Optimal Battery Participation in Frequency Regulation Markets

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Abstract—Battery participants in performance-based frequency regulation markets must consider the cost of battery aging in their operating strategies to maximize market profits. In this paper we solve this problem by proposing an optimal control policy and an optimal bidding policy based on realistic market settings and an accurate battery aging model. The proposed control policy has a threshold structure and achieves near-optimal performance with respect to an offline controller that has complete future information. The proposed bidding policy considers the optimal control policy to maximize market profits while satisfying the market performance requirement through a chance-constraint. It factors the value of performance and supports a trade-off between higher profits and a lower risk of violating performance requirements. We demonstrate the optimality of both policies using simulations. A case study based on the PJM regulation market shows that our approach is effective at maximizing operating profits.

Index Terms—Battery energy storage, degradation, frequency regulation, power system economics

I. INTRODUCTION

The share of battery energy storage (BES) in the frequency regulation markets is increasing rapidly [11]. In the PJM market, the BES capacity has increased from zero in 2005 to over 280 MW in 2017, making up 41% of its regulation procurement capacity [2]. This growth rate was made possible by the rapid decrease in battery cell manufacturing cost [3], the increase in renewable penetration [4], as well as changes in market rules that have lowered the barrier for batteries to provide frequency regulation. Following the Federal Energy Regulatory Commission (FERC) Orders 755 [5] and 784 [6], all Independent System Operators (ISO) and Regional Transmission Organizations (RTO) in the U.S. have implemented pay-for-performance regulation markets, and improved the automatic generation control (AGC) framework to account for the state of charge (SoC) constraint of energy storage in their regulation dispatch [7]. In these revised markets, BES participants are rewarded for their fast and accurate regulation response. They are also able to offer a very small energy capacity (15 minutes at minimum) to provide regulation products specially designed for energy storage.

These regulation market reforms have encouraged numerous BES projects targeted solely at the provision of regulation. These BES units often behave as price-takers by submitting offers at zero price into the market and accepting any clearing prices. If the market prices for regulation are high, these participants can earn considerable profits with naive control strategies. However, since the amount of regulation capacity needed is limited, a market can easily become saturated by too many price-taking participants. Operational evidence shows that the PJM RegD market became saturated in 2016 and its market clearing prices have dropped by two-thirds since 2014 [2, 8], making BES operation hardly profitable with naive bidding and operating strategies.

BES participants must employ more advanced operating and bidding strategies to secure operational profits against dropping market prices and the cost of battery degradation. Previous studies have assessed how providing regulation with batteries affects their aging characteristic under simple SoC control strategies [8]–[19]. However, few studies have examined battery aging as part of operation or bidding optimization. References [20], [21] take into account the battery lifespan in regulation control optimization, using an aging model that is too simple to reflect properly the complex battery cycle aging mechanisms. The results in [22] incorporate the battery aging cost into regulation bidding strategies, but the proposed method does not optimize real-time operations.

Since regulation instructions are highly stochastic and occur at a very high time resolution, accounting for them explicitly in an optimization problems is computationally challenging. It is thus crucial to reduce the formulation complexity using statistical and analytical derivations, and to jointly optimize the bidding and real-time control strategies. In this work, we incorporate battery cycle aging into a real-time control policy that ensures profitability under any market clearing result. In addition, we introduce a bidding policy based on the proposed control policy that maximizes profits while satisfying the market performance requirement. The main contributions of this paper are as follows:

• We formulate battery bidding and operation in a pay-for-performance frequency regulation market as a chance-constrained profit maximization problem that includes an accurate battery cycle aging model.
• We propose a real-time control policy that balances the battery cycle aging cost and the regulation mismatch penalty. This policy achieves near-optimal control results with a bounded gap with respect to an optimal offline policy that has full information about future regulation signals.
• We prove that under the proposed control policy, the optimal battery regulation problem is reduced to finding the highest capacity possible while ensuring the required...
regulation performance. We therefore propose a bidding policy that ensures regulation performance while maximizing the operational profit by factoring the value of performance in the frequency regulation market.

- Numerical examples and case studies using real market data are given to demonstrate the effectiveness of the proposed policies.

The rest of this paper is organized as follows. Section II describes the formulation of the optimization problem. Section III introduces the proposed control and bidding policies. Section IV describes and discusses the simulation results. Section V concludes the paper.

