On the Role of Large Nuclear Gravity in Understanding Strong Coupling Constant, Nuclear Stability Range, Binding Energy of Isotopes and Magic proton numbers – A Critical Review

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ABSTRACT

With reference to our earlier published views on large nuclear gravitational constant $G_s$, nuclear elementary charge $e_s$ and strong coupling constant $\alpha_s \equiv \left(\frac{e_s}{e} \right)^2$, in this paper, we present simple relations for nuclear stability range, binding energy of isotopes and magic proton numbers. Even though ‘speculative’ in nature, proposed concepts are simple to understand, easy to implement, result oriented, effective and unified. Our proposed model seems to span across the Planck scale and nuclear scale and can be called as SPAN model (STRANGE* physics of atomic nucleus).

Summary: Probable range of stable mass numbers can be estimated with

$$A_s \approx \frac{Z}{\alpha} + \pm \left( \frac{1}{\alpha} \right)$$

where $x \equiv 1.2$ for $Z = (3$ to $100)$ and $x \approx 1.19$ for $Z \geq 100$. $A_s$ can also be expressed as,

$$A_s \approx 2Z + kZ'$$

where $k \equiv \left( \frac{4\pi\varepsilon_0 (\hbar/2)}{m_e c^2} \right) \approx 10.06$ MeV, for $Z \approx (5$ to $118)$, nuclear binding energy can be understood/fitted with two terms as,

$$B_A \approx A - \left( \frac{kA Z/2.531 + 3.531}{10.06} \right) \times 10.06$$

where

$$\ln \left( 1/k \right) \approx (m_e - m_p) / m_p \approx 2.531.$$ 

By considering a third term of the form $A - A' / A$, binding energy of isotopes of $Z$ can be fitted approximately. It needs further investigation. See section12 for an in-depth discussion.

Keywords: Strong nuclear gravity, Nuclear elementary charge, Strong coupling constant, Nuclear stability range, Binding energy of isotopes, Magic proton numbers

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1. Introduction

With reference to ‘Strong (nuclear) gravity’ [1-20], if $G_s \approx 10^{38} G_N$, and with reference to our recent symposium proceedings and journal publications [21-37], we try to refine our proposed concepts with the following three assumptions for a better understanding on nuclear stability range, binding energy of isotopes and magic proton numbers. We consider,

$$G_s \approx G_s \approx \frac{4\pi\varepsilon_0 \hbar^2 c^2 m_p}{e^2 m_p} \approx 3.329561 \times 10^{28} \text{ m}^2\text{kg}^{-1}\text{sec}^{-2}.$$ 

2. Assumptions

1) Nuclear charge radius can be expressed as,

$$R_{0} \approx \frac{2G_s m_p}{c^2} \approx 1.23929083 \text{ fm}$$

2) Strong coupling constant can be expressed as,

$$\alpha_s \approx \left( \frac{\hbar c}{G_s m_p^2} \right)^2 \approx 0.1151937353$$

3) There exists a nuclear elementary charge,

$$e_s \approx \frac{e}{\alpha_s} \approx \frac{G_s m_p^2}{\hbar c} \approx 4.720586027 \times 10^{-19} \text{ C}$$

3. Semi Empirical Relations and Applications

1) Proton magnetic moment can be expressed as

$$\mu_p \approx \frac{e \hbar}{2m_p} \approx \frac{e^2 m_p}{2c} \approx 1.488142 \times 10^{-26} \text{ J.T}^{-1}$$

2) Ignoring the negative sign, neutron magnetic moment can be expressed as

$$\mu_n \approx \left( \frac{e_s - e}{2m_n} \right) \approx 9.817102 \times 10^{-27} \text{ J.T}^{-1}.$$ 

3) Nuclear unit radius can be expressed as,
4. Neutron-proton Mass Difference

Neutron-proton mass difference can be understood with:

\[
\left( \frac{m_n c^2 - m_p c^2}{m_c^2} \right) = \ln \left( \frac{E_{(n2)}}{m_c^2} \right) \ln \left( \frac{4 \varepsilon G m_p^3}{4 \varepsilon_0 h^2 c^2} \right) \quad (4)
\]

5. Neutron Life Time

Neutron life time \( t_n \) can be understood with the following relation:

\[
t_n \cong \exp \left( \frac{E_{(n2)}}{(m_n - m_p) c^2} \right) \left( \frac{h}{m_c} \right) \cong 877.3 \text{ sec} \quad (5)
\]

This value can be compared with recommended value of \((878.5 \pm 0.8)\) sec.

6. Understanding Proton-neutron Stability

Let, \( \left( \frac{m_n c^2}{E_{(n2)}} \right) = \left( \frac{4 \varepsilon \varepsilon_0 h^2 m_p^3}{4 \varepsilon_0 h^2 c^2} \right) \cong k \cong 0.0063326 \quad (6)\)

Quantitatively, we noticed that,

\[
\frac{e^2}{4 \pi \varepsilon_0 G m_n m_p} \cong 4 \pi \cong \frac{1}{4k} \quad (7)
\]

The new factor \( k \) needs a clear interpretation and we are working on that for its scope and applicability. It can be considered as a result oriented number connected with nuclear stability and binding energy.

Stable mass number \( A_s \) of Z can be estimated with the following simple relations [38],

\[ A_s \cong (N_s + Z) \cong 2Z + kZ^2 \cong 2Z + 0.0063326(Z)^2 \quad (8) \]

\[ A_s \cong \left[ Z + \sqrt{\left( \frac{1}{1/\alpha_s} \right)^2} \right]^2 \cong [Z + 2.9463]^2 \quad (9) \]

where \( (e/e_p)(1/k)^{1/4} \cong (\alpha_s)^{1/2} (1/k)^{1/4} \cong 1.2 \). It can be called as 'power factor of stability'.

