Factorization and resummation for single color-octet scalar production at the LHC

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Abstract

Heavy colored scalar particles appear in a variety of new physics (NP) models and could be produced at the Large Hadron Collider (LHC). Knowing the total production cross section is important for searching for these states and establishing bounds on their masses and couplings. Using soft-collinear effective theory, we derive a factorization theorem for the process \( pp \rightarrow SX \), where \( S \) is a color-octet scalar, that is applicable to any NP model provided the dominant production mechanism is gluon-gluon fusion. The factorized result for the inclusive cross section is similar to that for the Standard Model Higgs production, however, differences arise due to color exchange between initial and final states. We provide formulae for the total cross section with large (partonic) threshold logarithms resummed to next-to-leading logarithm (NLL) accuracy. The resulting \( K \)-factors are similar to those found in Higgs production. We apply our formalism to the Manohar-Wise model and find that the NLL cross section is roughly 2 times (3 times) as large as the leading order cross section for a color-octet scalar of mass of 500 GeV (3 TeV). A similar enhancement should appear in any NP model with color-octet scalars.

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Discovering the Higgs particle and the mechanism of electroweak symmetry breaking is one of the major goals of the Large Hadron Collider (LHC). It is well-known that the main Higgs production mechanism is the gluon-gluon fusion process. An important issue in determining the total production cross section is the large perturbative corrections in the threshold region, defined by \( z \to 1 \), where \( z = m_H^2 / \hat{s} \), where \( m_H \) is the Higgs mass and \( \hat{s} \) is the partonic center of mass energy squared. The leading corrections are enhanced by factors of \( \log(1 - z) / (1 - z) \) and invalidate fixed order perturbation theory in the threshold region. These corrections can significantly affect the normalization of the total cross section even though the total cross section receives contributions from a range of \( \hat{s} \) [1, 2, 3, 4]. In the threshold region, the inclusive scattering cross section \( \sigma(pp \to HX) \) can be factorized (at leading twist) into a hard part, soft part, and parton distribution function (PDF) of gluons inside the proton [5], and the renormalization group equations (RGE) for these parts can be used to resum the large threshold corrections. For the Higgs production cross section the calculations up to the next-to-next-leading logarithm (NNLL) accuracy have already been performed [5, 6] and give total cross section about three times bigger than predicted at leading order.

Obviously, properly incorporating these effects will be important for other heavy particles predicted in theories of New Physics (NP) that may be observed at the LHC. In this paper we will focus on the production of a heavy color-octet scalar, which appears in a number of NP models such as grand unified theories [7, 8, 9], supersymmetric theories [10, 11], Pati-Salam unification [12, 13], chiral color [14], and topcolor [15]. We will calculate the cross section for the Manohar-Wise model [16] of color-octet scalars which is consistent with the principle of Minimal Flavor Violation (MFV) [17, 18]. Performing the resummation for color-octet scalars is very similar to the resummation for Higgs [1, 3, 4]. Similar resummations have been performed for squark-antisquark and gluino-pair production cross sections in Refs. [19, 20]. We will use Soft-Collinear Effective Theory (SCET) [21, 22, 23] to derive a factorization theorem for the cross section. Gluon-gluon fusion cross sections in the full theory are matched onto SCET operators. The matching coefficients for these operators will differ between various models, but the structure of these operators is universal. The cross section computed with the SCET operators factors into correlation functions of SCET collinear and soft fields, and renormalization group equations for these correlation functions can be used to perform the resummation of the threshold logarithms. This resummation procedure is independent of the NP model.

Before we discuss the factorization theorem, we will describe some details of the color-octet scalar model we will be focusing on. The principle of MFV requires that the Yukawa couplings of the color-octet scalars be proportional to the Yukawa matrices in the Standard Model. The Standard Model Yukawa couplings are

\[
\mathcal{L} = -g_D^D \bar{d}_{Ri} H^\dagger Q_{Lj} - g_U^U \bar{u}_{Ri} \epsilon H^\ast Q_{Lj} + h.c.,
\]

where \( Q_L \) is the doublet of lefthanded quarks, \( u_R \) and \( d_R \) are the righthanded up and down
quarks, respectively, \( i \) and \( j \) are flavor indices, and \( H \) is the Higgs doublet:

\[
H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad \epsilon H^* = \begin{pmatrix} H^{0*} \\ -H^- \end{pmatrix}.
\] (2)

If the color-octet scalar Yukawa couplings are

\[
\mathcal{L} = -\lambda^{\text{U,D}}_{ij} \bar{d}_{Ri} S^a T^a Q_{Lj} - \lambda^{\text{U,D}}_{ij} \bar{u}_{Ri} \epsilon S^a T^a Q_{Lj} + \text{h.c.},
\] (3)

then the MFV hypothesis requires \( \lambda^{\text{U,D}}_{ij} = \eta^{\text{U,D}}_{ij} g^{\text{U,D}}_{ij} \), where \( \eta^{\text{U,D}}_{ij} \) are constants. This eliminates tree-level flavor changing neutral currents and ensures that experimental constraints from flavor physics are not violated. Note that this also implies that the \( S^a \) couple most strongly to the third generation of quarks. The color-octet scalars have gauge couplings to gluons and a gauge invariant mass term

\[
\mathcal{L}_{\text{QCD}} = -\frac{1}{2} S^a (D^2)^{ac} S^c - \frac{1}{2} m^2_S S^a S^a,
\] (4)

where \( D_{\mu}^{ac} = \partial_{\mu} \delta^{ac} + g f^{abc} A^b_{\mu} \). Finally, there is a scalar potential for the \( S^a \) and the Higgs that can be found in Ref. [16].

Though it may seem ad hoc to impose \( \lambda^{\text{U,D}}_{ij} \propto g^{\text{U,D}}_{ij} \), this can arise naturally in certain models. For example, consider the chiral color model of Ref. [14]. In this model, the gauge group is enlarged to \( SU(3)_L \times SU(3)_R \times SU(2)_L \times U(1)_Y \), and the chiral color group \( SU(3)_L \times SU(3)_R \) breaks down to \( SU(3)_c \) at some high energy scale. If the righthanded quarks are placed in the \((1, 3, 1)\) representation of \( SU(3)_L \times SU(3)_R \times SU(2)_L \), and the lefthanded quarks are placed in the \((3, 1, 2)\), then quarks can obtain masses from the following Yukawa couplings

\[
\mathcal{L} = -\sqrt{3} g^{\text{U,D}}_{ij} \bar{d}_{Ri} \Phi^i Q_{Lj} - \sqrt{3} g^{\text{U,D}}_{ij} \bar{u}_{Ri} \epsilon \Phi^i Q_{Lj} + \text{h.c.},
\] (5)

where \( \Phi \) and \( \Phi' \) transform in the \((3, 3, 2)\) and \((\bar{3}, 3, 2)\), respectively. The fields \( \Phi \), \( \Phi' \) can be decomposed into singlet and octet scalars under the unbroken \( SU(3)_c \).

