Magnon Kerr Effect in a Cavity Quantum Electrodynamics System

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We experimentally demonstrate magnon Kerr effect in a cavity quantum electrodynamics (QED) system, where magnons in a small yttrium iron garnet (YIG) sphere are strongly but dispersively coupled to the photons in a three-dimensional cavity. When the YIG sphere is pumped to generate considerable magnons, the Kerr effect yields a perceptible shift of the cavity central frequency and more appreciable shifts of the magnon modes. We derive an analytical relation between the magnon frequency shift and the drive power for the uniformly magnetized YIG sphere and find that it agrees very well with the experimental results of the Kittel mode. Our study paves the way to explore nonlinear effects in the cavity QED system with magnons.

Hybridizing two or more quantum systems can harness the distinct advantages of different systems to implement quantum information processors (see, e.g., [1, 2]). Recently, cavity spintronics has attracted considerable attention [3–9], because of the enhanced coupling between magnons in yttrium iron garnet (YIG) single crystal and microwave photons in a high-finesse cavity. Based on the cavity-spintronics system, coherent interaction between a magnon and a superconducting qubit was realized [10], and magnon dark modes in a magnon gradient memory [11] were utilized to store quantum information. When combined with spin pumping techniques, this cavity-spintronics system provides a new platform to explore the physics of spintronics and to design useful functional devices [7, 9]. Potentially acting as a quantum information transducer, microwave-to-optical frequency conversion between microwave photons generated by a superconducting circuit and optical photons of a whispering gallery mode supported by a YIG microsphere was also explored [12–15]. Furthermore, coherent phonon-magnon interactions relying on the effect of magnetostrictive deformation in a YIG sphere was demonstrated [16]. Now, a versatile quantum information processing platform based on the coherent couplings among magnons, microwave photons, optical photons, phonons, and superconducting qubits is being established.

In this Letter, we report the first experimental demonstration of the magnon Kerr effect in a cavity quantum electrodynamics (QED) system, where magnons in a small YIG sphere are strongly but dispersively coupled to the microwave photons in a three-dimensional (3D) cavity. When considerable magnons are generated by pumping the YIG sphere, the Kerr effect gives rise to a shift of the cavity central frequency and yields more appreciable shifts of the magnon modes, including the Kittel mode [17], which holds homogeneous magnetization, and the magnetostatic (MS) modes [18–20], which have inhomogeneous magnetization. We derive an analytical relation between the magnon frequency shift and the pumping power for a uniformly magnetized YIG sphere and find that it agrees very well with the experimental results of the Kittel mode. In contrast, the experimental results of MS modes deviate from this relation, which confirms the deviation of the MS modes from homogeneous magnetization. To enhance the magnon Kerr effect, the pumping field is designed to directly drive the YIG sphere and its coupling to the magnons is strengthened using a loop antenna. Moreover, this field is tuned very off-resonance with the cavity mode to avoid producing any appreciable effects on the cavity. These make it experimentally feasible to demonstrate the cavity QED in the presence of the magnon Kerr effect. Our work is the first convincing study of a cavity QED system with magnon Kerr effect and paves the way to experimentally explore nonlinear effects in this tunable cavity QED system with magnons.

The experimental setup is diagrammatically shown in Fig. 1(a). The 3D cavity is made of oxygen-free copper with inner dimensions of 44.0 × 20.0 × 6.0 mm3 and contains three ports labeled as 1, 2 and 3 (here ports 1 and 2 are used for transmission spectroscopy and port 3 is for loading the drive field). The frequency of the cavity mode TE102 that we use is ωc/2π = 10.1 GHz. A small YIG sphere of diameter 1 mm is glued on an inner wall of the cavity at the magnetic-field antinode of the TE102 mode [c.f. the magnetic-field intensity distribution of this mode marked by colored shades in Fig. 1(a)]. We apply a static magnetic field generated by a superconducting magnet to magnetize the YIG sphere. This bias magnetic field is tunable in the range of 0−1 T, so the given frequency of the Kittel mode (i.e., the ferromagnetic resonance mode) ranges from several hundreds of MHz to 28 GHz. The cavity is placed in a BlueFors LD-400 dilution refrigerator under a cryogenic temperature of 22 mK. The spectroscopic measurement is carried out with a vector network analyzer by probing the transmission of the cavity. A drive tone supplied by a microwave source can directly drive the YIG sphere via a superconducting microwave line going through the port 3. Moreover, a loop antenna is attached to the end of the superconducting microwave line near the YIG sphere, so as to strengthen the coupling between the drive field and the YIG sphere [21]. Here we have the driving magnetic field Bd, the bias magnetic field Bb, and the driving power Pd.

