Research on Multi-uav Cooperative Tracking Target Based on PSO Predictive Control Algorithm

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Abstract. In the multiple unmanned aerial vehicle (multi-uav) cooperative tracking target control system, the control system is extremely complex due to the nonlinear control model and constraints. In this paper, multi-uav cooperative tracking target motion modeling is adopted, optimization performance indicators is established, and multi-objective rolling optimization is transformed to single-objective optimization. Finally, particle swarm optimization (PSO) prediction control algorithm is adopted for optimization solution, realizing real-time control of multi-uav cooperative tracking target, and its feasibility is verified by simulation.

1. Introduction
Multi-uav cooperative tracking target control not only considers the relative motion between uavs and targets, but also considers the relative motion between uavs. Therefore, while improving the flight control accuracy, the balance between the two control targets should also be taken into account. As a result, in the control system, not only the control model is nonlinear, but also the constraint is nonlinear[1]. To solve this problem, this paper conducts multi-uav cooperative tracking target motion modeling, establishes optimization performance index, converts multi-objective rolling optimization to single-objective optimization, and finally proposes PSO prediction control algorithm for optimization solution, realizing real-time control of multi-uav cooperative tracking target.

2. Multi-uav cooperative tracking target flight motion modeling
With the ground coordinate system as the reference coordinate system, the target tracking of multi-uav is usually at the same height, so they are projected onto the two-dimensional plane for research[2], the specific motion analysis diagram is shown in figure 1.

![Figure 1. Schematic diagram of cooperative tracking target movement](image-url)
In the figure, \( v_i \) is the target velocity, and the included Angle with the X-axis of the ground coordinate system is \( \phi_i \); \( v_i \) is the velocity of UAV1, and the included Angle with the X-axis of the ground coordinate system is \( \phi_i \); \( v_w \) is the velocity of UAV2, and the included Angle with the X-axis of the ground coordinate system is \( \phi_i \); \( \omega \) is the yaw angular velocity.

Suppose UAV1 coordinate is \((x_i, y_i)\), UAV2 coordinate is \((x_w, y_w)\), and target coordinate is \((x_t, y_t)\), and the UAV1 motion model is as follows.

\[
\dot{x}_i = v_i \cos(\phi_i) \\
\dot{y}_i = v_i \sin(\phi_i)
\]

(1)

(2)

The UAV2 motion model is as follows.

\[
\dot{x}_w = v_w \cos(\phi_w) \\
\dot{y}_w = v_w \sin(\phi_w) \\
\dot{\phi}_w = \omega
\]

(3)

(4)

(5)

The target motion model is as follows.

\[
\dot{x}_t = v_t \cos(\phi_t) \\
\dot{y}_t = v_t \sin(\phi_t)
\]

(6)

(7)

3. Discrete model and predictive control analysis of multi-uav cooperative tracking target

Discretization of the motion equation of UAV1.

\[
x_i(k+1) = x_i(k) + v_i(k) \cos(\phi_i(k)) \Delta T \\
y_i(k+1) = y_i(k) + v_i(k) \sin(\phi_i(k)) \Delta T
\]

(8)

(9)

Discretization of the motion equation of UAV2.

\[
x_w(k+1) = x_w(k) + v_w(k) \cos(\phi_w(k)) \Delta T \\
y_w(k+1) = y_w(k) + v_w(k) \sin(\phi_w(k)) \Delta T \\
\phi_w(k+1) = \phi_w(k) + \omega \Delta T
\]

(10)

(11)

(12)

\[
r_2(k+1) = \sqrt{(x_i(k+1) - x_i(k))^2 + (y_i(k+1) - y_i(k))^2}
\]

(13)

\[
\theta(k+1) = \arctan\left(\frac{y_t(k+1) - y_t(k)}{x_t(k+1) - x_t(k)}\right) - \arctan\left(\frac{y_w(k+1) - y_w(k)}{x_w(k+1) - x_w(k)}\right)
\]

