Non-critical fluctuations of (net) charges and (net) protons from iEBE-VISHNU hybrid model

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In this paper, we investigate the non-critical fluctuations of (net) charges and (net) protons in Au+Au collisions at \( \sqrt{s_{NN}} = 7.7, 39 \) and 200 GeV, using iEBE-VISHNU hybrid model with Poisson fluctuations added in the particle event generator between hydrodynamics and UrQMD. Various effects, such as volume fluctuations, hadronic evolution and scatterings, resonance decays, as well as realistic centrality cuts and acceptance cuts have been embedded in our model calculations. With properly tuned parameters, iEBE-VISHNU roughly describe the centrality dependent moments and cumulants of (net) charges and (net) protons measured in experiment. Further comparison simulations show that the volume fluctuation is the dominant factor to influence the multiplicity fluctuations, which makes the higher moments of (net) charges largely deviate from the Poison baselines. We also find that the effects from hadronic evolutions and resonance decays are pretty small or even negligible for the multiplicity fluctuations of both (net) charges and (net) protons.

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I. INTRODUCTION

Exploring the QCD structure phase of the strongly interacting matter is one of the major goals of relativistic heavy-ion collisions (RHIC) [1–4]. As a unique feature of the phase diagram, the QCD critical point has attracted particular interests from both theoretical and experimental sides [5–7]. It is proposed that the higher moments of conserved quantities are sensitive observable to probe the QCD critical point [8, 9]. The recent Beam Energy Scan (BES) program has systematically measured the higher moments (cumulants) of net charges and net protons in Au+Au collisions at \( \sqrt{s_{NN}} = 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4 \) and 200 GeV [10–12]. It was found that the cumulant ratio \( \kappa_2/\sigma^2 \) of net protons obviously deviates from the Poisson baselines and shows a non-monantoic behavior at lower collision energies, which indicates the potential of discovery the QCD critical point in experiments [13].

Besides studying the critical fluctuations, it is important to systematically investigate the non-critical/thermal fluctuations of produced hadrons for the location of the QCD critical point [6, 7, 14–30]. In traditional Hadron Resonance Gas (HRG) model with Boltzmann approximations, the thermal fluctuations are governed by the Poisson statistics [16–18]. Correspondingly, the Poisson expectations are served as the basic thermal fluctuations baselines, which have been widely used in both experimental analysis and theoretical study [10–13, 29–32]. However, a realistic heavy ion collision involves many complicated processes. Many factors could make the measured multiplicity fluctuations deviate from the Poisson baselines. For example, the system size of the collision systems within a centrality bin fluctuate event by event, the related volume fluctuations could change the cumulants of the multiplicity distributions [33–36]. Meanwhile, the finite acceptance window and acceptance efficiency also bias the multiplicity distributions measured in experiments [37–40]. In Ref. [41–43], it was found the global conservation of baryon number, strangeness number and electric number modify the cumulants of net charges and net baryons. Besides, the isospin-randomization progress [44, 45], the weak decay and other related progress also influence the fluctuations of the final produced hadrons to some extend (For related review on non-critical fluctuations, please refer to [6, 7]).

Many past research of HRG model and Lattice QCD simulations assume the system is static and in global chemical and thermal equilibrium [17–27]. However, the QGP fireball created in a relativistic heavy ion collisions is a dynamically evolving system, where the chemical and thermal equilibrium can not be maintained during the late hadronic evolution [46–50]. Within the framework of UrQMD [51], Luo and his collaborators have systematically calculated the thermal fluctuation baselines of final produced hadrons in Au+Au collisions at \( \sqrt{s_{NN}} = 7.7 – 200 \) GeV. However, their model simulations assumed that the created systems are purely hadronic at various collision energies, which neglected the collective expansion of the QGP phase at higher collision energies.

