Non-Critical Type 0 String Theories and Their Field Theory Duals

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Abstract

In this paper we continue the study of the non-critical type 0 string and its field theory duals. We begin by reviewing some facts and conjectures about these theories. We move on to our proposal for the type 0 effective action in any dimension, its RR fields and their Chern–Simons couplings. We then focus on the case without compact dimensions and study its field theory duals. We show that one can parameterize all dual physical quantities in terms of a finite number of unknown parameters. By making some further assumptions on the tachyon couplings, one can still make some “model independent” statements.

This work is dedicated to the memory of Mauri Miettinen.

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1 Introduction

Polyakov’s proposal [1] of using type 0A/0B string theory for describing non-supersymmetric Yang-Mills theories is a promising and novel attempt at formulating the QCD string and provides an alternative way of extending the AdS/CFT correspondence [2, 3, 4] beyond the realm of supersymmetric field theories.

Simply put, the proposal postulates using a closed string theory in $d \leq 10$ space-time dimensions with world-sheet supersymmetry and a diagonal GSO projection that removes all space-time fermions, thus yielding a non-supersymmetric theory in target space. One can refer to such theories for which $d < 10$ as “non-critical” although the name can be confusing because Weyl invariance is recovered once the conformal factor of the world-sheet metric is counted among the other space-time coordinates. Throughout this paper, $d$ will always include such Liouville mode and hence Weyl invariance is retained, at least at the lowest level in the sigma-model expansion.

So far, most of the literature following [1] has analyzed the most conservative case of non-supersymmetric theories in $d = 10$ [5, 6, 7, 8, 9, 10]. Such theories [11, 12] are often referred to as type 0A or 0B, depending on the particular GSO projection employed. Their open string descendants were analyzed in [13, 14, 15, 16].

However, it is our opinion that the “non-critical” scenario [17, 18, 19, 20, 21] must be taken seriously and will give rise to additional interesting models that are not accessible at $d = 10$. Throughout the paper we shall continue to refer to these lower dimensional theories as type 0A or 0B in even dimensions, depending on the choice of chirality in the GSO projection. In odd space-time dimensions there is only one such theory due to the lack of chirality. We shall refer to it as type 0AB.

Although the full conformal field theory corresponding to the above non-critical string theory has not yet been constructed, there are some indications that such a construction is indeed possible:

Consider the issue of modular invariance. Define the fermionic traces for the pair of Majorana–Weyl fermions $\psi(z)$, $\bar{\psi}(z)$ as\(^4\)

$$Z_0^0(\tau) = \text{tr}_{\text{NS}} \left( q^N \right)$$

\(^4\)We use the notation of [23]. Here $q = \exp(2\pi i \tau)$. 

1
\[ Z_1^0(\tau) = \text{tr}_{\text{NS}} \left( (-)^F q^N \right) \]
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\[ Z_1^1(\tau) = \text{tr} \left( (-)^F q^N \right) \]

(1)

In the ordinary type II string theory in \( d \) dimensions, modular invariance requires that the holomorphic contribution of the fermions to the partition function

\[ Z_0^0(\tau)^{(d-2)/2} - Z_1^0(\tau)^{(d-2)/2} - Z_0^1(\tau)^{(d-2)/2} \pm Z_1^1(\tau)^{(d-2)/2} \]

(2)

and the analogous antiholomorphic contribution be separately modular invariant up to an overall opposite phase. It is well known that the lowest dimension where the product of the holomorphic contribution and the antiholomorphic contribution is modular invariant is \( d = 10 \).

In type 0 theories, the joined contributions of the holomorphic and the antiholomorphic sectors give rise to

\[ |Z_0^0(\tau)|^{d-2} + |Z_0^1(\tau)|^{d-2} + |Z_1^0(\tau)|^{d-2} \pm |Z_1^1(\tau)|^{d-2}, \]

(3)

which is modular invariant for any \( d \). Of course, for \( d \neq 10 \) the explicit expressions for the fermionic traces will be modified because of the changes in the spectrum, but we view the above as an indication that the continuation “off-criticality” is more likely to work for the type 0 string than for the usual type II.

Another objection that needs to be addressed is the “\( d = 2 \) barrier”. Let us briefly recall the physics behind this problem as presented e.g. in [23]. Because we do not yet know how to describe RR fields at the level of the sigma-model we are forced to discuss this argument in the case of the bosonic theory. In this context it is well know that there exists an exact CFT solution in any \( d \) in which the only non-zero background fields are a flat metric \( g_{MN} \), a linearly rising dilaton \( \Phi \) and an exponential tachyon \( T \)

\[ g_{MN} = \eta_{MN}, \quad \Phi = \sqrt{\frac{26 - d}{6}} X^1, \quad T = \exp \left( \left( \sqrt{\frac{26 - d}{6}} - \sqrt{\frac{2 - d}{6}} \right) X^1 \right) \]

(4)
For \( d \leq 2 \) the background tachyon is exponentially rising, preventing the string from entering the region of strong coupling. Moreover, fluctuations around the tachyon background are stable and thus the theory is well defined. On the contrary, for \( d > 2 \), the background tachyon oscillates. In principle this could still act as a cutoff for the string coupling but the fluctuations have some negative frequency square modes and the theory becomes unstable.

As stressed in \[1, 24\] we should not think of the “\( d = 2 \) barrier” as a no-go theorem but rather as an indication that solutions for \( d > 2 \) will necessarily involve a curved space-time metric. This is the type of situation that is of interest in the connection with gauge theory so, in a sense, it is to be expected that flat space-time be ruled out. Unfortunately, we do not yet have an example of an exact CFT of this type and we are forced to work order by order in \( \alpha' \) at the level of the effective action. But it should be clear that there are no a priori reasons for why there should not exist an exact solution.

A third encouraging sign comes from the analysis of the RR sector performed in Section 3. By making some plausible assumptions about the massless degrees of freedom it is possible to construct a rather compelling picture of the RR sectors in various dimensions and their couplings, including Chern–Simons terms. For instance, the necessity of doubling the RR spectrum in \( d = 4 \) or \( d = 8 \) Minkowski space-time is seen as coming from the fact that there are no real self-dual forms in these dimensions.

Finally, let us note that considering \( d < 10 \) from the sigma model point of view is a very natural thing to do if the string theory has a perturbative tachyon. The only way for a theory with a perturbative tachyon to make sense is if there exists a mechanism through which the tachyon field condenses by acquiring a vacuum expectation value. The tachyon potential at that point will then give rise to a tree-level contribution to the cosmological constant by shifting the central charge. Since the effective central charge is going to be different from zero anyway\footnote{It seems unnatural and there is no symmetry argument for which the tachyon potential should vanish at that point.}, one is led to consider the theory with the most general value for the effective central charge

\[
c_{\text{eff.}} = 10 - d - \frac{1}{2} V \langle \langle T \rangle \rangle.
\] (5)

From the target space point of view this acts as a contribution to the cosmological constant.
logical constant and thus shows that, for $d > 2$, one should look at curved space-times.

