Modulation stabilization of Bloch oscillations of two-component Bose–Einstein condensates in optical lattices

Huai-Qiang Gu1, Jun-Hong An2 and Kang Jin3

1 School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, People’s Republic of China
2 Center for Interdisciplinary Studies, Lanzhou University, Lanzhou 730000, People’s Republic of China
3 Department of Physics, Northwest University, Xi’an 710118, People’s Republic of China
E-mail: guhq06@lzu.edu.cn

Received 18 June 2010, in final form 6 August 2010
Published 5 October 2010
Online at stacks.iop.org/JPhysB/43/205308

Abstract

We study the Bloch oscillations (BOs) of two-component Bose–Einstein condensates (BECs) trapped in spin-dependent optical lattices. Based on the derived equations of motion of the wave packet in the basis of localized wavefunctions of the lattice sites, the damping effect induced by the intercomponent and intracomponent interactions to the BOs is explored analytically and numerically. We also show that such damping of the BOs can be suppressed entirely if all the atom–atom interactions are modulated synchronously and harmonically in time with suitable frequency via the Feshbach resonance. When the intercomponent and the intracomponent interactions have opposite signs, we find that the long-lived BOs and even the revival of the BOs can be achieved via only statically modulating the configuration of optical lattices. The results provide a valuable guidance for achieving long-lived BOs in the two-component BEC system by Feshbach resonances and manipulating the configuration of the optical lattices.

1. Introduction

The system of ultracold atomic gases in optical lattices has become a nice experimental platform to simulate effects in condensed matter physics [1]. The high controllability of such a system allows many sophisticated effects in condensed matter physics to be well studied in this system [2]. The Bloch oscillation (BO) is the oscillatory motion of a quantum particle in a periodic potential when it is subjected to an external force. It was originally predicted in a solid-state system where the motions of the electrons in tilted periodic potentials undergo coherent oscillations [3]. The formal resemblance between electrons in crystals and Bose–Einstein condensates (BECs) in optical lattices has inspired extensive interest in exploring BOs in an optical lattice system. Successful observations of BOs have been reported for atoms in interacting BECs [4–7] and for ensembles of noninteracting quantum-degenerate fermions [8] in tilted optical lattices.

However, perfect BOs can only be available in the ideal case where there are no interactions among the atoms of the BECs. In practice, due to the intrinsically weak interactions of atoms, the momentum distribution of the BECs will show a rapid broadening, which causes the atoms of the BECs to lose their phase coherence, i.e. the dephasing effect [9,10]. Consequently, the BOs in the BECs cannot persist for a long time. In the framework of the mean-field treatment, the motion of the BECs can be well described by the so-called Gross–Pitaevskii equation (GPE) with a nonlinear term. It is believed that such nonlinearity induced by the atom–atom interactions generally leads to a breakdown of the BOs, as recently studied experimentally [11] and theoretically [12–14]. Therefore, a
natural question is: is it possible to prolong or even stabilize the BOs by some active control methods?

Addressing this question, some progress, especially in single-component BECs, has been made. It has been found that a long-lived BO can be induced by properly designing the spatial dependence of the scattering length [15] and the configuration of the optical lattices [16]. Gustavsson et al [11] showed experimental evidence that the dephasing time of the BOs can be much enhanced by decreasing the interaction strengths via the Feshbach resonance. However, in many situations, such finite enhancement to the dephasing time of the BOs is not enough, and one desires to preserve the BOs indefinitely. Recently, Gual et al have reported that a persistent BO of single-component BECs can be obtained by modulating the interaction harmonically in time with a suitable frequency and phase [10, 18], which can be easily realized by means of Feshbach resonance.

So far, most of the studies of the stabilization control to the BO of the BECs in an optical lattice have been based on the single-component BEC case. Compared with the single-component BEC, a two-component BEC system may exhibit more novel physical effects due to the condensate mixtures [19–22]. In this system, both intracomponent and intercomponent atom–atom interactions affect the nonlinear behaviour of the BECs. The two-component BECs can be experimentally realized by spin-dependent optical lattices, which hold the BECs with two components composed of two distinct hyperfine states of the same atomic species [23].