In this paper, we will investigate the multiplicity fluctuations of (net) charges and (net) protons, using iEBE-VISHNU hybrid model that combines viscous hydrodynamics for the expansion of the QGP with a hadron cascade model for the evolution of the hadronic matter. Compared with other dynamical model simulations, such as UrQMD [51–54] and JAM [55], we input Poisson fluctuations in the particle event generator between the hydrodynamics and hadron cascade simulations. We focus on investigating how various effects, such as volume fluctuations, hadronic scatterings, resonance decays, centrality cut and acceptance cut, etc., influence the multiplicity fluctuations of final produced hadrons. Consider-
ing that the event-by-event simulations of iEBE-VISHNU hybrid model are time-consuming, we only perform the simulations at three selected collision energies, $\sqrt{s_{NN}} = 200, 39$ and $7.7$ GeV, where the net baryon density gradually increases from higher to lower collision energies. The following related calculations will show that effects from hadronic evolution are pretty small or even negligible for the multiplicity fluctuations of (net) charges and (net) protons, which may help to largely increase the numerical efficiency for the massive data simulations in the near future.

The paper is organized as follows: Sec. II and Sec. III introduce iEBE-VISHNU hybrid model, the observables of multiplicity fluctuations and the set-ups of calculations. In Sec. IV, we present the calculations for the moments and moment products of (net) charges and (net)-protons in Au+Au collisions at 7.7, 39 and 200 GeV from iEBE-VISHNU, together with a comparison to the STAR data. We also investigate the effects from volume fluctuations, resonance decays and hadronic evolution for the multiplicity fluctuations of (net) charges and (net)-protons. In Sec. V, we briefly summarize this paper.

II. THE MODEL AND SETUPS

In this paper, we investigate the non-critical multiplicity fluctuation of (net) charges and (net) protons in Au+Au collisions at $\sqrt{s_{NN}} = 7.7, 39$ and $200$ GeV, using iEBE-VISHNU hybrid model. iEBE-VISHNU [56] is an event-by-event version of VISHNU, which combines viscous hydrodynamics for the QGP expansion with a hadron cascade model for the hadronic evolution [57]. It contains four main components to simulate different stages of a relativistic heavy ion collision: (1) the initial conditions, which are generated by some initial condition models, such as Monte-Carlo Glauber model (MC-Glauber) [58], Monte-Carlo KLN model (MC-KLN) [59, 60], TRENTo model [61], AMPT [62], etc. (2) the macroscopic expansion of the QGP fluid, which is simulated by a $(2+1)$-dimensional viscous hydrodynamics VISH2+1 [63, 64]. (3) the switching between the hydrodynamics and the succeeding hadron cascade simulations, which is realized by a Monte-Carlo event generator that samples particles on the switching hyper-surface with the Cooper-Frye formula [57]. (4) The microscopic evolution and decoupling of the hadron resonance gas, which is simulated by Ultra-relativistic Quantum Molecular Dynamics (UrQMD) hadron cascade model [65, 66].

In the following text, we will introduce step (3) in more details since it is directly related to the thermal fluctuations investigated in this paper. For other details of iEBE-VISHNU hybrid model, please refer to [56, 67, 68]. From the macroscopic hydrodynamics to the microscopic UrQMD simulations, the thermal hadrons emitted from the switching hyper-surface are sampled according to the differential Cooper-Frye formula [57, 69]:

$$E \frac{d^3N_i}{dp_T^3}(x) = \frac{g_i}{(2\pi)^3} \rho^0 d^3\sigma_{\rho_i}(x) f(x, p),$$

where $x = (\tau, \vec{x}, \eta_i), p = (E, \vec{p}_T, \gamma)$ are position and 4-momentum of the emitted hadrons, $d^3\sigma_{\rho_i}$ is the surface element of the switching hyper-surface $\Sigma_i$, and $g_i$ is the spin degeneracy of the $i$th hadrons. The distribution function $f(x, p) = f_0 + \delta f$, where $f_0$ is the equilibrium distribution function and $\delta f = \frac{p^\mu}{\rho^0} z_i f_0(1 \mp 0)$ is the corresponding viscous correction [63, 64]. Following [70], the equilibrium distribution function is taken the form: $f_0 = 1/(\gamma_s \delta p^\mu \rho_0(\vec{c} \cdot x)^0 \pm 1)$, where $\gamma_s$ is the strangeness saturation factor and $[S_i]$ the total number of strange and anti-strange quarks of hadron species $i$, $\vec{c}_i = (B_i, Q_i, S_i)$ are the corresponding conserved charges. In many traditional hybrid model simulations [71–73], $\mu_B, \mu_S$ and $\mu_Q$ are all set to zero. Here, these additional tunable parameters help to achieve a nice description of the mean values of positive (negative) charges and (anti-)protons at various centralities and collision energies, which are important for the investigations of multiplicity fluctuations.

