Effect of Stochastic Loading on Tensile Damage and Fracture of Fiber-Reinforced Ceramic-Matrix Composites

Longbiao Li

College of Civil Aviation, Nanjing University of Aeronautics and Astronautics, No.29 Yudao St., Nanjing 210016, China; llb451@nuaa.edu.cn

Received: 27 April 2020; Accepted: 26 May 2020; Published: 28 May 2020

Abstract: In this paper, the effect of stochastic loading on tensile damage and fracture of fiber-reinforced ceramic-matrix composites (CMCs) is investigated. A micromechanical constitutive model is developed considering multiple damage mechanisms under tensile loading. The relationship between stochastic stress, tangent modulus, interface debonding and fiber broken is established. The effects of the fiber volume, interface shear stress, interface debonding energy, saturation matrix crack spacing and fiber strength on tensile stress–strain curve, tangent modulus, interface debonding fraction and fiber broken fraction are analyzed. The experimental tensile damage and fracture of unidirectional and 2D SiC/SiC composites subjected to different stochastic loading stress are predicted. When fiber volume increases, the initial composite strain decreases, the initial tangent modulus increases, the transition stress for interface debonding decreases and the initial fiber broken fraction decreases. When fiber strength increases, the initial composite strain and fiber broken fraction decrease.

Keywords: ceramic-matrix composites (CMCs); Stochastic loading; tensile; matrix cracking; interface debonding; fiber failure

1. Introduction

Ceramic-matrix composites (CMCs) have the advantages of high-temperature resistance, corrosion resistance, low density, high specific strength and high specific modulus [1]. The fabrication methods of CMCs include the chemical vapor infiltration (CVI), polymer infiltration and pyrolysis process (PIP) and melt infiltration (MI). The mechanical performance of CMCs depends on the fabrication method. To ensure the reliability and safety of fiber-reinforced CMCs used in hot-section components of an aero engine, it is necessary to develop performance evaluation, damage evolution, strength and life prediction tools for airworthiness certification [2]. Since the applications of fiber-reinforced CMCs involve components with lives that are measured in tens of thousands of hours, the successful design and implementation of CMC components depend on the knowledge of the material behavior over periods of time comparable to the expected service life of the component [3].

Under tensile loading, multiple damage mechanisms of matrix cracking, interface debonding and fiber failure occur [4–8]. The tensile stress–strain curves can be divided into four stages, including:

1. Stage I, linear-elastic region.
2. Stage II, matrix cracking and interface debonding region. Matrix micro cracking occurs first and the fiber debonding from the matrix, leading to nonlinear behavior of CMCs.
3. Stage III, saturation matrix cracking region. Matrix cracking approaches saturation with complete debonding of the fiber from the matrix.
4. Stage IV, fiber failure region. The fiber gradually fractures with increasing applied stress.
The composite elastic modulus, proportional limit stress, ultimate tensile strength and the fracture strain can be obtained from the tensile stress–strain curve. For the SiC/SiC composite fabricated using the MI method, the fracture strength and strain of Hi-Nicalon SiC/SiC composite is much higher than that of Tyranno SA3 SiC/SiC composite; however, the initial elastic modulus and proportional limit stress of Hi-Nicalon SiC/SiC composite is lower than that of Tyranno SA3 SiC/SiC composite at the same fiber volume of 34.8% and the composite tensile strength increases with the fiber volume [9]. Marshall et al. [10] and Zok and Spearing [11] investigated the first matrix cracking and matrix multiple cracking evolution in fiber-reinforced CMCs using the fracture mechanics approach. Curtin [12] investigated multiple matrix cracking evolution of CMCs considering matrix internal flaws. Evans [13] developed an approach for design and life prediction issues for fiber-reinforced CMCs and established the relationship between macro mechanical behavior and constituent properties of CMCs. McNulty and Zok [14] investigated the low-cycle fatigue damage mechanism and established the damage models for predicting the low-cycle fatigue life of CMCs. Naslain et al. [15] investigated the monotonic and cyclic tensile behavior of Nicalon™ SiC/SiC minicomposite at room temperature. The relationship between the statistical parameters of both the fiber and the matrix and the fiber/matrix interfacial parameters and the effect of environment and the tensile curves has been established. Goto and Kagawa [16] investigated the tensile and fracture behavior of a bi-directional woven Nicalon™ SiC/SiC composite at room temperature. The tensile stress–strain curve shows nonlinear behavior above 70 MPa and the composite effective Young’s modulus at fracture was about 40% of the initial value. Guo and Kagawa [6] investigated the tensile fracture behavior and tensile mechanical properties of Nicalon™ and Hi-Nicalon™ SiC/BN/SiC composites at temperature between 298 and 1400 K in air atmosphere. The tensile strength dropped from 140 MPa at 800 K to 41 MPa at 1200 K. Meyer and Waas [17] investigated tensile response of notched SiC/SiC composite at elevated temperature using a novel digital image correlation technique. Li et al. [18–20] developed a micromechanical approach to predict the tensile behavior of unidirectional, cross-ply, 2D and 2.5D woven CMCs considering the damage mechanisms of matrix cracking, interface debonding and fiber failure. However, the effect of stochastic loading on tensile damage and fracture of fiber-reinforced CMCs has not been investigated.

The objective of this paper is to investigate the effect of stochastic loading on tensile damage and fracture of fiber-reinforced CMCs for the first time. A micromechanical constitutive model is developed considering multiple damage mechanisms under tensile loading. The relationship between stochastic stress, tangent modulus, interface debonding and fiber broken is established. The effects of fiber volume, interface shear stress, interface debonding energy, saturation matrix crack spacing and fiber strength on tensile stress–strain curve, tangent modulus, interface debonding fraction and fiber broken fraction are analyzed. The experimental tensile damage and fracture of unidirectional and 2D SiC/SiC composites subjected to different stochastic loading stress are predicted.

2. Theoretical Model

When stochastic stress occurs under tensile loading, matrix cracking, interface debonding, and fiber failure occur. Multiple stochastic loading sequence is shown in Figure 1. The shear-lag model is used to analyze micro stress filed of damaged composite. A unit cell is extracted from the ceramic composite system, as shown in Figure 2. The fiber axial stress in different damage regions is given by Equation (1).

\[
\sigma_f(x) = \begin{cases} 
\Phi - \frac{2\tau_i}{r_f}x, & x \in [0, l_d] \\
\sigma_{fo} + \left(\Phi - \sigma_{fo} - 2\frac{l_d}{r_f}\tau_i\right) \exp\left(-\rho \frac{x - l_d}{l_c}\right), & x \in [l_d, \frac{l_c}{2}]
\end{cases}
\]

where \(\Phi\) is the fiber intact stress, \(\tau_i\) is the interface shear stress, \(r_f\) is the fiber radius, \(\sigma_{fo}\) is the fiber axial stress in the interface bonding region, \(l_d\) is the interface debonding length, \(l_c\) is the matrix crack spacing and \(\rho\) is the shear-lag model parameter.
The interface debonding length is determined by Equation (3).

\[ l_d = \frac{r_f}{2} \left( \frac{V_m E_m \sigma}{V_f E_c \tau_i} - \frac{1}{\rho} \right) - \sqrt{\left( \frac{r_f}{2\rho} \right)^2 + \frac{r_f V_m E_m E_t}{E_c \tau_i^2} \zeta_d} \]  

(3)

where \( l_s \) is the saturation matrix crack spacing, \( \sigma_m \) is the matrix stress, \( \sigma_R \) is matrix cracking characteristic stress and \( m \) is the matrix Weibull modulus.

The fracture mechanics approach is used to determine the fiber/matrix interface debonding length. The interface debonding length is determined by Equation (3).

\[ l_d = \frac{r_f}{2} \left( \frac{V_m E_m \sigma}{V_f E_c \tau_i} - \frac{1}{\rho} \right) - \sqrt{\left( \frac{r_f}{2\rho} \right)^2 + \frac{r_f V_m E_m E_t}{E_c \tau_i^2} \zeta_d} \]  

(3)

where \( V_f \) and \( V_m \) are the fiber and matrix volume fraction, respectively, \( E_m \) and \( E_c \) are the matrix and composite elastic modulus, respectively, and \( \zeta_d \) is the interface debonding energy.

The Global Load Sharing (GLS) criterion is used to determine the intact fiber stress. The relationship between the fiber intact stress, fiber failure probability and fiber pullout length is given by Equation (4) [21].

\[ \frac{\sigma}{V_f} = \Phi(1 - P(\Phi)) + \frac{2\pi}{r_f} (L) P(\Phi) \]  

(4)
where $<L>$ is the average fiber pullout length, and $P(\Phi)$ is the fiber failure probability, which is obtained by Equation (5).

$$P(\Phi) = 1 - \exp\left(-\frac{(\Phi)^{m_f+1}}{\sigma_c}\right)$$

where $\sigma_c$ is the fiber characteristic strength, and $m_f$ is the fiber Weibull modulus.

