Harnessing coherent scattering of waves underpins a wide range of phenomena but backscattering on structural defects is at the same time the main limiting factor for photonic-crystal technologies because it turns waveguides into random cavities. Recent developments in topological photonics have led to the vision of backscattering-protected optical waveguides in the form of topological interface modes, but previous works considered only artificial definitions of disorder, such as deliberately introduced sharp bends or restricted classes of perturbations. Here we measure the propagation losses due to structural disorder in the slow-light regime of valley-Hall topological waveguides. Our experiments confirm the protection from backscattering on sharp bends but we find no improvement in the propagation losses relative to topologically trivial waveguide modes with the same group index, even for state-of-the-art silicon photonics. We image the light scattered out of the plane of the topological waveguides and observe clear signatures of randomly localized cavity modes. Since the interfaces between valley-Hall photonic topological insulators are currently the only viable the light scattered out of the plane of the topological waveguides and observe clear signatures of randomly localized cavity modes. Since the interfaces between valley-Hall photonic topological insulators are currently the only viable contenders for backscattering-protected topological photonic waveguides without absorption or out-of-plane losses, our work raises fundamental questions about the existence of topological protection against real-world disorder in time-reversal-symmetric photonics.

Planar nanostructures built with high-index dielectric materials using top-down nanofabrication techniques have enabled precise control of the spatial and spectral properties of electromagnetic fields at optical frequencies, fostering the development of integrated photonic devices such as quantum light sources [1], programmable photonics [2], nanolasers [3], and optical communication technology [4]. While highly optimized performance can be achieved with aperiodic structures [5], periodicity offers simple building blocks that may be readily scaled to larger architectures. In particular, periodic structures allow tailoring the dispersion relation of light far beyond simple translationally invariant systems. In addition, deliberately introduced regions that break the periodicity allows building high-Q optical cavities [6] or waveguides that slow light by several orders of magnitude [7]. While introducing such defects can greatly enhance the interaction of light with other degrees of freedom [1, 8, 9], random structural fluctuations effectively destroy long-range translational symmetry. Improvements in nanoscale fabrication down to nanometer tolerances can reduce the magnitude of this disorder, but stochastic deviations from the designed structures are inherent to any fabrication method and can never be completely eliminated. This disorder results in extrinsic scattering events for light, whose cross-section is, unfortunately, often enhanced at the same spectro-spatial locations targeted at the design stage. A well-known example is that of slow light in photonic-crystal waveguides, where disorder incurs substantial propagation losses [10] and ultimately limits the maximal slowdown by localization of the light field induced by multiple coherent backscattering [11, 12]. This has long been recognized as a primary obstacle to the application of photonic crystals [13].

A possible solution to this problem has sought inspiration from solid-state systems, for which the quantum Hall effect offers unidirectional propagation, i.e., completely suppressed backscattering, by breaking time-reversal symmetry. These quantum states are related to an underlying mathematical concept, the wavevector-space topology of the Bloch eigenstates, which can equally be explored and exploited for photonic-crystal structures, indicating a deep analogy between nanophotonics and solid-state physics [14]. This naturally led to the development of topological photonics [15, 16] and to the demonstration of one-way robust electromagnetic waveguides [17], i.e., photonic topological insulator (PTI) waveguides. While early attempts relied on real magnetic fields and non-reciprocal magneto-optical materials to generate non-trivial topologies, further realizations were achieved by effective magnetic fields through time-modulated media [18]. However, such approaches to combat backscattering have seen only microwave-domain implementations that use intrinsically lossy materials [19] or complex active schemes of difficult practical implementation [20]. PTIs that instead rely on breaking spatial symmetries to emulate pseudospins akin to that in quantum spin-Hall (QSH) and quantum valley-Hall (VH) solid-state topological insulators have been predicted [21, 22] and demonstrated [23, 24]. The resulting interface states, albeit not unidirectional owing to reciprocity, can in principle exhibit robustness to a certain class of perturbations [25]. Both QSH and VH interface states in high-index dielectric photonic-crystal slabs have been observed at telecom wavelengths, but the former support states above the light line [26] which are intrinsically lossy [27], making VH topological interface states particularly attractive to test and exploit topological protection against backscattering.

Observation of strong backscattering in valley-Hall topological interface modes

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Disorder length-scales in photonic-crystal structures

Much of the existing work on disorder in topological photonics has explored topological protection against defects on scales relevant to their electronic counterpart. Topological quantum states of electrons can travel unhindered along paths prone to crystallographic defects such as vacancies, interstitials, or dislocations [28], which are all on the scale of one to a few crystal unit cells. Such lattice-scale disorder has been mimicked in PTIs, with the most paradigmatic case being that of sharp Z- or Ω-shaped bends. Unlike in conventional line-defect photonic-crystal waveguides as that shown in Fig. 1a [29], suppressed back-reflection through sharp bends over a large bandwidth has been demonstrated for topological interface states [24, 30], enabling flexibly-shaped photonic circuits like ring cavities [31]. Nanophotonic waveguides are, however, prone to nanometer-scale roughness in the etched sidewalls as exemplified in Fig. 1b. Such structural disorder occurs at a scale significantly smaller than the unit cell and is consistently present across the entire crystal, thus questioning how the notions of topological protection can be directly transferred to the interface states in PTIs. Recent numerical works have addressed this question, but the studies are limited to effective disorder models in two-dimensional crystals [32], semi-analytical models [33], or single-event incoherent scattering theory [34]. More accurate modelling of the propagation losses including coherent multiple scattering has been applied to conventional monomode slow-light waveguides [35, 36], but such studies are still lacking for topological interface states. Ultimately, the subtle interplay between out-of-plane losses and backscattering in photonic-crystal slab waveguides calls for experiments that directly compare the propagation losses of topological and conventional slow-light waveguides subject to equivalent disorder. Here, we address this by fabricating and characterizing a set of suspended silicon VH PTI waveguides [37] that support both a topologically protected and a topologically trivial guided mode with nearly identical group indices. We characterize waveguides with and without sharp bends, cf. Figs. 1c and 1d. As shown in previous work [30], the difference between the two guided modes is profoundly manifest when introducing four sharp turns in the waveguide path. They effectively suppress the transmittance in the trivial mode, while leaving transmission through the topological mode essentially unaffected as shown in Fig. 1e. This is a striking demonstration of topological protection, but it offers no evidence of the applicability of PTIs for protecting against backscattering from fabrication imperfections. It is clear from simple experiments, e.g., comparing the transmittance of short and long waveguides as shown in Fig. 1f, that the propagation loss of the topological mode studied here is nonzero. However, assessing topological protection against backscattering requires the precise extraction and modeling of the propagation losses to disentangle out-of-plane radiation losses from backscattering, the careful study of which has so far been absent from literature.

