Interacting Chaplygin gas cosmology

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Abstract. In this paper, we study in FRW universe the interaction between the viscous Chaplygin gas and the dark energy in the modified gravity theory F(R,T), where T and R are the trace of the energy-momentum tensor and is the Ricci scalar respectively. In this model, the universe is dominated by both components Chaplygin gas and dark energy. We discuss the effect of the bulk viscosity on the dark energy state parameter, this study was carried for two interactions after obtaining the modified Friedmann equations. Finally, we found that both interaction models lead to quintessence phase of the dark energy.

1. Introduction
The current accelerated expansion of the universe indicated by different recent observations like the supernovae type Ia (SNe Ia) [1, 2], gravitational lensing statistics [3], the cosmic microwave background radiation (CMBR) [4] is created from a mysterious invisible cosmic fluid with negative pressure called dark energy, which constitutes about 70% of the total energy of the universe. The first and simplest model proposed to explain this expansion is the one introducing an cosmological constant Λ as done by Einstein within General Relativity in 1917 as an additional term to his theory in order to make the universe static.

Various alternative candidates have been proposed to describe dark energy scenario such as quintessence which behaves like a vacuum field energy providing the negative pressure (anti-gravitational character) [5], phantom energy which is a perfect fluid with EoS parameter less than −1[6,7], quintom which is another hypothetical scenario for dark energy, with a time-varying equation for the state parameter, which can cross the phantom division[8]. Negative pressure which leads to an accelerated universe can also be obtained in the cosmology of Chaplygin gas[9,10,11,12,13,14,15,16,17].

Chaplygin proposed an exotic equation of state that has emerged for the first time to describe the lifting force on the wing of the aeroplane [18]. It is widely used as a perfect fluid to describe the expansion of the universe obeying an exotic equation of state:

\[ p = -\frac{B}{\rho} \]

Chaplygin gas behaves like a cosmological constant at a later stage and a pressureless fluid for a precocious universe which can be one of among ways to achieve the unification of dark energy and dark matter. Chaplygin gas has been extended to other more flexible models from the point of view of...
of comparison with observational data: generalised Chaplygin gas, modified Chaplygin gas and extended Chaplygin gas.

Moreover, some other methods are introduced to describe the universe’s accelerated expansion, where the advantage of these theories is consistent with recent observations concerning the universe in late stage of acceleration from dark energy. These theories are introduced under the names of gravity F (R), F (T) and F (G), which are respectively an arbitrary function of the Ricci scalar R, the scalar torsion T and the Gauss-Bonnet term G [19]. Harko et al [19] suggested the modified gravity theory F(R,T) which is one of the modified gravity theories proposed as a generalization of general relativity [20,21,22], the theory consists of modifying the Ricci scalar R in Einstein - Hilbert action to an arbitrary function F (R,T) where R is the Ricci scalar and T is the trace of the energy-impulse tensor. Several theoretical models can be obtained corresponding to each choice of F (R, T), functional forms of F (R, T) have been widely used to obtain cosmological solutions; in this work, we have used the simplest and most used form [23]:

\[
\mathcal{F}(R, T) = +2(T)
\]

where \( F(T) \) is an arbitrary function of the trace of the energy-momentum tensor.

In fact, elementary fluid mechanics indicates that a real fluid is never perfect and so we expect that the concept of viscosity will also be important in cosmology. Viscous cosmology is an important theory to describe the evolution of the universe [24]. This means that the presence of viscosity in the fluid introduces many interesting images into the dynamics of homogeneous cosmological models, which is used to study the evolution of the universe. Two main formalisms have been used in the literature: the Eckart formalism, which supposes that the viscous fluid reaches instantaneously the equilibrium, and the causal Muller-Israel-Stewart formalism, which induces a relaxation time [25,26]. In some viscous models and in the absence of a baryonic component, the equations found are equivalent to those of the generalized Chaplygin gas model. Therefore, the viscous model may be a candidate for the unified model for the dark sector and be related to a grand unified theory phase transition [27,28].

With little knowledge of the nature of dark energy and dark matter, it seems unlikely that these two sectors do not interact. So, when trying to explain the accelerated expansion of the universe on a large scale, one of the central ideas of modern cosmology is the non-gravitational interaction between dark energy and dark matter which only interacts by gravity with ordinary matter. On the other hand, there is no restriction on the existence of an interaction between other sources of energy participating to the dynamics of the universe.

So, inspired by all these researches, we study in this work the effect of the viscosity on the interaction between the dark energy and the Chaplygin gas in the F(R, T) gravity theory formalism.

The paper is organized as follows: in the next section we review F(R, T) gravity model and we formulate the modified field equations for a particular model and in the presence of viscosity. Section 3 explores the interaction between Chaplygin gas and dark energy. Finally, we conclude by summarizing the results.

