Effective $\bar{K}N$ interaction with strong $\pi\Sigma$ dynamics constrained by chiral SU(3) symmetry

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Abstract We derive a single-channel effective $\bar{K}N$ interaction from chiral SU(3) coupled-channel dynamics, emphasizing the important role of the $\pi\Sigma$ channel and the structure of the $\Lambda(1405)$ resonance. The chiral low energy theorem requires strongly attractive interaction not only in the $\bar{K}N$ channel but also in the $\pi\Sigma$ channel. As a consequence of the strong $\pi\Sigma$ dynamics, the equivalent potential in single $\bar{K}N$ channel turns out to be less attractive than the one used in a purely phenomenological approach.

Keywords Chiral SU(3) dynamics · $\Lambda(1405)$ resonance · kaonic nuclei

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1 Introduction

The study of $\bar{K}$-nuclear systems (kaonic nuclei) attracts considerable interest in nuclear and hadron physics. It was suggested in a phenomenological approach [1] that the strongly attractive $\bar{K}N$ interaction could form $\bar{K}$-nuclear bound systems with interesting properties. Concerning the simplest $K^-pp$ system, an experiment has performed to search for the possible bound state [2] while the interpretation of the results is still controversial [3]. In recent experimental studies [4, 5], a broad peak structure in the $\Lambda N$ invariant mass spectrum is reported.

For a quantitative estimation of the properties of the antikaon binding in few-nucleon systems, we need to perform rigorous few-body calculation using realistic potentials. The $K^-pp$ system has recently been studied in two approaches: Faddeev calculations in Refs. [6, 7] and a variational calculation with local potentials in Ref. [8]. For a variational calculation of the few-body kaonic nuclei, it is desired to construct a realistic $\bar{K}N$ interaction which incorporates the dynamics of the $\pi\Sigma$ channel. At this
point, it should be noted that the relevant energy region for the study of kaonic nuclei is far below the $\bar{K}N$ threshold, so the result would be sensitive to the extrapolation of the $\bar{K}N$ interaction into this energy region. The only available information is the invariant mass spectrum of $\pi\Sigma$ channel, which is dominated by the $\Lambda(1405)$ resonance. These facts indicate that we need a careful assessment of the single-channel $\bar{K}N$ interaction together with a proper treatment of the $\Lambda(1405)$ resonance.

For this purpose, we rely upon the following guiding principles for the description of the $\bar{K}N$ scattering: chiral symmetry and coupled-channel dynamics. Chiral symmetry of QCD determines the low energy interaction between the pseudoscalar meson (the Nambu-Goldstone boson) and any target hadron [9], and the importance of the coupled-channel dynamics has been emphasized in the phenomenological study of $\bar{K}N$ scattering [10]. Theoretical framework based on these principles has been developed as the chiral coupled-channel approach, reproducing successfully the $\bar{K}N$ scattering data and the properties of the $\Lambda(1405)$ resonance [11]. Here we derive an effective $\bar{K}N$ interaction based on chiral dynamics [12], and discuss its phenomenological consequence in the study of $K^-pp$ system [13].

2 Chiral low energy interaction and coupled-channel approach

In the leading order of chiral perturbation theory, meson-baryon s-wave interaction at total energy $\sqrt{s}$ from channel $j$ to $i$ reads

$$ V_{ij}(\sqrt{s}) = -C_{ij} \frac{4}{f^2} (2\sqrt{s} - M_i - M_j) \sqrt{\frac{E_i + M_i}{2M_i}} \sqrt{\frac{E_j + M_j}{2M_j}} \sim -C_{ij} \frac{4}{f^2} (\omega_i + \omega_j), \quad (1) $$

where $f$ is the pseudoscalar meson decay constant, $M_i$, $E_i$, and $\omega_i$ are the mass of the baryon, the energy of the baryon, and the energy of the meson in channel $i$, respectively. The coupling strengths $C_{ij}$ are collected in the matrix