None of the above points constitute a proof that conformally invariant solutions to the type 0A/B string exist for arbitrary $d$ but we view them as strong indications that such construction is possible. Having taken this as our basic assumption, throughout the paper we work at the level of the effective action to one loop in $\alpha'$, i.e. the gravity level without higher order corrections.

The form of the action of the type 0 gravity, can be determined perturbatively, with a certain number of ambiguities involving the tachyon potential and the coupling to the RR fields, which are essentially there because of lack of supersymmetry on the target space. However, supersymmetry on the world sheet allows one to make some statements on these terms, as we will see in Section 3. We will stay generic and will be able to show that in any dimension there exists a set of exact solutions of the classical equations of motion, which give AdS metric and involve a non-zero RR field, other than constant dilaton and tachyon. Such solutions depend only on a finite number of parameters for which a string-theoretical derivation is still lacking.

The solutions found represent Polyakov’s conformal fixed points in the dual gauge theory — they support a condensed tachyon, but the first issue to be addressed is the stability against quantum fluctuations of the fields. Because of the mixing of several of these fields it is not enough to analyze the tachyon itself, one has to disentangle the full set of fluctuations [25]. For simplicity, we restrict to the case where the space is $d$-dimensional AdS, so that there are no KK modes to be worried about. There the analysis simplifies considerably, but it is still non trivial because of dilaton-tachyon mixing.

Another issue that was raised in [1] is the field theory interpretation of these solutions. There it was claimed that they may represent an interacting UV conformal point. To address this issue one needs to show that these gravity solutions represent in fact a point in wider space, that is actually the RG phase diagram of the field theories we have at hand. In this enlarged theory space there may be more than one conformal solution and there exist trajectories which interpolate between these points.

These considerations have already been realized in the framework of type IIB supergravity, where the RG flow is believed to be driven by operators dual to scalar fields coming from KK reduction in the compact directions [26].
or by the running of the dilaton \[30, 31, 32, 33\]. It turns out that type 0 gravity provides an example of this general feature of the AdS/CFT correspondence, as well. In fact, the physics is already captured by the set of solutions involving no compact space. Even without including KK modes, the tachyon will mix with the dilaton field, and generate on the field theory side a RG flow that connects interacting conformal fixed points.

2 A look at the type 0A/B theory in \(d = 10\)

In this section we briefly summarize some known facts about the perturbative properties of these theories. We will thus set \(d = 10\) throughout this section and consider perturbation theory around the (unstable) vacuum. We will, however, point out the various places where modifications occur when one is considering \(d \neq 10\). We will return to these changes in Section 3 where we present the detailed structure of the RR terms in \(d \leq 10\).

In the notation of \[23\], there are (up to equivalences) only four consistent ways of combining the various sectors of the \(\mathcal{N} = (1, 1)\) NSR closed oriented string in \(d = 10\):

\[
\begin{align*}
\text{IIA} & : (NS+, NS+) \oplus (NS+, R+) \oplus (R-, NS+) \oplus (R+, R-) \\
\text{IIB} & : (NS+, NS+) \oplus (NS+, R+) \oplus (R+, NS+) \oplus (R+, R+) \\
\text{0A} & : (NS+, NS+) \oplus (NS-, NS-) \oplus (R+, R- \oplus (R-, R+) \\
\text{0B} & : (NS+, NS+) \oplus (NS-, NS-) \oplus (R+, R+) \oplus (R-, R-). (6)
\end{align*}
\]

The first two are the usual type IIA/B superstring and the last two are those of interest here. The massless fields of the theory are the dilaton \(\Phi\), graviton \(g_{MN}\) and two-form \(B_{MN}\) coming from the \((NS+, NS+)\) sector and twice as many RR fields coming from the doubled RR sectors. There is a tachyon \(T\) from the \((NS-, NS-)\) and no fermionic mode at all. All other modes are massive.

2.1 Selection rules for \(d = 10\)

The lack of space-time fermions means that the theory has no space-time supersymmetry; however, the presence of \(\mathcal{N} = (1, 1)\) world-sheet supersymmetry has the following simplifying features \[5\]:

\[\]
a) All tree-level correlators involving an odd number of tachyons and only \((NS+, NS+)\) vertex operators are zero. This can be seen from the explicit form of the vertex operators. The vertex operator for the massless \((NS+, NS+)\) states and for the tachyon in the \((NS-, NS-)\) sector are, again in the notation of [23]

\[
\begin{align*}
V_{(NS+, NS+)}^{0,0} &= -(i\partial X^M + k \cdot \psi \bar{\psi}^M)(i\partial X^N + k \cdot \bar{\psi} \bar{\psi}^N) e^{ik \cdot X} \\
V_{(NS+, NS+)}^{-1,-1} &= e^{-\phi - \bar{\phi}} \psi^M \bar{\psi}^N e^{ik \cdot X} \\
V_{(NS-, NS-)}^{0,0} &= k \cdot \psi k \cdot \bar{\psi} e^{ik \cdot X} \\
V_{(NS-, NS-)}^{-1,-1} &= e^{-\phi - \bar{\phi}} e^{ik \cdot X}.
\end{align*}
\] (7)

Since on the sphere we need to take any two of the above in the \((-1, -1)\) picture and all the rest in the \((0, 0)\) picture it is clear that we will always end up with the correlation function of an odd number of \(\psi\)'s which vanishes. In particular, notice that in this vacuum \(\langle T \rangle = 0\) whereas we expect a tachyon condensate in the true vacuum.

b) All tree-level correlators involving an odd number of tachyons, any number of fields from the \((NS+, NS+)\) and two RR fields from the same sector must vanish. For instance, if the amplitude with one tachyon did not vanish there would be a tachyon pole in some tree-level correlation function of the corresponding type II theory. The vanishing of the amplitude can also be seen at the level of the correlation functions of the vertex operators. We can always use the vertex operators for the RR fields in the \((-1/2, -1/2)\) picture – written as a bispinor it reads

\[
V_{(R^\pm, R^\pm)}^{-1/2, -1/2} = e^{-\phi/2 - \bar{\phi}/2} \Theta_\alpha \bar{\Theta}_\beta e^{ik \cdot X}.
\] (8)

In a correlator that involves two such vertex operators, one remaining \((NS \pm, NS \pm)\) vertex \([7]\) needs to be taken in the \((-1, -1)\) picture yielding an overall even number of \(\psi\)'s. Schematically, this part of the correlator is proportional to \(C T^{2n}\) and only connects spinors of opposite chirality because the charge conjugation matrix \(C\) anticommutes with the chirality matrix \(\Gamma_\chi\). This is also true in \(d = 6\) whereas in \(d = 4\) and \(d = 8\) the opposite is true, since now the matrix \(C\) commutes with \(\Gamma_\chi\). Hence, in \(d = 4\) and \(d = 8\) we expect to find a coupling between an odd number of tachyons and RR fields from the same sector and none
between an odd number of tachyons and RR fields from the opposite sector.

c) The reverse statement holds for an even number of tachyons (in particular no tachyon at all). The correlators will now involve an odd number of \( \psi \)'s and these vanish in \( d = 10 \) between spinors of opposite chirality. Again, it is natural to expect that such a statement will hold in \( d = 6 \) and be reversed in \( d = 4 \) and \( d = 8 \).

