Influence of asymmetric factor on spur gears to dynamic bending stress

Agus Sigit Pramono*, Mohamad Zainulloh Rizal
Mechanical Engineering Department, Institut Teknologi Sepuluh Nopember
Kampus ITS Reputih Sukolilo Surabaya Indonesia
*Email : pramono@me.its.ac.id

Abstract. Asymmetric spur gear is used in one-way rotation transmission. This spur gear has a higher bending strength than symmetric spur gear. In this research, the effect of asymmetrical spur gear level on bending stresses is studied dynamically. The asymmetric spur gear level is expressed as asymmetric factor (K). Asymmetric factors K = 1, K = 1.05, K = 1.06, K = 1.07 and K = 1.08 are used. Dynamic analysis is performed by explicit dynamics numerical modeling. Material used for five asymmetric factor is AISI 1045. Two variables applied on this spur gears are torque loads and rotational speeds to investigate that effect on bending stress. Torque loads varied from 100 Nm to 250 Nm with 50 Nm increment and rotational speeds varied from 1000 rpm to 2500 rpm with 500 rpm increment. The results of this study show that the asymmetric factor can reduce more significantly bending stress cause of torque loads than cause of rotational speeds.

Keywords: spur gears, symmetric gear, asymmetric gear, torque, rotational speed, bending stress, explicit dynamics.

1. Introduction
Symmetric spur gear is the type of gear most often used in transmission systems. One of the advantages of this gear is that it can be used to rotate clockwise (CW) or counter clockwise (CCW). In industrial or automotive applications often symmetric gear is operated in only one direction. This is of course less optimal use because of a shorter service life. There are several ways to decrease the bending stress of the symmetric spur gear. Four of them are teeth geometries [1,2], the materials [3-5], the lubrications [6,7], and backlash selections [8]. Based on Lewis Equation, geometries selection of spur gear consists of sizes of the teeth (module/diametral pitch), contacts angle, teeth width, and profile shapes. Besides those factors, another factor is the installation factor, namely misalignment will increase the bending stress [9].

There are a lot of researches that have been done regarding the modification of the dimensions of the gear to decrease bending stress. However, those studies are only limited to gears with symmetric involute gear profiles. Asymmetric involute spur gears can be formed from two involute profiles with different contact angles. For asymmetric involute spur gear studies, several researchers have performed. They worked to derive mathematical formulation, tools
design to get asymmetric involute profile, testing in laboratory specimen, and practical implementation. These are some of their works [10-12] where they worked not present numerical dynamic modeling yet.

1.1. Asymmetric factor
Asymmetric factor (K) is a formulation that is used to determine how much the level of asymmetric. Symmetric involute profile side is involute profile that is used on a standard profile where contact angle are 14.5°, 20°, and 25°. For this study, contact angle 14.5° is used. The K value is obtained by comparing the base diameter of the asymmetric side with the symmetric side base diameter or comparing the asymmetric side contact angle with the symmetric side contact angle. For this study, asymmetric side profile formed from contact angles of 22.77°, 24.03°, 25.20°, 26.31°.

![Figure 1. Asymmetric gears](image)

So K is formulated as follows:

\[ K = \frac{d_{ba}}{d_{bs}} = \frac{\cos \theta_a}{\cos \theta_s} \]  

(1)

where

- \( K \) : asymmetric factor
- \( d_p \) : pitch diameter
- \( d_{ba} \) : base diameter on the asymmetric side
- \( d_{bs} \) : base diameter on symmetric side
- \( \theta_a \) : contact angle on asymmetric side
- \( \theta_s \) : contact angle on symmetric side

1.2. Dynamic factor on bending stress
The bending stress analysis of the gears is based on the static analysis of the Lewis Equation while other factors are obtained by correction factors and expressed on AGMA equation.

\[ \sigma_b = \frac{F_t}{b.m.Y} \]  

(Lewis Equation)

(2)

where

- \( \sigma_b \) : bending stress
- \( b \) : face width
- \( Y \) : Lewis Factor
- \( F_t \) : tangential force
- \( m \) : module
One of them is speed factor or dynamic factor. The magnitude of these factors is obtained empirically and experimentally. For AGMA velocity factor is expressed in Barth equation (2). But in this study, dynamic factor is performed by numeric explicit dynamics model.

$$K_v = \frac{6.1 + V}{6.1}$$  \hspace{1cm} \text{(cut or milled profile)}  \hspace{1cm} (3)$$

where

- \(K_v\) : velocity factor
- \(V\) : pitch line velocity

1.3. Dynamic interference
When high torque loads applied with high rotational speed, deflection of gear tooth becomes higher than static condition. This event will cause insufficient available backlash so that dynamic interference occurs. In addition, due to the flexibility tooth of the gear, the gear mesh will not occur continuously along the tooth profile but becomes intermittent [13]. Dynamic interference will cause bending stress to increase sharply.