In the present paper, we study systematically the modulation stabilization of BOs of two-component BECs in optical lattices. We mainly use two control methods: one is by modulating periodically the interactions via the Feshbach resonance; the other is by tuning statically the parameters of the optical lattices. We will show that stable BOs can be obtained when the interactions are modulated synchronously and harmonically in time with suitable frequencies. Moreover, if the intercomponent and the intracomponent interactions have opposite signs, the long-lived BOs and even the revival of the BOs can be achieved via only tuning the relative separation between lattices.

The paper is organized as follows. In section 2, we discuss the methods and formalism used in this work. In section 3, we explore quantum manipulation of the BOs from two aspects according to the signs of the intracomponent atom–atom interactions. Finally, a summary is given in section 4.

2. Model and formulation

2.1. Gross–Pitaevskii equations for two-component BECs in an optical lattice

We consider two-component BECs which are composed of bosonic atoms of the same isotope but having different internal spin states, e.g. $^87$Rb atoms in hyperfine states $|F = 2, m_F = 2\rangle$ and $|F = 1, m_F = -1\rangle$ [24]. The BECs are trapped in spin-dependent optical lattices. The dynamics of the system is governed by coupled GPEs under the mean-field approximation:

$$
i h \partial_t \Phi_i = \left[-\frac{\hbar^2}{2M} \nabla^2 + V_i + \sum_{j=1}^2 g_{ij}(t)|\Phi_j|^2\right] \Phi_i, \quad (1)$$

where $\Phi_i (i = 1, 2)$ is the macroscopic condensate order parameter of the $i$th component with identical mass $M$. The time-dependent interaction coefficients are given by $g_{ij}(t) = 4\pi \hbar^2 a_{ij}(t)/M$, with $a_{ij}(t)$ being the s-wave scattering length which can be controlled via the Feshbach resonance induced by the modulated magnetic field. The external potential felt by the $i$th component can be decomposed into $V_i = V_c + V_{Li}$, where $V_c = f_z$ is a linear potential induced by a constant force $f$ and $V_{Li}$ is the trapping potential of the optical lattice. The additional weak potential $V_c$ tilts the optical potentials and drives coherent oscillations [5]. The trapping potentials for different components can be explicitly written as $V_{L1} = U_p \sin^2(k_L z + \frac{\theta}{2})$ and $V_{L2} = U_p \sin^2(k_L z - \frac{\theta}{2})$, where $U_p$ is the depth of the 1D lattice potentials, $k_L$ is the wave-vector of the lasers used to construct the optical lattice and $\theta$ is the polarization angle of the two counter-propagating laser beams to form the standing wave configuration of the optical lattice [20, 23]. By changing $\theta$, one can also control the separation between the two potentials.

When the linear field is too weak to induce Landau–Zener tunnelling [25, 26], a BO can be described by adiabatic evolution of the BECs in the lowest lattice band. In collective coordinates [9], the condensate order parameter $\Phi_i (r, t)$ can be expanded as a linear combination of the wave packets localized at the individual lattice sites, i.e., the Wannier wavefunctions $\phi_{n_i}(r)$, as

$$\Phi_i (r, t) = \sqrt{N_i} \sum_{n_i} \psi_{i,n_i}(t) \phi_{n_i}(r). \quad (2)$$

where $N_i$ is the total number of particles of the $i$th component, and the Wannier wavefunction $\phi_{n_i}$ satisfies the orthogonality condition $\int d\mathbf{r} \phi_{n_i} \phi_{n_i+\pm 1} = 0$ and the normalization condition $\int d\mathbf{r} \phi_{n_i}^2 = 1$. $\psi_{i,n_i}(t) = \sqrt{N_{i,n_i}(t)/N_i} e^{i \theta_{i,n_i}(t)}$, where $N_{i,n_i}(t)$ and $\theta_{i,n_i}(t)$ are the number of particles and phase, respectively, is the amplitude of the $i$th component trapped in the site $n_i$. In the following, we assume that the two components have the same total number of particles, i.e. $N_1 = N_2$. Substituting equation (2) into equations (1), we can discretize equations (1) into a set of coupled nonlinear equations with respect to a different lattice site $n_i$:

$$i \psi_{i,n_i} = \frac{-\psi_{i,n_i-1} + \psi_{i,n_i+1}}{2} + \left[\epsilon_{i,n_i} + \Lambda_{i,i}(t)\right] \psi_{i,n_i}^2 + \Lambda_{ij}(t) (\eta_{i-1,n_i+1}^2 + \eta_{i,n_i+1}^2) \psi_{j,n_i+1}, \quad (3)$$

