Many-body effects in a model of electromagnetically induced transparency

Jose Reslen

Department of Physics, National University of Singapore, Science Drive 3, Blk 12, Singapore 117543
E-mail: reslenjo@yahoo.com

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Abstract
We study the spectral properties of a many-body system under a regime of electromagnetically induced transparency. A semi-classical model is proposed to incorporate the effect of inter-band interactions on an otherwise single-body scheme. We use a Hamiltonian with non-Hermitian terms to account for the effect of particle decay from excited levels. We explore the system response as a result of varying the interaction parameter. Then we focus on the highly interacting case, also known as the blockade regime. In this latter case, we present a perturbative development that allows us to get the transmission profile for a wide range of values of the system parameters. We observe a reduction of transmission when interaction increases and show how this property is linked to the generation of a strongly correlated many-body state. We study the characteristics of such a state and explore the mechanisms giving rise to various interesting features.

(Some figures in this article are in colour only in the electronic version)
discussed in [5], the quantum superposition can be seen as involving states where, due to dipole–dipole interactions, only a single atom can be excited to the interacting band and therefore only one atom is involved in EIT. This results in a reduction of transparency as the possibilities of interference of excitation pathways are diminished. Such reduction shows neither resonance shift nor linewidth broadening. Some of the experimental results cannot be explained via mean-field approximations and full quantum models must be employed to reproduce the observed features [6]. In [20], it was shown that two-photon correlations lead to an enhanced attenuation of the probe beam for strong intensities and in this way a detailed description of the experiment in [5] is achieved. When emphasis is made on the propagation of the probe laser through the medium as in [21], it is shown that the blockade mechanism gives rise to a highly non-local response in addition to nonlinearities.

Many-body effects in an ensemble of interacting atoms can be studied using the master equation formalism [22]. In this case, the number of coefficients necessary to describe the density matrix is proportional to the square of the total number of elements in the Hilbert space. Instead, we probe the advantages of introducing decay factors as imaginary elements in the Hamiltonian, so that the whole analysis can be carried in terms of state functions, providing insight into the development of the many-body scenario as well as the statistical effects that arise due to the bosonic nature of the particles. Non-Hermitian Hamiltonians (NHH) have proved useful in several studies, e.g. single-particle models of EIT [1], STIRAP in the presence of degenerate product states [23, 24] and the dynamics of Bose–Hubbard dimers [25] among many others. This approach is reasonable since we want to explore the case of a high number of particles where standard approximations have displayed mixed results [26]. Below we show that the proposed methodology produce consistent results and is especially suitable to develop a perturbative approach which is valid over a wide range of parameters. The insight acquired in this way is less accessible using a fully numerical approach since several tunable parameters must be considered.

In our proposal, we assume that probe and coupling lasers induce particle exchange among three energy levels as in a ladder EIT scheme [5, 17] (figure 1) and that two-body interactions take place only in the interaction band. Such two-body exchange is mediated by a constant parameter \( U \). In general, the interaction depends on the inter-atomic distance and the Rydberg principal quantum number [5, 19]. Using a semi-classical approach and incorporating the rotating wave approximation in addition to non-Hermitian decay terms, the NHH reads

\[
\hat{H} = -\Omega_e e^{-i\omega_e t} \hat{a}_R^\dagger \hat{a}_e - \Omega_p e^{-i\omega_p t} \hat{a}_L^\dagger \hat{a}_e + \text{H.c.}
\]

\[
+ \frac{U}{2} \hat{a}_R^\dagger \hat{a}_R (\hat{a}_L^\dagger \hat{a}_R - 1) - i\Gamma_R \hat{a}_R^\dagger \hat{a}_R - i\Gamma_e \hat{a}_e^\dagger \hat{a}_e
\]

\[
+ E_e \hat{a}_R^\dagger \hat{a}_R + E_R \hat{a}_R^\dagger \hat{a}_R + E_e \hat{a}_e^\dagger \hat{a}_e.
\]  

(1)

