Charm and Beauty in a Hot Environment*

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Abstract:

We discuss the spectral analysis of quarkonium states in a hot medium of deconfined quarks and gluons, and we show that such an analysis provides a way to determine the thermal properties of the quark-gluon plasma.

1. Introduction

Since some thirty years it is known that besides the almost massless \textit{up} and \textit{down} quarks of the everyday world, and the still relatively light \textit{strange} quarks required to account for the strange mesons and hyperons observed in hadron collisions, there are heavy quarks at the other end of the scale, whose bare masses alone are larger than those of most of the normal hadrons. These heavy quarks first showed up in the discovery of the $J/\psi$ meson \cite{1}, of mass of 3.1 GeV; it is a bound state of a \textit{charm} quark ($c$) and its antiquark ($\bar{c}$), each having a mass of some 1.2–1.5 GeV. On the next level there is the $\Upsilon$ meson \cite{2}, with a mass of about 9.5 GeV, made up of a \textit{bottom} or \textit{beauty} quark-antiquark pair ($b\bar{b}$), with each quark here having a mass around 4.5 - 4.8 GeV. Both charm and bottom quarks can of course also bind with normal light quarks, giving rise to open charm ($D$) and open beauty ($B$) mesons. The lightest of these ‘light-heavy’ mesons have masses of about 1.9 GeV and 5.3 GeV, respectively.

The bound states of a heavy quark $Q$ and its antiquark $\bar{Q}$ are generally referred to as quarkonia. Besides the initially discovered vector ground states $J/\psi$ and $\Upsilon$, both the $c\bar{c}$ and the $b\bar{b}$ systems give rise to a number of other \textit{stable} bound states of different quantum numbers. They are stable in the sense that their mass is less than that of two light-heavy mesons, so that strong decays are forbidden. The measured stable charmonium spectrum contains the $1S$ scalar $\eta_c$ and vector $J/\psi$, three $1P$ states $\chi'_c$ (scalar, vector and tensor), and the $2S$ vector state $\psi'$, whose mass is just below the open charm threshold. There are further charmonium states above the $\psi'$; these can decay into $D\bar{D}$ pairs, and we shall here restrict our considerations only to quarkonia stable under strong interactions.

* Dedicated to Adriano Di Giacomo on the occasion of his 70th birthday
The study of quarkonia has played and continues to play a major role in many aspects of QCD: in spectroscopy, in production and decay, and last but not least as probe of hot QCD media \[3\]. And in the development of our understanding of and our feeling for the charm and the beauty of strong interactions, Adriano Di Giacomo has played and certainly will continue to play a major role. Neither my competence nor the space and time available here allow me to give an overview of these contributions; so let me just cite a few recent ones which are of particular importance to what I want to discuss here \[4, 5\].

Quarkonia are rather unusual hadrons. The masses of the light hadrons, in particular those of the non-strange mesons and baryons, arise almost entirely from the interaction energy of their nearly massless quark constituents. In contrast, the quarkonium masses are largely determined by the bare charm and bottom quark masses. These large quark masses allow a very straightforward calculation of many basic quarkonium properties, using non-relativistic potential theory. It is found that, in particular, the ground states and the lower excitation levels of quarkonia are very much smaller than the normal hadrons, and that they are very tightly bound. Now deconfinement is a matter of scales: when the separation between normal hadrons becomes much less than the size of these hadrons, they melt to form the quark-gluon plasma. What happens at this point to the much smaller quarkonia? When do they become dissociated? That is the main question we want to address here. We shall show that the disappearance of specific quarkonia signals the presence of a deconfined medium of a specific temperature \[6\]. Thus the study of the quarkonium spectrum in a given medium is akin to the spectral analysis of stellar media, where the presence or absence of specific excitation lines allows a determination of the temperature of the stellar interior.

