Scissors mode of a rotating Bose-Einstein condensate

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A scissors mode of a rotating Bose-Einstein condensate is investigated both theoretically and experimentally. The condensate is confined in an axi-symmetric harmonic trap, superimposed with a small rotating deformation. For angular velocities larger than \( \omega_///\sqrt{2} \), where \( \omega_// \) is the radial trap frequency, the frequency of the scissors mode is predicted to vanish like the square root of the deformation, due to the tendency of the system to exhibit spontaneous rotational symmetry breaking. Measurements of the frequency confirm the predictions of theory. Accompanying characteristic oscillations of the internal shape of the condensate are also calculated and observed experimentally.

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Bose-Einstein condensates rotating at high angular velocity exhibit spontaneous breaking of rotational symmetry \([1]\). This phenomenon, which is the consequence of two-body repulsive interactions, shows up in the occurrence of considerable deformations of the trapped atomic cloud in the plane normal to the rotation axis. These configurations have been recently observed experimentally using a nearly axi-symmetric harmonic trap, with a small deformation of the trapping potential rotating around the \( z \) axis \([2]\).

Under such conditions the collective modes of the system exhibit new interesting features. In particular one mode in the transverse plane (orthogonal to the rotation axis) corresponds to a shape-preserving oscillation of the atomic cloud with respect to the principal axes \( xy \) of the rotating trap. The frequency \( \omega \) of this scissors-type motion is much smaller than the mean transverse trap frequency \( \omega_// \). This surprisingly low value originates from the fact that the restoring force of the oscillation is proportional to the small trap deformation, while the moment of inertia of the condensate remains finite due to its considerable deformation. This represents a major difference with respect to the traditional scissors mode \([3,4]\) of a non-rotating condensate where both the restoring force and the moment of inertia vanish in the limit of an axi-symmetric trap.

The purpose of this work is to provide both a theoretical and experimental investigation of the problem. We first derive the relevant equations describing the scissors mode. For a rotation frequency \( \Omega \) larger than the critical value \( \omega_///\sqrt{2} \), we show that the scissors mode frequency \( \omega \) vanishes like the square root of the trap deformation.

We then report on the experimental observation of this scissors mode, and we present results which confirm the theoretical predictions with good accuracy. The atoms of mass \( m \) are confined in the transverse plane by the potential

\[
V_//(x, y) = m(\omega_{x,x}^2 + \omega_{y,y}^2)/2 ,
\]

where \( \omega_{x,y}^2 = \omega_1^2(1 \pm \varepsilon) \). The \((x, y)\) coordinates in the rotating frame are deduced from the coordinates in the laboratory frame by a rotation \( \Omega t \) and the trap deformation \( \varepsilon \) is positive. A simple description of the collective oscillations of a rotating condensate is provided by the hydrodynamic equations of superfluids evaluated in the rotating frame:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho(\mathbf{v} - \mathbf{\Omega} \times \mathbf{r})] = 0 ,
\]

\[
m\frac{\partial \mathbf{v}}{\partial t} + \nabla \left[ \frac{mv^2}{2} + V + g\rho - m\mathbf{v} \cdot (\mathbf{\Omega} \times \mathbf{r}) \right] = 0 ,
\]

where \( \rho(\mathbf{r}, t) \) is the spatial density, \( \mathbf{v}(\mathbf{r}, t) \) is the velocity field in the laboratory frame, and \( \mathbf{\Omega} = \mathbf{\Omega}_{//}(u_z \text{ unit vector along the } z \text{ axis}) \). The harmonic confining potential \( V(\mathbf{r}) = V_//(x, y) + m\omega_//^2 z^2/2 \) is time independent in the rotating frame. The parameter \( g \) characterizes the strength of the interatomic interactions and is related to the \( s \)-wave scattering length \( a \) by \( g = 4\pi\hbar^2a/m \). These equations are valid in the Thomas-Fermi limit, where the so-called quantum pressure term can be neglected in \([3,4]\). We shall be interested here in vortex-free solutions for which \( \nabla \times \mathbf{v} = 0 \).

