Gravity, higher dimensions, nanotechnology and particle physics

Masato Ito
Aichi University of Education, Department of Physics, Kariya, 448-8542, Japan
E-mail: mito@euecc.aichi-edu.ac.jp

Abstract. The paper indicates that there is profound connection between the surface technology and the particle physics. Historically the Casimir effect tied up three different researches, the particle physics and nano/micro technology and number theory. Such as the Casimir effect, it is expected that the surface technologies are necessary for the experimental verification of the higher dimensional theories proposed by the particle physics. Recently, the particle physics is suffering from the experimental verifications. From the point of view of the elementary particle physics, let us explain the importance of the surface technologies.

1. The great achievement of Casimir effect
In 1948, Casimir predicted a macroscopic force due to quantum fluctuations [1]. He calculated the vacuum energy of electro-magnetic field between the parallel and perfectly conducting plates in vacuum. The prescription of the calculation is the renormalization scheme proposed by Feynmann, Schwinger and Tomonaga. The Casimir’s approach is shown below.

The vacuum energy of electro-magnetic field in free space is divergent quantity by the sum over all possible modes. When locating the parallel and perfectly two conducting plates in free space, the electro-magnetic field takes the Dirichlet boundary condition on the plates. In the case with plates, the vacuum energy is divergent quantity because of the same reason above. However, taking the difference between the vacuum energy energy with and without the plates, Casimir had confirmed that the energy becomes finite.

Let us suppose that the two parallel plates are located at $z = 0$ and $z = a$. The vacuum energy per unit volume is given by

$$E = \frac{1}{L^2 a} \sum \frac{1}{2} \hbar \omega \bigg|_{\text{with plates}} - \frac{1}{L^3} \sum \frac{1}{2} \hbar \omega \bigg|_{\text{without plates}}$$

$$= \frac{1}{2} \hbar c \left( \frac{1}{2\pi} \right)^2 \frac{1}{a} \int_{-\infty}^{\infty} dk_x dk_y \left( \sqrt{k_x^2 + k_y^2} + 2 \sum_{n=1}^{\infty} \sqrt{k_x^2 + k_y^2 + \left( n \frac{\pi}{a} \right)^2} \right)$$

$$- \frac{1}{2} \hbar c \left( \frac{1}{2\pi} \right)^3 \int_{-\infty}^{\infty} dk_x dk_y dk_z \int_{-\infty}^{\infty} \sqrt{k_x^2 + k_y^2 + k_z^2} + 2 \sqrt{k_x^2 + k_y^2 + k_z^2},$$

where $k_x, k_y, k_z$ are the components of the wave number of each direction, respectively. Notice that Dirichlet boundary conditions at $z = 0, a$ are imposed in the case with plates, and the plane wave in free space is subject to the periodic boundary condition of length $L$. The polar
coordinate of \((k_x, k_y)\) and \(\kappa = \sqrt{k_x^2 + k_y^2}\) can be taken. In order to make clear the idea of the renormalization, we introduce the upper limit \(\Lambda\) of integration with respect to \(\kappa\). Here \(\Lambda\) is called cutoff momentum in the terminology of particle physics, \(\Lambda\) takes the limit of infinity due to the uncertainty principle of quantum mechanics. After the troublesome calculation we can get

\[
E = \frac{hc}{4\pi a} \left( \int_0^\Lambda \kappa^2 d\kappa + 2 \sum_{n=1}^\infty \int_0^\Lambda d\kappa \kappa \sqrt{\kappa^2 + \left(\frac{n}{a}\right)^2} \right) - \frac{hc}{2\pi^2} \int_0^\Lambda \kappa d\kappa \int_0^\infty dk_z \sqrt{\kappa^2 + k_z^2}
\]

\[
= \frac{hc}{4\pi a} \left\{ \frac{\Lambda^3}{3} + \frac{2}{3} \Lambda^3 \left( \sum_{n=1}^\infty 1 - \int_0^\infty dn \right) - \frac{2\pi^3}{3a^3} \left( \sum_{n=1}^\infty n^3 - \int_0^\infty n^3 dn \right) \right\}.
\] (2)

