Why a homogeneous dual readout calorimeter won’t work

Don Groom
Lawrence Berkeley National Laboratory, Berkeley CA 94720
E-mail: degroom@lbl.gov

Abstract. If the response to a hadronic shower in a semi-infinite uniform calorimeter structure is \( S \) relative to the electronic response, then \( S/E = [f_{em} + (1 - f_{em})(h/e)] \), where \( E \) is the incident hadron energy, \( f_{em} \) is the electronic shower fraction, and \( h/e \) is the hadron/electron response ratio. In conventional calorimeters the resolution is dominated by the stochastic variable \( f_{em} \), whose broad, skewed pdf has an energy-dependent mean. The slow increase of the mean with \( E \) is responsible for response nonlinearity and the skewness results in a non-Gaussian response. If the cascade is observed in two channels with different values of \( h/e \) (typically scintillator (S) and Cherenkov (C)), \( f_{em} \) can be eliminated. An energy estimator, linear in \( C \) and \( S \), is obtained which is proportional to the incident hadron’s energy. The resolution depends upon the contrast in \( h/e \) between the two channels. The Cherenkov \( h/e \) will be 0.20–0.25. In sampling calorimeters, \( h/e \) can be increased to about 0.7 by arranging for preferential absorption of the electromagnetic (EM) shower energy in the absorber (decreasing \( e \)) and using a hydrogenous detector (organic scintillator) to enhance \( h \) through the contribution of recoil protons in \( n-p \) scattering. Neither mechanism is available in a homogeneous crystal or glass scintillator, where \( h/e \) is expected to be in the vicinity of 0.4 because of invisible hadronic energy loss and other effects. The \( h/e \) contrast is very likely too small to provide the needed energy resolution. We support this conclusion with simple Monte Carlo simulations.

1. Introduction
A homogeneous dual readout hadron calorimeter has been proposed for the future linear collider. The machine will probably be an \( e^+e^- \) collider with a long bunch spacing (100’s of ns), so that detectors with time constants in this range can be used. Discrimination between the Cherenkov signal (C) and scintillator (S) optical signals is expected to be made using a combination of timing and color. For the best candidate materials (inorganic crystals and glasses), \( \lambda_I \geq 20 \) cm. A hadron calorimeter \( \geq 5\lambda_I \) thick lying outside the tracking region and (probably) outside of a large solenoid will have a volume of several hundred cubic meters, setting stringent price as well as properties requirements on the material.

For the purposes of this analysis we will assume that corrections for leakage, cracks, etc. have been made correctly, and consider the energy resolution of a semi-infinite calorimeter with uniform structure that is either fine-sampling or homogeneous.

Mostly via \( \pi^0 \rightarrow 2\gamma \) in a sequence of high-energy hadronic collisions, a fraction \( f_{em} \) of the incident energy is deposited by EM showers. The mean, \( \langle f_{em} \rangle \), is \( \approx 0.5 \) for 100–150 GeV incident pions. The hadronic response \( S \) to an incident hadron with energy \( E \) is

\[
S = E[f_{em} + (1 - f_{em})(h/e)].
\]
(Here and elsewhere the energy $E$ is normalized to the electron response.) The EM deposit is
detected with relative efficiency $e$, and the hadronic energy deposit with relative efficiency $h$.
Both vary from event to event, but the variance of $h$ is much larger than the variance of $e$. It
makes sense to treat $h/e$ as the stochastic variable, since the distribution of the more usual ratio
$e/h$ is fairly pathological.

The energy-independent ratio $\langle h/e \rangle$ depends upon calorimeter composition and structure,
and also depends upon readout—an organic scintillator readout is somewhat sensitive to the
otherwise-invisible neutron content of the cascade, and a Cherenkov readout is relatively blind
to the hadronic content. $\langle \eta_C \rangle$ and $\langle \eta_S \rangle^1$ can be found from fits to the average $\pi^-/e$ response at
a sequence of test-beam energies.

