Uplink Scheduling Strategy Based on A Population Game in Vehicular Sensor Networks

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Abstract—Recent advances in the integration of vehicular sensor network (VSN) technology, and crowd sensing leveraging pervasive sensors called onboard units (OBUs), like smartphones and radio frequency IDentifications to provide sensing services, have attracted increasing attention from both industry and academy. Nowadays, existing vehicular sensing applications lack good mechanisms to improve the maximum achievable throughput and minimizing service time of participating sensing OBUs in vehicular sensor networks. To fill these gaps, in this paper, first, we introduce real imperfect link states to the calculation of Markov chains. Second, we incorporate the result of different link states for multiple types of vehicles with the calculations of uplink throughput and service time. Third, in order to accurately calculate the service time of an OBU, we introduce the steady state probability to calculate the exact time of a duration for back-off decrement, rather than using the traditional relative probability. Additionally, to our best knowledge, we first explore a multichannel scheduling strategy of uplink data access in a single roadside unit (RSU) by using a non-cooperative game in a RSU coverage region to maximize the uplink throughput and minimize service time under saturated and unsaturated traffic loads. To this end, we conduct a theoretical analysis and find the equilibrium point of the scheduling. The numerical results show that the solution of the equilibrium points are consistent with optimization problems.

I. INTRODUCTION

Vehicular Sensor Networks connected to the Internet backbone or various other application servers is emerging as a new network paradigm for sensed information sharing in urban environments [1]. With the advent of 4G networks and more powerful processors, smartphones have received a lot of attention for their potential as portable vehicular urban sensing platforms, as they are equipped with a variety of environment and motion sensors (e.g., audio/video, accelerometer, and GPS) and multiple wireless interfaces (e.g., WiFi, Bluetooth and 2/3G). The ability to take a smartphone on board to complement the sensors of the latter with advanced smartphone capabilities is of immense interest to the industry [2]. Recent these advances make it possible for most of OBUs on the roadway to a wide range of applications.

Thus, in vehicular networks, V2R communications are the preferred way for OBUs on the roadway to a wide range of applications.

Recently, many researchers has focused on performance analysis of V2R communications, since these RSUs like 802.11(WiFi) can provide data transfers of broadband speeds for OBUs in its coverage region including highly mobile users traveling in cars [4]. Currently, several works have experimentally validated the feasibility of using 802.11(WiFi) based RSUs communications at vehicular speeds.

As is well known, in static single-hop wireless LAN, the network performance under saturation and non-saturated assumption has been greatly enhanced [5], [6], [7], [8], [9], [10], [11]. However, in the high speed vehicle networks, this mobility greatly increases the collisions between simultaneous transmissions of vehicles contending for the access to the same RSU, resulting in significant deterioration of the performance of V2R communications. Therefore, how to assure the reliability and QoS of safety and non-safety services in this type of V2R communications is a common concern. To this end, some researchers start to focus on IEEE 802.11 DCF scheme in V2R communications [12], [3], [13], [14], [15]. From these existing works, we know that most of them focus on a very sparse OBUs in the coverage region of a single RSU. However, in the high-speed mobile dense scenario, V2R communications utilizing 802.11 DCF scheme face the following new challenges as follows:

- The inaccuracy in modeling the back-off process about 802.11 DCF scheme is one of the main reasons for deterioration of the communication performance in V2R communications. If we are able to highly accurately calculate the back-off time by some models, the data transfer can be reasonably scheduled, thereby, the performance in V2R communications will be greatly enhanced.
- Real traffic flows are abrupt and do not generate true saturation in V2R communications. OBUs with the transient empty queue are also counted in the scheduling algorithm. This is an urgent need for an adaptive algorithm to solve this real unsaturated flow impact on the performance.
- This single-channel data transfers are difficult to adapt to the growing demands in V2R communications.
With the emergence of low-cost 802.11-based WiFi devices and the advances of multichannel wireless technology, it is reasonable to expect to enjoy data transfers at broadband speeds by connecting to multichannel wireless devices [4], [16], [17], [18]. Thus, each RSU can provide the Internet access to dense vehicles simultaneously and efficiently.

In addition, to meet demands in V2R communications, the U.S Federal Communication Commission (FCC) allocates 75MHz of spectrum ranges between 5.850 to 5.925 GHz band for VSN. Further, the 75MHz spectrum ranges are partitioned into seven non-overlapping channels by category, one for safety application control channel (CCH) and the others for service channel (SCH) providing non-safety data transmissions such as Internet services and video on demand run on. Further, the authors of [16] provides guidelines for the design of an efficient MAC for single cells employing MIDU nodes. More importantly, it scales very easily to MIMO systems and provides large self-interference cancelation no matter transmission or reception is performed simultaneously, thereby make the co-existence of MIMO with full duplex possible. Recent works [17], [18], have implemented Multi-User MIMO schemes, in which an RSU can communicate with a number of OBUs simultaneously by utilizing the antennas that belong to a group of OBUs.

Therefore, in this paper, we mainly focus on situations where the overall loads generated by high density vehicles are too heavy, i.e., packet collisions are too many, thus many traffic applications cannot be supported satisfactorily. To avoid such problems, we need to provide an alternative mechanism which is able to guarantee maximizing their individual throughput and minimizing the service time. Specifically speaking, our main results and contributions are summarized as follows:

- To our best knowledge, we are the first attempt to explore a multichannel scheduling strategy of uplink data access in a single RSU by using a non-cooperative game in V2R communications to maximize the uplink throughput and minimize service time under saturated and unsaturated traffic loads. To this end, we conduct a theoretical analysis to find the balance point of the scheduling. The numerical results show that the solution of the equilibrium points are consistent with optimization problems. Finally, Section VII presents concluding remarks.

- In order to accurately calculate the service time of an OBU, we introduce the steady state probability to calculate the exact time of a duration for back-off decrement, rather than using the traditional relative probability. Further, we explore saturated and unsaturated traffic loads to accurately estimate the MAC-layer uplink throughput and service time by a calculation of back-off freezing probability for an arbitrary buffer size under multichannel conditions.

- We first introduce real imperfect link states to the calculation of our vehicle model for Markov chains. Eventually, the whole system throughput and the service time are accurately calculated.

- We incorporate the result of different link states for multiple types of vehicles with the calculations of uplink throughput and service time under multichannel conditions in a dense traffic scenario.

