Extracting Event Dynamics from Event-by-Event Analysis†

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ABSTRACT

The problem of eliminating the statistical fluctuations and extracting the event dynamics from event-by-event analysis is discussed. New moments $G_p$ (for continuous distribution), and $G_{q,p}$ (for anomalous distribution) are proposed, which are experimentally measurable and can eliminate the Poissonian type statistical fluctuations to recover the dynamical moments $C_p$ and $C_{q,p}$. In this way, the dynamical distribution of the event-averaged transverse momentum $\bar{p}_t$ can be extracted, and the anomalous scaling of dynamical distribution, if exists, can be recovered, through event-by-event analysis of experimental data.

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1 Introduction

In the conventional investigation of high energy multiparticle production, all the events in a collision process are taken as a whole and the distributions, fluctuations and correlations inside the sample are studied without distinguishing the individual events. This kind of study is usually referred to as inclusive one. In the 80’s – 90’s of last century, motivated by the experimental observation [1] [2] and theoretical perspective [3], people started to carry on the study event by event [4] [5] [6] [7], with the aim of exploring the possible existence of new physics, such as quark-gluon plasma (QGP) [8] or non-linear dynamics [9].

One of the differences in these two kinds of study is that, the inclusive dynamics can readily be extracted from the experimental measurement through averaging over a large number of events, while the extracting of event dynamics is complicated due to the statistical fluctuations coming from the limited number of particles in a single event.

For concreteness, let us consider the event-by-event fluctuation of the transverse momentum distribution $p(p_t)$, a knowledge of which is important for the understanding of the basic collision dynamics [10]. Usually, it is convenient to “coarse-grain” $p(p_t)$, i.e. to divide the phase space region $\Delta$ of $p_t$ into $M$ bins and integrate $p(p_t)$ over $p_t$ in the $m$th bin $\delta_m$

$$p_m = \int_{\delta_m} p(p_t) dp_t, \quad (m = 1, 2, \ldots, M).$$

(1)

The set

$$p_1, p_2, \ldots, p_M$$

(2)

is the “coarse-grained” distribution [9]. When $M \to \infty$ ($\delta_m \to 0$) it recovers the original distribution $p(p_t)$ [11]. If $p(p_t)$ is continuous this process is convergent and a good approximation for $p(p_t)$ could be obtained for not very large $M$.

The realization of $p(p_t)$ in experiment is the distribution of the total number $N$ of particles in the $p_t$ region $\Delta$, and the expression

$$q_m = N_m/N \quad (m = 1, 2, \ldots, M)$$

(3)

is used to evaluate $p_m$, where $N_m$ is the number of particle falling into the $m$th bin. This is, however, exact only when $N \to \infty$. Just at this point appears the difference between the inclusive and event-by-event studies.
In an inclusive study, what is under consideration is the transverse momentum distribution $p_{\text{incl}}(p_t)$ in the event sample. In this case, the number $N$ in Eq. (3) is the total number of particles in the whole sample and could be made arbitrarily large through increasing the number of events in the experiment and thus the effect of statistical fluctuation can be gotten rid of through averaging over a sufficiently large event sample. On the contrary, in an event-by-event study the distribution $p(p_t)$ in consideration is the transverse momentum distribution in a single event and the number $N$ in Eq. (3) is the number $n$ of particles in the event, which is limited by energy conservation, and the statistical fluctuation inevitably comes in.

Various methods have been proposed to eliminate the influence of statistical fluctuation and evaluate the dynamical ones. Most of them are based on the comparison of the measured fluctuation with the expectation of statistically independent particle emission. For example, in Ref. [6] the results from mixed events are considered as the baseline for the random distribution and the difference in the fluctuation from a random distribution defined as

$$d = \omega_{\text{data}} - \omega_{\text{baseline}}$$

is taken as a measure of the dynamical fluctuation. In Eq. (4)

$$\omega = \frac{\sqrt{\langle \bar{p}_t^2 \rangle - \langle \bar{p}_t \rangle^2}}{\langle \bar{p}_t \rangle} = \frac{\sqrt{\sigma_{\bar{p}_t}^2}}{\langle \bar{p}_t \rangle},$$

$\bar{p}_t$ is the mean transverse momentum in a single event and $\langle \cdots \rangle$ denotes the average over event sample.

