ASYMPTOTIC PROFILE OF GROUND STATES
FOR THE SCHRÖDINGER-POISSON-SLATER EQUATION

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Abstract. We study the Schrödinger-Poisson-Slater equation
\[-\Delta u + u + \lambda (I_2 \ast |u|^2) u = |u|^{p-2} u \quad \text{in } \mathbb{R}^3,\]
where \( p \in (3,6) \) and \( \lambda > 0 \). By using direct variational analysis based on the comparison of the ground state energy levels, we obtain a characterization of the limit profile of the positive ground states for \( \lambda \to \infty \).

1. Introduction

We are concerned with the asymptotic profiles of positive ground state solutions of a Schrödinger-Poisson-Slater equation
\[(P_{\lambda}) \quad -\Delta u + u + \lambda (I_2 \ast |u|^2) u = |u|^{p-2} u \quad \text{in } \mathbb{R}^3,\]
where \( p > 2 \) and \( \lambda > 0 \). Here \( I_2(x) := ((4\pi|x|)^{-1})^2 \) is the Riesz potential and \( \ast \) denotes the standard convolution in \( \mathbb{R}^3 \). By a ground state solution of \((P_{\lambda})\) we understand a weak solution \( u_0 \in H^1(\mathbb{R}^3) \setminus \{0\} \) which has a minimal energy amongst all nontrivial solutions of \((P_{\lambda})\), namely \( I_\lambda(u_0) \leq I_\lambda(u) \) for any solution \( u \) of \((P_{\lambda})\), where \( I_\lambda : H^1(\mathbb{R}^3) \to \mathbb{R} \) is the corresponding functional of \((P_{\lambda})\) defined as
\[I_\lambda(u) = \frac{1}{2} \int_{\mathbb{R}^3} |\nabla u|^2 dx + \frac{1}{2} \int_{\mathbb{R}^3} |u|^2 dx + \frac{\lambda}{4} \int_{\mathbb{R}^3} (I_2 \ast |u|^2)|u|^2 dx - \frac{1}{p} \int_{\mathbb{R}^3} |u|^p dx, \quad u \in H^1(\mathbb{R}^3).\]

Equation \((P_{\lambda})\) appears in quantum mechanics as an approximation of the Hartree-Fock model of a quantum many-body system of electrons \([6,7,15] \), and in semi-conductor theory \([4]\) (under the name of the Schrödinger-Maxwell equation). From a mathematical point of view, as pointed out in \([26]\), this model presents a combination of repulsive forces (given by the nonlocal term) and attractive forces (given by the nonlinearity). The interaction between the two forces gives rise to unexpected situations concerning the existence, non-existence and multiplicity of solutions, and their qualitative behavior, see e.g. \([1,3,5,11,17,18,20,22,25,26,27,29]\) and the references therein.

It is shown in \([25]\) that for \( p \in (2,3) \) equation \((P_{\lambda})\) has no solutions for \( \lambda \geq 1/4 \) and has at least two positive radial solutions for small \( \lambda > 0 \). One of the solutions is a ground state and a local minimizer, another is a higher energy mountain pass type solution. For \( p \in (3,6) \) equation \((P_{\lambda})\) has a positive radial ground state for all \( \lambda > 0 \) \([3,25]\). For \( p = 3 \) there is at least one radial positive solution for small \( \lambda > 0 \) and no positive solutions for \( \lambda \geq 1/4 \) \([25]\).

Our goal in this work is to describe the asymptotic profile of the ground state solutions of \((P_{\lambda})\) when \( p \in (3,6) \) and \( \lambda \to \infty \). Observe that for \( p \neq 3 \) the rescaling
\[(1.1) \quad v(x) = \lambda^{\frac{4}{p-2}} u(|x|^{\frac{4}{p-2}})\]
transforms \((P_{\lambda})\) into the equation
\[(1.2) \quad -\Delta v + \lambda^{\frac{4}{p-2}} v + (I_2 \ast |v|^2)v = |v|^{p-2} v \quad \text{in } \mathbb{R}^3.\]

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Moreover, in [26] the author studied the profile of the radial global minimizer for $v_{p}$ since for $p \in (18/7, 3)$ and $\lambda \to 0$, we can work in the radially symmetric settings step by step.

