}(1)$.

Since the dominant production mechanism of S-wave excited meson states at the large transverse momentum region is due to fragmentation, we can identify the quantities $L$ and $T,$ defined in Section I, to be the first moments $D^L(1)$ and $D^T(1),$ respectively. The various spin asymmetry parameters introduced in Eqns. (1) - (2) can be expressed as ratios of the first moments of the fragmentation functions as follows:

\begin{align*}
\xi &= \frac{D^T(1)}{D^T(1) + D^L(1)}, \\
\alpha &= \frac{2D^L(1) - D^T(1)}{D^T(1)}, \\
\mathcal{W} &= \frac{D^T(1)}{D^T(1) + 2D^L(1)}. 
\end{align*}

(37)

(38)

(39)

Note that these spin asymmetry parameters depend only on the parameter $r$ and do not depend upon the overall constant $N$ and the evolution scale $\mu.$

Before we proceed further, it is instructive to repeat here the usual arguments of how the heavy quark spin information is lost during hadronization of the heavy quark into a S-wave heavy-light meson. Suppose a spin down heavy quark $Q^\downarrow$ combines with a spin up or spin down light anti-quark $\bar{q}$ forming the state $Q^\downarrow \bar{q}^\uparrow$ or $Q^\downarrow \bar{q}^\downarrow.$ Since parity is conserved in the fragmentation process, these two states must occur with equal probability. While $Q^\downarrow \bar{q}^\downarrow$ is an eigenstate of the total spin $S = 1,$ $Q^\downarrow \bar{q}^\uparrow$ is a mixture of the spin states $S = 0$ and $S = 1.$
One can then decompose the state $Q_1 \bar{q}^\uparrow$ into a sum of eigenstates of the total spin $S = 0$ and $S = 1$:  
\[ Q_1 \bar{q}^\uparrow = \frac{1}{\sqrt{2}} \left( Q_1 \bar{q}^\uparrow - Q_1 \bar{q}^\downarrow \right) + \frac{1}{\sqrt{2}} \left( Q_1 \bar{q}^\uparrow + Q_1 \bar{q}^\downarrow \right). \]  
(40)

These $S = 0$ and $S = 1$ components are identified as the pseudoscalar $P$ and vector meson $V^*$, respectively. In the limit of $m_Q \to \infty$, these two states $P$ and $V^*$ are degenerate. They will then have the same time evolution and will propagate coherently. The spin wavefunction will remain the same as $Q_1 \bar{q}^\uparrow$ and the coherent superposition of the two meson states will preserve the heavy quark spin forever. In reality, $m_Q \neq \infty$; the pseudoscalar $P$ and vector meson $V^*$ have a slight mass difference $\Delta M$ and hence different time evolutions. In most cases, like the heavy-light mesons and the $(\bar{b}c)$ mesons, the finite mass difference $\Delta M$ is considerably larger than the decay rate $\Gamma_{V^*}$ of $V^* \to P + X$, where $X$ denotes a photon or a pion. At a time $t \sim \Delta M^{-1}$, the $S = 0$ and $S = 1$ components become completely out of phase and incoherent before any decay actually occurs. Thus the heavy quark spin is depolarized over a period of time characterized by the chromo-magnetic dipole moment, which is also responsible to the finite mass difference $\Delta M$. At a later time $t \sim \Gamma_{V^*}^{-1}$, the vector meson $V^*$ decays into the pseudoscalar $P$. The spin information of the heavy quark is carried in the relative population of the helicity $+1$ and helicity $-1$ components of the vector meson. If the decay proceeds through electromagnetic or strong interactions, the parity invariance combined with rotational symmetry implies that the spin information is lost in the angular distribution of the final state particles. Nevertheless, the angular distribution carries important information about the fragmentation process through the spin alignment variables, which measure the relative population of the helicity 0 and helicity $\pm 1$ components of the vector meson.

In the heavy quark mass limit of $m_b \gg m_c \gg \Lambda_{QCD}$, i.e. $r \to 0$, the spin asymmetry parameters obtained in Eqns.(37)-(39) take the following simple forms
\[ \xi - \frac{2}{3} = \frac{5}{18} r + \mathcal{O}(r^2), \]  
(41)
\[ \alpha = \frac{5}{4} r + \mathcal{O}(r^2) , \quad (42) \]
\[ 2W - 1 = \frac{5}{8} r + \mathcal{O}(r^2) . \quad (43) \]

