The role of the inner disk in mass accretion to the star in the early phase of star formation

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Abstract

A physical mechanism that drives FU Orionis-type outbursts is reconsidered. We study the effect of the inner part of a circumstellar disk covering a region from near the central star to a radius of approximately 5 au (hereafter, the inner disk). Using the fluctuating mass accretion rate onto the inner disk \( \dot{M}_{\text{out}} \), we consider the viscous evolution of the inner disk and the time variability of the mass accretion rate onto the central star \( \dot{M}_{\text{in}} \) by means of numerical calculation of an unsteady viscous accretion disk in a one-dimensional axisymmetric model. First, we calculate the evolution of the inner disk assuming an oscillating \( \dot{M}_{\text{out}} \). It is shown that the time variability of \( \dot{M}_{\text{in}} \) does not coincide with \( \dot{M}_{\text{out}} \) due to viscous diffusion. Second, we investigate the properties of spontaneous outbursts with temporally constant \( \dot{M}_{\text{out}} \). Outbursts occur only in a limited range of mass accretion rates onto the inner disk of \( 10^{-10} < \dot{M}_{\text{out}} < 3 \times 10^{-6} \dot{M}_\odot \text{yr}^{-1} \) due to the gravo-magneto limit cycle (GML). Finally, we discuss the case with a combination of episodic \( \dot{M}_{\text{out}} \) and accretion outbursts caused by the GML in the inner disk. The GML can drive accretion outbursts onto the star even for the case of fluctuating \( \dot{M}_{\text{out}} \), although fluctuations of \( \dot{M} \) decay while transmitting the inner disk inwards. We have newly identified two modes of outburst, one spontaneous and one stimulated. In the stimulated mode of outburst, \( \dot{M}_{\text{out}} \) appears directly in \( \dot{M}_{\text{in}} \) (the latter defining the stellar accretion luminosity). In the spontaneous mode of outburst, \( \dot{M}_{\text{out}} \) appears as the interval between outbursts.

Key words: accretion, accretion disks — stars: formation — stars: pre-main sequence — stars: protostars

1 Introduction

In the early phase of star formation, it is thought that mass is accreted by the central star. FU Ori-type objects provide direct observational evidence for mass outbursts with short duration time (\( \sim 100 \text{yr} \)). A number of theoretical models have been proposed to explain FU Ori-type outbursts. There are three famous types of model, including the thermal instability (TI) model (Bell & Lin 1994), the
gravitational instability (GI) model (Vorobyov & Basu 2005, 2006), and the gravo-magneto limit cycle (GML) model (Armitage et al. 2001; Zhu et al. 2009; Martin & Lubow 2011).

Bell and Lin (1994) discussed whether thermal instability in the inner region of the disk can explain outbursts. They obtained the steady state solutions of the accretion disk, and they showed the results on the $M$ versus $\Sigma$ plane. In their model, they succeeded in explaining the timescales of outbursts, taking into account vertical convection of the disk and radial heat flux. They predicted that outbursts occur in the region $r < 0.1$ au and $T \sim 10^4$ K. However, the temperature at which TI occurs is too high for a protostellar disk if we consider the early phase of disk formation. Zhu et al. (2007, 2008) pointed out that the TI model cannot work for FU Ori because the region with a high accretion rate is required to extend to 0.5 au.

Using two-dimensional simulations of star and disk formation, Vorobyov and Basu (2005, 2006) discussed the GI model. It was shown that clumps formed via disk gravitational fragmentation at $\sim 100$ au in the early phase of the disk formation can be driven into the central region and cause the mass accretion outbursts. However, due to the use of a sink cell, Vorobyov and Basu (2005, 2006) could not follow the disk evolution in the inner 5 au. Although they assume the same mass accretion rate onto their sink cell as onto the star, the validity of this assumption is unclear. To help resolve this problem, in this paper we consider the inner region of the disk (hereafter the inner disk).

In the third GML model of outbursts in the inner region of the disk, magneto-rotational instability (MRI) is used to explain outbursts (Armitage et al. 2001; Zhu et al. 2009, 2010a, 2010b; Martin & Lubow 2011). In the GML model, the disk has two states: the MRI active state when $T > T_{\text{crit}}$ and the MRI inactive state when $T < T_{\text{crit}}$. In the GML model, spontaneous outbursts are driven by the state transition between the MRI active and inactive states. The transition is caused by mismatches of mass accretion rate between the inner and outer regions of the disk. From the viewpoint of the angular momentum transport, MRI is dominant in the inner region ($\sim 0.2$ au) and GI is dominant in the outer region ($\sim 1$ au) of the disk. The GML model of outbursts is demonstrated by numerical calculation, where the outburst is associated with the time variation of the MRI active/inactive state in the inner region of the disk (Armitage et al. 2001; Zhu et al. 2009, 2010a, 2010b).

Previous studies of the GML model have been limited to the case of a temporally constant mass accretion rate onto the inner disk. According to studies of the GI model, the mass accretion rate onto the inner disk is expected to vary in time. In this paper, we consider the role of the inner disk in time variation of the mass accretion rate onto the inner disk. The viscous evolution of the inner disk (0.2–5.0 au) is solved by taking into account both the outbursts driven by GML and the time-variable mass accretion rate onto the inner disk. In order to understand the time variability of the mass accretion rate onto the star, first we consider the above two processes independently. In section 2, using a simple model of the viscous accretion disk, we show the accretion rate near the surface of the protostar as a response to the episodic accretion at the outer boundary. In section 3, features of outbursts driven by the GML are shown with a temporally constant accretion rate onto the outer boundary of the inner disk. In section 4, we obtain the time variability of the accretion rate as a result of a combination of the outer episodic accretion and the GML. We discuss the implications of our results and future work in section 5. Our results are summarized in section 6.

2 Response to oscillating accretion onto the inner disk

2.1 Assumptions and basic equations

The accretion disk model that we use in this paper describes the unsteady, viscous accretion process of the circumstellar disk by assuming that the disk is axisymmetric and geometrically thin. All the equations are integrated in the vertical direction in cylindrical coordinates ($r, \phi, z$). The system is described by a one-dimensional equation dependent on time $t$ and radial distance $r$. We apply this disk model to the inner disk. First, we investigate the simple response of the inner disk to the oscillating accretion onto the outer boundary of the inner disk. Note that the mass accretion rate onto the outer region in our model corresponds to the accretion rate onto the sink cell in previous works on the GI model (e.g., Vorobyov & Basu 2010).