(14)

Discretization of the motion equation of the target.

\[
x_t(k+1) = x_t(k) + v_t(k) \cos(\phi_t(k))
\]

(15)

\[
x_t(k+1) = x_t(k) + v_t(k) \cos(\phi_t(k))
\]

(16)

Will type (8), (9), (10), (11), (15), (16) into (13), (14)

\[
r_2(k+1) = \sqrt{(x_w(k) + v_w(k) \cos(\phi_w(k)) \Delta T - x_i(k+1))^2 - (y_w(k) + v_w(k) \sin(\phi_w(k)) \Delta T - y_i(k+1))^2}
\]

(17)

\[
\theta(k+1) = \arctan\left(\frac{y_t(k+1) + v_t(k) \sin(\phi_t(k)) \Delta T - y_t(k)}{x_t(k+1) + v_t(k) \cos(\phi_t(k)) \Delta T - x_t(k)}\right) - \arctan\left(\frac{y_w(k+1) + v_w(k) \sin(\phi_w(k)) \Delta T - y_w(k)}{x_w(k+1) + v_w(k) \cos(\phi_w(k)) \Delta T - x_w(k)}\right)
\]

(18)

Where \( \omega \) is the angular velocity of yaw, \( r_i \) is the distance between UAV1 and the target, \( r_2 \) is the distance between UAV2 and the target, and \( \theta \) is the included Angle between UAV1 and UAV2 and the target.
In the rolling optimization process, the sampling period is relatively short. Assuming that the speed and yaw Angle of UAV1 in the sampling period remain unchanged, that is, the wind speed remains unchanged[3], then:

\[ v_r(k+i) = v_r(k+i-1) = v_r(k) \]  \hfill (19)

\[ \phi_r(k+i) = \phi_r(k+i-1) = \phi_r(k) \]  \hfill (20)

You can get:

\[ r_2(k+2) = \frac{\left(x_u(k+1) + v_u(k+1) \cos(\phi_r(k+1)) \Delta T - \left(x_u(k+1) + v_u(k+1) \cos(\phi_r(k+1))\right)\Delta T \right)^2 - \left(y_u(k) + v_u(k) \sin(\phi_r(k)) \Delta T + v_u(k) \sin(\phi_r(k)) \Delta T\right)^2}{\left(x_u(k+1) + v_u(k+1) \sin(\phi_r(k+1)) \Delta T - \left(x_u(k+1) + v_u(k+1) \sin(\phi_r(k+1))\right)\Delta T \right)^2 - \left(y_u(k) + v_u(k) \sin(\phi_r(k)) \Delta T + v_u(k) \sin(\phi_r(k)) \Delta T\right)^2} \]

\[ \theta(k+2) = \arctan \frac{v_r(k+1) + v_r(k+1) \sin(\phi_r(k+1)) \Delta T - (y_r(k+1) + v_r(k+1) \sin(\phi_r(k+1)) \Delta T)}{x_r(k+1) + v_r(k+1) \cos(\phi_r(k+1)) \Delta T - (x_r(k+1) + v_r(k+1) \cos(\phi_r(k+1)) \Delta T)} \]

\[ r_2(k+3), r_2(k+4), r_2(k+5), ..., r_2(k+N-1), r_2(k+N) \]  \hfill (23)

\[ \theta(k+3), \theta(k+4), \theta(k+5), ..., \theta(k+N-1), \theta(k+N) \]  \hfill (24)

As you can see, the predicted output sequence is a function based on future control variables \(v_u(k), v_u(k+1), v_{u+1}, v_{u+1+1}, v_{u+1+2}, ..., v_{u+N-1}, v_{u+N}\), which is obviously a multi-input multi-output nonlinear control problem[4]. The values of \(v_u(k), v_u(k+1), v_{u+1}, v_{u+1+1}, v_{u+1+2}, ..., v_{u+N-1}, v_{u+N}\) can be solved by the optimization algorithm, which can be used as the output of the control quantity at the next moment to perform the rolling optimization solution successively. The specific idea is shown in figure 2.

