Optimal Radio Resource Allocation for Hybrid Traffic in Cellular Networks: Traffic Analysis and Implementation

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Abstract—In part I of this paper, a distributed and a centralized architecture for an optimal radio resource allocation aware of the traffic delay-tolerance nature, user subscription type, and application usage variations were developed. In the current article, a transmission overhead analysis of the aforementioned distributed and centralized architectures is investigated and it is proved that the centralized scheme endures a significantly lower transmission overhead than does the distributed approach. Furthermore, the lower bounds of the transmission overhead for both the centralized and the distributed architectures are derived. Moreover, a sensitivity analysis of the resource allocation procedures of the aforementioned centralized and distributed architectures to the changes in the number of users in the system is presented. Besides, a sensitivity analysis of the centralized and distributed approaches to the temporal changes in application usages are investigated. Ultimately, the transmission overhead and sensitivity relevant statements are verified through appropriate simulations. And last but not the least, a real-world implementation of the resource allocation methods developed in Part I is provided.

Index Terms—Utility function, hybrid traffic, convex optimization, centralized algorithm, distributed algorithm, optimal resource allocation, implementation

I. INTRODUCTION

Resource allocation methods aiming at fulfilling the quality of service (QoS) requirements of current-day cellular networks have been the focus of many research studies. In fact, cellular communication systems nowadays contain smartphones capable of running several applications simultaneously. Since the applications have miscellaneous QoS criteria based on the type of the traffic that they deal with, many of the resource allocation techniques have to account for the application types in order to incorporate QoS into their operation. Furthermore, the diversity of the users has to be included in the resource allocation methods implemented in networks catering to a wide variety of subscribers. Besides, concurrent running of applications on the smartphones motivates incorporating the application usage temporal changes into the resource allocation schemes. Much of the state of the art in single carrier resource allocation ([2]–[5]) considers either traffic or subscriber types into their formulation. However, some of these work fail to provide optimality for the allocated rates or do not account for all the aforementioned QoS-related issues at the same time. Many other multi carrier resource allocation studies ([6]–[9]) also have been done, which do not address all the aforesaid QoS-related concerns.

A single carrier optimal radio resource allocation with utility proportional fairness formulation was presented in part I [10] of the article at hand. The aforementioned formulation was cast into a centralized and a distributed architecture in the part I of this paper [10], and also included algorithms to solve them. The current article is part II of the resource allocation paper presented in [10] and tries to address the QoS-related issues mentioned above with regard to the method presented in [10]. Hereafter, a referring to “part II” indicates part II of the resource allocation paper [10], referred to as “part I”, unless otherwise is explicitly stated. Part II first presents a traffic analysis of the centralized and distributed resource allocation architectures that were introduced in part I.

The traffic analysis includes a transmission overhead analysis of the aforesaid architectures and obtains lower bounds for the transmission overheads. Furthermore, part II contains a sensitivity analysis of the centralized and distributed architectures presented in the part I [10] of the current article. The sensitivity analysis studies whether the user equipment (UE) bids and their allocated rates remain optimal when the number of UEs or their application usage percentages change in the system. The sensitivity analysis also investigates the variations of the transmission overhead of the centralized and distributed architectures, developed in part I [10], when the UE quantity or application usage percentages change and under the existence or lack of a rebidding procedure in response to the aforesaid changes. Then, simulations are done to confirm to-be-derived predicates relevant to the transmission overhead and sensitivity analyses. The traffic and sensitivity analysis deductions, applicable to part I, can be applied to the resource block allocation formulations in ([11], [12]) as well as to the radar-spectrum shared resource allocation work in [13]. Ultimately, a real-world network is developed on which the implementation results reveal that applying the algorithms improves the QoS for the applications. The current article is intended to complement the part I of the paper.

A. Contributions

The contributions in the current paper are summarized below.

- We analyze the sensitivity of the distributed and centralized resource allocations developed in part I to the
of the first predicate explains that when more rate is assigned to the applications, they obtain a higher QoS satisfaction. The second predicate indicates that the utility functions are continuous. The delay-tolerant and real-time applications are respectively modelled by normalized logarithmic and sigmoidal utility functions represented in equations (1) and (2) in that order [15].

\[
U(r) = c \left( \frac{1}{1 + e^{-a(r-b)}} - d \right)
\]  

(1)

Here, \( c = \frac{k + e^b}{e^b} \) and \( d = \frac{1}{1 + e^{a+b}} \). It can be easily verified that \( U(0) = 0 \) and \( U(\infty) = 1 \), where the former is one of the previously mentioned utility function properties and the latter indicates that assigning infinite resources ensues 100% QoS satisfaction. Furthermore, it is easily derivable that the inflection point of the sigmoidal utility function in equation (1) occurs at \( r = r^{\text{inf}} = b \), where the superscript “inf” stands for inflection. In equation (2), \( r^{\text{max}} \) is the maximum rate at which the application QoS achieves 100% utility satisfaction percentage and \( k \) is the utility function increase with augmented allocated rate \( r \). It can be easily verified that \( U(0) = 0 \) and \( U(r^{\text{max}}) = 1 \), where the former is the property of the utility functions and the latter implies that \( r = r^{\text{max}} \) achieves a 100% QoS satisfaction. Moreover, the inflection point of normalized logarithmic function is at \( r = r^{\text{inf}} = 0 \).

\[
U(r) = \frac{\log(1 + kr)}{\log(1 + kr^{\text{max}})}
\]  

(2)

The objective is to determine the optimal rates that the eNBs should be allocating to their UE applications to ensure that Real-time applications are rendered priority over delay-tolerant ones, no user is dropped, applications temporal usage variations are accounted for, and subscription-based treatments are honored. We assume that UEs run multiple real-time and delay-tolerant applications, mathematically represented by sigmoidal and logarithmic utility functions. To obtain the optimal rates and fulfill the above requirements, part I of the paper developed the resource allocation algorithms cast under a centralized architecture, which is displayed again for the readers’ convenience.