2. Methodology
The method performed in this study is numerical modeling by explicit dynamics with the finite element method. Geometries of asymmetric mating spur gear used in this study are the same pitch diameters 120 mm, the number of teeth 30, the module 4 and the contact angle of the symmetrical side is 14.5\(^\circ\). Detail of the asymmetric mating spur gear geometry could be seen in the following table:

| No | Parameter                                      | Unit | Value  |
|----|-----------------------------------------------|------|--------|
| 1  | Asymmetric factor                             | Unit | Value  |
| 2  | Number of teeth                                | 30   | 30     |
| 3  | Module                                        | \(\text{mm}\) | 4     |
| 4  | Pressure angle for symmetric side              | \(\text{deg}\) | 14.5  |
| 5  | Pressure angle for asymmetric side             | \(\text{deg}\) | -     |
| 6  | Pitch circle diameter                          | \(\text{mm}\) | 120   |
| 7  | Base diameter for symmetric side               | \(\text{mm}\) | 116.1 |
| 8  | Base diameter for asymmetric side              | \(\text{mm}\) | -     |
| 9  | Addendum                                      | \(\text{mm}\) | 4.00  |
| 10 | Dedendum                                      | \(\text{mm}\) | 3.46  |
| 11 | Bottom clearance                               | \(\text{mm}\) | 0.20  |
| 12 | Center of distance                             | \(\text{mm}\) | 120   |

Asymmetric factor (K) performed is shown in 2D on figure 2.a and its comparison can be seen in figure 2.b.
Figure 2. Asymmetric factor and profiles comparison

From the figure 2.b above it can be seen that the asymmetric factor K is getting bigger, then the tooth root width is also getting wider. Thus it will cause bending stress on the tooth root to decrease due to increased width of the tooth root. On the contrary the tip of the upper teeth is getting smaller thus increasing the backlash width which can reduce dynamic interference. The material used is AISI 1045 with detail it properties can be seen in table 2 below.

Table 2. Material Properties

| Material Property                  | Unit                        |
|-----------------------------------|-----------------------------|
| Material                          | AISI 1045                   |
| Density                           | (7.7-8.03)1000 kg/m³        |
| Coefficient of Thermal Expansion  | $1.2 \times 10^{-5}$ °C     |
| Reference Temperature             | 22 °C                       |
| Young Modulus                     | 200 GPa                     |
| Poisson’s ratio                   | 0.27-0.30                   |
| Bulk Modulus                      | 167.67 GPa                  |
| Shear Modulus                     | 76.92 GPa                   |
| Interpolation                     | Semi log                    |
| Strength Coefficient              | 920 MPa                     |
| Strength Exponent                 | -0.106                      |
| Ductility Coefficient             | 0.213                       |
| Ductility Exponent                | -0.47                       |
| Yield Strength                    | 505 MPa                     |
| Compressive Yield Strength        | 250 MPa                     |
| Tensile Ultimate Strength         | 250 MPa                     |
| Hardness                          | 170 BHN                     |
| Elasticity Modulus                | 190 GPa                     |

The next step in modeling after getting the geometry image and material data is meshing. The following is the meshing process carried out on a model that has been designed. Because of the model is very complex shape, the meshing method used is the sweep method with explicit physic preference and relevance 40. The definition of contact is given on all of teeth surface profiles that allow to contact / impact between surfaces. The type of contact used is frictional with symmetric behavior, whereas the joint is put on shaft part of each spur gear.
In this simulation the connection used is a revolute body to ground connection. So by using this connection, it is assumed that the gears are attached to the rotating shaft on the bearing. The next parameter is applying the loads on the model.

The loading process is the process of applying the loads on the model. The loading used are rotational speed (rpm), torque and standard earth gravity (acceleration of gravity). Based on the loading, this study consists of two types of simulation. The first simulation are performed where torque loads applied at constant rotational velocity (1000 rpm) and the second simulation are performed where rotational speed loads applied at torque load constant (100 Nm). Rotational speeds varied from 1000 rpm to 2500 rpm with 500 rpm increment, while torques varied from 100 Nm to 250 Nm with 50 Nm increment.

