On astrophysical explanations due to cosmological inhomogeneities for the observational acceleration

Kenji Tomita
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
(Dated: June 7, 2009)

We review various cosmological models with a local underdense region (local void) and the averaged models with the backreaction of inhomogeneities, which have been proposed to explain (without assuming a positive cosmological constant) the observed accelerating behaviors appearing in the magnitude-redshift relation of SNIa. To clarify their reality, we consider their consistency with the other observational studies such as CMB temperature anisotropy, baryon acoustic oscillation, kinematic Sunyaev-Zeldovich effect, and so on. It is found as a result that many inhomogeneous models seem to be ruled out and only models with the parameters in the narrow range remain to be examined, and that, unless we assume very high amplitudes of perturbations or gravitational energies, the averaged models cannot have the accelerated expansion and the fitted effective $\Lambda$ has not the value necessary for the observed acceleration.

PACS numbers: 98.80.-k, 98.70.Vc, 04.25.Nx

I. INTRODUCTION

The observational magnitude ($m$) - redshift ($z$) relation for distant supernovas of type Ia (SNIa) have been studied by the High-z SN Search Team[1, 2, 3] and the Supernova Cosmology Project[4], and among the Friedman-Lemaitre-Robertson-Walker cosmological models, the $\Lambda$-dominated models have been selected as the best models which can explain their accelerating behavior. At present, the model parameters ($\Omega_M, \Omega_\Lambda$) $\simeq (0.25, 0.75)$ are regarded as the representative one.

About 40 years ago, the cosmological constant has been paid attention to avoid the contradiction between the age of oldest stars and the cosmic age[5, 6, 7, 8], but its value has been uncertain, since the Hubble constant was inaccurate. But recently the existence of $\Lambda$ has been realistic and reduced to the above value.

The $\Lambda$-dominated models with the above parameters have recently been found to be consistent with all results of many observations such as CMB temperature anisotropies[9, 10], the baryon acoustic oscillation (BAO)[11, 12, 13, 14, 15], statistics of cluster abundance[16, 17, 18] and the inverse Sachs-Wolfe effect[19, 20], and so it is regarded as a concordant model.

In these models, however, there are well-known unsolved problems: the cosmological-constant problem and the incidence problem. The former problem shows that the standard value of the cosmological constant is too small ($\sim 10^{-120}$), compared with a constant appearing as the zero-point fluctuation of a field. The latter shows that the role of the cosmological constant is important only at the later stage of $z < 1$, when structure formations are remarkable. Moreover we know the low-$l$ anomaly in the CMB temperature anisotropies which remain to be unsolved[21, 22].

In view of these problems, on the other hand, astrophysical explanations for the accelerating property of SNIa were tried by considering the optical role of inhomogeneous matter distribution which may cause the apparent cosmological acceleration without $\Lambda$ or dark energy. The explanations are divided into the following two types by what inhomogeneity is assumed:

1) Non-Copernican inhomogeneity. We assume to live in a special point of the zero $\Lambda$ universe with a spherically symmetric underdense local inhomogeneity, which is called a local void. The apparent accelerating behavior of SNIa is obtained by deriving the $m$-$z$ relation in this inhomogeneous model.

2) Copernican inhomogeneity. The universe is assumed to have an uniform distribution of density perturbations with zero $\Lambda$. The accelerating behavior is obtained by deriving the effective $\Lambda$ from them in the two procedures:

a) Averaging and b) Backreaction. The averaged model with the effective $\Lambda = 0$. First the average model with the effective $\Lambda$ is derived and the $m$-$z$ relation is obtained to examine the accelerating behavior.

b) Fitting. By comparing the $m$-$z$ relation in a perturbed model with zero $\Lambda$ and that in an unperturbed model with nonzero $\Lambda$, the best fitted value of effective $\Lambda$ is obtained.

The excellent review about the works on (1) and (2)a), b) was written by Céleri[23] in 2007. In this note we have a review of the works which include more recent theoretical and observational contributions.
II. A LOCAL INHOMOGENEITY AND SNIA

Soon after the contributions by the High-z SN Search Team\textsuperscript{[1, 2, 3]} and the Supernova Cosmology Project\textsuperscript{[4]}, another possibility was suggested independently by Céléri\textsuperscript{[24]}, Goodwin et al. (unpublished)\textsuperscript{[25]}, and me\textsuperscript{[26, 27]}. It was that the $m-z$ relation of SNIa may be reproduced also in \textit{non-Copernican} inhomogeneous cosmological models with a central underdense region.

