Vacuum fluctuation effects on hyperonic neutron star matter

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The vacuum fluctuation (VF) effects on the properties of the hyperonic neutron star matter are investigated in the framework of the relativistic mean field (RMF) theory. The VF corrections result in the density dependence of in-medium baryon and meson masses. We compare our results obtained by adopting three kinds of meson-hyperon couplings. The introduction of both hyperons and VF corrections soften the equation of state (EoS) for the hyperonic neutron star matter and hence reduce hyperonic neutron star masses. The presence of the $\delta$ field enlarges the masses and radii of hyperonic neutron stars. Taking into account the uncertainty of meson-hyperon couplings, the obtained maximum masses of hyperonic neutron stars are in the range of $1.33 M_\odot \sim 1.55 M_\odot$.

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I. Introduction

The nonlinear Walecka model (NLWM), based on the relativistic mean field (RMF) theory, has been successfully used in the study of nuclear matter and neutron stars\textsuperscript{[1, 2, 3, 4, 5, 6, 7]}. Neutron stars are composed of highly compressed matter. Nuclear matter at high densities exhibits a new degree of freedom: strangeness. Hyperons, kaon condensation, and quarks may appear in neutron stars, and these complicated compositions of neutron stars have attracted much attention. The important property of a neutron star is characterized by its mass and radius, which can be obtained from the appropriate equation of state (EoS) at high densities. The RMF model was first used to investigate the properties of hyperonic neutron stars ($npe\mu H$) ($H$ denotes hyperons throughout this paper) in 1980s\textsuperscript{[8, 9]}. Recently, in some studies based on the RMF model it has been indicated that the hyperons play an important role in the neutron star matter\textsuperscript{[10, 11, 12]}. In recent years it has been stressed that the inclusion of the $\delta$ field is important in the study of the asymmetric nuclear matter\textsuperscript{[6, 7, 12, 13, 14]}. The inclusion of the $\delta$ field leads to the structure of relativistic interactions, where a balance between an attractive (scalar) and a repulsive (vector) potential exists. The $\delta$ field plays a role in the isospin channel and mainly affects the behavior of the system in the high density regions and so is of great interest in nuclear astrophysics. The influence of the $\delta$ field on the properties of hyperonic neutron stars has been investigated based on the RMF model\textsuperscript{[12, 15, 16]}.

In Ref.\textsuperscript{[17]}, the vacuum fluctuation (VF) corrections were taken into account to study the properties of nuclear matter. In our recent paper\textsuperscript{[18]}, we developed the VF-RMF model by including the isovector...
mesons ($\rho$ and $\delta$) to investigate the properties of the asymmetric nuclear matter and neutron stars. The VF effects lead to the dependence of in-medium hadron masses on the total baryon density. In this work, we will extend the VF-RMF model to hyperon-rich matter in neutron stars by including the hyperons and leptons in the relativistic Lagrangian density. The VF effects will be introduced by considering loop corrections in the self-energies of in-medium baryons and mesons as in Ref. [18]. The VF effects on the properties of the hyperonic neutron star matter will be studied.

This article is organized as follows. In Sec. II, we derive the in-medium masses of baryons and mesons and the EoS for the hyperonic neutron star matter in the VF-RMF model. Sec. III is devoted to our results and discussions. In Sec. IV, a brief summary is presented.

II. The baryon octet VF-RMF model

The relativistic Lagrangian density with the baryon octet and free leptons used in this work reads

$$\mathcal{L} = \sum_B \bar{\psi}_B [i\gamma_\mu \partial^\mu - (M_B - g_{\sigma B} \phi - g_{\omega B} \tilde{\omega}_B \cdot \tilde{\delta}) - g_{\omega B} \gamma_\mu \omega^\mu - g_{\rho B} \gamma_\mu \rho^\mu \tilde{t}_B \cdot \tilde{b}_B] \psi_B$$

$$+ \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m_\sigma^2 \phi^2) - U(\phi) + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu + \frac{1}{2} m_\rho^2 \tilde{b}_B \cdot \tilde{b}_B$$

