Reionization Bias in High Redshift Quasar Near-Zones

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\textbf{ABSTRACT}

Absorption spectra of high redshift quasars exhibit an increasingly thick Ly\textalpha forest, suggesting the fraction of neutral hydrogen in the intergalactic medium (IGM) is increasing towards \( z \sim 6 \). However, the interpretation of these spectra is complicated by the fact that the Ly\textalpha optical depth is already large for neutral hydrogen fractions in excess of \( 10^{-4} \), and also because quasars are expected to reside in dense regions of the IGM. We present a model for the evolution of the ionization state of the IGM which is applicable to the dense, biased regions around high-redshift quasars as well as more typical regions in the IGM. We employ a cold dark matter (CDM) based model in which the ionizing photons for reionization are produced by star formation in dark matter halos spanning a wide range of masses, combined with numerical radiative transfer simulations which model the resulting opacity distribution in quasar absorption spectra. With an appropriate choice for the parameter which controls the star formation efficiency, our model is able to simultaneously reproduce the observed Ly\textalpha forest opacity at \( 4 < z < 6 \), the ionizing photon mean-free-path at \( z \sim 4 \) and the rapid evolution of highly ionized near-zone sizes around high-redshift quasars at \( 5.8 < z < 6.4 \). In our model, reionization extends over a wide redshift range, starting at \( z \gtrsim 10 \) and completing as HII regions overlap at \( z \sim 6 - 7 \). We find that within 5 physical Mpc of a high redshift quasar, the evolution of the ionization state of the IGM precedes that in more typical regions by around 0.3 redshift units. More importantly, when combined with the rapid increase in the ionizing photon mean-free-path expected shortly after overlap, this offset results in an ionizing background near the quasar which exceeds the value in the rest of the IGM by a factor of \( \sim 2 - 3 \). We further find that in the post-overlap phase of reionization the size of the observed quasar near-zones is not directly sensitive to the neutral hydrogen fraction of the IGM. Instead, these sizes probe the level of the background ionization rate and the temperature of the surrounding IGM. The observed rapid evolution of the quasar near-zone sizes at \( 5.8 < z < 6.4 \) can thus be explained by the rapid evolution of the ionizing background, which in our model is caused by the completion of overlap at the end of reionization by \( 6 \lesssim z \lesssim 7 \).

\textbf{Key words:} cosmology: theory - galaxies: formation - intergalactic medium

\section{1 INTRODUCTION}

The reionization of cosmic hydrogen, which is commonly believed to have been due to ultraviolet photons produced by the first stars and quasars (Barkana & Loeb 2001), was an important milestone in the history of the Universe. The recent discovery of very distant quasars has enabled detailed studies of the ionization state of the high redshift IGM to be made at a time when the universe was less than a billion years old (Fan et al. 2006; White et al. 2003). Several studies have used the evolution of the ionizing background inferred from these spectra to argue that the reionization of cosmic hydrogen was completed just beyond \( z \sim 6 \) (White et al. 2003; Fan et al. 2006; Gnedin & Fan 2006). However other authors have claimed that the evidence for this rapid change becomes significantly weaker for a different choice density distribution in the IGM (Becker et al. 2007). One reason for the ambiguity in interpreting these absorption spectra is that Ly\textalpha absorption can only be used to probe
neutral fractions that are larger than $10^{-4}$, owing to the large cross-section of the Ly$\alpha$ resonance.

Several of the most distant quasars exhibit complete Gunn-Peterson troughs, the red edges of which do not extend as far as the redshifted Ly$\alpha$ wavelength. A possible interpretation of the observation of a Gunn-Peterson trough is the presence of an HII region in a partially neutral IGM (e.g. Cen & Haiman 2000, Madau & Rees 2000). This interpretation has been used in different ways to argue that there is a significant fraction of neutral hydrogen in the IGM beyond $z \approx 6$ (Wyithe & Loeb 2004; Mesinger & Haiman 2004; Wyithe, Loeb & Carilli 2005), with a lower limit on the hydrogen neutral fraction that is as much as two orders of magnitude larger than constraints available from direct absorption studies. Bolton & Haehnelt (2007a), Maselli et al. (2007) and Lidz et al. (2007), however, have argued that this interpretation is uncertain and that these observed regions of transmission could either be due to an HII region or due to a classical proximity zone.

Given this uncertainty, and in order to connect the evolution of the IGM ionization state at redshifts both above and below those where the Gunn-Peterson troughs have been observed, Fan et al. (2006) proposed defining the sizes of these regions of transmission as the distance where the smoothed spectrum first drops below the level where 10 per cent of the flux is transmitted. Bolton & Haehnelt (2007a) refer to this region as the highly ionized near-zone, and Fan et al. (2006) argued that its size should reflect the ionization state of the IGM in a simple way. Using this definition, Fan et al. (2006) found a rapid trend of near-zone size with redshift, from which they inferred an order of magnitude increase in the neutral fraction of the IGM over the observed range of $5.8 \leq z \leq 6.4$. However Bolton & Haehnelt (2007a) demonstrated that the near-zone size becomes independent of the neutral hydrogen fraction when the IGM is highly ionized.

In this paper we shall address two points regarding the interpretation of the near-zones observed in high redshift quasar absorption spectra. As pointed out recently by Lidz et al. (2007) and Alvarez & Abel (2007), the quasars observed prior to $z \sim 6$ are likely to have formed in regions substantially preionized by nearby clustered galaxies. We therefore first discuss the extent to which regions of the IGM observed close to high redshift quasars in absorption spectra are representative of the Universe on average, using a density dependent semi-analytic model for the reionization history. This model is carefully calibrated to be consistent with the current observational data, and as such provides a consistent theoretical framework for modeling quasar near-zones in highly biased regions of the IGM. We then use this model to predict the sizes of quasar near-zones and their evolution with redshift, both semi-analytically and using detailed simulations of radiative transfer.

We begin by describing our density dependent semi-analytic model for the reionization history (§2), including our procedures for the calculation of the ionizing background. We then present results for the bias of reionization near quasars in §3 before discussing the evolution of the observed high redshift quasar near-zones using the semi-analytic model in §4. We then explore this model in more detail with full radiative transfer simulations in §5.

We present our conclusions in §6. Throughout the paper we adopt the set of cosmological parameters determined by WMAP (Spergel et al. 2007) for a flat ΛCDM universe.

2 SEMI-ANALYTIC MODEL FOR REIONIZATION

Miralda-Escude et al. (2000) presented a model which allows the calculation of an effective recombination rate in an inhomogeneous universe by assuming a maximum overdensity ($\Delta_\text{c}$) which may be penetrated by ionizing photons within HII regions. The model assumes that reionization progresses rapidly through islands of lower density prior to the overlap of individual ionized regions. Following overlap, the remaining regions of high density are gradually ionized. It is therefore hypothesised that at any time, regions with gas below some critical normalised density threshold $\Delta_\text{i} \equiv \rho_\text{i}/\langle \rho \rangle$ are ionized while regions of higher density are not. In what follows, we draw primarily from their prescription and refer the reader to the original paper for a detailed discussion of its motivations and assumptions. Wyithe & Loeb (2003) employed this prescription within a semi-analytic model of reionization. In the current work we limit our attention to reionization due to population-II stars (Gnedin & Fan 2006; Srbinovsky & Wyithe 2006), which govern the final stages of reionization even in the presence of an earlier partial or full reionization by population III stars (e.g. Wyithe & Loeb 2003).

Within the model of Miralda-Escude et al. (2000) we describe the post-overlap evolution of the IGM by computing the evolution of $dF_\text{M}\left(\Delta_i\right)/dz$, where

$$F_\text{M}\left(\Delta_i\right) = \int_{0}^{\Delta_i} d\Delta P_\text{V}(\Delta)\Delta$$

is the fraction of mass in regions with overdensity below $\Delta_i$, and $P_\text{V}(\Delta)$ is the volume weighted probability distribution for $\Delta$. In a region of large scale over density $\delta$ at $z_\text{obs}$, the mass fraction $F_\text{M}\left(\Delta_i\right)$ (or equivalently $\Delta_i$) therefore evolves according to the equation

$$\frac{dF_\text{M}\left(\Delta_i\right)}{dz} = \frac{1}{n_0} \frac{dn_\text{lo}(\delta)}{dz} - \alpha_B \frac{R(\Delta_i)}{a^4} \left[1 + \delta D(z)\right] \frac{D(z)}{D(z_\text{obs})} \frac{dt}{dz},$$

(2)

where $D$ is the growth factor, $\alpha_B$ is the case B recombination coefficient, $n_0$ is the comoving electron density, and $R(\Delta_i)$ is the effective clumping factor of the IGM (see below). This equation was described in Wyithe & Loeb (2003), but it is generalised in this work to apply to regions of large scale overdensity $\delta$ that differ from the average IGM.

