Dense baryonic matter in the hidden local symmetry approach:  
Half-skyrmions and nucleon mass

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Hadron properties in dense medium are treated in a unified way in a skyrmion model constructed with an effective Lagrangian, in which the ρ and ω vector mesons are introduced as hidden gauge bosons, valid up to $O(p^4)$ terms in chiral expansion including the homogeneous Wess-Zumino terms. All the low energy constants of the Lagrangian – apart from the pion decay constant and the vector meson mass – are fixed by the master formula derived from the relation between the five-dimensional holographic QCD and the four-dimensional hidden local symmetry Lagrangian. This Lagrangian allows one to pin down the density $n_{1/2}$ at which the skyrmions in medium fractionize into half-skyrmions, bringing in a drastic change in the equation of state of dense baryonic matter. We find that the $U(1)$ field that figures in the Chern-Simons term in the five-dimensional holographic QCD action or equivalently the ω field in the homogeneous Wess-Zumino term in the dimensionally reduced hidden local symmetry action plays a crucial role in the half-skyrmion phase. The importance of the ω degree of freedom may be connected to what happens in the instanton structure of elementary baryon noticed in holographic QCD. The most striking and intriguing in what is found in the model is that the pion decay constant that smoothly drops with increasing density in the skyrmion phase stops decreasing at $n_{1/2}$ and remains nearly constant in the half-skyrmion phase. In accordance with the large $N_c$ consideration, the baryon mass also stays nonscaling in the half-skyrmion phase. This feature which is reminiscent of the parity-doublet baryon model with a chirally invariant mass $m_0$ is supported by the nuclear effective field theory with the parameters of the Lagrangian scaling modified at the skyrmion–half-skyrmion phase transition. It also matches with one-loop renormalization group analysis based on hidden local symmetry. A link between a nonvanishing $m_0$ and the origin of nucleon mass distinctive from dynamically generated mass is suggested. We briefly discuss the possible consequences of the topology change found in this paper on the forthcoming experiments at the rare isotope beam machines under construction.

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I. INTRODUCTION

Understanding the QCD phase structure at high density is a wide-open problem in nuclear and particle physics as it involves the highly nonperturbative region of strong interactions so far unaccessible by lattice gauge calculations. Any theoretical investigation of the QCD phase structure, whether it is at the level of models or effective field theories or more fundamental nature, would be highly desirable. In this paper, we investigate what happens to hadrons at high density in the theoretical framework developed in Refs. 1,2. To access dense baryonic matter, we exploit the approach proposed in Ref. 3, where both the elementary baryons and multibaryon systems are described in a unified way with a single effective mesonic Lagrangian with appropriate symmetries of QCD, the most prominent of which is chiral symmetry, from which baryons are generated as solitons (called skyrmions) and many-body interactions are taken into account with the given chiral Lagrangian.

In the past, for example in Ref. 4, dense matter was studied with the Skyrme model put on crystal lattice with pion fields only 5 or minimally implemented with the lowest-lying vector mesons $\rho$ and $\omega$ that will be referred to in what follows as HLS$_{\text{min}}(\pi, \rho, \omega)$. There are, however, no reasons to think that such models would succeed to describe adequately, not to mention quantitatively, dense baryonic matter. The Skyrme model has the leading current algebra term which can account for very low-energy dynamics but the Skyrme quartic term that is crucial in stabilizing the soliton is neither unique nor sufficient. Given other degrees of freedom that can enter, there can be a variety of different ways of stabilizing

1 This terminology will be defined precisely below.
the vector meson mass $m$.

A great advantage: Once the pion decay constant $f_\pi$ is known, the approximation used to simplify a part of the hidden local symmetry Lagrangian, namely the homogeneous Wess-Zumino (hWZ) term, is unjustified when the vector meson mass could decrease in density, which is the basic premise of hidden local symmetry (HLS) theory [6, 7].

In this paper, we approach the problem with the five-dimensional Yang-Mills action in curved space plus the Chern-Simons term that has been arrived at both dimensionally deconstructed, bottom-up, starting from the current algebra term [6] and, top-down, from gravity-gauge duality in string theory [10, 11]. Kaluza-Klein-reduced to four dimensions, this theory consists of an infinite tower of hidden local symmetric gauge fields. Our assumption is that this is a theory of QCD type valid in the large $N_c$ limit and in the case of the top-down model, large $\lambda$ Hooft $\lambda \equiv g^2 N_c/\text{limit}$. One may ultimately use the five-dimensional action for dense baryonic matter as was done in Refs. [12–14] for the single baryon generated as an instanton. It is not yet known how to do this for dense baryonic matter although there has been an attempt toward that direction [15, 16]. As the first step, we follow the strategy of Refs. [17, 18] to integrate out all but the lowest vector mesons $\rho$ and $\omega$ of the infinite tower of vector and axial vector mesons, making sure that hidden local symmetry is preserved up to $O(p^3)$ in derivative expansion. The structure of single baryon has been worked out in Refs. [1, 2] using the holographic QCD model of Sakai and Sugimoto (SS for short) [10, 11] and also with flat space in the SS model. This brings about a drastic simplification from the infinite-tower theory but it has a great advantage: Once the pion decay constant $f_\pi$ and the vector meson mass $m_\pi$ are fixed from experiments, there are no undetermined parameters. As explained in Ref. [2], this comes about by the “master formula” which allows us to connect all the low-energy constants (LECs) of the (chiral) Lagrangian to the two constants $f_\pi$ and $m_\pi$.

The most important feature that is found in Refs. [1, 2] was that the $O(p^4)$ terms in the Lagrangian, in particular the hWZ terms that carry information on the $\omega$ meson, could not be approximated by only a few terms – not to mention ignored – for the nucleon structure. This observation suggested that the role of vector mesons, particularly the $\omega$, would be crucial in dense baryonic matter. Indeed this will be what we will find in the present work [1].

The objective of this paper is to put the skyrmions constructed as described in Refs. [1, 2] on the crystal lattice as was done for the Skyrme model. Before stating our principal result, we should mention a possible shortcoming in our model. When an HLS Lagrangian is implemented with baryons (that we shall refer to as BHLS), the theory in principle should give at the mean field approximation a relativistic mean-field model that describes nuclear matter properties as in the Walecka mean-field model. However, as is well known, it is essential that there be a scalar meson in the theory giving rise to the necessary attraction for binding [19]. (See Ref. [20] and references given therein.) Such a scalar, identified in Ref. [20] as “dilaton,” is missing in the HLS framework used here. How to introduce a scalar in the theory is highly problematic so we are ignoring it in this paper but it may not be justified as we know from the original Skyrme model that a correct nucleon mass can be obtained only when the Casimir contribution – which is $O(N_c^0)$ and captured in chiral perturbation theory at higher loop orders in the scalar channel – is taken into account. As we will discuss, some of the attraction is generated by the $\omega$-$\rho$ coupling in the crystal. However the missing dilaton degree of freedom may still be needed for more reliable calculations. We will return to this matter in the last section.

