Progress and prospects in magnetic topological materials

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Magnetic topological materials represent a class of compounds with properties that are strongly influenced by the topology of their electronic wavefunctions coupled with the magnetic spin configuration. Such materials can support chiral electronic channels of perfect conduction, and can be used for an array of applications, from information storage and control to dissipationless spin and charge transport. Here we review the theoretical and experimental progress achieved in the field of magnetic topological materials, beginning with the theoretical prediction of the quantum anomalous Hall effect without Landau levels, and leading to the recent discoveries of magnetic Weyl semimetals and antiferromagnetic topological insulators. We outline recent theoretical progress that has resulted in the tabulation of, for the first time, all magnetic symmetry group representations and topology. We describe several experiments realizing Chern insulators, Weyl and Dirac magnetic semimetals, and an array of axionic and higher-order topological phases of matter, and we survey future perspectives.

Topological insulators and topological semimetals, first predicted 15 years ago1, can exhibit robust boundary states, quantized bulk responses and exotic transport properties. They represent a possible route towards manipulating quantum information3, coherent spin transport4 and high-efficiency catalysis5. Although a myriad of insulating and (semi)metallic non-magnetic topological phases have now been predicted, characterized and measured, magnetic materials have so far been scarce. The interesting nature of these materials renders their theoretical prediction more difficult than that of their non-magnetic counterparts, yet they are experimentally attractive because magnetism potentially offers greater opportunity for manipulation of topological states. In the past three years, theoretical and experimental advances in topological magnetic materials have precipitated6–11.

For background, in Box 1 we give an overview of the general steps needed for the high-throughput theoretical screening of magnetic topological materials (further technical details are available in Supplementary Information A). Once a candidate topological material has been synthesized (itself a challenge), various experimental tools must be marshalled to measure their electronic band structures and their transport properties, and identify topological features: these are summarized in Box 2.

The main purpose of this Review is to discuss recent theoretical and experimental progress in this area by reviewing the rich phenomenology predicted and detected in a variety of compounds. We elaborate on two examples, one from each of the two main material classes—magnetic topological insulators and magnetic topological semimetals. Specifically, we will discuss the characteristic band structure features and transport phenomena of such systems based on two of the most well studied magnetic topological materials: the van der Waals antiferromagnetic topological insulator MnBi2Te4, and the kagome ferromagnetic Weyl semimetal Co3S6S2. Other magnetic topological possibilities exist that are linked to orbital magnetism instead of spin magnetism; we will defer such discussion to Supplementary Information B.

Finally, we will conclude with a brief discussion of the opportunities for further theoretical exploration and experimental discovery in this space, and what they might mean for both fundamental studies and practical applications.

Theory of magnetic topological insulators

The very first magnetic topological insulator is the (integer and fractional) quantum Hall effect, the discovery and theoretical explanation of which resulted in two Nobel prizes, in 1985 and 1998. It was subsequently realized12 by Haldane that an applied magnetic field is not necessary to realize a magnetic topological insulator, and research over the past ten years theoretically uncovered other insulating phases of matter, the properties of which are defined by magnetic group topology.

The quantum anomalous Hall effect (AHE)12 has been realized by opening a magnetic mass gap (by either magnetic impurities13,14 or intrinsic magnetic order15,16) in the Dirac cone at the surface of a thin three-dimensional (3D) topological insulator film. This state is, however, the axion (higher-order) insulator with a chiral hinge mode, where the sample has been thinned to quasi-two-dimensional (2D) with magnetism added on the surface: it is not the original, stoichiometric, purely 2D Chern insulator12. The magnetic order mechanism in these samples is under debate17,18, between Ruderman–Kittel–Kasuya–Yosida (RKKY) (Mn-doped Bi2Te3)19 and Van Vleck20 (Cr-doped (Bi,Sb)2Te3)20.

