Analysis of the influence of vertical sound field distribution on acoustic testing in shallow sea environment

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Abstract. The vertical sound field distribution in shallow seas is similar to the standing wave field, and the single-frequency sound source will have antinodes and nodes in the vertical direction. Therefore, the acoustic test system will have a greater impact on the test results when the acoustic test system is placed at different depths. In this paper, the fast field method is used to solve the spatial sound field distribution of the single-frequency line spectrum and the one-third octave broadband spectrum. The calculation results of the sound source fixed depth receiving point at a certain depth are given, and the different sound velocity gradients and acoustics are analyzed. The impact of the test system receiving depth changes on the sound field strength has certain engineering guiding significance for the actual underwater acoustic test.

1. Introduction
When testing the underwater radiated noise of a real ship, most of them are carried out in shallow seas[1]. Under this condition, the sound waves are restricted by the interface and form a standing wave field in the vertical direction. The typical characteristics of a standing wave field are antinodes and nodes[2][3]. Different deployment depths of underwater acoustic test systems will have a greater impact on the test results. In this paper, the fast field method is used to solve the spatial sound field distribution of single frequency line spectrum and one-third octave broadband spectrum under different sound velocity gradients. The influence of constant sound velocity, thermocline layer, and negative gradient on the test results and the calculation results of the sound source set depth and the test system change at a certain depth are given. It has certain engineering guiding significance for actual underwater acoustic testing.

2. Fast field model
The fast field theory separates the wave equation parameters according to the normal wave approximation method[4][5]. The normal wave approximation is given by the Hankel transform of the Helmholtz equation with respect to distance, and the fast Fourier transform algorithm is used to digitally estimate the transform result. At present, the models based on the fast field theory are: FFP, PRESS, SCOOTER, SPARC and OASES[6][7]. In this paper, FFP algorithm is used to calculate the vertical sound field characteristics of shallow sea. The theory of FFP algorithm is as follows.

A cylindrical coordinate system was established which is shown in Figure 1, and set z=0 at the sea surface and the positive direction of the z-axis under the orientation. Let the density of the middle layer and the speed of sound be \( \rho \) and \( c \), respectively. The layer thickness is \( H \). The density of the lower medium and the longitudinal sound velocity are \( \rho_1 \) and \( c_1 \) respectively. The longitudinal wave...
attenuation coefficient is $\alpha$ and continues to infinity. $z_0$ represents the depth position of the sound source.

\[
\begin{array}{c|cc}
\rho_1 c_1 & 0 & z=0 \\
\rho \ c & z_0 & \\
\rho_2 c_2 & \alpha/ & z=H \\
\end{array}
\]

Figure 1. Medium environment of two-layer fluid

The wave equation of the sound field is:

\[
\frac{\partial^2 p}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial r^2} + \frac{1}{2 r} \frac{\partial p}{\partial r} + \frac{1}{2 r^2} \frac{\partial^2 p}{\partial \xi^2} + k^2(z)p = 0
\]

(2)

Then there is:

\[
p(r, z) = \frac{1}{2\pi} \int_0^\infty P(z, \xi) H^0(\xi r) \xi d\xi
\]

(3)

Get:

\[
p(r, z) = \sqrt{\frac{1}{2\pi r}} \int_0^\infty P(z, \xi) \xi e^{i\xi r} d\xi
\]

(4)

From the wave equation, we know that the unknown function $P(z, \xi)$ in the above integral formula should satisfy the differential equation:

\[
\frac{d^2 P}{dz^2} + (k^2 - \xi^2) P = -2\delta(z - z_0) \quad 0 \leq z < H
\]

(5)

\[
\frac{d^2 P}{dz^2} + (k^2 - \xi^2) P = 0 \quad H < z
\]

(6)

Let:

\[
\beta = \sqrt{k^2 - \xi^2} \quad \beta_1 = \sqrt{k_1^2 - \xi^2} \quad b = \rho / \rho_2
\]

(7)

If $0 \leq z < z_0$:

\[
P(z, \xi) = \frac{2\sin \beta z}{\beta_1} \left[ \frac{\beta_1 \cos \beta (H - z) - ib \beta \sin \beta (H - z_0)}{\beta \cos \beta H - ib \beta \sin \beta H} \right]
\]

(8)

Else if $z_0 < z < H$

\[
P(z, \xi) = \frac{2\sin \beta z}{\beta_1} \left[ \frac{\beta_1 \cos \beta (H - z) - ib \beta \sin \beta (H - z)}{\beta \cos \beta H - ib \beta \sin \beta H} \right]
\]

(9)

Truncate the integral interval on the right side of equation (4) to $(0, \xi_{\text{max}})$, then equation (4) becomes:

\[
p(r, z) = \sqrt{\frac{1}{2\pi r}} \int_0^{\xi_{\text{max}}} P(z, \xi) \xi e^{i\xi r} d\xi
\]