The principal finding in this work is that the effective pion decay constant in medium $f_\pi^2$ decreases smoothly up to the density $n_{1/2}$ at which a skyrmion that has a baryon number one configuration fractionizes into two half-skyrmions each of which has half a baryon number, and then stays nonzero, more or less constant up to the chiral restoration density $n_c$. The notable consequence is that the effective nucleon mass $m_N^*$ also stops scaling for $n > n_{1/2}$. This is consistent with the large $N_c$ consideration. In the large $N_c$ limit, a nucleon in medium as a skyrmion scales as $f_\pi^2$ and hence remains more or less constant up to $n_c$. What this means is that the nucleon mass could have a component that remains nonvanishing up to the chiral transition, which is reminiscent of the parity doublet picture of the baryon [21] with the nucleon mass given by

$$m_N = m_0 + \Delta(\langle\bar{q}q\rangle), \tag{1}$$

where $m_0$ is the chiral invariant mass and $\Delta$ is the part of the mass that vanishes as $\langle\bar{q}q\rangle \to 0$ (in the chiral limit) for $n \to n_c$. The same result was obtained in a renormalization group (RG) analysis of hidden local symmetry Lagrangian with baryons in Ref. [20] and also in a phenomenological study of the equation of state (EoS) for dense compact-star matter in Ref. [22].

That $m_0 \neq 0$ implies that there is a part of the nucleon mass that is not generated by spontaneous breaking of

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2 Note that the curvature is fixed by the gravity in the latter while undetermined in the former. As will be specified, we will deal with a “master formula” for any curvature.

3 As will be shown explicitly, the minimal version of the HLS model

HLS$_{\text{min}}(\pi, \rho, \omega)$ exemplifies that it is a poor approximation to pick only one term out of three that are in the hWZ action.
chiral symmetry. The question then would be whether $m_0$ reflects a fundamental feature of QCD as suggested in Refs. \[23\] or an emergent symmetry via correlations in medium as in condensed matter as indicated in this crystal calculation. This raises the issue of the origin of the nucleon mass as we shall discuss below.

This paper is organized as follows. In Sec. II we briefly sketch the derivation of the HLS Lagrangian from holographic models that we are using in the exploration of dense matter. In Sec. III we first discuss the main considerations in the investigation of the dense skyrmion matter using the face-centered cubic (FCC) crystal in the framework of HLS, then provide our numerical results of dense skyrmion matter and discuss their physical implications. We study the in-medium properties of hadrons with FCC crystal as the background in Sec. IV. In Sec. V we match our results to those obtained in (1) nuclear effective field theory (EFT) focused on the EoS for compact star matter and (2) RG-analysis of HLS theory with baryons. Further remarks relevant to the physics of rare isotope beam (RIB) facilities and conclusions are given in Sec. VI.

II. HIDDEN LOCAL SYMMETRY AND SINGLE SKYRMION SOLUTION

We briefly describe the HLS Lagrangian that we shall use in exploring dense matter. The full symmetry group of the effective Lagrangian is $G_{\text{full}} = [SU(2)_L \times SU(2)_R]_{\text{chiral}} \times [U(2)]_{\text{HLS}}$. Here, $U(2)]_{\text{HLS}}$ is adopted as the “hidden local symmetry” to incorporate the $\rho$ and $\omega$ vector mesons. The building blocks of the HLS Lagrangian are the two 1-forms $\hat{\alpha}_{\parallel \mu}$ and $\hat{\alpha}_{\perp \mu}$ defined by

\[
\hat{\alpha}_{\parallel \mu} = \frac{1}{2i} (D_\mu \xi_R \xi_1^R + D_\mu \xi_L \xi_1^L), \\
\hat{\alpha}_{\perp \mu} = \frac{1}{2i} (D_\mu \xi_R \xi_1^R - D_\mu \xi_L \xi_1^L),
\]

with the chiral fields $\xi_L$ and $\xi_R$, which are expressed in the unitary gauge as

\[
\xi^\dagger_L = \xi_R = e^{i\pi/2f_\pi} \equiv \xi \quad \text{with} \quad \pi = \pi \cdot \tau,
\]

where $\tau$'s are the Pauli matrices. The covariant derivative is defined as

\[
D_\mu \xi_{R,L} = (\partial_\mu - iV_\mu)\xi_{R,L}
\]

with $V_\mu$ being the gauge boson of the HLS. This is the way to introduce vector mesons in the HLS, where the vector meson field $V_\mu$ is

\[
V_\mu = \frac{g}{2} (\omega_\mu + \rho_\mu)
\]

and $\rho_\mu = \rho \cdot \tau \equiv \left( \frac{\rho^0}{\sqrt{2}\rho^\tau} \sqrt{2}\rho^\tau - \rho^0 \right)$. Up to $O(p^4)$ including the hWZ terms, the most general HLS Lagrangian can be expressed as

\[
\mathcal{L}_{\text{HLS}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_{\text{anom}},
\]

with

\[
\mathcal{L}_2 = \frac{f_\pi^2}{2} \text{Tr} (\hat{\alpha}_{\perp \mu} \hat{\alpha}_{\parallel \mu}) + af_\pi^2 \text{Tr} (\hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \mu})
\]

\[
- \frac{1}{2g^2} \text{Tr} (V_{\mu \nu} V^{\mu \nu}),
\]

where $f_\pi$ is the pion decay constant, $a$ is the parameter of the HLS, $g$ is the vector meson coupling constant, and the field-strength tensor of the vector meson is

\[
V_{\mu \nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu].
\]

In the most general form of the $O(p^4)$ Lagrangian there are several terms that include two traces in the flavor space such as the $y_{10}^\mu y_{18}^\nu$ terms listed in Ref. \[8\]. These terms are suppressed by $N_c$ compared to the other terms in the Lagrangian and are not considered in the present work. Then the $O(p^4)$ Lagrangian which we study in this paper is given by

\[
\mathcal{L}_4 = \mathcal{L}_4(y) + \mathcal{L}_4(z),
\]

where

\[
\mathcal{L}_4(y) = y_1 \text{Tr} \left[ \hat{\alpha}_{\perp \mu} \hat{\alpha}_{\perp \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} + y_2 \text{Tr} \left[ \hat{\alpha}_{\perp \mu} \hat{\alpha}_{\perp \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} \right] + y_3 \text{Tr} \left[ \hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} \right] + y_4 \text{Tr} \left[ \hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} \right]
\]

\[
+ y_5 \text{Tr} \left[ \hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} \right] + y_6 \text{Tr} \left[ \hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} \right] + y_7 \text{Tr} \left[ \hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} \right] + y_8 \left\{ \text{Tr} \left[ \hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} \right] \right\} + y_9 \text{Tr} \left[ \hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} \right],
\]

\[
\mathcal{L}_4(z) = z_1 \text{Tr} \left[ \hat{\alpha}_{\parallel \mu} \hat{\alpha}_{\parallel \nu} \hat{\alpha}_{\parallel \rho} \hat{\alpha}_{\parallel \sigma} \right],
\]

in what we obtain here.