Integration of equation (2) requires knowledge of $P_\text{V}(\Delta)$. Miralda-Escude et al. (2000) found that a good fit to the volume weighted probability distribution for the density as seen in cosmological hydrodynamic simulations has the functional form

$$P_\text{V}(\Delta) d\Delta = A \exp\left[-\frac{(\Delta^{-2/3} - C_0)^2}{2(2C_0)^{2/3}}\right] \Delta^{-\beta} d\Delta,$$

(3)

where $\delta_0 = 7.61/(1 + z)$ and $\beta = 2.23, 2.35$ and 2.48, and $C_0 = 0.558, 0.599$ and 0.611 at $z = 2, 3$ and 4. At $z = 6$ they assume $\beta = 2.5$ and solve for $A$ and $C_0$ by requiring the mass and volume to be normalised to unity. We repeat this
procedure to find $F_{\nu}(\Delta)$ at higher redshifts. The proportionality of $\delta$ to the scale factor is expected for the growth of structure in an $\Omega_m = 1$ universe or at high redshift otherwise, and its amplitude should depend on the amplitude of the power-spectrum. The simulations on which the distribution in Miralda-Escude et al. (2000) was based assumed $\Omega_m = 0.4, \Omega_\Lambda = 0.6$ and $\sigma_8 = 0.79$, close to the values used in this paper. The above probability distribution remains a reasonable description at high redshift when confronted with a more modern cosmology and updated simulations, although the addition of an analytical approximation for the high density tail of the distribution remains necessary as a best guess at correcting for numerical resolution (Bolton & Haehnelt 2007b).

Equation (2) provides a good description of the evolution of the ionization fraction following the overlap of individual ionized bubbles because the ionization fronts are exposed to the mean ionizing radiation field. However prior to overlap, the prescription is inadequate due to the large fluctuations in the intensity of the ionizing radiation. A more accurate model to describe the evolution of the ionized volume prior to overlap was suggested by Miralda-Escude et al. (2000). In our notation the appropriate equation is

$$\frac{d\langle Q_{M, \delta}(\Delta_i) \rangle}{dz} = \frac{1}{n^{\delta}} \frac{dn_{\delta}(\delta)}{dz} R(\Delta_{\text{crit}}) n_z \left( 1 + \frac{\delta}{D(z)} \right) \frac{Q_t}{dz} - \alpha_\delta (1 + z)^3 R(\Delta_{\text{crit}}) n_z \left( 1 + \frac{\delta}{D(z)} \right) \frac{dt}{dz}. \quad (4)$$

or

$$\frac{dQ_{t}}{dz} = \frac{d\langle Q_{M, \delta}(\Delta_i) \rangle}{dz} \frac{dn_{\delta}(\delta)}{dz} - \left[ \alpha_\delta (1 + z)^3 R(\Delta_{\text{crit}}) n_z \left( 1 + \frac{\delta}{D(z)} \right) \frac{dt}{dz} \right] + \frac{dF_{M}(\Delta_i)}{dz} \frac{Q_t}{F_{M}(\Delta_i)} \quad (5)$$

In this expression, $Q_t$ is redefined to be the volume filling factor within which all matter at densities below $\Delta_i$ has been ionized. The effective clumping of the IGM can now be written

$$R(\Delta_i) \equiv Q_t \frac{(\rho^2)}{\langle \rho^2 \rangle} = \int_{0}^{\Delta_i} d\Delta P_{\nu}(\Delta) \Delta^2, \quad (6)$$

where $P_{\nu}(\Delta)$ is the fraction of the volume with $\Delta < \Delta_i$.

Within this formalism, the epoch of overlap is precisely defined as the time when $Q_t$ reaches unity. However, we have only a single equation to describe the evolution of two independent quantities, $Q_t$ and $F_{M}$. The relative growth of these depends on the luminosity function and spatial distribution of the sources. The appropriate value of $\Delta_{\text{crit}}$ is set by the mean separation of the ionizing photon sources. More numerous sources can attain overlap for smaller values of $\Delta_{\text{crit}}$.

We assume $\Delta_{\text{crit}}$ to be constant with redshift before overlap with values of $\Delta_{\text{crit}} = 20$ and $\Delta_{\text{crit}} = 5$ (which lie in the range for galaxies) in this paper, and show examples for both.

Our approach is to compute a reionization history given a particular value of $\Delta_{\text{crit}}$, combined with assumed values for the efficiency of star-formation and the fraction of ionizing photons that escape from galaxies. With this history in place we then compute the evolution of the background radiation field due to these same sources. Post overlap, ionizing photons will experience attenuation due to residual over-dense pockets of H\textsc{i} gas. We use the description of Miralda-Escude et al. (2000) to estimate the ionizing photon mean-free-path, and subsequently derive the attenuation of ionizing photons. We then compute the flux at the Lyman limit ($\nu_{\text{LL}} = 3.29 \times 10^{15}$Hz) in the IGM due to sources immediate to each epoch, in addition to redshifted contributions from earlier epochs. This calculation follows the model described in Srianovskiy & Wyithe (2006) [see also Choudhury & Ferrara 2005] but is summarised below.

We note that Hz Lyman limit photons (13.6 eV) are incapable of ionizing helium (He) [24.6 eV (Heii), 54.4 eV (Heiii)]. We therefore neglect He when computing the intensity of the ionizing background in our semi-analytic model.

### 2.1 Evaluating flux at the Lyman Limit

In this section we review the process for calculating the ionizing background flux at a particular redshift. We assume the post overlap ionizing background to be generated by population-II stars alone following the results of Srianovskiy & Wyithe (2006). We first define $z_0$ to be the redshift at which the flux is to be evaluated. The flux at the Lyman limit is normalised in physical units of $J_{21}$ (10$^{-21}$ergs/sec/Hz/cm$^2$/sr), and at redshift $z_0$ it is related to the energy density by

$$J_{21}(z_0) = \frac{c}{4\pi} \frac{d^2E_{\text{tot}}(z_0)}{d\nu dz}, \quad (7)$$

where $c$ is the speed of light and $d^2E_{\text{tot}}(z_0)/d\nu dz$ is the energy per unit frequency interval per unit co-moving volume at frequency $\nu_z$ and redshift $z_0$. Contributions to the radiation field at $\nu_z$ (and $z_0$) from sources at redshift $z$ were emitted at frequency $\nu_e = 1 + \frac{z}{1+z_0}$. Note that $\nu_e$ remains below the He Lyman limit (24.6 eV) whilst $\frac{z}{1+z_0} \ll 2$. The semi-analytic model of the reionization history predicts an average redshift of overlap, which we refer to as $z_{\text{ol}}$.

The ionizing background flux contains contributions from sources at $z_0$ in addition to redshifted fluxes from sources at higher redshift. To compute the ionizing background flux we consider contributions from sources with redshifts $z > z_0$.

The total co-moving energy density at redshift $z_0$ is

$$\frac{d^2E_{\text{tot}}(z_0)}{d\nu dz} = \int_{0}^{z_0} dz \frac{d^3E_{\text{tot}}(z)}{d\nu dz dt} \frac{Q_t}{1 + z} \left( \frac{1 + z}{1 + z_0} \right) e^{-\tau(z,z_0)} \frac{dt}{dz}. \quad (8)$$

The term $d^3E_{\text{tot}}(z)/d\nu dz dt$ represents the frequency dependent (co-moving) energy density per unit time generated by ionizing sources, and $\tau$ is the optical depth for ionizing photons between $z$ and $z_0$. Rather than assume the complete attenuation of ionizing photons emitted prior to the redshift of overlap, we instead assume attenuation due to the mean-free-path evaluated in ionized regions prior to overlap and weight the contribution to the energy density by $Q_t$. This approach should approximate the contribution of sources to the ionizing background that emit at redshifts late in the overlap epoch when the typical bubble size is evolving rapidly, and is much larger than the ionizing photon mean-free-path (Furlanetto, Zaldarriaga & Hernquist 2004).

Finally, the photoionization rate may be related to the

\[ \dot{n}_{\text{phot}} = \frac{d^2E_{\text{tot}}(z_0)}{d\nu dz} \frac{Q_t}{1 + z} \left( \frac{1 + z}{1 + z_0} \right) e^{-\tau(z,z_0)} \frac{dt}{dz}, \]
observed ionizing background flux using

$$\Gamma = 4\pi \int_{v_c}^{\infty} J_\nu \frac{\sigma_v}{h_\nu} dv,$$

where $\sigma_v$ is the HI photo-ionization cross-section and $h_\nu$ is Planck’s constant. Using equation (9) and the spectra describing our sources, we find $\Gamma_{12} = 2.82J_{21}$ (where $\Gamma_{12}$ is the ionizing rate in units of $10^{-14} \text{s}^{-1}$) for a background in which stars are the dominant source. This last step does not account for the variation of the spectral shape (due to redshifting) from that emitted at a constant time.

