Generalized Nesterov Accelerated Conjugate Gradient Algorithm for a Compressively Sampled MR Imaging Reconstruction

XIUHAN LI, WEI WANG, SONGSHENG ZHU, WENTAO XIANG, AND XIAOLING WU
Key Laboratory of Clinical Engineering, School of Biomedical Engineering and Informatics, Nanjing Medical University, Nanjing 211166, China
Corresponding author: Wei Wang (bmeww@njmu.edu.cn)

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ABSTRACT To accelerate the scanning speed of magnetic resonance imaging (MRI) and improve the quality of magnetic resonance (MR) image reconstruction, a fast MRI technology based on compressed sensing is proposed. Nesterov’s accelerated gradient descent (NAG) algorithm uses Nesterov acceleration to optimize the gradient descent (GD) method. However, this form of acceleration factor uses a fixed iteration curve update and cannot adapt to different iteration processes. A generalized Nesterov acceleration concept is proposed. Combining the total variation model, a generalized Nesterov accelerated conjugate gradient based on total variation (GNACG_TV) algorithm is proposed. It extends the acceleration factor in a generalized manner, introducing the Frobenius norm of the objective function as a parameter, so that the acceleration factor is related not only to the number of iterations but also to the iteration process and guarantees the convergence of the iterative process. Experiments on three MR images (abdomen, head, and ankles) at different sampling ratios show that the proposed GNACG_TV algorithm compares favorably with conjugate gradient (CG), conjugate gradient based on total variation (CG_TV), Nesterov accelerated conjugate gradient based on total variation (NACG_TV), and conjugate gradient based on adaptive moment estimation (ADAMCG) algorithms in the MSE, PSNR and SSIM exhibit better performance and robustness in denoising performance for the proposed algorithm. Comparing with the result of qualitative and quantitative analysis, it was concluded that the proposed method can better reconstruct under-sampled MR images than other 4 methods. GNACG_TV can further improve the convergence speed based on Nesterov acceleration and get better reconstruction performance.

INDEX TERMS Compressed sensing, conjugate gradient, generalized Nesterov acceleration, MR image reconstruction.

I. INTRODUCTION
Magnetic resonance imaging (MRI) technology is a noninvasive and nonray examination process. It offers the advantages of high contrast resolution, multidirectional capabilities, multiparameter acquisition and functional imaging [1]. It provides soft tissue contrast ability that is unmatched by most other imaging methods. It is possible to use noninvasive examination methods to examine and quantify the metabolic and physiological characteristics of tissues, thereby providing valuable information about pathology, which has become an important clinical examination method. However, the imaging speed of MRI technology is relatively slow, and the data acquisition time is too long; MR images are susceptible to motion-related artifacts, and the relatively low time resolution limits its use on body parts (such as the abdomen and heart) that move with breathing [2], [3]. The shortcomings of MRI have become its main obstacles in practical applications.

In 2006, Donoho [4] proposed the compressed sensing (CS) theory based on previous studies. According to this theory, if the image to be reconstructed is sparse in a certain transform domain, it can pass low sample data in a way that is at or far below the Nyquist sampling standard and reconstruct the image with high probability.
Magnetic resonance images are suitable for compressed sensing theory because of their own sparseness in a specific transform domain [5]. In the MRI application process, according to compressed sensing theory, the data required for MR image reconstruction come from the undersampling of k-space data and selecting the appropriate undersampling method to accurately recover the required magnetic resonance image with a small amount of data [6].

MRI reconstruction methods mainly include the most basic gradient descent series algorithm, as well as the derived threshold contraction series algorithm [7]–[9] and split Bregman iteration [10]. The threshold contraction series algorithm uses a contraction operator to solve the L1 optimization problem to improve the iteration efficiency, and the essence of the contraction operator [11] is gradient descent. Split Bregman iteration splits the multiconstraint problem into L1 and L2 optimization problems, which can be solved alternately using the contraction operator and Gauss-Seychelles iteration [10], [12], which can effectively solve the multiconstraint reconstruction problem.

However, as the basis of many algorithms, the gradient descent method is still researched and applied by many scholars, especially in the fields of signal denoising [13], signal reconstruction [14], [15], machine learning [16], [17] and deep learning [18]–[20]. Therefore, continuous improvement of the gradient descent algorithm is necessary. Commonly used gradient descent methods include conjugate gradient (CG) [21], conjugate gradient based on total variation (CG_TV) [22], Nesterov accelerated conjugate gradient based on total variation (NACG_TV) [23], [24], and conjugate gradient based on adaptive moment estimation (ADAMCG) [25], [26].

