STUDY OF ISOSPIN EIGENSTATES OF THE PENTAQUARK STATES WITH STRANGENESS

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Abstract

In the present work, we construct eight color singlet-singlet type five-quark currents with strangeness to study the pentaquark states of $\bar{D}\Xi^*_c$, $\bar{D}'\Xi^*_c$, $\bar{D}'\Xi^*_c$ and $\bar{D}'\Xi^*_c$ via the QCD sum rules. The considered states are isospin eigenstates with isospin quantum number $I = 1$ or $I = 0$. Numerical results show that the central value of the masses extracted from the Borel platforms with high isospin are slightly above the thresholds of the meson and baryon constituents and the low isospin ones are a few dozens of MeV below, what’s more, the present study supports the assignment of $P_{cs}(4459)$ as $\bar{D}'\Xi^*_c$ molecular state with the $IJ^P = 0(\frac{3}{2})^-$. Technically, besides the analyses of the uncertainties of the masses and pole residues, detailed discussion about the contribution of vacuum condensates proportional to the mass of $s$ quark are conducted which may present a reference for the technical application of QCD sum rules in the future work.

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1 Introduction

In the past few decades, many exotic $X$, $Y$, $Z$ particles had been observed at the Belle, BaBar, BESIII and LHCb collaborations [1], the intriguing fact is that many of their masses are near the hadron–hadron thresholds which shed light on the possible hadronic molecule interpretations [2]. In 2015, the LHCb collaboration observed two hidden-charm pentaquarks in the $\Lambda^0_b \rightarrow J/\psi pK^-$ decay process [3], namely, $P_c(4380)$ and $P_c(4450)$. In 2019, the observation was updated and $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ were reported by the LHCb collaborations [4], it reported that $P_c(4450)$ is actually the overlapping peak of $P_c(4440)$ and $P_c(4457)$. In the present work, we focus on the observation reported by the LHCb collaborations in 2020 for the hidden-charm strange pentaquark $P_{cs}(4459)$ in the $J/\psi \Lambda$ mass spectrum from amplitude analysis of $\Xi^0_b \rightarrow J/\psi \Lambda K^-$ decay [5], the state mass and width are $4458.8 \pm 2.9_{-1.1}^{+4.7}$ MeV and $17.3 \pm 6.5_{-5.7}^{+8.0}$ MeV, respectively.

Due to the exotic hadronic structures, the $P_c$ states have been attracting lots of interests in strong interaction area [2, 6–8]. Now, for these $P_c$ states, a typical interpretation is that they are the S-wave hidden-charm meson–baryon molecules with definite isospin $I$, spin $J$ and parity $P$ [9–13]. Inspired by the interpretation of exotic $P_c$ states, many theoretic groups interpret the newly discovered $P_{cs}(4459)$ in a similar way. For example, in Ref. [14], the authors assume the $P_{cs}(4459)$ as $\bar{D}'\Xi^*_c$ molecular state and study its strong decay via the consideration of its $J^P$ as $\frac{3}{2}^-$ and $\frac{5}{2}^-$. In the framework of QCD sum rules [15], the study support the assignment of $P_{cs}(4459)$ as the $\bar{D}'\Xi^*_c$ hadronic molecular state of either $J^P = (\frac{3}{2})^-$ or $(\frac{5}{2})^-$. Applying the quasi potential Bethe-Salpeter equation approach [16], the $P_{cs}(4459)$ is interpreted as $\bar{D}'\Xi^*_c$ molecule with the $J^P = (\frac{3}{2})^-$. Under the one-boson-exchange model, the authors conclude that this exotic state is not the pure molecular state [17]. As for the other arguments about the properties of $P_{cs}(4459)$, one can consult the Refs. [18–22] and so on.

Since the $J^P$ of $P_{cs}(4459)$ is not determined yet in experiment, as the above introduction indicates, the nature of this exotic state is still under debate. In Ref. [23], our group apply the color-singlet-color-singlet type pentaquark currents to study the $P_c$ and $P_{cs}$ states in a systemic

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way via the QCD sum rules, and assign $P_{cs}(4459)$ with the $J^P$ either $(\frac{1}{2})^-$ or $(\frac{3}{2})^-$, in that paper, the color-singlet-color-singlet type pentaquark currents of the isospin eigenstates are proposed. In Ref. [24], the isospins are unambiguously distinguished to study the hadron-hadron molecules in the framework of QCD sum rules for the first time and the $P_c(4312)$, $P_c(4380)$, $P_c(4440)$ and $P_c(4457)$ are assigned as the $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecules with low isospin $I = \frac{1}{2}$ in detail, intrigued by our previous works, we are very interested to investigate the present topic: what about the situation if we differentiate the isospins for the pentaquark states with strangeness? Can we clearly determine the nature of $P_{cs}(4459)$? What about the predictions of properties of the other possible pentaquark states with strangeness?

