Superfield Derivation of the Low-Energy Effective Theory of
Softly Broken Supersymmetry

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Abstract

We analyze the soft supersymmetry breaking parameters obtained in grand unified theories after integrating out the heavy GUT-states. The superfield formalism greatly simplifies the calculations and allows us to derive the low-energy effective theory in the general case of non-universal and non-proportional soft terms by means of a few Feynman diagrams. We find new contributions not considered before. We discuss the implications for the destabilization of the gauge hierarchy, flavor violating processes and the non-unification of sparticle masses in unified theories.

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I. INTRODUCTION

The idea of gauge unification in the minimal supersymmetric model (MSSM) \cite{1,2} seems to be supported by the recent experimental data \cite{3}. The unification scale $M_G \approx 2 \times 10^{16}$ GeV is just below the Planck scale $M_P \approx 2 \times 10^{18}$ GeV but much higher than the weak scale $M_Z$. Supersymmetry plays the role of stabilizing this gauge hierarchy against radiative corrections. Even if supersymmetry is broken, the hierarchy can be kept stable if the breaking arises only from soft terms (these are terms that do not reintroduce quadratic divergencies) \cite{1,4}: 

$$- \mathcal{L}_{soft} = m_{ij}^2 \phi_i^* \phi_j + \left( \frac{1}{6} A_{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} B_{ij} \phi_i \phi_j + \frac{1}{2} \tilde{M}_i \tilde{\lambda}_i^2 + h.c. \right),$$ \hspace{1cm} (1)

where $\phi_i (\tilde{\lambda}_i)$ are the scalar (gaugino) fields of the theory. Eq. (1) parametrizes the most general soft supersymmetry breaking (SSB) terms. In order to maintain the gauge hierarchy, the scale of supersymmetry breaking, $m_S$, must be close to $M_Z$.

In supergravity theories, where supersymmetry is assumed to be broken in a hidden sector which couples only gravitationally to the observable sector, the SSB terms of eq. (1) are generated at the Planck scale \cite{5,6}. In some supergravity models, one can obtain relations between the SSB parameters and then reduce the number of independent parameters. For example, in minimal supergravity theories where the Kähler potential is flat, one finds that the SSB parameters have universal values at $M_P$ \cite{3,6}, i.e.,

$$m_{ij}^2 \equiv m_0^2, \quad B_{ij} \equiv B_0 M_{ij}, \quad A_{ijk} \equiv A_0 Y_{ijk}, \quad \tilde{M}_i \equiv \tilde{M}_{1/2},$$ \hspace{1cm} (2)

where $Y_{ijk}(M_{ij})$ are the trilinear (bilinear) couplings in the superpotential. Other examples in which relations between the SSB parameters can be derived, can be found in superstring theories \cite{7}.

At lower energy scales, however, the SSB parameters deviate from their initial values at $M_P$ according to the renormalization group equations (RGEs) of the corresponding effective theory. In grand unified theories (GUTs), the SSB parameters will evolve from $M_P$ to $M_G$. 

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according to the RGEs of the GUT. At $M_G$, one has to integrate out the heavy particles [of masses of $\mathcal{O}(M_G)$] and evolve again the SSB parameters from $M_G$ to $M_Z \sim m_s$ with the RGEs of the MSSM. In these two processes, running and integrating out, the SSB parameters can be shifted from their initial values at $M_P$. Since the low-energy sparticle spectrum depend on the SSB parameters, the study of these effects is crucial for the phenomenology of the MSSM. Nevertheless, to compute the effects one has to specify the GUT, rendering the studies very model-dependent. Partial analysis can be found in refs. \cite{RGEs, MSSM}.

The purpose of this paper is to carry out a general study of how the SSB parameters are modified when the heavy particles of a GUT or a flavor theory at a high scale $M_G$, are integrated out at tree-level. We will consider the most general softly broken supersymmetric theory and will use superfield techniques. Previous analysis \cite{RGEs, MSSM} have been carried out in component fields instead of superfields, requiring lengthy calculations. Moreover, conditions such as universality \cite{RGEs, MSSM} or proportionality \cite{RGEs, MSSM} have been assumed in order to simplify the calculations. Here we will reproduce these previous results in an easier way using superfield techniques, generalize them and further pursue their phenomenological implications. As we will see, integrating out the heavy modes can be easily accomplished by Feynman diagrams.

Softly broken supersymmetric theories can be formulated in the superfield formalism by using a spurion external field, $\eta$ \cite{Superfield}. Supersymmetry is broken by giving to this superfield a $\theta$-dependent value, $\eta \equiv m_s \theta^2$. Then, the most general Lagrangian describing a softly broken supersymmetric theory can be written as\footnote{We do not include terms with only gauge vector-superfields since they are not relevant for our analysis. We assume that the gauge group of the GUT is simple and that the chiral superfields are non-singlets under the GUT-group \cite{GUT}. Our notation and conventions follow ref. \cite{Notation}.}

\begin{equation}
\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}
\end{equation}

where

\begin{equation}
\mathcal{L}_{\text{SUSY}} = \int d^4 \theta \Phi^\dagger e^{2gV} \Phi + \left( \int d^2 \theta W(\Phi) + h.c. \right),
\end{equation}
\[ \mathcal{L}_{soft} = \int d^4 \theta \Phi^\dagger \Theta_s^{\Gamma^*} + \eta \Gamma - \bar{\eta} \eta Z_e^{2gV} \Phi - \left( \int d^4 \theta \bar{\eta} \eta \Phi T \frac{A}{2} \Phi + \int d^2 \theta \eta W' (\Phi) + h.c. \right), \] (5)