We assume that the pressure gradient force is negligible, and the accretion speed within the disk is sufficiently slow, i.e.,

$$\left| \frac{c_s^2}{\Sigma} \frac{\partial \Sigma}{\partial r} \right| \frac{GM}{r^2} \ll 1 \quad \text{and} \quad |v_r| \ll v_\phi, \quad (1)$$

where $\Sigma$ is the surface density, $v_r$ is the radial velocity, and $v_\phi$ is the azimuthal velocity, which is assumed to be Keplerian ($v_\phi = \sqrt{GM_*/r}$). As a result, the viscous
evolution of the surface density $\Sigma$ is described by

$$\frac{\partial \Sigma}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{M(r)}{2\pi} \right) = 0, \quad (2)$$

with

$$M(r) = -\frac{2\pi}{\partial (\rho v)} \frac{\partial}{\partial r} \left( v \Sigma r^3 \frac{\partial \Omega}{\partial r} \right), \quad (3)$$

where $\Omega = v_0/r$ is the angular velocity and $v$ is the viscous coefficient (e.g., Pringle 1981 and subsection 7.2 in Hartmann 2008). A hydrostatic equilibrium in the $z$-direction is assumed. Then the scale height $h$ is given by $b = c_s/\Omega$. As angular momentum transport, the standard $\alpha$ prescription for a viscous disk (Pringle 1981; Shakura & Sunyaev 1973) is employed. By this prescription, the viscous coefficient $v$ in equation (3) is given by $v = \alpha c_s b$. In this section, we use the simplified treatment of $\alpha$ and $c_s$ with a temporally and spatially constant $\alpha = 0.1$, and temporally constant $c_s = 0.63(r/1\text{ au})^{-1/4} \text{[km s}^{-1}\text{]}$.

We solve equation (2) numerically for the time evolution of $\Sigma$ and $M(r)$ in $r_{in} < r < r_{out}$, where $r_{in} = 0.2$ au and $r_{out} = 5$ au. The mass of the central star $M_* = 0.5 M_\odot$. The diffusion time $t_{diff}(r) = r^2/v(r)$ at $r_{out} = 5$ au is about 17500 yr in this case. For the mass accretion rate onto the disk outer edge, supposedly mimicking a periodic effect of the GI in the outer disk, we adopt the following form:

$$M_{out}(t) = M_0 \left[ \cos \left( \frac{2\pi}{t_{osci}} t \right) + (1 + \delta) \right], \quad (4)$$

where $M_0$, $t_{osci}$, and $\delta$ are constant parameters. Using equation (4), the time-averaged value of the mass accretion rate at the outer boundary in one period of oscillation is given as $\langle M(t_{out}) \rangle_{osci} = M_0/\sqrt{2\delta}$, where $\langle M(t_{out}) \rangle_{osci} = M_0/\sqrt{2\delta}$. Equation (4) includes the case of constant $M_{out} = M_0/2$ with $t_{osci} = \infty$ and $\delta \ll 1$. Equation (4) with $M_0 = 5 \times 10^{-8} M_\odot \text{ yr}^{-1}$ and $\delta = 0.01$ is used in this section. As the inner boundary condition, we use a free boundary, $\partial \Sigma/\partial r = 0$ at $r = r_{in}$. We set the grid number $N_r = 37$, $\Delta r$ equally divided in logarithmic space, and $dt = C_{safe} \Delta t_{diff} = C_{safe} (\Delta r)^2/\nu$, where $C_{safe} = 10^{-2}$. For the initial conditions, we set power law distributions for $\Sigma = \Sigma_0 r^{-1}$. However, the initial conditions are unimportant because the inner disk forgets them after several diffusion timescales.

2.2 Result

The time evolution of the mass accretion rate at $r_{out}$ and $r_{in}$ is shown in figure 1 for the cases with $t_{osci} = 2500$ yr (a) and $t_{osci} = 125000$ yr (b). From figure 1a, where $t_{osci} < t_{diff}(r_{out})$, it can be seen that the mass accretion rate at $r_{out}$, $(dM/dt)_{out}$ (dotted line), has a large oscillation amplitude while the mass accretion rate at $r_{in}$, $(dM/dt)_{in}$ (solid line), has a small amplitude. This means that the inner disk acts to smooth out oscillations in the mass accretion onto the disk’s outer edge and the true accretion rate onto the star exhibits little time variability. From figure 1a, it can also be seen that the mass accretion rate at $r_{in}$ is almost the same as the time-averaged value of the mass accretion rate at the outer boundary $\langle M(r_{out}) \rangle_{osci} = M_0/\sqrt{2\delta}$, rather than $M(r_{out})$. This means that if $t_{osci} < t_{diff}(r_{out})$, $M(r)$ approaches $\langle M(r_{out}) \rangle_{osci}$ in the small $r$ limit. In figure 1b, where $t_{diff}(r_{out}) < t_{osci}$, it can be seen that the oscillation amplitude of the mass accretion rate is similar at the inner and outer boundaries of the inner disk, although the oscillation of the mass accretion rate at the inner boundary slightly decays due to viscous diffusion in the inner disk. In summary, the mass accretion rate onto the protostar tends to be different from that onto the sink cell due to viscous diffusion. The mass accretion rate onto the protostar approaches the time-averaged value $\langle M_{out} \rangle_{osci}$

![Fig. 1. Simple responses of the mass accretion rate at $r = r_{in}$ to the oscillated mass accretion at the outer boundary for the cases with $t_{osci} = 2500$ yr (a) and 125000 yr (b). Constant $\alpha = 0.1$ and temporally constant $c_s = 0.63(r/1\text{ au})^{-1/4} \text{[km s}^{-1}\text{]}$ are used. For these parameters, $t_{osc}(r_{out}) = 17500$ yr.](https://academic.oup.com/pasj/article-abstract/66/6/112/1574878)
when \( t_{\text{osci}} < t_{\text{diff}} \) and the time variability of the mass accretion rate is weakly affected when \( t_{\text{diff}} < t_{\text{osci}} \).

3 Spontaneous outburst with steady \( M_{\text{out}} \)

3.1 Formulation

In this section, we investigate features of outbursts driven by the GML model. Time variability of the mass accretion rate at \( r_{\text{in}} \) is investigated using a steady mass accretion rate onto the outer boundary \( r_{\text{out}} \) of the inner disk \( M_{\text{out}} \) by solving equations (2) and (3). In this subsection, the effects of MRI and GI are effectively included in \( \alpha \) as

\[
\alpha = \alpha_{\text{M}} + \alpha_{\text{G}},
\]

where \( \alpha_{\text{M}} \) is a viscous parameter which mimics the angular momentum transport induced by the MRI, and \( \alpha_{\text{G}} \) is a viscous parameter which mimics the angular momentum transport induced by gravitational instabilities. As for \( \alpha_{\text{M}} \), we take into account the MRI active/inactive states by \( \alpha_{\text{M, on}} \) and \( \alpha_{\text{M, off}} \):

\[
\alpha_{\text{M}} = \begin{cases} 
\alpha_{\text{M, on}} & \text{if } T > T_{\text{crit}} \\
\alpha_{\text{M, off}} & \text{if } T < T_{\text{crit}},
\end{cases}
\]

