Exclusive scalar $f_0(1500)$ meson production for energy ranges available at the GSI Facility for Antiproton and Ion Research (GSI-FAIR) and at the Japan Proton Accelerator Research Complex (J-PARC)

A. Szczurek$^{1,2,*}$ and P. Lebiedowicz$^{1,2,†}$

$^1$Institute of Nuclear Physics PAN, PL-31-342 Cracow, Poland
$^2$University of Rzeszów, PL-35-959 Rzeszów, Poland

(Dated: June 1, 2009)

Abstract

We evaluate differential distributions for exclusive scalar $f_0(1500)$ meson (glueball candidate) production for $p\bar{p} \to N_1 N_2 f_0$ (FAIR@GSI) and $pp \to pp f_0$ (J-PARC@Tokai). Both QCD diffractive, pion-pion meson exchange current (MEC) components as well as double-diffractive mechanism with intermediate pionic loop are calculated for the first time in the literature. The pion-pion component, which can be reliably calculated, dominates close to the threshold while the diffractive component may take over only for larger energies. At the moment only upper limit for the QCD-diffractive component can be obtained. The diffractive component is calculated based on two-gluon impact factors as well as in the framework of Khoze-Martin-Ryskin approach proposed for diffractive Higgs boson production. Different unintegrated gluon distribution functions (UGDFs) from the literature are used. Rather large cross sections due to pion-pion fusion are predicted for PANDA energies, where the gluonic mechanism is shown to be negligible. The production of $f_0(1500)$ close to threshold could limit the so-called $\pi NN$ form factor in the region of larger pion virtualities. We discuss in detail the two-pion background to the production of the $f_0(1500)$ meson.

PACS numbers: 12.38.-t, 12.39.Mk, 14.40.Cs

Keywords: exclusive production, pion-pion fusion, diffractive mechanisms, differential cross sections

$^*$Electronic address: antoni.szczurek@ifj.edu.pl
$^†$Electronic address: piotr.lebiedowicz@ifj.edu.pl
I. INTRODUCTION

Many theoretical calculations, including lattice QCD, predicted existence of glueballs (particles dominantly made of gluons) with masses $M > 1.5$ GeV. No one of them was up to now unambiguously identified. The nature of scalar mesons below 2 GeV is also not well understood. Lattice QCD approach with quenched quarks find a scalar gluonium (glueball) at approximately 1.6 GeV \cite{1}. Also the analyses in the framework of chiral Lagrangians \cite{3, 4} indicate that $f_0(1500)$ is dominantly gluonium state. The QCD Sum Rules \cite{5, 6, 7, 8, 9, 10} suggest that the states at approximately 1 GeV and 1.5-1.6 GeV are admixtures of gluonium and $q\bar{q}$ states. A recent analysis in the framework of Gaussian QCD Sum Rules \cite{11}, which is well suited to $q\bar{q}$ - gluonium mixing, find that the states at about 1 GeV ($f_0(980)$) and at about 1.4 GeV are strongly mixed with the preference of the higher-mass state to have slightly larger gluonium admixture. Summarizing this discussion, it may be very difficult to find a clear signal of gluonium. Further studies of the scalar meson production in several processes may shed more light on the quite complicated problem.

The lowest mass meson considered as a glueball candidate is a scalar $f_0(1500)$ \cite{12} discovered by the Crystall Barrel Collaboration in proton-antiproton annihilation \cite{13}. The branching fractions are consistent with the dominant glueball component \cite{14}. It was next observed by the WA102 Collaboration in central production in proton-proton collisions in two-pion \cite{15} and four-pion \cite{16} decay channels at $\sqrt{s} \approx 30$ GeV \cite{2}. Close and Kirk \cite{17} proposed a phenomenological model of central exclusive $f_0(1500)$ production. In their language the pomerons (transverse and longitudinal) are the effective (phenomenological) degrees of freedom \cite{18}. The Close-Kirk amplitude was parameterized as \cite{3}

$$\mathcal{M}(t_1, t_2, \phi') = a_T \exp \left( \frac{b_T}{2} (t_1 + t_2) \right) + a_L \sqrt{t_1 t_2} \exp \left( \frac{b_L}{2} (t_1 + t_2) \right) \cos(\phi') . \quad (1.1)$$

In their approach there is no explicit $f_0(1500)$-rapidity dependence of the corresponding amplitude. Since the parameters were rather fitted to the not-normalized WA102 data \cite{15} no absolute normalization can be obtained within this approach. Furthermore the parameterization is not giving energy dependence of the cross section, so predictions for other (not-measured) energies are not possible. In the present paper we will investigate rather a QCD-inspired approach. It provides absolute normalization \cite{4}, energy dependence and dependence on meson rapidity (or equivalently on $x_F$ of the meson).

The nature of the $f_0(1500)$ meson still remains rather unclear. New large-scale devices being completed (J-PARC at Tokai) or planned in the future (FAIR at GSI) may open a new possibility to study the production of $f_0(1500)$ in more details.

In the present analysis we shall concentrate on exclusive production of scalar $f_0(1500)$ in

\footnote{The approaches with dynamical quarks find relatively large mixing with $q\bar{q}$ states \cite{2}.}
\footnote{No absolute normalization of the corresponding experimental cross section was available. Only two-pion or four-pion invariant mass spectra were discussed.}
\footnote{The $\phi'$ dependence applies in the meson rest frame (current-current c.m.).}
\footnote{As will be discussed later it is rather upper limit which can be easily obtained.}
the following reactions:

\[
p + p \rightarrow p + f_0(1500) + p , \\
p + \bar{p} \rightarrow p + f_0(1500) + \bar{p} , \\
p + \bar{p} \rightarrow n + f_0(1500) + \bar{n} .
\] (1.2)

While the first process can be measured at J-PARC, the latter two reactions could be measured by the PANDA Collaboration at the new complex FAIR planned in GSI Darmstadt. The combination of these processes could shed more light on the mechanism of \(f_0(1500)\) production as well as on its nature.

If \(f_0(1500)\) is a glueball (or has a strong glueball component \[19\]) then the mechanism shown in Fig. 1 may be important, at least in the high-energy regime. This mechanism is often considered as the dominant mechanism of exclusive Higgs boson \[20\] and \(\chi_c(0^+)\) meson \[21\] production at high energies. There is a hope to measure these processes at LHC in some future when forward detectors will be completed. At intermediate energies the same mechanism is, however, not able to explain large cross section for exclusive \(\eta'\) production \[22\] as measured by the WA102 Collaboration. Explanation of this fact is not clear to us in the moment.

At lower energies (\(\sqrt{s} < 20\) GeV) other processes may become important as well. Since the two-pion channel is one of the dominant decay channels of \(f_0(1500)\) (34.9 ± 2.3 %) \[23\] one may expect the two-pion fusion (see Fig. 2) to be one of the dominant mechanisms of exclusive \(f_0(1500)\) production at the FAIR energies. The two-pion fusion can be also relatively reliably calculated in the framework of meson exchange theory. The pion coupling to the nucleon is well known \[24\]. The \(\pi NN\) form factor for larger pion virtualities is somewhat less known. This may limit our predictions close to the threshold, where rather large virtualities are involved due to specific kinematics. At largest HESR (antiproton ring) energy, as will be discussed in the present paper, this is no longer a limiting factor as average pion virtualities are rather small.

In this paper we concentrate on the mechanism of the reaction. Our aim here is to explore a possibility of studying exclusive \(f_0(1500)\) meson production in the FAIR and J-
PARC energy range and explore the potential of these facilities. While their are some ideas about the reaction mechanism at higher energies, the mechanism at lower energies was never studied. We shall investigate new mechanisms of pion-pion fusion shown in Fig. 2, the QCD mechanism shown in Fig. 1 and a mechanism with intermediate pionic loop shown in Fig. 3. The second (QCD) mechanism is typical for high energies but here we wish to investigate its role at intermediate energies and in particular its vanishing at low energies and the interplay with the pion-pion fusion mechanism.

FIG. 2: The sketch of the pion-pion MEC mechanism. Form factors appearing in different vertices and kinematical variables are shown explicitly.

FIG. 3: The sketch of the double-diffractive mechanism with pionic loop for exclusive production of the glueball candidate $f_0(1500)$. The stars attached to $\pi$ mesons denote the fact that they are off-mass-shell.
II. EXCLUSIVE PROCESSES

A. Cross section and phase space

The cross section for a general 3-body reaction $pp \rightarrow ppf_0(1500)$ can be written as

$$d\sigma_{pp \rightarrow ppM} = \frac{1}{2\sqrt{s(s-4m^2)}}|\mathcal{M}|^2 \cdot d^3PS.$$  \hspace{1cm} (2.1)

Above $m$ is the mass of the nucleon.

The three-body phase space volume element reads

$$d^3PS = \frac{d^3p'_1}{2E'_1(2\pi)^3} \frac{d^3p'_2}{2E'_2(2\pi)^3} \frac{d^3P_M}{2E_M(2\pi)^3} \cdot (2\pi)^4 \delta^4(p_1 + p_2 - p'_1 - p'_2 - P_M).$$  \hspace{1cm} (2.2)

At high energies and small momentum transfers the phase space volume element can be written as \cite{25}

$$d^3PS \approx \frac{1}{2\pi^4} dt_1 dt_2 d\xi_1 d\xi_2 d\phi \left| s(1 - \xi_1)(1 - \xi_2) - M^2 \right|,$$  \hspace{1cm} (2.3)

where $\xi_1, \xi_2$ are longitudinal momentum fractions carried by outgoing protons with respect to their parent protons and the relative angle between outgoing protons $\phi \in (0, 2\pi)$. Changing variables $(\xi_1, \xi_2) \rightarrow (x_F, M^2)$ one gets

$$d^3PS \approx \frac{1}{2\pi^4} \frac{dx_F}{s \sqrt{x_F^2 + 4(M^2 + |P_{M,t}|^2)/s}} d\phi.$$  \hspace{1cm} (2.4)

The high-energy formulas (2.3) and (2.4) break close to the meson production threshold. Then exact phase space formula (2.2) must be taken and another choice of variables is more appropriate. We choose transverse momenta of the outgoing nucleons ($p'_{1t}, p'_{2t}$), azimuthal angle between outgoing nucleons ($\phi$) and rapidity of the meson ($y$) as independent kinematically complete variables. Then the cross section can be calculated as:

$$d\sigma = \sum_k J^{-1}(p_{1t},p_{2t},\phi,y) \left| \frac{\mathcal{M}(p'_{1t},p'_{2t},\phi,y)^2}{2\sqrt{s(s-4m^2)}} \right| \frac{2\pi}{(2\pi)^5} \frac{1}{2E'_1} \frac{1}{2E'_2} \frac{1}{2} p_{1t} p_{2t} dp_{1t} dp_{2t} d\phi dy,$$  \hspace{1cm} (2.5)

where $k$ denotes symbolically discrete solutions of the set of equations for $p'_{1z}$ and $p'_{2z}$:

$$\begin{cases} \sqrt{s} - E_M = \sqrt{m^2_{1z} + p^2_{1z}} + \sqrt{m^2_{2z} + p^2_{2z}}, \\ -P_{Mz} = p'_{1z} + p'_{2z}, \end{cases}$$  \hspace{1cm} (2.6)

where $m_{1t}$ and $m_{2t}$ are transverse masses of outgoing nucleons. The solutions of Eq. (2.6) depend on the values of integration variables: $p'_{1z} = p'_{1z}(p'_{1t},p'_{2t},\phi,y)$ and $p'_{2z} = p'_{2z}(p'_{1t},p'_{2t},\phi,y)$. The extra Jacobian reads:

$$J_k = \left| \frac{p'_{1z}(k)}{\sqrt{m^2_{1t} + p^2_{1z}(k)^2}} - \frac{p'_{2z}(k)}{\sqrt{m^2_{2t} + p^2_{2z}(k)^2}} \right|.$$  \hspace{1cm} (2.7)

In the limit of high energies and central production, i.e. $p'_{1z} \gg 0$ (very forward nucleon1), $-p'_{2z} \gg 0$ (very backward nucleon2) the Jacobian becomes a constant $J \rightarrow \frac{1}{2}$.

