CONSTRANTS ON THE ABUNDANCE OF HIGHLY IONIZED PROTOCLUSTER REGIONS FROM THE ABSENCE OF LARGE VOIDS IN THE $\text{Ly}_\alpha$ FOREST

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ABSTRACT

Energetic feedback processes during the formation of galaxy clusters may have heated and ionized the majority of the intergalactic gas in protocluster regions. When such a highly ionized superbubble falls along the sight line to a background quasar, it would be seen as a large void with little or no absorption in the $\text{Ly}_\alpha$ forest. We examine the spectra of 137 quasars in the Sloan Digital Sky Survey to search for such voids and find no clear evidence of their existence. The size distribution of voids in the range $5 \AA \leq D_L \leq 70 \AA$ (corresponding to physical sizes of $3 \ h^{-1} \ \text{Mpc}$) is consistent with the standard model for the $\text{Ly}_\alpha$ forest without additional ionized bubbles. We adapt a physical model for H ii bubble growth during cosmological reionization to describe the expected size distribution of ionized superbubbles at $z \sim 3$. This model incorporates the conjoining of bubbles around individual neighboring galaxies. Using the nondetection of voids, we find that models in which the volume filling factor of ionized bubbles exceeds $\sim 20\%$ at $z \sim 3$ can be ruled out, primarily because they overproduce the number of large ($40 \sim 50 \ \AA$) voids. We conclude that any preheating mechanism that explains galaxy cluster observations must avoid heating the low-density gas in the protocluster regions, either by operating relatively recently ($z \leq 3$) or by increasing entropy primarily in high-density regions.

Subject headings: intergalactic medium — large-scale structure of universe — methods: data analysis — quasars: absorption lines

1. INTRODUCTION

Observations of galaxy clusters suggest that feedback played an important role during their formation. The simplest models for galaxy clusters neglect feedback and assume the gravitational collapse of a dark matter halo accompanied by gas infall. These models fail to reproduce either the observed scaling relations between bulk characteristics or the structural properties of individual clusters (e.g., Bialek et al. 2001; Voit et al. 2002). For example, the self-similar gas distribution expected in this model predicts the relation between X-ray luminosity and temperature, $L_X \propto T^2$ (Kaiser 1986), whereas observations find a steeper relation, closer to $L \propto T^{3.2}$ (e.g., David et al. 1993; Arnaud & Evrard 1999; Helsdon & Ponman 2000). A compelling suggestion to explain the discrepancy is that the intracluster gas has been preheated, i.e., raised to a higher adiabat, at an early stage in the formation of the cluster. The resulting so-called entropy floor would then preferentially affect low-mass clusters. Indeed, the observed $L_X - T$ and related scaling relations are well reproduced in models that simply endow the gas with extra entropy, of order $\sim 100 \ \text{keV cm}^{-2}$, before its collapse (Voit & Bryan 2001; Bialek et al. 2001; Voit et al. 2002; McCarthy et al. 2003a, 2003b). The physical mechanism responsible for the preheating could be supernova-driven galactic winds or the radiation output of active galactic nuclei (AGNs).

The simplest form of this preheating model does not appear to provide an acceptable fit to the detailed cluster profiles (Pratt & Arnaud 2005; Pratt et al. 2006; Younger & Bryan 2007). Nevertheless, the broader idea, namely, that energetic processes strongly influenced at least parts of the intergalactic medium (IGM), corresponding to protocluster regions at early times, remains viable. Indeed, there is considerable empirical support for this broader picture. The global star formation rate, inferred from galaxies discovered at redshift $z \sim 3$, such as the Lyman break galaxies (LBGs), appears significantly higher than the star formation rate in the local universe. Energetic superwinds from LBGs at $z \sim 3$ have been inferred directly from their UV spectra, showing several hundred km s$^{-1}$ offsets between stellar and interstellar lines (Heckman et al. 2000; Pettini et al. 2001). Similar winds are known to accompany nearby starbursts (e.g., Heckman et al. 1990), and such winds would be natural candidates for large-scale feedback at earlier times.

Indeed, various studies have suggested that winds from galaxies can affect not only the galaxy itself, but also the surrounding IGM out to a distance approaching $\sim 1 \ (\text{comoving}) \ h^{-1} \ \text{Mpc}$, which may affect global $\text{Ly}_\alpha$ absorption statistics (e.g., Croft et al. 2002a; Kollmeier et al. 2003; Bruscoli et al. 2003; Desjacques et al. 2004; McDonald et al. 2005; Fang et al. 2005; Kollmeier et al. 2006). Recent works have focused on interpreting observations of $\text{Ly}_\alpha$ absorption statistics in quasar spectra with sight lines passing near LBGs. The observations possibly indicate a reduced level of absorption within $\sim 1 \ h^{-1} \ \text{Mpc}$ of LBGs (Adelberger et al. 2003, 2005), which may be attributable to the impact of these galaxies on the ambient IGM (but see, e.g., a recent study by Desjacques et al. 2006). In the context of the LBGs, several groups have used numerical simulations to study the metal enrichment of the IGM by galactic outflows and the corresponding impact on the global $\text{Ly}_\alpha$ absorption statistics (e.g., Theuns et al. 2002).

The present study is motivated by the related suggestion of Theuns et al. (2001), namely, that the preheating of large protocluster regions may leave a direct imprint on the global $\text{Ly}_\alpha$ forest absorption statistics. Irrespective of the physical mechanism, the
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preheating would likely ionize the hydrogen in the protocluster region, and the resulting ionized bubble would be optically thin in Lyman line absorption. Theuns et al. (2001) proposed that if the suggested entropy level does exist, the highly ionized protocluster regions could produce large voids, stretches of wavelength as long as ~20 Å with little or no absorption.

