With data from Pantheon, we have at our disposal a sample of more than a thousand supernovae Ia covering a wide range of redshifts with good precision. Here we make fits to the corresponding Hubble–Lemaître diagram with various cosmological models, with intergalactic extinction, evolution of the luminosity of supernovae, and redshift components due to partially non-cosmological factors. The data are well fitted by the standard model to include dark energy, but there is a degeneracy of solutions with several other variables. Therefore, the Hubble–Lemaître diagram of SNe Ia cannot be used alone to infer the existence of the accelerated expansion scenario with dark energy.

Within this degeneracy, models that give good fits to the data include the following alternative solutions: Einstein–de Sitter with gray extinction $a_V = 1.2 \times 10^{-4}$ Mpc$^{-1}$; linear Hubble–Lemaître law static Euclidean with gray extinction $a_V = 0.4 \times 10^{-4}$ Mpc$^{-1}$; Static Euclidean with tired light and gray extinction $a_V = 2.8 \times 10^{-4}$ Mpc$^{-1}$; Einstein–de Sitter with absolute magnitude evolution $\alpha = -0.10$ mag Gyr$^{-1}$; Friedmann model with $\Omega_M = 0.07 - 0.29$, $\Omega_\Lambda = 0$ and partially non-cosmological tired-light redshifts/blueshift with attenuation/enhancement $|K_i| < 2.2 \times 10^{-4}$ Mpc$^{-1}$ (although requiring calibration of $M$ incompatible with local SNe measurements).

Keywords: cosmology; dark energy; supernovae Ia

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1. Introduction

In 1998 and 1999 Riess et al.\textsuperscript{1} and Perlmutter et al.\textsuperscript{2} published papers establishing the existence of an accelerated expansion from the analyses of type Ia supernovae (SNe Ia). Their sample contained few stars; for instance, Riess et al. analyzed data from 10 type Ia supernovae (SNe Ia) with redshifts between 0.16 and 0.62. Along with earlier work, they augmented the sample with 34 other nearby supernovae and 16 more distant ones. The luminosity distance measured with supernovae exceeded by up to 0.25 to 0.28 magnitudes that expected in the standard model in the early 1990s (open universe with matter density $\Omega_M \sim 0.2$). This excess could be explained by adding a positive cosmological constant. They saw that the fit determined an
accelerating expanding universe \([q_0 \leq 0; q_0 = \frac{\Omega_M}{2} - \Omega_\Lambda]\) with a confidence level in the range 99.5–99.9 percent. The best fits gave the value of the parameters \(\Omega_M = 0.24^{+0.56}_{-0.24}, \Omega_\Lambda = 0.72^{+0.48}_{-0.72},\) and \(H_0 = 65.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1} ,\) where the newly introduced \(\Omega_\Lambda \neq 0\) stands for a cosmological constant or quintessence component. With more recent, larger, and more accurate sample of data by Ref. 3 (Pantheon data), the results of Riess et al. and Perlmutter et al. for a universe with a positive cosmological constant are confirmed with much lower error bars (see §2).

There are other ways to obtain a fit to the data of SNe Ia. We now analyze the different fitting alternatives, focusing on scenarios where no \(\Lambda\) term is included.

2. SNe Ia data: Pantheon

The 'Pantheon' sample of SNe Ia\(^3\) is currently one of the largest samples with highest redshift supernova data. Its redshift range spans between \(z = 0.001\) and \(z = 2.3\), thus extending the range far beyond \(z = 1\). We use the 2018 version of this sample with 1048 SNe. Just after we did our analyses, a more recent version was released (Pantheon+\(^4,5\)) with 1550 SNe Ia, most of the new sources being low \(z\) SNe; the maximum redshift is not higher. In any case, for our purposes, the Pantheon sample is more than enough to test those cosmological scenarios that are more sensitive to the high \(z\) sources.

Unlike previous samples where the \(\Delta M\) errors (where \(M\) is the assumed absolute magnitude of SNe Ia) were given, Pantheon provides only the redshifts and the distance moduli with their uncertainty along with the covariance matrix. Here we simply use the redshifts and apparent magnitudes of SNe Ia to produce the fits detailed in the following sections. There is some discussion on the discrepancies of redshifts and magnitudes of Pantheon with other catalogues and on how systematic errors in redshift or magnitude measurements can affect the measured parameters\(^6,7\). We do not enter in that discussion here; we simply assume that the numbers in the Pantheon table are correct, and we question whether other cosmological scenarios without dark energy can reproduce these data with the same accuracy as the official published results of Pantheon SNe analyses.