The matrix element depends on the process and is a function of kinematical variables. The mechanism of the exclusive production of $f_0(1500)$ close to the threshold is not known. We shall address this issue here. Therefore different mechanisms will be considered and the corresponding cross sections will be calculated.
B. Diffractive QCD amplitude

According to Khoze-Martin-Ryskin approach (KMR)\(^\text{[20]}\), the amplitude of exclusive double diffractive colour singlet production \(pp \rightarrow ppf_0(1500)\) can be written as

\[
\mathcal{M}^{gg^*} = \frac{s}{2} \pi \frac{\delta_{cic}}{2N_c^2 - 1} \int d^2q_{0,t} V_{f_0}^{cic} f_{g,1}^{0ff}(x_1, x'_1, q_{0,t}^2, q_{1,t}^2, t_1) f_{g,2}^{0ff}(x_2, x'_2, q_{0,t}^2, q_{2,t}^2, t_2) \frac{1}{q_{0,t}^2 q_{1,t}^2 q_{2,t}^2} \tag{2.8}
\]

The normalization of this amplitude differs from the KMR one\(^\text{[20]}\) by the factor \(s/2\) and coincides with the normalization in our previous work on exclusive \(\eta^\prime\)-production\(^\text{[22]}\). The amplitude is averaged over the colour indices and over two transverse polarisations of the incoming gluons\(^\text{[20]}\). The bare amplitude above is subjected to absorption corrections which depend on collision energy (the bigger the energy, the bigger the absorption corrections). We shall discuss this issue shortly when presenting our results.

The vertex factor \(V_{f_0}^{cic}\) in expression (2.8) describes the coupling of two virtual gluons to \(f_0(1500)\) meson. Recently the vertex was obtained for off-shell values of \(q_{1,t}\) and \(q_{2,t}\) in the case of \(\chi_{c}(0)\) exclusive production\(^\text{[21]}\). An almost alternative way to describe the vertex is to express it via partial decay width \(\Gamma(M \rightarrow gg)\).\(^5\) The latter (approximate) method can be used also for the \(f_0(1500)\) meson production.

In the original Khoze-Martin-Ryskin (KMR) approach\(^\text{[20]}\) the amplitude is written as

\[
\mathcal{M} = N \int \frac{d^2q_{0,t}}{q_{0,t}^2 q_{1,t}^2 q_{2,t}^2} P[f_0(1500)] f_g^{KMR}(x, x', Q_{1,t}^2, \mu^2; t_1) f_g^{KMR}(x, x', Q_{2,t}^2, \mu^2; t_2) \tag{2.9}
\]

where only one transverse momentum is taken into account somewhat arbitrarily as

\[
Q_{1,t}^2 = \min\{q_{0,t}^2, q_{1,t}^2\} \quad \text{and} \quad Q_{2,t}^2 = \min\{q_{0,t}^2, q_{2,t}^2\} \tag{2.10}
\]

and the normalization factor \(N\) can be written in terms of the \(f_0(1500) \rightarrow gg\) decay width (see below).

In the KMR approach the large meson mass approximation \(M \gg |q_{1,t}|, |q_{2,t}|\) is adopted, so the gluon virtualities are neglected in the vertex factor

\[
P[f_0(1500)] \simeq (q_{1,t} + q_{2,t})(q_{0,t} + q'_{1,t}); \tag{2.11}
\]

The KMR UGDFs are written in the factorized form:

\[
f_g^{KMR}(x, x', Q_{t}^2, \mu^2; t) = f_g^{KMR}(x, x', Q_{t}^2, \mu^2) \exp(b_0 t) \tag{2.12}
\]

with \(b_0 = 2 \text{ GeV}^{-2}\)\(^\text{[20]}\). In our approach we use somewhat different parameterization of the \(t\)-dependent isoscalar form factors.

Please note that the KMR and our (general) skewed UGDFs have different number of arguments. In the KMR approach there is only one effective gluon transverse momentum (see Eq. (2.10)) compared to two independent transverse momenta in general case (see Eq. (2.16)).

\(^5\) The last value is not so well known. We shall take \(\Gamma(M \rightarrow gg) = \Gamma_{tot}^{\text{int}}\). This will give us an upper estimate. As a consequence this will allow us to show that the gluonic component is negligible for future experiments with the PANDA detector.
The KMR skewed distributions are given in terms of conventional integrated densities $g$ and the so-called Sudakov form factor $T$ as follows:

$$f^{\text{KMR}}_g(x, x', Q_i^2, \mu^2) = R_g \frac{\partial}{\partial \ln Q_i^2} \left[ \sqrt{T(Q_i^2, \mu^2)} x g(x, Q_i^2) \right].$$

(2.13)

The square root here was taken using arguments that only survival probability for hard gluons is relevant. It is not so-obvious if this approximation is reliable for light meson production. The factor $R_g$ in the KMR approach approximately accounts for the single log $Q^2$ skewed effect [20]. Please note also that in contrast to our approach the skewed KMR UGDF does not explicitly depend on $x'$ (assuming $x' \ll x \ll 1$). Usually this factor is estimated to be 1.3–1.5. In our evaluations here we take it to be equal 1 to avoid further uncertainties. Following now the KMR notations we write the total amplitude (2.8) (averaged over colour and polarisation states of incoming gluons) in the limit $M \gg q_{1,t}, q_{2,t}$ as

$$\mathcal{M} = A \pi^2 s \int d^2 q_{0,t} P[f_0(1500)] f_{g,1}^{\text{off}}(x_1, x'_1, q_{0,t}^2, q_{1,t}^2, t_1) f_{g,2}^{\text{off}}(x_2, x'_2, q_{0,t}^2, q_{2,t}^2, t_2),$$

(2.14)

where the normalization constant is

$$A^2 = \frac{64\pi \Gamma(f_0(1500) \to gg)}{(N_c^2 - 1) M_{f_0}^3}.$$  

(2.15)

In addition to the standard KMR approach we could use other off-diagonal distributions (for details and a discussion see [21, 22]). In the present work we shall use a few sets of unintegrated gluon distributions which aim at the description of phenomena where small gluon transverse momenta are involved. Some details concerning the distributions can be found in Ref. [26]. We shall follow the notation there.

In the general case we do not know off-diagonal UGDFs very well. In [21, 22] we have proposed a prescription how to calculate the off-diagonal UGDFs:

$$f_{g,1}^{\text{off}} = \sqrt{\int g^{(1)}(x'_1, q_{0,t}^2, \mu_0^2) \cdot f_g^{(1)}(x_1, q_{1,t}^2, \mu^2) \cdot F_1(t_1)},$$

$$f_{g,2}^{\text{off}} = \sqrt{\int g^{(2)}(x'_2, q_{0,t}^2, \mu_0^2) \cdot f_g^{(2)}(x_2, q_{2,t}^2, \mu^2) \cdot F_1(t_2)},$$

(2.16)

where $F_1(t_1)$ and $F_1(t_2)$ are isoscalar nucleon form factors. They can be parameterized as [21]

$$F_1(t_{1,2}) = \frac{4m_p^2 - 2.79 t_{1,2}}{(4m_p^2 - t_{1,2})(1 - t_{1,2}/0.71)^2}.$$  

(2.17)

In the following for brevity we shall use notation $t_{1,2}$ which means $t_1$ or $t_2$. Above $t_1$ and $t_2$ are total four-momentum transfers in the first and second proton line, respectively. While in the emission line the choice of the scale is rather natural, there is no so-clear situation for the second screening-gluon exchange [22].

Even at intermediate energies ($W = 10$–$50$ GeV) typical $x'_1 = x'_2$ are relatively small ($\sim 0.01$). However, characteristic $x_1, x_2 \sim M_{f_0}/\sqrt{s}$ are not too small (typically $> 10^{-1}$).
Therefore here we cannot use the small-\(x\) models of UGDFs. In the latter case a Gaussian smearing of the collinear distribution seems a reasonable solution:

\[
f_g^{Gauss}(x, k_t^2, \mu_r^2) = x g^{coll}(x, \mu_r^2) \cdot F_{Gauss}(k_t^2; \sigma_0) ,
\]

where \(g^{coll}(x, \mu_r^2)\) are standard collinear (integrated) gluon distribution and \(F_{Gauss}(k_t^2; \sigma_0)\) is a Gaussian two-dimensional function

\[
F_{Gauss}(k_t^2, \sigma_0) = \frac{1}{2\pi\sigma_0^2} \exp \left( -k_t^2 / 2\sigma_0^2 \right) / \pi .
\]

Above \(\sigma_0\) is a free parameter which one can expect to be of the order of 1 GeV. Based on our experience in \[22\] we expect strong sensitivity to the actual value of the parameter \(\sigma_0\). Summarizing, a following prescription for the off-diagonal UGDF seems reasonable:

\[
f(x, x', k_t^2, k_t'^2, t) = \sqrt{f_{small-x}(x', k_t'^2)f_g^{Gauss}(x, k_t^2, \mu^2)} \cdot F(t) ,
\]

where \(f_{small-x}(x', k_t'^2)\) is one of the typical small-\(x\) UGDFs (see e.g.\[26\]). So exemplary combinations are: KL \(\otimes\) Gauss, BFKL \(\otimes\) Gauss, GBW \(\otimes\) Gauss (for notation see \[26\]). The natural choice of the scale is \(\mu^2 = M_{f_0}^2\). This relatively low scale is possible with the GRV-type of PDF parameterization \[27\]. We shall call \(2.20\) a ”mixed prescription” for brevity.

