Abstract

We introduce and define operatively in a model independent way a new “heavy” b-vertex parameter, $\eta_b$, that can be derived from the measurement of a special polarization asymmetry for production of b-quarks on Z resonance. We show that the combination of the measurement of $\eta_b$ with that of a second and previously defined “heavy” b-vertex parameter $\delta_{bV}$ can discriminate a number of models of New Physics that remain associated to different “trajectories” in the plane of the variations of the two parameters. This is shown in particular for some popular SUSY and technicolor-type models. In general, this discrimination is possible if a measurement of both parameters is performed.

\footnote{Work partially supported by NATO.}
1 Introduction

In the first four years of running at LEP1, a remarkable experimental effort has allowed to collect a number of events that begins to approach the $10^7$ limit, that was once considered as nothing more than an optimistic dream. This is the result of a number of machines’s modifications or improvements, whose main features can be found in several recent publications or in the Proceedings of dedicated Workshops.

Meanwhile, on the other side, the theoretical approach to the interpretation of this huge amount of data has also been adapted and improved. In fact, in very recent years it has become clear that, to a certain extent, the comparison of the various results with the Minimal Standard Model (MSM) predictions, and the consequent search of possible signals of New Physics through small deviations due to one-loop effects, can be performed in a rigorously model-independent way. In particular, it has been stressed [1] that the leptonic charged processes can be “read” in terms of two parameters, originally called $\epsilon_{1,3}$ in ref.[1], in a totally unbiased way, that is for models of New Physics that are willing, or able, to modify any of the three classes (self-energies, vertices, boxes) of one-loop radiative effects (in practice, owing to their intrinsic irrelevance for LEP 1 Physics at the starting MSM level, boxes are usually neglected for this kind of search).

The generalization of the previous philosophy to hadron production requires some preliminary choice. In fact, the extra vertex corrections that enter the theoretical expressions are not universal and introduce new unwanted degrees of freedom of both “light” (in practice, massless) and “heavy” quark type. The latter effect is, for the specific case of $e^+e^-$ Physics on Z resonance, entirely due, in the MSM, to that component of the $Zb\bar{b}$ vertex due to the charged would-be Goldstone exchange that behaves as $m_t^2$ for large top masses, as it has been exhaustively shown in the literature [2]. Since various models of New Physics generally contribute either the light quark and lepton or the heavy quark degrees of freedom but not both, it becomes necessary to develop an appropriate strategy
to perform a satisfactory search of New Physics effects.

A first possible attitude is that of only considering those models that would not contribute the lepton and light quark vertices. Then, one only has to add to the "canonical" quantities $\epsilon_{1, 3}$ one extra parameter. For the latter, an operational definition should now be provided. The original proposal [3], [4], to which we shall stick in this paper, was to define the vertex correction $\delta_{bV}$ from the ratio of the $Zb\bar{b}$ and $Zs\bar{s}$ partial widths i.e.

$$\frac{\Gamma_b}{\Gamma_s} \equiv 1 + \delta_{bV}$$

(1)

where the physical b width (we follow in fact the slightly modified version given in ref.[4]) should be taken.

Once the definition eq.(1) is chosen, a systematic analysis of all LEP 1 data that includes both leptonic and hadronic channels can be performed in terms of three parameters e.g. $\epsilon_1, \epsilon_3, \delta_{bV}$ or $\Delta\rho, \Delta_{3Q}, \delta_{bV}$ in the notation of ref.[4], for the previously selected set of models of New Physics. This was proposed in ref.[4] and also in another series of papers [5], where an essentially similar $Zb\bar{b}$ vertex parameter was introduced (and defined $\epsilon_b$). Without entering the details of the methods, it should be stressed that the parameter $\delta_{bV}$ as defined in eq.(1) is operationally connected to the experimentally measured ratio $R_b = \frac{\Gamma_b}{\Gamma_h}$ by the relation (valid in the considered class of models)

$$\frac{\Gamma_b}{\Gamma_h} \equiv R_b = \frac{13}{59}(1 + \frac{46}{59}\delta_{bV} - \frac{23}{59}(\delta_1 - \delta_2) + \frac{2}{65}\Delta\kappa' + 0.1\frac{\alpha_s(M_Z^2)}{\pi} + \text{"negligible"})$$

(2)

Here $\Delta\kappa'$ is a radiative correction entirely fixed by the measurements at LEP1 (SLC) of the effective angle $s_{EFF}^2(M_Z^2)$ (which can be identified for practical purposes with each of the existing popular definitions [6])

$$s_{EFF}^2(M_Z^2) = s^2(1 + \Delta\kappa') \quad s^2 \simeq 0.231$$

(3)

and the weight of $\alpha_s(M_Z^2)$ is practically irrelevant. The parameters $\delta_{1, 2}$ are certain combinations of leptonic and light quark vertices, whose (small) numerical value can be exactly
computed in the MSM; their definition has been given in a previous paper [7], to whose
notations we shall stick. Thus, if New Physics does not affect the light fermion vertices,
$R_b$ can provide the unbiased value of $\delta_{bV}$, to be compared with the MSM prediction.