3. Fit with the standard \(\Lambda\)CDM model

By knowing the intrinsic luminosity \(L\) of a supernova and measuring the observed flux \(F\), we can obtain an estimate of the distance-luminosity to each of these supernovae.

\[
D_L = \left( \frac{L}{4\pi F} \right)^{(1/2)}
\]  

(1)

Considering as cosmological parameters the Hubble constant, \(H_0\), the mass density, \(\Omega_M\), and the density due to dark energy, \(\Omega_\Lambda\), the expression for the luminosity distance in a Friedmann–Lemaître–Robertson–Walker (FLRW) Universe without
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The luminosity distance is:

\[
D_L = \frac{c}{H_0} (1 + z) |\Omega_K|^{-1/2} \sinh \left[ \Omega_K^{1/2} \int_0^z \frac{dx}{E(x)} \right], \tag{2}
\]

\[
E(z) = \sqrt{\Omega_M (1 + z)^3 + \Omega_K (1 + z)^2 + \Omega_\Lambda},
\]

where \( \Omega_K = 1 - \Omega_M - \Omega_\Lambda \) and \( \sinh \) is \( \sinh \) for \( \Omega_K \geq 0 \), \( \sin \) for \( \Omega_K \leq 0 \). If we give this distance in Mpc, we have the following distance modulus:

\[
\mu_p = 5 \log D_L + 25 \tag{3}
\]

There is a degeneracy in the values of \( H_0 \) and in the absolute magnitude \( M \). To avoid this degeneracy, we set an equivalent formulation with a single free parameter \( A \) containing the two previous ones. The function to be fitted is:

\[
y = 10^{0.2m} = A D_{L70}, \tag{4}
\]

\[
A = \frac{70 \text{ km s}^{-1}\text{Mpc}^{-1}}{H_0} 10^{0.2M+5}, \tag{5}
\]

where \( D_{L70} \) is the calculated luminosity distance for \( H_0 = 70 \text{ km s}^{-1}\text{Mpc}^{-1} \). Thus, both \( H_0 \) and \( M \) (absolute magnitude) are included in \( A \).

The fit is done through a minimization of \( \chi^2 \), minimizing the differences of \( y \) between the data and the theoretical models. For the standard \( \Lambda CDM \) model, \( \Omega_K = 0 \), leaving \( A \) and \( \Omega_M \) as free parameters. Results of the fit are shown in Table 1 which are compatible with those obtained by Ref. 3. We also provide the values of the probabilities \( Q \) associated with the \( \chi^2 \) values for the corresponding degrees of freedom (=1048 - number of free parameters). In Fig. 1 we show the logarithm of the quantity \( y \) versus the log of \( z \).

The chi-square test already takes into account the difference in the number of free parameters by reducing the degrees of freedom, and the \( Q \) probability depends on these degrees of freedom. Information Criterion methods such as AIC, BIC or KIC present different estimators with a somewhat higher dependence on the number of free parameters, but no major differences arise when \( N \) is a very large number. They are equivalent to a modification of \( \chi^2 \), \( \chi^2_{mod} = \chi^2 + u \), where \( p \) is the number of free parameters and \( u \) is a parameter that depends on the information criterion.$^8$ \( u = 1 \) for the conventional minimization of \( \chi^2 \); \( u = 2 \) for AIC, \( u = 3 \) for KIC; and \( u = ln(N) \) for BIC. Given than \( N \) is much larger than \( u p \) for any of the methods, the numbers of \( \chi^2_{mod} \) are almost independent of the criterion.

4. Alternative cosmologies

In this section we will fit Pantheon data for alternative cosmologies different from the standard \( \Lambda CDM \) model.$^9,10$ Results in Table 1.
Fig. 1. Fit of Pantheon data with ΛCDM, Ω_M = 0.287.

**FLRW with curvature and dark energy**: We repeat the fit of the standard model, but for a model in which we do not impose a flat universe. We obtain a slightly better fit and a significantly greater value for Ω_M. We obtain as best fit a curvature parameter of Ω_K = -0.19. However, although the fit to the supernova data is better, other observations show that the universe has effectively zero curvature, as shown, for example, by Cosmic Microwave Background Radiation (CMBR) analyses.[11]

**FLRW with curvature without dark energy**: We now consider an FLRW cosmology without dark energy and satisfying Ω_K = 1 - Ω_M, leaving both A and Ω_M as free parameters. In this way, we obtain a negative-curvature cosmology (flat if Ω_M = 1). In the fit we obtain a value of practically zero got Ω_M. The fit is worse than in the case of the ΛCDM and FLRW model without curvature and dark energy. This is equivalent to the Milne Universe that we will comment later. The low value of probability Q almost completely rules out this model.

