Joint halo mass function for modified gravity and massive neutrinos I: simulations and cosmological forecasts

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ABSTRACT

We present a halo mass function accurate over the full relevant Hu-Sawicki f(R) parameter space based on spherical collapse calculations and calibrated to a suite of modified gravity N-body simulations that include massive neutrinos. We investigate the ability of current and forthcoming galaxy cluster observations to detect deviations from general relativity while constraining the total neutrino mass and including systematic uncertainties. Our results indicate that the degeneracy between massive neutrino and modify gravity effects is a limiting factor for the current searches for new gravitational physics with clusters of galaxies, but future surveys will be able to break the degeneracy.

Key words: clusters of galaxies – large-scale structure of Universe – modified gravity – neutrinos

1 INTRODUCTION

One of the goals in modern cosmology is to understand the underlying dynamics and statistics of the cosmic density field. Clusters of galaxies trace the highest of its peaks, and theory predicts their abundance to depend exponentially on the amplitude of the matter power spectrum (Press & Schechter 1974; Bond et al. 1991; Sheth & Tormen 2002) which turns them into a formidable probe of cosmological parameters (Allen et al. 2011; Kravtsov & Borgani 2012).

Studying the cosmic density field is especially of interest because it might reveal the mechanism for the observed accelerated expansion of the Universe. It can either be explained by introducing a smooth dark energy component to the universe’s energy budget, or by modifying gravity itself. Both scenarios can potentially be tested via their imprint on the abundance of clusters (Battye & Weller 2003; Mohr et al. 2003), but in this paper we will focus on the latter.

Because general relativity (GR) is the unique theory of gravity in 1 + 3 dimensions under very general assumptions (Lovelock 1972), any modifications introduce new physical degrees of freedom. While these modifications can give rise to accelerated expansion, they also tend to enhance gravity at the perturbative level. One example discussed in this paper are the f(R) scalar-tensor theories, which generalise the Einstein-Hilbert action by adding a non-linear function of the Ricci scalar $R$.

The enhancement of gravity tends to result in an increased abundance of clusters, and several approaches to model the halo mass function in modified gravity exist (Kopp et al. 2013; Cataneo et al. 2016; von Braun-Bates et al. 2017). But all of these studies were performed within a one-parameter extension of the minimal ΛCDM standard model, and a natural extension is the inclusion of massive neutrinos which form a small, but unknown fraction of cosmological dark matter. The detection of a non-zero neutrino mass is firmly established by particle physics as a consequence of neutrino flavour oscillations (Araki et al. 2005) and in cosmology the neutrino background can be measured in both the cosmic microwave background (Sellentin & Durrer 2015) and the large scale structure (Baumann et al. 2018). Even though the mass scale is still uncertain, neutrinos lead to a suppression of structure growth below their free-streaming scale (Lesgourgues & Pastor 2006a). This then leads to the question: Can neutrinos mask modified gravity effects in the large scale structure? Are constraints obtained on f(R) theories from cluster number counts (Schmidt et al. 2009; Lombriser et al. 2012; Cataneo et al. 2015) then still valid when including massive neutrinos into the analysis? And on a more fundamental level, how can the joint effects of neutrinos and modified gravity be included in the theoretical prediction of cluster abundance?

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Early investigations of these issues have been presented by Baldi et al. (2014), who performed the first N-body simulations of \( f(R) \) gravity in the presence of massive neutrinos, clearly demonstrating a strong degeneracy between their effects on the abundance of gravitationally bound systems. More recently, Giocoli et al. (2018) and Peel et al. (2018) explored the same degeneracies based on a combination of cluster counts and weak lensing statistics along the past light cone. In this work, we continue investigating the combined effects of \( f(R) \) and massive neutrinos by developing a theoretical model of the joint halo mass function, calibrated to a suite of specifically designed N-body simulations.

We start with a brief summary of \( f(R) \) gravity in Sec. 2 and present the simulation suite used to explore joint effects of modified gravity and neutrinos in Sec. 3. In Sec. 4 we introduce the joint mass function and apply our framework to forecast the ability of current and future surveys to constrain \( f(R) \) theories in Sec. 5. We summarise our results in Sec. 6.

### 2 REVIEW OF \( f(R) \) GRAVITY

We start from the modified Einstein-Hilbert action

\[
S = \int \mathrm{d}^4x \sqrt{-g} \left( \frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right),
\]

with the Lagrangian of the matter fields \( \mathcal{L}_m \). We adopt the functional form proposed by Hu & Sawicki (2007)

\[
f(R) = -2\Lambda - \frac{R}{R + m^2},
\]

with a constant \( \Lambda \) and the curvature scale \( m^2 \). Note that \( f(R) \rightarrow 0 \) for \( R \rightarrow 0 \), in that sense the model does not contain a cosmological constant. For \( m^2 \ll R \), the function can be expanded to get

\[
f(R) \approx -2\Lambda - f_{R0} \frac{R^2}{R^0} - \frac{R^2}{R},
\]

where \( R^0 \) is the Ricci scalar today, overbars denote background quantities and we introduced the dimensionless parameter \( f_{R0} = -2\Lambda m^2/R^0 \). To recover the well-measured CDM expansion history, we fix the first term to the cosmological constant in GR \( \Lambda = \Lambda_{GR} \) and \( f_{R0} \ll 1 \) is the only remaining free parameter of the model. This implies that background quantities are indistinguishable from CDM.

The modified Einstein equations are obtained by variation of Eq. 1 with respect to the metric \( g_{\mu\nu} \)

\[
G_{\mu\nu} - f_R R_{\mu\nu} - \left( \frac{f}{2} - \Box f_R \right) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R = 8\pi G T_{\mu\nu},
\]

with the new scalar degree of freedom \( f_R \equiv df/\partial R \). The trace of Eq. 4 leads to an equation of motion for the scalar field \( f_R \)

\[
\nabla^2 f_R = \frac{f}{3} (\delta R(f_R) - 8\pi G \delta \rho_m),
\]

where we adopted the quasi-static approximation and consider small perturbations on a smooth background, i.e. the quantities \( \delta x \equiv x - \bar{x} \). The time-time component of the modified Einstein equations gives a Poisson-like equation for the scalar metric perturbation \( \psi = \delta_{00}/\delta_0 \)

\[
\nabla^2 \psi = \frac{16\pi G}{3} a^2 \rho_m - \frac{a^2}{6} \delta R(f_R),
\]

which can still be identified with the Newtonian potential but has contributions from both the matter density \( \rho_m \) and the scalar field via \( \delta R(f_R) \). Eqs. 5 and 6 are non-linear and thus we will later resort to N-body simulations to solve them in general, but two limiting cases are insightful:

For large field values \( |f_{R0}| \gg |\psi| \) we can linearise

\[
\delta R \approx \frac{dR}{df_R}|_{R=R_0} \delta f_R,
\]

and the Fourier-space solution of Eqs. 5 and 6 becomes

\[
k^2 \psi(k) = -4\pi G \left( \frac{4}{3} - \frac{1}{3} k^2 + \frac{\mu^2 a^2}{2} \right) a^2 \delta \rho_m(k),
\]

where we introduced the Compton wavelength of the scalar field \( \mu^{-1} = (3d_{fR}/dR)^{-1/2} \). On small scales \( k > \mu \) this leads to a Poisson equation with an additional factor \( 4/3 \). For scales larger than the Compton wavelength the additional contribution vanishes and we recover behaviour as in general relativity.

In the opposite limit of small field values \( |f_{R0}| \ll |\psi| \) the two contributions in Eq. 5 approximately cancel, therefore

\[
\delta R \approx 8\pi G \delta \rho_m
\]

and Eq. 6 turns into the usual Poisson equation. This is the screened regime.

