NANO bT CLOSED SET IN NANO TOPOLOGICAL SPACES

Krishnaveni, K.* and M. Vigneshwaran

Department of Mathematics, Kongunadu Arts and Science College (Autonomous), Coimbatore-641029.

*E.mail: krishnavenikaliswami@gmail.com

ABSTRACT

In this paper, we introduce a new class of set namely nano bT-closed sets in nano topological space. We also discussed some properties of nano bT-closed set.

Keywords: Nano T closed and nano bT-closed.

1. INTRODUCTION

Nano topological space was introduced (Lellis Thivagar and Camel Richard, 2013a, b) with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. He also established certain weak forms of nano open sets such as nano α-open sets, nano semi-open sets and nano pre-open sets. b-open sets in topological spaces was introduced and studied (Andrijevic, 1996). Several properties of a new type of sets called supra T-closed set and supra T-continuous maps was studied (Arockiarani and Trintia Pricilla, 2011). Also a new class of set called nano bT-closed set was introduced and studied (Krishnaveni and Vigneshwaran, 2013). In this paper, we introduced a new class of set called nano bT-closed sets and study its basic properties.

2. PRELIMINARIES

2.1. Definition

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be approximation space. Let X ⊆ U.

(i) The lower approximation of X with respect to R is the set of all objects, which can be classified as X with respect to R and it is denoted by Lₐ(X). That is Lₐ(X) = x ∈ U : R(x) ⊆ X, where R(x) denotes the equivalence class determined by x ∈ U.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by Uₐ(X).

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by Bₐ(X).

That is, Bₐ(X) = Uₐ(X) - Lₐ(X).

2.2. Property

If (U, R) is an approximation space and X, Y ⊆ U, then

(i) Lₐ(X) ⊆ X ⊆ Uₐ(X),
(ii) L(∅) = U(∅) = ∅ and L(U) = U(U) = U,
(iii) Uₐ(X ∪ Y) = Uₐ(X) ∪ Uₐ(Y),
(iv) Uₐ(X ∩ Y) = Uₐ(X) ∩ Uₐ(Y),
(v) Lₐ(X ∪ Y) = Lₐ(X) ∪ Lₐ(Y),
(vi) Lₐ(X ∩ Y) = Lₐ(X) ∩ Lₐ(Y),
(vii) Lₐ(X) ⊆ Lₐ(Y) and Uₐ(X) ⊆ Uₐ(Y)
whenever X ⊆ Y.

(iii) Uₐ(X ∩ c) = [Lₐ(X)] c and Lₐ(X ∩ c) = [Uₐ(X)] c.
(ix) Uₐ Uₐ(X) = Lₐ Uₐ(X) = Uₐ(X).
(x) Lₐ Lₐ(X) = Uₐ Lₐ(X) = Lₐ(X).

2.3. Definition

Let U be the universe, R be an equivalence relation on U and τₐ(X) = {Uₐ, Lₐ(X), Uₐ(X), Bₐ(X)}, where X ⊆ U. Then by property 2.2, τₐ(X) satisfies the following axioms:

(i) U and φ ∈ τₐ(X).
(ii) The union of the elements of any subcollection of τₐ(X) is in τₐ(X).
(iii) The intersection of the elements of any finite subcollection of τₐ(X) is in τₐ(X).
That is, \( \tau_R(X) \) is a topology on \( U \) called the nano topology on \( U \) with respect to \( X \). We call \( (U, \tau_R(X)) \) as the nano topological space. The elements of \( \tau_R(X) \) are called as nano open sets.

2.4. Definition

If \( (U, \tau_R(X)) \) is a nano topological space with respect to \( X \) where \( X \subseteq U \) and if \( A \subseteq U \), then the nano interior of \( A \) is defined as the union of all nano open subsets of \( A \) and it is denoted by \( \text{Nint}(A) \). That is, \( \text{Nint}(A) \) is the largest nano open subset of \( A \). The nano closure of \( A \) is defined as the intersection of all nano closed sets containing \( A \) and it is denoted by \( \text{Ncl}(A) \).