Design of a valley-Hall slow-light waveguide

The VH PTI waveguides explored here rely on a photonic crystal that emulates graphene with two equilateral triangular holes arranged in a honeycomb structure, the unit cell of which is shown in the inset of Fig. 2a. For identical triangles, $s_1 = s_2$, the crystal dispersion exhibits a Dirac cone for transverse-electric-like (TE-like) modes at the K-point in the Brillouin zone. Breaking inversion symmetry such that $s_1 \neq s_2$ lifts the degeneracy and opens a band gap, as shown in Fig. 2a. The non-trivial geometrical structure in momentum space of the wave functions of the air and dielectric bands of the crystal results in non-vanishing Berry curvatures [38] at the K and K′ points, as shown in Fig. 2b. In this case, the bulk-edge correspondence theorem [22] ensures the existence of two degenerate counterpropagating interface states at the domain wall between two such crystals with inverted symmetry. The particular geometry explored here (Fig. 2c) uses a bearded interface [16] between two mutually inverted VH crystals. Since this particular interface obeys a combination of mirror and translation symmetry, i.e., a glide symmetry, a degeneracy is enforced at the edge of the Brillouin zone [39] leading to the existence of two guided modes (Fig. 2d) along the interface. The superior transmission through sharp bends at wavelengths corresponding to the low-energy band relative to those in the high-energy band, shown in Fig. 1e, indicates that the former is topological (see Supplementary Section 6).
**Characterization of optical propagation losses**

We characterize the dispersive propagation losses of the suspended silicon photonic-crystal waveguides fabricated from a silicon-on-insulator wafer with a 220 nm-thick silicon device layer. This is achieved by measuring the optical transmission of suspended photonic circuits where waveguides of varying length, $L$, from 250$a_0$ to 1750$a_0$, with $a_0 = 512$ nm denoting the lattice constant, are embedded. Scanning electron microscopy (SEM) images of characteristic devices are shown in Fig. 3a. The circuits are comprised of input and output free-space broadband grating couplers, strip silicon waveguides to direct light into the region of interest (Fig. 3b), and intermediate waveguides [40] (Fig. 3c) to couple to the VH interface modes (Fig. 3d) with high efficiency (87 %, see Supplementary Section 3). Figure 3e shows a high-magnification SEM image, which reveals the presence of roughness along the sidewalls. In principle, the roughness could be measured and used to calculate the scattering but, in practice, such a procedure is experimentally unfeasible and numerically intractable [32, 41]. Therefore, we instead benchmark our VH waveguides against conventional line-defect W1 waveguides fabricated on the same chip such that the structural disorder is approximately statistically identical. We extract a minimum propagation loss of $(1 \pm 1)$ dB/cm in the non-dispersive region of the W1-waveguide, which constitutes a record-low value for suspended silicon photonics (see Supplementary Section 5) and shows that our nanofabrication [5] provides an ideal testing ground for measuring VH-waveguides with the lowest level of roughness realized to date.

The circuit transmittance for a single VH device for each waveguide length is shown in Fig. 3f. We convolute the raw spectra with a Gaussian kernel (standard deviation $\sigma = 2.5$ nm) to simultaneously remove Fabry-Pérot fringes resulting from reflections at the grating couplers and account for possible systematic structure-to-structure variations (see Supplementary Sections 1 and 2). Inside the transmission band, we observe that the loss is largest around $\lambda = 1515$ nm, which is a clear spectral indication of the $n_g$-peak shown in Fig. 2d. We account for the stochastic nature of the sidewall roughness by studying averaged quantities obtained from measurements done over 3 nominally identical circuits for each waveguide length. We find that the ensemble-averaged transmission intensity can be described by an exponential spatial decay, with an attenuation coefficient $\alpha(\lambda)$.

$$\langle \ln T(\lambda) \rangle = -\frac{L}{\alpha(\lambda)} + \ln T_0(\lambda)$$

(1)

where additional losses in the circuit are cast into $T_0$ (see Supplementary Section 3). Characteristic fits to the ensemble-averaged data are shown in Fig. 3g. While such Beer-Lambert-like attenuation has been theoretically shown to fail for particular periodic monomode waveguides [42], the moderate values of $n_g$ explored here and the state-of-the-art nanofabrication process justify the model. The same arguments also support the use of 3 devices per length since the variance of the stochastically distributed transmission, which depends on the loss pathway, group index, and waveguide length [36], is low as confirmed by data subsets with increasing number of nominally identical devices (see Supplementary Section 3).

**Group-index dependence**

The propagation loss over a wide wavelength range is shown in Fig. 4a and it exhibits a prominent dispersion across the maximal group index around $\lambda = 1515$ nm. As a consequence of the strong dispersion, the extracted propagation loss depends on the width of the filtering kernel so the values shown in Fig. 4a constitute a lower bound to the propagation losses. Broadly speaking, the dependence of the propagation loss on wavelength reflects the physics underlying several distinct
scattering mechanisms [43]. The losses of a propagating mode in a photonic-crystal waveguide can be classified into intrinsic losses, $\varepsilon^{-1}(\lambda)$, scattering into radiation modes in the cladding, $\varepsilon^{-1}_{\text{out}}(\lambda)$, and inter- or intramodal scattering into other slab or waveguide modes. Intrinsic losses include absorption material losses, which can be neglected in crystalline silicon at telecom wavelengths, except for two-photon absorption in high-$Q$ cavities [44] and intrinsic radiation losses when operating above the light line [27]. In the case of a monomode photonic-crystal waveguide with vertical sidewalls and operated at wavelengths within the bulk band gap, all sources of inter- and intramodal scattering except for backscattering, $\varepsilon^{-1}_{\text{out}}(\lambda)$, are strongly suppressed. This holds provided that the disorder levels are perturbative and that the number of unit cells in the direction perpendicular to the waveguide axis, here 16, significantly exceeds the Bragg length. All such conditions are satisfied for the VH waveguides explored here, and the remaining loss lengths add up reciprocally to the propagation length,

$$\varepsilon^{-1}_L(\lambda) = \varepsilon^{-1}_{\text{in}}(\lambda) + \varepsilon^{-1}_{\text{out}}(\lambda).$$