To estimate where the transition occurs, we can formally solve Eq. 5 using the Green’s function of the Laplacian

\[
\delta f_R(r) = \frac{1}{4\pi r} \int_0^r \mathrm{d}r' 8\pi G \left( \delta \rho - \frac{\delta R}{8\pi G} \right)
\]

\[
= \frac{2}{3} \frac{G M_{\text{eff}}(r)}{r},
\]

with an effective mass \( M_{\text{eff}} \) as the source for field fluctuations \( \delta f_R \) (Schmidt 2010). Note that \( M_{\text{eff}}(r) \leq M(r) \) and equality holds in the unscreened regime where we get \( \delta f_R = \frac{2}{3} \psi_N \) with the Newtonian potential of a spherical overdensity \( \psi_N = GM/r \). Because the fluctuation in \( f_R \) is by definition smaller than its background value \( \delta f_R \leq f_R \), this translates to

\[
|f_{R0}| \leq \frac{2}{3} \psi_N(r),
\]

thus the additional force is only sourced by mass outside of the radius where this condition is met.

To summarise, the theory is identical to CDM on the background level, but perturbatively yields a maximum enhancement of gravity by 1/3 on scales smaller than the Compton wavelength \( \mu^{-1} \). It also includes a screening mechanism that restores GR in regions of high density and its onset is given by the typical depth of cosmological potential wells \( \phi \sim 10^{-5} - 10^{-6} \), so that \( f_{R0} \sim 10^{-3} - 10^{-6} \) is the relevant parameter space where this mechanism can function. Values of \( f_{R0} \) below this threshold are always screened, and therefore phenomenologically uninteresting.
3 THE DUSTGRAIN-PATHFINDER SIMULATIONS

For our analysis we make use of the halo catalogues extracted from the DUSTGRAIN-pathfinder simulations (see Giocoli et al. 2018, for a detailed description), a suite of cosmological N-body simulations designed to investigate the possible observational degeneracies between $f(R)$ gravity and massive neutrinos by sampling their joint parameter space. The simulations have a periodic box size of 750 Mpc/h per side filled with 768³ dark matter particles of mass $m_{\text{cdm}} = 8.1 \times 10^{10} M_{\odot}/h$ (for the case of $m_\nu = 0$) and with as many neutrino particles (for the case of $m_\nu > 0$). The particles are moving under the effect of an $f(R)$ gravitational interaction mediated by the scalar potential $\psi$ satisfying Eq. 6 above.

The DUSTGRAIN-pathfinder runs have been performed with the MG-Gadget code (Puchwein et al. 2013) – a modified version of the GADGET code (Springel 2005) for $f(R)$ gravity theories – combined with the particle-based implementation of massive neutrinos developed by Viel et al. (2010), and already employed in Baldi et al. (2014). The MG-Gadget $f(R)$ solver has been thoroughly tested (see e.g. Winther et al. 2015) and already used for several applications in cosmology ranging from pure collisionless simulations (Baldi & Villaescusa-Navarro 2018; Arnold et al. 2018) to hydrodynamical simulations (Arnold et al. 2015; Roncarelli et al. 2018), to zoomed simulations of Milky Way-sized objects (Arnold et al. 2016; Naik et al. 2018).

Initial conditions have been produced by generating two separate but fully correlated random realisations of the linear density power spectrum for CDM and massive neutrino particles as computed by the Einstein-Boltzmann code CAMB (Lewis et al. 2000) at the starting redshift of the simulation $z_i = 99$. Following the approach of e.g. Zennaro et al. (2017); Villaescusa-Navarro et al. (2017), neutrino gravitational velocities are calculated based on the scale-dependent growth rate $D(z, k)$ for the neutrino component. On top of these, neutrino particles also receive an additional thermal velocity extracted from the neutrino momentum distribution for each value of neutrino mass under consideration.

In the present work – which is the third in a series of papers making use of the DUSTGRAIN-pathfinder simulations after Giocoli et al. (2018) and Peel et al. (2018) – we restrict our focus on a subset of the full simulations suite consisting of nine runs whose parameters are summarised in Table 1. All simulations share the same standard cosmological parameters which are set in accordance with the Planck 2015 constraints (Planck Collaboration et al. 2016a), namely $\Omega_m = 0.124 + \Omega_b + \Omega_{\Lambda} = 0.31345$, $\Omega_b = 0.0481$, $\Omega_{\Lambda} = 0.68655$, $H_0 = 67.31$ km s$^{-1}$ Mpc$^{-1}$, $A_s = 2.199 \times 10^{-9}$, $n_s = 0.9658$.

For all simulations we have identified collapsed CDM structures in each comoving snapshot by means of a Friends-of-Friends algorithm (FoF hereafter, see Davis et al. 1985) on the CDM particles with linking length $l = 0.16h^{-1}$d where $d$ is the mean inter-particle separation, retaining only structures with more than 32 particles. On top of such FoF catalogue we have run the SUBFIND algorithm (Springel et al. 2001) to identify gravitationally bound structures and to associate standard quantities such as the mass and the radius to the main substructure of each FoF group. The latter quantities are computed in the usual way by growing spheres of radius $R$ around the most-bound particle of each main substructure enclosing a total mass $M$ until the condition

$$4/3 \pi R_{200m}^3 \times 200 \times \rho_{200m} \Omega_m = M_{200m}$$

is fulfilled for $R = R_{200m}$ and $M = M_{200m}$, where $\rho_{200m} = 3H^2/8\pi G$ is the critical density of the universe.

4 joint mass function

Dark matter halos form from collapsing regions that decouple from the background expansion. Their abundance can be related to the volume fraction of the Gaussian density field $\delta_R$ smoothed on a radius $R$ above a critical collapse threshold $\delta_c$. (Press & Schechter 1974). This yields the number density of halos within a mass interval $[M, M + dM]$, the halo mass function:

$$\frac{dn}{dM} = f(\sigma) \rho_m \frac{d \ln \sigma^{-1}}{M^2} \frac{d \ln M}{d \ln \sigma}$$

where $\rho_m = \rho_{m200c}$ is the mean density of the Universe and $f(\sigma)$ is the multiplicity function related to the collapsed volume fraction $F(M)$ occupied by halos over mass $M$ by

$$f(\sigma) = 2\sigma^2 \frac{dF}{d\sigma^2}.$$  

It depends on the variance of the linear density field $S \equiv \sigma^2(R(M), z) = \int \frac{dk}{k} k^3 P(k, z) \frac{W^2(kR(M))}{2\pi^2}$

It is contained in the filter with the mass $M = 4/3\pi R^3 \rho_m$. The variables $M$, $R$ and $\sigma^2$ are monotonous functions of each other and can therefore be used interchangeably.

Note that even though $\sigma$ is often thought of as growing with cosmic time $\sigma(z) = D(z)\sigma_0$, in the framework of spherical collapse it is instructive to consider the threshold $\delta_c(z) = \delta_c(D(z))$ as the dynamical quantity. At early times, the density field is Gaussian and completely characterised by its variance alone. The collapse criterion is then really a criterion imposed on the initial conditions.

If we assume a top-hat filter in Fourier space $W = \theta(k - 1/R)$, each new mode of the density field entering the filter is independent and the smoothed field performs a random walk with $R$ (or equivalently $S$) as a time variable. The problem can then be rephrased: when does a trajectory $\delta(S)$ first cross the threshold $\delta_c$ (Bardeen et al. 1986; Bond et al. 1991)?

