Productions of $f_1(1420)$ in pion and kaon induced reactions

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The $f_1(1420)$ productions via pion and kaon induced reactions on a proton target are investigated in an effective Lagrangian approach. Two treatments of the $t$-channel Born term, the Feynman model and the Reggeized model, are introduced to calculate total and differential cross sections of $\pi p \rightarrow f_1(1420)n$ and $K^- p \rightarrow f_1(1420)\Lambda$. The numerical results indicate that the experimental data of total cross section of $\pi p \rightarrow f_1(1420)n$ reaction can be reproduced within both the Feynman model or the Reggeized model. Based on the parameters determined in the pion induced reaction, the cross sections of the $K^- p \rightarrow f_1(1420)\Lambda$ reaction, about which few data can be found in literature, is predicted in the same energy region. It is found that the shape of total cross section obtained in kaon reduced reaction with two treatments are quite different. The cross section for both reactions are at an order of magnitude of $\mu$b, or larger, at energies up to 10 GeV. The differential cross sections for both pion and kaon induced reactions are also present. It is found that with the Reggeized model the $t$ channel will provide a sharp increase at extreme forward angles. The results suggest that experimental study of the $f_1(1420)$ in kaon induced reaction on a proton target is promising as in pion induced reaction. Such experimental measurement is also very helpful to clarify the production mechanism of the $f_1(1420)$.

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I. INTRODUCTION

The light meson is an important way to understand the nonperturbative QCD. Many light mesons have been observed and listed in the Review of Particle Physics (PDG) [1]. However, the internal structure of light meson is still a confusing problem due to large nonperturbative effect in light flavor sector. Currently, the electron-positron collision is the most important way to study light meson. It will be very helpful to study the production and properties of light meson in different reactions. A new detector, glueX, was equipped at CEBAF after 12 GeV upgraded, which will focus on light meson spectrum [2]. The pion-induced light meson production is very important in the history of discovery of many light mesons. The secondary pion beam is accessible at J-PARC [3] and COMPASS [4] with high precision. The kaon beam can be also used to study light meson, which is available at OKA@U-70 [5] and SPS@CERN [6], and J-PARC [7]. The data from future experiments at those facilities will provide a good opportunity to deep our understanding of internal structure of light meson.

In the PDG, the $f_1(1420)$ is listed as an axial-vector state with quantum number $I^G(J^{PC}) = 0^+(1^{++})$ with a suggested mass of 1426.4± 0.9 MeV and a suggested width of 54.9±2.6 MeV [1]. The $f_1(1420)$ meson was first observed in pion-nucleon interaction in the Lawrence Radiation Laboratory in 1967 [8], and confirmed in other experiments with pion beams about the year 1980 [9–11]. The $f_1(1420)$ was also observed in recent experiments in $e^+e^-$ and $J/\psi$ decays [12–14]. Though the $f_1(1420)$ is well established in experiment as a resonance structure, the internal structure of the $f_1(1420)$ is still unclear up to now. In the conventional $q\bar{q}$ picture, the $f_1(1420)$ can be classified as a partner to the $f_1(1285)$ in the $^3P_1$ nonet of axial mesons, and the mixture of nonstrange $f_{1q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and strange $f_{1s} = s\bar{s}$ was also discussed in the literature [15–17]. However, a recent study in Ref. [18] suggested that the $f_1(1420)$ is not a genuine resonance but from the decay modes of the $f_1(1285)$ in $K^+\bar{K}$ and $\pi\alpha_0(980)$.

To determine origin of the $f_1(1420)$, more precise experimental data are required. In this work, based on the existent old data we will analyze the $f_1(1420)$ production in pion induced reactions in an effective Lagrangian approach. The kaon induced production will be discussed based on the results of pion induced interaction, which will be helpful to future high-precision experimental studies. Since in the current work we focus on the production mechanism of $f_1(1420)$, the coupling constants are still determined assuming the $f_1(1420)$ as a genuine resonance [1].

This paper is organized as follows. After introduction, we present formalism including Lagrangians and amplitudes of the $f_1(1420)$ production in Section II. The numerical results of cross section follow in Sec. III. Finally, the paper ends with a brief summary.

