Analysis of the surfaces and gravitational fields of planets using robust modeling methods

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Abstract. The work focuses on the development of methods and algorithms for the simulation of celestial bodies’ dynamic and static states. There are 2 types of models: global and local ones. The creation of local simulation models (LSM) is complicated as the emergent properties of the system are getting lost. Nevertheless, LSM are of great importance when analyzing planetology processes. When areas are covered with reference points unevenly and the distance between them increases, the method used in the work provides the higher accuracy of model forecasting compared to the use of interpolation formulas. LSM allow investigating both the whole planetary system and its local parts and are aimed at the predictive determination of topographic, gravimetric, and magnetic parameters.

1. Introduction

Analyzing structure and evolution of celestial bodies involves various statistical and multiparametric methods [1–3]. Currently, some of the promising directions of studying the structure, materials, and properties of various natural objects are robust methods.

It should be noted that analysis of complex physical systems using the robust method allows for an estimation of their parameters. Thus, the use of robust analysis in studying physical celestial objects and processes is an important direction [4, 5]. In these studies, the invariant properties of robust analysis are very important too. The inhomogeneity of nonlinear processes and complex topographic systems may be investigated by obtaining robust estimates of the required parameters.

Physical processes, including those related to the Earth, can be successfully studied using the robust analysis. In the future, the use of robust analysis for space measurements reducing will surely bring some interesting results which will allow for a solution of a number of space geodesy problems [6, 7].

2. Method of the multiparametric harmonic analysis

Using harmonic analysis of topographic data, obtained during space missions, expansion in a series of spherical functions, the topographic models of celestial bodies (Mars, Venus, Earth, Moon) were constructed [8]:

\[ h(\varphi, \lambda) = \sum_{n=0}^{N} \sum_{m=0}^{n} (c_{nm} \cos m\lambda + s_{nm} \sin m\lambda) \cdot \tilde{p}_{nm}(\cos \varphi) + \varepsilon, \] (1)

where \( h(\varphi, \lambda) \) is function of altitude; \( \varphi, \lambda \) are latitude and longitude (known parameters);
\( \bar{C}_{nm}, \bar{S}_{nm} \) are normalized harmonic amplitudes;
\( \bar{P}_{nm} \) are normalized associated Legendre functions;
\( \varepsilon \) is random regression error.

This equation was also used for the analysis of positional observations taken at Engelhardt Astronomical Observatory [9].

On the basis of gravimetric measurements, harmonic models of celestial bodies’ gravitational potentials were constructed [10]:

\[
U(\varphi, \lambda, r) = \frac{GM}{r} \left[ 1 + \sum_{n=0}^{\infty} \left( \frac{R_0}{r} \right)^n \sum_{m=0}^{n} (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \cdot \bar{P}_{nm}(\sin \varphi) \right] + \varepsilon, \tag{2}
\]

where \( G \) is the gravitational constant;
\( GM \) is gravitational field of Mars (42 828.314 km\(^3\)/sec\(^2\));
\( \varphi, \lambda, r \) are spherical coordinates of a reference point;
\( R_0 \) is equatorial radius of planet.

To assess these models, the advanced forecasting algorithm is used. Forecast module allows tracking lowering and elevation on the initial data, if 10 or more measurement points are involved.

Analysis of a local surface area model implies using a subroutine dividing harmonic of higher and lower orders.

Much attention in the development of the existing version of the Automatic System of Scientific Research (ASSR-2018) was given to the capabilities related to building graphic maps on the basis of the output data for visual forecasting during these simulations. ASSR-2018 includes the developed software for contour mapping with the scalability, the tone coloring of the isolines, and image export. The program shell of ASSR-2018 allow analysis the constructed model use user-friendly interface. Identification and evaluation of contour mapping accuracy are provided. There is also a 3D visualization mode.

Using the latest version of the software, a global relief model of the planetary surface [11], anomalies of the Earth’s gravitational field [12], a number of regional models of the gravitational anomalies of the planets [13], anomalies of the geomagnetic field, and distribution of the induced polarization of the regional sections are built.

To increase the data processing accuracy of this method from 2 to 10 times and to expand the capabilities of processing a large number of data, new production engineering is going to be used.

The cluster based on Gentoo Linux operating system together are used. For programming languages C/C++ and Fortran compiling programs GNU Compiler Collection and Sun Studio are used. Pascal compiling programs are Free Pascal Compiler and GNU Pascal. The software is free, i.e. installation, launch, use, study, extending, and improving of the programs are allowed. The dilated program cluster can have of three and more (to 13) the computing laboratories. In turn, each laboratory can have 13 analogous computers for parallel computing.