Such hot protocluster bubbles could be an order of magnitude larger (in linear size) than the ~1 h^{-1} Mpc ionized bubbles that may envelop individual LBGs. These large protocluster bubbles may, of course, correspond to a clustered group of bubbles around LBGs. On the other hand, they could be produced by the collective effect of a group of galaxies that are much smaller and/or formed earlier than the known population of LBGs. In this case, the large voids may be more readily identified by studying the global Lyα forest statistics.

In the standard model for the Lyα forest, the absorption lines are produced by fluctuations in the density field. Observed statistics, such as the column density distribution and evolution and the spatial distribution of the absorbers, are consistent with a model in which the gas traces the primordial dark matter fluctuations and is kept photoionized by a uniform metagalactic radiation (Zhang et al. 1995; Hemquist et al. 1996; Miralda-Escudé et al. 1996; Hui & Gnedin 1997). The expected autocorrelation functions of Lyα absorption lines along the line of sight have been computed in hydrodynamic simulations by, e.g., Cen et al. (1998; see their Figs. 7 and 8). They found little evidence for clustering for strong (signal-to-noise ratio [S/N] = 50) absorption lines at separations of Δv ≥ 300 km s^{-1}, corresponding to ~5 Å at z ~ 3; weaker lines are expected to cluster even less strongly. Most subsequent simulation work on the Lyα forest has treated the transmitted flux as a continuous one-dimensional function (rather than a set of discrete lines), but the results are generally consistent with little clustering on scales larger than ~300 km s^{-1} (see, e.g., Figs. 13 and 14 in McDonald et al. 2000). Empirical studies of a handful of high-resolution quasar spectra led to similar conclusions, namely, that the two-point correlation function of the absorbers at line-of-sight separations of ≥300 km s^{-1} is insignificant (see, e.g., Cristiani et al. 1997 and references therein).

Historically, several authors have searched for large voids in the Lyα forest (Atwood et al. 1985; Crotts 1987; Ostriker et al. 1988; Duncan et al. 1989; Dobrzycki & Bechtold 1991). These studies have identified only a handful of candidates for large voids that were discrepant with a Poisson distribution (and not associated with the proximity effect of the background quasar itself), but none of these have been confirmed at high statistical significance. These works justify our null hypothesis below, namely, that on scales corresponding to ~5 Å at z ~ 3, the Lyα lines are randomly distributed, and in the absence of nongravitational effects, they should obey a Poisson distribution.

In this paper, we perform a new search for large voids in the Lyα forest. Our analysis differs from existing studies in two important ways. First, we use a larger sample of quasar spectra, available from the Sloan Digital Sky Survey (SDSS). Second, while we adopt the same null hypothesis as previous works (i.e., a Poisson distribution for the absorbers), we use a new physical model for the bubble distribution that tracks the conjoining of bubbles around individual galaxies. These mergers between bubbles are important when their volume filling factor rises above a few percent: galaxies are clustered in space, and a single large void will typically contain many galaxies. As a result, mergers are a way to produce larger, possibly detectable voids.

The rest of this paper is organized as follows. In § 2, we explain how we model the mass function of highly ionized regions. In § 3, we briefly describe the observational data that we used. In § 4, we introduce our approach of statistically comparing the predicted and observed void distribution in the Lyα forest. In § 5, we present our main results. In § 6, we discuss the limitations of our approach, as well as possible future improvements. In § 7, we briefly summarize our conclusions and the implications of this work. Throughout this paper, we adopt a spatially flat universe dominated by a cosmological constant and cold dark matter (CDM), with the following set of cosmological parameters: Ω_m = 0.3, Ω_Λ = 0.7, σ_8 = 0.9, and H_0 = 70 km s^{-1} Mpc^{-1}. These values are consistent with measurements by the Wilkinson Microwave Anisotropy Probe (WMAP) experiment (Spergel et al. 2003, 2007; we include a discussion of the sensitivity of our results to the choice of σ_8 below).

2. MODELING THE DISTRIBUTION OF THE HIGHLY IONIZED REGIONS

Our main task is to model the abundance and size distribution of highly ionized protocluster regions. In the back-of-the-envelope-style estimate in Theuns et al. (2001), a protocluster that later develops into a cluster of mass M was treated as a uniform sphere containing the same amount of fully ionized gas at some fixed overdensity δ relative to the cosmic mean gas density at redshift z. Here z and δ are both free parameters, the relevant values of which would need to be estimated from some further modeling. Assuming that preheating operates at redshift z ~ 3 and that the mean overdensity in the protocluster region is δ ~ 1, they calculated the protocluster size distribution from the known mass function of galaxy clusters. They concluded that the typical size of a void at this redshift should be a few × 10 Å, which would appear prominent in high-redshift (z ≥ 3) quasar spectra. Based on the local abundance of massive clusters, they estimated that there should be approximately 1 such void per unit redshift.

An obvious refinement of this simple estimate is to make a connection between redshift and overdensity by using the spherical collapse model. Given the current overdensity of clusters and their collapse redshift, the cluster’s expected overdensity at some higher redshift can be obtained directly. This approach would eliminate one free parameter, but would still miss an important ingredient of cluster formation: mergers. In the hierarchical structure formation scenario, protocluster are more likely to be made up of many smaller clumps that would later merge together. Star formation and AGN activity in these clumps (which could represent an individual galaxy or a small group of highly clustered galaxies) could then ionize gas in their vicinity. Hereafter we refer to the area ionized around a single progenitor clump as an “ionized bubble.” For concreteness, we implicitly assume that the bubble is photoionized by star formation or quasar activity in the host galaxy. However, we emphasize that our modeling does not require us to specify the physical mechanism of ionization, and our results below will be applicable to other scenarios. For example, the gas may becollisionally ionized due to shock heating by outflows, in which case our modeling would still be applicable, with a suitable reinterpretation of the efficiency parameter ζ defined below. (In a future detection of a Lyα void, a measurement of the corresponding He i/He ii absorption should help constrain the physical mechanism creating the ionization; see more discussion in § 6.) Several ionized bubbles, initially generated independently in different collapsed regions, may overlap with each other and form a larger superbubble. In order to compute the distribution of voids in the Lyα forest, we first need to get the mass function of these superbubbles.