### A. Centralized Resource Allocation

Here, the application rates are directly assigned by the eNBs in a singular stage. The formulation of the centralized algorithm is illustrated in equation (3). Here, \( M \) UEs are covered by an eNB and \( r = [r_1, r_2, \ldots, r_M] \) is the UE allocated rate vector, whose \( i^{\text{th}} \) element is the rate allocated to the \( i^{\text{th}} \) UE. Besides, \( r_{ij} \), \( U_{ij}(r_{ij}) \), and \( \alpha_{ij} \) respectively represent the rate allocation, application utility function, and application usage percentage of the \( i^{\text{th}} \) application running on the \( j^{\text{th}} \) UE. Hence, we can write \( \sum_{j=1}^{N_i} \alpha_{ij} = 1 \) which states that the addition of the usage percentage of the application running on the \( j^{\text{th}} \) UE is 100%. Furthermore, we can write \( r_i = \sum_{j=1}^{N_i} r_{ij} \) where \( N_i \) is the number of coevally running applications on the \( i^{\text{th}} \) UE. The former of the afore-written equations , and the letter one implies that the the \( i^{\text{th}} \) UE rate is the augmentation of all its \( N_i \) applications resources assignments. Also, \( R \) is

![Fig. 1. System Model: Single cell, within the cellular network, with an eNB covering \( M = 6 \) UEs each with simultaneously running delay-tolerant and relay-time applications represented by logarithmic and sigmoidal utility functions respectively.](image)
the maximum bandwidth available to the eNB and $\beta_i$ is a subscription-dependent weight for the $i^{th}$ UE. Regarding the centralized architecture, part I of this article does the following:

- Part I proves that the resource allocation in equation (3) is convex and has a tractable solution.
- Part I proves that the resource allocation in equation (4) refrains from dropping UEs, prioritizes real-time applications, considers application usage variations, and accounts for UE priorities.
- Part I provides with algorithms, implemented in the UEs and eNBs, to obtain the optimal application rates.

$$\max_{\mathbf{r}} \quad \prod_{i=1}^{M} \left( \prod_{j=1}^{N_i} U_{ij}^{\alpha_{ij}}(r_{ij}) \right)^{\beta_i}$$

subject to

$$\sum_{i=1}^{M} \sum_{j=1}^{N_i} r_{ij} \leq R,$$

$$r_{ij} \geq 0, \quad i = 1, 2, ..., M, \quad j = 1, 2, ..., N_i$$

More details about the solution algorithms for the distributed architecture can be found in part I. In addition, part I of the paper developed the resource allocation algorithms cast under a distributed architecture, which is displayed below for the readers' convenience.

B. Distributed Resource Allocation

In the distributed architecture, there are two optimization problems, the first of which concerns the UE optimal rates allocated by the eNBs via collaborations between the eNBs and the UEs that they are covering. This stage is referred to as external UE resource allocation (EURA) and is written as equation (4), where for $M$ UEs $V_i(r_i) = \prod_{j=1}^{N_i} U_{ij}^{\alpha_{ij}}(r_{ij})$ is the aggregated utility function for the $i^{th}$ UE. $\mathbf{r} = [r_1, r_2, ..., r_M]$ is the UE rate vector whose $i^{th}$ component represents the rate assigned by the eNB to the $i^{th}$ UE. The second optimization focuses on distributing the application rates by their host UEs and is performed internally to the UEs. This stage is denoted as the internal UE resource allocation (IURA) and is written in equation (5). Here, $\mathbf{r}_i = [r_{i1}, r_{i2}, ..., r_{iN_i}]$ is the application rate allocation vector such that its $j^{th}$ component indicates the bandwidth allotted by the $i^{th}$ UE to its $j^{th}$ application, $r_{i}^{\text{opt}}$ is the $i^{th}$ UE rate allocated by eNB via solving the EURA optimization in equation (4), and $N_i$ is the number of applications running in the $i^{th}$ UE. Superscript "opt" indicates the optimality of the UE rates which was proved in part I [10], which also proved that there exists a tractable global optimal solution to the IURA optimization problem in equation (5) and also provided the solving algorithm thereof. In summary, regarding the distributed architecture, part I of the article does the following:

- Part I proves that the EURA and IURA in equations (4) and (5) are convex and have tractable solutions.
- Part I proves that the EURA and IURA in equations (4) and (5) refrain from dropping UEs, IURA prioritize real-time applications, IURA considers application usage variations, and EURA accounts for UE priorities.

- Part I provides with algorithms, implemented in the UEs and eNBs, to obtain the optimal application rates.

$$\max_{\mathbf{r}} \quad \prod_{i=1}^{M} V_i^{\beta_i}(r_i)$$

subject to

$$\sum_{i=1}^{M} r_i \leq R,$$

$$r_i \geq 0, \quad i = 1, 2, ..., M.$$
1) Distributed Algorithm with Rebidding: For every UEs entering/leaving the cell, we assume that prior UEs (i.e. users already in the cell and having been allotted optimal rates) will rebid for resources as the system has changed. In this case, proposition III.2 is considerable with regard to the transmission overhead.

**Proposition III.2.** The transmission overhead of the distributed resource allocation algorithm with rebidding is maximum and greater than $2M_2 + 2 - M_1$.

**Proof:** When $M_2 > M_1$, i.e. the UE quantity increases, new UEs send their initial bids requiring $M_2 - M_1$ transmissions to the eNB. If $\beta_i$ is available at the UEs, the eNB broadcasts a new shadow price in $M_2 - M_1 + 1$ transmissions. Next, the $M_2$ UEs transmit the eNB their new bids followed by the eNB sending a modified shadow price. This is repeated $k$ times until an optimal rate allocation occurs which amounts to the transmission overhead $2M_2 + 2 - M_1$ for one iteration ($k = 1$). On the other hand, if $\beta_i$ is only available at the eNB, it transmits a modified shadow price $\frac{p_2}{\beta_i}$ to the UEs under its coverage. Thus, the transmission overhead will be $(M_2 - M_1) + M_2 + 2kM_2 = (2k + 2)M_2 - M_1$, corresponding minimally to $M_2 + 2 - M_1$ for one iteration ($k = 1$). Hence, the transmission overhead is greater than $2M_2 + 2 - M_1$.

