Information transfer in leaky atom-cavity systems

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Abstract

We consider first a system of two entangled cavities and a single two-level atom passing through one of them. A “monogamy” inequality for this tripartite system is quantitatively studied and verified in the presence of cavity leakage. We next consider the simultaneous passage of two-level atoms through both the cavities. Entanglement swapping is observed between the the two-cavity and the two-atom system. Cavity dissipation leads to the quantitative reduction of information transfer though preserving the basic swapping property.

1 Introduction

Quantum entanglement is endowed with certain curious features. Unlike classical correlations, quantum entanglement cannot be freely shared among many quantum systems. It has been observed that a quantum system being entangled with another one limits its possible entanglement with a third system. This has been dubbed the “monogamous nature of entanglement” which was first proposed by Bennett\textsuperscript{1}. If a pair of two-level quantum systems $A$ and $B$ have a perfect quantum correlation, namely, if they are in a maximally entangled state $\Psi^-= (|01\rangle - |10\rangle)/\sqrt{2}$, then the system $A$ cannot be entangled to a third system $C$. This indicates that there is a limitation in the distribution of entanglement, and several efforts have been devoted to capture this unique property of “monogamy of quantum entanglement” in a quantitative way for tripartite and multipartite systems\textsuperscript{2, 3, 4}. Another distinctive property of quantum entanglement for multipartite systems is the possibility of entanglement swapping between two or more pairs of qubits. Using this property, two parties that never interacted in their history can be entangled\textsuperscript{5}. There may indeed exist a deeper connection between the characteristics of “monogamy” and entanglement swapping since the features of

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the distribution and transfer of quantum information is essentially reflected in the both these properties.

Practical realization of various features of quantum entanglement are obtained in atom-photon interactions in optical and microwave cavities. Recently, some studies have been performed to quantify the entanglement obtained between atoms through atom-photon interactions in cavities. An important attribute of real devices in the ubiquitous presence of dissipative effects in them. These have to be monitored in order for the effects of quantum correlations to survive till detection. The consequences of cavity leakage on information transfer in the micromaser has been quantified recently. It is natural to expect the other characteristics of entanglement such as its “monogamous” nature, and also its exchange or swapping to be affected by dissipative processes. Atom-photon interactions in cavities are a sound arena for the quantitative investigations of different aspects of quantum entanglement in realistic situations.

With the above motivation we perform a quantitative study of the monogamy of quantum entanglement and its swapping in dissipative atom-photon interactions in microwave cavities. We focus on a system of two entangled single-mode cavities which are empty initially. We then consider the passage of a two-level atom through either of both of them. In the next section we first consider a tripartite pure system (two ideal cavities and one atom) and study the features of “monogamy” exhibited between the atom-cavity and the cavity-cavity entanglements. In particular, we demonstrate the applicability of the Coffman-Kundu-Wootters (CKW) “monogamy” inequality to this system. We next consider a realistic cavity with photon leakage, and repeat the above analysis keeping in mind the recently conjectured validity of the CKW inequality extended for mixed states. We find that cavity dissipation could lead to interesting possibilities, such as the enhancement of the entanglement between the atom and the cavity mode that it interacts with, a feature that could be understood by the “monogamous” behaviour of entanglement. In section IV we consider a four-qubit system (two cavities and two atoms) where our goal is to observe entanglement swapping, or the transfer of entanglement from the initially entangled two cavities to the two atoms. Here again, we first perform the analysis with ideal cavities, and then consider the effects of cavity leakage on entanglement swapping. We present some concluding remarks in section V.
2 Monogamy of entanglement in a system of two cavities and a single atom

2.1 Pure state of three qubits

We first consider two ideal cavities which can be maximally entangled by sending a single circular Rydberg atom prepared in the excited state through two identical and initially empty high-Q cavities ($C_1$ and $C_2$). The initial state of the two-cavity entangled system can be written as

$$|\Psi\rangle_{C_1C_2} = \frac{1}{\sqrt{2}}(|0_11_2\rangle + |1_10_2\rangle),$$

where the index 1 and 2 refer to the first and second cavity, respectively. In this set-up we consider the passage of a two-level Rydberg atom $A_1$ prepared in the ground state $|g\rangle$ through the cavity $C_1$. We are considering the resonant interaction between the two-level atom and cavity mode frequency. The interaction Hamiltonian in the rotating frame approximation for the atom-cavity system is