$$+ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu}$$

$$+ \sum_l \bar{\psi}_l (i\gamma_\mu \partial^\mu - m_l) \psi_l + \delta \mathcal{L}, \quad (1)$$

where the sum on $B$ is over all the states of the lowest baryon octet (with mass $M_B$) ($B = n, p, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0$) and the sum on $l$ is over the free leptons (with mass $m_l$) ($l = e^-, \mu^-$); $\phi$, $\omega$, $\tilde{\omega}$ (with masses $m_\sigma$, $m_\omega$, $m_\rho$, respectively) represent $\sigma$, $\omega$, and $\rho$ meson fields, respectively; $\tilde{t}_B$ represents the isospin generator matrix for the baryon $B$; $U(\phi) = \frac{1}{4} a \phi^4 + \frac{1}{2} b \phi^3$ is the nonlinear potential of the $\sigma$ meson, $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $\tilde{G}_{\mu\nu} \equiv \partial_\mu \tilde{b}_\nu - \partial_\nu \tilde{b}_\mu$; the counterterm for the Lagrangian density, $\delta \mathcal{L}$, has the same form as that in Ref. [18].

The field equation for the baryon $B$ in the RMF approximation is given by

$$[i\gamma_\mu \partial^\mu - (M_B - g_{\sigma B} \phi - g_{\omega B} t_3 B \delta_3) - g_{\omega B} \gamma_\mu \omega_0 - g_{\rho B} \gamma_\mu t_3 B b_0] \psi_B = 0, \quad (2)$$

with

$$\phi = \frac{1}{m_\sigma^2} \left( \sum_B g_{\sigma B} \rho B - a \phi^2 - b \phi^3 \right),$$

$$\omega_0 = \frac{1}{m_\omega^2} \sum_B g_{\omega B} \rho B,$$

$$b_0 = \frac{1}{m_\rho^2} \sum_B g_{\rho B} t_3 B \rho B,$$

$$\delta_3 = \frac{1}{m_\delta^2} \sum_B g_{\delta B} t_3 B \delta B.$$

(3)

where $t_3 B$ is the third direction projection of the $\tilde{t}_B$ for baryon $B$. $\rho B$ and $\rho_B^3$ are the number and
scalar densities of the baryon $B$, which are given in the following respectively,

$$
\rho_B = \frac{k_{FB}^3}{3\pi^2},
$$

and

$$
\rho_B^s = -i \int \frac{d^4k}{(2\pi)^4} \text{Tr} G^B_{FB}(k),
$$

where $k_{FB}$ is the Fermi momentum of the baryon $B$ and $G^B_{FB}(k)$ is the propagator of the baryon $B$ in the VF-RMF model:

$$
G^B(k) = (\gamma_\mu k^\mu + M_B^*) \left[ \frac{1}{k^2 - M_B^*} + \frac{i\pi}{E_{FB}^*} \delta(k^0 - E_{FB}) \theta(k_F - |k|) \right]
$$

$$
\equiv G^B_{FB}(k) + G^B_D(k),
$$

where $M_B^*$ is the effective mass of the baryon $B$, $E_{FB}^* = \sqrt{k^2_{FB} + M_B^{*(s)}^2}$ and $\eta$ is infinitesimal.

![Diagram](FIG_1.png)

FIG. 1: Loop-diagram corrections to the self-energy of baryon octet states (a) and mesons (b) in medium, where $B^{(s)}$ denotes baryons and $k$ is the four momentum of the meson.

In the present work, we only consider the dominant VF contributions from the tadpole diagrams to the self-energies of baryon octet states. Thus, when the VF corrections are introduced through Fig. 1(a), the effective mass of the baryon $B$ can be written as:

$$
M_B^* = M_B + ig_{\sigma B} \sum_{B'} \frac{g_{sB'}}{m_{\sigma B}^2} \int \frac{d^4k}{(2\pi)^4} \text{Tr} G_{FB}^{B'}(k)
$$

$$
+ ig_{B' t_{3B}} \sum_{B'} \frac{g_{sB'}}{m_{\delta B}^2} \int \frac{d^4k}{(2\pi)^4} \text{Tr} G_{FB}^{B'}(k)
$$

$$
+ \frac{g_{sB}}{m_{\sigma B}^2} (a\phi^2 + b\phi^3),
$$

where $m_j^* (j = \sigma, \omega, \rho, \delta$ throughout this paper) are the off-shell in-medium meson masses.

