Valence and sea contributions to the nucleon spin

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Abstract

The first moments of the polarized valence parton distribution functions (PDFs) truncated to the wide Bjorken $x$ region $0.004 < x < 0.7$ are directly (without any fitting procedure) extracted in the next to leading order (NLO) QCD from both COMPASS and HERMES data on pion production in polarized semi-inclusive DIS (SIDIS) experiments. The COMPASS and HERMES data are combined in two ways and two scenarios for the fragmentation functions (FFs) are considered. Two procedures are proposed for an estimation of light sea quark contributions to the proton spin. Both of them lead to the conclusion that these contributions are compatible with zero within the errors.

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Longitudinally polarized parton distribution functions (PDFs), and especially their first moments, which directly compose the nucleon spin together with the orbital parton momenta, are of crucial importance for solution of the proton spin puzzle, attracting great both theoretical and experimental efforts during many years. Nowadays, there is a huge growth of interest to the semi-inclusive DIS (SIDIS) experiments with longitudinally polarized beam and target such as SMC \cite{1}, HERMES \cite{2}, COMPASS \cite{3} (see, for instance, \cite{4} for review). It is of importance that the SIDIS experiments, where one identifies the hadron in the final state, provide us with the additional information on the partonic spin structure in comparison with the usual DIS experiments. Namely, in contrast to the DIS data, the SIDIS data allows us to find the sea and valence contributions to the nucleon spin in separation. In this paper we just focus on this important task. To this end we apply the original procedure of the polarized SIDIS data analysis in NLO QCD, elaborated in the sequel of papers \cite{5,6,7} (see also \cite{4} for more details). It is of importance (especially for analysis of still relatively poor SIDIS data we deal with) that this alternative procedure allows to extract the truncated to the accessible for measurement Bjorken $x$ region moments of polarized PDFs directly, without any fitting procedure (with unavoidable arbitrariness in the choice of functional form of $\Delta q$ at initial $Q^2$ and a lot of free varying parameters): within this procedure the central values of asymmetries and their uncertainties directly propagate to the extracted values of moments and their errors. Of importance also that we use in our analysis the difference asymmetries (just as in Refs. \cite{5,7}), that allows to avoid the application of poorly known FFs (such as $D_{u}^{K_{L}^{\pm}}$ and $D_{g}^{h}$ FFs) – see \cite{5,7} and also \cite{4} for review.

Within this paper we use two NLO parametrizations AKK08 \cite{8} and DSS \cite{9} for FFs, which differ quite essentially in the pion sector. AKK08 parametrization corresponds to unbroken $SU_f(2)$ symmetry:

$$D_1 \equiv D_{u}^{+} \equiv D_{u}^{-} \equiv SU(2) D_{d}^{+} \equiv D_{d}^{-},$$

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\[ D_2 \equiv D_u^{\pi^+} = D_u^{\pi^-} = D_u^{SU(2)} = D_d^{\pi^+} = D_d^{\pi^-}. \]  

(2)

On the other hand, DSS parametrization [9] allows violation of \( SU_f(2) \) symmetry in the sector of the favored pion FFs:

\[ D_1 \equiv D_u^{\pi^+} = D_u^{\pi^-}, \]  

(3)

\[ \tilde{D}_1 \equiv D_d^{\pi^+} = D_d^{\pi^-}, \]  

(4)

\[ D_2 \equiv D_u^{\pi^+} = D_u^{\pi^-} = D_u^{SU(2)} = D_d^{\pi^+} = D_d^{\pi^-}, \]  

(5)

so that the favored FFs \( D_1 \) and \( \tilde{D}_1 \) of \( u \) and \( d \) quarks are not equal to each other.

For the scenario preserving \( SU_f(2) \) symmetry (AKK08 parametrization for FFs here) the procedure of direct extraction in NLO QCD of \( n \)-th moments of the valence PDFs from the measured difference asymmetries is described in Refs. [5], [7] in details. Let us recall the key necessary equations.

The theoretical expressions for the difference asymmetries in NLO QCD look as

\[ A_{p}^{\pi^+\pi^-}(x, Q^2) \bigg|_Z = (1 + R) \frac{\int_1^{z_h} dz_h (4 \Delta u_V - \Delta d_V) \Delta \hat{K}(D_1 - D_2)}{\int_1^{z_h} dz_h (4u_V - d_V) \hat{K}(D_1 - D_2)}, \]  

(6)

for the proton target, and

\[ A_{d}^{\pi^+\pi^-}(x, Q^2) \bigg|_Z = (1 + R)(1 - \frac{3}{2} \omega_D) \frac{\int_1^{z_h} dz_h (\Delta u_V + \Delta d_V) \Delta \hat{K}(D_1 - D_2)}{\int_1^{z_h} dz_h (u_V + d_V) \hat{K}(D_1 - D_2)}, \]  

(7)

for the deuteron target. Here, for brevity, we have introduced the operator notation \( \hat{K} \) and \( \Delta \hat{K} \):

\[ \hat{K} \equiv 1 + \otimes \frac{\alpha_s}{2\pi} C_{qq}^2 \otimes, \quad \Delta \hat{K} \equiv 1 + \otimes \frac{\alpha_s}{2\pi} \Delta C_{qq} \otimes, \]  

(8)

so that

\[ [q \hat{K} D](x, z_h) = q(x) D(z_h) + \frac{\alpha_s}{2\pi} \int_x^{z_h} \frac{dx'}{x'} \int_z^{z'} \frac{dz'}{z'} q \left( \frac{x}{x'} \right) C_{qq}^2(x', z') D \left( \frac{z_h}{z'} \right), \]  

