Scalable Decision-Focused Learning in Restless Multi-Armed Bandits with Application to Maternal and Child Care

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Abstract

This paper studies restless multi-armed bandit (RMAB) problems with unknown arm transition dynamics but with known correlated arm features. The goal is to learn a model to predict transition dynamics given features, where the Whittle index policy solves the RMAB problems using predicted transitions. However, prior works often learn the model by maximizing the predictive accuracy instead of final RMAB solution quality, causing a mismatch between training and evaluation objectives. To address this shortcoming, we propose a novel approach for decision-focused learning in RMAB that directly trains the predictive model to maximize the Whittle index solution quality. We present three key contributions: (i) we establish differentiability of the Whittle index policy to support decision-focused learning; (ii) we significantly improve the scalability of decision-focused learning approaches in sequential problems, specifically RMAB problems; (iii) we apply our algorithm to the service call scheduling problem on a real-world maternal and child health domain. Our algorithm is the first for decision-focused learning in RMAB that scales to large-scale real-world problems.

1 Introduction

Restless multi-armed bandits (RMABs) are composed of a set of heterogeneous arms and a planner who can pull multiple arms under budget constraint at each time step to collect rewards. Different from the classic stochastic multi-armed bandits, the state of each arm in an RMAB can change even...

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when the arm is not pulled, where each arm follows a Markovian process to transition between different states with transition probabilities dependent on arms and the pulling decision. Rewards are associated with different arm states, where the planner’s goal is to plan a sequential pulling policy to maximize the total reward received from all arms. RMABs are commonly used to model sequential scheduling problems where limited resources must be strategically assigned to different tasks sequentially to maximize performance. Examples include machine maintenance Glazebrook et al. (2006), cognitive radio sensing problem Bagheri and Scaglione (2015), and healthcare Mate et al. (2022).

In this paper, we study offline RMAB problems with unknown transition dynamics but with given arm features. The goal is to learn a mapping from arm features to transition dynamics, which can be used to infer the dynamics of unseen RMAB problems to plan accordingly. Prior works Mate et al. (2022), Sun et al. (2018) often learn the transition dynamics from the historical pulling data by maximizing the predictive accuracy. However, RMAB performance is evaluated by its solution quality derived from the predicted transition dynamics, which leads to a mismatch in the training objective and the evaluation objective. Previously, decision-focused learning Wilder et al. (2019) has been proposed to directly optimize the solution quality rather than predictive accuracy, by integrating the one-shot optimization problem Donti et al. (2017), Perrault et al. (2020) or sequential problems Wang et al. (2021), Futoma et al. (2020) as a differentiable layer in the training pipeline. Unfortunately, while decision-focused learning can successfully optimize the evaluation objective, it is computationally extremely expensive due to the presence of the optimization problems in the training process. Specifically, for RMAB problems, the computation cost of decision-focused learning arises from the complexity of the sequential problems formulated as Markov decision processes (MDPs), which limits the applicability to RMAB problems due to the PSPACE hardness of finding the optimal solution Papadimitriou and Tsitsiklis (1994).

Our main contribution is a novel and more efficient approach for decision-focused learning in RMAB problems using Whittle index policy Whittle (1988), where Whittle index policy is commonly used as a computationally efficient approach in RMAB; it also provides approximation guarantees under indexability Weber and Weiss (1990). Our three key contributions are (i) we establish the differentiability of Whittle index policy to support decision-focused learning to directly optimize the final RMAB solution quality; (ii) we show that our approach of differentiating through Whittle index policy improves the scalability of decision-focused learning in RMAB; (iii) we apply our algorithm to the service call scheduling problem on a real-world maternal and child health domain operated by ARMMAN (2022).

We establish the differentiability of Whittle index by showing that Whittle index can be expressed as a unique solution to a selected subset of Bellman equations with transition dynamics as entries, which allows us to compute the derivative of Whittle index with respect to transition dynamics. From Whittle indices to Whittle index policy, a strict top-k selection process is commonly used to select arms with top-k Whittle indices to pull. We relax this non-differentiable process by using a differentiable soft top-k selection to establish differentiability. Our differentiable Whittle index policy enables decision-focused learning in RMAB problems to backpropagate from final policy performance to the predictive model. An additional benefit from the use of differentiable Whittle index policy is that our algorithm scales linearly in the number of arms and polynomially in the number of states, while previous decision-focused learning approaches in sequential problems scale exponentially in the number of arms. This significant reduction in computation cost is crucial for extending decision-focused learning to large-scale RMAB problems with large number of arms.

In our experiments, we apply decision-focused learning to RMAB problems to optimize an importance sampling-based evaluation on synthetic datasets as well as a real RMAB dataset about scheduling service calls to improve maternal and child health operated by ARMMAN (2022). We compare our algorithm with the two-stage method that trains to minimize the predictive loss. The two-stage method achieves the best predictive loss but significantly degraded solution quality. In contrast, decision-focused learning reaches a slightly worse predictive loss but with a much better importance sampling-based solution quality evaluation and the improvement generalizes to the simulation-based evaluation that is built from the data. Lastly, the scalability improvement is the crux of applying decision-focused learning to real-world RMAB problems: we show our scalability by applying our algorithm to large-scale maternal and child health problem with hundreds of arms, whereas state of the art is a 100-fold slower even with 20 arms and grows exponentially worse.
Related Work

**Restless multi-armed bandits with given transition dynamics** This line of research primarily focuses on solving RMAB problems to get a sequential policy. The complexity of solving RMAB problems optimally is known to be PSPACE hard [Papadimitriou and Tsitsiklis (1994)]. One approximate solution is proposed by Whittle (1988), where they use Lagrangian relaxation to decompose arms and compute the associated (what are now called) Whittle indices to compute a policy. Specifically, the indexability condition [Akbarzadeh and Mahajan (2019), Wang et al. (2019)] guarantees this Whittle index policy to be asymptotically optimal Weber and Weiss (1990). Although indexability in general remains open, in practice, Whittle index policy usually provides a near-optimal solution to RMAB problems.

**Restless multi-armed bandits with missing transition dynamics** When the transition dynamics are unknown in RMAB problems but an interactive environment is available, prior works [Tekin and Liu (2012), Liu et al. (2012), Oksanen and Kouvunen (2015), Dai et al. (2011)] consider this as an online learning problem that aims to maximize the expected reward by balancing between exploration and exploitation. However, these approaches become infeasible when interacting with the environment is expensive, e.g., healthcare [Mate et al. (2022)] or wildlife conservation problems [Qian et al. (2016)]. In this work, we consider the case where repeated interactions are expensive, but instead each arm comes with an arm feature that is correlated to the transition dynamics. As noted below, our problem is an offline RMAB problem.

