Abstract

Based on well-known properties of semi-classical black holes, we show that weakly-coupled string theory can be viewed as a theory of $N = 1/g_s^2$ particle species. This statement is a string theoretic realization of the fact that the fundamental scale in any consistent $D$-dimensional theory of gravity is not the Planck length $l_D$, but rather the species scale $L_N = N^{\frac{1}{D-2}} l_D$. Using this fact, we derive the bound on semi-classical black hole entropy in any consistent theory of gravity as $S > N$, which when applied to string theory provides additional evidence for the former relation. This counting also shows that the Bekenstein-Hawking entropy can be viewed as the entanglement entropy, without encountering any puzzle of species. We demonstrate that the counting of species extends to the $M$-theory limit. The role of the species scale is now played by the eleven-dimensional Planck length, beyond which resolution of distances is gravitationally-impossible. The conclusion is, that string theory is a theory of species and gets replaced by a pure gravitational theory in the limit when species become strongly coupled and decouple.
1 The Species Scale

Consider a consistent quantum field theory in $D = 4 + d$ space-time dimensions in which gravity flows to the Einsteinian theory in deep infra-red (IR). This condition implies that at large distances the gravitational force is mediated by a $D$-dimensional massless particle of spin-2, the graviton $h_{\mu\nu}$. The corresponding Newton’s constant we shall denote by $G_D \equiv M_D^{-(D-2)}$, where $M_D$ is a $D$-dimensional Planck mass. Let $L_s$ be a length scale defining the gravitational ultra-violet (UV) cutoff of the theory, beyond which the quantum gravitational effects can no longer be ignored. In particular, $L_s$ marks the size (generalized Schwarzschild radius) of a smallest possible semi-classical black hole (BH). We shall assume, that around and below the scale $M_s$ there are $N$ different elementary particle species. To be more precise, $N$ counts all the weakly-coupled elementary particles with the decay width much less than their mass.

Previous studies [2–4] have shown, that in any consistent theory satisfying the above conditions there is an absolute lower bound on the scale $L_s$, given by,

$$L_s > L_N \equiv N^{\frac{1}{D-2}} l_D,$$

(1)

where, $l_D \equiv M_D^{-1}$ is the $D$-dimensional Planck length.

Below we shall refer to $L_N$ as the species scale. The aim of the present paper is to show that $L_N$ plays a fundamental role in any consistent theory of gravity. Namely, $L_N$ is an absolute shortest distance, beyond which resolution of species is impossible, in principle. In particular, gravitational consistency of string theory demands that $L_N$ sets the larger of the two scales, the string length, $l_s$, or $l_D$. As we shall see, in a weakly coupled string theory we have $l_s = L_N$, whereas in the strongly coupled $M$-theory limit, in which $l_s \ll l_{11}$, the role of $L_N$ is assumed by the Planck length, $L_N = l_{11}$.

We shall reproduce two different derivations of the bound (1), and show, that when applied to string theory, they reveal that string theory is a theory of species with their effective number given by,

$$N_{eff} = \frac{1}{g_s^2}.$$  

(2)

This may come as a surprise, since it is known that the number of string Regge excitations diverges exponentially with the mass level. So one expects that from the effective field theoretic point of view at high-energies the string theory for all the practical purposes should behave as a theory of exponentially large number of species. Contrary to this
expectation, the effective number of species resolved by gravity appears to be much smaller and finite. This point of view was already suggested by the findings of [7], which indicated that effective number of string resonances participating in the evaporation of semi-classical BHs is not exponentially large, and is limited by $1/g_s^2$. Our present analysis reinforces this conclusion, but without any reference to the details of the BH and string dynamics in UV. We shall rather show that it follows solely from the extremely well-understood properties of semi-classical BHs and the known relations among the scales in string theory.

The conclusion that will emerge from our analysis is, that string theory is theory of species coupled to gravity and this statement can be extended beyond the weak coupling. In the strongly coupled limit of $M$-theory in which string species decouple, we are left with pure supergravity theory in 11 dimensions with a single propagating graviton multiplet. In full accordance with [1], the role of $L_N$ is then taken by the 11-dimensional Planck length $l_{11}$, which sets the absolute bound on the resolution capacity of any detector.

In addition, we derive the bound on entropy in any theory of the above sort as [12],

$$S > N.$$  \hspace{1cm} (3)

When applied to string theory this bound again translates as the relation [2].

