NEWS FROM THE LATTICE

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ABSTRACT

A summary of some recent results from lattice simulations is presented. These include first calculations of the strangeness magnetic moment of the nucleon, three new studies of the gluon propagator, flux tube attraction in $U(1)$ gauge theory, and the static-quark potential and its gluonic excitations.

1. Introduction

Lattice simulations afford a means of studying confinement from first principles. Faster computers and more efficient simulation techniques (duality transformations, improved actions, anisotropic lattices) are allowing access to physics in previously unexplored territories: the strangeness content of the nucleon, the infrared region of the Landau-gauge gluon propagator, the long flux in $U(1)$, the rich glueball spectrum, and the static-quark potential and its gluon excitations for quark-antiquark separations as large as 4 fm. In this talk, the new lattice results presented at this conference are summarized.

2. Strangeness magnetic moment of the nucleon

The strangeness content of the nucleon is currently of much interest. A significant $\bar{s}s$ content in the nucleon can resolve the long-standing discrepancy between the pion-nucleon sigma term extracted from low-energy pion-nucleon scattering and that from the octet baryon masses. A large and negative strange-quark polarization has emerged from spin structure function studies at CERN and SLAC, combined with neutron and hyperon $\beta$ decays. Substantial contributions from strange quarks in the axial-vector and scalar current matrix elements would seem to imply similar such contributions to the vector current, but this would jeopardize the $SU(6)$ prediction of the neutron to proton magnetic moment ratio, which lends credence to the valence quark picture. The SAMPLE collaboration has recently measured the neutral weak magnetic form factor in elastic parity-violating electron scattering at backward angles, and obtained the strangeness magnetic form factor by subtracting out the nucleon magnetic form factor. Elastic $ep$ and $e\,^4He$ parity-violation experiments are currently planned at TJNAF to measure the asymmetry at forward angles to extract the strangeness electric mean-square radius.

Williams presented results from the first lattice calculation of the strangeness magnetic $G_M^S(q^2)$ and electric $G_E^S(q^2)$ form factors, where $q$ is the four-momentum
transfer. These form factors were obtained from ratios of two- and three-point correlation functions involving the vector current. The simulations were carried out on a $16^3 \times 24$ lattice with an inverse spacing $a^{-1} = 1.72(4)$ GeV and were possible only because of recent developments in $Z_2$ noise techniques for quark propagators. The calculations were done in the quenched approximation; contributions due to sea quarks were explicitly included at one fermion-loop order (the disconnected insertion diagram). The connected insertion diagram yields the contributions from the valence quarks and their $Z$ graphs. Three values for the Wilson hopping parameter were used, corresponding to quark masses of 120, 200, and 360 MeV.

Results for $G_M^p(q^2)$ and $G_E^p(q^2)$ were obtained at four nonzero values of $q^2$. Ex-
trapolating to the limit $q^2 \to 0$ using a single pole form yielded $G_M^s(0) = -0.36 \pm 0.20$ and a value for $G_E^s(0)$ consistent with zero. The electric mean-square radius was $\langle r_E^2 \rangle = -0.061 \pm 0.003 \text{fm}^2$. For the $u$ and $d$ quarks in the disconnected insertion diagram, $G_{M,\text{dis}}^{u/d}(0) = -0.65 \pm 0.30$ was found. Adding all of the sea quark contributions from the disconnected insertion diagram gives $\mu_\text{dis} = (2/3G_M^{u}\text{dis}(0) - 1/3G_M^{d}\text{dis}(0) - 1/3G_M^{s}(0))\mu_N = -0.097 \pm 0.037\mu_N$, which, when combined with the connected insertion diagram, brings the ratio of neutron to proton magnetic moments $\mu_n/\mu_p = -0.68 \pm 0.04$ into agreement with the experimental value of 0.685. The accidental cancellation between the disconnected insertion diagram and the rest-frame $Z$-graph component of the connected insertion diagram leaves the nonrelativistic valence contribution dominant, which explains why the $SU(6)$ valence quark picture works well for the $\mu_n/\mu_p$ ratio; $SU(6)$ fails in the axial-vector and scalar cases because this cancellation does not occur. The results for the form factors are shown in Fig. 1. Contributions from the strange quarks are not particularly small, but they are largely cancelled by the $u$ and $d$ quark contributions, resulting in a small overall effect to the form factors (the effect is greatest in the nucleon electric form factor). Issues related to the quenched approximation, chiral extrapolations, finite lattice-spacing artifacts, and finite volume errors must still be explored.

