Correlation and diffusion breaking in the failure process of composite materials

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Abstract: Fiber bundle model is one of the important statistical and theoretical physics approaches to investigate the fracture and breakdown of heterogeneous material extensively used both by the engineering and physics community. In this paper, using the correlation function and its properties, we study the diffusion of the failure process in composite materials that consist of a set of fibers loaded by an external constant total load. The investigation is based on fiber bundle model where the fibers are randomly oriented; each fiber is subject only to the cosine component of the external load, and when it fails, its load is shared by surviving neighboring fibers according to the local load sharing rule (LLS). The failure process ensures an advancing interfacial fracture and the areas of the damaged regions increases with time until a final crack of material. By using the correlation function of the fraction of broken fibers, we study the diffusion properties of micro cracks created in the composite materials. The results show that the correlation decreases exponentially with time and decreases with both of applied load and temperature. We calculate then the effective diffusion coefficient cracks which increase exponentially with applied load and linearly with the temperature. The failure time $t_f$ of the materials decrease with temperature as a power law. Obtained results are compared with the one of the classical model consist of a set of parallel fibers.

1. Introduction

The failure of composite materials has recently attracted much technological and industrial interest, it has been widely studied in statistical and theoretical physics, several authors propose that the failure of a composite materials subjected to an increasing external load shares many features with thermodynamic and statistic phase transitions [1-2]. In particular, a stressed composite can be considered to be in a metastable state and the point of global failure can be seen as a nucleation process in a first order transition near a spinodal [2-3]. Thus, the power law behavior observed experimentally in the acoustic emission during failure [4] has been compared with the mean-field scaling expected close to a spinodal point [5]. In analogy with spinodal nucleation [6], scaling behavior can only be seen when long-range interactions are present, as it is the case for elasticity but should not be observable when the stress transfer function is short ranged. This observation is confirmed in statistical fracture models with short-range elastic forces which usually do not show scaling [1;7;8].

The fiber bundle models (FBM) is a very important statistical models of material failure [9;10], which have been extensively studied during the past years. This model consists of a set of parallel fibers having statistically distributed strength. The sample is loaded parallel to the fibers direction, and the fibers fail if the load on them exceeds their threshold value. In stress controlled experiments, Each fiber is linearly elastic up to a threshold length which is usually taken randomly from some distribution function: Uniform, Weibull, Gaussian, Gumbel [11-12], when a fiber fails, the load carried is redistributed among the intact ones following two different transfer load type GLS and LLS [7;11;12]. Among the several
theoretical approaches, one simplification that makes the problem analytically tractable is the assumption of global load-sharing (GLS), which means that after each fiber breaking the stress is equally distributed on the intact fibers neglecting stress enhancement in the vicinity of failed regions [13-15]. The second is local load-sharing (LLS) model, the released stress is distributed equally only to the nearest surviving neighbors [16-17].

The relevance of FBM is manifold, in spite of their simplicity, these models capture the most important aspects of material damage and due to the analytic solutions, and they provide a deeper understanding of the fracture process. Furthermore, they serve as a basis for more realistic damage models having also practical importance [1;3;14]. The very successful micro mechanical models of fiber reinforced composites are improved variants of FBM taking into account stress localization (LLS) the effect of matrix material between fibers , and possible nonlinear behavior of fibers [15-21].

Previous studies of FBM addressed the macroscopic constitutive behavior, the reliability and size scaling of the global material strength, and the avalanches of fiber breaks preceding ultimate failure [21-23]. In classical fiber bundle model, the fibers have the same direction as the applied load. This choice is made in order to simplify the calculations [3;24]. Nevertheless, in and reality, it not reflects the anisotropy of the composite materials.

In order to check the effect of the anisotropy on failure process, we have considered, in this paper, a package of fibers randomly arranged in all directions according to the vertical one [1;24]. These materials are subject to a constant external load in the framework of a local load-sharing rule in two dimensions. Throughout the paper, we have used uniform distribution for breaking elongation threshold of each fiber.

In this investigation, we propose to study the failure process by using the correlation function associated to the number of broken fiber per time unity [3], and we evaluate behavior of the diffusion failure coefficient as function of two physics parameter, load and temperature, we also study failure time behavior versus temperature. Each result will be compared with those obtained in the classical model when the external load has the same direction to fibers [3].

