Response of the integrals in the Tremaine–Weinberg method to multiple pattern speeds: a counter-rotating inner bar in NGC 2950?

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ABSTRACT

When integrals in the standard Tremaine–Weinberg method are evaluated for the case of a realistic model of a doubly barred galaxy, their modifications introduced by the second rotating pattern are in accord with what can be derived from a simple extension of that method, based on separation of tracer’s density. This extension yields a qualitative argument that discriminates between prograde and retrograde inner bars. However, the estimate of the value of inner bar’s pattern speed requires further assumptions. When this extension of the Tremaine–Weinberg method is applied to the recent observation of the doubly barred galaxy NGC 2950, it indicates that the inner bar there is counter-rotating, possibly with the pattern speed of $-140 \pm 50 \, \text{km} \, \text{s}^{-1} \, \text{arcsec}^{-1}$. The occurrence of counter-rotating inner bars can constrain theories of galaxy formation.

Key words: galaxies: individual: NGC 2950 – galaxies: kinematics and dynamics – galaxies: structure.

1 INTRODUCTION

Bars within bars appear to be a common phenomenon in galaxies. Recent surveys indicate that up to 30 per cent of early-type barred galaxies contain such double bars (Erwin & Sparke 2002; Laine et al. 2002). Inner bars remain distinct in near infrared (Wozniak et al. 1995); therefore fairly old stars must contribute to their light. The relative orientation of the two bars in doubly barred galaxies is random; therefore it is likely that the bars rotate with different pattern speeds.

The origin of multiply barred systems remains unclear. A bar takes away angular momentum from gas very efficiently, and in the young Universe bars might have been responsible for the early rapid growth of the massive black holes (Begelman, Volonteri & Rees 2006). If the innermost parts of galactic discs formed first, then early instabilities there might have led to the formation of small-scale bars, which may be surviving in the present-day Universe as nuclear bars, often nested inside larger, outer bars, that might have formed later. Studying the dynamics of nested bars can therefore help us to understand the formation of galaxies and of their central massive black holes.

Our understanding of how double bars are sustained has significantly improved in the recent years. Maciejewski & Sparke (1997, 2000) have developed a formalism that enables finding families of stable regular orbits in such systems. Stable regular orbits are robust structures, which define the shape of the galaxy, and therefore they can serve as a backbone for double bars. So far, they have been analysed for the case when the pattern speed of the inner bar is higher than that of the outer bar (Maciejewski & Sparke 2000). Using N-body simulations, Rautiainen, Salo & Laurikainen (2002) confirmed that stars get trapped around these orbits, and form long-lasting doubly barred systems. However, in this scenario the kinematics of the inner bar are not a scaled-down copy of that of the main bar, since the inner bar cannot extend to its corotation. Other numerical simulations have shown that systems of two counter-rotating bars are also possible (Sellwood & Merritt 1994; Friedli 1996), and systems with secondary bars rotating slower than the outer, main bars, have never been excluded on theoretical grounds.

Various dynamical scenarios for doubly barred galaxies may have different implications for the evolution of the galactic centres. In order to discriminate between them, one should measure the pattern speed of the inner bar. If only one pattern speed is present in the system, the Tremaine & Weinberg (1984) method allows us to derive it using a set of simple kinematical measurements. Recent kinematical observations of the doubly barred galaxy NGC 2950 (Corsini, Debattista & Aguerri 2003; hereafter CDA03) are inconsistent with one pattern speed there. The observations are suggestive of another pattern speed in the area of the inner bar. CDA03 attempted to estimate it, but they concluded a wide range of values, consistent with a fast-rotating prograde secondary bar as well as with a retrograde one.

In this paper, I show that a simple extension of the Tremaine–Weinberg method to multiple pattern speeds, based on the separation of tracer’s density, is sufficient to discriminate between prograde and retrograde inner bars. Similar extension has already been...
2 A SIMPLE EXTENSION OF THE TREMAINE–WEINBERG METHOD TO MULTIPLE PATTERN SPEEDS

The Tremaine–Weinberg method is designed for one pattern speed, which it derives from the luminosity centroid and the luminosity-weighted line-of-sight velocity of a chosen tracer moving in the galaxy’s potential. The method rests on three assumptions: the disc of the galaxy is flat, it has a well-defined pattern speed and the tracer obeys the continuity equation. Under these assumptions, the surface density of the tracer, $\Sigma(x, y, t)$, can be written as

$$\Sigma(x, y, t) = \Sigma(R, \varphi - \Omega_P t),$$

where $\Omega_P$ is the pattern speed, $(x, y)$ are Cartesian coordinates in the disc plane, $(R, \varphi)$ are polar coordinates there, centred on the galactic centre, and $t$ is time.