Under these assumptions individual trajectories follow a Langevin equation

$$\frac{\partial \delta}{\partial S} = \eta(S),$$

with a stochastic driving term $\eta$ defined by its mean $\langle \eta \rangle = 0$ and variance $\langle \eta^2 \rangle = \delta_\eta (S - S')$. The probability distribution $\Pi$ of trajectories then evolves according to the corresponding Fokker-Planck equation

$$\frac{\partial \Pi}{\partial S} = \frac{1}{2} \frac{\partial^2 \Pi}{\partial \delta^2},$$

with the boundary condition $\Pi(\delta, S = 0) = \delta_\Pi(\delta)$ because the Universe is homogeneous on large scales. However, trajectories can cross the barrier more than once leading to double-counting of halos. To solve this, one demands the additional boundary condition (an absorbing barrier) $\Pi(\delta = \delta_c, S = 0)$.
The solution to Eq. 18 is then given by (Bond et al. 1991)
\[
\Pi(\delta, \sigma^2) = \frac{1}{2\pi \sigma^2} \left( e^{-\delta^2/2\sigma^2} - e^{-(2\delta_c - \delta)^2/2\sigma^2} \right),
\]
(19)
where the second Gaussian term reflects the fact that trajectories end at the barrier. Omitting it lead to the missing normalisation factor 2 of the Press & Schechter (1974) prediction.

With the boundary condition the distribution function vanishes for \( \delta > \delta_c \), so we express \( F(S) \) by subtracting the fraction of trajectories that did not yet cross the threshold
\[
F(\sigma^2) = 1 - \int_{-\infty}^{\delta_c} \Pi(\delta, \sigma^2) d\delta,
\]
(20)
from which we can derive the multiplicity function \( f(\sigma) \) by using Eq. 15 to get the mass function by Press & Schechter (1974)
\[
f_k(\sigma) = \sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma} e^{-\frac{\delta_c^2}{2\sigma^2}},
\]
(21)
with the correct normalisation. Note that we indicate solutions from non-correlated random walks (using a \( k \)-space top-hat) with subscript \( k \).

This approach works reasonably well, but has several shortcomings:

(i) Collapse in a Gaussian random field does not occur spherically. In the Zel’dovich approximation, the eigenvalues \( \lambda_i \) of the deformation tensor follow the joint probability distribution (Doroshkevich 1970)
\[
p(\lambda_1, \lambda_2, \lambda_3) = \frac{15^3}{8 \pi^4 \sqrt{5} \sigma^6} \exp \left( -\frac{3\lambda_1^2 + 15\lambda_2}{2\sigma^2} \right)
\times |(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)|,
\]
(22)
with \( I_1 = \lambda_1 + \lambda_2 + \lambda_3 \) and \( I_2 = (\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3) \). Isotropic collapse with \( \lambda_1 = \lambda_2 = \lambda_3 \) therefore does not occur. Instead the Zel’dovich picture suggests a collapse into subsequently walls, sheets, filaments and halos, where the last step occurs typically along a filament in an ellipsoidal fashion. This is fully consistent with structure formation observed in \( N \)-body simulations.

(ii) Real halos do not form out of sharp \( k \)-space top-hats. Usually one assumes rather a real-space top-hat as initial condition for the spherical collapse. This leads to coupling of Fourier modes and introduces correlations between steps of the random walk.

### Table 1.
The subset of the DUSTGRAIN-pathfinder simulations considered in this work with their specific parameters.

| Simulation Name | Gravity type | \( |f_{\delta_0}| \) | \( \sum m_{\nu} [\text{eV}] \) | \( \Omega_{\text{cdm}} \) | \( \Omega_{\gamma} \) | \( n_0^p [M_{\odot}/h] \) | \( m_0^p [M_{\odot}/h] \) |
|-----------------|--------------|------------------|------------------|--------------|--------------|-----------------|-----------------|
| ΛCDM           | GR           | –                | –                | 0.31345      | –            | 8.1 \times 10^{10} | –               |
| fR4            | \( f(R) \)   | 10^{-4}         | –                | 0.31345      | –            | 8.1 \times 10^{10} | –               |
| fR5            | \( f(R) \)   | 10^{-5}         | –                | 0.31345      | –            | 8.1 \times 10^{10} | –               |
| fR6            | \( f(R) \)   | 10^{-6}         | –                | 0.31345      | –            | 8.1 \times 10^{10} | –               |
| fR4-0.3eV      | \( f(R) \)   | 10^{-4}         | 0.3              | 0.30630      | 0.00715      | 7.92 \times 10^{10} | 1.85 \times 10^{9} |
| fR5-0.15eV     | \( f(R) \)   | 10^{-5}         | 0.15             | 0.30987      | 0.00358      | 8.01 \times 10^{10} | 9.25 \times 10^{8} |
| fR5-0.1eV      | \( f(R) \)   | 10^{-5}         | 0.1              | 0.31107      | 0.00238      | 8.04 \times 10^{10} | 6.16 \times 10^{8} |
| fR6-0.06eV     | \( f(R) \)   | 10^{-6}         | 0.6              | 0.31202      | 0.00143      | 8.07 \times 10^{10} | 3.7 \times 10^{8} |

**Figure 1.** Effect on the halo mass function when changing the barrier width \( D_B \) around the fiducial value \( D_B = 0.4 \). A broader barrier leads to a smaller suppression of the exponential tail of the mass function and therefore more high-mass objects.

### 4.1 Diffusing, drifting barrier

The non-spherical collapse dynamics can be addressed by modifying the collapse barrier. The main motivation is that low-mass (high \( \sigma \)) halos are more ellipsoidal, while the largest objects are approximately spherical. Ellipsoidal patches collapse later because they have to get rid of angular momentum, which leads to an effective higher threshold. There are various ways to extend the excursion set formalism to account for this, and here we follow Kopp et al. (2013) and introduce a scale-dependent barrier of the form
\[
B = \delta_c + B(S),
\]
(24)
that tends to the spherical collapse threshold \( \delta_c \) for high-mass halos \( \sigma \ll 1 \). Even though more general forms for the ellipsoidal collapse barrier \( B \) can be found in the literature (e.g. \( B = \delta_c + B(S) \); see Sheth & Tormen 2002), the linear approximation adopted in this work is sufficient for typical cluster abundance studies using clusters of mass \( M \gtrsim 10^{13.5} M_{\odot}/h \).

In addition to the barrier drift, the collapse dynamics themselves are complicated by environmental effects and fuzzy halo definitions. In Maggiore & Riotto (2010b) this was taken into account by turning the barrier itself into a Gaussian stochastic variable with a mean \( B = \delta_c + B(S) \) and width \( D_B \). Both the trajectories and the barrier itself perform a random walk, and the joint probability distribution
is obtained from a 2D Fokker-Planck equation (Maggiore & Riotto 2010b; Corasaniti & Achitouv 2011)
\[ \frac{\partial \Pi}{\partial S} = \frac{1}{2} \frac{\partial^2 \Pi}{\partial \delta^2} + \frac{D_B}{2} \frac{\partial^2 \Pi}{\partial B^2}, \]
which is solved by
\[ f_k(\sigma) = \frac{2a}{\kappa \sigma} e^{-a\sigma^2/(2\sigma^2)} \left[ \bar{B} - \kappa^2 \frac{\partial B}{\partial \sigma^2} \right], \]
where \( a \equiv 1/(1 + D_B) \). Using Eq. 24, this reduces to a Press-Schechter like solution with the constant threshold \( \delta_c \) replaced by the full barrier:
\[ f_k(\sigma) = \frac{2a}{\kappa \sigma} e^{-a(\delta_c + \beta R^2)^2/(2\sigma^2)}. \]

The effect of \( D_B \) is demonstrated in Fig. 1: a broader barrier leads to a smaller factor in the exponential, boosting the abundance of high-mass clusters because those rare trajectories can cross the threshold easier.