The 3D antiferromagnetic topological insulator (AFMTI)21,22 is the only other magnetic topological insulator that has been realized21, in MnBi2Te4 in type-IV magnetic space group R3c (no. 167.108)23. Figure 1 summarizes the different magnetic topological phases observed and predicted in MnBi2Te4 and related compounds. An AFMTI is the result

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of turning on the antiferromagnetism in a non-magnetic time-reversal-invariant topological insulator. Forcing doubling of the unit cell, the symmetry group becomes a combination \((\mathbb{I}T\bar{0}0\bar{2}1)\) of time reversal \((\mathbb{T})\) and half a lattice translation \((\bar{0}0\bar{2}1)\). Before turning on the antiferromagnetism, the strong topological insulator exhibited a Dirac cone on each surface; the antiferromagnetism gaps the Dirac cone on the surface perpendicular to the half lattice translation. On the side surfaces, the \((\mathbb{I}T\bar{0}0\bar{2}1)\) enforces Kramers degeneracy at \(k_z = 0\) (but not at \(k_z = \pi\)). The Dirac node sits at \(k_z = 0\) in the antiferromagnetic Brillouin zone, irrespective of its location (at \(k_z = 0\) or \(\pi\)) in the larger non-magnetic Brillouin zone (see Fig. 1), as previously reported\(^{46}\).

Thin samples of MnBi, Te produce a quantum AHE expected to be more robust than that of magnetic-impurity-doped topological insulators\(^{43-45}\). When a Dirac fermion (exhibiting \(\pi\) Berry phase around the Fermi surface) is gapped, the resulting insulator has an integrated Berry curvature—Chern number \(\mathcal{C} = 1\). The lower and upper surfaces can then either subtract (to lead to \(\mathcal{C} = 0\)) or add, to lead to \(\mathcal{C} = 1\) quantum AHE. Whether they add or subtract depends on the surface exchange field. Even (odd) number of non-magnetic unit cells will experience opposite (equal) exchange field on the top and bottom surface, giving rise to \(\mathcal{C} = 0\) (1). Furthermore, the application of a magnetic field can drive the magnetic moments ferromagnetically. Theoretically, ferromagnetic MnBi, Te\(_z\) (with magnetic space group \(R3\) (no. 166.101)) is a bulk Weyl semimetal with one pair of Weyl points along \(\Gamma-Z\) (refs. \(^{26,27}\)). In between the two Weyl points, in each \(k_z\) plane, the Chern number is \(1\) per \(k_z\) momentum. For a thin-film material of layers, this momentum is quantized in units of \(2\pi N\), which, for small \(N\), gaps the Weyl nodes, owing to quantum confinement. Hence the finite-layer thin film of ferromagnetic MnBi, Te\(_z\) can exhibit a quantum AHE with a Chern number greater than \(1\). In ref. \(^{28}\), a quantum confinement \(5\) meV gap is induced at the \(\Gamma\) point and \(\mathcal{C} = 2\).

The \(\mathcal{C} = 1\) quantum AHE state obtained by gapping a thin film of topological insulator is a `static' axion insulator, a state of matter with the \(\theta\) angle—coefficient of the Chern–Simons form of the Berry potential \(A(k)\) —equal to \(\pi\) (ref. \(^{30}\)). The discovery of higher-order topological insulators (HOTIs)\(^{30}\) resolved the link between the axion insulator and the quantum AHE. \(\mathcal{T}\) symmetry breaking can drive a 3D topological insulator into an axion insulator phase with gapped surface states and gapless (chiral) hinge modes\(^{29,31-37}\) which carry the Chern number of the quantum AHE. Inversion-symmetric topological insulators can be axion insulators and at the same time magnetic HOTIs with `intrinsically' hinge states\(^{31,35-40}\) (see Fig. 1). Their bulk topology may be inferred from the Fu–Kane parity formula\(^{31,32,41}\); all of their surface states and Wilson loops are gapped. Gapless hinges provide chiral spectral flow\(^{30,32}\).

A large number of theoretically predicted but experimentally undiscovered topological states exist (see Fig. 1). Although the static axion insulator has been observed\(^{30,31,34-45}\), it was predicted\(^{46}\) that a `dynamical' mode of the axion insulator (phason as the dynamical mode) can arise from gapping two Weyl nodes spontaneously by charge-density wave formation at the wave vector between the nodes. This proposal remains unrealized, although its time-reversal counterpart (charge-density wave in a non-centrosymmetric time-reversal-invariant Weyl) has been predicted and discovered in \((TaSe_4)_2I\)\(^{47,48}\). In magnetic space groups, axion insulator phases can be protected by other bulk symmetries such the product of twofold rotation and \(T\) (Refs. \(^{42,43}\)) (see Fig. 1). The magnetic Möbius topological crystalline insulators\(^{4,52}\) host unpaired Dirac-cone surface states—similar to those of 3D topological insulators—appearing along surface glide lines and have been predicted in \(\text{MnBi}_n\) \(\text{Te}_{3n-1}\)\(^{53,54}\) with canted magnetic moments. They are the \(T\) breaking analogue of the hourglass topological crystalline insulators\(^{56-59}\) in \(\text{KmgSb}\). Glide mirror on some surfaces allows for a degeneracy along mirror symmetric lines in the Brillouin zone (see Fig. 1).