When another pattern speed is introduced, the two patterns cannot rotate rigidly one through another (Louis & Gerhard 1988; Sridhar 1989; Maciejewski & Sparke 2000), and in principle one cannot split the right-hand side of (1) into components with different pattern speeds. However, if a rough separation of patterns is possible, and if one neglects secular evolution, this system is periodic with period $P = \pi/(\Omega_S - \Omega_B)$, where $\Omega_S$ and $\Omega_B$ are pattern speeds of the two bars. Consequently, the surface density of the tracer can generally be written as

$$\Sigma(x, y, t) = \Sigma_B(R, \varphi - \Omega_B t, t | P) + \Sigma_S(R, \varphi - \Omega_S t, t | P),$$

where $t | P$ depends on dependence on time with periodicity $P$. In Appendix A, I evaluate correction to the Tremaine–Weinberg integrals that arises because of this periodic oscillation of realistic double bars. This correction turns out to be small, and I will neglect it in the following argument.

Once periodic oscillations of nested bars are neglected, (2) gets reduced to

$$\Sigma(x, y, t) = \Sigma_B(R, \varphi - \Omega_B t) + \Sigma_S(R, \varphi - \Omega_S t).$$

Twofold interpretation of such a separation of tracer’s density is possible. Either the tracer in the disc (e.g. stars) can be divided into subgroups belonging to two unchanging patterns, each rotating with a constant pattern speed, or one can separate radial zones in the galactic disc that rotate with constant pattern speeds, with no net exchange of the tracer between the zones. Neither of these interpretations is consistent with the dynamics of galaxies, because patterns rotating one through another change with time, and tracers of each pattern are often present at the same radius. However, as I show in Appendix A, these inconsistencies are likely to be small, and the separation (3) can still be approximately valid.

Below, I will show that although the extension of the Tremaine–Weinberg method based on the separation of tracer’s density (3) cannot recover the two pattern speeds, it can yield a qualitative prediction of how the integrals in the standard Tremaine–Weinberg method change when a second rotating pattern is introduced. In Section 3, I will show that this change, calculated properly for a realistic model of doubly barred galaxy, with bars oscillating in time and with tracers of each bar overlapping, is the same as predicted by the extended method proposed here. Thus, the separation of tracer’s density may help in interpreting the Tremaine–Weinberg integrals in the models and in the observed galaxies.

Once tracer’s density is separated according to (3), one can write two separate continuity equations. Further derivation is identical to the one performed by Tremaine & Weinberg (1984), and it can be conducted for each tracer’s density separately, leading to two equations:

$$\Omega_B \sin i \int_{-\infty}^{+\infty} \Sigma_B(X, Y_{\text{dis}}) X \, dX$$

$$= \int_{-\infty}^{+\infty} \Sigma_B(X, Y_{\text{dis}}) V_{\text{los}}^B(X, Y_{\text{dis}}) \, dX,$$

(4)

$$\Omega_S \sin i \int_{-\infty}^{+\infty} \Sigma_S(X, Y_{\text{dis}}) X \, dX$$

$$= \int_{-\infty}^{+\infty} \Sigma_S(X, Y_{\text{dis}}) V_{\text{los}}^S(X, Y_{\text{dis}}) \, dX,$$

(5)

where $(X, Y_{\text{dis}})$ are coordinates on the sky ($X$ running parallel to the line of nodes and $Y_{\text{dis}}$ being the offset, perpendicular to the line of nodes, of the slit along which the integration is performed), $V_{\text{los}}$ is the observed line-of-sight velocity and $i$ is the inclination of the disc. Let the tracer be made of stellar-light emission; then one can define for each bar component, as well as for the whole galaxy, the luminosity–density

$$L_\odot = \int_{-\infty}^{+\infty} \Sigma_\odot(X, Y_{\text{dis}}) \, dX,$$

the luminosity centroid

$$X_\odot = \frac{1}{L_\odot} \int_{-\infty}^{+\infty} \Sigma_\odot(X, Y_{\text{dis}}) X \, dX$$

and the luminosity-weighted line-of-sight velocity

$$V_\odot = \frac{1}{L_\odot} \int_{-\infty}^{+\infty} \Sigma_\odot(X, Y_{\text{dis}}) V_{\text{los}}^\odot(X, Y_{\text{dis}}) \, dX$$

where the index $\odot$ is $B$ for the outer bar, $S$ for the inner bar and $\text{tot}$ for the whole galaxy. Obviously, only values with index $\text{tot}$ can be observed, for example, by placing a slit parallel to the line of nodes (Fig. 1), and deriving along it the values of luminosity, centroid and line-of-sight velocity, where all integrals include integration over the width of the slit. With the above definitions, the sum of (4) and (5) takes a form

$$\Omega_B \sin i F_B X_B + \Omega_S \sin i F_S X_S = F_B V_B + F_S V_S,$$

(6)

