The Relationship between Fungal Growth Rate and Temperature and Humidity

Zhichao Zhan\textsuperscript{1}, Meiling Xu\textsuperscript{2}, Yue Li\textsuperscript{3} and Meihua Dong\textsuperscript{4}

\textsuperscript{1}Student, College of Science, Yanbian University, Yanji, P.R., CHINA
\textsuperscript{2}Student, College of Science, Yanbian University, Yanji, P.R., CHINA
\textsuperscript{3}Student, College of Science, Yanbian University, Yanji, P.R., CHINA
\textsuperscript{4}Senior Lecturer and Master Tutor, Department of Mathematics, School of Science, Yanbian University, CHINA

\textsuperscript{1}Corresponding Author: zzc010324@163.com

\section*{ABSTRACT}

In order to determine the relation among the three factors of wood fiber decomposition rate, mycelial elongation and moisture resistance, our team resorted to the Monod equation and the modified Logistic equation. Combing with the kinetic principle and the law of mass action, the equations between the decomposition rate of wood fiber, the elongation rate of mycelium and the moisture resistance were established. In the course of solving the model, we found that when the temperature ranges from 24\textdegree C to 28\textdegree C and the relative humidity from 60\% to 75\%, the growth rate of fungi is the fastest.

\textit{Keywords}— Monod Equation, The Modified Logistic Equation, The Law Of Mass Action

\section*{I. INTRODUCTION}

The carbon cycle is the foundation of life. Suffice it to say, once the carbon cycle suspends, the biological system will immediately collapse. However, the carbon cycle is closely related to the decomposition of wood fiber. On top of that, fungi, which plays a pivotal role in the decomposition of wood fiber, must be taken seriously. Therefore, it is more than paramount to figure out that under what circumstances the fungi can decompose wood fibers most effectively. But in what environment will the fungus exert its greatest effect?

Our team introduced the Monod equation and the modified Logistic equation to study the rate of fungi decomposing lignocellulosic fibers, and further obtain the optimal growth temperature and humidity.

\section*{II. DECOMPOSITION OF WOOD FIBER}

\begin{equation}
X + S \rightarrow C \rightarrow (1 + Y_{S})X
\end{equation}

\subsection*{2.1 Modelling}

The main challenge of simulated decomposition is to determine which fungal parameters and mechanisms matter in predicting lignocellulosic fibers. Based on the exploration and integration of most data, we found that there are two main factors affecting fungal decomposition of lignocellulosic fibers. One is fungal hyphae extension rate, that is, the growth rate of mycelium. The other is the moisture resistance of fungi, which means that fungi have different growth rates in environments with different humidity. Meanwhile, the growth of fungi is also affected by temperature. The elongation rate of mycelium of fungi is the main factor in the decomposition of wood fiber. The faster the mycelium is extended, the faster the decomposition of wood fiber will be.

First, we use first-order Monod kinetics to describe the large-scale flow of wood fiber through various nutritional pathways. In this way, the relationship between growth rate and moisture resistance is established. At the same time, it is postulated that the decomposition rate of wood fiber is directly proportional to the growth rate of fungi, and fungal biomass is still an integral part of the decomposition model. Then use a two-level model to describe the influence of different growth environment factors (temperature) on fungal growth.

\subsection*{2.2 Factoring: Fungal Growth Rate & Moisture Resistance}

We use Monod equation to describe the specific growth of fungi in a specific litter environment. However, wood fibers are usually dispersed in the continuous phase surrounding the fungus. Once the substrate reaches the fungal body through the transport mechanism, the metabolic process leads to the formation of new fungi through the reproduction mechanism. In this way, the Monod equation is based on the following reaction format [2]:

(1.)
Where $X$ is the fungal cell, $S$ is the wood fiber, and $C$ is the complex formed by the absorbed wood fiber and the enzyme contained in the fungal cell body. $k_c$ is the rate constant of complex formation, and $k_x$ is the rate constant of fungal cell formation. Besides, the parameter $Y_{x/s}$ is the consumption coefficient of the substrate to the fungal cells. Although the kinetic scheme is a simple view of the mechanism involved in the growth of fungal cells from a depleted substrate, the scheme provides a basis for the systematic derivation of the Monod equation. The first kinetic step reflects the transportation of wood fiber from a large number of fungi to the cell surface/large number of substrate/enzyme complexes; the second kinetic step involves metabolic processes that result in the formation of new microbial cells from substrate consumption. From the law of conservation of mass, the following expression can be obtained:

\[ \frac{dX}{dt} = -k_cXS + (1 + Y_{x/s})k_xC \]
\[ \frac{dS}{dt} = -k_cXS \]
\[ \frac{dC}{dt} = k_cXS - k_xC \]

