Research on triaxial array induction logging response in inclined anisotropic formation

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Abstract: The logging response calculation and fast inversion are the key and difficulties for logging data interpretation and formation parameter estimation of the triaxial array induction logging. We transform the formation conductivity from media coordinate to instrument coordinate based on coordinate transformation theory. This paper examines anisotropic response of the triaxial array induction in inclined anisotropic formation by finite element method. The greater the anisotropy coefficient is, the more obvious the curve separation. The apparent conductivity decreases with the increase of dip angle for the coaxial system, but the coplanar system increases. The research results show that the apparent conductivity of the coplanar system appears as obviously positive and negative horns on the horizontal interfaces, which is generated by the accumulated surface charge at the boundary of the layer.

1. Introduction

Electric anisotropy reservoir has become an important exploration direction at present, which is an urgent resource to be exploited in the late stage of oil field development. The coil probe of triaxial array induction logging is different from that of traditional array induction logging, which consists of 3 orthogonal transmitting coils and 3 orthogonal receiving coils [1]. The normal of the transmitter coil coincided with that of the receiver coil, which is called a coaxial coil system. The surfaces of the transmitter coil and the receiver coil are in a plane, which is called a coplanar coil system [2]. The horizontal and vertical resistivity of the formation can be obtained, that their ratio describes the formation electrical anisotropy characteristics accurately. These provide important basis for the correct evaluation of reservoir. The response of multicomponent induction logging tool in the cylindrically stratified show that the response of coplanar coil system is often more complex and more sensitive to the change in drilling liquid conductivity, invasion zone's conductivity, formation conductivity, and formation anisotropy, even cause sign flip in the response under many conditions, which can be solved by analytical method or semi-analytical method [3]. The inclined well model with borehole belongs to full 3D model, which can be determined by 3D numerical techniques [4]. In this paper, the diagonal conductivity tensor in the medium coordinate system is transformed into the complete conductivity tensor in the instrument coordinate system by the coordinate transformation theory. Based on the finite element method, the response characteristics of triaxial array induction logging are simulated and analysed in the inclined well model. The effect of borehole inclination angle and anisotropy
coefficient is discussed on logging response in anisotropic formation. Meanwhile, we identify the reasons for the difference in response characteristics between the coplanar system and coaxial system.

2. Inclined borehole in anisotropic formation

2.1 Basic theory of anisotropy

Consider a borehole electromagnetic induction system equipped with transverse coil or longitudinal coil located on the origin, the coil can be equivalent to a magnetic dipole source \( M \) because its radius is much smaller than its distance from the receiver coil when an alternating current passes through it during induction logging \(^5\). Figure 1 shows magnetic dipole on the origin with conductivity of the horizontal and vertical denoted by \( \sigma_h \) and \( \sigma_v \) respectively.

The electrical anisotropy means the different conductivity of the formation in different directions, including horizontal resistivity in the horizontal direction (its reciprocal is the horizontal conductivity \( \sigma_h \)) and vertical resistivity (its reciprocal is the horizontal conductivity \( \sigma_v \)) in the vertical direction. It is proved in the published classical literature that the magnetic moment consisting of a Hertz vector of a dipole emitter has the form of \( M = (M_x, M_y, M_z) \) \(^6\).

The magnetic moment of the horizontal Hertz vectors are \(^7\)

\[
M_x = M_y = M_z = 0
\]

The Hertz vector of horizontal magnetic dipole can be expressed as \(^7\)

\[
\Pi_x = \frac{M_x e^{i\lambda r}}{4\pi \lambda s} \\
\Pi_y = 0
\]

\[
\Pi_z = \frac{M_z}{4\pi \rho^2} \left[ \frac{\lambda z e^{ik_x s} - e^{ik_x r}}{s} - \frac{z e^{ik_y r}}{r} \right]
\]

The Hertz scalar of horizontal magnetic dipole is \(^7\)

\[
\Phi_x = \frac{M_x}{4\pi} \frac{i k_x x}{\rho^2} \left[ e^{ik_x s} - e^{ik_x r} + \frac{\rho^2}{r^2} \left( 1 - \frac{1}{i k_x r} \right) e^{ik_y r} \right]
\]

The field in which direction-y magnetic dipole is produced were similar to those of the direction-x magnetic dipole, so that won't be covered again here \(^8\). The magnetic moment of the vertical Hertz vectors are

\[
M_z = M_x = M_y = 0
\]

The Hertz vector of vertical magnetic dipole can be expressed as

\[
\Pi_x = 0 \\
\Pi_y = 0
\]
\[
\Pi_z = \frac{M_z}{4\pi} e^{ik_z r}
\]

The Hertz scalar of vertical magnetic dipole is
\[
\Phi_z = \frac{M_z}{4\pi} \frac{ik_z z}{\rho^2} \left( 1 - \frac{1}{ik_z r} \right) e^{ik_z r}
\]  
(6)

where \( \rho = \sqrt{x^2 + y^2} \), \( s = \sqrt{\rho^2 + z^2} \), \( \lambda_z = \frac{\sigma_h}{\sigma_v} \), \( r = \sqrt{\rho^2 + z^2} \), \( k_h^2 = i\omega \mu \sigma_h \), \( k_v^2 = i\omega \mu \sigma_v \).

\( \mu = 4\pi \times 10^{-7} \text{ H/m} \).