where \( T_{\text{crit}} = 1400 \text{ K} \) (\( c_{\text{v,crit}} = 2.6 \text{ km s}^{-1} \)) is the critical temperature (see also Zhu et al. 2009). A small but nonzero value of \( \alpha_{\text{M, off}} \) mimics a finite amount of angular momentum transport in the MRI active surface layer around the dead zone in the MRI inactive region. Since the exact value of \( \alpha_{\text{M, off}} \) is not well known, we assume this value, which is sufficiently smaller than \( \alpha_{\text{M, on}} \). The viscous parameter \( \alpha_{\text{G}} \) is assumed to be

\[
\alpha_{\text{G}} = \begin{cases} 
\eta \left( \frac{Q^2}{Q_c^2} - 1 \right) & \text{if } Q < Q_c, \\
0 & \text{if } Q \geq Q_c,
\end{cases}
\]

where \( Q \equiv (c_s \kappa_{\text{ep}})/(\pi G \Sigma) \) is Toomre’s Q-value (Toomre 1964), \( \kappa_{\text{ep}} = \Omega \) is the epicyclic frequency for a Keplerian disk, \( Q_c \) is the critical Q-value below which gravitational torque acts, and \( \eta < 1 \) is the dimensionless number which represents the efficiency of the angular momentum transport (Lin & Pringle 1990). According to the three-dimensional hydrodynamical calculations in Kratter et al. (2010), \( \alpha_{\text{G}} \approx 1 \) is indicated when the disk is gravitationally unstable. Boily et al. (2006) indicate that angular momentum transport by gravitational torque is important when \( Q < 1.4 \). In this paper, \( Q_c = 1.4 \) is adopted, and several cases with \( \eta = 10^{-3} - 10^{-1} \) are considered.

To calculate the temperature, local thermal equilibrium between the viscous heating rate \( Q_{\text{visc}} \) and radiative cooling rate \( Q_{\text{cool}} \) is assumed, i.e.,

\[
Q_{\text{visc}} = Q_{\text{cool}}.
\]

Each term is given by

\[
Q_{\text{visc}} = \frac{9}{8} \nu \Sigma \left( \frac{d\Omega}{dR} \right)^2, \quad Q_{\text{cool}} = \frac{32}{3} \sigma T^4 \Sigma \kappa.
\]

with Stefan–Boltzmann constant \( \sigma \) and opacity \( \kappa \). For simplicity, we use the constant opacity \( \kappa = 3.0 \text{ cm}^2 \text{ g}^{-1} \), which is the averaged value of \( \kappa \) between several \( 10^2 \) to \( 10^3 \text{ K} \) in Pollack et al. (1994). The sound speed \( c_s \) and viscous parameter \( \alpha \) are obtained when we solve the system of equations (5) and (8), using the description of \( v = \alpha c_s b, Q = (c_s \Omega)/(\pi G \Sigma) \), and \( c_s^2 = (\gamma k T)/(\mu m_H) \), where \( \gamma = 7/5 \) is the specific heat ratio, \( k \) is Boltzmann’s constant, \( \mu = 2.35 \) is the molecular weight, and \( m_H \) is the mass of hydrogen.

Equations (5) and (8) constitute a nonlinear set of equations with respect to \( \alpha \) and \( c_s \). We solve \( \alpha \) and \( c_s \) iteratively using Newton’s method. In this section, equation (4) with \( t_{\text{osci}} = \infty \) is used as the outer boundary condition. The initial conditions, inner boundary conditions, and the grid number are the same as in section 2, except that the boundary conditions for \( c_s \), are treated as \( \partial c_s / \partial \theta = 0 \) at \( r = r_{\text{in}} \) and \( r = r_{\text{out}} \).

We investigate such features as the interval and duration of outbursts driven by the GML model, assuming that the mass accretion rate onto the inner disk is temporally constant. In table 1, the model parameters for this section are summarized. We terminate our calculation after outbursts occur a few dozen times or when the inner disk becomes a steady state without outbursts.

3.2 Result

We investigate the time variability of the mass accretion rate at \( r_{\text{in}} \) for several cases with different \( M_{\text{out}} \). The time evolution of the mass accretion rate at \( r_{\text{in}} \) for several cases with constant \( M_{\text{out}} = \dot{M}_{0}/2 \) and \( \eta = 10^{-2} \) are shown in figure 2.

In figure 2, it can be seen that for the cases with \( M_{\text{out}} = 10^{-5} \) and \( 10^{-10} \text{ M}_\odot \text{ yr}^{-1} \), the mass accretion rate at \( r_{\text{in}} \) is temporally constant without outbursts in the late
phase of calculation. In figure 2, only the period of 70 kyr in the late phase is plotted for $M_{\text{out}} = 10^{-10}$ and $10^{-5}$. In this phase, it is also found that $M_{\text{in}} = M_{\text{out}}$. In these two cases, the evolution is well described by a steady state model. In the case with $M_{\text{out}} = 10^{-10} M_{\odot} \text{ yr}^{-1}$, $\alpha_{M}$ stays equal to $\alpha_{M_{\text{off}}}$ and $\alpha_{G}$ equal to zero. This steady state with low $M$ (LS state) appears when $M_{\text{out}}$ is sufficiently small. In the case with $M_{\text{out}} = 10^{-5} M_{\odot} \text{ yr}^{-1}$, the disk is in a steady state with $\alpha_{M} = \alpha_{M_{\text{on}}}$ and $\alpha_{G} = 0$. This steady state with high $M$ (HS state) appears when $M_{\text{out}}$ is sufficiently large. Using equations (5), (8), and $M \propto \sqrt{\Sigma_{r}}$, the relation between $Q$, $\alpha_{r}$, and $M$ is given as $Q \propto M^{2/5} \Sigma^{7/10}$. In the case with small $M$, $Q$ is large and it is found that $Q > Q_{c}$ for $\alpha = \alpha_{M_{\text{off}}} = 10^{-5}$ when $M$ is smaller than $10^{-10} M_{\odot} \text{ yr}^{-1}$. In the case with $M_{\text{out}} = 10^{-5} M_{\odot} \text{ yr}^{-1}$, although $M$ is $10^{5}$ times larger, $Q$ is only $10^{11/10} \sim 1.26$ times larger for $\alpha = \alpha_{M_{\text{on}}} = 10^{-2} = 10^{1} \alpha_{\text{off}}$. This makes $Q > Q_{c}$ even in the case with $M_{\text{out}} = 10^{-5} M_{\odot} \text{ yr}^{-1}$. Thus, $\alpha_{G} = 0$ in both of LS and HS states.

