Model of P controller's effect on tension of UAV motor

Tiansong Li¹, Haoqiang Zhang¹*, Qinghua Liu¹ and Xiaoyan Zhou¹

¹ School of Institute of Information and Communication, Guilin University of Electronic Technology, Guilin, Guangxi, 541004, China

*Corresponding author’s e-mail: huanghunxiao0607@163.com

Abstract. The tuning of the multi-rotor UAV PID parameters is a long and iterative process, and sometimes it is impossible to find a suitable magnitude. To improve the setting efficiency of control parameters, the classic PID control algorithm is combined with the dynamic model of the UAV to build a mathematical model that describes the effect about parameters of P controller on the tension of brushless motor. Computer simulation and open-loop experimental verification were conducted under the change of parameter of P controller. The simulation and experimental results show that this mathematical model describes the effect of parameters of P controller on the UAV motor's tension under different attitude angle errors. When the parameters of the P controller are input to the UAV which has determined the parameters of the power system, the motor’s tension can be calculated directly from this model, and this model can assist in determining the magnitude of parameters of the P controller.

1. Introduction

With the gradual maturity of strapdown inertial navigation technology, multi-sensor fusion technology, and modern control theory, UAVs (unmanned aerial vehicle) are increasingly being used in human production and life. UAV stability control is the bottleneck technology for the development of drones. The main control algorithms of UAV are: classic PID control, neural network control, auto-disturbance control, sliding mode control, etc. PID controllers are still widely used due to their simple structure and good robustness.

At present, the PID parameter adjustment of multi-rotor aircraft is mainly divided into the general adjustment method and the Ziegler-Nichols adjustment method[1]. The general adjustment method is a common method of PID tuning. This method is not easy to find the appropriate parameters of P and D, and requires a lot of trial and error. The Ziegler-Nichols adjustment method is a loop tuning technique invented by John Ziegler and Nathaniel Nichols. The core difficulty of this method is to construct a closed-loop control loop. For multi-rotor UAV, the quantitative relationship between control parameters and the motor is a key step in building a closed-loop control loop. The multi-rotor UAV’s power system has been established in multi-rotor propellers, motors, electronic speed controllers (ESC), batteries and other models, and the power system matching based on the parameters of the finished power system components has been relatively mature[2]. The purpose of this article is to combine the PID controller with the power system model of the UAV, and seek the quantitative relationship between the parameters of P controller and the single motor’s tension, to assist in the estimation of PID parameters of UAV, improve the parameter tuning efficiency.
2. PID control principle of multi-rotor UAV

PID controller, namely proportional, integral and differential controller, is a closed-loop feedback adjustment method, and it is the most common control method in classic control theory. When the PID algorithm is executed on a computer, the integral and differential terms need to be discretized, that is:

\[ u(k) = k_p e(k) + k_i \sum_{j=0}^{k} e(j) + k_d [e(k) - e(k-1)] \]  

In the formula, \( k \) represents a time series.

As mentioned above, the value output by the PID controller can be sent directly to the actuator, that is:

\[ C = u(k) \]  

In the following, quad-rotor UAV is taken as an example, and the control principle of PID to the UAV will be specifically described. As shown in Figure 1, the UAV has a roll angle change from the initial state (default is 0°) from left to right at an update time. The angle of change is \( \gamma \), then \( e(k) = \gamma \), and substitute equation (1) for calculate. The calculated result is directly used as the throttle value of the motor to control the motor.

Since the UAV is a coupling system, the control effect of each attitude angle change generated by the PID controller on each motor needs to be superimposed.

3. Modeling of power system for multi-rotor UAV

The power system model of a multi-rotor UAV generally includes propeller modeling, motor modeling, ESC modeling, and battery modeling. In order to ensure the universality of the mathematical model, the device parameters used in this section are parameters that are easy to find in the manual provided by the manufacturer, and the mathematical model of the three links of propeller, motor, and ESC is mainly described according to literature [3].

3.1 Propeller modeling

The multi-rotor usually adopts fixed-pitch propellers, so according to the literature [3] [4] [5], the formulas for the calculation of the tension and torque of the propeller are as follows:

\[ T = C_T \rho \left( \frac{N}{60} \right)^2 D_p^3 \]  

\[ M = C_M \rho \left( \frac{N}{60} \right)^2 D_p^5 \]  

Where \( T \) is the propeller tension, \( M \) is the torque, \( N \) is the propeller speed, \( D_p \) is the propeller diameter, and \( C_T \) is the tension coefficient, \( C_M \) is the torque coefficient, \( \rho \) is the air density of the flying environment, and \( \rho \) can be used as a constant input model.