2. Fiber bundle model

As mentioned before, in our approach we use the fiber bundle model with other arrangement fibers. We consider composite materials represented as a discrete set of N fibers randomly oriented in all directions according to the vertical one, placed on a regular square lattice of size \( L \). Each fiber \( i \) is inclined according to the vertical axis with a fixed angle \( \theta_i \), \( 0 < |\theta_i| < \frac{\pi}{2} \). \( \theta_i \) is randomly dispersed with the probability density \( p(x) \) and the uniform distribution \( P(x) \) [1-3]:

\[
P(\theta) = \int_{0}^{\theta} p(y) dy = \theta
\]  

(1)

The fibers, which are placed vertically (\( \theta = 0 \)) have the same initial length \( l_0 \) and are mounted with well-fixed clamps belonging to the two lateral surfaces of the system. However, the ones, which are weakly inclined, \( \{0 < |\theta_i| < \frac{\pi}{2}\} \) have different initial lengths, which are chosen in such a way that these fibers also remain attached to the two lateral surfaces by clamps: \( l_0 = l_i \cos \theta_i \). Finally, the fibers, which are strongly inclined, \( \{\frac{\pi}{2} < |\theta_i| < \pi\} \) have the same initial length \( l_0 \). The fibers are mounted between two hard clamps and the matrix ensures the cohesion of the system, we suppose that each fiber keep the same direction during the application of load (\( \theta_i \) is constant) [8]. The fibers can solely support longitudinal deformation under an external total constant load parallel to the vertical direction [1]. Thus, each fiber is subjected to an imposed tensile constant load \( f \), which leads to an axial longitudinal displacement, by the effect of its axial component \( f_{\theta_i} \):

\[
f_{\theta_i} = f \cos \theta_i
\]  

(2)

This cosine component ensures an axial elongation \( x \), which is linked by the Hooke’s law:
where $k$ denotes the stiffness, which is the same for all the fibers. However, the sinus component is mitigated by the matrix-fiber interface reaction. For this reason, the meaning action is arisen from the cosine force component [1].

On the other hand, in order to take into account of temperature effect, we introduce the corresponding local elongation $\zeta_i$ of each fiber:

$$\zeta_i = \gamma_i \sqrt{K_B T}$$

$K_B$ is the Boltzmann constant, and $\gamma$ is coefficient of proportionality between elongation and square root of one-dimensional thermal energy (fiber). $l_i$ is the initial length of fiber $i$.

The temperature has a quasi-static effect on the failure process in the composite material, yet that will affect the lifetime of the system [25-27]. Then, the actual total elongation arising $x$ of each fiber is written as:

$$x = x_i + \zeta$$

If the arising elongation $x$ of fiber $i$ exceeds the broken threshold elongation $x_c$, the fiber cracks and its load is shared by its neighboring remains fibers and so one [1;4]. In this paper the broken threshold elongation $x_c$ is randomly chosen and given with the probability density $p(x)$ and the uniform distribution $P(x)$ [1]:

$$P(x) = \int_0^x p(y)dy = x \quad \text{with} \quad p(x) = 1$$

The fibers behave linearly elastic until they fail at a failure threshold elongation. The randomness of breaking thresholds is assumed to represent the disorder of heterogeneous materials. During this process when some threshold value of elongation is exceeded the stress carried by the broken fibers is shared by the intact neighboring fibers and so on. Thus each of the other fibers undergoes another elongation due to the force released by the LLS share and the expression of total elongation $x'$ of each fiber becomes:

$$x' = x + \delta$$

In our model the mean threshold elongation is $<x_c> \approx 0.1 l_i$. Our calculations are done for a load range $0 < f \leq 0.65 f_c$ and temperature range $0.025T_c \leq T \leq 0.05T_c$ where $f_c$ represents the mean critical load at which the system crack totally and $T_c$ represents a critical temperature value at zero applied load ($f=0$) [1].

We propose a quantitative study of failure diffusion in composite material by using the correlation function associated to number of broken fiber per time unity [3]:

$$G(\tau) = < \frac{dN_b}{dt}(t + \tau) \frac{dN_b}{dt}(t) >$$

$N_b(t)$ is the number of broken fibers at time $t$. The basic idea is that there is an analogy between the crack diffusion process and the adatoms diffusion in the scaling tunneling microscopy (STM) one [27-30], so we propose to evaluate variation of the diffusion coefficient of failure as function of two different physics parameter force and temperature. The results will be compared with those finding in the classical fiber bundle model [3].