Proton number \( Z \) associated with stable \( A_s \) can be estimated with the following simple relations,

\[ Z \cong \frac{\sqrt{1 + kA_s} - 1}{k} \quad \text{Or} \quad Z \cong \frac{A_s}{1 + \sqrt{1 + kA_s}} \quad (10) \]

7. Understanding Proton-neutron Stability Range

Considering relation (8), it seems possible to find the best possible range of \( A_s \). We noticed that,

\[ (A_s)_{\text{low}} \cong \left[ Z + \sqrt{\left( \frac{1}{1/\alpha_s} \right)^2} \right] \]

\[ (A_s)_{\text{up}} \cong \left[ Z + \left( \frac{1}{1/\alpha_s} \right) + 1 \right] \]

(11)

Lower stable \( A_s \) can be estimated with,

\[ (A_s)_{\text{low}} \cong \left[ Z + \left( \frac{1}{1/\alpha_s} - 1 \right) \right] \]

\[ \cong [Z + 1.9463]^2 \quad (12) \]

Upper stable \( A_s \) can be estimated with,

\[ (A_s)_{\text{up}} \cong \left[ Z + \left( \frac{1}{1/\alpha_s} + 1 \right) \right] \]

\[ \cong [Z + 3.9463]^2 \quad (13) \]
See Table 1 for the estimated range of stable mass numbers for Z=3 to 100. With even-odd corrections data can be refined.

Considering a factor of 1.19 in place of 1.2, stable mass numbers of super heavy elements can be fitted. For Z=116, estimated stable mass number range seems to be 292 to 298 and its experimental mass range is 291 to 294 [39]. See Table 2 for a comparison.

Table 1: Estimated range of stable mass numbers for Z=3 to 100 with a power factor of 1.20

| Z  | \(A_{\text{low}}\) | \(A_{\text{mean}}\) | \(A_{\text{up}}\) | Main Isotope range |
|----|------------------|-----------------|-----------------|------------------|
| 3  | 7                | 8               | 10              | 6 to 7           |
| 4  | 8                | 10              | 12              | 7 to 10          |
| 5  | 10               | 12              | 14              | 10 to 11         |
| 6  | 12               | 14              | 16              | 11 to 14         |
| 7  | 14               | 16              | 18              | 13 to 15         |
| 8  | 16               | 18              | 20              | 16 to 18         |
| 9  | 18               | 20              | 22              | 18 to 19         |
| 10 | 20               | 22              | 24              | 20 to 22         |
| 11 | 22               | 24              | 26              | 22 to 24         |
| 12 | 24               | 26              | 28              | 24 to 26         |
| 13 | 26               | 28              | 30              | 26 to 27         |
| 14 | 28               | 30              | 32              | 28 to 32         |
| 15 | 30               | 32              | 34              | 31 to 33         |
| 16 | 32               | 34              | 36              | 32 to 36         |
| 17 | 34               | 36              | 38              | 35 to 37         |
| 18 | 36               | 38              | 41              | 36 to 42         |
| 19 | 38               | 41              | 43              | 39 to 41         |
| 20 | 41               | 43              | 45              | 40 to 48         |
| 21 | 43               | 45              | 47              | 44 to 48         |
| 22 | 45               | 47              | 50              | 46 to 50         |
| 23 | 47               | 50              | 52              | 48 to 51         |
| 24 | 50               | 52              | 54              | 50 to 54         |
| 25 | 52               | 54              | 57              | 52 to 55         |
| 26 | 54               | 57              | 59              | 54 to 60         |
| 27 | 57               | 59              | 61              | 56 to 60         |
| 28 | 59               | 61              | 64              | 58 to 64         |
| 29 | 61               | 64              | 66              | 63 to 67         |
| 30 | 64               | 66              | 69              | 64 to 72         |
| 31 | 66               | 69              | 71              | 66 to 73         |
| 32 | 69               | 71              | 74              | 68 to 76         |
| 33 | 71               | 74              | 76              | 73 to 75         |
| 34 | 74               | 76              | 79              | 72 to 82         |
| 35 | 76               | 79              | 81              | 79,81            |
| 36 | 79               | 81              | 84              | 78 to 86         |
| 37 | 81               | 84              | 86              | 83 to 87         |
| 38 | 84               | 86              | 89              | 82 to 88         |
| 39 | 86               | 89              | 91              | 87 to 91         |
| 40 | 89               | 91              | 94              | 88 to 96         |

41  91  94  96  90 to 96
42  94  96  99  92 to 100
43  96  99 101  95 to 99
44  99 101 104  96 to 106
45 101 104 107  99 to 105
46 104 107 109 100 to 110
47 107 109 112 105 to 111
48 109 112 114 106 to 116
49 112 114 117 113,115
50 114 117 120 112 to 126
51 117 120 122 121 to 125
52 120 122 125 120 to 130
53 122 125 128 123 to 135
54 125 128 131 124 to 136
55 128 131 133 133 to 137
56 131 133 136 130 to 138
57 133 136 139 137 to 139
58 136 139 141 134 to 144
59 139 141 144 141 to 143
60 141 144 147 142 to 150
61 144 147 150 145 to 147
62 147 150 152 144 to 154
63 150 152 155 150 to 155
64 152 155 158 148 to 160
65 155 158 161 157 to 159
66 158 161 164 154 to 164
67 161 164 166 163 to 167
68 164 166 169 160 to 172
69 166 169 172 167 to 171
70 169 172 175 166 to 177
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79 195 198 201 195 to 199
80 198 201 204 194 to 204
81 201 204 207 204 to 210
82 204 207 209 202 to 214
83 207 209 212 207 to 210
84 209 212 215 208 to 210
85 212 215 218 209 to 211
86 215 218 221 218 to 224
87 218 221 224 221 to 223
88 221 224 227 223 to 228
89 224 227 230 225 to 227
Table 2: Estimated range of stable mass numbers for Z=101 to 118 with a power factor of 1.19