Creation and annihilation operators introduce boson-like statistics through the commutation relations \([\hat{a}_L, \hat{a}_e^\dagger] = 1\) for \( L = g, e, R \). \( \Omega_e \) and \( \Omega_p \) are the probe- and coupling Rabi frequencies, respectively. Similarly, \( \omega_p \) and \( \omega_e \) are the laser frequencies while \( E_e, E_R \) and \( E_e \) represent the energies of the corresponding bands. The intensity of particle decay from the interaction- and excited bands is controlled via \( \Gamma_R \) and \( \Gamma_e \), respectively. Both constants are positive definite. This approach is valid in the limit of weak coupling between the ground and excited bands [1]. In our analysis, we set \( \hbar = 1 \) and measure all parameters in the recoil energies \( E_r = \hbar^2 K^2 / (2m) \). The energy structure of the system corresponds to a three-level EIT picture where \( E_R > E_e > E_e \) (figure 1). Although not explicit in (1), the total number of particles is \( M \), which accounts for the number of atoms in a blockade sphere. The proposed scheme can be realized by projecting counter-propagating lasers onto a cloud of ultracold atoms—for example, \(^{87}\text{Rb}\). These lasers provide the probe- and coupling frequencies in our proposal. The number of atoms can be controlled by pumping atoms into energy levels that do not couple to the probe- or coupling lasers. The energy spectrum of the gas contains bands that can stand for the ground and excited bands of figure 1. The gas also contains highly excited atoms which display interaction intensities much larger than atoms in the ground or excited levels, so that it is valid to neglect interaction effects on these bands. The intensity of the interaction can be tuned using Feshbach resonance [27]. Actual experiments implementing this approach have been reported in several works, as for instance in [5, 6]. As a result of the inclusion of non-Hermitian terms in equation (1), the wavefunction norm is no longer preserved and therefore the quantum state must be normalized in anticipation to any explicit calculation.

After an appropriate transformation, we get the following dressed NHH in the interaction picture:

\[
\hat{H}_D = -\Omega_e (\hat{a}_R^\dagger \hat{a}_R + \hat{a}_e^\dagger \hat{a}_e) - \Omega_p (\hat{a}_L^\dagger \hat{a}_L + \hat{a}_R^\dagger \hat{a}_L)
\]

\[
+ \frac{U}{2} \hat{a}_R^\dagger \hat{a}_R (\hat{a}_L^\dagger \hat{a}_R - 1) - i\Gamma_R \hat{a}_R^\dagger \hat{a}_R - i\Gamma_e \hat{a}_e^\dagger \hat{a}_e
\]

\[
+ \delta \hat{a}_R^\dagger \hat{a}_R + (E_R - \omega_e) \hat{a}_R^\dagger \hat{a}_R + E_e \hat{a}_e^\dagger \hat{a}_e,
\]  

(2)

where \( \delta = E_e - \omega_e \). For simplicity, we have assumed single-photon resonance and without loss of generality we set
$E_R - \omega c = E_e = 0$. Here we are mainly concerned with the magnitude of the coupling between the atoms and the probe laser, i.e. the atomic susceptibility. Hence, we focus on the mean value

$$\chi_M^{(n)} = \frac{\langle (\hat{g}_a^\dagger \hat{a}_i)^n \rangle}{M^n}.$$  \hfill (3)

Imaginary and real parts of $\chi_M^{(n)}$ account for different orders of absorption and refraction, respectively. Figure 2 presents the behaviour of $\text{Im}(\chi_M^{(1)})$ as various parameters are tuned. In all cases, the state of the system is given by the right eigenstate of equation (2) corresponding to the eigenvalue with the highest imaginary part (the less decaying state). The NHH (2) conserves the total number of particles so that the full dimension of the Hilbert space is $(M + 2)(M + 1)/2$. The pattern shown in figure 2 is proportional to the transmission profile of the probe laser. It indicates a characteristic window of transparency that results from the combined action of the incident lasers. In the absence of interaction, the dip in absorption can be ascribed to the interference of pathways with opposite phases, i.e. the process by which one atom goes from $|g\rangle$ to $|e\rangle$ is cancelled out by the process by which the atom goes from $|e\rangle$ to $|R\rangle$ and then all the way back from $|R\rangle$ to $|e\rangle$ to $|g\rangle$ again. As can be seen, the case $U = 0$, which is equivalent to the single-particle case, displays the maximum interference. As the interaction is gradually turned on, the two-photon resonance is shifted to the right due to the energy increase of the interaction band produced by the repulsion among particles. $\text{Im}(\chi_M^{(1)})$ continues to grow until it reaches a saturation value, but without completely suppressing transparency. $\text{Im}(\chi_M^{(1)})$ also grows with $M$, but no saturation is visible over values of $M$ less than 100. In general, we can see that the insertion of particles causes a reduction of quantum interference. Such an effect can be enhanced either by increasing $U$ or $M$. However, in every case the physical response is different. While changing $U$ affects the intensity of two-body interactions, adding particles produces a (sometimes steep) rearrangement of the quantum state.

