FORTRAN-codes for an analysis of the ultrashort pulse propagation

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Abstract
Short description of the FORTRAN-codes for an analysis of the ultrashort pulse dynamics is presented. We consider: 1) the aberration-less approximation and the momentum method for the search of the single pulse stability regions in the laser with the soft-aperture Kerr-lens mode locking; 2) the distributed complex Ginzburg-Landau model for the same aim; 3) the generalized Schrödinger equation for the analysis of the femtosecond pulse propagation in the tapered and photonic crystal fibers.

1 Introduction
Analysis of the intra- and extra-cavity pulse dynamics has the common features: it can be based on the distributed models of the complex Ginzburg-Landau type [1] or the generalized Schrödinger type [2]. The basic difference of ultrashort pulse laser dynamics from, for example, intra-fiber propagation of the femtosecond pulses is the essential contribution of the dissipation into former. However, for the direct simulation the appearance of the non-Hamiltonian part in the dynamical equation doesn’t cause some problems. It allows considering the distributed models on the common basis (some physical and formal backgrounds can be found in [3]).

The main problem results from the transition to the higher-dimensional models. The trivial dimension is 1 + 1, i.e. propagation distance (or cavity transit number for a laser) + local time (for an analysis of the ultrashort pulse dynamics). Our experience proved that such dimension is quite sufficient for the solution of the various problems, for example, spectral continuum generation in the fibers or multiple pulse operation of the femtosecond lasers. This approach adequately describes the pulse dynamics and at the same time is not too complicated to remain physically meaningful. The minimum set of the governing parameters allows the overall optimization by the direct scanning of the parametrical space [4, 5].

However, a consideration of the self-start ability of the soft-aperture Kerr-lens mode-locked lasers (see [2]) and an analysis of the real-world laser config-
urations requires at the least $1 + 2$ dimensions, where one transverse spatial dimension appears as a result of the rotational symmetry. The direct simulations on this way exceed the resources of the desk-top computer, which is common tool for the laser community. Therefore we have to reduce the dimension, for example, by the means of the so-called aberration-less approximation. This allows performing the simulation not on the time-spatial grid but on the grid formed by the parametrical set defining the trial (aberration-less) solution.

Below we consider some examples of $1 + 1$ and $1 + 2$ dimensional models. The latter model has in fact reduced dimension due to the use of the momentum method. It should be noted that the analytical computations underlying the numerical codes were realized in the MAPLE computer algebra system \[^6\].

2 Passive mode-locking and nonlinear complex Ginzburg-Landau equation

This approach is based on $1+1$ dimensional model in the framework of the so-called nonlinear Ginzburg-Landau equation, which describes the Kerr-lens mode locking as an action of the fast saturable absorber governed by the few physically meaningful parameters, viz., its modulation depth $\gamma$ and the inverse saturation intensity $\sigma$.

The master equation describing the ultrashort pulse generation in the Kerr-lens mode-locked solid-state laser is:

$$
\frac{\partial a(z,t)}{\partial z} = \left[ \alpha - \rho + t_f^2 \frac{\partial^2}{\partial t^2} - \frac{\gamma}{1 + \sigma |a(z,t)|^2} \right] a(z,t) - i \left\{ \sum_{m=2}^{N} \frac{(-i)^m \beta_m}{m!} \frac{\partial^m}{\partial t^m} + \delta \left( |a(z,t)|^2 - \frac{1}{\omega_0} \frac{\partial}{\partial t} |a(z,t)|^2 \right) \right\} a(z,t),
$$

where $a(z,t)$ is the field amplitude (so that $|a|^2$ has a dimension of the intensity), $z$ is the longitudinal coordinate normalized to the cavity length, $t$ is the local time, $\alpha$ is the saturated gain coefficient, $\rho$ is the linear net-loss coefficient taking into account the intracavity and output losses, $t_f$ is the group delay caused by the spectral filtering within the cavity, $\beta_m$ are the $m$-order group-delay dispersion coefficients, $\delta = l_g n_2 \omega_0 / c = 2\pi n_2 l_g / (\lambda_0 n)$ is the self-phase modulation coefficient, $\omega_0$ and $\lambda_0$ are the frequency and wavelength corresponding to the minimum spectral loss, $n$ and $n_2$ are the linear and nonlinear refraction coefficients, respectively, $l_g$ is the double length of the gain medium (we suppose that the gain medium gives a main contribution to the self-phase modulation). The last term in Eq. (1) describes the self-steepening effect and for the simplification will be not taken into account in the simulations. As an additional simplification we neglect the stimulated Raman scattering in the active medium \[^7\].

For the numerical simulations in the framework of the distributed model it is convenient to normalize the time and the intensity to $t_f = \lambda_0^2 / (\Delta \lambda c)$ and $1/\delta$,.
respectively ($\Delta \lambda$ is the gain bandwidth). The simulation were performed on the $2^{12} \times 10^4$ mesh. Only steady-state pulses were considered. As the criterion of the steady-state operation we chose the peak intensity change less than 1% over last 1000 cavity transits.

Note that the local time interval, which is equal to the cavity period $\approx 10$ nanoseconds, is not covered in our case ($2^{12} \times t_f \approx 20 \div 100$ picoseconds). This puts the questions about stability against the multipulsing with the large inter-pulse separations. Additionally we can not be sure in the ability of the spontaneous appearance of the mode locking (problem of the mode locking self-start ability).

The solution of Eq. (1) is based on the fast Fourier-transform split-step method (see Appendix 1). We symmetrized non-Hamiltonian (square brackets in Eq. (1)) and Hamiltonian part (braces in Eq. (1)) separately. The mode locking in the considered model is governed by the only four basic parameters: $\alpha - \rho$, $\beta_2$, $\gamma$, and $\sigma$. This allows unambiguous multiparametric optimization. In the presence of the higher-order dispersions, the additional $\beta_m$ parameters appear. This complicates the optimization procedure, but keeps its physical clarity. As an initial condition we take the analytical solution of the cubic Ginzburg-Landau or Schrödinger equation [8].

Some results obtained on the basis of this model are presented in [4, 5, 7].

3 Spectral continuum generation in the tapered fiber

Generation of spectral supercontinuum became a hot topic in optics in recent years [8]. In the crystal or tapered fiber the propagating field has a comparatively determinate spatial structure due to a strong confinement. Therefore, the $1+1$-dimensional simulations give a quite thorough result. However, the nonlinearity in such fibers is enhanced by their small core size. As a result, a set of the nonlinearities is wider than that described by the Hamiltonian part of Eq. (1), which is the high-order nonlinear Schrödinger equation. The non-trivial generalization can be obtain due to taking into account the stimulated Raman scattering [9]:

$$i \frac{\partial a}{\partial z} + \sum_{m \geq 2} \frac{i^m \beta_m}{m!} \frac{\partial^m a}{\partial t^m} = -\delta (1 - f) |a|^2 a$$

$$-\delta f a \int_{-\infty}^{t} R(t) |a(z, t - \tau)|^2 \, d\tau,$$

where $\delta = n_2 \omega_0 / c$ is the self-phase modulation coefficient, $\beta_m$ is the $m$th-order group-velocity dispersion coefficient, $f$ is the fraction of the stimulated Raman scattering contribution to the nonlinear refractive index of the fiber,
\[ R(t) = \frac{T_1^2 + T_2^2}{T_1^2 + T_2^2} \exp \left(-\frac{t}{T_1} \right) \sin \left(\frac{t}{T_2} \right) \] is the Raman response function \[ \text{(10) (11).} \]

\( T_1 = 12.2 \text{ fs} \) and \( T_2 = 32 \text{ fs} \) define the phonon oscillation period and its dumping time, respectively.

We normalized \( t \) to 1 fs (the normalization to the initial pulse width is convenient, too), \( z \) to the nonlinear length \( L_{nl} = (\omega_0 n_2 I_0/c)^{-1} \) defined by the initial pulse intensity \( I_0 \). The simulations were carried out on the mesh with the time step 1 fs (2\(^{13}\) points) and the spatial step \( 10^{-3} L_{nl} \). The solution of Eq. (2) was based on the fast Fourier-transform split-step method with the evaluation of the Raman response in the time domain (see Appendix 2).

The analysis of the pulse propagation in the tapered fibers requires the attention to the so-called transient sectors: before (after) the tapered sector with the almost constant waist there is the convergent (divergent) sector, where the fiber characteristics change from those in the single-mode fiber to those in the tapered fiber (or vice versa). The exact law of these changes is unknown, but we used the simple linear approximation for the evolution of the intensity and the dispersion coefficients. Note, that the normalization of the intensity was defined through the parameters of the tapered sector.