From Maxwell' equation, we know
\[
H = i\omega \mu_0 \sigma_h \Pi_m + \nabla \Phi
\]  
(7)

Through proper simplification, the following equations are obtained
\[
H_{xx} = \frac{e^{ik_z r}}{4\pi} \left[ \frac{i\omega \mu_0 \sigma_h}{\lambda s} + \frac{ik_z s - k_h x^2}{s \rho^2} - \frac{2ik_h x^2}{\rho^4} \right] - \frac{e^{ik_z r}}{4\pi} \left[ \frac{ik_h r - k_h^2 x^2}{r \rho^2} - \frac{2ik_h x^2}{\rho^4} + \frac{2ik_h x^2}{r^2} + \frac{(k_h^2 x^2 + 1)}{r^3} + \frac{3ik_h x^2}{r^4} - \frac{3x^2}{r^5} \right]
\]  
(8)

\[
H_{zz} = \frac{e^{ik_z r}}{4\pi r} \left[ i\omega \mu_0 \sigma_h + \frac{ik_h}{r} - \frac{(k_h^2 z^2 + 1)}{r^2} - \frac{3ik_h z^2}{r^3} + \frac{3z^2}{r^4} \right]
\]  
(9)

The correspondingly magnetic functional is given based on the variational principle. The solution satisfying the boundary condition will be obtained.
\[
F(H) = \frac{1}{2} \iint_V \left[ \frac{1}{\varepsilon} (\nabla \times H)(\nabla \times H) - k_h^2 \mu_0 H \cdot H \right] dV + \iint_{S_1} \left[ \frac{\gamma_h}{2} (\tilde{n} \times H)(\tilde{n} \times H) + H \cdot \nabla \right] dS - \iint_{S_2} H \cdot \left( \nabla \times \frac{1}{\varepsilon} J \right) dV
\]  
(10)

where \( \gamma_h \) and \( V \) are known parameter and vector, \( \tilde{n} \) is the external normal unit vector of the medium, \( J \) is impressed or source current , \( \varepsilon \) and \( \mu_0 \) are permittivity and permeability, respectively.

2.2 Coordinate transformation approach
The model of the triaxial array induction logging in 3D coordinate system is showed in Figure.2. The stratigraphic reference systems is \( xyz \), the tool reference systems is \( x'y'z' \) and the axis of the instrument is parallel to the axis- \( z'' \), the distance between transmitter and receiver is \( L \), the conductivity in all directions are \( \sigma_x \), \( \sigma_y \), \( \sigma_z \) respectively, the instrument is located in deviated borehole with inclination angle \( \alpha \). Suppose that the instrument rotates \( \beta \) by rolling during the measurement. The new coordinate system is adapted \( x'y'z' \), so \( z' \) and \( z'' \) coincide.
Figure 2. Induction logging tool model in 3D coordinate system

The rotation transformation matrix from the formation conductivity tensor spindle to the instrument reference coordinate system is

\[
R_\alpha = \begin{bmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{bmatrix}
\] (11)

The rotation transformation matrix from the instrument reference coordinate system to the current position of the instrument is

\[
R_\beta = \begin{bmatrix}
\cos \beta & \sin \beta & 0 \\
-\sin \beta & \cos \beta & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (12)

The formation conductivity in the instrument coordinate system is obtained from the stratum model in the usual geodetic coordinate system through two coordinate rotation transformations.

\[
\tilde{\sigma} = R^{-1} \eta R
\]

where

\[
R = \begin{bmatrix}
\cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\
-\sin \beta & \cos \beta & 0 \\
\sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha
\end{bmatrix}
\] (13)

3. Results and discussion

For the investigation of the influence of anisotropic on the coaxial system and coplanar system of the triaxial induction logging, we suppose that the model is composed of three layers, the middle thickness is 4m, the formation transverse and longitudinal conductivity are 1.0S/m, 0.25S/m, 1.0S/m and 1.0S/m, 0.1S/m, 0.5S/m, instrument dip angle are 0°, 15°, 30° and 45° respectively when the operating frequency is 13 kHz. Table 1 lists parameters for each subarray. Figure 3 and Figure 4 show the numerical results in the above model.

Table 1. Parameters of subarray in the triaxial induction logging tool.

| Subarray | Receiver L/m | Bucking receiver L/m |
|----------|--------------|----------------------|
| 1        | 0.1500       | 0.0750               |
| 2        | 0.2250       | 0.1125               |
| 3        | 0.3000       | 0.1500               |
| 4        | 0.3750       | 0.1875               |
| 5        | 0.5250       | 0.2625               |
| 6        | 0.6750       | 0.3375               |
| 7        | 0.9750       | 0.4875               |
| 8        | 1.8000       | 0.9000               |
From the results, we can see that the response curves almost coincide for the coaxial subarray in the isotropic formation, which are not affected by the angle of the instrument. The response curves are separated from each other in anisotropic formation, which is influenced by anisotropy and skin effect. The greater difference between vertical conductivity and horizontal conductivity, the longer the array spacing, the more obvious the curve separation. The apparent conductivity decreases with the increase of inclined angle.

Unlike the coaxial system, Figure 4 shows the apparent conductivity appears as obviously positive and negative horns on the horizontal interfaces, which are generated by accumulated charge near the boundary of the layer for the coplanar subarray. The horns appear and become more and more obvious with the increase of array space. The apparent conductivity increases with the increase of inclined angle.

The effect of curve splitting is similar to that of coaxial coil system, but it is different of inclination effect on apparent conductivity for different coil system configuration.

Figure 3. Anisotropy influence on coaxial system at different dip angle.
4. Conclusions
The coplanar component of triaxial array induction logging response is more complex than that of the coaxial component, which is generated by the accumulated surface charge at the boundary of the layer. Borehole inclination and anisotropy coefficient influence logging response in anisotropic formation simultaneously. The greater the anisotropy coefficient is, the more obvious the curve separation. The apparent conductivity decreases with the increase of dip angle, but that of the coplanar system increases. The different influence of subarrays also relates to their spacing.

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