Two parentheses in the equation above are evaluated by using Euler-Maclaurin formula as follows,

\[
\frac{1}{2} F(0) + \sum_{n=1}^\infty F(n) - \int_0^\infty F(n) dn = -\frac{1}{12} F'(0) + \frac{1}{24 \cdot 30} F'''(0) + \cdots.
\]

Speaking strictly, \(F(n)\) must be the decreasing function for large \(n\) in the Euler-Maclaurin formula. To extract the finite value by using the formula, the cutoff function which is exponential decreasing function for large \(\kappa\) must be inserted in first line of Eq.(2). However, we don’t introduce the cutoff function in this calculation. Because the cutoff function is independent of the physics of the Casimir effect, we can formally calculate the Eq.(2) in the absence of cutoff function. Therefore, formally we can get

\[
\sum_{n=1}^\infty 1 - \int_0^\infty dn ='' 1 + 1 + 1 + \cdots'' = -\frac{1}{2}
\]

\[
\sum_{n=1}^\infty n^3 - \int_0^\infty n^3 dn ='' 1^3 + 2^3 + 3^3 + \cdots'' = \frac{1}{120}.
\] (3)

As mentioned later, Eq.(3) corresponds to the zeta function and "\(\cdots''\) means the regularized sum. Substituting Eq.(3) into Eq.(2), the divergence terms including \(\Lambda\) are cancelled excellently. Eq.(2) is defined as the Casimir energy, it is given by

\[
E = -\frac{h\epsilon\pi^2}{720a^4}.
\] (4)

The Casimir force per unit area is given by

\[
F = -\frac{\partial}{\partial a} (a \cdot E) = -\frac{h\epsilon\pi^2}{240a^4}.
\] (5)

Thus we can see the subtraction of infinity in process of calculation. This procedure corresponds to the renormalization scheme in quantum field theory. After the prediction of Casimir, the Casimir force was confirmed by the experiment with the geometry of sphere-plate [2]. Up to now, the Casimir forces are measured in various setups [3, 4, 5].

As the other important point, the existence of Casimir force is closely related to zeta function playing the important role in the number theory. For every prime number \(p\) \((p \neq 1)\),

\[
\frac{1}{1 - \frac{1}{p^s}} = 1 + \frac{1}{p^s} + \frac{1}{p^{2s}} + \cdots
\] (6)
where $s > 1$. Multiplying Eq.(6) for all primes, one has

$$
\prod_p \frac{1}{1 - \frac{1}{p^s}} = \prod_p \left(1 + \frac{1}{p^s} + \frac{1}{p^{2s}} + \cdots\right) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots.
$$

(7)

In 1859, Riemann had realized that the only way to gain access to the features of the distribution of prime numbers was to turn $s$ into complex variables [6]. He defined r.h.s of Eq.(7) as the zeta function

$$
\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad s \in \mathbb{C}, \Re s > 1.
$$

(8)

By using the analytic continuation of the complex analysis, we can transform the zeta function of $s > 1$ to $s < 0$. In number theory, the important equation of zeta function is known:

$$
\pi^{-s/2} \Gamma\left(\frac{s}{2}\right) \zeta(s) = \pi^{-(1-s)/2} \Gamma\left(\frac{1-s}{2}\right) \zeta(1-s).
$$

(9)

Notice that $\pi^{-s/2} \Gamma(s/2)\zeta(s)$ is invariant under $s \leftrightarrow 1 - s$ and the property is called duality transformation. When $s = 1/2$, the equation becomes the self-dual relationship and the fact is related to the Riemann’s conjecture which is open question. Therefore $\zeta(-1)$ and $\zeta(-3)$ are evaluated by $\zeta(2) = \pi^2/6$ and $\zeta(4) = \pi^4/90$, respectively. That is, Eq.(9) leads to

$$
\zeta(-1) = 1 + 2 + 3 + \cdots = -\frac{1}{12}, \quad \zeta(-3) = 1^3 + 2^3 + 3^3 + \cdots = \frac{1}{120}.
$$

(10)

Equations above are the same as Eq.(3). Although it seems that the sums are infinity, these can be made finite value. In number theory, the procedure is called regularization. In fact, the finite comes from the subtraction of infinity. From Eq.(2) and (3), we can see that the infinity coming from the sums over all modes is cancelled by the value of $\zeta(-1)$ and the physical meaningful value of the Casimir force stems from the value of $\zeta(-3)$.