3. Using robust analysis for estimations planetary parameters

Within the regression modeling approach, the overdetermined system (1) was solved for various sources of hypsometric information. Along with the usual stages (postulating the models (1, 2) and the amplitudes \( \bar{C}_{nm}, \bar{S}_{nm} \) assessing), the approach involves using a number of quality statistics including external measures:

- the diagnostics of the basic LSM conditions observance. As LSM computational schemes, Gauss-Jordan and Householder ones were used;

- adaptation in case of their violation. The main violation of LSM application conditions to processing (1) is the presence of:
- surplus (noise) harmonics which lead to decrease in the accuracy of predicting single altitudes and isohypses;
- harmonic amplitudes correlating with each other, when (1) is applied to describe the relief on a part of the sphere or when the objects are distributed unevenly; in this case the digital model (a set of LSM-estimates amplitudes) should be considered incorrect.

The adaptation to these 2 violations was made possible by the application of step by step regression which is well-known procedure of regression analysis. It appeared to be efficient enough for the model (1) parameters if:

a. The objects are distributed over the entire sphere even though unevenly.
b. The order of expansion determined by the amount of points should be relatively small (n<15); otherwise, the calculation time increases rapidly. In case when the objects are distributed evenly over the entire sphere, it is only necessary to eliminate statistically insignificant harmonics and recalculate.

To obtain the expansion (1) in spherical harmonics in order to form a digital terrestrial model, an automated research system “SFERA” was used. “SFERA” is designed to distribute various properties (relief, gravitational, magnetic and other kinds of potential fields) description over the sphere and its parts by their values measured in the points with known coordinates. Using this software, the models of (1) type can be formed, predictions in the form of cross-sections, isolines, tones, and three-dimensional representation of property values distributions may be implemented. The models (1) formation is accompanied by their quality control and observance of LSM conditions. In case of their violation, the corresponding adaptation methods were applied. “SFERA” software in “split” mode may be used for models of higher orders during parallel processing. Owing to the fact of using the expansion in harmonic functions and other properties described above, the application of “SFERA” allows for 40% increase in description and prediction accuracy compared to the popular software package “SURFER”.

The corresponding internal criteria characterizing the estimation accuracy and statistical significance of both single coefficients and the whole model in general were determined simultaneously with the harmonic coefficients. The step by step regression procedure (inclusion-exclusion principle) at significance level α=0.05 (statistically significant dependence is used for observance of regression analysis and LSM estimation for the postulated and optimal model) was used to form an optimal structure of a model according to t-criterion (Student’s criterion). It was noted that coefficients values at N=40 almost coincided with the values at N=70. That confirms the correctness of basic calculations. The regression analysis assumptions (LSM) compliance led to the conclusions as follows: the model contained about 30% of statistically insignificant terms; paired correlation coefficients r<0.3, which indicates the practical orthogonality of the expansion; on the basis of residue analysis it was concluded that there had been some violation of normality conditions; Durbin-Watson criterion was 0.58 which meant there had been some autocorrelation of the first order. As a result, the digital model of the Earth was created.

In Automated System for Robust Modeling the innovative informational and mathematical production engineering were applied:

- SNOR (System of navigation of optimum regressions); it is intended to obtain an optimum model of the data handling used for the forecasting. SNOR has a rather rational structure including: 1) the control module; 2) the inquiry creation unit; 3) library of the functional procedures; 4) the scenario unit; 5) the system tuner; 6) the unit data editor; 7) the tables forming unit; 8) the quick reference.
- ASSE “Orb” (the Automated system of scientific examinations); the developed automated system is a specialized software for implementing the strategy of statistical (regression) modeling to solve a number of problems related to mathematical exposition of a landform, gravitational, and other potential fields of planets. The system’s main role is obtaining regression models of processes or appearances with their subsequent use to forecast the output characteristics (responses) and implement some control functions in interactive (display) and package operational modes. The need for an availability of the similar automated system is caused by a number of difficulties with the similar
operations implementation requiring both multivariant approach to calculations and application of various methods of parameters estimation and structural identification.

- PES (parametric estimation system); the developed software package is a specialized system for the implementation of the strategy of statistical (regression) modeling for solving the problem of parameters estimation. Its basic assignment is the creating parametric regression models. These models are not intended to forecast. Their arm is the description of the cause-effect relationships of the studied phenomenon.

- AC DRM (the Automated system of dynamic regression modeling). The developed software is intended to analyze samplings, indexes of processes (time series) varying in time. The correspondence to the prescribed accuracy (the compliance of mathematical modeling accuracy to measurement accuracy) is going to be achieved by application and development of the adaptive statistical (regression) modeling approach implying as follows:

  - At the postulation stage use of the model adequate to the observations as a first approximation (expansion in orthogonal and scalene basis’).
  - Application of the accurate Gauss-Markov computational scheme at the stage of the model parameters assessing.
  - For adapting to violations of Gauss-Markov conditions were used application of multifactor or dynamic model with the best linear estimators of some highly accurate methods and algorithms of a structural and parametric samplings (a heckle of Sampling, robust methods, a method of oblate estimates, etc.; exhaustive search of structures, regular and random search, Staircase Methods (Up-And-Down)).

