A Rewriting Logic Semantics and Statistical Analysis for Probabilistic Event-B

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Abstract. Probabilistic specifications are fast gaining ground as a tool for statistical modeling of probabilistic systems. One of the main goals of formal methods in this domain is to ensure that specific behavior is present or absent in the system, up to a certain confidence threshold, regardless of the way it operates amid uncertain information. This paper presents a rewriting logic semantics for a probabilistic extension of Event-B, a proof-based formal method for discrete systems modeling. The proposed semantics adequately captures the three sources of probabilistic behavior, namely, probabilistic assignments, parameters, and concurrency. Hence, simulation and probabilistic temporal verification become automatically available for probabilistic Event-B models. The approach takes as input a probabilistic Event-B specification, and outputs a probabilistic rewrite theory that is fully executable in PMaude and can be statistically tested against quantitative metrics. The approach is illustrated with examples in the paper.

Keywords: Probabilistic Event-B · Statistical model checking · PVeStA

1 Introduction

For many systems, there is an obvious need for using specialized formal methods in the spirit of formalisms, inference systems, and simulation techniques for selected tasks. When properly combined, formal methods have a great potential to become more useful in practice, and scale up because of modularization and specialization of needs. In the realm of probabilistic systems, where a vast number of randomized algorithms and protocols fall, both inference- and algorithmic-based analysis techniques are needed to answer the key question of whether such systems are correct. Probabilistic systems must behave properly, up to a confidence threshold, regardless of the way they operate amid uncertain information. A challenging task is to find combinations of formalisms, inference systems, and simulation techniques for verifying such systems.

This paper focuses on probabilistic simulation and statistical analysis for a probabilistic extension of Event-B [1], a formal method for system-level modeling and analysis. Event-B uses set theory as a modeling notation, refinement to represent systems at different abstraction levels, and mathematical proofs to verify consistency between refinement levels (and other proof obligations). The probabilistic extension of Event-B proposed by [6] is based on three mechanisms. First, interaction with the external environment is governed by a probabilistic choice; that is, external inputs are chosen uniformly from finite sets representing the potential values for the variables under the
control of the external environment. Second, the extension includes probabilistic assignment to variables (representing the state of the system) by uniform choice from finite sets. This is useful to handle the uncertainty associated to “external” data. Third, all concurrent transitions in a system are probabilistic. That is, in the presence of the two first mechanisms, concurrency yields a probabilistic transition system where each possible transition from a state is weighted by a probability measure. The authors of [8] have proposed an inference system for reasoning within this probabilistic extension of Event-B. The purpose of the presented paper is to complement the work in [8] by enabling algorithmic simulation and statistical model checking.

This paper develops a rewriting logic semantics for the above-mentioned probabilistic extension of Event-B. It takes as input a probabilistic Event-B model and outputs a probabilistic rewrite theory [4]. The mapping implementing this translation is explained in detail, including its soundness and completeness properties for simulation in relation to the given model. The resulting probabilistic specification is executable in Maude [11] and it is amenable to simulation-based verification such as, e.g., statistical model checking with help of the PVeStA model checker tool [5]. The translation has been fully automated and it supports a wide range of Event-B operators.

The approach presented here can be seen as a complement to the proof-theoretic techniques developed in [8]. It allows system designers to experiment with the system via simulation and automatically verify system’s properties via (stochastic) model checking, thus gaining more confidence on the model before embarking on a proving task. It is known that Event-B offers both proof-theoretic methods, and simulation and model checking tools (via, e.g., the ProB animator and model checker in the Rodin platform [1,14]) for system modeling. These latter features do not exist, to the best of the authors’ knowledge, for probabilistic Event-B models. Ultimately, the developments presented here open new opportunities for incarnating the more ambitious long-term ideal of combining formalisms, inference systems, and simulation techniques for verifying probabilistic systems.

Outline. Section 2 presents an overview of the probabilistic extension of Event-B. Section 3 develops the mapping from Event-B models to probabilistic rewrite theories and studies its main formal properties. Section 4 showcases an example of the transformation and its statistical analysis. The web page of the companion tool [17] of this paper presents other case studies and provides some experimental results. Finally, Section 5 concludes the paper.

2 Probabilistic Event-B in a Nutshell

This section presents an overview of the probabilistic extension of Event-B proposed by [8]. As it is the case for the non-probabilistic case, a probabilistic Event-B model consists of a context (§2.1) and a machine (§2.2).

2.1 Contexts

A context in Event-B describes the constant part of the system, including the definition of deferred sets and constants. More precisely, a context is a triple $\mathcal{C} = \langle SET_1 \cup SET_n, CTE \rangle$ where $SET_1$ is a set of set declarations of the form $S : \{id_1, \ldots, id_m\}$, $SET_n$
is a collection of set declarations of the form \( S : n \) (where \( n \) is a natural number), and \( CTE \) is a set of constant definitions of the form \( c : type := v \).

**Deferred sets.** A typical context can define any number of deferred sets, and axioms indicating either that any such a set is finite (but its cardinality is unknown) or has a fixed cardinality \( n \). The former construction is not considered here, while the latter is supported via the \( SET_n \) component. For instance, the declaration \( ENUM:3 \) generates three constant symbols (e.g., \( ENUM_i \), for \( 1 \leq i \leq 3 \)) that inhabit the set \( ENUM \). Moreover, the deferred set declaration \( STATE:\{ \text{open}, \text{close} \} \) defines the set \( STATE \) inhabited exactly by the two (distinct) constant symbols \( \text{open} \) and \( \text{close} \).

**Types, values, and constants.** The syntax \( \text{DEFAULT}:STATE := \text{open} \) defines the constant \( \text{DEFAULT} \) of type \( STATE \) with value \( \text{open} \). Basic types include deferred sets, Boolean values, and integer numbers. Types can be also built via Cartesian products \( (A \times B) \) and powersets \( \text{POW}(A) \). From those constructions, relations and (partial, total, injective, etc.) functions can be defined as expected. For instance, the set-theoretic language of Event-B reduces \( r \in A \leftrightarrow B \) \( (r \) is a relation from \( A \) to \( B \)) into \( r \subseteq A \times B \) and finally into \( r \in \text{POW}(A \times B) \). For succinctness, \( A \) and \( B \) in \( A \times B \) are restricted here to be basic types.

According to the types above, the values for constants can be elements of a deferred set, numbers, Booleans, pairs \( (a \mapsto b) \), set of values as in \( \{a, b, c\} \), or integer intervals as in \( 1..42 \). The complete grammar can be found in [17].