Furlanetto et al. (2004) have studied an analogous problem for the mergers of ionized bubbles at higher redshifts in the context of cosmological reionization. Here we adopt their formalism and
apply it to the superbubbles at lower redshift. In the context of reionization, the formalism has been compared to numerical simulations of the growth of ionized bubbles and was found to accurately reproduce the size distribution and large-scale clustering properties of ionized bubbles (Zahn et al. 2007). We caution, however, that a similar test against simulations has not yet been performed at lower redshifts (see discussion below). The formalism is based on the simple assumption that a collapsed dark matter (DM) halo can ionize a region whose mass is proportional to the halo’s own mass. The effective proportionality coefficient, denoted as $\zeta$, 

$$m_{\text{ion}} = \zeta m_{\text{col}},$$

depends, in the context of reionization, on the efficiency of ionizing photon production, the escape fraction of these photons from the host galaxy, the star formation efficiency, and the mean number of recombinations. In our case, we define an analogous coefficient between the mass of a bubble and the mass of a halo,

$$m_{\text{bubble}} = \zeta m_{\text{col}}.$$ 

The value of this coefficient should depend on the velocity, temperature, and typical age of galactic winds, or, alternatively, on similar parameters for the typical AGN outflows. However, this simplified description could plausibly describe other scenarios as well (e.g., a simple photoionization proximity effect).

The condition $\zeta > 1$ has to be satisfied in order for the winds to propagate outside the DM halos and generate ionized bubbles in the IGM. In this case, there is a chance that different bubbles can overlap and unite into a larger superbubble. The statistics of the superbubble size distribution is, in general, then driven by this overlap, which, in turn, is governed by the large-scale density fluctuations. In order to avoid modeling the complex process of overlap, Furlanetto et al. (2004) propose utilizing the following relation, which must be satisfied for every superbubble:

$$f_{\text{coll}} \geq \zeta^{-1},$$

where $f_{\text{coll}}$ is the collapsed fraction (the ratio of mass residing in collapsed halos to the total mass inside the superbubble) and is determined using the extended Press-Schechter model. In this approach, $f_{\text{coll}}$ depends on the mean linear overdensity $\delta_m$ inside the superbubble. The excursion set formalism can be used to find the largest region surrounding an arbitrary point in space, where the relation above is satisfied. The final result for the mass function of superbubbles is

$$\frac{dn}{dm} = \frac{\sqrt{2}}{\pi} \frac{\rho}{m} \frac{d}{dm} \left[ B_0 \frac{B^2(m, z)}{2\sigma^2(m)} \exp \left[ -B^2(m, z) \right] \right],$$

where $\sigma^2(m)$ is the variance of density fluctuations on the scale of mass $m$ and $B$ is the critical overdensity. If the mean density within a region of mass $m$ is higher than $B$, then it is ionized; $B_0$ is the limiting value of $B$ as $m \rightarrow \infty$. The expression is analogous to the Press-Schechter mass function, except that the value of the critical overdensity is different and mass dependent. This formalism requires us to specify the parameter $M_{\text{min}}$, the mass of the smallest collapsed halo that can produce winds or an ionized bubble. Our fiducial value of $M_{\text{min}}$ throughout this paper is set to $10^{11} M_\odot$, but we also consider a range of values $M_{\text{min}} = 10^9, 10^{10}, \text{or} 10^{12} M_\odot$. The choice for the lowest value is motivated by the expectation that the cooling and collapse of gas, and therefore star formation in smaller halos, is prevented by the UV background (Efstathiou 1992; Thoul & Weinberg 1996; Dijkstra et al. 2004), whereas the highest value corresponds roughly to the largest masses considered for LBGs at $z \sim 3$ (Somerville et al. 2001).

We assume that the temperature in these ionized bubbles is sufficiently high ($\geq 5 \times 10^4$ K) for hydrogen to be essentially completely ionized and that these regions therefore produce negligible Ly$\alpha$ absorption in the spectra of background quasars. The signature of such an ionized bubble intersecting a quasar sight line would therefore be a void in the Ly$\alpha$ forest. We are now in the position to compute the size distribution of these voids; the results are explicitly calculated and shown in § 4 below.

### 3. OBSERVATIONAL DATA

In this section, we briefly describe the data we used for our analysis. The spectra were selected from the SDSS Data Release 4 (DR4; Adelman-McCarthy et al. 2006). We examined 137 quasar spectra with redshifts in the range $3.5 < z < 4$. We selected these high-quality spectra by hand from 798 among the brightest quasars in DR4 in this redshift range. Spectra were selected so that the S/N is greater than 8, and the spectral resolution of SDSS data is $R = 1800$, which implies that the FWHM resolution in the wavelength range that we are interested in is about 3 Å. The data are oversampled into $\approx 1$ Å pixels (equivalent to 60 km s$^{-1}$ at $z \approx 3$). There are at least 4 pixels in the voids that we study. From every spectrum, we used the Ly$\alpha$ region only from 1025(1 + $z$) to 1215(1 + $z$) Å, discarding shorter wavelengths subject to additional Ly$\beta$ absorption. In order to avoid having to model the proximity effect, we also excised the wavelength range corresponding to radial separations of $\leq 10$ Mpc from the source quasar. The total wavelength range we analyzed is 112203 Å, which is equivalent to an effective redshift range of $\Delta z = 92.3$. The median redshift of the wavelength range we utilized is $\bar{z} = 3.3$. For illustration, in Figure 1 we show the spectrum of a typical source.

