Consequences of Triplet Seesaw for Leptogenesis

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Abstract

We present the various leptogenesis scenarios which may occur if, in addition to the ordinary heavy right-handed neutrinos, there exists a heavy scalar $SU(2)_L$ triplet coupled to leptons. We show that the contributions of the right-handed neutrinos and the triplet to the lepton asymmetry are proportional to their respective contributions to the neutrino mass matrix. A consequence of the triplet contribution to the lepton asymmetry is that there is no more upper bound on the neutrino masses from leptogenesis due to the fact that the neutrino mass constraints do not necessarily induce asymmetry washout effects. We also show how such a triplet leptogenesis mechanism may emerge naturally in the framework of the left-right symmetric theories, such as Pati-Salam or $SO(10)$.

1 Introduction

Recent evidence for neutrino masses, and the belief that these masses are presumably associated to lepton number violation, have established the leptogenesis mechanism\textsuperscript{1} as today’s favorite explanation of the baryon asymmetry of the universe. In the usual seesaw\textsuperscript{2} mechanism the lepton asymmetry is generated through the decay of heavy Majorana right-handed neutrinos, the same ones that lead to small neutrino masses. This scenario is particularly natural in the framework of theories that predict the existence of heavy right-handed neutrinos, such as $SO(10)$\textsuperscript{3}, Pati-Salam\textsuperscript{4} and left-right symmetric theories\textsuperscript{4,5} in general.

A possible loophole in the ordinary seesaw mechanism is that in the framework of these unified theories, the heavy neutrinos provide neither the only possible source of light neutrino masses nor the only possible source of lepton asymmetry generation. In a large class of renormalizable left-right symmetric theories, it is well known that the existence of triplet Higgses also induces neutrino masses via the so-called type II seesaw mechanism\textsuperscript{6}. This alternative can naturally provide a connection\textsuperscript{7} between the large atmospheric
mixing and $b - \tau$ unification in the context of the minimal supersymmetric $SO(10)$ theory.

In this letter we analyze the effects of the triplet for the leptogenesis mechanism. We show that if the triplet type II seesaw dominates the neutrino masses, diagrams involving the triplet will also in general dominate the leptogenesis. Even if the triplet is heavier than the lightest right-handed neutrino this will be the case via the decay of this lightest right-handed neutrino and a diagram involving a virtual triplet. If instead, the triplet is lighter than all the right-handed neutrinos then it is the decay of the triplet to two leptons involving virtual right-handed neutrinos which will dominate. The suggestion that the triplet could be important for leptogenesis, in the context of the left-right model, was already made in Ref. [9] where also the relevant diagrams were exhibited. The diagrams for the decay of the right-handed neutrinos were also exhibited and estimated in Ref. [10]. Here we calculate the corresponding CP asymmetries and relate them to the neutrino mass constraints.

2 The three types of lepton asymmetries

It is useful to start with the minimal situation where in addition to the SM particles there are three heavy right-handed neutrinos and one heavy $SU(2)_L$ Higgs triplet. From the asymmetries we will obtain in this case, one can derive easily the corresponding asymmetries in the more realistic left-right or $SO(10)$ models. The relevant Lagrangian of this model reads:

$$
\mathcal{L} \ni - \frac{1}{2} M_{N_i} \bar{N}^T_R C N_R i - M^2_\Delta T \tau \Delta^T_L - H^\dagger \bar{N}_R (Y_N)_{ij} \psi^T_j L
$$

with $\psi_i = (\nu_i, l_i)^T$, $H = (H^0, H^-)^T$ and

$$
\Delta_L = \begin{pmatrix} \frac{1}{\sqrt{2}} \delta^+ & \delta^{++} \\ \delta^0 & -\frac{1}{\sqrt{2}} \delta^+ \end{pmatrix}.
$$

In the presence of these interactions, the triplet acquires a vev which will be naturally seesaw suppressed if the triplet is heavy: $\langle \delta^0 \rangle = v_L \simeq \mu^* v^2 / M^2_\Delta$. Due to this vev there are now in general two sources of neutrino masses:

$$
M_\nu = -Y^T_N M^{-1}_N Y_N v^2 + 2Y_L v_L
$$

where the first term is the ordinary “type I” seesaw induced by the right-handed neutrinos and the second term is the triplet “type II” seesaw mass term, with $v = 174$ GeV.

