AC susceptibility studies of grain-aligned superconductors by grained Bean model

Nobuyoshi Sakamoto, Tadahiro Akune and Yasukuni Matsumoto

1 Department of Electrical Engineering and Information Technology, Kyushu Sangyo University, 2-3-1 Matsukadai, Fukuoka, Japan
2 Department of Electrical Engineering, Fukuoka University, 8-19-1 Nanakuma Fukuoka, Japan
E-mail: saka@te.kyusan-u.ac.jp

Abstract. AC susceptibility of low-\(T_c\) metallic superconductors shows smooth transition in the in-phase \(\chi'\) and a peak in the out-phase \(\chi''\). High-\(T_c\) oxide superconductors with anisotropic and grain-textured structures show deformed complex characteristics, such as double peaks in \(\chi''\) and shoulders in \(\chi'\). Instead of simple Bean model, a grained Bean model, where the superconducting grains is immersed in weak superconducting matrix, are proposed. The susceptibilities numerically analyzed using the grained Bean model show varied and deformed curves as observed in the high-\(T_c\) superconductors. From the dependence of \(\chi'\) and \(\chi''\) on temperatures \(T\) and DC magnetic fields \(B_{dc}\) in grain-aligned Hg(Re)-1223 superconductors, textures of grains and interconnecting links and their grain-aligned nature can be estimated.

1. Introduction
Sintered high-\(T_c\) superconductors are considered to be a granular superconductor. They are polycrystalline, with grains that are coupled together. In such materials, the coupling component transports supercurrents and has its own effective critical temperature \(T_{c\ell}\), critical current density \(J_{c\ell}\) and pinning penetration depth \(B_{p\ell}\). The situation is less certain, but lack of stoichiometry at the grain boundaries and microbridges between grains give rise to a proximity-effect coupling. The coupling region shows a weak superconductivity and the field penetrates more freely through them compared to the grain region. Then the surface field of the grain is determined by the coupling matrix superconducting region. The intrinsic superconductors with high pinning penetration field \(B_{pg}\) are considered to be immersed in the weak matrix with low \(B_{p\ell}\)[1,2]. Field distribution and magnetization in the multi-phase structure are calculated by Bean model [3]. Fourier integration of magnetization is carried out numerically and gives rise to the real part \(\chi'\) and the imaginary part \(\chi''\) of AC susceptibilities. Measured results of Re-doped Hg-1223 superconductors [4] are compared with the simulated results to reveal the link characteristics.

2. Grained Bean Model
Matrix superconductor with a penetration field \(B_{p\ell}\) determines the magnetic fields at the \(i\)-th grain surface with the penetration field \(B_{pgi}\) as shown in figure 1. Field distribution \(B_{oi}\) outside the grain is given by using \(B_{p\ell i}\) following the Bean model and the field \(B_{gi}\) inside the grain with the size \(d_{gi}\) by using \(B_{pgi}\) and \(B_{oi}\).

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2.1. Magnetization

Average of the magnetic flux density \( \langle B \rangle \) in the increasing period for the superconductor which consists of the matrix and \( n_g \) grains with grain interval \( d_{vi} \) is given by

\[
\langle B \rangle = B_o - \frac{B_{pl}}{2} + \sum_{i=1}^{n_g} \left( \frac{d_{gi}}{D} \right) m_{gi}, \tag{1}
\]

where \( m_{gi} \) is the magnetization of the \( i \)-th grain. In the case of uniform grain structure with the constant grain size \( d_g (= d_{gi} \) for all \( i \)), the intergrain distance \( d_v (= d_{vi} \) for all \( i \)) and the penetration field \( B_{pg} (= B_{pgi} \) for all \( i \)), the magnetization \( M \) of the superconductor at the field \( B_o \) for increasing magnetization processes is given by

\[
M = \langle B \rangle - B_o = -\frac{B_{pl}}{2} + \frac{f_g}{n_g} \sum_{i=1}^{n_g} m_{gi}, \tag{2}
\]

where \( f_g (= d_g/d_v) \) is the grain content factor.