The rest of the paper is organized as follows. Section II briefly discusses the related work. In Section III, we present our system model and related definitions. We briefly discuss concepts of finite non-cooperative games and population games. In Section IV, we first analyze the the actual link state for V2R communications, and then present the mathematical development of a single type of vehicles model and multiple types of vehicles model. Second, we incorporate the actual link state and multiple types of vehicles model with IEEE 802.11p Markov chain based on different regions. Additionally, we develop the expressions of throughput and service time by using the conflict probability. Section IV we form a non-cooperative game problem, and then we conduct a theoretical analysis, and find the balance point of the scheduling. In Section V we study the dynamics of the system in a non-cooperative scenario. The idea here is to show that the system is stable using Lyapunov techniques. We next study the efficiency of such an equilibrium and show that the Wardrop equilibrium is efficient. In Section VI we make a case study. The numerical results show that the solution of the equilibrium points are consistent with optimization problems. Finally, Section VII presents concluding remarks.

II. BACKGROUND AND RELATED WORK

The popular IEEE 802.11 wireless LAN using a CSMA/CA mechanism called the Distributed Coordination Function (DCF) is studied extensively in the literature. The authors of [5] focus on obtaining the system throughput and average long term metrics such as saturation throughput by using a bi-dimensional discrete-time Markov-chain model, while two important features specified by IEEE 802.11b standard, which are retransmission limits and back-off counter freezing, are not taken into account. The authors of [6] used a renewal theory to develop a fixed-point formulation relating the per-station attempt rate with the collision probability of a packet under saturation.

 Compared with the traditional decoupling saturation assumption, non-saturated models differ in approach and scope, but they are all in some way derived from a saturated fixed-point formulation. The authors of [9] modified a saturated fixed-point formulation to overcome these difficulties under non-saturated conditions. The authors in [9] analyzed a three-dimensional extension of the Bianchi Markov chain that explicitly tracked the buffer state of a station, as well as the number of other stations with a nonempty buffer. The authors in [9], [19] dispense with the decoupling assumption for the collision probability. The authors in [10] use a three-way fixed point to model the node behavior with Bernoulli packet arrivals and determine closed form expressions for the distribution of the time spent between two successful transmissions in an isolated network. The authors of [8] present an accurate non-saturated model on the saturated renewal process of [6] and extend the buffer size to an arbitrary value for the non-saturated attempt rate. However, they are only confined to using the relative probability to calculate the freezing time of a duration for back-off decrement, which leads to inaccurate calculation of the frame service time. The authors of [7] apply...
the idea of the steady probability to calculate transmission probability for unsaturated traffic cases, while they do not take the exact calculation of a duration for back-off decrement into account.

In V2R communications, the authors of [12] are the first to introduce the traditional decoupling saturation assumption to analyze the maximum achievable throughput when multiple vehicles simultaneously share the bandwidth of the same in a given mobility scenario. The authors of [13] are the first to model time division between CCH and SCH with multiple traffic combinations/classes in the non-saturation regime under a single channel, assuming that an OBU’s buffer has infinite capacity to model time division between CCH and SCH with multiple locations and environmental dynamism.

In fact, the wireless link states can indeed vary with their conditions and environmental dynamism. Not taking the impact of link-state sending rate into account. Assuming that there are N link states, corresponding to the saturated or not. These OBUs are deployed on a road segment. Access under multichannel conditions whether the nodes are different types of vehicles (cars, trucks, buses, etc.) can occur in a single RSU coverage region. All vehicles have the same speed mobile model and different speed parameters.

A. System Model

In this work we take different type OBUs in the coverage region for a single RSU into account, which operate uplink data access under multichannel conditions whether the nodes are saturated or not. These OBUs are deployed on a road segment. Assuming that there are N links states, corresponding to the N non-overlapping regions by thresholds \( \Gamma_f (f \in \{1, \cdots , N\}) \). In each region \( f \) within the RSU coverage region, OBUs have different link qualities resulting in different transmission rates according to the signal-to-noise ratio (SNR) at the receiver to RSU. Fig. 1 shows our system model in detail. For analytical convenience, we make the following approximations:

- Synchronization delays for the OBUs and the RSU can not occur. Time is slotted with slot length and the back-off process will be completed in the vicinity of a single RSU. Different channels are orthogonal and non-interfering.
- Different types of vehicles (cars, trucks, buses, etc.) can occur in a single RSU coverage region. All vehicles have the same speed model and different speed parameters.

B. A Population Game Theory

A population game with \( C \) continuous populations is defined by a mass and a strategy set for each population class and a payoff function for each strategy, where the set of population classes \( C = \{1, \cdots , C\} \), each of which corresponds to the same type of OBUs with the same channel conditions \( \{L_i, \epsilon_1^i, \cdots , \epsilon_f^i\} \), where these OBUs can choose a channel from the same channel set \( \{\epsilon_1^i, \cdots , \epsilon_f^i\} \), and the set of strategies corresponds to the set of \( L \) independent channels which are orthogonal and so do not interfere with each other, \( \mathcal{S} = \{1, \cdots , L\} \). Strategies of these (population \( i \)) OBUs lead to a strategy distribution \( \mathcal{X}_i = \{x_i \in R_L^c : \sum_{j \in S} x_{ij} = n_i\} \).

The \( n_i \) denotes the number of OBUs belonging to the i-th type. As such, the overall strategy distributions is denoted as \( \mathcal{X} = \{x = (x_1, \cdots , x_C) : x_i \in \mathcal{X}_i\} \).

**Definition 1 (Potential Game).** A potential game holds: There exists a \( C \times 1 \) potential function of the game \( f : \mathcal{X} \to R \) such that \( \frac{\partial f}{\partial x_c}(x) = F_c^i(x) \) for all \( x \in \mathcal{X} \), \( l \in \mathcal{S} \), and \( c \in C \) [20], where \( f \) is a continuously differentiable function which is unique up to an additive constant, and \( F \) is the payoff vector equaling \( f \)’s gradient.

**Definition 2 (Nash Equilibrium).** A Nash equilibrium is a state whose support consists solely of best responses to itself. At a Nash equilibrium, no OBU can unilaterally improve his payoffs.

**Definition 3 (Wardrop Equilibrium).** A state \( \hat{x} \) is a Wardrop equilibrium if \( S \subset \mathcal{S} \), \( F_c^i(\hat{x}) \geq F_c^j(\hat{x}), \forall l \in \mathcal{S} \) and \( l' \in \mathcal{S} \)

**Definition 4 (Positive Correlation).** The dynamics \( \dot{x} = V(x) \) is called positive correlation (PC) if \( V(x) \cdot F(x) = \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{S}} V^i_j(x)F^i_j(x) > 0, V(x) \neq 0 \) [20] [27].