Céléri\textsuperscript{[24]} discussed the accelerated expansion from the qualitative viewpoint with general inhomogeneous models, and Goodwin et al.\textsuperscript{[25]} discussed at the similar epoch with a physical analysis of SNIa result due to the local to global Hubble-constant ratio.

In my first model\textsuperscript{[26, 27]}, the inhomogeneity is described by an open FRW solution (in the inner underdense region) and a self-similar solution (in the outer overdense region). The latter is a special case of Lemaître-Tolman-Bondi (LTB) solutions which is connected with the former open FRW solution at a boundary, in order to avoid the central singularity, and tends to the Einstein-de Sitter (EdS) solution in the limit of radial infinity. It was found in this model that the theoretical $m-z$ relation can be consistent with the observed relation, by adjusting the Hubble constants ($H_0$) and the total density parameters ($\Omega_{M0}$) in the two regions, in the case when the boundary is at the distance of $z \approx 0.07$ from the center. The super-horizon version of this model, in which the distance of the boundary is larger than the horizon size, was once proposed in 1995\textsuperscript{[28]}.

In my second model\textsuperscript{[29]}, two FRW models with $\Lambda = 0$ are used to build an inhomogeneous model, that is, an open FRW model (in the inner underdense region) and a flat FRW model (in the outer overdense region) connected with a discontinuous, sharp boundary with $z \approx 0.07$. The angular diameter distance $d_A$ is derived, solving the Dyer-Roeder equation, the theoretical $m-z$ relation was derived, and it was shown that the observed $m-z$ relation can be reproduced similarly by adjusting the values of $H_0$ and $\Omega_{M0}$ in the two regions.

Moreover, in my next paper\textsuperscript{[30]}, the quantitative comparison between the observed and theoretical $m-z$ relations was done using the second inhomogeneous model and the best model parameters were estimated.

Subsequently, Iguchi, Nakamura and Nakao\textsuperscript{[31]} analyzed general inhomogeneous models using LTB solutions. They reproduced the $m-z$ relation in the concordant model, and it was shown that, at an intermediate radius (with $z < 1.7$), a critical point appears in the models when we assume the uniform big-bang time and asymptotic vanishing spatial curvature.

The geometrical structure of LTB models mimicking $\Lambda$ or dark energy was analyzed by Vanderveld et al.\textsuperscript{[32]} in detail and it was shown that a weak central singularity and a critical point appear generally. Recently Yoo, Kai and Nakao\textsuperscript{[33]} have showed that the LTB models, which reproduce the $m-z$ relation in the concordant model and have no critical point, can be obtained only under the condition of uniform big-bang time, by improving Iguchi et al.’s model, and Clifton et al.\textsuperscript{[34]} derived LTB models without central weak singularity and critical point and discussed the reproduction of the observed $m-z$ relation in their model.

The works on reproducing the observed $m-z$ relation in LTB models were done successively by Alnes, Amarzguioui and Grøn\textsuperscript{[35, 36]}, Alnes and Amarzguioui\textsuperscript{[37, 38]}, Mansouri\textsuperscript{[39, 40]}, Moffat\textsuperscript{[41, 42]}, Biswas et al.\textsuperscript{[43]}, and Alexander et al.\textsuperscript{[44]}.

On the other hand, Kasai\textsuperscript{45} found from SN data themselves that they can be divided into the low $z$ group (with $z < 0.2$) and the high $z$ group (with $z > 0.3$), which correspond to higher and lower Hubble constants, and that the different trend of the data with respect to redshifts may represent the inhomogeneity of cosmological models.

III. CONSISTENCY OF INHOMOGENEOUS LOCAL-VOID MODELS WITH THE OTHER OBSERVATIONS

In order that the inhomogeneous local-void cosmological models may be realistic, they must be consistent with not only SNIa data, but also the other observations such as CMB temperature anisotropies, BAO, the kinematic Sunyaev-Zeldovich effect, and so on. In the following, let us review the works investigating the consistency with these observations.

A. CMB temperature anisotropies

We can see several acoustic peaks in the correlation ($C_l$) - multipole ($l$) diagram for $l > 200$, given with the WMAP data. Since the inhomogeneous models are equal or nearly equal to the Einstein-de Sitter (EdS) model at the recombination epoch, the EdS model must be consistent with the observed property of CMB anisotropies. First, Alnes et al.\textsuperscript{35} discussed the consistency for the first peak and Alexander et al.\textsuperscript{44} showed that their
model (the Minimum model) could give the consistent first and second peaks by assuming a value of the Hubble constant (in the outer region) significantly lower than the conventionally accepted value (70). The consistency with CMB was discussed also by Blanchard et al.\[46\] and Kundu and Sarkar\[47\] in small values of the Hubble constant (\(\sim 47\)) in the outer region. 