where the overdot denotes the time derivative and $i \neq j$ labels the two different components of the BECs. $\epsilon_{i,n_i} = \frac{1}{2M} \int d\mathbf{r} \left[\hbar^2 \nabla \psi_{n_i}^2 + V_i \phi_{n_i}^2\right]$, with $J = -\frac{\hbar^2}{2M} \nabla \psi_{n_i} \nabla \phi_{n_i+1} + \phi_{n_i} V_i \phi_{n_i+1}$ being the tunnel parameter. $\Lambda_{ii} = \frac{\hbar^2}{2M} \int d\mathbf{r} \phi_{n_i}^2$ describes the intracomponent atom–atom interaction strength. $\Lambda_{ij} = \frac{\hbar^2}{2M} \int d\mathbf{r} \phi_{n_i}^2 \phi_{n_i+1}$ multiplying with $\eta_{i-1,n_i+1}^2 = \left|\int d\mathbf{r} \phi_{n_i}^2 \phi_{n_i+1}\right|^2$ describe the intracomponent atom–atom interaction strengths, where $\eta_{i}$ and $\eta_{i-1}$ stem from the
overlaps of the wavefunctions of the two components. \( \tau \) (in the lattice unit) is determined by the relative separation between the two nearest neighbouring spin-dependent potentials, which can be controlled by the polarization angle \( \theta \). It is noted that not only all the interactions are time dependent but also intercomponent nonlinear interactions depend on \( \eta_2 \) and \( \eta_{2-1} \). In equation (3) the time has been rescaled to be dimensionless as \( t \rightarrow \hbar t/2J \).

In 1D optical potentials, we can denote the Wannier wavefunction as \( \phi_0(r) = \phi(x, y)\phi_0(z) \). The transverse wavefunction can be expressed as a 2D Gaussian profile \( \phi(x, y) = \phi_0(x)\phi_0(y) \), where \( \phi_0(\alpha) = \frac{1}{\sqrt{2\pi\sigma_d^2}} \exp\left(-\frac{x^2}{2\sigma_d^2}\right) \), with \( \sigma_d \) being the Gaussian widths in the \( x \) = \( y \) directions. The wavefunction along the direction of the optical lattice can be denoted as \( \phi_0(z = n_d) \), with \( d = \pi/k_L \) being the lattice constant. Based on the variational ansatz for \( \phi_0(z) \), a minimum energy can be obtained when the width of \( \phi_0(z) \) equals \( \sigma_z = \frac{\pi}{2}\sqrt{U_0/\kappa_{\text{rec}}} \), where \( \kappa_{\text{rec}} \) is the recoil energy [25]. Under this consideration, the parameters in equation (3) can be determined explicitly as

\[
\epsilon_{1,n_i} = \omega m + \frac{\theta^2 U_p}{8}, \quad \epsilon_{2,n_2} = \omega m - \frac{\theta^2 U_p}{8},
\]

\[
\Lambda_{ij}(t) = \frac{1}{2J}\sqrt{2\pi}\sum_{\ell \neq n}^{-1} \sigma_i \sigma_j \frac{\gamma_{ij}(t)}{\kappa_{\text{abs}}}, \quad \eta_2 = \exp\left(-\frac{\tau^2}{2\sigma_d^2}\right),
\]

\[
J = \exp\left(\frac{d^2}{4\sigma_z^2}\right) \frac{\hbar^2}{2M} \frac{2\sigma_a^2}{\sigma_z^2} - \frac{U_p}{2} \left[1 + \exp\left(-\frac{\pi^2\sigma_a^2}{\sigma_z^2}\right)\right],
\]

where \( \omega = \frac{\hbar^2}{2M} \) and \( f \) corresponds to the weak atomic gravity.