From Eq.(1), one could obtain a fixed (mean) value of $dN_i/dy$ for each hadron species $i$. In the past simulations [71, 72], the Monte-Carlo event generator simultaneously generates many profiles from one switching hyper-surface with fixed number of multiplicity $N_i = w_i * dN_i/dy$ for each hadron species $i$ (where $w_i$ is the width of the rapidity window), which are then input into UrQMD for the succeeding evolution of the hadronic matter. The multiplicity fluctuations of the final produced hadrons are mainly come from the initial state fluctuations and the fluctuations from the evolution, scatterings and decays of the hadronic matter.

In this paper, we assume the emitted hadrons from the hydrodynamic switching hyper-surface contain additional thermal fluctuations that obey the Poisson distribution:

$$P_i(k) = \frac{\lambda_i^k e^{-\lambda_i}}{k!}.$$

Here, $\lambda_i = N_i$, which is the mean value (multiplicity) of the hadron species $i$. Note that the Cooper-fryer freeze-out of hydrodynamics is belong to the framework of statistical hadronization with grand canonical ensemble. For heavier particles like protons, the equilibrium distribution $f_0(x, p)$ is very close to the Boltzmann distribution near $T_{exp}$, which leads to an approximately Poisson distribution after considering the related thermal fluctuations. We have also realized that other factors, such as the non-equilibrium distribution function $\delta f$, the Bose-Einstein distributions for light hadrons, etc. could break such Poisson distribution to some extend. As pointed out in [56, 67], an exact implementation of the realistic thermal fluctuations in iEBE-VISHNU is non-trivial. Here, we take such distribution in Eq.(2) as a basic assumption, and then focus on investigating how the effects of volume fluctuations, hadronic evolution, resonance decays, etc., influence the multiplicity fluctuations of final produced hadrons.

For the investigation of multiplicity fluctuations, the numerical efficiency is one of the most important factors to be considered since millions of final particle profiles are needed for
the analysis of higher moments of final produced hadrons. In our simulations, we first run MC-Glb model to generate millions of initial profiles, and then cut the “centralities” according to the distributions of total initial entropy \( s_0 \). Here, we divide the initial profiles into 20 “centrality bins”. For each unit bin, we run one hydrodynamic simulation with a smoothed initial entropy density averaged from \( N (N = 100 \sim 1000) \) MC-Glb profiles within that “centrality”, which then follows with \( N \) UrQMD simulations for the succeeding hadronic evolution. In more details, one hydrodynamic simulation generates one switching hyper-surface, which gives the mean multiplicity \( \bar{N}_i \) for each thermal hadron species. For the \( \alpha \)'s initial profile within a specific “centrality”, we do not run the time-consuming hydrodynamic simulation, but estimate the mean multiplicity of each hadron species by \( s_0 \bar{N}_i / \bar{s} \) considering that the multiplicity is approximately proportional to the initial entropy for a hydrodynamic system (where \( \bar{s} \) is the total entropy of the event-averaged initial profiles and \( s_0 \) is total entropy of the \( \alpha \)-th event). Before the succeeding UrQMD simulations for the \( \alpha \)'s event, we add additional Poisson fluctuations for each thermal hadron species through Eq.(2) with the Poisson parameter set to \( \lambda_\alpha = s_0 \bar{N}_i / \bar{s} \). With such method and a properly chosen acceptance cut for final produced particles, the effect of volume fluctuations [33–36] have been included in our model simulations.

In the following calculations, we set \( \tau_0 = 0.6 \) fm/c, \( \eta/s = 0.08 \), and neglect the bulk viscosity and heat conductivity [68, 74]. We use the multiplicities, particle ratios, and the mean values of (anti-)protons, (negative) positive charges [11, 12, 70] to fix the freeze-out parameters in Eq.(1), including the chemical freeze-out temperature \( T_{ch} \), baryon chemical potential \( \mu_B \), strangeness chemical potential \( \mu_S \), charge chemical potential \( \mu_Q \), and strangeness saturation factor \( \gamma_S \). These fine tuned parameters are listed in Table I.

