Driving a mechanical resonator into coherent states via random measurements

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Abstract

We propose dynamical schemes to engineer coherent states of a mechanical resonator (MR) coupled to an ancillary, superconducting flux qubit. The flux qubit, when repeatedly projected on to its ground state, drives the MR into a coherent state in probabilistic, albeit heralded fashion. Assuming no operations on the state of the MR during the protocol, coherent states are successfully generated only up to a certain value of the displacement parameter. This restriction can be overcome at the cost of a one-time operation on the initial state of the MR. We discuss the possibility of experimental realization of the presented schemes.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The last decade has been witness to incredible progress in the fabrication of high quality factor mechanical resonators (MRs) in the nano-scale characterized by fundamental frequencies in the 100 MHz–1 GHz range. Control of such systems in the quantum limit has emerged as a very active field of research with implications for ultra-sensitive sensing technologies, quantum information processing as well as fundamental tests of quantum mechanics [1, 2]. A key required feature for applications in the quantum limit is the ability to engineer well-defined quantum states of the MR. The properties of a thermal MR can be manipulated through controlled quantum mechanical coupling with electromagnetic radiation. Indeed, this forms a powerful avenue to cool the mechanical systems, coherently control their evolution and measure them.
Recently, various techniques have been proposed to cool the MRs to their motional ground states, such as bang–bang cooling [3], single-shot state-swapping cooling via a superconductor [4], side-band laser cooling [5–12], electromagnetically-induced-transparency cooling [13], dynamic dissipative cooling [14], and through repeated projective measurements on an auxiliary flux qubit [15]. In this paper, we consider protocols for preparation of pure coherent states, instead of ground states, of MRs from initial general states (e.g., thermal states). Such coherent states can be used as inputs, e.g. for schemes to generate superpositions of spatially separated quantum states (so-called Schrödinger cat states) that can be used to study quantum behaviour, as well as the transition from quantum to classical behaviour, in macroscopically populated (phonon) states of MRs [16]. We note here that a scheme for generating Schrödinger cats in a mechanical system was discussed in [17] in the case of a MR capacitively coupled to a Cooper-pair box, which assumes the availability of an initial coherent state without providing an explicit method of engineering it.

In contrast with the conventional resolved side-band cooling scheme where the MR reaches the steady state, our previous work shows that the MR cooling can be realized by repeated projective measurements on the auxiliary flux qubit [15]. Here, instead of ground-state cooling of MR, we propose a cooling scheme to drive the MR directly into a coherent state by repeated random measurements.

2. The driving protocol: random projective measurements

We consider here a doubly clamped MR coupled to a gradiometer-type superconducting flux qubit [18, 19] (the schemes presented here formally work for coupling to any two level system). The simplest such system can be described by the effective Hamiltonian [20]:

\[ H = H_q + \hbar \omega_m a^\dagger a - \hbar g (a + a^\dagger) \sigma_z. \] (1)

Here \( H_q \) is the free Hamiltonian of the flux qubit, the second term represents the free Hamiltonian \( H_{\text{MR}} \) of the MR with fundamental frequency \( \omega_m \), while the third term describes the interaction between these two systems \( H_{\text{int}} \). In the Pauli spin-1/2 representation, the flux-qubit Hamiltonian in the basis of equal superpositions of persistent currents \( |\uparrow\rangle \pm |\downarrow\rangle \) is described by a bias energy term \( -\sigma_z \), that is set to zero by trapping a half-flux quantum in the superconducting loop [18, 23], and a tunnelling term between the persistent current states, i.e. \( H_q = \frac{\hbar \Delta}{2} \sigma_z \). We shall consider the weak coupling limit, i.e., \( g \ll \omega_m \). In this regime, the rotating wave approximation applies and the system is described by the Jaynes–Cummings (JC) model:

\[ H_{\text{JC}} = H_q + H_{\text{MR}} - \hbar g (a^\dagger a + a^\dagger a^\dagger) \] (2)

where \( \sigma^\pm = (\sigma_x \pm i\sigma_y)/2 \). In a previous paper by some of us [15], based on the system described by the above JC Hamiltonian (2), a feasible scheme for the ground-state cooling of a MR was achieved via projective measurements on an auxiliary flux qubit which interacts with it. For the protocols to generate coherent states, we shall require the MR to be displaced in space by \( \alpha \). To this end, let us assume a flux-qubit–MR model whose original Hamiltonian is given through a displacement transformation \( D(\alpha) = \exp(aa^\dagger - h.c.) \) to the JC Hamiltonian (2) (up to a constant term)

\[ H_{\text{effective}} = H_{\text{JC}} - \alpha \sqrt{2\hbar \omega_m} (a + a^\dagger) - g \alpha \sigma_z. \] (3)