In fact, an overall analysis of data is more elaborated and includes other variables as
well. The full details can be found in refs.[4] and [5]; the point that we want to stress
here is that, after the most recent LEP1 communicated data [8], this type of investigation
leads to the conclusion that $\epsilon_1, \epsilon_3, \Delta \rho, \Delta_{3Q}$ (or $\Delta_{3Q}$ in the notation of ref.[4]) are now perfectly
consistent with the MSM predictions. This means that the small discrepancy that might
have been present in the previous determinations of $\epsilon_3$ ($\Delta_{3Q}$) has now been (almost)
completely washed out. On the contrary, the possibility of a small deviation is still
allowed in the heavy vertex parameter $\delta_{bV}$, since one has now [9]:

$$\delta_{bV} = (-12 \pm 10) \times 10^{-3}$$  \hspace{2cm} (4)

and the MSM tolerance region (corresponding to the last bound $m_t \geq 113$ GeV [10]) is

$$\delta_{bV}^{MSM} \leq -0.016$$  \hspace{2cm} (5)

One possible question that becomes relevant at this stage is whether the assumption that
the light fermion vertices remain unaffected has some experimental support. To answer
this question one should identify (at least) one quantity that is only reacting to such kind
of New Physics effect. In fact, this “light vertex indicator” has been proposed in ref.[7] as a
certain combination of hadronic and leptonic widths and of $s_{EFP}^2(M_Z^2)$, and defined $D$. At
one loop, it is only affected by a certain combination of light fermion vertices parameters
(different from that entering $R_b$ eq.(4)). For that combination, the experimental data
show a very good agreement with the MSM predictions, as fully discussed in ref.[7].

If one believes that a small discrepancy is still present in $R_b$ eq.(2), two attitudes
become possible. One is that of addressing the full responsibility to the heavy $b$ vertex
parameter $\delta_{bV}$. The other one is that of thinking that an effect of the light vertex type
could modify the combination entering $R_b$ (with $\delta_{bV}$ unaffected), but not that contained in $D$. Although a priori no possibility should be discarded, we feel that the second choice appears somehow unnatural. Therefore, we shall first concentrate on the more plausible solution, in which New Physics only affects $\delta_{bV}$ as a direct consequence of the fact that the $b$ quark is, for a certain type of effects, to be considered as a member of a “heavy” doublet.

In terms of shifts in the (conventionally defined) vector and axial vector $Zb\bar{b}$ couplings, the effect of New Physics on $\delta_{bV}$ can be parametrized as

$$\delta_{bV}^{NP} = \frac{-4}{1 + b^2} [b \delta g^H_{Vb} + \delta g^H_{Ab}]$$  \hspace{1cm} (6)

where

$$b = 1 - \frac{4}{3} s^2$$  \hspace{1cm} (7)

and $s^2$ is defined by eq.(3). The subscript “H” denotes the fact that we are now considering “heavy” quark type of effects.

For the purposes of our search, it would be extremely useful to define and to measure a certain experimental quantity where a different combination of shifts in $g_{Vb}, g_{Ab}$ enters. In fact, such a quantity exists and has been proposed a few years ago [11]. It was defined as the “longitudinally polarized forward-backward $b\bar{b}$ asymmetry” and usually called $A_b$

$$A_b = \frac{\sigma(e^-_L \to bF) - \sigma(e^-_R \to bF) - \sigma(e^-_L \to bB) + \sigma(e^-_R \to bB)}{\sigma(e^-_L \to bF) + \sigma(e^-_R \to bF) + \sigma(e^-_L \to bB) + \sigma(e^-_R \to bB)}$$  \hspace{1cm} (8)

and, as one sees, it requires the availability of longitudinally polarized electron beams.

The remarkable feature of $A_b$ is that of only depending on the couplings of $Z$ to $b$, as it was stressed in Ref.[11]. This explains the great potential interest of its measurement, that will be performed in a very near future at SLC if the very encouraging trend of recent progress in the machine performance is (hopefully) going to continue [12], and might also be performed in a not too far future at LEP if a phase with polarized beams became
operative [13]. If this were the case, an extremely fruitful combination with the results on $R_b$ obtained by unpolarized measurements at LEP1 would become possible, which could allow to draw unexpected conclusions on this fascinating and still existing possibility of small MSM failures.

This short paper is dedicated to the study and to the exploitation of the possible theoretical consequences of a combined determination of $R_b$ and $A_b$. In Section 2, we shall very briefly recall the needed definitions and the relevant theoretical expressions. In Section 3, an investigation of the possible combined effects on the two heavy vertex measurable combinations of some models of New Physics will be performed, showing that there would be distinct “trajectories” in the $(\delta R_b, \delta A_b)$ plane in correspondence to different models, and also a brief discussion of some “unnatural” possibility of light vertex-type effects will be given, before drawing the final conclusions. A short Appendix will be devoted to the derivation of some mass relationships in one of the considered models, where one extra U(1) is involved.