If $\lambda > 0$ is well-posed in $H^{1}(\mathbb{R}^{3})$. Since $E(\mathbb{R}^{3}) \subsetneq H^{1}(\mathbb{R}^{3})$, small perturbation arguments in the spirit of the Lyapunov-Schmidt reduction are not directly applicable to the family $\tilde{I}_\lambda$ in the limit $\lambda \to \infty$. Using direct variational analysis based on the comparison of the ground state energy levels for two problems, we establish the following result.

**Theorem 1.1.** Let $3 < p < 6$. Then for any sequence $\{\lambda_n\}$ with $\lambda_n \to \infty$ as $n \to \infty$, there exists $\{\xi_{\lambda_n}\} \subset \mathbb{R}^{3}$ such that the rescaled family of ground states of $(P_{\lambda})$

$$v_{\lambda_n}(x) := \lambda_n^{-\frac{2}{6-p}}u_{\lambda_n}(\lambda_n^{-\frac{2}{6-p}}(x + \xi_{\lambda_n}))$$

converges in $E(\mathbb{R}^{3})$ to a positive ground state solution $\tau_{\infty}$ of the formal limit equation (1.3). Moreover, $\lambda_n^{-\frac{2}{6-p}}\|v_{\lambda_n}\|_{2}^{2} \to 0$ as $\lambda \to \infty$.

We remark that our strategy is quite different from [26], and follows from the ideas of [17, 21]. In [24] the author studied the profile of the radial global minimizer for $p \in (18/7, 3)$ as $\lambda \to 0$. Since for $p \in (18/7, 3)$ the energy functional $\tilde{I}_\infty$ is coercive, the approach of [24] is not applicable for $p \in (3, 6)$. Note that Theorem 1.1 remains valid also for the radial ground state solutions since we can work in the radially symmetric settings step by step.

Our results do not rely and do not require the uniqueness or non-degeneracy of the ground-states of (1.3). We also point out that since $E(\mathbb{R}^{3}) \subsetneq H^{1}(\mathbb{R}^{3})$, it is crucial to know that the ground state solution $v_{\infty}$ of the formal limit equation (1.3) has exponential decay at infinity.
Lemma 2.1. Assume that $\|\cdot\|_E$ is a norm, and $(E(\mathbb{R}^3), \|\cdot\|_E)$ is a uniformly convex Banach space. Moreover, $C^\infty(\mathbb{R}^3)$ is dense in $E(\mathbb{R}^3)$ and also $C^\infty_0(\mathbb{R}^3)$ is dense in $E_r(\mathbb{R}^3)$.

Let us define $\phi_u = I_2 * |u|^2$, then $u \in E(\mathbb{R}^3)$ if and only if $u$ and $\phi_u$ belong to $D^{1,2}(\mathbb{R}^3)$. The characterizations of the convergences in $E(\mathbb{R}^3)$ is given by the following proposition.

Proposition 2.2. Given a sequence $\{u_n\}$ in $E(\mathbb{R}^3)$, $u_n \to u$ in $E(\mathbb{R}^3)$ if and only if $u_n \to u$ and $\phi_{u_n} \to \phi_u$ in $D^{1,2}(\mathbb{R}^3)$.

Moreover, $u_n \to u$ in $E(\mathbb{R}^3)$ if and only if $u_n \to u$ in $D^{1,2}(\mathbb{R}^3)$ and $\int_{\mathbb{R}^3} (I_2 * |u|^2)|u|^2dx$ is bounded. In such case, $\phi_{u_n} \to \phi_u$ in $D^{1,2}(\mathbb{R}^3)$.

It is also proved in [26, Theorem 1.2] that $E(\mathbb{R}^3) \hookrightarrow L^q(\mathbb{R}^3)$ continuously for $q \in [3, 6]$, $E_r(\mathbb{R}^3) \hookrightarrow L^q(\mathbb{R}^3)$ continuously for $q \in (18/7, 6]$, and the inclusion is compact for $q \in (18/7, 6)$. As in [13], we define $M : E(\mathbb{R}^3) \to \mathbb{R}$ as

$$M(u) = \int_{\mathbb{R}^3} |\nabla u|^2dx + \int_{\mathbb{R}^3} (I_2 * |u|^2)|u|^2dx,$$

then we can easily check that for any $u \in E(\mathbb{R}^3)$,

$$\frac{1}{2} \|u\|_E^2 \leq M(u) \leq \|u\|_E^2.$$ 

The following estimate on $M(u)$ is given by [13, Lemma 3.1].