In total, four different topological phases (three are shown in Fig. 1e) can be (theoretically) realized in the same compound formula, corresponding to different topological classifications: AFMTI (collinear antiferromagnetic state), high-order Möbius insulator (in canted antiferromagnetic state) and mirror topological crystalline insulators (in-plane ferromagnetism), as well as a ferromagnetic axion (out-of-plane ferromagnetism) in \(\text{MnBi}_n\) \(\text{Te}_{3n-1}\). The difference in their topological classifications is a clear example of how different magnetic symmetry groups give rise to different topology.

Magnetic topological states implied by symmetry eigenvalues have recently been classified\(^{46}\), even though material predictions are scarce. Of these, spinful helical magnetic HOTI phases are related to rotational anomalies and exhibit trivial axion angles \(\theta = 0\) (mod \(2\pi\)). When terminated (see Fig. 1) in nanorod geometries, the helical magnetic HOTIs...
generically exhibit even numbers of massless twofold surface Dirac cones (see Fig. 1f) on surfaces perpendicular to a rotation axis similar to those in ref. 61. On their side surfaces, domain walls between surfaces with oppositely signed masses bind mirror-protected helical hinge states (Fig. 1g).

### Materials for magnetic topological insulators

There are four main routes for turning a topological insulator into a magnetic quantum AHE system, sketched in Fig. 2: (1) extrinsic deposition of magnetic layers onto the surfaces of the topological insulator.
In 201572 the quantum AHE was demonstrated in V-doped (Bi$_{1-x}$Sb$_x$)$_2$Te$_3$ thin films with a larger coercive field and in higher temperatures up to 100 mK. Several attempts made to fabricate hybrid heterostructures of magnetic elements (Fig. 2b); (3) interleaving magnetic layers into the topological insulator unit cell (Fig. 2c); and (4) identifying intrinsic magnetically ordered topological insulator states (Fig. 2d). The last two may introduce magnetic symmetries that directly affect the topological classification. Several attempts made to fabricate hybrid heterostructures of magnetic elements on topological insulator surfaces such as EuS over Bi$_2$Se$_3$, but could not demonstrate quantized anomalous Hall conductance. An alternative approach, inspired by semiconductor spintronics, is doping of the canonical topological insulators Bi$_2$Se$_3$, Bi$_2$Te$_3$, or HgTe with magnetic ions (for example, V, Mn, Cr, Sm) depicted in Fig. 2b. In 2013, the quantum AHE was measured in thin films of Cr-doped (Bi$_{1-x}$Sb$_x$)$_2$Te$_3$ with a quantized Hall resistance $\rho_H$ observed up to temperatures of 30 mK. In 2015, the quantum AHE was demonstrated in V-doped (Bi$_{1-x}$Sb$_x$)$_2$Te$_3$ with a larger coercive field and in higher temperatures up to 100 mK. Despite promising transport experiments—although limited to cryogenic temperatures—the spectroscopic investigation of the energy gap of the corresponding surface states has so far been inconclusive. Low Curie temperatures, and the risk of inhomogeneous clustering of dopants thus gave way to intrinsic magnetic topological insulators. Recently, a new family of intrinsic antiferromagnetic topological insulators was discovered with the general composition MnTe(Bi$_2$Te$_3$)$_n$ (Fig. 2e, f). MnBi$_2$Te$_4$ is the first member of the family with a Néel temperature $T_N$ of 20 K (ref. 10). MnBi$_2$Te$_4$ is a natural heterostructure of MnTe and Bi$_2$Te$_3$. The compound topology (band inversion) is akin to the Bi$_2$Te$_3$ quintuple layer, whereas the magnetism is related to MnTe. The combined symmetry of time reversal and half a unit cell translation protects the Dirac states parallel to the antiferromagnetic order above the Neel temperature. Spectroscopic reports have been thus far inconclusive on the formation of a magnetic gap at the surface Dirac points: some angle-resolved photoemission spectroscopy (ARPES) measurements find a gapped surface spectrum, other image gapless Dirac bands with weak response to lifting of the antiferromagnetic order above the Neel temperature. Local spectroscopic mappings in scanning tunnelling microscopy (STM) visualize a high level of substitutional Mn atoms on Bi sites, posing a similar challenge to that encountered with magnetically doped topological insulators. Thin-film quantization was shown to give rise to either a quantum AHE, or an axion insulator (Fig. 2g, h, respectively). Accordingly, the MnTe(Bi$_2$Te$_3$)$_n$ family will undoubtedly open many new opportunities for magnetic Weyl semimetals, and beyond.