4 Aberration-less approximation: analysis of the real-world laser configurations

Above the problem of the ultrashort pulse stability in the mode-locked laser were formulated on the basis of the distributed 1 + 1-dimensional Ginzburg-Landau model. We noted also some problems of such model. Here we present an analysis on the basis of the time-spatial model. The spatial distribution for the laser beam is assumed to be Gaussian that reduces the problem to 1 + 2-dimensions. The free-space propagation of the Gaussian beam can be considered on the basis of the usual ABCD-matrix formalism \[ \text{(12),} \] while the propagation inside the nonlinear active medium is described by the following equation:

\[
\frac{\partial a(z,r,t)}{\partial z} - \frac{2i r^2 a(z,r,t)}{w_p^2} - \frac{\partial^2 a(z,r,t)}{2kr} + \beta'_2 \frac{\partial^2 a(z,r,t)}{\partial t^2} + \frac{t'_f}{2} \frac{\partial^2 a(z,r,t)}{\partial t^2} = 0
\]

Here \( \beta'_2 \) and \( t'_f \) are the group-velocity dispersion and the inverse group-velocity delay coefficients (for ZnSe laser we used \( \beta'_2 = 2054 \text{ fs}^2/\text{cm} \) and \( t'_f = 13 \text{ fs/cm} \)). The left-hand side of Eq. (3) describes the non-dissipative factors: thermo-lensing \((\partial = k \frac{dn}{dT} \Pi a \exp (\zeta z) / (4\pi n_0 \kappa_{th})\), \( k \) is the wave number, \( \frac{dn}{dT} \) is the coefficient of the refractive index thermo-dependence \(5.35 \times 10^{-5} \text{K}^{-1} \) for ZnSe), \( \zeta \) is the loss coefficient at the pump wavelength, \( \Pi a \) is the pump power, \( \kappa_{th} \) is the thermo-conductivity coefficient \(0.172 \text{W/K}^{-1} \text{cm}^{-1} \) for ZnSe); diffraction
(in the cylindrically symmetrical case); group-velocity dispersion and self-phase modulation (providing self-focusing for radially varying beam, $\chi = n_2 k / n_0$). The right-hand side of Eq. (3) describes the dissipative factors inside the gain medium: radially varying gain (providing gain guiding and soft aperture action, $\alpha$ and $w_p$ are the saturated gain coefficient and the pump beam size, respectively); spectral filtering caused by the gain band profile. The saturated gain can be expressed in the following way:

$$\alpha = \frac{2 \alpha_{\text{max}} \sigma_a P_g T_{r}}{\hbar \omega_p \pi w_p^2 \left( \frac{2 \sigma_a P_g T_r}{\hbar \omega_p \pi w_p^2} + \frac{2 \nu P_g \tau_p}{\pi w^2 I_{s} \tau_p} + 1 \right)},$$

where $\nu = E \pi w^2 / (2 P_g \tau_p)$ ($P_g$ is the generation power, $E$ is the generation energy, $\omega_p$ is the pump frequency, $\tau_p$ is the pulse width, $T_{\text{cav}}$ is the cavity period, $\sigma_a$ is the absorption cross-section, $I_s$ is the gain saturation energy), $w$ is the generation mode beam size. $\nu = \sqrt{\pi / 2}$ for the pulse with the Gaussian time-profile, 2 for the $\text{sech}$-shaped pulse and 1 for the CW (in the latter case $\tau_p = T_{\text{cav}}$). The approximated solution of Eq. (3) is based on the so-called aberration-less approximation: the propagating field has the invariable spatial-time profile, which is described by the set of the $z$-dependent parameters. In the non-dissipative case this approximation allows the variational approach providing rigorous description of the Gaussian beam propagation outside the parabolical approximation [13].

In the dissipative case we use the momentum method [14] and consider the momentums resulting from the symmetries of Eq. (3). The $a \rightarrow a \exp(i\phi)$ invariance, the transverse and time translating invariance suggest the following momentums [15]:

$$T_{m,n} = \int \int \infty r^m t^n |a|^2 \, dr \, dt,$$

$$J_{m,n} = \int \int \infty r^m t^n \left( a \frac{\partial a^*}{\partial t} - a^* \frac{\partial a}{\partial t} \right) \, dr \, dt,$$

$$M_{m,n} = \int \int \infty r^m t^n \left( a \frac{\partial a^*}{\partial r} - a^* \frac{\partial a}{\partial r} \right) \, dr \, dt.$$

Like the variational approach we can substitute to Eqs. (3, 5) the trial solution describing the ultrashort pulse. If we take the Gaussian time-spatial profile $a(z,r,t) = W(r) \exp \left( G(r) - \frac{\nu^2}{2 w^2} + ib(r)^2 - \frac{\nu^2}{2 \tau^2} + i \psi(r)t^2 \right)$ ($W(r)$ is the complex amplitude, $2w^2 = w_0^2$, $G(r)$ is the pulse amplification parameter excepting the geometrical magnification for the Gaussian beam), the equations describing the evolution of the pulse and beam parameters are [16]:

5
\begin{equation}
\frac{dw'}{dz} = -\frac{2}{k} w'(z) b(z) - \frac{2 \alpha w'(z)^3}{w_p^2 \left(1 + \frac{2 \delta w'(z)^2}{w_p^2}\right)^{3/2}},
\end{equation}

\frac{d\tau}{dz} = \left[ \frac{2}{\tau(z)} - 2 \tau(z)^3 \psi(z)^2 \right] t_\tau^2 + 2 \beta_2^2 \tau(z) \psi(z),

\frac{dG}{dz} = \frac{\alpha}{1 + \frac{2 \delta w'(z)^2}{w_p^2}} - \frac{2 \beta_2^2 \psi(z) \tau(z)^2}{\tau(z)^2} - \beta_2^2 \psi(z),

\frac{db}{dz} = \frac{2 \phi}{w_p^2} + \frac{2 b(z)^2}{k} + \frac{\sqrt{2} P_0 e^{2G(z)}}{\pi P_{cr} kw(z)^4} - \frac{1}{2 \kappa w'(z)^4},

\frac{d\psi}{dz} = 2 \beta_2^2 \left( \frac{1}{\tau(z)^4} - \psi(z)^2 \right) - \frac{8 t_\tau^2 \psi(z)^2}{\tau(z)^2} + \frac{2 P_0 e^{2G(z)}}{\pi P_{cr} w(z)^4 \tau(z)^2},

P_g = P_0 e^{2G(z)},

where $P_0$ and $w'_0$ are the power and the beam size before the active medium, respectively. This system can be solved on the basis of the fourth-order Runge-Kutta method (see Appendix 3; $\beta, \alpha, \gamma, \delta$ and $\psi$ correspond to the right-hand side of Eqs. (6) for $b, w, G, \tau$ and $\psi$, respectively).

5 Conclusion

Above considered models allow the different generalizations. For example, in the framework of the Ginzburg-Landau model the stimulated Raman scattering inside the active medium can be taken into account immediately (by analogy with the fiber optics). The high-order nonlinear Schrödinger equation can be generalized in order to take into account the birefringence. The momentum method requires an additional analysis for the dissipative propagation. Nevertheless, in the presented form it can be used for the Kerr-lens mode-locked lasers optimization. Outside the aberration-less approximation for the field time-profile, the model allows most adequate description of the mode-locked lasers, but the computational time can be enormous in this case (for up-to-date desk-top computers).

The considered numerical codes were prepared as a result of the preliminary analysis in the framework of the computer algebra system MAPLE (see http://www.geocities.com/optomaplev).

6 Acknowledgments

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Appendix 1:

ULTRASHORT PULSE STABILITY
IN THE KERR-LENS MODE-LOCKED LASER:
ANALYSIS ON THE BASIS OF THE COMPLEX
GINZBURG-LANDAU EQUATION

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PARAMETERS:

- \( \alpha \) is the saturated net-gain;
- \( \beta \) is the group-delay dispersion;
- \( D3 \) is the third-order dispersion;
- \( \gamma \) is the fast absorber modulation depth;
- \( \delta \) is the fast absorber saturation parameter.
- \( V_r \) and \( V_i \) are the real and imaginary parts of the field, respectively;
- \( V \) is the field intensity; \( VV \) is the spectral intensity;
- \( V_{\text{max}} \) is the maximum pulse intensity;
- \( En \) is the generation energy;
- \( width \) is the pulse width;
- \( shift \) is the spectrum maximum shift.