The introduction of the density dependence of the in-medium meson masses is the critical effect of VF corrections. Because the meson propagators in the baryon self-energies carry zero four-momenta, we must use the off-shell ($q^\mu = 0$) meson masses in the tadpole loop calculations for self consistency.
We calculate the in-medium meson masses in the random-phase approximation (RPA) \cite{17, 19}, see Fig. 1(b). The obtained off-shell effective mass of the $\sigma$ meson is given by

$$m_\sigma^\star = m_\sigma^2 + \Pi_\sigma(q^\mu = 0),$$

(8)

where

$$\Pi_\sigma(q^\mu = 0) = -i \sum_B g_\sigma^2 B \int \frac{d^4k}{(2\pi)^4} \text{Tr}[G^B(k + q)G^B(k)].$$

(9)

The off-shell effective mass of the $\delta$ meson is obtained as follows:

$$m_\delta^\star = m_\delta^2 + \Pi_\delta(q^\mu = 0),$$

(10)

where

$$\Pi_\delta(q^\mu = 0) = -i \sum_B g_\delta^2 B \int \frac{d^4k}{(2\pi)^4} \text{Tr}[G^B(k + q)G^B(k)].$$

(11)

The off-shell effective mass of the $\omega$ meson is given by

$$m_\omega^\star = m_\omega^2 + \Pi_\omega T(q^\mu = 0),$$

(12)

where $\Pi_\omega T$ is the transverse part of the following polarization tensor:

$$\Pi_\omega^{\mu\nu}(q^\mu = 0) = -i \sum_B g_\omega^2 B \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\gamma^\mu G^B(k + q)\gamma^\nu G^B(k)].$$

(13)

The off-shell effective mass of the $\rho$ meson is given by

$$m_\rho^\star = m_\rho^2 + \Pi_\rho T(q^\mu = 0),$$

(14)

where $\Pi_\rho T$ is the transverse part of the following polarization tensor:

$$\Pi_\rho^{\mu\nu}(q^\mu = 0) = -i \sum_B g_\rho^2 B \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\gamma^\mu G^B(k + q)\gamma^\nu G^B(k)].$$

(15)

Obviously, the modification of the in-medium hadron masses will affect the properties of the hyperonic neutron star matter. The meson masses appearing in the Lagrangian density should be replaced by the off-shell in-medium meson masses in our calculations. Therefore, the energy-momentum tensor in the VF-RMF model can be expressed as

$$T_{\mu\nu} = \sum_B i \bar{\psi}_B \gamma_\mu \partial_\nu \psi_B + \sum_i i \bar{\psi}_i \gamma_\mu \partial_\nu \psi_i + \sum_B g_{\mu\nu} \left[ \frac{1}{2} m_\sigma^2 \phi^2 + U(\phi) - \frac{1}{2} m_\omega^2 \omega_\lambda \omega^\lambda - \frac{1}{2} m_\rho^2 b_\lambda b^\lambda + \frac{1}{2} m_\delta^2 \delta^\lambda \delta_\lambda \right].$$

(16)

The EoS for the hyperonic neutron star matter is given by the diagonal components of the energy-momentum tensor. Thus we have the energy density as follows:

$$\epsilon = -i \sum_B \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\gamma^0 G^B(k)] k^0$$

$$+ \sum_i \frac{1}{8\pi^2} \left[ k_F \tilde{E}_F (m_i^2 + 2k_F^2) - m_i^4 \ln \left( \frac{k_F + \tilde{E}_F}{m_i} \right) \right]$$

$$+ \frac{1}{2} m_\sigma^2 \phi^2 + U(\phi) + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 b_0^2 + \frac{1}{2} m_\delta^2 \delta_0^2,$$