In figure 2, it can also be seen that outbursts occur in the case with $M_{\text{out}} = 3 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$. We set the time as $t = 0$ when the first outburst starts to occur. In figure 2, only $t > 50$ kyr is shown because during $t < 50$ kyr the results are affected by the initial conditions. In figure 2, it can be seen that the time variability of $M_{\text{in}}$ does not coincide with $M_{\text{out}}$. In this case, the inner disk is not in a steady state, and the viscous parameters $\alpha_{M}$ and $\alpha_{G}$ are time dependent. In figure 2, at $t = 60$ kyr, the second outburst is seen. We pick up this outburst to study its features. Subsequent outbursts have the same features. Outbursts are characterized by duration of outburst $t_{\text{dur}} = 1.2 \times 10^{3}$ yr, the interval between outbursts $t_{\text{int}} = 5 \times 10^{3}$ yr, the maximum of the mass accretion rate during the outburst $M_{\text{FU}} = 5.6 \times 10^{-5} M_{\odot} \text{ yr}^{-1}$, and the mass accretion rate in the quiescent phase between bursts $M_{\text{TT}} = (4.7) \times 10^{-12} M_{\odot} \text{ yr}^{-1}$. These characteristic values appear periodically until the end of our calculation at 10$^{6}$ yr. Note that $M_{\text{FU}}$ is larger than $M_{\text{out}}$ in these bursts. $M_{\text{FU}}$ is not driven by $M_{\text{out}}$ but is driven by MRI with large $T > T_{\text{cri}}$ and $\alpha_{M_{\text{on}}}$.

In addition, we investigated the time variability of the mass accretion rate $M_{\text{in}}$ at $r_{\text{in}}$ for different values of $\eta = 10^{-3}$ and $10^{-1}$. The results were similar. It was found that $t_{\text{dur}}$, $M_{\text{FU}}$, and $M_{\text{TT}}$ are nearly independent of $\eta$, and the difference in $t_{\text{int}}$ is within 3%. It was also found that $\alpha_{G}$ is a non-zero value and does not depend on $\eta$. When we use a small initial $\alpha_{G}$ with $\eta$ smaller than 0.02, the angular momentum transport at the outer region of the disk is initially less efficient. Because $M_{\text{out}}$ is the same value, $\Sigma$ becomes larger and the $Q$-value become smaller. The viscous parameter $\alpha_{G}$ increases due to this disk evolution. On the other hand, when we use a larger $\eta$ and large initial $\alpha_{G}$, the angular momentum transport at the outer region of the disk becomes more efficient, and $\alpha_{G}$ decreases. This behavior indicates that the value of $\alpha_{G}$ with different $\eta$ is self-regulated by the $Q$-value so that $\alpha_{G}$ stays nearly the same (see also Takahashi et al. 2013).

In table 2, our results for several cases with $M_{\text{out}}$ and $\eta$ are summarized from the viewpoint of whether outbursts of mass accretion rate occur or not in the inner disk. Whether outbursts occur or not is nearly independent of $\eta$, and strongly dependent on $M_{\text{out}}$. It is seen that outbursts occur in a limited range of mass accretion rates onto the inner disk $10^{-10} < M_{\text{in}} < 3 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$. In addition, we calculated models with $\alpha_{G} = \exp (-Q^{4})$ as used in previous works (e.g., Takahashi et al. 2013; Zhu et al. 2009, 2010a, 2010b) instead of equation (7). We obtained almost the same range of $M_{\text{out}}$ for outbursts. It is found that outbursts occur when $\alpha_{G}$ has a value $\alpha_{M_{\text{off}}} < \alpha_{G} < \alpha_{M_{\text{on}}}$.

In figure 3, radial profiles of surface density $\Sigma$ and temperature $T$ in several epochs around $t_{\text{peak}} = 59396$ yr during one burst are shown for the case with $M_{\text{out}} = 3 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$. Each time instance in figure 3 is indicated by an asterisk in figure 2. In figure 2, the asterisk at $t = t_{\text{peak}} = 250$ yr overlaps with that at $t = t_{\text{peak}} = 60$ yr. At $t = t_{\text{peak}} = 250$ yr, from figure 2 it is seen that the inner disk is in a quiescent state. In the right diagram of figure 3, it is seen that $T < T_{\text{cri}}$ and the inner disk is in the MRI inactive state. However, the matter is piling up due to the mismatch in the mass accretion rate between the inner region.

**Table 2. Conditions for spontaneous outburst to occur.**

| $M_{\text{out}}$ [M$_{\odot}$ yr$^{-1}$] | $\eta = 10^{-3}$ | $\eta = 10^{-2}$ | $\eta = 10^{-1}$ |
|----------------------------------------|------------------|------------------|------------------|
| $3 \times 10^{-6} \leq M_{\text{out}}$ | High M Steady state (HS state) | burst | |
| $10^{-10} < M_{\text{out}} < 3 \times 10^{-6}$ | Low M Steady state (LS state) | |

$M_{\text{off}} = 3 \times 10^{-5} M_{\odot} \text{ yr}^{-1}$
(r < 2.5 au) and the outer region (r > 2.5 au), and the temperature at r = 2.5 au is increasing. At \( t = t_{\text{peak}} - 60 \) yr, it is seen that the region with \( T > T_{\text{crit}} \) and \( \alpha = \alpha_{\text{M,off}} = 10^{-2} \) (MRI active region) appears. After the MRI active region with \( \alpha_{\text{M,off}} \) appears at \( r = 2.5 \) au, rapid accretion to the inner region and diffusion to the outer region occur, since the angular momentum transport in the active region is more efficient than in the other region. The surface density and temperature of the outer (r > 4.5 au) and the inner regions (r < 2 au) increase due to this mass redistribution. The MRI active region (\( T > T_{\text{crit}} \)) propagates to both the inner and the outer regions of the inner disk. At \( t = t_{\text{peak}} \), the MRI active region reaches the center and the whole disk becomes MRI active. Indeed, both \( \Sigma \) and \( T \) are high due to mass redistribution from the MRI active region around \( r = 2.5 \) au. In figure 2, it can be seen that \( M_{\text{in}} = 5.6 \times 10^{-7} \, M_{\odot} \, \text{yr}^{-1} \) at \( t = t_{\text{peak}} \) is higher than \( M_{\text{out}} \). The matter in the inner disk is accreted onto the star faster than it is replenished due to accretion from the outer region of the inner disk, and the surface density and temperature in the inner disk start to decrease. As a result, at \( t_{\text{peak}} + 1100 \) yr, a large amount of mass is accreted, and \( \Sigma \) becomes low. The temperature is lower than \( T_{\text{crit}} \), and thus the inner disk is in the MRI inactive state.

In figure 4, the time evolution of the mass accretion rate at \( r_{\text{in}} \) for the cases with \( M_{\text{out}} = 1 \times 10^{-6} \) and \( 3 \times 10^{-7} \, M_{\odot} \, \text{yr}^{-1} \) is shown. It can be seen that the maximum of \( \dot{M} \) at \( r_{\text{in}} \) (\( M_{\text{FU}} \)) and the duration of the outburst \( t_{\text{dur}} \) are almost the same in two cases. On the other hand, the interval between the outbursts is quite different. Differences in the interval for two cases are summarized in table 3. It can be seen that the interval \( t_{\text{int}} \) for the case with \( M_{\text{out}} = 1 \times 10^{-6} \, M_{\odot} \, \text{yr}^{-1} \) is shorter than that for \( M_{\text{out}} = 3 \times 10^{-7} \, M_{\odot} \, \text{yr}^{-1} \). This is because it takes a longer time to replenish the disk with matter until \( T > T_{\text{crit}} \) in the case with \( M_{\text{out}} = 3 \times 10^{-7} \, M_{\odot} \, \text{yr}^{-1} \) than in the case with \( M_{\text{out}} = 1 \times 10^{-6} \, M_{\odot} \, \text{yr}^{-1} \).

