Lowest Order Constrained Variational Calculation of the Polarized Nuclear Matter with the Modern $AV_{18}$ Potential

G.H. Bordbar *† and M. Bigdeli

Department of Physics, Shiraz University, Shiraz 71454, Iran‡

and

Research Institute for Astronomy and Astrophysics of Maragha,

P.O. Box 55134-441, Maragha, Iran

Abstract

The lowest order constrained variational method is applied to calculate the polarized symmetrical nuclear matter properties with the modern $AV_{18}$ potential performing microscopic calculations. Results based on the consideration of magnetic properties show no sign of phase transition to a ferromagnetic phase.

*Corresponding author
†E-mail: Bordbar@physics.susc.ac.ir
‡Permanent address
1 Introduction

The properties of dense matter is a subject that theoretical physicists have desired to study. The magnetic property of nucleon matter is of special interest in nuclear and astrophysics which can be related directly with magnetic source of pulsars, rapidly rotating neutron stars with strong surface magnetic fields in the range of $10^{12} - 10^{13}$ Gauss [1, 2, 3]. The most interesting and stimulating mechanisms that have been suggested is the possible existence of a phase transition to a ferromagnetic state at densities corresponding to the theoretically stable neutron stars and, therefore, of a ferromagnetic core in the liquid interior of such compact objects. Such a possibility has been studied by several authors using different theoretical approaches [4-24], but the results are still contradictory. Vidana et al. [21], Vidana and Bombaci [22] have considered properties of spin polarized neutron matter and polarized isospin asymmetric nuclear matter using the Brueckner-Hartree-Fock (BHF) approximation by employing three realistic nucleon-nucleon interactions, Nijmegen II, Reid93 and NSC97e respectively. Zuo et al. [25] have also obtained properties of spin polarized neutron and symmetric nuclear matter using same method with $AV_{18}$ potential. The results of those calculations show no indication of ferromagnetic transition at any density for neutron and asymmetrical nuclear matter. Fantoni et al. [20] have calculated spin susceptibility of neutron matter using the Auxiliary Field Diffusion Monte Carlo (AFDMC) method employing the AU6 + UIX three-body potential, and have found that the magnetic susceptibility of neutron matter shows a strong reduction of about a factor 3 with respect to its Fermi gas value. Baldo et al. [26],
Akmal et al. [27] and Engvik et al. [28] have considered properties of neutron matter with $AV_{18}$ potential using BHF approximation both for continuous choice (BHFC) and standard choice (BHFG), variational chain summation (VCS) method and lowest order Brueckner (LOB) respectively. On the other hand some calculations, like for instance the ones based on Skyrmelike interactions predict the transition to occur at densities in the range $(1-4)\rho_0$ ($\rho_0 = 0.16 fm^{-3}$) [29]. This transition could have important consequences for the evolution of a protoneutron star, in particular for the spin correlations in the medium which do strongly affect the neutrino cross section and the neutrino mean free path inside the star [30].

Recently, we have used the lowest order constrained variational (LOCV) method [31] to calculate the equation of state of symmetrical and asymmetrical nuclear matter and some of their properties such as symmetry energy, pressure, etc. [32-35]. We have also obtained the properties of spin polarized liquid $^3He$ [36] using this method. The LOCV method is a useful tool for the determination of the properties of neutron, nuclear and asymmetric nuclear matter at zero and finite temperature. It is a fully self-consistent formalism which does not bring any free parameters into calculation. It employs a normalization constraint to keep the higher order term as small as possible [31]. The functional minimization procedure represents an enormous computational simplification over unconstrained methods that attempt to go beyond lowest order.

In our pervious work, we have developed the LOCV method to compute the properties of polarized neutron matter such as total energy, magnetic susceptibility, pressure, etc. [37], and have seen that the spontaneous phase transition to a ferromagnetic state in the
neutron matter does not occur. In the present work, we intend to calculate the polarized symmetrical nuclear matter properties using the LOCV method with the modern AV$_{18}$ potential [38] employing microscopic calculations.