### Fig. 1 | Interplay between magnetic orders and topology.

Depending on the spin configuration, the MnBi$_2$Te$_4$ system is predicted to be one of the following. a, For an AFMTI, with a single gapless Dirac cone protected by $T_0$ symmetry on the symmetry-preserving (010) (or (100)) surface, whereas the symmetry-non-preserving (001) surface is gapped. b, In a thin 2D sample with only a few layers, a quantum AHE state with $C = 1$ (axion insulator) or $C = 2$, depending on the number of layers, and with Chiral edge states. c, A Möbius insulator in a canted antiferromagnetic state which respects glide mirror symmetry with $M_x$ mirror followed by half-lattice translation. The insulator shows surface states on the symmetry-preserving (010) surface but not on the (100) and (001) surfaces. Two opposite (010) surfaces are linked together by one-dimensional chiral hinge states, manifesting the higher-order nature (HOTI) of the system. The surface state is a Dirac cone, whose position is on the $T$–$Z$ line, and their mirror eigenvalues, proportional to $\exp(i k_z/2)$, require two Brillouin zones to return to themselves, hence the name Möbius. d, A topological crystalline insulator phase for in-plane ferromagnetism, where the glide mirror $M_z(001)$ is promoted to a mirror $M_x$, and now a surface state appears on both symmetry-preserving (010) and (001) surfaces. Parts of a–d are reprinted with permission from ref. 10 by the American Physical Society; part of a is reprinted with permission from ref. 73 by the American Physical Society.
Theory of magnetic topological semimetals

More than 100 years ago Edwin Hall realized that all ferromagnetic semimetals and metals exhibit an anomalously large Hall effect (AHE). Because the Hall resistivity versus an applied external magnetic field behaves similarly to the magnetization versus the external magnetic field, it was concluded that the AHE is proportional to the magnetization. Nowadays it is established that the Berry curvature has an important role in determining the AHE in ferromagnetic semimetals and metals. Berry noted that an energy-level crossing leads to a physical prediction of the presence of Dirac states on surfaces that preserve combined time-reversal and half-unit-cell-translation operation (right) and massive ones on those that break it (left). Spectroscopic ARPES measurement images the Dirac surface states with possible induced gap at the Dirac node. The magnetoresistive responses show prominent magnetic topological response. For even septuple-layer thin film the quantum AHE is observed with quantized Hall resistance, whereas for odd septuple-layer axion response is found with null Hall response. Figure 2d, e, f are reprinted by permission from Springer Nature, ref. 15, g is reprinted with permission from AAAS; h is reprinted by permission from Springer Nature, ref. 15.

Meanwhile, an increasing number of intrinsic magnetic compounds (Fig. 2d), are identified and investigated in transport and spectroscopy in search of clear signatures of topology induced by broken time-reversal symmetry. These include the ferromagnetic AHE Fe₃GaTe₂, the AFMTI EuCd₂As₂, and the antiferromagnetic topological crystalline insulator (and possibly a magnetic HOTI) EuIn₂As₂, which we briefly discuss. Large anomalous Hall and Nernst signals were detected in the ferromagnetic Fe₃GaTe₂, with a critical temperature Tₘ higher than room temperature in the few-layer limit of gate devices. The anomalous Hall behaviour is believed to originate from the intrinsic Berry curvature contribution due to gapping of a nodal line semimetallic state. No clear spectroscopic evidence of topological states has been provided yet. EuCd₂As₂ is predicted to turn from a paramagnetic narrow-gap semiconductor to an AFMTI below Tₘ = 10 K with the easy axis perpendicular to the layers. The combined non-symmetric time-reversal symmetry protects the Dirac surface states on the side surfaces that respect it, and the top and bottom facets are gapped forming Chern or axion insulating states that have been reported to exist by ARPES. EuIn₂As₂ is predicted to turn from a paramagnetic narrow gap semiconductor into a type-A antiferromagnetic axion insulator or rather a magnetic HOTI below Tₘ = 10 K, neither of which is established spectroscopically. A third classification arises when its antiferromagnetic order aligns parallel to the layers and a magnetic mirror symmetry is restored, classifying the electronic phase as an antiferromagnetic topological crystalline insulator.

