Measures of azimuthal correlations in relativistic heavy ion collisions

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Based on the initial state geometrical symmetry for collisions between two identical heavy ions at high energy, the general form for the one- and two-particle azimuthal distributions is deduced. Relation between these distributions and the usual flow parameters is discussed. New measures for the azimuthal correlations are suggested. Some numerical results on the values of the measures are shown from an event generator for Au+Au collisions with different colliding centralities at 200 GeV.
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I. INTRODUCTION

Particle correlation is always one of hot topics of high energy physics for studying the interactions among the colliding particles. Among various correlations, azimuthal correlation is crucial in ultra-relativistic $pp$, $pA$ and $AA$ collisions for determining the event shape and extracting collective information about the produced medium, such as the equation of state [1]. In a collision between two identical nuclei with a finite impact parameter, the overlap region is approximately an oblong shape in the transverse plane. In an intuitive picture, if the interacting system reaches an approximate local equilibrium and expands according to (viscous) hydrodynamics, the geometrical elliptic asymmetry in the initial state will be transformed during the collective expansion into an asymmetry in the final state momentum distribution of the detected particles. The efficiency of this transformation is sensitive to medium collective properties such as viscosity. Thus the azimuthal distribution of the produced particles can tell us important information about the dynamics in the collisions. Since the first data were taken at RHIC, one of the most important experimental observations [2, 3] has been the azimuthal anisotropy of the detected particles. In particular, the large value of the so-called elliptic flow observable [4], which indicates strong collective behavior of the produced system, has been one of the most important and most frequently studied measurements. The measurement of the collective flow parameters provided one of the strongest pieces of evidence for the creation of a strongly-coupled, low-viscous QGP medium in these collisions. With other measurements, those parameters have the potential to provide tight constraints on models on extracting precise quantitative properties of the QGP, as well as to shed light on the non-equilibrium QCD dynamics of the initial stage of the collision, which are poorly understood up to now.

Because of fluctuations in the colliding system’s evolution and particle production processes and due to the lack of solid theoretical ground for calculating the correlation functions from first principles, our understanding of the correlations is quite limited up to now to some model analysis. Related with the azimuthal correlations, azimuthal flow effect is one of the centers in both experimental and theoretical studies of particle production mechanism and interactions of various components in the system.

In this paper, more measures for the collective flow effect are suggested from the two-particle azimuthal correlations. These measures are proposed based on some initial geometrical symmetry and thus are very general, independent of the interactions during the evolution of the produced partons and hadrons. Relation with the usual flow measurement is discussed also. Though the discussion focuses mainly on correlations in the mid-rapidity region, extension to other rapidity region is straightforward. The measures discussed here may be used to constrain theoretical models for relativistic heavy ion collisions. By using an event generator, some numerical results are obtained for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with different centralities.

This paper is organized as follows. In section II the most general form for the one- and two-particle distributions are deduced based on the (event averaged) geometrical symmetry. Section III is for numerical results for the coefficients in the distributions from a Monte Carlo model. Section IV is for conclusions and discussions.

II. GENERAL FORM OF ONE- AND TWO-PARTICLE AZIMUTHAL DISTRIBUTIONS

A. For the case with all particles in the same kinematic region

Let us first study particles in some prefixed kinematic region. Consider an event with $\tilde{N}$ final state particles in that kinematic region, for example with rapidity $|y| < 0.5$ and $p_T < 1 \text{GeV}/c$, with azimuthal angles $\varphi_1, \varphi_2, \ldots, \varphi_{\tilde{N}}$ in the reaction plane of the colliding system. The one- and two-particle azimuthal distributions for the event can be written as

$$\tilde{p}_1(\Phi) = \sum_{i=1}^{\tilde{N}} \delta(\Phi - \varphi_i),$$

(1)
\[
\bar{\rho}_2(\Phi_1, \Phi_2) = \sum_{i \neq j = 1}^{N} \delta(\Phi_1 - \varphi_i)\delta(\Phi_2 - \varphi_j). \tag{2}
\]

