V-shaped sliding bearings using micropolar lubricants caused by a melt accounting for the dependence of lubricant viscosity and porous lauer permeability on pressure

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Abstract. The paper presents a method for formation of an exact self-similar solution of the hydrodynamic calculation of a wedge-shaped slip bearing (slide, guide) using a micropolar liquid lubricant caused by a melt of the guide taking into account the dependence of lubricant viscosity and porous lauer permeability on pressure. Based on the system of motion of a viscous incompressible fluid of a micropolar lubricant for the “thin layer” taking into account the dependence of viscosity characteristics of the micropolar lubricant and permeability of the porous layer on pressure, continuity equations and equations for the dissipation rate of mechanical energy, an analytical dependence for the molten surface profile was obtained. In addition, performance characteristics of the friction pair were determined. The influence of parameters caused by the melt of the guide, structural-viscous parameters of the micropolar liquid lubricant and permeability of the porous layer on the bearing capacity and friction force was estimated.

1. Introduction
New machines are designed to increase static and shock loads acting on sliding bearings. This is determined by modern engineering tasks. One of the most important equal-right structural elements of fluid-friction bearings is a lubricating medium.

One of the methods for solving design and operational problems may be the use of melt lubrication for low-melting bearing coatings.

Lubrication with liquid metals is used at temperatures at which conventional lubricating environments undergo irreversible physical and chemical changes. Melt lubrication has been studied in many applied studies, in particular, in the processes of forming and cutting metals [1–7]. Hydrodynamic calculation of a system consisting of a “slider - guide”, when the slider is located at an angle to the guide surface, in the absence of a lubricant, and taking into account the dependence of lubricant viscosity on pressure was studied in [8–13]. A significant drawback of the friction pair which uses melt lubrication is a low bearing capacity and ignorance of rheological properties of non-Newtonian lubricants. In addition, the process of lubricating plastic lubricant is not self-sustaining.

Thus, the development of a mathematical design model of sliding bearings in the presence of a lubricant having micropolar rheological properties, as well as a melt guide covered with a low-melting metal melt and permeability of the porous layer is a promising area of modern tribology.
2. Problem statement

A wedge-shaped support consisting of a “slider-guide” system is analyzed. It is assumed that the surfaces of the slider with a porous coating and a guide covered with a low-melting metal melt are separated by a layer of lubricant having micropolar properties; the slide is stationary, and the guide made of a material with a low melting point moves towards the narrowing gap with speed \( \dot{u} \) (Figure 1).

\[
y' = h_0 + x'tg\alpha, \quad y' = -\eta f'(x'). \tag{1}
\]

We assume that viscosity characteristics of a micropolar liquid lubricant and permeability of a porous coating depend on the pressure according to the law

\[
\mu' = \mu_0 e^{\beta p'}, \quad \kappa' = \kappa_0 e^{\beta p'}, \quad \gamma' = \gamma_0 e^{\beta p'}, \quad K' = k_0 e^{\beta p'}, \tag{2}
\]

where \( \mu' \) – lubricant dynamic viscosity coefficient; \( \kappa' \), \( \gamma' \) – micropolar lubricant viscosity coefficients; \( \mu_0 \) – characteristic viscosity of a newtonian lubricant; \( p' \) – hydrodynamic pressure in the lubricant layer; \( \alpha \) – experimental constant, \( k_0 \) – characteristic permeability of the porous layer; \( k' \) – permeability of the porous layer.

The conditions of motion are considered under the following assumptions:

1. Liquid medium is a viscous incompressible fluid.
2. All heat released in the lubricating film goes to the melting of the surface of the material of the guide.

Initial equations and boundary conditions. Initial equations are the system of equations of motion of a lubricant having micropolar properties for the “thin layer”, the continuity equation, and the Darcy equation.

\[
\frac{1}{2}(2\mu + \kappa) \frac{\partial^2 u'}{\partial y'^2} + \kappa \frac{\partial^2 u'}{\partial y'^2} = \frac{dp'}{dx'}; \quad \frac{\partial^2 v'}{\partial y'^2} + 2\kappa\gamma' - \kappa \frac{\partial^2 u'}{\partial y'^2} = 0; \quad \frac{\partial u'}{\partial y'} + \frac{\partial u'}{\partial x'} = 0; \quad \frac{\partial P'}{\partial x'} + \frac{\partial P'}{\partial y'} = 0. \tag{3}
\]

\( u', v' \) – components of the velocity vector of the lubricating medium; \( p' \) – hydrodynamic pressure in the lubricant layer; \( u' \) – micro rotation speed.

