Radiative Leptonic $B_c$ Decays in Effective Field Theory

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Abstract

The recent discovery of the $B_c$ meson by the CDF collaboration and proposed new experiments at Fermilab and CERN motivate new theoretical studies of the $B_c$ system. Here we investigate the radiative leptonic decay $B_c \to \gamma l\nu$. This process is an important background to the annihilation process $B_c \to l\nu$, which will be used to extract the $B_c$ decay constant. We perform a model-independent calculation, based on QCD, of the partial width and various kinematic distributions. We also examine the decay within the framework of NRQCD, an effective field theory of nonrelativistic quarks, generalizing the NRQCD Lagrangian to include external sources for the weak and electromagnetic currents. Finally, we will show how NRQCD reproduces the correct position of the $B_c^*$ pole in the emission of very soft photons.
I. INTRODUCTION

The $B_c$ is probably the final long-lived pseudoscalar meson which will be found in our lifetimes. Its recent discovery at CDF [1], and the future prospect of thorough experimental studies at LHC and BTeV, motivates extensive theoretical studies of this system [2]. In particular, the $B_c$ offers a unique possibility to study the effects of weak interactions in a quarkonium-like environment. Since it is composed of heavy quarks of two different flavors, the $B_c$ is stable against strong annihilation decays and its dynamics approaches a simple perturbative limit as $m_c,m_b \to \infty$. In this limit, the $B_c$ is a very compact bound state of a $c$ and $\bar{b}$ quark, with a small admixture of non-perturbative higher Fock states containing gauge bosons and light quark-antiquark pairs. This admixture is small because soft nonperturbative gluons, with large Compton wavelengths, have little overlap with the compact $B_c$ state. It is useful to study the $B_c$ system, like quarkonium, in the framework of an effective nonrelativistic quantum field theory [3].

The advantages of an effective field theory approach are both conceptual and quantitative. While one would intuit that the $m_c,m_b \to \infty$ limit is one in which a description based on a constituent quark model should work reasonably well, an effective field theory based on the operator product expansion puts this intuition on a rigorous basis. It is unsurprising, of course, that the results we obtain in this paper are similar in structure to those of the quark model, since the quark model does respect the symmetries of the heavy quark limit. What an effective field theory does, that the quark model does not, is provide an organized expansion within which the leading corrections to this limit may be accommodated. While we will not compute higher order corrections in this paper, we will use the power counting of the effective field theory to estimate their size, thereby casting light on the accuracy of our results.

One basic characteristic of the $B_c$ meson is its decay constant, defined by

$$\langle 0 | \bar{b} \gamma^\mu \gamma^5 c | B_c(p) \rangle = i f_{B_c} p^\mu .$$

This is a QCD definition, although in quark models and in the nonrelativistic limit $f_{B_c}$ may be identified with the value of the wave function at the origin. The decay constant probes the strong QCD dynamics which is responsible for the binding of the quark-antiquark state. The most straightforward way to determine $f_{B_c}$ would be to measure the purely leptonic partial widths, such as $B_c \to l \nu_l$, where $l = e, \mu, \tau$ [4]. The rate for this process is given by

$$\Gamma(B_c \to l \nu) = \frac{G_F^2 |V_{cb}|^2 f_{B_c}^2 m_{B_c}^3}{8\pi^2 m_{B_c}^3} \frac{m_l^2}{m_{B_c}^2} \left(1 - \frac{m_l^2}{m_{B_c}^2}\right)^2 .$$

However, the practical usefulness of this method is limited, because the decaying meson is spinless and this mode is helicity suppressed. A helicity flip on an external lepton line is required, leading to a suppression of the rate by an additional factor $m_l^2/m_{B_c}^2$. This suppression is $6 \times 10^{-8}$ for $B_c \to e \nu_e$, and $3 \times 10^{-4}$ for $B_c \to \mu \nu_\mu$. The only leptonic mode with a substantial branching fraction is $B_c \to \tau \nu_\tau$, but this is difficult to observe because the $\tau$ must be reconstructed from its decay products.

The helicity suppression can be overcome if there is third particle in the final state. In particular, adding a photon does not change the fact that the decay rate is proportional
to the decay constant. A naive estimate suggests that for \( l = \mu \) the additional electromagnetic coupling is effectively compensated by the lifting of the helicity suppression, since \((\alpha/4\pi)/(m_{B_c}^2/m_\mu^2) \sim 2\). For \( l = e \), the same counting suggests that the radiative leptonic mode dominates the purely leptonic decay. If branching ratios as small as that for \( B_c \to \mu\nu_\mu \) are eventually measured, then there are two consequences. First, \( B_c \to \gamma\mu\nu_\mu \) will be an important background (or tool) for the extraction of \( f_{B_c} \). Second, \( B_c \to \gamma e\nu_e \) will be observable even though \( B_c \to e\nu_e \) is not. For both of these reasons, it is important to understand the radiative decay process.

Radiative leptonic \( B_c \) decays already have been studied using quark potential models \(^5\) and QCD sum rules \(^6\). We will comment on these approaches later and relate them to our own, which will be based on QCD and its nonrelativistic expansion, NRQCD. The corresponding decay for systems with one heavy and one light quark, \( B \to \gamma l\nu_l \), also has been examined \(^7\), employing insofar as possible a heavy fermion expansion for the \( b \) quark. Unfortunately, that decay is dominated by photons radiated from the \( u \), the effect of which is impossible to compute model-independently. The advantage of the \( B_c \) system is that it can be treated systematically in an expansion in \( v \), the nonrelativistic three-velocity of the \( \bar{b} \) and the \( c \) in their mutual bound state. The expansion may be used to justify the application of perturbative QCD to this decay \(^8\). In addition, we note that, in contrast to \( B \to l\nu, \gamma l\nu \), the decays \( B_c \to \nu_l, \gamma l\nu \) are not CKM-suppressed.