The simplest topological semimetal, without time-reversal or crystalline symmetry is the solid-state realization of conventional Weyl fermions—twofold degeneracies appearing when two singly degenerate bands cross, at any point in the Brillouin zone, and exhibiting linear dispersion away from the degeneracy point. Weyl fermions carry a nontrivial topological invariant, the Chern number |C| = 1 evaluated on a sphere at energy Eₘ around the Weyl point. Magnetic Weyl semimetals are common: every crossing point in the band structure of a ferromagnetic centrosymmetric compound is related to nodal lines or Weyl points.

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Constant). By the Mott relation $\sigma_w = \frac{n^2}{e^2 c} \sigma_w(e)$, the anomalous Nernst effect is also expected to be large. The $Z$-valued Chern number of the Weyl points reflects the difference in the Chern number of 2D Brillouin zone planes above and below the Weyl point. Each Brillouin zone plane carrying nonzero Chern numbers projects on surfaces of the crystal to give rise to quantum Hall-like edge states, summing up into surface Fermi arcs spanning the momentum space between the projections of the bulk Weyl points. Higher charge Weyl points appear when two or more Weyl nodes are pinned together by a crystalline rotation symmetry. The first prediction, still unrealized, of a $C = 2$ Weyl node, stabilized by $C_2$, was in the magnetic phase of HgCr$_2$Se$_4$ in the magnetic space group $H41/amd'$ (no. 141.557). In which a strong AHE was reported.

A series of experimentally promising antiferromagnetic topological semimetals has been predicted on the basis of a search of large AHE in Mn$_x$ (X = Sn, Ge and Ir) and on direct ab initio calculations in Mn$_x$Sn and Mn$_x$Ge with kagome layers Mn atoms. The non-collinear magnet Mn$_x$Sn in magnetic space group $Cmc/m'$ (no. 63.463) is a magnetic Weyl semimetal candidate with six pairs of Weyl points. Under rigorous magnetic topological quantum chemistry principles, it was found that these Weyl points are accidental: if the six Weyl points reported in ref. 108 in one half of the Brillouin zone were pairwise-annihilated without closing a gap at the inversion-invariant momenta, the gapped phase would either be an axion insulator or a 3D quantum anomalous Hall state.

CuMnAs and CuMnP have been proposed to exhibit Dirac points. Their antiferromagnetic order maintains the type-III symmetry $I_7^{23}$ leading to doubly degenerate bands at each $k$ in Brillouin zone. Two pairs of these bands cross and their Dirac degenerate point is protected by a non-symmetric $C_{2z}$, $C_0$. EuCu$_2$As$_2$ was also proposed as a Dirac semimetal in a type-IV magnetic space group $(D_{4d} \oplus D_{4d})^{23}$. Doubly degenerate bands exist owing to $I_7(0, 0, 0)$, and symmetry, and two pairs can cross with the Dirac point stabilized by $C_{2z}$. When threefold rotation symmetry $C_{2z}$ is broken, the Dirac semimetal phase can evolve into the AFMTI phase. Magnetic nodal line semimetals were predicted in the layered system Fe$_3$GeTe$_2$, without spin-orbit coupling SOC. Similarly, in the ferromagnetic Co$_3$MnGa with space $p$ group $Fm3m$ (no. 225) two majority spin bands near the Fermi level cross on the mirror planes stabilized by mirror symmetry. The nodal lines gap when the SOC is present, although in reality the SOC is negligible. Proposals of nodal line semimetals in ferromagnetic phases of LaCl (LaBr) have not been realized; we believe that these materials are most probably non-magnetic.

