Two-dimensional higher-derivative gravity and conformal transformations

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Abstract

We consider the lagrangian \( L = F(R) \) in classical (=non-quantized) two-dimensional fourth-order gravity and give new relations to Einstein’s theory with a non-minimally coupled scalar field.

We distinguish between scale-invariant lagrangians and scale-invariant field equations. \( L \) is scale-invariant for \( F = c_1 R^{k+1} \) and a divergence for \( F = c_2 R \). The field equation is scale-invariant not only for the sum of them, but also for \( F = R \ln R \). We prove this to be the only exception and show in which sense it is the limit of \( \frac{1}{k} R^{k+1} \) as \( k \to 0 \). More generally: Let \( H \) be a divergence and \( F \) a scale-invariant lagrangian, then \( L = H \ln F \) has a scale-invariant field equation.

Further, we comment on the known generalized Birkhoff theorem and exact solutions including black holes.

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1 Introduction

In recent years there has been a great interest in two-dimensional theories of gravity [1-10], due in part to their connection with string theories [11-15]. However, two-dimensional gravity models have a great interest in themselves, since their qualitative features are similar to those of general relativity, even if the mathematical structure is much simpler. They can therefore be used to gain some insight on the four-dimensional theory.

The essential property which distinguishes the 2-dimensional theory from the higher-dimensional ones is the fact that the Einstein-Hilbert lagrangian is a total derivative in two dimensions. This problem is usually circumvented by introducing a scalar field (sometimes called dilaton) non-minimally coupled to the Ricci scalar [1, 2].

The action is however not uniquely defined in this way, essentially because of the freedom in the choice of the kinetic and potential terms for the scalar field. Thus one can generate a large class of models, by simply requiring the renormalizability of the theory [11, 12]. Some special examples are given by the Jackiw-Teitelboim theory [1, 2], the tree-level string lagrangian [13-15], and the 2-dimensional limit of general relativity [16-18]. A one-parameter class of models with constant potential containing these special cases has been studied in [8-9].

A different solution to the problem of defining a suitable action for 2-dimensional gravity is given by higher derivative theories. In this case one defines a lagrangian which is a non-linear function of the Ricci scalar, avoiding in this way the problems found with the Einstein lagrangian in 2 dimensions [5, 6].

As is well-known, in dimensions higher than two, higher-derivative models following from a non-linear lagrangian $F(R)$ are conformally equivalent to general relativity minimally coupled with a self-interacting scalar field [19-23].
In two dimensions, since it is not possible to define a minimally coupled theory, the situation is more subtle. The existence of an equivalence between higher-derivative and gravity-scalar theories has been noticed by several authors [7, 10, 11, 28]. However, no general formulation of the equivalence is available in the literature. Moreover, its relation with conformal transformations of the metric has not been stated explicitly.

In this paper, we give an explicit classification of the gravity-scalar actions which are equivalent to higher-derivative actions up to conformal transformations. The existence of a non-trivial special case leads us to discuss the nature of scale-invariance for two-dimensional theories. Moreover, we briefly discuss the significance of the Birkhoff theorem in this context and the black hole solutions of the theory.

Some further discussion on different aspects of two-dimensional gravity can be found [24-32]. In [33], also two-dimensional gravity is considered, but they apply independent variation with respect to metric and connection, so the results are not directly comparable. In [34], there is observed a universal behaviour in the process of forming a two-dimensional black hole. Ref. [35] deals with the evaporation of two-dimensional black holes, where $N$ scalar fields have been added as source.