From equation (3) and the canonical equation \( \dot{\psi}_i = \frac{\partial H}{\partial \psi_i^*} \), the Hamiltonian functions can be obtained:

\[
\mathcal{H}_i = \sum_{n_i, n_{2-1}} \left[ -\psi_{i,n_i} \psi_{i,n_i}^* \psi_{i,n_{2-1}} \psi_{i,n_{2-1}}^* + \psi_{i,n_{2-1}}^* \psi_{i,n_i} \right]
\]

\[
+ \left[ \epsilon_{i,n_i} + \frac{\Lambda_{ij}(t)}{2} \left| \psi_{i,n_i} \right|^2 + \Lambda_{ij}(t)\eta_2 \left| \psi_{j,n_{2-1}} \right|^2
\]

\[
+ \eta_{2-1} \left| \psi_{j,n_{2-1}} \right|^2 \right] \left| \psi_{i,n_i} \right|^2 \right],
\]

where both the Hamiltonian \( \mathcal{H}_i \) and the norm \( \sum_{n_i} \left| \psi_{i,n_i} \right|^2 = N_i \) are conserved.

2.2. The dynamics of the wave packet: Bloch oscillations

In order to analyse how the interactions affect the BOs of the two-component BECs, we parameterize the Gaussian profile wave packet for the 4th component [9]

\[
\psi_{i,n_i} = \sqrt{K_i} \exp\left[-\frac{(n_i - \xi_i)^2}{\gamma_i^2} + ip_i (n_i - \xi_i) + i\frac{\delta_i}{2} (n_i - \xi_i)^2\right],
\]

where \( K_i = \left(\frac{1}{\sqrt{2\pi\gamma_i}}\right)^{1/2} \) is a normalization factor. The Gaussian wave packet is parameterized by four types of quantities: the centre-of-mass position \( \xi_i(t) \), the width of the wave packet described by \( \gamma_i(t) \), the linear phase \( p_i(t) \) describing the group velocity of the wave packet and the quadratic phase \( \delta_i(t) \) over the wave packet. The latter phase allows us, on the one hand, to describe the linear evolution of the wave packet for which the quadratic dispersion in momentum space directly translates into a quadratic phase in real space. On the other hand, the nonlinearity due to the atom–atom interactions also leads to a quadratic phase since the density near the Gaussian maximum is quadratic [1]. Such a Gaussian profile wave packet was used to successfully explain the BO in the Anderson–Kasevich experiment [5].

The dynamical evolution of the wave packet can be obtained by a variational principle from the Lagrangian \( \mathcal{L}_i = \frac{1}{2} \sum_{n_i} \psi_{i,n_i} \psi_{i,n_i}^* - \mathcal{H}_i \). After some algebra, the Lagrangian can be achieved:

\[
\mathcal{L}_i = p_i \hat{\xi}_i - \frac{\gamma_i^2 \delta_i}{8} + e^{-\gamma_i} \cos \alpha_i \mathcal{H}_i \frac{\Delta_{ij}(t)}{2\sqrt{\gamma_i}} - \mathcal{H}_j(t) - \frac{\Lambda_{ij}(t)}{2\sqrt{\gamma_i}} \eta_2 - \frac{\Lambda_{ij}(t)\kappa}{\sqrt{\gamma_i}} (\eta_2 e^{-\mu_2} + \eta_{2-1} e^{-\mu_2-1}),
\]

where \( \gamma_2 = \frac{1}{\sqrt{2}} + \frac{\gamma_2^8}{\gamma_2}, \quad v_i = K_i \int d\epsilon \epsilon \psi_{i,n_i} \exp\left[\frac{-\epsilon^2}{2\gamma_i^2}\right], \quad \kappa = \frac{\gamma_2^8}{\gamma_2}, \quad \gamma_2^8 = \gamma_2^8 - \frac{\gamma_2^8}{\gamma_2} \), and \( \mu_2 = \frac{\gamma_2^8}{\gamma_2} \), with \( \xi_2 = \xi_2 - \xi_2 + \tau \). It is noted that in our calculation the summation over \( n_i \) has been replaced by integration when the widths \( \gamma_i \) are not too small [9, 22]. The equations of motion of the collective coordinates can be obtained from the Euler–Lagrange equations:

\[
\dot{\xi}_i = e^{-\gamma_i} \sin \alpha_i p_i,
\]

\[
\dot{\gamma}_i = \eta_2 \delta_i e^{-\gamma_i} \cos \alpha_i,
\]

\[
p_i = \frac{2\kappa^2 \Delta_{ij}(t)}{\sqrt{\gamma_i}} [\eta_2 \xi_2 e^{-\mu_2} + \eta_{2-1} \xi_2 e^{-\mu_2-1}] - \alpha_i,
\]

\[
\dot{\delta}_i = \left(\frac{4}{\gamma_i^8 - \delta_i^2}\right) e^{-\gamma_i} \cos \alpha_i p_i + \frac{2\Lambda_{ij}(t)}{\sqrt{\gamma_i}} \kappa \gamma_2 \Delta_{ij}(t) + \frac{\kappa^2 \gamma_2 \Delta_{ij}(t)}{\sqrt{\gamma_i}}.
\]