**Lemma 1.** If \( F \) is a potential game and \( V \) satisfies PC, then the potential function of \( F \) is a global Lyapunov function and all Wardrop equilibria of \( F \) are the stationary points for \( \dot{x} = V(x) \).

**Lemma 2.** A potential game \( F \), with dynamics \( V(x) \) that are PC, has asymptotically stable stationary points [27].

According to lemma 1 and lemma 2 the dynamics \( \dot{x} = V(x) \) would converge to either a Wardrop equilibrium or a boundary point of the set \( B \).

**Definition 5 (Brown-von Neumann-Nash Dynamics).** The Brownvon NeumannCNash (BNN) dynamics is defined [22] as

\[ \dot{x}_c = V(x) = n_ck_c^i - x_c^l \sum_{c' \in \mathcal{S}} k_{c'} \]

where \( k_c = \max \{F_c^i(x) - \frac{1}{n_c} \sum_{j \in \mathcal{S}} x_{ji}F_j^i(x), 0\} \).

**Lemma 3.** The system with BNN dynamics satisfies PC. The complete proof that BNN dynamics are PC is present in [20].

As such, if each class follows BNN dynamics, the system is PC according to lemma 2. The detailed proof can be seen in [21].

IV. ANALYTICAL MODEL

In this section, we formalize present and develop relative results used in Section VII.

A. Link State Model

The wireless links between a RSU and an OBU is modeled as a finite-state Markov chain (FSMC). In a region \( f \), we assume that \( \mathcal{C}^c_f \) is the diagonal probability matrix corresponding to the transmission of \( c \) packets from a RSU...
to an OBU at location $L_t$ of the region $f$. The elements of this matrix are denoted as $C^f_t(f, f)$, which is defined as the probability that $c_f$ packets are successfully transmitted when the channel changes from state $f$ in the current transmission period to state $f$ in the following transmission period, i.e., the OBU is in the region $f$, can be given by \cite{23} as follows:

$$
\hat{C}^f_t(f, f) \begin{cases} 
\sum_{c \in C^f} \binom{c_f}{c} (P_s)^c (1 - P_s)^{c_f - c} & u \geq 1 \\
0 & \text{otherwise}
\end{cases}
$$

where $u \in \{0, 1, \cdots, R_{max}\}$ is the amount of rate demand for real wireless transmission and $P_s$ is defined in Section \ref{V-B}. $R_{max}$ is the maximum rate from the RSU in the region $f$.

From \cite{24, 23, 25}, the average packet transfer rate from an OBU at region $f$ to the RSU, denoted by $C_t$, can be obtained as

$$
C_t = \sum_{f=1}^{F} C_f(\alpha_t C^f_t 1)
$$

where $\alpha_t$ denotes the steady-state probability that the channel is in state $f$. Our highway mobility model is illustrated in Fig. \ref{Fig.1} where a single V2R system with one coverage region shown as a disk.

**Definition 6 (The probability of an OBU).** Let $D$ be the distance between a RSU and an OBU, and let $P_f$ be the probability of an OBU in link data rate $C_f$. According to Figure \ref{Fig.7} $P_f$ is

$$
P_f = \begin{cases} 
P(r_f < D \leq r_f), & \text{for } f = 1, 2, \cdots, N - 1 \\
P(0 < D \leq r_f), & \text{for } f = N
\end{cases}
$$

where $r_f$ is depicted in Figure \ref{Fig.7}

**Lemma 4 (The cumulative distribution function).** The cumulative distribution function of an OBU moving to the position $(x, 0)$ with length $d$ is

$$
F_X(x) = P(X \leq x) = \begin{cases} 
\frac{2x^2}{d^2} (\ln d - \ln x) + \frac{x^2}{d^2}, & 0 < x \leq d \\
0, & x \leq 0 \\
1, & x > d
\end{cases}
$$

The detailed proof of lemma \ref{Lemma 4} can be seen in \cite{15}. From \cite{4} and \cite{5}, we obtain the probability of an OBU as follows:

$$
P_f(f \neq N) = \frac{(1 + 2n \ln d)(s_f^2 - s_{f+1}^2) - 2s_f^2 \ln s_f - 2s_{f+1}^2 \ln s_{f+1} + (1 + 2n \ln d)(s_{2N-f}^2 - s_{2N-f+1}^2) - 2s_{2N-f}^2 \ln s_{2N-f} - 2s_{2N-f+1}^2 \ln s_{2N-f+1}}{d^2}
$$

$$
P_f(f = N) = \frac{(1 + 2n \ln d)(s_f^2 - s_{N+1}^2) - 2s_f^2 \ln s_f - 2s_{N+1}^2 \ln s_{N+1}}{d^2}
$$

Let $l_f$ denote variable frame size in the region $f$, which includes payload, MAC and physical layer header. According to \cite{6} and \cite{7}, the PGF for frame size within the transmission range of the RSU is:

$$
S_f(z) = \sum_{f=1}^{N} \frac{l_f + \text{ACK}}{z^{r_{sifs} + sifs}}
$$

where sifs and ack is denoted as duration of the SIFS and ACK period in slots, respectively. Additionally, $P_f$ is the average transmission rate obtained by eq \cite{3} and $P_f$ can be obtained from the expression \cite{6} and \cite{7}. In a RTS/CTS model, the PGF for the collision period is given by $C_f(z) = z^{r_{sifs} + sifs}$.

**B. Distribution of Vehicles**

We now assume that $n$ OBUs move at a speed $V$ on a straight line highway segment with length $d$. There are $n_i$ OBUs for type $i$ in the coverage area of the RSU ($i = 1, 2, \cdots, C$). Let $X_{n_i}$ be the distance between the $n$-th and the $(n+1)$-th vehicle of type $i$ that entered the APs coverage area.

In Table \ref{Table I} we summarize the various quantities and notations we will use throughout the paper.

