Non-Perturbative Entangling Gates between Distant Qubits using Uniform Cold Atom Chains

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We propose a new fast scalable method for achieving a two-qubit entangling gate between arbitrary distant qubits in a network by exploiting dispersionless propagation in uniform chains. This is achieved dynamically by switching on a strong interaction between the qubits and a bus formed by a non-engineered chain of interacting qubits. The quality of the gate scales very efficiently with qubit separations. Surprisingly, a sudden switching of the coupling is not necessary and our gate mechanism is not altered by a possibly gradual switching. The bus is also naturally reset to its initial state making the complex resetting procedure unnecessary after each application of the gate. Moreover, we propose a possible experimental realization in cold atoms trapped in optical lattices and near field Fresnel trapping potentials, which are both accessible to current technology.

Introduction:—Universal quantum computation can be achieved by arbitrary local operations on single qubit and one two-qubit entangling gate [1]. While single qubit operations are easily achieved by local actions, the story is very different for the two qubit gate. In an array of spins an entangling gate between neighboring qubits can be accomplished by letting them interact. However, for non-neighboring qubits, a direct interaction is normally not possible unless there is a separate common bus mode [2] or flying qubits. In realizations without an additional bus mode, such as with cold atoms in optical lattices, one cannot choose an arbitrary pair of atomic qubits for a gate operation and usually gates parallelly occur between all neighboring pairs [3]. Thus, designing bus modes for logic gates between arbitrary and distant pairs of qubits is of utmost importance in any physical realizations and various unconventional examples of buses are continuously being proposed [4,5]. One possible realization is to have both the qubits and the bus composed of the same physical objects, generally called spin chains. The quality of an unmodulated spin chain, even as a data-bus, is affected by dispersion [6]. Thus, in order to have a quantum gate between two qubits through such buses [5,7,9], delocalized encodings over several spins [10], delicately engineered couplings [11] or very weak couplings between qubits and the bus [5] is thought to be necessary. Recently, a new scheme based on tuning the couplings between qubits and the bus has been proposed [12] for fast and high-quality state transmission, which we here exploit for achieving an entangling quantum gate between arbitrarily distant qubits.

Cold atoms in optical lattices are now an established field for testing many-body physics [13–17]. In particular, chains of atoms in Mott insulator regime (one atom per site) are being built experimentally [16,17], paving the way for realizing spin Hamiltonians [18]. With recent cooling methods, the required temperatures for observing magnetic quantum phases has become reachable [19]. In this framework, series of multiple two-qubit gates, acting globally and simultaneously, have been proposed [20] and realized [15]. Could the same framework solve the problem of realizing quantum gates between any two selected neutral atom qubits? This is still an outstanding problem, unless one uses the physical movements of neutral atoms to each other’s proximity [21] which has its own complexity.

Recently, single site addressing in an optical lattice setup has been experimentally achieved [17]. Furthermore, local traps have been proposed for individual atoms using Near Field Fresnel Diffraction (NFFD) light [22]. A new approach for scalable quantum computation has been suggested [23] through a combination of local NFFD traps, for qubits, and an empty optical lattice, for mediating interaction between them. Since the interaction is achieved through controlled collisions between delocalized atoms it may suffer a high decoherence when qubits, on which the gate is applied, are far apart [15].

In this letter we put forward a scalable, non-perturbative (i.e. not relying on weak couplings) dynamical scheme for achieving high-quality entangling gates between two arbitrarily distant qubits, suitable for subsequent uses without resetting. Unlike previous proposals, we do not demand encoding, engineering or weak couplings: we only need switchable couplings between qubits and the bus. We also propose an application, based on a combination of NFFD traps and optical lattices, which is robust against possible imperfections.

Introducing the model:—Let us describe our bus as a chain of spin 1/2 particles interacting through

\[ H_M = J \sum_{n=1}^{N-1} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \lambda \sigma_n^z \sigma_{n+1}^z), \]

(1)

where \( \sigma_n^\alpha \) (\( \alpha = x, y, z \)) are Pauli operators acting on site \( n \), \( J \) is the exchange energy and \( \lambda \) is the anisotropy. The qubits \( A \) and \( B \), on which the gate acts, sit at the opposite sides of the bus, labeled by site 0 and \( N + 1 \) respectively. The interaction between the bus and the qubits is

\[ H_I = J_0 \sum_{n=0,N} (\sigma_n^x \sigma_n^x + \sigma_n^y \sigma_n^y + \lambda \sigma_n^z \sigma_n^z), \]

(2)

where the coupling \( J_0 \) can be switched on/off. For the moment the anisotropy \( \lambda \) is set to zero. Initially the qubits are prepared in the states \( |\psi_A\rangle \) and \( |\psi_B\rangle \) and decoupled from the bus.
which is in the state $|\psi_M\rangle$, an eigenstate of $H_M$, for instance the ground state. Since $H_M$ commutes with the parity operator $\Pi_{n=1}^N(\sigma^z_n)^\epsilon$ and with the mirror inversion operator, the state $|\psi_M\rangle$ has a definite parity $(-1)^p$, for some integer $p$, and is mirror symmetric. At time $t = 0$ the coupling $J_0$ is switched on and the whole system evolves under the effect of the total Hamiltonian $H = H_M + H_I$, i.e. $|\Psi(t)\rangle = e^{-itH}|\psi_M\rangle|\psi_M\rangle|\psi_M\rangle$.

