GRAIN DYNAMICS IN MAGNETIZED INTERSTELLAR GAS

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ABSTRACT

The interstellar medium is turbulent, and this induces relative motions of dust grains. We calculate relative velocities of charged grains in a partially ionized magnetized gas. We account for the anisotropy of magnetohydrodynamic (MHD) turbulence, grain coupling with magnetic field, and the turbulence cutoff arising from the ambipolar drag. We obtain grain velocities for turbulence with parameters consistent with those in H I and dark clouds. These velocities are smaller than those in earlier papers, in which MHD effects were disregarded. Finally, we consider grain velocities arising from photoelectric emission, radiation pressure, and H2 thrust. These are still lower than the relative velocities induced by turbulence. We conclude that turbulence should prevent these mechanisms from segregating grains by size.

Subject headings: dust, extinction — ISM: kinematics and dynamics — magnetic fields

1. INTRODUCTION

Dust is an important constituent of the interstellar medium (ISM). It interferes with observations in the optical range, but it provides an insight into star formation activity through far-infrared radiation. It also enables molecular hydrogen formation and traces the magnetic field via emission and extinction polarization. The basic properties of dust (optical, alignment, etc.) strongly depend on its size distribution. The latter evolves as the result of grain collisions, whose frequency and consequences depend on grain-relative velocities.

Various processes can affect the velocities of dust grains. Radiation, ambipolar diffusion, and gravitational sedimentation all can bring about a dispersion in grain velocities. It was speculated in de Oliveira-Costa et al. (2002) that starlight radiation can produce the segregation of different-sized grains that is necessary to explain a poor correlation of the microwave and 100 μm signals of the foreground emission (Mukherjee et al. 2001). If true, it has big implications for the cosmic microwave background foreground studies. However, the efficiency of this segregation depends on random grain velocities, which we study in this Letter.

Interstellar gas is turbulent (see Arons & Max 1975). Turbulence was invoked by a number of authors (see Kusaka, Nakano, & Hayashi 1970; Volk et al. 1980; Draine 1985; Ossenkopf 1993; Weidenschilling & Ruzmaikina 1994) to provide substantial relative motions of dust particles. However, they discussed hydrodynamic turbulence. It is clear that this picture cannot be applied to the magnetized ISM since the magnetic fields substantially affect fluid dynamics. Moreover, dust grains are charged, and their interactions with magnetized turbulence is very different from the hydrodynamic case. This unsatisfactory situation motivates us to revisit the problem and calculate the relative grain motions in the magnetized ISM. In what follows, we use the model of MHD turbulence by Goldreich & Sridhar (1995, hereafter GS95), which is supported by recent numerical simulations (Cho & Vishniac 2000; Maron & Goldreich 2001; Cho, Lazarian, & Vishniac 2002a, hereafter CLV02). We apply our results to the cold neutral medium (CNM) and a dark cloud to estimate the efficiency of the coagulation, shattering, and segregation of grains.

2. MHD AND GRAIN MOTION

Unlike hydrodynamic turbulence, MHD turbulence is anisotropic, with eddies elongated along the magnetic field. This happens because it is easier to mix the magnetic field lines perpendicular to their direction than it is to bend them. The energy of eddies drops with the decrease of eddy size (e.g., \( v_p \sim l^{2/3} \) for the Kolmogorov turbulence), and it becomes more difficult for smaller eddies to bend the magnetic field lines. Therefore, the eddies get more and more anisotropic as their sizes decrease. As eddies mix the magnetic field lines at the rate of \( k \cdot v_p \), where \( k \) is a wavenumber measured in the direction perpendicular to the local magnetic field and \( v_p \) is the mixing velocity at this scale, the magnetic perturbations propagate along the magnetic field lines at the rate \( k_\parallel V_\parallel \), where \( k_\parallel \) is the parallel wavenumber and \( V_\parallel \) is the Alfvén velocity. The cornerstone of the GS95 model is a critical balance between those rates, i.e., \( k \cdot v_p \sim k_\parallel V_\parallel \), which may also be viewed as the coupling of eddies and wavelike motions. Mixing motions perpendicular to the magnetic field lines are essentially hydrodynamic (see CLV02), and therefore it is not surprising that GS95 predicted the Kolmogorov one-dimensional energy spectrum in terms of \( k_\perp \), i.e., \( E(k_\perp) \sim k_\perp^{-5/3} \) (see review by Cho, Lazarian, & Yan 2002, hereafter CLY02).

