D2 brane as the wormhole and the number of the universes

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Abstract
We construct wormhole-like solutions in the type IIA string theory. These solutions represent wormholes in four-dimensions and are given by the D2 branes within appropriated backgrounds fields. We present the conditions on these fields which lead to the four-dimensional wormholes. In the special case we show how the particular solution in the type IIA theory leads to the dynamic wormhole. We also speculate about the number of universes and the cosmological constant.

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1 Introduction
Generally, a wormhole is constructed by imposing the geometrical requirement on spacetime that there exists a throat but no horizon. Thus the wormholes can be interpreted as connections between different universes or topological handles between distant parts of the same universe. Hence the spacetime with the wormholes has the nontrivial topology. The conception of the wormhole was first introduced by Einstein and Rosen in 1935 [1]. Moreover there are proposals how to distinguish (astrophysical) black holes from wormholes [2, 3]. It is known that wormholes usually cannot occur as classical solutions of gravity due to violations of the energy conditions [4-8] thus the non-exotic matter produces only spacetime with trivial topology.

It is known that the source of backgrounds which have geometries with non-trivial topology are space-time extended objects called branes. Hence one can try to obtain the wormhole solution in such backgrounds. Among the branes are D-branes which are the degrees of freedom in string theory. On the classical level a D-brane is a submanifold \( \mathcal{M} \) on which the ends of the string terminated. Hence the D-brane corresponds to the Dirchlet boundary conditions that have been put on the open string. The dimensions of the D-branes depend on the string theory considered. From the other side the branes are solutions of the low energy approximation of the string theory by a field theory (supergravity).
this approximation the equations of motion involve fields from Neveu-Schwarz (NSNS) and Ramond-Ramond (RR) sectors: the graviton, the dilaton and the other various antisymmetric tensor fields. The solutions of these equations form backgrounds in which the string and D-branes are propagating. In the supergravity approximation of the string theory the evolution of a D-brane is described by the non-linear Dirac-Born-Infeld (DBI) action which consists of the background fields which are pulled-back on the world-volume of the D-brane. Hence the evolution of such D-branes depend on these fields and the internal geometry of the D-brane. Thus the D-brane is propagating in the background of branes and fields which are field theoretic solutions. We ask: can the equations governing the D-brane evolution be interpreted as the equations for the wormhole in four dimensions? We will show that such an interpretation is possible for some backgrounds. As a result we will obtain the Lorentzian wormhole. In string theory the Euclidean wormholes were considered in \[9-12\].

The aim of this paper is to relate four-dimensional wormholes to the solutions of string theory. We present backgrounds with branes which lead to the wormholes in four dimensional spacetime.

In section 2 we recall the construction of the wormholes and equations of motion. In section 3 we reproduce these equations from DBI action and consider a type IIA string background with a D2 brane. This D2 brane can be interpreted as the dynamic wormhole. As an example we consider a special background which leads to the wormhole. In section 4 we speculate about the wormholes, the number of universes and the observed value of the cosmological constant \( \Lambda \simeq 1.21 \times 10^{-52} [1/m^2] \) in the observed universe. Section 5 is devoted to conclusions.

## 2 Wormholes

In order to obtain a space-time with a wormhole one does a so-called surgery procedure (see \[4, 5\]). In the four dimensions one starts by taking two copies \( M_4 \) of the same spacetime and remove a four dimensional region \( \Omega \) from the one \( M_4 \) and from the second \( M_4 \). As a result one obtains two spacetimes \( M^+ \) and \( M^- \). These spacetimes \( M^+ \) and \( M^- \) are joining together along the three-dimensional boundary \( \partial \Omega = \Sigma = \mathbb{R}^1 \times \Sigma_2 \), where \( \Sigma_2 \) is a two-dimensional surface, so a new spacetime

\[
M = M^+ \cup_{\mathbb{R}^1 \times \Sigma_2} M^-
\]

is obtained. In the general case the linear size \( \rho \) of \( \Sigma_2 \) is time-dependent \( \rho = \rho(\tau) \), where \( \tau \) is the proper time on \( \Omega \). The spacetime \( M \) is the manifold without boundary that has a “kink” in the geometry at \( \rho(\tau) \). The joining can be made in two ways:

1. if one joins together the two external regions \( r \in (\rho(\tau), \infty) \), then the result is a wormhole spacetime \( M_w \) with two asymptotic regions.
2. if one joins together the two internal regions \( r \in (0, \rho(\tau)) \), then the result is a closed baby universe \( M_b \).
In the case of the wormhole spacetime $M_w$ with the spherical symmetry the surface $\Sigma_2$ is two dimensional sphere with a radius $\rho$ and $\Sigma_2$ satisfies the flare-out condition: in both sides of $M_4$, surfaces of constant $r$ increase their areas as one moves away from $\Sigma_2$; thus one says that $M_w$ has a throat at $r = \rho$, which can be time-dependent. So the topology of the dynamic wormhole $S$ is:

$$S \approx \mathbb{R}^1 \times \Sigma_2.$$  

As is well-known the spherically symmetric Lorentzian wormhole, a la Morris and Thorne [4], is defined through the specification of two arbitrary functions $b( r)$ and $\varphi ( r)$ with a line element $ds^2 = g_{\mu \nu} dx^\mu dx^\nu$:

$$ds^2 = -e^{2\varphi(r)}dt^2 + \frac{1}{1-b( r)/r}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \quad (2.1)$$