2 LOCV FORMALISM

We consider a cluster expansion of the energy functional up to the two-body term,

$$E([f]) = \frac{1}{A} \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} = E_1 + E_2. \quad (1)$$

The smallness of the three-body cluster energy has been discussed in Ref. [32], where it is shown that our cluster expansion converges reasonably and it is a good approximation to stop after the two-body energy term. This property can also be predicted by looking at the correlation between the particles which will be discussed in the next section.

The one-body term $E_1$ can be written as Fermi momentum functional ($k_F^i = (3\pi^2 \rho(i))^{\frac{1}{3}}$),

$$E_1 = \sum_{i=1,2} \frac{3\hbar^2 k_F^i}{5} \frac{\rho(i)}{2m} \rho. \quad (2)$$

Labels 1 and 2 are used instead of spin up and spin down nucleons, respectively, and $\rho = \rho^1 + \rho^2$ is the total nuclear matter density. The two-body energy $E_2$ is

$$E_2 = \frac{1}{2A} \sum_{ij} \langle ij | \nu(12) | ij - ji \rangle, \quad (3)$$

where

$$\nu(12) = -\frac{\hbar^2}{2m} [f(12), [\nabla_{12}^2, f(12)]] + f(12)V(12)f(12) \quad (4)$$
f(12) and V(12) are the two-body correlation and potential, respectively. For the two-body correlation function, f(12), we consider the following form [32, 33]:

\[ f(12) = \sum_{k=1}^{3} f^{(k)}(12) O^{(k)}(12), \quad (5) \]

where, the operators \(O^{(k)}(12)\) are given by

\[ O^{(k=1-3)}(12) = 1, \left(\frac{2}{3} + \frac{1}{6}S_{12}\right), \left(\frac{1}{3} - \frac{1}{6}S_{12}\right), \quad (6) \]

and \(S_{12}\) is the tensor operator. A complete discussion of correlation function and especially its form are given in Ref. [31].

After doing some algebra, we find the following equation for the two-body energy of the polarized symmetrical nuclear matter,

\[
E_2 = \frac{2}{\pi^2 \rho} \left(\frac{\hbar^2}{2m}\right) \sum_{JLTSS_z} \frac{(2J + 1)(2T + 1)}{2(2S + 1)} \left[ 1 - (-1)^{L+S+T} \right] \left| \langle \frac{1}{2} \sigma_{z1} \frac{1}{2} \sigma_{z2} | SS_z \rangle \right|^2 \\
\times \int dr \left\{ \left( f^{(1)}_\alpha^2 a^{(1)}_\alpha^2 (k, r) + \frac{2m}{\hbar^2} (\{V_c - 3V_\sigma + (V_\tau - 3V_\sigma\tau)(4T - 3) \right. \\
+ (V_T - 3V_\sigma)(4T) \} a^{(1)}_\alpha^2 (k, r)[V_{i2} - 3V_{i2\sigma}] \\
+ (V_{i2\tau} - 3V_{i2\sigma\tau})(4T - 3)] c^{(i)}_\alpha^2 (k, r) \right) \left( f^{(1)}_\alpha \right)^2 \right\} \\
+ \sum_{k=2,3} \left[ f^{(k)}_\alpha^2 a^{(k)}_\alpha^2 + \frac{2m}{\hbar^2} (\{V_c + V_\sigma + (-6k + 14)V_t + -(k - 1)V_{is} \right. \\
+ [V_T + V_\sigma + (-6k + 14)V_{it} - (k - 1)V_{is\tau}])(4T - 3) \\
+ [V_T + V_\sigma + (-6k + 14)V_{it}][4T] \} a^{(i)}_\alpha^2 (k, r) \\
+ [V_{i2} + V_{i2\sigma} + (V_{i2\tau} + V_{i2\sigma\tau})(4T - 3)] c^{(i)}_\alpha^2 (k, r) \\
+ [(V_{i2s} + V_{i2s\tau})(4T - 3)] d^{(k)}_\alpha^2 (k, r) \right) \left( f^{(k)}_\alpha \right)^2 \right\} \\
+ \frac{2m}{\hbar^2} [[[V_{is\tau} - 2(V_{i2\sigma\tau} + V_{i2\tau} - 3V_{i2\sigma\tau})(4T - 3)] \\
+ (V_{i2\sigma\tau} - 3V_{i2\sigma\tau})(4T - 3)] \right] \\
\]

