Distribution Principle of Bone Tissue

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Using the analytic and experimental techniques we present an exploratory study of the mass distribution features of the high coincidence of centre of mass of heterogeneous bone tissue in vivo and its centroid of geometry position. A geometric concept of the average distribution radius of bone issue is proposed and functional relation of this geometric distribution feature between the partition density and its relative tissue average distribution radius is observed. Based upon the mass distribution feature, our results suggest a relative distance assessment index between the center of mass of cortical bone and the bone center of mass and establish a bone strength equation. Analysing the data of human foot in vivo, we notice that the mass and geometric distribution laws have expanded the connotation of Wolff’s law, which implies a leap towards the quantitative description of bone strength. We finally conclude that this will not only make a positive contribution to help assess osteoporosis, but will also provide guidance to exercise prescription to the osteoporosis patients.

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Bone tissue structure and function are largely associated with its mechanical and biological environment 1, 2, 3, 4, 5. Growth, modeling and remodeling are the basic physiological features of bone. Biomechanically, bone growth is defined as mass changes 6, and mechanical force and movement play a role in bone growth 7. When the skeleton bears loads externally, bone tissue will undergo adaptive changes such as reabsorption or remodeling 8, and point-to-point changes to the material property by changing mass distribution 9, 10, 11 will maximize its external loads. Individual bone growth indicates that external force has great effect on cross-sectional geometry and internal anatomy 12. Bone structure is an optimization of stress transformation 13 and it is an adaptive response to incorporation of minimal weight to maximal strength by some special rules 13. Bone physiological activity is regarded as a process of optimization 14, 15, 16. Consequently, stress has caused adaptive changes of bone shape and structure, which involve constant optimization of structures. But it remains unclear what distribution principle these changes follow.

From the biomechanical perspective, osteoporosis means a sharp drop of bone mass and strength and they cannot meet the demands of adaptive strength and movement load 17. Many mechanical models adequately represent the relation between the bone geometry and its strength 18, 19, 20, as well as the correlation between bone density and its strength 21. While the phenomenological models may often be helpful in obtaining a qualitative understanding of the data, microscopically these models do not provide a trustworthy guide into unknown territory of an accurate relation between the distribution of bone tissue and its strength. Our approach will be to use a combination analysis of analytic and experimental techniques to examine the above two uncertain areas by setting up a bone strength equation to obtain a quantitative description macroscopically.

Centroid of geometry (hereinafter referred to as COG) and the center of mass (hereinafter referred to as COM) of the homogeneous materials are coincide, whereas in most cases those of heterogeneous do not. Using CT (computed tomography) scan technology, we conducted the analysis to explore the relation between COM and COG of bone in the physiological activities, such as continuous modeling and remodeling in its adaptive mechanical condition. At small enough CT resolution rate and its slice distance, the bone tissue density of infinitesimal bone volume segmentation, dV, can be regraded as continuous. Its point density ρi can be taken as that of the micro-element, dm = ρi dV, thereby approximating the bone as a collection of particles. To find the optimal program for heterogeneous bone tissue mass distribution, we minimize

\[
\text{min } \Psi(p_c) = \sum_i \rho_i \Delta V \left( (x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2 \right),
\]

where \( p_c(x_c, y_c, z_c) \) and \( p_i(x_i, y_i, z_i) \) refer to the relative locations of coordinators of bone COM and random point related to CT image, respectively. The series \( \sum \rho_i \Delta V, \sum ( | x_i - x_c | + | y_i - y_c | + | z_i - z_c | ), \sum \rho_i \Delta V ( | x_i - x_c | + | y_i - y_c | + | z_i - z_c | ) \) are all convergent. Thus in the limit \( \Delta V \to 0 \) (\( \Delta V = abc \),

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and COG.

The behaviour seen in Fig. 1 confirms our signature that the optimal program for the heterogeneous bone tissue mass distribution should be the coincidence of its COM and COG. This signature does not, however, indicate a similar behaviour of the tissue distribution of bone in vivo. In order to study the relation between the COM and COG of bone in vivo, a CT scanning is conducted to the foot of eight volleyballers, eight classical wrestlers and two senior females. The position of COG of bone in vivo is independent of the changes in the geometric distribution of bone tissue simultaneously, we segment the density \( \rho_i \) and use \( \mathbf{r}_i = (x_i, y_i, z_i) \) as the CT image resolution, \( c \) the slice distance.

Fig. 1 collects and displays our data shown in Table I. The coincidence seen in Fig. 1 confirms our signature that the high coincidence of the position of COM and COG of heterogeneous bone in vivo is independent of the changing process of bone in its adaptive mechanical environment and that the bone tissue mass distribution observes the optimal principle with a coincidence between COM and COG.