The number of the considered steady-state pulses is defined by
the number counter

All values are dimensionless: the intensity is normalized to the self-
phase modulation coefficient, the time is normalized to the inverse
bandwidth of the spectral filter, the propagation distance is normalized
to the cavity length

REAL*8 Vr(4096),Vi(4096),V(4096),VV(4096),alpha,beta,gamma,del
Ele
common /comin/ts(4096),ntab

DOUBLE PRECISION rho,rho1,rho2,tau,C,S,X,Y,Argum,Fout,Omega,Vm
DATA Nt,Nst,Pi/4096,12,3.14159265358979323846264338d0/

epsilon = 1.e-4
open(1,file='Landau_G.dat')
write(1,*)'gamma, alpha, beta, D3, delta, Vmax, En, width, shift'
gamma = 0.05
D3 = -150.
do j2=1,10
   alpha=(gamma-epsilon)*j2/10d0
do j3=1,100
   beta=-j3
do j4=1,50
   delta=10**(-2d0+4d0*j4/50d0)
Fout = 1d0/Nt
c Initialization of the initial field
   K = 0
tau=sqrt(gamma-alpha)
rho1 = sqrt(2d0)*tau/sqrt(gamma*delta) !from cubic Landau-Ginzburg
rho2 = sqrt(-beta)*tau ! from Schrödinger
if(rho1.gt.rho2)then
   rho=rho1
else
   rho=rho2
end if
do j=1,Nt
   Vr(j)=rho/cosh((j-2048)*tau)
   Vi(j)=0.
end do
1 K = K+1
c DISSIPATIVE PART
c First amplification step
do I=1,Nt
   Vr(i) = Vr(i)*exp(0.5d0*alpha)
   Vi(i) = Vi(i)*exp(0.5d0*alpha)
end do
c First filter action
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
do I=1,Nt
   IF(I.LE.Nt/2+1)Is=I-1
   IF(I.GE.Nt/2+2)Is=I-1-Nt
   Omega=2.*Pi*Is*Fout
   Argum=Omega**2 ! gain profile and action
   Vr(I)=Vr(I)*exp(-0.5d0*Argum)
   Vi(I)=Vi(I)*exp(-0.5d0*Argum)
end do
call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time
c First nonlinear part’s action
do I=1,Nt
   Argum=gamma/(1d0 + delta*(Vr(I)**2+Vi(I)**2))
   Vr(I)=Vr(I)*exp(-0.5d0*Argum)
   Vi(I)=Vi(I)*exp(-0.5d0*Argum)
end do
c Second amplification step
do I=1,Nt
   Vr(I)= Vr(I)*exp(0.5d0*alpha)
   Vi(I)= Vi(I)*exp(0.5d0*alpha)
end do
c Second filter action
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
do I=1,Nt
   IF(I.LE.Nt/2+1)Is=I-1
   IF(I.GE.Nt/2+2)Is=I-1-Nt
   Omega=2.*Pi*Is*Fout
   Argum=Omega**2 ! gain profile and action
   Vr(I)=Vr(I)*exp(-0.5d0*Argum)
   Vi(I)=Vi(I)*exp(-0.5d0*Argum)
end do
call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time
c Second nonlinear part’s action
do I=1,Nt
   Argum=gamma/(1d0 + delta*(Vr(I)**2+Vi(I)**2))
   Vr(I)=Vr(I)*exp(-0.5d0*Argum)
   Vi(I)=Vi(I)*exp(-0.5d0*Argum)
end do
c HAMILTONIAN PART
c First dispersion step
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
do I=1,Nt
   IF(I.LE.Nt/2+1)Is=I-1
   IF(I.GE.Nt/2+2)Is=I-1-Nt
   Omega=2.*Pi*Is*Fout
   C=cos(0.5d0*((beta/2d0)*Omega**2 - (D3/6d0)*Omega**3))
   S=sin(0.5d0*((beta/2d0)*Omega**2 - (D3/6d0)*Omega**3))
   X=Vr(I)
   Y=Vi(I)
   Vr(I)=X*C+Y*S
   Vi(I)=Y*C-X*S
end do
call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time
c First self-phase modulation step
do I=1,Nt
Argum=0.5d0*(Vr(I)**2+Vi(I)**2)
   C=cos(Argum)
   S=sin(Argum)
   X=Vr(I)
   Y=Vi(I)
   Vr(I)=X*C+Y*S
   Vi(I)=Y*C-X*S
end do

c Second dispersion step
  call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
  do I=1,Nt
    IF(I.LE.Nt/2+1)Is=I-1
    IF(I.GE.Nt/2+2)Is=I-1-Nt
    Omega=2.*Pi*Is*Fout
    C=cos(0.5d0*((beta/2d0)*Omega**2 - (D3/6d0)*Omega**3))
    S=sin(0.5d0*((beta/2d0)*Omega**2 - (D3/6d0)*Omega**3))
    X=Vr(I)
    Y=Vi(I)
    Vr(I)=X*C+Y*S
    Vi(I)=Y*C-X*S
  end do
  call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time

  c Second self-phase modulation step
  do I=1,Nt
    Argum=0.5d0*(Vr(I)**2+Vi(I)**2)
    C=cos(Argum)
    S=sin(Argum)
    X=Vr(I)
    Y=Vi(I)
    Vr(I)=X*C+Y*S
    Vi(I)=Y*C-X*S
  end do

  c Analysis of Generation Field
  call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
  do i=1,nt
    if(i.le.nt/2)then
      j=nt/2+i
      VV(j)=Vr(i)**2 + Vi(i)**2
    else
      j=i-nt/2
      VV(j)=Vr(i)**2 + Vi(i)**2
    end if
  end do
  call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time
  Sm=VV(1)
  do 6 I=1,NT
  10
if(VV(I).LT.Sm)goto 6
Sm=VV(I)
Ism=I
6 continue
shift=Ism-Nt/2
do I=1,NT
V(I)=Vr(I)**2+Vi(I)**2
end do

Vm=V(1)
do 2 I=1,NT
if(V(I).LT.Vm)goto 2
Vm=V(I)
Im=I
2 continue
if(k.eq.9000)contp=Vm
if(Vm.lt.1e-10)goto 5
if(k.lt.10000)goto 1
c Analysis of Generation Field
do I=1,NT
IF(V(I).ge.Vm/2.and.V(I-1).le.Vm/2)h1=(2.*I-1)/2.
IF(V(I).le.Vm/2.and.V(I-1).ge.Vm/2)h2=(2.*I-1)/2.
end do
En = 0.
do I=1,Nt
En=En+V(i)
end do
if(Vm.gt.0.)then
stab=abs(contp-Vm)/Vm
else
stab=1.
end if
Vmax=Vm
c PULSE NUMBER COUNTER
call fftin(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
c Filter
do I=1,NT
IF(I.LE.Nt/2+1)Is=I-1
IF(I.GE.Nt/2+2)Is=I-1-Nt
Omega=2.*Pi*Is*Fout
Argum=(0.1*(h2-h1))**2*Omega**2
Vr(I)=Vr(I)*exp(-Argum)
Vi(I)=Vi(I)*exp(-Argum)
end do
call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time

do I=1,NT
V(I)=Vr(I)**2+Vi(I)**2
end do
Vm=V(1)
do 3 I=1,NT
if(V(I).LT.Vm)goto 3
Vm=V(I)
Im=I
3 continue

nm=0
do i=2,nt-1
if(V(i).ge.V(i-1).and.V(i).gt.Vm/10.)then
if(V(i+1).lt.V(i))nm=nm+1
else
end if
end do
width=(h2-h1)
if(nm.eq.1.and.stab.lt.0.01)then
write(1,fmt=4)gamma,alpha,beta,D3,delta,Vmax,En,width,shift
4 format(f4.2,1x,f5.3,1x,f6.1,1x,f6.1,1x,f6.1,1x,f6.1,1x,f6.1,1x,f6.1,1x,f6.1,1x,f6.1,1x,f7.4,1x,f7.3,1x,
#f7.1,1x,f7.1)
else
end if
5 end do
end do
end do