The GS95 model describes incompressible MHD turbulence. Recent research suggests that the scaling is approximately true for the dominant Alfvénic modes in a compressible medium with Mach numbers \( M \equiv V/\dot{C}_s \) of the order of unity (Lithwick & Goldreich 2001, hereafter LG01; CLY02; J. Cho & A. Lazarian 2002, in preparation), which is also consistent with the analysis of observational data (Lazarian & Pogosyan 2000; Stanimirovic & Lazarian 2001; CLY02). In what follows we apply the GS95 scaling to handle the problem of grain motions.

Because of turbulence anisotropy, it is convenient to consider separately grain motions parallel and perpendicular to the magnetic field. The motions perpendicular to the magnetic field are influenced by Alfvén modes, while those parallel to the magnetic field are subject to the magnetosonic modes. The scaling relation for perpendicular motion is \( v_p \propto k^{-5/3}_\perp (GS95) \). As the eddy turnover time is \( \tau_e \propto (k \cdot v_p)^{-1/3} \), the velocity may be expressed as \( v_p \approx v_{max} (\tau_e/\tau_{max})^{1/3} \), where \( \tau_{max} = l_{max}/v_{max} \) is the timescale for the largest eddies, for which we adopt the fiducial values \( l_{max} = 10 \) pc and \( v_{max} = 5 \) km s\(^{-1}\).
Grains are charged and coupled with the magnetic field. If the Larmor time $\tau_L = 2 m_e c / q B$ is shorter than the gas-drag time $t_{\text{drag}}$, then grain perpendicular motions are constrained by the magnetic field. In this case, grains have a velocity dispersion determined by the turbulence eddy, whose turnover period is approximately $\tau_L$, while grains move with the magnetic field on longer timescales. Since the turbulence velocity grows with the eddy size, the largest velocity difference occurs on the largest scale at which grains are still decoupled. Thus, following the approach in Draine (1985), we can estimate the characteristic grain velocity relative to the fluid as the velocity of the eddy with a turnover time equal to $\tau_L$:

$$v_i(a) = \frac{v_{\text{max}}}{l_{\text{max}}} \left( \frac{\rho_g}{\rho_a} \right)^{1/3} \left( \frac{8 \pi^2 c}{3 q B} \right)^{1/2} a^{3/2}. \quad (1)$$

The relative velocity of grains to each other should be approximately equal to the larger one of the grains’ velocities, i.e., the larger grain’s velocity,

$$\delta v_i(a_1, a_2) = \frac{v_{\text{max}}}{l_{\text{max}}} \left( \frac{\rho_g}{\rho_a} \right)^{1/3} \left( \frac{8 \pi^2 c}{3 q B} \right)^{1/2} \max(a_1, a_2)^{1/2}$$

$$= 1.4 \times 10^5 \text{ cm s}^{-1} (v_a a_s) \sqrt{q / l_{\text{drag}}, B},$$

in which $v_a = v_{\text{max}} / l_{\text{drag}}$ cm s$^{-1}$, $a_s = a / 10^{-5}$ cm, $q_e = q / 1$ electron, $l_{\text{drag}} = l_{\text{drag}} / 10$ pc, $B = 1 \mu G$, and the grain density is assumed to be $\rho_g = 2.6 \text{ g cm}^{-3}$.

