Critical scaling in random-field systems: 2 or 3 independent exponents?

Gilles Tarjus1, Ivan Balog1,2 and Matthieu Tissier1

1 LPTMC, CNRS-UMR 7600, Université Pierre et Marie Curie - boîte 121, 4 Pl. Jussieu, 75252 Paris cédex 05, France, EU
2 Institute of Physics - P.O. Box 304, Bijenicka cesta 46, HR-10001 Zagreb, Croatia

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Abstract – We show that the critical scaling behavior of random-field systems with short-range interactions and disorder correlations cannot be described in general by only two independent exponents, contrary to previous claims. This conclusion is based on a theoretical description of the whole \((d, N)\) domain of the \(d\)-dimensional random-field \(O(N)\) model (RF\(O(N)\)M) and points to the role of rare events that are overlooked by the proposed derivations of two-exponent scaling. Quite strikingly, however, the numerical estimates of the critical exponents of the random-field Ising model are extremely close to the predictions of the two-exponent scaling in \(d = 3\) and \(d = 4\), so that the issue cannot be decided only on the basis of numerical simulations in these spatial dimensions.

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Introduction. – The critical behavior of models in the presence of a quenched random field has attracted a lot of attention over the past decades [1,2]. Beyond the experimental interest, such models provide a rich playground to investigate the influence of quenched disorder on the long-distance properties of a system. Their equilibrium critical point, separating a disordered, paramagnetic phase from an ordered, ferromagnetic one, is known to be controlled by a zero-temperature fixed point, with temperature being an irrelevant variable in the renormalization group sense. Among the new features brought by such a fixed point is the violation of the “hyperscaling relation” between critical exponents. The space dimension \(d\) that appears in this relation must be corrected by the exponent \(\theta\) describing the flow of the renormalized temperature to zero, thereby leading to an apparent dimensional reduction from \(d\) to \(d - \theta\) [2–4].

Whereas phenomenological theories take \(\theta\) as an independent exponent [3,4], which implies that equilibrium scaling behavior is described by three independent exponents in place of the usual two-exponent scaling for finite-temperature fixed points, two different approaches have claimed that \(\theta\) is actually fixed. One is the dimensional-reduction prediction, based either on perturbation theory [5–7] or on the Parisi-Sourlas supersymmetric formulation [8]. It states that \(\theta = 2\) and that the exponents of the random-field model are those of the corresponding pure model in two dimensions less. This prediction has been rigorously proven to be wrong in low enough dimension [9,10], and we have recently provided a complete theoretical explanation of dimensional reduction and its breakdown through a nonperturbative functional renormalization group [11–14].

Another line of argument has been put forward by Schwartz and coworkers [15,16]. The claim is that \(\theta = 2 - \eta\), with \(\eta\) the anomalous dimension of the (order parameter) field, so that scaling is described by only two independent exponents, e.g., \(\eta\) and the correlation length exponent \(\nu\). The derivation involves general manipulations and the result is supposed to hold for the Ising as well as the continuous version with \(O(N)\) symmetry. It is also supported by heuristic considerations [16,17].

The problem with the above derivations is that they rely on formal manipulations that are blind to the presence of rare events or rare regions, such as avalanches or droplets, or that overlook the presence of multiple metastable states. This is known to be the reason for the failure of the simple supersymmetric formulation [13,14,18]. This also casts some doubt on the general validity of the order-of-magnitude estimates of the relative
strength of the fluctuations carried out in Schwartz’s arguments [15,16].

How to conclude on the validity of the two-exponent scenario proposed by Schwartz and coworkers? The question is actually quite subtle as the description is exact at the lower and the higher critical dimensions of the random-field models and it appears to be numerically very-well verified in computer simulations, high-temperature expansions or approximate renormalization group treatments, at least in $d = 3$. However, error bars may always blur the conclusion. On the other hand, with the help of the functional renormalization group (FRG), the problem can be studied for continuous values of the dimension $d$ and of the number of components $N$ of the field. We can then show that the two-exponent scenario cannot be right in general. This implies that the corresponding prediction for the random-field Ising model in $d = 3$ is a very, or even extremely [19], good but by no means exact, result. We suggest that extensive simulations of the RFIM in $d = 5$, where we predict that the violations of the dimensional-reduction property is very small, could settle the issue on the numerical side.