In the above expressions, a tilde is used to represent distributions fluctuating from event to event, including variations of multiplicity and azimuthal angles of particles in an event. These two distributions are normalized to \(\tilde{N}(\tilde{N} - 1)\), respectively,

\[
\int_0^{2\pi} d\Phi \bar{\rho}_1 = \tilde{N}, \quad \int_0^{2\pi} d\Phi_1 d\Phi_2 \bar{\rho}_2(\Phi_1, \Phi_2) = \tilde{N}(\tilde{N} - 1). \tag{3}
\]

From the definition of \(\bar{\rho}_2\), one can observe the exchange symmetry \(\bar{\rho}_2(\Phi_1, \Phi_2) = \bar{\rho}_2(\Phi_2, \Phi_1)\).

One can rewrite the above distributions as sums of infinite set of sine and cosine functions

\[
\bar{\rho}_1(\Phi) = \sum_{k=0}^{\infty} (\tilde{s}_k \sin(k\Phi) + \tilde{c}_k \cos(k\Phi)),
\]

\[
\bar{\rho}_2(\Phi_1, \Phi_2) = \sum_{k,l=0}^{\infty} (\tilde{s}_{k,l} \sin(k\Phi_1) \sin(l\Phi_2) + \tilde{d}_{k,l} \sin(k\Phi_1) \cos(l\Phi_2) + \cos(k\Phi_1) \sin(l\Phi_2) + \tilde{c}_{k,l} \cos(k\Phi_1) \cos(l\Phi_2)). \tag{5}
\]

with \(\tilde{s}_{k,l} = \tilde{s}_{l,k}, \tilde{d}_{k,l} = \tilde{d}_{l,k}, \tilde{c}_{k,l} = \tilde{c}_{l,k}\). From the normalization conditions in Eq. (3), one gets \(\tilde{c}_0 = \tilde{N}/(2\pi), \tilde{c}_{0,0} = \tilde{N}(\tilde{N} - 1)/(4\pi^2)\).

Theoretically and experimentally, one can investigate

\[
\rho_1(\Phi) = \sum_{k=0}^{\infty} (s_k \sin(k\Phi) + c_k \cos(k\Phi)),
\]

\[
\rho_2(\Phi_1, \Phi_2) = \sum_{k,l=0}^{\infty} (s_{k,l} \sin(k\Phi_1) \sin(l\Phi_2) + d_{k,l} \sin(k\Phi_1) \cos(l\Phi_2) + \cos(k\Phi_1) \sin(l\Phi_2) + c_{k,l} \cos(k\Phi_1) \cos(l\Phi_2)). \tag{7}
\]

Therefore, \(\rho_1\) and \(\rho_2\) can be rewritten as

\[
\rho_1(\Phi) = c_0 + \sum_{k=1}^{\infty} c_{2k} \cos 2k\Phi, \tag{10}
\]

\[
\rho_2(\Phi_1, \Phi_2) = \Delta_{0,0} + \sum_{k,l} \left[ 2\Delta_+^{k,l} \cos(k\Phi_1 + l\Phi_2) + 2\Delta_-^{k,l} \cos(k\Phi_1 - l\Phi_2) \right]. \tag{11}
\]

where \(\sum_{k,l}'\) means that the summation should be performed for all non-negative integers \(k\) and \(l\) with \(k + l\) an even number larger than zero, or in other words, non-negative \(k\) and \(l\) should be both odd or even in every term and at least one of them is positive. Eq. (11) is the usual expression for the inclusive azimuthal distribution, and the coefficients \(c_{2k}\) are related to the flow measures \(v_{2k}\) by \(v_{2k} = c_{2k}/(2c_0)\) for \(k = 1, 2, \cdots\). Because of the exchange symmetry \(\rho_2(\Phi_1, \Phi_2) = \rho_2(\Phi_2, \Phi_1)\) for pairs of particles in the same kinematic region, \(\Delta_{k,l}^+\) are symmetric under exchange of their indices

\[
\Delta_{k,l}^\pm = \Delta_{l,k}^\pm. \tag{12}
\]
FIG. 1. A schematic illustration for the geometric symmetry of the colliding system in the transverse plane at mid-rapidity or in the whole rapidity region. The $x$ axis is along the direction of the impact parameter.