When $M_1 > M_2$, i.e. the UE quantity decreases, the exiting UEs send service termination signals to the eNB in $M_1 - M_2$ transmissions. If $\beta_i$ is available at the UEs, the eNB broadcasts a new shadow price in $M_1 - M_2 + 1$ transmissions. Next, the $M_2$ UEs transmit the eNB their new bids followed by the eNB sending another shadow price. This process is repeated $k$ times until an optimal rate allocation occurs which amounts to the transmission overhead $(M_1 - M_2) + M_2 + 2kM_2 = (k - 1)M_2 + k + 1 - M_1$, corresponding minimally to $2M_2 + 2 - M_1$ for one iteration ($k = 1$). On the other hand, if $\beta_i$ is only available at the eNB, it transmits a modified shadow price $\frac{p_2}{\beta_i}$ to the UEs under its coverage. Thus, the transmission overhead becomes $(M_1 - M_2) + M_2 + 2kM_2 = 2kM_2 + M_1$ transmissions, corresponding minimally to $2M_2 + M_1$ for one iteration ($k = 1$). Hence, the transmission overhead of the distributed approach with rebidding is no less than $2M_2 + 2 - M_1$.

For the distributed scheme with rebidding, the following proposition III.3 discusses a sensitivity of rates and bids to UE quantity variations.

**Proposition III.3.** The allocated rates and pledged bids with rebidding remain optimal in the distributed resource algorithm under UE quantity changes.

**Proof:** Without loss of generality, we let the number of UEs change from $M_1$ to $M_2$, denoted as $M_1 \rightarrow M_2$. Assuming the distributed algorithm arrives at the steady state according to the equation (7), theorem III.2 and corollary III.3 conclude that the EURA and IURA optimizations with $M = M_1$ are optimal so that for $M_1$ UEs before the time slot $n_1$, we have:

$$r_1(n_1) = S^{-1}_1(p(n_1))$$
$$w_i(n_1) = p(n_1)S^{-1}_i(p(n_1))$$

where $p(n_1) = \sum_{R_i} w_i(n_1)$ and the allocations are done in accordance UE EURA, eNB EURA, and IURA algorithms with initial bids $w_i(1) = 1$. Likewise, theorem III.2 and corollary III.3 conclude that the EURA and IURA optimizations with $M = M_2$ are optimal when the users rebid for resources under UE quantity changes so that for $M_2$ UEs before time slot $n_2$, we have:

$$r_1(n_2) = S^{-1}_1(p(n_2))$$
$$w_i(n_2) = p(n_2)S^{-1}_i(p(n_2))$$

where $p(n_2) = \sum_{R_i} w_i(n_2)$ and the reallocations are done in accordance UE EURA, eNB EURA, and IURA algorithms with initial bids $w_i(n_1)$. Therefore, the rates and bids remain optimal for the distributed scheme with rebidding once the UE quantity changes.

Next, section III.4 analyzes the sensitivity of bid and rates of the distributed approach to UE quantity changes when no-rebidding happens.

2) Distributed Algorithm without rebidding: For every UEs entering/leaving the cell, we assume that prior UEs (i.e. users already in the cell and having been allotted optimal rates) will not rebid for resources so that priori UEs bids do not vary from those in the allocation before time slot $n = n_1$, when UE quantity alters. It is shown in proposition III.4 that the transmission overhead is less than the situation where users rebid for resources.

**Proposition III.4.** The transmission overhead for the distributed resource allocation with no rebidding is not larger than the one with rebidding and is $M_2 + 1 - M_1$ for $M_2 > M_1$ and $M_1 - M_2$ for $M_2 > M_1$ at minimum.

**Proof:** When $M_2 > M_1$, i.e. the UE quantity increases, new UEs send their initial bids requiring $M_2 - M_1$ transmissions to the eNB which broadcasts a new shadow price only to the new UEs if $\beta_i$ is available at the UEs. This is repeated $k$ times until an optimal rate allocation is reached which amounts to the transmission overhead $k(M_2 - M_1) + k$, corresponding minimally to $M_2 - M_1 + 1$ for one iteration ($k = 1$). On the other hand, if $\beta_i$ is only available at the eNB, it transmits a modified shadow price $\frac{p_2}{\beta_i}$ to the $M_2 - M_1$ UEs in $2(M_2 - M_1) + 2k(M_2 - M_1) = (2k + 2)(M_2 - M_1)$ transmissions, corresponding minimally to $4(M_2 - M_1)$ for one iteration ($k = 1$).

When $M_1 > M_2$, i.e. the UE quantity decreases, exiting UEs send service termination signals to the eNB in $M_1 - M_2$ transmissions. There are no further transmissions and the transmission overhead becomes $M_1 - M_2$. Therefore, the minimum transmission overhead is $M_2 + 1 - M_1$ for $M_2 > M_1$ and $M_1 - M_2$ for $M_1 > M_2$, both less than the minimum $2M_2 + 2 - M_1$ transmissions of the distributed scheme with rebidding proved in proposition III.2.

Next, lemma III.4 proves that the rates and bids are not optimal for the distributed scheme with no rebidding when
Lemma III.5. The allocated rates and bids without rebidding are not optimal for the distributed resource allocation under UE quantity changes.

Proof: Assuming a rate and bid optimal convergence for the $M_1$ UEs according to equation (7) before the time slot $n_1$, we have:

$$r_i(n_1) = S_i^{-1}(p(n_1)), \quad w_i(n_1) = p(n_1)S_i^{-1}(p(n_1)),$$

where $p(n_1) = \sum_{i=1}^{M_1} w_i(n_1)$ and the allocations are done in accordance with UE EURA, eNB EURA, and IURA algorithms with initial bids $w_i(1) = 1$.

When $M_2 > M_1$, i.e. the UE quantity increases, prior users bids ($i \in \{1, 2, ..., M_1\}$) remain fixed for $n > n_1$ and the shadow price can be written as:

$$p(n > n_1) = \frac{\sum_{i=1}^{M_1} w_i(n_1) + \sum_{i=M_1+1}^{M_2} w_i(n > n_1)}{R},$$

and user bids can be expressed as equation (12).

$$w_i(n > n_1) = \begin{cases} p(n_1)S_i^{-1}(p(n_1)) & ; \text{i} \in \{1, 2, ..., M_1\} \\
p(n > n_1)S_i^{-1}(p(n > n_1)) & ; \text{i} \in \{M_1+1, M_1+2, ..., M_2\} \end{cases}$$

Assuming a convergence for the $M_2$ UEs according to equation (7) before the time slot $n_2$, the optimal bids and rates can be written as equation (13):

$$w_i(n_2) = \begin{cases} p(n_1)S_i^{-1}(p(n_1)) & ; \text{i} \in \{1, 2, ..., M_1\} \\
p(n_2)S_i^{-1}(p(n_2)) & ; \text{i} \in \{M_1+1, M_1+2, ..., M_2\} \end{cases}$$

where $p(n_1) = \sum_{i=1}^{M_1} w_i(n_1)$ and $p(n_2) = p(n_1) + \sum_{i=M_1+1}^{M_2} w_i(n_2)$ and the reallocation is done in accordance with UE EURA, eNB EURA, and IURA algorithms with the initial bids $w_i(n_1)$. Contrasting equations (13) and (9), we observe the rates and bids are non-optimal.

