Nucleon Magnetic Moments in the Quasipotential Quark Model
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Abstract

Calculation of the nucleon magnetic moments was performed in the quasipotential quark model. The main contribution into the nucleon magnetic moments is determined by the $SU(6)$ symmetrical spin-isospin part of the wave function and is equal to 3 and -2 (in the units of nuclear magnetons) for proton and neutron, respectively. Corrections are explained by relative quark motion and are equal to $-4a/3$ and $+a$ for proton and neutron, respectively, where the dimensionless value $a$ depends on the dimensionless ratio of the oscillator parameter and quark mass in case of the oscillator interaction. It is shown that the independence of the nucleon magnetic moments on nucleon mass is a consequence of relativistic kinematics. The invariant dimensionless parameter is introduced to estimate the violation of current conservation. By means of this parameter it is shown that the current leakage is negligibly small in the model.

Introduction.

This paper is dedicated to calculation of nucleon magnetic moments in the relativistic quasipotential quark model [1]. In the nonrelativistic quark model the nucleon magnetic moments are given by: $\mu^p = M/m$, $\mu^n = (-2/3)M/m$ (in units of the nuclear magnetons $e/2M$, $e$ electron charge, $M$ nucleon mass). To obtain the experimental values of the nucleon magnetic moments it is needed to introduce the relation of nucleon and quark mass $M \approx 3m$. In [2] the expressions $\mu^p = (1 - b)M/E_0$, $\mu^n = (-2/3)(1 - b)M/E_0$, containing the ratio of the nucleon quark mass and level of quark energy $E_0$, are obtained on the base of the relativistic shell model of quasiindependent quarks being in the potential hole. The model of quasiindependent quarks solves the confinement problem by means of a big quark mass and big bound energy. But in this case a quark has a big anomalous magnetic moment $(M/m - M/E_0)$ and relation with deep inelastic processes is lost. In the bag quark model [3] it is shown that dirac particle, closed in the bag with radius $R$, has not such big anomalous magnetic moments because of the big quark energy. In this model the quark and proton magnetic moments are proportional to the bag radius ($\mu^p \approx 0.4RM$) and the quark mass may be small. The relativistic approach in paper [4] proposes the assumption of stationarity of the one particle energy in a system of interactive quarks. In this case the nucleon magnetic moments depend on ”effective quark mass” $\varepsilon_q$. In light-front quark model [5, 6] the nucleon magnetic moments are proportional to ratio $M/m$.

The relative quark motion in nucleon is defined by at least two parameters: constituent quark parameter $(m, E_0, \varepsilon_q)$ and energy scale parameter $\gamma$ ($\gamma = 1/R$ in case of the quark bag model). The nucleon mass is an additional parameter. In all models considered above, as in other relativistic models (see for example [4, 5, 6, 7, 8]), the dimensionless nucleon magnetic moments (in units of the nuclear magnetons) are defined by two dimensionless parameters: combination of the model relative motion parameters $(m/\gamma)$ and ratio of the one relative motion parameter and the nucleon mass $(M/m, M/E_0, M/\varepsilon_q)$. This work will show that in the quasipotential model the nucleon magnetic moments don’t depend on the nucleon mass.
1. Basic relations of the model.

In work [1] a structure of the three quark nucleon wave function is considered in the framework of the quasipotential approach. In particular the relation of the three quark component of the nucleon vector state decomposition in the Fock-momentum space was obtained (summary of notations will be done in the end of this section):

\[
\langle \{ p_k \lambda_{sk} \lambda_{tk} \} | K \lambda_J \lambda_T \rangle = (2\pi)^2 \delta^{(3)} \left( \sum_{k=1}^{3} p_k^o \right) \left[ \prod_{k=1}^{3} D_{\lambda_{sk} \lambda_{tk}}^{(1/2)} (L_k^{-1}, p_k) \right] \Phi^{(+)}_{M \lambda_J \lambda_T (\lambda_{sk} \lambda_{tk})} (\{ p_k^o \}).
\]