Lemma 2.1. Assume that $p \in (3, 6)$. Then there exists $C > 0$ such that $\|u\|_p^p \leq CM(u)^{3p-2}$ for all $u \in E$.

To finish this section, we state a Pohožaev type identity, see [13, 25].

Proposition 2.3. Assume that $p \in (2, 6)$.

1. Let $u \in H^1(\mathbb{R}^3)$ be a weak solution of (1.2), then

$$\frac{1}{2} \int_{\mathbb{R}^3} |\nabla u|^2dx + \frac{3\lambda_{E(\mathbb{R}^3)}}{2} \int_{\mathbb{R}^3} |u|^2dx + \frac{5}{4} \int_{\mathbb{R}^3} (I_2 * |u|^2)|u|^2dx - \frac{3}{p} \int_{\mathbb{R}^3} |u|^pdx = 0.$$

2. Let $u \in E(\mathbb{R}^3) \cap H^2_{\text{loc}}(\mathbb{R}^3)$ be a weak solution of (1.3), then

$$\frac{1}{2} \int_{\mathbb{R}^3} |\nabla u|^2dx + \frac{5}{4} \int_{\mathbb{R}^3} (I_2 * |u|^2)|u|^2dx - \frac{3}{p} \int_{\mathbb{R}^3} |u|^pdx = 0.$$
3. Asymptotic profiles of the rescaled ground state

It is well known that in [3] the authors have obtained the existence of ground states $u_\lambda$ of (1.2) when $p \in (3, 6)$, which is a mountain pass type solution. Then the rescaling

$$v_\lambda(x) = \lambda^{\frac{3}{4p-3}} u_\lambda(\lambda^{\frac{3}{4p-3}} x)$$

is a ground state of (1.2) and corresponding to the rescaled minimization problem

$$m_\lambda = \inf_{u \in \mathcal{P}_\lambda} \tilde{I}_\lambda(u), \quad \mathcal{P}_\lambda := \{ u \in H^1(\mathbb{R}^3) \setminus \{0\} : \mathcal{P}_\lambda(u) = 0 \},$$

where $\mathcal{P}_\lambda : H^1(\mathbb{R}^3) \to \mathbb{R}$ is defined by

$$\mathcal{P}_\lambda(u) = \frac{3}{2} \int_{\mathbb{R}^3} |\nabla u|^2 dx + \frac{\lambda^{\frac{3}{4p-3}} \int_{\mathbb{R}^3} |u|^2 dx + \frac{3}{4} \int_{\mathbb{R}^3} (I_2 * |u|^2)|u|^2 dx - \frac{2p-3}{p} \int_{\mathbb{R}^3} |u|^p dx.$$

Observe that if $u \in H^1(\mathbb{R}^3)$ is a critical point of $\tilde{I}_\lambda$, then $u \in \mathcal{P}_\lambda$ since $\mathcal{P}_\lambda(u) = 0$ is nothing but the combination of $\langle \tilde{I}_\lambda(u), u \rangle = 0$ and the Pohozaev type identity. For each $u \in H^1(\mathbb{R}^3) \setminus \{0\}$, set

$$u_\lambda(x) := t^2 u(tx).$$

Then

$$m_\lambda = \inf_{u \in \tilde{\mathcal{I}}_\lambda} \tilde{I}_\lambda(u), \quad \tilde{\mathcal{I}}_\lambda := \{ u \in E(\mathbb{R}^3) \setminus \{0\} : \tilde{\mathcal{I}}_\lambda(u) = 0 \},$$

where $\mathcal{E}_\lambda : E(\mathbb{R}^3) \to \mathbb{R}$ is defined by

$$\mathcal{E}_\lambda(u) = \frac{3}{2} \int_{\mathbb{R}^3} |\nabla u|^2 dx + \frac{3}{4} \int_{\mathbb{R}^3} (I_2 * |u|^2)|u|^2 dx - \frac{2p-3}{p} \int_{\mathbb{R}^3} |u|^p dx.$$

Similarly we can show that $\mathcal{E}_\lambda \neq \emptyset$ and $\mathcal{E}_\lambda$ has the same properties as $\mathcal{P}_\lambda$.

In this section, we are going to show that $v_\lambda$ converges to a positive ground-state of the formal limit equation (1.3). Similar to the proof of [13] Corollary 3.2, by using Lemma 2.1 we obtain a lower bound on $M(u)$ for all $u \in \mathcal{E}_\lambda$.