Hence the evolution of the wormhole $S$ is determined by the Einstein equations projected on $\mathbb{R}^1 \times \Sigma_2$, that is, by the Lanczos equations:

$$-[K_{\alpha \beta}] + \{K\} \tilde{g}_{\alpha \beta} = 8\pi G S_{\alpha \beta}, \quad (2.2)$$

where on the left hand of the equation the symbols have meaning: $\tilde{g}_{\alpha \beta}$ is the metric on $S$, the bracket $[K_{\alpha \beta}]$ denotes the jump: $[K_{\alpha \beta}] = K_{\alpha \beta}^+ - K_{\alpha \beta}^-$ across the hypersurface $\mathbb{R}^1 \times \Sigma_2$ and $[K] = \tilde{g}^{\alpha \beta} [K_{\alpha \beta}]$, where $K_{\alpha \beta}^\pm$ is the extrinsic curvature tensor of $S$ in $M^\pm$. On the right side of the eq. (2.2) $S_{\alpha \beta}$ is the surface stress-energy tensor on $S$.

In the special case the metric (2.1) takes the form:

$$ds^2 = -V^2 (r) dt^2 + V^{-2} (r) dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad (2.3)$$

and the wormhole $S$ is the 3-dimensional spacetime with the topology: $\mathbb{R}^1 \times S^2$ which is embedded in $M_4$ as follows:

$$X : \mathbb{R}^1 \times S^2 \rightarrow S \subset M_4 \quad (2.4)$$

where the embedding $X$ is:

$$X( \tau, \theta, \phi) = (t( \tau), \rho( \tau), \theta, \phi) \in M_4, \quad (2.5)$$

the time-like coordinate $t$ and the radius $\rho$ of the sphere $S^2$ depend on proper time $\tau$ and $(\theta, \phi) \in S^2$. The induced metric $\tilde{g}_{\alpha \beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu \nu}$ on $S$ is (where $\alpha = \tau, \theta, \phi$):

$$\tilde{g}_{\tau \tau} = -t V^2(\rho(\tau)) + r^2 V^{-2}(\rho(\tau)),$$

$$\tilde{g}_{\theta \theta} = \rho^2(\tau), \quad \tilde{g}_{\phi \phi} = \rho^2(\tau) \sin^2 \theta \quad (2.6a)$$

and "dot" means differentiation on $\tau$ e.g. $\dot{\rho} = d\rho/d\tau$. The manifold $S$ has three unit tangent vectors $T(\alpha) = \partial_\alpha X^\mu \partial_\mu$: one is time-like: $T(\tau) = \partial_\tau X^\mu \partial_\mu$:

$$T(\tau) = \frac{1}{V^2(\rho)} \sqrt{V^2(\rho) + \rho^2 \frac{\partial}{\partial \tau} + \rho \frac{\partial}{\partial r}},$$

$$T(\tau) \cdot T(\tau) = -1, \quad (2.7)$$
and other ones are space-like:

\[\begin{align*}
T_\theta &= \frac{1}{\rho} \frac{\partial}{\partial \theta}, \quad T_\phi = \frac{1}{\rho \sin \theta} \frac{\partial}{\partial \phi}, \\
T_\theta \cdot T_\theta &= T_\phi \cdot T_\phi = +1.
\end{align*}\] (2.8)

The condition \(T_\tau \cdot T_\tau = -1\) leads to that

\[\dot{t}^2 = V^{-1} \left( V^2 + \rho^2 \right)\] so the induced metric \(\bar{g}_{\alpha\beta}\) is:

\[d\bar{s}^2 = \bar{g}_{\alpha\beta} dy^\alpha dy^\beta = -d\tau^2 + \rho^2 (\tau) \left( d\theta^2 + \sin^2 \theta d\phi^2 \right).\] (2.9)

There are two unit normal space-like vectors \(n_\pm = \pm n_\mu \partial_\mu\) to \(S\), one is inward (belongs to \(M^+\)) and second is outward (belongs to \(M^-\)):

\[\begin{align*}
n_\pm &= \pm \left( \frac{\dot{\rho}}{V^2 (\rho)} \frac{\partial}{\partial \tau} + \sqrt{\frac{V^2 (\rho)}{\rho^2}} \frac{\partial}{\partial \rho} \right), \\
n_\pm \cdot n_\pm &= +1, \quad n_\pm \cdot T_\tau = n_\pm \cdot T_\theta = n_\pm \cdot T_\phi = 0.\end{align*}\] (2.10)