c Fast Fourier transformation
SUBROUTINE FFTINN(XB,XM,NT,NST,SIGN)
REAL*8 XB(NT),XM(NT)
COMMON/COMIN/TS(4096),NTAB
DOUBLE PRECISION XB1,XB2,XM1,XM2,PRIR,W1,W2,FP,TS
IF(NTAB.GT.0)GOTO 30
NTAB=4096
PRIR=3.14159265358979323846264338d0/NTAB
DO 20 I=1,NTAB
FP=PRIR*(I-1)
TS(I)=SIN(FP)
20 continue
30 CONTINUE
LEC1=NT-1
LEC2=NT/2
J=1
DO 10 I=1,LEC1
IF(I.GE.J)GO TO 8
XB1=XB(J)
XM1=XM(J)
XB(J)=XB(I)
XM(J)=XM(I)
XB(I)=XB1
XM(I)=XM1
8 L=LEC2
9 IF(L.GE.J)GOTO 10
J=J-L
L=L/2
GO TO 9
10 J=J+L
INCP=NTAB*2
NSDV=NTAB/2
JLI=1
JKI=1
KLI=NT
DO 3 I=1,NST
JKI=JKI+JKI
KLI=KLI/2
INCP=INCP/2
INI=1
DO 2 J=1,JLI
LEC1=J
W2=TS(INI)
IF(INI-NSDV)5,5,6
5 W1=TS(INI+NSDV)
GO TO 7
6 W1=-TS(INI-NSDV)
7 IF(SIGN.GT.0.)W2=-W2
DO 1 K=1,KLI
LEC2=LEC1+JLI
XB1=XB(LEC1)
XM1=XM(LEC1)
XB2=W1*XB(LEC2)-W2*XM(LEC2)
XM2=W1*XM(LEC2)+W2*XB(LEC2)
XB(LEC1)=XB1+XB2
XM(LEC1)=XM1+XM2
XB(LEC2)=XB1-XB2
XM(LEC2)=XM1-XM2
13
1 LEC1 = LEC1 + JKI
2 INI = INI + INCP
3 JLI = JLI + JLI
IF (SIGN.LT.0.) RETURN
FP = 1./NT
DO 4 I = 1, NT
XB(I) = XB(I) * FP
4 XM(I) = XM(I) * FP
RETURN
END

8 Appendix 2:

SPECTRAL CONTINUUM GENERATION IN THE TAPERED FIBER:
GENERALIZED SCHROEDINGER EQUATION

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PARAMETERS:

Amp is the initial pulse intensity (1 for the tapered section)
ar_begin is defined by the ration of the tapered fiber and single-mode fiber cross-sections
width is the initial pulse width (in fs)
D_begin and D3_begin are the group-velocity and third-order dispersions of the single-mode fiber
D_end and D3_end are the group-velocity and third-order dispersions of the tapered fiber
All dispersions are defined through the dispersion lengths normalized to the nonlinear length in the tapered fiber
N_steps and N_t are 1000*L/Lnl, where L is the length of the transient fiber sector or the tapered sector, respectively, Lnl is the nonlinear length of the single-mode or tapered fiber, respectively

REAL*8 Vr(8192), Vi(8192), V(8192), R(8192)
REAL*8 Vr_out(8192), Vi_out(8192)
INTEGER N_steps, N_t
common /comintab/

DOUBLE PRECISION Amp,width,C,S,X,Y,Argum,Fout,Omega,Vm,n2eff
DATA Nt,Nst,Pi,Dx/8192,13,3.14159265358979323846264338d0,1d-3/
DATA Fr,T1,T2/0.15d0,12.2d0,32d0/

open(1, file='phase.dat')
open(2, file='spectrum.dat')
open(3, file='intensity.dat')
Fout = 1d0/Nt

c Raman responce function
do i=1,Nt
   R(i) = ((T1**2+T2**2)/(T1*T2**2))*exp(-(i-1)/T2)*sin((i-1)/T1)
end do
c Initialization of the initial field
   K = 0
   width=28.36d0

   D_begin = -width**2/7.55d0
   D_end = width**2/0.53d0
   D3_begin = -0.58d0*D_begin*width
   D3_end = -0.78d0*D_end*width
   ar_begin = 0.0149
   N_steps = 3774
   NT = 16992
   AAA = ar_begin**(-1./(2.*N_steps))
   D = D_begin
   D3 = D3_begin
   Amp = sqrt(ar_begin)

do j=1,Nt
   Vr(j)=Amp/cosh((j-4096)/width)
   Vi(j)=0.
end do
do j=1,Nt
   write(3,*)k,j,Vr(j)**2+Vi(j)**2
end do

call fftin(Vr,Vi,NT,Nst,1.) ! from Time to Frequency
   do i=1,nt
      if(i.le.nt/2)then
         j=nt/2+i
      end if
      V(j)=Vr(i)**2 + Vi(i)**2
\[ V_{r_{\text{out}}}^{(j)} = V_r^{(i)} \]
\[ V_{i_{\text{out}}}^{(j)} = V_i^{(i)} \]

else
\[ j = i - \text{nt}/2 \]
\[ V^{(j)} = V_r^{(i)}^{\ast} + V_i^{(i)}^{\ast} \]
\[ V_{r_{\text{out}}}^{(j)} = V_r^{(i)} \]
\[ V_{i_{\text{out}}}^{(j)} = V_i^{(i)} \]
end if
end do

end do

call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time
do i=1,Nt
write(1,*) k,i,Vr_{\text{out}}^{(i)},Vi_{\text{out}}^{(i)}
end do
do j=1,Nt
write(2,*) k,j,V(j)
end do

c FIRST TRANSIENT SECTOR

1
K = K+1
write(*,*) K

c First dispersion step
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
do I=1,Nt
IF(I.LE.Nt/2+1) Is = I-1
IF(I.GE.Nt/2+2) Is = I-1-Nt
Omega = 2.*Pi*Is*Fout
C = cos(Dx*0.5d0*((D/2d0)*Omega^{\ast}2 - (D3/6d0)*Omega^{\ast}3))
S = sin(Dx*0.5d0*((D/2d0)*Omega^{\ast}2 - (D3/6d0)*Omega^{\ast}3))
X = Vr(I)
Y = Vi(I)
Vr(I) = X*C - Y*S
Vi(I) = Y*C + X*S
end do
call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time
c First self-phase modulation step
do I=1,Nt
Argum = Dx*0.5d0*(1d0-Fr)*(Vr(I)^{\ast}2 + Vi(I)^{\ast}2)
C = cos(Argum)
S = sin(Argum)
X = Vr(I)
Y = Vi(I)
Vr(I) = X*C - Y*S
Vi(I) = Y*C + X*S
end do
c First Raman step
do I=1,Nt
Argum = 0d0
    do J=1,I
        Argum=Argum + R(I-J+1)*(Vr(J)**2+Vi(J)**2)
    end do
    C=cos(Dx*0.5d0*Fr*Argum)
    S=sin(Dx*0.5d0*Fr*Argum)
    X=Vr(I)
    Y=Vi(I)
    Vr(I)=X*C - Y*S
    Vi(I)=Y*C + X*S
end do

Second dispersion step
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
do I=1,Nt
    IF(I.LE.Nt/2+1)Is=I-1
    IF(I.GE.Nt/2+2)Is=I-1-Nt
    Omega=2.*Pi*Is*Fout
    C=cos(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
    S=sin(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
    X=Vr(I)
    Y=Vi(I)
    Vr(I)= X*C - Y*S
    Vi(I)= Y*C + X*S
end do
call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time

Second self-phase modulation step
do I=1,Nt
    Argum=Dx*0.5d0*(1d0-Fr)*(Vr(I)**2+Vi(I)**2)
    C=cos(Argum)
    S=sin(Argum)
    X=Vr(I)
    Y=Vi(I)
    Vr(I)=X*C - Y*S
    Vi(I)= Y*C + X*S
end do

Second Raman step
do I=1,Nt
    Argum = 0d0
    do J=1,I
        Argum=Argum + R(I-J+1)*(Vr(J)**2+Vi(J)**2)
    end do
    C=cos(Dx*0.5d0*Fr*Argum)
    S=sin(Dx*0.5d0*Fr*Argum)
\[
X = \text{Vr}(I) \\
Y = \text{Vi}(I) \\
\text{Vr}(I) = X \times C - Y \times S \\
\text{Vi}(I) = Y \times C + X \times S \\
\]

end do

do I=1,Nt
\[
\text{Vr}(i) = \text{Vr}(i) \times \text{AAA} \\
\text{Vi}(i) = \text{Vi}(i) \times \text{AAA} \\
\]
end do

\[
D = D_{\text{begin}} + K \times (D_{\text{end}} - D_{\text{begin}}) / N_{\text{steps}} \\
D3 = D3_{\text{begin}} + K \times (D3_{\text{end}} - D3_{\text{begin}}) / N_{\text{steps}} \\
\]

if((K/1000)*1000.eq.K)then

do j=1,Nt
write(3,*) k,j, \text{Vr}(j)**2+\text{Vi}(j)**2
end do

\]
call fftinn(\text{Vr},\text{Vi},Nt,Nst,1.) ! from Time to Frequency

do i=1,nt

if(i.le.nt/2)then
\[
j = nt/2 + i \\
\text{V}(j) = \text{Vr}(i)**2 + \text{Vi}(i)**2 \\
\text{Vr}_{\text{out}}(j) = \text{Vr}(i) \\
\text{Vi}_{\text{out}}(j) = \text{Vi}(i) \\
\]
else
\[
j = i - nt/2 \\
\text{V}(j) = \text{Vr}(i)**2 + \text{Vi}(i)**2 \\
\text{Vr}_{\text{out}}(j) = \text{Vr}(i) \\
\text{Vi}_{\text{out}}(j) = \text{Vi}(i) \\
\]
end if
end do