In summary, in this section we have shown that the spontaneous outburst occurs when \( 10^{-10} < M_{\text{out}} < 3 \times 10^{-6} \, M_{\odot} \, \text{yr}^{-1} \) and that the value of \( M_{\text{out}} \) affects the interval between the bursts \( t_{\text{int}} \) but not the value of \( M_{\text{in}} \). Note that our results suggest that time variability of \( M_{\text{in}} \) is possible without time variability of \( M_{\text{out}} \), and that the features of outburst, i.e., \( t_{\text{dur}}, t_{\text{int}}, M_{\text{FU}}, \) and \( M_{T} \) are nearly independent of \( M_{\text{out}} \) except for \( t_{\text{int}}. \) \( M_{\text{FU}} \) is larger than \( M_{\text{out}} \). The low boundary for the steady accretion rate is sensitive to \( \alpha_{\text{M,off}} \), which we know little about.

![Figure 3. Radial profiles of surface density \( \Sigma \) (left diagram) and temperature \( T \) (right diagram) in several epochs at \( t_{\text{peak}} - 250 \) yr, \( t_{\text{peak}} - 60 \) yr, \( t_{\text{peak}} \), and \( t_{\text{peak}} + 1100 \) yr.](image)

![Figure 4. Time variability of mass accretion rate at \( r_{\text{in}} \) for several cases with \( M_{\text{out}} \).](image)

**Table 3. Difference in the interval between outbursts.**

| \( M_{\text{out}} \) [\( M_{\odot} \, \text{yr}^{-1} \)] | Interval [yr] |
|---|---|
| \( 1 \times 10^{-6} \) | \( 1.2 \times 10^{4} \) |
| \( 3 \times 10^{-7} \) | \( 5 \times 10^{4} \) |
4 Combination of episodic accretion and GML

In this section, we investigate the time variability of the accretion rate at \( r_{\text{in}} \) due to a combination of episodic accretion imposed onto disk’s outer edge and the GML operating within the disk. Viscous evolution between \( r_{\text{in}} \) and \( r_{\text{out}} \) is calculated by taking into account \( \alpha_{G} \) and \( \alpha_{M} \) as well as the episodic accretion rate onto the inner disk \( \dot{M}_{\text{out}}(t) \) by solving equations (2), (3), (5), and (8) with the parameters given in table 1. For the pattern of episodic accretion, we consider two types of \( \dot{M}_{\text{out}}(t) \). One is based on an analytic formula, and the other is based on numerical hydrodynamical calculations. The inner boundary condition is the same as in sections 2 and 3.

4.1 Analytic \( \dot{M}_{\text{out}}(t) \) with a single period

In this subsection, as the outer boundary condition we use the same formula as equation (4) in subsection 2.1. In equation (4), we use \( \eta = 10^{-3} \), \( \dot{M}_{0} = 2 \times 10^{-5} \, M_{\odot} \, \text{yr}^{-1} \), and \( \delta = 2 \times 10^{-3} \). By this combination of \( \dot{M}_{0} \) and \( \delta \), the mean accretion rate in a single oscillation period is given by \( \langle \dot{M}_{\text{out}}(t) \rangle_{\text{osci}} = 3 \times 10^{-7} \, M_{\odot} \, \text{yr}^{-1} \), which is the same as the value of the constant \( \dot{M}_{\text{out}}(t) \) in section 3. We use a finite \( t_{\text{osci}} \), the value of which is different from that of section 3.

In figures 5a and 5b, the time evolution of the mass accretion rate at \( r_{\text{out}} \) and \( r_{\text{in}} \) for the cases with \( t_{\text{osci}} = 5 \, \text{kyr} \) and \( t_{\text{osci}} = 100 \, \text{kyr} \) are shown, respectively. In figure 5a, it can be seen that \( \dot{M}_{\text{in}} \) (solid line) does not coincide with \( \dot{M}_{\text{out}} \) (dotted line) and that outbursts of \( \dot{M}_{\text{in}} \) occur at \( t = 105 \, \text{kyr} \) and \( t = 155 \, \text{kyr} \). This is because the inner disk acts to damp the oscillation in \( \dot{M}_{\text{out}} \) when transporting the accreted matter from the outer to the inner boundary, as explained in subsection 2.2, and because the GML drives outbursts. The outbursts have the following properties: \( t_{\text{dur}} = 1.2 \times 10^{3} \, \text{yr} \), \( \tau_{\text{int}} = 5 \times 10^{4} \, \text{yr} \), \( \dot{M}_{\text{FU}} = 5.6 \times 10^{-5} \, M_{\odot} \, \text{yr}^{-1} \), and \( \dot{M}_{\text{TT}} = (4–7) \times 10^{-12} \, M_{\odot} \, \text{yr}^{-1} \). These values are similar to the case with constant \( \dot{M}_{\text{out}} = 3 \times 10^{-7} \, M_{\odot} \, \text{yr}^{-1} \) in figure 4 in section 3. By comparing figures 4 and 5a, it is found that the resulting \( \dot{M}_{\text{in}} \) are similar although \( \dot{M}_{\text{out}} \) in figure 4 is temporally constant and \( \dot{M}_{\text{out}} \) in figure 5a includes time dependence. Thus, the outbursts in figure 5a can be regarded as the same mode as in section 3.

For the case with \( t_{\text{osci}} = 100 \, \text{kyr} > t_{\text{int,c}} \) in figure 5b, it can be seen that the period of \( \dot{M}_{\text{in}} \) (solid line) is the same as the period of \( \dot{M}_{\text{out}} \) (dotted line), although \( \dot{M}_{\text{in}} \) is different from \( \dot{M}_{\text{out}} \). It can be also seen that \( \dot{M}_{\text{in}} \) has outburst features \( t_{\text{dur}} \sim 3.5 \times 10^{3} \, \text{yr} \), \( \tau_{\text{int}} = 10^{5} \, \text{yr} \), \( \dot{M}_{\text{FU}} = 1 \times 10^{-5} \, M_{\odot} \, \text{yr}^{-1} \), and \( \dot{M}_{\text{TT}} = (4–10) \times 10^{-12} \, M_{\odot} \, \text{yr}^{-1} \). These features are different from the figures 4 and 5a, i.e., \( t_{\text{dur}} \) in figure 5b is three times longer, \( \tau_{\text{int}} \) is twice as long, \( \dot{M}_{\text{FU}} \) is five times smaller, and the range of \( \dot{M}_{\text{TT}} \) is twice as broad as in figures 4 and 5a, respectively. Thus, outbursts in figure 5b should be regarded as a different mode from those of section 3 and figure 5a. In this new mode, the time-dependent \( \dot{M}_{\text{out}} \) affects \( \dot{M}_{\text{in}}(t) \). We call this a stimulated mode of outburst. Furthermore, in figure 5b, it can be seen that at the burst phase at \( t = 50 \, \text{kyr} \), \( \dot{M}_{\text{in}} \) is three times smaller, and the range of \( \dot{M}_{\text{TT}} \) is twice as broad as in figures 4 and 5a, respectively. Thus, the agreement between \( \dot{M}_{\text{in}} \) and \( \dot{M}_{\text{out}} \) at and near the burst phase occurs when the roughly constant \( \dot{M}_{\text{out}} > 3 \times 10^{-6} \, M_{\odot} \, \text{yr}^{-1} \) (\( \dot{M}_{\text{out}} \) for the HS state) is kept for a time longer than the diffusion.
time $t_{\text{diff}}$ in the MRI active state (several $10^3$ yr) because information at $r_{\text{out}}$ reaches $r_{\text{in}}$ in $t_{\text{diff}}$.