**III- Results and Discussions**

We consider a composite material of square size $L$ with $N_0 = L \times L$ intact fibers randomly oriented in different directions according to the (OZ) axis. Each fiber $i$ is randomly oriented with a fixed angle $\theta_i$
and supports an external constant load $f$. Initially, the material resists to the applied load, but at time $t_r$ when the elongation $x$ for a fiber become greater than their critical elongation, micro cracks appear in the system [1]. Nevertheless the length $l(i)$ of the other intact fiber $i$ varies differently with time according to the local load sharing (LLS) of load. When a fiber breaks, the load carried by it is redistributed equally among its neighbors, then this breaking and redistribution dynamics continues. In general, whenever a fiber breaks, the load carried by that fiber is equally shared by the surviving fiber(s) nearest to it [1;16;17].

On the other hand, Figure 1 presents an image system (3D) calculated for material of size $L = 20$ at time $t = 0.25t_f$. The applied load and the considered temperature are respectively $f/f_c = 0.3$ and $T/T_c = 0.03$. The blue spots represent the broken fibers [1].

As mentioned before, by correlation function associated to number of broken fiber per time unity, defined in equation (6), we study the diffusion of failure process in a composite material with randomly oriented fibers [3 ;30].

Firstly, we have calculated the time evolution of the intact fibers fraction $\rho_A = \frac{N_A}{N}$, where $N_A$ is the number of surviving fibers. Results are plotted in figure 2. The fraction $\rho_A$ is saturated the vicinity of 0.5, for this system with randomly oriented fibers, all the considered applied stress of range values $(0 < f / f_c < 0.65)$ cannot break totally the system in spite of that the load will be shared equally by the nearest surviving fiber(s) and so on. These load shares accumulate on all the fibers on the perimeter and grow with time since their respective appearance on the perimeter of the damaged regions, in addition, when the applied load value increases, the saturated intact fiber density decreases [1], but not reaching the value 0, because the fibers which are more inclined $\left[0 < |\theta| < \frac{\pi}{4}\right]$ resist to the applied load and cannot cracked. In contrary, in the FBM with parallel fibers the system crack totally and the broken fraction tend to the value 1 [1;17;31].

Nevertheless, the fibers which are less inclined $\left[0 < |\theta| < \frac{\pi}{4}\right]$ are the most affected by the applied load than the others ones more inclined which can undergo any load. For this last case The only felled charges are transferred to the fibers by the matrix under LLS mechanism [1]. The heterogeneous material with randomly oriented fibers is more resistant than the one of the classical model where the fibers have a same direction and the corresponding observed avalanche breaking is characterized by two successive regimes [1;13;17].
We analyze the correlation process observed in the failure process for the considered materials by using the equation (6). Figure 3 presents time variation of the normalized crack correlation function for different values of applied load calculated for system size $L=500$ at temperature $T/T_c = 0.03$. The correlation function $G(t)$ decreases exponentially with time as:

$$G(t) = G_0 \exp\left(-\frac{t}{\tau}\right)$$

(8)

where $\tau$ is the characteristic time of the correlation function which corresponds to the $0.37G_0$.

The correlation tends to approximately 0.5. By analogy, this behavior of $G(t)$ is also observed in the surface adatoms diffusion process by using the scaling tunneling microscopy (STM) [26-28].

Also, the same behavior was found in the classical model [3], but the correlation function of the latter case exponentially decreases going through great values and tends to 0. Comparing the two curves, we remark that in the system with randomly oriented fibers, the correlation function decreases rapidly in time compared with that found in the classical model. This strong fall of $G(t)$ reflects a weak correlation between broken fibers, in other words, broken fibers which are randomly directed are less correlated in their breaks compared to those having the same direction in the classical system [3]. This can be explained on the one hand by the fact that inclined fibers resist more to the force, on the other hand, they are not all subject to force.

In order to check the effect of the external load on the correlation function, we have calculated for the same intervals of time, the normalized correlation function $G(t)$ versus applied load for system size $L=500$ at temperature $T/T_c = 0.03$. The corresponding results are plotted in figure 4.

Correlation function decreases with applied load with a less impressive monotony even if the load is applied on all the system and even if this correlation decrepit the total undamaged region of system, the same curve shape as that found in the classical case but with small correlation values which correspond to large force values [3], which confirm that the anisotropy in the directions of fibers increases resistance of the system and paralyzes the diffusion of the crack in the material.
Diffusion of micro cracks in a composite material is one of interesting process to understand the behavior of the failure process. The effective diffusion coefficient $D_{\text{eff}}$ can be calculated by using the characteristic time $\tau$ of the crack correlation function $G$. The method is based on the proportionality relation between $D_{\text{eff}}$ and $\tau$ used in the fluctuations of the adatoms surface in the scanning tunneling microscopy investigations [28-29]:

$$D_{\text{eff}} \propto \frac{1}{\tau} \quad (9)$$

We have study the behavior of The effective diffusion coefficient as function of external load for system size $L=500$ at temperature $T/T_c = 0.03$, corresponding results are plotted in figure 5. The effective diffusion coefficient increase exponentially with applied load:
However, for the classical model [3], the effective diffusion coefficient increases as a power law with applied load, that either lead to breakdown of the whole bundle:

$$D_{\text{eff}} \propto \exp \left( \frac{f}{f_c} \right)$$  \hspace{1cm} (10)