If $\langle h/e \rangle$ (≡ $\langle \eta \rangle$) is not unity, then the broad, skewed $f_{em}$ probability distribution function
(p.d.f.) significantly degrades and skews the distribution of $S$. It in fact usually dominates,
producing the familiar wide, non-Gaussian energy distributions, as well as the non-linear
response via its energy-dependent mean. If $f_{em}$ could be measured for each event, then an energy
estimator distribution could be obtained that has a much narrower Gaussian distribution and a
mean proportional to the energy. This was realized more than 3 decades ago, when the physics
summarized in Eq. 1 was first understood.

One approach is to obtain two signals for each event, one preferentially sensitive to the
EM content (invariably Cherenkov readout) and one as sensitive as possible to the hadronic
content (scintillator). This could be done either by readout of separate elements, as so
successfully demonstrated by the DREAM collaboration[1, 2], or by separation via color and
timing information in a single material[3].

This study is motivated by renewed interest in the latter approach, in particular a
homogeneous dual-readout heavy crystal (or glass) calorimeter for use at a future linear collider.
The collider’s bunch spacing is to be in the 100–500 $\mu$s range, a good match to the properties
of many high-$\langle Z \rangle$ inorganic crystals and glasses.

While the desired signal separation has already been demonstrated by Akchurin, et al.[4, 5],
at least for BGO and PbWO$_4$, we are concerned that low $h/e$ for the scintillator component ($\eta_S$)
will prevent adequate hadronic energy resolution. In this paper we explore the likely resolution
as a function of energy and $\eta_S$ using resolution contributions based on published homogeneous
crystal and sampling calorimeter performance. Simple, transparent Monte Carlo’s (MC’s) are
used by preference, to make the physics more transparent than if a sophisticated MC such as
GEANT4 were used. The probability distribution function (p.d.f.) of $f_{em}$ is approximated with
some care, while other resolution contributions are taken to be Gaussian.

2. $\langle h/e \rangle$ in a high-density crystal or glass scintillator

In an EM cascade relativistic electrons deposit most of the energy. The result is a response very
nearly linear in the incident electron or photon energy.

Hadronic interactions deposit energy in a variety of ways (See Table 1 (by Gabriel and
Schmidt) in Ref. [6]; detailed discussions can be found in Refs. [7, 8] and other recent reviews).
20–40% of the energy goes to nuclear dissociation and is “invisible.” Much or most of the neutron
energy is lost. Low-energy protons and charged fission fragments produce saturated signals in
scintillator[9]. All of these factors result in low visible response to the hadronic component of
the cascade relative to response to the EM component.

Detection of recoil protons in neutron scattering in hydrogenic detectors increases $h$. A
disproportionate fraction of the EM energy is deposited in the higher-$Z$ absorber, decreasing $e$.
Both of these effects increase $h/e$. In practical sampling calorimeters $\langle h/e \rangle$ is typically 0.7, and
can be made to approach unity with careful design.

1 We introduce the less cumbersome notation $\eta = h/e$. 

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Figure 1. Energy-independent event locus in the C/E–S/E plane. With increased energy, resolution improves and the mean moves upward along the locus.

Neither mechanism for increasing h/e is available to a high-density homogenous calorimeter. The resolution is dependent on the “h/e contrast,” the difference between the values of h/e for the Cherenkov (\(\eta_C\)) and scintillation(\(\eta_S\)) readouts.

Based on experience with quartz-fiber readout calorimeters, \(\langle \eta_C \rangle = 0.20–0.25[1, 2, 10]\). There are few data concerning \(\langle h/e | S \rangle\) in a homogeneous calorimeter, but there is no way to hide EM energy in absorber and there is very little neutron sensitivity. We might expect \(\sim 40\%\) of the hadron energy to be expended on nuclear dissociation and therefore invisible, and 15%–20% to be carried by neutrons. These alone would result in \(\langle \eta_S \rangle \approx 0.5\). There are other effects, such as incomplete Cherenkov-scintillator separation and saturated response to highly ionizing particles, so we might expect \(\langle \eta_S \rangle = 0.35–0.5\). This is corroborated by a measurement reported in Ref. [5]: “The e/h value of ECAL [PbWO\(_4\)] as a scintillation device is much larger than for the Cu/plastic sampling structure in DREAM: 2.4 vs. 1.3.” (h/e = 0.43 vs 0.7).