If an observer is off-center, he observes the dipole component of CMB temperature anisotropies, proportional to the distance \(r\) from the center of the models. So this value \(r\) is constrained by the observed upper limit of the dipole anisotropy and the upper limit of \(r\) is about 15 Mpc\[38, 41, 48\].

\[B. BAO\]

The above acoustic peaks (representing an oscillation of photon-baryon fluid around and before the recombination epoch) appear also as the baryon acoustic oscillation (BAO), which is an oscillation of baryons (at the later stage) imprinted in the matter spectrum.

Recently it was detected in the SDSS and 2dFGRS surveys, and the BAO scale has been regarded as a standard ruler\[11, 12, 13, 14, 15\]. The characteristic (BAO) scale is the sound horizon at the recombination epoch:

\[r_s(z_{rec}) = \int_{z_{rec}}^{\infty} dz \frac{C_s(z)}{H(z)}, \quad (3.1)\]

where \(C_s(z)\) is the sound speed at redshift \(z\) and \(z_{rec}\) is the redshift at the recombination epoch. This scale is approximately expressed as

\[r_s \approx 147(\Omega_0 h^2/0.13)^{-0.5}(\Omega_b h^2/0.024)^{-0.08} \text{ Mpc}, \quad (3.2)\]

where the density and Hubble parameters have the values at the place we notice.

The observed scales of BAO brought from the galaxy samples in the above surveys are used to constrain the values of the distance measure \(d_V(z) \equiv [(1 + z)^2(d_A)^2cz/H(z)]^{1/3}\). Here \(d_A\) is the angular diameter distance and \(H(z)\) is the Hubble parameter at redshift \(z\) and \(d_V\) is the average of the scales perpendicular and parallel to the line-of-sight directions. Percival et al.\[13\] have recently obtained the following relations at \(z = 0.2\) and \(0.35\):

\[r_s/d_V(0.2) = 0.1980 \pm 0.0058\]
\[r_s/d_V(0.35) = 0.1094 \pm 0.0033. \quad (3.3)\]

In the \(\Lambda\) dominated concordant models, \(r_s/d_V(0.2)\) and \(r_s/d_V(0.35)\) can be reproduced approximately (in about 95%). In my second models with a local void, on the other hand, these two ratios are about 80% of the observed values. This is because the epochs \(z = 0.2\) and \(0.35\) belong to the outer region with the EdS model. So my second models cannot represent them, and they are therefore ruled out. In my first model and the Minimum model given by Alexander et al.\[14\] also, the situation is similar and they are ruled out, as long as the boundary is of the order of 300 Mpc.

In order that inhomogeneous models may not contradict with the BAO observation, the epoch \(z = 0.35\) must belong to the inner underdense region, so that the scale of the inhomogeneous models must be Gpc size. From this viewpoint, the Gpc-size inhomogeneous (LTB) models have recently been studied by Clifton et al.\[34\] and García-Bellido and Haugbølle\[49\].

It has recently been found that the scale of BAO in the parallel to line-of-sight or in the radial direction (the radial BAO scale) impose on the models a more stringent condition than the BAO in the perpendicular direction. RBAO has been studied by Gaztaña\[50\] and it was found that the concordant models are consistent with the observational result of RBAO. On the other hand, Zibin et al.\[51\] showed that RBAO imposes stringent conditions to Gpc-size inhomogeneous models, using Gaztaña et al.'s data at \(z = 0.24\) and 0.43. They considered the two types of models: a constrained model and an unconstrained model, which were made so as to reproduce the recent SN data and WMAP data with the boundary of \(z \approx 1\), and investigated the consistency with RBAO for the above two redshifts. The constraint condition is \(\int \delta \rho(t_i)^2 dr \leq 0\). Their result is

(1) the unconstrained model is consistent with the RBAO data, but \(H_0\) (the present Hubble constant) must be \(\sim 44\), so that this model is ruled out, and

(2) the constrained model is approximately consistent with the RBAO data and \(H_0\) is \(\sim 60\), so that this model is not ruled out at present. However the consistency is not so good as the concordant models.
C. Kinematic Sunyaev-Zeldovich effect

The Sunyaev-Zeldovich (SZ) effect is a small spectral distortion of the CMB radiation caused by the collisions of CMB photons with hot thermal electrons. CMB photons passing through the hot center of massive clusters interact with their electrons and take a small distortion in the CMB spectrum due to the inverse-Compton scattering.

