Transport theory is an efficient approach to derive an effective theory for the soft modes of QCD at high temperature. It is known that the leading order operators of this theory can be obtained from (semi-classical) kinetic equations of quasiparticles carrying classical or quantum color charges. Higher order operators can also be obtained. Discrepancy between these quasiparticle models starts for dimension 4 operators, which converge in the limit of high dimensional color representations. These models are reviewed and compared.

1. Introduction

The study of the high temperature phase of QCD is necessary to characterize the properties of the quark-gluon plasma (QGP), relevant for the phenomenology of heavy ion collisions. A serious obstacle to make robust predictions of its properties is the non-perturbative character of the long distance physics. The standard Monte Carlo techniques can only be used to get the static properties of the system, either by performing 4d lattice or (dimensional reduced) 3d lattice simulations. Unfortunately, a similar powerful approach to study the dynamics in this phase is still missing. Efforts to develop non-perturbative tools for the QGP are certainly required.

One approach to study dynamical properties of non-Abelian plasmas is based on the fact that the low energy modes behave classically. It is possible to construct effective classical field theories which are amenable to numerical treatment. These theories treat the hard modes as quasiparticles moving in the background of soft classical fields, and they are modeled with

*Work supported by the Generalitat Valenciana, grant ctidia/2002/5.
a simple transport equation. Previous sessions of SEWM were intensively
devoted to the construction of this program, and I refer to their proceedings
for the basic references. These effective models allowed one to evaluate
transport coefficients to leading order in $g$, the gauge coupling constant.

It seems reasonable to ask at which point the predictions of kinetic
theory depart from those of quantum field theory. If not, it would be
interesting to see whether the analysis explained above could be pushed
beyond leading order in $g$. With this aim I will review the predictions of
kinetic theory for systems close to thermal equilibrium, comparing them
with those from quantum field theory. This approach is constructive, as
the intention is not to derive transport equations from quantum field theory
(while this is possible!), but rather to see if a simple quasiparticle model
describes correctly the QCD predictions for the long distance physics. This
is based on work done in collaboration with Mikko Laine$^1$ and Stanislaw
Mrówczyński$^2$. A similar effort in that direction was first carried out in Ref.$^3$.

There are two different kinetic equations to describe classical colored
particles$^4$. One of them describes color as a classical degree of freedom.
The other one describes color as a quantum degree of freedom, represented
by a matrix in a certain representation $R$ of the gauge group $SU(N_c)$. While
the two different kinetic equations predict the same low order operators in
these effective theories, they do not for higher order operators.

2. Transport Equations for Classical Color

Let us consider a non-relativistic particle of mass $m$ carrying a classical
color charge $Q^a$, where $a = 1$ to $N_c - 1$ for a $SU(N_c)$ group. The
Hamiltonian for this particle is

$$H = \frac{1}{2m} (\mathbf{k} - gA^a Q_a)^2 + gA^a\xi_Q,$$

where $A^a_\mu$ is the non-Abelian vector gauge field. The color charge is a
dynamical variable, which evolves in time $t$, as the canonical momentum $\mathbf{k}$
and position $\mathbf{x}$ of the particle. Its phase-space is then enlarged by adding the
color charge variables. The Poisson brackets (PB) of this classical system
are also modified$^5$. Hamiltonian equations of motion can be derived with
the help of these Poisson brackets, and they correspond to the well-known
Wong equations$^6$.

A statistical description of a system of classical colored particles starts
with the definition of the one-particle distribution function $f(\mathbf{x}, \mathbf{k}, Q; t)$. In
the absence of collisions it evolves according to the Hamiltonian dynamics, 
\[ \frac{df}{dt} = \{H, f\}_PB. \]  
A formulation of the transport equation in terms of the canonical momentum (the phase-space variable) yields a gauge dependent equation\(^a\). This problem can be overcome by rewriting the equation in terms of the (gauge invariant) kinetic momentum \( p = k - gA^aQ_a \), as this is the variable associated to the particle velocity \( v = p/m \). Then the equation reads

\[ v^\mu \left( D^\mu - gQ_a F_{\mu\nu}^{\alpha} \frac{\partial}{\partial p^\nu} \right) f(x, p, Q) = 0, \quad (2) \]

where \( v^\mu = (1, v) \), \( D^\mu = \partial^\mu - gf^{abc} A^b_\mu Q^c \partial/\partial Q^a \), and \( p_0 = p^2/2m \) for a non-relativistic particle.

A Hamiltonian formulation of the dynamics is very useful. In particular, it allows one to recognize constants of motion, and thus getting exact solutions of the collisionless transport equation. If the system is invariant under space translations in the direction \( n \), then it is easy to prove that \( n \cdot k \) is a constant of motion. While this is not a gauge invariant quantity, it is in the type of gauges that are respectful with the translation invariance of the system, \( n^i \partial_i A^{\mu}_a = 0 \). Thus, an exact solution to (2) is written as \( f(n \cdot (p + gA_a Q^a)) \), for an arbitrary function \( f \), which should be fixed with a boundary condition.

