Charge-exchange-induced perturbations of ion and atom distribution functions in the heliospheric interface

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Abstract. Different hydrodynamic models of the heliospheric interface have been presented meanwhile, numerically simulating the interaction of the solar wind plasma bubble with the counterringing partially ionized interstellar medium. In these model approaches the resulting interface flows are found by the use of hydrodynamic simulation codes trying to consistently describe the dynamic and thermodynamic coupling of the different interacting fluids of protons, H-atoms and pick-up ions. Within such approaches, the fluids are generally expected to be correctly described by the three lowest velocity moments, i.e., by shifted Maxwellians. We shall show that in these approaches the charge-exchange-induced momentum coupling is treated in an unsatisfactory representation valid only at supersonic differential flow speeds. Though this flaw can be removed by an improved coupling term, we shall further demonstrate that the assumption of shifted Maxwellians in some regions of the interface is insufficiently well fulfilled both for H-atoms and protons. Using a Boltzmann-kinetic description of the proton- and H-atom- distribution functions coupled by charge exchange processes we emphasize the fact that non-negligible deviations from shifted Maxwellians are generated in the interface. This has to be taken into account when interpreting inner heliospheric measurements in terms of interstellar parameters.

INTRODUCTION

The mutual interaction of plasma and H-atom gas flows in the heliosheath, in view of large Knudsen numbers $K_n = \lambda_{ex}/L$, needs a kinetic treatment of charge-exchange induced coupling processes with typical mean free paths $\lambda_{ex}$ larger than typical structure scales $L$, as was already emphasized by Osterbart and Fahr (1992) or Baranov and Malama (1993). In kinetic approaches the distribution function of the H-atom gas needs to be described by a Boltzmann-Vlasov integro-differential equation (see, e.g., Ripken and Fahr, 1983; Osterbart and Fahr, 1992; Baranov and Malama, 1993; Pauls and Zank, 1996; Fahr, 1996; McNutt et al. 1998,1999; Bzowski et al. 1997, 2000).

Generally spoken, for time-independent problems one would have to start from the following typical Boltzmann equations:

$$\begin{align*}
(\vec{v} \cdot \nabla_r) f_i + \left( \vec{F} \cdot \nabla_v \right) f_i &= f_j(\vec{r}, \vec{v}) \int 3 f_i(\vec{r}, \vec{v}') v_{rel}(\vec{v}, \vec{v}') \sigma(v_{rel}) d^3v' - \\
&- f_i(\vec{r}, \vec{v}) \int 3 f_j(\vec{r}, \vec{v}') v_{rel}(\vec{v}, \vec{v}') \sigma(v_{rel}) d^3v
\end{align*}$$

(1)
where indices $i, j$ can be used to denote $f_H(\vec{r}, \vec{v})$ and $f_p(\vec{r}, \vec{v})$ as the velocity distribution functions of the H-atoms and the protons, respectively, $\vec{r}$ and $\vec{v}$ are the relevant phase-space variables, $\vec{F}$ are forces per mass, $v_{\text{rel}}$ denotes the relative velocity between collision partners of velocities $\vec{v}$ and $\vec{v}'$, and $\sigma(v_{\text{rel}})$ is the velocity-dependent charge exchange cross-section. Due to the fairly laborious mathematical tractability of the above Boltzmann equation, many authors have preferred to change over from Eq. (1) to a set of hydrodynamic moment equations (for a review see Zank, 1999) thereby only admitting for the lowest moments of the distribution function to be different from zero, i.e. density $n_i$, bulk velocity $\vec{U}_i$, and scalar pressure $n_iKT_i = P_i$. For the case of stationary ionospheric plasma-gas couplings this had led Banks and Holzer (1968) to the following equation of motion:

\[
\left( \vec{C}_i \cdot \nabla \right) \vec{C}_i + \frac{1}{n_im_i} \left( \nabla \cdot \vec{\Psi}_i \right) - \langle \vec{F}_i \rangle = \frac{\vec{A}_{ij}}{n_im_j} = -\frac{KT_i}{m_i} \vec{C}_i + \vec{D}_{ij}
\]  

(2)

where $\vec{C}_{ij} = \vec{U}_j - \vec{U}_i$ is the differential drift velocity between the fluids $i$ and $j$, $\langle \vec{F}_i \rangle$ is the velocity-average of the external forces, $\vec{A}_{ij}$ are forces due to charge-exchange induced momentum transfers, and $D_{ij}$ is the ion-neutral diffusion coefficient given by:

\[
D_{ij} = \frac{3\sqrt{8\pi} T_i}{8n_j \sigma_{\text{ex}} \sqrt{T_i + T_j}}
\]  

(3)

One should, however, clearly keep in mind, that the above form of a macroscopic equation of motion was derived under the simplifying assumption that $C_{ij}/\sqrt{2KT_i/m_i} \ll 1$ (i.e., that highly subsonic differential drifts prevail) and that the charge exchange cross section $\sigma_{\text{ex}}$ can be taken as independent of velocity.