Both contributions are generally dispersive and may vary independently of each other based on geometry, material properties, disorder, and wavelength. As a consequence, the precise scaling with $n_g$ is far from trivial [36]. Nevertheless, based on the experimental observations and perturbation theory [43], we model $\varepsilon^{-1}_L(\lambda)$ by

$$\varepsilon^{-1}_L(\lambda) = \beta n_g^2(\lambda - \Delta \lambda) + \gamma n_g(\lambda - \Delta \lambda)$$

where $n_g(\lambda)$ is the theoretical group index shown in Fig. 2d. The coefficients $\beta$ and $\gamma$ describe the loss due to backscattering and out-of-plane radiation. Furthermore, to account for the observed spectral shifts of about 20 nm between the calculated $n_g$ and the observed loss peak, we introduce an additional model parameter, $\Delta \lambda$, describing a linear spectral shift between theory and experiment. The fit is shown in Fig. 4a and agrees well with the experiment, which shows that Eq. (3) describes the measured losses well, irrespective of the transition between topological and trivial modes. The fitted coefficients are $\beta = (0.151 \pm 0.003) \, \text{dB/cm}$ and $\gamma = (0.39 \pm 0.07) \, \text{dB/cm}$. The band structure is shifted by $\Delta \lambda = (20.22 \pm 0.05) \, \text{nm}$, which accounts minor deviations in average dimensions between model and samples, and this shift is henceforth applied to all theoretical quantities, placing the degeneracy point at $(1517.38 \pm 0.05) \, \text{nm}$. The calculated intrinsic radiation losses (see Methods) of the topological mode above the light line is included for reference in Fig. 4a and shows good agreement with the measured propagation loss up to a small wavelength shift. Using the fitted parameters,
We now turn to exploring the physics of backscattering in the waveguides [36]. By converting wavelength into group index, we infer that backscattering losses dominate over out-of-plane radiation losses. The vertical red line indicates the group index above which backscattering losses above the light line is included for reference (solid green line). Dependence of the loss on the group index, $n_g$, where the wavelength to group-index conversion is derived from the fit in a. The vertical line above the group index indicates the group index above which backscattering losses dominate out-of-plane radiation losses.

Observation of coherent backscattering
We now turn to exploring the physics of backscattering in the topological waveguide beyond the ensemble behaviour. We image the vertically scattered far-fields from single waveguide realizations ($L = 1750a_0$) using a tunable laser and an imaging system with a near-infrared camera. For these measurements, we use a different sample with propagation losses comparable to those reported in Fig. 3 (see Supplementary Section 8). Since, it is challenging to distinguish between the topological and the trivial mode from far-field measurements on straight waveguides (see Supplementary Section 8), we employ sharply-bent waveguides, which act as highly efficient (see Supplementary Section 7) modal filters that only allow the topological mode to pass [37]. Figure 5a shows a microscope image of a device, overlayed with the scattered far-field at a wavelength well within the topological band. In addition to scattering at the interface between the strip waveguide and the intermediate waveguide and at the two vertices of the bend, we observe spatially varying scattering in a finite region of the waveguide close to the second corner. This is a clear fingerprint of strong coherent backscattering leading to complex interference patterns in the near-field of the propagating mode and projected into the far-field by out-of-plane radiation losses [11]. Coherent backscattering ultimately leads to spectral and spatial localization of the light field and one-dimensional Anderson localization [12]. In a single realization, this corresponds to the formation of random optical cavities with distinct spatial patterns (Fig. 5b) and quality factors reaching $Q \simeq 2 \cdot 10^5$ (Fig. 5c). The observation of random cavities with high quality factors corroborates that coherent backscattering is the dominant source of loss at the imaged wavelengths, i.e., that the inverse loss length reported in Fig. 4 may be interpreted as the inverse localization length [45]. Finally, we step the laser wavelength across a wide wavelength range around the degeneracy point and study the transition from the topological to the trivial mode. The spectro-spatial map in Fig. 5d depicts the acquired intensities after the second corner and along the axis of the waveguide. It reveals the presence of multiple spectral resonances associated with spatially localized far-field patterns, whose spatial extent generally increases with wavelength, as expected from the behaviour of the ensemble-averaged loss length shown in Fig. 4. The modes labelled A–E correspond to the images in Figs. 5a and b and are not visible in the transmission spectrum of the circuit, which further evidences their localized nature. In addition, a close-up of the bend (Fig. 5e) unveils the topological or trivial nature of the propagating modes. For all wavelengths below $\lambda = 1554$ nm, we observe a single scattering spot at the first corner (Fig. 5e) and no emission from the waveguide (Fig. 5d), indicating wavelengths within or very near the trivial band. At the resonant wavelength of the mode E, light is scattered strongly at the first corner but this could be due to a higher group index or proximity to the trivial band and is not necessarily an indication that the wavelength lies outside the topological band (see Supplementary Section 7). At the wavelengths of the modes labelled A–D, the radiation losses in both corners are not only suppressed but also evenly distributed, confirming the existence of localized modes within the topological band.

Fig. 4 | Dispersive propagation losses in a slow-light glide-symmetric valley-Hall waveguide. a. Wavelength dependence of the measured propagation loss (solid black line) obtained from the fit coefficients shown Fig. 3g, with the shaded area indicating the standard error. The propagation loss is fitted with the model in Eq. (3) (red line) and the trivial-topological transition shaded according to the found wavelength offset. The wider segments of the fit line indicate the intervals that were fitted. The calculated intrinsic loss above the light line is included for reference (solid green line). b. Dependence of the loss on the group index, $n_g$, where the wavelength to group-index conversion is derived from the fit in a. The vertical line above the group index indicates the group index above which backscattering losses dominate out-of-plane radiation losses.

Conclusion and outlook
The experiments presented here, both the dependence of the measured loss length on the group index as well as the scattered light observed via far-field imaging, establishes a consistent picture of the transmission and scattering of slow light in VH PTI interface modes: Backscattering dominates over out-of-plane losses and is sufficiently strong to induce random cavities with high quality factors. Additionally, we observe no difference between the dependence of the loss length on group index for topological and trivial modes.

Obviously, our experiments do not rule out the existence
of backscattering resilience in other time-reversal-invariant PTIs with different symmetries, interfaces, or disorder levels. Even if structural disorder eventually destroys the crystal symmetry behind non-trivial topologies, backscattering might still be suppressed for limited disorder [32]. To approach that regime, we have employed a bearded VH interface, which has been theoretically shown to be more robust than other types of interfaces [33]. In addition, our record-low-loss W1 waveguides show that we are probing the lowest levels of disorder realized in silicon photonics so far. Even so, we do not observe any signature of reduced backscattering and our results therefore cast doubts on whether any topological protection of interfaces [33]. In addition, our record-low-loss W1 waveguides show that we are probing the lowest levels of disorder realized in silicon photonics so far. Even so, we do not observe any signature of reduced backscattering and our results therefore cast doubts on whether any topological protection of interfaces [33].

