What is the optimal schedule for the UEFA Champions League groups?

László Csató*  Roland Molontay†  József Pintér‡

9th May 2022

“Teams and leagues want to optimize their investments by playing a good schedule which seeks to meet various criteria. Good fixtures are important in order to maximize revenues, ensure the attractiveness of the games, and to keep the interest of both the media and the fans.”

Abstract

In a sports competition, a team might lose a powerful incentive to exert full effort if its final rank does not depend on the outcome of the matches still to be played. Therefore, the organiser should reduce the probability of such a situation to the extent possible. Our paper provides a classification scheme to identify these weakly (where one team is indifferent) or strongly (where both teams are indifferent) stakeless games. A statistical model is estimated to simulate the UEFA Champions League groups and compare the candidate schedules used in the 2021/22 season according to the competitiveness of the matches played in the last round(s). The option followed in four of the eight groups is found to be optimal under a wide set of parameters. Minimising the number of strongly stakeless matches is verified to be a likely goal in the computer draw of the fixture that remains hidden from the public.

Keywords: OR in sports; football; simulation; sports scheduling; tournament design

MSC class: 62F07, 90-10, 90B35, 90B90

JEL classification number: C44, C63, Z20

* Corresponding author. Email: laszlo.csato@sztaki.hu
Institute for Computer Science and Control (SZTAKI), Eötvös Loránd Research Network (ELKH), Laboratory on Engineering and Management Intelligence, Research Group of Operations Research and Decision Systems, Budapest, Hungary
Corvinus University of Budapest (BCE), Department of Operations Research and Actuarial Sciences, Budapest, Hungary

† Email: molontay@math.bme.hu
Budapest University of Technology and Economics, Department of Stochastics, Institute of Mathematics, Műegyetem rkp. 3., H-1111 Budapest, Hungary
MTA-BME Stochastics Research Group, Műegyetem rkp. 3., H-1111 Budapest, Hungary

‡ Email: pinterj@edu.bme.hu
Budapest University of Technology and Economics, Department of Management and Business Economics, Faculty of Economics and Social Sciences, Műegyetem rkp. 3., H-1111 Budapest, Hungary

1 Source: Kendall et al. (2010, p. 1).
1 Introduction

The UEFA Champions League is probably the most prestigious annual association football (henceforth football) club competition around the world. Since the 2003/04 season, the tournament contains a single group stage with 32 teams, divided into eight groups of four. This phase is played in a double round-robin format, that is, each team meets the other three teams in its group once home and once away. The top two clubs from each group progress to the Round of 16. The third-placed clubs go to the UEFA Europa League, the second-tier competition of European club football, while the fourth-placed clubs are eliminated.

One of the most important responsibilities of sports governing bodies is to set the right incentives for the contestants (Szymanski, 2003). At first sight, they are almost guaranteed in the group stage of the Champions League since—according to the rules described above—every team benefits from being ranked higher in its group. However, the situation is not so simple as the following illustrations reveal.

Table 1: Ranking in Group B of the 2021/22 UEFA Champions League after Matchday 4

| Pos | Team                   | W | D | L | GF | GA | GD | Pts |
|-----|------------------------|---|---|---|----|----|----|-----|
| 1   | Liverpool FC           | 4 | 0 | 0 | 13 | 5  | +8 | 12  |
| 2   | FC Porto               | 1 | 2 | 1 | 3  | 6  | -3 | 5   |
| 3   | Club Atlético de Madrid| 1 | 1 | 2 | 4  | 6  | -2 | 4   |
| 4   | AC Milan               | 0 | 1 | 4 | 4  | 7  | -3 | 1   |

Pos = Position; W = Won; D = Drawn; L = Lost; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played four matches.

Example 1. Table 1 presents the standing of Group B in the 2021/22 Champions League with two rounds still to be played. Liverpool leads by seven points over Porto, therefore, it will certainly win the group. Thus, Liverpool is probable to play with little enthusiasm against its opponents on the last two matchdays, Porto (home) and Milan (away).

Table 2: Ranking in Group C of the 2021/22 UEFA Champions League after Matchday 5

| Pos | Team                        | W | D | L | GF | GA | GD | Pts |
|-----|------------------------------|---|---|---|----|----|----|-----|
| 1   | AFC Ajax                     | 5 | 0 | 0 | 16 | 3  | +13| 15  |
| 2   | Sporting Clube de Portugal   | 3 | 0 | 2 | 12 | 8  | +4 | 9   |
| 3   | Borussia Dortmund            | 2 | 0 | 3 | 5  | 11 | -6 | 6   |
| 4   | Beşiktaş JK                  | 0 | 0 | 5 | 3  | 14 | -11| 0   |

Pos = Position; W = Won; D = Drawn; L = Lost; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played five matches.

Example 2. Table 2 presents the standing of Group C in the 2021/22 Champions League with one round still to be played. If two or more teams are equal on points on completion of the group matches, their ranking is determined by higher number of points obtained in the matches played among the teams in question, followed by superior goal difference from the group matches played among the teams in question (UEFA, 2021, Article 17.01). Since the result of Borussia Dortmund vs. Sporting CP (Sporting CP vs. Borussia Dortmund)
has been 1-0 (3-1), and Sporting CP has an advantage of three points over Borussia Dortmund, Sporting CP is guaranteed to be the runner-up. Furthermore, Ajax is the group winner and Besiktas is the fourth-placed team. Consequently, the outcomes of the games played in the last round do not influence the group ranking at all.