**Figure 2.** Rolling optimization process diagram

4. Determination of predictive performance indicators
For the problem of multi-uav cooperative tracking target and prediction control, its rolling optimization goal is to find appropriate \( v_w(k), v_w(k+1), L, v_w(k+N-1) \) and \( \omega_w(k), \omega_w(k+1), L, \omega_w(k+N-1) \), so that the prediction output in the whole optimization time domain is as close as possible to the reference trajectory. In this paper, the target predetermined value is used as the reference trajectory, and the error square of the target predetermined value and the predicted output in the time domain is used as the prediction performance index.

Assuming that \( \theta_a \) is the included angle between the two UAV and the target, and \( r_c \) is the distance between UAV2 and the target, the following performance indicators can be obtained:

\[
J_1 = \sum_{i=1}^{N} \left( r_2^2 (k + i) - r_c^2 \right)^2
\]

\[
J_2 = \sum_{i=1}^{N} \left( \tan(\theta_2 (k + i) - \tan \theta_c) \right)^2
\]

(25)

\( J_1 \) and \( J_2 \) are functions based on future control variables \( v_w(k), v_w(k+1), L, v_w(k+N-1) \) and \( \omega_w(k), \omega_w(k+1), L, \omega_w(k+N-1) \). Optimization constraints of control variables are introduced as follows:

\[
\begin{align*}
 v_w(k+i-1) - \Delta v < & v_w(k+i) < v_w(k+i-1) + \Delta v & i & \in 0, 1, 2L, N-1 \\
 \omega_{\text{min}} < & \omega_w(k+i) < \omega_{\text{max}} & i & \in 0, 1, 2L, N-1 \\
 v_{\text{min}} < & v_w(k+i) < v_{\text{max}} & i & \in 0, 1, 2L, N-1
\end{align*}
\]

(26)

This problem becomes a multi-objective nonlinear optimization problem with constraints.

5. The transformation from multi-objective optimization to single-objective optimization

In the process of multi-uav cooperative target tracking, it is not necessary to accurately control the relative distance between the uav and the target to a constant value, as long as the output fluctuates within a small range around the set value, the set value target is considered to have been achieved[5]. First, the performance index \( J_1 \) of relative distance is optimized by a single objective to make UAV2 reach the radius required for tracking target. Then, \( J_1 \) is disturbed by a small target by setting the radius to find out the feasible solution space. Then, the optimized value of \( J_2 \) is found in the feasible solution to achieve hierarchical multi-objective optimization.

Specific ideas are as follows:

If \( r_2(k) > r_c + r_{\text{error}} \) or \( r_2(k) < r_c - r_{\text{error}} \), then:

\[
\min(J_1) = \min \left( \sum_{i=1}^{N} \left( r_2^2 (k + i) - r_c^2 \right)^2 \right)
\]

St.

\[
\begin{align*}
 v_w(k+i-1) - \Delta v < & v(k+i) < v_w(k+i-1) + \Delta v & i & \in 0, 1, 2L, N-1 \\
 \omega_{\text{min}} < & \omega_w(k+i) < \omega_{\text{max}} & i & \in 0, 1, 2L, N-1 \\
 v_{\text{min}} < & v_w(k+i) < v_{\text{max}} & i & \in 0, 1, 2L, N-1
\end{align*}
\]

(27)

If \( r_c - r_{\text{error}} \leq r_2(k) \leq r_c + r_{\text{error}} \), add a slight disturbance error, and the value is positive \( r_{\text{error}} \), then:

\[
\sum_{i=1}^{N} \left( r_2^2 (k + i) - r_c^2 \right)^2 < r_{\text{error}}^2 \times N
\]

St.
Solve a set \( M \) of \((v, \omega)\), and get:

\[
\min(J_2) = \min\left(\sum_{i=1}^{N} (\tan(\theta_j(k + i) - \tan \theta_j)^2)\right)
\]

St..