Let $\epsilon_X = k_x^{-1}$, and $\epsilon_X$ is the time constant of microorganism reproduction. Therefore, equation set (2) can be regarded as a singularly perturbed system with singular parameters $\epsilon_X$. Output is as follows:

\[ \frac{dX}{dt} = \mu_{max} \frac{XS}{K_M + S} \]

Among them, $K_M = \frac{k_x}{1 + Y_{x/s}}$ is the Monod constant, which reflects the ratio between the rate of complex formation of cells and the rate of complex formation. $\mu_{max} = \frac{Y_{x/s}k_x}{1 + Y_{x/s}}$ is the maximum specific growth rate of fungi. So the specific growth rate of autocatalysis $\mu = \frac{1}{X \frac{dX}{dt}} = \frac{\mu_{max}S}{K_M + S}$.

From this we get the joint dynamic model of fungus and wood fiber as follows:

\[ \frac{dX}{dt} = \mu_{max} \frac{XS}{K_M + S} \]
\[ \frac{dS}{dt} = -\mu_{max} \frac{XS}{K_M + S} \]
\[ Y = \frac{X - X_0}{S_0 - S} \]

Among them, $X_0$ and $S_0$ are the initial values of fungus and wood fiber. In order to make the calculation simple and in line with common sense, we set $X(0) = X_0 > 0, S(0) = S_0 > 0$.

Since the stronger the moisture resistance of the fungus, the greater the decomposition rate of wood fiber,

\[ \mu_{max} = \left(1 + \frac{k_xK_s}{S_0k_x + X_0}\right) \frac{X}{X_0} - \frac{K_x}{S_0k_x + X_0} \frac{1}{S_0} \frac{1}{X_0} \frac{1}{S_0} \]
\[ \mu_{max} = \left(1 + \frac{k_xK_s}{S_0k_x + X_0}\right) \frac{X}{X_0} - \frac{K_x}{S_0k_x + X_0} \frac{1}{S_0} \frac{1}{X_0} \frac{1}{S_0} \]

We use the modified Logistic equation to describe the relationship between microbial growth and time under certain culture conditions, through which we can get [3]
Where \( X_t \) is the number of microorganisms at time \( t \), \( X_0 \) is the initial number of microorganisms, \( X_{max} \) is the maximum number of microorganisms, \( \mu_{max} \) is the maximum specific growth rate of microorganisms, \( t_i \) is the time to reach \( 1/2X_{max} \), and \( t_{lag} \) is the lag time, and \( t \) is the time.

By solving the model, we can get the relation between fungal growth rate and fungal moisture resistance as

\[
\lambda = \frac{k_1 \times K_s + b_1}{K_s^2 + k_2 \times K_s + b_2}
\]

Where \( \lambda \) is the growth rate of the fungus, \( k, b \) are the parameters to be tested, and \( K_s \) is the humidity resistance factor.

### 2.3 The Introduction of Environmental Factors

The growth of fungi is also affected by temperature. So what is the relation between temperature and fungal growth rate? We use the established first-order equation to fit the relationship between temperature and specific growth rate at this temperature is 0), and \( T \) is the current temperature

The quadratic polynomial model:

\[
\mu_{max} = a_\mu \times T^2 + b_\mu \times T + c_{lag}
\]

\[
t_{lag} = a_{lag} \times T^2 + b_{lag} \times T + c_{lag}
\]

By comprehensively solving equations (11), (12) and (1), the relation between temperature and fungal hyphae elongation can be obtained as

\[
\lambda = a_1 \sin(b_1 T + c_1) + a_2 \sin(b_2 T + c_2)
\]

Among them are \( a_1, a_2, b_1, b_2, c, c_2 \), parameters to be determined, \( T \) is temperature, and \( T \) is the experimental temperature.

### 2.4 The Relationship between Decomposition Rate and Fungal Growth Rate

According to the law of conservation of mass, we can find:

\[
\left\{ \begin{array}{l}
\lambda = \frac{X S}{K_M + S} \\
\theta = \frac{X S}{K_M + S}
\end{array} \right.
\]
Where $\lambda$ is the elongation rate of fungal hyphae, $\theta$ is the substrate decomposition rate, and $a$ is the undetermined coefficient.