**Decision-focused learning** The predict-then-optimize framework [Elmachtoub and Grigas (2021)] is composed of a predictive problem that makes predictions on the parameters of the later optimization problem, and an optimization problem that uses the predicted parameters to come up with a solution, where the overall objective is the solution quality of the proposed solution. Standard two-stage learning method solves the predictive and optimization problems separately, leading to a mismatch of the predictive loss and the evaluation metric [Huang et al. (2019), Lambert et al. (2020), Johnson and Khoshgoftaar (2019)]. In contrast, decision-focused learning [Wilder et al. (2019), Mandi et al. (2020), Elmachtoub et al. (2020)] learns the predictive model to directly optimize the solution quality by integrating the optimization problem as a differentiable layer [Amos and Kolter (2017), Agrawal et al. (2019)] in the training pipeline. Our offline RMAB problem is a predict-then-optimize problem, where we first (offline) learn a mapping from arm features to transition dynamics from the historical data [Mate et al. (2022), Sun et al. (2018)], and the RMAB problem is solved using the predicted transition dynamics accordingly. Prior work [Mate et al. (2022)] is limited to using two-stage learning to solve the offline RMAB problems. While decision-focused learning in sequential problems were primarily studied in the context of MDPs [Wang et al. (2021), Futoma et al. (2020)] they come with an expensive computation cost that immediately becomes infeasible in large RMAB problems.

2 Model: Restless Multi-armed Bandit

An instance of the restless multi-armed bandit (RMAB) problem is composed of a set of \( N \) arms, each is modeled as an independent Markov decision process (MDP). The \( i \)-th arm in a RMAB problem is defined by a tuple \( (S, A, R_i, P_i) \). \( S \) and \( A \) are the identical state and action spaces across all arms. \( R_i, P_i : S \times A \times S \to \mathbb{R} \) are the reward and transition functions associated to arm \( i \). We consider finite state space \( S = \{s^{(1)}, s^{(2)}, \ldots, s^{(M)}\} \) with \( |S| = M \) fully observable\(^3\) states, and action set \( A = \{0, 1\} \) corresponding to not pulling or pulling the arm, respectively. For each arm \( i \), the reward function is defined by \( R_i(s_i, a_i, s'_i) = R(s_i) \), i.e., the reward \( R(s_i) \) only depends on the current state \( s_i \), where \( R : S \to \mathbb{R} \) is a vector of size \( M \). Given the current state \( s_i \) and action \( a_i \), \( P_i(s_i, a_i, s'_i) = [P_i(s_i, a_i, s'_i)]_{s'_i \in S} \in \mathbb{R}^M \) defines the probability distribution of transitioning to all possible next states \( s'_i \in S \).

In a RMAB problem, at each time step \( t \in [T] \), the learner observes \( s_t = [s_{t,i}]_{i \in [N]} \in S^N \), the states of all arms. The learner then chooses action \( a_t = [a_{t,i}]_{i \in [N]} \in A^N \) denoting the pulling actions of all arms, which has to satisfy a budget constraint \( \sum_{i \in [N]} a_{t,i} \leq K \), i.e., the learner

\(^3\)Fully observable RMAB can be relaxed to partially observable RMAB with finite time horizon as seen in Section 4.6 by using belief states.
can pull at most $K$ arms at each time step. Once the action is chosen, arms receive action $a_t$ and transitions under $P$ with rewards $r_t = [r_{t,i}]_{i \in [N]}$ accordingly. We denote a full trajectory by $\tau = (s_1, a_1, r_1, \ldots, s_T, a_T, r_T)$. The total reward is defined by the summation of the discounted reward across $T$ time steps and $N$ arms, i.e., $\sum_{t=1}^{T} \gamma^{t-1} \sum_{i \in [N]} r_{t,i}$, where $0 < \gamma \leq 1$ is the discount factor.

A policy is denoted by $\pi$, where $\pi(a \mid s)$ defines the probability of choosing action $a$ given state $s$. Additionally, we define $\pi(a_i = 1 \mid s)$ to be the marginal probability of pulling arm $i$ given state $s$, where $\pi(s) = \{\pi(a_i = 1 \mid s)\}_{i \in [N]} \in \mathbb{R}^N$ to be a vector of arm pulling probabilities. Specifically, we use $\pi^*$ to denote the optimal policy that optimizes the cumulative reward, while $\pi_{\text{solver}}$ to denote a near-optimal policy solver.

## 3 Problem Statement

This paper studies the RMAB problem where we do not know the transition probabilities $P = \{P_i\}_{i \in [N]}$ in advance. Instead, we are given a set of features $x = \{x_i \in \mathcal{X}\}_{i \in [N]}$, each corresponding to one arm. The goal is to learn a mapping $f_w : \mathcal{X} \to \mathcal{P}$, parameterized by weights $w$, to make predictions on the transition probabilities $P = \{f_w(x_i)\}_{i \in [N]}$. The predicted transition probabilities are later used to solve the RMAB problem to derive a policy $\pi = \pi_{\text{solver}}(f_w(x))$. The performance of the model $f$ is evaluated by the performance of the proposed policy $\pi$.

### 3.1 Training and Testing Datasets

To learn the mapping $f_w$, we are given a set of RMAB instances as training examples $D_{\text{train}} = \{(x, \tau)\}$, where each instance is composed of a RMAB problem with feature $x$ that is correlated to the unknown transition probabilities $P$, and a set of realized trajectories $\tau = \{\tau^{(j)}\}_{j \in J}$ generated from a given behavior policy $\pi_{\text{beh}}$ that determined how to pull arms in the past. The testing set $D_{\text{test}}$ is defined similarly but hidden at training time.

### 3.2 Evaluation Metrics

**Predictive loss** To measure the correctness of transition probabilities $P = \{P_i\}_{i \in [N]}$, we define the predictive loss as the average negative log-likelihood of seeing the given trajectories $\tau$, i.e., $L(P, \tau) := -\log \Pr(\tau \mid P) = -\mathbb{E}_{\tau \sim \mathcal{T}} \sum_{t \in [T]} \log P(s_t, a_t, s_{t+1})$. Therefore, we can define the predictive loss of a model $f_w$ on dataset $D$ by:

$$E_{(x, \tau) \sim D} L(f_w(x), \tau)$$

**Policy evaluation** On the other hand, given transition probabilities $P$, we can solve the RMAB problem to derive a policy $\pi_{\text{solver}}(P)$. We can use the historical trajectories $\tau$ to evaluate how good the policy performs, denoted by $\text{Eval}(\pi_{\text{solver}}(P), \tau)$. Given dataset $D$, we can evaluate the predictive model $f_w$ on dataset $D$ by:

$$E_{(x, \tau) \sim D} \text{Eval}(\pi_{\text{solver}}(f_w(x)), \tau)$$

Two common types of policy evaluation are importance sampling-based off-policy policy evaluation and simulation-based evaluation, which will be discussed in Section 5.