Those $\Delta_{\pm l}^k$ contain more information on the azimuthal distributions of the final state particles than the flow parameters. Thus they are new measures for collective effect as

$$
\Delta_{2n,0}/\Delta_{0,0} = \langle \cos(2n\Phi_1) \rangle = v_{2n},
$$

$$
\Delta_{\pm l,0}^k/\Delta_{0,0} = \langle \cos(k\Phi_1 \pm l\Phi_2) \rangle.
$$

Here $\langle \cdots \rangle$ means average over the corresponding distributions.

It is interesting to note that $\Delta_{11}^+/\Delta_{00} = \langle \cos(\Phi_1 + \Phi_2) \rangle$ has been suggested in [2] as a quantity to detect the presence of $CP$-odd domains [2] in the deconfined QCD vacuum. Also, the new expressions, Eqs. (10) and (11) are in agreement with the latest results [3] on the absence of directed flow in the central rapidity region.

For azimuthal distributions in the forward/backward rapidity region or for collisions between two non-identical nuclei, $\rho_1(\Phi)$ and $\rho_2(\Phi_1, \Phi_2)$ satisfy the symmetry conditions

$$
\rho_1(\Phi) = \rho_1(-\Phi) \neq \rho_1(\pi + \Phi),
$$

$$
\rho_2(\Phi_1, \Phi_2) = \rho_2(-\Phi_1, -\Phi_2)
\neq \rho_2(\pi + \Phi_1, \pi + \Phi_2),
$$

then there are odd harmonic terms $\cos(2k + 1)\Phi$ in the Fourier expansions of $\rho_1$ and terms with odd $k + l$ in $\rho_2$. This can be concluded from results in [1].

Very often, one needs the distribution $P(\Delta \Phi)$ of $\Delta \Phi = \Phi_1 - \Phi_2$, which can be obtained without determination of the reaction plane. From the expression for $\rho_2$, one readily gets

$$
P(\Delta \Phi) = \int d\Phi_1 d\Phi_2 \rho_2(\Phi_1, \Phi_2) \delta(\Delta \Phi - \Phi_1 + \Phi_2)
= 2\pi \left( \Delta_{0,0} + \sum_{k=1}^{\infty} 2\Delta_{-k}^k \cos(k\Delta \Phi) \right).
$$

In this expression, $k$ can be odd and even. From this expression, one can see that generally $P(\Delta \Phi = 0) \neq P(\Delta \Phi = \pi)$, as observed experimentally [8]. For high-$p_T$ particles such phenomena have been explained in the framework of jet quenching [9]. By comparing coefficients in $\rho_2(\Phi_1, \Phi_2)$ and $P(\Delta \Phi)$, one can see easily that some correlation information in $\rho_2(\Phi_1, \Phi_2)$ is washed out in obtaining the $\Delta \Phi$ distribution from $\rho_2(\Phi_1, \Phi_2)$. However, the above expression is different from the usual parametrization for $P(\Delta \Phi)$ used by experimentalists, where $P(\Delta \Phi)$ was parameterized by only terms of $\cos 2k\Delta \Phi$ with $k$ an integer number. In our new expression, $\cos(2k + 1)\Delta \Phi$ terms are present. In consequence, the presence of those odd terms in $P(\Delta \Phi)$, $P(\Delta \Phi) \neq P(\pi - \Delta \Phi)$, $P(\Delta \Phi) \neq P(\pi + \Delta \Phi)$. These two consequences can be tested easily in experiments.

If there were no azimuthal correlations among the produced particles, the same information about flow would be contained in $\rho_2(\Phi_1, \Phi_2)$ and $p_1(\Phi_1)p_1(\Phi_2)$, then $\rho_2$ would be factorized. Such a factorization has been used experimentally to determine the elliptic flow coefficient $v_2$ [2]. Such a factorization may be expected when the soft particles are emitted from thermalized medium independently. If this factorization is true, one can get

$$
\Delta_{2k,2l}^+ = c_{2k}c_{2l}, \Delta_{2k+1,2l+1}^\pm = 0.
$$

In particular, for $k = l = 0$, the above condition reads $\langle N(N - 1) \rangle = \langle N \rangle^2$, thus the multiplicity fluctuation must be of Poissonian. Under the above condition, the $\Delta \Phi$ distribution could be obtained then from products of $p_1$’s as