For odd space-time dimensions, both statements b) and c) are empty due to the lack of chirality and we do not expect any particular symmetry of the couplings. Statement c) is also consistent with the fact that in \( d = 10 \) all correlation functions between \((NS+,NS+)\) fields and fields from one given RR sector are the same as in type II theory, which is obvious because the vertex operators and their correlation functions are precisely the same and there is no loop in which other modes could propagate. In particular, it indicates that there are Chern–Simons terms in the effective action and that the various field strengths need to be modified by shifting them with a \( B_{MN} \) dependent transformation just as in type II supergravity. It seems natural to postulate that such terms will be present also in lower dimensions in the cases where they are allowed by the symmetries of the problem.

These simple properties and some explicit tree-level computations allow one to write down, for the critical case, to first non-trivial order in \( \alpha' \), an effective action up to terms quadratic in the gauge fields. Below we shall present our proposed generalization of this action in any dimension including Chern–Simons terms.

### 2.2 D-branes in the \( d = 10 \) theory

We conclude this section with a look at the type of D-branes in this theory. In this subsection we always work in \( d = 10 \).

Due to the doubling of the RR fields, there are twice as many D-branes as in the corresponding type II theory. Let us denote by \( F \) and \( F' \) the RR fields from the two RR sectors of \( \mathbb{R} \) respectively. For a given \( p \) we have:

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\( ^6 \)We refrain from using the notation \( F \) and \( \bar{F} \) used in \( \mathbb{R} \) because we reserve such notation for the \( d = 4 \) and \( d = 8 \) case where such fields are complex conjugate of each other.
thus four types of elementary branes (counting the anti-branes) with charges $(Q = 1, Q' = 1)$, $(Q = 1, Q' = -1)$, $(Q = -1, Q' = 1)$, $(Q = -1, Q' = -1)$. Branes charged only with respect to, say, $F$ are possible but carry charge $(Q = 2n, Q' = 0)$ in these units and thus can be built from the four constituents above.

A quick way to understand the spectrum of massless excitations living on the world-volume of a stack of such branes is to consider the closed string exchange between two such branes, perform a modular transformation and read off the spectrum from the open string sector.

Let us denote the usual open string traces in the various sectors as

\begin{align}
\text{Tr}_{\text{NS}}(q^N) &= \left(\frac{f_3(q)}{f_1(q)}\right)^8 \\
\text{Tr}_{\text{NS}}((-)^F q^N) &= \left(\frac{f_4(q)}{f_1(q)}\right)^8 \\
\text{Tr}_{\text{R}}(q^N) &= \left(\frac{f_2(q)}{f_1(q)}\right)^8 ,
\end{align}

(9)

where, as usual,

\begin{align}
f_1(q) &= q^{1/12}\Pi(1-q^{2n}) \\
f_2(q) &= \sqrt{2}q^{1/12}\Pi(1+q^{2n}) \\
f_3(q) &= q^{-1/24}\Pi(1+q^{2n-1}) \\
f_4(q) &= q^{-1/24}\Pi(1-q^{2n-1}).
\end{align}

(10)

The closed string exchange can be written in terms of these functions by making a modular transformation $\tilde{q} = e^{-\pi^2} \rightarrow q = e^{-\pi/\tilde{t}} = e^{-\pi t}$. Let us denote by $H$ the light-cone oscillator part of the closed string Hamiltonian, and use the boundary states $|B\rangle$ to denote the branes (cf. \cite{35, 36} for a complete discussion of the boundary state formalism). Let us also introduce the shorthand notation

\begin{equation}
[N_{S+}, N_{S+}] = \langle B| e^{-iH} |B\rangle_{(N_{S+}, N_{S+})},
\end{equation}

(11)

\footnote{We use the symbol $\text{Tr}$ to denote the sum over all eight transverse bosonic and fermionic components, to distinguish it from the trace $\text{tr}$ in (1).}
and similar expressions for the other sectors. Then, after a modular transformation:

\[
\begin{align*}
[NS+, NS+] & \rightarrow \frac{1}{2} \left( \text{Tr}_{NS} \left( q^N \right) - \text{Tr}_{R} \left( q^N \right) \right) = \frac{1}{2} \left( \frac{f_3(q)^8 - f_2(q)^8}{f_1(q)^8} \right) \\
[NS-, NS-] & \rightarrow \frac{1}{2} \left( \text{Tr}_{NS} \left( q^N \right) + \text{Tr}_{R} \left( q^N \right) \right) = \frac{1}{2} \left( \frac{f_3(q)^8 + f_2(q)^8}{f_1(q)^8} \right) \\
[R\pm, R\pm] & \rightarrow -\frac{1}{2} \text{Tr}_{NS} \left( (-)^F q^N \right) = -\frac{1}{2} \left( \frac{f_4(q)^8}{f_1(q)^8} \right).
\end{align*}
\]

(12)

All RR sectors give the same contribution.

There are three elementary cases to consider. First, consider the case of two like-like charged branes \((Q_1 = 1, Q'_1 = 1)\) and \((Q_2 = 1, Q'_2 = 1)\). Let us consider the type 0B case for definitiveness; all considerations apply to the type 0A case as well. In this case we have the following situation:

\[
[NS+, NS+] + [NS-, NS-] + [R+, R+] + [R-, R-] \rightarrow \frac{f_3(q)^8 - f_4(q)^8}{f_1(q)^8}.
\]

From (14) we read off, with the wisdom of [37, 38], that the world-sheet theory has no tachyon, no fermions, and the same massless bosons as pure \(d = 10\) Yang-Mills theory dimensionally reduced to \(p + 1\) dimensions.

Second, consider the case \((Q_1 = 1, Q'_1 = 1)\) and \((Q_2 = 1, Q'_2 = -1)\). Here the exchange is

\[
[NS+, NS+] - [NS-, NS-] + [R+, R+] - [R-, R-] \rightarrow -\frac{f_2(q)^8}{f_1(q)^8}.
\]

(15)

Perhaps the only subtlety is the minus sign in front of the \((NS-, NS-)\) exchange. This sign is fixed by looking at the coupling of the tachyon to the branes and noticing [3] that it has a coupling constant proportional to the product of the charges of the brane, thus yielding \(Q_1Q'_1 \times Q_2Q'_2 = -1\). From (15) we see that this brane configuration has only fermions on the world-volume. These two configurations were studied in [4].

Finally the brane/anti-brane case is given by \((Q_1 = 1, Q'_1 = 1)\) and
\[Q_2 = -1, Q'_2 = -1\) and corresponds to

\[\{NS^+, \bar{NS}^+\} + \{NS^-, \bar{NS}^-\} - \{R^+, \bar{R}^+\} - \{R^-, \bar{R}^-\} \to \frac{f_3(q)^8 + f_4(q)^8}{f_1(q)^8}.\]

(16)

There is an open string tachyon in this theory, just as in [39, 40], signaling an instability of the theory.

