Multi-focus Image Fusion Method based on Wavelet Transform

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Abstract. In this paper, the wavelet decomposition of multi-focus images is used to fuse the low-frequency components and high-frequency components obtained after decomposition. In order to make more information of the original image in the merged image, the method of maximizing the coefficient and the method of maximizing the variance of the region mean when the high frequency coefficient is selected, and combining the average and the selection in the selection of the low frequency coefficient. The simulation experiment results and the objective evaluation index values show that the modification method can make up for the defects of the two original images and obtain a relatively complete and clear image.

1. Introduction
Wavelet transform is a powerful tool for modern signal processing and analysis. The wavelet transform image can decompose the detail information in three different directions on each scale, and the transformed information has no redundancy compared with the original image information. Wavelet transform is widely used in signal processing, image analysis, computer applications and other fields in the field of image processing [1,2]. Graphic fusion is the synthesis of multiple images acquired by different sensors through the same target or the same scene. The synthesized image cannot be acquired by a single sensor. The composite image obtained after image fusion can maintain key information in multiple images, providing more precise conditions for target analysis. The traditional pixel-level image fusion method ignores the mutual relationship between pixels, resulting in poor image quality after fusion, and the visualization effect is not ideal. In this paper, wavelet transform is introduced into the image fusion algorithm, because the image can be separated by high-frequency components and low-frequency components after wavelet decomposition, and then the high-low frequency components are fused using different fusion rules in different frequency domains. This method can improve the target detection resolution and suppress the detection noise of different sensors at the same time[3].

2. Theoretical Basis

2.1 Discrete wavelet transform
The process of wavelet transform essentially refers to the signal being projected onto a set of wavelet functions, and then the signal is decomposed into a series of wavelet functions. Wavelet transform is an integral transform. The function f(t) in any $L_2(\mathbb{R})$ space is expanded under wavelet basis $\varphi_{\alpha,\tau}(t)$ as follows [4]:
\[ WT_f(a, \tau) = \langle f(t), \varphi_{a,\tau}(t) \rangle = \frac{1}{\sqrt{a}} \int_{\tau} f(t) \overline{\varphi\left(\frac{t-\tau}{a}\right)} \, dt \quad (1) \]

Among them, the scale factor and translation factor of the wavelet base are two important parameters of the wavelet base. The process of projecting a time function onto a two-dimensional time-scale phase plane is a wavelet basis expansion process. In wavelet transform, when analyzing low frequency signals, the time window is very large. When the high frequency signal is analyzed, the time window decreases.

In the continuous wavelet transform, the wavelet basis functions \( \varphi_{a,\tau}(t) \) have a large correlation under the continuous scale \( a \) and time \( \tau \) values. This leads to redundant information in coefficients of continuous wavelet transform \( WT_f(a, \tau) \). Redundancy, in order to minimize the redundancy of the wavelet transform without losing the original signal information, the \( a, \tau \) of the wavelet basis function is limited to some discrete points. The scale \( a \) is discretized in power series, taking \( a_m = a_0^m \), where \( m \) is an integer. The time \( \tau \) is uniformly and discretely taken to cover the entire time axis. Therefore, \( \varphi_{a,\tau}(t) \) is expressed as:

\[ \varphi_{m,n}(t) = 2^{-m/2} \varphi(2^{-m}t - n), m, n \in \mathbb{Z} \quad (3) \]

The wavelet transform of arbitrary signal \( f(t) \) can be expressed as:

\[ WT_f(m, n) = \int_{\tau} f(t) \overline{\varphi_{m,n}(t)} \, dt \quad (4) \]

### 2.2 Fast Algorithm for Discrete Wavelet Transform

For any function \( f(t) \in V_0 \), it can be decomposed into detail part \( W_1 \) and large-scale approximation part \( V_1 \), and then the large-scale approximation part \( V_1 \) is further decomposed. By repeating this, the approximation part and the detail part at any scale can be obtained [5].

Let \( f^j(t) \) be the profile signal at the \( j \) scale obtained after the function \( f(t) \) is projected onto the scale space \( V_j \).

\[ f^j(t) = \sum_k c_{j,k} \phi_{j,k}(2^{-j}t) = \sum_k c_{j,k} \phi_{j,k}(t) \quad (5) \]

Where \( c_{j,k} \) is the scale expansion coefficient.