Note that, to order \( r \), the subleading terms \( \mathcal{O}(r \log r) \) in Eqns. (35) and (36) cancel in these quantities. These limits make it clear that the angular distribution given in Eqns. (4)–(6) is isotropic to leading order in \( r \). This result is in accord with the recent general analysis of Falk and Peskin [7] using heavy quark symmetry. As a matter of fact, in the heavy quark limit the spin asymmetry parameter \( W \) coincides with the Falk-Peskin variable \( w_{1/2} \), which is the conditional probability for a heavy quark fragmenting into a heavy-light meson with a spin \( \frac{1}{2} \) light degree of freedom to be in the helicity states \( h_l = +\frac{1}{2} \) or \( -\frac{1}{2} \). Light degree of freedom denotes collectively the light quark plus all the soft gluons that combine with the static heavy quark to form the color singlet heavy-light system. Parity invariance implies these two helicity configurations \( (h_l = \pm \frac{1}{2}) \) of the spin \( \frac{1}{2} \) light degree of freedom must occur with equal probability [7]. Thus \( 2W \to w_{1/2} = 1 \) in the heavy quark limit. Consequently, the anisotropy of the decay product of the excited S-wave heavy-light meson arises entirely from the spin-flavor symmetry breaking effects of the heavy quark. Our results of the polarized fragmentation functions and the spin asymmetry parameters obtained above allow us to study these symmetry breaking effects as a function of \( r \).

We plot these three spin asymmetry parameters versus \( r \) in Fig. 3(a), (b), and (c) respectively. The anisotropy of 5.7\% [12] in the decay distribution of \( J/\psi \to \ell \bar{\ell} \), where \( \ell = e^- \) or \( \mu^- \), can be immediately evaluated from the values of \( \xi, \alpha, \) or \( W \) at \( r = 1/2 \) in Figs. 3. It means that the decay distribution behaves like \( (1 + 0.057 \cos^2 \theta) \), where \( \theta \) is the angle between the outgoing lepton and the polarization axis of the \( J/\psi \). In principle, a 5.7\% asymmetry is measurable, but it is seriously contaminated by the \( J/\psi \)'s coming from \( B \) decays. This anisotropic distribution is also true for \( \Upsilon \to \ell \bar{\ell} \). Surprisingly, the anisotropy in \( B_c^* \to B_c \gamma \) is only \( \sim 5.8\% \), almost the same as in the equal mass quarkonium case. But it is relatively clean because the production rates for the \( 2S, 2P, 3P, \) and \( 3D (\bar{b}c) \) states which
can contaminate the direct $\bar{b} \to B_c^*$ fragmentation by their hadronic cascades or radiative decays into $B_c^*$ are in general small. The production rates for the $2S$ ($\bar{b}c$) states are about 60% of the corresponding $1S$ states \[9\]. The fragmentation probabilities for $\bar{b} \to P$-wave ($\bar{b}c$) states are only about 10% of the S-wave case \[19\]. While the fragmentation functions for $\bar{b} \to D$-wave ($\bar{b}c$) states are not known, they are not expected to be large. In order to measure the asymmetry, one still has to disentangle the issue of whether the $B_c^*$ is coming via direct $\bar{b}$ fragmentation or cascade from higher excitations. The anisotropies in the $D^*$ and $B^*$ systems are 5.1% and 2.6% respectively.

Since the $z$-integrated spin asymmetry parameters in general imply small anisotropies, one might try to get more sensitivity by studying the $z$-dependent spin asymmetry parameters defined by:

$$\xi(z, \mu) = \frac{D^T(z, \mu)}{D^T(z, \mu) + D^L(z, \mu)}, \quad (44)$$

$$\alpha(z, \mu) = \frac{2D^L(z, \mu) - D^T(z, \mu)}{D^T(z, \mu)}, \quad (45)$$

$$\mathcal{W}(z, \mu) = \frac{D^T(z, \mu)}{D^T(z, \mu) + 2D^L(z, \mu)}. \quad (46)$$

These $z$-dependent spin asymmetry parameters are necessarily $\mu$ dependent because the fragmentation functions depend on the scale $\mu$. They are shown respectively in Fig. 4(a), (b), and (c) at the corresponding initial scale $\mu_0$ and $r$ for $J/\psi(r = 0.5)$, $B_c^*(r = 0.23)$, $D^*(r = 0.17)$, and $B^*(r = 0.058)$. We have chosen $\mu_0$ as the sum of the heavy constituent quark mass and double the light constituent quark mass for each bound state, and $r$ to be the ratio of the light constituent quark mass to the meson mass. We also take the light $u$ or $d$ constituent quark masses in the $B^*$ and $D^*$ mesons to be 0.3 GeV. The dependence of $\xi(z)$, $\alpha(z)$, and $\mathcal{W}(z)$ on $z$ in Fig. 4 shows that the maximum asymmetry occurs at $z$ around 0.6 – 0.8. We note that despite having a different initial scale $\mu_0$, the spin asymmetry parameters for the $Y$ are the same as those for the $J/\psi$.