(9)

\[ [\Delta q \Delta \hat{K} D](x, z_h) = \Delta q(x) D(z_h) + \frac{\alpha_s}{2\pi} \int_x^{z_h} \frac{dx'}{x'} \int_z^{z'} \frac{dz'}{z'} \Delta q \left( \frac{x}{x'} \right) \Delta C_{qq}(x', z') D \left( \frac{z_h}{z'} \right). \]  

(10)

In Eqs. (6), (7) \((1 - 1.5\omega_D), \ \omega_D = 0.05 \pm 0.01, \) is the factor accounting for the deuteron D-state contribution, the quantity \( R = \sigma_L / \sigma_T \) is taken from Ref. [12], \( \Delta C_{qq} \) is the Wilson coefficient entering NLO QCD expression for the polarized SIDIS structure function \( g_1^B \) and can be found, for instance, in [13], \( C_{qq}^2 = C_{qq}^4 + C_{qq}^6 \) is the Wilson coefficient entering the NLO QCD expression for the unpolarized SIDIS structure function \( F_2^B \), while the coefficients \( C_{qq}^4 \) and \( C_{qq}^6 \) (entering \( F_1^B \) and \( F_1^B \)) also can be found in [13]. In our subsequent calculations we also use NLO parametrization GJR08 [14] for unpolarized PDFs.

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4 Since namely \( F_2 \) function is measured in experiment and then is used to parameterize unpolarized PDFs, the form of these equations used here is the most convenient to properly account for correction due to the factor \( R = \sigma_L / \sigma_T \) (see discussion around Eq. (10) in [16], around Eq. (12) in [11] and references therein).
The equations allowing to find from the data on difference asymmetries the $n$-th moments $\Delta_n q\equiv \int_a^b dx x^{n-1} q(x)$ of valence PDFs truncated to the accessible for measurement $x$ region $[a, b]$ look as

$$\Delta'_n u_V \simeq \frac{1}{5} \frac{A_p^{(n)} + A_d^{(n)}}{L_{(n)1} - L_{(n)2}}; \quad \Delta'_n d_V \simeq \frac{1}{5} \frac{4A_d^{(n)} - A_p^{(n)}}{L_{(n)1} - L_{(n)2}},$$

where

$$A_p^{(n)} = \sum_{i=1}^{N_{n,\pi^+}} A_{p_i}^{+\pi^-}(\langle x_i \rangle) \int_{x_i}^{x_i^s} dx x^{n-1} (1 + R)^{-1} \int_0^{1} dz_k [(4u_V - d_V)\hat{K}(D_1 - D_2)],$$

$$A_d^{(n)} = (1 - 1.5 \omega_D)^{-1} \times \sum_{i=1}^{N_{n,\pi^+}} A_{d_i}^{-\pi^+}(\langle x_i \rangle) \int_{x_i}^{x_i^s} dx x^{n-1} (1 + R)^{-1} \int_0^{1} dz_k [(u_V + d_V)\hat{K}(D_1 - D_2)].$$

The quantities $L_{(n)1}$, $L_{(n)2}$ are defined as

$$L_{(n)1} \equiv L_{(n)u}^\pi = L_{(n)d}^\pi = L_{(n)d}^{-\pi},$$

$$L_{(n)2} \equiv L_{(n)d}^{\pi^+} = L_{(n)u}^{-\pi} = L_{(n)d}^{-\pi},$$

$$L_{(n)q}^h = \int_0^{1} dz_k \left[ D_q^h(z_k) + \frac{\alpha_s}{2\pi} \int_{z_k}^{1} \frac{dz'}{z'} \Delta_n C_{qq}(z') D_q^h(z_k/z') \right],$$

where

$$\Delta_n C_{qq}(z) \equiv \int_0^{1} dx x^{n-1} \Delta C_{qq}(x, z).$$

A lot of numerical tests performed in Refs. [5, 7] have shown that Eqs. (11) allow to reconstruct the truncated moments of polarized valence PDFs with a high precision even in the case of rather narrow HERMES $x$ region $0.023 < x < 0.6$, divided on relatively small number of bins (nine bins only).

It is easy to see that for scenario with the broken $SU_f(2)$ symmetry one has instead of (9), (7) the equations

$$A_{p_i}^{+\pi-} = (1 + R) \frac{(4u_V - d_V)\Delta\hat{K}(D_1 - D_2) + \Delta d_V \Delta\hat{K}(D_1 - \hat{D}_1)}{(4u_V - d_V)\hat{K}(D_1 - D_2) + d_V \hat{K}(D_1 - D_1)},$$

$$A_{d_i}^{-\pi^+} = (1 + R) (1 - \frac{3}{2}\omega_D) \frac{(4u_V + d_V)\Delta\hat{K}((D_1 - D_2) + \frac{1}{3}(D_1 - \hat{D}_1))}{(u_V + d_V)\hat{K}((D_1 - D_2) + \frac{1}{3}(D_1 - \hat{D}_1))}. $$

The generalization of Eqs. (11)-(14) on the case of broken $SU_f(2)$ symmetry (Eqs. (3)-(5)) is also straightforward.

Both COMPASS [13, 16] and HERMES [2] collaborations published the data only on asymmetries $A_{p,d}^{\pi^+\pi^-}$, while the published data on the pion difference asymmetries $A_{p,d}^{\pi^+\pi^-}$ are still

\footnote{At present this work is in preparation at COMPASS.}
absent. That is why the special procedure was applied in [7] to construct asymmetries $A_{p,d}^{\pi^+-\pi^-}$ from the HERMES data on pion production, and we repeat here this procedure for the COMPASS case. Namely, in each $i$-th bin the pion difference asymmetries can be rewritten as

$$A_{\pi^+-\pi^-}^i(x_i) = \frac{R_i^{+/+}}{R_i^{+-}-1}A_{\pi^+}^i(x_i) - \frac{1}{R_i^{+-}-1}A_{\pi^-}^i(x_i),$$

where $R_i^{+/+}$ is the ratio of unpolarized cross-sections for $\pi^+$ and $\pi^-$ production: $R_i^{+/+} = \sigma_{\pi^+}^\text{unpol}(x_i)/\sigma_{\pi^-}^\text{unpol}(x_i) = N_i^{\pi^+}/N_i^{\pi^-}$. As it was argued in Ref. [7] this relative quantity is very well reproduced by the the LEPTO generator of unpolarized events [17], which gives a good description of the fragmentation processes. So, we again use here the LEPTO generator for this purpose.