TABLE I: Basic Information of the subjects.

| Sample size | Wrestlers | Volleyballers | Seniors |
|-------------|-----------|---------------|---------|
| Age(year)   | 21.00 ± 2.78 | 21.88 ± 0.99 | 64.50 ± 4.95 |
| Height(cm)  | 168.00 ± 5.68 | 183.94 ± 3.90 | 150.50 ± 3.54 |
| Body mass(kg)| 65.52 ± 5.16 | 71.80 ± 5.20 | 52.88 ± 3.15 |
| Calcaneus volume(cm³) | 71.79 ± 7.86 | 81.79 ± 4.26 | 49.43 ± 5.22 |
| Calcaneus density(g/ml) | 1.47 ± 0.04 | 1.49 ± 0.05 | 1.28 ± 0.03 |

where we have used \( \mathbf{p}(x, y, z) \) for the COG. This shows that the optimal program for the heterogeneous bone tissue mass distribution should be the coincidence of its COM and COG. Using \( \sqrt{a^2 + b^2 + c^2} \) and the above signature brings us to the issue of mass distribution index of bone tissue. The mechanical properties reveal that the elastic property and pressure-bearing strength of cortical bone is several times more than those of the same-volume spongial bone. Using the mechanical insight, we divide the tissue continuous density into three parts; bone marrow, spongial bone and compact bone with their COM and COG highly coincident as indicated by the observed fact \( \mathbf{p} = \mathbf{p}_c \). Using \( \sqrt{(x_{ci} - x_c)^2 + (y_{ci} - y_c)^2 + (z_{ci} - z_c)^2} \), where \((x_{ci}, y_{ci}, z_{ci})\) refer to COM of bone tissue and \((x_c, y_c, z_c)\) refers to COM of calcaneus, we calculate the distance between each individual tissue COM and that of the calcaneus. In order to avoid the effect from the size of the subject’s calcaneus, we standardize the average distribution radius of calcaneus so as to compare calcaneus of different volume, thereby establishing an analtz for the distribution index:

\[
ID = \frac{\sum_1^n \sqrt{(x_{ji} - x_i)^2 + (y_{ji} - y_i)^2 + (z_{ji} - z_i)^2}}{\sum_1^n \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}},
\]

where \( a = \frac{1}{X} \), \( b = \frac{1}{Y} \) \( XY \) the CT image resolution, \( c \) the slice distance.

From Fig. 2 it is clear that the position of compact bone COM of the volleyballers is closest to that of the calcaneus COM. Distinct difference exists between the volleyballers and wrestlers, so is the difference between the senior females and the volleyballers and wrestlers. The distance of the senior females is the largest. If such a trend continues for larger sample sizes, it will add one more quantitative evaluation index while diagnosing osteoporosis.

To see whether the changes of BMC (bone mineral content) and BMD (bone mineral density) bring about changes in the geometric distribution of bone tissue simultaneously, we segment the density \( \rho_i \) and use \( \mathbf{r}_i = (x_i, y_i, z_i) \) as the CT image resolution, \( c \) the slice distance.
and distribution radius standardized by $\rho$

A typical value of correlation coefficient is developed. After an analysis of the fitting function, we established a functional relationship between the two for an athlete. When the continuity of segment density is guaranteed and the calcaneus density variation ranges have been determined, the tissue geometric distribution will determine the strength of bone tissue. The moment of inertia of bone is an important index to reflect bone strength. For the heterogeneous materials the bone density and intensity follow a non-linear relationship. We introduce a coefficient $e^{\rho_k}$ and combine the segmented density and intensity to calculate the segmented strength on the basis of moment of inertia

$$M_i r_i^2 e^{\rho_k} = \rho_i V_i r_i^2 e^{\rho_k}$$

and establish a functional relation between the calcaneus bone strength and the segmented density as

$$\sigma = \int_a^b f(\rho) d\rho, \tag{4}$$

where $\sigma$ refers to strength of calcaneus, $f(\rho) = \rho V r_i^2(\rho)e^{\rho_k}$. When $\Delta \rho$ is small enough, and $f(a) \neq f(b)$, Eq. 4 can be approximated by

$$\sigma = (\rho_{max} - \rho_{min}) \sum \rho_i V_i r_i^2 \exp(\rho_k) \sum 1$$

$\rho_{max}$ and $\rho_{min}$ refer to the maximal value of compact bone density and the minimal value of spongial bone density, respectively and $\exp(\rho_k)$ is the coefficient parameter of bone density and strength. Note that the above equation holds for continuous heterogeneous material only.

Fig. 4 shows that when the bone tissue density of wrestlers and volleyballers is greater than 1.8, the difference in intensity between the two grows and reaches its maximum in the range of 2.4 – 2.5. On the other hand, the bone intensity of the senior females begins to show a larger difference with that of the athletes for density $> 1.4$.