The paper is organized as follows: in section 2 we review 2-dimensional higher-derivative theories and discuss their connection with the more common approach given by the addition of a non-minimally coupled scalar field. Moreover, we study the action of a conformal transformation on the lagrangian. In section 3 we clarify the rôle of scale transformations for the lagrangian and the field equations. Section 4 is devoted to a review of the Birkhoff theorem in the context of two-dimensional gravity. In section 5 we compare the exact solutions of the theory in various gauges. We discuss the results in the final section 6.
2 Transformation from fourth to second order

As is by now well known, higher-derivative gravity models in dimensions $D > 2$ can be reduced by means of a conformal transformation to Einstein’s theory minimally coupled to a scalar field [19-23]. Consider for example the $D$-dimensional action

$$I = \int L(R)\sqrt{|g|}d^Dx$$  (2.1)

where $L(R) = R^{k+1}$, $k \neq 0$, $-1$ and $R \neq 0$. For simplicity, we write the next formulas for the region $R > 0$ only, the other sign gives analogous ones. If one defines the scalar field $\sigma$ by

$$e^{-2\sigma} \equiv \frac{dL}{dR} = (k+1)R^k$$

and performs a conformal transformation

$$\tilde{g}_{ij} = e^{-2n\sigma}g_{ij}$$

where $n$ is a parameter to be fixed, one obtains the action

$$I = \int e^{-[2+(D-2)n]\sigma} \left[ \tilde{R} + 2n(D-1)\tilde{\nabla}^2\sigma ight. - n^2(D-1)(D-2)(\tilde{\nabla}\sigma)^2 \left. - \Lambda \exp\left\{-2\left(\frac{k+1}{k} + nD\right)\sigma\right\}\sqrt{|\tilde{g}|}d^Dx \right.$$}

where $\Lambda = k/(k+1)^{1+1/k}$. In particular, only if one chooses $n = -\frac{2}{D-2}$, the scalar field is minimally coupled to the Einstein action as follows:

$$I = \int \left[ \tilde{R} - \frac{D-1}{D-2}(\tilde{\nabla}\sigma)^2 - \Lambda \exp\left\{-2\left(\frac{k+1}{k} - \frac{D}{D-2}\right)\sigma\right\}\right]\sqrt{|\tilde{g}|}d^Dx$$

This choice of $n$ is of course singular for dimension $D = 2$. This is due to the fact that in 2 dimensions $R\sqrt{g}$ is a total derivative and therefore no analogue of the higher-dimensional minimally coupled action exists. It is in
fact necessary to make use of a non-minimally coupled scalar field $\sigma$ and define an action of the kind

$$\int e^{-2\sigma} R \sqrt{|g|} d^2x$$

Actually, in 2 dimensions, it is not even necessary to perform a conformal transformation in order to get a linear lagrangian from eq. (2.1). In fact, if one defines as before $e^{-2\sigma} = \frac{dL}{dR} = (k+1)R^k$ one gets

$$I = \int [Re^{-2\sigma} - \Lambda \exp\{-2\frac{k+1}{k}\sigma\}]\sqrt{|g|} d^2x \quad (2.2)$$

This is a perfectly well defined action for 2-dimensional gravity. If one performs a conformal transformation on eq. (2.2) $\tilde{g}_{ij} = e^{-2n\sigma}g_{ij}$, one gets

$$I = \int e^{-2\sigma} [\tilde{R} + 4n(\tilde{\nabla}\sigma)^2 - \Lambda \exp\{-2\frac{1}{k}n\sigma\}]\sqrt{|\tilde{g}|} d^2x \quad (2.3)$$

so that the gravitational part is unchanged, while the scalar field acquires a kinetic term. All the actions (2.3) are conformally equivalent in the sense that if $g_{ij}$ is a stationary point of action (2.2) then $\tilde{g}_{ij}$ is one of (2.3). In particular, for $n = 1$ one obtains the well-known ”string-like” action [7]:

$$I = \int e^{-2\sigma} [\tilde{R} + 4(\tilde{\nabla}\sigma)^2 - \Lambda \exp\{-2\frac{1}{k}n\sigma\}]\sqrt{|\tilde{g}|} d^2x$$

whose solutions are given by $\tilde{g}_{ij} = e^{-2\sigma}g_{ij}$.

The previous discussion can be generalized to the case when the lagrangian is a generic function $L = F(R)$ of the curvature. In this case, one defines $e^{-2\sigma} = G$ where $G(R) = \frac{dF(R)}{dR}$ and the most general action related to $L = F(R)$ by a conformal transformation $\tilde{g}_{ij} = e^{-2n\sigma}g_{ij}$ of the two-dimensional metric takes the form

$$I = \int \{e^{-2\sigma}[\tilde{R} + 4n(\tilde{\nabla}\sigma)^2] - V(\sigma)\}\sqrt{|\tilde{g}|} d^2x \quad (2.4)$$

where $n$ is a free parameter and

$$V(\sigma) = (RG - F)e^{-2n\sigma} \quad (2.5)$$
For $n = 0$, this is found in [28].