2 Definition of the second heavy quark vertex parameter

An immediate and natural way of defining a new heavy b vertex parameter is to follow the philosophy that led to eq.(1) in the case of $\delta_{bV}$ and to introduce the quantity $\eta_b$ as

$$A_b = A_s(1 + \eta_b) \quad (9)$$

i.e. as the ratio of the longitudinal polarization forward-backward asymmetries for b and s-type quarks. The asymmetry $A_s$ (which corresponds mathematically to that of practically massless b quarks) can be written in a form similar to that of eq.(2):

$$A_s = 0.703(1 - 0.158(\Delta\kappa' + \delta_s') - \Delta_{QCD}\frac{\alpha_s}{\pi} + "negligible") \quad (10)$$
in which $\delta_s'$ is a vertex correction defined in [7] and $\Delta_{QCD}$ is a QCD factor of order one. With this choice, one can easily see that the expression of $\eta_b$ becomes:

$$\eta_b = -\frac{2(1 - b^2)}{b(1 + b^2)}[\delta g_{Vb}^H - b \delta g_{Ab}^H]$$  \hspace{1cm} (11)

The shifts $\delta g_{Vb,Ab}^H$ in eq.(11) take into account in the MSM the effect of the would-be Goldstone exchange in the $Zb\bar{b}$ vertex and also QCD effects due to the not negligible $b$-mass, whose complete calculation has been given elsewhere [14] and that are, as such, supposedly known. The important feature is that, in the MSM (but not a priori in the models of New Physics that we shall consider) the effect on $\eta_b$ of the charged would-be Goldstone boson (that is proportional to $m_t^2$ in $\delta_{bV}$) is practically negligible, owing to the fact that it gives the same contributions to $\delta g_{Vb}$ and to $\delta g_{Ab}$, that are nearly cancelling in the combination of eq.(11). Thus, in the MSM prediction for $A_b$, the “heavy” $b$ vertex component $\sim m_t^2$ can be ignored and the relevant expression does only contain universal self-energies and light vertices (and known QCD corrections). Obviously, this property is a priori no longer verified as soon as one considers models of New Physics, for which the relative role of $\eta_b$ could be much more relevant or fundamental.

To make the previous statement more illustrative, it is convenient to reexpress the shifts of $\delta_{bV}$ and $\eta_b$, rather than in the $(g_V, g_A)$ basis, in that provided by the (conventionally defined) $(g_L, g_R)$ parameters. In that case, one can write:

$$\delta_{bV} = -\frac{4(1 + b)}{(1 + b^2)} \left[ \delta g_{bL}^H - \frac{(1 - b)}{(1 + b)} \delta g_{bR}^H \right]$$  \hspace{1cm} (12)$$

$$\eta_b = -\frac{2(1 - b)}{b} \left[ \delta g_{bR}^H + \frac{(1 - b)}{(1 + b)} \delta g_{bL}^H \right]$$  \hspace{1cm} (13)

As one sees, in the (L,R) basis the two shifts are orthogonal, which means that effects that would not contribute one observable will be revealed by the other one, and conversely.
To the previous remarks one can still add a property of $\eta_b$ that is a direct consequence of our chosen definition eq.(9). In fact, if one eliminates $\delta g_{bL}$ in eq.(12), one obtains:

$$\eta_b = \frac{2(1 - b)}{b} \left[ \frac{2(1 + b^2)}{(1 + b^2)} \delta g_{bR}^H - \frac{(1 - b)(1 + b^2)}{4(1 + b)^2} \delta_{bV} \right]$$

(14)

and, to a very good approximation, this becomes:

$$\eta_b = - \frac{1}{25} \delta_{bV}$$

(15)

showing that, once $\delta_{bV}$ is experimentally known, the measurement of $\eta_b$ fixes unambiguously the pure right-handed contributions from various models to the “heavy” $Zb\bar{b}$ vertex.

After these preliminary definitions, all the necessary ingredients to formulate an unbiased search of New Physics effects in the “heavy” quark vertex sector are at our disposal. One only has to take eqs.(12), (13), insert a “New Physics” apex to both the right and the left-hand side, and choose a set of interesting models to be examined. This will be done in the forthcoming Section 3.

3 Survey of models affecting the heavy $b$ vertex

The simplest known example of a model that contributes the heavy $b$ vertex is that with just one extra Higgs doublet. In this case both the charged and the neutral higgses will have to be considered. The charged contribution can be decomposed into two terms. The first one essentially reproduces that of the MSM (i.e. $\sim \delta g_{bL}$) with the same kind of $m_t$ dependence (weighted by a factor $\sim \cos \theta^2 \beta$ where $\tan \beta$ is the ratio of the two VEV’s); the second one is proportional to the product of $m_b^2$ and $\tan^2 \beta$. As such, it can only be relevant for very large values of $\tan \beta \approx m_t/m_b$. Since it only modifies the right-handed $Zb\bar{b}$ coupling, it will generate a suppressed effect in $\delta_{bV}$ (again, of the same sign as that of the MSM). More interestingly, it will also be able to affect $\eta_b$. The neutral higgses sector
is described by a larger set of parameters, and is therefore more model dependent than
the charged one. In general, it will affect both $\delta g_{bL}$ and $\delta g_{bR}$ with terms proportional to
$m_b^2$ and will consequently be only relevant if some enhancement factor can be adjusted.
In particular, this can be achieved when the value of $\tan\beta$ becomes very large. In this
case, its contribution to $\delta_{bV}$ can be of opposite sign to that of the MSM [15].