That is, \( \text{Ncl}(A) \) is the smallest nano closed set containing \( A \).

2.5. Definition

A subset \( A \) of a topological space \( (X,\tau) \) is said to be \( b \)-open if \( A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A)) \). The complement of \( b \)-open set is called a \( b \)-closed set.

2.6. Definition

A set \( A \) of \( X \) is called generalized \( b \)-closed set (simply \( gb \)-closed) if \( \text{bcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open. The complement of generalized \( b \)-closed set is generalized \( b \)-open set.

2.7. Definition

Let \( (X,\mu) \) is a supra topological spaces. A subset \( A \) of \( (X,\mu) \) is called \( \tau_R(X) \)-closed set if \( \text{bcl}^\mu(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \tau \)-open in \( (X,\mu) \). The complement of \( \tau_R(X) \)-closed set is called \( \tau \)-open set.

2.8. Definition

A subset \( A \) of a topological space \( (X,\tau) \) is called regular open if \( A = \text{cl}(\text{int}(A)) \). The complement of regular open set is called regular closed set.

2.9. Definition

A subset \( A \) of a topological space \( (X,\tau) \) is called generalized \( b \)-regular closed set if \( \text{bcl}(A) \subseteq U \) and whenever \( A \subseteq U \) and \( U \) is regular open of \( (X,\tau) \). The complement of generalized \( b \)-regular closed set is called generalized \( b \)-regular open set.

2.10. Definition

A subset \( A \) of a supra topological space \( (X,\mu) \) is called \( b\mu \)-closed set (Krishnaveni and Vigneshwaran, 2013) if \( \text{bcl}^\mu(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is \( \tau_R(X) \)-open in \( (X,\mu) \).

3. NANO \( bT \)-CLOSED SET

3.1. Definition

Let \( (U, \tau_R(X)) \) be a nano topological space. A subset \( A \) of \( (U, \tau_R(X)) \) is called nano \( T \)-closed set if \( \text{Nbcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is nano \( gb \)-open in \( (U, \tau_R(X)) \).

3.2. Example

Let \( U = \{a,b,c,d\} \) with \( U/R = \{\{a\},\{d\},\{b,c\}\} \) and \( X = \{a,c\} \). Then the nano topology \( \tau_R(X) \) is \( \{U,\phi,\{a\},\{b,c\},\{a,b,c\}\} \). The nano \( T \)-closed sets are \( U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,d\}, \{b,c\}, \{c,d\}, \{b,c,d\}, \{a,c,d\}, \{a,b,d\} \) and \( \{a,b,c\} \).

3.3. Definition

Let \( (U, \tau_R(X)) \) be a nano topological space. A subset \( A \) of \( (U, \tau_R(X)) \) is called nano \( bT \)-closed set if \( \text{Nbcl}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is nano \( T \)-open in \( (U, \tau_R(X)) \). The complement of nano \( bT \)-closed set is called nano \( bT \)-open set.

3.4. Example

Let \( U = \{a,b,c,d\} \) with \( U/R = \{\{a\},\{d\},\{b,c\}\} \) and \( X = \{a,c\} \). Then the nano topology \( \tau_R(X) \) is \( \{U,\phi,\{a\},\{b,c\},\{a,b,c\}\} \). The nano \( bT \)-closed sets are \( U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,d\}, \{b,c\}, \{c,d\}, \{b,c,d\}, \{a,c,d\}, \{a,b,d\} \) and \( \{a,b,c\} \).

3.5. Theorem

Every nano closed set is nano \( bT \)-closed.

Proof Let \( A \subseteq U \) and \( U \) is nano \( T \)-open set, since \( A \) is nano closed then \( \text{Ncl}(A) = A \subseteq U \). We know that \( \text{Nbcl}(A) \subseteq \text{Ncl}(A) \subseteq U \), implies \( \text{Nbcl}(A) \subseteq U \). Therefore \( A \) is nano \( bT \)-closed.