According to Examples 1 and 2, a club might lose a powerful incentive to exert full effort in some matches towards the end of the competition, especially if it focuses mainly on qualification. Therefore, team $i$ may field weaker players and take into account other factors such as resting before the next match in its domestic championship. In addition, such a game offers an opportunity for *quid pro quo* behaviour as demonstrated in sumo wrestling (Duggan and Levitt, 2002) and football (Elaad et al., 2018). Since this would be unfair for the teams that played against team $i$ when it had stronger incentives to win, the organiser—the Union of European Football Associations (UEFA)—should avoid these games to the extent possible.

This study aims to find the schedule for the group stage of the UEFA Champions League that is optimal for competitiveness by minimising the probability of stakeless games where one or both clubs cannot achieve a higher rank. Our main contributions can be summarised as follows:

- A reasonable statistical method is chosen to simulate group matches in the Champions League (Section 3.2);
- Games are classified into three categories based on their level of competitiveness (Section 3.3);
- In the lack of further information, five candidate schedules are identified by “reverse engineering” in the 2021/22 Champions League (Section 3.4);
- The alternative schedules are compared with respect to the probability of each match type (Section 4).

The remainder of the paper is organised as follows. Section 2 gives a concise overview of literature. The theoretical background is detailed in Section 3. Section 4 provides the results of the simulations, while Section 5 contains conclusions and reflections.

## 2 Related literature

The Operational Research (OR) community devotes increasing attention to optimising the design of sports tournaments (Csató, 2021; Kendall and Lenten, 2017; Lenten and Kendall, 2021; Wright, 2014). One of the challenges is choosing a schedule that is fair for all contestants both before and after the matches are played (Goossens et al., 2020). The traditional issues of fairness in scheduling are the number of breaks (two consecutive home or away games), the carry-over effect (which is related to the previous game of the opponent), and the number of rest days between consecutive games. They are discussed in several survey articles (Goossens and Spieksma, 2012; Kendall et al., 2010; Rasmussen and Trick, 2008; Ribeiro, 2012).

The referred studies usually consider the teams as nodes in graphs. However, they are strategic actors and should allocate their limited effort throughout the contest, or even across several contests. Researchers have recently begun to take similar considerations into account. Krumer et al. (2017) investigate round-robin tournaments with a single prize
and either three or four symmetric players. In the subgame perfect equilibrium of the contest with three players, the probability of winning is maximised for the player who competes in the first and the last rounds. This result holds independently of whether the asymmetry is weak or strong, but the probability of winning is the highest for the player who competes in the second and the third rounds if there are two prizes (Krumer et al., 2020). In the subgame perfect equilibrium of the contest with four players, the probability of winning is maximised for the player who competes in the first game of both rounds. These theoretical findings are reinforced by an empirical analysis, which includes the FIFA World Cups and the UEFA European Championships, as well as two Olympic wrestling events (Krumer and Lechner, 2017).

Some papers have attempted to determine the best schedule for the FIFA World Cup, a global sporting event that attracts one of the highest audiences. Stronka (2020) focuses on the temptation to lose, resulting from the desire to play against a weaker opponent in the first round of the knockout stage. This danger is found to be the lowest if the strongest and the weakest competitors meet in the last (third) round. Inspired by the format of the 2026 FIFA World Cup, Guyon (2020) quantifies the risk of collusion in groups of three teams, where the two teams playing the last game know exactly what results let them advance. The author identifies the match sequence that minimises the risk of collusion. Chater et al. (2021) develop a general method to evaluate the probability of any situation in which the two opposing teams do not play competitively, and apply it to the current format of the FIFA World Cup (a single round-robin contest of four teams). The scheduling of matches, in particular, the choice of teams playing each other in the last round, turns out to be crucial for obtaining exciting and fair games.

Analogously, the design of the UEFA Champions League has been the subject of several academic studies. Scarf et al. (2009) compare alternative formats with 32 teams via simulations. Klößner and Becker (2013) assess the financial consequences of the distorted mechanism used for the Round of 16 draw such that the strengths of the teams are measured by the UEFA club coefficients. Dagaev and Rudyak (2019) estimate the competitiveness changes caused by the seeding reform in the Champions League from the 2015/16 season. Corona et al. (2019) follow a Bayesian approach to uncover how the new seeding regime has increased the uncertainty over progression to the knockout stage. Csató (2022b) analyses the impact of changing the Champions League qualification system from the 2018/19 season.

