Abstract—In many cases, such as trajectories clustering and classification, we often divide a trajectory into segments as preprocessing. In this paper, we propose a trajectory semantic segmentation method based on learned behavior models. In the proposed method, we learn some behavior models from video sequences. Next, using learned behavior models and a hidden Markov model, we segment a trajectory into semantic segments. Comparing with the Ramer-Douglas-Peucker algorithm, we show the effectiveness of the proposed method.

1. Introduction

Analyzing behavior and trajectories of pedestrians captured by video cameras are one of important topics in computer vision, which has been widely studied over the decades. In such studies, segmentation of trajectories often is performed for reducing computation cost and extracting local information. There are three typical approaches [1]:

- Temporal segmentation: splitting a trajectory at points where two observed locations are temporally away from each other.
- Shape-based segmentation: splitting at points of larger curvature indicating that the target may change its direction at that point. This is used for simplifying the shape of trajectories, and the Ramer-Douglas-Peucker algorithm [2], [3] is a famous approach.
- Semantic segmentation: dividing a whole trajectory into semantically meaningful segments, and many methods have been proposed for different tasks [4], [5], [6], [7], [8].

In this paper we focus on the third type, semantic segmentation, of trajectories based on models of human behavior (or agents). It would be very beneficial if we could have segments related to behavior, however no segmentation methods of trajectories have been proposed for the task of human behavior analysis. Our proposed method first estimates agent models by Mixture model of Dynamic pedestrian Agents (MDA) [9], then segment trajectories with the learned agent models by using Hidden Markov Models (HMM) [10], [11].

2. Related work

The Ramer-Douglas-Peucker algorithm [2], [3] is often used for trajectory simplification. It segments a trajectory while preserving important points in order to keep the trajectory shape as much as possible. First the start and end points of a trajectory are kept. Next it finds the most far point away from the line between two kept points, and keep it if the distance is larger than threshold $\epsilon$. This process iterates recursively until no further points are kept. Finally, all of the kept points are used to segment the trajectory. This method is simple and preserve the rough shape of the trajectory, while an appropriate value of $\epsilon$ has to be specified.

Task oriented methods are also proposed. Yuan et al. [4] propose a system called T-Finder, which recommends to taxi drivers places where as many potential customers exist as possible, and to end users places where taxis are expected to be find. To this end, they estimate taxi locations based on taxi driving trajectories and segments the trajectories as preprocessing. Lee et al. [5] proposes TRAOD, an algorithm for finding outliers in trajectories based on segmentation by using the Minimum Description Length (MDL) principle. Zheng et al. estimates Transportation Mode [6], [7], [8] such as walk, car, bus, and bike used for semantic segmentation in terms of a mode of transportation.

In contrast, our proposed method uses semantic human behavior models, called agent models, learned from pedestrian trajectories in videos.

3. Mixture model of Dynamic pedestrian Agents

In this section, we briefly describe Mixture model of Dynamic pedestrian Agents (MDA) [9] proposed by Zhou et al. for learning behavior models, or agents. MDA is a hierarchical Bayesian model that represents pedestrian trajectories by a mixture model of Dynamic pedestrian Agents (MDA) [9], then segment trajectories with the learned agent models by using Hidden Markov Models (HMM) [10], [11].

3.1. Formulation

Let $y_t \in \mathbb{R}^2$ be two-dimensional coordinates of a pedestrian at time $t$, and $x_t \in \mathbb{R}^2$ be the corresponding
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That is, belief is represented as a two-dimensional scene. Belief describes the starting point and to where it is going. The state of the following linear dynamic system:

\[ x_t \sim P(x_t | x_{t-1}) = N(x_t | Ax_{t-1} + b, Q) \]  \hspace{1cm} (1)

\[ y_t \sim P(y_t | x_t) = N(y_t | x_t, R), \]  \hspace{1cm} (2)

where \( N(\cdot) \) represents a normal distribution with covariance matrices \( Q, R \in \mathbb{R}^{2 \times 2} \), and \( A \in \mathbb{R}^{2 \times 2} \) is state transition matrix and \( b \in \mathbb{R}^2 \) is translation vector, assuming the state transition be a similar transformation. In this paper we use explicitly use the translation vector for similar transformation, while Zhou et al. \([9]\) used homogeneous coordinates for their formulation.