$$ C^{I=0}_{ij} = \begin{pmatrix} 3 - \sqrt{2} & 3 & 0 \\ -\sqrt{2} & 4 & 0 \\ 0 & 0 & -\sqrt{2} \end{pmatrix}, $$

for the $S = -1$ and $I = 0$ channels in the following order: $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$, and $K\Xi$. The important point is that the properties of the interaction—sign, strength, and energy dependence—are strictly governed by the chiral theorem. One observes that the interactions in both $\bar{K}N$ and $\pi\Sigma$ channels are attractive, which is inevitable as far as we respect chiral symmetry. As we will see below, these attractive forces are so strong that pole singularities of the amplitude are generated for both channels.

Since the system is strongly interacting, we need to perform nonperturbative resummation. In Refs. [11,12] this has been achieved by solving the Bethe-Salpeter equation

$$ T_{ij}(\sqrt{s}) = V_{ij}(\sqrt{s}) + V_{il}(\sqrt{s}) G_l(\sqrt{s}) T_{lj}(\sqrt{s}), \quad (2) $$

with the interaction kernel $V_{ij}$ in Eq. (1) and the meson-baryon loop integral $G_i$ in dimensional regularization. The solution of Eq. (2) is given in matrix form by $T = [V^{-1} - G]^{-1}$ under the on-shell factorization. This form of the amplitude is also obtained in the N/D method by neglecting the contributions from the left-hand cut. This solution guarantees the unitarity of the scattering amplitude.
3 Structure of the $\Lambda(1405)$ resonance

It has been shown that the amplitude in this framework reproduces the experimental observables of $\bar{K}N$ scattering very well, by properly choosing the subtraction constants in the loop function \[11, 14\]. The dynamically generated resonance is then expressed by a pole of the scattering amplitude in the complex energy plane. An interesting observation is that the $\Lambda(1405)$ resonance is associated by two poles \[15\]. Using the model given in Refs. \[14\], we find the poles at

\[z_1 = 1428 - 17i \text{ MeV}, \quad z_2 = 1400 - 76i \text{ MeV},\]

which appear above $\pi\Sigma$ threshold and below $\bar{K}N$ threshold. Since the two poles locate close to each other, the observed spectrum exhibits only one bump structure, which was interpreted as a single resonance, the $\Lambda(1405)$. The coupling strengths of the poles to the $\pi\Sigma$ and $\bar{K}N$ channels are different from one to the other. Therefore, these poles contribute to the $\bar{K}N$ and $\pi\Sigma$ amplitudes with different weights, leading to the different spectral shapes of two amplitudes \[15\].

Here we study the origin of this interesting structure. In order to isolate the contribution of each channel, we perform the resummation of the single-channel interaction by switching off the couplings to the other channels. This single channel $\bar{K}N$ interaction generates a relatively weak bound state below threshold, while the $\pi\Sigma$ amplitude exhibits a broad resonance above threshold:

\[z_1(\bar{K}N \text{ only}) = 1427 \text{ MeV}, \quad z_2(\pi\Sigma \text{ only}) = 1388 - 96i \text{ MeV}.\]

In this way, the attractive forces in diagonal $\bar{K}N$ and $\pi\Sigma$ channels already generate two poles between thresholds. We plot the positions of these poles in Fig. 1 together with the poles in the full amplitude in the coupled-channel framework. The figure obviously suggests that the pole $z_1(\bar{K}N \text{ only})$ is the origin of the pole $z_1$, whereas $z_2(\pi\Sigma \text{ only})$ evolves to the pole $z_2$. This observation agrees with the qualitative behavior discussed in Ref. \[15\]: the pole $z_1$ strongly couples to the $\bar{K}N$ channel and the pole $z_2$ to the $\pi\Sigma$ channel.