| Z  | (A)_low | (A)_mean | (A)_up  | Current synthetic isotopes range    |
|----|--------|---------|--------|-----------------------------------|
| 101| 248    | 251     | 254    | 257 to 260                        |
| 102| 251    | 254     | 257    | 253 to 259                        |
| 103| 254    | 257     | 260    | 254 to 266                        |
| 104| 257    | 260     | 263    | 261 to 267                        |
| 105| 260    | 263     | 266    | 262 to 270                        |
| 106| 263    | 266     | 269    | 265 to 271                        |
| 107| 266    | 269     | 271    | 267 to 278                        |
| 108| 269    | 271     | 274    | 269 to 271                        |
| 109| 271    | 274     | 277    | 274 to 282                        |
| 110| 274    | 277     | 280    | 279 to 281                        |
| 111| 277    | 280     | 283    | 279 to 286                        |
| 112| 280    | 283     | 286    | 277 to 285                        |
| 113| 283    | 286     | 289    | 278 to 290                        |
| 114| 286    | 289     | 292    | 284 to 290                        |
| 115| 289    | 292     | 295    | 287 to 290                        |
| 116| 292    | 295     | 298    | 290 to 294                        |
| 117| 295    | 298     | 301    | 293, 294                          |
| 118| 298    | 301     | 304    | 294, 295                          |

8. Nuclear Binding Energy Close to Stable Mass Numbers

Based on the new integrated model proposed by N. Ghahramany et al [40,41],

\[
B(Z, N) = A - \left( \frac{N^2 - Z^2}{3Z} + \delta(N - Z) + 3 \right) \frac{m_e c^2}{\gamma} \quad (14)
\]

where, \( \gamma \) = Adjusting coefficient \( \approx (90 \text{ to } 100) \).

If \( N = Z \), \( \delta(N - Z) = 0 \) and if \( N = Z \), \( \delta(N - Z) = 1 \).

Thus, binding energy can be fitted with,

\[
B_k \approx \left[ A - \frac{k A Z}{2.531} + 3.531 \right] \times 10.06 \text{ MeV} \quad (20)
\]

See the following Figure 1. Dotted red curve plotted with relations (7) and (20) can be compared with the green curve plotted with the standard semi empirical mass formula (SEMF) [38,42].

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Data has been taken from https://en.wikipedia.org/wiki/Isotope

Readers are encouraged to see references there in [40,41] for derivation part. Point to be noted is that, close to the beta stability line, the effects of coulombic and asymmetric effects. In this context, we propose that,

\[
\frac{m_e c^2}{\gamma} \approx \frac{m_e c^2}{(90 \text{ to } 100)} \approx \text{Constant}
\]

\[
\cong \frac{e^2}{8\pi \varepsilon_0 \left( G m_p / c^2 \right)} \cong 10.09 \text{MeV}
\]

(15)

Proceeding further, with reference to relation (7), it is also possible to show that, for \( Z \approx (40 \text{ to } 83) \), close to the beta stability line,

\[
\left( \frac{N^2 - Z^2}{3Z} \right) \cong k A Z
\]

(16)

\[
\left( \frac{N^2 - Z^2}{3Z} \right) \cong \frac{k A Z}{3}
\]

(17)

Based on the above relations and close to the stable mass numbers of \( Z \approx 5 \), with a common energy coefficient of 10.06 MeV, we propose two terms for fitting and understanding nuclear binding energy.

First term helps in increasing the binding energy and can be considered as,

\[
\text{Term}_1 = A \times 10.06 \text{ MeV}
\]

(18)

Second term helps in decreasing the binding energy and can be considered as,

\[
\text{Term}_2 = \left( \frac{k A Z}{2.531} + 3.531 \right) \times 10.06 \text{ MeV}
\]

(19)

where

\[
3.531 \cong 1 + 2.531 \cong 1 + \ln \left( \frac{1}{\sqrt{k}} \right)
\]

Thus, binding energy can be fitted with,

\[
B_k \approx \left[ A - \frac{k A Z}{2.531} + 3.531 \right] \times 10.06 \text{ MeV}
\]

(20)
Figure 1: Binding energy per nucleon close to stable mass numbers of $Z = 5$ to 118

For medium and heavy atomic nuclides, fit is excellent. It seems that some correction is required for light atoms. See Table 3 for the estimated data.