(1)

If we perform summation of the quark spin by scheme $$[[1+2]+3]$$ and assume that the orbital angular momentum of the system $$[1+2]$$ is equal to zero in $$[1+2]$$ center-of-mass system (c.m.s.) and the orbital angular momentum of the third quark relatively the [1+2] system in the nucleon c.m.s. is equal to zero, then function $$\Phi^{(+)}$$ has the following form:

\[
\Phi^{(+)}_{\{ \lambda_{sk} \lambda_{tk} \}} (\{ p_k^o \}) = (2\pi)^3 \left[ D_{\lambda_{sk} \lambda_{tk}}^{(1/2)} (L_{\bf{p}_{12}}, p_1^o) D_{\lambda_{sk} \lambda_{tk}}^{(1/2)} (L_{\bf{p}_{12}}, p_2^o) \right] \delta_{\lambda_{sk} \lambda_{tk}} \chi_{\{ \lambda_{sk} \lambda_{tk} \}} \varphi_M (\{ p_k^o \}),
\]

(2)

where $$\chi_{\{ \lambda_{sk} \lambda_{tk} \}} = [\xi_{\{ \lambda_{sk} \}} \eta_{\{ \lambda_{tk} \}} + \epsilon_{\{ \lambda_{sk} \}} \eta_{\{ \lambda_{tk} \}}] / \sqrt{2}$$ - SU(6) nucleon wave function is symmetrical relatively the quark permutations [3]. We will omit indices $$\lambda_J$$ and $$\lambda_T$$ anywhere supposing $$\lambda_J = +1/2$$, and we will indicate the proton as index $$p$$ ($$\lambda_T = +1/2$$) or the neutron as index $$n$$ ($$\lambda_T = -1/2$$). Three Wigner rotation matrices $$D$$ in (1) are related with transition from the laboratory system to c.m.s. of nucleon. Two Wigner rotation matricies $$D$$ in expression (2) are related with transition from the rest frame system $$[1+2]$$ to c.m.s. of the nucleon. Function $$\varphi_M$$ is normalized according to the relation:

\[
\int d\Omega_{p_1} d\Omega_{p_2} \left| \varphi_M (M_0 (\{ p_k^o \})) \right|^2 / E_{p_3} = 1.
\]

(3)

In work [1] the relativistic three-particle oscillator was proposed on the basis of the quasipotential equation and its approximate solution was obtained:

\[
\varphi^{osc}_{M} (\{ p_k^o \}) \approx N \exp \left[ -m \sum_{k=1}^{3} E_{p_k}^o - 3m/\gamma^2 \right].
\]

(4)

If the effective mass approximation is better then the approximation in (4) is also better. The effective mass approximation supposes that $$\delta^{(k)}_E \equiv m^{eff}_k - \langle E_{p_k} \rangle$$ is small, where $$m^{eff}_k = \sqrt{m^2 + \langle p_k^2 \rangle}$$, $$\langle p_k^2 \rangle = \int d\Omega_{p_1} d\Omega_{p_2} \langle \varphi_M | p_k^2 | \varphi_M \rangle / E_{p_3}$$, $$\langle E_{p_k} \rangle = \int d\Omega_{p_1} d\Omega_{p_2} \langle \varphi_M | E_{p_k} | \varphi_M \rangle / E_{p_3}$$.

We suppose that quarks have equal masses. Symmetry relatively the quark permutations in normalization condition (3) leads to the following: $$m^{eff}_k, \langle E_{p_k} \rangle, \delta^{(k)}_E$$ don’t depend on quark number $$k$$. This symmetry relatively the quark permutations will be important to obtain the main expression of this paper (22)-(24).

