Eternal Inflation: Prohibited by Quantum Gravity?

Yi Wang

Institute of Theoretical Physics, CAS, Beijing 100080, P.R.China
Interdisciplinary Center of Theoretical Studies, USTC, Hefei, Anhui 230026, P.R.China

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We investigate how eternal inflation is affected by quantum gravity effects. We consider general features of quantum gravity, such as renormalizability, complementarity, minimal length, definition of observables, and weak gravity conjecture. We also consider phenomenological models such as ghost inflation, non-commutative inflation, brane inflation, k-inflation and resonant tunneling. We find that all these features and models do not support eternal inflation. These evidences show hints that eternal inflation is prohibited by quantum gravity.

I. INTRODUCTION

It has been widely accepted that the observable stage of inflation provides the initial condition for a flat universe, as well as the cosmic microwave background (CMB) and the large scale structure (LSS). Based on this, it is natural to ask what provides the initial condition for the observable inflation, or in other words, where our universe comes from. This selection of original universe program [1, 2] is hopeful to answer some profound questions in physics, such as to determine the fundamental constants and matter contents of nature.

One possible mechanism to provide the initial condition for the observable inflation is eternal inflation. Eternal inflation indicates that inflation in the universe is eternal to the future, and we live in a locally thermalized bubble embedded in the inflating background.

There are typically two ways to realize eternal inflation, namely, the slow roll eternal inflation [3, 4] and the false vacuum eternal inflation [5]. The slow roll eternal inflation originates from the comparison between the quantum fluctuation and the classical motion of the inflaton per Hubble time. If the quantum fluctuation is comparable with or larger than the classical motion, inflation runs into a self-reproducing process and become eternal. The false vacuum eternal inflation originates from a smaller decay rate of the false vacuum compared with the inflationary Hubble constant. In this case, the physical spatial volume during inflation increases and inflation becomes eternal. If the vacuum structure of the real world is very complicated, such as the string landscape, then eternal inflation can happen as a combination of these two types.

Eternal inflation is inevitable semi-classically if the inflaton potential satisfies the conditions for eternal inflation. However, as we will discuss in the following sections, eternal inflation becomes problematic when quantum gravity effects are taken into consideration. Whether eternal inflation happens or not has deep implications in cosmology, deciding whether the best information for the initial condition of our universe is probabilistic or deterministic.

If eternal inflation happens, it is widely believed that the deterministic initial condition for the creation of our universe is diluted. In the best case, if there are some exponentially favored vacua, we can determine the initial condition by constructing measures for eternal inflation. Otherwise, it may be hopeless to determine the initial condition for our universe.

However, if eternal inflation is prohibited in the quantum gravity level, then it becomes possible that the initial condition for our universe can be calculated from the first principle. In this case, the information for creation of the universe is available, and can be verified by experiments.

In this paper, we consider how eternal inflation is affected by quantum gravity effects, from both general arguments and phenomenological models. In Section 2, we argue that the prohibition of eternal inflation is rather general in quantum gravity. We conclude in Section 3.

II. QUANTUM GRAVITY PHENOMENOLOGY

As we shall discuss, quantum gravity is cried for during eternal inflation. But unfortunately, we do not have a full quantum gravity theory for cosmology so far. In this section, we shall discuss some general features and phenomenological models of quantum gravity.

A. General Arguments

One of the key problems in quantum gravity is the non-renormalizable nature of gravity. In order to have a renormalizable or finite theory for gravity, one need to suppress the quantum fluctuations in the high energy regime. On the other hand, the slow roll eternal inflation needs large quantum fluctuations. So it is likely for quantum theory effects to kill slow roll eternal inflation. One explicit example of this general argument is shown in Subsection D of this section.

It is well known that it is very difficult to construct a measure for eternal inflation. Two classes of measures are considered in the literature, namely, the global [6, 7] and

*Electronic address: wangyi@itp.ac.cn
local \[8, 9, 10\] measures. However, regardless of technical difficulties such as divergences or gauge dependence, both the global and local measures suffer problems of the nature of quantum gravity.

Global measures are weighted by the spatial volume or the number of the bubbles of a given kind. These attempts to describe spatial regions divided by future event horizon, and violates directly the cosmic complementary principle. So local measures suffer quantum gravity problems globally [7].

Local measures are weighted by how long or how many times a given vacuum is accessed by an eternal co-moving observer. However, if inflation has happened for a sufficiently long time, and we track different co-moving observers back in time, these co-moving observers run into a single Planck volume. Thus it does not make sense to distinguish these observers. So local measures suffer quantum gravity problems locally [2, 10].

As we have seen, none of these measures are self-consistent in quantum gravity. Keeping this problem in mind, it is natural to take one step back to conjecture that this inconsistency is evidence for the absence of eternal inflation.

Another hint for the prohibition of eternal inflation is the difficulty in defining the observables in the asymptotically de Sitter space. One solution for this problem is to embed the de Sitter inflating patch into an anti-de Sitter background. However, if eternal inflation takes place, then the causal structure of the spacetime is changed and the above picture no longer applies [11].