Among the popular theoretic methods, the QCD sum rules [25, 26] is a powerful tool to study the hadronic interaction. It has achieved many successful descriptions, such as the tetraquark states [27, 28, 29, 30, 31], tetraquark molecular states [32, 33], pentaquark states [34, 35], pentaquark molecular states [36, 37, 38, 39, 40], dibaryon and baryonium [41, 42, 43, 44, 45] and so on. However, the isospins of the states are seldom differentiated except our previous calculation [24], it shows that mass of the high isospin state is a few dozens of MeV above that of the low one. As is known, deviation of a few dozens of MeV is enough to confuse the assignment of the state, thus we argue that the differentiation of the isospin may be one of the key preconditions for the accurate assignment.

The article is organized as follows: in Sect.2, the QCD sum rules for the pentaquark states is derived; the numerical results and discussions are given in Sect.3; Sect.4 is reserved for our conclusions.

2 QCD sum rules for the pentaquark states

In the isospin space, the $u$ and $d$ quarks have the isospin eigenvalues $\frac{1}{2}$ and $-\frac{1}{2}$, respectively, thus the $\bar{D}^0$, $\bar{D}^{*0}$, $\bar{D}^-$, $\bar{D}^{*-}$, $\bar{\Xi}^0$, $\bar{\Xi}^{*0}$, $\bar{\Xi}^+$ and $\bar{\Xi}^{*+}$ correspond to the isospin eigenstates $|\frac{1}{2}, \frac{1}{2} \rangle$, $|\frac{1}{2}, -\frac{1}{2} \rangle$, $|\frac{1}{2}, -\frac{1}{2} \rangle$, $|\frac{1}{2}, -\frac{1}{2} \rangle$, $|\frac{1}{2}, -\frac{1}{2} \rangle$, $|\frac{1}{2}, -\frac{1}{2} \rangle$, $|\frac{1}{2}, -\frac{1}{2} \rangle$, respectively. We can apply the following color-singlet currents to interpolate the above mesons and baryons,

$$
J_{\mu}^{\bar{D}^0}(x) = \bar{c}(x)i\gamma_\mu u(x), \\
J_{\mu}^{\bar{D}^-}(x) = \bar{c}(x)i\gamma_\mu d(x), \\
J_{\mu}^{\bar{D}^{*0}}(x) = \bar{c}(x)\gamma_\mu u(x), \\
J_{\mu}^{\bar{D}^{*-}}(x) = \bar{c}(x)\gamma_\mu d(x), \\
J_{\mu}^{\bar{\Xi}^0}(x) = \varepsilon^{ijk} d^T(x) C\gamma_\mu s^j(x)\gamma^k\gamma_5 c^k(x), \\
J_{\mu}^{\bar{\Xi}^{*0}}(x) = \varepsilon^{ijk} d^T(x) C\gamma_\mu s^j(x)c^k(x), \\
J_{\mu}^{\bar{\Xi}^+}(x) = \varepsilon^{ijk} u^T(x) C\gamma_\mu s^j(x)\gamma^k\gamma_5 c^k(x), \\
J_{\mu}^{\bar{\Xi}^{*+}}(x) = \varepsilon^{ijk} u^T(x) C\gamma_\mu s^j(x)c^k(x),
$$

where, the superscripts $i, j, k$ are color indices and the $C$ represents the charge conjugation matrix. Based on the above currents of the mesons and baryons, we construct the color-singlet-color-singlet
type five-quark currents to study the $\bar{D}\Xi$, $\bar{D}^*\Xi$, $\bar{D}^*\Xi$ and $\bar{D}^*\Xi$ pentaquarks, states with not only the negative but also the positive parity. At the hadron side, we isolate the five-quark currents to study $\bar{\psi}\bar{\psi}$, where, the subscripts 0 and 1 mean the isospins $P\bar{\psi}$ and $J\bar{\psi}$, respectively. Note that, the currents $\bar{\psi}\gamma_\mu J\bar{\psi}$, for which $\bar{\psi}\bar{\psi}$ is the Dirac spinor, one can show that the above eight currents have the negative parity. The two-point correlation functions are then written as,

$$
P(0) = i \int d^4xe^{ipx} \langle 0| T \{ J(x), J(0) \} | 0 \rangle, 
$$

$$
P_{\mu\nu}(0) = i \int d^4xe^{ipx} \langle 0| T \{ J_\mu(x), J_\nu(0) \} | 0 \rangle, 
$$

$$
P_{\mu\nu\alpha\beta}(0) = i \int d^4xe^{ipx} \langle 0| T \{ J_{\mu\nu}(x), J_{\alpha\beta}(0) \} | 0 \rangle, 
$$

where, the subscripts 0 and 1 mean the isospins $I = 0$ or 1 $[23]$, and these currents are isospin eigenstates, either $|0, 0 \rangle$ or $|1, 0 \rangle$. Consider the parity operator $\hat{P}$, since $\hat{P}\psi\hat{P}^{-1} = \psi_{-1}$, for which $\psi$ is the Dirac spinor, one can show that the above eight currents have the negative parity. The two-point correlation functions are then written as,

