A pedestal temperature model with self-consistent calculation of safety factor and magnetic shear

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Abstract. A pedestal model based on theory-motivated models for the pedestal width and the pedestal pressure gradient is developed for the temperature at the top of the \(H\)-mode pedestal. The pedestal width model based on magnetic shear and flow shear stabilization is used in this study, where the pedestal pressure gradient is assumed to be limited by first stability of infinite \(n\) ballooning mode instability. This pedestal model is implemented in the 1.5D BALDUR integrated predictive modeling code, where the safety factor and magnetic shear are solved self-consistently in both core and pedestal regions. With the self-consistently approach for calculating safety factor and magnetic shear, the effect of bootstrap current can be correctly included in the pedestal model. The pedestal model is used to provide the boundary conditions in the simulations and the Multi-mode core transport model is used to describe the core transport. This new integrated modeling procedure of the BALDUR code is used to predict the temperature and density profiles of 26 \(H\)-mode discharges. Simulations are carried out for 13 discharges in the Joint European Torus and 13 discharges in the DIII-D tokamak. The average root-mean-square deviation between experimental data and the predicted profiles of the temperature and the density, normalized by their central values, is found to be about 14%.

1. Introduction

It is known that when a plasma discharge is heated with sufficiently high heating power, it exhibits a spontaneous transition from a regime of confinement called low-mode (\(L\)-mode) to a regime with markedly improved confinement called high-mode (\(H\)-mode) \([1]\). The change in the plasma first appears at the edge of the plasma, where a region of steep gradients in temperature and density is formed. This steep gradient region, called pedestal, is caused by a transport barrier that forms near the edge of the plasma. The pedestal region is important because the height of the pedestal strongly influences the confinement of the core plasma. It has been found in integrated modeling simulations that the same core transport model used in simulations of \(L\)-mode discharges can also be used in simulations of \(H\)-mode discharges, with equally good agreement with experimental data, as long as the simulation boundary conditions are given at the top of the pedestal at the edge of the \(H\)-mode discharges \([2]\). It has also been found in both experiments and simulations of \(H\)-mode plasmas that the height of the pedestal has a significant impact on the shape of the temperature and density profiles and, consequently, a large effect on the global confinement scaling \([3-5]\). However, the need to use experimental data to provide temperature and density at the boundary of the integrated
modeling simulations reduces the predictive capability of these simulations. Therefore, models are needed for the pedestal boundary conditions in order to make the integrated modeling simulations more predictive. It is important to develop more completely predictive integrated modeling codes in order to understand plasma confinement in present day tokamaks and to simulate new fusion experiments and future fusion reactor designs such as ITER.

Models that can be used to predict the temperature and density at the top of the pedestal have recently been developed and calibrated against experimental data [6-9]. In reference [6], several pedestal temperature models were developed using a theoretical approach. The magnetic shear and safety factor are calculated at one pedestal width away from the separatrix, where the safety factor profile is assumed. The predictions of the best model agree with experiment data with 30% RMSE. Note that the definition of RMSE can be found in reference [6]. Later, in reference [10], one of the best pedestal temperature models in reference [6] was chosen to combine with predictive models for the core plasma to simulate existing tokamak experimental results. That pedestal temperature model is based on magnetic and flow shear stabilization width model. This pedestal model was implemented in the BALDUR integrated modeling code [11] using an automated procedure to simulate the transition from the L-mode to H-mode, once the heating power rises above the H-mode threshold. When the plasma is in the H-mode state, the predicted values for the temperature and density at the top of the pedestal are used as boundary conditions in the simulations. The average relative rms deviation between experimental data and the predicted profiles of temperature and density, normalized by central values, is found to be about 10%. Even though the procedure developed in reference [10] successfully reproduced various experimental data, the confidence in its predictions, especially when it is applied to a new plasma regime such as burning plasma experiment like ITER, is quite limited. One weak point of this procedure in reference [10] is the estimation of magnetic shear and safety factor in the pedestal temperature calculation, which is different from that used in the rest of the code. Note that the magnetic shear and safety factor play important roles in the maximum pedestal pressure gradient estimation. In addition, the pedestal temperature models in reference [6] were compared directly with the pedestal experimental data, which it is known to contain high error values, in order to determine a pedestal width constant. In this work, we examine a new approach for calculating magnetic shear and safety factor to be used in an estimation of a pedestal temperature. The pedestal model used in this experiment is one of the best pedestal temperature models in reference [6]. This pedestal temperature model is derived based on the “magnetic shear and flow shear stabilization width concept” and the “1st stability regime of infinite n ballooning mode pressure gradient limit concept”. The magnetic shear and safety factor used in the pedestal model are taken directly from the BALDUR prediction, where the time-dependent equilibrium and plasma profile effects are properly included. Moreover, the pedestal width constant will be determined by minimizing the average relative rms deviation of the ion core temperature profile, in which tends to be more accurate measurements than the pedestal measurements.