It is interesting to note that the higher energy $\bar{K}N$ channel has stronger attraction ($\sim 3\omega_{K}$) to generate a bound state, and the lower energy $\pi\Sigma$ channel shows the relatively weaker attraction ($\sim 4\omega_{\pi}$), which is nevertheless strong enough to create a resonance. The appearance of the two poles in this energy region is caused by the
valence of the two attractive forces. As we emphasized in the previous section, the meson-baryon interaction is governed by the chiral low energy theorem. Hence, we consider that the two-pole structure of the $\Lambda (1405)$ is a natural consequence of chiral symmetry.

4 Effective single-channel interaction

Keeping the structure of the $\Lambda (1405)$ in mind, we construct an effective single-channel $\bar{K}N$ interaction which incorporates the dynamics of the other channels 2-4 ($\pi\Sigma$, $\eta N$, and $K\Xi$). We would like to obtain the solution $T_{11}$ of Eq. (2) by solving a single-channel equation with kernel interaction $V^{\text{eff}}$, namely,

$$T^{\text{eff}} = V^{\text{eff}} + V^{\text{eff}} G_1 T^{\text{eff}} = T_{11}. $$

Consistency with Eq. (2) requires that $V^{\text{eff}}$ be the sum of the bare interaction in channel 1 and the contribution $\tilde{V}_{11}$ from other channels:

$$V^{\text{eff}} = V_{11} + \tilde{V}_{11}, \quad \tilde{V}_{11} = \sum_{m=2}^{4} V_{1m} G_m V_{m1} + \sum_{m,l=2}^{4} V_{1m} G_m T_{m}^{(3)} G_l V_{l1}, \quad (3)$$

where $T_{m}^{(3)}$ is the $3 \times 3$ matrix with indices 2-4, and expresses the resummation of interactions other than channel 1. Note that $\tilde{V}_{11}$ includes iterations of one-loop terms in channels 2-4 to all orders, stemming from the coupled-channel dynamics. This is an exact transformation, as far as the $\bar{K}N$ scattering amplitude is concerned.

The effective $\bar{K}N$ interaction $V^{\text{eff}}$ is calculated within a chiral coupled-channel model [14]. It turns out that the $\pi\Sigma$ and other coupled channels enhance the strength of the interaction at low energy, although not by a large amount. The primary effect of the coupled channels is found in the energy dependence of the interaction kernel. In the left panel of Fig. 2 we show the result of $\bar{K}N$ scattering amplitude $T^{\text{eff}}$, which is obtained by solving the single-channel scattering equation with $V^{\text{eff}}$. The full amplitude in the $\pi\Sigma$ channel is plotted in the right panel for comparison. It is remarkable that the resonance structure in the $\bar{K}N$ channel is observed at around 1420 MeV, higher than the nominal position of the $\Lambda (1405)$. What is experimentally observed is the spectrum...
in the $\pi\Sigma$ channel, whose peak is in fact located close to 1405 MeV. The discrepancy of the shapes of the two amplitudes is a consequence of the existence of two poles with different coupling strengths [15].

The deviation of the resonance position has a large impact to the single-channel $\bar{K}N$ potential, which should be constructed so as to reproduce the scattering amplitude in $\bar{K}N$ channel. Measuring from the $\bar{K}N$ threshold ($\sim 1435$ MeV) we find the “binding energy” of the $\Lambda(1405)$ as $\sim 15$ MeV, not as the nominal value of $\sim 30$ MeV. This shift of the binding energy apparently reduces the strength of the potential.

5 Equivalent local potential

Next we construct an equivalent local $\bar{K}N$ potential in coordinate space. We consider an $s$-wave antikaon-nucleon system in nonrelativistic quantum mechanics, whose radial wave function $u(r)$ follows the Schrödinger equation with the potential $U(r, E)$. As explained in detail in Ref. [12], the local potential $U(r, E)$ has been constructed such that the scattering amplitude in coupled-channel approach is reproduced in this system. Note that this is not an exact transformation, since it is not guaranteed that a simple local potential can reproduce the complicated coupled-channel dynamics. Nevertheless, we have constructed a complex and energy-dependent $\bar{K}N$ potential with the gaussian form of the spatial distribution, which well reproduces the coupled-channel results.