Alternatively, in Ref. [14] the statistical variance of event mean $p_t$, under the assumption of independent particle production, is estimated as

$$\sigma_{\bar{p}_t,\text{stat}}^2 = \frac{\sigma_{\bar{p}_t,\text{incl}}^2}{\langle n \rangle},$$

and the difference between the variances of $\bar{p}_t$ obtained from data and “stat” is taken as the dynamical variance

$$\sigma_{\bar{p}_t,\text{dynam}}^2 = \sigma_{\bar{p}_t,\text{data}}^2 - \sigma_{\bar{p}_t,\text{stat}}^2 = \frac{\sigma_{\bar{p}_t,\text{data}}^2 - \sigma_{\bar{p}_t,\text{incl}}^2}{\langle n \rangle}.$$  

A widely used measure for the non-statistical mean $p_t$ fluctuation is the $\Phi_{p_t}$ proposed in Ref. [12]

$$\Phi_{p_t} \equiv \sqrt{\langle Z^2 \rangle / \langle n \rangle} - \sqrt{\langle z^2 \rangle},$$

(8)
where $z$ and $Z$ are defined as $z \equiv p_t - \langle p_t \rangle$ for each particle and $Z \equiv \sum_{i=1}^{n} z_i = n(\bar{p}_t - \langle p_t \rangle)$ for each event, respectively [13]. The second term of the r.h.s. of Eq.(8) is the square root of the inclusive variance $\sigma^2_{p_t \text{incl}} = \langle (p_t - \langle p_t \rangle)^2 \rangle$. Assuming that the multiplicity fluctuation is uncorrelated with the $p_t$ fluctuation, we get from Eq.(8)

$$\Phi_{p_t} = \sqrt{\langle n^2 \rangle / \langle n \rangle \sigma_{p_t \text{data}} - \sigma_{p_t \text{incl}}}.$$  

This equation is evidently similar to Eq.(7), both have the same structure as Eq.(4), being based on a subtraction procedure, i.e. to subtract the variance of $\bar{p}_t$ or a quantity related to it, that will be expected from pure statistical fluctuation, from the same quantity obtained in experiment. These measures will, of course, vanish for a pure statistical system, and a non-vanishing value of them will indicate the existence of dynamical effect. Therefore, the measures based on the subtraction procedure, as those listed above, will at least qualitatively measures the effect of dynamical fluctuation.

The aim of the present paper is to develop a systematic method, which is able to eliminate the statistical fluctuations directly from the experimental event-by-event analysis and extract quantitatively the dynamical $\bar{p}_t$ moment of any positive integer order, under the assumption that the statistical fluctuations are of the Poisson type, i.e. due to uncorrelated random particle emission.

In Section II a theorem will be proved which is the basis of the elimination of Poissonian statistical fluctuations. In Section III the theorem is applied to the event-by-event analysis of transverse momentum distribution. In Section IV the event-by-event fluctuation of the non-linear fractal property will be discussed and the proposed method will be applied to this case to extract the dynamical fluctuation of fractal property. Section V is the conclusions.

## 2 The elimination of Poissonian statistical fluctuation

Divide the transverse momentum region $\Delta$ into $M$ bins. Let $p_m$ be the event dynamical probability of $p_t$ in the $m$th bin, cf. Eq.(1), and $n_m$ the number of particle in the event
with \( p_t \) lying in the \( m\)th bin, then we have the theorem:

\[
\left\langle \sum_{m=1}^{M} f_m p_m^p \right\rangle = \left\langle \sum_{m=1}^{M} f_m \frac{n_m(n_m-1)\cdots(n_m-p+1)}{\langle n \rangle^p} \right\rangle,
\]

where \( f_m \) is an arbitrary variable depending on \( m \).