In our case, the crack diffusion process is less accelerated due to anisotropy in the directions of the fibers which further amortize the diffusion of the crack within the system. These results are in agreement with the experimental results [31].

Temperature has a very important effect on the failure process in composite materials. The influence of this parameter on the failure in fiber bundle model has been widely studied numerically [1;32;33], it catalysis appearance micro cracks in the material. These defects or micro cracks induce strong fluctuations on material strength, which influence lifetime of the system and its transition from elastic mode to plastic one [1].

The strength of each fiber is characterized by a critical length which is randomly varies. The variation of temperature drives the cracking process. To make in evidence this point, we have investigated the time evolution of the crack correlation function for system size of $L = 500$ subject to the load value $f / f_c = 0.3$ and for a two different values of temperature (figure 6).

The quasi-static effect of temperature accelerates the decrease of the fibers breaks correlation, the elongation of the fibers reaches its critical elongation more quickly, leading more to a reduction of the number of intact fibers, which affects the correlation of the fibers that break. Thus, an increase in temperature activates the spread of cracking and the correlation between fibers decreases faster over time but without reaching the total destruction of the material. The similar result is observed for the classical model but with decreasing monotony slower until the total failure of the system [3].
Figure 6: time variation of correlation function for different temperature value

Figure 7 presents the variation of the effective diffusion coefficient $D_{\text{eff}}$ with temperature for system size of $L = 500$ subject to the load value $f / f_c = 0.3$. The obtained profile shows that the crack diffusion process increases linearly with thermal noise as:

$$D_{\text{eff}} \propto \frac{2}{3} \left( \frac{T}{T_c} \right)$$

proving our precedent remark. However, this variation is different comparing to the classical model, where $D_{\text{eff}}$ exhibits a height increase with avalanche phenomena due to the avalanche breaking of the fibers, that lead to breakdown of the whole bundle [3].

Figure 7: diffusion coefficient behavior with temperature
The lifetime one of the more investigated parameter in the failure process of composite materials. In figure 8, we present the variation of this parameter versus temperature under an external load \( f = 0.3 f_c \), obtained results are fitted by a linear function indicating that the lifetime deceases with the temperature as power law:

\[
\frac{L}{T/T_c} = \left( \frac{t_f}{t_c} \right)^{-\frac{1}{2}}
\]

(13)

where \( t_c \) is the critical time when the cracks start. This scaling law is observed at each value in the used range of applied load which proves its universal character [1]. However, for the classical model with local load transfer rule LLS, the lifetime material decreases linearly with the temperature [3]. For the classical model with global load transfer rule GLS, Pradhan and al found that the lifetime decreases exponentially with temperature [34]. It means that temperature effect is influenced by applied load transfer rule and fibers direction.

Figure 8: Lifetime as function of temperature

3. Conclusion

In summary, the fiber bundle model of composite material shows some precursors which can help to study the failure phenomena loading situations. Here we use an approach for describing and study the diffusion of failure process within composite materials with randomly oriented fibers. The investigation is based on the calculation of the crack correlation function using the fiber bundle model. Each fiber is subject only the cosine component of the external load, and when it fails, its applied load is shared by surviving neighboring one according to the local load sharing rule (LLS) [8]. The results show that the correlation decreases exponentially with time and decreases with both applied load and temperature. The effective diffusion coefficient \( D_{eff} \) of the cracks increase exponentially with applied load and linearly with and temperature. However, for the classical model, this coefficient increases with the applied load and exhibits a power law, on the other hand, versus temperature, \( D_{eff} \) exhibits an increase avalanche phenomena due to the avalanche breaking of the fibers. Life time \( t_f \) decrease with temperature following a power law. However, for the classical model with local load transfer rule LLS, this time decreases linearly with the temperature following a linear law.
Yet, for the classical model with global load transfer rule GLS, Pradhan and al found that failure time decrease exponentially with temperature in GLS sharing load, which confirm that the temperature effect is influenced by applied load transfer rule and fibers direction.

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