One might worry about how the scale $\mu$ affects the shapes of $\xi(z, \mu)$, $\alpha(z, \mu)$, and $\mathcal{W}(z, \mu)$. Here we remind the readers that the $z$-integrated $\xi$, $\alpha$, and $\mathcal{W}$ are independent of the scale.
We can evolve the polarized fragmentation functions by solving the Altarelli-Parisi evolution equation \[20\]
\[\mu \frac{\partial}{\partial \mu} D_{L,T}^L(z, \mu) = \int_z^1 \frac{dy}{y} P_{qq}(\frac{z}{y}) D_{L,T}^L(y, \mu), \tag{47}\]
where \(P_{qq}(x) = \frac{\alpha_s(\mu)}{\pi} C_F [(1 + x^2)/(1 - x)]_+\) is the usual quark-quark Altarelli-Parisi splitting function, and the plus distribution is defined by \(f(x)_+ = f(x) - \delta(1-x) \int_0^1 dx' f(x')\). When the initial heavy quark is polarized, we should use the polarized Altarelli-Parisi splitting function \(\Delta P_{qq}(x)\), which happens to be the same as \(P_{qq}(x)\), in Eqn. (47). However, in the small \(z\) region, the Altarelli-Parisi evolution equation does not handle the threshold effect properly \[21\]. Unphysical behaviors occur at the small \(z\) region for the fragmentation functions and hence for the spin asymmetry parameters, as one evolves the scale up to, say, \(\mu = m_Z/2\). Therefore, we can only discuss the evolution behaviors at the large \(z\) region with confidence.

We found that in the large \(z\) region the curves for the spin asymmetry parameters at the scale \(\mu = m_Z/2\) do not differ significantly from those given in Fig. 4, both in shape and magnitude. In the future when detailed fragmentation data for the polarized \(D^*\) and \(B^*\) become available at LEP, it will be very interesting to compare our theoretical predictions of the spin asymmetry parameters in the large \(z\) region given in Fig. 4 to the experimental results.

A recent set of experimental data on the spin alignment of \(D^*\) meson was from the CLEO detector operating at a center-of-mass energy of 10.5 GeV \[22\]. The data set given in Table I of Ref. \[22\] is for the spin asymmetry parameter \(\alpha(z)\). The scale of the data set is taken to be half of the center-of-mass energy, i.e., 5.25 GeV, which is not very far from the \(\mu_0 = 2.1\) GeV of the \(D^*\) meson. We therefore ignore the evolution effects and directly compare the \(D^*\) curve in Fig. 4(b) with the data from the CLEO measurements. Unfortunately, since the experimental data for \(\alpha(z)\) had very large error bars, we have to show the comparison on another graph (Fig. 5) with a larger vertical scale.

While our model always predicts a slightly negative value of \(\alpha(z)\) for all values of \(z\), the CLEO data are rather scattered with statistical tendency toward the positive side, as
indicated by the mean value $\langle \alpha(z) \rangle = 0.08 \pm 0.07 \text{(stat)} \pm 0.04 \text{(sys)}$. As shown in Fig. 5, the agreement is good because our curve $\alpha(z)$ is within 2$\sigma$ of all the data points. In the CLEO analysis, they also concluded that the data was marginally consistent with zero.

In this Section, we have shown that both the $z$-integrated and the $z$-dependent spin asymmetry parameters indicate a small misalignment in the two polarization states of the excited S-wave heavy mesons produced by heavy quark fragmentation. As indicated by the parameter $\alpha(z)$, one predicts that the transverse states produced by heavy quark fragmentation should be populated slightly more than would be given by naive spin counting over the entire physical region of $z$.

V. Mean Longitudinal Momentum Fraction

Another useful observable, the mean longitudinal momentum fraction $\langle z \rangle$, of the $B_c^*$ (or the heavy-light mesons $D^*$ and $B^*$, or the heavy quarkonia) at the scale $\mu$ is defined as

$$\langle z \rangle = \frac{\int_0^1 dz z D(z, \mu)}{\int_0^1 dz D(z, \mu)}.$$  \hspace{1cm} (48)

In other words, $\langle z \rangle$ is the ratio of the second to the first moment of the fragmentation function at the scale $\mu$. Since the anomalous dimensions of all the moments of the fragmentation function are known explicitly, the scaling behavior of $\langle z \rangle$ can be determined as

$$\langle z \rangle = \frac{D(2, \mu)}{D(1)} = \langle z \rangle_0 \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{2\gamma_b / b},$$ \hspace{1cm} (49)

where $\langle z \rangle_0 = D(2, \mu_0)/D(1)$, $\gamma = -4C_F/3$, and $b = (11N_c - 2n_f)/3$. The first moments $D_L^L(1)$ of the longitudinally and transversely polarized fragmentation functions are given in Eqns. (53) - (54). The second moments $D_L^L(2, \mu_0)$ at the scale $\mu_0$ are given by

$$D_L^L(2, \mu_0) = \int_0^1 dz z D_L^L(z, \mu_0)$$

$$= 2N \left[ \frac{12 + 97r - 453r^2 + 252r^3 + 437r^4 + 15r^5}{15r^6} \right.$$

$$\left. + \frac{r(5 - 11r - 15r^2 + 33r^3 + 12r^4)}{r^7}\log(r) \right],$$ \hspace{1cm} (50)
\[ D^T(2, \mu_0) = \int_0^1 d\frac{dz}{z}D^T(z, \mu_0) \]
\[ = 4N \left[ \frac{12 + 112r - 33r^2 + 252r^3 + 17r^4}{15r^6} + \frac{r(5 + r + 9r^2 + 9r^3)}{r^7} \log(r) \right] . \]