The prediction of the Casimir force depends on the finite of $\zeta(-1)$ and $\zeta(-3)$, and the Casimir force was confirmed by various experiments. Namely, it implies that the finite of $\zeta(-1)$ and $\zeta(-3)$ were confirmed by the experiments. Thus Casimir effect tied up three different researches, that is, the particle physics and the nano/micro technology and number theory. In this point, the achievement of Casimir is great.

Such as the Casimir effect, the connection between nano/micro technology and the particle physics will be described.

2. The higher dimensions and gravity

The purpose of the particle physics is to establish a ultimate theory unifying four forces. Furthermore the particle physics tries to provide the explanations for why mankind lives in 4 dimensions or how our universe generates.

The four forces are electro-magnetic, weak, strong and gravity. The weak force gives rise to the beta decay, which turns neutron into proton. The strong force plays the role of glues between quarks in nuclei. The theory of strong force is described by the quantum chromodynamics (QCD). The theory of gravity is the general relativity proposed by A.Einstein. Historically, S.Weinberg, A.Salam and S.Glashow had achieved the unification of the electro-magnetic force and weak force. The electro-weak theory won Nobel prize in 1979. The unification of the electro-weak theory and QCD is called the Standard model, which is supported by the various high
energy accelerator experiments. The Standard model formulated by the quantum gauge theory is renormalizable theory, namely, the divergence of quantum effects can be removed. However the Standard model and general relativity don’t be unified. The reason is that when calculating the quantum effects of gravity, the degree of divergence of Feynmann diagram increases. Namely the quantum general relativity is nonrenormalizable theory.

In order to realize the unification, the concept of point particle which acts each other through interactions was changed. The innovative idea that everything is made of the string with vibration is the foundation of string theory. String theory was energetically studied from about 1985 [7]. String theory is one of the most familiar candidates of a unification including the gravity. However we must accept the strange thing with respect to the dimensions. In order to quantize the string theory with supersymmetry (symmetry of interchanging boson and fermion), the string must live in 10 dimensions. Namely the string theory is higher dimensional theory. Higher dimensions are the four dimensional spacetime and extra spatial dimensions.

In the string theory, the extra invisible six dimensions must be compactified at radii of Planck size. But the string theory is still incomplete theory. Because the string theory can not necessarily explain all phenomena of the high energy accelerator physics. Moreover, since it is known that the string theory has five kinds of theories (Type I, Type IIA, Type IIB, Heterotic SO(32), Heterotic E8), string theory is insufficient as only one unification theory. Due to the recent developments of the string theory, the different five string theories unified to the M-theory in eleven dimensions. However the M-theory has the various problems. Since it is outside the scope of this paper, we don’t discuss it. Although the string theory has beautiful elegant mathematical structure, the experimental verification is rather difficult or may be impossible. Under the present circumstances, the string theory doesn’t have almost experimental prediction.

But it is believed that the difficulty toward the unification is solved by the introduction of the higher dimensional theory.

Recently, alternative remarkable higher dimensional theories had been proposed. One is the braneworld. The main assumption of the braneworld is that we live in the four dimensional membrane (so we called brane) embedded in higher dimensions and gravity propagates in higher dimensional world. The braneworld was proposed by [8, 9]. Especially, as shown in Fig.1, the model of [9] assumes that our world is a brane embedded in five dimensional Anti-de Sitter (AdS) space with a negative cosmological constant and the fifth direction \(y\) is infinite (de-compactification). The model provided the explanation for the reason why the gravity is extremely small compared with other three interactions. According to [9], by integrating the \(y\) direction of the five dimensional Einstein action, the four dimensional Planck mass \(M_p\) is given by

\[
M_p^2 \sim \frac{l}{G_5},
\]

(11)

where \(l\) is the radius of AdS and \(G_5\) is the gravitational constant of the five dimensions. It implies that the Planck mass is controlled by the geometry of higher dimensional world. This model insists that extreme smallness of gravity stems from the geometry of higher dimensional world. Moreover the static gravitational potential on the brane is calculated by [9] and it is given by

\[
V(r) \sim G \frac{m_1 m_2}{r} \left(1 + \frac{l^2}{r^2}\right),
\]

(12)

where \(m_1\) and \(m_2\) are masses of two particle on the brane. The first term is the usual Newtonian law due to massless graviton and the second term is correction term due to massive graviton in AdS. For large distance \(r \gg l\), the first term is dominated and the four dimensional observer on
the brane can detect the usual Newtonian law ($V \sim r^{-1}$). For small distance $r \ll l$, the observer can detect the unusual gravitational law ($V \sim l^3 r^{-3}$). Thus the experimental verification of the braneworld is probably performed by measuring the behaviour of power law.