### 4.2 Non-Markovian Corrections

Accounting for realistic filter functions makes it necessary to consider the deviations from an uncorrelated random walk. Halos form from regions that resemble spherical patches in the initial conditions and several possible window functions to capture the correct form of these proto-halos exist (Bond et al. 1991). Here we assume a real space top-hat, which in Fourier space turns into
\[ W(k) = \frac{3j_1(x)}{x}, \]
with the spherical Bessel function \( j_n \), which we use from here on to calculate the variation of the density field \( S \) in Eq. 16. In Maggiore & Riotto (2010a) the authors calculated the corrections induced by correlations between the variance \( S \) smoothed at different radii \( R \) for this choice of smoothing filter. The general two-point correlation function can be written as
\[ \langle \delta_1 \delta_2 \rangle = \min(S_1, S_2) + \Delta(S_1, S_2), \]
where we introduced the shorthand \( \delta_1 = \delta(R_1) \), and the first term expresses the Markov dynamics leading to the Press-Schechter result with a general barrier in Eq. 27. The correction is of the form
\[ \Delta(S_1, S_2) = \kappa \frac{S_1(S_2 - S_1)}{S_2}, \]
with the coefficient
\[ \kappa(R) \approx 0.459 - 0.003R, \]
and has a weak dependence on cosmology via the power spectrum. As pointed out above, we deal with a purely Gaussian field in the initial conditions here, and all correlations are introduced by the filter and not by later non-linear mode coupling. This also means that \( \kappa \) should be calculated from the ΛCDM relation in Eq. 31 even within a modified gravity model. We will return to this point when discussing the modified gravity mass function.

This leads to the real-space top-hat multiplicity function \( f_x \), to first order in \( \kappa \) (Maggiore & Riotto 2010a; Kopp et al. 2013):
\[ f_x(\sigma) = f_k(\sigma) + f_{m-m}(\sigma) + f_{m-m}(\sigma) + f_{m-m}(\sigma) \]

### Table 2. Fiducial values for the GR mass function barrier shape and the virial \( f(R) \) collapse threshold Eq. 36.

| GR   | \( f(R) \) |
|------|------------|
| \( \beta \) | \( \mu_1 \) | \( \mu_2 \) |
| 0.4   | 0.12       | 0.11       |
| 2.7 \times 10^{-3} | 1.99   | 26.21      |

with the Markovian term \( f_k \) for a diffusive, drifting barrier given by Eq. 27 and corrections
\[ f_{m-m}(\sigma) = \frac{a}{\sigma^2} \left[ e^{-a\sigma^2/2\sigma^2} - \frac{1}{2} \Gamma \left( 0, \frac{a\sigma^2}{2\sigma^2} \right) \right], \]
\[ f_{m-m}(\sigma) = -\alpha \beta \beta \left[ e^{-\alpha \beta \beta^2/2\sigma^2} + \frac{a}{\sigma^2} \right], \]
\[ f_{m-m}(\sigma) = -\beta \frac{\beta}{2} \sigma^2 e^{-m-m}(\sigma) + \delta_c f_{m-m}(\sigma)^{-m-m}(\sigma). \]

### 4.3 Spherical collapse in modified gravity

As for the ΛCDM case, the starting point of our analysis is spherical collapse. Kopp et al. (2013) numerically solved the full modified Einstein, scalar field and non-linear fluid equations to obtain \( \delta_c \) in \( f(R) \) gravity, and they parameterised their solution for the threshold by
\[ \delta_c(M) \sim \delta_c^{GR}(M) \times \Delta(M). \]
where the deviation from GR is captured by the correction factor
\[ \Delta(M) = 1 + b_2(1 + z)^{-a_2} \left[ m_b - \sqrt{m_b^2 + 1} \right] \]
\[ b_2 = 0.0166 \]
\[ m_b = (1 + z)^{a_1} \log_{10} M - m_1(1 + z)^{-a_1} \]
\[ m_1(1 + z)^{a_1} \log_{10} M + m_2 \]
The parameterisation converges to the GR limit \( \delta_c^{GR} \) separately for high \( z \) and \( f(R) \to 0 \), which is well approximated by (Nakamura & Suto 1997)
\[ \delta_c^{GR}(z) = \frac{3(12\pi^2)^{2/3}}{20} \left( 1 - 0.0123 \log_{10} \left( 1 + \frac{\Omega_m - 1}{1 + z} \right) \right)^{2/3}. \]

The coefficients \( a_1, b_2, \mu_1, \mu_2 \) from Kopp et al. (2013) are given in Tab. 2 which were fitted to numerical solutions and should be regarded as prediction of their spherical collapse model. Here we want to bring this model closer to data before we consider possible constraints from cluster abundance. The crucial ingredient of the model is \( m_b \), which sets the transition mass where screening sets in. We will express this scale as the screening mass \( M_{\text{screen}} \), defined by \( m_b(M_{\text{screen}}) = 0 \). In Fig. 2 we show the connection between the threshold and the cluster abundance: up to \( M_{\text{screen}} \) the
threshold grows linearly with $\log M$ and afterwards it starts reverting to the fiducial GR value. In the mass function, this scale corresponds to a characteristic peak in the additional relative abundance. Note that the negative relative abundance for lower masses shown in the plot is physical because of mass conservation: additional high-mass objects form from low-mass halos.

For $m_b = 0$, the threshold is given by

$$\delta_c = \delta_c^{GR} (1 + b_2(1 + z)^{-\alpha_3}) - b_3,$$

(40)

and because a lower $\delta_c$ leads to a higher cluster abundance, $b_2$ and $b_3$ set the height of the additional abundance peak, $a_3$ and $a_2$ control the redshift evolution of the screening mass, and $\mu_1, \mu_2$ determine how quickly the model reverts to GR when changing $f_{R0}$.

Fig. 3 shows the variation of the threshold as a function of redshift and the $f_{R0}$ parameter for a halo of mass $M_{200} = 10^{14} M_{\odot}/h$. Considering this mass representative of the lightest objects entering a cosmological cluster catalogue, the leftmost line indicates the limit of cluster abundance studies to constrain the theory at a given redshift where the deviation in $\delta_c$ is of order 1%.

To write the multiplicity function for $f(R)$ including non-Markovian corrections, we assume that the correlation between steps behaves similar for modified gravity and GR. This is justified because we measure the correlation in the initial conditions where the density fields in both theories are identical – all modifications to the time evolution are absorbed into the threshold $\delta_c(f_{R0}, M, z)$. Therefore we write (Kopp et al. 2013)

$$f_{x}^{R}(\sigma) = f^{GR}_{x} (\sigma) \frac{f^{R}_{x}(\sigma)}{f^{GR}_{x}(\sigma)}$$

(41)

with the Markovian multiplicity function $f^{R}_{x}(\sigma)$ derived from the modified gravity barrier $\tilde{B} = \delta_c(f_{R0}, M, z) + \beta \sigma^2$ given in

$$f_{k}^{R}(\sigma) = \frac{\sqrt{2\sigma^2 \pi^{-3/2} \exp[-\tilde{B}^2/(2\sigma^2)]}}{\sqrt{2\sigma^2 \pi^{-3/2} \exp[-B^2/(2\sigma^2)]}} \left[ \frac{\partial \tilde{B}}{\partial \ln \sigma} - \frac{3M}{2} \frac{\partial \delta_c^{GR}}{\partial M} \frac{\partial \ln R}{\partial \ln \sigma} \right].$$

(42)

Together with $f^{GR}_{k}$ (Eq. 27) and $f^{GR}_{x}$ (Eq. 32), this defines the full modified gravity multiplicity function (Eq. 41), and yields the halo mass function via Eq. 14. We emphasize again that all expressions are defined for the smoothed density field $\sigma^{GR}$ calculated in a standard cosmology – as already discussed, the threshold is imposed on the initial conditions, and all subsequent effects of modified gravity are encapsulated in the dynamics of the barrier.