The boundary conditions will be written as:

\[
u' = -u', \quad v' = 0, \quad v' = 0 \quad \text{if} \quad y' = -\eta f'(x');
\]

\[
u' = 0, \quad v' = 0, \quad v' = 0, \quad \text{if} \quad y' = h_0 + x'tg\alpha;
\]

\[
p'(0) = p'(l) = p_0; \quad \eta f'(x') = \Phi(x) \quad \text{if} \quad x' = 0.
\]
where $\Phi(x)$ – melted film thickness function. To determine $\Phi(x)$, caused by the molten guide, we use the formula for the rate of dissipation of energies

$$\frac{dn_f^*(x')}{dx'} \cdot u' L' = 2\mu \int_{-\Phi(x)}^{h + x\tan \alpha} \left( \frac{\partial u}{\partial y'} \right)^2 dy',$$

(5)

where $L'$ – specific heat of fusion per unit volume.

Dimensional quantities are related by the corresponding dimensionless quantities to the following relations:

$$u' = u u; \quad v' = u' v; \quad \nu' = \nu' v; \quad p' = p^* p; \quad y' = h_0 y; \quad \mu' = \mu_0 \mu; \quad \kappa' = \kappa_0 \kappa; \quad \gamma' = \gamma_0 \gamma; \quad N^2 = \frac{\kappa}{2\mu + \kappa};$$

$$N_1 = \frac{2\mu^2}{\kappa h_0^2}; \quad I^2 = \frac{\gamma}{4\mu}; \quad \varepsilon = \frac{h_0}{L}; \quad \nu' = \frac{u'}{2h_0}; \quad p^* = \frac{(2\mu_0 + \kappa_0)Lu^*}{2h_0}; \quad x' = lx. \quad \bar{a} = \frac{a}{p};$$

(6)

In the porous layer:

$$x' = lx'; \quad y' = ly'; \quad P' = p^* P; \quad \kappa' = \kappa_0 \kappa;$$

(7)

Taking into account the transition to dimensionless variables in the lubricating layer, we have the following system of equations:

$$\frac{\partial^2 u}{\partial y'^2} + N^2 \frac{\partial u}{\partial y} = -\frac{dp}{dx}, \quad \frac{\partial^2 v}{\partial y'^2} = \frac{\nu}{N_1} \frac{1}{\partial y}, \quad \frac{\partial \bar{h}}{\partial x} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = 0;$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial P}{\partial y^2} = 0, \quad \frac{d\Phi(x)}{dx} = K \int_{-\Phi(x)}^{h(x)} \left( \frac{\partial u}{\partial y} \right)^2 dy.$$

where $K = \frac{2\mu u^*}{h_0 L'}$ and boundary conditions:

$$\nu = 0, \quad v = 0, \quad u = 0, \quad \text{if} \quad y = 1 + \eta x = h(x);$$

$$\nu = 0, \quad v = 0, \quad u = -1, \quad \text{if} \quad y = -\Phi(x);$$

$$p(0) = P(1) = \frac{P_0}{p^*}; \quad \Phi(x) = \Phi_h = Kg_0 \quad \text{if} \quad x = 0.$$  

(8)

$$p = P \quad \text{if} \quad y' = 1 + \eta x + \frac{\bar{H}}{h_0}; \quad u = \bar{M} \frac{\partial P}{\partial y} \quad \text{if} \quad y' = 1 + \eta x + \frac{\bar{H}}{h_0}; \quad \frac{\partial P}{\partial y} = 0 \quad \text{if} \quad y' = 1 + \eta x;$$

where $\bar{M} = \frac{k l}{h_0}, \quad \eta = \frac{\mu \tan \alpha}{h_0}.$

Given the smallness of the gap, as well as equality $\nu = 0$ on moving and stationary surfaces, let us average the second equation of system (7) by the thickness of the lubricating layer:

$$\frac{1}{h(x) + \Phi(x)} \int_{-\Phi(x)}^{h(x)} \frac{\partial^2 u}{\partial y'^2} dy = \frac{1}{N_1} \int_{-\Phi(x)}^{h(x)} \left( \frac{\partial u}{\partial y} \right)^2 dy + \frac{1}{h(x) + \Phi(x)} \int_{-\Phi(x)}^{h(x)} \left( \frac{\partial u}{\partial y} \right)^2 dy.$$  