\[
\Phi^{(i)} = \frac{1}{\sqrt{3}} H^{(i)} + S^{a(i)} T^a
\] (6)

and \( \lambda^{\text{U,D}}_{ij} = \sqrt{3} g^{\text{U,D}}_{ij} \) at tree level. Note that this model is different from the Manohar-Wise model since there are two distinct color-octet scalars, \( S^a \) and \( S^{a'} \). If the gluon-gluon fusion production of a single color-octet scalar proceeds through a top-quark loop, this mechanism will predominantly produce \( S^{a'} \). We do not wish to study this model further, but mention it to show that the MFV constraint \( \lambda^{\text{U,D}}_{ij} \propto g^{\text{U,D}}_{ij} \) could emerge as a consequence of a symmetry of the underlying theory.

The color-octet scalars can be produced via the pair production cross section, \( gg \to SS \) which proceeds through gauge couplings [16]. Constraints on pair production followed by

\[1\] Note that an additional scalar in the \((1, 1, 2)\) is required to give masses to leptons.
decay to heavy quarks has been used to establish a lower bound on $m_S$ of about 200 GeV in Ref. [25]. Neutral color-octet scalars can also be produced singly via gluon-gluon fusion [24]. This proceeds through loop diagrams containing quarks, of which the top quark gives by far the dominant contribution, and loops with color-octet scalars. The relative size of the top quark and scalar loop contributions is determined by $\eta_U$ and other parameters in the scalar potential. If these parameters are all taken to be of order unity then the top quark loop is the largest contribution. In this case the production mechanism is very similar to that for a single Higgs, and hence threshold corrections are expected to be significant. At the LHC the production cross section for single $S$ production is larger than pair production when the mass of $S$ is larger than 1 TeV [24].

When applying SCET to color-octet scalar production we first match full QCD onto SCET operators at the hard scale $\mu_h \sim m_S$, where $m_S$ is the mass of the color-octet field. SCET is formulated as an expansion in $\lambda \sim \sqrt{\mu_s/\mu_h}$, where $\mu_s \sim m_S(1-z)$. The allowed SCET operators are constrained by SCET gauge symmetries [26]. At leading order in $\lambda$, we find two dimension-5 operators with different color structures. Subleading operators will be constrained by the requirement of reparametrization invariance [27].

Our result for the factorized scattering cross section is

$$\sigma(pp \rightarrow SX) = \tau H(m_S, \mu_f) \int_\tau^1 \frac{dz}{z} \tilde{S}(m_S(1-z), \mu_f) F(\tau/z, \mu_f).$$  \hspace{1cm} (7)$$

Here $\tau = m_S^2/s$ where $s$ is the center of mass energy squared at the LHC, $H(m_S, \mu)$ and $\tilde{S}(m_S(1-z), \mu)$ are the hard and soft functions respectively, and $F(x, \mu)$ is the following convolution of the gluon PDF's:

$$F(x, \mu_f) = \int_x^1 \frac{dy}{y} f_{g/P}(y, \mu_f) f_{g/P}(x/y, \mu_f).$$  \hspace{1cm} (8)$$

Renormalization group equations for the $H(m_S, \mu_f)$, $\tilde{S}(m_S(1-z), \mu_f)$, and $F(\tau/z, \mu_f)$ can be used to resum large threshold corrections as will be discussed below.

In order to prove the factorization theorem in Eq. (7) we need to construct the SCET operators composed of the collinear gluons from the initial state hadrons, soft gluons, and a heavy color-octet scalar field. In the center of the mass frame, the incoming gluons from the two incoming protons are described as $n$ and $\bar{n}$-collinear fields where the light-cone vectors satisfy $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$. The lowest dimension operator with a single $n$-collinear gluon that is $n$-collinear gauge invariant is $W_n^\dagger G_n^{\mu\nu} W_n$. Here $G_\mu^{\nu} = G_a^{\mu\nu} T^a$ is a SCET gluon field strength tensor, and $W_n$ is the collinear Wilson line

$$W_n(x) = P \exp \left( i g \int_{-\infty}^x ds \; \bar{n} \cdot A_n^a(s\bar{n}^\prime)^T T^a \right).$$  \hspace{1cm} (9)$$

Similarly, the lowest dimension $\bar{n}$-collinear gauge invariant operator is $W_{\bar{n}}^\dagger G_{\bar{n}}^{\mu\nu} W_{\bar{n}}$. Combining $W_n^\dagger G_n^{\mu\nu} W_n$ and $W_{\bar{n}}^\dagger G_{\bar{n}}^{\mu\nu} W_{\bar{n}}$ into a Lorentz scalar and then expanding to lowest order in

\footnote{If the color-octet is a pseudoscalar only the top quark loop contributes.}
\[ (W_n^\dagger G_{n}^{\mu\nu}W_n)_{\alpha\beta} = \left( W_n^\dagger G_{n,\mu}^\pi W_n \right)_{\gamma\delta} = - \frac{1}{g^2} \left( W_n^\dagger \left[ iD_{n,\mu}, iD_{n,\nu}^\pi \right] W_n \right)_{\alpha\beta} \left( W_n^\dagger \left[ iD_{\pi,\mu}, iD_{\pi,\nu}^\prime \right] W_n \right)_{\gamma\delta} \]
\[ = - \left( B_{n,\mu}^{\perp} \right)_{\alpha\beta} \left( B_{n,\mu}^{\perp} \right)_{\gamma\delta} + \mathcal{O}(\lambda). \]

Here \( \alpha, \beta, \gamma, \) and \( \delta \) are color indices in the fundamental representation and \( B_{n,\mu}^{\perp} \) is

\[ B_{n,\mu}^{\perp} = \frac{1}{g} \left[ \overline{\pi} \cdot \mathcal{P} W_n^\dagger iD_{n,\mu} W_n \right], \]

where the derivative operator \( \mathcal{P}^\mu \) returns the large label momentum and only acts on collinear fields within the brackets \( [\cdots] \). It will be convenient to write the field \( B_{n,\mu}^{\perp} \) in terms of the Wilson line in the adjoint representation (i.e. with color generator \( (t^a)_{bc} = - if^{abc} \)). Defining \( B_{n,\mu}^{\perp} = B_{n,\mu}^{\perp,T^a} \), \( B_{n,\mu}^{a,\mu} \) is given by

\[ B_{n,\perp}^{a,\mu} = \alpha \overline{\pi} g^{\mu \nu} W_n^\dagger G_{n,\mu \nu}^{b} = \alpha \overline{\pi} g^{\mu \nu} G_{n,\mu \nu}^{b} W_n^\dagger, \]

where \( W_n \) is the collinear Wilson line in the adjoint representation, and we used the relation \( W_n^{ab} = W_n^{bca} \) for the second equality. For the \( \overline{\pi} \)-collinear fields, \( B_{n,\perp}^{a,\mu} \) and \( B_{n,\mu}^{a,\mu} \) are identical to \( B_{n,\mu}^{\perp} \) and \( B_{n,\mu}^{a,\mu} \), respectively, after interchanging \( n \) and \( \overline{\pi} \).

