Scalar and pseudoscalar correlators in Resonance Chiral Theory

J.J. Sanz-Cillero

Abstract

The SU(3) octet $SS - PP$ correlator and the difference of the singlet and octet scalar correlators are computed within Resonance Chiral Theory. The calculation is carried out on the one-loop level, i.e., up to next-to-leading order in the $1/N_C$ expansion. Using the resonance expressions as interpolators between long and short distances, we demand the correlators to follow the high-energy power behavior prescribed by the operator product expansion and extract predictions for the low-energy constants. By adding more and more complicated operators to the hadronic action, our description is progressively improved, producing for the $SS - PP$ correlator the chiral coupling estimates for each separate scalar form-factor and preliminary expressions for the $L_6(\mu)$ low-energy constant.

Keywords:
Chiral Lagrangians, $1/N_C$ expansion

1. Introduction

In this talk we discuss the results from a previous work on SS-PP correlator together with some new estimates for the OZI suppressed difference of the SU(3) singlet and octet scalar correlators ($S_1 S_0 - S_8 S_0$). We will work with a chiral invariant action for the chiral pseudo-Goldstones and the meson resonances, namely, resonance chiral theory (RχT). The large-$N_C$ limit and the $1/N_C$ expansion are taken as guiding principles to sort out the different contributions, being the leading order (LO) provided by the tree-level diagrams and the meson loops suppressed by $1/N_C$. This framework ensures the proper recovery of chiral perturbation theory (χPT) at long distances.

The procedure followed in the analysis of both correlators is identical. First, the structure of the amplitude is constrained by imposing the power behaviour prescribed by the operator product expansion (OPE) for deep Euclidean momentum $p^2 \to -\infty$. After that, one is ready to go to the low-energy regime to make predictions on the $\chi$PT couplings, using the RχT expression as an interpolator between short and long distances. The phenomenological determinations for the LEC $L_8(\mu)$ and $C_{38}(\mu)$, which rule the SS-PP correlator at long distances, were found in good agreement with previous results in the bibliography.

This same machinery has been also applied to the $S_1 S_1 - S_8 S_8$ correlator. It is governed at low energies by the $L_6(\mu)$ low-energy constant (LEC), which happens to be suppressed in the large-$N_C$ limit. The study of this amplitude casts some first interesting results on the relation between singlet and octet scalar resonances, the asymptotic short-distance behaviour of the scalar form-factors and some preliminary results on $L_6(\mu)$.

2. Octet SS − PP correlator

The two-point Green function $SS - PP$ is defined as

$$\Pi_{SS-P}(p) = i \int d^4x e^{ipx} \langle 0|[S^a(x)S^b(0) - P^a(x)P^b(0)]|0\rangle = \delta^{ab}\Pi(p^2),$$

(1)

with the SU(3) octet densities $S^a = \hat{q}^{\lambda}_{\mu\nu}q$ and $P^a = i\hat{q}^{\lambda}_{\mu\nu}\gamma_5 q$, being $\lambda_a$ the Gellmann matrices ($a = 1, \ldots , 8$).
In the chiral limit, assumed all along the paper, the low-energy expansion of the correlator in $\chi$PT up to $O(p^6)$ has the form [12].

\[
\frac{1}{B_0^2}\Pi(p^2) = \frac{2F^2}{p^2} + 32L_8'(\mu_c) + \frac{\Gamma_s}{\pi^2}\left(1 - \frac{p^2}{M_s^2}\right) + \frac{p^2}{F^2}\frac{16C_{38}}{\pi^2}\left(1 - \frac{p^2}{M_s^2}\right) + O(N_c^3) \tag{2}
\]

where $\Gamma_s = 5/48 \ [3/16]$ and $\Gamma_{sl} = -5L_5/6 \ [3L_5/2]$ in $SU(3)-\chi$PT $[18,13]$. The coupling $L_8$ is quite relevant in quantum chromodynamics (QCD) phenomenology, as it is one of the $O(p^4)$ LEC that rule the relation between the quark and the pseudo-Goldstone masses [8].

Within the large–$N_C$ limit, the amplitudes is given by tree-level meson exchanges:

\[
\frac{1}{B_0^2}\Pi(p^2) = \frac{2F^2}{p^2} + 16 \sum_i \left( \frac{d_{m,i}}{M_{s,i}^2 - p^2} - \frac{d_{m,i}}{M_{p,i}^2 - p^2} \right) \tag{3}
\]

where the sum goes over the different resonance multiplets.