The function \( f(t) \) is projected onto the wavelet space \( W_j \) of different scales, and the detailed information \( f_d^j(t) \) at different scales is obtained.

\[ f_d^j(t) = \sum_k d_{j,k} \psi_{j,k}(2^{-j}t) = \sum_k d_{j,k} \psi_{j,k}(t) \quad (6) \]

Where \( d_{j,k} \) is the wavelet expansion coefficient.

Expand \( f(t) \in L^2(\mathbb{R}) \) according to equation (7), then the function \( f(t) \) is expressed as equation (8).

\[ L^2(\mathbb{R}) = \sum_{j=-\infty}^{\infty} W_j \Theta V_j \quad (7) \]

\[ f(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d_{j,k} \psi_{j,k}(t) + \sum_{k=-\infty}^{\infty} c_{j,k} \phi_{j,k}(t) \quad (8) \]

when \( j \to \infty \).

Equation (8) is transformed into.

\[ f(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d_{j,k} \psi_{j,k}(t) \quad (9) \]

Equation (8) (9) is a comprehensive formula of discrete orthogonal wavelet transform.

### 3. Image Fusion Algorithms

The original image is decomposed into high-frequency and low-frequency regions by wavelet transform, and uses different fusion rules in different frequency domains to preserve the salient features of the original image in different frequency domains [6].

Wavelet transform is used in image fusion algorithm. Firstly, multi-source images are geometrically accurately registered, and appropriate wavelet bases and decomposition layers are selected. The
original image is decomposed into approximate coefficients and detail coefficients by multi-level wavelet decomposition. The fusion wavelet coefficients are fused by different wavelet fusion rules. Finally, the fused image is obtained by inverse transformation of the wavelet coefficients. The principle of the fusion algorithm is shown in Figure 1.

![Figure 1. Fusion Algorithm](image)

### 3.1 Wavelet high frequency coefficient fusion method
In this paper, large absolute coefficients and maximization of regional mean variance are used as fusion criteria for high frequency coefficients. Taking the fusion of two images as an example, the fused image of image 1 and image 2 is image 3. The wavelet coefficients of fused images are Formula 10.

\[
\begin{align*}
D_{N,3}^i &= D_{N,1}^i & \text{if } |D_{N,1}^i| \geq |D_{N,2}^i| \\
D_{N,3}^i &= D_{N,2}^i & \text{else}
\end{align*}
\]

(10)

In the formula, \(D_{N,1}^i\) and \(D_{N,2}^i\) represent the wavelet coefficients of image 1 and 2 in the direction \(i\) on the wavelet decomposition scale \(N\).

In the middle decomposition layer, the wavelet coefficients of image 1 or image 2 are taken as the wavelet coefficients of fused image, that is, the mean variance of region \(3 \times 3\) is the largest.

\[
\begin{align*}
D_{N,3}^i &= D_{N,1}^i & \text{if } \text{MSE1} \geq \text{MSE2} \\
D_{N,3}^i &= D_{N,2}^i & \text{else}
\end{align*}
\]

(11)

MSE1, MSE2 represents the local variance of image 1 and image 2 on the decomposition scale, and the mean variance MSE is defined as:

\[
\text{MSE} = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (X_{i,j} - \bar{X})^2
\]

(12)

Where \(M\) represents the number of rows in the region, \(N\) represents the number of columns in the region, \(X_{i,j}\) represents the gray value of the local region, and \(\bar{X}\) represents the average gray value of the local region.