Eternal inflation also runs into problems of Boltzmann brains. As discussed in [12], if we are typical observers, and the universe eternally inflates, then we should find ourselves to be Boltzmann brains instead of human observers. The simplest solution to this problem is that eternal inflation can not happen. For other possible solutions, see [13, 14].

B. Weak Gravity Conjecture (WGC)

It is conjectured in [15] that gravity should be the weakest force in a self-consistent quantum gravity theory. This conjecture leads to a UV cut-off for the effective field theory of the inflaton. In [16], we have shown that eternal inflation is prohibited by WGC for \( m^2 \varphi^2 + \lambda \varphi^4 \) single field assisted inflation. Take the \( n \) field \( \lambda \varphi^4 \) potential for example, by comparing the interaction strength for the inflaton self coupling with the gravitational coupling, WGC gives a cut-off for the Hubble parameter

\[
H < \Lambda = \sqrt{\lambda} M_P .
\]

In order for inflation to take place, the kinetic energy for inflaton should be smaller than the potential energy,

\[
n(\partial_\mu \varphi)^2 \sim n H^4 < V .
\]

The eternal inflation condition takes the form

\[
\sqrt{n} \delta_\eta \varphi > n \delta_c \varphi ,
\]

where \( \delta_\eta \varphi \sim H/(2\pi) \) is the quantum fluctuation of each inflaton, and \( \delta_c \varphi \) is the classical motion of each inflaton during one Hubble time.

One can show that [11, 20] and [13] can not be satisfied simultaneously when \( \lambda < 1 \) and \( n \geq 1 \). So this kind of slow roll eternal inflation is prohibited by WGC. Similarly, we can prove the same result for the \( m^2 \varphi^2 \) potential.

C. Ghost Inflation

Ghost inflation [18] is proposed as an IR modification of gravity. In this subsection, we shall show that slow roll eternal inflation is absent in ghost inflation. In ghost inflation, the inflaton background moves with a constant velocity \( \langle \varphi \rangle = M^2 \). The perturbation \( \pi \) around the background is defined as \( \varphi = M^2 t + \pi \), with Lagrangian

\[
S = \int d^4 x \left[ \frac{1}{2} \pi^2 - \frac{\alpha^2}{2M^2} (\nabla \pi)^2 - \frac{\beta}{2M^2} (\nabla^2 \pi)^2 + \cdots \right] ,
\]

where \( \alpha \) and \( \beta \) are order one constants. The condition for the effective field theory to hold is

\[
H \ll m .
\]

In [18], it is shown by scaling arguments that the quantum and classical fluctuations take the form

\[
\delta_\eta \varphi \sim (HM^3)^{1/4} , \quad \delta_c \varphi \sim \varphi / H \sim M^2 / H .
\]

So the eternal inflation condition \( \delta_\eta \varphi > \delta_c \varphi \) takes the form \( H > M \). This directly violates [19], thus slow roll eternal inflation is prohibited.

[18] also discussed two corrections for the above amplitude for quantum fluctuations, one from the parameter \( \alpha \) and the other from tilting the potential. However, on condition that high order terms are important, \( \alpha \) should not be much smaller than one, so the \( \alpha \) correction does not change the result.

For tilting the potential, it can also be shown that eternal inflation is absent. In order to stay within the effective field theory, the classical motion of inflaton remains almost the same. When the potential for the perturbation satisfies \( -V' > 3H^2 M \), the quantum fluctuation is significantly modified [18]. However, it can be shown that the new expression for quantum fluctuation also prohibits the slow roll eternal inflation.

So we conclude that after the corrections are taken into consideration, ghost inflation still can not be eternal.

D. Spacetime Non-Commutativity

In this section, we consider the spacetime non-commutativity inspired by string theory [19]. The spacetime non-commutativity takes the form

\[
[t_{\text{phys}}, x_{\text{phys}}] = i M_N^{-2} ,
\]

where \( M_N \) is the Planck mass in the non-commutative theory.
where $M_N$ is the non-commutative scale.

It is shown in [21] that when the non-commutative effect is strong, the inflaton quantum fluctuation per Hubble time takes the form

$$\delta q \varphi \simeq \frac{1}{2\pi} \frac{M_N^2}{H}.$$ (8)

Comparing this result with the classical motion $\delta_c \varphi$, we conclude that for $\lambda M_p^{2-p} \varphi^p$ potential ($p \geq 2$), eternal inflation will never happen when

$$M_N < (p^p \lambda)^{1/2p} M_p.$$ (9)

For example, for the $m^2 \varphi^2$ potential, eternal inflation will never happen when

$$M_N < \sqrt{mM_p}.$$ (10)

Plugging in the experimental value $m \sim 10^{-6}$, we find that when $M_N < 10^{-3} M_p$, eternal inflation can not happen.