Section III-3 discusses the sensitivity of the centralized resource allocation to UE quantity alterations.

3) Sensitivity of the Centralized Algorithm: A transmission overhead analysis of the centralized resource allocation, assigning optimal rates after one algorithm iteration, is explained in proposition III.6.

Proposition III.6. The centralized resource allocation scheme assigns optimal rates with a minimum transmission overhead $2M_2 - M_1$ and $M_1$ under increasing and decreasing UE quantities, respectively.

Proof: When $M_2 > M_1$, i.e. the UE quantity increases, the new UEs send their utility function parameters in $M_2 - M_1$ transmissions to the eNB which returns application rates for the new $M_2$ UEs. Thus, the transmission overhead becomes $2M_2 - M_1$. On the other hand, for $M_1 > M_2$, i.e. the UE quantity decreases, the exiting UEs transmit service termination signals in $M_1 - M_2$ transmissions to the eNB which sends the rates for the new $M_2$ UEs. There are no further transmissions and the transmission overhead becomes $M_1 - M_2 + M_2 = M_1$.

Next, section IV presents a sensitivity analysis of the distributed and centralized resource allocations to temporal application usage changes.

IV. TRAFFIC ANALYSIS UNDER APPLICATION USAGE DYNAMICS

We consider a cell of $M$ UEs and an eNB as in part I. At time slot $n_1$, the $i^{th}$ user has the aggregated utility function $V_i(r_i)$. At time instant $n_1 + 1$, $M'$ of the $M$ UEs ($M' < M$) change their aggregated utility, e.g. the $i^{th}$ UE of the $M'$ users gets the aggregated utility function $V_i'(r_i)$. Thus, the time-dependent aggregated utility function for the $i^{th}$ UE of the $M'$ users, denoted as $V_i'(r_i, n)$, can be written as equation (14).

$$V_i(r_i, n) = \begin{cases} V_i(r_i) & ; n \leq n_1 \\
V_i'(r_i) & ; n > n_1 \end{cases}$$

Furthermore, we assume that a steady state rate allocation is achieved for the aggregated utility functions $V_i(r_i)$ and $V_i'(r_i)$ at time slots $n_1$ and $n_2$ respectively so that the steady state rate vector can be expressed as equation (15).

$$r^s = \begin{cases} r(n_1) & ; n \leq n_1 \\
r(n_2) & ; n > n_1 \end{cases}$$

Like section III in addition to the centralized scheme, we consider the distributed algorithms with and without rebidding, of which the former is discussed in section IV.4.

4) Distributed Algorithm with Rebidding: For every UEs altering their application usage percentages, which in turn change the corresponding aggregated utility functions, fixed-utility users (UEs with unvaried aggregated utility and optimal rates) rebid for resources. The eNB initiates the allocation by either sending (in case $\beta_i$ is only available at the eNB) or broadcasting (in case $\beta_i$ is available at the UEs) the shadow price. Proposition IV.1 proves the transmission overhead maximum. Moreover, the rates and bids remain optimal as explained in the proposition IV.2.

Proposition IV.1. The transmission overhead of the distributed resource allocation with rebidding is maximum and greater than $M + M' + 2$ under application usage changes.

Proof: The $M'$ UEs changing their aggregated utility send their initial bids in $M'$ transmissions to the eNB. If $\beta_i$ is available at the UEs, the eNB broadcasts a new shadow price to the devices followed by $M$ UEs transmit new bids to the eNB. This process is repeated $k$ times until optimal rates are achieved and the transmission overhead becomes $M' + 1 +$.
\[ kM + k, \text{ corresponding minimally } M + M' + 2 \text{ for one iteration } \]
\[(k = 1). \text{ On the other hand, if } \beta_i \text{ is only available at the eNB, it transmits a modified shadow price } \frac{\beta_i}{\beta_n} \text{ in } M \text{ transmissions to the UEs under its coverage. The } M \text{ UEs send new bids to which eNB and this process is iterated } k \text{ times until optimal rates are achieved. The transmission overhead becomes } M' + M + 2kM \text{ minimally corresponding to } 3M + M' \text{ for one iteration } \]
\[(k = 1). \text{ Contrasting minimal overheads in both cases, the minimum transmission overhead is } M + M' + 2. \]

**Proposition IV.2.** The allocated rates and bids of the distributed resource allocation with rebidding remain optimal under application usage changes.

**Proof:** Theorem III.2 and corollary III.3 concludes that the new EURA and IURA optimizations are optimal for \( M \) users. Assuming the aggregated UE utility \( V_i(r_i) \) and slope curvature function \( S_i(r_i) \), the optimal rates and bids at time slot \( n_1 \) can be expressed as equation set (18).

\[
\begin{align*}
  r_i(n_1) & = S_i^{-1}(p(n_1)) \\
  w_i(n_1) & = p(n_1)S_i^{-1}(p(n_1))
\end{align*}
\]

where \( p(n_1) = \sum_{i=1}^{M} w_i(n_1) \) and allocations are done in accordance with UE EURA, eNB EURA, and IURA algorithms with initial bids \( w_i(1) = 1 \). Once \( M' \) UEs alter the usage percentage of at least one of their applications leading to their aggregated utility functions \( V_i'(r_i) \) and slope curvature functions \( S_i'(r_i) \), the \( M - M' \) UEs keep their aggregated utilities \( V_i(r_i) \) and slope curvature functions \( S_i(r_i) \). Thus, optimal rates and bids at time slot \( n_2 \) can be expressed as:

\[
\begin{align*}
  w_i(n_2) & = \begin{cases} \\
  p(n_1)S_i^{-1}(p(n_2)) ; & i = \{1, 2, ..., M'\} \\
  p(n_2)S_i^{-1}(p(n_2)) ; & i = \{M' + 1, M' + 2, ..., M\}
\end{cases}
\end{align*}
\]

\[
\begin{align*}
  r_i(n_2) & = \begin{cases} \\
  \frac{p(n_1)}{p(n_2)}S_i^{-1}(p(n_1)) ; & i = \{1, 2, ..., M_1\}, \\
  S_i^{-1}(p(n_2)) ; & i = \{1, 2, ..., M'\}, \\
  S_i^{-1}(p(n_2)) ; & i = \{M' + 1, M' + 2, ..., M\}
\end{cases}
\end{align*}
\]

where \( p(n_2) = \sum_{i=1}^{M'} w_i(n_2) \) and the reallocation procedure is done in accordance with UE EURA, eNB EURA, and IURA algorithms with initial bids \( w_i(n_2) \). Contrasting equations (17) and (4), we observe that the rates and bids are optimal under application usage changes.

