Twist-3 Formalism for Single Transverse Spin Asymmetry Reexamined: Semi-Inclusive Deep Inelastic Scattering

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Abstract

We study the single spin asymmetry (SSA) for the pion production in semi-inclusive deep inelastic scattering, $e\mu^+ \rightarrow e\pi X$, in the framework of the collinear factorization. We derive the complete cross section formula associated with the twist-3 quark-gluon correlation functions for the transversely polarized nucleon, including all types of pole (hard-pole, soft-fermion-pole and soft-gluon-pole) contributions which produce the strong interaction phase necessary for SSA. We prove that the partonic hard part from each pole contribution satisfies certain constraints from Ward identities for color gauge invariance. We demonstrate that the use of these new constraints is crucial to reorganize the collinear expansion of the Feynman diagrams into manifestly gauge-invariant form so as to obtain the factorization formula for the cross section in terms of a complete set of the twist-3 distributions without any double counting. It also provides a simpler method for the actual calculation. We also present a simple estimate of SSA based on our cross section formula, using a model for the “soft-gluon-pole function” that represents the relevant twist-3 quark-gluon correlation, and compare the magnitude of the terms involving the derivative of the soft-gluon-pole function with that of the “non-derivative” terms.
1 Introduction

Understanding of the large single transverse spin asymmetries (SSAs) observed in $pp$ collisions in 70s and 80s [1, 2] has been a big challenge for QCD theorists. (See [3] for a review.) Recent data on SSA at higher energies in $pp$ collisions at RHIC [4, 5, 6], and SSAs in semi-inclusive deep-inelastic scattering (SIDIS) [7, 8] have been providing even more exciting opportunities to clarify fundamental mechanisms of SSA as well as the relevant hadron structures. Time-reversal invariance in QCD implies that the nonzero SSA is caused by the interference of the amplitudes which have different phases. In addition, the nonzero SSA requires a helicity-flip mechanism induced by certain mass scale. In the framework of the factorization formula with conventional twist-2 parton distribution functions and fragmentation functions, the corresponding effects are provided through purely perturbative mechanism and is known to give only tiny SSA [9]. Therefore, the observed large SSAs require the extension of the factorization formula for high energy semi-inclusive reactions to incorporate novel nonperturbative functions beyond conventional twist-2 distribution/fragmentation functions, and those new functions should represent the single helicity flip mechanism reflecting the dynamics of chiral symmetry breaking.

In the literature, two QCD mechanisms have been developed to explain the large SSAs, in conjunction with the corresponding novel nonperturbative functions. One is based on the use of so-called “T-odd” distribution and fragmentation functions which have explicit intrinsic transverse momentum $\vec{k}_⊥$ of partons inside hadrons [10, 11]. This mechanism describes the SSA as a leading twist effect in the region of small transverse momentum $p_T$ of the produced hadron. Factorization formula with the use of $\vec{k}_⊥$-dependent parton distribution functions and fragmentation functions has been extended to SIDIS [12], applying the method used for $e^+e^-$ annihilation [13] and the Drell-Yan process [14], and the universality property of those $k_⊥$-dependent functions have been examined in great details [15, 16, 17, 18, 19]. Phenomenological applications of the “T-odd” functions have been also performed to interpret the existing data for SSAs [20].

The other mechanism describes the SSA as a twist-3 effect in the collinear factorization [21, 22], and is suited for describing SSA in the large $p_T$ region. In this framework, SSA is connected to particular quark-gluon correlation functions on the lightcone. This formalism has been applied to SSA in $pp$ collisions, such as direct photon production $p^i p \to \gamma X$ [22], pion production $p^i p \to \pi X$ [23, 24, 25, 26], the hyperon polarization $pp \to \Lambda^+X$ [27, 28], and SIDIS $e p^\uparrow \to e\pi X$ [29]. Although the above two mechanisms describe SSA in different kinematic regions, it has been known that the soft-gluon-pole function appearing in the twist-3 mechanism is connected to a $\vec{k}_⊥$ moment of a “T-odd” function [17, 30]. More recently, it has been also shown in refs. [31, 32] that the two mechanisms give the identical SSA in the intermediate $p_T$ region for the Drell-Yan process as well as for the SIDIS, unifying the two mechanisms.

In our previous paper [29], we studied the SSA in SIDIS, $e p^\uparrow \to e\pi X$, applying the twist-3 mechanism. The purpose of that study was to derive the cross section formula which is valid in the region of the large transverse momentum of the pion, i.e. $p_T \sim Q \gg \Lambda_{QCD}$ with $Q$ the virtuality of the exchanged virtual photon, and to present an estimate of SSA in
SIDIS, in comparison with the similar twist-3 mechanism that reproduces the asymmetry $A_N$ observed for $pp \uparrow \rightarrow \pi X$. There we identified two kinds of terms in the twist-3 cross section, contributing to SSA in SIDIS:

$$
(A) \quad G_F(x_1, x_2) \otimes D(z) \otimes \hat{\sigma}_A,
$$

$$
(B) \quad \delta q(x) \otimes \hat{E}_F(z_1, z_2) \otimes \hat{\sigma}_B,
$$

where $G_F(x_1, x_2)$ and $\hat{E}_F(z_1, z_2)$ are, respectively, the twist-3 distribution function for the transversely polarized nucleon and the twist-3 fragmentation function for the pion. $\delta q(x)$ is the quark transversity distribution of the nucleon, and $D(z)$ is the usual quark/gluon fragmentation function for the pion. In [29], we focussed on the “derivative term” of the soft-gluon-pole (SGP) contributions of (A) and (B) terms, which is associated with the derivative of the corresponding twist-3 distribution function $G_F$ or fragmentation function $\hat{E}_F$ and is expected to be dominant in the large-$x_{bj}$ or large-$z_f$ region. Within this approximation, we also presented a simple estimate of the (A) and (B) contributions: We fix the relevant nonperturbative functions involved in the (A) and (B) contributions so that the similar twist-3 mechanism associated with those functions reproduces the data on $A_N$ in $pp \uparrow \rightarrow \pi X$.

It turned out that the (B) term is negligible compared with the (A) term for SSA in SIDIS, even if we require that each of the corresponding (A)- and (B)-type contributions can independently reproduce $A_N$ in $pp \uparrow \rightarrow \pi X$.

A formalism of the twist-3 mechanism for SSA in the framework of collinear factorization was first presented by Qiu and Sterman for the direct photon production [22], and it was applied to various other processes [23, 24, 25, 26, 27, 28, 29, 31, 32]. In this formalism, a coherent gluon field $A^\mu$ generated from quark-gluon correlation inside transversely polarized nucleon participates into the hard scattering subprocess. A systematic procedure for extracting the associated twist-3 effect is the collinear expansion of the relevant Feynman diagrams in terms of the parton’s transverse momenta $k_\perp$. Thus we have to perform the collinear expansion of a set of Feynman diagrams for the partonic hard scattering with the extra gluon field $A^\mu$ attached. On the other hand, gauge-invariant twist-3 quark-gluon correlation functions appearing in the factorization formula of the cross section are eventually expressed in terms of either the transverse components of the covariant derivative, $D^\alpha = \partial^\alpha - igA^\alpha$ with $\alpha = \perp$, or those from the gluon field strength tensor, $F^{\alpha+} = \partial^\alpha A^+ - \partial^+ A^\alpha - ig[A^\alpha, A^+]$, see (2), (3) below. Therefore, a crucial step for the calculation is to reorganize the terms from the collinear expansion of the diagrams in terms of the gauge-invariant combination corresponding to $D^\alpha$ or $F^{\alpha+}$ with $\alpha = \perp$ in the twist-3 accuracy. In [22], the calculation was performed in the Feynman gauge, and the combination $\partial^+ A^+$ resulting from the collinear expansion is identified as a part of $F^{\perp+} = \partial^\perp A^+ - \partial^+ A^\perp + \ldots$, without any assurance about the gauge invariance of the procedure nor the uniqueness of the obtained results. To justify this procedure, it is necessary to show that “$-\partial^+ A^+$” also appears with exactly the same coefficient as that for $\partial^+ A^+$. Note that, in the Feynman gauge, the matrix element of the type $\langle p\, S|\tilde{\psi}\partial^+ A^+ \psi|p\, S \rangle$ is equally important as $\langle p\, S|\tilde{\psi}\partial^+ A^+ \psi|p\, S \rangle$, and both matrix elements have to be treated as the same order in the collinear expansion. This is because, even though $\langle p\, S|\tilde{\psi}A^+ \psi|p\, S \rangle \gg \langle p\, S|\tilde{\psi}A^+ \psi|p\, S \rangle$ in
the Feynman gauge in a frame with $p^+ \gg p^-, p^\perp$, acting $\partial^\perp$ to the coherent gluon field in the former matrix element brings relative suppression compared with the latter with $\partial^+$ acting on the gluon field. Accordingly, it is necessary to identify and keep the $\partial^+ A^\perp$ contribution to perform the calculation in a consistent way, taking into account all terms in the collinear expansion up to the retained order, which was missed in the literature. This eventually allows us to establish for the first time that the corresponding factorization formula indeed holds for the twist-3 mechanism of SSA, guaranteeing its gauge invariance and uniqueness. Through this development, we can also address the relations among possible different expressions for the cross section.

The purpose of writing this paper is twofold: First of all, we present all necessary ingredients to establish a systematic collinear-expansion formalism for the twist-3 calculation for SIDIS, $ep^\uparrow \to e\pi X$. We will clarify the relation between different definitions of twist-3 distributions, and identify and classify all pole contributions that can generate the interfering phase for the partonic subprocess. Then we will pay particular attention to the consistency relations for the hard part of the cross section which are required from gauge invariance. Those relations express sufficient condition to allow us to reorganize the next-to-leading terms in the collinear expansion in terms of the combination $\partial^\perp A^+ - \partial^+ A^\perp$ in a unique way, so that eventually justify the identification of $\partial^\perp A^+ \to F^\perp$ taken in [22]. This is a nontrivial issue since the “$\perp$” components of a four-vector are treated in a completely different way from its “$+$” component in the collinear expansion. We will next show that all kinds of pole contributions for $ep^\uparrow \to e\pi X$ indeed satisfy the above-mentioned consistency relations, respectively. Based on this development, we derive the complete cross section formula corresponding to the (A) term in (1) including all pole contributions that produce SSA. The obtained cross section formula is valid in all kinematic region for large $p_T$ pion production, and has a phenomenological relevance for, e.g., the eRHIC experiment [33].

The remainder of this paper is organized as follows: In section 2, after recalling the basic properties of a complete set of the twist-3 quark-gluon correlation functions for the transversely polarized nucleon and the kinematics of SIDIS, we present our formalism of the twist-3 calculation of SSA in SIDIS. We keep all the subleading contributions of twist-3 from all diagrams for the cross section, and show that they can be expressed in terms of the twist-3 gauge-invariant correlation functions introduced earlier if the corresponding hard part satisfies certain constraints. The twist-2 cross section formula for the unpolarized SIDIS is also given for completeness. In section 3, we derive the complete result for the factorization formula of the twist-3 cross section corresponding to the (A) term in (1). To this end, we shall prove that all kinds of the pole contributions indeed satisfy the constraints discussed in section 2 by using the Ward identities. We also provide an economic way of actual calculation, and discuss the relation between different expressions for the cross section. In section 4, we will present a numerical estimate of SSA in SIDIS, using our cross section formula combined with a simple model for SGP functions. We will include all terms from the SGP contributions, the so-called “derivative” and “non-derivative” terms of the SGP functions, to see the importance of the latter contribution which was not included in our previous study [29]. Brief summary and conclusion are presented in section 5.
2 Formalism for the Twist-3 mechanism in QCD

2.1 A complete set of Twist-3 distribution functions

We first recall from [29] the definition and the basic properties of the twist-3 distribution functions for the transversely polarized nucleon. They are characterized by the explicit participation of the gluon field in the lightcone correlation functions for the nucleon. For the transversely polarized nucleon, it is known that there are two independent twist-3 quark-gluon correlation functions [34]. Actually, two types of different definitions for those functions have been used in the literature, depending on whether the explicit gluon is included or not.