Table 3: Nuclear Binding energy close to stable mass numbers of $Z = 5$ to 118

| Proton number | Mass number | Estd. BE (MeV) | SEMF BE (MeV) | Error (MeV) |
|---------------|-------------|----------------|---------------|-------------|
| 5             | 10          | 63.8           | 62.3          | -1.53       |
| 6             | 12          | 83.4           | 87.4          | 4.01        |
| 7             | 14          | 102.9          | 98.8          | -4.04       |
| 8             | 16          | 122.2          | 123.2         | 1.03        |
| 9             | 19          | 151.3          | 148.9         | -2.46       |
| 10            | 21          | 170.5          | 167.5         | -2.94       |
| 11            | 23          | 189.5          | 186.1         | -3.35       |
| 12            | 25          | 208.4          | 204.7         | -3.71       |
| 13            | 27          | 227.3          | 223.2         | -4.04       |
| 14            | 29          | 246.0          | 241.6         | -4.35       |
| 15            | 31          | 264.6          | 260.0         | -4.65       |
| 16            | 34          | 292.8          | 290.8         | -2.06       |
| 17            | 36          | 311.2          | 305.1         | -6.18       |
| 18            | 38          | 329.5          | 327.2         | -2.32       |
| 19            | 40          | 347.7          | 341.5         | -6.27       |
| 20            | 43          | 375.4          | 371.6         | -3.84       |
| 21            | 45          | 393.4          | 389.6         | -3.80       |
| 22            | 47          | 411.3          | 407.5         | -3.80       |
| 23            | 49          | 429.1          | 425.2         | -3.85       |
| 24            | 52          | 456.2          | 454.6         | -1.61       |
| 25            | 54          | 473.7          | 468.9         | -4.85       |
| 26            | 56          | 491.2          | 489.6         | -1.61       |
| 27            | 59          | 517.9          | 515.2         | -2.72       |
| 28            | 61          | 535.1          | 532.5         | -2.63       |
| 29            | 63          | 552.3          | 549.7         | -2.61       |
| 30            | 66          | 578.6          | 577.9         | -0.67       |
| 31            | 68          | 595.5          | 592.0         | -3.52       |

32  70  612.3  611.7  -0.60
33  73  638.2  636.6  -1.60
34  75  654.8  653.3  -1.52
35  78  680.4  677.9  -2.56
36  80  696.8  697.0  0.26
37  83  722.2  721.3  -0.84
38  85  738.3  737.6  -0.69
39  88  763.4  761.6  -1.80
40  90  779.3  780.2  0.93
41  93  804.1  803.9  -0.21
42  95  819.7  819.7  0.00
43  98  844.3  843.2  -1.13
44 100  859.7  861.2  1.52
45 103  884.0  884.4  0.38
46 105  899.2  899.8  0.62
47 108  923.2  922.7  -0.49
48 111  947.0  947.6  0.62
49 113  961.9  962.8  0.96
50 116  985.5  987.5  2.03
51 118 1000.1 1000.2  0.16
52 121 1023.4 1024.6  1.22
53 124 1046.5 1046.5  0.05
54 126 1060.8 1063.4  2.62
55 129 1083.6 1085.1  1.47
56 132 1106.3 1108.7  2.38
57 135 1128.9 1130.1  1.17
58 137 1142.7 1144.4  1.73
59 140 1165.0 1165.6  0.58
60 143 1187.1 1188.5  1.42
61 146 1209.1 1209.3  0.23
62 148 1222.4 1225.3  2.91
63 151 1244.1 1245.9  1.77
64 154 1265.6 1268.2  2.56
65 157 1287.0 1288.4  1.41
66 160 1308.3 1310.4  2.16
67 162 1321.0 1322.1  1.14
68 165 1342.0 1343.9  1.94
69 168 1362.8 1363.6  0.86
70 171 1383.4 1385.1  1.64
71 174 1404.0 1404.5  0.58
72 177 1424.3 1425.7  1.34
73 180 1444.5 1444.8  0.30
74 183 1464.6 1465.7  1.06
75 186 1484.5 1484.6  0.06
76 189 1504.3 1505.1  0.82
77 192 1523.9 1523.7  -0.14
78 195 1543.3 1544.0  0.64
79 198 1562.6 1562.4  -0.27
80 201 1581.8 1582.3  0.54
9. Nuclear Binding Energy of Isotopes of Z

We are working on understanding and estimating the binding energy of mass numbers above and below the stable mass numbers. With trial and error, we have developed a third term of the form

\[ \frac{\left( A - \frac{kAZ}{2.531} + 3.531 \right)}{A} - \left( \frac{(A - A_s)^2}{A_s} \right) \times 10.06 \text{ MeV}. \]

Using this term, approximately, it is possible to fit the binding energy of isotopes in following way.

\[ B_s \cong \left[ A - \left( \frac{kAZ}{2.531} + 3.531 \right) \right] - \left[ \frac{(A - A_s)^2}{A_s} \right] \times 10.06 \text{ MeV} \]

(21)

See Figure 2 and Table 4 for the estimated isotopic binding energy of Z=50. Dashed red curve plotted with relations (7) and (21) can be compared with the green curve plotted with total binding energy of Thomas-Fermi model [42].

For Z=50 and A=100 to 130, with reference to total binding energy of Thomas-Fermi model [42], there is no much more difference in the estimation of binding energy. When (A > 130), binding energy seems to be increasing and when (A > 170), binding energy seems to be decreasing rapidly. It needs further study and refinement.

See Figures 3 to 10 for the estimated isotopic binding energies of Z=22, 32, 42, 52, 62, 72, 82 and 92. Dashed red curve plotted with relations (7) and (21) can be compared with the green curve plotted with the semi empirical formula.