As a running example, consider the controller for a landing gear system modeled in Event-B in [9] and probabilized in [6]. When landing, the following sequence of actions occur: the doors of the system are opened, the landing gears are extended, and then the doors are closed. Similarly, after taking off, the doors are opened, the gears retracted, and the doors are closed. The pilot may initiate and interrupt these sequences with a handle that can be in two positions: \( \text{up} \) (executing the retracting sequence) and \( \text{down} \) (extending sequence). The context \( \text{GEAR_CTX} \) defines the three needed deferred sets and a constant of type \( \text{Nat} \) that will be used in the next section.

### 2.2 Machines

A **machine** in Event-B specifies a set of variables, defining the state of the system, and the system’s actions, called **events**. More precisely, a machine is a structure of the form \( M = \langle G, \bar{x}, I, \Delta, \text{init} \rangle \) where \( G \) is a context, \( \bar{x} \) is a set of variables typed by the invariant \( I \), \( \Delta \) is a set of probabilistic events, and \( \text{init} \) is the initialization event.

**State of the machine.** In an Event-B specification, the section SEES of a machine determines the context \( G \) accessible by the model. For simplicity, it is assumed here that each machine sees exactly one context. The section VARIABLES contains a list of identifiers. The types (or domains) for each variable (e.g., \( \text{currentState}:STATE \)) are defined in the section INvariants. Besides typing information, Event-B models include also invariant properties that must be preserved along the system’s transitions. The translation in Section[3] requires only the types for the variables and hence, for the moment, only invariants of the form \( x : A \) (the type of \( x \) is \( A \)) are considered. Section[2] shows how the infrastructure presented here can be used to verify some other properties.
Events. The machine initialization is deterministic and the section INITIALISATION assigns values (of the appropriate type) to all the variables (e.g., currentState := open). Such values are built from elements of deferred sets, constants in the context, standard arithmetic and Boolean operations, and expressions on sets and relations (see [1] Chapter 6) for the complete set-theoretic language.

The system’s actions are defined by events, which are composed of guards and actions. In the case of probabilistic models, each event is also assigned a weight. At a given state, the numerical expression \( W \) determines the weight of the event with identifier \( ID \). Such an expression is built from constants, the machine’s variables, and arithmetic operations. As explained later in the semantics, events with higher weights are more likely to be chosen for execution.

The variables declared in the optional section ANY are called parameters; they represent an interaction with an external environment, which is not under the control of the system. Hence, the behavior of the event is not fixed but it may have different outcomes depending on the values chosen for those parameters. The expressions \( y \in S \) states that the parameter \( y \) may take values from the non-empty and finite set \( S \). The set \( S \) can be an arbitrary expression returning a set, including for instance: set comprehension \( \{ x \in S : P(x) \} \), elements of \( S \) that satisfy the predicate \( P \) and \( \{ x \in S : F(x) \} \) defining the set \( \{ F(x) : x \in S \} \); the Cartesian product; domain of a relation; range restriction \( (r \triangleright S = \{ x \mapsto y \in r : y \in S \}) \); domain subtraction \( (S \triangleleft r = \{ x \mapsto y \in r : x \not\in S \}) \); over-riding \( (r \triangleright s = s \cup (\mathrm{dom}(s) \triangleleft r)) \); etc. The complete list of expressions involving set/relations (union, membership, range restriction, cardinality, etc.) currently supported by our tool can be found at [17]. Semantically, in \( y \in S \), a value from \( S \) is chosen with a uniform probability distribution and assigned to the parameter \( y \).

The Boolean expression \( G \) in the WHERE section determines whether the event can be fired at a given state or not. The action of the event determines the new state and it is specified by a list of simultaneous assignments in the THEN section. It is assumed that the right hand side (RHS) of an assignment is an expression of the appropriate type including constants and variables (considering the values before the assignment), as well as the event’s parameters. When the RHS is a list of expressions, a uniform probabilistic distribution is assumed for them. Moreover, it is also possible to use an enumerated probabilistic assignment as in \( x : = \{ \text{open} @ 0.7, \text{close} @ 0.3 \} \), specifying that \( x \) may take the value \( \text{open} \) (resp., \( \text{close} \)) with probability 0.7 (resp., 0.3).

In the running example, the pilot may use the handle to initiate and interrupt the extending and retracting sequences. The machine GEAR uses three variables to observe the current state of the handle, the gear, and the doors. The variable cmd controls the number of times the pilot has initiated the sequence.

The event extend in Figure [I] models the extension of the gear when the handle is down and the doors are opened. As a model of failures, there is a 10% of risk that the gears do not react correctly to their command. The event retract can be explained similarly. The state of the doors are controlled by the events open and close. For in-
Fig. 1: Events controlling the gear and the doors

stance, if the doors are closed, the gear retracted, and the current command is extend (handle=down), the doors are opened with probability 0.9.

The interface of the system with the pilot is modeled with the event pcmd below. The requirements of the system impose that: the pilot cannot command the handle more than a fixed number of times before one of the sequences begins; and consecutive uses of the handle must decrease the priority of using it again. Since the pilot may interrupt an already started sequence, this event modifies the state of the handle with equal probability to up and down. Due to the weight of the event, such changes are allowed only up to FCMD times (the constant defined in the context GEAR_CTX).

2.3 Probabilistic Semantics

The state of a machine is a valuation \( s \) mapping variables and constants to values of the appropriate type. For a variable or constant \( x \), \( s(x) \) denotes the value of \( x \) in \( s \). Given an expression \( E(\vec{x}) \) that may depend on the (list of) variables \( \vec{x} \), the evaluation of \( E \) in the context \( s \) is denoted as \( E(\vec{x})[s] \). Moreover, for an event \( e \in \mathcal{E} \), \( \text{var}(e) \) is the set of variables that appear on the left hand side (LHS) of the assignments in the action of the event. In each event, a variable can appear only once as LHS in the list of assignments.

An event \( e = \langle w(\vec{x}), \vec{y}, G(\vec{x}, \vec{y}), \text{act} \rangle \in \mathcal{E} \) is \textit{enabled} at state \( s \) if \( w(\vec{x})[s] > 0 \) and there exists a valuation \( \sigma \) for the parameters \( \vec{y} \) making the guard true, i.e., \( G(\vec{x}, \vec{y})[s][\sigma] = \text{true} \). Then, the action \( \text{act} \) can be performed. Given a state \( s \), \( \mathcal{E}(s) \) denotes the set of enabled events at \( s \). The notation \( w_e \) is used to denote the weight expression of the event \( e \); similarly for the other components of the event.