One of the direct consequences of our species counting, is the full equivalence between Bekenstein-Hawking and entanglement entropies without encountering any puzzle of species.

## 2 Bound on Species Scale from Black Hole Evaporation

We shall define the effective number of species using the properties of semi-classical black holes. Our focus will be on classically-static, neutral, non-rotating BHs on asymptotically flat $D$-dimensional space. We shall assume, that the BH in question is by design neutral under all possible long-range gauge fields, and the only characteristic quantum number is its mass $M$, which determines its Schwarzschild radius $R_g(M)$. Notice, that as shown in [3], in any ghost-free $D$-dimensional theory, for given $M$, $R_g$ is minimal in a pure-Einsteinian gravity, in which the only propagating $D$-dimensional degree of freedom is massless spin-2. The relation between $R_g$ and $M$ is then given by (we shall ignore the
factors of order one throughout the paper),

\[ R_g^{D-3} = M G_D . \]  

(4)

Any departure from \( D\)-dimensional Einstein at short distances can only increase \( R_g \) per given \( M \). This is because the addition of any extra gravitational degrees of freedom can only strengthen the gravitational attraction in the classical domain. Thus, any consistent deviation from the Einstein gravity increases the value of \( R_g \) relative to Einstein. Other than this constraint, we shall keep the relation between \( R_g \) and \( M \) completely general. The crucial property, however, is that the Hawking temperature of a semi-classical BH is always \( T = R_g^{-1} \).

Under the above conditions, the evaporation of a semi-classical black hole obeys the following law of the temperature-change,

\[ \frac{1}{T} \frac{dT}{T} = \Gamma_{\text{total}} , \]

(5)

where \( \Gamma_{\text{total}} \) is the total rate of the emission of all the particle species. The evaporation rate into heavy species of mass \( m \), is suppressed by the Boltzmann factor \( e^{-\frac{m}{T}} \).

So, for defining the effective number of species at the given temperature, it is most useful to normalize the emission rate to that of a massless graviton. The latter rate by general covariance is given by,

\[ \Gamma_{gr} = T \left( \frac{T}{M_D} \right)^{D-2} . \]

(6)

We can thus introduce the dependence on the number of species in BH evaporation rate in the following way,

\[ \frac{1}{T} \frac{dT}{T} = \Gamma_{gr} N . \]

(7)

Thus, \( N \) counts the BH evaporation rate in the units of the graviton emission rate \( \Gamma_{gr} \).

Using the explicit form of (6) for \( \Gamma_{gr} \) we can rewrite (7) as,

\[ \frac{1}{T^2} \frac{dT}{T} = \left( \frac{T}{M_D} \right)^{D-2} N . \]

(8)

The usefulness of the latter equation is that the left hand side of it represents a semi-classicality parameter (we shall call it \( \xi \)),

\[ \xi(T) \equiv \frac{1}{T^2} \frac{dT}{T} , \]

(9)
which defines the semi-classicality condition of the BH through the inequality,

$$\xi(T) \ll 1.$$  \hspace{1cm} (10)

Any classically-stable static non-rotating neutral BH must satisfy the above condition. Violation of it means, that the BH is out of the semi-classical regime. The lowest temperature $T_*$ for which the condition (10) is violated, marks the scale of the gravitational cutoff for the classical theory. That is $T_* = M_*$. The corresponding BH size, $T_*^{-1}$, defines the shortest length-scale of semi-classical gravity. The value of the cutoff $M_*$ is thus uniquely determined by the value of the temperature $T_*$ for which $\xi(T_*) = 1$. This leads to the bound (1) as well as to a maximum BH temperature $T_* = 1/L_N$. As we will discuss in the next section, this bound on temperature naturally fits with the Hagedorn bound in string theory.

Notice, that although for our derivation it is enough that the Schwarzschild radius is given by the usual $D$-dimensional Einstein relation (11), the bound on $L_N$ is in fact absolute. It cannot be avoided even if, let us say, the laws of classical gravity start to depart from $D$-dimensional Einstein’s general relativity at short distances. As explained above, any modification of Einsteinian gravity in the weakly-coupled classical domain can only strengthen the gravitational interaction relative to Einstein [11]. From here it follows [1], that Einsteinian gravity has the smallest horizon per given mass of a BH. Thus, any hypothetical deviation of classical gravity from $D$-dimensional Einstein’s theory, can only increase the species scale relative to the value of $L_N$ given by (1), but can never make it shorter.