C. Two-gluon impact factor approach for subasymptotic energies

The amplitude in the previous section, written in terms of off-diagonal UGDFs, was constructed for rather large energies. The smaller the energy the shorter the QCD ladder. It is not obvious how to extrapolate the diffractive amplitude down to lower (close-to-threshold) energies. Here we present slightly different method which seems more adequate at lower energies.

At not too large energies the amplitude of elastic scattering can be written as amplitude for two-gluon exchange \[28,29\]

\[
\mathcal{M}_{pp \rightarrow pp}(s, t) = i s \frac{N_c^2 - 1}{N_c^2} \int d^2 k_t \alpha_s(k_{1t}^2)\alpha_s(k_{2t}^2) \frac{3F(k_{1t}, k_{2t})3F(k_{1t}, k_{2t})}{(k_{1t}^2 + \mu_0^2)(k_{2t}^2 + \mu_0^2)} .
\]

In analogy to dipole-dipole or pion-pion scattering (see e.g. \[29\]) the impact factor can be parameterized as:

\[
F(k_{1t}, k_{2t}) = \frac{A^2}{A^2 + (k_{1t} + k_{2t})^2} - \frac{A^2}{A^2 + (k_{1t} - k_{2t})^2} .
\]

At high energy the net four-momentum transfer: \(t = -(k_{1t} + k_{2t})^2\). \(A\) in Eq.\(2.22\) is a free parameter which can be adjusted to elastic scattering. For our rough estimate we take \(A = m_\rho\).

Generalizing, the amplitude for exclusive \(f_0(1500)\) production can be written as the amplitude for three-gluon exchange shown in Fig.\[1\]

\[
\mathcal{M}_{pp \rightarrow ppf_0(1500)}(s, y, t_1, t_2, \phi) = i s \frac{N_c^2 - 1}{N_c^2} \int d^2 k_{0t} \left( \alpha_s(k_{0t}^2)\alpha_s(k_{1t}^2) \right)^{1/2} \left( \alpha_s(k_{0t}^2)\alpha_s(k_{2t}^2) \right)^{1/2} \times \frac{3F(k_{0t}, k_{1t})3F(k_{0t}, k_{2t})}{(k_{0t}^2 + \mu_0^2)(k_{1t}^2 + \mu_0^2)(k_{2t}^2 + \mu_0^2)} V_{gg \rightarrow f_0(1500)}(k_{1t}, k_{2t}) .
\]
At high energy and $y \approx 0$ the four-momentum transfers can be calculated as:
\[ t_1 = -(k_{0t} + k_{1t})^2, \quad t_2 = -(k_{0t} - k_{2t})^2. \]
At low energy and/or $y \neq 0$ the kinematics is slightly more complicated. Let us define effective four-vector transfers:
\[ q_1 = (p'_1 - p_1) = (q_{10}, q_{1x}, q_{1y}, q_{1z}), \]
\[ q_2 = (p'_2 - p_2) = (q_{20}, q_{2x}, q_{2y}, q_{2z}). \]
(2.24)

Then $t_1 \equiv q_1^2 = q_{10}^2 + q_{1t}^2$ and $t_2 \equiv q_2^2 = q_{20}^2 + q_{2t}^2$. Close to threshold the longitudinal components $q_{1t} = q_{10} - q_{1z} \ll 0$ and $q_{2t} = q_{20} - q_{2z} \ll 0$. Then the amplitude (2.23) must be corrected. Then also four-vectors of exchanged gluons ($k_0, k_1$ and $k_2$) cannot be purely transverse and longitudinal components must be included as well. To estimate the effect we use formula (2.23) but modify the transferred four momenta of gluons entering the $g^*g^* \rightarrow f_0(1500)$ production vertex:
\[ k_1 = (0, k_{1t}, 0) \rightarrow (q_{10}, k_{1t}, q_{1z}), \]
\[ k_2 = (0, k_{2t}, 0) \rightarrow (q_{20}, k_{2t}, q_{2z}) \]
(2.25)
and leave $k_0$ purely transverse. This procedure is a bit arbitrary but comparing results obtained with formula (2.23) with that from the formula with modified four-momenta would allow to estimate related uncertainties.

We write the vertex function $gg \rightarrow f_0(1500)$ in the following tensorial form:
\[ V(k_1, k_2, p_M) = C_{f_0(1500)\rightarrow gg} g_{\mu\nu} k_{1\mu} k_{2\nu}. \]
(2.26)

---

6 It would be more appropriate to calculate in this case a four-dimensional integral instead of the two-dimensional one.

7 In general, another tensorial forms are also possible. This may depend on the structure of the considered meson. In principle, the details depend on the form of the vertex. To avoid uncertainties in the $k_t$-factorization approach we work in the on-shell approximation. In the on-shell approximation (or infinitely heavy meson approximation) the vertex is expressed through decay width and all vertices should be equivalent. Even if the off-shell effects are included we do not expect very different energy dependence of the cross section for different tensorial forms as due to kinematics only small virtualities of gluons enter into game.
The normalization factor is obtained from the decay of \( f_0(1500) \) into two soft gluons:

\[
|C_{f_0(1500)\rightarrow gg}|^2 = \frac{64\pi}{M_{f_0}^3(N_c^2 - 1)} \Gamma_{f_0(1500)\rightarrow gg}.
\]  \(2.27\)

Of course the partial decay width is limited from above:

\[
\Gamma_{f_0(1500)\rightarrow gg} < \Gamma_{\text{tot}}.
\]  \(2.28\)

The amplitudes discussed here involve transverse momenta in the infra-red region. Then a prescription how to extend the perturbative \( \alpha_s(k_t^2) \) dependence to a nonperturbative region of small gluon virtualities is unavoidable. In the following \( \alpha_s(k_t^2) \) is obtained from an analytic freezing proposed by Shirkov and Solovtsev \[30\].

### D. Double-diffractive mechanism with intermediate pionic triangle

The \( f_0(1500) \to \pi\pi \) is the second most probable decay channel \[23\]. As a consequence the mechanism shown in Fig.5 may play important role in the exclusive production of \( f_0(1500) \) \[31\]. It is relatively easy to estimate the contribution of this mechanism at high energies \[31\]. In this paper we shall make an estimate of the corresponding cross section not far from the threshold, where the situation is slightly more complicated.

![FIG. 5: A sketch of the double-diffractive mechanism with pionic loop for exclusive production of the glueball candidate \( f_0(1500) \). Some kinematical variables are shown explicitly.](image)

The amplitude of the process \( pp \to ppf_0(1500) \) sketched in Fig.3 can be written in a simplified form \[31\] as:

\[
\mathcal{M}_{\lambda_1\lambda_2\rightarrow\lambda_1'\lambda_2'}(y, p_{1t}, p_{2t}, \phi) \approx \tilde{T}_{IP}F_{0}(q_1, q_2, p_{f0}) \delta_{\lambda_1\lambda_1'} F_{\text{cut}}(s_{1,\text{eff}}) \delta_{\lambda_2\lambda_2'} F_{\text{cut}}(s_{2,\text{eff}})
\]

\[
\times \left( is_{1,\text{eff}} C^{IP}_{\pi p} \left( \frac{s_{1,\text{eff}}}{s_0} \right)^{\alpha_{R}(t_1) - 1} e^{-\frac{B_s}{2} t_1} + \eta_f s_{1,\text{eff}} C^{IP}_{\pi p} \left( \frac{s_{1,\text{eff}}}{s_0} \right)^{\alpha_{R}(t_1) - 1} e^{-\frac{B_s}{2} t_1} \right)
\]

\[
\times \left( is_{2,\text{eff}} C^{IP}_{\pi p} \left( \frac{s_{2,\text{eff}}}{s_0} \right)^{\alpha_{R}(t_2) - 1} e^{-\frac{B_s}{2} t_2} + \eta_f s_{2,\text{eff}} C^{IP}_{\pi p} \left( \frac{s_{2,\text{eff}}}{s_0} \right)^{\alpha_{R}(t_2) - 1} e^{-\frac{B_s}{2} t_2} \right).
\]  \(2.29\)

The delta functions are related to helicity conservation in hadronic processes. While the pomeron (sub)amplitudes are dominantly imaginary, the reggeon (sub)amplitudes have both
real and imaginary parts. The factor \( \eta_f \approx i - 1 \). In the formula above \( \alpha_P(t_{1,2}) = \alpha_R(0) + \alpha'_R \cdot t_{1,2} \) and \( \alpha_R(t_{1,2}) = \alpha_R(0) + \alpha'_R \cdot t_{1,2} \) are a so-called reggeon trajectories. For brevity we use notation \( t_{1,2} \) which means \( t_1 \) or \( t_2 \). We take from the phenomenology: \( \alpha_R(0) = 1.0808, \alpha'_R = 0.25 \text{ GeV}^{-2}, \alpha_R(0) = 0.5475 \) and \( \alpha'_R = 0.93 \text{ GeV}^{-2} [32] \). The strength parameters for the \( \pi N \) scattering fitted to the corresponding total cross sections [32]: \( C_{\pi p} = 13.63 \text{ mb} \) and \( C_{\pi N} = (27.56 + 36.02)/2 \text{ mb} \). At not too high energies the slope parameter \( B_{\pi N} \approx 6 \text{ GeV}^{-2} \).

The subchannel Mandelstam variable \( s_{1, \text{eff}} \) and \( s_{2, \text{eff}} \) are related to center-of-mass energies of relevant pion-nucleon subsystem. In principle, they are functions of pion-four momenta \( p_1 \) and close to kinematical threshold \( \hat{s} \) where \( \hat{s} \) gives the position of the cut and parameter \( \alpha_f \) describes how sharp is the cut off. The latter parameter can have significant influence on the numerics. We shall take \( \alpha_f = 2 \text{ GeV} \) and \( \alpha' = 0.1 \text{ GeV} \). For large energies \( F_{\text{cut}}(\hat{s}) \approx 1 \) and close to kinematical threshold \( \hat{W} = m_\pi + M_N : F_{\text{cut}}(\hat{s}) \approx 0. This means that we limit to double-diffractive contribution only.