(9)

The solution of equation (9) is

$$\nu = A_1(x) x^2 + A_2(x) x + A_3(x).$$  

(10)

Taking into account the boundary conditions (8), we have
\[ v = A_i(x) \cdot \left( y^2 - (h(x) - \Phi(x))y - \Phi(x)h(x) \right). \]  

(11)

Inserting (11) in (9) with accuracy to \( O(\Phi/N_1) \), \( O(1/N_2) \), we have:

\[ v = \frac{1}{2N_hh(x)}(y^2 - h(x)y), \quad \frac{\partial v}{\partial y} = \frac{1}{2N_hh(x)}(2y - h(x)), \quad A_i = \frac{1}{2N_hh(x)}. \]  

(12)

Given (12), the system (7) is

\[ \frac{\partial^2 u}{\partial y^2} + \frac{N^2}{2N_hh(x)}(2y - h(x)) = e^{-\alpha y} \frac{dp}{dx}, \quad v = \frac{1}{2N_hh(x)}(y^2 - h(x)y), \]

\[ \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{\partial^2 p}{\partial x^2} + \frac{\partial p}{\partial y^2} = 0; \quad \frac{d\Phi(x)}{dx} = K \int_{\Phi(x)}^{\Phi(x)} \left( \frac{\partial u}{\partial y} \right)^2 dy. \]  

(13)

Let us introduce \( z = e^{-\alpha y} \). Let us differentiate both sides of the equation:

\[ \frac{dz}{dx} = -ae^{-\alpha y} \frac{dp}{dx} \text{ or } e^{-\alpha y} \frac{dp}{dx} = -\frac{dz}{a dx}, \]

equation (13) and the corresponding boundary conditions will be:

\[ \frac{\partial^2 u}{\partial y^2} + \frac{N^2}{2N_hh(x)}(2y - h(x)) = -\frac{1}{a} \frac{dz}{dx}, \quad v = \frac{1}{2N_hh(x)}(y^2 - h(x)y), \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{\partial^2 p}{\partial x^2} + \frac{\partial p}{\partial y^2} = 0, \quad z \frac{d\Phi(x)}{dx} = K \int_{\Phi(x)}^{\Phi(x)} \left( \frac{\partial u}{\partial y} \right)^2 dy. \]  

(14)

\[ v = 0, \quad v = 0, \quad u = 0 \quad \text{if} \quad y = 1 + \eta x = h(x); \]

\[ v = 0, \quad u = -1, \quad v = 0 \quad \text{if} \quad y = -\Phi(x); \quad z(0) = z(1) = e^{-\alpha y}. \]  

(15)

Taking \( K \) as a small parameter caused by the melt and the rate of energy dissipation, \( \Phi(x) \) can be written as

\[ \Phi(x) = -K\Phi_1(x) - K^2\Phi_2(x) - K^3\Phi_3(x) - \ldots = H. \]  

(16)

Boundary conditions for dimensionless velocity components \( u \) and \( v \) on the contour \( y = 0 - \Phi(x) \) can be written as

\[ v(0 - H(x)) = v(0) \cdot \left[ \frac{\partial v}{\partial y} \right]_{y=0} H(x) \left[ \frac{\partial^2 v}{\partial y^2} \right]_{y=0} H^2(x) - \ldots = 0; \]

\[ u(0 - H(x)) = u(0) \cdot \left[ \frac{\partial u}{\partial y} \right]_{y=0} H(x) \left[ \frac{\partial^2 u}{\partial y^2} \right]_{y=0} H^2(x) - \ldots = -1. \]  

(17)