The raw data include the flux $f_i$ and the noise $n_i$ for each $\sim 1$ Å wide wavelength bin centered at $\lambda_i$. We first fit the continuum for every spectrum, using the same clipped-variance estimator continuum technique employed for the SDSS absorption-line catalog (e.g., York et al. 2005). We then search the Ly$\alpha$ part of the spectrum for voids larger than 5 Å. We neglected smaller voids because of the limitation from spectrum resolution, and in order to avoid small-scale correlations between pixels induced by large-scale structures (the size of the smallest void we consider, $\sim 3 h^{-1}$ Mpc, exceeds the correlation length of the absorption lines by a factor
of \( \approx 10 \); e.g., Cristiani et al. 1997). Here we define a void to be a contiguous range of neighboring pixels where the flux-to-continuum ratio exceeds a certain threshold. The threshold could, in principle, be very high (\( \geq 99\% \)) from the simple theoretical speculation above. However, noise in the data limits our choice for the threshold to be smaller than or equal to 80\% (see a more detailed discussion in § 4 below). This, means that, in effect, we allow ionized bubbles to contain some residual neutral hydrogen causing \( \sim 20\% \) absorption. We create a size distribution of the voids, i.e., a histogram using discrete wavelength bins of width in the range 5 \( \AA \leq \Delta \lambda \leq 70 \AA \) in increments of 5 \( \AA \). We generated mock histograms using different fitting functions, derived from models with or without ionized bubbles (see § 4). The goodness of fit and the likelihood of each model is obtained from the usual \( \chi^2 \) statistic,

\[
\chi^2 = \sum_i \frac{(N_i - n_i)^2}{n_i},
\]

where \( N_i \) and \( n_i \) are the number of observed versus predicted voids in bin \( i \), respectively.

4. STATISTICAL ANALYSIS

In § 2, we explained how we calculated the bubble size distribution. In this section, we convert this bubble size distribution into the Ly\( \alpha \) void distribution that could be compared to the observational data. In general, voids in the observed absorption spectrum could be produced in two ways. First, in the usual picture for the undisturbed Ly\( \alpha \) forest, due to the density fluctuations, the IGM will contain low-density regions that produce little absorption. Second, the presence of ionized bubbles can produce additional voids, as we explained above. For clarity, we refer to these two kinds of voids as “density voids” and “ionization voids,” respectively. In the first half of this section, we discuss the void size distribution, including the nontrivial overlap between individual voids, ignoring, for simplicity, the presence of noise. In the second half of this section, we discuss the impact of noise on our predicted void size distributions.

4.1. The Expected Noise-Free Void Size Distribution

The total number of voids is not simply the sum of density voids and ionization voids, since these voids can mix in a nontrivial way. For example, an ionized bubble may be expanding into a low-density region in the IGM, producing an ionization void that connects with an adjacent density void, forming a single, larger apparent void. Bearing this in mind, we first calculate the size distribution of ionization voids. We neglect peculiar velocities in our analysis. Typical peculiar velocities at \( z \approx 3 \) on the relevant large scales are \( \leq 100 \text{ km s}^{-1} \) (e.g., Gnedin & Hamilton 2002), which would correspond to \( \sim 1.6 \AA \) shifts in the apparent spectrum. This is a small fraction of the smallest void size we consider. Furthermore, peculiar velocities are proportional to the overdensity and will be smaller for spectral pixels of interest that have lower than usual absorption. In this case, the size of an ionization void is determined solely by the Hubble flow, which in turn scales directly with the size of the ionized bubbles. For simplicity, and consistent with the model assumptions in § 3, we further assume that the bubbles are spherical. For a given mass \( m \), the volume of a bubble is then

\[
V = \frac{m}{\rho(1 + \delta_m)},
\]

where \( \rho \) is the mean density of the universe and \( \delta_m \) is the overdensity within the bubble. By construction, in the ionized bubbles \( \delta_m \) is equal to \( B \) in equation (4). Let as assume that the line of sight (LOS) intersects a bubble of radius \( R = R(m) \) at an impact parameter \( 0 \leq b \leq R \) (defined as the distance of closest approach between the center of the ionized bubble and the LOS). The length of LOS within the ionized bubble is then \( l = 2(R^2 - b^2)^{1/2} \), and the Hubble velocity across this region is

\[
v_H = H(z)l = 2H(z)\sqrt{R^2 - b^2}.
\]

The ionized region produces a void covering the range of observed wavelengths

\[
\Delta \lambda = \lambda_{\alpha}(1 + z)^\frac{v_H}{c} = K\sqrt{R^2 - b^2},
\]

where \( \lambda_{\alpha} = 1215 \AA \) is the rest-frame Ly\( \alpha \) wavelength, \( c \) is the speed of light, and in the last step we have defined \( K = 2\lambda_{\alpha}(1 + z)c^{-1}H(z) \). The number density of voids of size \( \Delta \lambda \) along a given LOS per unit redshift and unit size is obtained directly from the mass function through the equation

\[
\frac{d^2N}{d\Delta \lambda dz}(z, \Delta \lambda) = \int_{m_{\text{min}}}^{\infty} d\rho \int_0^R 2\pi bdb(\Delta \lambda - K\sqrt{R^2 - b^2}) \times \frac{dn}{d\rho} \frac{c(1 + z)^2}{H(z)}.
\]