Before going on to prove this theorem, let us notice that the symbol \( \langle \cdots \rangle \) in the two sides of Eq.(10), despite of both being the average over event sample, have different meanings. In the l.h.s. it is simply the average over dynamical probability distribution, while in the r.h.s. it includes also the average over Poisson distribution of particle number.

\[
\left\langle \sum_{m=1}^{M} f_m p_m^p \right\rangle = \int \sum_{m=1}^{M} f_m p_m^p P(p_m) dp_m,
\]

\[
\left\langle \sum_{m=1}^{M} f_m \frac{n_m(n_m-1)\cdots(n_m-p+1)}{\langle n \rangle^p} \right\rangle = \int \sum_{m=1}^{\infty} \sum_{n_m=0}^{\infty} f_m \frac{n_m(n_m-1)\cdots(n_m-p+1)}{\langle n \rangle^p} \cdot \frac{e^{-p_m(n)} (p_m(n))^{n_m}}{n_m!} P(p_m) dp_m,
\]

where \( P(p_m) \) is the dynamical probability distribution of \( p_m \) in the event space, \( p_m \langle n \rangle = \langle n_m \rangle \) is the average multiplicity in the \( m\)th bin. Using the normalization condition of Poisson distribution, it is easy to see that the r.h.s. of the above two equations are equal and Eq.(10) follows.

Note that a simplified version of the above theorem with \( f_m = 1 \) in Eq.(10) has been proved in Ref. [9] and has been widely used in the intermittency study [4]. Eq.(10) is a more general theorem with a factor \( f_m \) included, which is essential in the application of this theorem to the elimination of statistical fluctuation in event-by-event analysis.

### 3 The elimination of statistical fluctuation in event-by-event transverse momentum fluctuation

Let \( \bar{p}_t \) and \( \bar{p}_{t\text{exp}} \) be, respectively, the dynamical and experimentally measured values of the event-averaged \( p_t \),

\[
\bar{p}_t = \int_{\Delta} p_t p(p_t) dp_n = \sum_{m=1}^{M} (p_t)_m p_m,
\]

\[
\bar{p}_{t\text{exp}} = \sum_{m=1}^{M} (p_t)_m q_m = \sum_{m=1}^{M} (p_t)_m \frac{n_m}{n},
\]
where \((p_t)_m\) is the \(p_t\) value in the \(m\)th bin. The event-space moments of \(\bar{p}_t\) and \(\bar{p}_{t\exp}\) are

\[
C_p(\bar{p}_t) = \langle \bar{p}_t^p \rangle = \left\langle \left( \sum_{m=1}^{M} (p_t)_m P_m \right)^p \right\rangle, \tag{15}
\]

\[
C_{p\exp}(\bar{p}_t) = \langle (\bar{p}_{t\exp})^p \rangle = \left\langle \left( \sum_{m=1}^{M} (p_t)_m \frac{n_m}{n} \right)^p \right\rangle, \tag{16}
\]

respectively.

Let us first consider the elimination of statistical fluctuations in the second and third order moments. When \(p = 2, 3\) we have

\[
C_2(\bar{p}_t) = \left\langle \left( \sum_{m=1}^{M} (p_t)_m P_m \right)^2 \right\rangle
= \left\langle \sum_{m=1}^{M} (p_t)_m P_m^2 \right\rangle + \left\langle \sum_{m \neq m'} (p_t)_m (p_t)_{m'} P_m P_{m'} \right\rangle, \tag{17}
\]

\[
C_3(\bar{p}_t) = \left\langle \left( \sum_{m=1}^{M} (p_t)_m P_m \right)^3 \right\rangle
= \left\langle \sum_{m=1}^{M} (p_t)_m P_m^3 \right\rangle + 3 \left\langle \sum_{m \neq m'} (p_t)_m (p_t)_{m'} P_m P_{m'}^2 \right\rangle
+ \left\langle \sum_{m \neq m' \neq m''} (p_t)_m (p_t)_{m'} (p_t)_{m''} P_m P_{m'} P_{m''} \right\rangle. \tag{18}
\]

