Scaling Relationships of Dissipation-Induced Pavement–Vehicle Interactions

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Rolling resistance is one of the key factors that affect the fuel efficiency of the national pavement system. In addition to pavement texture and pavement roughness, the dissipation of mechanical work provided by the vehicle because of viscous deformation within the pavement structure has been recognized as a relevant factor contributing to the environmental footprint of pavement systems. This dissipation depends on material and structural parameters that can be optimized to increase the fuel efficiency of pavements. Identifying the key material and structural parameters that drive this dissipation is the focus of this paper. This identification is achieved by a combination of dimensional analysis and model-based simulations of the dissipation of a viscoelastic beam on an elastic foundation. For linear viscoelastic systems, the dissipation is found to scale with the square of the vehicle weight and with the inverse of the viscous relaxation time, in addition to distinct power relations of top-layer stiffness, thickness, and subgrade modulus. These scaling relations can be used by pavement engineers to reduce such pavement-inherent dissipation mechanisms and increase the fuel efficiency of a pavement design. An example shows the application of these scaling relations with data extracted from FHWA’s Long-Term Pavement Performance database for seven road classes. The scaling relations provide a means for evaluating the performance of the various road classes in terms of the fuel efficiency related to dissipation.

The need to enhance the sustainability of the pavement system requires quantitative engineering models that relate pavement structure, pavement condition, and materials to rolling resistance, fuel consumption, and related greenhouse gas emissions. Besides pavement texture and pavement roughness, the dissipation of mechanical work provided by the vehicle because of viscous deformation within the pavement structure was recognized some 40 years ago as a relevant factor contributing to rolling resistance. In fact, in his famous book Viscoelasticity, in the conclusion of his analysis of the viscoelastic response of a Kelvin beam on an elastic foundation to a moving load, showing that the vehicle load is on an upward slope, Wilhelm Flügge noted that “the load moving with the velocity \( c \) has to do work,” and that the associated horizontal force “supplies the energy needed for the viscoelastic deformation” (1, pp. 92–93). He continued that “this phenomenon, well known and occurring in various situations, does not stand in common text books.”

The phenomenon has indeed been observed both experimentally and theoretically in many pavement and railroad mechanics studies (2–4). It has gained new attention more recently in the context of the development of engineering methods for the sustainable design of pavements, accounting for and eventually reducing the generation of greenhouse gas emissions during the use phase of pavements (5–7), especially for roads with high traffic volume. In contrast to the impact of roughness-induced pavement–vehicle interactions (PVI) that depend primarily on vehicle characteristics (8), excess fuel consumption caused by viscous energy dissipation depends on material and structural parameters that can be influenced by the pavement engineer. The study presented in this paper derives scaling relations from a combination of dimensional analysis and model-based simulations of a simple viscoelastic beam model.

THEORETICAL BACKGROUND

Consider a pavement structure subjected to a load \( P \) moving at a constant speed \( c \) in the \( x \)-direction (Figure 1). For any irreversible deformation that takes place in the pavement structure, energy is dissipated; this energy needs to be compensated by additional vehicle power and hence fuel consumption. There are a priori two ways of evaluating the dissipation rate \( \mathcal{D} \), that is, the amount of work rate \( \delta W \) that is not stored into recoverable (elastic) energy but is dissipated into heat form. One approach is based on use of a fixed reference frame. This approach, used by, for example, Pouget et al. (5), typically employs finite elements for estimating the time history of the displacement field in a sufficiently large block of pavement (to minimize the effects of boundary conditions). With the classical finite element procedure, stresses and viscoelastic strains are determined, and the dissipated energy is obtained by integrating over the entire pavement system. The second approach considers a moving coordinate frame that moves at the speed of the vehicle. In the vein of Flügge’s conjecture, because of the presence of a dissipative mechanism in the system, the vehicle is always on an uphill slope, leading to an additional horizontal force supplied by the vehicle, which is added to the rolling resistance and thus to fuel consumption. The dissipation rate \( \mathcal{D} \) in this approach is obtained from writing the external work rate of the force in the moving coordinate system, \( X = x - ct \), where \( x \) and \( t \) are space and time variables, respectively, in the fixed coordinate system, and by assuming steady state conditions under constant speed \( c \) [i.e., Lagrangian derivative, \( \partial \omega / \partial t = -c \partial \omega / \partial X \)], so that (6, 9)

\[
\mathcal{D} = \delta W = P \frac{dw}{dt} = -cP \frac{dw}{dX} \geq 0 \tag{1}
\]
where

\[ P = \text{axle load}, \]
\[ w = \text{deflection at point of load application}, \]
\[ \frac{dw}{dX} = \text{slope}. \]