where the column vector \( \Phi = (\Phi_1, \Phi_2, \ldots)^T \) denotes the chiral superfields of the theory, \( V \equiv T_A V_A \) are the vector superfields and the matrices \( \Gamma, Z, \Lambda \) denote dimensionless SSB parameters. Eq. (5) leads, in component fields \( \text{[and after replacing } \eta (\bar{\eta}) \text{ by } m_S \theta^2 (\bar{\theta}^2) \text{]}, \) to eq. (1) with

\[
A_{ijk} = m_S \left[ Y'_{ijk} + Y_{ijk} \Gamma_{ii} + Y_{ijk} \Gamma_{ij} + Y_{ijk} \Gamma_{ik} \right], \\
B_{ij} = m_S \left[ M'_{ij} + M_{ij} \Gamma_{ii} + M_{ij} \Gamma_{ij} + m_S \Lambda_{ij} \right], \\
m^2_{ij} = m^2_{S} \left[ Z_{ij} + \Gamma^*_i \Gamma_{ij} \right],
\] (6)

where \( Y'_{ijk} (M'_{ij}) \) are the trilinear (bilinear) couplings in \( W'(\Phi) \). This function \( W'(\Phi) \) is in principle a general holomorphic function of the superfields different from the superpotential \( W(\Phi) \). In supergravity theories, however, where supersymmetry is broken by a hidden sector that does not couple to the observable sector in the superpotential, one has that

\[ W'(\Phi) = a W(\Phi), \] (7)

where \( a \) is a constant. Eq. (7) will be referred as the \( W \)-proportionality condition. It is crucial to note that \( \int d^2 \theta \eta W' \) does not renormalize. It is due to the non-renormalization theorem which states that terms under the integral \( \int d^2 \theta \) do not receive radiative corrections \[21\]. Then, if the proportionality of \( W' \) to \( W \) is satisfied at the scale where supersymmetry is broken, \textit{i.e.}, Planck scale in supergravity, it will be satisfied at any lower scale.

From eq. (6), we see that the universal conditions for \( m^2_{ij} \) and \( A_{ijk} \) \textit{[eq. (2)]} are satisfied when eq. (7) holds and

\[ \Gamma, \ Z \propto 1. \] (8)

Nevertheless, even if eq. (8) holds at the scale where the SSB terms are generated \( \sim M_P \), renormalization effects modify \( \Gamma \) and \( Z \). (The renormalization of \( \Gamma \) and \( Z \) can be found in ref. \[22\].) At \( M_G \), deviations from universality can be sizeable \[12 \ 14\].
One of the main motivations for the analysis in refs. [6, 9–11] was to see whether integrating out the heavy GUT-modes, generates SSB terms of $O(M_Gm_S)$ in the low-energy theory that could destabilize the gauge hierarchy. It was shown in refs. [9, 11] that in fact such terms can be present if the SSB parameters are non-universal. Here we will show that the superfield formalism allows us to understand easily the effects that destabilize the gauge hierarchy.

The effects of integrating out the heavy GUT-modes are also very important for phenomenological purposes. They modify the SSB parameters and consequently the sparticle spectrum. Two types of effects in the sparticle masses are of special interest. Effects that lead to FCNC [8, 14] and effects that modify mass GUT-relations [11, 15–17]:

1. **FCNC (Horizontal effects):** For arbitrary values of $\sim O(1 \text{ TeV})$ for $m_{ij}$ and $A_{ijk}$, the squark and slepton contribution to FCNC processes typically exceed the experimental bounds [1]. These processes put severe constraints on the masses of the first and second family of squarks and sleptons [23]. These constraints can be satisfied if we demand

   (a) universal soft masses for the squarks and sleptons [condition (8)] [1],

   (b) proportionality between $A$-terms and the corresponding Yukawa-terms, i.e.,

   $A_{ijk} \propto Y_{ijk}$ [W-proportionality and condition (8)].

Grand unified models which as a result of integrating out the heavy GUT-modes, generate large deviations from (7) and (8) for the squarks and sleptons will not lead to viable low-energy theories. A priori, it seems that this is the case in GUTs or flavor models that have the different families of quarks and leptons couple to different Higgs representation above $M_G$. Since the gauge renormalization of the the trilinear $A$-terms depend on the GUT-representation of the fields, these corrections will not be universal in flavor space. We will show, however, that once the heavy states are integrated out, such effects cancel out from the low-energy effective theory.

2. **Unification of soft masses (Vertical effects):** In GUTs, where quarks and leptons are embedded in fewer multiplets, one expects that, due to the GUT-symmetry, the number
of independent SSB parameters is reduced. For example, in the minimal SU(5) one has

\[ \{Q, U, E\} \in 10, \{L, D\} \in \bar{5}, \]  

(9)

where \( Q \) (\( L \)) and \( U \), \( D \) (\( E \)) are respectively the quark (lepton) SU(2)\(_L\)-doublet and singlets. Thus, one expects at \( M_G \)

\[ m_Q^2 = m_U^2 = m_E^2 = m_{10}^2, \quad m_L^2 = m_D^2 = m_{\bar{5}}^2. \]  

(10)

Nevertheless, such GUT-relations can be modified by GUT-effects even at tree level. When the heavy GUT-modes are integrated out, the soft masses of the sparticles that belong to the same GUT-multiplet can be split. This is known to happen, for example, in SU(5) theories \([15]\) if more than three families are present at \( M_G \) or in SO(10) theories where \( D \)-term contributions split the soft masses of the matter fields in the 16 representation \([17]\).

