ON THE STABILITY OF QUASI-EQUILIBRIUM SELF-GRAVITATING CONFIGURATIONS IN A TIDAL FIELD

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ABSTRACT

The possibility that quasi-equilibrium self-gravitating galaxy-like configurations exist in a tidal field is analyzed in this paper. More specifically, we address the question of how to predict initial configurations modeling galaxies that are able to survive environmental effects in a dense environment for roughly one Hubble time, provided that dynamical friction is neglected. For simplicity, the configurations in the tidal field have been taken initially to be spherically symmetric and to have an isotropic velocity dispersion tensor (\(t\)-limited King spheres); they orbit inside steady state, spherical halos, as those that presumably surround compact galaxy groups and galaxy clusters. Both circular and eccentric orbits have been considered. In both cases, the initial quasi-equilibrium configurations have been built up taking into account the external tidal field produced by the halo. It modifies the escape velocity field of the configuration, compared with isolated configurations. The survival of the configurations as they orbit inside the halos has been studied through \(N\)-body simulations. As a general result, it has been found that the bulk of the models are conserved along 12.5 Gyr of evolution and that the low rates of mass loss they experience are consistent with those expected when the adiabatic protection hypothesis is at work. So, solutions for galaxy configurations in tidal quasi-equilibrium have been found, showing that tidal stripping in quiescent phases does not seem to be very important, unless the density of the galaxy environment at its formation had been much lower than that of the galaxy environment at the point of its orbit where the tidal perturbation is maximum.

Subject headings: celestial mechanics, stellar dynamics — galaxies: kinematics and dynamics — methods: analytical

1. INTRODUCTION

Dense halos of dark matter were first detected in galaxy clusters and then in many galaxy groups, through observations of X-ray diffuse emission of the gas component surrounding the member galaxies (Ponman et al. 1996; Boehringer 1997). Also, some small dwarf galaxies are known to be orbiting inside the dark halos of larger ones (see, e.g., Mateo 1998). Interactions with such dense environments can produce several dynamical effects on these galaxies (i.e., tidal heating, mass loss from tidal stripping, energy loss from dynamical friction, among others) that could result in severe modifications, relative to isolated galaxies, of the evolutionary history of both individual galaxies and the systems they form. In fact, some characteristic times and scales playing relevant roles in astrophysical processes, such as the typical times for decay to the center of the halos, the rates of background enrichment by processed gas, and so on, could change appreciably, depending on the characteristics of the environment (see, e.g., Gunn & Gott 1972; Merritt 1985; Moore, Lake, & Katz 1998).

The modeling of environmental interactions has been mainly carried out through \(N\)-body simulations of the evolution of a theoretical galaxy model moving in an external field (Barnes 1985; Bode, Cohn, & Lugger 1993; Funato, Makino, & Ebisuzaki 1993; Bode et al. 1994; Garcia-Gómez, Athanassoula, & Garijo 1996; Athanassoula, Makino, & Bosma 1997 and references therein). One of the main shortcomings of galaxy models appearing in the literature is that, in most cases, galaxies are built up as if they were isolated. However, to properly quantify the effects of environmental interactions, it would be more convenient if the galaxy model, at the beginning of the simulation, took into account the external forces. Otherwise, it is difficult to disentangle which effects are effectively due to interactions and which ones are spurious, due to an incorrect choice of the initial galaxy model (see Gómez-Flechoso & Domínguez-Tenreiro 2001, in preparation, for a discussion). In this paper, we will focus on the choice of the initial galaxy model and on how this choice affects the later evolution of the galaxy as it orbits inside a halo. The effects of the dynamical friction and the interactions between galaxies in groups and clusters will be tackled in forthcoming papers.

King models (King 1965, 1966) provide a good framework to study equilibrium self-gravitating configurations. They are based on distribution functions that depend on the potential \(\Phi\) causing the forces felt by the constituent particles. So, if the body is isolated, the potential that enters in King distribution functions is \(\Phi = \Phi_\text{S}\), where \(\Phi_\text{S}\) is due to the mass distribution of the self-gravitating configuration (hereafter the satellite); however, when it moves through an external halo, then \(\Phi = \Phi_\text{S} + \Phi_\text{ext}\) must be used instead, where \(\Phi_\text{ext}\) is the potential causing the external force seen by the constituent particles. This force also determines the limiting or tidal radius of a spherical configuration, \(r_t\), that can be defined as the asymptotic distance at which constituent particles remain stably bound to the satellite. In spite of this, most King models found in the literature are constructed on the assumption that \(\Phi = \Phi_\text{S}\) and take \(r_t\) as a free parameter, even if the body is not isolated (Meylan & Heggie 1997; but see also Heggie & Ramamani 1995). The tidal radius is a fundamental parameter of King models describing spherically symmetric satellites, when, as is right, the effects of the external field are explicitly taken into account.

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account. These models will be termed hereafter $t$-limited King models.

The stability of self-gravitating configurations relative to tidal perturbations is equivalent to the stability of the orbits of its constituent particles. After the pioneering work by King (1962), the problem of tidal limitations imposed on such configurations has been studied in models provided by the circular and elliptical restricted three-body problems (Keenan & Innanen 1975; Jefferys 1976; Keenan 1981a, 1981b). When the configuration is on a circular orbit, the problem can be worked out in some detail, since the equations of motion of their constituent particles have one integral of motion, the Jacobi integral, $E_\phi$ that can be used to define zero-velocity surfaces in the configuration space. Stability against escape for a given orbit is assumed when the corresponding zero-velocity surface is closed (Spitzer 1987; note that the zero-velocity surface can be open, and, nevertheless, the particle does not escape over a given time interval). And so, the limiting radius, $r_l$, has been defined in terms of the distance, $x_o$, from the satellite center to the inner Lagrangian point of the potential $\Phi = \Phi_s + \Phi_{ext}$. However, in a tidal field zero-velocity surfaces are not spherically symmetric, and then an ambiguity arises when one intends to determine the radius of a spherically symmetric body embedded in a tidal field that does not have this symmetry. So different authors make different choices. King (1962) takes $r_l = x_o$, while Keenan (1981b) suggests that $r_l = 2x_o/3$ is preferable (see also Innanen, Harris, & Webbink 1983; Spitzer 1987; Lee 1990; Heggie & Ramaprilani 1995); these choices correspond to two semiaxes of the Roche surface.

