Dimension Driven Accelerating Universe

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Abstract

The current acceleration of the universe leads us to investigate higher dimensional gravity theory, which is able to explain acceleration from a theoretical viewpoint without the need of introducing dark energy by hand. We argue that the terms containing higher dimensional metric coefficients produce an extra negative pressure that apparently drives an acceleration of the 3D space, tempting us to suggest that the accelerating universe seems to act as a window to the existence of extra spatial dimensions. Interesting to point out that in this case our cosmology apparently mimics the well known quintessence scenario fuelled by a generalised Chaplygin-type of fluid where a smooth transition from a dust dominated model to a de Sitter like one takes place. Correspondence to models generated by a tachyonic form of matter is also briefly discussed.

Keywords : cosmology; higher dimensions; accelerating universe
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1 INTRODUCTION

There are growing evidences today that the current expansion of the universe is accelerating. It follows directly from the findings of Ia Supernovae and indirectly from CMBR fluctuations. If we put faith in FRW type of models then General Relativity is unambiguous about the need for some sort of dark energy source to explain the acceleration, which should behave like a fluid with a large negative pressure in the form of a time dependent cosmological constant or an evolving scalar field called quintessence. But it is very difficult to construct a theoretical basis for the origin of this exotic matter, which is seen precisely at the current epoch when one needs the source for cosmic acceleration.

So there has been a resurgence of interests among relativists, field theorists, astrophysicists and people doing astroparticle physics both at theoretical and experimental levels to address the problems coming out of the recent extra galactic observations (for a lucid and fairly exhaustive exposition of some of these ideas one is referred to \cite{1} and references therein ) without involving a mysterious form of scalar field by hand but looking for alternative approaches \cite{2} based on sound physical principles. Alternatives include, for example, modification of the Einstein-Hilbert action through the introduction of additional curvature terms, $R^m + R^n$ ($m > 0$, $n < 0$ and not necessarily integer) in the Lagrangian \cite{3}. The effective

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Friedmann equations contain extra terms coming from higher curvatures which may be viewed as a fluid, responsible for the current acceleration. However the resulting field equations are extremely difficult to solve and moreover, the cosmology is mostly unstable against perturbations.

On the other hand, serious attempts are recently being made \[4\] to incorporate the phenomenon of accelerating universe within the framework of higher dimensional space time \[5\] itself without involving any mysterious scalar field with large negative pressure by hand. The realisation that the higher dimensional corrections to the Einstein’s field equations can be viewed as an effective fluid which can emulate the action of the homogeneous part of the quintessence field has recently renewed interests in higher dimensional model. So in quintessential scenario what we observe as a new component of cosmic energy density is, so to say, an effect of higher dimensional corrections to the Einstein-Hilbert action. This approach has definite advantage over the standard quintessence scenario because we do not need to search for the quintessence scalar field and pick them by hand. On the contrary the extra fluid responsible for the acceleration is geometrical in origin having strong physical foundation and also in line with the spirit of general relativity as proposed by Einstein and others \[6\]. Assuming a shear free expansion we get a form of solution where an additional free parameter appearing in the expression of the scale factor characterizes the form of the matter field similar to the well known form of the generalized Chaplygin gas for quintessential models. The resulting energy momentum tensor behaves like a mixture of a cosmological constant and a perfect fluid obeying higher dimensional equation of state. When the cosmological radius is small the matter field in the form of dust (for example) predominates giving a decelerating expansion till the cosmological term takes over effecting a smooth transition to the current accelerating phase, while in the intermediate stage our cosmology interpolates between different phases of the universe. This phenomena has been exhaustively discussed in the context of quintessence in 4D spacetime. However we are not aware models of similar kind in higher dimensional spacetime, that too without assuming by hand any form of an extraneous scalar field with mysterious properties.