$$
P(\Delta \Phi) = 2\pi \left( c_0^2 + \sum_{k=1}^{\infty} 2c_{2k}^2 \cos(2k\Delta \Phi) \right),
$$

with the coefficient ratios $c_{2k}^2/c_0^2 = v_{2k}^2$. The above equation has been used in [2] to measure the flow coefficients for Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV. However the validity of the above expression has never been proved. In fact, some experimental data have shown that some correlation variables are not the same at $\Delta \Phi = 0$ and $\pi$, as in the $\Delta \Phi$ dependence of the joint autocorrelations at $\Delta \eta = 0$ in [10], in the charged di-hadron distribution in the $\Delta \eta - \Delta \Phi$ plane in [11, 12], and in the correlation structure shown in Fig. 3 in [12]. From Fig. 3 in [12], one can see clearly that the correlation structure is not the same at $\Delta \Phi = 0$ and $\pi$. From the same figure, one can see also that even for $p + p$ collisions the correlation structure from PYTHIA simulation can not be well described by expressions with only $\cos 2k\Delta \Phi$ terms, like Eq. (19).

By comparing Eq. (19) with Eq. (17), one can see that terms with odd $k$ in the Fourier expansion of $P(\Delta \Phi)$, Eq. (17), are absent in the factorization scheme. Thus the presence of $\cos(2k + 1)\Delta \Phi$ terms in the $\Delta \Phi$ distribution disqualifies the factorization of the two-particle azimuthal angle distribution.
B. For the case with particles within different kinematic regions

Now we turn to the measure of correlations between two sets of particles selected from two different kinematic regions. As an example, let one set of particles come from low \( p_T \) (\( \leq 1 \text{GeV/c} \)) region (soft particles), another from high \( p_T \) (\( \geq 2 \text{GeV/c} \)) region (hard particles). The high \( p_T \) particles are frequently called triggers which can be neutral pions, photons, etc. Then one should use two one-particle distributions for the soft and hard particles, respectively. The azimuthal distributions are

\[
\tilde{\rho}_{1s}(\Phi_1) = \sum_{i} \delta(\Phi_1 - \varphi_{1i}), \quad \tilde{\rho}_{1h}(\Phi_2) = \sum_{i} \delta(\Phi_2 - \varphi_{2i}), \quad \tilde{\rho}_{2}(\Phi_1, \Phi_2) = \sum_{i} \sum_{j} \delta(\Phi_1 - \varphi_{1i}) \delta(\Phi_2 - \varphi_{2j}),
\]

where \( \tilde{N}_s \) and \( \tilde{N}_h \) are multiplicities of the soft and hard particles in an event.

One can perform Fourier decomposition to the distributions and obtain expressions similar to Eqs. (20) with coefficients fluctuating from event to event. After averaging over many events, smooth distributions similar to Eqs. (21) can also be obtained at mid-rapidity as

\[
\rho_{1s}(\Phi_1) = \frac{\langle N_s \rangle}{2\pi} \left( 1 + \sum_{k=1}^{\infty} 2\nu_{2k}^s \cos(2k\Phi_1) \right), \quad \rho_{1h}(\Phi_2) = \frac{\langle N_h \rangle}{2\pi} \left( 1 + \sum_{k=1}^{\infty} 2\nu_{2k}^h \cos(2k\Phi_2) \right), \quad \rho_{2}(\Phi_1, \Phi_2) = \frac{\langle N_s N_h \rangle}{4\pi^2} + \sum_{k=1}^{\infty} \sum_{l=0}^{L_\Phi(k)} \Delta_{k,l}^s \cos(k\Phi_1 + l\Phi_2) + 2\Delta_{k,l}^h \cos(k\Phi_1 - l\Phi_2)) .
\]

In the above equations, \( \langle \cdots \rangle \) represents average over many events.