4 Here for simplicity we are assuming that flavor $U(2)$ symmetry applies to the $\rho$ and $\omega$ in medium as it seems to do in free space. It is found by the RG analysis in Ref. \[23\] that the $U(2)$ must be broken down appreciably in medium, so there may be a caveat

5 Another example of this kind is $\text{Tr} \left[ \hat{\alpha}_{\parallel \mu}^\dagger \text{Tr} \left[ \hat{\alpha}_{\parallel \mu} \right] \right]$ that generates the mass difference between the $\rho$ and $\omega$ mesons.
Finally, the anomalous parity hWZ terms \( \mathcal{L}_{\text{anom}} \) are written as

\[
\Gamma_{\text{hWZ}} = \int d^4x \mathcal{L}_{\text{anom}} = \frac{N_c}{16\pi^2} \int_M \sum_{i=1}^3 c_i \mathcal{L}_i, \tag{13}
\]

where \( M^4 \) stands for the four-dimensional Minkowski space and

\[
\mathcal{L}_1 = i \text{Tr} \left[ \hat{\alpha}_L^2 \hat{\alpha}_R - \hat{\alpha}_R^2 \hat{\alpha}_L \right], \tag{14a}
\]
\[
\mathcal{L}_2 = i \text{Tr} \left[ \hat{\alpha}_L \hat{\alpha}_R \hat{\alpha}_L \hat{\alpha}_R \right], \tag{14b}
\]
\[
\mathcal{L}_3 = \text{Tr} \left[ F_V (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L) \right], \tag{14c}
\]

in the 1-form and 2-form notations with

\[
\hat{\alpha}_L = \hat{\alpha}_\parallel - \hat{\alpha}_\perp, \quad \hat{\alpha}_R = \hat{\alpha}_\parallel + \hat{\alpha}_\perp, \quad F_V = dV - iV^2. \tag{15}
\]

As for the low energy constants \( f_\pi, a, g, y_1, z_1, \) and \( c_i \), we use the general “master formula” proposed in Refs. \[1, 2\] from a class of holographic QCD models, namely,

\[
f_\pi^2 = N_c G_{\text{YM}} M_{KK}^2 \int dz K_2(z) \left( \bar{\psi}_0(z) \right)^2, \tag{16a}
\]
\[
a f_\rho^2 = N_c G_{\text{YM}} M_{KK}^2 \lambda_1 (\bar{\psi}_1), \tag{16b}
\]
\[
\frac{1}{g^2} = N_c G_{\text{YM}} (\bar{\psi}_1), \tag{16c}
\]
\[
y_1 = -y_2 = -N_c G_{\text{YM}} \left( 1 + \psi_1 - \psi_0^2 \right), \tag{16d}
\]
\[
y_3 = -y_4 = -N_c G_{\text{YM}} \left( \psi_1^2 + 1 + \psi_1^2 \right), \tag{16e}
\]
\[
y_5 = 2y_8 = -y_9 = -2N_c G_{\text{YM}} \left( \psi_1^2 \psi_0^2 \right), \tag{16f}
\]
\[
y_6 = - (y_3 + y_4), \tag{16g}
\]
\[
y_7 = 2N_c G_{\text{YM}} \left( \psi_1 (1 + \psi_1) (1 + \psi_1 - \psi_0^2) \right), \tag{16h}
\]
\[
z_4 = 2N_c G_{\text{YM}} \left( \psi_1 (1 + \psi_1 - \psi_0^2) \right), \tag{16i}
\]
\[
z_5 = -2N_c G_{\text{YM}} \left( \psi_1^2 (1 + \psi_1) \right), \tag{16j}
\]
\[
c_1 = \left( \bar{\psi}_0 \psi_0 \left( \frac{1}{2} \bar{\psi}_0^2 + \frac{1}{6} \psi_1^2 - 1 \right) \right), \tag{16k}
\]
\[
c_2 = \left( \bar{\psi}_0 \psi_0 \left( - \frac{1}{2} \bar{\psi}_0^2 + \frac{1}{6} \psi_1^2 + \frac{1}{2} \psi_1 + \frac{1}{2} \right) \right), \tag{16l}
\]
\[
c_3 = \left( \frac{1}{2} \bar{\psi}_0^2 \right), \tag{16m}
\]

where the smallest nonzero eigenvalue \( \lambda_1 \) and its corresponding eigenfunction \( \psi_1(z) \) satisfy the eigenvalue equation

\[
- K_1^{-1}(z) \partial_z \left[ K_2(z) \partial_z \psi_n(z) \right] = \lambda_n \psi_n(z), \tag{17}
\]

with \( K_1(z) \) and \( K_2(z) \) being the warping factors of the five-dimensional space-time. In Eq. (16) we have made the following definitions:

\[
\langle A \rangle = \int_{-\infty}^{\infty} dz K_1(z) A(z), \tag{18a}
\]
\[
\langle \langle A \rangle \rangle = \int_{-\infty}^{\infty} dz A(z). \tag{18b}
\]

In the present work, we take as in Refs. \[1, 2\] \( K_1(z) = K^{-1/2}(z) \) and \( K_2(z) = K(z) \) with \( K(z) = 1 + z^2 \) corresponding to the Sakai-Sugimoto model \[10, 11\]. Therefore, once the hQCD parameters \( M_{KK} \) and \( G_{YM} \) are determined by using the real world parameters, for example, such as \( f_\pi \) and \( m_\rho \), all the rest of the parameters of the effective Lagrangian except the parameter \( a \) are fixed through the master formula given by Eq. (10). In this work, we will work with the parameter set determined by the empirical values,

\[
f_\pi = 92.4 \text{ MeV}, \quad m_\rho = 775.5 \text{ MeV}. \tag{19}
\]

Note that due to the relations between \( y_i \)'s, the \( \omega \) meson can couple to the others only through the hWZ terms. We will denote the \( \omega \)-decoupled HLS Lagrangian with \( c_i = 0 \) for all \( i \) as HLS(\( \pi, \rho \)) in order to distinguish it from the full Lagrangian HLS(\( \pi, \rho, \omega \)). If we keep only the terms up to \( \mathcal{O}(p^2) \) and only one term among the hWZ action that results if one takes the equation of motion for the \( \rho \) meson and the large \( m_\rho \) limit in the hWZ action in Eq. (14) and use the same values for the parameters, the Lagrangian is reduced to the “minimum model” adopted in Ref. \[27\], which is denoted as HLS_{\text{min}}(\( \pi, \rho, \omega \)).

In Refs. \[1, 2\], the single skyrmion solution was found under the hedgehog configuration for various parameter sets. For HLS(\( \pi, \rho, \omega \)) with the parameters determined by Eq. (14), the soliton mass \( M_{\text{sol}} \) is 1184 MeV and the root mean square winding number radius \( \sqrt{r^2_W} \) is 0.433 fm. Compared to the HLS_{\text{min}}(\( \pi, \rho, \omega \)) which yields \( M_{\text{sol}} = 1407 \) MeV and \( \sqrt{r^2_W} = 0.540 \) fm, we can see that the \( \mathcal{O}(p^4) \) terms provide about 300 MeV reduction in the soliton mass and about 0.1 fm reduction in the size. This means that the \( \pi-\rho \) interactions incorporated through the \( \mathcal{O}(p^4) \) terms and the \( \pi-\rho-\omega \) interactions through the hWZ terms provide such an effective attraction. As we will see later, the reduction in the size of the skyrmion plays the most important role in determining the critical density \( n_{1/2} \), where the half-skyrmion phase transition happens.