II. FORMALISM

The reaction mechanism is illustrated in Figs. 1. Usually, the contribution from $s$ channel with nucleon pole is expected to be very small, and will be neglected in the current calculation. The $u$-channel contribution is usually small and negligible at low momentum [19]. Considering the experiment data points at high beam momentum are obtained by continuation of the $t$-channel contribution at very forward angles to all an-
gles, it is reasonable to calculate the cross section only with the $t$-channel contribution. Hence, in the present work, we do not include the contributions from the nucleon resonances in the $s$ or $u$ channel.

\[
\begin{array}{ccc}
\pi^- & \rightarrow & f_1 \\
K^- & \rightarrow & f_1 \\
p & \rightarrow & a_0 \\
\end{array}
\]

\[
K^+ \rightarrow \Lambda
\]

FIG. 1. (Color online) Feynman diagrams for the $\pi^- p \rightarrow f_1(1420)n$ reaction (left) and for the $K^- p \rightarrow f_1(1420)\Lambda$ (right) reactions.

Since dominant decay of the $f'(1420)$ is $K\bar{K}^*$, it is reasonable to take the $K^*$ exchange as the dominant contribution in the $t$ channel of kaon induced production. For pion induced production, we need vertex for decay of the $f_1(1420)$ with a pion. In Ref. [20], a branch ratio about 5% was reported in the $a_0\pi$ channel. Hence, in the current work, we adopt $a_0$ exchange in the $t$-channel of pion induced production as in the case of pion-induced $f_1(1285)$ production [19].

A. Lagrangians

For pion induced production of $f_1(1420)$, to describe the $t$-channel $a_0(980)$ ($\equiv a_0$) exchange we needs following Lagrangians [21–24]

\[
\mathcal{L}_{a_0NN} = g_{a_0NN}\bar{N}(\gamma^\mu a_0)N, \tag{1}
\]

\[
\mathcal{L}_{f_1a_0\pi} = -g_{f_1a_0\pi}f_1^\mu a_0\partial_\mu\pi, \tag{2}
\]

where $N$, $f_1$, $a_0$ and $\pi$ are the nucleon, $f_1(1420)$, $a_0(980)$ and $\pi$ meson fields, respectively. As suggested in PDG [1], the average width of $f_1(1420)$ is 54.9 MeV. In addition, the branching fraction of $f_1(1420)$ decay to $a_0\pi$ was determined to be 4% in Ref. [20]. Thus, one gets $\Gamma_{f_1\rightarrow a_0\pi} \approx 2.2$ MeV, and obtains $g_{f_1a_0\pi} \approx 2.72$. In order to reduce the number of free parameter, we take the best fitting value $g_{a_0NN} = 28.44$ in the pion-induced $f_1(1285)$ production [19].

For kaon induced production, the relevant Lagrangians for the $t$ channel read as following [24–26],

\[
\mathcal{L}_{f_1K^-K} = \frac{g_{f_1K^-K}}{m_{f_1}}(\partial_\mu K^-\partial^\mu K^*_{f_1} - \partial_\mu K^*_{f_1}\partial^\mu K^-), \tag{3}
\]

\[
\mathcal{L}_{K^-NN} = -g_{K^-NN}\bar{N}\left(K^* - \frac{\kappa_{K^-NN}}{2m_N}\sigma_{\mu\nu}\partial^\mu K^-\right)N + h.c., \tag{4}
\]

where $m_{f_1}$ is the mass of $f_1$ meson, $K$, $K^*$, $f_1$, $N$ and $\Lambda$ are the kaon, $K^*$, $f_1(1420)$, $\Lambda$ and nucleon fields, respectively. Here, we adopt coupling constants $g_{K^-NN} = -4.26$ and $\kappa_{K^-NN} = 2.66$ calculated by the Nijmegen potential [32].

The value of $g_{f_1K^-K}$ can be determined from the decay width

\[
\Gamma_{f_1\rightarrow K^-K} = \frac{g_{f_1K^-K}^2 |\vec{p}_{K^-K}|}{24\pi m_{f_1}^2} \times \left[\frac{(m_{f_1}^2 - m_{K^-}^2 - m_{K^*}^2)^2}{2} + m_{K^-}^2 E_K^2\right], \tag{5}
\]

with

\[
|\vec{p}_{K^-K}| = \frac{\lambda(m_{K^-}^2, m_{K^*}^2, m_{f_1}^2)}{2m_{f_1}}, \tag{6}
\]

\[
E_K = \sqrt{|\vec{p}_{K^-K}|^2 + m_{K^*}^2}, \tag{7}
\]

where $\lambda$ is the Källen function with a definition of $\lambda(x, y, z) = \sqrt{(x - y - z)^2 - 4yz}$. Since the branching fraction of $f_1(1420)$ decay to $K^*K$ was suggested to be 96% in Ref. [20], one gets $g_{f_1K^-K}/m_{f_1} \approx 8.36$ by taking $\Gamma_{f_1\rightarrow K^-K} \approx 52.7$ MeV [1, 20].