Finally, we need to include fields for the color-octet scalar. The strong interactions of this field are described by Eq. (4). At scales well below \( m_s \), the strong interactions of the heavy color-octet scalar simplify because the scalar is slowly moving. In the threshold region, the \( S^a \) is produced nearly at rest (in the parton center-of-mass frame) and heavy particle effective theory techniques can be applied. We use a heavy scalar effective theory (HSET), similar to heavy quark effective theory (HQET). In HSET, the scalar momentum is decomposed into large and small parts: \( p_s^\mu = m_s v^\mu + k^\mu \), where \( v^\mu \) is the static four-velocity and \( k^\mu \) represents fluctuations of \( O(\mu_s) \). In order for derivatives in HSET to bring factors of \( k^\mu \) rather than the total momentum, we use the standard rephasing trick to relate full theory and HSET fields:

\[ S^a(x) = \frac{1}{\sqrt{2m_S}} \left( e^{-im_s v \cdot x} S_v^a(x) + e^{im_s v \cdot x} S_s^a(x) \right). \]

The HSET Lagrangian is obtained by plugging Eq. (13) into Eq. (4) and taking the large \( m_s \) limit, in which only the surviving terms are those for which the phase factor cancels. We find

\[ \mathcal{L}_{\text{HSET}} = S_s^a(v \cdot D^\mu)_{ac} S_v^c - \frac{1}{2m_S} S_v^a(D^\mu)_s S_s^c, \]

where \( v^\mu \) is the four-velocity and the covariant derivative, \( D^\mu_s \), involves only soft gluons. The first term in Eq. (14) gives the leading interactions, and the second term is suppressed by \( 1/m_S \) and so can be neglected.

In our SCET-HSET operators, the soft gluons appear in the soft Wilson lines,

\[ \mathcal{Y}_v(x) = P \exp \left( ig \int_{-\infty}^{x} ds \ v \cdot A_s^a(sv^\mu) t^a \right), \]
where $v^\mu$ can be either $n^\mu$, $\pi\mu$, or $\nu^\mu$. These soft Wilson lines arise when we decouple the leading soft interactions from the collinear and heavy scalar fields by the field redefinitions \(^{22}\)

\[
\begin{align*}
A_{\alpha }^{a,\mu } & \rightarrow \gamma _{\mu }^{n}A_{\alpha }^{a,\mu }, & W_{n}^{(\mu )ab} & \rightarrow \gamma _{\mu }^{n}W_{n}^{(\mu )ab} \\
A_{\alpha }^{a,\mu } & \rightarrow \gamma _{\mu }^{\pi }A_{\alpha }^{a,\mu }, & W_{\pi }^{(\mu )ab} & \rightarrow \gamma _{\mu }^{\pi }W_{\pi }^{(\mu )ab} \\
S_{\nu }^{a} & \rightarrow \gamma _{\nu }^{a}S_{\nu }^{b}, & S_{\nu }^{ab} & \rightarrow \gamma _{\nu }^{ab}S_{\nu }^{ab}.
\end{align*}
\]

After this field redefinition, the collinear fields and HSET fields do not interact with the soft particles. Note that after the field redefinition, $L_{\text{HSET}} = S_{\nu }^{a*}(v \cdot i\partial)S_{\nu }^{a} + O(1/m_{S})$, so the strong interactions vanish at leading order in $O(1/m_{S})$. The heavy scalar’s interaction with soft gluons can be reproduced by a soft Wilson line. This replacement simplifies the derivation of the factorization theorem as we show below.

Using $B_{\nu }^{\pm \mu }$, $B_{\pi }^{\pm \mu }$, $S_{\nu }^{a}$, and the soft Wilson lines we can construct the effective Lagrangian for color-octet production at leading order in $\lambda$,

\[
L_{\text{SCET}} = C_{S}(\mu)O_{S}(\mu) + C_{P}(\mu)O_{P}(\mu),
\]

where the effective theory operators $O_{S}$ and $O_{P}$ are for color-octet scalars $(S_{S})$ and pseudoscalars $(S_{P})$ respectively. Those operators have different color structures and are

\[
\begin{align*}
O_{S} & = \frac{d_{abc}}{\sqrt{2m_{S}}} (S_{\nu }^{a}Y_{\nu }^{\dagger})^{a}(B_{\nu }^{\pm \mu }Y_{\nu }^{\dagger})^{b}(Y_{\nu }^{\dagger}B_{\nu }^{\pm \mu })^{c}, \\
O_{P} & = \frac{i f_{abc}}{\sqrt{2m_{S}}} \epsilon_{\nu \mu}^{\pm} (S_{\nu }^{a}Y_{\nu }^{\dagger})^{a}(B_{\nu }^{\pm \mu }Y_{\nu }^{\dagger})^{b}(Y_{\nu }^{\dagger}B_{\nu }^{\pm \mu })^{c},
\end{align*}
\]

where $\epsilon_{\nu \mu}^{\pm} = \epsilon_{\nu \rho \sigma}n^{\rho}n^{\sigma}/2$. Note that the strong interaction Lagrangian for the pseudoscalar is the same as the scalar. Since the $n$-collinear, $\pi$-collinear, and the soft fields are decoupled, the renormalization of the both operators will be a simple product given by

\[
Z_{S,P} = Z_{Y}Z_{n}Z_{\pi},
\]

where $Z_{n}$ and $Z_{\pi}$ are the renormalization factors for the collinear parts $B_{\nu }^{\pm \mu }$ and $B_{\pi }^{\pm \mu }$, respectively, and $Z_{Y}$ is the renormalization factor for the three soft Wilson lines. So the renormalizations of $O_{S}$ and $O_{P}$ are the same and do not depend on either color structure constants or the Lorentz structure. Therefore, below we only consider the renormalization of the scalar operators, however our results hold for pseudoscalar operators as well.