2 The FIELD EQUATIONS

We begin with considering a 5-dimensional line-element

$$ds^2 = dt^2 - R^2 \left( \frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) - A^2 dy^2$$

(1)

where the 3D and extra dimensional scale factors $R$ and $A$ depend on time only and $y$ is the fifth dimensional co-ordinate and $K$ is the 3D curvature. For our manifold $M^1 \times S^3 \times S^1$ the symmetry group of the spatial section is $O(4) \times O(2)$. The stress tensor whose form will be dictated by Einstein’s equations must have the same invariance leading to the energy momentum tensor as

$$T_{00} = \rho, \quad T_{ij} = p(t)g_{ij}, \quad T_{55} = p_5(t)g_{55}$$

(2)

where the rest of the components vanish. Here $p$ is the isotropic 3-pressure and $p_5$, that in the fifth dimension. The independent field equations for our metric (1) are
\[
\frac{3\dot{R}^2 + K}{R^2} + \frac{3\dot{R}\dot{A}}{RA} = \rho \\
\frac{2\ddot{R}}{R} + \frac{\dot{R}^2 + K}{R^2} + \frac{\ddot{A}}{A} + 2\frac{\dot{R}\ddot{A}}{RA} = -p \\
3\frac{\dddot{R}}{R} + 3\frac{\dot{R}^2 + K}{R^2} = -p_5
\]

Here we have five unknowns (A, R, \(\rho\), p and \(p_5\)) with three independent equations and we are at liberty to choose two connecting equations. We first assume \(p = p_5\) such that the field equations (K = 0) give

\[
\ddot{A} + 2\frac{\dot{R}}{R}\dot{A} - \left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2}\right)A = 0
\]

Assuming a form of the deceleration parameter we have rigorously solved the above equation, amenable to early deceleration followed by late acceleration to account for the current observations. But we defer discussion of the above work for a future publication. We see that \(A = R\) is a particular solution of the equation (6). Though simple, in what follows, we shall presently see that this choice is rich with various possibilities in interpreting our matter field as also in comparing the evolution of the universe with a Chaplygin type of fluid.

As our second assumption we take

\[
A = R = sinh^n\omega t
\]

(where \(n\) is an arbitrary constant for the present but we shall presently see it has strong physical significance) such that

\[
\rho = 6n^2\omega^2 + \frac{6n^2\omega^2}{sinh^{2n}\omega t} = \Lambda + \frac{B}{R^{2n}}
\]

\[
p = -\Lambda - \frac{2n - 1}{2n} \frac{B}{R^{2n}}
\]

where \(A = 6n^2\omega^2\) and \(B = \Lambda R_0^{2n}\). A little algebra shows that for a perfect fluid in five dimension with \(p = p_5\) \(n = \frac{1}{4}\) corresponds to a stiff fluid with \(\rho \sim \frac{1}{R^6}\) and \(n = \frac{2}{5}\) to a radiation dominated phase with \(\rho \sim \frac{1}{R^4}\) and lastly \(n = \frac{1}{2}\) to a matter dominated model with \(\rho \sim \frac{1}{R^2}\). Thus, interesting to point out that the exponent \(n\) characterizes the nature of the fluid we are dealing with. So we get the following cases of matter field

a. \((n = \frac{1}{4})\) (stiff fluid)

\[
\rho = \Lambda + \frac{B}{R^8} \text{ and } p = -\Lambda + \frac{B}{R^8}
\]

b. \((n = \frac{2}{5})\) (radiation)

\[
\rho = \Lambda + \frac{B}{R^5} \text{ and } p = -\Lambda + \frac{B}{4R^5}
\]
c. $(n = \frac{1}{2})$ (dust)

\[ \rho = \Lambda + \frac{B}{R^4} \]  \quad \text{and} \quad \rho = 0 \tag{12} \]