Initially, we assume that the flux-qubit+MR system is prepared in the product state \( |g\rangle \langle g| \otimes \rho_{\text{in}} \), where \( |g\rangle = (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2} \) is the ground state of the free flux-qubit Hamiltonian, while the MR is in some general state with non-zero overlap with the coherent state we want to prepare.
The basic constituent of the protocols we present, for driving the MR to a coherent state, is a quantum-Zeno type effect: the flux-qubit+MR is allowed to undergo dynamic evolution under the Hamiltonian (3) which is interrupted by a series of randomly timed projected measurements on to the ground state of the flux qubit $O = \sigma_0 |g\rangle \langle g| \otimes 1$. We show below that after $N$ consecutive measurements on the qubit yielding outcomes $|g\rangle$, the state of the MR will approach the coherent state $|\alpha\rangle$ with fidelity increasing to 1 as $N$ increases. Notice that the effective evolution operator pertaining to the MR encompassing a single measurement at time $\tau$ can be disentangled to the form

\[ \langle g| e^{-i\hat{H}_{\text{interaction}} \tau} |g\rangle = D(\alpha) V^\dagger_g(\tau) D(\alpha), \]

where from (3) $V^\dagger_g(\tau) \equiv \langle g| e^{-i\hat{H}_{\text{interaction}} \tau / \hbar} |g\rangle$ and furthermore $D(\alpha) = \exp(\alpha (a^\dagger - a))$ is the displacement operator [21]. After one measurement

\[ \mathcal{O}_U(|g\rangle \otimes \rho_m) U^\dagger \mathcal{O}^\dagger = |g\rangle \otimes \rho_m(1) \]

\[ \rho_m^\dagger(1) = D(\alpha) V^\dagger_g(\tau) D(\alpha) \rho_m D(\alpha) V^\dagger_g(\tau) D(\alpha). \]

Thus after $N$ consecutive measurements of the flux-qubit ground state performed at times $\{\tau_1, \tau_2, \ldots, \tau_N\}$, the density matrix of the MR is

\[ \rho_m^N(N) = \frac{D(\alpha) V^\dagger_g(\tau_1) \rho^\text{eff}_m V^\dagger_g(\tau_1) D(\alpha)}{P^\tau_g(N)}, \]

where $V^\dagger_g(\tau) = V^\dagger_g(\tau_1) V^\dagger_g(\tau_2) \ldots V^\dagger_g(\tau_N)$ and $\rho^\text{eff}_m = D^\dagger(\alpha) \rho_m D(\alpha)$. The success probability of this process is

\[ P^\tau_g(N) = \text{Tr} \left( D(\alpha) V^\dagger_g(\tau_1) \rho^\text{eff}_m V^\dagger_g(\tau_1) D(\alpha) \right). \]

We first analyse the properties of the above success probability. The Jaynes–Cummings evolution $\exp(-i\hat{H}_{\text{JC}} \tau / \hbar)$, due to the symmetry associated with conservation of total number of excitations $a^\dagger a + |e\rangle \langle e|$, where $|e\rangle = |\phi\rangle + |\psi\rangle$ is the excited state of the flux qubit, is easily decomposed into two-dimensional sectors, apart from the space of 0 excitations which is one-dimensional [15, 22]. The conditional evolution operator $V^\dagger_g(\tau)$ therefore is diagonal in the phonon basis $|n\rangle$ of the MR (corresponding to $H_{\text{MR}}$):

\[ V^\dagger_g(\tau) = \sum_n \lambda_n |n\rangle \langle n| \]

with complex eigenvalues $\lambda_0 = e^{i\Delta \tau / 2}$ and for $n > 1$, $\lambda_n = e^{-i(n-1/2)\Omega_n \tau} (\cos \Omega_n \tau + i \sin \Omega_n \tau \cos 2\theta_n)$ where $\Omega_n = \sqrt{(\Delta - \alpha_0)^2 + g^2 n}$ is the Rabi frequency. The parameter $\theta_n$ measures the ratio of the effective interaction energy scale to the deviation from resonance, according to the equation: $\tan 2\theta_n = 2g\sqrt{n} / (\Delta - \alpha_0)$. Note that $|\lambda_0| = 1$ is conserved due to effective decoupling of the zero phonon state during the conditional evolution, while for random times $\tau$ the coefficients $|\lambda_n| < 1$ for higher phonon states $n > 1$.