3. Dual-readout hadron calorimetry
The proposed scenario is a Cherenkov readout (C) that is fairly blind to hadronic activity and a scintillator readout (S) with optimized hadronic response. Eq. 1 is replaced by

\[
S = E[f_{em} + (1 - f_{em})(\eta_S)] ; \quad C = E[f_{em} + (1 - f_{em})(\eta_C)] . \tag{2}
\]

In parametric form, Eqs. (2) describe a straight line-segment event locus in the (C/S) (or (C/E–S/E)) plane, as illustrated in Fig. 1. If the cascade is “all electromagnetic” (\(f_{em} = 1\)), then \(S/E = C/E = 1\). If the cascade is “all hadronic” (\(f_{em} = 0\)), then \(S/E = \eta_S\) and \(C/E = \eta_C\).

Equations (2) are linear in 1/E and \(f_{em}\), with solutions[11, 12]

\[
E = \frac{S(1 - \eta_C) - C(1 - \eta_S)}{\eta_S - \eta_C} ; \quad f_{em} = \frac{C \eta_S - S \eta_C}{S(1 - \eta_C) - C(1 - \eta_S)} . \tag{3}
\]

There is an important difference between Eqs. 2 and Eqs. 3: Eqs. 2 give estimators of the response, given the incident energy, \(f_{em}\), and \(\eta_S\), or \(\eta_C\). In contrast, Eq. 3 provides estimators of
the energy and \( f_{em} \) given \( S, C \), \( \eta_S \) and \( \eta_C \). Here \( \eta_C \) and \( \eta_S \) are the values peculiar to that event. These are unknown—and unknowable, until some way of tagging the hadronic content becomes available. But in an experimental situation, an estimator of the energy must be established for a given event. There is little choice but to replace \( \eta_C \) and \( \eta_S \) by their means, \( \langle \eta_C \rangle \) and \( \langle \eta_S \rangle \), for this purpose.

It should be evident from all of these relationships, especially Eq. 3, that the estimator \( E \) is exceedingly sensitive to the difference between \( \eta_S \) and \( \eta_C \), both of which are subject to large event-to-event fluctuations.

4. Electromagnetic fraction

The \( f_{em} \) p.d.f. has an energy-dependent mean (near 0.5 at 100–150 GeV and approaching unity as \( E \to \infty \)). Its standard deviation is about 11%, depending only weakly on energy, and it is skewed to the large-\( f_{em} \) side[7, 11]. For the present study, the mean and fractional standard deviation of \( f_{em} \) as parameterized in Ref. [11] for cascades in a large lead cylinder are used. A skewness \( \gamma_1 = \mu_3/\sigma^3 = 0.6 \) was assumed. In Ref. [11] it was also pointed out that the Beta p.d.f., \( B(x; \alpha, \beta) \propto x^{\alpha-1}(1-x)^{\beta-1} \), is very similar to \( f_{em} \)'s p.d.f. with proper choice of \( \alpha \) and \( \beta \), except for the position of the mean. To construct the p.d.f. for \( f_{em} \), \( \alpha \) and \( \beta \) are found which give the desired variance and skewness. The distribution is then displaced so that the mean is at \( \langle f_{em} \rangle \).

5. Resolution contributions

Except in a few compensated sampling calorimeter cases, the \( f_{em} \) distribution dominates single-readout calorimeter resolution. In a dual-readout calorimeter \( f_{em} \) has been elevated from a stochastic quantity to a measured quantity.

The \( C \) and \( S \) distributions are broadened by a variety of instrumental effects, which we assume are correctable. Dominant remaining stochastic contributions include photoelectron (p.e.) statistics and intrinsic fluctuations that contribute differently to the two channels:

(i) Cherenkov p.e.: Based upon the best Pb glass detectors (e.g. OPAL[13]), the contribution is assumed to be \( 10–20%/\sqrt{C} \).

(ii) Scintillator p.e.: For good signal separation, the scintillator response cannot be very large compared with the Cherenkov signal. We assume a small multiple of the Cherenkov detector's p.e. contribution, scaling as \( 1/\sqrt{S} \).