**The critical behavior of random-field systems.** – The long-distance behavior of the random-field $O(N)$ model is described by the following Hamiltonian or bare action:

$$S[\phi, h] = \int x \left\{ \frac{1}{2} |\partial_\phi(x)|^2 + U_B(|\phi(x)|^2) - h(x) \cdot \phi(x) \right\},$$

where $\int \equiv \int d^d x$, $\phi(x)$ is an $N$-component field, $U_B(|\phi|^2) = (\tau/2)|\phi|^2 + (u/4!)(|\phi|^2)^2$, and $h(x)$ is a random source (a random magnetic field in the language of magnetic systems) with zero mean and a variance

$$\overline{h^\mu(x)h^\nu(y)} = \Delta_B \delta_{\mu\nu} \delta^{(d)}(x-y),$$

where $\mu, \nu = 1, \ldots, N$ and the overline denotes an average over the random field. An ultra-violet (UV) momentum cutoff $\Lambda$, associated with an inverse microscopic length scale such as a lattice spacing, is also implicitly considered. We focus here on the short-range version defined above. We shall briefly comment on the long-range version in the conclusion.

Due to the presence of the random field, one needs to consider two different types of pair correlation functions of the $\phi$ field: the so-called connected one, $G^{\mu\nu}_{con}(x-y) = \langle \phi^\mu(x)\phi^\nu(y) \rangle - \langle \phi^\mu(x) \rangle \langle \phi^\nu(y) \rangle$ and the disconnected one, $G^{\mu\nu}_{dis}(x-y) = \langle \phi^\mu(x) \rangle \langle \phi^\nu(y) \rangle - \langle \phi^\mu(x) \rangle \langle \phi^\nu(y) \rangle$. At the critical point $T_c$, the two correlation functions behave as

$$G^{\mu\nu}_{con}(x-y) \sim \frac{T^\theta_{\mu\nu}}{|x-y|^{d-2+\eta}},$$

$$G^{\mu\nu}_{dis}(x-y) \sim \frac{\delta_{\mu\nu}}{|x-y|^{d-4+\eta}},$$

where $\eta$ is the usual anomalous dimension of the field and $\eta$ is a new exponent. Accordingly, one can also define two types of susceptibilities that diverge as one approaches the critical point from above as

$$x_{con} = \int x G_{con}(x) \sim (T - T_c)^{-\gamma},$$

$$x_{dis} = \int x G_{dis}(x) \sim (T - T_c)^{-\gamma},$$

the former one being the usual magnetic susceptibility. (The component indices have been dropped as the functions are then proportional to $\delta_{\mu\nu}$ as in eq. (3).) The exponents $\eta$ and $\tilde{\eta}$ are related via $\gamma = (2 - \eta)\nu$ and $\tilde{\gamma} = (4 - \eta)\nu$, with $\nu$ the correlation length exponent.

The renormalized temperature is irrelevant at the fixed point controlling the critical behavior [3,4] and it flows to zero with an exponent $\theta$. As a result, the hyperscaling relation has an unusual form

$$2 - \alpha = (d - \theta)\nu,$$

where $\alpha$ is the specific-heat exponent. The exponents $\theta$, $\eta$, and $\tilde{\eta}$ are related through

$$\theta = 2 + \eta - \tilde{\eta},$$

so that the scaling around the critical point is a priori described by three independent exponents, e.g., $\eta$, $\nu$, and $\theta$ or $\tilde{\eta}$.