2. Viscous modified gravity cosmology

The action for F(R, T) gravity is given as [19]:

\[
S = \frac{1}{16\pi} \int F(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x
\]

where \( F(R, T) \) is an arbitrary function of the Ricci scalar \( R \), \( L_m \) the Lagrangian density and \( T \) is the trace of the energy-momentum tensor of the matter \( T_{\mu\nu} \) which is given by:

\[
T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \sqrt{-g} L_m}{\delta g^{\mu\nu}}
\]

The Variation of the action \( S \) with respect to the metric tensor components \( g_{\mu\nu} \) is:
\[
\delta = \frac{1}{16\pi} \int \left[ F_R(\ ,\ ) \delta + F_T(\ ,\ ) \frac{\partial^T}{\partial g_{\mu\nu}} \delta g^{\mu\nu} - \frac{1}{2} g^{\mu\nu} (\ ,\ ) \delta g^{\mu\nu} + 16\pi \frac{\delta \sqrt{g} L_m}{\sqrt{-g}} \right] \sqrt{-g} d^4x \tag{3}
\]

where \( F_R(R,T) \) and \( F_T(R,T) \) are written as:

\[
F_R(R,T) = \frac{\partial F(R,T)}{\partial R} \tag{4}
\]

\[
F_T(R,T) = \frac{\partial F(R,T)}{\partial T} \tag{5}
\]

We obtain the following gravitational field equations form:

\[
F_R(R,T) R_{\mu\nu} - \frac{1}{2} F(R,T) g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_{\mu} \nabla_{\nu}) F_R(R,T) = 8\pi T_{\mu\nu} - F_T(R,T) T_{\mu\nu} - F(R,T) \Theta_{\mu\nu} \tag{6}
\]

where:

\[
\Theta_{\mu\nu} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g_{\mu\nu}} = -2 T_{\mu\nu} + g_{\mu\nu} L_m - 2 g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g^{\nu\alpha} \partial g^{\mu\beta}} \tag{7}
\]

The field equations become for the simplest and most used form of \( F(R,T) \) mentioned in the introduction:

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} - 2 F_T(T_{\mu\nu} + \Theta_{\mu\nu}) + g_{\mu\nu} (\ ) \tag{8}
\]

Therefore, the modified Friedmann equations are as follows:

\[
3H^2 = \rho + ( ) + 2 F_T(\rho + ) \tag{9}
\]

\[
-2 \dot{H} - 3H^2 = p - F(T) \tag{10}
\]

The energy-momentum tensor for the viscous fluid is given by:

\[
T_{\mu\nu} = (\rho + p_{eff}) u_\mu u_\nu - p_{eff} g_{\mu\nu} \tag{11}
\]

The effective pressure is written as:

\[
p_{eff} = + \Pi \tag{12}
\]

where \( \Pi \) is the bulk viscous pressure in terms of bulk viscous coefficient \( \xi \):

\[
\Pi = -3\xi H \tag{12}
\]

The modified Friedmann equations become as follows:

\[
3H^2 = \rho + ( ) + 2 F_T(\rho + p_{eff}) = \rho_{tot} \tag{13}
\]

\[
-2 \dot{H} - 3H^2 = p_{eff} - F(T) = p_{tot} \tag{14}
\]

3. Interacting Chaplygin gas
We study in the framework of FRW universe, a model based on a Chaplygin gas with the following equation of state \[ p = -\frac{B}{\rho} \] (15)

where \( B \) is positive constant. This type of fluid was first used in fluid mechanics and describes the airflow near the wing of an aircraft. It is important to mention a remarkable characteristic of the Chaplygin gas, namely that the squared speed of sound:

\[ v_s^2 = \frac{\partial p}{\partial \rho} = \frac{B}{\rho^2} \] (16)

is positive. This is a non-trivial fact for fluids with negative pressure. Indeed, Chaplygin gas has a good link with string theory and can be obtained from the Nambu-Goto action for d-branes moving in a \((d + 2)\) dimensional space-time in the parametrization of the light cone [30].

We now consider a model of universe containing both Chaplygin gas and dark energy components:

\[ \rho_{\text{tot}} = \rho + \rho_{\text{DE}} \] (17)

\[ p_{\text{tot}} = p_{\rho} + p_{\text{DE}} \] (18)

Consequently, within the last modified Friedmann equations, energy density and pressure of dark energy become:

\[ \rho_{\text{DE}} = \rho_{\text{eff}} + 2F_T(\rho + p_{\text{eff}}) \] (19)

\[ p_{\text{DE}} = -F(T) - 3\xi H \] (20)

Then, the dark energy EoS becomes:

\[ \omega_{\text{DE}} = \frac{p_{\text{DE}}}{\rho_{\text{DE}}} = \frac{-F(T) - 3\xi H}{\rho_{\text{eff}} + 2F_T(\rho + p_{\text{eff}})} \] (21)

3.1. Interaction \( Q = 3Hb^2\rho \)

The continuity equations are:

\[ \dot{\rho}_{\text{DE}} + 3H(\rho_{\text{DE}} + \rho_{\text{eff}}) = -Q \] (22)

\[ \rho = 3(\rho + \rho) \] (23)

where \( Q \) the interaction term between the universe components which can take different forms, and \( b \) is the coupling constant [31].