### III. OBSERVABLES

To evaluate the multiplicity fluctuations, one calculates the cumulants of the multiplicity distributions of final produced particles:

\[
\begin{align*}
    c_1 &= \langle N \rangle = M, \\
    c_2 &= \langle (N - \langle N \rangle)^2 \rangle = \sigma^2, \\
    c_3 &= \langle (N - \langle N \rangle)^3 \rangle = S \sigma^3, \\
    c_4 &= \langle (N - \langle N \rangle)^4 \rangle - 3c_2^2 = \kappa \sigma^4,
\end{align*}
\]

where \( \Delta N = N - \langle N \rangle \), \( N \) is the multiplicity of the particle of interest, and \( \langle \ldots \rangle \) denotes the event average. Here, \( M, \sigma, S \) and \( \kappa \) are the mean value, standard variance, skewness and kurtosis of the probability distribution. In order to partially remove the volume effects, one calculates the moment products \( \sigma^2/M, S \sigma \) and \( \kappa \sigma^2 \), which can be expressed as the cumulant ratios:

\[
\frac{\sigma^2/M}{C_2/C_1} = \frac{\sigma}{C_2/C_1} = \frac{S}{C_2/C_1} = \frac{\kappa \sigma^2}{C_2/C_1} = \frac{C_4}{C_2}.
\]

In experiments, the multiplicity fluctuations of (net) charges and (net) protons are measured within certain centrality bin and acceptance window. For positive and negative charges, one first cuts the centrality bin by the total number of all charged hadrons within the pseudo-rapidity window \( 0.5 < |y| < 1.0 \), and then measures the event-by-event multiplicity distributions of positive and negative charges within the acceptance cut \( |y| < 0.5 \) and \( 0.2 < p_T < 2.0 \) GeV for each centrality bin. For the case of protons and anti-protons, the centrality bin is cut by the total number of pions and kaons within the pseudo-rapidity window \( |y| < 1.0 \). For each centrality, the multiplicity fluctuations of protons and anti-protons are analyzed within the mid-rapidity \( |y| < 0.5 \) and transverse momentum \( 0.4 < p_T < 0.8 \) GeV. Follow the exper-

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1 The “centrality” cut here are different from the centrality definition of final multiplicities, since the Poisson distribution in Eq.(2) brings additional multiplicity fluctuations, which influences the centrality cut of final produced hadrons within an acceptance window.
In general, the multiplicity fluctuations are not directly measured within a wide centrality bin (e.g., 0-5%, 10-20%, etc.), which associates with the wide centrality bin effects that can distort the imprinted fluctuations [75]. To reduce such effects, one divides a wide centrality bin into many fine bins, and then calculates the total moments within that wide centrality bin from the moments of each fine bin with some weight:

\[ X = \frac{\sum n_i X_i}{\sum n_i} \]

where \( X \) represents the total moment within a wide centrality, \( X_i \) is the moment of the fine centrality bin \( i \), \( n_i \) is the number of events in the \( i^{th} \) bin, and \( \sum n_i \) is the total number of events in the wide centrality bin. Following [76, 77], we choose each reference multiplicity to define a fine centrality bin [11], and then implement the Delta theorem to calculate the statistical errors for the moments and moment products.

IV. RESULTS AND DISCUSSIONS

A. Comparisons with the STAR data

Fig. 1 shows the centrality dependent moments (mean value \( M \), standard deviation \( \sigma \), skewness \( S \) and kurtosis \( \kappa \)) of positive and negative charges in Au + Au collisions at \( \sqrt{s_{NN}} = 7.7, 39 \) and 200 GeV. In our model calculations, we first tune the related parameters in iEBE–VISHNU to fit the mean values of positive and negative charges, and then predict other moments at these selected collision energies (please refer to Sec. II for details). For the centrality-dependent standard variance \( \sigma \), iEBE–VISHNU nicely describe the data of both positive and negative charges, except for the most-central collisions. Meanwhile, our model calculations show certain deviations from the Poisson baselines for various moments, including the standard deviation \( \sigma \), skewness \( S \) and kurtosis \( \kappa \).

In iEBE–VISHNU calculations, there are many factors that could influence the multiplicity fluctuations of final produced hadrons, which include the initial state fluctuations, the poisson fluctuations for the emitted hadrons on the hydrodynamic switching surface, the hadronic scatterings and resonance decays in UrQMD, as well as the centrality and acceptance cuts.
for the particle of interest. In the following Sec IV B, we will show that the effects from the hadronic evolution and decays are pretty small or even negligible for the multiplicity fluctuations of (net) charges and (net) protons. For a certain centrality bin, the combined effects of initial entropy fluctuations and the Poison fluctuations on the switching surface are similar to the volume fluctuations/corrections as investigated in the early paper [34, 35], which are the dominant factors to influence the multiplicity fluctuations and make them deviate from the Poison Baselines (please refer to Sec. IV B for details).