From the equations of motion, we can extract the expression

$$\frac{d\hat{a}_k^\dagger}{dt} = i\Omega_R \hat{a}_k^\dagger - i\delta \hat{a}_k^\dagger,$$ \hfill (4)

where $\hat{a}_n^\dagger = e^{-\delta \hat{a}_R^\dagger} \hat{a}_n^\dagger e^{\delta \hat{a}_R^\dagger}$ for $n = g, R, e$. Since $\Omega_R \ll \Omega_c$ we can assume that $\hat{a}_R^\dagger$ does not greatly influence the evolution of $\hat{a}_R^\dagger$ and $\hat{a}_e^\dagger$, so that $\hat{a}_e^\dagger$ can be treated as a function of time. Thus, it follows that

$$\hat{a}_e^\dagger(t) = i\Omega_R e^{-\delta t} \int_0^t e^{\delta \hat{T}} \hat{a}_e^\dagger(T) dT.$$ \hfill (5)

Under such an assumption, in the blockade regime, equation (2) can be reduced to a Jaynes–Cummings-like Hamiltonian on the interaction- and excited bands

$$\hat{H}_{\text{Re}} = -\Omega_c (\hat{\sigma}^+ \hat{a}_e + \hat{\sigma}^- \hat{a}_e^\dagger) - i\Gamma_R \left( \hat{a}_e^\dagger + 1 \right) \hat{a}_e + i\Gamma_R \hat{a}_e^\dagger \hat{a}_e.$$ \hfill (6)
where we have employed Pauli matrices to account for the dynamics of the interaction band. The eigensystem of matrix (6) is given by

\[ E = 0, \quad |0, 0>, \quad |0, 0|, \]

\[ E_n^\pm = -i(\Gamma_n + \Gamma_R + \Omega_n \sqrt{n + 1} e^{\pm i\theta_n}), \]

\[ |E_n^\pm> = (|1, n> + i e^{\pm i\theta_n} |0, n + 1>) / Z_n^\pm, \]

\[ \langle E_n^\pm| = (\langle 1, n| + i e^{\pm i\theta_n} \langle 0, n + 1|) / Z_n^\pm, \]

where

\[ Z_n^\pm = \sqrt{1 - e^{\pm 2i\theta_n}}, \]

\[ \cos \theta_n = \frac{\Gamma_n - \Gamma_R}{2 \Omega_n \sqrt{n + 1}}. \]

The first and second integers in a ket correspond to the number of particles in the interaction and excited band, respectively. The angle \( \theta_n \) and the eigenvalues of the NHH are in general complex. In the form in which they appear above, the eigenvectors satisfy \( \langle E_k^+| E_k^-> = \delta_k^+ \delta_k^- \). This eigensystem can be used to write

\[ \hat{H}_{\text{BE}} = \sum_{n=0}^{M-1} E_n^+ |E_n^+><E_n^+| E_n^-|E_n^-><E_n^-|. \]

The spectrum of the NHH is well defined except when \( \cos \theta_n = 1 \), in which case the eigenvectors (9) and (10) become indeterminate but one can still recover the NHH as the limit of equation (13). These singularities therefore produce removable discontinuities leading to no divergence in the mean value of the system’s observables. Now we project on the subspace associated with the NHH:

\[ \hat{a}_k^+ = \hat{a}_k^+ \left\{ \sum_{n=0}^{M-1} |E_n^+><E_n^+| + |0, 0><0, 0| \right\}. \]