This paper is organized as follows: Brief descriptions for a BALDUR integrated predictive modeling code, Multi-mode transport models, and pedestal models are given in section 2. A statistical analysis of the temperature and density profiles produced by simulation, using the MMM95 and the pedestal model together, compared with experimental data, is presented in section 3, while conclusions are given in section 4.

2. BALDUR integrated predictive modeling code

The BALDUR integrated predictive modeling code is used to compute the time evolution of plasma profiles including electron and ion temperatures, deuterium and tritium densities, helium and impurity densities, current density, neutrals, and fast ions. These time-evolving profiles are computed in the BALDUR integrated predictive modeling code by combining the effects of many physical processes self-consistently, including the effects of transport, plasma heating, particle influx, boundary conditions, the plasma equilibrium shape, and sawtooth oscillations. Fusion heating and helium ash accumulation are computed self-consistently. The BALDUR simulations have been intensively
compared against various plasma experiments, which yield an over all agreement with 10% relative rms deviation \cite{2,12}. In the BALDUR code, fusion heating power is determined using the nuclear reaction rates and a Fokker Planck package to compute the slowing down spectrum of fast alpha particles on each flux surface in the plasma \cite{11}. The fusion heating component of the BALDUR code also computes the rate of the production of thermal helium ions and the rate of the depletion of deuterium and tritium ions within the plasma core. In this work, an MMM95 anomalous core transport model in BALDUR will be used to carry out simulations. The descriptions of this core transport model and the pedestal temperature model are described below.

2.1. Multi-mode core transport model
The MMM95 (Multi-Mode Model 95) \cite{13} is a combination of theory-based transport models which consists of the Weiland model for the ion temperature gradient (ITG) and trapped electron modes (TEM), the Guzdar–Drake model for drift-resistive ballooning modes, as well as a smaller contribution from kinetic ballooning modes. The Weiland model for drift modes such as ITG and TEM modes usually provides the largest contribution to the MMM95 transport model in most of the plasma core. The Weiland model is derived by linearizing the fluid equations, with magnetic drifts for each plasma species. Eigenvalues and eigenvectors computed from these fluid equations are then used to compute a quasilinear approximation for the thermal and particle transport fluxes. The Weiland model includes many different physical phenomena such as effects of trapped electrons, $T_e/T_i$ ratio, impurities, fast ions, and finite $\beta$. The resistive ballooning model in MMM95 transport model is based on the $E\times B$ drift-resistive ballooning mode model by Guzdar–Drake at 1993, in which the transport is proportional to the pressure gradient and collisionality. The details of these models can be found in reference \cite{13}. The contribution from the resistive ballooning model usually dominates the transport near the plasma edge. Finally, the kinetic ballooning model is a semi-empirical model, which usually provides a small contribution to the total diffusivity throughout the plasma, except near the magnetic axis. This model is an approximation to the first ballooning mode stability limit. All the anomalous transport contributions to the MMM95 transport model are multiplied by $k^{-4}$, since the models were originally derived for circular plasmas. More information about inclusion of elongation effect can be obtained in reference \cite{13}.

2.2. Pedestal models
A complete description of the pedestal width model, based on the magnetic and flow shear stabilization, that is used in this paper to predict the temperature at the top of the pedestal at the edge of the type I ELMy H-mode plasmas is given in reference \cite{14}. In the development of the pedestal temperature models described in reference \cite{6}, two ingredients are required: the pedestal width ($\Delta$) and the pedestal pressure gradient ($\partial p/\partial r$). If the pedestal density ($n_{ped}$) is known, the temperature at the top of the pedestal ($T_{ped}$) can be estimated as

$$T_{ped} = \frac{1}{2n_{ped}k} \left[ \frac{\partial p}{\partial r} \right] \Delta = \frac{\Delta}{2kn_{ped}} \frac{\alpha_c B_T^2}{2\mu_0 R q^2},$$

where $k$ is the Boltzmann constant, $\mu_0$ is the permeability of free space, $\alpha_c$ is the normalized critical pressure gradient, $B_T$ is the magnetic field, $R$ is the major radius and $q$ is the safety factor. The width of the pedestal is assumed to be determined by a combination of magnetic and flow shear stabilization of drift modes \cite{14}:

$$\Delta = C_{\mu} \rho s^2,$$

where $C_{\mu}$ is a constant of proportionality, $\rho$ is the ion gyroradius, and $s$ is the magnetic shear. Note that the magnetic shear in equation (2) is calculated at the top of the pedestal. The pedestal pressure
gradient is assumed to be uniform throughout the pedestal region and it is limited by the first stability limit of infinite n ballooning mode, so that the normalized critical pressure gradient for the pedestal region is given by

\[ \alpha_c = 0.4s(1 + \kappa_{95}^2(1 + 5\delta_{95}^2)) \]

where \( \kappa_{95} \) is the elongation at the 95% flux surface, and \( \delta_{95} \) is the triangularity at the 95% flux surface. Therefore, the pedestal temperature takes the following form:

\[ T_{ped}(keV) = 0.323C_W^2\left( \frac{B_T}{q^2} \right)\left( \frac{M_i}{R^2} \right)\left( \frac{\alpha_c}{n_{ped,19}} \right)^2s^4, \]

where \( n_{ped,19} \) is the electron density at the top of the pedestal in units of \( 10^{19} m^{-3} \), and \( M_i \) is the ion mass. The constant \( C_W \) is the constant of proportionality in equation (2). The constant \( C_W \) is chosen so as to optimize the agreement between the experimental data of ion temperature profile and the simulation results for ion temperature profile by minimizing the average relative rms. This pedestal model is implemented in the 1.5D BALDUR integrated predictive modeling code, where the safety factor and magnetic shear are solved self-consistently in both core and pedestal regions. With the self-consistently approach for calculating safety factor and magnetic shear, the effect of bootstrap current can be correctly included in the pedestal model.

3. Simulation results and discussion

BALDUR integrated modeling simulations were carried out for the 26 H-mode discharges (13 JET and 13 DIII-D) in various scans, using a combination of the new pedestal temperature model together with the MMM95 core transport model. These discharges are taken from the International Profile Database [15]. In this work, the electron pedestal temperature is assumed to be equal to the ion pedestal temperature. The pedestal density is taken directly from the experiment. For each of the profiles (ion temperature, electron temperature, and electron density), the normalized deviation, \( \varepsilon_j \), between the \( j^{th} \) experimental data point \( X_j^{exp} \) and the simulation result \( X_j^{sim}(R_j) \) at the major radius \( R_j \) of the corresponding experimental data point is defined as

\[ \varepsilon_j = \frac{X_j^{sim}(R_j) - X_j^{exp}}{X_{Max}^{exp}}. \]

In order to give all the deviations across each profile equal weight, each deviation is normalized by the maximum experimentally measured value for that given profile, \( X_{Max}^{exp} \). An alternative choice of normalizing each temperature deviation by the local temperature, for example, would magnify deviations near the edge of the plasma, where temperatures are lower. The relative rms deviation \( \sigma \) and the offset \( f \) between the profile resulting from each simulation and the corresponding experimental data is defined as

\[ \sigma = \sqrt{\frac{1}{N} \sum_{j=1}^{N} \varepsilon_j^2}, \]

and

\[ f = \frac{1}{N} \sum_{j=1}^{N} \varepsilon_j, \]
where \( N \) is the number of experimental data points in a profile. The relative rms deviation (\( \sigma \)) and offset are evaluated for each of the temperature profiles for the discharges in consideration.

In figure 1, the average relative rms deviations for the ion temperature profiles of 26 \( H \)-mode discharges are plotted as a function of the pedestal width constant \( C_W \). It can be seen that this function is close to a quadratic equation. The best fit with the quadratic equation is found to be:

\[
\sigma(C_W) = 60.0C_W^{-2} - 56.1C_W - 27.4.
\]

The minimum point for the function in equation (8) can be found at the pedestal width constant of 0.47 and the lowest average rms deviation is about 14.4%.

![Figure 1](image)

**Figure 1.** The average relative rms deviation for the ion temperature is plotted as a function of the pedestal width constant \( C_W \).