To improve the reconstruction speed of magnetic resonance images, researchers have combined new algorithm concepts on the basis of the conjugate gradient method to form a variety of conjugate gradient method variants. Ajiz and Jennings [27] proposed an incomplete Choleski conjugate gradient algorithm (ICCG) to improve the convergence speed when solving large sparse simultaneous equations. The results showed that ICCG is suitable for any problem where the coefficient matrix is symmetric and positively definite and has a reliably good convergence speed. Zhang et al. [28] proposed an improved Polak–Ribiére–Polyak (PRP) conjugate gradient method. The method follows the falling direction of the objective function. The results show that the improved PRP method exhibits effectiveness and global convergence. Xiao and Zhu [21] proposed an improved CG_DESCENT method to solve monotone equations with large nonlinear convex constraints. The results showed that this method has global convergence, is effective, and has the potential to solve large-scale inhomogeneities.

However, MR image reconstruction with compression sampling by the above algorithm still has defects. To quickly converge in the early iterations and further improve the resolution of image reconstruction, based on the conjugate gradient algorithm, the TV model and the improved Nesterov [29] acceleration concept are introduced, and a generalized Nesterov accelerated conjugate gradient based on total variation (GNACG_TV) algorithm is proposed.

GNACG_TV uses Nesterov acceleration to optimize the gradient descent method, which is generally called NAG [30], but this form of acceleration factor t is not controlled; the following problems exist: 1. the convergence curve is not necessarily optimal, and 2. the acceleration factor is not controlled and is only related to the number of iterations. To solve this problem, generalized Nesterov acceleration is proposed. It extends t in a generalized manner by introducing the Frobenius norm of the objective function as a parameter so that t is related not only to the number of iterations but also to the iteration process, guaranteeing the convergence of the iterative process. The generalized NAG iteration formula is adjusted as shown in section B below.

II. THEORY

A. CS-MRI
In magnetic resonance imaging based on compressed sensing, the basic model of undersampling magnetic resonance image reconstruction is:

$$\min \|X\|_0 \quad \text{s.t.} \quad Y = \Phi X$$

where $X$ is the reconstructed MR image, $Y$ is the undersampled MR image data in k-space, and $\Phi$ is the observation matrix.

$\Phi$ is expressed as:

$$\Phi = \mu F$$

where $F$ is a two-dimensional Fourier transform, and $\mu$ represents an undersampling mode. Under the condition of satisfying the constraint isometricity, the image can be accurately reconstructed.

The above equation is an underdetermined problem, and the sparsest solution is found through norm regularization. L0-norm optimization is an NP-hard problem. The common way is to convert it to its optimal convex approximation L1-norm optimization where $X$ is sparsely expressed as:

$$X = \Psi^T \theta$$

where $\Psi = [\psi_1, \psi_2, \ldots, \psi_N] \in R^{N \times N}$ is sparse basis and $\theta$ is the projection coefficient matrix, where $\Phi \Psi^T$ is the encoding matrix, and

$$A = \Phi \Psi^T$$

Considering the influence of noise in the imaging process, the above model is transformed into:

$$\min \| \theta \|_1 \quad \text{s.t.} \quad \|A\theta - Y\|_2 < \varepsilon$$

where the threshold parameter $\varepsilon$ controls the fidelity of the reconstructed image to the measurement data, and the value of $\varepsilon$ is usually set to be lower than the expected noise level.
Converting the L1-norm to a simpler L2-norm, the MR image can be reconstructed by optimizing the following problems:

$$\min_{\theta} \frac{1}{2} \|Y - A\theta\|^2_2 + \lambda \|\theta\|_1$$

where $\lambda$ is the regularization parameter, which is used to balance the proportion of the fidelity term and regular term. The optimal solution can be obtained by solving Eq. (6), and the optimal solution can be brought into Eq. (3) to reconstruct the original magnetic resonance image.

**B. GENERALIZED NESTEROV ACCELERATED CONJUGATE GRADIENT BASED ON TOTAL VARIATION ALGORITHM**

Based on the conjugate gradient method, this article introduces the concept of the TV model and generalized Nesterov acceleration algorithm for CS-MRI reconstruction. The specific solution process is as follows:

The TV regularization term is added to the CS-MR image reconstruction model, and the model of Eq. (6) can be changed to:

$$\min_{\theta} \frac{1}{2} \|Y - A\theta\|^2_2 + \lambda_1 \|\Phi X\|_1 + \lambda_2 \|TV(\theta)\|_1$$

The introduction of regularization of the total variation can better preserve the edge information of the image, the regularization parameters $\lambda_1, \lambda_2$ are used to adjust the weight of regular terms, and the selection of the appropriate regularization parameter value can achieve the ideal reconstruction effect.