We see that for small $R$ the equation in case $a$ is approximated by $\rho = \frac{\Lambda}{R^3}$, which corresponds to a universe dominated by a stiff fluid in 5D spacetime. Similarly the case $b$ and case $c$ refer to radiation dominated and dust dominated universe respectively. On the other hand for a large value of the cosmological radius we see that the above equations suggest that $\rho = \Lambda$ and $p = -\Lambda$ which, in turn, corresponds to an empty universe with a cosmological constant $\Lambda$ (i.e., a de Sitter universe)

Thus equations (10-12) describe the mixture of a cosmological constant with a type of fluid obeying some equation of state. The last case known as ‘stiff fluid’ characterized by the equation of state, $p = \rho$ is particularly interesting. Note that a massless scalar field is a particular instance of stiff matter. Therefore, in a generic situation, our cosmology may be looked upon as interpolating between different phases of the universe from a stiff fluid, radiation or dust dominated universe to a de Sitter one passing through an intermediate phase which is a mixture just mentioned above. The interesting point, however, is that such an evolution may be accounted for by using one fluid only as opposed to the earlier works [7] representing simple two fluid model. Correspondence to models driven by a generalized Chaplygin type of fluid [8] described by an equation of state

\[ \rho = \left( \Lambda + \frac{B}{R^{3(1+\alpha)}} \right) \frac{1}{1+\alpha} \tag{13} \]

is only too apparent although here, as mentioned before we do not need to hypothesize the existence of a mysterious type of fluid to explain the observations. Here $\alpha$ is an additional free parameter to play with to fit the observational data.

Moreover we know that for a sheer-free evolution, if the temporal dependence of the scale factor is given, one can construct a potential for a minimally coupled scalar field which would simulate the evolution as with a perfect fluid. Let us illustrate the situation in our model. For the Lagrangian

\[ L(\phi) = \frac{1}{2} \dot{\phi}^2 - V(\phi) \tag{14} \]

we get the analogous energy density as

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \Lambda + \frac{B}{R^2} \tag{15} \]

and the corresponding ‘pressure’ as

\[ p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi) = -\Lambda - \frac{B}{2n R^2} + \frac{B}{2n R^2} \tag{16} \]

such that

\[ \dot{\phi}^2 = \frac{B}{2n R^2} \tag{17} \]

which, in turn, gives via equation (5) for flat 4D space
\[ \phi' = \sqrt{\frac{3B}{n}} \frac{1}{R\sqrt{\lambda R^{\frac{2}{n}} + B}} \]  

(18)

where \( \phi' \) denotes differentiation w. r. t. the scale factor \( R \). Integrating we get,

\[ \phi = \sqrt{\frac{3n}{4}} \ln \frac{\sqrt{B} - \sqrt{\lambda R^{\frac{2}{n}} + B}}{\sqrt{B} + \sqrt{\lambda R^{\frac{2}{n}} + B}} \]  

(19)

Using equation (7) we finally get

\[ \phi = \sqrt{3n} \ln \tanh \frac{\omega t}{2} \]  

(20)

On the other hand simple algebra shows that

\[ V(\phi) = \lambda \left( 1 + \frac{1}{2 \sinh^2 \omega t} \right) \]  

(21)

For the dust case (\( n = \frac{1}{2} \))

\[ V(\phi) = \lambda \left( 1 + \frac{1}{2 \sinh^2 \omega t} \right) \]  

(22)

while for the analogous stiff fluid case (\( n = \frac{1}{4} \)) yields a constant potential \( V(\phi) = \Lambda = V_0 \)

It may not be out of place to call attention to a quintessential model driven by a tachyonic scalar field \[7\] with a potential in 4D space time

\[ V(T) = \frac{\Lambda}{\sin^2 \left( \frac{3\sqrt{\lambda(1+k)}}{2} T \right)} \left( 1 - (1 + k) \cos^2 \left( \frac{3\sqrt{\lambda(1+k)}}{2} T \right) \right) \]  

(23)

\( (T \) is a tachyonic scalar field) giving the cosmological evolution as

\[ R(t) = R_0 \left( \sinh \frac{3\sqrt{\lambda(1+k)t}}{2} \right)^{\frac{2}{(1+k)}} \]  

