Application of simulation techniques in supernova physics

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Abstract. In supernova (SN) physics, the responses of nuclear neutrino-detectors to SN neutrino-spectra, are studied by convoluting original theoretical cross sections with well known SN (anti)neutrino-energy distributions (such distributions are the two-parameter Fermi-Dirac and Power-Law distributions). Also, the interpretation of SN neutrino signals, created at various nuclear $\nu$-detectors, is explored by applying simulation techniques in low-energy $\bar{\nu}_e$ anti-neutrino spectra of boosted $\beta$-radioactive $^6$He ions (beta-beam neutrinos). In the present paper, we employ simulation techniques to analyze original SN anti-neutrino signals by using synthetic beta-beam spectra (defined as linear combinations of boosted beta-beam spectra of $^6$He). The quality of the fits, obtained by using the MERLIN optimization package, is in general good. From a nuclear theory point of view, the resulted nuclear responses reflect the effectiveness of some detector materials as SN neutrino detectors (COBRA, CUORE, ICARUS experiments).

1. Introduction
The convolution (or folding) needed in the present work, from a mathematical point of view, is a well known basic operation applied in many areas of research (physics, mathematics, engineering, informatics, etc.) to extract information from images [1]. This operation takes two functions, $f$ and $g$, and produces a third function $C$ that, in a sense, represents the amount of overlap between $f$ and a translated version of $g$. Thus, for two continuous one-dimensional (simplest case) integrable functions $f$ and $g$, the convolution is defined by the integral [1]

$$C(t) \equiv (f \ast g)(t) = \int_{-\infty}^{\infty} f(t')g(t-t')dt'.$$

(1)

One can notice that, the function $g$ in the integrand is shifted. Equation (1) shows that, the convolution constitutes a particular kind of integral transform which blends one function with another and expresses the overlap of the function $g$ as it is shifted over the function $f$. The integration range in Eq. (1), depends on the domain on which $f$ and $g$ are defined and it could be a finite range, but often we take $\alpha = -\infty$ and $\beta = +\infty$ by extending either periodically or with zeros the integration range in both directions. It is worth mentioning the specific type of convolution which is defined between spectral distributions very oftenly used in physics, as we do in the present paper where we utilize Fermi-Dirac (FD) and Power-law (PL) distributions (many times 3-dimensional convolutions are needed) [1, 2].

In this paper, through the energy-spectrum, $\eta(\varepsilon_\nu)$, of a SN neutrino source, represented by Fermi-Dirac (FD), Power-Law (PL), Maxwell-Boltzman (MB), etc. distributions, theoretical $\nu$-nucleus cross sections (of neutral- and charged-current reactions) computed in the context of a
nuclear model, are connected via the convolution procedure with physical observables measured in $\nu$-detection experiments. Then, the signals created at the nuclear detectors can be analysed by utilizing the convolution (folding) method described below.

2. Convolution (folding) procedure for $\nu$-detection studies

For the purposes of the present work, convoluted double differential cross sections as well as convoluted single differential and flux-averaged total (theoretical) cross sections are needed.

In the case of the original double differential $\nu$-nucleus cross section, $d^2\sigma(\theta,\omega,\varepsilon_\nu)/d\Theta d\omega$, where $d\Theta = \sin\theta d\theta$ [3], the convolution with a spectral $\nu$-energy distribution, $\eta(\varepsilon_\nu)$, is defined by the expression [2]

$$
\left[\frac{d^2\sigma(\Theta,\omega)}{d\Theta d\omega}\right]_{\text{fold}} = \int_0^\infty \frac{d^2\sigma(\Theta,\omega,\varepsilon_\nu)}{d\Theta d\omega} \eta(\varepsilon_\nu) d\varepsilon_\nu ,
$$

(2)

where the energy $\omega$ in the lower limit of the integral denotes that incoming neutrinos with energy $\varepsilon_\nu$ cause nuclear transitions for which the excitation energy $\omega$ satisfies the condition $\omega \leq \varepsilon_\nu$ [3].