3. Gluon propagator

The connection between confinement and the gauge-dependent gluon propagator is unclear. However, knowledge of the infrared behaviour of the gluon propagator is important for various approaches to modelling confinement. Some studies based on Dyson–Schwinger equations claim that an infrared enhanced (such as $1/q^4$) propagator is required for confinement; others assume that a dynamically generated gluon mass leads to an infrared finite behaviour. Lattice simulations can tell us the true infrared behaviour of the gluon propagator from first principles. However, large finite-volume effects have prevented previous lattice studies from accessing the momentum region where the relevant nonperturbative behaviour is expected. At this conference, three groups reported on new lattice studies of the gluon propagator; two worked in the Landau gauge and one used the maximally-abelian gauge in $SU(2)$ with residual $U(1)$ Landau gauge fixing.

Williams presented results, shown in Fig. 2(a), revealing new structure in the gluon propagator for momenta as small as 0.4 GeV. Three separate simulations were done in order to control systematic errors from finite lattice spacing and finite volume. They obtained results in the quenched approximation on a large $(32^3 \times 64)$ lattice for a lattice spacing near 0.1 fm, which enabled them to probe momenta as small as 400 MeV. Gribov copies were ignored. They also verified the tensor structure of the propagator, given in the continuum by

$$D_{\mu\nu}^{ab}(q) = \delta^{ab} \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) D(q^2),$$

where $q$ is the four-momentum of the gluon, $a$ and $b$ are colour indices, and $\mu$ and $\nu$ are the space-time indices. One of their main findings was to rule out a $1/q^4$ infrared
 enhancement (for the case when quark-antiquark pair creation is neglected). They fit several phenomenological functions to their data, but most resulted in very large $\chi^2$ values. In particular, the Gribov, Stingl, Marenzoni, and Cornwall forms were all shown to fail. One of the few functions to provide a satisfactory description of the data over a wide range of momenta was found empirically to be

$$D(q^2) = Z \left( \frac{A}{(q^2)^{1+\alpha} + (M_{IR}^2)^{1+\alpha}} + \frac{1}{q^2 + M_{UV}^2} \left(\frac{1}{2} \ln \frac{q^2 + M_{UV}^2}{M_{UV}^2}\right)^{-d_D} \right),$$

where $Z = 1.78^{+45}_{-20}$, $A = 0.49^{+17}_{-6}$, $aM_{UV} = 0.20^{+37}_{-19}$, $aM_{IR} = 0.43^{+5}_{-1}$, and $\alpha = 0.95^{+7}_{-5}$ were the fit parameters, and $a^{-1} = 1.885$ GeV and $d_D = 13/44$. This function does incorporate some ultraviolet information from perturbation theory. In the future, these authors intend to use an improved-gauge field action and increase the lattice volume in order to probe deeper into the infrared.

Results for the $SU(2)$ gluon propagator in the maximally-abelian gauge (with residual $U(1)$ Landau gauge fixing) were presented by Suganuma and are shown in Fig. 2(b). Two simulations on a $12^3 \times 24$ lattice were done, but the lattice spacings were too similar to allow any serious study of systematic errors from finite spacing and finite volume. Their focus was to demonstrate that the colour off-diagonal component has a Yukawa fall-off corresponding to a large mass so that long-range physics ($r > 0.35$ fm) must be dominated by the colour diagonal component. This, they claimed, is the origin of abelian dominance in nonperturbative QCD. The dependence of the off-diagonal gluon’s effective mass on the residual $U(1)$ gauge fixing was not studied. It should be noted here that abelian dominance of nonperturbative QCD has not yet been convincingly established, and that confinement is much more than the area law of the Wilson loop.
Fig. 3. Results from dual simulations of flux tubes in $U(1)$. (a) Ratio of the string tension of a doubly-charged toroidal flux tube over that of a singly-charged flux tube with respect to the coupling $\beta$. (b) Longitudinal electric field profile of two interacting flux tubes in the symmetry plane ($E_\parallel$, solid line) for $\beta = 0.96$. The equal charges are separated by $4a$, where $a$ is the lattice spacing. Dotted lines are field profiles $E_\parallel_1$ and $E_\parallel_2$ for single flux tubes at $x = -2a$ and $x = +2a$. The noninteracting superposition $E_\parallel_1 + E_\parallel_2$ is shown as a dashed line.

Nakajima and Furui presented preliminary results from not only quenched simulations, but also from Langevin simulations including dynamical fermions (for three values of the Wilson hopping parameter $\kappa = 0.1, 0.15, \text{and } 0.2$). Smeared gauge fixing was used to resolve Gribov ambiguities. Their lattices were very small ($4^3 \times 8$) and much more work is needed to control systematic errors before any conclusions can be drawn on the role of quark-antiquark pair creation in the gluon propagator.