The effective Regge parametrizations of \( \pi N \) interactions [32] are for both colliding particles being on-mass-shell. In our case the triangle pions are off-mass-shell. We correct the Regge strength parameters by multiplying by two vertex form factors \( F_{\text{off}}(k, k_i) \) (see Fig5). We take them in the following factorized form:

\[
F_{\text{cut}}(\hat{s}) = \frac{\exp \left( \frac{\hat{W} - W_{\text{cut}}}{a_{\text{cut}}} \right)}{1 + \exp \left( \frac{\hat{W} - W_{\text{cut}}}{a_{\text{cut}}} \right)},
\]

where \( \hat{W} = \sqrt{\hat{s}} \). The parameter \( W_{\text{cut}} \) gives the position of the cut and parameter \( a_{\text{cut}} \) describes how sharp is the cut off. The latter parameter can have significant influence on the numerics. We shall take \( W_{\text{cut}} = 2 \text{ GeV} \) and \( a_{\text{cut}} = 0.1 \text{ GeV} \). For large energies \( F_{\text{cut}}(\hat{s}) \approx 1 \) and close to kinematical threshold \( \hat{W} = m_\pi + M_N : F_{\text{cut}}(\hat{s}) \approx 0. This means that we limit to double-diffractive contribution only.

The effective Regge parametrizations of \( \pi N \) interactions [32] are for both colliding particles being on-mass-shell. In our case the triangle pions are off-mass-shell. We correct the Regge strength parameters by multiplying by two vertex form factors \( F_{\text{off}}(k, k_i) \) (see Fig5).

We take them in the following factorized form:

\[
F_{\text{off}}(k, k_i) = \exp \left( -|k^2 - m_{\pi}^2|/\Lambda_{\text{off}}^2 \right) \cdot \exp \left( -|k_i^2 - m_{\pi}^2|/\Lambda_{\text{off}}^2 \right).
\]

\( \Lambda_{\text{off}} \) is in principle a free parameter. In the calculation shown in the result section we shall take \( \Lambda_{\text{off}} = 1 \text{ GeV} \). The dependence on triangle four-momenta forces us to merge the form factors inside the triangle integration which leads to a modified pion-triangle function:

\[
\tilde{T}_{P'P'f_0}(q_1, q_2, p_{f_0}) = \int \frac{d^4k}{(2\pi)^4} \tilde{T}(k; q_1, q_2, p_{f_0}) F_{\text{off}}(k, k_1) F_{\text{off}}(k, k_2),
\]

\[8 \text{ We take average value for } \pi^+p \text{ and } \pi^-p \text{ scattering.} \]
where standard triangle integrand $\hat{T}(k; q_1, q_2, p_{f_0})$ reads:

$$
\hat{T}(k; q_1, q_2, p_{f_0}) = F(q_1, k_1, k) \frac{1}{(q_1 - k)^2 - m_\pi^2 + i\epsilon} \\
\times F(q_2, k_2, k) \frac{1}{(q_2 + k)^2 - m_\pi^2 + i\epsilon} \\
\times F(k_1, k_2, p_{f_0}) \frac{1}{k^2 - m_\pi^2 + i\epsilon}.
$$

In addition to three pion propagators we have written three vertex form factors which are functions of four momenta of corresponding legs. In principle, these functions are relatively well known for space-like pions. We parametrize the triangle-vertex form factors in the following factorized exponential form:

$$
F(q_1, k_1, k) = \exp \left( -\frac{|k_1^2 - m_\pi^2|}{\Lambda_\pi^2} \right) \cdot \exp \left( -\frac{|k^2 - m_\pi^2|}{\Lambda_\pi^2} \right),
$$

$$
F(q_2, k_2, k) = \exp \left( -\frac{|k_2^2 - m_\pi^2|}{\Lambda_\pi^2} \right) \cdot \exp \left( -\frac{|k^2 - m_\pi^2|}{\Lambda_\pi^2} \right),
$$

$$
F(k_1, k_2, p_{f_0}) = \exp \left( -\frac{|k_1^2 - m_\pi^2|}{\Lambda_\pi^2} \right) \cdot \exp \left( -\frac{|k_2^2 - m_\pi^2|}{\Lambda_\pi^2} \right).
$$

In this factorized form each exponent is associated with individual leg in the vertex. Such form factors (exponents) are normalized to unity when pions in the loop are on-mass shell. Please note that we symmetrically (modulus in (2.35)) damp configurations above and below pion-mass shell. $\Lambda_\pi$ is related to the size of the pions in the triangle. It is natural to expect: $\Lambda_\pi < \Lambda_{\text{off}}$. In the calculation presented here we shall take $\Lambda_\pi = 0.5$ GeV. Since the configurations close to the mass shells give the biggest contributions the sensitivity to the actual value of the form factor $F$ (see Eqs. (2.34) and (2.35)) is not substantial. The $g_{f_0\pi\pi} = g_{\pi\pi f_0}$ coupling constant can be calculated from the corresponding partial decay width [31].

Calculating the triangle function for running kinematics of the $h_1 h_2 \rightarrow h'_1 h'_2 f_0(1500)$ process (each point of the phase space) is in practice impossible. We calculate numerically the triangle function for:

$$
q_1 \rightarrow \left( \langle q_{10} \rangle_{y=0}, 0, 0, \langle q_{1z} \rangle_{y=0} \right),
$$

$$
q_2 \rightarrow \left( \langle q_{20} \rangle_{y=0}, 0, 0, \langle q_{2z} \rangle_{y=0} \right).
$$

Transverse components are on average small and are neglected in the present approximation. Close to threshold $| \langle q_{10} \rangle_{y=0} | \neq | \langle q_{1z} \rangle_{y=0} |$ and $| \langle q_{20} \rangle_{y=0} | \neq | \langle q_{2z} \rangle_{y=0} |$.

### E. Pion-pion MEC amplitude

It is straightforward to evaluate the pion-pion meson exchange current contribution shown in Fig[2]. If we assume the $i\gamma_5$ type coupling of the pion to the nucleon then the Born
amplitude squared and averaged over initial and summed over final spin polarizations reads:

\[
|\mathcal{M}|^2 = \frac{1}{4} \left[ (E_1 + m)(E'_1 + m) \left( \frac{\mathbf{p}_1^2}{(E_1 + m)^2} + \frac{\mathbf{p}_1'^2}{(E'_1 + m)^2} - \frac{2\mathbf{p}_1 \cdot \mathbf{p}_1'}{(E_1 + m)(E'_1 + m)} \right) \right] \times 2 \\
\times \frac{g_{\pi NN}^2 T_k}{(t_1 - m_\pi^2)^2} F_{\pi NN}^2(t_1) \times |C_{f_0(1500)\to \pi\pi}|^2 V_{\pi\pi\to f_0(1500)}^2(t_1, t_2) \times \frac{g_{\pi NN}^2 T_k}{(t_2 - m_\pi^2)^2} F_{\pi NN}^2(t_2) \\
\times \left[ (E_2 + m)(E'_2 + m) \left( \frac{\mathbf{p}_2^2}{(E_2 + m)^2} + \frac{\mathbf{p}_2'^2}{(E'_2 + m)^2} - \frac{2\mathbf{p}_2 \cdot \mathbf{p}_2'}{(E_2 + m)(E'_2 + m)} \right) \right] \times 2 .
\]

(2.37)

In the formula above \( m \) is the mass of the nucleon, \( E_1, E_2 \) and \( E'_1, E'_2 \) are energies of initial and outgoing nucleons, \( \mathbf{p}_1, \mathbf{p}_2 \) and \( \mathbf{p}_1', \mathbf{p}_2' \) are the corresponding three-momenta and \( m_\pi \) is the pion mass. The factor \( g_{\pi NN} \) is the familiar pion-nucleon coupling constant and is relatively well known \([35]\) \( (\frac{g_{\pi NN}}{4\pi} = 13.5 - 14.6) \). In our calculations we take \( \frac{g_{\pi NN}}{4\pi} = 13.5 \). The isospin factor \( T_k \) equals 1 for the \( \pi^0\pi^0 \) fusion and equals 2 for the \( \pi^+\pi^- \) fusion. Limiting to nucleons in the final state, in the case of proton-proton collisions only \( ppf_0(1500) \) final state channel is possible and therefore the \( \pi^0\pi^0 \) fusion is allowed while in the case of proton-antiproton collisions both \( p\bar{p}f_0(1500) \) and \( n\bar{n}f_0(1500) \) final state channels are possible, i.e. both \( \pi^0\pi^0 \) and \( \pi^+\pi^- \) MEC are allowed. In the case of central heavy meson production rather large transferred four-momenta squared \( t_1 \) and \( t_2 \) are involved and one has to include extended nature of the particles involved in corresponding vertices. This is incorporated via \( F_{\pi NN}(t_1) \) or \( F_{\pi NN}(t_2) \) vertex form factors. The influence of the t-dependence of the form factors will be discussed in the result section. In the meson exchange approach \([36]\) they are parameterized in the monopole form as

\[
F_{\pi NN}(t) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - t}.
\]

(2.38)

Typical values of the form factor parameter are \( \Lambda = 1.2 - 1.4 \) GeV \([36]\), however the Gottfried Sum Rule violation prefers smaller \( \Lambda \approx 0.8 \) GeV \([37]\).

The normalization constant \(|C|^2\) in \((2.37)\) can be calculated from the partial decay width as

\[
|C_{f_0(1500)\to \pi\pi}|^2 = \frac{8\pi}{\sqrt{M_{f_0}^2 - 4m_\pi^2}} \frac{2M_{f_0}^2 \Gamma_{f_0(1500)\to \pi^0\pi^0}}{\sqrt{M_{f_0}^2 - 4m_\pi^2}},
\]

(2.39)

where \( \Gamma_{f_0(1500)\to \pi^0\pi^0} = 0.109 \cdot BR(f_0(1500) \to \pi\pi) \cdot 0.5 \) GeV. The branching ratio is \( BR(f_0(1500) \to \pi\pi) = 0.349 \) \([23]\). The off-shellness of pions is also included for the \( \pi\pi \to f_0(1500) \) transition through the extra \( V_{\pi\pi\to f_0(1500)}(t_1, t_2) \) form factor which we take in the factorized form:

\[
V_{\pi\pi\to f_0(1500)}(t_1, t_2) = \frac{\Lambda_{\pi\pi f_0}^2 - m_\pi^2}{\Lambda_{\pi\pi f_0}^2 - t_1} \cdot \frac{\Lambda_{\pi\pi f_0}^2 - m_\pi^2}{\Lambda_{\pi\pi f_0}^2 - t_2}.
\]

(2.40)

It is normalized to unity when both pions are on mass shell

\[
V(t_1 = m_\pi^2, t_2 = m_\pi^2) = 1.
\]

(2.41)

In the present calculation we shall take \( \Lambda_{\pi\pi f_0} = 1.0 \) GeV.
III. RESULTS

A. Gluonic QCD mechanism

Let us start with the QCD mechanism relevant at higher energies. We wish to present differential distributions in $x_F$, $t_1$ or $t_2$ and relative azimuthal angle $\phi$. In the following we shall assume: $\Gamma_{f_0(1500) \rightarrow gg} = \Gamma_{f_0(1500)}^{\text{tot}}$. This assumption means that our differential distributions mean upper limit of the cross section. If the fractional branching ratio is known, our results should be multiplied by its value. There are almost no absolutely normalized experimental data on exclusive $f_0(1500)$ production in the literature, except of Ref. [38]. The absolutely normalized data of the ABCDHW Collaboration [39] put emphasis rather on $f_2(1270)$ production. In principle, some (model-dependent) information on glueball wave function could be obtained from radiative decays $J/\psi \rightarrow \gamma f_0(1500)$ and $\Upsilon \rightarrow \gamma f_0(1500)$ [40]. The present data are not good enough to provide a detailed information on coupling of gluons to $f_0(1500)$.