Hence after $N$ consecutive measurements of the flux-qubit ground state, the MR is driven into the state (7) given explicitly as

\[ \rho^\tau_g(N) = \sum_{n,k \geq 0} \tilde{\lambda}_n(\tau) \tilde{\lambda}_k^\dagger(\tau) \frac{|n\rangle \rho^\text{eff}_m |k\rangle D(\alpha) |n\rangle \langle k| D^\dagger(\alpha)}{P^\tau_g(N)}, \]

where $\tilde{\lambda}_n(\tau) = \prod_{j=1}^N \lambda_n(\tau_j)$. The success probability of realizing the conditional evolution (8) is thus:

\[ P^\tau_g(N) = \sum_{n,k \geq 0} \tilde{\lambda}_n(\tau) \tilde{\lambda}_k^\dagger(\tau) \rho^\text{eff}_m |k\rangle \langle k| \text{Tr}(D(\alpha) |n\rangle \langle n| D^\dagger(\alpha)) \]

\[ = \sum_{n \geq 0} |\tilde{\lambda}_n(\tau)|^2 \rho^\text{eff}_m \]

\[ \text{J. Phys. A: Math. Theor. 46 (2013) 485305} \]
where the second line comes about from the cyclic property of the trace and unitarity of the displacement operator $D(\alpha)$ and $\rho_m^{\text{eff}(n)} = |n\rangle \rho_m^{\text{eff}} |n\rangle$ is the diagonal matrix element of the effective input density matrix.

On the other hand, the fidelity of the state of the MR (10) with the coherent state $|\alpha\rangle = D(\alpha)|0\rangle$ is given by:

$$F_g^\tau(N) = \langle 0|D^\dagger(\alpha)\rho_m^{\text{eff}(n)}(N)D(\alpha)\rangle|0\rangle = \sum_{n,k \geq 0} \bar{\lambda}_m(\tau) \bar{\lambda}_k(\tau) \frac{|n\rangle \rho_m^{\text{eff}} |k\rangle \langle k| |0\rangle \langle 0|}{P_g^\tau(N)}$$

$$= \rho_m^{\text{eff}(0)} / P_g^\tau(N)$$

(12)

where in the last line we have used the fact that $|\bar{\lambda}_0(\tau)|^2 = 1$. This result shows that in order to obtain non-zero fidelity with a coherent state $|\alpha\rangle$, the effective input density matrix must have non-zero overlap with the phonon vacuum.

We now turn to the asymptotic characteristics of our method. Due to the properties of the eigenvalues $\lambda_m(\tau)$ mentioned earlier, the products over different times $|\bar{\lambda}_0(\tau)| \to 0$ as the number of measurements $N \to \infty$ for all $n > 0$ and only $|\bar{\lambda}_0(\tau)| = 1$. Thus asymptotically,

$$P_g^\tau(N) = \rho_m^{\text{eff}(0)} = |\alpha\rangle \rho_m |\alpha\rangle.$$  

(13)

Using this, we see from (12) that the fidelity asymptotically tends to 1, which shows that one effectively obtains the coherent state of the MR. Importantly, the results (12), (13) also imply, that by measuring the ground state of the ancillary system, in this case the flux qubit, many times, one basically remotely implements the coherent state projective measurement on the initial state of the MR, which is hard to implement directly. Indeed, (13) shows that the asymptotic success probability of the protocol is given by the probability of projecting the initial state on the target coherent state $|\alpha\rangle$.

The characteristics of the described process depend on the initial state $\rho_m$ of the MR. In what follows, we consider two, rather natural, possible initializations of the MR and compare the performance of the above described protocol.

3. Thermal state of the MR

Consider the initial state of the MR to be the thermal state $\rho_m = \exp(-\beta \hbar \omega_n a^\dagger a) / \text{Tr} \exp(-\beta \hbar \omega_n a^\dagger a) = \sum_n p(n) |n\rangle |n\rangle$, where the probability $p(n) = \bar{n}^n / (1 + \bar{n})^n$ with average phonon excitation at inverse temperature $\beta$ given by the Planck formula $\bar{n} = 1 / \exp(\beta \hbar \omega_m) + 1$ and $|n\rangle$ is the phonon state basis of the undisplaced MR. This can be obtained by allowing the MR to thermalize, while the interaction with the flux qubit ($H_{\text{int}}$) as well as the ‘displacement’ term for the MR, i.e., the second term in (3), is switched off. Then the interaction as well as the displacement term are switched on at time $t = 0$ followed subsequently by measurements of the qubit ground state.

