Hydrodynamics, thermodynamics, and acoustics of exponential growth of the vapor bubbles at saturated boiling

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Abstract. The results of earlier performed work are summarized. Formulae for the absolute values of the following variables: the radius of the vapour bubble, the velocity and acceleration of its growth, the specific and total heat flux through the interphase surface of the bubble, the liquid overheating and the heat transfer coefficient, sound pressure in one- and three-dimensional cases are presented. On their basis, the relationship between the relative values of the pairs of these variables because of time elimination is derived.

1. Introduction
The exponential dependence of the vapour bubble radius on time was theoretically obtained in the study [1] in three different ways. This dependence with high accuracy (± 1%) is confirmed by the results of experiments with saturated boiling in the volume of liquid, on a thin wire and on a flat surface [2], and describes well the bubble growth at surface boiling of liquids [3]. On its basis, the hydrodynamic, thermodynamic, acoustic dependences of the vapour bubble growth were obtained, and then the bound correlations of all these dependences were derived. This paper is a continuation and completion of the previous research [1-14].

2. Hydrodynamic, thermodynamic and acoustic dependencies
In [1-14], an analytically rigorous derivation of the formulae of absolute values of variables presented in Table 1 on the left is presented: the radius of the vapour bubble, the velocity and acceleration with its growth, the specific and total heat flux through the interphase surface of the bubble, the liquid overheating and the heat transfer coefficient, sound pressure for one- and three-dimensional cases.

3. Relative values
Each of the formulae (1), (2), ..., (9) in Table 1 on the right represents each of the dependencies (1a), (2a), ..., (9a), respectively on a relative scale. The substitution \( \exp \left( -\frac{t}{\tau_*} \right) = W \) is used. The corresponding bold letter indicates the ratio of the variable to its maximum value. The dependencies (5a), (8a) and (9a) have been investigated for extrema in order to determine the numerical factors for which the maxima (5), (8) and the positive maximum (9) are equal to 1.
Table 1. Formulas for the absolute values of the hydrodynamic, thermodynamic, and acoustical variables of the exponential bubble growth (on the left); the hydrodynamic, thermodynamic and acoustic dependences of the growth of vapour bubbles (on the right).

\[
R = R_0 \left[ 1 - \exp \left( -\frac{t}{\tau_*} \right) \right] \quad (1a) \quad R = 1 - W
\]

\[
R = \frac{R_0}{\tau_*} \exp \left( -\frac{t}{\tau_*} \right) \quad (2a) \quad R = W
\]

\[
R = -\frac{R_0}{\tau_*} \exp \left( -\frac{t}{\tau_*} \right) \quad (3a) \quad R = -W
\]

\[
q = \rho^* \frac{LR_0}{\tau_*} \exp \left( -\frac{t}{\tau_*} \right) \quad (4a) \quad q = W
\]

\[
Q = qS = \frac{4\pi \rho^* LR_0^2}{\tau_*} \left[ 1 - \exp \left( -\frac{t}{\tau_*} \right) \right]^2 \exp \left( -\frac{t}{\tau_*} \right) \quad (5a) \quad Q = 6.75W (1 - W)^2
\]

\[
\theta = \theta_0 \left( 1 - \left[ 1 - \exp \left( -\frac{t}{\tau_*} \right) \right]^3 \right) \quad (6a) \quad \theta = 1 - (1 - W)^3
\]

\[
\alpha = \frac{q}{\rho^* \frac{LR_0}{\theta_0 \tau_*} \left[ 1 - \exp \left( -\frac{t}{\tau_*} \right) \right]^3} \quad (7a) \quad \alpha = W / \left[ 1 - (1 - W)^3 \right]
\]

\[
p_1 = \frac{2\pi \rho^* cR_0^3}{\tau_*} \left[ \exp \left( -3\frac{t}{\tau_*} \right) - 2 \exp \left( -2\frac{t}{\tau_*} \right) + \exp \left( -\frac{t}{\tau_*} \right) \right] \quad (8a) \quad p_1 = 6.75W (1 - W)^2
\]

\[
p_2 = \frac{\rho^* LR_0^3}{\tau_*^2} \left[ -3 \exp \left( -3\frac{t}{\tau_*} \right) + 4 \exp \left( -2\frac{t}{\tau_*} \right) - \exp \left( -\frac{t}{\tau_*} \right) \right] \quad (9a) \quad p_2 = 4.26(-3W^3 + 4W^2 - W)
\]

4. Bound correlations
If the exponent of the relative time \( W \) of one of the formulae (1) - (9) is substituted into one of the other of these formulas, then we obtain an equation with the excluded time. This is the correlation of the corresponding variables, which is valid at any time. Such bound correlations are presented in Table 2. In this case, the numbers of the original formulae are given separated by commas in brackets.

5. Determination of the parameters of vapour bubble exponential growth by the acoustic method
As a result of the study of the equality (9a) on extrema and the use of the correlation (9), it is easy to obtain formulae that allow us to determine the main parameters \( \tau_* \) and \( R_* \) of the exponential growth of the bubble with respect to time \( t_{m1} \) and \( t_{m2} \), as well as amplitude \( P_{m1} \) and \( P_{m2} \) characteristics of the sound pulse generated by the bubble.

The constants \( \tau_{e1} \) and \( R_{e1} \) are determined on the basis of \( t_{m1} \) and \( P_{m1} \) only of the first maximum of the sound pulse, and \( \tau_{e2} \) and \( R_{e2} \) according to \( t_{m2} \) and \( P_{m2} \) of the negative maximum of this pulse. The most accurate result is \( \tau_* \) and \( R_* \) has been obtained with the use of all the data calculated with the application of experimental data \( t_{m1}, t_{m2}, P_{m1} \) and \( P_{m2} \). The discrepancy equals \( \pm \delta \), % has been
calculated with the application of experimental data of a direct experiment with simultaneous recording of changes in the radius of the spherical vapour bubble and the sound pulse.