Here we give a list of notations: $$d\Omega_{p_k} = dp_k/E_{p_k}; E_{p_k} = \sqrt{p_k^2 + m^2}; E_K = \sqrt{K^2 + M^2}; K$$ and $$p_k$$ are nucleon and quark momenta, $$M$$ and $$m$$ are nucleon and quark masses, $$U$$ and $$u$$ are nucleon and quark spinors, $$\{ p_k \lambda_{sk} \lambda_{tk} \} \equiv p_1 \lambda_{s1} \lambda_{t1} p_2 \lambda_{s2} \lambda_{t2} p_3 \lambda_{s3} \lambda_{t3}$$. The total spins and isospins of the nucleon and constituent quarks are not indicated and equal to $$1/2$$; $$\lambda_J$$, $$\lambda_{sk}$$ are the third projections of the nucleon and quark spins. We assume a sum over the repeated indices of the third projection of spin and isospin; $$\vec{p}_k = (L_K^{-1} p_k), \quad \vec{p}_k = (L^{-1}_K \vec{p}_k), \quad \vec{p}_{12} = \vec{p}_1 + \vec{p}_2$$.
nucleon and \([1+2]\) quarks. The normalization condition of vector states and spinors of nucleon and quarks are given by:

\[
\langle K'J'\lambda'_jT'\lambda'_T \mid KJ\lambda_jT\lambda_T \rangle = (2\pi)^3\delta^3(K' - K)\delta_{\lambda'_j\lambda_j}\delta_{\lambda'_T\lambda_T},
\]

\[
\langle p'_{k}\lambda'_k\omega'_{k}t_k \lambda'_k \mid p_k\lambda_k\omega_k t_k \lambda_k \rangle = (2\pi)^3E_{p_k}\delta^3(p_k' - p_k)\delta_{\lambda'_k\lambda_k}\delta_{\omega'_k\omega_k},
\]

\[
\bar{u}^{\lambda_k}(p)\gamma^{\lambda_k}(p) = m, \quad \bar{u}^{\lambda_j}(K)U^{\lambda_j}(K) = M/E^{M}_{K} \quad \text{(there is no sum over} \lambda_{sk} \text{and} \lambda_j). \quad (5)
\]

2. Nucleon electromagnetic current

We suppose that in the three quark model the nucleon current is expressed as a sum of quark currents \(j(x) = \sum_{k=1}^{3} j^{(k)}(x)\), and write the current matrix element between nucleon states in the rest frame of the initial nucleon:

\[
J_{\mu}^{\lambda_j\lambda'_j}(K) = \frac{1}{(2\pi)^4} \int \left[ \prod_{k=1}^{3} d\Omega_{p_k} d\Omega_{p'_k} \right] \times
\]

\[
\times \langle E^{M}_{K}K\lambda'_j\lambda'_T \mid j(0) \mid M0\lambda_j\lambda_T \rangle \rangle \langle \{p'_k\lambda'_k\omega'_k t_k \lambda'_k \} \rangle \langle \{p_k\lambda_k\omega_k t_k \lambda_k \} \rangle \mid M0\lambda_j\lambda_T \rangle.
\]

Since the three quark component of the nucleon vector state is symmetrical relatively the quark permutations, it is possible to replace \(\sum_{k=1}^{3} j^{(k)}(0)\) to \(3j^{(3)}(0)\) and it is possible to use the scheme of decomposition of the wave function (WF) over states with the definite angular momentum, assuming \(p_1\) and \(p_2\) as independent variables. Involving WF (1) in (6), we obtain:

\[
J_{\mu}(K, 0) = 3 \int d\Omega_{p_1} d\Omega_{p_2} \varphi(\vec{p}_1, \vec{p}_2, \vec{p}_3) \chi M^{(1)}M^{(2)}\Gamma^{(3)}_{\mu} \varphi(\vec{p}_1, \vec{p}_2, \vec{p}_3)/E_{p_1}E_{p_2}, \quad (7)
\]

where indices \(\lambda_j = \lambda'_j = +1/2\) are omitted and the following matrix notations are used:

\[
M^{(k)}_{\lambda'_k\lambda_k\lambda'_{sk}\lambda_{sk}} \equiv \delta_{\lambda'_k\lambda_k} T^{(1/2)}_{\lambda_{sk}\lambda'_{sk}} \left[ \frac{D^{(1/2)}_{\lambda_{sk}\lambda'_{sk}}(L_{p_1}, \vec{p}_k)}{D^{(1/2)}_{\lambda_{sk}\lambda'_{sk}}(L_{p_1}, \vec{p}_k)} D^{(1/2)}_{\lambda_{sk}\lambda'_{sk}}(L_{p_2}, \vec{p}'_k) \right]_{\lambda'_{sk}\lambda_{sk}}, \quad k = 1, 2;
\]

\[
\Gamma^{(3)}_{\mu, \lambda'_{sk}\lambda_{sk}\lambda'_{ts}\lambda_{ts}} \equiv D^{(1/2)}_{\lambda'_{sk}\lambda_{sk}}(L_{p_3}, \vec{p}'_3) J^{(3)}_{\mu, \lambda'_{sk}\lambda_{sk}}(p'_3, p_3) \delta_{\lambda'_{ts}\lambda_{ts}},
\]

\[
j^{(3)}_{\mu, \lambda'_{sk}\lambda_{sk}}(p'_3, p_3) \delta_{\lambda'_{ts}\lambda_{ts}} \equiv \langle p'_3\lambda'_{sk}\lambda'_{ts}\lambda_{ts} \rangle \mid \langle j^{(3)}(0) \rangle \mid p_3\lambda_{sk}\lambda_{ts} \rangle.
\]

The current normalization condition \(J_0(0, 0) = e_N \quad (e_N \text{ is the electron charge in units of the electron charge} e)\) leads to the WF normalization condition (3) \([1]\), which was obtained by the Green function method. Quark momenta are related by: \(p_3 = -p_1 - p_2, \quad \vec{p}'_3 = -\vec{p}'_1 - \vec{p}'_2, \quad p_1' = p_1, \quad p_2' = p_2, \quad p_k' = L^{-1}_{p_1} p_k, \quad p_{12} = p_1 + p_2\).

Let us consider the nucleon current conservation law. If \(s = 0\), then \(s = s_0 K/M\) for arbitrary 4-momentum \(s\). Assuming \(K = (E_K, K)\), \(K' = (M, 0)\), we have

\[
p_1 + p_2 + p'_3 = K(E_{p_1} + E_{p_2} + E_{p_3})/M, \quad p_1 + p_2 + p_3 = K'(E_{p_1} + E_{p_2} + E_{p_3})/M.
\]

For transfer momentum \(q = K' - K\), we obtain

\[
(E_{p'_1} + E_{p'_2} + E_{p'_3})q_{\mu}/M = (p'_3 - p_3)_{\mu} - (E_{p'_3} - E_{p_3})(1, 0)_{\mu}.
\]
If quark current is conserved, then the last term in (8) represents a deviation from the current conservation law in the model. To estimate the current conservation numerically, we introduce the dimensionless invariant ratio

$$\delta_J = \frac{|q_0 J_0 - q J|}{|q_0 J_0 + q J|} \times 100\%.$$  (9)

According to (8) the current (7) is conserved better if the effective mass approximation is satisfied better (see p.1 in [1]), where quark energies \(E_{\mathbf{p}_2}\) and \(E_{\mathbf{p}_3}'\) of the initial and final nucleons in the nucleon c.m.s. are replaced by effective quark mass \(m^{eff}\). The value \(\delta_J\) allows to estimate a part of the current lost in the model with fixed number of particles. It was obtained that \(\delta_J < 0.1\%\) for \(t \in [0.0001, 2] GeV^2\) (\(t = |q^2|\)) for ratio \(\gamma/m = 0.55\), which corresponds to the best fit of the nucleon magnetic moments. Thus, the current leakage which is related to the assumption of the fixed number of particles is negligibly small.