To conclude, we notice that the action (2.2) admits two well-known theories as special limiting cases. First, both for $k \to \infty$ and

for $k \to -\infty$ it reduces to the action of the Jackiw-Teitelboim theory

$$I = \int \Phi[R - \Lambda]\sqrt{|g|}d^2x$$

where we have put $\Phi = e^{-2\sigma}$. Second, it can be shown that the stationary points of (2.1) and (2.2) coincide in the limit $k \to 0$ with those of the tree-level string action. This limit is not at all trivial, since for $k = 0$, (2.1) is a total derivative, while (2.2) is not defined. As mentioned in [10, eq. (2.18)], the $k \to 0$ limit actually corresponds to the action

$$I = \int R \ln R \sqrt{|g|}d^2x$$

(2.6)

This is not fully trivial but can be understood starting from the well-known formula

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon}(e^{\epsilon x} - 1) = x$$

(2.7)

We insert $x = \ln R$, multiply by $R$ and get

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon}(R^{\epsilon+1} - R) = R \ln R$$

(2.8)

When inserted into the action (2.6), the $R$-term is a total derivative, so one has

$$\int R \ln R \sqrt{|g|}d^2x = \text{boundary terms} + \lim_{\epsilon \to 0} \frac{1}{\epsilon} \int R^{\epsilon+1} \sqrt{|g|}d^2x$$

(2.9)

There is an essential difference between the minimally and the non-minimally coupled scalar field: If the kinetic term $(\nabla \phi)^2$ is absent, then in the minimally coupled case no dynamics for $\phi$ exists at all, whereas in the non-minimally coupled case, the introduction of the kinetic term does not alter the order of the corresponding field equation. This is the reason for the possibility of actions (2.2)/(2.3) becoming equivalent. In formulas: For
\( L = F(\Phi, R) \) with \( G = \frac{\partial F}{\partial R} \) one gets \( 0 = \frac{\partial F}{\partial \Phi} \), \( F = GR + \Box G \) and the trace-free part of \( G_{;ij} \) has to vanish. In the non-minimally coupled case, i.e. \( \frac{\partial G}{\partial \Phi} \neq 0 \), the parts with \( G_{;ij} \) contain the dynamics for \( \Phi \).

3 On different notions of scale-invariance

The lagrangian density \( R \ln R \) eq. (2.8) possesses also some peculiar properties in relation with the scale invariance of the theory. For the notion of scale-invariance one has to specify to which situation it refers. Here, we distinguish two different notions for the following situation: We consider a two-dimensional Riemannian or Pseudoriemannian metric \( g_{ij} \) with curvature scalar \( R \). Let a Lagrangian \( L = F(R) \) be given where \( F \) is a sufficiently smooth function (three times differentiable is enough) and \( G = \frac{dF}{dR} \). The variational derivative of \( L \sqrt{|g|} \) with respect to \( g_{ij} \) gives a fourth order field equation. The trace of that equation reads

\[
0 = GR - F + \Box G
\]

(3.1)

The field equation is completed by requiring that the trace-free part of \( G_{;ij} \) vanishes.

First definition: Let \( \alpha \) be an arbitrary constant and let \( \tilde{g}_{ij} = e^{2\alpha} g_{ij} \). Then, e.g., \( \tilde{R} = e^{-2\alpha} R \) etc. The Lagrangian \( L \) is called scale-invariant if there exists a function \( f(\alpha) \) such that for all metrics it holds

\[
\tilde{L} = f(\alpha) L
\]

One can get some knowledge on the function \( f \) as follows: We apply the defining condition with \( \beta \) instead of \( \alpha \) and with \( \tilde{g}_{ij} = e^{2\beta} \tilde{g}_{ij} \). This leads to

\[
\tilde{L} = f(\beta)\tilde{L} = f(\beta)f(\alpha)L = f(\beta + \alpha)L
\]