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of inclination $\Omega_1$, with the counter-rotating secondary bar. The line of nodes passing through the centre of the galaxy sets a division of this galaxy into four quadrants. Consider, for example, the case when the two bars lie in opposite quadrants of the galaxy (Fig. 1). This is the case of NGC 2950. Even if we cannot measure $X_B$ or $X_S$, we know that they are always of opposite signs, no matter how the slit is placed (parallel to the line of nodes). Then, if $\Omega_B$ and $\Omega_S$ are of the same sign, adding the inner bar should bring the sum on the left-hand side of (6) closer to zero (or even through zero for strong fast inner bars) when compared to the contribution of the outer bar alone. This implies that the observed $|V_{tot}|$ should be smaller when entering the region of the inner bar than that interpolated from the outer bar. To the contrary, CDA03 observe $|V_{tot}|$ increasing around this transition region in NGC 2950, consistent with $\Omega_S$ having the sign opposite to $\Omega_B$, i.e. with the counter-rotating secondary bar.

2.1 Immediate condition for the sign of $\Omega_S - \Omega_B$

In the above argument, no reference point has been fixed, since the sums on each side of (6) get modified by the introduction of the secondary bar. We do not know the contribution from the outer bar in the region where the inner bar is present, and interpolation can be misleading. Here, I show that (6) allows to tell whether $\Omega_S > \Omega_B$ or $\Omega_S < \Omega_B$. Equation (6) can be rewritten as

$$\Omega_B X_{tot} + \Omega_S F_B (\Omega_S - \Omega_B) = V_{tot}/\sin i.$$  (7)

This form shows how the relation between the integrals $V_{tot}$ and $X_{tot}$ gets modified by the presence of the secondary bar: the second term in the sum is a correction term. For slits outside the inner bar, this correction is zero (because $\Sigma_S$ is zero there), and for those slits the observed values of $X_{tot}$ and $V_{tot}$ should lie on a straight line of inclination $\Omega_S \sin i$. When the slit passes through the inner bar, $X_S \neq 0$. If for such slits $|V_{tot}|$ is larger than the values given by the linear relation (7) without the correction factor, then the correction has to have the same sign as $\Omega_B X_{tot}$. For the bars that lie in opposite quadrants, like in NGC 2950 (Fig. 1), $X_B$ and $X_S$ are of opposite signs, and $X_{tot}$ has the same sign as $X_B$ throughout the galaxy (fig. 3 in CDA03). Therefore, the two components of the sum in (7) can only be of the same sign when $\Omega_B$ has opposite sign to $\Omega_B$. If we take a convention that $\Omega_B > 0$ then $\Omega_S < \Omega_B$.

This argument can be extended to the case when $X_{tot}$ changes sign for slits passing through the secondary bar. In NGC 2950, the values of $X_{tot}$ for these slits are very small, which is consistent with the observed geometry of bars. A similar argument can be applied to galaxies with bars in the same quadrants, where $X_{tot}$ for innermost slits is not that small. In this case, $X_B$, $X_S$ and $X_{tot}$ are all of the same sign. If an increase of $|V_{tot}|$ is observed in the region of the inner bar, $\Omega_B$ must now have the same sign as $\Omega_S - \Omega_B$, which means $\Omega_B > \Omega_B$ for the assumed $\Omega_B > 0$. Thus, we see that the same increase of $|V_{tot}|$ in the region of the inner bar can indicate the inner bar rotating slower or faster than the outer bar, depending on the relative orientation of the bars.

2.2 Condition for counter-rotation

In some cases, an argument can be made about corotation or counter-rotation of the inner bar with respect to the inertial frame. It relies on the variation of the integrals in the Tremaine–Weinberg method with the distance of the slit from the centre of the galaxy, $Y_{tot}$. This argument can be made under an assumption that $|X_B F_B|$ does not increase when we march with the slit through the galaxy towards the line of nodes (in the direction marked by an arrow on the left-hand side of Fig. 1). This is a justified assumption, since $F_B$ decreases inwards for the very reason of the introduction of the secondary bar, and $|X_B|$ should not go up inwards, since early-type galaxies like NGC 2950 have ‘flat’ bars with a nearly constant surface brightness as a function of radius (Emlmegren et al. 1996).