### 3.3 Learning Methods

**Two-stage learning** To learn the predictive model $f_w$, we can minimize Equation 1 by computing gradient $\frac{\partial L(f_w(x), \tau)}{\partial w}$ to run gradient descent. However, this training objective (Equation 1) differs from the evaluation objective (Equation 2), which often leads to suboptimal performance.

**Decision-focused learning** In contrast, we can directly run gradient ascent to maximize Equation 2 by computing the gradient $\frac{\partial \text{Eval}(\pi_{\text{solver}}(f_w(x)), \tau)}{\partial w}$. However, in order to compute the gradient, we need to differentiate through the policy solver $\pi_{\text{solver}}$ and the corresponding optimal solution. Unfortunately, finding the optimal policy in RMABs is expensive and the policy is high-dimensional. Both of these challenges prevent us from computing the gradient to achieve decision-focused learning.
4 Decision-focused Learning in RMABs

In this paper, instead of using the optimal policy, we propose to run decision-focused learning using Whittle index policy [Whittle (1988)] to solve the RMAB problem approximately. In this section, we show that Whittle index policy is not only easier to compute with good approximation but also easier to backpropagate using a novel fixed-point property.

4.1 Whittle Index and Whittle Index Policy

Whittle index is the amount of subsidy to provide to action 0 (not pulling the arm) so that pulling and not pulling an arm $i$ result in an identical expected future value. More precisely, for each arm $i$, we can define a new RMAB problem where a subsidy $m$ is provided to arm $i$ when the arm is not pulled (action 0). We can express the Bellman equation of the new RMAB problem with subsidy $m$ and the corresponding value function $V^m_i(s)$ of arm $i$ with arm state $s \in S$ by:

$$V^m_i(s) = \max \{V^m_i(s; a = 0), V^m_i(s; a = 1)\}$$

$$V^m_i(s; a) = m \mathbf{1}_{a=0} + R(s) + \gamma \sum_{s' \in S} P_i(s, a, s') V^m_i(s')$$

where $V^m_i(s; a)$ is the expected future value of arm $i$ of the corresponding pulling action $a$.

**Definition 4.1 (Whittle index).** Given $u \in S$, we define the Whittle index associated to state $u$ by:

$$W_i(u) = \inf_m \{V^m_i(u; a = 0) = V^m_i(u; a = 1)\}$$

(4)

Given Whittle indices $W = \{W_i(u)\}_{i \in [N], u \in S}$, the Whittle index policy is denoted by $\pi^{\text{whittle}}: S^N \rightarrow \{0, 1\}^N$, which takes the states of all arms as input to compute their Whittle indices and determine the probabilities of pulling arms. A common choice of Whittle index policy is defined by:

**Definition 4.2 (Strict Whittle index policy).**

$$\pi^{\text{strict}}_W(s) = \mathbf{1}_{\text{top-k}(W_i(s), i \in [N])} \subseteq \{0, 1\}^N$$

(5)

which selects arms with the top-k Whittle indices to pull.

4.2 Decision-focused Learning Using Whittle Index Policy

Instead of using the optimal policy $\pi^*$ to run decision-focused learning with expensive computation cost, we propose using Whittle index policy $\pi^{\text{whittle}}$, e.g., strict Whittle index policy $\pi^{\text{strict}}_W$ with Whittle index $W$, as an approximate solution to the RMAB problem to run decision-focused learning. More precisely, we can compute the derivative of the evaluation metric by chain rule:

$$\frac{d\text{Eval}(\pi^{\text{whittle}}, T)}{dW} = \frac{d\text{Eval}(\pi^{\text{whittle}}, T)}{d\pi^{\text{whittle}}} \frac{d\pi^{\text{whittle}}}{dW} \frac{dW}{dP} \frac{dP}{dw}$$

(6)

where $W$ is the precomputed Whittle indices of all states using the predicted transition probabilities $P$. $\pi^{\text{whittle}}$ is the Whittle index policy induced by $W$. The flowchart is illustrated in Figure 1.

There are two challenges associated to Equation (6) (i) how to relax the strict Whittle index policy $\pi^{\text{strict}}$ to make it differentiable and compute $\frac{d\pi^{\text{strict}}}{dW}$ (ii) how to differentiate through Whittle index computation to derive $\frac{dW}{dP}$.
4.3 Differentiability of Whittle Index Policy

The first challenge is the non-differentiability of the strict top-k operation in Whittle index policy, which prevents us from computing a meaningful estimate of \( \frac{d\pi_{\text{Whittle}}}{dW} \) in Equation 6. In this paper, we adopt the idea from [Xie et al. (2020)] to relax the top-k selection to a soft-top-k selection, which can be expressed as an optimal transport problem with regularization and become differentiable. We apply the technique of soft-top-k to define a new differentiable soft Whittle index policy:

**Definition 4.3 (Soft Whittle index policy).**

\[
\pi_{W\text{soft}}(s) = \text{soft-top-k}([W_j(s_i)]_{i \in [N]}) \in [0, 1]^N
\]  

Using the soft Whittle index policy, the policy becomes differentiable and we can compute \( \frac{d\pi_{\text{Whittle}}}{dW} \).

4.4 Differentiability of Whittle Index

The second challenge is the differentiability of Whittle index. Whittle indices are often computed using value iteration and binary search [Qian et al. (2016); Mate et al. (2020)]. However, these operations are not differentiable and we cannot compute the derivative \( \frac{dW}{dP} \) in Equation 6 directly.

**Selected Bellman equation** Let \( u = s^{(k)} \) and arm \( i \) be the target state and target arm to compute the Whittle index. We first use value iteration and binary search to precompute the Whittle index \( W_i(u) \) that satisfies Definition 4.1 and the corresponding value functions \( [V_i^m(s)]_{s \in S} \) when a subsidy \( m = W_i(u) \) is provided. The main idea is to express Whittle index and the value functions as a solution to a selected subset of the Bellman equations in Equation 3.