We also noticed that, in the most central collisions, the standard deviation $\sigma$ of iEBE-VISHNU is about 10% higher than the experimental data. In [34, 35], it was found that the effects of volume fluctuations are largely suppressed in the most-central collisions. Correspondingly, the multiplicity fluctuations there are more sensitive to other factors, such as the initial state fluctuations, resonance decays, and etc. In Fig. 1, the slightly over-predictions of the data at 0-5% centrality indicates that the used $K^0$-G1b initial conditions may not fully capture the fluctuation patterns as imprinted in nature. More sophisticated model calculations, especially for the most central collisions, are still needed which we would like to leave it to the future study.

The dashed black lines are the Skellam baselines.

Fig. 2 shows the centrality dependent moments of net-charges in Au+Au collisions at $\sqrt{s_{NN}} = 7.7$, 39 and 200 GeV. Although iEBE-VISHNU has archived an overall fit of the standard variance $\sigma$ for both positive and negative charges, it fails to nicely describe the corresponding $\sigma$ of net-charges, which shows certain deviations between model and data. For the skewness $S$ and kurtosis $\kappa$ of net-charges, iEBE-VISHNU quantitatively describes the data within the statistical errors. In contrast, the Skellam baselines (which come from the subtraction of two independent Poisson distributions) show certain deviations from the experimental data, especially at lower collision energies. In [36], it was found that the volume fluctuations/corrections are the dominant factors to influence the skewness $S$ and kurtosis $\kappa$ of net-charges, but are negligible for the corresponding standard variance $\sigma$. This leads to the different descriptions of $\sigma$, $S$ and $\kappa$ in our model calculations, which will be further discussed in the following Sec. IV B.

Besides the effects of volume fluctuations, the correlation between positive and negative charges is another important factor to influence the fluctuations of net-charges, which may even play a dominant role to affect the standard variance $\sigma$ of net charges. In iEBE-VISHNU model, such correlations mainly come from the resonance decays during the late hadronic evolution. However, some additional correlations, e.g. the correlations from the charge conservation laws, are still missing, which may largely influence the standard variance $\sigma$ of net charges and should be investigated in the near future.

FIG. 2: (Color online) Similar to Fig. 1, but for the moments of net-charges, calculated from iEBE-VISHNU and measured by STAR [12]. The dashed black lines are the Skellam baselines.

FIG. 3: (Color online) Centrality dependent moment products $\sigma_S^2/M$ Skellam, $\sigma_S^2$ and $\kappa_S^2$ of net-charges in Au+Au collisions at $\sqrt{s_{NN}} = 7.7$, 39.0 and 200 GeV, calculated from iEBE-VISHNU and measured by STAR [11, 12].
future.

Fig. 3 shows the centrality dependent moment products \(c_i^n/M\) and \(\kappa \sigma^2\) of net-charges in Au+Au collision at \(\sqrt{s_{NN}} = 7.7, 39\) and 200 GeV. In general, iEBE–VISHNU roughly describes these experimental data. The slight deviations mainly come from the over-predictions of the standard variance \(\sigma^2\) of net-charges (please refer to Fig. 2 and the related discussions). Note that part of the correlations between positive and negative charges, e.g. from resonance decays and hadronic scatterings, has been included in our calculations, which makes the iEBE–VISHNU results are more close to the STAR data than the negative binomial baselines 2.

With the same parameter sets, we calculate the cumulants \((C_i - C_j)/C_i\) of protons, anti-protons and net-protons in Au+Au collisions at \(\sqrt{s_{NN}} = 7.7, 39\) and 200 GeV. Fig. 4 and fig. 5 show that our iEBE–VISHNU results are pretty close to the experimental data, which all monotonically increase with the participant number \(\langle N_{\text{part}} \rangle\). We also notice that the difference between our model calculations and the Poisson baselines are pretty small, which indicates that various effects included in our model calculations, i.e., volume fluctuations, hadronic scatterings and resonance decays, do not significantly influence the cumulants of protons, anti-protons and net protons. For more detailed discussions, please also refer to Sec IV B.