It is instructive to compare our potential with the phenomenological Akaishi-Yamazaki (AY) potential [13], whose form in $\bar{K}N$ and $\pi\Sigma$ channels is given by

$$v_{ij}(r) = \begin{pmatrix} -436 & -412 \\ -412 & 0 \end{pmatrix} \exp[-(r/b)^2] \text{ [MeV]},$$

with $b \sim 0.66$ fm. The interaction is qualitatively different from the chiral interaction, namely, the absence of a direct $\pi\Sigma \rightarrow \pi\Sigma$ coupling. The strength of the phenomenological interaction is determined by assuming the resonance position in $\bar{K}N$ channel is the same with the PDG value of the $\Lambda(1405)$ resonance. In this framework, the $\Lambda(1405)$ is described as a Feshbach resonance: $\bar{K}N$ quasibound state embedded in the $\pi\Sigma$ continuum.

Chiral SU(3) dynamics, on the other hand, leads to the quasibound structure in the $\bar{K}N$ system at around 1420 MeV, because of the strong $\pi\Sigma$ diagonal interaction. As a result, the equivalent local $\bar{K}N$ potential is less attractive. This is inevitable, as the chiral low energy theorem restricts the structure of the meson-baryon interaction. In this framework, as in the AY potential, the driving force to generate the $\Lambda(1405)$ is the $\bar{K}N$ attraction, but the chiral low energy theorem requires the $\pi\Sigma$ continuum to be also strongly interacting. As a consequence, we observe an interesting structure: $\bar{K}N$ quasibound state embedded in the resonating $\pi\Sigma$ continuum.

It should be however noted that both chiral and phenomenological amplitudes behave similarly above the $\bar{K}N$ threshold, since the potentials are adjusted to describe experimental data. The difference appears in the treatment of the $\pi\Sigma$ interaction and extrapolation of the $\bar{K}N$ amplitude to the subthreshold energy region. In other words, the existing experimental database, by itself, is not sufficient to constrain the $\bar{K}N$ interaction in the energy region relevant for the antikaon-nucleon physics.

1 Strictly speaking, the observed $\pi\Sigma$ spectrum should be described by the sum of the possible initial states $\sum_i C_i |T_{i \rightarrow \pi\Sigma}|^2$ [13]. To obtain the relative weights $C_i$, we need to introduce a model for each reaction process as done in Refs. [13].
6 Summary

We have derived an effective $\bar{K}N$ interaction based on chiral low energy theorem and the coupled-channel dynamics. We show that the model-independent chiral interaction leads to the strongly interacting $\pi\Sigma$-$K\bar{N}$ system, in which the $\Lambda(1405)$ is described as the $\bar{K}N$ quasibound state embedded in the resonating $\pi\Sigma$ continuum. We construct an equivalent local potential in single $\bar{K}N$ channel, which represents the effect of coupled-channel dynamics through the imaginary part and energy dependence. As a consequence of the strong $\pi\Sigma$ dynamics, the resulting potential is less attractive than the purely phenomenological potential in the energy range relevant to the discussion of deeply bound kaonic nuclei.

It is worth emphasizing that there is no direct experimental constraint on the $\bar{K}N$ amplitude below threshold. We have to extrapolate $\bar{K}N$ interaction calibrated by scattering data above threshold, down to the relevant energy scale. Here we utilize the principle of chiral SU(3) symmetry in order to reduce the ambiguity of the extrapolation. The precise knowledge of the threshold $\bar{K}N$ data and the $\pi\Sigma$ mass spectrum will be important for the prediction of the kaonic nuclei.

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