Hence, for the case of an elastic material with no dissipation, the slope is \( \frac{dw}{dX} = 0 \), which means that the tire is at the bottom of the deflection basin. However, if dissipation occurs in the pavement structure (for instance, because of viscous deformation mechanisms), the nonnegativity of the dissipation, Equation 1, requires that \( \frac{dw}{dX} < 0 \). This is Flügge’s conjecture, which he based on solving the viscoelastic beam problem (Ⅰ, pp. 92–93), but which is also a thermodynamic requirement: where the load is applied, the beam has an upward slope (Figure 1). For practical purposes, it is often more useful to translate the dissipation rate \( D \) (of dimension energy/time) into the amount of excess energy per pavement length the vehicle needs to spend to maintain constant velocity. This is achieved by dividing \( D \) by \( c \), that is,

\[ \delta E = \frac{D}{c} = -P \frac{dw}{dX} \tag{2} \]

where \( E \) is Young’s modulus.

This approach, which considers the slope as added grade, has been used by Akbarian et al. (7) and is often referred to as the deflection-induced PVI approach. It is, however, strictly equivalent to the so-called dissipation-induced PVI approach. A previous work devoted to a first-order estimate of the dissipation estimated the slope \( \frac{dw}{dX} \) from the maximum deflection of a beam on a viscoelastic foundation, using

\[ \frac{dw}{dX} \approx \frac{w_{\text{max}}}{\pi \ell_s} \]

where

\[ \ell_s = \left( \frac{12}{Eh^3} \right)^{1/4} \]

is the Winkler length (with \( h \) the top layer thickness, and \( k \) the subgrade modulus) (7). This approach, although simple, failed to capture both the velocity and the temperature dependence of the rolling resistance as shown in some studies. The present contribution explicitly addresses this issue by considering the viscoelastic response of the top layer. In this way the question is addressed of how the dissipation rate, as expressed by Equation 1, scales with material and structural properties of the pavement.

**DIMENSIONAL ANALYSIS**

A convenient way to screen possible invariants of material and structural parameters is to perform a dimensional analysis. To reduce the complexity of the pavement system, consider the dissipation rate of a viscoelastic beam of width \( b \) on an elastic foundation (subgrade modulus \( k \)) subjected to moving load \( P \). The beam’s elastic response is described by the Winkler length

\[ \ell_s = \left( \frac{12}{Eh^3} \right)^{1/4} \]

and the viscoelastic response is captured by a relaxation time \( (\tau = \eta/E) \). The constant speed \( (c) \) at which the load moves is smaller than the critical velocity

\[ c_r = \ell_s \left( \frac{k}{m} \right)^{1/2} \]

(where \( m \) equals \( ph \), the surface mass density, and \( p \) is the volume mass density). The critical velocity is close to a multiplying factor, the critical resonant frequency. For the dimensional analysis, a relationship between dissipation rate \( \phi \), two load parameters \( (P, c) \), and five material-structural parameters is needed:

\[ \phi = f(P, c, b, \ell_s, k, c_r, \tau) \tag{3} \]

where \( b \) and \( \tau \) are beam width and relaxation time of the viscoelastic top layer, respectively. For the dimensional analysis, an extended base dimension system is adopted that renders an account of the difference of dimensions in a different direction (9). In this system, all quantities involved in Equation 2 are expressed in an \( L, L, L \), \( MT \)-base dimension system, where \( L \) stands for the length dimension \( L \) in the \( i = x, y, z \) directions, \( M \) is mass, and \( T \) is time. For instance, the load \( P \) is applied in the \( z \)-direction and has as dimension function \( [P] = L, MT^{-2} \). Similarly, in this base dimension system, \([c] = [c_r] = L, T^{-1}, [b] = L, [\ell_s] = L, [k] = (L, L, L)^{-1}MT^{-2} \), and \([\tau] = T \). For the determination of the dimension function of dissipation \( \phi \), Expression 1 is used, that is, \([\phi] = [c][P][w]/[X] = L^3, MT^{-3} \). The dimension functions in the form of the exponent matrix of dimension are summarized:

\[
\begin{bmatrix}
0 & [P] & [c] & [b] & [\ell_s] & [k] & [c_r] & [\tau]
\end{bmatrix}
\]

\[
L, 0 0 1 0 1 -1 1 0
\]

\[
L, 0 0 0 1 0 -1 0 0
\]

\[
L, 2 1 0 0 0 0 0 0
\]

\[
M, 1 1 0 0 0 1 0 0
\]

\[
T, -3 -2 -1 0 0 -2 -1 1
\]

The rank of the matrix is \( k = 5 \), which, according to the pi theorem (10), allows one to reduce the dimensional relation (Equation 3) of
$N + 1 = 8$ parameters to $N + 1 - k = 2 + 1$ dimensional relation. That is, if $(P, b, \ell_c, k, c_r)$ are chosen as dimensionally independent parameters, the remaining ones can be expressed as power functions of the former in a dimensionless form as

$$
\Pi = \frac{\partial \zeta^2 \beta k}{P c_r} = F \left( \Pi_{1} = \frac{c}{c_r}, \Pi_{2} = \frac{\tau c_r}{\ell_c} = \zeta \right)
$$

(5)

or in terms of the excess energy consumption $\delta E = \partial/c$:

$$
\Pi = \frac{\delta E^2 \beta k}{P c_r} = F \left( \Pi_{1} = \frac{c}{c_r}, \Pi_{2} = \frac{\tau c_r}{\ell_c} = \zeta \right)
$$

(6)

where $\zeta = \pi (k/m)^{1/2}$ is the damping ratio. A first observation from Equations 5 and 6 is that the excess energy scales as $\partial \propto P^2$. That is, if a force, $P_0$ (i.e., axle load or vehicle mass), is increased by a factor $\lambda$, such that $P = \lambda P_0$, the dissipation rate (respectively, the excess energy) is increased by $\partial = \lambda^2 \partial_0$. Other scaling relations, however, require a solution of the dimensionless function $F$, as shown below.

Finally, from a dimensional analysis point of view, the dimensionless scaling relations, Equations 5 and 6, still hold for an elastic beam on a viscoelastic foundation (7). However, by considering the distinct viscoelastic behavior of the top layer, it is possible to factor into the scaling of dissipation-related fuel consumption environmental effects, such as the dependence of the materials’ relaxation time on temperature (5, 6). This is shown in a later section.

**MODEL-BASED SIMULATIONS OF DISSIPATION CAUSED BY PVI**

Model-based simulations of a viscoelastic beam on an elastic foundation are used here (Figure 2a). Because the focus is on first-order scaling relations of the dissipation in terms of the dimensionless expression (Equation 5), the most simple viscoelastic model is used, that is, a linear viscoelastic Maxwell model of an elastic spring (stiffness $E$), in series with a dashpot (viscosity $\eta$), that define the relaxation time $\tau = \eta/E$ (Figure 2b). For the beam problem, recall the equation of motion for an infinite elastic beam on an elastic foundation in a moving coordinate system (11, 12):

$$
EI \left( \frac{\partial^4 \psi}{\partial X^4} + mc \left( \frac{\partial^2 \psi}{\partial X^2} \right) + kw \right) = P
$$

(7)

where $I = h^3/12$ and $S_c$ is the tire–pavement contact area ($p = P/S_c$ is the contact pressure, which in the moving coordinate system is a constant). Taking the Fourier transform of this differential equation yields

$$
\hat{\psi} = \hat{p} \left( E \lambda^4 - mc \lambda^2 + k \right)^{-1}
$$

(8)

where $\lambda$ is the transformed field of $X$. Evaluation of the deflection of a viscoelastic beam uses the elastic–viscoelastic correspondence principle (13, 14) and substitutes the complex modulus for its elastic