These features of the simplest model with one extra Higgs doublet remain essentially
unchanged if one embeds it in a supersymmetric picture, with the additional constraints
between the various couplings and the existence of other types of contributions to be taken
into account. This has been done in great detail in a number of previous papers [16] for
the specific case of the so-called “Minimal” Supersymmetric Standard Model (MSSM)
[17] for both small and large values of $\tan\beta$. The results of all analyses indicate that
in some cases the effects of the Higgs sector and of the genuine “soft” supersymmetric
sector can add up constructively, leading to possible effects of a few percent that should
be visible at future measurements of $\delta_{bV}, \eta_b$.

Among the configurations examined in ref.[16], that corresponding to large $\tan\beta$ values
was considered as a particularly interesting one. The main motivation is that, while
for small $\tan\beta$ values the model essentially contributes $\delta g_{bL}$ but not $\delta g_{bR}$, in the large
$\tan\beta$ case it can affect both $\delta g_{bL}$ and $\delta g_{bR}$. As a consequence of this, two independent
experimental tests would become available which would give rise to some implications. In
particular, one would be able to draw certain “trajectories” in the $(\eta_b, \delta_{bV})$ plane that
would correspond to, or identify, a certain model and could be experimentally “seen”, at
least in a certain part of the plane.

In the analyses of ref. [16], the contribution of the Higgs sector was calculated using the
SUSY mass relationships valid at tree level in the MSSM. Since it has become known [18]
that these relationships are appreciably modified at one loop, one might be interested in
evaluating the eventual modification of the relevant trajectories (that are certain functions
of the various higgses masses). Also, one might consider the effect of adding an extra neutral Higgs to the model since this seems to be a reasonable extension of the “minimal” picture.

In this paper, we have examined the two possibilities and considered as a tool model with one extra Higgs the so called $\eta$ model [19], whose mass relationships at tree level, that have been already examined in the literature [20], show several interesting differences with those of the MSSM. The results of our calculation will be only shown for the Higgs sector and for the related trajectories. The remaining contributions should be identical with those computed in ref.[16] in the MSSM case. For the $\eta$ model, a separate calculation of non Higgs effects should be performed. We believe, though, that the already existing limits on the mass of the extra $Z$ of this model, $M_{Z'} > 500$ GeV [7], pushing the involved soft masses to large values, limit somehow in this model their potential effect (that should not differ drastically, in any case, from the corresponding MSSM one).

The relevant diagrams containing the various Higgses contributions are shown in Fig.1; from these one derives compact expressions that have been already provided in the literature. Here we shall follow the notations of Ref.[15] that, in the large $\tan \beta$ configuration chosen by us produce the relatively simple formulae:

$$\delta g^H_{bbR} = \frac{\alpha}{16\pi s^2} \frac{m_b^2 \tan^2 \beta}{M_W^2} \left( (1 - \frac{4}{3}s^2) \rho_3[m_t, M_{H^+}, m_t, M_Z] \right)$$

$$- m_t^2 \rho_4[M_0[m_t, M_{H^+}, m_t, M_Z]] + (s^2 - c^2) \rho_4[M_{H^+}, m_t, M_{H^+}, M_Z]$$

$$+ (-1/2 + 1/3s^2) (\rho_3[m_b, M_A, m_b, M_Z] + \rho_3[m_b, M_h, m_b, M_Z])$$

$$- \frac{1}{2} \rho_4[M_h, m_b, M_A, M_Z] - \frac{1}{2} \rho_4[M_A, m_b, M_h, M_Z]$$

(16)
Here $\rho_{3,4}[m_1, m_2, m_3, M_Z]$ and $C_0[m_1, m_2, m_3, M_Z]$ are the functions introduced in the appendix of ref.[15]. The masses that appear in the previous expressions are those of the charged Higgs ($M_{H^+}$), of the CP-odd neutral Higgs ($M_A$) and of that CP-even neutral Higgs ($M_h$) whose mass is nearly degenerate with $M_A$ in the MSSM and in the $\eta$ model. Starting from the given expressions, one only has to insert, at a certain level of accuracy, the mass relationships of the various models that are, in general, not the same. In particular, the famous tree-level formulae of the MSSM and the corresponding ones of the $\eta$ model [20] can be substantially different. For example, one finds in the first case the equality:

$$M_{H^+}^2 = M_A^2 + M_W^2$$

whilst in the second model one has:

$$M_{H^+}^2 = M_A^2 + M_W^2 \left[ 1 - \frac{2\lambda^2}{g^2} \right]$$

where $\lambda$ is a free parameter. Also, one finds a bound for the lightest neutral in the MSSM, that becomes sensibly larger in the other case [20]. At one loop, extra not negligible differences can arise in both models, which could in principle give rise to observable effects.

