Modelling the hydrodynamics of a two-phase flow in the non-Newtonian fluid

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Abstract. The paper presents the results of modelling the flow of droplets in the flow of a non-Newtonian fluid. The task is solved in a nonstationary formulation. We determined the time of stabilization of the speed profile of the droplets and the length of the initial hydrodynamic sector. We compared the specified characteristics for the main flow of the non-Newtonian fluid and for the set of droplets in the flow. We show the coincidence of the lengths of the initial hydrodynamic sections determined by the stabilization of speeds for the droplets and the non-Newtonian medium.

1. Introduction
Among many works that are devoted to studying the behaviour of non-Newtonian fluids under different conditions of operation of heat engineering installations, [1-3] can be noted as fundamental ones. This is due to the essential dependence of the rheological properties of the non-Newtonian fluid on the speed of its shift. The change in the temperature of the medium further complicates the prediction of its behaviour, and, consequently, the operation modes of heat engineering devices and systems using non-Newtonian media.

The lengths of the initial hydrodynamic segment for such media and the usual ones subjected to Newton's law are different. The heat exchange process in the medium in the initial segment in scientific works and engineering techniques is considered to a lesser extent. The reason for this lies in the fact that the length of such a segment is sufficiently small, and the main process, to a greater extent affecting the work of the equipment, happens in the hydrodynamically established region, i.e. after it. And it is so for Newtonian media. With non-Newtonian media, the length of the initial segment may be much larger, and in this case, it is impossible not to take into account mechanisms different from the laws in the region of steady flow, i.e. after the initial segment. In addition, the specified circumstance can affect manufactured products or even the work of an industrial enterprise.

2. Statement of the problem
To analyze the hydrodynamic pattern, let us consider a pipe cross-section with a length of 30 m in which the non-Newtonian medium flows with the initial speed specified at the input $v_i = 0.2$ m/s (Figure 1). In many ways, non-Newtonian media flow is determined by their rheological characteristics. For an aqueous solution of surfactants, such characteristics were defined earlier: fluid
consistency coefficient $m = 9.3319$ Pa·s, flow behaviour index $n = 0.073$, lower shear rate limit $\dot{\gamma} = 0.01$ 1/s, reference shear rate $\dot{\gamma}_{ref} = 1$ 1/s [4]. To determine the speed profile of the non-Newtonian medium, we will use the analysis of the speed profile formed by the particles of the second phase, which represents the Newtonian medium in the form of watercolour paint with the following characteristics: the particle diameter $d_p = 0.3$ mm, the density $\rho_p = 1031$ kg/m$^3$, the dynamic viscosity of the particles $\mu_p = 4.5$ mPa·s, the particle surface tension $\sigma_p = 30$ mN/m.

At the initial time, 100 drops are located at the boundary of the input of the non-Newtonian medium into the channel (are shows conventionally in Figure 1). In a planned further experimental study, it will be possible to compare the resulting profile in this model with a steady speed profile of the drops of paint for various modes, pipe sections and time segments.

The problem was considered in a nonstationary statement for a laminar mode of fluid flow, taking into account the gravitational force. Paint particles were taken as drip liquid, and their volume fraction was less than 1%. The effect of particles on the main flow was taken as insignificant. At the initial time, the speed of the second phase (drops) was taken $v_p = 0$ m/s.

![Figure 1. Problem statement.](image)

Let us make a mathematical model of the movement of an aqueous solution of surfactants, i.e. non-Newtonian fluid, in a pipe. To do this, we use the system of equations consisting of the movement equation and the equation of continuity. In general, for the stationary problem, they look as follows

$$\rho \: \mathbf{u} \cdot \nabla \mathbf{u} = \nabla \cdot \left[ - \rho \mathbf{I} + \mathbf{K} \right] + \rho \mathbf{g},$$

$$\rho \nabla \cdot \mathbf{u} = 0.$$ (1)

Here, $\rho$ – the density; $\mathbf{u}$ – the speed vector; $\mathbf{I}$ – the unit vector.

$$\mathbf{K} = m \left( \frac{\dot{\gamma}}{\dot{\gamma}_{ref}} \right)^{n-1} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T),$$

$$\dot{\gamma} = \max(\sqrt{2S} : \mathbf{S}, \dot{\gamma}_{\min}),$$

$$S = 0.5 \left[ \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right].$$ (3)

Here, $m$ – the fluid consistency coefficient; $\dot{\gamma}_{\text{ref}}$ – the reference shear rate; $\dot{\gamma}_{\text{min}}$ – the lower shear rate limit. In equation (4) $\langle \rangle$ - the contraction operator defined as $a : b = \sum_n \sum_m a_{nm} b_{nm}$.

We describe the movement of the second phase - paint drops with the following system of differential equations of the first order for the nonstationary problem

$$m_p \frac{dv}{dt} = F_p,$$ (6)

$$\frac{dd_p}{dt} = \frac{2R}{\pi \rho_p d_p^2}.$$ (7)
Here, \( v = \frac{dq}{dt} \); \( \mathbf{q} \) – the vector of the coordinates of the particles; \( t \) – the time; \( d_p \) – the particle diameter; \( R \) – the accretion rate; \( \rho_p \) – the density of the particles.