**TABLE I: SUMMARY OF NOTATIONS**

| Variable         | Description                                           |
|------------------|-------------------------------------------------------|
| $\lambda^i$      | The average number of type $i$ vehicles                |
| $x_{m}^i$        | A minimum inter-vehicle distance constraint           |
| $q$              | The average number of vehicles per unit time           |
| $S(t)$           | The current back-off phase of the tagged node         |
| $B(t)$           | The back-off time of the tagged node                  |
| $s_{m}$          | The minimum inter-OBU distance                        |
| $\omega$         | The maximum number of OBUs                            |
| $d$              | The maximum length of road segment                    |
| $R_{jam}$        | The road capacity/length and $R_{jam} = d/\omega$    |
| $T_o$            | The transmission overhead in slots                    |
| $T_c$            | The RTS collision overhead in slots                   |
| $T_s$            | The payload transmission duration in slots            |

**Definition 7 (Renewal Process).** Let $X_{n_i}$ and $N^i(d)$ represent the distance between the $n$-th and the $(n+1)$-th OBU of
type $i$, and the number of vehicles of type $i$ over the length $d$ meters i.e. the RSU coverage region, respectively. If the sequence of nonnegative random variables $\{X_1^i, X_2^i, \cdots\}$ is independent and identically distributed, then the counting process $\{N^i(d), d \geq 0\}$ is said to be a renewal process.

From Definition 7 in vehicular traffic stream models, $N^i(d)$ is a renewal process. Let $D_n^i = \sum_{k=0}^n X_n^i$ denote the distance of the $n$th renewal, $N^i(d)$ may be written as $N^i(d) = \max\{n : D_n^i \leq d\}$ [26]. We obtain

$$P\{N^i(d) = n\} = F_n^i(d) - F_{n+1}^i(d) \tag{9}$$

where $F_n^i$ the $n$-fold convolution of $F^i$ with itself. It is well known that the cumulative distribution function of $X_n^i$, the detailed $F^i$ can be seen in [27]. $F_n^i$ from [28], [12] may be calculated as

$$F_n^i(d) = 1 - \sum_{j=0}^n \frac{(\lambda^i)^j}{j!} (d - nx^i_m)^n e^{-\lambda^i (d - nx^i_m)} \quad \tag{10}$$

Let $\pi^i(n)$ denote the probability having $n$ vehicles of type $i$ under the RSU coverage region. According to [9] and (10), when $d = L$, we have

$$\pi^i(n) = P\{N^i(d) = n\} = F_n^i(d) - F_{n+1}^i(d)$$

$$= \sum_{k=0}^n \frac{(\lambda^i)^k}{k!} (\omega^i - n - 1)x^i_m k e^{-\lambda^i (\omega^i - n - 1)x^i_m}$$

$$- \sum_{k=0}^{n-1} \frac{(\lambda^i)^k}{k!} (\omega^i - n)x^i_m k e^{-\lambda^i (\omega^i - n)x^i_m}$$

$$n < \omega^i - 1 \quad \tag{11}$$

$$\pi^i(n) = 1 - \sum_{k=0}^{n-1} \frac{(\lambda^i)^k}{k!} (x^i_m)^k e^{-\lambda^i x^i_m}, n = \omega^i - 1 \quad \tag{12}$$

$$\pi^i(n) = 0, n \geq \omega^i \quad \tag{13}$$

Furthermore, we define $\Gamma_n$ as $\Gamma_n = \{n : \sum_{i=1}^n n_i = n, 0 \leq n_i \leq \omega^i\}$, where $\omega_i$ denotes the maximum number for OBU\s with type $i$ in a single RSU coverage region and satisfies $d = \omega_i s^i_m$. Additionally, $s^i_m$, $\omega_i$ and $d$ are defined in Table [I]. Thus, from [12], the probability that there are $n$ generic OBU\s in a single RSU coverage region is calculated as

$$\tilde{\pi}_n = \sum_{n \in \Gamma_n} \pi(n) = \prod_{i=1}^n \pi^i(n_i).$$

C. Model of IEEE 802.11p Features

Let us assume that all OBU\s are identical, and analyzing the behavior of one node make it enough to predict the behavior of the other nodes and the channel performance. We denote this node as the tagged node. Like [7], the size of contention window at backoff stage $j$, $W_j$, is defined as

$$W_j = \begin{cases} 
2^j CW_{\text{min}} & 0 \leq j \leq m \\
2^m CW_{\text{min}} & m < j < M 
\end{cases} \tag{14}$$

where $CW_{\text{min}}$ denote the the minimum contention window size of nodes and $m = \log_2(CW_{\text{max}})$. $CW_{\text{max}}$ is the maximum contention window size.

D. Collision Probability

In this section, we introduce the fixed-point equation detailed in [5], [6], which controls the collision probability under saturation and non-saturation regimes. Let $\beta^c$ denote the average attempt rate when the buffer is not empty. The general attempt rate can be calculated by $\beta = (1 - p_0) \beta^c$, where $p_0$ denote the probability that the buffer is empty and computed in [7]. By using the results in [6], $\beta^c$ can be given by:

$$\beta^c(\gamma) = \frac{m_{i=0}^m \gamma^i}{\sum_{i=0}^m i \gamma^i} \quad \tag{15}$$

Thus, by substituting $\beta^c$ and $p_0$ into the above expression of $\beta$, the general collision probability $\gamma$ can be solved by the following fixed-point equation $\gamma = \Gamma(\beta)$,

$$\gamma = \Gamma((1 - p_0) \beta^c) \quad \tag{16}$$

E. Calculation for The Packet Service Time

Assuming $T^e$ to be the service time distribution (in slots) of a packet of a tagged node on the condition that the buffer is not empty. Let $\chi$ be a random variable representing the time (in slots) that elapses for one decrement of the back-off counter.

$$T_{bf} = \sum_{i=1}^\chi \sum_{j=0}^b \chi$$

where $T = \sum_{j=0}^b$, with the probability that the packet transmission finishes at the $k$th back-off stage, $p(\gamma, k), 0 \leq k \leq M - 1$, and $p(\gamma, k)$ is given by

$$p(\gamma, k) = \begin{cases} 
(1 - \gamma)^k & k = 0, \ldots, m \\
\gamma^{m+1} & k = m + 1 
\end{cases} \quad \tag{18}$$

The generic slot duration $\chi$ depends on whether a slot is idle or interrupted by a successful transmission or a collision. We define $\chi$ as

$$\chi = \begin{cases} 
\sigma, & w.p. P_i \\
T^i + T_o + \sigma, & w.p. P_s \\
T^i + \sigma, & w.p. P_C 
\end{cases} \quad \tag{19}$$

where $P_i$, $P_s$ and $P_C$ denote the steady state probabilities of the channel being in idle, successful or collision state, respectively, and “w.p.” means “with the probability”.

