Four Leptons Production in
\(e^- e^+\) Collisions from 3-3-1 Model

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Abstract
We study the process \(e^- + e^+ \rightarrow e^+ + e^+ + e^- + e^-\) for ILC and CLIC energy regimes, in the minimal 3-3-1 model framework. The main contributions, for the production of two bileptons each one decaying into two same-sign leptons, come from s-channel annihilation (via \(\gamma, Z\) and \(Z'\)) and t-channel electron exchange. We evaluate the number of events for an extra neutral gauge boson mass \(M_{Z'}\) in the range 1 TeV to 3 TeV. We compare some distributions from 3-3-1 model with the Standard Model background in order to extract a possible signature of the contribution of bilepton and \(Z'\) for the process. From our analysis we conclude that our results give clear signals for physics beyond the Standard Model in \(e^- e^+\) colliders.

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1 Introduction

The Standard Model (SM) of strong and electroweak interactions is extremely successful. Its predictions are consistent with all available experimental data. However, the new colliders generation will explore TeV energy regimes presenting the possibility of new findings. On the other hand, the theoretical extensions of the SM are motivated by attempting to understand features that are accommodated in the SM but not explained by it, such as the presence of more than one family in the model and the lack of bounds for the Weinberg angle \(\theta_W\).
The 3-3-1 model \[1, 2, 3\] predicts the existence of new heavy particles (fermions and bosons), and it offers an explanation of flavor by anomalies cancellation and clearly shows a limit for the Weinberg angle \[4\]. The model offers a possible first step to understand the flavor question, because the cancellation of anomalies is obtained by matching the number of families and the number of colors. Another feature of the model is that, to keep its perturbative character, the ratio between the gauge couplings forbids \(\sin^2\theta_W > 1/4\).

The 3-3-1 model has this name because it is based in the semi-simple gauge group \(SU(3)_C \times SU(3)_L \times U(1)_X\). It is a gauge theory based in a largest group of symmetry and, as a consequence, it predicts the existence of more particles than the SM: exotic quarks, new gauge bosons, neutral (\(Z'\)), bileptons (\(V^\pm, Y^{\pm\pm}\)) and a large number of Higgs. In the model, total lepton number is conserved but the separate flavors lepton numbers are violated. This leads to dramatic signature for double charged bileptons because they can decay into two same-sign leptons. Such evidence will be accessible by next generation of \(e^- e^+\) colliders with center of mass energy: \(\sqrt{s} = 1\) TeV ILC \[5\] and \(\sqrt{s} = 3\) TeV, CLIC \[6\] (CERN). In this work we will review the minimal version of model in the next section. Our results are presented in the section 3 followed by our conclusions in section 4.

## 2 The Model

The model has five additional gauge bosons beyond the SM ones. In its minimal version they are: a neutral \(Z'\) and four heavy charged bileptons, \(Y^{\pm\pm}, V^\pm\) with lepton number \(L = \mp 2\) (hence their name). For each generation, the usual doublet-singlet pattern per family of SM is replaced by one triplet of electroweak \(SU(3)_L\).

\[
\psi_L = \begin{pmatrix} \nu_{\ell} \\ \ell \\ \ell^c \end{pmatrix}_L \sim (1, 3, 0) ,
\]

where \(\ell^c\) is the charge conjugate of \(\ell\) (\(e, \mu, \tau\) field and in parenthesis are respectively the dimensions of the group representation of \(SU(3)_C, SU(3)_L\) and \(U(1)_X\) charge.
The first quark family,

\[ Q_{L1} = \begin{pmatrix} u \\ d \\ J_1 \end{pmatrix}_L \sim (3, +2/3, 0) \quad (2) \]

transforms as a triplet under \( SU(3)_L \) group, and the two other families,

\[ Q_{L2} = \begin{pmatrix} J_2 \\ c \\ s \end{pmatrix}_L \sim (\bar{3}, -1/3, 0), \quad Q_{L3} = \begin{pmatrix} J_3 \\ t \\ b \end{pmatrix}_L \sim (\bar{3}, -1/3, 0) \quad (3) \]
as anti-triplets. \( J_1, J_2 \) and \( J_3 \) are exotic quarks with respectively \( 5/3 \), \( -4/3 \) and \( -4/3 \) units of positron charge.