$$
P(0) = i \int d^4xe^{ipx} \langle 0| T \{ J(x), J(0) \} | 0 \rangle, 
$$

$$
P_{\mu\nu}(0) = i \int d^4xe^{ipx} \langle 0| T \{ J_\mu(x), J_\nu(0) \} | 0 \rangle, 
$$

$$
P_{\mu\nu\alpha\beta}(0) = i \int d^4xe^{ipx} \langle 0| T \{ J_{\mu\nu}(x), J_{\alpha\beta}(0) \} | 0 \rangle, 
$$

where the currents

$$
J(x) = J_0^{\bar{D}\Xi}(x), \quad J_1^{\bar{D}\Xi}(x), 
$$

$$
J_\mu(x) = J_{0,\mu}^{\bar{D}\Xi}(x), \quad J_{1,\mu}^{\bar{D}\Xi}(x), \quad J_{0,\mu}^{\bar{D}^*\Xi}(x), \quad J_{1,\mu}^{\bar{D}^*\Xi}(x), 
$$

$$
J_{\mu\nu}(x) = J_{0,\mu\nu}^{\bar{D}^*\Xi}(x), \quad J_{1,\mu\nu}^{\bar{D}^*\Xi}(x). 
$$

Note that, the currents $J(x)$, $J_\mu(x)$ and $J_{\mu\nu}(x)$ can couple potentially with the pentaquark states with not only the negative but also the positive parity. At the hadron side, we isolate the contribution of the ground state and write the correlation functions as,

$$
P(0) = (\lambda_2) \frac{2}{M^2 - p^2} \gamma_\mu + \frac{1}{M^2 - p^2} \gamma_\nu + \cdots, 
$$

$$
P_{\mu\nu}(0) = -\Pi_2(p^2) \gamma_\mu \gamma_\nu + \Pi_0(p^2) g_{\mu\nu} + \cdots, 
$$

$$
P_{\mu\nu\alpha\beta}(0) = -\Pi_2(p^2) \gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta + \Pi_0(p^2) g_{\mu\nu} g_{\alpha\beta} + \cdots, 
$$

where, $\lambda_2$ and $\lambda_0$ are the isospin eigenvalues of the pentaquark states.
\begin{align}
\Pi_{\mu\nu\alpha\beta}(p) &= \left(\lambda^+ - \frac{1}{2}\right)^2 \frac{p^2 + M^2 - p^2}{M^2 - p^2} (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + \left(\lambda^+ - \frac{1}{2}\right)^2 \frac{p^2 - M^2 + p^2}{M^2 - p^2} (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + \cdots , \\
&= \Pi^1_{\frac{1}{2}}(p^2) \hat{p} (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + \Pi^0_{\frac{1}{2}}(p^2) (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) + \cdots ,
\end{align}
where, the subscripts $\frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$ are the spins of the pentaquark states, the subscripts or superscripts $\pm$ of $\lambda$ and $M$ denote the positive-parity or negative-parity, respectively, $\lambda$ with subscripts and superscripts are the pole residues which are used for the calculation of decays in the framework of QCD sum rules. The isospin indexes are not displayed in the above expressions.

The correlation functions are contracted at the quark-level via the Wick theorem, thus, they are in the forms in terms of full quark propagators. Followed by the operator product expansion, complicated structures of the correlation functions are derived. Same as the method applied in our previous studies [23, 34], for the correlation functions $\Pi(p)$, we pick out the structures $\hat{p}$ and 1, thus, we apply current $J(x)$ to investigate the pentaquark states with the $J^P = (\frac{1}{2})^\mp$. We select the structures $\hat{p}g_{\mu\nu}$ and $g_{\mu\nu}$ for the correlation functions $\Pi_{\mu\nu}(p)$, then, the axis-vector currents $J_\mu(x)$ are used to couple the states with the $J^P = (\frac{1}{2})^\mp$. We choose the structures $\hat{p} (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha})$ and $g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}$ from the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$, in this way, the tensor currents $J_\mu$ are applied to study the states with the $J^P = (\frac{5}{2})^\mp$.

We carefully analyze the contributions of all the related terms of vacuum condensates after the operator product expansion. The highest dimension of vacuum condensates are determined by the lead order Feynman diagrams which are $(\frac{2}{\Lambda}GG)\bar{q}q^3$ and $(\frac{2}{\Lambda}gGqGq)\bar{q}q$ with the dimension 13. Vacuum condensates proportional to the strong fine-structure constant $\mathcal{O}(\alpha_s^k)$ are selected for calculation under $k < 1$ [40]. Thus, in this work, there are solid reasons for us to choose the terms $(\bar{q}q), (\frac{2}{\Lambda}GG), (\frac{2}{\Lambda}gGqGq), (\bar{q}q)^2, (\frac{2}{\Lambda}GG)\bar{q}q, (\frac{2}{\Lambda}gGqGq)(\bar{q}q), (\bar{q}q)^3, (\frac{2}{\Lambda}gGqGq)^2, (\frac{2}{\Lambda}GG)(\bar{q}q)^2, (\bar{q}gGqGq)(\bar{q}q)^2, (\bar{q}gGqGq)^2(\bar{q}q)$ and $(\frac{2}{\Lambda}GG)(\bar{q}q)^3$, where, $q = u, d$ and $s$. Consider the masses of the light quarks $u$ and $d$ are too tiny to make decent difference, we set their masses be zero and keep the terms of vacuum condensates proportional to $m_s$, where, $m_s$ is the mass of the $s$ quark, and we throw away the terms related to $m_s^k$ for $k \geq 2$ to avoid trivial calculation.