Regarding the identification of unwanted games, Faella and Sauro (2021) introduce the concept of irrelevant match—which does not influence the ultimate ranking of the teams involved—and prove that a contest always contains an irrelevant match if the schedule is static and there are at least five contestants. This notion is somewhat akin to our classes of matches discussed in Section 1. A more serious form of a match-fixing opportunity is incentive incompatibility when a team can be strictly better off by losing. The first example of such a rule has been provided in Dagaev and Sonin (2018). The same misallocation of vacant slots has been present in the UEFA Champions League qualification between the seasons of 2015/16 and 2017/18 (Csató, 2019) and ruins the seeding policy of the Champions League group stage since the 2015/16 season (Csató, 2020). Even though all these instances can be eliminated by minor modifications, the lack of strategy-proofness in the recent qualifications for the UEFA European Championship (Haugen and Krumer, 2021) and FIFA World Cup can be mitigated only by additional draw constraints (Csató, 2022a). The current paper joins this line of research by emphasising that a good schedule is also able to reduce the threat of tacit collusion.
3 Methodology

For any sports competition, historical data represent only a single realisation of several random factors. Therefore, the analysis of tournament designs usually starts by finding a simulation technique that can generate the required number of reasonable results (Scarf et al., 2009).

To that end, it is necessary to connect the teams playing in the tournament studied to the teams whose performance is already known. It is achieved by rating the teams, namely, by assigning a value to each team to measure its strength (Van Eetvelde and Ley, 2019). This approach allows the identification of the teams by their ratings instead of their names.

UEFA widely uses such a measure, the UEFA club coefficient, to determine seeding in its club competitions. First, we provide some information on this statistics and the draw of the UEFA Champions League group stage in Section 3.1. After that, Section 3.2 introduces and evaluates some simulation models, and Section 3.3 describes how the games suffering from potential incentive problems can be selected. Finally, Section 3.4 outlines some valid schedules of the Champions League groups.

3.1 Seeding in the UEFA Champions League

The UEFA club coefficient depends on the results achieved in the previous five seasons of the UEFA Champions League, the UEFA Europa League, and the UEFA Europa Conference League, including their qualifying (UEFA, 2018). In order to support emerging clubs, the coefficient equals the association coefficient over the same period if it is higher than the sum of all points won in the previous five years.

For the draw of the Champions League group stage, a seeding procedure is followed to ensure homogeneity across groups. The 32 clubs are divided into four pots and one team is assigned from each pot to a group, subject to some restrictions: two teams from the same national association cannot play against each other, certain clashes are prohibited due to political reasons, and some clubs from the same country play on separate days where possible (UEFA, 2021).

Seeding is based primarily on the UEFA club coefficients prior to the tournament. Before the 2015/16 season, Pot 1 consisted of the eight strongest teams according to the coefficients, Pot 2 contained the next eight, and so on. The only exception was the titleholder, guaranteed to be in Pot 1. In the three seasons between 2015/16 and 2017/18, the reigning champion and the champions of the top seven associations were in Pot 1. Since the 2018/19 season, Pot 1 contains the titleholders of both the Champions League and the Europa League, together with the champions of the top six associations. The other three pots are composed in accordance with the club coefficient ranking. The seeding rules are discussed in Csató (2020) and Csató (2021, Chapter 2.3). Engist et al. (2021) estimate the effect of seeding on tournament outcomes in European club football.

Figure 1 plots the club coefficients for the 32 teams that participated in the Champions League group stage in four different seasons. As it has already been mentioned, Pot 1 does not contain the teams with the highest ratings in the 2015/16 and 2021/22 seasons.

3.2 The simulation of match outcomes

In football, the number of goals scored is usually described by Poisson distribution (Maher, 1982; Van Eetvelde and Ley, 2019). Dagaev and Rudyak (2019) propose such a model to
evaluate the effects of the seeding system reform in the Champions League, introduced in 2015. Consider a single match between two clubs, and denote by $\lambda_{H,A}$ and $\lambda_{A,H}$ the expected number of goals scored by the home team $H$ and the away team $A$, respectively. The probability of team $H$ scoring $n_{H,A}$ goals against team $A$ is given by

$$P_H(n_{H,A}) = \frac{\lambda_{H,A}^{n_{H,A}} \exp(-\lambda_{H,A})}{n_{H,A}!},$$

whereas the probability of team $A$ scoring $n_{A,H}$ goals against team $H$ is

$$P_H(n_{A,H}) = \frac{\lambda_{A,H}^{n_{A,H}} \exp(-\lambda_{A,H})}{n_{A,H}!}.$$

In order to determine the outcome of the match, parameters $\lambda_{H,A}$ and $\lambda_{A,H}$ need to be estimated. Dagaev and Rudyak (2019) use the following specification:

$$\log(\lambda_{H,A}) = \alpha_H + \beta_H \cdot (R_H - \gamma_H R_A),$$

$$\log(\lambda_{A,H}) = \alpha_A + \beta_A \cdot (R_A - \gamma_A R_H),$$

with $R_H$ and $R_A$ being the UEFA club coefficients of the corresponding teams, and $\alpha_i, \beta_i, \gamma_i$ ($i \in \{H, A\}$) being parameters to be optimised on a historical sample. A simpler version containing four parameters can be derived by setting $\gamma_H = \gamma_A = 1$.