$$H_I = g(\sigma^+a + \sigma^-a^\dagger),$$

where $a^\dagger$ and $a$ are usual creation and destruction operators of the radiation field and $\sigma^+(\sigma^-)$ are atomic operators analogous to the Pauli spin raising and lowering operators obeying the commutation relation $[\sigma^+, \sigma^-] = 2\sigma_z$, where $\sigma_z = +1/2(-1/2)$ represents the atom in the upper (lower) state. $g$ is the atom-field interaction constant (or $gt$ the Rabi angle). The dynamics of the atom-photon interaction is governed by the equation

$$\dot{\rho} = -i[H_I, \rho]$$

with joint three-party initial ($t = 0$) state corresponding to

$$|\Psi(t = 0)\rangle_{C_1C_2A_1} = \frac{1}{\sqrt{2}}(|0_11_2\rangle + |1_10_2\rangle) \otimes |g_1\rangle$$

Hence, a two-level atom entering the empty cavity in the upper state ($|e\rangle$) evolves to

$$|\Psi_e(t)\rangle = e^{-iH_I t}|e, 0\rangle$$

$$= \cos(gt)|e, 0\rangle + \sin(gt)|g, 1\rangle$$
at some time \( t \), and similarly, a two-level Rydberg atom entering the one photon cavity in the ground state evolves to

\[
|\Psi_g(t)\rangle = e^{-iH_I t} |g, 1\rangle = \cos(gt) |g, 1\rangle - \sin(gt) |e, 0\rangle
\]  

(6)

\[
|\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left( |01_2 g_1\rangle + \cos(gt) |11_2 g_1\rangle - \sin(gt) |00_2 e_1\rangle \right)
\]  

(7)

\[
\rho(t)_{C1C2A1} = \text{Tr}_{A1} \rho(t)_{C1C2A1}
\]  

(8)

\[
\rho(t)_{C1C2} = \text{Tr}_{A1} \rho(t)_{C1C2A1}
\]  

\[
= \frac{1}{2} |01_2\rangle\langle 01_2| + \frac{\cos^2 gt}{2} |11_2\rangle\langle 11_2|
\]

\[
+ \frac{\sin^2 gt}{2} |01_2\rangle\langle 01_2| + \frac{\cos gt}{2} |01_2\rangle\langle 11_2|
\]

\[
+ \frac{\cos gt}{2} |11_2\rangle\langle 01_2|
\]  

(9)
\[ \rho(t)_{C_2A_1} = \text{Tr}_{C_1}(\rho(t)_{C_1C_2A_1}), \]
\[ = \frac{1}{2}|1_{2g_1}\rangle\langle 1_{2g_1}| + \frac{\cos^2 gt}{2}|0_{2g_1}\rangle\langle 0_{2g_1}| \]
\[ + \frac{\sin^2 gt}{2}|0_{2e_1}\rangle\langle 0_{2e_1}| - \frac{\sin gt}{2} |1_{2g_1}\rangle\langle 0_{2e_1}| \]
\[ - \frac{\sin gt}{2} |0_{2e_1}\rangle\langle 1_{2g_1}|. \quad (10) \]
\[ \rho(t)_{C_1A_1} = \text{Tr}_{C_2}(\rho(t)_{C_1C_2A_1}), \]
\[ = \frac{1}{2}|0_{1g_1}\rangle\langle 0_{1g_1}| + \frac{\cos^2 gt}{2}|1_{1g_1}\rangle\langle 1_{1g_1}| \]
\[ + \frac{\sin^2 gt}{2}|0_{1e_1}\rangle\langle 0_{1e_1}| - \frac{\sin gt \cos gt}{2} |1_{1g_1}\rangle\langle 0_{1e_1}| \]
\[ - \frac{\sin gt \cos gt}{2} |0_{1e_1}\rangle\langle 1_{1g_1}|. \quad (11) \]

We now compute the mixed-state bipartite entanglement measure (Concurrence)\[10\] for different pairs. These are given by
\[ C(\rho(t)_{C_1C_2}) = |\cos gt|, \quad (12) \]
\[ C(\rho(t)_{C_2A_1}) = |\sin gt|, \quad (13) \]
\[ C(\rho(t)_{C_1A_1}) = |\cos gt \sin gt| \quad (14) \]
and are plotted in Figure 2 for varying Rabi angle, clearly reflecting the monogamous nature of entanglement between \(C_1C_2\) and \(C_2A_1\).