**FIGURE 2**  (a) Viscoelastic beam on elastic foundation, (b) Maxwell model representing viscoelastic behavior of top layer, (c) deflection obtained from Equation 9, and (d) deflection under moving load for normalized velocities ($\Pi_1 = c/\ell_c$, $\Pi_2 = \zeta = \sqrt{k/m}^{1/2}$).
counterpart in Equation 7. For a Maxwell material with the constitutive equation relating the components of stress $\sigma$ and strain $\varepsilon$, $(\sigma + \tau d\sigma/dX)/E = \varepsilon$, there is the moving reference frame, $(\sigma - c\tau d\sigma/dX)/E = -c\tau d\varepsilon/dX$. Then, taking the Fourier transformation, that is, $\delta(1 - c\tau i\lambda)(1 - c\tau i\lambda) = -c\tau i\lambda$, whence, instead of the viscoelastic solution in Fourier domain,

$$\hat{\sigma} = \frac{c\tau i\lambda}{1 - c\tau i\lambda} 1 / E = \frac{-c\tau i\lambda}{1 - c\tau i\lambda}$$

where $\hat{\lambda} = \xi\lambda$ and $\varepsilon = c/c_r$, and $\zeta = \tau(k/m)^{1/2}$ are the invariants identified from dimensional analysis in Equation 5. The dissipation rate is then directly obtained from Equation 1 by inverse Fourier transformation $(F^{-1})$:

$$\varnothing = -cP \frac{d\tau}{dX} = -cP F^{-1}(i\hat{\lambda}\hat{\varepsilon})_{k=0} \geq 0$$ (10)

or, in terms of excess energy consumption, Equation 2:

$$\delta E = \frac{D}{c} = -P F^{-1}(i\hat{\lambda}\hat{\varepsilon})_{k=0} \geq 0$$ (11)

By way of application, Figure 2d displays the deflection field for a viscoelastic beam obtained from the inverse Fourier transform of Equation 9.

**SCALING OF DISSIPATION-INDUCED FUEL CONSUMPTION**

Figure 3a displays, in the dimensionless form of Equation 5, the dissipation in the function of the normalized velocity, $\Pi_1 = c/c_r$, for three fixed damping ratios, $\Pi_2 = \zeta = \tau(k/m)^{1/2} = 1, 5, 10$. For a given value of $\zeta$ (and thus relaxation time), after a first sharp increase at low velocity values, the dimensionless dissipation is almost constant over a wide range of velocities, $\Pi_1 = c/c_r$, before increasing to a vertical peak.

**FIGURE 3** Plots of dimensionless dissipation $\Pi = \Delta P^2/2k$ in function (a) of dimensionless velocity $\Pi_1 = c/c_r$ at damping ratios $\Pi_2 = \zeta = \tau(k/m)^{1/2} = 1, 5, 10$; (b) of damping ratio $\Pi_2 = \zeta = \tau(k/m)^{1/2}$ at velocities $\Pi_1 = c/c_r = 0.11, 0.50$ (dashed lines are functions fitted to data proportional to $1/\zeta$); and (c) of both invariants $\Pi_1 = c/c_r$ and $\Pi_2 = \zeta$; (d) absolute error of fitting function (14) for coefficients presented in Table 1.
asymptote at the critical velocity. In return, as shown in Figure 3b, for a given normalized velocity $c/c_{cr}$, the dissipation rate decreases as the damping ratio, and hence the viscous relaxation time $\tau$, increases. This means, as expected, that the faster the rate of viscous deformation (and thus the smaller $\tau$), the greater the dissipation rate.

Given these results, for a first-order back-of-the-envelope engineering estimate, the dissipation rate (respectively, the excess energy consumption) scales, for a wide range of relevant velocities, $0.2 < c/c_{cr} < 0.8$, with the viscous relaxation time $\tau$, vehicle load $P$, top layer stiffness $E$, top layer thickness $h$, and subgrade modulus $k$, as

$$\mathcal{D} \propto \tau^{-1} \times P^2 \times E^{-\beta \delta} \times h^{-\beta \delta} \times k^{-\beta \delta}$$  \hspace{1cm} (12)

or, in terms of the excess energy consumption,

$$\delta E = \frac{\mathcal{D}}{c} \propto (c\tau)^{-1} \times P^2 \times E^{-\beta \delta} \times h^{-\beta \delta} \times k^{-\beta \delta}$$  \hspace{1cm} (13)

The scaling relations of dissipation rate and excess energy consumption are consistent with a recent North American calibration of the World Bank’s HDM-4 model for vehicle operating costs (15), which reported statistically significant effects of surface texture for heavier trucks (i.e., $\delta E \propto P^2$) and at low speed (i.e., $\delta E \propto c^3$).