We define a matrix \( A \in \mathbb{R}^{(M+1) \times (2M)} \) with all zeros except for the following entries:

\[
\begin{cases} 
A_{j,j} = 1 & \text{if } V_i^m(s^{(j)}) = V_i^m(s^{(j)}; a = 0) \\
A_{j,M+j} = 1 & \text{else if } V_i^m(s^{(j)}) = V_i^m(s^{(j)}; a = 1) \\
\text{one equality for non-target state } s^{(j)} \neq s^{(k)}
\end{cases}
\]

\[
\begin{cases} 
A_{k,k} = 1 \\
A_{M+1,M+k} = 1 \\
\text{two equalities for target state } s^{(k)}
\end{cases}
\]

We use \( V_i^m := [V_i^m(s)]_{s \in S} \), \( R(S) = [R(s)]_{s \in S} \), and \( P_i(S, a, S) := [P_i(s, a, s')]_{s, s' \in S} \in \mathbb{R}^{M \times M} \) to simplify the Bellman equation in Equation 3 and Whittle index condition in Definition 4.1. We use the matrix \( A \) to extract the rows where equality holds:

\[
A \begin{bmatrix} I_M \\
0_M 
\end{bmatrix} \gamma P_i(S, a = 0, S) - I_M = A \begin{bmatrix} m \\
R(S) 
\end{bmatrix}
\]

Based on the choice of matrix \( A \), Equation 8 receives \( M+1 \) equalities from the Bellman equation with \( M+1 \) variables and \( M+1 \) constraints. Therefore, we can consider the Whittle index \( m = W_i(u) \) as a solution to the linear system in Equation 8. Since the solution to a linear system is differentiable, we can use it to compute the derivative \( \frac{dW_i(u)}{dP} \) to achieve the differentiability of Whittle index. More details can be found in the appendix.

This process is repeated for every arm \( i \in [N] \) and every state \( u = s^{(k)}, k \in [M] \). Figure 2 summarizes the differentiable Whittle index policy and the entire algorithm is shown in Algorithm 1.
Algorithm 1: Decision-focused Learning in RMAB

1: **Input:** training set $D_{\text{train}}$, learning rate $r$, model $f_w$
2: for epoch $= 1, 2, \ldots$ and $(x, T) \in D_{\text{train}}$ do
3: Predict $P = f_w(x)$ and compute Whittle indices $W(P)$.
4: Let $\pi_{\text{whittle}} = \pi_W$ and compute $\text{Eval}(\pi_{\text{whittle}}, T)$. 
5: Update $w = w + r \frac{\partial \text{Eval}(\pi_{\text{whittle}}, T)}{\partial w} dW$, where $dW$ is computed from Equation 8
6: end for
7: **Return:** predictive model $f_w$

### 4.5 Computation Cost and Backpropagation

It is well studied that Whittle index policy can be computed more efficiently than solving the RMAB problem as a large MDP problem. Here, we show that the use of Whittle index policy also demonstrates a large speed up in terms of backpropagating the gradient in decision-focused learning.

In order to use Equation 8 to compute the gradient of Whittle indices, we need to invert the left-hand-side of Equation 8 with dimensionality $M + 1$, which takes $O(M^c)$, where $c \approx 2.373$ [Alman and Williams (2021)] is the best known matrix inversion constant. Therefore, the overall computation of all $N$ arms and $M$ states is $O(NM^{c+1})$ per gradient step.

In contrast, the standard decision-focused learning differentiates through the optimal policy using the full Bellman equation with $O(M^N)$ variables, where inverting the large Bellman equation requires $O(M^{2N})$ cost per gradient step. Thus, our algorithm significantly reduces the computation cost to a linear dependency on the number of arms $N$. To the best of our knowledge, this is the first decision-focused learning in RMAB problems that scales linearly in the number of arms.

### 4.6 Extension to Partially Observable Restless Bandits

For partially observable RMAB problem, we focus on a subclass of RMAB problem known as collapsing bandits [Mate et al. (2020)]. In collapsing bandits, belief states [Monahan (1982)] are used to represent the posterior measure of the unobservable states. Specifically, for each arm $i$, we use $b_i \in B = \Delta(S) \subset [0, 1]^{M}$ to denote the posterior belief of an arm, where each entry $b_i(s_i)$ denotes the probability that the true state is $s_i \in S$. When arm $i$ is pulled, the current true state $s_t \sim b_i$ is revealed and drawn from the posterior belief with expected reward $b_i^\top R$, where we can define the transition probability on the belief states. This process reduces partially observable states to fully observable belief states with in total $MT$ states since the maximal horizon is $T$. Therefore, we can use the same technique to differentiate through Whittle indices defined on partially observable states.

### 5 Policy Evaluation Metrics

In this paper, we use two different variants of evaluation metric: importance sampling-based evaluation [Sutton et al. (1998)] and simulation-based (model-based) evaluation.

**Importance sampling-based Evaluation** We adopt Consistent Weighted Per-Decision Importance Sampling (CWPDIS) [Thomas (2015)] as our importance sampling-based evaluation. Given target policy $\pi$ and a trajectory $\tau = \{s_1, a_1, r_1, \ldots, s_T, a_T, r_T\}$ executed by the behavior policy $\pi_{\text{beh}}$, the importance sampling weight is defined by $\rho_t = \prod_{t'=1}^t \frac{\pi(a_{t'}, | s_{t'})}{\pi_{\text{beh}}(a_{t'}, | s_{t'})}$. We evaluate the policy $\pi$ by

$$\text{Eval}_{\text{IS}}(\pi, T) = \sum_{t \in [T], i \in [N]} \gamma^{t-1} \frac{E_{\tau \sim T} [r_t, \rho_t]}{E_{\tau \sim T} [\rho_t]}$$

Importance sampling-based evaluation is usually less biased but with a larger variance due to the unstable importance sampling weights. CWPDIS normalizes the importance sampling weights to achieve a consistent estimate.

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4This is slightly modified when there is only one trajectory. Please refer to Section D for more details.
We compare two-stage learning (TS) with our decision-focused learning (DF-Whittle) that optimizes importance sampling-based evaluation directly. We consider three different evaluation metrics including predictive loss, importance sampling evaluation, and an additional simulation-based evaluation to evaluate all learning methods. We perform experiments on three synthetic datasets including 2-state fully observable, 5-state fully observable, and 2-state partially observable RMAB problems. We also perform experiments on a real dataset on maternal and child health problem modelled as a 2-state fully observable RMAB problem with real features and historical trajectories. For each dataset, we use 70%, 10%, 20% of the RMAB problems as the training, validation, and testing sets, respectively. All experiments are averaged over 50 independent runs.

**Synthetic datasets** We consider RMAB problems composed of $N = 100$ arms, $M$ states, budget $K = 20$, and time horizon $T = 10$ with a discount rate of $\gamma = 0.99$. The reward function is given by $R = \frac{1}{T} \sum_{t=1}^{T} \mathbb{1}_{[a_t = 1]}$, while the transition probabilities are generated uniformly at random but with a constraint that pulling the arm ($a = 1$) is strictly better than not pulling the arm ($a = 0$) to ensure the benefit of pulling. To generate the arm features, we feed the transition probability of each arm to a randomly initialized neural network to generate fixed-length correlated features with size 16 per arm. The historical trajectories $T$ with $|T| = 10$ are produced by running a random behavior policy $\pi_{\text{beh}}$. The goal is to predict transition probabilities from the arm features and the training trajectories.