After the operator product expansion and chosen of the vacuum condensates, we solve the integrals in the coordinate space and momentum space, and then conduct Borel transform for the correlation functions in regards to $P^2 = -p^2$,
\begin{equation}
\hat{B}_{T^2}(P^2)\Pi(p) = \int_{\Delta^2}^\infty ds \rho_{QCD}(s) \exp \left(-\frac{s}{T^2}\right),
\end{equation}
where $\rho_{QCD}(s)$ is the QCD spectral density, $\Delta^2 = 4m_s^2$ in the present study, $T^2$ is the Borel parameter and $\hat{B}_{T^2}(P^2)$ is the Borel operator which is defined as,
\begin{equation}
\hat{B}_{T^2}(P^2) = \lim_{-p^2, n \to \infty} \frac{(-p^2)^{n+1} \cdot \frac{d^n}{dp^n}}{n!}.
\end{equation}

Note that, due to the structure of the correlation functions of the pentaquark currents, the QCD spectral densities $\rho_{QCD}(s)$ contain two parts at the quark-gluon level, marked as $\rho_{QCD}^1(s)$ from $\Pi^1(p^2)$ and $\rho_{QCD}^0(s)$ from $\Pi^0(p^2)$, respectively. In the present work, the parities of the constructed currents are all negative, thus, it is natural for us to choose the negative parity of the potential pentaquark states. We take the quark-hadron duality below the continuum thresholds $s_0$ and get the QCD sum rules for the pentaquark states,
\begin{equation}
2M_-(\lambda^+)^2 \exp \left(-M_+^2\tau\right) = \int_{4m_s^2}^{s_0} ds \left[\sqrt{s}\rho_{QCD}^1(s) + \rho_{QCD}^0(s)\right] \exp \left(-\tau s\right),
\end{equation}
\begin{equation}
M_+^2 = \frac{\frac{d}{ds} \int_{4m_s^2}^{s_0} ds \left[\sqrt{s}\rho_{QCD}^1(s) + \rho_{QCD}^0(s)\right] \exp \left(-\tau s\right)}{\int_{4m_s^2}^{s_0} ds \left[\sqrt{s}\rho_{QCD}^1(s) + \rho_{QCD}^0(s)\right] \exp \left(-\tau s\right)}.
\end{equation}
where, \( \tau = \frac{1}{T} \). For simplicity, the detailed expressions of the complicated spectral densities \( \rho_{QCD}(s) \) and \( \rho_{QCD}^0(s) \) are not shown here, one can contact us via Email.

3 Numerical results and discussions

We apply the standard values of the vacuum condensates \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3, \langle \bar{s}s \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle, \langle \bar{q}g,\sigma Gq \rangle = m_0^2\langle \bar{q}q \rangle, \langle \bar{q}g,\sigma Gs \rangle = m_0^2\langle \bar{s}s \rangle, m_0^2 = (0.8 \pm 0.1 \text{ GeV}^2, \langle \alpha_s(GG) \rangle = (0.012 \pm 0.004) \text{ GeV}^4 \) at the energy scale \( \mu = 1 \text{ GeV} \), and choose the \( \overline{MS} \) mass \( m_c(m_c) = (1.275 \pm 0.025) \text{ GeV} \) and \( m_s(\mu = 2 \text{ GeV}) = (0.095 \pm 0.005) \text{ GeV} \) from the Particle Data Group \cite{1}. We consider the energy-scale dependence of these parameters,

\[
\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}}, \\
\langle \bar{s}s \rangle(\mu) = \langle \bar{s}s \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{-2}{33-2n_f}}, \\
\langle \bar{q}g,\sigma Gq \rangle(\mu) = \langle \bar{q}g,\sigma Gq \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{-2}{33-2n_f}}, \\
\langle \bar{q}g,\sigma Gs \rangle(\mu) = \langle \bar{q}g,\sigma Gs \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{-2}{33-2n_f}}, \\
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{33-2n_f}}, \\
m_s(\mu) = m_s(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{12}{33-2n_f}}, \\
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0} + \frac{b_2 (\log^2 t - \log t - 1)}{b_0^2 t^2} \right],
\]