Taking the matrix elements of $C_{S}(\mu)O_{S}(\mu)$ in Eq. (17), we find the scattering cross section

\[
\sigma(pp \rightarrow S_{S}X) = \frac{\pi}{s} \sum_{X} \int d^{4}q\delta(q^{2} - m_{S}^{2})\delta(P_{n} + P_{\pi} - q - p_{X})
\times |C_{S}(m_{S},\mu)|^{2} \langle P_{n}P_{\pi}|O_{S}^{\dagger}(\mu)|S_{S}X\rangle \langle S_{S}X|O_{S}(\mu)|P_{n}P_{\pi}\rangle,
\]

where $P_{n}^{\mu}$ and $P_{\pi}^{\mu}$ are the momenta of the incoming protons which are $n$-collinear and $\pi$-collinear, respectively. Because the final state $X$ consists of $n(\pi)$-collinear and soft states in
the partonic threshold region, it is possible to rewrite the final state summation as \( \sum_X = \sum_{X_n} \sum_{X_p} \sum_{X_S} \) and the final state momentum \( p_X = p_{X_n} + p_{X_p} + p_{X_S} \). The momentum of the color-octet field is

\[
q = P_n + \frac{p_{X_p} - (p_{X_n} + p_{X_p} + p_{X_S})}{(p_{X_n} + p_{X_p})} = p_n + p_{\pi} - p_{X_S},
\]

where \( p_n = P_n - p_{X_n} \) and \( p_{\pi} = P_{\pi} - p_{X_\pi} \) are momenta of the partons in the two incoming protons. Then the argument in the first delta function in Eq. (21) is

\[
q^2 - m_S^2 = (p_n + p_{\pi} - p_{X_S})^2 - m_S^2 \sim (p_n + p_{\pi})^2 - 2p_{X_S} \cdot (p_n + p_{\pi}) - m_S^2
\]

(22)

where \( \eta = p_{X_S}^0 \) is the energy carried by final state soft particles in the partonic center-of-mass frame, and \( \hat{s} = \Pi \cdot P_n \cdot P_{\pi} = y_1 y_2 \), \( y_1, 2 \) are the large (collinear) momentum fractions of the incoming partons, defined by \( y_1 = \Pi \cdot P_n / \Pi \cdot P_n \) and \( y_2 = n \cdot P_{\pi} / n \cdot P_{\pi} \). When the momentum \( q \) is on-shell, Eq. (22) implies

\[
\eta = \frac{\hat{s} - m_S^2}{2 \hat{s}} = \frac{\hat{s}(1 - z)}{2} = \frac{m_S(1 - z)}{2 z^{1/2}} \sim \frac{m_S(1 - z)}{2}
\]

(23)

where the last approximation is valid in the limit \( z = m_S^2 / \hat{s} = \tau / (y_1 y_2) \to 1 \).

The cross section in Eq. (21) can be factorized by first inserting the identity

\[
1 = \int d\eta dy_1 dy_2 \delta(\eta + i \partial_0) \delta(y_1 - \Pi \cdot P / \Pi \cdot P_n) \delta(y_2 - n \cdot P / n \cdot P_{\pi}),
\]

(24)

where \( \Pi \cdot P \) is a label operator acting on \( n \)-collinear fields, \( n \cdot P \) is a label operator acting on \( \Pi \)-collinear fields, and the partial derivative, \( i \partial_0 \), acts only on soft fields. Then \( \sigma(pp \to S_X) \) can be written as

\[
\sigma(pp \to S_X) = \frac{\pi}{s} \int d\eta dy_1 dy_2 \delta(\eta - \frac{m_S(1 - z)}{2}) \frac{|C_S(m_S, \mu)|^2}{2 z^{1/2}}
\]

\[
\times \frac{d^{abc} d^{def}}{2m_S} \left( \langle P_n P_{\pi} | (\gamma_\nu S_{S\nu})^a (\gamma_n^\dagger B_n^\dagger)^b (\gamma_{\pi} B_{\pi \mu}^\perp)^c | S_X \rangle \right.
\]

\[
\times \left. \langle S_X | \delta(\eta + i \partial_0) (S^{\perp}_{S\nu \gamma_\nu} (\gamma_n^\dagger B_n^\dagger [y_2] \gamma_{\pi} B_{\pi \mu}^\perp | S^{\perp}_{S\nu \gamma_\nu} (\gamma_{\pi} B_{\pi \mu}^\perp [y_1] \gamma_n^\dagger B_n^\dagger | P_n P_{\pi}) \right). \]

(25)

The cross section \( \sigma(pp \to S_{P} X) \) is the same up to the color factor and the replacement \( C_S(m_S, \mu) \to C_P(m_S, \mu) \). Then we use \( S^a |S\rangle = \sqrt{2m_S} \epsilon^a \) and \( \langle S | S^b \rangle = \sqrt{2m_S} \epsilon^b \) to remove the color-octet scalar from the final state, where \( \epsilon^a \) are color polarization vectors satisfying the relations, \( \sum_{\mu \nu} \epsilon^a \epsilon^{\mu \nu} = \delta^{ab} \). Finally, we apply the completeness relation, \( 1 = \sum_X |X \rangle \langle X | \). After these manipulations, we find that \( \sigma(pp \to S_X) \) is given by

\[
\sigma(pp \to S_X) = \frac{\pi d^{abc} d^{def}}{s} \int d\eta dy_1 dy_2 \frac{|C_S(m_S, \mu)|^2}{2 s^{1/2}} \left( \langle P_n P_{\pi} | \gamma_v^{ak} B_{n \mu}^{\perp} \gamma_\nu B_{\pi \mu}^{\perp} | S_X \rangle \right.
\]

\[
\times \left. \delta \left( \frac{m_S(1 - z)}{2} + i \partial_0 \right) Y_v^{\perp, \mu} B_{n \mu}^{\perp} [y_2] Y_{\pi \nu} \gamma_{\pi} B_{n \mu}^{\perp} \gamma_{\pi} B_{\mu \nu}^{\perp} [y_1] \right) P_n P_{\pi} \right).
\]

(26)
To simplify the notation we have defined
\[ B_n^{-\mu,a}[y_1] = \left[ \delta(y_1 - \frac{n \cdot P}{n \cdot P_n}) B_n^{-\mu,a} \right], \quad B_n^{\mu,a}[y_2] = \left[ \delta(y_2 - \frac{n \cdot P}{n \cdot P_n}) B_n^{\mu,a} \right]. \] (27)