To highlight the essential physics, from the coupled equations (8)–(11), the equation of motion of the centre of the wave packet can be recast into

\[
\dot{\xi}_i + \alpha_i \dot{\xi}_i = \beta_i,
\]

where

\[
\alpha_i = \frac{\delta_i \Delta_{ij}(t)}{2\sqrt{\gamma_i}} - \frac{\kappa^2 \gamma_2 \delta_i \Delta_{ij}(t)}{4\sqrt{\gamma_i}},
\]

\[
\beta_i = p_i e^{-\gamma_i} \cos \alpha_i.
\]

It is noted that equation (12) can recover the equation of motion of the wave-packet centre for a single-component BEC under the condition \( \Lambda_{ij} = 0 \) as [9]

\[
\dot{\xi}_i + \alpha_i \dot{\xi}_i + \omega^2 \xi_i = \frac{\omega N_i}{2\sqrt{\gamma_i}} \left[\gamma_i^{-1}(0) - \gamma_i^{-1}(1)\right],
\]

which is a standard equation of motion for a harmonic oscillation with an effective damping rate \( \alpha(t) \). Under the ideal condition \( \Lambda_{ij} = 0, \alpha(t) = 0 \), and the system undergoes a perfect oscillation, which is perfect BOs with frequency \( \omega \) as a result of the driven field. The damping of the BOs is caused by the intracomponent interactions \( \Lambda_{ii} \), which
The initial conditions are set as $p_g \quad \text{mechanism that can be used to modulate experiments, the Feshbach resonance is a quite effective sign of these parameters sensitively influence the dynamical component BECs in the optical lattice. The magnitude and $\eta_i(\Lambda_{1ii}, \Lambda_{1ij})$ the interaction coefficients intracomponent interactions generally lead to the breakdown from the above analytical results, we can see that the perfect BOs of the system are distorted by the cooperative action of the effective damping rate $\alpha(t)$ and the effective driven force $\eta_i(\Lambda_{1ii}, \Lambda_{1ij})$. It is noted that the frequency $\omega$ of the BOs, which corresponds to the inverse of the right-hand side of equation (10), for the two-component case is slightly detuned from $\omega$ by the intercomponent interactions $\Lambda_{ij}$. Explicit analysis of the BOs governed by $\alpha(t)$ and $\eta_i(\Lambda_{1ii}, \Lambda_{1ij})$ will be shown by quantitative calculations with experimentally adjustable parameters in the following section.

3. Quantum manipulation of Bloch oscillations in the optical lattice

From the above analytical results, we can see that the nonlinearities contributed by both the intercomponent and intracomponent interactions generally lead to the breakdown of the BOs of the BECs in the optical lattice. Therefore, the interaction coefficients $g_{ij}$ of such nonlinearities, which are essentially determined by the $s$-wave scattering lengths $a_{ij}$, have a profound impact on the dynamics of the two-component BECs in the optical lattice. The magnitude and sign of these parameters sensitively influence the dynamical behaviour of this ultracold boson system. In cold-atom experiments, the Feshbach resonance is a quite effective mechanism that can be used to modulate $g_{ij}$. Inspired by a recent experimental investigation of ultracold molecule production via a sinusoidal magnetic field modulation to the interaction coefficient around the Feshbach resonance [27], we intend to explore the possibility of stabilizing the BOs via such periodic modulations to the interaction coefficients $g_{ij}$ in the following. Besides the magnetic-field-induced Feshbach resonance, another way to modulate the dynamics of the BECs in our system is to adjust the configuration of the optical lattices. The separation between the spin-dependent potentials felt by the two components of the BECs, which can be adjusted by the polarization angle $\theta$ of the lasers, essentially determines the intercomponent interactions of the BECs. We also examine the influence of the separation on the dynamics of the BOs. The combined effect of the Feshbach resonance and a periodic external potential has been widely studied [28–30].

