$\geq 4\mu$ signal from a vector-like lepton decaying to a muon-philic $Z'$ boson at the LHC

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Abstract

We propose a novel possibility to detect a very distinctive signal with more than four muons originating from pair-produced vector-like leptons decaying to a muon-philic $Z'$ boson. These new particles are good candidates to explain the anomalies in the muon anomalous magnetic moment and the $b \to s\ell\ell$ processes. The doublet (singlet) vector-like leptons lighter than 1.3 (1.0) TeV are excluded by the latest data at the LHC if $\text{BR}(E \to Z'\mu) = 1$. We also show that the excess in the signal region with more than five leptons can be explained by this scenario if the vector-like lepton is a weak singlet, with mass about 400 GeV and $\text{BR}(E \to Z'\mu) = 0.25$. The future prospects at the HL-LHC are discussed.

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1 Introduction

The Large Hadron Collider (LHC) explores new physics beyond the Standard Model (SM) at TeV-scale. The SM has been established as the theory just above the electroweak (EW) scale, particularly by the discovery of the 125 GeV Higgs boson at the LHC [1, 2]. Although most of the experiments are consistent with the predictions of the SM, there are 2-3 $\sigma$ discrepancies in the measurements of rare semi-leptonic $B$ meson decays [3–19], $b \to s\ell\ell$, and the 4.2$\sigma$ discrepancy in the anomalous magnetic moment of muon, $\Delta a_{\mu}$ [20–22]. An interesting coincidence here is that both anomalies are found in physics related to muons, and hence these could be explained by the same origin. One way to establish these discrepancies as evidence of new physics is to increase the significance by reducing the uncertainties in the experimental measurements and in the predictions of the SM. Another way is by directly discovering new particles at the LHC, which we pursue in this paper.

It was shown in Refs. [23,24] that both anomalies in $b \to s\ell\ell$ and $\Delta a_{\mu}$ are addressed by introducing vector-like (VL) fermions and a $Z'$ boson associated with an additional gauge symmetry $U(1)_{3}^{'}$. The former anomaly is explained by $Z'$ exchange at the tree-level \(^{3}\), while the latter is explained by loop corrections involving the VL leptons and the $Z'$ boson. In this paper, we point out the possibility that pair productions of VL leptons provide very distinctive signals with more than four muons. We shall discuss the current limits from the recent ATLAS data [46] and future prospects at the HL-LHC in a simplified model with a VL lepton and $Z'$ boson. We also discuss the possible explanation for the excess in the more than 5-lepton signal found in Ref. [46].

The rest of this paper is organized as follows. The simplified model is defined and then the relation to the anomalies are discussed in Section 2. In Section 3, we discuss limits from the high-multiplicity lepton signal at the LHC. Section 4 is devoted to summary. The model proposed in Refs. [23,24] are reviewed in Appendix A as a UV completion of the simplified model.

2 Simplified model

We shall consider the simplified model with a VL lepton $E$, which is weak singlet-like, $E_1$, or doublet-like, $L = (E_2, N)$, where $N$ is the $SU(2)_L$ partner of $E_2$. The $Z'$ boson couplings to the leptons are given by

\[ \mathcal{L}_{Z'} = Z'_{\mu} \left( \bar{p} E \right) \gamma^\mu \left[ \left( \begin{array}{c} g_{\mu E}^L \\ g_{\mu E}^R \end{array} \right) P_L + \left( \begin{array}{c} g_{\mu E}^R \\ g_{\mu E}^L \end{array} \right) P_R \right] \left( \begin{array}{c} \mu \\ E \end{array} \right) + Z'_{\mu} \left( \bar{\nu} N \right) \gamma^\mu \left[ \left( \begin{array}{c} g_{\nu N}^L \\ g_{\nu N}^L \end{array} \right) P_L + \left( \begin{array}{c} g_{\nu N}^R \\ g_{\nu N}^R \end{array} \right) P_R \right] \left( \begin{array}{c} \nu \\ N \end{array} \right), \]

where $E = E_1$ or $E_2$ and the interactions with $N$ in the second line are absent in the case of weak singlet VL lepton. We assume that the off-diagonal couplings of the SM bosons to the SM and

\(^{3}\)See Refs. [25–30] for models with VL fermions and $U(1)'$ for the anomalies. The VL lepton explanation for $\Delta a_{\mu}$ is studied in e.g. Refs. [31–36]

\(^{4}\)The $b \to s\ell\ell$ anomaly can be explained by loop corrections involving VL families [37–45].
VL leptons are negligible, such that the dominant decay modes of the VL leptons are the decays to a $Z'$ boson and SM lepton. In fact, this is achieved in the model proposed in Refs. [23, 24].

The loop corrections involving the VL leptons and the $Z'$ boson contribute to the anomalous magnetic moment of the muon. It is known that the chiral-flip effect should be sizable to explain the current discrepancy of $O(10^{-9})$ with the new particles above the EW scale. In models with VL leptons, the chiral-flip effects may come from the non-zero VEV of the SM Higgs doublet. Hence the size of the loop correction is estimated as

$$\Delta a_\mu \sim -\frac{m_\mu \kappa v H}{8 \pi^2 m_{Z'}^2} g_{\mu E}^L g_{\mu E}^R C_{\Delta a_\mu},$$

$$\sim 2.9 \times 10^{-9} \times \left(\frac{500 \text{ GeV}}{m_{Z'}}\right)^2 \left(\frac{\kappa}{0.5}\right)^2 \left(\frac{C_{\Delta a_\mu}}{0.1}\right),$$

where $\kappa$ is the Yukawa coupling constant for $\tilde{H}_L R E_L$. $C_{\Delta a_\mu}$ is the factor from loop functions which is typically of $O(0.1)$, see Appendix A for the explicit form in the example model.