From the last two equations, the energy density and pressure of Chaplygin gas can be written as:

\[ \rho = \left( c_0a^{ax} + \frac{B}{r} \right)^{\frac{1}{2}} \] (24)
\[ p = -\frac{b}{(a^{-6k} + b)^{\frac{1}{2}}} \]  \hspace{1cm} (25)

where:
\[ k = 1 - b \] \hspace{1cm} (26)

By substituting these equations in (19) and (20) and taking the particular case of the scale factor in terms of cosmic time as:
\[ \rho \propto a^\tau \] \hspace{1cm} (27)

and also writing:
\[ F(T) = \chi T^\lambda \] \hspace{1cm} (28)

we obtain the densities and the pressures as:
\[ \rho_{DE} = \chi (\rho - 3p + 9\xi H)^{\lambda-1} (\rho + 2\lambda + (2\lambda - 3)(-3\xi + )) \] \hspace{1cm} (29)
\[ p_{DE} = -\chi (\rho - 3p + 9\xi H)^\lambda - 3\xi H \] \hspace{1cm} (30)

The densities, pressures and EoS graphs versus the cosmic time for the first interaction model are shown in Figure 1,2 and 3.

**Figure 1.** Plot of dark energy density and pressure versus cosmic time for \( B = 2, c = 1, \lambda = 1, \chi = 1, k = 2, \tau = 2, \xi = 0.01\text{ (green line)}, \xi = 0.05\text{ (Red line)}, \xi = 0.09\text{ (black line)}.\)

We note that the total and dark energy densities decrease, whereas the total and dark energy pressures increase with time. We notice also that the dark energy density and the total density increase while the dark energy pressure and total pressure decrease with bulk viscosity.
From the stat parameter graphs, we see that even by increasing the model parameter (viscosity \( \xi \)), the graphs don’t cross the phantom division but stay in the phantom area, so we can conclude that this interaction model behaves as the phantom dark energy model.

**Figure 2.** Plot of density and total pressure as a function of cosmic time for \( \gamma = 2, \ \lambda = 1, \chi = 1, \ \tau = 2, \ \xi = 0.01 \) (green line), \( \xi = 0.05 \) and \( \xi = 0.09 \).

**Figure 3.** Plot of the state parameters versus cosmic time for \( \gamma = 2, \ \lambda = 1, \chi = 1, \ \tau = 2, \ \xi = 0.01 \) (green line), \( \xi = 0.05 \) and \( \xi = 0.09 \).
3.2. Interaction

Unlike the first interaction term which is either positive or negative and cannot change the sign, this is not the case with this form of $Q$ where $\mu$ and $b$ are constants, and $q$ the deceleration parameter given as:

$$\frac{-\ddot{a}}{aH^2}$$

This type of interaction can change the sign from negative to positive due to the change of the universe phase from a deceleration expansion to an acceleration expansion. [32,33].

From the continuity equations, the energy density and pressure of Chaplygin gas are:

$$\rho_{MCG} = \left( c_0 t \frac{-2a_1a_3}{a_1} + B \frac{a}{a_1} \right)$$

$$p_{MCG} = \frac{B}{\left( c_0 t \frac{-2a_1a_3}{a_1} + B \frac{a}{a_1} \right)^\frac{1}{2}}$$

where $a_1 = 1 + \mu - \frac{1}{\tau}$, $a_2 = 3\tau$ et $a_3 = 1 + b - \frac{b}{\tau}$. Using the same form for $F(R)$ and the scale factor $a(t)$, we get the EoS parameter.

The EoS parameter graphs versus the cosmic time for the second interaction model are shown in Figure 4. The density, pressure and EoS parameter of the dark matter, as well as the total density, total pressure, and total parameter and even the effect of the bulk viscosity, behave similarly as the first interaction model. The model stays in the phantom area even when increasing the bulk viscosity and it doesn't cross the phantom division line.

**Figure 4.** Plot of state parameter versus cosmic time for $\mu = 2, \lambda = 1, \chi = 1, \psi = 0.05$, and $\phi = 0.09$. (green line), (red line) $\xi = 0.01$ (green line), $\xi = 0.05$ (red line) $\xi = 0.09$ (red line).
4. Conclusion
In this paper, we have investigated the interaction between the dark energy and Chaplygin gas in the framework of the modified gravity $F(R,T)$ in the presence of bulk viscosity. For simplicity, the function $F(R,T)$ has been taken as $R+2F(T)$. Afterwards, we wrote the corresponding time-dependent pressure and density of the dark energy and Chaplygin gas for the power law of the scale factor after having gotten the modified Friedmann equations. And finally, based on the positive value of the density and negativity of pressure of the dark energy, we have discussed the effect of the bulk viscosity for two interacting models.

We have found that the increase in bulk viscosity leads to a decrease in dark energy pressure and an increase in dark energy density, while the total EoS parameter tends to the phantom phase for both interactions.

Finally, we conclude that the interaction between the dark energy and Chaplygin gas in the framework of the modified gravity $F(R,T)$ for the power law of the scale factor and the Friedmann–Robertson–Walker universe leads to the phantom phase of dark energy.

5. References
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