In summary, from figure 5, it is shown that the GML can drive outbursts even if the oscillation of $\dot{M}$ decays during transmission to the inner disk. We identified two different modes of outburst, a spontaneous one and a stimulated one.

### 4.2 Numerical $\dot{M}_{\text{out}}$ produced by hydrodynamic simulations

In subsection 4.1, we used an analytic formula [equation (4)] for $\dot{M}_{\text{out}}(t)$. In this subsection, we use a more realistic $\dot{M}_{\text{out}}(t)$ as the outer boundary conditions in our model. Equations (2), (3), (5), and (8) are calculated with an $\dot{M}_{\text{out}}$ which is taken from the two-dimensional numerical hydrodynamics simulation of Vorobyov and Basu (2010). They calculated the formation and global evolution of the circumstellar disk and dynamically in-falling envelope as well as the central protostar, a modification presented in Vorobyov et al. (2013), using the thin-disk approximation.

We used the mass accretion rate through the sink cell at 5 au from the central star during $0.3 < t < 0.5$ Myr after the beginning of the cloud core collapse. The mass of the central star in hydrodynamical simulations is $M_\star = 0.44 - 0.53 M_\odot$ during this time. We approximate it by a constant value $M_\star = 0.5 M_\odot$ in our calculation.

In figure 6, the time evolution of the mass accretion rate at $r_{\text{out}}$ and $r_{\text{in}}$ is shown. It can be seen that the time behavior of $\dot{M}_{\text{out}}$ (dotted line) is complicated but the typical timescale of fluctuation is shorter than several kiloyears. The time-averaged value of $\dot{M}_{\text{out}}(t)$ during several kiloyears is gradually decreasing. For example, $\langle \dot{M}_{\text{out}} \rangle \sim 5 \times 10^{-7} M_\odot$ yr$^{-1}$, and $3 \times 10^{-7} M_\odot$ yr$^{-1}$ during $315 < t < 415$ kyr, and $415 < t < 471$ kyr, respectively. During $315 < t < 500$ kyr, $\dot{M}_{\text{out}}$ fluctuates around several $10^{-7} M_\odot$ yr$^{-1}$ and does not exceed $3 \times 10^{-6} M_\odot$ yr$^{-1}$. Conversely, in the right panel of figure 6, it can be seen that $\dot{M}_{\text{out}}$ has a burst with a maximum value of a few $10^{-5} M_\odot$ yr$^{-1}$ at around $310$ kyr and $\langle \dot{M}_{\text{out}} \rangle$ is about $3 \times 10^{-6} M_\odot$ yr$^{-1}$ during $310 < t < 312$ kyr. In figure 6, many outbursts are seen in $\dot{M}_{\text{in}}$ (solid line) even with the realistic $\dot{M}_{\text{out}}(t)$. It can be seen that the time behavior of $\dot{M}_{\text{in}}$ generally differs from that of $\dot{M}_{\text{out}}$. However, $\dot{M}_{\text{in}}$ has values of about $10^{-6} - 10^{-5} M_\odot$ yr$^{-1}$, similar to those of $\dot{M}_{\text{out}}$ when the state with $\dot{M}_{\text{out}} > 10^{-6} M_\odot$ yr$^{-1}$ is kept at $t = 310 - 312$ kyr. $\dot{M}_{\text{in}}$ in this burst remarkably coincides with $\dot{M}_{\text{out}}$ (see figure 5b) which is the stimulated mode of outburst discussed in subsection 4.1. In the phase during $415 < t < 471$ kyr, it is seen that the interval between outbursts is about $5 \times 10^4$ yr, which is almost the same as that in figure 5a. In the phase during $315 < t < 415$ kyr, it is seen that the interval between outbursts becomes shorter, about $3 \times 10^4$ yr. This is because the time-averaged value of $\dot{M}_{\text{out}}$ during $315 < t < 415$ kyr is larger than that during $415 < t < 471$ kyr. The peak of $\dot{M}_{\text{in}}$ is larger than $\dot{M}_{\text{out}}$ in spontaneous outbursts. Outbursts at $t \sim 345$, 375, 415, and 465 kyr belong to the spontaneous mode of outburst discussed in subsection 4.1 because $\dot{M}_{\text{out}}$ is smaller than $3 \times 10^{-6} M_\odot$ yr$^{-1}$.

In summary, we investigated the time variability of $\dot{M}_{\text{in}}$ using realistic $\dot{M}_{\text{out}}$. Our results in figure 6 confirm our findings in sections 2, 3, and subsection 4.1 in the sense that outbursts are driven by the GML although the fluctuation in $\dot{M}$ may decay when passing through the inner disk and that two modes of outburst (a spontaneous one and a stimulated one) are proved to exist in the case with realistic $\dot{M}_{\text{out}}$.

### 4.3 Effect of full opacity table

Finally, we consider the effect of the full opacity table $\kappa$. In this subsection, we use the Rosseland mean opacity,
\(\kappa(\rho, T)\), which is approximated by using a power law as
\[
\kappa = \kappa_0 \rho^a T^b, \tag{10}
\]
where \(\rho\) is the volume density and the values of \(\kappa_0, a,\) and \(b\) are summarized in table 4 (see also Bell & Lin 1994; Cossins et al. 2010; Kimura & Tsuribe 2012). We use the energy equation instead of equation (8):
\[
\frac{dE}{dt} = \frac{1}{\Sigma} (Q_{\text{visc}} - Q_{\text{cool}}), \tag{11}
\]
where \(E = c_s^2/\gamma(\gamma - 1)\) is the specific internal energy. Differently from the previous sections, we use the radiative cooling rate as
\[
Q_{\text{cool}} = \frac{32 \sigma T^4}{3 \Sigma \kappa} \quad (\kappa \Sigma > 1), \quad \frac{8 \sigma T^4 \Sigma \kappa}{3} \quad (\kappa \Sigma < 1), \tag{12}
\]
because the inner disk may become optically thin. In order to see the effects of a different expression for \(\alpha_c\), we use
\[
\alpha_c = \exp(-Q'). \tag{13}
\]

Equations (2), (3), (5), (10), (11), and (13) are calculated with realistic \(M_{\text{out}}\), which is the same value used in subsection 4.2. As a test case, we calculated these equations with constant \(M_{\text{out}}\) and \(T_{\text{crit}} = 800\, K\), which is the critical temperature used in Armitage, Livio, and Pringle (2001). We found that outbursts occur in a range of \(10^{-5} < M_{\text{out}} < 3 \times 10^{-6} M_\odot \, \text{yr}^{-1}\) for the case with \(T_{\text{crit}} = 800\, K\). We confirmed that the critical mass accretion rate for achieving a steady accretion is almost the same as in the previous study (Armitage et al. 2001).