In this equation, \( dn/d\rho \) is the mass function of ionized bubbles, \( m_{\text{min}} = m_{\text{min}}(\Delta \lambda) \) is the smallest ionized bubble that can produce a void of length \( \Delta \lambda \) (at \( b = 0 \)), \( d^2V/d\rho d\Omega = cH^{-1}(z)(1 + z)^2d\Omega(z) \) is the comoving volume per unit redshift and solid angle, and \( 2\pi bdb \frac{d^2z}{dz^2} \) is the solid angle extended by a narrow circular annulus at impact parameter \( b \) and width \( db \). Note that the angular diameter distance, \( d_A(z) \), drops out of the equation. The Dirac delta function in the top row enforces the relation between bubble radius \( R \) and impact parameter \( b \) to produce a void of fixed length \( \Delta \lambda \). Performing the integral over the size distribution of ionized bubbles, \( m \) and \( \rho \), we find

\[
\frac{d^2N}{d\Delta \lambda dz}(z, \Delta \lambda) = \frac{\pi \rho^3}{2\lambda_{\alpha}^2 H^2(z)} \int_{m_{\text{min}}}^{\infty} \frac{dm}{d\rho} \frac{c(1 + z)^2}{H(z)}.
\]

To calculate the overall distribution of voids, we need to take into account both density voids and ionization voids and the list of possible overlaps between them. If there were only density voids, their distribution would obey a simple exponential. This follows directly from the Poisson distribution of absorption lines (e.g., Crotts 1987; Ostriker et al. 1988) and assumes that the spatial correlations between different absorption lines on the large scales of interest (\( \geq 10 \) times the correlation length of absorption lines; e.g., Cristiani et al. 1997) are negligible. The number of pure density voids of size \( \Delta \lambda \) per unit observed total wavelength range \( \Delta \lambda_{\text{tot}} \) and per unit void size \( \Delta \lambda \) is then given by

\[
\frac{d^2N_0}{d\Delta \lambda d\Delta \lambda_{\text{tot}}} = A \exp(-b\Delta \lambda),
\]

where \( b \) is related to the mean number of absorption lines per unit wavelength above a given threshold strength, weighted appropriately over redshift (see below), and \( A \) is a normalization factor that also depends on the number density of absorption lines and in general on \( \Delta \lambda_{\text{tot}} \). Note that \( A \) has units of \( \text{wavelength}^{-2} \). The simplest case is that of a single redshift, negligible noise, and absorption lines that do not blend together (we discuss the issue of noise further below). In this case, one can compute the total
number of expected voids and uniquely compute the normalization $A$ as a function of $b$ and $\Delta \lambda_{\text{tot}}; A = A(b, \Delta \lambda_{\text{tot}})$. Taking the limiting case of $\Delta \lambda_{\text{tot}} \to \infty$ and the width of individual absorption lines approaching zero, we find $A \to b^2$. This unique correspondence, however, is spoiled when the assumptions above are relaxed. In order to avoid addressing these issues or having to model the mean transmission of the forest and its evolution with redshift self-consistently, we treat $A$ as an independent, free parameter in our fitting procedure. It is worth noting, however, that further modeling could significantly tighten the constraints we derive below on the abundance of ionized bubbles. In practice, we find deviations of up to a factor of $\sim 2$ from the limiting formula $A = b^2$ above, implying that the simplifying assumptions above do not introduce gross errors.

The presence of ionization voids will disrupt the exponential distribution of the density voids. Let us first consider an observed void of size $\Delta \lambda$ that contains exactly one ionization void of some size $s \leq \Delta \lambda$, overlapping with zero, one, or two neighboring density voids whose sizes add up to $\Delta \lambda - s$. The number of such voids per unit observed total wavelength range $\Delta \lambda_{\text{tot}}$ and per unit void size $\Delta \lambda$ is given at some redshift $z$ by

$$\frac{d^2 N_1}{d \Delta \lambda d \Delta \lambda_{\text{tot}}} = \frac{A}{\lambda_\alpha} \int_0^{\Delta \lambda_{\text{tot}}} ds \int_s^{\Delta \lambda_{\text{tot}}} d\lambda \frac{d^2 N}{d \Delta \lambda d \lambda (s)} \times \exp \left[ -b(\Delta \lambda - s)/(\Delta \lambda - s) \right].$$

(12)

This equation follows from noting that (1) the center of the ionization void of size $s$ can be placed anywhere over an interval of length $\Delta \lambda - s$ and (2) none of the pixels in the remaining length $\Delta \lambda - s$ that are not covered by the ionization void should have an absorption line. Note that in the limit of $b \to \infty$ and $A \to b^2$, density voids will be rare, and equation (12) indeed reduces to the abundance of ionization voids (eq. [10]), as it should.

Similarly, the observed void of size $\Delta \lambda$ could contain two ionization voids, of sizes $s, u \leq \Delta \lambda$, connecting with neighboring density voids. The number of these cases is given by an argument analogous to the previous case, except that we need to enforce the condition that the two ionization bubbles are, by definition, disjoint. This can be achieved by the following procedure: (1) choose a size $0 < s < \Delta \lambda$ for the first ionization void, (2) then choose a location for this void, measured by the distance $s \leq t \leq \Delta \lambda$ of the right “edge” of the $s$ void from the left “edge” of the larger $\Delta \lambda$ void, and (3) finally place a second ionization void, of size $0 < u < \Delta \lambda - t$ anywhere in the remaining interval $\Delta \lambda - u - t$. We find, accordingly,

$$\frac{d^3 N_2}{d \Delta \lambda d \Delta \lambda_{\text{tot}}} = A \int_0^{\Delta \lambda_{\text{tot}}} ds \int_s^{\Delta \lambda_{\text{tot}}} d\lambda \int_0^{\Delta \lambda_{\text{tot}}} dt \int_0^{\Delta \lambda_{\text{tot}} - t} du \frac{d^2 N (s)}{d \Delta \lambda d \lambda (s)} \frac{d^2 N (u)}{d \Delta \lambda d \lambda (u)} \times \exp \left[ -b(\Delta \lambda - s - u)/(\Delta \lambda - u - t) \right].$$

(13)

The total number of voids of size $\Delta \lambda$ is then given by

$$\frac{d^2 N}{d \Delta \lambda d \Delta \lambda_{\text{tot}}} = \frac{d^2 N_0}{d \Delta \lambda d \Delta \lambda_{\text{tot}}} + \frac{d^2 N_1}{d \Delta \lambda d \Delta \lambda_{\text{tot}}} + \frac{d^2 N_2}{d \Delta \lambda d \Delta \lambda_{\text{tot}}} + \ldots$$

$$n = n_0 + n_1 + n_2 + \ldots,$$

(14)

where we introduced $n = d^2 N/d\Delta \lambda d\Delta \lambda_{\text{tot}}$ to simplify notation. In the rest of this paper, we omit terms in the above formula above second order (the two-ionization void case); we discuss the effect of higher order terms in § 6 below.