In Fig. 6 we show as example distribution in Feynman $x_F$ for Kharzeev-Levin UGDF (solid) and the mixed distribution KL $\otimes$ Gaussian (dashed) for several values of collision energy in the interval $W = 10$ – $50$ GeV. In general, the higher collision energy the larger cross section. With the rise of the initial energy the cross section becomes peaked more and more at $x_F \sim 0$. The mixed UGDF produces slightly broader distribution in $x_F$.

![FIG. 6: The distribution of $f_0(1500)$ in Feynman $x_F$ for $W = 10, 20, 30, 40, 50$ GeV (from bottom to top). In this calculation the Kharzeev-Levin UGDF (solid line) and the mixed distribution KL $\otimes$ Gaussian (dashed line) were used.](image)

In Fig. 6 we present corresponding distributions in $t = t_1 = t_2$. The slope depends on UGDF used, but for a given UGDF is almost energy independent.

Finally we present corresponding distributions in relative azimuthal angle between outgo-
FIG. 7: Distribution in $t = t_1 = t_2$ for Kharzeev-Levin UGDF for $W = 10$, 20, 30, 40, 50 GeV (from bottom to top). The notation here is the same as in Fig. 6.

The QCD gluonic mechanism is of course charge independent.

FIG. 8: Distribution in relative azimuthal angle for Kharzeev-Levin UGDF for $W = 10$, 20, 30, 40, 50 GeV (from bottom to top). The notation here is the same as in Fig. 6.

9 The QCD gluonic mechanism is of course charge independent.
B. Diffractive versus pion-pion mechanism

What about the pion-pion fusion mechanism? Can it dominate over the gluonic mechanism discussed in the previous subsection? In Fig.9 we show the integrated cross section for the exclusive $f_0(1500)$ elastic production

$$p\bar{p} \rightarrow pf_0(1500)\bar{p}$$  \hspace{1cm} (3.1)

and for double charge exchange reaction

$$p\bar{p} \rightarrow nf_0(1500)n.$$  \hspace{1cm} (3.2)

The thick solid line represents the pion-pion component calculated with monopole vertex form factors (2.38) with $\Lambda = 0.8$ GeV (lower) and $\Lambda = 1.2$ GeV (upper). The difference between the lower and upper curves represents uncertainties on the pion-pion component. The pion-pion contribution grows quickly from the threshold, takes maximum at $W \approx 6-7$ GeV and then slowly drops with increasing energy. The gluonic contribution calculated with unintegrated gluon distributions drops with decreasing energy towards the kinematical threshold and seems to be about order of magnitude smaller than the pion-pion component at $W = 10$ GeV. We show the result with Kharzeev-Levin UGDF (dashed line) which includes gluon saturation effects relevant for small-$x$, Khoze-Martin-Ryskin UGDF (dotted line) used for the exclusive production of the Higgs boson and the result with the ”mixed prescription” (KL $\otimes$ Gaussian) for different values of the $\sigma_0$ parameter: 0.5 GeV (upper thin solid line), 1.0 GeV (lower thin solid line). In the latter case results rather strongly depend on the value of the smearing parameter. The thick dash-dotted line corresponds to the second diffractive mechanism with pionic triangle. It is above the WA102 experimental data point. This is probably because of absorption effects not included in the present calculation. This contribution stays below the pion-pion fusion contribution at the GSI HESR energies. For comparison we show also experimental data point of the WA102 Collaboration from Ref.[38] which lies between the results obtained with “KL” and ”mixed” off-diagonal UGDFs. The thick long-dashed line corresponds to the second diffractive mechanism with pionic triangle. It is above the WA102 experimental data point. This is probably because of absorption effects not included in the present calculation. This contribution stays below the pion-pion fusion contribution at the GSI HESR energies.

We calculate the gluonic contribution down to $W = 10$ GeV. Extrapolating the gluonic component to even lower energies in terms of UGDFs seems rather unsure. At lower energies the two-gluon impact factor approach seems more relevant. The impact factor approach result is even order of magnitude smaller than that calculated in the KMR approach (see lowest dash-dotted (red) line in Fig. 9), so it seems that the diffractive contribution is completely negligible at the FAIR energies.

Our calculation suggests that quite different energy dependence of the cross section may be expected in elastic and charge-exchange channels. Experimental studies at FAIR and J-PARC could shed more light on the glueball production mechanism.

The smaller energies the larger values of $x_1$ and $x_2$ are involved. Many of unintegrated gluon distributions in the literature are formulated in the region of very small $x$. Extrapolation of the method down to small energies automatically means going to the region of large $x$. Below we wish to demonstrate this fact. In Fig.10 we show the ratio of the cross sections

$$R = \frac{\sigma(W; x_1 < x_0, x_2 < x_0)}{\sigma(W)},$$  \hspace{1cm} (3.3)
as a function of center-of-mass energy. Above $x_0$ was introduced to define the region of small/large $x$. The solid line corresponds to $x_0 = 0.1$ and the dashed line to $x_0 = 0.2$. Down to largest HESR energies one stays in the region of $x_1, x_2 < 0.2$.

C. Predictions for PANDA at HESR

Let us concentrate now on $p\bar{p}$ collisions at energies relevant for future experiments at HESR at the FAIR facility in GSI. Here the pion-pion MEC (see Fig. 2) seems to be the dominant mechanism, especially for the charge exchange reaction $p\bar{p} \to n\bar{n}f_0(1500)$. As discussed in the previous section the gluonic component can be there safely neglected.

In Fig. 11 we show average values of $t_1$ (or $t_2$) for the two-pion MEC as a function of the center of mass energy. Close to threshold $W = 2m_N + m_{f_0(1500)}$ the transferred four-momenta squared are the biggest, of the order of about $1.5 \text{ GeV}^2$. The bigger energy the smaller the transferred four-momenta squared. Therefore experiments close to threshold open a unique possibility to study physics of large transferred four-momenta squared at relatively small energies. This is a quite new region, which was not studied so far in the literature.

Below we shall present cross sections for the $p\bar{p} \to n\bar{n}f_0(1500)$ reaction. The cross section for the $p\bar{p} \to p\bar{p}f_0(1500)$ reaction can be obtained by rescaling by the factor of 1/4.

The maximal energy planned for HESR is $\sqrt{s} = 5.5 \text{ GeV}$. At this energy the phase space is still very limited. In Fig. 12 we show rapidity distribution of $f_0(1500)$ calculated including
FIG. 10: The ratio of the cross sections (see Eq. (3.3)) as a function of center-of-mass energy. The solid line corresponds to $x_0 = 0.1$ and the dashed line to $x_0 = 0.2$.

FIG. 11: (Color online.) Average value of $<t_1> = <t_2>$ as a function of the center-of-mass collision energy for the two-pion exchange mechanism. In this calculation $\Lambda = 0.8$ GeV.

pion-pion fusion only. For comparison the rapidity of incoming antiproton and proton is 1.74 and -1.74, respectively. This means that in the center-of-mass system the glueball is produced at midrapidities, on average between rapidities of outgoing nucleons.

In Fig. 13 we show transverse momentum distribution of neutrons or antineutrons produced in the reaction $p\bar{p} \rightarrow n\bar{n}f_0(1500)$. The distribution depends on the $\pi NN$ form factors $F_{\pi NN}(t_1)$ and $F_{\pi NN}(t_2)$ in formula (2.37).

In Fig. 14 we show azimuthal angle correlation between outgoing hadrons (in this case
FIG. 12: Rapidity distribution of $f_0(1500)$ ($\pi^+\pi^-$ fusion only) produced in the reaction $p\bar{p} \rightarrow n\bar{n}f_0(1500)$ for $W = 3.5, 4.0, 4.5, 5.0, 5.5$ GeV (from bottom to top). In this calculation $\Lambda = 0.8$ GeV.

neutron and antineutron). The preference for back-to-back configurations is caused merely by the limitations of the phase space close to the threshold (the matrix element for pion-pion fusion is $\phi$-independent). This correlation vanishes in the limit of infinite energy. At high energy, where the phase space limitations are small, the distributions are isotropic, there is no dependence on azimuthal angle. In practice far from the threshold the distribution

FIG. 13: Transverse momentum distribution of neutrons or antineutrons produced in the reaction $p\bar{p} \rightarrow n\bar{n}f_0(1500)$ ($\pi^+\pi^-$ fusion only) for $W = 3.5, 4.0, 4.5, 5.0, 5.5$ GeV (from bottom to top). In this calculation $\Lambda = 0.8$ GeV.
becomes almost constant in azimuth. This has to be contrasted with similar distributions for pomeron-pomeron fusion shown in Fig. 8 which are clearly peaked for the back-to-back configurations. Therefore a deviation from the constant distribution in relative azimuthal angle for the highest HESR energy of $W = 5.5$ GeV for $p\bar{p} \rightarrow pf_0(1500)\bar{p}$ can be a signal of the gluon induced processes and/or the presence of subleading reggeon exchanges, e.g. $\rho\rho$.

It is not well understood what happens with the gluon induced diffractive processes when going down to intermediate ($W = 5-10$ GeV) energies. Our calculations shows, however, that the diffractive component is negligible compared to the pion-pion fusion at $W < 10$ GeV. Possible future experiments performed at J-PARC could bring some new insights into this issue by studying distortions (probably very small) from the pion-pion fusion mechanism.