The minimum Higgs structure necessary for symmetry breaking and that gives quarks and leptons acceptable masses are (three triplets and one anti-sextet):

\[ \eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^+ \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho^+ \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^- \\ \chi^- \\ \chi^0 \end{pmatrix}, \quad S = \begin{pmatrix} \sigma^0 \\ h_1^+ \\ h_1^- \\ h_2^+ \\ h_2^- \end{pmatrix} \quad (4) \]

The breaking of 3-3-1 group to the SM one is accomplished because the neutral component of scalars acquires vacuum expectation value (VEV). These scalars will produce the following hierarchical symmetry breaking

\[ SU_L(3) \otimes U_X(1) \xrightarrow{<\chi>} SU_L(2) \otimes U_Y(1) \xrightarrow{<\rho, \eta, S>} U_{e.m}(1). \]

The consistency of the model with SM phenomenology is imposed by fixing a large scale for the VEV of the neutral \( \chi \) field \( (v_\chi \gg v_\rho, v_\eta) \).

One of the main features of the model comes from the relation between the \( SU_L(3) \) coupling, \( g \), and \( U_X(1) \) coupling, \( g' \) \( (g' / g = t \equiv \tan \theta_W) \) that fixes \( \sin^2 \theta_W < 1/4 \) and is expressed as:

\[ \frac{g'^2}{g^2} = \frac{\sin^2 \theta_W}{1 - 4 \sin^2 \theta_W}. \quad (5) \]

The gauge bosons are \( W^a_\mu \) \( (a = 1...8) \) in a octet representation of \( SU(3)_L \) and a singlet \( B_\mu \) of \( U(1)_X \). The charged and neutral gauge bosons are defined from the combinations:

\[ W^\pm_\mu = \frac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}}, \quad V^\pm_\mu = \frac{W^4_\mu \pm iW^5_\mu}{\sqrt{2}}, \quad Y^{\pm \pm}_\mu = \frac{W^6_\mu \pm iW^7_\mu}{\sqrt{2}}. \quad (6) \]
\[ A_\mu = h(t)^{-1/2} \left[ (W_\mu^3 - \sqrt{3} W_\mu^8) t + B_\mu \right], \]
\[ Z_\mu \simeq -h(t)^{-1/2} \left[ f(t)^{1/2} W_\mu^3 + f(t)^{-1/2} \left( \sqrt{3} t^2 W_\mu^8 - tB_\mu \right) \right], \]
\[ Z'_\mu \simeq f(t)^{-1/2} \left[ W_\mu^8 + \sqrt{3} t B_\mu \right], \] (7)

with \( h(t) = 1 + 4t^2 \) and \( f(t) = 1 + 3t^2 \).

The charged gauge boson masses as a function of VEV’s are:
\[ M_{W}^2 = \frac{1}{4} g^2 \left( v_\eta^2 + v_\rho^2 \right), \quad M_{V}^2 = \frac{1}{4} g^2 \left( v_\rho^2 + v_\chi^2 \right), \quad M_{Y}^2 = \frac{1}{4} g^2 \left( v_\eta^2 + v_\chi^2 \right). \] (8)

and the neutral gauge masses
\[ M_{\gamma}^2 = 0, \quad M_{Z}^2 \simeq \frac{g^2}{4} \frac{g^2 + 4g'^2}{g^2 + 3g'^2} \left( v_\eta^2 + v_\rho^2 \right), \quad M_{Z'}^2 \simeq \frac{1}{3} (g^2 + 3g'^2)v_\chi^2 \] (9)

The relation between \( Z' \) and \( Y \) masses [7, 8], in the minimal model is:
\[ \frac{M_Y}{M_{Z'}} = \frac{(3 - 12 \sin^2 \theta_W)^{1/2}}{2 \cos \theta_W}, \] (10)

which will be used in the present work.

For our purpose, let us consider only the charged current interactions of leptons with bileptons given by:
\[ \mathcal{L}^{CC} = -\frac{g}{\sqrt{2}} \left( \ell^T C \gamma^\mu \gamma^5 \ell Y_{\mu}^{++} \right), \] (11)

where \( C \) is the charge conjugation matrix.

The neutral interactions follow from the Lagrangian
\[ \mathcal{L}^{NC} = -\bar{\ell} \gamma^\mu \ell A_\mu - \frac{g M_Z}{4 M_W} [\bar{\ell} \gamma^\mu (v_\ell + a_\ell \gamma^5) \ell Z_\mu + \bar{\ell} \gamma^\mu (v'_\ell + a'_\ell \gamma^5) \ell Z'_\mu], \] (12)

with \( v_\ell = -1/h(t) \), \( a_\ell = 1 \), \( v'_\ell = -\sqrt{3}/h(t) \), \( a'_\ell = v'_\ell/3 \).