In this framework, depending on the values of the masses and couplings, the leptogenesis can be obtained either from the decay of the right-handed neutrinos or from the decay of the triplet. From the decay of the right-handed neutrinos to leptons and Higgs scalars the CP asymmetry is given as usual by:

$$
\varepsilon_{N_k} = \sum_i \frac{\Gamma(N_k \rightarrow l_i + H^*) - \Gamma(N_k \rightarrow \bar{l}_i + H)}{\Gamma(N_k \rightarrow l_i + H^*) + \Gamma(N_k \rightarrow \bar{l}_i + H)}.
$$
This asymmetry is given by the interference of the ordinary tree level decay with the 3 diagrams of Fig. 1. The first two diagrams are the ordinary vertex and self-energy diagrams involving another (virtual) right-handed neutrino and give

\[
\varepsilon_{N_k} = \frac{1}{8\pi} \sum_j \frac{\text{Im}[(Y_N Y_N^\dagger)^2_{kj}]}{\sum_i |(Y_N)_{ki}|^2} \sqrt{x_j} \left[1 - (1 + x_j) \log(1 + 1/x_j) + 1/(1 - x_j)\right],
\]

where \(x_j = M^2_{N_j}/M^2_{N_k}\). The third diagram of Fig. 1, which was already displayed without calculations in Ref. [9] and estimated in Ref. [10] involves a virtual triplet and is a new contribution. Calculating it we obtain

\[
\varepsilon_{\Delta N_k} = \frac{1}{2\pi} \sum_j \frac{\text{Im}[(Y_N)_{ki}(Y_N)_{kl}(Y_\Delta^\dagger)_{il}]}{\sum_i |(Y_N)_{ki}|^2 M_{N_k}} \left(1 - \frac{M^2_\Delta}{M^2_{N_k}} \log(1 + M^2_{N_k}/M^2_\Delta)\right).
\]

Note that the tree level decay width is not affected by the existence of the triplet:

\[
\Gamma_{N_k} = \frac{1}{8\pi} M_{N_k} \sum_i |(Y_N)_{ki}|^2.
\]

From the decay of the triplet to two leptons an asymmetry can also be produced. It is given by the interference of the tree level process with the one-loop vertex diagram, given in Fig. 2, involving a virtual right-handed neutrino [9]. Note that with one triplet alone there is no self-energy diagram, and therefore without at least one right-handed neutrino no asymmetry can be produced. At least two triplets are necessary in order to produce an asymmetry without right-handed neutrinos, in which case the asymmetry comes from self-energy diagrams as was shown in Refs. [11, 12] and also used in Ref. [13]. Here we will restrict ourselves to the case where there is only one \(SU(2)_L\) triplet coupled to leptons (as it is in general the case in left-right and \(SO(10)\) models, both ordinary and

\[
\varepsilon_{\Delta L} = \frac{1}{8\pi} \sum_j \frac{\text{Im}[(Y_N)_{ki}(Y_N)_{kl}(Y_\Delta^\dagger)_{il}]}{\sum_i |(Y_N)_{ki}|^2 M_{N_k}} \left(1 - \frac{M^2_\Delta}{M^2_{N_k}} \log(1 + M^2_{N_k}/M^2_\Delta)\right).
\]
supersymmetric). Calculating the asymmetry from Fig. 2 we obtain:

$$\varepsilon_{\Delta} = \frac{2 \cdot \Gamma(\Delta^L \rightarrow l + l) - \Gamma(\Delta_L \rightarrow \bar{l} + \bar{l})}{\Gamma(\Delta^L \rightarrow l + l) + \Gamma(\Delta_L \rightarrow \bar{l} + \bar{l})}$$

$$= \frac{1}{8\pi} \sum_k M_{N_k} \sum_{il} \text{Im}[(Y_N^*)_{ki}(Y_N^*)_{li}(Y_{\Delta})_{ii}] \log(1 + M^2_{\Delta}/M^2_{N_k})$$

while the triplet decay width to two leptons and two scalar doublets is given by:

$$\Gamma_{\Delta} = \frac{1}{8\pi} M_{\Delta} \left( \sum_{ij} |(Y_{\Delta})_{ij}|^2 + |\mu|^2 \right).$$

Note that there is such an asymmetry for each of the three components of the triplet. In the case where the lighter right-handed neutrino and the triplet have approximately the same mass and same order of magnitude couplings, all 3 types of asymmetries of Eqs. (5), (6) and (9) can play an important role. In the following we will discuss the limiting cases where one process dominates over the others. We will distinguish four such cases.

2.1 Case 1: $M_{N_1} \ll M_{\Delta}$ with a dominant contribution of the right-handed neutrinos to the light neutrino masses

In the limit where the triplet couplings to two leptons are negligible with respect to the leading right-handed neutrino Yukawa couplings, and with at least one right-handed neutrino much lighter than the triplet, the triplet has a negligible effect for both the neutrino masses and the leptogenesis. This is equivalent to the ordinary right-handed neutrino scenario without the triplet. Only the 2 diagrams of Fig. 1.a and Fig. 1.b have a non-negligible effect for leptogenesis. This case has been extensively studied in the literature (see e.g. [1], [14]-[30]) and we have nothing to add here to it.