We had defined quarkonia as bound states of heavy quarks which are stable under strong decay, i.e., \( M_{c\bar{c}} \leq 2M_D \) for charmonia and \( M_{b\bar{b}} \leq 2M_B \) for bottomonia. Since the quarks are heavy, with \( m_c \simeq 1.2 - 1.5 \text{ GeV} \) for the charm and \( m_b \simeq 4.5 - 4.8 \text{ GeV} \) for the bottom quark, quarkonium spectroscopy can be studied quite well in non-relativistic potential theory. The simplest (“Cornell”) confining potential \[7\] for a \( Q\bar{Q} \) at separation distance \( r \) has the form

\[
V(r) = \sigma r - \frac{\alpha}{r}
\]

with a string tension \( \sigma \simeq 0.2 \text{ GeV}^2 \) and a Coulomb-like term with a gauge coupling \( \alpha \simeq \pi/12 \). The corresponding Schrödinger equation

\[
\left\{ 2m_c - \frac{1}{m_c} \nabla^2 + V(r) \right\} \Phi_i(r) = M_i \Phi_i(r)
\]

then determines the bound state masses \( M_i \) and the wave functions \( \Phi_i(r) \), and with

\[
\langle r_i^2 \rangle = \frac{\int d^3r \ r^2 |\Phi_i(r)|^2}{\int d^3r |\Phi_i(r)|^2}.
\]

the latter in turn provide the (squared) average bound state “radii”, here defined as the \( Q\bar{Q} \) separation for the state in question.

The solution of eq. (2) gives in fact a very good account of the full (spin-averaged) quarkonium spectroscopy, as seen in Table 1 \[8\]. The line labelled \( \Delta M \) shows the differences between the experimental and the calculated values; they are in all cases less than 1 %. The
input parameters for these results were $m_c = 1.25$ GeV, $m_b = 4.65$ GeV, $\sqrt{\sigma} = 0.445$ GeV, $\alpha = \pi/12$. We see that in particular the $J/\psi$ and the lower-lying bottomonium states are very tightly bound ($2M_{D,B} - M_0 \gg l$) and of very small spatial size ($r_0 \ll 2r_h \simeq 2 \text{ fm}$). What happens to them in a hot and dense medium?

| state | $J/\psi$ | $\chi_c$ | $\psi'$ | $\Upsilon$ | $\chi_b$ | $\Upsilon'$ | $\chi_b'$ | $\Upsilon''$ |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| mass [GeV] | 3.10 | 3.53 | 3.68 | 9.46 | 9.99 | 10.02 | 10.26 | 10.36 |
| $\Delta E$ [GeV] | 0.64 | 0.20 | 0.05 | 1.10 | 0.67 | 0.54 | 0.31 | 0.20 |
| $\Delta M$ [GeV] | 0.02 | -0.03 | 0.03 | 0.06 | -0.06 | -0.08 | -0.07 |
| $r_0$ [fm] | 0.50 | 0.72 | 0.90 | 0.28 | 0.44 | 0.56 | 0.68 | 0.78 |

Table 1: Quarkonium Spectroscopy from Non-Relativistic Potential Theory

2. Interaction Range and Colour Screening

Consider a colour-singlet bound state of a heavy quark $Q$ and its antiquark $\bar{Q}$, put into the medium in such a way that we can measure the energy of the system as function of the $Q\bar{Q}$ separation $r$ (see Fig. 1). The quarks are assumed to be heavy so that they are static and any energy changes indicate changes in the binding energy. We consider first the case of vanishing baryon density; at $T = 0$, the box is therefore empty.

In vacuum, i.e., at $T = 0$, the free energy of the $Q\bar{Q}$ pair is assumed to have the string form

$$F(r) \sim \sigma r \quad (4)$$

where $\sigma \simeq 0.16 \text{ GeV}^2$ is the string tension as determined in the spectroscopy of heavy quark resonances (charmonium and bottomonium states). Thus $F(r)$ increases with separation distance; but when it reaches the value of a pair of dressed light quarks (about the mass of a $\rho$ meson), it becomes energetically favorable to produce a $q\bar{q}$ pair from the vacuum, break the string and form two light-heavy mesons ($Q\bar{q}$) and ($\bar{Q}q$). These can now be separated arbitrarily far without changing the energy of the system (Fig. 1).