The Dirac form factors of the nucleon are given by the expression:

$$J^{\lambda_1 \lambda_2}(\mathbf{K}, \mathbf{0}) = e_N \bar{\psi}(\mathbf{K}) \left\{ \gamma_\mu F_1(t) + \frac{iK}{2M} F_2(t) \sigma_{\mu \nu} q_{\nu} \right\} U^{\lambda_1 \lambda_2}(\mathbf{0}),$$  (10)

where \(\kappa\) is the nucleon anomalous magnetic moment, \(\sigma_{\mu \nu} = \frac{i}{2} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)\). For convenience we introduce the polarization 3-vectors \(\varepsilon_{(k)}\), which have components \(\varepsilon_{(k)i} = \delta_{ki}\). For the 3-current we have

$$\left( \varepsilon_{(k)} J^{\lambda_1 \lambda_2}(\mathbf{K}, \mathbf{0}) \right) = \frac{e_N}{\sqrt{2E_K^M (M + E_K^M)}} \left[ (\mathbf{K} \varepsilon_{(k)} G_E(t) + i(\sigma [\mathbf{K} \times \varepsilon_{(k)}]) G_M(t) \right]_{\lambda_1 \lambda_2}.$$  (11)

where the Sacks form factors have the form: \(G_E(t) = F_1(t) + F_2(t) t/4 M^2\), \(G_M(t) = F_1(t) + \kappa F_2(t)\). It has been numerically shown, that the imaginary part of form factors, defined by (11) for current (7) is negligibly small. So, the constructed current approximately satisfies \(T\)-invariance with the sufficient precision.

3. Nucleon magnetic moments.

To write the analitical expression for the magnetic moment, we differentiate (11) over \(\mathbf{K}_1\), assuming \(\mathbf{K} = 0\), \(\lambda'_1 = \lambda_2 = 1/2; k = 2\), \(G_M(0) = \mu_N\) and using (7):

$$\mu_N \frac{e_N}{2M} = -i \frac{d}{d\mathbf{K}_1} I|_{\mathbf{K}=0},$$  (12)

$$I = 3 \int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} \varphi(\mathbf{p}_1, \mathbf{p}_2) \left( \hat{\varphi}_3 G^{(3)}(\mathbf{p}_3) \varphi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) / E_{\mathbf{p}_1} E_{\mathbf{p}_2} \right) \hat{\chi}_3 G^{(3)}(\mathbf{p}_3) \hat{\varphi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) / E_{\mathbf{p}_1} E_{\mathbf{p}_2}.$$

$$\hat{\varphi}_3 G^{(3)}(\mathbf{p}_3) \equiv \varepsilon_{(2)k} \Gamma^{(3)}_{\lambda'_1 \lambda'_2} = D^{(1/2)}_{\lambda'_1 \lambda'_2} \left( L^{-1}_{\lambda'_1} \mathbf{P}_3, \mathbf{P}_3 \right) \right) \left( \varepsilon_{(2)k} \chi^{(3)}_{\lambda'_1} \right) \left( \mathbf{p}_3, \mathbf{p}_3 \right) \right) \left( \mathbf{p}_3, \mathbf{p}_3 \right) \right) \left( \chi M^{(1)} M^{(2)} \hat{\varphi}_3 G^{(3)}(\mathbf{p}_3) \hat{\varphi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) / E_{\mathbf{p}_1} E_{\mathbf{p}_2}.$$

$$\left( \chi M^{(1)} M^{(2)} \hat{\varphi}_3 G^{(3)}(\mathbf{p}_3) \hat{\varphi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) / E_{\mathbf{p}_1} E_{\mathbf{p}_2}.$$

$$\left( \chi M^{(1)} M^{(2)} \hat{\varphi}_3 G^{(3)}(\mathbf{p}_3) \hat{\varphi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) / E_{\mathbf{p}_1} E_{\mathbf{p}_2}.$$

