Drop deformation in two-roll mills considering wall effects

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Abstract. Experimental, theoretical and numerical results of dynamics of drop deformation in strong flows generated by a co-rotating two-roll mill and considering the influence of near rigid walls are presented. The drop dynamics is altered, with respect to a drop free of wall effects, by the proximity of the rigid boundaries as well as caused by a non-linear and non-uniform flow due to gradients of flow-type parameter and shear rate. Simulations were carried out using the Boundary Element Method (BEM). Since the inclusion of the whole boundaries (drop and rollers surfaces) is not an easy and trivial task, bi-dimensional numerical simulations was performed as a first approach. The experimental and numerical results were obtained for a flow type of $\alpha = 0.03$ and two values of viscosity ratio $\lambda = 0.012$ and 16. In general, numerical results for the stationary deformation parameters, up to intermediate confinements, are in agreement with the experiments, with and without wall effects. Since the case of drops with a high viscosity ratio did not match existing theoretical models, the wall-effect theory of Shapira and Haber was modified, considering Cox’s second-order theory as the converging theory without wall effects. From low to intermediate confinements, the new Cox-Shapira-Haber model fitted the observed experimental deformations.

1. Introduction

Deformation and breakup of a droplet in a continuous immiscible liquid phase under confined flow is of interest in several applications, such as microfluidics technologies and emulsion processing [1], among many other where the flow dynamics can be significantly altered by the presence of channel walls.

Most of previous studies have investigated the deformation of drops in confined simple shear flows (which corresponds to $\alpha = 0$) by the relative motion of parallel plates [2-4]. The experimental [2, 3] and numerical [4] results remit to Shapira and Haber theory (SH model) [5] that is valid for small deformations. This theory predicts the total deformation of a drop under simple shear flow given by:

$$w = D_0 \left( 1 + \left( \frac{R}{W} \right)^3 \frac{1+2.5\lambda}{1+\lambda} C_s \right),$$

(1)

where $D_0$ correspond to deformation parameter of Taylor [6] —being a function of the capillary number $Ca$—, $R$ is the initial radius of drop, $W$ is the separation between the walls, $\lambda$ is the viscosity ratio, and $C_s$ is the so-called shape factor, which depends on the position of the drop. Recently, Maffetone and Minale [7] adapted this model (we will call it SH-MM Model), where $D_0$ corresponds to the deformation of ellipsoidal drops proposed previously by them [8].
Experimental and numerical studies [2-4] have compared the SH theory and found it valid for small capillary numbers \((Ca < 0.3)\) and intermediate ratios of confinement \((2R/W < 0.35)\). Beyond these limits the shapes of stationary drops shift from ellipsoidal to sigmoidal.

In this work, the flows are non-linear, two-dimensional and capable of generating a wide spectrum of values of its kinematic parameters: in one extreme the flow is very close to simple shear, sweeping smoothly towards the other end where significant elongation effects can be present. The flow generated by two-roll mills (2RM’s), has been studied analytically and numerically in the past, with general solutions being available [9, 10, 11] and more recently experimentally [12] as well. The size of the cylinders and their separation determine the value of the flow-type parameter \(\alpha\) at the stagnation point, while the shear rate \(\gamma\) also at the stagnation point depends on the angular velocities of the cylinders. For 2RM’s, the shear rate has a minimum value at the stagnation point and a maximum value on the surface of the cylinders while the flow-type has a maximum value at the stagnation point and a minimum value on the cylinders surface [9, 10].

In order to study the effect of walls on drops, the size of the drop (diameter) must be comparable to the gap between the rolls. Under those conditions, the drop deforms in flow fields with gradients on the flow-type parameter \(\alpha\) and the shear rate \(\gamma\) around the stagnation point. This combination of features makes the dynamics of droplets influenced by wall effects a non-idealized system not fully analysed up to date.

Since the experimental and numerical results of this work did not match the space of parameters of existing studies, a model that preserves the dependence of the confinement factor on deformation (in the spirit of the initial model) is developed, assuming as well drops with high viscosity ratios. Thus, \(D_0\) corresponds now to the steady free deformation of Cox theory [13].

1. Experimental

An extensive description of the 2RM flow can be found in [9-12]. The radius of the cylinders was 49 mm, with a separation between their centers of 52 mm giving a gap \((g)\) of 3 mm. The experiments were carried out for two viscosity ratio \((\lambda = 0.012\) and 16). In both cases the suspending fluid was PolyDiMethylSiloxane (PDMS), DMS T35 from Gelest Inc., with viscosity \(\mu_m = 51\) Poise and relative density \(\rho_m = 0.973\) at 25 °C. For the low viscosity ratio system the drop fluid consisted of vegetable canola oil with a viscosity \(\mu_{d1} = 0.6\) Poise and a relative density \(\rho_{d1} = 0.917\) at 25 °C. The interfacial tension for this system was 2.7 mN/m. For the high viscosity ratio system the drop fluid consisted in PolyIsobutilene by Polysciences Inc., with a viscosity \(\mu_{d2} = 816\) Poise and a relative density \(\rho_{d2} = 0.92\) at 25° C. The corresponding interfacial tension was 3.5 mN/m. All experiments were performed at 25°C ± 0.1°C.

