The Quest for the Neutrino Mass Spectrum

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1. Introduction

Recently the particle physics community was shocked with breathtaking news from the neutrino sector: Neutrino oscillations have been confirmed finally in the Super-Kamiokande experiment. Now for the first time, ongoing and future experiments in neutrino oscillations (Super-Kamiokande, Borexino, SNO, MINOS, KAMLAND, MINIBOONE,...) and double beta decay (Heidelberg–Moscow, GENIUS,...) together can aim to solve the neutrino mass puzzle.

It was in 1930, when Wolfgang Pauli (fig. 1) wrote his famous letter addressed as Liebe radioaktive Damen und Herren (Dear radioactive Ladies and Gentlemen), where he informed the participants of a nuclear physics workshop in Tübingen about his absence (he preferred to participate in a dance party) and postulated the neutrino to solve the problem of energy nonconservation in the nuclear beta decay. In 1956 the neutrino was observed for the first time by Clyde Cowan and Fred Reines in Los Alamos, who originally planned to explode a nuclear bomb for their experiment [2]. Finally, two years ago, the Super-Kamiokande experiment, a 50,000 ton water tank viewed by more than 11,000 photo multipliers 1,000 meter underground below a holy mountain in Japan, announced a significant signal for neutrino oscillations and established a non-vanishing mass of the neutrino as the first experimental signal of physics beyond the standard model. However, in spite of these successes, entering a new millennium the neutrino is still the most mysterious of the known particles. Alternatingly compared with spaceships travelling through the universe, ghosts penetrating solid rocks and vampires missing a mirror image [3], it still inspires the phantasy of hundreds of adventurous particle, nuclear and astro physicists being motivated by the hope, the neutrino could act as a key to the old human dream of a final theory, describing all particles and forces in a unified framework, and to a deeper understanding of the fate of the universe.

The attributes, making the neutrino this kind of outlaw among the known particles, are the following:
Figure 1. The man who proposed it and an impression of its potential cosmological consequences: Wolfgang Pauli (nobel prize 1945, left panel [4]) first thought about a neutral light particle being emitted in nuclear beta decays. Clouds of interstellar gas (right panel [5]) act as a birthplace for new stars. Neutrinos may be important for the formation of structure in the early universe.

- The neutrino seems to possess an at least million times smaller mass than the lightest of the remaining particles, the electron. While in the standard model the neutrino was introduced as massless “by hand”, this feature is especially problematic in unified theories, where the common treatment of neutrinos and charged fermions in extended multiplets implies them to have (Dirac) mass terms of the same order of magnitude as the other fermions.

- Among all fundamental fermions the neutrino is the only one being electrically uncharged. Thus the neutrino interacts a billion times less often than an electron and may penetrate the entire earth without even be deviated. This is the reason why neutrinos, in spite of their tiny masses, may be that abundant that they contribute substantially to the mass of the universe, about twenty times more than the mass of all visible stars in the sky, and may influence the evolution of the universe, e.g., the growth of structures, in a significant way.

The basis for an understanding of these features relates them to each other and was proposed in 1933 by the Italian theoretician Ettore Majorana [6], one year before he disappeared under mysterious circumstances. Majorana found out that neutrinos, due to their neutral charge, can be identical with their antiparticles, triggered by a new, so-called Majorana
In 1979 T. Yanagida and independently Murray Gell-Mann (Nobel prize 1969), P. Ramond and R. Slansky found out that these additional Majorana mass terms may cancel almost totally the usual Dirac mass terms in the so-called “see-saw mechanism”, yielding a natural explanation of the tiny neutrino masses. This would require the existence of right-handed heavy neutrinos as they are naturally predicted in “left-right-symmetric” unified models. The exact value of the mass then is correlated with a higher energy scale predicted by the underlying unified gauge group, and offers one of the rare possibilities to test these theories, since most of the predictions are observable only at very high energies, which are lying beyond the reach of present and future accelerators (see fig. 2). The question to experimentalists thus remains: What is the mass of the neutrino? The following review will outline the way to answer this question, concentrating on two experimental approaches, yielding the complementary pieces to solve the puzzle: Only both neutrino oscillations and neutrinoless double beta decay together could solve this absolute neutrino mass problem.