No integral of motion exists when the satellite moves along a noncircular orbit, since the intensity of the tidal field changes periodically with time, being maximum at pericentric passage. The question then arises of whether or not the energies of most constituent particles are conserved to a good approximation so that local (i.e., depending on the satellite orbital phase) tidal radii could be defined, as in the circular motion case, which would translate into satellites in tidal quasi-equilibrium (note that, in the strict sense, equilibrium configurations do not exist in a tidal field because some degree of mass loss can never be avoided). We see that more complications are added on the theoretical side to the ambiguities appearing in the circular case. Observational data on the limiting radii of globular clusters cannot clarify the situation either, since no clear conclusion still has been reached on their possible dependence on the cluster orbital phase (Oh & Lin 1992; Oh, Lin, & Aarseth 1992; Meziane & Colin 1996; Brosche, Odenkirchen, & Geffert 1999). In any case, results on tidal equilibrium for globular clusters, where two-body heat conduction plays an important role, could be not valid for galaxy-like configurations, which are essentially collisionless systems.

The purpose of this paper is to deepen the understanding of tidal quasi-equilibrium for self-gravitating galaxy-like configurations, in particular to predict initial configurations for galaxy models that will survive environmental effects along roughly one Hubble time. The possibility of predicting self-gravitating spherical collisionless configurations in tidal quasi-equilibrium has been tested through Monte Carlo realizations of the $t$-limited King models. They are left to evolve in the corresponding external field, which has been described by an analytical expression. Various possibilities have been explored concerning the matching of the internal and external field of forces at the limiting radius of the galaxy model. Most of the previous works on tidal quasi-equilibrium referred to globular clusters moving in a galactic potential. Since here we are mainly concerned with the influence of dense environments (i.e., halos) on galaxy evolution, the halo density profiles used in our test are those of dark halos appearing in N-body simulations of hierarchically clustering universes (Navarro, Frenk, & White 1996), and the parameters of the galaxy models correspond to those of typical ellipticals.

The paper is organized as follows: in § 2 we give the general expression for the tidal field caused by a spherical static halo in the harmonic approximation. In § 3 we specify the models and parameters of halos, orbits, and galaxies used to make our study. The results of this study are presented in § 4. Finally, in § 5, the summary and conclusions of the work are given.

2. THE TIDAL FIELD CAUSED BY A SPHERICAL STATIC HALO: GENERAL EXPRESSION

As a first step to build up quasi-equilibrium initial configurations in a tidal field, in this section we derive the general expression for the tidal field caused by a spherical halo in the harmonic approximation. Let us consider a satellite of mass $M_s$ distributed according to a density profile $\rho_s(r,t)$ embedded in a static, spherically symmetric extended halo of total mass $M_H$ and density profile $\rho_H(R)$. The satellite is assumed to move on an orbit characterized by energy $E_H$ and orbital angular momentum per unit mass $L_H$, relative to an inertial system of reference. Let $R_s(t)$ and $V_s(t)$ be the instantaneous position and velocity vectors of the center of mass of the satellite, respectively, relative to an inertial system, $O_l$, with origin at the center of potential, $O$, of the halo. Relative to the center of mass of the system formed by both the satellite and the halo, the center of potential $O$ and the center of mass of the satellite $S$ have position vectors $R'_0 = \frac{-M_s/M_H}{1 + M_s/M_H} R_S = \mathcal{C}(M_s/M_H), R'_S = \frac{1}{1 + M_s/M_H} R_S = R_S + \mathcal{C}(M_s/M_H).$ (1)

Neglecting terms in $M_s/M_H$, the combined potential of the halo and the satellite has a spherical symmetry, and so $L_H$ is conserved in $S_O$. The satellite moves around the point $O$ with an instantaneous angular frequency $\Omega(t) = \frac{L_H}{R^2(t)}$ (which is constant for circular orbits). In a coordinate system, $S_O$, that rotates at an angular speed $\Omega(t)$ with respect to $S_O$, the equation of motion for the mass center of the satellite is

$$\frac{d^2 R_s}{dt^2} = -[\mathbf{\Omega} \times R_S + 2\mathbf{\Omega} \times \dot{R}_S + \mathbf{\Omega} \times (\mathbf{\Omega} \times R_S)] - V_H(R_s),$$

where the first, second, and third terms on the right-hand side of equation (2) are the inertial force of the rotation, the Coriolis force, and the centrifugal force at $R_s$, respectively, and the last term is the force caused by the mass distribution of the halo at point $R_s$

Let us now consider a bound particle $P$ belonging to the satellite, whose position relative to $S$ is $r$ and relative to $O$ is
$R_p (R_p = R_s + r)$. The equation of motion for $P$ in $S_0$ is
\[
\frac{d^2 R_p}{dt^2} = -[\Omega \times R_p + 2\Omega \times R_p + \Omega \times (\Omega \times R_p)] \\
- \nabla \Phi_H(R_p) - \nabla_r \Phi_S(R_p),
\]
where the last term is the force on $P$ caused by the mass distribution of the satellite.