2.2 Case 2: $M_{N_1} \ll M_{\Delta}$ with a dominant triplet contribution to the light neutrino masses

If $M_{N_1} \ll M_{\Delta}$, the decays of the right-handed neutrino(s) will dominate the lepton asymmetry production. Under the assumption that these neutrinos have a hierarchical pattern, only the decay of the lightest heavy neutrino $N_1$ is important for the asymmetry and Eq. (5) can be rewritten as

$$\varepsilon_{N_1} = \frac{3}{16\pi} \frac{M_{N_1}}{v^2} \frac{\sum_{i\ell} \text{Im}[(Y_N)_{ii}(Y_N)_{\ell\ell}(M_\nu^{I*})_{i\ell}]}{\sum_i |(Y_N)_{ii}|^2},$$

while under the assumption that the triplet is sizeably heavier than $N_1$, Eq. (6) can be rewritten as

$$\varepsilon_{N_1}^{\Delta} = \frac{1}{8\pi} \frac{M_{N_1}}{v^2} \frac{\sum_{i\ell} \text{Im}[(Y_N)_{ii}(Y_N)_{\ell\ell}(M_\nu^{I*})_{i\ell}]}{\sum_i |(Y_N)_{ii}|^2}.$$
neutrino mass matrix, it is very likely to dominate also the lepton asymmetry production through Eq. (12).\(^1\)

An interesting feature of this scenario with respect to the one of case 1 is that since the couplings responsible for the neutrino masses are not anymore responsible for the tree level decay, the neutrino mass constraints will not induce violation of the out-of-equilibrium condition

$$\Gamma_{N_1} < H(T)|_{T=M_{N_1}} = \sqrt{\frac{4\pi^3 g_*}{45}} \frac{T^2}{M_{\text{Planck}}}|_{T=M_{N_1}}. \quad (13)$$

where \(g_*\) is the number of degrees of freedom active at the temperature of the asymmetry production (\(g_* \sim 100\)). Therefore the washout effects from inverse decays and associated scatterings will be avoided.\(^2\) As a consequence there is no upper bound on the neutrino masses as in the usual type I seesaw model.\(^3\) Moreover, the \(Y_N\) couplings constrained by the out-of-equilibrium conditions for the decay width and scatterings can essentially cancel between the numerator and the denominator in each of Eqs. (6) and (12), leaving an asymmetry depending above all on the \(Y_{\Delta}\) and \(\mu\) couplings. Since the triplet is heavier than \(N_1\), the out-of-equilibrium conditions on these couplings are much weaker and therefore there is essentially no constraint from these conditions on the size of the asymmetry. Here, the larger the neutrino masses from the triplet, the larger the asymmetry and this is not accompanied by larger washout effects as in the usual case.

It is also interesting to see in this case what happens to the upper bound\([23, 24, 25]\) on the asymmetry (or equivalently lower bound on the right-handed neutrino masses) due to the fact that neutrino masses are bounded from above. From Eq. (12) we see that such a bound still exists since the asymmetry is directly proportional to the neutrino mass matrix generated by the triplet. If we assume a hierarchical pattern of light neutrino masses (i.e. \(m_{\nu}^{\text{max}} \simeq \sqrt{\delta m^2_{\text{atm}}} \simeq 0.05\) eV\([31]\)), this bound is of the same order as in the case 1, i.e. \(M_{N_1} > \text{few} \times 10^8\) GeV. Note that in this case where the triplet diagram dominates the asymmetry, this bound is an absolute bound unlike for the pure right-handed neutrino case, because in the (single) triplet case there is no self-energy diagram as the one of Fig. 1.b (which allows one to avoid this bound through a resonance behavior if the right-handed neutrino masses are degenerate\([17, 19]\)). Therefore the triplet doesn’t allow much progress in being able to lower substantially the leptogenesis scale, which would be welcome

\(^1\)There is however no one to one correspondence between the contribution to the neutrino masses and to leptogenesis since the neutrino mass matrix is not a number but a 3 by 3 complex matrix. For example imagine a case where both the \((Y_N)_{1j}\) as well as the \((Y_{\Delta})_{ji}\) and \(\mu\) have negligible phases with non-negligible phases for the \((Y_N)_{2j}\) and/or \((Y_N)_{3j}\). Then the two diagrams of Fig. 1.a.b can still dominate the asymmetry production. However in this case one must be careful because, if the \(Y_N\) couplings have a negligible contribution to the neutrino masses, the produced asymmetry could be easily too small.