Since these two sets of particles are from two different kinematic regions, no exchange symmetry can be expected for \( \rho_2 \). Because of the geometrical symmetry of the colliding system, the symmetry properties depicted in Eqs. (21) are still valid at mid-rapidity or for the whole rapidity region. Because of the absence of the exchange symmetry, \( \rho_2(\Phi_1, \Phi_2) \neq \rho_2(\Phi_2, \Phi_1) \), the coefficients in \( \rho_2(\Phi_1, \Phi_2) \) are not symmetric under the exchange of their indices, \( \Delta_{k,l}^s \neq \Delta_{l,k}^s \). Equations similar to Eqs. (23) and (24) can be written for the soft and hard particles.

Experimentally, the reaction plane in a nucleus-nucleus collision is not known and must be determined from the produced particles. Of course, the determined \( x \)-axis has an angle \( \Phi_{\text{rec}} \) relative to the true \( x \)-axis. Then for each final state particle, the azimuthal angle detected can be related to the true value \( \phi_1 \) by \( \phi_{1\text{, exp}} = \phi_1 - \Phi_{\text{rec}} \).

From this relation, the averages of \( \cos(k\Phi_1 \pm \Phi_2) \) from experiments can be written as

\[
\langle \cos(k\Phi_1 \pm \Phi_2) \rangle_{\text{exp}} = \langle \cos(k\Phi_1 \pm l\Phi_2 - (k \pm l)\Phi_{\text{rec}}) \rangle .
\]

If one assumes that the experimentally determined reaction plane is distributed symmetrically around the true one and accurate enough (see Ref. [13] for the state of the art on determining reaction plane and the flow coefficients), one has

\[
\langle \sin((k \pm l)\Phi_{\text{rec}}) \rangle = 0, \quad \langle \cos((k \pm l)\Phi_{\text{rec}}) \rangle > 0 .
\]

Then

\[
\langle \cos(k\Phi_1 \pm \Phi_2) \rangle_{\text{exp}} \propto \langle \cos(k\Phi_1 \pm l\Phi_2) \rangle .
\]

The two-particle correlation function has the same form as from our theoretical consideration. Thus one can observe similar two-particle correlation distributions from experimental data. The same conclusion can be claimed for the single-particle azimuthal distributions Eqs. (10), (23) and (24).

III. NUMERICAL RESULTS FROM A TRANSPORT MODEL

The above discussions are based on the geometrical symmetry properties of the colliding heavy ion systems, thus should be valid for both \( \text{Au+Au} \) and \( \text{Pb+Pb} \) collisions at all colliding energies and centralities. Before real experimental data is available to test the above conclusions, one can use a Monte Carlo event generator for producing “the experimental data” and calculating the coefficients in the relevant expressions.

For the purpose of generating “experimental data”, a transport model, AMPT [14], is used to generate \( \text{Au+Au} \) collision events at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) with different centralities. The AMPT model is a multi-phase transport model [17], which is constructed to describe nuclear collisions. It includes initial partonic and final hadronic interactions, and the transition between these two phases. The model consists of four main processes: the initial conditions, partonic interactions, conversion from partonic matter to the hadronic matter and the hadronic rescattering. There are two kinds of AMPT model: the default AMPT and the AMPT model with string melting. HIJING model [15] provides the initial momentum and spatial distribution of minijet partons and soft string excitations. Parton scattering in the AMPT model is implemented by using the ZPC model [16]. In the default version of AMPT model, partons are recombined with their parent strings when they stop interacting, and Lund string fragmentation model is used to convert the resulting strings to hadrons. In the string melting version, a quark coalescence model is used to combine partons into hadrons. The dynamics of hadronic matter is modelled by a relativistic hadronic transport model (ART) [17].
The string melting version of AMPT model was used in our study, since it can give a reasonable description of flow of Au+Au collisions \cite{18,21}. With the increase of impact parameter, the number of produced particles in an event decreases, thus more events need to be generated for the average of quantities to reduce the statistical fluctuation. In our calculation, about four hundred thousand to four million events are generated from the AMPT model for different centralities (from 10% to 90%) for the analysis in the following. In the analysis, only charged hadrons within rapidity region $|y| < 1.0$ are considered. Among those particles, hadrons with $p_T < 1\text{GeV}/c$ are referred as soft particles, and those with $p_T > 2\text{GeV}/c$ as hard particles. From the azimuthal angles of soft and hard particles one can calculate, for pairs of soft-soft, hard-hard and soft-hard in every event, contributions to the $\Delta_{k,l}$ for any chosen non-negative integer $k, l$, and then obtain the coefficients discussed in the last section.