On the other hand, if the \( \omega \) is decoupled, \( M_{\text{sol}} \) is reduced to 834 MeV and the skyrmion has a very small size as \( \sqrt{r^2_W} = 0.247 \) fm. This emphasizes the role of the repulsive \( \omega \) meson in the HLS Lagrangian.

\[ \text{As mentioned, this limit is at odds with the basic premise of HLS [3].} \]
III. SKYRMION MATTER FROM THE HIDDEN LOCAL SYMMETRY

Since the skyrmions are classical solitons, they provide a natural framework to study dense matter. We can make the dense skyrmion matter by putting more and more skyrmions into the space without worrying about such a serious problem as the sign problem in the lattice QCD. We shall consider an FCC crystal made up of skyrmions. Such crystal configuration is known to give the lowest ground-state energy among the crystal symmetries studied so far \cite{25,26}. In the lowest energy configuration, the skyrmion at each lattice site should be arranged in such a way that the nearest skyrmions have the maximum attraction. In the study with the original Skyrme model, it turns out that two skyrmions apart from each other have the maximum attraction when one of them is rotated in the isospin space with respect to the other by an angle $\pi$ about the axis perpendicular to the line joining them. In Ref. \cite{27}, it was shown in a simple approximation that such a configuration is also most attractive in the model of HLS$_{\text{min}}(\pi,\rho,\omega)$. If we assume that it holds in the case of HLS($\pi,\rho$) and HLS($\pi,\rho,\omega$), then the FCC crystal configuration for the $\pi,\rho,$ and $\omega$ should have the symmetries with respect to the reflection and translations as summarized in Table II.

The classical solutions for $\pi$, $\rho$, and $\omega$ mesons satisfying the symmetries for the maximum attraction and the constraints for each FCC box to carry a given baryon number can be obtained by using the Fourier expansion method developed in Refs. \cite{24,25,26} for the original Skyrme model with the pions only, then generalized in Ref. \cite{27} for the model with vector mesons. The main idea is to introduce the unnormalized fields $\bar{\phi}_M^\alpha (M = \pi, \rho)$, each of which can be expanded in terms of the Fourier series as

$$
\bar{\phi}_0^M = \sum_{abc} \beta_{abc}^M \cos(\pi ax/L) \cos(\pi by/L) \cos(\pi cz/L),
$$

$$
\bar{\phi}_1^M = \sum_{hkl} \alpha_{hkl}^M \sin(\pi hx/L) \cos(\pi ky/L) \cos(\pi lz/L),
$$

and thereafter normalized as

$$
\phi_\alpha^M = \frac{\bar{\phi}_\alpha^M}{\sqrt{\sum_\beta (\bar{\phi}_\beta^M)^2}},
$$

for each $M = \pi, \rho$. Here, $2L$ is the size of a single FCC box containing four skyrmions inside. Thus, the skyrmion number density $n$ is given by $n = 1/(2L^3)$. If we take skyrmions as nucleons, the normal nuclear matter density $n_0$ is 0.17 fm$^{-3}$ corresponds to $L \sim 1.43$ fm.

Finally, $U(\equiv \xi^2)$ and $\rho^a_\sigma$ can be expressed in terms of these normalized fields as

$$
U = \phi_0^\pi + i \tau \cdot \bar{\phi}^\pi, \quad g\rho^a_\sigma = \frac{\varepsilon_{abc} \phi^\pi_\sigma \partial_i \phi^\rho_\sigma}{1 + \phi_0^\rho}. \quad (22)
$$

The isoscalar field $\omega_\parallel$ has the same symmetry properties as the $\phi_0^\pi$ and therefore could be expanded as

$$
\omega_\parallel = \sum_{abc} \beta_{abc}^\omega \cos(\pi ax/L) \cos(\pi by/L) \cos(\pi cz/L) \cos(\pi \omega_\parallel/L),
$$

with the expansion coefficients $\beta_{abc}^\omega$.

The Fourier coefficients and their indices are subject to the topological structure and the constraints from the symmetry properties listed in Table II. From the symmetry properties one can easily check that in the above equations $(a,b,c)$ should be all even or all odd integers, and $(k,l)$ should be all even (odd) if $h$ is odd (even). Furthermore, to provide the correct topological structure (winding number) to the configuration, the expansion coefficients $\beta_{abc}^\sigma$ must satisfy the constraint,

$$
\sum_{\text{even}} \beta_{abc}^\omega = 0. \quad (24)
$$

As for the pion and $\rho$ field configurations, all the Fourier coefficients are determined by minimizing the skyrmion energy per baryon, $E/B$, which, from the Lagrangian \cite{27}, is expressed as

$$
E/B = (E/B)_{O(\rho^1)} + (E/B)_{O(\rho^2)} + (E/B)_{h\omega z},
$$

where

$$
(E/B)_{O(\rho^2)} = \frac{1}{4} \int_{\text{Box}} d^3 x \left[ \frac{f_2^2}{2} (\bar{\alpha}_{-i} \cdot \bar{\alpha}_{-i}) - \frac{m_2^2}{2} \omega^2 + \frac{m_2^2}{2g^2} (\bar{\alpha}_{||i} \cdot \bar{\alpha}_{||i}) - \frac{1}{2} \partial_j \omega_\parallel \partial_j \omega_\parallel + \frac{1}{2} g^2 V_{ij} \cdot V_{ij} \right], \quad (26)
$$

$$
(E/B)_{O(\rho^1)} = \frac{1}{4} \int_{\text{Box}} d^3 x \left[ \frac{y_1}{4} (\bar{\alpha}_{-i} \times \bar{\alpha}_{-j}) \cdot (\bar{\alpha}_{-i} \times \bar{\alpha}_{-j}) + \frac{y_3}{4} (\bar{\alpha}_{||i} \times \bar{\alpha}_{||j}) \cdot (\bar{\alpha}_{||i} \times \bar{\alpha}_{||j}) + \frac{y_5}{4} \left( (\bar{\alpha}_{-i} \times \bar{\alpha}_{||j}) \cdot (\bar{\alpha}_{-i} \times \bar{\alpha}_{||j}) - (\bar{\alpha}_{-i} \times \bar{\alpha}_{||j}) \cdot (\bar{\alpha}_{-i} \times \bar{\alpha}_{||j}) \right) \right.
$$

$$
+ \frac{y_4}{4} \left( (\bar{\alpha}_{-i} \times \bar{\alpha}_{-j}) \cdot (\bar{\alpha}_{||i} \times \bar{\alpha}_{||j}) - \frac{z_4}{2} V_{ij} \cdot (\bar{\alpha}_{-i} \times \bar{\alpha}_{-j}) - \frac{z_5}{2} V_{ij} \cdot (\bar{\alpha}_{||i} \times \bar{\alpha}_{||j}) \right), \quad (27)
$$

and

$$
\bar{\phi}_0^M = \sum_{hkl} \alpha_{hkl}^M \sin(\pi hx/L) \cos(\pi ky/L) \cos(\pi lz/L),
$$

$$
\bar{\phi}_1^M = \sum_{hkl} \alpha_{hkl}^M \sin(\pi hx/L) \cos(\pi ky/L) \cos(\pi lz/L).
with the integration being taken over the space inside an FCC box of size \( L \). Here, \( \alpha_{\perp i}, \alpha_{||i} \), and \( V_{ij} \) are expressed in terms of \( \phi_{\mu}^{\pi} \) as

\[
\tilde{a}_{\perp i}^{\alpha} = \left[ \partial_{i}\phi_{\alpha}^{\pi} - \phi_{\alpha}^{\pi} \frac{(1 + \phi_{\alpha}^{\rho})}{\phi_{\alpha}^{\rho}} \partial_{i}\phi_{\rho}^{\pi} \right]^{a}, \tag{29}
\]