3 Type 0 effective actions, Ramond–Ramond fields and Chern–Simons terms

In this section we present our proposal for the effective action of type 0 string theory in any dimension including the Chern–Simons couplings. Lacking a formulation from “prime principles”, the identification of the RR sectors and their couplings requires a certain amount of guesswork. The picture that emerges, however, is quite simple and satisfying. We shall see, for instance, that it gives support to the idea that the RR sectors must be doubled compared to the type II string.

3.1 The NS-NS sector

The NS-NS sector is common to all of these theories and can in principle be obtained from a sigma-model approach. It involves the massless fields of the \((\text{NS}^+, \text{NS}^+)\) sector (a dilaton \(\Phi\), a graviton \(g_{MN}\) and an antisymmetric tensor \(B_{MN}\)) and a tachyon \(T\) from the \((\text{NS}^-, \text{NS}^-)\) sector. The tachyon potential \(V(T)\) is an even function from the property a) of the previous section [5]. The relevant action is thus, in the string frame

\[S_{NS-NS} = \int d^4x \sqrt{-g} \left\{e^{-2\Phi} \left( R - \frac{1}{12} |dB|^2 + 4|d\Phi|^2 - \frac{1}{2} |dT|^2 - V(T) \right) \right\},\]

(17)

where it is natural to absorb the central charge deficit \(10 - d\) into the definition of the tachyon potential, i.e.

\[V(T) = -10 + d - \frac{d - 2}{8} T^2 + \cdots\]

(18)
It should be kept in mind that (17) is by no means unique. It suffers from the usual ambiguities that come from extrapolating on-shell data. In particular, there could be arbitrary (even) functions of $T$ multiplying the various kinetic terms in the Lagrangian. Up to this order in $\alpha'$ this is essentially all that can happen. As shown in the Appendix A of [5] however, precisely because of their ambiguous nature it is possible to redefine away some of them, such as the term $R T^2$. At the same time, terms of the type $T^2|dB|^2$ and their counterpart for the RR kinetic terms are needed and should be kept. In the following we will never need the field $B_{MN}$ and we will set the coefficients of $R$, $|dB|^2$ and $|dT|^2$ as in (17), the main conclusions being independent of the presence of such terms.

3.2 RR kinetic and Chern–Simons terms

One guiding principle [21] in the identification of the RR sector is the idea that for any $d$ there will still be massless excitations in the R sector of the open string and thus their on-shell degrees of freedom will fall into representations of the little group $SO(d - 2)$. The resulting situation is best summarized in the table below.

In ten dimensions one obtains massless RR fields in the type 0A theory by considering tensor products of spinors of different chiralities $(+–)$ and $(-+)$, and in the 0B theory of same chiralities $(++)$ and $(-–)$. This can be readily generalized for any non-critical even dimension, whereas it is not quite clear what the right generalization to odd dimensions is. Note, however, that an odd dimensional bispinor can be decomposed in terms of the lower even dimensional bispinors as $(++) \oplus (+–) \oplus (–+) \oplus (––)$, and that this is the sum of the field contents of the 0A and the 0B theories of one lower dimension. It thus seems reasonable to assume that a sum of two modular invariant sectors should yield a modular invariant theory in one dimension higher without doubling the RR spectrum by hand.

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8 Things become even more complex at the next order in $\alpha'$ — for instance there could be terms of the type $T^{2n}R^{MN}\partial_M T \partial_N T$.

9 A different point of view was taken in [21]. It will be interesting to find a resolution to this puzzle.
| $d$ | $SO(d-2)$ | Spin reps. | $R \times R$ sector(s) | Real off-shell fields |
|-----|------------|------------|-------------------------|----------------------|
| 4   | $U(1)$     | $1_{1/2}, 1_{-1/2}$ | $0A$ 0B $1_0 + 1_0$  $1_{-1} + 1_1$ | $2A$ $A_M$ |
| 5   | $SU(2)$    | 2          | 0AB $1 + 3$            | $A,A_M$              |
| 6   | $SU(2)^2$  | $(1,2), (2,1)$ | 0A 0B $2(2,2)$ $2(1,1) + (1,3) + (3,1)$ | $2A_M$ $2A, A_{MN}$ |
| 7   | $Sp(4)$    | 4          | 0AB $1 + 5 + 10$      | $A, A_M, A_{MN}$     |
| 8   | $SU(4)$    | 4, 4       | 0A 0B $2(1 + 15)$ $6 + 10 + 6 + 10$ | $2A, 2A_{MN}$ $2A_M, A_{MNR}$ |
| 9   | $SO(7)$    | 8          | 0AB $1 + 7 + 21 + 35$ | $A, A_M, A_{MN}, A_{MNR}$ |
| 10  | $SO(8)$    | $8_a, 8_c$ | 0A 0B $2(8 + 56)$ $2(1 + 28 + 35)$ | $2A_M, 2A_{MNR}$ $2A, 2A_{MN}, A_{MNRP}$ |

To understand the table consider for example $d = 8$. In the type 0B theory, the bispinor in the $(R+, R+)$ sector decomposes into $4 \times 4 = 6 + 10$. These are all complex representations that yield a complex vector and a complex three form with a self dual field strength. Notice that it is possible to construct a self dual form in $d = 8$ only if it is complex because $*^2 = -1$. The bispinor in the $(R-, R-)$ sector yields the complex conjugate fields. These two sets of fields can be combined into two real one-forms and one real three-form without any duality constraint. These are the fields written in the last column.

Notice that in odd dimensions we have only one version of the theory (0AB) without the restriction on the rank of the forms. In even dimensions the forms come in even or odd rank depending on the RR projection. In $d = 10$ and $d = 6$ the assignment is the familiar one (odd forms for type A and even for type B) whereas in $d = 8$ and $d = 4$ it is reversed. Of course, it is possible to dualize some of the fields to obtain the magnetically charged
branes. The unique form of degree \( d/2 - 1 \) for the type 0B case admits both electric and magnetic charges.

Notice that if it were not for the doubling of the RR sectors it would be impossible to write off-shell real fields for the 0B theory in \( d = 4 \) and \( d = 8 \) because, due to the Minkowski signature, it is impossible to impose either self-duality or anti-self-duality on real forms. We view this fact, together with the argument based on modular invariance, as yet another piece of evidence for the necessity of the presence of both RR sectors.