The $Z'$ boson couplings to muons, $g_{\mu \mu}^L$ and $g_{\mu \mu}^R$, directly relate to the Wilson coefficients for the $b \to s \ell \ell$ decay. The effective Hamiltonian is given by [47, 48]

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F \alpha e}{\sqrt{2}} \frac{1}{4 \pi} \frac{V_{tb} V_{ts}^*}{2 m_{Z'}^2} (C_9 \mathcal{O}_9 + C_{10} \mathcal{O}_{10}),$$

where

$$\mathcal{O}_9 := \left[ \gamma^\mu P_L b \right] \left[ \overline{\gamma}_\mu \gamma \mu \right], \quad \mathcal{O}_{10} := \left[ \gamma^\mu P_L b \right] \left[ \overline{\gamma}_\mu \gamma_5 \gamma \mu \right].$$

The coefficients in this model are given by

$$C_9 = -\frac{\sqrt{2}}{4G_F \alpha_e} \frac{1}{V_{tb} V_{ts}^*} g_{sb}^L \left( g_{\mu \mu}^R + g_{\mu \mu}^L \right),$$

$$C_{10} = -\frac{\sqrt{2}}{4G_F \alpha_e} \frac{1}{V_{tb} V_{ts}^*} g_{sb}^L \left( g_{\mu \mu}^R - g_{\mu \mu}^L \right),$$

where $g_{sb}^L$ is the $Z'$ couplings to $\overline{s} b$ in the left-current. The value of $C_9$ is estimated as

$$|C_9| \sim 0.87 \times \left(\frac{m_{Z'}}{500 \text{ GeV}}\right)^2 \left(\frac{g_{sb}^L}{0.0007}\right)^2 \left(\frac{g_{\mu \mu}^L + g_{\mu \mu}^R}{0.5}\right).$$

Note that the $Z'$ couplings to quarks are tiny to explain the $b \to s \ell \ell$ anomaly, while those to muons are large so that $\Delta a_\mu$ is explained when $g_{\mu \mu}^{L,R} \sim g_{\mu E}^{L,R}$; which is true in the sample model. This feature ensures that the $Z'$ mass of $O(100 \text{ GeV})$ is not excluded by the di-lepton resonance search at the LHC [49].

\[5\text{See Refs. [50, 51] for general discussions for } Z' \text{ boson responsible for } b \to s \ell \ell.\]
In the $Z'$ boson explanation, the ratio of the coefficients are given by

$$\frac{C_{10}}{C_9} = \frac{g_R^{\mu\mu} - g_L^{\mu\mu}}{g_R^{\mu\mu} + g_L^{\mu\mu}}.$$  \hspace{1cm} (2.8)

The recent analyses [52, 53] including the measurement of $R_K$ based on the full run-2 data at the LHCb [19] favor $C_9$-only, $C_{10}$-only and $C_9 = -C_{10}$ scenarios among the one dimensional analyses, which correspond to $g_R^{\mu\mu} = g_L^{\mu\mu}$, $g_R^{\mu\mu} = -g_L^{\mu\mu}$ and $g_R^{\mu\mu} = 0$, respectively. Among these three cases, the explanation by $C_9 = -C_{10}$ is not preferred to explain the $\Delta a_\mu$ anomaly because $\Delta a_\mu \propto g_L^{\mu\mu} g_R^{\mu\mu}$, see Eq. (2.2). From this observation, we shall consider the case with $|g_L^{\mu\mu}| = |g_R^{\mu\mu}|$ which predicts

$$\text{BR} \left( Z' \rightarrow \mu\mu \right) := \frac{\Gamma (Z' \rightarrow \mu\mu)}{\Gamma (Z' \rightarrow \nu\nu) + \Gamma (Z' \rightarrow \mu\mu)} \simeq \frac{|g_L^{\mu\mu}|^2 + |g_R^{\mu\mu}|^2}{2 |g_L^{\mu\mu}|^2 + |g_R^{\mu\mu}|^2} = \frac{2}{3}. \hspace{1cm} (2.9)$$

Here, we assume $g_L^{\mu\mu} = g_L^{\nu\nu}$ as expected from the $SU(2)_L$ symmetry and the $Z'$ boson decay to quarks are negligible as expected from Eq. (2.7). Studies for $|g_L^{\mu\mu}| \neq |g_R^{\mu\mu}|$, as preferred by the two dimensional analyses on $(C_9, C_{10})$ plane, are interesting but are beyond the scope of this paper.

### 3 LHC signals

In this paper, we study signals from pair produced VL leptons decaying to the second generation leptons and the $Z'$ boson. This can be realized when the VL leptons are heavier than the $Z'$ boson. If the $Z'$ boson is heavier, the VL lepton may decay to a SM boson and a lepton. The limits for VL leptons in such a case are studied in Refs. [71–77]. It is also possible that the VL lepton decays to a new boson, such as the physical mode of the $U(1)'$ breaking scalar. Thus we treat the branching fraction of $E/N \rightarrow Z'\mu/\nu$ as a free-parameter. We further assume $\text{BR} \left( N \rightarrow Z'\nu \right) = \text{BR} \left( E \rightarrow Z'\mu \right)$ for simplicity. Figure 1 shows the relevant processes which can generate signals for more than four muons. Only the left process is relevant for the singlet-like case.

We recast the limits obtained in Refs. [46] and [80]. The former searches for signals with more than four leptons, and the latter searches for signals with exactly two leptons with large

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6See Refs. [54–70] for the analyses before the Moriond 2021.

7We used TikZ-FeynHand to draw these figures [78,79]
missing transverse energy, $E_T^{\text{miss}}$. We have generated events using MadGraph5_aMC@NLO [81] based on a UFO [82] model file generated with FeynRules_2.3.43 [81, 83]. The events are showered with PYTHIA8 [84] and then run through the fast detector simulator Delphes3.4.2 [84]. We used the default ATLAS card for the detector simulation, but the threshold on $p_T$ for the muon efficiency formula is changed to 5 GeV from 10 GeV since muons with $p_T < 5$ GeV are counted as signal muons in Ref. [46].