It means that \(S\) is the time-like submanifold. In the orthonormal frame \(e_\mu^a\) the metric (2.1) takes the form:

\[ds^2 = e_\mu^a e_\nu^b \eta_{\mu\nu} dx^\mu dx^\nu,\] (2.11)

where the tetrad field \(e_\mu^a\) is:

\[e = \left( e_\mu^a \right) = diag \left( V, V^{-1}, r, r \sin \theta \right),\] (2.12)

and \((\eta_{\mu\nu}) = diag (-1, +1, +1, +1)\). The inverse tetrad \(e_\mu^a\) has the components:

\[e^{-1} = \left( e_\mu^a \right) = diag \left( V^{-1}, V^{-1}, r^{-1}, r^{-1} \sin^{-1} \theta \right).\] The function \(V (r)\) is positive for \(r > r_h > 0\) thus the radius of the throat \(\rho (\tau)\) of the wormhole is bigger then the biggest positive root of \(V (r)\). It means that the classical wormhole has not horizon. The eigenvalues of the operator of the second fundamental form \(b_\perp (\cdot) = -\nabla n\) of \(S\) give the matrix of the extrinsic curvature \(K^a_\mu\) in the orthonormal frame \(K^a_\mu = K_{\mu\nu} e_\mu^a e_\nu^b\). Since in our case are two normal vectors \(n_\pm\) thus we get two extrinsic curvature \(K^a_\pm\) jumping across \(S^2\):

\[b_{n\pm} (T_\alpha) = K^a_\pm T_\alpha,\] (2.13)

where \(a = \tau, \theta, \phi\). In order to obtain \(K^a_{\tau\tau}\) one can notice that for the metric (2.2) there is the time-like Killing vector \(V = \partial_\tau\) and the four-vector acceleration is proportional to the normal vector \(n: \nabla T_\tau T_\tau = An\). Thus on \(S\) one gets :

\[K^a_{\tau\tau} = -A = -\frac{1}{V^2 n_\pm^a d\tau} \left( V^2 T_\tau^\tau \right).\] (2.14)
For the above tangent and normal vectors the extrinsic curvature $K_{\hat{a}\hat{b}}^{\pm}$ is:

$$K_{\hat{a}\hat{b}}^{\pm} = \mp \frac{1}{\rho} \frac{d}{d\tau} \left( \sqrt{V^2(\rho) + \rho^2} \right) = \mp \frac{1}{2} \frac{\partial_{r}V^2}{\sqrt{V^2(\rho) + \rho^2}}. \quad (2.15)$$

One can rewrite $K_{\hat{a}\hat{b}}^{\pm}$ as follows:

$$K_{\hat{a}\hat{b}}^{\pm} = \left( \pm \frac{1}{\rho} \sqrt{V^2(\rho) + \rho^2} \right). \quad (2.16)$$

The surface stress-energy tensor $S_{\alpha\beta}$ (for a perfect fluid) in the orthonormal frame is:

$$(S_{\hat{a}\hat{b}}) = \text{diag} (\sigma, \eta, \eta), \quad (2.19)$$

where $\sigma$ and $\eta$ are an energy density and a surface tangential pressure, localized on $S$, respectively. Hence the Lanczos equations (2.2) take the form:

$$\frac{1}{\rho} \sqrt{V^2(\rho) + \rho^2} = 2\pi G\sigma \quad (2.20)$$

and

$$\frac{d}{d\tau} \left( \rho \sqrt{V^2(\rho) + \rho^2} \right) = 2\pi G\eta \frac{d}{d\tau} \left( \rho^2 \right). \quad (2.21)$$

From these equations one gets relation:

$$\sigma + \frac{\rho d\sigma}{2 d\rho} = \eta. \quad (2.22)$$

From the other side the above equations are obtained from the Hilbert-Einstein action using the thin shell formalism [5-7]. In this formalism the Riemann tensor in the vicinity of the thin shell given by the equation $W(x) = 0$ has the form:

$$R_{\mu\nu\rho\sigma} = - \left[ (K_{\mu\rho}) n_{\nu} n_{\sigma} - (K_{\nu\rho}) n_{\nu} n_{\sigma} + (K_{\mu\sigma}) n_{\mu} n_{\rho} - (K_{\nu\sigma}) n_{\mu} n_{\rho} \right] \delta (W) + \theta (W) R_{\mu\nu\rho\sigma}^{W} + \theta (-W) R_{\mu\nu\rho\sigma}^{W}. \quad (2.23)$$
Hence the Hilbert-Einstein action with a matter field $\Phi$ gives:

$$
S \left[ g, \Lambda, \Phi \right] = \sum_{a=\pm} \int_{\mathcal{M}(a)} d^4x(x) \sqrt{-g(a)} \left[ \frac{1}{2\kappa} \left( R(a) (g) - 2\Lambda(a) \right) + L_{(a)} \left( \Phi(a), g(a) \right) \right] + \\
+\frac{c^2}{8\pi G} \int_{\partial M = \{W(x)=0\}} d^3x \sqrt{g(3)} \text{Tr} \left( [K] \right).
$$

(2.23)

In our case $\partial M = S$ and $R(a) (g) - 4\Lambda(a) = -\kappa T^{(a)}$ thus the above action is:

$$
S \left[ g, \Lambda, \Phi \right] = \sum_{a=\pm} \int_{\mathcal{M}(a)} d^4x(x) \sqrt{-g(a)} \left[ \frac{1}{\kappa} \Lambda(a) - \frac{1}{2} T^{(a)} + L_{(a)} \left( \Phi(a), g(a) \right) \right] + \\
+\frac{c^2}{4G} \int d\tau \rho^2 (\tau) \left[ \frac{d}{d\tau} \sinh^{-1} \left( \frac{\rho}{V(\rho)} \right) + \partial_\rho V^2 \frac{dt}{d\tau} + \frac{4}{\rho} \sqrt{V^2(\rho) + \rho^2} \right].
$$