**Corollary 3.1.** Let $p \in (3, 6)$. There exists $\eta > 0$ such that $M(u) > \eta$ for any $u \in \mathcal{E}_\lambda$.

With Proposition 2.5 we could prove that $\mathcal{E}_\lambda$ is a natural constraint in the spirit of [3] and the proof will be skipped.

**Lemma 3.1.** Assume that $p \in (3, 6)$. Then $m_\lambda > 0$ and $\mathcal{E}_\lambda$ is a natural constraint.

**Remark 3.1.** [13] Theorem 1.1 shows that (1.3) admits a ground state $v_\infty \in \mathcal{E}_\lambda$ with $\tilde{I}_\lambda(v_\infty) = m_\infty$ when $p \in (3, 6)$. Moreover, each solution of (1.3) has an exponential decay at infinity if $p \in (3, 6)$ [5] Theorem 1.3, which means that they belong to $L^2(\mathbb{R}^3)$.

We now turn our attention to study the asymptotic profile of the rescaled ground state $v_\lambda$. As we mentioned in the introduction, we use direct variational analysis based on the comparison of the ground state energy levels for two problems, we begin by studying the convergence of $m_\lambda$ as $\lambda \to \infty$.

**Lemma 3.2.** Assume that $p \in (3, 6)$. Then $0 < m_\lambda - m_\infty \to 0$ as $\lambda \to \infty$. 
Corollary 3.2. Lemma 3.2 implies that there exists a unique t_λ such that t_λ^2 v_λ(x) ∈ P_∞, and we have

\[ m_{∞} ≤ \tilde{t}_∞(t_λ^2 v_λ(x)) = \frac{t_λ^3 (p - 3)}{2p - 3} \parallel ∇ v_λ \parallel_2^2 + \frac{(p - 3) t_λ^3}{2(2p - 3)} \int_{\mathbb{R}^3} (I_2 ∗ |v_λ|^2) |v_λ|^2 dx < \tilde{t}_λ(\lambda ) = m_λ, \]

which means m_∞ < m_λ.

To show that m_λ → m_∞ as λ → ∞ we shall use v_∞ as a test function for P_λ. Note that the ground state v_∞ of (1.3) has an exponential decay at infinity, hence v_∞ ∈ L^2(\mathbb{R}^3) (see Remark 3.1).

Since P_\lambda(v_∞) = \frac{\lambda^{\frac{p-2}{2}}}{2} \parallel v_∞ \parallel_2^2 > 0, there exists \tilde{\gamma}_λ > 1 such that \tilde{t}_λ^2 v_λ(\tilde{\gamma}_λ x) ∈ P_\lambda, i.e.,

\[ \frac{3 \tilde{t}_λ^2}{2} \parallel ∇ v_∞ \parallel_2^2 + \frac{\lambda^{\frac{p-2}{2}} \tilde{\gamma}_λ}{2} \parallel v_∞ \parallel_2^2 + \frac{3 \tilde{t}_λ^2}{4} \int_{\mathbb{R}^3} (I_2 ∗ |v_∞|^2) |v_∞|^2 dx = \frac{(2p - 3)(\tilde{t}_λ^2 - 1)}{p} \parallel v_∞ \parallel_p^p. \]

This, combined with P_\lambda(v_∞) = 0 and v_∞ ∈ L^2(\mathbb{R}^N), implies that

\[ \frac{(2p - 3)(\tilde{t}_λ^2 - 1)}{p} \parallel v_∞ \parallel_p^p = \frac{\lambda^{\frac{p-2}{2}} \tilde{\gamma}_λ}{2} \parallel v_∞ \parallel_2^2. \]

Therefore, \tilde{\gamma}_λ → 1 as λ → ∞. Moreover,

\[ \tilde{t}_λ ≤ 1 + C \lambda^{\frac{p-2}{2}}, \]

where C > 0 is independent of λ. Thus we have

\[ m_λ ≤ \tilde{t}_λ(\tilde{t}_λ^2 v_λ(\tilde{\gamma}_λ x)) ≤ \tilde{t}_∞(v_∞) + C(\tilde{t}_λ^2 - 1) + \frac{\lambda^{\frac{p-2}{2}} \tilde{\gamma}_λ}{2} \parallel v_∞ \parallel_2^2 \]

\[ ≤ m_{∞} + C \lambda^{\frac{p-2}{2}}. \]