We recast the experimental limits on the signal regions without $Z$ boson, b-jet and hadronic $\tau$ defined in Ref. [46]. These are named $\text{SR0}_{\text{loose}}$, $\text{SR0}_{\text{tight}}$ and $\text{SR5L}$. The requirements for the events, in addition to the b-jet veto and hadronic $\tau$-veto, common in the signal regions are as follows. To meet the trigger thresholds, $p_T$ of the leading muon, ordered by $p_T$, must be larger than 27 GeV, or $p_T$’s of the leading and next-to leading muons are required to be larger than (15, 15) GeV or (23, 9) GeV. If an opposite-sign (OS) muon pair whose invariant mass $m_{\text{OS}}$ is less than 4 GeV or 8.4 < $m_{\text{OS}}$ < 10.4 GeV, both leptons are discarded. If two muons are found in $\Delta R < 0.6$ and one of them has $p_T < 30$ GeV, both leptons are discarded. The first (second) $Z$ candidate is found from a pair of OS muons whose $m_{\text{OS}}$ is the (second) closest to the $Z$ boson mass $m_Z = 91.2$ GeV. A pair is identified as a $Z$ boson if $m_{\text{OS}} \in [81.2, 101.2]$ GeV. Further, the event is considered to have a $Z$ boson if any system of $\mu^+\mu^-\mu^+\mu^-$ or $\mu^+\mu^-\mu^+\mu^-$ has invariant mass in [81.1, 101.2] GeV. In the signal regions $\text{SR0}_{\text{loose}}$ and $\text{SR0}_{\text{tight}}$, there must be more than four muons after the selections above, and the event must not have any combinations of muons which is identified as a $Z$ boson. Further, the effective mass of the event $m_{\text{eff}}$, defined as the scalar sum of $E_T^{\text{miss}}$, $p_T$ of signal leptons and $p_T$ of the jets with $p_T > 40$ GeV, is required to be larger than 600 (1250) GeV in the $\text{SR0}_{\text{loose}}$ ($\text{SR0}_{\text{tight}}$). In the $\text{SR5L}$, the requirement is simply the lepton number to be larger than five, and no further selection applied.

We also study the limits from the SUSY slepton search [80] which requires exactly two leptons and large $E_T^{\text{miss}}$. The most relevant signal region for our scenario is with same flavor (SF) two leptons without any jet. There must be exactly two OSSF leptons, both with $p_T > 25$ GeV. Events are rejected if there are more muons with $p_T > 10$ GeV and $|\eta| < 2.7$ or the two leading leptons are not opposite sign. The missing energy $E_T^{\text{miss}}$ and invariant mass of two leptons $m_{\text{OS}}$ must be larger than 110 GeV and 121.2 GeV, respectively. The stransverse mass $m_{T2}$ [85, 86] is required to be larger than 160 GeV. We name this signal region as $\text{SR2L}$.

The number of observed events, fitted SM backgrounds and 95% C.L. upper bounds on the

Table 1: The number of events observed (data), fitted SM backgrounds (SM) and 95% C.L. upper bound on the number of signal events ($S^{95}$) in the signal regions [46,80].

|                | $\text{SR0}_{\text{loose}}$ | $\text{SR0}_{\text{tight}}$ | $\text{SR5L}$ | $\text{SR2L}$ |
|----------------|-----------------------------|-----------------------------|--------------|--------------|
| data           | 11                          | 1                           | 21           | 37           |
| SM             | $11.5^{+2.9}_{-2.2}$        | $3.5^{+2.0}_{-2.2}$         | $12.4 \pm 2.3$ | $37.3 \pm 3.0$ |
| $S^{95}$       | $9.79$                      | $3.87$                      | $17.88$      | $14.3$       |

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8We recast the analysis such that light leptons, (e,µ), in Ref. [46], for our simple model these are just muons.

9We used the code provided by Ref. [87] to calculate the stransverse mass.
signal events in the signal regions are shown in Table 1. We see that there is an excess over the SM background in SR5L for which the local significance is 1.9$\sigma$. Figure 2 shows the production cross sections of VL leptons at $\sqrt{s} = 13$ TeV and 14 TeV calculated by MadGraph5. We calculated the probability of how many events pass the cuts in each signal region from the VL lepton pair production at $\sqrt{s} = 13$ TeV by generating 25000 (50000) events at each point on the $(m_E, m_{Z'})$ plane for the singlet-like (doublet-like) VL leptons.

3.1 Current limits

Figures 3 and 4 show upper bounds on BR $(E \to Z'\mu)$ in the signal regions, where the signal cross section is proportional to the branching fraction squared. In the gray region, $m_{Z'} > m_E$ and hence the decay $E \to Z'\mu$ is kinematically forbidden. The white region is not excluded by the current data even if BR $(E \to Z'\mu) = 1$. For the doublet-like VL lepton, we see that $\text{SR}_{0}^{\text{tight, veto}}$ gives the strongest bound if BR $(E \to Z'\mu)$ $\gtrsim$ 0.2, because of fewer backgrounds satisfying the tighter $m_{\text{eff}}$ cut. The current limit is about 1350 GeV if BR $(E \to Z'\mu) = 1$. If the branching fraction is smaller, then $\text{SR}_{0}^{\text{loose, veto}}$ gives the stronger bound, since the cut by $m_{\text{eff}} > 1250$ GeV of the $\text{SR}_{0}^{\text{tight, veto}}$ is too tight for $m_{E2} \lesssim 600$ GeV. The limit from $\text{SR}_{5L}$ is weaker because of the excess and that from $\text{SR}_{2L}$ is also weaker due to the larger backgrounds. Note that the limits do not change much as the mass difference between the $Z'$ boson and VL lepton decreases since the signal muons can originate from $Z'$ decays.

For the singlet-like VL lepton, the strongest bound of about 1000 GeV is again from the $\text{SR}_{0}^{\text{tight, veto}}$ if BR $(E \to Z'\mu) = 1$. The $\text{SR}_{0}^{\text{tight, veto}}$ is not sensitive to the cases when BR $(E \to Z'\mu) \lesssim 0.3$. The
difference comes from smaller production cross section of the singlet-like case. The \text{SR}^{\text{loose}}_{\text{bveto}} gives the strongest constraint for smaller branching fractions. The limits from \text{SR}^5L and \text{SR}^2L are not shown, since the limits are much weaker than those from \text{SR}^{\text{tight}}_{\text{bveto}} and \text{SR}^{\text{loose}}_{\text{bveto}} for the same reason as the doublet-like case. In particular, \text{SR}^2L gives no bounds for the singlet-like case.