To obtain the complete form of the RR couplings up to two derivatives we need to address the issue of Chern–Simons terms. The terms of relevance here are those constructed with 3 gauge fields and 2 derivatives. Despite the lack of space-time supersymmetry, the Chern–Simons terms are present, at least in \( d = 10 \), because they are there in the type II theories. It thus seems that the presence of these terms is dictated more by world-sheet supersymmetry than by space-time supersymmetry and it is natural to assume that such terms are also present in lower dimensions. The presence of so many RR fields may seem to lead to difficulties in determining these terms. However, there are two simplifying features that we infer from the \( d = 10 \) case: First, there will not be terms involving only RR fields since they correspond to the correlation function of an odd number of spin fields. One of the three gauge fields must therefore be the \((\text{NS}+,\text{NS}+)\) two-form \( B_{MN} \). Second, applying the selection rules of the previous section, we see that the coupling will involve fields from the same RR sector in \( d = 6 \) and \( d = 10 \) and from the opposite sector in \( d = 4 \) and \( d = 8 \).

Let us start with the case of \( d \) odd. In this case our proposal for the RR part of the action is

\[
S^\text{0AB}_{d=5} = \int f(T)(F_1 \ast F_1 + F_2 \ast F_2) + BF_1F_2 \\
S^\text{0AB}_{d=7} = \int f(T)(F_1 \ast F_1 + F_2 \ast F_2 + \tilde{F}_3 \ast \tilde{F}_3) + BF_2F_3 \\
S^\text{0AB}_{d=9} = \int f(T)(F_1 \ast F_1 + F_2 \ast F_2 + \tilde{F}_3 \ast \tilde{F}_3 + \tilde{F}_4 \ast \tilde{F}_4) + BF_3F_4. (19)
\]

The forms \( F_n \) are the field strengths associated to the RR gauge potentials. In this section we use “index free” notation and redefine the normalization coefficients to one in order not to clutter the formulas too much. A wedge product between forms is always understood. The tilde above the forms
always indicates the “NS-NS shift”, e.g. $\tilde{F}_4 = F_4 + BF_2$, with the appropriate modified gauge transformation just as in type II supergravity. Notice that, once it is assumed that the Chern–Simons terms are present, the modification in the field strength must also be present for the action to transform correctly under electric/magnetic duality. $f(T)$ is a function of the tachyon whose first few coefficients in the Taylor expansion around zero can in principle be determined by extrapolating from the on-shell computation $[5]$ in $d = 10$. We shall see that the detailed form of these functions is not directly relevant for the computations of the properties of the dual field theories.

To write the actions for the even dimensional cases, let us denote the field strengths from the two RR sectors by $F$, $F'$ for $d = 6$ and $d = 10$ and by $F$, $\tilde{F}$ for $d = 4$ and $d = 8$. In the former case the field strengths are real whereas in the latter they are complex conjugates of each other. The form of highest degree in the type 0B case is special – it is self dual in the complex case and its vertex operator does not contain the chiral projection.

\[
\begin{align*}
S_{d=4}^{0A} &= \int f_{\text{even}}(T)(F_1 * F_1) + f_{\text{odd}}(T)(F_1 * F_1 + \tilde{F}_1 * \tilde{F}_1) + iBF_1 \tilde{F}_1 \\
S_{d=4}^{0B} &= \int f_{\text{even}}(T)(F_2 * F_2) + f_{\text{odd}}(T)(F_2 * F_2 + \tilde{F}_2 * \tilde{F}_2) \\
S_{d=6}^{0A} &= \int f_{\text{even}}(T)(F_2 * F_2 + F'_2 * F'_2) + f_{\text{odd}}(T)(F_2 * F'_2) + B(F_2 F_2 + F'_2 F'_2) \\
S_{d=6}^{0B} &= \int f_{\text{even}}(T)(F_1 * F_1 + F'_1 * F'_1 + \tilde{F}_3 * \tilde{F}_3) + f_{\text{odd}}(T)(F_1 * F'_1 + \tilde{F}_3 * \tilde{F}_3) + B(F_1 F_3 + F'_1 F'_3) \\
S_{d=8}^{0A} &= \int f_{\text{even}}(T)(F_3 * F_3 + \tilde{F}_3 * \tilde{F}_3) + f_{\text{odd}}(T)(F_1 * F_1 + \tilde{F}_1 * \tilde{F}_1 + \tilde{F}_3 * \tilde{F}_3) + iBF_3 \tilde{F}_3 \\
S_{d=8}^{0B} &= \int f_{\text{even}}(T)(F_2 * F_2 + \tilde{F}_4 * \tilde{F}_4) + f_{\text{odd}}(T)(F_2 * F_2 + \tilde{F}_2 * \tilde{F}_2) + B(F_2 \tilde{F}_4 + F_4 \tilde{F}_2) \\
S_{d=10}^{0A} &= \int f_{\text{even}}(T)(F_2 * F_2 + F'_2 * F'_2 + \tilde{F}_4 * \tilde{F}_4) + f_{\text{odd}}(T)(F_2 * F'_2 + \tilde{F}_4 * \tilde{F}_4) + B(F_4 F_2 + F'_4 F'_2) \\
S_{d=10}^{0B} &= \int f_{\text{even}}(T)(F_1 * F_1 + F'_1 * F'_1 + \tilde{F}_3 * \tilde{F}_3 + \tilde{F}_5 * \tilde{F}_5) + f_{\text{odd}}(T)(F_1 * F'_1 + \tilde{F}_3 * \tilde{F}_3 + \tilde{F}_5 * \tilde{F}_5) + B(F_3 F_5 + F'_3 F'_5). \tag{20}
\end{align*}
\]
We present the form of the actions in all their generality because it will be useful for future more detailed computations. For our present purposes however, it should be noticed that the kinetic terms in the actions can be diagonalized by letting $F_\pm = F \pm F'$ in $d = 6, 10$ and $F_\pm = F \pm i F$ in $d = 4, 8$.

### 3.3 Massive type 0 gravity

There is still one RR form field that can be added to the actions (19) and (20). In a $d$-dimensional space-time it is possible to introduce a rank $d - 1$ gauge potential coupling to a corresponding extended object. It carries no physical degrees of freedom and therefore it is not visible in the on-shell analysis of the previous subsection. Its rank $d$ field strength, however, carries an energy density and it does affect the physics. This form will be used in Section 5 when constructing various field theory duals. This case will provide the simplest example which displays most of the interesting physics, and it allows one to avoid the complications of disentangling the Kaluza-Klein modes.

In type IIA supergravity (and thus in $d = 10$ type 0A for each RR sector) it is well known how to introduce such a field [41, 42]. The required modifications in the bosonic sector are the addition of the terms

$$\int M F_{10} + \frac{1}{2} M^2 \ast 1$$

(21)

to the action, and a further shift of the 2- and the 4-form field strengths by $M B$ and $M B^2/2$, respectively. The gauge transformations are changed accordingly in order to re-ensure gauge invariance. Integrating over the gauge potential of $F_{10}$ imposes the constraint that $M$ be constant. Solving the equation of motion for $M$ establishes a connection between $M$ and $F_{10}$. In the case $B = 0$ – relevant to our analysis – they are simply Hodge duals of each other as is readily seen from (21).

