EOQ Model for Constant Demand Rate with Completely Backlogged and Shortages

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Abstract
This paper presents an EOQ model with constant demand rate for non deteriorating items where shortages are allowed. In this paper shortages are considered as completely backlogged. The production rate is assumed to be proportional to demand rate and finite. The optimal solution of the model has been done for finding optimal time, optimal average cost by considering four different situations. Numerical example and sensitivity analysis is given to illustrate the proposed model.

Keywords: Demand; Inventory; Shortage; Production; Economic order quantity

Introduction

In last few decades, Mathematical ideas have been developed in different areas in real life problems, particularly for controlling inventory. The most important concern of the management is to decide when and how much to order so that the total cost connected with the inventory system should be minimum. This paper deals with the problems of determining EOQ model for non deteriorating items and constant demand rate. A number of authors have discussed inventory models for non-deteriorating items.

However, there are certain substances in which deterioration plays an important role and items cannot be stored for a long time. When items are kept in stock as an inventory for fulfilling the future demand, there may be deterioration of items in the inventory system. In inventory models generally four types of demand are assumed: (a) constant demand (b) time-dependent demand (c) probabilistic demand and (d) stock-dependent demand, inventory models dealing with constant demand have more relevance and common in the present situations.

Initially, the demand rate of items was considered to be constant. However, in real life situations, demand varies with time and situations. Goyal [1], developed an economic order quantity under conditions of permissible delay in payments. In paper [1] Goyal considered that the demand rate is constant. Chung et al. [2], established the optimal inventory policies under permissible delay in payments depending on the order quantity and total minimum variable cost per unit of time is obtained. Mahata and Mahata [3], developed optimal retailer's ordering policies in the EOQ model for deteriorating items under trade credit financing in supply chain. In paper [3] minimum retailer's optimal ordering policies are obtained. Huang [4], modified the assumption that the retailers will adopt the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model. Huang [4], assumed that the inventory lend is depleted by customer's demand alone. This assumption is valid for non permissible or non deteriorating inventory items. The demand rate for a particular item is not always constant it vary with time. Tripathi [5], established an EOQ model with time dependent demand rate and time-dependent holding cost functions. Tripathi [6], also established an optimal policy for items having constant demand and constant deterioration rate with trade credit. In this paper [6] a model for a retailer to determine its optimal price and time is obtained. Tripathi and Kumar [7], developed credit financing in economic ordering policies of time-dependent deteriorating items. This paper [7] presented the economic ordering policies in the presence of trade credit using discounted cash flows (DCF) approach.

Furthermore, when the shortages occur, it is assumed that it is either completely backlogged or completely lost. But in real life some customers are willing to wait for backorder and others would turn to buy from other sellers. Uthaya and Geetha [8], developed a replenishment policy for non-instantaneous deteriorating inventory system with partial backlogging. This paper [8] investigated an instantaneous replenishment inventory model for the non-instantaneous deteriorating items under cost minimization. Tripathi and Mishra [9], established an inventory model with shortage, time-dependent demand rate and quantity dependent permissible delay in payment. Patra et al. [10], established an order level EOQ model for deteriorating items in a single warehouse system with price dependent demand in non-linear form. In this paper [10] demand rae is considered as a function of selling price and order level inventory model for deteriorating items with single warehouse is developed where shortages are taken into consideration and it is completely backlogged. An EOQ model for permissible items with power demand and partial backlogging was developed by Singh et al. [11]. In paper [11] backlogging rate is considered as variable and dependent on the waiting time for the next replenishment. Research such as Park [12], Hollier and Mark [13], and Wee [14] considered the constant partial backlogging rate during the shortage period in their inventory models. Akad [15], established an EOQ model allowing shortage and partial backlogging. The thime proportion partial backlogging rate was developed by Chang and Dye [16], Wang [17], Teng and Yang [18], Yang [19], Wu et al [20], Dye et al. [21] and so on.

Assumption and Notation
The following assumptions and notations are used to develop the...
proposed model:
- Demand rate $R(t)$ is constant. Let it be $D$.
- Deterioration rate is zero.
- Production rate is $P(t) = \lambda_0 R(t)$ where $(\lambda_0 > 1)$ a constant therefore $P(t) = R(t)$.
- Lead time is zero.
- Shortages are allowed and are completely backlogged.

In addition the following notations are being made throughout the paper:
- $C_1$ Carrying cost per unit per unit time.
- $C_2$ Shortage cost per unit per unit time.
- $C_3$ Setup cost per production run.
- $C_4$, $C_5$, $C_6$ are all assumed to be known and fixed during production cycle.

'AC' the total average cost for a production cycle.