![Azimuthal angle correlations between neutron and antineutron produced in the reaction $p\bar{p} \rightarrow n\bar{n}f_0(1500)$ ($\pi^+\pi^-$ fusion only) for $W = 3.5, 4.0, 4.5, 5.0, 5.5$ GeV (from bottom to top). In this calculation $\Lambda = 0.8$ GeV.](image)

Up to now we have neglected interference between pion-pion and pomeron-pomeron contributions (for the same final channel). This effect may be potentially important when both components are of the same order of magnitude. At J-PARC energies there could be, in principle, some small interference effect. While the pomeron-pomeron contribution is dominantly nucleon helicity preserving the situation for pion-pion fusion is more complicated. In the latter case we define 4 classes of contributions with respect to the nucleon helicities: $cc$ (both helicities conserved), $cf$ (first conserved, second flipped), $fc$ (first flipped, second conserved) and $ff$ (both helicities flipped). The corresponding ratios of individual contributions to the sum of all contributions are shown in Fig. 15. In practice, only the $cc$ contribution may potentially interfere with the gluonic one. From the figure one can conclude that this can happen only when both transverse momenta of the final nucleons are small. We shall leave numerical studies of the interference effect for future investigations, when experimental details of such measurements will be better known; but already now one

---

10 At the PANDA energies the problem is rather academic as the diffractive component can be neglected.
can expect them to be rather small.

\[ R_{cc}(\text{GeV}) \]

\[ R_{ff}(\text{GeV}) \]

\[ R_{cf}(\text{GeV}) \]

\[ R_{fc}(\text{GeV}) \]

**FIG. 15:** Helicity decomposition of the cross section on the \((p_{1t}, p_{2t})\) plane for \(W = 10\) GeV. \(R_{cc}\) (upper left), \(R_{ff}\) (upper right), \(R_{cf}\) (lower left), \(R_{fc}\) (lower right). The standard nucleon dipole form factor was used in this calculation.

Now we wish to show the size of the triangle-double-diffractive (TDD) component at the GSI HESR energy range. In Fig. 16 we compare it with the pion-pion fusion component. The TDD component vanishes quickly with decreasing energy and stays below the pion-pion fusion component for the HESR energy range. The quick decrease of the cross section is caused mainly by the \(F_{\text{cut}}(s_{1,\text{eff}})\) and \(F_{\text{cut}}(s_{2,\text{eff}})\) factors in Eq. (2.29) and (2.31) and reflects smallness of \(\pi N\) subchannel energies.

**IV. BACKGROUND FOR THE \(f_0(1500)\) PRODUCTION FOR \(p\bar{p} \to p\bar{p}f_0(1500)\) REACTION**

**A. Pion-pion rescattering background**

In the previous section we have shown that in the PANDA energy range the pion-pion fusion is the dominant reaction mechanism for the production of the glueball candidate
f_0(1500). Up to now we have calculated the cross section for production of f_0(1500) meson – a process with three particles (p, \bar{p} and f_0(1500)) in the final state.

In practice one must select a given decay channel of f_0(1500). There are a few options: (a) a two-pion decay (\pi^+\pi^- or \pi^0\pi^0), (b) a four-pion decay (c) a two-kaon decay. The first one is attractive due to its simplicity but may have a large background. The second requires more complicated analysis but may have smaller background. The branching fraction for the last option is smaller by a factor of about 5 than for the two-pion channel.

Let us consider now an estimate of the background to the pp \rightarrow pp\pi^+\pi^- reaction. In
FIG. 18: The $2 \rightarrow 4$ amplitudes with the intermediate $\rho$-meson (reggeon) exchange as an example of the background.

In Fig. 17 we present our reaction of interest – the reaction which proceeds through the scalar resonance $f_0(1500)$. This reaction is viewed now as a process with four particles ($p, \bar{p}, \pi^+, \pi^-$) in the final state. Unavoidably there exists a nonreduceable background to this process. In Fig. 18 we show an example of the background. We shall call these two complex diagrams as $\rho$-meson (reggeon) exchanges for brevity. The region of $W_{\pi\pi} \sim 1.5$ GeV is slightly above the region of application of the standard meson-exchange formalism and slightly below the region of application of high-energy Regge approach. In principle, one should consider both approaches.

In the $\rho$ meson-exchange formalism the reduced amplitude for the $\pi^0\pi^0 \rightarrow \pi^+\pi^-$ process can be written as:

$$\mathcal{M}_{\pi^0\pi^0 \rightarrow \pi^+\pi^-}^{\rho-\text{exch.}} = g_{\pi\pi\rho} F_{\pi\pi\rho}(\hat{t}) \left( \frac{q_1^\mu + p_3^\mu}{t - m_\rho^2 + i\Gamma_\rho m_\rho} g_{\pi\pi\rho} F_{\pi\pi\rho}(\hat{t}) + g_{\pi\pi\rho} F_{\pi\pi\rho}(\hat{u}) \left( \frac{q_1^\mu + p_3^\mu}{\hat{u} - m_\rho^2 + i\Gamma_\rho m_\rho} g_{\pi\pi\rho} F_{\pi\pi\rho}(\hat{u}) \right) \right).$$

(4.1)

Above

$$P_{\mu\nu}(k) = -g_{\mu\nu} + k_\mu k_\nu/m_\rho^2.$$  

(4.2)

The quantities $F_{\pi\pi\rho}(k^2)$ in (4.1) describe couplings of extended objects: pions and the exchanged $\rho$-meson. We parameterize them in the exponential form:

$$F_{\pi\pi\rho}(k^2) = \exp \left( \frac{k^2 - m_\rho^2}{\Lambda^2} \right) = \exp \left( \frac{B_{\pi\pi\rho}}{4} (k^2 - m_\rho^2) \right).$$

(4.3)

Consistent with the definition of the coupling constant the form factors are normalized to unity when $\rho$ meson is on-mass-shell. We take $g_{\pi\pi\rho}^2 = 2.6$, which reproduces the $\rho$ meson decay width [23], and $\Lambda = 1$ GeV ($B_{\pi\pi\rho} = 4$ GeV$^2$).

In the case of $\rho$-reggeon exchange the amplitude can be written as

$$\mathcal{M}_{\pi^0\pi^0 \rightarrow \pi^+\pi^-}^{\rho-\text{reggeon}} = s_{34} \eta_\rho(\hat{t}) C_{\pi\pi\rho} F(\hat{t}) \left( \frac{s_{34}}{s_0} \right)^{\alpha_\rho(\hat{t}) - 1} F(\hat{t}) + s_{34} \eta_\rho(\hat{u}) C_{\pi\pi\rho} F(\hat{u}) \left( \frac{s_{34}}{s_0} \right)^{\alpha_\rho(\hat{u}) - 1} F(\hat{u}).$$

(4.4)
We parameterize the vertex form factors in the standard exponential form used usually in the Regge phenomenology

\[ F(k^2) = \exp \left( \frac{B_{\pi\pi\rho}}{4} k^2 \right). \] (4.5)

In practical calculation we take \( B_{\pi\pi\rho} = 6 \text{ GeV}^{-2} \). In the formula (4.4) \( \eta_{\rho}(k^2) \) is a signature factor which we take here \( \eta_{\rho} \approx i + 1 \) and \( \alpha_{\rho}(t/u) = \alpha_{\rho}(0) + \alpha_{\rho}' \cdot t/u \) is a so-called reggeon trajectory. We take from the phenomenology: \( \alpha_{\rho}(0) = 0.5475 \) [32] and \( \alpha_{\rho}' = 0.9 \text{ GeV}^{-2} \). The strength parameter \( C_{\pi\pi\rho} \) can be obtained assuming Regge factorization (see e.g. [29]) and using the known strength parameters for the \( NN \) and \( \pi N \) scattering fitted to the corresponding total cross sections [32]. The simple Regge parameterizations apply for energies \( W > 2 \text{ GeV} \) (see e.g. [32]). In our case of pion-pion scattering energies of \( W \sim 1.5 \text{ GeV} \) are of interest. Here a small modification of the Regge formula (4.4) may be in order.

Consistent with meson-exchange formalism (spin-1 exchange) one may expect saturation of the \( \pi^0\pi^0 \rightarrow \pi^+\pi^- \) cross section at lower energies. The following freezing of the energy factor in (4.4) seems a reasonable correction:

\[ \left( \frac{s_{\pi\pi}}{s_0} \right)^{\alpha_{\rho}} \rightarrow \left( \frac{s_{\text{freez}}}{s_0} \right)^{\alpha_{\rho}}, \] (4.6)

where \( s_{\text{freez}} = W_{\text{freez}}^2 \). One may expect \( W_{\text{freez}} = 1.5 - 2.0 \text{ GeV} \). The compatibility of the Regge formalism with low-energy approaches for pion-pion scattering was discussed in Ref. [33].

The \( 2 \rightarrow 2 \) amplitudes (4.1) and (4.4) may be inserted into the \( 2 \rightarrow 4 \) amplitude of Fig.18. When doing so we include in addition the correction for off-shellness of incoming pions as was done for the \( f_0(1500) \) meson using exponential form factors (as in 4.9). Now we can perform a genuine \( 2 \rightarrow 4 \) calculation including four-body phase space.

Here we discuss the results with \( \rho \)-reggeon exchange only. We have checked that the \( \rho \)-meson exchange formalism discussed in this section provides approximately the same results at \( W_{\pi\pi} < 1.2 \text{ GeV} \) as the modified reggeon exchange with \( W_{\text{freez}} = 1.5 - 2.0 \text{ GeV} \) (see formula (4.6)). Therefore the modified reggeon-exchange calculation provides a realistic predictions in the broad range of pion-pion energies, both above and below the \( f_0(1500) \) resonance.

In Fig.19 we show two-pion invariant mass distribution. The solid line corresponds to our resonance contribution. The dashed lines correspond to the \( \rho \)-reggeon exchange contribution. Here the resonance contribution is much lower than the \( \rho \)-exchange background. In this calculation the integration over whole phase space was done.

In Fig.20 we show corresponding distributions in rapidity of \( \pi^+ \) or \( \pi^- \) (identical). In order to better see the overlap of the signal and background for the \( \rho \)-reggeon exchange we impose in addition \( 1.4 \text{ GeV} < M_{\pi\pi} < 1.6 \text{ GeV} \) (the region of the \( f_0(1500) \) resonance). Limiting to very small center-of-mass rapidities one can further improve the signal-to-background ratio. In Fig.21 we show two-pion invariant mass distribution with extra cuts: \( -0.5 < y_{\pi^+}, y_{\pi^-} < 0.5 \). While the \( f_0(1500) \) contribution is only slightly modified, the \( \rho \)-reggeon background contribution is reduced by more than order of magnitude. One can clearly see the signal over background in this case. Especially the high-energy side of the \( f_0(1500) \) meson is now free of the \( \rho \)-exchange background. A better separation can be done by using pion-pion partial wave analysis.