### 4.4 Neutrinos

As we have seen, the signal of modified gravity is a lower collapse threshold and a resulting higher abundance of clusters compared to $\Lambda$CDM. To set realistic limits on deviations from GR, we will now incorporate effects of massive neutrinos. As has been studied before (see e.g. Lesgourgues & Pastor 2006b) they suppress structure growth below the free-streaming scale which leads to a lower abundance of galaxy clusters, counteracting possible effects of $f(R)$. Constraining the neutrino mass is an important goal for cluster cosmology in its own right, but here we will focus on degeneracy with modified gravity effects.

Costanzi et al. (2013) showed that the effect of neutrinos on the cluster abundance can be well captured by rescaling the smoothed density field

$$\sigma^2 \rightarrow \sigma^2_{cdm}(z) = \int \frac{dk}{k} \frac{k^3 P_{cdm}(k, z)}{2\pi^2} W^2(kR),$$

(43)

with the cold dark matter power spectrum obtained by rescaling the total matter power spectrum $P_m$ with the respective transfer functions weighted by the density of each
we choose to keep the total matter density the effects of modified gravity and neutrinos on the threshold outlined above and it allows us to compare a random walk. This rescaling expresses the cold dark matter species thus assuming that neutrinos are distributed smoothly on cluster scales. The scale dependent growth caused by neutrinos for the other components is also accounted for by the transfer functions. Eq. 43 is expressed as a time-dependent rescaling, but we can also again think of the initial density field as fixed and map the change to the collapse threshold $\delta^c_f = \sigma_c(z) \sigma_{c,\text{CDM}}(z)$. 

(45)

In this picture, we account for the effect of neutrinos by introducing an appropriate shift in the time variable $\sigma^2$ of the random walk. This rescaling expresses the cold dark matter approximation outlined above and it allows us to compare the effects of modified gravity and neutrinos on the threshold directly. While there is some ambiguity how to compare cosmologies with and without neutrinos, in this paper we choose to keep the total matter density $\Omega_m$ fixed. Thus when adding neutrinos, we rescale the dark matter density by (Lesgourgues & Pastor 2006a)

(46)

In Fig. 4 we show the rescaled critical density for collapse $\delta^c_f$ and the resulting effect on the halo mass function. A larger $\delta_c$ leads to an increased exponential suppression of high mass halos in Eq. 26. Note that the scale dependent growth caused by neutrinos translates to a weak mass dependence of $\delta_c$. The higher threshold leads then to a stronger suppression in the exponential high mass tail of the mass function (bottom).

**Figure 4.** Top: Change in the collapse threshold $\delta^c/\delta_{GR}$ for different neutrino masses. The scale dependent growth in $\nu$CDM cosmologies translates to a slight mass dependence of $\delta_c$. The higher threshold leads then to a stronger suppression in the exponential high mass tail of the mass function (bottom).

**Figure 5.** Change in the effective collapse threshold at $z = 0$ induced by massive neutrinos with $\sum m_\nu = 0.3\, \text{eV}$ (blue), for $f_{R_0} = 10^{-5}$ (orange) and the combined effect. The grey shaded region shows a 5% deviation from the fiducial value. Over the mass range $M > 10^{14} M_\odot h^{-1}$ relevant for cluster abundance studies the effects of neutrinos and modified gravity are approximately degenerate.

### 4.5 Halo bias and cluster clustering

The mass function also allows us to derive the corresponding clustering bias. The Eulerian bias is given by the overabundance of objects in a region with an overdensity $\delta_0$ compared to the mean abundance

(48)

therefore the first order bias is the linear response of the halo field to changes in the underlying density field. For a fixed barrier, the conditional mass function $f(\sigma|\delta_0)$ simply involves a shift of the barrier $\delta_c \rightarrow \delta_c - \delta_0$, but for a generic barrier the situation is more complicated.

Achitouv et al. (2016) proposed the conditional mass function for a generic barrier

(49)

and found good agreement with Monte Carlo random walks for various barrier shapes. This yields the linear bias

(50)

with the same barrier $\hat{B}$ as used for the mass function, but the bias depends only mildly on the barrier width $\Delta_B$ and drift $\beta$ for the mass range we focus on in this work. It is mainly sensitive to the mean threshold $\delta_c$.

We show the changes in the bias induced by modified
gravity or massive neutrinos in Fig. 6 using the \( f(R) \) barrier \( B(M, z, f_{R0}) \). The lower threshold means that clusters form out of smaller overdensities compared to ΛCDM, so they are less biased tracers of the density field. This tendency is only enhanced the stronger the \( f(R) \) effect gets and the linear bias shrinks with larger values of \( f_{R0} \). For neutrinos, this effect is reversed: because the high-mass tail of the mass function is suppressed, massive clusters are less abundant overall and therefore only form in very overdense regions. However the absolute scale of the halo bias in ΛCDM is still uncertain (Baxter et al. 2016; Paech et al. 2017), making it very difficult to use this behaviour for constraints – both neutrinos and modified gravity lead to a lower bias of low-mass objects compared to high-mass objects. We therefore leave a forecast analysis also including the clustering of clusters for future work.

4.6 Calibration and Comparison

The excursion set framework predicts the mass function in terms of the halo mass at virialization \( \Delta M_{\text{vir}} \) since this is the time at which the halo stops to collapse. Moving to modified gravity, the virial overdensity is even more complicated. While constant in an Einstein-de-Sitter universe, we expect \( \Delta_{\text{vir}}(R) \) to evolve with both redshift and \( f_{R0} \).

From the observational point of view, however, the mass of a cluster is often defined as the mass inside a sphere encompassing an overdensity \( \Delta \) times a reference value. In this work we adopt \( \Delta_{\text{m}} = 200 \) with respect to the mean matter density as given in Eq. 13 to define our simulated catalogues and calibrate the mass function accordingly.

A first exemplary comparison between the fiducial barrier model Eq. 36 and our simulation is shown in Fig. 7 for \( f_{R0} = 10^{-5} \) and \( \sum m_{\nu} = 0.3 \) eV. As expected the virial mass function from Kopp et al. (2013) is a bad fit to the \( M_{200\text{m}} \) catalogue and we can see that the screening mass is offset, leading to a wrong position and amplitude of the \( f(R) \) bump. To calibrate the mass function to a new mass definition, we focus on the screening mass \( m_{\nu} \) in Eq. 38. We keep the functional form, but because the position and evolution of the screening mass scale is different for another mass definition, we re-fit the parameters \( \mu_1, \mu_2 \) to account for the evolution with \( f_{R0}, \beta_3 \) to adapt the height of the relative abundance peak and \( a_4 \) to adjust the redshift evolution. This is done via minimisation of the Gaussian log-likelihood

\[
\ln L = -\frac{1}{2} \left( \mathbf{N}^{\text{theo}} - \mathbf{N}^{\text{sim}} \right) \mathbf{C}^{-1} \left( \mathbf{N}^{\text{theo}} - \mathbf{N}^{\text{sim}} \right)^T - \frac{1}{2} \ln \det \mathbf{C}^{-1},
\]

where the covariance matrix consists of a Poissonian contribution and a sample variance term

\[
C_{ij}^{-1} = \delta_{ij} N_i^{\text{theo}} + b_i b_j N_i^{\text{theo}} N_j^{\text{theo}} \sigma(V_{\text{box}}) \]

with theoretical cluster counts \( N_i^{\text{theo}} \) per mass bin \( i \) and \( \sigma(V_{\text{box}}) \) is the variance of the density field computed inside the box. We calculate the mean bias averaged over a bin \( \Delta M_i \) as

\[
b_i = \int_{\Delta M_i} \frac{dm}{dM} \frac{dN}{dm} (M) \left( \int_{\Delta M_i} \frac{dm}{dM} \frac{dN}{dm} \right)^{-1},
\]

using Eq. 51 for the bias and Eq. 41 for the mass function. Note that the barrier shape given by \( D_2 \) and \( \beta \) is very important for the proper GR limit, but largely cancels in Eq. 41. The mass function ratio is therefore almost completely independent from the fiducial barrier values. So while we choose to work within a consistent framework with a mass function that is extended to \( f(R) \), one could also replace \( f^\text{GR} \) in Eq. 41 with another multiplicity function such as ones by Tinker et al. (2008) or Crocce et al. (2010) as long as it is also calibrated to \( M_{200\text{m}} \). We do not perform a comprehensive comparison of mass functions here, but we note that our results for bias and multiplicity agree within ~5% with those established results in the literature – a value we take as an estimate for current systematic effects on the halo mass function mainly due to differences in halo definition.