3 Species Scale as the Holographic Scale

An alternative derivation of the species scale [3] follows from the impossibility of resolving species identities down to arbitrarily short length scales. The fundamental obstacle is created by gravity, as we shall now explain. Physically, having $N$ distinct species means that one can label them, and distinguish these labels by physical measurements. Let $\Phi_j$ be particle species and $j = 1, 2, ..., N$ be their labels. One can talk about species as long as their identities can be resolved, at least in principle. Gravity puts an insuperable lower bound on the resolution capacity of any detector. To see this, let us assume that $L$ is the size of an elementary pixel of the detector that is decoding the species identities. Elemen-
tary act of recognizing the species label \( j \) is a scattering process in which an unknown particle scatters off an elementary pixel. In order to read the species label, the pixel must contain either a sample of all the \( N \) different species, or an equivalent information. This fact automatically limits the size of the pixel from below. Indeed, localization of each sample particle within the pixel of size \( L_{\text{pixel}} \) costs energy \( E = 1/L_{\text{pixel}} \), which implies that the total mass within the pixel is at least \( M_{\text{pixel}} = N/L_{\text{pixel}} \). The corresponding Schwarzschild radius is

\[
R_{\text{pixel}}^{D-3} = \frac{N}{L_{\text{pixel}}} G_D. 
\]  

The key point is, that such a pixel cannot be made arbitrarily small, since eventually its Schwarzschild radius will exceed the size of the pixel, and this will happen while the size is still larger than the Planck length \( l_D \)! This means that at the crossover size, the pixel will collapse into a \textit{classical} BH, and the information will take much longer time (if ever) to be retrieved. Notice that this lower bound is absolutely insensitive to the question of the information loss by the BH. Regardless, whether information is lost or not, once the pixel collapses into a semi-classical BH, one has to wait at least the evaporation time, which in any case exceeds \( L_N \).

Thus, for resolving the species identities on the scale \( L_{\text{pixel}} \) it is necessary that \( R_{\text{pixel}} < L_{\text{pixel}} \). This condition after taking into the account (11), gives the limiting size of the pixel as \( L_N \) given by (1).

The holographic \([1, 6]\) meaning of \( L_N \) is actually quite transparent. In fact \( L_N \) is the minimal size where we can store \( N \) bits of information. The reason this holographic scale coincides with the species scale is, that the physical resolution of species requires, as discussed above, at least a number of information bits equal to the number of species. The only assumption in this holographic argument is to identify, as it is customary in our present interpretation of holography, the gravitational Planck length with the holographic scale defined as the minimum size where a bit of information can be stored. The holographic bound on information storage implies that whenever we try to prove sub-Planckian scales by localizing a particle in a region of size \( l \) smaller than \( l_D \), the corresponding dual description of that sub-Planckian physics requires a number of holographic degrees of freedom equal to \((l/l_D)^{D-3} \). Gravity as the way to set the holographic scale leads to an IR/UV correspondence between the energy scales \( l \) we want to probe and the energy \((l/l_D)^{D-3} (l_{pl}/l) \) \((l_{pl} \text{ in four dimensions}) \) of the corresponding dual holographic
degrees of freedom.

4 Species Scale in String Theory

4.1 Breakdown of Black Hole Semi-Classicality in Ten Dimensions

We shall now show that the above facts lead us to the relation (2) in string theory. The first immediate evidence for this relation comes from applying (8) to the evaporation of the semi-classical BHs in ten dimensions. This gives,

$$\xi(T) = \left(\frac{T}{M_{10}}\right)^8 N.$$ (12)

Assuming that the BH evaporation stays semi-classical (implying $\xi(T) \ll 1$) at least till temperatures of order $T \lesssim M_s$, we get the bound

$$N < (M_{10}/M_s)^8,$$ (13)

which after substituting the well known relation between the string and the Planck scales,

$$M_{10}^8 = \frac{M_s^8}{g_s^2},$$ (14)

gives (2).

We see, that the requirement that ten-dimensional BHs should “afford” to stay in semi-classical regime till the temperature $l_s$, immediately implies the bound (2) on the number of species in string theory.