By reference [3]:

\[ C_T = 0.25 \pi \lambda^2 B_p K_o \frac{e \arctan \frac{H_p}{\pi D_p} - \alpha_o}{\pi A + K_o} \]
\[ C_M = \frac{1}{8A} \pi^2 C_d \xi^2 \lambda B_r^2 \] (6)

among them,

\[ C_d = C_{\alpha d} + \frac{\pi AK_e^2}{e} \left( \arctan \frac{H_p}{\pi D_p} - \alpha_0 \right)^2 \] (7)

In the formula, \( D_p \) is the diameter of the propeller, \( H_p \) is the pitch of the propeller, and other parameters are described in reference [3, p78]. According to the literature[6], the values of some of the above parameters are as follows:

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| \( A \)   | 5~8   | \( e \)   | 0.7~0.9 |
| \( \varepsilon \) | 0.85~0.95 | \( C_{\alpha d} \) | 0.015 |
| \( \lambda \) | 0.7~0.9 | \( \alpha_0 \) | \( -\frac{\pi}{36} \) |
| \( \xi \) | 0.4~0.7 | \( \kappa \) | 6.11 |

The remaining parameters are provided by the manufacturer and entered into the model.

3.2 Motor modeling

Brushless DC Motor (BLDC) used in multi-rotor can be equivalent to permanent magnet DC motor[7]. The main purpose of motor modeling is to solve the motor equivalent voltage \( U_w \) and equivalent current \( I_w \) based on the motor's load torque \( M \), motor speed \( N \), and some basic parameters of the motor. According to reference [3], there are:

\[ U_w = \left( \frac{MK}{9.55(U_{n0} - I_{n0}R_m)} + I_{n0} \right) R_m + \frac{U_{n0} - I_{n0}R_m N}{K_{V0}} \] (8)

\[ I_n = \left( \frac{MK}{9.55(U_{n0} - I_{n0}R_m)} + I_{n0} \right) \] (9)

Where \( K_{V0} \) is the nominal no-load KV of the motor, \( U_{n0} \) is the nominal no-load voltage of the motor, and \( I_{n0} \) is the nominal no-load of the motor Current, \( R_m \) is the internal resistance of the motor.

3.3 ESC modeling

The speed of the brushless motor under ESC modulation mainly depends on the motor load torque and battery voltage. The main purpose of ESC modeling is to use the motor's equivalent voltage \( U_w \) and equivalent current \( I_w \), and the main parameters of the ESC and battery to calculate the ESC's input throttle command \( \sigma \) (between 0 and 1), input current and input voltage. According to the literature [8]

\[ \sigma = \frac{U_w}{U_\phi} \approx \frac{U_{n0}}{U_b} \] (10)

Where \( U_\phi \) is the ESC input voltage, \( U_b \) is the battery voltage, and \( U_{n0} \) is the equivalent DC voltage after the ESC modulation, which can be expressed as

\[ U_{n0} = U_w + I_w R_e \] (11)

Where \( R_e \) is the internal resistance of the ESC. From (10) and (11), we can get

\[ \sigma = \frac{U_w + I_w R_e}{U_b} \] (12)
4. Mathematical modeling of the effect of parameters of P controller on motor’s tension

In this section, the PID algorithm and the multi-rotor UAV dynamic model are combined to obtain the quantitative description of the parameters of P controller for the single motor’s tension of the UAV.

The correct way from the PID parameters to the tension is: (1) The expected value of the attitude angle of the UAV is subtracted from the actual measured value to get the error angle input to the PID controller; (2) The control value calculated by the PID controller and the battery-related parameters are input Electronic governor model; (3) The motor equivalent voltage and motor equivalent current output by the electronic governor when the motor load and motor parameters are determined; (4) The motor output motor speed when the propeller parameters are determined To the propeller model; (5) the propeller outputs the tension under the condition of determining the air density. The block diagram of the model from the PID parameters to the propeller tension is shown below:

![Diagram](image)

Figure 2. Model block diagram of PID parameter to propeller’s tension.