(51)

In Fig. 6, we plot \( \langle z \rangle^{L,T} \) versus the scale \( \mu \) for the four meson systems \( J/\psi (r = 0.5), B^*_c (r = 0.23), D^*(r = 0.17), \) and \( B^*(r = 0.058) \) with the same input parameters defined in the previous Section. Due to slightly different value of \( \alpha_s(\mu_0) \), the corresponding curves of \( \langle z \rangle^{L,T} \) for the \( \Upsilon (r = 0.5) \) are slightly different from those of the \( J/\psi (r = 0.5) \). We will not present the \( \Upsilon \) curves here.

As the scale \( \mu \) increases across each heavy quark threshold \( (2m_c \text{ and } 2m_b) \), the number \( n_f \) of active flavors increases by one unit. The kinks on the curves at \( \mu = 2m_c \) and \( 2m_b \) in Fig. 6 are due to these threshold effects. Notice that only slightly noticeable differences between the longitudinal \( \langle z \rangle^L \) and transverse \( \langle z \rangle^T \) occur for the \( J/\psi \) and \( B^*_c \), but no differences can be seen for the \( D^* \) and \( B^* \) mesons. Hence, experimentally using \( \langle z \rangle^{L,T} \) to distinguish the polarizability in heavy quark fragmentation into the S-wave excited meson states is not plausible. We list the \( \langle z \rangle_0 \) at \( \mu = \mu_0 \) and \( \langle z \rangle \) at \( \mu = m_Z/2 \) for the four different mesons in Table 1. Numerically, there are no noticeable difference between the longitudinal and transverse polarizations for all the four meson systems. Therefore, to a good approximation, one can set these polarized \( \langle z \rangle^{L,T} \) values for each meson to be the unpolarized \( \langle z \rangle \) value of the corresponding meson. Experimentally, unpolarized \( \langle z \rangle \) are available from the LEP, CLEO, and ARGUS data. The measured quantities are \( \langle x_E \rangle_{c \rightarrow D^*}, \langle x_E \rangle_{c \rightarrow H_c}, \) and \( \langle x_E \rangle_{b \rightarrow H_b} \), where \( x_E \) is the energy fraction of the hadron or meson relative to one half of the center-of-mass energy of the machines, and \( H_c \) and \( H_b \) denote the charm and bottom hadrons, respectively. Because \( x_E \) is a good approximation to the fragmentation variable \( z \), we simply treat them to be the same in the following comparisons. Since the data for \( c \rightarrow D^* \) is available separately, we will use \( \langle x_E \rangle_{c \rightarrow D^*} \) instead of the inclusive \( \langle x_E \rangle_{c \rightarrow H_c} \). On the other hand, only the inclusive \( \langle x_E \rangle_{b \rightarrow H_b} \) has been reported at the LEP. Nevertheless, this inclusive value of \( \langle x_E \rangle_{b \rightarrow H_b} \) should
be close to the $\langle x_E \rangle_{b \to B^\ast}$, since $b \to B^\ast + X$ is expected to be the dominant fragmentation mode of the $b$-quark. In addition, we take the scale of the various measurements to be one half of the center-of-mass energies of the machines.

Next we will describe briefly how we obtain the average from the LEP, CLEO, and ARGUS data. For the LEP data measured values of $\langle x_E \rangle_{c \to D^\ast}$ are from OPAL (0.52±0.0316), ALEPH (0.504 ± 0.0188), and DELPHI (0.487 ± 0.0158) [23], in which we have already combined their systematic and statistical errors in quadrature if they are given separately. We then simply take the mean of the central values from the three experiments to be the average central value. For the combined error we add the absolute errors from the three experiments in quadrature and divide it by 3. We thus obtain $\langle x_E \rangle_{c \to D^\ast} = 0.504 \pm 0.0133$ by combining all three LEP data. Similarly, we have measured values of $\langle x_E \rangle_{b \to H_b}$ from OPAL (0.726 ± 0.023), ALEPH (0.67 ± 0.050), DELPHI (0.695 ± 0.0326), and L3 (0.686 ± 0.017) [24]. Repeating the same exercise, we obtain $\langle x_E \rangle_{b \to H_b} = 0.694 \pm 0.0166$.