where, \( t = \log \frac{\mu^2}{\Lambda_{QCD}^2} \), \( b_0 = \frac{33-2n_f}{12\pi}, b_1 = \frac{153-19n_f}{24\pi}, b_2 = \frac{-335}{128}\pi^2 \) and \( \Lambda_{QCD} = 213 \text{ MeV}, 296 \text{ MeV}, 339 \text{ MeV} \) for the flavors \( n_f = 5, 4, 3 \), respectively \cite{1 48}, since there are \( u, d, s \) and \( c \) quarks for the pentaquark states with strangeness in the present study, we set the flavor number \( n_f = 4 \). As for the energy scale, we take account of the light flavor \( SU(3) \) mass breaking effect and apply the modified energy scale formula \cite{23 49},

\[
\mu = \sqrt{\frac{M_{X/Y/Z/P}^2}{4M_c^2} - 4M_c^2} - kM_s, \tag{12}
\]

where, \( M_c \) represents the effective charm quark mass, we choose the updated value \( M_c = 1.85 \pm 0.01 \text{ GeV} \) \cite{23}, the \( s \) quark number \( k = 1 \) for the present study and \( M_s = 0.2 \text{ GeV} \). \cite{49}

The pole dominance and convergence of the operator product expansion are the basic criteria of the QCD sum rules. In order to testify whether the calculation satisfy these two basic criteria or not, we define the pole contributions (PC) and the contributions of the vacuum condensates of dimension \( n \), which are listed as,

\[
PC = \frac{\int_{4M_c}^{s_0} ds \left[ \sqrt{s} \rho_{QCD}(s) + \rho_{QCD}^0(s) \right] \exp \left( -\frac{s}{T} \right) \right|_{-\infty}^{s_0} \right|_{-\infty}^{s_0}, \tag{13}
\]

\[
D(n) = \frac{\int_{4M_c}^{s_0} ds \left[ \sqrt{s} \rho_{QCD,n}(s) + \rho_{QCD,n}^0(s) \right] \exp \left( -\frac{s}{T} \right) \right|_{-\infty}^{s_0} \right|_{-\infty}^{s_0}, \tag{14}
\]
where, the $\rho^1_{QCD:m_s}(s)$ and $\rho^0_{QCD:m_s}(s)$ are the spectral densities of the $n$ dimensional vacuum condensates picked out from $\rho^1_{QCD}(s)$ and $\rho^0_{QCD}(s)$, respectively. In this paper, we also make a detailed technical discussion about the contribution of the terms of vacuum condensates proportional to the $s$ quark mass $m_s$, we define $D(m_s)$ and $D(m_s,n)$ as,

$$D(m_s) = \frac{\int_{4m_s^2}^{s_0} ds \left[ \sqrt{s} \rho^1_{QCD:m_s}(s) + \rho^0_{QCD:m_s}(s) \right] \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_s^2}^{s_0} ds \left[ \sqrt{s} \rho^1_{QCD}(s) + \rho^0_{QCD}(s) \right] \exp\left(-\frac{s}{T^2}\right)},$$  \hspace{1cm} (15)

$$D(m_s,n) = \frac{\int_{4m_s^2}^{s_0} ds \left[ \sqrt{s} \rho^1_{QCD:n,m_s}(s) + \rho^0_{QCD:n,m_s}(s) \right] \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_s^2}^{s_0} ds \left[ \sqrt{s} \rho^1_{QCD}(s) + \rho^0_{QCD}(s) \right] \exp\left(-\frac{s}{T^2}\right)},$$  \hspace{1cm} (16)

where, $\rho^1_{QCD:m_s}$ and $\rho^0_{QCD:m_s}$ refer to the spectral densities proportional to $m_s$ picked out from $\rho^1_{QCD}$ and $\rho^0_{QCD}$, $\rho^1_{QCD:n,m_s}$ and $\rho^0_{QCD:n,m_s}$ are the spectral densities with $n$ dimensional vacuum condensates selected from $\rho^1_{QCD:m_s}$ and $\rho^0_{QCD:m_s}$, respectively.

As for the determination of the Borel platforms, we must select the suitable energy scales, continuum threshold parameters and Borel parameters. For the numerical results of the masses of the pentaquark states $D\Xi_u^*$, $\bar{D}\Xi_u^*$, $D^*\Xi_0^*$ and $D^*\Xi_0^*$ with high and low isospins, we find the masses with high isospin $I = 1$ are a few dozens of MeV above the low ones on the condition that we set the same input parameters including threshold parameter $s_0$, Borel parameter $T^2$, energy scale $\mu$ and so on, of course, applying the same parameters, although both the two obey the convergence of the operator product expansion at the same time, they can never uniformly satisfy the other basic requirements of the QCD sum rules, namely, the pole dominance criterion and satisfying the energy scale formula, thus, we slightly adjust parameters and determine them via trial and error, one can consult the detailed steps in Ref. [24].