When matrix cracking and interface debonding occur, the composite strain is given by Equation (6).

$$\varepsilon_c = \begin{cases} \frac{\eta}{\frac{V_m}{V_i}} \frac{L_d}{l_c} \eta - \frac{V_m}{V_i} \frac{l_d}{l_c} \frac{\eta}{\Phi} (1 - \eta) - \frac{2}{\Phi^2} \left( V_m \frac{\eta}{\Phi} \sigma_{io} - \eta \tau_i \right) \times \exp\left(-\frac{2}{\Phi^2} (1 - \eta)\right) - 1 - (\alpha_c - \alpha_f) \Delta T, \eta < 1 \\ \frac{\eta}{\frac{V_m}{V_i}} \frac{L_d}{l_c} \eta - (\alpha_c - \alpha_f) \Delta T, \eta = 1 \end{cases}$$

When fiber failure occurs, the composite strain is given by Equation (7).

$$\varepsilon_c = \begin{cases} \frac{\Phi}{\frac{V_m}{V_i}} \frac{L_d}{l_c} \eta - \frac{V_m}{V_i} \frac{l_d}{l_c} \frac{\eta}{\Phi} (1 - \eta) - \frac{2}{\Phi^2} \left( V_m \frac{\eta}{\Phi} \sigma_{io} - \eta \tau_i \right) \times \exp\left(-\frac{2}{\Phi^2} (1 - \eta)\right) - 1 - (\alpha_c - \alpha_f) \Delta T, \eta < 1 \\ \frac{\Phi}{\frac{V_m}{V_i}} \frac{L_d}{l_c} \eta - (\alpha_c - \alpha_f) \Delta T, \eta = 1 \end{cases}$$

where $\eta$ is the interface debonding fraction, as shown in Equation (8).

$$\eta = 2 \frac{L_d}{l_c}$$

The tangent modulus is defined by Equation (9).

$$E_F = \frac{d\sigma}{d\varepsilon}$$

3. Results and Discussion

Under stochastic loading, the damages of the matrix cracking, interface debonding and fiber failure occur. The micro stress filed of the damaged CMCs after stochastic loading is given by Equation (1). The fiber axial stress distribution is affected by the stochastic loading stress level, matrix cracking, interface debonding and fiber failure. The stochastic matrix cracking model is used to determine the matrix crack spacing at the applied stress level, as shown in Equation (2), and the fracture mechanics interface debonding criterion is used to determine the interface debonding length, and the GLS criterion is used to determine the load allocation between fracture and intact fibers. The micromechanical constitutive models for the conditions of the matrix cracking, interface debonding and fiber failure are given by Equations (6) and (7). Using the developed micromechanical constitutive models and damage models, the effects of the fiber volume, interface shear stress, interface debonding energy, matrix crack spacing and fiber strength on tensile stress–strain curve, tangent modulus, interface debonding fraction and fiber broken fraction of SiC/SiC composite subjected to different stochastic loading are analyzed. Li et al. [4] investigated the tensile behavior of 2D woven SiC/SiC composite at room temperature. The composite was fabricated using the CVI method with the PyC interphase. The tensile experiments were conducted under displacement control of 0.3 mm/min. The material properties are given by: $V_f = 0.2$, $E_f = 350$ GPa, $E_m = 300$ GPa, $\eta_f = 7.5$ $\mu$m, $\alpha_f = 4.0 \times 10^{-6} / ^\circ$C, $\sigma_m = 4.8 \times 10^{-6} / ^\circ$C, $\Delta T = -1000$ $^\circ$C, $\zeta_q = 0.5$ J/m$^2$, $\tau_i = 25$ MPa and $m_f = 3$. 
3.1. Effect of Fiber Volume on Tensile Damage and Fracture of SiC/SiC Composite with Stochastic Loading

The fiber volume affects the tensile behavior of CMCs. The fiber volume range of SiC/SiC composite is between \( V_f = 27.7 \) and 40\% [9]. In the present analysis, the effect of the fiber volume (i.e., \( V_f = 0.3 \) and 0.35) on the tensile stress–strain curves, tangent modulus, interface debonding fraction and broken fiber fraction of SiC/SiC composite subjected to the stochastic loading of \( \sigma_s = 200, 300 \) and 350 MPa are shown in Figure 3; Figure 4 and Table 1. When the fiber volume increases, the stress carried by the fiber increases, the initial composite strain decreases, the initial tangent modulus increases, the transition stress for the interface debonding decreases and the initial fiber broken fraction decreases.

![Figure 3.](image-url)
Figure 4. (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 200, 300$ and $350$ MPa when $V_f = 0.35$.

Table 1. The effect of the fiber volume ($V_f = 0.3$ and $0.35$) on tensile stress–strain curve, tangent modulus, interface debonding fraction and broken fiber fraction of SiC/SiC composite subjected to stochastic loading of $\sigma_s = 200, 300$ and $350$ MPa.

| $V_f$ | $\sigma_s$ (MPa) | $\epsilon_0$ (%) | $E_p$ (GPa) | $\sigma_{tr}$ (MPa) | $P$ (%) |
|-------|-----------------|-----------------|-------------|----------------------|--------|
| $0.3$ | $200$           | 0.00161         | 268         | 79.2                 | 0.005  |
|       | $300$           | 0.0098          | 229         | 178.2                | 0.028  |
|       | $350$           | 0.023           | 228         | 231                  | 0.060  |
| $0.35$| $200$           | 0.0009          | 281         | 55                   | 0.0027 |
|       | $300$           | 0.0047          | 249         | 156.2                | 0.014  |
|       | $350$           | 0.0095          | 248         | 206.8                | 0.028  |
When \( V_f = 0.3 \) under \( \sigma_s = 200 \text{ MPa} \), the damages of matrix cracking and interface debonding occur at \( \sigma_s = 200 \text{ MPa} \), leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is \( \varepsilon_0 = 0.00161\% \) due to the matrix cracking and interface debonding at \( \sigma_s = 200 \text{ MPa} \); the initial tangent modulus is \( E_p = 268 \text{ GPa} \), the degradation rate of the tangent modulus is 15\% compared with the original specimen and the fiber broken fraction is \( P = 0.005 \). With increasing stress to \( \sigma_{tr} = 79.2 \text{ MPa} \), the interface debonding fraction increases, the tangent modulus decreases to \( E_p = 235 \text{ GPa} \) corresponding to \( \eta = 0.081 \). Upon increasing stress from \( \sigma_{tr} = 79.2 \text{ MPa} \) to \( \sigma = 200 \text{ MPa} \), the tangent modulus remains constant of \( E_p = 264 \text{ GPa} \) with \( \eta = 0.084 \).

Under \( \sigma_s = 300 \text{ MPa} \), the initial composite strain is \( \varepsilon_0 = 0.00982\% \) due to the damages of the matrix cracking and interface debonding at \( \sigma_s = 300 \text{ MPa} \); the initial tangent modulus is \( E_p = 229 \text{ GPa} \), the degradation rate of tangent modulus is 28\% compared with the original specimen and the fiber broken fraction is \( P = 0.028 \). With increasing stress to \( \sigma_{tr} = 178.2 \text{ MPa} \), the interface debonding fraction increases, the tangent modulus decreases to \( E_p = 145 \text{ GPa} \) corresponding to \( \eta = 0.392 \). Upon increasing stress from \( \sigma_{tr} = 178.2 \text{ MPa} \) to \( \sigma = 300 \text{ MPa} \), the tangent modulus remains constant of \( E_p = 170.1 \text{ GPa} \) with \( \eta = 0.4 \).

Under \( \sigma_s = 350 \text{ MPa} \), the initial composite strain is \( \varepsilon_0 = 0.023\% \) due to the damages of the matrix cracking and interface debonding at \( \sigma_s = 350 \text{ MPa} \); the initial tangent modulus is \( E_p = 228 \text{ GPa} \), the degradation rate of tangent modulus is 28\% compared with the original specimen and the fiber broken fraction is \( P = 0.06 \). With increasing stress to \( \sigma_{tr} = 231 \text{ MPa} \), the interface debonding fraction increases, the tangent modulus decreases to \( E_p = 131 \text{ GPa} \) corresponding to \( \eta = 0.51 \). Upon increasing stress from \( \sigma_{tr} = 131 \text{ MPa} \) to \( \sigma = 350 \text{ MPa} \), the tangent modulus remains constant of \( E_p = 151 \text{ GPa} \) with \( \eta = 0.51 \).

When \( V_f = 0.35 \) under \( \sigma_s = 200 \text{ MPa} \), the damages of matrix cracking and interface debonding occur at \( \sigma_s = 200 \text{ MPa} \), leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is \( \varepsilon_0 = 0.0009\% \); the initial tangent modulus is \( E_p = 281 \text{ GPa} \), the degradation rate of tangent modulus is 12\% compared with original specimen and the fiber broken fraction is \( P = 0.0027 \). With increasing stress to \( \sigma_{tr} = 55 \text{ MPa} \), the interface debonding fraction increases, the tangent modulus decreases to \( E_p = 265 \text{ GPa} \) corresponding to \( \eta = 0.046 \). Upon increasing stress from \( \sigma_{tr} = 55 \text{ MPa} \) to \( \sigma = 200 \text{ MPa} \), the tangent modulus remains constant of \( E_p = 291 \text{ GPa} \) with \( \eta = 0.046 \).

Under \( \sigma_s = 300 \text{ MPa} \), the initial composite strain is \( \varepsilon_0 = 0.0047\% \) due to the damages of matrix cracking and interface debonding at \( \sigma_s = 300 \text{ MPa} \); the initial tangent modulus is \( E_p = 249 \text{ GPa} \), the degradation rate of tangent modulus is 22\% compared with original specimen and the fiber broken fraction is \( P = 0.014 \). With increasing stress to \( \sigma_{tr} = 156.2 \text{ MPa} \), the interface debonding fraction increases and the tangent modulus decreases to \( E_p = 201 \text{ GPa} \) corresponding to \( \eta = 0.27 \). When the stress increases from \( \sigma_{tr} = 156.2 \text{ MPa} \) to \( \sigma = 250 \text{ MPa} \), the tangent modulus remains constant of \( E_p = 216 \text{ GPa} \) with \( \eta = 0.27 \).