Although the braneworld is far away from the unification theory, the braneworld indicates that the difference between the gravity and other three forces is the dimensions in which the force lives. Up to now, various versions of the braneworld are applied to the models in the particle physics and cosmological models.

On the other hands, large extra dimension theory had been proposed by [10]. It is assumed that the fundamental theory of gravity is Einstein action in $4+n$ dimensions and extra $n$ dimensions are compactified. The image of the model is illustrated in Fig. 2. By integrating the extra dimensional part of the higher dimensional Einstein action, we can read off the four dimensional gravitational constant. According to [10],

$$\frac{1}{16\pi G_{(4+n)}} \int d^{4+n}x \sqrt{-g^{(4+n)}} R^{(4+n)} = \frac{1}{16\pi G_{(4+n)}} V_n \int d^4x \sqrt{-g^{(4)}} R^{(4)} + \cdots$$

(13)

where $G_{(4+n)}$ is the gravitational constant, $g^{(4+n)}$ is the metric and $R^{(4+n)}$ is the scalar curvature of $4+n$ dimensions. Here $V_n$ is the volume of extra $n$ dimensions. From the dimensional analysis, the $4+n$ dimensional Planck mass $M_{p(4+n)}$ is defined as

$$8\pi G_{(4+n)} = \left( M_{p(4+n)} \right)^{-n-2}.$$  

(14)

Since the r.h.s of Eq. (13) is identical to the four dimensional Einstein action, the four dimensional Planck mass is given by

$$\left( M_{p(4)} \right)^2 = \left( M_{p(4+n)} \right)^{n+2} V_n.$$  

(15)

The equation above isn’t new. The generalization of the Kaluza-Klein model (KK model [11]) which is the unification of electro-magnetic force and gravity in five dimensions corresponds to Eq. (15). In the KK model, this corresponds to the choices $n = 1$ and $M_{p(5)} = M_{p(4)}$. Namely, the size of an extra dimension should be Planck length ($\sim 10^{-33}$ cm), so that we can’t detect it by the gravitational experiments. Authors of [10] proposed a loophole against the KK model. The three forces except for gravity are confined in the four dimensions and the size of extra dimension is not necessarily required to be so very small. Requiring that the four dimensional fundamental scale $M_{p(4)} \sim 10^{18}$GeV is controlled by the higher dimensional fundamental scale
Figure 3. The current experimental range of $r^{-2}$ law: The arrow in the figure indicates the lower bound estimated by the various experimental method. The scale above is adjusted suitably. The usual Newtonian $r^{-2}$ law is confirmed by the range of the heavy line ($1\text{mm} < r < 10^{15}\text{m}$). The broken heavy line corresponds to the experiments with inadequate measurement accuracy. From nonscale to a space scale, the usual Newtonian law is not confirmed by experiment.

$M_p^{(4+n)}$ which is set to about 1 TeV of electroweak scale, Eq.(15) yields the typical size of an extra dimension $R$

$$R \sim 10^{\frac{30}{n}-17}\text{cm}, \quad (16)$$

where we used $V_n \sim R^n$. The idea of the model implies that four dimensional world is described by the effective theory from higher dimensional world and four dimensional fundamental scale is coming from higher dimensions. Namely this model provided a explanation for the reason why Planck scale is huge compared with the electroweak scale, what is called hierarchy problem. According to the assumption mentioned previously, since the three forces except for gravity propagate only in four dimensions, it is difficult for the extra dimensions to be detected by the usual high energy accelerator experiments. Since gravity feels the higher dimensions, the experimental verification of the model is probably performed by the gravitational experiments.