\((v, \omega) \in M\) \hspace{1cm} (29)

Unify the above two questions to get:

\[
\min(J_2) = \min\left(\sum_{i=1}^{N} (\tan(\theta_j(k + i) - \tan \theta_j)^2)\right)
\]

St..

\[
\begin{cases}
v_n(k+i-1) - \Delta v < v(k+i) < v_n(k+i-1) + \Delta v & i \in 0, 1, 2L \ N - 1 \\
\omega_{\min} < \omega_n(k+i) < \omega_{\max} & i \in 0, 1, 2L \ N - 1 \\
v_{\min} < v_n(k+i) < v_{\max} & i \in 0, 1, 2L \ N - 1 \\
\sum_{i=1}^{N} (r_i^2(k+i) - r_{\text{error}}^2)^2 < r_{\text{error}}^2 \times N & i \in 0, 1, 2L \ N - 1
\end{cases}
\]

Then the above problems become two nonlinear multivariable constrained single objective optimization problems.

6. Nonlinear rolling optimization based on PSO

According to the basic principle of pso, after proper constraint processing and local convergence processing, the nonlinear rolling optimization process based on pso is obtained as follows.

6.1. Particle swarm initialization and the determination of search termination conditions

The particle population size is set to 25, the initial position of the particle is set to 0, and the particle velocity component is represented by the control variables \( v_n(k), v_n(k+1), \ldots, v_n(k+N-1) \) and \( \omega_n(k), \omega_n(k+1), \ldots, \omega_n(k+N-1) \). The initial value of the particle is divided into the initial value setting of the optimization variable of the first sampling point and the initial value setting of the optimization variable of the non-first sampling point. The initial value of the velocity control variable \( v_n(k), v_n(k+1), \ldots, v_n(k+N-1) \) of the first sampling point particle swarm optimization algorithm is set as \( v_n(k-1) \), and the initial value of the angular velocity control variable \( \omega_n(k), \omega_n(k+1), \ldots, \omega_n(k+N-1) \) is set as 0. The control quantity value optimized by the former sampling point is taken as the initial value of the latter sampling point, and so on (as shown in figure 3).
The maximum number of iterations is taken as the termination condition of the convergence of PSO, the larger the number of iterations, the higher the control accuracy, and the larger the calculation amount, the specific value is determined according to the hardware of the controller.

6.2. Fitness function and particle swarm variation calculation

Optimization performance index $J$ can be used as the fitness of particle swarm, but it is easy to be locally optimal. Genetic algorithm is adopted for local optimal processing, and the variation calculation formula is as follows.

$$
\begin{align*}
\text{child}_1(x_i) &= p_i \text{parent}_1(x_i) + (1 - p_i) \text{parent}_2(x_i) \\
\text{child}_2(x_i) &= p_i \text{parent}_2(x_i) + (1 - p_i) \text{parent}_1(x_i) \\
\text{child}_1(V_i) &= \frac{\text{parent}_1(V_i) + \text{parent}_2(V_i)}{\text{parent}_1(V_i) + \text{parent}_2(V_i)} \|\text{parent}_1(V_i)\| \\
\text{child}_2(V_i) &= \frac{\text{parent}_1(V_i) + \text{parent}_2(V_i)}{\text{parent}_1(V_i) + \text{parent}_2(V_i)} \|\text{parent}_2(V_i)\| 
\end{align*}
$$

(31)

$p_i$ is a random number between [0,1] (empirical value is about 0.2), child means child particle, parent means parent particle[6].

6.3. Particle swarm selection, movement and evaluation

In the calculation of feasible solution and infeasible solution method, the calculation of constraint violation degree is very important. First, a formula of constraint violation degree must be given.