From equations (2) and (3) the equation of motion of $P$ in the coordinate system, $S$, centered at $S$ that rotates with instantaneous angular speed $\Omega(t)$ with respect to $S_0$, is
\[
\frac{d^2 r}{dt^2} = -[\dot{\Omega} \times r + 2\Omega \times \dot{r} + \Omega \times (\dot{\Omega} \times r)] \\
- \nabla_r \Phi_S(r) - \nabla \Phi_H(R_p) + \nabla \Phi_H(R_s),
\]
and a series development around $R_s$ gives, at first order in $r/R_s$
\[
\frac{d^2 r}{dt^2} = -[\dot{\Omega} \times r + 2\Omega \times \dot{r}] - \nabla_r \Phi_S(r) - \nabla \Phi_{\text{tidal}}(r),
\]
where
\[
\Phi_{\text{tidal}}(r; R_s, \Omega) = \beta r^2 + (\gamma - \beta) \left( \frac{r \cdot R_s}{R_s} \right)^2,
\]
and
\[
\alpha = \frac{1}{2} \left[ \Phi_H(R_s) - \Omega^2 \right] \\
= 2\pi G \left[ \rho_H(R_s) - \frac{2}{3} \bar{\rho}_H(R_s) \right] - \Omega^2 / 2,
\]
\[
\beta = \frac{1}{2} \left[ \Phi_H(R_s)/R_s - \Omega^2 \right] = \frac{1}{2} \left[ \frac{4\pi G}{3} \bar{\rho}_H(R_s) - \Omega^2 \right],
\]
\[
\gamma = \frac{\Phi_H(R_s)}{2R_s} = \frac{2\pi G}{3} \bar{\rho}_H(R_s),
\]
with $\bar{\rho}_H(R)$ the mean halo density in a sphere of radius $R$. The inertial force of rotation and the Coriolis force cannot be put as the gradient of a scalar potential. Equation (6) tells us that the tidal potential $\Phi_{\text{tidal}}(r)$ can be written as a contribution, $\Phi_{\text{tidal}}(r) \equiv \beta r^2$, that gives rise to an isotropic radial force, $F_{\text{radial}}$, and contributions giving rise to forces in the direction of $R_s$ and $\Omega$. Taking in $S_s$ a Cartesian coordinate system with the $X$-axis pointing toward $R_s$, $e_x = R_s/R_s$, the $Z$-axis defined by $e_z = \Omega/R_s$ and $e_y$ such that $e_x \times e_y = e_z$, the tidal potential can be written as a quadratic form
\[
\Phi_{\text{tidal}}(x, y, z) = ax^2 + \beta y^2 + \gamma z^2,
\]
giving rise to forces in the three orthogonal directions (hereafter $F_{\text{radial}}$, $F_{\text{radial}}$, and $F_{\text{radial}}$, respectively). In Figure 1 we represent the intensity of the three components of the tidal force for one of the models studied in this paper (see § 3 and Table 3) as function of $R_s$. If the satellite is in circular motion, then $\Omega^2 = \Phi_H(R_s)/R_s$ and $\Phi_{\text{tidal}}(r) = 0$. In the general case, $\beta < 0 (\beta > 0)$ at the pericenter (apocenter) of the satellite orbit, while $\alpha < 0$ and $\gamma > 0$ anywhere in the orbit. So, the $F_{\text{radial}}$ force changes its intensity, being always disruptive (compressive), while the radial tidal force changes its sign and intensity as the satellite travels, being maximally disruptive at the pericenter and maximally compressive at the apocenter.

**TABLE 1**

| Halo Parameters |
|-----------------|
| $\delta_c$ (kpc) | $R_c$ (kpc) | $R_{500}$ (kpc) | $M_{500}$ ($10^{13} M_\odot$) |
|-----------------|
| Compact group (g) | $1.9 \times 10^3$ | 40 | 620 | 2.1 |
| Galaxy cluster (c) | $9 \times 10^3$ | 600 | 3300 | 174 |

**TABLE 2**

| Orbit Parameters |
|-------------------|
| $R_s$ (kpc) | $R_p$ (kpc) | $R_0$ (kpc) | $v(R_0)$ (km s$^{-1}$) | $T$ (Gyr) | $L_m$ (kpc km s$^{-1}$) | $E_H$ (km s$^{-1}$)$^2$ |
|-----------------|
| Galaxy cluster (c) | 120 | 120 | 120 | 964 | 0.75 | 115,680 | $-4.65 \times 10^4$ |
| Compact group (g) | 100 | 35 | 69.2 | 265 | 0.60 | 26,500 | $-5.24 \times 10^5$ |

Note—$R_s$ is the radius of the circular c-type orbit or the radius of the circular orbit with the same energy as the eccentric g-type orbit.
Defining an effective potential \( \Phi_{\text{eff}}(r; R_S, \Omega) \) as the total potential felt by the \( P \) particle,

\[
\Phi_{\text{eff}}(r; R_S, \Omega) = \Phi_0(r) + \Phi_{\text{tidal}}(r; R_S, \Omega),
\]

then the energy (Jacobi integral)

\[
E_1 = \frac{1}{2} \left( \frac{d\mathbf{r}}{dt} \right)^2 + \Phi_{\text{eff}}(r; R_S, \Omega)
\]

is not in general conserved:

\[
\frac{dE_1}{dt} = -\dot{\mathbf{r}} \cdot (\Omega \times \mathbf{r}) + \frac{d\Phi_{\text{eff}}}{dt}.
\]

Note that in the reference system \( S_0 \), \( \frac{d\Phi_{\text{eff}}}{dt} \neq 0 \) because \( R_S(t) \neq 0 \). Then, if the satellite is in circular motion, \( R_S(t) = 0, \Omega = 0 \) and \( E_1 \) is conserved along the trajectory of the \( P \) particle.

The angular momentum of the \( P \) particle is in general not conserved in \( S_0 \),

\[
\frac{dL}{dt} = -r \times (\Omega \times \dot{r}) - 2r \times (\Omega \times \dot{r})
\]

\[+ 2 \beta - \frac{\alpha}{R_S^2} (\mathbf{r} \cdot \mathbf{R}) \mathbf{r} \times \mathbf{R}, \]

\[+ 2 \beta - \frac{\gamma}{\Omega_0^2} (\mathbf{r} \cdot \Omega) \mathbf{r} \times \Omega,
\]

except for particles that move in radial trajectories along the \( \Omega \)-axis.

### 3. HALOS, ORBITS, AND GALAXIES

As an external field we have taken the potential due to a massive halo, with a mass distribution corresponding to a density profile given by

\[
\rho_{\text{crit}}(R) = \frac{\delta_c}{(R/R_c)(1 + R/R_c)^2},
\]