\[ 6 \]
\[ +V_{ls} - 2(V_{l2} + V_{l2s}) - 3V_{ls2}b_{\alpha}^2(kfr)f^{(2)}_{\alpha}f^{(3)}_{\alpha} \\
+ \frac{1}{r^2}(f^{(2)}_{\alpha} - f^{(3)}_{\alpha})^2b_{\alpha}^2(kfr) \]  

(7)

where \( \alpha = \{J, L, S, S_z\} \) and the coefficient \( a_{\alpha}^{(1)2} \), etc. are defined as follows,

\[ a_{\alpha}^{(1)2}(x) = x^2I_{L,S_z}(x) \]  

(8)

\[ a_{\alpha}^{(2)2}(x) = x^2[\beta I_{J-1,S_z}(x) + \gamma I_{J+1,S_z}(x)] \]  

(9)

\[ a_{\alpha}^{(3)2}(x) = x^2[\gamma I_{J-1,S_z}(x) + \beta I_{J+1,S_z}(x)] \]  

(10)

\[ b_{\alpha}^{(2)}(x) = x^2[\beta_{23} I_{J-1,S_z}(x) - \beta_{23} I_{J+1,S_z}(x)] \]  

(11)

\[ c_{\alpha}^{(1)2}(x) = x^2\nu_1 I_{L,S_z}(x) \]  

(12)

\[ c_{\alpha}^{(2)2}(x) = x^2[\eta_2 I_{J-1,S_z}(x) + \nu_2 I_{J+1,S_z}(x)] \]  

(13)

\[ c_{\alpha}^{(3)2}(x) = x^2[\eta_3 I_{J-1,S_z}(x) + \nu_3 I_{J+1,S_z}(x)] \]  

(14)

\[ d_{\alpha}^{(2)2}(x) = x^2[\xi_2 I_{J-1,S_z}(x) + \lambda_2 I_{J+1,S_z}(x)] \]  

(15)

\[ d_{\alpha}^{(3)2}(x) = x^2[\xi_3 I_{J-1,S_z}(x) + \lambda_3 I_{J+1,S_z}(x)] \]  

(16)

with

\[ \beta = \frac{J + 1}{2J + 1}; \quad \gamma = \frac{J}{2J + 1}; \quad \beta_{23} = \frac{2J(J + 1)}{2J + 1} \]  

(17)
\[ \nu_1 = L(L + 1); \quad \nu_2 = \frac{J^2(J + 1)}{2J + 1}; \quad \nu_3 = \frac{J^3 + 2J^2 + 3J + 2}{2J + 1} \] (18)

\[ \eta_2 = \frac{J(J^2 + 2J + 1)}{2J + 1}; \quad \eta_3 = \frac{J(J^2 + J + 2)}{2J + 1} \] (19)

\[ \xi_2 = \frac{J^3 + 2J^2 + 2J + 1}{2J + 1}; \quad \xi_3 = \frac{J(J^2 + J + 4)}{2J + 1} \] (20)

\[ \lambda_2 = \frac{J(J^2 + J + 1)}{2J + 1}; \quad \lambda_3 = \frac{J^3 + 2J^2 + 5J + 4}{2J + 1} \] (21)

and

\[ I_{J,S_z}(x) = \int dq P_{S_z}(q) J_J^2(qx) \] (22)