For the transversely polarized nucleon, it is known that there are two independent twist-3 distribution functions. These four correlation functions, which are outgoing from the nucleon, are characterized by the explicit gluon field in the lightcone correlation functions for the nucleon. For the transversely polarized nucleon, they are given in [29] as the “F-type” functions,

\[
\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_\perp | \psi_j(0) [0, \mu n] g F^{\alpha\beta}(\mu n) n_\beta | \lambda n \rangle | p S_\perp \rangle
\]

\[
= \frac{M_N}{4} (\bar{\psi})_{ij} e^{\alpha \mu S_\perp} G_F(x_1, x_2) + i \frac{M_N}{4} (\gamma_5 \bar{\psi})_{ij} S_\perp^\alpha \tilde{G}_F(x_1, x_2) + \cdots,
\]

and the “D-type” functions,

\[
\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p S_\perp | \psi_j(0) [0, \mu n] D_\perp^{\alpha}(\mu n) [\mu n, \lambda n] \psi_i(\lambda n) | p S_\perp \rangle
\]

\[
= \frac{M_N}{4} (\bar{\psi})_{ij} e^{\alpha \mu S_\perp} G_D(x_1, x_2) + i \frac{M_N}{4} (\gamma_5 \bar{\psi})_{ij} S_\perp^\alpha \tilde{G}_D(x_1, x_2) + \cdots,
\]

where \( \psi \) is the quark field, the nucleon’s momentum \( p \) can be regarded as light-like \( (p^2 = 0) \) in the twist-3 accuracy, \( n^\mu \) is the light-like vector \( (n^2 = 0) \) with \( p \cdot n = 1 \), \( e^{\alpha \mu S_\perp} = e^{\alpha \mu n_\mu} p_\lambda n_\mu S_{\lambda \mu} \) with \( e_{0123} = 1 \), and the spin vector satisfies \( S_\perp^2 = -1 \), \( S_\perp \cdot p = S_\perp \cdot n = 0 \).

\([\mu n, \lambda n] = P \exp [ig \int_0^l dn \cdot \vec{A}(n)] \) represents the gauge-link which guarantees gauge invariance of the nonlocal lightcone operator, and “…” denotes twist-4 or higher-twist distributions. These four correlation functions, \( G_F(x_1, x_2), \tilde{G}_F(x_1, x_2), G_D(x_1, x_2), \tilde{G}_D(x_1, x_2) \), are defined as dimensionless by introducing the nucleon mass \( M_N \) which is a natural scale for chiral-symmetry breaking. Each correlation function depends on the two variables \( x_1 \) and \( x_2 \), where \( x_1 \) and \( x_2 - x_1 \) are the fractions of the lightcone momentum carried by the quark and the gluon, respectively, which are outgoing from the nucleon. From hermiticity, P- and T-invariance of QCD, these functions are real, and satisfy the symmetry properties

\[
G_F(x_1, x_2) = G_F(x_2, x_1), \quad \tilde{G}_F(x_1, x_2) = -\tilde{G}_F(x_2, x_1),
\]

\[
G_D(x_1, x_2) = -G_D(x_2, x_1), \quad \tilde{G}_D(x_1, x_2) = \tilde{G}_D(x_2, x_1).
\]

The basis of twist-3 quark-gluon correlation functions for the transversely polarized nucleon, defined as (2) and (3), is overcomplete. In [29], the relation between the F-type functions of
and the $D$-type functions of (3) was also established using the QCD equations motion. They are connected by the relations

$$G_D(x_1, x_2) = P \frac{1}{x_1 - x_2} G_F(x_1, x_2), \quad (6)$$

$$\tilde{G}_D(x_1, x_2) = \delta(x_1 - x_2) \tilde{g}(x_1) + P \frac{1}{x_1 - x_2} \tilde{G}_F(x_1, x_2), \quad (7)$$

where $P$ denotes the principal value, and $\tilde{g}(x_1)$ can be expressed in terms of the $F$-type distributions $G_F(x, y)$ and $\tilde{G}_F(x, y)$, and the twist-2 quark helicity distribution $\Delta q(x)$ of the nucleon (see [29] for the detail). In the RHS of (6), the term proportional to $\delta(x_1 - x_2)$ vanishes due to the anti-symmetry of $G_D(x_1, x_2)$ as given in (5).

It is convenient to choose the $F$-type functions \{$G_F(x_1, x_2)$, $\tilde{G}_F(x_1, x_2)$\} as a complete set of twist-3 quark-gluon correlation functions for the transversely polarized nucleon: From (6) and (7), we note that the $D$-type functions are more singular than the $F$-type functions at the soft gluon point $x_1 = x_2$, while they are proportional to each other for $x_1 \neq x_2$. As will be discussed in our analysis for $ep^\uparrow \rightarrow e\pi X$ below, the cross section receives contributions from the “soft gluon point”, $x_1 = x_2$, of the twist-3 distribution; we assume that there is no singularity in the $F$-type functions, in particular, that $G_F(x_1, x_2)$ is finite at $x_1 = x_2$ ($\tilde{G}_F(x_1, x_1) = 0$, due to (4)), and use the $F$-type functions to express those contributions in the cross section. Other terms which receive contributions with $x_1 \neq x_2$ are expressed in terms of either $D$-type or $F$-type functions without any subtlety.

### 2.2 Kinematics for $ep^\uparrow \rightarrow e\pi X$

Here we summarize the kinematics for the SIDIS, $e(\ell) + p(p, S_L) \rightarrow e(\ell') + \pi(P_h) + X$. 1 (See refs. [35, 36] for the detail.) We have five independent Lorentz invariants, $S_{ep}$, $x_{bj}$, $Q^2$, $z_f$, and $q_T^2$. The center of mass energy squared, $S_{ep}$, for the initial electron and the proton is

$$S_{ep} = (p + \ell)^2 \simeq 2p \cdot \ell, \quad (8)$$

ignoring masses. In the twist-3 accuracy, one can set the masses to be zero for all the particles in the initial and the final states. The conventional DIS variables are defined in terms of the virtual photon momentum $q = \ell - \ell'$ as

$$x_{bj} = \frac{Q^2}{2p \cdot q}, \quad Q^2 = -q^2 = -(\ell - \ell')^2. \quad (9)$$

For the final-state pion, we introduce the scaling variable

$$z_f = \frac{p \cdot P_h}{p \cdot q}. \quad (10)$$

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1In this paper, we use different notation for the kinematic variables compared with our previous paper [29].
Finally, we define the “transverse” component of \( q \), which is orthogonal to both \( p \) and \( P_h \):

\[
q_t^\mu = q^\mu - \frac{P_h \cdot q}{p \cdot P_h} p^\mu - \frac{p \cdot q}{p \cdot P_h} P_h^\mu .
\]  

(11)

\( q_t \) is a space-like vector, and we denote its magnitude by

\[
q_T = \sqrt{-q_t^2} .
\]  

(12)

To derive the cross section, we work in a frame where the 3-momenta \( \vec{q} \) and \( \vec{p} \) of the virtual photon and the initial proton are collinear, and we denote the azimuthal angle between the lepton plane and the hadron plane as \( \phi \).

For the actual calculation, it is convenient to specify the frame further. We work in the so-called hadron frame [35], which is the Breit frame of the virtual photon and the initial proton:

\[
q^\mu = (0, 0, 0, -Q) ,
\]  

(13)

\[
p^\mu = \left( \frac{Q}{2x_{bj}}, 0, 0, \frac{Q}{2x_{bj}} \right) .
\]  

(14)

In this frame the outgoing pion is taken to be in the \( xz \) plane:

\[
P_h^\mu = \frac{z_f Q}{2} \left( 1 + \frac{q_T^2}{Q^2}, \frac{2q_T}{Q}, 0, \frac{q_T^2}{Q^2} - 1 \right) .
\]  

(15)

As one can see, the transverse momentum of the pion in this frame is given by \( P_hT = z_f q_T \). This is true for any frame in which the 3-momenta \( \vec{q} \) and \( \vec{p} \) are collinear. The lepton momentum in this frame can be parameterized as

\[
\ell^\mu = \frac{Q}{2} \left( \cosh \psi, \sinh \psi \cos \phi, \sinh \psi \sin \phi, -1 \right) ,
\]  

(16)

where

\[
\cosh \psi = \frac{2x_{bj}S_{ep}^\perp}{Q^2} - 1 .
\]  

(17)

We parameterize the transverse spin vector of the initial proton \( S_\perp^\mu \) as

\[
S_\perp^\mu = (0, \cos \Phi_S, \sin \Phi_S, 0),
\]  

(18)

where \( \Phi_S \) represents the azimuthal angle of \( \vec{S}_\perp \) measured from the hadron plane. With the above definition, the cross section for \( ep^+ \rightarrow e\pi X \) can be expressed in terms of \( S_{ep}, x_{bj}, Q^2, z_f, q_T^2, \phi \) and \( \Phi_S \) in the hadron frame. Note that \( \phi \) and \( \Phi_S \) are invariant under boosts in the \( \vec{q} \)-direction, so that the cross section presented below is the same in any frame where \( \vec{q} \) and \( \vec{p} \) are collinear.
2.3 Collinear expansion and gauge invariance

The differential cross section for $ep^\uparrow \rightarrow e\pi X$ can be written as

$$d\sigma = \frac{1}{2S_{ep}} \frac{d^3\vec{P}_h}{(2\pi)^3} \frac{d^3\vec{P}}{(2\pi)^3} \frac{e^4}{2P_0} q^4 L_{\mu\nu}(\ell, \ell') W_{\mu\nu}(p, q, P_h),$$

(19)

where

$$L_{\mu\nu}(\ell, \ell') = 2(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu) - g_{\mu\nu} Q^2$$

(20)

is the leptonic tensor for the unpolarized electron and $W_{\mu\nu}$ is the hadronic tensor. In the present study we are interested in the contribution from the twist-3 distribution for the initial proton combined with the twist-2 unpolarized fragmentation function for the final pion ((A) term in (1)). Accordingly, it is straightforward to factorize the fragmentation function from the hadronic tensor as

$$W_{\mu\nu}(p, q, P_h) = \sum_{j=q, g} \int \frac{dz}{z^2} D_j(z) w_{\mu\nu}^j(p, q, P_h z),$$

(21)

where $D_j(z) (j = q, g)$ is the quark and gluon fragmentation functions for the pion, with $z$ being the momentum fraction. Since the calculational procedure is the same for the two terms in (21), we consider the case for the quark fragmentation in detail and omit the index $j$ from $w_{\mu\nu}^j$ below. To extract the twist-3 effect in $w_{\mu\nu}$ contributing to single-spin-dependent cross section, one needs to analyze the diagrams shown in Fig. 1 for the hadronic tensor.

To this end, we work in the Feynman gauge. In Fig. 1, the partons from the nucleon, which is represented by the lower blob, undergoes the hard interaction with the virtual photon in the middle blob, followed by the fragmentation into the pion as represented by the upper blob. Note that the fragmentation function corresponding to the upper blob has already been factorized as (21). Corresponding to (a) and (b) of Fig. 1, where the momenta for the parton lines connecting the blobs are assigned, respectively, $w_{\mu\nu}$ can be written as the sum of the two terms:

$$w_{\mu\nu}(p, q, P_h) = \int \frac{d^4k}{(2\pi)^4} \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \text{Tr} \left[ M^{(0)}(k) S^{(0)}(k, q, P_h) \right] + \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \text{Tr} \left[ M^{(1)}(k_1, k_2) S^{(1)}(k_1, k_2, q, P_h) \right],$$

(22)

where the superscripts "(0)" and "(1)" indicate the number of coherent gluon line connecting the lower and middle blobs. $M^{(0)}(k)$ and $M^{(1)}(k_1, k_2)$ represent the lower part of (a) and (b) of Fig. 1, respectively, and they are given as the nucleon matrix elements:

$$M^{(0)}_{ij}(k) = \int d^4\xi e^{ik\xi} \langle p S_\perp | \tilde{\psi}_j(0) \psi_i(\xi) | p S_\perp \rangle,$$

(23)

$$M^{(1)}_{ij}(k_1, k_2) = \int d^4\xi \int d^4\eta e^{ik_1\xi} e^{i(k_2 - k_1)\eta} \langle p S_\perp | \tilde{\psi}_j(0) gA^\sigma(\eta) \psi_i(\xi) | p S_\perp \rangle,$$

(24)
Figure 1: Generic diagrams for the hadronic tensor of $e p^+ \to e \pi X$, decomposed into the three blobs as nucleon matrix element (lower), pion matrix element (upper), and partonic hard scattering by the virtual photon (middle). The first two terms, (a) and (b), in the expansion by the number of partons connecting the middle and lower blobs are relevant to the twist-3 effect induced by the nucleon.

where the spinor indices $i$ and $j$ are shown explicitly; $S^{(0)}(k, q, P_h/z)$ and $S^{(1)}_{\sigma}(k_1, k_2, q, P_h/z)$ represent the corresponding hard parts with the Lorentz indices $\mu$ and $\nu$ suppressed here and below for simplicity, and are matrices in spinor space. $\text{Tr}[\cdots]$ indicates the trace over Dirac-spinor indices while the color trace is implicit. Since the twist-2 unpolarized fragmentation functions $D_j(z)$ in (21) are chiral-even quantities, only chiral-even Dirac matrix structures in $M^{(0)}(k)$ and $M^{(1)}_{\sigma}(k_1, k_2)$ give nonvanishing contribution to $\text{Tr}[\cdots]$.