(10)

Sampling the wave number interval $(0, \xi_{\text{max}})$:

\[
\xi_m = \xi_{\text{max}} + m\Delta \xi \quad m = 0, 1, 2, \cdots, (M - 1)
\]

(11)

By sampling the horizontal interval, we can get:

\[
r_n = r_{\text{min}} + n\Delta \xi \quad n = 0, 1, 2, \cdots, (M - 1)
\]

(12)

The sound pressure expression at the receiving point can be obtained:
The formula (13) conforms to the Fourier transform form, which can be easily calculated by FFT or IFFT, and a series of sound pressure values over discrete distances can be obtained at the same time, which is very convenient when calculating the propagation loss.

According to Cauchy’s theorem, the integral between two points in the complex plane does not change with the change of the integral contour. Therefore, formula (4) can be replaced by the following formula:

\[ p(r, z) = \sqrt{\frac{1}{2\pi r}} e^{-\frac{\Delta_x^2}{2}} \sum_{n=0}^{\infty} \left[ \rho(z, \xi) \sqrt{2 \pi} e^{j n \xi} \right] e^{-\Delta_{z,n}^2} \]  

(14)

Use FFT to discretize the integral, we get:

\[ p(r, z) = e^{\Delta_x^2} \sum_{n=0}^{\infty} \left[ \rho(z, \xi) \sqrt{2 \pi} e^{j n \xi} \right] e^{-\Delta_{z,n}^2} \]  

(15)

For most practical problems, 60dB attenuation of the aliasing effect can meet the requirements. From the equation 20\lg P e^{-HR} = 20\lg P - 20\lg e^{HR} , the corresponding offset of the contour integral is

\[ \mu = \frac{3}{R} \frac{2(\xi_{max} - \xi_{min})}{2\pi (M-1) \lg \epsilon} \]  

(16)

Thus, the expression of \( p(r,z) \) in equation (3) can be obtained.

3. Shallow sea sound field model

According to the environmental parameters of the shallow ocean waveguide field, a shallow ocean acoustic field model is established. Suppose the sea depth is \( H = 60 \text{m} \), the sea water density is \( \rho_1 = 1000 \text{kg/m}^3 \), the sea surface is a free and flat interface, the reflection coefficient is \( V_1 = -1 \), the seabed is flat sand, the seabed sound velocity is \( c_2 = 1753 \text{m/s} \), the seabed density is \( \rho_2 = 1957 \text{kg/m}^3 \), the seabed absorption coefficient is \( \alpha = 0.57 \), the depth single point sound source is \( z_0 = -25 \text{m} \), and the shallow sea sound field model is shown in Figure 2.

![Shallow sea sound field model and the environmental parameters](image)

a. isothermal layer  
b. thermocline layer  
c. negative velocity gradient

Figure 2. Shallow sea sound field model and the environmental parameters

4. Vertical sound field distribution of line spectrum

According to the fast field theory, the vertical sound field characteristics of the single frequency line spectrum of different frequencies at 50 meters and 100 meters to the sound source are calculated, and the results are shown in Figure 3.

By comparing the vertical sound field distribution of different frequency line spectra in Fig. 3 when the sound velocity gradient is isothermal layer, negative gradient and thermocline layer, it can be seen that:
1) The line spectrum fluctuates greatly in the vertical direction, and the maximum value exceeds tens of dB. The fluctuation of low frequency is smaller than that of high frequency. The reason is that the waveguide field is similar to the standing wave field in the vertical direction. As the frequency increases, the number of antinodes and nodes gradually increases, which causes the sound field to fluctuate rapidly in the vertical direction.

2) The fluctuation of the sound field at 100 meters is greater than that at 50 meters, and the fluctuation of the thermocline layer is slightly greater than the isothermal layer and negative gradient. Therefore, the sound velocity gradient has a greater impact on the line spectrum during the actual test, and the depth and lateral distance of the receiving point also have a greater impact on the test results.