The semantics of a machine \( \mathcal{M} \) is a probabilistic labeled transition system (PLTS) \( \langle S, s_0, \text{Acts}, T \rangle \) where \( S \) is the set of states, \( s_0 \) is the valuation obtained after executing the initialization, \( \text{Acts} \) is the set of labels of the events, and \( T : S \times \text{Acts} \times S \rightarrow [0, 1] \) is the transition probability function defined as follows:
\[(s, e, s') = \begin{cases} \frac{w_e[s]}{\sum_{e' \in E} w_e[s']} \times \sum_{\sigma \in \mathcal{T}(s, e)} \frac{1}{|\mathcal{T}(s, e)|} \times \prod_{x \in \text{val}(e)} \sum_{\sigma' \in \text{val}(s, s', \sigma, e)} P(E) & \text{if } s \not\rightarrow s' \\ 0 & \text{otherwise} \end{cases}\]

where: \(s \not\rightarrow s'\) means that either \(s' = \emptyset\) or \(x[s] \neq x[s']\) for a variable \(x\) not in \(\text{var}(e)\) (i.e., \(s\) and \(s'\) differ on a variable not modifiable by the event); \(\mathcal{T}(s, e)\) is the set of valuations for the parameters of the event \(e\) that make its guard true, i.e., \(\sigma \in \mathcal{T}(s, e)\) iff \(G_e(x, \bar{y})[s] = true\); \(\text{Val}(x, s, s', \sigma, e)\) is the set of expressions on the RHS of the assignment that assign to \(x\) its value in \(s',\) i.e., \(E \in \text{Val}(x, s, s', \sigma, e)\) iff \(E[s] = x[s']\); and \(P(E)\) is the probability of choosing the expression \(E\) among all the other expressions.

The work in [8] establishes conditions for the PLTS generated by an Event-B specification to be a Discrete Time Markov Chain (DTMC). For that, some proof obligations must be discarded: (1) the weight of the events are natural numbers; and (2) in a probabilistic assignment \(x := \{E_1@p_1, \ldots, E_n@p_n\}, 0 < p_i \leq 1\) and \(\sum p_i = 1\). In that case, it is possible to show that for all states \(s, \sum \{p \mid s \stackrel{(p,e)}{\rightarrow} s'\} = 1\). These conditions are assumed to be true in the rest of the paper for any machine \(\mathcal{M}\).

### 3 Translating Machines to Probabilistic Rewrite Theories

Rewriting logic (RL) [15] (see a survey in [16]) is a general model for concurrency where systems are declaratively specified using algebraic data types and (conditional) rewrite rules. This section presents a map \(\llbracket \cdot \rrbracket : \mathcal{M} \rightarrow \mathcal{R}_{\mathcal{M}}\) from a probabilistic Event-B machine to a probabilistic rewrite theory [4].

The rewrite theory \(\mathcal{R}_{\mathcal{M}}\) is obtained in two steps. First, it contains (as a subtheory) a rewrite theory \(\mathcal{R}\) defining sorts and operations to represent declared types/sets and constants of any context, as well as the infrastructure needed to encode the variables and the events of any probabilistic machine. Second, \(\mathcal{R}\) is extended with rules specific for the machine \(\mathcal{M}\). Computation with \(\mathcal{R}_{\mathcal{M}}\) is shown to be free of un-quantified non-determinism (a condition needed for statistical analysis [4]), as well as to be sound and complete w.r.t. the probabilistic semantics of \(\mathcal{M}\).

In the following sections, the background on RL needed to understand the translation is introduced gradually. In most of the cases, the notation of Maude [11], a high-level language that supports membership equational logic and rewriting logic specifications, will be adopted. This has the immediate effect of producing an executable specification. For the sake of readability, some details of the specification are omitted.
The complete specification of the theory $\mathcal{R}_m$ is available at [17] as well as a parser automatizing the translation $[\cdot]$.

3.1 Translating Contexts

A rewrite theory is a tuple $\mathcal{R} = (\Sigma, E \cup B, R)$. The static behavior of the system is modeled by the order-sorted equational theory $(\Sigma, E \cup B)$ and the dynamic behavior by the set of rewrite rules $R$ (more on this in §3.2). The signature $\Sigma$ defines a set of typed operators used to build the terms of the language (i.e. the syntax of the modeled system). $E$ is a set of (conditional) equations over $T_\Sigma$ (the set of terms built from $\Sigma$) of the form $t = t'$ if $\phi$. The equations specify the algebraic identities that terms of the language must satisfy (e.g., $x + 0 = x$). Moreover, $B$ is a set of structural axioms (associativity, commutativity, and identity, or combinations of them) over $T_\Sigma$ for which there is a finitary matching algorithm. The equational theory thus defines algebraic data types and deterministic and finite computations as in a functional programming language.

Let’s start defining appropriate sorts (types) and operators to specify an Event-B contexts $C = \langle \text{SETl} \cup \text{SETn}, \text{CTE} \rangle$. Natural and integer numbers, as well as Booleans, are mapped directly to the corresponding sorts in Maude. Any other constant symbol inhabiting the deferred sets $\text{SETl}$ and $\text{SETn}$ (e.g., open, close) is represented as a string.

The sort $\text{EBElt}$ below defines the values for basic types:

```
--- Functional module EBElt (equational theory)
sort EBElt .

--- Basic values
op elt : Int -> EBElt .
--- Nat and Int
op elt : Bool -> EBElt .
--- Boolean
op elt : String -> EBElt .
--- Elements of deferred sets
op _<_ : EBElt EBElt -> Bool .
--- Order on EBElt
[...]

--- Total order on EBElt
view EBElt< from STRICT-TOTAL-ORDER to EBElt is sort Elt to EBElt . endv
```

Terms $\text{elt(“open”)}$ and $\text{elt(9)}$ of sort $\text{EBElt}$ represent, respectively, the constant symbol $\text{open}$ in the deferred set $\text{SOC}$ and the number $9$. The equations defining the strict total order $<_<$ (underscores in Maude denote the position of the parameters in an operator) are defined as expected for elements of the same type (e.g., for two integers $n, m$, $\text{elt(n)} < \text{elt(m)} = n < m$). Although elements of different types are not supposed to be compared in (well-typed) machines, the definition of $<$ decrees that $\text{elt(s)} < \text{elt(b)}$ for any string $s$ and Boolean $b$ and $\text{elt(b)} < \text{elt(n)}$ for any number $n$.

```
--- Sets of EBElt
pr SORTABLE-LIST-AND-SET(EBElt<) * (sort Set(EBElt<)) to EBSet .

op .. .. : Int Int -> EBSet .
--- Building finite subsets of Int
op gen-set : String Nat -> EBSet .
--- Building indexed (string) constants
op gen-set : List(String) -> EBSet .
--- Building a EBSet from a list of strings
[...]
```

Sortable lists allow to uniquely represent sets as lists, which is key for having quantified non-determinism. The Maude’s theory $\text{SORTABLE-LIST-AND-SET}$ is instantiated with the total order $\text{EBElt<}$ defined above and the sort $\text{Set(EBElt<)}$ is renamed to $\text{EBSet}$. The defined operators, and their equations omitted here, are used to encode deferred sets by enumeration, cardinality or as integer intervals: $\text{gen-set(“open” “close”)}$ reduces to $\{\text{elt(“open”), elt(“close”)}\}$; $\text{gen-set(s,n)}$ reduces to the set $\{\text{elt(s1), ..., elt(sn)}\}$ and the term $n .. m$ reduces to $\{\text{elt(n), ..., elt(m)}\}$.