For a closer look, figure 6 plots the moment products \(c_i^n/M\) and \(\kappa \sigma^2\) of net protons, which presents certain deviations between the data and our model calculations. Compared with the Skellam baselines that are obtained from the two independent Poisson distributions of protons and anti-protons, most of the iEBE–VISHNU results and the experimental data are respectively below and above the baselines. Fig. 6 also shows that the gap between model calculations and Skellam baselines increase with the decrease of collision energies. As discussed in Ref. [36] and Sec IV B, the related effects of volume fluctuations are closely related to the value of \((M_+ - M_-)/(k + 1)\) (where \(M_+ - M_-\) is the mean value of net protons and \(k\) is the reference multiplicity), which increases with the decrease of collision energy and leads to the increasing gap between our model calculations and the Skellam baselines.

In many past research [31, 32], the cumulant ratios of net

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2 As shown in Ref. [35], negative binomial baselines can be obtained from the Poisson distribution after considering the effects of volume fluctuations with some approximations.
protons are expected as sensitive observables to probe the non-gaussian fluctuations of the QCD critical point and the first order phase transitions. Note that our model calculations only include the various effects of non-critical fluctuations. The failure of describing the cumulant ratios of net protons at lower collision energies indicates that other possible effects, such as baryon conservation laws, critical fluctuations and spinodial instabilities of the first order phase transitions, may largely influence the multiplicity fluctuations of net protons there, which are worthwhile to be further studied in the near future.

B. The effects of volume fluctuations, resonance decays and hadronic evolution

As we have mentioned, various effects such as volume fluctuations, hadronic scatterings, resonance decays, etc., could influence the multiplicity fluctuations of (net) charges and (net) protons to some extend. To further explore these effects, we perform the model simulations with three different cases:

(a) full iEBE-VISHNU simulations as did in Sec.IV A, where the hydrodynamic expansion is followed by a full UrQMD hadronic evolution with both hadronic scatterings and decays;

(b) modified iEBE-VISHNU simulations with hydrodynamics followed by resonance decays, but without the hadronic scatterings and evolution of UrQMD;

(c) pure hydrodynamic simulations with thermal hadrons directly emitted from the freeze-out hyper-surface with the imprinted Poisson fluctuations described by Eq. (2).

Fig. 7 presents the centrality dependent moment products $\frac{\sigma^2}{M}$, $\frac{\sigma^2}{\Delta M}$ and $\kappa \sigma^2$ of net-charges and net-protons in Au+Au collisions at $\sqrt{s_{NN}}=7.7, 39.0$ and 200 GeV, obtained from iEBE-VISHNU simulations with these three above modes. For the simulations with case (c), we focus on the effects of the volume fluctuations. More specifically, for a single hydrodynamic simulation, thermal hadrons directly emitted from the freeze-out surface satisfy the Poisson fluctuations according to Eq. (2), which also generates the corresponding Skellam baselines in Fig. 7. In the event-by-event simulations, a specific reference multiplicity $k$ (within a centrality cut window e.g. $0.5 < |y| < 1.0$) can be generated from many different hydrodynamic simulations with the initial state fluctuations and poisson fluctuations. The related multiplicity fluctuations (within an acceptance cut window e.g. $0 < |y| < 0.5$) for a given reference multiplicity bin thus surfer similar volume fluctuation effects as investigated in early paper [35, 36].