4.2 Chan Paton Factors: $D$-Branes as Species-Resolving Device

The same conclusion arises from considering the Chan Paton (CP) factors. These factors count the independent end-points of the open strings attached to $D$-branes. In this way CP factors label different zero mode species in the world volume theory of the $D$-branes. A stuck of $n$ $D$ branes results in $n^2$ CP factors and thus in $N = n^2$ zero mode species (to be more precise in $n^2$ super-species, the degeneracy factor within the supermultiplet will be ignored). So, naively it may seem that we can arbitrarily increase the number of species by piling up an arbitrarily large number of $D_9$-branes on top of each other. However, the number of species one may obtain in this way is limited by (2). The physical reason for this
limit can be understood from the fact, that the species label is the simplest bit in which one can encode information, and $D$-branes are devices for decoding this information. In other words a stuck of $n$ $D$-branes represents a device that can resolve $N = n^2$ species. One should be able to decode this information at least down to the distances of order the string length $l_s$. On the other hand, as we know, the minimum size of a pixel with the resolution capacity of $N$ species in any consistent theory of gravity is bounded by the scale $L_N$. An immediate connection between $N$ and $g_s$ then follows from the fact that we should be able to read CP factors at least down to the string length, and thus $L_N$ should not exceed $l_s$, which implies [2].

The gravitational reason for the limited capacity of $D$-branes to support large number of species has a clear physical meaning. Indeed, there is an obvious gravitational obstacle on how many CP factors we can generate by increasing the number of $D$-branes. This obstacle arises in the following way. Each $D_9$ brane carries a tension $T_9 = \frac{M_{10}^2}{g_s}$. So, the collective gravitational source created by a stuck of $n$ branes (the source of the ten-dimensional zero mode graviton) is

$$R^{-2} = nT_9 G_{10} = ng_s M_s^2.$$  \hspace{1cm} (15)

Requiring that this quantity cannot exceed the string tension $M_s^2$, leads to [2] as the bound on the number of CP species.

Note, that if instead we consider a stuck of $n$ $D_p$-branes, for the gravitational radius in terms of the ten dimensional species scale, $L_N$, we get

$$R(p, N) = \lambda^{\frac{p-3}{4p-3}} L_N = \lambda^\frac{1}{p} l_s,$$  \hspace{1cm} (16)

where $\lambda$ is the t’Hooft coupling. Only for the special case of $D_3$-branes this gravitational IR cutoff coincides with the species scale $L_N$. This agreement is another manifestation of the IR/UV correspondence for $D_3$ branes which is a crucial ingredient for AdS/CFT correspondence [3].

An interesting question is what is the role (if any) of supersymmetry and BPS properties of the $D$-branes in the species count. The reason why we are bringing up this question is, that for the BPS $D$-branes the world-volume metric is formally flat despite the existence of the gravitational tension. So naively, one could add arbitrary number of branes, without encountering an immediate problem with the horizon, and thus, with retrieving the information. Thus, naively one may think that BPS properties avoid the
holographic bound. Of course, this is not the case, since the evaporation rate of a semi-classical and neutral (and thus, non-BPS) BHs to the leading order is not sensitive to the supersymmetry properties of the asymptotic space. Obviously, as long as the background is asymptotically flat, such a non-BPS BH of size \( \gg l_s \) can always be formed regardless the supersymmetry properties of the asymptotic background. Then, it must evaporate democratically into all the species, according to law given by (12). This evaporation tells us, that if the stuck of D-branes violates the species bound (2), the BH must become non-semi-classical at distances much larger than the string length, which would be a clear inconsistency. So, the bound (2) must be respected. However, an interesting question is whether the system responds by some other dynamical effect preventing growth of CP-species beyond the bound (2), independent of the BH-argument.

### 4.3 Regge Species

We can distinguish the two notions of species in string theory. These are Regge excitations and CP-species, and they contribute differently into the species count. Although, formally the number of Regge resonances is exponentially large, they contribute at best the factor \( 1/g_s^2 \) into the species number. The reason is, that only a small fraction of the Regge resonances contributes into the evaporation of a semi-classical BH [7]. In fact, what we have discovered for the Regge excitations is, that when the ”species scale” evaluated with respect to the number of Regge excitations becomes equal to the string length, the string state itself becomes a black hole and the counting is self-truncated.