Under \( \sigma_s = 350 \text{ MPa} \), the initial composite strain is \( \varepsilon_0 = 0.0095\% \) due to the damages of the matrix cracking and interface debonding at \( \sigma_s = 350 \text{ MPa} \); the initial tangent modulus is \( E_p = 248 \text{ GPa} \), the degradation rate of tangent modulus is 22\% compared with original specimen and the fiber broken fraction is \( P = 0.028 \). With increasing stress to \( \sigma_{tr} = 206.8 \text{ MPa} \), the interface debonding fraction increases and the tangent modulus decreases to \( E_p = 191.9 \text{ GPa} \) corresponding to the interface debonding fraction \( \eta = 0.362 \). When the stress increases from \( \sigma_{tr} = 206.8 \text{ MPa} \) to \( \sigma = 300 \text{ MPa} \), the tangent modulus remains constant of \( E_p = 196.7 \text{ GPa} \) with \( \eta = 0.363 \).

3.2. Effect of Interface Shear Stress on Tensile Damage and Fracture of SiC/SiC Composite with Stochastic Loading

The interface shear stress transfers the load between the fiber and the matrix when the matrix cracking and interface debonding occur. For the weak interface bonding between the fiber and the
matrix of SiC/SiC composite, the value of the interface shear stress is between $\tau_i = 10$ and 30 MPa [22]. In the present analysis, the effect of the interface shear stress (i.e., $\tau_i = 15$ and 20 MPa) on the tensile stress–strain curves, tangent modulus, interface debonding fraction and broken fiber fraction of SiC/SiC composite subjected to stochastic loading of $\sigma_s = 200, 230$ and 250 MPa are shown in Figures 5 and 6 and Table 2. When the interface shear stress increases, the load transfer capacity between the fiber and the matrix increases, the initial composite strain remains the same, the initial tangent modulus increases, the transition stress for interface debonding remains the same and the initial fiber broken fraction remains the same.

![Figure 5](image1.png)

![Figure 6](image2.png)
The degradation rate of tangent modulus is 48% compared with original specimen and the fiber broken fraction is 0.028. With increasing stress to \( \sigma = 200 \) MPa, the damages of matrix cracking and interface debonding occur at \( \sigma = 200 \) MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is \( \epsilon_0 = 0.00943\% \); the initial tangent modulus is \( E_p = 195.6 \) GPa, the degradation rate of tangent modulus is 37% compared with original specimen and the fiber broken fraction is \( P = 0.028 \). With increasing stress to \( \sigma_{\text{tr}} = 125.4 \) MPa, the interface debonding fraction increases, the tangent modulus decreases to \( E_p = 108.1 \) GPa corresponding to \( \eta = 0.374 \). When the stress increases from \( \sigma_{\text{tr}} = 15 \) to 20 MPa, the tangent modulus remains constant of \( E_p = 131.3 \) GPa with \( \eta = 0.382 \).

When \( \tau_1 = 15 \) MPa under \( \sigma_s = 200 \) MPa, the damages of matrix cracking and interface debonding occur at \( \sigma_s = 200 \) MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is \( \epsilon_0 = 0.00943\% \); the initial tangent modulus is \( E_p = 195.6 \) GPa, the degradation rate of tangent modulus is 37% compared with original specimen and the fiber broken fraction is \( P = 0.028 \). With increasing stress to \( \sigma_{\text{tr}} = 125.4 \) MPa, the interface debonding fraction increases, the tangent modulus decreases to \( E_p = 108.1 \) GPa corresponding to \( \eta = 0.374 \). When the stress increases from \( \sigma_{\text{tr}} = 15 \) to 20 MPa, the tangent modulus remains constant of \( E_p = 131.3 \) GPa with \( \eta = 0.382 \).

When \( \tau_1 = 20 \) MPa under \( \sigma_s = 200 \) MPa, the damages of matrix cracking and interface debonding occur at \( \sigma_s = 200 \) MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is \( \epsilon_0 = 0.00943\% \); the initial tangent modulus is \( E_p = 195.6 \) GPa, the degradation rate of tangent modulus is 37% compared with original specimen and the fiber broken fraction is \( P = 0.028 \). With increasing stress to \( \sigma_{\text{tr}} = 125.4 \) MPa, the interface debonding fraction increases, the tangent modulus decreases to \( E_p = 108.1 \) GPa corresponding to \( \eta = 0.374 \). When the stress increases from \( \sigma_{\text{tr}} = 15 \) to 20 MPa, the tangent modulus remains constant of \( E_p = 131.3 \) GPa with \( \eta = 0.382 \).

When \( \tau_1 = 15 \) MPa under \( \sigma_s = 230 \) MPa, the damages of matrix cracking and interface debonding occur at \( \sigma_s = 230 \) MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is \( \epsilon_0 = 0.00943\% \); the initial tangent modulus is \( E_p = 195.6 \) GPa, the degradation rate of tangent modulus is 37% compared with original specimen and the fiber broken fraction is \( P = 0.028 \). With increasing stress to \( \sigma_{\text{tr}} = 125.4 \) MPa, the interface debonding fraction increases, the tangent modulus decreases to \( E_p = 108.1 \) GPa corresponding to \( \eta = 0.374 \). When the stress increases from \( \sigma_{\text{tr}} = 15 \) to 20 MPa, the tangent modulus remains constant of \( E_p = 131.3 \) GPa with \( \eta = 0.382 \).

When \( \tau_1 = 20 \) MPa under \( \sigma_s = 230 \) MPa, the damages of matrix cracking and interface debonding occur at \( \sigma_s = 230 \) MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is \( \epsilon_0 = 0.00943\% \); the initial tangent modulus is \( E_p = 195.6 \) GPa, the degradation rate of tangent modulus is 37% compared with original specimen and the fiber broken fraction is \( P = 0.028 \). With increasing stress to \( \sigma_{\text{tr}} = 125.4 \) MPa, the interface debonding fraction increases, the tangent modulus decreases to \( E_p = 108.1 \) GPa corresponding to \( \eta = 0.374 \). When the stress increases from \( \sigma_{\text{tr}} = 15 \) to 20 MPa, the tangent modulus remains constant of \( E_p = 131.3 \) GPa with \( \eta = 0.382 \).

Under \( \sigma_s = 230 \) MPa, the initial composite strain is \( \epsilon_0 = 0.021\% \) due to the damages of matrix cracking and interface debonding at \( \sigma_s = 230 \) MPa; the initial tangent modulus is \( E_p = 163 \) GPa, the degradation rate of tangent modulus is 48% compared with original specimen and the fiber broken fraction is \( P = 0.056 \). With increasing stress to \( \sigma_{\text{tr}} = 154 \) MPa, the interface debonding fraction increases.
When the interface debonding energy increases, the initial composite strain, tangent modulus and fracture stress increases and the tangent modulus decreases to $E_p = 60.7$ GPa corresponding to $\eta = 0.952$. When the stress increases from $\sigma_{tr} = 173.8$ MPa to $\sigma = 250$ MPa, the tangent modulus remains constant of $E_p = 70.4$ GPa with $\eta = 0.975$.

When $\sigma_s = 250$ MPa under $\sigma_s = 200$ MPa, the damages of matrix cracking and interface debonding occur at $\sigma_s = 200$ MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is $\epsilon_0 = 0.038\%$ due to the damages of the matrix cracking and interface debonding at $\sigma_s = 250$ MPa; the initial tangent modulus is $E_p = 149.5$ GPa, the degradation rate of tangent modulus is 52\% compared with original specimen and the fiber broken fraction is $P = 0.092$. With increasing stress to $\sigma_{tr} = 173.8$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_p = 130$ GPa corresponding to $\eta = 0.275$. When the stress increases from $\sigma_{tr} = 173.8$ MPa to $\sigma = 200$ MPa, the tangent modulus remains constant of $E_p = 152.9$ GPa with $\eta = 0.284$.

Under $\sigma_s = 230$ MPa, the initial composite strain is $\epsilon_0 = 0.021\%$ due to the damages of the matrix cracking and interface debonding at $\sigma_s = 230$ MPa; the initial tangent modulus is $E_p = 185$ GPa, the degradation rate of the tangent modulus is 40\% compared with original specimen and the fiber broken fraction is $P = 0.056$. With increasing stress to $\sigma_{tr} = 154$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_p = 88.6$ GPa corresponding to $\eta = 0.53$. When the stress increases from $\sigma_{tr} = 154$ MPa to $\sigma = 230$ MPa, the tangent modulus remains constant of $E_p = 105.5$ GPa with $\eta = 0.54$.

Under $\sigma_s = 250$ MPa, the initial composite strain is $\epsilon_0 = 0.038\%$ due to the damages of the matrix cracking and interface debonding; the initial tangent modulus is $E_p = 171.7$ GPa, the degradation rate of tangent modulus is 45\% compared with original specimen and the fiber broken fraction is $P = 0.092$. With increasing stress to $\sigma_{tr} = 173.8$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_p = 72.8$ GPa corresponding to $\eta = 0.714$. When the stress increases from $\sigma_{tr} = 173.8$ MPa to $\sigma = 250$ MPa, the tangent modulus remains constant of $E_p = 86.3$ GPa with $\eta = 0.73$.