In the leading order in QCD perturbation theory, the hard blob for $S^{(0)}(k, q, P_h/z)$ in Fig. 1 (a) stands for a set of cut Feynman diagrams for the partonic Born subprocess with a hard gluon or quark in the final state, while the additional, coherent gluon participates in those partonic processes for $S^{(1)}_{\sigma}(k_1, k_2, q, P_h/z)$ in Fig. 1 (b). A standard and systematic procedure to identify the twist-3 contributions to $w_{\mu\nu}$ of (22) utilizes the collinear expansion of the hard parts $S^{(0)}(k, q, P_h/z)$ and $S^{(1)}_{\sigma}(k_1, k_2, q, P_h/z)$ with respect to the partonic momenta $k$ and $k_{1,2}$ around the momentum $p$ of the parent nucleon. This reads for $S^{(0)}(k, q, P_h/z)$

$$S^{(0)}(k, q, P_h/z) = S^{(0)}(x p, q, P_h/z) + \frac{\partial S^{(0)}(k, q, P_h/z)}{\partial k^\alpha} \bigg|_{k=xp} \omega^\alpha_{\beta}(x) k^\beta + \cdots,$$

(25)

where $x = k \cdot n$, $\omega^\alpha_{\beta} \equiv g^\alpha_{\beta} - p^\alpha n^\beta$, and “…” denote the terms of $O((k_\perp)^2)$ which are twist-4 or higher. Substituting this expansion, the first term in the RHS of (22) becomes

$$\int dx \text{ Tr} \left[ M^{(0)}(x) S^{(0)}(x p, q, P_h/z) \right] + \int dx \text{ Tr} \left[ i \omega^\alpha_{\beta} M^{(0)}(x) \frac{\partial S^{(0)}(k, q, P_h/z)}{\partial k^\alpha} \right]_{k=xp},$$

(26)

where the integration over $k^-$ and $k^\perp$ has been performed, with $\omega^\alpha_{\beta}(x) k^\beta$ of (25) transformed into the derivative $i \omega^\alpha_{\beta}(x) \partial^\beta$ on the quark field of (23) by partial integration, and the relevant
nucleon matrix elements have now become the lightcone correlation functions:

\[ M^{(0)}_{ij}(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S_{\perp} | \bar{\psi}_j(0) \psi_i(\lambda n) | p S_{\perp} \rangle, \quad (27) \]

\[ M^{(0,\alpha)}_{ij}(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p S_{\perp} | \bar{\psi}_j(0) \partial^\alpha \psi_i(\lambda n) | p S_{\perp} \rangle. \quad (28) \]

Here the matrix elements \( M^{(0)}(x) \) and \( M^{(0,\alpha)}(x) \) for the transversely polarized nucleon can be decomposed into several independent Dirac matrix structures based on Lorentz, P- and T-invariance. Among them, we only need to consider the chiral-even Dirac matrix structures as noted above. We find \( M^{(0)}(x) = \gamma_5 S_{\perp} g_T(x)/2 + \ldots \) with the chiral-even twist-3 quark distribution \( g_T(x) \) [37], up to irrelevant (chiral-odd or twist-4) terms, but the first term in (26) with the substitution \( M^{(0)}(x) \rightarrow \gamma_5 S_{\perp} g_T(x)/2 \) does not contribute to the cross section: The spinor trace \( \text{Tr}[ \cdots ] \) involving \( \gamma_5 \) produces the factor \( i \) to the first term in (26) and the result is contracted with the real tensor \( L_{\mu\nu} \) of (20) to derive the contribution to (19); but this yields a real quantity for the cross section only when another factor \( i \) is provided by the hard part \( S^{(0)}(xp,q,P_h/z) \), which is impossible for the Born subprocess. Similarly, it is straightforward to see that the second term of (26) does not contribute to the cross section to twist-3 accuracy, noting that the second term of (26) contains the factor \( i \) and considering a similar decomposition of \( \omega^{\alpha\beta} M^{(0,\beta)}(x) \) into Dirac matrix structures as in (3).

Therefore the twist-3 cross section for \( ep^1 \rightarrow e\pi X \) arises solely from the second term of (22). To analyze it, we first note that the matrix element \( M^{(1)}_{\sigma=\perp}(24) \) is suppressed by a power of \( 1/p^+ \) compared to \( M^{(1)}_{\sigma=+} \), and gives subleading contribution to \( w_{\mu\nu} \). Accordingly, in the twist-3 accuracy, collinear expansion is necessary only for \( S^{(1)}_+ = S^{(1)}_\alpha p^\alpha / p^+ \) which couples with \( M^{(1)+} \) in (22). We define

\[ S^{(1)}_+(k_1, k_2, q, P_h/z) \equiv S^{(1)}_\sigma(k_1, k_2, q, P_h/z) p^\sigma. \quad (29) \]

Collinear expansion for \( S^{(1)}_+(k_1, k_2, q, P_h/z) \) with respect to \( k_{1,2} \) around \( p \) gives

\[ S^{(1)}_+(k_1, k_2, q, P_h/z) = S^{(1)}_+(x_1 p, x_2 p, q, P_h/z) + \frac{\partial S^{(1)}_+(k_1, k_2, q, P_h/z)}{\partial k^\alpha_1} \bigg|_{k_i=x_i,p} \omega^\alpha_2 k_1^\beta + \cdots, \quad (30) \]

where \( x_i = k_i \cdot n \) with \( i = 1, 2 \). Using (30) in the second term of (22), one obtains

\[ w_{\mu\nu} = \int dx_1 \int dx_2 \text{Tr} \left[ M^{(1)}_\sigma(x_1, x_2) n^\sigma S^{(1)}_+(x_1 p, x_2 p, q, P_h/z) \right] 
+ \int dx_1 \int dx_2 \text{Tr} \left[ \omega^\alpha_\beta M^{(1)}_{\alpha\beta}(x_1, x_2) S^{(1)}_\sigma(x_1 p, x_2 p, q, P_h/z) \right] 
+ \int dx_1 \int dx_2 \text{Tr} \left[ i \omega^\alpha_\beta M^{(1)}_{\alpha\beta}(x_1, x_2) \frac{\partial S^{(1)}_+(k_1, k_2, q, P_h/z)}{\partial k^\alpha_1} \bigg|_{k_i=x_i,p} \right], \]
We rewrite the remaining terms of (31) as

\[
+ \int dx_1 \int dx_2 \text{Tr} \left[ i \omega^\alpha_\beta M_{\partial 2}^{(1)\beta}(x_1, x_2) \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_2^\alpha} \bigg|_{k_i=x_i P} \right],
\] (31)

up to twist-3 contributions, and, similarly to (27) and (28) above, the relevant nucleon matrix elements have now become the correlation functions on the lightcone:

\[
M^{(1)\sigma}(x_1, x_2) = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1 e^{i\mu(x_2-x_1)}} \langle p S_\perp | \bar{\psi}(0) g A^\sigma(\mu n) \psi(\lambda n) | p S_\perp \rangle,
\] (32)

\[
M_{\partial 1}^{(1)\beta}(x_1, x_2) = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1 e^{i\mu(x_2-x_1)}} \langle p S_\perp | \bar{\psi}(0) g A^\sigma(\mu n) n_\sigma \partial^\beta \psi(\lambda n) | p S_\perp \rangle,
\] (33)

\[
M_{\partial 2}^{(1)\beta}(x_1, x_2) = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1 e^{i\mu(x_2-x_1)}} \langle p S_\perp | \bar{\psi}(0) \left\{ g \left( \partial^\beta A^\sigma(\mu n) n_\sigma \right) \psi(\lambda n)
\right.
\]
\[
\left. + g A^\sigma(\mu n) n_\sigma \partial^\beta \psi(\lambda n) \right\} | p S_\perp \rangle.
\] (34)

The first term of (31) does not contribute to the single-spin-dependent cross section by the same reason as the first term of (26): Actually the former is absorbed into the latter, providing the \(O(g)\) contribution of the gauge-link operator to be inserted in-between the quark fields in \(M^{(0)}(x)\) of (27), as can be seen straightforwardly using the Ward identity for the coupling of the scalar polarized, coherent gluon field \(n \cdot A\) into the corresponding hard part (see also [38]),

\[
S^{(1)}(x_1 p, x_2 p, q, P_h/z) = -\frac{S^{(0)}(x_2 p, q, P_h/z)}{x_1 - x_2 + i\epsilon} - \frac{S^{(0)}(x_1 p, q, P_h/z)}{x_2 - x_1 - i\epsilon}.
\] (35)

We rewrite the remaining terms of (31) as

\[
w_{\mu\nu} = \int dx_1 \int dx_2 \text{Tr} \left[ \omega^\alpha_\beta M^{(1)\beta}(x_1, x_2) S^{(1)}(x_1 p, x_2 p, q, P_h/z) \right]
\]

\[
+ \int dx_1 \int dx_2 \text{Tr} \left[ i \omega^\alpha_\beta \left( M_{\partial 2}^{(1)\beta}(x_1, x_2) - M_{\partial 1}^{(1)\beta}(x_1, x_2) \right) \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_2^\alpha} \bigg|_{k_i=x_i P} \right]
\]

\[
+ \int dx_1 \int dx_2 \text{Tr} \left[ i \omega^\alpha_\beta M_{\partial 1}^{(1)\beta}(x_1, x_2)
\right.
\]
\[
\left. \times \left( \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_1^\alpha} \bigg|_{k_i=x_i P} + \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_2^\alpha} \bigg|_{k_i=x_i P} \right) \right],
\] (36)

and further rewrite \(M_{\partial 2}^{(1)\beta} - M_{\partial 1}^{(1)\beta}\) in the second term on the RHS as

\[
M_{\partial 2}^{(1)\beta}(x_1, x_2) - M_{\partial 1}^{(1)\beta}(x_1, x_2) = M_F^{(1)\beta}(x_1, x_2) + \tilde{M}^{(1)\beta}(x_1, x_2),
\] (37)
with
\[ M^{(1)\beta}_F(x_1, x_2) = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \ S_\perp | \bar{\psi}(0) \rangle \times g \left\{ \left( \partial^\beta A^\sigma(\mu n) \right) - \left( \partial^\sigma A^\beta(\mu n) \right) \right\} n_\sigma \psi(\lambda n) | p \ S_\perp \rangle, \tag{38} \]
\[ \tilde{M}^{(1)\beta}(x_1, x_2) = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle p \ S_\perp | \bar{\psi}(0) gn_\sigma \left( \partial^\sigma A^\beta(\mu n) \right) \psi(\lambda n) | p \ S_\perp \rangle \]
\[ = -i(x_2 - x_1) M^{(1)\beta}(x_1, x_2), \tag{39} \]
where the second equality of (39) is obtained by partial integration. One should note the two terms in (38) contribute in the same order with respect to the power of the large scale \( p^+ \): Even though, schematically, in the power counting for \( p^+ \)
\[ \langle \bar{\psi} A^+ \psi \rangle \gg \langle \bar{\psi} A^\perp \psi \rangle, \tag{40} \]
by one power of \( p^+ \), one has
\[ \langle \bar{\psi} (\partial^\perp A^\perp) \psi \rangle \sim \langle \bar{\psi} (\partial^+ A^\perp) \psi \rangle, \tag{41} \]
since \( \partial^\perp \) acting on the coherent gluon field causes relative power-suppression \( \sim 1/p^+ \) compared with \( \partial^+ \). Using the relation (37) and (39) in the second term of (36), one obtains
\[
\begin{align*}
 w_{\mu\nu} &= \int dx_1 \int dx_2 \text{Tr} \left[ i \omega_{\beta}^\alpha M^{(1)\beta}_F(x_1, x_2) \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_2^\alpha} \bigg|_{k_i=x_i p} \right] \\
 &+ \int dx_1 \int dx_2 \text{Tr} \left[ \omega_{\alpha}^\beta M^{(1)\beta}(x_1, x_2) \left\{ (x_2 - x_1) \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_2^\alpha} \bigg|_{k_i=x_i p} \\
 + S^{(1)}_\alpha(x_1 p, x_2 p, q, P_h/z) \right\} \right] \\
 &+ \int dx_1 \int dx_2 \text{Tr} \left[ i \omega_{\beta}^\alpha M^{(1)\beta}_\delta(x_1, x_2) \right. \\
 &\left. \times \left( \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_1^\delta} \bigg|_{k_i=x_i p} + \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_2^\delta} \bigg|_{k_i=x_i p} \right) \right]. \tag{42} \end{align*}
\]
In our analysis with a single coherent gluon connecting the hard part and the soft (nucleon) part as in Fig. 1 (b), \( M^{(1)\beta}_F(x_1, x_2) \) in the first term in the RHS of (42) can be identified as the gauge-invariant correlation function defined as (2) with the gluon field strength tensor (see (38)). On the other hand, the other terms in (42) involve the matrix elements that are not directly related to (2). Accordingly, if those terms vanish, the first term of (42) gives
the cross section in terms of the $F$-type functions. In order that this is the case, we may simply require that the hard parts for each of the second and third terms of (42) vanish separately as

$$
(x_2 - x_1) \left. \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_2^\alpha} \right|_{k_i = x_ip} + S^{(1)}_\alpha(x_1p, x_2p, q, P_h/z) = 0,
$$

(43)

$$
\left. \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_1^\alpha} \right|_{k_i = x_ip} + \left. \frac{\partial S^{(1)}(k_1, k_2, q, P_h/z)}{\partial k_2^\alpha} \right|_{k_i = x_ip} = 0,
$$

(44)

for $\alpha = \perp$, up to the twist-4 corrections. These constitute the sufficient condition to obtain the cross section formula in terms of the gauge-invariant correlation functions. In Section 3, we will see that these relations are indeed satisfied by the hard part for the partonic subprocess relevant to SSA.