II. PROBLEM FORMULATION

A. Frequency Regulation Market Model

We consider a pay-for-performance regulation market in which a participant is rewarded based on the regulation capacity it provides, as well as on how much it is instructed to alter dispatch set-points by the system operator, and how accurately it follows the regulation instructions. Most system operators calculate use as a performance index the relative error between a participant’s regulation response and the regulation instructions [7]. PJM uses a more complicated calculation method [23].

Pay-for-performance regulation markets have two-part offer and payment designs. A participant submits a regulation capacity offer and a regulation mileage offer, and then the system operator unifies the two offer prices into a single modified offer based on the participant’s historical performance index and the expected dispatch regulation mileage. In the ex-post market settlement, the capacity payment is calculated using the capacity clearing price and the assigned regulation capacity. The mileage payment is calculated using the mileage clearing price and the instructed mileage. Depending on the market design, the performance index is used to penalize the entire regulation payment, or only the mileage payment. All pay-for-performance regulation markets also have a minimum performance requirement, in which a participant must reach a certain performance index to be eligible for receiving regulation payment, and must maintain a satisfactory performance history to be qualified to participate in the regulation market [8].

The objective of a regulation market participant is to maximize its operational profit, including the expected market payment and the battery aging cost. To formulate this problem, we start by considering a market period with finite discrete dispatch intervals \( t \in T = \{1, \ldots, T\} \) and a single market clearing price \( \lambda \) which is calculated based on the capacity clearing price, the mileage clearing prices, and the instructed mileage [7]. We assume that the participant receives regulation instructions that are proportionally scaled with respect to its regulation capacity \( C \) based on a set of normalized regulation signals \( \mathbf{r} = \{r_t \in [-1, 1]\} \). The participant receives the following payment \( \Pi^M(\mathbf{r}) \) from the regulation market:

\[
\Pi^M(C, \lambda, \mathbf{r}, \mathbf{b}) = P(C, \mathbf{r}, \mathbf{b}, \lambda) C \tag{1}
\]

where \( \mathbf{b} = \{b_t\} \) is the participant’s response to the regulation instruction, and \( P(\cdot) \) is the performance index calculation function. We define \( P(\cdot) \) as a linear function of the relative response error:

\[
P(C, \mathbf{b}) = 1 - \frac{||C r - b||_1}{C ||r||_1} \delta. \tag{2}
\]

where \( ||C r - b||_1 \) calculates the absolute error in the response, and \( C ||r||_1 \) is the total amount of the instructed regulation signal. \( \delta \in [0, 1] \) is the part of the regulation payment that is evaluated based on the relative error of the response. This is a unified performance model that fits all market rules. The value of \( \delta \) is specific to each market and can be found in each market manuals.

B. BES Operation and Cost Model

We define \( B(e_0) \subset \mathbb{R}^N \) as the set of all feasible BES active power dispatch operations with an initial battery energy level \( e_0 \). This set is defined by the following constraints

\[
-B \leq b_t \leq B \tag{3}
\]

\[
E \leq e_t \leq E \tag{4}
\]

\[
e_t - e_{t-1} = M\eta |b_t|^\theta - M[-b_t]^\theta / \eta \tag{5}
\]

where \( B \) is the BES power ratings, \( e = \{e_t\} \) is the battery energy level at step \( t \), \( M \) is the dispatch interval duration, \( \eta \) is the BES single-trip charge or discharge efficiency, \( E \) and \( E \) are the BES upper and lower energy limits in MWh, and \( |x|^\theta \) models the BES power rating, \( \theta \) models the BES storage limits, and \( \Phi \) models the evolution of the state of charge.

The aging cost model we adopt here considers battery cycle aging and relies on the rainfall method [24] for cycle identification. Cycling is a major factor in battery aging for grid-scale BES because these applications consist mostly of low current rate operations [25]. Battery cycle aging is highly nonlinear with respect to the cycle depth. For example, a Lithium Nickel Manganese Cobalt Oxide (NMC) cell can provide ten times more lifetime energy throughput if cycled at 10% depth (between 55% SoC and 65% SoC) than if cycled at 100% depth [26]. Since it is crucial to model accurately the cycle aging process, we use the rainfall method, which is the state-of-the-art for cycle identification from operational SoC profiles.