At large distance $r > R$, the observer lives in the four dimensions, the gravitational potential behaves $V \sim r^{-1}$. At small distance $r < R$, the observer can feel the higher dimensions, the potential behaves $V \sim r^{-n-1}$ as if non-Newtonian law. By measuring the power law of potential, the existence of extra dimension will be probably confirmed. From Eq.(16), if $n = 1$, we have $R \sim 10^{13}\text{cm} \sim 1$ astronomical unit, which is ruled out. If $n = 2$, we have $R \sim 10^{-2}\text{cm} < 1\text{mm}$. This case is not necessarily ruled out. Although the familiar Newtonian inverse-square law is actually confirmed up to $r > 1\text{mm}$, the possibility of non-Newtonian law cannot be denied at scale $r < 1\text{mm}$. Thus the size of extra dimension is accepted to be large size compared with Planck length. Therefore this model is called large extra dimension.

At present, both the braneworld and large extra dimension theory are far away from the unification theory. Recently it has been proposed that the theories may be deeper connection to the low energy effective theory of the string theory. It is expected as the two theories in which experiment verification is possible.

3. The search for the higher dimensions and the surface technologies

In order to verify the theories proposed by the particle physics, the gravitational experiment has to be performed at distance $r < 1\text{mm}$. The experiments of the short distance obviously depend on the surface technologies. If the non-Newtonian law is detected, it may mean the existence of the higher dimensional world. Fig.3 shows the current experimental range of Newtonian $r^{-2}$ law by the various methods. The usual Newtonian $r^{-2}$ law is confirmed by the range of the heavy line in Fig.3. The upper bound ($\sim 10^{15}\text{m}$) is determined by the planetary motion and
astronomical data. In 1798, the most famous Cavendish experiment had been performed in the range of \( \sim 1 \text{ cm} \). Up to the present, the high precision torsion pendulum (improvement of Cavendish experiment) has confirmed the \( r^{-2} \) law in the range of order of centimeter. As indicated the broken heavy line shown in Fig.3, it is generally believed that the measurement accuracy is insufficient in the range of submillimeter.

Two groups have challenged. Let us describe only the measurement results of two groups. The university of Washington group used the torsion pendulum with laser equipments, it said that \( r^{-2} \) law was confirmed up to \( \sim 56 \mu m \) \[12, 13\]. On the other hands, by using the micro-cantilever devise, Stanford university group performed the gravitational experiment \[14\]. According to \[14\], it was reported that \( r^{-2} \) law was confirmed up to \( \sim 20 \mu m \). However, there is not enough to measurement accuracy which generally accepts the measurement result of two groups.

The properties of gravity cause the difficulties of performing the high precision gravitational experiments of the short distance. The main reasons are (i) the extreme smallness of gravity, (ii) influences of other interactions (Casimir force, electo-magnetic force) and (iii) no method of screening gravity. In the future, it is expected that the developments of surface technologies solve the difficulties.

Furthermore, let us explain the important submillimeter scale appearing in recent cosmology. It is the dark energy scale shown in Fig.3. It has been announced that our universe is accelerating \[15, 16\]. It implies the existence of the unknown repulsive force overcoming the gravity. In cosmology, the energy generating the repulsive force is called dark energy. The ratio of the dark energy to whole energy of universe had been determined by the WMAP satellite \[17\]. WMAP indicated that the energy density of dark energy is approximately \( \rho_d \sim 3.8 \text{keV/cm}^3 \). This dark energy density corresponds to a scale \( \lambda_d = (\hbar c / \rho_d)^{1/4} \sim 85 \mu m \). Nobody knows what the dark energy is. If the dark energy regards as the quantum mechanical vacuum energy, the vacuum energy \( \rho_{\text{vacuum}} \) is roughly given by \( M_p^4 \), where \( M_p \sim 10^{18} \text{GeV} \) is Planck mass. This is because the fundamental scale of our four dimensions is the Planck scale which is the upper limit of integral of the zero point energy. Therefore, we have

\[
\rho_d \sim \left(10^{-3} \text{ eV}\right)^4 \ll \rho_{\text{vacuum}} \sim \left(10^{18} \text{ GeV}\right)^4 \Rightarrow \frac{\rho_{\text{vacuum}}}{\rho_d} \sim 10^{120} \gg 1 \quad (17)
\]

Thus the observed \( \rho_d \) is very small compared with theoretical value. Namely, the unnatural fine-tuning of 120 is needed in order to explain the smallness of \( \rho_d \). In the framework of the four dimensional known physics, there is no theory which can explain the reason why \( \rho_d \) is very tiny. In particle physics and cosmology, it is called the cosmological constant problem. If the result of the WMAP is believed, we have to accept the existence of new force which is still not detected. Assuming that the fundamental scale of the new physics is the dark energy scale, non-Newtonian law may emerge at \( r < \lambda_d \). However it is not yet discovered.