Grain motions parallel to the magnetic field are induced by the compressive component of the slow mode with $v_i \propto k_{\parallel}^{1/2}$ (CLV02; LG01; CLY02). The eddy turnover time is $\tau_L \propto (v_y a_y)^{-1}$, so the parallel velocity can be described as $v_i \approx v_{\text{max}} \tau_L / l_{\text{drag}}$. For grain motions parallel to the magnetic field, the Larmor precession is unimportant, and the gas-grain coupling takes place on the translational drag time $t_{\text{drag}}$. The drag time due to collisions with atoms is essentially the time for collision with the mass of gas equal to the mass of grain, $t_{\text{drag}} = (\rho_a n / \pi) \pi (m / 4 \mu B T)^{1/2}$, where $\mu$ is the mass of gas species. The ion-grain cross section due to the long-range Coulomb force is larger than the atom-grain cross section (Draine & Salpeter 1979). Therefore, in the presence of collisions with ions, the effective drag time decreases, $t_{\text{drag}} = \alpha t_{\text{drag}}$, where $\alpha < 1$ is the function of a particular ISM phase. The characteristic velocity of grain motions along the magnetic field is approximately equal to the parallel turbulent velocity of eddies with turnover time equal to $t_{\text{drag}}$,

$$v_i(a) = \frac{v_{\text{max}}^2}{l_{\text{max}}} \theta \left( \frac{\rho_g}{\rho_a} \right)^{1/3} \left( \frac{2 \pi}{\mu B T} \right)^{1/2} a,$$ \quad (3)$$

and the relative velocity of grains for $T_{\text{ion}} = T / 100$ K is

$$\delta v_i(a_1, a_2) = \frac{v_{\text{max}}^2}{l_{\text{max}}} \theta \left( \frac{\rho_g}{\rho_a} \right)^{1/3} \left( \frac{2 \pi}{\mu B T} \right)^{1/2} \max(a_1, a_2)$$

$$= 1.0 \times 10^6 \text{ cm s}^{-1} (\alpha a_s) (n l_{\text{ion}} T_{\text{ion}}^{1/2}).$$ \quad (4)$$

When $\tau_L > t_{\text{drag}}$, grains are no longer tied to the magnetic field. Since at a given scale, the largest velocity dispersion is perpendicular to the magnetic field direction, the velocity gradient over the grain mean free path is maximal in the direction perpendicular to the magnetic field direction. The corresponding scaling is analogous to the hydrodynamic case, which was discussed in Draine (1985): $\delta v_i(a_1, a_2) = v_{\text{max}} \theta (l_{\text{max}} / \tau_{\text{drag}})^{1/2}$, i.e.,

$$\delta v_i(a_1, a_2) = \alpha \theta \left( \frac{v_{\text{max}}}{l_{\text{max}}} \right)^{1/2} \left( \frac{\rho_g}{\rho_a} \right)^{1/3} \left( \frac{2 \pi}{\mu B T} \right)^{1/4} \max(a_1, a_2)^{1/2}. \quad (5)$$

Turbulence is damped because of viscosity when the cascading rate $v_j k_j$ equals the damping time $t_{\text{drag}}$ (see Cho, Lazarian, & Vishniac 2002b). If the mean free path for a neutral particle $l_n$ in a partially ionized gas with density $n_{\text{ion}} = n_i + n_e$ is much less than the size of the eddy in consideration, i.e., $l_n < 1$, the damping time is $t_{\text{drag}} \approx \nu_c k_j^2 \sim (n_{\text{ion}}/m_n) (l_{\text{drag}}/v_{\text{drag}})^{-1} k_j^2$, where $\nu_c$ is the effective viscosity produced by neutrals. In the present Letter we consider cold gas with low ionization; therefore, the influence of ions on $l_n$ is disregarded. Thus, the turbulence cutoff time in a neutral medium is

$$\tau_{\text{c}} \approx \frac{l_n}{v_n} \left( \frac{l_{\text{max}}}{l_{\text{drag}}} \right)^{1/2} \left( \frac{\nu_c}{l_{\max}} \right)^{1/2} \left( \frac{v_{\text{drag}}}{l_{\text{drag}}} \right),$$ \quad (6)$$