\(^2\)As well known (see e.g. \([21]\) in the type I seesaw model of neutrino masses and the corresponding leptogenesis, a value of \(m_{\nu} \lesssim 0.05\) eV\([31]\) requires a Yukawa coupling which if associated to the decaying right-handed neutrino gives \(\Gamma_{N_1}/H(M_{N_1}) \gtrsim 2 \times 10^{-3}M_{\text{Planck}}\sqrt{\delta m^2_{\text{atm}}/v^2} \sim 10 - 100\) independently of the mass of the right-handed neutrino, resulting in suppression effects of similar order. These suppression effects are not present in Eqs. (6) and (12).

\(^3\)This bound in the usual scenario (i.e. \(m_{\nu} \lesssim 0.12\) eV\([21]\)) is due to the fact that, the larger the (degenerate) light neutrino masses, the larger have to be some of the \((Y_N)_{1j}\) couplings, which induce more violation of the out-of-equilibrium condition \(\Gamma_{N_1} < H(M_{N_1})\) and hence more associated washout effects. This obviously doesn’t hold here.
for the eventual gravitino problem.\textsuperscript{4} The precise calculation of the asymmetries and of the corresponding $M_{N_1}$ lower limit in the supersymmetric case is called for and is left for a future publication.

In order to show quantitatively that leptogenesis can be easily generated in this case and in order to illustrate the above discussion, as an example, let us consider the case where the Yukawa couplings $(Y_N)_{ij}$ of $N_1$ gives $K_{N_1} = \Gamma_{N_1}/H(M_{N_1}) \sim 1/10$. In this case the out-of-equilibrium condition is well satisfied and there is no sizeable washout effect associated to the $(Y_N)_{ij}$ couplings. The contribution of the $(Y_N)_{ij}$ couplings to the neutrino masses does not exceed $M_\nu \sim (Y_N)_{ij}^2 v^2/M_{N_1} \sim (10^{-12} \text{ GeV}) K_{N_1} \sim 10^{-4} \text{ eV}$ and is therefore negligible with respect to $\sqrt{\Delta m^2_{\text{atm}}} \simeq 0.05 \text{ eV} \text{[31]}$ and $\sqrt{\Delta m^2_{\text{sol}}} \simeq 0.008 \text{ eV \text{[33]}}$. Let us assume in addition for simplicity that the Yukawa couplings of the heavier right-handed neutrinos $N_{2,3}$ are also relatively small and that the triplet is responsible for the neutrino masses (with for example $M_\Delta \sim (10 - 1000) M_{N_1}$). In this case there is no substantial washout from diagrams involving the triplet. This is due to the fact that the triplet doesn’t couple directly to $N_1$ and therefore there are no scatterings with a virtual triplet which enter into the Boltzmann equation ruling the number density of $N_1$. The only potentially dangerous scatterings enter into the Boltzmann equation ruling the lepton number density, for instance scatterings such as $l + l \to \Delta \to H + H$. These however are not relevant since, on the one hand, the on-shell triplet contribution is already largely Boltzmann suppressed at the temperature of the $N_1$ decay, and since on the other hand, the off-shell triplet contribution is also suppressed by mass and couplings factors. As a result, in this example, it is not necessary to consider the explicit Boltzmann equations for the calculation of the produced lepton asymmetry. The asymmetry will be given safely by $n_L/s \sim \varepsilon_{N_1}^\Delta /g_*$. For example the values $M_{N_1} = 10^{10} \text{ GeV}$, $\mu = 10^{11} \text{ GeV}$, $M_\Delta = 10^{12} \text{ GeV}$, $(Y_N)_{ij}^{\text{max}} = 2 \cdot 10^{-4}$, $(Y_\Delta)_{il}^{\text{max}} = 10^{-2}$ give $m_\nu^{\text{max}} \simeq \sqrt{\Delta m^2_{\text{atm}}}$ \text{[31]} and $\varepsilon_N^\Delta \simeq 10^{-6}$ assuming a maximal phase. Taking $g_* \simeq 10^2$ we obtain an asymmetry as large as $n_L/s \simeq 10^{-8}$ which gives more than enough freedom to get the needed value $n_L/s \sim 10^{-10}$ \text{[32]} (by reducing the phase for example). In this numerical example, an increase in $m_\nu^{\text{max}}$ (resulting from increasing the $Y_\Delta$ and $\mu$ couplings) results simply in a linear increase of the produced lepton asymmetry.