### 3.2 Explanation for the excess in \text{SR}^5L

Figure 5 shows the upper bound on the number of signal events in the \text{SR}^5L allowed by the limits from the other signal regions. The background colors represent maximum values of BR \((E \rightarrow Z'\mu)\).
Figure 4: Limits on BR $(E \to Z'\mu)$ for the singlet-like VL lepton.

Figure 5: Maximum values of the number of signal events in SR5L consistent with the limits of SR$^{\text{loose}}_{\text{bveto}}$ and SR$^{\text{tight}}_{\text{bveto}}$ for the doublet-like (singlet-like) VL lepton in the left (right) panel. Background colors are the BR $(E \to Z'\mu)$.

Since the limits from the four lepton signal regions are severe, the branching fraction should be so small that the limits from SR$^{\text{tight}}_{\text{bveto}}$ and SR$^{\text{loose}}_{\text{bveto}}$ are relaxed by the fewer events passing the $m_{\text{eff}}$ cut. The excess is explained on the solid green line when the SM background is at the central value shown in Table. 1. The yellow band corresponds to the uncertainty of the background estimation. The singlet-like VL lepton can more easily explain the excess. The limits from SR$^{\text{tight}}_{\text{bveto}}$ and SR$^{\text{loose}}_{\text{bveto}}$ are much stronger for the doublet-like case, since the production cross section is larger and there are fewer muon signals originating from the VL neutrino production.
The left panel of Fig. 6 shows the $E_T^{\text{miss}}$ distribution after the selection of SR5L at the benchmark point with $m_{Z'} = 390$ GeV, $m_{E_1} = 400$ GeV and BR ($E \rightarrow Z'\mu$) = 0.25. The SM contributions are represented by yellow bars and the black dots (bars) show the data (its error bar), which are read from Fig. 8 of Ref. [46]. We see that the $E_T^{\text{miss}}$ distribution is well described by our scenario. The benchmark point in our example model which realizes these masses and branching fractions and is also consistent with the muon anomalies at the same time is shown Appendix A.3. The right panel of Fig. 6 shows the $m_{\text{eff}}$ distribution after the selection of the number of muons to be larger than four. The values of the bars are normalized such that the sum of all the bins is unity. The green bars are for the same benchmark point as the left panel, and the red hatched bars are for another benchmark point with $m_{Z'} = 400$ GeV and $m_{E_1} = 1000$ GeV. We see that the peak of the distribution in $m_{\text{eff}}$ is about $2m_{E_1}$, and hence the strong constraint from SR0ightbveto, which requires $m_{\text{eff}} > 1250$ GeV, is avoided and the tightest bound of BR ($E \rightarrow Z'\mu$) ≲ 0.25 is from SROlooselveto.

We emphasize that this model can only explain the SR5L excess by muons. Thus the SR5L excess can not be explained, in this scenario, if it includes signals with electrons. The limits from the data would be significantly tightened if lepton flavors are specified in the signal regions. Thus the information of lepton flavor is crucial to test this model with a muon-philic $Z'$ and VL leptons.

The excess with electrons, might be explained by VL leptons decaying to $Z$ or $W$ boson, where the SM bosons decay leptonically. If $m_{Z'} > m_{E_1}$ which is the opposite case to our scenario, the VL lepton will decay to a SM boson, including the Higgs boson. In addition, electrons may come from the decays of the heavier VL leptons, such as $E_2 \rightarrow E_1 Z$. We note that the roughly degenerate mass of the VL leptons are favored to explain the sizable $\Delta a_{\mu}$ in the model [24]. These possibilities are interesting, but beyond the scope of this paper.
3.3 Future prospects

We shall discuss the discovery and exclusion potential at the HL-LHC with 3 ab$^{-1}$ data. We consider the two signal regions, $\text{SR}_{\text{0.tight bveto}}$ and $\text{SR}_{\text{5L}}$ and propose two more new signal regions, $\text{SR}_{\text{Zp}}$ and $\text{SR}_{\text{5L'}}$.

We rescale the backgrounds to $\text{SR}_{\text{0.tight bveto}}$ and $\text{SR}_{\text{5L}}$ by simply multiplying the ratio of integrated luminosity, 3000/139. For the signal events, we use the same efficiency times acceptance factor as those used in the analyses for the current limits. These are then multiplied by the integrated luminosity.

Figure 7: Future prospects of the upper bound on BR ($E \rightarrow Z'\mu$) for the doublet-like VL lepton at the HL-LHC.
Figure 8: Future prospects of the upper bound on BR ($E \rightarrow Z'\mu$) for the singlet-like VL lepton at the HL-LHC.

luminosity and the production cross section at $\sqrt{s} = 14$ TeV shown in Fig. 2 to calculate the number of signal events.

We define the two new signal regions, named SRZp and SR5L'. In the SRZp, at least four muons are required and the $Z$-veto is applied. Then, two $Z'$ candidates are chosen from any OS pair of muons, where the reference $Z'$ mass, $m_{\text{ref}}^{Z'}$, is set at 500 GeV, in the same manner as the $Z$ candidates. The (next-to) leading $Z'$ candidate, $m_{\text{OS}}$, is the (second) closest to $m_{\text{ref}}^{Z'}$, must satisfy $|m_{\text{OS}} - m_{\text{ref}}^{Z'}| < 100$ (250) GeV. We take relatively large range for the selection, because we do not know the $Z'$ mass. The sensitivity can be improved by requiring strict range for $|m_{\text{OS}} - m_{\text{ref}}^{Z'}|$ and
scan over $m_{Z'}^{\text{ref}}$ in the analysis, as in the $Z'$ searches [49,88]. In the SR5L', more than 5 muons are required. Then, the Z-veto for any OS pair of muons and $m_{\text{eff}} > 1000$ GeV cuts are applied. In these two signal regions, we assume that there are 10 SM background events per 3 ab$^{-1}$ data. This may be a conservative assumption, since the cut is very tight and not so many background events will survive, c.f. the rescaled backgrounds in SR0$^\text{tight}$ and SR5L are 76.6 and 272, respectively.