Close to the two-pions production threshold the Roper resonance excitation and its subsequent decay \( (N^*(1440) \rightarrow N\pi\pi) \) is known to give the dominant contribution to the
FIG. 19: Pion-pion invariant mass distribution for $f_0(1500)$ (solid line) and the $\rho$-reggeon exchange background (dashed lines) for naive (upper line) and corrected (two lower lines with $W_{f_{\text{freez}}} = 1.5, 2$ GeV) extrapolations to low energies. Here the full phase space has been included. The calculation was performed for the highest PANDA center-of-mass energy $W = 5.5$ GeV.

FIG. 20: Rapidity distribution of pions from the decay of the glueball candidate $f_0(1500)$ (solid line) and from the $\rho$-reggeon exchange background (dashed lines) for naive (upper line) and corrected (lower lines) extrapolations to low energies. For the background we impose in addition: $1.4 \text{ GeV} < M_{\pi\pi} < 1.6 \text{ GeV}$. The calculation was performed for the highest PANDA center-of-mass energy $W = 5.5$ GeV.
FIG. 21: Pion-pion invariant mass distribution for $f_0(1500)$ (solid line) and the $\rho$-reggeon exchange background (dashed lines) for naive (upper line) and corrected (lower lines with $W_{\text{freeze}} = 1.5, 2.0$ GeV) extrapolations to low energies. Here an additional condition on center-of-mass rapidities -0.5 < $y_\pi^+, y_\pi^- < 0.5$ has been imposed. The calculation was performed for the highest PANDA center-of-mass energy $W = 5.5$ GeV.

$pp \rightarrow pp\pi^+\pi^-$ reaction. The same may be expected also for the $p\bar{p} \rightarrow p\bar{p}\pi^+\pi^-$ reaction.

The Roper resonance produces the two-pions in dominantly the $l=0$ and $I=0$ state (the tail of the $\sigma$ meson), i.e. the strength is concentrated at $M_{\pi\pi}$ much lower than $f_0(1500)$. The kinematical constraint gives $M_{\pi\pi} < M_{N^*(1440)} - M_N \approx 0.5$ GeV. In addition, this contribution could be eliminated by extra cuts on invariant masses $M(p\pi^+\pi^-)$ and $M(\bar{p}\pi^+\pi^-)$. The same method can, at least in principle, be used to eliminate the double $\Delta, \bar{\Delta}$ excitations followed by their decays $\Delta \rightarrow \pi p$ and $\bar{\Delta} \rightarrow \pi \bar{p}$. In this sense the last two contributions (Roper and double isobar excitations) are reducible. To which extend precision of the real apparatus will allow such a reduction is a matter of further investigations.

Certainly complete analysis requires including more processes and an analysis of cuts allowing for improving the signal-to-background ratio. This certainly goes beyond the scope of the present paper where we only signal a huge increase of the $f_0(1500)$ meson production cross section in the PANDA energy range due to pion-pion fusion, the mechanism never discussed before in the literature.

The $p\bar{p} \rightarrow p\bar{p}\pi\pi\pi\pi$ reaction may be more favorable as far as the signal-to-background ratio is considered. Unfortunately theoretical calculation of background are not feasible in this case. It is not clear to us at present if the 6-body channel can be measured by the PANDA detector at FAIR.

11 Here $l$ is angular momentum between pions and $I$ is the total isospin of the pion pair.
12 We have checked that eliminating the region of double-Delta excitation at the highest PANDA energy $W = 5.5$ GeV reduces the signal by less than about 5%.
In the case of the $pp \rightarrow p\bar{p}K^+K^-$ reaction the relevant branching fraction is smaller, but not negligible ($f_0(1500) \rightarrow K\bar{K} = 8.6\%$ [23]). On the other hand the contribution from nucleon resonances is probably considerably smaller. There is, however, unreducible contribution from $K^*$ exchange in the $K^0\bar{K}^0 \rightarrow K^+K^-$ subprocess. The parameters for the latter reaction are much less known than those for the $\pi^0\pi^0 \rightarrow \pi^+\pi^-$ subprocess.

\section*{B. Double-diffractive two-pion background}

At high energies another two-pion continuum may be of interest. The underlying mechanism was proposed long ago in Ref.[41]. The general situation is sketched in Fig.22. The corresponding amplitude for the $pp \rightarrow pp\pi^+\pi^-$ process (with four-momenta $p_a + p_b \rightarrow p_1 + p_2 + p_3 + p_4$) can be written as

$$
\mathcal{M}^{pp \rightarrow pp\pi^+\pi^-} = M_{13}(t_1, s_{13}) F(t_a) \frac{1}{t_a - m^2_{\pi}} F(t_a) M_{24}(t_2, s_{24})
$$

$$
+ M_{14}(t_1, s_{14}) F(t_b) \frac{1}{t_b - m^2_{\pi}} F(t_b) M_{23}(t_2, s_{23}),
$$

(4.7)

where $M_{ik}$ denotes "interaction" between nucleon $i=1$ (forward nucleon) or $i=2$ (backward nucleon) and one of the two pions $k=3$ ($\pi^+$), $k=4$ ($\pi^-$). In the Regge phenomenology they
can be written as:

\[
M_{13}(t_1, s_{13}) = is_{13} \left( C_{13}^{13} \left( \frac{s_{13}}{s_0} \right) \frac{\alpha_{R}(t_1)-1}{e^{\frac{b_{sN}}{2} t_1} + C_{R}^{13} \left( \frac{s_{13}}{s_0} \right) \frac{\alpha_{F}(t_1)-1}{e^{\frac{b_{sN}}{2} t_1}} \right) \right),
\]

\[
M_{14}(t_1, s_{14}) = is_{14} \left( C_{14}^{14} \left( \frac{s_{14}}{s_0} \right) \frac{\alpha_{R}(t_1)-1}{e^{\frac{b_{sN}}{2} t_1} + C_{R}^{14} \left( \frac{s_{14}}{s_0} \right) \frac{\alpha_{F}(t_1)-1}{e^{\frac{b_{sN}}{2} t_1}} \right) \right),
\]

\[
M_{24}(t_2, s_{24}) = is_{24} \left( C_{24}^{24} \left( \frac{s_{24}}{s_0} \right) \frac{\alpha_{R}(t_2)-1}{e^{\frac{b_{sN}}{2} t_2} + C_{R}^{24} \left( \frac{s_{24}}{s_0} \right) \frac{\alpha_{F}(t_2)-1}{e^{\frac{b_{sN}}{2} t_2}} \right) \right),
\]

\[
M_{23}(t_2, s_{23}) = is_{23} \left( C_{23}^{23} \left( \frac{s_{23}}{s_0} \right) \frac{\alpha_{R}(t_2)-1}{e^{\frac{b_{sN}}{2} t_2} + C_{R}^{23} \left( \frac{s_{23}}{s_0} \right) \frac{\alpha_{F}(t_2)-1}{e^{\frac{b_{sN}}{2} t_2}} \right) \right). \tag{4.8}
\]

Above \( s_{ik} = W_{ik}^2 \), where \( W_{ik} \) is the center-of-mass energy in the \((i,k)\) subsystem. The first terms describe the subleading reggeon exchanges while the second terms describe exchange of the leading (pomeron) trajectory. We have neglected real parts of the reggeon exchanges amplitudes for simplicity. The strength parameters of the \( \pi N \) interaction are taken from Ref. [32].

The extra form factors \( F(t_a) \) and \( F(t_b) \) ”correct” for off-shellness of the intermediate pion in the middle of the diagrams shown in Fig. 22. In the following they are parametrized as

\[
F(t) = \exp \left( \frac{t - m_{\pi}^2}{\Lambda_{off}^2} \right), \tag{4.9}
\]

i.e. normalized to unity on the pion-mass-shell. We take \( \Lambda_{off} = 1 \) GeV. More details of the calculation will be presented elsewhere [42]. The \( 2 \to 4 \) amplitude (4.7) is used to calculate the corresponding cross section including limitations of the four-body phase-space.

To excludes resonance regions we ”correct” the Regge parametrization (4.8) by multiplying byfactors \( F_{cut}(s_{ik}) \) (as in 2.31). In Fig 23 we show the two-pion invariant mass distribution of the double-diffractive background together with the \( f_0(1500) \) signal (\( \pi \pi \) fusion). We show three curves corresponding to different cuts on both \( W_{\pi N} \): \( W_{\text{min}} = 2.0 \) (solid), 1.9 (dashed), 1.8 (dotted) GeV. The figure suggests that the double-diffractive background should not disturb observing the \( f_0(1500) \) signal at the PANDA experiment energies.

In this case, unlike for the two-pion involved \( \rho \) exchange discussed in the previous subsection, imposing cuts on pion rapidities would not be helpful as the double-diffractive contribution is concentrated at midrapidities as shown in Fig 24. At lower HESR energies the situation is better, the double-diffractive two-pion background is relatively smaller.

V. DISCUSSION AND CONCLUSIONS

For the first time in the literature we have estimated the cross section for exclusive \( f_0(1500) \) meson (glueball candidate) production not far from the threshold. We have included both gluon induced diffractive and triangle-double-diffractive mechanisms as well as the pion-pion exchange contributions.

The QCD diffractive component was obtained by extrapolating down the cross section in the Khoze-Martin-Ryskin approach with unintegrated gluon distributions from the literature
FIG. 23: Pion-pion invariant mass distribution for $f_0(1500)$ (solid line) and the double-diffractive background (dashed lines) for different (sharp) cut-off for $W_{\pi N}$. Here the full phase space has been included. The calculation was performed for the highest PANDA center-of-mass energy $W = 5.5$ GeV.

FIG. 24: Rapidity distribution of pions from the decay of the glueball candidate $f_0(1500)$ (solid line) and from the double-diffractive background (dashed lines) for different (sharp) cut-off for $W_{\pi N}$. For the background we impose in addition: $1.4 \text{ GeV} < M_{\pi\pi} < 1.6 \text{ GeV}$. The calculation was performed for the highest PANDA center-of-mass energy $W = 5.5$ GeV.

as well as using two-gluon impact factor approach. A rather large uncertainties are associated with the QCD diffractive component. At present only upper limit can be obtained for the diffractive component as the $f_0(1500) \rightarrow gg$ decay coupling constant remains unknown. The
coupling constant could be extracted only in high-energy exclusive production of $f_0(1500)$ where other mechanisms are negligible.

We have found rather large contribution of pionic-triangle-double-diffractive component at higher energies ($W > 10$ GeV). However, at the GSI HESR energies this contribution is strongly damped because of the phase space limitations on the $\pi N$ subchannel energies.

A future experiment at RHIC could contribute to shed some light on the competition of the both diffractive mechanisms.