In figure 7, time evolutions of the mass accretion rate at \(r_{\text{out}}\) (a) and \(r_{\text{in}}\) (b) are shown. It can be seen that outbursts occur even in the case with the full opacity table \(\kappa(\rho, T)\). Different from subsections 3.2, 4.1, and 4.2, it can be seen that the bursts have a shorter timescale. \(M_{\text{crit}}\) is in the range \(10^{-8} - 10^{-7} M_\odot \, \text{yr}^{-1}\), which is about \(10^3\) times larger, and \(t_{\text{out}} \sim 10^5\, \text{yr}\), which is 10 times shorter than the result in the previous sections, respectively. This is because the opacity \(\kappa\) becomes much smaller than the constant \(\kappa = 3\) when \(T > T_{\text{crit}}\). The radiative cooling rate \(Q_{\text{cool}}\) during the burst becomes larger than the case with constant \(\kappa = 3\). The burst is shorter, and a smaller amount of mass is accreted during the burst than the case with constant \(\kappa = 3\). From equations (8) and (9), \(\Sigma \propto \kappa^{-1/2}\) when \(T = T_{\text{crit}}\), thus surface density \(\Sigma\) after the burst is larger than the case with constant \(\kappa = 3\). We checked that the viscous parameter \(\nu = \alpha c_s^2/\Omega\) is almost the same in each case because the temperature after the burst is almost the same. Thus, \(M_{\text{out}} \propto \nu \Sigma\) becomes larger.

It is found that the behavior of \(M_{\text{in}}\) becomes more complicated than the case with constant \(\kappa = 3\) in the previous sections. However, we found that the results are qualitatively consistent. From figures 7a, 7b, and 7c, it can be seen that the time behavior of \(M_{\text{in}}\) does not coincide with \(M_{\text{out}}\). From figure 7c, it is also seen that bursts of \(M_{\text{in}}\) occur about 1 kyr after \(M_{\text{out}}\) reaches \(10^{-5} M_\odot \, \text{yr}^{-1}\). The mass accretion rate onto the disk \(M_{\text{out}}\) does not appear in \(M_{\text{in}}\) directly. The large size of \(M_{\text{out}}\) triggers the spontaneous outbursts in the inner disk.

### 5 Discussions

#### 5.1 Comparison with previous theoretical studies

Some hydrodynamical simulations of the formation of a disk by using sink cells claim that FU Ori-type outbursts are explained by disk fragmentation (Vorobyov & Basu 2006). The mass accretion rate onto the sink radii, \(M_{\text{sink}}\), fluctuates strongly in their simulation. Although they assume \(M_{\text{sink}}\) to be equivalent to the mass accretion rate onto the protostar \(M_{\ast}\), this assumption needs justification because they set sink cells whose radii are too large. We show that fluctuations of \(M_{\text{sink}}\) may decay when passing through the inner disk when the diffusion time of the inner disk is much larger than the timescale for oscillation of fluctuations (see section 2). This result clearly indicates that it is important to consider an inner disk within the sink cell. Our modeling also shows that a sharp increase in disk accretion at around 5 au, caused for example by a massive clump approaching and then disintegrating near the star, can trigger a true mass accretion burst onto the star by pushing the inner disk into the MRI active state.
Some research has shown that outbursts like FU Ori can be explained by the GML model using one-dimensional disk models with realistic opacities and realistic mechanisms of angular momentum transport (Armitage et al. 2001; Zhu et al. 2009; Martin & Lubow 2011). However, these models cannot include the fluctuation in the outer mass accretion rate induced by fragmentation of the disk. We calculate the mass accretion rate onto a protostar with a fluctuating outer mass accretion rate, and newly find that fluctuations of $\dot{M}_{\text{out}}$ affect the $t_{\text{int}}$ rather than the value of $\dot{M}_{\text{in}}$ itself. It is also found that $t_{\text{int}}$ is smaller for larger $\dot{M}_{\text{out}}$ owing to rapid mass loading from the outer region. These results show that fluctuations of $M_{\text{out}}$ have non-negligible influence on the mass accretion rate onto the protostar. Multi-dimensional calculations are necessary in order to discuss the properties of outbursts in further detail.

### 5.2 Comparison with observations

Constraints from observations about FU Ori-type outbursts are shown in table 5 (see also Bell & Lin 1994; Stahler & Palla 2004; Audard et al. 2014). Mass accretion rates in the FU Ori stage are about $\dot{M}_{\text{FU}} \sim 10^{-5}$–$10^{-4} M_\odot \, \text{yr}^{-1}$, which are roughly $10^2$–$10^3$ times higher than those in the T-Tauri phase, $\dot{M}_{\text{TT}} \sim 10^{-8}$–$10^{-7} M_\odot \, \text{yr}^{-1}$. Timescales of the duration of outbursts, $t_{\text{dur}}$, are estimated by the decreasing rate of luminosity as $t_{\text{dur}} \sim$ a few × 10–100 yr. The timescale of the interval between FU Ori events, $t_{\text{int}}$, is difficult to estimate because it cannot be directly observed. It is estimated statistically as $10^3$–$10^5$ yr.

The results in our model are also tabulated in table 5. These values are obtained with the parameters tabulated in table 1. The mass accretion rate onto the star in the T-Tauri phase and in the outburst phase are around $\dot{M}_{\text{TT}} \sim 10^{-12} M_\odot \, \text{yr}^{-1}$ and $\dot{M}_{\text{FU}} \sim 10^{-5} M_\odot \, \text{yr}^{-1}$, respectively. Note that these mass accretion rates are independent of the outer mass accretion rate $\dot{M}_{\text{out}}$ (see section 3 for more detail). The duration time of an outburst in our result is about $t_{\text{dur}} \simeq 1 \times 10^3$ yr, which is also nearly independent of $\dot{M}_{\text{out}}$. The time interval between outbursts is about $10^4$ yr and $10^6$ yr for the cases with $\dot{M}_{\text{out}} = 10^{-6} M_\odot \, \text{yr}^{-1}$ and $\dot{M}_{\text{out}} = 10^{-8} M_\odot \, \text{yr}^{-1}$, respectively. Although $t_{\text{int}}$ and $\dot{M}_{\text{FU}}$ in our model are not so...
5.3 Ignored processes and the future direction of this study

First, we have simplified the conditions as to whether MRI is active or not. In this paper, the condition for MRI activation is represented as $T \geq T_{\text{crit}}$ since we consider that collisional ionization is effective above $T_{\text{crit}}$ owing to dust sublimation. Actually, however, the ionization rate determines the activity of the MRI. Ionization rate has a strong gradient in the vertical direction because it is related to the density, temperature, and external ionizing sources (Fujii et al. 2011; Landry et al. 2013). This fact implies that $\alpha_{\text{MRI}}$ strongly depends on the vertical coordinate $z$. Some previous studies include this effect approximately by using the layered accretion model (e.g., Armitage et al. 2001; Zhu et al. 2009). However, we use a simpler treatment in which $\alpha_{\text{MRI}}$ is assumed to be constant for $z$ because the layered accretion is not an essential component for the outbursts in the GML model. We should include the vertical structure of the disks in order to treat the effects of MRI more precisely. This treatment requires at least two-dimensional calculation.