In the above equation, \( J_J(x) \) is the Bessel’s function and \( P_{S_z}(q) \) is defined as follows:

\[ P_{S_z}(q) = \frac{2}{3} \pi \left( (k_F^{\sigma_z 1})^3 + (k_F^{\sigma_z 2})^3 - \frac{3}{2} ((k_F^{\sigma_z 1})^2 + (k_F^{\sigma_z 2})^2)q \right. \]
\[ \left. - \frac{3}{16} ((k_F^{\sigma_z 1})^2 - (k_F^{\sigma_z 2})^2)q^{-1} + q^3 \right) \] (23)

for \( \frac{1}{2} |k_F^{\sigma_z 1} - k_F^{\sigma_z 2}| < q < \frac{1}{2} |k_F^{\sigma_z 1} + k_F^{\sigma_z 2}| \),

\[ P_{S_z}(q) = \frac{4}{3} \pi \min(k_F^{\sigma_z 1}, k_F^{\sigma_z 2}) \] (24)

for \( q < \frac{1}{2} |k_F^{\sigma_z 1} - k_F^{\sigma_z 2}| \) and

\[ P_{S_z}(q) = 0 \] (25)

for \( q > \frac{1}{2} |k_F^{\sigma_z 1} + k_F^{\sigma_z 2}| \), where \( \sigma_z 1 \) and \( \sigma_z 2 \) are equal to \( \frac{1}{2}, -\frac{1}{2} \) for spin up and spin down nucleons, respectively.
Now, we minimize the two-body energy, Eq.(7), with respect to the variations in the
correlation functions $f^{(k)}_{\alpha}$, but subject to the normalization constraint [33],

$$\frac{1}{A} \sum_{ij} \langle ij \mid h_{S_z}^2 - f^2(12) \mid ij \rangle_a = 0, \quad (26)$$

where in the case of spin polarized nuclear matter, the function $h_{S_z}(r)$ is defined as

$$h_{S_z}(r) = \begin{cases} 
1 - \frac{g}{2} \left( \frac{f^2\left(\hat{r} \cdot \mathbf{k}\right)}{\hat{r} \cdot \mathbf{k}} \right)^2 & S_z = \pm 1 \\
1 & S_z = 0
\end{cases} \quad (27)$$

From the minimization of the two-body cluster energy, we get a set of coupled and
uncoupled differential equations which are the same as presented in Ref. [33].

3 RESULTS

In Fig. 1, we have shown the correlation function versus the relative distance ($r$). Fig.
1 shows that the correlation between particles is short range and heals to 1 very quickly.
This means that the two-body term mainly contributes to the interaction of particles and
therefore higher order terms can be neglected.

The energy per particle of the polarized symmetrical nuclear matter versus density
for different values of the spin polarization have been shown in Fig. 2. This figure shows
that the low polarization gives more binding energy than the high polarization. It is also
seen that there is no crossing of the energy curves of different polarizations, vice versa
by increasing density, the difference between the energy of nuclear matter at different
polarization becomes more sizable. This shows that the spontaneous phase transition to a ferromagnetic state in the symmetrical nuclear matter does not occur.

In Fig. 3, we have plotted the quadratic spin polarization dependence $\delta^2$ of energy per particle at different densities. As can be seen from this figure, there are two points worth stressing. First the energy per particle of the polarized symmetrical nuclear matter increases as the polarization increases and the minimum value of energy occurs at $\delta = 0$ for all densities. This indicates that the ground state of symmetrical nuclear matter is paramagnetic. Second the variation of the energy of symmetrical nuclear matter versus $\delta^2$ is nearly linear,

$$E(\rho, \delta) = E(\rho, 0) + a_{\text{nucl}}(\rho)\delta^2. \quad (28)$$

In Fig. 3, the results of ZLS calculations [25] are also given for comparison. There is an agreement between our results and those of ZLS, specially at low densities.