2. Numerical method

The boundary integral equations take into account explicitly the presence of rigid surfaces and deformable fluid interfaces, simulating the deformation of a drop between two cylinders. The integral equation for the flow field within the drop, \(u_d(x)\), or exterior to the drop in the matrix fluid, \(u_f(x)\), will be dependent upon boundary integrals taken on the drop interface \(S_{\text{drop}}\) as well as on the cylinder surfaces \(S_{\text{rigid,1}}\) and \(S_{\text{rigid,2}}\). These integral equations can be found in reference [14].

In Figure 1, a schematic representation is depicted. We assume a Newtonian, inertial-less drop with initial radius \(R\) and viscosity \(\mu_{\text{drop}}\)—located between the two cylinders—and immersed in
a Newtonian matrix fluid of viscosity $\mu_m$. The viscosity ratio was $\lambda = \mu_{\text{drop}}/\mu_m$. The position of the stagnation point is located exactly halfway between the cylinders when the angular velocities are same.

3. Results

The numerical and experimental results obtained in a two-roll mill device for stationary drop deformation values are presented as a function of capillary number and also compared with theoretical models of: (a) small deformation Cox theory [13], (b) Chaffey and Brenner model [15], for angle orientation in the free case, and (c) SH-Cox model, adapted here in the case of confined drop deformation.

3.1. Stationary Drop deformation. The case of Low viscosity ratios

The lowest viscosity ratio ($\lambda = 0.012$) was studied experimental and numerically for different capillary numbers (between 0.05-0.8). Figure 2 shown the deformation and orientation for the stationary drop deformation as a function of capillary number for two confinement factors $d/g = \frac{2R}{g}$.

For weak confinement a good agreement was found between the experimental, numerical, and theoretical models. In the numerical results, when $Ca > 0.8$, no stationary deformation was observed (drop break up occurs), whereas in the experimental results the critical capillary was found around $Ca = 0.95$. In Figures 2c and 2d a good agreement between numerical and experimental results can be observed in the case of confined drops, whereas the SH-Cox model exhibited a higher deviation with respect to the experimental results for $Ca > 0.4$ whereas no change was observed in the final angle orientation due to confinement. It was corroborated that there is not difference in the use of the Taylor, SH-MM or Cox theories for these viscosity ratios. Such theories coincided and predicted a linear dependence between capillary number and the deformation parameter.

3.2. Stationary Drop deformation. High viscosity ratio case
For weaker confinements, \( \frac{d}{g} = 0.1 \), the numerical results agreed with the values obtained using SH-Cox model, as it can be seen in Figure 3a; however, numerical and theoretical results gave larger deformation values. This discrepancy might be the result of the experimental control system that tends to enhance the flow vorticity, thus inhibiting slightly the experimental drop deformation. On the other hand, good agreement for \( \frac{d}{g} = 0.205 \) can be observed in Figure 3b. Numerical results and the SH-Cox model were very close to experimental results. Under this condition, the non-linear effects of the walls may mitigate the non-idealities of the control scheme of the flow.

For a higher confinement, \( \frac{d}{g} = 0.337 \), a good agreement was also found, as it is shown in Figure 3c. However, the theoretical models predict larger deformation than experimental or numerical data. For more pronounced confinement effects, \( \frac{d}{g} = 0.433 \), Figure 3d is shown. In this last case the numerical simulations and theory gave deformations larger than those occurring in experimental results. Both, SH and SH-MM models fail to predict this asymptotic behavior observed for high viscosities, even in the no-wall effects case, that's because these model preserves in some way, the linear dependence on Ca. Alternatively it is possible use the Taylor asymptotic model [6] but this not describe the initial dependence on Ca.

![Figure 3. Stationary deformation versus capillary number. Comparison of experimental (■), numerical (●) and theoretical (—, SH-Cox model), (—, SH-MM model) for four confinements factors, \( \frac{d}{g} = 0.1 \) (a), \( \frac{d}{g} = 0.205 \) (b), \( \frac{d}{g} = 0.337 \) (c) and \( \frac{d}{g} = 0.433 \) (d).](image)

![Figure 4. Comparison of theoretical model (Cox theory) with numerical (a) and experimental (b) results obtained for the orientation angle of the stationary drop deformation in a flow field generated by the two-roll mill device for different confinement factors.](image)
The steady state orientation angle of the deformed drops did not manifest wall effects, for neither the experimental or numerical results. In fact, both methods agreed with Cox Theory [13] as it can be seen in Figure 4. A comparison of numerical, experimental and theoretical results for stationary drop deformation values, as a function of confinement for high viscosity ratio, is shown in Figure 5. It can be observed that the agreement with predictions of the SH-Cox model were mostly qualitative. This difference may be attributed to the flow dynamics of the non-linear flow of the 2RM’s.

4. Conclusions
Numerical simulations using BEM in the study of stationary drop deformation with confinement effects showed a noticeable agreement with experimental results obtained in a device able to generate non-idealized flows (non-linear flow fields with gradients in shear rate and flow type, and curved walls). All phenomena observed in the experiment were reproduced numerically. Despite the bi-dimensional simulations nature, the numerical model captured the basic hydrodynamics, which confirm its predictive nature, mainly due to the fact that drop deformation remain small for most experimental conditions, and the drop curvature is quasi-homogenous. This work is the first effort to encompass the full kinematics existing in the two roll mills in the study of confined drop deformations. The adapted model SH-Cox model predicts the drop deformation on intermediate confinements and small deformations, covering the full regime of high viscosities drop deformations under wall effects.

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5. References
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