On the increase with relative distances of light cone operator product of currents and related phenomena

B. Blok\textsuperscript{1*} and L. Frankfurt\textsuperscript{2†}

\textsuperscript{1}Department of Physics, Technion – Israel Institute of Technology, Haifa 32000, Israel

\textsuperscript{2}School of Physics and Astronomy, Faculty of Natural Sciences, Tel Aviv University, Israel.

Abstract

We show that in QCD in the leading twist approximation flavor singlet light cone current-current correlators increase with distances rapidly and without spatial oscillations.

\textsuperscript{*}E-mail: blok@physics.technion.ac.il

\textsuperscript{†}E-mail: frankfur@lev.tau.ac.il
I. INTRODUCTION

It has been suggested that increase with energy of cross sections of hard processes found in QCD is equivalent in coordinate space to the increase with distance of flavor independent correlators of currents on light cone ref. [1]. A number of simplifying approximations made in ref. [1] like double logarithmic approximation with fixed coupling constant precludes unambiguous conclusions. In this paper we show that increase of correlators of currents with distance is valid in the DGLAP, BFKL and resummation approximations. Space-time oscillations of correlators of currents which are present in the parton model [2], disappear in QCD for sufficiently large invariant length $p_y$.

Increase with energy of cross sections of hard processes can not continue forever because it contradicts to conservation of probability. [3] Application of LO DGLAP approximation to the interactions of colorless gluon dipole $gg$ with the virtuality $Q^2 \approx 10 GeV^2 \gg Q_{o}^2$ with a nucleon target reveals this conflict with probability conservation in the kinematics of $x = (Q^2/W^2 + Q^2) \leq x_{cr}(Q^2) \approx 10^{-5}$ achievable at LHC. [3,5].($Q^2 \approx 10 GeV^2$ was chosen to guarantee smallness of running coupling constant.) At the same time the amplitude of the scattering of $q\bar{q}$ dipole in this kinematics is far from S matrix unitarity limit. Onset of new pQCD regime at $Q^2 \approx 1 - 2 GeV^2$ for the interaction of $q\bar{q}$ dipole has been proposed in [6,7] and references therein.

Account of unitarity of S matrix, of energy-momentum conservation law and nonconservation of bare particles helps to show that limiting behavior of hard processes at sufficiently high energies is described by black disc with the radius increasing with energy=BDL regime. [4,5]. A lot of new hard phenomena which may appear effective tool to identify BDL regime can be observed at LHC. For the review and references see [5]. The aim of this paper is to demonstrate that color and flavor blind correlators of currents calculated within conventional approximations rapidly increase with relative distance. Such a behavior of correlators is due to Lorentz dilatation of life time of colorless dipole produced by current in the target rest frame. It resembles turbulence where correlators of velocities are increasing with relative distances [8]. Increase with distance of the correlators calculated in LT approximation and even faster increase with distances of HT effects suggests similarity between the transition to BDL and phase transition. Really it is well known that key property of a system near critical point for a second order phase transition is increase of correlators with distance [9]. Since our interest is in the dependence of commutator of currents on distance but not its absolute value we will not distinguish below between different correlators (different structure functions).

For definiteness we begin our discussion from the DIS lepton scattering off hadron target $T$: $e + T \rightarrow e + X$. In this case $Q^2$ is the virtuality of photon and $W$ is invariant energy of $\gamma^*T$ collision. Our interest is in the regime of small $x = Q^2/W^2 + Q^2$ which is however significantly larger than $x_{cr}(Q^2)$ characteristic for BDL. It is well known that total cross section of DIS can be calculated as matrix element of correlator of electromagnetic currents at light-cone . Thus knowledge of structure functions as a function of $x, Q^2$ is sufficient to reconstruct correlators of e.m. currents in coordinate space. B.Joffe [2] was the first to extract this correlator from experimental data on structure functions of DIS within the framework of parton model. In ref. [11] Fourier transform into coordinate space of variety of expressions for structure functions has been calculated.In this paper we restrict ourselves
by evaluation of commutator of currents at large invariant distances \((py) \to \infty\) and small but fixed \(y^2\). (Here \(y\) is relative space-time difference between currents within a correlator.) In this kinematics correlator is dominated by structure function in the kinematics \(x \ll 1\) and small but fixed \(Q^2\).