2. Neutrino oscillations

The fact that neutrinos are massive has finally been established by neutrino oscillation experiments. Neutrino oscillations are a quantum mechanical process based on mixing between the three neutrino flavors, which is possible if the flavor (interaction) eigenstates $\nu_\alpha$ do not coincide with the mass eigenstates $\nu_i$. The flavor eigenstates are thus given by a superposition of the mass eigenstates:

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} \nu_i$$

In that case a neutrino, which is emitted as a flavor eigenstate $\nu_\alpha$ in a weak reaction, propagates as a superposition of the three mass eigenstates. If these mass eigenstates are non-degenerate, they travel with different velocities and the composition in eq. (1) is getting out of phase. With a probability, which is a function of the mass squared differences $\Delta m^2 = m_i^2 - m_j^2$ and the mixing $U_{\alpha i}$, after a certain distance the neutrino interacts as a different mass eigenstate $\nu_\beta \neq \alpha$ (see fig. 3). Obviously neutrino oscillation experiments cannot give any information about the absolute mass scale in

* In fact also pure usual “Dirac” mass terms for the neutrino are possible but are disfavored in most fundamental theories.

† Alternative mechanisms motivate neutrino masses at the weak scale, a famous example is R-parity violating supersymmetry, see e.g. [8], where neutrino masses provide a window into deep relations of particles and forces. Also gravity induced non-renormalizable mass terms can play a role in string-motivated scenarios, see e.g. [9].
Figure 2. The light neutrino mass can be related to a corresponding large mass scale in unified theories, e.g. in the see-saw mechanism. This way the hunt for neutrino masses offers a unique possibility to access physics at energy scales beyond the reach of running and future accelerators, which has been realized only shortly after the big bang. A neutrino mass of 0.1 eV - 0.001 eV corresponds, via the mass of its heavy right-handed partner, to energies of $10^{10}$ GeV or more, tiny fractions of a second after the big bang (background from [10]).
the neutrino sector, but yield informations about mass (squared) differences, only. Since the probability oscillates with the propagation distance, this phenomenon, which was predicted by Bruno Pontecorvo, after he disappeared in 1950 from England and later showed up again in Russia, is called neutrino oscillations [11].

Up to now, hints for neutrino oscillations have been observed in solar and atmospheric neutrinos as well as the accelerator experiment LSND (for an overview see fig. 4).

A deficit of the number of solar neutrinos [12] being expected has been confirmed in many experiments [13] after the pioneering Chlor experiment [14] of Ray Davis in the Homestake mine. The oscillation mechanism of the solar $\nu_e$ in (as normally assumed) $\nu_\mu$ may be induced via two different mechanisms. The usual neutrino oscillation mechanism requires maximal mixing and suffers from the fact, that for this case the distance earth-sun has to be finetuned (vacuum oscillations). An alternative solution has been suggested by works of S. Mikheyev, Alexei Smirnov and L. Wolfenstein [15]: Resonant conversions, which are triggered by matter effects in the solar interior implying a level crossing of mass eigenstates, can cause the neutrino deficit. In this case both small as well as large mixing are allowed. The different solutions of the solar neutrino experiments correspond to different combinations of mass squared differences $\Delta m^2_{12}$ and mixing matrix elements $U_{12}$. They will be tested by ongoing and future experiments such as Super-Kamiokande, SNO and BOREXINO [18] in the next years.† Vacuum oscillations should lead to seasonal variations, the small mixing MSW solutions should imply distortions

$\dagger$ It should be stressed that besides neutrino oscillations also new interactions beyond the standard model may provide solutions to some of the neutrino anomalies, see [17].

§ An alternative would be a fourth sterile $\nu_s$, see section 7.

