Theory of Thermodynamic Variables of Rubber Band Heat Engine

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Abstract. Rubber band heat engine is a heat engine that is easily applied in the experiment. However, to get the data from the experimental results are required a formulation that is able to accommodate the data, so that it will be obtained an accurate value. We show and analyze the variables thermodynamic formulation of rubber band heat engine to accommodate the experimental data, so that the equation of state, heat, work and efficiency are not only studied theoretically but also experimentally. The engine’s efficiency is calculated for an idealized but reasonable model. The engine’s work cycle is compared with a Carnot cycle, and it is shown to be equivalent to the Carnot cycle as an extremely ideal limiting case. We measured the force law parameters for a working model, and we obtained the efficiency of this model.

Index Term: Thermodynamics, Rubber Band Heat Engine

1. Introduction
Rubber band have interesting thermodynamic characteristics that can be used to develop heat engine and refrigeration cycles [1]. If most of the solid material will extend when heated, then the rubber band shows different properties, which will contract when heated. By utilizing the thermal properties of the rubber band, then a lot of research is made to study rubber bands as material conversion of heat engine to change heat to mechanical energy [2-5].

As an ideal gas, rubber band also has a thermodynamic variables, such as equation of state, heat capacity, Carnot cycle, work, and efficiency. In the learning process, thermodynamic variables of rubber band have been researched [6-8]. However, they did not consider about technical matters that may occur in experiments to determine the formula of work and heat engine efficiency, so in this paper, we will be explained thermodynamic variables of rubber band by considering the technical aspects to obtain the value of work and optimum efficiency when the experiment is done.

2. Rubber Band Heat Engine
The basic design of rubber band heat engine using specific thermodynamic characteristics of a rubber band. To observe the process of converting heat into mechanical energy, then the heat engine is designed to rotate as in Figure 1.
As seen in Figure 1, the engine will rotate when the bulb lamp turned on. The heat from the bulb causes the change in tension of the balloons and changes the position of the center of mass of heat engine system. This process has resulted in an imbalance of heat engine system. As for the rotating direction of the engine shown in Figure 2.

\[
\tau = K T \left( \frac{L}{L_0} - \frac{L_0^2}{L^2} \right)
\]

(1)

with \( \tau \) is tension (N) and \( L_0 \) is initial length (m), \( L \) is final length (under tension), \( K \) is a constant that depends on the temperature of the elastic material (N / K), and \( T \) is the temperature (K).

First law of thermodynamics is used to get another thermodynamic variables, such as heat capacity, heat energy, and work of rubber band as Equation 2.
\[ dU = dQ + dW \tag{2} \]

and

\[ dW = -\tau \, dL \tag{3} \]

where \( dU \), \( dQ \) is heat energy, \( dW \) is the work, and \( dL \) is the change in length of rubber band when the temperature changed.

Equation (2) is the first law of thermodynamics is used to describe the rubber band heat engine system and Equation (3) describes the work done by the system. Equation (2) and (3) will produce two constants, i.e heat capacity at a constant length \( (C_L) \) and heat capacity at a constant tension \( (C_\tau) \).

\[ C_L = \left( \frac{\partial U}{\partial T} \right)_L \tag{4} \]

\[ C_\tau = C_L - \tau L \alpha \tag{5} \]

where \( \alpha \) is length extensivity.

4. Heat of Rubber Band Heat Engine

Heat energy is the energy that out and into the system. Heat energy from the heat reservoir with high temperature is called the heat in and the heat energy out of the system towards the cold reservoir with low temperature is called the heat out. To calculate the heat of each process, \( a \to b \), \( b \to c \), \( c \to d \) and \( d \to a \), as shown in Figure 2 for a rubber band heat engine, will be calculated heat values of the four processes that will be a heat engine cycle in the rubber band system. The processes that occur in this cycle is a quasistatic process like the Carnot cycle, i.e two adiabatics and two isothermal.