We hope that our work will motivate further research into robustness against real-world disorder as well as theoretical studies on the mechanisms behind Anderson localization [46] in systems with valley-momentum locking. The interplay between disorder and topology has surprising consequences, such as topological Anderson insulators [47], and our work takes the first steps into research at the nexus between Anderson localization, topology, and silicon photonics.

Methods

Sample fabrication. The measurements are performed on two samples denoted Sample 1 and Sample 2. The data in Figs. 1 and 5 are taken from Sample 1 and the data shown in Figs. 2–4 are from Sample 2. Both samples are fabricated from the same silicon-on-insulator substrate with a nominally 220 nm-thick silicon device layer. The fabrication process is detailed in Ref. [5] with some minor modifications. Sample 1 is fabricated using a high-resolution electron beam lithography process, the details of which may be found in Ref. [48]. Sample 2 is fabricated using a modified process, which introduces a silicon-chromium hardmask, following Ref. [49].

Optical spectral measurements. The transmission of each device is measured using a confocal free-space optical setup with cross-polarized and spatially offset excitation and collection achieved via orthogonal free-space grating couplers. The broadband optical characterization is performed using a fiber-coupled supercontinuum coherent white-light source (NKT Photonics, SuperK COMPACT) focused onto the input grating coupler using a long-working-distance apochromatic microscope objective (Mitutoyo Plan Apo NIR 20X, NA = 0.4, 10 mm effective focal length). The input power (typically 120 µW at the sample surface) is controlled using a half-wave plate and a polarizing beamsplitter and the excitation polarization selected.
with a half-wave plate. Light coupled out from the chip is collected using the same microscope objective, split via a 50:50 beamsplitter and filtered in polarization and space, respectively, using a linear polarizer and a single-mode fiber aligned to the output grating coupler. The light is then sent to an optical spectrum analyzer (Yokogawa AQ6370D, 2 nm resolution bandwidth) for retrieving the spectrum.

We simulate the optical eigenmodes of the perfect photonic-crystal slab numerically. Using the symmetry relative to the mid-plane of the slab and solve for the band structures in Fig. 1 and the imaginary part to extract the intrinsic losses above the light cone shown in green in Fig. 3a. The latter are obtained as $\epsilon^{-1} = 4.34 \times 2 \text{Im}(|\omega|)/|v_g|$, where $v_g$ is the group velocity. The transmission simulations shown in Supplementary Sections 3 and 7 are used for the band structures in Fig. 1 and the imaginary part to extract the intrinsic losses above the light cone shown in green in Fig. 3a. The latter are obtained as $\epsilon^{-1} = 4.34 \times 2 \text{Im}(|\omega|)/|v_g|$, where $v_g$ is the group velocity. The transmission simulations shown in Supplementary Sections 3 and 7 are solved as frequency-domain problems using the fundamental strip waveguide mode for both input and output ports. All simulations use the symmetry relative to the mid-plane of the slab and solve for transverse-electric-like electromagnetic fields. This relies on the assumption of vertical sidewalls, which we observe, as well as the absence of scattering into transverse-magnetic-like modes.

**Data availability**
The data that support the figures in this paper are available from the corresponding author upon request.

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**Author contributions**
C.A.R., G.A., S.S. designed and developed the experiment. C.A.R., G.A., A.V., M.A., B.V.L. performed the numerical design and analyses of the structures and device components. M.A. developed the nanofabrication process. C.A.R. fabricated the samples. C.A.R. and G.A. carried out the measurements and data analysis. C.A.R., G.A., A.V., and S.S. prepared the manuscript with input from all authors. S.S. conceived, initiated, and supervised the project with co-supervision by G.A. and R.E.C.

**Competing interests**
The authors declare no competing interests.

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Supplementary information: Observation of strong backscattering in valley-Hall topological interface modes

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1 Fabrication and overview of samples

The data underlying the analysis of the main text and here is derived from measurements on two samples, refered to as Sample 1 and Sample 2 (cf. Methods). For comparison, Fig. S1 shows both top-view and tilted scanning electron microscope (SEM) images of Sample 1 (a and b) and Sample 2 (c and d). These images evidence that the improved fabrication process of Sample 2 reduces sidewall roughness and eliminates the bright residues visible on Sample 1 which may result from the electron-beam resist and we expect to have a negligible effect on the optical properties of the waveguides. The electron-beam lithographic mask for both samples use the same nominal crystal geometry with lattice constant $a_0 = 512\text{nm}$ and hole sidelengths $s_1 = 0.7a_0/\sqrt{3} = 0.40a_0$ and $s_2 = 1.3/\sqrt{3} = 0.75a_0$ (cf. Fig. 2a of the main text).

![Fig. S1: Comparison between Sample 1 and 2. a,b. Close-up scanning electron microscope images of the etched triangular holes and top-down view of the interface for Sample 1. c,d. Same for Sample 2.](image)

Fig. S2: Example test circuit. a. Scanning electron microscope (SEM) image of a test circuit including a photonic topological insulator (PTI) waveguide of length $L = 1500a_0$ (indicated on the image) where $a_0$ is the lattice constant. b. Top-down SEM image of the circular grating coupler used to couple in and out of the circuit.

Both samples contain test circuits which connect to a photonic topological insulator (PTI) waveguide using free-space grating couplers [S1] and rectangular strip waveguides, an example of which is shown in Fig. S2a. A detail of the grating coupler is shown in Fig. S2b. The circuits are arranged on the chip in arrays, where the length $L$ is swept from $L = 250a_0$ to $L = 1250a_0$ on Sample
In order to accurately model the dispersion of the topological waveguides under investigation, we extract the geometry of the fabricated samples using SEM. Figure S4a and b show top-down SEM images of both Sample 1 and 2. We then analyze the images to extract the outlines and centroids of several holes. Absolute dimensions are fixed by correlating the known lattice constant ($a_0 = 512$ nm) with the lattice spanned by the centroids of the extracted features, since we consider this measure to be more accurate than the dimensions obtained by SEM (although we find that the two values do generally agree). To extract the average hole shape, the points from several holes are aggregated by shifting them to a common centroid and averaging points within 3 degree angles from the common centroid. This yields averaged hole outlines as displayed in Fig. S4c. This procedure is carried out for several SEM images on waveguides located at different positions across the whole chip. We use the traced hole outlines for the calculation of the band structures for both Samples 1 and 2. We assume vertical sidewalls and a nominal thickness of $h = 220$ nm. Figure S5 shows the band structures for both samples. The band diagram for Sample 1 (Fig. S5a) includes bands...
for both groups of circuits (see Section 1), showing a 4 nm difference in the degeneracy point, not far from the 6 nm we observe experimentally. For both samples, a small experiment-theory offset of around 6 and 20 nm still persists after the SEM analysis, which is much smaller than the one found when mask parameters are considered (about 110 nm for Sample 2). The analysis is therefore necessary since it gives validity to the rigid shift of the band structure we use in Eq. (3) in the main text.