Pairs of $\text{EBElt}$s as well as (sortable) lists and sets of pairs are built as follows:
The module \textsc{EBRELATION} implements most of the operations on sets and relations defined in the syntax of Event-B. However, as explained in Section 3.2, the operations and equations needed to specify expressions including set comprehensions ($\{x.S \mid P(x)\}$ and $\{x.S \mid F(x)\}$) are generated according to the Event-B model at hand.

Values for constants and variables are terms of sort \textsc{EBType} built from (possibly singleton) sets of \textsc{EBElt}s or \textsc{EBPair}s:

\begin{verbatim}
mod \textsc{EB-TYPE} is
  | pr \textsc{EBRELATION}.
  sort EBType .
  op val : EBSet -> EBType . --- Sets of basic types
  op val : EBRel -> EBType . --- Sets of pairs
endm
\end{verbatim}

Hence, the value of the Event-B constant \textsc{FCMD} is encoded as the term \texttt{val(elt(9))} while a function \texttt{f} of type $1 \times 2 \times \textsc{SOC}$ might be assigned the value \texttt{val( (elt(1) \mapsto \text{elt("open")}, elt(2) \mapsto \text{elt("close")} ) ).}

All Boolean and arithmetic operations, and operations on sets and relations are lifted to operators of sort \textsc{EBType} ad-hoc. For instance, \texttt{val(elt(2))+val(elt("open"))} reduces to \texttt{val(elt(3)+elt(2))} that, in turn, reduces to \texttt{val(elt(3+2))}[3]. The theory \textsc{EB-TYPE} thus defines an encoding $\llbracket \cdot \rrbracket_e$ from Event-B expressions into \textsc{EBType} expressions where Event-B (arithmetic, Boolean, relational, and set) operators and values are mapped into the corresponding terms defined in \textsc{EB-TYPE}. Most of the cases in the definition of $\llbracket \cdot \rrbracket_e$ are immediate. The interesting cases will be introduced gradually next.

An Event-B context is specified as a mapping from the identifiers of the deferred sets to \textsc{EBSet}s and context’s constants to their values in the sort \textsc{EBType}. This is the purpose of the following theory:

\begin{verbatim}
mod \textsc{EBCONTEXT} is
  | pr \textsc{CONFIGURATION} . pr \textsc{EB-TYPE} . pr MAP(Qid, EBSet) . pr MAP(Qid, EBType) .
  subsort Qid < Oid . --- Names for contexts
  op Context = : Cid . --- Class for contexts
  op sets :- : Map(Qid , EBSet) -> Attribute . --- Context’s user-defined sets
  op constants :- : Map(Qid , EBType) -> Attribute . --- Context’s constants
  op init-context : Qid -> Object . --- Building a context
  var Q : Qid .
  eq init-context(Q) = < Q : Context | sets : init-sets, constants : init-constants > .
  --- Operators to be instantiated by the implementation of the context
  op init-sets :- -> Map(Qid , EBSet) . --- Building deferred sets
  op init-constants :- -> Map(Qid , EBSet) . --- Initializing constants
endm
\end{verbatim}

It is customary in Maude to represent complex systems by using an object oriented notation (theory \textsc{CONFIGURATION}). An object \texttt{O} of a class \texttt{C} is represented by a record-like structure of the form $\langle O : C \mid a_1 : V_1, \ldots, a_n : V_n \rangle$, where $a_i$ are attribute identifiers.

---

3 Operators of sort \textsc{EBType} are certainly partial and a term such as $\texttt{val(elt(2))} + \texttt{val(elt("open"))}$ does not have any reduction. It is worth noticing that such terms cannot appear in the encoding of a well-typed Event-B model.
and \(v_i\) are terms that represent the current values of the attributes. The state of the system is then a multiset of objects called configuration. In the above theory, quoted identifiers (e.g., 'GEAR_CTX) are used as object identifiers (subsort Qid < Oid). Moreover, a new class identifier (Cid) is defined along with the needed attributes. The term init-context(Q) encodes a context with identifier Q and attributes sets and constants.

Logical variables as context(Q). The theory EBCONTEXT is common to any Event-B model and each particular context needs to extend it with equations populating the definition of the deferred sets and also initializing the constants. For instance, in the running example, the following equations need to be added:

\[
\begin{align*}
\text{eq init-sets} & = \{ \text{'SUD} \rightarrow \text{gen-set("up" "down")}, \\
& \quad \text{'SOC} \rightarrow \text{gen-set("open" "close")}, \\
& \quad \text{'SER} \rightarrow \text{gen-set("extended" "retracted")}, \\
\text{eq init-constants} & = \{ \text{'FCMD} \rightarrow \text{val(elt(9))} \}.
\end{align*}
\]

**Definition 1 (Encoding of Contexts).** Let \(G = \langle \text{SET}_1, \text{SET}_m, \text{CTE} \rangle\) be an Event-B context. The theory \(\mathcal{R}_G\) results from extending EBCONTEXT with the following equations:

\[
\begin{align*}
\text{init-sets} & = \{ \text{id}_1 \rightarrow \text{gen-set(id}_1, \ldots, \text{id}_n) \} \cup \{ \text{id}_k \rightarrow \text{gen-set(id}_k, k \in \text{SET}_k) \} \\
\text{init-constants} & = \{ c \mapsto [e]_c \mid (c : \text{type} := e) \in \text{CTE} \}.
\end{align*}
\]

### 3.2 Encoding Machines

The specification of a machine \(\mathcal{M}\) in the theory \(\mathcal{R}_G\) requires some extra sorts and equations, but also, (probabilistic) rewrite rules. Before continuing with the encoding, the meaning of the set of rewrite rules \(R\) in \((\Sigma, E \cup B, R)\) is explained.