**Einstein-de Sitter**: The Einstein-de Sitter (EdS) model is equivalent to an FLRW cosmology with Ω_M = 1 and Ω_Λ = 0. We can use the same fitting method but imposing those values, with a single free parameter (A). As expected,
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TABLE 1: Best fits of Pantheon data ($N = 1048$ supernovae Ia) with different cosmological models, with extinction, evolution, or a non-cosmological LSV redshift component. $Q$ is the probability associated with $\chi^2$ with degrees of freedom $= N - p$, where $p$ is the number of free parameters.

| Model                  | Free parameters                                                                 | $\chi^2$ | $Q$ |
|------------------------|-------------------------------------------------------------------------------|----------|-----|
| $\Lambda$CDM          | $A = 13.389 \pm 0.041$ , $\Omega_M = 0.287 \pm 0.012$ , $\Omega_\Lambda = 1 - \Omega_M$ | 1024.3   | 0.678 |
| FLRWcurv.A             | $A = 13.320 \pm 0.056$ , $\Omega_M = 0.355 \pm 0.039$ , $\Omega_\Lambda = 0.835 \pm 0.067$ | 1021.1   | 0.696 |
| FLRWcurvA = 0          | $A = 13.909 \pm 0.039$ , $\Omega_M = 0 \pm 0.024$                             | 1147.1   | 0.015 |
| EdS                    | $A = 14.832 \pm 0.043$                                                       | 2395.4   | 0    |
| QSSC                   | $A = 14.444 \pm 0.067$ , $\Omega_M = 1.439 \pm 0.068$ , $\Omega_\Lambda = 0 \pm 0.042$ , $\Omega_c = 1 - \Omega_M - \Omega_\Lambda$ | 1635.6   | 0    |
| $R_h = ct$             | $A = 14.091 \pm 0.030$                                                       | 1296.3   | 0    |
| Milne                  | $A = 13.909 \pm 0.029$                                                       | 1147.1   | 0.016 |
| Static.lin.Hub.        | $A = 14.047 \pm 0.029$                                                       | 1239.1   | 0    |
| Static tired light     | $A = 15.559 \pm 0.061$                                                       | 4374.1   | 0    |
| St.tir.l. Compton      | $A = 12.493 \pm 0.033$                                                       | 2021.0   | 0    |
| St.tir.l.tim.dil.      | $A = 14.091 \pm 0.030$                                                       | 1296.3   | 0    |
| EdS extinction         | $A = 13.487 \pm 0.045$ \hspace{1cm} $a_V = (1.156 \pm 0.031) \times 10^{-4}$ Mpc$^{-1}$ | 1050.8   | 0.453 |
| St.lin.Hub.ext.        | $A = 13.566 \pm 0.045$ \hspace{1cm} $a_V = (0.403 \pm 0.031) \times 10^{-4}$ Mpc$^{-1}$ | 1065.2   | 0.333 |
| St.tir.l.ext.          | $A = 12.959 \pm 0.051$ \hspace{1cm} $a_V = (2.775 \pm 0.048) \times 10^{-4}$ Mpc$^{-1}$ | 1072.9   | 0.275 |
| $\Lambda$CDM evol.     | $A = 13.257 \pm 0.084$ , $\Omega_M = 1 \pm 1.406$ , $\Omega_\Lambda = 1 - \Omega_M$ , $\alpha = -0.102 \pm 0.178$ Gyr$^{-1}$ | 1020.4   | 0.701 |
| FLRWcurv.A.evolut.     | $A = 13.268 \pm 0.084$ , $\Omega_M = 0.957 \pm 0.162$ , $\Omega_\Lambda = 0$ , $\alpha = -0.099 \pm 0.066$ Gyr$^{-1}$ | 1020.3   | 0.702 |
| EdS evol.              | $A = 13.257 \pm 0.048$ , $\alpha = -0.102 \pm 0.003$ Gyr$^{-1}$              | 1020.4   | 0.709 |
such a restricted model obtains a very poor fit and is completely discarded. As we will see later, if we add certain factors to this model (evolution, extinction, or partially non-cosmological redshifts) we can obtain much better fits.