As for CP-species, they represent the zero mode excitations and must contribute the same rate as the zero mode graviton into the evaporation of any semi-classical BH. However, the number of CP-species cannot be increased arbitrarily for free, since they require existence of \( n = \sqrt{N} \) background D-branes as their sources. But, the latter number is automatically limited by the requirement, that the gravitational tension of the system must not exceed the string scale. In other words, there is an inevitable conflict between increase of the CP-species and limiting their gravitational backreaction, which results in the bound on the number of species given by (2).
5 Entropy Bound and the String-Black Hole Correspondence

We shall now show that the scale $L_N$ implies the lower bound on the BH entropy,

$$S > N,$$  \hspace{1cm} (17)

in any consistent $D$-dimensional theory of gravity. The bound follows from the fact that the scale $L_N$ on one hand is the size of a smallest possible semi-classical BH and on the other hand is the size of a smallest possible pixel that carries the information about the $N$ particle species labels. Notice, that the species label is a simplest form of the information storage. Assuming that the occupation number of species corresponding to each label is 0 or 1, the number of possible states of such a pixel scales as $2^N$, which gives the minimal entropy of the BH to which such a pixel can collapse being $S > N$.

This bound gives yet another evidence for (2). Indeed, when the black hole reaches the size of the species scale $L_N$ their entropy saturates the bound. On the other hand a string state becomes a black hole, i.e., a state with gravitational radius equal to the string length and with the string entropy equal to the Bekenstein-Hawking entropy, for $S = \frac{1}{g_s}$. Actually if we assume string theory as the UV completion, we find $\frac{1}{g_s}$ as an absolute bound on the BH entropy reached when the BH becomes of string size. This bound on the BH entropy sets the bound on the BH size as the string length and the Hagedorn temperature as the corresponding maximum temperature. We can obviously map this string physics with the physics we get in any consistent theory of species and gravity. Namely the entropy bound $S > N$ maps into the stringy bound $S > \frac{1}{g_s}$ using (2). In the other words, what happens to a black hole when it reaches the species scale is a Horowitz-Polchinski transition into a string state of a string theory with $l_s = L_N$ and $N = \frac{1}{g_s}$.

We wish to briefly discuss now how our bound relates to a naive counting of string Regge excitations. In string theory we can divide species into two categories. These are, the zero modes, in particular given by the CP factors, and the string Regge resonances. The semi-classical BH evaporation argument in ten-dimensions, presented above, puts the absolute bound on the number of zero modes, since all of these modes have to participate in the evaporation of semi-classical BH. Once the BH reaches the string size, the Regge excitations set in. The naive counting gives, that the number of these excitations is
exponentially large. However, the BH evaporation and holographic arguments indicate that the effective number of these species must be bounded by (2). In the other words, most of the string-theoretic “species” are not species at all! The reason for this is as follows.

First, the Regge excitations above the certain oscillator number are simply equivalent to BHs, as it is indicated by string-BH correspondence. At high energies the number of states in string theory grows with energy as \( e^{l_s E} = e^{\sqrt{N_{osc}}} \), with \( N_{osc} \) being the oscillator level. This leads to a stringy entropy \( S \sim E l_s = \sqrt{N_{osc}} \) and therefore to a temperature \( T \sim \frac{1}{l_s} \). Moreover, this temperature, the Hagedorn temperature, is an absolute bound. Now, for Einsteinian black holes the entropy scales with energy as \( S \sim (E l_D)^{\frac{D-2}{D-3}} \) for \( D \) the number of space-time dimensions. The black hole becomes a string state when both entropies scale with the energy in the same way, i.e. in ten-dimensions this happens when \( E l_{10}^8 = l_s^7 \), which after using the relation between the string and Planck lengths implies \( \sqrt{N_{osc}} = \frac{1}{g_s^2} \). Thus, any string state corresponding to a higher oscillator number is no longer a particle, but rather a BH. In other words, starting from the opposite end, when the black hole reaches the species scale its energy is related to the species scale as the string energy is related to the string length,

\[
M = NL_N^{-1},
\]

(18)

with the number of species playing the role of \( \sqrt{N_{osc}} \). That is, the BH in question becomes a string state with string length being the species scale and the number of species related to the string coupling by (2). Thus, in a theory of gravity the species scale fixes a maximal ”Hagedorn” temperature equal to \( \frac{1}{L_N} \) as well as the analogous ”Hagedorn” transition.