Furthermore, we can introduce Channel State Markov Chain (CSMC) defined in [17] to calculate the transition probabilities $p_{es}, p_{es}, p_{es}, p_{is}, p_{ps}, p_{cs}, p_{sc}$, and $p_{cc}$. Thus, $P_i$, $P_s$ and $P_C$ are calculated as follows:

$$\begin{pmatrix} 
p_{es} & p_{es} & p_{cc} \\
p_{is} & p_{ps} & 0 \\
p_{cs} & p_{sc} & p_{cc} 
\end{pmatrix} \begin{pmatrix} 
P_i \\
P_s \\
P_C 
\end{pmatrix} = \begin{pmatrix} 
P_i \\
P_s \\
P_C 
\end{pmatrix}$$

Let $(b_j)$, $\chi$, $T$, and $T^{ser}$ denote the generating function of $b_j$, $\chi$, $T$ and $T^{ser}$ respectively, we have
\[(b_j)_g(z) = \begin{cases} \frac{1 - z^{CW_{j}}}{CW_w(1-z)}, & j = 0, \ldots, m \\ \frac{1 - z^{CW_m}}{CW_m(1-z)}, & j = m + 1, \ldots, M - 1 \end{cases} \] (21)

\[
\chi_g(z) = P_z z^\sigma + P_z T_s + T_o + P_c z^{T_c + \sigma},
\]

(22)

\[
T_g(z) = \sum_{k=0}^{M-1} [p(\gamma, k) \prod_{j=0}^{k} (b_j)_g(z)]
\]

(23)

\[
T_g^{\text{sm}}(z) = \sum_{f=1}^{N} P_f T_g(\chi_g(z))
\]

(24)

Assume \(S_k(T_s) = kT_s\), and then \(T_s, T_c, \text{and} T_o\) are defined in Table II. In the basic access (BA) case, the service time on the overall service requiring \(k\) attempts, is given by:

if \(0 \leq k \leq m\),

\[
T(k) = (k+1)T_o + T_s + S_k(T_s) + \sum_{k=0}^{m} [p(\gamma, k) \prod_{j=0}^{k} (b_j)_g(z)]
\]

if \(k = m + 1\),

\[
T(m+1) = (m+1)T_o + S_m(T_s) + \sum_{k=0}^{m} [p(\gamma, k) \prod_{j=0}^{k} (b_j)_g(z)]
\]

In the RTS/CTS case, \(T(k)\) and \(T(m+1)\) is obtained by:

if \(0 \leq k \leq m\),

\[
T(k) = kT_c + T_o + T_s + \sum_{k=0}^{m} [p(\gamma, k) \prod_{j=0}^{k} (b_j)_g(z)]
\]

if \(k = m + 1\),

\[
T(m+1) = (m+1)T_c + \sum_{k=0}^{m} [p(\gamma, k) \prod_{j=0}^{k} (b_j)_g(z)]
\]

From eq (22), (23) and (24), the Laplace transforms of the service time pdf, \(L_T(s)\), in the BA and RTS/CTS cases are, respectively, given by

\[
L_T(s) = \sum_{k=0}^{m} [1 - \gamma] \gamma^k e^{-s(k+1)T_o} \sum_{i=1}^{M} q_i e^{-s\varphi_k(s)} g_i^k(s)
\]

\[
+ \gamma^{m+1} e^{-s(m+1)T_c} \varphi_m(s) g_i^{m+1}(s)
\]

and

\[
L_T(s) = \sum_{k=0}^{m} [1 - \gamma] \gamma^k e^{-sT(k)} + \gamma^{m+1} e^{-sT(m+1)}
\]

\[
= \sum_{i=1}^{M} q_i e^{-s\varphi_i} \sum_{k=0}^{m} (1 - \gamma) \gamma^k e^{-s(T_o + kT_c)} \varphi_k(s)
\]

\[
+ \gamma^{m+1} e^{-s(m+1)T_c} \varphi_m(s)
\]

where \(g_i(s) = \sum_{j=1}^{M} e^{-s \max(n_j, a_j)} (\psi_j - \psi_{j-1})\), \(\psi_j = (1 - \tau + \tau Q_j)^{n_j-1}/(1 - (1 - \tau)^n)\) for \(j = 1, \ldots, M\) and \(\varphi_k(s) = \prod_{i=0}^{k} 1 - E[e^{-s\varphi_i}].\) By derivation of the above expression, the first order moment of \(T, E[T]_{s=0} = (1 - \gamma^{m+1})E[U]/\theta(n)\) (\(\theta(n)\) is given in the following section.)\(^{[29, 30]}\)

According to eq (23), the total average service time is obtained by:

\[
T_{ser} = \sum_{f=1}^{N} P_f (1 - \gamma^{m+1})n(1 + [1 - (1 - \beta)^n]T_c)
\]

\[
+ n(\beta(1 - \beta)^{n-1})(T_o - T_c) + n\beta(1 - \beta)^{n-1}(\sum_{i=1}^{N} 1/\lambda_i)\]

(27)

F. Throughput in The Same Channel

Let us now consider a simpler situation where all OBUs are the transmitter for a single flow and all packet lengths are equal to \(L\). The network throughput of the tagged OBU is given from \([6]\) at the top of the next page.

V. A MULTICHLANLPOPULATION GAME

A. Problem Formulation

As the above mentioned, in this paper the focus of our consideration is how to maximize the upload throughput and minimize service time in V2R communications. To this end, we introduce a previously mentioned population game \([20]\). Let \(n_c\) denote the number of OBUs belonging to class \(c\), we have \(n = \sum_{c=1}^{C} n_c\) and \(n^* = \sum_{c=1}^{C} n^*_c\) to denote the sum of all OBU’s demands and the number of a single channel’s active OBUs respectively. From eq (23), the throughput received by the total mass of users of class \(c\) connected to channel \(j\) is

\[
\theta_j^c(x^j) = \frac{x^j_c L_c}{k_0^j + n^j [\sum_{f=1}^{N} P_f \sum_{c=1}^{C} x^j_c L_c/C_f]} \]

\[
k_j^0 = \frac{1}{\beta (1 - \beta) n^{j-1}} + n^j(T_o - T_c)
\]

\[
+ \frac{1}{\beta (1 - \beta) n^{j-1} + (1 - \beta)} n^j T_c
\]