3 Main analysis

In this section, we will describe the steps of the analysis and provide additional details.

3.1 Correcting for background and normalizing transmittance data

The procedure for obtaining the optical spectra is described in Methods. For the measurements on Sample 2, this yields a set of 501 points describing the PSD, \(S_{\text{raw}}(\lambda)\), of the collected light sampled uniformly in wavelength, \(\lambda\), from 1480 nm to 1640 nm. Although light collected at the output path is spatially and polarization filtered, the raw PSD data includes some amount of light scattered from other spatial locations at the chip surface (or elsewhere in the free-space optical setup). Hence, we acquire a PSD spectrum, \(S_{\text{bg}}(\lambda)\), from a location on the sample without structures. We assume the minor oscillations in the signal to be uncorrelated with \(S_{\text{raw}}(\lambda)\), so we smooth \(S_{\text{bg}}(\lambda)\) using a wide (101 points \(\sim\) 30 nm) degree-2 Savitzky-Golay filter to remove them and obtain the smoothed background \(\tilde{S}_{\text{bg}}(\lambda)\). This is subtracted, isolating the background-corrected PSD as

\[
S_{\text{corr}}(\lambda) = S_{\text{raw}}(\lambda) - \tilde{S}_{\text{bg}}(\lambda). \tag{S1}
\]

The used background spectrum (before and after smoothing) as well as a characteristic circuit spectrum (before and after subtraction of the background) is shown in Fig. S6a. Comparing this background spectrum to the device spectrum outside the transmission band shows good agreement, indicating that this measured spectrum describes the measurement background well. To precisely know the circuit transmittance from input to output grating coupler, we account for the losses and possible dispersion in the optical elements of the measurement setup by acquiring a normalization PSD, \(S_{\text{mirror}}(\lambda)\), measured from a silver mirror (cf. Methods), which has an approximately dispersion-free reflectance in the band of interest as well as a weak dependence on the angle of incidence. We again assume the minor oscillations in the signal to be uncorrelated with \(S_{\text{raw}}(\lambda)\) and smooth \(S_{\text{mirror}}(\lambda)\) with a filter identical to the abovementioned to obtain \(\tilde{S}_{\text{mirror}}(\lambda)\). From there, the normalized circuit transmittance, \(T_{\text{norm}}(\lambda)\), is defined as

\[
T_{\text{norm}}(\lambda) = \frac{S_{\text{corr}}(\lambda)}{\tilde{S}_{\text{mirror}}(\lambda)} . \tag{S2}
\]

The quantities \(S_{\text{corr}}(\lambda)\), \(S_{\text{mirror}}(\lambda)\), \(\tilde{S}_{\text{mirror}}(\lambda)\), and \(T_{\text{norm}}(\lambda)\) are displayed in Fig. S6b. The procedure described in the previous paragraph is repeated for a measured PSD for every circuit, reusing the same \(S_{\text{bg}}(\lambda)\) and \(S_{\text{mirror}}(\lambda)\). The previously discussed example transmittance spectrum, as well as correspondingly treated spectra for PTI waveguides of length \(L = 1000a_0\) and \(L = 1750a_0\), are shown in Fig. S6c. Since the spectra exhibit significant fringes likely due to reflections in the circuit, we convolute each transmittance, \(T_{\text{norm}}(\lambda)\), with a normalized Gaussian kernel with standard deviation...
\( \sigma = 2.5 \) nm to preserve spectral resolution while reducing fringes. We thus obtain the smoothed normalized transmittances, \( T(\lambda) \), as the \( (\text{numerically evaluated}) \) integral
\[
T(\lambda) = \int d\lambda' \frac{N}{2\pi} e^{-(\lambda-\lambda')^2/2\sigma^2} T_{\text{norm}}(\lambda'),
\]
where the constant \( N \) is chosen to normalize the kernel numerically. Figure S6c illustrates the effect of this smoothing filter for three different waveguide lengths. A complete set of smoothed spectra are shown in Fig. 3 of the main text.

### 3.2 Extraction of the propagation loss

Referring to Fig. S2a and the normalization procedure described above, we model the transmittance for a specific realization of a circuit of length \( L \), \( T_{\text{circuit,}L}(\lambda) \), as the following product of transmittances:
\[
T_{\text{circuit,}L}(\lambda) = T_{\text{GC,} \text{out}}(\lambda) \times T_{\text{WG,} \text{out}}(\lambda) \times T_{\text{coupling}}(\lambda) \times T_{\text{prop,} \text{L}}(\lambda) \times T_{\text{WG,} \text{in}}(\lambda) \times T_{\text{GC,} \text{in}}(\lambda),
\]
where \( T_{\text{GC,} \text{out}}(\lambda) \) and \( T_{\text{GC,} \text{in}}(\lambda) \) describe the loss in the input and output free-space couplers, \( T_{\text{WG,} \text{in}}(\lambda) \) and \( T_{\text{WG,} \text{out}}(\lambda) \) describe the loss in the strip waveguide before and after the PTI waveguide, \( T_{\text{coupling}}(\lambda) \) describe the coupling loss into and out of the PTI waveguide, and \( T_{\text{prop,} \text{L}}(\lambda) \) describes the propagation loss in the PTI waveguide. For simplicity, we have neglected reflections. The transmittances of Eq. (S4) are all generally stochastic quantities which vary between even nominally identical circuits. We assume...
When fitting the transmittance of the range of PTI waveguide lengths, the constant insertion loss, we assume (and observe experimentally) the expectation value of the log-transmittance through the PTI waveguide to depend which are obtained through the filtering procedure described in Section 3.1. The data set has three independent datapoints per which term the constant insertion loss. Hence, we model the ensemble-averaged circuit transmittance as

\[ \langle \ln T_{\text{circuit},L}(\lambda) \rangle = -a(\lambda)L + \ln T_0(\lambda). \]  