Next, section IV.5 investigates the sensitivity of the distributed algorithm with no rebidding to application usage alterations.

5) Distributed Algorithm without Rebidding: For every UEs altering their application usage percentages, which in turn change the corresponding aggregated utility functions, fixed-utility users (UEs with unvaried aggregated utility and optimal rates) will not rebid for resources so that their rates and bids at time \( n_2 \) are identical to those of time slot \( n = n_1 \). In this case, the transmission overhead is less than the one with rebidding as shown in proposition IV.3. Besides, the rates and bids are not optimal as illustrated in lemma IV.4.

**Proposition IV.3.** The transmission overhead of the distributed resource allocation with no rebidding is not larger than the one with rebidding and equals \( M' + 1 \) at minimum.

**Proof:** \( M' \) UEs changing their aggregated utility functions send the initial bids in \( M' \) transmissions to the eNB. If \( \beta_i \) is available at the UEs, the eNB broadcasts a new shadow price to the UEs, the routine is repeated \( k \) times until optimal rates are achieved, and the transmission overhead becomes \( kM' + k \), corresponding minimally to \( M' + 1 \) for one iteration \( (k = 1) \). On the other hand, if \( \beta_i \) is only available at the eNB, it transmits a modified shadow price \( \frac{\beta_i}{\beta_n} \) to the \( M' \) UEs, the algorithm is iterated \( k \) times until optimal rates are achieved, and the transmission overhead becomes \( 2M' + 2kM' = (2k + 2)M' \), corresponding minimally to \( 4M' \) for one iteration \( (k = 1) \). So the minimum transmission overhead for the distributed algorithm with no-rebidding becomes \( M' + 1 \), which is less than overhead for the distributed scheme with rebidding (proposition IV.1).

**Lemma IV.4.** The allocated rates and bids of the distributed resource allocation without rebidding are not optimal under application usage changes.

**Proof:** Assuming convergence before the time slot \( n_1 \), the optimal rates and bids at time slot \( n_1 \) can be expressed as equation set (18) where \( p(n_1) = \sum_{i=1}^{M} w_i(n_1) \) and the allocation is done in accordance with UE EURA, eNB EURA, and IURA algorithms with initial bids \( w_i(1) = 1 \).

\[
\begin{align*}
  r_i(n_1) & = S_i^{-1}(p(n_1)) \\
  w_i(n_1) & = p(n_1)S_i^{-1}(p(n_1)).
\end{align*}
\]

The \( i^{th} \) UE bids (\( \{i|i = \{M' + 1, M' + 2, ..., M\}\} \)) is fixed for time slots \( n > n_1 \). After applications usage changes, we can write the shadow price as equation set (19).

\[
\begin{align*}
  p(n > n_1) & = \sum_{i=1}^{M'} w_i(n > n_1) + \sum_{i=1+M'}^{M} w_i(n_1) \\
  R & = p(n > n_1) + p(n_1).
\end{align*}
\]

And user bids can be expressed as equation (20).

\[
\begin{align*}
  w_i(n > n_1) & = \begin{cases} \\
  p(n > n_1)S_i^{-1}(p(n > n_1)) ; & i = \{1, 2, ..., M'\}, \\
  p(n_1)S_i^{-1}(p(n_1)) ; & i = \{M' + 1, M' + 2, ..., M\}.
\end{cases}
\end{align*}
\]

Assuming convergence to optimal values before the time slot \( n_2 \), final rates and bids can be expressed as:
where $p_{\text{transmission overhead}}$ is distributed approach in the Algorithms IV.3, IV.4, and IV.5.

Proposition IV.5. The centralized resource allocation algorithm assigns optimal rates with a minimum transmission overhead $M' + M$ when $M'$ UEs change their applications usage percentages.

Proof: The $M'$ UEs changing their aggregated utilities send their utility parameters in $M'$ transmissions to the eNB, which transmits new rates to the $M$ UEs. Therefore, the transmission overhead is $M' + M$. The optimality of the centralized algorithm has been proved by the corollary III.4 in part I; and, the proof is complete.

Next, we show the sensitivity of the algorithms to temporal alterations in application usage percentages.

V. COMPUTATIONAL COMPLEXITY AND POWER CONSUMPTION INSIGHTS

The centralized resource allocation undergoes less computational complexity on the UE side compared to the distributed one as it does not include the computations of UE EURA algorithm of the two-stage distributed scheme. As a result, the power consumption required for implementing the centralized scheme in the UE is less. Next, in the section VI we present simulation results for the resource allocation methods developed in Part I.

VI. SIMULATION RESULTS

The communication cell, in part I, with $M = 6$ UEs and an eNB is considered such that each UE concurrently runs a delay-tolerant and a real-time applications with utility parameters in Table I of part I, where we have mentioned that the real-time are voice-over-IP, standard video, and high definition video and the delay tolerant ones are FTPs. Next, section VI-A presents simulations germane to the sensitivity of the algorithms to the changes in the application usage percentage.

Figure 2 depicts a comparison between the transmission overhead incurred employing the centralized and the distributed resource allocation approaches, between which the former is concomitant with so significantly less overhead that the germane plot is much closer to the abscissa as opposed to the one for the latter case. This is in light of the fact that the centralized scheme subsumes less message exchanges between the UEs and eNB (one stage only) to assign optimal rates to the applications.

Next, we show the sensitivity of the algorithms to temporal alterations in application usage percentages.