Finally note that all the discussion above for this case remains true even for $M_\Delta \sim M_{N_1}$, as long as the triplet contribution dominates the neutrino masses as we assumed here. In particular, except if there are cancellations in the interplay of phases, $\varepsilon_{N_1}^\Delta$ remains dominant over $\varepsilon_{N_1}$ and even $\varepsilon_\Delta$. The only additional restriction is that the relation $\Gamma_\Delta < H(M_\Delta)$ will be violated by at least a factor $\sim 10 - 100$ due to neutrino mass constraints (i.e. $\Gamma_\Delta/H(M_\Delta) \gtrsim 2 \cdot 10^{-3} M_{\text{Planck}} \sqrt{\Delta m^2_{\text{atm}}}/v^2$). This induces a washout from scatterings such as $l + l \to \Delta \to H + H$ which will be effective at the temperature of the $M_{N_1}$ decay, but this washout will be quite moderate because it intervenes only in the Boltzmann equation of the lepton number density, not in the one of the $N_1$ number density. This leaves enough freedom to have successful leptogenesis even for neutrino masses well above the other cosmological bounds \text{[32]}.}

\textsuperscript{4}Note that, except if there are large cancellations between the type I and type II seesaw contributions to the neutrino masses, this bound on $M_{N_1}$ still holds in the case of a mixed scenario where both type I and type II contributions would be important for both neutrino masses and leptogenesis as long as the right-handed neutrinos are not closely degenerate in mass.
2.3 Case 3: $M_{N_1} >> M_\Delta$ with a dominant right-handed neutrino contribution to the neutrino masses

Let us now consider a possibility that the triplet is much lighter than all the right-handed neutrinos. In this case leptogenesis will be dominated by the decay of the triplet to two leptons and the one loop diagram is the one of Fig. 2. If in addition the type I seesaw mechanism dominates the neutrino masses, the asymmetry of Eq. (9) becomes

$$\varepsilon_\Delta = -\frac{1}{8\pi} \frac{M^2_\Delta}{(\sum_{ij} |(Y_\Delta)_{ij}|^2 M^2_\Delta + |\mu|^2)} \frac{1}{v^2} \Im[(M_\nu^{I*})_{il}(Y_\Delta)_{il}\mu^s].$$

(14)

This can perfectly well lead to the needed lepton asymmetry (this case being obtained from case 2 by inverting the role of the right-handed neutrinos and the triplet). As in the case 2, there is no upper bound on the neutrino masses from leptogenesis because the couplings responsible for the neutrino masses are not responsible for the tree level decay. Moreover, also as in the case 2, the couplings constrained by the out-of-equilibrium conditions for the decay width and associated scatterings can essentially cancel between the numerator and denominator in each of Eqs. (9) and (14), leaving an asymmetry depending above all on the $Y_N$ couplings which are much less constrained by the out-of-equilibrium conditions. Note however that, in contrast to the case 2, the decaying particle is not a $SU(2)_L \times U(1)$ gauge singlet. Therefore there is the additional constraint that the triplet has not to be kept in thermal equilibrium by the gauge scatterings down to a temperature sizeably below its mass (otherwise an asymmetry can be created only from a Boltzmann suppressed number density of triplets). This puts the lower limit on $M_\Delta$ above the lower limit on $M_{N_1}$ in the previous cases.

To illustrate this, one can consider a parameter configuration just opposite to the one of the example in the case 2, i.e. with suppressed triplet couplings and larger right-handed neutrino Yukawa couplings. Considering such a configuration, with for example $M_\Delta = 10^{13}$ GeV, one can show that a large enough lepton asymmetry can be produced. We estimate that a successful leptogenesis can also occur for $M_\Delta$ down to $\sim 10^{11-12}$ GeV\cite{12,35} for light neutrino masses below a few tenth of eV. An explicit calculation, beyond the scope of the present letter, with full Boltzmann equations would be necessary to see exactly down to which value of the triplet and right-handed neutrino masses leptogenesis can be successfully generated.

Note also that a lower bound on the triplet mass still holds here from neutrino constraints, in a similar way as in the case 2, but this one is not useful as it is below the lower bound on the triplet mass from the gauge scattering washout effects.

Note finally that if the right-handed neutrinos dominate the neutrino masses as in this case, even for $M_{N_1} \sim M_\Delta$, $\varepsilon_\Delta$ will still be important for leptogenesis as long as the triplet mass is large enough to avoid the gauge scattering effects. In this case $\varepsilon_\Delta$ is of order $\varepsilon_{N_1}$ and both dominate over $\varepsilon_\Delta$, see Eqs. (5), (6) and (9). The interplay of Boltzmann equations is quite involved in this case.

