Duality Phase Transition in Type I String Theory

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Abstract

We show that the duality phase transition in the unoriented type I theory of open and closed strings is first order. The order parameter is the semiclassical approximation to the heavy quark-antiquark potential at finite temperature, extracted from the covariant off-shell string amplitude with Wilson loop boundaries wrapped around the Euclidean time direction. Remarkably, precise calculations can be carried out on either side of the phase boundary at the string scale $T_C=1/2\pi\alpha'^1/2$ by utilizing the T-dual, type IB and type I’, descriptions of the short string gas of massless gluon radiation. We will calculate the change in the duality transition temperature in the presence of an electromagnetic background field.

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1 Introduction

The canonical ensembles of the T-dual type IB, and type I', unoriented open and closed string theories are especially interesting because of the ease with which the finite temperature Yang-Mills gauge theory limits can be extracted from the low energy approximation to the full string amplitude. Unlike the heterotic closed string theories, where the low energy limit yields a Yang-Mills gauge theory coupled to a supergravity, here we can identify a self-consistent limit of any perturbative string scattering amplitude which isolates pure nonabelian gauge theory physics. In this note, we will see how this observation can be applied to infer the temperature dependence of the semiclassical heavy quark-antiquark potential in the finite temperature nonabelian gauge theory. We will find clear evidence of a first order phase transition at the string scale, inferring the precise transition temperature in both the empty, flat target space background, as well as in the presence of a background electromagnetic field strength.

In a previous work [11], we have shown that the $T^2$ high temperature growth of the string free energy at temperatures above the string scale is compatible with a much more rapid $T^{10}$ growth at low temperatures, exactly as would be expected in a finite temperature nonabelian gauge theory. The latter result follows from isolating the leading contribution to the free energy in the string mass level expansion, having performed the one-loop modular integrals explicitly for this single term [11]. Could the fact that the free energy grows much more slowly at high temperatures far beyond the string scale be an indication of a thermal phase transition in the string ensemble, akin to the deconfining transition familiar from finite temperature gauge theory? And how could one hope to extract evidence for a first order phase transition from perturbative string theory, given that the amplitudes in any renormalizable and ultraviolet finite background are known to be analytic in their dependence on any moduli parameters, in this case, the inverse temperature $\beta$. The key hint in favor of the possibility of evidence for such a phase transition is the existence of distinct, T-dual string descriptions, type IB and type I', on either side of the phase boundary at $T_C$. Thus, a matching of their respective low-energy gauge theory limits at the phase boundary can indeed display a discontinuity of first order, in an appropriately chosen physical observable. The observable of interest will turn out to be the semi-classical heavy quark-antiquark potential.

Our strategy for exposing such a phase transition is as follows. Since the string one-loop vacuum amplitude has shown no sign of a discontinuity, or non-analyticity, as a function of temperature we must look at a different amplitude as plausible order parameter for the thermal phase transition. A natural choice suggested by the correspondence in the low energy limit to finite temperature Yang-Mills gauge theory, would be the string theory analog of the expectation value of a timelike Wilson-Polyakov-Susskind loop wrapping the Euclidean time direction. Namely, the change in the string free energy in the thermal vacuum due to the introduction of an external heavy quark, generally taken to be the order parameter for the deconfinement phase transition in finite temperature gauge theory [12]. The expectation value of a single Wilson loop is related to the disk amplitude in string theories [13]. We should note that this quantity is extremely sensitive to the infrared divergences of finite temperature gauge theory, necessitating ingenious techniques for a clear-cut study of the order parameter in both the lattice, or dual confinement model, approaches. It is generally considered easier in nonabelian gauge theory to extract the desired result from a computation of the
pair correlator of Polyakov-Susskind loops which yields the static heavy quark-antiquark potential in the thermal vacuum. This is a fortunate circumstance, since the corresponding string theory computation is readily accessible.