The details of the statistical analysis using relative rms deviations for simulations using the pedestal model based on the magnetic and flow shear stabilization with \( C_W = 0.47 \) for ion and electron temperatures are presented in figures 2 and 3, respectively. Note that the DIII-D discharges are from discharge number 77557 to discharge number 99411 and the JET discharges are from discharge number 32745 to discharge number 38415. It can be seen in figure 2 that the relative rms deviations for ion temperature vary from one discharge to another, and from the profile to another, with a minimum of about 3.56% and a maximum of about 37.82% for ion temperatures and with a minimum of about 5.13% and a maximum of about 51.42% for electron temperatures. The average relative rms deviation \( \sigma_{avg} \) — averaged over 26 discharges — for the ion temperature is about 14.39% for the ion temperature profiles and about 15.51% for the electron temperature profiles. It is worth noting that in this paper, an average relative rms is chosen to represent the results. However, a median of relative rms could be another good choice. The discussion of this can be made in the future report. It can be noticed that for DIII-D discharge 90108 and JET discharges 34340, 35171, and 35174, the relative rms deviation is higher than 20%. The predicted pedestal in DIII-D discharge 90108 and JET discharge 34340 are in the range of experimental value. However, the predictions of the central ion and electron temperatures are quite low. For JET discharges 35171 and 35174, the predictions of pedestal are significantly higher than those obtained in the corresponding experiment. This is subjected to a further investigation.
Figure 2. Relative rms deviations (%) for the ion temperature profiles produced by simulations using the MMM95 transport model combined with the pedestal model (using $C_W=0.47$) compared with experimental data for 26 $H$-mode discharges from DIII-D and JET experiments.

Figure 3. Relative rms deviations (%) for the electron temperature profiles produced by simulations using the MMM95 transport model combined with the pedestal model (using $C_W=0.47$) compared with experimental data for 26 $H$-mode discharges from DIII-D and JET experiments.

In figures 4 and 5, the relative offsets for simulations using the pedestal model with $C_W=0.47$ are presented for ion and electron temperatures, respectively. It can be seen in figure 4 that the offset for ion temperature also vary from one discharge to another, and from the profile to another, with a minimum of about -1.84% and a maximum of about 0.84% for ion temperatures and with a minimum of about -2.03% and a maximum of about 1.11% for electron temperatures. The average offset over 26 discharges for the ion temperature is about -0.20% for the ion temperature profiles and about 0.01% for the electron temperature profiles.
Ion temperature profiles are shown in figure 6 from the simulation using the new pedestal model based on magnetic and flow shear stabilization and the experimental data for a DIII-D discharge. It can be seen that the simulation profile yields good agreement with experimental data. For figure 7, ion temperature profile from the simulation using the new pedestal model is compared with experimental data for a JET discharge. It can be seen that the simulation profile also yields good agreement with experimental data as shown in figure 7 to figure 8, the conditions at the boundary of the simulations that used the new pedestal model nearly match the corresponding experimental data. The major cause of the difference between the simulated profiles and the experimental data is to be clarified in near future; core transport models, models of sources/sinks or the new pedestal model.

The summary agreement between the predicted pedestal and experiment values is shown in figure 8. It can be seen that the predictions are in the range of the experimental values. The model tends to over predict the pedestal values in DIII-D experiments and underpredict the pedestal values in JET experiments. This might due to different nature of the pedestal in those tokamaks. The RMSE of the predicted pedestal temperature and the experimental data for this set is about 43.5%, which is higher than that found in reference [10]. The increase of RMSE might due to the error in the calculation of magnetic shear and safety factor in BALDUR code. The pedestal prediction with this model depends sensitively on a calculation of magnetic shear and safety factor since both parameters appear in pedestal width and pedestal pressure estimations. However, the discussion on the issue of the
magnetic shear and safety factor calculation in BALDUR code is beyond the scope of this paper. It rather leaves this discussion for the future work.

**Figure 6.** The ion temperature profile is shown as a function of major radius for simulations of the low $\kappa$ DIII-D discharge 81499. The closed squares represent experimental data, while the solid curve is the result of simulation.

**Figure 7.** The ion temperature profile is shown as a function of major radius for simulations of the low $\beta$ JET discharge 38407.

**Figure 8.** Comparison between the predicted ion temperature and corresponding experimental values from 26 $H$-mode discharges.
4. Conclusions
A pedestal model based on theory-motivated models for the pedestal width and the pedestal pressure gradient is developed for the temperature at the top of the H-mode pedestal. The pedestal width model based on magnetic shear and flow shear stabilization is used in this study, where the pedestal pressure gradient is assumed to be limited by first stability of infinite n ballooning mode instability. This pedestal model is implemented in the 1.5D BALDUR integrated predictive modeling code, where the safety factor and magnetic shear are solved self-consistently in both core and pedestal regions. With the self-consistently approach for calculating safety factor and magnetic shear, the effect of bootstrap current can be correctly included in the pedestal model. The pedestal model is used to provide the boundary conditions in the simulations and the Multi-mode core transport model is used to describe the core transport. This new integrated modeling procedure of the BALDUR code is used to predict the temperature and density profiles of 26 H-mode discharges. Simulations are carried out for 13 discharges in the Joint European Torus and 13 discharges in the DIII-D tokamak. It is found that the average root-mean-square deviation between experimental data and the predicted profiles of the temperature and the density, normalized by their central values, is about 14%.

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