**Probabilistic rewrite rules.** A rewrite rule \(l(\vec{x}) \rightarrow r(\vec{x})\) if \(\vec{\phi}(\vec{x})\) specifies a pattern \(l(\vec{x})\) that can match some fragment of the system’s state \(t\) if there is a substitution \(\theta\) for the variables \(\vec{x}\) that makes \(\theta(l(\vec{x}))\) equal (modulo the set of structural axioms \(B\)) to that state fragment, changing it to the term \(\theta(r(\vec{x}))\) in a local transition if the condition \(\theta(\vec{\phi}(\vec{x}))\) is true. In a probabilistic rewrite theory \([4]\), rewrite rules can have the more general form \(l(\vec{x}) \rightarrow r(\vec{x}, \vec{y})\) if \(\vec{\phi}(\vec{x})\) with probability \(\vec{\pi}(\vec{x})\), where some new variables \(\vec{y}\) are present in the pattern \(r\). Due to the new variables \(\vec{y}\), the next state specified by such a rule is not uniquely determined: it depends on the choice of an additional substitution \(\rho\) for the variables \(\vec{y}\). In this case, the choice of \(\rho\) is made according to the family of probability functions \(\pi_{\vec{d}}\): one for each matching substitution \(\theta\) of the variables \(\vec{x}\).

Probabilistic rewrite theories can be simulated directly in Maude by sampling, from the corresponding probabilistic functions, the values for the variables \(\vec{y}\) appearing on the RHS. Moreover, using a Monte-Carlo simulation and the query language QuaTEx (Quantitative Temporal Expressions), it is possible to analyze quantitative properties of the system by statistical model checking (more details on §4).

**Machine’s variables and state.** The specification of a machine starts with a theory that extends EBCONTEXT with a mapping from the identifiers of the variables to their values:

```plaintext
mod EBMACHINE is
inc EBCONTEXT .
op Machine : -> Cid .
op init-variables : Map(Qid , EType) -> Attribute .
op init-machine : Map(Qid , EType) .
vars QM QC : Qid .
--- Context specification.
--- Class for machines
--- Instantiated by the machine at hand
--- Machine’s variables
--- Initial state of the machine
```
The specification of a machine \( M \) as the rewrite theory \( \mathcal{R}_M \) is obtained by extending the previous theory in two ways: (1) the initialization event of the machine gives rise to an equation that populates the mapping \( \text{init-variables} \). And (2), building from the infrastructure in \( \text{EB-TYPE} \), different equations and rewrite rules are added to \( \text{EBMACHINE} \) in order to encode the semantics of the machine’s events.

For the running example, (1) amounts to define the equation:

\[
eq \text{init-variables} = ('handle \mapsto \text{val(elt("up"))}), ('door \mapsto \text{val(elt("close"))}), ('gear \mapsto \text{val(elt("retracted"))}), ('cmd \mapsto \text{val(elt(0))}) .
\]

More generally, for any machine with initialization \( x_1 := E_1, \ldots, x_n := E_n \) the following equation needs to be added to \( \text{EBMACHINE} \):

\[
\text{init-variables} = x_1 \mapsto \left[ E_1 \right]_e, \ldots, x_n \mapsto \left[ E_n \right]_e
\]  

For (2), the theory must generate a purely probabilistic transition system without un-quantified non-determinism, thus guaranteeing that execution paths in the system form a measurable set \([4]\). Otherwise, it is not possible to perform (sound) statistical analyses \((\S 4)\). Hence, the theory \( \text{EBMACHINE} \) is extended with:

(i) A deterministic mechanism that, given the current state of the machine, determines whether an event is enabled or not.

(ii) A probabilistic rule that chooses, according to the weights of the enabled events, the next event to be executed. This rule is common to any Event-B model and thus defined directly in the theory \( \text{EBMACHINE} \).

(iii) For each event, a rule that chooses probabilistically the parameters of the event (if any) as well as the values for probabilistic assignments. Then, the state of the machine is updated accordingly. The application of this rule will correspond to an observable state transition of the machine \( M \).

**Events' state (i).** Starting with (i), the sort \( \text{EvState} \) (declared in \( \text{EBMACHINE} \)) defines four possible states for an event:

\[
\text{sort EvState}.
\]

\[
\text{ops} \text{ blocked unknown execute} \mapsto \text{ EvState} .
\]

\[
\text{op enable} : \text{ NzNat} \mapsto \text{ EvState} .
\]

and the object events (see \( \text{init-machine} \) in the module \( \text{EBMACHINE} \)) stores a list with the state of each event:

\[
\text{sort Event LEvent} .
\]

\[
\text{op ev} : \text{ Qid EvState} \mapsto \text{ Event} .
\]

\[
\text{op state} : \text{ LEvent} \mapsto \text{ Attribute} .
\]

\[
\text{op init-events} : \mapsto \text{ LEvent} .
\]

Initially, all the events are in state \textit{unknown}. Hence, for any machine with events \( e_1, \ldots, e_n \), \( \text{EBMACHINE} \) must be extended with the following equation

\[
\text{init-events} = \text{ev}(e_1, \text{unknown}) \cdots \text{ev}(e_n, \text{unknown})
\]
For each constant and variable in the model, a Maude variable of sort EBType is added. This facilitates the definition of the forthcoming equations and rules. In the running example: \( \text{vars} \ SFCMD \ $handle \ $gear \ $door \ $cmd : \text{EBType}. \) Such variables will appear in the objects Context and Machine. This allows for defining \( [x]_e = x \) when the Event-B variable or constant \( x \) appears in the context of an expression. The prefix “$” is added to avoid clash of names. Consistently, the following shorthands will be used (CC, MM, and EE in Maude’s snippets):

\[
\mathcal{C} = \langle C : \text{Context} \mid \text{sets} : \text{sets}, \text{constants} : \text{init-constants} \rangle
\]

\[
\mathcal{M} = \langle M : \text{Machine} \mid \text{variables} : \{ x \mapsto $x \mid x \in \mathcal{I} \} \rangle
\]

\[
\mathcal{E} = \langle E : \text{Events} \mid \text{state} : \text{init-events} \rangle
\]