Non-magnetic and magnetic symmetry groups allow 2, 3, 4, 6 and 8-fold degeneracy new fermions in Brillouin zone. In (type-I) space groups three-, four- and six-dimensional degeneracies can appear: type-II and type-IV groups support 8-fold double Dirac point degeneracies. The chiral antiferromagnetic phase of Mn$_x$IrSi is predicted to host spin-1 Weyl fermions with 3-fold degeneracies. Mn$_x$Ir$_2$Si$_2$ and Nd$_x$Si$_2$ are predicted to be chiral magnetic topological semimetals. The 4-, 6- and 8-fold new degeneracies are not protected.
by a Chern number (as in the case of Weyl semimetal) and hence do not exhibit Fermi arcs on surfaces; they are novel higher-order topological semimetals exhibiting ‘hinge’ arcs. A simple model for hinge arcs can be expressed as a \( k_z \) phase transition between a quadrupole insulator in ref. 128 and a trivial insulator. Related arguments show that both Dirac higher-order topological semimetals and 6-fold degeneracies universally host intrinsic hinge states.

### Materials for magnetic topological semimetals

#### Ferromagnetic compounds

The ferromagnetic Weyl semimetal Co\(_3\)Sn\(_2\)S\(_2\) in magnetic space group \( R\overline{3}m' \) (no. 166.101) has been extensively explored and characterized for its topological properties. Its crystal structure is composed of A–B stacked triangular layers of Sn and S and kagome layers of magnetic Co ions (Fig. 3, inset) captured in STM topography. The compound hosts one electron more than the semiconducting non-magnetic stack Co\(_2\)InSn\(_2\)S\(_2\), Co\(_2\)Sn\(_2\)S\(_2\) fully polarized spin (0.29 µB per Co; 1.28 Bohr magneton) leads to a half-metallic ferromagnet with a relative high Curie temperature of 177 K with its spins oriented out of plane. A single valence and conduction band cross the Fermi energy, leading to prediction of Weyl crossings, as shown in Fig. 3b. Experimental evidence for the bulk Weyl nodes close to the Fermi energy was provided by ARPES measurements (Fig. 3c), as well as confirmed by STM through quasi-particle interference (Fig. 3d).

Clear magnetotransport signatures of the magnetic topological state were reported prior to the spectroscopic verification. These include negative magnetoresistance under parallel current and magnetic field (Fig. 3g), potentially signifying chiral anomaly, high anomalous Hall conductivity and a much higher anomalous Nernst signal than conventional materials (Fig. 3f). STM further finds presence of linearly dispersing step-edge modes (Fig. 3g), and theory predicts that isolated Co\(_2\)Sn sheets will exhibit a quantum AHE. Furthermore, the kagome structure of the magnetic Co ions can host flat-band models owing to the line-graph property of the lattice. Intriguingly, a zero-bias conductance peak has been detected in STM (Fig. 3h) on the Co surface termination, with an unusual response to magnetic field. Bulk single crystals of Co\(_2\)Sn\(_2\)S\(_2\) have been even used for a proof-of-concept implementation of the efficiency towards water oxidation.

All these suggest a new direction to search and synthesize magnetic topological semimetals in kagome and honeycomb layer of 3d transition metal ions. This family of materials exhibit Weyl and Dirac fermions in both ferromagnetic and antiferromagnetic materials. Examples so far include FeSn\(_2\), Fe\(_3\)Sn\(_2\), Mn\(_2\)Sn\(_2\), MnGe, and CoSn\(_2\), as well as the RMn\(_3\)Sn\(_2\) family with \( R = \) Tb, Gd, Tm, Lu. Experimental signatures include a temperature-independent enhanced AHE up to room temperature in Fe\(_3\)Sn\(_2\) and a gapped 2D Dirac band close to the Fermi energy by ARPES. A giant spontaneous nematic energy shift, larger than any possible Zeeman splitting, hints at strong correlations in Fe\(_3\)Sn\(_2\). To reduce the dimensionality and increase correlations effects, FeSn—with decoupled iron layers—was identified as an ideal kagome lattice. Flat bands and fully spin-polarized surface states (confirmed by ARPES) suggest the presence of spatially decoupled kagome planes. For a summary of the anomalous transport properties of the kagome compounds and the corresponding synthesis methods, see Table 1.