Within our simulations, we find no preference for any
redshift evolution in the GR barrier parameters \(D_B\) and \(\beta\). We fit them to our \(\Lambda\)CDM simulations first and keep them fixed while calibrating the remaining \(f(R)\) parameters \(\alpha_3\), \(\beta_3\), \(\mu_1\), \(\mu_2\) to our \(fR4\), \(fR5\) and \(fR6\) simulations. The resulting best-fit values with statistical errors are shown in Tab. 3. For the ACDM barrier values we find qualitative agreement with previous similar studies (Maggiore & Riotto 2010b; Kopp et al. 2013; Achitouv et al. 2016) while the position and evolution of the screening mass \(m_R\) given by the other parameters deviates substantially from the virial mass function from Kopp et al. (2013). The results are compared to our simulated catalogues in Fig. 8 for a wide range of redshifts and values of \(f_{R0}\). We find that our model for the halo mass function can reproduce the simulated data by fitting only four parameters to account for the full non-linear behaviour of the modified gravity model.

Table 3. Best-fit parameters for the width \(D_B\) and the drift \(\beta\) of the fiducial GR barrier and the calibrated values for the modified gravity threshold \(\delta_c(f_{R0},z)\) in Eq. 36. The statistical uncertainty of the fit is the last significant digit.

| GR  | \(f(R)\) |
|-----|------------|
| \(D_B\) | \(\beta\) | \(\alpha_3\) | \(\beta_3\) | \(\mu_1\) | \(\mu_2\) |
| 0.37 | 0.11 | 0.067 | 5.6 \times 10^{-3} | 1.38 | 21.32 |

For completeness, we also compare our result to a previously used prescription for the modified gravity mass function in Fig. 7. In this ansatz proposed by Cataneo et al. (2015), the relative effect of \(f(R)\) is captured by a ratio of ellipsoidal collapse multiplicity functions

\[
\frac{f^{(R)}}{f^{GR}} = \frac{f^{ST_{GR}}(\sigma f^{(R)}), \delta_c^{(R)}}{f^{ST_{GR}}(\sigma c^{GR}), \delta_c^{GR}} ,
\]

where \(f^{ST}\) denotes the mass function by Sheth & Tormen (2002). The density variance is calculated using the linear power spectrum \(P(k)\) in the respective theory, and \(\delta_c^{GR}\) denotes the threshold for spherical collapse in case the theory is unscreened everywhere, i.e. gravity is enhanced by 4/3 (Schmidt et al. 2009)

\[
\delta_c^{f(R)}(z) = 1.7063 \left(1 - 0.0136 \log_{10} \left(1 + \Omega_m^{-1} - 1 \right) \right) ,
\]

which shares the functional form of Eq. 39 but differs in the numerical coefficients. In comparison to our simulations, we can see that this prescription fails to properly predict the onset and shape of the characteristic enhancement peak.

The next step is to test the inclusion of neutrinos into our framework via Eq. 47. We show the combined effect of neutrinos and modified gravity measured from our simulations in Fig. 9 - note that the simulations including neutrinos were not used to fit the mass function parameters. Both cosmologies show an approximate degeneracy leading to an abundance of clusters that is within 10% consistent with \(\Lambda\)CDM expectation at \(z = 0\), and the behaviour is well captured by our mass function. This cancellation weakly depends on redshift, so cosmologies with similar mass functions at \(z = 0\) will in general differ at earlier times. The precise degeneracy depends on the survey specifications such as redshift range and selection function, and we will return to this problem within the full cosmological parameter space in the next section.

5 FORECASTS

To assess if differences in the cluster abundance are measurable, it is important to consider the changes in the halo mass function in the context of a survey with a specific selection function.

We now show with two idealised test cases the consequences of our results for the ability of current and fu-
ture surveys to constrain $f_{R0}$. The abundance of clusters is mostly sensitive to $(\Omega_m, \sigma_8, \sum m_\nu, \log f_{R0})$; for the other relevant cosmological parameters we include priors from different probes. This has to be done with caution, because datasets might show different results when analysed in a $f(R)$ framework. We therefore make use of the fact that the model reproduces a $\Lambda$CDM expansion history and limit ourselves to geometrical probes.

We add baryon acoustic oscillation priors on the distance scale $D_V(z)$ based on BOSS DR12 data (Alam et al. 2017) at redshifts $z = 0.38, 0.51$ and 0.61. We centre them on our fiducial cosmology and assume pre-reconstruction errors are by far larger than statistical uncertainty for a cluster of mass $M$. We assume a log-normal distribution, which allows us to solve the integration over the observable to arrive at

$$\langle N_{ij} \rangle = \Omega \int_{\Delta z_i} \frac{dV}{dz_0} \int_0^{\infty} \frac{dM}{dM} \int_{\Delta z_j} d\Delta \Omega \cdot \rho(\Delta | M),$$

where the survey area $\Omega$ is fixed, and introduce the probability $\rho(\Delta | M)$ for a cluster of mass $M$ to be observed with a richness $\lambda$. We assume this data set with BAO. We denote this data set with BAO. Complementary BAO and BBN mock probes. We therefore make use of the fact that the model reproduces a $\Lambda$CDM expansion history and limit ourselves to geometrical probes.

We add baryon acoustic oscillation priors on the distance scale $D_V(z)$ based on BOSS DR12 data (Alam et al. 2017) at redshifts $z = 0.38, 0.51$ and 0.61. We centre them on our fiducial cosmology and assume pre-reconstruction errors are by far larger than statistical uncertainty for a cluster of mass $M$. We assume a log-normal distribution, which allows us to solve the integration over the observable to arrive at

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with

$$\lambda_{\min/\max} = \frac{\ln \lambda_{\min/\max} - (\ln \lambda)(M)}{\sqrt{\sigma^2_{\ln \lambda}}}.$$

We use the weak-lensing calibrated $M \sim \lambda$ relation measured by Murata et al. (2018) on SDSS clusters:

$$\langle \ln \lambda \rangle(M) = A + B \ln \left( \frac{M}{M^*} \right)$$

$$\sigma_{\ln \lambda}(M) = \sigma_0 + q \ln \left( \frac{M}{M^*} \right),$$

where $M^* = 3 \times 10^{14} M_\odot / h$ is the pivot mass of the relation and $A, B, \sigma_0$ and $q$ are free parameters varied within priors given by the measurements by Murata et al. (2018). Note that the weak lensing mass estimate of a given cluster is not affected by $f(R)$ because geodesics are unchanged up to a negligible factor $1 + f_{R0}$.