¶ If one allows for larger confidence belts a third MSW “LOW” solution appears, which can be tested via its strong day-night effect at low neutrino energies, observable at BOREXINO, LENS and the double beta and dark matter detector GENIUS (see below) [19].
Figure 4. Summary of neutrino anomalies. The solar neutrino deficit can be solved within the light blue regions of the small mixing and large mixing MSW oscillations, and the vacuum oscillations (at $\Delta m^2 \sim 10^{-9} - 10^{-10}$ eV$^2$, not shown in the figure). Only the large MSW solution is directly testable by KAMLAND. In the atmospheric favored region (light green) $\nu_\mu - \nu_e$ oscillations are excluded by CHOOZ. The K2K and MINOS experiments will test for $\nu_\mu \to \nu_\tau$ oscillations. The favored region of LSND is shown in light blue and can be tested in part by KARMEN and in total by MINIBOONE. Also shown is the sensitivity of GENIUS 1 ton and GENIUS 10 t or comparable double beta decay projects to $\nu_e \to \nu_{\mu,\tau,s}$ oscillations. The lines correspond to (from above) $m_{\nu_e}/m_{\nu_{\mu,\tau,s}} = \infty, 0.01, 0.1, 0.5$ (background from [22]).
of the energy spectrum and the large mixing angle solution should show a small spectral distortion, a day-night effect of the total rate and a disappearance signal in the long baseline reactor experiment KAMLAND \cite{21} just under construction.

- A similar effect has been observed in atmospheric neutrinos \cite{20}, which stem from the decay of the pions produced from cosmic ray interactions in the upper atmosphere and the following-up decays. Here Super-Kamiokande obtained a high precision result of a deficit of muon neutrinos compared to electron neutrinos. Even more convincing is the distortion observed for the zenith angle dependence of the muon neutrino flux, which provides a strong hint for $\nu_{\mu} \rightarrow \nu_{\tau}$ oscillations with maximal mixing and information about $\Delta m_{23}^2$ and $U_{23}^2$ \cite{1}. Future long baseline experiments, K2K (already running), MINOS, and CERN-Gran Sasso \cite{21}, looking for oscillations in accelerator produced neutrino beams over distances of several hundred kilometers will provide a check of this result by directly looking for $\nu_{\tau}$ appearance and have the possibility to search for small contributions of $\nu_{e} \rightarrow \nu_{\tau}$ oscillations.

- Also an accelerator experiment, LSND, has reported evidence for $\nu_{e} - \nu_{\mu}$ neutrino oscillations. However, this evidence is generally understood as the most ambiguous. The KARMEN experiment has excluded a large part of the favored region of LSND. Since only two experimental evidences may be fitted with only three neutrinos, the LSND result would require the existence of a fourth, sterile (i.e. not weakly interacting) neutrino (see section 7). A decisive test will be obtained from the MINIBOONE experiment \cite{16}.

3. Neutrinoless double beta decay

Double beta decay ($0\nu \beta \beta$) corresponds to two single beta decays occurring in one nucleus and converts a nucleus (Z,A) into a nucleus (Z+2,A) (see fig. 3). While the standard model (SM) allowed process emitting two antineutrinos

$$\frac{A}{2} X \rightarrow \frac{A}{2} + 2 X + e^- + 2 \nu_e$$

is the rarest process observed in nature with half lives in the region of $10^{21-24}$ years, more interesting is the search for the lepton number violating and thus SM forbidden neutrinoless mode,

$$\frac{A}{2} X \rightarrow \frac{A}{2} + 2 X + e^-$$

which has been proposed by W.H. Furry in 1939 \cite{22}. In this case the neutrino is exchanged between the vertices (see fig. 3), a process being only
allowed if the intermediate neutrino has a Majorana mass. Neutrinosless double beta decay, when observed, also does not measure directly the neutrino mass. Since the neutrino in the propagator is only virtual, it does not have a definite mass. Propagating in the nucleus is the flavor eigenstate with the so-called effective neutrino Majorana mass

\[
\langle m \rangle = \left| \sum_j |U_{ej}|^2 e^{i\phi_j} m_j \right|,
\]

which is a function of the mixing angles \( U_{ej} \), complex phases \( \phi_j \), which allow for cancellations of the entering masses, and the neutrino mass eigenvalues. This quantity has exciting connections to the observables in neutrino oscillation experiments. The most stringent limit on this quantity, \( \langle m \rangle < 0.35 \) eV, is obtained by the Heidelberg–Moscow experiment \[23\], which was initiated by one of the authors \[24\] and is running since 10 years in the Gran Sasso underground laboratory in Italy. An impressive breakthrough to \( 10^{-2} - 10^{-3} \) eV could be obtained realizing the GENIUS project proposed in 1997 \[25\], a further proposal of H.V. Klapdor-Kleingrothaus, operating 1-10 tons of enriched Germanium directly in a tank of 12 m diameter and height filled with liquid nitrogen.