If the clusters are moving with respect to the CMB rest frame, there is an additional spectral distortion (the kinematic Sunyaev-Zeldovich effect) due to the Doppler effect of the cluster velocities on the scattered CMB photons. If $v_{pec}$ is the component of the cluster velocity along the line of sight, then the Doppler effect leads to the following distortion of CMB spectrum:

$$\frac{\Delta T_{SZ}}{T_{CMB}} = \tau_e (v_{pec}/c),$$

(3.4)

where $\tau_e \equiv n_e \sigma T R$ ($n_e, \sigma T, R$ are the electron density, the Thompson scattering cross-section and the effective radius of a cluster).

In inhomogeneous models with a local void, the CMB photon received by a central observer is emitted at the recombination epoch in the outer EdS region, in which the Hubble constant $(H_0)_{eds}$ is $\approx 47 \text{ km/s/Mpc}$. In the regions inside and in the neighbourhood of the local void, the Hubble constant $(H_0)_{loc}$ is assumed to be larger than $(H_0)_{eds}$. So a cluster in the distance $r$ from the center of the inhomogeneous models has a velocity $(|H_0)_{loc} - (H_0)_{eds}|)r$, relative to the CMB rest frame. This velocity can be observed as a peculiar velocity ($v_{pec}$) of clusters in the kinematic Sunyaev-Zeldovich effect.

The possibility to observe the cluster velocities systematically and put a constraint on cosmological models was studied by Benson et al.\[52\]. Recently, García-Bellido and Haugbølle\[53\] have shown the constraint due to the kinematic Sunyaev-Zeldovich effect using observed 9 clusters, and shown that a strong constraint is given to the Gpc LTB models. It is found that the models with the local void region larger than $\sim 1.5 \text{ Gpc}$ are ruled out practically, and only special Gpc LTB models with a limited range of cosmological parameters may be allowed. But, if a systematic peculiar motion of clusters is discovered, this effect may support the local-void model.

D. Spectral distortion of CMB radiation in the reionized region

In inhomogeneous models with a local void, the inner void region is considered to be at the reionized stage. The ionized gas there is moving outward, relative to the CMB frame, and leads to the Doppler effect. When CMB photons are reflected by this ionized gas and reach an observer at the center of the void region, spectral distortion appears in the accepted CMB radiation owing to the Doppler anisotropy. Here the reionized matter plays a role of moving mirror. Caldwell and Stebbins\[54\] derived this distortion in the inhomogeneous models with a local void of various sizes and showed that the constraints for the models can be obtained from the observed upper limit to the distortion of CMB spectrum. As a result, they found that severe constraints can be obtained and the models with the local void of the largest size ($\sim 2.5 \text{ Gpc}$) are ruled out.

IV. UNIFORM DISTRIBUTIONS OF DENSITY PERTURBATIONS

The appearance of observational acceleration due to the Copernican uniform distribution of density perturbations has been studied by the following two different forms.

A. Averaging and backreaction of inhomogeneous models

Averaging and backreaction have been studied by many workers, who include Buchert\[55\], Buchert and Corfora\[56\], Ellis and Buchert\[57\], Kolb et al.\[58, 59, 60\], Kolb et al.\[61\], Kasai\[62, 63\], and Nambu\[64\] for various gauges, and Zalaletdinov\[65\] and Paranjape\[66\] for a covariant form.

In the Buchert formalism, we consider an inhomogeneous model with dust, and its metric is expressed in the comoving and synchronous gauge as

$$ds^2 = -dt^2 + q_{ij}(t, x^m)dx^i dx^j.$$

(4.1)
For averaging, we specify a hypersurface $\Sigma$ of constant $t$, and take a compact region $D$ of $\Sigma$. If the volume of $D$ is $V_D$ and $\psi$ is a scalar variable, the average $<\psi>_D$ of $\psi$ over $D$ is

$$<\psi>_D \equiv \frac{1}{V_D} \int_D \psi d\Sigma. \quad (4.2)$$

Especially, for the matter density we have

$$<\rho>_D \equiv \frac{1}{V_D} \int_D \rho d\Sigma. \quad (4.3)$$

The averaged scale factor $a_D$ is defined by

$$a_D \equiv (V_D)^{1/3}. \quad (4.4)$$

Then we obtain from the Einstein equation

$$3 \frac{\ddot{a}_D}{a_D} = -\frac{\kappa^2}{2} <\rho>_D + Q_D,$$

$$3 \left( \frac{\dot{a}_D}{a_D} \right)^2 = \kappa^2 <\rho>_D - \frac{1}{2} <\mathcal{R}>_D - \frac{1}{2} Q_D \quad (4.5)$$