In general, there is a reduced knowledge on the spectrum at high energies and just the lightest resonances are moderately well known. Thus, one is forced in many cases to truncate the infinite tower of large–$N_C$ states and to work with a finite number of them or even within the single resonance approximation, as we did in the present work [11]. Still, one may think about interpolating this rational approximation between low-energies -ruled by $\chi$PT- and the deep Euclidean domain at $p^2 \to -\infty$ -governed by the OPE- [14]. After demanding for the short-distance OPE behaviour [4,9,15,16], this minimal hadronical approximation produces the large–$N_C$ Weinberg sum-rules (WSR) [17]

\[
2F^2 + 16d_m^2 - 16c_m^2 = 0, \quad 16d_m^2M_p^2 - 16c_m^2M_s^2 = \epsilon, \tag{4}
\]

where the tiny contribution $\epsilon$ from the dimension four condensate [18] is usually neglected, as we do in the present work [1].

However, there are several caveats that may be raised to this kind of procedures since the truncation might, in principle, induce important uncertainties in the LEC determinations [19]. Indeed, if one analyzes a growing number of observables eventually one reaches inconsistencies between high-energy constraints from different amplitudes [20,21]. Thus, the meson couplings of the truncated theory cannot be the same as in full QCD, being closer for the lightest states but, possibly, very different for the heaviest ones [14]. The conclusion is that there is always a price to pay when applying high-energy constraints to truncated large–$N_C$ amplitudes [22].

But the price can be paid in different ways and one can check out the approximations by observing how much the outcomes differ between one approach and another, e.g., between the determination from Ref. [10] and this work [11]. Thus, the $SS - PP$ dispersive calculation carried out in Ref. [10] was repeated here by explicitly computing the different diagrammatic contributions up to one loop [11]. The main difference was that now we demanded high-energy constraints for the whole correlator $\Pi(s)$ and not for each separate two-meson cut. Likewise, We did not consider short-distance conditions from other amplitudes, as it was done in Ref. [10], where constraints from the $VV - AA$ correlator and the vector and scalar form-factors were taken into account.

The starting point of all the analysis is the LO Lagrangian, which in addition to the leading tree-level amplitudes (LO in $1/N_C$), also generates the leading one-loop diagrams (next-to-leading order in $1/N_C$). The full large–$N_C$ generating functional is then approached by the addition of more and more complicated terms to the $\chi$PT action. First we consider the simplest operators $L_C$ and $L_R$, with, respectively, only Goldstones and one resonance field together with any number of Goldstones [3]. This is the relevant Lagrangian if one remains at tree-level. But at one-loop one needs to provide also a good description of the cuts with one resonance and one chiral Goldstone. One has then to consider the next relevant operators,

\[
\mathcal{L}_{RR'} = i\lambda_1^{PV}(\{\nabla^\mu P, V_{\nu}\mu\}u^\nu) + \lambda_3^{SA}(\{\nabla^\mu S, A_{\mu\nu}\}u^\nu) + \lambda_3^{SP}(\{\nabla^\mu P, S\mu\nu\}u^\nu), \tag{5}
\]

which contain Goldstones and two-resonance fields (one of them can be in the $s$-channel connected with the external scalar and pseudoscalar sources). In principle, one could go further and include also operators with three resonance fields to mend the two-resonance cuts. Nonetheless, we already found a good convergence to our final result without them. The $RR'$ channels have their thresholds at $(M_P + M_R) \sim 2$ GeV and are suppressed enough at low energies for our current level of precision [21].

In general, at next-to-leading order in $1/N_C$ (NLO) the perturbative calculation of the $SS - PP$ correlator $\Pi(p^2)$ shows the generic structure [11]

\[
\frac{1}{B_0^2}\Pi(p^2) = \tag{6}
\]
These relations allow us to fix the renormalized couplings $c_m$, $d_m$, $M_S$, $M_P$ and $L_S$. Notice that other possible NLO operators of the $\chi$PT Lagrangian that could contribute to $\Pi(s)$ are proportional to the equations of motion and have been already removed from the action for the computation of Eq. (6), by means of convenient meson field redefinitions \cite{1, 6, 7, 23}. Their information is then encoded in the remaining surviving couplings $c_m$, $d_m$, $M_S$, $M_P$ and $L_S$.