4. Dual lattice simulations of $U(1)$ flux tubes

The study of confinement in the strong-coupling phase of compact $U(1)$ in four dimensions can be viewed as a stepping stone on the way to understanding confinement in QCD. Confinement in the $U(1)$ theory is driven by magnetic monopoles whose currents squeeze the electric field into flux tubes. A dual transformation of the $U(1)$ path integral exists so that simulations can be carried out much more efficiently in the dual theory, which corresponds to a certain limit of a dual Higgs model. Results obtained in the dually-transformed theory were presented at this conference, along with some first computations in the dual Abelian Higgs model. This study focussed on the question of whether the vacuum corresponds to a type-I or a type-II superconductor, and the role of quantum fluctuations of the dual degrees of freedom were investigated.

First, it was shown that the string tension scales proportionally to the charge rather than the square of the charge in the confinement phase. This is shown in Fig. 3(a) in which the string tension of a closed double flux tube is compared to that of a single closed flux tube as a function of the coupling $\beta$. These results were obtained on a $8^3 \times 16$ lattice. In the Coulomb phase (large $\beta$ values), the ratio of the string tension of the double flux tube to that of the single flux tube is 4 in agreement with the expected result for the energy of a homogeneous field of double strength; below the transition in the confinement phase, the ratio is 2, in agreement with the expectations
for a dual superconductor. Flux tube attraction in the vicinity of the phase transition was confirmed by an examination of the longitudinal electric field profile at $\beta = 0.96$ as shown in Fig. 3(b). Two parallel flux tubes of length $22a$, where $a$ is the lattice spacing, were brought next to each other, a transverse distance $4a$ apart, and the resulting longitudinal electric field in the symmetry plane was measured. The profile differed dramatically from the noninteracting superposition of two single flux tubes; the formation of one flux tube was observed, in analogy to a type-I superconductor. These authors also compared their simulation results to the predictions from a dual London equation. Agreement was found for small charge distances, but not for distances greater than ten lattice spacings (they probed lengths up to $20a$). They interpreted this as further evidence that the confining $U(1)$ behaves as an effective type-I superconductor.

5. Static-quark potentials

Three groups reported on calculations of static quark-antiquark potentials; four-quark energies in $SU(2)$ lattice gauge theory were also presented.

The static-quark potential and its gluon excitations are very useful probes of confinement. It is generally believed that at large quark-antiquark separation $r$, the linearly-growing ground-state energy of the glue is the manifestation of the confining flux whose fluctuations can be described in terms of an effective string theory. The lowest-lying excitations are then the Goldstone modes associated with spontaneously-broken transverse translational symmetry. Expectations are less clear for small $r$. The gluon excitations of the static-quark potential are also useful for studying hybrid heavy-quark mesons using a Born-Oppenheimer expansion.

The first comprehensive determination of the low-lying spectrum of gluonic excitations in the presence of a static quark-antiquark pair was presented at this conference. The glue energies for $r$ from 0.1 to 4 fm were extracted from Monte Carlo estimates of generalized Wilson loops in eight simulations using an improved gauge-field action. The use of anisotropic lattices in which the temporal lattice spacing $a_t$ was much smaller than the spatial spacing $a_s$ was crucial for resolving the glue spectrum, particularly for large $r$. Finite volume errors were shown to be negligible. Particular attention was paid to the volumes for very large $r$: not only were checks carried out using additional simulations, but also by studying volume effects for a naive Nambu-Goto string. Agreement of energies obtained using different quark-antiquark orientations on the lattice was used to check the smallness of finite-spacing errors and to help identify the continuum rotational quantum numbers corresponding to each level (there are only discrete symmetries on the lattice). Results for lattice spacings ranging from 0.12 to 0.29 fm were obtained to facilitate extrapolation to the continuum limit. The hadronic scale parameter $r_0 \approx 0.5$ fm was used to set the scale.