The asymptotic solution of the system of differential equations (7) taking into account boundary conditions (8) and (17) is

\[ v = v_0(x, y) + K\Phi_1(x, y) + K^2v_2(x, y) + \ldots, \]

\[ u = u_0(x, y) + K\Phi_1(x, y) + K^2u_2(x, y) + \ldots, \]

\[ \Phi(x) = -K\Phi_1(x) - K^2\Phi_2(x) - K^3\Phi_3(x) - \ldots, \]

\[ z = z_0 + Kz_1(x) + K^2z_2(x) + K^3z_3(x) - \ldots. \]  

(18)

Substituting (18) into the system of differential equations (14) taking into account boundary conditions (15), we have:
– for the zero approximation:
\[
\frac{\partial^2 u_0}{\partial y^2} + \frac{N^2}{2N_i h(x)} (2y - h(x)) = -\frac{1}{\alpha} \frac{dz_0}{dx}, \quad \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} = 0.
\]
\[
\frac{\partial^2 P_0}{\partial x^2} + \frac{\partial P_0}{\partial y^2} = 0
\]
(19)
with boundary conditions:
\[
K\Phi_0(0) = Kg_0; \quad z_0(0) = z_0(1) = e^{-\frac{\alpha L_0}{h_0}};
\]
\[
v_0 = 0, \quad v_0 = 0, \quad u_0 = 0 \quad \text{if} \quad y = 1 + \eta x;
\]
\[
v_0 = 0, \quad u_0 = -1, \quad v_0 = 0 \quad \text{if} \quad y = 0;
\]
\[
p_0 = P_0 \quad \text{if} \quad y^* = 1 + \eta x + \frac{H}{h_0}; \quad u_0 = \tilde{M} \frac{\partial P_0}{\partial y} \quad \text{if} \quad y^* = 1 + \eta x + \frac{H}{h_0}; \quad \frac{\partial P_0}{\partial y} = 0 \quad \text{if} \quad y^* = 1 + \eta x; \quad (20)
\]
– for the first approximation:
\[
\frac{\partial^2 u_1}{\partial y^2} = -\frac{1}{\alpha} \frac{dz_1}{dx}, \quad \frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} = 0, \quad z_0 \frac{d\Phi_1(x)}{dx} = K \int_{-\phi_0}^{1+\eta x} \left( \frac{\partial u_0}{\partial y} \right)^2 dy; \quad \frac{\partial^2 P}{\partial x^2} + \frac{\partial P}{\partial y^2} = 0
\]
(21)
with boundary conditions:
\[
v_1 = \left( \frac{\partial v_0}{\partial y} \right)_{y=0} \cdot \Phi_1(x); \quad u_1 = \left( \frac{\partial u_0}{\partial y} \right)_{y=0} \cdot \Phi_1(x);
\]
\[
v_1 = 0, \quad v_1 = 0, \quad u_1 = 0 \quad \text{if} \quad y = 1 + \eta x;
\]
\[
z_1(0) = z_1(1) = 0, \quad K\Phi_1(0) = K\alpha^*; \quad \Phi(0) = \Phi(1) = \alpha^*.
\]
\[
p_1 = P_1 \quad \text{if} \quad y^* = 1 + \eta x + \frac{H}{h_0}; \quad u_1 = \tilde{M} \frac{\partial P}{\partial y} \quad \text{при} \quad y^* = 1 + \eta x + \frac{H}{h_0};
\]
\[
\frac{\partial P}{\partial y} = 0 \quad \text{if} \quad y^* = 1 + \eta x;
\]
(22)

### 3. Exact solution

The exact solution of the problem for the zero approximation in the adopted approximation up to
\[O(\eta/N_1)\]
is
\[
u_0(x, y) = \frac{\partial \psi_0(x, y)}{\partial x} + U_0(x, y); \quad v_0(x, y) = -\frac{\partial \psi_0(x, y)}{\partial y} + V_0(x, y); \quad \psi_0(x, y) = \tilde{\psi}_0(\xi); \quad \xi = \frac{y}{h(x)};
\]
\[
V_0(x, y) = -\tilde{v}(\xi) \cdot h'(x); \quad U_0(x, y) = \tilde{u}_0(\xi); \quad \tilde{u}_0(\xi) + \xi \tilde{y}_0(\xi) = 0.
\]
(23)