(2.24)

where $1/(2\kappa) = c^4/(16\pi G)$ and $T$ is the trace of the energy-momentum tensor for the matter field: $T = g^{\mu\nu} T_{\mu\nu}$. The boundary term integrated by parts gives:

$$
S \left[ g, \Lambda, \Phi \right] = S_m - \frac{c^2}{G} \int d\tau \rho \sinh^{-1} \left( \frac{\rho V(\rho)}{\rho} \right) - \rho \sqrt{V^2(\rho) + \rho^2} - \frac{1}{4} \partial_\rho V^2 \frac{dt}{d\tau}.
$$

(2.25)

The last two terms can be rewritten as follows:

$$
\rho \sqrt{V^2(\rho) + \rho^2} - \frac{1}{4} \partial_\rho V^2 \frac{dt}{d\tau} = \left( \rho + \frac{1}{2V^2} \partial_\rho V^2 \right) \sqrt{V^2(\rho) + \rho^2}.
$$

(2.26)

The special form of $V$ is considered in [8].

In the next section we will be interpreting the wormhole as a D2 brane with the Dirac-Born-Infeld (DBI) action in the background fields of the type IIA string theory. It means that D2 brane connects two copies of the four dimensional spacetime $M_4$. From the other side the consistent spacetimes obtained in the type IIA are Minkowski or anti-de Sitter. Thus the form of $V^2$ is: $V^2 = +1$ for Minkowski and $V^2 (r) = 1 + \frac{4\Lambda}{3} r^2$ for anti-de-Sitter.

### 3 DBI action for D2-brane and wormhole

The background fields of Type IIA consist of: the metric $g_{MN}$, the two-form $B = B_{MN} dx^M \wedge dx^N$ and the dilaton $\Phi$ and the gauge fields $C_{(i)}$ (they are $i$-forms) with $i = 1, 3$. These gauge fields are coupled to the D2 and D4 branes. The action for the one D2 brane is given by the Dirac-Born-Infeld (DBI) action:

$$
S = -T_2 \int_{M_3} e^{-\Phi} \left( -\det \left( \gamma_{\alpha\beta} + 2\pi \alpha' F_{\alpha\beta} + b_{\alpha\beta} \right) \right)^{1/2} d^3\xi + \\
T_2 \int_{M_3} \sum_i c_{(i)} \wedge \exp \left( 2\pi \alpha' F + b \right),
$$

(3.1)
where $\mathcal{M}_3$ is the world-volume of D2-brane embedded into a background manifold $M_{10}$:

$$X : \mathcal{M}_3 \to M_{10}.$$  \hfill (3.2)

The signature of the metric $g_{MN}$ is $(-, +, +, +)$. All the fields in the DBI action are the pull-back of the background fields by the embedding $X$:

$$\gamma = X^* (g), \ b = X^* (B), \ c_{(i)} = X^* (C_{(i)})$$  \hfill (3.3)

except for the abelian field $F_{\alpha\beta}$ which is the gauge field on the D2-brane. Hence the Wess-Zumino (WZ) term is:

$$\int_{\mathcal{M}_3} \sum_i c_{(i)} \wedge \exp (2\pi \alpha' F + b) = \int_{\mathcal{M}_3} [c_{(1)} \wedge (2\pi \alpha' F + b) + c_{(3)}].$$  \hfill (3.4)

The term $c_{(1)} \wedge (2\pi \alpha' F + b)$ has the form of the Chern-Simons term. Assuming that D2-brane is spherically symmetric with the topology $\mathbb{R}^1 \times S^2$ the embedding $X$ takes the form:

$$X (\tau, \theta, \phi) = (t(\tau), r(\tau), \theta, \phi, X_4^0, ..., X_9^0),$$  \hfill (3.5)

where $\tau \in \mathbb{R}^1$. Thus the fields on the world volume have the form:

$$\gamma_{\tau\tau} = t^2 g_{tt} + r^2 g_{rr}, \ \gamma_{\theta\theta} = g_{\theta\theta}, \ \gamma_{\phi\phi} = g_{\phi\phi},$$  \hfill (3.6)

$$b_{\tau\theta} = t B_{\theta\theta} + r B_{r\theta}, \ b_{\tau\phi} = t B_{\phi\theta} + r B_{r\phi}, \ b_{\theta\phi} = B_{\theta\phi},$$  \hfill (3.7)

$$c_{(1)} = \left( C_{(1)\tau} t + C_{(1)\tau} r^2 \right) d\tau + C_{(1)\theta} d\theta + C_{(1)\phi} d\phi,$$  \hfill (3.8)

$$c_{(3)} = \left( C_{(3)\theta\phi} t + C_{(3)\theta\phi} r \right) d\tau \wedge d\theta \wedge d\phi,$$  \hfill (3.9)

$$F = F_{\tau\phi} d\tau \wedge d\theta + F_{\tau\phi} d\tau \wedge d\phi + F_{\theta\phi} d\theta \wedge d\phi.$$  \hfill (3.10)

Here we are looking for a such background for D2 brane which will produce the action obtained from the Hilbert-Einstein action and leads to the equation of motion for the throat radius $\rho$. In the Appendix the explicit form of DBI action is presented. In order to obtain the action (2.25) one needs to put constraints on the background fields. Since in this action no terms linear in $\rho$ and $t$ nor $\rho t$ thus one can see that (see Appendix):

$$\chi_t = \chi_r = \Sigma_{tr} = 0.$$  \hfill (3.11)