The rainfall method decomposes a set of normalized battery SoC measurements into a combination of independent cycles as follows:

\[
u = \text{Rainflow}(e/E) \tag{6}
\]

where \( u \) is the set of all identified cycle depths, \( e \) is the time series of BES energy levels , \( E \) is the BES rated energy capacity. \( e/E \) is thus the normalized SoC series. After identifying the cycles, a cycle depth stress function calculates the incremental battery cycle aging caused by a particular cycle based on the cycle depth as \( \Phi(u) \). The total cycle life loss is calculated by summing the life loss from all identified cycles from the SoC series. The cycle depth stress can be obtained from lab test results [27]–[29] which are described in the manufacturer’s warranty. Since the battery cycle aging cost prorates the battery cell replacement cost to the cycle life...
loss, the the battery aging cost function $A(\cdot)$ is defined as follows:

$$A(b) = ER \sum_{i=1}^{u} \Phi(u_i)$$  \hspace{1cm} (7)

where $|u|$ is the cardinal of $u$, $R$ is the battery cell replacement cost in $\$/MWh. $\Phi(u)$ is the cycle depth aging stress function. The aging cost function $A$ is written as a function of $b$ because the battery SoC is a linear combination of the active power. For example, if the cell replacement cost of a 1 MWh NMC BES is 300K$, then a 10% depth–0.1 MWh energy throughput cycle costs 3$, while a 100% depth–1 MWh energy throughput cycle costs 300$.

### C. Optimization Problem

If the market clearing price $\lambda$ and the regulation signal realization $r$ are known, the participant can find the optimal regulation capacity $C$ and the optimal BES dispatch $b$ which ensure the minimal performance requirement by solving the following profit maximization problem: \[30\]

$$\max_{C, b} \Pi(C, \lambda, r, b) := \Pi^{\lambda}(C, \lambda, r, b) - A(b)$$  \hspace{1cm} (8)

s.t. $P(Cr, b) \geq \rho^{\min}$

$$C \in [0, B], \quad b \in B(e_0).$$

However, this problem cannot be solved in practice because the realization of $r$ is not known in advance. The participant must therefore decide on a policy $g$ that determines $b_t$ at time step $t$ based only on past information. The expected profit corresponding to a particular operational policy and regulation capacity $(g, C)$ is defined as:

$$J(g, C) = \mathbb{E} \left[ \Pi(C, \lambda, r, b^g) \right]$$

$$= \mathbb{E} \left[ \lambda C - \delta \lambda \left| \frac{|Cr - b^g|}{||r||} \right| - A(b^g) \right],$$  \hspace{1cm} (9)

where the superscript $g$ is included to indicate the dependence on the control policy $g$. However, the exact value of $\lambda$ and $||r||$ can only be known after the regulation provision, hence we approximate the formulation using the following expectations

$$\mu_{\lambda} = \mathbb{E}[\lambda], \quad \mu_{||r||} = \mathbb{E}[||r||].$$  \hspace{1cm} (10)

To solve this problem, we transform the minimum performance requirement into a chance constraint, and rewrite (8) as a chance-constrained stochastic programming problem

$$\max_{g, C} J(g, C) = \mu_{\lambda} C - \mathbb{E} \left[ \frac{\mu_{\lambda}}{\mu_{||r||}} |Cr - b^g|/||r|| - A(b^g) \right]$$

s.t. $\text{Prob}[P(Cr, b) \geq \rho^{\min}] \geq \xi$

$$g \in \mathcal{G}, \quad C \in [0, B].$$  \hspace{1cm} (11)

where $\xi$ is the confidence level that the performance requirement will be satisfied, and $\mathcal{G}$ is the set of all feasible operation policies $g$ that satisfy (3)–(5) and the causality condition.