The data from CLEO [25] and ARGUS [26] were given in form of fragmentation functions. We need to calculate $\langle x_E \rangle_{c \to D^\ast}$ from their fragmentation data. In the CLEO paper, the fragmentation function of $c \to D^\ast$ were given in Table I (a) and (b) of Ref. [25], which correspond to two different detection channels. We combine the two tables with the $y$-value of each bin equal to the mean of the two, and the error of the $y$-value of each bin equal to one half of the two errors added in quadrature. Then the value $\langle x_E \rangle$ is obtained by evaluating the two integrals in the ratio $\int_{0.37}^{1} d x_E x_E D(x_E) / \int_{0.37}^{1} d x E D(x_E)$ by the method of discrete sums. Assuming the error is only in the $y$-value of each bin, we obtain $\langle x_E \rangle_{c \to D^\ast} = 0.654 \pm 0.0563$, where the error is obtained by adding the errors from each bin in quadrature. Similarly, from the ARGUS data we obtain $\langle x_E \rangle_{c \to D^\ast} = 0.642 \pm 0.067$. Finally, we combine the two averages from CLEO and ARGUS and obtain $\langle x_E \rangle_{c \to D^\ast} = 0.648 \pm 0.0438$. Since the CLEO and ARGUS operating center-of-mass energies were so close to each other (10.55 GeV for CLEO and 10.6 GeV for ARGUS), we assume the scale of both measurements to be 5.3 GeV.

The LEP average for $\langle x_E \rangle_{c \to D^\ast}$ and $\langle x_E \rangle_{b \to H_b}$ at $\mu = m_Z/2$, as well as the combined
CLEO and ARGUS average for $\langle x_E \rangle_{c \rightarrow D^*}$ at $\mu = 5.3$ GeV are shown in Fig. 3. Excellent agreement between our predictions and the data is demonstrated. Here we remind the readers that for the $B^*$ and $D^*$ mesons, we have assumed a nonrelativistic bound state picture with the light constituent quark masses being set to be 0.3 GeV. Uncertainties arising from the overall normalization of the fragmentation functions are cancelled, as indicated by Eqn. (48). Therefore, the uncertainties in the heavy-light meson systems come only from how we define the strong coupling constant $\alpha_s$, the initial scale $\mu_0$ for the fragmentation functions, and what values we choose for the light constituent quark masses. Summing up, the mean longitudinal momentum fractions $\langle z \rangle^{L,T}$ for a heavy quark to fragment into polarized vector mesons do not show any measurable difference between the longitudinal and transverse components. Nevertheless, the predictions of $\langle z \rangle$ by our fragmentation functions for $D^*$ and $B^*$ mesons at different scales are in excellent agreement with the measured data from the LEP, CLEO, and ARGUS detectors.

VI. Transverse Momentum $p_\perp$ Dependence

Sections III and IV showed that the ratio of longitudinal-to-transverse populations from $\bar{b}$-quark fragmentation into polarized $B_c^*$ mesons is only marginally different from that given by the naive spin counting (i.e. the heavy quark mass limit), as indicated by both the $z$-integrated and $z$-dependent spin asymmetry parameters (see Figs. 3 and 4). Similar conclusions hold for the other mesons $J/\psi$, $\Upsilon$, $D^*$, and $B^*$. As a consequence, the two body decays of the excited S-wave mesons into the corresponding pseudoscalar ground states with emission of photon or pion are almost isotropic. In addition, we showed in Section V that it is not feasible to use the mean longitudinal momentum fractions $\langle z \rangle^{L,T}$ to distinguish the longitudinal and transverse polarizations of the excited S-wave mesons produced by fragmentation of heavy quarks (see Fig. 5). Hence, information about the spin of a heavy quark is not easy to extract from the fragmentation data of the heavy quark into S-wave excited mesons [27].
So far, we have only investigated the $z$-dependence of the fragmentation functions and of the spin asymmetry parameters. All the dependence on the motion of the meson perpendicular to the fragmentation axis has been integrated out. In this Section we will investigate the dependence of the fragmentation functions on the $p_{\perp} = |\vec{p}_{\perp}|$, where $\vec{p}_{\perp} = (p_1, p_2)$ is the transverse momentum of the meson with respect to the fragmentation axis. Recall that from Eqn. (15), we have

$$p_{\perp}^2 = z(1-z)\left(s - \frac{M^2}{z} - \frac{m_r^2}{1-z}\right).$$

(52)

Introducing the dimensionless variable $t = p_{\perp}/M$ and trading the variable $s$ to $t$ from Eqn.(52), we can define the fragmentation functions $D(z,t)$ and $D(t)$ according to

$$\int_0^\infty dt D(t) = \int_0^1 dz \int_0^\infty dt D(z,t),$$

$$= \int_0^1 dz \int ds \theta \left(s - \frac{M^2}{z} - \frac{m_r^2}{1-z}\right) D(z,s).$$

(53)

This implies

$$D(z,t) = \frac{2M^2t}{z(1-z)} D(z,s), \quad \text{with} \quad s = M^2\left[\frac{1+t^2}{z} + \frac{r^2 + t^2}{1-z}\right].$$

(54)

To our knowledge, QCD evolution equation for the fragmentation function $D(z,t)$ that depends on both the longitudinal momentum fraction $z$ and the rescaled transverse momentum $t$ has not been written down. But formalism, like those in Refs. [28], which dealt with similar issue in the parton distribution functions, may apply for fragmentation functions as well. For the following we will ignore the issue of QCD evolution effects in the fragmentation functions with explicit $t$-dependence.