The SN neutrinos studied here, have rather broad energy distribution, $\eta(\varepsilon_\nu) \equiv dN_\nu(\varepsilon_\nu)/d\varepsilon_\nu$, where $N_\nu$ denotes the number of neutrinos reaching the detector, characteristic of the considered neutrino or anti-neutrino type. The distributions $\eta(\varepsilon_\nu)$ are usually normalized as

$$
\int_0^\infty \eta(\varepsilon_\nu) d\varepsilon_\nu = 1 .
$$

(3)

A realistic energy distribution for SN neutrinos results by inserting the chemical potential $\mu$, in the black body shape distribution. Then, the well-known two-parameter Fermi-Dirac (FD) distribution is obtained which is written as [2]

$$
\eta_{FD}(T, n_{dg}; \varepsilon_\nu) = \left[ \frac{x^2}{e^{x-n_{dg}+1}} \right]^{-1} \left( \frac{\varepsilon_\nu^2/T^3}{1 + \varepsilon_\nu/T} \right) ,
$$

(4)

where $T$ (in MeV) and $n_{dg} = \mu/T$, are the neutrino temperature and the known as degeneracy parameter, respectively.

Alternatively, the SN-$\nu$ energy spectrum can also be described with the analytically simpler two-parameter Power-Law (PL) energy distribution of the form [2]

$$
\eta_{PL}(\varepsilon_\nu), \alpha; \varepsilon_\nu = \frac{(\alpha + 1)^{\alpha + 1}}{\Gamma(\alpha + 1)} \left( \frac{\varepsilon_\nu}{\varepsilon_\nu^\alpha} \right)^{\alpha + 1} e^{-(\alpha + 1)(\varepsilon_\nu/\langle \varepsilon_\nu \rangle)} ,
$$

(5)

where $\langle \varepsilon_\nu \rangle$ is the average neutrino energy. The parameter $\alpha$ adjusts the width of the distribution (pinching parameter). In Fig. 1(a) we show some FD and PL neutrino spectral distributions.

Recently, some accelerated (in storage rings) $\beta$-radioactive nuclei have been proposed as sources of laboratory $\nu$-beams (beta-beam neutrinos) [4, 5]. Such facilities may produce pure beam neutrinos in which the possible neutrino flavors are either the $\nu_e$ (for $\beta^+$-decaying ions) or the $\bar{\nu}_e$ (for $\beta^-$-decaying ions). The laboratory $\nu$-beams are used to search for standard and non-standard neutrino physics at low and intermediate energies ($\nu$-nucleus interactions, neutrino properties, neutrino oscillations, etc.) and measure $\nu$-nucleus scattering cross sections [4].

In the laboratory-frame, the energy distribution of $\beta$-beam neutrinos, assuming that the decaying nuclei are moving with velocity $u = c \sqrt{\gamma^2 - 1}/\gamma$, reads [5]

$$
\eta(\varepsilon_\nu) = \frac{\ln 2}{m_e^2(\gamma)} \frac{F(\pm Z, \varepsilon_\nu) E_e p_e \varepsilon_\nu^2}{2\gamma^3(1 + u)(1 - u^2)} ,
$$

(6)
Figure 1. (a) Equivalent Fermi-Dirac (FD) and Power-law (PL) $\nu$-energy distributions for various values of their parameters. (b) Normalized $\beta$-beam anti-neutrino spectra originating from boosted radioactive $^6$He nuclei. The boost factors are integers between $\gamma=3$ and $\gamma=15$. ($\gamma$ denotes the known Lorentz factor and $c$ the speed of light). In the latter expression, $m_e$, $E_e$ and $p_e$ are the mass, energy and momentum, respectively, of the outgoing $e^{-}$ (or $e^+$) of the reaction [see Fig. 1(b)]. ($ft$) is the known ft-value and $F(\pm Z, \varepsilon_e)$ denotes the Coulomb correction function (Fermi function) which accounts for the electromagnetic interaction between the emitted $e^{-}$ (or $e^+$) and the charge distribution of the daughter nucleus (final state interaction) [2].

By exploiting low-energy $\beta$-beam $\nu$-spectra of the form of Eq. (6), corresponding to different boost velocities ($\gamma$-factors), one could construct normalized synthetic neutrino energy distributions $\eta_{bb}(\varepsilon_\nu)$ given by linear combinations as

$$\eta_{bb}(\varepsilon_\nu) = \sum_{j=1}^{N} \alpha_j \eta_{\gamma_j}(\varepsilon_\nu),$$

(7)

where $N$ is the number of different $\gamma$ boosted spectra included in the synthetic spectrum [2, 4].