(iii) Cherenkov intrinsic fluctuations, presumably dominated by fluctuations in the number of relativistic pions: Following Wigmans's estimates in Ref. [7], \( \sigma_{h,C} = 170%/\sqrt{E} \) is assumed. (This has a small effect because \( \eta_C \) is small.)

(iv) Scintillator intrinsic fluctuations: Based on best compensated sampling calorimeter results coupled with separation of sampling and intrinsic contributions by Drews et al.[14], \( \sigma_{h,S} = 11%/\sqrt{E} \) is assumed.

Except possibly for \( \sigma_{p.e.} \), all of the standard deviations described in this section are highly uncertain, and can be used only as guides. Since the energy distributions reported for compensating calorimeters are all consistent with Gaussian distributions, it is evidently valid to consider these as Gaussian as well.

6. Simulation of the energy estimator distribution

Equation 3 describes the estimator of incident pion energy as obtained from the observed scintillation and Cherenkov signals. Using the above estimates, our elementary simulation proceeds as follows:
(i) Choose the incident energy $E$ and the detection efficiency ratios $\langle \eta_S \rangle$ and $\langle \eta_C \rangle$.

(ii) Choose the resolution parameters, as estimated above.

(iii) Generate $N$ values of $f_{em}$ chosen from the displaced Beta distribution for energy $E$.

(iv) Generate $N$ values of $\eta_S$ from a normal distribution with mean $\langle \eta_S \rangle$ and fractional standard deviation $\sigma_{h,S}$. Similarly, generate $N$ values of $\eta_C$ from a normal distribution with mean $\langle \eta_C \rangle$ and standard deviation $\sigma_{h,C}$.

(v) Construct the corresponding $S$ and $C$ arrays via Eqs. 2.

(vi) Replace each $S$ and $C$ as calculated in step 4 with values chosen from normal distributions with means $S$ and $C$ and standard deviations $\sigma_{p.e.,S}\sqrt{S}$ and $\xi\sigma_{p.e.,S}\sqrt{C}$, respectively. The resulting $S$ and $C$ include p.e. statistics, and are used for subsequent “data analysis.”

(vii) Find the $N$ energy estimators via Eq. 3, using the mean values of $\eta_C$ an $\eta_S$.

The results of four representative simulations are shown in Fig. 2, at 75 and 200 GeV using realistic (0.4) and optimistic (0.6) values of $\eta_S$. In every case the mean value of the estimator of $E$ scaled by the beam energy is 1.00, and the distribution is Gaussian, as shown by the smooth curves drawn over the $E$ histograms. In nearly all cases $|\gamma_1| \leq 0.05$ for 10000 event samples, where $\gamma_1$ is the third moment about the mean divided by $\sigma^3$.

The fractional standard deviation of the energy estimator scales as $1/\sqrt{E}$. The coefficient (resolution at 1 GeV) as a function of $h/e|_S$ is shown in Fig. 3 for two values of $h/e|_C$ that bracket the range found in SiO$_2$ calorimeters.

7. Discussion and conclusions

While the input variances are subject to question, the overall conclusion remains: in a non-hydrogeous homogeneous hadron calorimeter, the $h/e$ contrast between the Cherenkov and scintillator readouts is insufficient to obtain the needed resolution. On the other hand, the mean of the energy estimator distribution is the beam energy, and the distribution is Gaussian.

Acknowledgments

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Figure 2. Monte Carlo distributions of $C$, $S$, and the estimator of $E$. (a) and (c) are for beam energies of 75 GeV; for (b) and (c) the beam energy is 200 GeV. For (a) and (b) $\eta_S = 0.40$, while for (c) and (d) $\eta_S = 0.60$. In all cases $\eta_C = 0.25$. Gaussians with the “measured” $\sigma_E$ and mean relative to beam energy are shown as dotted black lines.

Figure 3. The coefficient of $1/\sqrt{E}$ upon $\eta_S \equiv h/e|_S$ at two values of $\eta_C \equiv h/e|_C$. The results for $\eta_C = 0.20$ are essentially those for $\eta_C = 0.25$ displaced to the left by 0.05; it is the contrast between $\eta_C$ and $\eta_S$ that determines the resolution.