The success probability of $N$ consecutive measurements (11) in this case becomes

$$P_g^\tau(N) = \sum_{n,j \geq 0} \prod_{i=1}^N |\lambda^2_m(\tau_i) p(i) |(iD(\alpha)|n\rangle)^2$$

where the matrix elements of the displacement operator $D^{(i,n)}(\alpha) = \langle i|D(\alpha)|n\rangle$ read

$$D^{(i,n)}(\alpha) = \frac{1}{\sqrt{n!}} |(iD(\alpha)a^\dagger|^n|0\rangle$$

$$= \frac{1}{\sqrt{n!}} |(i(a^\dagger - \alpha)^n|\alpha\rangle$$
probability is simply
\[ \alpha(\tau) = \exp(-|\alpha|^2) \frac{\alpha^{i-m}}{\sqrt{(i-m)!}} \]  

which is simply the displacement of the thermal state of the (undisplaced) MR. The success state \( \alpha \) only for coherent parameters below some critical value \( c \). In this scenario, the input state is the thermal state of the displaced MR, interaction is abruptly switched on followed by implementation of the driving protocol. In this setup where the MR is first displaced, followed by a period of thermalization, after which the corresponding to the displaced MR. Unlike in the previous scenario, we therefore consider a using a scheme based on a different input state. Indeed, consider instead the thermal state \( \rho_0 = \sum |n\rangle \langle n| \). For a given temperature, the restriction on the coherent state parameter can be avoided, decreasing overlap at a given temperature of the phonon number probability distribution \( p(n) \). This stems from the phonon number distribution of the coherent state \( p(n) \) of the thermal state, which is geometric with maximum value for the vacuum, and the phonon excitation value \( \alpha^2 \). Thus to drive the MR into coherent states with higher average phonon numbers will require higher temperatures.

\[ P_g^{(1)}(N) = \exp(-|\alpha|^2) \sum_{n} \prod_{i=1}^{N} \left[ \frac{\alpha^2}{2} \right] \sum_{m=0}^{\min(n,i)} \frac{(-1)^m \alpha^{n+i-2m}}{m! (n-m)! (i-m)!} ^2 \]  

On the other hand, the fidelity (12) reads
\[ F_g^{(1)}(N) = \sum_i p(i) p^\alpha(i)/P_g^{(1)}(N) \]  

where \( p^\alpha(i) = \frac{\exp(-|\alpha|^2)/\alpha^2}{i} \) is the phonon number probability distribution in a coherent state \( \alpha \).

We show below that, for this class of states, the MR can be driven into a coherent state only for coherent parameters below some critical value \( c \), which depends on the temperature or equivalently the average phonon number of the thermal MR. Indeed, this stems from the decreasing overlap at a given temperature of the phonon number probability distribution \( p(i) \) of the thermal state, which is geometric with maximum value for the vacuum, and the phonon number distribution of the coherent state \( p^\alpha(i) \) which is Poissonian and centred at the average phonon excitation value \( \alpha^2 \). Thus to drive the MR into coherent states with higher average phonon numbers will require higher temperatures.

4. Displaced thermal states of the MR

For a given temperature, the restriction on the coherent state parameter can be avoided, using a scheme based on a different input state. Indeed, consider instead the thermal state corresponding to the displaced MR. Unlike in the previous scenario, we therefore consider a setup where the MR is first displaced, followed by a period of thermalization, after which the interaction is abruptly switched on followed by implementation of the driving protocol. In this scenario, the input state is the thermal state of the displaced MR, \( \rho_m^{\text{New}} = \exp[-\beta / \hbar \omega_0 ((\alpha a^\dagger a - \alpha (a + a^\dagger)))/Z = D(\alpha) \rho_m D^\dagger(\alpha) \), where \( Z = Tr[\exp[-\beta / \hbar \omega_0 ((\alpha a^\dagger a - \alpha (a + a^\dagger)))] \) and \( \rho_m \) is the thermal state of the undisplaced MR, as given in the previous section.