Now as for the Regge excitations with the lower oscillator number, although their formal number is exponentially large, majority of these states cannot contribute into the evaporation of any BH that can be treated semi-classically. This was illustrated by the analysis of [7]. The reason is the suppression of the emission due to the size of a BH, which can also be connected to Veneziano-type softening of the string interactions at short distances.

To summarize, the reason why number of Regge resonances contributes at best \( \sim 1/g_s^2 \) to the species count is, that the majority of these resonances are either BHs or are not produced in the evaporation of the semi-classical BHs.
6 Entanglement Entropy

We now wish to show that the relation (2) independently arises from the interpretation of the Bekenstein-Hawking entropy ($S_{BH}$) as the entanglement entropy $S_{ent}$. Let us compute an entanglement entropy of a $D$-dimensional Einsteinian BH of area $A$. As it is well known [8], $S_{ent}$, just as $S_{BH}$, scales as area. But, unlike the latter entropy, $S_{ent}$ depends both on the cutoff as well as on the number of species,

$$S_{ent} = AL_{s}^{2-D}N.$$  \hspace{1cm} (19)

This dependence for some-time was a source of a seeming discrepancy that goes under the name of the species puzzle (see [9] for a review). In reality, however, this puzzle does not exist [10] in the view of the bound (1).

The cutoff $L_s$ that must enter in the computation of $S_{ent}$ is exactly the species scale $L_N$, since beyond this scale species cannot be resolved in principle. Once this is taken into the account the discrepancy between the scalings of the two entropies disappears. Let us now apply this reasoning to the ten dimensional BHs. Eq (19) can then be rewritten in the following way,

$$S_{ent} = \frac{A}{L_{s}^{N}}N.$$  \hspace{1cm} (20)

Demanding that $L_N = l_s$ and equating the resulting $S_{ent}$ to the $S_{BH}$, we get (2).

In fact, we can turn the latter result around and use (2), which is evident from the independent arguments presented earlier in this paper, to show that the entanglement entropy computed with the obvious UV cutoff, $l_s$, exactly reproduces the Bekenstein-Hawking entropy in ten-dimensions.

Moreover, for any local quantum field theory with $N$ elementary species the UV cutoff dependent part of the entanglement entropy can be always expressed as,

$$S_{ent} = S_{BH} \left( \frac{L_N}{L_{UV}} \right)^2,$$ \hspace{1cm} (21)

with $S_{BH}$ the entropy of a black hole with the horizon area equal to the boundary of the region we are tracing, and $L_{UV}$ the UV cutoff length of the theory without gravity.

If we now assume, in accordance with the holographic principle, that the BH entropy is the maximum entropy we can associate with a given region, we could conclude that $L_{UV}$ should be necessarily bounded by the species scale. In other words, we could conclude that if a local quantum field theory with $N$ elementary species can be consistently coupled
to gravity, then necessarily this local quantum field theory must posses an UV completion that can be identified , in the large $N$ limit, with a string theory. Moreover, since we can always define a gravity decoupling limit,

$$N \to \infty, \quad M_D \to \infty$$  

keeping the UV scale $L_N$ finite, we can tentatively interpret this argument as an indirect way to point out to the dynamical generation of a (string) scale in the limit of infinite number of species.

7 Species Scale in $M$-Theory

We shall now ask, how does the species-count extend beyond the weakly-coupled string theory? Of course, whenever a dual weakly-coupled description is known, the counting translates trivially in terms of the bound on the number of (weakly-coupled) species in that new description. We shall focus instead on a well known example in which no dual weakly coupled description is known, and instead one has to go to $M$-theory.

As it is well known [15], the strong coupling limit of type IIA string theory takes us to $M$ theory, with the low energy description being 11-dimensional gravity theory. The 11-th dimension opens up in the limit $g_s \to \infty$ in which $D_0$-branes become massless and give rise to the perturbative states, which from the point of view of the low energy theory are Kaluza-Klein (KK) excitations of 11-dimensional gravity compactified to 10-dimensions. We shall now show, that the species-counting given by (1) again works, but the difference is, that the scale $L_N$ is now determined by the 11-dimensional Planck length. It is now $L_{11}$ and not $l_s$ that sets the absolute limit on the resolution capacity of any detector.