We quantify the future discovery and exclusion limits by the $p$-values proposed in Ref. [89],

$$p_{\text{disc}} = \frac{\gamma(s + b, b)}{\Gamma(s + b)}, \quad p_{\text{excl}} = \frac{\Gamma(b + 1, s + b)}{\Gamma(b + 1)}, \quad (3.1)$$

where $s$ and $b$ are the number of signals and backgrounds. $\Gamma(z), \gamma(a, z)$ and $\Gamma(a, z)$ are the ordinary, lower incomplete and upper complete Gamma functions. The discovery (exclusion) limit corresponds to $p_{\text{disc}} < 2.867 \times 10^{-7}$ ($p_{\text{excl}} < 0.05$) where the significance is $> 5 (> 1.645)$. Here, we do not consider uncertainties in the signals and backgrounds for simplicity.

The future prospects at the HL-LHC for the doublet-like and the singlet-like VL leptons are shown in Fig. 7 and 8, respectively. The background colors are the exclusion limits ($p_{\text{excl}} < 0.05$) for the branching fraction $\text{BR} (E \rightarrow Z'\mu)$. The white lines show the discovery potential for a given branching fraction as labeled on the lines ($p_{\text{disc}} < 2.867 \times 10^{-7}$). Assuming $\text{BR} (E \rightarrow Z'\mu) = 1$, the doublet-like (singlet-like) VL lepton will be discovered up to $m_{E} \lesssim 1.5 (1.15)$ TeV by SR0$^\text{tight}$. The limits from SR5L are weaker. The SRZp may cover a wider parameter range than that of SR0$^\text{tight}$ at $m_{Z'} \sim 500$ GeV even if we set the relatively large range for $|m_{OS} - m_{Z'}^{\text{ref}}|$. The SR5L' may also cover a wider parameter range independent of the $Z'$ mass, based on our assumption of the background. It is interesting that the entire parameter range with $\text{BR} (E \rightarrow Z'\mu) \sim 0.25$, which can explain the excess in SR5L, can be discovered in SR5L, SRZp and SR5L'.

4 Discussions

In this paper, we study the signal with more than four muons originating from the pair-production of VL leptons decaying to a $Z'$ boson which couples to muons and/or muon neutrinos. These particles may provide a way to resolve the tensions in the $b \rightarrow s\ell\ell$ decays and $\Delta a_{\mu}$. The current limits can reach about 1 TeV when the VL lepton decays to the $Z'$ boson exclusively owing to the very low backgrounds. We showed that the excess in the signal region with five leptons or more may be explained in our model if the excess is given only by muons. A benchmark point in our example model is given in Appendix A.3 which simultaneously explains the muon anomalies as well as the excess in SR5L. If the excess is not only muons, then the cascade decay of the heavier VL lepton might be a nice candidate to explain the excess with electrons and muons in this kind of model. The information of lepton flavor is crucial to test these new physics models which explain the muon anomalies.

Note added. While finalizing this manuscript, the new experimental data of the muon anomalous magnetic moment was announced from the FNAL. The discrepancy from the SM prediction reaches to $4.2 \sigma$ [22]. Models with VL leptons and a $Z'$ boson coupling to muons nicely explains the discrepancy of $\Delta a_{\mu}$ as well as the anomalies in $b \rightarrow s\ell\ell$. These new particles could be confirmed by the LHC as discussed in this paper.
Table 2: Matter contents. Electric charge of fermion $f$ is $Q_f = T^3_f + Y_f/2$.

|        | $\ell_L$ | $\bar{\nu}_R$ | $H$ | $L_L$ | $\bar{E}_R$ | $\bar{\nu}_R$ | $E_L$ | $Z'$ | $\Phi$ |
|--------|----------|----------------|-----|-------|-------------|------------|-------|------|-------|
| $SU(2)_L$ | 2        | 1             | 2   | 2     | 1           | 1          | 1     | 1    | 1     |
| $U(1)_Y$   | -1       | 2             | -1  | -1    | 2           | 1          | -2    | 0    | 0     |
| $U(1)'_L$  | 0        | 0             | 0   | -1    | 1           | 1          | -1    | 0    | -1    |

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A Review of vector-like $U(1)'$ model

In this Appendix, we review the model proposed in Refs. [23,24] as an example of a UV completion of the simplified model. The matter contents of our model is given by Table 2. The $SU(2)_L$ doublets are defined as

\[
\ell_L = (\nu_L, \mu_L), \quad H = (H_0, H_-), \quad L_L = (N'_L, E'_L), \quad \bar{\nu}_R = (-\bar{E}_R, \bar{\nu}_R).
\] (A.1)

We only consider muons, and assume that the couplings with the other leptons are negligible for simplicity. The masses of VL states and Yukawa interactions are given by

\[
\mathcal{L} \supset -m_L \bar{\nu}_R L_L - m_E \bar{E}_R E_L + y_{\mu} \bar{\nu}_R \ell_L H + \kappa' \bar{E}_R L_L H - \kappa \bar{\nu}_R \tilde{H} E_L + \lambda_L \Phi \bar{\nu}_R \ell_L - \lambda_E \Phi \bar{\nu}_R E_L + h.c.,
\] (A.2)

where $\tilde{H} := i\sigma_2 H^* = (H_0^*, -H_-^*)$. The $SU(2)_L$ indices are contracted via $i\sigma_2$. After the symmetry breaking by $v_H := \langle H_0 \rangle$ and $v_\Phi := \langle \Phi \rangle$, the mass matrix for the leptons are given by

\[
\bar{\nu}_R \mathcal{M}_\nu e_L := \left(\begin{array}{c} m_R \bar{E}_R E_R' \\
\nu_L v_H & 0 & \lambda_E v_\Phi \\
0 & -\kappa' v_H & m_E \\
\lambda_L v_\Phi & m_L & \kappa v_H \\
\end{array}\right) \left(\begin{array}{c} \mu_L \\
E'_L \\
E_L \\
\end{array}\right),
\] (A.3)

\[
\bar{n}_R \mathcal{M}_n n_L := \bar{N}'_R \left(\begin{array}{c} \nu_L v_\Phi & m_L \\
\lambda_L v_\Phi \end{array}\right) \left(\begin{array}{c} \nu_L \\
N'_L \\
\end{array}\right).
\] (A.4)