### 3.3. Effect of Interface Debonding Energy on Tensile Damage and Fracture of SiC/SiC Composite with Stochastic Loading

The interface debonding energy is a key interface property of CMCs. Domergue et al. [23] estimated the interface debonding energy of unidirectional SiC/CAS composite by analyzing the hysteresis loops and obtained the interface debonding energy is in the range of $\zeta_d = 0.1 - 0.8$ J/m². The effect of the interface debonding energy (i.e., $\zeta_d = 0.1$ and $0.3$ J/m²) on the tensile stress–strain curves, tangent modulus, interface debonding fraction and broken fiber fraction of SiC/SiC composite subjected to stochastic loading of $\sigma_s = 180, 220$ and $250$ MPa are shown in Figures 7 and 8 and Table 3. When the interface debonding energy increases, the initial composite strain, tangent modulus and broken fiber fraction remain the same, the transition stress for interface debonding decreases.
Figure 7. (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 180, 220$ and $250$ MPa when $\zeta_d = 0.1$ J/m$^2$.

Figure 8. Cont.
With increasing stress to $\sigma$, modulus decreases to $E$ to $\sigma$ compared with original specimen and the fiber broken fraction is $P$. The initial composite strain is $\varepsilon_0 = 0.00537$% due to the damages of the matrix cracking and interface debonding; the initial tangent modulus is $E_T = 251.5$ GPa, the degradation rate of the tangent modulus is 19% compared with original specimen and the fiber broken fraction is $P = 0.018$. With increasing stress to $\sigma_T = 140.8$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_T = 182.6$ GPa corresponding to $\eta = 0.167$. When the stress increases from $\sigma_T = 140.8$ MPa to $\sigma = 180$ MPa, the tangent modulus remains constant of $E_T = 191$ GPa with $\eta = 0.171$.

When $\zeta_d = 0.1$ J/m$^2$ under $\sigma_s = 180$ MPa, the damages of matrix cracking and interface debonding occur at $\sigma_s = 180$ MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is $\varepsilon_0 = 0.00537$%; the initial tangent modulus is $E_T = 251.5$ GPa, the degradation rate of the tangent modulus is 19% compared with original specimen and the fiber broken fraction is $P = 0.018$. With increasing stress to $\sigma_T = 140.8$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_T = 171.6$ GPa corresponding to $\eta = 0.167$. When the stress increases from $\sigma_T = 140.8$ MPa to $\sigma = 180$ MPa, the tangent modulus remains constant of $E_T = 191$ GPa with $\eta = 0.171$.

Under $\sigma_s = 220$ MPa, the initial composite strain is $\varepsilon_0 = 0.01611$% due to the damages of the matrix cracking and interface debonding; the initial tangent modulus is $E_T = 209$ GPa, the degradation rate of the tangent modulus is 33% compared with original specimen and the fiber broken fraction is $P = 0.045$. With increasing stress to $\sigma_T = 182.6$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_T = 103.6$ GPa corresponding to $\eta = 0.446$. When the stress increases from $\sigma_T = 182.6$ MPa to $\sigma = 220$ MPa, the tangent modulus remains constant of $E_T = 118.4$ GPa with $\eta = 0.451$.

![Figure 8](image_url)

**Figure 8.** (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 180, 220$ and $250$ MPa when $\zeta_d = 0.3$ J/m$^2$.

| $\sigma_s$ (MPa) | $\varepsilon_0$(%) | $E_T$(GPa) | $\sigma_T$(MPa) | $P$(%) |
|------------------|--------------------|------------|----------------|-------|
| 180              | 0.00537            | 251.5      | 140.8          | 0.018 |
| 220              | 0.0161             | 209        | 182.6          | 0.045 |
| 250              | 0.038              | 188.5      | 211.2          | 0.092 |
| 180              | 0.00537            | 251.5      | 118.8          | 0.018 |
| 220              | 0.0161             | 209        | 158.4          | 0.045 |
| 250              | 0.038              | 188.5      | 189.2          | 0.092 |

**Table 3.** The effect of interface debonding energy ($\zeta_d = 0.1$ and $0.3$ J/m$^2$) on tensile stress–strain curve, tangent modulus, interface debonding fraction and broken fiber fraction of SiC/SiC composite subjected to stochastic loading of $\sigma_s = 180, 220$ and $250$ MPa.
Under $\sigma_s = 250$ MPa, the initial composite strain is $\varepsilon_0 = 0.038\%$ due to the damages of the matrix cracking and interface debonding; the initial tangent modulus is $E_p = 188.5$ GPa, the degradation rate of the tangent modulus is 40% compared with the original specimen and the fiber broken fraction is $P = 0.092$. With increasing stress to $\sigma_{tr} = 211.2$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_p = 77$ GPa corresponding to $\eta = 0.694$. When the stress increases from $\sigma_{tr} = 211.2$ MPa to $\sigma = 250$ MPa, the tangent modulus remains constant of $E_p = 88.2$ GPa with $\eta = 0.705$.

When $\zeta_d = 0.3$ J/m$^2$ under $\sigma_s = 180$ MPa, the damages of matrix cracking and interface debonding occur at $\sigma_s = 180$ MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is $\varepsilon_0 = 0.00537\%$; the initial tangent modulus is $E_p = 251.5$ GPa, the degradation rate of the tangent modulus is 19% compared with the original specimen and the fiber broken fraction is $P = 0.018$. With increasing stress to $\sigma_{tr} = 118.8$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_p = 180.6$ GPa corresponding to $\eta = 0.141$. When the stress increases from $\sigma_{tr} = 118.8$ MPa to $\sigma = 180$ MPa, the tangent modulus remains constant of $E_p = 202$ GPa with $\eta = 0.144$.

Under $\sigma_s = 220$ MPa, the initial composite strain is $\varepsilon_0 = 0.0161\%$; the initial tangent modulus is $E_p = 209$ GPa, the degradation rate of tangent modulus is 33% compared with original specimen and the fiber broken fraction is $P = 0.045$. With increasing stress to $\sigma_{tr} = 158.4$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_p = 110.4$ GPa corresponding to $\eta = 0.387$. When the stress increases from $\sigma_{tr} = 158.4$ MPa to $\sigma = 220$ MPa, the tangent modulus remains constant of $E_p = 127.6$ GPa with $\eta = 0.396$.

Under $\sigma_s = 250$ MPa, the initial composite strain is $\varepsilon_0 = 0.038\%$ due to the damages of the matrix cracking and interface debonding; the initial tangent modulus is $E_p = 188.5$ GPa, the degradation rate of tangent modulus is 40% compared with the original specimen and the fiber broken fraction is $P = 0.092$. With increasing stress to $\sigma_{tr} = 189.2$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_p = 82$ GPa corresponding to $\eta = 0.622$. When the stress increases from $\sigma_{tr} = 189.2$ MPa to $\sigma = 250$ MPa, the tangent modulus remains constant of $E_p = 95.1$ GPa with $\eta = 0.63$.

3.4. Effect of Saturation Matrix Crack Spacing on Tensile Damage and Fracture of SiC/SiC Composite with Stochastic Loading

Li [24] investigated multiple matrix cracking of CMCs with different fiber preforms and found that the saturation matrix cracking spacing is in the range of $l_s = 100$ and 500 $\mu$m. In the present analysis, the effect of the saturation matrix crack spacing (i.e., $l_s = 200$ and 250 $\mu$m) on the tensile stress–strain curves, tangent modulus, interface debonding fraction and broken fiber fraction of SiC/SiC composite subjected to stochastic loading of $\sigma_s = 180, 220$ and 250 MPa are shown in Figures 9 and 10 and Table 4. When saturation matrix crack spacing increases, the initial composite strain decreases, the initial tangent modulus increases, the transition stress for interface debonding and initial fiber broken fraction remain the same.

![Figure 9. Cont.](image-url)
Table 4. The effect of the saturation matrix crack spacing ($l_s = 200$ and $250$ μm) on tensile stress–strain curve, tangent modulus, interface debonding fraction and broken fiber fraction of SiC/SiC composite subjected to stochastic loading of $\sigma_s = 180, 220$ and $250$ MPa.

| $l_s$ (μm) | $\sigma_s$ (MPa) | $\varepsilon_0$ (%) | $E_p$ (GPa) | $\sigma_{tr}$ (MPa) | $P$ (%) |
|------------|------------------|---------------------|-------------|---------------------|--------|
| 200        | 180              | 0.00567             | 229.8       | 103.4               | 0.018  |

Figure 9. (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 180, 220$ and $250$ MPa when $l_s = 200$ μm.

Figure 10. (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 180, 220$ and $250$ MPa when $l_s = 250$ μm.
When $l_s = 200 \, \mu m$ under $\sigma_s = 180$ MPa, the damages of matrix cracking and interface debonding occur at $\sigma_t = 200$ MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is $\varepsilon_0 = 0.00567\%$ due to the damages of the matrix cracking and interface debonding; the initial tangent modulus is $E_t = 229.8$ GPa, the degradation rate of tangent modulus is 26% compared with original specimen and the fiber broken fraction is $P = 0.018$. With increasing stress to $\sigma_t = 103.4$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_t = 180.3$ GPa corresponding to $\eta = 0.184$. When the stress increases from $\sigma_t = 103.4$ MPa to $\sigma = 180$ MPa, the tangent modulus remains constant of $E_t = 182.3$ GPa with $\eta = 0.189$.

Under $\sigma_s = 220$ MPa, the initial composite strain is $\varepsilon_0 = 0.0168\%$ due to the damages of the matrix cracking and the interface debonding; the initial tangent modulus is $E_t = 180.3$ GPa, the degradation rate of tangent modulus is 42% compared with original specimen and the fiber broken fraction is $P = 0.045$. With increasing stress to $\sigma_t = 143$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_t = 134$ GPa corresponding to $\eta = 0.524$. When the stress increases from $\sigma_t = 143$ MPa to $\sigma = 220$ MPa, the tangent modulus remains constant of $E_t = 105.2$ GPa with $\eta = 0.536$.

