A simplified connection between constituent quark and parton

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Abstract

We propose a simple way to connect Gell-Mann constituent quark model and Feynman parton model for the nucleon. Thus, we can dynamically understand a large amount of data for high energy hadronic processes starting from the first QCD principle.

\textbf{keywords:} Constituent quark model, parton model, perturbative QCD, nonperturbative QCD
The consistent quark model (CQM) and the parton model (PM) successively described two different faces of a true nucleon: one is that the nucleon is consisted of limited number of massive (dressed) quarks and other regards the proton as a cluster of an infinite number of massless partons (quarks and gluons). Unfortunately, their connection is still an open question.

Experiments at high energy have accumulated a large amount of data about parton distributions in the nucleon. According to the factorization theorem [1], if we have measured parton distributions over a range \( x_0 \leq x \leq 1 \) at one value of \( Q_0^2 \), we can use perturbative QCD (pQCD) to predict parton distributions for \( Q^2 > Q_0^2 \) over the same range in the Bjorken variable \( x \). The variation of the parton distributions with \( Q^2 \) is known as their evolution. A selected value of \( Q_0^2 \) in this example is called as the factorization scale and the parton distribution \( f(x, Q_0^2) \) as the starting parton distribution. Usually, \( Q_0^2 > 1 \text{ GeV}^2 \) and \( f(x, Q_0^2) \) contains limited number of intrinsic (valence-like) quarks and infinite gluons and seq quarks. The application of the factorization theorem further assumes there is a special factorization scale \( \mu^2 \), where all the long-distance dependence resides in the nonperturbative input parton distributions \( f_{\text{IP}}(x, \mu^2) \), while all short-distance \((Q^2)\) dependence is in the evolvable perturbative parton distributions. According to this definition, the input parton distribution relates to a finite number of intrinsic partons. We will show that only the input parton distribution suitable to connect with the following constituent quark distribution.

Recently the nonperturbative QCD research (Schwinger-Dyson equation [2] and light front QCD [3]) reproduced the Gell-Mann picture about hadrons, which are mainly composed of three or two massive constituent quarks, or adding a limited number of intrinsic quark-antiquark pair. The results can describe the hadron spectra and the various form factors.
A following key step is to connect the wave function of constituent quarks with the input parton distribution. Once we found a "bridge" connecting these two distributions, one can predict a lot of hadronic processes at high energy starting from the first principle. Although several of starting distributions for partons at $Q^2_0 > 1 \text{ GeV}^2$ have been extracted from the experimental data with the linear DGLAP equation at the twist-2 level [4-6], however, these starting distributions always mix with the infinite number of gluons and sea quarks, they cannot correspond to the finite intrinsic components of the input distributions.

A nucleon is naively consisted of three constituents without the probing scale $Q^2$. A natural attempt beginning from 1976, is to assume that the nucleon has three valence quarks at a low starting point $\mu^2$ (but still in the perturbative region $\alpha_s(\mu^2)/2\pi < 1$ and $\mu > \Lambda_{QCD}$), and the gluons and sea quarks are radioactively produced at $Q^2 > \mu^2$ [7-9]. However, such natural input fails due to overly steep behavior of the predicted parton distributions at the small $x$. Instead of this natural input, Reya, Glück and Vogt (GRV) [10] added the valance-like sea quarks and valence-like gluon distributions to the input parton distribution. These valence-like components can slow down the evolution of the DGLAP equation at low $Q^2$ and reach agreement with experimental results, since the evolution region of the valence-like distributions is sizeably larger. However, either proton or neutron is not a hybrid hadron containing the intrinsic gluons. Therefore, it is difficult to connect the GRV-input with the constituent quark distribution.

As we know that the contributions of the parton recombination corrections become important at $Q^2 < 1 \text{ GeV}^2$, which are neglected in the DGLAP equation. The negative corrections of the parton recombination should slow down the parton evolution. These nonlinear effects can be calculated by pQCD at the twist-4 level. Such nonlinear corrections of the gluon recombination to the DGLAP equation were firstly derived by
Gribov, Levin and Ryskin [11] and Mueller and Qiu [12] in the double leading logarithmic (\( DLL(1/x, Q^2) \)) approximation. This evolution equation was re-derived by Zhu, Ruan and Shen [13-15]. The ZRS-version of this equation restores the momentum conservation, takes the leading logarithmic (\( LL(Q^2) \)) approximation and includes all parton recombination, therefore, it can naturally connect with the DGLAP equation and works in the whole \( x \) range. Based on the ZRS version of this equation, the possible available input distributions for the nucleon have been proposed [16-19], where the shadowing effect replaces the corrections of the valence-like gluon, the input contains only three valence quarks if without the flavor-asymmetric sea components, or adds the small amount of valence-like sea quarks if considering the flavor-asymmetry in the sea quark distributions.

