Precision calculations in the MSSM
Higgs-boson sector with FeynHiggs 2.14

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Abstract

We present an overview of the status and recent developments of FeynHiggs (current version: 2.14.3) since version 2.12.2. The main purpose of FeynHiggs is the calculation of the Higgs-boson masses and other physical observables in the MSSM. For a precise prediction of the Higgs-boson masses for low and high SUSY scales, state-of-the-art fixed-order and effective-field-theory calculations are combined. We first discuss improvements of the fixed-order calculation, namely an optional DR renormalization of the stop sector and a renormalization of the Higgs sector ensuring the chosen input mass to be equivalent with the corresponding physical mass. Second, we describe improvements of the EFT calculation, i.e. an implementation of non-degenerate threshold corrections as well as an interpolation for complex parameters. Lastly, we highlight some improvements of the code structure easing future extensions of FeynHiggs to models beyond the MSSM.

Keywords: MSSM, Higgs-boson observables;

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Preprint submitted to Computer Physics Communications March 2, 2022
New version program summary

Program Title: FeynHiggs

Licensing provisions: GPLv3

Programming language: Fortran, C, Mathematica

Journal reference of previous version: Comput. Phys. Comm. 180 (2009) 1426

Does the new version supersede the previous version? Yes.

Reasons for the new version: Improved calculations and code structure.

Summary of revisions: Apart from improvements discussed in other publications: implementation of optional DR renormalization of stop sector, adapted two-loop Higgs sector renormalization, implementation of full non-degenerate threshold corrections, interpolation of EFT calculation for complex parameters, better code structure.

Nature of problem: The Minimal Supersymmetric Standard Model (MSSM) allows predictions for the masses and mixings of the Higgs bosons in terms of a few relevant parameters. Therefore, comparisons to experimental data provide constraints on the parameter space. To fully profit from the experimental precision, a comparable level of precision is needed for the theoretical prediction.

Solution method: State-of-the-art fixed-order and effective-field-theory calculations are combined to obtain a precise prediction for small as well as large supersymmetry scales.
1. Introduction

While the gauge sector of the Standard Model (SM) is well investigated, the experimental precision in the Higgs sector [1–4] leaves significant room for physics beyond the SM (BSM). One of the most frequently discussed BSM theories containing an extended Higgs sector is the Minimal Supersymmetric Standard Model (MSSM) [5–8] based upon the concept of supersymmetry (SUSY). Apart from adding a superpartner to every SM degree of freedom, the MSSM also introduces a second Higgs doublet resulting in five physical Higgs bosons: at the tree level, these are the $\mathcal{CP}$-even $h$ and $H$ bosons, the $\mathcal{CP}$-odd $A$ boson as well as the charged $H^\pm$ bosons. Owing to the underlying supersymmetry, the Higgs sector is determined by only two additional non-SM parameters at the tree level. Conventionally, they are chosen as the ratio of the vacuum expectation values (vevs) of the two doublets, $\tan\beta = v_2/v_1$, and the mass of the $A$-boson, $M_A$ (or the mass of the charged bosons, $M_{H^\pm}$).

Consequently, the Higgs sector of the MSSM is highly predictive, i.e. the increasingly precise measurements of the properties of the Higgs boson discovered by the ATLAS and CMS collaborations [1,2] at the Large Hadron Collider (LHC) allow one to efficiently probe the parameter space of the MSSM by comparing the high-precision measurements of Higgs-boson properties to the corresponding theoretical predictions.
The most precisely measured quantity of the Higgs boson is its mass. In order to perform a meaningful comparison between the measured value and the theoretical prediction, it is essential to take into account radiative corrections to the theory predictions, since these have a large impact on the Higgs sector of the MSSM. To obtain a prediction with an uncertainty comparable to the experimental precision, much work has been dedicated to the calculation of these corrections (for the MSSM with real parameters see [9–70], for the MSSM with complex parameters see [45,51,71–85]).

Apart from tackling the actual calculations, there has also been a major effort to make the results publicly available by providing them in terms of easily usable computer programs.

