Pairing interactions and vanishing pairing correlations in hot nuclei

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Finite temperature Hartree-Fock-Bogoliubov calculations are performed in Sn isotopes using Skyrme and zero-range, density-dependent pairing interactions. For both stable and very neutron-rich nuclei the critical temperature at which pairing correlations vanish is independent of the volume/surface nature of the pairing interaction. The value of the critical temperature follows approximately the empirical rule $T_c \approx 0.5 \Delta T=0$ for all the calculated isotopes, showing that the critical temperature could be deduced from the pairing gap at zero temperature. On the other hand, the pairing gap at temperatures just below $T_c$ is strongly sensitive to the volume/surface nature of the pairing interaction.

I. INTRODUCTION

The competition between the temperature and pairing correlations in hot nuclei has been studied for more than four decades. The first studies were based on the BCS approximation \cite{1} but later on more involved calculations based on the Bogoliubov approach have been performed \cite{2, 3}. More recently the Bogoliubov approach has been employed together with self-consistent Hartree-Fock mean fields in order to study the pairing properties of hot nuclei. One of the first finite-temperature HFB (FT-HFB) calculations was based on a finite-range force of Gogny type, which is used for describing both the mean field and the pairing properties of hot nuclei \cite{4}. FT-HFB calculations using zero-range forces have been done for hot nuclei \cite{5} and for the inner crust matter of neutron stars \cite{6}. In the latter case the mean field is obtained by using a Skyrme force while the pairing correlations are calculated with a density-dependent delta interaction. Also, shell-model approaches \cite{7, 8} have been used in order to probe the impact of the temperature on both pairing and deformation degrees of freedom.

The interplay between temperature and pairing correlations was also studied intensively in nuclear and neutron matter \cite{9, 10}. Typically, the pairing gaps are calculated in the BCS approximation and using single-particle states determined by self-consistent Brueckner-Hartree-Fock or Green’s function methods (see \cite{11, 12} and references therein). In more fundamental approaches, which go beyond BCS approximation, is still unclear how much the pairing correlations are affected by the medium dependence of the nucleon-nucleon interaction (see \cite{13, 14} and references therein).

One open issue in current HFB calculations is how much the form of the pairing interaction affects the properties of nuclei, especially when one approaches the drip lines. It is also not clear yet if one really needs to introduce an explicit density dependence in the pairing interaction in order to enforce a pairing field evenly distributed in the nucleus ("volume type pairing") or strongly localized in the surface region ("surface type pairing"). Since a realistic pairing force derived from first principles is missing, one hopes to disentangle between various types of pairing forces by analyzing their consequences on measurable quantities. However, up to now these studies are not conclusive. For instance, in Ref. \cite{15} a mixed surface-volume pairing interaction is considered to better explain the odd-even mass differences of some isotopic chains, whereas in Ref. \cite{16} the surface or the volume type of the pairing interaction is found to be not so relevant for the neutron separation energies. It is also worth stressing that the pair density, which gives indications upon the localization of pair correlations in finite nuclei, is not strongly correlated to the surface or volume character of the pairing force but rather to the localization of the single-particle states close to the chemical potential \cite{17}.

Apart from the effects mentioned above, the type of the pairing force could also affect the vanishing of pairing correlations in hot nuclei. Besides constant G studies there have not been such systematic studies with effective density-dependent pairing interactions. It is known that, in a simple BCS approach with a constant pairing G, the vanishing of pairing correlations is expected to occur at $T_c \approx 0.5 \Delta T=0$ \cite{2}. The aim of the present work is to analyze if the volume or surface character of the pairing force could significantly influence vanishing pairing correlations using density-dependent pairing interactions. It should be noted that experimentally, the critical temperature could be extracted from the change of the specific heat in the vanishing pairing correlations region, using level densities measurements, as shown in Refs. \cite{18, 19}.
II. FINITE-TEMPERATURE HARTREE-FOCK-BOGOLIUBOV WITH SKYRME INTERACTIONS