2.4 Case 4: $M_{N_1} >> M_\Delta$ case with dominance of the triplet contribution to the neutrino masses

In this case too an asymmetry can be produced from the decay of the triplet to two leptons, i.e. the diagram of Fig. 2 and Eq. (9). In terms of the neutrino mass matrix generated by
the triplet, with \( M_\Delta << M_{N_1} \), \( \varepsilon_\Delta \) can be rewritten as

\[
\varepsilon_\Delta = \frac{1}{16\pi M_{N_1} v^2} \frac{\Im[(M_\Delta'^T)_{di}(Y_{N}^*)_{k_i}(Y_{N}^*)_{k_i}]}{\left(\sum_{ij} \left|(Y_\Delta)_{ij}\right|^2 + \frac{|\mu|^2}{M_\Delta^2}\right)}.
\]

(15)

There are quite a few constraints here:

- First, in this case the asymmetry will be proportional to the \( Y_N \) couplings which do not dominate the neutrino masses, and in this sense it is suppressed with respect to all other cases; compare Eq. (15) with Eqs. (12) and (14).

- Second, as in the case 3, the triplet needs to have a mass large enough to avoid large damping effects from gauge scatterings.

- In addition, due to the fact that the couplings responsible for the decay width are also responsible for the neutrino masses, the condition \( \Gamma_\Delta < H(M_\Delta) \) will be violated by at least a factor \( \sim 10^{-100} \) due to the atmospheric neutrino constraints, similarly to the case 1 discussed in the footnote 2 (i.e. \( \Gamma_\Delta/H(M_\Delta) \gtrsim 2 \cdot 10^{-3} M_{Planck} \sqrt{\delta m^2_{atm}}/v^2 \)). In fact for this condition there is even less freedom than in the case 1 because in the case 1 the condition \( \Gamma_{N_1} < H(M_{N_1}) \) can still be satisfied in the hierarchical case or inverse hierarchical case by assuming that \( N_1 \) has small Yukawa couplings (the heaviest light neutrino masses being induced by \( N_{2,3} \)). Since there is only one triplet, we don’t have this freedom and this condition is always violated.

- Moreover, due to this fact that the couplings responsible for the decay width are also responsible for the neutrino masses, there is now as in the case 1 an upper bound on neutrino masses.

Altogether this leads to a case which can still successfully generate the leptogenesis although within a narrower range of parameters than the other cases. The upper bound on neutrino masses and the lower bound on \( M_\Delta \) depend highly on how negligible we assume the type I seesaw contribution for the neutrino masses. They result from a rather involved interplay of Boltzmann equations beyond the scope of this letter.

3 Implications from and for the left-right model

A heavy \( SU(2)_L \) triplet \( \Delta_L \) emerges naturally in any left-right symmetric theory with a renormalizable see-saw mechanism. We discuss this generic feature in a prototype \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) model, but it equally well applies to theories such as Pati-Salam or \( SO(10) \). In this theory, the leptons are left and right-handed doublets \( \psi_{iL,R} = (\nu_i, l_i)^T \) and the analog of the standard model Higgs \( H \) is a bi-doublet \( \Phi \) (consisting of two \( SU(2)_L \) doublets). The right-handed neutrinos receive their masses through the \( SU(2)_R \) triplet \( \Delta_R \); this in turn, due to \( L-R \) symmetry necessarily implies the existence of our \( SU(2)_L \) triplet \( \Delta_L \). The relevant Yukawa and Higgs potential terms which reproduce the Lagrangian of Eq. (1) now read as

\[
\mathcal{L} \ni - \sum_{ij} \left[ (Y_{\Delta})_{ij} \psi^T_{iL} C i \tau_2 \Delta_L \psi_{jL} + \psi^T_{iR} C i \tau_2 \Delta_R \psi_{jR} \right] + Y_{ij} \tilde{\psi}_{iR} \Phi_1 \psi_{jL} - Y_{ij} \tilde{\psi}_{iR} \Phi_2 \psi_{jL} + \lambda_{ij} Tr \Delta^A_{R} \Phi_1 \Delta_L \Phi_j^T + h.c.
\]

(16)
with
\[
\Phi = \left( \begin{array}{c} \phi_1^0 \\
\phi_2^0 \\ \phi_1^+ \\
\phi_2^+ \end{array} \right), \quad \Delta_{L,R} = \left( \begin{array}{cc} \frac{1}{\sqrt{2}} \delta^+ & \delta^0 \\
\frac{1}{\sqrt{2}} \delta^0 & -\frac{1}{\sqrt{2}} \delta^+ \end{array} \right)_{L,R},
\]
and \(\Phi_1 = \Phi, \Phi_2 = \tau_2 \Phi^* \tau_2\). In writing Eq. (10) we have used the following \(SU(2)_L \times SU(2)_R\) transformations properties: \(\psi_{L,R} \rightarrow U_{L,R} \psi_{L,R}; \Delta_{L,R} \rightarrow U_{L,R} \Delta_{L,R} U_{L,R}^\dagger; \Phi \rightarrow U R \Phi U_L^\dagger\). We have also used the left-right symmetry, i.e. symmetry under: \(\psi_L \leftrightarrow \psi_R, \Delta_L \leftrightarrow \Delta_R, \Phi \leftrightarrow \Phi^\dagger\).