We have studied two options for quantifying the strength of a club: the UEFA club coefficient and seeding pot from which the team is drawn in the Champions League group stage. The latter can take only four different values. Furthermore, each group is guaranteed to consist of one team from each pot, consequently, the dataset contains the same number
Table 3: Model parameters estimated by the maximum likelihood method on the basis of Champions League seasons between 2003/04 and 2019/20

| Model       | $\alpha_H$ | $\alpha_A$ | $\beta_H$ | $\beta_A$ | $\gamma_H$ | $\gamma_A$ | $c$  |
|-------------|-------------|-------------|------------|------------|-------------|-------------|------|
| 6p coeff    | 0.335       | 0.087       | 0.006      | 0.006      | 0.833       | 0.963       | —    |
| 4p coeff    | 0.409       | 0.102       | 0.006      | 0.006      | —           | —           | —    |
| 6p pot      | 0.464       | 0.143       | -0.177     | -0.182     | 0.91        | 0.922       | —    |
| 4p pot      | 0.424       | 0.108       | -0.169     | -0.175     | —           | —           | —    |
| Bivariate   | 0.335       | 0.087       | 0.006      | 0.006      | 0.833       | 0.963       | $\exp(-12.458)$ |

of teams for each possible value, as well as the same number of matches for any pair of ratings.

Assuming the scores to be independent may be too restrictive because the two opposing teams compete against each other. Thus, if one team scores, then the other will exert more effort into scoring (Karlis and Ntzoufras, 2003). This correlation between the number of goals scored can be accounted for by bivariate Poisson distribution, which introduces an additional covariance parameter $c$ that reflects the connection between the scores of teams $H$ and $A$ (Van Eetvelde and Ley, 2019).

To sum up, five model variants are considered:

- 6-parameter Poisson model based on UEFA club coefficients (6p coeff);
- 4-parameter Poisson model based on UEFA club coefficients (4p coeff);
- 6-parameter Poisson model based on pot allocation (6p pot);
- 4-parameter Poisson model based on pot allocation (4p pot);
- 7-parameter bivariate Poisson model based on UEFA club coefficients (Bivariate).

All parameters have been estimated by the maximum likelihood approach on the set of $8 \times 12 \times 17 = 1632$ matches played in the 17 seasons from 2003/04 to 2019/20. They are presented in Table 3. The optimal value of $c$, the correlation parameter of the bivariate model is positive but almost zero, hence, the bivariate Poisson model does not improve accuracy. This is in accordance with the finding of Chater et al. (2021) for the group stage of the FIFA World Cup. The reason is that the bivariate Poisson model is not able to grab a negative correlation between its components, however, the goals scored by home and away teams are slightly negatively correlated in our dataset.

The performance of the models has been evaluated on two disjoint test sets, the seasons of 2020/21 and 2021/22. They are treated separately because most games in the 2020/21 edition were played behind closed doors owing to the COVID-19 pandemic, which might significantly affect home advantage (Benz and Lopez, 2021; Bryson et al., 2021; Fischer and Haucap, 2021).

Two metrics have been calculated to compare the statistical models. Average hit probability measures how accurately a model can determine the exact score of a match: we compute the probability of the actual outcome, sum up these probabilities across all matches in the investigated dataset, and normalise this value by the number of seasons. A simple baseline model serves as a benchmark, where the chances are determined by relative frequencies in the seasons from 2003/04 to 2018/19.
The results are provided in Table 4. The baseline model shows the worst performance, which is a basic criterion for the validity of the proposed methods. The bivariate Poisson variant does not outperform the 6-parameter Poisson based on UEFA club coefficients. Even though the club coefficient provides a finer measure of strength than the pot allocation, it does not result in a substantial improvement with respect to average hit probability.

The average hit probability does not count whether the prediction fails by a small margin (the forecast is 2-2 and the actual result is 1-1) or it is completely wrong (the forecast is 4-0 and the actual result is 1-3). However, there exists no straightforward “distance” among the possible outcomes. If the differences in the predicted and actual goals scored by the home and away teams are simply added, then the result of 2-2 will be farther from 1-1 than 2-1. But 1-1 and 2-2 are more similar than 1-1 and 2-1 from a sporting perspective since both 1-1 and 2-2 represent a draw. To resolve this issue, we have devised a distance metric for the outcome of the matches generated by the scalar product with a specific matrix, which has been inspired by the concept of Mahalanobis distance (De Maesschalck et al., 2000).

Let the final score of the game be \( R_1 = (h_1, a_1) \), where \( h_1 \) is the number of goals for the home team, and \( a_1 \) is the number of goals for the away team. Analogously, denote by \( R_2 = (h_2, a_2) \) the predicted result of this game. The distance between the two outcomes

| Model          | 2003/04-2019/20 | Season(s)     |
|----------------|-----------------|---------------|
|                | 2020/21         | 2021/22       |
| 6p coeff       | 7.016 (1)       | 6.682 (2)     | 6.148 (3)     |
| 6p pot         | 6.869 (4)       | 6.443 (5)     | 6.297 (1)     |
| 4p coeff       | 7.012 (3)       | 6.683 (1)     | 6.146 (5)     |
| 4p pot         | 6.868 (5)       | 6.456 (4)     | 6.297 (1)     |
| Bivariate      | 7.016 (1)       | 6.682 (2)     | 6.148 (3)     |
| Baseline       | 6.123 (6)       | 5.482 (6)     | 5.485 (6)     |

