Chance-constrained optimal location of damping control actuators under wind power variability

Horacio Silva-Saravia¹, Héctor Pulgar-Painemal¹, Russell Zaretzki²
Department of Electrical Engineering and Computer Science¹, Business Analytics & Statistics²
University of Tennessee, Knoxville, TN, 37996
Email: hsilvasa@vols.utk.edu, hpulgar@utk.edu, rzaretzk@utk.edu

Abstract—This paper proposes a new probabilistic energy-based method to determine the optimal installation location of electronically-interfaced resources (EIRs) considering dynamic reinforcement under wind variability in systems with high penetration of wind power. The oscillation energy and total action are used to compare the dynamic performance for different EIR locations. A linear approximation of the total action critically reduces the computational time from hours to minutes. Simulating an IEEE-39 bus system with 30% of power generation sourced from wind, a chance-constrained optimization is carried out to decide the location of an energy storage system (ESS) adding damping to the system oscillations. The results show that the proposed method, selecting the bus location that guarantees the best dynamic performance with highest probability, is superior to both traditional dominant mode analysis and arbitrary benchmarks for damping ratios.

Index Terms—oscillation energy, inter-area oscillations, small-signal stability, chance-constrained optimization, energy storage, renewable energy, wind power, random variable.

I. INTRODUCTION

The increasing penetration of variable renewable energy (VRE) into power systems, such as solar and wind, creates operational and technical challenges. Among the technical challenges, low frequency oscillations are particularly affected. The causes include (1) the reduction of relative inertia added by electronically-interfaced generation and (2) the variability of the injected power, which creates power imbalance disturbances and unusual power flows. These power flows determine new uncommon equilibrium points, affecting power system dynamics and deteriorating small signal stability.

Traditionally, small signal stability analysis with sources of uncertainty has been modeled with probabilistic approaches by using eigenvalue sensitivities. Linear approximation of the relationship between system eigenvalues and a random system element, such as load or generation, allows one to characterize the distribution of system eigenvalues, given the probability density function (pdf) or, more generally, the distribution function of the random variables of interest [1]. Analytical methods have been investigated to construct the pdf of critical modes, determining the probability of system instability under scenarios of wind generation [2]. Recently, nonlinear relationships have also been analyzed to obtain the cumulative distribution function (cdf) of the damping ratio of a dominant mode [3]. The stochastic information of system eigenvalues can be used to control the system, for example, through optimal tuning of actuators by defining probabilistic objective functions of all system eigenvalues [4]. Although these analyses provide tools to study and control specific operational requirements of a system, acceptable damping ratios for instance, these requirements, together with the study of a dominant mode, are usually arbitrary and fall short by not guaranteeing optimal system performance. From a practical perspective, we find a lack of a probabilistic analysis that considers a system’s overall dynamic performance and creates an index that combines all system modes in a meaningful way.

The use of energy functions is attractive to provide a meaningful study of system oscillations. By analyzing the kinetic energy oscillation, a weighted combination of the system eigenvalues can be obtained. This weighted combination of eigenvalues has been used to determine energy exchange paths [5], [6] and can be defined as a system performance index for control [7]. In particular, this kind of energy-based index can be utilized to solve a new problem related to the planning of the deployment of electronically-interfaced resources (EIR) such as non-conventional renewable generating systems (NRGS) or energy storage systems (ESS). The problem consists on finding the optimal location in the system to install EIR or ESS for dynamic reinforcement planning (DRP), i.e., damping electro-mechanical oscillations [8]–[10]. The optimal location has been studied by deterministic analysis and by relating system inertia distribution and residue index [11], [12]. However, the study assumes a single very low damped inter-area mode, which is uncommon in large scale systems, like in the U.S. where the existence of several critical modes with similar damping ratios is usual. Unlike this approach, an energy-based system performance index has been shown to be suitable for capturing the overall system dynamic behavior considering all system eigenvalues [7]. This index is based on the concept of total action and the optimal location is found by minimizing the area under the oscillation energy curve for a given disturbance; this dependency on the disturbance can be overcome by using a probabilistic framework.