2.2. AC susceptibility

Fourier integrals of the magnetization \( M \) gives the fundamental AC susceptibilities \( \chi' \) and \( \chi'' \) under an AC field \( B_a \cos \omega t \) as \(^5\)

\[
\chi = \chi' + i\chi'' = \frac{1}{\pi B_a} \int_{-\pi}^{\pi} M(\omega t) \exp(i\omega t) d\omega t. \tag{3}
\]

Numerical integration of eq. (3) in the case of \( B_{pl}/B_{pg} = 0.01 \) is carried out and the imaginary part \( \chi'' \) and the real part \( \chi' \) are plotted in figure 2 as a function of the amplitude \( B_a \) normalized by the link penetration field \( B_{pl} \). When the content factor \( f_g \) increases, new peak appears corresponding to the grain magnetization and the existing peak at low field begins to shrink. The peak height of \( \chi'' \) shifts smoothly from low link component to high grain one.
Figure 3. AC susceptibilities $\chi'$ and $\chi''$ as a function of AC field $B_a$ normalized by the grain penetration field $B_{pg}$. With increasing $B_{pl}$, small low field peak gradually shifts to high field, unites to single peak and large peak moves to higher field.

The effect of link penetration field $B_{pl}$ at the grain volume ratio $f_g = 0.5$ is shown in figure 3 for the low value $B_{pl} = 0.01B_{pg}$. The typical double peak characteristics appear in the imaginary part $\chi''$ and double transition in the real part $\chi'$. With increasing $B_{pl}$, lower peak begins to move higher temperature region, unite to be single large peak and the large peak moves to higher field.

Figure 4. AC susceptibilities $\chi'$ and $\chi''$ as a function of temperature $T$ reduced by the grain temperature $T_{cg}$. New peak appears with higher $B_{pg}(0)$. Temperature indices are $n_g = 2$ and $n_1 = 6$.

Figure 5. (a) Temperature dependence of $\chi'$ and $\chi''$ at dc fields of 0.1 ~ 1.0 T as a function of temperature $T$ reduced by $T_c(B_{dc})$ for Hg-Re-15ab sample. (b) Numerically computed AC susceptibilities as a function of $T/T_{cg}(B_{dc})$ using the grained Bean model.
Temperature dependence of $\chi'$ and $\chi''$ can be obtained by introducing the temperature variation of the penetration fields of $B_{p\ell}$ and $B_{pg}$. If these penetration fields have a usual parabolic dependence of the form as, $B_{pz} = B_{pz}(0)(1 - (T/T_{cz})^2)^{n_z}$ ($z = g$ or $\ell$), where the pinning effect for the fluxoid motion disappears over the temperatures $T_{cz}$ and $T_{cg}$ in the link and the grain region, respectively. The effect of $B_{pg}$ are shown in figure 4 at $f_g = 0.5$, AC field amplitude $B_d/B_p(0) = 0.2$, $n_g = 2$ and $n_\ell = 6$. The imaginary peaks decrease with increasing $B_{pg}(0)$ and new peaks appear and coexist at higher temperatures. This type of double peaks is often observed in high-$T_c$ materials [6].

3. Grain and link characteristics in Hg-1223 superconductors

The double peak characteristics of HgBa$_2$Ca$_2$Cu$_3$Re$_{0.15}$O$_{8+\delta}$ (Hg-Re-15ab) powdered sample [4] are shown in figure 5(a), where temperatures are normalized by the critical temperature $T_c(B_{dc})$ at the applied dc magnetic field $B_{dc}$. The computed results by the grained Bean model are plotted in figure 5(b) and fairly agree with the experimental curves. The fitting parameters are plotted in figure 6 together with previous data under the fields parallel to c-axis [4]. Grain penetration fields $B_{pg}$ are larger than the link penetration fields $B_{p\ell}$ under the fields parallel to $ab$-plane. Comparison of the values of the penetration fields is not suitable since the configurations of the grains are uncertain. Anisotropic nature of the Hg-system superconductor should be considered to make a substantial contribution of this difference.

4. Conclusions

Textures of grains and interconnecting links in high-$T_c$ superconductors are simulated by the grained Bean model, where the superconducting regions are divided two parts; grains and interconnecting links. A variety of characteristics, double peak and their movement in the imaginary part of AC susceptibility appear in the computed results. Double peak characteristics were successfully analyzed by the grained Bean model. Measured difference of AC susceptibilities under applied field orientation between parallel to c-axis and to $ab$-plane are well traced with the change by adjusting the penetration fields of grain and interconnecting link.

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Figure 6. Pinning penetration field $B_{pg}(0)$ and $B_{p\ell}(0)$ as a function of dc magnetic field $B_{dc}$ in Re added Hg-1223 superconductors. Grain penetration fields $B_{pg}$ are larger than the link penetration fields $B_{p\ell}$ under the fields parallel to $ab$-plane.
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