Let us now discuss the question of $Q^2$ dependence of asymmetries and its influence on the final results. The point is that both DIS and SIDIS asymmetries very weakly depend on $Q^2$ (see, for instance, Fig. 5 in Ref. [18]), so that the approximation

$$A(x_i,Q_i^2) \simeq A(x_i,Q_i^2_0)$$

is commonly used (see, for example, Refs. [1][2][16]) for analysis of the DIS and SIDIS asymmetries. Nevertheless, for more comprehensive analysis, it is useful to account for the corrections caused by the weak $Q^2$ dependence of the difference asymmetries, i.e., to estimate the shifts

$$\delta_iA_{p,d}^{\pi^+-\pi^-} = A_{p,d}^{\pi^+-\pi^-}(x_i,Q_i^2) - A_{p,d}^{\pi^+-\pi^-}(x_i,Q_i^2_0)$$

in the difference asymmetries and their influence on the moments of valence PDFs. To this end we first approximate r.h.s of Eq. (19) by the respective difference of “theoretical” asymmetries calculated with substitution of two novel parametrizations [19][11] on polarized PDFs (elaborated with application of both DIS and SIDIS data) into the NLO QCD equations (6), (7) (or (15)-(16)), and then average the obtained results on $\delta_iA_{p,d}^{\pi^+-\pi^-}$. Adding the calculated in this way $\delta_iA_{p,d}^{\pi^+-\pi^-}$ to the initial experimental asymmetries $A_{p,d}^{\pi^+-\pi^-}(x_i,Q_i^2_0)$, we estimate the evolved from $Q_i^2$ to $Q_i^2_0$ asymmetries $A_{p,d}^{\pi^+-\pi^-}(x_i,Q_i^2)$.

The evolved to $Q^2 = 10 GeV^2$ pion difference asymmetries constructed with Eq. (17) from the COMPASS [13][16] and HERMES [2] data on $A_{p,d}^{\pi^\pm}$ SIDIS asymmetries are presented in Figs. 1 and 2 respectively.

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Notice that the shifts in asymmetries as well as in the final results on the moments of valence PDFs obtained with two applied parametrizations differ very insignificantly from each other.

Notice that the considered procedure of the asymmetry evolution is quite similar to the procedure used by SMC for the $\Gamma_{1p(d)}$ reconstruction (see Section VB in Ref. [20]). Notice also that the original procedure of $A_1$ evolution was elaborated in Refs. [21][22].
We perform the combined analysis of COMPASS \cite{16, 15} and HERMES \cite{2} data on pion production with both proton and deuteron targets. COMPASS collaboration published their data on $A^{\pi^+\pi^-}_{p,d}$ in the Bjorken $x$ ranges $0.004 < x < 0.7$ and $0.004 < x < 0.3$ for the proton and deuteron targets, respectively, while the HERMES data on these asymmetries were presented in the range $0.023 < x < 0.6$ for both targets. The statistical summation of asymmetries $A^{\pi^+\pi^-}_{p,d}$ (constructed with (17)) is performed in accordance with the standard formulas

$$A^h_N|_{averaged} = \frac{A^h_N|_{\text{exp1}}/(\delta A^h_N|_{\text{exp1}})^2 + A^h_N|_{\text{exp2}}/(\delta A^h_N|_{\text{exp2}})^2}{1/(\delta A^h_N|_{\text{exp1}})^2 + 1/(\delta A^h_N|_{\text{exp2}})^2},$$

(20)

$$(\delta A^h_N|_{\text{averaged}})^2 = \frac{1}{1/(\delta A^h_N|_{\text{exp1}})^2 + 1/(\delta A^h_N|_{\text{exp2}})^2}.$$  

(21)

At the same time one can apply Eqs. (20), (21) directly only for coinciding $x$ bins of different experiments. However, this is the case only for last three bins of COMPASS and HERMES experiments we deal with (after proper extrapolation of HERMES data in the last bin from $0.6$ to $0.7$ upper $x$ value). Besides, notice that for last two bins the COMPASS published SIDIS data for deuteron target are still absent. That is why it is of especial importance (and we do it first of all) to include in the analysis of COMPASS data the HERMES data in the region $0.2 < x < 0.6$ (last three bins of HERMES). The results on the difference asymmetries obtained in such a way are presented in Fig. 3.

The respective results on the moments of polarized valence PDFs are presented in Table 1.