![Figure 2: Binding energy of isotopes of Z=50](image-url)

Table 4: Binding energy of isotopes of Z=50

| Proton number | Mass number | Estd. BE (MeV) | Total BE (MeV) | Error (MeV) |
|---------------|-------------|---------------|----------------|-------------|
| 50            | 100         | 822.4         | 826.0          | 3.6         |
| 50            | 101         | 833.9         | 837.2          | 3.3         |
| 50            | 102         | 845.2         | 850.7          | 5.4         |
| 50            | 103         | 856.4         | 860.7          | 4.3         |
| 50            | 104         | 867.3         | 873.1          | 5.8         |
| 50            | 105         | 878.1         | 882.7          | 4.6         |
| 50            | 106         | 888.8         | 894.6          | 5.8         |
| 50            | 107         | 899.2         | 903.5          | 4.3         |
| 50            | 108         | 909.5         | 914.9          | 5.5         |
| 50            | 109         | 919.6         | 923.5          | 3.9         |
50 110 929.5 934.7 5.2
50 111 939.3 942.9 3.6
50 112 948.9 953.5 4.6
50 113 958.3 961.1 2.8
50 114 967.5 971.4 3.9
50 115 976.6 978.7 2.2
50 116 985.5 988.5 3.0
50 117 994.2 995.4 1.3
50 118 1002.7 1004.7 2.0
50 119 1011.1 1011.3 0.2
50 120 1019.3 1020.3 1.1
50 121 1027.3 1026.8 −0.5
50 122 1035.1 1035.5 0.4
50 123 1042.8 1041.5 −1.3
50 124 1050.3 1050.1 −0.2
50 125 1057.6 1055.8 −1.8
50 126 1064.8 1064.1 −0.7
50 127 1071.8 1069.6 −2.2
50 128 1078.6 1077.5 −1.0
50 129 1085.2 1082.8 −2.4
50 130 1091.7 1090.5 −1.2
50 131 1098.0 1095.61 −2.3
50 132 1104.1 1102.6 −1.5
50 133 1110.0 1105.2 −4.8
50 134 1115.8 1109.5 −6.2
50 135 1121.4 1111.4 −9.9
50 136 1126.8 1115.2 −11.6
50 137 1132.0 1116.9 −15.1
50 138 1137.1 1120.5 −16.6
50 139 1142.0 1121.9 −20.1
50 140 1146.7 1125.3 −21.4

See Table 5 for the estimated and total binding energies of $A=2Z$ nuclides starting from $Z=20$ to 50.

**Table 5: Binding energy of $A=2Z$ nuclides**

| Proton number | Mass number | Est. BE (MeV) | Exp. BE(MeV) [40,42] | Error (MeV) |
|---------------|-------------|---------------|----------------------|-------------|
| 20            | 40          | 344.6         | 342.1                | −2.6        |
| 22            | 44          | 380.8         | 375.5                | −5.3        |
| 24            | 48          | 415.3         | 411.5                | −3.8        |
| 26            | 52          | 450.7         | 447.7                | −3.0        |
| 28            | 56          | 484.2         | 484.0                | −0.3        |
| 30            | 60          | 517.3         | 515.0                | −2.3        |
| 32            | 64          | 551.6         | 546.0                | −5.6        |

Figure 3: Binding energy of isotopes of $Z=22$

Figure 4: Binding energy of isotopes of $Z=32$

Figure 5: Binding energy of isotopes of $Z=42$
10. Understanding the Binding Energy of Light Atomic Nuclides

It is well established that, in light atomic nuclides, coulombic interaction seems to play a key role in reducing the binding energy. Based on this concept, starting from \( Z = 2 \) to \( Z = 30 \), close to stable mass numbers, binding energy can be expressed by the following relations.

\[
\begin{align*}
B_A & \approx - \left( A_1 - A_2 \right) (10.06 - 0.71) \text{ MeV} \\
& \approx \left( A_1 - A_2 \right) 9.35 \text{ MeV}
\end{align*}
\]  

(22)

See the following Table 6.

Table 6: Binding energy of \( Z = 2 \) to 30 based on coulombic correction

| Proton number | Mass number | Est. BE (MeV) | SEMF BE (MeV) [38] | Error (MeV) |
|---------------|-------------|---------------|---------------------|-------------|
| 2             | 4           | 22.6          | 22.0                | -0.5        |
| 3             | 6           | 39.1          | 26.9                | -12.2       |
| 4             | 8           | 56.1          | 52.9                | -3.2        |
| 5             | 10          | 73.4          | 62.3                | -11.1       |
| 6             | 12          | 90.8          | 87.4                | -3.4        |
| 7             | 14          | 108.4         | 98.8                | -9.6        |
| 8             | 16          | 126.0         | 123.2               | -2.8        |
| 9             | 19          | 152.7         | 148.9               | -3.8        |
| 10            | 21          | 170.6         | 167.5               | -3.0        |
| 11            | 23          | 188.5         | 186.1               | -2.3        |
| 12            | 25          | 206.4         | 204.7               | -1.7        |
| 13            | 27          | 224.4         | 223.2               | -1.2        |
| 14            | 29          | 242.4         | 241.6               | -0.8        |
| 15            | 31          | 260.5         | 260.0               | -0.5        |
| 16            | 34          | 287.6         | 290.8               | 3.2         |
| 17            | 36          | 305.7         | 305.1               | -0.7        |
| 18            | 38          | 323.9         | 327.2               | 3.4         |
11. Understanding Magic Proton Numbers

It may be noted that, the nuclear magic numbers, as we know in stable and naturally occurring nuclei, consist of two different series of numbers. The first series $-2, 8, 20$ is attributed to the harmonic-oscillator (HO) potential, while the second one $-28, 50, 82$ and $126$ is due to the spin–orbit (SO) coupling force [43-46]. In this context, our bold idea is that, atoms are exceptionally stable when their nuclear binding energy approaches,

$$B_n \approx \left[2.531 \left(n + \frac{1}{2}\right)^2\right] 10.06 \text{ MeV}$$

Based on point 5 of section-3, close to stable mass numbers of $Z \approx (2$ to $100)$, magnitude of nuclear binding energy can be expressed by a relation of following form.