**Table 2.** The formulae for the absolute values of the hydrodynamic, thermodynamic, and acoustic variables of the exponential bubble growth.

| Term                  | Formula                                                                 |
|-----------------------|--------------------------------------------------------------------------|
| Hydrodynamic          |                                                                         |
| \( R = 1 - R \)       | (2.1) \( R = R - 1 \)                                                   |
| \( \theta = 1 - (1 - q)^3 \) | (5.4) \( \theta = 1 - R^3 \)                                         |
| \( \alpha = \frac{q}{\sqrt[3]{1 - \theta}} \) | (7.4) \( \alpha = R/\sqrt{1 + R^3} \)                                |
| Thermodynamic         |                                                                         |
| \( q = 1 - R \)       | (4.1) \( Q = 6.75q(1 - R)^2 \)                                       |
| \( \theta = 1 - (1 - q)^3 \) | (5.1) \( \theta = 1 - R^3 \)                                         |
| \( \alpha = R/\sqrt{1 - R^3} \) | (6.1) \( \alpha = R/\sqrt{1 - R^3} \)                                |
| Hydrothermodynamic    |                                                                         |
| \( q = 1 - R \)       | (6.2) \( Q = 6.75(1 - R)^2 \)                                         |
| \( \theta = 1 - R^3 \) | (7.2) \( \theta = 1 - R^3 \)                                         |
| \( \alpha = R/\sqrt{1 - R^3} \) | (8.1) \( \alpha = R/\sqrt{1 - R^3} \)                                |
| Hydroacoustic         |                                                                         |
| \( p_1 = 6.75(1 - R)^2 \) | (8.2) \( p_1 = 6.75(1 - R)^2 \)                                       |
| \( p_3 = 4.26(3R^3 - 5R^2 + 2R) \) | (9.1) \( p_3 = 4.26(3R^3 + 4R^2 - R) \)                             |
| Thermoacoustic        |                                                                         |
| \( p_1 = 6.75q(1 - q)^2 \) | (8.4) \( p_1 = 6.75q(1 - q)^2 \)                                      |
| \( p_3 = 4.26(3R^3 - 5R^2 + 2R) \) | (9.2) \( p_3 = 4.26(3R^3 + 4R^2 - R) \)                             |

**Table 3.** Determination of the parameters \( \tau \) and \( R \) by the sound-ranging method.

| Term                  | Formula                                                                 |
|-----------------------|--------------------------------------------------------------------------|
| \( \tau_1 = 3.2976t_{m1} \) | (3.5917) \( \frac{P_{m1}t_{m1}^2}{\rho^{\prime}} \) \( \delta = \pm 2.0\% \) |
| \( \tau_2 = 0.5280t_{m2} \) | (1.5843) \( \frac{P_{m2}t_{m2}^2}{\rho^{\prime}} \) \( \delta = \pm 0.5\% \) |
| \( \tau_3 = 0.6286(t_{m2} - t_{m1}) \) | (1.0904) \( \frac{(P_{m1} - P_{m2})(t_{m2} - t_{m1})}{\rho^{\prime}} \) \( \delta = \pm 0.7\% \) |

6. Conclusion
All the results obtained in the paper are a consequence solving the differential equation of the vapour bubble growth dynamics at boiling. This solution is the exponential dependence of the bubble radius on time.

The application of relative values of the change in the radius of the bubble, velocity and acceleration with its growth, superheating of the liquid, heat flux and its density, the heat transfer
coefficient, and also the sound pressure generated by the bubble, made it possible analytically to obtain a series (30) of simple universal bound correlations of these values.

The deduced hydrodynamic and acoustic dependences are the basis of the sound-ranging method of experimental determination of the main parameters at the exponential growth of the vapour bubble. This method, according to the results of acoustic measurements, makes it possible to investigate the hydrodynamic and thermodynamic dependences of this growth quantitatively, and to study the possibilities of acoustic diagnostics of heat transfer at boiling.

**Nomenclature**

- \( c \) – sound velocity, m/s;
- \( c' \) – specific heat of the fluid, J/(kg⋅K);
- \( L \) – specific heat of vaporization, J/kg;
- \( m' \) – mass of the superheated liquid around the bubble, kg;
- \( p_{1,3} \) – transient pressure in dimensional cases 1 and 3, Pa;
- \( q \) – heat flow density, W/m²;
- \( q' \) – relative density of heat flow;
- \( Q \) – heat flow, J/m²;
- \( Q' \) – relative heat flow;
- \( r \) – distance between the vapor bubble center and the point of sound pressure determination, m;
- \( R \) – radius of the vapor bubble, m;
- \( R_* \) – parameter of exponential dependence of the vapor bubble growth, m;
- \( S \) – interphase surface of the bubble, m²;
- \( t \) – time, s;
- \( \alpha \) – heat transfer coefficient, W/(m²⋅K);
- \( \alpha' \) – relative heat transfer coefficient;
- \( \theta \) – superheat of liquid, K;
- \( \theta_0 \) – Initial superheat, K;
- \( \theta' \) – relative superheat;
- \( \rho' \) – fluid density, kg/m³;
- \( \rho'' \) – vapor density, kg/m³;
- \( \tau \) – parameter of exponential dependence of the vapor bubble growth, s.

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