Table 1: Four first moments of polarized valence PDFs truncated to the region $0.004 < x < 0.7$ are presented at $Q^2 = 10 \text{ GeV}^2$. The moments are obtained as a result of NLO QCD analysis of the combined data on $A^{\pi^+\pi^-}_{p,d}$ (see Fig. 3), constructed with Eq. (17) from the COMPASS data on $A^{\pi^+\pi^-}_{p,d}$ in the regions $0.004 < x < 0.7$ (proton target), $0.004 < x < 0.3$, (deuteron target), and HERMES data on $A^{\pi^+\pi^-}_{p,d}$ in the region $0.2 < x < 0.6$ (last three bins of HERMES). Capital letters A and B correspond to the application of AKK08 and DSS parametrizations for FFs, respectively. Rome numbers I and II correspond to the moments uncorrected and corrected due to evolution, respectively. Besides, the relative corrections $\delta_r(\Delta'_u qV) \equiv \delta(\Delta'_u qV) / \Delta'_u qV$ caused by evolution are presented here.

| $n$  | $A_I$     | $A_{II}$  | $\delta_r(\Delta'_u qV)$ | $B_I$     | $B_{II}$  | $\delta_r(\Delta'_u qV)$ |
|------|-----------|-----------|---------------------------|-----------|-----------|---------------------------|
| 1    | 0.731 ± 0.087 | 0.695 ± 0.087 | -3.8%                      | 0.693 ± 0.084 | 0.713 ± 0.084 | 2.8%                      |
| 2    | 0.166 ± 0.024 | 0.167 ± 0.024 | 0.8%                       | 0.155 ± 0.024 | 0.158 ± 0.024 | 1.6%                      |
| 3    | 0.055 ± 0.010 | 0.055 ± 0.010 | 1.3%                       | 0.052 ± 0.010 | 0.052 ± 0.010 | 1.8%                      |
| 4    | 0.022 ± 0.005 | 0.022 ± 0.005 | 1.5%                       | 0.021 ± 0.005 | 0.021 ± 0.005 | 2.0%                      |

| $n$  | $\Delta'_u dV$ | $\Delta'_d dV$ | $\delta_r(\Delta'_u dV)$ | $B_I$     | $B_{II}$  | $\delta_r(\Delta'_u dV)$ |
|------|----------------|----------------|---------------------------|-----------|-----------|---------------------------|
| 1    | -0.519 ± 0.162 | -0.524 ± 0.162 | 0.9%                      | -0.473 ± 0.157 | -0.481 ± 0.157 | 1.7%                      |
| 2    | -0.100 ± 0.054 | -0.102 ± 0.054 | 1.8%                      | -0.090 ± 0.051 | -0.092 ± 0.051 | 2.7%                      |
| 3    | -0.029 ± 0.023 | -0.030 ± 0.023 | 2.5%                      | -0.026 ± 0.022 | -0.027 ± 0.022 | 3.7%                      |
| 4    | -0.011 ± 0.011 | -0.011 ± 0.011 | 3.1%                      | -0.010 ± 0.010 | -0.010 ± 0.010 | 4.4%                      |

$^a$Our experience show that this leads to negligible changes in the final results, irrespective of the choice of the extrapolation procedure. So, we apply here the simplest way of extrapolation, prescribing to $A^{\pi^+\pi^-}_{p,d}|_{\text{HERMES}}$ to be in the extended region $[0.4, 0.7]$ the same as in the last HERMES $x$-bin $[0.4, 0.6]$. 

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On the other hand, to maximally increase the available statistics it is, certainly, very desirable to perform the combined analysis, using the COMPASS and HERMES data taken in the entire \(x\) regions accessible for these experiments. To this end we have elaborated the special procedure allowing to combine the data on asymmetries coming from experiments with different binnings – see the Appendix. The results on the difference asymmetries obtained in such a way are presented in Fig. 3.

The respective results on the moments of polarized valence PDFs are listed in Table 2.

### Table 2: Four first moments of polarized valence PDFs truncated to the region 0.004 < \(x\) < 0.7 are presented at \(Q^2 = 10\text{ GeV}^2\). The moments are obtained as a result of NLO QCD analysis of the combined data on \(A^\pi_{p,d}\) (see Fig. 1), constructed with Eq. (17) from the COMPASS and HERMES data on \(A^\pi_{p,d}\) in the entire \(x\)-regions accessible for measurement (0.004 < \(x\) < 0.7, 0.004 < \(x\) < 0.3 for COMPASS and 0.023 < \(x\) < 0.6 for HERMES). Capital letters A and B correspond to the application of AKK08 and DSS parametrizations for FFs, respectively. Rome numbers I and II correspond to the moments uncorrected and corrected due to evolution, respectively. Besides, the relative corrections \(\delta_r(\Delta'_uq_v) = \delta(\Delta'_uq_v)/\Delta'_uq_v\) caused by evolution are presented here.

| \(n\) | \(A_I\)  | \(A_{II}\) | \(\delta_r(\Delta'_uq_v)\) | \(B_I\)  | \(B_{II}\) | \(\delta_r(\Delta'_uq_v)\) |
|------|--------|--------|----------------|--------|--------|----------------|
| 1    | 0.712 ± 0.078 | 0.660 ± 0.078 | 0.683 ± 0.076 | 0.711 ± 0.076 | 4.0%   |
| 2    | 0.166 ± 0.023 | 0.168 ± 0.023 | 0.156 ± 0.024 | 0.159 ± 0.024 | 1.7%   |
| 3    | 0.055 ± 0.010 | 0.056 ± 0.010 | 0.052 ± 0.010 | 0.053 ± 0.010 | 1.8%   |
| 4    | 0.022 ± 0.005 | 0.022 ± 0.005 | 0.021 ± 0.005 | 0.021 ± 0.005 | 2.0%   |

| \(n\) | \(A_I\)  | \(A_{II}\) | \(\delta_r(\Delta'_dq_v)\) | \(B_I\)  | \(B_{II}\) | \(\delta_r(\Delta'_dq_v)\) |
|------|--------|--------|----------------|--------|--------|----------------|
| 1    | -0.414 ± 0.149 | -0.427 ± 0.149 | 0.30 ± 0.145 | -0.376 ± 0.145 | -0.381 ± 0.145 | 1.4% |
| 2    | -0.087 ± 0.053 | -0.089 ± 0.053 | 0.078 ± 0.051 | -0.080 ± 0.051 | 2.7%   |
| 3    | -0.027 ± 0.023 | -0.028 ± 0.023 | 0.024 ± 0.022 | -0.025 ± 0.022 | 3.8%   |
| 4    | -0.010 ± 0.011 | -0.011 ± 0.011 | 0.009 ± 0.010 | -0.010 ± 0.010 | 4.6%   |

In Tables 1 and 2 both scenarios Eqs. (11), (12) (AKK08 parametrization) and Eqs. (3)-5 (DSS parametrization) for FFs are considered. One can see that the results are consistent within the errors. Besides, for each scenario we present the results obtained with and without corrections due to evolution of asymmetries. It is seen that the difference is not too significant (the relative corrections \(\delta_r(\Delta'_uq_v)/\Delta'_uq_v\) take the small values).