Second, the efficiency of the angular momentum transport by GI is controversial. There are some formulas for $\alpha_{\text{GI}}$ based on analytic considerations (Lin & Pringle 1990) or fitting to numerical results (Kratter et al. 2008). However, Balbus and Papaloizou (1999) claimed that gravitational torque cannot be expressed in the alpha prescription, which is founded on the assumption that the torque acts as local stress. Since gravity is a long-range force, the assumption is not well satisfied. In order to establish the GML model as a robust mechanism for the outbursts, it is necessary to check whether the gravitational torque could be represented by the alpha prescription or not. To investigate this problem, non-axially symmetric calculations are necessary.

Finally, we fixed the outer radius $r_{\text{out}} = 5$ au, which is the sink radius of Vorobyov et al. (2013). According to their simulation, clumps made around 100 au are able to arrive at the sink radius. They are expected to be destroyed by the tidal force from the protostar if they approach closer to it than the tidal radius $r_{\text{tid}}$ (Nayakshin 2010; Tsukamoto et al. 2013). In this paper, we adopt a one-dimensional model with axial symmetry to describe the inner disk. In the situation that clumps exist, axial symmetry is not valid, and our model should not be used. Once clumps are destroyed, it is expected that their remnants spread around the protostar to form a disk that is moderately axially symmetric. Our model is likely to be available for $r \lesssim r_{\text{tid}}$, and thus we should set $r_{\text{out}} = r_{\text{tid}}$. If the tidal radius $r_{\text{tid}}$ is smaller than the radius of the protostar, the GI model could be feasible to explain outbursts because the clump is expected to fall onto the protostar directly. A reasonable estimation of the tidal radius for each clump is important to determine the mechanism that triggers the outbursts. This problem also needs non-axially symmetric calculations. To improve all the physical processes we discussed above, we need three-dimensional calculations in principle.

6 Summary

In this paper, we have investigated the role of the inner disk $r \lesssim 5$ au from the central star in time-dependent mass accretion flow. In order to understand the properties of the time variation of the mass accretion rate onto the central star, we considered viscous evolution of the inner disk taking into account both the gravo-magneto limit cycle (GML) and a time-variable mass accretion rate onto the inner disk. We assumed the $\alpha$-description in order to treat the transport of angular momentum driven by both GI and MRI. Our results and findings are summarized as follows:

(i) Mass accretion rate onto the protostar tends to be different from that onto the inner disk partly due to the viscous diffusion. In the case with a temporarily constant viscous parameter $\nu$, when $t_{\text{osci}} < t_{\text{diff}}$ the mass accretion rate $M(r)$ approaches the time-averaged value of $M_{\text{out}}$ in a single period $M_{\text{out}}/t_{\text{osci}}$ as $r \to 0$, and when $t_{\text{diff}} < t_{\text{osci}}$ the time variability of the mass accretion rate remains.

| Table 5. Outburst properties. | Observations | Constant opacity $\kappa = 3$ | Full opacity table |
|------------------------------|--------------|-----------------------------|-------------------|
| $M_{\text{TT}}$ [M$_{\odot}$ yr$^{-1}$] | $10^{-5}$−$10^{-7}$ | $10^{-12}$ | $10^{-8}$ |
| $M_{\text{FU}}$ [M$_{\odot}$ yr$^{-1}$] | $10^{-5}$−$10^{-4}$ | $10^{-5}$ | $10^{-4}$ |
| $t_{\text{dur}}$ [yr] | $10^2$−$10^3$ | $10^3$ | $10^2$ |
| $t_{\text{int}}$ [yr] | $10^3$−$10^4$ | $10^6$−$10^6$ |
(ii) Outbursts driven by the GML can occur under the condition that the constant mass accretion rate onto the inner disk is $10^{-10} < \dot{M}_{\text{out}} < 3 \times 10^{-6} M_\odot \text{yr}^{-1}$. The low boundary for the steady accretion rate is sensitive to $\alpha_{\text{M, off}}$, which we know little about. In this range of $\dot{M}_{\text{out}}$, among the features of outbursts such as $t_{\text{dur}}$, $t_{\text{int}}$, $M_{\text{FU}}$, and $M_{\text{TT}}$, only the interval between outbursts $t_{\text{int}}$ is a function of $\dot{M}_{\text{out}}$. Differences in $\dot{M}_{\text{out}}$ do not affect $M_{\text{FU}}$, but do affect $t_{\text{int}}$. Large $\dot{M}_{\text{out}}$ results in short $t_{\text{int}}$. Large $\dot{M}_{\text{out}}$ does not appear directly in the amplitude of $\dot{M}_{\text{in}}$.

(iii) Even with a fluctuating mass accretion rate onto the inner disk at $r_{\text{out}}$, the GML can drive outbursts although fluctuations of $\dot{M}$ may decay when passing the inner disk inwards. We have newly identified two modes of outburst: a spontaneous one and a stimulated one. In the case with $\langle \dot{M}_{\text{out}} \rangle$ smaller than $3 \times 10^{-6} M_\odot \text{yr}^{-1}$, the outburst is a spontaneous one in which $\dot{M}_{\text{in}} > \dot{M}_{\text{out}}$, and in the case with the $\langle \dot{M}_{\text{out}} \rangle$ greater than $3 \times 10^{-6} M_\odot \text{yr}^{-1}$, the outburst is a stimulated one in which $\dot{M}_{\text{in}} \sim \dot{M}_{\text{out}}$, where $\langle \dot{M}_{\text{out}} \rangle$ is the averaged value of $\dot{M}_{\text{out}}$ over several kyr.

The important results in the present paper are that the mass accretion rate onto the sink cell does not always appear directly in the mass accretion rate onto the star (the latter determining the accretion luminosity), but we suggest that the interval between outbursts can possibly be used as a probe for mass accretion rate.

Although we used many simplified treatments in the model, we believe our results help understand the role of the inner disk in the mass accretion rate onto the star in the early phase of star formation.

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