The Value Range of Contact Stiffness Factor between Pile and Soil Based on Penalty Function

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Abstract. The value range of contact stiffness factor based on penalty function is studied when we use finite element software ANSYS to analyze contact problems, take single pile and soil of a certain project for example, the normal contact between pile and soil is analyzed with 2D simplified model in horizontal load. The study shows that when adopting linear elastic model to simulate soil, the maximum contact pressure and penetration approach steady value as the contact stiffness factor increases. The reasonable value range of contact stiffness factor reduces as the underlying element thickness decreases, but the rule reverses when refers to the soil stiffness. If choose DP model to simulate soil, the stiffness factor should be magnified 100 times compares to the elastic model regardless of the soil bears small force and still in elastic deformation stage or into the plastic deformation stage. When the soil bears big force and into plastic deformation stage, the value range of stiffness factor relates to the plastic strain range of the soil, and reduces as the horizontal load increases.

1. Introduction

With the continuous development of finite element technique, ANSYS software is widely used in engineering analysis. For the pile foundation engineering, consider the interface between pile and soil reasonable or not has an apparent effect on the results. Contact element is introduced in finite element to deal with it, and the main contact calculation methods are penalty function method, the Lagrange multiplier method, and augmented Lagrange method. Contact force and contact penetration are established linear relationship by contact stiffness in Penalty function method, which is widely used because of its simple principle.

The core problem is to define the contact stiffness in finite element calculation and two kinds of methods are used. One is directly given the contact stiffness (KN) value and the other is to input contact stiffness factor (FKN) which satisfies formula \( FKN = \frac{KN \times E}{T_{\text{max}}} \). \( E \) is the elasticity modulus of soil, \( T_{\text{max}} \) represents maximum allowable penetration, which satisfies formula \( T_{\text{max}} = f_{\text{toln}} \times h / 2 \), \( f_{\text{toln}} \) is penetration tolerance factor with default value 0.1 in penalty function method. Contact element attaches to the soil element surface which adjacent piles. \( h \) is the soil element thickness called lower unit thickness which adjacent pile and parallel to the normal direction of pile. The value range of FKN is recommend from 0.01~10 in ANSYS software. Therefore, specific
value problem is existed whether is directly given \( KN \) or input \( FKN \), which will affect the calculation results.

In fact, in order to achieve the real contact, the contact stiffness should be infinite to realize zero penetration. But excessive contact stiffness may cause ill-conditioned matrix, too much iterations, and lead to difficulty convergence. On the contrary, too small value will cause excessive penetration and deviate from the truth although easy to convergence. In order to obtain the appropriate contact stiffness, balance should be sought between the penetration and the number of iterations. In this paper, rational \( FKN \) value range are explored with lateral loads at the top of the single pile by tentative calculation, using the pile and soil parameters of the transport junction of Tianjin station.

2. Penalty function method
Penalty function method\(^5\) in the application of elastic contact problem considers the non intrusive of constraint as a penalty term is introduced into system of total potential energy of the system. Spring is introduced to the gauss point of each contact element in finite element and the stiffness of the spring is the contact stiffness. The relationship of contact pressure \( (F) \) and contact penetration \( (\Delta) \) can be expressed as formula \( F = K \Delta \).

2.1. Finite element model
The pile and soil often consider to two-dimension model because of the complex structure and contact nonlinear between them\(^6,7\). Pile and soil parameters of the transport junction of Tianjin station are used to analysis the influence of different \( FKN \) value to the contact pressure and penetration. Two-dimensional finite element model is established and the soil is considered to homogeneous material. Pile can be simplified to the equivalent plane strain model which is proposed by Desai\(^8\). The principle is to reduce the elastic modulus of the sheet pile in the longitudinal direction at intervals arranged in a certain distance of the pile and make it equal to the total vertical stiffness of the Pile.

![Figure 1. Finite element model](image)

The pile and soil are simulated by PLANE42 unit. Contact pairs are constituted by TARGET169 and CONTA171 to simulate the pile-soil interaction while the bottom nodes of the pile and soil are coupled together. The length of the pile is 56 m and diameter is 2.2 m. The horizontal size of the soil on both sides of the pile is 44 m and the bottom size under pile is 12 m reaching the bedrock. Both the bottom and two broadsides of the soil are fixed constraints. The finite element model is shown as the figure 1.

2.2. Model material parameters
The piles of the transport junction of Tianjin station apply concrete of c30, and the distance between piles is 9 m. According to the method proposed by Desai, it needs to meet the condition $e_{SS} = 0$. The total vertical stiffness of the original pile: $S = \sum_{i=1}^{n} \frac{A_i E_i}{L_i}$, the total stiffness of vertical sheet piles in equivalent model: $S_e = \sum_{i=1}^{m} \frac{A_{ie} E_{ie}}{L_{ie}}$, the thickness of the sheet pile considers the same as the pile diameter. The piles and the soil parameters of two-dimensional model are shown in table 1.

| Name | Elastic Modulus | Poisson's ratio | Density | Cohesion | friction angle |
|------|-----------------|----------------|---------|----------|---------------|
| Pile | 5800 /aMP       | 0.2            | 2500 /aKP | 0        | 18 /a°        |
| Soil | 60 /aMP         | 0.4            | 2050 /aKP | 30       | 18 /a°        |

2.3. Selection of constitutive model

Linear elastic constitutive model is used to simulate the pile and soil. In addition, ideal elastic-plastic DP model \(^9\) is also used to simulate the soil in order to compare the different results of the two models.