Substituting (23) into the system of differential equations (19) taking into account boundary conditions (20), we have the following system of differential equations:
\[
\tilde{\psi}_0''(\xi) = \tilde{C}_2; \quad \tilde{u}_0''(\xi) = \tilde{C}_1 - \frac{N^2}{2N_i} (2\xi - 1); \quad \frac{dz_0}{dx} = -\alpha \left( \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} \right);
\]
and boundary conditions:
\[
\tilde{\psi}_0'(0) = 0, \quad \tilde{\psi}_0'(1) = 0; \quad \tilde{u}_0'(1) = 0; \quad \tilde{v}_0(1) = 0; \quad \nu(0) = \nu(1) = 0,
\]
\[
\tilde{u}_0(0) = 1, \quad \tilde{v}_0(0) = 0, \quad \int_0^1 \tilde{u}_0(\xi) d\xi = 0, \quad z_0(0) = z_0(1) = e^{-\frac{\alpha L_0}{h_0}}.
\]
By using the direct integration, we have

\[ \psi_0'(\xi) = \frac{\tilde{C}_1}{2} (\xi^2 - \xi), \quad \tilde{C}_1 = 6, \quad \tilde{u}_0(\xi) = \tilde{C}_1 \frac{\xi^2}{2} - \frac{N^2}{2N_1} \left( \frac{\xi}{3} - \frac{\xi^2}{2} \right) - \left( \frac{N^2}{12N_1} + \frac{\tilde{C}_1}{2} + 1 \right) \xi + 1. \]  

Let us determine the pressure in the contact zone from the equation:

\[ \frac{dx_0}{dx} = -\alpha \left( \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} \right) \]  

From the boundary condition \( z_0(0) = z_0(1) = e^{-\frac{a_p}{\rho}} \) it follows \( \tilde{C}_2 = -\tilde{C}_1 \left( 1 + \frac{1}{2} \eta \right) \).

For function \( z_0 \) we have \( z_0 = \alpha \tilde{C}_1 \frac{\eta}{2} (x - x^2) + e^{-\frac{a_p}{\rho}} \).

Using the asymptotic expansion \( e^x = 1 + x + \frac{x^2}{2} + \ldots \) to determine pressure \( P \) we have:

\[ \alpha p^2 - 2p_0 + \left[ \tilde{C}_1 \frac{\eta}{2} (x^2 - x) \right] + 2p_x - \alpha \left( \frac{p_x}{p} \right)^2 = 0 \]  

Finding hydrodynamic pressure accurate to \( O(\eta^2) \), \( O\left( \frac{p_x}{p} \right)^3 \) we have:

\[ p_0 = \left( 1 + \alpha \frac{p_x}{p} - \frac{\alpha^2}{2} \left( \frac{p_x}{p} \right)^2 \right) \left( \frac{\tilde{C}_1}{2} \frac{\eta}{2} (x^2 - x) \right) + \frac{p_x}{p} \]  

Given (31) the solution of the Darcy equation is

\[ P_0(x, y^*) = R_0(y^*) + \frac{\eta}{2} \left( 1 + \alpha \frac{p_x}{p} - \frac{\alpha^2}{2} \left( \frac{p_x}{p} \right)^2 \right) \tilde{C}_1 = \tilde{C}_1 + p_x \]  

Substituting equation (32) into the Darcy equation of system (19) for function we have:

\[ R_0'(y^*) + \frac{\eta}{2} \left( 1 + \alpha \frac{p_x}{p} - \frac{\alpha^2}{2} \left( \frac{p_x}{p} \right)^2 \right) \tilde{C}_1 = 0 \]  

Solution (33)-(34) is:

\[ R(y^*) = -\tilde{C}_1 \eta \left( 1 + \alpha \frac{p_x}{p} - \frac{\alpha^2}{2} \left( \frac{p_x}{p} \right)^2 \right) \left( \frac{y^*}{2} + \left( 1 + \eta x + \frac{\tilde{H}}{h_0} \right) y^* + \frac{p_x}{p} \right) \]  