and

$$(a_D^6 Q_D) : + a_D^4 (a_D^2 <\mathcal{R}>_D) : = 0, \quad (4.6)$$

where $\mathcal{R}$ is the scalar curvature of $\Sigma$,

$$Q_D \equiv \frac{2}{3} (<\theta^2>_D - <\theta>_D^2) - (\sigma_{ij} \sigma^{ij})_D, \quad (4.7)$$

and $\theta$ is the expansion of the world line of the dust fluid. In the derivation of these equations, the incommutability between the time derivative and the averaging was used:

$$<\psi>_D = <\dot{\psi}>_D + <\theta \psi>_D - <\theta>_D <\psi>_D, \quad (4.8)$$

where $<\dot{\psi}>_D \equiv \partial \psi / \partial t$ and $<\psi>_D \equiv \partial <\psi>_D / \partial t$. The condition of averaged acceleration $\ddot{a}_D > 0$ is given by

$$Q_D > \frac{1}{2} \kappa^2 <\rho>_D. \quad (4.9)$$

If this condition is satisfied, it seems that the averaged acceleration may be realized, and so it has been studied by many workers under what situation it is satisfied.

Nambu and Tanimoto\cite{71} considered the region $D$ which consists of many homogeneous and isotropic small regions with different scale factors. Then it was shown that, even if we have deceleration in each region, the average scale factor $a_D$ can have acceleration such as $\ddot{a}_D > 0$. That is, the deceleration observed in each region can be compatible with the averaged acceleration. Ishibashi and Wald\cite{72} discussed this situation and found that the Buchert averaging procedure has ambiguity both with regard to the choice of time slicing and the choice of domain $D$, and that their special choices can artificially derive the averaged acceleration.

Independently of Nambu and Tanimoto, Räisänen\cite{73} considered a model with two disjoint regions consisting of an overdense region and a completely empty region. In both regions we have separately FRW spacetimes with scale factors $a_1$ and $a_2$, where $a_1 \propto (1 - \cos u)$ and $t \propto (u - \sin u)$ and $a_2 \propto t$. If we define the averaged scale factor $a$ by $a^3 \equiv a_1^3 + a_2^3$, then the averaged deceleration parameter $q \equiv (\ddot{a}a) / \dot{a}^2$ becomes negative and so accelerating, when the overdense region turns around and starts collapse. However, Räisänen’s model is physically incomplete, because the junction condition is neither used nor satisfied. Paranjape and Singh\cite{81}, on the other hand, derived a LTB solution corresponding to Räisänen’s model, in which there are three regions corresponding to Räisänen’s two regions and a medium region. As a result, they found that $q < 0$ cannot be realized and so we have not obtained the averaged acceleration in the physically adjusted solution.

Next, Ishibashi and Wald\cite{72} insisted that in the Newtonian gauge the perturbed universes can be expressed by the metric

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(t)(1 - 2\Psi)\gamma_{ij} dx^i dx^j, \quad (4.10)$$
where $\gamma_{ij}$ denotes the metric of constant-curvature space. The potential $\Psi$ is related to the matter density by the cosmological Poisson equation

$$a^{-2} \Delta_{(3)} \Psi = \frac{1}{2} \kappa^2 \delta \rho = \frac{1}{2} \kappa^2 (\rho - \bar{\rho}),$$

(4.11)

where $\Delta_{(3)} \equiv \gamma^{ij} D_i D_j$ and $D_i$ denotes the derivative corresponding to $\gamma_{ij}$. Here $\Psi$ satisfies $|\Psi| << 1$, $(\partial \Psi / \partial t)^2 << a^{-2} D^i \Psi D_i \Psi$, and $(D^i \Psi D_i \Psi)^2 << (D^i D_i \Psi)^2 D_i \Psi$. In this expression of spacetimes, the deviation of perturbed universes from the FLRW spacetime is very small, even if there are nonlinear perturbations like galaxies, clusters, voids and superclusters. This is because $\Psi \sim G \delta M / (c^2 R) << 1$ for such perturbations, where $\delta M \approx \delta \rho R^2$ and $R$ is the radii of these structures.

The above analyses in the Newtonian gauge have a long history. Nariai and Ueno\cite{74} and Irvine\cite{75} derived cosmological equations in the cosmological Newtonian approximation, and they have been applied to the nonlinear treatment of matter evolution and the N-body simulation. Subsequently the formulation in the cosmological post-Newtonian approximation also was treated\cite{76, 77, 78, 79}.