Stationary solutions of these equations can be obtained in the form \( \mathbf{v}_0(\mathbf{r}) = \alpha \nabla (xy) \) for the velocity field and \( \rho_0(\mathbf{r}) = \left[ \mu - m(\omega_//^2x^2 + \omega_{x,y}^2y^2 + \omega_{x,y}^2z^2) / 2 \right] / g \) for the density profile. The density profile has the form of an inverted parabola whose parameters \( (\mu, \omega_{x,y}, \omega_{x,y}) \) are determined in a self-consistent way as a function of \( \Omega \) \([5]\). The value of the parameter \( \alpha \) is determined by the solution(s) of the cubic equation \( \alpha^3 + \alpha(\omega_//^2 - 2\Omega^2) + \omega_//^2 \Omega^2 = 0 \). It is related to the deformation \( \delta = (y^2 - x^2)/(x^2 + y^2) \) of the atomic cloud by the simple expression \( \alpha = -\delta \Omega \). In the present work we will be interested in the so-called normal branch \([6]\), corresponding to the stationary solutions which can be obtained by an adiabatic increase of the angular velocity of the trap, starting from \( \Omega = 0 \) (see Fig. \([6]\)). For \( \Omega \geq \omega_///\sqrt{2} \) these solutions exhibit large

\[
\omega_// = \sqrt{\omega_{x,y}^2 + \omega_{x,y}^2} .
\]
values of the cloud deformation ($\delta \sim 1$) even if the trap deformation $\epsilon$ is much smaller than 1.

An important class of collective oscillations can be derived on top of such stationary solutions by looking for time dependent solutions of the form $\delta \rho(r) = a_0 + a_x x^2 + a_y y^2 + a_z z^2 + a_{xy} xy$ and $\delta v(r) = \nabla(\alpha_x x^2 + \alpha_y y^2 + \alpha_z z^2 + \alpha_{xy} xy)$ where $a_i$ and $\alpha_i$ are time dependent parameters to be determined by solving Eqs. (3) and (4). In the linear limit, one can look for solutions varying in time like $e^{-i\omega t}$. The collective frequencies $\omega$ are then found to obey the equation

$$\omega^8 + c_3 \omega^6 + c_2 \omega^4 + c_1 \omega^2 + c_0 = 0 \quad (4)$$

with the coefficients $c_i$ given in the appendix. In [2] it has been shown that these collective oscillations are dynamically stable when evaluated on the normal branch. The stability conditions for higher multipole oscillations have been studied in [2], where it has been shown that the normal branch becomes dynamically unstable against the production of such excitations for $\Omega \lesssim 0.8 \omega_\perp$ when $\epsilon \to 0$.

For the two external branches (see Fig. 1), one finds that the coefficient $c_0$ in (4) vanishes linearly with $\epsilon$ as $\epsilon \to 0$. In this limit, one of the eigenfrequencies of (4) tends to 0, while the others remain finite. The frequency $\omega$ of the corresponding “soft” mode is:

$$\omega^2 = -\frac{c_0}{c_1} = \pm \frac{10 \omega_\perp^2}{3 \omega_\perp^4 + 3 \omega_\perp^2 \Omega^2 + 2 \Omega^4} \frac{\Omega(\omega_\perp^2 - \Omega^2) \sqrt{2 \Omega^2 - \omega_\perp^2}}{\Omega^2} \epsilon. \quad (5)$$

The positive value for $\omega^2$ corresponds to the normal branch while the negative value, which is a signature of dynamical instability, corresponds to the opposite branch. For the normal branch, the frequency $\omega$ of the low energy mode vanishes like the square root of the trap deformation $\epsilon$, as announced in the introduction of this paper. It is worth noticing that this frequency does not depend on the value of $\omega_\perp$. By looking at the corresponding form of the solution for $\delta \rho$ and $\delta v$, one can show that it corresponds to a scissors mode, i.e. to the oscillation of the angle $\theta$ between the axes of the condensate and the principal axes $x, y$ of the trap. The oscillation of $\theta$ is accompanied by an oscillation of the condensate deformation $\delta$ around the equilibrium value $\delta_0$. This oscillation of $\delta$ is dephased by $\pi/2$ with respect to the one of the angle $\theta$. For small trap deformations we find

$$\delta(t) - \delta_0 = \frac{\omega_\perp^2}{ \Omega^2 \sqrt{2 \Omega^2 - \omega_\perp^2} } \sin \frac{\theta}{\Omega_\perp}. \quad (6)$$

The fact that the frequency of the scissors mode vanishes when $\epsilon = 0$ (see Eq. (5)) reflects the rotational invariance of the Hamiltonian. This behavior deeply differs from the scissors mode in a non rotating condensate, whose frequency is given by $\omega_\perp \sqrt{2}$ and does not vanish in the limit of a symmetric trap [3]. The result (5) holds for a small amplitude $\Delta \rho$ of the oscillatory motion of $\theta$. To study motions with a larger amplitude, one has to solve numerically the hydrodynamic equations (2) and (3). We find that the frequency of the motion decreases as $\Delta \rho$ increases, as for a simple gravitational pendulum.