**Real dataset** The Maternal and Child Healthcare Mobile Health program operated by ARMMAN (2022) aims to improve dissemination of health information to pregnant women and mothers with an aim to reduce maternal, neonatal and child mortality and morbidity. ARMMAN serves expectant/new mothers in disadvantaged communities with median daily family income of $3.22 per day which is seen to be below the world bank poverty line (World Bank, 2020). The program is composed of multiple enrolled beneficiaries and a planner who schedules service calls to improve the overall engagement of beneficiaries; engagement is measured in terms of total number of automated voice (health related) messages that the beneficiary engaged with. More precisely, this problem is modelled as a $M = 2$-state fully observable RMAB problem where each beneficiary’s behavior is governed by an MDP with two states - Engaging and Non-Engaging state; engagement is determined by whether the beneficiary listens to an automated voice message (average length 1 minute) for more than 30 seconds. The planner’s task is to recommend a subset of beneficiaries every week to receive service calls from health workers to further improve their engagement behavior. We do not know the transition dynamics, but we are given beneficiaries’ socio-demographic features to make predictions on transition dynamics.

Simulation-based Evaluation An alternative way is to use the given trajectories to construct an empirical transition probability $\hat{P}$ to build a simulator and evaluate the target policy $\pi$. The variance of simulation-based evaluation is small, but it may require additional assumptions on the missing transition when the empirical transition $\hat{P}$ is not fully reconstructed.

6 Experiments

We evaluate all learning methods. We perform experiments on all synthetic domains and the real ARMMAN dataset. For the evaluation metrics, we plot the improvement against the no-action baseline that does not pull any arm. Although two-stage method achieves the smallest predictive loss, decision-focused learning consistently outperforms two-stage method in both solution quality evaluation metrics across all domains.

![Figure 3: Comparison of predictive loss, importance sampling-based evaluation, and simulation-based evaluation on all synthetic domains and the real ARMMAN dataset.](image)

For the evaluation metrics across all domains.
We use a subset of data from the large-scale anonymized quality improvement study performed by ARMMAN for $T = 7$ weeks, obtained from Mate et al. (2022), with beneficiary consent. In the study, a cohort of beneficiaries received Round-Robin policy, scheduling service calls in a fixed order, with a single trajectory $|\mathcal{T}| = 1$ per beneficiary that documents the calling decisions and the engagement behavior in the past. We randomly split the entire cohort into 8 training groups, 1 validation group, and 3 testing groups each with $N = 639$ beneficiaries and $K = 18$ budget formulated as an RMAB problem. The socio-demographic features of beneficiaries are used as arm features to infer missing transition dynamics.

7 Experimental Results

Performance improvement and justification of objective mismatch
In Figure 3, we show the performance of random policy, two-stage, and decision-focused learning (DF-Whittle) on three evaluation metrics - predictive loss, importance sampling-based evaluation and simulation-based evaluation for all domains. For the evaluation metrics, we plot the improvement against the no-action baseline that does not pull any arms throughout the entire RMAB problem. We observe that two-stage learning consistently converges to a slightly smaller predictive loss, while DF-Whittle outperforms two-stage on all solution quality evaluation metrics significantly (p-value < 0.05) by alleviating the objective mismatch issue. This result also provides evidence of aforementioned objective mismatch, where the advantage of two-stage in the predictive loss does not result in improved solution quality.

Significance in maternal and child care domain
In the ARMMAN data in Figure 3, due to the limited resources, we can only select 18 out of 638 beneficiaries to make service call per week. Both random and two-stage method lead to around 15 more (IS-based evaluation) listening to automated voice messages among all beneficiaries throughout the 7-week program by $18 \times 7 = 126$ service calls, when compared to not scheduling any service call; this low improvement also reflects the hardness of maximizing the effectiveness of service calls. In contrast, decision-focused learning achieves an increase of beneficiaries listening to 50 more voice messages overall; DF-Whittle achieves this much higher increase by strategically assigning the limited service calls with the right objective in the learning method. We find this improvement to be statistically significant (p-value < 0.05).

Examining the difference between those selected for service call in two-stage and DF-Whittle, we observe that there are some interesting differences. For example, DF-Whittle chooses to do service calls to expectant mothers earlier in gestational age (22% vs 37%), and to a lower proportion of those who have already given birth (2.8% vs 13%) compared to two-stage, but in terms of the income level, 94% of the mothers selected by both methods are below the poverty line World Bank (2020).

Impact of Limited Data
Figure 4 shows the improvement between decision-focused learning and two-stage method with varying number of trajectories given to evaluate the impact of limited data. We notice that a larger improvement between decision-focused and two-stage learning is observed when fewer trajectories are available. We hypothesize that less samples implies larger predictive error and more discrepancy between the loss metric and the evaluation metric.

Computation cost comparison
Figure 5a compares the computation cost per gradient step of our Whittle index-based decision-focused learning and other baselines in decision-focused learning Wang et al. (2021), Futoma et al. (2020) by changing $N$ (the number of arms) in $M = 2$-state RMAB
problem. The other baselines fail to run with $N = 30$ arms and do not scale to larger problems like maternal and child care with more than 600 people enrolled, while our approach is 100x faster than the baselines as shown in Figure 5a and with a linear dependency on the number of arms $N$.

In Figure 5b, we compare the empirical computation cost of our algorithm with the theoretical computation complexity $O(NM^{\omega+1})$ in $N$ arms and $M$ states RMAB problems. The empirical computation cost matches with the linear trend in $N$. Our computation cost significantly improves the computation cost $O(M^{\omega})$ of previous work as discussed in Section 4.5.

8 Conclusion and Broader Impacts

This paper presents the first decision-focused learning in RMAB problems that is scalable for potential real-world deployment. We establish the differentiability of Whittle index policy in RMAB by providing new method to differentiate through Whittle index and using soft-top-k to relax the arm selection process. We show the significant performance improvement and impact of decision-focused learning on both the synthetic and real data. We plan to carefully monitor the impact of decision-focused learning in partnership with ARMMAN before moving towards deployment.

References

Agrawal, A., Amos, B., Barratt, S., Boyd, S., Diamond, S., and Kolter, Z. (2019). Differentiable convex optimization layers. arXiv preprint arXiv:1910.12430.

Akbarzadeh, N. and Mahajan, A. (2019). Restless bandits with controlled restarts: Indexability and computation of whittle index. In 2019 IEEE 58th Conference on Decision and Control (CDC), pages 7294–7300. IEEE.