In this article, we use a simpler CS-MR image reconstruction model, that is, we set the weight coefficient of the sparse regular term in Eq. (7) to 0, which can be simplified to:

$$f(\theta) = \min_{\theta} \frac{1}{2} \|Y - A\theta\|^2_2 + \lambda \|TV(\theta)\|_1$$

The gradient formula of TV is as follows:

$$\text{grad}_{TV} = v_{i,j} = \frac{\partial \|\tilde{x}\|_{TV}}{\partial x_{i,j}}$$

$$\|\tilde{x}\|_{TV}$$ is expressed as:

$$\|\tilde{x}\|_{TV} = \sum_{i,j} \sqrt{(x_{i,j} - x_{i-1,j})^2 + (x_{i,j} - x_{i,j-1})^2}$$

where $x_{i,j}$ is the value of a pixel in the image, and $v_{i,j}$ is the gradient of the TV at the $x_{i,j}$ position.

CS-MR image reconstruction can be achieved by optimizing the above model based on the existing conjugate gradient method and proposing the concept of the generalized Nesterov acceleration algorithm to solve this problem.

In the CG method, the formula of the $k + 1$th iteration is:

$$x_{k+1} = x_k + a_k d_k$$

where $x_k$ is the current iteration point, $a_k$ is the step size, also known as the learning rate, and $d_k$ is the search direction.

During each iteration process, the iteration point of each iteration of the CG method is the function position updated during the previous iteration process. According to Eq. (11), the determination of the step size and the search direction is to find a suitable function point position, so determining the appropriate point for the next iteration is also an important step for the conjugate gradient method.

Combined with the concept of the Nesterov acceleration algorithm, using the position of the current iteration point and the previous iteration point, the conjugate gradient method is modified to update Eq. (11).

$$x_{k+1} = z_{k+1} + a_k d_k$$

where $z_{k+1}$ is:

$$z_{k+1} = x_k + (\frac{n_k}{t_k+1})(x_k - x_{k-1})$$

where $t_k+1$ is:

$$t_k+1 = 1 + \sqrt{1 + \frac{n_k^2 n_k}{t_k}}$$

where $n_k$ is:

$$n_k = 2 \cdot (1 - \frac{\|f(\theta)_{k-1}\|_F - \|f(\theta)_k\|_F}{\|f(\theta)_{k-1}\|_F})^h$$

where $\|f(\theta)_{k-1}\|_F$ and $\|f(\theta)_k\|_F$ are the Frobenius norms of the objective function in the $k$ and $k - 1$ iterations. $h$ is an adjustable weight, and the $k$ value range is $k > 3$.

As the number of iterations increases, $n_k \approx 2$, and then $t_{k+1} \approx (1 + \sqrt{1 + 4n_k^2}) / 2$, which is consistent with the $t_{k+1}$ form in the NAG algorithm [29].

**C. MR IMAGE RECONSTRUCTION WITH THE GNACG_TV**

Compressed sensing theory can reconstruct images from sparse images in the transform domain. In this article, observations are obtained through the propeller sampling trajectory, and GNACG_TV is used to reconstruct the image. Compared with the conventional FSE (Fast Spin Echo) acquisition sequence, the propeller sampling trajectory has significantly reduced motion artifacts. Similar to a simple radial trajectory, there are still radial stripes in the image field of view. The sampling trajectory is shown in Fig. 1. The complete algorithm flow of GNACG_TV is as follows:
### Algorithm 1 CS-MR Image Reconstruction Based on the GNACG_TV Algorithm

**Inputs:**
- $Y$: undersampled MR image data in k-space
- $\psi$: Daubechies wavelet basis

**Initialization:**
- $k = 1, n_1 = 0.2, r_1 = 1, z_1 = 0, \lambda = 0.01, h = 5$

**While** $k < \text{max}_\text{iter}$
- $k > 3$
- compute $n_k$ by Eq. (15);
- compute $t_{k+1}$ by Eq. (14);
- compute $z_{k+1}$ by Eq. (13);
- compute $x_{k+1}$ and update the search direction by Eq. (12);
- $k = k + 1$

**Output:** reconstructed MR image $X = \Psi^T \theta$

### III. RESULTS

As the sample sampling ratio increased, the quality of MR image reconstruction also improved. Samples were selected at a 20% sampling ratio for the experiment, and the proposed method was compared with existing methods, such as the conjugate gradient (CG) algorithm, the conjugate gradient based on total variation (CG_TV) algorithm, the Nesterov accelerated conjugate gradient based on total variation (NACG_TV) algorithm, and the conjugate gradient based on adaptive moment estimation (ADAMCG) algorithm.