This procedure is facilitated by introducing the coefficients validating the following identities:

\[ \hat{a}_k^+ |0, 0> = c_0^+ |E_0^+> + c_0^- |E_0^->, \]

\[ \hat{a}_k^+ |E_n^+> = c_{n+1}^+ |E_{n+1}^+> + c_{n+1}^- |E_{n+1}^->, \]

\[ \hat{a}_k^+ |E_n^-> = d_{n+1}^+ |E_{n+1}^+> + d_{n+1}^- |E_{n+1}^->. \]

Once \( \hat{a}_k^+ \) is written in terms of the eigenvectors of the NHH, we can calculate \( \hat{a} \) and replace it in equation (5) where we can now carry on the integration in terms of time. The result can be written in the form

\[ \hat{a}_k^+(t) = \hat{a}_k^+ + \Omega_p \hat{k}(0) - e^{-i\hat{k}} \hat{k}(t), \]

where

\[ \hat{k}(t) = c_0^+ e^{-i(E_0^- - \delta)} |E_0^+><0, 0| + c_0^- e^{-i(E_0^- - \delta)} |E_0^-><0, 0| \]

\[ + \sum_{n=0}^{M-2} \left( c_{n+1}^+ e^{-i(E_{n+1}^- - \delta)} |E_{n+1}^+><E_n^-> + E_{n+1}^-|E_n^+><E_{n+1}^-| \right) \]

\[ + d_{n+1}^+ e^{-i(E_{n+1}^- - \delta)} |E_{n+1}^+><E_n^-> + d_{n+1}^- e^{-i(E_{n+1}^- - \delta)} |E_{n+1}^-><E_n^+|. \]

This is found to satisfy

\[ \langle \hat{a}_k^+(\infty)|0\rangle = \sum_{k=0}^{M} \left( \frac{M}{k} \right)^M \langle \hat{a}_k^+ \rangle^{M-k} (\Omega_p \hat{k}(0))^k |0\rangle. \]

The operator \( \hat{k} \) couples the representations of \( \hat{H}_{\text{BE}} \) in such a way that

\[ \hat{k}(0)^k |0\rangle = v_k Z_{k-1}^+ |E_{k-1}^+> + w_k Z_{k-1}^- |E_{k-1}^->, \quad k = 1, \ldots, M. \]

By replacing (21) in (20), one obtains

\[ \langle M00| + \sum_{k=1}^{M} \sqrt{\frac{M}{k}} \langle \Omega_p ((v_k + w_k)|M - k, 1, k - 1) \]

\[ + \hat{a}_k^+ |v_k e^{\hat{p}^k} + w_k e^{-\hat{p}^k} - M, k, 0, k\rangle. \]

The set of coefficients \( v_k, w_k \) are connected by the recursion expression

\[ \begin{pmatrix} v_k \\ w_k \end{pmatrix} = \hat{B}_k \begin{pmatrix} v_{k-1} \\ w_{k-1} \end{pmatrix}, \quad k = 2, \ldots, M. \]

From an induction argument, \( \hat{B}_k \) is found to satisfy

\[ \hat{B}_k 2 i \sqrt{k} \sin \theta_{k-1} = \frac{\sqrt{\frac{k}{2}(E_{k-1}^+-\delta)E_{k-1}^-} + \sqrt{\frac{k}{2}(E_{k-1}^+-\delta)E_{k-1}^+}}{\sqrt{\frac{k}{2}(E_{k-1}^--\delta)E_{k-1}^-} + \sqrt{\frac{k}{2}(E_{k-1}^-\delta)E_{k-1}^+}}. \]

and the first elements are given by

\[ v_k = \frac{-1}{2(E_k^+-\delta) \sin \theta_0}, \quad w_k = \frac{1}{2(E_0^+-\delta) \sin \theta_0}. \]