Integrating over $z$ in Eqn. (54), we obtain the $t$-dependent polarized fragmentation functions $D^{L,T}(t)$:

$$D^L(t,\mu_0) = \frac{Nr}{2r^6} \frac{1}{t^6(1+t^2)^2} \left\{24r^2t^7(1+t^2)^2 \log(r) + 12t(1+t^2) \left[4r^3 - r(2 + r + 2r^2)t^2 + r^2t^6\right] \log\left(\frac{1 + \frac{t^2}{r^2}}{1 + t^2}\right)\right\}$$

23
\[
D^T(t, \mu_0) = \frac{N_r}{t^6} \left\{ \frac{1}{(1 + t^2)^2} \left[ 12rt(1 + t^2)^2 \left[ 4r^2 - (2 + 3r)t^2 \right] \log \left( \frac{1 + t^2}{1 + t^2} \right) \right. \right. \\
+ \left. \left. 3(1 + t^2)^2 \left[ 10r^4 - r^2(33 + 23r - 11r^2)t^2 + (3 + 9r + 3r^2 + 5r^3)t^4 \right. \right. \\
\left. \left. + (2 + r)t^6 \right] \arctan \left( \frac{\bar{r}t}{r + t^2} \right) \right. \right. \\
- \left. \left. \bar{r}t \left[ 30r^3 - r(61 + 31r - 83r^2)t^2 + (5 - 122r - 53r^2 + 71r^3)t^4 \right. \right. \\
\left. \left. + (13 - 64r - 16r^2 + 16r^3)t^6 + 3(2 + r)t^8 \right] \right\} . \tag{56}
\]

As a cross check, one can integrate \( D^{L,T}(t, \mu_0) \) over \( t \) from 0 to \( \infty \), and get back the total fragmentation probabilities given in Eqns. \((33)\) and \((36)\).

A couple of asymptotic behaviors of \( D^{L,T}(t, \mu_0) \) are in order. As \( t \to 0 \), \( D^{L,T}(t, \mu_0) \) vanishes linearly in \( t \):

\[
D^L(t, \mu_0) \to \frac{2Nrt}{\bar{r}^4} \left[ 6 \log(r) + \frac{\bar{r}}{35r^3}(8 - 46r + 160r^2 + 101r^3 - 13r^4) \right], \tag{57}
\]

\[
D^T(t, \mu_0) \to \frac{8(4 + 3r)Nt}{35r^2}. \tag{58}
\]

As \( t \to \infty \), \( D^{L,T}(t, \mu_0) \) falls off like \( 1/t^3 \) according to

\[
D^L(t, \mu_0) \to \frac{N_r}{\bar{r}^6 t^3} \left[ 12r(2 + r + 2r^2) \log(r) + \bar{r}(1 + r)(6 + 20r + r^2 + 4r^3 - r^4) \right], \tag{59}
\]

\[
D^T(t, \mu_0) \to \frac{2N_r}{\bar{r}^6 t^3} \left[ 12r(2 + 3r) \log(r) + \bar{r}(3 + 47r + 11r^2 - r^3) \right]. \tag{60}
\]

The dependence of the fragmentation functions \( D^{L,T}_{b \to B_c^*}(t, \mu_0) \) on \( t \) at the initial scale \( \mu_0 \) are shown in Fig. 7. Both functions peak at \( t \approx 0.2 \), i.e., \( p_\perp \approx 1.3 \) GeV for \( B_c^* \) meson.
At large $p_\perp$, one sees that the transverse $D^T(t)$ falls off more rapidly than the longitudinal $D^L(t)$. Explicitly, the curves for the longitudinal and transverse polarizations cross over at $t \approx 1.5$, i.e., $p_\perp \approx 10$ GeV for $B_c^*$ mesons. This is a very clean sign of difference between the longitudinally and transversely polarized states of $B_c^*$. Overall, the transverse states are populated about twice as much as the longitudinal one, but the longitudinal component becomes dominant beyond the cross-over. Unfortunately, it might be hard to observe the cross-over experimentally because the cross-over lies well beyond their peak values. Both the longitudinal and transverse spectra have dropped substantially by the time they reach the cross-over, which implies very small cross sections for $p_\perp > 10$ GeV. In addition, there are large uncertainties in determining the fragmentation axis of a jet and consequently also the value of $p_\perp$ for the $B_c^*$. Nevertheless, in principle, imposing a high $p_\perp$ cut can eliminate a large sample of the transverse population of the $B_c^*$.