These system of equations (3.11) has one simple solution: $B_{t\phi} = B_{r\phi} = B_{t\theta} = B_{r\theta} = 0$. Hence the DBI action takes the form:

$$S = -2\pi T_2 \int_{R^1 \times [0, \pi]} d\tau d\theta e^{-\Phi} \left[ t^2 \left( 1 + A^2 \right) g_{tt} - r^2 \left( 1 + A^2 \right) g_{rr} - f^2 \right]^{1/2} +$$

$$+ 2\pi T_2 \int_{R^1 \times [0, \pi]} \left[ t \Psi_t + r \Psi_r + \Psi_F \right] d\tau d\theta,$$  \hfill (3.12)
where $A, f$ and $\Psi_t, \Psi_r, \Psi_F$ are given in the Appendix. If one puts the gauge $\gamma_{\tau\tau} = -1$ in (3.6), than the above action becomes:

$$
S = -2\pi T_2 \int_{R^1 \times [0,\pi]} d\tau d\theta e^{-\Phi} \left[ 1 + A^2 - f^2 \right]^{1/2} +
$$

$$
+ 2\pi T_2 \int_{R^1 \times [0,\pi]} \left[ \pm \Psi_t \sqrt{|g_{tt}|} \left( \sqrt{\frac{1}{g_{rr}} + r^2} + r \Psi_r + \Psi_F \right) \right] d\tau d\theta. \quad (3.13)
$$

The signs $\pm$ follow from the two solutions of the equation $\gamma_{\tau\tau} = -1$ with respect to $t$. The sign $+$ corresponds to increasing of the coordinate time $t$. Here we chose the sign $+$ but we remember that there is the second solution with decreasing time $t$. One can see that the first part of the DBI action can be interpreted as the lagrangian for the matter on the throat of the four dimensional wormhole where $A$ corresponds to the "magnetic" component of $F$ and $B$ while $f$ is related to the "electric" part of $F$. The WZ term generates dynamics of D2 brane described by the one degree of freedom $r$ which corresponds to the radius of the throat. From the above action with the Lagrangian $L$:

$$
L(r, \dot{r}) = + \Psi_t \sqrt{|g_{tt}|} \left( \sqrt{\frac{1}{g_{rr}} + r^2} + r \Psi_r + \Psi_F - e^{-\Phi} \left[ 1 + A^2 - f^2 \right]^{1/2} \right) \quad (3.14)
$$

we get the momentum $p_r$ conjugated to $r$:

$$
p_r = \Psi_t \sqrt{|g_{tt}|} \left( \frac{r^2}{\sqrt{1 / g_{rr} + r^2}} + \Psi_r \right).
$$

Thus the Hamiltonian $H = \dot{r}p_r - L$ expressed in $r$ and $\dot{r}$ is:

$$
H(r, \dot{r}) = - \frac{\Psi_t \sqrt{|g_{tt}|} \left( \frac{1}{g_{rr}} + \dot{r}^2 \right)}{\sqrt{|g_{tt}|} \sqrt{1 / g_{rr} + \dot{r}^2}} + e^{-\Phi} \left[ 1 + A^2 - f^2 \right]^{1/2} - \Psi_F. \quad (3.15)
$$

From the other side it is known that in the reparametrization invariant theories Hamiltonian is the constraint: $H = 0$. Hence we obtain the equation:

$$
\sqrt{|g_{tt}|} \sqrt{1 / g_{rr} + \dot{r}^2} = \frac{\Psi_t e^\Phi}{[1 + A^2 - f^2]^{1/2} - \Psi_F e^\Phi}. \quad (3.16)
$$

This equation is the generalization of the (2.20) where the density of the energy $\sigma$ is replaced by the energy density $\varepsilon$ on the D2-brane:

$$
\varepsilon = \frac{\Psi_t e^\Phi}{[1 + A^2 - f^2]^{1/2} - \Psi_F e^\Phi}. \quad (3.17)
$$

Because the energy density $\varepsilon$ should be positive we get the following constraints:

$$
e^{-\Phi} \left[ 1 + A^2 - f^2 \right]^{1/2} > \Psi_F, \quad \Psi_t \geq 0. \quad (3.18)
$$
This is the general condition on the background fields in which the D2-brane can be interpreted as the wormhole in the type IIA.

Next we consider the background produced by the spherically symmetric Dp-brane [13-15] which leads to the wormhole interpretation of the D2-brane. The background metric $g_{MN}$ is:

$$ds^2 = g_{MN}dX^M dX^N = Z_p^{-1/2} (r) (-K (r) dt^2 + dx^2) + Z_p^{1/2} (r) (K^{-1} (r) dr^2 + r^2 d\Omega_{8-p}^2), \quad (3.19)$$

where $dx^2 = dx_i dx^i$ and $i = 1, ..., p$. The metric on a unit round sphere $S^{8-p}$ is denoted as: $d\Omega_{8-p}^2$. The functions $Z_p$ and $K$ are:

$$Z_p (r) = 1 + \alpha_p \left( \frac{r}{r_H} \right)^{7-p}, \quad K (r) = 1 - \left( \frac{r_H}{r} \right)^{7-p}, \quad (3.20)$$

where

$$\alpha_p = \left[ 1 + (r_H/r_p)^2 (7-p)/2 \right]^{1/2} - (r_H/r_p)^{(7-p)/2} \quad (3.21)$$

and

$$r_p^{7-p} = d_p (2\pi)^{p-2} g_s \alpha^{(7-p)/2} \quad (3.22)$$

with the numerical factor $d_p = 2^{7-2p} \pi^{(9-3p)/2} \Gamma ((7 - p)/2)$. There is the dilaton $\Phi$:

$$\exp (2\Phi) = g_s^2 Z_p^{(3-p)/2} \quad (3.23)$$

and the antisymmetric field $B = 0$. The number $N$ is the Dp-brane R–R charge of the p-form $C_{(p+1)}$:

$$C_{(p+1)} = \frac{1}{g_s} \left[ \frac{1}{Z_p (r)} - 1 \right] dX^0 \wedge dX^1 \wedge ... \wedge dX^p. \quad (3.24)$$