(17)
and the pressure is given by

\[
P = -\frac{i}{3} \sum_{B,i} \int \frac{d^4k}{(2\pi)^4} \text{Tr}[\gamma^i G^B(k)] k^i
\]

\[+ \sum_l \frac{1}{8\pi^2} \left[m_l^4 \ln \left(\frac{k_{F_l} + E_{F_l}^*}{m_l}\right) - E_{F_l}^* k_{F_l} \left(m_l^2 - \frac{2}{3} k_{F_l}^2\right)\right] \]

\[-\frac{1}{2} m_\sigma^2 \phi^2 - U(\phi) + \frac{1}{2} m_b^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \omega_0^2 - \frac{1}{2} m_\delta^2 \delta_3^2, \quad (18)\]

where the sum on \(i\) is over the space components of \(\gamma\) and \(k\), \(k_{F_l}\) is the Fermion momentum of free leptons and \(E_{F_l}^* = \sqrt{m_l^2 + k_{F_l}^2}\).

Hyperonic neutron stars are neutral charged objects in \(\beta\) equilibrium. For the hyperonic neutron star matter, the chemical potential of baryon octet states and leptons are constrained by the baryon number and electric charge conservation:

\[\mu_B = \mu_e, \quad (19)\]

\[\mu_p = \mu_n - \mu_e, \quad (20)\]

\[\mu_\Lambda = \mu_\Sigma^0 = \mu_\Xi^0 = \mu_n, \quad (21)\]

\[\mu_\Sigma^- = \mu_\Xi^- = \mu_n + \mu_e, \quad (22)\]

\[\mu_\Sigma^+ = \mu_p = \mu_n - \mu_e, \quad (23)\]

where \(\mu_n\) and \(\mu_e\) are the independent neutron and electron chemical potentials, respectively, where the chemical potentials of the baryon \(B\) and the lepton \(l\) are given by, respectively,

\[\mu_B = \sqrt{k_B^2 + M_B^2 + g_\omega B \omega_0 + g_\rho B t_3 b_0}, \quad (24)\]

\[\mu_l = \sqrt{k_{F_l}^2 + m_l^2} . \quad (25)\]

The neutral charged condition of the hyperonic neutron star matter can be expressed as:

\[\rho_p + \rho_\Sigma^+ - \rho_\Sigma^- - \rho_\Xi^- = \rho_e^- + \rho_\mu^- . \quad (26)\]

The properties of hyperonic neutron stars can be obtained by solving Tolmann-Oppenheimer-Volkov (TOV) equations with the derived EoS as the input.

**III. Results and discussions**

In this work, the meson-nucleon coupling constants are fixed by the same saturation properties of the nuclear matter as in Ref. 18. In general, the interactions between different meson and hyperon states should be different. We prefer to adopt the hyperon potentials to determine the \(\sigma\) meson-hyperon coupling constants with the vector and isovector meson-hyperon couplings fixed by SU(6) quark symmetry because such meson-hyperon coupling choice can reflect the different interactions between meson and hyperon states to some degree. For the meson-hyperon coupling constants, it is convenient to define \(x_{jH} = g_{jH}/g_{jN}\) \((N = n, p\) throughout this paper). The \(\sigma\) meson-hyperon coupling constants are fixed by the corresponding hyperon potentials, \(U_H = x_{\omega H} V - x_{\sigma H} S\), where \(V = g_\omega N \omega_0\) and \(S = g_{\sigma N} \phi\) are the \(\omega\) and \(\sigma\) field strengths at the saturation density. As discussed in
Table I. The $\sigma$ meson-hyperon coupling constants obtained from hyperon potentials in the VF-RMF and NL-RMF models.

| parameters | VF − RMF model | NL − RMF model |
|------------|----------------|----------------|
|            | $V_F$ $\rho$  | $V_F$ $\rho$ $\delta$ | $N_L$ $\rho$ | $N_L$ $\rho$ $\delta$ |
| $x_{\sigma \Lambda}$ | 0.62 | 0.64 | 0.61 | 0.62 |
| $x_{\sigma \Sigma}$ | 0.37 | 0.38 | 0.36 | 0.37 |
| $x_{\sigma \Xi}$ | 0.33 | 0.34 | 0.32 | 0.33 |

