Low field critical current density of titanium sheathed magnesium diboride wires

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Abstract. Magnesium diboride (MgB\textsubscript{2}) is replacing some of the conventional superconductors due to its low cost and availability in kilometer lengths. MgB\textsubscript{2} has also been considered for AC applications. In order to model the AC losses and the critical currents of the applications, intrinsic \( J_c(B) \)-dependence is an important factor also at low fields. In this work \( J_c(B) \)-dependence of an MgB\textsubscript{2} sample is extracted from the standard in field voltage–current measurements. The proposed method is applied to a non magnetic titanium sheathed sample at 16 and 20 K and a simple formula for \( J_c(B) \) aligns with the measurements. In the fitting process, the critical current distribution inside the wire is numerically simulated in order to take the self field of the sample into account. Moreover, the same formula aligns with measurements of a different sample. These critical current measurements, performed at 4.2 K, were based on magnetization. In the self field computations, the superconducting cross section must be determined accurately. Therefore, we tailored an image processing tool for MgB\textsubscript{2} wires to obtain the geometry from a photograph.

1. Introduction
Since the superconductivity of magnesium diboride (MgB\textsubscript{2}) was discovered in 2001 [1], the pursuit for high performance conductors started and currently these conductors are commercially available in kilometer lengths. The advantages of these wires are relatively low price and their suitability for cryocooled applications whose temperature range is between 12 to 30 K [2]. Therefore, they are expected to replace some of the conventional superconductors. The wires are becoming more attractive also for AC use, which is due to the higher operation temperature, and thus improved stability compared to LTS wires.

In case of MgB\textsubscript{2}, there are plenty of suitable candidates for cheap matrix metals, which do not react with MgB\textsubscript{2}. However, they must also fulfill the mechanical, thermal, and electromagnetic requirements for a specific application. For AC applications, hysteretic materials, for example Iron and Nickel, are excluded. However, Titanium is a promising nonmagnetic sheath material, because of its mechanically excellent properties and the lightness [3, 4].

The AC losses depend on the intrinsic \( J_c(B) \)-characteristics of superconductor [5]. Unlike in DC applications, the \( J_c(B) \)-dependence has to be taken into account for the whole AC cycle, also at the low field values. In addition, \( J_c(B) \)-curves provide valuable information on the quality of superconductor and on its pinning properties [6]. Therefore, we apply and develop
a tool for non magnetic MgB$_2$ wires to determine $J_c(B)$-characteristics from standard voltage–current measurements. The method takes into account the sample’s self field, that is computed numerically. Similar approach has also been applied for BSCCO tapes [7], whose self field had been computed by FEM, and YBCO tapes [8], whose field had been computed based on integral element method [9].

First, we computed $J_c(B)$-dependence of Titanium based MgB$_2$ samples from the in-field critical current measurements. The cross section of the sample was determined directly from an image file with self written image processing algorithm. A simple model was found to describe $J_c(B)$-characteristics of the measured wires. Therefore, this model was tested with another monocore sample, which $J_c(B)$-curve was determined from magnetization measurements.

2. Numerical model

The numerical model aims to find $J_c(B)$-dependence from in-field critical current measurements. First, for the critical current density, we choose some general function $J_c(B, c)$. In addition to magnetic flux density, $B$, it depends on a finite number of parameters, that are the elements of vector $c$. For example, $J_c(B, c)$ can be Kim-Anderson model [10] or a linear combination of triangular functions.

The problem is to determine $c$ so that it fits to the measured data. This can be formulated as a minimization problem,

$$\min_c f(c) = \min_c \sum_i [I_{cm}(B_i) - I_{cc}(B_i, c)]^2,$$

where $B_i$ is the external magnetic flux density of $i$th measurement. $I_{cm}$ and $I_{cc}$ are the measured and computed critical currents, respectively. $c$ was solved numerically in the same way as described in reference [8].

Note, that in the equation 1, $J_c(B, c)$ is not explicitly written, because especially at low fields, the critical current of the sample can not be directly computed so that the cross sectional area of the superconductor, $S$, is multiplied by $J_c$. In fact, neither $B$ nor $J_c$ are constants at the sample’s cross section, and therefore, the current distribution was numerically computed and integrated to get $I_{cc}(B_i, c)$. In practice, during every step of the numerical optimization, the critical current distribution was computed for every $i$ based on the reference [9].

3. Wire model and image processing

The precise cross section of the superconductor, including the shape, is the key for the critical current density calculations. In order to determine the cross sectional geometry directly from an image file, we tailored an image processing algorithm for MgB$_2$.