The background metric has the horizon given by $r_H$ related to the ADM mass of the Dp-brane and the singularity is at $r = 0$. Since we are in the frame of the type IIA the dimensions of Dp-branes are: $p = 0, 2, 4, 6$. The only non vanishing component of $C_{(p+1)}$ is:

$$C_{(p+1)\ell \ldots \ell} = \frac{1}{g_s} \left[ \frac{1}{Z_p (r)} - 1 \right]. \quad (3.25)$$

For the D2-brane in the above background we get (because $B = 0$):

$$\Psi_t = AC_{(1)\ell} \quad \text{and} \quad \Psi_F = \Psi_\tau = 0, \quad (3.26)$$

where $A = 2\pi \alpha' F_{\theta\phi}$ is the magnetic field on the D2-brane. Hence one can see that the non-trivial background p-form $C_{(p)}$ is for $p = 0$. The metric (3.19) for this background is:

$$ds^2 = -Z_0^{-1/2} (r) K (r) dt^2 + Z_0^{1/2} (r) K^{-1} (r) dr^2 + r^2 Z_0^{1/2} (r) d\Omega_6^2.$$
Thus the action (3.13) takes the form:

\[
S = -2\pi^2T_2 \int_{R^1} dr e^{-\Phi} \left[ 1 + A^2 - f^2 \right]^{1/2} \\
+ 2\pi^2T_2 \int_{R^1} AC(1)T \frac{Z_0^{1/2}}{K(r)} \sqrt{\frac{K(r)}{Z_0^{1/2}}} + \dot{r}^2 \, dr, \tag{3.27}
\]

where

\[
C(1)T = -\frac{\alpha_0r_0^7}{g_s(r + \alpha_0r_0^7)} \tag{3.28}
\]

and \(r_0^7 = 60\pi^3g_s(\alpha')^{7/2}N\). The expression \(\alpha_0r_0^7\) is the function of \(r_H\) and \(N\):

\[
\alpha_0r_0^7 = \frac{1}{2}\sqrt{4r_0^{14} + r_H^{14}} - \frac{1}{2}r_H^7 \tag{3.29}
\]

The equation (3.16) makes sense in this background only if \(AC(1)T > 0\). Thus one needs that the magnetic field \(A = 2\pi\alpha'F_{\theta\phi}\) is negative. We obtain from (3.16) the following equation on \(r\):

\[
\dot{r}^2 = \frac{1}{\sqrt{1 + \frac{\alpha_0r_0^7}{r}}} \left(-1 + \frac{r_H^7}{r^7} + \frac{A^2\alpha_0^2}{s^{14}} \right), \tag{3.30}
\]

where \(s = 1 + A^2 - f^2\). We will introduce a new dimensionless variable \(x\) as follows: \(r^7 = r_0^7x\). Hence the equation (3.30) becomes:

\[
\dot{x}^2 = V(x), \tag{3.31}
\]

where:

\[
V(x) = \frac{49}{r_0^7} \frac{(x_+ - x)(x - x_-)}{\sqrt{\alpha_0 + x}} x^{3/14}, \tag{3.32}
\]

and:

\[
x_\pm = -\frac{1}{2} \left( \alpha_0 - \frac{1}{\alpha_0} \right) \pm \frac{1}{2} \sqrt{\left( \alpha_0 - \frac{1}{\alpha_0} \right)^2 + 4 \frac{A^2}{s}\alpha_0^2}. \tag{3.33}
\]

In order to get the above formula we used the equation (3.29) which leads to the relation: \(r_H^7 = r_0^7 \left( 1 - \alpha_0^2 \right)/\alpha_0\). Since \(r_H \geq 0\) we obtain that \(\alpha_0 \in (0, 1]\) and \(x_- < 0\). Thus the equation (3.30) makes sense if \(V(x) \geq 0\). In this way we obtained that the evolution of the radius of the throat \(r\) corresponds to a particle motion in the potential \(-V\). From the other side the background metric (3.19) has the horizon given by \(r_H\). This horizon corresponds to \(x_H = (1/\alpha_0) - \alpha_0 > 0\) and \(x_+ > x_H\). Thus the wormhole would be visible for a observer at infinity if the motion is confined to the interval: \([x_H, x_+]\). The equation (3.31) is highly complicated. In order to get some prediction concerning to behavior of \(x\) we approximate \(V\) by a square polynomial \(v(x) = ax^2 + bx + c\). The coefficients \(a, b\) and \(c\) are obtained from the relations:

\[
v(x_m) = V(x_m), \quad v'(x_m) = 0 \text{ and } v''(x_m) = V''(x_m), \tag{3.34}
\]

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where \( x_m \) is the maximum of the function: \( V'(x_m) = 0 \) and \( V''(x_m) < 0 \).