We can also define the average transverse momentum $\langle p_\perp \rangle$ by

$$\frac{\langle p_\perp \rangle}{M} = \langle t \rangle = \frac{\int_0^\infty dt t D(t)}{\int_0^\infty dt D(t)} .$$

(61)

Numerically, the average $\langle t \rangle$ for the longitudinal and transverse $B_c^*$ are 1.1 and 0.61 respectively, and so the corresponding average $\langle p_\perp \rangle$ are 7.0 GeV and 3.9 GeV, respectively. Unlike $\langle z \rangle$, there is a big difference in $\langle p_\perp \rangle$ between the longitudinal and transverse states of $B_c^*$. The cross-over, and the average $\langle t \rangle$ and $\langle p_\perp \rangle$ for the $B_c^*$, $J/\psi$, $D^*$, and $B^*$ are all summarized in Table 2. Entries for $\Upsilon$ are the same as those for $J/\psi$, except that the average $p_\perp$ for $\Upsilon$ is larger by a factor of $m_b/m_c$. A significant difference in $\langle t \rangle$ between the longitudinal and transverse states persist in $J/\psi$, $\Upsilon$, $D^*$, and $B^*$ systems.

Despite the difficulties in their experimental measurements, it is instructive to evaluate the $t$-dependent spin asymmetry parameters defined by

$$\xi(t) = \frac{D^T(t)}{D^T(t) + D^L(t)} ,$$

$$\alpha(t) = \frac{2D^L(t) - D^T(t)}{D^T(t)} ,$$

(62) (63)
\[ W(t) = \frac{D_T(t)}{D_T(t) + 2D_L(t)}. \] (64)

In Fig. 8, we plot the three spin asymmetry parameters \( \xi(t) \), \( \alpha(t) \), and \( W(t) \) as functions of \( t \) for the mesons that we are considering. The curves in Fig. 8 vary far more dramatically than the corresponding curves in Fig. 4, because the longitudinal and transverse fragmentation functions show a very different dependence on \( p_\perp \). For all the vector meson systems that we are considering, the \( t \)-dependent longitudinal and transverse fragmentation functions cross over at \( t \approx 1.5 - 2.0 \). Consequently, the corresponding spin asymmetry parameters also change very rapidly at \( t \approx 1.5 - 2.0 \).

With \( D_{L,T}(t) \) replaced by \( D_{L,T}(z, t) \) in Eqns. (62) - (64), one can also introduce the spin asymmetry parameters \( \xi(z, p_\perp) \), \( \alpha(z, p_\perp) \), and \( W(z, p_\perp) \) that depend on both the variables \( z \) and \( p_\perp \). It may be possible to measure such spin asymmetry parameters, we will not discuss them any further here.

VII. Conclusions

In this paper, we have used PQCD to derive the polarized fragmentation functions to leading order in \( \alpha_s \) for a \( \bar{b} \) antiquark fragmenting into the longitudinally and transversely polarized \( B_{c}^* \) states. Parity invariance also implies that the polarized heavy quark fragmentation functions \( D_L(z) \) and \( D_T(z) \) derived in this paper are the same whether the initial heavy quark is polarized or unpolarized. These polarized fragmentation functions can be used to define various spin asymmetry parameters that can be determined experimentally.

In this work, we have also used the PQCD fragmentation functions to study the spin alignment of the other S-wave excited mesons that carry charm or beauty. The spin asymmetry parameter \( \alpha(z) \) for \( c \to D^* \) is consistent with the measurements by the CLEO detector. The mean longitudinal momentum fractions \( \langle z \rangle \) predicted for \( D^* \) and \( B^* \) are also in excellent agreement with the measurements of the LEP, CLEO, and ARGUS detectors. We also demonstrate a very interesting dependence of the polarized heavy quark fragmentation
functions and the spin asymmetry parameters on the transverse momentum $p_\perp$ of the vector meson relative to the fragmentation axis. Longitudinally polarized vector mesons have a harder $p_\perp$ spectrum than the transversely polarized states.

The perturbative QCD fragmentation functions that we obtained in this paper and in Refs. [29, 9] are expected to work well for $c \to \eta_c, J/\psi$, $b \to \eta_b, \Upsilon$, and $\bar{b} \to B_c, B^*_c$ in any high energy processes with large transverse momentum $p_T$. They are rigorously correct in the limit that the heavy quark masses $m_c$ and $m_b$ are much larger than $\Lambda_{\text{QCD}}$. Corrections to the fragmentation contributions are of order $M^2/p_T^2$ ($M$ is the mass of the meson) and therefore small at large enough $p_T$.

Our fragmentation functions also seem to provide a successful phenomenological model for describing the spin dependence of charm and bottom fragmentation into heavy-light mesons like $D^*$ and $B^*$. Furthermore, our fragmentation functions are consistent with heavy quark symmetry. Dominant error comes from the neglect of relativistic corrections and higher order perturbative corrections. We conclude that the PQCD-inspired fragmentation functions could be useful in describing charm and bottom fragmentation in the future experiments.

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Figure Captions

1. The leading order Feynman diagram contributing to the fragmentation process $\bar{b} \rightarrow B^*_c$.

2. The polarized fragmentation functions $D^L_{\bar{b} \rightarrow B^*_c}(z)$ and $D^T_{\bar{b} \rightarrow B^*_c}(z)$ versus $z$ at the initial scale $\mu_0$. The sum of the first two terms in the heavy quark mass expansion are also shown for the longitudinal and transverse polarizations.