In Ref. [12] it was shown that by tuning $J_0$ to an optimal non-perturbative value $J_{0}^{opt} = 1.05\sqrt{JN^{-1/2}}$ the mirror-inversion condition (7) is nearly satisfied resulting in a fast high-quality transmission. In fact, when $|\psi_M\rangle$ is initialized in either $|0\rangle \equiv |\uparrow\rangle$ or $|1\rangle \equiv |\downarrow\rangle$ an arbitrary quantum state of $A$ is transmitted almost perfectly to $B$ after time $t^* = (0.25N + 0.52N^{1/3})/J$.

The Hamiltonian $H$ is mapped to a free fermionic model by Jordan-Wigner transformation $c_n = \sum_{i=1}^{N}\{(-\sigma^z_i)^\epsilon\}e^{-itH}\sum_{i=1}^{N}\{\sigma^z_i\}$ followed by a unitary transformation $d_k = \sum_{n}\sqrt{g_n}c_n$. The total Hamiltonian finally reads $H = \sum_k\omega_k d_k^\dagger d_k$ where the explicit form of $g_n$ and $\omega_n$ are given in [24, 25]. The dynamics in the Heisenberg picture is given by $c_n(t) = \sum_{m}U_{nm}(t)c_m$ where $U_{nm}(t) = \sum_k\sqrt{g_n}g_m e^{-i\omega_n t}$. When the perfect transmission condition, i.e. $J_0 = J_{0}^{opt}$, is satisfied we have $|U_{0,N+1}(t^*)| = 1$ and thus we set $U_{0,N+1}(t^*) = e^{i\alpha_w}$. Notice that in any transmission problem there always might be an overall phase which is irrelevant to the quality of transmission. However, exploiting this phase is the heart of our proposal for obtaining an entangling two-qubit gate between $A$ and $B$. We define $|\Psi_{ab}\rangle = |\Psi(0)\rangle$ with $|\psi_A\rangle = |\alpha\rangle$ and $|\psi_B\rangle = |\beta\rangle$ where $a, b = 0, 1$. When $J_0$ is switched on the whole system evolves and at $t = t^*$ the states of $A$ and $B$ are swapped, while the bus takes its initial state $|\psi_M\rangle$, as a result of the mirror inverting dynamics. Therefore, an almost perfect transmission is achieved with an overall phase $\phi_{ab}$, namely $e^{-i\alpha_w}|\Psi_{ab}\rangle \approx e^{i\phi_{ab}}|\Psi_{ba}\rangle$. The explicit form of $\phi_{ab}$ follows from the dynamics depicted above with the freedom of setting $\phi_{00} = 0$. For instance to get $\phi_{10}$ we have

$$e^{-i\alpha_w}|\Psi_{10}\rangle = e^{-i\alpha_w}c_0|\Psi_{00}\rangle \approx U_{0,N+1}(-t^*)c_{N+1}|\Psi_{00}\rangle = \left((-1)^{N+1}e^{-i\alpha_w}|\Psi_{01}\rangle \equiv e^{i\phi_{10}}|\Psi_{01}\rangle. \right.$$  (3)

This defines $\phi_{10} = (p + 1)\pi - \alpha_w$ while $\phi_{01} = \phi_{10}$ due to the symmetry of the system. With similar argument we get $\phi_{11} = \pi - 2\alpha_w$. Therefore, the ideal mirror-inverting dynamics defines a quantum gate $G$ between $A$ and $B$, which reads $G(ab) = e^{i\phi_{ab}}|ba\rangle$ in the computational basis. Independent of the value of $\alpha_w$ when the pair $A, B$ is initially in the state of $|++\rangle$, where $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, the application of the gate $G$ results in a maximally entangled state between $A$ and $B$. Furthermore, the phase $\alpha_w$ is found to be equal to $\alpha_w(N+1)$. Since the dynamics is not perfectly dispersionless, $|U_{0,N+1}(t^*)|$ is not exactly 1, gate $G$ is not a perfect unitary operation. In the details of the dynamics of the qubits is described by a completely positive map, $\rho_{0,N+1}(t) = E_{[n=1]^N}(\rho_{0,N+1}(0))$, which can be written in components as $\langle\psi|e^{i\epsilon_t}\langle\psi|\rangle = \sum_{i,l}E_{i,l,k}(t)|k\rangle\langle\psi|\rho_{0,N+1}(0)|l\rangle$. To quantify the quality of the gate we calculate average gate fidelity $F_G(t) = \int d\psi \langle\psi|G^\dagger E_{[0]}(\psi)|\psi\rangle |G\psi\rangle$ where the integration is over all possible two-qubit pure states. Using the results of Ref. [26, 27] we get