**The two-exponent scaling description.** – The $d \rightarrow d - 2$ dimensional reduction predicts that $\theta = 2$, i.e., $\tilde{\eta} = \eta$, and furthermore that all the critical exponents are given by those of the pure model in ($d - 2$). On the other hand, the two-exponent scenario put forward by Schwartz and coworkers [15,16] states that the exponents obey the following relations:

$$\theta = 2 - \eta, \quad \tilde{\eta} = 2\eta.$$

The derivation actually also implies that the disconnected and connected correlation functions in Fourier space are related through

$$G_{dis}(q) = \frac{\Delta_B}{T^2} G_{con}(q)^2$$

at criticality and when $q \rightarrow 0$. Equation (8) implies that the second cumulant of the random field is not renormalized and stays fixed to its bare value $\Delta_B$.

A stronger claim was initially made by Schwartz [15], who suggested that all the exponents of a random-field system in dimension $d$ are the same as those of its pure counterpart in a reduced dimension $d - 2 + \eta(d)$. This prediction was however soon shown by Bray and Moore [20] to be already wrong for the exponent $\nu$ in the RFIM near its lower critical dimension 2 at first order in $\epsilon = d - 2$. To the best of our knowledge, it was subsequently abandoned.

It is also worth mentioning that various heuristic arguments have been used to derive the above two-exponent
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scaling [16,17]. For instance, one such argument states that the magnetization per spin in a finite-size system of linear size \( L \) (we consider here the RFIM for simplicity), which scales as \( L^{-d(d+4+\eta)/2} \), is given for a typical realization of the random field by the average magnetic susceptibility \( \chi_{\text{con.L}} \sim L^{2-\eta} \) times the mean random field \( \bar{h}_L \) which scales as \( L^{-d/2} \):

\[
m_L \sim L^{-(d+4+\eta)/2} \sim \chi_{\text{con.L}} \bar{h}_L \sim L^{2-\eta} L^{-d/2}.
\]

It immediately results that \( \bar{\eta} = 2\eta \).

In the case of the RFIM, the two-exponent scenario is exact at and near the lower critical dimension in first order in \( \epsilon = d - 2 \), as shown in ref. [20], and it is also somewhat trivially expected at the upper dimension \( d = 6 \) and in first order in \( \epsilon = 6 - d \) as both \( \eta \) and \( \bar{\eta} \) are zero and the mean-field result \( \bar{\eta} = 2 \) still applies. In between, numerical studies via high-temperature expansions in \( d = 3, 4, 5, 21 \) and computer simulations in \( d = 3, 27 \), confirm that \( \bar{\gamma} \) is very close to \( 2\gamma \), and \( \bar{\eta} \) to \( 2\eta \), certainly within the accuracy of the methods.

However, as mentioned in the introduction, a fundamental problem remains that the proposed derivations of the two-exponent scaling involve, beyond formal manipulations, estimates of the relative order of magnitude of the fluctuations that essentially rely on factorization approximations and the central-limit theorem in the limit of large system size (see e.g. the above heuristic argument). All of the derivations are therefore blind to rare events, rare regions or rare samples, which, precisely, have been shown to be crucial in disordered systems near zero-temperature fixed points [3,4,12-14,22-30].

In the absence of rigorous derivations, numerical evidence, no matter how good, is insufficient to establish the validity of the scenario, because of unavoidable uncertainty. Quite the contrary, we show below that the two-exponent description, which is claimed to apply to random-field systems below their upper critical dimension irrespective of the number of components \( N \) and of the dimension \( d \), cannot be valid in general and, as a consequence, has no rigorous foundations at this point.

The RFO(N)M near \( d = 4 \). – Near the lower critical dimension for ferromagnetism, which is equal to \( d = 4 \) in this case, the long-distance physics of the RFO(N)M is captured by a nonlinear \( \sigma \) model that in turn can be studied through a perturbative but functional RG [31]. The resulting FRG flow equations in \( d = 4 + \epsilon \) have been derived and studied by several groups, at one- [12,31,32] and two- [33,34] loop order.