Consider again equation (6). Normally, for slits that avoid the secondary bar, $|V_{tot}|$ decreases inwards (i.e. when shifting the slit in the direction given by the arrow in Fig. 1). This is also the case in NGC 2950 (fig. 3 in CDA03). If then $|V_{tot}|$ increases inwards when the slit reaches the secondary bar (like in NGC 2950), this can be caused by either of the two components of the sum on the left-hand side of (6). I already argued that the first component cannot be the cause, since $F_B$ decreases, $|X_B|$ is unlikely to increase and $\Omega_B = \text{const}$. The second component can only cause the increase of the sum when it is of the same sign as the first one. But again, $X_B$ and $X_S$ are of opposite sign. Therefore, $\Omega_S$ and $\Omega_B$ have to be of opposite sign, too.

One can repeat this reasoning for a galaxy with bars in the same quadrants to show that in that case $|V_{tot}|$ increasing in the region of the inner bar indicates that the inner bar is corotating in the inertial frame. Similar to Section 2.1, the conclusion about the sense of rotation of the inner bar depends on the relative orientation of the bars.

3 INTEGRALS IN THE TREMAINE–WEINBERG METHOD

CALCULATED FOR A REALISTIC MODEL OF A DOUBLY BARRED GALAXY

Regular motion of a particle in a potential of a doubly barred galaxy has two frequencies associated with it, each related to one of the bars, in addition to the frequency of its free oscillations (Maciejewski & Sparke 1997). In the linear approximation, this motion corresponds
to epicyclic oscillations with these frequencies around the guiding radius. Particles with the same guiding radii are bound to closed curves (loops) that oscillate in the pulsating potential of double bars. Loops are also observed in non-linear analysis (Maciejewski & Sparke 2000; Maciejewski and Athanassoula, in preparation).

The orbital approach directly indicates that the motion of a particle associated with one bar has always a component coming from the other bar. Amplitudes of the oscillations can be easily evaluated in the linear approximation (Maciejewski 2003), giving particle’s position and velocity at each relative position of the bars. In Fig. 2, I plot the loops populated by particles moving in the potential of a doubly barred galaxy constructed by Maciejewski & Sparke (2000; Model 2). This is a realistic potential, since it admits orbits that support the outer bar, as well as orbits supporting the inner bar, throughout the extent of each bar. The two bars in this model rotate in the same direction with pattern speeds $\Omega_2 = 35$ km s$^{-1}$ kpc$^{-1}$ and $\Omega_3 = 110$ km s$^{-1}$ kpc$^{-1}$. I plot the loops for two relative orientations of the bars: the bars parallel (Fig. 2, left-hand side), and the bars at the angle of $5\pi/8$ rad, the value similar to that observed in NGC 2950 after deprojection (Fig. 2, right-hand side). The loops change shapes as the bars rotate one with respect to another, and loops associated with one bar intersect the loops associated with the other bar. Thus, formally one cannot perform here the separation of tracer’s density postulated in (3).

However, changes in the shapes of the bars, and zones where both tracers coexist, may be small, and then, what formally prohibits the separation, may turn into a higher order correction to it (see also Appendix A). In order to check whether it is the case for the realistic Model 2 (Maciejewski & Sparke 2000), I calculated the positions and velocities of some $10^5$ points on 200 loops in that model, from which I obtained the centroids $X_{\text{tot}}$ and the line-of-sight velocities $V_{\text{tot}}$, as a function of the same offset of the slit from the galaxy centre, $Y_{\text{slit}}$. When the inner bar is placed at the position similar to the one observed in NGC 2950, the value of $X_{\text{tot}}$ (solid line) is closer to zero in the region of the inner bar ($|Y_{\text{slit}}| < 1$), than when the inner bar is absent (dotted line). However, $X_{\text{tot}}$ does not change the sign in the region of the inner bar, which is also observed in NGC 2950 (Fig. 3 in CDA03). This indicates that the light integrated along a slit passing through the inner bar is still dominated by the outer bar. In the case of the two bars parallel (Fig. 2, left-hand panel) there is no such decrease of $X_{\text{tot}}$ in the region of the inner bar (dashed line in Fig. 3b).

In Fig. 3(b), I plot the value of the centroid, $X_{\text{tot}}$, as a function of the same offset of the slit from the galaxy centre, $Y_{\text{slit}}$, as in Fig. 3(b). When the inner bar, rotating in the same direction as the outer bar, but faster, is placed at the position similar to the one observed in NGC 2950, the value of $V_{\text{tot}}$ (solid line) rapidly approaches zero in the region of the inner bar ($|Y_{\text{slit}}| < 1$), and in most of this region it has the sign opposite to the value of $V_{\text{tot}}$ for slits not passing through the inner bar ($|Y_{\text{slit}}| > 1$). This behaviour of $V_{\text{tot}}$ is opposite to that observed in NGC 2950 by CDA03. For the model of galaxy with $\Omega_2 > \Omega_3$, changes of $V_{\text{tot}}$ with $Y_{\text{slit}}$ observed by CDA03 can only be reproduced when both bars lie in the same quadrants of the galaxy. For the particular case of bars parallel, presented here, $|V_{\text{tot}}|$ increases in the region of the inner bar (dashed line in Fig. 3a), relative to the case when there is no inner bar (dotted-dashed line). This dependence of deviations in $V_{\text{tot}}$ on the relative position of the two bars is exactly as expected from the simple extension of the Tremaine–Weinberg method derived Section 2, based on the separation of tracer’s density (3). Namely, if the inner bar rotates faster than the outer bar then, when both bars are in the same quadrants (defined by the line of nodes and the centre of the galaxy), $|V_{\text{tot}}|$ increases in the slits passing through the inner bar, but it decreases, when the bars are in the opposite quadrants.