### 3. Results and Discussions

From the results of explicite dynamics numerical calculations obtained stress distribution as a function of time. Because of the time of calculation is very long, the modeling is only performed when only one pair of teeth starts to contact until leaving the contact. Stress distribution on mating spur gears at certain time can be seen in figure 5.a where a large amount of stress occurs at the root of the tooth due to bending and the contact surface. For one contact periode of one pair teeth of spur gear from initial contact until leaving contacts are shown graphically on figure 5.b.
Because of flexibility of spur gear tooth, it will be deflected and penetration occurs on contact area (figure 6 a). Then period of relaxation occurs so that the pair of teeth lose contact with each other for a moment (figure 6 b) and then they occur contact again. During initial contact until the end of contact occurs several times contact and release contact. Otherwise, if modeling using rigid material, the contact will be smooth from the initial contact to the end of the contact.

For modeling comparisons, calculations are performed statically with Lewis equations at $K = 1$ or symmetric gear. The comparison is only done at a torque load of 100 Nm with a rotating speed of 1000 rpm. With the geometry used, the Lewis Factor ($Y$) is 0.359. While due to a torque load of 100 Nm and a pitch diameter of 120 mm, the tangential force is obtained $F_t = 1667$ Newton. With a rotational speed of 1000 rpm using the Barth's equation the dynamic factor $K_v = 2$. With a spur gear width of 14 mm using the Lewis equation obtained Bending Stress 165.86 MPa. While the simulation results obtained 174.9 MPa. Thus there is a difference of 5.27%. Considering that what is analyzed is bending strength, the maximum bending stress taken for all variations as can be seen in the graph on figure 7 and figure 8.

3.1 **Effect of torque loads on constant rotational speed 1000 rpm to bending stresses**

The influence of the magnitude of the torque load on bending stress is seen in the figure 7 where the torque load is on the abscissa axis and the bending stress is on the ordinate axis whereas the asymmetric factor is expressed in color.
At \( K = 1 \) or symmetry spur gear, due to torque loads from 100 Nm to 150 Nm, the bending stress also rises but with a low gradient, but at a load of 200 Nm and 250 Nm bending stress rises with a higher gradient. This is because the tooth deflection has grown so that the available backlash is smaller than the deflection that occurs resulting in dynamic interference. At \( K = 1.05 \) and \( K = 1.06 \) the bending stress that occurs decreases compared to \( K = 1 \). This is because the width of the root tooth wider. With the increase in torque load, the bending stress also increases but with a gentle gradient to the torque load. This shows that dynamic interference has not yet occurred. At \( K = 1.07 \) there is also a decrease in bending stress compared to lower \( K \). Dynamic interference began to occur at a load of 200 Nm where bending stress gradient increase sharply and at \( K = 1.08 \), bending stress is also still decrease and is lower than others \( K \). Interference occurred at a load of 250 Nm.

The magnitude of the reduction in bending stress in the percentage of 100 Nm of torque load when compared with \( K = 1 \), for each was \( K = 1.05 \) by 19.9%, \( K = 1.06 \) by 30.4%, \( K = 1.07 \) by 70.2% and \( K = 1.08 \) by 88.9%. If seen decrease still proportionally due to pure increase in the width of the root of tooth. At a load of 150 Nm with the same calculation resulting in a decrease in bending stress respectively 15.2%, 34.8%, 64.1%, 81.8%. Like a decrease in 100 Nm of torque, the decrease is still proportional. At a load of 200 Nm, the reduction in bending stress was 31.8%, 44.4%, 45.6% and 84.1%, respectively. At \( K = 1.06 \) and \( K = 1.07 \) decrease in bending stress is relatively the same because interference starts at \( K = 1.07 \). And finally for 250 Nm of torque load, there was a decrease in bending stress 38.9%, 39.7%, 51.9% and 57.8%. Because all levels of interference have occurred, the strain reduction is relatively not much different.

So it can be concluded that bending stress will increase with increasing torque loads and increasing of asymmetric factor could reduce the bending stress.

### 3.2 Effect of rotational speed on constant torque load 100 Nm to bending stress

The result of the effect of rotating speed on a constant load (100 Nm) is graphed as shown figure 12 where the abscissa axis is the rotational speed and the ordinate axis is the bending stress. Asymmetric levels are expressed in color. The rotation is varied from 1000 rpm to 2500 rpm with a constant increment of 500 rpm. With the increasing of rotational speed at constant torque, the...
power generated will also increase so that the bending stress that occurs also increases for all asymmetric levels.