### 2.2 Lyman limit photons from Stars

We next describe the contribution to the ionizing background flux made by population-II stars. To begin, we describe the spectral energy distribution (SED) of population-II star forming galaxies, using the model presented in Leitherer et al. (1999). The SED has the form

$$\text{II star forming galaxies, using the model presented in Lei-}$$

We next describe the contribution to the ionizing background flux from stars. To begin, we describe the spectral energy distribution (SED) of population-II star forming galaxies, using the model presented in Leitherer et al. (1999). The SED has the form $d\nu d\nu d\nu M$, where $M$ is expressed in units of (baryonic) solar masses per year. In the instance that ionizing photons are produced primarily in star bursts, with lifetimes much shorter than the Hubble time, we may express the star formation rate per unit time as

$$\frac{dM}{dV}(z) = f^* dF(\delta, z) d\nu d\nu d\nu \rho_b,$$

where $\rho_b$ is the co-moving baryonic mass density, and $F(\delta, z)$ is the density dependent collapsed fraction of mass in halos above a critical mass at $z$. The factor $f^*$ (the star formation efficiency) describes the fraction of collapsed matter that participates in star formation. This fraction is largely unknown.

In a region of co-moving radius $R$ and mean overdensity $\delta(z) = \delta D(z) / D(z_0)$ [specified at redshift $z$ instead of the usual $z = 0$], the relevant collapsed fraction is obtained from the extended Press-Schechter (1974) model (Bond et al. 1991) as

$$F(\delta, R, z) = \text{erfc} \left( \frac{\delta_c - \delta(z)}{\sqrt{2 (\sigma_{gal}^2 - \sigma(R)^2)}} \right).$$

where $\text{erfc}(x)$ is the error function, $\sigma(R)$ is the variance of the density field smoothed on a scale $R$, and $\sigma_{gal}$ is the variance of the density field smoothed on a scale $R_{gal}$, corresponding to a mass scale of $M_{min}$ or $M_{son}$ (both evaluated at redshift $z$ rather than at $z = 0$). In this expression, the critical linear overdensity for the collapse of a spherical top-hat density perturbation is $\delta_c = 1.69$.

In a cold neutral IGM beyond the redshift of reionization, the collapsed fraction should be computed for halos of sufficient mass to initiate star formation. The critical virial temperature is set by the temperature ($T_N \sim 10^4 \text{K}$) above which efficient atomic hydrogen cooling promotes star formation. Following the reionization of a region, the Jeans mass in the heated IGM limits accretion to halos above $T_\text{i} \sim 10^7 \text{K}$ (Efstathiou 1992; Thoul & Weinberg 1996; Dijkstra et al. 2004). We may therefore write the time derivative of the collapsed fraction

$$\frac{dF}{dt_{\text{year}}}(z) = \left[ Q_{\text{in}}(z) \frac{dF(z, T_i)}{dz} + [1 - Q_{\text{in}}(z)] \frac{dF(z, T_0)}{dz} \right] \times \frac{dz}{dt_{\text{year}}},$$

where $Q_{\text{in}}$ is the ionized mass fraction in the universe.

To describe the ionizing flux from stars we require one further parameter. Only a fraction of ionizing photons produced by stars may enter the IGM. Therefore an additional factor of $f_{\text{esc}}$ (the escape fraction) must be included when computing the emissivity due to stars. Ciardi & Ferrara (2005) include a review of existing constraints on $f_{\text{esc}}$ which suggests that its value is $\lesssim 15$ per cent. The star formation efficiency and escape fraction may be combined into a single free parameter $(f_{\text{esc}})$ to describe the contribution of stars in our model.

Finally the energy density due to population-II stars at $z_0$ may be computed using equation (8) with $\frac{d^3 E_{\nu}(z)}{dV d\nu dt}$ given by

$$\frac{d^3 E_{\nu}(z)}{dV d\nu dt} = \frac{d^3 E_{\nu}(z)}{dV d\nu dt M} \frac{dF(z)}{dt_{\text{year}}} \rho_b f_{\text{esc}}.$$

This energy density may then be converted to a flux.

### 2.3 Attenuation of ionizing photons

Having computed a luminosity density we may now compute a value for the ionizing background using equations (7), following the calculation of the optical depth (τ). We fix $\Delta_c$ prior to overlap, and then follow the evolution of $\Delta_c$ following overlap. In each case we use the approach of Miralda-Escude et al. (2000) to estimate the mean-free-path (λ) as a function of redshift (z):

$$\lambda_i = \lambda_0 (1 - F_\nu)^{-2/3}.$$  

Here

$$F_\nu = \int_0^{\Delta_{\text{max}}(z)} d\Delta P(\Delta)$$

is the volume filling factor of ionized regions, computed using the volume weighted probability distribution $[P(\Delta)]$ of the overdensity (Miralda-Escude et al. 2000), and $\Delta_{\text{max}}$ is equal to $\Delta_c$ and $\Delta_1$ prior to and following overlap respectively. We note that prior to overlap the mean-free-path is ill-defined, and we calculate the mean-free-path for photons in ionized regions prior to overlap. The product $\lambda_0 H = 60 \text{kms}^{-1}$ was obtained from comparison to the scales of Lyα forest structures in simulations, at $z = 3$ (Miralda-Escude et al. 2000). Finally the mean-free-path as a function of redshift may be used to compute the attenuation of ionizing photons between redshifts $z$ and $z_0$. The resulting optical depth is

$$\tau(z, z_0) = \int_{z_0}^{z_{\text{in}}} dz \frac{dt}{dz} \frac{1}{\lambda_i}.$$  

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1 This conversion was reported incorrectly in Srbinovsky & Wyithe (2006) as $\Gamma_{12} = 1.69J_{21}$.  

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2.4 The overdensity near massive halos

Strong clustering of massive sources implies that these sources should trace the higher density regions of IGM. In this section we compute the distribution of overdensities on a scale $R$ that are centered on halos of mass $M$. Our aim is to calculate the distribution of over densities surrounding high redshift quasars, which are expected to be larger compared to an average region of the IGM (e.g. Guimaraes et al. 2007, Faucher-Giguere et al. 2007).

The likelihood of observing a galaxy at a random location is proportional to the number density of galaxies. At small values of the large scale overdensity $\delta$, this density is proportional to $[1 + \delta b(M, z)]$, where $b$ is the galaxy bias (Mo & White 1996; Sheth, Mo & Tormen 2002). More generally, given a large scale over density $\delta$ on a scale $R$, the likelihood of observing a galaxy may be estimated from the Sheth-Tormen (2002) mass function as

$$L_b(\delta) = \frac{(1 + \delta)\nu(1 + \nu^{-2p})e^{-\nu^2/2}}{\nu(1 + \nu^{-2p})e^{-\nu^2/2}},$$

where $\nu = (\delta_b - \delta)/[\sigma(M)]$ and $\delta_b = \delta_b/[\sigma(M)]$. Here $\sigma(M)$ is the variance of the density field smoothed with a top-hat window on a mass scale $M$ at redshift $z$, and $a = 0.707$ and $p = 0.3$ are constants. Note that here as elsewhere in this paper, we work with overdensities and variances computed at the redshift of interest (i.e. not extrapolated to $z = 0$). Utilising Bayes theorem, we find the a-posteriori probability distribution for the overdensity $\delta$ on the scale $R$ given the locations defined by a galaxy population. We obtain

$$\frac{dP}{d\delta}|_{\text{gal}} \propto L_b(\delta) \frac{dP_{\text{gal}}}{d\delta},$$

where $dP_{\text{gal}}/d\delta$ is the prior probability distribution for $\delta$, which is described by a Gaussian of variance $\sigma(R)$. We may use this probability for the overdensity within $R$ of a luminous high redshift quasar to calculate the enhancement in the recombination and star-formation rates. Thus, through integration of equations (2) and (5), we may compute the reionization history within typical regions surrounding luminous quasars, and compare these with the reionization history of the mean IGM ($\delta = 0$).

3 RESULTS

The model described in the previous section has two free parameters $\Delta_c$ and $f_{\text{esc}}^\ast$. For any combination of these parameters we are able to compute a reionization history, the evolution of the mean-free-path, and the evolution of the ionizing background as a function of the local overdensity. In this section we first summarise the observations to which we compare our model before describing the results of this comparison.