From a gray scale image, MgB$_2$ can be distinguished from the matrix metals as dark areas. One way to determine the cross section, is to predefined a threshold, $\tau \in [0, 1]$, for an intensity value of a pixel and then classify the dark pixels, having the intensity value below $\tau$, as a superconductor [11]. Unfortunately, especially in the case of low quality images, the black spots due to the noise are often interpreted as superconductor as well as the inhomogeneities or cracks in the matrix metal. In addition, the dark background causes a problem, because it should be separated from the superconductor manually.

In order to cope with these problems, we used so called region growing technique [12], which benefit is that it assigns indices for all of the distinct and dark segments and thus they can be treated separately. In our case, the image’s background is dark, and thus the largest segment is neglected as well as the tiny ones.

At first, the gray scale image is resampled to lower resolution in order to adjust the number of elements. Then the distinct segments are located by the region growing subroutine. The
Figure 1. The stages of the image processing algorithm. (a) An original image. (b) First, the image was resampled to 200 times 200 resolution. (c) Then, the region growing subroutine was executed with $\tau = 0.3$. The dark segments are shown in gray color whereas black represents the boundaries of the segments. (d) Finally, the background and noise were excluded and the filaments were labeled. Here they had to cover at least 0.5% of the image, for the noise reduction, but less than 25%, for the background removal. The maximum deviation of the filament sizes is 6.2%. This Ni sheathed MgB$_2$ wire was used just as an example.

figure 1 illustrates the stages of the algorithm. As a side product, one gets the edges of the filaments, which can be later used as input for a mesh generator, for example. In addition, the fill factor and even the sizes of the individual filaments can be solved for the needs of the quality control. In the computations, presented later in this paper, we use rectangular mesh, which is provided directly by the image processing algorithm.

In order to scale the mesh to the correct size, the real width, $w$, and height, $h$, of the cross section, which is covered by the image, are needed. Here they were computed based on the sample’s cross sectional area, $S_w$, and the fraction of the background, $F_b$, as

$$
\begin{align*}
  w &= \sqrt{\frac{S_w}{(1 - F_b)}}, \\
  h &= aw
\end{align*}
$$

$F_b$ is provided directly by the region growing subroutine and the aspect ratio of the image, $a$,
Figure 2. The cross section of the Titanium sheathed wire. The colored area represents the computed self field critical current distribution at 16 K with the solved $J_c(B)$-dependence. The lowest current density of $4.1 \cdot 10^8$ A m$^{-2}$ is on the filament edges whereas the maximum of $5.1 \cdot 10^8$ A m$^{-2}$ is located in the middle of the filament.

which is its height divided by the width, was obtained from the image file.

4. Experimental and fitting examples

The transport critical currents were measured from a Titanium sheathed monocore wire (see figure 2), that was manufactured with ex-situ process. The sample was measured at 16 and 20 K and also at low fields from 0 to 200 mT. Therefore, the wire was stranded to have a small superconducting cross section, $S \approx 0.24 \text{ mm}^2$, because the maximum current of the measurement system is 500 A.

The measurement system is conduction cooled and the cryostat is located between two conventional copper magnets whose field was boosted with iron yoke. The accuracy of the external field was verified before each measurement with a Hall-probe that was attached close to the sample. It was difficult to mount the sample, because the solder does not stick directly to Titanium. The problem was avoided with an electrodeposited Copper coating. Finally, the Copper was removed with Nitric acid around the voltage taps. In these measurements, the challenge is to stabilize the temperature of the sample [13]. Here we used thermally conducting grease (see figure 3) and relatively high ramp rate of 3.9 A s$^{-1}$.

The critical current measurements were fitted to a computed curve based on a following function:

$$J_c(B) = J_{c0} \cdot \frac{1 - B/B_1}{(1 + B/B_0)^\alpha}.$$  \hspace{1cm} (3)

Here parameter, $B_1$, represents the field in which the transport current drops to zero and $J_{c0}$ is the critical current density at zero field. Parameters $\alpha$ and $B_0$ both describe the steepness of $J_c(B)$-curve, which makes them more sensitive to measurement error than the other parameters in the optimization. Function 3 resembles Kim-Anderson model [10], but has two differences: The use of exponent $\alpha$ allows a wide range of $J_c(B)$-dependencies from Bean model ($\alpha = 0$) to the original Kim model ($\alpha = 1$) [7]. Moreover, we assumed that $J_c(B)$ drops gradually to zero,
Solders for voltage taps

*Figure 3.* The sample holder for the voltage–current measurements. The current leads are separated with fiberglass to ensure that the current flows solely in the sample. The copper coatings enable the soldering.