The dimensionless second cumulant of the renormalized random field is a function \( \Delta(z) \) (where \( z \) is the cosine of the angle between fields in two different copies of the system [12,31]) and it obeys the following RG equation at one loop:

\[
k \partial_k \Delta_k(z) = \epsilon \Delta_k(z) - \left[ z \Delta_k(z)^2 + (N - 3) \Delta_k(1) \Delta_k(z) + (N - 3 + 4 \epsilon^2) \Delta_k(z) \Delta_k'(z) - (N + 1) z \Delta_k(1) \Delta_k'(z) - z(1 - z^2) \Delta_k(z) \Delta_k''(z) + (1 - z^2) \Delta_k(1) \Delta_k''(z) - 3z(1 - z^2) \Delta_k'(z)^2 + (1 - z^2)^2 \Delta_k'(z) \Delta_k''(z) \right],
\]

where \( k \) is the running infrared (IR) momentum cutoff, a prime denotes a derivative, and \( \Delta_k(z) \) (which, up to a multiplicative constant, was denoted \( R'(z) \) in previous publications) is of order \( \epsilon \) near the fixed point. One can moreover define two running exponents \( \eta_k \) and \( \bar{\eta}_k \) as follows:

\[
\eta_k = \Delta_k(1), \quad \bar{\eta}_k = -\epsilon + (N - 1) \Delta_k(1).
\]

They converge to the fixed-point values \( \eta, \bar{\eta} \) when \( k \to 0 \). The corresponding equations at two-loop order are given in ref. [34] and are not reproduced here.

A numerical and analytical investigation of these FRG equations [12,34,35] shows that above a critical value of the number of components, \( N_{\text{DR}} \approx 18 - \frac{42}{\epsilon^2} \), there exists a fixed point that corresponds to the \( d \to d - 2 \) dimensional reduction, with \( \bar{\eta} = \eta = \frac{\Delta_4(1)}{N - 1} \). Above a slightly higher value, \( N_{\text{cusp}} = 2(4 + 3\sqrt{3}) - 3(2 + 3\sqrt{3})\epsilon/2 [35] \), this fixed point is stable and describes the critical behavior of the RFO(N)M (see also ref. [36]). In this domain of \( N \), critical scaling is therefore described by \( \bar{\eta} = \eta \) and \( \theta = 2 \), which contradicts the predictions of Schwartz and coworkers [15,16]. Below \( N_{\text{cusp}} \), the stable fixed point is now characterized by a nonanalyticity in the functional dependence of the renormalized disorder cumulant \( \Delta_4(z) \), \( \bar{\Delta}_4(1) \sim \sqrt{1 - z} \) when \( z \to 1 \), that is strong enough to break the dimensionless-reduction prediction, with therefore \( \bar{\eta} > \eta \). However, the possibility that this nonanalytic “cuspy” fixed point is characterized by the equality \( \bar{\eta} = 2\eta \) is invalidated by the results. We display the ratio \( (2\eta - \bar{\eta})/\eta \) as a function of \( N \) at one-loop order in fig. 1. (The results are confirmed at two-loop order.) The ratio is equal to 1 above \( N_{\text{cusp}} \approx 18.3923 \) and decreases continuously as \( N \) decreases. At the endpoint value of which \( \Delta_4(1) \) diverges, \( N_{\nu} = 2.8347 \ldots \), it reaches a strictly positive value of \( 3 - N_{\nu} \approx 0.1653 \ldots \) We stress that the output can be obtained with an arbitrary precision, which is quite different from simulations.