Another large family of half metallic Co\(_2\)-Heusler compounds holds great potential as magnetic Weyl candidate materials because of their tunability. They were proposed in Co\(_2\)YZ (\( Y = Zr, Nb, Ti, Hf; Z = Si, Ge, Sn \)) and Co\(_2\)MnZ (\( Z = Ga, Al \)) and can be grown in bulk and thin films. A leading magnetic topological semimetal candidate is Co\(_2\)MnGa, which was verified spectroscopically in ARPES as a nodal line semimetal hosting drum head surface states. Strong AHE, indicating interplay between nodal lines and their partial gapping into Weyl points close to the Fermi energy, were reported in it and in Co\(_2\)MnAl. The AHE in Co\(_2\)MnGa and Co\(_2\)MnAl is even larger than that magnetically induced in GdPtBi, leading to a Hall angle of 12 % and 21 % respectively. The synthesis methods, and the transport properties of GdPtBi and Co\(_2\)MnZ are summarized in Table 1. Additionally, the large signal of the anomalous Nernst effect is achievable at lower magnetic fields as it scales beyond the magnetization due to the Berry phase contribution. In Co\(_2\)MnGa, an anomalous Nernst effect material with a remarkably high value Seebeck coefficient (SA\(_{\text{N}} \)) of approximately 6.0 µV K\(^{-1} \) at room temperature, (an order of magnitude higher than for conventional ferromagnetics), was reported. High-throughput searches for large Berry phase contributions close to the Fermi energy have identified several magnetic compounds with the naturally abundant and low-cost element iron, such as the nodal line compounds Fe\(_3\)Sn and Fe\(_3\)Al. The expected high efficiency of lateral thin film devices may pave the way for new large-area energy harvesting technology.

#### Antiferromagnetic compounds

The first magnetically induced Weyl semimetal was realized in the antiferromagnetic half-Heusler compound GdPtBi. GdPtBi and NdPtBi become Weyl semimetals only in applied fields of the order of 2 T (refs. 141–143). Strong signatures typical for magnetic Weyl semimetals were observed in both compounds including the chiral anomaly, the gravitational anomaly, a large non-saturated negative quadratic magnetoresistance for fields of up to 60 T, an unusual intrinsic AHE, and planar Hall effect. In most antiferromagnetic compounds the
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Magnetic ordering is unknown, because large single crystals are needed for neutron scattering. Considering various antiferromagnetic orders enables the construction of a generalized Kane model with a resulting rich phase diagram ranging from Dirac, Weyl and nodal semimetal phases, type-B triple-point phases, topological mirror (or glide), and an antiferromagnetic topological insulating phase. The topological nature of Heusler half-metallic compounds such as MnPtSb and MnPtBi, with a Curie temperature up to $100\,\text{K}$, is still unexplored.

Antiferromagnetic order offers an even richer magnetic phase diagram than does ferromagnetic order. Collinear antiferromagnets with a zero net magnetic moment must have a net-zero Berry curvature, although crossings can be sometimes observed in the band structure of an antiferromagnetic metal. In agreement with this simplified picture, the AHE is absent in nearly all antiferromagnetic metals that have zero magnetization. The three systems, hexagonal Mn$_3$Sn$^{16-19}$, Mn$_3$Ge$^{16-19,131}$ and cubic Mn$_3$Ir$^{16-19}$ have non-collinear triangular antiferromagnetic arrangements, which is the origin of a non-vanishing Berry curvature. Mn$_3$Sn and Mn$_3$Ge have Weyl points close to the Fermi energy, and show the predicted properties of an AHE even at room temperature$^{19,131}$ and exhibit complex Fermi arcs in qualitative agreement with theory$^{19,131}$. The chiral anomaly was also reported in the anisotropic compound Mn$_3$Sn$^{19}$ as was a large anomalous Nernst effect$^{19}$ and magneto-optical Kerr effect$^{19}$. The strong response of the Weyl semimetal compounds to external stimuli makes them promising candidates for topological antiferromagnetic spintronics$^{19}$. Another family of compounds exhibiting an intricate antiferromagnetic order and showing promising experimental signatures in spectroscopy and magnetotransport, such as a singular angular magnetoresistance (SAMR), exceeding 1,000% per radian, is RAIGe with $R = \text{Ce}^{19,131,132}$, Pr$^{15}$ and the non-magnetic La$^{16-19}$. Lastly, we note that the pyroclore iridate family of materials, in which topological Weyl semimetals were first predicted$^{19}$, has remained largely unexplored. Their tetrahedral spin configuration gives rise to rich antiferromagnetic orders$^{19}$ coupled to magnetic topological phases$^{160-162}$ as well as correlated antiferromagnetic Mott states$^{161-162}$. Remarkably, metallic modes$^{19}$ on antiferromagnetic domain walls, thought to be a precursor of Fermi-arc states$^{19}$, were imaged within the otherwise Mott insulating state.