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MS modes in the YIG sphere. The coupling strength between them. Some other small splittings where the Kittel mode is resonant with the cavity mode $TE_{102}$ by vector network analyzer [see Fig. 1(b)]. At the point $B_0$ and the cavity mode $TE_{102}$ mode orthogonal to each other at the site of the YIG sphere. Also, a series of attenuators and isolators are used to prevent thermal noise from reaching the sample and the output signal is amplified by two low-noise amplifiers at the stages of 4K and room temperature, respectively.

We first measure the transmission spectrum of the cavity containing the YIG sphere, without applying a drive field on the YIG sphere. The transmission spectrum as a function of the probe microwave frequency and $B_0$ is recorded by vector network analyzer [see Fig. 1(b)]. At the point where the Kittel mode is resonant with the cavity mode $TE_{102}$, a distinct anti-crossing of the two modes occurs, indicating strong coupling between them. Some other small splittings are due to the couplings between the cavity mode and the MS modes in the YIG sphere. The coupling strength between the Kittel mode and the cavity mode $TE_{102}$ is found to be $g_m/2\pi = 42$ MHz from the magnon polaron splitting at the resonance point [see the red curve in Fig. 1(c)].

By fitting the measured transmission spectrum [8], the cavity-mode linewidth $\kappa/2\pi \equiv (k_1 + k_2 + k_{int})/2\pi$ and the Kittel-mode linewidth $\gamma_m/2\pi$ are determined to be 2.87 MHz and 24.3 MHz, respectively. Here $k_1$ ($k_2$) is the loss rate due to the port 1 (2) and $k_{int}$ is due to the intrinsic loss of the cavity. The obvious increase of the Kittel-mode damping rate compared with the previous work [4, 5] is due to the antenna close to the YIG sphere, which acts as an additional decay channel. Note that all the linewidths throughout the paper are defined as the full width at half maximum (FWHM). Because $g_m > \kappa, \gamma_m$, the hybrid system falls in the strong coupling regime with a cooperativity $C \equiv 4g_m^2/\kappa \gamma_m = 101$.

Then, we perform the dispersive measurement. We tune the static bias magnetic field $B_0$ to 346.8 mT, yielding about 9.55 GHz for the frequency of the Kittel mode. As shown in Fig. 2(a), we first measure the transmission spectrum of the cavity (i.e., the black curve) by tuning the frequency of the probe field, but without the drive field on the YIG sphere. The measured central frequency of the cavity mode is 10.1035 GHz, which has a frequency shift of about 3 MHz compared with the intrinsic frequency 10.1003 GHz of the $TE_{102}$ mode of an empty cavity. This cavity mode has a detuning $\Delta/2\pi \approx 550$ MHz from the Kittel mode. Because $\Delta > 10g_m$, the coupled hybrid system is in the dispersive regime. We then measure the transmission spectrum of the cavity by both tuning the frequency of the probe field and applying a drive field on the YIG sphere in resonance with the Kittel mode. The measured red curve corresponds to the drive power of 11 dBm. This transmission spectrum has a central frequency of 10.1042 GHz, with a frequency shift of about 0.7 MHz from the measured central frequency without applying a drive field.

Figures 2(b) and 2(c) show the measured transmission spectra by tuning the frequency of the drive field, where the frequency of the probe field is fixed at the central frequency $10.1035$ GHz of the cavity containing the YIG sphere. The probe field power is -129 dBm. The corresponding average cavity probe photon number can be estimated by [22] $\bar{n} = \gamma_P / (\hbar \omega_P [\Delta^2 + (\kappa/2)^2])$, where $P_p$ is the probe field power and $\Delta_p = \omega_p - \omega_c$. In our experiment, it is measured that $\kappa/2\pi = 0.70$ MHz, i.e., $\kappa \approx \kappa/4$. Also, the probe field frequency $\omega_p$ is tuned in resonance with the cavity mode $TE_{102}$. Then, the average cavity probe photon number is reduced to $\bar{n} = P_p / (\hbar \omega_P \kappa) \approx 1$. Here the probe tone is chosen extremely weak, so as to avoid producing any appreciable effects on the system. In Fig. 2(b), the power of the drive field is 11 dBm. It can be seen that when the frequency of the drive microwave field is resonant with the Kittel mode, the transmission coefficient has a large decrease at 9.59 GHz (see the main dip indicated by a blue arrow), caused by the shift of the central frequency of the cavity mode. The dips indicated by orange and purple arrows correspond to two different MS modes. In addition, we vary the power of the drive field from -5 dBm.
The Hamiltonian of the YIG sphere in the magnetic field $B_0$, including the Zeeman energy and the demagnetizing term, is given by (setting $\hbar = 1$) [21]