Evaluating the dissipated energy with Equation 11 is complex and therefore not appropriate for engineering purposes. A detailed method and calculations are available elsewhere (9). For practical use, the log of dimensionless Equation 5 (Figure 3c) was fitted to a two-dimensional surface that fits very well the discrete data (Figure 3d) for $0.03 < c/c_{cr} < 0.5$ and $0.0001 \leq \zeta \leq 12,000$:

$$\log_{10}(\Pi) = \log_{10}\left(\frac{\mathcal{D}^2 \beta k}{P^2 c_{cr}}\right) = \log_{10} F \left(\Pi_1 = \frac{c}{c_{cr}}, \Pi_2 = \zeta\right)$$

$$= \sum_{i=0}^{3} \sum_{j=0}^{3} p_{ij} \Pi_1^i \times \log_{10}(\Pi_2)^j$$  \hspace{1cm} (14)

Coefficients $p_{ij}$ (coefficient of determination of $R^2 = 972$) are tabulated in Table 1. With the material and structural properties of a pavement in hand, one can use Equation 14 to readily evaluate the dissipated energy and fuel consumption. This approach is used here to evaluate fuel consumption on the U.S. roadway network.

### EFFECT OF TEMPERATURE ON DISSIPATION

A simple means of accounting for the temperature dependence of the viscoelastic response is by using—analogous to the Arrhenius concept—the activation energy concept for the relaxation time:

$$\tau(T) = \tau(T_{ref}) \times a_r(T)$$  \hspace{1cm} (15)

where $a_r(T)$ is the shift factor accounting for the acceleration or deceleration of the relaxation time when the temperature $T$ is different from a reference temperature $T_{ref}$. For concrete, the classical activation energy concept has been shown to hold (16):

$$\log a_r(T) = \frac{1}{U_i} \left(\frac{1}{T} - \frac{1}{T_{ref}}\right)$$  \hspace{1cm} (16)

where $U_i = 2,700$ K. (Temperatures are reported in Kelvin.) In return, for asphalt mixes, following the suggestion of Pouget et al. (5), the William–Landel–Ferry law (17) has been shown to capture the time shift well:

$$\log a_r(T) = \frac{-c_1 (T - T_{ref})}{c_2 + (T - T_{ref})}$$  \hspace{1cm} (17)

where $C_1, C_2$ are empirical constants. Typical values for bituminous asphalt mixes reported by Pouget et al. are $C_1 = 34$ and $C_2 = 203$ K for a reference temperature of $T_{ref} = 283$ K (10°C).

### MODEL CALIBRATION AND VALIDATION

In contrast to previous approaches, the model presented here requires as input the characteristic relaxation time, $\tau(T_{ref})$, representative of the viscoelastic response of the constituent material. To calibrate the model, this study considers the results reported by Pouget et al. (5), in which the dissipated energy in an asphalt concrete layer subjected to a moving truck of 40 tons on three axles, as illustrated in Figure 4a, was calculated, by means of finite elements, considering a generalized Kelvin chain to represent asphalt’s viscoelastic response, at various temperatures and two speeds, $c = 100$ km/h and $c = 50$ km/h. The same pavement structure and material properties are modeled by means of a viscoelastic beam model representation: $E = 40,114$ MPa,

| Coefficient (95% confidence bounds), by Value of i | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $j$ | $(-1.922, -1.915)$ | $(4.379, 4.596)$ | $(-20.64, -18.44)$ | $(54.61, 64.55)$ | $(-102.6, -82.39)$ | $(48.63, 63.83)$ |
| $\log_{10}(\Pi)$ | 4.487 | $(-19.54)$ | 59.58 | $(-92.51)$ | 56.23 |
| $\log_{10}(\Pi_1)$ | $(-0.4123)$ | $(-1.802)$ | 4.014 | $(-4.628)$ | 1.375 |
| $\log_{10}(\Pi_2)$ | $(-0.4113, -0.4111)$ | $(-1.824, -1.78)$ | $(3.864, 4.163)$ | $(-5.04, -4.217)$ | $(0.9895, 1.761)$ |
| $p_{ij}$ | $(-0.06942, -0.06915)$ | $(0.2111, 0.2194)$ | $(-0.8794, -0.8441)$ | $(0.7124, 0.7563)$ | na |
| $\log_{10}(\Pi_1)$ | $(-0.00965, -0.009495)$ | $(0.0196, 0.021)$ | $(0.04542, 0.04797)$ | na |