where $v_n$ and $\nu_c$ are the velocity of a neutral and Alfvén velocity, respectively. It is easy to see that for $\tau_{\text{c}}$ longer than either $t_{\text{drag}}$ or $t_{\text{L}}$, the grain motions get modified. A grain samples only a part of the eddy before gaining the velocity of the ambient gas. In GS95’s picture, the shear rate $\partial v / \partial t$ increases with the decrease of eddy size. Thus, for $\tau_{\text{c}} > \tau_{\text{L}}$, these smallest available eddies are the most important for grain acceleration. Consider first the perpendicular motions. If $v_i$ is the velocity of the critically damped eddy, the distance traveled by the grain is $\Delta l \sim v_i \min(t_{\text{drag}}, \tau_{\text{e}})$. Thus, the grain experiences the velocity difference $\Delta l (\partial v / \partial t) \sim v_i \min(t_{\text{drag}}, \tau_{\text{e}}) / \tau_{\text{c}}$. Due to the critical balance in GS95’s model, the shear rate along the magnetic field is $\partial v / \partial t = v_i k_j \sim v_i l_{\text{drag}} / (v_{\text{drag}} \tau_{\text{e}})$. Therefore, the grain experiences a velocity difference $v_{\text{drag}} / \tau_{\text{c}}$ times smaller, i.e., $\sim v_{\text{drag}}^2 / \tau_{\text{c}}$.

3. Discussion

3.1. Shattering and Coagulation

Consider the CNM with temperature $T = 100$ K, density $n_n = 30$ cm$^{-3}$, electron density $n_e = 0.045$ cm$^{-3}$, and magnetic field $B \sim 1.3 \times 10^{-5}$ G (Weingartner & Draine 2001a, hereafter WD01a). To account for the Coulomb drag, we use the results of WD01a and get the modified drag time $t_{\text{drag}} = \alpha t_{\text{drag}}$. Using the electric potentials in Weingartner & Draine (2001b), we get grain charge and $\tau_{\text{c}}$.

For the parameters given above, we find that $t_{\text{drag}}$ is larger than $\tau_{\text{c}}$ for grains larger than $10^{-6}$ cm and that $\tau_{\text{c}}$ is smaller than $\tau_{\text{L}}$ even for grains as large as $10^{-5}$ cm. Here we only consider grains larger than $10^{-6}$ cm, which carry most of the grain mass (~80%) in the ISM, so we can still use equation (3) to calculate grain parallel velocities and equation (1) to get the perpendicular velocity for grains larger than $10^{-5}$ cm. Nevertheless, the perpendicular velocities of grains smaller than $10^{-5}$ cm should be estimated as $v_i(a) = v_i (\tau_{\text{L}} / \tau_{\text{drag}}) = v_{\text{max}} (\tau_{\text{L}} / \tau_{\text{drag}})^{1/2} (\tau_{\text{L}} / \tau_{\text{L}})^{1/2} v_i(a)$ (given by equation (1)). The results are shown in Figure 1.

The critical sticking velocity was calculated in Chokshi, Tielens, & Hollenbach (1993; see also Dominik & Tielens...
fig. 1.—Grain velocities as a function of radii (solid line) in the CNM. The dash-dotted line represents parallel velocity due to the drag by compressible modes, and the dotted line refers to perpendicular velocity from the contribution of the drag by Alfven mode; also plotted is the hydrodynamic result (dashed line). The change of the slope is caused by the cutoff of turbulence by ambipolar diffusion. The grain velocity driven by H2 thrust is plotted to illustrate the issue of grain segregation in the CNM (see text). The part marked by open circles is nonphysical because thermal flipping is not taken into account.

1997). However, experimental work by Blum (2000) shows that the critical velocity is an order of magnitude larger than the theoretical calculation. Thus, the collisions can result in coagulation for small silicate grains ($\lesssim 3 \times 10^{-6}$ cm$^{-3}$).

