A New Statistic for the Detection of Long Strings in Microwave Background Maps.

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Abstract

A new statistic is briefly reviewed, designed to detect isolated coherent step-like discontinuities produced by cosmic strings present at late times. As a background I superpose a scale invariant Gaussian random field which could have been produced by a superposition of seeds on all scales and/or by inhomogeneous quantum fluctuations. The statistical variable considered is the Sample Mean Difference (SMD) between large neighbouring sectors of CMB maps, separated by lines in two dimensional maps and points in one dimensional maps. I find that the SMD statistics can detect at the 1 level the presence of a long string with $G(v_{s}) = \frac{1}{8} (\frac{T}{T_{m,s}})^{2} \cdot 0.5 \cdot 10^{7}$ while more conventional statistics like the skewness or the kurtosis require a value of $G$ almost an order of magnitude larger for detectability at a comparable level.

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1 Introduction

The purpose of this talk is to briefly review a new statistic [1] which is optimized to detect the large scale non-Gaussian coherence induced by late long strings on CMB maps. The statistical variable to use is the Sample Mean Difference that is the difference of the mean $\frac{1}{T}$ between two large neighbouring regions of CMB maps. I will first discuss the main predictions of models for CMB fluctuations. Models based on inflation predict generically the existence of scale invariant CMB fluctuations with Gaussian statistics which emerge as a superposition of plane waves with random phases. On the other hand, models based on defects (for pedagogic reviews see e.g. [2, 3]), CMB fluctuations are produced by a superposition of seeds and are scale invariant [4, 5] but non-Gaussian [6, 7, 8, 9, 10]. Observations have indicated that the spectrum of fluctuations is scale invariant [11] on scales larger than about $2^2$, the recombination scale, while there seem to be Doppler peaks on smaller scales. These results are consistent with predictions of both inflation [12] and defect models ([2, 3] and references therein) even though there has been some debate about the model dependence of Doppler peaks in the case of defects (see the contributions of N. Turok and of R. Durrer in these proceedings).

Inflation also predicts Gaussian statistics in CMB maps for both large and small scales and this is in agreement with Gaussianity tests made on large scale data so far. On small angular scale maps where the number of superposed seeds per pixel is small, topological defect models predict non-Gaussian statistics. This non-Gaussianity however depends sensitively on both, the details of the defect network [13, 14] at the time of recombination $t_{\text{rec}}$ and on the physical processes taking place at $t_{\text{rec}}$. On the other hand, the large number of superposed seeds on large angular scales leads by the Central Limit Theorem to Gaussian statistics for angular resolution larger than $2^2$. This statement however, ignores the large scale coherence induced by large scale seeds present at late times. This large scale coherence can induce specific non-Gaussian features even on large angular scales. The question of how Gaussian are the topological defect fluctuations on large angular scales will be the focus of this talk.

The reason that the defect-induced fluctuations appear Gaussian in maps with large resolution angle is the large number of seeds superposed on each pixel of the map. This, by the Central Limit Theorem, leads to a Gaussian probability distribution for the fluctuations $\frac{1}{T}$. Non-Gaussianity can man-
Ifest itself on small angular scales comparable to the minimum correlation length between the seeds.

These arguments have led most efforts for the detection of defect induced non-Gaussianity towards CMB maps with resolution angle less than 1° [3, 14, 4]. There is however a loophole in these arguments. They ignore the large scale coherence induced by the latest seeds. Such large scale seeds must exist due to the scale invariance and they induce certain types of large scale coherence in CMB maps. This coherence manifests itself as a special type of non-Gaussianity which can be picked up only by specially optimized statistical tests. Thus a defect induced CMB fluctuation pattern can be decomposed into two parts. A Gaussian contribution \( \frac{T}{T_g} \) produced mainly by the superposition of seeds on small scales and possibly by in stationary fluctuations and a coherent contribution \( \frac{T}{T_c} \) induced by the latest seeds.

The question that we want to address is: What is the minimum ratio \( \frac{T}{T_g} \) of the last seed contribution on \( \frac{T}{T} \) over the corresponding Gaussian contribution that is detectable at the 1σ to 2σ level.

## 2 Cosmic Strings

I will focus on the case of cosmic strings. In this case the contribution of the latest long string comes in the form of a step-like discontinuity [13, 14] coherent on large angular scales. As a toy model we may first consider a one dimensional pixel array of standardized, scale invariant Gaussian fluctuations with a superposed temperature discontinuity of amplitude 2° [1].