Via trial and error, the Borel platforms are determined which are shown in the Fig. 4, and the numerical results extracted from the Borel platforms are displayed in the Table I in this table, one can clearly find the thresholds of the open meson and baryon pairs. The present study belongs to the systematic research of the color singlet-singlet-type pentaquark states under the eigenstates of isospins, the results of $P_{\pi}$ states from our previous study [24] are also attached in the Table I for the sake of our whole physical perspectives of the S-wave meson-baryon molecule under the eigenstates of isospins for the pentaquark states with and without strangeness. The dimensional contribution of the Borel platforms’ centers are shown in the Fig. 4 what’s more, we draw the dimensional contribution of vacuum condensates proportional to $m_s$ in the Fig. 2. As for Fig. 3 it is used for the analyses of uncertainties of the masses and pole residues of the chosen examples, $D\Xi_u^*$ pentaquark states with high and low isospins.

From Table I the pole contribution of all the considered states are around (40 – 60)% which means the pole dominance criterion is very well satisfied. In the Fig. 1, one can find the high dimensional contribution of the vacuum condensation play a tiny role, data show that $|D(12)|$ and $|D(13)|$ are less than 0.7% for these eight states, thus, the convergence of the operator product expansion also holds well. The most important contribution come from the leading order, $\langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^2$, where, $q = u, d$ and $s$. Contribution of the gluon condensate $\langle \sigma_G \bar{q}q \rangle$ is small, for all the states, $|D(4)| < 4\%$. We make a detailed calculation about the contribution of the terms which are proportional to $m_s$ picked out from the spectral densities, and we find $D(m_s)$ of these eight states are all around 5%, thus it is accurate enough for us to consider the vacuum condensation proportional to $m_s^k$ up to $k = 1$. Interestingly, for those terms of vacuum condensates proportional to $m_s$, $|D(m_s, 4)|$, $|D(m_s, 7)|$, $|D(m_s, 9)|$, $|D(m_s, 10)|$, $|D(m_s, 11)|$, $|D(m_s, 12)|$ and $|D(m_s, 13)|$ are less than 0.5%, especially for the high dimensional terms, their contributions are even smaller, thus, it is safe to simplify the operator product expansion by neglecting s quark mass for those terms which can save much work from trivial calculation. For the terms proportional to $m_s$, the worthy calculation are from the leading order, $\langle \bar{q}q \rangle$, $\langle \bar{q}g, \sigma Gq \rangle$, $\langle \bar{q}q \rangle^2$ and $\langle \bar{q}q \rangle (\bar{q}g, \sigma Gq)$, where, $q = u, d$ and $s$. 


The numerical results of the masses and pole residues are shown in the Table 1, and their uncertainties are calculated via the standard error analysis formula,

\[ \delta_i = \sqrt{\left( \frac{\partial f}{\partial x_i} \right)^2 (\bar{x}_i - \bar{x}_i)^2} \approx \sqrt{\left| f(\bar{x}_i + \Delta x_i) - f(\bar{x}_i) \right|^2}, \]

where, \( f \) mean the mass or pole residue, \( x_i \) represent the central value of the input parameters \( s_0, \langle \bar{q}q \rangle, m_c(m_c), m_0, M_c, m_s, \langle \bar{q}qGG \rangle \) and \( \langle \bar{s}s \rangle, \Delta x_i \) are the corresponding uncertainties of \( x_i \), respectively. As is known, the uncertainties are of great importance for the judgement of the states, it should be meaningful to figure out the contribution of the uncertainty due to each individual input parameter. We define the relative uncertainty due to each individual parameter \( x_i \) as,

\[ \delta'(x_i^\pm) = \frac{|f(x \pm \Delta x_i) - f(x_i)|}{\delta_f}, \]

where, the \( \pm \) and \( - \) refer to the upper and lower bounds of the uncertainties, \( \delta_f^\pm \) mean the corresponding upper and lower bounds of uncertainties of the mass and pole residue, respectively. For convenience of discussion, the relative uncertainties due to the individual parameter are normalized as,

\[ \delta(x_i^\pm) = \frac{\delta'(x_i^\pm)}{\sum \delta'(x_i^\pm)}. \]

We show the numerical results of \( \delta(x_i^\pm) \) of the \( \bar{D}\Xi_i^0 \) pentaquark states with isospin \( I = 0 \) and \( I = 1 \) in the Fig. 3. One can find that the uncertainties are mainly from the threshold parameter \( s_0 \), \( \langle \bar{q}q \rangle, m_c(m_c), m_0 \) and \( M_c \) account for around \((10-20)\%\). As is expected, the uncertainties of the upper and lower bounds do not have much difference. The uncertainties of \( \langle \bar{q}qGG \rangle \) and \( m_s \) play tiny role for both the masses and pole residues, and it is reasonable to understand the phenomena based on the analyses of dimensional contribution. Interestingly, similar conclusion can be derived for the other six states in this aspect.

