Determination of muscle effort at the proximal femur rotation osteotomy

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Abstract. The paper formulates the problem of biomechanics of a new method for treatment of Legg–Calve–Perthes disease. Numerical calculations of the rotational flexion osteotomy have been carried out for a constructed mathematical model of the hip joint, taking into account the main set of muscles. The work presents the results of the calculations and their analysis. The results have been compared with the clinical data. The calculations of the reactive forces arising in the acetabulum and the proximal part of the femur allowed us to reveal that this reactive force changes both in value and direction. These data may be useful for assessing the stiffness of an external fixation device used in orthopedic intervention and for evaluating the compression in the joint.

1. Introduction

In surgery it is important to know biomechanical changes. Usually in orthopedic operations it is the most important to know how changes muscles. Large biomechanical changes in muscles leads to increasing the isometric force. That means that bio-system will continue changes and usually not the way surgeon wants [1]. During the rotational osteotomy, the femur is cut by a parabolic surface around the Adams arc, and the rotation of the acetabular component is performed with respect to the axis R with the help of an external fixation device [2] (see figure 1). Lengthening and shortening of the muscles lead to a redistribution of forces in the joint, and this can cause an increase in joint compression and worsen quality of health. Therefore, evaluation of the permissible angle of rotation is an important task.

2. Another section of your paper

The rotational osteotomy involves cutting of the femur by a parabolic surface around the Adams arc, and rotation of the acetabular component relative to the axis R (see figure 1) using an external fixation device. Due to the lengthening and shortening of the muscles, a redistribution of forces in the joint takes place. When studying the stress-strain state (SSS) of the hip joint (HJ) under rotational osteotomy, the list of the main muscles having a major influence was determined in the first place: mm. piriformis, rectus femoris, iliopsoas, obturator internus, gluteus minimus, medius et maximus [1, 2]. The existing parametric model of the hip joint [3–6] was used for modeling. The following anatomical characteristics of the joint and femur were taken as the main parameters in this case: the
acetabular angle, the Wiberg angle, the cervical and diaphyseal angle, the length of the proximal part of the femur, the diameter of the acetabulum and the femoral head. To formulate the problem of HJ stress-strain, the following simplifications were introduced: the material of the pelvis and femur is isotropic and obeys Hooke's law. Since the rotation for tissues is instantaneous, Hooke's law is also applicable for muscles, but the muscles in this case work only in tension. The condition of frictionless contact was set on the cutting surface and the acetabulum. The rotation was simulated by kinematic boundary conditions imposed on the free surface of the proximal part cut off.

Figure 1. Calculation scheme.

The formulation considers three bodies: the pelvis – I, the proximal part of the femur – II, and the rest part of the femur – III. Let \( V_I, V_{II}, V_{III} \) denote the volumes of the respective bodies and \( \partial V_I, \partial V_{II}, \partial V_{III} \) – the boundaries of these sets (free surface), \( V = V_I \cup V_{II} \cup V_{III} \). The mechanics of a system that occupies the area \( V \) in \( \mathbb{R}^3 \) with the boundary \( \partial V \) is described within the linear elastic theory by the following system of equations:

\[
\nabla \cdot \sigma = 0, \quad \forall \mathbf{x} \in V^\circ, \tag{1}
\]

\[
\varepsilon = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T), \quad \forall \mathbf{x} \in V^\circ, \tag{2}
\]

\[
\sigma = \tilde{E} : \tilde{\varepsilon}, \quad \forall \mathbf{x} \in V^\circ, \tag{3}
\]

where \( V^\circ = V \cup \partial V \), \( \mathbf{u} \) is the displacement vector, \( \sigma \) is the stress tensor, \( \varepsilon \) is the tensor of elastic deformations, \( \tilde{E} \) is the tensor of elastic properties. This formulation is fulfilled for all three bodies.

Let us consider the boundary conditions and introduce a set of surfaces for this purpose. Let \( S_{mm} \) denote the surfaces of Body I, to which the muscles will be attached. Their position and size were identified according to the anatomical atlas. Those muscles which are attached to the bones absent in the model will be defined as the set of points \( S_{mm} \). The femoral muscles are attached to the greater trochanter – the surface \( S_{max} \), and the small trochanter – the surface \( S_{min} \), as well as to the distal part – the surface \( S_d \). Contact interaction between the bodies is possible on two surfaces: the dissected parabolic surface under the small trochanter – \( S_{conf} \), and the surface of the pelvic acetabulum and the femoral head – \( S_{conp} \). To determine the kinematic boundary conditions, let us introduce a) the surface \( S_{pk} \) denoting the surface of the pelvis, which is attached to the sacrum, b) the surface of rotation, which
is defined as $S_{\text{rot}} = \partial V_{\text{III}} \setminus S_{\text{conf}}$ (external surface of the proximal part of the femur excluding the surface being dissected), and c) $\partial V_{\text{III}}$ denoting the external surface of the body $V_{\text{III}}$.

$$\ddot{u} = 0, \quad \forall \mathbf{x} \in \partial V_{\text{III}} \cup S_{\text{rot}},$$

where the function $f(x, y, z)$ defines the rotation $S_{\text{rot}}$ relative to the axis $R$.

The influence of the muscles was determined by the following relation:

$$\sigma_i = E_i \cdot \varepsilon_i \cdot \frac{1 + \text{sign} \varepsilon_i}{2}, \quad \forall \mathbf{x} \in L_i,$$

where $L_i$ is the line defining the position of the $i$-th muscle; the attachment of all muscles was determined in the above introduced areas; all the muscles were introduced according to the anatomical atlas (figure 1 shows the illustration of only one muscle for convenience). Here $\sigma_i$ and $\varepsilon_i$ are the normal tensions and deformations in the muscle. As seen from the law, the muscles were defined by the elastic law without regard to compression; $E_i$ is the reduced elastic modulus of the $i$-th muscle.