The value of NRQCD is that it provides a rigorous counting of powers of \( v \), which may be applied to expectation values of heavy quark operators in external states \( Q\bar{Q} \) \(^3\) where \( m_Q, m_Q' \gg \Lambda_{QCD} \). This power counting is related to the heavy quark effective theory (HQET) derivative expansion in powers of \( D_\mu/m_Q \), although it differs in some details. What NRQCD and HQET share is a “spin symmetry”, by which the magnetic interactions of the heavy quarks decouple as \( 1/m_Q \). In general, the power of \( v \) which NRQCD assigns to an operator depends on the matrix element which is being taken. Therefore the NRQCD expansion must be performed on the matrix elements, not just on the operators. By contrast, the HQET power counting is identical for all bound states to which it is applied, which are of the form \( Q\bar{q} \), where \( m_q \lesssim \Lambda_{QCD} \). Although the HQET expansion is technically also an expansion of matrix elements, there is therefore no confusion in thinking of it equally as applying to the operators themselves.

In fact, we will be able to treat our application of NRQCD to the \( B_c \) system as an operator expansion as well, because of a number of simplifications which apply to the process under consideration. First, we will work only to leading order in the NRQCD expansion, with no dependence on the \( |\bar{b}cg\rangle \) higher Fock components for which the power counting in \( v \) can be subtle (and, of course, interesting). Second, the only external states which will be important are the \( B_c \) and the \( B_c^* \), both of which are dominated by the same spatial configuration of the \( \bar{b}c \) pair. Therefore, the relevant leading matrix elements are related by heavy quark spin symmetry. The only matrix element which will appear is the one which defines \( f_{B_c} \) in NRQCD, namely

\[
im B_c f_{B_c} = \langle 0 | \chi_b^\dagger \psi_c | B_c \rangle, \tag{1.3}\]

where \( \chi_b \) and \( \psi_c \) are the two-component NRQCD field operators. Third, the final state consists of no particles which are strongly interacting. As a result, the operator product
expansion, which NRQCD helps to organize in the case of hadronic decays, is here a trivial affair.

We will first study the decay $B_c \rightarrow l\nu\gamma$ in perturbative QCD, which is fairly simple since there are no infrared divergences or other subtleties. We will then match the QCD result onto NRQCD and identify the leading operator which contributes to the annihilation of the $B_c$. The expansion turns out to be straightforward, but it is useful to see how the parton model result emerges in the nonrelativistic limit. Finally, we will show how NRQCD reproduces the correct position of the $B_c^*$ pole in the emission of very soft photons.

II. THE DECAY IN PERTURBATIVE QCD

We begin by calculating the rate for $B_c \rightarrow l\nu\gamma$ perturbatively in full QCD. Since there are no strongly interacting particles in the final state, the QCD operator product expansion is trivial. The matrix element for $B_c \rightarrow l\nu\gamma$ is governed by the matrix element for $\bar{b}c \rightarrow l\nu\gamma$, followed by the projection of $\bar{b}c$ onto the $B_c$ state. At leading order, we take the quarks to be in the (leading) $^1S_0$ Fock configuration.

The computation is simple, involving the set of diagrams presented in Fig. 1. The decay amplitude can be written as

$$A(\bar{b}c[^1S_0] \rightarrow l\nu\gamma) = \frac{G_F}{\sqrt{2}}\epsilon_{\nu}(p_{\nu})\gamma_{\mu}(1 - \gamma_5)v(p_l)\langle 0|\bar{b}\gamma_{\alpha}(1 - \gamma_5)c|\bar{b}c\rangle$$

$$\times \left\{ \left[ \frac{eQ_c p_\nu{\nu}}{p_c k} - \frac{eQ_b p_\nu{\nu}}{p_b k} + \frac{eQ_l p_\nu{\nu}}{p_l k} \right] g^{\mu\alpha} - \left[ \frac{eQ_c}{p_c k}\Gamma^{\mu\nu\alpha} - \frac{eQ_b}{p_b k}\Gamma^{\nu\mu\alpha} - \frac{eQ_l}{p_l k}\Gamma^{\alpha\nu\mu} \right] \right\},$$

where

$$\Gamma^{\mu\nu\alpha} = k^\mu g^{\nu\alpha} + k^\nu g^{\mu\alpha} - k^\alpha g^{\mu\nu} - i\epsilon^{\mu\nu\alpha\beta}k_\beta.$$  

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Here $e^\mu$ and $k^\mu$ are the polarization and momentum of the photon. We now anticipate the results of the next section, in which we will verify that the current $\bar{b}\gamma_\alpha(1 - \gamma_5)c$ is the leading contribution to the decay, to justify the evaluation of the $B_c$ matrix element $(1.1)$. Defining the leptonic matrix element $l_\mu = \bar{u}\gamma_\mu(1 - \gamma_5)v$, we find

$$A(B_c \to l\nu\gamma) = -iV_{cb}\frac{G_F}{\sqrt{2}}f_{B_c}m_{B_c}l_\mu\epsilon_\nu\left\{\frac{eQ_c}{p_\ell k}v^\mu p_\ell^\nu + \left[\frac{eQ_c\beta^\nu}{p_\ell k} - \frac{eQ_b\beta^\nu}{p_b k}\right]v^\mu\right\}
$$

$$+ B_1(kv g^{\mu\nu} - k^\mu v^\nu) - B_2 i\epsilon^{\mu\nu\beta\gamma}v_\alpha k_\beta,$$

where $v^\mu$ is the four-velocity of the $B_c$, $B_1 = eQ_c/2p_\ell k - eQ_b/2p_b k + eQ_\ell/2p_\ell k$, and $B_2 = eQ_c/2p_\ell k + eQ_b/2p_b k + eQ_\ell/2p_\ell k$. This expression simplifies somewhat if we choose the second gauge condition $\epsilon \cdot v = 0$, which we can do since the photon is on shell.