The plot of $V_{\text{tot}}$ as a function of $X_{\text{tot}}$ is presented in Fig. 3(c). One may attempt to fit a second straight line in the region of the inner bar, but this fit will not yield the actual pattern speed of the inner bar. In Fig. 3(d), I plot the ratios of $V_{\text{tot}}/X_{\text{tot}}$ as estimators of the derived $\Omega_{\text{pattern}}$. For a single bar (dotted-dashed line), there is one pattern speed independent of the offset of the slit, and equal to the one assumed in the model. If there are two pattern speeds in the system, the inner bar does not significantly alter the pattern speed.
Figure 3. Values of the integrals in the Tremaine–Weinberg method, calculated from the epicyclic formulae for the positions and velocities of particles in the realistic model of doubly barred galaxy from Fig. 2. The dot–dashed line corresponds to the model without the inner bar, the dashed line is for the model with two bars aligned, as in the left-hand panel of Fig. 2, while the solid line is for the model with two bars located as in the right-hand panel of Fig. 2. This figure can be directly compared with Fig. 3 in CDA03. (a) The kinematic integrals $V_{\text{tot}}$, defined in Section 2, as a function of the slit offset $Y_{\text{slit}}$ with respect to the centre of the galaxy. (b) The photometric integrals $X_{\text{tot}}$, defined in Section 2, as a function of the slit offset $Y_{\text{slit}}$. (c) $V_{\text{tot}}$ as a function of $X_{\text{tot}}$. (d) Pattern speed in the model, $\Omega_{\text{pattern}} = V_{\text{tot}}/X_{\text{tot}}$, derived separately for each position of the slit $Y_{\text{slit}}$. However, the induced deviations are systematic: pattern speed of the outer bar is spuriously increased when the bars lie in the same quadrants, and decreased when they lie in quadrants opposite. The effect is small though, below 5 per cent. For the slits passing through the inner bar, the derived $\Omega_{\text{pattern}}$ rapidly decreases if the bars lie in opposite quadrants (solid line), and it becomes negative, not giving any information about the pattern speed of the inner bar. If the bars lie in the same quadrants (dashed line), the derived $\Omega_{\text{pattern}}$ is larger than $\Omega_B$ for the slits passing through the inner bar, but it varies with $Y_{\text{slit}}$, and never reaches the value of $\Omega_S = 110 \text{ km s}^{-1} \text{ kpc}^{-1}$ assumed in the model, remaining at slightly over half of that value.

4 A METHOD TO DERIVE THE VALUE OF $\Omega_S$

The main goal of this paper is to show that with a simple extension of the Tremaine–Weinberg method to multiple pattern speeds, one can derive at least rough qualitative information about the secondary pattern rotation. For the data on NGC 2950 presented by CDA03, this information is that the inner bar rotates in the opposite direction than the outer bar. On the other hand, getting the numerical value of the pattern speed of the secondary bar, $\Omega_S$, may not be possible without making additional assumptions. Here, I analyse one possible method to calculate $\Omega_S$.

If the tracer of the secondary bar does not extend beyond some radius $R_0$ in the galaxy plane (outlined in projection by the dotted ellipse in Fig. 1), then (5) can be rewritten as

$$\Omega_S \sin i \int_{-X_0}^{X_0} \Sigma_S(X, Y_{\text{slit}}) X \, dX = \int_{-X_0}^{X_0} \Sigma_S(X, Y_{\text{slit}}) V_{\text{los}}(X, Y_{\text{slit}}) \, dX, \quad (8)$$

where $R_0^2 = (Y_{\text{slit}}/\cos i)^2 + X_0^2$. Summing (4) and (8) leads to

$$\Omega_B X_{\text{tot}} + \frac{\Omega_S - \Omega_B}{L_{\text{tot}}} \int_{-X_0}^{X_0} \Sigma_S(X, Y_{\text{slit}}) X \, dX = \frac{V_{\text{tot}}}{\sin i}. \quad (9)$$