with \( \rho_{\text{crit}} \) the critical energy density corresponding to a flat geometry. This is an accurate analytical fit over 2 decades in radius and 4 orders of magnitude in mass to the equilibrium density profiles of dark matter halos that form in high-resolution \( N \)-body simulations in hierarchically clustering universes (Navarro et al. 1996). They are characterized by two parameters: a scale radius, \( R_c \), and a characteristic dimensionless density, \( \delta_c \), which in turn are correlated. Note that \( \overline{\rho}_{\text{crit}}(R) \) takes values in the interval \( (1.50, 3.17) \) for \( 0 \leq (R/R_c) \leq 2.5 \). This form of the profile has been chosen because we intend to analytically describe halos with different masses, so that different physical situations in which tidal forces play an important role can be considered globally. Specifically, in this work we will study orbits inside halos typically corresponding to galaxy clusters and galaxy compact groups (c and g halos, respectively). In Table 1 we give the particular parameter values we have used. These correspond to a mass inside the virial radius, \( M_{200} \) (the radius inside which the overdensity is 200), of \( M_{200} = 1.74 \times 10^{15} M_\odot \) and \( M_{200} = 2.1 \times 10^{15} M_\odot \), that is, about the typical mass of a galaxy cluster and a compact group, respectively. Note that the tidal field (and its radial gradient) produced by a compact grouplike halo, at its characteristic length (\( R_c = 40 \) kpc), is stronger than that produced by a galaxy cluster-like halo at its typical \( R_c = 600 \) kpc. Dark matter halos have been represented analytically by a continuous function because we are mainly interested in exploring the possibility that tidal quasi-equilibrium configurations are realized in nature and not in studying the effects of the dynamical friction force between the galaxy and the halo, which the fluctuating forces arising from their discrete character would cause. Otherwise, tidal and dynamical friction effects could not have been properly disentangled, as is often the case in the literature.

The definition of the tidal radius is more sound from a physical point of view when the satellite is in uniform circular motion (see § 4). So, as a first test, satellites have been put on circular orbits for halos corresponding to galaxy clusters (c-model). Small \( R_S \) values have been selected because the tidal effects are stronger in the central regions of the halo. Also, general orbits have been considered, in this case for compact grouplike halos (g-model). Parameters characterizing these orbits are listed in Table 2.

Concerning galaxies, numerical simulations of the gravitational collapse in a cosmological framework show that collapsed bodies are spherically symmetric in their relaxed central zones. So, as a simplifying hypothesis, we will assume that the satellite galaxy is spherically symmetric and has an isotropic velocity dispersion tensor. Note, however, that these symmetries will only approximately hold for satellite particles that move according to equation (5), particularly those whose apocenters lie in the outskirts of the configuration.

Initially, self-gravitating spherically symmetric configurations will be taken to be \( t \)-limited King spheres with an isotropic velocity dispersion tensor, i.e., King spheres with their tidal radius determined by the tidal field. They are based on the so-called King-Michie distribution function (DF), \( f(r, E) \), which is an approximately stationary solution to the Boltzmann equation with a Fokker-Planck collision term (Michie 1963; King 1965, 1966). These DFs are lowered Maxwellians, with a cutoff at the escape velocity to the border of the configuration for the less bound particles at each position. This escape velocity can be written as

\[
v_{\text{esc}}^2(r) = 2[K - \Phi(r)],
\]

where \( \Phi(r, R_S) \) is the total potential felt by the satellite particles and \( K \) is a constant defining the zero point of the potential. In terms of the shifted energy, \( \epsilon(r) = \Phi(r, R_S) + v^2/2 - K \), the King-Michie DF is zero for \( \epsilon(r) > 0 \), and for \( \epsilon(r) \leq 0 \) it is given by

\[
f(r, v) = k \exp \left[ W(r) - W_0 \right]
\]

\[\times \left\{ \exp \left( -j^2 v^2 \right) - \exp \left[ -j^2 v_{\text{esc}}^2(r) \right] \right\},
\]

where \( j^2 = 1/2\sigma_0^2 \), \( \sigma_0 \) is an approximation to the one-dimensional velocity dispersion at the center of the configuration, \( W(r) = 2j^2[K - \Phi(r, R_S)] \) is the dimensionless potential, \( W_0 \equiv W(0) \) is a parameter of the model, and \( k \) is a normalization constant. Standard King-Michie spheres

| Model | \( K \) or \( \alpha \) | \( \text{Kee or } \beta \) | \( \gamma \) | Monopolar |
|-------|----------------|----------------|--------|--------|
| c     | 22.69          | 15.28          | 13.75  | 15.28  |
| g     | 9.85           | 7.24           | 6.33   | 7.08   |
| g-R   | 15.78          | 10.56          | 9.85   | 10.80  |
| g'    | 25.82          | 16.10          | 15.74  | 16.99  |
take $\Phi(r, R_S) = \Phi_S(r)$, and they have $r_i$ as a free parameter. Other free parameters for these isolated spheres are the dimensionless central potential, $W_0$, the approximate central velocity dispersion, $\sigma_0$, the core radius, $r_0$, and the total mass, $M_S$. Note that only three of them are independent (see, e.g., Binney & Tremaine 1987).

In an external field, $r_i$ is determined by the external forces, and only two more parameters for t-limited King models are left free. To our knowledge, no description of the method to build up King spheres with a prefixed $r_i$ can be found in the literature, so we will briefly comment on it. First, we note that inside a spherically symmetric satellite we must have an isotropic potential. However, the tidal potential is not spherically symmetric; therefore we need to approximate the inner tidal potential field, $\Phi_{\text{tidal}}(r; R_S)$ (eq. [15]) to an isotropic field, $\Phi_{\text{radial}}(r; R_S)$ (see § 4.1 for a discussion on the approximation). Now, to build up these t-limited King spheres, the Poisson equation for the satellite potential, $\Phi_S(r, R_S)$, must be solved with a density given by the King-Michie DF (eq. [15]), which, on its turn, depends on the total potential $\Phi(r, R_S) = \Phi_S(r, R_S) + \Phi_{\text{radial}}(r; R_S)$,

$$\rho_S(r) = \int df(r, E) = \frac{\rho_0}{\Gamma(5/2, W_0)} \exp\left[ W(r) - W_0 \right] \Gamma[5/2, W(r)] ,$$

where $\rho_0 = \rho_S(0)$ and $\Gamma(\alpha, W)$ is the incomplete gamma function.