3. FKN value range when the soil constitutive considers the elastic model

Different soil stiffness and lower unit thickness are adopted to study the relationship between FKN and contact pressure and penetration. At the same time, it combines the relationship between the number of iterations and the FKN value.

According to the actual force situation of the pile, the finite element analysis process is divided into two steps. Vertical force of $2 \times 10^5$ N and horizontal force of $10^5$ N are exerted on the top of the pile step by step.

Twice pile diameter size of soil near the left and right sides of the pile considers as the local study region. That is $h_0 = 4.4m$, which is divided into two, three and four equal parts to achieve respective reasonable FKN value. Also, the elastic modulus of the soil respectively value 30 $MP_a$, 60 $MP_a$ and 90 $MP_a$.

When the soil elastic modulus values 30 $MP_a$ and the lower unit thickness is $h_0 / 3$, FKN values from 0.1 and increases gradually, contact pressure and penetration under different FKN value through trial are shown in figure 2.
The left coordinate of figure 2 represents contact pressure and the right one on behalf of penetration. It suggests that contact pressure increases with FKN increases and gradually stabilized. At the same time, the penetration gradually decreases and tends to zero. Figure 3 shows the relationship between FKN and iterations. It suggests that iterations increase slightly when FKN values less than 2.8, the number of iterations are less than 30 times. But it increases to 72 times suddenly when FKN values 3.0.

Generally speaking, big contact stiffness causes big contact pressure and small penetration, which is more in line with the actual situation. On the other hand, big contact stiffness often needs more iteration and causes ill-condition matrix leading convergence difficulties. The FKN setting principle should be the maximum contact stiffness, the smallest penetration and the least amount of iterations. Then FKN of 2.8 can be regarded as the reasonable upper limit. Suppose that contact pressure and the upper limit error within 5% for the control criteria which can meet the engineering requirements, the lower limit FKN value is 0.5. Therefore, the reasonable FKN range is 0.5 to 2.8. Likewise, respective reasonable FKN range of different lower unit thickness and soil stiffness are shown in Table 2.

### Table 2. Reasonable FKN range

| Thickness of lower unit | Elastic modulus of soil (MPa) |
|-------------------------|------------------------------|
|                         | 30  | 60  | 90  |
| 0.5h₀                   | 0.5 | 2.8 |
| h₀/₃                    | 0.5 | 2.8 | 0.3 | 1.8 |
| 0.25h₀                  |     |     | 0.3 | 1.6 |

In conclusion, when the soil considers the elastic model, decrease of the underlying element thickness and increase of the soil stiffness will lead to greater contact stiffness; reasonable stiffness factor values range will reduce.

4. FKN value range when the soil constitutive considers the elastic model

When the DP model is used to simulate the soil constitutive, the soil elastic modulus still values 30 MPa and the lower unit thickness is h₀/₃, the load situation is the same as the elastic model, the results of the finite element post-processing shows that the soil still in elastic stage. In ANSYS program, if the underlying element is simulated by plasticity model including DP model from the nonlinear material properties table, the actual normal contact stiffness will be decreased 100 times as computation value by the program automatically. In order to compare with the situation that the soil constitutive is used elastic model, the corresponding FKN values are multiplied 100 times. Contact pressure and penetration under different FKN value through trial are shown in figure 4 and relationship between FKN and iterations are shown in figure 5.

By comparing figure 2 and figure 4, as well as figure 3 and figure 5, it can be seen that the corresponding contact pressure, penetration and the number of iterations are equal. Therefore, when the DP model is used to simulate the soil constitutive and still in elastic stage, the FKN range should be multiplied 100 times. That is to say, the reasonable FKN value range inputting in the program is 50 to 280. If FKN range still values 0.01 to 10 which is suggested by the program will brings a large error.
It can be found that the soil enters into plastic deformation stage when the top of the pile bears horizontal force of $10^3 N$ through trial. Contact pressure and penetration under different FKN value are shown in figure 6 and relationship between FKN and iterations are shown in figure 7.

Figure 6 and figure 7 suggest that the reasonable FKN value range inputting in the program is 50 to 200. The value range is 50 to 180 when the horizontal force increases to $1.3 \times 10^4 N$. Therefore, when the soil enters into the plastic stage, the reasonable FKN range is associated with the scope of the plastic strain of soil, and the range reduces as horizontal force increases.

5. Conclusion
This paper studies the FKN value range between pile and soil of two-dimensional contact problem. The soil constitutive use linear elastic model and DP model to simulate. Reasonable FKN value range under different lower unit thickness and soil stiffness are studied, drawing conclusions as follow.

The maximum contact pressure and penetration approach steady value as the FKN increases, sudden increase phenomenon appears in the number of iterations after some FKN value.

When the soil constitutive considers the elastic model, the reasonable FKN range usually values the top one-third of the recommend one in the software. It reduces as the underlying element thickness decreases, but the rule reverses when refers to the soil stiffness. If choose DP model to simulate soil, the stiffness factor should be magnify 100 times compares to the elastic model. If the soil bears big force and into plastic deformation stage, the value range of stiffness factor also relates to the plastic strain range of the soil, and reduces as the horizontal load increases.

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