Because \( U = U(T, L) \), then Equation (2) and (3) can be rewritten as in equation (6).

\[ dQ = \left( \frac{\partial U}{\partial T} \right)_L \, dT + \left( \frac{\partial U}{\partial L} \right)_T \, dL - \tau dL \tag{6} \]

For the adiabatic process, because \( dL \equiv 0 \) then Equation (6) becomes,

\[ Q_{ab} = \int_{T_a}^{T_b} C_L \, dT \tag{7} \]

\[ Q_{cd} = \int_{T_c}^{T_d} C_L \, dT \tag{8} \]

For isothermal process, because \( dT = 0 \) and tension \( (\tau) \) is changed, so that Equation (6) becomes,

\[ Q = -\int \tau dL \tag{9} \]

Because \( L = L(T, \tau) \), then \( dL \) can be defined as

\[ dL = \left( \frac{\partial L}{\partial T} \right)_\tau \, dT + \left( \frac{\partial L}{\partial \tau} \right)_T \, d\tau \tag{10} \]

Substituting Equation (1) and (10) to Equation (3), it is obtained,

\[ Q = \int \tau(L,T) \left[ \left( \frac{\partial L}{\partial T} \right)_\tau \, dT + \left( \frac{\partial L}{\partial \tau} \right)_T \, d\tau \right] \tag{11} \]

Equation (11) is a general equation that shows the heat received by the system in the elastic material, while isothermal process is a process where the value of \( dT = 0 \), so that equation (11) becomes,
\[Q = \int \left( \frac{\partial L}{\partial \tau} \right) d\tau\]  \hspace{1cm} (12)

Substituting Equation (10) to Equation (12) then we make integral of it, will be obtained a heat value of isothermal process.

\[Q = \frac{L}{2AY} \left( \tau_f^2 - \tau_i^2 \right)\]  \hspace{1cm} (13)

\(Y\) is the Young's modulus and the magnitude is

\[Y = \frac{\tau}{A} + \frac{3KL_0^2}{AL^2}\]  \hspace{1cm} (14)

Taking the assumption that the value of \(Y = Y_0 =\) constant, where \(Y_0\) is the Young's modulus at \(\tau = 0\), then

\[Y = \frac{3KT}{A}\]  \hspace{1cm} (15)

From the above, the heat occurs in the process bc and da is,

\[Q_{bc} = \frac{L}{6KT_H} \left( \tau_c^2 - \tau_b^2 \right)\]  \hspace{1cm} (16)

\[Q_{da} = \frac{L}{6KT_C} \left( \tau_a^2 - \tau_d^2 \right)\]  \hspace{1cm} (17)

5. Work of Rubber Band Heat Engine
Assuming that no change of energy in one cycle in the system, then the work of rubber band heat engine can be written

\[W_{ab} = -\int_{\tau_b}^{\tau_c} C_L dT \]  \hspace{1cm} (18)

\[W_{cd} = -\int_{\tau_d}^{\tau_c} C_L dT \]  \hspace{1cm} (19)

\[W_{bc} = -\frac{L}{6KT_H} \left( \tau_c^2 - \tau_b^2 \right)\]  \hspace{1cm} (20)

\[W_{da} = -\frac{L}{6KT_C} \left( \tau_a^2 - \tau_d^2 \right)\]  \hspace{1cm} (21)

6. Efficiency of Rubber Band Heat Engine
The efficiency of heat engine is defined as,

\[\eta = \frac{W}{Q_{in}}\]  \hspace{1cm} (22)

With \(W = W_{ab} + W_{bc} + W_{cd} + W_{da}\) and \(Q_{in} = Q_{ab} + Q_{bc}\), we obtained

\[\eta = \frac{W_{ab} + W_{bc} + W_{cd} + W_{da}}{Q_{ab} + Q_{bc}}\]  \hspace{1cm} (23)
\[ \eta = \frac{-\frac{L}{6KT_H}(\tau_c^{-2} - \tau_b^{-2}) - \frac{L}{6KT_c}(\tau_a^{-2} - \tau_d^{-2}) + \int_{\tau_c}^{\tau_d} C_V dT + \frac{L}{6KT_H}(\tau_c^{-2} - \tau_b^{-2})}{\tau_c} \]  \tag{24}

7. Comparison with Past Rubber Band Heat Engine
Both equations of work and efficiency of a rubber band heat engine obtained is the equations whose data is more easily observed through experimentation. So the value of work and efficiency of the calculation results will be more accurate. Unlike the case with employment equality and efficiency are calculated by Mullen [4] for which data are difficult to observe experimentally, so the equations are more theoretical ones.

8. Conclusion
We have obtained the equations of thermodynamic variables of rubber band heat engine. The equations make the data from the experiment by using rubber band heat engine is more easily to be calculated.

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