The contact interaction is defined by the expressions:

$$\left( \bar{u}_{\text{II}} - \bar{t}_{\text{II}}^{(0)} \right) \cdot \hat{n} = (\bar{u}_{\text{II}} - \bar{u}_{\text{II}}) \cdot \hat{n}, \quad \forall \mathbf{x} \in S_{\text{comp}}$$

$$\left( \bar{u}_{\text{conf}}^{(0)} - \bar{t}_{\text{II}} \right) \cdot \hat{n} = (\bar{u}_{\text{II}} - \bar{u}_{\text{II}}) \cdot \hat{n}, \quad \forall \mathbf{x} \in S_{\text{conf}}$$

This technique used for calculating the contact interaction is described in detail in [3-5].

To perform a numerical investigation based on the parametric model, we built a finite-element model[6-8]. A four-node tetrahedral finite element with linear approximation was used for discretization of the volumes $V_I$, $V_{\text{II}}$, $V_{\text{III}}$. The characteristics of bone tissue were taken equal to $E = 6$ GPa, $\nu = 0.3$. We used a one-dimensional finite element for muscle modeling [9-13]. Equation (9) was used to determine the reduced Young's modulus:

$$E = 0,111 \cdot \frac{F_{\text{iso}}}{F_{\text{nm}}},$$

where $F_{\text{iso}}$ is the isometric force of the muscle.

According to the published studies on determining the maximum isometric force of a muscle, we defined the muscle stiffness and presented the results in Table 1, where $\bar{F}_{\text{iso}}$ is the mean value of the maximum isometric force of the muscle from the study [14]. An additional coordinate system was introduced for rotation modeling, where the $z$ axis coincided with the direction of the rotation axis (the line $R$ in figure 1). The kinematic boundary conditions of rotation for the surface $\partial V_{\text{II}}$ were set in this coordinate system.

### Table 1. Values of isometric force of the muscle and the muscle stiffness.

| Muscle name       | $\bar{F}_{\text{iso}}$, N | E, KPa |
|-------------------|--------------------------|--------|
| Iliacus           | 615,30                    | 56,18  |
| Obturator externus| 341,80                    | 101,13 |
| Obturator internus| 425,80                    | 81,18  |
| Piriformis        | 175,88                    | 196,55 |
| Gluteus minimus   | 629,00                    | 54,96  |
| Gluteus medius    | 1225,15                   | 28,22  |
| Gluteus maximus   | 1747,18                   | 19,79  |
| Tensor fascia lata| 310,75                    | 111,24 |
| Rectus femoris    | 508,75                    | 67,95  |
We set the instantaneous rotation angle and then performed calculation of the model. It consisted of several stages. At the first stage, the calculations were carried out within a range of the rotation angle between $-50^\circ$ and $50^\circ$, where the minus sign means back rotation. As a result, we determined the elongations and efforts in the muscles.

3. Results and discussion

The paper discusses the results of the first calculation stage, which involved the determination of elongations and efforts in the muscles. For the obtained values of efforts in the muscles under rotation, we built the dependences of the force in the muscle, related to the muscle isometric force, under rotation (example of the results for two muscles is shown in figure 2).

![Figure 2. Variation of force in the muscle, related to the muscle isometric force, under rotation.](image)

The analysis of the results was then carried out for all muscles that have efforts, i.e. can lengthen during rotation. The dependences of the relative (to the isometric force) increase in the force at a given angle of rotation were constructed. This dependence was nearly linear for some muscles, and therefore, it made it possible to construct a linear approximation according to the least-square method. The values calculated are shown in Table 1, which specifies the coefficients of increase in the force relative to the initial isometric force for each muscle, depending on the direction of rotation, and shows the approximation error $R^2$ by the least-square method.

| Muscles       | $K$     | $R^2$ |
|---------------|---------|-------|
| **Back rotation** |         |       |
| iliopsoas     | 0.1728  | 0.99  |
| gluteus medius| 0.0078  | 0.95  |
| gluteus maximus| 0.0041 | 0.98  |
| **Forward rotation** |   |       |
| piriformis    | 1.1664  | 0.99  |
| rectus femoris| Nonlinear behavior |   |
| iliopsoas     | 0.0458  | 0.97  |
The obtained dependences allow us to perform a qualitative assessment of the possible effort in the muscle under rotation at a given angle:

\[ F(r) = F_{iso} (r \cdot K_m + 1) \]

where \( r \) is the rotation angle, \( F_{iso} \) is the muscle isometric force, \( K_m \) is the coefficient of the relative grow of effort for the muscle (see Table 2).

We obtained the picture of pressure distribution in the joint for different values of rotation. It is significant that at different angles and directions of rotation, the reactive force in the joint, which characterizes the compression, changes both in value and direction. We also determined the reactive force arising in the proximal part of the femur. Assessment of the reactive force is necessary for defining the stiffness of the external fixation device, by which rotation is carried out in practice [5].

4. Conclusions

The paper shows the formulation of the problem of modeling the biomechanics of an orthopedic method for treatment of Legg–Calve–Perthes disease. We carried out the numerical calculations of rotational flexion osteotomy for the constructed mathematical model of the hip joint, taking into account the main muscle groups. As a result of the calculations, we determined the efforts arising in the muscles at different rotations. The results were analyzed and used to obtain linear functions for estimating the efforts in the muscles depending on the angle of rotation and the isometric force. The calculations of the reactive forces arising in the acetabulum and the proximal part of the femur allowed us to reveal that this reactive force changes both in value and direction. These data may be useful for assessing the stiffness of an external fixation device used in orthopedic intervention and for evaluating the compression in the joint.

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