However, both the GRV-input and the input based on the GLR-MQ-ZRS equation encountered the following disputes. (i) Is pQCD valid at \( Q^2 < 1 \) GeV? (ii) What is the conversion between the constituent quark and the parton at low \( Q^2 \)? The purpose of this letter is try to answer the above questions. We first focus on the corrections of the intrinsic quark mass to the QCD evolution equation when the equation works at the low \( Q^2 \) range. We find that the mass-effect of the intrinsic partons may freeze the pQCD evolution at \( Q^2 < M_{\text{eff}}^2 \), \( M_{\text{eff}} \sim 300 \) MeV is a simplified common mass-scale for the dressed light-quarks and dressed gluons. While this mass-effect gradually disappears at \( Q^2 > 1 \) GeV even if \( M_{\text{eff}} \neq 0 \). The parton distributions in the transition range \( M_{\text{eff}}^2 < Q^2 < 1 \) GeV will evolve along a special path \( \tilde{Q}^2 = Q^2 + M_{\text{eff}}^2 \).

A main difference between the constituent quark and the parton is that the former is massive \( M \sim m_p/3 \), while the later is regarded as massless. The mass \( M \) may be arisen from the propagation of bare-quark (even bare-gluon) in the strongly-coupled non-Abelian gauge field [20]. The QCD evolution equation is derived in the perturbative domain, where all partons are massless. We consider the corrections of the mass effect
to the GLR-MQ-ZRS equation at the low $Q^2$ range. For the sake of simplicity, we only write it for the flavor singlet quarks, their evolution reads

$$Q^2 \frac{dq^s(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} [P_{qq} \otimes q^s + P_{qg} \otimes g]$$

$$- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{gg \rightarrow q}(x, y) [yyg(y, Q^2)]^2$$

$$+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{x} \frac{dy}{y} x P_{gg \rightarrow q}(x, y) [yyg(y, Q^2)]^2, (i f \ x \leq 1/2),$$

$$Q^2 \frac{dq^s(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} [P_{qq} \otimes q^s + P_{qg} \otimes g]$$

$$+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{1/2} \frac{dy}{y} x P_{gg \rightarrow q}(x, y) [yyg(y, Q^2)]^2, (i f \ 1/2 \leq x \leq 1).$$

(1)

The DGLAP splitting function for $q(l) \rightarrow q(k) + g(l')$ is [5]

$$P_{qq} \frac{dk_{T}^2}{k_{T}^4} = \frac{E_k}{E_l} |M_{l \rightarrow k'l'}|^2 \left[ \frac{1}{E_{l} - E_{k} - E_{l'}} \right]^2 \left[ \frac{1}{2E_k} \right]^2 \frac{d^3l'}{(2\pi)^{3}E_{l'}}. \quad (2)$$

The momentum of partons are written in the infinite momentum frame, they are

$$l = [x_1p, 0, x_1p]; \quad k = \left[ x_2p + \frac{k_T^2}{2x_2p}, k_T, x_2p \right]; \quad l' = \left[ x_3p + \frac{k_T^2}{2x_3p}, -k_T, x_3p \right]. \quad (3)$$

The modification of the massive propagator leads to the following change in the $k_T$ dependent part of $P_{qq},$

$$\frac{k_T^2 dk_T^2}{k_T^4} \rightarrow \frac{k_T^2 dk_T^2}{(k_T^2 + M_{eff}^2)^2} \approx \frac{d(k_T^2 + M_{eff}^2)}{k_T^2 + M_{eff}^2} \equiv \frac{d\tilde{k}_T^2}{\tilde{k}_T^2}. \quad (4)$$

The $k_T^4$-factor in the denominator of first formula origins from the energy defect (i.e., the propagator in the time ordered perturbative theory form) in Eq. (2), therefore, it is
replaced by $k_T^2 + M_{\text{eff}}^2$. While the $k_T^2$-factor in the numerator of first formula is the result of the matrix $M_{l \rightarrow kl'}$, which is consisted of bare-vertex and is irrelevant to $M_{\text{eff}}$. The approximation in Eq. (4) may cause about maximum double deviation from a correct increment of the evolution at $k_T^2 \simeq M_{\text{eff}}^2$, although this deviation fast disappears at $k_T^2 \gg \mu^2$. However, the evolution increment at such lower $k_T$-scale is small since it is thin gluon environment. Therefore, we neglect the above deviation.