The method of moments or Wilson operator expansion for the product of currents is convenient to evaluate the behavior of the current-current commutator in the coordinate space.

\[
<N|j_{\mu}(y)j_{\nu}(o)|N> = (1/y^2)^2 \sum_n p_{\mu}p_{\nu}(py)^n <N|O_n(0)|N> + \text{NLT terms}
\]

\[
= p_{\mu}p_{\nu}(py)f(py, y^2)/(y^2)^2 + \text{NLT terms}
\]

(1.1)

where \((py)\) is a kinematical factor. In this paper we shall work with

\[
G(py, y^2) = \int d^4qG(x, q^2) \exp(iqy)
\]

(1.2)

where \(G(x, Q^2)\) is a structure function.

Another useful quantity is the Fourier transform of gluon density which can be interpreted as \(\propto\) effective colourless ”potential” \(D\) of dipole-target interaction.

\[
D(py, y^2) = \int d^4q \frac{G(x, q^2)}{2pq} \exp(iqy)
\]

(1.3)

This ”potential” is a Fourier transform of a dipole-target cross-section, and \(xG\) is a corresponding gluon structure function. (We use here conventional notations for structure functions. The name ”potential” is used because within the nonrelativistic approximation \(D\) has meaning of potential.) \(D\) is often used in the eikonal models for the structure functions in the regime of small \(x\) as ”potential” [6,7].

We have also studied the closely related problem of coherence length in QCD. We explain that coherence length which measures longitudinal length of virtual photon wave function in the target rest frame is \(l_c \approx 1/2m_Nx\), far from BDL. However near BDL coherence length becomes \(\propto x^{1-\mu} \ll 1/2m_Nx\) because of a more rapid increase with energy of the interaction of smaller dipole with a target. \(\mu \approx 0.2\) as known from theoretical and experimental investigation of \(xG_p(x, Q^2)\). This is because of the enhancement of contribution of small transverse size configurations in the wave function of photon whose interaction increases with energy faster than for average configurations.

Our results suggest that near BDL the dipole wave function should be modified due to the long range dipole-target interactions discussed above. Significant change of the wave function occurs in the kinematics where leading twist approximation is violated.

The paper is organized in the following way. In section 2 we consider the correlators of currents in the leading log DGLAP approximation. Then we explain that similar behaviour arises in other pQCD approaches also. In section 3 we discuss the coherence length in QCD far and near BDL. In section 4 we discuss the deformation of the dipole wave function. Our conclusions are in the section 5.
II. CURRENT-CURRENT CORRELATORS IN THE LEADING LOG DGLAP APPROXIMATION.

The DGLAP evolution equation in the leading log approximation is

\[ Q^2 \frac{d}{dQ^2} G(x, Q^2) = (\alpha_s/(2\pi)) \int_1^1 (dx'/x') \gamma_G(x/x') G(x', Q^2). \]  

(2.1)

Here \( \gamma_G \) is the kernel in the QCD evolution equation.

Consider first the case of a frozen coupling constant (considered in ref. [1]). In this case the structure function is given by

\[ G(x, Q^2) = \int_C \frac{dj}{2\pi i} x^{-j} D(j, \alpha_s \log(Q^2/Q_0^2)) \]  

(2.2)

The contour of integration over \( j \) (we denote it \( C \)) runs along a straight line parallel to the imaginary axis to the right of all singularities of the integral. We use the notation \( Q^2 = -q^2 \) if \( q^2 \leq 0 \) and \( Q^2 = q^2 \) if \( q^2 \geq 0 \). The Bjorken scaling variable is defined in a usual way:

\[ x = -q^2/(2pq). \]  

(2.3)