Then, we can obtain the throughput and service time per unit mass respectively,

\[
e_j^c(x^j) = \frac{L_c}{k_0^j + \sum_{f=1}^{N} P_f \sum_{c=1}^{C} x^j_c L_c/C_f}
\]

\[
T_{ser}^j = (1 - \gamma^{m+1})(k_0^j \beta^j (1 - \beta^j)^{n_j-1} + n^j \beta^j (1 - \beta^j)^{n_j-1} \sum_{f=1}^{N} P_f \sum_{c=1}^{C} x^j_c L_c/C_f]
\]

\[
T_{ser}^j = k_1^j + k_2^j \sum_{f=1}^{N} P_f \sum_{c=1}^{C} x^j_c (L_c/C_f)
\]

where \(k_1^j = (1 - (\gamma^{m+1})k_0^j \beta^j (1 - \beta^j)^{n_j-1} \text{ and } k_2^j = (1 - \gamma^{m+1})n^j \beta^j (1 - \beta^j)^{n_j-1}.\)
\[ \theta(n) = \frac{L_i}{\beta(n-\beta)} + n(T_n - T_c) + \frac{1}{\beta(n-\beta)} + (1 - \frac{1}{\beta})[T_c + n[\sum_{i=1}^{n} \frac{1}{\beta(n-\beta)}(\sum_{j=1}^{L_i} p_{t,c} x_i)] (28) \]

Thus, our problem is described in the following expression.

\[ \max_{x} \left( \sum_{i=1}^{\omega} \frac{n}{1 - \pi_0} \sum_{j=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_i^j e_i^j(x) \right. \]

\[ \left. - \sum_{i=1}^{\omega} \frac{n}{1 - \pi_0} \sum_{j=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_i^j T_{ser}(x_i^j) \right) \]

subject to \( \sum_{j=1}^{L} x_i^j = n_i, \forall i \in \{1, \ldots, C\}; x_i^j \geq 0 \) and \( x_i^j = 0 \) if channel \( j \) is not provided to OBUs of class \( i \).

To obtain the optimal solution satisfying the expression, let us see the following Lemma 5. In order to facilitate the description, we let \( \Phi = \sum_{i=1}^{\omega} \frac{n}{1 - \pi_0} \). The calculations of the following variables, are given in Section IV-B.

**Lemma 5.** A game \( F \) potential function can be given by

\[ \Theta(x) = \Phi \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_i^j e_i^j(x) \]

\[ - \Phi \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_i^j T_{ser}(x_i^j) \]

with \( x_i^j = 0 \) and \( \zeta \) is a weight that provides influence to service time versus the throughput for the tagged OBU, if channel \( j \) is not available to \( i \) class OBUs.

**Proof:** The proof of the lemma can be found in Appendix. Obviously, the above results to meet Definition 1. Thus, The function \( \Theta(x) \) is a potential function for the game \( F \).

From the above proof, we obtain the payoff function per unit mass for OBUs of class \( c \) in channel \( l \), \( F_c^l = \sum_{i=1}^{\omega} \frac{n}{1 - \pi_0} (\pi^c(n_c) e_c^l(x) - \psi_c^l(x)) \sum_{i=1}^{C} \pi^i(n_i) \theta_i^l(x) - k_i^l \frac{L_c}{n^l C^l} \sum_{i=1}^{1} \pi_i(n_i) x_i^l - \zeta_c \pi_c(n_c) T_{ser}^l). \)

**Lemma 6.** \( x^* = (x^{(1)}^*, \ldots, x^{(C^*)}) \) is called a Wardrop equilibrium if the payoff function per unit mass for OBUs of class \( c \) in channel \( l \) of the non-cooperative game \( F \) is given by

\[ F_c^l(x) = \sum_{i=1}^{\omega} \frac{n}{1 - \pi_0} \sum_{j=1}^{L} x_i^j e_i^j(x) - \sum_{i=1}^{C} \pi^i(n_i) \theta_i^l(x) + k_i^l \frac{L_c}{n^l C^l} \sum_{i=1}^{1} \pi_i(n_i) x_i^l - \zeta_c \pi_c(n_c) T_{ser}^l), \]

where for each \( c \) we have

\[ (x_i^j)^* \geq 0, \forall j, c \sum_{j=1}^{L} (x_i^j)^* = n_c, \forall c \]

**Proof:** From Definition 3, it holds evidently.

Further, it is obvious that all obtained vectors \( x \) have equal payoffs if eq (35) take the zero value in all channels \( l \). As such, a Wardrop equilibrium is obtained. Take the previous mentioned BNN dynamics into account, we have the following Lemma.

**Lemma 7.** The potential game \( F \) equilibrium satisfies the equation (33).

**Proof:** For Definition 5, we know BNN dynamics and its PC 21. Thus, \( F_c^l(x) = \frac{1}{n^l} \sum_{j=1}^{L} x_i^j e_i^j(x) \) or \( x_i^j = 0 \). Then we can construct an optimization problem as follows:

\[ \min_{x} \max_{\eta} \sum_{i=1}^{\omega} \frac{n}{1 - \pi_0} \sum_{j=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_i^j e_i^j(x) \]

\[ - \sum_{i=1}^{\omega} \frac{n}{1 - \pi_0} \sum_{j=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_i^j T_{ser}(x_i^j) \]

the solution of \( \eta_i \) satisfies \( \eta_i = F_i(x) = \frac{1}{\pi_i} \sum_{j=1}^{L} x_i^j e_i^j(x) \). Therefore, the solution of the potential game \( F \) satisfies the expression (36). We denote it as \( \Theta(x^*) \). Since the expression (33) is not concave, the solutions of the expression (36) and (33) are equal.

From Lemma 5, 6 we can know that selfish OBUs can attain the maximum throughput and the minimum service time by selecting one of those channels hold \( F_c^l(x) = 0 \). Lemma 7 shows that our solution satisfying a Wardrop equilibrium condition also is the optimal solution.

**B. A Game Among the OBUs and RSU**

We now define the price vector as \( p = \langle p_1, \ldots, p_L \rangle \), which is the price the RSU provides for different type OBUs. Contrary to 21, our policy is oriented for different type OBUs, instead of different channels. The reason that our policy is reasonable is that high-speed moving type OBUs should be different from low-speed OBUs like walkers.