Hence we obtain:

\[
v(x) = \frac{1}{2} V''_m x^2 - V'_m x_m x + \left( V_m + \frac{1}{2} V''_m x_m^2 \right), \tag{3.35}
\]

where \( V''_m = V''(x_m) \) and \( V_m = V(x_m) \). Moreover we demand that:

\[
V(0) = v(0) = 0 \text{ and } V(x_+) = v(x_+) = 0.
\]

so: \( x_m = x_+/2 \) and \( V''_m = -8V_m/x_+^2 < 0 \) since \( V_m > 0 \). Thus in the above approximation the equation (3.31) reads:

\[
\dot{x}^2 = V_m \left[ 1 - \frac{4}{x_+^2} \left( x - \frac{x_+}{2} \right)^2 \right], \tag{3.36}
\]

where:

\[
V_m = \frac{49}{4\alpha_0^2} \frac{x_+ (x_+ - 2x_-)}{\sqrt{\alpha_0 + x_+^2}} \left( \frac{x_+}{2} \right)^{3/14} > 0 \tag{3.37}
\]

The picture of the function \( V \) (for \( \alpha_0 = 1/2 \) and \( A^2/s = 1 \)) presented by the solid black line and the quadratic polynomial \( v(x) = 4V_m (x_+ - x) x/x_+^2 \) (where \( x_+ = (3 + \sqrt{13})/4 \) and \( 4V_m/x_+^2 \simeq 0.984 \)) presented by the dots red line is showed below:

Location of the horizon corresponds to \( x_H = 3/2 \).

Thus in this quadratic approximation the equation (3.36) with the initial condition: \( x(0) = x_H \) has the solution:

\[
x(\tau) = x_+ \cos^2 \left[ \tau \sqrt{\frac{V_m}{2x_+} - \phi_0} \right], \tag{3.38}
\]
where:
\[ \phi_0 = \arccos \sqrt{\frac{x_H}{x_+}} < \pi \]  
(3.39)

and \(0 < x_H/x_+ < 1\). In this way we obtained the oscillating wormhole, which appears on the horizon \(x_H\) at proper time \(\tau = 0\) next reaching maximal size \(x_M = x_+\) at \(\tau_M\):

\[ \tau_M = \sqrt{\frac{2x_+}{V_m}} \left( \pi - \arccos \sqrt{\frac{x_H}{x_+}} \right) \]  
(3.40)

and vanishing under the horizon \(x_H\) at time \(\tau_f\):

\[ \tau_f = 2 \sqrt{\frac{2x_+}{V_m}} \arccos \sqrt{\frac{x_H}{x_+}} . \]  
(3.41)

Hence the evolution of the throat radius \(r\) is confined to the domain:

\[ r_+ \geq r > r_H, \]  
(3.42)

where: \(r_+ = r_0 (x_+)^{-1/7}\) and:

\[ r(\tau) = r_0 \sqrt[7]{x(\tau)}. \]  
(3.43)

The requirement \(r > r_H\) is implicated by the fact that \(r_H\) is the horizon in the background metric (3.19). In otherwise the D2 wormhole has the throat under the horizon (with the radius \(r_H\)) of the background metric and becomes "invisible" for an observer at infinity. This condition is accomplished by the above solution. Hence we obtained in the IIA string theory the transient wormhole with the time dependent throat \(r\) given by (3.43).

### 4 Vacuum energy and wormholes

The vacuum energy obtained from the quantum field theory (QFT) depends on the energy scale up to which we trust the theory. The cutoffs of the energy changing from the order of supersymmetry breaking scale (which is probably of the order 10-100 TeV or bigger) to the Planck scale. Hence the vacuum energy density is changing from \(10^{-64}\) to 1 in the Planck units.

From the other side the observation of the apparent luminosity of distant supernovae [16, 17] indicates that the expansion of the universe has recently begun to accelerate. In order to explains this unexpected acceleration one introduced so called "dark energy". The observation leads to value of the dark energy density which is equal to \((1.35 \pm 0.15) \times 10^{-123}\). Next this energy density is identify with the cosmological constant \(\Lambda\). Expressing \(\Lambda\) in meter\(^{-2}\) one obtains the value \(\Lambda \simeq 1.21 \times 10^{-52} [1/m^2]\). Moreover as turn outs the dark energy is experimentally indistinguishable from vacuum energy. But as one can see the theoretical vacuum energy (obtained from the quantum field theory) is in huge discrepancy with the observed dark energy represented by the cosmological constant.
We propose a mechanism for an explanation of this discrepancy. This mechanism is based on the assumption that there are wormholes of the Planck size and that the vacuum energy is flowed across these wormholes. But not all energy can be drained by the wormholes. Since the throats $r_w$ of wormholes are the size of the Planck length $l_{Pl}$ the energy which can be flowed by the one wormhole is: $E_w = \hbar c/r_w$. Hence we get the following equation:

$$E_c - E_o = NE_w,$$  \hspace{1cm} (4.1)

where $E_c$ is the vacuum energy in the volume $V$ obtained from QFT, $E_o$ is the vacuum energy in this same volume $V$ obtained from the cosmological observations: $E_o = \Lambda V c^4/(4\pi G)$ and $N$ is the number of the wormholes in $V$. Thus the density number $n = N/V$ of wormholes expressed in units $[1/m^3]$ is:

$$\frac{1}{4\pi} \frac{r_w \Lambda}{l_{Pl}^2} \left( \frac{E_c}{E_o} - 1 \right) = n.$$ \hspace{1cm} (4.2)