4. Summary and conclusions
The developed methodology and software are important for planetary studies. Gravity measurements, revealing features of the gravitational field, play an important role in oil exploration on the Earth reducing its cost. The graphical fieldwork is a generalization of the gravitational anomaly maps whose construction is carried out using the presented software package with the feature of choosing one of the interpolation formulas. At the same time, the improvements of capacities of taking graphic measurements and generalizing mathematical models of the trend are intended to form a regional reference surface [14].

Prospects for the development of this system include the use of the new techniques based on sampling and ellipsoidal functions, the study of gravitational field of the Earth and other planets of the solar system, and the development of the ASSR-2018 new versions which are designed to effectively increase the speed and accuracy of the orthogonal expansion and elimination of noise components of the analyzed model within the adaptive modeling approach.

Let us also note that the solution of modern space research objectives requires the creation of the latest technologies and materials relying on the development of new manufacturing processes, software packages, and methods. It is required as follows [15]:

1. Update of the data on the solar system bodies and their internal structure, topography, etc.
2. Adding the feature of working with planetary data and telemetry applied in the international projects as well as with NASA, ESA, and JAXA standards, which is going to allow for a creation of the proper system interface.

As for the scientific aspect, the present paper deals with a number of interesting research problems of both fundamental and applied nature. A wide range of sciences are involved: physics, mathematics, space science, satellite control, etc. Working on this at Kazan Federal University (KFU), we are going to get colleagues from SAI MSU, MGTU of Bauman, MPTI (Moscow) involved. The themes of possible investigations include the theory of astronomical reading systems, geodynamics, terrestrial and planetary atmospheres, interplanetary plasma and its influence on distribution of radio and optical signals, theory of space flights, VLBI supervision, and many other branches of space studies. The updated database on all the components of space projects implementation and the corresponding
methods based on the new technologies are required for the successful work [16–18]. Besides, it is necessary to create analytical tools to work with the data (components, solutions, etc.).

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References
[1] Valeev S G 2001 Regression Modelling in Observations Treatment (Kazan: FAN) p 296
[2] Demin S A, Panischev O Y and Nefediyev Y A 2014 Kinemat. Phys. Celest. Bodies 30 63
[3] Demin S A, Panischev O Y and Nefediyev Y A 2015 J. Phys. Conf. Ser. 661 012003
[4] Nefediyev Y, Valeev S, Miikev R, Varaksina N and Andreev A 2012 Adv. Space Res. 50 1564
[5] Lapaeva V V, Meregin V P and Nefedjev Y A 2005 Geophys. Res. Lett. 32 L24304
[6] Nefedjev Yu A and Rizvanov N G 2002 Astron. Nachr. 323 135
[7] Rizvanov N G and Nefedjev Yu A 2005 Astron. Astrophys. 444 625
[8] Varaksina N Y, Nefediyev Y A, Churkin K O, Zabbarova R R and Demin S A 2015 J. Phys. Conf. Ser. 661 012014
[9] Sokolova M G, Nefediyev Y A and Varaksina N Y 2014 Adv. Space Res. 54 2415
[10] Valeev S G and Faskhutdinova V A 2008 Problems of Contemporary Science and Practice. Vernadsky University 2 64
[11] Sokolova M, Nefediyev Y, Sergienko M, Demina N and Andreev A 2016 Adv. Space Res. 58 541
[12] Usanin V, Nefedjev Y and Andreev A 2016 Adv. Space Res. 58 2400
[13] Sokolova M G, Kondratyeva E D and Nefediyev Y A 2013 Adv. Space Res. 52 1217
[14] Varaksina N Y, Nefediyev Y A, Churkin K O, Zabbarova R R and Demin S A 2015 J. Phys. Conf. Ser. 661 012015
[15] Nefedjev Y A and Nefedjeva A I 2005 Astron. Nachr. 326 773
[16] Nefediyev Y A, Bezmenov V M, Demin S A, Andreev A O and Demina N Y 2016 Nonl. Phen. Compl. Syst. 19 102
[17] Demina N Y, Andreev A O, Demin S A and Nefediyev Y A 2017 J. Phys. Conf. Ser. 929 012013
[18] Andreev A O, Demina N Y, Nefediyev Y A, Demin S A and Zagidullin A A 2018 J. Phys. Conf. Ser. 1038 012003