Ref. [23], $\Lambda$ is known to experience an attractive potential, $U_\Lambda = -28 \, MeV$, in hypernuclear matter. Recently, some authors suggested that $\Sigma^{-}$ may feel repulsive potential at high densities [24, 25, 26], which was supported by the absence of bound states in a recent $\Sigma$ hypernuclear search [27]. Therefore, the repulsive potential of $\Sigma$, $U_\Sigma = 30 \, MeV$, is adopted in our calculations as in [28]. The attractive potential of $\Xi$, $U_\Xi = -18 \, MeV$, is adopted from $\Xi$-$N$ interaction [15]. The obtained $\sigma$ meson-hyperon coupling constants are listed in Table I. As mentioned before, the vector and isovector meson-hyperon couplings are fixed by SU(6) quark symmetry [29]:

$$g_{\omega \Lambda} = g_{\omega \Sigma} = 2g_{\omega \Xi} = \frac{2}{3}g_{\omega N}, \quad (27)$$
$$g_{\rho \Lambda} = 0, \quad (28)$$
$$g_{\rho \Sigma} = 2g_{\rho \Xi} = 2g_{\rho N}, \quad (29)$$
$$g_{\delta \Lambda} = 0, \quad (30)$$
$$g_{\delta \Sigma} = 2g_{\delta \Xi} = 2g_{\delta N}. \quad (31)$$

The in-medium masses of baryons and mesons can be obtained by calculating the loop corrections to their self-energies, see Fig. 1. As discussed before, because the meson propagators appearing in the baryon self-energies are calculated at zero four-momentum transfer [see Fig. 1(a)], we have to use the off-shell in-medium meson masses in the tadpole loop calculations for self consistency. The off-shell in-medium meson masses in the hyperonic neutron star matter are shown in Fig. 2. It is found that the off-shell in-medium meson masses increase with the increase of the total baryon density (the sum of the densities of $n$, $p$, $\Lambda$, $\Sigma^{-}$, $\Sigma^{0}$, $\Sigma^{+}$, $\Xi^{-}$, $\Xi^{0}$). The introduction of the density dependence of in-medium meson masses is the critical effect of VF corrections.

In this work, hyperons are included in the VF-RMF model. The in-medium masses of hyperons play an important role in the calculations of EoS. Fig. 3 shows the in-medium masses of $\Lambda$ and $\Xi^{-}$ as a function of the total baryon density in different models for a comparison. It is found that the VF corrections soften the decrease of the in-medium hyperon masses at high densities. This implies the softer EoS for the hyperonic neutron star matter obtained in the VF-RMF model than that in the NL-RMF model.

Now we define the relative population of the baryon $B$ as the ratio of the density of $B$ and the total baryon density. Fig. 4 and Fig. 5 show the relative populations as a function of the total baryon density with the meson-hyperon couplings fixed by hyperon potentials and SU(6) quark symmetry in the NL-RMF model and the VF-RMF model, respectively. Comparing these two figures, we find that
FIG. 2: Off-shell in-medium meson masses ($m_*^j$) in the hyperonic neutron star matter as a function of the total baryon density with the meson-hyperon couplings fixed by hyperon potentials and SU(6) quark symmetry in the VF-RMF model.

FIG. 3: The in-medium masses of hyperons, $M_H^*$ ($H = \Lambda, \Xi^-$), as a function of the total baryon density with the meson-hyperon couplings by hyperon potentials and SU(6) quark symmetry in different models.

the VF corrections lead to later emergence of all hyperons. Meanwhile, the VF corrections reduce the population of $\Xi^0$ while they enlarge the population of leptons. We can see that $\Sigma^{(\pm,0)}$ experience such a strong repulsion that they do not appear at all in the density range found in the neutron stars. This is consistent with that $\Sigma^-$ is hardly stabilized in the hypernuclear matter [28]. In the VF-RMF model, the $\delta$ field effects shift the thresholds of all the hyperons to lower densities. On the other hand,
FIG. 4: The relative populations as a function of the total baryon density with the meson-hyperon couplings fixed by hyperon potentials and SU(6) quark symmetry in the NL-RMF model.

the $\delta$ field effects shift the thresholds of $\Lambda$ and $\Xi^-$ to lower densities while shifting the threshold of $\Xi^0$ to higher density in the NL-RMF model. The $\delta$ field effects on the population of baryons and leptons are not apparent in both models.