The numerical results of the masses and their uncertainties shown in the Table 1 are extracted from the centers of the corresponding Borel platforms, the data of the masses reveal that the state masses with isospin \( I = 0 \) are slightly below the related thresholds of the meson and baryon constitutes, for the ones with isospin \( I = 1 \), they are a few dozens of MeV above the corresponding thresholds of the meson and baryon pairs. Consider upper and lower bounds of the uncertainties of the masses are around \( 70 \) MeV, compare the state masses with thresholds of the meson and baryon pairs shown in Table 1, we can only conclude that those states with \( I = 0 \) are possible molecular states and the states with \( I = 1 \) are most likely to be the resonance states. Especially, for the pentaquark exotic states \( P_{cs}(4459) \), since the masses of \( \bar{D}^\pm \Xi_i^0 \) are over \( 0.1 \) GeV above and around \( 0.2 \) GeV for \( \bar{D}^\pm \Xi_i^0 \), it is unlikely to be the molecular state of \( \bar{D}^\pm \Xi_i^0 \) or \( \bar{D}^\pm \Xi_i^0 \). The mass of \( \bar{D}^\pm \Xi_i^0 \) with isospin \( I = 1 \) is \( 4.45^{+0.07}_{-0.08} \) which is near \( 4459 \) MeV, but this state is slightly above the meson and baryon threshold, it is reasonable for us to assign it as the resonance state, more significantly, from the observed decay mode \[ 3 \], the isospin of the exotic state \( P_{cs}(4459) \) is zero. Numerical result of the mass of \( \bar{D}^\pm \Xi_i^0 \) with \( I = 1 \) is \( 4.53^{+0.07}_{-0.07}\) GeV, it is about \( 70 \) MeV above \( 4459 \) MeV, even if we consider its lower bound of uncertainty, its mass is still slightly above \( P_{cs}(4459) \), what’s more its isospin \( I = 1 \). The mass of \( \bar{D}^\pm \Xi_i^0 \) with low isospin is \( 4.46 \) GeV which is in good agreement with the experimental result of \( P_{cs}(4459) \), thus, it is nice for us to assign \( P_{cs}(4459) \) as the molecular state of \( \bar{D}^\pm \Xi_i^0 \) with isospin \( I = 0 \), then we determine the \( J^P \) of \( P_{cs}(4459) \) as \( (\frac{2}{-})^- \), and its bounding energy is about \( 50 \) MeV.

The pole residues of the considered states have the order of magnitude of \( 10^{-3} \) GeV, as the detailed numbers are listed in the Table 3, one can find the relative uncertainty of each pole residue is around \( 12\% \). The pole residues can be used to calculate the decays of these pentaquark states.
via the QCD sum rules in the consideration of three-point correlation functions, which may be the challenge research of our future work. In the present paper, we do not show the $\lambda - T^2$ curves, one can reach them via Email.

4 Conclusions

In the present work, we construct the color-singlet-color-singlet type five-quark currents with strangeness to study the pentaquark states under the isospin eigenstates $I = 0$ and $I = 1$ via the QCD sum rules. We make the detailed technical discussion about the contribution of the vacuum condensates proportional to $m_s$. Data show that it is accurate enough to neglect the terms related to $m_s^k$ for $k \geq 2$, what’s more, only several low dimensional terms are worthy calculation in this aspect, this conclusion is helpful to avoid trivial job for future study. We analyze the uncertainties of both the masses and pole residues, and find the main contribution of the uncertainties are from the threshold parameter $s_0$ which account for around 50% for the considered states, as for the parameters $(\bar{q}q)$, $m_c(m_c)$, $m_0$ and $M_c$, they contribute around $(10 - 20)$% for the uncertainties, and the others play tiny role. This may give us the reference to make the calculation even more accurate for the future work. We find the masses of the $\bar{D} \Xi'$, $\bar{D} \Xi'_c$, $D^* \Xi'_c$ and $D^* \Xi^*_c$ pentaquarks with isospins $I = 0$ and $I = 1$, results show that the masses with low isospin are a few dozens of MeV below the high isospin ones, what’s more, they are slightly below the corresponding thresholds of the meson and baryon pairs, thus we assign these low isospin states as the possible molecular states. Masses with high isospin are all above the corresponding thresholds of the meson and baryon constitutes, and they are assigned as the possible resonance states. The mass of $\bar{D} \Xi^*_c$ with isospin quantum number $I = 0$ coincides well with that of the exotic state $P_{cs}(4459)$, we analyze the results and consider it is natural and reasonable to assign $P_{cs}(4459)$ as the $\bar{D} \Xi^*_c$ molecular state with the $IJ^P = 0(\frac{3}{2})^-$, thus we figure out the bounding energy of this molecular state, around 50 MeV. The numerical results and conclusion of this paper may present a reference for the experimental search of the other pentaquark states with strangeness except $P_{cs}(4459)$, they may be confronted to the future experimental observation via the search of $J/\psi \Lambda$ invariant mass spectrum, if the future experimental search for the other pentaquark states with strangeness predicted in the present work except $P_{cs}(4459)$ hold, this will in return solidify the assignment of $P_{cs}(4459)$ and shed light on the low-energy QCD dynamics.