A similar result satisfies the recombination function in Eq. (1). Thus we have [12]

$$P_{gg \rightarrow q} \frac{dk_T^2}{k_T^4} = \frac{E_k}{E_l + E_2} |M_{p_1p_1 \rightarrow kl'}|^2 \left[ \frac{1}{E_1 + E_1 - E_k - E_{l'}} \right]^2 \left[ \frac{1}{2E_k} \right]^2 \frac{d^3q}{(2\pi)^2 E_{l'}}.$$  \hspace{1cm} (5)

Corresponding to Eq. (4),

$$\frac{(k_T^2/k_T^4)dk_T^2}{k_T^4} \rightarrow \frac{k_T^2dk_T^2}{(k_T^2 + M_{\text{eff}}^2)^3} \approx \frac{d(k_T^2 + M_{\text{eff}}^2)}{(k_T^2 + M_{\text{eff}}^2)^2} = \frac{d\tilde{k}_T^2}{k_T^4}. \hspace{1cm} (6)$$

Note that $k_T^2/k_T^4 \rightarrow k_T^2/(k_T^2 + M_{\text{eff}}^2)$ since the matrix $|M_{p_1p_1 \rightarrow kl'}|^2$ contributes a pair of massive propagators. According to work [21], the the modified running coupling due to the mass effect is $\alpha_s(k_T^2 + M_{\text{eff}}^2)$. The suppression of the running-coupling near the $\mu^2$-scale helps improve the convergence of perturbative expansion.

The renormalization group theory is the basis of the standard QCD evolution equation. Comparing with the renormalization group equation for the moments of the structure function, one needs to set $k_T^2 \rightarrow Q^2$ in the splitting function (2) and recombination function (5) when constructing Eq. (1) [6]. Only in this way, Eqs. (2) and (5) can play the role of the evolution kernels. Thus, the GLR-MQ-ZRS equation (1) including massive corrections Eqs. (4) and (6) can be rewritten as

$$\frac{Q^2 dxq(x, Q^2)}{dQ^2}$$
\[
\alpha_s(Q^2) \frac{1}{2\pi} [P_{qq} \otimes q + P_{qg} \otimes g] \\
- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{gg \rightarrow q}(x, y) [yg(y, \sqrt{Q^2})]^2 \\
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{x} \frac{dy}{y} x P_{gg \rightarrow q}(x, y) [yg(y, \sqrt{Q^2})]^2 \right), \text{if } x \leq 1/2,
\]

\[
\frac{\overline{Q}^2 dxq(x, \overline{Q}^2)}{d\overline{Q}^2} = \alpha_s(Q^2) \frac{1}{2\pi} [P_{qq} \otimes q + P_{qg} \otimes g] \\
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{gg \rightarrow q}(x, y) [yg(y, \sqrt{Q^2})]^2, \text{if } 1/2 \leq x \leq 1.
\]

which evolves with \(Q^2\) along a following special path

\[
\overline{Q}^2 = Q^2 + M_{eff}^2,
\]

where \(M_{eff} \simeq 300\) MeV is a common scale for either the constituent quarks and dressed gluons. Figure 1 presents the relation between \(\overline{Q}^2 \sim Q^2\). On can find that \(\overline{Q}^2\) gradually approaches to a lower limit \(M_{eff}\) with \(Q^2 \rightarrow 0\). We freeze the perturbative QCD evolution at \(Q^2 \ll \mu^2\) because where \(d\ln Q^2 \simeq 0\). This is consistent with the definition of the factorization scale \(\mu^2\).