As a first check, to see whether ionized bubbles have a conspicuous effect on the Ly$_\alpha$ spectrum, we fit the observed histogram for $n(\Delta \lambda)$ using the exponential function $n_0$ alone. The exponential fit turns out to be adequate, immediately revealing that we cannot rule out the null hypothesis that the data contain no ionized bubbles. Then we used our model formula in equation (14) to fit the data. For a given $\zeta$, we adjusted our free parameters $A$ and $b$ to minimize $\chi^2$ and obtain the maximum probability. We start with $\zeta = 1$ and increase $\zeta$, gradually in increments of 0.01, until the fit breaks down, i.e., until the maximum probability is smaller than a threshold value. We chose the fiducial value of $10^{-3}$ for this threshold probability in our analysis (see discussion below).

The procedure outlined above yields an upper limit of $\zeta$ that corresponds to the threshold probability. Equivalently, we can convert $\zeta$ to a corresponding upper limit on the global volume filling factor $Q$ of the ionized bubbles, using the equation

$$Q = \int_{\zeta M_{\text{tot}}}^{\infty} \frac{dn}{dm} \frac{m}{[1 + B(m, z)]} dm.$$ 

(15)

4.2. The Impact of Spectral Noise

Before describing our results, we discuss the choice of our absorption threshold for defining a void in the data and the impact of noise in the spectra. The average S/N in the data we utilize is $S/N = 9.94$, which, essentially, forced us to select a low threshold in our definition of a void. Note that the relatively lenient observational threshold means that some residual absorption is allowed to take place in the ionized bubbles. In our simple treatment, Ly$_\alpha$ forest absorption lines are randomly distributed, yielding the exponential distribution of the density voids. However, spectral noise tends to produce some additional, fake absorption lines, which will break up true voids. Fortunately, noise at different pixels can be treated as uncorrelated over scales of more than a few pixels. If S/N were a constant over the whole wavelength range, the exponential void distribution would remain valid—the effect of the noise could be absorbed in the uninteresting (for us) constants $A$ and $b$.

Unfortunately, in practice, the S/N is usually larger in pixels where the flux is larger, which, in general, would necessitate further modeling. However, this complication can be avoided by choosing a sufficiently low threshold in defining a void, such that the number of fake absorption lines in the wavelength range we analyze is small. For a rough illustration, let us assume that the noise is Gaussian, with 1 $\sigma$ values corresponding to $\approx 10\%$ of the unabsorbed flux. Since we use $\approx 10^5$ independent pixels in our analysis, we expect roughly 2250, 600, 135, 25, and 3 pixels with absorption lines extending below thresholds of 80, 75, 70, 65, and 60 percent of the continuum, respectively. As we find below, there are 3632, 4064, 4345, 4573, and 4667 voids in these cases, occupying a total number of 37545, 44900, 51558, 58303, and 64833 pixels, respectively. Therefore, the fractional increase in the number of voids in these cases, assuming each fake absorption line results in one extra void, is $(2250/112203)(37545/3632) = 0.21$, etc. Clearly, this fractional increase is small for thresholds below 70%.

Next, let us consider including the effect of noise explicitly in our model-fitting function (14). Rather than modeling the noise in the data, we added noise to our theoretical void distribution, before comparing it with the data. The effect of uncorrelated constant S/N noise on the density void distribution $n_0$ is automatically absorbed into the free parameters $A$ and $b$ (i.e., the noise changes only the values of $A$ and $b$, which are free parameters in
gives the probability that a given subvoid’s length is exactly \( y \). Finally, we multiply this last factor by the number of subvoids, \( i + 1 \), and sum over all the possible \( i \).

As an illustration of the impact of noise, in Figure 2 we consider a 40 pixel void, which is allowed to be subdivided by noise. The four different panels correspond to different choices for the threshold to define a void. In the absence of noise, we would have a single 40 pixel void in each panel. The effect of noise is to produce a new distribution of smaller voids, which can be regarded as an asymmetric kernel, by which the actual noise-free distribution \( \rho \) should be convolved. As the figure shows, when the threshold is chosen to be lower than 2.5 \( \sigma \) (or at \( \sim 75\% \) of the continuum in our case), the noise has little effect on the void size distribution. Finally, we note a complication that arises during void mixing. When noise is ignored, any ionization void can connect with density voids on both sides. But if an ionization void divided, the subvoids can connect on only one side (if the subvoid is on the edge of the original void) or neither side (if the subvoid is flanked on both sides by noise spikes). In our numerical calculation of the noisy void size distribution, we kept track of these different types of voids and treated them accordingly. This entails modifying equations (12) and (13), which describe only the case in which ionization voids connect on both sides with density voids; in the interest of brevity, we do not list these modified equations here.

5. RESULTS

We first list, in Table 1, the results of the exponential fit. As we can see, the \( \chi^2 \) likelihoods in this table, for all five choices of the threshold, are \( \gtrsim 20\% \). This means that the exponential fits are acceptable, and there is no statistical evidence for ionized bubbles, or for any voids beyond those found in the usual fluctuating Gunn-Peterson absorption picture for the \( \text{Ly} \alpha \) forest.