### III. OPTIMAL CONTROL AND BIDDING POLICY

We propose an online control policy and a bidding policy that solve problem (11). Control and bidding in the regulation market are part of a sequential decision-making process because the regulation capacity $C$ is cleared before the actual dispatch. We therefore work backwards by first determining the optimal online control policy $g^\ast$ with respect to any cleared regulation capacity (Fig. 1)

$$g^\ast \in \arg\min_{g \in \mathcal{G}} J(g, C)$$  \hspace{1cm} (12)

We then we derive the optimal bidding policy $C^\ast$ based on the optimal control policy:

$$C^\ast \in \arg\min_{C \in [0, B]} \left\{ \min_{g \in \mathcal{G}} J(g, C) \right\}$$

s.t. $\text{Prob}[P(Cr, b) \geq \rho^{\min}] \geq \xi.$  \hspace{1cm} (13)

We consider the performance chance constraint at the bidding stage because it depends primarily on the regulation capacity and the battery energy capacity (see Section III-B). We assume that $r$ is energy zero-mean (including efficiency losses) because either the system operator or the participant can employ strategies for controlling the average BES SoC and these strategies do not affect a participant’s objective of minimizing its operating cost.

#### A. Optimal Control Policy

The online regulation response policy balances the cost of deviating from the regulation signal and the cycle aging cost of batteries while satisfying operating constraints. This policy takes a threshold form and achieves an optimality gap that is independent of the total number of time steps. Therefore in term of regret (consequence of decision-making under uncertainty) (11), this policy achieves the strongest possible result: the regret do not grow with time. The key part of the control policy is to calculate thresholds that bound the SoC of the battery as functions of the deviation penalty and degradation cost. The bound on the SoC $\hat{u}$ is given by:

$$\hat{u} = \varphi^{-1} \left( \frac{\eta^2 + 1}{\eta R} \right), \quad \pi = \delta \frac{\mu_{\lambda}}{\mu_{||r||}} \mu_{\lambda}$$  \hspace{1cm} (14)
Specifically, if at a particular control interval \( t \), the policy keeps track of the real-time maximum and minimum SoC level. When the distance between them reaches the calculated threshold \( \hat{u} \), the policy starts to constrain the response. (a) dispatch instruction vs. response; (b) Controlled vs. uncontrolled SoC profile.

Algorithm 1: Optimal Battery Regulation Response Policy

**Result:** Determine battery dispatch points \( b_t \)

```plaintext
// calculate optimal cycle depth
set \( \hat{u} \rightarrow \varphi^{-1}\left(\frac{\varphi^2 + 1}{\varphi R} \eta\right) \);

// set current max/min energy level to \( e_0 \)
set \( e_0 \rightarrow e_0^{\max}, e_0 \rightarrow e_0^{\min} \);

while \( n \leq N \) do
    // read \( \epsilon_t \) and update min/max energy level
    set \( \max\{e_{t-1}^{\max}, \epsilon_t\} \rightarrow e_t^{\max}, \min\{e_{t-1}^{\min}, \epsilon_t\} \rightarrow e_t^{\min} \);
    // activate energy level bounds if either reached \( \hat{u} \) or constraint \( \epsilon_t \)
    set \( \min\{e_t^{\min}, \hat{u}E\} \rightarrow \tau_t^g \);
    set \( \max\{e_t^{\max} - \hat{u}E\} \rightarrow \tau_t^g \);
    // read \( \epsilon_t \) and enforce soc bounds
    if \( C_{r_t} \geq 0 \) then
        set \( \min\left\{\frac{1}{\tau_t}\left(\tau_t^g - \epsilon_t\right), C_{r_t}\right\} \rightarrow b_t \);
    else
        set \( \max\left\{\frac{\hat{u}}{\tau_t}\left(\epsilon_t - \tau_t^g\right), C_{r_t}\right\} \rightarrow b_t \);
    end
    // wait until the next control interval
    set \( n + 1 \rightarrow n \);
end
```

where \( \varphi^{-1}(\cdot) \) is the inverse function of the derivative of the cycle stress function \( \varphi(x) = d\Phi(x)/dx \).

Algorithm 1 summarizes this control policy, and Fig. 2 shows an example of control based on this policy, where the battery follows the dispatch instructions until the distance between its maximum and minimum SoC reaches \( \hat{u} \). Specifically, if at a particular control interval \( t \), \( \epsilon_t \) and \( r_t \) are observed, the proposed dispatch policy has the following form: \( g_t(\epsilon_t, \pi, r_t) = b_t^* \). Let \( g(\epsilon_0, \pi, C_r) = b^* \) denote the BES response profile to the regulation instruction \( C_r \) subject to an initial energy level \( e_0 \) and a penalty price \( \pi \).