Motivated by the previous argument, we have calculated eqs.(16), (17) inserting the one-loop mass relationships of the two models. For the MSSM, these are known and can be found in the literature [18]. For the $\eta$ model, in the chosen configuration, they are given in the short Appendix. The numerical values of $\delta_{bV}$ and $\eta_b$ are shown in the following Figures. They will depend on $m_t$ (from the charged sector), on $m_b\tan\beta$ (from both...
sectors) and on one residual neutral mass chosen to be \( M_A \approx M_h \). The value of \( M_{H^+} \) remains fixed by the choice of the configuration, as shown in Appendix, for the MSSM. In the case of the \( \eta \) model, for which extra parameters exist, we have chosen the situation that optimizes the effect and thus the related figures are actually showing the maximal deviations that the model can produce. All the numerical results are given for \( m_b = 5 \) GeV, \( \tan \beta = 70 \), following the approach of Ref. [16].

To get a qualitative feeling of the differences obtained by using the modified mass relationships, we show in Figs. 2, 3 the trajectories corresponding to the MSSM with mass constraints at tree level, eq. (18), and at one loop. One sees that one effect is that of “smoothening” the \( m_t \) dependence, particularly in the heavy mass region, say, between 150 GeV and 200 GeV (intermediate and upper lines) (this is a consequence of the fact that in the charged Higgs contribution this dependence is now weakened in the relevant ratio between the top and the Higgs masses). Also, one notices a systematic (small) decrease in \( \eta_b \), compensated by a corresponding (small) increase in \( \delta_{bV} \).

In fact, the compensation between \( \eta_b \) and \( \delta_{bV} \) is quite general, in the sense that for small \( M_A \) values the full (positive) effect is on the second parameter, while for large \( M_A \) only the first one is modified. This is related to the fact that \( \eta_b \) is dominated by right-handed effects, that are peculiar of the charged Higgs contribution whose decoupling is slower than that of the neutral ones (that give the important effect on \( \delta_{bV} \)).

If we accept the experimental available indications [8] that seem to prefer positive (or, at least, not too negative) \( \delta_{bV} \) shifts, we conclude that the most relevant part of the Higgs sector trajectory of this model lies in the positive \( \eta_b \) region of the plane (with the exception of the fraction that would correspond to substantial \( \delta_{bV} \) effects (larger than, say, two percent) i.e. to very small \( M_A \) values , where the shift on \( \eta_b \) could be negative). Since the same feature seems to be valid for the remaining genuinely supersymmetric contributions of the model [16], we conclude that the simultaneous observation of (small)
positive deviations in either $\delta_{bV}$ or $\eta_b$, or possibly in both, could be interpreted as the experimental evidence for this model in the considered region of its parameter space. This would require a precision of the two measurements of the order of a relative one percent, although in certain favourable cases the shifts could be larger than that, particularly if the effects from the Higgs and the genuine SUSY sector added in a substantial way as they seem to be willing to do.

The case of the $\eta$ model is illustrated in Fig.4, only showing the situation where the mass constraints are used at one loop. As one sees, the results for the Higgs sector are very similar to those of the previous example, with a small general increase of $\eta_b$ and practically no change in $\delta_{bV}$. Since we expect that other contributions are somehow depressed in this case, we would conclude that the trajectories of this model are qualitatively similar to those of the MSSM (with possibly smaller overall effects); in other words, the presence of one more neutral scalar does not affect the trajectory in this case. Whether this is a general feature of SUSY models with one extra (singlet) scalar remains to be investigated; we postpone the discussion of this point to a next forthcoming paper.

It can be interesting to remark that in the “orthogonal” case of Technicolor-type modifications of the MSM, the associated trajectories would be completely different for a wide class of models. This can be deduced from the analysis presented in reference [21] where the contributions to $\delta_{bV}$ were computed. In fact, for a class of “walking technicolor” cases the effect on $\delta_{bV}$ was negative and of purely left-handed type, leading in any case to negative corrections to $\eta_b$ as one can easily verify from the defining eqs.(12), (13). The exception to this statement would be represented by a class of special models where fermion masses are due to the presence and mixing of technibaryons [22], that produce positive shifts in $\delta_{bV}$. But for these models, the shift in $\eta_b$ can be written to a good approximation, using again eqs.(12), (13) as follows:

$$\eta_b \simeq \delta_{bV} \frac{1 - 5c^2}{5(5 + c^2)}$$ (20)
where \( c^2 = \sin^2 \alpha / \sin^2 \beta' \) and \( \alpha, \beta' \) are the two mixing angles of the model. Varying this ratio from zero to infinity fixes \( \eta_b \) in a region between, practically, zero and \(-\delta_{bV}\) as shown in the next Fig. 5. Thus, the observation of two small effects of opposite sign with a negative \( \eta_b \) would provide a rather peculiar evidence for this special model.