From the viewpoint of Newtonian approximation, it is expected that the averaging and backreaction of sub-horizon perturbations have only very small contributions to the background. Kasai et al.\cite{80} derived the no-go theorem in the Newtonian gauge that the nonlinear backreaction neither accelerates nor decelerates the cosmic expansion, because the cosmic averaged acceleration $\ddot{a} / a$ is determined merely by the mean density as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} < \rho >$$

(4.12)

and the nonlinear backreaction reduces the expansion rate $\dot{a} / a$ as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} < \rho > - \frac{1}{9a^2} \frac{\delta M}{D_i \Psi}$$

(4.13)

Paranjape and Singh\cite{81} and Siegel and Fry\cite{82} also analyzed the possibility of averaged expansion in the Newtonian gauge and obtained the negative result.

In the super-horizon case, Kolb et al.\cite{58, 60} and Barausse et al.\cite{83} studied the possibility of averaged acceleration which is caused by second-order super-horizon perturbations associated with primordial inflation, but Flanagan\cite{84} and Geshnizjani et al.\cite{85} obtained the negative results because of the incompleteness in their second-order analysis and the constraints from the CMB anisotropies.

On the other hand, Kai et al.\cite{86} studied the possibility of accelerated expansion by use of the LTB solution and found that, in a strong inhomogeneous condition such as in Räsänen’s one, the acceleration of cosmic volume expansion is realized only in some cases when the size of the perturbed region is comparable to the horizon radius of the EdS universe.

From the above studies, it seems difficult, therefore, that the accelerated expansion is caused not only by the averaging and backreaction of sub-horizon perturbations, but also by those of super-horizon perturbations.

Wiltshire\cite{87, 88, 89} studied the difference of gravitational energies and their influence to the clock rates in the different regions such as the averagedly expanding region, voids and walls (finite infinity), and considers that this difference brings the observed accelerated expansion. Here we take notice of the definition of the gravitational energy and why it can be so large as to change remarkably the clock rates in different regions.

### B. Fitting

Without averaging and backreaction of inhomogeneous models, we can consider the accelerating effect of inhomogeneities by calculating directly the observational quantities such as the angular diameter distance and the redshift in inhomogeneous models and comparing them with the counterparts in FLRW models with nonzero $\Lambda$. That is, by fitting these two models, we can obtain an effective $\Lambda$.

Vanderveld et al.\cite{90} studied this fitting by using the post-Newtonian approximation and assuming the linear perturbations which are normalized with respect to CMB temperature anisotropies and cluster statistics. Then they obtained an important result that the effective $\Lambda$ is about 0.004, which is very small, compared with the value required by the realization of the observed acceleration.

Recently, Marra et al.\cite{91, 92} studied the fitting by assuming nonlinear perturbations whose amplitudes are much larger than those expected from the CMB normalization and which are given using many arranged swiss-cheese models on scales of several hundred Mpc. From these models they could obtain the effective $\Lambda$ which is comparable with the value necessary for the observed acceleration. At present, however, we do not know how these perturbations with such high amplitudes can exist, without contradicting with the observed CMB temperature anisotropies and the other observations.
V. CONCLUDING REMARKS

Various inhomogeneous cosmological models with a local void have so far been proposed to explain the observed accelerating property in the \( m-z \) relation of SNIa, without a cosmological constant, and their geometrical structure of the models obtained using LTB solutions also has been studied in details. At present, on the other hand, there is no data for SNIa with \( z > 1.7 \). We should notice that this is very important for the model selection, especially to clarify which of the concordant model or local-void models are better with respect to this observation.

Recently the studies of their consistency with observations such as BAO (especially RBAS), kinematic Sunyaev-Zeldovich effect, and spectral distortion at the reionization stage have shown that many models with a local void are ruled out, and we find that only Gps models with the narrow range of parameters remain to be examined. So at present the possibility for these models to survive is small, but may not be zero.

Moreover, the averaging and backreaction of inhomogeneous models and the fitting with nonzero \( \Lambda \) models have been studied. At present, however, it seems difficult to obtain accelerated expansion and the expected effective \( \Lambda \), unless we assume perturbations with amplitudes much larger than those corresponding to the CMB normalization or high amplitudes of gravitational energies included within structures like clusters and voids.

In these analyses, we have assumed the models based on the Einstein gravitational theory and the existence of an inflationary early stage. If the models should be derived, however, from the other gravitational theories, such as the superstring theory, the cosmological situation may be quite different, because we do not know how the inflation arises, what inflation we have, and whether the expected cosmological constant can exist or not. Then the inhomogeneous models with a local void may play some more role to explain the observed accelerating behavior.