Note: na = not applicable.
$h = 0.22$ m, and $k = 35$ MPa/m. For determining the subgrade modulus, an empirical relationship developed by Khazanovich et al. is used \((18)\); it links the subgrade elastic modulus $E_s$ to the subgrade stiffness $k = \alpha E_s$, where $\alpha = 0.29/m$. The model calibration and validation are done as follows:

- For the calibration, the reported dissipated energy values for $c = 100$ km/h at various temperatures \((10^\circ C, 30^\circ C, 40^\circ C, 50^\circ C, 55^\circ C, \text{ and } 60^\circ C)\) are considered. By minimizing the quadratic error between these model predictions and the model predictions reported by Pouget et al. \((5)\), the relaxation time $\tau(T_{ref} = 10^\circ C) = 0.0083$ s is calibrated. The fit is shown in Figure 4b.

- For the validation, model predictions are compared with the reported dissipated energy values for $c = 50$ km/h at two temperatures \((40^\circ C \text{ and } 60^\circ C)\). This comparison is shown in Figure 4c.

The successful validation and calibration shows that the presented model can capture the sensitivity of the pavement’s material–structural response to both temperature and speed. Given the simplicity of the model \(a\) Maxwell model, this result may be unexpected. However, at typical vehicle speeds the characteristic time of loading of the material is relatively short, so that one viscous relaxation time may be sufficient for capturing the viscous response for the estimation of the dissipated energy in a moving coordinate frame. Further validation based on measuring the energy dissipation caused by a moving load at various temperatures and speeds is needed for confirming this conjecture, and such validation will be reported in the future.

### Application of Scaling with Long-Term Pavement Performance Database

The model is applied with data extracted from FHWA’s Long-Term Pavement Performance (LTPP) program \((19)\). Specifically, the data available through the General Pavement Studies (GPS) program are considered; these data are recorded from in-service pavement test sections either in their original design phase or in their first overlay phase \((19)\). Under the GPS program, more than 800 test sections were established on in-service pavements in all 50 states and in Canada. The GPS sections generally represent pavements that incorporate materials and structural designs used in standard engineering practices in the United States and in Canada. Each GPS section is 152 m \((500 \text{ ft})\) long and is located in the outside traffic lane. The data collected at the GPS sections include climatic, material properties, traffic frequency, deflection profile, distress, and friction data. The GPS section categories considered in the investigation are listed in Table 2.

### Input Data

For each GPS section class, distributions of top layer thickness $h$, top layer stiffness $E$, and subgrade modulus $k$ are determined with a consistent calibration–validation method \((7)\) that for each section involves:

- Calibration of $(E, k)$ with wave propagation characteristics of falling weight deflectometer time history data recorded by FHWA’s LTPP program,
• Validation of calibrated \((E, k)\) data against deflection values from the falling weight deflectometer tests at various distances from the loading point, and
• Comparison of the study’s values with top layer and subgrade modulus values reported in the LTPP database.

The distributions of top layer modulus \(E\), subgrade stiffness, and top layer thickness are displayed in Figure 5 for the two asphalt concrete pavement systems (GPS-1, GPS-2), portland cement concrete systems (GPS-3, GPS-4, GPS-5), and composite systems (GPS-6, GPS-7). The general trend is that the distributions are lognormal. The lognormal mean and standard deviations are provided in Table 3. As detailed below, these distributions are the input for Monte Carlo

TABLE 2  GPS Section Categories Based on Pavement Type Considered in Investigation (19)

| GPS Number | Pavement Type                                      |
|------------|---------------------------------------------------|
| GPS-1      | Asphalt concrete pavement on granular base        |
| GPS-2      | Asphalt concrete pavement on stabilized base      |
| GPS-3      | Jointed plain concrete pavement                   |
| GPS-4      | Jointed reinforced concrete pavement               |
| GPS-5      | Continuously reinforced concrete pavement         |
| GPS-6      | Asphalt concrete overlay of asphalt concrete pavement |
| GPS-7      | Asphalt concrete overlay of portland cement concrete pavement |