**SUPersonic AND QUASI-SONIC DIFFERENTIAL DRIFTS**

Since in the heliospheric interface, due to the fact that here $C_{ij}/\sqrt{2KT_i/m_i} \geq 1$ prevails, these above made assumptions are not fulfilled and charge exchange coupling of the two fluids "i" and "j" has to be treated in a different manner. Again using shifted Maxwellians with isotropic temperatures $T_p$ and $T_H$ (as done by Holzer, 1972; Fahr, 1973; Holzer and Leer, 1973; Ripken and Fahr, 1983; Isenberg, 1986; Fahr, 1996; Lee, 1997) and taking velocity-independent cross sections $\sigma_{\text{ex}}$ then suggests to present the above mentioned charge exchange momentum coupling term $\vec{A}_{ij}$ in the following form:

\[
\vec{A}_{ij} = \sigma_{\text{rel}} \langle v_{\text{rel}} \rangle m_in_in_j(\vec{U}_j - \vec{U}_i) = \Gamma \vec{C}_{ij} = \vec{Q}
\]  

(4)

with $\langle v_{\text{rel}} \rangle$ being the double-Maxwellian average of the relative speed between protons and H-atoms as given, e.g., by Holzer (1972) in the form:

\[
\langle v_{\text{rel}} \rangle = \sqrt{\frac{128}{9\pi} \left( \frac{P_p}{\rho_p} + \frac{P_H}{\rho_H} \right) + \left( \vec{U}_H - \vec{U}_p \right)^2}.
\]  

(5)
The problem appearing with this approach when treating the passage of neutral interstellar gas (LISM H-atoms) through the plasma interface ahead of the solar system can easily be identified: The problem essentially is comparable to the passage of an H-atom gas flow through a predetermined quasistatic plasma structure simulating the region downstream of the expected outer interstellar bow shock and ahead of the stagnation point at the heliopause. In a one-dimensional approach for the region along the stagnation line (z-axis!) this LISM plasma ahead of the heliopause, due to its very low sonic Mach number, can be taken as quasi-incompressible and nearly stagnating. To describe the charge exchange imprint of this pre-heliopause plasma sheath on the H-atom flow at its penetration through this wall one traditionally uses the following set of equations (see Holzer, 1972):

\[ \frac{d}{dz}(\rho_H V_H) = 0, \]  

\[ \rho_H U_H \frac{d}{dz} U_H = -\frac{d}{dz} P_H - \sigma_{rel} V_{rel} n_p \rho_H U_H, \]  

\[ \frac{d}{dz} \left[ U_H \left( \frac{\rho_H U_H^2}{2} + \frac{\gamma P_H}{\gamma - 1} \right) \right] = \sigma_{rel} V_{rel} n_p \rho_H \left[ \frac{1}{\gamma - 1} \left( \frac{P_p}{\rho_p} - \frac{P_H}{\rho_H} \right) - \frac{U_H^2}{2} \right], \]  

where \( \gamma \) is the polytropic index taken as identical for both protons and H-atoms. As seen from Eq. (6), the H-atom mass flow is constant yielding \( C_0 = \rho_0 U_{H0} = \rho_H U_H \). In addition with introduction of the normalized space coordinate \( \xi \) defined by \( z = \xi D \) (\( D \) being the linear extent of the plasma wall and the quantity \( \Lambda = D/\lambda = D \sigma_{rel} n_p \) (\( \xi = 0 \) and \( \xi = 1 \) mark inner and outer border of the plasma wall) one then obtains the following characteristic equation (see Fahr, 2003):

\[ \frac{d}{d\xi} U_H = \frac{V_{rel} \Lambda (\Delta \rho P_p - P_H + \frac{1}{2} C_0 U_H (\gamma + 1))}{\gamma P_H - C_0 U_H}, \]  

where \( \Delta \rho = \rho_p/\rho_H \) is used.