Next, we constrain the abundance of ionized bubbles using our fitting formula in equation (14), modified to include the effects of constant Gaussian noise as discussed \( \S \, 4.2 \). Table 2 gives the maximum values of \( \zeta \) and the corresponding maximum allowed ionized bubble volume filling factors \( Q \), with \( \chi^2 \) likelihoods at 10^{-3}.

### Table 1

| Threshold (%) | \( A \)  | \( b \)  | \( \chi^2 \) | Likelihood |
|--------------|--------|--------|-------------|------------|
| 80           | 0.0192 | 0.198  | 1.093       | 0.198      |
| 75           | 0.0164 | 0.173  | 0.437       | 0.823      |
| 70           | 0.0136 | 0.150  | 0.529       | 0.757      |
| 65           | 0.0114 | 0.132  | 0.812       | 0.481      |
| 60           | 0.0092 | 0.114  | 0.762       | 0.539      |

**Notes:** Models of the \( \text{Ly} \alpha \) forest have no ionized bubbles. Fits are made to the observed histogram of void sizes. Five different thresholds are considered for defining voids.
6. DISCUSSION

It is interesting to ask whether the constraints we obtained above, using 137 quasars, could become significantly tighter by simply increasing the number of the spectra. The SDSS database (up to DR4) contains approximately 30,000 quasar spectra at redshifts $z > 2.3$. At a fixed value of $A$ and $b$, $\chi^2$ will increase roughly in proportion to the number of quasar spectra. To quantify the effect of this increase on the upper limit on $Q$, we generated mock void size histograms implementing realizations of the exact exponential void distribution with $A$ and $b$ chosen to be the best-fit values from Table 1. As a test of the method, we first generated histograms corresponding to 137 quasar spectra. When fitting our model to these mock data, we found an upper limit $Q < 16.9\%$ at the threshold of 70%, in agreement with the results in Table 2 using the actual spectra. Next, we generated mock histograms for a hypothetical 13,700 quasar spectra. We found that this 100-fold increase in the number of quasars improved the upper limit on the volume filling factor to $Q < 6.6\%$.

Another way to improve the sensitivity of the constraints would be to use spectra with higher signal-to-noise ratio and/or with higher resolution. To illustrate the impact of noise, we assumed noise is negligible (a more precise requirement, given that we use $\sim10^5$ pixels in our analysis, is that the noise is smaller than...
one-fourth of the decrement between the chosen threshold and the continuum) and repeated our analysis for the 80% threshold case. We found that the upper limits on \(\zeta\) and \(Q\) improve from (5.70, 0.226) to (4.67, 0.155), respectively. We expect the limits to tighten further if we raise our threshold, as would be possible if the spectral noise was indeed very small. We expect the main improvement allowed by higher resolution spectra to be oversimplifications. First, we treated \(\zeta\) as a constant, while it is possible that it could be strongly dependent on the mass and environment of the collapsed halo and could also evolve with redshift. For example, the star formation rate may scale roughly as \(z^{-2/3(1+z)}\), making it more difficult for winds to escape from larger halos. Furthermore, in its original context of reionization, the bubble-merger model we adopted was motivated by the reasonable assumption that the merger of two photoionized bubbles conserves the total ionized mass. If the ionized bubbles are produced by overlapping galactic winds (rather than photoionization), then this assumption is likely to be much less accurate. When two winds overlap, they will interact dynamically, and the winds will not instantaneously propagate to the edge of the joint bubble, to conserve mass. As a result, it is likely that the effective value of \(\zeta\) will further decrease as the overlap between winds becomes more significant. It would be possible to incorporate an \(M\) - and \(z\)-dependence, \(\zeta = \zeta(M, z)\), in our modeling, as well as a further decrease in \(\zeta\) that depends on the number of galaxies per bubble, but we leave such improvements to future work. We also note that if a void is identified in future data, then the simultaneous measurement of the corresponding He \(\text{ii}\) absorption should help constrain the physical mechanism creating the ionization. In particular, one does not expect helium to be overionized if the hydrogen Ly\(\alpha\) void is created by excess photoionization by soft stellar sources, whereas it is expected to be overionized if the void is created by strong shocks or photoionization by harder sources, such as AGNs.

Our modeling also assumes spherical hollow bubbles and ignores their inner structures. It is possible for bubbles to be quite nonspherical; this would be similar to smoothing the size distribution with an appropriate scatter, representing the dispersion in radial extent when the line of sight crosses a bubble in different directions. The impact of such a scatter would be to increase the number of large voids. Since the upper limit we found is driven by the predicted number of such large voids (Fig. 3), these upper limits should be improved if the scatter due to nonsphericity was included. On the other hand, it is also possible that there is residual neutral hydrogen within the ionized bubbles, so the voids are not completely empty, either due to incomplete mixing between hot wind material and the ambient IGM or due to radiative transfer effects (if bubble heating is due to photoionization). Peculiar velocities could exacerbate this problem by making the absorption by neutral hydrogen appear more concentrated in frequency space than in real space, possibly producing apparent absorption spikes that go below the defined void threshold (for example, Kollmeier et al. [2003] find in their simulations that a small amount of neutral gas near the turnaround radius can cause nearly total absorption at the frequency of the galaxy). Taking precise account of this effect requires more sophisticated modeling, which we defer to future work. Some numerical simulations also indicate that the galactic winds tend to expand preferentially toward lower density regions and leave the relatively more overdense filaments, which produce the deeper Ly\(\alpha\) absorption lines, intact (e.g., Theuns et al. 2002; Bruscoli et al. 2003; McDonald et al. 2005; Kollmeier et al. 2006). In particular, Theuns et al. (2002) explicitly show that in their models for galactic winds, the winds produce no discernible effect on the column density distribution of absorption lines, even down to column densities below \(10^{12}\) cm\(^{-2}\). In this case, the winds would produce very few, if any, new voids in Ly\(\alpha\) spectra, even at stringent thresholds, although they may still cause a change in the effective void threshold. This conclusion, however, may not be generic—it must depend on the spatial distribution of the sources and the nature and geometry of the winds, as well as on the filling factor of winds (e.g., we expect winds to ultimately penetrate the denser regions, if their filling factor is high).