Table 4: Average hit probability for the statistical models (%)
Table 6: Average distance of match scores for the statistical models

| Model   | 2003/04-2019/20 | 2020/21   | 2021/22   |
|---------|----------------|-----------|-----------|
| 6p coeff| 2.041 (3)      | 2.199 (3) | 2.163 (3) |
| 6p pot  | 1.958 (2)      | 2.068 (2) | 2.013 (2) |
| 4p coeff| 2.054 (5)      | 2.218 (5) | 2.175 (5) |
| 4p pot  | 1.957 (1)      | 2.066 (1) | 2.012 (1) |
| Bivariate| 2.041 (3)    | 2.199 (3) | 2.163 (3) |
| Baseline| 2.095 (6)      | 2.247 (6) | 2.21 (6)  |

Baseline model: The probability of any match outcome is determined by the relative frequency of this result in the training set (all seasons between 2003/04 and 2019/20).

The ranks of the models are indicated in bracket.

equals

$$\Delta(R_1, R_2) = \sqrt{[h_1 - h_2 \ a_1 - a_2 \ [1 \ -9/10 \ -9/10 \ 1 \ [h_1 - h_2 \ a_1 - a_2 \ \text{]} \ \text{]} \ \text{]} \ \text{]}}^2$$

For instance, with the final score of 2-0 and the forecast of 1-2, $h_1 - h_2 = 1$ and $a_1 - a_2 = -2$, which leads to

$$\Delta(R_1, R_2) = \sqrt{[1 \ -2 \ [1 + 2 \times 0.9 \ -0.9 \ -2 \times 1 \ \text{]} \ \text{]} \ \text{]}} = \sqrt{2.8 - 2 \times (-2.9)} = \sqrt{8.6} \approx 2.933.$$
3.3 Classification of games

As has been presented in Section 1, a team might be indifferent with respect to the outcome of the match(es) played on the last matchday(s) since its position in the final ranking is already secured. In the group stage of the UEFA Champions League, there are six matchdays. The position of a team in the group ranking can be known first after Matchday 4. In particular, a club is guaranteed to win its group if it has at least seven points more than the runner-up or if

- it leads by six points over the runner-up; and
- it leads by at least seven points over the third-placed team; and
- it has played two matches against the runner-up.

The second- and third-placed clubs cannot be fixed after Matchday 4. A club will certainly be fourth in the final ranking if it has at least seven points less than the third-placed team or if

- it has six points less than the third-placed team; and
- it has at least seven points less than the runner-up; and
- it has played two matches against the third-placed team.

Note that these definitions are more complicated than the ones appearing in the previous works (Chater et al., 2021; Guyon, 2020) because of two reasons: (a) there are more matches due to organising the groups in a double round-robin format; and (b) tie-breaking is based on head-to-head results instead of goal difference.
Table 7: Number of matches with a given outcome in the sample

(a) Seasons between 2003/04 and 2019/20

| Final score | 0   | 1   | 2   | 3   | 4   |
|-------------|-----|-----|-----|-----|-----|
| 0           | 115 | 109 | 83  | 48  | 20  |
| 1           | 157 | 175 | 96  | 38  | 19  |
| 2           | 138 | 139 | 76  | 25  | 5   |
| 3           | 84  | 72  | 36  | 14  | 2   |
| 4           | 44  | 22  | 18  | 5   | 2   |

(b) Season 2020/21

| Final score | 0   | 1   | 2   | 3   | 4   |
|-------------|-----|-----|-----|-----|-----|
| 0           | 5   | 3   | 8   | 4   | 4   |
| 1           | 6   | 9   | 7   | 3   | 1   |
| 2           | 7   | 5   | 6   | 2   | 0   |
| 3           | 7   | 5   | 4   | 0   | 1   |
| 4           | 2   | 1   | 0   | 0   | 0   |

(c) Season 2021/22

| Final score | 0   | 1   | 2   | 3   | 4   |
|-------------|-----|-----|-----|-----|-----|
| 0           | 6   | 5   | 1   | 3   | 1   |
| 1           | 9   | 7   | 8   | 4   | 2   |
| 2           | 10  | 7   | 3   | 2   | 0   |
| 3           | 2   | 3   | 3   | 2   | 0   |
| 4           | 5   | 2   | 2   | 0   | 0   |

Goals scored by the home team are in the rows, goals scored by the away team are in the columns. Games where one team scored at least five goals are not presented.

It would be difficult to determine all possible cases by similar criteria after Matchday 5. Hence, we consider only the extreme cases as follows. The results of the two games played on Matchday 6 are assumed to be: (a) M-0, M-0; (b) M-0, 0-M; (c) 0-M, M-0; and (d) 0-M, 0-M, where M is a high number. The position of a team is known if it is the same in all scenarios (a) to (d).

Depending on whether the final position of a team is already secured or not, three categories of matches can be distinguished:

- **Competitive game**: Neither team is indifferent because they can achieve a higher rank through a better performance on the field with a positive probability.