In string theory, it turns out that the Polyakov path integral summing surfaces with the topology of an annulus and with boundaries mapped to a pair of fixed curves, $C_1, C_2$, in the embedding target spacetime, wrapping the Euclidean time coordinate, and with fixed spatial separation, $R$, can be computed from first principles using Riemann surface methodology [1], and is unambiguously normalized unlike the disk amplitude [8]. The requisite analysis is an interesting extension of Polchinski's well-known covariant, one-loop vacuum amplitude computation based on the Polyakov path integral [2, 3, 8]. The amplitude we will compute can be interpreted as an off-shell closed string tree propagator, and the result in closed bosonic string theory, but only in the limit that the macroscopic boundaries, $C_1, C_2$, were point-like, was first obtained by Cohen, Moore, Nelson, and Polchinski in 1986 [1]. Their bosonic string theory analysis was adapted by myself, in collaboration with Yujun Chen and Eric Novak in [4], to address the limit of large macroscopic loop length of interest here. The extension to the macroscopic loop amplitude of the type I and type II superstring theories with Dbranes by Novak and myself appears in [5, 6]. It should be noted that further extension to generic Wilson loop computations should be possible, and our results by no means the end of this fascinating application to nonabelian gauge theory.

We will calculate the pair correlator of a pair of Polyakov-Susskind loops wrapping the Euclidean time coordinate, extracting the low energy gauge theory limit of the resulting expression where the contribution from massive string modes has been suppressed. Notice that in the limit of vanishing spatial separation, $R \to 0$, the amplitude will be dominated by the shortest open strings, namely, the gauge theory modes in the massless open string spectrum. We can analyze this limit of the expression for its dependence on temperature. We find clear evidence for a thermal phase transition in the gauge theory at the self-dual temperature: $T_C = 1/(2\pi \alpha')^{1/2}$. The temperature dependence in the limit of vanishing spatial separation transitions from a $O(1/T^2)$ fall at low temperatures to an $O(T^2)$ growth at high temperatures above the string scale. The argument is as follows. A Euclidean T-duality transformation on our expression for the macroscopic loop amplitude in the type IB string theory conveniently maps it to an expression for a corresponding amplitude in the type I' string theory. That expression will be well-defined in the temperature regime above $T_C$, or vice versa, and the low energy gauge theory limit can be easily taken as before. Thus, the existence of T-dual type IB, and type I', descriptions of the thermal ground state enable precise computations to be made in the low energy gauge theory limit on either side of the phase boundary at $T_C$.

The intuition that a gas of short open strings transitions into a high temperature long string phase is an old piece of string folklore, and could find important application in both gauge theory and early universe cosmology, including the physics of cosmic strings [15]. However, as is clear from the detailed analysis given by us in [11], despite the exponential growth in the degeneracies of states with high mass level number in the string spectrum, the free energy itself is free of both divergences, and of non-analyticity! In particular, it is not true that the superstring canonical ensemble "breaks down" above a string scale "limiting" temperature; the canonical ensemble is well-defined on both sides of the phase boundary at $T_C$. Rather, it is necessary to carry out a T-duality transformation to probe the novel high temperature behavior.
2 Evidence for the Long String Phase Transition

A plausible order parameter signalling a thermal phase transition in the type IB string theory at a temperature of order the string mass scale is suggested by the correspondence with the low energy finite temperature gauge theory limit. It is well known that the order parameter signalling the thermal deconfinement phase transition in a nonabelian gauge theory at high temperatures is the expectation value of a closed timelike Wilson-Polyakov-Susskind loop [12]. We wish to investigate evidence for a thermal phase transition in the low energy gauge theory limit of type I string theory at a temperature of order the string scale. Such a phase transition has long been conjectured to arise in the massless radiation gas of short open strings, characterized plausibly by long string formation in the high temperature phase [15], and often interchangeably referred to as a “Hagedorn” phase transition.

Since the one-loop free energy in the type IB thermal vacuum displays no non-analyticity, or discontinuities, as a function of temperature as shown by us in [11], it is natural to look for evidence in a different string amplitude. A natural choice suggested by the correspondence in the low energy limit to finite temperature Yang-Mills gauge theory, would be the string theory analog of the expectation value of a timelike Wilson loop wrapping the Euclidean time direction. Namely, the change in the free energy in the thermal vacuum due to the introduction of an external heavy quark, generally taken to be the order parameter for the deconfinement phase transition in finite temperature gauge theory, pointed out, independently, by both Polyakov and Susskind [12].