Define

\[
G_2(\bar{p}_t) = \left\langle \sum_{m=1}^{M} (p_t)_m^2 \frac{n_m(n_m - 1)}{(n)^2} \right\rangle + \left\langle \sum_{m \neq m'} (p_t)_m (p_t)_{m'} \frac{n_m n_{m'}}{(n)^2} \right\rangle, \tag{19}
\]

\[
G_3(\bar{p}_t) = \left\langle \sum_{m=1}^{M} (p_t)_m^3 \frac{n_m(n_m - 1)(n_m - 2)}{(n)^3} \right\rangle + 3 \left\langle \sum_{m \neq m'} (p_t)_m (p_t)_{m'}^2 \frac{n_m n_{m'}(n_{m'} - 1)}{(n)^3} \right\rangle
+ \left\langle \sum_{m \neq m' \neq m''} (p_t)_m (p_t)_{m'} (p_t)_{m''} \frac{n_m n_{m'} n_{m''}}{(n)^3} \right\rangle. \tag{20}
\]

It is easy to see, using Eq.(10), that \(G_2(\bar{p}_t) = C_2(\bar{p}_t), G_3(\bar{p}_t) = C_3(\bar{p}_t)\), provided the statistical fluctuations are Poissonian. Thus the dynamical moments \(C_2, C_3\) can be extracted from the experimental measurement by using \(G_2, G_3\) and the statistical fluctuations have been eliminated. The elimination of statistical fluctuations in higher order moments can be proceeded in a similar manner.
Figure 1: Variance $\sigma^2_{\tilde{p}_n}$ of $\tilde{p}_n$ distribution
Let us demonstrate the above results with a toy-model Monte Carlo simulation. In this model the distribution of $p_t$ is taken as

$$p(p_t) = \frac{4}{a^2} p_t e^{-2p_t/a},$$

(21)

with a Gaussian distributed parameter $a \ (\sigma^2(a) = 0.24)$. In total 1,000,000 events have been generated. The resulting variance $\sigma^2_{\bar{p}_t} = \langle \bar{p}_t^2 \rangle - \langle \bar{p}_t \rangle^2$ for $C$, $C^{\text{exp}}$ and $G$ are plotted in Fig.1 as upward triangles, full circles and downward triangles, respectively. It can be seen from the figure that the width $\sqrt{\sigma^2_{\bar{p}_t}}$ of experimentally measured $\bar{p}_t$ distribution is wider than that of the dynamical distribution, especially when the average multiplicity is low, while the width calculated from $G$ coincides with the dynamical width, $\sigma^2_{G_{\bar{p}_t}} = \sigma^2_{C_{\bar{p}_t}}$, where $\sigma^2_{G_{\bar{p}_t}} = G_2(\bar{p}_t) - (G_1(\bar{p}_t))^2$ and $G_1(\bar{p}_t) = \langle \sum_m (p_t)_m n_m \rangle / \langle n \rangle$. Therefore, using $G_{p_t}$ the Poissonian statistical fluctuations are eliminated thoroughly and the event dynamics is successfully extracted.

4 The elimination of statistical fluctuation in event-by-event analysis of fractal property

In the above discussion the dynamical event-distributions of phase space variables —— rapidity $y$, transverse momentum $p_t$, azimuthal angle $\varphi$ —— are implicitly assumed to be continuous functions, i.e. fulfil the condition

$$\delta x \to 0 \implies \delta p(x) \to 0, \quad x = y, p_t, \varphi,$$

(22)

which means that when the bin size decreases the probabilities in neighboring bins tend to be equal to each other.