Fig. 7 shows that, the volume fluctuations/corrections for $\frac{\sigma^2}{M}$ of net charges are pretty small or even negligible, but very large for $S \sigma$ and $\kappa \sigma^2$ which make them obviously deviate from the Skellam baselines. For net protons, the volume...
fluctuations/corrections for $\sigma^2/M$, $S\sigma$ and $k\sigma^2$ are all pretty small, but also gradually increase with the decrease of collision energy. In [36], it was found that the volume fluctuations/corrections for the standard variance $\sigma$ of net charges (protons) are approximately proportional to $(M_+ - M_-)/(k+1)$, where $M_+$ and $M_-$ are the mean multiplicity of positive and negative charges (protons) and $k$ is the reference multiplicity for the centrality cut. Note that, for both net charges and net protons $(M_+ - M_-) \propto (k+1)$. This largely suppresses the related volume fluctuations of $\sigma$, which is also directly demonstrated in our model calculations of Fig. 7. For the skewness $S$ and kurtosis $\kappa$, the volume correction is not only dependent on $(M_+ - M_-)/(k+1)$, but also depend on $(M_+ + M_-)/(k+1)$ [36]. For net charges, $(M_+ + M_-)/(k+1) \sim 1$, we thus observe large volume fluctuations for both $S\sigma$ and $k\sigma^2$ in Fig. 7 (left). For the case of net protons, $(M_+ + M_-)/(k+1)$ are still pretty small, but gradually increase with the decrease of collision energy. Correspondingly, we find the iEBE-VISHNU results with only volume fluctuations are still pretty close to the Skellam baselines for both $S\sigma$ and $k\sigma^2$. Meanwhile, the increased mean values of protons and anti-protons also leads to slightly larger deviations from the Skellam baselines for $S\sigma$ and $k\sigma^2$ at lower collision energies.

Fig. 7 also compares the model simulations with case (a) (b) and (c). For $S\sigma$ and $k\sigma^2$ of net-charges, the results from the three comparison runs almost overlap within error bars, which also obviously deviate from the Skellam baselines. This indicates that the volume fluctuations are the dominant factors to influence these two moment products of net-charges. For $\sigma^2/M$, the volume fluctu are largely suppressed as discussed above. Fig. 7 shows that resonance decays become the dominant roles to influence $\sigma^2/M$, while the effects from the hadronic evolution and scatterings are pretty small. For the moment products of net protons in Fig. 7 (right), both the resonance decays and hadronic evolution do not significantly influence the values of $\sigma^2/M$, $S\sigma$ and $k\sigma^2$. In general, the effects of volume fluctuations are also pretty small (except for 7.7 GeV), which indicates that the multiplicity fluctuations of (net) protons keep the main features of the Poisson fluctuations imprinted in our model calculations.

C. The dependence on centrality and acceptance cuts

In Fig. 8, we study the acceptance dependence of the moments products of net-charges and net-protons, using the iEBE-VISHNU simulations with different pseudo-rapidity and transverse momentum cut. We find broader acceptance windows lead to larger deviations from the Skellam baselines. As discussed above, the volume fluctuation in our model calculations is the main factor to influence the multiplicity fluctuations of net charges and net protons, especially for moments products of Skewness and Kurtosis. For a broader acceptance window, the mean values of net charges and net protons $(M_+ - M_-)$ increases. This leads to larger values of $(M_+ - M_-)/(k+1)$ for a fixed reference multiplicity bin $k$, which enhances the corresponding volume fluctuations for $\sigma^2/M$, $S\sigma$ and $k\sigma^2$, making them deviate from the Skellam baselines.

We have also noticed that the measured acceptance dependence for the moments products of net protons present different behaviors, when compared with our calculations. For example, $\sigma^2/M$ of net protons with an acceptance cut $0.8 < p_T < 2.0$ GeV is obviously below the Skellam baselines [35], while our model calculations are above the Skellam baselines. For Au+Au collisions at $\sqrt{s_{NN}}=7.7$ GeV, the mea-
FIG. 8: (Color online) Acceptance cut dependence of the moment products $\sigma^2/M_{\text{Skellam}}$ and $\kappa^2$ in Au+Au collisions at $\sqrt{s_{\text{NN}}}=7.7$, 39 and 200 GeV, calculated from iEBE-VISHNU with the acceptance cut set to $|y|<0.1$ and $|y|<0.5$ (with $0.4<p_T<2$ GeV) for net charges and $0.4<p_T<0.8$ GeV and $0.4<p_T<2$ GeV (with $|y|<0.5$) for net protons.

FIG. 9: (Color online) Centrality cut dependence of the moment products $\sigma^2/M_{\text{Skellam}}$, $\sigma^2$ and $\kappa^2$ in Au+Au collisions at $\sqrt{s_{\text{NN}}}=7.7$, 39 and 200 GeV, calculated from iEBE-VISHNU with the centrality defined by total charges within $0.5<|y|<1.0$ or $0.5<|y|<2.0$ (left panels, for net charges) and by total pions and kaons within $|y|<1.0$ or $|y|<2.0$ (right panels, for net protons).