MDA represents pedestrian trajectories by a mixture of dynamics \( D \) and belief \( B \). Here, dynamics \( D = (A, b, Q, R) \) describes dynamics of human movement in the two-dimensional scene. Belief \( B \) describes the starting point \( x_s \) and end point \( x_e \) of the trajectory, each represented by normal distributions as follows:

\[ x_s \sim p(x_s) = N(x_s | \mu_s, \Phi_s) \]  \hspace{1cm} (3)

\[ x_e \sim p(x_e) = N(x_e | \mu_e, \Phi_e). \]  \hspace{1cm} (4)

That is, belief is represented as \( B = (\mu_s, \Phi_s, \mu_e, \Phi_e) \), describing where it starts and to where it is going. The mixture weights are written as \( \pi_m = p(z = m) \) where hidden variable \( z \) represents that the trajectory is generated by agent \( m \).

Furthermore, observation \( y = \{y_1, y_2, \ldots, y_\tau\} \) of length \( \tau \) is assumed not to start and end at the exact start and end points \( x_s \) and \( x_e \) of the agent, that is, states of the trajectory exists before and after the observed points of the trajectory:

\[ x = \{x_s = x_{-t_s}, x_{-t_s+1}, \ldots, x_0, \]  \hspace{1cm} (5)

\[ x_1, x_2, \ldots, x_\tau, \]  \hspace{1cm}

\[ x_{\tau+1}, \ldots, x_{\tau+t_e} = x_e \}. \]

Hereafter, \( x_{1:T} \) denotes the sequence of states \( x \) except \( x_s \) and \( x_e \). Figure 1 shows the state space model of MDA.

### 3.2. Learning

Given \( K \) trajectories \( Y = \{y^k\} \) MDA estimates \( M \) agents \( \Theta = \{(D_m, B_m, \pi_m)\} \) by maximizing the following log likelihood:

\[ L = \sum_k \log p(y^k | x^k, z^k, t^k_s, t^k_e, \Theta) \]  \hspace{1cm} (6)

This can be rewritten by replacing hidden variables \( Z = \{z^k\}, T = \{t^k_s, t^k_e\} \) with \( H = \{Z, T\}, h^k = \{z^k, t^k_s, t^k_e\} \) as follows:

\[ L = \sum_k \log p(y^k | x^k, h^k, \Theta) \]  \hspace{1cm} (7)

The EM algorithm estimates iteratively as \( H \) is not observed.

#### 3.2.1. E step.

\[ Q(\Theta, \hat{\Theta}) = E_{X,H|Y,\hat{\Theta}}[L] \]  \hspace{1cm} (8)

\[ = E_{H|X,Y,\hat{\Theta}}[E_{X|Y,H,\hat{\Theta}}[L]] \]  \hspace{1cm} (9)

\[ = \sum_k \sum_{h^k} \gamma_k E_{x^k|y^k, h^k}[L] \]  \hspace{1cm} (10)

Hereafter, \( E_{x^k|y^k, h^k}[x^k] = \hat{x}^k \) is denoted as \( x^k \), which is computed by the modified Kalman filter \([12]\).

Weights are given as follows:

\[ \gamma_k = \frac{p(h^k | \hat{x}^k_{1:T}, \hat{\Theta}) p(y^k | x^k, h^k, \hat{x}^k_{1:T}, \hat{\Theta})}{p(\hat{x}^k_{1:T}, \hat{\Theta})} \frac{p(\hat{x}^k_s, \hat{x}^k_e)}{p(\hat{x}^k_s, \hat{x}^k_e)} \]  \hspace{1cm} (11)

Note that we assume the conditional independence between \( y^k \) and \( x^k_s, \hat{x}^k_e \), and between \( x^k_s, \hat{x}^k_e \) and \( h^k, x^k_{1:T} \).