A comment on the previous works is in order regarding the relation (43). A formalism using the Feynman gauge for the twist-3 calculation of SSA was first presented in [22]. In [22] and all the subsequent works [23, 24, 25, 26, 27, 28, 29, 31, 32], the identification of $\partial_{\perp} A^+ \rightarrow F_{\perp}^+$ in the matrix element was done in the course of the collinear expansion ignoring other terms in $F_{\perp}^+$. In [39], the relation (40) was correctly recognized (see the argument following eq. (86) of [39]) but the relation (41) was not noticed and the $\partial^\perp A^\perp$ term in $F_{\perp}^+$ was discarded. (see eqs. (27a) and (27b) of [39]). Clearly this procedure is not justified as was emphasized around (40) and (41). It is this fact that forces us to reorganize the first and second terms of (36) into those of (42) and require eventually (43). Therefore, we emphasize the condition (43) is crucial to justify the above identification, $\partial_{\perp} A^+ \rightarrow F_{\perp}^+$.

2.4 Pole contribution to the cross section

As has been known from the previous studies [21, 22], the single-spin-dependent cross section occurs as pole contributions of the internal propagators in the partonic hard cross section. This can be seen as follows: Since the leptonic tensor (20) for the unpolarized lepton is real, the cross section receives the contributions from the real part of the hadronic tensor $W_{\mu\nu}$ of (21). When (43) and (44) are satisfied, the hadronic tensor is given by the first term of (42) which is proportional to the factor $i$. Substituting (2) into this term, we immediately see that the trace $\text{Tr}[\cdots]$ of the relevant Dirac matrices, combined with $\slash{p}$ and $i\gamma_5\slash{p}$ of the two terms of (2), produces a real factor, so that the hadronic tensor receives the real contributions only from the imaginary part of $\partial S^{(1)}/\partial k_2^\alpha|_{k_i = x_ip}$. This requires to pick up imaginary part from a pole of an internal propagator in $\partial S^{(1)}/\partial k_2^\alpha|_{k_i = x_ip}$.

Analyzing the cut diagrams for the hard blob in Fig. 1(b), which represents the partonic Born subprocess with the additional, coherent gluon attached, it is easy to see that, when the momenta of “external” partons are in the collinear configuration (i.e. $k_{i,2} = x_{1,2}\slash{p}$), some internal lines can go on shell, with the propagators reducing to one of the following three factors:

$$
(1) \frac{1}{x_1 - x_{bj} + i\varepsilon} = P \frac{1}{x_1 - x_{bj}} - i\pi\delta(x_1 - x_{bj}),
$$
or \[ \frac{1}{x_2 - x_{bj} - i\varepsilon} = P \frac{1}{x_2 - x_{bj}} + i\pi\delta(x_1 - x_{bj}), \] (45)

(II) \[ \frac{1}{x_1 - i\varepsilon} = P \frac{1}{x_1} + i\pi\delta(x_1), \quad \text{or} \quad \frac{1}{x_2 + i\varepsilon} = P \frac{1}{x_2} - i\pi\delta(x_2), \] (46)

(III) \[ \frac{1}{x_1 - x_2 + i\varepsilon} = P \frac{1}{x_1 - x_2} - i\pi\delta(x_1 - x_2). \] (47)

These (I), (II), and (III) produce the pole contributions called hard pole (HP) [40], soft-fermion pole (SFP) [22] and soft-gluon pole (SGP) [22], respectively, when integrated over the parton momentum fraction. For \( \partial S^{(1)} / \partial k_2 |_{k_i = x_{bj}} \) of (42), the derivative of the internal propagators may contribute as double pole as well as single pole, but the position of the double pole is the same as that of (45)-(47). In Section 3 we shall calculate all pole contributions to the cross section.

### 2.5 Unpolarized cross section

Here we present a brief description of the calculational procedure for SIDIS. See [35, 36] for the detail. Following [35], we introduce the following 4 vectors which are orthogonal to each other [35]:

\[
T^\mu = \frac{1}{Q} \left( q^\mu + 2x_{bj}p^\mu \right),
\]

\[
X^\mu = \frac{1}{q_T} \left\{ \frac{P_h^\mu}{z_f} - q^\mu - \left( 1 + \frac{q_T^2}{Q^2} \right) x_{bj}p^\mu \right\},
\]

\[
Y^\mu = \epsilon^{\mu\nu\rho\sigma} Z_\nu X_\rho T_\sigma,
\]

\[
Z^\mu = -\frac{q^\mu}{Q}. \tag{48}
\]

These vectors become \( T^\mu = (1, 0, 0, 0), X^\mu = (0, 1, 0, 0), Y^\mu = (0, 0, 1, 0), Z^\mu = (0, 0, 0, 1) \) in the hadron frame. Note that \( T, X \) and \( Z \) are vectors, while \( Y \) is an axial vector. Since \( W^{\mu\nu} \) satisfies the current conservation, \( q_\nu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0 \), \( W^{\mu\nu} \) can be expanded in terms of 9 independent tensors, for which one can employ the following:

\[
\mathcal{V}_1^{\mu\nu} = X^\mu X^\nu + Y^\mu Y^\nu, \quad \mathcal{V}_2^{\mu\nu} = g^{\mu\nu} + Z^\mu Z^\nu, \quad \mathcal{V}_3^{\mu\nu} = T^\mu X^\nu + X^\mu T^\nu,
\]

\[
\mathcal{V}_4^{\mu\nu} = X^\mu X^\nu - Y^\mu Y^\nu, \quad \mathcal{V}_5^{\mu\nu} = i(T^\mu X^\nu - X^\mu T^\nu), \quad \mathcal{V}_6^{\mu\nu} = i(X^\mu Y^\nu - Y^\mu X^\nu),
\]

\[
\mathcal{V}_7^{\mu\nu} = i(T^\mu Y^\nu - Y^\mu T^\nu), \quad \mathcal{V}_8^{\mu\nu} = T^\mu Y^\nu + Y^\mu T^\nu, \quad \mathcal{V}_9^{\mu\nu} = X^\mu Y^\nu + Y^\mu X^\nu. \tag{49}
\]

For both unpolarized twist-2 cross section and single-spin-dependent twist-3 cross section, the corresponding hadronic tensor \( W^{\mu\nu} \) is symmetric. In addition, for both cases, pseudotensor \( \mathcal{V}_{8,9} \) do not contribute to the expansion of \( W^{\mu\nu} \), so that \( W^{\mu\nu} \) can be expanded in terms
of $V_i$ ($i = 1, \cdots, 4$). The latter fact can be seen by explicit calculation, or by inspection: For the single-spin-dependent case, substituting (2) into the first term of (42), the trace $\text{Tr}[\cdots]$ of the relevant Dirac matrices gives the factor $\epsilon^{\rho h} S_{\perp} \propto \vec{S}_\perp \cdot (\vec{p} \times \vec{P}_h)$ characteristic of SSA, which behaves as scalar under parity transformation.

The expansion coefficients of $W^{\mu \nu}$ for these tensors are easily obtained by using the inverse tensors $\tilde{V}_k$ for $V_k$ as

$$W^{\mu \nu} = \sum_{k=1}^{4} V_k^{\mu \nu} \left[ W_{\rho \sigma} \tilde{V}_k^{\rho \sigma} \right],$$  \hspace{1cm} (50)

where

$$\tilde{V}_1^{\mu \nu} = \frac{1}{2} (2T^\mu T^\nu + X^\mu X^\nu + Y^\mu Y^\nu), \hspace{1cm} \tilde{V}_2^{\mu \nu} = T^\mu T^\nu,$$

$$\tilde{V}_3^{\mu \nu} = -\frac{1}{2} (T^\mu X^\nu + X^\mu T^\nu), \hspace{1cm} \tilde{V}_4^{\mu \nu} = \frac{1}{2} (X^\mu X^\nu - Y^\mu Y^\nu).$$  \hspace{1cm} (51)

With these $V_k^{\mu \nu}$, we can decompose the $\phi$-dependence of the cross sections by the contraction with $L^{\mu \nu}$ in (20). Define $A_k$ ($k = 1, \cdots, 4$) as

$$A_k = \frac{1}{Q^2} L_{\mu \nu} V_k^{\mu \nu},$$  \hspace{1cm} (52)

then one obtains

$$A_1 = 1 + \cosh^2 \psi,$$

$$A_2 = -2,$$

$$A_3 = -\cos \phi \sinh 2\psi,$$

$$A_4 = \cos 2\phi \sinh^2 \psi.$$  \hspace{1cm} (53)

With this method, the unpolarized cross section of twist-2 for $ep \rightarrow e\pi X$ was derived some time ago [41, 35, 36]. It reads

$$\frac{d^5 \sigma}{dx_{bj}dQ^2dz_f dq^2_f d\phi} = \frac{\alpha^2_{em} \alpha_s}{8\pi x_{bj}^2 S^2_{ep} Q^2} \sum_{k=1}^{4} A_k \int_{x_{min}}^{1} \frac{dx}{x} \int_{z_{min}}^{1} \frac{dz}{z}$$

$$\times \sum_a e_a^2 [f_a(x) D_a(z) \tilde{\sigma}_k^{qq} + f_g(x) D_a(z) \tilde{\sigma}_k^{qg} + f_a(x) D_g(z) \tilde{\sigma}_k^{gq}]$$

$$\times \delta \left( \frac{q^2}{Q^2} - \left( \frac{1}{x} - 1 \right) \left( \frac{1}{z} - 1 \right) \right),$$  \hspace{1cm} (54)

where $\alpha_{em} = e^2/4\pi$ is the QED coupling constant, $f_a(x)$ and $D_a(z)$ are, respectively, unpolarized quark distribution and fragmentation functions with quark flavor $a$ and summation
\[ \sum_a \text{ is over all quark and anti-quark flavors. } f_g(x) \text{ and } D_g(z) \text{ are the gluon distribution and fragmentation functions, respectively. In (54), we have introduced the variables} \]
\[ \hat{x} = \frac{x_{bj}}{x}, \quad \hat{z} = \frac{z_f}{z}, \quad (55) \]
and
\[ x_{min} = x_{bj} \left( 1 + \frac{z_f q_T^2}{1 - z_f Q^2} \right), \quad z_{min} = z_f \left( 1 + \frac{x_{bj} q_T^2}{1 - x_{bj} Q^2} \right). \quad (56) \]

The partonic hard cross sections in (54) are defined as \( (C_F = (N_c^2 - 1)/(2N_c)) \)
\[ \hat{\sigma}_{1q} = 2C_F \hat{x} \hat{z} \left\{ \frac{1}{Q^2 q_T^2} \left( \frac{Q^4}{\hat{x}^2 \hat{z}^2} + \left( Q^2 - q_T^2 \right)^2 \right) + 6 \right\}, \]
\[ \hat{\sigma}_{2q} = 2\hat{\sigma}_{4q} = 8C_F \hat{x} \hat{z}, \]
\[ \hat{\sigma}_{3q} = 4C_F \hat{x} \hat{z} \frac{1}{Q q_T} (Q^2 + q_T^2), \quad (57) \]
\[ \hat{\sigma}_{1g} = \hat{x}(1 - \hat{x}) \left\{ \frac{Q^2}{q_T^2} \left( \frac{1}{\hat{x}^2 \hat{z}^2} - \frac{2}{\hat{x} \hat{z}} + 2 \right) + 10 - 2 \frac{1}{\hat{x}} - 2 \frac{1}{\hat{z}} \right\}, \]
\[ \hat{\sigma}_{2g} = 2\hat{\sigma}_{4g} = 8\hat{x}(1 - \hat{x}), \]
\[ \hat{\sigma}_{3g} = \hat{x}(1 - \hat{x}) \frac{2}{Q q_T} \left\{ 2(Q^2 + q_T^2) - \frac{Q^2}{\hat{x} \hat{z}} \right\}, \quad (58) \]
\[ \hat{\sigma}_{1g} = 2C_F \hat{x}(1 - \hat{z}) \left\{ \frac{1}{Q^2 q_T^2} \left( \frac{Q^4}{\hat{x}^2 \hat{z}^2} + \left( 1 - \hat{z} \right)^2 \left( Q^2 - \frac{z_T^2 q_T^2}{(1 - \hat{z})^2} \right)^2 \right) + 6 \right\}, \]
\[ \hat{\sigma}_{2g} = 2\hat{\sigma}_{4g} = 8C_F \hat{x}(1 - \hat{z}), \]
\[ \hat{\sigma}_{3g} = -4C_F \hat{x}(1 - \hat{z})^2 \frac{1}{\hat{z} q_T} \left\{ Q^2 + \frac{z_T^2 q_T^2}{(1 - \hat{z})^2} \right\}. \quad (59) \]

For a given \( S_{ep}, Q^2 \) and \( q_T \), the kinematic constraints for \( x_{bj} \) and \( z_f \) are
\[ \frac{Q^2}{S_{ep}} < x_{bj} < 1, \quad (60) \]
\[ 0 < z_f < \frac{1 - x_{bj}}{1 - x_{bj} + x_{bj} q_T^2/Q^2}. \quad (61) \]
Consequently, $q_T$ is limited by

$$0 < q_T < Q \sqrt{\left( \frac{1}{x_{bj}} - 1 \right) \left( \frac{1}{z_f} - 1 \right)}.$$  \hspace{1cm} (62)

In a frame where $\vec{q}$ and $\vec{p}$ are collinear, the transverse momentum, $P_{hT}$, obeys

$$P_{hT} = z_f q_T < z_f Q \sqrt{\left( \frac{1}{x_{bj}} - 1 \right) \left( \frac{1}{z_f} - 1 \right)}.$$  \hspace{1cm} (63)

3 Calculation of the single-spin-dependent cross section

In Section 2, we have seen that the single-spin-dependent cross section arises from the pole part of the internal propagators in the partonic hard cross sections. When the conditions (43) and (44) for the hard part are satisfied in the Feynman gauge calculation, the color gauge invariance of the cross section and the consistency of the calculational procedure are guaranteed. In this section, we shall show that these conditions are truly satisfied by all types of pole contributions in the hard part, and derive the twist-3 factorization formula for the single-spin-dependent cross section. Because the relevant three types of poles arise at the different position in the phase space as in (45)-(47), the conditions (43) and (44) should be satisfied separately by the corresponding hard part that arises from each pole contribution. We first discuss in detail each pole contribution in the hard part for the quark fragmentation channel, in particular, those associated with the quark-gluon correlation function $G_F$ for the nucleon in Section 3.1. The results for the gluon fragmentation channel as well as the contributions associated with the correlation function $\tilde{G}_F$ will be presented in the subsequent sections 3.2, 3.3.