A. Sensitivity to Application Usage Changes

In the following simulations, we set the termination threshold as $\delta = 10^{-3}$ and the total achievable rate at the eNB as $R = 180$. The, we measure the sensitivity of the algorithm to changes in the usage percentages of the applications running on the UEs. Based on the formulation in part I, the usage percentage is represented as an application-status differentiation weight, and the $M$ UEs switch between their applications with the usage percentages as equation 22 whose values are taken from the first five rows in Table 1. As an illustration, during the simulation time $0 < t < 100$, $\alpha = \alpha_t = \{0.1, 0.5, 0.9, 0.1, 0.5, 0.9, 0.9, 0.5, 0.1, 0.9, 0.5, 0.1\}$, and so forth. It is noteworthy that the temporal application usage percentage $\alpha_t = \{0.1, 0.5, 0.9, 0.1, 0.5, 0.9, 0.9, 0.5, 0.1, 0.9, 0.5, 0.1\}$ indicates that the $1^{st}$ utilizes its real-time and delay-tolerant applications 90% and 10% of the $0 < t < 100$ simulation time respectively, and likewise for other UEs and applications.
TABLE I
APPLICATIONS STATUS DIFFERENTIATION

| Applications Usage-Percentage |
|-----------------------------|
| \( \alpha_1 \)          | \{0.1, 0.5, 0.9, 0.1, 0.5, 0.9, 0.9, 0.5, 0.1, 0.9, 0.5, 0.1\} |
| \( \alpha_2 \)          | \{0.5, 0.3, 0.2, 0.5, 0.3, 0.2, 0.5, 0.7, 0.8, 0.5, 0.7, 0.8\} |
| \( \alpha_3 \)          | \{0.5, 0.9, 0.8, 0.5, 0.9, 0.8, 0.5, 0.1, 0.2, 0.5, 0.1, 0.2\} |
| \( \alpha_4 \)          | \{0.5, 0.3, 0.2, 0.5, 0.3, 0.2, 0.5, 0.7, 0.8, 0.5, 0.7, 0.8\} |
| \( \alpha_5 \)          | \{0.5, 0.9, 0.8, 0.5, 0.9, 0.8, 0.5, 0.1, 0.2, 0.5, 0.1, 0.2\} |
| \( \alpha_a \)         | \{0.1, 0.5, 0.9, 0.1, 0.5, 0.9, 0.9, 0.5, 0.1, 0.9, 0.5, 0.0\} |
| \( \alpha_b \)         | \{0.1, 0.5, 0.9, 0.1, 0.5, 0.9, 0.9, 0.5, 0.1, 0.9, 0.5, 0.1\} |

The sensitivity of the algorithms as for UE rate allocations and bids is depicted in Figure 6(a), where the users rates, e.g. \( r_i \) for the \( i^{th} \) UE, changes with time for the changing usage percentages in accordance with \( \alpha(t) \); they also converge to optimal rates during each time interval as the resource allocation algorithms execute. Furthermore, Figure 6(b) portrays the UE bids, e.g. \( w_i \) for the \( i^{th} \) UE, as they change during the different time intervals during which application usage percentages alter; however, the bids also converge with time for the changing usage percentages. In particular, we observe that at the commencement of the algorithm the jumps between the bids are larger, whereas these jumps significantly shrink as time proceeds and the rates/bids converge to their optimal values.

In Figure 4, we show the changes in the shadow price \( p \) vs. time when the application usage percentages change. Here, we can observe a shrinkage of the shadow price variations as the algorithms execute in time. Furthermore, higher usage corresponds to a higher shadow price. Moreover, a comparison between the centralized and distributed algorithms transmission overhead whilst switching the application-usage percentages is illustrated in Figure 7 which shows that the centralized method incurs a significantly lower transmission overhead due to the lesser number of transmissions it needs as opposed to those of the distribution approach. In fact, the centralized scheme transmission overhead plot is very close to the abscissa in view of the drastic less message exchanges it incurs, and the transmission overhead is independent of the termination threshold \( \delta \). On the flip side, the lower termination thresholds augments the transmission overhead for the distributed resource allocation in that the smaller thresholds make the algorithm execute more iterations before it converges to optimal rates and the more iterations are tantamount to a larger number of messages exchanged between the eNB and UEs, which in turn escalates the transmission overhead dramatically.

\[
\alpha(t) = \begin{cases} 
\alpha_1 & ; 0 < t \leq 100 \\
\alpha_2 & ; 100 < t \leq 200 \\
\alpha_3 & ; 200 < t \leq 300 \\
\alpha_4 & ; 300 < t \leq 400 \\
\alpha_5 & ; 400 < t \leq 500 
\end{cases}
\]  

Next, section VI-B discusses the sensitivity of the resource allocation schemes to changes in the number of UEs in the system.

B. Sensitivity to Changes in the UE Quantity

Similarly to the simulations in section VI-A we set \( \delta = 10^{-3} \) and \( R = 180 \) and measure the sensitivity of the proposed system.
algorithms to changes in the number of the UEs in the system, where new users enter the cell so that the UE quantity at time slot $n_1 = 100$ is $M_1 = 5$ and at time instant $n_1 + 1$ the is $M_2 = 6$. Then, the instantaneous number of users, denoted as $M(n)$, is given by the equation (23).

$$M(n) = \begin{cases} M_1 ; & n \leq n_1 \\ M_2 ; & n > n_1 \end{cases}$$ (23)

Furthermore, we set the temporal application status differentiation $\alpha$ as equation (24).

$$\alpha(t) = \begin{cases} \alpha_a ; & 0 < t \leq 100 \\ \alpha_b ; & 100 < t \leq 200 \end{cases}$$ (24)

Our experiments show that the distributed algorithm without rebidding as well as the centralized approach deviate from optimal rates, bids, and shadow price under UE quantity changes. Figures 6(a) and 6(b) respectively show the errors in UE rates and bids, i.e. $|r_i - r_{i, opt}|$ and $|w_i - w_{i, opt}|$ for the $i^{th}$ UE, when a new user (here UE6) enters the cell. As we can observe, the centralized algorithm retains its optimal UE rate assignment and bid pledging so that the errors become 0, whereas the distributed method is concomitant with peak errors before converging to the steady state values. Besides, looking at the different UE rate and bid errors, we observe that the UEs whose real-time applications have higher QoS requirements, tantamount to larger inflection points in their respective sigmoidal utility functions, undergo larger errors vis-a-vis the other UEs.

Last, Figure 7 depicts the error in the eNB announced shadow price, i.e. $|p - p_{opt}|$, when UE6 enters the cell. As we can see, the centralized algorithm boasts a zero pricing error, whereas the distributed one is susceptible to about 30% shadow price error.