(3.2)
This last equality can be fulfilled only if there exists a constant real number $m$ such that

$$f(\alpha) = e^{2\alpha m}$$

Our definition is therefore equivalent to:

The Lagrangian $L$ is called scale-invariant if there exists a constant $m$ such that for all metrics it holds

$$\tilde{L} = e^{2\alpha m} L$$

We want to find out all scale-invariant Lagrangians. To this end we insert $R = 1$ into eq. (3.3), i.e. into

$$F(e^{-2\alpha} R) = e^{2\alpha m} F(R)$$

We use $x = e^{-2\alpha}$ and $c = F(1)$. We get $F(x) = cx^{-m}$. This is the sense in which usually $L = R^{k+1}$ is called the scale-invariant gravitational Lagrangian.

$L$ is a divergence iff the field equation is identically fulfilled. For the situation considered here this takes place if and only if $F(R) = cR$ with a constant $c$.

Let us now come to the second definition: Let $\alpha$ be an arbitrary constant and let $\tilde{g}_{ij} = e^{2\alpha} g_{ij}$. The field equation following from the Lagrangian $L$ is called scale-invariant if there exist functions $f(\alpha)$ and $g(\alpha)$ such that for all metrics it holds

$$\tilde{L} = f(\alpha) L + g(\alpha) R$$

This definition is equivalent to: The field equation following from $L$ is called scale-invariant iff $L$ is scale-invariant up to a divergence. It is motivated by the fact that for a scale-invariant field equation and one of its solutions $g_{ij}$, the homothetically transformed $\tilde{g}_{ij}$ is a solution, too.

To find out all scale-invariant field equations, we write the analogue to eq. (3.2), i.e.

$$0 = [f(\alpha + \beta) - f(\alpha)f(\beta)] F(R) +$$
\[ [g(\alpha + \beta) - g(\alpha)f(\beta) - g(\beta)e^{-2\alpha}]R \]

A linear function \( F(R) \) gives always rise to a scale-invariant field equation. For non-linear functions \( F(R) \), however, both lines of the above equation must vanish separately. The vanishing of the first line gives again \( f(\alpha) = e^{2\alpha m} \). We insert this into the second line and get

\[ g(\alpha + \beta) = g(\alpha)e^{2m\beta} + g(\beta)e^{-2\alpha} \]

To solve this equation it proves useful to define \( h(\alpha) = g(\alpha)e^{2\alpha} \) leading to

\[ h(\alpha + \beta) = h(\alpha)e^{2(m+1)\beta} + h(\beta) \]

1. case: \( m \neq -1 \): After some calculus one gets \( F(R) = cR^{-m} + kR \), just the expected sum.

2. case: \( m = -1 \): Then there exists a constant \( c \) such that \( h(\alpha) = c \cdot \alpha \), i.e., \( g(\alpha) = c \cdot \alpha e^{-2\alpha} \). To find the corresponding \( F(R) \) we have to solve

\[ F(e^{-2\alpha}R) = e^{-2\alpha}[F(R) + c\alpha R] \]

which is done by

\[ F(R) = -\frac{c}{2}R \ln R + kR \]  \hspace{1cm} (3.4)

\( k \) being a constant. So we see: \( L = R \ln R \) is not a scale-invariant lagrangian but it has a scale-invariant field equation and one learns: To find out all lagrangians being ”scale-invariant up to a divergence” it does not suffice to add all possible divergencies (here: \( kR \), \( k \) being constant) to all scale-invariant lagrangians (here: \( L = cR^{-m} \)).

The distinction made here can analogously be formulated for higher dimensions. One gets the following: Let \( H \) be a divergence and \( F \) be a scale-invariant lagrangian, then \( L = H \ln F \) gives rise to a scale-invariant field equation. This covers the above example for \( D = 2 \) with \( H = F = R \).