A statistical variable designed to pick up the presence of this step is the Sample Mean Difference (SMD) \( Y_k \) which assigns to each pixel of the map the difference of the mean of pixels \( 1: k \) minus the mean \( \frac{T}{T} \) of pixels \( k+1:n \). It is straightforward to show [3] that

\[
Y_k = X_k + 2 \frac{n}{n-k} \frac{i_0}{k} \quad k \geq 1, i_0 
\]

\[
Y_k = X_k + 2 \frac{i_0}{k} \quad k \geq 2, i_0, n 
\]

where \( k \) labels the \( k^{th} \) out of the \( n \) random variables of the pixel map and \( i_0 \) is the location of the superposed coherent discontinuity. \( X_k \) is the SMD
of the underlying scale invariant Gaussian background. The SM D average statistic \( Z \) is defined as the average of \( Y_k \) over all \( k \)

\[
Z = \frac{1}{n} \sum_{k=1}^{n} Y_k
\]  

(3)

It is straightforward to show that the mean \( Z \) over many realizations and locations of the step function is \( Z = 1 \) and the variance of \( Z \) depends both on the number of pixels \( n \) and on the step function amplitude

\[
\frac{\sigma^2}{n} = \frac{2 \ln n}{n} + \frac{1}{3}
\]  

(4)

The condition for detectability of the coherent step discontinuity at 1 level is that \( Z \) is larger than the standard deviation of \( Z \) which implies that \( Z > 0.2 \) for \( n > 0 \left(10^3\right) \) where \( \sigma \) is measured in units of the standard deviation of the underlying Gaussian map. It is straightforward to apply a similar analysis for the more conventional statistics skewness and kurtosis. That analysis [1] shows that the minimum value of detectable at the 1 level is about an order of magnitude larger. It is therefore clear that SM D statistical variable is particularly effective in detecting coherent step-like discontinuities superposed on Gaussian CMB maps.

A detailed understanding of the effectiveness of the SM D statistic requires the use of Monte Carlo simulations. In order to verify the analytical results for the mean and variance of the SM D variable I first applied this statistic on one-dimensional Monte Carlo maps of scale invariant Gaussian fluctuations with step function superposed. The results were in good agreement with the analytical predictions shown above and are described in detail in [1]. Here I will only discuss the two-dimensional Monte Carlo simulations.

Figures 1 and 2 show 30 x 30 pixel maps of standardised Gaussian scale invariant fluctuations without (Fig. 1) and with (Fig. 2) a coherent step function superposed. The amplitude of the superposed coherent seed is \( = 0.5 \).

Uncorrelated noise has also been included with noise to signal ratio of 0.5. The scale invariant background map \( X(i,j) \) was constructed in the usual way by taking its Fourier transform \( g(k_1,k_2) \) to be a Gaussian complex random variable. Its phase was taken to be random with a uniform distribution and its magnitude was a Gaussian random variable with 0 mean and variance equal
**Figure 1:** A standardized two dimensional pixel array of scale invariant Gaussian fluctuations. No step function has been superposed.

**Figure 2:** The two dimensional array of Figure 2 with a superposed coherent step-discontinuity of amplitude $a = 0.5$ defined by the random points $(x_1, y_1) = (13.5; 18.1)$ and $(x_2, y_2) = (9.4; 20.4)$
to a scale invariant power spectrum. The SMD was obtained by randomly dividing the map in two sectors and taking the difference of the means of the two sectors. The SMD average was then obtained by averaging over many randomly chosen divisions for each map realization. Using 50 such map realizations I obtained the mean and the standard deviation of the statistics skewness and SMD average for several values of . The results are shown on Table 1 and indicate that the statistics skewness and kurtosis can not identify a coherent discontinuity of amplitude \( < 1 \) but would require a much larger amplitude for such identification.

Table 1: A comparison of the effectiveness of the statistics considered in detecting the presence of a coherent step discontinuity with amplitude 2 relative to the standard deviation of the underlying Gaussian map. The SMD average was obtained after ignoring 150 pixels on each boundary of the Monte Carlo maps. The discontinuities were also excluded from these 300 pixels. This significantly improved the sensitivity of the SMD test.

| Skewness | Kurtosis | SM D Average |
|----------|----------|--------------|
| 0.00     | 0.01 0.10| 2.96 0.15    |
| 0.25     | 0.01 0.09| 2.95 0.15    |
| 0.50     | 0.02 0.14| 2.94 0.18    |
| 1.00     | 0.03 0.30| 2.78 0.30    |

On the other hand the SMD statistic can identify a coherent discontinuity at the 1 to 2 level with \( \sigma = 0.5 \). For \( \frac{i}{T_{\text{rms}}} \) \( G v_s s > 4 \times 10^7 \) where \( i \) is the mass per unit length of the string, \( v_s \) is its velocity and \( s \) is the relativistic Lorenz factor.

The main points I wanted to stress in this talk are the following:

The detection of non-Gaussianity induced by cosmic strings on large angular scales is possible.

For this a special statistic is needed optimized to pick up the large scale structure of the latest seeds.

Such a statistic is the average of the Sample Mean Difference which can pick up non-Gaussian features of long strings with \( G v_s s > 4 \times 10^7 \).
in maps with about $10^3$ pixels and with a noise to signal ratio of about 0.5 in a Gaussian background with $\frac{T}{T_{ms}} \sim 10^5$.

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