The analyzed data set consists of measurements of circuit transmittance, \( T(\lambda, L) \), as a function of length, \( L \), and wavelength, \( \lambda \), which are obtained through the filtering procedure described in Section 3.1. The data set has three independent datapoints per length-wavelength combination. For each wavelength, we obtain the linear least-squares fit to \( \langle T(\lambda, L) \rangle \), assuming it to follow the relation

\[ \langle \ln T(\lambda, L) \rangle = -a(\lambda)L + \ln T_0(\lambda). \]  

To illustrate the effect of the number of nominally identical copies, Fig. S6d shows the main fitted propagation length as well as the resulting propagation length where the fitted data has been artificially restricted to only one or two copies of the test circuit array. The similarity between three curves evidences that the extracted propagation loss is only weakly dependent on the number of instances even when these are only a few, implying a narrow probability density function for the waveguide transmission as well as highly reproducible fabrication and optical alignment. The figure also shows the propagation length extracted from the data of the full three copies but without any smoothing applied to the transmittance spectra. The presence of the fringes in the raw data is visible.

### 3.3 Coupling loss into the photonic topological insulator waveguides

When fitting the transmittance of the range of PTI waveguide lengths, the constant insertion loss, \( T_0 \), describes the transmittance of the test circuit with all propagation loss in the PTI waveguide removed and is displayed in Fig. S7a (solid black line). On the same sample, next to the circuits with PTI waveguides, we include also shunt circuits, equivalent to the test circuits except for the removal of the PTI waveguide and its coupling regions. We measure and transform the transmittance spectra of three nominally identical shunts (displayed in Fig. S7a) and compute their normalized transmittance spectra, \( T^{(\text{shunt})}(\lambda) \), as

\[ T^{(\text{shunt})}(\lambda) = \int d\lambda' \, N \, e^{-i(\lambda - \lambda')^2/2\sigma^2} \frac{S^{(\text{raw})}_\text{shunt}(\lambda') - \tilde{S}_\text{bg}(\lambda')}{\tilde{S}_\text{mirror}(\lambda')} , \]  

where \( S^{(\text{shunt})}_\text{raw}(\lambda) \) denotes the measured PSD from a shunt and the rest of the symbols have the same meaning as in Section 3.1. The quantity \( T^{(\text{shunt})}(\lambda) \) for each of the three measured shunts as well as their average, \( \langle T^{(\text{shunt})}(\lambda) \rangle \), is displayed in Fig. S7a. By comparison of the fitted constant insertion loss, \( T_0(\lambda) \), of a PTI test circuits to the average transmittance of the shunt circuits, we may obtain the coupling efficiency, \( \eta_{\text{coupling}}(\lambda) \), as

\[ \eta_{\text{coupling}}(\lambda) = \sqrt{\frac{T_0(\lambda)}{\langle T^{(\text{shunt})}(\lambda) \rangle}} , \]  

and subsequently the coupling loss as

\[ \langle T_{\text{coupling}}(\lambda) \rangle = 1 - \eta_{\text{coupling}}(\lambda) . \]  

The extracted coupling loss from the strip to the PTI waveguide is shown in Fig. S7b and reach values as low as 0.6 dB. The measured coupling loss exhibits some fringes between 1500 nm and 1520 nm which are likely due to internal reflections. Fig. S7b also includes the coupling loss \( 1 - \eta_{\text{coupling, \text{sim}}}(\lambda) \) calculated using three-dimensional frequency-domain finite-element simulations (see Methods), showing good quantitative agreement. Practically, we model a short section of waveguide (11.5 lattice periods) connected to the intermediate coupling waveguides of length 5 lattice periods (as in the fabricated devices), which are in turn connected to rectangular strip waveguides of width 400 nm, and assume the input and output coupling efficiencies to be equal, i.e. \( \eta_{\text{coupling, \text{sim}}}(\lambda) = \sqrt{T^{\text{sim}}(\lambda)} \). The critical sizes correspond to those in the fabricated structure. The numerical coupling loss
Fig. S7: Coupling loss into photonic topological insulator waveguides. a, Constant insertion loss, $T_i$, of PTI circuits extracted as the constant term in the fit of the propagation length (black line) with standard error (gray shaded area). The transmittance of three shunt circuits is measured (thin lines, see legend) and the average shunt transmittance (red line) is computed. b, Coupling loss (black line) derived from the data in a and coupling loss obtained from numerical simulations (connected blue dots). Simulated wavelengths have been offset by 15.4 nm to align the degeneracy points between simulation and experiment. The points corresponding to the simulations shown in c and d are indicated by arrows. The uncertainty in the measurement is indicated by the gray shading. c, Simulated geometry and electric field amplitude at $\lambda = 1515.4$ nm (offset from 1500 nm) corresponding to $n_g = 25$. d, Simulated geometry and electric field amplitude at $\lambda = 1550$ nm (offset from 1550 nm) corresponding to $n_g = 7$. c,d The displayed field strength assumes an injected power of 1 W.

are offset in wavelength by 15.4 nm to align the degeneracy point of the simulated structure with that found in the experiment (see Section 2). Fig. S7c and d show the resulting simulated electric field strength for the two wavelengths highlighted with arrows in Fig. S7b. We note that the spectral shift is different from the one obtained for the propagation loss. This is likely due to perturbations of the transmittance by geometrical variations and/or the surface oxide, which is expected to be dispersive due to the significant differences in the spatial structure of the modes at different wavelengths.