Figure 8. Rotational speeds vs bending stresses

In symmetric gear (K = 1), for all rotational speed gradients have approach the same at each rotational speed increase. This is in accordance with Barth Equation. With a radius of 58.05 mm, the \( K_r \) are 2; 2.5; 3.0; 0.3.5 respectively for each rotational speed. According to Lewis Equation, for \( b, Y \) and \( F_t \) are constants, the bending stress to rise by 2; 2.5, 3.0, 3.5 of the static conditions (\( K_r=1 \)). However, the modeling that has been done is in dynamic conditions so the reference is rotational speed at 1000 rpm with \( K_r=2 \). Thus, because of the increasing of rotational speed is 500 rpm, so the increase in the speed factor (\( \Delta K_r \)) is the same value 0.5 or constant. The consequence is that the increase in bending stress is proportional to the rotational speed.

At asymmetric \( K = 1.05 \) the bending stress drop is quite significant from the symmetrical gear \( K = 1 \). This is due to a significant decrease in base diameter from 116.1 mm to 110.6 mm or 5.5 mm reduction, whereas at \( K = 1.06, K = 1.07 \) and \( K = 1.08 \) where the asymmetric factor increase the same increment 0.01 there is a decrease in the diameter of the base of 1 mm, 1.02 mm, 1.01 mm respectively so that the decrease in bending stress is not significant.

Due to asymmetric factor, the decrease in bending stress at rotational speed 1000 rpm is from 33.26% to 46.64%, at 1500 rpm is from 19.53% to 29.82%, at 2000 rpm torque is from 29.93% to 34.36 % and at 2500 rpm is from 34.48% to 39.11%. Overall, the reduction in bending stress that occurred from 19.53% to 39.11%.

4. Conclusion
The modelling of asymmetric spur gear have performed successfully in explicit dynamics with \( K=1, K=1.05, K=1.06, K=1.07, K=1.08 \). Based on the results that have been obtained can be concluded

- Torque load can reduce significantly bending stresses on root tooth from 15.2% to 88.9% depend on torque load and asymmetric factor.
- Rotation speed can reduce bending stresses but not significantly like torque load from19.53% to 39.11%.

Although the above results have not been validated by experiment, but it has been proven that asymmetric can reduce bending stress in accordance with the hypothesis and qualitatively the reduction in bending stress is significant enough.
5. References

[1] Yilmaz T G and Karpat F, 2018 Influence of root geometry on bending stress for Involute spur Gears Proceedings of the 4th World Congress on Mechanical, Chemical, and Material Engineering (MCM’18) 124 1-8.

[2] Murali M.V and Ajit Prasad S.L, 2016 International Journal of Mechanical Engineering and Robotics Research 5(3) 224-228, Influence of module and pressure angle on contact stresses in spur gears.

[3] Townsend D.P, Zaretsky E.V, 1980 Endurance and failure characteristics of modified vascox-2CBS 600 and AISI 9310 spur gears”, NASA TM 81421.

[4] Townsend D.P, 1985, Surface fatigue life and failure characteristics of EX-53, CBS 1000 M, and AISI 9310 gear materials”, NASA TP 2513.

[5] Gorla C at al, 2017 International Journal of Applied Engineering Research 12 11306-11322, Bending fatigue strength of case carburized and nitrided gear steel for aeronautical applications.

[6] Lubrecht A. A, et el, 2009 Film thickness calculation in elastohydrodynamics lubricated line and elliptical contact, Proceeding. Inst. Mech. Engineering. Part J 223 511-515.

[7] Li. S and Kahraman A., 2011 Journal Engineering Tribology 225 740-753, Influence of dynamic behaviour on elastohydrodynamics lubrication of spur gears.

[8] Shinde D and Mangrulkar K.S, 2016 International Journal of Scientific Development and Research 1(7) 349-354, Effect of backlash on bending stress in spur gears.

[9] Prabhakaran S, Balaji D S, and Joel C, 2014 International Journal of Applied Engineering Research 9(22) 13061-13071, Stress analysis and effect of misalignment in spur gear.

[10] Kapelevich A , 2009 Direct design of asymmetric gears: approach and application, JSME International Conference on Motion and Power Transmissions, 87-91.

[11] Kapelevich A, 2016 Theory and Practice of Gearing and Transmissions, Mechanisms and Machine Science, Direct gear design for asymmetric tooth Gears, Springer International Publishing Switzerland.

[12] Alipiev, O., 2010 Geometric design of involute spur gear drives with symmetric and asymmetric teeth using the Realized Potential method, Mechanism and Machine Theory 46 10-32, Elsevier.

[13] Gourinat Yves and Pramono Agus Sigit, 1999 Educational application of photoelastodynamics for solid mechanics and dynamics of structures, Machine Graphics and Vision 8 (4) 1-13.