The CKW inequality\[3\] for the tripartite pure state \(\rho(t)_{C_2C_1A_1}\):
\[ C_{C_2C_1}^2 + C_{C_2A_1}^2 \leq C_{C_2(C_1A_1)}^2 \]
reduces to \(\cos^2 gt + \sin^2 gt = 1\) in this case.

2.2 Effects of cavity dissipation on entanglement

Let us now investigate the above case in presence of the cavity dissipation. Since the lifetime of a two-level Rydberg atom is usually much longer compared to the atom-cavity interaction time, we can safely neglect the atomic dissipation. The dynamics of the flight of the atom is governed by the evolution equation
\[ \dot{\rho} = \dot{\rho}_{\text{atom-field}} + \rho_{\text{field-reservoir}}, \quad (15) \]
Figure 2: $C(\rho(t)_{C_1C_2})$ (solid line), $C(\rho(t)_{C_2A_1})$, (dotted line), $C(\rho_{C_1A_1})$ (broken line) plotted with respect to the Rabi angle $\gamma t$.

where the strength of the couplings are given by the parameters $\kappa$ (the cavity leakage constant) and $g$ (the atom-field interaction constant). At temperature $T = 0K$ the average thermal photon number is zero, and hence one has\cite{11}

$$\dot{\rho}_{\text{field-reservoir}} = -\kappa (a^\dagger a \rho - 2 a \rho a^\dagger + \rho a^\dagger a).$$  \hspace{1cm} (16)

When $g \gg \kappa$, it is possible to make a secular approximation\cite{12} while solving the complete evolution equation by combining Eqs.(3) and (16) in order to get the density elements of $\rho(t)_{C_1C_2A_1}$. We also work under a further approximation (that is justified when the cavity is close to $0K$) that the probability of getting two or more photons inside the cavities is zero, or in other words, a cavity always remains in the two level state comprising of $|0>$ and $|1>$. The tripartite (mixed) state is then obtained to be

$$\rho(t)_{C_1C_2A_1} = \alpha_1 |0_11_2g_1\rangle\langle 0_11_2g_1|$$
$$+ \alpha_2 |1_10_2g_1\rangle\langle 1_10_2g_1|$$
$$+ \alpha_3 |0_10_2e_1\rangle\langle 0_10_2e_1|$$
$$+ \alpha_4 |0_11_2g_1\rangle\langle 1_10_2g_1|$$
$$+ \alpha_5 |1_10_2g_1\rangle\langle 0_11_2g_1|$$
$$+ \alpha_6 |0_10_2e_1\rangle\langle 1_10_2g_1|$$
$$- \alpha_5 |0_10_2e_1\rangle\langle 1_10_2g_1|$$
$$- \alpha_6 |0_11_2g_1\rangle\langle 0_10_2e_1|.$$ \hspace{1cm} (17)
where the $\alpha_i$ are given by

\[
\begin{align*}
\alpha_1 &= (1 - \frac{e^{-\kappa_1 t}}{2})e^{-2\kappa_2 t}, \\
\alpha_2 &= (\cos^2 gt)e^{-\kappa_1 t}(1 - \frac{e^{-2\kappa_2 t}}{2}), \\
\alpha_3 &= (\sin^2 gt)e^{-\kappa_1 t}(1 - \frac{e^{-2\kappa_2 t}}{2}), \\
\alpha_4 &= \frac{(\cos gt)e^{-\kappa_1 t}}{2}e^{-\kappa_2 t}, \\
\alpha_5 &= i(\sin 2gt)e^{-\kappa_1 t}(1 - \frac{e^{-2\kappa_2 t}}{2}), \\
\alpha_6 &= i(\frac{\alpha_2 - \frac{\kappa_1 e^{-\kappa_1 t} \cos gt}{4g}}{2} + \frac{\kappa_1 \alpha_3 e^{-\kappa_1 t}}{4g})e^{-\kappa_2 t},
\end{align*}
\]