Under $\sigma_s = 250$ MPa, the initial composite strain is $\varepsilon_0 = 0.038\%$ due to the damages of the matrix cracking and the interface debonding; the initial tangent modulus is $E_t = 157.7$ GPa, the degradation rate of tangent modulus is 50% compared with original specimen and the fiber broken fraction is $P = 0.092$. With increasing stress to $\sigma_t = 173.8$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_t = 64$ GPa corresponding to $\eta = 0.857$. When the stress increases from $\sigma_t = 173.8$ MPa to $\sigma = 250$ MPa, the tangent modulus remains constant of $E_t = 75.3$ GPa with $\eta = 0.869$.

When $l_s = 250 \, \mu m$ under $\sigma_s = 180$ MPa, the damages of matrix cracking and interface debonding occur at $\sigma_t = 180$ MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is $\varepsilon_0 = 0.00549\%$ due to the damages of the matrix cracking and the interface debonding; the initial tangent modulus is $E_t = 242.3$ GPa, the degradation rate of tangent modulus is 22% compared with original specimen and the fiber broken fraction is $P = 0.018$. With increasing stress to $\sigma_t = 103.4$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_t = 173.7$ GPa corresponding to $\eta = 0.147$.

| $l_s$ = 200 $\mu m$ | $\sigma_s$ = 180 MPa | $\varepsilon_0$/($\%$) | $E_t$/(GPa) | $\sigma_t$/(MPa) | $P$/($\%$) |
|-------------------|------------------|----------------|----------|----------------|---------|
| 0.00567 | 229.8 | 103.4 | 0.018 |

| $l_s$ = 250 $\mu m$ | $\sigma_s$ = 180 MPa | $\varepsilon_0$/($\%$) | $E_t$/(GPa) | $\sigma_t$/(MPa) | $P$/($\%$) |
|-------------------|------------------|----------------|----------|----------------|---------|
| 0.0168 | 180.3 | 143 | 0.045 |

| $l_s$ = 200 $\mu m$ | $\sigma_s$ = 220 MPa | $\varepsilon_0$/($\%$) | $E_t$/(GPa) | $\sigma_t$/(MPa) | $P$/($\%$) |
|-------------------|------------------|----------------|----------|----------------|---------|
| 0.00567 | 229.8 | 103.4 | 0.018 |

| $l_s$ = 250 $\mu m$ | $\sigma_s$ = 220 MPa | $\varepsilon_0$/($\%$) | $E_t$/(GPa) | $\sigma_t$/(MPa) | $P$/($\%$) |
|-------------------|------------------|----------------|----------|----------------|---------|
| 0.0168 | 180.3 | 143 | 0.045 |

| $l_s$ = 200 $\mu m$ | $\sigma_s$ = 250 MPa | $\varepsilon_0$/($\%$) | $E_t$/(GPa) | $\sigma_t$/(MPa) | $P$/($\%$) |
|-------------------|------------------|----------------|----------|----------------|---------|
| 0.038 | 157.7 | 173.8 | 0.092 |

| $l_s$ = 250 $\mu m$ | $\sigma_s$ = 250 MPa | $\varepsilon_0$/($\%$) | $E_t$/(GPa) | $\sigma_t$/(MPa) | $P$/($\%$) |
|-------------------|------------------|----------------|----------|----------------|---------|
| 0.038 | 174.8 | 173.8 | 0.092 |
When the stress increases from $\sigma_{tr} = 103.45$ MPa to $\sigma = 180$ MPa, the tangent modulus remains constant of $E_p = 198.7$ GPa with $\eta = 0.151$.

Under $\sigma_s = 220$ MPa, the initial composite strain is $\varepsilon_0 = 0.0164\%$ due to the damages of the matrix cracking and the interface debonding; the initial tangent modulus is $E_p = 196.8$ GPa, the degradation rate of tangent modulus is 37% compared with original specimen and the fiber broken fraction is $P = 0.045$. With increasing stress to $\sigma_{tr} = 143$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_p = 102.8$ GPa corresponding to $\eta = 0.42$. When the stress increases from $\sigma_{tr} = 143$ MPa to $\sigma = 220$ MPa, the tangent modulus remains constant of $E_p = 121.2$ GPa with $\eta = 0.43$.

Under $\sigma_s = 250$ MPa, the initial composite strain is $\varepsilon_0 = 0.038\%$ due to the damages of the matrix cracking and the interface debonding; the initial tangent modulus is $E_p = 174.8$ GPa, the degradation rate of tangent modulus is 44% compared with original specimen and the fiber broken fraction is $P = 0.092$. With increasing stress to $\sigma_{tr} = 173.8$ MPa, the interface debonding fraction increases and the tangent modulus decreases to $E_p = 75$ GPa corresponding to $\eta = 0.685$. When the stress increases from $\sigma_{tr} = 173.8$ MPa to $\sigma = 250$ MPa, the tangent modulus remains constant of $E_p = 88.6$ GPa with $\eta = 0.695$.

### 3.5. Effect of Fiber Strength on Tensile Damage and Fracture of SiC/SiC Composite with Stochastic Loading

Guo et al. [25] investigated the SiC fiber strength and found that the SiC fiber strength is in the range between $\sigma_c = 2.3$ and 3.7 GPa. In the present analysis, the effect of the fiber strength (i.e., $\sigma_c = 2.0$ and 2.5 GPa) on the tensile stress–strain curves, tangent modulus, interface debonding fraction and broken fiber fraction of SiC/SiC composite subjected to stochastic loading of $\sigma_s = 180, 220$ and 250 MPa are shown in Figures 11 and 12 and Table 5. When the fiber strength increases, the initial composite strain and fiber broken fraction decrease and the initial tangent composite modulus and transition stress for interface debonding remains the same.

![Figure 11](image)

**Figure 11.** (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 180, 220$ and 250 MPa when $\sigma_c = 2.0$ GPa.
Figure 12. (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 180, 220$ and $250$ MPa when $\sigma_c = 2.5$ GPa.

Table 5. The effect of the fiber strength ($\sigma_c = 2.0$ and 2.5 GPa) on tensile stress–strain curve, tangent modulus, interface debonding fraction and broken fiber fraction of SiC/SiC composite subjected to stochastic loading of $\sigma_s = 180, 220$ and $250$ MPa.

| $\sigma_c = 2.0$ GPa | $\sigma_s$ = 180 MPa | $\varepsilon_0$(%)$|$ $E_p$(GPa)$|$ $\sigma_{tr}$(MPa)$|$ $P$(%)
|---|---|---|---|---|
| $\sigma_s$ = 180 MPa | 0.0065 | 251.5 | 103.4 | 0.022 |
| $\sigma_s$ = 220 MPa | 0.02 | 209.6 | 143 | 0.058 |
| $\sigma_s$ = 250 MPa | 0.059 | 188.5 | 173.8 | 0.138 |

| $\sigma_c = 2.5$ GPa | $\sigma_s$ = 180 MPa | $\varepsilon_0$(%)$|$ $E_p$(GPa)$|$ $\sigma_{tr}$(MPa)$|$ $P$(%)
|---|---|---|---|---|
| $\sigma_s$ = 180 MPa | 0.028 | 251.5 | 103.4 | 0.0087 |
| $\sigma_s$ = 220 MPa | 0.007 | 209.6 | 143 | 0.02 |
| $\sigma_s$ = 250 MPa | 0.015 | 188.5 | 173.8 | 0.036 |
When \( \sigma_c = 2.0 \) GPa under \( \sigma_s = 180 \) MPa, the damages of matrix cracking and interface debonding occur at \( \sigma_t = 180 \) MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The composite initial strain is \( \varepsilon_0 = 0.0065\% \) due to the damages of the matrix cracking and the interface debonding; the initial tangent modulus is \( E_p = 251.5 \) GPa, the degradation rate of tangent modulus is 19\% compared with original specimen and the fiber broken fraction is \( P = 0.022 \). With increasing stress to \( \sigma_t = 103.4 \) MPa, the interface debonding fraction increases and the tangent modulus decreases to \( E_p = 187.4 \) GPa corresponding to \( \eta = 0.122 \). When the stress increases from \( \sigma_t = 103.4 \) MPa to \( \sigma = 180 \) MPa, the tangent modulus remains constant of \( E_p = 211.3 \) GPa with \( \eta = 0.126 \).

Under \( \sigma_s = 220 \) MPa, the initial composite strain is \( \varepsilon_0 = 0.02\% \) due to the damages of the matrix cracking and the interface debonding; the initial tangent modulus is \( E_p = 209.6 \) GPa, the degradation rate of tangent modulus is 33\% compared with original specimen and the fiber broken fraction is \( P = 0.058 \). With increasing stress to \( \sigma_t = 143 \) MPa, the interface debonding fraction increases and the tangent modulus decreases to \( E_p = 115.7 \) GPa corresponding to \( \eta = 0.349 \). When the stress increases from \( \sigma_t = 143 \) MPa to \( \sigma = 220 \) MPa, the tangent modulus remains constant of \( E_p = 134.9 \) GPa with \( \eta = 0.357 \).

Under \( \sigma_s = 250 \) MPa, the initial composite strain is \( \varepsilon_0 = 0.059\% \); the initial tangent modulus is \( E_p = 188.5 \) GPa, the degradation rate of tangent modulus is 40\% compared with original specimen and the fiber broken fraction is \( P = 0.138 \). With increasing stress to \( \sigma_t = 173.8 \) MPa, the interface debonding fraction increases and the tangent modulus decreases to \( E_p = 86 \) GPa corresponding to \( \eta = 0.571 \). When the stress increases from \( \sigma_t = 173.8 \) MPa to \( \sigma = 250 \) MPa, the tangent modulus remains constant of \( E_p = 100.6 \) GPa with \( \eta = 0.579 \).