As mentioned in the introduction, a convenient starting point is the Polyakov path integral summing surfaces with the topology of an annulus and with boundaries mapped to a pair of fixed curves, \(C_1, C_2\), in the embedding target spacetime, wrapping the Euclidean time coordinate, and with fixed spatial separation, \(R\). This macroscopic loop amplitude can also be computed from first principles using Riemann surface methodology, an extension of the covariant one-loop string vacuum amplitude derived by Polchinski in [2, 8, 3]. The macroscopic loop amplitude is an off-shell closed string tree propagator, and the result in closed bosonic string theory, but only in the limit that the macroscopic boundaries, \(C_1, C_2\), were point-like, was first obtained by Cohen, Moore, Nelson, and Polchinski [1], and extended to include the limit of large macroscopic loop lengths of interest here by myself in collaboration with Yujun Chen and Eric Novak in [4]. The low energy gauge theory limit of the macroscopic amplitude yields the pair correlator of Wilson loops wrapping the Euclidean time coordinate in the finite temperature nonabelian gauge theory living on the worldvolume of D9branes, with the contribution from massive string modes suppressed. Notice that in the limit of vanishing spatial separation, \(R \to 0\), the target spacetime bosonic contribution to the string one-loop amplitude will be dominated by the shortest (bosonic) open strings, namely, the gauge theory modes in the massless open string spectrum. We will analyze this field theoretic limit of the macroscopic loop amplitude for its dependence on temperature.

Consider, therefore, the pair correlation function of a pair of Polyakov-Susskind loops lying within the worldvolume of the D9branes, and with fixed spatial separation \(R\) in a direction transverse to compactified Euclidean time, \(X^0\). Recall that the boundaries of the worldsheet are the closed “world-histories” of the open string endpoint, which couples to the gauge fields living on the worldvolume of the Dbranes. The endpoint states are in the fundamental, and anti-fundamental,
representations of the gauge group. Thus, when the closed worldlines are constrained to coincide with closed timelike loops in the embedding spacetime, the resulting string amplitude has a precise correspondence in the low energy gauge theory limit to the correlation function of two closed timelike loops representing the spacetime histories of a static, heavy "quark–antiquark" pair with fixed spatial separation. Since we wish to probe the high temperature behavior of the low energy gauge theory limit, we should use the Euclidean T-dual type I' description of the thermal vacuum.

The result for the pair correlator of temporal Wilson loops in the Euclidean T-dual type I' vacuum, \( W^{(2)}_I \), derived from first principles from an extension of the ordinary Polyakov path integral in the references [1, 4], takes the remarkably simple form [9]:

\[
W^{(2)}_I(R, \beta) = \lim_{\beta \to 0} \int_0^\infty dt \frac{e^{-R^2 t / 2 \pi \alpha'}}{\eta(it)^8} \sum_{n \in \mathbb{Z}} q^{n^2 \beta^2 / 4 \pi^2 \alpha'}
\times \left[ \left( \frac{\Theta_{00}(it; 0)}{\eta(it)} \right)^4 - \left( \frac{\Theta_{01}(it; 0)}{\eta(it)} \right)^4 \right]
\]

The summation variable, \( n \), labels closed string winding modes in this expression, each of which wraps around the Euclidean time-like coordinate \( X^0' \). The analysis in [11] includes both half integer and integer moding in the thermal winding spectrum, but let us restrict ourselves to just the integer modes in this calculation; the half-integer moded spectrum makes a similar contribution.

We need to identify a suitable physical observable that can be extracted from this calculation. An observable of considerable significance in nonabelian gauge theory is the semiclassical heavy quark-antiquark potential. The low energy finite temperature gauge theory limit of the macroscopic loop amplitude yields the temperature dependence of the static heavy quark potential as follows. Let us set:

\[
W^{(2)}_I(\beta) = \lim_{s \to \infty} \int_{-s}^{+s} ds V[R, \beta]
\]

where \( s \) is the proper time parameterizing the worldlines of the quark-antiquark pair, wrapping the Euclidean time coordinate. We can invert this relation to express \( V[R, \beta] \) as an integral over the modular parameter \( t \) [7, 8]. Consider the \( q \) expansion of the integrand valid for \( t \to \infty \), where the shortest open strings dominate the modular integral. The low temperature behavior is best extracted from the type I' amplitude, with its spectrum of thermal winding modes. Retaining the leading terms in the \( q \) expansion and performing explicit term-by-term integration over the worldsheet modulus, \( t \), isolates the following interaction potential at temperatures below the string mass scale [9, 4]:

\[
V(R, \beta)|_{\beta >> \beta_C} = (8\pi^2 \alpha')^{-1/2} \int_0^\infty dt e^{-R^2 t / 2 \pi \alpha'} t^{1/2} \sum_{n=-\infty}^{\infty} \left[ 16 + O(e^{-\pi t}) \right] q^{n^2 \beta^2 / 4 \pi^2 \alpha'}
\]

\[
\approx (8\pi^2 \alpha')^{-1/2} \Gamma(3/2)(2\pi \alpha')^{3/2} \frac{2^4}{R^3} \left[ 1 - \frac{3}{2} \sum_{n=-\infty}^{\infty} \frac{\beta^2 n^2}{R^2} \right]
\]

\[
\equiv 16\pi^{1/2} \alpha' \Gamma(3/2) \frac{1}{R^3} \left[ 1 - 3 \zeta(-2, 0) \frac{\beta^2}{R^2} \right].
\]

Note that at low temperatures, \( \beta >> \beta_C \), the power series expansion gives an \( O(\beta^2 / R^5) \) correction to the zero temperature static potential. Here, \( \beta_C \) is the inverse self-dual temperature. Above \( T_C \),
we can extract the correct form of the potential from the low energy gauge theory limit of the
T-dual type IB vacuum amplitude. The corrections to the zero temperature static potential take
the form of a power series in \((\beta^4/\beta^2 R^2)\) at high temperature:

\[
V(R, \beta)|_{\beta<\beta_C} = (8\pi^2 \alpha')^{-1/2} \int_0^\infty dt e^{-R^2 t/2\pi \alpha'} t^{1/2} \\
\times \sum_{n=-\infty}^\infty \left[ 16 + O(e^{-\pi t}) \right] q^{4\pi^2 \alpha' n^2/\beta^2} + \ldots
\]

\[
\simeq (8\pi^2 \alpha')^{-1/2} \Gamma(3/2)(2\pi \alpha')^{3/2} \frac{2^4}{R^3} \left[ 1 - \frac{3}{2} \sum_{n=-\infty}^\infty \frac{(4\pi^2 \alpha')^2 n^2}{\beta^2 R^2} \right]
\]

\[
\equiv 16\pi^{1/2} \alpha' \Gamma(3/2) \frac{1}{R^3} \left[ 1 - 3\zeta(-2,0) \frac{\beta^4}{\beta^2 R^2} \right],
\]

Notice that the discontinuity is in the first derivative with respect to temperature. Remarkably,
precise computations can nevertheless be carried out on either side of the phase boundary by
utilizing, respectively, the low energy gauge theory limits of the pair of thermal dual string theories,
type IB and type I'. The result clarifies that the thermal phase transition at \(T_C = 1/2\pi \alpha'^{1/2}\) in
the gauge theory is first order.

Finally, it is helpful to note that in the presence of a background electromagnetic field, the result
for the transition temperature is simply altered to \(T_D = 1/2\pi \alpha'^{1/2} u\), where \(F_{\theta j} = \tanh^{-1} u\) is
the constant part of the magnetic field strength, and \(j\) labels any other spatial coordinate. This fact
was noted by us in [9, 10], but without appreciation of the full significance of the thermal duality
transition.

3 Conclusions

We have shown that the duality phase transition in the unoriented type I theory of open and closed
strings is first order. The order parameter is the semiclassical approximation to the heavy quark-
antiquark potential at finite temperature, extracted from the covariant off-shell string amplitude
with Wilson loop boundaries wrapped around the Euclidean time direction. Remarkably, precise
calculations can nevertheless be carried out on either side of the phase boundary by utilizing
the respective low energy limits of the T-dual, type IB and type I', string theories. Either describes
the short string gas of massless gluon radiation, but on one, or other, side of the phase boundary.
We will leave discussion of the applications of this result in both gauge theory and cosmology [15] to
future work.

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