- **Weakly stakeless game**: One of the teams is completely indifferent as its position in the final group ranking is independent of the outcomes of the matches still to be played. However, it has a positive probability that the other team can obtain a higher rank through a better performance on the field.

- **Strongly stakeless game**: Both teams are completely indifferent since their positions
and one away match on the first and last two matchdays.

more than two home or two away matches in a row and each club plays one home match

Matchday 2: 1 v 2, 3 v 4;
Matchday 1: 2 v 3, 4 v 1;

The regulation of the UEFA Champions League provides surprisingly little information on

A club does not play

of group matches used in the 2021/22 Champions League are regarded as valid solutions

in the final group ranking are not influenced by the results of the remaining matches.

In the situation outlined in Example 1, Liverpool is indifferent in its last two matches, thus at least two weakly stakeless games will be played in the group. In Example 2, all teams are indifferent before Matchday 6, hence, there will be two strongly stakeless games.

Our classification differs from the definitions given in the existing literature. For example, Chater et al. (2021) call a match stakeless if at least one team becomes indifferent between winning, drawing, or even losing by 5 goals difference with respect to qualification. This notion does not consider the incentives of the opponent, which is an important factor for the competitiveness of the game.

3.4 Candidate schedules

The regulation of the UEFA Champions League provides surprisingly little information on how the group matches are scheduled (UEFA, 2021, Article 16.02): “A club does not play more than two home or two away matches in a row and each club plays one home match and one away match on the first and last two matchdays.”3 Therefore, the eight schedules of group matches used in the 2021/22 Champions League are regarded as valid solutions

(a) Seasons between 2003/04 and 2019/20

| Final score | 0  | 1  | 2  | 3  | 4  |
|-------------|----|----|----|----|----|
| 0           | 103.0 ± 9.8 | 124.6 ± 10.7 | 81.2 ± 8.7 | 38.1 ± 6.0 | 14.0 ± 3.7 |
| 1           | 159.4 ± 12.0 | 175.8 ± 12.5 | 106.4 ± 9.9 | 46.8 ± 6.7 | 16.6 ± 4.0 |
| 2           | 133.8 ± 11.0 | 135.2 ± 11.1 | 74.7 ± 8.4 | 30.3 ± 5.4 | 9.9 ± 3.1  |
| 3           | 80.5 ± 8.6  | 75.6 ± 8.5  | 38.5 ± 6.1  | 14.3 ± 3.8 | 4.1 ± 2.0  |
| 4           | 38.9 ± 6.1  | 34.0 ± 5.7  | 16.0 ± 4.0  | 5.4 ± 2.3  | 1.6 ± 1.3  |

(b) Seasons 2020/21 and 2021/22

| Final score | 0  | 1  | 2  | 3  | 4  |
|-------------|----|----|----|----|----|
| 0           | 6.1 ± 2.4 | 7.3 ± 2.6 | 4.8 ± 2.1 | 2.2 ± 1.5 | 0.8 ± 0.9 |
| 1           | 9.4 ± 2.9 | 10.3 ± 3.0 | 6.3 ± 2.4 | 2.8 ± 1.6 | 1.0 ± 1.0 |
| 2           | 7.9 ± 2.7 | 8.0 ± 2.7 | 4.4 ± 2.0 | 1.8 ± 1.3 | 0.6 ± 0.8 |
| 3           | 4.7 ± 2.1 | 4.4 ± 2.1 | 2.3 ± 1.5 | 0.8 ± 0.9 | 0.2 ± 0.5 |
| 4           | 2.3 ± 1.5 | 2.0 ± 1.4 | 0.9 ± 1.0 | 0.3 ± 0.6 | 0.1 ± 0.3 |

The numbers indicate the average number of occurrences based on simulations ± standard deviations.

Goals scored by the home team are in the rows, goals scored by the away team are in the columns.

Games where one team scored at least five goals are not presented.

In the situation outlined in Example 1, Liverpool is indifferent in its last two matches, thus at least two weakly stakeless games will be played in the group. In Example 2, all teams are indifferent before Matchday 6, hence, there will be two strongly stakeless games.

Our classification differs from the definitions given in the existing literature. For example, Chater et al. (2021) call a match stakeless if at least one team becomes indifferent between winning, drawing, or even losing by 5 goals difference with respect to qualification. This notion does not consider the incentives of the opponent, which is an important factor for the competitiveness of the game.