The continuum-limit extrapolations are shown in Fig. 4. The energies are labelled by the magnitude $\Lambda$ of the projection of the total angular momentum of the gluons onto the molecular axis, and the symmetry $\eta_{CP}$ under spatial inversion with charge conjugation. The capital Greek letters $\Sigma, \Pi, \Delta, \Phi, \ldots$ are used to indicate states with $\Lambda = 0, 1, 2, 3, \ldots$, respectively. States with $\eta_{CP} = 1(-1)$ are denoted by the subscripts $g$ ($u$). The $\Sigma$ states are additionally labelled by a superscript $+$ ($-$) depending on whether they are even (odd) under a reflection in a plane containing the molecular axis. The
ground-state $\Sigma_g^+$ is the familiar static-quark potential. The lowest-lying excitation is the $\Pi_u$. There is definite evidence of a band structure at large $r$: the $\Sigma_g^-, \Pi_g$, and $\Delta_g$ form the first band above the $\Pi_u$; the $\Sigma_u^+, \Sigma_u^-, \Pi_u^+/\Phi_u$, and $\Delta_u$ form another band; and the $\Sigma_u^-$ is the highest level. The level orderings and approximate degeneracies of the gluon energies at large $r$ match, without exception, those expected of the Goldstone excitations which are a universal feature of any effective string theory description of the long confining flux. However, the precise $m\pi/r$ gap behaviour is not observed. For separations less than 2 fm, the gluon energies lie well below the Goldstone energies and the Goldstone degeneracies are no longer observed. The two $\Sigma^-$ states are in violent disagreement with expectations from a fluctuating string. These somewhat surprising results cast serious doubts on the validity of treating glue in terms of a fluctuating string for quark-antiquark separations less than 2 fm.

Some authors claim that the glue can be described merely as a mathematical string using the Nambu-Goto action (turning a blind eye to the fact that the Nambu-Goto string cannot be consistently quantized in four dimensions). These results clearly contradict this claim. For $r$ greater than 1 fm, the $\Pi_u$ and $\Pi_g$ energies disagree the least with the naive Nambu-Goto energies. In all other cases, the discrepancies are significant. The $\Sigma_u^-$ and $\Sigma_g^-$ levels look nothing like the predictions from the naive Nambu-Goto string. The departures of these levels from the Nambu-Goto energies are so severe that explanations in terms of mixings with other string modes or consequences
of small-$r$ symmetry requirements are difficult to accept. Rather, such behaviour signals the complete failure of the Nambu-Goto interpretation.

Baker \cite{Baker} presented a comparison of results on the central, spin- and momentum-dependent interquark potentials from a dual superconductor model with those determined from first principles using lattice simulations. The model is an effective theory of long-distance Yang-Mills in which an octet of dual potentials are coupled minimally to three octets of scalar Higgs fields carrying colour magnetic charge. Monopole condensation takes place and the dual potentials couple to a quark-antiquark pair via a Dirac string connecting the pair. Chromoelectric flux is confined as the quark-antiquark separation increases as a result of the dual Meissner effect. Assuming a particular spontaneous symmetry breaking sequence, the Lagrangian of the model can be replaced by an Abelian Higgs Lagrangian. The model has two parameters, a coupling constant $\alpha_s$ and the vacuum expectation value of the dual Higgs field $\phi_0$. The interquark potentials were calculated from the Wilson loop of the dual theory. The two parameters of the model were determined by fitting to the lattice results for the central potential. The remaining nine potentials are then uniquely determined and were compared to the lattice data. They were shown to be in fair agreement for $r$ greater than 0.2 fm; the agreement was aided by large uncertainties in the simulation results.

Klindworth \cite{Klindworth} calculated the static quark potential in transverse light-front QCD. In this approach, the two light-cone coordinates $x^\pm = (x^0 \pm x^3)/\sqrt{2}$ are continuous and their associated degrees of freedom are non-compact, while the other two transverse directions $x_\perp$ are discretized and the gauge fields associated with these directions are expressed in terms of compact, non-unitary link variables. The calculation was carried out in the large $N_c$ limit, where $N_c$ is the number of colours. In so doing, quark-antiquark pair creation could be ignored. The static quark potential was then calculated using Lanczos diagonalization techniques which necessitated a Fock-space truncation. A linearly confining potential was found, and an excited potential was also determined. The calculation served mainly as an initial test of this new regularization and computational scheme.

Furui \cite{Furui} presented results for four-quark energies from quenched $SU(2)$ simulations. The aim of the analysis was to obtain a compact expression for the four-quark interactions for an arbitrary spatial configuration of the quarks. They have examined square, rectangular, tilted rectangular, linear, quadrilateral, non-planar (at least one link on axis), and tetrahedral (two links off axis) geometries on a $16^3 \times 32$ lattice at two different lattice spacings. Their results suffer from significant lattice artifacts.

6. Conclusion

A plethora of new results from lattice simulations are shedding light in some of the previously dark corridors of the labyrinth surrounding confinement. Computer simulations of gluons and quarks remain an important tool for navigating this maze.

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