Quasi-steady state: The Steady State model establishes a universe that, in addition to the cosmological principle (homogeneity and spatial isotropy), is homogeneous in time. To compensate for the expansion of the universe, there is a field that continuously creates matter to keep the density constant. This model presented several problems and was later modified to become the quasi-steady state (QSSC) model. In addition to the matter creation field, we have an expansion with an oscillatory term. In this model, calling \( \Omega_c \) the matter creation field we have:

\[
D_L(z) = \frac{c}{H_0} (1 + z) \int_0^z \frac{dz}{\sqrt{\Omega_c (1 + z)^4 + \Omega_m (1 + z)^3 + \Omega_\Lambda}} \tag{6}
\]

We see that the term \( \Omega_c \) behaves like a radiation term in FLRW, which also goes as \((1 + z)^4\). A fundamental problem with QSSC is that no galaxies should be observed at \( z > 6 \), but galaxies have been observed with redshifts as high as \( z = 8.6 \). With the Pantheon data and without taking into account other factors, we obtain a fit that completely rules out this model. There have been previous attempts to fit supernova Ia data with this model with apparently very good results, but in that paper they also take dust extinction into account.

\( R_h = ct \): This is an FLRW model in which we have an equation of state, \( \rho + 3p = 0 \),
thus leading to expansion, \( R_h = ct \). The luminosity distance is:

\[
D_L(z) = \frac{c}{H_0} (1 + z) \ln(1 + z).
\]  

(7)

The fit, being better than the QSSC and the Einstein–de Sitter models, is still ruled out by the Pantheon data. Other authors\(^{[43]}\) have stated that this model fits the supernovae diagram, but only after re-evaluating the calibration of their luminosities and putting their absolute magnitude as a function of several adjustable parameters instead of being constant.

**Milne Universe:** This model is a special case of the FLRW metric in which we consider zero density, pressure, and cosmological constant. This results in a linear time dependence of the scale factor. We have the following expression for the luminosity distance:

\[
D_L(z) = \frac{c}{H_0} (1 + z) \sinh[\ln(1 + z)].
\]  

(8)

Despite being a model with very restricted parameters, the fit is better than that of the other models considered. However, Milne’s universe is unable to explain other contrasting facts of cosmology and such as the CMBR, the abundance of light elements.

**Static Euclidean with linear Hubble-Lemaître law:** In this case, we consider a static universe in which we have a redshift term due to energy loss without expansion (no time dilation), and the linear Hubble–Lemaître law \( cz = H_0 D \) is maintained even at high redshift. The luminosity distance is:

\[
D_L(z) = \frac{c}{H_0} \sqrt{(1 + z)z}.
\]  

(9)

The factor \( \sqrt{(1 + z)} \) stems from the loss of energy of photons due to non-cosmological redshift. We obtain a setting that rules it out. Moreover, although it is not the subject of this work, a simple static model has many other problems.

**Static Euclidean with tired light:** This variation considers that photons lose energy in their path by some interaction, and that this energy loss is proportional to the length traveled, \( \frac{dE}{dr} = -\frac{H_0}{c} E \). This modifies the luminous distance as follows:

\[
D_L(z) = \frac{c}{H_0} \sqrt{(1 + z)\ln(1 + z)}.
\]  

(10)

In view of the results, this consideration of energy loss is totally incompatible with the Pantheon data and greatly worsens the fit of the simplest linear model.

**Static Euclidean with tired light-plasma/Compton:** First, we assume a Compton scattering factor, modifying the expression to become:

\[
D_L(z) = \frac{c}{H_0} (1 + z)^{3/2} \ln(1 + z).
\]  

(11)
Although it represents an improvement in the fit with respect to the tired light model, it is still far from the simple linear model, which had already been discarded.

**Static Euclidean with tired light-plasma/time dilation:** Let us try another tired photon model with a scattering that gives rise to a time dilation that broadens the light curves of supernovae with redshift:

$$D_L(z) = \frac{c}{H_0}(1 + z)\ln(1 + z).$$

(12)

In this case we have a much better fit that is still, however, ruled out by the data.

5. **Including extinction**

A progressive flux reduction with redshift is seen in the supernova data. This can be explained by a universe with a positive cosmological constant (dark energy). Another explanation is the presence of dust particles in the medium that passes through the light. This dust absorbs light in the optical and re-emits it in the far infrared. Riess et al. state that the effect of extinction is too small to be taken into account in the analysis. Still, with more recent data, with more supernovae measured with greater precision, it is important to reanalyze this effect to see whether it is really minimal, or whether there is simply no need for a dark energy term. Ref. [15] consider three types of extinction:

**Large gray intergalactic dust grains:** Following a model of Ref. [16], large grains are considered because the smaller ones have been destroyed. Ref. [15] perform a Monte Carlo simulation to obtain an adjustment of the cosmological parameters while taking into account this extinction factor (whose values depend, in turn, on the cosmological model itself).