The convolution of the theoretical cross sections [with $\nu$-spectra of the form of Eqs. (4)-(7)], provides estimates of nuclear-detector responses to energy-distributions of the observed neutrinos. The features of the arriving at the detector SN-$\nu$ flux are encoded in the nuclear response of the detector which theoretically could be reproduced by convoluted cross sections calculations [1, 3]. As an example, using Eqs. (6) and (7), we apply the convolution method to simulate SN-$\nu$ signals recorded at a nuclear detector with detector medium the $^{66}$Zn isotope [5].

3. Application of simulation techniques in SN-$\nu$ detection

The distributions of synthetic spectra $\eta_{bb}(\varepsilon_\nu)$, may be used to fit original SN-$\nu$ energy spectra, $\eta_{SN}(\varepsilon_\nu)$, reaching terrestrial $\nu$-detectors. This may be achieved by adjusting the weight parameters $\alpha_j$ of Eq. (7) through a minimization procedure which determines the participation of each of the $\gamma_j$ components in the linear combination $\eta_{bb}(\varepsilon_\nu)$ [4]. The best fit of $\eta_{bb}(\varepsilon_\nu)$ to an original SN-$\nu$ spectrum $\eta_{SN}(\varepsilon_\nu)$, representing the signal recorded at the $\nu$-detector, results by minimizing the integral [4, 5]

$$Q = \int_0^\infty \left| \sum_{j=1}^{N} \alpha_j \eta_{\gamma_j}(\varepsilon_\nu) - \eta_{SN}(\varepsilon_\nu) \right| d\varepsilon_\nu.$$

(8)

For the simulations performed in the present paper, we employ various SN-$\nu$ spectral distributions. The minimization of the integral in Eq. (8), was achieved by utilizing the optimization algorithm MERLIN [2, 6].
Figure 2. Folded total cross-section (in $10^{-40}$ cm$^2$) $\sigma_{\text{fold}}^{9\gamma}$ (full line) for $^{66}\text{Zn}$. They resulted by using a synthetic spectrum with 9 Lorentz factors $\gamma = 5, 6, \ldots, 13$. The assumed SN-$\nu$ signal, $\sigma_{\text{fold}}^{\text{sign}}$ (dash-dotted line), is well reproduced [w = 0.8 in (a) and w = 0.9 in (b), see text]. The illustrated normalized original SN-neutrino spectrum, $\eta_F \epsilon (\nu)$, is in $10^{-2}$ MeV$^{-1}$ [2].

In Fig. 2 (full line), the folded total cross-section of a signal, $\sigma_{\text{fold}}^{\text{sign}}(\nu)$, recorded at the $^{66}\text{Zn}$ detector [5], is compared with the corresponding (fitted) folded cross section, $\sigma_{\text{fold}}^{bb}(\nu)$. The latter is obtained by using the synthetic spectrum $\eta_{bb}(\nu)$ with nine components ($N=9$), i.e. beta-beam anti-neutrino spectra with Lorentz factors $\gamma = 5, 6, \ldots, 13$ (see dashed line of Fig. 2). The assumed original SN-$\nu$ spectra that create the signal on the detector, are described by two-parameter FD distributions. Their parametrization is described in Fig. 2, where we give also the width parameter $w$ defined in Eq. (19) of Ref. [2], namely $w = 0.8$ Fig. 2(a), and $w = 0.9$ Fig. 2(b). As can be seen, in all cases the peak of the synthetic spectrum, $\eta_{bb}(\nu)$, is located at lower energies as compared to the folded total cross-section $\sigma_{\text{fold}}^{bb}(\nu)$. Concerning the quality of the fitting in Fig. 2, even thought the MERLIN package is a very effective algorithm [6], it is not very good due to the fact that the curves for $\gamma = 3, \ldots, 7$ have noticeably different shapes than those for $\gamma > 7$. We mention that, in Ref. [4] authors utilized a weighted factor of the order of 2 for values $\gamma > 7$ which improves significantly the quality of the fit.

4. Conclusions
We concentrated on the convolution method used in nuclear and astro-nuclear physics studies and, in particular, in the investigation of nuclear responses of various neutrino detectors to the energy-spectra of SN-$\nu$. By using the minimization procedure utilized recently to fit the synthetic beta-beam spectra on original SN-$\nu$ spectra we employed various boosted beta-beam spectra of $\beta$-radioactive $^6\text{He}$ ions to simulate specific SN-$\nu$ scenarios.

References
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