The RFIM near \( d_{\text{DR}} \approx 5.1 \). – The main result indicating that the two-exponent scaling is generically inexact in the short-range RFIM is that there exists a range of dimension below the upper critical one, \( d = 6 \), for which the \( d \to d - 2 \) dimensional reduction is valid and therefore for which \( \bar{\eta} = \eta \neq 2\eta \) and \( \theta = 2 \neq 2 - \eta \). The arguments establishing this result are as follows:

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In a small interval to a nonanalytic, “cuspy” perturbation. Points coexist, but the dimensional-reduction one is unstable. Below $N_{\text{DR}} = 18$ (marked by the leftmost vertical dashed line), the critical behavior is described by a “cuspy” fixed point and dimensional reduction breaks down but $(2\eta - \bar{\eta})/\bar{\eta} = 1$. The fixed point associated with dimensional reduction [11,13] is stable, the eigenvalue associated with a nonanalytic, “pless” perturbations that indeed prove to be irrelevant at and in the vicinity of $d = 6$ and can be numerically obtained below 6. The perturbation is then found to be irrelevant at and in the vicinity of $d = 6$: the eigenvalue is equal to 1 in $d = 6$ and continuously decreases as one lowers $d$ slightly below 6; physically, this corresponds to a situation where the effect of the avalanches is only subdominant at the critical point [14].

By means of a nonperturbative truncation of the exact hierarchy of FRG equations (NP-FRG) for the cumulants of the renormalized disorder, we have located the limit of existence of the dimensional-reduction fixed point at $d_{\text{DR}} \simeq 5.1$ [13]. There is thus a nonzero range of dimension above $d_{\text{DR}}$ and below 6 in which $\bar{\eta} = \eta \neq 0$ and $\theta = 2$, which contradicts the claim of the two-exponent scaling scenario.

\footnote{For a similar result in the context of elastic interfaces in a random medium, see refs. [24–30].}

\footnote{Note that in the case of the RFIM, $d_{\text{DR}}$ and $d_{\text{cusp}}$ appear essentially indistinguishable [35]. The difference could only affect a}

\textbf{Fig. 1:} (Colour on-line) Ratio $(2\eta - \bar{\eta})/\eta$ as a function of $N$ for the RFO(N)/M in $d = 4 + \epsilon$ at one-loop order. Above $N_{\text{cusp}} = 2(4 + 3\sqrt{3}) \simeq 18.3923$... the stable fixed point is associated with the $d \rightarrow d - 2$ dimensional reduction and $(2\eta - \bar{\eta})/\bar{\eta} = 1$. Below $N_{\text{DR}} = 18$ (marked by the leftmost vertical dashed line), the critical behavior is described by a “cuspy” fixed point and dimensional reduction breaks down but $(2\eta - \bar{\eta})/\bar{\eta}$ is generically $> 0$. In a small interval $N_{\text{DR}} < N < N_{\text{cusp}}$ the two fixed points coexist, but the dimensional-reduction one is unstable to a nonanalytic, “cuspy” perturbation.

\textbf{i)} The fixed point associated with dimensional reduction proceeds continuously from the Gaussian fixed point in $d = 6$ as one lowers $d$. It is characterized by the absence of a linear cusp in the functional dependence of the cumulants of the renormalized random field. (It can indeed be shown by the FRG that only such a cusp can lead to a breakdown of dimensional reduction [11,13].)

\textbf{ii)} That this dimensional-reduction fixed point is stable for some interval of $d$ below 6 can furthermore be seen by studying, in addition to all possible “cuspy” perturbations that indeed prove to be irrelevant, the eigenvalue associated with a nonanalytic, “cuspy” perturbation around this fixed point. This can be done exactly in $d = 6$ and can be numerically obtained below 6. The perturbation is then found to be irrelevant at and in the vicinity of $d = 6$: the eigenvalue is equal to 1 in $d = 6$ and continuously decreases as one lowers $d$ slightly below 6; physically, this corresponds to a situation where the effect of the avalanches is only subdominant at the critical point [14].

\textbf{In addition, we have investigated in more detail the “cuspy” fixed point corresponding to a breakdown of the $d \rightarrow d - 2$ dimensional reduction below $d_{\text{DR}}$.} Our purpose was to show that this fixed point is described by three independent exponents in general. The NP-FRG equations that must be numerically solved are given in ref. [13], together with technical comments, and this is not reproduced here.