However, there is evidences showing that this may not be true in some cases. The first experimental evidence is from a JACEE event in 1983\[1\], in which the total multiplicity is about one thousand and the multiplicity fluctuations in a small rapidity bin are still $2 \times$ the average. Similar phenomena have also been observed afterwards in accelerator experiments with local fluctuations up to $60 \times$ the average \[2\]. Obviously, this kind of fluctuations is out of the usual statistical ones, indicating the existence of self-similar fractal property in phase space distribution.
A characteristic phenomenon of self-similar fractal is the anomalous scaling of the normalized probability moments (NPM):

\[ C_q(M) \equiv M^{-1} \left( \sum_{m=1}^{M} (Mp_m)^q \right) \propto M^{\phi_q}. \tag{23} \]

Such an anomalous scaling property can be demonstrated using a toy model for self-similar fractal —— random cascading \( \alpha \) model \[9\] [15]. This model describes each multiparticle event as a series of steps, in which the initial phase space region \( \Delta \) is repeatedly partitioned into \( \lambda = 2 \) parts. After \( \nu \) steps we get \( M = 2^\nu \) sub-cells of size \( \delta = \Delta / M \). At each step \( \nu \) the probability in each of the two parts is obtained by multiplication of the probability in the step \( \nu - 1 \) by a particular value of the random variable \( \omega^{(\nu)}_{j_\nu} \), where \( j_\nu \) is the position of a sub-cell at the \( \nu \)th step \((1 \leq j_\nu \leq 2^\nu)\). The elementary fluctuation probability \( \omega \) can be chosen in various ways. The simplest way is to choose it as \[15\]

\[ \omega^{(\nu)}_{2j-1} = \frac{1}{2} (1 + \alpha r), \quad \omega^{(\nu)}_{2j} = \frac{1}{2} (1 - \alpha r), \tag{24} \]

where \( r \) is a uniformly-distributed random number in the interval \([-1, 1]\), \( j \) is an integer \((1 \leq j \leq 2^{\nu-1})\), \( \alpha \) is a characteristic parameter of the model taking value in the interval \([0,1]\).

In this model, after \( \nu \) steps of partition, the NPM defined in Eq.(23) becomes

\[ C_q(M) = M^q \langle \omega^q (1) \cdots \omega^q (\nu) \rangle. \tag{25} \]

Since \( \langle \omega \rangle = 1/2 \), we have the anomalous scaling of NPM, cf. Eq.(23),

\[ C_q(M) = M^q \langle \omega^q \rangle^\nu = M^q e^{\nu \ln \langle \omega^q \rangle} = M^{\phi_q}, \tag{26} \]

with \( M = 2^\nu = e^{\nu \ln 2}, \phi_q = q + \ln \langle \omega^q \rangle / \ln 2 \).

It is easy to see using Eq.(10) that the NPM \( C_q \) can be extracted from experimental data through the normalized factorial moments (NFM)

\[ F_q(M) = M^{q-1} \sum_{m=1}^{M} \frac{n_m(n_m-1)\cdots(n_m-q+1)}{\langle n \rangle^q}, \tag{27} \]

The anomalous scaling of NFM, usually referred to as intermittency \[9\] [11], has been observed in hadron-hadron \[16\] [17] and \( e^+e^- \) experiments, providing a first signal for the non-linear fractal property of strong interaction dynamics.
The npm defined in Eq.(23) can be viewed as the sample average of “event-probability moment” (EPM)

\[ C_q^{(e)}(M) = M^{-1} \sum_{m=1}^{M} (Mp_m)^q. \] (28)