Figure 1: String breaking for a $Q\bar{Q}$ system
The string breaking energy for charm quarks is found to be
\[ F_0 = 2(M_D - m_c) \simeq 1.2 \text{ GeV}; \] (5)
for bottom quarks, one obtains the same value,
\[ F_0 = 2(M_B - m_b) \simeq 1.2 \text{ GeV}, \] (6)
using in both cases the quark mass values obtained in the solution leading to Table 3. Hence the onset of string breaking is evidently a property of the vacuum as a medium. It occurs when the two heavy quarks are separated by a distance
\[ r_0 \simeq 1.2 \text{ GeV}/\sigma \simeq 1.5 \text{ fm}, \] (7)
independent of the mass of the (heavy) quarks connected by the string.

If we heat the system to get \( T > 0 \), the medium begins to contain light mesons, and the large distance \( Q\bar{Q} \) potential \( F(\infty, T) \) decreases, since we can use these light hadrons to achieve an earlier string breaking through a kind of flip-flop recoupling of quark constituents [9], resulting in an effective screening of the interquark force (see Fig. 2). Near the deconfinement point, the hadron density increases rapidly, and hence the recoupling dissociation becomes much more effective, causing a considerable decrease of \( F(\infty, T) \).

![Figure 2: In-medium string breaking through recoupling](image)

A further increase of \( T \) will eventually bring the medium to the deconfinement point \( T_c \), at which chiral symmetry restoration causes a rather abrupt drop of the light quark dressing (equivalently, of the constituent quark mass), increasing strongly the density of constituents. As a consequence, \( F(\infty, T) \) now continues to drop sharply. Above \( T_c \), light quarks and gluons become deconfined colour charges, and this quark-gluon plasma leads to a colour screening, which limits the range of the strong interaction. The colour screening radius \( r_D \), which determines this range, is inversely proportional to the density of charges, so that it decreases with increasing temperature. As a result, the \( Q\bar{Q} \) interaction becomes more and more short-ranged.

In summary, starting from \( T = 0 \), the \( Q\bar{Q} \) probe first tests vacuum string breaking, then a screening-like dissociation through recoupling of constituent quarks, and finally genuine colour screening. In Fig. 3 we show the behaviour obtained in full two-flavour QCD for the colour-singlet \( Q\bar{Q} \) free energy as a function of \( r \) for different \( T \) [10].

It is evident in Fig. 3 that the asymptotic value \( F(\infty, T) \), i.e., the energy needed to separate the \( Q\bar{Q} \) pair, decreases with increasing temperature, as does the separation
distance at which “the string breaks”. For the moment we consider the latter to be defined by the point beyond which the free energy remains constant within errors, returning in section 4.3 to a more precise definition. The behaviour of both quantities is shown in Fig. 4. Deconfinement is thus reflected very clearly in the temperature behaviour of the heavy quark potential: both the string breaking energy and the interaction range drop sharply around $T_c$. The latter decreases from hadronic size in the confinement region to much smaller values in the deconfined medium, where colour screening is operative.

The in-medium behaviour of heavy quark bound states thus does serve quite well as probe of the state of matter in QCD thermodynamics. We had so far just considered $Q\bar{Q}$ bound states in general. Let us now turn to a specific state such as the $J/\psi$. What happens when the range of the binding force becomes smaller than the radius of the state? Since the $c$ and the $\bar{c}$ can now no longer see each other, the $J/\psi$ must dissociate for temperatures above this point. Hence the dissociation points of the different quarkonium states provide a way to measure the temperature of the medium. The effect is illustrated schematically in Fig. 5, showing how with increasing temperature the different charmonium states “melt” sequentially as function of their binding strength; the most loosely bound state disappears first, the ground state last.

Moreover, since finite temperature lattice QCD also provides the temperature dependence of the energy density, the melting of the different charmonia or bottomonia can be
specified as well in terms of $\epsilon$. In Fig 6 we illustrate this, combining $\epsilon(T)$ with the force radii shown in Fig. 4. It is evident that although $\psi'$ and $\chi_c$ are both expected to melt around $T_c$, the corresponding dissociation energy densities could still differ.