Wave function \(\varphi_M\) is 3-scalar and depends on \(\mathbf{p}_k^2\) and \(\mathbf{p}_k\) \((k, s = 1, 2, 3)\) is the quark number). From such function we have

$$\int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} \varphi(\mathbf{p}_k) |\mathbf{p}_k|^2 \mathbf{p}_k^i \mathbf{p}_k^j \sim \delta_{ij}, \quad \int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} \varphi(\mathbf{p}_k) \nabla^i \mathbf{p}_k \varphi(\mathbf{p}_k) \sim \delta_{ij}. \quad (14)$$
Taking into account (14) it is followed from (12) that

\[ \mu_N \frac{\epsilon_N}{2M} = 3 \int d\Omega_{p_1}d\Omega_{p_2}|\varphi(\{p_k\})|^2(\epsilon \frac{d}{dK_1}(\chi M^{(1)}M^{(2)}e_3G^{(3)}\chi)|_{K=0}. \quad (15) \]

Dependence on \( K \) in (15) under the derivative sign is contained in form of a complicated function in the relative quark momenta in the nucleon c.m.s., in the Wigner rotation matrices and in the quark current. Using the explicit form of \( \hat{p}_k \), \( D \)-matrices and quark current, we will show below, that the right-handed side of equation (15) is proportional to the first order of ratio \( 1/M \). So, \( \mu_N \) in (15) does not depend on the nucleon mass, and it is a consequence of relativistic kinematics.

The quark current with charge operator \( \hat{e}_3 \) has the form: \( j^{(3)}_\mu(p_3', p_3) = \hat{e}_3 \bar{u}(p'_3)\gamma_\mu u(p_3) \).

Let us rewrite it in the 2-spinors:

\[ \langle \epsilon(2)j^{(3)}(p'_3, p_3) \rangle = \frac{\hat{e}_3}{2} \left[ (\sigma(3)\epsilon(2))(\sigma(3)p_3) \sqrt{(E_{p'_3} + m)/(E_{p_3} + m)} + (\sigma(3)p'_3)(\sigma(3)\epsilon(2)) \sqrt{(E_{p_3} + m)/(E_{p'_3} + m)} \right]. \quad (16) \]

The upper index of Pauli matrices \( \sigma^{(k)} \) indicates the quark number. Using the explicit form of Lorentz transformation, we obtain following relations in first order of \( \sigma \) in the first order of \( \epsilon \):

\[ \text{Dependence on } K \text{ in (15) under the derivative sign is contained in form of a complicated function in the relative quark momenta in the nucleon c.m.s., in the Wigner rotation matrices and in the quark current. Using the explicit form of } \hat{p}_k, D \text{-matrices and quark current, we will show below, that the right-handed side of equation (15) is proportional to the first order of ratio } \frac{1}{M}. \text{ So, } \mu_N \text{ in (15) does not depend on the nucleon mass, and it is a consequence of relativistic kinematics.}

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\[ \langle \epsilon(2)j^{(3)}(p'_3, p_3) \rangle = \frac{\hat{e}_3}{2} \left[ (\sigma(3)\epsilon(2))(\sigma(3)p_3) \sqrt{(E_{p'_3} + m)/(E_{p_3} + m)} + (\sigma(3)p'_3)(\sigma(3)\epsilon(2)) \sqrt{(E_{p_3} + m)/(E_{p'_3} + m)} \right]. \quad (16) \]