From the generated events, the flow coefficients $v_n$ for $n = 2, 4, 6$ and the correlation coefficients $\Delta_{k,l}^{\pm}$ for particle pairs are calculated. For the soft particles, the transverse momentum averaged flow coefficients are $v_2 = 0.039$, $v_4 = 0.0019$, $v_6 = 1.02 \times 10^{-4}$ while $v_1 \approx v_3 \approx 0$ for centrality 30-40%. The results for $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle = \Delta_{k,l}^{\pm}/\Delta_{0,0}$ at the same colliding centrality are tabulated in TABLE I. Comparing $\Delta_{k,l}^{\pm}/\Delta_{0,0}$ and $v_k v_l$, one can see clearly that the magnitude of the later is much smaller than the former, indicating that the factorization of $p_2(\Phi_1, \Phi_2)$ into product of $p_1$ is not satisfied. $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle = \Delta_{k,l}^{\pm}/\Delta_{0,0}$ for soft particles at other colliding centralities are shown in Fig. 2 as functions of the impact parameter $b$.

If all the selected particles are produced from jets or within the high $p_T$ region, one can obtain expressions and exchange symmetries for the two-particle distributions in the same form as for the soft particles, considering the geometrical symmetry of the colliding system. For this case, however, two-particle distribution can never be expressed as product of two one-particle distributions, since jets are produced in pairs almost back to back and particles in one jet are correlated. The azimuthal asymmetry for high $p_T$ particles comes from the interaction of jets and the produced medium, or in other words by jet quenching \cite{9}. When the number of jets is huge in almost every event, the back-to-back correlation plays an unimportant role, then the factorization may be valid approximately.
With the events generated with AMPT, the flow coefficients $v_{2n}$ for $n = 1, 2$ and $3$ and $\Delta_{k,l}^\pm$ in the Fourier expansion of the two-particle distribution for hard particles can be calculated, as for the soft particles considered in the above. We get $v_2 = 0.115$, $v_4 = 0.00107$, and $v_6 = 1.37 \times 10^{-3}$ for hard particles at colliding centrality 30–40%. The corresponding values of $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle = \Delta_{k,l}^\pm / \Delta_{0,0}$ at the same centrality are tabulated in TABLE II. Values of $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle$ for hard particles are much larger in magnitude than products of $v_n$, as for soft particles. Also $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle$ for hard particles are much larger than those for the soft particles discussed in the above. $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle = \Delta_{k,l}^\pm / \Delta_{0,0}$ for hard particles at other colliding centralities are shown in Fig. 5.

With the choice of soft and hard particles as in the above, the coefficients $\Delta_{k,l}$ for soft-hard correlation in Eq. 24 are calculated from the events generated by using AMPT, as used in the above. The results for $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle = \Delta_{k,l}^\pm / \Delta_{0,0}$ for centrality 30–40% are tabulated in TABLE III, and are shown in Fig. 4 for other centralities.

![Figure 4](image_url)

**Fig. 4.** (Color online) Centrality dependence of $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle$ for different combinations of $(k,l)$ in Eq. (11) for soft-hard particle correlation function in Eq. (25). The soft and hard particle correlation functions can be calculated, as for the soft particles considered in the above. We get $v_2 = 0.115$, $v_4 = 0.00107$, and $v_6 = 1.37 \times 10^{-3}$ for hard particles at colliding centrality 30–40%. The corresponding values of $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle = \Delta_{k,l}^\pm / \Delta_{0,0}$ at the same centrality are tabulated in TABLE II. Values of $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle$ for hard particles are much larger in magnitude than products of $v_n$, as for soft particles. Also $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle$ for hard particles are much larger than those for the soft particles discussed in the above. $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle = \Delta_{k,l}^\pm / \Delta_{0,0}$ for hard particles at other colliding centralities are shown in Fig. 5.