The calculation of the MEC contribution requires introducing extra vertex form factors. At largest PANDA energies they are relatively well known and the pion-pion fusion can be reliably calculated. The situation becomes more complicated very close to the threshold where rather large $|t_1|$ and $|t_2|$ are involved. The cross section for energies close to the threshold is very sensitive to the functional form and parameters of vertex form factor. Therefore a measurement of $f_0(1500)$ close to its production threshold could limit the so-called $\pi NN$ form factors in the region of exchanged four-momenta never tested before.

We predict the dominance of the pion-pion contribution close to the threshold. Our calculation shows that the diffractive components (in fact its upper limit for the QCD mechanism) are by more than order of magnitude smaller than the pion-pion fusion component in the energy region of future PANDA experiment.

The diffractive components may dominate over the pion-pion component only for center-of-mass energies $W > 15$ GeV. Taking into account rather large uncertainties the predictions of this component should be taken with some grain of salt. Clearly an experimental program is required to disentangle the reaction mechanism at energies $W > 15$ GeV.

Disentangling the mechanism of the exclusive $f_0(1500)$ production not far from the meson production threshold would require study of the $p\bar{p} \rightarrow p\bar{p}f_0(1500)$, $p\bar{p} \rightarrow n\bar{n}f_0(1500)$ processes with the PANDA detector at FAIR and $pp \rightarrow ppf_0(1500)$ reaction at J-PARC. In the case the pion exchange mechanism is a dominant process one expects: $\sigma(p\bar{p} \rightarrow n\bar{n}f_0(1500)) = 4 \times \sigma(p\bar{p} \rightarrow ppf_0(1500))$. At high energies, when the gluonic, or double-diffractive with intermediate triangle, components dominate over MEC components $\sigma(p\bar{p} \rightarrow p\bar{p}f_0(1500)) > \sigma(p\bar{p} \rightarrow n\bar{n}f_0(1500))$.

At intermediate energies one cannot exclude a priori subleading reggeon exchanges like $\rho\rho$ for instance. However, we do not know how to reliably calculate them from first principles. We believe that the distortions from the pion-pion at low energies and/or distortions from the QCD gluonic mechanism at high energy may tell us more and allow for a phenomenological analysis taking into account the $\rho\rho$ component explicitly. We leave this problem for a future analysis when experimental data will be available.

Only a careful studies of different final channels in the broad range of energies could help to shed light on coupling of (nonperturbative) gluons to $f_0(1500)$ and therefore would give a new hint on its nature. The experimental studies of exclusive production of $f_0(1500)$ are not easy at all as in the $\pi\pi$ decay channel one expects a large continuum. We have performed an involved calculation of the four-body $pp\pi^+\pi^-$ background. Our calculation shows that imposing extra cuts should allow to extract the signal of the glueball $f_0(1500)$ candidate at the highest PANDA energy. A partial wave $\pi\pi$ analysis should be helpful in this context. The two-pion continuum will be studied in more detail in our future work. A smaller continuum may be expected in the $K\bar{K}$ or four-pion $f_0(1500)$ decay channel. This requires, however, a good geometrical (full solid angle) coverage and high registration efficiencies. PANDA detector seems to fulfill these requirements, but planning real experiment requires a dedicated Monte Carlo simulation of the apparatus.
It is a central problem of our field if $f_0(1500)$ is a $q\bar{q}$ or glueball type. Unfortunately, our analysis does not allow to give a definite answer to this important question. Some information on baryon-baryon correlation may be helpful but certainly not decisive.

If the cross section at high energies (where the contribution of subleading reggeon exchanges may be neglected) is much smaller than predicted based on the KMR method it means that gluons only weakly couple to $f_0(1500)$. This could provide some indirect information on the $f_0(1500)$ structure. A direct comparison of the shape of differential distributions at high energies may provide a valuable test of the KMR method originally proposed for exclusive Higgs production (the latter experiment is very difficult as very small statistics is predicted). A possible disagreement with the prediction for exclusive $f_0(1500)$ production at high energies could put into question the KMR approach, at present state of art in the field. Experiments at RHIC could be useful in this context and could shed light on the nonperturbative coupling of gluons to $f_0(1500)$.

Acknowledgements We are indebted to Roman Pasechnik, Wolfgang Schäfer, Oleg Teryaev, Leonard Leśniak and Andrew Kirk for a discussion and Tomasz Pietrycki for a help in preparing some diagrams.

[1] C.J. Morningstar, M. Peardon, Phys. Rev. D60 (1999) 034509;
    A. Vaccarino, D. Weingarten, Phys. Rev. D60 (1999) 114501;
    Y. Chen et al, Phys. Rev. D73 (2006) 014516.
[2] A. Hart, C. McNeile, C. Michael, J. Pickavance, (UKQCD Collaboration), Phys. Rev. D74 (2006) 114504.
[3] A.H. Fariborz, Phys. Rev. D74 (2006) 054030;
    A.H. Fariborz, Int. J. Mod. Phys. A19 (2004) 2095.
[4] M. Albaladejo, J.A. Oller, Phys. Rev. Lett. 101 (2008) 252002, [arXiv:hep-ph/0801.4929v2].
[5] S. Narison, G. Veneziano, Int. J. Mod. Phys. A4 (1989) 2751;
    S. Narison, Nucl. Phys. B509 (1998) 312;
    S. Narison, Phys. Rev. D73 (2006) 114024.
[6] Tao Huang, Hong Ying Jin and Ai-lin Zhang, Phys. Rev. D59 (1998) 034026.
[7] L.S. Kisslinger, J. Gardner and C. Vanderstraeten, Phys. Lett. B410 (1997) 1.
[8] D. Harnett, T.G. Steele, Nucl. Phys. A695 (2001) 205.
[9] G. Orlandini, T.G. Steele, D. Harnett, Nucl.Phys. A686 (2001) 261.
[10] T.G. Steele, D. Harnett, G. Orlandini, AIP Conf. Proc. 688 (2004) 128, [arXiv:hep-ph/0308074].
[11] D. Harnett, K. Moats, T.G. Steele, [arXiv:hep-ph/0804.2195].
[12] C. Amsler and F.E. Close, Phys. Rev. D53 (1996) 295;
    F.E. Close, Acta Phys.Polon. B31 (2000) 2557.
[13] C. Amsler et al. (Crystal Barrel Collaboration), Phys. Lett. B327 (1994) 425;
    C. Amsler et al. (Crystal Barrel Collaboration), Phys. Lett. B333 (1994) 277;
    C. Amsler et al. (Crystal Barrel Collaboration), Phys. Lett. B340 (1994) 259.
[14] V.V. Anisovich, Phys. Lett. B364 (1995) 195.
[15] D. Barberis et al. (WA102 Collaboration), Phys. Lett. B462 (1999) 279.
[16] D. Barberis et al. (WA102 Collaboration), [arXiv:hep-ex/0001017].
[17] F.E. Close and A. Kirk, Phys. Lett. B397 (1997) 333;
    F.E. Close, A. Kirk and G. Schuler, Phys. Lett. B477 (2000) 13, arXiv:hep-ph/0001158.
[18] F.E. Close and G.A. Schuler, Phys. Lett. B458 (1999) 127;
    F.E. Close and G.A. Schuler, Phys. Lett. B464 (1999) 279.
[19] F.E. Close and Q. Zhao, Phys. Rev. D71 (2005) 094022.
[20] V.A. Khoze, A.D. Martin and M.G. Ryskin, Phys. Lett. B401 (1997) 330;
    V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C23 (2002) 311;
    A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C31 (2003) 387,
    arXiv:hep-ph/0307064;
    A.B. Kaidalov, V.A. Khoze, A.D. Martin and M.G. Ryskin, Eur. Phys. J. C33 (2004) 261;
    V.A. Khoze, A.D. Martin, M.G. Ryskin and W.J. Stirling, Eur. Phys. J. C35 (2004) 211.
[21] R. S. Pasechnik, A. Szczurek and O. V. Teryaev, Phys. Rev. D78 (2008) 014007, arXiv:hep-ph/0709.0857.
[22] A. Szczurek, R. S. Pasechnik and O. V. Teryaev, Phys. Rev. D75 (2007) 054021,
    arXiv:hep-ph/0608302.
[23] W. M. Yao et al. (Particle Data Group), Jour. Phys. G33 (2006) 1.
[24] T. Ericson and A. Thomas, Pions and Nuclei, Oxford University Press, 1988.
[25] N.I. Kochelev, T. Morii and A.V. Vinnikov, Phys. Lett. B457 (1999) 202.
[26] M. Luszczak and A. Szczurek, Phys. Rev. D73 (2006) 054028.
[27] M. Glück, E. Reya and A. Vogt, Z. Phys. C67 (1995) 433;
    M. Glück, E. Reya and A. Vogt, Eur. Phys. J. C5 (1998) 461.
[28] J.F. Gunion and D.E. Soper, Phys. Rev. D15 (1977) 2617;
    E.M. Levin and M.G. Ryskin, Sov. J. Nucl. Phys. 34 (1981) 619.
[29] A. Szczurek, N.N. Nikolaev and J. Speth, Phys. Rev. C66 (2002) 055206.
[30] D.V. Shirkov and I.L. Solovtsov, Phys. Rev. Lett. 79 (1997) 1209.
[31] P. Lebiedowicz and A. Szczurek, a paper in preparation.
[32] A. Donnachie and P.V. Landshoff, Phys. Lett. B296 (1992) 227.
[33] J.R. Pelaez and F.J. Yndurain, Phys. Rev. D69 (2004) 114001.
[34] L. Alvarez-Ruso, E. Oset and E. Hernandez, Nucl. Phys. A633 (1998) 519.
[35] T.E.O. Ericson, B. Loiseau and A.W. Thomas, Phys. Rev. C66 (2002) 014005,
    arXiv:hep-ph/0009312.
[36] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. 149 (1987) 1.
[37] A. Szczurek and J. Speth, Nucl. Phys. A555 (1993) 249;
    B. C. Pearce, J. Speth and A. Szczurek, Phys. Rep. 242 (1994) 193;
    J. Speth and A.W. Thomas, Adv. Nucl. Phys. 24 (1997) 83.
[38] A. Kirk, Phys. Lett. B489 (2000) 29.
[39] A. Breakstone et al. (ABCDHW Collaboration), Z. Phys. C31 (1986) 185.
[40] M. Melis, F. Murgia and J. Parisi, Phys. Rev. D70 (2004) 034021.
[41] J. Pumplin and F.S. Henyey, Nucl. Phys. B117 (1976) 377.
[42] P. Lebiedowicz and A. Szczurek, a paper in preparation.