![Figure 5: Charmonium spectra at different temperatures](image)

![Figure 6: Charmonium dissociation vs. temperature and energy density](image)

To make these considerations quantitative, we thus have to find a way to determine the in-medium melting points of the different quarkonium states. This problem has been addressed in three different approaches:

- Model the heavy quark potential as function of the temperature, $V(r, T)$, and solve the resulting Schrödinger equation (2).

- Determine the internal energy $U(r, T)$ of a $Q\bar{Q}$ pair at separation distance $r$ from lattice results for the corresponding free energy $F(r, T)$, using the thermodynamic relation

$$U(r, T) = -T^2 \left( \frac{\partial [F(r, T)/T]}{\partial T} \right) = F(r, T) - T \left( \frac{\partial F(r, T)}{\partial T} \right), \quad (8)$$

and solve the Schrödinger equation with $V(r, T) = U(r, T)$ as the binding potential.

- Calculate the quarkonium spectrum directly in finite temperature lattice QCD.
Clearly the last is the only model-independent way, and it will in the long run provide the decisive determination. However, the direct lattice study of charmonium spectra has become possible only quite recently, and so far most results are obtained in quenched QCD (no dynamical light quarks); corresponding studies for bottomonia are still more difficult. Hence much of what is known so far is based on Schrödinger equation studies with different model inputs. To illustrate the model-dependence of the dissociation parameters, we first cite some early work using different models for the temperature dependence of \( V(r, T) \), then some recent studies based on lattice results for \( F(r, T) \), and finally summarize the present state of direct lattice calculations of charmonia in finite temperature media.

3. Potential Model Studies

The first quantitative studies of finite temperature charmonium dissociation [12] were based on screening in the form obtained in one-dimensional QED, the so-called Schwinger model. The confining part of the Cornell potential \( V(r) \sim \sigma r \), is the solution of the Laplace equation in one space dimension. In this case, Debye-screening leads to \[ V(r, T) \sim \sigma r \left\{ 1 - \frac{e^{-\mu r}}{\mu r} \right\}, \tag{9} \]
where \( \mu(T) \) denotes the screening mass (inverse Debye radius) for the medium at temperature \( T \). This form reproduces at least qualitatively the convergence to a finite large distance value \( V(\infty, T) = \sigma / \mu(T) \), and since \( \mu(T) \) increases with \( T \), it also gives the expected decrease of the potential with increasing temperature. Combining this with the usual Debye screening for the \( 1/r \) part of eq. (1) then leads to
\[ V(r, T) \sim \sigma r \left\{ 1 - \frac{e^{-\mu r}}{\mu r} \right\} - \frac{\alpha}{r} e^{-\mu r} = \sigma \left\{ 1 - e^{-\mu r} \right\} - \frac{\alpha}{r} e^{-\mu r} \tag{10} \]
for the screened Cornell potential. In [12], the screening mass was assumed to have the form \( \mu(T) \approx 4 T \), as obtained in first lattice estimates of screening in high temperature \( SU(N) \) gauge theory. Solving the Schrödinger equation with these inputs, one found that both the \( \psi' \) and the \( \chi_c \) become dissociated essentially at \( T \approx T_c \), while the \( J/\psi \) persisted up to about \( 1.2 T_c \). Note that as function of the energy density \( \epsilon \sim T^4 \), this meant that the \( J/\psi \) really survives up to much higher \( \epsilon \).

This approach has two basic shortcomings:
- The Schwinger form [12] corresponds to the screening of \( \sigma r \) in one space dimension; the correct result in three space dimensions is different [13].
- The screening mass \( \mu(T) \) is assumed in its high energy form; lattice studies show today that its behaviour near \( T_c \) is quite different [14].

While the overall behaviour of this approach provides some first insight into the problem, quantitative aspects require a more careful treatment.