This is the leading result in the nonrelativistic expansion. It differs from existing calculations in two respects [33]. First, it is model-independent. Second, radiation from the lepton leg is included, which is required for gauge invariance of the amplitude. At this point, it is straightforward to derive the total rate and the photon and lepton energy distributions. Clearly, they depend only on the one parameter $f_{B_c}$. Since the same parameter enters the expression for the rate of $B_c \to l\nu$, we will normalize the rate to that for $B_c \to l\nu$, and keep the dependence on the fermion charges explicit so that the various contributions can be examined. Summing over the photon and lepton polarizations and integrating over the phase space, we find the decay rate

$$\frac{\Gamma(B_c \to l\nu\gamma)}{\Gamma(B_c \to l\nu)} = \left[\frac{\alpha}{4\pi}\frac{m_{B_c}^2}{m_l^2}\right]m_{B_c}^2\left[\frac{1}{9}\frac{Q_b^2}{m_b^2} + \frac{1}{9}\frac{Q_c^2}{m_c^2} - \frac{2}{9}\frac{Q_b Q_\ell}{m_b m_{B_c}} + \frac{2}{9}\frac{Q_c Q_\ell}{m_c m_{B_c}} + \frac{2}{9}\frac{Q_\ell^2}{m_{B_c}^2}\right].$$

We see that there is no interference between the photon emitted from the charm and bottom legs. This is a consequence of the anticorrelation of the spins of the two quarks in the pseudoscalar $B_c$. The dimensionless coefficient $r_\ell = (\alpha/\pi)(m_{B_c}^2/m_l^2)^2$ is $r_\mu = 2.1$ for $l = \mu$ and $r_e = 8.8 \times 10^5$ for $l = e$. Taking $m_b = 4.8$ GeV, $m_c = 1.5$ GeV and $m_{B_c} = 6.3$ GeV, we find

$$\frac{\Gamma(B_c \to l\nu\gamma)}{\Gamma(B_c \to l\nu)} = r_\ell \left(0.19 Q_b^2 + 1.96 Q_c^2 - 0.29 Q_b Q_\ell + 0.93 Q_c Q_\ell + 0.22 Q_\ell^2\right)$$

$$= r_\ell \left(0.02 + 0.87 - 0.10 - 0.62 + 0.22\right)$$

$$= 0.40 r_\ell.$$  

Hence the rate for $B_c \to \mu\nu\gamma$ is approximately 80% of that for $B_c \to \mu\nu$, while $B_c \to e\nu\gamma$ dominates $B_c \to e\nu$. The doubly differential spectrum in $x = 2E_\nu/m_{B_c}$ and $y = 2E_/m_{B_c}$ is

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx dy} = 18(1 - x)x^{-2}\left(\mu_b^2(1 - 6\mu_b) + 2\mu_b^2(2 - 6\mu_c + 9\mu_c^2)\right)^{-1}\left[\mu_c^2(1 - 6\mu_b)(1 - y)\right]^2$$

$$+ 4\mu_b^2(1 - 3\mu_c)(1 - x - y)^2 + 9\mu_b^2\mu_c^2(2 - 2x + x^2 - 4y + 2xy + 2y^2),$$

where $\mu_b = m_b/m_{B_c}$ and $\mu_c = m_c/m_{B_c}$. The normalized photon energy spectrum follows the simple shape
FIG. 2. Photon energy spectrum in $B_c \rightarrow l\nu\gamma$ decay.

The photon energy spectrum is shown in Fig. 2. Note that there is no soft divergence in the limit $E_\gamma \rightarrow 0$. This is because the helicity suppression for $B_c$ decay can only be lifted by recoil effects or a heavy quark spin flip from a magnetic transition, both of which vanish with $E_\gamma$. The lepton energy spectrum, whose analytic form is

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} = 6x(1-x). \quad (2.7)$$

is shown in Fig. 3. Unlike the photon energy spectrum, the lepton energy spectrum depends on the mass ratios $\mu_c$ and $\mu_b$. Finally, we compute the branching ratio for $B_c \rightarrow l\nu\gamma$.

Taking for the $B_c$ lifetime the CDF central value $\tau(B_c) = 0.46$ ps \[1\], and using the estimate $f_{B_c} = 375$ MeV \[4\], we obtain

$$\mathcal{B}(B_c \rightarrow e\nu\gamma) \simeq \mathcal{B}(B_c \rightarrow \mu\nu\gamma) \simeq 4.4 \times 10^{-5}. \quad (2.9)$$

It is certainly a challenge to observe such rare processes, even with the large $B_c$ samples which one hopes will be available at future experiments such as BTeV and LHC.

This result is similar to what one would find using a constituent quark model. However, in a number of respects it goes beyond the quark model framework. First, the result holds even when the photon is hard and the virtual $c$ or $b$ quark is far from its mass shell. Second, the it is rigorously true in the limit $m_c, m_b \rightarrow \infty$. Third, it is possible to improve systematically the accuracy of this result by including both QCD radiative corrections and higher dimension
operators in NRQCD. The QCD calculation provides insight into both the success and the limitations of the quark model picture.

Note that for very soft photons, the shape of the spectrum is dominated by the nearest $\bar{b}c$ resonance, the $B_c^*$, and the parton-hadron duality which underlies the operator product expansion breaks down. (Since the splitting between the $B_c$ and the $B_c^*$ is caused by a hyperfine interaction, it is much smaller than the splittings between the $B_c$ and all other $B_c^{**}$ excitations.) This corner of phase space will be examined in more detail in Section IV. At larger photon energies, all virtual $B_c^{**}$ states contribute equally to the shape, but the integration over the momenta of the leptons smears the effect of the resonances into a smooth result. By the usual application of global parton-hadron duality, the smeared spectrum should be reproduced by perturbative QCD (supplemented by a hierarchy of nonperturbative contributions from operators of higher dimension). Note that the kinematics forbids the production of on-shell $B_c^{**}$ intermediate states. Nor do on-shell $\bar{c}c$ bound states contribute to this exclusive process via the diagram in Fig. [Ia, since the physical photon has $k^2 = 0$.