To conclude our investigation, we have considered the (less attractive, in our opinion) possibility that the origin of small discrepancies in \( \frac{\delta R_b}{R_b} \) and \( \frac{\delta A_b}{A_b} \) is due to effects of light-fermion type. Firstly, we have considered the class of models with one extra \( Z' \) of \( E_6 \) origin that has been often considered in the literature [19]. For these models, strong experimental constraints on the mixing angle exist [7] that limit its modulus to be less than, say, one percent. Using this extreme value as the tolerated limit for every single model (which is somehow optimistic) we obtain the effects shown in Fig. 6. As expected, the possible effects of this kind are always below the one percent level and are spread in the \( (\frac{\delta R_b}{R_b}, \frac{\delta A_b}{A_b}) \) plane. In other words, the existing limits on the mixing angle seem to prevent interesting effects from these models. Note, accidentally, that the contribution coming from the \( \eta \) model (that would belong in the chosen configuration of large \( \tan \beta \) values to positive mixing angles) goes in the opposite direction to that of the Higgs sector, which represents a negative feature of the model. We have repeated our analysis for an extra \( Z' \) predicted by Left-Right symmetry models and for higher vector bosons predicted by various types of different models, in particular compositeness inspired models \( (Y, Y_L, Z^*) \) [24] and alternative symmetry breaking models \( (Z_V) \) [25]. As in the first case, the limits imposed by precision tests in the light fermion sector prevent from getting large effect on \( R_b \) and \( A_b \) as one can see in Fig. 6.

We can summarize the results of this preliminary investigation as follows. Assuming as a realistic goal a final experimental accuracy on the measurements of both \( A_b \) and \( R_b \) of a relative one percent, the best chances of providing visible signals seem to belong to models of New Physics that can affect the “heavy” b-vertex component. Among these,
we have seen that those of SUSY type are associated to trajectories in the plane of the variations of $\delta_{bV}$ and $\eta_b$ that differ substantially from those of technicolor type. We stress the fact that this differentiation is made possible by the combined measurements of the two observables; for instance, the discovery of a positive effect in $\delta_{bV}$ could not discriminate the models of Figs.2, 3, 4 from that of Fig.5. Should this effect (that is apparently not disallowed by the existing data) survive in the future, the role of a high precision measurement of $\eta_b$ would become, least to say, fundamental.

Before concluding this paper we would like to make a rather speculative remark concerning the possibility that a positive shift of $R_b$ is observed with no effect on $A_b$. From a purely technical point of view, it might be possible to explain this effect in a picture where the MSM calculation is still valid, but where the effective axial coupling of $Z$ to the top is slightly decreased. In fact, in the large $m_t$ limit, the dominant contribution to $\delta g_{bL}$ can be expressed in the form:

$$\delta g_{bL} \simeq \frac{\alpha}{8\pi s^2 M_W^2} g_{t,A}$$

and values of $g_{t,A}$ slightly smaller than one-half (with no effect on the corresponding b-vertex) could provide this possible deviation, thus motivating searches of reasonable models where the axial “form factors” of heavy quarks can be possibly modified [23].

**Acknowledgements**

One of us (C.V.) acknowledges some useful discussions with B.W.Lynn and H.Schwarz during a stay at UCLA in summer 1993.
Appendix

In this Appendix we give the expressions of the relevant radiative corrections (R.C.) to eq.(18) in the MSSM and to eq.(19) in the $\eta$ model. The Higgs sector of the MSSM at tree level is described by two parameters, $\tan\beta$ and $M_A$; when we include the radiative corrections all the parameters which describe the spectrum of the theory enter in the mass formulae. The most important contributions to the R.C. come from the stop-sbottom sector, so we must fix: the soft squark masses ($m_{\tilde{t},\tilde{b},\tilde{q}} \approx 1$ TeV); the trilinear SUSY breaking parameters ($A_t = A_b = 100$ GeV) ; the SUSY $H_1H_2$ coupling $\mu$ and of course the top mass. In the large $\tan\beta$ limit the one loop mass relationships read [18]:

$$M^2_{H^+} = M^2_A + M^2_W + \Delta M^2_{H^+} \quad (22)$$

where

$$\Delta M^2_{H^+} = \frac{3g^2}{32\pi^2M^4_W}[2m_t^2m_b^2\tan^2\beta - M^2_W(m_t^2 + m_b^2\tan^2\beta)]$$

$$+ \frac{2}{3}M^4_W \log \frac{m^2_{\tilde{q}}}{m^2_t} + \frac{3g^2}{96\pi^2}[m_t^2(\frac{\mu^2 - 2A^2_t}{m^2_{\tilde{q}}})]$$

$$+ m_b^2\tan^2\beta \left(\frac{\mu^2 - 2A^2_b}{m^2_{\tilde{q}}} \right) + \frac{3g^2}{64\pi^2M^2_W} [m_b^2m_t^2\tan^2\beta \left(\frac{A_t + A_b}{m^2_{\tilde{q}}} \right)^2$$

$$- \frac{\mu^2}{m^2_{\tilde{q}}} \left(\frac{m_b^2 + m_t^2\tan^2\beta}{M^2_W} \right)^2] = \frac{3g^2}{192\pi^2M^2_W} \left(\frac{A_tA_b - \mu^2}{m^2_{\tilde{q}}} \right)^2 \quad (23)$$

The radiatively corrected mass $M_h$ of the CP-even neutral Higgs which runs into the loop of Fig.1 is always nearly equal to $M_A$.