$$B_n \approx \left[Z - \sqrt{\ln(Z)} \frac{e^2}{4\pi\varepsilon_0 \left(G_m p^2/c^3\right)} \right] \pm 10.06 \text{ MeV}$$

$$\approx \left[Z - \sqrt{\ln(Z)} \ast 20.12 \text{ MeV}\right] \pm 10.06 \text{ MeV}$$

where $A_i \approx 2Z + 0.0063326(Z)^2$

$$B_n \approx \left[2.531 \left(n + \frac{1}{2}\right)^2\right] 10.06 \text{ MeV}$$

Let $M_n$ be a possible magic proton number. Considering relations (23) and (24), it is possible to develop a relation of the following form having a factor $(1/2)$.

$$M_n \approx \left[\frac{1}{2} \left[2.531 \left(n + \frac{1}{2}\right)^2 + 1\right] + \Delta\right]$$

$$\approx \left[3.203 \left(n + \frac{1}{2}\right)^2 + 1\right] + \Delta$$

if, $\left[\frac{1}{2} \left[2.531 \left(n + \frac{1}{2}\right)^2 + 1\right]\right]$ is Odd, $\Delta = \pm 1$

if, $\left[\frac{1}{2} \left[2.531 \left(n + \frac{1}{2}\right)^2 + 1\right]\right]$ is Even, $\Delta = \pm 2$

See the following Table 7. It is possible to say that,

1) Magic proton numbers $2, (6), (14), 28, 50, 82, 114...$ etc [44-46] can be shown to be $n^\text{th}$ levels.

2) Magic proton numbers $2, 8, 20, 40...$ can be shown to be $\left(n + \frac{1}{2}\right)$ levels.

Table 7: To understand the magic proton numbers

| $n + \frac{1}{2}$ | Round off | $3.203 \left(n + \frac{1}{2}\right)^2 + 1$ | $M_n$ |
|------------------|-----------|------------------------------------------|-------|
| 0                | 1         | 1.2                                      | 1.2   |
| 0.5              | 2         | 2.4                                      | 2.4   |
| 1                | 4         | 2.46                                     | 2.46  |
| 1.5              | 8         | 6.8,10                                   | 6.8,10|
| 2                | 14        | 12.14,16                                 | 12.14,16|
| 2.5              | 21        | 20.21,22                                 | 20.21,22|
| 3                | 30        | 28.30,32                                 | 28.30,32|
| 3.5              | 40        | 38.40,42                                 | 38.40,42|
| 4                | 52        | 50,52,54                                 | 50,52,54|
| 4.5              | 66        | 64,66,68                                 | 64,66,68|
| 5                | 81        | 80,81,82                                 | 80,81,82|
| 5.5              | 98        | 96,98,100                                | 96,98,100|
| 6                | 116       | 114,116,118                              | 114,116,118|
| 6.5              | 136       | 134,136,138                              | 134,136,138|
| 7                | 158       | 156,158,160                              | 156,158,160|
| 7.5              | 181       | 180,181,182                              | 180,181,182|
| 8                | 206       | 204,206,208                              | 204,206,208|

See the following Figure 11 for the plotted (dotted) black curve compared with SEMF green curve.

Figure 11: Nuclear Binding energy close to stable mass numbers of $Z = 2$ to $100$
12. Discussion

11) With reference to the proposed characteristic mass unit of $\sqrt{\hbar c/G_\alpha} \approx 546.62$ MeV/$c^2$, basic baryonic mass spectrum can be fitted with the following relation,

$$m_{\rho c}c^2 \approx \left( \frac{n}{\alpha_r} \right)^{\frac{1}{2}} \left( \frac{\hbar c^2}{G_\alpha} \right) \approx \left( \frac{n}{\alpha_r} \right)^{\frac{1}{2}} 546.6\text{MeV} \quad (26)$$

where $n = 1, 2, 3, \ldots$

See Table 8. For further details, readers are encouraged to see our published paper [33].

Table 8: Estimated basic baryons rest energy

| $n$ | Baryon rest energy (MeV) | $n$ | Baryon rest energy (MeV) |
|-----|--------------------------|-----|--------------------------|
| 1   | 938.3                    | 11  | 1708.7                   |
| 2   | 1115.8                   | 12  | 1746.3                   |
| 3   | 1234.8                   | 13  | 1781.6                   |
| 4   | 1326.9                   | 14  | 1814.9                   |
| 5   | 1403.0                   | 15  | 1846.5                   |
| 6   | 1468.5                   | 16  | 1876.5                   |
| 7   | 1526.1                   | 17  | 1905.2                   |
| 8   | 1578.0                   | 18  | 1932.6                   |
| 9   | 1625.1                   | 19  | 1958.9                   |
| 10  | 1668.5                   | 20  | 1984.2                   |

2) So far no model could succeed in understanding nuclear binding energy with gravity [19]. It can be confirmed from main stream literature [1-20].

3) So far no model could address or succeed in implementing strong coupling constant in low energy nuclear physics.

4) So far no model could attempt to understand nuclear stability and binding energy with the combined effects of strong nuclear gravity and strong nuclear charge.

5) Understanding nuclear binding energy with a single energy coefficient of magnitude $8 \pi \varepsilon_0 (G m_p / c^2) \approx 10.09$ MeV is a challenging task and so far, except Ghahramany et al, no one could attempt to do that. It may also be noted that, in Ghahramany’s model, energy constant is a variable [47] and in our model energy constant remains same for any nuclide.

6) Estimation of nucleon stability range is simple in our model compared to SEMF and Ghahramany’s model. Interesting point to be noted is that, in our model, nucleon stability range or stable mass numbers can be estimated without considering the binding energy formula. We have provided different relations for understanding nucleon stability.