4. Summary
The extra dimensions are not yet discovered by the experiments. Under the present circumstance, whether the theories are correct or not can not determined. What the theoretical physicists expect is that the surface technologies developed by the promotion of friction science detect the extra dimensions or the new physics. Although it may be dream, the particle physics expects it.

5. Acknowledgments
The author would like to thank Prof. Miura for giving the opportunity for announcing this problem in the poster session.
6. References

[1] H.B.G. Casimir, Proc.Kon. Nederl. Akad. Wet., 51, 793(1948).

[2] S. K. Lamoreaux, “Demonstration of the Casimir force in the 0.6 to 6 micrometers range,” Phys. Rev. Lett. 78, 5 (1997).

[3] U. Mohideen and A. Roy, “Precision measurement of the Casimir force from 0.1 to 0.9 μm,” Phys. Rev. Lett. 81, 4549 (1998) [arXiv:physics/9805038].

[4] G. Bressi, G. Carugno, R. Onofrio and G. Ruoso, “Measurement of the Casimir force between parallel metallic surfaces,” Phys. Rev. Lett. 88, 041804 (2002) [arXiv:quant-ph/0203002].

[5] R. S. Decca, E. Fischbach, G. L. Klimchitskaya, D. E. Krause, D. L. Lopez and V. M. Mostepanenko, “Improved tests of extra-dimensional physics and thermal quantum field theory from new Casimir force measurements,” Phys. Rev. D 68, 116003 (2003) [arXiv:hep-ph/0310157].

[6] B. Riemann; Gesammelte Mathematische Werke, Wissenschaftlicher Nachlass und Nachträge, Collected Papers, R. Narasimhan, Springer Verlag-BSG B.G. Verlagsgesellschaft (1990)

[7] M.B. Green, J.H. Schwarz, E. Witten, “Superstring theory vol.1, vol.2”, Cambridge Monographs on Mathematical Physics.

[8] L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension,” Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221].

[9] L. Randall and R. Sundrum, “An alternative to compactification,” Phys. Rev. Lett. 83, 4690 (1999) [arXiv:hep-th/9906064].

[10] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, “The hierarchy problem and new dimensions at a millimeter,” Phys. Lett. B 429, 263 (1998) [arXiv:hep-ph/9803315].

[11] Theodor Kaluza, On the problem of unity in physics, Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math. Phys.) 966-972 (1921), Oskar Klein, Quantum theory and five dimensional theory of relativity, Z. Phys. 37 895-906 (1926).

[12] E. G. Adelberger, B. R. Heckel, S. Hoedl, C. D. Hoyle, D. J. Kapner and A. Upadhye, “Constraints on exotic interactions from a recent test of the gravitational inverse square law,” Phys. Rev. Lett. 98, 131104 (2007) [arXiv:hep-ph/0611223].

[13] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, “Tests of the gravitational inverse-square law below the dark-energy length scale,” Phys. Rev. Lett. 98, 021101 (2007) [arXiv:hep-ph/0611184].

[14] J. Chiaverini, S. J. Smullin, A. A. Geraci, D. M. Weld and A. Kapitulnik, “New experimental constraints on non-Newtonian forces below 100-μm-m,” Phys. Rev. Lett. 90, 151101 (2003) [arXiv:hep-ph/0209325].

[15] A. G. Riess et al. [Supernova Search Team Collaboration], “Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant,” Astron. J. 116, 1009 (1998) [arXiv:astro-ph/9805201].

[16] S. Perlmutter et al. [Supernova Cosmology Project Collaboration], “Measurements of Omega and Lambda from 42 High-Redshift Supernovae,” Astrophys. J. 517 (1999) 565 [arXiv:astro-ph/9812133].

[17] C. L. Bennett et al. [WMAP Collaboration], “First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results,” Astrophys. J. Suppl. 148 (2003) 1 [arXiv:astro-ph/0302207].