The PDF for the \( n \)-collinear proton in terms of SCET fields is
\[ f_{g/P_n}(x) = \frac{1}{2 \pi x \bar{p} \cdot P_n} \int \frac{dn \cdot z}{2} e^{-ix \bar{p} \cdot P_n n \cdot z/2} \]
\[ \times \bar{p} \cdot P_n \delta^{\mu \nu} \int d^3 g \left\langle \bar{P}_n \left[ G_{n,\alpha \mu} \left( \frac{n \cdot z}{2} \right) W^{\nu \beta}_n \left[ \frac{n \cdot z}{2}, 0 \right] G^\dagger_{n,\beta \nu}(n, \beta \nu) \right] P_n \right\rangle \]
\[ = \frac{1}{\pi \bar{p} \cdot P_n} \bar{p} \delta^{\mu \nu} \left\langle P_n \left| B_{n,\mu}^{0} \delta \left( x - \frac{n \cdot P}{n \cdot P_n} \right) B_{n,\mu}^{0} \right| P_n \right\rangle \]
where \( P_n^\mu \) is the momentum of the proton. Here, we have defined \( W_n^{\nu \beta}(z, 0) = W_n^{\nu \beta}(z) W_n^{\mu \nu}(0) \) and used Eq. (12) for the third equality. A similar set of manipulations yields the PDF for the \( \bar{n} \)-collinear proton, and since \( f_{g/P_n}(x) = f_{g/P_n}(x) \), we will drop the subscripts \( n \) and \( \bar{n} \) in what follows. Therefore, after averaging over proton spins in Eq. (28), we find
\[ \left\langle P_n \left| B_{n,\mu}^{0,\mu} \right| P_n \right\rangle = g^{\mu \nu} \delta^{\mu a} \frac{y_1 (\bar{p} \cdot P_n)^2}{2(N_c^2 - 1)} f_{g/P}(y_1), \] (29)
\[ \left\langle P_n \left| B_{\bar{n},\mu}^{0,\mu} \right| P_n \right\rangle = g^{\mu \nu} \delta^{\mu b} \frac{y_2 (n \cdot P_n)^2}{2(N_c^2 - 1)} f_{g/P}(y_2). \]

The soft function \( \bar{S}_S (\bar{S}_P) \) for the scalar (pseudoscalar) production is defined to be
\[ \bar{S}_S (m_S (1 - z)) = \frac{3 d^{abc} d^{def}}{40} \left\langle 0 \right| \gamma^{nak} \gamma^{nb} \gamma^{m} \delta \left( 1 - z + \frac{2 i \partial_0}{m_S} \right) \gamma^{\nu \beta} \gamma^{\tau \mu} \gamma^{\mu \nu} \delta \left( 1 - z + \frac{2 i \partial_0}{m_S} \right) \gamma^{\nu \beta} \gamma^{\tau \mu} \gamma^{\mu \nu} \left| 0 \right\rangle, \] (30)
\[ \bar{S}_P (m_S (1 - z)) = \frac{3 f^{abc} f^{def}}{24} \left\langle 0 \right| \gamma^{nak} \gamma^{nb} \gamma^{m} \delta \left( 1 - z + \frac{2 i \partial_0}{m_S} \right) \gamma^{\nu \beta} \gamma^{\tau \mu} \gamma^{\mu \nu} \delta \left( 1 - z + \frac{2 i \partial_0}{m_S} \right) \gamma^{\nu \beta} \gamma^{\tau \mu} \gamma^{\mu \nu} \left| 0 \right\rangle. \] (31)

Here, the soft functions are normalized to \( \delta(1 - z) \) at lowest order.

Using these definitions we obtain the factorized scattering cross sections which are given by
\[ \sigma(pp \rightarrow S_i X) = \tau H_i(m_S, \mu_f) \int_0^1 \frac{dz}{z} \bar{S}_i (m_S (1 - z), \mu_f) F(\tau/z, \mu_f), \quad i = S, P, \] (32)
where the hard coefficients \( H_i \) are
\[ H_S (m_S, \mu_f) = \frac{5 \pi |C_S (m_S, \mu_f)|^2}{48}, \quad H_P (m_S, \mu_f) = \frac{3 \pi |C_P (m_S, \mu_f)|^2}{16}. \] (33)

This factorization theorem is our main result. When we consider the renormalization group evolution effects, we will calculate \( H_{S,P}(m_S, \mu) (\bar{S}_{S,P}(m_S (1 - z), \mu)) \) at the scale \( \mu_h (\mu_s) \) and then evolve them to the factorization scale \( \mu_f \). At the leading order (LO) in \( \alpha_s \), the cross section is
\[ \sigma(pp \rightarrow S_i X) = \tau H_i^{(0)} F(\tau), \quad i = S, P, \] (34)
where $H_i^{(0)}$ are the hard coefficient at the lowest order and are equal to the scattering cross section at the Born level.

Next we discuss the RGE evolution of the hard and soft parts and the resummation of the cross section. Once the evolution for the coefficient functions, $C_{S,P}(\mu)$, is determined, the evolution equation for the soft function can be easily derived, since the evolution equations for $f_{g/p}(x)$ are known. The evolution of the soft functions can be done in momentum space as in the analysis of Higgs production and Drell-Yan in Refs. [2, 3, 4]. We follow this approach in this paper. Alternatively, one can solve evolution equations for the moments of the soft functions and PDF’s, and then take an inverse Mellin transform to obtain the resummed cross section. Resummed expressions for the moments of the cross section are given in the Appendix.

To determine the one-loop anomalous dimensions of $O_{S,P}$, we need to consider the Feynman diagrams in Fig. 1 as well as the wavefunction renormalization graphs. We regulate ultra-violet (UV) divergences using dimensional regularization and the infrared (IR) divergences by taking the external legs to be off-shell. It is then straightforward to extract the UV divergences and we find

$$Z_S = Z_P = 1 + \frac{\alpha_s}{4\pi} \frac{1}{\epsilon_{\text{UV}}} \left[ N_c \left( \frac{2}{\epsilon_{\text{UV}}} + 2 \ln \frac{\mu^2}{m_S^2} + \frac{14}{3} + i\pi \right) - \frac{2}{3} n_f \right],$$  \hspace{1cm} (35)

where $N_c$ and $n_f$ are the number of colors and flavors, respectively. From $Z_{S,P}$ we obtain the anomalous dimension for $O_S$ and $O_P$,

$$\gamma_{S,P} = \left( \mu \frac{\partial}{\partial \mu} + \beta g \frac{\partial}{\partial g} \right) \ln Z_{S,P} = -\frac{\alpha_s}{\pi} \left[ N_c \left( \ln \frac{\mu^2}{m_S^2} + \frac{7}{3} + i\pi \right) - \frac{n_f}{3} \right].$$  \hspace{1cm} (36)

Note that

$$\ln \frac{\mu^2}{m_S^2} + i\pi = \frac{1}{2} \ln \frac{\mu^2}{-m_S^2 - i\epsilon} + \frac{1}{2} \ln \frac{\mu^2}{m_S^2},$$  \hspace{1cm} (37)

so logarithms of both $+m_S^2$ and $-m_S^2$ appear. This is because there are corrections coming from soft exchanges between the two initial state particles, similar to Drell-Yan, which give rise to logs of $-m_S^2$, and soft exchanges between initial and final state particles, similar to