The upper index of Pauli matrices \( \sigma^{(k)} \) indicates the quark number. Using the explicit form of Lorentz transformation, we obtain following relations in first order of \( K, \): \( p'_3 \approx p_3 + (E_{p_1} + E_{p_2} + E_{p_3})K/M, \ E_{p'_3} \approx E_{p_3} + (Kp_3)(E_{p_1} + E_{p_2} + E_{p_3})/ME_{p_1}. \) As a result in the first order of \( K \) assuming \( K = (K, 0, 0) \), we obtain the following from (16):

\[ \langle \epsilon(2)j^{(3)}(p'_3, p_3) \rangle = i \frac{\hat{e}_3}{2M}(\sigma(3)[K \times \epsilon(2)])(E_{p_1} + E_{p_2} + E_{p_3}) - \frac{1}{\sqrt{2M}}(\sigma(3)p_3 \times \epsilon(2)))(Kp_3)(E_{p_1} + E_{p_2} + E_{p_3}) + \hat{e}_3(\epsilon(2)p_3) + O(K^2). \quad (17) \]

From the terms of expression (17) we have three contributions into the nucleon magnetic moment:

\[ \mu_N = \mu_1 + \mu_2 + \mu_3. \quad (18) \]

According to (17) the first term of (18) contains \( (\sigma(3)[K \times \epsilon(2)]) \) and corresponds to the quark motion as a part of nucleon with momentum \( K \). The second term contains \( (\sigma(3)p_3 \times \epsilon(2)) \) and takes into account the relative quark motion with momentum \( p_3 \). The third term, as it is shown below, takes into account the contribution of the relativistic Wigner rotation matrices of the quark spin.

The first term in (18) has the form:

\[ \mu_1 = 3(\hat{e}_3\sigma_3^{(3)}\chi) \int d\Omega_{p_1}d\Omega_{p_2}|\varphi(p_1, p_2, p_3)|^2(E_{p_1} + E_{p_2} + E_{p_3})/E_{p_3}^2. \quad (19) \]

Charge coefficients \( (\eta^{MS}, \hat{e}_3\eta^{MS}) \) and \( (\eta^{MA}, \hat{e}_3\eta^{MA}) \) are equal to 0 and 2/3, respectively, for proton and 1/3 and -1/3 for neutron; spin coefficients are as follows: \( (\xi^{MA}\sigma_3^{(3)}\xi^{MA}) = 1, (\xi^{MS}\sigma_3^{(3)}\xi^{MS}) = 1/3 \). Isospin-spin coefficients in (19) are: \( (\chi\hat{e}_3\sigma_3^{(3)}\chi)^p = 1/3, (\chi\hat{e}_3\sigma_3^{(3)}\chi)^n = -2/9 \).

Using (14), from (17) and (15) for \( \mu_2 \) we have:

\[ \mu_2 = -\frac{1}{3}(\hat{e}_3\sigma_3^{(3)}\chi) \int d\Omega_{p_1}d\Omega_{p_2}|\varphi(p_1, p_2, p_3)|^2(E_{p_1} + E_{p_2} + E_{p_3})(E_{p_3} - m)/E_{p_3}^3. \quad (20) \]

The term \( \mu_3 \) is represented as a sum of two parts \( \mu_3 = \mu_3^{(1)} + \mu_3^{(2)} \), where \( \mu_3^{(1)} \) originates from differentiation of Wigner rotation matrices \( D^{(1/2)}(L_{K}^{-1}, p_k) \) \( (k = 1, 2, 3) \), \( \mu_3^{(2)} \)
parameters of the oscillator-like model (4). For average values \( a_0 \) in the constructed quasipotential model the main contribution in the nucleon magnetic moments. Indeed, we perform differentiation under the first component of \( \mathbf{K} \) momentum, but the third term in (17) contains the second component of \( \mathbf{p}_3 \)-momentum. This leads to expressions, which are proportional to \( \delta_{12} = 0 \), if to apply (14). Taking into account that \( \mathbf{p}_1 = - \mathbf{p}_2 \), it is possible to show that \( \mu_3^{(2)} = 0 \). So, we have \( \mu_3 = \mu_3^{(1)} \).