When lattice results for the heavy quark free energy as function of the temperature first became available, an alternative description appeared [15]. It assumed that in the thermodynamic relation [18] the entropy term \(-T(\partial F/\partial T)\) could be neglected, thus equating binding potential and free energy,
\[ V(r, T) = F(r, T) - T(\partial F/\partial T) \simeq F(r, T). \tag{11} \]
Using this potential in the Schrödinger equation specifies the temperature dependence of the different charmonium masses. On the other hand, the large distance limit of \( V(r, T) \) determines the temperature variation of the open charm meson \( D \),

\[
2M_D(T) \simeq 2m_c + V(\infty, T)
\]  

(12)

In fig. 7 we compare the resulting open and hidden charm masses as function of temperature. It is seen that the \( \psi' \) mass falls below \( 2M_D \) around 0.2 \( T_c \), that of the \( \chi_c \) at about 0.8 \( T_c \); hence these states disappear by strong decay at the quoted temperatures. Only the ground state \( J/\psi \) survives up to \( T_c \) and perhaps slightly above; the lattice data available at the time did not extend above \( T_c \), so further predictions were not possible.

![Figure 7: Temperature dependence of open and hidden charm masses](image)

The main shortcoming of this approach is quite evident. The neglect of the entropy term in the potential reduces \( V(r, T) \) and hence the binding. As a result, the \( D \) mass drops faster with temperature than that of the charmonium states, and it is this effect which leads to the early charmonium dissociation. Moreover, in the lattice studies used here, only the colour averaged free energy was calculated, which leads to a further reduction of the binding force.

We conclude from these attempts that for a quantitative potential theory study, the free energy has to be formulated in the correct three-dimensional screened Cornell form, and it then has to be checked against the space- and temperature-dependence of the corresponding colour singlet quantity obtained in lattice QCD.

4. Screening Theory

The modification of the interaction between two charges immersed in a dilute medium of charged constituents is provided by Debye-Hückel theory, which for the Coulomb potential in three space dimensions leads to the well-known Debye screening,

\[
\frac{1}{r} \rightarrow \frac{1}{r} e^{-\mu r},
\]

(13)

where \( r_D = 1/\mu \) defines the screening radius. Screening can be evaluated more generally for a given free energy \( F(r) \sim r^q \) in \( d \) space dimensions, with an arbitrary number \( q \). We shall here apply this to the two terms of the Cornell form, with \( q = 1 \) for the string term, \( q = -1 \) for the gauge term, in \( d = 3 \) space dimensions.
We thus assume that the screening effect can be calculated separately for each term, so that the screened free energy becomes

\[ F(r, T) = F_s(r, T) + F_c(r, T) = \sigma rf_s(r, T) - \frac{\alpha}{r} f_c(r, T). \]  

(14)

The screening functions \( f_s(r, T) \) and \( f_c(r, T) \) must satisfy

\[ f_s(r, T) = f_c(r, T) = 1 \quad \text{for} \quad T \to 0, \]

\[ f_s(r, T) = f_c(r, T) = 1 \quad \text{for} \quad r \to 0, \]  

(15)

since at \( T = 0 \) there is no medium, while in the short-distance limit \( T^{-1} \gg r \to 0 \), the medium has no effect. The resulting forms are

\[ F_c(r, T) = -\frac{\alpha}{r} \left[ e^{-\mu r} + \mu r \right] \]  

(16)

for the gauge term, and

\[ F_s(r, T) = \frac{\sigma}{\mu} \left[ \frac{\Gamma(1/4)}{2^{3/4} \pi^{3/4}} - \frac{\sqrt{\mu r}}{2^{1/4} \pi^{1/2}} K_{1/4}((\mu r)^{1/2}) \right] \]  

(17)

for the string term. The first term in eq. (17) gives the constant large distance limit due to colour screening; the second provides a Gaussian cut-off in \( x = \mu r \), since \( K_{1/4}(x^2) \sim \exp\{-x^2\} \); this is in contrast to the exponential cut-off given by the Schwinger form (10), but in accord with string breaking arguments of Di Giacomo et al. [5].