From the string theory point of view [37], the natural generalization of the RR $\beta$-function equations implies $d \ast F_d = 0$ and $d F_d = 0$, as the top-form, too, appears in the reduction of an even dimensional type 0A bispinor into antisymmetric tensor representations. In our case, we must also include the coupling with the tachyon. The relevant addition is

$$- \frac{1}{2} \int f(T) F_d \ast F_d$$

(22)

that we assume be present in any dimension.
4 Classical solutions

In what follows we shall show that the above described low energy theories allow Freund–Rubin type solutions [43], where the dilaton and the tachyon are constant, the space-time factorizes into a product of an AdS space and a sphere, and the only nontrivial form-field is a RR field. Such types of solutions are very familiar from the supergravity literature, see e.g. [44, 45, 46, 47, 48].

It is sufficient to consider the Einstein frame action

\[ S = \int d^dx \sqrt{-g} \left\{ R - \frac{1}{2} (\partial_M \Phi)^2 - \frac{1}{2} (\partial_M T)^2 - V(T) \ e^{a\Phi} \right. \]

\[ \left. - \frac{1}{2 (p+2)!} f(T) \ e^{b\Phi} \left( F_{M_1 \ldots M_{p+2}} \right)^2 \right\}, \]

where \( V(T) \) is the sum of the tachyon potential and the central charge deficit, and \( f(T) \) is the coupling between the \((p+2)\)-dimensional RR form \( F \) and the tachyon. The RR gauge field is the appropriate linear combination of some of the fields of the previous section in such a way that the kinetic terms are diagonal. After diagonalization, \( f(T) \) no longer has any particular symmetry property.

The coefficients \( a \) and \( b \) are

\[ a = \sqrt{\frac{2}{d-2}} \]

\[ b = \frac{1}{2} (d-2p-4) \sqrt{\frac{2}{d-2}}. \]

The field \( B_{MN} \) that we are setting to zero here may appear linearly in the full action only in the Chern–Simons term, but in that case multiplied by \( F \wedge F \), which will vanish in the Freund–Rubin ansatz.

The equations of motion can be summarized as follows:

\[ \Box \Phi = aV(T) \ e^{a\Phi} + \frac{b}{2} f(T) \ e^{b\Phi} \frac{1}{(p+2)!} \left( F_{M_1 \ldots M_{p+2}} \right)^2 \]

\[ 1^{10} \text{We now switch to component notation for clarity and reinstate all the appropriate normalizations. Note that our normalization of the tachyon differs by a factor of } \sqrt{2} \text{ from that of most of the recent literature.} \]
\[ \Box T = V'(T) e^{\Phi} + \frac{1}{2} f'(T) e^{b\Phi} \frac{1}{(p+2)!} \left( F_{M_1\ldots M_{p+2}} \right)^2 \] (27)

\[ R_{MN} = \frac{1}{2} \partial_M \Phi \partial_N \Phi + \frac{1}{2} \partial_M T \partial_N T + \frac{1}{d-2} g_{MN} V(T) e^{a\Phi} + \frac{1}{2} f(T) e^{b\Phi} T_{MN} \] (28)

\[ 0 = \nabla^N \left( f(T) e^{b\Phi} F_{N,M_1\ldots M_{p+1}} \right). \] (29)

The tensor \( T_{MN} \) is shorthand for the (trace subtracted) stress energy tensor

\[ T_{MN} = \frac{1}{(p+1)!} \left( F_{MK_1\ldots K_{p+1}} F^N_{K_1\ldots K_{p+1}} - \frac{(p+1) g_{MN}}{(p+2)(d-2)} \left( F_{K_1\ldots K_{p+2}} \right)^2 \right) \] (30)

Again, we have ignored potential contributions from Chern–Simons terms because they vanish for the classical solution. They do contribute to the analysis of the fluctuations in the general case and also for this reason, in the next section, when computing the critical properties of the field theory duals we restrict to the simple case \( d = p + 2 \) where such complications do not arise. We hope to return to the general case in a later paper [23].

These equations of motion have a solution with constant dilaton \( \Phi = \Phi_0 \) and tachyon \( T = T_0 \) in the gravity background of a product space

\[ \text{AdS}_{p+2} \times S^{d-p-2}. \] (31)

The size of the two maximally symmetric spaces is determined by setting\( ^\dagger \)

\[ R_{\mu\nu\rho\lambda} = -\frac{1}{R_0^2} (g_{\mu\nu} g_{\rho\lambda} - g_{\mu\lambda} g_{\nu\rho}) \quad R_{ijkl} = \frac{1}{L_0^2} (g_{ik} g_{jl} - g_{il} g_{jk}). \] (32)

Finally, the RR field is set proportional to the volume-form of the anti-de Sitter space, and hence its only nontrivial components are

\[ F_{\mu_1\ldots \mu_{p+2}} = F_0 \sqrt{-g(\text{AdS}_{p+2})} \epsilon_{\mu_1\ldots \mu_{p+2}}, \] (33)

where the constant \( F_0 \) is related to the conserved charge \( k \) by

\[ k = f(T_0) e^{b\Phi_0} F_0. \] (34)

\( ^\dagger \)The Greek indices refer to the AdS space and the Latin indices to the sphere.
Given the two functions $V(T)$ and $f(T)$, the tachyon and the dilaton vacuum expectation values are determined from Eqs. (26), (27) and (34). The tachyon $T_0$ can be expressed implicitly, as the solution of an algebraic equation, namely

$$\frac{f''(T_0)}{f(T_0)} = \frac{1}{2}(d - 2p - 4) \frac{V'(T_0)}{V(T_0)}.$$  \hfill (35)

The dilaton $\Phi_0$ can then be readily obtained from

$$e^{(a+b)\Phi_0} = \frac{(d - 2p - 4) k^2}{4 \ f(T_0) V(T_0)}.$$  \hfill (36)

The radii of the anti-de Sitter space $R_0$ and that of the sphere $L_0$ can be solved from the Einstein equations (28)

$$R_0^2 = (p + 1)(d - 2p - 4) e^{-a\Phi_0} \frac{1}{V(T_0)}.$$  \hfill (37)

$$L_0^2 = (d - p - 3)(d - 2p - 4) e^{-a\Phi_0} \frac{1}{V(T_0)}.$$  \hfill (38)

In the derivation we assumed $k \neq 0$. Also three special dimensionalities were excluded for compactness:

a) The case $d = p + 3$ leads to an infinite radius in the AdS space-time, i.e. the flat Minkowski space times a circle, and is not considered in what follows.

b) For $d = 2p + 2$ the dilaton becomes a free parameter. Rather than its vacuum expectation value $\Phi_0$, the charge $k$ is determined from the equation of motion (26)

$$k^2 = -2V(T_0) f(T_0).$$  \hfill (39)

The rest of the formulae (33), (37) and (38) are still valid.

c) We assumed that $V(T_0) \neq 0$. In addition to some completely Ricci flat solutions this condition also excludes the middle dimensional branes, for which we have $d = 2p + 4$. In these dimensions the radii are

$$R_0^2 = L_0^2 = 4 (p + 1) \frac{f(T_0)}{k^2}.$$  \hfill (40)
Now $T_0$ is determined from $V(T_0) = 0$ (not $V'(T_0) = 0$) and $\Phi_0$ from

$$V'(T_0)e^{\alpha\Phi_0} - \frac{k^2}{2} \frac{f'(T_0)}{f(T_0)^2} = 0.$$  \hspace{1cm} (41)

The solutions discussed above are physically acceptable only for $f(T_0) > 0$.