For the anomalous dimension we have:

\[ V_{\pm}(j) = 0.5(V_F(j) + V_G(j)) \pm \sqrt{(V_F(j) - V_G(j))^2 + 24 \Phi^F(j) \Phi^G(j)}, \]  

(2.4)

where

\[ V_F(j) = -C_2(4\psi(j + 1) + 4\gamma_E - 3 - 2/(j(j + 1))), \]  

(2.5)

\[ V_G(j) = -4N_c(\psi(j + 1) + \gamma_E) + 11N_c/3 - 2 + 8N_c(j^2 + j + 1)/(j(j^2 - 1)(j + 2)), \]  

(2.6)

\[ \Phi^F(j) = 2C_2(j^2 + j + 2)/(j(j^2 - 1)), \]  

(2.7)

\[ \Phi^G(j) = (j^2 + j + 2)/(j(j + 1)(j + 2)). \]  

(2.8)

The function \( \psi(j) \) is the usual logarithmic derivative of Gamma function

\[ \psi(j) = \frac{d\Gamma(j)}{dj} \]

and

\[ \gamma_G(j) \equiv V_+(j) \]  

(2.9)

Let us begin calculations within the simplifying assumption of frozen coupling constant and then to analyse realistic case. It is easy to carry out the integration over \( d^4q \) in the Fourier transform. We use the formulae [1]:

3
\[(q^2)^\beta F(x) \rightarrow \pi \Gamma(\beta + 2) \frac{(py)^{\beta+1}}{(y^2)^{\beta+2}} \int_0^1 F(x)x^\beta \cos(x(py) + \beta \pi /2)dx\]  \hspace{1cm} (2.10)

if the latter integral converges. Here \(u_+ = \theta(u)u\). For the case of the power like behavior of \(F(x)\) this integration can be performed analytically, and in the limit of large invariant length \((py) \gg 1\) one obtains power like asymptotics:

\[(q^2)^\beta x^\alpha \rightarrow \pi \Gamma(\beta + 2) \Gamma(\alpha + \beta + 1) \cos((2\alpha + \beta)\pi/2)((py)^\alpha) y^{\beta+2}\]  \hspace{1cm} (2.11)

In our case \(\beta = \gamma(j), \alpha = -j\), so

\[G(y^2, py) = (\pi/2) \int d\gamma(j)^{+j} \frac{1}{(y^2)^{\gamma(j)+2}} \Gamma(\gamma(j) + 2) \Gamma(1-j + \gamma(j)) \sin((-j + 2\gamma(j))\pi/2)\]  \hspace{1cm} (2.12)

We now use the saddle point method to integrate over \(j\).

\[\frac{\log(y_0^2/y^2)}{\log(py)} = \alpha_s \frac{d\gamma(j)}{dj}\]  \hspace{1cm} (2.13)

The saddle point can be found numerically. The derivative of the DGLAP anomalous dimension can be approximated with good accuracy by

\[\gamma'(n) \sim -12/(n-1)^2\]

for \(1 \leq n \leq 2.5\) and

\[\gamma' \sim -5/(n-1)\]

for \(n \geq 3\). This means that the double log expression works quite good numerically for

\[\log(py) \geq \alpha_s \log(y_0^2/y^2)\]  \hspace{1cm} (2.14)

Using the latter numerical approximations for anomalous dimension, one obtains the relevant saddle point:

\[j = 1 + \sqrt{12\alpha_s \log(y_0^2/y^2)/\log(py)}\]  \hspace{1cm} (2.15)

i.e. we have the double log expression. However now we are permitted to take into account also the terms that contain only one big logarithm. For \(\log(py) \geq \alpha_s \log(1/y^2)\), one has

\[G(py, y^2) = \frac{\log(Q_0^2y^2)^{1/4}}{\log(py)^{3/4}} \frac{1}{(y^2)^2} \exp\left[\sqrt{4\alpha_s N_c/\pi} \log(py) \log(Q_0^2y^2)\right]\]