$s_2^2$ presents an obvious centrality dependence, which dramatically deviates from the Skellam baseline in the most central collisions [35]. In contrast, our model calculations show weak centrality dependence for $\kappa^2$ at 7.7 GeV. Again, our model simulations do not include all the possible effects as imprinted in nature, such as the conservation laws of net charges and net baryons, critical fluctuations, the spinodal instabilities of the first order phase transitions, etc., which are worthwhile to be further explored in the future.

Besides the acceptance cuts, different centrality definitions also influence the volume fluctuations and the measured fluctuations of (net) charges and (net) protons. In Fig. 9, we calculate the moment products of net-charges and net-protons from iEBE-VISHNU with different centrality cuts. In more de-
tails, the centrality bins are defined by the total charges within pseudo-rapidity cuts $0.5 < |\eta| < 1.0$ or $0.5 < |\eta| < 2.0$ (Fig. 9, left). For the case of net protons, the centralities are cut by the number of $\pi$, $K$ within $|\eta| < 1.0$ or $|\eta| < 2.0$ (Fig. 9, right). In contrast to acceptance cut dependence shown in Fig.8, broader acceptance windows for the reference particles in the centrality cut lead to smaller deviations from the Skellam baselines here. As mentioned above, the volume fluctuations for Skewness and Kurtosis are dependent on $(M_3 - M_\epsilon)/(k + 1)$. A border acceptance cut during the centrality definition leads to a larger value of $k$ of the reference particles, which reduces the related effects of volume fluctuations. In Ref. [76], such effects are also mentioned as centrality resolution effects. In order to reduce the effects from volume fluctuations with a better centrality resolution, a larger acceptance cut for the reference particles in the centrality definition is preferred [76].

V. SUMMARY

In this paper, we investigated the multiplicity fluctuations of (net) charges and (net) protons in Au+Au collisions at $\sqrt{s_{NN}} = 7.7$, 39 and 200 GeV, using the iEBE-VISHNU hybrid model that combines 2+1-d viscous hydrodynamics with the UrQMD hadron cascade model. With a modified Cooper-Fryer freeze-out procedure, the Poisson fluctuations have been added in the Monte-Carlo event generator that samples thermal hadrons on the switching hypersurface between hydrodynamics and UrQMD. In our investigations, the moments/cumulants of (net) charges and (net) protons are calculated with the same centrality and acceptance cut as used in experiments, which also account various effects of initial state fluctuations, volume fluctuations, hadronic scatterings and evolution, resonance decays, etc.

With well tuned parameters that fit the mean values of pions, kaons and protons in Au+Au collisions at $\sqrt{s_{NN}} = 7.7$, 39 and 200 GeV, iEBE-VISHNU roughly described the centrality dependent moments and cumulants of (net) charges and (net) protons measured in experiment. We also found that the iEBE-VISHNU results are largely deviated from the Poisson/Skellam baselines for (net) charges, but are pretty close to the Poisson/Skellam baselines for (net) protons, especially at higher collision energy. Further comparison simulations have shown that the volume fluctuations play dominant roles to influence the higher moments of (net) charges, but do not significantly affect the multiplicity fluctuations of (net) protons. We also found that the effects from hadronic evolutions are pretty small or even negligible for the multiplicity distributions of (net) charges and (net) protons. Considering that full UrQMD hadronic evolution consumes a great portion of the calculation time in iEBE-VISHNU, such finding may help to largely improve the numerical efficiency for the massive data simulations in the near future, which makes it possible to calculate more realistic thermal fluctuation baselines with a realistic but simplified dynamical model.

Finally, we would like to emphasize that, although part of particle correlations have been included through the hadronic scatterings and resonance decays, our iEBE-VISHNU calculations are still belong to framework of independent production since various thermal hadrons are independently emitted from the switching hyper-surface according to the Cooper-Frye formula without further considering the conservation laws and other sources of correlations. Correspondingly, our calculations failed to nicely describe the standard variation $\sigma$ of net charges, which showed a certain gap between model and data. For an improved description of the data at higher collision energies and realistic predictions of the non-critical fluctuation baselines for the BES program, more effects, e.g. the conservation laws of net charges and net baryons [42], thermal fluctuations within hydrodynamics [78], improved initial state fluctuations, etc., should be further considered in our model calculations in the future.

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