3.1 $G_F$ contribution: quark fragmentation channel

A. Hard pole

The diagrams for partonic subprocesses which produce hard-pole (HP) contributions are shown in Fig. 2, with $k_{1,2}$ the momenta of “external quarks” as was defined in Fig. 1(b). The momentum flowing through the quark line marked by a cross is $k_1 + q$, and the corresponding quark propagator produces the HP at $(k_1 + q)^2 = 0$, which reduces to the pole at $x_1 = x_{bj}$ after the collinear expansion, $k_1 \to x_1 p$ (see (45)). The corresponding HP contribution can be evaluated as the imaginary part of the quark propagator through the distribution identity,

$$\frac{1}{(k_1 + q)^2 + i\varepsilon} = P \left( \frac{1}{(k_1 + q)^2} \right) - i\pi \delta \left( (k_1 + q)^2 \right),$$ \hspace{1cm} (64)

before the collinear expansion. In the following, we assume that the diagrams with a cross represent the corresponding Feynman amplitude only with its pole contribution for the crossed propagator.
Figure 2: Feynman diagrams which give rise to the HP contributions in the quark fragmentation channel, where the hard quark fragments into final-state pion and the hard gluon goes into unobserved final state. The cross $\times$ denotes the quark propagator which gives the HP contribution. Mirror diagrams also contribute.

Figure 3: The definition of $F^{HP}_{\rho;\alpha}$, as the sum of three types of diagrams which appear in the LHS of the cut in Fig. 2.

Fig. 3 shows the three types of different diagrams appearing in the LHS of the cut of the diagrams in Fig. 2, and we define $F^{HP}_{\rho;\alpha}(k_1, k_2, q, P_h/z)$ as the sum of the three diagrams in Fig. 3, where the factors for the external lines are amputated, $\rho$ is the Lorentz index for the coherent gluon from the nucleon, and $\alpha$ is the Lorentz index for the hard gluon emitted into the final state. $F^{HP}_{\rho;\alpha}(k_1, k_2, q, P_h/z)$ contains the factor $\delta ((k_1 + q)^2)$ from the second term of (64). Due to the tree level Ward identity shown in Fig. 4, $F^{HP}_{\rho;\alpha}(k_1, k_2, q, P_h/z)$ satisfies the relation

$$(k_2 - k_1)^\rho \bar{u}(P_h/z) F^{HP}_{\rho;\alpha}(k_1, k_2, q, P_h/z) \epsilon^{\alpha}_{(\lambda)}(k_2 + q - P_h/z) \delta \left( (k_2 + q - P_h/z)^2 \right) = 0, \quad (65)$$
where \( \bar{u}(P_h/z) \) is the spinor for the quark fragmenting into pion, \( \epsilon^\alpha_\lambda(k_2 + q - P_h/z) \) is the polarization vector for the hard gluon with the momentum \( k_2 + q - P_h/z \) and the physical polarization \( \lambda \), and \( \delta \left((k_2 + q - P_h/z)^2\right) \) comes from the on-shell condition for the final gluon. We also define the amputated amplitude \( f_\beta(k_2, q, P_h/z) \) for the sum of the diagrams shown in Fig. 5, which represent two types of diagrams in the RHS of the cut in Fig. 2. Combining \( F_{\rho\alpha}^{HP}(k_1, k_2, q, P_h/z) \) and \( f_\beta(k_2, q, P_h/z) \), the sum of the six diagrams in Fig. 2 can be written as

\[
S_{\rho}^{HP}(k_1, k_2, q, P_h/z) = f_\beta(k_2, q, P_h/z) \frac{P_h}{z} F_{\rho\alpha}^{HP}(k_1, k_2, q, P_h/z) \mathcal{P}^{\alpha\beta}(k_2 + q - P_h/z) 2\pi \delta \left((k_2 + q - P_h/z)^2\right),
\]

where we have used the relation \( \sum_{\text{spins}} u(P_h/z)\bar{u}(P_h/z) = P_h/z \), and

\[
\mathcal{P}^{\alpha\beta}(k) \equiv \sum_\lambda \epsilon^*_\alpha_\lambda(k)\epsilon^\beta_\lambda(k)
\]

is the polarization tensor for the on-shell \( (k^2 = 0) \) hard gluon with the sum of \( \lambda \) restricted over the physical polarizations. Note that (66) is exactly the part of \( S^{(1)}_h(k_1, k_2, q, P_h/z) \) appearing in (42), which comes from the HP contribution. Owing to the relation (65),

\[
\sum_{\text{spins}} u(P_h/z)\bar{u}(P_h/z) = P_h/z,
\]
\( S^\text{HP}_\rho(k_1, k_2, q, P_h/z) \) satisfies
\[
(k_2 - k_1)\rho S^\text{HP}_\rho(k_1, k_2, q, P_h/z) = 0.
\]

We now take the derivative of (68) with respect to \( k_i^\sigma \) \((i = 1, 2; \sigma = \perp)\) and set \( k_i = x_i p \). One then obtains
\[
(x_2 - x_1) \frac{\partial S^\text{HP}_\rho(k_1, k_2, q, P_h/z)p^\rho}{\partial k_2^\sigma} \bigg|_{k_i = x_i p} + S^\text{HP}_\sigma(x_1 p, x_2 p, q, P_h/z) = 0,
\]
\[
(x_2 - x_1) \frac{\partial S^\text{HP}_\rho(k_1, k_2, q, P_h/z)p^\rho}{\partial k_1^\sigma} \bigg|_{k_i = x_i p} - S^\text{HP}_\sigma(x_1 p, x_2 p, q, P_h/z) = 0.
\]
This proves that the relation (43) holds for the HP contribution. Since \( x_1 \neq x_2 \) for the HP contribution, the relations (69) and (70) can be written as
\[
\frac{\partial S^\text{HP}_\rho(k_1, k_2, q, P_h/z)p^\rho}{\partial k_2^\sigma} \bigg|_{k_i = x_i p} = -\frac{\partial S^\text{HP}_\rho(k_1, k_2, q, P_h/z)p^\rho}{\partial k_1^\sigma} \bigg|_{k_i = x_i p}
\]
\[
= \frac{1}{x_1 - x_2} S^\text{HP}_\sigma(x_1 p, x_2 p, q, P_h/z),
\]
which proves that the condition (44) is satisfied.

Furthermore, the relation (71) provides a useful method to calculate the corresponding hard cross section: Instead of calculating the derivative \( \frac{\partial S^\text{HP}_\rho(k_1, k_2, q, P_h/z)p^\rho}{\partial k_2^\sigma} \bigg|_{k_i = x_i p} \) for the first term of (42), one simply needs to calculate \( S^\text{HP}_\sigma(x_1 p, x_2 p, q, P_h/z)/(x_1 - x_2) \), the RHS of (71). \( S^\text{HP}_\rho(k_1, k_2, q, P_h/z)p^\rho \) represents the sum of the six diagrams in Fig. 2, and, as was noticed in (65), (66) above, the contribution from every diagram contains the product of two delta functions as \( \delta ((k_1 + q)^2) \delta ((k_2 + q - P_h/z)^2) \). Accordingly, the calculation of the derivative \( \frac{\partial S^\text{HP}_\rho(k_1, k_2, q, P_h/z)p^\rho}{\partial k_2^\sigma} \bigg|_{k_i = x_i p} \) produces the derivative of these delta functions. However, these terms with the derivative of delta functions eventually cancel among each other, since \( S^\text{HP}_\sigma(x_1 p, x_2 p, q, P_h/z)/(x_1 - x_2) \) does not contain such derivatives. This explains the absence of the derivatives of delta functions in the HP contribution, which has been observed in the literature. \(^2\) Therefore, it is much more convenient to calculate \( S^\text{HP}_\sigma(x_1 p, x_2 p, q, P_h/z)/(x_1 - x_2) \) to obtain the hard cross section for the HP contribution.

We also note that the calculation of \( S^\text{HP}_\sigma(x_1 p, x_2 p, q, P_h/z) \) can be further simplified. It is defined as (66) in terms of the physical polarization tensor (67) which can be expressed as
\[
\mathcal{P}^{\alpha\beta}(k) = -g^{\alpha\beta} + \frac{k^\alpha h^\beta + h^\alpha k^\beta}{h \cdot k},
\]
\(^2\)The authors of [31, 32] calculated \( \partial S^\text{HP}_\rho(k_1, k_2, q, P_h/z)p^\rho/\partial k_2^\sigma \bigg|_{k_i = x_i p} \) directly to obtain the HP contributions for Drell-Yan and SIDIS, and found the cancellation among the terms with the derivative of the delta functions.
where $h^\alpha$ is an arbitrary light-like vector ($h^2 = 0$) satisfying $h \cdot k \neq 0$. However, for $S_{\sigma}^{HP}(x_1 p, x_2 p, q, P_h/z)$ with the transverse Lorentz index $\sigma = \perp$, which guarantees the physical polarization for the coherent gluon with the momentum $(x_2 - x_1)p$ in the diagrams in Fig. 2, one can make the replacement $\mathcal{P}^{\alpha\beta}(x_2 p + q - P_h/z) \rightarrow -g^{\alpha\beta}$ because the contribution of the second term of (72) vanishes by the Ward identities owing to the gauge invariance for the sum of the relevant diagrams.

It is straightforward to check that electromagnetic gauge invariance, $q_\mu w^{\mu\nu} = q_\nu w^{\mu\nu} = 0$ for our hadronic tensor (42), is satisfied by the HP contributions. Substituting (71) and (2) into the first term of (42), the relevant hard cross section associated with $G_F(x_1, x_2)$ is proportional to $\text{Tr} \left[ S_{\sigma}^{HP}(x_1 p, x_2 p, q, P_h/z) \slashed{\psi} \right] / (x_1 - x_2)$. Using the Ward identities of QED, the sum of the diagrams in Fig. 2 for $\text{Tr} \left[ S_{\sigma}^{HP}(x_1 p, x_2 p, q, P_h/z) \slashed{\psi} \right]$ vanishes, when a virtual photon vertex is contracted by $q_\mu$ or $q_\nu$.

By the method described above, it is now straightforward to calculate the HP contribution to the cross section.

B. Soft-fermion pole

The diagrams which give rise to the soft-fermion-pole (SFP) contributions are shown in Fig. 6, similarly to those for the HP in Fig. 2. The internal quark propagator which produces the SFP is indicated by a cross in each diagram of Fig. 6. The momentum flowing through this line is $k_1 - k_2 + P_h/z - q$, so that the SFP arises at $(k_1 - k_2 + P_h/z - q)^2 = 0$. Since each graph accompanies the factor $\delta ((k_2 + q - P_h/z)^2)$ coming from the cut propagator for the
hard gluon, this pole reduces to that at $x_1 = 0$ as in (46) in the collinear limit $k_{1,2} \rightarrow x_{1,2} p$. The SFP contribution can be evaluated as the imaginary part $\propto \delta ((k_1 - k_2 + P_h/z - q)^2)$ of the corresponding propagator before the collinear expansion. As in the case of the Ward identity (65) for the HP contribution, the relevant Ward identity for the present case is shown in Fig. 7. Following the same step as for the HP contribution, it is easy to prove that the conditions (43) and (44) are satisfied by the SFP contribution, and also (71) with $HP \rightarrow SFP$ holds. Accordingly, the SFP contribution can be calculated via $S_{\perp SFP}(x_1 p, x_2 p, q, P_h/z)/(x_1 - x_2)$ and $\mathcal{P}^{\alpha\beta} \rightarrow -g^{\alpha\beta}$, i.e., in the same economic way as in the HP contribution. It is also straightforward to check that electromagnetic gauge invariance is satisfied by the SFP contributions using the Ward identities of QED, i.e., the sum of the diagrams in Fig. 6 for the calculation of $\text{Tr} \left[ S_{\perp SFP}(x_1 p, x_2 p, q, P_h/z)\phi \right]/(x_1 - x_2)$ vanishes when a virtual photon vertex is contracted by $q_\mu$ or $q_\nu$.