Mathematical Formulation

Inventory level $I(t)$ is zero at time $t=0$. The shortages starts at $t=0$ and accumulates up to the level $A$ at time $t=T_1$. The production starts at $t=T_1$ and backlog is cleared at $t=T_2$. The stock-level reaches at level $B$ at $t=T_3$. The production stops at level $t=T_3$. Thus the inventory level decreases gradually due to demand and becomes zero at $t=T_4$. The cycle completed and repeals itself. $I(t)$ is the inventory level at any time $t$ $(0 \leq t \leq T_4)$.

The differential equations of states are given by

\[
\frac{dI(t)}{dt} = -R(t), \quad 0 \leq t \leq T_1
\]

\[
\frac{dI(t)}{dt} = K(t) - R(t), \quad T_1 \leq t \leq T_2
\]

\[
\frac{dI(t)}{dt} = K(t) - R(t), \quad T_2 \leq t \leq T_3
\]

\[
\frac{dI(t)}{dt} = -R(t), \quad T_3 \leq t \leq T_4
\]

With boundary conditions

$I(0)=0, I(T_3)=A, I(T_1)=0, I(T_4)=B, I(T_3)=0$ (5)

Substituting $R(t)=D$ and $K(t)=\lambda_0 R(t)$ in the equations (1)-(4) and solving then using the boundary conditions (5), we get the following solutions:

$I(t) = -Dt$, \quad $0 \leq t \leq T_1$ (6)

$I(t) = (\lambda_0 - 1)D(t - T_1)$, \quad $T_1 \leq t \leq T_2$ (7)

$I(t) = (\lambda_0 - 1)D(t - T_1)$, \quad $T_2 \leq t \leq T_3$ (8)

$I(t) = DT_1$, \quad $T_3 \leq t \leq T_4$ (9)

Using the condition $I(t)=-A$ and $t=T_1$ in (6) and (7) and then equating the two values of $A$, we get

$T_1 = \frac{\lambda_0 - 1}{\lambda_0} T_2$ (10)

Again using $I(t)=B$ and $t=T_4$ in (8) and (9) and then equating the two values of $B$, we get

$T_3 = \frac{(\lambda_0 - 1)T_2 + T_4}{\lambda_0}$ (11)

Now, we shall try to find the different costs involved in the system.

The total shortage cost in the system is

\[
SC = \int_0^T I(t)dt + \int_{T_3}^T I(t) dt = \frac{C_1D}{2} \left[ (\lambda_0 - 1)(T_2 - T_3) + (T_4 - T_3) \right] (12)
\]

The total inventory holding cost in the system is

\[
HC = \int_0^T I(t)dt + \int_{T_3}^T I(t)dt = \frac{C_2D}{2} \left[ (\lambda_0 - 1)(T_2 - T_3) + (T_4 - T_3) \right] (13)
\]

Hence total average cost in the system is,

\[
AC = \frac{SC + HC + C_1}{T_i} (14)
\]

For finding optimal values of $T_1$ and $T_4$ in terms of $T_2$ and $T_4$

\[
T_1 = \frac{1}{\lambda_4} \left[ \left(1 - \frac{1}{\lambda_3}\right) T_2^2 + \frac{C_4D}{2} \left(\lambda_0 - 1\right)(T_2 - 2T_3 + \lambda_0 T_3^2) + C_1 \right] (15)
\]

The optimal value of $T_2$ and $T_4$ is obtained by solving \(\frac{\partial AC}{\partial T_2} = 0\) and \(\frac{\partial AC}{\partial T_4} = 0\) we get,

\[
\lambda_0(C_1 + C_2)T_2 - C_4T_4 = 0 (16)
\]

\[
(\lambda_0 - 1)\left[C_1DT_2^2 - (C_1 + C_2)DT_3^2\right] - 2\lambda_0 C_4 = 0 (17)
\]

Since, $\frac{\partial^2 AC}{\partial T_2^2} > 0$ and $\frac{\partial^2 AC}{\partial T_4^2} > 0$, and $\frac{\partial^2 AC}{\partial T_2 \partial T_4} < 0$

This shows that average cost (AC) will be minimum.

Solving equations (16) and (17), we get different value of $T_2$ and $T_4$ for changing different parameters.

Numerical Example

Let $\lambda_0 = 1.05, C_1 = 20, C_2 = 30, C_3 = 40, D = 100$ in appropriate units from (16) and (17), we obtain the optimal values of $T_2$ and $T_4$. Putting the optimal value of $T_2$ and $T_4$ in equation (10) and (11), we obtain the optimal values of $T_1$ and $T_4$ respectively.

We get, $T_1 = 0.2726$, $T_2 = 0.5725$, $T_3 = 1.4576$, $T_4 = 1.5029$ and Total average cost 'AC' = $602.37$.