![Graphs showing distributions for considered GPS systems: (a) top layer stiffness, \(E\); (b) subgrade stiffness, \(E_s\); and (c) top layer thickness, \(h\) (PDF = probability density function).](image-url)
TABLE 3  Mean Value and Standard Deviation for Lognormal Distribution of Top Layer Stiffness, Subgrade Shear Modulus, and Top Layer Thickness for GPS Section Categories

| GPS Number | \(\mu_{\ln} (E)\) | \(\sigma_{\ln} (E)\) | \(\mu_{\ln} (G_s)\) | \(\sigma_{\ln} (G_s)\) | \(\mu_{\ln} (h)\) | \(\sigma_{\ln} (h)\) |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| GPS-1      | 9.4941          | 0.6632          | 5.4393          | 0.5285          | -1.8524         | 0.5718          |
| GPS-2      | 9.0151          | 0.6378          | 5.7375          | 0.4997          | -1.7556         | 0.5618          |
| GPS-3      | 10.4086         | 0.2540          | 5.3498          | 0.3705          | -1.4048         | 0.1079          |
| GPS-4      | 10.4603         | 0.1588          | 5.4723          | 0.2636          | -1.3684         | 0.0515          |
| GPS-5      | 10.3903         | 0.2485          | 5.5017          | 0.3805          | -1.5135         | 0.0892          |
| GPS-6      | 9.0928          | 0.9506          | 6.0642          | 0.6308          | -1.4827         | 0.5278          |
| GPS-7      | 9.1184          | 0.8796          | 6.7312          | 0.7609          | -1.4297         | 0.2233          |

Note: For translation of the subgrade shear modulus \(G_s\) to the subgrade modulus \(k\) (or coefficient of subgrade reaction), an empirical relation developed in Khazanovich et al. (18) is used: \(k = 2 \alpha (1 + v) G_s\), where \(\alpha = 0.29/m, v e = 0.4\). (Poisson’s ratio of subgrade is assumed constant for all sections.)

Monte Carlo Approach

A Monte Carlo approach is implemented to evaluate, for each GPS section category in the LTPP data set and a given vehicle class (i.e., axle load \(P\)), the excess energy consumption from Equation 6, that is,

\[
\delta E = \frac{Q}{c} = \frac{P^2}{V^2 b} \times \frac{c_r}{c} \times F \left( \Pi_1 = \frac{c}{c_r}, \Pi_2 = \zeta = \tau \left( \frac{k}{m} \right)^{\frac{1}{2}} \right) \tag{18}
\]

where the value of the dimensionless dissipation rate, \(\Pi = F(\Pi_1, \Pi_2)\), is estimated from Equation 14.

In the Monte Carlo simulations, for each GPS section type, the distributions of top layer stiffness, top layer thickness, and subgrade modulus are considered for estimating the distribution of the excess energy consumption. In addition, in the Monte Carlo simulations seasonal temperature variations can be considered as well through consideration of the annual mean temperature and the standard variation.

The Monte Carlo approach is based on the ergodicity principle: the average values over time of physical quantities (here, dissipated energy) that characterize a system (here, pavement) are equal to the statistical average values of the same quantities realized by a large amount of possible configurations (20). In this sense, the Monte Carlo approach is well suited to capture the excess energy consumption of the network represented by the LTPP database.

Results

Figure 6, a through c, displays the dissipated energy of an HS20-44 truck shown in Figure 4a with axle loads \(P_1 = 36.29\) kN (8,000 lb), \(P_2 = 145.15\) kN (32,000 lb) at a constant speed \(c = 100\) km/h and a temperature of 10°C ± 5°C, for the seven GPS section types, regrouped for comparison in three categories: asphalt concrete pavements (GPS-1, GPS-2, Figure 6a), concrete pavement (GPS-3, GPS-4, GPS-5, Figure 6b), and composite pavement structures (GPS-6, GPS-7, Figure 6c). Given the lognormal distribution of the input parameters, the excess energy caused by dissipation follows as well a lognormal distribution. The following observations deserve attention:

- For asphalt concrete pavements (Figure 6a), a statistically significant effect of the subgrade on the dissipated energy is recognized: a stabilized base can reduce the dissipated energy on average by 25% compared with a granular base. In return, the distribution of the dissipated energy is wide for both GPS-1 and GPS-2, spanning more than three orders of magnitude.
- The dissipated energy of concrete pavement structures (Figure 6b) exhibits a very narrow distribution and is almost insensitive to the presence of joints and reinforcement (GPS-3, GPS-4, GPS-5).
- Composite sections (Figure 6c) achieve a peak distribution similar to concrete pavements, although with a wider distribution, reminiscent of that of asphalt pavements. There is a statistically significant difference between GPS-6 and GPS-7 that relates to the stiffness of the pavement on which the asphalt overlay is built (asphalt concrete for GPS-6, Portland cement concrete for GPS-7).

The results so far presented considered a constant speed of \(c = 100\) km/h and a moderate temperature of 10°C ± 5°C (mean ± standard deviation). Showing the impact of both a varying speed and temperature, Figure 7 displays the dissipated energy as functions of temperature and speed for three of the seven GPS systems. To account for the difference in distributions, the dissipated energy is evaluated at the 95% confidence level. As expected, the specific temperature and rate sensitivity of the top-layer material translate into a sensitivity of the dissipated energy.

CONCLUSIONS

The need to enhance the sustainability of the pavement system requires quantitative engineering models that can capture—at least in first order—the impact of pavement structure and constitutive materials on fuel consumption and related greenhouse gas emissions. The model presented here, which refines previous work (7), aims at just this. The following points deserve attention:

1. To maintain a constant speed, the vehicle’s engine must supply additional energy to compensate for the energy that is dissipated in the pavement structure. This excess energy depends on structural and material properties of the pavement, temperature, and vehicle speed.
2. The complexity of the viscoelastic phenomena involved can be reduced to a few dimensionless quantities that combine materials and structural parameters. In this dimensionless space, the excess energy demand can be evaluated by means of mechanistic models. Here, the simplest model, a viscoelastic beam on an elastic foundation, was chosen to capture the dissipative response of the system in a dimensionless form that can be easily used for first-order evaluations of the dissipated energy. The dimensionless form, however, can also be used with other, more comprehensive models based on plate theory or continuum theory that are currently in development. In the model developed here, it is assumed that the pavement material is viscoelastic. That is, although the energy dissipation is temperature dependent because of the viscous component of the pavement model, no irreversible deflection is considered. That is, the dissipation of energy caused by the plasticity of the pavement arising in extreme temperature and stress levels is not taken into account.
3. Besides elastic and structural data that are (mostly) readily available, the model requires calibration of a viscous relaxation time. Here, this relaxation time was calibrated for asphalt against literature...
data. Further research is needed to link this relaxation time to classical material test results. Given the relatively short residence time of a moving vehicle on a specific place, it appears that the entire frequency domain is not required for capturing the time-dependent response of the constituent materials.

4. The application of the model with data from the LTPP database shows that the dissipation and its distribution are temperature, speed, and material dependent. For a chosen or given pavement structure, it becomes possible to evaluate the excess fuel consumption with a minimum of material and structural parameters and to use dissipated energy as a design criterion for minimizing the carbon footprint of the pavement system.

Although small for a single vehicle, the increasing average annual daily truck traffic (AADTT) adds up to a significant amount of excess fuel consumption. For instance, for AADTT = 10,000 (an average value representative of the traffic volume of the GPS pavement systems), together with EPA’s miles per gallon of gasoline equivalent...
rating (according to which 1 L of fuel equates to 32.05 MJ), the total energy dissipated per day at the 95% confidence level amounts to 178 to 468 L/km of excess fuel consumption per day for GPS-1, 11 to 38 L/km for GPS-3, and 44 to 139 L/km for GPS-7; the lower value corresponds to $c = 100 \text{ km/h}$ and $10^{\circ} \text{C}$, and the upper value corresponds to either lower speeds ($c = 20 \text{ km/h}$ and $10^{\circ} \text{C}$) or higher temperature ($c = 100 \text{ km/h}$ and $20^{\circ} \text{C}$). That is, there are significant opportunities for reducing the environmental impact of the pavement system.

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