**Theorem 1:** Suppose the battery cycle aging stress function \( \Phi(\cdot) \) is strictly convex. The proposed control strategy \( g(\cdot) \) has a worst-case optimality gap (regret) \( \epsilon \) that is independent of the operation time duration \( T_N \):

\[
\pi ||C_r - b^*||_1 - A(b^*) \leq \epsilon
\]

where \( b^* \in \text{arg min}_{b \in [e_0, E]} \pi ||C_r - b||_1 - A(b) \).

**Proof:** A proof of Theorem 1 can be found in [32].

The intuition for this theorem is as follows. After a particular cycle of depth \( u \), the energy discharged from this cycle is \( uE \). If the BES follows the regulation signal during this cycle, it avoids a penalty cost \( uE\pi \), but bears an aging cost \( ER\Phi(u) \). Since the penalty cost is linear with respect to \( u \) while the aging cost is super-linear (strictly convex), then there must exists an optimal depth \( \hat{u} \) that trades off the penalty and aging cost. Because \( \hat{u} \) applies to all cycles, then the battery SoC profile must contain no cycles larger than \( \hat{u} \), which, according to the rainflow method, is equivalent to defining upper and lower SoC bounds as in Algorithm 1.

Based on realistic battery costs and market prices, the value of \( \epsilon \) should be less than one dollar. Section IV-B also shows that \( \epsilon \) is small and negligible compared to the total regulation operating cost. \( g \) therefore approximates the optimal regulation policy \( g^* \).

### B. Optimal Regulation Capacity

We first show that under the optimal control policy and when relaxing the minimum performance constraint, a participant should adopt a price-taker bidding strategy, i.e., it should provide the maximum possible regulation capacity under any market prices. We then incorporate the performance chance constraint and show that the performance-constrained optimal capacity can be explicitly characterized based on statistics of historical regulation signals.

**Theorem 2:** When relaxing the minimum performance requirement, the optimal regulation capacity \( C^* \) is equal to the BES power rating \( B \) when using the proposed optimal response policy \( g^* \).

**Proof:** See Appendix.

The intuition for this theorem is that the proposed policy guarantees optimal operating profit under any regulation capacity and signal realization. A higher regulation capacity offers higher profit potential since market payment is capped by \( \lambda C \). Therefore under optimal real-time control a participant will never earn less profit with a higher capacity because the penalty cost never exceeds the market payment.

Stemming from Theorem 2, the solution of (13) is reduced to finding the maximum \( C \) while satisfying the performance chance-constraint. The challenge is that the regulation signal realization is unknown when determining the regulation capacity. We must therefore characterize a probabilistic function that correlates the performance index \( \rho \) with the regulation
capacity $C$. In addition, since the battery regulation response $b^g$ under the proposed policy $g^*$ is price responsive, we must also take the expected clearing price $\mu_\lambda$ into consideration.

We start with the performance index calculation with the regulation signal realization $r$ and show that it can be reformed into a function with respect to $b^g/C$, which is the normalized battery response

$$P(Cr, b) = 1 - \frac{||Cr - b^g||_1}{C||r||_1} = 1 - \frac{||r - b^g/C||_1}{||r||_1}. \quad (16)$$

Recall that in the proposed policy the battery energy level is constrained between $\tau^g$ and $\xi^g$ that are calculated from battery data and the penalty price. We substitute $b^g/C$ into the proposed policy and show that $b^g/C$ only depends on $\tau^g/C$ and $\xi^g/C$ since the battery operation constraints (3–5) are linear

$$\frac{b^g}{C} = \begin{cases} \min \left\{ \frac{\left(\tau^g - e_t\right)/(M\eta C), r_t \right\} & \text{if } r_t \geq 0 \\ \max \left\{ \frac{\left(\xi^g - e_t\right)/(M\eta C), r_t \right\} & \text{if } r_t < 0. \end{cases} \quad (17)$$

We omit the effect of the battery initial energy level $e_0$ on the performance index because $r$ has a zero mean (i.e. it is energy neutral). The performance index thus depends only on the ratio between the usable energy capacity of the battery $\tau^g - \xi^g$ and the regulation capacity $C$. We can therefore define this ratio as the normalized regulation energy capacity $v$

$$v = \frac{\tau^g - \xi^g}{C} = \min \left\{ \frac{E - E_\lambda, \hat{u}E}{\hat{u}E} \right\}. \quad (18)$$

where we substitute $\hat{u}$ into $\tau^g - \xi^g$ as in Algorithm 1. Recall that $\hat{u}$ is calculated from the expected market clearing price, hence we represent $v$ as a function of $\mu_\lambda$ and $C$.