\[\times \cos\{\sqrt{\alpha_s N_c/\pi} (\sqrt{\log(Q_0^2y^2)/\log(py)} + 2\sqrt{\log(py)/\log(Q_0^2y^2)})\pi/2\}\]

\[\times \Gamma\{\sqrt{\alpha_s N_c/\pi} (\sqrt{\log(Q_0^2y^2)/\log(py)} + \sqrt{\log(py)/\log(Q_0^2y^2)})\}\]

\[\times \Gamma\{2 + \sqrt{\alpha_s N_c/\pi} (\log(py)/\log(Q_0^2y^2))\}\]  \hspace{1cm} (2.16)
Consider now the realistic case of the running coupling constant. In this case [15,17,16] the gluon structure function is given by

\[ G(x, Q^2) = \int_C \frac{dj}{(2\pi i)} x^{-j} D(j, \xi) \]  

(2.17)

Here

\[ D(j, \xi) = \frac{(V_+(j) - V_0(j)}{V_+(j) - V_-(j)} \exp(V_+(j)\xi), \]  

(2.18)

where

\[ \xi = \frac{1}{b} \log(\alpha_s(Q^2_0)/\alpha_s(Q^2)), \]  

(2.19)

and

\[ b = 11 - \frac{2}{3} N_F = 9, \quad N_F = 3 \]  

(2.20)

The calculations one needs to carry through for the realistic case of the running structure constant are completely similar to the calculations with the freezed coupling constant. The only change in the calculation procedure is the substitution in all calculations of the terms like \( \log(Q^2/Q_0^2) \) by \( \log(y_0^2/y^2) \). Consequently, we have to substitute \( \alpha_s \log(y_0^2/y^2) \) by

\[ \xi(y^2) = \frac{1}{b} \log(\alpha_s(1/y^2)/\alpha_s(1/y_0^2)). \]  

(2.21)

The saddle point will be in the point

\[ j = 1 + \sqrt{\frac{12\xi(y^2)}{\log(py)}} \]  

(2.22)

and we obtain

\[ G(py, y^2) = \log(Q_0^2 y^2)^{1/4} 1 \left( \frac{\log(py)}{y^2} \right)^{3/4} \frac{exp(\sqrt{12\xi(y^2) log(py)})}{(y^2)^2} \times \cos((\sqrt{\xi(y^2)/\log(py)})) \times \Gamma((\sqrt{\xi(y^2)/\log(py)})) \]  

(2.23)

Similar calculations can be carried for an opposite case \( \log(py) \leq \alpha_s \log(y_0^2/y^2) \), and one can easily see that the correlator increases with the increase of the invariant length in this case also. Similarly it is easy to derive the increase with distance of the effective dipole-target potential.

One of the important features of the correlators in the parton model is their space-time oscillations [2]. We will show that oscillations at large distances in the correlator and "potential" for dipole-target interactions disappear in pQCD.
For fixed $y^2$ and frozen coupling constant the correlators and the effective dipole-target potentials derived above oscillate, as shows the existence of the cos term. The period of oscillations can be determined from the condition that the argument of the cosinus is $n\pi$ where $n$ is the integer number.

$$\gamma(j_0) + j_0 = n$$ (2.24)

where $j_0$ is the saddle point. We immediately obtain the condition

$$\sqrt{\alpha_s N_c / \pi} (\sqrt{\log(1/y^2)/\log(py)} + \sqrt{\log(py)/\log(1/y^2)}) = n$$ (2.25)

In our approximation the second term is bigger than the first and we obtain that the oscillation period is

$$py \sim (1/y^2)^{n^2/(\alpha_s N_c / \pi)}$$ (2.26)

Thus oscillations exist in the case of frozen coupling constant for large $py$. This analysis can be easily upgraded to analyze the case of the running coupling constant. In this case the relevant condition is

$$\xi(y^2)/\log(py) = n^2$$ (2.27)

Hence

$$py \sim \exp(\xi(y^2)/n^2)$$ (2.28)

Thus taking into account of running coupling constant changes dramatically the oscillation period: instead of increasing with $n$ it now decreases. Therefore oscillations are present at small $(py)$ which are beyond of region of applicability of our consideration.