$\kappa_1$ and $\kappa_2$ are the leakage constants for cavity $C_1$ and $C_2$ respectively. The reduced density states of the pairs $C_1C_2$, $C_2A_1$, $C_1A_1$ are thus given by

\[
\begin{align*}
\rho(t)_{C_1C_2} &= \text{Tr}_{A_1}(\rho_{C_1C_2A_1}), \\
&= \alpha_1|0_11_2\rangle\langle 0_11_2| + \alpha_2|1_10_2\rangle\langle 1_10_2| \\
&+ \alpha_3|0_10_2\rangle\langle 0_10_2| + \alpha_4|0_11_2\rangle\langle 1_10_2| \\
&+ \alpha_4|1_10_2\rangle\langle 0_11_2|. \quad (18)
\end{align*}
\]

\[
\begin{align*}
\rho(t)_{C_2A_1} &= \text{Tr}_{C_1}(\rho_{C_1C_2A_1}), \\
&= \alpha_1|1_2g_1\rangle\langle 1_2g_1| + \alpha_2|0_2g_1\rangle\langle 0_2g_1| \\
&+ \alpha_3|0_2e_1\rangle\langle 0_2e_1| - \alpha_6|1_2g_1\rangle\langle 0_2e_1| \\
&+ \alpha_6|0_2e_1\rangle\langle 1_2g_1|. \quad (19)
\end{align*}
\]

\[
\begin{align*}
\rho(t)_{C_1A_1} &= \text{Tr}_{C_2}(\rho_{C_1C_2A_1}), \\
&= \alpha_1|0_1g_1\rangle\langle 0_1g_1| + \alpha_2|1_1g_1\rangle\langle 1_1g_1| \\
&+ \alpha_3|0_1e_1\rangle\langle 0_1e_1| + \alpha_5|1_1g_1\rangle\langle 0_1e_1| \\
&- \alpha_5|0_1e_1\rangle\langle 1_1g_1|. \quad (20)
\end{align*}
\]

and one can obtain the respective concurrences. These, namely, $C(\rho(t)_{C_1C_2})$, $C(\rho(t)_{C_1A_1})$, and $C(\rho(t)_{C_2A_1})$ are plotted with respect to the Rabi angle $gt$. 

7
in Figure 3. As expected, dissipation reduces the respective concurrences. However, the “monogamous” character, or the ‘complementarity’ between $C(\rho(t)_{C_1C_2})$ and $C(\rho(t)_{C_2A_1})$ is maintained even with cavity leakage.

Figure 3: $C(\rho(t)_{C_1C_2})$ (solid line), $C(\rho(t)_{C_2A_1})$, (dotted line), $C(\rho(t)_{C_1A_1})$ (broken line) plotted with respect to the Rabi angle $gt$. $\frac{\kappa_1}{g} = \frac{\kappa_2}{g} = 0.1$.

To verify the CKW inequality for the mixed state $\rho(t)_{C_1C_2A_1}$, one has to average $C(\rho(t)_{C_2(C_1A_1)})$ over all pure state decompositions. We however, adopt an utilitarian point of view, and for small $\kappa$ take $C(\rho(t)_{C_2(C_1A_1)}) \approx 2\sqrt{\det\rho_{C_2}}$. Note that this result holds exactly for a pure state. Nevertheless, for a small value of $\kappa$ and for a bipartite photon field, one stays very close to a pure state. In Figure 4 we plot the left and the right hand sides ($C^2_{C_2C_1} + C^2_{C_2A_1}$ and $C^2_{C_2(C_1A_1)}$ respectively), of the corresponding CKW inequality and observe that it always holds under the above approximation.

An interesting feature of the entanglement obtained between the atom $A_1$ and the cavity $C_1$ through which it interacts directly is displayed in Figure 5 where $C(\rho(t)_{A_1C_1})$ is plotted versus the dissipation parameter $\kappa$. Note that the concurrence increases for increasing cavity loss. This happens because the cavity leakage reduces the initial entanglement between $C_1$ and
$C_2$, and hence makes room for the subsequent entanglement between $C_1$ and $A_1$ to form. The dissipative mechanism is thus a striking confirmation of the “monogamous” character of entanglement. The role of the dissipative environment in creating desired forms of entanglement has been revealed earlier in the literature[13]. The present case can be also viewed as a further example of this kind.

3 Entanglement swapping in a system of two cavities and two atoms

3.1 Ideal case of four qubits

In this section we will consider a few aspects of entanglement swapping or the transfer of entanglement from the two-cavity to the two-atom system. Such a scheme can be affected by sending two Rydberg atoms $A_1, A_2$ prepared in their ground states $g_1, g_2$ through two maximally entangled cavities $C_1, C_2$ respectively. The time of flights for the atoms through the cavities are same.
Figure 5: $C(\rho(t)_{A_1C_1})$ (solid line) for $gt = \pi/4$, $C(\rho(t)_{A_1C_1})$ (broken line) for $gt = 3\pi/4$, $C(\rho(t)_{A_1C_1})$, (dotted line) for $gt = 5\pi/4$ plotted with respect to $\log(\kappa/g)$, where $\kappa/g = \kappa_1/g = \kappa_2/g$.