In section II we will calculate in the superfield formalism the shifts in the SSB parameters of the low-energy theory induced when heavy modes are integrated out. We will consider non-universal SSB parameters (\( \Gamma \) and \( Z \) will be arbitrary matrices). We will first assume \( W \)-proportionality (sections IIA and IIB), and subsequently will analyze the effects of relaxing this assumption in section IIC. In section III we will study the phenomenological implications focusing on the stability of the gauge hierarchy, FCNC and the unification of the soft masses at \( M_G \). In section IV we will present our conclusions.

**II. INTEGRATING OUT THE HEAVY SUPERFIELDS**

Eqs. (4) and (5) parametrize the most general softly broken supersymmetric theory above \( M_G \). At \( M_G \), some of the chiral superfields get vacuum expectation values (VEVs) breaking the gauge symmetry of the GUT to SU(3)\( \times \) SU(2)\(_L\)\( \times \)U(1)\(_Y\), the MSSM group. We can rotate the superfield \( \Phi - \langle \Phi \rangle \) to a basis where the supersymmetric mass matrix is diagonal (SUSY-physical basis) and separate the superfields in lights and heavies:

\[ \Phi \rightarrow \Phi' = \langle \Phi \rangle + \Phi + \phi \equiv \begin{pmatrix} \langle \Phi_i \rangle \\ 0 \end{pmatrix} + \begin{pmatrix} \Phi_i \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \phi_{\alpha} \end{pmatrix}, \]  

(11)
where \( \langle \Phi \rangle \) is of \( \mathcal{O}(M_G) \) and\(^\dagger\)

\[\Phi_i : \text{Heavy superfields of mass (but no VEV) of } \mathcal{O}(M_G),\]
\[\phi_\alpha : \text{Light superfields of mass of } \mathcal{O}(\lesssim m_S).\]  

Assuming \( W \)-proportionality [eq. (7)], the SSB terms (5) are given by

\[
\mathcal{L}_{\text{soft}} = \int d^4 \theta \left\{ \langle \Phi^\dagger \rangle \left[ \bar{\eta} \Gamma^* + \eta \Gamma - \bar{\eta} \eta Z \right] e^{2gV} \langle \Phi \rangle + \langle \Phi^\dagger \rangle \left[ \bar{\eta} \Gamma^* + \eta \Gamma - \bar{\eta} \eta Z \right] e^{2gV} \Phi - \langle \Phi^T \rangle \bar{\eta} \eta \Lambda \Phi + h.c. \right\} + \left( \Phi^\dagger + \phi^\dagger \right) \left[ \bar{\eta} \Gamma^* + \eta \Gamma - \bar{\eta} \eta Z \right] e^{2gV} \left( \Phi + \phi \right) - \left\{ \left( \Phi^T + \phi^T \right) \bar{\eta} \eta \Lambda \frac{1}{2} \left( \Phi + \phi \right) + h.c. \right\} \right) + \left( \int d^2 \theta \bar{\eta} \eta aW + h.c. \right),
\]

where now \( \Gamma, Z \) and \( \Lambda \) define the SSB parameters in the new basis (11). In this basis, it is obvious that the superpotential does not contain either mass terms mixing heavy with light superfields or linear terms with the heavy superfields. Since we assumed \( W \)-proportionality at \( M_P \), the non-renormalization theorem guarantees that such terms are also absent in eq. (14) at \( M_G \). We have also assumed that there are not light MSSM-singlets in the model. We can easily see that if light singlets are present, a term

\[
\int d^4 \theta \langle \Phi^\dagger \rangle \bar{\eta} \Gamma^* \phi \sim M_G m_S \left( \frac{\partial W}{\partial \phi} \right)^*,
\]

can be induced and spoil the gauge hierarchy\(^\ddagger\) [24].

We are now ready to integrate out the heavy superfields. We will first consider the heavy chiral-superfields \( (g = 0) \), leaving the gauge sector for later.

\(^\dagger\) Greek (latin) letters denote light (heavy) superfields.

\(^\ddagger\) There are different possibilities to suppress eq. (15) and allow light singlets [24]. We will not consider such alternatives here.
A. Integrating out the heavy chiral-sector

To integrate out the heavy chiral-superfields at tree-level, we compute their equations of motion and use them to write the heavy superfields as a function of the light and spurion superfields, \( i.e., \Phi_i = f(\phi_\alpha, \eta) \). When one derives the equations of motion by the variational principle, one must take into account the fact that the chiral superfields are subject to the constraints \( \bar{D}\Phi = 0 \) where \( D = \partial_\theta + i(\sigma^\mu\bar{\theta})\partial_\mu \) is the supersymmetric covariant derivative. However, using the relation

\[
\int d^4\theta F(\Phi, \Phi^\dagger) = -\int d^2\theta \frac{\bar{D}^2\eta}{4} F(\Phi, \Phi^\dagger),
\]

one can write the action as an integral over the chiral superspace (\( \int d^2\theta \)) where the chiral superfields are unconstrained and calculate the equations of motion by the variational principle \[25\]. Hence

\[
\delta \frac{\delta}{\delta \Phi_j} \left\{ \int d^2\theta W(\Phi, \phi)(1 - a\eta) - \int d^2\theta \frac{\bar{D}^2\eta}{4} \left( \langle \Phi^\dagger \rangle [\Gamma^* - \eta Z] \Phi - \langle \Phi^T \rangle \eta \Lambda \Phi \right) + \ldots \right\} = 0,
\]