III. THE NRQCD EXPANSION FOR THE $B_C$

We now turn to the nonrelativistic limit of the QCD answer, matching onto a tower of NRQCD operators. The purpose is to identify the leading contribution to the $B_c$ matrix elements which contribute to $B_c \rightarrow \gamma l\nu$. (The result, which turns out to be simple, was already anticipated in the previous section.) We will write our expansion in terms of effective fields $\Psi(x)$, which are related to the usual QCD fields $Q(x)$ by a Foldy-Wouthuysen transformation,

$$Q(x) = \exp\{i\not{\mathbf{D}}_\perp/2m_Q\} \times \Psi(x) = \exp\{i\not{\mathbf{D}} \cdot \not{\gamma}/2m_Q\} \times \Psi(x) , \quad (3.1)$$

FIG. 3. Lepton energy spectrum in $B_c \rightarrow l\nu\gamma$ decay.
where $D_\mu^\perp = D^\mu - v^\mu v \cdot D$. In the rest frame of the $B_c$ and in the Dirac representation, $\Psi$ may be decomposed as

$$\Psi = \begin{pmatrix} \psi \\ \chi \end{pmatrix},$$

(3.2)

where $\psi$ and $\chi$ are the two-component quark and antiquark fields, which the Foldy-Wouthuysen transformation disentangles to order $1/m_Q$. The field $\psi$ annihilates quark states, while $\chi$ creates antiquark states. Note that the projection operators $P_\pm = (1 \pm \gamma^5)/2$, where $v^\mu = (1, 0, 0, 0)$ is the four-velocity of the $B_c$, project out separately the $\psi$ and $\chi$ parts of $\Psi$. It is often convenient to use this covariant (and representation-independent) form of the projection.

The transformation (3.1) can be used to rewrite QCD operators in terms of the effective fields $\Psi$. This is one step in the matching of QCD onto NRQCD. For example consider the expansion of the weak current $\bar{b} \Gamma_\mu c = \bar{b} \gamma_\mu (1 - \gamma^5) c$ to order $1/m_{c,b}$. In the four-component notation, we find

$$\bar{b} \gamma_\mu (1 - \gamma^5) c \rightarrow -g_\mu^0 \left[ \chi^\dagger_b \psi^c + \frac{1}{2m_c} \chi^\dagger_b \frac{e^{iD^j\psi^c}}{2} \right] + \frac{1}{2m_b} \chi^\dagger_b \sigma^i \psi^c \rightarrow -g_\mu^j \left[ \chi^\dagger_b \sigma^j \psi^c + \frac{1}{2m_c} \chi^\dagger_b \sigma^j iD^j\psi^c \right] - \frac{1}{2m_b} i\epsilon^{jkl} \chi^\dagger_b \sigma^k \psi^c + \frac{1}{2m_c} i\epsilon^{jkl} \chi^\dagger_b D^k \sigma^i \psi^c,$$

(3.4)

which is somewhat more cumbersome but also more explicit. These leading terms in the expansion in $1/m_{b,c}$ are also the leading terms in the velocity expansion; the first omitted terms are of order $v^2$. A more complete calculation would include radiative corrections as well, in which case the various operators which appear on the right hand sides of Eqs. (3.3) and (3.4) would develop a dependence on the renormalization scale. Note that the covariant derivative $D_\mu$ includes both gluon and photon gauge fields.

The effective fields $\Psi$ realize explicitly the heavy quark spin symmetry which emerges as the magnetic interactions of $Q$ decouple in the limit $m_Q \to \infty$. An immediate implication of this symmetry in NRQCD is that the $B_c$ and the $B_c^*$ are degenerate. Another is the equality of their decay constants,

$$im_{B_c} f_{B_c} = \langle 0 | \chi^\dagger_b \psi^c | B_c \rangle = \eta^i \langle 0 | \chi^\dagger_b \sigma^i \psi^c | B_c^* \rangle + O(v^2).$$

(3.5)

A particularly nice realization of this symmetry is available in terms of the four-component effective fields. One can assemble the $B_c$ and the $B_c^*$ into a single “superfield” $H_{B_c}$

$$H_{B_c} = \frac{1 + \gamma^5}{2} \left[ B_c^{*\mu} \gamma_\mu - B_c \gamma^5 \right].$$

(3.6)
Matrix elements which conserve the spin symmetry then take the form
\[
\langle 0| \bar{b} \Gamma c B_c^s \rangle = \frac{i}{2} f_{B_c} m_{B_c} \text{Tr} [\Gamma H_{B_c}],
\]
\[
\langle B_c^s| \bar{c} \Gamma c B_c^s \rangle = -m_{B_c} \text{Tr} [H_{B_c} \Gamma H_{B_c}],
\]
\[
\langle B_c^s| \bar{b} \Gamma b B_c^s \rangle = m_{B_c} \text{Tr} [H_{B_c} \Gamma H_{B_c}].
\]
(3.7)

The leading contribution to the mass difference between the $B_c$ and the $B_c^*$ comes from the chromomagnetic operator
\[
O_2 = \frac{g_s}{4m_c} \bar{c} \sigma^{\alpha \beta} G_{\alpha \beta} c + \frac{g_s}{4m_b} \bar{b} \sigma^{\alpha \beta} G_{\alpha \beta} b.
\]
(3.8)

The mass difference $\Delta = m_{B_c^*} - m_{B_c}$ may be written in terms of the matrix elements of $O_2$,
\[
\Delta = \langle B_c^*|O_2|B_c^*\rangle - \langle B_c|O_2|B_c\rangle.
\]
(3.9)

This relation will be useful later, when we consider the effect of the $B_c^*$ pole.

A. NRQCD calculation and matching

Due to the simplicity of the final state, we can match the currents of QCD directly onto NRQCD, rather than the decay rates. For example, Lorentz invariance restricts the form of the NRQCD current which annihilates the $B_c$ to
\[
\langle 0|j^\mu|B_c\rangle = -g^{\mu 0} \left\{ C_1 \langle 0|\bar{\chi}_b \gamma_\mu \psi_c|B_c\rangle + \frac{C_2}{m_{red}^2} \langle 0|\bar{\chi}_b \tilde{\nabla}^2 \psi_c|B_c\rangle + \ldots \right\},
\]
(3.10)
where $m_{red} = m_b m_c / (m_b + m_c)$ is the reduced mass of the $\bar{b} c$ pair. At leading order, this reduces to the matrix element of a single universal NRQCD operator which can be either fixed by other experimental measurements or related to the $B_c$ wave function at the origin [10].