**Dust in galaxies along the line of sight:** In this case, they perform several ray-tracing Monte Carlo simulations to estimate the probability, as a function of $z$, of a ray passing close to the center of a galaxy. Again, without going into much detail, they find that the probability of a supernova at $z \sim 1$ being obscured by plus 0.02 mag is only 0.33%. They claim that this value is highly dependent on the dust density normalization itself. On the other hand, a galaxy acts as a gravitational lens that would have the opposite effect of extinction dimming.

**Extinction in the host galaxy:** Since an SN Ia occurs in both early- and late-type galaxies, obscuration due to dust from the host galaxy would have to be considered in the late-type case. By modeling the galaxy and estimating at 1/8 the probability of the supernova occurring in the bulge rather than in the disk, the simulations performed conclude that about 2600 out of 10000 sources would be obscured by more than 0.02 mag by dust from the host galaxy. Nonetheless, attention to new analyses that show that Type
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Ia Supernova brightness correlates with host galaxy dust content, which might be interpreted as an indication that dust in host galaxies is more important than we thought.

Ref. [15] conclude that the overall effect of extinction would be very small, since only 1% of the supernovae at $z = 1$ would be obscured by more than 0.02 mag (the detection limit of the SNAP probe).

5.1. Fits including extinction

Introducing an extinction term will make very distant galaxies appear less luminous. This effect can counteract that of dark energy and thus provide a good fit. Assuming a constant comoving dust density term, $\kappa$ being the absorption coefficient per unit mass, we have the following expression for luminosity:

$$L_{V,\text{rest}} = 4\pi F_{V,\text{rest}}D_L^2 e^{\rho_{\text{dust}} \int_0^d \kappa [\lambda_V (1+z)/(1+z)]} \left[ \frac{1+z}{1+z} \right]^m, \quad (13)$$

where the comoving distance $d$ in the corresponding cosmology associated with the redshift of the supernova. We assume a absorption coefficient with a wavelength dependence,

$$\kappa(\lambda) = \kappa(\lambda_V) \left( \frac{\lambda}{\lambda_V} \right)^{-\beta}, \quad (14)$$

and adopt $\beta = 2$.[8] With this we obtain:

$$L_{V,\text{rest}} = 4\pi F_{V,\text{rest}}D_L^2 e^{\frac{a_V}{m(1+z)^m} \left[ (1+z)^m - (1+z)^{-\beta} \right]}, \quad (15)$$

$a_V \equiv \kappa(\lambda_V) \rho_{\text{dust}}$ being the absorption in V per unit length. Together with $A$, we shall fit the parameter $a_V$. Regarding the other parameters, $m = 1$ for the Einstein-de–Sitter and Linear Static models; and $m = 0$ for the static model with tired light. See the best fits in Fig. 2. The attenuations $a_V = 0.4 - 2.8 \times 10^{-4}$ Mpc$^{-1}$ obtained are within the possible range of values. Assuming $\kappa(\lambda_V) \sim 10^5$ cm$^2$/gr,[13] the value for the dust density necessary to produce such an extinction would be $\rho_{\text{dust}} \sim 10^{-34} - 10^{-35}$ g/cm$^3$, which is within the range of possible values. Ref. [19] allow values as high as $\rho_{\text{dust}} \sim 10^{-35}$ g/cm$^3$ for the high $z$ IGM in the standard concordance cosmology. For comparison, the average baryonic density of the Universe (taking $\Omega_b = 0.042$) is $\rho_b = 3.9 \times 10^{-31}$ g/cm$^3$, so this would mean that IGM dust constitutes 0.025–0.25% of the total baryonic matter (reasonable amounts). Results are shown in Table 1.

**Einstein-de Sitter with extinction** We see a substantial improvement in the fit over the model without extinction. If we consider only the Pantheon data, this would be a model to consider.

**Static Euclidean with linear Hubble law with extinction:** We see that the fit is also considerably better when an extinction term is included.
Static Euclidean with tired light with extinction: Despite being a worse fit than the previous two, compared to the model without extinction, the improvement is great indeed (going from $\chi^2 = 4374.1$ to 1072.9).

6. Including evolution

One of the assumptions made in the work of Riess et al. was that the shape of supernova light curves has negligible variation with redshift, so that well-defined models of nearby supernovae could be applied to distant supernovae. With more recent data, reaching higher redshift, perhaps this effect can no longer be neglected completely. At higher redshift we have more massive progenitor stars with lower metallicity. This influences the composition of their degenerate CO core. This variation may have implications for ignition due to accretion, so that the supernova light curve varies, having a difference of 0.2 mag for the luminosity maximum, comparable to the brightness variation indicating dark energy.\(^{20}\) Or there may possibly be evolution of the luminosity if there is an evolution of the physical constants.\(^{21}\)