Let us consider the action of the two forces on the particles: resistance and gravity. Let us describe the law of resistance to the movement of particles with the following equation

\[
F_D = \frac{1}{\tau_p} m_p (u - v),
\]

where \( u \) and \( v \) - the speed of the first and second phase, respectively.

The characteristic time of the particle response, determining the resistance as a function of the particle’s properties and the particle’s number of Reynolds, is determined by the following dependence

\[
\tau_p = \frac{4 \rho_p d_p^2}{3 \mu C_D R e_r},
\]

(9)

The coefficient of resistance is determined by the Hadamard-Rybczynski formula

\[
C_D = \frac{8}{R e_r} \frac{2 + 3 \kappa}{1 + \kappa},
\]

(10)

Here, the particle’s number of Reynolds \( R e_r = \frac{\rho |u - v| d_p}{\mu} \), \( \mu \) – the dynamic viscosity of the main non-Newtonian medium. \( \kappa = \frac{\mu_p}{\mu} \), \( \mu_p \) – the dynamic viscosity of the second phase, i.e. of the particle.

We determine the gravity force from the following equation

\[
F_g = m_p g \frac{\rho_p - \rho}{\rho_p},
\]

(11)

where \( \rho_p \) – the density of the particles.

3. Solution

3.1. Modelling the movement of particles in the flow of an aqueous solution of surfactants

Let us solve the task in two stages. At the first stage, using the system of equations for the movement of the first phase (non-Newtonian medium) (1)-(2), we define its fields of pressures and speeds for the steady flow. At the second stage, we solve the problem of movement of the second phase (Newtonian medium) in the nonstationary formulation using (6)-(11). To solve this, we use one of the numerical packages for engineering analysis of hydrodynamic systems. To determine the stabilization of the speed profile, we draw it for sections from 10 mm to 3000 mm. Figure 2 shows the dynamics of changes in the speed profile of the main flow. The speed changing along the three central lines is represented in the logarithmic coordinates for better visual assessment of the stabilization of speeds. Moreover, the axial line corresponds to the line in the center of the flow, and two more lines are located at distances corresponding to distances of 21 and 79 drops from the central one. From both drawings, we can see that the speed is stabilized at a distance of about 1000 mm from the input. This value corresponds to the length of the initial hydrodynamic segment.
In order to visually identify this profile in the planned experimental study, we compare the dependencies obtained with similar dependencies for the drops. To do this, we estimate the speed of several drops in the vicinity of the central flow lines and determine the length of the initial hydrodynamic segment. This can be determined by the drop’s speed stabilization segment in Figure 3. The first of them shows that the speed stabilization time is about 5 s. Using this time value, we can see that the distance that the drops passed during this time is about 1000 mm. This value practically coincides with the value for the main flow.

For a number of drops from the central set, i.e. from drop 21 to drop 79, we define the speed of each of them when the cross-section is located at a distance equal to the length of the initial hydrodynamic segment for the non-Newtonian medium (1000 mm). To do this, we use the results of the solution of the non-stationary problem of the flow of drops in the non-Newtonian medium. We find the speed for each of the drops using the speed determination itself

$$v_{pn} = \frac{1}{t_n}$$  \hspace{1cm} (12)

where $t_n$ – the time, which it takes the $n$-th drop to reach 1 m distance.

Calculating the speed value for each of the drops by formula (12), we can construct a speed profile for the section at a distance of 1000 mm from the input of the pipe.
3.2. Findings
The speed profiles for each of the phases of the flow are parabolic. This does not contradict the classical ideas of the hydrodynamic pattern of the flow of the liquid medium [5]. The comparative analysis shows that their difference is insignificant. The nature of the stabilization of the speed of the first and second phases for the central line and extreme central flow lines have differences. This is explained by the fact that the speed on the peripheral lines of the central flow slows down due to an increase in the thickness of the boundary layer. However, the speeds for all the lines of the central flow are stabilized at a distance of about 1000 mm. The speed value along the specified lines increases and takes the value of 0.214 m/s, i.e. more than the value that is set at the input of the pipe. After passing 1000 mm, the speed of the central flow in the pipe does not change. Thus, it can be assumed that the length of the initial hydrodynamic area for this non-Newtonian fluid is 1000 mm.

4. Conclusion
The results of the study of the two-phase flow of non-Newtonian fluid show that to identify the hydrodynamic pattern of the flow in a round tube, we can use the second phase in the form of drops of paint. The calculated length of the initial hydrodynamic segment, estimated by the parameters of the main flow (non-Newtonian fluid) and droplets (Newtonian fluid) is the same. And such characteristics of the non-stationary flow process as the time and the distance passed by the drops in the future physical experiment will allow us to evaluate the adequacy of the presented mathematical model.

References
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