The converse of the above theorem need not be true as seen from the following example.

3.6. Example

Let \( U = \{a,b,c,d\} \) with \( U/R = \{\{a\},\{d\},\{b,c\}\} \) and \( X = \{a,c\} \). Then the nano topology \( \tau_R(X) \) is \( \{U,\phi,\{a\},\{b,c\},\{a,b,c\}\} \). The nano closed sets are \( U,\phi,\{b,c,d\},\{a,d\} \) and \( \{d\} \).
The nano bT closed sets are U, ϕ, {a}, {b}, {c}, {d}, {a,d}, {b,c}, {c,d}, {b,d}, {b,c,d}, {a,c,d}, {a,b,d} and {a,b,c}. The nano gb closed sets are U, ϕ, {a,b,c}, {c,d}, {a,b}, {a,c}, {a,d}, {b,c}, {a,b,c} and {d}. Here the sets (b), {a}, {c}, {d}, {b,c}, {c,d}, {a,c,d}, {a,b,d} and {a,b,c} are nano bT closed sets but not in nano closed sets.

3.7. Theorem

Every nano b closed set is nano bT closed.

Proof Let A ⊆ U and U is nano T-open set. Since A is nano b closed then Nbcl (A) ⊆ U. Therefore A is nano bT-closed.

The converse of the above theorem need not be true as seen from the following example.

3.8. Example

Let U = {a,b,c,d} with U/R = {{{a},{c},{b,d}}} and X = {a,c}. Then the nano topology τ R (X) = {U,ϕ,{a},{b,c},{a,b,c}}. The nano b closed sets are U,ϕ,{{a},{c},{b,d}},{{a,c},{b,d}},{{a,b,c},{d}},{{a,b,c},{d}}. The nano gb closed sets are U,ϕ,{{a,b,c},{d}},{{a,b,c},{d}},{{a,b,c},{d}}.

Theorem

Every nano bT-closed set is nano gbT-closed.

Proof Let A ⊆ U and U is nano open set. We know that every nano open set is nano T-open set, then U is nano T-open set. Since A is nano bT-closed set, we have Nbcl (A) ⊆ U. Therefore A is nano gbT-closed set.

3.9. Example

Let U = {a,b,c,d} with U/R = {{{a},{c},{b,d}}} and X = {b,d}. Then the nano topology τ R (X) = {U,ϕ,{{b,d}}}. The nano gb closed sets are U,ϕ,{{a},{c},{b,d}}, {{a,b,c},{d}},{{a,b,c},{d}},{{a,b,c},{d}}. The nano gbT closed sets are U,ϕ,{{a},{c},{b,d}},{{a,b,c},{d}},{{a,b,c},{d}}.

Theorem

Every nano T-open set is nano gbT-closed.

Proof Let U be nano T-open in set (U,X). Then Nbcl (A) - A does not contain any non empty nano T-closed set.

Proof: Necessity Let A be nano T-closed set. Suppose F ≠ ϕ is a nano T-closed set of Nbcl (A) - A. Then F ⊆ Nbcl (A) - A implies F ⊆ Nbcl (A) and A. This implies A ⊆ F. Since A is nano bT-closed set, Nbcl (A) ⊆ F. Consequently, F ⊆ Nbcl (A). Hence F ⊆ Nbcl (A) ∩ Nbcl (A) = ϕ. Therefore F is empty, a contradiction.

Sufficiency: Suppose A ⊆ U and that U is nano T-open. If N bcl (A) ⊆ U. Then Nbcl (A) ∩ U is a not empty nano T-closed subset of Nbcl (A) - A.

Hence Nbcl (A) ∩ U is F and N bcl (A) ⊆ U. Therefore A is nano bT-closed set.