As a matter of fact, there is a systematic difference of $\sim 0.14$ mag between supernovae whose host galaxies are of very early and very late type.\(^{22}\) In addition, there is a correlation between the mass of the host galaxy and the brightness of the SNe Ia, suggesting that in less massive galaxies we have $\sim 0.1$ mag lower brightness than in massive galaxies. There is also a correlation indicating that in high star formation environments, SNe Ia are less bright than in more passive environments. All of these correlations actually indicate a relationship between supernova brightness, and the age and metallicity of the progenitor. We have a brightness variation due to evolution that is comparable to the margin indicated by a positive cosmological constant ($\sim 20\%$). This has led several researchers to point out that an evolution of SNe Ia luminosity can fit their Hubble–Lemaître diagram without dark energy.\(^{22}27\)

6.1. Fits including evolution

In our analysis, we use a simple expression to model cosmic time-dependent evolution and a parameter $\alpha$. We fit cosmological models without dark energy. We assume that the absolute magnitude of supernovae varies over cosmological time, using the simple evolution equation

$$M = M_0 - \alpha [t(0) - t(z)],$$

(16)

where $M_0$ is the absolute magnitude without taking evolution into account and $t(z)$ is the age of the universe at a given $z$ [$t(0)$ for $z = 0$]. We calculate that age for each given $z$ and incorporate the new $M$ into the formula for the $A$ parameter fit:

$$A_{\text{evol}} = A \times 10^{-0.2\alpha [t(0) - t(z)]},$$

(17)

where

$$t(z) = \frac{1}{H_0} \int_z^\infty (1 + x)^{-1} E(x) dx.$$  

(18)
Results in Table 1.

**ΛCDM with evolution:** We repeat the setting for the standard model but with a magnitude that takes into account the evolution with redshift. We get a small improvement in the value of $\chi^2$. But the most striking thing is that we obtain a value of $\Omega_M$ of practically 1, although with a very large error. That value would produce (for a flat metric) $\Omega_\Lambda \sim 0$. That is, introducing an evolutionary term allows us to fit the data without the need for dark energy. Refs. 26, 27 obtained a correlation magnitude–age of galaxies (early type galaxies having a passive evolution, which is supposed to trace the age of the universe) with lower $\alpha$ for $\Omega_m = 0.27$, $\Omega_\Lambda = 0$, and for a lower redshift sample: $\alpha$ between -0.06 and -0.04 Gyr$^{-1}$.

**Friedmann with curvature and dark energy and with evolution:** We do the same but in this case we do not impose a null curvature. We impose a starting value of zero for dark energy (since we obtained zero for the ΛCDM case without curvature) to see if evolution can fit the data well without taking it into consideration. By adjusting one parameter less, we have a lower uncertainty for the value of $\Omega_M$. This means that the value is very close to unity, but somewhat smaller. Since $\Omega_\Lambda = 0$, that implies a positive curvature of $\Omega_K = 0.043$. Regarding the fit itself, we get a slightly better value. That is, with the Pantheon data, we get better fits by introducing evolution (and also with a very simple model) than when considering the dark energy term alone.

**Einstein–de Sitter with evolution:** Finally we fit to an Einstein–de Sitter model with evolution, which is equivalent to a ΛCDM model but imposing $\Omega_M = 1$ and $\Omega_\Lambda = 0$. We obtain a very good fit. This is the model that gives a better value for the $Q$ probability. In addition, we have smaller uncertainties by fitting only two parameters ($A$ and $\alpha$).

We see, then, that with the Pantheon data a model with evolution is just as valid (or better) than one with dark energy. Moreover, when trying to fit with both terms, the best value of $Q$ is for the one that gives $\Omega_\Lambda \sim 0$. See the best fit in Figure 3.

As said by Ref. 23, the result is that cosmological models and evolution are highly degenerate with one another, so that the incorporation of evolution into even very simple models renders it virtually impossible to pin down the values of $\Omega_M$ and $\Omega_\Lambda$.

### 7. Partially non-cosmological redshifts

If we add a partially non-cosmological redshift or blueshift we will obtain supernova distances that are different from those obtained in equation (2). The non-cosmological redshift component may be due to the non-conservation of the energy-momentum tensor of a photon propagating through electromagnetic fields [Lorentz–
Poincaré Symmetry Violation (LSV)\cite{28} or Mach effects that relate tired light with the mass of the Universe\cite{29,30} or other non-standard effects. The Hubble-Lemaître diagram could be fitted without the need for dark energy. We now study four simple non-cosmological redshift models.

7.1. Fits including non-cosmological redshifts

We consider models in which the total redshift is the sum of the redshift due to expansion plus a non-cosmological term.