Figure 3. Different frequency vertical sound field distribution of line spectrum

5. Vertical sound field distribution of one-third octave broadband spectrum
Figure 4. Different frequency vertical sound field distribution of broadband spectrum

c. 500Hz-distribution of vertical sound field
d. 1000Hz-distribution of vertical sound field
e. 20Hz to 10kHz overall SLP-distribution of vertical sound field

Table 1. The difference between the isothermal layer and negative gradient of the vertical sound field at a distance of 50 meters to the sound source (dB)

| f(Hz)/Depth(m) | 5  | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|----------------|----|----|----|----|----|----|----|----|----|----|----|----|
| 20Hz           | 0.2| 0.2| -1.6| 1.6| -0.5| 0.4| -1.0| 0.3| 1.5| 0.0| 1.3| -0.4|
| 100Hz          | 0.7| 0.5| 0.4 | 0.5 | -0.9| -1.5| -1.1| 0.7 | 0.6 | 0.5 | 0.2 | 0.3 |
| 500Hz          | 1.5| -1.3| -0.2| 0.6 | 0.4 | -0.2| 0.2 | 2.2 | -0.1 | -0.1 | 0.5 | -0.4 |
| 1kHz           | 0.2| 0.2| 0.1 | 0.9 | 0.6 | 0.0 | 1.5 | 0.9 | 0.3 | -0.5 | 0.2 | -0.4 |
| 20Hz-10kHz     | 0.1| 0.2| -0.1| 0.4 | 0.1 | 0.2 | 0.1 | 0.1 | 0.3 | 0.0 | 0.1 | -0.4 |

Table 2. The difference between the isothermal layer and the thermocline layer of the vertical sound field at a distance of 50 meters to the sound source (dB)

| f(Hz)/Depth(m) | 5  | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|----------------|----|----|----|----|----|----|----|----|----|----|----|----|
| 20Hz           | 1.7| 1.2| 1.2 | 1.0 | -0.5| -1.2| -0.9| 0.1 | 1.1 | 1.9 | 2.2 | 6.9 |
| 100Hz          | 1.1| -0.1| -3.1| 3.1 | 0.1 | 0.1 | -1.7| -0.6 | 2.3 | 2.6 | 1.7 | 8.2 |
| 500Hz          | 3.1| -2.2| 0.2 | -1.0| 1.4 | 0.2 | 0.1 | 0.9 | -0.8| 0.7 | 2.0 | 8.0 |
| 1kHz           | 1.2| -0.9| -0.4| -0.5| 0.2 | 0.4 | 1.4 | 0.6 | 0.8 | 0.4 | -0.6| 8.1 |
| 20Hz-10kHz     | 0.3| 0.4 | -0.4| 0.2 | -0.2| 0.1 | 0.0 | 0.4 | 0.7 | 0.2 | 1.1 | 8.0 |

According to the fast field theory, the vertical sound field characteristics of different one-third octave broadband spectrum and 20Hz to 10kHz total sound level at 50 meters and 100 meters to the sound source are calculated, and the results are shown in Figure 4, Table 1 and Table 2.

By comparing the results in Figure 4, Table 1 and Table 2, the following conclusions can be drawn:
1) The vertical fluctuation of the sound field is larger at low frequencies and smaller at high frequencies. When approaching the sea surface and the seabed, the fluctuation of the sound field is significantly larger than the middle position. The reason is that the reflected and refracted sound rays are denser on the sea surface and the seabed, while the density of the reflected sound rays is relatively small at the middle position of the water layer. At a position 50 meters away from the sound source, the vertical fluctuation of the sound field is less than 100 meters. The reason is that the direct sound plays a greater role at close range, and the multi-path interference gradually strengthens at a distance which cause the vertical distribution intensity of the sound field changes greatly.

2) The sound velocity gradient has a certain influence on the vertical distribution of the sound field. At a certain distance from the sound source, the difference in the vertical sound field caused by different sound velocity gradients has a greater impact on the 100 meters than at 50 meters. At 50 meters, the sound field difference between the isothermal layer and the negative gradient condition is small, and the difference between the two is almost less than 3dB. The difference between the isothermal layer and the thermocline layer is greater than the difference between the isothermal layer and the negative gradient. The locations where the difference between the two are greater than 3dB are mainly concentrated within 10 meters from the water surface and within 10 meters from the seabed. The main reason is the difference caused by interface reflection. Comparing the results of the total sound level, the difference between the isothermal layer and the negative gradient is less than 1dB at the full depth at 50 meters from the sound source. The isothermal layer and the thermocline layer only differ greatly at the bottom of the sea, and the difference at other depths is also less than 1dB. The above analysis shows that at the distance and depth of the near-field test on a actual ship, the difference in sound velocity gradient has little effect on the test results of broadband noise (one-third octave bandwidth sound level and total sound level).

### 6. Conclusion

Through the above analysis, the following conclusions can be drawn:

1) Because the line spectrum results fluctuate greatly in the vertical direction, it is difficult to grasp its regularity. In combination with the actual ship evaluation, the one-third octave broadband spectrum is mainly used. It is recommended that the one-third octave broadband spectrum should be used when studying the characteristics of the radiation noise of the actual ship.

2) In the actual noise test, the acoustic test system should be placed at a certain distance from the water surface and the bottom to avoid the boundary effect caused by the seabed and the sea surface, so as to better ensure the consistency of the test results.

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