Sensitivity Analysis

All the observations from table 1 to 4 can be sum up as follows:

(i) From table 1, it can be easily seen that increase in the value of parameter $C_i$ will results increase in $T_1$, $T_2$ but decrease in $T_3$, $T_4$ and increase in average cost.
Table 1: Variation of $C_j$ keeping other parameters same as mentioned in numerical example.

| $C_j$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | AC (in dollars) |
|-------|-------|-------|-------|-------|-----------------|
| 30    | 0.0272| 0.5725| 1.4576| 1.5029| 602.37          |
| 35    | 0.0235| 0.4941| 1.3622| 1.4267| 632.33          |
| 40    | 0.0206| 0.4344| 1.3240| 1.3685| 657.51          |
| 45    | 0.0184| 0.3876| 1.2762| 1.3227| 678.82          |
| 50    | 0.0166| 0.3498| 1.2411| 1.2857| 697.25          |
| 55    | 0.0151| 0.3187| 1.2106| 1.2552| 713.24          |
| 60    | 0.0139| 0.2927| 1.1850| 1.2296| 727.20          |
| 65    | 0.0128| 0.2706| 1.1632| 1.2078| 739.52          |
| 70    | 0.0122| 0.2516| 1.1447| 1.1891| 750.55          |
| 75    | 0.0110| 0.2206| 1.1138| 1.1658| 769.28          |
| 80    | 0.0093| 0.1964| 1.0899| 1.1345| 784.56          |
| 85    | 0.0084| 0.1770| 1.0706| 1.1153| 797.36          |

(ii) From table 2, increase in the value of parameter $C_j$ will results in decrease in $T_1$, $T_2$, $T_3$ and $T_4$ but increase in average cost.

(iii) From table 3, increase in the value $C_j$ of will results in increase in $T_1$, $T_2$, $T_3$, $T_4$ and average cost AC.

(iv) From table 4, increase in the value $\lambda_0$ of will results in decrease in $T_1$, $T_2$, $T_3$, $T_4$ and average cost AC.

Table 2: Variation of keeping other parameters same as mentioned in numerical example.

| $C_3$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | AC (in dollars) |
|-------|-------|-------|-------|-------|-----------------|
| 20    | 0.0272| 0.5725| 1.4576| 1.5029| 602.37          |
| 25    | 0.0300| 0.6320| 1.4206| 1.4601| 623.60          |
| 30    | 0.0325| 0.6831| 1.3967| 1.4345| 637.91          |
| 35    | 0.0346| 0.7274| 1.3655| 1.4184| 648.29          |
| 40    | 0.0364| 0.7660| 1.3771| 1.4077| 656.38          |
| 45    | 0.0380| 0.8000| 1.3714| 1.4000| 662.54          |
| 50    | 0.0394| 0.8297| 1.3671| 1.3940| 669.27          |
| 55    | 0.0407| 0.8559| 1.3635| 1.3889| 672.58          |
| 60    | 0.0418| 0.8788| 1.3601| 1.3842| 677.02          |

Table 3: Variation of keeping other parameters same as mentioned in numerical example.

| $C_2$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | AC (in dollars) |
|-------|-------|-------|-------|-------|-----------------|
| 1.1   | 0.0186| 0.3912| 1.6930| 1.0758| 476.74          |
| 1.2   | 0.0124| 0.2818| 0.7606| 0.7855| 407.07          |
| 1.3   | 0.0097| 0.2049| 0.6442| 0.6662| 387.47          |
| 1.4   | 0.0081| 0.1716| 0.5804| 0.6008| 381.45          |
| 1.5   | 0.0071| 0.1494| 0.5407| 0.5603| 380.87          |

(ii) From table 4, increase in the value of parameter $C_2$ will results in decrease in $T_1$, $T_2$, $T_3$, $T_4$ and average cost AC.

Table 4: Variation of keeping other parameters same as mentioned in numerical example.

| $\lambda_0$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | AC (in dollars) |
|-------------|-------|-------|-------|-------|-----------------|
| 1.1         | 0.0186| 0.3912| 1.6930| 1.0758| 476.74          |
| 1.2         | 0.0124| 0.2818| 0.7606| 0.7855| 407.07          |
| 1.3         | 0.0097| 0.2049| 0.6442| 0.6662| 387.47          |
| 1.4         | 0.0081| 0.1716| 0.5804| 0.6008| 381.45          |
| 1.5         | 0.0071| 0.1494| 0.5407| 0.5603| 380.87          |

Conclusion

The model developed in this paper assumed that demand rate of a product is constant, shortages are allowed and replenishment rate is infinite, production rate is greater than demand rate and proportional to the demand rate. The effect of changing parameters is quite sensitive and makes significant difference to the changes. Numerical example is given to illustrate the model. Sensitivity analysis has been also given for changing the various parameters.

This model can be extended for several ways. For instance, we may extend the model for time-dependent demand rate. We could also generalised the model for cash discount, inflation etc.

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