---

FIG. 1: One-loop renormalization of $O_{S,P}$. Here the curly lines with the straight lines are $n(\pi)$-collinear gluons and the only curly lines denote the soft gluons coming from the soft Wilson lines. Double line denotes outgoing color-octet field.
deep inelastic scattering, which give rise to logs of $+m_S^2$. From Eq. (36) we can infer the double logarithms in the $O(\alpha_s)$ corrections to $C_i(\mu)$, $i = S, P$,

$$C_i(\mu) = C_i^{(0)} \left[ 1 - \frac{C_A}{4\pi} \left( \frac{1}{2} \log^2 \left( \frac{-m_S^2 - i\epsilon}{\mu^2} \right) + \frac{1}{2} \log^2 \left( \frac{m_S^2}{\mu^2} \right) + \ldots \right) \right], \quad (38)$$

where $...$ denotes terms without double logs. From this we see that if $\mu = m_S$, $C_i(m_S)$ gets a $\pi^2$-enhanced contribution: $C_i(m_S) = C_i^{(0)} (1 + C_A \alpha_s(m_S) \pi/8)$. For the range of $m_S$ considered in this paper, $\alpha_s(m_S) \leq 0.1$. If $\alpha_s = 0.1$, this $\pi^2$-enhanced correction increases the cross section by about 24%, and is half as big as the corresponding $\pi^2$-enhanced contribution to Higgs production. Refs. [3, 4] argued that the $\pi^2$-enhanced terms dominate the fixed-order corrections to Higgs production, and that these terms can be resummed to all orders by evolving the hard function from the scale $m_H^2$ to the scale $-m_H^2$. They also showed that the leading terms exponentiate. In our case, setting $\mu_j^2 = -m_S^2$ does not remove the factor of $\pi^2$ in the hard coefficient. The double logs vanish if

$$\mu^2 = e^{\pi(\pm 1-i)/2} m_S^2, \quad (39)$$

but it is not clear that evolving to this complex scale will give a sensible resummation the $\pi^2$-enhanced contribution. Below we will calculate the $K$-factor with the next-to-leading order (NLO) $\pi^2$-enhanced correction. Even if the $\pi^2$-enhanced corrections exponentiate, the NLO correction should be a good approximation to the resummed result since $1 + C_A \alpha_s \pi/4$ and $\exp(C_A \alpha_s \pi/4)$ differ by less than 3% for $\alpha_s = 0.1$.

The soft functions in Eqs. (30) and (31) can be computed perturbatively when $\mu_s \sim m_S(1-z) \gg \Lambda_{\text{QCD}}$. The Feynman diagrams in Fig. 2 give us the $O(\alpha_s)$ corrections to $S_S(m_S(1-z), \mu)$ and $S_P(m_S(1-z), \mu)$:

$$\bar{S}_S^{(1)}(m_S(1-z), \mu) = \bar{S}_P^{(1)}(m_S(1-z), \mu) \quad (40)$$

$$= \frac{\alpha_s}{\pi} N_c \left\{ \frac{\delta(1-z)}{\varepsilon_{\text{UV}}} + \frac{1}{\varepsilon_{\text{UV}}} \left( \frac{1}{2} + \ln \frac{\mu^2}{m_S^2} \right) + 1 - \frac{\pi^2}{4} + \frac{1}{2} \ln \frac{\mu^2}{m_S^2} + \frac{1}{2} \ln^2 \frac{\mu^2}{m_S^2} \right\}$$

$$- \left( \frac{2}{\varepsilon_{\text{UV}}} + 1 + 2 \ln \frac{\mu^2}{m_S^2} \right) \frac{1}{1-z} + 4 \left( \frac{\ln(1-z)}{1-z} \right)^+ \right\},$$

where the plus distributions are defined in the standard way. Note that there is no IR divergence in the sum of the real and virtual diagrams in Fig. 2. The IR finiteness of the soft function can be easily understood in SCET because the soft function is just the Wilson coefficient obtained at the second-step matching.

To obtain the resummed cross section we employ the method of momentum space resummation developed in Refs. [2, 3, 4, 29, 30]. The resummed result can be written as

$$\sigma(pp \to S_i X) = \tau \int_z^1 \frac{dz}{z} V_i(z, m_S, \mu_f) F(\tau/z, \mu_f), \quad i = S, P \quad (41)$$

where the resummation function $V_i(z, m_S, \mu_f)$ is given by

$$V_{S,P}(z, m_S, \mu_f) = H_{S,P}(m_S, \mu_h) U(\mu_h, m_S, \mu_f) \left( \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \right) \bar{S}_{S,P}(\partial^\eta, \mu_s) e^{-2\gamma_E \eta} \Gamma(2\eta). \quad (42)$$

FIG. 2: One loop corrections to the soft function. The dashed line represents the cut. The diagram (a) and its Hermitian conjugate (b) describe the virtual soft gluon radiation and the diagram (c) denotes real soft gluon radiation.

Here $\tilde{S}_{S,P}(\partial_\eta, \mu_s)$ is defined in terms of the Laplace transform of the soft functions, and the evolution function $U(\mu_h, \mu_s, \mu_f)$ is a product of terms obtained from evolving the hard function to the scale $\mu_h$ and the Laplace transform soft function to the scale $\mu_s$. To NLL accuracy, we find

$$\ln U(\mu_h, \mu_s, \mu_f) = \ln \left[ 4SU_{NLL}(m_S, \mu_s) + \frac{B_S^\eta}{\beta_0} \ln \frac{\alpha_s(\mu_s)}{\alpha_s(m_S)} + \frac{B_g^\eta}{\beta_0} \ln \frac{\alpha_s(\mu_f)}{\alpha_s(\mu_s)} \right],$$

where $SU_{NLL}$ is

$$SU_{NLL}(\mu_1, \mu_2) = \frac{A_1}{4\beta^2_0} \left[ \frac{4\pi}{\alpha_s(\mu_1)} \right] \left( 1 - \frac{1 - r - \ln r}{r} \right) + \left( \frac{A_2}{A_1} - \frac{\beta_1}{\beta_0} \right) (1 - r + \ln r) + \frac{\beta_1}{2\beta_0} \ln^2 r,$$

where $r = \alpha_s(\mu_2)/\alpha_s(\mu_1)$. The parameters $A_1$, $A_2$, $B_S^\eta$, and $B_g^\eta$ appear in the anomalous dimensions of the hard and soft functions, and are given in the Appendix. The parameter $\eta$ is defined in terms of an integral over the cusp anomalous dimension (see Ref. [29]) and in our calculation $\eta = (A_1/\beta_0) \ln(\alpha_s(\mu_f)/\alpha_s(\mu_s))$. In our case, $\eta < 0$ since $\mu_s < \mu_f$ and hence the integral in Eq. (41) is singular. The integral is then defined by analytic continuation from positive $\eta$. We will choose $\mu_h = \mu_f = m_S$. For this choice of $\mu_h$ there are no large logs of $m_S^2/\mu^2$ in $H_{S,P}(m_S, \mu_h)$ and $H_{S,P}(m_S, \mu_h) = H_{S,P}^{(0)}(m_S, \mu_h)$ to NLL accuracy. In order to resum logarithms of $1 - z$ we should choose the scale $\mu_s = m_S(1 - z)$, however, this will lead to divergences in the $z$ integral as the running coupling will cross the Landau pole as $z \to 1$. Practically, it is simpler to choose $\mu_s$ to be a scale parametrically smaller than $\mu_h$.