Alman, J. and Williams, V. V. (2021). A refined laser method and faster matrix multiplication. In Proceedings of the 2021 ACM-SIAM Symposium on Discrete Algorithms (SODA), pages 522–539. SIAM.

Amos, B. and Kolter, J. Z. (2017). Optnet: Differentiable optimization as a layer in neural networks. In International Conference on Machine Learning, pages 136–145. PMLR.

ARMMAN (2022). ARMMAN helping mothers and children. https://armman.org/. Accessed: 2022-05-19.

Bagheri, S. and Scaglione, A. (2015). The restless multi-armed bandit formulation of the cognitive compressive sensing problem. IEEE Transactions on Signal Processing, 63(5):1183–1198.

Bai, S., Kolter, J. Z., and Koltun, V. (2019). Deep equilibrium models. arXiv preprint arXiv:1909.01377.

Benamou, J.-D., Carlier, G., Cuturi, M., Nenna, L., and Peyré, G. (2015). Iterative bregman projections for regularized transportation problems. SIAM Journal on Scientific Computing, 37(2):A1111–A1138.

Bubeck, S. and Cesa-Bianchi, N. (2012). Regret analysis of stochastic and nonstochastic multi-armed bandit problems. arXiv preprint arXiv:1204.5721.

Dai, W., Gai, Y., Krishnamachari, B., and Zhao, Q. (2011). The non-bayesian restless multi-armed bandit: A case of near-logarithmic regret. In 2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pages 2940–2943. IEEE.

Donti, P. L., Amos, B., and Kolter, J. Z. (2017). Task-based end-to-end model learning in stochastic optimization. arXiv preprint arXiv:1703.04529.

Elmachtoub, A., Liang, J. C. N., and McNellis, R. (2020). Decision trees for decision-making under the predict-then-optimize framework. In International Conference on Machine Learning, pages 2858–2867. PMLR.

Elmachtoub, A. N. and Grigas, P. (2021). Smart “predict, then optimize”. Management Science.
Futoma, J., Hughes, M. C., and Doshi-Velez, F. (2020). Popcorn: Partially observed prediction constrained reinforcement learning. arXiv preprint arXiv:2001.04032.

Gittins, J., Glazebrook, K., and Weber, R. (2011). Multi-armed bandit allocation indices. John Wiley & Sons.

Glazebrook, K. D., Ruiz-Hernandez, D., and Kirkbride, C. (2006). Some indexable families of restless bandit problems. Advances in Applied Probability, 38(3):643–672.

Huang, C., Zhai, S., Talbott, W., Martin, M. B., Sun, S.-Y., Guestrin, C., and Susskind, J. (2019). Addressing the loss-metric mismatch with adaptive loss alignment. In International Conference on Machine Learning, pages 2891–2900. PMLR.

Jiang, S., Song, Z., Weinstein, O., and Zhang, H. (2020). Faster dynamic matrix inverse for faster lps. arXiv preprint arXiv:2004.07470.

Johnson, J. M. and Khoshgoftaar, T. M. (2019). Survey on deep learning with class imbalance. Journal of Big Data, 6(1):1–54.

Lambert, N., Amos, B., Yadan, O., and Calandra, R. (2020). Objective mismatch in model-based reinforcement learning. arXiv preprint arXiv:2002.04523.

Liu, H., Liu, K., and Zhao, Q. (2012). Learning in a changing world: Restless multiarmed bandit with unknown dynamics. IEEE Transactions on Information Theory, 59(3):1902–1916.

Mandi, J., Stuckey, P. J., Guns, T., et al. (2020). Smart predict-and-optimize for hard combinatorial optimization problems. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 34, pages 4678–4691.

Mate, A., Killian, J. A., Xu, H., Perrault, A., and Tambe, M. (2020). Collapsing bandits and their application to public health intervention. In NeurIPS.

Mate, A., Madaan, L., Taneja, A., Madhiwalla, N., Verma, S., Singh, G., Hegde, A., Varakantham, P., and Tambe, M. (2022). Field study in deploying restless multi-armed bandits: Assisting non-profits in improving maternal and child health. In Proceedings of the AAAI Conference on Artificial Intelligence.

Monahan, G. E. (1982). State of the art—a survey of partially observable markov decision processes: theory, models, and algorithms. Management science, 28(1):1–16.

Oksanen, J. and Koivunen, V. (2015). An order optimal policy for exploiting idle spectrum in cognitive radio networks. IEEE Transactions on Signal Processing, 63(5):1214–1227.

Papadimitriou, C. H. and Tsitsiklis, J. N. (1994). The complexity of optimal queueing network control. In Proceedings of IEEE 9th Annual Conference on Structure in Complexity Theory, pages 318–322. IEEE.

Perrault, A., Wilder, B., Ewing, E., Mate, A., Dilkina, B., and Tambe, M. (2020). End-to-end game-focused learning of adversary behavior in security games. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 34, pages 1378–1386.

Qian, Y., Zhang, C., Krishnamachari, B., and Tambe, M. (2016). Restless poachers: Handling exploration-exploitation tradeoffs in security domains. In Proceedings of the 2016 International Conference on Autonomous Agents & Multiagent Systems, pages 123–131.

Sun, Y., Feng, G., Qin, S., and Sun, S. (2018). Cell association with user behavior awareness in heterogeneous cellular networks. IEEE Transactions on Vehicular Technology, 67(5):4589–4601.

Sutton, R. S., Barto, A. G., et al. (1998). Introduction to reinforcement learning, volume 135. MIT press Cambridge.

Tekin, C. and Liu, M. (2012). Online learning of rested and restless bandits. IEEE Transactions on Information Theory, 58(8):5588–5611.

Thomas, P. S. (2015). Safe reinforcement learning. PhD thesis, University of Massachusetts Libraries.
Wang, K., Shah, S., Chen, H., Perrault, A., Doshi-Velez, F., and Tambe, M. (2021). Learning mdps from features: Predict-then-optimize for sequential decision making by reinforcement learning. *Advances in Neural Information Processing Systems*, 34.

Wang, K., Yu, J., Chen, L., Zhou, P., Ge, X., and Win, M. Z. (2019). Opportunistic scheduling revisited using restless bandits: Indexability and index policy. *IEEE Transactions on Wireless Communications*, 18(10):4997–5010.

Weber, R. R. and Weiss, G. (1990). On an index policy for restless bandits. *Journal of applied probability*, 27(3):637–648.

Whittle, P. (1988). Restless bandits: Activity allocation in a changing world. *Journal of applied probability*, 25(A):287–298.

Wilder, B., Dilkina, B., and Tambe, M. (2019). Melding the data-decisions pipeline: Decision-focused learning for combinatorial optimization. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 33, pages 1658–1665.

World Bank. (2020). *Poverty and shared prosperity 2020: Reversals of fortune*. The World Bank.