The method discussed above gives the HP and SFP contributions in terms of the $F$-type functions of (2) based on (42)-(44). It is instructive to note that the HP and SFP contributions can be rewritten in terms of the $D$-type distributions of (3) using the relation (6). In the new expression, $S_{\perp Hp,SFP}(x_1 p, x_2 p, q, P_h/z)$ can be regarded as the hard part associated with the $D$-type distributions.

In this connection, we note that the authors of [22], in their study on $p^+ p \rightarrow \gamma X$, employed a procedure of calculating $S_{\perp SFP}(x_1 p, x_2 p, q, P_h/z)$ directly as the hard cross section corresponding to the $D$-type distributions, and eventually obtained the results which are formally the same as those obtained by the above mentioned rewriting of the $F$-type functions in terms of the $D$-type functions. However, the logic for this procedure in [22] is not based on the relations like (71), (6), (7) in contrast to our method. In fact, the authors of [22] tried to find the hard cross sections corresponding to the $D$-type and $F$-type distributions independently, and relied on their claim that the $F$-type functions do not receive the SFP contribution (see Section 3.3 of [22]). But this statement cannot be consistent with the relation (6) which was not noticed in [22]: If this statement were true, it would mean that the $D$-type functions do not receive the SFP contribution either, since the $F$-type and $D$-type functions are proportional to each other as shown in (6) and (7) for $x_1 \neq x_2$. Their argument leading to the above claim was the following: The $A^+$ component of the coherent gluon field represents the scalar-polarized component at the relevant accuracy in the collinear expansion, and thus decouples from a gauge-invariant set of diagrams in the hard part by means of the Ward identity, so that the hard part for the $F$-type function,
Figure 8: Same as Fig. 2, but for the SGP contributions in the quark fragmentation channel.

\[
\left\{ \partial S^{(1)}_{\rho}(k_1, k_2, q, P_h/z) p^\rho / \partial k_2^+ - \partial S^{(1)}_{\rho}(k_1, k_2, q, P_h/z) p^\rho / \partial k_1^+ \right\}_{k_i=x_i p},
\]
identified as the coefficient of the combination \( \partial^+ A^+ \), should vanish at \( x_1 \neq x_2 \). (And the hard part for the \( F \)-type function can survive only for the SGP contributions with \( x_1 = x_2 \), where \( A^+ \) does not represent a scalar-polarized component.) However, the correct relations from the Ward identity are those as given in (69) and (70), which are different from their relation. Our demonstration leading to (69) and (70) indicates that it is not allowed to apply the Ward identities naively to the “derivative of the amplitude” which appears as a next-to-leading order term in the collinear expansion. We emphasize that both ingredients —— (i) the relations (6), (7) among the nonperturbative functions, and (ii) the correct relations (69), (70) from the Ward identities for the partonic subprocess —— are essential to guarantee the consistency of the calculation. Using only one of them would lead to contradiction or double counting.

C. Soft-gluon pole

The diagrams which give rise to the soft-gluon-pole (SGP) contributions are shown in Fig. 8, where the quark line which produces the pole is indicated by a cross. The momentum flowing through this line is \( k_1 - k_2 + P_h/z \), so that the SGP arises at \( (k_1 - k_2 + P_h/z)^2 = 0 \), which reduces to the pole at \( x_1 = x_2 \) as in (47) for \( k_{1,2} \rightarrow x_{1,2} p \). The SGP contribution can be evaluated as the imaginary part \( \propto \delta ((k_1 - k_2 + P_h/z)^2) \) of the corresponding propagator, i.e., taking the quark line on-shell.

Define \( S^{\rho}_{\ell}(k_1, k_2, q, P_h/z) \) as the sum of the diagrams shown in Fig. 8, where the factors for the external lines are amputated, the index \( \rho \) is for the coherent gluon, and
Likewise we can also define $S^L_{\rho}(k_1, k_2, q, P_h/z)$ corresponding to the mirror diagrams of Fig. 8. $S^L_{\rho}(k_1, k_2, q, P_h/z)$ represent the SGP contributions in the corresponding partonic hard scattering as directed to take the quark line on-shell by the crosses in Fig. 8. It also has the delta function as the cut propagator for the on-shell hard gluon. Therefore $S^L_{\rho}(k_1, k_2, q, P_h/z)$ is proportional to the product of the two delta functions as

$$S^L_{\rho}(k_1, k_2, q, P_h/z) \sim \delta \left((k_1 - k_2 + P_h/z)^2\right) \delta \left((k_2 + q - P_h/z)^2\right).$$

(73)

Notice that we are considering $S^L_{\rho}(k_1, k_2, q, P_h/z)$ with parton momenta before collinear expansion, as in the HP and SFP cases discussed above. Since the quark-gluon vertex attaching the coherent gluon is sandwiched by the on-shell quark lines, the tree level Ward identity implies

$$(k_2 - k_1)^\sigma S^L_{\rho}(k_1, k_2, q, P_h/z) = 0.$$  

(74)

Actually, the Ward identity of this type holds for each diagram in Fig. 8. By taking the derivative of (74) with respect to $k_i^\sigma (i = 1, 2; \sigma = \perp)$ and setting $k_i = x_ip$, one arrives at

$$(x_2 - x_1) \frac{\partial S^L_{\rho}(k_1, k_2, q, P_h/z) p^\rho}{\partial k_2^\sigma} \bigg|_{k_i = x_ip} + S^L_{\sigma}(x_1p, x_2p, q, P_h/z) = 0,$$

(75)

$$(x_2 - x_1) \frac{\partial S^L_{\rho}(k_1, k_2, q, P_h/z) p^\rho}{\partial k_1^\sigma} \bigg|_{k_i = x_ip} - S^L_{\sigma}(x_1p, x_2p, q, P_h/z) = 0.$$  

(76)

Apparently the similar relations hold for $S^R_{\rho}(k_1, k_2, q, P_h/z)$, too. Noting that $S^L_{\rho}(k_1, k_2, q, P_h/z) + S^R_{\rho}(k_1, k_2, q, P_h/z)$ is the part of $S^{(1)}_{\rho}(k_1, k_2, q, P_h/z)$ appearing in (42), which comes from the SGP contribution, the above equation (75) shows that the required consistency condition (43) is satisfied.

So far, the logic used for deriving (75) and (76) is the same as that used for the HP and SFP cases. However, there is an important difference between (75), (76) and (69), (70). Namely, we cannot derive the relation similar to (71) using (75) and (76), because the first term of (75), (76) contains singularity at $x_1 = x_2$ due to the delta function. Owing to (73), $\partial S^L_{\rho}(k_1, k_2, q, P_h/z) p^\rho/\partial k_2^\sigma \big|_{k_i = x_ip}$ consists of the terms with $\delta'(x_1 - x_2) \delta ((x_2p + q - P_h/z)^2)$, $\delta(x_1 - x_2) \delta'((x_2p + q - P_h/z)^2)$ or $\delta(x_1 - x_2) \delta((x_2p + q - P_h/z)^2)$. Because of the factor $x_2 - x_1$ in the first term of (75), the calculation of $S^L_{\rho}(x_1p, x_2p, q, P_h/z)$ misses the terms in $\partial S^L_{\rho}(k_1, k_2, q, P_h/z) p^\rho/\partial k_2^\sigma \big|_{k_i = x_ip}$, which vanish due to $(x_1 - x_2)\delta(x_1 - x_2) = 0$. Therefore it is necessary to calculate $\partial S^L_{\rho}(k_1, k_2, q, P_h/z) p^\rho/\partial k_2^\sigma \big|_{k_i = x_ip}$ directly to get the SGP contribution to the cross section correctly.

This also explains why it is difficult to obtain the SGP cross section in the lightcone gauge, $A^+ = 0$. In this gauge, one would identify the contribution of the $F$-type functions as $A^+ \rightarrow iF^{1n}/(x_1 - x_2)$ and calculate $S^L_{\rho}(x_1p, x_2p, q, P_h/z)$ to get the cross section. But a naive calculation of $S^L_{\rho}(x_1p, x_2p, q, P_h/z)$ would miss some parts of the SGP cross section as was noted above.
Because of the lacking of the analogue of (71), we have to rely on direct inspection of the diagrams in Fig. 8 and their mirror diagrams in order to show that the condition (44) is realized for the SGP contribution. We first note that, when the derivative $\partial/\partial k_{1,2}$ hits an internal propagator which depends on $k_1$ and $k_2$ only through $k_1 - k_2$, the condition (44) is automatically satisfied for that contribution. For other cases, one needs to consider a combination of some diagrams in Fig. 8 and their mirror diagrams. As an example, consider a pair of the diagrams shown in Fig. 9. The momentum flowing through the quark line with a cross are, respectively, $k_1 - k_2 + P_h/z$ and $k_2 - k_1 + P_h/z$ for Fig. 9 (a) and (b), and they give SGPs which have opposite signs of the residue in the collinear limit, $k_i \to x_i p$ ($i = 1, 2$). On the other hand, Fig. 9 (a) and (b), respectively, contain $\delta ((k_2 + q - P_h/z)^2)$ and $\delta ((k_1 + q - P_h/z)^2)$ as the cut propagator for the on-shell hard gluon. Therefore, for the sum of these two diagrams in Fig. 9, the derivative hitting on these delta functions satisfies (44) at the SGP. If the derivative hits other internal propagator such as that with momentum $k_1 + q$, or $k_2 + q$, in Fig. 9, the corresponding contribution simply cancel between the diagrams, because the SGPs have opposite signs of the residue between the diagrams (a) and (b) in Fig. 9. It is straightforward to see that the action of the derivative $\partial/\partial k_{1,2}$ can be classified into one of these three cases for all diagrams in Fig. 8. Therefore, the condition (44) is also satisfied for the SGP contribution by taking into account all the graphs including the mirror diagrams. This is in contrast to the HP and SFP cases, where the condition (44) is already satisfied by summing the diagrams which have a coherent-gluon attachment on one side of the cut.

Here we mention the form of the polarization tensor from the polarization sum (67) of the final-state hard gluon in $S^{L,R}_\rho(k_1, k_2, q, P_h/z)$. In principle, $\mathcal{P}^{\alpha\beta}(k_2 + q - P_h/z)$ with (72) should be used to calculate $S^L_\rho(k_1, k_2, q, P_h/z)$, and likewise for $S^R_\rho(k_1, k_2, q, P_h/z)$. However, one is allowed to make the replacement $\mathcal{P}^{\alpha\beta}(k_2 + q - P_h/z) \to -g^{\alpha\beta}$. This is formally similar to the HP and SFP cases, but the logic to allow the replacement is somewhat sophisticated compared with those cases because we have to treat the scalar polarized coherent-gluon in the present case. For the proof, we first note that, in calculating the hard cross section.
associated with the quark-gluon correlation function $G_F$, based on (42), the corresponding spinor trace in the first term of (42) is taken with the insertion of the Dirac structure $\gamma$ of the first term of (2). We consider the following replacement for this $\gamma$ in the calculation:

$$\gamma = \frac{1}{2} \gamma \gamma \rightarrow \frac{1}{2x_1x_2} \gamma \gamma \gamma \gamma,$$  \hspace{1cm} (77)