With our input parameters, grains do not shatter if the shattering threshold for silicate is $2.7 \, \text{km s}^{-1}$ as in Jones, Tielens, & Hollenbach (1996). Nevertheless, the grain velocities strongly depend on $v_{\text{max}}$ at the injection scale. For instance, we will get a cutoff of $6 \times 10^{-3}$ cm because of shattering if $v_{\text{max}} = 10 \, \text{km s}^{-1}$.

For a dark cloud, the situation is different. As the density increases, the drag by gas becomes stronger. Consider a typical dark cloud with temperature $T = 20 \, \text{K}$, density $n_H = 10^6 \, \text{cm}^{-3}$ (Chokshi et al. 1993), and magnetic field $B \sim 2.3 \times 10^{-4}$ G. Assuming that dark clouds are shielded from radiation, grains get charged by collisions with electrons: $\langle q \rangle = 0.3(r/10^3 \, \text{cm})e$. The ionization in the cloud is $n_H/n_{\text{act}} \sim 10^{-6}$, and the drag by neutral atoms is dominant. From equation (6) and the expression for the drag time and the Larmor time, we find $\tau_L < \tau_{\text{drag}}$ for grains of sizes between $10^{-6}$ and $4 \times 10^{-6}$ cm, and $\tau_{\text{drag}} < \tau_L$ for grains larger than $4 \times 10^{-6}$ cm. In both cases, the turbulence cutoff $\tau_\text{cutoff}$ is smaller than $\tau_{\text{drag}}$ and $\tau_L$.

Thus for the smaller grains, we use equations (1) and (3) to estimate grain velocities. For larger grains, grain velocities are given by equation (5).

Our results for dark clouds show only a slight difference from the earlier hydrodynamic estimates. Since the drag time $\tau_{\text{drag}} \propto n^{-1}$ and Larmor time $\tau_L \propto B^{-1} \propto n^{-1/2}$, the grain motions are less affected by the magnetic field as the cloud becomes denser. Thus, we agree with the conclusion of Chokshi et al. (1993) that densities well in excess of $10^4 \, \text{cm}^{-3}$ are required for coagulation to occur. Shattering will not happen because the velocities are small, so there are more large grains in dark clouds. This agrees with observations (see Mathis 1990).

3.2. Grain Segregation and Turbulent Mixing

Our results are also relevant to grain segregation. Grains are the major carrier of heavy elements in the ISM. The issue of grain segregation may have a significant influence on the ISM metallicity. Subjected to external forcing, e.g., because of radiation pressure, grains gain size-dependent velocities with respect to gas. WD01a have considered the forces on dust grains exposed to anisotropic interstellar radiation fields. They included photoelectric emission, photodesorption, as well as radiation pressure and calculated the drift velocity for grains of different sizes. The velocities they got for silicate grains in the CNM range from $0.1$ to $10^3 \, \text{cm s}^{-1}$. Figure 1 shows that the turbulence produces larger velocity dispersions. Thus, the grain segregation of very small and large grains speculated in de Oliveira-Costa et al. (2002) is unlikely to happen for typical CNM conditions.

A different mechanism of driving grain motions is a residual imbalance in “rocket thrust” between the opposite surfaces of a rotating grain (Purcell 1979). This mechanism can provide grain-relative motions and preferentially move grains into molecular clouds. It is easy to see that due to averaging caused by grain rotation, the rocket thrust is parallel to the rotation axis. Three causes for the thrust were suggested by Purcell (1979): the spatial variation of the accommodation coefficient for impinging atoms, photoelectric emission, and H formation. The latter was shown to be the strongest among the three. The uncompensated force in this case arises from the difference of the number of catalytic active sites for H formation on the opposite grain surfaces. The nascent H atoms leave the active sites with kinetic energy $E$, and the grain experiences a push in the opposite direction. The number of active sites varies from one grain to another, and we should deal with the expectation value of the force for a given distribution of active sites.