where

\[
W(\Phi, \phi) = \frac{1}{2} M_{ij} \Phi_i \Phi_j + \frac{1}{2} Y_{i\alpha\beta} \Phi_i \phi_\alpha \phi_\beta + \ldots
\]

is the superpotential in the SUSY-physical basis \[11\]. We have kept not only the dominant terms of \( O(M_G) \) but also terms of \( O(1) \) that involve a heavy superfield and two light superfields. It will become clear later, that the terms neglected do not lead to any contribution in the limit \( M_G \gg m_S \). Eq. \[17\] leads to

\[
\Phi_i \approx -\frac{1}{M_{ij}} \left[ \frac{1}{2} Y_{j\alpha\beta} \phi_\alpha \phi_\beta - \frac{\bar{D}^2\eta}{4(1 - a\eta)} \right] \left( \langle \Phi^\dagger \rangle [\Gamma^* - \eta Z] - \langle \Phi^T \rangle \eta \Lambda \Phi \right) \]

Eliminating the heavy superfields from the effective Lagrangian through the above equations of motion, and using eq. \[16\], we get

\[
\mathcal{L}_{\text{eff}}(\phi, \eta) = \mathcal{L}(\Phi = 0) - \int d^4\theta \left\{ \langle \Phi^\dagger \rangle [\bar{\eta} \Gamma^* - \bar{\eta} \eta Z] - \langle \Phi^T \rangle \bar{\eta} \eta \Lambda \right\} \frac{Y_{i\alpha\beta} \phi_\alpha \phi_\beta}{2 M_{ij}} + h.c.
\]
Replacing \( \bar{\eta} \to m_S \bar{\theta}^2 \), the second term of the r.h.s. of eq. (20) gives a mass term of \( \mathcal{O}(m_S) \) to the superpotential of the light superfields (a \( \mu \)-term for the light Higgs doublets of the MSSM can be induced in this way \([6]\)). Note, however, that this term is only generated if there is a trilinear coupling between a heavy MSSM-singlet and two light superfields.

Had we worked in component fields instead of superfields, the process of integrating out the heavy modes would have been much more complicated since we have to deal with the scalar fields and auxiliary fields independently with complex equations of motion \([6,9–11]\). Eq. (20) gives the result of ref. \([3]\) obtained here in a much simpler way using superfields techniques. Furthermore, it is valid for a more general class of theories, since universality is not assumed.

A set of simple rules to obtain the terms (20) can be easily derived. These rules are:

(i) Draw all possible Feynman diagrams with heavy superfields in the internal lines and light and spurion superfields in the external lines.

(ii) For each external line, write the corresponding superfield \( \phi_\alpha, \eta \) (or \( \phi_\alpha^\dagger, \bar{\eta} \)).

(iii) For each \( \langle \Phi_i \Phi_j \rangle \) propagator, write \(-1/M_{ij}\).

(iv) Vertices are read directly from the Lagrangian eqs. (4), (13) and (14).

(v) Integrate over \( \int d^4\theta \) \( (\int d^2\theta) \) if at least one vertex (none of the vertices) comes from a \( D \)-term.

Following the rules above, we have that the only diagrams that do not go to zero in the heavy limit \( M_{ij} \gg m_S \) are those given in fig. 1 and they give the contribution eq. (20).

**B. Integrating out the heavy gauge vector-sector**

If the gauge symmetry of the GUT is broken by the VEVs of the chiral superfields, the vector superfields associated with the broken generators get a mass term of \( \mathcal{O}(M_G^2) \):

\[
2g^2 \int d^4\theta \langle \Phi^\dagger \rangle T_A T_B \langle \Phi \rangle V_A V_B .
\]
One can perform a rotation in the $V_A$ and work in a basis where the mass matrix of the heavy vector-superfield is diagonal. In the supersymmetric limit, each broken generator has a chiral superfield associated, $\langle \Phi^\dagger \rangle T_A \Phi$, that contains the Goldstone boson and its superpartners. We will work in the super-unitary gauge \cite{26} where these chiral superfields have been gauged away:

$$\langle \Phi^\dagger \rangle T_A \Phi = 0. \quad (22)$$

Expanding the exponentials in eqs. \cite{11} and \cite{13} and using eqs. \cite{11}, \cite{22} and the condition that supersymmetry is not broken by the observable sector (that the $D$-terms do not get VEVs),

$$\langle \Phi^\dagger \rangle T_A \langle \Phi \rangle = 0, \quad (23)$$

we have

$$\mathcal{L}_{SUSY} = 2 \int d^4 \theta \left\{ \phi^\dagger g T_A \phi V_A + \left[ \frac{1}{2} \phi^\dagger g^2 T_A T_B \phi + \langle \Phi^\dagger \rangle g^2 T_A \Phi + \phi \right] + \text{h.c.} \right\} V_A V_B$$
$$+ \frac{1}{2} M_A^2 V_A^2 + \frac{2}{3} \langle \Phi^\dagger \rangle g^3 T_A T_B T_C \langle \Phi \rangle V_A V_B V_C \right\} + \ldots,$$

$$\mathcal{L}_{soft} = 2 \int d^4 \theta \left\{ V_A \eta \left[ \langle \Phi^\dagger \rangle g T_A \Gamma \langle \Phi \rangle + \langle \Phi^\dagger \rangle g T_A \Gamma \langle \Phi + \phi \rangle \right]$$
$$+ (\Phi^\dagger + \phi^\dagger) g T_A \Gamma \langle \Phi \rangle + \phi^\dagger g T_A \Gamma \phi \right\} + \eta \langle \Phi^\dagger \rangle g^2 T_A T_B \Gamma \langle \Phi \rangle V_A V_B + \text{h.c.} \right\}$$
$$- 2 \int d^4 \theta V_A \eta \bar{\eta} \langle \Phi^\dagger \rangle g T_A Z \langle \Phi \rangle + \ldots, \quad (24)$$