The mass basis is defined as

\[
\hat{\nu}_L := U_L^{\dagger} \nu_L, \quad \hat{\bar{\nu}}_R := U_R^{\dagger} \bar{\nu}_R, \quad \hat{n}_L := V_L^{\dagger} n_L, \quad \hat{n}_R := n_R,
\] (A.5)
where unitary matrices diagonalize the mass matrices as

\[ U_R^\dagger M_e U_L = \text{diag} \left( m_\mu, m_{E_2}, m_{E_1} \right), \quad M_n V_L = \begin{pmatrix} 0 & m_N \end{pmatrix}, \tag{A.6} \]

where \( E_1 \) (\( E_2 \)) is the singlet-like (doublet-like) VL lepton\(^{10}\). The non-zero mass of the SM neutrino will be explained by introducing the right-handed counterparts, but these are irrelevant for the present discussion.

We define the Dirac fermions as

\[ \mathbf{e} := (\mu, E_2, E_1), \quad \mathbf{n} := (\nu, N), \tag{A.7} \]

where

\[ [\mathbf{e}]_i := ([\hat{e}_L]_i, [\hat{e}_R]_i), \quad [\mathbf{n}]_i := ([\hat{n}_L]_i, 0), \quad N := ([\hat{n}_L]_2, N'_R), \tag{A.8} \]

with \( i = 1, 2, 3 \).

### A.1 Interactions

The gauge interactions with the \( Z' \) boson in the mass basis are defined as

\[ \mathcal{L}_V = Z'_{\mu} \sum_{f=e,n} \bar{f} \gamma^\mu \left( g^Z_{Lf} P_L + g^Z_{Rf} P_R \right) f, \tag{A.9} \]

where the coupling matrices are given by

\[ g^Z_{fL} = g' U_R^\dagger Q'_f U_L, \quad g^Z_{fR} = g' U_R^\dagger Q'_f U_R, \quad g^n_{fL} = g' V_L^\dagger Q'_n V_L, \quad g^n_{fR} = g' Q'_n. \tag{A.10} \]

\( P_L \) (\( P_R \)) are the chiral projections onto the left- (right-)handed fermions. \( g' \) is the gauge coupling constant for \( U(1)' \).

We expand the neutral scalar fields as

\[ H_0 = v_H + \frac{1}{\sqrt{2}} (h + ia_h), \quad \Phi = v_\Phi + \frac{1}{\sqrt{2}} (\chi + ia_\chi), \tag{A.11} \]

where \( h \) and \( \chi \) are the physical real scalar fields, while the pseudo-scalar components \( a_h \) and \( a_\chi \) are absorbed by the \( Z \) and \( Z' \) bosons, respectively. The Yukawa interactions are given by

\[ -\mathcal{L}_Y = \frac{1}{\sqrt{2}} \sum_{S=h,\chi} \sum_{f=e,n} S \bar{f} Y^S_{f} P_L f + h.c., \tag{A.12} \]

where

\[ Y^h_{\mathbf{e}} = U_R^\dagger \begin{pmatrix} y_\mu & 0 & 0 \\ 0 & \kappa' & 0 \\ 0 & 0 & \kappa \end{pmatrix} U_L, \quad Y^h_{\mathbf{n}} = U_R^\dagger \begin{pmatrix} 0 & 0 & \lambda_E \\ 0 & 0 & 0 \\ \lambda_L & 0 & 0 \end{pmatrix} U_L, \quad Y^h_{\mathbf{n}} = 0_{2 \times 2}, \quad Y^\chi_{\mathbf{n}} = \begin{pmatrix} 0 & 0 \\ \lambda_L & 0 \end{pmatrix} V_L. \tag{A.13} \]

\(^{10}\)We restrict cases which \( m_{E_1} \ll m_{E_2} \) or \( m_{E_1} \gg m_{E_2} \), so we can always identify the VL lepton is singlet-like or doublet-like.
Let us define the approximate masses of the VL leptons as
\[ M_L := \sqrt{m_{2L}^2 + \lambda_L^2 v_H^2}, \quad M_E := \sqrt{m_{2E}^2 + \lambda_E^2 v_H^2}. \] (A.14)

Assuming \( \kappa v_H \ll |M_L - M_E| \), the diagonalization matrices for the charged lepton mass matrix are given by
\[
U_L \sim \begin{pmatrix} c_L & s_L - \delta_L s_L & 0 \\ -s_L & c_L - \delta_L c_L & \delta_L \\ 0 & 0 & 1 \end{pmatrix} + \mathcal{O}(\delta_L^2), \quad U_R \sim \begin{pmatrix} c_R & s_R \delta_R & s_R \\ -s_R & c_R \delta_R & c_R \\ 0 & 1 & -\delta_R \end{pmatrix} + \mathcal{O}(\delta_R^2),
\] (A.15)

where
\[
\begin{align*}
c_L &:= \frac{m_L}{M_L}, & s_L &:= \frac{\lambda_L v_H}{M_L}, & \delta_L &:= \frac{\kappa v_H M_L}{M_L^2 - M_E^2}, \\
c_R &:= \frac{m_E}{M_E}, & s_R &:= \frac{\lambda_E v_H}{M_E}, & \delta_R &:= \frac{\kappa v_H M_E}{M_L^2 - M_E^2}.
\end{align*}
\] (A.16) (A.17)

The diagonalized mass matrix is given by
\[
U_R^\dagger M_e U_L = \begin{pmatrix} y_\mu c_L c_R + \kappa' s_L s_R v_H & 0 & 0 \\ 0 & M_L + \mathcal{O}(\kappa v_H \delta_L, \delta_R) & 0 \\ \mathcal{O}(m_\mu) & \mathcal{O}(\kappa v_H \delta_L^2) & M_E + \mathcal{O}(\kappa v_H \delta_L, \delta_R) \end{pmatrix} + \mathcal{O}(m_\mu \delta_L, \delta_R),
\] (A.18)

where we assume \( y_\mu v_H, \kappa' v_H \lesssim m_\mu \) to explain the muon mass without fine-tuning.