In this work we employ the FT-HFB approach with zero-range forces. Details can be found elsewhere [5, 6] and we recall only the main equations. The FT-HFB equations, in coordinate representation, have the following form:

\[
\begin{pmatrix}
    h_T(r) - \lambda & \Delta_T(r) \\
    \Delta_T(r) & -h_T(r) + \lambda
\end{pmatrix}
\begin{pmatrix}
    U_i(r) \\
    V_i(r)
\end{pmatrix}
= E_i
\begin{pmatrix}
    U_i(r) \\
    V_i(r)
\end{pmatrix},
\]

where \( E_i \) is the quasiparticle energy, \( U_i, V_i \) are the components of the radial FT-HFB wave function and \( \lambda \) is the chemical potential. The quantity \( h_T(r) \) is the thermal averaged mean field Hamiltonian and \( \Delta_T(r) \) is the thermal averaged pairing field. The latter is calculated with a density-dependent contact force of the following form [20]:

\[
V(r - r') = V_0[1 - \eta(\rho(r)/\rho_0)^\alpha]\delta(r - r') \equiv V_{eff}(\rho(r))\delta(r - r'),
\]

where \( \rho(r) \) is the baryonic density. With this force the thermal averaged pairing field is local and is given by:

\[
\Delta_T(r) = \frac{1}{2}V_{eff}(\rho(r))\frac{1}{4\pi}\sum_i(2j_i + 1)U_i^*(r)V_i(r)(1 - 2f_i)
\]

\[
\equiv \frac{1}{2}V_{eff}(\rho(r))\kappa_T(r),
\]

where \( \kappa_T(r) \) is the thermal averaged pairing tensor. Due to the density dependence of the pairing force, the thermal averaged mean field Hamiltonian \( h_T(r) \) depends also on \( \kappa_T \). In addition, the averaged mean field Hamiltonian depends on thermal averaged particle density, spin density and kinetic energy density. The thermal averaged particle density is given by:

\[
\rho_T(r) = \frac{1}{4\pi}\sum_i(2j_i + 1)[V_i^*(r)V_i(r)(1 - f_i) + U_i^*(r)U_i(r)f_i]
\]

where \( f_i = [1 + \exp(E_i/k_BT)]^{-1} \) is the Fermi distribution, \( k_B \) is the Boltzmann constant and \( T \) is the temperature. For the expressions of other densities see Ref. [6].

The FT-HFB equations [1] are solved under a spherical symmetry assumption. The Hartree-Fock mean field is determined by using the Skyrme force SLy4 [21]. For the pairing interaction we choose: \( \rho_0=0.16 \text{ fm}^{-3} \) and \( \alpha=1 \). The parameter \( \eta \) controls the spatial localization of the pairing force. A volume pairing interaction corresponds to \( \eta=0 \), whereas a surface type is given by \( \eta=1 \). The pairing force is used with a quasiparticle energy cutoff equal to 60 MeV and the calculations are performed in a box of radius \( R=18 \text{ fm} \).

III. VANISHING PAIRING CORRELATIONS IN SN AND NI ISOTOPES

The study is performed for the case of Sn isotopes. In what follows we present the results of the FT-HFB calculations, first in some stable isotopes and then for isotopes closer to the neutron-drip line.

Figure 1 shows the pairing field obtained in \(^{124}\text{Sn}\) with surface and volume pairing interactions. In the volume pairing case, the pairing field has similar values around the center and the surface, with a depression in the middle of the nucleus. With a surface pairing interaction, the pairing field exhibits a large peak in the surface of the nucleus. To analyze how the pairing correlations are destroyed by the temperature one usually studies the temperature dependence of the pairing gap. Since in the present calculations the gap is state dependent, one can use as order parameter the averaged gap. Another choice is to use the pairing energy as order parameter. The two alternatives are compatible if the averaged gap is defined by:

\[
\langle \Delta_n \rangle_\kappa = \frac{\int d\kappa T(r)\Delta_{T,r}(r)}{\int d\kappa T(r)}
\]

Figure 2 shows the thermal evolution of the mean neutron gap in \(^{124}\text{Sn}\), in the case of a surface pairing interaction. The critical temperature above which pairing correlations vanish is \( T_c=0.7 \text{ MeV} \). It should be noted that the \( T_c \approx 0.5 \Delta_{T=0} \) rule is still qualitatively verified.
An alternative definition of the pairing gap is to use the particle density $\rho_T$ instead of the pairing density $\kappa_T$:

$$\langle \Delta_n \rangle_{\rho} = \frac{\int d^{3}r \rho_T(r) \Delta_{T,\rho}(r)}{\int d^{3}r \rho_T(r)} \tag{6}$$

However, in this case a too large weight is put on states located deeply below the Fermi level where pairing effects are small. Such a mean pairing gap is displayed in Fig. 2. The value of the gap at T=0 is significantly smaller than with $\kappa$. The volume averaged gap does not follow the BCS rule $T_c \sim 0.5 \Delta_{T=0}$, and it is much closer to $T_c = \Delta_{T=0}$. It shows that using the adequate definition $\Delta_\kappa$ of the mean pairing gap is necessary when predicting temperature effects related to pairing in nuclei. It should be noted that this deviation of $\Delta_\kappa$ from the standard $T_c \sim 0.5 \Delta_{T=0}$ rule is only observed for surface pairing. In the case of a volume pairing interaction, $\Delta_\rho$ and $\Delta_\kappa$ are similar and are both compatible with the $T_c \sim 0.5 \Delta_{T=0}$ rule.