We first present our results in terms of a $K$-factor, defined as the ratio of leading order and NLL resummed cross sections, which is given by

$$K_{S,P}(m_S^2, \tau) = \int_z^1 \frac{dz}{z} V_{S,P}(z, m_S, \mu_f) F(\tau/z, \mu_f) \left( H_{S,P}^{(0)}(m_S, \mu_f) \int_z^1 \frac{dz}{z} F_{LO}(\tau/z, \mu_f) \right),$$

where $F_{LO}$ is a convolution of PDFs at LO. This result is universal in that it is independent of the NP model. $O(\alpha_s(m_S))$ corrections to the hard coefficient can depend on the NP model but this beyond the accuracy we are working. For our numerical calculations, we use the LO
\[ \alpha_s, \text{ setting } \alpha_s(M_Z) = 0.1205 \text{ and } m_t = 170.9 \text{ GeV}. \]

For the gluon PDF’s we use CTEQ5 at NLO \[31\]. In order to determine \( \mu_s \), we follow the procedure of Ref. \[2\] and calculate the convolution of the one-loop expression for \( \bar{S}_i(m_S(1-z), \mu) \) in Eq. (40) with \( F(\tau/z, \mu) \) in Eq. (7). The scale \( \mu_s \) is chosen so that the higher order corrections to the soft function are under perturbative control. This is accomplished by specifying \( \mu_s^I \) and \( \mu_s^{II} \): The scale \( \mu_s^I \) is defined by starting with \( \mu_s = \mu_h \) and lowering \( \mu_s \) until the \( O(\alpha_S) \) correction is less than 15%. The scale \( \mu_s^{II} \) is chosen so that the one-loop correction is minimized. We use the average \( (\mu_s^I + \mu_s^{II})/2 \) in Fig. 3. The solid line is the result for the \( K \)-factor with the \( \pi^2 \)-enhanced correction included. The \( K \)-factor varies from about 2.4 for \( m_S = 500 \) GeV to about 3.6 for \( m_S = 3 \) TeV. As expected, the resummation of threshold corrections significantly enhances the cross section and becomes more important as \( m_S \) increases. The dashed line in Fig. 3 is the result without the \( \pi^2 \)-enhanced correction. This correction increases the \( K \)-factor by 25% and is independent of \( m_S \). In Fig. 4 we show the variation in the prediction as \( \mu_s \) is varied between \( \mu_s^I \) and \( \mu_s^{II} \). The uncertainty from the choice of \( \mu_s \) is \( \pm 15\% \) for \( m_S = 500 \) GeV, and decreases with increasing \( m_S \). The variation with the choice of \( \mu_f \) is also shown in Fig. 4. The sensitivity to the choice of \( \mu_f \) is greater and introduces an uncertainty of \( \pm 25\% \). The dependence on the scales \( \mu_s \) and \( \mu_f \) should decrease when higher order corrections are included.

In Fig. 5 we show our calculation of the color-octet scalar production cross section in the Manohar-Wise Model \[16\]. In this model, the two real components of the complex color-octet scalars are denoted \( S_R^0 \) and \( S_I^0 \), where \( S_R^0 \) is a scalar and \( S_I^0 \) is a pseudoscalar if \( \eta_U \) is chosen to be real. The LO calculation of their production cross sections from Ref. \[24\] are the dashed lines in Fig. 4 and our NLL results are the solid lines. At \( m_S = 1 \) TeV, we obtain...
\[
\sqrt{s} = 14 \text{ TeV, } M_s/2 \leq \mu_s \leq M_s
\]

FIG. 4: Scale dependences of the \(K\)-factor.

\[
\sqrt{s} = 14 \text{ TeV}
\]

\[
\sigma_{NLL}(pp \rightarrow S_R X) = 57 \text{ fb and } \sigma_{NLL}(pp \rightarrow S_I X) = 73 \text{ fb.}
\]

We have fixed the parameters \(\eta_U = 1\) and \(\lambda_{4,5} = 1\) as in Ref. [24]. The NLL results are almost 3 times as large as the LO results, which are \(\sigma_{LO}(pp \rightarrow S_R X) = 21 \text{ fb and } \sigma_{LO}(pp \rightarrow S_I X) = 26 \text{ fb.}\)

In summary we have used SCET to derive a factorization theorem for color-octet scalar production at the LHC. The factorization theorem can be used to resum large threshold corrections which have a significant impact on the total cross section. It is universal in the sense that all details dependent on NP models are encoded in the Wilson coefficients. The factorization theorem is similar to Higgs production, however, some details are different because the final state particle is colored. Because there are both soft exchanges between initial state partons as well as between partons in the initial and final states, the structure of double logarithms and corresponding \(\pi^2\)-enhanced corrections is different. We obtained a resummed calculation of \(\sigma(pp \rightarrow SX)\) to NLL accuracy. The resummed cross sections are 2-4 times larger than the LO cross section, depending on the mass of the color-octet scalar. Uncertainties from varying \(\mu_S\) and \(\mu_f\) in these calculations are \(\pm 15\%\) and \(\pm 25\%\), respectively.
NNLL log resummation and higher order perturbative corrections will be required to reduce scale dependence of the resummed cross section. Further development of the factorization theorem to account for scales besides $m_S$ and $m_S(1-z)$ maybe required if color-octet scalars are actually discovered. For example, precision measurements of the mass may require taking into account the width of the color-octet scalar, as is required for determining the top quark mass [32].