The RSU expects to provide the bandwidth for the types with the maximum price, so its gain function \( \Psi = \sum_{j=1}^{L} \sum_{c=1}^{C} \pi^c(n_i) x_i^j \) \( p_\beta \). However, the OBUs want to give the minimum price to meet their demands. Thus, a game exists among the OBUs and RSU. Consequently, they converge to the equilibrium of the system and the corresponding to potential function is defined as follows:
Lemma 8. A game \( F \) potential function can be given by
\[
\Theta(x) = \Phi \sum_{i=1}^{C} \pi^n_i(n_i) \sum_{j=1}^{L} x^i_j e^i_j(x) - \Phi \sum_{i=1}^{C} \sum_{j=1}^{L} \zeta^i \pi^n_i(n_i) x^i_j T_{ser}(x^i_j) - \Phi \sum_{j=1}^{L} \sum_{i=1}^{C} \zeta^i \pi^n_i(n_i) x^i_j p_i
\]

(37)

with \( x^i_j = 0 \) if channel \( j \) is not available to \( i \) class OBUs.

Proof: Certification process is similar to that in Lemma 5.

Further, taking the previous mentioned BNN dynamics into account, we have the following Lemma.

Lemma 9. The potential game \( F \) equilibrium can be calculated by the following equation
\[
\max_x (\Phi \sum_{i=1}^{C} \pi^n_i(n_i) \sum_{j=1}^{L} x^i_j e^i_j(x) - \Phi \sum_{i=1}^{C} \sum_{j=1}^{L} \zeta^i \pi^n_i(n_i) x^i_j T_{ser}(x^i_j) - \Phi \sum_{j=1}^{L} \sum_{i=1}^{C} \zeta^i \pi^n_i(n_i) x^i_j p_i)
\]

subject to
\[
\sum_{j=1}^{L} x^i_j = n_i, \forall i \in \{1, \cdots, C\}; x^i_{1j} \geq 0
\]

and \( x^i_j = 0 \) if channel \( j \) is not provided to OBUs of class \( i \).

Proof: Certification process is similar to that in Lemma 7.

From Lemma 8 and 9, the RSU can attain the optimal performance by selecting the price vector that makes the maximum throughput and minimum service time.

VI. CASE STUDY AND SIMULATIONS

In this section, we test our channel scheduling performance for our uplink data access and validate successfully with the NS-2 simulator. In our scenario, there are three wireless channels, i.e. 802.11a, 802.11b and 802.11g, and a highway model of length 1.2Km, with two lanes. all OBUs compete for the channel applying IEEE 802.11 DCF. The values for the parameters of vehicular velocity and delay are illustrated in Table II.

| Table II | THE VELOCITY AND DEADLINE PARAMETERS OF OBUS |
|-----------|------------------------------------------------|
| Variable  | Value  |
| \( v_{\text{max}} \) (m/s) | 35 |
| \( v_{\text{min}} \) (m/s) | 10 |
| an OBU deadline belonging to type 1 (ms) | 0.2 |
| an OBU deadline belonging to type 2 (ms) | 0.35 |

According to the parameters from Table II, firstly, we use VanetMobiSim simulator to produce a TCL script about vehicular mobility. Secondly we add a fixed RSU to the TCL script. Thirdly, we explore the vehicle density impact on throughput and the number of data transferred respectively by using the modified script as the input of NS2. Finally, according to experimental results, we make analysis about our optimal policy.

In Fig. 2 we fix the density of OBU type 2 as \( \lambda^2 = 0.03 \) and assume the density of OBU type 1 \( \lambda^1(\lambda^2 = 0.03) \) changes from 0 to 2\( \rho \), to which is the results of \( \lambda^2 \) are similar. This is the result of a vehicle simulation scenarios construction, and further research is beyond the scope of our discussion. Fig. 3 shows our model predictions are accurate.

Fig. 2 shows our model predictions are accurate. In particular, we have observed that there is a deviation in the
OBUs versus time in fixed throughput than [13]. We also observed that under the three density of OBUs, our Wardrop equilibrium policy has better steady state than the algorithm in [13].

In Fig. 4, we test the throughput of OBUs versus time in fixed $\lambda^2 = 0.03$, $\lambda^1 = 0.05$, $\lambda^1 = 0.1$ and $\lambda^1 = 0.2$. (a) $\lambda^1 = 0.05(\lambda^2 = 0.03)$; (b) $\lambda^1 = 0.1(\lambda^2 = 0.03)$; (c) $\lambda^2 = 0.2(\lambda^2 = 0.03)$.

In Fig. 5, we test the service time of data packets versus time in $\lambda^2 = 0.03$, $\lambda^1 = 0.05$, $\lambda^1 = 0.1$ and $\lambda^1 = 0.2$. (a) $\lambda^1 = 0.05(\lambda^2 = 0.03)$; (b) $\lambda^1 = 0.1(\lambda^2 = 0.03)$; (c) $\lambda^1 = 0.2(\lambda^2 = 0.03)$.

throughput performance, it is because an OBU through the RSU coverage region needs some during to reach the steady state of a backoff process. It is such a transition time caused the overall throughput performance deviation. The experiments of both groups have demonstrated the network throughput and service time calculated from the expressions are very similar to the simulated environment results based on the case for the distribution of Fig. 3. Further, our optimization policy based on throughput performance and service time are compared with [13].

In Fig. 4 we test the throughput of OBUs versus time in fixed $\lambda^2 = 0.03$, $\lambda^1 = 0.05$, $\lambda^1 = 0.1$ and $\lambda^1 = 0.2$ respectively. Experimental results show that under the three density of OBUs, our Wardrop equilibrium policy has better steady performance than the algorithm in [13]. We also observed that as the vehicle density increases, our policy has greater total throughput than [13].

In Fig. 5 we test the service time of data packets from OBUs versus time in fixed $\lambda^2 = 0.03$, $\lambda^1 = 0.05$, $\lambda^1 = 0.1$ and $\lambda^1 = 0.2$ respectively. Experimental results show that under the three density of OBUs, our Wardrop equilibrium policy makes the service time of the data packets to attain an earlier steady state than the algorithm in [13]. We also observed that as the vehicle density increases, the service time of the data packets has a slighter increase. Even in the increase case, our policy also can ensure inclusive service to basically meet delay requirements of different type OBUs.