Starting the integration at \( \xi = 0 \) with supersonic H-atom inflow velocities, i.e., with \( U_{H0}^2 \geq \gamma P_{H0}/\rho_{H0} \), one first obtains physically meaningfull results for \( U_H \) and \( P_H \) with increasing values of \( \xi \). At a critical point \( \xi = \xi_c \geq 0 \), however, where locally the equality \( \gamma P_{Hc} = C_0 U_{Hc} \) is reached, the integration of the upper system of differential equations cannot be continued, since a singularity of an O-type appears which cannot be avoided (see, e.g., Kopp and Holzer, 1976). This “neuralgic” point can, however, be eliminated when instead of Eq. (4) a more refined expression for the term of the charge-exchange induced momentum exchange between plasma and H-atom flow is used which considers the velocity-dependence of \( \sigma_{ex} = (A + B \log (v/v_0))^2 \), A and B being constants, and the individual relative velocities \( v_{rel} \) as, e.g., carried out by Williams et al. (1997), McNutt et al. (1998, 1999) or Fahr (2003). As shown by the latter author, the corresponding expression, which is valid for moderate and small Mach numbers \( M_H \), while Eq. (4) is only justified for large Mach numbers, can be brought into the following form:

\[ \tilde{Q}_{sub} = \Pi \left[ \frac{7}{3} g_{H1} M_H - \sqrt{\pi} \left( -9 g_{H1} M_H - 2 g_{2M_H} + 5 \alpha g_{H1} M_H + 2 \alpha g_{1M_H}^3 \right) \right] \left( \bar{u}_H / u_H \right). \]
Here it should be noted that, compared to the above expression, the usually applied expression used in Equ.(7), is only justified for the case of high Mach numbers $M_H$, and when written in a manner analogous to Equ.(10) using the above introduced quantities, attains the following form:

$$\vec{Q}_{\text{super}} = -\Pi M_H \sqrt{\frac{4\pi}{9}}(1 + \alpha) + \alpha M_H^2 \left(\bar{u}_H/u_H\right)$$

(11)

In the above expressions the following notations have been used: $\alpha = T_H/T_p$; $M_H^2 = \rho_H U_H/\gamma P_H$; $g_1 = \frac{1}{15}(1 + \frac{B}{\sqrt{\sigma_{\text{rel}}}})$; $g_2 = g_1 - \sqrt{\pi - \frac{2b}{\sqrt{\sigma_{\text{rel}}}}}$; $\Pi = \frac{2}{\sqrt{\pi}} n_p n_H \sigma_{\text{rel}} \sqrt{2KT_p/m} \sqrt{2KT_H/m}$.

This evidently means that depending on prevailing Mach numbers $M_H$ the plasma-gas friction force, based in the past on Holzer’s term, was either over- or underestimated in published hydrodynamical theories (see Fahr, 2003, for details).

With the newly derived expression one obtains, instead of Equ.(9), the following characteristic equation:

$$\frac{dU_H}{d\xi} = \Lambda V_{\text{rel}} \left(\Delta \rho P_p - P_H - \frac{1}{2}C_0 U_H(\gamma - 1)\right) + \gamma D U_H Q_{1,\text{sub}}(U_H, P_H)$$

\[
\gamma P_H - C_0 U_H
\]

(12)

which now has an X-type critical point with an avoidable singularity condition requiring simultaneous vanishing of the numerator and the denominator. Solutions of the set of Equs.(6) through (8) with application of the newly derived expression (10) instead of (11) are shown by Fahr (2003).

**CONCLUSIONS AND OUTLOOK**

It is clearly manifest from the study of works by Baranov and Malama (1993), Baranov et al. (1997), Fahr (2000), Fahr et al. (2000), Mueller et al. (2000) or Izmodenov (2000, 2001) that the locally prevailing Mach numbers $M_H$ of the relative flows between the LISM proton plasma and the LISM H-atom fluid in the heliosheath region (i.e., post-bow-shock Mach numbers $M_H$) generally are found to be smaller than $M_H = 2$. Consequently and strictly speaking, in this Mach number range, the adequate momentum exchange term for hydrodynamic approaches must be taken in the newly derived form given by Equ. (10), instead in the conventionally used form given by Equ. (11). Since the effective momentum exchange rate described by this new expression, depending on prevailing local Mach numbers is smaller or greater than that taken into account by the conventionally used term, one can presume that the adaptation of the LISM H-atom flow to the bow-shocked LISM proton plasma in the interface region ahead of the heliopause operates differently from what is described up to now.