The \( Z' \) couplings in the mass basis are approximately given by
\[
\begin{align*}
g_{eL}^{Z'} &= -g' \begin{pmatrix} s_L^2 & -c_L s_L & -c_L s_L \delta_L \\ -c_L s_L & c_L^2 & s_L^2 \delta_L \\ c_L s_L \delta_L & s_L^2 \delta_L & 1 \end{pmatrix} + \mathcal{O}(\delta_L^2), \\
g_{eR}^{Z'} &= -g' \begin{pmatrix} s_R^2 & -c_R s_R \delta_R & s_R \delta_R \\ -c_R s_R \delta_R & 1 & -s_R^2 \delta_R \\ -s_R^2 \delta_R & s_R^2 \delta_R & c_R^2 \end{pmatrix} + \mathcal{O}(\delta_R^2).
\end{align*}
\] (A.19) (A.20)

Hence, the effective couplings defined in Eq. (2.1) are given by
\[
\begin{pmatrix} g_{\mu\mu}^L & g_{\mu E}^L \\ g_{\mu E}^L & g_{EE}^L \end{pmatrix} \sim -g' \begin{pmatrix} s_L^2 & -s_L c_L \\ -s_L c_L & c_L^2 \end{pmatrix}, \quad \begin{pmatrix} g_{\mu\mu}^R & g_{\mu E}^R \\ g_{\mu E}^R & g_{EE}^R \end{pmatrix} \sim -g' \begin{pmatrix} s_R^2 & -c_R s_R \delta_R \\ -c_R s_R \delta_R & 1 \end{pmatrix},
\] (A.21)
in the doublet-like case, and these are given by

\[
\begin{pmatrix}
g_{\mu\mu}^L & g_{\mu E}^L \\
g_{\mu E}^L & g_{\mu E}^E
\end{pmatrix} \sim -g' \begin{pmatrix} s_L^2 & c_L s_L \delta_L \\ c_L s_L \delta_L & 1 \end{pmatrix}, \quad \begin{pmatrix}
g_{\mu\mu}^R & g_{\mu E}^R \\
g_{\mu E}^R & g_{\mu E}^E
\end{pmatrix} \sim -g' \begin{pmatrix} s_R^2 & -c_R s_R \\ -c_R s_R & c_R^2 \end{pmatrix},
\]

(A.22)
in the singlet-like case.

The Yukawa couplings with \( \chi \) are given by

\[
Y_e^{\chi} \sim \begin{pmatrix} 0 & \lambda_{ECR} \delta_L & \lambda_{ECR} \\ \lambda_{LCL} & \lambda_{L SL} & \lambda_{ESR} \delta_R - \lambda_{LSL} \delta_L \\ -\lambda_{LCR} \delta_R - \lambda_{ESR} \delta_L & \lambda_{ESR} \end{pmatrix}, \quad Y_\mu^{\chi} \sim \lambda_L \begin{pmatrix} 0 \\ 0 \\ c_L s_L \end{pmatrix},
\]

(A.23)
where the coupling of \( \chi \) with \( \mu \mu \) is as small as \( m_\mu / m_E \). The couplings to the SM bosons are the SM-like up to \( O(m_\mu/m_E) \).

### A.2 Muon anomalies

The \( Z' \) and \( \chi \) boson contribution to \( \Delta a_\mu \) is given by [33,90]

\[
\Delta a_\mu \sim \frac{m_\mu k \kappa H}{64 \pi^2 v_\Phi^2} s_{2L} s_{2R} C_{LR},
\]

(A.24)
with

\[
C_{LR} := \sqrt{x_L x_E} \frac{G_Z(x_L) - G_Z(x_E)}{x_L - x_E} + \frac{1}{2} \sqrt{y_L y_R} \frac{y_L G_S(y_L) - y_R G_S(y_R)}{y_L - y_R},
\]

(A.25)
where \( x_L := M_L^2 / m_{Z'}^2, x_E := M_E^2 / m_{Z'}^2, y_L := M_L^2 / m_\chi^2 \) and \( y_E := M_E^2 / m_\chi^2 \). Here, \( m_{Z'}^2 = 2g^2 v_\Phi^2 \) is used. The loop functions are given by

\[
G_Z(x) := \frac{x^3 + 3x - 6x \ln(x) - 4}{2(1 - x)^3}, \quad G_S(y) := \frac{y^2 - 4y + 2 \ln(y) + 3}{(1 - y)^3}.
\]

(A.26)
The contribution from the scalar \( \chi \) is included since it is sizable unless \( m_\chi \) is very heavy which requires very large quartic couplings.

For the \( b \to s \mu \mu \) anomaly, the Wilson coefficients are given by

\[
C_9 \sim -\frac{\sqrt{3}}{4 G_F} \frac{4 \pi}{\alpha_e} \frac{1}{V_{ib} V_{ts}^*} \frac{1}{4 v_\Phi^2} (s_R^2 + s_L^2) \epsilon_Q \epsilon_Q,
\]

(A.27)
\[
C_{10} \sim -\frac{\sqrt{3}}{4 G_F} \frac{4 \pi}{\alpha_e} \frac{1}{V_{ib} V_{ts}^*} \frac{1}{4 v_\Phi^2} (s_R^2 - s_L^2) \epsilon_Q \epsilon_Q,
\]

(A.28)
where the \( Z' \) boson couplings to the SM doublet quarks are parametrized as

\[
\begin{pmatrix}
g_{d s}^Z \end{pmatrix}_{ij} \sim \begin{pmatrix} g_{d s L}^Z \end{pmatrix}_{ij} \sim -g' \epsilon_{Q_i} \epsilon_{Q_j},
\]

(A.29)
$\epsilon_Q$, is the similar quantity as $s_L := \lambda_L v_\Phi/m_L$, but we now consider the couplings with the second and third generation quarks and these are typically small in contrast to that for muon.