$$F_G(t) = \frac{\sum_{i,j,k}G^\dagger_{i,k}E_{j,l}(t)G_{i,k} + 4}{20},$$  (4)

where $E_{i,j,k}(t) = \langle i|E_{[i]}(|k\rangle\langle l|)j\rangle$ are numerically evaluated.

In Fig. 1(a) we plot the time evolution of the average gate fidelity for a bus of length $N = 100$ initially in its ground state: $F_G(t)$ displays a marked peak at $t = t^*$. To show the scaling of the gate fidelity we plot $F_G(t^*)$ as a function of $N$ in Fig. 1(b) where we remarkably see that $F_G(t^*)$ exceed 0.9 even for chains up to $N = 100$ and decays very slowly with $N$. Moreover, as shown in the inset of Fig. 1(b) and unlike the perturbative schemes proposed in Refs. [5, 24] our dynamics is fast.

Our dynamical gate works properly for arbitrary initial states of the bus with fixed parity. Ideally after each gate application the parity of the bus remains unchanged making it perfect for reusing. However, initialization in an eigenstate of
TABLE I: $T_G(k')$ and $F_M(k')$ for up to 8 subsequent uses of the bus of length $N = 8$ without resetting.

| $k$ | $T_G(k')$ | $F_M(k')$ |
|-----|-----------|-----------|
| 1   | 0.984     | 0.966     |
| 2   | 0.961     | 0.926     |
| 3   | 0.939     | 0.840     |
| 4   | 0.918     | 0.795     |
| 5   | 0.898     | 0.748     |
| 6   | 0.879     | 0.701     |
| 7   | 0.861     | 0.654     |
| 8   | 0.844     |           |

$H_M$, besides automatically fixing the parity, has the advantage of simplicity for preparation. Let us initially set the bus in its ground state and define $F_M(t)$ as the fidelity between the ground state of $H_M$ and the density matrix of the bus at time $t$. To see how the quality of the gate operation is affected by $k$ subsequent uses of the bus, we compute $T_G(k')$ and $F_M(k')$ which are shown in TABLE I for $k = 1, \ldots, 8$ subsequent uses.

**Application:**—We now propose an application of the above gate mechanism for a scalable neutral atom quantum computer with qubits held in static traps. We consider a network of qubits each encoded in two degenerate hyperfine levels of a neutral atom, cooled and localized in a separate NFFD trap [22]. In Fig. 2(a) we show a single atom confined in a NFFD trap. The position of the minimum of the trapping potential is controlled by varying the aperture radius [22] through micro electro mechanical system technology, as proposed in [23]. Local unitary operation on each qubit may be applied through an extra fiber, along with the NFFD trapping fiber [23], as show in Fig. 2(a). The qubits in the network are connected by a bus realized by cold atoms in an optical lattice, prepared in the Mott insulator regime [16, 17]. The polarization and intensity of lasers are tuned so that one ends up with an effective Hamiltonian of Eq. (1). For the moment we assume that the distance between the two qubits, on which we want to apply the gate, is equal to the length of the lattice such that the two qubits interact with the atoms in the ending sites of the lattice, as shown in Fig. 2(b). To switch on the interaction $H_I$ between the qubits and the bus we have to move the minimum of NFFD trapping potential slightly higher such that the qubits move upwards and sit at a certain distance from the ends of the lattice. By controlling such distance one can tune the interaction coupling to be $J_{0}^{opt}$. In order to simultaneously obtain interactions effectively described by $H_M$ and $H_I$ we have to use the same spin dependent trapping laser beams in both NFFD traps and optical lattice.