We now turn to the experimental observation of this scissors mode. We use a $^{87}$Rb gas in a Ioffe-Pritchard magnetic trap, with frequencies $\omega_x/2\pi = 182$ Hz, $\omega_y \simeq \omega_x$, and $\omega_z/2\pi = 11.7$ Hz. The cloud is pre-cooled using optical molasses to a temperature $\sim 100 \mu$K. The gas is further cooled by radio-frequency evaporation to a temperature around 50 nK, corresponding to a quasi-pure condensate with $10^5$ atoms. We denote by $t_0$ the time at which the evaporation phase ends. For time $t > t_0$, the atomic cloud is stirred by a focused laser beam of wavelength 852 nm and waist $w_0 = 20 \mu$m, whose position is controlled using acousto-optic modulators [4]. This laser beam creates a rotating optical-dipole potential which is harmonic over the extension of the cloud. The superposition of this dipole potential and the magnetic one creates a transverse trapping potential identical to (4). The trap deformation $\epsilon$ is proportional to the laser intensity $I_L$ and can be adjusted between 0 and 4 %.

The displacement of the center of the trap due to gravity and slight asymmetries in the trapping geometry produce an additional static deformation which has been estimated to be $\sim 1\%$ by measuring the splitting between the center of mass frequencies along the $x$ or $y$ axes. This static anisotropy plays a minor role in the present study and has been neglected in the analysis above. The frequency $\Omega(t)$ of the stirrer is first varied linearly during the time interval $(t_0, t_1)$, starting from $\Omega(t_0) = 0$ up to $\Omega(t_1) = 2\pi \times 139$ Hz (so that $\Omega(t_1) \sim 0.76 \omega_\perp$). The stirring frequency then stays constant in the time interval $(t_1, t_2)$. At time $t_2$ we switch off the magnetic trap and the laser stirrer, allow for a 25 ms free-fall, and image the absorption of a resonant laser beam propagating along $z$. We measure in this way the transverse density profile of the atom cloud, which we fit assuming a parabolic shape.

![Graph showing deformation $\delta$ as a function of angular velocity $\Omega$](image)
We extract from the fit the long and short diameters in the transverse plane (hence the deformation $\delta$), and the orientation $\theta(t)$ of the condensate axes with respect to the rotating frame of the laser stirrer.

The excitation of the scissors mode arises directly from the small non-adiabatic character of the condensate evolution as $\Omega(t)$ increases, during the time interval $(t_0,t_1)$. At time $t_1$, the state of the condensate slightly differs from the steady-state corresponding to $\Omega(t_1)$. Consequently the state of the condensate still evolves in the rotating frame in the time interval $(t_1,t_2)$, even though the characteristics of the stirrer do not change anymore.

We have plotted in Fig. 2 the angle $\theta$ and the condensate deformation $\delta$ as a function of the time $\tau = t_2 - t_1$. These data have been obtained for a ramping time $t_1 - t_0 = 300$ ms and a spoon anisotropy $\epsilon = 0.017(6)$. The angle $\theta$ oscillates with a frequency $\omega/2\pi \approx 11.6 \pm 0.1$ Hz, in good agreement with the value $11.4(\pm 2.1)$ Hz expected from (5). The initial amplitude $\Delta \theta$ is 43 degrees, and the oscillation is damped with a time constant $\sim 110$ ms. The deformation $\delta$ of the condensate also exhibits a small oscillatory motion with an amplitude $\Delta \delta = 0.16$ around the mean value $\delta_0 = 0.51$. This motion has the same frequency as that of $\theta$, is phase shifted by $\sim \pi/2$, as expected from (4) for the scissors mode, and it is damped with a similar time constant.