A difference between the original hard cross section,

$$\frac{\partial}{\partial k_2^z} \text{Tr} \left[ S^{L,R}_{\rho}(k_1, k_2, q, P_h/z)p^\rho p^\rho \right] \bigg|_{k_i = x_i p},$$  \hspace{1cm} (78)

and the new one with (77),

$$\frac{\partial}{\partial k_2^z} \text{Tr} \left[ S^{L,R}_{\rho}(k_1, k_2, q, P_h/z)p^\rho \frac{1}{2x_1x_2} \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma 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\gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma 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tion for the pion can be obtained as

\[
\frac{d^5 \sigma^V}{dx_{bj}dQ^2dz_Jdq_T^2d\phi} = \frac{\alpha_s^2 \alpha_S}{8\pi^2 c_s^2 c_p Q^2} \sum_{k=1}^{4} A_k \left( \frac{-\pi M_N}{4} \right) \sin \Phi_S \int_{x_{min}}^{1} \frac{dx}{x} \int_{z_{min}}^{1} \frac{dz}{z} 
\]

\[
\times \sum_a e_a^2 \delta_a \left[ \hat{\sigma}_D^V (x \frac{d}{dx} G_F^a (x, x)) + \hat{\sigma}_G^V (x, x) 
\right] + \hat{\sigma}_H^V G_F^a (x, x) + \hat{\sigma}_F^V G_F^a (0, x) \right] D_a(z) \delta \left( \frac{q_T^2}{Q^2} - \left( 1 - \frac{1}{x} \right) \left( 1 - \frac{1}{z} \right) \right), \tag{80} \]

where $G_F^a$ represents $G_F$ for quark-flavor $a$ with electric charge $e_a$, $\delta_a$ is defined as $\delta_a = 1$ for quark and $\delta_a = -1$ for anti-quark, and the summation $\sum_a$ is over all quark and anti-quarks. The factor $-\pi M_N/4$ arises as the product of the factor $i$ in the first term of (42), $M_N/4$ in (2), and $i\pi$ associated with evaluation of the pole contributions. $\hat{\sigma}_D^V$, $\hat{\sigma}_G^V$, $\hat{\sigma}_H^V$, and $\hat{\sigma}_F^V$ are the hard cross sections corresponding, respectively, to the derivative term for the SGP contribution, non-derivative term for the SGP contribution, HP contribution, and SFP contributions. They are given as

\[
\hat{\sigma}_D^V = -\frac{4C_q q_T}{Q^2} \frac{x}{1-z} \hat{\sigma}_U^V, 
\]

\[
\hat{\sigma}_U^V = \begin{cases} 
\hat{\sigma}_U^1 = 2\hat{x}\hat{z} \left[ \frac{1}{Q^2 q_T^2} \left( \frac{Q^4}{\hat{x}^2 \hat{z}^2} + (Q^2 - q_T^2)^2 \right) \right] + 6, \\
\hat{\sigma}_U^2 = \hat{\sigma}_U^4 = 8\hat{x}\hat{z}, \\
\hat{\sigma}_U^3 = 4\hat{x}\hat{z} \frac{1}{Q_T^2} (Q^2 + q_T^2), 
\end{cases} \tag{81} 
\]

\[
\hat{\sigma}_G^V = \begin{cases} 
\hat{\sigma}_G^1 = \frac{8C_q q_T}{Q^2} \hat{x} \left[ \frac{1 + \hat{z}^2}{(1 - \hat{x})^2 (1 - \hat{z})^2} \right] + \frac{2\hat{z}(2 - 3\hat{z}) + (1 - 2\hat{x})(1 + 6\hat{z}^2 - 6\hat{z})}{(1 - \hat{z})^2}, \\
\hat{\sigma}_G^2 = 2\hat{\sigma}_G^4 = \frac{64C_q q_T}{Q^2} \hat{x}^2 \hat{z}, \\
\hat{\sigma}_G^3 = \frac{8C_q}{Q} \hat{x} \left[ \frac{\hat{x}\hat{z}(5 - 4\hat{x})}{(1 - \hat{x})(1 - \hat{z})} + 3 - 4\hat{x} \right], 
\end{cases} \tag{82} 
\]
The twist-3 distribution also with the gluon fragmentation function for the pion, \( \hat{G} \), and \( \hat{G}^2 \sigma \) agree with those in \([32]\). All the other results in (82)-(84) are new.

\[
\begin{align*}
\hat{\sigma}_{H1}^V &= \frac{8q_T}{Q^2} \left( C_F \hat{z} + \frac{1}{2N_c} \right) \frac{\hat{x}(1+\hat{x} \hat{z}^2)}{(1-\hat{x})^2(1-\hat{z})^2}, \\
\hat{\sigma}_{H2}^V &= 0, \\
\hat{\sigma}_{H3}^V &= \frac{8}{Q} \left( C_F \hat{z} + \frac{1}{2N_c} \right) \frac{\hat{x} \hat{z}}{(1-\hat{x})(1-\hat{z})}, \\
\hat{\sigma}_{H4}^V &= \frac{8q_T}{Q^2} \left( C_F \hat{z} + \frac{1}{2N_c} \right) \frac{\hat{x} \hat{z}}{(1-\hat{x})(1-\hat{z})}, \\
\hat{\sigma}_{F1}^V &= -\frac{8C_q q_T}{Q^2} \left[ \frac{-8\hat{x}^2 \hat{z}}{1-\hat{z}} + \hat{x}(2\hat{x} - 1) + \frac{\hat{x}^2 \hat{z}^2(2\hat{x}^2 - 3\hat{x} + 1)}{(1-\hat{x})^2(1-\hat{z})^2} \right], \\
\hat{\sigma}_{F2}^V &= \frac{64C_q q_T}{Q^2} \frac{\hat{x}^2 \hat{z}}{1-\hat{z}}, \\
\hat{\sigma}_{F3}^V &= -\frac{8C_q}{Q} \left[ \hat{x}(4\hat{x} - 3) - \frac{\hat{x} \hat{z}(4\hat{x} - 1)}{1-\hat{z}} \right], \\
\hat{\sigma}_{F4}^V &= -\frac{8C_q q_T}{Q^2} \frac{\hat{x} \hat{z}(1-2\hat{x})^2}{(1-\hat{x})(1-\hat{z})},
\end{align*}
\]

where \( C_q = -1/2N_c \) and \( \hat{\sigma}_{Uk}^V \) in (81) is the same as (57) for the unpolarized cross section (54) except for the color factor. \( \hat{\sigma}_{H1}^V \) was already given in \([29]\). The above result for \( \hat{\sigma}_{H1}^V \) and \( \hat{\sigma}_{H1}^V \) also agree with those in \([32]\). All the other results in (82)-(84) are new.

### 3.2 \( G_F \) contribution: gluon fragmentation channel

The twist-3 distribution \( G_F(x,y) \) contributes to the single-spin-dependent cross section also with the gluon fragmentation function for the pion, \( D_g(z) \). Again, this is classified into the HP, SFP and SGP contributions. The diagrams corresponding to the HP and SFP contributions are given by the same diagrams as shown Figs. 2 and 6, respectively, except that the hard gluon fragments into final-state pion and the hard quark goes into unobserved final state. The diagrams for the SGP contributions are shown in Fig. 10. The method of the calculation and the proof of the conditions (43) and (44) are the same as that described in the previous Section 3.1. So we shall not repeat it here. The result is obtained as follows:

\[
\begin{align*}
\frac{d^5\sigma^V}{dx_{bj}dQ^2dz_fdq_T^2d\phi} &= \frac{\alpha^2_m \alpha_S}{8\pi x_{bj} S_{ep}^2 Q^2} \sum_{k=1}^4 A_k \left( \frac{-\pi M_N}{4} \right) \sin \Phi_S \int_{x_{min}}^{1} \frac{dx}{x} \int_{z_{min}}^{1} \frac{dz}{z} \\
&\times \sum_a c_a \left[ \hat{\sigma}^V_{Dk} \left( x \frac{d}{dx} G^a_F(x,x) \right) + \hat{\sigma}^V_{Gk} G^a_F(x,x) \right]
\end{align*}
\]
Figure 10: Same as Fig. 2, but for the SGP contributions in the gluon fragmentation channel where the hard gluon fragments into final-state pion and the hard quark goes into unobserved final state.

\[ + \delta_a \hat{\sigma}^V g D_k G^a_F(x_b j, x) + \delta_a \hat{\sigma}^V g G^a_L(0, x) \right] D_g(z) \left( \frac{q_T^2}{Q^2} - \frac{1}{\hat{x}^2} \left( 1 - \frac{1}{\hat{x}} \right) \left( 1 - \frac{1}{\hat{z}} \right) \right), \]

where each hard cross section is given by

\[ \sigma^V g = -\frac{4C_g q_T}{Q^2} \frac{\hat{x}}{1 - \hat{z}} \hat{\sigma}^V g, \]

\[
\left\{
\begin{array}{l}
\hat{\sigma}^V g = 2\hat{x}(1 - \hat{z}) \left\{ \frac{1}{Q^2 q_T^2} \left( \frac{Q^4}{\hat{x}^2 z^2} + \frac{(1 - \hat{z})^2}{\hat{z}^2} \left( Q^2 - \frac{\hat{z}^2 q_T^2}{(1 - \hat{z})^2} \right)^2 \right) + 6 \right\}, \\
\hat{\sigma}^V U_1 = 2\hat{x}(1 - \hat{z}), \\
\hat{\sigma}^V U_2 = 2\hat{\sigma}^V U_4 = 8\hat{x}(1 - \hat{z}), \\
\hat{\sigma}^V U_3 = -4\hat{x}(1 - \hat{z})^2 \frac{1}{\hat{z}Q q_T} \left\{ Q^2 + \frac{\hat{z}^2 q_T^2}{(1 - \hat{z})^2} \right\},
\end{array}
\right.
\]

(85)
\[
\begin{align*}
\hat{\sigma}_{G1}^V & = -C_g \frac{8q_T}{Q^2 (1 - \hat{x})^2 (1 - \hat{z}) \hat{z}} \hat{x} \\
& \quad \times \left[2(6\hat{z}^2 - 6\hat{z} + 1)\hat{x}^3 + (-24\hat{z}^2 + 22\hat{z} - 3)\hat{x}^2 + 4\hat{z}(3\hat{z} - 2)\hat{x} - \hat{z}^2 - 1\right], \\
\hat{\sigma}_{G2}^V & = 2\hat{\sigma}_{G4}^V = C_g \frac{64q_T}{Q^2} \hat{x}^2, \\
\hat{\sigma}_{G3}^V & = -C_g \frac{8}{Q (1 - \hat{x}) \hat{z}} \left[(8\hat{z} - 4)\hat{x}^2 + (5 - 12\hat{z})\hat{x} + 3\hat{z}\right], \\
\hat{\sigma}_{H1}^V & = \frac{8q_T}{Q^2} \left(C_F(\hat{z} - 1) - \frac{1}{2N_c}\right) \frac{\hat{x}(1 + \hat{x}(1 - \hat{z})^2)}{(1 - \hat{x})^2 (1 - \hat{z}) \hat{z}} \\
\hat{\sigma}_{H2}^V & = 0, \\
\hat{\sigma}_{H3}^V & = \frac{8}{Q} \left(C_F(\hat{z} - 1) - \frac{1}{2N_c}\right) \frac{\hat{x}(\hat{z} - 1)}{(1 - \hat{x}) \hat{z}}, \\
\hat{\sigma}_{H4}^V & = \frac{8q_T}{Q^2} \left(C_F(\hat{z} - 1) - \frac{1}{2N_c}\right) \frac{\hat{x}}{1 - \hat{x}}, \\
\hat{\sigma}_{F1}^V & = -\frac{8C_q q_T}{Q^2} \frac{\hat{x}}{(1 - \hat{x})(1 - \hat{z}) \hat{z}} \left[2(6\hat{z}^2 - 6\hat{z} + 1)\hat{x}^2 + (12\hat{z}^2 + 10\hat{z} - 1)\hat{x} + \hat{z}^2\right], \\
\hat{\sigma}_{F2}^V & = -\frac{64C_q q_T}{Q^2} \hat{x}^2, \\
\hat{\sigma}_{F3}^V & = -\frac{8C_q}{Q} \frac{\hat{x}}{\hat{z}} [-4\hat{z} + \hat{x}(8\hat{x} - 4) + 1], \\
\hat{\sigma}_{F4}^V & = \frac{8C_q q_T}{Q^2} \frac{(1 - 2\hat{x})^2 \hat{x}}{1 - \hat{x}},
\end{align*}
\]

where \(C_g = N_c/2\). We remind that the hard cross section for the derivative term, (86), was also derived in [29] and \(\hat{\sigma}_{V_{uk}}^V\) is the same as (59) for the unpolarized cross section (54) except for the color factor.