3.1. $a_{ii}(0) > 0$, $a_{ij}(0) > 0$

In this case, both the intercomponent and intracomponent interactions are repulsive. Without loss of generality, we assume that the intercomponent interactions $\Lambda_{ij}$ relate to the intracomponent interactions $\Lambda_{ii}$ as $\Lambda_{12} = \Lambda_{21} = \sqrt{\Lambda_{11}\Lambda_{22}}$ in our numerical simulations.

To see the effect of nonlinearities on the BOs, we plot in figure 1 the attenuations of the BOs without modulations. From the time-dependent behaviour of the wave-packet centres $\xi_i$ (figure 1(a)), we can see clearly the breakdown of the BOs with time evolution. Such damping oscillations are manifested by the behaviour of the effective damping rates $\alpha_i$ (figure 1(c)) and the driven forces $\eta_i$ (figure 1(f)), where $\alpha_i$ are always positive and increase with time, and $\eta_i$ tend to constants after the same cycles as $\xi_i$. Compared with the case for the single-component BECs [9], the presence of the intercomponent interactions, sharing the same sign with the intracomponent ones here, plays the role of additional nonlinearities and speeds up the collapse in our two-component situation. So, the stronger the intercomponent interactions, the faster the BOs

![Figure 1](image-url)
damp. Figure 1(d) shows that $p_i$, the associated momenta of $\xi_i$, increase linearly with time. This can be understood from the analysis of equation (10). The first term on the right-hand side of equation (10), which is contributed by the intercomponent interactions, is much smaller than the second term, which is contributed by the linear potential. Consequently, the time-dependent behaviours of $p_i$ are dominated by the second term, i.e. $p_i(t) \approx p_i(0) - \omega t$. Figure 1(b) shows that the widths $\gamma_i$ undergo breathing oscillations and soon approach constants. The associated momenta $\delta_i$ are also divergent, as shown in figure 1(e). All these time-dependent behaviours indicate that the system is set into a macroscopical quantum self-trapping mode [31–33] due to the nonlinearities from the intercomponent and intracomponent interactions.

Now we focus on how to stabilize the BOs in our system. We mainly use the method of modulating periodically the interactions via a magnetic field.

In figure 2, we plot the time-dependent behaviour of the wave-packet variables under $\cos(\omega t)$, where $\omega$ is the frequency of the BOs, modulation to all the interactions. We can see that the damping of the BOs can be fully stabilized by this modulation, and perfect oscillations are thus obtained. Such persistent phenomena can also be explained straightforwardly by studying the time dependence of the damping coefficients $\alpha_i$ (figure 2(c)) and the effective driven forces $\eta_i$ (figure 2(f)). In contrast to the positivity in the full evolution range in figure 1(c), $\alpha_i$ in this case exhibit periodic oscillations between positive and negative values with definite amplitudes, which characterizes well the lossless BOs of $\xi_i$. It means that the effective damping coefficients with alternate signs drive the system itself to guarantee the stabilizations of the BOs. Besides, we find that the driven forces in figure 2(f) are replaced by constant values completely after such modulation. It has been proven that there is a family of stable solutions in terms of collective coordinates in a single-component BEC system when all the interactions are modulated by $\cos(\omega t)$ [10]. In fact, in the two-component ones, the coupled terms in equations (1) can be regarded as additional nonlinearities, which possess the same time dependence as the intracomponent interactions so long as all interactions are modulating harmonically in time with the same suitable frequencies. As viewed from the mean field, the two components can be reduced to two independent single ones. So it is understandable that such modulation stabilization is also present in our two-component BECs system.

It is noted that the effect of the modulation on the BOs is sensitively dependent on the forms of the modulating field we used. In figure 3, we plot the numerical simulation when all the interactions are modulated by a $\sin(\omega t)$ field. It is found that the damping of the BOs manifested by $\xi_i$ is not suppressed, and the BOs are destroyed after several rounds of oscillation. The wave-packet widths $\gamma_i$ reduce their amplitudes quickly. So the modulation has no effect in this case.

In the present case, the intercomponent interactions share the same sign as that of the intracomponent ones, so the tuning of the relative separation $\tau$ does not have constructive action to suppress the damping of the BOs. However, things are changed dramatically when the intercomponent interactions have opposite signs to the intracomponent ones, as discussed in the following.