By further assuming the independence among hidden variables \( z, t_s, t_e \) and the conditional independence between \( x_s^k \) and \( \Theta \), we have

\[ p(h^k | \hat{x}^k_s, \hat{\Theta}) = p(h^k) = p(z^k, t^k_s, t^k_e) = p(z^k) p(t^k_s) p(t^k_e) \]  \hspace{1cm} (12)

By removing \( t_s, t_e \) by assuming those be uniform, we have

\[ \gamma_k = \frac{p(z^k) p(y^k | x^k, \hat{x}^k_{1:T}, \hat{\Theta}) p(\hat{x}^k_s)}{\sum_{h^k} p(z^k) p(y^k | h^k, \hat{x}^k_{1:T}, \hat{\Theta}) p(\hat{x}^k_s) p(\hat{x}^k_e)} \]  \hspace{1cm} (13)

where \( p(y^k | h^k, \hat{x}^k_{1:T}, \hat{\Theta}) \) is computed by the modified Kalman filter \([12]\) considering unobserved states \( \{x_{-t_s}, x_{-t_{s+1}}, \ldots, x_0, x_{t+1}, \ldots, x_{t+t_e}\} \).

#### 3.2.2. M step.

Next we find \( \hat{\Theta} = \arg \max_\Theta Q(\Theta, \hat{\Theta}) \) by solving a system of equations obtained by differentiating \( Q \) with respect to \( \Theta \), resulting in the following analytical solutions:

#### 3.2.3. Implementation.

In summary, the EM algorithm iterates the following two steps:

1) for each trajectory \( y^k \), for all \( h^k = (z^k, t^k_s, t^k_e) \) the modified Kalman filter is applied to estimate \( \hat{x}^k \) and \( \gamma_k \).

2) Update \( \Theta \).
3.3. HMM and Switching Kalman Filter

A Hidden Markov Model (HMM) shown in Figure 2 has discrete latent variables \( z \). By using the Baum-Welch algorithm [10], HMM learns parameters from training data, then infers unobserved states \( Z = \{ z_n \}_{n=1}^N \) from observations \( X = \{ x_n \}_{n=1}^N \) using the Viterbi algorithm [11].

A possible extension of MDA to segmentation is shown in Figure 3 where \( z_n \) is assigned to each state \( x_n \) by using HMM, where \( x_n \) is a state of Kalman filter, and \( z_n \) is a hidden variable indicating which agent the observation is generated. Both \( x_n \) and \( z_n \) are dependent on previous variables \( x_{n-1} \) and \( z_{n-1} \) as in Fig. 5 which is known as Switching Kalman Filter [13], [14].

Switching Kalman Filter [13], [14] is a dynamic system model having parameters that depend on hidden variables. State \( x_n \) and observation \( y_n \) at time \( n \) is given by

\[
x_n \sim P(x_n | x_{n-1}) = N(x_n | A_n x_{n-1}, Q_n)
\]

\[
y_n \sim P(y_n | x_n) = N(y_n | C_n x_n, R_n)
\]

where \( A_n = A[z_n] C_n = C[z_n] Q_n = Q[z_n] R_n = R[z_n] \) are parameters that are switched by the value of hidden variable \( z_n \).

Switching Kalman Filter is a useful model, however, needs state transition probabilities to be given [14], therefore is not applicable to the task presented here. We instead propose to separate MDA agent estimation from HMM inference to make the whole procedure to work.

4. Proposed method

Figure 4 shows the overview of the proposed method. First we learn multiple agent models of trajectories from videos by using MDA. Then we segment trajectories by using HMM based on the learned agents.

4.1. Agent estimation by MDA

Let \( M \) agents be \( D_m = (A_m, b_m, Q_m, R_m) \), \( B_m = (\mu_{s,m}, \Phi_{s,m}, \mu_{e,m}, \Phi_{e,m}) \), and \( \pi_m \). Then all agents are denoted as

\[
\Theta = \{ (D_m, B_m, \pi_m) \}_{m=1}^M = \{ \omega_m \}_{m=1}^M
\]

and these are estimated as shown section 3.