Having shown that the performance index only depends on $v$, we now define a probabilistic function $P^*_\xi(v)$ of $v$, which means that a battery with a normalized regulation energy capacity of $v$ is $\xi$ certain to reach a performance score of $P^*_\xi(v)$. Hence we can rewrite the performance chance-constraint as

$$\text{Prob}[P(Cr, b^g) \geq P^*_\xi(v)] = \xi \quad (19)$$

and $P^*_\xi(v)$ can be determined by simulating historical regulation signals.

**Lemma 1:** $P^*_\xi(v)$ is monotonic over $P^*_\xi(v) \in (1 - \delta, 1)$ and has an inverse function $[P^*_\xi]^{-1}(\rho)$ over $\rho \in (1 - \delta, 1)$.

**Proof:** This lemma is trivial. First we consider a regulation signal realization set $r$, the minimum performance index that a battery can possibly score is $1 - \delta$ because the rest is not dependent on the battery’s response. Before reaching the perfect performance of 1, an increment in $v$ must result in an improvement in the performance index due to that $v$ is the only constraining factor. This is true for any realizations of $r$, hence it is trivial that $P^*_\xi(v)$ is monotonic over $P^*_\xi(v) \in (1 - \delta, 1)$.

Following Lemma 1, if a participant wishes to reach a performance score $\rho \in (1 - \delta, 1)$ with exactly $\xi$ confidence, it must use the following value of $v$:

$$v = [P^*_\xi]^{-1}(\rho), \quad \rho \in (1 - \delta, 1) \quad (20)$$

We now combine Theorem 2, (18) and (14) to characterize analytically the optimal regulation capacity when using the proposed optimal response policy:

$$C^*(\mu_\lambda) = \min \left\{ \frac{B, \min \left\{ \frac{E - E_\lambda, \hat{u}E}{\hat{u}E} \right\}}{[P^*_\xi]^{-1}(\rho_{\min})} \right\}$$

where $\hat{u} = \varphi^{-1} \left( \eta^2 + 1 \right) \eta R\mu_\lambda M \delta_{\mu_\lambda} \lambda \lambda$ (21)$$

The only variable in this equation is $\mu_\lambda$. The rests are either based on the market policy or the BES design. It is easy to see that if $\Phi$ is strictly convex, $C^*(\mu_\lambda)$ is monotonic over the following range

$$C \in (0, \overline{C})$$

where $\overline{C} = \min \left\{ \frac{B, \min \left\{ \frac{E - E_\lambda, \hat{u}E}{\hat{u}E} \right\}}{[P^*_\xi]^{-1}(\rho_{\min})} \right\}$ (22)

and $\overline{C}$ is the maximum regulation capacity a participant could provide in order to reach the performance index $\rho_{\min}$ with a confidence of at least $\xi$.

C. Optimal Bidding Policy in Regulation Markets

**Algorithm 2:** Optimal Battery Regulation Bidding Policy

**Result:** Determine the regulation offer price $\lambda^b$ associated with capacity segment $C^b$

// starts from the first (cheapest) bid segment set $1 \rightarrow J$

// goes through each segment until reaching $\overline{C}$

while $\sum_{j=1}^{J} C^b_j \leq \overline{C}$ do

// total capacity offered so far set $\sum_{j=1}^{J} C^b_j \rightarrow C^\text{total}$;

// total payment price expected set $[C^*]^{-1}(C^\text{total}) \rightarrow \lambda^\text{total}$;

// calculate the segment offer price set $\left( \lambda^\text{total} C^\text{total} - \sum_{j=1}^{J-1} \lambda^b_j \right) / C^b \rightarrow \lambda^b_j$;

// go to the next capacity segment set $J + 1 \rightarrow J$;

end

At the bidding stage in the regulation market, each participant must submit a segment bidding curve that specifies how much regulation capacity that it is willing to provide at a given market price. We denote this set $(\lambda^b_j, C^b_j) = \{(\lambda^b_j, C^b_j) \in \mathbb{R}_+^2 | j \in \mathbb{N}\}$. Therefore, each participant needs a bidding policy to calculate $\lambda^b$ with respect to the regulation capacity segments $C^b$.