In the $\eta$ model the tree level Higgs sector is defined by 4 parameters: $\tan\beta, M_A, x, \lambda$. The new parameter $x$ is the VEV of the extra complex Higgs field $N$ and fixes the scale of the breaking of the extra U(1) gauge group, so naturally $x \gg v_1, v_2$. In this large
limit the Higgs sector, that is described by 3 CP-even, 1 CP-odd and 1 charged-state, effectively reduces, at the $M_Z$ scale, to that of the MSSM with the following identifications:

$$\mu = \lambda x, m_3^2 = \lambda A_\lambda x$$

($m_3^2$ is the soft SUSY breaking term of the operators $H_1 H_2$ in the MSSM, $A_\lambda$ is the trilinear soft term which multiplies the product $N H_1 H_2$ in the potential). When R.C. are evaluated, besides the parameters of the MSSM there is another Yukawa coupling $h_E$ of the exotic quark sector ($m_\tilde{E} = m_{\tilde{q}}, A_E = A_t$). So finally the extra new parameters are $\lambda, x$ and $h_E$. We fix $x$ via the mass of the extra $Z'$ boson:

$$M_{Z'} = 25/18 g_1^2 x^2 = 0(1 \text{ TeV})$$

The exotic Yukawa coupling gives very little contributions (some GeV) to the “standard” Higgs sector and can be safely fixed to 1. The Higgs spectrum is at the contrary very sensitive to the $\lambda$ parameter: this strong dependence is exhibited by the charged Higgs sector (see eq.(19)) and by the lightest CP-even mass. As shown in ref.[20] (for values of $M_A < M_{Z'}$) the lightest Higgs mass ($M_t$) is a convex parabola in the $\lambda^2$--$M_t$ plain. The imposition of the experimental bound $M_t \geq 60$ GeV gives a very strong upper limit on $\lambda$ (tipically $\lambda < 0.4$); therefore the difference between the charged Higgs mass (for fixed $M_A$) in the two models cannot be arbitrarily large. The mass $M_h$ is again nearly equal to $M_A$. So, the only effective difference between the $\eta$ model and the MSSM, in this region of the space parameters, is contained in the relation $M_{H^+} - M_A$:

$$M_{H^+}^2 = M_A^2 + M_W^2 (1 - \frac{2\lambda^2}{g^2}) + \Delta M_{H^+}^2 + \Delta' M_{H^+}^2$$

(24)

where $\Delta M_{H^+}^2$ is the same as in eq.(20) with the suitable identifications and $\Delta' M_{H^+}^2$ is the small contribution of the exotic sector:

$$\Delta' M_{H^+}^2 = -\frac{3}{8\pi^2} M_W^2 \frac{\lambda^2}{g^2} h_E^2 \left[ \log \frac{m_{\tilde{q}}^2 + m_E^2}{M_Z^2} - \frac{1}{6m_{\tilde{q}}^2 + m_E^2} A_E^2 m_E^2 \right]$$

(25)

In general when $\lambda \to 0$ we have the same relationships $M_{H^+} - M_A$ as in the MSSM and the trajectories in the plane $(\delta_{bV}, \eta_b)$ are the same. What we have shown in Fig.4 are the trajectories with the maximum value of $\lambda$ such that the neutral Higgs sector is beyond
the present experimental bound.
References

[1] G.Altarelli and R.Barbieri, Phys. Lett. B253, 161 (1990).

[2] A.Akhundov, D.Bardin and T.Riemann, Nucl. Phys. B276, 1 (1986); W. Beenakker and W.Hollik, Z. Phys. C40, 141 (1988); B.W.Lynn and R.G.Stuart, Phys. Lett. B252, 676 (1990); J.Bernabeu, A.Pich and A. Santamaria, Nucl. Phys. B363, 326 (1991).

[3] A.Blondel, A.Djouadi and C. Verzegnassi, Phys. Lett. B293, 253 (1992).

[4] A.Blondel and C. Verzegnassi, Phys. Lett. B311, 346 (1993).

[5] G.Altarelli, R.Barbieri, F.Caravaglios, Phys. Lett. B314, 357 (1993).

[6] D.C.Kennedy and B.W.Lynn, Nucl. Phys. B253, 216 (1985); A.Sirlin Phys. Lett. B232, 971 (1989); M.Consoli and W.Hollik in “Physics at LEP I”, G.Altarelli, R.Kleiss, C.Verzegnassi eds. CERN 89-09 (1989).

[7] J.Layssac, F.M. Renard and C. Verzegnassi preprint PM/93-13 UTS-DFT-93-13 (to appear on Phys. Rev.D).

[8] We use the data comunicated by the various LEP collaborations at the EPS Conference, Marseille, July 1993.

[9] A.Blondel and C. Verzegnassi, seminar presented at the 1993 Marseille Conference, to appear on those proceedings.

[10] The CDF Collaboration, A.Barbaro-Galtieri, Proceedings of the EPS Conference on High Energy Physics, Marseille, France (1993).

