MATHEMATICAL MODELLING OF OPTIMIZATION OF STRUCTURES OF MONOLITHIC COVERINGS BASED ON LIQUID RUBBERS

R Kh Turgumbayeva¹, M N Abdikarimov², R Mussabekov², D T Sartayev³

¹ Kazakh national pedagogical university named after Abai (KazNPU), 13, Dostyk str., Almaty, 050010, Republic of Kazakhstan
² Almaty University of Power Engineering&Telecommunications, 126, Baytursunov str., Almaty, 050013, Republic of Kazakhstan
³ Kazakh national technical research university named after K.I. Satpayev (KazNTU), 22, Satpayev str., Almaty, 050022, Republic of Kazakhstan, mn.abdikarimov@mail.ru
E-mail: rturgumbayeva@mail.ru

Abstract. The paper considers optimization of monolithic coatings compositions using a computer and MPE methods. The goal of the paper was to construct a mathematical model of the complete factorial experiment taking into account its plan and conditions. Several regression equations were received. Dependence between content components and parameters of rubber, as well as the quantity of a rubber crumb, was considered. An optimal composition for manufacturing the material of monolithic coatings compositions was recommended based on experimental data.

1. Introduction
Optimization of the compositions of monolithic coatings on the basis of liquid rubbers were carried out using a computer [1]. The mathematical design of the experiment (MPE) is understood as the formulation of experiments using a pre-compiled scheme that has optimal properties in terms of the volume of experimental work and statistical requirements [2-22]. The theory of experiment planning is based on the probability-statistical methods that allows one to theoretically substantiate the minimum number and composition of experiments, and the order of their carrying out to obtain quantitative dependencies between the studied parameter and the factors influencing it. The application of MPE methods allows one to significantly reduce the amount of experimental work in the study and management of multifactor dependencies and offers extensive use of computers, although relatively simple problems can be successfully solved with the help of conventional counting devices.

2. Methods
In the present studies, the goal was to construct a mathematical model of rebound elasticity, Shore A hardness, tensile strength, elongation and residual elongation, depending on the content of the components in the feed mixture (rubber, epoxy, PVAD (vinyl acetate dispersion), rubber chips) for two types of liquid rubber (SKN-26-1A and SKD-1A).
The experiments were carried out in accordance with the plan of the complete factorial experiment (Table 1).

### Table 1. Plan of complete factorial experiment

| Points | FACTORS |
|--------|---------|
|        | X₁ | X₂ | X₃ | X₄ |
| 1      | +1 | +1 | +1 | +1 |
| 2      | +1 | +1 | +1 | -1 |
| 3      | +1 | +1 | -1 | +1 |
| 4      | +1 | +1 | -1 | -1 |
| 5      | +1 | -1 | +1 | +1 |
| 6      | +1 | -1 | +1 | -1 |
| 7      | +1 | -1 | -1 | +1 |
| 8      | +1 | -1 | -1 | -1 |
| 9      | -1 | +1 | +1 | +1 |
| 10     | -1 | +1 | +1 | -1 |
| 11     | -1 | +1 | -1 | +1 |
| 12     | -1 | +1 | -1 | -1 |
| 13     | -1 | -1 | +1 | +1 |
| 14     | -1 | -1 | +1 | -1 |
| 15     | -1 | -1 | -1 | +1 |
| 16     | -1 | -1 | -1 | -1 |

Conditions of planning of experiment are given in table 2.

### Table 2. Experiment planning conditions

| FACTORS | Variation of level | Interval variations |
|---------|--------------------|---------------------|
| The natural look | The coded look | -1 | 0 | +1 |
| PVAD | x₁ | 0 | 10 | 20 | 10 |
| Cut. crumb | x₂ | 0 | 20 | 40 | 20 |
| CKH-26-1A or SKD-1A | x₃ | 10 | 50 | 90 | 40 |
| ED-20 | x₄ | 5 | 20 | 35 | 15 |

For receiving the equations of regression, the program in the FORTRAN language was made.