where $M_A^2 = 2 g^2 \langle \Phi^\dagger \rangle T_A^2 \langle \Phi \rangle$. It will be justified a posteriori, when we use Feynman diagrams to integrate out the heavy vector superfields, why only the terms kept in eq. \cite{24} are relevant for the calculation. Gauge invariance implies $[T_A, \Gamma] = [T_A, Z] = 0$. It is easy to check using eqs. \cite{22} and \cite{23} that if $\Gamma, Z \propto 1$, the equations of motion for the vector superfields are simple $V_A = 0 + \mathcal{O}(1/M_X^2)$ and they decouple from the effective theory in agreement with ref. \cite{6}. For general SSB parameters, this will not be the case.

Let us first consider the case where the $V_A$ are not singlets under the MSSM gauge group. This is always the case if the rank of the GUT-group is not larger than the rank of the MSSM.
group (e.g., SU(5)). One then has that the linear terms with \( V_A \) in (24) vanish. Thus, the equation of motion is given by
\[
V_A = -\frac{g}{M_A^2} \left( \eta \langle \Phi \rangle^T T_A \Gamma \phi + \eta \phi^T T_A \Gamma \langle \Phi \rangle + h.c. \right) + O\left( \frac{1}{M_G^2} \right),
\]
that inserting it back in eq. (24) gives a new effective term
\[
\mathcal{L}_{\text{eff}}' = -\int d^4 \theta \frac{g^2}{M_A^2} \left( \eta \langle \Phi \rangle^T T_A \Gamma \phi + \eta \phi^T T_A \Gamma \langle \Phi \rangle + h.c. \right)^2.
\]
The above term can be easily obtained by applying the rules (i)–(v) with
\[(vi)\] For each \( \langle V_A V_A \rangle \) propagator, write \( -1/(2M_A^2) \).

The only possible Feynman diagram is given in fig. 2 and gives the contribution obtained in eq. (26).

Finally, let us consider the case where \( V_A \) can also be a MSSM-singlet. Applying the rules (i)–(vi), we have the Feynman diagrams of fig. 3 that give the new contributions
\[
\mathcal{L}_{\text{eff}}'' = -2 \int d^4 \theta \left( \eta \phi^T g T_A \phi K_A + h.c. \right)
\]
\[
-2 \int d^4 \theta \eta \left\{ - \{ \langle \Phi \rangle \}^2 \{ T_A, T_B \} K_B K_C^* \right\} \frac{Y_{\alpha \beta}}{2M_{ij}} \phi_\alpha \phi_\beta
\]
\[
+ \phi^T g T_A \phi \left( \frac{\langle \Phi \rangle^3 \{ T_A, T_B \} \{ T_C \} \text{sym} (\Phi) K_B K_C^*}{3M_A^2} \right) - \frac{\langle \Phi \rangle^2 \{ T_A, T_B \} \Gamma \langle \Phi \rangle K_B^*}{M_A^2} - \frac{\langle \Phi \rangle \{ T_A \} \Gamma \langle \Phi \rangle}{2M_A^2}
\]
\[
- \frac{1}{2} \phi^T g^2 \{ T_A, T_B \} \phi K_B K_B^* + \phi^T g T_A \Gamma \phi K_A^*
\]
\[
- \frac{1}{M_A^2} \left( \langle \Phi \rangle \{ T_A, T_B \} \phi K_B^* + \phi^T g T_A \Gamma \langle \Phi \rangle K_B^* + h.c. \right) \langle \Phi \rangle \frac{g^2 \{ T_A, T_B \} \phi + h.c.}{M_A^2}
\]
\[
+ \frac{1}{M_A^2} \left( \langle \Phi \rangle \frac{g^2 \{ T_A, T_B \} \phi + h.c.}{M_A^2} \right) \langle \Phi \rangle \{ T_A, T_C \} \phi + h.c. \right) K_B K_C^*,
\]
where \( K_A = \langle \Phi \rangle \{ T_A \} \Gamma \langle \Phi \rangle / M_A^2 \) and \( \{ T_A T_B T_C \} \text{sym} \) is the symmetrization of the product \( T_A T_B T_C \) under the indices \( A, B \) and \( C \).

Eqs. (27) and (28) together with eqs. (24) [from integrating out the heavy chiral sector] and (26) [from integrating out the heavy MSSM-non-singlet vectors] give the full low-energy effective theory of SSB terms below \( M_G \).
C. Case with $W'(\Phi, \phi) \neq a W(\Phi, \phi)$

To derive the effective Lagrangian in the previous sections, we have assumed that $W' \propto W$ ($W$-proportionality). Here we want to study the implications of relaxing such a proportionality. This is the case of the most general softly broken supersymmetric theory. It could also be the case that $W' = aW$ holds at tree-level but gravitational (or string [27]) corrections alter this relation (by the non-renormalization theorem this relation cannot be modified by radiative corrections from the GUT).

When $W' \propto W$, the mass matrix in $W'$ is not diagonal in the SUSY-physical basis and then terms of the order $M_G \Phi \Phi$ and $M_G^2 \Phi$ can appear in $W'$. These terms induce new contributions to the SSB parameters of the light superfields that have not been considered before in the literature. In order to reduce the number of new diagrams that can now be generated, we can perform a superfield redefinition, $\Phi \rightarrow \Phi - \eta \Gamma \Phi$, in eq. (5) such that the terms proportional to $\Gamma$ can be absorbed in $W'$ without changing $\mathcal{L}_{SUSY}$ [22]\textsuperscript{**}. Also the terms proportional to $\Lambda$ can be absorbed in $W'$.