Under this assumption, the new density operator matrix after \( N \) consecutive successful measurements is
\[ \rho^{(1)}(N) = \sum |g\rangle \langle g| \otimes D(\alpha) V_g^{N}(\tau) \rho_m V_g^{N}(\tau) D^\dagger(\alpha) \]  

which is simply the displacement of the thermal state of the (undisplaced) MR. The success probability is simply
which is independent of the coherent state parameter $\alpha$ and asymptotically (13) is $\langle 0 | \rho_m | 0 \rangle$, i.e. the occupation probability of the vacuum state of the bare MR at the given temperature.

5. Numerical results

We now study quantitative characteristics of the two presented driving protocols numerically. We consider a realistic $2\pi \times 100$ MHz nano-MR with quality factor $Q_m = 10^5$, coupled near-resonantly to a flux qubit non-resonantly with a tunnelling splitting $\Delta_1 = 1.1 \omega_m$. We first consider the scheme described in section 3. The MR is taken to be in equilibrium at the ambient temperature $T = 20$ mK, which corresponds to a mean phonon number of 3.69. The coupling constant is assumed $g = 0.04 \omega_m$ and the measurement times $\tau_j$ are randomly selected. Figure 1 shows the evolution of the fidelity (12) with increasing number of projective measurements for different values of $\alpha$ up to $\alpha \leq \alpha_{\text{max}} = 5.1$ above which the asymptotic success probability $\leq 8 \times 10^{-4}$. The fidelity approaches the asymptotic value of 1 for all $\alpha$s within 30 measurement steps. In these cases, the MR is driven close to the coherent state $|\alpha\rangle$ as can be seen also by considering the average phonon number $\langle n \rangle = \delta n + |\alpha|^2$ which is the sum of thermal-like and coherent-like contributions. The thermal-like contribution $\delta n$ can be calculated by subtracting the coherent like contribution $|\alpha|^2$ from the average phonon number $\langle n \rangle$ in the calculated final state (10). The thermal-like contribution after 30 measurements is $\delta n = 2 \times 10^{-2}$. In figure 2, we consider the variation of the success probability of obtaining $N$ consecutive ground-state measurements of the flux qubit. For $\alpha = 2$ (red curve), this probability changes very slowly after 20 measurements showing that the coherent state has practically been achieved. On the other hand, as we approach the value $\alpha = 5.1$ the survival probability becomes very small, i.e., we practically no longer can obtain the flux qubit in its ground state. Indeed for $\alpha = 5$ (green curve) the survival probability saturates to a very small value within ten measurements. From the results above, the asymptotic probability of success (with number of measurements
Figure 2. The success probability $P^{(1)}_g(N)$ for different values of $\alpha$. Red triangles denote the case $\alpha = 2$. Green triangles denote the case $\alpha = 5$. The controllable parameters are the same as in figure 1.

Figure 3. Asymptotic value of the success probability depending on the displacement $\alpha$ that is sought to be obtained.

$N \rightarrow \infty$ is seen to be a measure of obtaining a coherent state with fidelity 1. It decreases to zero with increase of the coherent displacement parameter $\alpha$ as shown in figure 3.

We now consider the scheme described in section 4. Figure 4 presents the same physical quantities as figure 1 and figure 2 for a thermal state of the physically displaced MR. We consider a higher bath temperature $T = 40$ mK with mean phonon number 7.84. After 30 measurements, the thermal-like contribution is $\delta n \simeq 10^{-3}$, thus, the average phonon number is $\langle n \rangle \simeq |\alpha|^2$. It is noticeable that in this framework the coherent state is always reached after 30 measurements. As mentioned in the previous section, the characteristics of the protocol including the fidelity with the coherent state (see (18)) are independent of the displacement parameter.
Figure 4. (a) Survival probability after $N$ measurements. (b) Fidelity. The parameters are $T = 40$ mK and $\tau = 8/\omega$.

6. Conclusions

We would like to remark that though the results for the protocols of sections 3 and 4 are obtained for the case of randomly timed measurements, they are equally valid if one performs equal or fixed time measurements. Exact timing of measurements is however difficult to implement experimentally and indeed unfavourable. As shown in [15], randomly timed measurements result in faster dynamics towards asymptotic states of the MR.

In conclusion, we have demonstrated that quantum-Zeno effect-type schemes consisting of repeated measurements on an auxiliary two level system can drive a nano-MR into a coherent state. We propose two different ways to realize a coherent state: the first one is efficient for small values of $\alpha$, while the second one, with a displaced initial state of the MR, is robust for any value of the coherent parameter. The schemes work when the flux qubit and MR are coupled both resonantly or off-resonantly. We showed that the driving process is akin to performing a projective measurement on the MR without directly acting on it.

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