How are the results in double beta decay and neutrino oscillations related? In a recent work \[26\] the authors of this article in collaboration with Alexei Smirnov from the ICTP Trieste were studying the relations of the neutrino oscillation parameters and the effective Majorana mass in the several possible neutrino mass scenarios and settled the conditions under which the neutrino mass spectrum can be reconstructed with future projects (see fig. 5). In the following we will concentrate on three extreme cases as examples, the hierarchical spectrum, the degenerate scheme and the inverse hierarchical scheme.

4. Hierarchical schemes

Hierarchical spectra (fig. 6)

\[
m_1 \ll m_2 \ll m_3
\]

can be motivated by analogies with the quark sector and the simplest seesaw models. In these models the contribution of \( m_1 \) to the double beta decay observable \( \langle m \rangle \) is small. The main contribution is obtained from \( m_2 \) or \( m_3 \), depending on the solution of the solar neutrino deficit.

If the small mixing angle solution is realized in solar neutrinos (i.e. small \( \nu_e - \nu_\mu \) mixing), the contribution of \( m_2 \) is small due to the small admixture \( U_{e2} \). The same is true for vacuum oscillations, where \( U_{e2} \) is maximal but
Figure 5. Schematic views of three experiments which will provide important complementary pieces of information about the neutrino mass. The Super-Kamiokande neutrino oscillation experiment, the GENIUS double beta decay project (successor of the Heidelberg-Moscow experiment) and the Planck cosmic microwave satellite [28].
Figure 6. Neutrino masses and mixings in the scheme with mass hierarchy, shown is the example of small solar neutrino mixing. Coloured bars correspond to flavor admixtures in the mass eigenstates $\nu_1$, $\nu_2$, $\nu_3$. The quantity $\langle m \rangle$ is determined by the dark blue bars denoting the admixture of the electron neutrino $U_{ei}$.

Figure 7. Double beta decay observable $\langle m \rangle$ and oscillation parameters: The case of hierarchical schemes with either the MSW small mixing solution or vacuum solution. Shown is the dominant contribution of the third state to $\langle m \rangle$ which is constrained by the CHOOZ experiment, excluding the region to the upper right. Further informations can be obtained from the long baseline project MINOS and future double beta decay experiments [26].

The mass of the second state is tiny. In these cases the main contribution to $\langle m \rangle$ comes from $m_3$. The contribution of the latter is shown in fig. 7. Here lines of constant $\langle m \rangle$ are shown as functions of the oscillation parameters...
Figure 8. Double beta decay observable $\langle m \rangle$ and oscillation parameters: The case for the MSW large mixing solution of the solar neutrino deficit, where the dominant contribution to $\langle m \rangle$ comes from the second state. Shown are lines of constant $\langle m \rangle$ (diagonal) and constant day-night asymmetry (almost horizontal) [26]. The closed line shows the region allowed by present solar neutrino experiments. Complementary informations can be obtained from double beta decay and the search for a day-night effect in future solar neutrino experiments.

$\Delta m^2_{13}$ and $U_{13}$, parametrized by $\sin^2 2\theta_{13}$. The shaded areas show the mass $m_3 \approx \sqrt{\Delta m^2_{13}}$ favored by atmospheric neutrinos with the horizontal line indicating the best fit value. The region to the upper right is excluded by the nuclear reactor experiment CHOOZ [29], implying $\langle m \rangle < 2 \cdot 10^{-3}$ eV in the range favored by atmospheric neutrinos. Obviously in this case only the 10 ton GENIUS experiment could observe a positive 0νββ decay signal. A coincidence of such a measurement with a signal of $\nu_e \rightarrow \nu_\tau$ oscillations at MINOS and a confirmation of solar vacuum or small mixing MSW oscillations by solar neutrino experiments would be a strong hint for this scheme.