**Table 1.** The fitted parameters for three samples annealed in different temperatures. Parameter $\alpha$ was fixed to 1.

|        | 700 °C | 800 °C | 850 °C |
|--------|--------|--------|--------|
| $J_{c0}$ [A cm$^{-2}$] | $1.29 \times 10^9$ | $3.15 \times 10^9$ | $9.44 \times 10^9$ |
| $B_1$ [T]     | 6.15   | 7.98   | 9.58   |
| $B_0$ [T]     | 0.653  | 0.557  | 0.359  |

whereas in HTS materials, which have very high upper critical field value, this approach is not needed. Also $J_{c0}(B)$-dependence is isotropic at the measured magnetic fields. This was based on reference [14], which reports minor anisotropy originated from the grain orientation due to the flattening of the wire, but in our sample, the aspect ratio was close to one.

The measurements and the computed values, based on the function with the optimized parameters, are summarized in figure 4. At 16 K the fitted $I_c(B)$-curve aligns with the measurements, but at 20 K, the expected self-field effect was not so visible and thus the fitting was difficult. Therefore, computations were repeated so that the first measurement point was neglected, and after that, this point was roughly 4 A higher than the computed value, whereas the theoretical critical current, which neglects the self field, was about 17 A higher.

High noise level in these measurements was expected, because non stabilized monocore wires are usually tricky to measure. In all of these measurements, the transition was very steep and the wires practically quenched right after the critical current was achieved. At the fields higher than 1.2 T, the transition was somehow visible. This was due to the low transport currents and thus reduced thermal effects. However, additional measurements are needed to scrutinize the error sources and to get an accurate overview on the critical surface. Also multifilament wires, with better stability and thus lower thermal effects, would give more precise answers. These kind of wires are required for AC applications in any case.

Equation 3 also qualified to model $J_c(B)$-curves based on magnetization measurements of three different samples at 4.2 K. In this case, the alignment was excellent as shown in figure 5. There is some difference at high fields, where the model predicts more pessimistic values than the measurements that are described in reference [15]. These samples were manufactured with ex-situ and in-situ mixture doped by SiC in Ti/Cu sheath. They were finally annealed in Argon at 700, 800, and 850 °C. Surprisingly, in the fitting process, the parameter $\alpha$ could be fixed to one, which corresponds directly to Kim-Anderson model. In these measurements, the self field contribution had been negligible, and thus the equation 3 was directly fitted with the parameter values listed in the table 1.

In the previous example cases, the direct comparison between the solved parameters is difficult because of the different measurement temperature and manufacturing process, which means that quality of grain-connectivity has been different. In these two cases, the parameter values differs
Figure 4. Measured (dots) and computed (solid line) critical currents. Part of the same data, where the self field matters, is magnified in the inset. Red extensions show theoretical critical currents without self field effect, which is $J_c(B) \cdot S$. The neglected measurement is highlighted with red dot. At 16 K, the parameters were $J_{c0} \approx 5.20 \cdot 10^8$ A m$^{-2}$, $B_1 \approx 3.55$ T, $\alpha \approx 0.261$, and $B_0 \approx 56.8$ mT. At 20 K, these values were $4.71 \cdot 10^8$ A m$^{-2}$, 2.91 T, 0.232, and 33.9 mT, respectively.

substantially. In the transport current case, due to the high temperature, it is obvious that the parameters, $J_{c0}$ and $B_1$, have lower values, but also $\alpha$ and $B_0$ are substantially lower. However for the both cases, equation 3 can be fitted to the measurements in the wide range of magnetic field and can be used as a basis for the AC loss computations.

5. Conclusions
This work showed a way to find an intrinsic $J_c(B)$-dependence of non magnetic MgB$_2$ wires. This approach is also applicable at low fields, where the sample's self field matters, because it was taken into account with numerical computations. The main idea was that the $J_c(B)$-dependence should produce the same computed critical current values that have been actually measured. However, some general $J_c(B)$-model, which depends on the finite number of parameters, had to be assumed first. As a starting point, we tried a simple four parameter function, from which the parameters were solved with numerical optimization. In this case, the computed critical currents aligned with the measured ones, which suggest that the function qualifies to describe $J_c(B)$-dependence for a wide field range. The critical currents were measured, between 0 and 1.8 T at 16 and 20 K, from the Ti sheathed monocore wire. Furthermore, the same function aligned also with three different Ti sheathed samples measured at 4.2 K. In this case, the critical current density was measured based on magnetization measurements.

The found $J_c(B)$-dependence can be later used as a basis for AC loss computations for the true cross section obtained from the image file. However, in these first experiments, the critical currents were measured from monocore tape, which was problematic to measure due to their weak thermal stability, especially at low fields. As a next step, we will study multi filament wires, which are better stabilized and suitable for AC use. These measurements should be repeated in numerous temperatures and they would help to get an overview on the critical surface of MgB$_2$ in the wires.
Figure 5. Critical current, based on the magnetization measurements (blue), aligns with the model (red) described by equation 3. Three samples were annealed in 700, 800, and 850 °C. The inset shows the difference, which can be better distinguished with a logarithmic plot.

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