For the $t$ channel exchange [23], the form factor $F(q^2) = (\lambda^2 - m^2)/(\lambda^2 - q^2)$ is taken into account. Here, $q$ and $m$ are four-momentum and mass of the exchanged meson, respectively. The value of cutoff $\Lambda$ will be determined by fitting experimental data.

B. Amplitudes

According to the above Lagrangians, the scattering amplitude of the $\pi^- p \rightarrow f_1(1420)n$ or $K^- p \rightarrow f_1(1420)\Lambda$ process can be written as

\[
-iM = \epsilon_{f_1}^\mu(k_2)\bar{u}(p_2)\mathcal{A}_\mu(p_1), \tag{8}
\]

where $\epsilon_{f_1}^\mu$ is the polarization vector of $f_1$ meson, and $\bar{u}$ or $u$ is the Dirac spinor of nucleon or $\Lambda$ baryon.

For the $\pi^- p \rightarrow f_1(1420)n$ reaction, the reduced amplitude $\mathcal{A}_\mu(p_1)$ reads

\[
\mathcal{A}_\mu^{(a_0)} = i\sqrt{2}g_{a_0NN}g_{f_1a_0\pi}F(q^2)\left(\gamma_2 - \frac{f_{K^-NN}}{2m_N}\gamma_\nu\gamma_K\right)\frac{1}{t - m_{K^-}^2}\bar{k}_\mu, \tag{9}
\]

where $t = (k_1 - k_2)^2$ is the Mandelstam variables. The coupling constants are fixed with the experimental data and the fitting the pion decued $f_1(1285)$ production as addressed above.

For the process of $K^- p \rightarrow f_1(1420)\Lambda$, the reduced amplitude $\mathcal{A}_\mu(p)$ is written as

\[
\mathcal{A}_\mu^{(K^-)} = ig_{K^-NN}g_{f_1K^-K}m_{f_1}\left(\gamma_2 - \frac{f_{K^-NN}}{2m_N}\gamma_\nu\gamma_K\right)\frac{q^\mu}{t - m_{K^-}^2}\left[(k_1 - k_2) \cdot k_1g_{\mu\nu} - (k_1 - k_2)\mu \cdot k_1\nu\right], \tag{10}
\]

with

\[
q^\mu = i\left(g_{\sigma\kappa\gamma} + q_{\kappa}^\gamma q_{\kappa}^\gamma/m_{K^-}^2\right), \tag{11}
\]

Here, the coupling constants are also fixed as in the pion induced production. Hence, the only free parameter are the cut-off in form factor.
C. Reggeized $t$-channel

To analyze hadron production at high energies, a more economical approach may be furnished by a Reggeized treatment [25–30]. In our previous works [19, 28, 31], an interpolating Regge treatment was introduced to interpolate the Regge trajectories smoothly to the Feynman propagator at low energy as proposed in Ref. [33]. Because there are only 4 data points, we do not adopt the interpolating Reggeized treatment, but discuss both the Feynman model and the Regge model. For the Feynman model, the $t$-channel amplitude in Eqs. (9 and 10) is applied directly. The Regge model can be introduced by replacing the $t$-channel Feynman propagator with the Regge propagator follows:

$$\frac{1}{t - m^2_{\alpha_0}} \to \left( \frac{s}{s_{\text{scale}}} \right)^{\alpha_0(t)} \frac{\pi \alpha'_m}{\Gamma[1 + \alpha_0(t)] \sin[\pi \alpha_0(t)]},$$

(12)

$$\frac{1}{t - m^2_{\alpha_K}} \to \left( \frac{s}{s_{\text{scale}}} \right)^{\alpha_K(t)-1} \frac{\pi \alpha'_{K}}{\Gamma[\alpha_K(t)] \sin[\pi \alpha_K(t)]},$$

(13)

The scale factor $s_{\text{scale}}$ is fixed at 1 GeV. In addition, the Regge trajectories $\alpha_0(t)$ and $\alpha_K(t)$ read as [29, 30],

$$\alpha_0(t) = -0.5 + 0.6t, \quad \alpha_K(t) = 1 + 0.85(t - m^2_{K^*}).$$

(14)

After the Reggeized treatment introduced, no additional parameter is introduced.