Thus, we estimated in NLO QCD the contributions of valence quarks (first moments of polarized valence PDFs) to the nucleon spin. Let us now estimate the respective contributions of light sea quarks. We do it in two different ways.

Within the first procedure one first of all uses some NLO QCD parametrization on the polarized PDFs to estimate the quantities \(\Delta'_lq + \Delta'_l\bar{q} (q = u, d)\). Since the sums \(\Delta q(x) + \Delta \bar{q}(x) (q = u, d)\) are well fitted by the precise purely inclusive DIS data (these quantities are considered as relatively well known and practically are the same for the different modern parametrizations) it is not especially important which parametrization one applies for this purpose (here we use the most popular and widely cited DSSV [19] parametrization). Then, having both \(\Delta'_lq + \Delta'_l\bar{q}\) and \(\Delta'_lq_v (q = u, d)\) quantities one easily gets the truncated first moments of sea \(u\) and \(d\) quarks, applying the obvious relation

\[
\Delta'_l\bar{q} = \frac{1}{2}[\Delta'_lq + \Delta'_l\bar{q}] - \Delta'_lq_v. \tag{22}
\]
The first moments $\Delta'_t\bar{u}$, $\Delta'_t\bar{d}$ obtained in this way, as well as their differences and sums are presented in Tables 3 and 4.

Table 3: First moments of polarized sea PDFs truncated to the region $0.004 < x < 0.7$ are presented at $Q^2 = 10\,GeV^2$, as well as their sums and differences. The moments are obtained with application of Eq. (22), where DSSV parametrization is used to estimate $(\Delta'_t q + \Delta'_t \bar{q})|_{\text{parametrization}}$, while the first moments of valence PDFs are taken from Table 1 (HERMES data only from three last bins are applied). Capital letters A and B correspond to the application of AKK08 and DSS parametrizations for FFs, respectively. Rome numbers I and II correspond to the moments uncorrected and corrected due to evolution, respectively.

|                | $A_I$        | $A_{II}$       | $B_I$      | $B_{II}$     |
|----------------|--------------|----------------|------------|--------------|
| $\Delta'_t\bar{u}$ | 0.018 ± 0.044 | 0.036 ± 0.044  | 0.037 ± 0.042 | 0.027 ± 0.042 |
| $\Delta'_t\bar{d}$ | 0.065 ± 0.081  | 0.067 ± 0.081  | 0.042 ± 0.079 | 0.046 ± 0.079 |
| $\Delta'_t\bar{u} + \Delta'_t\bar{d}$ | 0.082 ± 0.092  | 0.102 ± 0.092  | 0.078 ± 0.089 | 0.072 ± 0.089 |
| $\Delta'_t\bar{u} - \Delta'_t\bar{d}$ | -0.047 ± 0.092 | -0.032 ± 0.092 | -0.005 ± 0.089 | -0.019 ± 0.089 |

Table 4: First moments of polarized sea PDFs truncated to the region $0.004 < x < 0.7$ are presented at $Q^2 = 10\,GeV^2$, as well as their sums and differences. The moments are obtained with application of Eq. (22), where DSSV parametrization is used to estimate $(\Delta'_t q + \Delta'_t \bar{q})|_{\text{parametrization}}$, while the first moments of valence PDFs are taken from Table 2 (all data on pion production of COMPASS and HERMES are combined). Capital letters A and B correspond to the application of AKK08 and DSS parametrizations for FFs, respectively. Rome numbers I and II correspond to the moments uncorrected and corrected due to evolution, respectively.

|                | $A_I$        | $A_{II}$       | $B_I$      | $B_{II}$     |
|----------------|--------------|----------------|------------|--------------|
| $\Delta'_t\bar{u}$ | 0.027 ± 0.039 | 0.053 ± 0.039  | 0.042 ± 0.038 | 0.028 ± 0.038 |
| $\Delta'_t\bar{d}$ | 0.012 ± 0.075  | 0.019 ± 0.075  | -0.007 ± 0.073 | -0.004 ± 0.073 |
| $\Delta'_t\bar{u} + \Delta'_t\bar{d}$ | 0.039 ± 0.084  | 0.072 ± 0.084  | 0.035 ± 0.082 | 0.023 ± 0.082 |
| $\Delta'_t\bar{u} - \Delta'_t\bar{d}$ | 0.015 ± 0.084  | 0.034 ± 0.084  | 0.048 ± 0.082 | 0.032 ± 0.082 |

The idea of alternative procedure for investigation of the sea contributions to the nucleon spin in NLO QCD is based on the proper application of $SU_f(2)$ (Bjorken sum rule) and $SU_f(3)$ sum rules:

$$a_3 \equiv (\Delta_1 u + \Delta_1 \bar{u}) - (\Delta_1 d + \Delta_1 \bar{d}) = \frac{|g_A|}{|g_V|} = F + D = 1.2670 \pm 0.0035,$$

$$a_8 \equiv \Delta_1 u + \Delta_1 \bar{u} + \Delta_1 d + \Delta_1 \bar{d} - 2(\Delta_1 s + \Delta_1 \bar{s}) = 3F - D = 0.585 \pm 0.025.$$  