5 Dual field theory interpretation

In this section we finally make contact with the conjectured gravity/field theory duality by studying the field theory duals of the type 0 theories for the simple case of $d = p + 2$. This case already contains all the relevant qualitative features of the most general one, without the complication of the Kaluza–Klein analysis.

Let us state the logic of the approach. The classical solutions on the gravity side correspond to fixed points on the field theory side. There is a geodesic flow that relates these classical solutions, which is interpreted as the renormalization group flow connecting different fixed points. Fluctuation modes with positive, vanishing, and negative mass square correspond to irrelevant, marginal, and relevant deformations. The critical exponents can be obtained from these masses and they depend on a finite set of undetermined parameters due to the arbitrariness of the tachyon couplings. These parameters should be fixed by comparing some universal quantities with experiment which leads to a prediction for the remaining quantities.

5.1 Stability of the AdS$_d$ solutions

Classical solutions can serve as sound vacua for a quantum theory only if small fluctuations around the solutions are stable. In Minkowski space this implies that tachyonic fluctuation modes are forbidden. In an AdS background this requirement can be relaxed, and one finds the bound \[49, 50, 51, 52\]

$$m^2 \geq -\frac{(d-1)^2}{4} \frac{1}{R_0^2}.$$  \hspace{1cm} (42)

for the masses of the scalar fluctuation modes.
The first source of these instabilities near the solutions found in the previous section are obviously fluctuations in the tachyon field $T$. Tachyonic instabilities may enter also through the various scalar fields that appear on the AdS space, as the fields are compactified on the sphere $S^{d-p-2}$. In order to show that the theory is stable against these perturbations, one has to linearize the full set of equations of motion around the classical solution, and check that no mode violates the bound (42).

This can be done, but the physically relevant features already appear in the case where the transverse sphere is absent. We shall discuss this example below in detail.

As $d = p+2$, the nontrivial RR field is the top-form, dual to a cosmological constant
\[ F_{\mu_1 \cdots \mu_{p+2}} = F \sqrt{-g} \epsilon_{\mu_1 \cdots \mu_{p+2}}. \] Note, that this is no longer the Freund–Rubin ansatz, but the RR field is a priori entirely general and unconstrained. This is the field discussed in Section 3.3. The equation of motion (29) becomes in this case a constraint, and it turns out that the conserved charge is
\[ k = f(T) e^{b\Phi} F. \] With the help of (44), the equations of motion for the other fields reduce to a Hamiltonian form
\[ \Box \Phi = -\frac{\partial}{\partial \Phi} V(\Phi, T) \] \[ \Box T = -\frac{\partial}{\partial T} V(\Phi, T) \]
\[ R_{\mu\nu} = \frac{1}{2} \partial_{\mu} \Phi \partial_{\nu} \Phi + \frac{1}{2} \partial_{\mu} T \partial_{\nu} T - \frac{1}{d-2} V(\Phi, T) g_{\mu\nu}. \]
where the effective potential is
\[ V(\Phi, T) = -V(T)e^{a\Phi} - \frac{1}{2} \frac{k^2}{f(T)} e^{-b\Phi}. \]
\[ \Phi = \Phi_0 + \varphi \]
\[ T = T_0 + t \]
\[ g_{\mu\nu} = \hat{g}_{\mu\nu} + h_{\mu\nu}. \]
In order to do this, we need some knowledge of the functions \( V(T) \) and \( f(T) \). The only characteristics of these functions that will enter the stability analysis are the coefficients

\[
x = \frac{V'(T_0)}{V(T_0)}, \quad y = \frac{V''(T_0)}{V(T_0)}, \quad \text{and} \quad z = \frac{f''(T_0)}{f(T_0)}.
\]  

Perturbative string theory analysis around \( T = 0 \) yields

\[
V(T) = d - 10 - \frac{d - 2}{8} T^2 + \mathcal{O}(T^4) \tag{53}
\]

\[
f(T) = 1 + T + \frac{1}{2} T^2 + \mathcal{O}(T^3). \tag{54}
\]

This is not enough to determine the coefficients \((52)\), and they should indeed be treated as free parameters of the theory. Including other unknown functions (see discussion in Section 3.1) would give rise to more than three such parameters, but the analysis performed here would still have the same qualitative features.

The fact that the graviton fluctuations actually decouple completely from those of the scalars simplifies the calculations: The graviton equations of motion can, in fact, be derived to first order from the effective action

\[
S_h = \int d^d x \sqrt{-(\hat{g} + h)} \left\{ R(\hat{g}_{\mu\nu} + h_{\mu\nu}) + V(\Phi_0, T_0) \right\}. \tag{55}
\]

The scalar fluctuations obey

\[
\left( -\Box + \mathcal{M} \right) \begin{pmatrix} \varphi \\ t \end{pmatrix} = 0 \tag{56}
\]

where the mass matrix is

\[
\mathcal{M} = d(d - 1) R_0^{-2} \begin{pmatrix} \frac{1}{\sqrt{d-2} x} & \sqrt{\frac{d-2}{2}} x \\ \sqrt{\frac{d-2}{2}} x & d x^2 - y - \frac{2}{3} z \end{pmatrix}. \tag{57}
\]

The mass eigenvalues are

\[
m_{1,2}^2 = d(d - 1) R_0^{-2} \left( 1 + \frac{\tau}{2} \pm \frac{1}{2} \sqrt{\tau^2 + (2d - 4)x^2} \right). \tag{58}
\]
where

\[ \tau = d \frac{x^2 - 2z}{d} - y - 1. \]  \hspace{1cm} (59)

Note that the masses depend only on two independent parameters \(x\) and \(\tau\).

If we assume, following [6], that \(f(T) = \exp(T)\), then the equations of motion give \(x = -2/d\), and we can easily extract some interesting qualitative features as the only undetermined parameter is \(\tau\).

In this case there turns out to be three different, continuously connected phases: First, there can be two particles, both with positive mass squared. Second, there can be a particle and a tachyon that obeys the bound (42). Third, there can be a tachyon that makes the vacuum unstable. In AdS/CFT correspondence this translates into the statement that there can be at most one relevant operator in the infrared near the fixed point described by this theory.

### 5.2 Solutions connecting conformal fixed points

The stability analysis as applied to the critical points of the potential yields local information about the behavior of the dual field theory near its critical points. Depending on the form of the potential, there may exist gravity solutions that interpolate between different critical points. These solutions should be interpreted on the field theory side as RG trajectories between conformal points.