The optimal bidding policy is straightforwardly based on (21) and (22): a participant uses the inverse function of $C^*(\cdot)$ to calculate the offer price associated with each capacity segment, while the total offered capacity must be smaller than $\overline{C}$ in order to satisfy the performance requirement. This optimal bidding policy is described in Algorithm 2.
IV. SIMULATION

A. Data and Setting

We use the following parameters for the battery energy storage in simulations unless otherwise specified:

- Charging and discharging power rating: 10 MW
- Energy capacity: 3 MWh
- Charging and discharging efficiency: 95%
- Maximum state of charge: 95%
- Minimum state of charge: 10%
- Round-trip efficiency: 92%
- Battery cycle life: 1000 cycles at 80% cycle depth
- Battery shelf life: 10 years
- Cell temperature: maintained at 25°C
- Battery cycle life: 1000 cycles at 80% cycle depth
- Battery pack replacement cost: 300 $/kWh
- Li(NiMnCo)O₂-based 18650 lithium-ion battery cells

These cells have a near-quadratic cycle depth stress function [27].

\[ \Phi(\delta) = (1.57E-3)\delta^{2.03}. \]  

(23)

We use a simple control policy as a benchmark in all of the following simulations. The simple policy uses all battery energy capacity \([E, T]\) for operation and does not consider the penalty price or the cycle aging cost, hence it is equivalent to fixing \(\hat{u} = 1\) in the proposed policy. Although this naive control approach, it has been used in most stochastic real-time battery operation studies [14], [33], [34].

B. Illustration of Theorem 1

| Case | \(\pi\) [$/MWh] | \(\eta\) [%] | \(T\) | \(\hat{u}\) | Maximum regret [\$] |
|------|-----------------|----------------|--------|----------|-----------------|
| 1    | 50              | 100            | 100    | 11.1     | 0.00 183.9      |
| 2    | 100             | 100            | 100    | 21.9     | 0.00 127.5      |
| 3    | 200             | 100            | 100    | 42.8     | 0.00 47.9       |
| 4    | 50              | 92             | 100    | 11.2     | 0.06 184.9      |
| 5    | 50              | 92             | 200    | 11.2     | 0.06 408.8      |

We illustrate Theorem 1 by comparing the results of the proposed control policy with those of the simple control policy. Table I summarizes the simulation results. In each case the battery was subjected to 100 simulated regulation signals. Each case has a different simulation duration \(T\), or penalty price \(\pi\), or efficiency \(\eta\). Control regrets are calculated by comparing the results from the control policy with the offline optimal control where all regulation signals are known. The proposed control policy achieves negligible regrets in all cases, while the simple policy control result has a significantly higher regrets. A further discussion of these results can be found in [32].

C. Illustration of Theorem 2

We illustrate Theorem 2 by simulating a 1 MWh BES providing a full-year regulation service with varying regulation capacities in the PJM market using price and signal data from 03/2016 to 02/2017. The minimum performance requirement is not enforced in this example. Fig. 3 shows the simulation result for the proposed policy and the simple policy to demonstrate the effectiveness of our method.

The result in Fig. 3 includes the aging cost, the performance penalty cost, and the operational profit for regulation capacities up to 10 MW. With the proposed policy, the profits increase with the regulation capacity, which validates Theorem 2. With the simple policy, the BES operation is somewhat profitable when the regulation capacity is below 4 MW, but this profitability disappears completely once the regulation capacity is greater than 6 MW.

D. Case Study

We simulate a BES providing dynamic regulation (RegD) service in the PJM regulation market using signal and price data for a full-year (from 03/2016 to 02/2017). The minimum performance index requirement for this market is 0.7, and the settlement period in one hour. The regulation signal is slightly biased to make it energy zero-mean, a realistic assumption in the current market structure [8]. Parameters of the bidding policy, including the value of \(\mu_r\) and the function \(P_\xi\), are determined using the PJM RegD signal from 06/2013 to 05/2014. The value of \(\mu_\lambda\) is calculated from the PJM capacity and performance clearing prices using a mileage ratio of 3 based on the PJM market clearing manual [7]. We consider 10 1 MW regulation capacity offer segments for a 10MW/3MWh BES. The BES thus offers a total of 10 MW regulation capacity. The offer price associated with each segment is calculated according to the proposed bidding policy using the performance index simulation result from 2013, as shown in Fig. 4a. The performance confidence function concluded from 2016 data is slightly different from 2013, in particular, the energy capacity requirement at high confidence levels are higher, possibly due to changes in the dispatch algorithm, unexpected weather conditions, or increased renewable penetration.