The excessive dimming of very distant galaxies can be attributed to this combined redshift without the need to include a dark energy term. For a SN Ia we have the relation:

\[ m = M + 5 \log_{10}(d_L(z_c)(\text{Mpc})) + 25 \]  

Where \( z_c \) is the cosmological redshift and \( z_{LSV} \) is the non-cosmological redshift, which depends on the model. We have four different models, as seen in Table 2.

### TABLE 2: Four different models with non-cosmological redshifts.

| Type | 1 | 2 | 3 | 4 |
|------|---|---|---|---|
| \( d_L \) | \( k_1 \nu e^{k_1 r} \) | \( k_2 \nu e^{k_2 r} \) | \( k_3 r \) | \( k_4 \nu_0 e^{k_4 r} \) |
| \( \nu_0 \) | \( \nu e^{k_5 r} \) | \( \nu e(1 + k_2 r) \) | \( \nu e + k_3 r \) | \( \frac{r}{1 - k_4 r} \) |
| \( z_{LSV} \) | \( e^{-k_1 r} - 1 \) | \( -\frac{k_2 r}{1 + k_2} \) | \( -\frac{k_3 r}{\nu e + k_3 r} \) | \( -\frac{k_4 r}{1 - k_4 r} \) |
| \( r_{LSV} \) | \( \frac{z_{LSV}}{k_5} \) | \( \frac{z_{LSV}^2}{k_5(1 + z_{LSV})} \) | \( \frac{z_{LSV}^3}{k_5(1 + k_{LSV})} \) | \( \frac{z_{LSV}^4}{k_5(1 + k_{LSV})^2} \) |

In model 1, the variation in frequency is proportional to the instantaneous frequency and the distance; in model 2, to the emitted frequency (\( \nu e \)) and the distance; in model 3 only to the distance; and in model 4, to the observed frequency (\( \nu_0 \)). We see that model 3 is equivalent to model 2 if we make \( k_3 = \nu e k_2 \). Since the Pantheon data are all in the B-band, we assume the value \( \nu e = 6.74 \times 10^{14} \text{ s}^{-1} \). We need an iterative function to obtain the values of \( z_c \), \( r \), and \( z_{LSV} \), since they depend on each other. For this we will use the equations (19), (20), and the relations in Table 2.

\[ r(z_c) = c[t(0) - t(z_c)], \]  

where \( t(z) \) is the age of the universe at redshift \( z \), given by Equation (18). Positive values for \( k \) indicate a blueshift with a small, or even decreasing, change in the amplitude \( A \). This non-cosmological blueshift increases the magnitude of the SNe Ia at high redshift. With negative values, we obtain a non-cosmological redshift that increases the value of \( A \) and a reduction in photon frequency with distance traveled.

Results of best fits are shown in Table 1.
Einstein–de Sitter + LSV non-cosmological redshift: With model 1, by including the non-cosmological redshift we obtain a better fit than the original one, although still far from that obtained by including extinction or evolution. In view of the results, the Pantheon data would rule out this model. The negative value of $k_1$ indicates that we have redshift. Model 2 and 3 produce a better fit than model 1, but are also practically ruled out ($Q = 0.0008$). Model 4 gives a result very similar to that of model 1, so it is also discarded. Similar results were obtained with the Pantheon sample by Ref. 31.

FLRW with curvature, $\Omega_\Lambda = 0 +$ LSV non-cosmological redshift: Good fits for models 1–3, and a very good fit for model 4, with a quite remarkable best free parameter $\Omega_M = 0.26 \pm 0.03$ (hence, $\Omega_K = 0.74 \pm 0.03$). However, model 4 gives an amplitude $A = 3.0 \pm 1.0$, which is $> 4$ lower than the value for $A$ with $\Lambda$CDM. According to Eq. (5), this would mean that the absolute magnitude of SNe Ia is 3 magnitudes brighter than the local calibrated ones, which is an absurd result. For models 1–3 the discrepancy among the $A$ values is also high, much larger than the inaccuracies in the measurement of $M$ for local SNe Ia. Moreover, this cosmological model with such high curvature is also discarded by many other kinds of observational cosmological data. Nonetheless, the mathematical fit, especially with model 4, for whatever reason, is remarkably good. Results with the Pantheon sample of Ref. 31 were similar, although with an unclear dependence of $A$ given that the type of fit was different.

8. Conclusions

The original 1998–1999 work of Refs. 1,2 relied on data from few supernovae with a small range of redshifts. Even so, those papers served to establish the dark energy model with an accelerating expanding universe ($\Lambda$CDM). It is cautiously stressed that, owing to the paucity of data in the sample and the uncertainties involved, such a model cannot be taken as established beyond doubt. In subsequent years, with a larger amount of data and coverage of a wide range of redshifts, the fits have been repeated. Many of these papers confirmed the curveless dark energy model as the best model to fit these data, to the point of being considered the standard model of cosmology.