A complete structure function for the nucleon can be perturbatively expanded on the twist

\[
F_N(x, Q^2) = F_N^{(2)}(x, Q^2) + \sum_{n=2}^{\infty} F_N^{(2n)}(x, Q^2),
\]

where the leading twist \(F_N^{(2)}\) is generally described by the parton model. As we known that in history the parton description was once considered valid only at the Bjorken limit \(Q^2 \gg m_N^2\), for say, at last at \(Q^2 > 10\) GeV\(^2\). However, the experimental data show that the DGLAP equation at the twist-2 level is already effective enough at \(Q^2 > 1\) GeV. This
fact is called as the precocity of the parton model due to asymptotic freedom of QCD, i.e., \( \alpha_s(Q^2)/\pi \ll 1 \) at \( Q^2 > 1 \text{ GeV}^2 \). Taking this result, a following twist-4 correction is either necessary and sufficient in the range \( 0.1 \text{ GeV}^2 < Q^2 < 1 \text{ GeV}^2 \). Besides, the suppression of the running-coupling with the mass-effect \( \alpha_s(Q^2 + M_{\text{eff}}^2) \) at low \( Q^2 \) helps improve the convergence of perturbative expansion. If we further consider that the evolution is freezeed at \( Q^2 < M_{\text{eff}}^2 \), where the complex nonperterbative effects are covered, the questions about the validity of the GLR-MQ-ZRS equation at the range \( Q^2 > \mu^2 \) have a positive answer (see Fig. 2).

In order to check above our understanding, we take an input parton distribution based on the GLR-MQ-ZRS equation from Ref. [16]

\[
\begin{align*}
   xu_{IP}(x, \mu^2) &= \frac{2}{B(1.98, 3.06)} x^{1.98} (1-x)^{2.06}, \\
   xd_{IP}(x, \mu^2) &= \frac{1}{B(1.31, 5.8)} x^{1.31} (1-x)^{4.8},
\end{align*}
\]

(10)

which is extracted from a globe fitting by the GLR-MQ-ZRS equation; \( B \) is Beta function. Note that in work [17] the suppression of \( \alpha_s \) at \( \mu^2 < Q^2 < 1 \text{ GeV}^2 \) is neglected but the evolution is freezeed at \( Q^2 < \mu^2 = 0.064 \text{ GeV}^2 \) (see the broken curve in Fig. 1). For the sake of simplicity, we assume that \( \mu^2 = M_{\text{eff}}^2 \) since they are of the same order of magnitude.

Recently, the BLFQ collaboration [22,23] gives the constituent quark distribution \( f_{CQ}(x) \) in the proton using the light-front (LF) QCD. We take it as an example of the constituent quark model and plot two distributions \( f_{CQ}(x) \) and \( f_{ZRS}(x, \mu^2) \) \( (f = u, d) \) in Fig. 3. One can find that two distributions are close, however, there is a not negligible difference between them. It seems a part of momentum fraction transfers from \( d \)-quark to \( u \)-quark in the proton at \( Q^2 < \mu^2 \). This process is nonperturbative since the perturbative evolution has been freezeed below the factorization scale \( \mu \). We try to understand it as
follows. The asymmetry \( u \)- and \( d \)-Coulomb potential \( V_{ud} + V_{uu} > 2V_{du} \) in the proton of the constituent quark model is negligible comparing with the strong QCD interaction since \( \alpha_{em} \ll \alpha_s \). However, a quark is knocked out a nucleon by impulse at \( Q^2 = \mu^2 \) in deep inelastic scattering, it transits from bound state to a free one. Although we don’t know the details of the proton fragmentation, according to the parton model, the struck quark keeps its original distribution if without extra interaction. However, the interaction of the Coulomb field in this case is highlighted due to the QCD de-confinement. The average momentum of the proton will redistributed between \( u \)- and \( d \)-CQs, i.e., a part of momentum fraction will transfer from \( d \)-quark to \( u \)-quark since the total momentum of quarks are conservation. We write it as

\[
2 < u(\mu^2) >_2 = 2 < u >_2 + \Delta x,
\]

\[
< d(\mu^2) >_2 = < d >_2 - \Delta x,
\]

where \(< ... >_2\) is the second moment of the distribution. This nonperturbative deformation of the quark distributions keeps the number of the quarks and their total momentum,

\[
\int_0^1 dx \sum_{f=u,d} f_c(x) = \int_0^1 dx \sum_{f=u,d} f_{IP}(x, \mu^2) = 1.
\]

\[
\int_0^1 dx u_c(x) = \int_0^1 dx u_{IP}(x, \mu^2) = 2,
\]

and

\[
\int_0^1 dx d_c(x) = \int_0^1 dx d_{IP}(x, \mu^2) = 1.
\]