Figures 3-5 show the images of the abdomen, head and ankle reconstructed with different algorithms at a 20% sampling ratio. Figures 3-5(a)-(e) show the reconstructed MR images using the gradient descent algorithm with CG, CG_TV, NACG_TV, ADAMCG and GNACG_TV. Figures 3-5(f)-(j) show the local details of the above five algorithms to reconstruct the images. Figures 3-5(k)-(o) show the reconstruction error of the above five algorithms used to reconstruct the images. To facilitate observation, each error image was enhanced 5 times.

Table I lists the final convergence values of MSE, PSNR and SSIM of the abdomen, head and ankle MRI data using CG, CG_TV, NACG_TV, ADAMCG and GNACG_TV at a sample ratio of 20%. Table II lists the running time of the abdomen, head and ankle MRI data reconstruction using CG, CG_TV, NACG_TV, ADAMCG and GNACG_TV algorithms for 50 iterations at a sampling ratio of 20%, the time unit is second.

Figures 6-8 show the convergence curves of the MSE, PSNR and SSIM evaluation parameters after forcing 20 iterations using different algorithms at a 20% sampling ratio.

To verify the reconstruction effect of the proposed algorithm at different sampling ratios, the reconstruction effects of the abdomen MR images at other sampling ratios were compared. Figure 9 shows the reconstructed image, detailed image and residual image of the human abdomen at a sampling ratio of 40%. Figure 10 shows the differences in the evaluation indexes of reconstruction image quality at 10%, 30% and 40% sampling ratios in the abdomen.
Rician noise is added to the magnetic resonance image, and the noise level is measured by the standard deviation $\sigma$. Taking the abdomen image as an example at a 20% sampling ratio, Fig. 11 shows the denoising effect at a Rician noise level $s = 5 \sim 25$ and a detailed image at $s = 5 \sim 10$ and $s = 20 \sim 25$.
FIGURE 5. Reconstruction, local details and error images of different algorithms for ankle MR images at a 20% sampling ratio.

TABLE 1. The performance of the reconstructed image at a sampling ratio of 20%.

|       | Image | CG     | CG_TV  | NACG_TV | ADAMCG  | GNACG_TV |
|-------|-------|--------|--------|----------|----------|-----------|
| MSE   | abdomen | 55.70460 | 50.17655 | 47.73134 | 45.20953 | 44.52495  |
|       | head    | 59.55567 | 53.75051 | 51.73549 | 51.05112 | 49.66116  |
|       | ankle   | 51.61771 | 45.48582 | 41.73092 | 39.71958 | 36.31885  |
| PSNR  | abdomen | 36.72649 | 37.18039 | 37.39736 | 37.63310 | 37.69937  |
|       | head    | 36.43617 | 36.88157 | 37.04751 | 37.10535 | 37.22523  |
|       | ankle   | 37.05741 | 37.60664 | 37.98082 | 38.19535 | 38.58408  |
| SSIM  | abdomen | 0.99884 | 0.99907 | 0.99943 | 0.99990 | 0.99990   |
|       | head    | 0.99952 | 0.99968 | 0.99988 | 0.99998 | 0.99999   |
|       | ankle   | 0.98504 | 0.98487 | 0.98688 | 0.98809 | 0.98955   |

IV. DISCUSSION

In this article, the MSE, PSNR, SSIM and denoising effects are used to qualitatively and quantitatively analyze the GNACG_TV algorithm. The proposed algorithm is compared with the CG, CG_TV, NACG_TV and ADAMCG algorithms to prove the effectiveness of the GNACG_TV algorithm.

Figures 3-5(a)-(e) show the reconstructed images for the abdomen, head and ankle of the human body by the CG, CG_TV, NACG_TV, ADAMCG and GNACG_TV algorithms at a 20% sampling ratio. Figures 3-5(f)-(j) show the local details of the reconstructed image. Compared with the other four algorithms, the GNACG_TV algorithm can reduce Gibbs artifacts and show more local details and
The MSE, PSNR and SSIM of reconstructed abdomen MR images with different algorithms at a 20% sampling ratio.