Note that for \( t^2 = 11/6 \), \( v_\ell \) and \( a_\ell \) have the same values as the SM couplings.
3 Results

Let us first consider the production of two real bileptons in \( e^- + e^+ \rightarrow Y^{++} + Y^{--} \). For this calculation and in order to show the relevant distributions, we have used the package CompHep [9]. The contributions are from \( \gamma, Z \) and \( Z' \) (s-channel) and \( e \) exchange (t-channel). In Fig. 1 we show the total cross section for some pairs of \( Z' \) and \( Y^{\pm\pm} \) masses, as a function of \( \sqrt{s} \). A resonance peak is observed for \( M_{Z'} = 1 \) TeV (\( \Gamma_{Z'} = 164 \) GeV) and \( \sqrt{s} = 1 \) TeV. For higher masses and widths, and for \( \sqrt{s} = 3 \) TeV, the peak disappears.

For an annual integrated luminosity from \( \mathcal{L}_{int} = 100 \) fb\(^{-1}\) to 500 fb\(^{-1}\) the number of events with two final bileptons is in the range \( 10^4 \) to \( 10^5 \). There is no similar process in the SM producing two very massive gauge bosons. However one can estimate the order of magnitude for four leptons production by considering \( e^- + e^+ \rightarrow Z + Z \) that, for \( \mathcal{L}_{int} = 100 \) fb\(^{-1}\), produces around \( 10^4 \) events for \( \sqrt{s} = 1 \) TeV and this value reduces to \( 10^3 \) for \( \sqrt{s} = 3 \) TeV.

![Figure 1: Total cross section for \( e^- + e^+ \rightarrow Y^{++} + Y^{--} \) versus \( \sqrt{s} \).](image)

Another indication of the presence of heavy boson exchange is the forward-backward asymmetry (\( A_{FB} \)), obtained from the angular distribution of \( Y^{++} \) relative to the direction of the initial electron, shown in Fig. 2. We note again the resonant peak, originated from \( Z' \) exchange, only for \( M_{Z'} = 1 \) TeV.
TeV. For higher masses this effect is not present because $Z'$ gets broader, as shown in Table I.

From the $Y^{++}$ transverse momentum distribution, depicted in the Fig. 3, we observed that the bileptons are produced with $p_t \simeq 50$ GeV for $M_{Z'} = 1$ TeV and $\sqrt{s} = 1$ TeV. This value increases to $\simeq 150$ GeV when $M_{Z'} = 1$ TeV and $\sqrt{s} = 3$ TeV. This behavior is inherent to heavy particle production. This important observation allows one to introduce a minimal cut for the sum of transverse momenta for each pair of same-sign leptons produced when the bilepton are off shell ($p_{ti} + p_{jt} \simeq 50$ GeV) for $\sqrt{s} = 1$ TeV and 150 GeV for $\sqrt{s} = 3$ TeV.

![Figure 2: The forward-backward asymmetry extracted from the angular distribution of one bilepton relative to the direction of the initial electron versus $\sqrt{s}$.](image)

Figure 2: The forward-backward asymmetry extracted from the angular distribution of one bilepton relative to the direction of the initial electron $versus$ $\sqrt{s}$. 
This process was also calculated in [10], where the authors only calculated total cross section arriving to same order of magnitude than we obtained. In addition, in the present paper, we calculated distributions: that allow one to introduce kinematic cuts to be used in bilepton off-shell calculation and show a signature for heavy neutral boson exchange.

Next we consider the complete process $e^{-} + e^{+} \rightarrow e^{+} + e^{-} + e^{+} + e^{-}$, experimentally accessible, not yet calculated in literature, for which the gauge bosons are off-shell. In this case the number of tree diagrams increases from 36 corresponding to SM contributions to 133 diagrams for 3-3-1 model. Notice that 97 diagrams involve $Z'$, $Y^{\pm\pm}$ and their combinations. Due to the calculation complexity to obtain cross sections and distributions it is mandatory to use again CompHep package [9].

We adopted, for the detector acceptance, an angular cut of $|\cos \theta| \leq 0.995$ for the direction of final leptons relative to the beam, and energy cut of 5 GeV for final leptons [11, 12, 13]. As we are interested to show a signature for the bilepton existence, we have also selected an invariant mass cut of two same-sign leptons $|M_{ee} - M_{Y^{\pm\pm}}| \simeq \Gamma_{Y^{\pm\pm}}$, where the $Y^{\pm\pm}$ widths are presented in Table I.