This paper provides a new probabilistic measure to determine the optimal location of EIR in a DRP problem considering variability in systems with high penetration of
wind power. The problem is solved by chance-constrained optimization, using a linear estimation of the total action as a system performance index in the objective function, and the probability of a set of disturbances. Simulations are performed in the IEEE-39 bus system with 30% of power derived from wind penetration. Monte Carlo simulations show the computational advantage of the linear approximation compared to the exact calculation for analyzing the location of an ESS. The optimization results show that the best location for ESS differs from that obtained with traditional stochastic analysis of the dominant mode or an arbitrary benchmark for damping ratios. This occurs because the proposed approach chooses the location that maximizes the probability that the best system dynamic behavior is guaranteed. The paper is structured as follows. Section II describes the oscillation energy, the total action and its linear estimation. Section III formulates the chance-constrained optimization problem. Simulation results are presented in Section IV. Finally, Section V provides the conclusions.

II. OSCILLATION ENERGY ANALYSIS

A. Oscillation energy and action

Electromechanical oscillations occur as a result of the kinetic energy exchange between different group of synchronous generators. The kinetic energy of each machine will oscillate depending the constant of inertia for each machine and the oscillation modes. The sum of the oscillation kinetic energy over all frequencies and all machines, which is the system oscillation energy, can be used as a system wide dynamic performance index because of its intuitive physical interpretation and convenient representation in terms of all modes [7]. Consider the linearized power system equations with system matrix $A$, $p$ synchronous generators and $n$ state variables. By similarity transformation, the state variable vector $\Delta x$ and the transformed state variable vector $\Delta z$ are related as $\Delta x = M \Delta z$, where $M = \{v_1, v_2, ... v_p\}$ is the matrix of right eigenvectors, $\Lambda = M^{-1} A M = \text{diag}\{\lambda_i\}$, and $\lambda_i$ is the $i$-th system eigenvalue. Thus, for a given initial value $\Delta x(0) = \Delta x_0$ at time $t = 0$:

$$
\Delta \dot{x} = A \Delta x
\Delta x(0) = \Delta x_0 \quad \Rightarrow \quad \Delta \dot{z} = \Lambda \Delta z = \Delta z
\Delta z(0) = \Delta z_0 = M^{-1} \Delta x_0
$$

(1)

The system oscillation energy is as follows:

$$
E_k(t) = \sum_{j=1}^{p} \frac{1}{2} I_j \Delta \omega_j^2 = \frac{1}{2} \Delta x^T J \Delta x
$$

$$
= -\frac{1}{2} \Delta z^T G \Delta z \in \mathbb{R}
$$

(2)

where $J$ and $G = M^T J M$ are the inertia and the transformed inertia matrices, respectively (see reference [7]), and $\omega_j$ is the rotational speed in per unit of $j$th generator. Consider now the system action ($S$), which is the integral over time of the system kinetic energy:

$$
S(\tau) = \int_0^\tau E_k(t) dt = \int_0^\tau \frac{1}{2} (\Delta z^T G \Delta z) dt \in \mathbb{R}
$$

(4)

$$
S(\tau) = \frac{1}{2} \sum_{j=1}^{p} \sum_{i=1}^{n} \frac{e^{(\lambda_i + \lambda_j) t}}{(\lambda_i + \lambda_j)} z_{0i} z_{0j} g_{ij} \bigg|_0^\tau
$$

(5)

where $z_{0i}$ is the $i$-th element of $\Delta z_0 = M^{-1} \Delta x_0$ and $g_{ij}$ is the entry in the $i$-th row and $j$-th column of $G$. Assuming stability, the total action is defined as,

$$
S_\infty = \lim_{\tau \to \infty} S(\tau) = -\frac{1}{2} \sum_{j=1}^{p} \sum_{i=1}^{n} z_{0i} z_{0j} g_{ij}
$$

(6)

Note that $E_k \to 0$ as $t \to \infty$ and $E_k(t) > 0, \forall t$. The best dynamic performance occurs when $E_k$ quickly approaches zero, which is equivalent to the case when the total action is minimized.