The MSE, PSNR and SSIM of reconstructed head MR images with different algorithms at a 20% sampling ratio.

The MSE, PSNR and SSIM of reconstructed ankle MR images with different algorithms at a 20% sampling ratio.

Table 2. The running time of the five algorithms are iterated 50 times at a sampling ratio of 20%.

| Image   | Time of CG | Time of CG_TV | Time of NACG_TV | Time of ADAMCG | Time of GNACG_TV |
|---------|------------|---------------|------------------|----------------|------------------|
| abdomen | 9.8261     | 9.7661        | 9.8069          | 10.4688        | 9.8371           |
| head    | 9.9009     | 9.7915        | 9.8341          | 10.3039        | 9.8470           |
| ankle   | 10.0810    | 10.1403       | 10.2525         | 10.8487        | 10.2242          |

clearer contours. Figures 3-5(k)-(o) show the errors of the reconstructed image. The proposed algorithm can generate fewer error points and more accurately reconstruct the image. Figure 9 shows that at a higher sampling ratio, the proposed algorithm can provide richer and more accurate reconstructed image details.

To compare the performance of the five algorithms, the evaluation parameters were calculated in iterations. Figures 6-8 show that as the number of iterations increases, the MSE of the five algorithms gradually decreases, and PSNR and SSIM gradually increase. However, the convergence speed of the five algorithms is obviously different, and the convergence speed of the proposed algorithm is obviously faster than that of the other three algorithms. The proposed algorithm exhibits better performance than the other four algorithms.

Tables I shows that the algorithm demonstrates good stability and good reconstruction performance for MR images of different data sources and different parts of the human body.

Table II shows that the running time of the GNACG_TV algorithm is not significantly different from the CG, CG_TV, NACG_TV algorithms. The running time of the ADAMCG algorithm is the longest.
Figure 11 shows the denoising effect at different Rician noise levels. The denoising ability of the proposed algorithm is superior to that of the other four algorithms and exhibits better robustness performance.
V. CONCLUSION
In this article, we introduced a GNACG_TV algorithm for CS-MRI reconstruction. In terms of quantitative and qualitative analysis, compared with CG, CG_TV, NACG_TV and ADAMCG, GNACG_TV has good reconstruction efficiency and stability, higher performance in MSE, PSNR and SSIM, and better robustness in denoising performance. It was concluded that the proposed method can better reconstruct under-sampled MR images than other four methods, and can further improve the convergence speed based on Nesterov acceleration and get better reconstruction performance.

The contribution of this article is that we extend the acceleration factor in a generalized manner, so that the acceleration factor is related not only to the number of iterations but also to the iteration process. So as to achieve quickly converge in the early iterations and further improve the detail expressiveness of image reconstruction.

The proposed algorithm contains some parameters, the parameters are selected based on experience. Different choices affect the performance of the algorithm. In the future, the empirical parameters can be turned into formulas related to the iterative process to help reduce the uncertainty brought by the empirical parameters to the algorithm.

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XIUHAN LI received the M.S. degree in biomedical engineering from Nanjing Medical University, Nanjing, China. He is currently with the School of Biomedical Engineering and Informatics, Nanjing Medical University. His research interests include medical image processing and research on intelligent operation planning.

WEI WANG received the Ph.D. degree in communication engineering from Hohai University. He is currently with the School of Biomedical Engineering and Informatics, Nanjing Medical University, Nanjing, China. His research interests include biomedical signal and image processing, advanced medical equipment, and artificial intelligence.

SONGSHENG ZHU received the Ph.D. degree from Nanjing Normal University. He is currently with the School of Biomedical Engineering and Informatics, Nanjing Medical University, Nanjing, China. His research interests include biomedical signal analysis, medical electronics, and biomedical devices or instruments.

WEINTAO XIANG received the Ph.D. degree in signal, image, and vision from the University of Rennes 1, Rennes, France, in 2019. He is currently with the School of Biomedical Engineering and Informatics, Nanjing Medical University, Nanjing, China. His research interests include biomedical signal processing, pattern recognition, brain connectivity, and neural network analysis.

XIAOLING WU received the Ph.D. degree in condensed matter physics from Nanjing University. She is currently with the School of Biomedical Engineering and Informatics, Nanjing Medical University, Nanjing, China. Her research interests include nanomagnetic materials and biomedical signal processing.