Fig. 6 shows the EoS, pressure vs. the total baryon density, for the hyperonic neutron star matter with the meson-hyperon couplings fixed by hyperon potentials and SU(6) quark symmetry in different models. The insert of Fig. 6 presents the EoS for the nucleonic ($npe\mu$) neutron star matter for a comparison. We can see that the introduction of hyperons and VF corrections soften the EoS greatly. Unlike the case of the nucleonic neutron star matter, the presence of the $\delta$ field stiffen the EoS at first, and then from the appearance of $\Lambda$ and $\Xi^-$ till higher densities soften the EoS for the hyperonic neutron star matter. This is because that the attractive effects of $\Lambda$ and $\Xi^-$ are larger than the repulsive effects of the $\delta$ field. Such effects of the $\delta$ field reflect the complicated nature of interactions between mesons and hyperons in the hyperonic neutron star matter, which needs deeper study in the future.

The properties of neutron stars can be calculated by solving TOV equations. Fig. 7 shows the correlation between the neutron star masses and the corresponding radii for hyperonic and nucleonic neutron stars with meson-hyperon couplings obtained by hyperon potentials and SU(6) quark symmetry in different models. The obtained maximum masses, the corresponding radii and the central densities are presented in Table II. As pointed out in Ref. [30], the introduction of hyperons leads
FIG. 5: The relative populations as a function of the total baryon density with the meson-hyperon couplings fixed by hyperon potentials and SU(6) quark symmetry in the VF-RMF model.

Table II. The maximum masses ($M_S$ in unit of $M_\odot$), the corresponding radii and the central densities of hyperonic neutron stars and nucleonic neutron stars. The meson-hyperon couplings are fixed by hyperon potentials and SU(6) quark symmetry.

| Neutron Star      | Properties | VF-RMF model | NL-RMF model |
|-------------------|------------|--------------|--------------|
| Hyperonic neutron star | M_S/M_\odot | VF_p  | VF_pδ | NL_p | NL_pδ |
| npeµH             | R(km)      | 12.37 | 13.13 | 12.92 | 13.036 |
|                   | \rho_c/\rho_0 | 4.91 | 4.89 | 5.76 | 4.63 |
| Nucleonic neutron star | M_S/M_\odot | VF_p  | VF_pδ | NL_p | NL_pδ |
| npe\mu            | R(km)      | 10.85 | 11.82 | 10.93 | 11.37 |
|                   | \rho_c/\rho_0 | 7.58 | 6.36 | 6.66 | 6.29 |

to the reduction of the maximum neutron star masses due to the Pauli principle effects. We can see from Table II that our results are consistent with this statement. Furthermore, we can see that the VF corrections also result in the reduction of the maximum masses of neutron stars. The presence of the \( \delta \) field enlarges the maximum masses and radii of neutron stars.

In literatures, there are various approaches to determine the meson-hyperon couplings [9, 29, 31].
FIG. 6: The EoS for the hyperonic neutron star matter with the meson-hyperon couplings fixed by hyperon potentials and SU(6) quark symmetry. The insert is the EoS for the nucleonic neutron star matter.

FIG. 7: The masses of neutron stars as a function of the radii of neutron stars in the VF-RMF model. The meson-hyperon couplings are fixed by hyperon potentials and SU(6) quark symmetry.
order to see the dependence of our results on the meson-hyperon couplings, we also compare our results by adopting the meson-hyperon couplings derived from the quark counting method, \( x_{jH} = \sqrt{2/3} \) \( \text{[31]} \), and the universal meson-hyperon couplings, \( x_{jH} = 1 \) \( \text{[9]} \).