Therefore, according to the above formula, we can obtain a linear relation between the substrate decomposition rate $\theta$ and the fungal hyphae elongation rate, indicating that the higher the fungal hyphae elongation rate, the faster the substrate consumption.

### III. SOLVING

#### 3.1 Temperature

Considering there are many fungi working simultaneously in wood decomposition, we will first study the effect of a single fungus on the rate of substrate consumption, and then go deep. According to the information available, there are four main types of fungi that decompose wood fibers: white rot fungi, brown rot fungi, soft rot fungi and agaric wood rot fungi. From our experimental data, we can get the decomposition rates under different humidity conditions.

![Figure 1: White-Rot Fungus](image)

![Figure 2: Brown-Rot Fungus](image)

![Figure 3: Soft-Rot Fungus](image)

![Figure 4: Umbrella-Wood-Rot Fungus](image)

Figure 1-4 shows the growth of white rot fungi, brown rot fungi, soft rot fungi and agaric wood rot fungi under the same temperature and different relative humidity, also known as, the decomposition rate. With the increase of humidity, the decomposition rate of wood fiber by the four fungi increased slowly first, and then decreased rapidly after reaching the peak. The decomposition rate of wood fiber can be higher when the relative humidity ranges from 60% to 75%. The error square sum $R^2 = [0.9492, 0.9549, 0.9658, 0.9633]$ are all small, indicating that the modified model can better describe the decomposition rate of fungi under different humidity conditions.

#### 3.2 Moisture Resistance

By means of Matlab, we can get the relation between different temperatures and the decomposition rate of fungi under the same humidity.
It can be seen from the above figure that the decomposition rate of white rot fungi continues to increase with temperature. When it gets below 0°C, the decomposition rate of white rot fungi initially decreases slightly, while then continues to increase. At 22°C, the decomposition rate of white rot reaches the highest. Subsequently, as the temperature increases, the decomposition rate of white rot fungi slides; while the decomposition rate of brown rot fungi rises with temperature. Between 2 °C and 8 °C, the decomposition rate of white rot fungi falls, and then goes up, reaching the highest decomposition rate around 25 °C. After this, the decomposition rate of brown rot fungi continues to decrease, and then its decomposition rate continues to increase, at 52 °C. Due to the increase in temperature, the decomposition rate of soft rot fungi also increased first and then gets lower, and reached the highest decomposition rate at about 20 °C; the decomposition rate of umbrella-shaped wood rot fungi also increased with temperature. The constant rises, there have been two cases of first rising and then lowering, and reached the highest decomposition rate near 22 °C. And the temperature is between [24°C, 28°C], the decomposition rate of wood fiber is higher. Their sum of square error $R^2 = [0.9474, 0.9497, 0.9415, 0.9373]$, which are all close to 1, indicating that the modified model can better describe the decomposition rate of fungi under different temperature conditions.

IV. CONCLUSION

In the course of solving the model, we found that when the temperature ranges from 24 °C to 28 °C and the relative humidity from 60% to75%, the growth rate of fungi is the fastest. In the case of controlling temperature and humidity changes and humidity and temperature changes, the decomposition rate of fungi increases first and then decreases. Moreover, the interaction of two different fungi has a certain promotion effect on decomposition. As time goes by, the promotion effect gradually weakens, and their interaction has certain fluctuations. At the same time, we can know the advantages and disadvantages of the fungal combination. The growth trend of fungi is increasing in the long-term and short-term. The decomposition rate of fungi varies in different environments, and the decomposition rate of different fungi is gradually reduced, which reflects The diversity of species.
REFERENCES

[1] Zhao Xiuyun, Zhao Xinyu, Yang Jinjin, Li Shaokang, Lu Xiangxin, & Li Xiang. (2021). Research progress on lignin degradation mechanism and influencing factors during composting process [J/OL]. Environmental Engineering, 1-17.

[2] Jose Alvarez-Ramirez, M. & Meraz, E. (2019). Jaime vernon-carter. A theoretical derivation of the monod equation with a kinetics sense. Biochemical Engineering Journal, 150.

[3] Guo Mingjuan, Guang Cui'e, Gan Jianping, & Fan Zhiwen. (2020). Comparison of growth kinetic models of Staphylococcus aureus in coconut milk at different temperatures. Journal of Food and Biotechnology, 39(02): 89-96.