3 At first sight, one might conclude that UEFA has fixed the schedule of group matches until the 2020/21 season. For instance, the regulation of the competition for 2020/21 (UEFA, 2020, Article 16.02) says that:

“The following match sequence applies:

Matchday 1: 2 v 3, 4 v 1;
Matchday 2: 1 v 2, 3 v 4;”
Table 9: Group schedules in the 2021/22 UEFA Champions League

| Gr. A | Gr. B | Gr. C | Gr. D | Gr. E | Gr. F | Gr. G | Gr. H |
|-------|-------|-------|-------|-------|-------|-------|-------|
| H A   | H A   | H A   | H A   | H A   | H A   | H A   | H A   |
| Matchday 1  | 1 3 1 3 1 3 1 3 1 2 2 1 1 3 1 4 1 3 | 4 2 2 4 2 4 3 4 4 2 2 3 4 2 |
| Matchday 2  | 2 1 4 1 2 1 3 1 1 4 2 1 3 1 2 1 | 3 4 3 2 3 4 2 4 3 2 3 4 4 2 3 4 |
| Matchday 3  | 4 1 1 2 4 1 1 4 3 1 4 1 2 1 4 1 | 2 3 3 4 3 2 3 2 2 4 2 3 4 3 2 2 |
| Matchday 4  | 1 4 2 1 1 4 4 1 1 3 1 4 2 1 4 1 | 3 2 4 3 2 3 2 3 4 2 3 2 4 3 2 3 |
| Matchday 5  | 1 2 1 4 1 2 1 3 4 1 1 2 1 3 1 2 | 4 3 2 3 4 3 4 2 2 3 4 3 2 4 3 |
| Matchday 6  | 3 1 3 1 3 1 2 1 1 2 3 1 4 1 3 1 | 2 4 4 2 2 4 3 4 3 4 2 4 3 2 2 4 |

The numbers indicate the pots from which the teams are drawn.

and options available for the tournament organiser. Since each group consists of one team from each of the four pots, the clubs are identified by their pot in the following, that is, team \( i \) represents the team drawn from Pot \( i \).

Table 9 outlines these alternatives.\(^4\) From our perspective, five different patterns exist as the prediction of match outcome does not depend on the schedule, and game classification starts after Matchday 4:

- The schedules of Groups A and C differ only in one game played on Matchdays 3 and (consequently) 4;
- The schedules of Groups A and F coincide;
- The schedules of Groups A and H differ in the two games played on Matchdays 3 and (consequently) 4.

On the other hand, the schedules of Groups A, B, D, E, and G are worth assessing for the frequency of weakly and strongly stakeless games.

The five scheduling options are summarised in Table 10. Note that only the last two matchdays count from our perspective.

\(^4\) In the Champions League seasons from 2003/04 to 2020/21, Matchday 4/5/6 was the mirror image of Matchday 3/1/2, respectively. Consequently, the same two teams played at home on the first and last matchday in the previous seasons. This arrangement has been changed in the 2021/22 season such that Matchday 4/5/6 is the mirror image of Matchday 3/2/1, see Table 9.
Table 10: Candidate schedules from the 2021/22 UEFA Champions League

|                | Matchday 5 |          | Matchday 6 |          |
|----------------|------------|----------|------------|----------|
|                | Home       | Away     | Home       | Away     |
| Schedule A     | Pot 1      | Pot 2    | Pot 3      | Pot 1    |
|                | Pot 4      | Pot 3    | Pot 2      | Pot 4    |
| Schedule B     | Pot 1      | Pot 4    | Pot 3      | Pot 1    |
|                | Pot 2      | Pot 3    | Pot 4      | Pot 2    |
| Schedule D     | Pot 1      | Pot 3    | Pot 2      | Pot 1    |
|                | Pot 4      | Pot 2    | Pot 3      | Pot 4    |
| Schedule E     | Pot 4      | Pot 1    | Pot 1      | Pot 2    |
|                | Pot 2      | Pot 3    | Pot 3      | Pot 4    |
| Schedule G     | Pot 1      | Pot 3    | Pot 4      | Pot 1    |
|                | Pot 2      | Pot 4    | Pot 3      | Pot 2    |

Figure 3: The probability of a weakly stakeless game on Matchday 5

4 Results

We focus on the probability of a stakeless game as the function of the group schedule. For sample size $N$, the error of a simulated probability $P$ is $\sqrt{P(1-P)/N}$. Since even the smallest $P$ exceeds 2.5% and 1 million simulation runs are implemented, the error always remains below 0.016%. Therefore, confidence intervals will not be provided because the averages differ reliably between the candidate schedules.

Figure 3 plots the likelihood of a weakly stakeless game at the first point where it might occur, on Matchday 5. The probability varies between 2.5% and 4%, it is the lowest for schedules A and D, while schedules B and G are poor choices to avoid these matches.

The probability of a weakly stakeless game in the last round is given in Figure 4. The solutions differ to a high degree, the worst schedules (B and G) increase the danger of a weakly stakeless match by 35% (more than 10 percentage points). The most widely used option, schedule A—followed in four groups of the 2021/22 Champions League—becomes
Figure 4: The probability of a weakly stakeless game on Matchday 6

Figure 5: The probability of a strongly stakeless game on Matchday 6

unfavourable from this point of view.

However, according to Figure 5, schedule A is the best alternative to minimise the chance of strongly stakeless games that are totally unimportant with respect to the group ranking. Now the scheduling options vary less in absolute terms, the probability of such a situation remains between 8% and 10.5%.

Consequently, there are three objectives to be optimised, depicted in Figures 3–5. While schedule A dominates both schedules B and G, the remaining three alternatives can be optimal depending on the preferences of the decision-maker. In order to evaluate them, it is worth considering a weighting scheme. The cost of a weakly stakeless game played on Matchday 6 can be fixed at 1 without losing generality. It is reasonable to
assume that the cost of a weakly stakeless game played on Matchday 5 is not lower than 1. Analogously, a strongly stakeless game is certainly more threatening than a weakly stakeless game, perhaps even by an order of magnitude.