To confirm that the oscillatory motion shown in Fig. 2 indeed corresponds to the scissors mode described above, we have measured the variation of the frequency $\omega$ as a function of the laser intensity, which is itself proportional to the trap anisotropy $\epsilon$. The data are plotted in Fig. 3. They clearly show the expected dependence $\omega^2 \propto \epsilon$. Actually, this scissors mode constitutes a very precise way to measure $\epsilon$, once the frequencies $\omega_\perp$ and $\Omega$ are known with sufficient precision.

We have also compared the measured amplitudes $\Delta \theta(t_2)$ and $\Delta \delta(t_2)$ with the results from a numerical integration of the equations of motion for the coefficients $a_i$ and $\alpha_i$ when $\Omega$ is ramped from 0 up to its final value $\Omega(t_1)$. The calculated results reproduce the behavior found experimentally. The amplitude of the scissors mode decreases when the ramping time $t_1 - t_0$ or the trap anisotropy $\epsilon$ are increased. The physical reason for this variation is clear: the evolution of the condensate during the time interval $(t_0,t_1)$ is closer to adiabatic following, and the condensate is left at time $t_1$ in a state closer to the stationary state expected for $\Omega = \Omega(t_1)$. However we could not reach a strict quantitative agreement between the calculated amplitudes and the measured ones (typical deviation of 50%). We think that this is due to the damping of the scissors mode, present in the experiment (see Fig. 2) and neglected in our simple theoretical model. A proper description of this damping could be obtained using a formalism similar to (7), where the damping of the scissors mode in a static trap is investigated.

Our final study concerns the behavior of the ratio $\Delta \delta / \Delta \theta$ at $t_2$, where $\Delta \delta$ and $\Delta \theta$ are the oscillation amplitudes for the deformation and for the angle respectively. The results, plotted in Fig. 4, show that $\Delta \delta / \Delta \theta$ varies linearly with $\omega/2\pi$ with a slope of $3.2 \times 10^{-4}$ [degree Hz]$^{-1}$ (i.e. $\Delta \delta / (\omega \Delta \theta) \approx 2.9 \times 10^{-3}$ s/radian$^2$). This is in good agreement with the prediction of Eq. (8) which gives an expected slope of $4.0 \times 10^{-4}$ [degree Hz]$^{-1}$. The small de-
frequency. Here we have been mostly interested in the
small oscillations of the condensate around its rotating
steady-state. A natural extension of this work consists in
studying the non-linear regime of the system, where a
chaotic dynamics can emerge.

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Appendix

The values of the coefficients $c_i$ entering into (6) are:

$$
c_3 = -\omega ^2 \{ 8(1 + \tilde{\Omega}^2) + 3 \lambda ^2 \},
$$

$$
c_2 = \omega ^4 \{ 4(5 - 2 \epsilon ^2) + 16 \tilde{\Omega}^2 + 16 \tilde{\Omega}^4
-4[\alpha ^2(2 \tilde{\Omega}^2 - 1) - 3 \tilde{\Omega} \epsilon] + 2 \lambda ^2(11 - \alpha ^2 + 12 \tilde{\Omega}^2) \},
$$

$$
c_1 = -\omega ^6 \{ 16[(2 \tilde{\Omega}^2 - 1)(2 \tilde{\Omega}^2 - 1 + \epsilon ^2)
-\alpha ^2(2 \tilde{\Omega}^2 - 1 + 3 \epsilon ^2) - \epsilon \tilde{\Omega}(5 - 4 \tilde{\Omega}^2) - 3 \tilde{\Omega}^2 \epsilon)] + 4 \lambda ^2[(13 - 5 \epsilon ^2) + 8 \tilde{\Omega}^2 + 12 \tilde{\Omega}^4
-\alpha ^2(2 \tilde{\Omega}^2 - 1) + 3 \tilde{\Omega} \epsilon] \},
$$

$$
c_0 = \omega ^8 \{ 40 \lambda ^2[(2 \tilde{\Omega}^2 - 1)(2 \tilde{\Omega}^2 - 1 + \epsilon ^2)
-\alpha ^2(2 \tilde{\Omega}^2 - 1 + 3 \epsilon ^2) - \epsilon \tilde{\Omega}(5 - 4 \tilde{\Omega}^2) - 3 \tilde{\Omega}^2 \epsilon)] \},
$$

where we have introduced the reduced quantities $	ilde{\Omega} = \Omega / \omega \lambda$, $\lambda = \omega / \omega \lambda$, $\omega_\perp$, $\lambda$ and $\tilde{\alpha} = \alpha / \omega \perp$.

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