\[
\tilde{a}_{||i}^{\alpha} = \left[ \frac{(\partial_{i}\phi_{\alpha}^{\pi} \times \phi_{\alpha}^{\pi})}{(1 + \phi_{\alpha}^{\rho})} - \frac{(\partial_{i}\phi_{\alpha}^{\rho} \times \phi_{\alpha}^{\rho})}{(1 + \phi_{\alpha}^{\rho})} \right]^{a}, \tag{30}
\]

\[
V_{ij}^{\omega} = \frac{1}{2} \left\{ - \frac{\partial_{i}\phi_{\rho}^{\pi}}{1 + \phi_{\rho}^{\rho}} (g_{p_{j}}^{\pi} + g_{p_{j}}^{\rho}) + \frac{\partial_{j}\phi_{\rho}^{\pi}}{1 + \phi_{\rho}^{\rho}} (g_{p_{i}}^{\pi} + g_{p_{i}}^{\rho}) - \frac{2 (\partial_{i}\phi_{\rho}^{\pi} \times \partial_{j}\phi_{\rho}^{\rho})}{1 + \phi_{\rho}^{\rho}} + [(g_{p_{i}}^{\rho} \times (g_{p_{j}}^{\rho}) \right]^{a}. \tag{31}
\]

On the other hand, the simple minimization procedure of \( E/B \) to find the expansion coefficients \( \beta_{abc}^{\omega} \) would lead to the trivial solution \( \omega_{0} = 0 \). This is because there are no constraints for the \( \beta_{abc}^{\omega} \)'s, and the \( \omega \) always provides repulsive interactions. Therefore, the \( \omega \) expansion coefficients can be determined by satisfying the equation of motion for \( \omega(=\omega_{0}/f_{\pi}) \): viz.

\[
(-\partial_{i}\partial_{i} + m_{\omega}^{2}) \omega(r) = J^{\omega} - \partial_{i}J_{i}^{\omega}, \tag{32}
\]

where \( J^{\omega} \) and \( \partial_{i}J_{i}^{\omega} \) coming from the hWZ terms are

\[
J^{\omega} = -\frac{gNc}{32\pi^{2}} \varepsilon_{ijk} \left[ (c_{1} + c_{2}) \tilde{a}_{\perp i} \cdot (\tilde{a}_{||j} \times \tilde{a}_{||k}) + (c_{1} - c_{2}) \tilde{a}_{\perp i} \cdot (\tilde{a}_{||j} \times \tilde{a}_{\perp k}) \right. \nonumber \\
- c_{2} \omega_{0} \tilde{a}_{\perp i} \cdot (\tilde{a}_{||j} \times \tilde{a}_{||k}) - \tilde{a}_{\perp i} \cdot (\tilde{a}_{\perp j} \times \tilde{a}_{\perp k}) \left. \right]. \tag{33}
\]

Due to the crystal structure, the source term can be expanded in terms of the Fourier series as

\[
J^{\omega} - \partial_{i}J_{i}^{\omega} = \sum_{abc} \gamma_{abc} \cos(\pi ax/L) \cos(\pi by/L) \cos(\pi cz/L). \tag{34}
\]

Then, thanks to the linear nature of the equation of motion for the \( \omega \), \( \beta_{abc}^{\omega} \) can be easily obtained as

\[
\beta_{abc}^{\omega} = \frac{\gamma_{abc}}{(\pi/L)^{2}(a^{2} + b^{2} + c^{2}) + m_{\omega}^{2}}. \tag{35}
\]

Thus, in the numerical minimizing processes only the \( \alpha_{hkl}^{\pi,\rho} \) and \( \beta_{abc}^{\pi,\rho} \) are taken as adjustable parameters.

Shown in Fig. 1 are the numerical results for the density \( \rho \), \( \omega \) in HLS(\( \pi, \rho, \omega \)) and HLS(\( \pi, \rho \)) phase transition takes place close to the normal nuclear phase transition occurs at \( L \). This analysis is supported by the results from HLS(\( \pi, \rho, \omega \)), where the half-skyrmion phase transition occurs at \( L \sim 0.8 \) fm, i.e., \( n_{1/2} \sim 6 \sigma_{0} \), much higher than with the \( \omega \). This also accounts that in HLS(\( \pi, \rho \)), the absence of the \( \omega \) reduces the skyrmion size to nearly half of that of HLS(\( \pi, \rho \)).

The density \( n_{\text{min}} \) at which \( E/B \) is the minimum turns out to be larger than \( n_{1/2} \) for all three models considered in the present work. As one can see from Fig. 1 the minimum is achieved at \( n_{\text{min}}/n_{1/2} \sim 2 \) in HLS(\( \pi, \rho \)) and in HLS(\( \pi, \rho, \omega \)), but at \( n_{\text{min}}/n_{1/2} \sim n_{\text{min}}/n_{1/2} \sim \)
to the Bogomolnyi bound. However, when the lowest skyrmion carries an excess energy of about 25% relative to the ground state, it is shown that, when only the pions are considered, the skyrmion is transformed to the skyrmion. It is true that, in the formalism, in particular in medium, the dominance, a powerful notion in hadron physics, manifests itself. As emphasized and proved in Refs. [1, 2], our calculation of the energy versus density curve. The same argument can be made here, i.e., $E/B$ and the expectation value $\langle \sigma \rangle$ are independent of the choice on the parameter $\alpha$. Again, as in Refs. [1, 2], we could confirm the $\alpha$ independence of $E/B$ and $\langle \sigma \rangle$ in the present work.

Figure 2 presents the contribution of each term in $E/B$ given in Eq. (25). Here, $(E/B)_{O(p^2)\pi,\rho}$, $(E/B)_{O(p^4)\pi,\rho}$, and $(E/B)_{\omega}$ respectively.

In standard HLS, $\alpha$ is a free parameter which is normally taken to be $1 \leq \alpha \leq 2$ [6, 8]. In free space $\alpha \approx 2$ is preferred, but in a hadronic medium at high temperature and/or density, one gets $\alpha \approx 1$ [5]. That the parameter $\alpha$ does not figure in physical observables in this theory raises the question as to how the vector mesons obstructed by the $U(1)$ vector meson $\omega$ (and its tower). It is this $\omega$ that plays a crucial role in nuclear dynamics as will be explained below.

As emphasized and proved in Refs. [1, 2], our calculation of the energy versus density curve. The same argument can be made here, i.e., $E/B$ and the expectation value $\langle \sigma \rangle$ are independent of the choice on the parameter $\alpha$. Again, as in Refs. [1, 2], we could confirm the $\alpha$ independence of $E/B$ and $\langle \sigma \rangle$ in the present work.

1.5 in HLS($\pi, \rho$). The binding energy per baryon at this minimum energy configuration can be also read from Fig. 1 as $\sim 150$ MeV, $\sim 100$ MeV, and $\sim 50$ MeV for HLS$_{\pi, \rho, \omega}$ ($\pi, \rho, \omega$), HLS$_{\pi, \rho}$, and HLS$_{\pi, \rho}$, respectively. As a complete theory of nuclear matter, the minimum at $n_{\text{min}}$ should represent the nuclear matter ground state. That $n_{\text{min}} > n_0$ and the binding energy at $n_{\text{min}}$ is larger than the empirical value $\sim 16$ MeV per baryon signals that there is something missing in the present skyrmion crystal description for the EoS of nuclear matter. We will discuss the plausible causes for this defect and possible remedies in the last section.