In fig. 4 we show the new Feynman diagrams due to the new couplings in $W'$. We denote by “•” a vertex arising from $\int d^2 \theta \eta W'$. The internal line with a “×” denotes a $\langle \Phi \Phi \rangle$ propagator. Although it goes like $\sim M_G^{-2}$, it can be compensated by powers of $M_G$ arising from the new vertices in $\int d^2 \theta \eta W'$ and give a non-negligible contribution. The explicit contributions can be obtained from the above rules (i)–(vi) together with

(vii) For each $\langle \Phi_i \Phi_i^\dagger \rangle$ propagator, write $-\bar{D}^2/(4M_{ij}^2)$.

The diagrams of fig. 4a induce new trilinear SSB terms, the diagrams of fig. 4b–c induce bilinear SSB terms and the diagrams of fig. 4d–e induce scalar soft masses. In the next

\textsuperscript{**}We have not done such a redefinition in the previous sections, since we wanted to maintain the proportionality between $W$ and $W'$. Of course, this redefinition had changed the Feynman diagrams but not the final result.
section we will present some examples of models where such diagrams are generated after integrating out the heavy modes.

III. PHENOMENOLOGICAL IMPLICATIONS

A. Hierarchy destabilization

In the absence of light MSSM-singlets and assuming $W$-proportionality, the effective theory of SSB terms is given by eqs. (20) and (26)–(28). Inspection of these terms reveals that there are not terms of $\mathcal{O}(M_G m_S)$. One can also see this from dimensional analysis of the diagrams of figs. 1–3. Thus, the stability of the hierarchy, after integrating the heavy modes, is guaranteed for a general SSB terms if $W' \propto W$ even in the absence of universality. Same conclusions have been reached in ref. [9,11] in component fields.

Nevertheless, if $W$-proportionality does not hold at $M_P$, we have that new SSB terms are generated (such as the diagram of fig. 4b) that can be of $\mathcal{O}(M_G m_S)$ and spoil the hierarchy. One example where this occurs, is the minimal SU(5) model [1]. The Higgs sector of the model consists of three supermultiplets, a Higgs fiveplet $H$ and antifiveplet $\bar{H}$ and the adjoint $24$. The superpotential is given by

$$W = M_H \bar{H} H + \lambda H 24 \bar{H} + W(24).$$

In the supersymmetric limit the $24$ develops a VEV of $\mathcal{O}(M_G)$,

$$\langle 24 \rangle = V_{24} Y \equiv V_{24} \text{diag}(2, 2, 2, -3, -3),$$

that breaks SU(5) down to the MSSM gauge group. To keep light the Higgs SU(2)$_L$-doublets embedded in the $\bar{H}$ and $H$, we need the fine-tuning

$$M_H - 3\lambda V_{24} \lesssim \mathcal{O}(M_Z).$$

$^\dagger$Higher-dimensional operators suppressed by powers of $M_P^{-1}$, could also be present in $W$. In this case, we would also need to fine-tune these operators to preserve the light Higgs of the MSSM.
However, if the last SSB term of (3),
\[
\int d^4\theta \eta W'(\Phi) = \int d^4\theta \eta \left[ M'_H \overline{H} H + \lambda' 24 \overline{H} H + W'(24) \right],
\]
is present in the model with \( \lambda' \neq \lambda \) or \( M'_H \neq M_H \), a mass term of \( \mathcal{O}(M_G m_S) \) for the MSSM Higgs doublets is induced. This term is not cancelled by the fine-tuning (31) and destabilize the gauge hierarchy.

It is important to notice that the destabilization of the hierarchy by the SSB parameters arises because a heavy superfield couples linearly to light superfields (see diagram of fig. 4b), and not because we fine-tuned parameters. In models without these couplings the hierarchy will be stable even if \( W \)-proportionality does not hold. This is the case for models where the doublet-triplet splitting is obtained by the missing partner or missing VEV mechanisms [28,29].

**B. A-terms, soft masses and FCNC**

As we discussed in section I, deviations from \( W \)-proportionality and/or eq. (8) for the squark and slepton SSB parameters can have implications in FCNC. In the low-energy effective theory the SSB parameters can be modified by (1) renormalization effects or (2) integrating out the heavy modes. Renormalization effects can cause \( \Gamma_{a\beta} \) and \( Z_{a\beta} \) [in \( \mathcal{L}_{soft}(\Phi = 0, V_A = 0) \)] to deviate from universality if the two light generations have different Yukawa couplings. Such contributions, however, can be suppressed if one assumes small Yukawa couplings for the first and second family or a flavor symmetry. Renormalization effects from the top Yukawa can also modify the SSB parameters of the light generations and enhance the supersymmetric-contribution to FCNC processes [14].