The following is derived from the PID parameters, battery, ESC, motor, propeller and other parameters to the mathematical model of the propeller’s tension.

Because the I controller and D controller involve integration and differentiation processes, it is necessary to observe the attitude change process of the UAV. This article only deduces the influence of parameters of P controller on the motor’s tension. which is: \( k_i = 0, \ k_d = 0 \). Equation(1) becomes

\[
\sigma = u(k) = k_c e(k) \tag{13}
\]

First, when the P controller parameter \( k_p \) is determined, equation (13) will output a control value \( u(k) \) every time the time period \( k \) is updated and an attitude angle error amount is input. The relationship between the control value and the throttle amount is

\[
\sigma = \frac{u(k)}{1000} \tag{14}
\]

Then calculate the motor speed \( N \), and substitute equation(4) into (8) and (9) to get

\[
U_m = \left\{ \frac{C_d \rho \left( \frac{N}{60} \right)^2}{9.55(U_{m0} - I_{w0}R_m) + I_{w0}} + I_{w0} \right\} R_m + \frac{U_{m0} - I_{w0}R_m}{K_f U_{m0}} N \tag{15}
\]

\[
I_a = \frac{C_d \rho \left( \frac{N}{60} \right)^2}{9.55(U_{m0} - I_{w0}R_m) + I_{w0}} + I_{w0} \tag{16}
\]

Then take (14), (15) into (12), get

\[
\sigma U_a = \left\{ \frac{C_d \rho \left( \frac{N}{60} \right)^2}{9.55(U_{m0} - I_{w0}R_m) + I_{w0}} + I_{w0} \right\} R_m + \frac{U_{m0} - I_{w0}R_m}{K_f U_{m0}} N + \left\{ \frac{C_d \rho \left( \frac{N}{60} \right)^2}{9.55(U_{m0} - I_{w0}R_m) + I_{w0}} + I_{w0} \right\} R_n \tag{17}
\]

The value of \( C_d \) can be obtained from (6), (7), then (6), (8), (14), (17) are combined, and the battery, ESC, motor, propeller, air density and other physical parameters under known conditions, the rotation speed \( N \) of the motor and the propeller can be obtained, \( N \geq 0 \).

Substituting rotation speed \( N \) and formula (5) into formula (3), we get
\[
T = 0.25\pi^2\lambda \xi^2 B_p K_\phi \rho \frac{\arctan \frac{H_p}{\pi D_p} - \alpha_0}{\pi A + K_0} \left( \frac{N}{60} \right)^2 D_p^2
\]  

(18)

Then the equations composed of equations (6), (7), (13), (14), (17), (18) are mathematical models of the effects of parameters of P controller on the UAV motor's tension.

5. Experimental, simulation and data analysis

This section conducts computer simulation and experimental verification of the mathematical model of the parameters of P controller for the motor’s tension. The parameters are selected according to the experimental conditions: This experiment uses APC’s 8x3.8-inch two-bladed blade, according to literature [3, p67], blade parameters: \( D_p = 0.2032 \), \( H_p = 0.0965 \), \( B_p = 2 \), \( A = 5 \), \( \varepsilon = 0.85 \), \( \lambda = 0.75 \), \( \xi = 0.55 \), \( e = 0.83 \), \( C_{mf} = 0.015 \), \( \alpha_0 = 0 \). This experiment uses SUNNYSKY Angel series motors: A2212, 1250KV. According to the official parameters provided by SUNNYSKY, \( K_{Vp} = 1250 \), \( U_{w0} = 11.1 \), \( I_{w0} = 0.9 \), \( R_m = 0.1 \). In addition, this experiment uses the HOBBYWING XRotor series 40A ESC, Quan Sheng Electronics 3S, 5200mAh, 35C lithium polymer battery. In this experiment, \( k_p = 10, 12.5, 15 \), \( k_i = 0 \), \( k_d = 0 \). The parameter values are assigned to variables and simulated. In the case of different \( k_p \), the change of the tension with the attitude angle is shown in Figure 3.

![Figure 3. Influence of PID value on tension](image)

As shown in Figure 3, the horizontal axis is the error value of the expected angle minus the measured angle, and the vertical axis is the tension. The \( k_p \) value is changed for three simulations. It can be seen from the figure that the tension increases with the error of the attitude angle, and the tension increases with the increase of the P controller coefficient at the same attitude angle.