So at $t = 0$, the state of the total system is

$$|\Psi(t = 0)\rangle_{C_1C_2A_1A_2} = \frac{1}{\sqrt{2}}(|0_11_2\rangle + |1_10_2\rangle) \otimes |g_1g_2\rangle$$

(21)

Figure 6: Two Rydberg atoms $A_1, A_2$ prepared in the ground states $g_1, g_2$ through two maximally entangled cavities $C_1, C_2$ respectively.

For any interaction time $t$ the evolved state is

$$|\Psi(t)\rangle_{C_1C_2A_1A_2} = \frac{1}{\sqrt{2}}(|0_11_2g_1g_2\rangle - \sin gt|0_10_2g_1e_2\rangle + \cos gt|1_10_2g_1g_2\rangle - \sin gt|0_10_2e_1g_2\rangle)$$

(22)
The reduced density states of the pairs $C_1C_2, A_1A_2$ are given by
\[
\rho(t)_{C_1C_2} = |\Psi(t)_{C_1C_2A_1A_2}\rangle\langle\Psi(t)_{C_1C_2A_1A_2}|
\]
(23)

\[
\rho(t)_{C_1C_2} = \text{Tr}_{A_1A_2}(\rho(t)_{C_1C_2A_1A_2}),
\]
\[
= \frac{\cos^2 gt}{2}|0_11_2\rangle\langle 0_11_2| + \frac{\cos^2 gt}{2}|1_10_2\rangle\langle 1_10_2|
\]
\[
+ \sin^2 gt|0_10_2\rangle\langle 0_10_2| + \frac{\cos^2 gt}{2}|0_11_2\rangle\langle 1_10_2|
\]
\[
+ \frac{\cos^2 gt}{2}|1_10_2\rangle\langle 0_11_2|. \quad (24)
\]

\[
\rho(t)_{A_1A_2} = \text{Tr}_{C_1C_2}(\rho_{C_1C_2A_1A_2}),
\]
\[
= \cos^2 gt|g_1g_2\rangle\langle g_1g_2| + \frac{\sin^2 gt}{2}|g_1e_2\rangle\langle g_1e_2|
\]
\[
+ \frac{\sin^2 gt}{2}|e_1g_2\rangle\langle e_1g_2| + \frac{\sin^2 gt}{2}|g_1e_2\rangle\langle e_1g_2|
\]
\[
+ \frac{\sin^2 gt}{2}|e_1g_2\rangle\langle g_1e_2|. \quad (25)
\]

The concurrences for the pairs $C_1C_2$ and $A_1A_2$ are plotted in the Figure 7. One sees that the entanglement between two cavities are swapped by two atoms for the interaction times $gt = (2n + 1)\pi/2$, $(n = 0, 1, 2, \ldots)$.

3.2 Information transfer with cavity dissipation

Finally, we consider the effect of cavity leakage on the transfer of information from the two-cavity to the two-atom system. Under the secular approximation and the approximation of a two-level cavity, one can solve the master equation to obtain the four-party density matrix which can be formally expressed as
\[
\rho(t)_{C_1C_2A_1A_2} = \alpha_1|0_11_2g_1g_2\rangle\langle 0_11_2g_1g_2|
\]
Figure 7: $C(\rho(t)_{c_1c_2})$ (solid line), $C(\rho(t)_{A_1A_2})$, (dotted line) plotted with respect to the Rabi angle $gt$.

\[ +\alpha_2|0_1 0_2 g_1 e_2\rangle\langle 0_1 0_2 g_1 e_2| \\
+\alpha_3|1_1 0_2 g_1 g_2\rangle\langle 1_1 0_2 g_1 g_2| \\
+\alpha_4|0_1 0_2 e_1 g_2\rangle\langle 0_1 0_2 e_1 g_2| \\
+\alpha_5|0_1 1_2 g_1 g_2\rangle\langle 1_1 0_2 g_1 g_2| \\
+\alpha_5|1_1 0_2 g_1 g_2\rangle\langle 0_1 1_2 g_1 g_2| \\
+\alpha_6|0_1 0_2 g_1 e_2\rangle\langle 0_1 0_2 e_1 g_2| \\
+\alpha_6|0_1 0_2 e_1 g_2\rangle\langle 0_1 0_2 g_1 e_2| \\
+ \ldots \]  