If the large mixing solution of the solar neutrino deficit is realized, the contribution of $m_2$ becomes large due to the almost maximal $U_{e2}$, now. Fig. 8 shows values of $\langle m \rangle$ in the range of the large mixing angle solution (closed line). The almost horizontal lines correspond to constant day-night
Figure 9. Neutrino masses and mixings in the degenerate scheme, here the example with large solar neutrino mixing.

asymmetries. A coincident measurement of $\langle m \rangle \simeq 10^{-3}$ eV, a day-night asymmetry of 0.07 at future oscillation experiments and a confirmation of the large mixing angle solution by KAMLAND would identify a single point in the large mixing angle MSW solution (in this example near the present best-fit point) and provide a strong hint for this scheme.

5. Degenerate schemes

Degenerate schemes (fig. 9)

$$m_1 \simeq m_2 \simeq m_3 \simeq m_0$$  \hspace{1cm} (6)

require a more general (and more complicated) form of the see-saw mechanism [30]. One of their motivations is also, that a large overall mass scale allows neutrinos to be cosmologically significant. Neutrinos with an overall mass scale of a few eV could play an important role as “hot dark matter” component of the universe. When structures were formed in the early universe, overdense regions of (cold) dark matter provide the seeds of the large scale structure, which later formed galaxies and clusters. A small “hot” (relativistic) component could prevent an overproduction of structure at small scales. Since structures redshift photons, this should imply also imprints on the cosmic microwave background (CMB), which could be measured by the future satellite experiments MAP and Planck [31].

In degenerate schemes the mass differences are not significant. Since the contribution of $m_3$ is strongly bounded by CHOOZ again, the main contributions to $\langle m \rangle$ come from $m_1$ and $m_2$. The relative contributions of these states depend on their admixture of the electron flavor, which is determined by the solution of the solar neutrino deficit.
Figure 10. Double beta decay observable $\langle m \rangle$ and oscillation parameters: The case for degenerate neutrinos. Plotted on the axes are the overall scale of neutrino masses $m_0$ and the mixing $\sin^2 \theta_{12}$. Allowed values for $\langle m \rangle$ for a given $m_0$ correspond to the regions between $m_0$ and the corresponding curved line. Also shown are informations which could be obtained from cosmological fits (see text) and the expected sensitivity of the satellite experiments MAP and Planck. A value of $\langle m \rangle = 0.1$ eV in the case of small solar neutrino mixing would be in the range to be explored by MAP and Planck [26].

In fig. [14] lines of constant double beta decay observables (solid curved lines) are shown together with information from cosmological observations about the overall mass scale (horizontal lines). Shown are best fits to the CMB and the large scale structure of Galaxy surveys in different cosmological models as well as the sensitivity of MAP and Planck. E.g., a $\Lambda$CDM model with a total $\Omega_m = 0.5$ of both cold and hot dark matter as well as a cosmological constant, and a Hubble constant of $h = 0.6$ would imply an overall mass scale of about 0.5 eV. However, the contributions of different mass eigenstates are in the same order of magnitude and may cancel, now. Assuming a mixing corresponding to the best fit of solar large mixing MSW or vacuum oscillations this yields $\langle m \rangle = 0.2 - 0.5$ eV, just in the range of the recent half life limit of the Heidelberg–Moscow experiment. If even larger mixing turns out to be realized in the solution of the solar neutrino deficit,
this allows for a larger cancellation. A coincidence of the absolute mass scale reconstructed from double beta decay and neutrino oscillations with a direct measurement of the neutrino mass in tritium beta decay spectra or its derivation from cosmological parameters determined from the CMB in the satellite experiments MAP and Planck would prove this scheme to be realized in nature. To establish this triple evidence however is difficult due to the restricted sensitivity of the latter approaches. Future tritium experiments aim at a sensitivity down to $O(0.1 \text{ eV})$ and MAP and Planck have been estimated to be sensitive to $\sum m_\nu = 0.5 - 0.25 \text{ eV}$. Thus for neutrino mass scales below $m_0 < 0.1 \text{ eV}$ only a range for the absolute mass scale can be fixed by solar neutrino experiments and double beta decay.

6. Inverse Hierarchy

A further possibility is an inverse hierarchical spectrum (fig. 11)

$$m_3 \simeq m_2 \gg m_1$$

where the heaviest state with mass $m_3$ is mainly the electron neutrino, now.