Acknowledgments

I would like to thank H. Kodama and A. Ishibashi for their kind organization of the KEK Cosmophysics Workshop DE2008 on “Is our Universe really undergoing an accelerating expansion?” held in KEK (Institute of High Energy Physics, Tukuba, Japan) in Dec. 8-12, 2008. I am grateful to the participants A. Starobinski, R.A. Vandervelt, A. Notari, D. Wiltshire, M. Kasai, K. Nakao, Y. Nambu, J. Soda and K.T. Inoue for their helpful discussions.

[1] B.P. Schmidt et al., Astrophys. J. 507, 46 (1998).
[2] A.G. Riess et al., Astron. J. 116, 1009 (1998).
[3] A.G. Riess et al. Astron. J. 118, 2668 (1999).
[4] S. Perlmutter et al., Astrophys. J. 517, 565 (1999).
[5] K. Tomita and C. Hayashi, Prog. Theor. Phys. 30, 691 (1963).
[6] R. Stabell and S. Refsdal, MNRAS 132, 379 (1966).
[7] S. Refsdal et al., Mem. RAS 71, 143 (1967).
[8] J.-E. Solheim, MNRAS 133, 321 (1966).
[9] C.L. Bennet et al., Astrophys. J. Suppl. 148, 1 (2003).
[10] D.N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003).
[11] D.J. Eisenstein et al., Astrophys. J. 633, 560 (2005).
[12] H.-J. Seo and D.J. Eisenstein, Astrophys. J. 598, 720 (2003).
[13] W.J. Percival et al., MNRAS 381, 1053 (2007).
[14] W.J. Percival et al., Astrophys. J. 657, 51 (2007).
[15] W.J. Percival et al., Astrophys. J. 657, 645 (2007).
[16] N.A. Bahcall and X. Fan, Proc. Nat. Acad. Sci., 95, 5956 (1998).
[17] N.A. Bahcall, X. Fan and R. Cen, Astrophys. J. 485, L53 (1997).
[18] L. Wang and P.J. Steinhardt, Astrophys. J. 508, 483 (1998).
[19] B.R. Granett, M.C. Neyrinck and I. Szapudi, Astrophys. J. 683, L99 (2008).
[20] K. Tomita and K. T. Inoue, Phys. Rev. D77, 103522 (2008). See also Phys. Rev. D79, 103505 (2009).
[21] A. de Oliveira-Costa et al., Phys. Rev. D69, 063516 (2004).
[22] C.R. Contaldi et al., J. Cosmol. Astropart. Phys. 7, 2 (2003).
[23] M.N. Célérier, New Adv. Phys. 1, 29 (2007).
[24] M.N. Célérier, Astron. Astrophys. 353, 63 (2000); astro-ph/9907206.
[25] S.P. Goodwin, A.J. Barber, J. Griffin and L.I. Onuora, astro-ph/9906187.
[26] K. Tomita, Astrophys. J. 529, 26 (2000); astro-ph/9905287.
[27] K. Tomita, Astrophys. J. 529, 38 (2000); astro-ph/9906027.
[28] K. Tomita, Astrophys. J. 451, 1 (1995); Astrophys. J. 461, 507 (1996); Erratum: Astrophys. J. 464, 1054 (1996).
[29] K. Tomita, MNRAS 326, 287 (2001); astro-ph/0011484.
[30] K. Tomita, Prog. Theor. Phys. 106, 929 (2001); astro-ph/0104141.
[31] H. Iguchi, T. Nakamura and K. Nakao, Prog. Theor. Phys. 108, 809 (2002).
[32] R.A. Vanderveld, et al., Phys. Rev. D74, 023506 (2006).
[33] C.-M. Yoo, T. Kai and K. Nakao, Prog. Theor. Phys. 120, 937 (2008).
[34] T. Clifton, P.G. Ferreira and K. Land, Phys. Rev. Lett. 101, 131302 (2008).
[35] H. Alnes, M. Amarzguioui and O. Grøn, Phys. Rev. D73, 083519 (2006).
[36] H. Alnes, M. Amarzguioui and O. Grøn, J. Cosmol. Astropart. Phys. 1, 7 (2007).
[37] H. Alnes and M. Amarzguioui, Phys. Rev. D75, 023506 (2007).
[38] H. Alnes and M. Amarzguioui, Phys. Rev. D74, 103520 (2006).
[39] R. Mansouri, astro-ph/0011484.
[40] R. Mansouri, astro-ph/0104141.
[41] J.W. Moffat, J. Cosmol. Astropart. Phys. 10, 12 (2005).