B. Linear estimation of the total action

Consider eigenvalue displacements caused by changes in an operational parameter such as the injected wind power $\Delta P_w = P_w - P_{w0}$, a random variable. As the total action must be calculated under the new operating conditions after the changes, to reduce computational burden, an approximation of the total action as a function of the random variable is of interest. By linearizing Equation (6) around initial eigenvalues $\lambda_0$, the following expression is obtained:

$$
\Delta S_\infty \approx \sum_{i=1}^{n} \beta_i \frac{\partial S_\infty}{\partial \lambda_i} \Delta P_w = \left( \sum_{i=1}^{n} \beta_i \frac{\partial \lambda_i}{\partial P_w} \right) \Delta P_w
$$

(7)

where

$$
\beta_i = \frac{\partial S_\infty}{\partial \lambda_i} = \sum_{j=1}^{p} \frac{z_{0i} z_{0j} g_{ij}}{(\lambda_i^0 + \lambda_j^0)^2}
$$

(8)

$$
\frac{\partial \lambda_i}{\partial P_w} = l_i^T \frac{\partial A}{\partial P_w} v_i
$$

(9)

Here $l_i$ and $v_i$ are the left and right column-eigenvectors associated with $\lambda_i$, respectively. Note that $\Delta S_\infty$ is a real number, although $\beta_i$ and $\partial \lambda_i/\partial P_w$ are all complex quantities. Based on preliminary evaluations, for an important range of operating conditions, the terms $\partial \lambda_i/\partial P_w$ can be assumed constant and equal to those calculated at the initial condition.

Assume now that we are interested in comparing the total action when different EIR locations are chosen to improve the system oscillations under wind power variability. If $k$ is the bus where the EIR is placed, the total action becomes:

$$
S_\infty^k \approx S_\infty^0 + \gamma_k \Delta P_w
$$

(10)

where $S_\infty^k$ is the total action estimated for EIR at bus $k$, $S_\infty^0$ is the initial total action for bus $k$ at the initial wind power $P_{w0}$ and $\gamma_k$ is the linear coefficient of equation (7) when EIR is connected at bus $k$. 
III. CHANCE CONSTRAINED OPTIMIZATION

A. Formulation

Because the wind power injection $\Delta P_w$ is a random variable, the system matrix $A$, $\Delta \lambda_i \forall i$ and the total action in Equation (6) become random variables as well. Conditional probabilities associated with the total action can be computed for a given disturbance. By modeling $x_0 = \Delta x_0$ as a random variable, and obtaining some probability density function of common disturbances based on system data, the total probability of the total action can be calculated using Bayes’ rule. Using this information and recalling that the total action ability of the total action can be calculated using Bayes’ rule.

$$k^* = \arg \max_{k \in \mathcal{K}} \Phi_k$$

where,

$$\Phi_k = \sum_{x_0 \in X_0} P(S_k^1 = \min_{i \in \mathcal{K}} S_i^1|x_0) \times P(x_0)$$

$$= \sum_{x_0 \in X_0} P(S_k^\infty \leq S_1^\infty \ldots \leq S_m^\infty|x_0) \times P(x_0)$$

Here $X_0$ is the set of initial disturbances and $\mathcal{K}$, with $|\mathcal{K}| = m$, is the set of bus candidates to connect the EIR. Algorithm 1 shows the procedure to solve this chance-constrained optimization using the linear estimation of the total action.