A generalization of the approach for relativistic systems is straightforward. All what changes in the final equations is the relativistic, rather than non-relativistic, dispersion relation. Using this philosophy exact solutions to the relativistic transport equations in static systems were found in Ref.\(^1\). The imposed boundary condition was that \( f \) should reduce to the Fermi-Dirac or Bose-Einstein equilibrium functions when \( g \to 0 \), which then fixes the form of \( f \). One can also prove that these distribution functions describe equilibrium solutions in the presence of background fields (that is, with no entropy production). The colored current in the plasma is written in terms of the external fields, and one can then compute the associated effective action. When compared to the effective action of a system in the background of static fields as computed in quantum field theory, one notices that it matches for the low dimensional operators (those proportional to the quadratic and cubic Casimirs), but there is a numerical discrepancy in the coefficient of the dimension 4 operator, unless the particles are in a high dimensional color representation. It is possible to identify the reason of this

\(^a\)The same problem reappears in quantum field theory derivations of transport equations!
discrepancy in the classical color algebra, and the fact that the $Q$ charges are commuting objects. Since quarks and gluons are in low dimensional representations, a quantum treatment of color then seems mandatory for QCD, at least to reach agreement between the two theories for all the static operators.

3. Transport Equations for Quantum Color

A (first) quantized treatment of the system described in (1) amounts to replace c-numbers by operators in a Hilbert space. For a particle with color charge in a representation $R$ of $SU(N_c)$, the color charge $Q_a$ is replaced by $T_a$, where $T_a$ is a matrix, a generator of $SU(N_c)$ in the representation $R$. Poisson brackets are replaced by commutators. In a Heisenberg representation, one can define the operators of position, momentum and color charge, and derive their equations of motion, according to the Hamiltonian dynamics. These equations reduce to the classical Wong equations in the limit where all operators commute.

The statistical treatment of the system starts with the Wigner operator, defined as a certain Fourier transform of the density matrix. The Wigner operator is also a matrix in color space, whose dimension depends on the representation $R$. In a collisionless situation, its dynamical evolution is governed by the Hamiltonian. In the limit where spatial derivatives of higher order can be neglected (that is, performing a gradient expansion), it reduces to

$$ [v^\mu D_\mu, W(p, x)] - \frac{g}{2} v^\mu \left\{ F_{\mu\nu}(x), \frac{\partial W(p, x)}{\partial p_\nu} \right\} = 0 , \quad (3) $$

where $D_\mu = \partial_\mu - igA_\mu^a T^a$. The equation is the same for non-relativistic and relativistic systems, all what changes is the dispersion relation. These transport equations were first derived from QCD in a gradient expansion in Refs. 7,8, while here they have been obtained from a simple quasiparticle model.

In Ref. 2 exact solutions to (3) were found for systems with a translation invariance in the direction $n^\mu$. One could expect that the form of the solutions to be $W(p, x) = f(n \cdot (p + gA^a T^a))$. However, this is only the case when $[D(n \cdot A), n \cdot A] = 0$. If not, gradient (and non-local) terms correct this expression. For static systems, this implies that an equilibrium solution in the presence of background fields is only possible if those are in the direction of the Cartan subalgebra of $SU(N_c)$.

One can compare the results arising from a quasiparticle model, made
up of quarks, antiquarks and gluons, and those of QCD. One finds a perfect matching in the static limit with constant background fields\textsuperscript{9}. A discrepancy is found when compared with dimensional reduced effective theories\textsuperscript{10}, if the vector gauge fields do not commute. There is also a subtlety concerning the gluons. The transport equation for the hard gluons needs an infrared cutoff, so as to avoid a double counting of the soft modes (as quasiparticles, as soft classical fields). This cutoff is also needed in order to avoid that the distribution function of the hard gluons becomes negative, loosing its probabilistic interpretation. A matching procedure with the soft classical field modes allows one to eliminate the cutoff dependence in all final physical quantities.

4. Conclusions

We have pointed out where the classical and quantum color quasiparticle models agree and disagree. While we have not discussed the structure of the collision terms, a similar discrepancy beyond leading order in $g$ might be expected there.

I would like to emphasize that it is worthwhile to explore these quasiparticle models, not only for the reasons mentioned in the introduction, but also because they might teach us a lot about fluctuations around global and local equilibrium in a quark-gluon plasma.

References

1. M. Laine and C. Manuel, Phys. Rev. D \textbf{65}, 077902 (2002).
2. C. Manuel and S. Mrowczynski, arXiv:hep-ph/0206209, to be published in Phys. Rev. D.
3. D. Bödeker and M. Laine, JHEP \textbf{0109}, 029 (2001).
4. U. Heinz, Phys. Rev. Lett. \textbf{51}, 351 (1983); Ann. Phys. (N.Y.) \textbf{161}, 48 (1985).
5. P. F. Kelly, Q. Liu, C. Lucchesi, and C. Manuel, Phys. Rev. Lett. \textbf{72}, 3461 (1994); Phys. Rev. D \textbf{50}, 4209 (1994).
6. S.K. Wong, Nuovo Cim. \textbf{A65}, 689 (1970).
7. H. T. Elze, M. Gyulassy, and D. Vasak, Phys. Lett. B \textbf{177}, 402 (1986).
8. H. T. Elze, Z. Phys. C \textbf{38}, 211 (1988); St. Mrówecki, Phys. Rev. D \textbf{39}, 1940 (1989).
9. D. J. Gross, R. D. Pisarski and L. G. Yaffe, Rev. Mod. Phys. \textbf{53}, 43 (1981).
10. A. Hart, M. Laine, and O. Philipsen, Nucl. Phys. B \textbf{586}, 443 (2000).