The reason for such dramatic influence of asymptotic freedom is evident in the procedure used to calculate the integral over $q^2 = -Q^2$. This integral is the product of rapidly oscillating $\cos(q^2(t-r)/x_B)$ and $F(q^2)$ [2]

$$G(px,x^2) = -\frac{\pi}{2m(py)} \int dq^2 \int dx_B G(x_B, Q^2)(q^2/x_B^2) \sin(q^2/(m^2 x_B)x^2/(py) + (py)x_B)$$ (2.29)

In the case of frozen coupling constant the function $G$ behaves as the power of $Q^2$, and rapidly changes within the period of oscillations of cosinus. Thus exact calculation of the integral produces terms proportional to $\cos(V(j)\pi/2$, or corresponding sinuses because of $F(Q^2) \sim Q^{2V(j)}$. This gives rise to the above result. However in the case of the running coupling constant the function $F(Q^2)$ depends on $Q^2$ more slowly, due to the presence of the additional logarithm. Indeed, the characteristic period of oscillations is $\sim x(py)/y^2$. However the characteristic $x$ is connected to characteristic $(py)$ (coherence length) as $py \sim 1/(x)^\beta$ where $\beta$ is a number close but smaller than one. Then the period oscillations is $\sim (py)^{1-1/\beta}/y^2$. Thus the period of oscillations decreases with increasing $(py)$. Evidently, the function $\xi(y^2)$ on this scales can be considered constant, and one can safely carry out
ξ(q^2) from the integral and substitute it by ξ(y^2). The oscillations die down for running coupling constant and large (py). The behavior for the case of log(py) ≤ α_s log(1/y^2) is qualitatively similar. Thus the oscillations we found within the DGLAP approach are qualitatively different from the oscillations in the parton model found in ref. [2].

Similar analysis can be carried through for the BFKL case [18] by substituting q_o = 1/(t - z) in the Fourier transform as it is legitimate in the leading α_s ln(x_0/x) approximation.

Formally, we use for the BFKL case the contour integral representation Let us remind that in the BFKL case the structure function G can be represented as

$$G(x, Q^2) = \int (dM/2\pi) \exp(Mt + \alpha_s \chi(M)u)$$

(2.30)

where t = log(Q^2/Q_0^2), u = log(1/x). In the BFKL case the coupling constant α_s is freezeed. The integration contour is defined in the same way as in the DGLAP case. For the Regge limit the integral can be taken using saddle point method. The result is the energy independent saddle point M_0 = 1/2. It is easy to take the Fourier transform and obtain according to the formulae above,

$$G(py, y^2) = \frac{1}{y^{5/2}} \exp(\alpha_s \chi(1/2) \log(py))$$

(2.31)

Here the BFKL anomalous dimension χ is given by

$$\chi(M) = \frac{3\alpha_s}{\pi} (2\psi(1) - \psi(M) - \psi(1 - M))$$

(2.32)

with

$$\chi(1/2) = \frac{12}{\pi} \log(2)$$

(2.33)

We obtain that the correlators are quickly increasing with the distance and the oscillations are absent.

Within the resummation models of the ABF [12] and Ciafaloni et al [13] it was argued that the behavior of the structure functions in these models for the wide range of x is the same as in leading + plus nonleading order DGLAP. Numerically this behavior is qualitatively quite similar to the leading order DGLAP, except that the splitting functions is 15-30 % lower than the leading order DGLAP, in the wide range of Bjorken x. At quite low x (say, x ~ 10^{-6} for Q^2 ~ 4.5 GeV), the DGLAP behavior of the kernel switches to BFKL type rise.(Note [14] that the switch point depends on Q^2 very weakly, x_c ~ 1/√α_s(Q^2)). This rise, although its analytic form is unknown leads to the rapid rise of the correlator without oscillations

$$G \sim (py)^{α_P - 1}$$

(2.34)