It is of theoretical, though perhaps academic, interest that the $E/B$ versus density of HLS($\pi, \rho$) is (nearly) flat. One can easily understand this feature, which arises from the important role of the vector mesons in nuclear structure. In Refs. [28, 29], the five-dimensional BPS theory – which can be considered as the Sakai-Sugimoto hQCD model with flat metric – is transformed to a four-dimensional infinite-tower meson theory and the instanton in that theory that satisfies the Bogomolnyi bound condition is then transformed to the skyrmion. It is shown that, when only the pions are considered, the skyrmion carries an excess energy of about 25% relative to the Bogomolnyi bound. However, when the lowest SU(2) vector meson, the $\rho$ meson, is included, the energy is drastically reduced, arriving almost at the Bogomolnyi bound. The next vector meson, which is the axial $a_1$, brings it even closer to the bound. What is remarkable – and highly relevant to our work here – is that the lowest vector meson $\rho$ alone brings the energy nearly to the bound. This is highly reminiscent of the vector dominance of the nucleon isovector electromagnetic form factor by the $\rho$ (with a small correction from the next vector meson $\rho'$) [12, 13, 14, 15, 50]. Since the HLS Lagrangian used in our work is constructed from the five-dimensional Sakai-Sugimoto model in a similar way, the difference being the background curvature, the inclusion of the $\rho$, and the higher order terms in the Lagrangian makes the single skyrmion mass almost saturate the Bogomolnyi bound. Consequently, there is very little interaction energy left in the skyrmion made of the pion and the $\rho$ to disturb the energy versus density curve.

We should stress that the approach to the Bogomolnyi bound by the $\rho$ – and higher isovector vector mesons is obstructed by the $U(1)$ vector meson $\omega$ (and its tower). It is this $\omega$ that plays a crucial role in nuclear dynamics as will be explained below.

As emphasized and proved in Refs. [1, 2], our calculation of the energy versus density curve. The same argument can be made here, i.e., $E/B$ and the expectation value $\langle \sigma \rangle$ are independent of the choice on the parameter $\alpha$. Again, as in Refs. [1, 2], we could confirm the $\alpha$ independence of $E/B$ and $\langle \sigma \rangle$ in the present work.

Figure 2 presents the contribution of each term in $E/B$ given in Eq. (25). Here, $(E/B)_{O(p^2)\pi,\rho}$, $(E/B)_{O(p^4)\pi,\rho}$, and $(E/B)_{\omega}$ respectively.
and \((E/B)_{\omega}\) are given by the solid, dashed, and dash-dotted lines, respectively. First of all, one can find that \((E/B)_{O(p^2)^{\pi,\rho}}\) is negative for all density range involved, showing clearly that the account of the entire \(O(p^4)\) terms and vector mesons brings in a sizable attraction, a feature that has not been exposed before in the literature. What is more striking is its weak density dependence compared to the other two contributions. Next, \((E/B)_{O(p^2)^{\pi,\rho}}\) and \((E/B)_{\omega}\) show distinctly different density dependence in the skyrmion phase and in the half-skyrmion phase. In the single skyrmion phase, \((E/B)_{O(p^2)^{\pi,\rho}}\) dominates over \((E/B)_{\omega}\), with their ratio \((E/B)_{O(p^2)^{\pi,\rho}}/(E/B)_{\omega}\) even increasing from \(\sim 2\) to \(\sim 3\) as the density increases whereas in the half-skyrmion phase, \((E/B)_{O(p^2)^{\pi,\rho}}\) starts to decrease while \((E/B)_{\omega}\) starts to increase. Then, at some density, the repulsive interaction due to the \(\omega\) becomes dominant.

IV. HADRON PROPERTIES IN THE FCC CRYSTAL BACKGROUND

A. In-medium modification of meson properties

As stressed, the power of the skyrmion-matter model is that it describes baryons and mesons in a unified way. This, therefore, makes it a natural framework to investigate the in-medium modification of meson properties \([3, 31]\). The Lagrangian \((7)\) itself could describe the meson dynamics in baryon free space. If we expand the Lagrangian in terms of the fluctuating meson fields about their vacuum values, that is, \(U = 1\), \(\rho^a_\mu = \omega_\mu = 0\), the Lagrangian \((7)\) becomes

\[
\mathcal{L} = \frac{1}{2} \partial_\mu \pi^a \partial^\mu \pi^a - \frac{1}{4} \rho^a_\mu \rho^{a\mu} + \frac{1}{2} m^2 \rho^a_\mu \rho^{a\mu} - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m^2 \omega_{\mu\nu} \omega^{\mu\nu} + \text{interaction terms},
\]

where we have used \(ag^2 f^2_\pi = m^2_\rho = m^2_\omega\).

On the other hand, what we have obtained in the previous section is the classical solutions for \(U\), \(\rho^a_\mu\), and \(\omega_\mu\), which are the minimum energy configurations for a given baryon number density. If we incorporate the fluctuations with respect to these classical solutions, then the fluctuating fields describe the corresponding mesons and their dynamics in dense baryonic matter, which is the main idea of Refs. \([3, 31]\). We denote the minimum energy solutions as \(\xi_{(0)}^L = \xi_{(0)}^R, \rho^{a(0)}_\mu, \text{ and } \omega^{(0)}_\mu\), and introduce the fluctuating fields on top of the classical solutions as

\[
\xi_{L,R} = \xi_{(0)}^{L,R} \delta_{L,R}, \quad \rho^a_\mu = \rho^{a(0)}_\mu + \tilde{\rho}^a_\mu, \quad \omega_\mu = \omega^{(0)}_\mu + \tilde{\omega}_\mu,
\]

where \(\tilde{\xi}^{L,R} = \tilde{\mu} = \exp(\imath \tau_a \pi_a / 2 f^*_\pi), \tilde{\rho}^a_\mu, \text{ and } \tilde{\omega}_\mu\) stand for the corresponding fluctuating fields. By substituting these fields into the HLS Lagrangian and taking the space average for the background fields configurations as denoted by \(\langle \cdot \rangle\), we are led to

\[
\mathcal{L} = \frac{1}{2} \int \left[ -\langle \partial_\mu \pi^a \partial^\mu \pi^a \rangle - \frac{1}{4} \rho^a_\mu \rho^{a\mu} + \frac{1}{2} m^2 \rho^a_\mu \rho^{a\mu} - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m^2 \omega_{\mu\nu} \omega^{\mu\nu} + \cdots \right],
\]

where we have considered the \(O(p^2)\) terms only. The extra factor in front of the pion kinetic term can be absorbed into the redefinition of the pion fields as

\[
\tilde{\pi}^*_a = f^*_\pi \pi^*_a = \sqrt{1 - (a - 1) \frac{2}{3} \left(1 - \langle \sigma^2_0 \rangle \right)} \tilde{\pi}^*_a,
\]

with

\[
\frac{f^*_\pi}{f^*_{\pi}} = \sqrt{1 - (a - 1) \frac{2}{3} \left(1 - \langle \sigma^2_0 \rangle \right)}.
\]

Since we have considered only the \(O(p^2)\) terms, there does not appear any modification factor for the \(\rho\) meson and \(f^*_\pi / f^*_{\pi}\) becomes dependent on the parameter \(a\). If we include \(O(p^4)\) terms and one-loop corrections from the \(O(p^2)\) terms correctly, we believe that such an \(a\)-dependence will disappear or be weakened.