Hereby the question how to apply the above derived momentum exchange expression (10) – even though the real interface plasma is not stagnating, but is a two-dimensional plasma flow with locally variable properties – is relatively easy to answer: Imagine that the plasma flow and the H-atom flow in the interface in nearly all simulation codes conventionally first are calculated without taking into account the effect of charge exchange interactions by only integrating the relevant hydrodynamical differential equations for
mass, momentum, and energy flow conservation without coupling terms on an appropriate spatial grid system. Then in a second step of the integrations as usually practiced at all grid points the resulting charge exchange interaction terms are evaluated, describing local exchanges of momentum and energy per unit of volume and of time. Instead of using the conventionally used momentum interaction term given by Equ.(11), one now applies the newly derived term given by Equ. (10) which when evaluated in the local rest frame comoving with the local plasma bulk flow attains just the form identical with Equ. (10) and thus given by:

$$\vec{Q}_{sub}^* = \Pi \vec{M}_H^* \left( \frac{7}{3} g_1 - \sqrt{\pi} (-9 g_1 - 2 g_2 + 5 \alpha_1 + 2 \alpha_1 M_H^2) \right)$$

(13)

besides the fact that $\vec{M}_H^*$ now has to be taken as defined by: $\vec{M}_H^* = \left( \vec{U}_p - \vec{U}_H \right) / \sqrt{2 KT_H / m}$.

Since forces are invariant under Galilean transformations (i.e., $U_p \ll c$), one can simply, in the next run of integrations, now take into account the ad-hoc forces $\vec{Q}_{sub}^* = \vec{Q}_{sub}$ to remodel the H-atom fluid, and $-\vec{Q}_{sub}$ to remodel the proton fluid. This procedure as usual can then be iterated until convergence is achieved. In a future work we are going to apply this above mentioned procedure within the frame of our existing five-fluid simulation code to better model the interface flows (see Fahr et al., 2000).

An additional flaw in the hydrodynamic simulation codes is due to the assumption of highly relaxed distribution functions $f_p$ and $f_H$ in the form of shifted Maxweillians. This assumption has been shown to be not sufficiently well fulfilled in the heliospheric plasma interface since rapid charge-exchange-induced injections of new particles permanently keep the resulting distribution functions away from relaxed hydrodynamical ones (see Fahr and Bzowski, 2004). Even the proton distributions in some regions of the interface develop pronounced non-equilibrium features, especially in regions where the proton densities are low and proton temperatures are high, i.e., where Coulomb relaxation processes have typical periods $\tau_{pp}$ larger than the injection periods $\tau_{ex}$. While we refer for details to the paper by Fahr and Bzowski (2004), we here may give helpful estimates to characterize the resulting deviations to be expected: Representing the actual proton distribution function by $f_p^* = f_{0p} + f_{1p}$, with $f_{0p}$ being the hydrodynamic “core distribution” delivered by the hydrodynamic multifluid code, one can integrate the total rate $\delta n_{1p}$ of protons relaxing by means of pp-Coulomb collisions per unit of time and volume towards the core distribution by:

$$\delta n_{1p} \simeq \frac{1}{\tau_{pp}} \int d^3 v (f_p^* - f_{0p}) = \frac{n_{1p}}{\tau_{pp}}$$

(14)

where $n_{1p}$ is the total density described by $f_{1p}$. To achieve stationary conditions, this rate must just be balanced by the charge exchange injection rate thus yielding the following relation:

$$\zeta_p = \frac{n_{1p}}{n_{0p}} \simeq n_{0H} \sigma_{ex} \langle v_{rel,p,H} \rangle \tau_{pp} = \frac{\tau_{pp}}{\tau_{ex}}$$

(15)

In the outer interface, where $\tau_{pp}$ is of the order $2 \cdot 10^7$ s while $\tau_{ex}$ is or the order of $2 \cdot 10^9$ s (see Fahr and Bzowski, 2004), one thus obtains $\zeta_p \simeq 10^{-2}$. In the inner interface
inside the heliopause, however, where $\tau_{\text{ex}} \approx 4 \cdot 10^8$ s and $\tau_{\text{pp}} \sim (T_p^{3/2}/n_p) \approx 10^{12}$ s should be valid, fairly strong perturbations of the relaxated distribution functions must be expected.

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