![Figure 5](image_url)

**Fig. 5.** $\Delta \Phi$ distributions for soft-soft, hard-hard and soft-hard particle pairs in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at centrality 30–40%.

**Table II.** $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle$ for $k + l \leq 6$ in Eq. (11) for all final state hard hadrons in the $p_T$ larger than 2 GeV/c for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV with centrality 30–40%.

| $(k,l)$ | $\langle \cos(k\Phi_1 - l\Phi_2) \rangle$ | $\langle \cos(k\Phi_1 + l\Phi_2) \rangle$ |
|--------|---------------------------------|---------------------------------|
| (1, 1) | $-1.01 \times 10^{-4}$          | $-6.43 \times 10^{-4}$         |
| (1, 3) | $3.88 \times 10^{-6}$          | $-1.66 \times 10^{-4}$         |
| (2, 2) | $2.38 \times 10^{-2}$          | $1.32 \times 10^{-2}$          |
| (1, 5) | $-2.70 \times 10^{-4}$          | $-8.75 \times 10^{-6}$         |
| (2, 4) | $3.12 \times 10^{-3}$          | $1.15 \times 10^{-3}$          |
| (3, 3) | $8.53 \times 10^{-3}$          | $2.37 \times 10^{-4}$          |

**Table III.** Averages of $\langle \cos(k\Phi_1 \pm l\Phi_2) \rangle$ for $k + l \leq 6$ for soft-hard particle correlation function in Eq. (25). The soft and hard particles are the same as in TABLE I and II, respectively.

| $(k,l)$ | $\langle \cos(k\Phi_1 - l\Phi_2) \rangle$ | $\langle \cos(k\Phi_1 + l\Phi_2) \rangle$ |
|--------|---------------------------------|---------------------------------|
| (1, 1) | $-9.25 \times 10^{-4}$          | $9.58 \times 10^{-5}$          |
| (1, 3) | $-7.58 \times 10^{-5}$          | $5.23 \times 10^{-6}$          |
| (1, 5) | $5.47 \times 10^{-6}$          | $9.22 \times 10^{-7}$          |
| (2, 2) | $7.38 \times 10^{-3}$          | $4.47 \times 10^{-3}$          |
| (2, 4) | $1.04 \times 10^{-3}$          | $4.68 \times 10^{-3}$          |
| (3, 1) | $-2.85 \times 10^{-5}$          | $-1.38 \times 10^{-5}$         |
| (3, 3) | $1.29 \times 10^{-3}$          | $9.92 \times 10^{-7}$          |
| (4, 1) | $4.89 \times 10^{-4}$          | $2.14 \times 10^{-4}$          |
| (5, 1) | $7.86 \times 10^{-6}$          | $7.01 \times 10^{-6}$          |
particle pairs, one can plot the $\Delta \Phi$ distributions $P(\Delta \Phi)$ in the same figure for the three sets of particle pairs. The results are shown in Fig. 5. Because of the fact that $\Delta_{k,k}$ for soft-soft and soft-hard pairs are very small compared to $\Delta_{0,0}$, the $\Delta \Phi$ distributions for those pairs are quite flat. For hard-hard particle pairs, it is a quite different case. A peak can be seen at $\Delta \Phi = \pi$ in the distribution. This is not surprising, because when a trigger particle is found with a high $p_T$, it is much more possible that hard particles appear in the opposite direction in $\phi$, because jets are produced almost back-to-back in heavy ion collisions. Such a correlation can survive through the averaging process over many collision events. The asymmetry of $P(\Delta \Phi)$ is not as obvious as observed in [2, 3], because in this paper the transverse momenta for the triggers and the associated particles are much smaller and there are flat contributions from soft particles.

IV. SUMMARY

In this paper, the general form of two-particle azimuthal distribution is studied for high energy heavy ion collisions. New variables are suggested for describing the collective behavior in the final state of the collisions, and the azimuthal correlation function is re-expressed in terms of those collective variables. Connection between those variables and the well-studied flow parameters is discussed. Some numerical results on the variables are presented for Au+Au collisions at \( \sqrt{s_{NN}} = 200\text{GeV/c} \) at different centralities from an event generator AMPT.

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