Investigating the output characteristics of Ring Laser in Laser Dynamic Goniometer

N.A. Eno¹, P.A. Pavlov¹, J.T. Bagshaw¹
¹Laser measurement and Navigation System Dept., Saint Petersburg Electrotechnical University “LETI”, Saint Petersburg 197376, Russia.

Abstract: Lately, it becomes a pressing need for increasing the accuracy in measurements and calibration of optical polygon using laser dynamic goniometer, therefore the need to understudy the output characteristics of Ring laser which is used as the reference scale during this measurement of refractive indices, calibration of optical polygon and in building of laser dynamic goniometer using high precision goniometer along with the optical angle encoder. This paper covers algorithms, procedures of operation, and results of the investigations, as well as the calculations of coefficient characterizing the null shift of ring laser output, the scale factor, coefficient characterizing the nonlinearity of ring laser output and graph of dependency. This article also shows that, the algorithm for compensating of the ring laser generalized zero shifts increases the accuracy of the angular measurements and makes it possible to determine the coefficients of the ring laser output characteristic.

1. Introduction

RL is He-Ne laser of GL-1 (He-Ne laser plasma tube of type ‘GL-111’ made in the former USSR. Helium Neon Gas Laser with 12V DC Power Supply) type in a ring form, despite its name, it’s often in square form or triangular cavity, two light waves of the same wavelength traveling in opposite direction to each other is generated in RL and interference pattern is observed that is used to determine rotational speed; RL is use as a reference scale in the building of LDGs due to its high rate of accuracy and speed of operations. LDG is a device used for calibration of angles, measurement of Refractive indices [2] and so on, in general, our final objective is creating possible methods of increasing accuracy of LDG, as it has been seen that the use of gyros reduces the time of measurement and increases accuracy, reliability and measurement re-productivity [3-6]. Systematic errors of RL make up the systematic errors encountered during operation in LDG, hence a strong need to investigate RL output characteristics. Using high precision DG made it possible for this understudy [1-7]. Algorithms and equation were created following procedures based on the already created algorithms. processing of measurements results using a special software like Mathlab, originlab etc., in order to obtain the respective values of the three component of the output characteristics of RL which are: coefficient characterizing the null shift of the RL output characteristic, scale factor, coefficient characterizing the nonlinearity of the RL output and plotting a dependency graph of $N2\pi$ (RL pulse) against $T$ (time of RL complete revolution)

2. Methods

Applying a D.C voltage to the motor of the DG enable the spindle to rotate, note that the speed of rotation depends largely on the voltage. The spindle of a dynamic goniometer rotates in bearings with optical angle encoder and RL attached to it. Spindle is brought into rotation by a drive consisting of a motor and a motor control system. output signals from the optical angle encoder and RL are fed to the electronics unit that performs pre-processing of data and their transfer to the computer. The data obtained from the computer are post processed to generate two columns of data consisting of time recorded by the counter and the RL pulses. These two columns is use to calculate rotation of angles. The investigation took place within the angular velocity of rotation range of 360 degs clockwise and counterclockwise, it was carried out with the period of 25 revolutions. The output signal from optical encoder passed through the counting frequency divider and produced the RL output signal phase reading pulse ($Ni$) and time moment reading pulse ($ti$). Readable pulses were produced at the interval of 1 deg. The measurements were taken within the period of not less than 10 rotations of RL. Nonlinear output characteristics decrease with increase of the speed of revolution.
3. Algorithm and Equation

From the RL pulling zone or lock in zone, the frequency of the output RL signal is proportional to the input as written below:

$$v(t) = K_1 \Omega + K_0 + K_{-1} / \Omega \cdot E(t)$$  \hspace{1cm} (1)$$

where $K_0$ is the coefficient characterizing the null shift of the RL output characteristic, $k_1$ is the RL scale number, $K_{-1}$ is the coefficient characterizing the nonlinearity of the RL output characteristic, $\Omega$ and $\Omega^*_{E}$ are RL rotation velocity in the laboratory system of coordinates and the vertical component of the Earth rotation velocity respectively.