III. NUMERICAL RESULTS

With the preparation in the previous section, the cross section of the $\pi^+ p \to f_1(1420)n$ and $K^- p \to f_1(1420)\Lambda$ reactions will be calculated and compared with experimental data [8–11]. The differential cross section in the center of mass (c.m.) frame is written as

$$\frac{d\sigma}{d\cos \theta} = \frac{1}{32\pi s} \left( \frac{k_{1\text{c.m.}}^2}{k_{2\text{c.m.}}^2} \right)^2 \left( \frac{1}{2} \sum_{\lambda} |M|_{\lambda}^2 \right),$$

(15)

where $s = (k_1 + p_1)^2$, and $\theta$ denotes the angle of the outgoing $f_1(1420)$ meson relative to $\pi/K$ beam direction in the c.m. frame. $k_{1\text{c.m.}}$ and $k_{2\text{c.m.}}$ are the three-momenta of initial $\pi/K$ beam and final $f_1(1420)$, respectively.

A. Cross section of the $\pi^+ p \to f_1(1420)n$ reaction

Here, we minimize $\chi^2$ per degree of freedom (d.o.f.) for the experimental data of the total cross section by fitting the cutoff parameter $\Lambda$ using a total of 4 data points at the beam momentum $P_{\text{Lab}}$ from 3.1 to 13.5 GeV. The fitted cutoff and the $\chi^2$/d.o.f. are listed in Table I.

From Fig. 2, it is found that the experimental data of total cross section of $\pi^+ p \to f_1(1420)n$ reaction can be reproduced using the Feynman model or with the traditional Reggeized treatment. The shapes of total cross section in both models are analogous. The total cross section increases sharply near the threshold and reaches maximum at a momentum about 3 GeV. The Regge model gives a little larger cross section than the Feynman model in this energy region. The total cross section decreases in both Feynman and Regge model at momenta larger than 3 GeV. In this energy region, the Feynman model give a larger total cross section. The $\chi^2$/d.o.f. are 1.68 and 1.93 for the Feynman and the Regge model, respectively. The current results suggested that both models can describe the existing rude data rudely. If mixing of two models, that is, the interpolating Regge treatment [19, 28, 31, 33], is introduced, the data can be described better, but more precise data are required. It is obvious that the $\chi^2$ is mainly from the two data point around 4 GeV. In both models, the upper data point is suggested, which can be checked in future high-precision experiment.

![Graph](image)

**FIG. 2.** (Color online) Total cross section for $\pi^+ p \to f_1(1420)n$ reaction. The Full (red) and dashed (blue) lines are for the Feynman model and Regge model, respectively. The bands stand for the error bar of cutoff $\Lambda$.

In Fig. 3, we present the prediction of differential cross section of $\pi^+ p \to f_1(1420)n$ reaction in two schemes at different beam momentum. It can be seen that the differences between the Regge and the Feynman model at low energies are small but become large at higher energies. With increases of the beam energy, the slope of curve in the Regge model is steeper than that in the Feynman model at forward angles, which can be tested by further experiment to clarify the role of the Reggeized treatment.

| $\Lambda$  | $\chi^2$/d.o.f. |
|-----------|----------------|
| Feynman   | 1.18 ± 0.01    | 1.68       |
| Regge     | 1.60 ± 0.03    | 1.93       |

**TABLE I.** The fitted value of free parameter $\Lambda$, in the unit of GeV.
B. Cross section of the $K^- p \to f_1(1420)\Lambda$ reaction

Since there does not exist the experimental data for the $K^- p \to f_1(1420)\Lambda$ reaction, here we give the prediction of the cross section of the $K^- p \to f_1(1420)\Lambda$ reaction. In Ref. [11], the experiment shown that $\sigma(K^- p \to f_1(1420)\Lambda)/\sigma(\pi^- p \to f_1(1420)n) > 10$ at beam momentum $P_{\text{Lab}} = 32.5$ GeV. In our above calculation, the $\sigma(\pi^- p \to f_1(1420)n) \approx 0.017$ (µb) at $P_{\text{Lab}} = 32.5$ GeV by taking $\Lambda = 1.6$ GeV in the Regge model. For the $K^- p \to f_1(1420)\Lambda$ reaction, by taking $\Lambda = 1.6$, one can get a value of total cross section about $0.14$ µb in the Regge model, which means that the $\Lambda = 1.6$ GeV is a relatively reasonable value to calculate the cross section of $K^- p \to f_1(1420)\Lambda$ reaction in the Regge model. In Fig. 4 we present the total cross section of $K^- p \to f_1(1420)\Lambda$ reaction within the Regge model.