One might have got the impression that if a scale-invariant lagrangian is rewritten with a conformally transformed metric then the resulting field
equation remains essentially the same. But this is not always the case. The
typical example is: Take the Einstein-Hilbert action
\[ I = \int R \sqrt{|g|} d^D x \]
and define \( \hat{g}_{ij} = R^m g_{ij} \) for \( R > 0 \). Then \( \sqrt{|\hat{g}|} = R^{Dm/2} \sqrt{|g|} \). For \( Dm = 2 \),
\[ I = \int \sqrt{|\hat{g}|} d^D x, \]
so only for \( Dm \neq 2 \) the corresponding field equations become equivalent.

4 The generalized Birkhoff theorem

In [5] and [11] the following was shown: Let \( L = F(R) \) be a non-linear
Lagrangian in two dimensions and \( G = \frac{dF}{dR} \); then \( \Theta^i = \epsilon^{ij} G_j \) is a Killing
vector. This result is called "generalized Birkhoff theorem" for its type being
"a spherically symmetric vacuum solution has an additional Killing vector";
in fact, in one spatial dimension, the assumption of spherical symmetry is
empty.

To know whether the existence of a Killing vector implies a local sym-
metry, one must be sure that it does not vanish. Supposed, \( \Theta^i \) identically
vanishes, then \( G \) must be a constant, and so \( R \) is a constant. Then the space
is of constant curvature and a non-vanishing non-lightlike Killing vector ex-
ists. Supposed, \( \Theta^i \) is a non-vanishing null vector, then again, the space turns
out to be of constant curvature. So the only possibility for \( \Theta^i \) becoming
lightlike is at a line (the horizon) where it changes its signature. These are
the solutions being known under the name "two-dimensional black holes".

Generically (meaning here: in a region where the Killing vector is non-
lightlike) one can always write the solution as
\[ ds^2 = A^2(x)dx^2 \pm B^2(x)dy^2 \]  (4.1)

As usual, the free transformation of \( x \) can be used to eliminate \( A \) or \( B \);
especially the condition \( AB = 1 \) leads to generalized Schwarzschild coordi-
nates. What is essential for eq. (4.1): The change between Euclidean and
Lorentzian signature is possible by the complex rotation $y \rightarrow iy$. This is of course only local and generically, so that the global topology may be (and indeed, is) different, but in higher dimensions such a relation does not need to take place even locally. (The reason is: in two dimensions, a Killing vector is automatically hypersurface-orthogonal.)

This generalized Birkhoff theorem has the consequence that special solutions having symmetries (see sect. 5 below) found in the past already cover the whole space of solutions. Of course, the theorem can be extended to the gravity-scalar theories, owing to their equivalence with higher-derivative theories.

## 5 Exact solutions

The solution of the field equations stemming from eqs. (2.1)/(2.3) have been found in [5, 6] and, in a conformal gauge, in [7]. We shortly discuss them in this section.

For $k \neq -1/2$, the Lorentzian signature solutions can be written in the so-called Schwarzschild gauge as [5, 6]:

$$ds^2 = -A^2(x)dt^2 + A^{-2}(x)dx^2$$

with

$$A^2(x) = -C + |x|^{2+1/k}$$

while for $k = -1/2$,

$$A^2(x) = -C + \ln |x|$$

where $C$ is a free parameter, proportional to the mass of the solution. In particular, for positive $C$ one gets in general black hole solutions, while for negative $C$ one has naked singularities. $C = 0$ corresponds to the self-similar
ground state of the theory. The conformal gauge solutions found in \cite{7} can be obtained from (5.1) for $k \neq -1/2$ by the coordinate transformation

$$\rho = \int dx [-C + |x|^{2+1/k}]^{-1}$$

In particular, if $C = 0$, $\rho = x^{-(1+1/k)}$, and

$$ds^2 = \rho^{-(2k+1)/(k+1)}(d\rho^2 - dt^2).$$

Let us discuss in some detail the properties of the solutions: in the Schwarzschild gauge the curvature is simply given by: $R = -d^2(A^2)/dx^2$. Thus one sees that a singularity (in the sense of a diverging curvature scalar $R$) is present at the origin only if $k$ is negative. Moreover, for positive $C$, a horizon is present at $x = C^{k/(k+2)}$ for any $k$. The horizon is absent if $C$ is negative.