Bjorken sum rule (23) rewritten in terms of valence and sea distributions produces quite good approximation (see Ref. [4] for review)

$$\Delta'_t \bar{u} - \Delta'_t \bar{d} \approx \frac{1}{2} \frac{|g_A|}{|g_V|} - \frac{1}{2} (\Delta'_t u_V - \Delta'_t d_V)$$

(25)

for the difference of full (not truncated!) moments $\Delta'_t \bar{u}$ and $\Delta'_t \bar{d}$ even in the case of rather narrow HERMES $x$-range, while for the wide COMPASS $x$-range we deal with here this approximation works very well – see the respective numerical tests in Ref. [5]. The point is that since the valence PDFs (contrary to the sea PDFs) are suppressed near the low boundary $x = 0$ (so that $\Delta q = \Delta q_V + \Delta \bar{q} \rightarrow \Delta \bar{q}$ as $x \rightarrow 0$), the omitted in r.h.s. of Eq. (25) term $\int_0^a dx (\Delta u_V - \Delta d_V)$ is small even for HERMES low $x$ boundary $a = 0.023$, and becomes really negligible for COMPASS
respectively. Roman numbers I and II correspond to the moments uncorrected and corrected due to evolution, respectively. Capital letters A and B correspond to the application of AKK08 and DSS parametrizations for FFs, respectively. Roman numbers I and II correspond to the moments uncorrected and corrected due to evolution, respectively.

\[
\Gamma_1^N \equiv (1 - 1.5 \omega_D)^{-1} \Gamma_1^d = \frac{1}{2} (\Gamma_1^u + \Gamma_1^d) = \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right) \left(\frac{1}{36} a_8 + \frac{1}{9} \Delta_1 \Sigma(Q^2)\right),
\]

which produces very good approximation for \(\Delta_1 \bar{u} + \Delta_1 \bar{d}\):

\[
\Delta_1 \bar{u} + \Delta_1 \bar{d} \simeq \left(3 \left(1 + \frac{\alpha_s}{\pi}\right) \Gamma_1^u + \frac{1}{12} a_8\right) - \frac{1}{2} (\Delta'_1 u_V + \Delta'_1 d_V),
\]

where we again omitted the small contributions of valence PDFs \(\int_{0,7}^1 dx (\Delta u_V + \Delta d_V)\) and \(\int_{0,7}^1 dx (\Delta u_V + \Delta d_V)\). We use Eq. (27) the numerical value of \(\Gamma_1^N\) taken from the COMPASS paper [10]:

\[
\Gamma_1^N = 0.051 \pm 0.003 \pm 0.006.
\]

Table 5: Sums and differences of the first moments of polarized sea PDFs, as well as on the moments themselves in separation are presented in Tables 5 and 6.

|          | \(A_I\)         | \(A_{II}\)        | \(B_I\)         | \(B_{II}\)        |
|----------|-----------------|-------------------|-----------------|-------------------|
| \(\Delta_1 \bar{u}\) | 0.059 ± 0.045   | 0.077 ± 0.045     | 0.078 ± 0.043   | 0.068 ± 0.043     |
| \(\Delta_1 \bar{d}\) | 0.050 ± 0.082   | 0.053 ± 0.082     | 0.027 ± 0.079   | 0.031 ± 0.079     |
| \(\Delta_1 \bar{u} + \Delta_1 \bar{d}\) | 0.109 ± 0.095   | 0.129 ± 0.095     | 0.105 ± 0.092   | 0.099 ± 0.092     |
| \(\Delta_1 \bar{u} - \Delta_1 \bar{d}\) | 0.009 ± 0.092   | 0.024 ± 0.092     | 0.051 ± 0.089   | 0.037 ± 0.089     |

Table 6: Sums and differences of the first moments of polarized sea PDFs, as well as on the moments themselves in NLO QCD at \(Q^2 = 10 GeV^2\) within approximations (25) and (27). The truncated first moments of valence PDFs are taken from the Table 11 (HERMES data only from three last bins are applied). Capital letters A and B correspond to the application of AKK08 and DSS parametrizations for FFs, respectively. Rome numbers I and II correspond to the moments uncorrected and corrected due to evolution, respectively.

|          | \(A_I\)         | \(A_{II}\)        | \(B_I\)         | \(B_{II}\)        |
|----------|-----------------|-------------------|-----------------|-------------------|
| \(\Delta_1 \bar{u}\) | 0.068 ± 0.041   | 0.094 ± 0.041     | 0.083 ± 0.040   | 0.069 ± 0.040     |
| \(\Delta_1 \bar{d}\) | -0.002 ± 0.075  | 0.004 ± 0.075     | -0.021 ± 0.073  | -0.019 ± 0.073    |
| \(\Delta_1 \bar{u} + \Delta_1 \bar{d}\) | 0.066 ± 0.087   | 0.099 ± 0.087     | 0.061 ± 0.085   | 0.050 ± 0.085     |
| \(\Delta_1 \bar{u} - \Delta_1 \bar{d}\) | 0.071 ± 0.084   | 0.090 ± 0.084     | 0.104 ± 0.082   | 0.087 ± 0.082     |
Looking at Tables 3, 4, and Tables 5, 6 one can draw an unexpected conclusion, that irrespective of the procedure used in the SIDIS data analysis the first moments of sea PDFs are consistent with zero within the errors. In particular, in contrast with the different model predictions \[24, 25, 26, 27, 28, 29\] the polarized sea asymmetry \(\Delta_1 \bar{u} - \Delta_1 \bar{d}\) appear to be just zero within the errors. It is of importance, because some of these models (see Refs. \[25, 27\] and references therein) predict that this asymmetry is even larger in absolute value than the unpolarized sea asymmetry \(\bar{u} - \bar{d}\) (which takes quite considerable value allowing to explain a large violation of the Gottfried sum rule).