The second type of effects (those from integrating out the heavy modes) can arise from the rotation of the superfields to the SUSY-physical basis and/or from the diagrams of fig. 1–4. The first ones are analyzed in ref. [13,16]. Here we will analyze the effects from diagrams of fig. 1–4; their explicit expressions are given in eqs. (20) and (26)–(28). For the case \( W' \propto W \), only eq. (27) can induce a trilinear SSB term.
\[ K_A \int d^4\theta \bar{\eta} \bar{\phi} T_A \phi \rightarrow K_A \phi^\ast (T_A)_{\alpha \beta} \left( \frac{\partial W_{\text{eff}}(\phi)}{\partial \phi_\beta} \right)^\ast, \]  

which, as mentioned before, arises only if \( T_A \) commutes with the unbroken GUT-generators. However, from the gauge invariance of \( W \) one has

\[ \phi^\ast (T_A)_{\alpha \beta} \left( \frac{\partial W}{\partial \phi_\beta} \right)^\ast + \phi^\ast (T_A)_{\alpha i} \left( \frac{\partial W}{\partial \Phi_i} \right)^\ast = 0, \]  

and then if \( (T_A)_{\alpha i} = 0 \), e.g., \( T_A \) is a diagonal generator, the contribution from eq. (33) vanish. Therefore, no trilinear SSB parameter is induced from the diagrams of fig. 1-3 in GUTs such as SU(5) and SO(10) which do not contain broken generators that are singlet under the MSSM group and non-diagonal. This leads to a surprising consequence. Consider a GUT, such as the Georgi-Jarlskog model [30], in which the different families couple above \( M_G \) to Higgs with different gauge quantum numbers. As we said in section I, one would expect that in such theories the \textit{gauge} corrections spoil the proportionality \( A_{ijk} \propto Y_{ijk} \). Nevertheless, these effects decouple from the low-energy effective theory; the \( A \)-terms for the light fields arise only from \( \mathcal{L}_{\text{soft}}(\Phi = 0, V_A = 0) \). For example, in the MSSM the trilinear term for \( Q_\alpha U_\beta H \) is given by

\[ A_{Q_\alpha U_\beta H} = m_S \left[ a Y_{Q_\alpha U_\beta H} + Y_{Q_u U_\beta H} \Gamma_{Q_u Q_\alpha} + Y_{Q_u U_\gamma H} \Gamma_{U_\gamma U_\beta} + Y_{Q_\alpha U_\beta H} \Gamma_{HH} \right], \]  

independently of the physics above \( M_G \). In the MSSM only one Higgs couples to the \( Q \) and \( U \), and then corrections to \( \Gamma_{HH} \) are universal in flavor space. Note that the breaking of universality in \( \Gamma_{Q_u Q_\alpha} \) and \( \Gamma_{U_\gamma U_\beta} \) can only arise from Yukawa corrections but not from gauge corrections.

For the light scalar soft-masses, we have contributions from eqs. (27) and (28). In particular, the terms of (28) proportional to \( \phi^\dagger T_A \phi \) (diagrams of fig. 3c) are the so-called \( D \)-term contributions analyzed in ref. [17]. These terms can be dangerous since they can split the squark masses if the three generations have different transformation properties under the broken generators. This is the case in most of the flavor theories based on a horizontal gauge symmetry.
Let us turn to the case $W' \not\propto W$. From eq. (6), we see that now $A_{ijk}$ are not proportional to $Y_{ijk}$. Thus, large deviations from $W$-proportionality are not allowed by FCNC constraints. It is important to notice that even if the breaking of $W$-proportionality arises only in the bilinear terms ($B_{ij} \not\propto M_{ij}$) of the heavy fields, diagrams like those of fig. 4a can induce a breaking of proportionality in the trilinear terms and lead to FCNC. An example of a model where this can occur is the Georgi-Jarlskog model [30]. In this model the quarks and leptons couple to different Higgs representations such that below $M_G$, only two linear combinations of them are light (the MSSM Higgs doublets). Since the rotation that diagonalize the Higgs mass matrix in $W$ does not diagonalize the mass matrix in $W'$, mixing mass terms of $O(M_G)$ between the heavy and the light Higgs will be present in $W'$. These terms will induce diagrams like those in fig. 4a, breaking the proportionality $A_{ijk} \propto Y_{ijk}$ and inducing dangerous flavor violations.

C. Unification of soft masses at $M_G$

It has been recently shown in ref. [15] that GUT-relations such as eq. (10) in SU(5) can be altered if the light quarks and leptons come from different linear combinations of a pair of GUT-multiplets. In our formalism this effect can be understood as follows. If the rotation (11) depends on the VEVs of the fields that break the GUT-group, the matrices $Z$ and $\Gamma$ after such a rotation do not have to preserve the GUT-symmetry. Thus, fields embedded in a same GUT-representation can have different soft masses. Of course, if $\Gamma, Z \propto 1$, the rotation (11) will not change $\Gamma, Z$ and universality will be maintained.

In the SUSY-physical basis, there are also effects arising from the tree-level diagrams of fig. 2–4 that can induce mass-splittings. If $W' \propto W$, the only soft mass term induced in SU(5) (or in any GUTs that does not contain heavy MSSM-singlet vectors) is eq. (26) – diagram of fig. 2. Note, however, that this contribution is non-zero only if the coupling $\eta\langle\Phi^\dagger\rangle T_A\Gamma\phi$ exists. This coupling is present if the GUT-multiplet that contains light superfields mixes at $M_G$ with a GUT-multiplet that gets a VEV of $O(M_G)$. This condition is not satisfied...
in simple models. In the minimal SU(5) model, the matter fields are in the $\bar{5}$ and $10$ representations and cannot mix with the $24$ that contains the MSSM-singlet. Thus, the absence of the term (26) is guaranteed and the relation (10) is not modified. A different situation can occur for a SU(5)$\times$U(1) model where

$$\{Q,D,\nu\} \in 10, \{L,U\} \in 5, E \in 1.$$  \hspace{1cm} (36)

Now, the $10$ contains a singlet under the MSSM, the neutrino $\nu$, and for $\langle \nu \rangle \sim M_G$ the diagram of fig. 2 can induce different soft masses for the $Q$ and $D$.