4.2. HMM parameter estimation

Figure 5 shows the model of the proposed method. Agents transit from one another according to state transition matrix \( A \), and state \( X \) is generated based on output probability matrix \( B \). We use the Baum-Welch algorithm [10] to estimate initial probability distributions \( \rho \) of learned \( M \) agents, as well as matrices \( A \) and \( B \).

We assume that an agent may switch to other agent at each step, and observation \( Y_t \in R^k \) of one step consists of successive three coordinates \( y_{t1}, y_{t2}, y_{t3} \in R^2 \) in a trajectory, as shown in Fig. 5. State \( X_t \) is considered to be generated by an agent specified \( z_t \) associated to step \( t \). Hence a trajectory is represented by hidden variables \( Z = \{ z_t \}_{t=1}^T \), states \( X = \{ X_t \}_{t=1}^T \), and observations \( Y = \{ Y_t \}_{t=1}^T \).

Here let \( \rho \) be an \( M \)-dimensional vector whose \( m \)-th element represents the initial distribution \( \rho_m \) of agent \( \omega_m \), and \( A \) by an \( M \times M \) matrix whose \( (i,j) \) element is transition probability \( a(i,j) \) from agent \( \omega_i \) to agent \( \omega_j \). Output distribution of agent \( \omega_m \) is assumed to be normal \( N(\mu_m, \Sigma_m) \), and let \( B \) a vector whose \( m \)-th element is output probability of agent \( \omega_m \). Also we denote probability that agent \( \omega_j \) outputs state \( X_t \) by

\[
b(j, t) \sim p(X_t | \omega_j) = N(\mu_j, \Sigma_j)
\]

Denoting HMM parameters \( \Theta, A, B \) to be estimated by \( \Theta = (\rho, A, B) \), we maximize the following log likelihood to estimate \( \Theta \) given \( K \) trajectories;

\[
Q(\Theta, \Theta^{old}) = \sum_k \sum_Z p(Z | X, \Theta^{old}) \ln p(X, Z; \Theta)
\]

by using the EM algorithm.

4.3. Semantic segmentation

A trajectory is segmented by applying Viterbi algorithm [11] with the learned HMM parameters \( \Theta \), that is, sequences of hidden variables \( Z^* \) and agents \( \Omega^* \);

\[
Z^* = \{ i_1, i_2, \cdots, i_n \}
\]

\[
\Omega^* = \{ \omega_{i_1}, \omega_{i_2}, \cdots, \omega_{i_n} \}
\]
5. Experiments

We compare the proposed method, denoted by MDA+HMM in the following, with the Ramer-Douglas-Peucker (RDP) algorithm \cite{2,3} in terms of segmentation accuracy. Trajectories in the Pedestrian Walking Path Dataset \cite{15} are used for this experiments. This dataset has a large number of pedestrian trajectories in videos of size 1920 x 1080 pixels. First we evaluate methods with synthetic trajectories generated from the dataset for performance comparison, then with real trajectories of the dataset.

5.1. Metrics

Evaluation metrics used in this experiments are Positional error and Step error defined in Algorithm 1. Note that $N_{est}$ and $N_{gt}$ are numbers of estimated and actual segmentation points in a trajectory.

5.2. Synthetic data

In order to compare methods with a large number of trajectories, we generate 20,000 trajectories from MDA agent models learned from the dataset. By assuming that transition probabilities are uniform, these trajectories are sampled from the linear system of Eqs. (1) and (2). In the following, we use 10,000 trajectories for HMM training (parameter estimation), and the other 10,000 trajectories for HMM inference (segmentation). Segmentation points are ones where agent models are switched from one another.

For the RDP method, we segment trajectories by changing the parameter values $\epsilon$, then choose the best one. In this case, $\epsilon = 69$ and $\epsilon = 80$ minimize each error.

For the proposed method, we choose different number of agents for segmentation (between 5 and 10) for HMM parameter estimation and inference. Because 10 agents were learned by MDA, we perform the same procedure for 10 times (except the case of using all 10 agents) then report averaged results.