With $R$ denoting the radius of 11-th dimension, the basic defining relations of $M$-theory are:

$$R = g_s l_s$$  \hspace{1cm} (23)

and

$$M_{10}^8 = R M_{11}^9,$$  \hspace{1cm} (24)

where the first relation follows from identifying the $D_0$-branes (with mass $g_s l_s$) with the first KK excitations of mass $1/R$. The equation (24) is the standard geometric relation between the 10 and 11-dimensional Planck scales. Equations (23) and (24), together with
the basic relation \( l_{10} = g_s^{\frac{1}{2}} l_s \), lead to the following relation between the string scale and the eleven dimensional Planck length,

\[
l_{11} = g_s^{\frac{1}{4}} l_s.
\]  

(25)

The above equations are already enough to understand how the bound (1) is saturated in the \( M \)-theory limit. Indeed, for \( g_s \to \infty \), string excitations become strongly coupled and no longer fall under our definition of perturbative species. Instead, the role of the perturbative states is fulfilled by the KK excitations that originate from \( D_0 \)-branes. Their number is given by \( N = M_{11} R \). Taking this into the account, we realize that the equation (24) is nothing else but the saturation of the bound (1), where the role of \( L_N \) is played by \( L_{11} \). The latter fact has a clear physical meaning, since from equation (25) it follows that \( l_s \) is much shorter than \( l_{11} \). Thus, one hits the strong gravity scale way before there is any chance to probe the string length physics. As a result it is \( L_{11} \) and not \( l_s \) that plays the role of the species scale. In fact, physics at distances shorter than \( L_{11} \) cannot be resolved, in principle.

In order to follow how the \( L_N \) interpolates between \( l_s \) and \( L_{11} \) in two different descriptions, let us evaluate some useful relations for the two regimes. Let us start with a weakly-coupled description and consider \( n \) \( D_0 \) branes in \( D = 10 \). The mass of this stuck is

\[
M = \frac{n}{g_s l_s}.
\]  

(26)

The corresponding gravitational radius in \( 10D \) is

\[
R_g^7 = (g_s n) l_s^7.
\]  

(27)

that leads to the standard bound,

\[
g_s < \frac{1}{n},
\]  

(28)

as long as we require \( R_g \) to be smaller than the string length. Noticing that \( D_0 \) brane CP factors in \( 10D \) create \( N = n^2 \) different types of stringy species in the string weak coupling regime, the latter equation reproduces the bound (2).

For strong string coupling the gravitational radius of \( n \) \( D_0 \) branes will be bigger than the string length, meaning that the CP factors can no longer be resolved at the string length scale. This is the gravitational reason why string excitations can no longer be considered as perturbative species.
Let us consider now a graviton in eleven dimensions with the 11th component of the momentum being $P_{11} = \frac{N}{R}$. The corresponding gravitational radius in eleven dimensions is

$$R^8_g = \frac{P_{11}^9 N}{R^9}.$$  \hspace{1cm} (29)

Notice, that for any $N < (RM_{11})$, this gravitational radius is $R_g < l_{11}$. The value of $N$ that saturates this inequality has a clear physical meaning. From the point of view of an 11-dimensional observer this is a maximal value of the graviton momentum before the latter collapses into a BH. From the point of view of a 10-dimensional observer this is a maximal number of species compatible with the bound (1). The reason why the role of $L_N$ is played by $l_{11}$ is also clear, since this is the energy scale above which the Compton wave-length of the 11-dimensional graviton crosses over below its Schwarzschild horizon.

The counting of species matches from both points of view. This agrees with the idea that the eleven dimensional graviton is holographically described by $N D_0$ branes. This was the M(atrix) conjecture [16].

What happens when we push $g_s$ into the strong coupling regime? Using the standard KK definition of the number of species $n$ as

$$n = \frac{R}{l_{11}},$$  \hspace{1cm} (30)

we get using (23) and (25)

$$n = g_{s^2}^{\frac{3}{4}}.$$  \hspace{1cm} (31)

Moreover from (25) we get

$$l_{11} = g_s^{\frac{1}{4}} l_{10},$$  \hspace{1cm} (32)

for $l_{10}$, the ten dimensional Planck length. This together with (31) leads to

$$l_{11} = n^{\frac{1}{16}} l_{10}$$  \hspace{1cm} (33)

that is the standard definition of the species scale in 10 D.