FIG. 8: The relative populations as a function of the total baryon density with quark counting meson-hyperon couplings in the VF-RMF model.

Fig. 8 and Fig. 9 show the relative populations as a function of the total baryon density with the quark counting and universal meson-hyperon coupling choices, respectively. We can see that \( \Sigma^- \) is the first hyperon to appear due to its low mass and favored charge \( \text{[32]} \). For both of the two kinds of meson-hyperon couplings, the presence of \( \delta \) field leads to earlier emergence of hyperons. For both the quark counting and the universal meson-hyperon coupling choices, the populations of nucleons dominate in the whole density region. However, as it can be seen from Fig. 5, the population of \( \Lambda \) exceeds that of the proton at high densities when we adopt the meson-hyperon couplings fixed by hyperon potentials and SU(6) quark symmetry. Comparing Fig. 8 and Fig. 9 with Fig. 5, we can see that the onset of \( \Xi^- \) is greatly reduced in Fig. 5 to compensate the absence of \( \Sigma^- \) in order to keep charge neutrality \( \text{[9]} \). We note that \( \Xi^0 \) and \( \Sigma^+ \) will appear beyond the maximum density considered here, \( 9 \rho_0 \), when we adopt the quark counting and universal meson-hyperon coupling choices.

Fig. 10 shows the masses of hyperonic neutron stars as a function of the neutron star radii with the quark counting and universal meson-hyperon couplings in the VF-RMF model. Comparing Fig. 10 with Fig. 7, it is obvious that the properties of hyperonic neutron stars are sensitive to the meson-hyperon couplings. We can see that the weaker the meson-hyperon couplings the lower the masses and
FIG. 9: The relative populations as a function of the total baryon density with the universal meson-hyperon coupling choice in the VF-RMF model.

FIG. 10: The masses of hyperonic neutron stars as a function of the radii of hyperonic neutron stars with the quark counting and universal meson-hyperon couplings in the VF-RMF model.
Table III. The maximum mass \( (M_S \text{ in unit of } M_\odot) \), the corresponding radii and the central densities of hyperonic neutron stars with the quark counting and universal meson-hyperon coupling choices in the VF-RMF model.

| meson-hyperon couplings properties | VF-RMF model |
|-----------------------------------|--------------|
|                                   | \( VF_\rho \) | \( VF_\rho_\delta \) |
| \( x_{jH} = \sqrt{2/3} \)        |              |                |
| \( M_S/M_\odot \)                | 1.43         | 1.55           |
| \( R(km) \)                      | 12.98        | 13.98          |
| \( \rho_c/\rho_0 \)              | 4.71         | 3.98           |
| \( x_{jH} = 1 \)                 |              |                |
| \( M_S/M_\odot \)                | 1.69         | 1.80           |
| \( R(km) \)                      | 13.59        | 14.50          |
| \( \rho_c/\rho_0 \)              | 4.48         | 3.89           |

The radii of hyperonic neutron stars are obtained. As in Fig. 7, the presence of the \( \delta \) field also increases the masses and radii of hyperonic neutron stars.

Table III displays the properties of hyperonic neutron stars with the quark counting and universal meson-hyperon couplings in the VF-RMF model. Comparing Table III with Table II, we can see that the influence of meson-hyperon couplings on our results are distinct. With the adoption of the universal meson-hyperon couplings, the obtained maximum hyperonic neutron star masses are higher than those obtained by adopting the other two kinds of meson-hyperon couplings and even higher than the maximum masses obtained in the case of nucleonic neutron stars. As discussed in Refs. \[21, 30\], the conversion of nucleons to hyperons are energetically favored. The inclusion of hyperons softens the EoS and consequently reduces the maximum masses of neutron stars because Pauli principle minimizes the total energy at a given density. Based on the above discussion, the choice of universal hyperon couplings is not appropriate for our model. It is found that the weaker the meson-hyperon couplings the lower maximum masses and the corresponding radii of hyperonic neutron stars are obtained. The adoption of meson-hyperon couplings fixed by the hyperon potentials and SU(6) quark symmetry results in the softest EoS of the hyperonic neutron matter and hence leads to the lowest maximum masses of hyperonic neutron stars. We also find that the maximum masses and radii of hyperonic neutron stars increase when the \( \delta \) field presents in the VF-RMF model.