On measuring the angular output signal of RL periods which integrated within time interval formed by output pulses of optical encoder. Integrating (1) gives the number of period $N_1$ in the time interval from $0 \ldots \ldots t_i$

The angle $\phi_1$ which RL rotates can be written as:

$$\phi_1 = 2\pi N_1 / N_{2\pi}$$  \hspace{1cm} (2)$$

where $N_1 = \frac{1}{2\pi} \int_0^t v(t) dt$; and $N_{2\pi} = \frac{1}{2\pi} \int_0^t v(t) dt$

$N_1$ and $N_{2\pi}$ are the numbers of periods in the RL output signal occurring in a time interval defined by the angle and during a complete rotation respectively; $v(t)$ is the output signal frequency.

t = time of measurement of angle $\phi_1$

$2\pi$ is the number of RL output signal periods within the complete revolution.

$$\int_0^T v \, dt = \frac{T}{O} K_1 \Omega \, dt + \frac{T}{O} K_0 \, dt + \frac{T}{O} K_{-1} \, \frac{1}{\Omega} \, dt$$  \hspace{1cm} (3)$$

From equation (2)

$$\phi_1 = \phi_i + \frac{k_0 + k_1 \Omega^*_{E}}{K_1} \chi(\omega t - \omega i) \phi_i \frac{k_{-1}}{k_1} (J_i - J_1 \frac{\phi_i}{2\pi})$$  \hspace{1cm} (4)$$

But

$$J_i = \frac{ti}{\omega_{E}(t) + \Omega^*_{E}}$$  \hspace{1cm} (5)$$

$$\phi_1 = \omega i . t_1$$

Where $i$ is $\phi_1$ measurement time

Component proportionality to the vertical component of the Earth rotation velocity is the major contribution

$$\Delta \phi = \phi_i - \phi = \frac{k_0 + k_1 \Omega^*_{E}}{K_1} \chi(\omega t - \omega i) \phi_i \frac{k_{-1}}{k_1} (J_i - J_1 \frac{\phi_i}{2\pi})$$  \hspace{1cm} (6)$$

As

$$k_1 \Omega^*_{E} \geq K_0, \frac{K_{-1}}{\Omega 0}$$

It implies that the major contribution to the measurement error is the vertical component of the earth rotation since the latitude of the site is well known, the systematic error from the earth rotation can be reduced logically

$$\phi_1^* = 2\pi \frac{N_1 \pm N_{2\pi} \Omega^*_{E}. t_i / 2\pi}{N_{2\pi} (1 \pm \Omega^*_{E} / 2.\pi)}$$  \hspace{1cm} (7)$$
Measurement equation with compensation of the earth component and it’s called phase–time angle measurement technique algorithm with compensation of the vertical component of the earth’s rotational speed. The “plus” and “minus” sign depends on the rotational direction. “Plus” is the clockwise direction of the RL, while “minus” is the anticlockwise direction.

After compensation of the earth rotation, this eliminates the instability of the RL rotational speed

\[ F = \left( \frac{1}{2\pi} \right) (K_o + K_2 \Omega^* E) \]  

where \( F = \) general zero shift, describing the corresponding general null shift of the output characteristics of RL.

### 3.1 Equation of measurement with compensation of generalized zero shift

By the \( F \) we mean the sum of the terms characterizing the vertical component of the earth's rotation speed and the actual zero shift of the output characteristic of the RL

\[ F' = \frac{1}{2\pi} (K_o + K_2 \Omega^* E) \]  

We introduce a new measurement equation

\[ \tilde{\phi}_i = 2\pi \frac{N_i \pm F'_i}{N_{2\pi} \pm F T} \]  

Where the sign "plus" or "minus" is determined by the direction of rotation RL

For definiteness, in what follows we shall take the minus sign and consider terms of the first order of smallness with respect to \( \Delta \phi \)

Then the expression (1.32) takes the form:

\[ \dot{\phi}_i^{**} = \dot{\phi}_i (1 - \frac{2\pi}{N_{2\pi}} F'_{1, T} \frac{\omega_T - \omega_i}{\varpi_i \sigma_T}) \approx \dot{\phi}_i (1 - \frac{2\pi}{K_1} \frac{\omega_T - \omega_i}{\varpi_i \sigma_T}) \]  

Substituting the expression \( \tilde{\phi}_i \) in (1.33) [7], we obtain:

\[ \Delta \phi_i^{**} = \tilde{\phi}_i^{**} - \phi_i = \frac{1}{\omega_0} K_1 \left[ \varphi_i \left( \frac{\sigma_i - \sigma_T}{\varpi_i \sigma_T} \right) + \frac{1}{\omega_0} (\Delta J_i - \frac{\varphi_i}{2\pi} \Delta J_T) \right] \]  

Thus, the error in measuring the angle using the measurement equation with compensation of the generalized zero shift is determined by the instability of the rotational speed and the value of the nonlinearity coefficient in the output characteristic of the RL. From comparison of expressions (6) and (10) it follows that the use of an algorithm with compensation of generalized zero shifts reduces the error of angular measurements by a ring laser.