When \( \sigma_c = 2.5 \) GPa under \( \sigma_s = 180 \) MPa, the damages of matrix cracking and interface debonding occur at \( \sigma_t = 180 \) MPa, leading to the increase of the composite initial strain, decreasing of the tangent modulus and increase of the broken fiber fraction. The initial composite strain is \( \varepsilon_0 = 0.0028\% \) due to the damages of the matrix cracking and the interface debonding; the initial tangent modulus is \( E_p = 251.5 \) GPa, the degradation rate of tangent modulus is 19\% compared with original specimen and the fiber broken fraction is \( P = 0.0087 \). With increasing stress to \( \sigma_t = 103.4 \) MPa, the interface debonding fraction increases and the tangent modulus decreases to \( E_p = 115.7 \) GPa corresponding to \( \eta = 0.122 \). When the stress increases from \( \sigma_t = 103.4 \) MPa to \( \sigma = 180 \) MPa, the tangent modulus remains constant of \( E_p = 211.3 \) GPa with \( \eta = 0.126 \).

Under \( \sigma_s = 220 \) MPa, the initial composite strain is \( \varepsilon_0 = 0.007\% \) due to the damages of the matrix cracking and the interface debonding; the initial tangent modulus is \( E_p = 209.6 \) GPa, the degradation rate of tangent modulus is 33\% compared with original specimen and the fiber broken fraction is \( P = 0.02 \). With increasing stress to \( \sigma_t = 143 \) MPa, the interface debonding fraction increases and the tangent modulus decreases to \( E_p = 115.7 \) GPa corresponding to \( \eta = 0.344 \). When the stress increases from \( \sigma_t = 143 \) MPa to \( \sigma = 220 \) MPa, the tangent modulus remains constant of \( E_p = 134.9 \) GPa with \( \eta = 0.357 \).

Under \( \sigma_s = 250 \) MPa, the initial composite strain is \( \varepsilon_0 = 0.015\% \); the initial tangent modulus is \( E_p = 188.5 \) GPa, the degradation rate of the tangent modulus is 40\% compared with original specimen and the fiber broken fraction is \( P = 0.036 \). With increasing stress to \( \sigma_t = 173.8 \) MPa, the interface debonding fraction increases and the tangent modulus decreases to \( E_p = 86 \) GPa corresponding to \( \eta = 0.571 \). When the stress increases from \( \sigma_t = 173.8 \) MPa to \( \sigma = 250 \) MPa, the tangent modulus remains constant of \( E_p = 100.6 \) GPa with \( \eta = 0.579 \).

4. Experimental Comparisons

Li et al. [4], Liu [5], Guo and Kagawa [6] and Morscher [7] investigated tensile behavior of unidirectional and 2D SiC/SiC composites at room temperature. In this section, using the developed damage models and micromechanical constitutive models for the conditions of matrix cracking, interface debonding and fiber failure, the experimental tensile stress–strain curves are predicted.
The comparisons between tensile stress–strain curves with and without stochastic loading are analyzed. The relationships between the stochastic loading stress levels, tangent modulus, interface debonding fraction and fiber broken fraction are established.

4.1. 2D SiC/SiC under Stochastic Loading of 140, 180, 200 and 240 MPa

Li et al. [4] investigated the tensile behavior of 2D SiC/SiC composite at room temperature. The composite was fabricated using chemical vapor infiltration (CVI) method. The tensile test was performed under displacement control with the speed of 0.3 mm/min. The experimental tensile stress–strain curves, tangent modulus versus strain curves, interface debonding fraction and broken fiber fraction versus stress curves of 2D SiC/SiC composite without stochastic loading and with stochastic loading at \( \sigma_s = 140, 180, 200 \) and 240 MPa at room temperature are shown in Figure 13 and Table 6. When stochastic loading stress increases, the initial composite strain increases, the initial tangent modulus decreases, the transition stress for interface debonding increases and the initial fiber broken fraction increases.

![Figure 13. (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of 2D SiC/SiC composite for conditions without stochastic loading and with stochastic loading of \( \sigma_s = 140, 180, 200 \) and 240 MPa.](image-url)
Table 6. The tensile stress–strain curve, tangent modulus, interface debonding fraction and broken fiber fraction of 2D SiC/SiC composite subjected to stochastic loading of $\sigma_s = 140, 180, 200$ and 240 MPa.

| $\sigma_s$ (MPa) | $\varepsilon_0$ (%) | $E_p$ (GPa) | $\sigma_{tr}$ (MPa) | $P$ |
|------------------|---------------------|-------------|---------------------|-----|
| 140              | 0.001               | 288         | 63.8                | 0.006 |
| 180              | 0.005               | 251         | 103.4               | 0.018 |
| 180              | 0.009               | 229.5       | 123.2               | 0.028 |
| 180              | 0.028               | 194.2       | 162.8               | 0.078 |

Under $\sigma_s = 140$ MPa, the initial strain is $\varepsilon_0 = 0.001\%$; the initial tangent modulus is $E_p = 288$ GPa, the degradation rate of tangent modulus is 7% compared with original specimen and the fiber broken fraction is $P = 0.006$. With increasing stress to $\sigma_{tr} = 63.8$ MPa, the tangent modulus decreases to $E_p = 267.4$ GPa corresponding to $\eta = 0.024$. When the stress increases from $\sigma_{tr} = 63.8$ MPa to $\sigma = 140$ MPa, the tangent modulus remains constant of $E_p = 282.3$ GPa with $\eta = 0.025$.

Under $\sigma_s = 180$ MPa, the initial strain is $\varepsilon_0 = 0.005\%$; the initial tangent modulus is $E_p = 251$ GPa, the degradation rate of tangent modulus is 19% compared with original specimen and the fiber broken fraction is $P = 0.018$. With increasing stress to $\sigma_{tr} = 103.4$ MPa, the tangent modulus decreases to $E_p = 187.4$ GPa corresponding to $\eta = 0.122$. When the stress increases from $\sigma_{tr} = 103.4$ MPa to $\sigma = 180$ MPa, the tangent modulus remains constant of $E_p = 211.3$ GPa with $\eta = 0.126$.

Under $\sigma_s = 200$ MPa, the initial strain is $\varepsilon_0 = 0.009\%$; the initial tangent modulus is $E_p = 229.5$ GPa, the degradation rate of tangent modulus is 26% compared with original specimen and the fiber broken fraction is $P = 0.028$. With increasing stress to $\sigma_{tr} = 123.2$ MPa, the tangent modulus decreases to $E_p = 147.1$ GPa corresponding to $\eta = 0.22$. When the stress increases from $\sigma_{tr} = 123.2$ MPa to $\sigma = 200$ MPa, the tangent modulus remains constant of $E_p = 169.7$ GPa with $\eta = 0.226$.

Under $\sigma_s = 240$ MPa, the initial strain is $\varepsilon_0 = 0.028\%$; the initial tangent modulus is $E_p = 194.2$ GPa, the degradation rate of tangent modulus is 38% compared with original specimen and the fiber broken fraction is $P = 0.078$. With increasing stress to $\sigma_{tr} = 162.8$ MPa, the tangent modulus decreases to $E_p = 94$ GPa corresponding to $\eta = 0.49$. When the stress increases from $\sigma_{tr} = 162.8$ MPa to $\sigma = 240$ MPa, the tangent modulus remains constant of $E_p = 109.8$ GPa with $\eta = 0.5$.

4.2. UD and 2D SiC/SiC under Stochastic Loading

Liu [5] investigated the tensile behavior of unidirectional and 2D SiC/SiC composites at room temperature. The tensile test was performed under displacement control with the loading rate of 0.2 mm/min.

For unidirectional SiC/SiC composite, the tensile stress–strain curves, tangent modulus versus strain curves, interface debonding fraction and broken fiber fraction versus stress curves without stochastic loading and with stochastic loading at $\sigma_s = 140$, 180, 200 and 220 MPa at room temperature are shown in Figure 14 and Table 7.
temperature. The tensile test was performed under displacement control with the loading rate of 0.2
strain curves, interface debonding fraction and broken fiber fraction versus stress curves without
stochastic loading and with stochastic loading at $E_p = 187.4 \text{ GPa}$ corresponding to
are shown in Figure 14 and Table 7.

The degradation rate of tangent modulus is 1% compared with original specimen and the fiber broken
fraction is $P = 0.01$. With increasing stress to $E_p = 235.9 \text{ GPa}$, the tangent modulus remains constant of

Under $\sigma_s = 140 \text{ MPa}$, the initial strain is $\epsilon_0 = 0.002\%$; the initial tangent modulus is $E_p = 235.9 \text{ GPa}$, the
degradation rate of tangent modulus is 1% compared with original specimen and the fiber broken
fraction is $P = 0.01$. With increasing stress to $E_p = 58.8 \text{ MPa}$, the tangent modulus decreases to
$E_p = 235.7$ GPa corresponding to $\eta = 0.0001$. When the stress increases from $\sigma_{tr} = 58.8$ MPa to $\sigma = 140$ MPa, the tangent modulus remains constant of $E_p = 235.8$ GPa with $\eta = 0.0001$

Under $\sigma_s = 200$ MPa, the initial strain is $\epsilon_0 = 0.0174\%$; the initial tangent modulus is $E_p = 229.6$ GPa, the degradation rate of tangent modulus is 2.8% compared with original specimen and the fiber broken fraction is $P = 0.023$. With increasing stress to $\sigma_{tr} = 118.8$ MPa, the tangent modulus decreases to $E_p = 215.4$ GPa corresponding to $\eta = 0.023$. When the stress increases from $\sigma_{tr} = 118.8$ MPa to $\sigma = 200$ MPa, the tangent modulus remains constant of $E_p = 217.8$ GPa with $\eta = 0.023$.