\( \tilde{C}_1 \) is determined from the continuity equations ranging from 0 to 1:

\[ \tilde{M} \left. \frac{\partial P_0}{\partial y} \right|_{y^* = 1 + \eta x + \frac{p_x}{h_0}} = \int_0^1 \tilde{u}_0(\xi) d\xi \]  

given (26), (32) and (35) for \( \tilde{C}_1 \) we have:
\[
\tilde{c}_1 = \frac{6}{-12\tilde{M} \eta \left(1 + \alpha \frac{p_a}{p} - \frac{\alpha^2}{2} \left(\frac{p_a}{p}\right)^2\right) \left(2 + \frac{H}{h_0}\right) + 1}
\]

for the hydrodynamic pressure for the zero approximation, we have the following expression:

\[
P_0 = \frac{3\eta (x^2 - x)}{-12\tilde{M} \eta \left(1 + \alpha \frac{p_a}{p} - \frac{\alpha^2}{2} \left(\frac{p_a}{p}\right)^2\right) \left(2 + \frac{H}{h_0}\right) + 1} + \frac{p_a}{p}
\]

Given (36) for we have:

\[
z_0 = \frac{3\eta (x^2 - x)}{-12\tilde{M} \eta \left(1 + \alpha \frac{p_a}{p} - \frac{\alpha^2}{2} \left(\frac{p_a}{p}\right)^2\right) \left(2 + \frac{H}{h_0}\right) + 1} + e^{-\frac{p_a}{p}}
\]

To determine \( \Phi_1(x) \) accounting for (38), we have:

\[
\frac{d\Phi_1(x)}{dx} = \frac{h(x)}{z_0} \int_0^x \left(\psi_0^\prime(\xi) + \bar{u}_1^\prime(\xi)\right) d\xi.
\]

Integrating equation (39), we have:

\[
\Phi_1(x) = \frac{1}{z_0} \left[\frac{\Delta_1}{h^2(x)} + \int_0^x \frac{\Delta_2}{h^2(x)} + \int_0^x \frac{\Delta_3}{h(x)}\right],
\]

where

\[
\Delta_1 = \int_0^1 \left(\psi^\prime(\xi)\right)^2 d\xi = \frac{\tilde{c}_1^2}{12}; \quad \Delta_2 = \int_0^1 2\psi^\prime(\xi) \bar{u}^\prime(\xi) d\xi = -\frac{1}{6} \tilde{c}_1 \tilde{c}_2; \quad \Delta_3 = \int_0^1 \left(\bar{u}^\prime(\xi)\right)^2 d\xi = 4 + \frac{N^4}{720N_i^2}.
\]

Solving equation (40) given (41) and \( K\Phi_1(0) = K\alpha^* \), we have:

\[
\Phi_1(x) = \frac{1}{z_0} \left[\frac{\tilde{c}_1^2}{12} \left(-x + \frac{\pi}{2} x^2\right) + \left(4 + \frac{N^4}{720N_i^2}\right) \left(x - \frac{\pi}{2} x^2\right)\right] + \alpha^*.
\]

The exact auto-similar solution for the first approximation is

\[
u_1(x,y) = \frac{\partial \psi_1(x,y)}{\partial x} + U_1(x,y); \quad v_1(x,y) = -\frac{\partial \psi_1(x,y)}{\partial y} + V_1(x,y); \quad \psi_1(x,y) = \bar{\psi}_1(\xi); \quad \xi = -\frac{y}{h(x)};
\]

\[
V_1(x,y) = -\bar{u}(\xi) \cdot h'(x); \quad U_1(x,y) = \bar{u}(\xi).
\]

Substituting (43) into the system of differential equations (21) taking into account boundary conditions (22), we have the following system of differential equations:

\[
\bar{\psi}_1''(\xi) = \tilde{c}_2, \quad \bar{u}_1''(\xi) = \tilde{c}_1, \quad \bar{u}_1''(\xi) + \xi \bar{v}_1''(\xi) = 0, \quad \frac{d\xi}{dx} = -\alpha \left[\frac{\tilde{c}_1}{h^2(x)} + \frac{\tilde{c}_2}{h'(x)}\right],
\]

boundary conditions
\[ \psi'(0) = 0, \quad \psi'(1) = 0; \quad \bar{u}_i(0) = 0, \quad \bar{v}_i(0) = 0; \quad \nu_i(0) = \nu(1) = 0, \]
\[ \bar{u}_i(0) = M, \quad \bar{v}_i(0) = 0, \quad \int_0^1 \bar{u}_i(\xi) d\xi = 0, \quad z_i(0) = z_i(1) = 0. \]
\[ u_i = \bar{M} \frac{\partial P}{\partial y^*}; \quad \text{if} \quad y^* = 1 + \eta x + \frac{\bar{H}}{h_0}; \quad \frac{\partial P}{\partial y^*} = 0; \quad \text{if} \quad y^* = 1 + \eta x; \]
\[ p_i = P_i \quad \text{if} \quad y^* = 1 + \eta x + \frac{\bar{H}}{h_0}; \]