The magnetic susceptibility, $\chi$, which characterizes the response of a system to the magnetic field and gives a measure of the energy required to produce a net spin alignment in the direction of the magnetic field, is defined by

$$\chi = \left( \frac{\partial M}{\partial H} \right)_{H=0}, \quad (29)$$

where $M$ is the magnetization of the system per unit volume and $H$ is the magnetic field. We have calculated the magnetic susceptibility of the polarized symmetrical nuclear matter in the ratio $\chi/\chi_F$ form. By using the Eq. 29 and some simplification, the ratio of
χ to the magnetic susceptibility for a degenerate free Fermi gas \( \chi_F \) can be written as

\[
\frac{\chi}{\chi_F} = \frac{2}{3} \frac{E_F}{\left( \frac{\partial^2 (E/N)}{\partial \delta^2} \right)_{\delta=0}},
\]  

(30)

where \( E_F = \hbar^2 k_F^2 / 2m \) is the Fermi energy and \( k_F = (3/2\pi^2 \rho)^{1/3} \) is Fermi momentum.

Our results for magnetic susceptibility are displayed as a function of density in Fig. 4. As can be seen from Fig. 4, this ratio changes continuously for all densities and decreases as the density increases. Therefore, the ferromagnetic phase transition is not predicted by our calculation. For comparison, we have also shown the results of ZLS [25] in this figure which shows good agreement with our results.

By differentiating symmetrical nuclear matter energy curve at each polarization (\( \delta \)) with respect to the density we can evaluate the corresponding pressure,

\[
P(\rho, \delta) = \rho^2 \frac{\partial E(\rho, \delta)}{\partial \rho},
\]  

(31)

In Fig. 5, we have shown the pressure of polarized symmetrical nuclear matter as a function of density \( \rho \) for various polarizations. We see that equation of state of polarized symmetrical nuclear matter, \( P(\rho, \delta) \), becomes stiffer by increasing the polarization in the density range which was considered.

In Fig. 6, we have also presented the Landau parameter, \( G_0 \), which describes the spin density fluctuation in the effective interaction, versus density. It is seen that the value of \( G_0 \) is always positive and monotonically increasing up to highest density and does not show any magnetic instability for the neutron matter. A magnetic instability would require \( G_0 < -1 \).
4 Summary and Conclusions

We have computed the magnetic properties of polarized symmetrical nuclear matter, that is related directly with magnetic source of pulsars, and other properties using the lowest order constrained variational (LOCV) method with with $AV_{18}$ potential. We have studied the total energy per particle of nuclear matter as a function of density and the spin polarizations $\delta$. We have found that in the range of densities explored, difference between the energy of polarized nuclear matter at different polarization becomes more appreciable. We have also seen that total energy per particle is parabolic on the spin polarization $\delta$ in a very good approximation up to full polarization for all densities. Magnetic susceptibility, which characterizes the response of the system to the magnetic field was calculated for the system under consideration and was found that it changes continuously for all densities. There is an overall agreement between our result and those of Zuo et al. [25]. In conclusion, we see that equation of state of polarized symmetrical nuclear matter becomes stiffer by increasing the polarization in the density range which was considered. The Landau parameter, $G_0$ has been considered and it was seen that the value of $G_0$ is always positive and monotonically increasing up to high densities. Finally, our results have shown no phase transition to ferromagnetic state.

Acknowledgements

This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha, and Shiraz University Research Council.
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Figure 1: The two-body correlation functions of the full polarized nuclear matter as a function of relative distance at $\rho = 0.67 \, fm^{-3}$. 

[Graph showing the correlation functions $f_1(r)$, $f_2(r)$, and $f_3(r)$ as a function of $r(fm)$]
Figure 2: The energy per particle of the polarized symmetrical nuclear matter versus density ($\rho$) for different values of the spin polarization ($\delta$).

Figure 3: Our results (full curves) for the energy difference of polarized and unpolarized cases versus quadratic spin polarization ($\delta^2$) for different values of the density ($\rho$) of the neutron matter. The results of ZLS [25] (dashed curves) are also presented for comparison.
Figure 4: Our result (full curve) for the magnetic susceptibility of the polarized symmetric nuclear matter as the function of density ($\rho$). The results of ZLS [25] (dashed curves) are also given for comparison.
Figure 5: The equation of state of polarized symmetrical nuclear matter for different values of the spin polarization ($\delta$).

Figure 6: Our result for the Landau parameter, $G_0$, as function of density ($\rho$).