A weakly stakeless game has the same cost on both Matchdays 5 and 6

The cost ratio of a strongly and a weakly stakeless game played on Matchday 6

A weakly stakeless game has the double cost on Matchday 5 compared to Matchday 6

The cost ratio of a strongly and a weakly stakeless game played on Matchday 6

Figure 6: The weighted cost of stakeless games
Figure 6 calculates the price of candidate schedules as the function of the cost ratio between a strongly and weakly stakeless game played in the last round. The price of a weakly stakeless game on Matchday 5 is 1 in the first chart and 2 in the second chart. Schedule E is the best alternative if the relative cost of a strongly stakeless game is moderated, namely, at most 5, but schedule A should be chosen if this ratio is higher. If the goal is to avoid strongly stakeless games, schedules D and E are unfavourable options.

The key findings can be summed up as follows:

- Schedule A dominates schedules B and G with respect to stakeless games;
- Schedules B and G have approximately the same cost, which can be explained by their similarity on the last two matchdays, where only the positions of the teams drawn from Pots 3 and 4 are interchanged (see Table 10);
- Schedule E is preferred to schedule D except for the case when an unlikely high weight is assigned to weakly stakeless games played on Matchday 5;
- Schedule E (and, to some extent, D) should be followed if strongly stakeless games are not judged to be substantially more harmful than weakly stakeless games;
- Schedule A can decrease the probability of a strongly stakeless game by at least 10%, hence, it needs to be implemented to reduce the number of matches that do not affect the final group ranking.

In the 2021/22 Champions League, UEFA has essentially used schedule A in four groups and schedules B, D, E, and G in one group, respectively. Therefore, it is likely that minimising the number of strongly stakeless games has been a crucial goal in the computer draw of the fixture, which remains unknown to the public.

5 Discussion

Motivated by recent examples from the most prestigious European club football competition, this paper has proposed a novel classification method for games played in a round-robin tournament. The selection criterion is whether the position of a team in the final ranking is already known, independently of the outcomes of matches still to be played. A game is called (1) competitive if neither opposing team is indifferent; (2) weakly stakeless if exactly one of the opposing teams is indifferent; or (3) strongly stakeless if both teams are indifferent. Avoiding stakeless game should be an imperative aim of the organiser because a team might play with little enthusiasm if the outcome of the match cannot affect its final position.

We have built a simulation model to compare some sequences for the group matches of the UEFA Champions League according to the probability of games where one or both clubs cannot achieve a higher rank. Five candidate schedules have been identified based on the 2021/22 season. The prevailing schedule—applied in half of the eight groups—is optimal under a wide set of costs assigned to different stakeless games. Reducing the number of strongly stakeless games has probably been a crucial goal in the computer draw of the fixture.

Our study has some limitations. The simulation model may be refined. There are other aspects of scheduling, for example, selecting the kick-off time, which might influence the performance of teams (Krumer, 2020). The suggested classification scheme does not
deal with the sequence of matches played in the first four rounds. Furthermore, stakeless games are identified in a deterministic framework but a team may exert lower effort still if its position is known with a high probability. The number of stakeless games can also be reduced through more radical changes in the tournament design: for example, according to the recent proposal of Guyon (2022), teams that have performed best during the preliminary group stage can choose their opponents during the subsequent knockout stage, which provides a strong incentive to exert full effort even if the position of the team in the final ranking is already known.

Finally, it has not been taken into account that the group fixtures cannot be determined independently of each other. For example, FC Internazionale Milano (drawn from Pot 1 to Group D) and AC Milan (drawn from Pot 4 to Group B) share the same stadium, but both of them should have played at home on Matchday 5 if their groups would have been organised according to schedule A, which is clearly impossible. Similar constraints might prevent choosing the optimal schedule in all groups, and can (partially) explain the variance of schedules used in the 2021/22 season.

UEFA is strongly encouraged to increase the transparency of how its competitions are scheduled by announcing these restrictions. Then some of them can probably be addressed by the draw procedure: Csató (2022a) has recently demonstrated the role of draw constraints in avoiding unfair situations that might include stakeless games. Despite these caveats, the current work has hopefully managed to uncover some important aspects of tournament design and can inspire further research by scheduling experts to optimise various aspects of competitiveness beyond the classical criteria of fairness.

Acknowledgements

This paper could not have been written without the father of the first author (also called László Csató), who has helped to code the simulations in Python. We are grateful to Dries Goossens, Alex Krumer, and Stephan Westphal for useful advice. We are indebted to the Wikipedia community for summarising important details of the sports competition discussed in the paper. The research was supported by the MTA Premium Postdoctoral Research Program grant PPD2019-9/2019. The research reported in this paper is part of project no. BME-NVA-02, implemented with the support provided by the Ministry of Innovation and Technology of Hungary from the National Research, Development and Innovation Fund, financed under the TKP2021 funding scheme. The work of Roland Molontay is supported by the NKFIH K123782 research grant.

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