Denote, next, \(<\phi_1^0> = v_1\), and introduce the notation \(H_i = (\phi_1^0, \phi_1^-)^T\). It is easy to show that \(H = (v_1 H_1 + v_2 H_2)/\sqrt{v_1^2 + v_2^2}\) corresponds to the standard model Higgs doublet, whereas \(H' = (v_2 H_1 - v_1 H_2)/\sqrt{v_1^2 + v_2^2}\) has zero vev and picks up a mass proportional to \(M_R = <\Delta_R>\) and decouples from the low energy sector. Comparison with Eq. (11), says that after diagonalizing \(Y_\Delta\),
\[
M_{N_i} = 2(Y_{\Delta}^{\text{diag}})_{ii} M_R
\]
gives Dirac neutrino Yukawa couplings as a function of \(Y^{(1)}\) and \(Y^{(2)}\)
\[
Y_N = \frac{Y^{(1)} v_1 + Y^{(2)} v_2}{\sqrt{v_1^2 + v_2^2}},
\]
and
\[
\mu = \frac{(\lambda_{11} + \lambda_{22}) v_1 v_2 + \lambda_{12} v_1^2 + \lambda_{21} v_2^2}{v_1^2 + v_2^2} M_R.
\]

The main message is very simple. For large \(M_R\), we have effectively the situation as in section 2, but with one important proviso for the type II seesaw: right and left-handed neutrino masses are proportional to each other (given by \(Y_\Delta\) and the \(<\Delta_R>\) and \(<\Delta_L>\) respectively) and so are their eigenvalues. This must be taken into account for a quantitative analysis of the asymmetry.

Three more comments are in order:

- First, in the \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) theory the right-handed neutrino masses, and \(M_{\Delta_L}\) and \(\mu \sim \lambda <\Delta_R>\) are clearly not predictable and there is no way of knowing which particle is lightest and therefore responsible for leptogenesis. A natural prejudice, encouraged by what we know in the standard model, is a hierarchy of \(M_{N_i}\) and not so light scalar particles, (i.e. \(M_{\Delta_L} \gg M_{N_i}\)). This, to us, makes the cases 1 and 2 more plausible than 3 and 4, although not necessarily correct. Keep also in mind that a heavy \(\Delta_L\) can still naturally dominate the light neutrino masses; after all Yukawa couplings \(Y_N\) could be much smaller than \(Y_\Delta\). In this sense the case 2 is not less plausible than the case 1.

- Second, we don’t know the mass of the second doublet \(H'\) in the bi-doublet \(\Phi\). As we said, unless we fine tune it, it is naturally of order \(M_R\), but it is not clear whether it lies above or below the mass of decaying particle responsible for the leptogenesis. We assume here that \(H'\) is heavy enough not to affect leptogenesis. We do this only for the reason of simplicity, but it is straightforward to generalize it to the case of a lighter \(H'\); one must only take into account the fact that \(\mu\) becomes a \(2 \times 2\) matrix and keep an index \(i\) on \(Y_N^{(i)}\), \(i = 1, 2\).
Third, for the case where \(M_{N_1} < M_\Delta\), we assume that the \(SU(2)_R\) gauge bosons are heavy enough to avoid too large damping effects from the corresponding gauge scatterings. To this end, it has been estimated that these interactions are out-of-equilibrium for \(M_{W_R} > (2 \cdot 10^5 \text{ GeV}) (M_{N_1}/10^2 \text{ GeV})^{3/4}\). This should be the object of a more detailed study. It is likely that for specific sets of parameters, leptogenesis can be successfully generated for even smaller values of \(M_{W_R}\).