For each event \( e \), EBMACHINE is extended with an equation that (deterministically) updates the state of \( e \) from unknown to blocked or to enabled. More precisely, consider an event \( e \in \mathcal{E} \) with guard \( e_G \), weight \( e_W \), and set of parameter declarations \( e_{\text{any}} \). Let \( W = \text{ebtype2nat}(e_W)_e \) and \( B = \text{ebtype2bool}(e_G)_e \) where \( \text{ebtype2nat}(\text{val}(\text{elt}(n))) = n \) (and similarly for \( \text{ebtype2bool} \)). The needed equation is the following:

\[
\mathcal{C} \triangleright \langle E : \text{Events} \mid \text{state}:\text{LE} \; \text{ev}(e, \text{unknown}) \text{ LE} \rangle = \mathcal{C} \triangleright \langle E : \text{Events} \mid \text{state}:\text{LE} \; \text{NS} \text{t} \text{ LE} \rangle \tag{3}
\]

where \( \text{NS}t \) is the expression

\[
\text{if } B \wedge W > 0 \land \left( \bigwedge_{(j \in \mathcal{E}) \in e_{\text{any}}} \text{not-empty}(\mathcal{I}_j)_e \right) \quad \text{then } \text{ev}(e, \text{enable}(W)) \quad \text{else } \text{ev}(e, \text{blocked}) \quad \text{fi}
\]

The constants of the model (\( \mathcal{C} \)) and the current values of the variables (\( \mathcal{M} \)) are used to evaluate the Boolean expression of the guard (\( B \)) and the integer expression of the event’s weight (\( W \)). If \( B \) holds, \( W > 0 \), and the set of possible values for each parameters is not empty, the event becomes enabled with weight \( W \). Otherwise, it is blocked. The variables \( \text{LE} \) and \( \text{LE}' \) of sort \( \text{LE}vent \), allow to apply this equation at any position of the list of events.

**Next event (ii).** The next step is to add to \( \text{EBMACHINE} \) the probabilistic rule

\[
\text{if } \phi(\text{LE}) \text{ with probability } p : = \pi(\text{LE}) \quad \text{then } \text{LE}' = \text{filter}(\text{LE}) \quad \text{do not change the state of the }} \text{LE' with weights } w_1, \cdots, w_n. \text{ Moreover, let } \text{LE}_A = \text{acc}(\text{LE}') \text{ be as } \text{LE}' \text{ where the weight of the } i \text{th event in } \text{LE}_A \text{ is } \sum_{1 \leq j \leq i} w_j. \text{ Then, pick}(\text{LE}_A, p) = e_k \text{ iff } e_k \text{ is the first event in } \text{LE}_A \text{ whose weight is strictly greater than } p. \text{ Intuitively, the rule is enabled only when the state of all the events is different from unknown and at least one event is enabled (otherwise, the system is in a deadlock). The enabled events are filtered and their weights accumulated (LE}_A). \text{ Hence, an enabled event } e \text{ with weight } w \text{ is chosen for execution with probability } w/W, \text{ where } W \text{ is the sum of the weights of the enabled events at the current state.} \text{ Following [4], the probabilistic rule [4] can be written in Maude by generating random numbers appropriately:}
\]
Here, \texttt{max-value} returns the weight of the last element in LEA (i.e., the sum \(W\) of the weights of the enabled events). A random number in the interval \([0, W]\) is generated and the choice of the next event follows the distribution \(\pi(LE)\) as explained above.

**Actions in events (iii).** Step (iii) consists in defining a rule, for each event \(e\), that updates the state of the machine when such a rule is applied:

\[
\mathcal{C} \ M \langle \text{events:Events} \mid \text{state: ev(e, execute)} \rangle \Rightarrow \mathcal{C} \ M' \ E
\]

where \(M' = \{M : \text{Machine} \mid \text{variables:}\{x \mapsto $x \mid x \in \bar{x} \} \cup \{x \mapsto [\bar{e}_i]/e \mid x := e, e \in e_{act}\}\}\) and \(\bar{x}\) is the set of variables that do not appear in the LHS of the set of actions/assignments \(e_{act}\) of the event. This rule can be fired only if the event has been chosen for execution. The expression \([\bar{e}_i]/e\), updating the state of the variable \(x\), is built as before but new cases need to be considered:

- \([y]/e = \text{choice(makeList}([\bar{e}_i]/e)\) if \(y \in e\) is a parameter of the model;
- \([\{E\}]/e = \text{choice}($\bar{E}/e)\); and\)
- \([\{E_1@p_1,...,E_n@p_n\}]/e = \text{choice}($\bar{E}/e@p_1,...,[\bar{E}_n]/e@p_n)\).

The set of values a parameter can take and the (set) expression \(E\) in the probabilistic assignment \(x := \{E\}\) are converted into lists of \(EBTypes\). Since the elements of \(EBSet\) and \(EBRel\) are sortable, these lists are always generated in the same order. This guarantees that there is no un-quantified non-determinism due to the way the elements of the set are arranged. Similar to the definition of the rule \([\text{next-event}]\), random numbers are used to realize the needed probabilistic rewrite rule. More precisely, the function \(\text{choice}(L)\) generates a random number \(0 \leq i < \text{size}(L)\) and returns the \(i\)th element of the list \(L\), thus following a uniform probabilistic distribution. The \(i\)th element of \(L' = \text{accumulate}(\{E_1@p_1,...,E_n@p_n\})\) is \(E_i@\sum_{0 \leq j \leq p_i}\). In this case, the function \(\text{choice}\) generates a random number \(0 \leq R < 1\) and returns the first element \(E_i@p_i\) in \(L'\) where \(R < p_i\). For illustration, the tool generates the following rule for the event \(pcmd:\)

\[
rl [pcmd] : \begin{array}{l}
\text{cc } \text{MM } < \text{events : Events } \mid \text{state: ( ev('pcmd, execute) ) } > \\
\text{cc } \text{mm } : \text{Machine } \mid \text{variables:} \\
\{\text{handle } \mapsto \text{choice( makeList}([\text{val}($\text{elt("up")}$)], \text{val}($\text{elt("down")}$))), \}
\{\text{cmd } \mapsto \text{($\text{cmd}$) + ($\text{val}($\text{elt(1)$})$), "door" } \mapsto \text{$\text{door}$} ) \rangle \Rightarrow \text{ EE } .
\end{array}
\]

**Handling set comprehension.** Sets in parameters and RHS in assignments can be built using set comprehension. Such expressions must be evaluated in both, the equation determining the state of the event (checking whether the set values for a parameter is empty or not) and the rule specifying the state transition. The expression \(\{x.S \mid P(x)\}\) (resp., \(\{x.S \mid F(x)\}\)) can be thought of as the higher order function \texttt{filter} (resp., \texttt{map}) in functional languages. Even though it is possible to use the reflective capabilities of rewriting logic and meta-programming in Maude to encode higher-order functions [12], a simpler path is followed here. For each expression of the form \(\{x.S \mid P(x)\}\), the tool generates a new operator and two equations of the following form:
The first parameter corresponds to the (encoding of the) set of values $S$ and the second is the configuration giving meaning to the constants and variables of the model. The term $\mathit{exp-P}$ is the $\mathit{EBType}$ expression encoding the predicate $P$ and the element $E$ is discarded ($\mathit{union\ val(\mathit{empty})}$) if such an expression evaluates to $\mathit{val(elt(false))}$. Hence, $\{\{\mathit{x.S} | P(x)\}\}_e = \mathit{filter-id}([\{\mathit{S}\}_e, \mathit{C}\mathit{M}])$. A similar strategy is used to encode the set $\{\mathit{x.S} | F(x)\}$.