This, together with (3.5), means that m_λ - m_∞ → 0 as λ → ∞.

\[ \Box \]

Corollary 3.2. Let p ∈ (3, 6). Then the quantities

\[ \parallel ∇ v_λ \parallel_2^2, \ \lambda^{\frac{p-2}{2}} \parallel v_λ \parallel_2^2, \ \parallel v_λ \parallel_q^p, \ \int_{\mathbb{R}^3} (I_2 ∗ |v_λ|^2) |v_λ|^2 dx, \]

are uniformly bounded as λ → ∞.

\[ \Box \]

Proof. From v_λ ∈ P_λ and Lemma 3.2 we have

\[ m_∞ + o(1) = m_λ = \tilde{t}_λ(v_λ(x)) \]

\[ = \frac{p - 3}{2p - 3} \parallel ∇ v_λ \parallel_2^2 + \frac{p - 3}{2p - 3} \lambda^{\frac{p-2}{2}} \parallel v_λ \parallel_2^2 + \frac{p - 3}{2(2p - 3)} \int_{\mathbb{R}^3} (I_2 ∗ |v_λ|^2) |v_λ|^2 dx. \]

Therefore,

\[ \parallel ∇ v_λ \parallel_2^2, \ \lambda^{\frac{p-2}{2}} \parallel v_λ \parallel_2^2, \ \parallel v_λ \parallel_q^p, \ \int_{\mathbb{R}^3} (I_2 ∗ |v_λ|^2) |v_λ|^2 dx, \]

are uniformly bounded as λ → ∞.

\[ \Box \]

Lemma 3.3. Let p ∈ (3, 6). Then \lambda^{\frac{p-2}{2}} \parallel v_λ \parallel_2^2 → 0 as λ → ∞.

\[ \Box \]

Proof. Lemma 3.2 implies that there exists a unique t_λ ∈ (0, 1) such that t_λ^2 v_λ(t_λ x) ∈ P_∞. Indeed, assume that t_λ → t_∞ < 1 as λ → ∞. Then by (3.3) we have, as λ → ∞,

\[ m_∞ ≤ \tilde{t}_∞(t_λ^2 v_λ(t_λ x)) = \frac{t_λ^3 (p - 3)}{2p - 3} \parallel ∇ v_λ \parallel_2^2 + \frac{t_λ^3 (p - 3)}{2(2p - 3)} \int_{\mathbb{R}^3} (I_2 ∗ |v_λ|^2) |v_λ|^2 dx \]

\[ < \frac{t_λ^3}{2} \tilde{t}_λ(\lambda ) = t_λ^3 m_λ + t_λ^3 m_∞ < m_∞. \]
which is a contradiction. Therefore $t_\lambda \to 1$ as $\lambda \to \infty$. Using $\mathcal{P}_\infty(t^2_\lambda v_\lambda(t_\lambda x)) = 0$ again, we see that

$$0 = \frac{3t^2_\lambda}{2} \|\nabla v_\lambda\|^2_2 + \frac{3t^2_\lambda}{4} \int_{\mathbb{R}^3} (I_2 * |v_\lambda|^2)|v_\lambda|^2 \, dx - \frac{(2p - 3)t^2_\lambda}{p} \|v_\lambda\|^p_p - \mathcal{P}_\lambda(v_\lambda) = \frac{\lambda^\frac{p-2}{2} t_\lambda}{2} \|v_\lambda\|^2_2 + \frac{3(t^2_\lambda - 1)}{2} \|\nabla v_\lambda\|^2_2 + \frac{3t^2_\lambda}{4} \int_{\mathbb{R}^3} (I_2 * |v_\lambda|^2)|v_\lambda|^2 \, dx - \frac{(2p - 3)(t^2_\lambda - 1)}{p} \|v_\lambda\|^p_p.$$ 

This, together with Corollary 3.2 and $\mathcal{P}_\lambda(v_\lambda) = 0$, implies that $\lambda^\frac{p-2}{2} t_\lambda \|v_\lambda\|^2_2 \to 0$ as $\lambda \to \infty$. \hfill \Box

**Proof of Theorem 1.1.** From Corollary 3.2, Lemma 3.3 and (3.6), we see that the first author is partially supported by Natural Science Foundation of China (Nos. 11901418, 11771319, 12171470).

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