For comparison, we include a benchmark strategy, where the 10MW/3MWh BES participates the regulation market as
TABLE II
CASE STUDY RESULTS IN PJM REGD MARKET 03/2016-02/2017.

|                           | Benchmark  | Optimal participation under ξ performance confidence |
|---------------------------|------------|-----------------------------------------------------|
|                           | ξ = 99%    | ξ = 95%    | ξ = 90%    | ξ = 85%    | ξ = 75%    | ξ = 50%    |
| Market income [k$]        | 1472.9     | 494.8      | 823.2      | 958.8      | 1035.9     | 1164.9     | 1266.8     |
| Aging cost [k$]           | 1091.8     | 67.5       | 165.8      | 213.0      | 240.3      | 286        | 326.3      |
| Prorated operating profit [k$] | 381.1 | 427.3      | 657.4      | 745.7      | 795.6      | 876.8      | 940.5      |
| Battery cell life expectancy [month] | 9 | 37 | 18 | 15 | 13 | 11 | 10 |
| Annual average performance | 0.99 | 0.99 | 0.96 | 0.95 | 0.93 | 0.9 | 0.84 |
| Hours of under-performance | 6 | 81 | 204 | 246 | 297 | 426 | 1022 |
| Total regulation capacity cleared [MW·h] | 87600 | 14991 | 29051 | 36838 | 42166 | 54087 | 73504 |

Fig. 4. PJM case study. (a) The relative regulation capacity v vs. different ξ from simulating historical regulation signals in 2013 and 2016; (b) Histogram of all cleared regulation capacities under different confidence ξ.

V. CONCLUSION
In this paper, we proposed profit-maximizing control and bidding policies for battery energy storage participating in performance-based regulation market. These policies consider the cycle aging mechanism of electrochemical battery cells, and is adaptive to realistic market settings. We validate the optimality of the proposed policies using simulations, and demonstrate the effectiveness of our approach using a case study based on the PJM regulation market.

Performance-based frequency regulation markets have been proposed for around five years and their designs are still maturing. With an increasing number of battery participants, frequency regulation markets are becoming increasingly competitive. As exemplified by the ongoing market revisions in PJM, system operators may also tighten the performance requirements. The proposed approach ensures not only profitability but also the satisfaction of the performance requirements.

APPENDIX
A. Supplement Material to Theorem 1
ε from Theorem 1 is calculated as [30]

\[ \epsilon = 2J_w(\hat{u}) + J_v(\hat{u}) - 2J_w(\hat{w}) - J_v(\hat{v}) \]  (24)
where
\[
\begin{align*}
J_r(v) &= (1/2) E R \phi(v) + \left( E / \eta_R \right)_\pi v \\
J_w(w) &= (1/2) E R \phi(w) + E \eta_D \pi w \\
\hat{v} &= \varphi^{-1}(\pi/\eta_R) , \quad \hat{w} = \varphi^{-1}(\pi/\eta_D) .
\end{align*}
\]

\[\text{(25)}\]

\[\text{(26)}\]

\[\text{(27)}\]

\[\text{B. Proof for Theorem 2}\]

Following the proposed control policy in Algorithm \[1\] we can rewrite the penalty function as

\[|C r - b^p|_1 = C |r|_1 - |b^p|_1,\]

\[\text{(28)}\]

because \(|b_1| \leq |C r_1|\) and \(b_t\) always has the same sign as \(r_t\). It follows that

\[
\frac{\partial J(g, C)}{\partial C} = \mu_C - E \left[ \delta \mu_\lambda |r|_1 \right] = \mu_\lambda - \delta \mu_\lambda \frac{E |r|_1}{\mu_r} = 1 - \delta \frac{\mu_\lambda}{\mu_r} \geq 0
\]

\[\text{(29)}\]

because \(\mu_r = E |r|_1\) and \(\delta \in [0, 1]\) and \(\mu_\lambda \geq 0\). Hence it proves Theorem 2.

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