The following equations of regression were received:

1. For CKH-26-1A rubber

   \[
   y₁ = 10.29 - 2.396*x₁ + 2.01*x₂ + 1.496*x₃ - 3.01*x₄ - \\
   1.39*x₁*x₂ - 0.38*x₁*x₃ - 0.39*x₁*x₄ + 1.01*x₂*x₃ + \\
   + 0.779*x₂*x₄ + 2.258*x₃*x₄
   \]  

2. \( y₂ = 1.761 + 0.189*x₁ - 0.267*x₂ - 0.171*x₃ + 1.061*x₄ - \\
   - 0.371*x₁*x₃ + 0.226*x₁*x₄ + 0.271*x₂*x₃ + \\
   + 0.168*x₂*x₄ - 0.249*x₃*x₄ \)

3. \( y₃ = 32.958 + 5.321*x₁ - 1.478*x₂ - 16.561*x₃ + 16.559*x₄ - \\
   0.601*x₁*x₂ - 1.777*x₁*x₃ - 1.579*x₁*x₄ - \\
   6.155*x₂*x₃ + 0.803*x₂*x₄ - 7.531*x₃*x₄ \)

4. \( y₄ = 2.053 - 0.298*x₁ + 0.374*x₃ - 0.653*x₄ + 0.188*x₁*x₂ + \\
   + 0.188*x₁*x₃ - 0.582*x₁*x₄ - 0.249*x₂*x₃ + \\
   - 0.347*x₂*x₄ - 0.188*x₃*x₄ \)
The increase in elasticity is also possible if the maintenance of a rubber crumb and rubber is at the same time at the top or lower level. Thus, the received systems of the linear equations also provide the maximum value.

II. For SKD-1A rubber

\[
y_1 = 8.997 - 0.878x_1 + 1.001x_2 - 0.252x_3 - 0.753x_4 + 0.125x_1x_2 - 0.627x_1x_3 + 1.376x_1x_4 - 0.245x_2x_3 + 1.251x_2x_4 - 0.997x_3x_4
\]

\[
y_2 = 41.354 + 2.662x_1 - 2.098x_2 - 10.348x_3 + 8.773x_4 - 2.669x_1x_2 + 1.331x_1x_3 + 0.785x_1x_4 + 0.109x_2x_3 + 4.704x_2x_4 - 9.791x_3x_4
\]

\[
y_3 = 3.928 + 0.40x_1 - 1.019x_2 - 1.839x_3 + 2.998x_4 - 1.042x_1x_2 - 0.753x_1x_3 + 2.04x_2x_3 - 0.319x_2x_4 - 1.506x_3x_4
\]

\[
y_4 = 1.393 - 0.145x_2 + 0.318x_3 - 0.368x_4 + 0.339x_1x_2 + 0.476x_1x_3 + 0.249x_1x_4 + 0.055x_2x_3 + 0.620x_2x_4
\]

\[
y_5 = 0.337 + 0.072x_1 - 0.024x_2 - 0.113x_3 - 0.113x_4 + 0.029x_1x_3 + 0.103x_1x_3 - 0.042x_1x_4 - 0.089x_2x_3 + 0.042x_2x_4
\]

For the definition of a combination of the factors providing the maximum value of the output parameter, the extremum is found by consistent differentiation on x1, x2, x3, x4. Then the solution of the received systems of the linear equations also provides the extreme value.

Analyzing the equations of regression, it is possible to tell that in case of using CKH-26-1A and SKD-1A rubber, elasticity on a rebound increases with an increase in the quantity of a rubber crumb and rubber, elasticity decreases with growth of quantity of ED-20 (provided that other factors are accepted at the main level). The increase in elasticity is also possible if the maintenance of a rubber crumb and rubber is at the same time at the top or lower level; other components of raw mix have to be at different levels. The maximum elasticity in this case is reached with the following content of components:

- CKH-26-1A – 62%
- ED-20 – 3%
- PVAD – 2%
- Rubber crumb – 27%

and

- SKD-1A – 66%
- ED-20 – 2%
- PVAD – 3%
- Rubber crumb – 23%

2) Hardness on Shora And increases during an increase in quantities of PVAD and ED-20 and decreases with increase of quantity of a rubber crumb and rubber. In case of using CKH-26-1A rubber, the increase in hardness on Shora And is possible if the content of PVAD and ED-20, a rubber crumb and rubber is at one level, and factors PVAD and CKH-26-1A are at different levels. The coefficient
of interaction of factors of PVAD and SKN-26-1A was insignificant. In case of SKD-1A rubber use, the increase in hardness is also possible if the content of PVAD and ED-20, PVAD and SKD-1A, a rubber crumb and SKD-1A, a rubber crumb and ED-20 is at one level and factors of PVAD and a rubber crumb, SKD-1A and ED-20 are at different levels.

The maximum hardness is reached with the following content of components:

- CKH-26-1A – 10%
- ED-20 – 41%
- PVAD – 19%
- Rubber crumb – 24%
- SKD-1A – 11%
- ED-20 – 42%
- PVAD – 18%
- Rubber crumb – 23%

3) Strength on a gap increases with an increase in quantity of PVAD and ED-20 and a reduction of the content of a rubber crumb and rubber. In case of using CKH-26-1A rubber to increase durability factors - a rubber crumb and ED-20 - it is necessary to accept them at one level. All other interactions of factors should be referred to different levels.

In case of using SKD-1A rubber to increase strength, it is necessary to accept a rubber crumb and rubber at one level. All other interactions of factors consist in the fact that they have to be at different levels. The coefficient of interaction of factors of PVAD and ED-20 was insignificant.

Maximum value of strength in a gap is reached with the following content of components:

- CKH-26-1A – 15%
- ED-20 – 50%
- PVAD – 29%
- Rubber crumb – 1%
- SKD-1A – 16%
- ED-20 – 45%
- PVAD – 28%
- Rubber crumb – 2%

4) Relative lengthening increases with an increase in the amount of rubber and decreases with an increase in quantities of PVAD, rubber crumb, ED-20, and in case of using SKD-1A rubber the coefficient at PVAD was small. The increase in relative lengthening is possible if factors of PVAD and a rubber crumb, PVAD and rubber, a rubber crumb and rubber are at one level, and factors such as PVAD and ED-20, a rubber crumb and ED-20, rubber and ED-20 are at the different levels.

The maximum relative lengthening is reached with the following content of components:

- CKH-26-1A – 61%
- ED-20 – 3%
- PVAD – 10%
- Rubber crumb – 20%
- SKD-1A – 62%
- ED-20 – 2,5%
- PVAD – 9%
- Rubber crumb – 21%

5) Residual lengthening increases with an increase in PVAD and decreases with an increase in the amount of a rubber crumb, rubber and ED-20. The increase in residual lengthening is possible, if factors: PVAD and a rubber crumb, PVAD and rubber, a rubber crumb and rubber, are at one level, and factors: PVAD and ED-20, a rubber crumb and ED-20, rubber and ED-20 – at different levels.

The maximum residual lengthening is reached with the following content of components:

- CKH-26-1A – 21%
ED-20 – 40%
PVAD – 20%
Rubber crumb – 12%
and
SKD-1A – 22%
ED-20 – 39%
PVAD – 18%
Rubber crumb – 13%

The data obtained by the method of mathematical planning of experiments were used by optimization of structures of monolithic sports coverings. Complex research of physical and chemical, physico-mechanical parameters, operational characteristics and sports and technical indicators is caused by possibility of development of technology of monolithic coverings for different types of sports [2-5].

For development of new types of a monolithic covering for the plane sports constructions (PSC), the authors conducted research on studying compositions based on the rubbers modified by epoxies. Design of experiments and different ways of modeling have been studied [6-23].

3. Conclusion
Samples of a rubber-epoxy mixture containing 28, 50, 70% of the filler were made. Samples containing 50 and 70% of the filler were very hard, non-elastic. The best results were obtained for samples containing 25% of the filler. Thus, based on the experimental data, the following composition is recommended for fabrication of the material: ED-20: rubber: PEPA: fillers = 20: 80: 10: 25.

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