$$H_m = -\gamma B_0 S_z + \frac{\mu_0}{6} \frac{\gamma^2}{V_m} S_z^2,$$

(1)

where $\gamma/2\pi = 28$ GHz/T is the gyromagnetic ratio, $\mu_0$ is the vacuum permeability, and $S_z = M_z V_m / \gamma$ is the macrospin operator of the YIG sphere, with $V_m$ being the volume of the YIG sample. The macrospin operator $S_z$ is related to the magnon operators via the Holstein-Primakoff transformation [24]: $S_z = S - b^\dagger b$, where $b^\dagger (b)$ is the magnon creation (annihilation) operator.

When including the drive field, the cavity mode, and the interaction between the cavity photon and magnon, the total Hamiltonian of the coupled hybrid system is [21]

$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + K b^\dagger b b^\dagger b + g_m (a^\dagger b + ab^\dagger) + \Omega_d (b^\dagger e^{-i\omega_d t} + be^{i\omega_d t}),$$

(2)

where $\hat{a}^\dagger (\hat{a})$ is the creation (annihilation) operator of the cavity photons at frequency $\omega_c$, $K b^\dagger b b^\dagger b$, with $K = \mu_0 \gamma^2 / (6V_m)$, represents the Kerr effect of magnons owing to the demagnetization in the YIG sphere, $\Omega_d$ (i.e., the Rabi frequency) denotes the strength of the drive field, and $\omega_m$ is the drive field frequency. Thus, our experimental setup provides a cavity QED system with magnon Kerr effect, which is an extension of the magnon-cavity system without the nonlinear effect [25]. Note that $K$ is inversely proportional to $V_m$, so the Kerr effect can become important when using a small YIG sphere.

In the dispersive regime, when considerable magnons are generated by the drive field, the effective Hamiltonian of the cavity QED system can be written as [21]

$$H_{\text{eff}} = \left[ \omega_c + \frac{g_m^2}{\Delta} + \frac{2g_m^2}{\Delta^2} K(b^\dagger b) \right] a^\dagger a + \left[ \omega_m + \frac{g_m^2}{\Delta} + \left( 1 - \frac{g_m^2}{\Delta^2} \right) K(b^\dagger b) \right] b^\dagger b + \Omega_d^\prime (b^\dagger e^{-i\omega_d t} + be^{i\omega_d t}),$$

(3)

with the effective Rabi frequency $\Omega_d^\prime$ given by

$$\Omega_d^\prime = \left[ 1 - \frac{1}{2(\omega_c - \omega_d)} \left( \frac{g_m^2}{\Delta} + \frac{2g_m^2}{\Delta^2} K(b^\dagger b) \right) \right] \Omega_d,$$

(4)

where $\Delta = \omega_c - \omega_m$. Because of the coupling between the cavity and the YIG sphere, the cavity frequency shifts from the intrinsic cavity mode frequency $\omega_c + g_m^2 / \Delta$, with $g_m^2 / \Delta$ being the dispersive shift. The measured central frequency $10.1035 \text{ GHz}$ corresponds to $\omega_c + g_m^2 / \Delta$. When pumping the YIG sphere with a drive field, the magnon number $\langle b^\dagger b \rangle$ increases. Then the cavity frequency has an additional blue shift of $\Delta_c = (2g_m^2 / \Delta^2)K(b^\dagger b)$ due to the Kerr effect. Also, the Kerr effect yields a blue shift to the magnon frequency, $\Delta_m = (1 - 2g_m^2 / \Delta^2)K(b^\dagger b) \approx K(b^\dagger b)$. Both cavity frequency shift and magnon frequency shift due to the Kerr effect have a

FIG. 2. (color online). (a) Central frequency shift of the cavity mode TE102 when the drive field is on (red curve) and off (black curve), respectively. (b) Transmission spectrum of the cavity measured as a function of the drive-field frequency. The blue arrow indicates the response of the Kittel mode, while the orange and purple arrows indicate the MS modes 1 and 2, respectively. (c) Transmission spectrum of the cavity measured as a function of the drive frequency by successively increasing the driving power. The probe field is fixed at $10.1035 \text{ GHz}$ in both (b) and (c).
similar trend depending on $\langle b^\dagger b \rangle$, which is related to the drive power.