In conclusion, let us briefly discuss the obtained results.

The first moments of the polarized valence PDFs truncated to the wide Bjorken \(x\) region \(0.004 < x < 0.7\) are directly (without any fitting procedure) extracted in NLO QCD from both COMPASS and HERMES polarized SIDIS data. To this end we apply two scenarios for the fragmentation functions and two ways to combine the COMPASS and HERMES data on pion production (one of them is based on the proposed special procedure, allowing to combine all data coming from experiments with different binnings). In turn, the obtained results on the valence PDFs have allowed us to estimate in two ways the contributions of light sea quarks to the proton spin which, surprisingly, occur compatible with zero within the errors. Certainly, this conclusion should be considered as still preliminary, since the results on the sea contributions are obtained for the restricted Bjorken \(x\) region. Nevertheless, its degree of reliability is high enough due to the discussed above advantage of approximations \[25, 27\] to the full moments, which become especially good for the wide COMPASS \(x\) region we deal with. Having in mind the surprisingly small values of \(\Delta_1 G\) and \(\Delta_1 s\) it seems that we now became still more close to the minimal “retro” picture of the proton spin puzzle, where only helicity PDFs and orbital moments of valence quarks compose the nucleon spin.

Now we are waiting for the new COMPASS data with the planned \(180\, \text{GeV}\) muon beam, which should allow to reach still smaller \(x\) values (we expect for the bottom boundary about \(a = 0.003\) instead of \(a = 0.004\)) and, thereby, increase the reliability of the presented results.

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### Appendix

Combined analysis of data coming from experiments with different binnings

For brevity and readability we clarify our procedure of combined analysis on a simple example.

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9While the values of \(\Delta_1 d\) listed in in Tables 5, 6 are just zeros within the errors, the values of \(\Delta_1 \bar{u}\) are compatible with zero within \(2\sigma\).

10Certainly, because of evolution these quantities still can deviate from zero at values of \(Q^2\) distinct from considered here \(Q^2 = 10\, \text{GeV}^2\). However, in the wide range of \(Q^2\) really available to experiment they are still negligible.

11Notice that similar conclusion was made in the recent COMPASS paper \[16\], where the truncated moment of sea \(u\) quark is zero within the errors, while the moment of \(d\) quark very slightly differs from zero. However, first, the analysis in \[16\] was performed only in LO QCD. Second, the moments studied there were truncated to the rather narrow region \(0.004 < x < 0.3\) (because of lack of the deuteron data in the last two bins). Third, all set of FFs has been used, while we apply only well known pion FFs. Fourth, the corrections due to evolution were not taken into account in \[16\] (approximation \[18\] was applied).
Let us suppose that we would like to combine the data on the DIS virtual photon spin asymmetry

\[ A_1 \simeq g_1/F_1, \]

coming from two independent experiments \((exp1\ and\ exp2)\) with different binnings. Of importance is that our final destination is the integral quantity – truncated first moment –

\[ \Gamma_{[a,b]} = \int_a^b dx \, g_1 = \int_a^b dx \, A_1 F_1 \quad (A.1) \]

defined in the structure function \(g_1\), where the unpolarized structure function \(F_1 = F_2/2x(1 + R)\) is considered as an already known \(^{12}\) continuous function of \(x\); \(a\) and \(b\) are the lowest and highest \(x\) boundaries available at least to one of the considered experiments (\(a = 0.004, b = 0.7\) for the combined analysis of COMPASS and HERMES data we deal with). Of importance is also that, just as in Ref. \([7]\), we apply the advanced approximation of the integral by the sum over bins, which reproduces much better the real integral value than the “middle point” approximation (see discussion around Eq. (16) in Ref. \([7]\)). For the example considered here, it means that we approximate by a constant within the bin only the measured asymmetry \(A_1\):

\[ A_1(x) = \sum_{i=1}^{N_{\text{bins}}} A_1(\langle x_i \rangle) \theta(x - x_{i-1})\theta(x_i - x), \]

where \(A_1(\langle x_i \rangle)\) is the value of asymmetry in \(i\)-th bin, \(x_0 = a, x_{N_{\text{bins}}} = b\) and \(\theta(x)\) is the usual step function, while the known unpolarized input \(F_1(x)\) remains a continuous function within each bin. Thus, instead of the rather crude approximation (“middle point” approximation)

\[ \Gamma_{[a,b]} \simeq \sum_{i=1}^{N_{\text{bins}}} A_1(\langle x_i \rangle) F_1(\langle x_i \rangle) \]

to Eq. (A.1) we use the improved approximation

\[ \Gamma_{[a,b]} = \sum_{i=1}^{N_{\text{bins}}} A_1(\langle x_i \rangle) \int_{x_{i-1}}^{x_i} dx \, F_1(x). \quad (A.2) \]

Let us now consider the case where some bin \([\alpha, \beta]\) of \(exp1\) is covered by two bins \([\alpha, \gamma]\) and \([\gamma, \beta]\) of \(exp2\). Then, to safely combine the respective data with Eqs. (20), (21) we apply the following procedure. First, we artificially divide bin \([\alpha, \beta]\) of \(exp1\) into two bins \([\alpha, \gamma]\), \([\gamma, \beta]\) and assign to each of the two new bins a pseudo-measurement of the asymmetry \(A_1\): \(\tilde{A}_1\). These are \(\tilde{A}_1 \pm \delta \tilde{A}_1\) and \(\tilde{A}_1 \pm \delta \tilde{A}_1\) for the artificial bins \([\alpha, \gamma]\) and \([\gamma, \beta]\), respectively, while the measured asymmetry in the initial real bin \([\alpha, \beta]\) of \(exp1\) is just \(A \pm \delta A\). Then, we impose the natural requirement on our imaginary measurements:

\[ \tilde{A} = \tilde{A}_1 = A. \quad (A.3) \]