Xie, Y., Dai, H., Chen, M., Dai, B., Zhao, T., Zha, H., Wei, W., and Pfister, T. (2020). Differentiable top-k operator with optimal transport. *arXiv preprint arXiv:2002.06504*. 


Appendix

A Hyperparameter Setting and Computation Infrastructure

We run both Decision Focused Learning and Two-Stage Learning for 50 epochs in 2-state and 5-state synthetic domain problems, 30 epochs in ARMMAN domain and 18 epochs in 2-state partially observable setting. The learning rate $\gamma$ is kept at 0.01 and $\gamma = 0.99$ is used in all experiments. All the experiments are performed on an Intel Xeon CPU with 64 cores and 128 GB memory.

Neural Network Structure

The predictive model $f_w$ we use to predict the transition probability is a neural network with an intermediate layer of size 64 with ReLU activation function, and an output layer of size of the transition probability followed by a softmax layer to match probability distribution. Dropout layers are added to avoid overfitting. The same neural network structure is applied to all domains and all training methods.

In the synthetic datasets, given the generated transition probabilities, we feed the transition probability of each arm into a randomly initialized neural network with two intermediate layers each with 64 neurons, and an output dimension size 16 to generate a feature vector of size 16. The randomly initiated neural network uses ReLU layers as nonlinearity followed by a linear layer in the end.

B Real ARMMAN Dataset

The large-scale quality improvement study conducted by ARMMAN [2022] contains 7668 beneficiaries in the Round Robin Group. Over a duration of 7 weeks, 20% of the beneficiaries receive at least one active action (LIVE service call). We randomly split the 7668 beneficiaries into 12 groups while preserving the proportion of beneficiaries who received at least one active action. There are 44 features available for every beneficiary which describe characteristics such as age, income, education level, call slot preference, language preference, phone ownership etc.

C Societal Impacts and Limitations

C.1 Societal Impacts

The improvement shown in the real dataset directly reflects the number of engagements improved by our algorithm under different evaluation metrics. On the other hand, because of the use of demographic features to predict the engagement behavior, we must carefully compare the models learned by standard two-stage approach and our decision-focused learning to further examine whether there is any bias or discrimination concern.

Specifically, the data is collected by ARMMAN, an India non-government organization, to help mothers during their pregnancy. The ARMMAN dataset we use in the paper does not contain information related to race, religion, caste or other sensitive features; this information is not available to the machine learning algorithm. Furthermore, examination by ARMMAN staff of the mothers selected for service calls by our algorithm did not reveal any specific bias related to these features. In particular, the program run by ARMMAN targets mothers in economically disadvantaged communities; the majority of the participants (94%) are below the international poverty line determined by The World Bank [World Bank 2020]. To compare the models learned by two-stage and DF-Whittle approach, we further examine the difference between those mothers who are selected for service call in two-stage and DF-Whittle, respectively. We observe that there are some interesting differences. For example, DF-Whittle chooses to do service calls to expectant mothers earlier in gestational age (22% vs 37%), and to a lower proportion of those who have already given birth (2.8% vs 13%) compared to two-stage, but in terms of the income level, 94% of the mothers selected by both methods are below the poverty line. This suggests that our approach is not biased based on income level, especially when the entire population is coming from economically disadvantaged communities. Our model can identify other features of mothers who are actually in need of service calls.
C.2 Limitations

Impact of limited data and the strength of decision-focused learning  As shown in Section 4.5, we notice a smaller improvement between decision-focused learning and two-stage approach when there is sufficient data available in the training set. This is because the data is sufficient enough to train a predictive model with small predictive loss, which implies that the predicted transition probabilities and the true transition probabilities are also close enough with similar Whittle indices and Whittle index policy. In this case with sufficient data, there is less discrepancy between predictive loss and the evaluation metrics, which suggests less improvement led by fixing the discrepancy using decision-focused learning. Compared to two-stage approach, decision-focused learning is still more expensive to run. Therefore, when data is sufficient, two-stage may be sufficient to achieve comparable performance while maintaining a low training cost.

On the other hand, we notice a larger improvement between decision-focused learning and two-stage approach when data is limited. When data is limited, predictive loss is less representative with a larger mismatch compared to the evaluation metrics. Therefore, fixing the objective mismatch issue using decision-focused learning becomes more prominent. Therefore, decision-focused learning may be adopted in the limited data case to significantly improve the performance.

Computation cost  As we have shown in Section 4.5, our approach improves the computation cost of decision-focused learning from $O(M^{2N})$ to $O(NM^{2N+1})$, where $N$ is the number of arms and $M$ is the number of states. This computation cost is linear in the number of arms $N$, allowing us to scale up to large real-world deployment of RMAB applications with larger number of arms involved in the problem. Nonetheless, the extension in terms of the number of states $M$ is not cheap. The computation cost still grows between cubic and biquadratic as shown in Figure 6. This is particularly significant when working on partially observable RMAB problems, where the partially observable problems are reduced to fully observable problems with larger number of states. There is room for improving the computation cost in terms of the number of states to make decision-focused learning more scalable to real-world applications.

D Importance Sampling-based Evaluations for ARMMAN Dataset with Single Trajectory

Unlike the synthetic datasets that we can produce multiple trajectories of an RMAB problem, in the real problem of service call scheduling problem operated by ARMMAN, there is only one trajectory available to us for every RMAB problem. Due to the specialty of the maternal and child health domain, it is unlikely to have the exactly same set of the pregnant mothers participating in the service call scheduling program at different times and under the same engagement behavior.

Given this restriction, we must evaluate the performance of a newly proposed policy using the only available trajectory. Unfortunately, the standard CWPDIS in Equation 9 does not work because the CWPDIS estimator is canceled out when there is only one trajectory:

$$\text{Eval}_{IS}(\pi, T) = \sum_{t \in [T], i \in [N]} \gamma^{t-1} \frac{E_{\pi \sim \tau} [r_{t,i}\rho_{t,i}(\tau)]}{E_{\pi \sim \tau} [\rho_{t,i}(\tau)]} = \sum_{t \in [T], i \in [N]} \gamma^{t-1} \frac{r_{t,i} \rho_{t,i}(\tau)}{\rho_{t,i}(\tau)} = \sum_{t \in [T], i \in [N]} \gamma^{t-1} r_{t,i}$$

which is fixed regardless what target policy $\pi$ is used and the associated importance sampling weights $\frac{\pi(a_{t,i} | s_{t})}{\pi_{\text{na}}(a_{t,i} | s_{t})}$ and $\rho_{t,i} = \prod_{t' = 1}^{T} \frac{\pi(a_{t',i} | s_{t'})}{\pi_{\text{na}}(a_{t',i} | s_{t'})}$. This implies that we cannot use CWPDIS to evaluate the target policy when there is only one trajectory.