This experiment uses the Cortex-M3 architecture-based processor: STM32F103, attitude sensor: MPU6050, 5kg pressure sensor, conversion module: HX711AD, for hardware design, reasonable installation of propeller, brushless motors, etc. Get attitude angle after attitude solution and complementary filtering[9], and substituted it into the P controller, and then the output value is sent to the ESC to drive the brushless motor. The serial port will output the tension and attitude angle in real time.

The following are the tension’s data collected at \( k_p = 10 \), \( k_p = 12.5 \), and \( k_p = 15 \), and compared with the simulation values.
Figures 4, 5, and 6 show that the collected tension values basically reflect the magnitude and changes of the tension. It can be seen that the amount of tension is generally smaller than the simulation value at the current attitude angle. Because the experimental environment is not standard atmospheric pressure, and the resistance of the wires and solder joints and the reverse airflow of the propeller are affected by the factors[10], it is normal that the tension value does not reach the ideal state. In addition, it is obtained from the figure that the tension value is 0 when the attitude angle tilt is small. In actual verification, the motor cannot start when the throttle amount is too small, and it needs to reach a certain throttle amount, after the spin, the tension will gradually approach the simulation value.

6. Conclusion
This article takes the improvement of PID parameter tuning efficiency of multi-rotor UAV as a starting point, combines the classic PID algorithm with the dynamic model of multi-rotor UAV, establishes a mathematical model of the effects of parameters of P controller on tension of the UAV motor, and performs a computer simulation and experimental verification. Simulation and experiments prove that the tension value increases with the increase of the attitude angle deviation, and the tension value increases with the increase of the P controller coefficient at the same attitude angle. And this model can basically accurately describe the quantitative change of the motor’s tension with the attitude angle error under the given conditions of the P controller parameters. Because the brushless motor’s tension can be calculated more accurately, this study will be able to assist in finding the appropriate P controller parameter magnitude of the UAV to improve the tuning efficiency of the PID parameters of the multi-rotor UAV.

Acknowledgments
This work was financially supported by the following funds: 1. Major Science and Technology Project in Guangxi (No. AA17204093). 2. Ministry of Education Key Lab. of Cognitive Radio and Information Processing’s Director Fund Project in 2019 (No. CRKL190108).
References

[1] Ziegler J. G., Nichols N. B., (1942) Optimum settings for automatic controllers. Trans ASME., 64(8): 759-768.

[2] Zhang H., Song B. F., Wang H. F., (2019) Wang G., Modeling and optimal design of power system for electric fixed-wing quadrotor hybrid unmanned aerial vehicle. Journal of Aerospace Power, 34(06): 1311-1321.

[3] Quan Q., (2018) Design and control of multi-rotor aircraft. Beijing. Publishing House of Electronics Industry, 64-84.

[4] Moffnt B. A., Bradley T. H., Parekh D. E., (2008) Validation of vortex propeller theory for UAV design with uncertainty analysis. In: Proc. 46th AIAA Aerospace Sciences Meeting and Exhibit, AIAA 2008-406.

[5] Merchant M. P., Miller L. S., (2006) Propeller performance measurement for low Reynolds number UAV applications. In: 44th AIAA Aerospace sciences meeting and exhibit. Reno, Nevada, AIAA 2006-1127.

[6] Chen J., Yang S. X., Mo L., (2009) Modeling and experimental analysis of UAV electric propulsion system. Journal of Aerospace Power, 24(06): 1339-1344.

[7] Bangura M., Lim H, Kim H. J., (2014) Aerodynamic power control for multirotor aerial vehicles. In: Proc. IEEE International Conference on Robotics and Automation (ICRA), pp 529-536.

[8] Lindahl P., Moog E., Shaw S. R., (2012) Simulation, Design, and Validation of an UAV SOFC Propulsion System. IEEE Transactions on Aerospace and Electronic Systems, 48(2): 2582-2593.

[9] Mahony R., Hamel T., Pflimlin J. M., (2008) Nonlinear Complementary Filters on the Special Orthogonal Group. IEEE Transactions on Automatic Control, 53(3): 1203-1218.

[10] Gao Y., Chao X. L., Li Q. F., (2018) Validation of the Propeller tension Calculation and the Parameter Sensitivity Analysis. Aeronautical Science & Technology, 29(06): 37-41.