In Fig. 3, we extract the Kerr-effect-induced frequency shifts of the Kittel mode and two MS modes as well as the central frequency shift of the cavity mode at each given drive power $P$. From Fig. 3(a), it is clear that both the Kittel-mode frequency shift and the cavity central frequency shift indeed have similar behaviors depending on the drive power, as predicted above. We also see that all the frequency shifts exhibit nonlinear dependence on the drive power. As given in [21], we derive an analytical relation between the magnon frequency shift $\Delta_m$ and the drive power $P$ using a Langevin equation approach,

$$\left[\Delta_m^2 + \left(\frac{\gamma_m}{2}\right)^2\right]\Delta_m - cP = 0,$$  \hspace{1cm} (5)

where $c$ is a characteristic parameter reflecting the coupling strength of the drive field with the magnon mode. For the Kittel mode, we have already measured its linewidth $\gamma_m/2\pi = 24.3$ MHz. We use Eq. (5) to fit the experimental results of the Kettle mode. As shown in Fig. 3(a), the obtained theoretical (blue) curve fits very well with the experimental data, where $c = (2\pi)^3 \times 4.7 \times 10^{24}$ kg$^{-1}$m$^{-2}$. Note that when the drive power is small, $\gamma_m \gg \Delta_m$, so Eq. (5) reduces to $(\frac{\gamma_m}{2})^2\Delta_m = cP = 0$, i.e., the magnon frequency shift depends linearly on the drive power in the small drive power limit. When the drive power becomes sufficiently large, $\Delta_m \gg \gamma_m$, and then Eq. (5) reduces to $\Delta_m^3 = cP$. It yields $\Delta_m = (cP)^{1/3}$, i.e., in the large drive power limit, the magnon frequency shift depends linearly on the cubic root of the drive power. These limit results are consistent with the previous work in [26], where there is a threshold power separating the small and large driving power regions. A dispersive measurement on the cavity transmission was also implemented in [27], but the cavity central frequency shift due to the magnon Kerr effect was not observed. Different from our setup in which the YIG sphere is directly pumped by the drive field and the nonlinear effect of large-amplitude spin waves can be induced [28], the drive field was applied on the cavity in [27]. In the dispersive regime, this gives rise to a weak effective drive field on the YIG sample and thus only a small number of magnons can be generated.

For the MS modes, we have two unknown parameters, the MS mode linewidth $\gamma_m$ and the parameter $c$. We manage to fit the experimental data in Fig. 3(b) with $\gamma_m = 15$ MHz and $c = (2\pi)^3 \times 1.35 \times 10^{24}$ kg$^{-1}$m$^{-2}$ for MS mode 1 (orange curve), and with $\gamma_m = 30$ MHz and $c = (2\pi)^3 \times 6 \times 10^{24}$ kg$^{-1}$m$^{-2}$ for MS mode 2 (purple curve). Note that the theoretical curves do not fit the experimental data of the MS modes so well as those of the Kittel mode, especially in the region around the threshold power mentioned above [see the region of 1-3 mW in Fig. 3(b)]. In fact, as a collective mode of spins with zero wavevector, the Kittel mode is uniform precession mode with homogeneous magnetization, while the MS modes are nonuniform precession modes holding inhomogeneous magnetization and have a spatial variation comparable to the sample dimensions [19, 20, 29]. The appreciable deviations of the experimental data from the theoretical fitting curves are due to the inhomogeneous magnetization of the MS modes.

In conclusion, we have realized a cavity QED system with magnon Kerr effect. By directly pumping the YIG sphere with a drive field, we have demonstrated the Kerr-effect-induced central frequency shift of the cavity mode as well as the frequency shifts of the Kittel mode and MS modes. An analytical relation between the magnon frequency shift and the pumping power for a uniformly magnetized YIG sphere is derived, which agrees very well with the experimental results of the Kittel mode. In contrast, the experimental results of MS modes deviate from this relation owing to the spatial variations of the MS modes over the sample. We can use this relation to characterize the degrees of deviation of the MS modes from the homogeneous magnetization. Our setup can provide a flexible and tunable platform to further explore nonlinear effects of magnons in the cavity QED system.

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