At temperatures \( T > T_c \), when the medium really consists of unbound colour charges, we thus expect the free energy \( F(r, T) \) to have the form (14), with the two screened terms given by eqs. (16) and (17). In Fig. 8a, it is seen that the results for the colour singlet free energy calculated in two-flavour QCD [17] are indeed described very well by this form, with \( m_c = 1.25 \text{ GeV} \) and \( \sqrt{\sigma} = 0.445 \text{ GeV} \), as above. The only parameter to be determined, the screening mass \( \mu \), is shown in Fig. 9, and as expected, it first increases rapidly in the transition region and then turns into the perturbative form \( \mu \sim T \).
The behaviour of $Q\bar{Q}$ binding in a plasma of unbound quarks and gluons is thus well described by colour screening. Such a description is in fact found to work well also for $T < T_c$, when quark recombination leads to an effective screening-like reduction of the interaction range, provided one allows higher order contributions in $x = \mu r$ in the Bessel function $K_{1/4}(x^2)$ governing string screening [14]. The resulting fit to the two-flavour colour singlet free energy below $T_c$ is shown in Fig. 8, using $K_{1/4}(x^2 + (1/2)x^4)$ in eq. (17); the corresponding values for the screening mass are included in Fig. 9.

With the free energy $F(r,T)$ of a heavy quark-antiquark pair given in terms of the screening form obtained from Debye-Hückel theory, the internal energy $U(r,T)$ can now be obtained through the thermodynamic relation [8], and this then provides the binding potential $V(r,T)$ for the temperature dependent version of the Schrödinger equation [2]. The resulting solution then specifies the temperature-dependence of charmonium binding as based on the correct heavy quark potential [8]. Let us briefly comment on the thermodynamic basis of this approach. The pressure $P$ of a thermodynamic system is given by the free energy, $P = -F = -U + TS$; it is determined by the kinetic energy $TS$ at temperature $T$ and entropy $S(T)$, reduced by the potential energy $U(T)$ between the constituents. In our case, all quantities give the difference between a thermodynamic system containing a $Q\bar{Q}$ pair and the corresponding system without such a pair. The potential energy of the $Q\bar{Q}$ pair, due both to the attraction of $Q$ and $\bar{Q}$ and to the modification which the pair causes to the internal energy of the other constituents of the medium, is therefore given by $U$. To determine the dissociation points for the different quarkonium states, we thus have to solve the Schrödinger equation [2] with $V(r,T) = U(r,T)$.

From eqs. (16) and (17) we get for the $Q\bar{Q}$ potential

$$V(r,T) = V(\infty,T) + \tilde{V}(r,T),$$

with

$$V(\infty,T) = c_1 \sigma \mu - \alpha \mu + T \frac{d\mu}{dT}[c_1 \sigma \mu^2 + \alpha],$$

and $c_1 = \Gamma(1/4)/2^{3/2}\Gamma(3/4)$, and where $\tilde{V}(r,T)$ contains the part of the potential which vanishes for $r \to \infty$. The behaviour of $V(\infty,T)$ as function of the temperature is shown in Fig. 10. It measures (twice) the energy of the cloud of light quarks and gluons around
an isolated heavy quark, of an extension determined by the screening radius, relative to the energy contained in such a cloud of the same size without a heavy quark. This energy difference arises from the interaction of the heavy quark with light quarks and gluons of the medium, and from the modification of the interaction between the light constituents themselves, caused by the presence of the heavy charge. – The behaviour of \( \tilde{V}(r,T) \) is shown in Fig. 11 for three different values of the temperature. It is seen that with increasing \( T \), screening reduces the range of the potential.

\[
\left\{ \frac{1}{m_c} \nabla^2 - \tilde{V}(r,T) \right\} \Phi_i(r) = \Delta E_i(T) \Phi_i(r) \tag{20}
\]

where

\[
\Delta E_i(T) = V(\infty,T) - M_i - 2m_c \tag{21}
\]

is the binding energy of charmonium state \( i \) at temperature \( T \). When it vanishes, the bound state \( i \) no longer exists, so that \( \Delta E_i(T) = 0 \) determines the dissociation temperature \( T_i \) for that state. The temperature enters only through the \( T \)-dependence of the screening mass \( \mu(T) \), as obtained from the analysis of the lattice results for \( F(r,T) \). – Equivalently, the divergence of the binding radii, \( r_i(T) \to \infty \), can be used to define the dissociation points \( T_i \).