Summing up, Event-B models are represented as a term of sort $\mathit{Configuration}$ with three objects: $\langle \mathit{C:Context} | \mathit{CAts} \rangle \langle \mathit{M:Machine} | \mathit{MAts} \rangle \langle \mathit{E:Events} | \mathit{EAts} \rangle$ where $\mathit{CAts}$, $\mathit{MAts}$, and $\mathit{EAts}$ are the attributes explained above. Moreover, Maude’s variables appear in $\mathit{MAts}$ and $\mathit{CAts}$ in all the LHS of equations and rules, thus making available the values of the model to encode arbitrary Event-B expressions as $\mathit{EBType}$ expressions.

**Definition 2 (Encoding).** Let $\mathcal{M} = \langle \mathit{C}, \mathit{\bar{x}}, \mathit{\bar{c}}, \mathit{\bar{E}}, \mathit{\mathit{init}} \rangle$. The rewrite theory $\mathcal{R}_\mathcal{M}$ specifying the behavior of $\mathcal{M}$ is obtained by extending $\mathit{EBContext}$ as in Definition 1 and extending the theory $\mathit{EBMachine}$ with: the equations (1), (2), (3); the rule (5) (for each event); and the operators and equations defining filter- and map-like expressions. Given a state $s$ of the machine $\mathcal{M}$, $[[\mathcal{M}]_s]$ denotes the term $\mathcal{C}_s\ \mathit{M}_s\ E$ where $\mathcal{C}_s$ is as $\mathcal{E}$ but each variable $\mathit{$C_i$}$ is replaced with $[[\mathit{s(c_i)}]]_e$ and $\mathit{M}_s$ replaces each variable $\mathit{$x_i$}$ with $[[\mathit{s(x_i)}]]_e$.

Assuming that the specification of the arithmetic, Boolean, and relational operators in $\mathcal{R}_\mathcal{M}$ is adequate with respect to the corresponding semantics for the Event-B operators, it is possible to show that the specification is correct in the following sense.

**Theorem 1 (adequacy).** Let $\mathcal{M}$ be an Event-B machine and $\mathcal{R}_\mathcal{M}$ be as in Definition 2. Hence, $s \xrightarrow{\mathcal{P},e} s'$ iff $[[\mathcal{M}]_s] \rightarrow_{\mathcal{P}} [[\mathcal{M}]_s']$, where $\rightarrow_{\mathcal{P}}$ denotes one-step rewriting in $\mathcal{R}_\mathcal{M}$ with probability $p$.

**Proof.** (sketch). In what follows, $\rightarrow$ denotes equational reduction in $\mathcal{R}_\mathcal{M}$. Note that $p$ must be of the form $p_w \times q$ where $p_w$ is the first term (weight) in the transition probability function (2.3) and $q$ is the rest of the expression (resulting probability for parameters and assignments). $[[\mathcal{M}]_s]$ necessarily exhibits the following reductions: $[[\mathcal{M}]_s] = \mathcal{C}_s\ \mathit{M}_s\ E \rightarrow_{\mathcal{P}} \mathcal{C}_s\ \mathit{M}_s'\ E' \rightarrow_{\mathcal{P}} \mathcal{C}_s'\ \mathit{M}_s'\ E'' \rightarrow_{\mathcal{P}} \mathcal{C}_s'\ \mathit{M}_s'\ E$ where: $E'$ is the (unique) normal form of $E$ where the state of the events are either blocked or enabled; $E''$ results from an application of the rule next-event, choosing probabilistically one of the enabled events; and $\mathit{M_s'}$ is the new state after the application of the rule corresponding to the (unique) chosen event. If the semantics of the operators in Event-B agrees with the one defined for terms of sort $\mathit{EBType}$, (i.e., the Event-B expression $e$ reduces to the value $v$ if $[[e]]_e \rightarrow^* [[v]]_e$) the set of enabled events in $E''$ must necessarily coincide with those enabled in state $s$ and necessarily $p_w = p_w'$. Under the same assumption, the set of possible values for the parameters are the same in $s$ and $\mathit{M_s'}$. Therefore, $q = q'$ and the term $\mathcal{C}_s\ \mathit{M}_s'\ E$ necessarily corresponds to $[[\mathcal{M}]_s']$. 

\[\text{op \filter-id : \text{EBType Configuration} \rightarrow \text{EBType}}\] 
\[\text{eq \filter-id(val(\emptyset)), \mathcal{C}) = val(\emptyset). \quad \text{--- Base case} \] 
\[\text{eq \filter-id(val(\mathit{E}), \mathcal{S}), (\langle \mathit{SNAME} : \text{Context} | \mathit{sets} : (\langle \mathit{CNAME} \rightarrow \mathit{\bar{c}} | \ldots \rangle \rangle)\rangle) = \] 
\[\\text{eq \filter-id(val(\mathit{S}), (\langle \mathit{SNAME} : \text{Machine} | \mathit{variables} : (\langle \mathit{VNAME} \rightarrow \mathit{\bar{v}} | \ldots \rangle \rangle)\rangle) = \] 
\[\text{if \mathit{ebtype2bool}(\mathit{exp-P}(\mathit{val}(\mathit{E}))) \text{ then } val(\mathcal{E}) \text{ else } val(\emptyset) \end{equation}\]
4 Case Studies and Experiments

Given a model $\mathcal{M}$, the theory $\mathcal{R}_\mathcal{M}$ generated by the tool [17] can be used to perform statistical analysis. The resulting Maude’s module includes a template to ease the specification of properties as formulas in the Quantitative Temporal Expressions language (QuaTEx) [4]. QuaTEx expressions can query the expected value of any expression, thus generalizing probabilistic computation tree logic (PCTL) [13].