Algorithm 1 Chance constrained optimization

1: Get random vector with $N$ number of samples $P_w$
2: for each disturbance $x_0 \in X_0$ do
3: for each actuator $k \in \mathcal{K}$ do
4: Calculate $\gamma_k$ and $S_{0k}^i$
5: Compute $S_k^i$ according to (10)
6: end for
7: for each actuator $k \in \mathcal{K}$ do
8: Determine $P_k = P(S_k^i = \min_{i \in \mathcal{K}} S_i^i|x_0)$
9: Update $\Phi_k^{\text{new}} = \Phi_k^{\text{old}} + P_k \times P(x_0)$
10: end for
11: end for
12: Obtain $k^*$ according to (11)

IV. CASE STUDY

The IEEE 39-bus system is employed for simulations. Each synchronous generator is represented by a 6th order model with an IEEE type-1 exciter and an IEEE G1 governor. The wind power variability is studied by adding an equivalent 1,000 MW wind turbine connected at bus 16 in Figure 1 ($\approx 30\%$ of the system load). The equivalent wind turbine is modeled as a static generator with fixed dispatched power, i.e., no additional dynamics of the wind turbine are included. The set of generator buses $\mathcal{K} = \{39, 31, 32, \ldots, 30\}$ corresponds to the set of bus candidates to connect the EIR. For simulation the EIR used in this paper corresponds to a 200 MW ESS (battery). The damping control consists of a proportional gain between the frequency changes at the connection bus and the reference active power. Data for controllers, system parameters and the full battery model are obtained from the simulation library in DlgSILENT PowerFactory.

A. Preliminary deterministic analysis

To first get insights of the chance-constrained optimization stated in section III a parameterized analysis is performed to calculate the exact total action for a disturbance in machine speed $\omega_1$—a short circuit at bus 39. The injected power of the wind turbine is used as a parameter, which is varied from 0 to 1,000 MW. For each prospective ESS location, the parameterized analysis is performed. At each value of wind power generation, the power system equations are linearized, eigenvalues and right/left eigenvector of the system matrix $A$ calculated, and the total action defined in Equation (6) determined. Figure 2 shows the total action as a function of the wind power for each ESS location connected at bus $k$.

Fig. 1: IEEE 39-bus test system

Fig. 2: Parameterized total action for different ESS location.

The results show that for $P_w = 0$ the best bus to place an ESS-based damping control corresponds to bus 36—bus of
transformation; parameters can be found in [13].

of the wind speed, and (c) the probability density function

Figure 4 shows (a) the deterministic relationship between

power generated by a Weibull distribution of wind speed. (b) the probability density function

sample points $P$ are obtained from the pdf of the wind

$P_{w}$ is increased. The

Histograms of the total action when the ESS is connected

at bus 36: (a) exact total action, (b) estimated total action.

Fig. 4: (a) Wind speed-power characteristic, (b) Weibull dis-

Fig. 5: Histograms of total action when the ESS is connected

at bus 36: (a) exact total action, (b) estimated total action.

of the approximation is the reduction in computational time,

which is about 2 minutes for all the bus candidates—including

the calculation of $\gamma_k$—while the exact calculation computing
eigenvalues is around 4 hours using an Intel® Core™ i7-4790

CPU @3.6 GHz processor. Consider now $N = 10^6$ sample

points of $P_{w}$ to solve the chance-constrained problem using

the linear estimation, and the set of equally likely disturbances

in (11) can be computed. Table II shows the results. Note that

the chance-constrained optimization in (11) is needed to solve the DRP problem. Figure 2 is

obtained using the exact calculation of the total action for

the ESS connected at each bus at a time. This calculation

requires significant computational resources, and therefore,
it is impractical in real systems. Consider instead the lin-

ear approximation in Equation (10). Note that this estimate

requires significant computational resources, and therefore,
it is impractical in real systems. Consider instead the lin-

ear approximation in Equation (10). Note that this estimate

assumes eigenvalue trajectories to be linear with respect to
changes in the wind power. Figure 3 verifies this assumption,
which, due to space limitations, focuses only the base case
without ESS. The arrows in Figure 3 represent the direction of
eigenvalue trajectories for the electromechanical modes when
the parameter $P_{w}$ is increased.