However, aside for $\Omega_S$, we still do not know the integral in (9). It can be approximated when one assumes that only tracers of the inner bar and of the axisymmetric component are present inside the dotted ellipse in Fig. 1, i.e. that the outer bar is almost axisymmetric in this region. Note that this is a strong and poorly founded assumption. However, if we take it, and since the contribution of the axisymmetric component to the integral in (9) cancels out, one can substitute there the observable $\Sigma_{\text{tot}}$ for the unknown $\Sigma_S$, and rewrite (9) as a linear regression of $\Omega_S - \Omega_B$:

$$\frac{\Omega_S - \Omega_B}{L_{\text{tot}}} \int_{-X_0}^{X_0} \Sigma_{\text{tot}}(X, Y_{\text{slit}}) X \, dX = \frac{V_{\text{tot}}}{\sin i} - \Omega_B X_{\text{tot}}. \quad (10)$$
Thus, $\Omega_S - \Omega_L$ can be obtained as a slope of a straight line fitted to the data from slits passing through the inner bar. A similar equation has been used by CDA03 to estimate $\Omega_S$, but here the coefficient at $\Omega_S - \Omega_L$ is defined differently.

I tested equation (10) on the model examined in Section 3, for the position angles of the bars like the ones observed in NGC 2950. As in Section 3, in the model I use deprojected velocities, which replaces $V_{\text{tot}}$ with $V_{\text{tot}}/\sin i$ by $V_{\text{tot}}$, and I measure $Y_{\text{slit}}$ in the plane of the galaxy. On the basis of the shapes of the loops, I chose $R_0 = 1.1$ kpc for this model, and fixed $\Omega_L = 35 \text{ km s}^{-1} \text{ kpc}^{-1}$. In Fig. 4, I plot the data points for the regression (10). They follow a straight line well, except for the points around zero values. However, these points are plotted against $V_{\text{tot}} - \Omega_B X_{\text{tot}}$.

In order to apply the same method to the observed data on NGC 2950 from CDA03, the value of $R_0$ can be estimated by checking how the integral in (10) changes with varying $R_0$ – it should have a plateau in the transition area between the bars. I used the $R$-band image of NGC 2950 (Erwin & Sparke 2003) to calculate this integral and $L_{\text{tot}}$ for each of the slits passing through the secondary bar, placed at positions reported by CDA03. The values of $\int_{X_0}^{X_0} \Sigma_{\text{tot}}(X, Y_{\text{slit}})X dX$ as a function of $R_0$ are plotted in Fig. 5. All curves have indeed a plateau around the same $R_0$ of 5–6 arcsec. It is located just outside a nuclear stellar ring at $\sim 4.2 \text{ arcsec}$, reported by Erwin & Sparke (2003), which is likely circular in the plane of the galaxy. Thus, the method proposed here may be particularly well suited for NGC 2950.

Each curve in Fig. 5 has a characteristic U shape, whose depth depends on the relative contribution of the inner bar to the integral in (10), and therefore it vanishes outside the inner bar, and also very close to the line of nodes, where the integral tends to be zero. From the $R$-band image, the depth is largest for slits offset by $\sim 1 \text{ arcsec}$. The linear fit to (10), with the data for $V_{\text{tot}}$ and $X_{\text{tot}}$ from each slit, passing through the inner bar provided by CDA03, reveals a large negative value of $\Omega_S - \Omega_L$, about $-150 \pm 50 \text{ km s}^{-1} \text{ arcsec}^{-1}$. For $\Omega_L \approx 11 \text{ km s}^{-1} \text{ arcsec}^{-1}$, measured by CDA03, this gives $\Omega_S \approx -140 \pm 50 \text{ km s}^{-1} \text{ arcsec}^{-1}$. This result, although with much poorer grounds than the qualitative arguments from Section 2, reaffirms counter-rotation of the inner bar. Note that the unrealistically large value of $\Omega_S$ derived with this method is likely an overestimate: In order to get it, the integral $\int_{X_0}^{X_0} \Sigma_{\text{tot}}(X, Y_{\text{slit}})X dX$ has been substituted for $\int_{X_0}^{X_0} \Sigma_{\text{tot}}(X, Y_{\text{slit}})X dX$. The substituted integral is most likely smaller than the original one, because the contributions of the two bars are of opposite signs.