Concerning bone tissue distribution, our study confirms that the high coincidence of COM and COG of heterogeneous material. This coincidence is the prerequisite to meet the requirement of max-min-principle and forms the bases for developing an evaluation index. We noticed that there is no difference in spongial bone between the wrestlers and volleyballers, whereas there is obvious difference in compact bone. This would explain the movement of the compact bone COM towards the calcaneus COM, which enables the calcaneus structure to bear greater stress. This can also be a representation that bone can yield adaptive changes functionally.

What’s more significant is the fact that the compact bone COM of the senior females moves away from the calcaneus COM. If a larger size sample can verify this, it will bring greater significance to the clinical practice.

One of the main aims of bone study is to conduct qualitative analysis to bone strength. Bone strength relies on

![FIG. 2: Relative positions of bone tissue COM and the calcaneus COM.](image)

![FIG. 3: Relation between bone tissue density and its average distribution radius. Density is converted by $\rho$ and distribution radius standardized by $\rho_i$.](image)
FIG. 4: Geometric distribution of bone strength. The tissue volume of the segmented density has been normalized by $V_i = \sum_1^i \Delta V_i$ and its distribution radius standardized by $r_i = \sum_1^i \Delta r \sum_1^i 1$. The typical value of parameter $k$ is chosen to be 1.68.

bone mass, but the uni-index of bone mass cannot paint a holistic and realistic picture of bone intensity objectively [20, 21, 22]. Modeling analysis reveals that the main factors that determine the structure strength include mass distribution, geometric distribution and moment of inertia of various tissues [24, 25]. Eq. (4) has successfully combined those factors and from a mathematical perspective it has illustrated that volume, mass, density, distribution radius and moment of inertia of bone tissue cannot be employed individually to assess bone strength. Eq. (4) also reveals that criteria of selecting a training approach to increase or improve bone mass of compact bone and its distribution radius that are clinically significant. The bone tissue geometric distribution principle sheds light on the effect to bone structure from different types of training. A geometrically-distributed bone strength equation can mirror the effects to bone strength from various factors. When the physiological bone mass decrease has become unavoidable, will exercise patterns be able to change its geometric distribution? If yes, Eq. (4) will undoubtedly provide some guidance to the development of exercise prescription and it can also be employed as an important evidence to examine and modify exercise prescription.

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[1] J. Wolff, Das Gesetz der Transformation der Knochen. Berlin: A. Hirchwild, 1892. [Macquet P, Furlong R, translators. The law of bone remodeling. Springer Berlin, pp110-157 (1986)].
[2] C.S. Chen, M. Mrksich, S. Huang, et al., Science 276, 1425 (1997).
[3] C. Rubin, A.S. Turner, S. Bain, et al., Nature 412, 603 (2001).
[4] M. Rusconi, Phys. Rev. Lett. 100, 128101(2008).
[5] M. Norbert, Phys. Rev. D 79, 021903(2009).
[6] Y.C. Fung Biomechanics: Motion, Flow, Stress, and Growth. New York: Springer, pp499-532 (1990).
[7] J.A. Buckwalter, M.J. Glimcher, R.R. Cooper, et al., Instr Course Lect 45, 371 (1996).
[8] E.H. Burger and J. Klein-Nulend, FASEB. J. 13, S101 (1999).
[9] T.P. Harrigan, R.W. Mann, J. Mater. Sci. 19, 761(1984).
[10] A. Odgaard, J. Kabel, B. vanRietbergen, et al., J. Biomech. 30, 487 (1997).
[11] M. Bagge, J. Biomech. 33, 1349 (2000).
[12] C. Ruff, J. Hum. Evol. 45, 317 (2003).
[13] H. Roesler, J. Biomech. 20, 1025 (1987).
[14] C.H. Turner, J. Biomech. 25, 1 (1992).
[15] R. Huiskes and S.J. Hollister, J. Biomech. Eng-T. Asme 115, 520 (1993).
[16] T.P. Harrigan and J.J. Hamilton, J. Biomech. 27, 323 (1994).
[17] H.M. Frost, Bone 20, 385 (1997).
[18] H. Gemunu, Phys. Rev. Lett. 88, 068101 (2002).
[19] R. Weinkamer, Phys. Rev. Lett. 93, 228102 (2004).
[20] C.D. Rubin, Curr. Med. Res. Opin. 21, 1049 (2005).
[21] R.P. Crawford, W.S. Rosenberg, and T.M. Keaveny, J. Biomech. Eng-T. Asme 125, 434 (2003).
[22] G.H. Hardy, A Course of Pure Mathematics. 10th Ed. New York: Cambridge University Press, pp394 (2002).
[23] C.M. Bagi, N. Hanson, C. Andresen, et al., Bone 38, 136 (2006).
[24] M. Hudelmaier, A. Kollstedt, E.M. Lochmiller, et al., Osteoporos Int. 16, 1124 (2005).
[25] E. Mittra, C. Rubin, B. Gruber, et al., J. Biomech. 41, 368 (2008).