The asymptotic properties of the solutions are also interesting: for negative $k$ the curvature vanishes at infinity, but only in the limit case $k = 0$ the solutions are asymptotically flat in the usual sense (i.e. $A \to 1$ at infinity). For positive $k$ the curvature diverges at infinity. Finally, in the limit $k \to \pm \infty$ (Jackiw-Teitelboim theory), the solutions are asymptotically anti-de-Sitter.

The limit case $k \to 0$ has been studied in \cite{5}. In this case the solutions coincide with the ”stringy” solutions found in \cite{13, 14} and with a solution of Liouville gravity (see \cite{30-32} for details):

$$A^2(x) = 1 - Ce^x$$

and describe asymptotically flat black holes.

To summarize, regular black hole solutions are found only for $k \leq 0$ and positive $C$.

In a similar manner, one can discuss the solutions of the Euclidean theory. Apart from the horizon, theses are simply obtained by setting $t \to it$. In the black hole case, the conical singularity at the origin (i.e., the point corresponding to the horizon) can be removed by a standard procedure, requiring
that the Euclidean time has periodicity $\beta$ which is related to the temperature $T$ of the black hole via

$$T = \beta^{-1} = \frac{2k + 1}{4\pi k} C^{(k+1)/(2k+1)}$$

6 Discussion

In this paper, we considered several types of two-dimensional theories of gravity. We restricted to the classical (= non-quantum) case; the metric and one scalar field are the only ingredients (no torsion, no further matter). The aim of the paper was to clarify the conformal relation between different versions of the theory; especially, we carefully distinguished between transformations on the lagrangian and on the field equation’s level.

Section 2 dealt with the conformal transformation from a non-linear lagrangian $L(R)$ (corresponding to a fourth-order field equation) to Einstein’s theory with one additional scalar field. To simplify the formulas we first considered the case $L(R) = R^{k+1}$, ($k \neq 0, -1$) to show how the transformation breaks down for dimension $D = 2$ if the scalar field is required to be minimally coupled. The reason is that for $D = 2$ the curvature scalar is a divergence. So, for $D = 2$, the conformal equivalence becomes possible for a non-minimally coupled scalar field only. We showed this in two steps: first for $L(R) = R^{k+1}$, and second, eqs. (2.4, 2.5), the one-parameter set (the parameter is $n$) of conformal transformations from a general non-linear $L(R)$ to Einstein’s theory with a non-minimally coupled scalar field. (The points where this transformation becomes singular are not explicitly written down but become clear from the formulas.) Only few special cases of this result can be found in the literature. From eq. (2.4) it becomes clear that the kinetic term of the scalar field vanishes for $n = 0$. This does not destroy the equivalence because the dynamics of the scalar field now comes from the
non-minimal coupling to \( R \). So, the change from \( n = 0 \) to \( n \neq 0 \) represents a conformal transformation of a scalar field without to a scalar field with kinetic term. This generalizes the class of conformal transformation of [12] relating between

\[
L = \frac{1}{2} (\nabla \Phi)^2 + F(\Phi)R + U(\Phi)
\]

and

\[
L = \frac{1}{2} (\nabla \phi)^2 + \frac{q}{2} R + V(\phi)
\]

To avoid possible misunderstandings: Some papers do not have the factor 4 in front of the kinetic term as we have. In [10], e.g., one has

\[
L = e^\Phi [R + (\nabla \Phi)^2 + \lambda]
\]

If one inserts \( \Phi = \pm 2\phi \) then one gets

\[
L = e^{\pm 2\phi}[R + 4(\nabla \phi)^2 + \lambda]
\]

so this is only a notational difference. A further misunderstanding can appear by noting that \( R^{k+1} \) tends to \( R \ln R \) as \( k \to 0 \). In eqs. (2.6 - 2.9) we clarified in which sense this is a mathematically correct statement.