3.2. $a_{ii}(0) > 0$, $a_{ij}(0) < 0$

In this case, the original intracomponent interactions are repulsive, while the original intercomponent ones are attractive. Unlike the above case, there are many interesting effects induced from the competition between these two kinds of atom–atom interactions. For example, the stability of static solitonic excitations in the two-component BECs has been analysed within the Gross–Pitaevskii approximation [34]. For convenience, we assume the intercomponent atom–atom interactions to be $\Lambda_{12} = \Lambda_{21} = -\sqrt{\Lambda_{11}\Lambda_{22}}$ in our numerical simulations.

As analysed in the above case, the BOs are destroyed by the nonlinearities. From this point, the dynamical behaviour of the wave packet in the present case shows no difference to the above one. However, we can prolong the coherent time of the BOs dramatically by tuning the relative separation $\tau$. 
Figure 3. Attenuations of the BOs with modulating interactions by sin(ωt). τ = 0.1 and the same parameters and notations are as used in figure 1.

Figure 4. Attenuations of the BOs without modulating interactions for different potential separations τ. The solid and dashed lines indicate two individual components with the intracomponent interactions as Λ11 = 20 and Λ12 = 18, respectively. The other parameters are chosen as $U_p = 16E_{\text{rec}}$, $\omega = 2$, and $\tau = 0.5$ for (a) and (d); $\tau = 0.1$ for (b) and (e); and $\tau = 0.05$ for (c) and (f). The initial conditions are set as $p_1(0) = p_2(0) = 0$, $\delta_1(0) = \delta_2(0) = 0$, $\xi_1(0) = \xi_2(0) = 0$ and $\gamma_1(0) = \gamma_2(0) = 10$.

of the potentials felt by the two components in the present case. To confirm this, we plot the time evolutions of the wave-packet centres $\xi_i$ and widths $\gamma_i$ in figure 4 for different relative separations. A large τ means a large distances between the nearest neighbours of the Wannier wavefunctions, which in turn induces a small intercomponent interaction rate $\eta_\tau$. Figures 4(a) and (d) show the breakdown of the BOs when the intercomponent interactions are small for large τ. With the decrease of τ, the intercomponent interactions get stronger. The damping of the BOs is obviously slowed down (figures 4(b) and (e)). In particular, it is noticed that $\gamma_i$ show a revival at about $t = 150$, see the dashed line in figure 4(e). It provides valuable evidence that the dynamics of the system would show revival in this case. If the relative separation τ is further reduced so that the two lattices are extremely close, the BOs show obvious revival (figures 4(c) and (f)). This phenomenon is caused by the competition between the intercomponent and intracomponent interactions. Because the intracomponent interactions have opposite signs to those of the intercomponent ones, the nonlinearities contributed from the intracomponent interactions are partially counteracted by the intercomponent ones. For the full overlap at $\tau = 0$, the two components perfectly mix together and attenuations of the BOs reappear. To sum up, the coherent time of the BOs can be much enhanced by only tuning the relative separation τ of the optical lattices.

However, in many situations, such finite enhancement to the dephasing time of the BOs is not enough, and one desires to
preserve the BOs indefinitely. In fact, this can also be achieved by modulating the interactions by a suitable time-dependent field. Figure 5 shows that the BOs are entirely stabilized by modulating interactions harmonically in time with the same frequency as that of the BOs. Such stabilization is independent of the magnitudes of the nonlinearities, so the behaviours for different \( \tau \) under the modulation are the same, as shown in figures 5(a) and (b).

4. Conclusions

In summary, we have studied analytically and numerically the dynamical behaviour of BOs for two-component BECs trapped in combined potentials consisting of linear potentials and spin-dependent optical lattices. We found that the damped BOs can be stabilized when all the atom–atom interactions are modulated synchronously and harmonically in time with the Bloch frequency. Moreover, it has been shown that if the intercomponent and the intracomponent interactions have opposite signs, then the dephasing time of the BOs can be much enhanced by decreasing the relative separation of the two potentials felt by the two components. Our results provide valuable guidance for achieving long-lived BOs in the two-component BEC system by Feshbach resonances and manipulating the configuration of the optical lattices.