We would like to generalize the effective Lagrangian of NRQCD to include the effects of the weak interactions. This can be achieved by introducing external sources. The effective NRQCD Lagrangian can be separated into two sectors representing flavor conserving ($L_{FCon}$) and flavor changing ($L_{FCh}$) interactions, as well as a source part $L_S$,
\[
L_{NRQCD} = L_{FCon} + L_{FCh} + L_S.
\]
(3.11)

The sources represent external perturbatively interacting fields. In addition to the electromagnetic field, in this calculation it is convenient to treat the leptonic current as an external source field responsible for the flavor changing interactions. Each part of this Lagrangian can be organized by the velocity expansion. At leading order, the flavor conserving part can be written
\[
L_{FCon}^L = \chi_\alpha^\dagger \left( i D_\mu - \tilde{\nabla}^2 / 2m_\alpha \right) \chi_\alpha + \psi_\alpha^\dagger \left( i D_\mu + \tilde{\nabla}^2 / 2m_\alpha \right) \psi_\alpha
+ \frac{\alpha_s^2}{2m_\alpha} \left( \psi_\alpha^\dagger \bar{\sigma} \cdot \bar{\nabla}_2 \psi_\alpha - \chi_\alpha^\dagger \bar{\sigma} \cdot \bar{\nabla}_2 \chi_\alpha \right),
\]
(3.12)
where the covariant derivatives include external scalar $s$, $D_t = \partial_t + a_0^a s$ and vector $\vec{v}_1$, $\vec{D} = \vec{\partial} + ia_0^a \vec{v}_1$ sources, and summation over quark flavors $\alpha = \{c, b\}$ is understood. Here $a_i^\alpha$ represent the coefficients of NRQCD operators, which are normalized such that $a_i^\alpha = 1 + O(\alpha_s)$. Note that in this notation, the field $\chi$ creates an antiquark, so if the Lagrangian (3.12) were normal ordered then the kinetic term for the antiquark field would have the opposite sign. At leading order, the flavor changing part of Eq. (3.11) reads

$$L_{FCh}^\alpha = -a_3 \chi^\dagger_\alpha \psi_\beta S_1 - a_4 \chi^\dagger_\alpha \vec{\sigma} \cdot \vec{V}_1 \psi_\beta + \text{h.c.,}$$

(3.13)

where $S_1$ and $\vec{V}_1$ represent flavor-changing external sources. It is important to remember that the power counting rules for the Lagrangian with external sources are different from the usual NRQCD Lagrangian, in that one counts only the powers of quark fields. Therefore, the next-to-leading Lagrangian contains covariant derivative insertions into the leading order Lagrangian,

$$L_{FCh}^{NL} = \frac{b_1^a}{8m_2^a} \left[ \chi^\dagger_\alpha \bar{D}^i D^j \chi_\alpha + \psi^\dagger_i D^j \psi_\alpha \right] v_3 + \frac{b_2^a}{8m_2^a} \left[ \chi^\dagger_\alpha (\vec{D} \times \vec{\sigma})^i \chi_\alpha + \psi^\dagger_i (\vec{D} \times \vec{\sigma})^i \psi_\alpha \right] v_3. \quad (3.14)$$

For the external electromagnetic field one identifies $s_1$ and $\vec{v}_1$ with the external electromagnetic potentials, while $\vec{v}_2$ and $\vec{v}_3$ represent external magnetic and electric fields, $v_2^i = eQ_\alpha B^i = \frac{1}{2} eQ_\alpha \epsilon^{ijk} F^j k$ and $v_3^i = eQ_\alpha E^i = eQ_\alpha F^{0i}$. At next-to-leading order, the flavor changing Lagrangian is

$$L_{FCh}^{NL} = -\frac{b_3^a}{2} \chi^\dagger_\alpha \left( \frac{i\bar{D}}{m_\alpha} + \frac{iD^i}{m_\beta} \right) \sigma^i \psi_\beta S_2 - \frac{b_4^a}{2} \chi^\dagger_\alpha \left( \frac{i\bar{D}}{m_\alpha} + \frac{iD^i}{m_\beta} \right) \psi_\beta V_2^i$$

$$+ \frac{b_5^a}{2} \epsilon^{ikl} \chi^\dagger_\alpha \left( \frac{\bar{D}}{m_\alpha} - \frac{D^i}{m_\beta} \right) \sigma^k \psi_\beta V_2^l + \text{h.c.} \quad (3.15)$$

We will identify $S_i$ and $\vec{V}_i$ with the time and space components of the leptonic current $L_\mu = V_{cb} (G_F / \sqrt{2}) \bar{u} \gamma_\mu (1 - \gamma_5) v = V_{cb} (G_F / \sqrt{2}) I_\mu$. As defined in Eqs. (3.14) and (3.15), the coefficients satisfy $b_i^a = 1 + O(\alpha_s)$.

A set of NRQCD Feynman rules follows directly from this Lagrangian and can be used to calculate the set of diagrams of Fig. 1. The calculation of diagram Fig. 1c is identical to the one in QCD and yields

$$A_c = a_3 V_{cb} \frac{G_F e Q_1}{2\sqrt{2} (p_c k)} \left[ 2 \nu^\mu \nu^\nu + (k v) g^{\mu \nu} - k^\mu v^\nu - i \epsilon^{\mu \alpha \nu \beta} v_\alpha k_\beta \right] \epsilon^\dagger_\mu \left( \langle 0 | \chi^\dagger_\beta \psi_\beta | b c \rangle (1 S_0) \right). \quad (3.16)$$

The amplitudes of diagrams Fig. 1a and Fig. 1b can be computed using Feynman rules derived from Eqs. (3.12) and (3.13),

$$A_{a,b} = -ia_2 a_4 V_{cb} \frac{G_F}{\sqrt{2}} \epsilon^{ikj} k^i j^j \epsilon^{*k} \left( \langle 0 | \chi^\dagger_\beta \psi_\beta | b c \rangle (1 S_0) \right)$$

$$\times \left[ \frac{eQ_b}{2m_b (p_1^0 - \bar{p}_1^2 / 2m_b)} + \frac{eQ_c}{2m_c (p_2^0 - \bar{p}_2^2 / 2m_c)} \right], \quad (3.17)$$

9
where \( p_0^1 = E_b + E_\gamma, \vec{p}_1 = \vec{p}_b + \vec{k}, p_2^0 = E_c - E_\gamma, \vec{p}_2 = \vec{p}_c - \vec{k} \).