For 2D SiC/SiC composite, the tensile stress–strain curves, tangent modulus versus strain curves, interface debonding fraction and broken fiber fraction versus stress curves without stochastic loading and with stochastic loading at $\sigma_s = 80$, 100 and 120 MPa at room temperature are shown in Figure 15 and Table 8.

![Figure 15](image-url)

**Figure 15.** (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of 2D SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 80$, 100 and 120 MPa.
Table 8. The tensile stress–strain curve, tangent modulus, interface debonding fraction and broken fiber fraction of 2D SiC/SiC composite subjected to stochastic loading of $\sigma_s = 80, 100$ and 120 MPa.

| $\sigma_s$ = 80 MPa | $\varepsilon_0$/($) | $E_p$/(GPa) | $\sigma_{tr}$/MPa | $P$  |
|---------------------|---------------------|-------------|-------------------|------|
| 0.002               | 138.3               | 13.2        | 0.01              |

| $\sigma_s$ = 100 MPa | $\varepsilon_0$/($) | $E_p$/(GPa) | $\sigma_{tr}$/MPa | $P$  |
|---------------------|---------------------|-------------|-------------------|------|
| 0.008               | 135.6               | 32.4        | 0.027             |

| $\sigma_s$ = 120 MPa | $\varepsilon_0$/($) | $E_p$/(GPa) | $\sigma_{tr}$/MPa | $P$  |
|---------------------|---------------------|-------------|-------------------|------|
| 0.025               | 131.9               | 52.8        | 0.068             |

Under $\sigma_s = 80$ MPa, the initial strain is $\varepsilon_0 = 0.002\%$; the initial tangent modulus is $E_p = 138.3 \text{ GPa}$, the degradation rate of tangent modulus is 1.2% compared with original specimen and the fiber broken fraction is $P = 0.01$. With increasing stress to $\sigma_{tr} = 13.2 \text{ MPa}$, the tangent modulus decreases to $E_p = 137.6 \text{ GPa}$ corresponding to $\eta = 0.001$. When stress increases from $\sigma_{tr} = 13.2 \text{ MPa}$ to $\sigma = 80 \text{ MPa}$, the tangent modulus remains constant of $E_p = 137.7 \text{ GPa}$ with $\eta = 0.002$.

Under $\sigma_s = 100$ MPa, the initial strain is $\varepsilon_0 = 0.008\%$; the initial tangent modulus is $E_p = 135.6 \text{ GPa}$, the degradation rate of tangent modulus is 3.1% compared with original specimen and the fiber broken fraction is $P = 0.027$. With increasing stress to $\sigma_{tr} = 32.4 \text{ MPa}$, the tangent modulus decreases to $E_p = 131 \text{ GPa}$ corresponding to $\eta = 0.012$. When stress increases from $\sigma_{tr} = 32.4 \text{ MPa}$ to $\sigma = 100 \text{ MPa}$, the tangent modulus remains constant of $E_p = 131.3 \text{ GPa}$ with $\eta = 0.013$.

Under $\sigma_s = 120$ MPa, the initial strain is $\varepsilon_0 = 0.025\%$; the initial tangent modulus is $E_p = 131.9 \text{ GPa}$, the degradation rate of tangent modulus is 5.8% compared with original specimen and the fiber broken fraction is $P = 0.068$. With increasing stress to $\sigma_{tr} = 52.8 \text{ MPa}$, the tangent modulus decreases to $E_p = 119.5 \text{ GPa}$ corresponding to $\eta = 0.038$. When stress increases from $\sigma_{tr} = 52.8 \text{ MPa}$ to $\sigma = 120 \text{ MPa}$, the tangent modulus remains constant of $E_p = 119.5 \text{ GPa}$ with $\eta = 0.039$.

4.3. 2D SiC/SiC under Stochastic Loading of 80, 100 and 120 MPa

Guo and Kagawa [6] investigated the tensile behavior of 2D plain-woven fabric SiC/SiC composite fabricated by the PIP process. The quasi-static tensile test was conducted under displacement control with the rate of 0.5 mm/min. The tensile stress–strain curves, tangent modulus versus strain curves, interface debonding fraction and broken fiber fraction versus stress curves without stochastic loading and with stochastic loading at $\sigma_s = 80, 100$ and 120 MPa at room temperature are shown in Figure 16 and Table 9.
The degradation rate of tangent modulus is 11% compared with original specimen and the fiber broken fraction is $P = 0.068$. With increasing stress to $P = 0.01$. With increasing stress to $p = 13.2$ GPa, the tangent modulus decreases to $p = 49.4$ GPa corresponding to $\eta = 0.07$. When stress increases from $p = 33.6$ MPa to $\sigma = 80$ MPa, the tangent modulus remains constant of $E_p = 50.4$ GPa with $\eta = 0.074$.

Under $p = 100$ MPa, the initial strain is $\varepsilon_0 = 0.005$; the initial tangent modulus is $E_p = 56$ GPa, the degradation rate of tangent modulus is 6.7% compared with original specimen and the fiber broken fraction is $P = 0.01$. With increasing stress to $p = 33.6$ MPa, the tangent modulus decreases to $E_p = 49.4$ GPa corresponding to $\eta = 0.07$. When stress increases from $p = 33.6$ MPa to $\sigma = 80$ MPa, the tangent modulus remains constant of $E_p = 50.4$ GPa with $\eta = 0.074$.

Under $p = 120$ MPa, the initial strain is $\varepsilon_0 = 0.016$; the initial tangent modulus is $E_p = 53.5$ GPa, the degradation rate of tangent modulus is 11% compared with original specimen and the fiber broken fraction is $P = 0.027$. With increasing stress to $p = 52.8$ MPa, the tangent modulus decreases to

**Figure 16.** (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of 2D SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 80, 100$ and 120 MPa.

**Table 9.** The tensile stress–strain curve, tangent modulus, interface debonding fraction and broken fiber fraction of 2D SiC/SiC composite subjected to stochastic loading of $\sigma_s = 80, 100$ and 120 MPa.

| $\sigma_s$ (MPa) | $\varepsilon_0$(%) | $E_p$(GPa) | $\sigma_tr$(MPa) | $P$  |
|-----------------|---------------------|------------|-----------------|------|
| 80              | 0.005               | 56         | 33.6            | 0.01 |
| 100             | 0.016               | 53.5       | 52.8            | 0.027|
| 120             | 0.046               | 51         | 73.2            | 0.068|
The tensile test was conducted under load control. The tensile stress–strain curves, tangent modulus $E$ versus strain curves, interface debonding fraction and broken fiber fraction versus stress curves without stochastic loading and with stochastic loading are shown in Figure 17 and Table 10.

$E_p = 39.7$ GPa corresponding to $\eta = 0.19$. When stress increases from $\sigma_{tr} = 52.8$ MPa to $\sigma = 100$ MPa, the tangent modulus remains constant of $E_p = 40.7$ GPa with $\eta = 0.2$.

Under $\sigma_s = 120$ MPa, the initial strain is $\varepsilon_0 = 0.046%$; the initial tangent modulus is $E_p = 51$ GPa, the degradation rate of tangent modulus is $15\%$ compared with original specimen and the fiber broken fraction is $P = 0.068$. With increasing stress to $\sigma_{tr} = 73.2$ MPa, the tangent modulus decreases to $E_p = 30.8$ GPa corresponding to $\eta = 0.39$. When stress increases from $\sigma_{tr} = 73.2$ MPa to $\sigma = 120$ MPa, the tangent modulus remains constant of $E_p = 31.7$ GPa with $\eta = 0.395$.

4.4. 2D SiC/SiC under Stochastic Loading of 180, 220, 260 and 300 MPa

Morscher [7] investigated the tensile behavior of 2D SiC/SiC composite at room temperature. The tensile test was conducted under load control. The tensile stress–strain curves, tangent modulus versus strain curves, interface debonding fraction and broken fiber fraction versus stress curves without stochastic loading and with stochastic loading at $\sigma_s = 180, 220, 260$ and $300$ MPa are shown in Figure 17 and Table 10.

Figure 17. (a) The tensile stress–strain curves; (b) the tangent modulus versus strain curves; (c) the interface debonding fraction versus stress curves; and (d) the broken fibers fraction versus stress curves of 2D SiC/SiC composite for conditions without stochastic loading and with stochastic loading of $\sigma_s = 180, 220, 260$ and $300$ MPa.
Table 10. The tensile stress–strain curve, tangent modulus, interface debonding fraction and broken fiber fraction of 2D SiC/SiC composite subjected to stochastic loading of $\sigma_s = 180, 220, 260$ and 300 MPa.

| $\sigma_s$ (MPa) | $\varepsilon_0$ (%) | $E_p$ (GPa) | $\sigma_{tr}$ (MPa) | $P$ |
|-----------------|---------------------|-------------|-------------------|-----|
| 180             | 0.003               | 142.3       | 122               | 0.004 |
| 220             | 0.007               | 131.4       | 162               | 0.01  |
| 260             | 0.014               | 128.7       | 150               | 0.021 |
| 300             | 0.029               | 128.3       | 148               | 0.04  |

Under $\sigma_s = 180$ MPa, the initial strain is $\varepsilon_0 = 0.003\%$; the initial tangent modulus is $E_p = 142.3$ GPa, the degradation rate of tangent modulus is 50% compared with original specimen and the fiber broken fraction is $P = 0.004$. With increasing stress to $\sigma_{tr} = 122$ MPa, the tangent modulus decreases to $E_p = 53.9$ GPa corresponding to $\eta = 0.63$. When stress increases from $\sigma_{tr} = 122$ MPa to $\sigma = 180$ MPa, the tangent modulus remains constant of $E_p = 63.9$ GPa with $\eta = 0.638$.