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APPENDIX A: LARGE N RESUMMATION IN MOMENT SPACE

Here we present our results for the resummed cross section in moment space. All the results below are taken in the large $N$ limit. For the renormalized soft function $S_N$ we find

$$\bar{S}^N_{S,P}(\mu) = \int_0^1 z^{N-1} \bar{S}_{S,P}(m_S(1-z), \mu) = 1 + \frac{\alpha_s}{\pi} N_c \left( \frac{1}{2} \ln^2 \frac{\mu^2 N^2}{m_S^2} + \frac{1}{2} \ln \frac{\mu^2 N^2}{m_S^2} + 1 + \frac{\pi^2}{12} \right) + O(\alpha_s^2),$$  \hfill (A1)

where $\bar{N} = N e^{\gamma_E}$. From Eq. (A1), we notice that the choice $\mu = m_S/\bar{N}$ minimizes the large logarithms. The $\mu$-independence of cross section implies the following RGE:

$$\frac{d}{d\mu} \bar{S}^N_i(\mu) = \left( 2 \gamma^N_g - 2 \text{Re}[\gamma_i] \right) \bar{S}^N_i(\mu), \quad i = S, P,$$  \hfill (A2)

where $\gamma^N_g$ is the well-known Altarell-Parisi evolution kernel for the gluon PDF in the moment space,

$$\gamma^N_g = \frac{\alpha_s}{\pi} C_A \left[ 2 \ln \bar{N} - \left( \frac{11}{6} - \frac{n_f}{9} \right) \right] + O(\alpha_s^2).$$  \hfill (A3)

From our results, Eqs. (36), (A1), and (A3) we can easily see that Eq. (A2) is satisfied to first order in $\alpha_s$. If we take the moments of $\sigma(pp \rightarrow S_i X)$ in Eq. (32), the result is

$$\sigma_N(pp \rightarrow S_i X) = \int_0^1 d\tau \tau^{N-1} \sigma(pp \rightarrow S_i X) = H_i(m_S, \mu_s) S^N_i(\mu_s, \mu_f)[f^N_{S,P}(\mu_f)]^2 + O\left( \frac{1}{\bar{N}} \right),$$  \hfill (A4)

where we identified $\mu_h = m_S$. Here we employed the two-step matching: the hard coefficient $H_{S,P}(m_S)$ at the scale $m_S$ is evolved down to the soft scale $\mu_s$ and then the soft function $S(\mu_s)$ obtained at $\mu_s$ can be evolved to the factorization scale $\mu_f$. This is equivalent to the scaling evolution realized in Eq. (42), where the hard and soft function are evolved from $\mu_h$ and $\mu_s$ to $\mu_f$ respectively, but the soft function’s renormalization behavior compensates the evolution of the hard function from $\mu_s$ to $\mu_f$. So the exponentiated matching coefficients
$C_{S,P}$ and $S_{S,P}^N$ are given by

$$C_{S,P}(m_S, \mu_s) = C_{S,P}(m_S) e^{-I_1(m_S,\mu_s)} = C_{S,P}(m_S) \exp \left[ - \int_{\mu_s}^{m_S} \frac{d\mu}{\mu} \gamma_{S,P}(\mu) \right].$$

(A5)

$$S_{S,P}^N(\mu_s, \mu_f) = S_{S,P}^N(\mu_s) e^{-I_2(\mu_s,\mu_f)} = S_{S,P}^N(\mu_s) \exp \left[ -2 \int_{\mu_f}^{\mu_s} \frac{d\mu}{\mu} \gamma_{g}^N(\mu) \right].$$

(A6)

Therefore Eq. (A4) can be rewritten as

$$\sigma_N(pp \to S_iX) = H_i(m_S) e^{-2Re[I_1(m_S,\mu_s)]} \left[ S_{1}^N(\mu_s) e^{-I_2(\mu_s,\mu_f)} \right] \left[ f_{g/p}(\mu_f) \right]^2,$$

(A7)

$$= H_i(m_S) \exp [G(m_S, \mu_f)] \left[ f_{g/p}(\mu_f) \right]^2,$$

(A8)

where we set $\mu_s = m_S/\bar{N}$ in the second equality, and then the exponential factor $G_S$ can be expanded as

$$G(m_S, \mu_f) = \ln \bar{N} g^{(0)}_S + g^{(1)}_S(m_S, \mu_f) + \alpha_s(m_S) g^{(2)}_S(m_S, \mu_f) + \cdots.$$  

(A9)

Here each of the coefficients $g^{(i)}_S$, $i = 0, 1, 2$ are correspond to the resummed results at LL, NLL, and NNLL accuracies, respectively.

For the computation of the exponentiation factor up to NLL accuracy, we need to expand $\text{Re}[\gamma_{S,P}]$ and $\gamma_{g}^N$ up to second order in $\alpha_s$

$$\text{Re}[\gamma_S] = \text{Re}[\gamma_P] = -\frac{\alpha_s}{4\pi} \left[ A_1 \ln \frac{\mu^2}{m_S^2} + B_1^S \right] - \left( \frac{\alpha_s}{4\pi} \right)^2 A_2 \ln \frac{\mu^2}{m_S^2} + \mathcal{O}(\alpha_s^3),$$

(A10)

$$\gamma_{g}^N = \frac{\alpha_s}{4\pi} \left[ A_1 \ln \bar{N}^2 - B_1^g \right] + \left( \frac{\alpha_s}{4\pi} \right)^2 A_2 \ln \bar{N}^2 + \mathcal{O}(\alpha_s^3),$$

(A11)

where $\alpha_s \ln(\mu/m_S) \sim \alpha_s \ln \bar{N}$ are treated as $\mathcal{O}(1)$, and the coefficients of the large logarithms, $A_1$ denote the coefficients of the cusp anomalous dimension $[33]$. In the above equations, $A_1$, $B_1^S$, and $B_1^g$ were already given in Eqs. (36) and (A3), and $A_2$ is $8N_c[(67/18-\pi^2/6)N_c-5n_f/9]$ $[33]$.

After a brief calculation using Eqs. (A10) and (A11), we find

$$g^{(0)}_S = \frac{A_1}{\lambda \beta_0} \left[ 2\lambda + (1 - 2\lambda) \ln(1 - 2\lambda) \right],$$

(A12)

$$g^{(1)}_S = \frac{1}{\beta_0} \left[ (B_1^g - B_1^S) \ln(1 - 2\lambda) + 2A_1 \lambda \ln \frac{\mu_f^2}{m_S^2} - \frac{A_2}{\beta_0} \left[ 2\lambda + \ln(1 - 2\lambda) \right] \right.\left. + \frac{\beta_1 A_1}{2\beta_0^2} \left[ 4\lambda + (2 + \ln(1 - 2\lambda) \ln(1 - 2\lambda) \right],\right.$$

(A13)

where $\lambda = \alpha_s \ln \bar{N}/(4\pi)$, and $\beta_{0,1}$ are the first two coefficients of the QCD $\beta$ function. Here note that we have set $\mu_f \sim m_S$. However, if we choose $\mu_f$ as $\mu_f \leq \mu_s \sim m_S/\bar{N}$, the logarithm $\ln(\mu_f/m_S)$ should be power-counted as $\mathcal{O}(\ln \bar{N})$.

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