call fftinn(\text{Vr},\text{Vi},Nt,Nst,-1.) ! from Frequency to Time

do i=1,Nt
write(1,*) k,i, \text{Vr}_{\text{out}}(i),\text{Vi}_{\text{out}}(i)
end do

do j=1,Nt
write(2,*) k,j, \text{V}(j)
end do
else
endif

if(k.lt.N_{\text{steps}}) goto 1

c TAPERED SECTOR

K = K+1

write(*,*) K
\[
D = D_{\text{end}} \\
D3 = D3_{\text{end}} \\
\]
c First dispersion step
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
    do I=1,Nt
        IF(I.LE.Nt/2+1)Is=I-1
        IF(I.GE.Nt/2+2)Is=I-1-Nt
        Omega=2.*Pi*Is*Fout
        C=cos(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
        S=sin(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
        X=Vr(I)
        Y=Vi(I)
        Vr(I)= X*C - Y*S
        Vi(I)= Y*C + X*S
    end do
    call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time
    c First self-phase modulation step
    do I=1,Nt
        Argum=Dx*0.5d0*(1d0-Fr)*(Vr(I)**2+Vi(I)**2)
        C=cos(Argum)
        S=sin(Argum)
        X=Vr(I)
        Y=Vi(I)
        Vr(I)=X*C - Y*S
        Vi(I)=Y*C + X*S
    end do
    c First Raman step
    do I=1,Nt
        Argum = 0d0
        do J=1,I
            Argum=Argum + R(I-J+1)*(Vr(J)**2+Vi(J)**2)
        end do
        C=cos(Dx*0.5d0*Fr*Argum)
        S=sin(Dx*0.5d0*Fr*Argum)
        X=Vr(I)
        Y=Vi(I)
        Vr(I)=X*C - Y*S
        Vi(I)=Y*C + X*S
    end do
    c Second dispersion step
    call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
    do I=1,Nt
        IF(I.LE.Nt/2+1)Is=I-1
        IF(I.GE.Nt/2+2)Is=I-1-Nt
        Omega=2.*Pi*Is*Fout
        C=cos(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
        S=sin(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
\[ X = V_r(I) \]
\[ Y = V_i(I) \]
\[ V_r(I) = X \cdot C - Y \cdot S \]
\[ V_i(I) = Y \cdot C + X \cdot S \]

end do

call fftnn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time

c Second self-phase modulation step

do I=1,Nt
   \[ \text{Argum} = D_x \cdot 0.5d0 \cdot (1d0 - Fr) \cdot (V_r(I)^2 + V_i(I)^2) \]
   \[ C = \cos(\text{Argum}) \]
   \[ S = \sin(\text{Argum}) \]
   \[ X = V_r(I) \]
   \[ Y = V_i(I) \]
   \[ V_r(I) = X \cdot C - Y \cdot S \]
   \[ V_i(I) = Y \cdot C + X \cdot S \]
end do

c Second Raman step

do I=1,Nt
   \[ \text{Argum} = 0d0 \]
   do J=1,I
      \[ \text{Argum} = \text{Argum} + R(I-J+1) \cdot (V_r(J)^2 + V_i(J)^2) \]
   end do
   \[ C = \cos(D_x \cdot 0.5d0 \cdot Fr \cdot \text{Argum}) \]
   \[ S = \sin(D_x \cdot 0.5d0 \cdot Fr \cdot \text{Argum}) \]
   \[ X = V_r(I) \]
   \[ Y = V_i(I) \]
   \[ V_r(I) = X \cdot C - Y \cdot S \]
   \[ V_i(I) = Y \cdot C + X \cdot S \]
end do

if((K/1000)*1000.eq.K)then
   do j=1,Nt
      write(3,*)k,j,Vr(j)**2 + Vi(j)**2
   end do
end if

call fftnn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency

do i=1,nt
   if(i.le.nt/2)then
      j=nt/2+i
      \[ V(j) = V_r(i)^2 + V_i(i)^2 \]
      \[ Vr\_out(j) = V_r(i) \]
      \[ V_i\_out(j) = V_i(i) \]
   else
      j=i-nt/2
      \[ V(j) = V_r(i)^2 + V_i(i)^2 \]
      \[ Vr\_out(j) = V_r(i) \]
      \[ V_i\_out(j) = V_i(i) \]
   end if
end do
end if
end do

call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time
do i=1,Nt
write(1,*)k,i,Vr(i),Vi(i)
end do
do j=1,Nt
write(2,*)k,j,V(j)
end do
else
end if
if(k.lt.Nsteps+Nt)goto 2

SECOND TRANSIENT SECTOR

K = K+1
write(*,*)K

First dispersion step
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
do I=1,Nt
IF(I.LT.Nt/2+1)Is=I-1
IF(I.GT.Nt/2+2)Is=I-1-Nt
Omega=2.*Pi*Is*Fout
C=cos(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
S=sin(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
X=Vr(I)
Y=Vi(I)
Vr(I)= X*C - Y*S
Vi(I)= Y*C + X*S
end do
call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time

First self-phase modulation step
do I=1,Nt
Argum=Dx*0.5d0*(1d0-Fr)*(Vr(I)**2+Vi(I)**2)
C=cos(Argum)
S=sin(Argum)
X=Vr(I)
Y=Vi(I)
Vr(I)=X*C - Y*S
Vi(I)= Y*C + X*S
end do
c First Raman step
do I=1,Nt
Argum = 0d0
do J=1,I
Argum=Argum + R(I-J+1)*(Vr(J)**2+Vi(J)**2)
end do
C=cos(Dx*0.5d0*Fr*Argum)
S=sin(Dx*0.5d0*Fr*Argum)
X=Vr(I)
Y=Vi(I)
Vr(I)=X*C - Y*S
Vi(I)=Y*C + X*S

end do

c Second dispersion step
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
do I=1,Nt
IF(I.LE.Nt/2+1)Is=I-1
IF(I.GE.Nt/2+2)Is=I-1-Nt
Omega=2.*Pi*Is*Fout
C=cos(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
S=sin(Dx*0.5d0*((D/2d0)*Omega**2 - (D3/6d0)*Omega**3))
X=Vr(I)
Y=Vi(I)
Vr(I)= X*C - Y*S
Vi(I)= Y*C + X*S
end do
call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time

c Second self-phase modulation step
do I=1,Nt
Argum=Dx*0.5d0*(1d0-Fr)*(Vr(I)**2+Vi(I)**2)
C=cos(Argum)
S=sin(Argum)
X=Vr(I)
Y=Vi(I)
Vr(I)=X*C - Y*S
Vi(I)=Y*C + X*S
end do

c Second Raman step
do I=1,Nt
Argum = 0d0
do J=1,I
Argum=Argum + R(I-J+1)*(Vr(J)**2+Vi(J)**2)
end do
C=cos(Dx*0.5d0*Fr*Argum)
S=sin(Dx*0.5d0*Fr*Argum)
X=Vr(I)
Y=Vi(I)
Vr(I)=X*C - Y*S
Vi(I)=Y*C + X*S
end do
do I=1,Nt
Vr(i) = Vr(i)/AAA
Vi(i) = Vi(i)/AAA
end do

D = D_end + (K-N_steps-N_t)*(D_begin - D_end)/N_steps
D3 = D3_end + (K-N_steps-N_t)*(D3_begin - D3_end)/N_steps

if((K/1000)*1000.eq.K)then
do j=1,Nt
write(3,*)k,j,Vr(j)**2+Vi(j)**2
end do
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
do i=1,Nt
if(i.le.nt/2)then
j=nt/2+i
V(j)=Vr(i)**2 + Vi(i)**2
Vr_out(j)=Vr(i)
Vi_out(j)=Vi(i)
else
j=i-nt/2
V(j)=Vr(i)**2 + Vi(i)**2
Vr_out(j)=Vr(i)
Vi_out(j)=Vi(i)
end if
doi=1,nt
write(1,*)k,i,Vr_out(i),Vi_out(i)
end do
do j=1,Nt
write(2,*)k,j,V(j)
end do
else
end if
if(k.lt.N_t + 2*N_steps)goto 3
do j=1,Nt
write(3,*)k,j,Vr(j)**2+Vi(j)**2
end do
call fftinn(Vr,Vi,Nt,Nst,1.) ! from Time to Frequency
do i=1,nt
if(i.le.nt/2)then
j=nt/2+i
V(j)=Vr(i)**2 + Vi(i)**2
Vr_out(j)=Vr(i)
Vi_out(j)=Vi(i)
else
end if
\[ j = \text{i-nt}/2 \]
\[ V(j) = V_r(i)^2 + V_i(i)^2 \]
\[ V_{r\text{out}}(j) = V_r(i) \]
\[ V_{i\text{out}}(j) = V_i(i) \]
end if
end do
call fftinn(Vr,Vi,Nt,Nst,-1.) ! from Frequency to Time
do i=1,Nt
write(1,*)(k,i,Vr_out(i),Vi_out(i))
end do
do j=1,Nt
write(2,*)(k,j,V(j))
end do

close(1)
close(2)
close(3)
end
c Fast Fourier transformation
see Appendix 1

9 Appendix 3

ULTRASHORT PULSE STABILITY IN THE KERR-LENS
MODE-LOCKED LASER: ANALYSIS ON THE BASIS OF
THE MOMENTUM METHOD

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This program is realized on the basis of the computer algebra approach
(the corresponding description can be found on [http://www.geocities.com/optomaplev](http://www.geocities.com/optomaplev)/
[http://www.geocities.com/](http://www.geocities.com/)