5 DISCUSSION

The original Tremaine–Weinberg method, and its extension proposed here, are based on the continuity equation applied to a tracer moving in the gravitational field of the galaxy. If old stars are taken as the tracer, the continuity equation is well satisfied globally. If tracers associated with each bar are separated as proposed in (2), continuity of each tracer can only be violated when there is a net flux of stars from one bar to the other, consistent over many rotations of the bars. This corresponds to a secular strengthening of one bar at the cost of the other, hence such a system is no longer periodic. However, if the mass transfer is slow, the analysis presented in this paper is still applicable. This can be supported by an argument similar to that presented in Appendix A, but for densities of the bars monotonically changing. Moreover, studies of orbits in self-consistent models of double bars show that mass transfer between the bars is likely to be small, because orbits that are trapped and oscillate around one or the other bar, populate large fraction of phase space (Maciejewski & Sparke 2000; Maciejewski and Athanassoula, in preparation).

Note that the separation of tracers in (2) is only formal, and (2) applies to any system that is periodic with period $P$. However, in order to follow the Tremaine–Weinberg formalism for each tracer, one has to approximate each tracer as a solid-body rotator, which leads...
Viceful way of detecting counter-rotation in galaxies with nested bars. Thus far, all detections of counter-rotation in galaxies come directly from the observed velocity fields obtained with the long-slit (e.g. Kuijken, Fisher & Merrifield 1996) or integral-field (e.g. Emsellem et al. 2004) spectroscopy. However, if the counter-rotating population is small, it may not be recognized with those methods. On the other hand, the Tremaine–Weinberg method goes beyond the raw observed velocity field by finding integrals that are useful in detecting a rotating pattern. In the presence of rotating patterns, counter-rotation should be spotted more easily with this method.

The use of this method, like of the original Tremaine–Weinberg method, is limited to galaxies with bars considerably inclined to both the major and the minor axes of the disc. NGC 2950 fulfils this requirement particularly well, but there are a number of other galaxies to which this method can be readily applied. For example, NGC 3368, 3941, 5365, 5850 and 6684 have the position angles of both bars separated by $20^\circ - 70^\circ$ from the position angle of the line of nodes [see the catalogue by Erwin (2004) for the parameters]. The bar inside the oval in NGC 3081 has also parameters favourable for this method. In addition, a modification of the Tremaine–Weinberg method along the lines proposed in Section 4 can be applied to the interiors of nuclear rings that often host nuclear bars (e.g. NGC 1097, 6782).

The fraction of counter-rotating inner bars can constrain theories of galaxy formation and evolution. Currently the most accepted view on the origin of the inner bar is that it forms through instabilities in gas inflowing along the outer bar (Shlosman, Frank & Begelman 1989). However, if inner bars form on early stages of galaxy assembling, and outer bars form from material that settled on the galaxy later, the spins of the two bars may be unrelated, leading to a large fraction of counter-rotating inner bars. Determining this fraction may help to distinguish between these two evolutionary scenarios.

6 CONCLUSIONS

In this paper, I presented an argument that a simple extension of the Tremaine–Weinberg method to multiple pattern speeds can provide an information about the sense of rotation of the inner bar in doubly barred galaxies. The extended formula advocated here links the relative position of the bars to the sign of rotation of the inner bar. This extension cannot be as rigorous as the original method, because it assumes that the patterns do not change as they rotate one through another, which is not true in general. However, it predicts the same deviations of the integrals in the Tremaine–Weinberg method from their values for the single rotating pattern, as in the orbital model of a realistic doubly barred galaxy, which does not involve this assumption. This indicates that the degree of change that the patterns rotating one through another undergo is small, and that it does not significantly affect the results of the extended method proposed here.

Application of the extended method to NGC 2950 implies that the inner bar there counter-rotates with respect to the outer bar and to the large-scale disc. Since a retrograde bar is unlikely to be supported in a prograde disc, a retrograde inner disc may be hiding in the central kiloparsecs of NGC 2950.

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APPENDIX A: CORRECTION TERMS IN TREMAINE–WEINBERG INTEGRALS FOR A PULSATING FERRERS’ BAR

Consider a Ferrers’ bar with major and minor axes a and b. Its surface density can be written in the Cartesian coordinates (x, y) in the plane of the galaxy as
\[
\Sigma(x, y) = \begin{cases} 
\Sigma_{OB} \left( 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right) & \text{if } \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \\
0 & \text{if } \frac{x^2}{a^2} + \frac{y^2}{b^2} > 1,
\end{cases}
\]
where \(\Sigma_{OB}\) is the central density of the bar. Pulsation of the bar can be described by periodic variation of the length of its axes
\[
a^2(t) = a_0^2 / g_1(t \mid P), \quad b^2(t) = b_0^2 / g_2(t \mid P),
\]
where \(g_1\) and \(g_2\) are periodic functions of time with period \(P\). Thus, non-zero density, corresponding to the top line in (A1), of a pulsating bar that rotates with pattern speed \(\Omega_b\), can be written in polar coordinates \((R, \phi)\) as
\[
\Sigma(x, y, t) = \Sigma_b (R, \phi - \Omega_b t, t) = \Sigma_{OB} \times \left[ 1 - f_1(R, \phi - \Omega_b t, t) / g_1(t \mid P) - f_2(R, \phi - \Omega_b t, t) / g_2(t \mid P) \right],
\]
where
\[
f_1(R, \phi) = \frac{r^2 \cos^2 \phi}{a_0^2}, \quad f_2(R, \phi) = \frac{r^2 \sin^2 \phi}{b_0^2}.
\]
Note that (A2) is one of the components of (2) for the particular case of a Ferrers’ bar.