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| \(D\Sigma\) & \(IJ^P\) & \(T^2(GeV^2)\) & \(\sqrt{s_0}(GeV)\) & \(\mu(\text{GeV})\) & PC & \(M(\text{GeV})\) & \(\lambda(10^{-3}\text{GeV}^6)\) & Assignments | Thresholds (MeV) |
|---|---|---|---|---|---|---|---|---|---|
| \(D\Sigma_c\) & 0(\(1\)) & 3.4 & 5.12 ± 0.10 & 2.2 & (41 & 58)% & 4.43 & 0.06 & 3.02 & 0.39 & molecular state & 4446 |
| \(D\Sigma_u\) & 1(\(1\)) & 3.2 & 5.14 ± 0.10 & 2.3 & (43 & 61)% & 4.45 & 0.06 & 2.50 & 0.33 & resonance state & 4446 |
| \(D\Sigma_s\) & 0(\(1\)) & 3.4 & 5.15 ± 0.10 & 2.3 & (43 & 60)% & 4.46 & 0.06 & 1.71 & 0.21 & \(P_{ss}(4459)\) & 4513 |
| \(D\Sigma_u\) & 1(\(1\)) & 3.3 & 5.22 ± 0.10 & 2.4 & (44 & 62)% & 4.53 & 0.07 & 1.56 & 0.19 & resonance state & 4513 |
| \(D\Sigma_s\) & 0(\(1\)) & 3.5 & 5.26 ± 0.10 & 2.5 & (42 & 59)% & 4.57 & 0.07 & 3.41 & 0.43 & molecular state & 4588 |
| \(D\Sigma_u\) & 1(\(1\)) & 3.4 & 5.31 ± 0.10 & 2.6 & (43 & 60)% & 4.62 & 0.08 & 3.05 & 0.37 & resonance state & 4588 |
| \(D\Sigma_s\) & 0(\(1\)) & 3.2 & 5.31 ± 0.10 & 2.6 & (42 & 58)% & 4.64 & 0.07 & 4.36 & 0.51 & molecular state & 4655 |
| \(D\Sigma_u\) & 1(\(1\)) & 3.4 & 5.35 ± 0.10 & 2.6 & (44 & 61)% & 4.67 & 0.08 & 3.25 & 0.39 & resonance state & 4655 |

Table 1: Numerical results extracted from the Borel windows and assignments of the pentaquark states with strangeness for high and low isospins, numerical results and assignments of pentaquark states without strangeness for high and low isospins are also attached [24], thresholds of the meson-baryon pairs are listed.
Figure 1: The absolute value of the dimensional contribution extracted from the centers of Borel platforms, where A, B, C, D, E, F, G and H denote the pentaquark states $\bar{D}\Xi_c^*$ with $I = 0$, $\bar{D}\Xi_c^*$ with $I = 1$, $\bar{D}\Xi_c^*$ with $I = 0$, $\bar{D}\Xi_c^*$ with $I = 1$, $\bar{D}\Xi_c^*$ with $I = 0$ and $\bar{D}\Xi_c^*$ with $I = 1$, respectively.

Figure 2: The absolute value of dimensional contribution due to the vacuum condensates proportional to the s quark mass $m_s$, where A, B, C, D, E, F, G and H denote the pentaquark states $\bar{D}\Xi_c^*$ with $I = 0$, $\bar{D}\Xi_c^*$ with $I = 1$, $\bar{D}\Xi_c^*$ with $I = 0$, $\bar{D}\Xi_c^*$ with $I = 1$, $\bar{D}\Xi_c^*$ with $I = 0$, $\bar{D}\Xi_c^*$ with $I = 1$, $\bar{D}\Xi_c^*$ with $I = 0$ and $\bar{D}\Xi_c^*$ with $I = 1$, respectively.

Figure 3: The normalized relative uncertainties of $\bar{D}\Xi_c^*$ pentaquark states with $I = 0$ (left) and $I = 1$ (right) due to the individual parameter, where $a$, $b$, $c$ and $d$ represent the upper bound of mass, lower bound of mass, upper bound of pole residue and lower bound of pole residue, $x_i$ for $i = 1, 2, \cdots, 8$ are the input parameters $s_0$, $\langle \bar{q}q \rangle$, $m_c(m_c)$, $m_0$, $M_c$, $m_s$, $\langle \alpha_s G \rangle$ and $\langle s\bar{s} \rangle$, respectively.
Figure 4: The $M_i - T^2$ curves, where $M_i (i = 1, 2, \cdots , 8)$ denote the masses of $\bar{D}\Xi_c^\prime$ with $I = 0$, $\bar{D}\Xi_c^\ast$ with $I = 1$, $\bar{D}\Xi_c^\prime$ with $I = 0$, $\bar{D}\Xi_c^\ast$ with $I = 1$, $\bar{D^\ast}\Xi_c^\prime$ with $I = 0$ and $\bar{D^\ast}\Xi_c^\ast$ with $I = 1$, respectively.