[42] J.W. Moffat, J. Cosmol. Astropart. Phys. 5, 1 (2006).
[43] T. Biswas, R. Mansouri and A. Notari, J. Cosmol. Astropart. Phys. 12, 017 (2007).
[44] S. Alexander, T. Biswas, A. Notari and D. Vaid, arXiv:0712.0370.
[45] M. Kasai, Prog. Theor. Phys. 007, 1067 (2007).
[46] A. Blanchard et al., Astron. Astrophys. 412, 35 (2003).
[47] P. Hunt and S. Sarkar, Phys. Rev. D76, 123504 (2007); arXiv:0706.2443.
[48] K. Tomita, Astrophys. J. 584, 580 (2003).
[49] J. García-Bellido and T. Haugbølle, J. Cosmol. Astropart. Phys. 4, 3 (2008).
[50] E. Gaztañaga, R. Miquel and E. Sánchez, arXiv:0808.1921.
[51] J.P. Zibin, A. Moss and D. Scott, Phys. Rev. Lett. 101, 251303 (2008).
[52] B.A. Benson et al., arXiv:astro-ph/0303510.
[53] R. Mansouri, astro-ph/0011484.
[54] R. Mansouri, astro-ph/0011484.
[55] J. García-Bellido and T. Haugbølle, arXiv:0807.1326.
[56] R.R. Caldwell and A. Stebbins, arXiv:0711.3459.
[57] T. Buchert, Gen. Rel. Grav. 32, 105 (2000).
[58] T. Buchert and M. Carfire, Phys. Rev. Lett. 90, 031101 (2003).
[59] G.F. Ellis and T. Buchert, gr-qc/0506106.
[60] E.W. Kolb, S. Matarrese, A. Notari and A. Riotto, Phys. Rev. D71, 023524 (2005).
[61] E.W. Kolb, S. Matarrese, A. Notari and A. Riotto, hep-th/0503117.
[62] M. Kasai, Phys. Rev. Lett. 69, 2330 (1992).
[63] M. Kasai, Phys. Rev. D47, 3214 (1993).
[64] M. Kasai, Phys. Rev. D52, 5605 (1995).
[65] Y. Nambu, Phys. Rev. D62, 104010 (2000).
[66] Y. Nambu, Phys. Rev. D63, 044013 (2001).
[67] Y. Nambu, Phys. Rev. D65, 104012 (2002).
[68] Y. Nambu, Phys. Rev. D71, 084016 (2005).
[69] R. Zalaletdinov, Intern. J. Mod. Phys. A23, a1173 (2008).
[70] A. Paranjape, Phys. Rev. D78, 063522 (2008).
[71] Y. Nambu and M. Tanimoto, gr-qc/0507057.
[72] A. Ishibashi and R.M. Wald, Class. Quant. Grav. 23, 235 (2006).
[73] S. Räsänen, JCAP 11, 003 (2006); Int. J. Mod. Phys. D15, 2141 (2006).
[74] H. Nariai and Y. Ueno, Prog. Theor. Phys. 23, 305 (1960).
[75] W.M. Irvine, Ann. Phys. 32, 322 (1965).
[76] K. Tomita, Prog. Theor. Phys. 85, 1041 (1991).
[77] T. Futamase, Prog. Theor. Phys. 86, 389 (1991).
[78] M. Shibata and H. Asada, Prog. Theor. Phys. 94, 11 (1995).
[79] S. Matarrese and D. Terranova, MNRAS 283, 400 (1996).
[80] M. Kasai, H. Asada and T. Futamase, Prog. Theor. Phys. 115, 827 (2006).
[81] A. Paranjape and T.P. Singh, JCAP 3, 023 (2008).
[82] E.R. Siegel and J.N. Fry, Astrophys. J. 628, L1 (2005).
[83] E. Barausse, S. Matarrese and A. Riotto, Phys. Rev. D71, 063537 (2005).
[84] E.E. Flanagan, Phys. Rev. D71, 103521 (2005).
[85] G. Geshnizjani, D.J.H. Chung and N. Afshordi, Phys. Rev. D72, 023517 (2005).
[86] T. Kai, H. Kozuki, K. Nakao, Y. Nambu and C.-M. Yoo, Prog. Theor. Phys. 117, 229 (2007).
[87] D.L. Wiltshire, New J. Phys. 3, 377 (2007).
[88] D.L. Wiltshire, Astrophys. J. 672, L91 (2008).
[9] D.L. Wiltshire, Phys. Rev. D78, 084232 (2008).
[90] R. A. Vanderveld, É.É. Flanagan and I. Wasserman, Phys. Rev. D76, 083504 (2007).
[91] V. Marra, E.W. Kolb and S. Matarrese, Phys. Rev. D76, 123004 (2007).
[92] V. Marra, E.W. Kolb and S. Matarrese, Phys. Rev. D77, 023003 (2008).