In Fig. 4, we present \(f^*_\pi / f^*_{\pi}\) as a function of the FCC box size, where we have fixed the parameter \(a = 2\)

Again, the density dependence of \(f^*_\pi / f^*_{\pi}\) shows the different behavior in the single skyrmion phase and in the half-skyrmion phase. In the single skyrmion phase, \(f^*_\pi / f^*_{\pi}\)

\[8\] We include only the \(O(p^2)\) terms, which can reasonably reproduce the meson properties only when we take \(a = 2\).
decreases as the baryon number density increases. On the other hand, in the half-skyrmion phase, it stays in a nonvanishing value around 0.65. Due to this nonvanishing $f_\pi^*$, the half-skyrmion phase was interpreted as a kind of pseudogap phase in Ref. [32].

B. In-medium modification of baryon properties

In our HLS Lagrangian, all the parameters are determined by the master formula (16) given in the matter-free space. As the simplest approximation in applying it to dense matter, we shall assume that the in-medium modification of all the parameters involved, such as the $O(p^4)$ terms, can be calculated in terms of the in-medium constants $f_\pi^*$ and $m_\rho^*$. We denote them with asterisk. Now given these starred parameters, we can solve for the single skyrmion solution as in Ref. [2] to obtain the in-medium modification of the baryon mass in dense medium. To get the leading corrections, it suffices to use the $O(p^2)$ Lagrangian with $a = 2$ as suggested by phenomenology to calculate the two in-medium constants, in which case the most important effect is lodged in the pion-decay constant. The effects of the $O(p^4)$ terms so calculated can then be straightforwardly incorporated with the starred quantities. In principle, we should rewrite the $O(p^4)$ terms in medium to obtain a relation which may be different from the free-space master formula, but this will be complicated considering that Lorentz invariance is broken in medium. Intuitively, however, we do not expect the result to be significantly modified.

Shown in Fig. 4 is the in-medium single skyrmion masses calculated in the approximation described above as a function of the FCC size parameter $L$. We also present the $\Delta$-$N$ mass difference in Fig. 5 and the soliton root-mean-square (rms) radii in Fig. 6.

Note that the behavior of the crystal size dependence of the soliton mass shown in Fig. 4 is the same as the behavior of $f_\pi^*$ shown in Fig. 3. This can be understood by that in the skyrmion model the soliton mass scales roughly as $M_{\text{sol}} \sim f_\pi^*$, which follows simply from the scaling of the nucleon mass in the large $N_c$ limit as in matter-free space. Figure 5 shows that the cry-
tial size dependence of the $\Delta N$ difference $\Delta M$ changes a lot depending on the existence of the $\omega$ meson. In the calculation with the $\omega$ meson, $\Delta M$ decreases up to $n_{1/2}$ beyond which $\Delta M$ becomes nearly constant. However, in the case of HLS($\pi, \rho$), $\Delta M$ is unscaled up to $n_{1/2}$ but, after $n_{1/2}$, $\Delta M$ increases dramatically. Our finding that the nucleon mass, which decreases with increasing density in the skyrmion phase, stops dropping at $n_{1/2}$ and stays constant in the half-skyrmion phase is the most significant result of this paper and, as far as we are aware, has not been obtained in other models. In the case of HLS($\pi, \rho, \omega$), the in-medium soliton mass approaches $\sim 0.6 M_{\omega\pi}$. As will be discussed in the next section, this finding will get some support from other approaches, both theoretical and phenomenological, and implies consequences on the upcoming experiments.

Similarly, the skyrmion radius is strongly influenced by the presence of the $\omega$ meson as seen in Fig. 1. In the case of HLS($\pi, \rho$), the skyrmion radius is small and constant at varying density. This is simply because in the absence of the $\omega$, the skyrmion is almost pointlike and remains stable with the crystal size. This feature was already obvious with the instantons in the hQCD model.

It is noteworthy that, while the impacts of the skyrmion–half-skyrmion transition can be seen in all three models for the masses and splittings indicating the effect of topology change in medium, it does not affect the skyrmion size when the $\omega$ is absent, which is in a stark contrast to the case when the $\omega$ is present.

V. ORIGIN OF NUCLEON MASS

The principal finding in this paper, as mentioned before, is that simulated on the crystal lattice, the nucleon mass decreases smoothly as density increases up to $n_{1/2}$ at which skyrmions fractionize into half-skyrmions and, in the half-skyrmion phase, the dropping of the nucleon mass stops and the mass remains constant going toward the chiral restoration. In the skyrmion phase with $n < n_{1/2}$, chiral symmetry is broken with the order parameter $(\bar{q}q) \neq 0$. Hadrons are massive apart from the Nambu-Goldstone bosons, the pions. In the half-skyrmion phase, $(\bar{q}q) = 0$ on the average, but chiral symmetry is not restored since hadrons are still massive and there exist pions. There seems to be no obvious order parameter characterizing this state apart from the presence of the half-skyrmion structure, the changeover involving a sort of topology change.

Apart from the changes in the baryon properties discussed above, there are several other remarkable happenings at $n_{1/2}$. For instance, in Ref. [33] was found that an antikaon $K^-$ propagating in the skyrmion matter undergoes propitious drop in its effective mass at $n_{1/2}$, which may have an influence on kaon condensation in compact-star matter. Another phenomenon concerns the symmetry energy of nuclear matter. At $n_{1/2}$, the symmetry energy is found to develop a cusp structure which leads to drastic modification of the nuclear tensor forces [34] that contribute importantly to the structure of asymmetric nuclei and large mass neutron stars [22]. We believe that all these phenomena are closely linked together and indicate the possible source of the proton mass.

There are several indications that the nucleon mass does stay constant going toward the chiral transition point. It points to a possible origin of the proton mass. The first indication comes from a phenomenological application of the skyrmion–half-skyrmion transition to the EoS of nuclear and neutron matter [22]. The idea is to translate what is observed in the skyrmion description to the density dependence of the parameters of the HLS Lagrangian, namely the masses and coupling constants scaling with density, with which a nuclear effective field theory is constructed. The topology change at $n_{1/2}$ impacts the changes in the parameters of the EFT Lagrangian, and the drastic effects mentioned above reflect on nuclear forces, in particular the tensor forces. It is observed in the EoS at high density that the nucleon mass should stabilize at $n_{1/2}$ to $m_N \approx 0.8 m_N$, while the effective $\omega NN$ coupling $g_{\omega NN}^*$ can drop but very slowly in density, constrained by the constant nucleon mass. Nature seems to be exposing an intricate interplay between $m_N$ and the coupling $g_{\omega NN}^*$ or more generally the $\omega$ degree of freedom.

Another indication comes from an RG analysis of nucleon properties in HLS with baryons, i.e., BHLS, as is changed. It is found in Ref. [20] that as one approaches the dilaton limit fixed point at which the nonlinear sigma-model phrased in HLS transforms to a linear sigma model of Gell-Mann-Lévy type, the nucleon mass that falls in dilute environment stops falling at $n_A$ and then stays constant afterwards, strongly correlated with the $\omega NN$ coupling. It is suggestive that $n_A$ corresponds to $n_{1/2}$ in the crystal description.