From Eq. (A.24),

$$\Delta a_\mu \sim 2.9 \times 10^{-9} \times \left( \frac{1.0 \text{ TeV}}{v_\phi} \right)^2 \left( \frac{\kappa}{1.0} \right) \left( \frac{s_{2L}s_{2R}}{1.0} \right) \left( \frac{C_{LR}}{0.1} \right).$$  \hspace{1cm} (A.30)

For the $b \to s\mu\mu$ anomaly,

$$C_9 \sim -0.62 \times \left( \frac{1.0 \text{ TeV}}{v_\phi} \right)^2 \left( \frac{s_L^2 + s_R^2}{1} \right) \left( \frac{\epsilon_{Q_2}\epsilon_{Q_3}}{-0.002} \right).$$  \hspace{1cm} (A.31)

Assuming $s_L = s_R = 1/\sqrt{2}$, i.e. $\lambda_L v_\Phi = m_L$ and $\lambda_E v_\Phi = m_E$, the quark mixing angles are given by

$$\epsilon_{Q_2}\epsilon_{Q_3} \sim -0.003 \times \left( \frac{C_9}{-0.82} \right) \left( \frac{2.51 \times 10^{-9}}{\Delta a_\mu} \right) \left( \frac{\kappa}{1.0} \right) \left( \frac{C_{LR}}{0.1} \right),$$  \hspace{1cm} (A.32)

when the both anomalies are explained. With such small couplings with quarks, $Z'$ boson is sufficiently suppressed to be consistent with the constraints from the resonant di-lepton signal search at the LHC, unless $\epsilon_{Q_2} \sim 1$ or $\epsilon_{Q_3} \sim 1$ to have large production cross section from $s\overline{s}$ or $b\overline{b}$, respectively.

### A.3 Benchmark

We show a benchmark scenario which explains the anomalies in $\Delta a_\mu$ and $b \to s\ell\ell$ and the excess in SR5L simultaneously. As discussed in the main text, singlet-like VL lepton is more suitable to explain the excess in SR5L. We take \(^{11}\)

\begin{align*}
m_{Z'} & = 390 \text{ GeV}, \quad M_L = 1.1 \text{ TeV}, \quad M_E = 404 \text{ GeV}, \quad m_\chi = 365 \text{ GeV}, \quad (A.33) \\
s_L = s_R = 1/\sqrt{2}, \quad y_\mu v_H = 2m_\mu, \quad \kappa' = 0, \quad \kappa = -0.821, \quad g' = 0.25.
\end{align*}

The VL lepton masses are 400 and 1111 GeV. The correction to the anomalous magnetic moment of the muon is $\Delta a_\mu = 2.51 \times 10^{-9}$. $s_L = s_R$ realizes the $C_9$-only scenario, and $C_9 \sim -0.81$ is explained if $\epsilon_{Q_2}\epsilon_{Q_3} \sim -0.0032$.

The partial decay widths of the singlet VL lepton $E_1$ are approximately given by

\begin{align*}
\Gamma (E_1 \to Z'\mu) & \sim \frac{M_E^3}{64\pi v_\Phi^2} c_R^2 s_R^2 (1 - z)^2 (1 + 2z), \quad (A.34) \\
\Gamma (E_1 \to \chi\mu) & \sim \frac{M_E^3}{64\pi v_\Phi^2} c_R^2 s_R^2 (1 - x)^2, \quad (A.35)
\end{align*}

\(^{11}\)With these values, $v_\Phi = 1103$ GeV which is sufficiently large to evade the bound from the neutrino trident process [91–96].
where \( z := \frac{m^2_{Z'}}{M^2_E} \) and \( x := \frac{m^2_\chi}{M^2_E} \). Hence the branching fraction of \( E_1 \), assuming no other decay modes, is approximately given by

\[
\text{BR} \left( E_1 \to Z'\mu \right) \sim \frac{(1 - z)^2(1 + 2z)}{(1 - z)^2(1 + 2z) + (1 - x)^2}.
\] (A.36)

At the benchmark point, \( \text{BR} \left( E_1 \to Z'\mu \right) \simeq 0.25 \). The \( \chi \) boson predominantly decays to VL fermions as far as these are kinematically allowed. If this is not the case, it should decay to a pair of SM leptons or quarks. For the lepton coupling, as seen from Eq. (A.23), the coupling to 2 muons are strongly suppressed by the muon mass. Hence, the dominant decay mode of \( \chi \) may be to a pair of top quarks due to the weaker suppression if \( m_\chi > 2m_t \) which is true at the benchmark point. In this case, the processes with \( \chi \) decays will not contribute to the \text{SR}_0^{\text{tight}}\text{bveto} \) and \text{SR}_0^{\text{loose}}\text{bveto} \) due to the b-jet veto, and thus the results in the main text will not be changed. If \( m_\chi < 2m_t \), \( \chi \) decays to a pair of bottom quarks, where the relevant Yukawa is estimated as \( \sim \epsilon_3 m_b / M_Q \sim 10^{-4} \) for \( \epsilon_3 \sim 0.1 \) and \( M_Q \sim 4 \text{ TeV} \). This would be comparable to the decay to a pair of muons which the relevant Yukawa coupling is estimated as \( \sim m_\mu / M_L \sim 10^{-4} \) for \( M_L \sim 1 \text{ TeV} \). Thus, there will be additional contributions with six muons on top of the decays from \( Z' \) boson.
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