The effects of Final State Interactions (FSI), Colour Reconnection (CR) and Bose–Einstein Correlations (BEC) at small relative momenta of particles are discussed. The short review of the LEP results on BEC at Z-peak, on BEC between particles from different Ws in $e^+e^- \rightarrow W^+W^-$ events, and the W-mass systematics due to BEC is given.

1. Bose-Einstein Correlations at LEP

The observation of the enhanced production of like–sign particle pairs by LEP experiments at Z does not insure that the observed correlations owe their origin to Bose-Einstein correlations (BEC). They could, for example, arise from final state interactions (FSI) between pions with small relative momenta. (we recommend not to call BEC and/or Colour Reconnection as 'FSI effects' as it is generally used). It turns out that it is possible to discount the possibility of visible FSI effects by virtue of the smaller correlations between unlike-sign particles, which could arise from final state interaction effects too. As was shown at LEP, the unlike-sign particle configurations show also a correlations. The like-sign pions necessarily are in an isospin $I=2$ state and the unlike-sign pions are dominantly in the isospin $I=0$ state. The scattering length for $I=2$ is more than a factor three smaller than the one for $I=0$. This insures that the FSI effects for like-sign pion pairs are expected to be nearly an order of magnitude smaller than those observed in unlike-sign pions. Even if one would attribute all the correlations observed for unlike-sign pions to final state effects, then essentially none of
the correlation effects observed in like-sign pairs can therefore be attributed to final state interactions. As for Colour Reconnection(CR) effects, there is no evidence that any of CR model predicts visible correlations between pion pairs at small relative momenta.

On the other hand, it is known that at LEP energies for high multiplicity events (like $e^+e^- \rightarrow Z \rightarrow \text{hadrons}$ and $e^+e^- \rightarrow W^+W^- \rightarrow \text{hadrons}$), as well as a clear effect on like-sign particle correlations, the BEC affect unlike-sign particle pairs as well. These so-called 'residual BEC', discussed in details in $^1$, were observed at Z energies by three LEP experiments. (The distortion of the $\rho^0(770)$ line–shape in the mode $\rho^0(770) \rightarrow \pi^+\pi^-$ was observed. These measurements are model independent.) The influence of BEC to unlike-sign particles is also predicted by LUBOEI $^2$ and by a 'reweighting' BEC model $^1$. From the generalised Bose statistics and isospin invariance follows also that the pairs with different charges which belong to the same isospin multiplet may show BEC enhancement. Thus, one may conclude that the observed at LEP correlations for like-sign as well for unlike-sign particle pairs can be ascribed to Bose–Einstein Correlations.

As a consequence, we recommend do not divide(or subtract) the plots of like–sign and unlike–sign particle configurations in analysis of the Bose–Einstein Correlations effects in $e^+e^- \rightarrow Z$ and $e^+e^- \rightarrow W^+W^-$ events.

2. LEP Results on Correlations Between Particles from different Ws in $e^+e^- \rightarrow W^+W^-$ Events

2.1. The DELPHI results

The ratio $D(Q)$ for DELPHI data obtained using practically all available statistics, for model with full and inside Ws BEC using LUBOEI BE$_{32}$ $^2$, are shown in Fig. 1 for like–sign and for unlike–sign pairs $^3$. The $D$ is the ratio of plots of WW fully hadronic to mixed semileptonic events. The $Q$ is defined as $Q^2=M_{\pi\pi}^2-4m_\pi^2$. The alternative independent analysis $^3$ of the same DELPHI data using a different event selections and mixing method yielded practically identical measurements of the $D(Q)$.

DELPHI reports an effect of inter–W correlations at the level of $2.8\sigma$ for like–sign particle pairs $\Lambda(\text{DELPHI})=0.142\pm0.049(\text{stat})\pm0.015(\text{syst})$. This value of $\Lambda(\text{DELPHI})$ has been obtained by fitting the $D(Q)$ plot in range of $Q=0–4.0$ GeV using the equation $D(Q) = N (1 + \delta Q) (1 + \Lambda e^{-RQ})$. The parameters $N$, $\delta$ and $\Lambda$ were fit parameters, while the parameter $R$ were fixed to value $R=0.82$ fm, as was obtained from the model with full BEC tuned to the data at Z-peak. The value $\Lambda$ for the LUBOEI BE$_{32}$ with full
BEC was $\Lambda(\text{full BEC}) = 0.241 \pm 0.009$.

DELPHI reports also an effect of inter–W correlations at the level of $2.0\sigma$ for unlike–sign particle pairs. The value $\Lambda(\text{DELPHI})$ for unlike–sign particle pairs was in agreement with prediction of the LUBOEI BE32 with full BEC.

![Figure 1. The DELPHI $D(Q)$ plots (see text).](image)

**2.2. The L3 results**

The L3 results using all available statistics are presented in $^4$. The measured value of $\Lambda(\text{L3})$ for like–sign pairs is consistent with zero, i.e. absence of inter–W correlations: $\Lambda(\text{L3}) = 0.008 \pm 0.018(\text{stat}) \pm 0.012(\text{syst})$. The value for model with full BEC L3 quotes is $\Lambda(\text{full BEC}) = 0.098 \pm 0.008$. L3 measured also the $\Delta \rho(Q)$ distributions.

The value $\Lambda(\text{L3})$ has been obtained from the fits of the $D'(Q)$ plot in range of $Q=0–1.4$ GeV by the equation $D'(Q) = N \left(1 + \delta Q \right) \left(1 + \Delta e^{-k^2 Q^2} \right)$. We noted that the two intriguing assumptions have been made in $^4$: (a) the parameter $k$ was fit parameter, (b) the normalization $N$ was fixed to unity which does not allow the number of pairs
to vary. We performed the new test fits to L3 data plots in range of $Q=0$–
1.4 GeV (using the values and errors as presented in 4) to estimate the
effects of (a) and (b). We found the remarkable differences in values of $\Lambda$,
particulary, the statistical error in case of UNFIXED $N$ was significantly
higher than in case of fixed $N=1$, also significantly higher than it is pre-
sented in published number of $\Lambda(L3)=0.008 \pm 0.018(stat) \pm 0.012(syst)$.
It is important particulary for comparison with model predictions.

3. Summary

The LEP data and theoretical predictions suggest that the observed correla-
tions between both, like-sign and unlike-sign particle pairs at small relative
momenta are due to BEC.

The results by DELPHI 3 indicate clearly (2.8$\sigma$ effect for like-sign and
2.0$\sigma$ effect for unlike-sign particle pairs) the presence of BEC between par-
ticles from different Ws in $e^+e^-\rightarrow W^+W^-$ events, while the L3 analysis in
4 shows no effect. The DELPHI and L3 analysis use all available statistics
and the event mixing method which, as generaly agreed, is the best method
for the measuring of this effect. We have found that there are uncertainties
in main measured quantities in L3 analyses 4. No WW BEC measurements
exist yet for ALEPH and OPAL using all available statistics.

Although there were many attempts to implement various approaches
of BEC effect in the models, LUBOEI model 2 is the only BEC model
the predictions of which were compared with the LEP correlation data in
detail. The model describes the data reasonably well. The latest version of
the model, the LUBOEI BE$\chi$ code 2 with BEC included between particles
from different Ws in WW fully hadronic channel yields about $-$35 MeV
shift of W-mass due to BEC.

To quote Prof. Bo Andersson (Datong, 2001): if pions from different
Ws are produced close in phase space, they should show BEC, whether or
not coming from the same or different Ws; that is Quantum Mechanics.

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