Future directions

Using high-throughput searches$^6$, a more systemic search for magnetic topological materials with high Curie temperatures is important for quantum (computing, sensors) and classical (thermoelectrics, Hall sensors, efficient catalysts) applications. These following additional characteristics should be considered for material selection: (1) topological magnets with high Curie temperatures, which enable a high anomalous Nernst effect signal close to room temperature; (2) low magnetic moment for eliminating stray magnetic fields in devices; (3) hard magnets favourable for AHE and anomalous Nernst effect at zero magnetic field; (4) low-dimensional crystal structure and electronic structures for quantum confinement; and, finally, (5) frustrated atomic arrangement such as kagome lattices for flat bands and non-collinear spin structures. The design of a material that exhibits a high-temperature quantum AHE via quantum confinement of a magnetic Weyl semimetal and its integration into quantum devices is desired. Indeed, several magnetic topological semimetals and insulators are predicted to realize the quantum AHE in the thin-film limit$^{160-162,168}$. The realization of the quantum AHE at room temperature would be revolutionary, overcoming the limitations of data-based technologies, which are affected by large electron-scattering-induced power losses. This would pave the way to new generations of low-energy-consuming quantum electronic and spintronic devices.

Magnetic topological systems are a fertile field for further theoretical discoveries. Although the complete stable topological indices of magnetic and non-magnetic topological insulators, topological crystalline insulators, and topological semimetals have been computed$^{8,9,19,19}$, the magnetic fragile topological indices remain an outstanding problem$^{170}$. A classification of the magnetic obstructed atomic insulators—phases of matter described by bands, which are not topological in the sense that they admit a local Wannier description, but whose Wannier centres do not locate at the atom positions$^{71}$—is still outstanding.

A further fundamental breakthrough would be the development of a framework to predict crossing points of Weyl semimetals or Dirac semimetals that are extremely close to the Fermi energy. The study of topological materials that display incommensurate magnetism is almost non-existent at the present time and should be pursued; it is unclear if in this case other topological classes besides the Chern quantum AHE class exists.

In parallel, an understanding of the magnetic topological responses must also be developed. Although for non-magnetic systems we understand a series of responses such as chiral and gravitational anomaly, quantized photo-galvanic effects in Weyl, and topological defects, we do not yet understand any specific responses that are unique to magnetic systems. A full classification of magnetic topological defects in crystalline topological insulators is absolutely necessary. Predictions of magneto-optical responses are needed, especially in new, rotation-anomaly magnetic topological insulators. Predictions of quantized responses are particularly desired.

The field of magnetic topological superconductivity is also completely open. Magnetic materials already have time-reversal breaking, and so they might exhibit topological excitations such as Majorana zero modes without the need for applying a magnetic field, thereby rendering these systems useful for practical applications. Although study of the types of phases that one obtains by proximitizing the surface of non-magnetic topological systems is mature, similar studies for all magnetic crystalline topological insulators are absent.

The next step is the theoretical introduction of bulk–surface interactions in topological materials. In the bulk, topological metals can give rise to many-body states by tuning interactions. The simplest example is a Weyl charge-density wave axionic insulator, mirroring the non-magnetic experiment$^{12}$. Many-body effects on the surface of magnetic topological insulators could give rise to topologically ordered states of matter.

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Acknowledgements Work from B.A.B. on magnetic topology is mainly supported by DOE grant no. DE-SC0016239. Further support comes from the Schmidt Fund for Innovative Research, Simons Investigator grant no. 404513, the Packard Foundation, the Gordon and Betty Moore Foundation through grant no. GBMF8685 towards the Princeton theory programme, the NSF-EAGER no. DMR-1643312, NSF-MRSEC nos DMR-1420541 and DMR2011750, ONR no. N00014-20-1-2303, BSF Israel US Foundation no. 2018228, and the Princeton Global Network Funds. C.F. was supported by the ERC Advanced grant no. 742068 “TOPMAT” and by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy through the Würzburg–Dresden Cluster of Excellence on Complexity and Topology in Quantum Matter—ct.qmat (EXC 2147, project-id 390858490). H.B. acknowledges support from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement no. 678702) and the German–Israel Foundation (GIF, 1-1364-303.7/2016).

Author contributions B.A.B., C.F. and H.B. wrote the review.

Competing interests The authors declare no competing interests.

Additional information Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s41586-021-04105-x.

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Peer review information Nature thanks Shuang Jia and the other, anonymous, reviewer(s) for their contribution to the peer review of this work.

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