Integrating the equation, we have:
\[ \tilde{\psi}'(\xi) = \frac{\tilde{C}}{2}(\xi^2 - \xi), \quad \tilde{u}_i(\xi) = \tilde{C}_1 \frac{\xi^2}{2} - \left( \frac{\tilde{C}_i}{2} + M \right) \xi + M. \] 
where
\[ M = \left| \sup_{x \in [0,1]} \left( \frac{\tilde{C}_i}{2} \right) \cdot \Phi(x) \right| = \]
\[ = \sup_{x \in [0,1]} \left\{ - \frac{N^2}{2N_i} \left(1 + \eta x \right) - \frac{n}{4} (2x - 1) \tilde{C}_i \right\} \left[ \frac{1}{\tilde{C}_i} \left( \tilde{C}_2 \left( -x^2 + \frac{\eta}{2} x^2 \right) + \left( 4 + \frac{N^4}{720N_i^2} \right) x - \frac{\eta}{2} x^2 \right) \right] + \alpha \]

Let us determine pressure in the contact zone for the first approximation from the equation
\[ \frac{dz_i}{dx} = -\alpha \left( \frac{\tilde{C}_1}{h^2(x)} + \frac{\tilde{C}_2}{h^3(x)} \right) \]
\[ z_i = -\alpha \left( \tilde{C}_1 \left( x - \eta x^2 \right) + \tilde{C}_2 \left( x - \frac{3}{2} \eta x^2 \right) \right) \]

Given boundary conditions \( z_i(0) = z_i(1) = 0 \)
\[ \tilde{C}_2 = -\tilde{C}_1 \left( 1 + \frac{1}{2} \eta \right) \]

For \( z_i \) we have
\[ z_i = \alpha \tilde{C}_1 \frac{\eta}{2} (x - x^2) \]

Using the asymptotic expansion to determine \( p_1 \), we have the following approximate equation:
\[ 1 - \alpha p_1 + \frac{\alpha^2}{2} x^* = \alpha \tilde{C}_1 \frac{\eta}{2} (x - x^2) \]

Determining the hydrodynamic pressure accurate to the members \( O(\eta^2) \) we have:
\[ p_i = \frac{\eta}{2} \tilde{C}_1 (x - x^2) \]

Given (50), solution of the Darcy equation is
\[ P_i(x, y^*) = R_i(y^*) + \frac{\eta}{2} \tilde{C}_1 (x - x^2). \]

Substituting expression (51) into the Darcy equation of system (21) for function \( R_i(y^*) \), we have
\[ R'_i(y^*) - \eta \tilde{C}_i = 0 \]  

(52)

With boundary conditions

\[ R_i(0) = 0, \quad \frac{\partial R_i}{\partial y} = 0 \text{ if } y^* = 1 + \eta x + \frac{\tilde{H}}{h_0} \]  

(53)

The solution of equations (52)-(53) is

\[ R_i(y^*) = \eta \tilde{C}_i \left( \frac{y^*}{2} + \left(1 + \eta x + \frac{\tilde{H}}{h_0}\right) y^* \right), \]  

(54)

\( \tilde{C}_i \) is determined from the continuity equations ranging from 0 to 1

\[ \tilde{M} \frac{\partial P}{\partial y} \bigg|_{y^*=1+\eta x} = \int_0^1 \tilde{u}_i(\xi) d\xi \]  

(55)

Given (46), (51) and (54), for \( \tilde{C}_i \) we have

\[ \tilde{C}_i = \frac{6M}{12\tilde{M} \eta \left( 2 + \frac{\tilde{H}}{h_0} \right)} + 1 \]  

