3D line matching model and optimization for rigid pipeline assembly

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Abstract. Rigid pipeline assembly via welding has rich applications in mechanical engineering and civil engineering. The quality of assembly directly influences the reliability, performance, and lifecycle of products. In the assembly process, two ends of a rigid pipeline are required to be matched with another two ends of existing pipelines. Due to the installation errors, two ends of a rigid pipe may not match their corresponding targets. To mitigate the dismatch, the following two issues need to be addressed: (i) matching the axis lines of the ends of the moving pipe and the axis lines of the ends of the existing pipes; and (ii) maximizing the overlap areas of each end of the moving pipe and its corresponding target on an existing pipe. The mathematical model of rigid pipeline assembly problem is established and a method for computing a nice initial position of the moving pipeline with respect to installed pipes is proposed. An iterative optimization algorithm are elaborated to address the problems. Using a running example, it is shown that our proposed methods are efficient for solving the problems.

1. Introduction
Rigid pipeline assembly via welding is widely used in the fabrication processes of aircrafts, ships, and pressure vessels and so on [1, 2]. The quality of rigid pipeline assembly affects the reliability, performance and lifecycle of the assembled products. For simplicity, we abuse the notation a bit by calling the ends of the installed (existing) pipeline interfaces, while the ends of a rigidly moving pipeline orifices. In a typical installation, see Fig. 1, the orifices of a pipeline are installed onto a pair of interfaces (located on the main body of a device). However, due to the installation errors, the interfaces may differ from their ideal designated positions. As a result, the orifices cannot match their corresponding interfaces perfectly. See Fig. 1 (b) for an illustration, as the interfaces differ by a small angle, the orifices on the rigid pipeline cannot match with the interfaces through the design positions.
A point matching optimization algorithm has been used in the field of the lightweight space-frame-structures which are increasingly used in the automobile and aircraft industry today [3]. The key geometric data between the actual-profiles derived from on-site measurement data and the target-profiles from the CAD model can be available. With the implemented optimization tool, an analysis of the measurement data can be performed and a flexible strategy optimization for the space-frame structure can be carried out [4, 5]. Observe that the interfaces and the orifices are cylinders; their matching can be simplified as the matching of the axis lines of the cylinders. Formally, this problem is called line matching [6]. Line matching is a basic tool in computer vision that has applications in scene registration, object localization, motion estimation and so on. Faugeras and Hebert [7] present the first algorithm for matching 3D line sets using an objective function that is a sum of line orientations and locations. Zhang and Faugeras [8, 9] subsequently proposed an improved algorithm for motion estimation based on Kalman filter and quaternion. Walker et al. [10] and Daniilidis [11] also proposed a solution based on dual quaternion for motion estimation and robot Hand-Eye Calibration respectively.

Fig.1 An illustration of rigid pipe assembly: (a) a real aerospace engine; (b) the CAD model

Throughout the paper, assume without loss of generality that the radius of the cross-section of the moving pipe is equal to that of the fixed pipe. See Fig.2, for the welding purpose, normally the ends of the rigidly moving pipe are longer than the design model, therefore some portion of each end is to be cut off in order to match the corresponding interface of the installed pipe. See Fig.2, the angular disparity between the axis lines of the pipes $\theta$ and the region to be cut off (shown in hatched) together affect the matching effect on the welding plane. Formally, welding a rigidly moving pipe to as fixed pipe involves the following two sub-problems:

(i) Matching the axes of the interfaces and those of the orifices;
(ii) Matching the ends of interfaces and those of the orifices on the welding plane.

Fig.2 Matching an end of a rigidly moving pipe to match an interface of an installed pipe.
In this paper, we shall provide an iterative optimization algorithm for solving these problems. Different from existing methods that only focus on (i), our objective involves maximizing the overlap areas of the interfaces and orifices. The remainder of this paper is organized as follows: The mathematical model of rigid pipeline assembly problem is established in Section 2; in Section 3, a method for computing a nice initial position of the pipeline with respect to the interfaces is proposed and an iterative optimization algorithm is proposed; Section 4 concludes the paper with some discussions.

2. Mathematical models for rigid pipeline assembly problem
For ease of description, a set of parameters are defined as follows (Fig.3):

- \( a \): the intersection of interface A and fixed axis A;
- \( a_0 \): a point other than \( a \) on fixed axis A that is interior to the fixed pipe;
- \( b \): the intersection of interface B and fixed axis B;
- \( b_0 \): a point other than \( b \) on fixed axis B that is interior to the fixed pipe;
- \( c_0 \): the intersection of orifice A and moving axis A;
- \( c \): a point other than \( c \) on moving axis A that is interior to the moving pipe;
- \( d_0 \): the intersection of orifice B and moving axis B;
- \( d \): a point other than \( d_0 \) on moving axis B that is interior to the moving pipe;

![Fig.3 A set of parameters defined on the pipes.](image)

The rigidly moving pipe has 6 degrees of freedom including rotations and translations; its current position \( P' \) upon a rotation \( R \) and translation \( T \) from its initial position \( P \) can be expressed as \( P'=RP+T \), where \( R \) is a standard 3×3 rotation matrix, and \( T \) is a 3D translation vector.