To summarize, the following picture emerges. In the weak string-coupling limit, ten-dimensional species correspond to the string vibration modes, and the role of $L_N$ is played by the string scale $l_s$. The latter scale is the first barrier that one encounters in trying to resolve the short distance scales, way before reaching the Planck length. Saturation of the bound (1) then implies (2).
In the strongly coupled limit, string excitations can no longer be regarded as perturbative states. Instead, the role of the latter states is played by KK species of the compact 11-th dimension. The role of the species scale now is played by $l_{11}$, since this is the largest length scale beyond which resolution of states is gravitationally impossible. The bound (1) then emerges as the well-known geometric relation (24).

### 7.1 Formulation in Terms of the Dual Coupling

We can present the bound (2) in completely general terms, both in the weak coupling and in the strong coupling regimes, provided in the strong coupling regime we replace $g_s$ by its corresponding dual,

$$g_s \rightarrow g_d.$$  \hspace{1cm} (34)

We wish to illustrate this on the example of $M$-theory. As discussed above, in the weak coupling regime of ten-dimensional string theory the number of species is defined by the CP factors attached to $n D_0$-branes, i.e. $N = n^2$, which according to (28) is bounded by $1/g_s^2$. When we move to the strong coupling limit, the dual light degrees of freedom become the $n D_0$-branes and we should expect that the number of species is now $N = n$, as it indeed is the case (see equation (33)). If we think in terms of a dual coupling ($g_d$) we should generalize the bound (2) in the strong coupling regime as

$$N_{\text{eff}} = \frac{1}{g_d^2}.$$  \hspace{1cm} (35)

But then, according to (31) we must conclude that the dual coupling is

$$g_d = g_s^{-\frac{1}{3}}.$$  \hspace{1cm} (36)

The latter expression for the dual coupling is easy to understand using the dynamics of $D_0$-branes. In fact, the dynamics of $D_0$ branes is the $0 + 1$ - dimensional quantum mechanics obtained by the dimensional reduction of ten-dimensional Yang Mills, i.e. (for the bosonic components),

$$\int dt \frac{1}{g_s} Tr F_{\mu\nu} F^{\mu\nu}.$$  \hspace{1cm} (37)

This gives the following interaction term for the scalar components $\phi_i i = 1, \ldots, 9$,

$$\frac{1}{g_s} |\phi_i \times \phi_j|^2.$$  \hspace{1cm} (38)

Now, since $l_{11} = g_s^{\frac{2}{3}} l_s$, we can move from string units into M-theory units by replacing,

$$\phi_i \rightarrow g_s^{\frac{1}{3}} \phi_i.$$  \hspace{1cm} (39)
and transforming (38) into
\[ g_d^2 |\phi_i \times \phi_j|^2. \] (40)

Reading the above expression as the definition of the dual coupling,
\[ \frac{1}{g_d^2} |\phi_i \times \phi_j|^2, \] (41)

leads to the desired relation (36) and to the species bound in strong coupling regime defined by (35) for \( N = n \), the number of \( D_0 \)-branes.

8 Final Comments

In this note we have collected some evidence in the direction of mapping any consistent theory of gravity and \( N \) weakly coupled species into a string theory. A theory of species is characterized by two fundamental lengths scales; the Planck length setting the intensity of gravity and the species scale given by (I) that sets the minimal length where resolution of species is physically possible. The ultimate reason for the existence of these two scales is the holographic meaning of gravity and the information content of species. Non BPS black holes with negative specific heat are the natural candidates to probe physics near the species scale. We have observed that at this scale black holes behave as string states for a weakly coupled string theory under the correspondence: \( l_s \to L_N \), \( N \to \frac{1}{g_d^2} \). In this setup the weakly coupled string theory completes in the UV the theory of species, filling the gap between \( L_N \) and the Planck scale. However, consistency of this interpretation of string theory at strong string coupling forces the existence of a weakly coupled dual description with the number of effective dual species again related to the dual string coupling by the same fundamental relation. We have presented some evidence that this is in fact the case for the M-theory dual description of type IIA strings. In summary, we have attempted to map string theory as a theory of two length scales, the string scale and the Planck scale, into its most natural cousin a theory with species coupled to gravity. Hopefully this correspondence could shed some novel light in order to foresee what is string theory.

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