B. Stochastic analysis

The chance-constrained DRP problem is solved through
Monte Carlo simulations using the linear approximation of
the total action for an initial operation at $P_{w0} = 0$. The
sample points $P_{w}$ are obtained from the pdf of the wind
power generated by a Weibull distribution of wind speed.
Figure 4 shows (a) the deterministic relationship between
wind speed and power, (b) the probability density function
of the wind speed, and (c) the probability density function
of the wind power. The latter is obtained by random variable
transformation; parameters can be found in [13].

Before applying Algorithm 1, a total of 1,000 sample points
were employed to compare the results from the exact and
estimated calculation of the total action. Figure 5 shows the
histograms of the total action when the ESS is connected
at bus 36 for a disturbance in the speed of generator 1. This
preliminary result shows agreement between the distributions
of the exact and approximate total action. The big advantage

of the approximation is the reduction in computational time,

which is about 2 minutes for all the bus candidates—including
the calculation of $\gamma_k$—while the exact calculation computing
eigenvalues is around 4 hours using an Intel® Core™ i7-4790

CPU @3.6 GHz processor. Consider now $N = 10^6$ sample

points of $P_{w}$ to solve the chance-constrained problem using

the linear estimation, and the set of equally likely disturbances

$X_0 = \{x_{01}, x_{02}, \ldots, x_{0s}\}$, where $\Delta \omega_i = 0.01$ for each

disturbance case. Table II shows the disturbances and the probabilities $P_k$, which corresponds to the probability of the ESS at bus $k$ providing the minimum total action among all candidates buses over all wind power scenarios.

Using the results given in Table I, the objective function $\Phi_k$ in (11) can be computed. Table II shows the results. Note that
the chance-constrained optimization determines that bus 36,
TABLE I: Disturbances and probabilities of optimal location for each disturbance

| Δω | P(S_{S_k} = \min S_{S_k}) |
|-----|-----------------------------|
| x_0 | Δω_1                       | P_{S_35} = 0.1, P_{S_36} = 0.9 |
| x_1 | Δω_1, Δω_2, Δω_{10}        | P_{S_35} = 0.07, P_{S_36} = 0.93 |
| x_2 | Δω_1, Δω_2, Δω_{10}, Δω_5  | P_{S_32} = 0.05, P_{S_36} = 0.95 |
| x_3 | Δω_{10}, Δω_9, Δω_{10}     | P_{S_38} = 0.01, P_{S_30} = 0.99 |
| x_4 | Δω_1, Δω_2, Δω_3, Δω_{10}  | P_{S_35} = 0.05, P_{S_36} = 0.95 |
| x_5 | Δω_2, Δω_3                 | P_{S_32} = 1 |
| x_6 | Δω_9                       | P_{S_38} = 1 |

TABLE II: Results of the probability that each bus provides the best dynamic reinforcement.

| Φ_{39} | Φ_{31} | Φ_{32} | Φ_{33} | Φ_{34} |
|--------|--------|--------|--------|--------|
| 0      | 0.03   | 0.53   | 0      | 0.14   | 0.14   |

TABLE III: Methods comparison

| Optimization method       | Optimal solution |
|---------------------------|------------------|
| (a) Benchmark for dominant mode | k = 30           |
| (b) Benchmark for all modes    | k = 30           |
| (c) Total action and disturbance based | k = 36           |

V. CONCLUSION

This paper describes a novel approach to guarantee the optimal EIR installation location considering dynamic reinforcement under wind power variability. The proposed chance-constrained optimization uses an energy-based index—total action—to measure system dynamic performance by combining all system eigenvalues rather than the study of a dominant mode or an arbitrary operational benchmark for damping ratios. Additionally, this method benefits from the treatment of disturbance probabilities. Simulations are implemented in the IEEE-39 bus system with 30% of wind penetration, and the location of an ESS is analyzed with a linear estimation of the total action. Results show the linear estimation drastically reduces the computation time from 4 hours to 2 minutes. Moreover, the comparison of the results with traditional approaches demonstrates its superiority by choosing a location that maximizes the probability of having the best performance, while the solutions obtained by other methods lead to inefficient installations.

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