4 Imaging the vertically scattered light in straight photonic topological insulator waveguides

In addition to the far-field imaging in waveguides including sharp bends, we also perform measurements on the straight waveguides used for the propagation loss extraction (Sample 2). Figure S8 shows the emitted light collected along the waveguide axis. Figs. S8c and d evidence the formation of tightly localized and high-$Q$ optical modes at wavelengths around the group index maximum for one particular waveguide of length $L = 1750a_0$ (with $a_0 = 512$ nm). For clarity, the spectro-spatial maps along the waveguide axis for the three nominally identical waveguides are given in Figs. S8e–g. We observe distinct localized optical modes with decreasing extent towards the group index maxima where the propagation losses are shown to peak (see Fig. 3a in the main text and red solid lines included in the maps). The transition from the topological to the trivial branch is also indicated, using the same approach as in the main text. These measurements already evidence strong coherent backscattering leading to complex interference patterns for the topological interface state and even more strongly localized than those shown in Fig. 4 in the main text. However, the exact wavelength for the degeneracy point might vary from waveguide to waveguide (cf. Section 1), so we consider that
Fig. S8: Vertically scattered light in straight photonic topological insulator waveguides. a. Scanning electron microscope image of example waveguide (its total length is 1750a₀). Insets show details of coupling region. b. Optical near-infrared image of the example circuit with the input free-space coupler indicated. The black box indicates the rectangle to which the images shown in c are cropped. c. Imaged scattered light from the waveguide at different wavelengths showing localized resonances of high Q-factor. The modes labeled A and B are from the same circuit, while C and D are from a different. Frequency dependent intensity of modes imaged in c. The plotted intensities are extracted from a single bright pixel near the labels in c. d-f. Wavelength-dependent maps of the imaged intensity on a horizontal line along the waveguide center for the three fabricated devices with total waveguide length 1750a₀. The red line indicates the loss length, l_L(λ), plotted as a function of wavelength, found by the analysis described in the main text. The blue line indicates the transition between topological and trivial bands as found by fitting the model given by Eq. (3) in the main text.

Imaging after a sharp bend constitutes a more clear fingerprint of strong backscattering unequivocally occurring for the topological mode, since the bend establishes the topological nature of the transmitted mode (see Section 7).

5 Reference measurements on W1 waveguides

Structural defects are extremely hard and in practice impossible to quantify because the structural and chemical composition would need to be mapped with atomic resolution across hundreds of microns of waveguide. And even if possible, such a procedure has never been attempted and such a measurement would therefore not enable a direct comparison with the levels of disorder present in previously published works in nanophotonics. We therefore benchmark the structural disorder in our samples indirectly by measuring the losses in conventional W1 photonic-crystal waveguides [S2] fabricated on the same chips and batches as for the PTI devices. On both samples, we include a set of W1 waveguides formed by removing a row of holes in a triangular lattice with lattice constant a₀ = 400 nm and hole radius r = 0.3a₀. On Sample 2, we additionally include a W1 waveguide of similar
Fig. S9: Design and dispersion of W1 waveguides. a. Schematic showing the geometry of a single periodic unit of the W1 waveguides used here. The geometrical parameters (lattice constant $a_0$, hole radius $r$, and membrane thickness $h$) are indicated. b. Dispersion (left) and group index (right) computed for the supercell with $a_0 = 400\,\text{nm}$, $h = 220\,\text{nm}$, and $r = 0.3a_0$. c. Dispersion (left) and group index (right) computed for the supercell with $a_0 = 420\,\text{nm}$, $h = 220\,\text{nm}$, and $r = 0.3a_0$.

As described in the main text, the proposed waveguide design supports two guided modes within the band gap of the bounding PTIs. We perform a linear regression to extract the propagation loss, yielding the wavelength-dependent propagation loss shown in the supplementary information. The same conclusions about the fabrication process as those drawn from Fig. S1 can be drawn from the W1 waveguides.

We characterize the propagation loss of the W1 waveguides using the same procedure as for the PTI waveguides. The transmittance spectra for a single array of devices of each kind are shown in Fig. S10b, e, and h. All spectra evidence a transmission band of approximately 70 nm. When the wavelength approaches the band edge, the group index diverges in theory (see Fig. S9b and d), which is not reached in experiment because the backscattering also scales with the group index and the system enters the regime of Anderson localization [S4–S6]. This causes a large increase in propagation loss for all waveguides. On the other side of the transmission band (toward lower wavelengths), the dispersion crosses the light line and the waveguide mode becomes leaky, which is also visible as a large increase in propagation loss. For each wavelength in the spectra of Fig. S10b, e, and h, we perform a linear regression to extract the propagation loss, yielding the wavelength-dependent propagation loss shown in Fig. S10c, f, and i, respectively. For the waveguides with $a_0 = 420\,\text{nm}$ (Sample 2), we observe losses below $1\pm1\,\text{dB/cm}$ in the linear part of the dispersion, i.e., when spectrally far from the slow-light regime. The losses are slightly higher for the waveguides with $a_0 = 400\,\text{nm}$ (Sample 2) and still larger for the W1 waveguides in Sample 1. While the first may be due to either the slight differences in group index in the linear regime or different field intensities at the hole boundaries, the higher losses for Sample 1 are very likely due to the differing fabrication processes, which is consistent with the higher level of sidewall roughness directly visible in the SEM (see Fig. S10a, d, and g). For W1 waveguides, the previously reported state-of-the-art propagation losses in that region and at wavelengths around 1550 nm include 24 dB/cm [S7], 5 dB/cm [S8], 4.1 dB/cm [S9], and 2 dB/cm [S10, S11]. This means that, despite the differences observed for the losses in the different W1 waveguides, they all exhibit state-of-the-art loss values and by extension, since the W1 waveguides are fabricated alongside the PTI waveguides, that the conclusions drawn from the measurements on the VH PTI waveguides hold for state-of-the-art silicon nanofabrication.

6 Unraveling the topological and trivial nature of the two modes

As described in the main text, the proposed waveguide design supports two guided modes within the band gap of the bounding PTIs. In accordance with the bulk-edge correspondence theorem for systems exhibiting time-reversal symmetry, the difference between the number of forward and backward-propagating topological interface states is equal to the difference between the valley-Chern numbers of the corresponding energy levels for the unit-cell bands of each PTI [S12, S13]. This indicates that only one of the two guided modes is topological. In Fig. 1 in the main text, we identify the topological mode via transmission across Z-shaped bends (see also Sections 7 and 8). We independently confirm this distinction by relying on the underlying properties of topological modes, which are preserved under continuous deformations of the associated differential operator, as long as the bandgap remains open. In Ref. [S14], it was demonstrated that the trivial mode disappears under a continuous transformation to a different topological interface [S15] while the topological mode is kept intact. Since our geometry differs slightly in refractive index and geometry, we repeat the calculation with a geometry more closely matching that used in our experiments, and reach the same
Fig. S10: Benchmark measurements on W1 waveguides. a, Scanning electron microscope (SEM) image of the W1 waveguide on Sample 1 with the inset showing the details of a single hole. b, Transmission measurements of a single array of W1 waveguides on Sample 1, with lengths from 250$a_0$ to 1500$a_0$ (light to darker, see legend). c, Wavelength-dependent propagation loss for W1 waveguides on Sample 1, fitted from the decrease in transmission as a function of length obtained from measurements on three sets of waveguides like the one shown in b. The gray shaded area indicates the standard error on the fits. d,g, SEM images of the waveguides on Sample 2 with lattice constants $a_0 = 400$ nm and $a_0 = 420$ nm respectively. The insets show single holes in the respective devices. e,h, Normalized measured transmittance spectra for a single array of test circuits on Sample 2 with W1 waveguides with lattice constants $a_0 = 400$ nm and $a_0 = 420$ nm, respectively. The measured circuit lengths span from $L = 250a_0$ to $L = 1750a_0$ (see legend). f,i, The measured propagation loss obtained from measurements of three arrays of W1 waveguide test circuits with lattice constants $a_0 = 400$ nm and $a_0 = 420$ nm, respectively. The gray shaded area indicates the standard error on the fits.

collection: The deformation, which changes the size of a single triangle in the unit cell, is displayed in Fig. S11 and causes the trivial mode to merge with the upper bulk while the topological mode remains between the bulk bands. This confirms that only the latter is topological.