Under $\sigma_s = 220$ MPa, the initial strain is $\varepsilon_0 = 0.007\%$; the initial tangent modulus is $E_p = 131.4$ GPa, the degradation rate of tangent modulus is 54% compared with original specimen and the fiber broken fraction is $P = 0.01$. With increasing stress to $\sigma_{tr} = 162$ MPa, the tangent modulus decreases to $E_p = 42$ GPa corresponding to $\eta = 0.63$. When stress increases from $\sigma_{tr} = 122$ MPa to $\sigma = 180$ MPa, the tangent modulus remains constant of $E_p = 63.9$ GPa with $\eta = 0.97$.

Under $\sigma_s = 260$ MPa, the initial strain is $\varepsilon_0 = 0.014\%$; the initial tangent modulus is $E_p = 128.7$ GPa, the degradation rate of tangent modulus is 55% compared with original specimen and the fiber broken fraction is $P = 0.021$. With increasing stress to $\sigma_{tr} = 150$ MPa, the tangent modulus decreases to $E_p = 41.3$ GPa corresponding to $\eta = 0.93$. When stress increases from $\sigma_{tr} = 150$ MPa to $\sigma = 260$ MPa, the tangent modulus remains constant of $E_p = 46$ GPa with $\eta = 1.0$.

Under $\sigma_s = 300$ MPa, the initial strain is $\varepsilon_0 = 0.029\%$; the initial tangent modulus is $E_p = 128.3$ GPa, the degradation rate of tangent modulus is 55.1% compared with original specimen and the fiber broken fraction is $P = 0.04$. With increasing stress to $\sigma_{tr} = 148$ MPa, the tangent modulus decreases to $E_p = 41.3$ GPa corresponding to $\eta = 0.93$. When stress increases from $\sigma_{tr} = 148$ MPa to $\sigma = 300$ MPa, the tangent modulus remains constant of $E_p = 46$ GPa with $\eta = 1.0$.

5. Conclusions

In this paper, the effect of stochastic loading on tensile damage and fracture of fiber-reinforced CMCs is investigated. A micromechanical constitutive model is developed considering multiple damage mechanisms under tensile loading. The relationship between stochastic stress, tangent modulus, interface debonding and fiber broken is established. The effects of fiber volume, interface shear stress, interface debonding energy, saturation matrix crack spacing and fiber strength on tensile stress–strain curve, tangent modulus, interface debonding fraction and fiber broken fraction are analyzed. The experimental tensile damage and fracture of unidirectional and 2D SiC/SiC composites subjected to different stochastic loading stress are predicted.
When fiber volume increases, the initial composite strain decreases, the initial tangent modulus increases, the transition stress for interface debonding decreases and the initial fiber broken fraction decreases;

(2) When the interface shear stress increases, the initial composite strain remains the same, the initial tangent modulus increases, the transition stress for interface debonding remains the same and the initial fiber broken fraction remains the same;

(3) When the interface debonding energy increases, the initial composite strain, tangent modulus and broken fiber fraction remain the same and the transition stress for interface debonding decreases;

(4) When saturation matrix crack spacing increases, the initial composite strain decreases, the initial tangent modulus increases and the transition stress for interface debonding and initial fiber broken fraction remain the same;

(5) When the fiber strength increases, the initial composite strain and fiber broken fraction decrease and the initial tangent composite modulus and transition stress for interface debonding remains the same.

Funding: This research was funded by the Fundamental Research Funds for the Central Universities grant number [NS2019038].

Acknowledgments: The author also wishes to thank three anonymous reviewers and editors for their helpful comments on an earlier version of the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Naslain, R. Design, Preparation and properties of non-oxide CMCs for application in engines and nuclear reactors: An overview. Compos. Sci. Technol. 2004, 64, 155–170. [CrossRef]

2. Li, L.B.; Reynaud, P.; Fantozzi, G. Cyclic-dependent damage evolution in self-healing woven SiC/[Si-B-C] ceramic-matrix composites at elevated temperatures. Materials 2020, 13, 1478. [CrossRef] [PubMed]

3. Ruggles-Wrenn, M.B.; Christensen, D.T.; Chamberlain, A.L.; Lane, J.E.; Cook, T.S. Effect of frequency and environment on fatigue behavior of a CVI SiC/SiC ceramic matrix composite at 1200 °C. Compos. Sci. Technol. 2011, 71, 190–196. [CrossRef]

4. Li, P.; Wang, B.; Zhen, W.; Jiao, G. Tensile loading/unloading stress–strain behavior of 2D-SiC/SiC composites. Fuhe Cailiao Xuebao/acta Materiae Compositae Sinica 2014, 31, 676–682.

5. Liu, Y. Study on damage failure mechanism of SiC/SiC ceramic matrix composites. Master Thesis, Harbin Institute of Technology, Harbin, China, 2019.

6. Guo, S.; Kagawa, Y. Tensile fracture behavior of continuous SiC fiber-reinforced SiC matrix composites at elevated temperatures and correlation to in situ constituent properties. J. Euro Ceram. Soc. 2002, 22, 2349–2356. [CrossRef]

7. Morscher, G.N. Modal acoustic emission of damage accumulation in a woven SiC/SiC composite. Compos. Sci Technol. 1999, 59, 687–697. [CrossRef]

8. Marsili, R.; Rossi, G.; Speranzini, E. Fibre bragg gratings for the monitoring of wooden structures. Materials 2018, 11, 7. [CrossRef]

9. Morscher, G.N.; Pujar, V.V. Design guidance for in-plane mechanical properties of SiC fiber-reinforced melt-infiltrated SiC composites. Int J. Appl Ceram. Technol 2009, 6, 151–163. [CrossRef]

10. Marshall, D.B.; Cox, B.N.; Evans, A.G. The mechanics of matrix cracking in brittle-matrix fiber composites. Acta Metall. 1985, 33, 2013–2021. [CrossRef]

11. Zok, F.W.; Spearing, S.M. Matrix crack spacing in brittle matrix composites. Acta Metall. Mater. 1992, 40, 2033–2043. [CrossRef]

12. Curtin, W.A. Multiple matrix cracking in brittle matrix composites. Acta Metall. Mater. 1993, 41, 1369–1377. [CrossRef]

13. Evans, A.G. Design and life prediction issues for high-temperature engineering ceramics and their composites. Acta Mater. 1997, 45, 23–40. [CrossRef]
14. McNulty, J.C.; Zok, F.W. Low-cycle fatigue of Nicalon-fiber-reinforced ceramic composites. Compos. Sci. Technol. 1999, 59, 1597–1607. [CrossRef]

15. Naslain, R.; Lamon, J.; Pailler, R.; Bourrat, X.; Guette, A.; Langlais, F. Micro/minicomposites: A useful approach to the design and development of non-oxide CMCs. Compos. Part A 1999, 30, 537–547. [CrossRef]

16. Goto, K.; Kagawa, Y. Fracture behavior and toughness of a plain-woven SiC fiber-reinforced SiC matrix composite. Mater. Sci. Eng. A. 1996, 211, 72–81. [CrossRef]

17. Meyer, P.; Wass, A.M. Experimental results on the elevated temperature tensile response of SiC/SiC ceramic matrix notched composites. Compos. Part B 2018, 143, 269–281. [CrossRef]

18. Li, L.B.; Song, Y.; Sun, Y. Modeling the tensile behavior of unidirectional C/SiC ceramic-matrix composites. Mech Compos. Mater. 2013, 49, 659–672. [CrossRef]

19. Li, L.B.; Song, Y.; Sun, Y. Modeling the tensile behavior of cross-ply C/SiC ceramic-matrix composites. Mech Compos. Mater. 2015, 51, 359–376. [CrossRef]

20. Li, L.B. Modeling the monotonic and cyclic tensile stress–strain behavior of 2D and 2.5D woven C/SiC ceramic-matrix composites. Mech Compos. Mater. 2018, 54, 165–178. [CrossRef]

21. Curtin, W.A. Theory of mechanical properties of ceramic-matrix composites. J. Am. Ceram. Soc. 1991, 74, 2837–2845. [CrossRef]

22. Li, L.B. Assessment of the interfacial properties from fatigue hysteresis loss energy in ceramic-matrix composites with different fiber preforms at room and elevated temperatures. Mater. Sci. Eng. A 2014, 613, 17–36.

23. Domergue, J.C.; Vagaggini, E.; Evans, A.G. Relationships between hysteresis measurements and the constituent properties of ceramic matrix composites: II, Experimental studies on unidirectional materials. J. Am. Ceram. Soc. 1995, 78, 2721–2731. [CrossRef]

24. Li, L.B. Modeling matrix fracture in fiber-reinforced ceramic-matrix composites with different fiber preforms. Text. Res. J. 2020, 90, 909–924. [CrossRef]

25. Guo, S.Q.; Kagawa, Y.; Tanaka, Y.; Masuda, C. Microstructure and role of outermost coating for tensile strength of SiC fiber. Acta Mater. 1998, 46, 4941–4954. [CrossRef]

© 2020 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).