Now the pseudo-errors \(\delta \tilde{A}, \delta \tilde{A}_1\) should be properly adjusted. We do it by imposing the necessary condition that the constructed system of pseudo-measurements must be statistically equivalent to the system of real measurements within \(exp1\). Thus, a first relation connecting the errors \(\delta A\) and \((\delta \tilde{A}, \delta \tilde{A}_1)\) is obtained by applying equation (21):

\[ (\delta A)^2 = \frac{1}{1/(\delta \tilde{A})^2 + 1/(\delta \tilde{A}_1)^2}. \quad (A.4) \]

\(^{12}\)It is calculated in NLO QCD by using the known expression for DIS Wilson coefficients, some parametrization on unpolarized PDFs and the known parameterization on \(R\).
At the same time one can see that due to Eq. \((A.3)\) the second equation \((20)\) of the statistical addition just transforms into identity.

A second relation for the errors is obtained by taking into account that the requirement of statistical equivalence should also be satisfied in the calculation of the integral quantity \(\Gamma_{[\alpha,\beta]}\) we are interested in. In the example considered here it means that the quantity (entering the sum in Eq. \((A.2)\)) \(\Gamma_{[\alpha,\beta]}|_I = \tilde{A} \int_\alpha^\beta dx F_1\) and its error \(\delta\Gamma_{[\alpha,\beta]}|_I\) should not change when the bin \([\alpha, \beta]\) of \(exp1\) is divided into two pseudo-bins \([\alpha, \gamma]\) and \([\gamma, \beta]\), from which the same quantity is derived:

\[
\Gamma_{[\alpha,\beta]}|_II = \tilde{\tilde{A}} \int_\gamma^\beta dx F_1 + \tilde{\tilde{A}} \int_\gamma^\beta dx F_1 \pm \delta\Gamma_{[\alpha,\beta]}|_II. \tag{A.5}
\]

The requirement of the central values equality

\[
\Gamma_{[\alpha,\beta]}|_I = \Gamma_{[\alpha,\beta]}|_II \tag{A.6}
\]

is satisfied automatically by virtue of Eq. \((A.3)\), while the requirement

\[
\delta\Gamma_{[\alpha,\beta]}|_I = \delta\Gamma_{[\alpha,\beta]}|_II, \tag{A.7}
\]

is written as

\[
\left(\delta A \int_\alpha^\beta dx F_1\right)^2 = \left(\delta \tilde{A} \int_\gamma^\beta dx F_1\right)^2 + \left(\delta \tilde{\tilde{A}} \int_\gamma^\beta dx F_1\right)^2. \tag{A.8}
\]

Solving the system \((A.4), (A.8)\) one obtains

\[
\delta \tilde{\tilde{A}} = \delta A \sqrt{\frac{\int_\alpha^\beta dx F_1}{\int_\alpha^\beta dx F_1}}, \quad \delta \tilde{A} = \delta A \sqrt{\frac{\int_\gamma^\beta dx F_1}{\int_\gamma^\beta dx F_1}}. \tag{A.9}
\]

We operate in the same way every time when some bins of \(exp1\) and \(exp2\) do not coincide with each other and, after that, safely apply Eqs. \((20), (21)\) to combine the data of both experiments.

It is easy to see that the generalization of the proposed procedure is straightforward in the case of difference SIDIS asymmetries (or in the case of any other asymmetry we deal with). For these asymmetries the role of the integral quantity \(\Gamma\) in the above procedure is played by the integral quantities \(A^{(n)}_p, A^{(n)}_d\) (see \((12)\)), giving direct access to the \(n\)-th moments of valence PDFs we are interested in (see \((11)\)).

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Figure 1: Pion difference asymmetries $A^{+}_{p} - A^{-}_{p}$ and $A^{+}_{d} - A^{-}_{d}$ at $Q^2 = 10\text{GeV}^2$, constructed with Eq. (17) from the COMPASS data on $A^{+}_{p,d}$ in the regions $0.004 < x < 0.7$ and $0.004 < x < 0.3$, respectively.

Figure 2: Pion difference asymmetries $A^{+}_{p} - A^{-}_{p}$ and $A^{+}_{d} - A^{-}_{d}$ at $Q^2 = 10\text{GeV}^2$, constructed with Eq. (17) from the HERMES data on $A^{+}_{p,d}$ in the region $0.023 < x < 0.6$. 
Figure 3: Pion difference asymmetries \( A_{p}^{\pi^+ - \pi^-} \) and \( A_{d}^{\pi^+ - \pi^-} \) at \( Q^2 = 10 \text{ GeV}^2 \), constructed with Eq. \[\text{Eq. (17)}\] from the COMPASS data on \( A_{p,d}^{\pi^\pm} \) in the regions 0.004 < \( x \) < 0.7, 0.004 < \( x \) < 0.3 (for proton and deuteron targets, respectively), and HERMES data on \( A_{p,d}^{\pi^\pm} \) in the region 0.2 < \( x \) < 0.6 (last three bins of HERMES).

Figure 4: Pion difference asymmetries \( A_{p}^{\pi^+ - \pi^-} \) and \( A_{d}^{\pi^+ - \pi^-} \) at \( Q^2 = 10 \text{ GeV} \), constructed with Eq. \[\text{Eq. (17)}\] from the COMPASS and HERMES data on \( A_{p,d}^{\pi^\pm} \) in the entire \( x \)-regions accessible for measurement (0.004 < \( x \) < 0.7, 0.004 < \( x \) < 0.3 for COMPASS and 0.023 < \( x \) < 0.6 for HERMES).