PARAMETERS:

a is the distance from the out-put plane mirror to the first folding
mirror;
c is the distance from the totally reflecting plane mirror to the
second folding mirror;
b is the distance between folding mirrors;
c bs and bf give the limits of the scanning on b;
c b1 is the distance of the active medium facet from the first folding
c mirror;
c b1s is the starting b1

c f is the focus length of the folding mirrors;
c z is the active medium length;
c n is the refractive index of the active medium;
c lambda is the generation wavelength;
c eps1 gives the criterion of the convergence to the steady-state
c solution;
c P is the pump power in watts;
c wp is the pump beam size;
c l is the loss coefficient on the pump wavelength (in 1/cm);
c loss is the out-put loss coefficient;
c am is the maximum gain coefficient;
c Pcr is the critical power of the self-focusing in the active medium (in watts);
c Dam is the group-velocity dispersion of the active medium (in fs^-2/cm,
c normal dispersion has a negative sign);
c Is is the gain saturation intensity;
c
q is the complex Gaussian beam parameter;
c Pw is the pulse power;
c delta is the pulse width (for Gaussian pulse);
c psi is the pulse chirp;
c alpha and beta is the generation beam parameters (see description)
c
Lengths are given in centimeters; powers are given in watts

COMPLEX*16 i,qs,q(15),qq,t ! i is the imaginary unit, qs is the initial

REAL*8 a,c,z,f,n,eps1/30d0,60d0,0.28d0,15d0,2.442d0,1d-3/
DATA b1s,bs,bf,wp,P,loss/0d0,20d0,55d0,100d-4,1.5d0,6d0,5d-2/
DATA Pi,lambda/3.14159265358979323846264338d0,2.5d-4/
DATA am,Pcr,cv,Dam,Is/9d0,965d3,3d10,-2054d0,1d4/
i=dcmplx(0.,1.)
! w0=100 mkm is the initial size of the plane wave:
qs = (0..7957747152)
! wmax=10 cm is maximum size of the simulated mode
eps2 = 1d2*Pi/lambda
k = 2d0*Pi/lambda ! is the wave number
! It takes into account the pump wave damping in the active medium:
kappa = .5506035739d0*P

S = 2d0*0.48d0*P*.2900302115d-4/Pi/wp**2 ! is the pump parameter
  ! sigma_a*T_r/h/nu_p=.2900302115d-4.
  ! 0.48 - averaging along propagation axis.
  ! sigma_a, T_r, nu_p are the absorption
  ! cross section of the active medium,
  ! the gain relaxation time, and the gain
  ! wavelength, respectively
x = z/n ! is the optical length of the gain medium
tg = 4d0/z ! is the gain band width in fs/cm
dx = x/1d3 ! is the step size
DO I1=1,201 ! scanning on b
  b = bs + (bf-bs)*(I1-1)/200d0
DO I2=1,201 ! scanning on b1
  b1 = b1s + (b1f-b1s)*(I2-1)/200d0
  b1f = b-z
b2 = b-b1-z
Lcav = a + b + c ! is the cavity length
Tcav = (2d0*Lcav/cv)*1d15 ! is the cavity period fs
write(*,*)I1,I2 ! just step numbers!
gamma0 = 0d0
Pw = 10d0 ! initial pulse power in watts
delta0 = 1d3 ! initial pulse width in fs
psi0 = 0d0 ! initial pulse chirp

DO 2 continue
Num = 0
qq = qs
2 continue
Num = Num + 1
Pwold = Pw

q(1) = qq
q(2) = q(1) + a
q(3) = 1d0/(1d0/q(2) - 1d0/f)
q(4) = q(3) + b1

if(-1d0/dimag(1d0/q(4))).lt.0d0 goto 1
beta0 = -k*dreal(1d0/q(4))/2d0
alpha0 = dsqrt(-1d0/dimag(1d0/q(4))/k)
g = am*S/(1d0 + S + #Pw*delta0*dsqrt(Pi/2d0)/(Pi*alpha0**2*Is*Tcav)) ! saturated gain

c  Active medium (Runge-Kutta fourth-order method)

DO J=1,1000
  ka(1) = (-0.5d0/alpha0**4 + 2d0*beta0**2 + #2d0*k*kappa*dexp(-l*dx*(J-1))/wp**2)/k +
  #dsqrt(2d0)/Pi)*(Pw/Pcr)*dexp(2*gamma0)/alpha0**4/k !beta
  qu(1) = - 2d0*beta0*alpha0/k -
  #2d0*g*alpha0**3/((1d0+2d0*alpha0**2/wp**2)**(3/2)/wp**2) !alpha
  tau(1) = g/(1d0 + 2d0*alpha0**2/wp**2) - 2d0*tg**2/delta0**2 +
  Dam*psi0
  delta(1) = 2d0*tg**2*(1d0/delta0 - delta0**3*psi0**2) - !delta
  #2d0*Dam*delta0*psi0
  psi(1) = 2d0*Dam*(psi0**2 - 1d0/delta0**4) - 8d0*tg**2*psi0/ !psi
  #delta0**2 + (2d0/Pi)*(Pw/Pcr)*dexp(2*gamma0)/alpha0**2/delta0**2

  ka(2) = (-0.5d0/(alpha0+0.5d0*dx*ka(1))**4 +
  #2d0*(beta0+0.5d0*dx*ka(1))**2 +
  #2d0*k*kappa*dexp(-l*dx*(J-0.5d0))/wp**2)/k +
  #dsqrt(2d0)/Pi)*(Pw/Pcr)*dexp(2*gamma0+0.5d0*dx*tau(1))/
  #alpha0+0.5d0*dx*qu(1))**4/k
  qu(2) = - 2d0*(beta0+0.5d0*dx*ka(1))*(alpha0+0.5d0*dx*qu(1))/k -
  #2d0*gamma0*alpha0+0.5d0*dx*qu(1))**3/(1d0+
  #2d0*(alpha0+0.5d0*dx*qu(1))**2/wp**2)**(3/2)/wp**2
  tau(2) = g/(1d0 + 2d0*(alpha0+0.5d0*dx*qu(1))**2/wp**2) -
  #2d0*tg**2/(delta0+0.5d0*dx*delta(1))**2+Dam*(psi0+0.5d0*dx*psi(1))
  delta(2) = 2d0*tg**2*(1d0/(delta0+0.5d0*dx*delta(1)) -
  #delta0+0.5d0*dx*delta(1))**3*(psi0+0.5d0*dx*psi(1))**2 -
  #2d0*Dam*(delta0+0.5d0*dx*delta(1))**2*(psi0+0.5d0*dx*psi(1))
  psi(2) = 2d0*Dam*(psi0+0.5d0*dx*psi(1))**2 -
  #1d0/(delta0+0.5d0*dx*delta(1))**4 -
  #8d0*tg**2*(psi0+0.5d0*dx*psi(1))/
  #delta0+0.5d0*dx*delta(1))**2 + (2d0/Pi)*(Pw/Pcr)*
  #dexp(2*gamma0+0.5d0*dx*tau(1))/(alpha0+0.5d0*dx*qu(1))**2/
  #delta0+0.5d0*dx*delta(1))**2
  ka(3) = (-0.5d0/(alpha0+0.5d0*dx*ka(2))**4 +
  #2d0*(beta0+0.5d0*dx*ka(2))**2 +
  #2d0*k*kappa*dexp(-l*dx*(J-0.5d0))/wp**2)/k +
  #dsqrt(2d0)/Pi)*(Pw/Pcr)*dexp(2*gamma0+0.5d0*dx*tau(2))/
  #alpha0+0.5d0*dx*qu(2))**4/k
  qu(3) = - 2d0*(beta0+0.5d0*dx*ka(2))*(alpha0+0.5d0*dx*qu(2))/k -
  #2d0*g*(alpha0+0.5d0*dx*qu(2))**3/(1d0+ 