VII. IMPLEMENTATION RESULTS

In this section, we implement the resource allocation developed in Part I on a real-world network. The implementation system model is depicted in Figure 8 and subsumes UEs WiFi-connected to the Internet through a network outfitted with a resource broker (RB) unit, containing the resource allocation code introduced in Part I, installed on a router which shapes the traffic based on application rates assigned by the RB entity in accordance with the resource allocation method in Part I.

In order to implement the scenario in Figure 8 on a real-world network, we leverage a personal computer (PC) to configure the network in a distributed manner to decrease the processing load through the virtual machine (VM) architecture in Figure 9. Here, we have used a single-socket IBM
x3250 M4 server \[16\] with two physical and two Peripheral Component Interconnect (PCI)-enabled ports to create two three-interfaced VMs, between which one hosted the RB entity and the other formed a router including an enforcement engine to manage rate assignments via an onboard Linux router traffic control, and another dedicated file server VM. The two VMs (not the file server one) are annotated as "Guest 1" and "Guest 2" in Figure 9 where the router and RB sit on Guest 1 and Guest 2, respectively. Besides, we create three virtual switches intended for the phone network, another for office-Internet-connected external devices, and one for network maintenance/ operation issues. The testbed phones and their wireless access point (WAP) are on their own private network (i.e. the Ethernet network with internet protocol (IP) address 192.168.2.1), router gateway settings enable connections to the office network (intranet) and Internet, and finally smartphones running Youtube and Hyper Text Transfer Protocol (HTTP) download applications are deployed. The traffic generated by the real-time/delay-tolerant Youtube/HTTP applications is inelastic/elastic, and we apply the resource allocation procedure in Part I to obtain application rates (throughput) and germane subjective QoE, reflected by buffering occurrences for the inelastic Youtube traffic and incomplete downloads for the elastic HTTP traffic. The dearth of the aforesaid events indicates an acceptable QoE for the application users and effectiveness of the resource allocation method in Part I.

For the test platform in Figure 9 the small number of phones working in a high throughput WiFi network is an object of concern as it provides too ideal of a data transfer environment to be able to appropriately illustrate the benefits that may emerge from an RB-induced traffic shaping, i.e. from the resource allocation method in Part I. To observe the traffic shaping effects, we should impose a higher load on the network. We do this simply by restricting the overall network bandwidth to 1 Mega bits per second (Mbps). In order to make a comparison between the scenarios with and without the resource allocation algorithm of Part I, we first apply the rate assignment scheme to the network in Figure 9 under no bandwidth constraints; then, we introduce an overall network constraint with no traffic shaping, and ultimately contrast non-shaped throughput and QoE observation to a situation where the RB operates under a 1 Mbps network constraint. The Youtube and HTTP applications throughput under neither network constraints nor the resource allocation application is shown in Table II where a low/high rate Youtube application "YouTube 1"/"YouTube 2" and a small/large file download application "HTTP 1"/"HTTP 2" (obtained from content providers of the VM-created file server) run on three UEs in the network whose speed under the absence of applications is measured at 32 Mbps. For instance, the 1st scenario in Table III indicates that two phones run Youtube 1 and the average bandwidth consumption is $R_{avg} = 3.492$ Mbps, while the single-UUE 3rd scenario incurs a much lower bandwidth consumption $R_{avg} = 0.951$ Mbps due to the low rate Youtube 1 application. On the other hand, the bandwidth consumption significantly increased to $R_{avg} = 21.11$ Mbps due to the single-UUE high rate Youtube 2 application in the 4th scenario, and a similar rate rise is apparent in the transition from the 5th scenario to the 6th one where the high rate HTTP 2 replaces the low rate HTTP 1 at the 2nd phone amounting to 24.866 Mbps bandwidth consumption. The last scenario is concomitant with a lower rate $R_{avg} = 13.747$ Mbps as opposed to the 6th scenario ($R_{avg} = 24.866$ Mbps) in spite of augmenting a Youtube 1 application to phone 3 in the latter case. This can be explained by the need to transfer more bits which happen over a longer time interval vis-a-vis that of the 6th case, thereby the throughput slashes down relative to the 6th configuration.

To see the effect of the resource allocation developed in Part I and implemented in the RB entity, we throttle the overall network bandwidth to $R = 1$ Mbps. Without loss of generality, we focus on deploying only two phones in the network in Figure 2 with IP addresses and a Youtube 1 and HTTP 1 application as illustrated in Table III. Running the experiment with no resource allocation applied, we observe...
that YouTube 1 incurred multiple buffering periods indicating a poor QoE from its user’s perspective, the overall average bandwidth usage was 0.963 Mbps, and HTTP 1 download completed in 1200 seconds (s) (IV). Using WireShark [17], we obtain the traces in Figure 10, where both applications sharing the total 1 Mbps bandwidth annotated on the black curve alternate in bursty transmission intervals. In particular, at certain times, the HTTP 1 application (the red curve) utilizes the entire available bandwidth, shown by the red curve reaching the black curve, which simultaneously zeros the YouTube 1 throughput illustrated by the green curve lowering to the abscissa (time axis). This behavior adversely affects the QoE for the UE 1.

The same situation of the network in Figure 9 with the parameters in Table III is repeated when the resource allocation method in Part I is applied to have the router shape the applications traffic by optimally assigning them rates based on their requirements. To apply the resource allocation method developed in Part I, the phones UE 1 and UE 2 register with the RB entity, where the rate allocation code runs and calculates rates to be enforced at the router. The average bit rates for the YouTube 1 and HTTP 1 are respectively 731 and 267 kbps, the convergence time for the algorithm is measured at 528 ms, and the overall throughput becomes 0.758 Mbps which is less than the maximum 1 Mbps available capacity due to periods over which no YouTube traffic load is present on the network. Using Wireshark, the rates of the YouTube 1 and HTTP 1 applications when the resource allocation is leveraged in the network under overall \( R = 1 \) Mbps constraint are depicted in Figure 11.