Its mass is mainly determined by the atmospheric neutrinos, $m_3 \simeq \Delta m_{23}$. Thus for the case of the small mixing angle solution one gets a unique prediction of $\langle m \rangle = (5 - 8) \cdot 10^{-2} \text{ eV}$, which could be tested by the 1 ton version of GENIUS. For the vacuum or large mixing MSW solution cancellations of the two heavy states become possible and $\langle m \rangle < 8 \cdot 10^{-2} \text{ eV}$. A test of the inverse hierarchy is possible in matter effects of neutrino oscillations. For this case the MSW level crossing happens for antiparticles rather than for particles. Effects could be observable in long baseline experiments and in the neutrino spectra of supernovae.
7. Four neutrinos

Sterile neutrinos, which do not couple to the weak interactions, can easily be motivated in superstring inspired models: multidimensional candidates for a final “Theory of Everything”, in which the fundamental constituents of matter have a string rather than a particle character. Such theories could accommodate for additional neutrinos in different ways. Examples are extended gauge groups, fermions living in extra (compactified) dimensions as well as a mirror world, which contains a complete duplicate of matter and forces building the universe, interacting only via gravity. In the latter case ⟨m⟩ = 0.002 eV is predicted [33].

If the four neutrinos are arranged as two pairs of degenerate states (mainly νe − νs for solar and νµ − ντ for atmospheric neutrinos) separated by a LSND gap, all three neutrino anomalies can be solved and the two heavy states can account for the hot dark matter. The main contribution to ⟨m⟩ comes from the heavy states, then, and can be derived from the LSND result. Depending on the phase of these two contributions ⟨m⟩ can be as large as O(10⁻³ eV). A strong hint for the scheme would be a coincidence of the Δm² favored in LSND and possibly MINIBOONE, cosmological observations and double beta decay, together with the discovery of sterile neutrinos in solar neutrino oscillations by SNO.

8. Summary

The recent years brought exciting developments in neutrino physics. Neutrino oscillations have finally been confirmed in atmospheric neutrinos and at the same time double beta decay experiments realized for the first time a sensitivity, leading to strong implications on the neutrino mass spectrum and cosmological parameters. After this particle physics now seems to enter its “neutrino epoche”: The neutrino mass spectrum and its absolute mass
scale offer unique possibilities to provide crucial information for cosmology and theories beyond the standard model. Only both neutrino oscillations and neutrinoless double beta decay together have the chance to solve this neutrino mass problem (see also, e.g. [4]) and to set the absolute scale in the neutrino sector: If the solution of the solar neutrino deficit and the character of hierarchy (direct or inverse) is determined in neutrino oscillation experiments, the following informations will be obtained from a future double beta decay project:

For the case of direct/normal hierarchy, a confirmation of the small mixing MSW solution would mean: If double beta decay would be measured with \( \langle m \rangle > 0.1 \text{ eV} \) this would establish a degenerate spectrum with a fixed mass scale. If \( \langle m \rangle \) is measured in the range \((0.5\times3)\times10^{-2} \text{ eV} \) a partially degenerate spectrum, \( m_1 \simeq m_2 \ll m_3 \), with fixed mass scale is realized in nature. For \( \langle m \rangle < 2 \cdot 10^{-3} \text{ eV} \) a hierarchical spectrum exists in nature. For the large mixing MSW solution a value of \( \langle m \rangle > 3 \cdot 10^{-2} \text{ eV} \) implies a degenerate spectrum with a region for the mass scale determined by the solar mixing angle. For \( \langle m \rangle < 2 \cdot 10^{-2} \text{ eV} \) a partially degenerate or hierarchical spectrum is realized in nature and a region for the mass scale is set by the solar mixing angle. If \( \langle m \rangle < 2 \cdot 10^{-3} \text{ eV} \) is measured the spectrum is hierarchical. If vacuum oscillations are the correct solution for the solar neutrino deficit a value of \( \langle m \rangle > 3 \cdot 10^{-2} \text{ eV} \) implies degeneracy, \( \langle m \rangle > 2 \cdot 10^{-3} \text{ eV} \) partial degeneracy and \( \langle m \rangle < 2 \cdot 10^{-3} \text{ eV} \) hierarchy, but no information about the absolute mass scale is obtained.