Other authors, obtaining equally good fits without considering a dark energy term (with more modern data samples) were more critical. Only a year after Riess’s work, there were already authors who performed their own study of the data by wondering whether the extinction due to intergalactic dust was sufficient without the need for dark energy. Others considered possible time evolution of type Ia supernovae. Numerous papers have also been published that examine other cosmologies that could fit the data, including exotic cases such as the Carmeli cosmology, the Lemaître–Tolman–Bondi solution, and relativistic effects on the
In this work we have focused on well-known cosmologies, as well as on simple models for extinction, evolution, and partial redshift not due to expansion. As expected, the standard model continues to fit the data excellently, just as it did prior to the small SNe Ia samples used by Riess et al. and Perlmutter et al. It is very interesting to see, however, that other models, discarded as a basis for current cosmological study, also fit the supernova data.

There are also other evidences that favor the ΛCDM model with Λ non-zero over other models, namely, the cosmic microwave background radiation (CMBR), the large-scale structure of the distribution of galaxies, and the measurement of the age of the oldest objects in the universe. Cosmologies such as the static universe and Einstein–de Sitter have trouble explaining some (if not all) of them; more theoretical developments would be needed to modify such cosmologies to explain these lines of evidence. It is also true that a standard model including dark energy has numerous problems and difficulties with no clear solutions in sight. The nature of dark energy is also a mystery. Associating it with the energy of the quantum vacuum leads to one of the greatest contradictions in physics (over 120 orders of magnitude). In any case, the discussion of the different cosmological tests and the theoretical meaning of dark energy term are not included in the topics discussed here, but only the fit of the Hubble-Lemaître diagram for SNe Ia concerning us here.

Listed below are the models discussed in this paper that fit the Pantheon data well (Q indicates the probability), with values of A similar to the standard model (thus, the value of M is not discrepant with the local SNe Ia measurements):

- ΛCDM, $\Omega_M = 0.287$ ($Q = 0.678$).
- FLRW with curvature, $\Omega_M = 0.355$, $\Omega_\Lambda = 0.835$ ($Q = 0.696$).
- Einstein–de Sitter with extinction ($Q = 0.453$).
- Linear Hubble–Lemaître law static Euclidean with extinction ($Q = 0.333$).
- Static Euclidean with tired light with extinction ($Q = 0.275$).
- FLRW with curvature and with evolution $\Omega_M = 0.957$, $\Omega_\Lambda \approx 0$ ($Q = 0.702$).
- Einstein–de Sitter with evolution $\Omega_M = 1$, $\Omega_\Lambda = 0$ ($Q = 0.709$).

Other cosmologies with a significant non-zero probability (although very low and practically ruled out) and/or values of A very different from the standard one are: FLRW with curvature and $\Lambda = 0$ ($Q = 0.015$) Milne Universe ($Q = 0.016$); and several combinations of cosmological model+partial non-cosmological redshift.

The most interesting result of this analysis is that we have that a model with the same probability as that of the standard model or even slightly higher is the Einstein–de Sitter model with linear time evolution in the absolute magnitude of SNe Ia, with only one free parameter other than amplitude A. Similarly, when adding evolution to ΛCDM-type models with free parameters, the best fit is precisely that with $\Omega_M \sim 1$ and $\Omega_\Lambda = 0$. Obviously, with the values of Q shown in the list, we cannot be sure that the Einstein–de Sitter model is confirmed by the data above...
the ΛCDM (which also has a very high $Q$ value), but it is clear that there is still much to be said about the cosmological parameter fit from distant SNe Ia data.

The inclusion of dark energy (and thus accelerated expansion of the universe) is not necessary in view of this analysis. There is degeneracy in several variables: dark energy, extinction, evolution, partially non-cosmological redshifts (although requiring calibration of $M$ far from compatibility with local SNe measurements), and possibly other parameters that we have not explored here.

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Fig. 2. Fit of Pantheon data with Einstein de–Sitter, Static Euclidean with linear Hubble law, Static Euclidean with tired light cosmological models including $a_V = 1.156 \times 10^{-4}$, $0.403 \times 10^{-4}$, $2.775 \times 10^{-4}$ Mpc$^{-1}$ respectively.
Fig. 3. Einstein–de Sitter with evolution: $\Omega_M = 1$, $\Omega_\Lambda = 0$, $\alpha = -0.102 \text{ Gyr}^{-1}$.