Accordingly, we use the following variant to evaluate the performance:

$$\text{Eval}_{IS}(\pi, T) = \sum_{i \in [N], t \in [T]} \gamma^{t-1} \frac{r_{t,i}\rho'_{t,i}(\tau)}{E_{\pi' \sim \tau} [\rho'_{t,i}(\tau)]}$$

where the new importance sampling weights are defined by $\rho'_{t,i}(\tau) = \frac{\pi(a_{t,i} | s_{t})}{\pi_{\text{na}}(a_{t,i} | s_{t})}$, which is not multiplicative compared to the original ones.
The main motivation of this new evaluation metric is to decompose the given trajectory into a set of length-1 trajectories. We can apply CWPDIS to the newly generated length-1 trajectories to compute a meaningful estimate because we have more than one trajectory now. This slight modification requires adjustment in the definition of the importance sampling weights, where since all trajectories are of length-1, there is no multiplicative factor involved in computing the importance sampling weights. Empirically, we noticed that this temporal decomposition helps define a meaningful importance sampling-based evaluation with the consistency benefit brought by CWPDIS.

E Computation Cost Analysis of Decision-focused Learning

We have shown the computation cost of backpropagating through Whittle indices in Section 4.5. This section covers the remaining computation cost associated to other components, including the computation cost of Whittle indices in the forward pass, and the computation cost of constructing soft Whittle index policy using soft-top-k operator.

E.1 Solving Whittle Index (Forward Pass)

In this section, we discuss the cost of computing Whittle index in the forward pass. In the work by Qian et al. (2016), they propose to use value iteration and binary search to solve the Bellman equation with \( M \) states. Therefore, every value iteration requires updating the current value functions of \( M \) states by considering all the possible \( M^2 \) transitions between states, which results in a computation cost of \( O(M^2) \) per value iteration. The value iteration is run for a constant number of iterations, and the binary search is run for \( O(\log \frac{1}{\epsilon}) \) iterations to get a precision of order \( \epsilon \). In total, the computation cost is of order \( O(M^2 \log \frac{1}{\epsilon}) = O(M^2) \) where we simply use a fixed precision to ignore the dependency on \( \epsilon \).

On the other hand, there is a faster way to compute the value function by solving linear program with \( M \) variables directly. The Bellman equation can be expressed as a linear program where all the \( M \) variables are the value functions. The best known complexity of solving a linear program with \( M \) variables is \( O(M^2 + \frac{1}{L}) \) by Jiang et al. (2020). Notice that this complexity is slightly larger than the one in value iteration because (i) value iteration does not guarantee convergence in a constant iterations (ii) the constant associated to the number of value iterations is large.

In total, we need to compute the Whittle index of \( N \) arms and for \( M \) possible states in \( S \). The total complexity of value iteration and linear program are \( O(NM^3) \) with a large constant and \( O(NM^3 + \frac{1}{L}) \), respectively. In any cases, the cost of computing all Whittle indices in the forward pass is still smaller than \( O(NM^{1+\omega}) \), the cost of backpropagating through all the Whittle indices in the backward pass. Therefore, the backward pass is the bottleneck of the entire process.

E.2 Soft-top-k Operators

In Section E.1 and Section 4.5, we analyze the cost of computing and backpropagating through Whittle indices of all states and all arms. In this section, we discuss the cost of computing the soft Whittle index policy from the given Whittle indices using soft-top-k operator.

Soft-top-k operators Xie et al. (2020) reduces top-k selection problem to an optimal transport problem that transports a uniform distribution across all input elements with size \( N \) to a distribution where the elements with the highest-k values are assigned probability 1 and all the others are assigned 0.

This optimal transport problem with \( N \) elements can be efficiently solved by using Bregman projections Benamou et al. (2015) with complexity \( O(LN) \), where \( L \) is the number of iterations used to run Bregman projections. In the backward pass, Xie et al. (2020) shows that the technique of differentiating through the fixed point equation Bai et al. (2019); Amos and Kolter (2017) also applies, but the naive implementation requires computation cost \( O(N^2) \). Therefore, Xie et al. (2020) provides a faster computation approach by leveraging the associate rule in matrix multiplication to lower the backward complexity to \( O(N) \).

In summary, a single soft-top-k operator requires \( O(LN) \) to compute the result in the forward pass, and \( O(N) \) to compute the derivative in the backward pass. In our case, we need to apply
one soft-top-k operator for every time step in $T$ and for every trajectory in $\mathcal{T}$. Therefore, the total computation cost of computing a soft Whittle index policy and the associated importance sampling-based evaluation metric is bounded by $O(LNT|\mathcal{T}|)$, which is linear in the number of arms $N$, but still significantly smaller than $O(NM^{\omega+1})$, the cost of backpropagating through all Whittle indices as shown in Section 4.5. Therefore, we just need to concern the computation cost of Whittle indices in decision-focused learning.

### E.3 Computation Cost Dependency on the Number of States

Figure 6 compares the computation cost of our algorithm, DF-Whittle, and the theoretical computation cost $O(NM^{\omega+1})$. We vary the number of states $M$ in Figure 6 and we can see that the computation cost of our algorithm matches the theoretical guarantee on the computation cost. In contrast to the prior work with computation cost $O(M^\omega N)$, our algorithm significantly improves the computation cost of running decision-focused learning on RMAB problems.

### F Additional Experimental Results

We provide the learning curves of fully observable 2-state RMAB, fully observable 5-state RMAB, partially observable 2-state RMAB, and the real ARMMAN fully observable 2-state RMAB problems in Figure 7, 8, 9, 10, respectively. Across all domains, two-stage method consistently converges to a lower predictive loss faster than decision-focused learning in Figure 7a, 8a, 9a, 10a. However, the learned model does not produce a policy with good performance in the importance sampling-based evaluation metric in Figure 7b, 8b, 9b, 10b and similarly in the simulation-based evaluation metric in Figure 7c, 8c, 9c, 10c.

Figure 7: Comparison between two-stage and decision-focused in the synthetic fully observable 2-state RMAB problems.
Figure 8: Comparison between two-stage and decision-focused learning for fully observable 5-state RMAB problems.

Figure 9: Comparison between two-stage and decision-focused learning for 2-state partially observable RMAB problems.

Figure 10: Comparison between two-stage and decision-focused learning in the real ARMMAN service call scheduling problem. The pulling action in the real dataset is much sparser, leading to a larger mismatch between predictive loss and evaluation metrics. Two-stage overfits to the predictive loss drastically with no improvement in evaluation metrics. In contrast, decision-focused learning can directly optimize the evaluation metric to avoid the objective mismatch issue.