For problem (i), the objective function of minimizing the maximum angular disparity of the axis lines is defined as follows:

\[
M^\theta(R) = \min \max \left( M^\theta_A(R), M^\theta_B(R) \right)
\]  
(1)

Where, 
\[
M^\theta_A(R) = \arccos \left( \frac{\vec{a}_0 \cdot (R \vec{c}_0 \vec{c})}{\|\vec{a}_0\| \|R \vec{c}_0 \vec{c}\|} \right),
M^\theta_B(R) = \arccos \left( \frac{\vec{b}_0 \cdot (R \vec{d}_0 \vec{d})}{\|\vec{b}_0\| \|R \vec{d}_0 \vec{d}\|} \right)
\]

Problem (ii) focuses on local deviation of the dismatch between the two pairs of interfaces and orifices. The objective function can be modeled by the following two models.

**Model 1: minimizing dismatch on the welding plane**

**Model 2: maximizing overlap area on the welding plane**

Assume for simplicity that each pair of fixed axis and moving axis are coplanar, a simplified model is illustrated in Fig. 4(a). See Fig. 4(b), on a welding plane, a pair of interface and the orifice may not
perfectly overlap each other. Instead of reducing the amount of dismatch, we try to maximize the overlap area between the interface and the orifice.

![Diagram](image)

**Fig. 4** Pipeline assembly problem models: (a) Minimizing the dismatch; (b) maximizing overlap area

### 3. Iterative optimization algorithm

In this section, we show how to compute the relative position of the moving pipe and the fixed pipe in order to solve problems (i) and (ii) mentioned above. We begin with an initialization of rotation matrix $R_0$ and translation vector $T_0$, and then to minimize the objective function, an iterative optimization algorithm is proposed.

#### 3.1. Initialization of Rotation matrix $R_0$

See Fig. 5 (a), by translation, the origins of the unit vectors of moving axes and the origins of the unit vectors of fixed axes can be coincident at a common point, e.g., the origin of the coordinate frame. $M^0$ can be determined by angular bisector and the normal of the plane as shown in Fig. 5 (b). Fig. 5 (c) shows the resulting position of the moving pipe after applying the rotation matrix $R_0$. It can be seen that the moving axes are almost parallel to the fixed ones.

![Diagram](image)

**Fig. 5** The relative position of the fixed pipe and moving pipe after using rotation matrix $R_0$

#### 3.2. Initialization of Translation vector $T_0$

After applying the rotation matrix $R_0$, the orifices might not overlap any orifices. In this section, we present a translation vector $T_0$ that guarantees each moving axis being as close to the "center" of its corresponding interface (a or b) as possible.
After applying $R_0$ and $T_0$ to the initial position of the moving pipe as shown in Fig.6(a), the resulting position of the moving pipe is shown in Fig.6(b).

### 3.3. Iterative optimization algorithm

After applying rotation matrix $R_0$ and translation vector $T_0$ obtained in section 3.1 and 3.2 respectively, each moving axis is almost parallel to its fixed counterpart and the "center" of the orifice is close to the "center" of its corresponding interface. From Fig.6, we can see that there is still a large mismatch between a pair of interface and orifice on their welding plane. A close look of the welding plane $A$ is shown in Fig.7.

![Image](image.png)

**Fig.6** The interfaces and orifice after using rotation matrix $R_0$ and translation vector $T_0$

In this section, we shall show how to minimize this mismatch using an iterative algorithm. by using a randomly perturbation matrix $R$ to refine the position of the moving pipe. Fig.8 shows that the relative position of the fixed pipe and moving pipe after running. Formally, the iterative procedure is presented as follows:

Procedure: $\text{Iterative\_Procedure}(R_0, T_0)$

Step 0: $R^0 := R_0$, $T^0 := T_0$, $R^{best} := R^0$, $T^{best} := T^0$, $i := 0$, count := 0, $\text{count\_max} := 20$, $\text{max\_iter} := 200$, $R_C := 0.001$:

Compute $M^{area}(R^0, T^0)$;

Step 1: if $i = \text{max\_iter}$ or $\text{count} = \text{count\_max}$ then goto Step 5;

else

![Image](image.png)

**Fig.7** The detail geometric information on the welding plane $A$
i:=i+1, randomly generate $\alpha, \beta, \gamma\ (0<\alpha, \beta, \gamma<R_D)$, $R^i:= R^{i-1} \ast R_\nu$;

Step 2: Compute $M^i(R^i)$;
if $M^i(R^i)> M^i(R_0) + R_C$ then goto Step 2;
Step 3: Compute $M^\text{area}(R^i, T^i)$;
if $M^\text{area}(R^i, T^i) < M^\text{area}(R_{\text{best}}, T_{\text{best}})$ then
$R^i := R_{\text{best}}$, $T^i := T_{\text{best}}$, count := count + 1;
else count:= 0;
go to step 2;
Step 4: return $R=R_{\text{best}}, T=T_{\text{best}}$.

![Diagram showing relative position of fixed and moving pipes](image)

**Fig. 8** Shows that the relative position of the fixed pipe and moving pipe after running procedure

4. **Conclusion**

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Rigid pipeline assembly via welding is an important problem with lots of applications. Due to the installation errors, the orifices cannot be assembled to their corresponding interfaces correctly since the interfaces may differ a bit from their design positions. The methods proposed in this paper efficiently solve the problem by aligning the axis lines two pairs of orifices and interfaces, as well as the overlap areas of the two pairs. A running example is studied, and it is shown that the proposed methods are quite efficient for solving the problems.

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