7) Proposed new and result oriented number $k \approx \left( \frac{4 \pi \varepsilon_0 \hbar m_e c^2}{4 e^2 G m_p} \right) \approx 0.0063326$ seems to play a key role in understanding nuclear stability and binding energy vide relations (6), (7), (8), (9), (10), (16) and (20).

8) Proposed first term is not new and proposed second term $\left[(kA/Z/2.531) + 3.531\times10.06 \text{MeV} \right]$ seems to play an excellent role in fitting and understanding the binding energy of medium and heavy stable nuclides. It can be evidenced form Table 3. Correction seems to be required for light atomic nuclides. It needs further study.

9) Proposed third term $\left[(A - A' \sqrt{2}) / A' \right] \times 10.06 \text{MeV}$ seems to be approximate in fitting and understanding the binding energy of isotopes. We are working on it for its validity and better alternative with respect correct stable mass number of $Z$. For example, see the following Table 9.

Table 9: Binding energy of isotopes of $Z = 8, 10$ and 20

| Proton number | Mass number | Est. BE (MeV) | Total BE (MeV) | Error (MeV) |
|---------------|-------------|---------------|----------------|-------------|
| 8             | 14          | 100.0         | 98.7352        | -1.25       |
| 8             | 15          | 111.7         | 111.9576       | 0.23        |
| 8             | 16          | 122.2         | 127.6211       | 5.40        |
| 8             | 17          | 131.4         | 131.7646       | 0.32        |
| 8             | 18          | 139.4         | 139.8091       | 0.39        |
| 8             | 19          | 146.1         | 143.7665       | -2.37       |
| 10            | 17          | 123.6         | 112.9107       | -10.64      |
| 10            | 18          | 136.7         | 132.1432       | -4.57       |
| 10            | 19          | 148.9         | 143.7827       | -5.14       |
| 10            | 20          | 160.2         | 160.6521       | 0.49        |
| 10            | 21          | 170.5         | 167.4136       | -3.04       |
| 10            | 22          | 179.8         | 177.7751       | -2.01       |
| 10            | 23          | 188.2         | 182.9756       | -5.18       |
| 10            | 24          | 195.6         | 191.841        | -3.72       |
| 20            | 36          | 297.1         | 281.3644       | -15.69      |
| 20            | 37          | 309.6         | 296.1548       | -13.50      |
| 20            | 38          | 321.8         | 313.1263       | -8.65       |
| 20            | 39          | 333.4         | 326.4138       | -7.03       |
| 20            | 40          | 344.6         | 342.0563       | -2.58       |
| 20            | 41          | 355.4         | 350.4187       | -4.94       |
| 20            | 42          | 365.6         | 361.9002       | -3.72       |
| 20            | 43          | 375.4         | 369.8327       | -5.58       |
10) In deuteron, binding energy seems to be proportional to $e^2$ and in other atomic nuclides, binding energy seems to be proportional to $e_i^2$.

11) Considering the average of $(e^2, e_i^2)$ and without considering 0.71 MeV (as there exists only one proton), based on relation (22), binding energies of $^3H$ and $^1H$ nuclides can be estimated as, $2 - 2^{-1}1.56 \equiv 4.15$ MeV and $3 - 3^{-11}5.6 \equiv 8.72$ MeV respectively.

12) Considering the average of $(e_i^2, e_i^2)$ and considering 0.71 MeV (since there exists two protons), based on relation (22), binding energy of $^3He$ can be estimated as, $3 - 3^{-11}4.9 \equiv 7.63$ MeV.

13) Coulombic energy coefficient being 0.7 MeV, with reference to $\ln \left( \frac{e^2}{4\pi\varepsilon_0 G_m m_p m_\gamma} \right) \equiv 1.515$,

volume or surface energy coefficient can be expressed as $1.515 \times 10.09 = 15.3$ MeV and asymmetric energy coefficient can be expressed as, $1.515 \times 15.3 = 23.0$ MeV. Thus, 10.09 MeV, 15.3 MeV and 23.0 MeV seem to follow a geometric series with a geometric ratio of 1.515. For $Z \geq 10$, binding energy can also be estimated with,

$$B_s \cong (A - A^{221} - 1) \times 15.3\text{MeV} - \frac{Z^2}{A^2} \times 0.7\text{MeV} - \left( \frac{A - 2Z}{A} \right)^2 \times 23.0\text{MeV}$$

(27)

14) With advanced research in high energy nuclear physics, hadronic melting points can be understood and bare quarks can be made identifiable.

15) With further research in nuclear astrophysics, it is certainly possible to understand the combined effects of Newtonian gravitational constant and proposed nuclear gravitational constant. Considering the ratio of nuclear gravitational constant and Newtonian gravitational constant, estimated masses of white dwarfs, neutron stars and black holes [48,49], can be fitted approximately. For example,

$$M_x \approx \left( \frac{G_x}{G_n} \right) \sqrt{\frac{e^2}{4\pi\varepsilon_0 G_n}} \approx 0.473M_\odot$$

$$M_x \approx \left( \frac{G_x}{G_n} \right) \sqrt{\frac{e_i^2}{4\pi\varepsilon_0 G_n}} \approx 1.373M_\odot$$

$$M_x \approx \left( \frac{G_x}{G_n} \right) \frac{\hbar c}{G_N} \approx 5.456M_\odot$$

(28)

$$M_x \approx \left( \frac{G_x}{G_n} \right) \sqrt{\frac{e_i^2}{4\pi\varepsilon_0 G_n m_p}} \approx 0.2M_\odot$$

$$M_x \approx \left( \frac{G_x}{G_n} \right) \frac{\hbar c}{G_N m_\gamma} \approx 3.174M_\odot$$