VII. CONCLUSIONS AND FUTURE WORK

With the development of wireless technology and popularity of roadside multi-channel WiFi devices, more and more passengers in the vehicle expect the enjoyment of high-bandwidth data transmissions from the multi-channel wireless devices. So such high data throughput and low latency scheduling problem for car passengers are common concerns of industrial and academic fields. This paper presents our system model and related definitions, and performs a brief discussion of the non-cooperative games and population game. Then, we analyze the actual link state communication model of OBUs and a RSU. A single OBU type in a mathematical model is extended to multiple types for OBUs. Further, we will combine the actually link status and a Markov multi-type model based on different regions. The throughput and service time expressions are further developed by applying the collision probability. Finally, we have formed a non-cooperative game problem. Theoretically we proved that the solution of the balance point and the optimization problem is the same. Further simulations also show that the solution of the equilibrium point meets the requirements of the maximum throughput and service time.

What’s more, in future work, in order to protect the car users to enjoy multi-hop scenario, high-bandwidth data transmission, we will further study the timeliness issues of data transmission and scheduling in a multi-hop scene.
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APPENDIX

PROOF OF LEMMA 5

A game $F$ potential function can be given by

$$\Theta(x) = \Phi \sum_{i=1}^{C} \pi^i \left( n_i \right) \sum_{j=1}^{L} x_i^j e_j^i(x)$$

$$- \Phi \sum_{j=1}^{C} \zeta \sum_{i=1}^{C} \pi^i \left( n_i \right) \sum_{j=1}^{L} x_i^j T_{ser}(x_j^i)$$

with $x_i^j = 0$ and $\zeta$ is a weight that provides influence to service time versus the throughput for the tagged OBU, if channel $j$ is not available to $i$ class OBUs.

Proof: From eq (30) and (32), we obtain

$$\frac{\partial \Theta(x)}{\partial x_i^j} = \frac{\partial}{\partial x_i^j} \left[ \Phi \sum_{i=1}^{C} \pi^i \left( n_i \right) \sum_{j=1}^{L} x_i^j e_j^i(x) \right]$$

$$- \frac{\partial}{\partial x_i^j} \left[ \Phi \sum_{j=1}^{C} \zeta \sum_{i=1}^{C} \pi^i \left( n_i \right) \sum_{j=1}^{L} x_i^j T_{ser}(x_j^i) \right]$$
where

\[
\frac{\partial}{\partial x_c} \left[ \Phi \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j L_i \right] = \frac{\partial}{\partial x_c} \left[ \Phi \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j L_i \right] \]

\[
= \Phi \left[ \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j L_i \right] - \frac{\sum_{j=1}^{L} x_c^j L_i}{k_0 + \sum_{j=1}^{L} \frac{x_c^j}{C_{j,i}}} \]

\[
= \Phi \left[ \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j L_i \right] - \frac{\sum_{j=1}^{L} x_c^j L_i}{k_0 + \sum_{j=1}^{L} \frac{x_c^j}{C_{j,i}}} \]

\[
= \Phi \left[ \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j L_i \right] - \frac{\sum_{j=1}^{L} x_c^j L_i}{k_0 + \sum_{j=1}^{L} \frac{x_c^j}{C_{j,i}}} \]

where this occupancy factor of per unit mass is \( \psi_c^l(x) = \frac{\sum_{j=1}^{L} x_c^j L_i}{k_0 + \sum_{j=1}^{L} \frac{x_c^j}{C_{j,i}}} \). In fact, if we let \( C^l_c(x) = \psi_c^l(x) \sum_{i=1}^{C} \pi_i(n_i) \theta^i_c(x) \) in the above expression. From the RSUs aspect, \( C^l_c(x) \) denotes the cost of a unit mass of OBUs of class \( l \). Further,

\[
\frac{\partial}{\partial x_c} \left[ \Phi \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j T_{ser}(x_c^j) \right] = \frac{\partial}{\partial x_c} \left[ \Phi \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j T_{ser}(x_c^j) \right]
\]

\[
= \Phi \left[ \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j T_{ser}(x_c^j) \right] - \frac{\sum_{j=1}^{L} x_c^j T_{ser}(x_c^j)}{k_0 + \sum_{j=1}^{L} \frac{x_c^j}{C_{j,i}}} \]

\[
= \Phi \left[ \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j T_{ser}(x_c^j) \right] - \frac{\sum_{j=1}^{L} x_c^j T_{ser}(x_c^j)}{k_0 + \sum_{j=1}^{L} \frac{x_c^j}{C_{j,i}}} \]

\[
= \Phi \left[ \sum_{i=1}^{C} \pi^i(n_i) \sum_{j=1}^{L} x_c^j T_{ser}(x_c^j) \right] - \frac{\sum_{j=1}^{L} x_c^j T_{ser}(x_c^j)}{k_0 + \sum_{j=1}^{L} \frac{x_c^j}{C_{j,i}}} \]

Therefore,

\[
\frac{\partial \Theta(x)}{\partial x_c} = \Phi(x)^c(n_c)^c(x_l) - \psi_c^l(x) \sum_{i=1}^{C} \pi^i(n_i) x_c^i T_{ser} \]

\[
= \sum_{i=1}^{C} \pi^i(n_i) x_c^i L_i - k_2 \frac{L_c}{n^l C_l} \sum_{i=1}^{C} \pi_c(n_c) x_c^i \]

\[
= \sum_{i=1}^{C} \pi^i(n_i) x_c^i L_i - k_2 \frac{L_c}{n^l C_l} \sum_{i=1}^{C} \pi_c(n_c) x_c^i \]

\[
= \sum_{i=1}^{C} \pi^i(n_i) x_c^i L_i - k_2 \frac{L_c}{n^l C_l} \sum_{i=1}^{C} \pi_c(n_c) x_c^i \]

\[
= \sum_{i=1}^{C} \pi^i(n_i) x_c^i L_i - k_2 \frac{L_c}{n^l C_l} \sum_{i=1}^{C} \pi_c(n_c) x_c^i \]

\[
= \sum_{i=1}^{C} \pi^i(n_i) x_c^i L_i - k_2 \frac{L_c}{n^l C_l} \sum_{i=1}^{C} \pi_c(n_c) x_c^i \]

\[
= \sum_{i=1}^{C} \pi^i(n_i) x_c^i L_i - k_2 \frac{L_c}{n^l C_l} \sum_{i=1}^{C} \pi_c(n_c) x_c^i \]

Obviously, the above results to meet Definition [1]. Thus, The function \( \Theta(x) \) is a potential function for the game \( F_c \).