It is natural to consider the distribution of EPM itself instead of only its average [19]. This distribution can be characterized by the \( p \)th order event-space moment \( C_{q,p} \) of the \( q \)th order event-moment \( C_q^{(e)} \)

\[ C_{q,p}(M) = \left( \left( C_q^{(e)}(M) \right)^p \right) = \left( \left( M^{-1} \sum_{m=1}^{M} (Mp_m)^q \right)^p \right) \] (29)

and the corresponding normalized moment

\[ C_{q,p}^{(norm)} = C_{q,p}/(C_{q,1})^p. \] (30)

In order to measure \( C_{q,p} \) in real experiments, the \( p \)th order event-space moment of the \( q \)th order event-factorial-moment

\[ F_{q,p} = \left( M^q-1 \sum_{m=1}^{M} \frac{n_m(n_m - 1) \cdots (n_M - q + 1)}{\langle n \rangle^q} \right)^p \] (31)

has been proposed [19]. The scaling property of \( F_{q,p} \) with the increase of \( M \) is referred to as erraticity [19] and has been observed in experiments [20][21]. However, when \( p \neq 1 \), \( F_{q,p} \) contains statistical fluctuations [22] and is unequal to \( C_{q,p} \), despite of the factorial moments apparently used. The observed erraticity phenomena have been shown to be dominated by statistical fluctuations and the dynamical effect is hidden [22][23].

In order to be able to observe the dynamical “erraticity”, the statistical fluctuation should be eliminated first. The elimination of (Poissonian) statistical fluctuation for the case \( p = 1 \) is straightforward using Eq.(10). The result is just Eq.(31) with \( p = 1 \), or Eq.(27). Let us consider the elimination of statistical fluctuations for the cases \( q = 2, \ p = 2, 3 \). We have

\[ C_{2,2} = \left( \left( M \sum_{m=1}^{M} p_m^2 \right)^2 \right) = M^2 \left( \sum_{m=1}^{M} p_m^4 + \sum_{m \neq m'} p_m^2 p_{m'}^2 \right), \] (32)

\[ C_{2,3} = \left( \left( M \sum_{m=1}^{M} p_m^2 \right)^3 \right) = M^3 \left( \sum_{m=1}^{M} p_m^6 + 3 \sum_{m \neq m'} p_m^2 p_{m'}^4 + \sum_{m \neq m' \neq m''} p_m^2 p_{m'}^2 p_{m''}^2 \right). \] (33)

Define

\[ G_{2,2} = M^2 \left( \sum_{m=1}^{M} \frac{n_m \cdots (n_m - 3)}{\langle n \rangle^4} \right) + M^2 \left( \sum_{m \neq m'} \frac{n_m(n_m - 1)n_{m'}(n_{m'} - 1)}{\langle n \rangle^4} \right), \] (34)
\[ G_{2,3} = M^3 \left( \sum_{m=1}^{M} \frac{n_m \cdots (n_m - 5)}{\langle n \rangle^6} \right) + 3M^3 \left( \sum_{m \neq m'} \frac{n_m(n_m - 1)n_m'(n_m' - 1)n_m''(n_m'' - 1)}{\langle n \rangle^6} \right) \]

\[ + M^3 \left( \sum_{m \neq m' \neq m''} \frac{n_m(n_m - 1)n_m'(n_m' - 1)n_m''(n_m'' - 1)}{\langle n \rangle^6} \right). \] (35)

Then, utilizing Eq.(10), it is ready to show that \( G_{2,2} = C_{2,2} \), \( G_{2,3} = C_{2,3} \), provided the statistical fluctuations are Poissonian. This means that using \( G_{2,2}, G_{2,3} \) instead of \( F_{2,2}, F_{2,3} \) to measure the dynamical moments \( C_{2,2}, C_{2,3} \), the (Poissonian) statistical fluctuations are eliminated and the real dynamical “erraticity” is observed. This method can be extended to any positive integer orders \( q \) and \( p \).