To compare Eq. (3.17) to the corresponding expression in full QCD, it is convenient to adopt a covariant notation and choose the gauge \( \epsilon \cdot v = 0 \). Then it can be rewritten as

\[
A_{a,b} = \frac{i a_2 a_4}{\sqrt{2} k v} G_F \left[ \frac{e Q_b}{2 m_b} + \frac{e Q_c}{2 m_c} \right] \epsilon^{\mu \nu \alpha \beta} l_\mu e^*_\nu v_\alpha k_\beta \langle 0 | \chi_b^\dagger \psi_c | \bar{b} c (1 S_0) \rangle.
\]

In addition, there is also a local interaction coming from a \( b_4 \) term of the flavor changing part of the Lagrangian (3.15),

\[
A_s = -b_4 V_{cb} G_F \frac{i}{\sqrt{2}} \frac{g^\mu}{2 m_b} \left( \frac{\bar{D}_i}{m_b} + \frac{D_i}{m_c} \right) \epsilon_\mu \langle 0 | \chi_b^\dagger \psi_c | \bar{b} c (1 S_0) \rangle.
\]

This contribution involves operators with covariant derivatives. Recalling that covariant derivatives contain electromagnetic source fields whose couplings can be treated perturbatively, we obtain

\[
A_s = -b_4 V_{cb} G_F \frac{i}{\sqrt{2}} \frac{g^\mu}{2 m_b} \left( \frac{\bar{D}_i}{m_b} + \frac{D_i}{m_c} \right) \epsilon_\mu \langle 0 | \chi_b^\dagger \psi_c | \bar{b} c (1 S_0) \rangle.
\]

Note that this term has no dependence on the photon energy. Adding the contributions \( (3.16), (3.18) \) and \( (3.20) \) and comparing with the QCD result (2.1), we reproduce at leading order the anticipated matching conditions \( b_i^a = a_i^a = 1 \).

We hasten to add a remark here. At leading order, the only contribution that survives in the denominators of Eq. (3.17) is \( E_\gamma \equiv k v \), matching the propagator of full QCD. However, in NRQCD this calculation is only good for \( E_\gamma \ll m_{c,b} \), where the heavy quarks are close to their mass shell. There is a substantial portion of the final state phase space in which the propagating heavy quark is relativistic and therefore not described by the formalism of NRQCD at the leading order. In a formal sense, the sum of an infinite number of NRQCD operators is required to fully describe the propagation of the relativistic quark. We can separate the region in which NRQCD is valid from the one in which it is not by introducing a factorization scale \( \mu \ll m_{c,b} \). Then the amplitude (3.18) is generated by the NRQCD Feynman diagrams in Fig. 1 only for \( E_\gamma < \mu \). For the region \( E_\gamma > \mu \), we must add a local “operator” to the Lagrangian,

\[
- V_{cb} G_F \frac{1}{\sqrt{2} k v} \left( \frac{e Q_b}{2 m_b} + \frac{e Q_c}{2 m_b} \right) \epsilon^{ij} \psi_i^c \bar{b} c (1 S_0)
\]

To reproduce NRQCD amplitude in this kinematic region. This object is an operator in the NRQCD quark fields, but a c-number with respect to the external electromagnetic sources. Note that its inclusion is required by the matching conditions of QCD onto NRQCD, to account for the situation in which the intermediate heavy quark is far from its mass shell \( \Re \).

\(^1\text{Clearly, the matching procedure can be performed in any gauge; in NRQCD, this gauge choice is most convenient and quite common.}\)
This is entirely in accord with what one usually finds upon applying the operator product expansion, that highly virtual intermediate states generate operators of higher dimension in the low energy theory. Note that since the this new object is to be included only in the case of large photon energy \( E_\gamma > \mu \), it could equally well be written in terms of a local operator built out of the photon field strength \( F_{\mu \nu} \). We have chosen to write Eq. (3.21) in this form so that it may be compared transparently to the amplitudes which contribute at small \( E_\gamma \).

It is now clear that the leading contribution to the decay \( B_c \to \gamma l \nu \) indeed comes from the dimension three matrix element

\[
\langle 0 | \chi_b^\dagger \psi_c | B_c \rangle = i m_{B_c} f_{B_c},
\]

as anticipated in the previous section. This entirely unsurprising result has now been placed on firm theoretical footing, as the leading term in a systematic expansion. Furthermore, we are in a position to estimate the size of the most important corrections. From the expansion (3.10), we see that there is a relativistic correction to

\[
\langle 0 | \chi_b^\dagger \psi_c | B_c \rangle
\]

of relative order \( \nu^2 \) by NRQCD power counting \([3]\). This is a contribution to \( f_{B_c} \), however, which cancels in the ratio (2.5). The leading correction which is particular to \( B_c \to \gamma l \nu \) comes from the dimension five operator

\[
\langle 0 | \chi_b^\dagger \tilde{D}^2 \psi_c | B_c \rangle/2m_c,
\]

generated by attaching a soft gluon to the \( c \) quark line in Fig. 1a. This matrix element is of relative order \( \nu^4 \sim 5\% \). In NRQCD, matrix elements such as

\[
\langle 0 | \chi_b^\dagger \hat{D}^2 \psi_c | B_c \rangle
\]

and

\[
\langle 0 | \chi_b^\dagger g \tilde{\sigma} \cdot \vec{B} \psi_c | B_c \rangle
\]

are universal quantities which can, in principle, be extracted from other \( B_c \) decays. Note that the relativistic contributions are formally smaller than QCD radiative corrections of order \( \alpha_s \), which we have not included.

