Comparative Analysis of GPS Data

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Comparative Analysis of GPS Data

Abstract
The goal of this project is to calculate the distance traveled during a bike ride around the University of South Florida using information gathered by a GPS (Global Positioning System) application on a smartphone. We calculate the route distance in two ways: from latitudes and longitudes using the haversine formula and 2) from velocities and times using the trapezoidal rule. These computed distances were compared to the distance given by the smartphone application.

Keywords
Haversine formula, Trapezoidal rule, GPS Tracker, Travel distance

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The purpose of this project is to calculate the distance traveled on a bicycle around the University of South Florida (Figure 1) by using an open GPS tracker on an Android smartphone. The data was recorded in the format of a GPX file, which contains time, latitude, longitude, elevation and accuracy information of a data point. Data points were recorded every one to two seconds to ensure an accurate path.

Using this information, calculate the distance of the journey in two ways: 1) Use the Trapezoidal rule with the speeds and times of the trip and 2) Use the haversine formula with the latitude and longitude coordinates of the trip. Compare the results from these calculations with the distance given by the GPS tracker application to see if the methods agree.

Given that thirty-five percent of cell phone users owning smartphones, people have the ability to use GPS technology to better navigate unknown areas, map out rides, and integrate location information with social media (Smith 2011). Applications like Open GPS Tracker on the Android, record location data and displays along a path on a map of the area detailing speed, and distance traveled. GPS applications usually output location data as a GPX with latitude,
longitude and speed at a specific time. Our goal in this study is to find a way to calculate the
distance traveled with the GPX file to figure out how the application calculates distance traveled
and to find other ways to calculate distance using principles in calculus.

**MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH**

The smartphone GPS application outputs data captured over the bicycle ride around
campus in the form of a GPX file and the latitude, longitude, speed, and distance were converted
to an Excel spreadsheet. Assuming a linear change in speed between the sampled data points, the
total distance traveled equates to the area beneath the curve of the *Speed (MPH)* versus *Time (s)*
plot (see Fig. Figure 2: Speed (mph) versus Time graph for the entire journey.).

![Speed vs Time Graph](https://scholarcommons.usf.edu/ujmm/vol5/iss2/1)

**Figure 2:** Speed (mph) versus Time graph for the entire journey.

Using the *Trapezoidal* rule we can approximate the area under the curve as

$$\int_{a}^{b} f(x) \, dx \approx \frac{1}{2} \sum_{k=1}^{N} (x_{k+1} - x_{k}) (f(x_{k+1}) + f(x_{k}))$$  \hspace{1cm} (1)

where $x_{k}$ and $f(x_{k})$ are the time and speed for data point $k$, respectively (see Figure 3). Summing
the piecewise trapezoids for the entire journey yields a total distance of 1.173 miles for the
bicycle trip.
Figure 3: Area under the Speed vs. Time curve approximated piecewise as trapezoids for the first 8 seconds of the journey.

We now calculate the distance of the trip using latitudes and longitudes rather than speed and time. The haversine function is defined as

\[ \text{haversin } \theta = \sin^2 \left( \frac{\theta}{2} \right) \]  

(2)

and naturally arises in the study of spherical geometry. In particular, it can be used to calculate the circular distance \( d \) between two points embedded on the outside of a sphere. Given point \( p_1 \) at latitude \( \phi_1 \) and longitude \( \lambda_1 \) and point \( p_2 \) at latitude \( \phi_2 \) and longitude \( \lambda_2 \), the haversine formula for \( d \) is

\[ d = 2r \sin^{-1} \left( \sqrt{\text{haversin}(\phi_2 - \phi_1) + \cos(\phi_1) \cos(\phi_2) \text{haversin}(\lambda_2 - \lambda_1)} \right) \]  

(3)

where \( r \) is the radius of the sphere. In this case, the bicycle is moving around the outside of the earth, so we take \( r = 3,959 \) miles. Summing the length of the arcs between all the data points amounts to a total trip distance of 1.168 miles.
The distance of the bicycle ride was obtained in three different ways: the trapezoidal rule, the haversine formula, and the application available on the smartphone. Both calculated distances are in good agreement with the distance given by smartphone application, considering the amount of variables that can affect the accuracy of the data. The distances obtained from the haversine formula and trapezoidal rule deviate 0.001 miles and 0.006 miles respectively from that of the smartphone application (see Table 1).

| Method        | Miles |
|---------------|-------|
| Haversine Formula | 1.168 |
| Trapezoidal rule     | 1.173 |
| Application                | 1.167 |

Table 1: The results obtained from three different methods.

CONCLUSION AND RECOMMENDATIONS

We successfully found two methods to calculate distance from a GPX file by using the Haversine formula and the Trapezoidal rule. Both methods proved to be accurate within a 0.09 to 0.51 percent difference from the distance calculated by the smartphone application, but the haversine formula looks to be more consistent with the method used by the smartphone application.
NOMENCLATURE

| Symbol | Description | Value |
|--------|-------------|-------|
| $p_k$  | $k^{\text{th}}$ GPS point from GPX file | - |
| $x_k$  | Time at point $p_k$ | - |
| $f(x_k)$ | Velocity at point $p_k$ | - |
| $r$    | Radius of the Earth | 3,959 miles |
| $\phi_k$ | Latitude at point $p_k$ | - |
| $\lambda_k$ | Longitude at point $p_k$ | - |

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