In order to compare the results obtained with volume or surface pairing forces, one needs to give a reasonable prescription for fixing the strength of the interaction. The best choice would be to get for both forces the same odd-even mass difference. The alternative we have taken here was to choose, in a given nuclei, the strength of the prescription for fixing the strength of the interaction. The best choice would be to get for both forces the same $\Delta_\rho$ value.

Figure 3 displays the pairing gaps in $^{104,116,124,128}$Sn calculated with a volume and a surface pairing interaction. They generally exhibit slightly different values at T=0 (10% deviation). These differences depend on the intensity of pairing correlations: $V^{\text{Sur}}_0$ in $^{124}$Sn is somewhat smaller compared to other isotopes (Table I), whereas $V^{\text{Vol}}_0$ doesn’t change very much from $^{104}$Sn to $^{128}$Sn. Therefore the largest differences appear in $^{104}$Sn, where in average the pairing gaps are the largest. On the other hand, in $^{124}$Sn, in which the averaged pairing gap was reduced compared to the other isotopes, the volume and the surface pairing forces give practically the same energy gap at all temperatures. The interesting fact seen in Figure 3 is that the two gaps converge towards similar critical temperatures even in the case when at T=0 the gaps are not the same. One can also notice that for both surface and volume pairing forces the rule $T_c \sim 0.5 \Delta_{T=0}$ gets approximately fulfilled. However, for all nuclei except $^{124}$Sn the pairing gap at $T \leq T_c$ is significantly larger in the case of the surface pairing interaction than in the volume one. It should be noted that this dependence on the nature of the pairing interaction requires that the pairing correlations at T=0 are large enough, which is the case of $^{104,116,128}$Sn. In the recent study of Ref. [17] it has been shown that the pairing density depends on the angular momentum of the states close to the Fermi level. In contrast, Fig. 3 shows that the critical temperature is not sensitive to this feature since the nuclei under consideration correspond to various energy positions of the Fermi level in the N=50-82 shell.

In order to check the dependence of the critical temperature on the Skyrme interaction, FT-HFB calculations have been performed in $^{130}$Sn with the SGII interaction [22] (Fig. 4). No deviation is found in the thermal evolution of the mean gap, compared to the above-mentioned results obtained with the SLY4 interaction. This shows again the stability of the critical temperature. In this case, the surface pairing interaction gives also a larger averaged gap for $T \leq T_c$.

The average gap results from the contributions of various single quasiparticle states. One can have a more microscopic insight of vanishing pairing correlations by looking at the individual pairing gap values. Fig. 4 displays the evolution of the pairing gaps associated with the one quasiparticle levels of the neutron valence shell in $^{104}$Sn, using a surface pairing interaction. The various gap values at T=0 range from 1.3 MeV to 1.8 MeV. However, the behavior of the single quasiparticle gaps with respect to temperature are very similar: they all decrease so as to converge towards the same critical temperature $T_c \simeq 1$ MeV. Hence the critical temperature itself is independent of the l-value of the single quasiparticle state. We have checked the validity of this result on other nuclei such as $^{128}$Sn.

In order to investigate more deeply the role of the individual states, Fig. 5 shows the evolution of the neutron single quasiparticle energies with respect to the temperature in $^{104}$Sn. The relative positions of the energy levels are given by their vicinity to the chemical potential: the states close to the chemical potential have low single quasiparticle energies, and are more sensitive to the pairing gap. On the contrary, states located far from the chemical potential have larger quasiparticle energies and are less sensitive to the pairing gap evolution. One can see a general smooth decrease of the energies with increasing temperature, which is due to the decrease of the pairing gap (see Fig. 5). Around the critical temperature, only the lowest single quasiparticle states such as $2d_{5/2}$ also exhibit a strong decrease. At higher temperatures, the single quasiparticle energies follow the smooth increase of the single particle energies, due to the absence of pairing correlations. This interpretation is also valid for other nuclei, such as $^{128}$Sn.