IV. VERY SOFT PHOTONS AND THE \( B^*_c \) POLE

For weak radiative \( B_c \) decays, there is the possibility that the outgoing photon is very soft, so much so that there is a large time separation between the event where the photon is emitted and the event where quarks annihilate to the lepton pair. In this case, the physics is not adequately described by (even nonrelativistic) quark fields, and consideration of hadronic intermediate states is necessary. Let us study this part of the spectrum more carefully. The treatment in this section is similar to that of Ref. \([12]\) for the decay \( B \to \pi l \nu \).

The amplitude \( \mathcal{A}(B_c \to l \nu \gamma) \) is a second-order process involving contributions both from the electromagnetic part of the Hamiltonian \( H_{em} \) and from the weak part \( H_w \). For a sufficiently soft photon, in which case the recoil of the hadrons can be neglected, the amplitude is

\[
\mathcal{A}(B_c \to l \nu \gamma) = i \int d^4x e^{ikx} \langle l \nu \gamma | T \{ H_{el}(x), H_w(0) \} | B_c \rangle
\]

\[
= \sum_M \langle l \nu | H_w | M \rangle \frac{1}{\Delta E_M} \langle M \gamma | H_{el} | B_c \rangle + \text{local terms},
\]

(4.1)

where \( \Delta E_M \) is the energy difference between the \( B_c \) and the \( M \gamma \) intermediate state, and

\[
M = B_c, B^*_c, B^{**}_c, \ldots
\]

represents any meson of the \( B_c \) family. Heavy quark spin symmetry implies that the mesons \( M \) are found in degenerate pairs, of which the \( B_c \) and \( B^*_c \) comprise

---

\[2\]The leading radiative correction to \( B_c \to l \nu \) was computed in Ref. \([11]\).
the lightest. Heavy quark symmetry also implies that in the limit $E_\gamma \to 0$, the matrix element $\langle M \gamma | H_{el} | B_c \rangle$ vanishes except when $M$ is a member of the ground state doublet $(B_c, B_c^*)$. In this limit, this is a “zero-recoil” transition, such as at the $w = 1$ point in semileptonic $B$ decays. Therefore, for very small $E_\gamma$ the sum in Eq. (4.1) is dominated by the contributions from $B_c$ and $B_c^*$. Since the weak $B_c \to l\nu$ transition is helicity suppressed for $m_l = 0$, only the vector $B_c^*$ state survives the sum. This situation is represented in Fig. 4a.

We will neglect contributions from the graphs of Fig. 4b, corresponding to intermediate $\psi$ states. These graphs are proportional to $g_{\gamma\psi}/m_\psi^2$ and are formally suppressed compared to the $B_c^*$ pole. Moreover, since the $\psi$ states are far off shell, these contributions may be systematically accounted for by including a set of higher-order NRQCD operators. Note that the situation is completely different in $B_u \to l\nu\gamma$ decays, where the analogous contribution is from an intermediate $\rho$ meson and cannot be calculated in a model independent fashion in HQET, thus providing an intrinsic uncertainty. There are also local terms arising from the graphs like Fig. 4c. Concentrating on the “pole” part,

$$A(B_c \to l\nu\gamma) = \sum_M i\langle l\nu|H_w|B_c^*\rangle \frac{i}{2m_{B_c}v \cdot (-k)} \langle B_c^*\gamma|H_{el}|B_c\rangle + \ldots \quad (4.2)$$

Clearly, the position of the pole in Eq. (4.2), although consistent with a heavy quark symmetry expectations, is not correctly located at $E_\gamma = 0$. We now show how the leading chromomagnetic corrections move the pole to the right place.

At this stage it is convenient to adopt an HQET notation, in which the NRQCD operators are build from four-component spinors constrained by a set of on-shell conditions,

$$\gamma^\dagger \Psi_c = \Psi_c, \quad \bar{\Psi}_b \gamma^\dagger = -\bar{\Psi}_b. \quad (4.3)$$

In this formalism, the electromagnetic part of the Hamiltonian contains two parts, which can be identified with the spin-conserving electric and spin-flipping magnetic transitions,

$$H_{em} = \frac{eQ_c}{4m_c} \bar{\Psi}_c \sigma^{\alpha\beta} F_{\alpha\beta} \Psi_c + \frac{eQ_b}{4m_b} \bar{\Psi}_b \sigma^{\alpha\beta} F_{\alpha\beta} \Psi_b \quad (4.4)$$

where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the electromagnetic field strength tensor. Clearly, only the magnetic part of $H_{em}$ will contribute to the $B_c^*$ intermediate state in Eq. (4.2). The $\langle B_c^*\gamma|H_{em}|B_c\rangle$ matrix element is then

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{Pole structure in $B_c \to l\nu\gamma$, at leading order.}
\end{figure}
\begin{align}
\langle B^*_c|H_{em}|B_c \rangle &= -k_{\alpha} \epsilon^*_\beta \left[ \frac{eQ_c}{2m_c} \langle B^*_c|\bar{\Psi}_c \sigma^{\alpha\beta} \Psi_c|B_c \rangle + \frac{eQ_b}{2m_b} \langle B^*_c|\bar{\Psi}_b \sigma^{\alpha\beta} \Psi_b|B_c \rangle \right].
\end{align}

These matrix elements can be calculated using the trace formalism (3.7), yielding
\begin{align}
\langle B^*_c|\bar{\Psi}_q \sigma^{\alpha\beta} \Psi_q|B_c \rangle &= 2m_B \eta^*_\mu v_\mu \epsilon^{\mu \alpha \beta},
\end{align}
where \( \eta_\mu \) is the polarization vector of the \( B^*_c \) meson, and \( \Psi_q \) represents either heavy effective field.