Equations (25), (24), (23) and (22) can be used to numerically generate the stationary state for any set of parameters excluding singularities. Such a condition can be improved by introducing new discrete variables

\[ p_k = v_k + w_k, \quad q_k = \sin \theta_{k-1}(v_k - w_k). \]
These variables obey a recursion relation analogous to equation (23) with $u_k$ and $w_k$ replaced by $p_k$ and $q_k$, respectively, and $B_k$ replaced by

$$\lambda_k = \frac{1}{\sqrt{F_k}} \left( R_k\omega + S_k T_k \sin^2 \theta_{k-2} \quad R_k T_k + S_k \omega \right),$$

in such a way that

$$F_k = \omega^2 - T_k^2 \sin^2 \theta_{k-2},$$

$$R_k = i \Omega_c \sqrt{\frac{k}{k-1}} \cos \theta_{k-1} - \eta \sqrt{k-1},$$

$$S_k = -\Omega_c (2k-1),$$

$$r_k = \frac{i \eta \cos \theta_{k-1}}{\sqrt{k-1}} - \Omega_c \sqrt{k(k-1)} (\sin^2 \theta_{k-1} + \sin^2 \theta_{k-2}),$$

$$s_k = i \Omega_c \cos \theta_{k-1} - \eta \sqrt{k},$$

$$T_k = -2 \eta \Omega_c \sqrt{k-1},$$

$$\omega = \eta^2 - \Omega_c^2,$$

$$\eta = i \Gamma_c + \delta.$$

These expressions along with the initial coefficients

$$p_1 = \frac{\Omega_c}{(i \Gamma_R + \delta)(i \Gamma_c + \delta) - \Omega_c^2},$$

$$q_1 = \frac{(i \Gamma_c + \Gamma_R) + 2 \delta}{(i \Gamma_R + \delta)(i \Gamma_c + \delta) - \Omega_c^2}$$

can be integrated into a programming routine that recursively calculates the state coefficients and then $\chi^{(n)}$. We note that when $|\Gamma_c - \Gamma_R| \ll 2\Omega_c k$ the recursion matrix can be approximated as follows:

$$\lambda_k \approx \begin{pmatrix} \Omega_c & 0 \\ 0 & \Omega_c \end{pmatrix}.$$

Employing this $\lambda_k$, one finds a stationary state displaying the following susceptibility:

$$\chi^{(1)} = -\Omega_c \frac{i \Gamma_c}{(i \Gamma_c + \delta)}, \quad \chi^{(2)} = (\chi^{(1)})^2.$$

Similarly, from a direct calculation, the single-particle susceptibility is found to be

$$\chi^{(1)} = -\Omega_c \frac{(i \Gamma_R + \delta)}{(i \Gamma_R + \delta)(i \Gamma_c + \delta) - \Omega_c^2}.$$

As can be seen in figure 3, equations (38) and (39) are extreme cases of the numerical results obtained using the matrix $A_k$ recursively. Here we have chosen the case $\Gamma_R = 0$ because it is most close to the actual experimental case where the decay rate of the Rydberg state is usually negligible. The inclusion of more particles in the system induces a rise of $\text{Im}(\chi^{(1)})$ at zero detuning. In the same instance, $\text{Re}(\chi^{(2)})$ undergoes a change of sign as it dips from its peaking value.  

Figure 3. First-order absorption-profile (a) and second-order refraction-profile (b) for different $M$. The cases $M = 1$ and $M = \infty$ correspond to equations (38) and (39), respectively. Other cases shown are generated following the procedure sketched in the text. The parameters involved in the computations are $\Gamma_c = 2, \Gamma_R = 0, \Omega_c = 0.1$ and $\Omega_r = 1$. 

In order to facilitate the presentation of equations (38) and (39), we omit a normalization factor of order $\Omega_c^2$. This is not the case when the same formulæ are used in figure 3.
From figure 4, we can see that the absorption profile at $\delta = 0$ features nonlinear behaviour over a wide range of values of $M$ and then asymptotically converges towards the estimation given by equation (38). As we increase $\Gamma_e$, the growth speed decreases and the curves display stronger inflection. In every case, the absorption profile is characterized by a steep absorption growth. Such a nonlinear response is linked to a cooperative many-body state where the single-particle outcome is no longer dominant. As $M$ goes up further, the absorption value starts to saturate, suggesting that less particles are being integrated in the interaction process.