(24)

It behaves like a two fluid model where one of the fluids is a cosmological constant while the other obeys a state equation \( p = k\rho \), \((-1 < k < 1)\). Similarity of this evolution with our model is more than apparent except that we are dealing with a higher dimensional spacetime. To end the section a final remark may be in order. From the equations (9-10) we form a sort of equation of state as

\[ p = \frac{1 - 2n}{2n} \rho - \frac{\Lambda}{2n} \]  

(25)

such that the sound speed is given by

\[ C_s^2 = \frac{\delta p}{\delta \rho} = \frac{1 - 2n}{2n} \]  

(26)
which implies that to avoid imaginary value of the speed of sound \( n < \frac{1}{2} \). Evidently in the dust model \((n = \frac{1}{2})\) \( C_s \) vanishes as expected. This along with the requirement that \( C_s \) should never exceed the speed of light further restricts the range of \( n \) as \( \frac{1}{4} < n < \frac{1}{2} \).

3 Discussion

In this work we have discussed a 5D homogeneous model with maximally symmetric 3D space. We have taken only one extra spatial dimension but we believe most of the findings may be extended if we take a larger number of extra dimensions. The most important finding in this work, in our opinion, may be summarised as: we do not have to hypothesise the existence of an extraneous scalar field with mysterious properties of matter to achieve an accelerating universe. The extra matter field in our model is of geometrical origin which is, however, not very uncommon in the literature. Correspondence to curvature quintessence, Wesson’s induced matter theory as also the shadow matter concept of Frolov et al in the context of brane cosmology may be of some relevance here. Another point to note is that the whole exercise is based on assumption of a specific form of the deceleration parameter, which definitely suffers from the disqualification of a sort of ad-hocism. But it generates a matter field which is a mixture of perfect fluid obeying an equation of state as well as a cosmological constant with either term dominating at different phases of evolution allowing a smooth transition from a decelerating to an accelerating model. As a future exercise one should envisage an additional scenario with other inputs such that the currently observed acceleration is followed by a decelerating phase, which finally hits a big brake singularity.

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References

[1] T. Padmanabhan - ‘Understanding our Universe: Current status and open issues’, gr-qc / 0503107

[2] B. M. N. Carter, et al, ‘Type IA supernovae Tests of fractal bubble universe with no cosmic acceleration’, astro-ph / 0504192 ; Zong-Kuan Guo and Y. Z. Zhang, astro-ph / 0506091

[3] S. Das, N. Banerjee and N. K. Dadhich, ‘Curvature driven acceleration: a utopia or a reality?’, astro-ph / 0505096; Ujjaini Alam, Varun Sahni and A. A. Starobinsky, J. Cosm. and Astr. Part. Phy. 0406, 008 (2004)

[4] S. Chatterjee, A. Banerjee and Y. Z. Zhang, gr-qc 0509112; Int. J. Mod. Phy. A 21 4035(2006); B. Cuadros- Melgar and E. Papantonopoulos, Brazilian J. Phys. 35, 1117 (2005); Li Qiang, Yongge Ma, Moxin Han and Dan Yu, Phys. Rev.D71, 061501 (R), 2005.
[5] A. Banerjee, D. Panigrahi and S. Chatterjee, Class. Quantum Grav. **11**, 1405, (1994).

[6] A. Einstein, The Meaning of Relativity (Princeton Univ. Press, Princeton, 1956); J. A. Wheeler, ‘Einstein’s Vision’, Springer, Berlin (1968) bibitemrdS.

[7] V. Gorini, A. Kamenshchik, U. Moschella and V. Pasquier, ‘Tachyons, scalar fields and cosmology’, hep-th/0311111; J. D. Barrow, Phys. Lett. **B235**, 40 (1990)

[8] M. C. Bento, O. Bertolami and A. A. Sen, Phy. Rev. **D66**, 043507 (2002).