That the nucleon mass remains nonvanishing as one approaches the chiral transition point can be captured by the nucleon parity-doublet model [21] with the two-component nucleon mass [11] with a substantial $m_0 \sim (0.6 - 0.8) m_N$ that remains nonvanishing. In Ref. [20], this model was used to show that the RG flow to the infrared fixed point, i.e. dilaton-limit fixed point, does indeed indicate that there is a changeover at some density for the nucleon mass in a close interplay with the $\omega$-nucleon coupling. The nonzero $m_0$ can be interpreted as reflecting a symmetry intrinsic in QCD as suggested by Glozman et al. [23, 24]. This interpretation suggests a mechanism for a bulk of the mass generated differently from the spontaneous breaking of chiral symmetry, presumably tied to chiral invariant gluon interactions. The interesting question is whether one cannot identify the agent for the nonzero $m_0$ with one that triggers the spontaneous breaking of scale symmetry that makes the dilaton condense.

On the contrary, one can consider $m_0$ representing a symmetry that is “emerging” from strong correlations
in nuclear interactions, which has nothing to do with a fundamental symmetry of QCD. This is what is coming out of our skyrmion crystal picture. In either case, it is an interesting issue in nuclear physics, intimately linked to the origin of the nucleon mass.

VI. REMARKS AND DISCUSSIONS

In this paper, to investigate dense baryonic matter, we simulated on crystal lattice the HLS Lagrangian written to \( O(p^4) \) in the derivative expansion with all the parameters of the Lagrangian fixed except for the pion decay constant \( f_\pi \) and the \( \rho \) meson mass \( m_\rho \). This comes about in integrating out all vector mesons in the infinite tower of vector and axial vector mesons from a generic five-dimensional hQCD action in a curved space plus the Chern-Simons term, leaving for relevant degrees of freedom only the lowest vector mesons \( \rho \) and \( \omega \) in a hidden local symmetric form, obtaining a master formula that relates all \( O(p^4) \) low energy constants to two constants that figure in hQCD Lagrangian, i.e., \( f_\pi \) and \( m_\rho \). The dependence on the famous \( a \) parameter which figures importantly in the usual HLS theory gauge equivalent to nonlinear sigma model, such as the KSRF-type relation \( m_\pi^2 = a q^2 f_\pi^2 \), can be eliminated in a way analogous to a gauge parameter. Physical quantities are independent of \( a \).

The most notable findings in this work are (1) the prominent role of the \( \omega \) meson encapsulated in the homogeneous Wess-Zumino term (equivalently the \( U(1) \) gauge field in the Chern-Simons topological term in hQCD) in the structure of both elementary nucleon and dense baryonic matter and (2) the presence of topological change from skyrmions to half-skyrmions at a density relevant slightly above normal nuclear matter density and higher density.

The first which plays a crucial role in giving correct structure to the nucleon controls the location of the topology change \( n_{1/2} \) and accounts for the large constant mass to the nucleon that remains nonzero at the chiral restoration which is locked to the pion decay constant effective in the half-skyrmion phase. This feature combined with in-medium properties of mesons introduces a dramatic change in the nuclear tensor forces at a density above nuclear matter and gives rise to a stiff EoS that could accommodate large neutron star masses such as the 1.97\( M_\odot \) object found recently. Highly relevant to the forthcoming RIB machines is that the drastic change in the nuclear tensor forces caused by the topology change we found at a density near that of nuclear matter can have a strong consequence on the shell evolution of asymmetric nuclei. It has been shown that the monopole matrix element of the tensor forces follows closely the shell evolution of single-particle states. Thus any changes in the tensor forces could be probed by looking at the shell evolution.

We should point out a few caveats and approximations made in the paper that require further work. The most glaring defect of this work is that the minimum of the energy density of the system occurs at too high a density and with too large a binding energy. One reason may be that our approximations are anchored on large \( N_c \) considerations and \( 1/N_c \) corrections cannot be ignored in medium. This is a general problem with skyrmion physics which relies on the Lagrangian valid for large \( N_c \).

Perhaps more important is the mechanism that leads to a correct equilibrium density and binding in EFT approaches to nuclear matter such as RG \( V_{l_{\text{flow}}K}^{\text{low}} \). There one finds that without irreducible repulsive three-body forces, the system saturates at much higher density and with much larger binding energy, very similar to what is found in our theory. It is possible that the skyrmion crystal approach, while containing reducible many-body effects, misses a mechanism that corresponds to this irreducible three-body force effect.

Now one can ask whether the predictions made in a theory that fail to reproduce the correct properties of normal nuclear matter can be trusted when applied to higher density. There is no clear answer to this question for the moment. However it may be that the quantity that depends on topology is robust. It is seen in the calculation that although the location of \( n_{1/2} \) depends on dynamical details, the existence of the skyrmion–half-skyrmion transition itself is generic. The RG analysis of Ref. provides an independent support to this result. The issue then is where the transition takes place, and this can be ultimately checked by experiments. There is an indication that \( n_{1/2} \) is near \( n_0 \) and the phenomenology presented in Ref. hints that \( n_{1/2} \sim (1.5 - 2.0) n_0 \) is compatible with nature, a region that can be accessed by the future RIB machines and is also highly relevant to compact stars.

As already mentioned, one important missing ingredient is scalar degree(s) of freedom in the Lagrangian. In the Skyrme model (with pions only), the Casimir contribution to the nucleon mass which comes at \( O(N_c^0) \) is attractive and significant, of order of \( \sim -0.5 \) GeV. This cannot be calculated accurately, so is generally ignored in the community. In fact, it is the same \( O(N_c^0) \) order effect that gives rise to the binding of \( K^- \) to the skyrmion that describes the hyperons in the Callan-Klebanov model. It is very likely that this (or part of this) effect can be captured by a low-mass scalar as in Walecka’s mean field model for nuclear matter. We have seen in our model HLS(\( (\pi, \rho, \omega) \) that the \( \omega-\rho-\pi \) coupling generates — via hWZ terms — a significant attraction in the nucleon, but there could be further attraction that gets enhanced in dense matter. How to implement scalars into HLS(\( (\pi, \rho, \omega) \) remains a problem to be worked out. A scalar in the form of dilaton associated with spontaneously broken conformal symmetry as exploited in Ref. could provide an approach to the problem.

Among the approximations we made, we note that the crystal-size (or density) dependence of \( f_\pi \) is obtained.
with the $O(p^2)$ terms of HLS. A complete calculation of this quantity up to $O(p^4)$ should include not only the tree-level $O(p^4)$ contributions but also one-loop corrections with the $O(p^2)$ terms. But the latter are suppressed by the $N_c$ counting. Moreover, since we only consider fluctuations from the $O(p^2)$, both the $\rho$ and $\omega$ meson masses are the same as their vacuum values and independent of the density. In a complete calculation including the full $O(p^4)$ contributions which contains not only the tree-level $O(p^4)$ terms but also one-loop corrections from the $O(p^2)$ terms, these masses will change with density, although the changes will be suppressed by $1/N_c$.

Similarly for the density dependence of the soliton properties, only the in-medium modified $f_\pi^2$ is considered since the other input parameter $m_\rho$ is the same as its vacuum value in the present approximation. Therefore, the scaling behaviors of the soliton properties agree with the large $N_c$ argument. In a complete calculation up to $O(p^4)$ including the one-loop corrections from the $O(p^2)$ terms, these results might be changed but again they will be suppressed by the chiral counting or the $N_c$ counting.

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