The simple mathematical form of Eq.(10) allows us to determine the deformation form
using the minimum free parameters. The Regge exchange dominates the nonperturbative input distribution at \( x \to 0 \) [24]. The exchanged Regge trajectory is determined by the target quantum numbers, which are irrelevant to the deformation of the distributions. Therefore, we assume that two power indexes 1.98 and 1.31 in Eq.(10) are almost invariant in the deformation. Thus, we have only a free parameter \( \Delta x \). We take \( \Delta x = 0.08 \) and get the constituent quark distribution before the deformation \( f_{CQa}(x) \) using Eq.(10). The results are presented in Fig. 3 (dashed curves), they are parameterized as

\[
x u_{CQa}(x) = \frac{2}{B(1.98, 3.63)} x^{1.98} (1 - x)^{2.63},
\]
\[
x d_{CQa}(x) = \frac{1}{B(1.31, 3.14)} x^{1.31} (1 - x)^{2.14}.
\]

One can find that

\[
u_{CQa}(x) \simeq u_{CQ}(x); \quad d_{CQa}(x) \simeq d_{CQ}(x).
\]

To test the validity of the above treatment about the deformation of the quark distributions, we take a similar example in Fig. 4, where the distributions \( f_{NJL}(x) \) is provided by the Nambu-Jona-Lasinio model [25] and \( \Delta x = 0.06 \). The corresponding constituent quark distributions in the proton before the deformation are

\[
x u_{CQb}(x) = \frac{2}{B(1.98, 3.48)} x^{1.98} (1 - x)^{2.48},
\]
\[
x d_{CQb}(x) = \frac{1}{B(1.31, 3.47)} x^{1.31} (1 - x)^{2.47},
\]

They are consistent with the constituent quark distributions in the \( NJL \) model, i.e.,

\[
u_{CQb}(x) \simeq u_{NJL}(x); \quad d_{CQb}(x) \simeq d_{NJL}(x).
\]
The above discussions also satisfy the quark distributions in meson. In this case $\Delta x = 0$ because the symmetry of the Coulomb energy $V_{ud} = V_{du}$ or $V_{us} = V_{su}$, $s$ is strange quark.

A more accurate fitting between two distributions needs to consider the improvement of the QCD dynamic model and the higher-order corrections of perturbative calculation. Besides, the above discussions take the three quark approximation. We need consider the contributions of multi-quarks state in the constituent quark model and the asymmetry intrinsic sea quarks in the parton distributions. The GLR-MQ-ZRS equation has prepared such input [15,16]. By the way, the measured structure functions in deep inelastic scattering are not exactly the contributions of the parton distributions at very low $Q^2$, which is beyond the impulse approximation, the contributions of the handbag-diagram representation of the virtual-photon-pion forward Compton scattering amplitude should be noted when comparing the results with the experimental data. For a detailed discussion, see Ref. [26]. Thus, we realise a connection between a nonperturbative quark model and the perturbative parton model for the nucleon.

In summary, the constituent quark distribution and a set of input parton distribution in the proton are compared. We find the nonperturbative deformation of the quark distribution below the factorization scale $\mu^2$. A possible deformation mechanism and the quark mass effect at the transfer range are discussed. Based on the above discussions, a simplified connection between the constituent quark model and the parton model is established at the three quark configuration. The result is useful to realize the connection of the constituent quark model and the parton model.

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Figure 1: The new evolution path $Q^2 \sim Q^2$ due to the mass-effect. The result shows that the evolution is approximately saturated at $Q^2 < \mu^2$.

Figure 2: The schematic diagram for the applicable range of the perturbative expansion (9). (I) for $F^{(2)}(x)$, (II) for $F^{(2)}(x) + F^{(4)}(x)$, (III) $F^{(2)}(x) + F^{(4)}(x) + F^{(6)}(x)$, (X) where all perturbative evolutions are freezeed. A: A naive expectation, B: Precocious parton model, C: This work.
Figure 3: The input parton distribution in the proton $x f_{IP}(x, \mu^2)$ (solid curves) taken from Ref. [16]; Predicted constituent quark distribution $x f_{CQa}(x)$ (dashed curves), which compares with the constituent quark distribution $x f_{CQ}(x)$ (point curves) in the LFQCD model [24]. A free parameter $\Delta x = 0.08$. The above set is the u-quark distribution and the following set is the d-quark distribution.
Figure 4: Similar to Fig.3 but for the comparison with the constituent quark distribution $x f_{N, JL}(x)$ (point curves) in the NJL model [26]. A free parameter $\Delta x = 0.06$. 