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#2d0*(alpha0+0.5d0*dx*qu(2))**2/wp**2)**(3/2)/wp**2)
  tau(3) = g/(1d0 + 2d0*(alpha0+0.5d0*dx*qu(2))**2/wp**2) -
#2d0*taug**2/(delta0+0.5d0*dx*delta(2))**2+Dam*(psi0+0.5d0*dx*psi(2))
  delta(3) = 2d0*taug**2*(1d0/(delta0+0.5d0*dx*delta(2))) -
#(delta0+0.5d0*dx*delta(2))**3*(psi0+0.5d0*dx*psi(2))**2 -
#2d0*Dam*(delta0+0.5d0*dx*delta(2))*(psi0+0.5d0*dx*psi(2))
  psi(3) = 2d0*Dam*(psi0+0.5d0*dx*psi(2))**2 -
#1d0/(delta0+0.5d0*dx*delta(2))**4 -
#8d0*tg**2/(delta0+0.5d0*dx*delta(2))**2 + (2d0/Pi)*(Pw/Pcr)*
#dexp(2*(gamma0+0.5d0*dx*tau(2)))/(alpha0+0.5d0*dx*qu(2))**2/
#(delta0+0.5d0*dx*delta(2))**2
  ka(4) = (-0.5d0/(alpha0+dx*qu(3))**4 +
#2d0*(beta0+dx*ka(3))**2 +
#2d0*k*kappa*dexp(-l*dx*J)/wp**2)/k +
#(dsqrt(2d0)/Pi)*dexp(2*gamma0+dx*tau(3))/(alpha0+dx*qu(3))**2/
#(alpha0+dx*qu(3))**4/k
  qu(4) = -2d0*(beta0+dx*ka(3))*(alpha0+dx*qu(3))/k -
#2d0*g*(alpha0+dx*qu(3))**3/(1d0+
#2d0*(alpha0+dx*qu(3))**2/wp**2)**(3/2)/wp**2
  tau(4) = g/(1d0 + 2d0*(alpha0+dx*qu(3))**2/wp**2) -
#2d0*taug**2/(delta0+dx*delta(3))**2 + Dam*(psi0+dx*psi(3))
  delta(4) = 2d0*taug**2*(1d0/(delta0+dx*delta(3))) -
#(delta0+dx*delta(3))**3*(psi0+dx*psi(3))**2 -
#2d0*Dam*(delta0+dx*delta(3))*(psi0+dx*psi(3))
  psi(4) = 2d0*Dam*((psi0+dx*psi(3))**2 -
#1d0/(delta0+dx*delta(3))**4 -
#8d0*tg**2/(psi0+dx*psi(3)))/
#(delta0+dx*delta(3))**2 + (2d0/Pi)*(Pw/Pcr)*
#dexp(2*(gamma0+dx*tau(3)))/(alpha0+dx*qu(3))**2/
#(delta0+dx*delta(3))**2
  beta = beta0 + (dx/6d0)*(ka(1)+2d0*ka(2)+2d0*ka(3)+ka(4))
  alpha = alpha0 + (dx/6d0)*(qu(1)+2d0*qu(2)+2d0*qu(3)+qu(4))
  gamma = gamma0 + (dx/6d0)*(tau(1)+2d0*tau(2)+2d0*tau(3)+tau(4))
  delta = delta0+(dx/6d0)*((delta(1)+2d0*delta(2)+2d0*delta(3)+delta(4))
  ps = psi0 + (dx/6d0)*(psi(1)+2d0*psi(2)+2d0*psi(3)+psi(4))
if(alpha.lt.0.or.gamma.lt.0.or.delta.lt.0.
or.gamma.gt.10.or.delta.gt.10.
or.psi.lt.0.)goto 1
alpha0 = alpha
beta0 = beta
gamma0 = gamma
delta0 = del
psi0 = ps
END DO
Pw = Pw*dexp(2d0*gamma0)
gamma0 = 0d0
if(Pw.le.1d-10.or.Pw.gt.1d20)goto 1

q(5) = dcmplx(dreal(t),dimag(t))

---------------------------------------------------------------------
c ABCD-modul: active medium - second folding mirror - second flat mirror
(and backwards)
q(6) = q(5) + b2
q(7) = 1d0/(1d0/q(6) - 1d0/f)
q(8) = q(7) + c
q(9) = q(8) + c
q(10) = 1d0/(1d0/q(9) - 1d0/f)
q(11) = q(10) + b2

c---------------------------------------------------------------------
if(-1d0/dimag(1d0/q(11)).lt.0d0)goto 1
beta0 = -k*dreal(1d0/q(11))/2d0
alpha0 = dsqrt(-1d0/dimag(1d0/q(11))/k)

if(-1d0/dimag(1d0/q(11)).lt.0d0)goto 1
beta0 = -k*dreal(1d0/q(11))/2d0
alpha0 = dsqrt(-1d0/dimag(1d0/q(11))/k)

g = am*S/(1d0 + S + #Pw*delta0*dsqrt(Pi/2d0)/(Pi*alpha0**2*Is*Tcav))

---------------------------------------------------------------------
c Active medium (Runge-Kutta fourth-order method)
DO J=1,1000
ka(1) = (-0.5d0/alpha0**2 + 2d0*beta0**2 +
#2d0*k*kappa*dexp(-l*dx*(J-1))/wp**2)/k +
#(dsqrt(2d0)/Pi)*(Pw/Pcr)*dexp(2*gamma0)/alpha0**2/k
qu(1) = - 2d0*beta0*alpha0/k -
#2d0*g*alpha0**2/(1d0 + 2d0*alpha0**2/wp**2)**(3/2)/wp**2
tau(1) = g/(1d0 + 2d0*alpha0**2/wp**2) - 2d0*tau/alpha0**2 +
#Dam*psi0
delta(1) = 2d0*tau**2*(1d0/delta0 - delta0**3*psi0**2) -
#2d0*Dam*delta0*psi0
psi(1) = 2d0*Dam*(psi0**2 - 1d0/delta0**4) - 8d0*tau**2*psi0/
#delta0**2 + (2d0/Pi)*(Pw/Pcr)*dexp(2*gamma0)/alpha0**2/delta0**2