In order to study these interpolating solutions we consider the ansatz

\[ ds^2 = dy^2 + A^2(y) \frac{dx^2}{d} \]  \hspace{1cm} (60)

and allow the two scalars to depend on the Liouville coordinate \(y\). We already know from the previous sections that there are exact solutions of the form

\[ A(y) = e^{y/R}, \]  \hspace{1cm} (61)

where \(R\) is the radius of the pertinent AdS space.

The Einstein equation gives rise to two independent equations. Defining the following auxiliary function

\[ \gamma(y) = (d-1) \frac{d}{dy} \log(A), \]  \hspace{1cm} (62)
the full set of equations takes the form

\[
\frac{\ddot{A}}{A} + (d-2) \left( \frac{\dot{A}}{A} \right)^2 = \frac{1}{d-2} \nu \tag{63}
\]

\[
\ddot{\Phi} + \gamma \dot{\Phi} = -\nabla V \tag{64}
\]

\[
\dot{\gamma} = -\frac{d-1}{2(d-2)} (\dot{\Phi})^2 \leq 0 \tag{65}
\]

Here we denote derivatives with respect to \( y \) with a dot, and we have introduced the compact notation \( \Phi = (\Phi, T) \) for the two scalars.

Provided \( \gamma \geq 0 \), equation (64) has the physical interpretation of a particle moving on a plane in the potential \( V \), subject to a friction force. Let us assume that the potential has two critical points \( \Phi_1 \) and \( \Phi_2 \) that satisfy \( V(\Phi_1) > V(\Phi_2) \), and that there is at least one unstable direction at \( \Phi_1 \) for increasing \( y \) and, similarly, a stable direction for \( \Phi_2 \). This can always be arranged by choosing the \( \mathcal{O}(T^4) \) part in \( V(T) \) suitably. Due to the friction coefficient we expect our particle to roll down starting from the IR fixed point, and to converge in an infinite amount of time towards the lower UV fixed point. This happens, since \( \gamma \) is strictly positive: Indeed, at the critical points \( \gamma \) approaches the values

\[
\gamma \to \frac{d-1}{R_1} \quad \text{for } y \to -\infty \tag{66}
\]

\[
\gamma \to \frac{d-1}{R_2} \quad \text{for } y \to +\infty , \tag{67}
\]

and the friction coefficient decreases monotonously between them according to (65). This is consistent with the fact that

\[
R_{1,2}^2 = \frac{(d-1)(d-2)}{V(\Phi_{1,2})} \tag{68}
\]

as follows from (63).

The solution might be oscillatory near the UV critical point. Whether this happens depends on whether the friction is enough to stop the particle as it arrives at the lower point. Clearly, if one wants to interpret the result as an RG flow, the oscillatory behavior would be difficult to accommodate in
the field theory picture. The oscillatory solutions are exactly the solutions that would violate the bound \( (12) \), which is a necessary condition for the consistency of the system on the gravity side. Hence, quite remarkably, the stability in the gravity theory is dual to the consistency of the field theory interpretation.

Some universal information can be read from the local behavior of these solutions. In the spirit of the Wilsonian RG treatment, let us study the critical behavior near the two fixed points in the linearized approximation. We must first identify the appropriate coordinate which in field theory can be consistently interpreted as the energy scale and parameterizes the interpolating solution. Such a coordinate can be chosen to be

\[
U = \frac{A^2}{\dot{A}}
\]  

(69)

since at the critical points this reduces to \( U = R \frac{\psi''}{R} \), where the metric takes the standard form

\[
ds^2 = \frac{R^2}{U^2}dU^2 + \frac{U^2}{R^2}dx_\parallel^2.
\]  

(70)

Define

\[
\tilde{\Phi}(U) = \Phi_0 + \delta \tilde{\Phi}(U),
\]  

(71)

so that Eq. (56) takes the form

\[
\left[-\frac{1}{R^2}[d \partial_U + U^2 \partial_U^2] + \mathcal{M}\right] \delta \tilde{\Phi} = 0.
\]  

(72)

The eigenvalues of \( \mathcal{M} \), namely \( m_i^2 \) are found in Section 5.1, and for each of the two eigenvectors we get two linearly independent solutions

\[
\delta \tilde{\Phi}_i = A_i U^\lambda_i + B_i U^{-\lambda_i},
\]  

(73)

where

\[
\lambda_i = \frac{-(d-1) \pm \sqrt{(d-1)^2 + 4m_i^2 R^2}}{2}.
\]  

(74)

\footnote{This definition corresponds locally, near the fixed points, to the one used in \([4]\). However, there are alternative definitions. For instance choosing \( U = \dot{A} \) one obtains the holographic relation, cf. \([53]\). All of these definitions lead to the same universal quantities.}
Notice first that the stability condition \([42]\) ensures the reality of the roots and they only depend on the dimensionless parameters \(x, y,\) and \(z\).

The IR limit corresponds to taking \(U \to 0\), for which there must be at least one positive eigenvalue, say, \((m_{\text{IR}}^1)^2 > 0\). In order for the solution not to blow up at this point we must choose \(B_1^{\text{IR}} = B_2^{\text{IR}} = 0\). If \((m_{\text{IR}}^2)^2 < 0\) we must also set \(A_2^{\text{IR}} = 0\), otherwise, the trajectory may in general start with a linear combination of the two eigenvectors. The trajectory will then evolve to the UV fixed point as \(U \to \infty\) where there will be at least one negative mass eigenvalue, say \((m_{\text{UV}}^1)^2\). Generically, both the coefficients \(A_1^{\text{UV}}\) and \(B_1^{\text{UV}}\) will be non-zero and the root \(\lambda_{+}^1\) will dominate.

In the AdS/CFT correspondence it is common to identify \(g_{\text{YM}}^2 = e^\Phi\). This relation is plausible by considering the weak coupling expansion of the world-volume gauge theory living on the stack of D-branes. On the other hand, away from the Gaussian fixed point there is no a priori reason to make such an identification. Thus we will stay general and regard \(\Phi\) and \(T\) as the coupling constants.

We can now read off the leading order behavior of the \(\beta\)-functions near the fixed points:

\[
\beta_i(g_i) = U \frac{dg_i}{dU} = \lambda_{+}^i (g_i - g_i^*) + \ldots, \tag{75}
\]

where \(g_i = \tilde{\Phi}_i\). In particular, the conformal dimension of an operator coupled to the bulk field \(\tilde{\Phi}_i(U, x)\) (a linear combination of the original tachyon and dilaton) is \(\Delta_i = d - 1 + \lambda_{+}^i\) and the anomalous dimension is \(\lambda_{+}^i\).

Note that, if \(\lambda_{+}\) vanishes, then we should have \(A = 0\) as well. This represents a marginal operator within our approximation and the computation of the \(\beta\)-function would pick up the sub-leading contribution which in the UV is \(\lambda_{-} = -(d - 1)\). This is what happens in [30], where the RG flow is studied in the framework of type II supergravity.

One could also study confining and asymptotically free solutions of these models as well as extend to situations where Kaluza–Klein modes are present. We hope to return to some of these issues in the future.

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