Next, we demand \cite{3} to vanish like $1/p^6$ at high Euclidean momentum, as OPE dictates (after neglecting the tiny dimension four condensate \cite{17, 18}). At the one-loop level one has

$$
\Pi(p^2) = \sum_{k=0,1,2,\ldots} \left( \frac{1}{p^{2k}} \right)^{\lambda} \left( a^{(p)}_{2k} + a^{(f)}_{2k} \ln \frac{-p^2}{\mu^2} \right),
$$

where now there are $1/p^{2n}$ terms for $n = 0, 1, 2$ which provide three large--$N_C$ conditions $a_0^{(f)} = a_2^{(f)} = a_4^{(f)} = 0$. From the non-log constraints we find, on one hand, the value $\tilde{L}_S(\mu) = 0$ \cite{24} and, on the other, the WSRs of $\chi_T$ get now NLO corrections $A(\mu)$ and $B(\mu)$,

$$
2F^2 + 16d_m(\mu)^2 - 16c_m(\mu)^2 + A(\mu) = 0,
$$

$$
16d_m(\mu)^2M_P(\mu)^2 - 16c_m(\mu)^2M_S(\mu)^2 + B(\mu) = 0.
$$

These relations allow us to fix the renormalized couplings $c_m(\mu)$ and $d_m(\mu)$ up to NLO in $1/N_C$ \cite{1, 10}.

Now, we are in conditions to perform the low-energy limit. The first thing to notice is that one recovers exactly the coefficients of the chiral logs from $\chi$PT, independently of the value of the $\chi_T$ couplings. Thus, the matching with $\chi$PT is always possible and the running of the LEC can be always recovered \cite{24}, producing here the LEC predictions \cite{11}.

$$
L_8(\mu) = \frac{c_m(\mu)^2}{2M_S(\mu)^2} - \frac{d_m(\mu)^2}{2M_P^2} + \tilde{L}_S(\mu) + \xi_L(\mu),
$$

$$
C_{38}(\mu) = \frac{F^2c_m(\mu)^2}{2M_S(\mu)^2} - \frac{F^2d_m(\mu)^2}{2M_P(\mu)^2} + \xi_{C_m}(\mu).
$$

The leading terms, with $c_m^2$ and $d_m^2$, come from the low-energy expansion of the tree-level resonance exchanges, $\tilde{L}_8$ from the local $\chi_T$ contribution and $\xi_{L,C_m}$ from the low-energy expansion of renormalized one-loop diagrams. The last simplification comes after substituting the high-energy OPE constraints \cite{4, 8, 15}, which set $\tilde{L}_8(\mu) = 0$ \cite{1, 24}, fix $c_m(\mu)$ and $d_m(\mu)$ through the NLO WSR \cite{8}, and imposes the three logarithmic large--$N_C$ constraints $a_0^{(f)} = a_2^{(f)} = a_4^{(f)} = 0$ previously referred \cite{11}.

If we now have a look at the phenomenology and make a first large--$N_C$ estimate, we get \cite{11}.

$$
L_8 = (0.8 \pm 0.3) \cdot 10^{-3}, \quad C_{38} = (5 \pm 5) \cdot 10^{-6},
$$

with $M_S = 980 \pm 20$ MeV, $M_P = 1300 \pm 50$ MeV, $F = 90 \pm 2$ MeV and the error given essentially by the naive error in the saturation scale $\mu$, which was varied between 0.5 and 1 GeV.

At NLO in $1/N_C$ with the simplest Lagrangian with operators $L_G + L_R$, with at most one resonance field \cite{8}, the predictions go completely off, yielding $L_8(\mu_0) = (2.28 \pm 0.19) \cdot 10^{-3}$ and $C_{38}(\mu_0) = (26 \pm 4) \cdot 10^{-6}$ for the standard comparison scale $\mu_0 = 770$ MeV, really far away from the usual determinations in the bibliography \cite{8, 10, 11}. The predictions immediately move towards the right direction after including any of the three two-resonance operators $L_{BR}$ of \cite{8}. These new determinations converge into the values $L_8(\mu_0) = (1.1 \pm 0.3) \cdot 10^{-3}$ and $C_{38}(\mu_0) = (9 \pm 4) \cdot 10^{-6}$ if one includes the $L_{PV}$ and $L_{SP}$ operators from \cite{8}, which are essential for the description of the lightest thresholds ($\pi\pi$ and $\pi\rho$) beyond the $\pi\pi$ one. No relevant variation is found after adding to this the last operator $L_{SA}$ in Eq. (5), which is needed for the $A\pi$ channel. Thus, we obtain our final outcome \cite{11}.

$$
L_8(\mu_0) = (1.0 \pm 0.4) \cdot 10^{-3}, \quad C_{38}(\mu_0) = (8 \pm 5) \cdot 10^{-6},
$$

in relatively good agreement with former determinations \cite{8, 10, 11}.