ka(2) = (-0.5d0/(alpha0+0.5d0*dx*ka(1)))**2 +
#2d0*(beta0+0.5d0*dx*ka(1))**2 +
#2d0*k*kappa*dexp(-l*dx*(J-0.5d0))/wp**2)/k +
#(dsqrt(2d0)/Pi)*(Pw/Pcr)*dexp(2*gamma0+0.5d0*dx*tau(1))/
#(alpha0+0.5d0*dx*qu(1))**4/k
qu(2) = - 2d0*(beta0+0.5d0*dx*ka(1))*alpha0+0.5d0*dx*qu(1))/k -
#2d0*g*(alpha0+0.5d0*dx*qu(1))**3/(1d0+
#2d0*(alpha0+0.5d0*dx*qu(1))**2/wp**2)**(3/2)/wp**2
tau(2) = g/(1d0 + 2d0*(alpha0+0.5d0*dx*qu(1))**2/wp**2) -
\[ 2d0^*tg^**2/(\delta 0+0.5d0^*dx^*\delta(1))^**2+Dam^*(\psi 0+0.5d0^*dx^*\psi(1)) \]
\[ \delta(2) = 2d0^*tg**2*(1/d0/(\delta 0+0.5d0^*dx^*\delta(1)) - \]
\[ (\delta 0+0.5d0^*dx^*\delta(1))^**3*(\psi 0+0.5d0^*dx^*\psi(1))^**2) - \]
\[ 2d0^*Dam^*(\delta(0+0.5d0^*dx^*\delta(1))^*(\psi 0+0.5d0^*dx^*\psi(1)) \]
\[ psi(2) = 2d0^*Dam^*((\psi 0+0.5d0^*dx^*\psi(1))^**2 - \]
\[ 1d0/(\delta(0+0.5d0^*dx^*\delta(1))^**4) - \]
\[ 8d0^*tg**2*(\psi 0+0.5d0^*dx^*\psi(1))/ \]
\[ (\delta(0+0.5d0^*dx^*\delta(1))^**2 + (2d0/Pi)*(Pw/Pcr)^* \]
\[ dexp(2*(\gamma 0+0.5d0^*dx^*\tau(1)))/(\alpha 0+0.5d0^*dx^*\qu(1))^**2/ \]
\[ (\delta(0+0.5d0^*dx^*\delta(1))^**2 \]
\[ ka(3) = (-0.5d0/(\alpha 0+0.5d0^*dx^*\qu(2))^**4 + \]
\[ 2d0^*(\beta 0+0.5d0^*dx^*\ka(2))^**2 + \]
\[ 2d0^*k^*kappa^*dexp(-1^*dx^*(J-0.5d0))/wp^**2/k + \]
\[ (dsqrt(2d0)/Pi)^*(Pw/Pcr)^*dexp(2^*\gamma 0+0.5d0^*dx^*\tau(2))/ \]
\[ (\alpha 0+0.5d0^*dx^*\qu(2))^**4/k \]
\[ qu(3) = -2d0^*(\beta 0+0.5d0^*dx^*\ka(2))*(\alpha 0+0.5d0^*dx^*\qu(2))/k - \]
\[ 2d0^*g*(\alpha 0+0.5d0^*dx^*\qu(2))^**3/(1d0+ \]
\[ 2d0^*(\alpha 0+0.5d0^*dx^*\qu(2))^**2/wp^**2)^**2(3/2)/wp^**2 \]
\[ tau(3) = g/(1d0 + 2d0^*(\alpha 0+0.5d0^*dx^*\qu(2))^**2/wp^**2) - \]
\[ 2d0^*tg**2/(\delta(0+0.5d0^*dx^*\delta(2))^**2+Dam^*(\psi 0+0.5d0^*dx^*\psi(2)) \]
\[ delta(3) = 2d0^*tg**2*(1/d0/(\delta(0+0.5d0^*dx^*\delta(2)) - \]
\[ (\delta(0+0.5d0^*dx^*\delta(2))^**3*(\psi 0+0.5d0^*dx^*\psi(2))^**2 - \]
\[ 2d0^*Dam^*(\delta(0+0.5d0^*dx^*\delta(2))^*(\psi 0+0.5d0^*dx^*\psi(2)) \]
\[ psi(3) = 2d0^*Dam^*((\psi 0+0.5d0^*dx^*\psi(2))^**2 - \]
\[ 1d0/(\delta(0+0.5d0^*dx^*\delta(2))^**4) - \]
\[ 8d0^*tg**2*(\psi 0+0.5d0^*dx^*\psi(2))/ \]
\[ (\delta(0+0.5d0^*dx^*\delta(2))^**2 + (2d0/Pi)*(Pw/Pcr)^* \]
\[ dexp(2*(\gamma 0+0.5d0^*dx^*\tau(2)))/(\alpha 0+0.5d0^*dx^*\qu(2))^**2/ \]
\[ (\delta(0+0.5d0^*dx^*\delta(2))^**2 \]
\[ ka(4) = (-0.5d0/(\alpha 0+dx^*\qu(3))^**4 + \]
\[ 2d0^*(\beta 0+dx^*\ka(3))^**2 + \]
\[ 2d0^*k^*kappa^*dexp(-1^*dx^*J)/wp^**2)/k + \]
\[ (dsqrt(2d0)/Pi)^*(Pw/Pcr)^*dexp(2^*\gamma 0+dx^*\tau(3))/ \]
\[ (\alpha 0+dx^*\qu(3))^**4/k \]
\[ qu(4) = -2d0^*(\beta 0+dx^*\ka(3))*(\alpha 0+dx^*\qu(3))/k - \]
\[ 2d0^*g*(\alpha 0+dx^*\qu(3))^**3/(1d0+ \]
\[ 2d0^*(\alpha 0+dx^*\qu(3))^**2/wp^**2)^**2(3/2)/wp^**2 \]
\[ tau(4) = g/(1d0 + 2d0^*(\alpha 0+dx^*\qu(3))^**2/wp^**2) - \]
\[ 2d0^*tg**2/(\delta(0+dx^*\delta(3))^**2+Dam^*(\psi 0+dx^*\psi(3)) \]
\[ delta(4) = 2d0^*tg**2*(1/d0/(\delta(0+dx^*\delta(3)) - \]
\[ (\delta(0+dx^*\delta(3))^**3*(\psi 0+dx^*\psi(3))^**2 - \]
\[ 2d0^*Dam^*(\delta(0+dx^*\delta(3))^*(\psi 0+dx^*\psi(3)) \]
\[ psi(4) = 2d0^*Dam^*((\psi 0+dx^*\psi(3))^**2 - \]
\[ 1d0/(\delta(0+dx^*\delta(3))^**4) - \]
\[ 8d0^*tg**2*(\psi 0+dx^*\psi(3))/ \]
\[ (\delta(0+dx^*\delta(3))^**2 + (2d0/Pi)*(Pw/Pcr)^* \]

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\[ \frac{d\exp(2*(\gamma_0 + dx*\tau(3)))/((\alpha_0 + dx*\phi(3))^2)}{((\delta_0 + dx*\delta(3))^2)} \]

\begin{align*}
\text{beta} &= \text{beta}_0 + (dx/6d0)*(ka(1)+2d0*ka(2)+2d0*ka(3)+ka(4)) \\
\text{alpha} &= \text{alpha}_0 + (dx/6d0)*(\phi(1)+2d0*\phi(2)+2d0*\phi(3)+\phi(4)) \\
\text{gamma} &= \text{gamma}_0 + (dx/6d0)*(\tau(1)+2d0*\tau(2)+2d0*\tau(3)+\tau(4)) \\
\text{del} &= \text{delta}_0 + (dx/6d0)*(\delta(1)+2d0*\delta(2)+2d0*\delta(3)+\delta(4)) \\
\text{ps} &= \text{psi}_0 + (dx/6d0)*(\psi(1)+2d0*\psi(2)+2d0*\psi(3)+\psi(4))
\end{align*}

\[ \text{if(alpha.gt.10.or.gamma.lt.0.)} \text{goto 1} \]

\[
\text{alpha}_0 = \text{alpha} \\
\text{beta}_0 = \text{beta} \\
\text{gamma}_0 = \text{gamma} \\
\text{delta}_0 = \text{del} \\
\text{psi}_0 = \text{ps}
\]

\[ \text{END DO} \]

\begin{verbatim}
Pw = Pw*dexp(2d0*gamma0) \\
gamma0 = 0d0 \\
if(Pw.le.1d-10.or.Pw.gt.1d20) goto 1 \\
t = k/(-2d0*beta0 - i/alpha0**2) \\
q(12) = dcmplx(dreal(t),dimag(t)) \\
c ABCD-modul for the residuary propagation up to out-put mirror \\
q(13) = q(12) + b1 \\
q(14) = 1d0/(1d0/q(13) - 1d0/f) \\
q(15) = q(14) + a \\
Pw = Pw*dexp(-loss)
\end{verbatim}

\[ \text{if(Pw.le.1d-10.or.Pw.gt.1d20) goto 1} \] ! criteria for the power

\[ \text{if(Num.gt.5000) goto 1} \] ! and iteration number

\[ w^2*Pi/\lambda \text{ converts the initial part of the beam parameter to the beam size} \]

\[ \text{DO I4=1,15} \]

\[ \text{ro(I4)} = -1d0/dimag(1d0/q(I4)) \]

\[ \text{END DO} \]

\begin{verbatim}
c Beam is to have the positive size and hasn’t to be too large \\
if(ro(1).le.0..or.ro(2).le.0..or.ro(3).le.0..or. \\
2 ro(4).le.0..or.ro(5).le.0..or.ro(6).le.0..or. \\
3 ro(7).le.0..or.ro(8).le.0..or.ro(9).le.0..or. \\
4 ro(10).le.0..or.ro(11).le.0..or.ro(12).le.0..or. \\
5 ro(13).le.0..or.ro(14).le.0..or.ro(15).le.0.) goto 1 \\
if(ro(1).gt.eps2.or.ro(2).gt.eps2.or.ro(3).gt.eps2.or. \\
2 ro(4).gt.eps2.or.ro(5).gt.eps2.or.ro(6).gt.eps2.or. \\
3 ro(7).gt.eps2.or.ro(8).gt.eps2.or.ro(9).gt.eps2.or.) \\
\end{verbatim}

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4       ro(10).gt.eps2.or.ro(11).gt.eps2.or.ro(12).gt.eps2.or.
5       ro(13).gt.eps2.or.ro(14).gt.eps2.or.ro(15).gt.eps2)goto 1

    qq = q(15)

    c Pulse power stability
    if(abs(ro(15)/ro(1)-1d0).gt.eps1.
#or.abs(Pw/Pwold-1d0).gt.eps1)goto 2
    c Out-put for the stable pulse
    write(1,*)(b,b1,Pw,sqrt(ro(5)*lambda/Pi),delta0,Num
1     continue
    END DO
    END DO

    close(1)
    close(2)

STOP

END

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