We focus on the vicinity of $d_{\text{DR}}$, which is where the violation of the proposed relation $\bar{\eta} = 2\eta$ is unambiguous. We display in fig. 2 for illustration the RG flow of the running exponents $\eta_k$ and of $2\eta_k - \bar{\eta}_k$ as a function of $t = \log(k/\Lambda)$ for the RFIM in $d = 5$. The arrows indicate the flow toward the infrared ($k \rightarrow 0$). Note that one reaches $t \sim 19$, which means a length scale bigger than $10^8$ times the microscopic one. One is then clearly in the asymptotic regime as shown by comparing the curves with the symbols on the $y$-axis, which are the values obtained by directly solving the fixed-point equation. The two curves are superimposed, implying $\bar{\eta}_k = \eta_k$, until the Larkin scale (marked by a circle), at which point they separate.

We also plot in fig. 3 the fixed-point value of the ratio $(2\eta - \bar{\eta})/\eta$ as a function of $d$ for the RFIM in the vicinity of $d_{\text{DR}}$ from the NP-FRG equations\footnote{Note that in the case of the RFIM, $d_{\text{DR}}$ and $d_{\text{cusp}}$ appear essentially indistinguishable [35]. The difference could only affect a}. The crux of
the present demonstration is not the actual values of the exponents (which, however, are always within 10% of the best estimates), but the fact that, around $d_{DR}$, $\bar{\eta}$ and $\eta$ are equal or close to each other so that $(2\eta - \bar{\eta})/\eta$ stays near 1 and unambiguously violates the prediction of the two-exponent scenario. This is clearly seen in fig. 3.

This conclusion is similar to that reached above for the RFIM in the vicinity of $d_{DR} \simeq 5.1$ from the NP-FRG. Above $d_{DR}$, the ratio is exactly equal to 1 as the stable fixed point is associated with the $d \to d - 2$ dimensional reduction. For $d < d_{DR}$, the stable fixed point is characterized by the existence of a cusp in the cumulants of the renormalized random field. The ratio then decreases continuously from 1 as one decreases $d$. As in fig. 1, there is a finite domain of $d$ for which $2\eta - \bar{\eta} \neq 0$ with no ambiguity.

**Conclusion.** – In this paper, we have challenged the two-exponent scaling scenario of the critical behavior of random-field systems proposed by Schwartz and coworkers [12–16]. Due to the generality of its derivations, this scenario is supposed to apply to all random-field models, whether in the Ising or in the $O(N)$ version, and in all dimensions $d$ between the lower and the upper critical ones. We have however clearly proven that the predictions, most notably the relation between exponents $\eta = 2\bar{\eta}$, cannot be exact for generic random-field systems. Our demonstration is beyond error bars as it involves the fact that there is a whole domain of $d$ and $N$, including the Ising version, very small region near $d \simeq 5.1$ with no consequences on the present demonstration.

3On the other hand, the Schwartz-Soffer inequality, $2\eta - \bar{\eta} \geq 0$ [37], which has a rigorous basis, is of course not questioned here.