3.1 Observational estimates of the ionization rate

Fan et al. (2006) have analyzed the absorption spectra of 19 high redshift quasars. Based on this data Bolton & Haehnelt (2007b) present estimates of the ionization rate, $\Gamma_{12}$ and the neutral hydrogen fraction over a range of redshifts near the end of the reionization era using a suite of detailed numerical simulations. In Figures 1 and 2 we compare our model to these estimates to demonstrate that it is consistent with the available data.

3.2 Comparison of reionization near a quasar and in the general IGM

We now use the model developed earlier to address the level of bias in reionization near to a luminous high redshift quasar relative to the IGM as a whole. We show results for a particular model in which we assume that quasars are hosted by halos of mass $10^{16}M_\odot$ for $\Delta_c = 20$. Although we compare our model for the ionizing background to only three observational points at $z \gtrsim 4$ using two free parameters, $\Delta_c$ and $f_{\text{esc}}^\ast$, we find that these parameters are not degenerate following overlap. Larger values of $\Delta_c$ increase the volume of hydrogen that must be reionized prior to overlap, leading to a delay in the reionization redshift. However, following overlap, the mean-free-path increases rapidly, and hence $\Gamma_{12}$ at redshifts below $\sim 5$ is quite insensitive to $\Delta_c$. On the other hand, varying $f_{\text{esc}}^\ast$ strongly affects the overlap redshift, since the critical number of ionizing photons per baryon is achieved earlier. Following overlap, a larger $f_{\text{esc}}^\ast$ also increases the value of $\Gamma_{12}$, which is proportional to the local emissivity in addition to the mean-free-path. We therefore adjust the parameter $f_{\text{esc}}^\ast$ so that the model evaluated at the mean density of the Universe gives a good fit to the observed ionizing background (we find $f_{\text{esc}}^\ast = 0.0037$). The best fit to the ionization rate observed at $z \sim 6$ is provided by a value of $\Delta_c \sim 20$. Values of $\Delta_c \gtrsim 5$ also yield a satisfactory fit. The overlap redshifts corresponding to $(\Delta_c, f_{\text{esc}}^\ast) = (20, 0.0037)$ and $(\Delta_c, f_{\text{esc}}^\ast) = (5, 0.0037)$ are $z_{\text{ol}} = 6.2$ and $z_{\text{ol}} = 7.2$ respectively.

Figure 1 shows the redshift evolution of $Q_1$, $\lambda_1$, the volume averaged neutral fraction and the ionization rate for the best fit model. Reionization histories for four different values of overdensity were computed. These overdensities were evaluated in 5 physical Mpc spheres, and correspond respectively to the mean overdensity centered on a quasar at $z \sim 6$ (dark solid lines), and to the plus and minus 1-$\sigma$ fluctuations around that mean (dotted and dashed lines). We also show the case of the mean IGM with $\delta = 0$ (thick grey lines). The examples demonstrate the level of enhancement in the reionization process within 5 physical Mpc of a high redshift quasar, and the expected scatter around this enhancement.

In the top left panel of Figure 1 we show the fraction of the IGM yet to overlap according to the definition given in § 2. The more overdense regions experience overlap first due to the enhanced production of ionizing photons in these regions. In this model the mean IGM achieves overlap at $z \sim 6.1$. However within 5 physical Mpc of a quasar the overlap is achieved $\sim 0.25$ redshift units earlier on average, with a 1–$\sigma$ scatter of $\sim 0.2$ redshift units. These results are in quantitative agreement with the analytic calculation of Lidz et al. (2007; see their Figure 1). However our model allows the calculation of post-overlap properties such as the volume averaged neutral fraction, ionizing photon mean-free-path and ionization rate in addition to the pre-overlap mass-averaged ionization fraction.

In the central left panel of Figure 1 we show the corresponding value of the mean-free-path for ionizing photons, computed using the formalism in § 2. The model predicts a
Figure 1. The effect of overdensity on the redshift of overlap, and the subsequent ionization state of the IGM. Four cases are shown, corresponding to different overdensities evaluated in 5 physical Mpc spheres, including the average overdensity centered on a quasar at $z \sim 6$ (dark solid curves) and the plus and minus 1-$\sigma$ fluctuations around that mean (dotted and dashed curves). We also show the case of the mean IGM with $\delta = 0$ (thick grey lines). Top Left Panel: The fraction of the IGM yet to overlap according to the definition of overlap given in §2. Top Right Panel: The volume averaged filling factor of neutral gas. Central Left Panel: The mean-free-path for ionizing photons computed using the formalism in §2. Central Right Panel: The ionization rate. Lower Left Panel: The emissivity of ionizing sources. Also shown (thick dotted line) is the observed evolution of ionizing sources as estimated in Bolton & Haehnelt (2007b). Lower Right Panel: The ratio of the ionization rate at different overdensities relative to the average overdensity centered on a quasar host. In this example the halo mass was $M = 10^{13} M_\odot$ and $\Delta_c = 20$. The observational points in the top and middle right panel are also from Bolton & Haehnelt (2007b).

The rapid increase in the mean-free-path following overlap, which is also indicated from the results of numerical simulations, (e.g. Gnedin 2000). However, the offset in the overlap redshift induced by large scale overdensity leads to substantial differences in mean-free-path within regions of different average overdensities for a brief period following overlap. Once the rate of change of mean-free-path slows, then its dependence on overdensity is reduced.

In the central right panel of Figure 1 we show the dependence of the ionization rate on redshift. The rapid change in the mean-free-path following overlap leads to a corresponding rapid change in the ionization rate. Thus, for a brief period near overlap the model predicts large variations in the ionizing background within regions with different mean overdensities. Note that the rapid increase in $\Gamma_{12}$ is entirely due to the increasing mean-free-path, and not to an increase in the source emissivity which is shown in the lower left panel of Figure 1. Indeed, the radiative feedback that accompanies the end of the reionization era results in a slight reduction of the emissivity during the time when the ionizing background undergoes a large increase. For comparison the fit to the observationally derived ionizing emissivity from Bolton & Haehnelt (2007b) is shown by the thick dotted line. The model emissivity is close to the observationally derived estimate (with the offset due to the assumption of a somewhat harder spectrum here), and it has a similar (small) redshift
Figure 2. The effect of overdensity on the redshift of overlap, and the subsequent ionization state of the IGM. Five cases are shown, corresponding to overdensities evaluated within spheres with radii between 5 and 15 physical Mpc, centered on a quasar at $z \sim 6$. In each case we evaluate the reionization history assuming the mean overdensity surrounding the quasar. We also show the case of the mean IGM with $\delta = 0$ (thick grey lines). Top Left Panel: The fraction of the IGM yet to overlap according to the definition of overlap given in §2. Top Right Panel: The volume averaged filling factor of neutral gas. Central Left Panel: The mean-free-path for ionizing photons computed using the formalism in §2. Central Right Panel: The ionization rate. Lower Left Panel: The emissivity of ionizing sources. Also shown (thick dotted line) is the observed evolution of ionizing sources summarised in Bolton & Haehnelt (2007b). Lower Right Panel: The ratio of the ionization rate at different overdensities relative to the average overdensity centered on a quasar host. In this example the halo mass was $M = 10^{13} M_\odot$ and $\Delta_c = 20$. The observational points in the top and middle right panel are also from Bolton & Haehnelt (2007b).

dependence. In the upper right panel we show the volume averaged hydrogen neutral fraction. Again, near overlap the value of the neutral fraction inferred within the vicinity of high redshift quasars is significantly biased relative to the average IGM. To better illustrate the level and duration of the bias introduced by early overlap we also plot the ionization rate at different overdensities relative to the average overdensity centered on a quasar host halo (lower right hand panel of Figure 1). We see that at times near overlap, the ionizing background within 5 physical Mpc of a quasar can be as much as a factor of 3 in excess of the average ionizing background. We have also checked the enhancement of the ionizing background (at a fixed distance) near a quasar host following overlap for different values of the quasar host mass and $\Delta_c$. We find the enhancement is reduced for smaller values of the quasar host mass, but is not sensitive to the value of the critical overdensity $\Delta_c$.