IV. Summary

In this work, we investigate the properties of the hyperonic neutron star matter in the extended VF-RMF model. The interactive hyperons and free leptons are introduced into the relativistic Lagrangian density. The VF effects are included by taking into account the loop corrections in the hadron self-energies. In our calculations, we replace all the meson masses by the off-shell in-medium meson masses since the propagators of mesons in the tadpole diagrams of baryon self-energies carry zero-momenta. With the VF corrections the in-medium baryon and meson masses are dependent on the total baryon density.

In general, the interactions between different mesons and hyperons should be different. We prefer to adopt the \( \sigma \) meson-hyperon couplings derived from the hyperon potentials with the vector and isovector...
meson-hyperon couplings fixed by the SU(6) quark symmetry because this choice for the meson-
hyeron couplings can reflect such differences to a certain degree. We find that the off-shell in-medium
meson masses increase with the increase of total baryon density. The in-medium masses of hyperons
decrease slower at high densities when the VF corrections are introduced. The density dependence
of off-shell in-medium meson masses and in-medium hyperon masses influences the properties of the
EoS for the hayepronic neutron star matter directly. The introduction of hyperons soften the EoS since
Pauli principle minimizes the total energy at a given density. The results obtained in the VF-RMF
model are compared with those obtained in the NL-RMF model. It is found that the VF corrections
soften the EoS for the hyperonic neutron star matter and hence the maximum masses of hyperonic
neutron stars are reduced. Σ(±,0) are absent in the range of densities found in neutron stars because
they feel a strong repulsion in this meson-hyperon couplings choice.

Then, the dependence of our results on the meson-hyperon couplings is studied. We use other
two choices for meson-hyperon couplings for a comparison: one is derived from the quark counting
method, the other is the so-called universal couplings. It is found that the properties of hyperonic
neutron stars are sensitive to the meson-hyperon couplings. The weaker the meson-hyperon couplings
the lower the maximum masses of hyperonic neutron stars are obtained. For the quark counting and
universal coupling choices, Σ(±,0) are present and the populations of nucleons dominant in the whole
region of total baryon densities. The maximum masses of hyperonic neutron stars obtained with the
universal meson-hyperon couplings exceed the maximum masses of nucleonic neutron stars. Because
Pauli principle assures that the appearance of hyperons will lower the Fermi energy of baryons and
hence will lower the total energy at a given baryon density, the universal meson-hyperon couplings
is not appropriate for our model. Taking into account the uncertainty of meson-hyperon couplings,
the obtained maximum hyperonic neutron star masses are in the range of 1.33M⊙ ∼ 1.55M⊙ (M⊙
denotes the mass of the sun) in the VF-RMF model.

The effects of the δ field on hyperonic neutron stars are investigated in the VF-RMF model. Unlike
the case of the nucleonic neutron star matter, we find that the presence of the δ field stiffen the EoS at
first and then soften the EoS from the appearance of Λ and Ξ− till higher densities for the hyperonic
neutron star matter. Such effects of the δ field on the EoS reflect the complicated interactions in the
hyperonic neutron star matter. In addition, the presence of the δ field enlarges the maximum masses
and radii of hyperonic neutron stars. The effects of the δ field on neutron stars are more apparent
when the VF corrections are included.

As discussed in Ref. [2], the exchange diagram contributions only provide small corrections to the
EoS for nuclear matter in the RMF approach at high densities. We simply extend this statement to
the case of the hyperonic neutron star matter as the first step to study the VF effects by including
hyeron in our model. Consequently, we only consider the contributions from tadpole diagrams to
the baryon self-energies in the present work. It will be very interesting to study the VF effects on the
properties of hyperon-hyperon interaction, kaon condensation and unconfined quarks in the core of
neutron stars in the future.
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