In order to solve the Poisson equation, appropriated boundary conditions have to be specified. First, as usual, the net force at the center of the configuration must vanish, $[dW(x)/dx]_{x=0} = 0$, and $W(0) = W_0$. Moreover, given a satellite of mass $M_S$, the tidal field fixes its tidal radius, and one must have $M(x_i) = M_S$ (or equivalently $W(x_i) = 0$), where $x_i = r_i/r_0$ and

$$M(x_i) = 4\pi r_0^3 \int_0^{x_i} x^2 dx \rho_S(W(x)) .$$

Once an orbit has been selected, the t-limited King models corresponding to the tidal field at different points in the orbit have been obtained. The values of the satellite mass, $M_S$, and core radius, $r_0$, used as input are $M_S = 2.2 \times 10^{11} M_\odot$ and $r_0 = 2.4$ kpc for galaxy models on c-type orbits, and $M_S = 1.3 \times 10^{11} M_\odot$ and $r_0 = 1.2$ kpc for galaxy models on g-type orbits. Different models for the tidal radius have been considered (see § 4). In Table 3 we give these tidal radii, and in Table 4 we give their corresponding $W_0$ and $\sigma_0$ values. Note that both $\sigma_0$ and $M_S$ are within the observationally allowed ranges for typical elliptical galaxies.

Once the velocity DF (given by eq. [15] with the corresponding parameters) and the density profile (given by eq. [16]) have been determined, a galaxy represented by a Monte Carlo realization of this velocity DF and density profile, with 10,000 particles, has been built up. Galaxies are nonrotating in the $S_8$ rotating frame, and so the tidal potential is time-independent as long as $R_S$ is constant along the orbit. These systems have been left to evolve during a time interval of 12.5 Gyr, under the gravitational forces caused by both the particle configuration and the dark matter halo. A tree code algorithm (Hernquist 1987), modified to take into account the external force caused by the density distribution given by equation (13) acting on each satellite particle $P$, has been used to integrate the motion equations. Note that the approximations discussed in § 2 are not used at this stage; these approximations are only used to build up the initial configurations of the self-gravitating satellites (see § 4). The neglect of the stochastic forces caused by the discrete character of the halo is not likely to substantially modify the results we obtain. The study of the dynamical friction effects must be made for each particular case; however, no important effects can be expected for both a cluster-like halo of $M_\odot = 1.74 \times 10^{15}$ (see Fig. 7 of Klypin et al. 1999) or a compact grouplike halo when the velocity dispersion of the orbiting galaxies is of the order of that of the halo itself (see Tables 2 and 4 and eq. [53] of Domínguez-Tenreiro & Gómez-Flechoso 1998, where it is shown that, when this is the case, the dynamical friction timescales can be considerably longer than those predicted by the popular Chandrasekhar 1943 formula).

### 4. RESULTS

#### 4.1. Satellite in Uniform Circular Motion

In this case $R_S$ and $\Omega$ do not change, and $\alpha = 2\pi G[\rho_S(R_S) - \bar{\rho}_H(R_S)]$, $\beta = 0$, and $\gamma = 2\pi G\rho_S(R_S)/3$. The energy $E_j$ and, consequentially, $t_j = E_j - K_j$ are integrals of the motion for the satellite particles as it orbits inside the halo. The gradient of the tidal potential, $V_j\Phi_{\text{tidal}}(r; R_S, \Omega)$, makes a contribution to $V_j\Phi_{\text{tidal}}(r; R_S, \Omega)$ that has the same sign as $V_j\Phi_{\text{tidal}}(r; R_S, \Omega)$ along the $z$-direction, while it has the opposite sign.
along the $x$-direction (see eq. [8]). The effective potential has a saddle point at positions $I_{k}^{\pm} = (\pm x_{e}, 0, 0)$ where

$$x_{e}^{3} = -\frac{GM_{S}}{2x(R_{S})}.$$ 

These points are Lagrangian points, where the net force on a satellite particle vanishes when terms of second order in $r/R_{S}$ or higher are neglected, as in equation (5). The equipotential surface $\Phi_{eff}(r, R_{S}, \Omega) = \Phi_{eff}(x_{e}, 0, 0); R_{S}, \Omega$ is the corresponding Roche surface; this is the last closed zero-velocity surface relative to the conserved Jacobi integral $E_{J}$.

As pointed out in § 1, the limiting radius $r_{l}$ is usually defined in terms of the $x_{e}$ distance; while King (1962) takes $r_{l, K} = x_{e}$, Keenan (1981b) proposes $r_{l, Kee} = 2x_{e}/3$. As a first step to study quasi-equilibrium configurations, we have built up $t$-limited King spheres, with limiting radii equal to both $r_{l, K}$ and $r_{l, Kee}$ and moving along a circular orbit (with parameters as specified in Table 2) inside a cluster-like halo, characterized by parameters as specified in Table 1. The zero point of the potential for these spheres has been taken to be

$$K(R_{S}) = \Phi_{eff}(x_{e}, 0, 0); R_{S}, \Omega] = -\frac{3GM_{S}}{2x_{e}(R_{S})}.$$ 

In this way we ensure that the zero-velocity surface for $x_{e} = 0$ particles (i.e., the less bound ones) is also the limiting surface of the configuration (ideally defined as the equipotential surface where the less bound particles with minimum angular momentum have their apocenters; see Gómez-Flechoso 1997 for a discussion).

To test that the results on the evolution of the galaxy models presented in this paper are free from two-body effects, the evolution of isolated King models, corresponding to the galaxy models in Tables 3 and 4, has been followed in control simulations. After 12.5 Gyr, no two-body effects have been detected in any case at a significant level. As an illustration, in Figure 2 we plot the evolution of the radii enclosing 75% and 95% of the initial mass of a King model, normalized to the corresponding tidal radius corresponding to each model. Several galaxy models, moving on either circular (c) or eccentric (g) orbits, are shown (see Tables 3 and 4) and also galaxy models that evolve in isolation (iso).

Let us now turn to the behavior of the galaxy models orbiting on circular orbits inside the halo. In Figures 2a and 2b we show the evolution of the radii enclosing 75% and 95% of the initial mass, $M_{S}$, for c-Kee and c-K galaxy models, normalized to the corresponding tidal radii. These radii are referred to the center of density of the galaxy for one c-model (Figs. 2a and 2b) and one g-model (Figs. 2e and 2f).