(29)

16) At the moment of a neutron star's birth, the nucleons that compose it have a temperature of around $10^{15}$ to $10^{13}$ K [50]. Equating a black hole's mass-energy density and thermal energy density [51,52,53], it is possible to show that,

$$T_B \equiv \frac{\hbar c}{k_B G_N M_B M_{pl}}$$

(30)

where, $M_{pl} \equiv \frac{\hbar c}{G_N M_\odot} \approx 2.176 \times 10^4$ kg

$M_\odot$ Mass of blackhole

$T_B \equiv$ Temperature of blackhole

This just resembles famous Hawking's Black hole temperature formula [54] with a change in its effective mass $\sqrt{M_B M_{pl}}$. With reference to relation (29), considering $M_x$ as a critical mass for neutron stars and black holes, corresponding critical temperature can be fitted with,

$$T_x \equiv \frac{\hbar c}{8\pi k_B G_N M_B M_{pl}}$$

(31)

Quantitatively, Fermi's weak coupling constant [55] and electron rest mass can be fitted with the following relations.

$$G_F \approx \left( \frac{m_e}{m_p} \right)^2 \frac{\hbar c}{G_N M_\odot} \approx 4G_F^2 m_e^2 \hbar$$

$$m_e \approx \frac{4G_F^2 m_e^2 \hbar}{\sqrt{4G_N^2 \hbar}}$$

(32)

$$m_e \approx \frac{4G_F^2 m_e^2 \hbar}{\sqrt{4G_N^2 \hbar}}$$

(33)

18) In a theoretical and verifiable approach, magnitude of the Newtonian gravitational constant can be estimated with nuclear elementary physical constants [56, 57].
For example, with reference to Planck scale, we noticed that,
\[
\frac{\pi R_0^2}{\pi R_0^2 c^2} \approx \frac{G_s m_p^2}{G_N \hbar c} \approx \left(\frac{m_p}{m_e}\right)^{12}
\]
(34)
where, \( R_0 \approx \frac{2G_N m_p}{\hbar c} \), \( R_0 \approx \frac{2G_s M_{10}}{c^2} \approx 2 \sqrt{\frac{G_N \hbar}{c^3}} \).

19) Another very interesting relation is,
\[
\left(\frac{m_p}{m_e}\right)^{12} \approx \left(\frac{G_s m_p^2}{G_N \hbar c}ight) G_s
\]
(35)
\[
G_s \approx \left(\frac{G_s m_p^2}{G_N \hbar c}ight) \frac{1}{12} \left(\frac{m_p}{m_e}\right)^{12}
\]
(36)
\[
G_F \approx \left(\frac{G_s m_p^2}{G_N \hbar c}\right) \frac{1}{4} \frac{1}{12} \frac{c^2}{m_p}
\]
(37)
\[
\left(\frac{m_p}{m_e}\right)^{10} \approx \exp\left(\frac{1}{\alpha_s}\right)
\]
(38)
19) Another very interesting relation is,
\[
\left(\frac{m_p}{m_e}\right)^{10} \approx \exp\left(\frac{1}{\alpha_s}\right)
\]
(39)

20) If, \( G_s \approx \frac{4\pi\varepsilon^2 h^2 c^2 m_p^3}{e^2 m_p^3} \approx 3.329561 \times 10^{-28} \text{ m}^3 \text{ kg} \cdot \text{sec}^2 \)

\[
\begin{align*}
\alpha_s & \approx 0.115194 \\
G_s & \approx 1.44021 \times 10^{-62} \text{ J} \cdot \text{m}^3 \\
G_N & \approx 6.679856 \times 10^{11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}
\end{align*}
\]
(40)
21) If \( \alpha_s \approx \ln\left(\left(\frac{m_p}{m_e}\right)^{10}\right) \approx 0.1153515 \)

\[
\begin{align*}
G_s & \approx 3.327283 \times 10^{28} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2} \\
G_F & \approx 1.43824 \times 10^{-62} \text{ J} \cdot \text{m}^3 \\
G_N & \approx 6.670719 \times 10^{11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}
\end{align*}
\]
(41)
22) With reference to the macroscopic Planck’s constant and microscopic strong coupling constant, average values seem to be:

\[
\begin{align*}
\alpha_s & \approx 0.115273 \\
G_s & \approx 3.32842 \times 10^{28} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2} \\
G_F & \approx 1.43922 \times 10^{-62} \text{ J} \cdot \text{m}^3 \\
G_N & \approx 6.675285 \times 10^{11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}
\end{align*}
\]
(42)
23) Relations (34), (35) and (38) seem to indicate the direct role of \( G_s \) in microscopic physics. We are working on understanding their physical significance with respect to proton-electron mass ratio.
24) Our proposed assumptions seem to ease the way of understanding and refining the basic concepts of final unification [58, 59, 60].

**Conclusion**

Liquid drop model, Fermi gas model, quantum chromodynamics and string theory models are lagging in implementing the strong coupling constant and gravity in basic nuclear structure. In this context, understanding and estimating nuclear binding energy with ‘strong interaction’ and ‘unification’ concepts seem to be quite interesting and needs a serious consideration at basic level. Even though they are semi empirical, section (3) and relations (6), (7), (8), (9), (10), (11), (20), (21), (24), (26), (27), (28), (29), (30), (31), (34), (35), (38), (39) and (42) can be considered as favorable or supporting tools for our proposed model. One very interesting point to be noted is that, our proposed model seems to span across the Fermi scale and Planck scale. With further research, mystery of magic numbers can be understood and a unified model of nuclear binding energy and stability scheme pertaining to high and low energy nuclear physics can be developed.

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