In addition to these observational uncertainties, also the mass function measured in simulations shows systematic scatter. This is mainly caused by ambiguities in the halo definition, so even an identical underlying dark matter field can result in slightly different halo statistics. Typically, different halo finders vary in the resulting amplitude and tilt of the mass function (Knebe et al. 2011), so we assume

$$\frac{dn}{dM} \rightarrow \frac{dn}{dM} \left( 1 + \gamma \log \left( \frac{M}{M^*} \right) \right)$$

with $\gamma$ and $q$ free to vary with Gaussian priors with width $\sigma = 0.05$ centred at 1 and 0 respectively. Because these systematic errors are by far larger than statistical uncertainty in our fit of barrier parameters, we keep the latter fixed.

The selection function is crucial for the specific degeneracy between parameters, so we distinguish two cases: Either a large, shallow layout or a deeper survey focused on a smaller sky area.

For the shallow case, we assume an area of $10^4$ deg$^2$ with eight richness bins as in Murata et al. (2018) $\lambda \in [0.05, 0.55]$. For the deeper case, we assume an area of $10^5$ deg$^2$ with 50 richness bins as in Murata et al. (2018) $\lambda \in [0.05, 0.55]$. We assume that the weak lensing mass estimate of a given cluster is not affected by $f(R)$ because geodesics are unchanged up to a negligible factor $1 + f_{R0}$.
Shallow + BAO + BBN
Deep + BAO + BBN

Figure 10. Expected constraints on \( f_{\text{R0}} \) from a shallow or deep optical cluster survey as described in the text. The cases shown here keep the neutrino mass fixed.

[20, 25, 30, 35, 40, 47.5, 55, 77.5, 100] and one redshift bin \( z \in [0.1, 0.3] \). This translates to an approximately flat limiting mass of \( M_{\text{min}} \sim 10^{14.4}M_\odot/h \). All bins are well populated with over 100 clusters so we assume a Gaussian likelihood as in Eq. 52. This mock survey is combined with either CMB or BAO + BBN priors as given in Tab. 4 and we evaluate the resulting likelihood using the Monte Carlo Markov Chain code \texttt{MontePython} (Audren et al. 2013; Brinckmann & Lesgourgues 2018).

We show the cluster count distribution in redshift and richness for a shallow survey in Fig. 11. For the given selection function, at low redshifts the effects of neutrinos and modified gravity are almost completely degenerate. Both roughly translate into a shift in the overall amplitude which is also easily mimicked by the amplitude of the \( M-\lambda \) relation. The richness information does help to break this degeneracy slightly because neutrinos tend to cause a strong suppression of very massive clusters while modified gravity leads to a higher abundance of low- and intermediate mass objects. The resulting limits on \( f_{\text{R0}} \) that can be achieved with such a survey are shown in Tab. 5. If cluster counts are only combined with BAO information, the limits are rather weak and when adding neutrinos we find no relevant upper bound. Adding the CMB improves the situation by pinning down the other cosmological parameters, but even then adding neutrinos weakens the bounds considerably. Note that there is a small additional effect due to broader CMB constraints on other parameters in a \texttt{LCDM} cosmology, but this mostly extends the contours in the direction of larger allowed \( \Omega_m \) values while \( f_{\text{R0}} \) is anti-correlated with the matter density.

For the deep survey, we take an area of 5000 deg\(^2\) – the total area that will be covered by the Dark Energy Survey\(^2\) – and bins in richness \( \lambda \in \{20, 30, 45, 60, 200\} \) and redshift \( z \in \{0.2, 0.35, 0.5, 0.65, 0.8\} \). The resulting cluster counts for this configuration in redshift and richness are shown in Fig. 12. Information about the abundance at higher redshifts helps in breaking the degeneracy, because while neutrinos suppress the population there, the \( f(R) \) mass function reverts to GR for \( z > 0.5 \). Even though modified gravity boosts the abundance of high mass clusters at low redshifts as shown in Fig. 8, integrated over \( z \) the effect on low-richness clusters is dominant as shown on the right panel of Fig. 12. Neutrinos on the other hand suppress the high-mass end of the halo mass function, so that – when combined – the two effects largely break the degeneracy between \( f(R) \) and neutrinos. Even without adding CMB information, such a survey can constrain \( f_{\text{R0}} \) down to the effective cluster floor of \( \sim 10^{-6} \) independent of neutrinos. We show the resulting posterior from both surveys combined with BAO and BBN priors for vanishing neutrino mass in Fig. 10.

### Table 5. Forecasted constraints from optical cluster surveys in various configurations as described in the text.

| Probes | Limit (95 \%) |
|--------|--------------|
| Shallow + BAO + BBN | \( |f_{\text{R0}}| < 8.1 \times 10^{-4} \) |
| Shallow + BAO + BBN + \( \nu \) | \( |f_{\text{R0}}| < 7.6 \times 10^{-4} \) |
| Shallow + BAO + BBN + CMB | \( |f_{\text{R0}}| < 1.5 \times 10^{-4} \) |
| Shallow + BAO + BBN + CMB + \( \nu \) | \( |f_{\text{R0}}| < 2.0 \times 10^{-6} \) |
| Deep + BAO + BBN + \( \nu \) | \( |f_{\text{R0}}| < 1.8 \times 10^{-6} \) |

\(^2\) https://www.darkenergysurvey.org
between 1 in the screened regime and 4/3 for the unscreened case. From this we can derive the weighted average
\[
\bar{g} = \frac{\int dr r^2 w(r) g(r)}{\int dr r^2 w(r)}.
\]

(66)

with the weighting function
\[
w(r) = \rho(r) r \frac{\partial \phi}{\partial r}
\]

(67)

which corresponds to the averaged additional potential energy. We follow Schmidt (2010) and make the simplified as-

Figure 11. Left: Evolution of cluster counts with redshift for the shallow optical cluster survey described in the text. Grey shaded bands indicate 1σ uncertainty in the mass-richness-relation. The bottom plot shows relative deviations caused by modified gravity (orange), massive neutrinos (blue), or both (violet). For low redshifts with the given selection function, both effects are approximately a shift in total amplitude of the counts. Right: Richness distribution of cluster counts. The bottom plot shows relative deviations.

Figure 12. Left: Evolution of cluster counts with redshift for the deep optical cluster survey described in the text. Grey shaded bands indicate 1σ uncertainty in the mass-richness-relation. The bottom plot shows relative deviations caused by modified gravity (orange), massive neutrinos (blue), or both (violet). While degenerate at low redshifts, neutrino effects are more pronounced at high z (where the \( f(R) \) mass function reverts to GR). Right: Richness distribution of cluster counts. The bottom plot shows relative deviations. The degeneracy here crucially depends on the position of the \( f(R) \) peak in the relative abundance.
assumption that the fifth force is only sourced by mass outside of the radius given by Eq. 12. Therefore we write
\[ g(r) = 1 + \frac{M(< r) - M(< r_{\text{screen}})}{\frac{2}{3} M(< r)}, \]
where \( r_{\text{screen}} \) is the radius where the equality in Eq. 12 holds. The time evolution of \( r_{\text{screen}} \) and subsequently \( g \) is induced by the background evolution of \( f_R \)
\[ f_R(z) = f_{R0} \frac{1 + \frac{\Omega_m}{\Omega_\Lambda}}{(1+z)^3 + 4 \frac{\Omega_m}{\Omega_\Lambda}}, \]
and the integrals in Eq. 66 can be solved by assuming NFW profiles so both density and potential are determined. Note that \( g \) is only very weakly sensitive to the concentration of the profiles, so we fix the relation to the results of Bullock et al. (2001). Even though halos tend to be more concentrated in \( f(R) \), this does not change our qualitative argument.