For the case of inverse hierarchy the situation is more predictive. For the small mixing angle MSW solution \( \langle m \rangle \equiv (5 \pm 8) \cdot 10^{-2} \text{ eV} \) is expected. For large mixing angle MSW or vacuum oscillations one awaits \( \langle m \rangle < 8 \cdot 10^{-2} \text{ eV} \), above this value the scheme approaches the degenerate case.

In four neutrino schemes \( \langle m \rangle \) can be as large as \( \mathcal{O}(10^{-3}) \text{ eV} \). A coincidence of a double beta decay signal with the \( \Delta m^2 \) favored in LSND and possibly in MINIBOONE, an imprint of neutrinos as hot dark matter in the CMB as well as the discovery of sterile neutrinos in SNO would prove the scheme and fix the mass scale.

This outcome will be a large step both towards the understanding of the evolution of the universe and towards the dream of a unified theoretical description of nature. We are entering an exciting decade!

References

[1] Y. Fukuda et al. (Super-Kamiokande Collab.) Phys. Rev. Lett. 81 (1998) 1562
[2] F. Reines, C. Cowan, Phys. Rev. 113 (1959) 273
[3] C. Sutton, Spaceship Neutrino, Cambridge University Press 1992
[22] W.H. Furry, Phys. Rev. 56 (1939) 1184
[23] Heidelberg–Moscow Collab. (L. Baudis et al.), Phys. Rev. Lett. 83 (1999) 41 and preprint, June 2000
[24] H.V. Klapdor–Kleingrothaus, Heidelberg-Moscow proposal, MPI H V17 1987; H.V. Klapdor–Kleingrothaus, Springer Tracts in Modern Physics, 163 (2000) 2000
[25] H.V. Klapdor–Kleingrothaus, in Beyond the Desert '97 – Accelerator and Non-Accelerator Approaches (Eds. H.V. Klapdor–Kleingrothaus, H. Päş). Proc. Int. Workshop on Particle Physics beyond the Standard Model, Castle Ringberg, June 8-14, 1997, IOP Publ., Bristol, Philadelphia, p. 485 and Int. J. Mod. Phys. A 13 (1998) 3953;
J. Hellmig, H.V. Klapdor–Kleingrothaus, Z. Phys. A 359 (1997) 351;
H.V. Klapdor–Kleingrothaus, M. Hirsch, Z. Phys. A 359 (1997) 361;
H.V. Klapdor–Kleingrothaus, J. Hellmig, M. Hirsch, J. Phys. G 24 (1998) 483;
H.V. Klapdor–Kleingrothaus, L. Baudis, G. Heusser, B. Majorovits, H. Päş, hep-ph/9910203
[26] H.V. Klapdor–Kleingrothaus, H. Päş, A.Yu. Smirnov, hep-ph/0003219
[27] L. Di Lella, hep-ex/9912010
[28] http://www.phys.washington.edu/~superk
[29] M. Apollonio et al. (CHOOZ collab.), Phys. Lett. B 466 (1999) 415-430
[30] R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912
R.N. Mohapatra, hep-ph/9702229, Proc. Neutrino 1996, Helsinki
[31] R.E. Lopez, astro-ph/9909414
E. Gawiser, J. Silk, Science 280 (1998) 1405-1411;
J.R. Primack, M.A.K. Gross, astro-ph/9810204, Proc. of the Xth Rencontres de Blois “The birth of galaxies”, June 28 - July 4 1998;
D.J. Eisenstein, W. Hu, M. Tegmark astro-ph/9807130, Submitted to ApJ.
[32] A.S. Dighe, A.Yu. Smirnov, hep-ph/9907423
[33] R.N. Mohapatra, hep-ph/9903261
R.N. Mohapatra, A. Perez-Lorenzana, hep-ph/9910474
R.N. Mohapatra, in Proc. Erice school on neutrinos 1997, Progr. Part. Nucl. Phys. 40 1998
[34] S.M. Bilenky, C. Giunti, W. Grimus, B. Kayser, S.T. Petcov, Phys.Lett. B465 (1999) 193-202
V. Barger, K. Whisnant, Phys. Lett. B 456 (1999) 194-200
F. Vissani, JHEP 9906 (1999) 022