3.10. Theorem

If A is nano bT-closed set in a supra topological space (U,X) and A ⊆ B ⊆ Nbcl (A) then B is also nano bT-closed set.

Proof Let U be nano T-open in set (U,X) such that B ⊆ U. Since A ⊆ B ⇒ A ⊆ U and since A is nano...
3.16. **Theorem**

Let A be nano \( bT \)-closed set then A is nano b-closed if \( Nbcl(A) \subseteq A \). Since \( B \subseteq Nbcl(A) \), then \( Nbcl(B) \subseteq U \). Therefore B is also nano \( bT \)-closed set in \((U, X)\).

**Proof** Let A be nano \( bT \)-closed set. If A is nano b-closed, then \( Nbcl(A) \subseteq A \). Conversely, let \( Nbcl(A) \subseteq A \) be nano \( bT \)-closed. Then by the theorem 3.13, \( Nbcl(A) - A \) does not contain any non-empty nano T-closed and \( Nbcl(A) - A = \emptyset \). Hence A is nano b-closed.

3.17. **Theorem**

A subset \( A \subseteq X \) is nano \( bT \)-open iff \( F \subseteq Nbint(A) \) whenever \( F \) is nano T-closed and \( F \subseteq A \).

**Proof** Let A be nano \( bT \)-open set and suppose \( F \subseteq A \), where \( F \) is nano T-closed. Then \( X - A \) is nano \( bT \)-closed set contained in the nano \( T \)-open set \( X - F \). Hence \( Nbcl(X - A) \subseteq X - F \). Consequently, if \( F \) is nano \( T \)-closed set with \( F \subseteq Nbint(A) \) and \( F \subseteq A \), then \( X - Nbint(A) \subseteq X - F \). This implies that \( Nbcl(X - A) \subseteq X - F \). Hence \( X - A \) is nano \( bT \)-closed. Therefore A is nano \( bT \)-open set.

3.18. **Theorem**

If B is nano T-open and nano \( bT \)-closed set in X, then B is nano b-closed.

**Proof** Since B is nano T-open and nano \( bT \)-closed then \( Nbcl(B) \subseteq B \), but \( B \subseteq Nbcl(B) \). Therefore \( B = Nbcl(B) \). Hence B is nano b-closed.

3.19. **Corollary**

If B is nano open and nano \( bT \)-closed set in X. Then B is nano b-closed.

3.20. **Theorem**

Let A be nano g b-open and nano \( bT \)-closed set. Then \( A \cap F \) is nano T-closed whenever \( F \) is nano b-closed.

**Proof** Let A be nano g b-open and nano \( bT \)-closed set then \( Nbcl(A) \subseteq A \) and also \( A \subseteq Nbcl(A) \). Therefore \( Nbcl(A) = A \). Hence A is nano b-closed. Since \( F \) is nano b-closed. Therefore \( A \cap F \) is nano b-closed in X. Hence \( A \cap F \) is nano T-closed in X.

From the above theorem and example we have the following diagram:

\[
\begin{array}{c}
\text{nano closed} \\
\downarrow \\
\text{nano b-closed} \\
\text{nano bT-closed} \\
\downarrow \\
\text{nano gb-closed} \\
\text{nano gbr-closed}
\end{array}
\]

**REFERENCES**

Andrijevic, D. (1996). On b-open sets, Mat. Vesnik 48(1-2): 59-64.

Arockiarani, I. and M. Trinita Pricilla, (2011). On supra T-closed sets, Int. J. Math. Arch., 2(8): 1376-1380.

Krishnaveni, K. and M. Vigneshwaran, (2013). On \( bT \)-Closed sets in supra topological spaces. Int. J. Math. Arch., 4(2): 1-6.

Leilis Thivagar and Camel Richard, (2013a). On nano continuity. Mathematical theory and modeling, 7: 32-37.

Leilis Thivagar and Camel Richard, (2013b). On nano forms of weakly open sets. Int. J. Math. Stat. Inv., 1(1): 31-37.