### 4. Results

Due to the uncertainty of the generalized zero shift, before practical use of the measurement equation (11), it is necessary to determine experimentally the value of the generalized zero shift (GNS), which, in general, is not a constant value. The generalized zero shift (GNS) is found from the dependence of the number of periods of the output signal of the RL on the angle 2\( \pi \) on the turn-over time \( T \). Indeed, from expression (12) under the assumption that \( \omega(t) \gg \Omega^* E \), \( \omega(t) \equiv \text{const} \) follows that

\[ N_{2\pi} = K_1 + F T + K' T^2 \]
Where \( K'_{-1} = \frac{1}{(2\pi)^2} K_{-1} \). \( F' = F / 2\pi \)

To find the value of GNS, it is necessary, using the experimental data, to construct the dependence of \( N_{2\pi} = f(T) \), from which the value of GNS is determined by regression analysis. It is known \[7\] that when using the method of least squares, the error in finding the regression coefficients decreases with increasing data array involved in processing. Therefore, it is expedient to use the so-called "sliding" values of \( N_{2\pi} \), which are obtained at \( 2\pi \), but with a shift less than \( 2\pi \):

\[
N_{2\pi} = N_{i+k} - N_i
\]

Where \( k \) is the number of uniform shifts within the turnover.

One of the ways of realizations of GNS measurements with slightly accelerated rotation is shown in Figure 4(a) below. Approximation of the dependence presented in Fig. 4(a) with the use of the method of least squares allowed us to determine \( K_i = 987922.935 \pm 0.001; F = -10.033 \pm 0.001s^{-1}; K = 0.01759 \pm 0.0001s^{-2} \).

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Figure 4.1 (a, b). (a) slightly accelerated rotation (b) un-stabilized rotation speed

Figure 4.1 (a) Dependence of the number of periods of RL on the angle of \( 2\pi \) from the period of revolution with slightly accelerated rotation.

This example shows that in the regime with uniformly accelerated rotation of the RL, the GNS can be determined with high accuracy. However, its implementation requires an additional experiment, which is not always possible with the practical use of the laser goniometric system (LGS).

4.1 Determination of the generalized zero shift without additional experiment

In real LGS, there is always instability in the rotational speed of the RL, which makes it possible to construct the dependence \( N_{2\pi} = f(T) \) without resorting to an additional experiment. Figure 4(b) shows the characteristic dependence of \( N_{2\pi} = f(T) \) for the LGS without stabilizing the rotational speed of the RL, from which the GNS was found.

Figure 4(b) Dependence of the number of periods of RL on the angle of \( 2\pi \) from the period of revolution with un-stabilized rotation speed.

From Figure 4(b), the range of change in the revolution period is not as great as in the case of accelerated rotation, so the nonlinearity of the output characteristic is not manifested. In this case, linear approximation was used to find the GNS. The result of the calculations is \( F = -5.94 +/- 0.01s^{-1} \).
From the results of the definition of GNS, it follows that; the error in measuring the GNS is smaller, the larger the change in the period of the RL revolution, i.e. the greater the instability of the rotation speed.

Conclusions

The phase-time measurement method, consisting of simultaneous measurement of the phase of the output signal of the RL and its measurement time, allow the use of measurement equations that reduce the error caused by the instability of the RL rotational speed. The highest measurement accuracy was obtained by using the equation with compensation of the generalized zero shift. The considered method of finding a GNS is based on the use of the experimental dependence $N_{2x}(T)$, from which the regression analysis using the dependence $N_{2x} = K_1 + F'T + K'_2T^2$ determines the value of the GNS. The error in the result of the measurements of the GNS decreases with the use of "sliding" values of $N_{2x}$ and an increase in the range of the change in the RL. Values of $K_0$, $K_1$ were calculated, from our results it can be stated nonlinearity output of RL ($K_{-1}$) could not be calculated due to non-detection, was smaller the noise. Average RL velocity $\Omega$ as 13arc/seconds means that we had a stable zero shifts for RL output, after knowing the accurate latitude of the site, and from the graph we can see the dependency of the output characteristics as the scale factor increases with time(s).

It is of utmost importance to understudy the output characteristics of RL from time to time taking into account the vertical component of the earth rotation.

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