Here, the black curve indicates the overall available bandwidth and the ordinate (throughput axis) shows that the network bandwidth is confined to 1 Mbps. As we can see, YouTube 1 (green curve) consumes more resources per the rate allocation interval than does the HTTP 1 application, whose download time expectedly takes longer to be completed at 2650 s (IV). Interestingly, there are intervals where YouTube 1 rate becomes 0, over which HTTP 1 obtains more bandwidth; this zero-grounding behavior is analogously observed for the HTTP 1 download. In this experiment, we observe no YouTube buffering occurrences, thereby it provisions a better video watching experience for the user as opposed to the unshaped traffic scenario depicted in Figure 10. Such a dearth of buffering speaks directly to the speculated QoE in that the real-time YouTube 1 application is provided with a consistent rate assignment such that it is able to fill the buffer and does not require any more bandwidth usage.

As we observed in this real-world implementation of the resource allocation algorithm presented in Part I of this paper, the resource allocation elevates the QoE despite the fact that less resources are consumed. This directly helps communications carriers to reduce their operation expenditure (OPEX) and customer churn. As a case in point, the algorithm-induced traffic shaping decreased the bandwidth consumption from 0.963 Mbps to 0.758 Mbps, thereby a 0.205 Mbps reduction of resource consumption is stemmed which has desirable monetary indications by using up less resources without degrading users’ QoE. As such, an QoE remedial measurement is implied in light of employing the developed resource allocation scheme.

---

**TABLE II**

| Scenario | Phone 1        | Phone 2        | Phone 3 | \( R_{\text{avg}} \) Mbps |
|----------|----------------|----------------|---------|--------------------------|
| 1        | Youtube 1      | Youtube 2      | -       | 3.492                    |
| 2        | Youtube 1      | HTTP 1         | -       | 4.050                    |
| 3        | Youtube 1      | -              | -       | 0.951                    |
| 4        | Youtube 2      | -              | -       | 2.11                     |
| 5        | HTTP 1         | HTTP 1         | -       | 4.262                    |
| 6        | HTTP 1         | HTTP 2         | -       | 24.866                   |
| 7        | HTTP 1         | HTTP 1         | Youtube 1 | 13.747                  |

Network Speed with neither Application nor Rate Constraints: 32 Mbps

**TABLE III**

| Phone | IP           | Traffic Type | Application |
|-------|--------------|--------------|-------------|
| UE 1  | 192.168.2.57 | Inelastic    | YouTube 1   |
| UE 2  | 192.168.2.98 | Elastic      | HTTP 1      |

---

![Fig. 10. Wireshark Throughput Analysis without Applying the Resource Allocation in Part I](image-url)

![Fig. 11. Wireshark Throughput Analysis with Applying the Resource Allocation in Part I](image-url)
TABLE IV
BANDWIDTH CONSUMPTION AND DOWNLOAD TIME.

| Performance              | Shaped Traffic | Unshaped Traffic |
|--------------------------|----------------|------------------|
| HTTP 1 Download Time (s) | 1200           | 2650             |
| Youtube 1 Buffering      | No             | Yes              |
| Total Bandwidth (kbps)   | 758            | 951              |

VIII. Conclusion

In part I of this paper, we introduced a novel QoS-minded centralized and a distributed algorithm for the resource allocation within the cells of cellular communications systems. We formulated the centralized and distributed approaches as respectively a singular and double utility proportional fairness optimization problems, where the former allocated running applications rates directly by the allocation entity such as an eNB in response to the UE utility parameters sent, whereas the latter assigned the UE rates by the eNB in its first stage followed by the application rate allocation by the UEs in its second stage. Users ran both delay-tolerant and real-time applications mathematically modelled correspondingly as logarithmic and sigmoidal utility functions, where the function values represented the applications QoS percentage. Both of the proposed resource allocation formulations incorporated the service differentiation, application status differentiation, and subscriber differentiations amongst subscribers priority within networks into their formulation. Not only did we prove that the proposed resource allocation problems were convex and solved them through Lagrangian of their dual problems, but also we proved the optimality of the rate assignments and the mathematical equivalence of the proposed distributed and centralized resource allocation schemes.

Furthermore, we analyzed the algorithm convergence under varying sums of resources available to the eNB and introduced robustness into the distributed algorithm by incorporating decay functions into the aforementioned algorithm so that it converged to optimal rates for both high and low traffic loads occurring during the day by damping the rate assignment fluctuations resulting from scarcity of resources that appeared particularly in peak-traffic circumstances.

In part II, we proved that the centralized resource allocation algorithm incurs the minimum possible transmission overhead during the rate assignment procedure. Moreover, we derived the transmission overhead based on changes in the number of UEs in the cell under the existence of the dearth a rebidding policy. We proved that the absence of a rebidding policy entails a transmission overhead less than or equal to that of the one with presence of a rebidding policy. In addition, we proved that the rate assignments remain optimal for variations in the network’s UE quantity for the distributed resource allocation approach under a rebidding policy, whilst the rates are not optimal when the number of UEs change for the distributed approach with a no-rebidding policy and also for the centralized resource allocation scheme. Besides, we proved the rate assignments remain optimal when the application usage percentages changes in case of the centralized resource allocation scheme, but it is not optimal when the aforesaid parameter varies in case of the distributed resource allocation method. We also analyzed the transmission overhead and its changes for both the centralized and distributed method when the application usage percentages vary in the network.

Furthermore, we provisioned a traffic-dependent pricing technique for network providers using which they could flatten the traffic loads during peak traffic hours. The devised pricing could be as well employed by network subscribers to potentially decrease the cost of utilizing the network by using services at low-cost (low-traffic) time periods. Ultimately, through simulations, we showed that the rate assignment, bid pledging, and pricing errors for the centralized approach is 0 while those of the distributed approach converge to their steady state values different from the optimal ones.

Ultimately, we set up a real-world network to evaluate the QoE users perceive when the procedure is implemented in practice. The large-scale network configuration included Youtube and HTTP applications on UEs connected to the Internet through their WAP and a router which ran the resource allocation routine and enforced the rates. We realized that the absence of the resource allocation algorithm in the network caused multiple buffering instances of the real-time Youtube application, thereby the users QoE was adversely undermined. On the other hand, the presence of the algorithm, through which the rates were assigned to the applications and enforced at the gateway router, eliminated any Youtube buffering at the expense of lengthening the duration of delay-tolerant downloads. Therefore, applying the resource allocation method significantly escalated the users QoE without hurting QoS requirements of applications. Finally, we realized that despite the dearth of Youtube buffering under the presence of the resource allocation algorithm, the applications consumed less resources as opposed to the QoE-hurting algorithm-absent scenario. Consequently, the bandwidth conservation yields in a lower OPEX for the network as less to-be-paid resources are consumed.

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