Here α_P is the renormalized perturbative Pomeron slope of the resummation models, which is quite close (effectively α_P ~ 1.25). Note that the oscillations are absent starting from sufficiently large py already in the DGLAP curve.
III. COHERENCE LENGTH IN QCD

Let us now evaluate coherence length $l_c$ for the total cross section of DIS evaluated within DGLAP approximation. We use here the same definition as B.Joffe length within parton model [2,19] and perform the inverse Fourier transform:

$$xG(x, Q^2) = 2\nu \int_0^\infty dy^2 \int_0^\infty du F(y^2, u) \cos(\nu y^2/(2u) - xu),$$

(3.1)

(the latter equation defining function $F$ [19]). Let us carry on firstly the integration over $y^2$. The relevant scale for the integration over $y^2$ is $1/Q^2$. Thus within the LO DGLAP approximation: one may substitute $y^2 \rightarrow 1/Q^2$, so that the integral takes the form:

$$xG(x, Q^2) = \int_0^\infty 4udu F(1/Q^2, u) \sin(1/2xu - xu)$$

(3.2)

The coherence length is defined for given $Q^2, x$ as the typical $u$ in the integral. Such $u$ is either determined by the maximum of the function $F$, or by the radius of oscillations in the sinus multiplier. (For the another definition of $l_c$ and therefore different dependence on $x$ see Y.Kovchegov and M.Strikman [20].)

$l_c$ within a parton model was determined by B.Ioffe who assumed that

$$F(u) \rightarrow \text{const.}$$

(3.3)

for $u \rightarrow \infty$. If so essential $u$ in the integral are completely determined by the oscillating multiplier , i.e.

$$u_I(x, Q^2) = 1/x$$

(3.4)

The same behaviour is expected within DGLAP, LO+NLO BFKL, resummation approximations. This is because in all approaches $l_c$ is effectively life time of dipole with mass $M$ where $M^2 = k_t^2/z(1 - z)$. Here $k_t$ is transverse momentum of quark within dipole and $z$ is the fraction of dipole momentum carried by quark. Thus $l_c \approx q_o/Q^2 + M^2$ and $Q^2 \approx M^2$. Therefore $l_c \approx 1/2m_Nx$.

Near unitarity limit fast increase with energy of the coherence length should somewhat slow down. This is because the fast increase of amplitude with energy leads to the related increase of probability of configurations of constituents with large $k_t$. Let us consider for definitness the case of longitudinally polarized photon where maximal important $k_t$ can be evaluated from pQCD formulae as: $Q^2/k_t^21/x^{\mu(k_t^2)} \approx 1$ So near unitarity limit: $l_c \approx q_o/M^2 \propto 1/x^1-\mu(k_t^2) \ll 1/2m_Nx$.

IV. ON THE DISTORSION OF DIPOLE WAVE FUNCTION NEAR UNITARITY LIMIT.

Another consequence of the increase of the correlators with the invariant length py is the deformation of the wave function of the dipole near BDL.
We found that fast increase with energy of LT contribution transforms in coordinate space into fast increase with distance of correlators of currents. The contribution of higher twist effects is increasing with x decrease even faster: \( \propto 1/x^{n\mu(Q^2)} \) where \( n=1 \) for LT \( n=2 \) for first HT etc. Performing the same Fourier transform of this formulae into coordinate space as for LT term will produce even faster increase with distance of the contribution of HT effects as \( \propto (py)^{1+n\mu(Q^2)} \). Thus dipole-target interaction near unitarity limit is characterized by important role of long range interactions of dipole with target (which however are significantly smaller than B.Joffe length). This long-range target-dipole interaction effectively leads to the interaction between the components of the dipole. Indeed, at moderately small x the contribution of dipole configurations with \( z \approx 1/2 \) is multiplied in the cross-section by the small factor -square of the dipole moment \( d^2 \). Near BDL this suppression disappears and the dominant configurations are \( z \sim 1/2 \). Thus effective wave function of the dipole changes significantly near the unitarity limit.