3.3 The dependence of reionization on the distance from a quasar

In the previous section we saw that the early reionization of overdense regions will lead to an ionizing background within 5 physical Mpc of a high redshift quasar that is in excess of the average IGM by a factor of $\sim 3$. In this section, we explore the variation of this enhancement with distance from
the quasar. As before, in Figure 2 we plot the effect of overdensity on the redshift of overlap, and the subsequent ionization state of the IGM. We again assume the case of \( \Delta_e = 20 \) and a quasar host mass of \( 10^{13} M_\odot \). Five cases are shown, corresponding to overdensities evaluated within spheres with radii between 5 and 15 physical Mpc, centered on a quasar at \( z \sim 6 \). In each case we evaluate the history assuming the mean overdensity surrounding the quasar host of mass \( M = 10^{13} M_\odot \). We also show the case of the mean IGM with \( \delta = 0 \) (thick grey lines).

In the top left panel of Figure 2 we show the fraction of the IGM that has yet to overlap. Reionization will occur earlier in regions that are closer to the quasar, where the overdensity is typically higher. In the central left panel we plot the mean-free-path for ionizing photons. At a fixed time around overlap we expect that the mean-free-path will be larger closer to the quasar. It is important to note that throughout the epoch where the evolution of the mean-free-path is rapid, the mean-free-path is smaller than the distances from the quasar under consideration. In the upper right and central right hand panels we plot the volume averaged neutral fraction of hydrogen and the ionization rate. As before we also plot the model and observed emissivity of ionizing sources in the lower left panel. Finally, in the lower right hand panel we show the ratio of the average ionization rate within different radii relative to the the value calculated at the average overdensity centered on a quasar host. As expected, we find that the level of enhancement decreases as the distance from the quasar increases.

To better see the dependence of the ionization rate on radius, we plot its enhancement within a spherical region of radius \( R \) surrounding a quasar relative to the mean IGM (Figure 3). The values are shown for a range of redshift bins near overlap. The enhancement in the ionizing background is greatest close to the quasar, where the overdensity is largest. We find that the enhancement drops below 10 per cent beyond distances of 15 physical Mpc. At all scales the enhancement is largest at a time shortly after the overlap of the average IGM. The enhancement more than halves in value by around half a redshift unit after overlap, and has almost disappeared by 1 redshift unit following overlap.

Before proceeding we note that we have not included the effect of the quasar in our calculation of the enhancement of the ionizing background near the quasar. The additional ionizing flux from the quasar would increase the local value of the ionizing photon mean-free-path, and therefore increase the value of the ionization rate due to the stellar emissivity. The enhancements described in Figure 2 should therefore be considered as lower limits.

4 THE EVOLUTION OF QUASAR NEAR-ZONES

We now turn to the prediction of Ly\( \alpha \) near zone sizes. Bolton & Haehnelt (2007a), Masetti et al. (2007) and Lidz et al. (2007) have shown that the interpretation of near-zone sizes with respect to the neutral hydrogen fraction in the IGM is difficult. Indeed, the data may approximately correspond to the edge of an HII region surrounded by a neutral IGM or instead correspond to a classical proximity zone in an highly ionized IGM. As discussed in the introduction, Fan et al. (2006) proposed defining the sizes of these regions of transmission as the distance where the smoothed spectrum first drops below the level where 10 per cent of the flux is transmitted. Using this definition, Fan et al. (2006) found a rapid trend of near-zone size with redshift, from which they inferred an order of magnitude increase in the neutral fraction of the IGM over the observed range \( 5.8 \lesssim z \lesssim 6.4 \). However, the detailed modeling of Bolton & Haehnelt (2007a) showed that the evolution of the near-zone size is not a simple function of the neutral hydrogen fraction. Their modeling suggested that in the limit of a classical proximity zone the rapid evolution of near-zone sizes could be explained by a relatively modest evolution in the neutral fraction, with the rapid evolution being due to the approach of the background transmission reaching 10 per cent.

The evolution of the ionization state of the IGM is thus difficult to extract from the observed evolution of the near-zone size. In this section we begin by discussing a calculation of quasar near-zone sizes using our semi-analytic reionization model. We then describe the observed relation between near-zone size and redshift before comparing our model with the observed evolution of quasar near-zone size.

4.1 Semi-analytic estimates for the evolution of near-zone sizes

In Figure 4 we plot the ionization rate due to galaxies in a biased, overdense region of the IGM surrounding a quasar. The ionization rates are displayed as a function of radius from the quasar. The overlap of the mean IGM in the model used (which was shown in Figures 1 and 2 and has \( \Delta_e = 20 \)) is at \( z = 6.2 \). The quasar host mass is \( 10^{13} M_\odot \), and its ionizing luminosity is \( N = 2 \times 10^{37} \text{s}^{-1} \). In the left-hand panel, we show the ionization rate due to galaxies calculated at the mean overdensity found within spheres of radius \( R \) surrounding a high redshift quasar. The curves are plotted at a range of redshifts between \( z = 5.6 \) and \( z = 6.4 \). The ionization rate due to the quasar emission assuming the IGM to be optically thin is also shown (thick grey line). We see that at redshifts near \( z \sim 5.8 \), the value of the ionization rate at distances corresponding to the observed quasar near-zone
Figure 4. The ionization rate as a function of radius from a quasar. **Left Panel:** The ionization rate due to galaxies calculated at the mean overdensity found within spheres of radius $R$ surrounding a high redshift quasar. The curves are plotted at a range of redshifts between $z = 5.6$ and $z = 6.4$. The ionization rate due to the quasar in a highly ionized IGM is also shown (thick grey line). **Right Panel:** The corresponding ionization rates due to galaxies plus the central quasar. The overlap of the mean IGM in this model (with $\Delta_c = 20$) is at $z = 6.2$. The quasars host masses were $10^{13} M_\odot$, and their ionizing luminosities were $\dot{N} = 2 \times 10^{57} \text{s}^{-1}$.

Figure 5. The predicted evolution of quasar near-zone sizes as a function of redshift. **Upper Left Hand Panel:** The evolution of the ionization rate corresponding to regions at the mean IGM density (solid line $\Delta_c = 20$; dashed line $\Delta_c = 5$). **Upper Right Hand Panel:** The predicted evolution of the near-zone size. The solid line corresponds to the size expected assuming the mean overdensity surrounding a quasar. The long-dashed line shows the near-zone evolution expected due to expansion of the Universe only. Also shown for comparison (thick grey line) is the observed near-zone size evolution, which may be parameterised by $R_{\text{max}} = 7.7 - 8.1(z - 6)$ physical Mpc (see section 4.2). The thin grey lines illustrate the level of observed scatter ($\pm1.2\text{Mpc}$). **Lower Left Hand Panel:** As before, except now dotted curves correspond to the near-zone size evolution computed at $-1\sigma$ and $+1\sigma$ fluctuations around the mean density for $\Delta_c = 20$. **Lower Right Hand Panel:** As before, but now the dotted lines correspond to the near-zone size evolution computed for $z_{10} = 5.0$ and 5.5, each for $\Delta_c = 20$. The overlap redshifts (with $f_{\text{esc}} = 0.0037$) of the mean IGM in these models are at $z = 6.2$ (with $\Delta_c = 20$) and $z = 7$ (with $\Delta_c = 5$). In all panels, the quasar host masses were assumed to be $10^{13} M_\odot$, and quasar ionizing luminosities were $\dot{N} = 2 \times 10^{57} \text{s}^{-1}$. 

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sizes (~10Mpc) is comparable to the ionization rate due to the quasar alone. Moreover, at this epoch, the value of the galaxy ionization rate increases rapidly with redshift due to the increasing mean-free-path at the end of the overlap epoch. In the right hand panel of Figure 4 we show the corresponding ionization rates due to the sum of the galaxies and the central quasar. Note that we have linearly added the ionization rates and thus do not account for the expected increase of the mean free-path due to the ionizing radiation of the quasar. We therefore still underestimate the ionization rate somewhat in the quasar near-zone in our model. The contribution to the ionization rate from galaxies flattens the radial power-law profile of the ionization rate at large distances surrounding the quasar. This flattening becomes more pronounced towards lower redshifts.

We now turn to an analytical estimate of quasar near-zone sizes within our reionization model. Adopting the near-zone size definition of Fan et al. (2006) discussed previously, we firstly need to estimate the ionization rate which leads to a transmission level of 10 per cent. Fan et al. (2006) smoothed their spectra on a 20Å scale to compute the near-zone size; an effective Lyα optical depth of \( \tau_{\text{eff}} = -\ln(0.1) = 2.3 \) is therefore an appropriate measure to estimate this quantity. The IGM exhibits an effective optical depth of 2.3 around \( z_{10} \sim 5.2 \) (Songaila 2004, Fan et al. 2006), at which point our semi-analytic reionization model predicts a value we shall denote as \( \Gamma(5.2) \). Therefore, the limiting value of the ionization rate which will correspond to a transmission level of 10 per cent at some redshift is approximately given by

\[
\Gamma_{\text{lim}} = \Gamma(5.2) \left( \frac{1 + z}{6.2} \right)^{9/2},
\]

since \( \Gamma \propto (1 + z)^{9/2} \) for a fixed optical depth (e.g. Bolton & Haehnelt 2007a). We may therefore estimate the near-zone size as the value of \( R \) at which the sum of the galaxy and quasar ionizing rates in Figure 4 drops below \( \Gamma_{\text{lim}} \). Note that while we are confident in our model's ability to estimate the background ionization rate, our analytic estimate of near-zone size is very approximate due to the neglect of several important effects. These effects include the attenuation of the quasars ionizing flux in the near-zone as well as the effect of discrete absorbers on the observed near-zone size (our analytic model computes only the mean properties of a clumpy IGM). We will address these issues using a numerical model of radiative transfer through a realistic simulation of the high redshift IGM in §5.