### 3.3 The \(\tilde{G}_F\) contribution

Another twist-3 distribution for the nucleon, \(\tilde{G}_F(x,y)\), also contributes to \(ep^\dagger \rightarrow e\pi X\). Again, in principle, the corresponding cross section arises as the HP, SFP and SGP contributions. The diagrams for those contributions are the same as in the \(G_F\) case for the
quark fragmentation channel as well as the gluon fragmentation channel, see Figs. 2, 6, 8, and 10. The calculation can be carried out in parallel with the case for $G_F$, by changing the relevant Dirac matrix structure to calculate the first term of (42) as $\not{\psi}e^{\alpha p S_\perp} \rightarrow i\gamma_5\not{\psi}S_\perp$ (see (2)). Owing to the anti-symmetry (4), one has $\tilde{G}_F(x, x) = 0$, so that there is no non-derivative term for the SGP contribution. In principle, $\frac{\partial \tilde{G}_F(x_1, x)}{\partial x_1} \bigg|_{x_1=x}$ can be nonzero and could contribute to the cross section as the derivative term for the SGP contribution. However, after summing all the diagrams for the SGP contributions, it turns out that no such derivative term survives. Therefore, there appears no SGP contribution, and the result for the single-spin-dependent cross section associated with $\tilde{G}_F(x, y)$ is obtained from the HP and SFP contributions as

$$\frac{d^5 \sigma^A}{dx_j dy dQ^2 dq_T^2 d\phi} = \frac{\alpha^2_{em} \alpha_s}{8\pi x_{bj}^2 S^2_{ep} Q^2} \sum_{k=1}^{4} \mathcal{A}_k \left( -\frac{\pi M_N}{4} \right) \sin \Phi_S \int_{x_{min}}^{1} \frac{dx}{x} \int_{z_{min}}^{1} \frac{dz}{z}$$

$$\times \sum_a e_a^2 \delta_a \left[ \hat{\sigma}^A_{Hk} \tilde{G}_F^a(x_{bj}, x) + \hat{\sigma}^A_{Fk} \tilde{G}_F^a(0, x) \right] D_a(z)$$

$$+ \left\{ \hat{\sigma}^A_{Hk} \tilde{G}_F^a(x_{bj}, x) + \hat{\sigma}^A_{Fk} \tilde{G}_F^a(0, x) \right\} D_g(z)$$

$$\times \delta \left( \frac{q_T^2}{Q^2} - \left( 1 - \frac{1}{\hat{x}} \right) \left( 1 - \frac{1}{\hat{z}} \right) \right),$$

(90)

where the hard cross sections for the quark fragmentation contribution are given by

$$\hat{\sigma}^A_{H1} = \frac{8q_T}{Q^2} \left( C_F \hat{z} + \frac{1}{2N_c} \right) \frac{\hat{x}(1 - \hat{x}\hat{z}^2)}{(1 - \hat{x})^2(1 - \hat{z})^2},$$

$$\hat{\sigma}^A_{H2} = 0,$$

$$\hat{\sigma}^A_{H3} = -\frac{8}{Q} \left( C_F \hat{z} + \frac{1}{2N_c} \right) \frac{\hat{z}\hat{x}}{(1 - \hat{x})(1 - \hat{z})},$$

$$\hat{\sigma}^A_{H4} = -\frac{8q_T}{Q^2} \left( C_F \hat{z} + \frac{1}{2N_c} \right) \frac{\hat{x}\hat{z}}{(1 - \hat{x})(1 - \hat{z})},$$

(91)
\[
\begin{aligned}
\hat{\sigma}_{F1}^A &= \frac{8C_q q_F}{Q^2} \frac{\hat{x}(1-\hat{z})^2 + \hat{x}(2\hat{z} - 1)}{(1-\hat{x})(1-\hat{z})^2}, \\
\hat{\sigma}_{F2}^A &= 0, \\
\hat{\sigma}_{F3}^A &= \frac{8C_q q_F}{Q} \frac{\hat{x}}{1-\hat{z}}, \\
\hat{\sigma}_{F4}^A &= \frac{8C_q q_F}{Q^2} \frac{\hat{x}\hat{z}}{(1-\hat{x})(1-\hat{z})},
\end{aligned}
\] 

(92)

and those for the gluon fragmentation contribution are given by

\[
\begin{aligned}
\hat{\sigma}_{H1}^g &= \frac{8q_T}{Q^2} \left( C_F(\hat{z} - 1) - \frac{1}{2N_c} \right) \frac{\hat{x}(1-\hat{x}(1-\hat{z})^2)}{(1-\hat{x})^2(1-\hat{z})\hat{z}}, \\
\hat{\sigma}_{H2}^g &= 0, \\
\hat{\sigma}_{H3}^g &= \frac{8}{Q} \left( C_F(\hat{z} - 1) - \frac{1}{2N_c} \right) \frac{\hat{x}(1-\hat{z})}{(1-\hat{x})\hat{z}}, \\
\hat{\sigma}_{H4}^g &= -\frac{8q_T}{Q^2} \left( C_F(\hat{z} - 1) - \frac{1}{2N_c} \right) \frac{\hat{x}}{1-\hat{x}},
\end{aligned}
\] 

(93)

\[
\begin{aligned}
\hat{\sigma}_{F1}^g &= \frac{8C_q q_T}{Q^2} \frac{(-\hat{z}^2 + 2\hat{z}\hat{z} - \hat{x})\hat{x}}{(1-\hat{x})(1-\hat{z})\hat{z}}, \\
\hat{\sigma}_{F2}^g &= 0, \\
\hat{\sigma}_{F3}^g &= \frac{8C_q q_T}{Q} \frac{\hat{x}}{\hat{z}}, \\
\hat{\sigma}_{F4}^g &= -\frac{8C_q q_T}{Q^2} \frac{\hat{x}}{1-\hat{x}}.
\end{aligned}
\] 

(94)

4 Azimuthal asymmetry

By summing all the pole contributions with \( G_F \) and \( \tilde{G}_F \) derived as (80), (85) and (90) in Section 3, one obtains the complete single-spin-dependent cross section for \( e p^+ \rightarrow e \pi X \) from the \( (A) \) term in (1). Its \( \phi \)-dependence can be decomposed as

\[
\frac{d^5 \sigma^{(A)}}{dx_b dQ^2 dz_f dq_T^2 d\phi} = \sin \Phi_S \left( \sigma_0^A + \sigma_1^A \cos(\phi) + \sigma_2^A \cos(2\phi) \right).
\] 

(95)
This should be compared with the similar decomposition of the twist-2 unpolarized cross section of (54) [41, 35]:

\[ \frac{d^5\sigma^{\text{unpol}}}{dx_{bj} dQ^2 dz_f dq_T^2 d\phi} = \sigma_0^U + \sigma_1^U \cos(\phi) + \sigma_2^U \cos(2\phi). \] (96)

If one uses the \textit{lepton plane} as a reference plane to define the azimuthal angle of the spin vector of the initial proton as \( \Phi_S = \phi_S - \phi_h \) and that of the hadron plane as \( \phi_h \), as employed in [7], one has the relation \( \Phi_S = \phi_S - \phi_h \) and \( \phi = -\phi_h \). Accordingly, the azimuthal dependence of the \( \sigma_0^A \) term in (95) is the same as the Sivers effect (\( \propto \sin(\phi_h - \phi_S) \)) and (95) contains the azimuthal components which are absent in the leading order Sivers effect.

We now present a simple estimate of SSA by using the obtained formulae. So far we don't have any definite information on the HP, SFP and SGP components of \( G_F^a \) and \( \tilde{G}_F^a \). In [29], we presented an estimate of SSA including only the derivative term of the SGP function, \( \frac{d}{dx} G_F^a(x, x) \), which is expected to be a good approximation at large \( x_{bj} \) or large \( z_f \). For comparison with that study, we will here include all the SGP contribution, the derivative and non-derivative terms of the SGP function. This calculation may be relevant, since we expect that there are much more soft gluons in the nucleon than the gluons with finite momentum.

In order to see the relative importance of the non-derivative term, we assume the same model for \( G_F^a(x, x) \) as in [29]: We take into account only the contributions from \( u \) and \( d \) quarks with the ansatz \( G_F^a(x, x) = K_a f_a(x) \) \( (a = u, d) \) [25] and the flavor-dependent constant \( K_u = -K_d = 0.07 \), which were shown to approximately reproduce the observed SSA of \( p^+ p \rightarrow \pi X \) [2, 4] as SGP contributions. As in [29], we calculate the azimuthal asymmetries normalized by the unpolarized cross section. We define \( \phi \)-integrated azimuthal asymmetries at \( \Phi_S = \pi/2 \) as,

\[
\left\langle 1 \right\rangle_N \equiv \frac{\sigma_0^A}{\sigma_0^U}, \quad \langle \cos \phi \rangle_N \equiv \frac{\sigma_1^A}{2\sigma_0^U}, \quad \langle \cos 2\phi \rangle_N \equiv \frac{\sigma_2^A}{2\sigma_0^U}. \] (97)

For the purpose of illustration, we choose two sets of kinematic variables: The first one is \( S_{ep} = 300 \text{ GeV}^2, Q^2 = 20 \text{ GeV}^2, x_{bj} = 0.1 \), which is close to the COMPASS kinematics. Another one is \( S_{ep} = 2500 \text{ GeV}^2, Q^2 = 50 \text{ GeV}^2 \) and \( x_{bj} = 0.03 \), which is in the region of planned eRHIC experiment [33]. Both sets give the same cosh \( \psi \) in (17). In our calculation, we have used GRV unpolarized parton distribution \( f_{a,g}(x) \) [42] and KKP pion fragmentation function \( D_{a,g}(z) \) [44] as in [29].

The results for the azimuthal asymmetries (97) for the \( \pi^0 \) production in SIDIS are shown in Figs. 11(a) and (b) for the COMPASS and the eRHIC kinematics, respectively. The asymmetries \( \left\langle 1 \right\rangle_N, \langle \cos \phi \rangle_N \), and \( \langle \cos 2\phi \rangle_N \) are shown by the solid, long-dashed, and dot-dashed curves, respectively, at \( q_T = 4 \text{ GeV} \) as a function of \( z_f \). For comparison, we also showed, by the short-dashed curve, the result for \( \left\langle 1 \right\rangle_N \) which was obtained only with the derivative term of the SGP contribution. One sees that \( \left\langle 1 \right\rangle_N \) is the order of 2 % for the COMPASS kinematics and is the order of 0.5 % for the eRHIC kinematics.

\[ ^5 \text{In [29], we made a sign mistake in this relation.} \]
Figure 11: The calculated azimuthal asymmetries (97) with all the SGP contributions for (a) the COMPASS kinematics and (b) the eRHIC kinematics. For \( \langle 1 \rangle_N \), we have also shown the result with only the derivative term of the SGP contribution.

while \( \langle \cos \phi \rangle_N \) and \( \langle \cos 2\phi \rangle_N \) are negligible for both kinematics. One also sees, comparing the solid and short-dashed curves, that the effect of the non-derivative terms of the SGP contribution is important in the small and intermediate \( z_f \) region, while the derivative term dominates in the large \( z_f \) region. In the large \( z_f \) region, where the large \( x \) region of the distribution function becomes important (see (56), (80), (85)), the derivative term \( (d/dx)G_F(x, x) \) together with our ansatz \( G_F(x, x) = K_f a(x) \) causes a steep rising behavior of the asymmetry. If one had used a model for \( G_F(x, x) \) which behaves as \( \sim (1 - x)^\beta \) at \( x \to 1 \) with larger \( \beta \) than the present model \( G_F(x, x) = K_f a(x) \), as suggested by a recent study on \( p^+p \to \pi X \) [26], one would not have got such a strongly rising behavior and would have obtained a negative asymmetry for \( \langle 1 \rangle_N \) in the whole \( z_f \) region. It is quite natural to expect that the negative trend of the asymmetry \( \langle 1 \rangle_N \) persists in the lower energy region, which is consistent with the recent HERMES data [7]. (Recall the difference between the present work and [7] in the sign convention of the relevant angles for the asymmetry, as stated just below (96).)

It has been known that the effect of the soft gluon radiation is important in the shown \( q_T \) region. However, recent study on these effects for the twist-2 double spin asymmetry based on the \( q_T \) resummation formalism shows that the resummation effect more or less cancels between the polarized and unpolarized cross sections and the fixed order perturbative result for the asymmetry gives a reasonable prediction for the asymmetry [45]. Although the resummation formalism for the twist-3 cross section has not yet been developed, it should be the one which resums logarithmically enhanced contributions due to the emission of the soft gluons, whose coupling is generically blind to spin degrees of freedom. Accordingly, we
expect the result shown in Fig. 11 provides a correct order of magnitude for the SSA (97).

5 Summary and conclusion

In this paper, we have studied the SSA for the pion production in SIDIS, \( ep^\uparrow \rightarrow e\pi X \), in the framework of the collinear factorization. This study is relevant for the pion production with large transverse momentum. In this framework, the SSA is a twist-3 observable, and the single-spin-dependent cross section occurs as pole contributions from internal propagators in the partonic subprocess, which produce the strong interaction phase necessary for SSA. We have derived the complete cross section formula associated with the twist-3 quark-gluon correlation functions for the transversely polarized nucleon, by including all types of pole (hard-pole, soft-fermion-pole and soft-gluon-pole) contributions. To accomplish this study, we have proved that the hard part from each pole contribution satisfies certain constraints from the Ward identities for color gauge invariance, and also clarified a complete set of the gauge-invariant, twist-3 correlation functions. These new developments are crucial to prove the factorization property for the twist-3 single-spin-dependent cross section in manifestly gauge-invariant form, and the “uniqueness” of the corresponding factorization formula, which was missing in the literature. Based on the soft-gluon-pole approximation, we have presented a simple estimate for the azimuthal asymmetry of SSA. It turned out that the non-derivative term of the SGP contribution causes sizable correction to the results using the derivative term only, for the small as well as moderate values of \( x_{bj} \) and \( z_f \).

Our formalism of the twist-3 calculation can be extended to study SSAs in other processes of current interest such as \( p^\uparrow p \rightarrow \gamma X, \ p^\uparrow p \rightarrow \pi X \), which will be reported in the future publication.

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