Table 1 show comparison results. The proposed method works better when the number of used agents is larger than eight.

| Method          | #Agent | Positional error | Step error |
|-----------------|--------|-----------------|------------|
| MDA+HMM         | 5      | 44.90 ± 6.25    | 2.35 ± 0.17|
| (Ours)          | 6      | 38.81 ± 2.70    | 2.10 ± 0.18|
|                 | 7      | 33.76 ± 3.01    | 1.81 ± 0.13|
|                 | 8      | 30.91 ± 4.09    | 1.57 ± 0.13|
|                 | 9      | 25.50 ± 3.34    | 1.32 ± 0.10|
|                 | 10     | 20.88           | 1.09       |
| RDP             | 69     | 33.69           | 1.84       |
|                 | 80     | 34.17           | 1.82       |

Algorithm 1: Calculate positional and step errors

1: function CALCERROR($S_1$, $S_2$)
2: $pos = stp = 0$
3: for i do
4:     if $S_1(i)$ is a Seg point then
5:         $j = \text{argmin}\ |j - i|$ if $S_2(j)$ is a segmentation point
6:         $pos + = \|obs(j) − obs(i)\|$ $stp + = |j − i|$ |
7:     end if
8: end for
9: return $pos, stp$
10: end function
11: $pos, stp = CALCERROR(est, gt) + CALCERROR(gt, est)$
12: $pos, stp/ = (N_{est} + N_{gt})$


### 5.3. Real data

For evaluating methods with the real dataset, we selected and manually annotated 104 trajectories so that trajectories are segmented at the point where pedestrians turn their walking directions.

For the RDP method, we segment all trajectories by changing the parameter values $\epsilon$, then choose the best one. In this case, $\epsilon = 38$ and $\epsilon = 29$ minimize each error.

For the proposed method, we choose different number of agents for segmentation (between 5 and 10), as we did in the previous section. For separating the dataset for HMM parameter estimation and inference, we perform four-fold cross validation.

Results are shown in table 2. Figures 6 to 9 visualize segmentation results by the proposed method. RDP errors are smaller than the proposed method, however it doesn’t provide any semantic information of segmentation. In contrast, the proposed method divides trajectories into semantically meaningful segments with associated agent models, which helps to understand the behavior of the pedestrians in the real scene.

### 6. Conclusions

In this paper we have proposed a semantic trajectory segmentation method by combining MDA and HMM to estimate agent models and segment trajectories according to the learned agents. Experimental results with synthetic trajectory dataset show that the proposed method works better than the baseline, the Ramer-Douglas-Peucker method. Errors of the proposed method on the real dataset are relatively large due to the fact that the HMM tends to infer multiple agents frequently at turning points of pedestrians. Our future work includes to overcome this issue.

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Figure 6. Segmentation of pedestrian 1. The arrow indicates the walking direction. (a), (b), and (c) indicate temporal order; in (a) the pedestrian belongs to an agent model going from right-bottom toward upper side, in (b) to a model toward upper-left, and in (c) to a model going left-side.

Figure 7. Segmentation of pedestrian 2. The arrow indicates the walking direction. (a), (b), and (c) indicate temporal order; in (a) the pedestrian belongs to an agent model going downward, in (b) to a model toward right-bottom, and in (c) to a model going right-side.

Figure 8. Segmentation of pedestrian 3. The arrow indicates the walking direction. (a), (b), and (c) indicate temporal order; in (a) the pedestrian belongs to an agent model going downward, in (b) to a model toward right, and in (c) to a model going downward. Notice that there exists many models around points where the pedestrian turns its direction abruptly.

Figure 9. Segmentation of pedestrian 4. The arrow indicates the walking direction. (a), (b), and (c) indicate temporal order; in (a) the pedestrian belongs to an agent model going downward, in (b) to a model toward left-bottom, and in (c) to a model going downward. Notice that there exists many models around points where the pedestrian turns its direction abruptly.