Let us see now what happens to \(\varepsilon_{N_1}^{\Delta}\) in this theory. From \((Y_N)_{ij} = \frac{1}{2} \delta_{ij} M_{N_i} M_{N_j} / M_\Delta\) in the basis of diagonal \(M_N\), one obtains from Eq. (6) for the case of hierarchical \(N\) and for \(M_\Delta \gg M_{N_1}\) (i.e. in the, to us, most interesting case 2):

\[
\varepsilon_{N_1}^{\Delta} \simeq - \frac{1}{8\pi} \sum_j M_{N_j} M_{N_1} \Im (Y_N)_{1j} (Y_N)_{1j}^\dagger M_R \sum_i |(Y_N)_{1i}|^2.
\]

This can be contrasted with \(\varepsilon_{N_1}\) in the same limit

\[
\varepsilon_{N_1} \simeq - \frac{3}{16\pi} \sum_j M_{N_j} \Im (\sum_j (Y_N)_{1j} (Y_N)_{1j}^\dagger) \sum_i |(Y_N)_{1i}|^2.
\]

Let us take for illustration the same example of the section 2 with hierarchical masses for the \(N\), say \(M_{N_1} \simeq 10^{10}\ \text{ GeV}, M_{N_2} \simeq 10^{12}\ \text{ GeV}, M_{N_3} \simeq 10^{13}\ \text{ GeV}\) and \(M_R \simeq 10^{15}\ \text{ GeV}\).

Clearly for \(Y_N^2 v^2 / M_N \lesssim \sqrt{\delta m_{sol}^2} \sim 0.008\ \text{ eV}\) as dictated by the dominance of the triplet seesaw, one gets \(\varepsilon_{N_1} \lesssim 10^{-7}\) to be compared with \(\varepsilon_{N_1}^{\Delta} \simeq 10^{-6}\).

Last but not least, note that the lower bound \(M_{N_1} \gtrsim \text{ few } 10^8\ \text{ GeV}\) in the case of hierarchical right-handed neutrinos translates in a lower limit about one order of magnitude higher on the scale of \(SU(2)_R\) symmetry breaking \(M_R\), or in other words on the mass of the right-handed gauge bosons.

4 Summary

The analysis of leptogenesis and neutrino masses with a triplet in addition to right-handed neutrinos is somewhat involved but still, a clear picture emerges. If \(M_\Delta > M_{N_1}\), the triplet will have in general a non-negligible contribution to the leptogenesis as soon as it has a non-negligible contribution to the neutrino masses, even if the triplet is much heavier than \(N_1\) (see the case 2). If \(M_\Delta < M_{N_1}\), with \(M_\Delta \gtrsim 10^{11-12}\ \text{ GeV}\), even with rather small couplings (i.e. not necessarily dominating the neutrino masses), the triplet diagrams will in general dominate the production of the asymmetry (see cases 3 and 4). If \(M_\Delta < M_{N_1}\), with \(M_\Delta \lesssim 10^{11-12}\ \text{ GeV}\), the triplet will not be able to produce the asymmetry due to gauge scattering effects and it must have in addition tiny couplings not to wash out the asymmetry (which in this case can be produced by the pure right-handed neutrino contribution). The situation which definitely allows most freedom in the parameter space is the one of the case 2 with triplet seesaw neutrino masses and leptogenesis from the decay of the right-handed neutrino(s) (i.e. with a triplet mass larger or of order the lightest right-handed neutrino mass). The most constrained situation is with the dominance of the triplet for the neutrino masses and a triplet mass lighter than all right-handed neutrino masses (case 4).
One of the consequences of the triplet contribution to leptogenesis is that there is no more an upper limit on neutrino masses from leptogenesis (see the cases 2 and 3). This is due to the fact that in $\epsilon^\Delta_N$ and $\epsilon^\Delta$, the couplings responsible for the tree level decay are not necessarily the ones dominating the neutrino masses. Therefore the neutrino mass constraints do not necessarily imply violation of the out-of-equilibrium conditions on the decay width and associated scatterings, even for degenerate light neutrino masses (see the cases 2 and 3). Moreover these couplings responsible for the tree level decay can essentially cancel in the numerator and denominator of the asymmetry, leaving an asymmetry depending above all on other couplings much less constrained by the out-of-equilibrium conditions. The size of the asymmetry is bounded from above only by the size of the neutrino masses (which results in the lower bound on $M_{N_1}$ similar to the one of the usual case without the triplet, i.e. $M_{N_1} \gtrsim \text{few } 10^8 \text{ GeV}$).

In left-right symmetric theories with the seesaw mechanism, the existence of the triplet $\Delta_L$ is a must if one sticks to renormalizability. Regarding neutrino masses, it is a priori impossible to know whether they originate from the triplet or right-handed neutrinos (or both). In this sense it is equally probable that the triplet or right-handed neutrinos play an important role for neutrino masses and therefore for leptogenesis. It is still to be seen what happens in more constrained theories such as the minimal renormalizable $SO(10)$ theory, where the small number of parameters allows one in principle to find out the origin of the seesaw mechanism and in turn the origin of the lepton asymmetry.

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