Suppose M constraints are: $g_i(x) \leq 0$, $i \in M$

Substitute the value of particle velocity into $g_i(x)$, if $g_i(x) > 0$, the violation degree of constraint increases by 1, otherwise it is 0, the constraint violation value converse is as follows.

$$
\text{converse} = \sum_{i=1}^{M} k_i \quad \text{if} \; g_i(x) > 0 \quad \text{else} \quad k_i = 0
$$

(33)

For particle selection strategy, the comparison criteria are:

- When both particles are feasible, their objective function value is compared, and the particle with small objective function value is better.
- When both particles are infeasible, if their constraint violation degree is less than the given threshold, the particle with small objective function value is optimal. If it is larger than the threshold, the particle with small constraint violation degree is preferred.
- When particle X is not feasible and particle Y is feasible, if the violation degree of partial constraints of particle X is less than the given threshold, the particle with small objective function value is optimal. Otherwise the Y particle is optimal.

In order to reserve a large number of unfeasible solution particles at the early stage of evolution and a small number of unfeasible solution particles at the later stage of evolution, the following threshold change formula is proposed.

\[
\epsilon_{\text{converse}} = (1 - \frac{N_{\text{now}}}{N_{\text{max}}}) \times \text{converse}_{\text{max}}
\]  

(34)

Where \( N_{\text{now}} \) is the current iteration number, \( N_{\text{max}} \) is the maximum iteration number, and \( \text{converse}_{\text{max}} \) is the maximum possible violation value.

6.4. Output of control quantity
The optimization results are obtained through the above steps, and the control quantity is output according to the optimization results.

7. The simulation results
Suppose the tracking scenario is: the target first moves at the angular velocity of \(-\pi/100 \text{ rad/s}\), after 100s, it turns to the X axis for straight motion, after 200s, it turns to the angle of \(\pi/3\) with the Y axis for straight motion, after 300s, it moves in the Y axis for straight motion, there is angular velocity noise of \(\pm 0.001 \text{ rad/s}\) when the target moves in a straight direction, and the target position has estimation error. The Initial conditions for cooperative tracking target is shown in table 1.

| Table 1. Initial conditions for cooperative tracking target |
|----------------------------------------------------------|
| Initial position and angle of the target | Initial position and estimation error of UAV1 | Initial position and angle of UAV2 |
| \((0,0,\pi/2)\) | \((20,0)\) | \(\pm 5\) | \((-2000,-1800,\pi/4)\) | \((-2000,-1900,\pi/4)\) |
| Initial velocity and angular velocity of UAV2 | Min and max cruising speed of UAV1 | Min and max cruising speed of UAV2 | Target tracking distance and target Angle |
| \((40,0)\) | \((35,45)\) | \((25,45)\) | \((1000,\pi/4)\) |

Prediction time domain N=5, uav angular velocity change rate is limited to (-0.1, 0.1), Matlab Simulink toolbox was used for simulation, and the simulation results are shown in figure 4-8.
Figure 4. Position of UAV1 relative to target.

Figure 5. Position of UAV2 relative to target.

Figure 6. The Angle between UAV2 and UAV1 and the target

Figure 7. The error between the relative distance between UAV1 and the target and the preset distance
Figure 8. The error between the relative distance between UAV2 and the target and the preset distance

Through simulation, it can be seen that when the target moves at a certain angular speed or suddenly turns, the tracking distance error of the UAV1 and the UAV2 is larger than that of the target when it moves in a straight line, and the included angle error between the two drones and the target is also larger. However, as long as the final target can move in a straight line and at a uniform speed, the error can finally be within the allowable range through the adjustment of the controller, and target tracking can be well completed. The angle between the following uav and the piloting uav will eventually remain at $\pi/4$. Moreover, the time of a rolling optimization of uav predictive control by the simulation program in ordinary PC is less than 0.2 seconds. When running in high-speed dedicated chip, the speed will only be faster, which can obviously meet the real-time requirements of control. Therefore, pso based predictive control algorithm for target tracking control can not only meet the control requirements, but also meet the real-time requirements.

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