So we conclude that the slow roll eternal inflation is prohibited by a low space spacetime non-commutativity. It is also interesting to investigate the possibility for the false vacuum eternal inflation. We find [22] that false vacuum eternal inflation with Hawking-Moss tunneling can happen with non-commutativity. This is not surprising, because the probability for Hawking-Moss tunneling is suppressed when quantum fluctuation is suppressed. It should be interesting to investigate the CDL tunneling with non-commutativity. We hope we can address this issue in future work.

E. Power Counting for Chaotic Inflation

Another piece of evidence against eternal inflation originating from power counting and validity for effective field theory is observed in [17]. We can write the power expansion for the inflaton potential as

$$V = V_0 \pm \frac{1}{2} m^2 \varphi^2 + M \varphi^3 + \frac{\lambda}{4} \varphi^4 + \sum_{d=5}^{\infty} \lambda_d \Lambda_d \varphi^d.$$ (11)

The terms in the summation ($d \geq 5$) can usually be neglected because they are suppressed by the cut-off of the effective field theory $\Lambda_{UV}$. However, in the chaotic inflation, $\varphi > M_p$, so the power expansion is not under control.

One can always shift $\varphi$ to make the above expansion work. However, in the chaotic inflation, the variation of the inflaton $\Delta \varphi \equiv \varphi_f - \varphi_i > M_p$. So we still can not get the effective field theory work along the whole inflation trajectory. As the most studied slow roll eternal inflation models are within the chaotic inflation framework, this problem for chaotic inflation also serves as a problem for slow roll eternal inflation. Similar conjecture that the inflaton trajectory should not be longer than $M_p$ is also proposed in [23, 24]. Finally, one should note that there are also arguments against this power counting reasoning, see, e.g. [25].

F. Inflation Models in String Theory

As is well known, it is very difficult to construct large field inflation models in string theory. This difficulty makes slow roll eternal inflation problematic in string theory. For example, in [26], it is shown that brane inflation can not be slow roll eternal inflation. (However, in [27], it is reported that by adding higher order polynomial corrections to the potential, eternal inflation can take place in brane inflation.) And it is noticed in [28] that when the sound speed of the inflaton perturbation has deviation from unity by $1 - c_s^2 > \epsilon - 2c_s/(Hc_s)$, then on condition that inflation happens in the perturbative regime, one obtains

$$c_s^4 > \frac{H^2}{M_p^2 c_s} \simeq P_\zeta,$$ (12)

where $P_\zeta$ is the power spectrum of the curvature perturbation on uniform density slice. So when $c_s < 1$ (as indeed the case in brane inflation and most k-inflation models), we have $P_\zeta < 1$. This leads to $\delta q \varphi < \delta_c \varphi$, so eternal inflation is prohibited by the small sound speed effect.

Another class of string inflation models is modular inflation. The existence of eternal inflation in modular inflation is in debate. On one hand, eternal inflation can happen in some effective field theory models derived from the reshaping of the compactified dimensions [29]. On the other hand, some people insist that no realization of slow roll eternal inflation has been found in string theory so far [11].

G. Rapid Tunneling in the Landscape

Most of the above discussions are related to the slow roll eternal inflation. In this subsection, we review briefly the rapid tunneling in the string landscape [30, 31, 32], which may prohibit the false vacuum eternal inflation.

One mechanism for the rapid tunneling is resonant tunneling. Resonant tunneling is a well known effect in quantum mechanics. Recently, resonant tunneling in quantum field theory is investigated [33]. It is shown in [30, 31] that the tunneling probability in the landscape with resonant tunneling takes the form

$$\Gamma \sim n^d \Gamma_0,$$ (13)

where $d$ is the dimension of the landscape, $\Gamma_0$ is the original tunneling rate without resonant tunneling, and $n$ is the number of effective steps of tunneling which is not much suppressed compared with $\Gamma_0$. It is noticed in [34, 35] that if the CMB power spectrum is produced by a chain of resonant tunneling, then the tunneling rate should satisfy $\Gamma \gg H$ during observable inflation. As observed in [31], the tunneling rate increases with inflationary energy scale, so it is natural that $\Gamma > H$ also holds for higher energy scale. Then the false vacuum eternal
inflation can not happen because the false vacuum decays too fast.

III. CONCLUSION

To conclude, we discussed some general features of quantum gravity, such as renormalizability, complementarity, minimal length, definition of observables, and weak gravity conjecture. We also considered phenomenological models such as ghost inflation, non-commutative inflation, brane inflation, k-inflation and resonant tunneling. All these effects and models show evidence that eternal inflation is prohibited by quantum gravity.

One should note that a full quantum gravitational treatment for eternal inflation is not so far available. The arguments and models considered above are inspired by some features of quantum gravity. We hope we can gather some information for eternal inflation from these arguments and models. It may turn out that some of these models are ruled out by experiments or theoretical developments in the future. On the other hand, some of these models require that the scale of quantum gravity is significantly lower than $10^{19}$ GeV, which needs some luck or fine-tuning. However, since all the above models are against eternal inflation, we infer that eternal inflation may be really problematic when quantum gravity effects are taken into consideration.

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