### 3.2 Wavelet low-frequency coefficient fusion method
In this paper, the selection of low-frequency scale coefficients is based on the combination of averaging and selection proposed by Burt, and \(\text{SA}(X, p)\) is used to represent the significance of the scale factor of image \(X\) at point \(p\).

\[
\text{SA}(X, p) = \sum_{q \in Q} w(q) C_N^2(X, q)
\]

(13)

\(Q\) represents the energy in a small area, \(w(q)\) is a weight, \(C_N^2(X, q)\) is a wavelet coefficient of the image \(X\) at the \(q\) point, and the matching matrix \(R\) is defined as:

\[
R(p) = \frac{2 \sum_{q \in Q} w(q) C_N(A,q) C_N(B,q)}{\text{SA}(A,p)+\text{SA}(B,p)}
\]

(14)
The R value is close to 1 the image correlation is high, when the R value is larger, the weighted scale coefficient of the fused image two image scaling coefficient average, fusion function can be expressed as:

\[ C_N(F, p) = W(A, p) \cdot C_N(A, p) + W(B, p) \cdot C_N(B, p) \] (15)

4. Evaluation Indicators

In order to measure whether the image fusion method has good reliability and robustness, the quality and performance of the fused image should be evaluated. In addition to subjective fused image quality evaluation, this paper uses three objective evaluation indicators: entropy (E), mutual information (MI) and average gradient (AG) are selected to evaluate the effect of fusion image.

Image entropy is an important index to evaluate the richness of image information. The size of the entropy reflects the size of the average information of the image. Let the gray distribution of the fused image be \( p = \{p_1, p_2, ..., p_i, ..., p_n\} \), \( p_i \) be the ratio of the gray value of the first pixel to the gray value of the total pixel, \( n \) be the total gray level, and the entropy be defined as:

\[ E = -\sum_{i=1}^{n} p_i \log_2 p_i \] (16)

The average gradient is a measure of image sharpness. The bigger the average gradient is, the higher the definition of the image is. The average gradient is defined as:

\[ AG = \frac{1}{(M-1)(N-1)} \sum_{i=1}^{M-1} \sum_{j=1}^{N-1} \sqrt{\frac{(F(i+1,j) - F(i,j))^2 + (F(i,j+1) - F(i,j))^2}{2}} \] (17)

If the gray level distribution of the original image 1 is \( q = \{q_1, q_2, ..., q_i, ..., q_n\} \) and the joint probability density function of the two images is \( \gamma_{ij} \), the mutual information between the fused image and the original image 1 can be expressed as follows:

\[ MI_1 = \sum_{i=1}^{n} \sum_{j=1}^{n} y_{ij} \log_2 \frac{y_{ij}}{p_i q_j} \] (18)

The mutual information between the fused image and the two original images is represented by MI.

\[ MI = MI_1 + MI_2 \] (19)

5. Experiment

To test the effectiveness of the proposed image fusion algorithm. In this paper, three groups of images with different focus are used for fusion experiments. The focus of each group of original images is distributed on different distant and near targets. The original images 1 and 2 in figure. 2, 3, 4 focus on the near and far respectively. The operation of two-dimensional wavelet decomposition, fusion and reconstruction is used to realize the fusion effect.

From the fusion effect of the three sets of images, the fusion algorithm used in this paper can perform better fusion processing on multifocal image. From the quality of the fused image, it can be seen that the image fusion method based on wavelet transform is better than that based on direct image fusion. Using wavelet transform to fuse the two original images can make up for the different defects of the two images, and obtains a relatively complete and clear image.

It can be seen from Table 1 that the fused image has a large entropy and an average gradient value, which indicates that the fused image contains rich information and has high definition. It can be seen from the mutual information value that the fusion image has a great correlation with the original image.
Figure 2. Image fusion experiment 1

Figure 3. Image fusion experiment 2

Figure 4. Image fusion experiment 3

Table 1. Evaluation results of fusion results

| Evaluation index         | E   | MI   | AG   |
|--------------------------|-----|------|------|
| Figure 2 fusion image    | 7.3101 | 6.2176 | 3.7805 |
| Figure 3 fusion image    | 7.1772 | 6.5183 | 3.4415 |
| Figure 4 fusion image    | 7.0561 | 6.3124 | 3.9451 |
6. Conclusion

Image fusion algorithm takes advantage of the human eye's sensitivity to local contrast transformation. According to certain fusion rules, some significant features are selected from multiple original images, and these features are retained in the final fused image. Different fusion algorithms are used for different decomposition layers and different frequency bands. The neighborhood-variance of wavelet coefficients is used to define the fusion factor, which can make good use of the time-frequency domain features of wavelet transform.

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