Now we consider the situation in which the optical lattice size is larger than the distance between the qubits $A$ and $B$ (see Fig. 2(c)). In this case if we simply switch on the interaction between qubits and two intermediate sites $(L, R)$ of the optical lattice, shown in Fig. 2(c), the two external parts of the lattice play the role of environment and deteriorate the quality of the gate. To preserve the gate quality we need to cut the lattice into three parts and separate the bus, extended from $L$ to $R$, from the rest of the optical lattice. This can be done by adiabatically shining a localized laser beam on the atoms sitting on sites $L − 1$ and $R + 1$ to drive them off resonance, as shown in Fig. 2(c). In this case driving the atom effectively generates a Stark shift between the two degenerate ground state through a highly detuned classical laser beam with strength $\Omega$ and detuning $\Delta \gg \Omega$. This provides an energy shift $\delta E = \Omega^{2}/\Delta$ between the two degenerate ground states, which can be treated as a local magnetic field in the $z$ direction on sites $L − 1$ and $R + 1$. Keeping $\Omega/\Delta$ small one can control the strength $\Omega$ and detuning $\Delta$ such that $\delta E$ becomes larger than $J$. When $\delta E \gg J$ the bus is separated from the external parts of the optical lattice. Moreover, as $\delta E$ adiabatically increases, the bus moves into its ground state, meanwhile splitting up from the rest. Despite the gapless nature of Hamiltonian (1) there is always a gap $\propto J/N$ due to the finite size of the bus which guarantee the success of the adiabatic evolution. In Fig. 3(a) we plot $F_{M}(t)$ over the course of adiabatic cutting when the whole lattice is initially in its ground state. In this adiabatic evolution $\delta E$ is linearly increased from 0 to $30J$ over the time interval of $100/J$. Once the bus been prepared in its ground state the gate operation can be accomplished as discussed above. After the operation of the gate one may want to glue the previously split optical lattice and bring it back into its ground state. This can be done easily by adiabatically switching off $\delta E$ as shown in Fig. 3(a) where the fidelity of the state of the whole optical lattice with its ground state is plotted.

**Time scale:**—We now give an estimation of $t^{*}$ in the worst case scenario where $A$ and $B$ sit on the boundary of the lattice, which typically consists of $N \approx 100$. The typical values for $J$ in optical lattices are few hundred Hertz (e.g. $J = 360$ Hz in [28]). From the inset of Fig. 1(b) we get $Jt^{*} \approx 30$ for $N = 100$ and thus $t^{*} \approx 13$ ms which is well below the typical decoherence time of the hyperfine levels ($\approx 10$ minutes [29]). Though there are some recent realizations of entangling gates [30], faster than ours, they are much less versatile as they design a single, very specific, isolated gate and do not construct the gate as part of an extended system. Considering this latter kind of architecture, our mechanism is much faster than the perturbative methods [5, 24], and operates at the time scale of $O(N/J)$ which is the best possible in any physical realization.

**Imperfections:**—Cold atom systems are usually clean and almost decoherence free; however, in the above proposed setup there might be some sources of destructive effects which may deteriorate the quality of our scheme. In particular, we consider: (i) gradual switching of $J_{0}$; (ii) imperfect cutting of the chain when $\delta E$ is not large enough; (iii) existence of interaction terms in the Hamiltonian which alter its non-interacting free-fermionic nature. In Fig. 3(b) we show $F_{G}(t^{*})$ when $J_{0}$ is gradually switched on from 0 to $J_{0}^{opt}$ according to $J_{0}(t) = J_{0}^{opt}t/\tau$, as a function of switching time $\tau$. It is indeed of general relevance that a plateau over which $F_{G}(t^{*})$ remains constant is observed, even for $\tau$ as long as 1/$J_{0}$. In Fig. 3(c) we plot $F_{G}(t^{*})$ as a function of the energy splitting $\delta E$ on which the cutting process is based. As it is clear from Fig. 3(c), when $\delta E > 10J$ the bus is well isolated from the external parts, which guarantees the high quality of the gate. We have also studied the effect of the anisotropy $\lambda$, possibly entering $H_M$ and $H_I$, due to imperfect tuning of laser parameters [18]. In Fig. 3(d) we plot $F_{G}(t^{*})$ as a function of $\lambda$ and observe
weak deterioration of the gate quality as far as $|\lambda| < 0.2$.

Conclusions:— In this letter, we have proposed a scalable scheme for realizing a two-qubit entangling gate between arbitrary distant qubits. In our proposal, qubits are made of localized objects which makes single qubit gates affordable. The qubits interact dynamically via an extended unmodulated bus which does not need being specifically engineered and, besides embodying a quantum channel, actively serves to operate the entangling gate. Moreover, thanks to the non-perturbative interaction between the qubits and the bus our dynamics is fast, which minimizes destructive decoherence effects. Provided the coupling between the qubits and the bus is properly tuned, the dynamical evolution of the whole system is essentially dispersionless, thus allowing several subsequent uses of the bus without resetting. Surprisingly, a sudden switching of the coupling is not necessary and our fast dynamical gate mechanism is not altered by a possibly gradual switching: this is of absolute relevance, not only from practical viewpoints but also in a theoretical perspective. Our proposal is general and can be implemented in various physical realizations. Specifically we have proposed an application based on neutral atom qubits in an array of separated NFFD traps connected by an optical lattice spin chain data bus, which both are accessible to the current technology.

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