7 Modeling light transmittance in sharp-bend geometries

We apply three-dimensional frequency-domain finite-element modelling to explore the transmission properties of PTI waveguides with sharp 120°-bends. Similar to the modeling described in Section 3.3, we simulate the fabricated geometry using a slightly simplified version of the outlines traced for Sample 1 (see Section 2; the inset of Fig. S12a shows dispersion relation) and a short waveguide of length 11.5$a_0$. We also simulate a comparable structure where two 120°-turns have been introduced inside the PTI crystal to offset the output waveguide interface by 10 lattice periods compared to its input $y$-location as in the fabricated structure. The transmittance and reflectance of the straight and bent waveguides are shown in Fig. S12a and b. We observe that the transmittance drops dramatically just before the interface transitions from topological to trivial, with a 25 dB extinction occurring within only 3 nm. Importantly, across these 3 nm, we observe a considerable drop of the transmittance with the group index value
inside the topological mode, as already hinted at in Ref. [S16]. This clearly confirms that the vertically scattered light seen after the sharp bend in Fig. 5 in the main text results from the topological mode. The simulated reflectance of both the straight and bent waveguide exhibit fringes associated to reflections at the different facets, although these are suppressed within the trivial band for the bent waveguide. This, in addition to the absolute reflectance value (−2.8 dB), indicates that a considerable amount of light is vertically scattered at the first corner. Simulated geometry and electric field amplitude for the bent waveguide at wavelengths in the trivial and topological band exhibiting comparable group indices of \(n_g = 31\) are shown in Fig. S12c and d. Fig. S12e and f show field amplitude for the straight waveguide at the same wavelengths, where the transmission is large and comparable for both modes.

8 Propagation losses for photonic topological insulator waveguides on Sample 1

The measurements of propagation propagation losses shown in Figs. 2 and 3 of the main text are performed on Sample 2. To establish that the topological waveguides on both samples operate in essentially the same way, we apply the analysis described in Section 3 to equivalent transmission measurements on the circuits on Sample 1 to obtain the propagation loss of these structures. Due to the systematic spectral shifts observed (see Section S3), we perform independent analyses of the measurements for the two spatially offset groups of test circuits. The normalized circuit transmittances (obtained as described in Section 3.1) for a single array in Group 1 and 2 are displayed in Fig. S13a and c, respectively. The transmittance spectra clearly exhibit two intervals of high transmittance separated by a region of low transmittance and low coupling efficiency, which we identify as the transition between topological and trivial bands. The fitted propagation length (Fig. S13b and d) exhibits more oscillations compared to Sample 2, which is likely due to the more limited span in waveguide lengths on Sample 1 (the length \(L\) is varied from 250\(a_0\) to 1250\(a_0\) compared to the variation from 250\(a_0\) to 1750\(a_0\) for Sample 2) exacerbating the effect of internal circuit reflections on the analysis. We also explore the propagation losses as measured from test circuits where two sets of two sharp turns near the beginning and end of the PTI waveguides are included. The central segments of PTI waveguide between the bends are varied in length as the straight waveguides, allowing for the same analysis to be performed on the topological band for these waveguides. The normalized transmittance spectra of a single array of bent waveguides waveguides is shown in Fig. S13e. We apply the equivalent analysis to a total of two arrays of test circuits to obtain the propagation loss shown in Fig. S13f, which is in very good quantitative agreement with the data in the straight waveguides for the topological mode.

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**Fig. S11**: Continuous transition between zigzag and bearded interface. **a**, Visualization of the geometry of the crystal interface as it is continuously deformed (left to right) from \(s_f = s_1 = 0.614a_0\) to \(s_f = s_2 = 0.7a_0/\sqrt{3} = 0.404a_0\). **b**, Dispersions computed for the zigzag (beige lines) and bearded (black lines), as well as the intermediate interfaces shown in a (purple and orange lines). The additional glide-plane symmetry of the bearded interface enforces a degeneracy at \(k_x = x/a_0\), which is lifted when \(s_f \neq s_1\). Under the deformation, the trivial band (dashed lines) shifts into the bulk bands whereas the topological band (solid lines) remains outside.

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S10
Fig. S12: Numerical transmittance and reflectance spectra through sharp bends. a, b, Transmittance and reflectance spectra of the straight waveguide (cyan) as well as the waveguide with sharp bends (red). The degeneracy point, $\lambda_D$, is marked by a vertical line. The group index, $n_g$, is overlayed in gray on both plots with a scale given by the right vertical axis. The inset of a shows the dispersion of the simulated crystal, where the topological (trivial) band is shown as a red (gray) line. c, Simulated geometry and electric field amplitude for the bent waveguide at a wavelength of $\lambda = 1545 \text{ nm}$ corresponding to the trivial band and a group index of $n_g = 31$. d, Simulated geometry and electric field amplitude for the bent waveguide at a wavelength of $\lambda = 1553 \text{ nm}$ corresponding to the topological band and a group index of $n_g = 31$. e, Simulated geometry and electric field amplitude for the straight waveguide at a wavelength of $\lambda = 1545 \text{ nm}$. f, Simulated geometry and electric field amplitude for the straight waveguide at a wavelength of $\lambda = 1553 \text{ nm}$. c,d,e,f, The displayed field strength assumes an injected power of 1 W.
Fig. S13: Propagation losses of photonic topological insulator waveguides on Sample 1. 

a, c. Measured normalized test circuit transmittance spectra for Group 1 and 2 circuits. 
b, d. Propagation loss as fitted from measurements performed on Group 1 and 2, respectively. 
e. Measured normalized transmittance spectra for test circuits which include sharp bends. 
f. Propagation loss as fitted from the transmittance measurements on test circuits that include sharp bends in the photonic topological insulator waveguide.