From Eq. 64 we therefore expect the mass estimate to be biased compared to GR by
\[ M_{\text{eff}} = g^3/5 M_{\text{true}}, \]
i.e. the SZ signal coming from an unscreened cluster of fixed mass is higher by a factor of \((4/3)^{3/5} \simeq 1.19\) compared to the GR expectation. Similar arguments have been used before to constrain \( f(R) \) by comparing lensing masses with X-ray (Wilcox et al. 2015) or dynamical mass estimates (Pizzuti et al. 2017). Here we want to incorporate the effect into a cluster abundance framework.

To illustrate the method, we consider the consequences for the Planck SZ cluster sample (Planck Collaboration et al. 2016b). There, the hydrostatic mass bias \((1-b)\) is introduced to account for the difference between masses inferred from lensing and the corresponding hydrostatic SZ signal. In \( f(R) \), we therefore expect \((1-b)\) to be modified by an additional factor \( g^{3/5} \). Because the mass definition used in SZ surveys is typically \( M_{500c} \), we calculate NFW potentials to determine \( g \) using this mass definition and we consider a cluster fully screened if the condition in Eq. 12 has not been met at \( R_{500c} \).

In Fig. 13 we show the resulting limiting mass for the Planck SZ selection function. Because the clusters in the sample are very massive, they are screened unless \( f_{R0} \) reaches quite high values \( \sim 10^{-4} \). However, if all clusters in the Planck sample are unscreened, this would be completely absorbed by the fiducial measurement of the bias factor - but because the lensing calibration is performed on very massive objects, smaller objects can still exhibit deviations. This is illustrated with the dot-dashed line for \( |f_{R0}| = 5 \times 10^{-5} \). The resulting Planck SZ cluster counts are shown in Fig. 14. Here we recalibrate our mass function to \( M_{500c} \) using the rescaling outlined in Hu & Kravtsov (2003). While this simplified procedure will not predict the position of the screening mass and the subsequent position of the \( f(R) \) peak in the mass function correctly, we just want to point out that the effect of the adjusted selection function can be quite powerful - in this case as important as the higher cluster abundance from the mass function itself.

The high mass scale for the Planck clusters limits the usefulness of this method here, but upcoming X-ray surveys such as \( e\text{Rosita} \) are expected to detect clusters and groups down to \( M \sim 10^{13}M_{\odot}/h \) where similar methods can be very powerful.

### 5.3 Searching for modified gravity with other parametrisations

The problem in searching for modifications of gravity is that theory space is enormous, and there are potentially many models to test. Current and future cosmological surveys are mostly designed to search for deviations in the dark energy equation of state \( w \) from \(-1\), so we might wonder if these standard searches are sufficient to detect deviations from \( \Lambda CDM \) without assuming a specific model. The hope is then that once an anomaly is detected (for example an equation of state \( w \neq -1 \)), one can resolve the tension in an extended model involving new physics.

---

**Figure 13.** Limiting mass using the Planck SZ cluster selection function for the fiducial case (grey), assuming all clusters are unscreened for a high value of \( f_{R0} = 10^{-4} \) (orange) and an intermediate case where parts of the sample are screened with \( f_{R0} = 5 \times 10^{-5} \) (red, dot-dashed).

**Figure 14.** Redshift distribution for the Planck cluster counts due the halo mass function into account (solid) and selection function effects (dot-dashed). Grey bands indicate a 10% uncertainty in the cluster mass scale \((1-b)\).
Figure 15. Posterior distribution of the main cosmological parameters for the shallow survey + BBN + BAO + CMB with a fiducial model generated using $f_{R0} = 10^{-4}$. All parameters are fully consistent with a vanilla $\Lambda$CDM model and none (including nuisance parameters not plotted here) show significant deviations $>1\sigma$ from their fiducial values indicated by dashed lines.

As a test case, we set $\sum m_{\nu} = 0$ and generate a fiducial cluster catalogue with $f_{R0} = 10^{-4}$ for the shallow optical cluster survey described above combined with CMB and BAO + BBN information. This value of $f_{R0}$ is larger than the 95% upper limit $f_{R0} < 7.2 \times 10^{-4}$ from the same combination of data sets given in Tab. 5. We then explore the posterior assuming a wCDM model and use the according CMB covariance matrix for our prior.

We find that the best-fit wCDM model does not show any significant deviations from the vanilla case. The full posterior distribution of the major cosmological parameters is shown in Fig. 15, and while there are small deviations in the nuisance parameters, all of them are within $1\sigma$ compatible with their standard values without any peculiar features.

We also compare the richness distribution of cluster counts for the best-fit model with the $f(R)$ mock data in Fig. 16 and find no significant deviations. The full parameter space (including the nuisance parameters described above) proves to be flexible enough to account even for a large value of $f_{R0}$ that could be detected if the correct model is assumed in the analysis. This indicates that wCDM might not be a good approach to search for generic deviations from $\Lambda$CDM for models that are not captured by this particular parametrisation.

Therefore we can not necessarily exclude modified gravity (or other) models just from the lack of tensions in the $\Lambda$CDM or wCDM analysis of cosmological surveys. Instead, it is necessary to consider the phenomenology of models individually in order to exclude them.

Figure 16. Bars show the binned richness distribution of clusters for the best-fit wCDM model compared to the fiducial data points generated with $f_{R0} = 10^{-4}$. All parameters for the best-fit model agree within $1\sigma$ with their fiducial values.

6 CONCLUSIONS

In this paper, we presented an accurate halo mass function based on a spherical collapse framework valid for modified gravity and neutrino cosmologies, and calibrated it to a suite of specifically-designed cosmological simulations, the DUSTGRAIN-pathfinder runs. This allows joint constraints from cluster abundance studies. We keep the additional relative change and the fiducial GR mass function separate, so our results can be used with any other mass function calibrated to our mass definition of $M_{200m}$.

The cluster mass definition is crucial to accurately predict the characteristic $f(R)$ peak in the relative abundance because it governs the onset of screening effects. Mass functions for other commonly used mass definitions such as $M_{500c}$ therefore require recalibration of the screening mass, which we refer to future work.

We also demonstrate that the inclusion of neutrinos via a rescaling of the density field Eq. 43 still holds in extended models, and we find a degeneracy between effects of $f(R)$ and massive neutrinos in the abundance of clusters that limits the ability of surveys with small redshift reach to disentangle them. This is likely to weaken existing limits on $f_{R0}$ from cluster abundance, and we will use the mass function for joint constraints using cluster data in a follow-up paper.

Deeper cluster surveys however can tell neutrinos and modified gravity reliably apart by their different redshift evolution, and future optical cluster samples will be able to probe the entire phenomenologically relevant parameter range of the model even when accounting for systematic uncertainties. This could be realised by the complete Dark Energy Survey, eRosita or Euclid\(^4\) cluster samples.

We also explore the possibility to include $f(R)$ effects in the selection function of SZ or X-ray surveys directly as proposed by Schmidt (2010) and we find potentially large effects if the sample can be extended to include nearby, intermediate and low mass objects with $M \lesssim 10^{14}M_\odot/h$. Even though neutrinos can mask the additional abundance in the

\(^4\) https://www.euclid-ec.org/
mass function at low redshifts, it is still possible to detect fifth forces through these selection effects. This allows to incorporate the limits on $f_{R0}$ from comparing lensing mass estimates and X-ray, SZ or dynamical mass estimates consistently into cluster abundance studies in a fully consistent framework.

Finally we find that generic searches for wCDM do not necessarily lead to significant tensions or conspicuous features when used to analyse mock $f(R)$ data – even if the value of $f_{R0}$ could be detected with the same data set in a dedicated analysis. This emphasizes the need to model phenomenology of ΛCDM extensions carefully. A lack of tensions within a parametrisation does not imply the absence of new physics.

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