(28)

where the $\alpha_i$ are given by

\[
\begin{align*}
\alpha_1 &= (1 - \frac{e^{-\kappa_1 t}}{2})e^{-\kappa_2 t}\cos^2 gt, \\
\alpha_2 &= (\sin^2 gt)e^{-\kappa_2 t}(1 - \frac{e^{-\kappa_1 t}}{2}), \\
\alpha_3 &= (\cos^2 gt)e^{-\kappa_1 t}(1 - \frac{e^{-\kappa_2 t}}{2}), \\
\alpha_4 &= (\sin^2 gt)e^{-\kappa_1 t}(1 - \frac{e^{-\kappa_2 t}}{2}), \\
\alpha_5 &= \frac{(\cos gt)e^{-\kappa_1 t/2}e^{-\kappa_2 t/2}}{2}, \\
\alpha_6 &= \frac{(e^{-\kappa_1 t/2}\sin gt - \frac{\kappa_1 e^{-\kappa_1 t/2}}{2g} + \frac{\kappa_2}{2g})}{2} \times 
\end{align*}
\]
\[
(e^{-\kappa_1 t/2} \sin gt - \frac{\kappa_2 e^{-\kappa_2 t/2}}{2g} + \frac{\kappa_2}{2g})
\]

Apart from the above eight terms no other term contributes to either of the reduced density states \(\rho_{C_1C_2}\) or \(\rho_{A_1A_2}\), which are given by

\[
\rho(t)_{C_1C_2} = \text{Tr}_{A_1A_2}(\rho(t)_{C_1C_2A_1A_2}),
\]
\[
= \alpha_1|0_11_2\rangle\langle 0_11_2| + \alpha_3|1_10_2\rangle\langle 1_10_2| + (\alpha_2 + \alpha_4)|0_10_2\rangle\langle 0_10_2| + \alpha_5|0_11_2\rangle\langle 1_10_2| + \alpha_5|1_10_2\rangle\langle 0_11_2|.
\]

(29)

\[
\rho(t)_{A_1A_2} = \text{Tr}_{C_1C_2}(\rho(t)_{C_1C_2A_1A_2}),
\]
\[
= (\alpha_1 + \alpha_3)|g_1g_2\rangle\langle g_1g_2| + \alpha_2|g_1e_2\rangle\langle g_1e_2| + \alpha_4|e_1g_2\rangle\langle e_1g_2| + \alpha_6|e_1e_2\rangle\langle e_1e_2|.
\]

(30)

Figure 8: \(C(\rho(t)_{C_1C_2})\) (solid line), \(C(\rho(t)_{A_1A_2})\), (dotted line) plotted with respect to the Rabi angle \(gt\). \(\kappa_1/g = \kappa_2/g = 0.1\)

Though the concurrences \(C(\rho(t)_{C_1C_2})\) and \(C(\rho(t)_{A_1A_2})\) are reduced by the loss of cavity photons, one sees from Figure 8 that perfect swapping is still
obtained for $gt = (2n + 1)\pi/2$. One of the basic features of information exchange between bipartite systems, represented by entanglement swapping, is thus seen to be preserved for mixed states too.

4 Conclusions

In this paper we have considered two important and interesting features of quantum entanglement, viz., “monogamy”, and entanglement swapping. We have used the set-up of two initially entangled cavities and a single Rydberg atom passing through one of them to study the quantitative manifestation of a “monogamy” inequality in atom-photon interactions. The unavoidable photon leakage exists in all real cavities used for the practical realization of quantum information transfer. The effects of such dissipation have been investigated on the “monogamous” nature of the entanglement between the two cavities, on one hand, and the atom and the second cavity on the other. We have found that the essential “monogamous” character is preserved even with cavity dissipation. We have further seen that the entanglement between the atom and the cavity through which it passes increases with larger dissipation, a feature that could be understood by invoking the “monogamous” character of entanglement. We have then considered a set-up involving two entangled cavities, and two Rydberg atoms. Entanglement swapping from the two cavities by the two atoms which never interact directly with each other is observed in this system. Cavity dissipation, of course reduces the total amount of information exchange, similar to the results obtained in the context of the single-atom micromaser. Moreover, here we have verified that the property of swapping is preserved with dissipation. Further studies on different quantitative manifestations of information transfer in the presence of dissipative effects might be useful for the construction of realistic devices implementing various protocols. Practical realization of two-cavity entanglement is in progress at the Ecole Normale Superieure.

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