To illustrate the above result by Monte Carlo simulation, the random cascading \( \alpha \) model described above is used with a Gaussian distributed parameter \( \alpha \) (mean=0.5, width=0.15). Particles are put into each event according to Poisson distribution with \( \langle n \rangle = 6 \). The results from 2,500,000 generated events are plotted in Fig.2. The coincidence of \( G_{q,p}^{\text{norm}} \) and \( C_{q,p}^{\text{norm}} \) is remarkable. It is to be contrasted with the strongly upward bending curves of \( \ln F_{q,p}^{\text{norm}} \) vs. \( \ln M \). This shows that the strongly upward-bending behavior of \( \ln F_{q,p}^{\text{norm}} \) vs. \( \ln M \) for positive \( p \) which is typical in the conventional erraticity analysis [19][20][21] is due to statistical fluctuations. The reason is: when the partition number \( M \) increases, the average multiplicity per bin becomes smaller and smaller and the statistical fluctuations get stronger and stronger. After eliminating the statistical fluctuations, the \( \ln C_{q,p}^{\text{norm}} \) vs. \( \ln M \) curve coincides with \( \ln C_{q,p}^{\text{norm}} \) vs. \( \ln M \) and recovers the anomalous scaling property of the event dynamics (dynamical erraticity) in the model.

V Conclusions

In this paper the problem of eliminating the random noise in event-by-event analysis is considered. New moments \( G_p \) (for continuous distribution), and \( G_{q,p} \) (for anomalous distribution) are proposed, which are experimentally measurable and can be used to eliminate the Poissonian type statistical fluctuations and recover the dynamical moments \( C_p \) and \( C_{q,p} \).

For a comparison of different methods we notice that most of the measures proposed in the market for the dynamical fluctuation of transverse momentum are based on a
subtraction procedure, i.e. to subtract the variance of $\bar{p}_t$ or a quantity related to it, that will be expected from pure statistical fluctuation, from the same quantity obtained in experiment. In the present paper we take another approach, i.e. not to evaluate the effect of pure statistical fluctuation and subtract it, but to eliminate the statistical fluctuations directly and quantitatively from the experimental data and recover the dynamical $\sigma^2_{\bar{p}_t}$ under the unique assumption that the statistical fluctuation is Poissonian, i.e. due to uncorrelated random particle emission.

The difference between these two approaches is two-fold.

Firstly, in the subtraction method how to get the “pure statistical” variance from experimental data is a big problem. The mixing-event method [6] can in principle get the statistical variance, but the accuracy depends on the mixing-procedure. The method proposed in Ref. [14] relies on the equality $\sigma^2_{\bar{p}_t,\text{stat}} = \sigma^2_{\bar{p}_t,\text{incl}}/\langle n \rangle$, cf. Eq.(6), but this equality holds only for a pure statistical system without any dynamical fluctuation. In experimental data sample there exist simultaneously dynamical and statistical fluctuations, and the statistical variance $\sigma^2_{\bar{p}_t,\text{stat}}$ included in the data sample is unequal to the inclusive variance

Figure 2: Scaling property of event-space moments
of the data sample over average multiplicity:

\[
\sigma_{\bar{p}_t \text{stat}}^2 \neq \sigma_{\bar{p}_t \text{incl}}^2 / \langle n \rangle.
\] (36)

The same holds also for the \( \Phi_{p_t} \) method [12].

Secondly, the variance \( \sigma_{\bar{p}_t}^2 \) contains the \( \bar{p}_t \) moment only up to second order, which only gives the width of the distribution of \( \bar{p}_t \) in the event space. Using the method proposed in the present paper the \( \bar{p}_t \) moment of any integer order can be extracted and the dynamical distribution of \( \bar{p}_t \) in the event space is portrayed in much more detail.

In the anomalous case, the difficulty coming from the dominance of statistical fluctuation in the theoretical and experimental studies of erraticity up to now can be overcome and the dynamical erraticity, if exists, can be extracted using the method proposed in the present paper.

Applying the proposed method to real experimental data is highly recommended.

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