Our analysis also neglects the correlations both among Ly\(\alpha\) lines and also between Ly\(\alpha\) lines and ionized bubbles. Both deep Ly\(\alpha\) absorption lines and ionized bubbles are inclined to appear at overdensity regions, so there should be a positive correlation between these two, which should be taken into account in future modeling. Finally, in our fitting procedure, we omit terms above second order in equation (14). Intuitively, one expects that higher order terms tend to produce even larger voids, and, as Figure 3 shows, it is these large predicted voids that yield our constraints. This expectation is borne out in Table 3, where we list the upper limits on \(\zeta\) when only the first-order term is retained (i.e., we use \(n_0 + n_1\) in eq. [14]). The table shows that omitting the second-order terms weakens the constraints.

Finally, we note that the three-year data from WMAP favors a lower value for the power spectrum normalization than the fiducial \(\sigma_8 = 0.9\) we adopted. We have explicitly verified, however, that this choice has no significant effect on our conclusions. In particular, we repeated the calculations from Table 2, with all parameters left unchanged, except replacing \(\sigma_8 = 0.9\) by \(\sigma_8 = 0.75\). We found that this changes the upper limits on the filling factor \(Q\) by less than 3%, although the corresponding values of the efficiency \(\zeta\) are increased by a factor of \(\sim 2\). This latter change is to be expected—the reduction in the underlying dark matter halo abundance implies that producing the same filling factor requires a higher efficiency.

Alternatively, the power spectrum of the Ly\(\alpha\) forest could provide a way to detect the effect of winds. Weinberg et al. (2003) examined a few wind models, two of which have detectable effects in the flux power spectrum, given the uncertainty in the observed spectrum (Croft et al. 2002b). Although their models are not exactly the same as ours, a rough comparison can still be made. In one of the models with detectable winds, they eliminated all neutral hydrogen within a radius of \(1.5\ h^{-1}\) Mpc around 40 high star formation rate galaxies. Dividing the total bubble volume by the volume of the simulation box, the volume filling factor is 5.2%. Since these bright simulated galaxies reside in dark matter halos with masses of \(\sim 10^{12}\ M_\odot\), the fairest comparison is with the rightmost columns in our Table 3, showing constraints on bubbles
generated by halos above this mass threshold. This comparison reveals that the power spectrum constraint is about a factor of ~2 more stringent than ours. Our semianalytical modeling, however, allows us to include lower mass halos and the mergers of bubbles around these halos to derive additional constraints on bubbles around these smaller halos. Weinberg et al. include smaller halos in an alternative wind model, in which 641 galaxies are surrounded by bubbles whose volume (rather than the bubble mass) is proportional to the mass of the galaxy and which they find produce a detectable effect in the power spectrum. We were not able to compare our results with this alternative model, because we lack the details of this model.\textsuperscript{1}

7. CONCLUSIONS

Motivated by the empirical evidence for significant preheating of at least parts of the IGM at $z \approx 3$, we made a simple model for the spatial distribution of preheated regions. The model assumes spherical ionized bubbles around collapsed dark matter halos and allows these spheres to merge into larger superbubbles. We predicted the number of voids that such ionized bubbles would produce in Ly$\alpha$ absorption spectra of background quasars.

Our comparison with the observed spectra of 137 quasars did not uncover evidence for ionized bubbles at $z \approx 3$. Instead, we found an upper limit on the volume filling factor of ionized bubbles, ranging from 11% to 25%, depending on the assumed size of the smallest halo that produces ionized bubbles. This is comparable to the fraction of the total mass in the present-day universe in low-mass clusters and groups, suggesting that the preheating at $z \approx 3$ may not have affected all the gas currently residing in these objects.

These constraints are complementary to studies of the impact of galactic winds on Ly$\alpha$ absorption spectra in the vicinity of known galaxies (LBGs). The latter approach is a more sensitive probe of the effects of the LBGs themselves, whereas searching the global statistics is sensitive to feedback from undetected galaxies whose spatial distribution is not strongly correlated with LBGs.

While the constraints we obtain are still relatively weak, they suggest that preheating, if it occurred, avoided heating the low-density gas in the protocluster regions, either by operating relatively recently ($z \approx 3$) or by increasing entropy preferentially in high-density regions. We expect that our constraints could be improved significantly by analyzing a larger number of quasar spectra and by improving on the simple model presented here.

\textsuperscript{1} We should also note that different data sets were used. The observed power spectrum that Weinberg et al. were comparing with were extracted from fewer spectra, yet with higher resolution and signal-to-noise ratio.

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**TABLE 3**

| Threshold (%) | $Q_{min}$ | $Q_{max}$ | $Q_{min}$ | $Q_{max}$ | $Q_{min}$ | $Q_{max}$ |
|---------------|-----------|-----------|-----------|-----------|-----------|-----------|
| 80............ | 2.39      | 0.314     | 3.28      | 0.278     | 5.70      | 0.226     |
| 75............ | 2.27      | 0.271     | 3.05      | 0.234     | 5.14      | 0.185     |
| 70............ | 2.20      | 0.249     | 2.94      | 0.215     | 4.89      | 0.169     |
| 65............ | 2.21      | 0.252     | 2.95      | 0.217     | 4.94      | 0.172     |
| 60............ | 2.38      | 0.310     | 3.27      | 0.276     | 5.77      | 0.231     | 18.06 | 0.170 |

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