**REFERENCES**

[1] IMRY Y. and MA S. K., Phys. Rev. Lett., 35 (1975) 1399.
[2] For a review, see NATTEMTANN T., Spin Glasses and Random Fields (World Scientific, Singapore) 1998, p. 277.
[3] VILLAIN J., Phys. Rev. Lett., 52 (1984) 1543.
[4] FISHER D. S., Phys. Rev. Lett., 56 (1986) 416.
[5] AHRONY A., IMRY Y. and MA S. K., Phys. Rev. Lett., 37 (1976) 1364.
[6] GRIENSTEIN G., Phys. Rev. Lett., 37 (1976) 944.
[7] YOUNG A. P., J. Phys. C, 10 (1977) L257.
[8] PARISI G. and SOURLAS N., Phys. Rev. Lett., 43 (1979) 744.
[9] IMRIE J. Z., Phys. Rev. Lett., 53 (1984) 1747.
[10] BRICMONT J. and KUPIANEK A., Phys. Rev. Lett., 59 (1987) 1829.
[11] TARJUS G. and TISSIER M., Phys. Rev. Lett., 93 (2004) 267008; Phys. Rev. B, 78 (2008) 024203.
[12] TISSIER M. and TARJUS G., Phys. Rev. Lett., 96 (2006) 087202; Phys. Rev. B, 78 (2008) 024204.
[13] TISSIER M. and TARJUS G., Phys. Rev. Lett., 107 (2011) 041601; Phys. Rev. B, 85 (2012) 104202; 104203.
[14] TARJUS G., BACZYK M. and TISSIER M., Phys. Rev. Lett., 110 (2013) 135703.
[15] Schwartz M., J. Phys. C, 18 (1985) 135.
[16] Schwartz M. and Soffer A., Phys. Rev. B, 33 (1986) 2059; Schwartz M., Gofman M. and Natterman T., Physica A, 178 (1991) 6.
[17] Eischhorn K. and Binder K., J. Phys.: Condens. Matter, 8 (1996) 5209; Vink R. L. C., Binder K. and Len H., J. Phys.: Condens. Matter, 20 (2008) 404222; Vink R. L. C., Fischer T. and Binder K., Phys. Rev. E, 82 (2010) 051134.
[18] Parisi G., in Proceedings of Les Houches 1982, Session XXXIX, edited by Zuber J. B. and Stora R. (North Holland, Amsterdam) 1984, p. 473.
[19] Fytas N. G. and Martin-Mayor V., preprint arXiv:1304.0318 (2013).
[20] Bray A. J. and Moore M. A., J. Phys. C, 18 (1985) L927.
[21] Gofman M., Adler J., Aharony A., Harris A. B. and Schwartz M., Phys. Rev. Lett., 71 (1993) 1569.
[22] Bray A. J. and Moore M. A., J. Phys. C, 17 (1984) L463.
[23] Fisher D. S. and Huse D. A., Phys. Rev. B, 38 (1988) 373; 386.
[24] Fisher D. S., Phys. Rev. Lett., 56 (1986) 1964; Narayan O. and Fisher D. S., Phys. Rev. B, 46 (1992) 11520.
[25] Balents L., Bouchaud J. P. and Mézard M., J. Phys. I, 6 (1996) 1007.
[26] Le Doussal P., Wiese K. J. and Chauve P., Phys. Rev. B, 66 (2002) 174201; Phys. Rev. E, 69 (2004) 026112.
[27] Balents L. and Ledoussal P., Ann. Phys. (N.Y.), 315 (2005) 213.
[28] Le Doussal P. and Wiese K. J., Phys. Rev. E, 79 (2009) 051106.
[29] Le Doussal P., Middleton A. A. and Wiese K. J., Phys. Rev. E, 79 (2009) 050101.
[30] Le Doussal P., Ann. Phys. (N.Y.), 325 (2010) 49.
[31] Fisher D. S., Phys. Rev. B, 31 (1985) 7233.
[32] Feldman D. E., Int. J. Mod. Phys. B, 15 (2001) 2945.
[33] Ledoussal P. and Wiese K. J., Phys. Rev. Lett., 96 (2006) 197202.
[34] Tissier M. and Tarjus G., Phys. Rev. B, 74 (2006) 214419.
[35] Baczyk M., Tissier M., Tarjus G. and Balog I., unpublished (2013).
[36] Sakamoto Y., Mukaida H. and Itoi C., Phys. Rev. B, 74 (2006) 064402.
[37] Schwartz M. and Soffer A., Phys. Rev. Lett., 55 (1985) 2499.
[38] Bray A. J., J. Phys. C, 19 (1986) 6225.
[39] Fedorenko A. A. and Kuehnel F., Phys. Rev. B, 75 (2007) 174206.
[40] Baczyk M., Tissier M., Tarjus G. and Sakamoto Y., Phys. Rev. B, 88 (2013) 014204.