The same formalism, with \( m_b = 4.65 \text{ GeV} \) replacing \( m_c \), leads to the bottomonium dissociation points. The combined quarkonium results are listed in Table 2. They agree quite well with those obtained in a very similar study based on corresponding free energies obtained in quenched lattice QCD [18], indicating that above \( T_c \) gluonic effects dominate. Using a parametrically generalized screened Coulomb potential obtained from lattice QCD results also leads to very compatible results for the \( N_f = 2 \) and quenched cases[19]. We recall here that the main underlying change, which is responsible for the much higher dissociation temperatures for the quarkonium ground states, is the use of the full internal energy [8], including the entropy term, as potential in the Schrödinger equation: this makes the binding much stronger.
We should note, however, that in all such potential studies it is not so clear what binding energies of less than a few MeV or bound state radii of several fermi can mean in a medium whose temperature is above 200 MeV and which leads to screening radii of less than 0.5 fm. In such a situation, thermal activation \cite{20} can easily dissociate the bound state.

| state | $J/\psi(1S)$ | $\chi_c(1P)$ | $\psi'(2S)$ | $\Upsilon(1S)$ | $\chi_b(1P)$ | $\Upsilon(2S)$ | $\chi_b(2P)$ | $\Upsilon(3S)$ |
|-------|---------------|---------------|--------------|----------------|---------------|----------------|---------------|----------------|
| $T_d/T_c$ | 2.10 | 1.16 | 1.12 | > 4.0 | 1.76 | 1.60 | 1.19 | 1.17 |

Table 2: Quarkonium Dissociation Temperatures \cite{8}

5. Charmonium Correlators

The direct spectral analysis of charmonia in finite temperature lattice has come within reach only in very recent years \cite{21}. It is possible now to evaluate the correlation functions $G_H(\tau, T)$ for hadronic quantum number channels $H$ in terms of the Euclidean time $\tau$ and the temperature $T$. These correlation functions are directly related to the corresponding spectral function $\sigma_H(M, T)$, which describe the distribution in mass $M$ at temperature $T$ for the channel in question. In Fig. 12 schematic results at different temperatures are shown for the $J/\psi$ and the $\chi_c$ channels. It is seen that the spectrum for the ground state $J/\psi$ remains essentially unchanged even at 1.5 $T_c$. At 3 $T_c$, however, it has disappeared; the remaining spectrum is that of the $c\bar{c}$ continuum of $J/\psi$ quantum numbers at that temperature. In contrast, the $\chi_c$ is already absent at 1.1 $T_c$, with only the corresponding continuum present.

Figure 12: $J/\psi$ and $\chi_c$ spectral functions at different temperatures

These results are clearly very promising: they show that in a foreseeable future, the dissociation parameters of quarkonia can be determined \textit{ab initio} in lattice QCD. For the moment, however, they remain indicative only, since the underlying calculations were generally performed in quenched QCD, i.e., without dynamical quark loops. Since such loops are crucial in the break-up of quarkonia into light-heavy mesons, final results require
calculations in full QCD. Some first calculations in two-flavour QCD have just appeared \[22\] and support the late dissociation of the $J/\psi$. The widths of the observed spectral signals are at present determined by the precision of the lattice calculations; to study the actual physical widths, much higher precision is needed. Finally, one has so far only first signals at a few selected points; a temperature scan also requires higher performance computational facilities. Since the next generation of computers, in the multi-Teraflops range, is presently going into operation, the next years should bring the desired results. So far, in view of the mentioned uncertainties in both approaches, the results from direct lattice studies and those from the potential model calculations of the previous section appear quite compatible.

6. Conclusions

The theoretical analysis of the in-medium behaviour of quarkonia has greatly advanced in the past decade. Potential model studies based on lattice results for the colour-singlet free energy appear to be converging to results from direct lattice calculations, and within a few more years, the dissociation temperatures for the different quarkonium states should be known precisely. Through corresponding calculations of the QCD equation of state, these temperatures provide the energy density values at which the dissociation occurs. In statistical QCD, quarkonia thus allow a spectral analysis of the quark-gluon plasma, as many of the observed features as possible.

Finally, it seems worthwhile to note that experimental measurements of the relative dissociation points of the different quarkonium states could be possible in the study of high energy nucleus-nucleus collisions. If they succeed, this might in fact be a unique chance to compare quantitative ab initio QCD predictions directly to data.

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