We now use heavy quark symmetry (3.3) to relate the decay constant of the \( B^*_c \) to that of the \( B_c \),
\begin{align}
\langle 0|\bar{\Psi}_b \gamma_\mu (1-\gamma_5) \Psi_c |B^*_c \rangle &= if_{B_c} m_{B_c} \eta_\mu.
\end{align}
Thus in the heavy quark limit and with \( m_l = 0 \) we obtain for the soft photon amplitude
\begin{align}
A(B_c \rightarrow l \nu \gamma) &= -\frac{V_{cb} G_F}{\sqrt{2}} f_{B_c} m_{B_c} \frac{\mu_{B_c}}{v} \epsilon^{\mu \nu \alpha \beta} l_\mu v_\nu \epsilon^*_\alpha k_\beta + \text{local term},
\end{align}
where we have introduced for convenience the perturbative \( B_c \) magnetic moment, \( \mu_{B_c} = eQ_b/2m_b + eQ_c/2m_c \). Since in the heavy quark limit the \( B_c \) and \( B^*_c \) are degenerate, the pole is at the “wrong” position \( E_\gamma = 0 \). Next we show how inclusion of the \( 1/m_{c,b} \) corrections removes the degeneracy and shifts the pole to \( E_\gamma = -\Delta = -(m_{B_c} - m_{B^*_c}) \), in the unphysical region where it belongs.

The leading \( 1/m_{c,b} \) corrections come from the insertion of the kinetic energy and chromomagnetic dipole operators, \( O_{kin} = \bar{\Psi}_q(iD)^2 \Psi_q/2m \) and \( O_{mag} = g_s \bar{\Psi}_q \sigma^{\alpha\beta} G_{\alpha\beta} \Psi_q/4m \). We use the customary definitions
\begin{align}
\langle B^{(*)}_c|\bar{\Psi}_q(iD)^2 \Psi_q|B^{(*)}_c \rangle &= 2m_{B_c} \lambda_1,
\langle B^{(*)}_c|\bar{\Psi}_q \sigma^{\alpha\beta} G_{\alpha\beta} \Psi_q|B^{(*)}_c \rangle &= 2m_{B_c} d^{(*)}_M \lambda_2,
\end{align}
where \( d_M = 3 \) and \( d^{(*)}_M = -1 \). Note that unlike in HQET, in NRQCD the matrix elements of these operators have different velocity powers; the first operator is there at leading order, while the second is suppressed by a power of \( v \).

Inserting \( O_{kin} \) and \( O_{mag} \) vertices on the \( B^*_c \) meson line gives rise to the double pole contribution
\begin{align}
A(B_c \rightarrow l \nu \gamma) &= \langle l \nu|H_{em}|B^{(*)}_c \rangle \langle B^{(*)}_c|O_{kin} + O_{mag}|B^{(*)}_c \rangle \langle B^{(*)}_c |H_{el}|B_c \rangle \left( \frac{1}{2m_{B_c} v \cdot k} \right)^2 \\
&= -\frac{\delta^*}{2m_{B_c} (v \cdot k)^2} \langle l \nu|H_{em}|B^{(*)}_c \rangle \langle B^{(*)}_c |H_{el}|B_c \rangle + \ldots,
\end{align}
where \( \delta^* = -(\lambda_1 + d^{(*)}_M \lambda_2)/2m_{red} \) and \( m_{red} = m_b m_c/(m_b + m_c) \). The same matrix elements contribute to \( 1/m_{c,b} \) corrections to the physical meson masses,
\begin{align}
m_{B^{(*)}_c} &= m_b + m_c - \frac{1}{2m_{red}} (\lambda_1 + d^{(*)}_M \lambda_2).
\end{align}
FIG. 5. Pole structure in $B_c \rightarrow l \nu \gamma$, at next-to-leading order. The $O_i$ represent insertions of the kinetic energy and chromomagnetic operators.

When $O_{\text{kin}}$ and $O_{\text{mag}}$ and inserted on the external line as shown in Fig. 4b, they modify the heavy meson propagator by shifting the position of the pole,

$$
\frac{i}{v \cdot k} \rightarrow \frac{i}{v \cdot k + \delta} = \frac{i}{v \cdot k} \left(1 - \frac{\delta}{v \cdot k} + \ldots\right),
$$

(4.12)

where $\delta = -(\lambda_1 + d_M \lambda_2)/2m_{\text{red}}$. Taking this modification into account in (4.8) and combining it with the double pole contribution (4.10), one sees that the terms proportional to $\lambda_1$ cancel but terms proportional to $\lambda_2$ don’t, since $d_M \neq d^*_M$. Then the propagator in (4.2) is replaced by

$$
\frac{i}{v \cdot (-k)} \rightarrow \frac{i}{v \cdot (-k)} \left[1 + \frac{\Delta}{v \cdot (-k)}\right] = -\frac{i}{v \cdot k + \Delta}.
$$

(4.13)

Hence we obtain

$$
A(B_c \rightarrow l \nu \gamma) = -\frac{G_F V_{cb}^*}{\sqrt{2}} f_{B_c} m_{B_c} \frac{\mu_{B_c}}{v \cdot k + \Delta} e^{\mu \nu \alpha \beta} \epsilon^*_{\nu \alpha \beta} \epsilon_{\mu \alpha \beta} + \text{local},
$$

(4.14)

with the pole now in the correct place. Since the photon energy spectrum is finite, and in fact vanishes, as $E_\gamma \rightarrow 0$, the shift in the position of the pole has little effect on the total rate.

V. SUMMARY

We have performed a model-independent study of the weak radiative $B_c$ decay in the framework of nonrelativistic QCD. This process is an important competitor to the annihilation process $B_c \rightarrow \mu \nu$, which eventually could be used to extract the $B_c$ decay constant. We found that the branching ratio for $B_c \rightarrow \gamma \mu \nu$ is of the same order of magnitude as the corresponding purely leptonic decay, while $B_c \rightarrow \gamma e \nu_e$ completely dominates its leptonic counterpart. We have generalized the NRQCD Lagrangian by introducing external sources for the electromagnetic and weak fields. At leading order, NRQCD gives a result similar
to what one might expect from a constituent quark model; at higher order, we estimate that the leading nonperturbative corrections to $\Gamma(B_c \to \gamma l\nu)/\Gamma(B_c \to l\nu)$ are at the level of 5%. Finally, we showed how the $B_c^*$ pole emerges for very soft photons. Unfortunately, the branching ratio for this process is small, $\mathcal{B}(B_c \to l\nu\gamma) \simeq 4.4 \times 10^{-5}$, and it will certainly be a challenge to observe this process even at future hadronic $B$ Factories.

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