The semi-analytic predictions for the evolution of quasar near-zone sizes as a function of redshift are shown in the upper right panel of Figure 4. The redshift evolution for sizes calculated at the mean overdensity surrounding a quasar is plotted as the solid line. We show two cases, \( \Delta_z = 20 \) (solid line) and \( \Delta_z = 5 \) (dashed line). The corresponding evolution of \( \Gamma(5.2) \) is compared to the observational data in the upper left panel of Figure 4. Figure 4 shows that the near-zone size should not evolve much prior to the redshift of overlap where the quasar dominates the ionizing flux in the near-zone. However, when the ionization rate due to galaxies approaches the level necessary to maintain a mean transmission of 10 per cent, the near-zone size starts to evolve rapidly until eventually the definition of the near-zone size introduced by Fan et al. (2006) breaks down. This is because the flux level in most of the spectrum eventually exceeds 10 per cent, and the edge of the near-zone becomes indistinguishable from the Lyα forest.

4.2 Observations of the near-zone size

We now discuss our predictions for the near-zone size evolution in comparison with the observational data. Fan et al. (2006) found a rapid evolution in the near-zone size with redshift. Their data show a strong correlation about a mean relation, with a scatter that is in excess of the uncertainty on individual parameters. To remove the dependence of near-zone size on quasar luminosity, \( L_\alpha \), Fan et al. (2006) rescaled their near-zone sizes to a common absolute magnitude of \( M_{1400} = -27 \) by assuming the sizes are proportional to \( L_\alpha^{1/3} \). However, we caution that the uncertainty in the interpretation of the nature of the near-zones (HII region vs classical proximity zone) makes the scaling of the near-zone size with the ionizing luminosity of the quasar rather uncertain (Bolton & Haehnelt 2007a).

In addition to the near-zone measurements of Fan et al. (2006), Willott et al. (2007) have recently measured the near-zones for two new high redshift quasars. In particular, they find sizes of 6.4Mpc at \( z = 6.12 \), and 6.3Mpc at \( z = 6.43 \). Willott et al. (2007) use the \( L_\alpha^{1/3} \) scaling suggested by Fan et al. (2006), and obtain revised values of 6.4Mpc at \( z = 6.12 \) and 10.8Mpc at \( z = 6.43 \), with the latter point in particular apparently weakening the claim for evolution of near-zone size with redshift (Willott et al. 2007). However Willott et al. (2007) used the definition of near-zone size suggested by Bolton & Haehnelt (2007a), who defined the near-zone size to be the last pixel at which the unsmoothed quasar spectrum drops below a transmission level of 10 per cent. This definition avoids smoothing out important features in the spectrum, but breaks down once the IGM as a whole becomes highly ionized and the edge of the near-zones becomes ambiguous. Here we use the definition of Fan et al. (2006) to facilitate a comparison with the published sizes of Fan et al. Inspection of the spectra published by Willott et al. (2007) shows that the near-zones are sub-

Figure 6. Constraints on the parameters \( \alpha \) and \( R_6 \) that describe the evolution of observed near-zone sizes with redshift, assuming scalings of \( L_\alpha^{1/2} \) (dotted contours) and \( L_\alpha^{1/3} \) (solid contours) respectively. In each case the extrema of the contours presented represent the 68, 84, 91 and 97 per cent bounds on single parameters.
stantially smaller when the Fan et al. (2006) definition is employed. Therefore, our revised values for the near-zone sizes of the Willott et al. (2007) quasars are \( \sim 4.5 \text{Mpc} \) at \( z = 6.12 \) and \( \sim 3.5 \text{Mpc} \) at \( z = 6.43 \). Rescaling these to account for luminosity differences (using \( L^{1/3} \)) we find scaled values of \( \sim 4.5 \text{Mpc} \) at \( z = 6.12 \) and \( \sim 6 \text{Mpc} \) at \( z = 6.43 \). Thus using a consistent near-zone definition, the near-zone sizes for the new quasars discovered by Willott et al. (2007) are consistent with the relation between near-zone size and redshift measured by Fan et al. (2006).

Following Fan et al. (2006) we quantify the evolution of the near-zone using a parameterised form for the evolution:

\[
R = R_0 - \alpha (z - 6),
\]

where \( R_0 \) is the value of the near-zone size at \( z = 6 \), and \( \alpha \) is the slope of the evolution in units of Mpc. Assuming the uncertainty (\( \sigma_i \)) for each of the \( N_i = 18 \) (including both the Fan et al. (2006) and Willott et al. (2007) samples) observed near-zone sizes (\( R_i \)) at redshift \( z_i \) to be smaller than the intrinsic scatter (\( \sigma_R \)), we performed a chi-squared fit minimising the variable

\[
\chi^2(R_0, \alpha) = \sum_{i=1}^{N_i} \frac{(R_i - R(z_i))^2}{\sigma_R^2},
\]

The intrinsic scatter is estimated by adjusting \( \sigma_R \) to a level that yields a minimum value of reduced-\( \chi^2 \) equal to unity.

The resulting constraints on \( \alpha \) and \( R_0 \) are presented in Figure 5 assuming scalings of \( L^{1/3} \) (Fan et al. 2006) and \( L^{1/2} \), as is appropriate if the near-zone is the observational signature of a classical proximity effect (Bolton & Haehnelt 2007a). The extrema of the contours shown represent the 68, 84, 91 and 97 per cent bounds on single parameters. We find values of \( R_0 = 7.7 \pm 0.75 \) and \( \alpha = 8.1 \pm 2 \) with an intrinsic scatter of \( \sigma_R = 1.2 \) assuming a scaling of \( L^{1/2} \) and values of \( R_0 = 7.9 \pm 1 \) and \( \alpha = 7.5 \pm 2.5 \) with an intrinsic scatter of \( \sigma_R = 1.5 \) assuming a scaling of \( L^{1/2} \). The resulting constraint on \( R_0 \) is consistent with that quoted by Fan et al. (2006). However their quoted slope of \( \alpha = 9 \) was somewhat higher than our revised value, partly due to neglect of intrinsic scatter, and partly due to the addition of the two additional quasars from Willott et al. (2007).

In the case of a uniformly ionized IGM, Bolton & Haehnelt (2007a) have emphasised that the size of the quasar near-zone is independent of both the neutral fraction of the surrounding IGM and the quasar’s lifetime, providing the background ionization rate at the near-zone edge is small in comparison to the quasar ionization rate. The near-zone size defined in this way is plotted as the dashed line in Figure 5 using the analytic model of Bolton & Haehnelt (2007a) [their equation (10) with \( \Delta_{\text{lim}} = 0.6 \)]. The figure shows that the evolution of near-zone size due to density evolution alone would yield a value of \( \alpha = 2 \), which is excluded by the data at more than 95 per cent. This disagreement implies that an additional effect is responsible for the observed evolution. As suggested by Bolton & Haehnelt (2007a), and as we shall argue in more detail here using a model consistent with recent observational constraints, this additional effect is most likely due to the increasing amplitude of the ionizing background towards lower redshifts which raises the overall transmission the IGM.

4.3 Comparison between the semi-analytic model and observations

For comparison with our semi-analytic model we show the observed evolution of the quasar near-zones with redshift (Figure 5 thick grey line). This evolution may be parameterised by \( R_{\text{max}} = 7.7 - 8.1(z - 6) \) physical Mpc (Figure 5). The thin lines illustrate the level of observed scatter. Both the predicted size and evolution of the quasar near-zones are similar to observations for this reionization model (which was not tuned beyond the requirement to fit the ionization rate at \( z \sim 4 - 6 \)).