\[
\mu_3 = - \int d\Omega_{\mathbf{p}_1}d\Omega_{\mathbf{p}_2}|\varphi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)|^2 \frac{1}{E_{\mathbf{p}_3}} \sum_{k=1}^{3} \left\{ \frac{\chi \bar{\epsilon}_3 \sigma_3^{(k)} \chi}{E_{\mathbf{p}_k} + m_1} \right\}. \tag{21}
\]

For proton and neutron we have used charge coefficients listed above. Spin coefficients \( (\xi^M \sigma_3^{(k)} \xi^M) \) for \( k = 1, 2 \) and 3 are equal to 0, 0 and 1, respectively, and \( (\xi^M S \sigma_3^{(k)} \xi^M) \) for \( k = 1, 2 \) and 3 are equal to \( 2/3, 2/3 \) and \(-1/3\). So, in (21) isospin-spin coefficients \( (\chi \bar{\epsilon}_3 \sigma_3^{(k)} \chi) \) are equal to 0, 0 and 1/3 for proton and 1/9, 1/9 and \(-2/9\) for neutron.

Using decomposition of the intergrand function in neighborhood of the effective mass and permutation symmetry of (4) in expressions (19)- (21), it is possible to change \( E_{\mathbf{p}_k} \) to \( E_{\mathbf{p}_3} \). Taking into account the wave normalization condition (3), we obtain a simple result, which approximates the expressions (18)- (21):

\[
\mu_p \approx 3 - 4a/3, \quad \mu_n \approx -2 + a; \tag{22}
\]
\[
\mu_1^p \approx 3, \quad \mu_1^n \approx -2; \quad \mu_2^p \approx -a, \quad \mu_2^n \approx -2a/3; \quad -\mu_3^p \approx \mu_3^n \approx a/3, \tag{23}
\]
\[
a = \int d\Omega_{\mathbf{p}_1}d\Omega_{\mathbf{p}_2}|\varphi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)|^2(E_{\mathbf{p}_3} - m)/E_{\mathbf{p}_3}^2 \tag{24}
\]

In the oscillator-like model (4) with \( \gamma/m = 0.55 \), corresponding to the best fit of the nucleon magnetic moments, the deviation of the approximate value from the exact value is less than 0.1% for \( \mu_1 \), less than 0.08% for \( \mu_2 \) and less than 0.003% for \( \mu_3 \) (percent is relative \( \mu_N \)).

Using the experimental data for the nucleon magnetic moments, we obtain \( a_p = 0.155 (\gamma/m = 0.64), a_n = 0.087 (\gamma/m = 0.45) \). In brackets we have listed corresponding parameters of the oscillator-like model (4). For average \( a = (a_p + a_n)/2 = 0.121 (\gamma/m = 0.55) \) we have:

\[
\mu_p = 2.854, 2.839, (2.793),
\]
\[
\mu_n = -1.890, -1.879, (-1.913). \tag{25}
\]

The first two values in the strings are exact (expressions (18)- (21)) and approximate values ( (22) for \( a = 0.121 \)), the experimental data are given in brackets.

**Conclusion.**

The nucleon electromagnetic current in the quasipotential model with the fixed number of particles is approximately conserved in the physically interesting region of the oscillator parameters.

The main result of the work is contained in formulae for the nucleon magnetic moments: exact (18)- (21) and approximate expressions (22)- (24). The expressions (22) show, that in the constructed quasipotential model the main contribution in the nucleon
magnetic moment is defined by the $SU(6)$ part of the nucleon wave function. The corrections have the right sign for proton and neutron, respectively. In the constructed model with point quarks without anomalous magnetic moments the nucleon magnetic moments are consistent with the experimental data with the error of 2%.

The independence of the nucleon magnetic moments on the nucleon mass means that using the experimental data it is possible to determine only ratio $\gamma/m$ (for the oscillator-like model). As a result the mass of the constituent quark is a free parameter. This conclusion is a consequence of relativistic kinematics and is not related with the property of the quasipotential model.

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