Another way to quantify the diffusion in position space is to analyze the evolution of $M(r, t)/M_{S}$, the mass inside radius $r$, normalized to the initial satellite mass, $M_{S}$. This is plotted in Figure 3 for the c-Kee model at $t = 0, 6.25$, and 12.5 Gyr. This figure indicates that at $t = 6.25$ Gyr, only 0.8% of the initially bound particles are beyond the tidal radius (a similar result is obtained for the c-K model). At the end of the simulation, 2.3% of the particles are at $r > r_{t}$. However, these minor changes result from two-body evolution, and, moreover, in any of the c-models, the innermost
volumes of the galaxies (inside a sphere of, say, $r/r_i \approx 0.3$) are absolutely not affected by the evolution.

Let us now analyze the evolution of the particle velocity distribution. The average velocity dispersion of particles that remain at the configuration does not appreciably change. In Figure 4a we represent $M(>v,t)/M_{d}(t)$, i.e., the fraction of particles hotter than a given $v$, versus $v$ normalized to the initial three-dimensional velocity dispersion at several times for the c-Kee prescription (escapers have not been included). No evolution is detected at a significant level for this model. The behavior of the c-Kee model is similar.

The anisotropy parameter for a given particle sample, $\beta_{an} = 1 - \sigma_{r}^2/\sigma_{\phi}^2$, where $\sigma_{r}$ ($\sigma_{\phi}$) is the tangential (radial) velocity dispersion for the sample in consideration, quantifies how much a given velocity distribution deviates from one with an isotropic velocity dispersion tensor, which would have $\beta_{an} = 0$. In Figure 5 we show the evolution of the $\beta_{an}$ parameter for configuration particles outside the radius enclosing 67% of the bound (i.e., excluding escapers) mass at each time. Both models exhibit similar amounts of anisotropy ($\approx 0$), indicating that the evolution of the external orbits (the most affected by the tidal field) is negligible.

The negligible amount of change along 12.5 Gyr of evolution indicates that both the c-K and c-Kee models are quasi-equilibrium configurations in the cluster tidal field. As explained above, $r_{K}$ and $r_{Kee}$ correspond to the semi-axes of the galaxy Roche surface along the $OX$ and $OY$ directions, respectively. To gain some insight into the physical basis of this quasi-stable configuration, let us recall that for a spherically symmetric system, the escape velocity to the border of the configuration at position $r$ can be written as

$$v_{esc}(r; R_S) = \left[2 |K(R_S) - \Phi_S(r) - \Phi_{\text{radial}}(r; R_S)| \right]^{1/2}$$

once the isotropic approximation $\Phi_{\text{radial}}(r; R_S)$ for the tidal potential is considered. A stable configuration needs to have zero escape velocity at its limiting radius, $v_{esc}(r_{\text{lim}}; R_S) = 0$. This condition ensures the stability of the configuration because the inner escape velocity field of the King model satisfies the boundary conditions imposed by the tidal field so that the effective potential vanishes at $r_{\text{lim}}$. Taking $r_{\text{lim}} = r_{K}$, the $v_{esc}(r_{\text{lim}}; R_S) = 0$ condition and equation (21) demand that at $r_{\text{lim}}$ the tidal potential is $\Phi_{\text{radial}}(r_{\text{lim}}; R_S) = \Phi_{\text{radial}}(r_{K}; R_S)$, with $\Phi_S(r)$ given by equation (19); taking $r_{\text{lim}} = r_{Kee}$, the tidal potential at $r_{\text{lim}}$ must be $\Phi_{\text{radial}}(r_{\text{lim}}; R_S) = \beta(R_S)r_{\text{lim}}^2 = \Phi_{\text{radial}}(r_{\text{lim}}; R_S)$ (0 for circular orbits). By continuity requirements, the limiting radii prescription in the c-K and c-Kee models can now be looked at as a result of two different choices of the $\Phi_{\text{radial}}(r; R_S)$ potential field under the condition that $v_{esc}(r; R_S) = 0$, namely, $\Phi_{\text{radial}}(r; R_S) = \Phi_{\text{radial}}(r_{K}; R_S) = \Phi_{\text{radial}}(r_{Kee}; R_S)$. A third possible choice for the $\Phi_{\text{radial}}(r; R_S)$ potential is to take $\Phi_{\text{radial}}(r; R_S) = \gamma(R_S)r^2$, which results, under the condition of zero escape velocity at $r_{\text{lim}}$, in a limiting radius, $r_{\text{lim}}$, corresponding to the semiaxis of the galaxy Roche surface along the $OZ$ direction. Hereafter the limiting radii satisfying the zero escape velocity condition for a given choice of $\Phi_{\text{radial}}(r; R_S)$ will be termed after this choice. In Table 3 (first row) we give the corresponding numerical values for models of the galaxy under consideration and of the halo and circular orbit as specified in Tables 1 and 2. Note that $r_{\text{lim}} > r_{K}$ (or $r_{\text{Kee}} > r_{\text{lim}}$). For completeness, in Table 3 we also give the limiting radii, $r_{\text{lim}}$, corresponding to the case where $\Phi_{\text{radial}}(r; R_S)$ is taken to be the monopole component of the tidal field potential expansion (eq. [6]) into spherical harmonics, namely,

$$\Phi_{\text{radial, M}}(r; R_S) = [\alpha(R_S) + \beta(R_S) + \gamma(R_S)]r^2/3.$$ 

The tidal radius $r_{\text{lim}}$ represents a kind of average radius of the Roche surface. The results described above concerning the near constancy of the configuration along 12.5 Gyr, both for the $r_{K}$ and $r_{Kee}$ choices, with mass losses compatible with that expected from two-body heating, suggests that the ambiguity in the precise prescription for $r_{\text{lim}}$ has no consequences, with the different choices being essentially equally good, provided that the initial configuration is a quasi-equilibrium one. This nonvarying character of quasi-equilibrium collisionless self-gravitating configurations as they move on circular orbits inside a tidal field can be easily understood in the framework of the adiabatic protection hypothesis (see § 4.2) because the time-independent character of the tidal field in the rotating $S_0$ frame would imply zero escape rates.
4.2. Satellite in General Motion

If the satellite is not in circular motion, the intensity of the tidal forces changes as the satellite orbits, being maximum (minimum) at pericenter (apocenter) passage (see Fig. 1 for an illustration). Moreover, in the $S_2$ frame, a force term in $F_0 = -\Omega \times r$ appears that cannot be expressed as the gradient of a potential; however, the disruptive effects of the tidal forces are more important than those of this term for almost all the satellite particles along the orbit and particularly so at pericenter where this term vanishes, so that it can be safely neglected in this work in what concerns the setup of the initial configurations (this term is not neglected in the numerical models, where, as explained above, the exact force produced by the halo on each satellite constituent particle has been considered).