Acknowledgments

This work was supported by the Fundamental Research Fund for Physics and Mathematics of Lanzhou University (grant LZULL200806). JHA thanks the support by the Fundamental Research Funds for the Central Universities under grant no Izujbky-2010-72 and the Gansu Provincial NSF of China under grant no 0803RJZA095.

References

[1] Morsch O and Oberthaler M 2006 Rev. Mod. Phys. 78 179
[2] Bloch I 2005 Nature Phys. 1 23
[3] Bloch F 1929 Z. Phys. 52 555
[4] Dahan M B, Peik E, Reichel J, Castin Y and Salomon C 1996 Phys. Rev. Lett. 76 4508
[5] Anderson B P and Kasevich M A 1998 Science 282 1686
[6] Morsch O, Müller J H, Cristiani M, Ciampini D and Arimondo E 2001 Phys. Rev. Lett. 87 140402
[7] Salger T, Ritt G, Geckeler C, Kling S and Weitz M 2009 Phys. Rev. A 79 011605
[8] Roati G, de Miranda E, Ferlaino F, Ott H, Modugno G and Inguscio M 2004 Phys. Rev. Lett. 92 230402
[9] Trombettoni A and Smerzi A 2001 Phys. Rev. Lett. 86 2353
[10] Gaul C, Lima R P A, Díaz E, Müller C A and Domínguez-Adame F 2009 Phys. Rev. Lett. 102 255303
[11] Gustavsson M, Haller E, Mark M J, Danzi J G, Rojas-Kopeining G and Nägerl H C 2008 Phys. Rev. Lett. 100 080404
[12] Gangardt D M and Kamenev A 2009 Phys. Rev. Lett. 102 070402
[13] Krimer D O, Khomeriki R and Flach S 2009 Phys. Rev. E 80 036201
[14] Kolovsky A R, Gómez E A and Korsch H J 2010 Phys. Rev. A 81 025603
[15] Salerno M, Konotop V V and Biadov Y V 2008 Phys. Rev. Lett. 101 030405
[16] Walter S, Schnelle D and Durst A C 2010 Phys. Rev. A 81 033623
[17] Donley E A, Claussen N R, Cornish S L, Roberts J L, Cornell E A and Wieman C E 2001 Nature 412 295
[18] Díaz E, Gaul C, Lima R P A, Domínguez-Adame F and Müller C A 2010 Phys. Rev. A 81 051607
[19] Pu H and Bigelow N P 1998 Phys. Rev. Lett. 80 1130
[20] Ostrovskaya E A and Kivshar Y S 2004 Phys. Rev. Lett. 92 180405
[21] Ma X, Xia L, Yang F, Zhou X, Wang Y, Guo H and Chen X 2006 Phys. Rev. A 73 013624
[22] Wang J-J, Zhang A-X, Zhang K-Z, Ma J and Xue J-K 2010 Phys. Rev. A 81 033607
[23] Mandel O, Greiner M, Widera A, Rom T, Hänsch T W and Bloch I 2003 Phys. Rev. Lett. 91 010407
[24] Schmaljohann H, Erhard M, Kronjäger J, Kottke M, van Staa S, Cacciapuoti L, Arlt J J, Bongs K and Sengstock K 2004 Phys. Rev. Lett. 92 040402
[25] Cristiani M, Morsch O, Müller J H, Ciampini D and Arimondo E 2002 Phys. Rev. A 65 063612
[26] Witthaut D, Werder M, Mossmann S and Korsch H J 2005 Phys. Rev. E 71 036625
[27] Thompson S T, Hodby E and Wieman C E 2005 Phys. Rev. Lett. 95 190404
[28] Abdullaev F K, Tsoy E N, Malomed B A and Kraenkel R A 2003 Phys. Rev. A 68 053606
[29] Abdullaev F K, Baizakov B B, Darmanyan S A, Konotop V V and Salerno M 2001 Phys. Rev. A 64 043606
[30] Brazhnyi V A and Konotop V V 2005 Phys. Rev. A 72 033615
[31] Smerzi A and Raghavan S 2000 Phys. Rev. A 61 063601
[32] Milburn G J, Corney J, Wright E M and Walls D F 1997 Phys. Rev. A 55 4318
[33] Alexander T J, Ostrovskaya E A and Kivshar Y S 2006 Phys. Rev. Lett. 96 040401
[34] Schumayer D and Apagyi B 2004 Phys. Rev. A 69 043620