QuaTEx formulas can be evaluated by performing a Monte-Carlo simulation with the aid of the PVeStA tool [5]. Hence, the expected value of a given QuaTEx expression can be obtained within a confidence interval according to a parameter $\alpha$. PVeStA defines the operator $\text{val}: \text{Nat Configuration} \rightarrow \text{Float}$ that identifies the QuaTEx expression (first parameter) and, given a configuration, returns the value of the expression as a float. The parser of the tool accepts a section `PROPERTIES`, at the end of the machine definition, including Event-B expressions that are later translated and used in equations giving meaning to $\text{val}$. In the running example, the expression $\text{door}=\text{open}$ is translated to the following equation that reduces to 1.0 when the doors are open:

$$\text{eq } \text{val}((\text{Conf < $MNAME$ : Machine | variables: ... }) = \text{toFloat}(((\text{door}) = b (\text{val}(\text{elt}(\text{"open").})))) \) .$$

The simulation shows that the expected value of such expression is 0.0 (the doors are always closed after finishing the maneuver). Also, it is possible to estimate that the probability of ending the sequence of actions with the gear retracted (property $\text{gear} = \text{retracted}$) is $0.49 \pm 0.01$. This is explained by the fact that the event $\text{pcmd}$ may change the value of the handle to $\text{up}$ and $\text{down}$ with equal probability.

As a more compelling example, consider the model of the P2P protocol in [8]. A file partitioned into $K$ blocks is to be downloaded by $N$ clients; a client can only download a block at a time. As a model of failure, some blocks may be lost and then retransmitted. The context of the model includes two constants $N$ and $K$ of type $\text{Nat}$. A deferred set $\text{STATE} = \{\text{emp}, \text{ok}, \text{downloading}\}$ defines the state of the blocks. The file is modeled as a variable of type $\text{POW}(\text{Nat} \times \text{State})$ and initialized with the expression $(0 .. (N \times K - 1)) * \{\text{emp}\}$. The pair $i \mapsto \{\text{emp}\}$ means that the block $i/N$ has not been downloaded by the client $i \mod N$. Below the events for receiving and sending blocks:

```
EVENT sent
  WEIGHT $N \times K - \text{card(file)} \Rightarrow \{\text{downloading}\}$
  ANY block $\in \{x . \text{dom(file)} \Rightarrow \{\text{emp}\} \mid ((x \mod N) \notin \{y . \text{dom(file)} \Rightarrow \{\text{downloading}\} \mid y \mod N\})\}$
  WHERE True
  THEN file := file $\Leftrightarrow \{\text{block} \Rightarrow \text{downloading}\}$
    $n := n + 1$
  END

EVENT receive
  WEIGHT $1 \Rightarrow \text{card(file)} \Rightarrow \{\text{ok}\}$
  ANY block $\in \text{dom(file)} \Rightarrow \{\text{downloading}\}$
  WHERE True
  THEN file := file $\Leftrightarrow \{\text{block} \Rightarrow \text{ok}\}$
```

The event $\text{sent}$ selects a block $x$ s.t. $\text{file}(x) = \text{emp}$ (range restriction $\Rightarrow$) and whose client ($x \mod N$) is not in the set of clients that are currently downloading blocks. Note the nested set comprehension expression. In that case, the variable $\text{file}$ is updated ($\Leftrightarrow$) by changing the state of the block to $\text{downloading}$ and incrementing the counter $n$. The event $\text{receive}$ selects one of the blocks in state $\text{downloading}$ and updates it to the state $\text{ok}$, signaling that it was successfully downloaded. The probability of sending blocks decrements according to the number ($\text{card}$) of blocks being downloaded.
The event fail leaves the file unchanged with probability 0.6 and, with probability 0.4, it changes the state of one downloading block to emp. In the second case, the block needs to be retransmitted. For instance, if \(N = 16\) and \(K = 30\), PVeStA reports that the expected value for \(n\) is 1554.56 (i.e., each block is transmitted, in average, 3.24 times). Other experiments for different values of \(N\) and \(K\) as well as more details about the simulations can be found at [17]. The site of the tool contains also other case studies including the probabilistic model for the emergency brake system described in [7] and the bounded re-transmission protocol modeled in Event-B in [1, Chapter 6].

5 Concluding Remarks

Combining formalisms in the context of the B-method has been explored in different directions, thus leading to more robust tools for system modeling and verification. The authors of [2] propose a correct-by-construction approach for hybrid systems in Event-B. The main idea is to move from an event-triggered to a time-triggered approach because of real-life scenarios. However, models become more difficult to verify. For this latter purpose, the authors use a dynamic logic for refinement relations on hybrid systems to prove that time-triggered models are refinements of event-triggered models. There is a wide-range effort by the ANR agency and its partners in the EBRP project [3] to enhance Event-B and the corresponding Rodin [1] toolset by defining extension mechanisms. Their main goal is to allow Event-B models to import and use externally defined domain theories via theory constructs already available in Event-B and implemented in Rodin as a plug-in. The work in [18] proposes an extension of Event-B to enable stochastic reasoning about dependability-related non-functional properties of cyclic systems. Such an extension integrates reasoning about functional correctness and stochastic modeling of non-functional characteristics. Recently, the authors in [10] have proposed B Maude, a prototype executable environment for the Abstract Machine Notation (AMN) implemented in the Maude language. It endows the B method with execution by rewriting, symbolic search with narrowing, and Linear Temporal Logic model checking of AMN descriptions.

Following the lines of the aforementioned works, this paper couple Event-B with further tools and reasoning techniques by proposing a rewriting logic semantics for a probabilistic extension of it. The translation from an Event-B model to a probabilistic rewrite theory has been fully automated, and it was shown to be sound and complete w.r.t. the semantics of the model. The translation supports a wide spectrum of operators and constructs usually present in Event-B specifications and all sources of probabilistic behavior as proposed in [8]. The resulting probabilistic rewrite theory can be executed in Maude [4] and statistically model checked with the PVeStA tool [5]. A case study has been presented to illustrate the encoding and how the statistical analysis enabled by the translation can complement the inference-based approach in Event-B.

Future work stems from different needs. It is worth investigating how to incorporate other distribution functions to govern probabilistic choices (including concurrency/interleaving in events) in Event-B models. As shown in §3.2 probabilistic rewrite theories
can incorporate arbitrary probabilistic distribution functions. Hence, the semantics proposed here can be used to experiment with different alternatives and propose suitable extensions for Event-B. Support for new operators and constructs (e.g., arbitrary types in Cartesian products) could be included in a new version of the translation. Furthermore, a plug-in to integrate the tool proposed here to Rodin is currently under development. The framework presented here supports also non-probabilistic Event-B models (see the bounded re-transmission protocol [1] in the tool’s site whose only source of probabilistic behavior is the weight on events). It will be interesting to investigate the use of symbolic techniques, such as rewriting modulo SMT, for synthesis of parameters in Event-B specifications (e.g., finding the range of values for constants that makes the invariants true).

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