Because $\Phi_{\text{eff}}(R_0)$ is not constant, neither $E_1$ nor $\varepsilon_j$ are conserved, as deduced from equation (11). The effective potential $\Phi_{\text{eff}}$ changes with a timescale given by the anomalistic period of the satellite, $T$ (i.e., the time interval between two successive pericenter passages).

When a particle of energy $E$ orbits inside a time-dependent potential with sidereal period $\tau$, its energy can be increased or decreased as time passes; in most cases, the average energy of a particle system in a time-dependent potential tends to increase (see Spitzer 1987). The relative energy change in a given orbit per revolution can be very small if $T/\tau > 1$. In fact, it has been shown (Kruskal 1962) that it goes to zero faster than any power of $T/\tau$, as for example, does an exponential function of $-AT/\tau$ with $A$ a dimensionless constant of order unity; see Spitzer 1987, provided that the $T$ and $\tau$ periods are not commensurate quantities, otherwise resonance phenomena can occur (Weinberg 1994). So, an energy gain occurs when $T < \tau$, and it would result in an orbit expansion. If this happens for an important fraction of the satellite constituent particle orbits, the satellite will expand and lose mass at a rate similar to the expansion rate. On the contrary, if $T > \tau$ for most of the constituent particles (or $T > t_{\text{dyn}}$, where $t_{\text{dyn}}$ is a dynamical time measuring an average period for the satellite constituent particles), then only moderate satellite heating can be expected; the system is said to be adiabatically protected. In any case, as mass-loss rates are similar to the expansion rate due to tidal heating, it can be expected that the relative mass losses of a galaxy with a dynamical timescale $t_{\text{dyn}}$ orbiting with a period $T$ go to zero approximately as $\exp(-AT/t_{\text{dyn}})$.

Now let us point out that only when mass losses are unimportant, that is, when the system is adiabatically protected, is quasi-equilibrium in a tidal field a physically sound concept; tidal quasi-equilibrium demands low mass loss as the satellite orbits. But in this case the energies of most satellite particles will be conserved to a good approximation, and then the equilibrium tidal radius can be calculated following the same physical reasoning as discussed in §4.1, namely, that $r_1(R_0)$ must be taken to be the radii of the Roche surface along the three axis or, equivalently, must be taken such that the escape velocity field of the King model satisfies the conditions imposed by the external tidal field ($v_{\text{esc}}[r(R_0); R_0] = 0$, with $v_{\text{esc}}(r; R_0)$ given by eq. [21] and with the different choices for the isotropic component of the tidal field as discussed in §4.1). Moreover, in this case, a new ambiguity appears concerning the matching procedure, since the external potentials and, consequently, $x_0$, depend on the satellite orbital phase $R_0$. The most natural choice is to take $R_{eq} = R_0$ ($R_{eq}$ is the point of the orbit where the initial configuration is built up) because the system suffers a kick at the pericenter passage. In this work other possibilities also have been considered to quantify how much the system evolution depends on the orbital point where the initial equilibrium configuration is built up: $R_{eq} = R_a$ (apocenter distance) and $R_{eq} = R_b$ (radius of the circular orbit with the same energy $E_a$ as the eccentric orbit under consideration). Table 3 summarizes the different possibilities that we have considered and gives the corresponding limiting radii $r_1^n$.

Note that several models in this table have similar $t_1$ values: (1) those with $r_1(R_{eq}^n) \approx r_1(R_p)(g-p-\beta, g-p-\gamma, \text{and } g-p-M \text{ models})$, (2) those with $r_1(R_{eq}^n) \approx 1.5r_1(R_p)(g-p-\alpha, g-R_0-\beta, g-R_0-\gamma, \text{and } g-R_0-M \text{ models})$, (3) those with $r_1(R_{eq}^n) \approx 2r_1(R_p) (g-R_0-\alpha, g-a-\beta, g-a-\gamma, \text{and } g-a-M \text{ models})$, and finally (3) the g-\alpha galaxy model whose $r_1(R_{eq}^n)$ is about 3.5$r_1(R_p)$. In Table 4 we give other galaxy parameters of interest (see §3) for the models in Table 3. In particular, an average dynamical timescale $t_{\text{dyn}} = (3\pi/16G\rho)^{1/2}$ (Binney & Tremaine 1987) for these models is given in this table.

Different models have been left to evolve as they orbit inside a compact grouplike halo (see Table 1) following an eccentric g-like orbit (see Table 2) during 12.5 Gyr. Accordingly, with the discussion above, it can be expected that the relative mass losses for these different models at any given time be approximately proportional to $\exp(-AT/t_{\text{dyn}})$ if adiabatic protection is at work in these simulations. To show that this is the case, in Figure 6 we have plotted $T/t_{\text{dyn}}$ versus the logarithm of $\Delta M_g/M_g$ after 12.5 Gyr of evolution for the g-model galaxies in Table 3, where $\Delta M_g$ is the total mass lost in escapers at 12.5 Gyr and $M_g$ is the initial satellite mass. A very good linear relation appears. It corroborates the adiabatic protection hypothesis. This hypothesis predicts zero mass losses for a galaxy on circular motion (because $T = \infty$), as we have found in §4.1.

Let us now describe different quantitative aspects of the satellite evolution. In Figure 7 we plot the $M_g(t)/M_g$ evolution for six g-models in Table 3 [$(M_g(t)$ is the bound mass at time $t$, defined as the mass of the system whose total internal
energy \( E < 0 \). Note that mass losses are quite linear and regular. No evidences of the perigalactic passages can be seen because of the small amount of mass loss at each passage. This figure is an illustration that g-models in Table 3 exhibit different degrees of mass loss according to the classification above: (1) those with \( r_t(R^3) \approx r_{t,p}(R) \) that lose very little mass in 12.5 Gyr of evolution (\( \approx 0.1\% \)); (2) those with \( r_t(R^3) \approx 1.5r_{t,p}(R) \) that have up to 5.8% of escapers; (3) those with \( r_t(R^3) \approx 2r_{t,p}(R) \), where the mass losses are not negligible (g- and g-\( \beta \) galaxy models with a 12.8% and 14.6% of escapers at 12.5 Gyr); and, finally, (4) the g-\( \alpha \) galaxy model, where mass losses are more significant (33.5%).