(56)

The hydrodynamic pressure for the first approximation is:

\[ P_i = \frac{3M\eta(\chi - \chi^2)}{12\tilde{M} \eta \left( 2 + \frac{\tilde{H}}{h_0} \right) + 1} \]  

(57)

Determination of the bearing capacity and the friction force. Taking into account (19), (21), (37) and (57) for the bearing capacity and the friction force, we have:

\[ W = p \int_0^1 \left[ \rho_0 + K p_j - \frac{p}{P} \right] dx = \frac{2p_0 + \kappa p}{4h_0^2} \left[ \frac{1 + a \frac{p}{\rho} - \alpha^2 \left( \frac{p}{p} \right)^2}{1 + a \frac{p}{\rho} - \alpha^2 \left( \frac{p}{p} \right)^2} \left( 2 + \frac{\tilde{H}}{h_0} \right) - 1 \right] + \frac{KM}{12 \tilde{M} \eta \left( 2 + \frac{\tilde{H}}{h_0} \right) + 1} \]  

(58)

4. Experimental studies

On the basis of the calculated models obtained in the theoretical part, an experimental study was conducted. The area of prospective operation of the developed tribosystem was determined in the form of a range of load-speed modes and basic tribological characteristics.

Considering the lubricant as part of a single tribosystem, we carried out research on the control of surface films formed between contacting surfaces as a result of tribochemical exchange between the lubricant and these surfaces.

An experimental study examined a thrust bearing with a slider coating and a low-melting metallic coating guide.

According to the results of the experiments, the value of the friction coefficient was determined. It allowed us to identify the hydrodynamic lubrication mode.
According to the results of experimental studies, the values of the coefficient of friction were determined. Their analysis shows that the load affects the coefficient of friction 2–5 times more intensively than speed.

Experimental studies confirmed the validity of the developed theoretical calculation models and data on their numerical analysis in the considered range of design and operational parameters of tribosystems with a porous and low-melting metallic coating as a result of satisfactory convergence of theoretical and experimental results.

Tribological tests on friction machines showed a significant (up to 25%) decrease in the wear spot and a longer retention of the lubricating film (up to 50%).

For the numerical analysis, the following parameters were used:

\[
\begin{align*}
\bar{H} &= 0.0055 \pm 0.0100, \\
\mu_0 &= 0.001022 \text{Ns/m}^2, \\
\eta &= 0.3 \ldots 1 \text{ m}, \\
l &= 0.1256 \ldots 0.1884 \text{ m}, \\
u^* &= 1.3 \text{ m/s}, \\
h_0 &= 10^{-7} \ldots 2 \cdot 10^{-6} \text{ m}, \\
K &= 0.0000022 \ldots 0.00052, \\
p_c &= 0.08 \pm 0.101325 \text{ MPa}, \\
L^* &= 35.33 \ldots 38.1 \text{ N/m}^2.
\end{align*}
\]

According to the results of numerical calculations (the average values of the measurement range were used), graphs presented in Figure 2–3 were plotted.

**Figure 2.** The dependence of the bearing capacity of parameter K caused by the melt and the rate of energy dissipation on parameter \( \bar{H} \) characterizing thickness of the porous coating.

**Figure 3.** The dependence of the bearing capacity of parameter K caused by the melt and the rate of energy dissipation on parameter \( \alpha \) characterizing the dependence of viscosity on pressure.

5. Conclusion

According to the results of numerical analysis, graphs were built (Figure 2-3).

1. A refined computational model of a wedge-shaped sliding bearing operating under hydrodynamic lubrication on a micropolar liquid lubricant caused by the melt of the surface of the low-melting coating of the guide surface was developed.

2. The significance of the following parameters was demonstrated: \( N_t \), characterizing the size of molecules of the lubricant, \( K \), caused by the surface melting, \( N \), and \( \alpha \) characterizing the dependence of viscosity on pressure.

3. A significant increase in the bearing capacity and a decrease in the friction force are due to the growth of structural-viscous parameters of the micropolar lubricant material (\( N \) and \( N_t \)), parameter \( K \) caused by the melt of the surface of the low-melting coating of the guide surface, and parameter \( \alpha \) characterizing the dependence of viscosity on pressure.
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