Let (A2), wherever larger than zero, describe non-zero surface density of a tracer in a rotating and pulsating bar. In the Tremaine–Weinberg method, the time derivative of this surface density enters the continuity equation. Simple partial derivation of (A2) gives
\[
\frac{\partial \Sigma}{\partial t} = -\Omega_b \frac{\partial \Sigma}{\partial \phi} - \Sigma_{OB} \frac{\partial f_1}{\partial t} g_1(t \mid P) - \Sigma_{OB} \frac{\partial f_2}{\partial t} g_2(t \mid P).
\]
Further on in the Tremaine–Weinberg method, the continuity equation is integrated over \(x\) and \(y\), which in the case considered here leads to
\[
\Omega_b \int_{-\infty}^{\infty} \Sigma(x, y) \, dx \, dy - \Sigma_{OB} \frac{\partial g_1}{\partial t} \int_{-\infty}^{\infty} f_1 \, dx \, dy
\]
where \(v_r\) is the velocity of the tracer along the y axis. Equation (A4) is the counterpart of (4) and (5), and since bar’s pulsation is explicitly included here, it contains two additional correction terms (second and third term), involving integration over \(f_1\) and \(f_2\). This integration, although formally extending to infinity, in this case is limited to the regions where bar’s surface density is non-zero. Thus, since \(f_1\) and \(f_2\) contain only rotated Cartesian coordinates, the integrals that involve them are finite, and will be denoted as \(I_{1B}\) and \(I_{2B}\). Moreover, let us represent the pulsation of the bar by a simple oscillatory form of \(g_1\) and \(g_2\):
\[
g_{1/2} = 1 \pm \epsilon g \sin \omega t, \quad \epsilon g, \quad A_5
\]
where \(\omega = 2\pi / P\), and \(\epsilon g\) controls the amplitude of the oscillation. Then, with the use of notation from Section 2, (A4) takes the form
\[
\Omega_b L_{B} X_{B} - \epsilon g \omega \cos \omega t \, \Sigma_{OB}(I_{1B} - I_{2B}) = L_{B} V_B / \sin i.
\]
Similar exercise can be done for the second bar, indexed by \(S\), leading to an equation being a counterpart to (A6)
\[
\Omega_s L_{S} X_{S} - \epsilon s \omega \cos \omega t \, \Sigma_{OB}(I_{1S} - I_{2S}) = L_{S} V_S / \sin i.
\]
The sum of these two equations gives (6), the equation of the extended Tremaine–Weinberg method, but here with correction terms, one for each bar. In both (A6) and (A7), the correction term is the second term.

For each bar, the integrals \(I_1\) and \(I_2\) can be evaluated explicitly, indicating that \(\Sigma_{OB}(I_{1B} - I_{2B}) \) is about twice smaller than \(\Sigma_{OB} L_{B} S\) throughout each bar. Numerical simulations indicate that the overall pulsation of the inner bar is more notable than that of the outer bar, hence the magnitude of the coefficient \(\epsilon\) should be larger for the inner bar. In the numerical N-body simulations by Rautiainen et al. (2002), the axial ratio of the inner bar, \(b/a\), varies roughly between 0.52 and 0.72. In the notation adopted here, this corresponds to \(\epsilon_s = 0.16\). Finally, \(\omega = 2(\Omega_s - \Omega_b)\) is always smaller than \(2\Omega_s\). Thus, the magnitude of the second, correction term in (A7) should not exceed about 15 per cent of the magnitude of the first, leading term in the case of the inner bar. For the outer bar, the ratio of \(\omega / \Omega_b\) is larger, but since the pulsation of that bar has smaller amplitude, \(\epsilon_B\) is significantly smaller.

This example shows that periodic changes in time of surface density of the tracer of each bar in this bar’s reference frame can be accommodated as correction terms in the extended Tremaine–Weinberg formalism proposed in this paper, given that the amplitude of these changes is sufficiently small. This amplitude, as observed in numerical simulations, is indeed small enough, and it leads to correction terms that are one order of magnitude smaller than the leading terms in the extended Tremaine–Weinberg equation (6).

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