Figure 2 for the g-\( p, p, \beta \), g-\( p, \alpha \), g-\( \alpha, \beta \), and g-\( \alpha, \alpha \) galaxy models (Figs. 2c, 2d, 2e, and 2f, respectively) indicate that the mass loss is maximum for the g-\( \alpha, \alpha \) model, where the shell corresponding to 25% of the outsiders is lost at \( t \approx 7 \) Gyr, and minimum for the g-\( p, \beta \) galaxy, where the radius enclosing 95% of the inner particles had not yet crossed \( r_{t, \beta} \) at \( t \approx 12.5 \) Gyr. The g-\( p, \alpha \) and g-\( \alpha, \beta \) models show an intermediate behavior. Other models, not shown in the Figure 2 (see Table 3), exhibit different evolutionary trends according with the classification above. These trends are confirmed by Figure 3, where we can see that the inner regions of the configuration (say, \( r/r_* < 0.2 \)) are not very much affected by the tidal forces for g-\( p, \alpha \) model.

The analysis in the velocity space indicates that \( \sigma \) for particles that remain at the configuration does not change significantly; also, the percentage of particles in each velocity bin remains roughly constant along the 12.5 Gyr of evolution. All the g-models in Table 3 have a similar qualitative behavior, but heating is maximum for the g-\( \alpha, \alpha \) model and minimum and negligible for the g-\( p, \beta \) model. An average behavior is exhibited by the g-\( p, \alpha \) model (see Fig. 4b). The anisotropy evolution (Fig. 5) confirms these findings and is, for any g-model in Table 4, always indicative that particles on more elongated orbits are more likely to escape (Keenan 1981a, 1981b).

The results so far described indicate that once a quasi-equilibrium galaxy model is built up at a given point of its orbit, the configuration does not change appreciably as the galaxy orbits, except for mass losses. The system does not relax toward the different equilibrium solutions corresponding to the different points of its trajectory.

5. SUMMARY AND CONCLUSIONS

In this paper we address the issue of the existence of quasi-equilibrium self-gravitating configurations in a quiescent tidal field, that is, the possibility that self-gravitating configurations exist that are able to survive for roughly one Hubble time in a given dense environment without being tidally stripped or disrupted. More specifically, we have tried to answer the question of how to build up such configurations with their limiting radius determined by the tidal field and their remaining characteristics described by parameters that can be fixed a priori.

As the simplest hypotheses, the tidal field produced by a static, spherically symmetric, dense, extended halo has been considered. Also for simplicity, the configurations have been taken initially to be spherically symmetric and to have an isotropic velocity tensor (\( t \)-limited King spheres). They orbit inside the halos. Both circular and eccentric orbits have been considered. In both cases, quasi-equilibrium self-gravitating configurations have been built up by taking as tidal radii the radii of their Roche surface along different axes or, equivalently, by defining the escape velocity field of the configuration taking into account the requirements imposed by the tidal field produced by the external halo. The gravitational field inside the configuration is spherically symmetric, while the tidal field has no symmetry, and one has to resort to a choice of its radial component. So, an ambiguity arises when matching the internal and external fields of forces at \( r_t \). Different possible choices have been considered. In the case of an eccentric orbit, the tidal field depends on the orbital phase, and a new ambiguity arises regarding the orbital position where the matching is made. Here also different possibilities have been explored.

To study the survival of the configurations, we have evolved Monte Carlo realizations of \( t \)-limited King galaxy models, orbiting in the tidal field produced by a dark cluster-like halo (for circular orbits) or galaxy grouplike (for eccentric orbits) halo. These galaxies have been taken to be collisionless systems, i.e., such that heat transport and relaxation cannot proceed through two-body encounters but through the oscillations of their collective self-consistent potential.

The general result of the simulations, irrespective on how the matching has been made, is that the bulk of the models are conserved along 12.5 Gyr of evolution for both circular and eccentric orbits, even if some mass losses occur in some cases. A good linear relation between the ratio of the galaxy anomalistic period to its dynamical timescale, \( T/t_{\text{dyn}} \), and the logarithm of the relative mass losses for the different galaxy models has been obtained, suggesting that adiabatic protection is at work in these simulations. In the case of galaxies on circular orbits, the adiabatic protection hypothesis predicts zero mass losses; the results of our simulations for circular motion are also in agreement with this prediction, once the almost imperceptible two-body effects are considered. In the case of eccentric orbits, if the galaxy configuration closely corresponds to the tidal equilibrium solution at its actual environment, no important oscillations of the potential produced by tidal forces can be expected, and, in fact, our simulations show that once the satellite particles are distributed in positions and velocities according to the equilibrium solution at one given point of the satellite orbit, they approximately remain so as the satellite orbits, except for the marginally bound particles, in which case most of
them are lost to the configuration. Even if continuous mass losses are important in some cases, the system does not relax toward the equilibrium solutions corresponding to the different positions of the satellite, presumably because these different equilibrium solutions are not far enough to produce strong collective potential oscillations.

Configurations corresponding to equilibrium at pericenter, where the tidal forces are maximum, are those that suffer from less escape (they are hyperstable solutions at the other points of the orbit) and anisotropy development. Among the simulations presented in this paper, the maximum mass losses occur when the component of the tidal force in the \( R \) direction is chosen as radial component of these forces and the initial configuration is prepared at the apocenter (\( \approx 33.5\% \) along 12.5 Gyr). But, as stated above, it is mainly the \( T/\tau_{\text{dyn}} \) ratio that determines the evolution rates and mass losses.

The results described so far suggest that the configuration of spheroidal galaxies is fixed at its formation, determined by its mass, energy content, and the environment at that moment. After formation, only moderate tidally induced evolution can be expected for a galaxy living in environments of density similar to that of its environment at formation. These results also suggest that a continuous slow mass loss along long periods can occur, without destroying the system, if the density of the environment at formation is lower than that of the environment at galaxy pericenter passage. The galaxy will be easily destroyed, however, should it be placed at an external field whose corresponding equilibrium tidal radii is much smaller than the limiting radius of the actual galaxy.

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