![Figure 3: Final bilepton transverse momentum distribution.](image)

As mentioned before, one is allowed to apply a cut for the sum of transverse momenta of same-sign leptons. The study of transverse momentum of a final
lepton can be done from Fig. 4 for two values of \( \sqrt{s} \). By this figure we observe that they are mainly produced with high \( p_t \), differently from those produced considering the SM. This is an indication that they are originated from heavy particle decay.

![Figure 4: Final lepton transverse distribution in 3-3-1 model for \( M_Y = 276 \) GeV and \( \sqrt{s} = 1 \) TeV.](image)

Another interesting distribution that reveals the existence of bileptons is Fig. 5 that shows the invariant mass distribution of two same-sign leptons (\( M_{ee} \)) for \( \sqrt{s} = 1 \) TeV. It is clear that one can identify the bilepton as a resonance by reconstructing it from same-sign final leptons momenta. The same behavior was observed for \( \sqrt{s} = 3 \) TeV. On the other hand this is not observed in SM that leads to a flat distribution.

The angular distribution between same-sign leptons obtained in 3-3-1 calculation is shown in Fig. 6. For an energy \( \sqrt{s} = 1 \) TeV, the angle between the same sign leptons is \( \approx 66^\circ \), decreases to \( 0^\circ \) for 3 TeV, this indicates a slow heavy particle decay.

Finally, in Fig. 7 we present the total cross section as a function of bilepton mass. This picture allows to know the accessible range for \( M_Y \), that can be explored in next linear colliders. Considering a luminosity \( \mathcal{L} = 100 \) fb\(^{-1}/y \), and a range 276 GeV \( \leq M_Y \leq 500 \) GeV, we obtained around \( 10^2 \) to \( 10^4 \) events.
Figure 5: Invariant mass distribution of two same-sign leptons in 3-3-1 model for $M_Y = 276$ GeV and $\sqrt{s} = 1$ TeV, a similar distribution was obtained for $\sqrt{s} = 3$ TeV.

for $\sqrt{s} = 1$ TeV. Increasing $\sqrt{s}$ to 3 TeV and exploring up to $M_Y = 800$ GeV, this range turns into $10^3$ to $10^2$ events, what is also a measurable yield.

3-3-1 model results ($M_{Z'} = 1$ TeV and $M_{Y^{\pm\pm}} = 276$ GeV) for total cross section versus $\sqrt{s}$ are approximately five orders of magnitude bigger than SM. The unitarity property of the total cross section is assured by the presence of the extra gauge boson $Z'$.

4 Conclusion

We explored the process $e^-e^+$ at energies corresponding to next generation of linear colliders in order to find signals of the existence of new gauge bosons, predicted by the chiral extension of the SM, called 3-3-1 model.

We have assumed a special constraint between bilepton and neutral gauge boson masses, respecting the experimental bounds that, even being a consequence of the model, it is not often used in the literature.
In the present paper, to find a signal of new physics, we concentrated our study in four charged leptons production ($e^{-}+e^{+}+e^{-}+e^{+}$). This process was selected because double charged bilepton mainly decays into two same-sign leptons leading to a non standard signature.

We started with a calculation of on-shell bilepton production as a guide to the complete calculation. In literature exist a similar calculation, but only for total cross section. Instead, our results lead us to implement a cut on the transverse momenta of final leptons when bileptons are off-shell. Another indication that followed from this calculation was a resonance peak in the forward-backward asymmetry related to $Z'$ exchange.

To perform the complete calculation of four leptons production, we used CompHep package. We have applied angular and energy cuts for the detector acceptance. In order to distinguish the signal from SM background, we introduced cuts in invariant mass and transverse momenta of final leptons.

We evaluate that the number of events produced by year, for a bilepton mass range from $M_{Y^{±±}} = 276$ GeV to $M_{Y^{±±}} \simeq 830$ GeV, and for $\sqrt{s} = 1$ TeV and $\sqrt{s} = 3$ TeV, varies from $10^4$ to $10^2$, a promising signature.
The next generation of linear colliders working with energy in scale of 1 TeV and beyond, is a good place to explore a new physics predicted by 3-3-1 model.

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Table 1: Some widths for new gauge bosons $Z'$ and bilepton $Y^{±±}$ in the minimal 3-3-1 model.

| $M_{Z'}$ (GeV) | $\Gamma_{Z'}$ (GeV) | $M_{Y^{±±}}$ (GeV) | $\Gamma_{Y^{±±}}$ (GeV) |
|----------------|----------------------|---------------------|-------------------------|
| 1000           | 164                  | 276                 | 2.33                    |
| 1800           | 788                  | 496                 | 4.2                     |
| 2200           | 998                  | 600                 | 6.4                     |
| 3000           | 1405                 | 826                 | 15.2                    |