V. CONCLUSION

The main aim of the present paper was to demonstrate that the correlators of currents fastly increase with distance in pQCD.

We have also studied the issue of the space-time oscillations of the correlator of currents and found no oscillations in QCD contrary to the parton model where the correlator of currents is \( \sim \cos(py) \). Within DGLAP approximation oscillations are decreasing with distance because of decrease of coupling constant with virtuality.

We estimate coherent length near BDL as \( l_c \propto x^{1-\lambda} \).

We argue existence of the deformation of the dipole wave function near BDL resulting from the long range interaction between dipole and target [1].

The most suggesting consequence of our results is that in condensed matter physics increase of correlations with distance usually leads to the phase transition. [8]

One of us (LF) is indebted to M.Strikman for the discussion of increase of correlators with distance which led to the understanding of the impact of increase of gluon distribution on the coherence length.
REFERENCES

[1] B. Blok and L. Frankfurt, Phys. Rev. D70 (2004) 094003.
[2] B. Ioffe, Phys. Lett., B30 (1969) 123.
[3] H. Abramowicz, L. Frankfurt, M. Strikman, Surveys High Energ. Phys., 11 (1997) 51.
[4] M. McDermott, L. Frankfurt, V. Guzey and M. Strikman Eur. Phys. J., C16 (2000) 641.
[5] L. Frankfurt, M. Strikman, C. Weiss, Annual Review of Particle and Nuclear Science, 2005, to be published.
[6] A.H. Mueller and A.I. Shoshi, Small x physics near the saturation regime, CU-TP-1113, May 2004. 5pp. Contributed to 39th Rencontres de Moriond on QCD and High-Energy Hadronic Interactions, La Thuile, Italy, 28 Mar - 4 Apr 2004, hep-ph/0405205.
[7] L. McLerran, Nucl. Phys., A752 (2005) 355-371.
[8] L. Landau, E. Lifshitz, Fluid mechanics, 2d edition, Pergamon, Oxford, 1987.
[9] L. Landau, E. Lifshitz, Statistical Mechanics, 2d edition, Pergamon, Oxford, 1986.
[10] L. Frankfurt, M. Strikman Phys. Rep., 160 (1988) 235.
[11] Y. Frishman, Phys. Reports, 13 (1974) 1.
[12] G. Altarelli, S. Forte, R.D. Ball, Nucl. Phys., B621 (2002) 359; B674 (2003) 459.
[13] M. Ciafaloni, P. Colferai, G.P. Salam, A. M. Stasto, Phys. Lett., 587 (2004) 87; Phys. Rev. D68 (2003) 114003.
[14] G.P. Salam, hep-ph/0501097, Talk presented at the QCD at the cosmic energies workshop, Ettore Majorana Centre, Erice, Italy, September 2004.
[15] G. Altarelli and G. Parisi, Nucl. Phys., B126 (1977) 298; V.N. Gribov and L. N. Lipatov, Sov. J. of Nucl. Phys., 15 (1972) 438, 672; Yu.L. Dokshitser, Sov. Phys. JETP 46 (1977) 641.
[16] Yu. L. Dokshitser et al., Basics of Perturbative QCD. Edition Frontieres, Gif-Sur-Yvette Cedex, France, 1991.
[17] Y. Dokshitser, D. Diakonov, S.I. Trojan, Phys. Reports 58 (1980) 269.
[18] I.I. Balitskii, L. Lipatov, Sov. J. Nucl. Phys., 28 (1978) 22, Fadin, E. Kuraev, L. Lipatov, Sov. PJys. JETP 44 (1976) 443; 45 (1977) 199.
[19] B.L. Ioffe, V.A. Khoze, L.N. Lipatov, Hard processes, North-Holland, 1984, Amsterdam.
[20] Yu. Kovchegov and M. Strikman, Phys. Lett. B516 (2004) 314.