In section 3 we distinguished different notions of scale-invariance. It turned out that two of them are essentially different: Scale-invariant lagrangians and scale-invariant field equations. It is trivial to see that the sum of a scale-invariant lagrangian and an arbitrary divergence gives rise to a scale-invariant field equation. Surprisingly, these sums do not yield all scale-invariant field equations. One (the only !) counterexample is the often discussed case \( L = R \ln R \).

In section 4 we discussed the fact that in the models under consideration a non-vanishing Killing vector always exists (generalized Birkhoff theorem). Here we want to emphasize: A) that this does not need the scale-invariance of the action (a case for which it is often formulated) but that it takes place for all models. B) The conformal transformation shows that the Birkhoff
Theorem is valid in all the versions of two-dimensional gravity under consideration, and C) it is just this Birkhoff theorem which makes possible (at least locally) the complex rotation from Euclidean signature to Lorentz signature solutions; the latter are discussed as two-dimensional black holes. Section 5 represents known exact solutions in a better readable form.

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References

[1] Jackiw R 1984 in Quantum Theory of gravity, Christensen S M ed. (Adam Hilger, Bristol) p. 403
[2] Teitelboim C 1983 Phys. Lett. B 126 41
[3] Mann R B, Shiekh A and Tarasov I 1990 Nucl. Phys. B 341 134
[4] Mann R B 1992 Gen. Rel. Grav. 24 433
[5] Schmidt H-J 1991 J. Math. Phys. 32 1562
[6] Schmidt H-J 1992, p. 330 in: Relativistic Astrophysics and Cosmology, Eds.: Gottlöber S Mücket J and Müller V, World Scientific, Singapore
[7] Mignemi S 1994 Phys. Rev. D 50 R4733
[8] Lemos J P L and Sá P M 1994 Phys. Rev. D 49 2997
[9] Cadoni M and Mignemi S 1994 preprints INFN-CA-20-93 and INFNCA-TH-94-4
[10] Frolov V P 1992 Phys. Rev. D 46 5383
[11] Banks T and Loughlin M O 1991 Nucl. Phys. B 362 649
[12] Russo J G and Tseytlin A A 1992 Nucl. Phys. B 382 259
[13] Witten E 1991 Phys. Rev. D 44 314
[14] Mandal G, Sengupta A M and Wadia S R 1991 Mod. Phys. Lett. A 6 1685
[15] Callan C G, Giddings S B, Harvey J and Strominger A 1992 Phys. Rev. D 45 1005
[16] Sikkema A E and Mann R B 1991 Class. Quantum Grav. 8 219
[17] Mann R B and Ross S F 1993 Class. Quantum Grav. 10 1405
[18] Lemos J P L and Sá P M 1994 Class. Quantum Grav. 11 L11
[19] H.-J. Schmidt 1987 Astron. Nachr. 308 183
[20] Magnano G, Ferraris M and Francaviglia M 1987 Gen. Rel. Grav. 19 465
[21] Barrow J and Cotsakis S 1988 Phys. Lett. B 214 515
[22] Schmidt H-J 1988 Astron. Nachr. 309 307
[23] Maeda K 1989 Phys. Rev. D 39 3159
[24] Katanaev M O, Volovich I V 1990 Ann. Phys. (NY) 197 1
[25] Kummer W and Schwarz D 1992 Nucl. Phys. B 382 171
[26] Grosse H, Kummer W, Presnajder P and Schwarz D 1992 J. Math. Phys. 33 3892
[27] Nojiri S and Oda I 1994 Mod. Phys. Lett. A 9 959
[28] Solodukhin S N 1994 preprint JINR E2-94-185
[29] Gegenberg J, Kunstatter G and Louis-Martinez D 1994 preprint gr-qc/9408015
[30] Christensen D and Mann R B 1992 Class. Quantum Grav. 9 1769
[31] Mann R B and Ross S F 1992 Class. Quantum Grav. 9 2335
[32] Mann R B, Morris M and Ross S F 1993 Class. Quantum Grav. 10 1477
[33] Ferraris M, Francaviglia M and Volovich I 1994 Class. Quantum Grav. 11 1505
[34] Strominger A and Thorlacius L 1994 Phys. Rev. Lett. 72 1584
[35] Hawking S 1992 Phys. Rev. Lett. 69 406