For the Feynman case, if we choose the cutoff as in pion induced case, the cross section of $K^- p \to f_1(1420)\Lambda$ reaction will increase continuously with the increase of the momentum $P_{\text{Lab}}$ in the energy region considered. Though it does not conflict with $\sigma(K^- p \to f_1(1420)\Lambda)/\sigma(\pi^- p \to f_1(1420)n) > 10$ as suggested in Ref. [11], it seems unnatural that total cross section will reach $100$ µb. Hence, here we chose a smaller value of cutoff $\Lambda = 1.0$ GeV.

Different from the pion induced case, from Fig. 4 the shape of curve in the Regge model is different from that in the Feynman model. For the Regge case, we notice that the line shape of the total cross section goes up very rapidly and has a peak around $P_{\text{Lab}} = 3.53$ GeV. For the Feynman case, it is seen that the value of total cross section is becoming higher and higher with the increase of the beam momentum up to 20 GeV. The monotonically increasing behavior may be caused by the $K^*$ exchange amplitude [30]. The differences between the Regge and the Feynman model will be useful in clarifying the role of the Reggeized treatment.

The differential cross section in two models are illustrated in Fig. 5, which show that the discrepancy of the differential cross sections of two models is small at low beam momenta but become large at higher beam momenta. From Fig. 5, one notice that, relative to the results within Feynman model, the differential cross section in Regge model is very sensitive to

FIG. 3. (Color online) The differential cross section $d\sigma/d\cos\theta$ of the $\pi^- p \to f_1(1420)n$ process as a function of $\cos\theta$. The Full (red) and dashed (blue) lines are for the results of Feynman model and Regge model, respectively.

FIG. 4. (Color online) Total cross section for $K^- p \to f_1(1420)\Lambda$ reaction. The Full (red) and dashed (blue) lines are for the results of Feynman model and Regge model, respectively. The bands stand for the error bar of cutoff $\Lambda$.

FIG. 5. (Color online) The differential cross section $d\sigma/d\cos\theta$ of the $K^- p \to f_1(1420)\Lambda$ process as a function of $\cos\theta$. The Full (red) and dashed (blue) lines are for the results of Feynman model and Regge model, respectively.
the θ angle and gives a considerable contribution at forward angles with the increases of beam momentum.

IV. SUMMARY AND DISCUSSION

We have studied the π⁻p → f₁(1420)n and K⁻p → f₁(1420)Λ reaction within the Feynman model and Regge model. For the π⁻p → f₁(1420)n reaction, both results calculated with the Regge and the Feynman model can reproduce the experimental data, but the differential cross sections at high energies are different. It is found that differential cross section for π⁻p → f₁(1420)n reaction within Regge model is very sensitive to the θ angle and gives a considerable contribution at forward angles, which can be checked by further experiment and may be an effective way to examine the validity of the Reggeized treatment.

For the K⁻p → f₁(1420)Λ reaction, the shapes of total cross sections obtained in both models are very different. The total cross section will increase continuously in Feynman model in the energy considered in the current work. If the cut-off in the pion induced production is adopted, the cross section will increase to 100 μb. In the Regge model it will decrease at momenta larger than P_{lab} ≈ 4 GeV. It is consistent with that the Regge model is more suitable to describe the behavior of cross section at high energies. The shape of differential cross section of K⁻p → f₁(1420)Λ reaction is similar to the result of π⁻p → f₁(1420)n reaction. With the Reggeized treatment, the t channel provides a sharp increase at extreme forward angles.

The pion and kaon beams can be provided at J-PARC and COMPASS. The precision of possible data in future experiments at these facilities will be much higher than old experiments. The above theoretical results may provide valuable information for possible experiment of searching for the f₁(1420) at these facilities.

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