The near-zone sizes measured by Fan et al. (2006) show significant scatter, whose source we now discuss. First, while quasars are typically found in overdense regions, there are fluctuations in the overdensity of the nearby IGM, and hence also fluctuations in the stage of reionization occurring at a fixed redshift around different quasars. In the lower left panel of Figure 5 we re-plot the \( \Delta_c = 20 \) case for the average overdensity surrounding a quasar, and also plot the curves corresponding to the \( -1\sigma \) and \( +1\sigma \) density fluctuations around the mean (dotted lines). Our model predicts an amount of scatter in the near-zone sizes due to this cosmic variance that is significantly smaller than the scatter in observed near-zone sizes. The observed scatter is therefore due to the differing properties of the density field along different quasar lines-of-sight as has been shown previously in numerical simulations (Bolton & Haehnelt 2007a; Maselli et al. 2007; Lidz et al. 2007). The variation along lines-of-sight, which must be described by a full numerical treatment is manifested as scatter in the value of ionization rate that allows a transmission of 10 per cent. We may therefore estimate the scatter in the near-zone size using our analytic model by computing the dependence of near-zone size on redshift over the range of \( z_{10} \) allowed by observation. This is shown in the lower-right panel of Figure 5 again for the \( \Delta_c = 20 \) case, with the mean shown by the solid line and the evolution for \( z_{10} = 5.0 \) and \( z_{10} = 5.5 \) shown by the dotted lines. The scatter introduced is comparable to that of the observations [note that we do not expect additional scatter from quasar luminosity which has been scaled out in the relation of Fan et al. (2006), or from quasar lifetime which does not effect the size of the near zone in a highly ionized IGM (Bolton & Haehnelt 2007a)]. There may, however, be additional scatter due to variations in the temperature of the surrounding IGM attributable to the temperature dependence of the recombination coefficient.

Finally, it is important to note that the qualitative behaviour of the near-zone size evolution shown in Figure 5 is independent of the details of the reionization model. The two cases shown in Figure 5 have overlap redshifts for the mean IGM at \( z = 6.2 \) (with \( \Delta_c = 20 \)) and \( z = 7 \) (with \( \Delta_c = 5 \)). The corresponding ionization rate as a function of redshift for these models is shown in the upper left-hand panel of Figure 5. While the evolution of the near-zone sizes can be successfully reproduced using an evolution for the ionization rate predicted by our model at the tail-end of the overlap phase, the near-zone sizes themselves cannot not be used as a probe of this process. This is because the rapid evolution in the near-zone sizes is attributable to a relatively small change in the IGM effective optical depth, and therefore is no more sensitive to the reionization history than the

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Gunn-Peterson trough. It is nevertheless gratifying that we can qualitatively reproduce the evolution of the near-zone sizes with our analytical model, which is calibrated with the evolution of the background ionization rate measured from the effective optical depth in typical regions of the spectra. Note, however, that our analytic model is likely to overestimate the near-zone size both because it does not account for attenuation of quasar flux due to intervening neutral clumps in the IGM, and because the near-zone size is computed for average absorption properties in a clumpy IGM. Full numerical modeling with radiative transfer is required to reliably interpret the spectra of individual high redshift quasars. We present such numerical simulations in the next section.

5 MODELING THE OBSERVED NEAR-ZONE SIZE EVOLUTION WITH RADIATIVE TRANSFER SIMULATIONS

We now combine our semi-analytical model for the ionization rate in the biased regions surrounding quasars with a radiative transfer implementation and realistic density distributions drawn from a large cosmological hydrodynamical simulation. These simulations have been discussed in detail in Bolton & Haehnelt (2007a), and their description is not reproduced here. However, we have made three important changes. Firstly, we include an evolving, density dependent ionizing background using our semi-analytic model, which has been calibrated to observations at lower redshift. This addition is important because we have argued that it is the increase in the intensity of the ionizing background which leads to the rapid increase observed in the quasar near-zone sizes at \( z < 6 \). Secondly, we construct the absorption spectra using an ionizing background computed as a function of proper time along the trajectory of a photon emitted by the quasar, rather than at the proper time of the quasar. This effect is appropriate when considering spectra at the end of the reionization era, when the ionizing background can evolve significantly during the light travel time across a quasar near-zone. Thirdly we have increased the temperature in the surrounding IGM within 15 proper Mpc of the quasar to \( \sim 40000 \) K to take account the possibility that the hydrogen and helium in the near-zone has been highly ionized by a hard spectrum. We will discuss this point in more detail below.

In Figure 7 we show the synthetic quasar absorption spectra used in this study. The redshifts and luminosities correspond to the 18 quasars analysed in the sample of Fan et al. (2006), plus the two additional quasars reported recently by Willott et al. (2007). The emission rate of ionizing photons by the quasars, \( \dot{N} \), is displayed in each panel of Figure 7 normalised by \( 10^{57} \) s\(^{-1} \). The thick solid lines show the synthetic spectra after processing them to resemble data obtained with the Keck telescope Echelle Spectrograph and Imager [see Bolton & Haehnelt (2007a) for details], while the underlying thin solid lines show these spectra after smoothing by a top-hat filter of width 20\( \AA \) bins to allow comparison with observational data. The vertical lines mark the measured near-zone size using the definition of Fan et al. (2006).

\[ \text{Figure 7. Synthetic Echelle Spectrograph and Imager (ESI) spectra of high redshift quasar near-zones. The thick solid lines show the spectra at the computed resolution and the thin solid lines show the spectra smoothed by a top-hat filter of width 20\( \AA \) bins to allow comparison with observational data. The vertical lines mark the measured near-zone size using the definition of Fan et al. (2006).} \]
assuming the near-zone size is proportional to $N^{1/3}$. There is very good agreement between the observed and simulated sample of spectra. We have performed the $\chi^2$ minimisation on the parameterised evolution for the sample of 18 simulated near-zones, with the resulting constraints plotted in Figure 9.

We find parameters describing the mean relation of $R_6 = 7.0 \pm 0.4$ and $\alpha = 6.9 \pm 2$. The intrinsic scatter is at a level of $\pm 1.2$Mpc. These near-zones are slightly smaller than those observed. The modeled near-zones show an evolution that is consistent with the observed rate of evolution. However as we found for the observed near-zones, the models are not consistent with evolution due to IGM density alone ($\alpha = 2$). The scatter of model near-zone sizes is consistent with the scatter among the observed near-zone sizes. Note, however, that any scatter due to expected variations in the quasar emission histories and the subsequent impact on the IGM has been neglected in the simulations.

We now return to the discussion regarding the temperature of the IGM we adopted when constructing the synthetic near-zone spectra. When computing $N$, each quasar is assumed to have a broken power law spectral energy distribution (SED) similar to that reported by Telfer et al. (2002),

$$\epsilon_{\nu} \propto \nu^{-0.5} \quad \text{for} \quad 1050 < \lambda < 1450 \, \text{Å},$$
$$\epsilon_{\nu} \propto \nu^{-1.5} \quad \text{for} \quad \lambda < 1050 \, \text{Å}$$  \hspace{1cm} (22)

Thus, $M_{1450} = -27$ corresponds to $N = 1.9 \times 10^{37}$s$^{-1}$, and the 18 quasars in the Fan et al. (2006) and Willott et al. (2007) samples span a range of $0.4 < N/10^{37}$s$^{-1} < 4.0$. In Bolton & Haehnelt (2007a) it was noted that, even with a suitable increase in the ionizing background towards lower redshift, these quasar luminosities appeared to be too low to reproduce the absolute sizes of the observed near-zones with the simulations when using the definition advocated by Fan et al. (2006), particularly at $z < 6$ where the near-zone sizes are increased by the additional transmission from the emerging Ly$\alpha$ forest.

Consequently, in order to reproduce the observational result of Fan et al. (2006), each quasar in the simulations of Bolton & Haehnelt (2007a) also had a value of $N$ which was increased by a factor of 2.5 relative to the value corresponding to $M_{1450} = -27$ for our adopted SED. Since a factor of 2.5 increase in the quasar luminosity is rather large, it was argued that this increase may be instead attributable to the increased mean free path for ionizing photons in the near-zone, which leads to an enhanced contribution from the ionizing background due to galaxies. In this work we have consistently modeled the ionizing background due to galaxies by including the effect of reionization bias in the near-zone in detail. Nevertheless, we find the enhancement of the ionization rate in the near-zone due to this effect alone is still not enough to resolve this difference in the simulations. Note that this is not in contradiction with our analytical model predictions for the near-zone sizes. As stressed previously, the analytical estimates are likely to overestimate the near-zone sizes. We have therefore re-examined the assumptions which go into our numerical modeling.