[11] A.Blondel, B.W.Lynn, F.M. Renard and C. Verzegnassi, Nucl. Phys. B304, 438 (1988).
For a recent review of the machine’s status, see e.g. C.Prescott, SLAC-PUB-66355, September 1993.

For a recent review on the subject, see e.g. A. Blondel CERN-93-125.

A.Djouadi, J.H. Kuhn and P. Zerwas, Z. Phys. C46, 411 (1990).

A. Denner, R. J. Guth, W. Hollik and J. H. Kuhn, Z. Phys. C51, 695 (1991).

A. Djouadi, J. L. Kneur and G. Moultaka, Phys. Lett. B242, 265 (1990); A. Djouadi, G. Girardi, W. Hollik, F. M. Renard and C. Verzegnassi, Nucl. Phys. B349, 48 (1991); M. Boulware and D. Finnel, Phys. Rev. D44, 2054 (1991); C. Sheng Li, J. Min Yang, B. Quan Hu and Y. Sheng Wei CQU-TH-92-17, Dec. 5, 1992.

L. E. Ibanez Phys. Lett. B118, 73 (1982); Nucl. Phys. B218, 514 (1983); R. Barbieri, S. Ferrara and C. Savoy, Phys. Lett. B119, 343 (1982); P. Nath, A. Arnowitt and A. Chamseddine, Phys. Lett. B49, 970 (1982).

J. F. Gunion and A. Turski, Phys. Rev. D40, 2325 (1989); M. A. Diaz and H. E. Haber, Phys. Rev. D45, 4246 (1992).

For a full discussion of this model (and similar ones) see e.g. L. Hewett and T. G. Rizzo, Phys. Rep. C183, 193 (1989).

H. E. Haber and M. Sher, Phys. Rev. D35, 2206 (1987); M. Drees, Phys. Rev. D35, 2910 (1987); D. Comelli, C. Verzegnassi, Phys. Lett. B303, 277 (1992).

R. S. Chivukula, S. B. Selipsky and E. H. Simmons, Phys. Rev. Lett. 69, 575 (1992).

D. B. Kaplan, Nucl. Phys. B365, 259 (1991).

The possibility that the axial form factors of the constituent quarks are modified in a rather different context has been already discussed in several papers. See e.g. S. Peris,
Phys. Lett. **B268**, 415 (1991) ; S.Weinberg, Phys. Rev. Lett. **67**, 3473 (1991) ; E. de Rafael and S.Peris, Phys. Lett. **B309**, 389 (1993) . The effects of anomalous top quark couplings on precision electroweak measurements have been also considered by R.D.Peccei and X.Zhang, Nucl. Phys. **B337**, 269 (1990) ; R.D.Peccei, S.Peris and X.Zhang, Nucl. Phys. **B349**, 305 (1991) ; M.Frigeni and R.Rattazzi, Phys. Lett. **B269**, 412 (1991) .

[24] M.Kuroda, D.Schildknecht and K.H.Schwarzer, Nucl. Phys. **B261**, 432 (1985) .

[25] R.Casalbuoni, S.De Curtis, D.Dominici and R.Gatto, Nucl. Phys. **B282**, 234 (1987) ; J.L.Kneur and D.Schildknecht, Nucl. Phys. **B357**, 357 (1991) ; J.L.Kneur, M.Kuroda and D.Schildknecht, Phys. Lett. **B262**, 93 (1991) .
**Figure Captions**

**Fig. 1**: Self energy and vertex corrections to the $Z\bar{b}b$ vertex

**Fig. 2**: Plot in the $(\delta b_V, \eta_b)$ plane of the corrections (in percent) in the MSSM case with the relationships $M_{H^+} - M_A$ at tree level (see eq. (18)). There are 16 points for each “curve”, each one corresponding to a given value of $M_A$, in particular (starting from the right to the left): $M_A = 40, 45, 50, 55, 60, 65, 70, 75, 80, 90, 100, 120, 140, 160, 180, 200$ GeV. The upper line corresponds to $m_t = 200$ GeV, the intermediate one to $m_t = 150$ GeV and the lowest one to $m_t = 110$ GeV.

**Fig. 3**: The same as before for the MSSM but with the mass relationships at one loop (see eq. (22)).

**Fig. 4**: The same as before but for the $\eta$ model and with the mass relationships at one loop (see eq. (24)).

**Fig. 5**: The set of allowed trajectories for the Kaplan model discussed in ref. [21,22] at variable ratio $c^2$ of the two mixing angles.

**Fig. 6**: Maximal allowed $Z - Z'$ mixing effects in the $(\delta R_b, \delta A_b)$ plane, from $E_6$ based models with $-1 \leq \cos(\beta) \leq +1$ (dashed), from L-R symmetry based models with $\sqrt{\frac{2}{3}} \leq \alpha_{LR} \leq \sqrt{\frac{2}{3}}$ (full), in both cases with $|\theta_M| = 0.01$. We have also indicated the trajectories or small domains allowed for various alternative models of higher vector bosons ($Y, Y_L, Z^*, Z_V$) taking into account the constraints established in ref. [7].