3. $S_1S_1 - S_8S_8$ correlator

We will have a look now at the connected correlator of two scalar densities of different flavour \cite{25}. In the chiral limit, this correlator can be written as the difference of the $SU(3)$ singlet and octet scalar correlators $\Pi_{1-s}(p^2) \equiv \Pi_{S,S}(p^2) - \Pi_{S,S}(p^2)$, being provided at $O(p^2)$ in $\chi$PT at low energies by \cite{25}

$$
\frac{1}{B_0} \Pi_{1-s}(p^2) = 96L_6(\mu_0) + \frac{3\Gamma_0}{\pi} \bigg( 1 - \ln \frac{-p^2}{\mu^2} \bigg),
$$

with $\Gamma_0^{-\mu(3)} = \frac{11}{144}$ and $\Gamma_6^{-\mu(3)} = \frac{5}{7}$.

In general, for the $\chi_T$ Lagrangians we considered (those from previous section), we found in all the allowed one loop diagrams the singlet--octet relation,

$$
\Pi_{S,S}(p^2)^{1-loop} = 2 \Pi_{S,S}(p^2)^{1-loop},
$$

which means that the two--meson cut spectral function obeys the positivity relation

$$
\text{Im} \Pi_{1-s}(t) = \text{Im} \Pi_{S,S}(t) \geq 0.
$$
In addition, the OPE tells us that $\Pi_{1\to s}(t)$ must vanish at short distances like $1/t^2$ [24]. Hence every separate two-meson contribution to $\text{Im}\Pi_{1\to s}(t)$ has to vanish like $1/t^2$ when $t \to \infty$. The reason for this factor–2 relation is the symmetric structure of the meson vertices. Thus, if the flavour flow of a general diagram in our $R_T$ calculation is analyzed, one explicitly finds this factor:

$$\langle T^a(T^a', T^b) \langle T^a', T^b' \rangle T^b \rangle = 6 (\delta^{ab} + \delta^{ab} \delta^{(0)})$$.

The octet generators would correspond to $T^a$ with $a = 1, \ldots, 8$ and the SU(3) singlet one would be $T^9$.

This leads to the constraints obtained in previous works for two-meson scalar form-factors [7, 10, 21]:

$$\frac{4e_\mu}{F^2} = 1 \ (\pi \pi \text{ channel}), \quad \lambda_1^{(P)} = \frac{d_m}{e_m} \ (P \pi \text{ channel}), \quad \lambda_1^{(A)} = 0 \ (A \pi \text{ channel}).$$

If we repeat the procedure applied to the $SS-PP$ correlator $\Pi(s)$ and impose the OPE high-energy behaviour on $\Pi_{1\to s}(s)$, we obtain a pretty simple prediction for the correlator at low energies and its corresponding LEC:

$$L_6(\mu)^{SU(3)} = e_m(\mu)^2 \left( \frac{1}{M_8^2(\mu)} - \frac{1}{M_8^2(\mu)} \right)$$

$$- \frac{d_m^2}{32\pi^2 F^2} \left( 1 + 2M_8^2 \left( \ln \frac{M_8^2}{m_0^2} + \frac{2304\pi^2}{\mu^2} \right) \right),$$

with the last term given by $(\lambda_1^{(A)} - \lambda_1^{(P)})/32\pi^2$ and the $\eta_1$ mass $m_0$, and providing the matching between $U(3)$ and SU(3) $\chi$PT [13]. The first bracket on the r.h.s. contains the tree-level and the one-loop $\pi\pi$ contribution. The $A\pi$ channel turns out to be exactly zero after performing the short-distance matching, which demands $\lambda_1^{(A)} = 0$. The $P\pi$ loops contribute through the term proportional to $d_m^2$ in Eq. (16).

4. Conclusions

The different caveats about the high-energy matching and the truncation of the large–$N_C$ spectrum [14, 19, 21] were confronted in Ref. [11]. Although we worked with a resonance theory with a finite number of mesons and terms in the Lagrangian, a nice convergence to our final outcomes was found as more and more operators were added to the $R_T$ action. Our result for the octet $SS-PP$ correlator was found to be consistent with those from previous analyses [8, 10, 11].

Following the same procedure, we have started the analysis of the difference $\Pi_{1\to s}(s)$ between the SU(3) singlet and octet scalar correlators, which also vanishes at short distances [25] and provides high-energy constraints. The structure of the vertices has led to a positivity relation for the two-meson spectral function. Hence, each separate two-meson cut and the corresponding scalar form-factors must vanish individually at $p^2 \to \infty$ [7, 11, 21]. This allows one to fix most of the resonance parameters entering in this amplitude and yields a pretty compact expression for the corresponding low-energy constant $L_6(\mu)$ in terms of $R_T$ parameters.

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