This number crucially depends on the cutoff of the energy scale via $E_c$. Because the vacuum energy density represented by $\Lambda$ is constant the excess of energy should flow to the other universes which are connected to our universe by these wormholes. If we assume that supersymmetry exists, then the number of other universes will be depend on the fundamental supersymmetry breaking scale $E_{SUSY}$. It means that the universes with different values of $E_{SUSY}$ will have different number connected universes by the wormholes. For example if the breaking scale is of order 100 TeV, then: $E_c/E_o \simeq 10^{60}$. Thus we get that the number density of the wormholes with the size of throats $l_{Pl}$ is equal to:

$$n \simeq 10^{42}.$$ \hspace{1cm} (4.3)

The observable universe has volume $10^{80}$ (in cubic meters $m^3$) so the total number $N$ of the wormholes in the observable universe is $10^{122}$. Hence the number of other universes connected to our by the wormholes is: $10^{122}$. It will be interesting relate this number to the number of vacua obtained from the string theory landscape.

## 5 Conclusions

The purpose of this paper was relate the four-dimensional lorentzian wormholes to the backgrounds of string theory. From the dimensional arguments we get that the appropriate backgrounds are given by the type IIA string theory. We related DBI action for D2 brane to the action of dynamic wormhole. This relation between these actions led us to the conditions on the backgrounds with the D2 branes which can be interpreted as the wormholes in the four dimensional spacetimes. These spacetimes had to be asymptotically Minkowski or anti-de Sitter. The background fields acquired interpretation of the matter on the throat of the wormhole. The dynamics of the throat is obtained from WZ term in the DBI action. In this way the four-dimensional wormhole took on the form
of D2 brane in the type IIA supergravity approximation of the string theory. The energy condition on the matter supporting the wormhole is given by eq. (3.18). As the example we considered special solution of IIA which produced the transient wormhole. We also speculated about the relation of number of universes, the value of the cosmological constant and supersymmetry breaking scale.

6 Appendix

6.1 DBI action

The determinant of the sum of a symmetric $g = (g_{ab})$ and an antisymmetric $B = (B_{ab})$ 3x3 matrices is:

$$\det (g + B) = \det g \det (1 + g^{-1}B).$$

For the diagonal matrix $g = \text{diag} (g_1, g_2, g_3)$ one gets:

$$\det (g + B) = g_1 g_2 g_3 + b_1^2 g_1 + b_2^2 g_2 + b_3^2 g_3,$$

where $b_a = \varepsilon_{abc} B_{bc}/2$: $b_1 = B_{23}$, $b_2 = -B_{13}$, $b_3 = B_{12}$.

The DBI action for a D2 brane with the background fields (3.6 - 3.10) is:

$$S = -T_2 \int_{R^1 \times S^2} d\tau d\theta d\phi e^{-\Phi} \left[ t^2 \Sigma_{tt} - 2t \Sigma_{tr} - \frac{1}{2} \Sigma_{rr} - 2t \chi_t - 2r \chi_r - f^2 \right]^{1/2} + T_2 \int_{R^1 \times S^2} \left[ i \Psi_t + i \Psi_r + \Psi_F \right] d\tau d\theta d\phi,$$

where:

$$\Sigma_{tt} = (1 + A^2) |g_{tt}| - g_{\theta \theta} B_{t \phi}^2 - g_{\phi \phi} B_{t \theta}^2,$$

$$\Sigma_{rr} = (1 + A^2) |g_{rr}| + g_{\theta \theta} B_{r \phi}^2 + g_{\phi \phi} B_{r \theta}^2,$$

$$\Sigma_{tr} = g_{\theta \theta} B_{t \phi} B_{r \phi} + g_{\phi \phi} B_{t \theta} B_{r \theta},$$

$$\chi_t = 2\pi \alpha' \left( g_{\theta \theta} B_{t \phi} F_{\tau \phi} + g_{\phi \phi} B_{t \theta} F_{\tau \theta} \right),$$

$$\chi_r = 2\pi \alpha' \left( g_{\theta \theta} B_{r \phi} F_{\tau \phi} + g_{\phi \phi} B_{r \theta} F_{\tau \theta} \right),$$

$$f^2 = (2\pi \alpha')^2 \left( g_{\theta \theta} F_{\tau \phi}^2 + g_{\phi \phi} F_{\tau \theta}^2 \right),$$

$$A = B_{t \phi} + 2\pi \alpha' F_{\phi \theta}.$$

The terms in WZ part reads:

$$\Psi_t = AC_{(1)t} - C_{(1)\theta} B_{t \phi} + C_{(1)\phi} B_{t \theta} + C_{(3)t \theta},$$

$$\Psi_r = AC_{(1)r} - C_{(1)\theta} B_{r \phi} + C_{(1)\phi} B_{r \theta} + C_{(3)r \theta},$$

$$\Psi_F = 2\pi \alpha' \left( -C_{(1)\theta} F_{\tau \phi} + C_{(1)\phi} F_{\tau \theta} \right).$$
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