3. The spin asymmetry parameters (a) $\xi$, (b) $\alpha$, and (c) $W$ versus the mass ratio parameter $r$.

4. The spin asymmetry parameters (a) $\xi(z,\mu)$, (b) $\alpha(z,\mu)$, and (c) $W(z,\mu)$ versus $z$ at the initial scale $\mu_0$ for $r = 0.5$ (solid), 0.23 (dash), 0.17 (dotdash), and 0.058 (dot) corresponding to $J/\psi$, $B^*_c$, $D^*$, and $B^*$, respectively.

5. A comparison between the spin asymmetry parameter $\alpha(z)$ for $c \rightarrow D^*$ predicted by our model and the experimental measurements by the CLEO Collaboration [22].

6. The mean longitudinal momentum fraction $\langle z \rangle^{L,T}$ versus the scale $\mu$ at $r = 0.5, 0.23, 0.17$, and 0.058 for the $J/\psi$, $B^*_c$, $D^*$, and $B^*$, respectively. The longitudinal curve is solid and the transverse one is dotted. The two measurements $\langle x_E \rangle_{c \rightarrow D^*}$ and $\langle x_E \rangle_{b \rightarrow H_b}$ from the LEP detectors are shown at $\mu = m_Z/2$, and the combined CLEO and ARGUS measurement $\langle x_E \rangle_{c \rightarrow D^*}$ is at $\mu = 5.3$ GeV.

7. $D^L_{\bar{b} \rightarrow B^*_c}(t)$ and $D^T_{\bar{b} \rightarrow B^*_c}(t)$ versus the rescaled transverse momentum $t = p_\perp/M$ of the $B^*_c$ meson at the initial scale $\mu_0$.

8. The spin asymmetry parameters (a) $\xi(t)$, (b) $\alpha(t)$, and (c) $W(t)$ versus $t = p_\perp/M$ at $r=0.5$ (solid), 0.23 (dash), 0.17 (dotdash), and 0.058 (dot), which correspond to $J/\psi$, $B^*_c$, $D^*$, and $B^*$ mesons, respectively.
Table 1: The mean longitudinal momentum fraction \( \langle z \rangle \) when a heavy quark fragments into a polarized heavy meson in the spin-orbital state \( ^3S_1 \) at the scales \( \mu = \mu_0 \) and \( \mu = m_Z/2 \). We take \( m_b = 4.9 \text{ GeV}, m_c = 1.5 \text{ GeV}, \) and the light \( u \) or \( d \) constituent quark masses to be 0.3 GeV. Here \( L \) stands for longitudinal and \( T \) for transverse.

| Meson     | \( r \) | \( \mu_0 \) | \( \langle z \rangle_{\mu=\mu_0} \) | \( \langle z \rangle_{\mu=m_Z/2} \) |
|-----------|---------|-------------|---------------------------------|---------------------------------|
|           |         |             | \( L \) | \( T \) | \( L \) | \( T \) |
| \( J/\psi \) | 0.50    | 4.5         | 0.61 | 0.62 | 0.48 | 0.48 |
| \( B^*_c \) | 0.23    | 7.9         | 0.73 | 0.73 | 0.61 | 0.61 |
| \( D^* \)   | 0.17    | 2.1         | 0.77 | 0.77 | 0.50 | 0.50 |
| \( B^* \)   | 0.058   | 5.5         | 0.87 | 0.87 | 0.70 | 0.70 |

Table 2: A table showing where the curves \( D^L(t) \) and \( D^T(t) \) cross-over, the average \( \langle t \rangle = \langle p_{\perp} \rangle/M \), and the average \( \langle p_{\perp} \rangle \) for the \( \bar{b} \) quark fragmenting into \( B^* \) and \( B^*_c \), and for the \( c \) quark fragmenting into \( D^* \) and \( J/\psi \). The mass of each meson is taken to be the sum of the constituent quark masses \( (m_b = 4.9, m_c = 1.5, m_{u,d} = 0.3 \text{ GeV}) \).

|          | \( r \) | \( t(\text{cross-over}) \) | \( \langle t \rangle \) | \( \langle p_{\perp} \rangle \) (GeV) |
|----------|---------|--------------------------|----------------------|---------------------------------|
|          |         | \( L \) | \( T \) | \( L \) | \( T \) |
| \( J/\psi \) | 0.5    | 1.6 | 1.8 | 0.86 | 5.5 | 2.6 |
| \( B^*_c \) | 0.23   | 1.5 | 1.1 | 0.61 | 7.0 | 3.9 |
| \( D^* \)   | 0.17   | 1.6 | 0.86 | 0.52 | 1.5 | 0.94 |
| \( B^* \)   | 0.058  | 2.0 | 0.41 | 0.30 | 2.1 | 1.5 |
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