The behavior of the vanishing of pairing correlations in exotic nuclei close to the drip-lines is an open question. We examine in Fig. 6 the pairing gap of the very neutron-rich nucleus $^{170}$Sn. The results are obtained with the SLY4 interaction and the surface pairing interaction is here adjusted to reproduce the T=0 gap value obtained with the volume type pairing. Once again the critical temperature is predicted to be the same for both volume and surface pairing interactions: $T_c = 0.8$ MeV. This result stresses again the strong stability of the critical temperature with
respect to pairing localization properties, also in very neutron-rich nuclei. There is, however, a difference in the thermal evolution of the two mean gaps in $^{170}$Sn, as noticed in previous Sn isotopes. For instance, at $T=0.7$ MeV the gap is two times higher for a surface pairing interaction than for a volume one. Hence pairing correlations at temperatures below $T_c$ in neutron-rich as well as in stable nuclei may provide information about the surface or volume type of the pairing interaction.

Calculations have also been performed in Ni isotopes, and the results are illustrated here for the case of the $^{84}$Ni nucleus. The $T_c \simeq 0.5\Delta_{T=0}$ rule is still verified, both with surface and volume type interactions. The surface and the volume pairing interactions lead to similar critical temperatures, with 100 keV variation between the two cases. The situation is analogous in $^{104}$Sn (Fig. 3) where the surface and volume interactions give the largest variation in $T_c$, namely 100 keV. This upper limit shows that the absolute value of the critical temperature itself is rather independent of the nature of the pairing interaction. However, if measurements can reach such a resolution it would be of high interest to measure pairing properties at temperatures located just below $T_c$: for instance, in $^{104}$Sn at $T=0.8$ MeV, the pairing gap is close to zero in the volume case whereas the gap remains $\Delta=1$ MeV in the surface case. This is also the case for $^{84}$Ni.

Fig. 7 displays the specific heat for $^{84}$Ni, obtained in the FT-HFB calculations. Due to the finite step in temperature used in the HFB calculations, the specific heat displays a kink at the critical temperature instead of the usual singularity. Actually, due to the finite size of the nucleus, the specific heat should have a smooth shape behavior around the critical temperature $T_c$. In order to get this behavior in the calculations, one needs to go beyond the HFB approach, e.g., projecting out the number of particles and taking into account the thermal fluctuations [23].

IV. CONCLUSION

In conclusion, the critical temperature for vanishing of pairing correlations in hot nuclei appears to be rather insensitive to the surface or volume localization of the pairing force used in FT-HFB calculations. For all the neutron-rich tin isotopes studied, we have found that the critical temperature is given approximatively by $T_c \simeq 0.5\Delta_{T=0}$ for both type of pairing forces and for stable and unstable nuclei. Hence, the critical temperature could be deduced from the gap value at zero temperature. On the other hand, the pairing gap is strongly sensitive to the nature of the pairing interaction for temperatures just below the vanishing of pairing correlations, for the large majority of nuclei studied. This result should open an experimental investigation for the pairing interaction in nuclei.

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TABLE I:

$V_0$ values of Eq. (2) corresponding to volume and surface pairing interactions, for the different nuclei considered

|       | $^{104}$Sn | $^{116}$Sn | $^{124}$Sn | $^{128}$Sn | $^{130}$Sn | $^{170}$Sn | $^{54}$Ni |
|-------|------------|------------|------------|------------|------------|------------|------------|
| $V_0^{Vol}$ (MeV.fm$^3$) | -220       | -200       | -197       | -220       | -220       | -220       | -240       |
| $V_0^{Surf}$ (MeV.fm$^3$) | -580       | -520       | -490       | -570       | -575       | -510       | -502       |

FIG. 1: Pairing field at T=0 in $^{124}$Sn, calculated with the HFB approach and the SLy4 interaction. Solid and dashed lines correspond to volume and surface interactions, respectively.

FIG. 2: Mean value of the neutron pairing gap in $^{124}$Sn, calculated with Eq. (5) (solid line) and Eq. (6) (dashed line).
FIG. 3: Mean value of the neutron pairing gap in $^{104,116,124,128}\text{Sn}$, calculated with a volume pairing interaction (solid line) and a surface one (dashed line).

FIG. 4: Mean value of the neutron pairing gap in $^{130}\text{Sn}$ (with SGII interaction) and $^{170}\text{Sn}$ (with SLy4 interaction), calculated with a volume pairing interaction (solid line) and a surface one (dashed line).

FIG. 5: Neutron pairing gaps in $^{104}\text{Sn}$ corresponding to the quasiparticle states of the N=50-82 valence shell.

FIG. 6: Neutron single quasiparticle energies in $^{104}\text{Sn}$ corresponding to the states of the N=50-82 valence shell.

FIG. 7: Specific heat in $^{84}\text{Ni}$ calculated with a surface pairing interaction.
$\Delta$ (MeV) vs. $T$ (MeV) for $^{130}$Sn and $^{170}$Sn.

- Solid line: Volume
- Dotted line: Surface