In the single-particle case, maximal transmission is achieved as a result of almost perfect quantum interference between excitation pathways. In this case, the only coupling mode involved in EIT in equations (8)–(12) is $n = 0$. A gradual increase of $\Omega_r$ allows more particles into the excited band and therefore more coupling modes participate in the EIT process. One important characteristic of EIT is that the reduction of absorption at zero detuning is accompanied by a peak in the refraction coefficient $\Gamma_e = 1$ because $\chi^{(2)}_1 = 0$. This means that the system’s refraction at $\delta = 0$ depends only on two- or more-body transitions, where several atoms are excited (or decay) simultaneously. In a sense, one can think of the process taking place at zero detuning as the one in which single-body transitions play a rather marginal role and instead the leading response is mediated by higher order transitions. Likewise, coupling modes involving more than one particle display less interference, as many-particle pathways between the ground- and excited-bands can only interfere partially with their blockade counterparts.

As even more atoms are integrated in the model, the possibilities of light being absorbed via particle excitation become higher and the characteristic profile of EIT finally fades away. The enhancement of absorption due to many-body effects is especially notorious in the range $M < 1000$. Since the peak of refraction at $M = 2$ is positive, one can find a value of $M$ ($M = 65$ for the values of figure 3) for which refraction is almost zero at zero-detuning, while absorption is still low. In this case, the system is almost unresponsive to the incoming radiation.

In order to check for the consistency of our approach we stress that our central assumption is the cancellation of $\hat{k}(t)$ in equation (19) as $t \to \infty$. This is indeed the case as long as all imaginary parts of the arguments of the exponentials in equation (19) turn out to be negative. Only the following arguments could become greater than zero:

Note that the peaking of refraction can only be seen for $M > 1$ because $\chi^{(2)}_1 = 0$. This means that the system’s refraction at $\delta = 0$ depends only on two- or more-body transitions, where several atoms are excited (or decay) simultaneously. In a sense, one can think of the process taking place at zero detuning as the one in which single-body transitions play a rather marginal role and instead the leading response is mediated by higher order transitions. Likewise, coupling modes involving more than one particle display less interference, as many-particle pathways between the ground- and excited-bands can only interfere partially with their blockade counterparts.
and
\[ \text{Im}(E_{n+1}^- - E_n^-) = \frac{1}{2} (D(n) - D(n+1)) \Gamma_e, \]
where
\[ D(n) = \text{Re} \left( (\Gamma_e - \Gamma_R)^2 - 4 \Omega^2 (n+1) \right) \]
for \( n = 2, 3, \ldots, \infty \).

If either (41) or (42) become positive, then operator \( \hat{k}(t) \) will display exponential growth and will dominate the stationary state. While it seems operationally possible to obtain such a state, this feature is less consistent with our initial consideration in which \( \Omega_p \) is a perturbative parameter and hence most particles remain in the ground band. Due to the form of \( D(n) \), the maximum value of (41) and (42) takes place at \( n = 2 \). Therefore, if (41) or (42) are positive for any \( n > 2 \), then they are positive for \( n = 2 \) as well. Hence, \( n = 2 \) is the only relevant mode in a relaxation analysis. We have depicted in the inset of figure 4 the relaxation map that results following any arbitrary set of realistic parameters in which the arguments discussed above. It is worth pointing out that then they are positive for \( \Gamma_e > \Gamma_R \) fall well inside the regular relaxation zone. It also becomes apparent that in the many-body case, the mere existence of a dark state does not guarantee the state convergence towards such a state.

We have investigated the reduction of transparency in an EIT setup as a result of the collective character developed by many-body matter. Results corresponding to a semi-classical model were obtained from numerical diagonalization as well as from a perturbative approach. In the latter case, we presented a semi-analytical procedure that can be used to find the stationary state of the system in the limit of small intensity of the probe laser. We have also studied the range of validity of our method and established explicit conditions for regular relaxation. The procedure itself shows interesting issues and is valid in the range of realistic experimental parameters. We have in this way presented an alternative analysis of the effects of the many-body interaction on EIT. As a prospect extension of this work, it would be interesting to introduce a light mode to describe the probe laser in order to explore the evolution of initially coherent states of light and the effect of particle interaction on photons.

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