In SO(10) models, since there is a broken U(1)-generator that commutes with the MSSM group generators, the diagrams of fig. 3 can induce shifts in the soft masses of the fields embedded in a single SO(10) representation. Note that only part of these contributions, diagrams of fig. 3c, is usually considered in the literature ($D$-term contribution [17]).

If $W'$ is not proportional to $W$, extra contributions to the soft masses come from the diagrams of fig. 4d. In the minimal SU(5), such diagrams do not arise since there are not couplings of a light matter superfield to two heavy superfields. In non-minimal SU(5), however, these couplings can be present and induce the diagrams of fig. 4d. We will study these contributions in the context of the model in ref. [15]. The model is a SU(5) GUT where the matter content is extended with an extra $5$ and $\bar{5}$. The superpotential (for only one light generation) is given by

$$W = 5_H [M \bar{5}_1 + \lambda 24 \bar{5}_2] + h 10 \bar{H} 5_1.$$  \hspace{1cm} (37)

In the supersymmetric limit, the $24$ gets a VEV given by eq. (30). One linear combination of $\bar{5}_1$ and $\bar{5}_2$ will acquire a large mass of order $\sim M_G$ and the orthogonal combination will be the light quarks and leptons. Because the hypercharges of the quark and lepton embedded in the $5$s are different, they will be different linear combinations of the corresponding states in $\bar{5}_1$ and $\bar{5}_2$:

$$\begin{pmatrix} D \\ L \end{pmatrix} = - \sin \theta_Y \bar{5}_1 + \cos \theta_Y \bar{5}_2,$$  \hspace{1cm} (38)
where

\[
\sin \theta_Y = \frac{\rho Y}{\sqrt{1 + \rho^2 Y^2}},
\]

with \( \rho = \lambda V_{24}/M \). In this model the diagrams of fig. 4d lead to extra contributions to the soft masses of the \( L \) and \( D \). For example, the first diagram of fig. 4d induce a soft mass

\[
m_D^2 \sim m_S^2 \frac{|-M's_D + \lambda' V_{24}c_D|^2}{M^2 + \lambda^2 V_{24}^2},
\]

\[
m_L^2 \sim m_S^2 \frac{|-M's_L + \lambda' V_{24}c_L|^2}{M^2 + \lambda^2 V_{24}^2},
\]

(40)

where \( s_a \) is given by

\[
s_a = \sin \theta_{Y_a} = \frac{\rho Y_a}{\sqrt{1 + \rho^2 Y_a^2}},
\]

and \( Y_a \) is the hypercharge of \( a \). Since squarks and sleptons have different hypercharge, these contributions break the degeneracy of their masses. These are extra contributions to those calculated in ref. [15]. They go to zero in the limit \( M' \to M \) and \( \lambda' \to \lambda \) (\( W \)-proportionality).

**IV. CONCLUSION**

The SSB parameters, if generated above \( M_G \), can provide us direct information about the physics at high-energy scales. In this paper, we have calculated the shifts induced in the SSB parameters of the effective theory when the heavy modes, coming from a GUT or a flavor theory at a high scale \( M_G \), are integrated out at tree-level\(^\ddagger\). We have considered the most general softly broken theory and worked within the superfield formalism. This formalism is very suitable for this purpose and the calculations can be easily done using Feynman diagrams. For models where supersymmetry is broken by a hidden sector, and therefore \( W \)-proportionality [eq. (7)] holds, the contributions to the SSB parameters of the MSSM are given by the diagrams of fig. 1–3 and can be easily calculated following the

\(^{\ddagger}\)One-loop corrections from the heavy modes could also be important for large couplings [15].
Feynman rules \((i)-(vi)\) given in section II. Some general statements can be inferred from the above analysis. For example, in SU(5) models we have found that the induced bilinear SSB terms are always of \(\mathcal{O}(m_S^2)\) [never of \(\mathcal{O}(M_G m_S)\)] and neither \(A\)-terms nor soft masses are generated. Nevertheless, mass-splittings can arise when the superfields are rotated to the physical basis \([15]\).

The pattern of the SSB parameters depends strongly on the \(W\)-proportionality condition. When it is relaxed, extra contributions to the \(A\)-terms and soft masses are induced. These contributions depend on ratios between VEVs and masses of the heavy superfields. Hence a hierarchy of soft masses can be generated in the low-energy spectrum even if there is an unique supersymmetric scale \(m_S\). Looking at complete GUT models (where the usual GUT problems, such as the doublet-triplet splitting or the fermion mass spectrum, are addressed) \([29]\), one finds that most of these contributions are present and can cause serious problems with flavor violations. The pattern of SSB parameters at \(M_G\), after integrating out the heavy states, can be completely different from that induced at \(M_P\).

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FIGURE CAPTIONS

FIG. 1: Tree-level diagrams generating a $D$-term for the light superfields from interchange of a heavy chiral-superfield.

FIG. 2: Tree-level diagram generating a $D$-term for the light superfields from interchange of a heavy gauge vector-superfield non-singlet under the MSSM.

FIG. 3: Tree-level diagrams generating a $D$-term for the light superfields from interchange of heavy gauge vector-superfields.

FIG. 4: Tree-level diagrams contributing to a $F$-term (fig. a–d) and D-term (fig. e) for the light superfields for the case $W' \neq aW$. We denote by an internal line with “×” a $\langle \Phi\Phi^\dagger \rangle$ propagator and denote by “•” a vertex arising from $\int d^2\theta \eta W'$. 
Fig. 1
Fig. 2
Fig. 3
Fig. 4