In the case of a classical proximity zone the neutral hydrogen fraction and thus the sizes of the near-zones do not only depend on the ionization rate but are also rather sensitive to the temperature of the IGM in the near-zone. This is due to the temperature dependence of the recombination rate for ionized hydrogen; in ionization equilibrium the neutral hydrogen density $n_H \propto T^{-2/7}/\Gamma_{12}$. Therefore, a factor of 2.5 increase in the ionization rate is equivalent to around a factor of four increase in the ambient IGM temperature. In Bolton and Haehnelt (2007a) the numerical simulations had a median IGM temperature of $\sim 15000$ K before the quasar turned on, with the gas assumed to be in ionization equilibrium with the ionizing background. However, since the cooling timescale for the low density IGM is typically

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**Figure 8.** The near-zone sizes measured from synthetic spectra, plotted as a function of redshift (filled circles), along with the observational data of Fan et al. (2006) (open diamonds) and Willott et al. (2007) (crosses). The temperature in the surrounding IGM within 15 proper Mpc of the quasar has been raised to $\sim 40,000$ K to take account the possibility that the helium in the near-zone has been highly ionized by a hard spectrum.

**Figure 9.** Constraints on the parameters $\alpha$ and $R_6$ that describe the evolution of model near-zone sizes with redshift. The model had $N = 2 \times 10^{37}$s$^{-1}$ and a large IGM temperature within 15Mpc of the quasar. In each case the extrema of the contours presented represent the 68, 84, 91 and 97 per cent bounds on single parameters. The observational constraints assuming a scaling of $L^{1/2}$ are reproduced (dotted contours).
of order a Hubble time at $z = 6$, changes in the gas temperature made during a putative earlier accretion phase by the quasar would still persist. For example, if the quasar reionized the hydrogen and helium around it during an earlier phase when it had a very hard spectral index and the IGM was largely neutral, radiative transfer effects can increase the temperature in the IGM to $\sim 30\,000 - 40\,000\,K$ (Abel & Haehnelt 1999; Bolton, Meiksin & White 2004; Tituley & Meiksin 2006). The IGM will then have only cooled marginally during the interval between active quasar phases. Although our radiative transfer scheme does correctly include these kind of effects, the heating boost is not present in our current simulations. This is because we assume the IGM is already highly ionized by our model ionizing background before the quasar turns on, and we do not model the previous emission history of the quasar, which would require substantially more detailed simulations.

Therefore, to mimic this effect, we have raised the temperature in our simulations within 15 Mpc of the quasar by a factor of 2.5, corresponding to a median value of $\sim 40\,000\,K$. Note that this temperature is smaller than the value of $\sim 60\,000\,K$ one would expect based according to the scaling relation discussed above, which is higher than can be easily achieved by photoionization. Nevertheless the near-zone sizes are in good agreement with that of the observed sample. The deviation from the expected scaling is due to the reionization bias which we had not taken into account in Bolton & Haehnelt (2007a). Our model for the increased ionizing background in the biased regions surrounding quasars helps to increase the near-zone sizes and drive their rapid evolution. As discussed above, our simulations provide a good description of the observed near-zone sizes with the observed luminosities and reasonable assumptions for the quasar SED, and may thus provide a hint that the IGM is substantially hotter than average around the highest redshift quasars.

There remains a small discrepancy between observed and modeled near-zone sizes (Figure 9). However, as already mentioned we have not modeled the effect of the quasar radiation on the mean free path in the near-zone. There is also the possibility that the discrepancy is partially due to a continuum on the observed spectra which has been placed too low, or perhaps a quasar SED which is different to the one we have assumed. Considering these uncertainties the agreement appears excellent.

In summary, our semi-analytic modeling of the history for the ionizing background, combined with numerical simulation of the Ly$\alpha$ transmission in quasar near-zones, is able to quantitatively describe the evolution of quasar near-zone sizes at $6.4 < z < 5.8$, and the evolution of the ionizing background at $z \sim 4 - 6$. We find that the rapid evolution observed in the sizes of the high redshift quasar near-zones is due to the increasing level of ionizing background near $z \sim 6$, with absolute sizes which suggest an increased IGM temperature in the vicinity of the quasars.

6 SUMMARY AND CONCLUSIONS

Absorption spectra of high redshift quasars show an increasingly thick Ly$\alpha$ forest, suggesting an increase in the fraction of neutral gas in the IGM approaching $z \sim 6$, culminating in the complete lack of detected flux in the Gunn-Peterson trough blueward of the Ly$\alpha$ line. However the meaning of the complete Gunn-Peterson troughs remains controversial, having been interpreted both as the completion of the reionization epoch, and as merely a gradual thickening of the Ly$\alpha$ forest. The inconclusive nature of the observations has its root in the very large cross-section for absorption of Ly$\alpha$ photons, which allow a direct probe of the neutral hydrogen content of the IGM only down to volume averaged neutral fractions of around $10^{-4}$.

The fact that quasars reside in massive halos places them in overdense and strongly biased regions of the IGM. This presents an additional challenge for the interpretation of regions in quasar absorption spectra close to the quasars intrinsic redshift, since one is using results from these biased regions to infer the properties of the mean IGM. In this work we have presented a model for the evolution of the emissivity of ionizing photons and the ionization state of the IGM that takes such a reionization bias in the environment of bright, high-redshift quasars into account. In our CDM based model the ionizing photons for reionization are produced by star formation in dark matter halos spanning a wide mass range. At redshifts both prior to, and post overlap, our model allows us to calculate values for the volume and mass-averaged neutral fractions, the ionizing photon mean-free-path and the evolution of the ionizing background. With appropriate choices for the parameters controlling the star formation efficiency, we are able to reproduce the ionizing emissivity at $4 < z < 6$ and the ionizing photon mean-free-path at $z \sim 4$ inferred from the observed opacity distribution in Ly$\alpha$forest absorption spectra. In our model the comoving ionizing emissivity is roughly constant with redshift and the volume filling factor of ionizing regions increases slowly from 10 to 90 per cent between $z \geq 10$ and $z \sim 6 - 7$, where reionization completes following the overlap of cosmological HII regions. Within 5 physical Mpc of a high redshift quasar, we find the evolution of the ionization state of the IGM precedes that of the mean universe by around 0.3 redshift units. During the period shortly after overlap the mean-free-path is seen to increase rapidly in the model. The offset in the redshift of overlap between overdense regions and the mean IGM therefore results in an ionizing background near high redshift quasars (excluding the flux of ionizing radiation from the quasar itself) that exceeds the value in the mean IGM by a factor of $\sim 2 - 3$ for a short period of time.

High redshift quasar spectra also show evidence for a highly ionized region immediately surrounding the quasar. Using our semi-analytic prescription for biased reionization, we model the evolution of these quasar near-zones. We find near-zone sizes which exhibit a redshift dependence which is qualitatively consistent with the current observational data. The model indicates that the rapid evolution in the near-zone sizes in the redshift range $5.8 < z < 6.4$ is due to the increasing background ionization rate originating from an almost constant galactic emissivity. Within our model, this rise in the background ionization rate is therefore due to the rapid rise in the mean free path for ionizing photons, which is expected near the tail-end of the reionization epoch as H$\alpha$ regions overlap.

In order to model the opacity distribution in spectra of bright high-redshift quasars in more detail we have com-
bined our model for the evolution of the metagalactic ionization rate in the biased environment surrounding quasars with detailed line-of-sight radiative transfer simulations in the inhomogeneous high-redshift IGM. In this way we were able to reproduce the sizes of the observed near-zones and their rapid evolution in the redshift range $5.8 < z < 6.4$ very well. Our numerical modeling also suggests that the rapid evolution is not due directly to large changes in the neutral hydrogen fraction of the IGM. Instead, as suggested by Bolton & Haehnelt (2007a), the near-zone sizes probe the temperature dependence of the recombination rate. The observed near-zone sizes therefore find that the absolute sizes of the near-zones are rather sensitive to the IGM temperature in the near-zones, due to the temperature dependence of the recombination rate. The observed near-zone sizes are reproduced given the temperature of the surrounding IGM is high ($\sim 40,000$ K). Heating to such high temperatures at distances of up to $10 - 15$ proper Mpc may be plausible if the hydrogen and helium in the near-zone have been highly ionized by a hard spectrum prior to the completion of reionization in the IGM as a whole. This requires the continuous or intermittent emission of a hard spectrum for an extended period of time. If the IGM temperature is not as hot as $40,000$ K, the differences found between modeled and observed near-zone sizes could be explained as at least partially being due to a continuum on the observed spectra which has been placed too low, or perhaps a quasar SED which is different to the one we have assumed.

There are still several issues to address regarding the sizes and redshift evolution of high redshift quasar near-zones. Larger samples will clarify the significance of the evolution observed in the current data and hopefully confirm the rapid evolution in the redshift range $5.8 < z < 6.4$. This scenario success-

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