Because of the internal relaxation of energy (see Lazarian & Draine 1999a, 1999b and a review by Lazarian 2000), the grain rotational axis tends to be perpendicular to the largest $b$-$b$ surface. Adopting the approach in Lazarian & Draine (1997), we get the mean square root force of H$_2$ thrust on a grain in the shape of a square prism with dimensions $b \times b \times a$ ($b > a$),

$$\langle F_{\text{H}_2} \rangle = r^{3/2}(r+1)^{1/2}\gamma(1-\gamma)n_Hv_Ha^2\left(\frac{2m_HE}{\nu}\right)^{1/2},$$

where $r = b/2a$, $n_H \equiv n(H) + 2n(H_2)$, $y = 2n(H_2)/n_H$ is the H$_2$ fraction, $\gamma$ is the fraction of impinging H atoms, and $\nu$ is the number of active sites over the grain surface. The expected grain velocity is $v = \langle F_{\text{H}_2} \rangle t_{\text{drag}}/m$. In the CNM we consider, $y = 0$, adopting the characteristic values in Lazarian & Draine (1997), $r = 1$, $\gamma = 0.2$, $E = 0.2 \, \text{eV}$, and the density of active sites is $10^{13} \, \text{cm}^{-2}$, so that $\nu = 80(a/10^{-3} \, \text{cm})^2(r+1)$, and we get the “optimistic” velocity shown in Figure 1. For maximal active site density $10^{15} \, \text{cm}^{-2}$, we get the lower boundary of

2 There are obvious misprints in the numerical coefficient of eq. (7) in Chokshi et al. (1993) and in the power index of Young’s modulus in eq. (28) of Dominik & Tielens (1997).
grain velocity \( v = 3.3 \times 10^{-5} \text{ cm/s} \). The scaling is approximate because of the complexity of the coefficient \( \alpha \) (see WD01a, Fig. 16).

Lazarian & Draine (1999a, 1999b) have shown that when subjected to \( \text{H}_2 \) torques alone, grains \( \leq 10^{-4} \text{ cm} \) should experience frequent thermal flipping, which means that the \( F_{\text{\tiny{H}}} \) fluctuates. This flipping results from the coupling of grain rotational and vibrational degrees of freedom through internal relaxation and would average out \( F_{\text{\tiny{H}}} \). However, the flipping rate depends on the value of the grain angular momentum (Lazarian & Draine 1999a). If a grain is already spun up to a sufficient velocity, it gets immune to thermal flipping. Radiative torques (Draine & Weingartner 1996) can provide efficient spin if the grain size is comparable to the wavelength. For a typical interstellar diffuse radiation field, the radiative torques are expected to spin up grains with sizes larger than \( \sim 4 \times 10^{-4} \text{ cm} \). They will also align grains with rotational axes parallel to the magnetic field. Thus, grains should acquire velocities along the magnetic field lines, and the corresponding velocities should be compared with those arising from turbulent motions parallel to the magnetic field. It is clear from Figure 1 that for the chosen set of parameters, the effect of \( \text{H}_2 \) thrust is limited. All in all, we conclude that the radiation effects and \( \text{H}_2 \) thrust are not efficient for segregating grains in typical ISM conditions.

4. SUMMARY

We have calculated the relative motions of dust grains in a magnetized turbulent fluid, taking into account turbulence anisotropy, turbulence damping, and grain coupling with the magnetic field. We find that these effects decrease the relative velocities of dust grains compared to the earlier hydrodynamically-based calculations. The difference is substantial in the CNM but less important for dark clouds. For the CNM we find that coagulations of silicate grains happen for sizes \( \leq 3 \times 10^{-4} \text{ cm} \). The force due to \( \text{H}_2 \) formation on the grain surface might drive small grains \( \leq 10^{-4} \text{ cm} \) to larger velocities, but the thermal flipping of grains suppresses the forces for grains less than \( 4 \times 10^{-4} \text{ cm} \). We conclude that radiation and \( \text{H}_2 \) thrust are not capable of segregating grains.

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