One such program is FeynHiggs [28,40,54,62,65,79,86,87], which is available at http://feynhiggs.de. Its main purpose is the calculation of the Higgs-boson masses in the MSSM. In addition, it provides precise predictions for various other phenomenologically relevant observables. It has become a standard tool that is used for instance by the LHC Higgs Cross-Section Working Group [88,89] for the calculation of the MSSM Higgs boson masses and branching ratios. For the calculation of the Higgs boson masses a combined approach of fixed-order Feynman-diagrammatic and effective-field-theory (EFT) calculations is employed. In this paper, we describe recent updates of the code released in versions 2.13.0 through 2.14.3. These mainly improve the calculation of the Higgs boson masses and self-energy corrections, which also enter the calculation of other observables like e.g. the Higgs branching ratios. We will therefore focus on the evaluation of these corrections in the present work. 1 The improvements affect the diagrammatic calculation (see Section 3) as well as the EFT calculation (see Section 4). Moreover, we review the observables calculated by FeynHiggs. We discuss improvements of the code structure (see Section 5) and give a short introduction to FeynHiggs and how to use it (see Section 2). In the recent version FeynHiggs 2.14.0, also the pole-mass-determination procedure was improved. This issue has been discussed in a separate publication [96].

2. FeynHiggs overview

In this section we give an overview about the various observables evaluated by FeynHiggs, together with the relevant references. The main emphasis is put on the Higgs-boson mass and self-energy calculations, which were in the focus of the updates of the recent years.

2.1. Higgs boson mass spectrum

One of the main purposes of FeynHiggs is to provide predictions for the Higgs-boson masses in the MSSM. The most direct approach is to calculate higher-order corrections

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1 A detailed comparison with other codes that evaluate the Higgs boson mass spectrum [60,91,92] or that can use the spectrum calculated by FeynHiggs to obtain the Higgs decay widths or production cross-sections [93,95] is beyond the scope of this paper.
to the propagators of the Higgs bosons performing a fixed-order Feynman-diagrammatic
calculation. FeynHiggs was originally developed around this approach: It incorporates
full one-loop contributions \([17,20,23]\) as well as the leading two-loop contributions\(^2\)
of \(\mathcal{O}(\alpha_t \alpha_s, \alpha_t \alpha_s, \alpha_t^2, \alpha_t \alpha_b, \alpha_b^2)\) \([28,31,36,37,40,42,47,79,82,86,87,97,98]\) to the Higgs
two-point functions. For these corrections, a mixed OS/DR renormalization scheme
is employed (see \([79]\) for more details). The bottom-type Yukawa couplings include a
resummation of the \(\tan \beta\)-enhanced terms (the "\(\Delta_b\) corrections") \([99–103]\) as detailed
in \([37,42,47]\) (see also \([104–106]\) for corresponding next-to-leading order (NLO) contributions). The diagrammatic calculation allows one to take into account complex
parameters fully at the one-loop level \([79]\) and at \(\mathcal{O}(\alpha_t \alpha_s, \alpha_t^2)\) \([80,82,98]\) at the two-
loop level (the phase dependences of the other two-loop corrections are interpolated).
Moreover, non-minimal flavour violation can be considered at the one-loop level \([107–109]\).

The diagrammatic calculation captures all contributions at a given order. This
result contains logarithms involving some SUSY mass divided by the mass of a SM
particle. For relatively low SUSY scales, these logarithms are small and the fixed-order
calculation is therefore expected to be precise. For a large separation between the
SUSY scale and the electroweak scale, however, these logarithms become large. Thus,
they can spoil the convergence of the perturbative expansion, rendering the fixed-order
calculation inaccurate.

Effective-field-theory (EFT) techniques provide a tool to resum these large loga-
rithmic contributions to all orders \([55,57,59,61,64,69,70,110]\). The main idea is to
integrate out some or all heavy SUSY particles at a high scale. The effective
couplings are then evolved down to the electroweak scale at which the Higgs mass (or
masses) are calculated, effectively resumming all large logarithms that emerged from
the masses of the heavy SUSY particles. A state-of-the-art EFT calculation is available
in FeynHiggs \([54,62,65]\): based upon the results of \([57,59,64]\), it includes full resum-
mation of leading and next-to-leading logarithms (NLL) as well as \(\mathcal{O}(\alpha_s, \alpha_t)\) resum-
mation of next-to-next-to-leading logarithms (NNLL). Moreover, it allows one to take
into account light electroweakinos and gluinos by implementing the corresponding low-
energy thresholds and RGEs. This logarithmic accuracy level ensures a high precision
for high SUSY scales. However, since no higher-dimensional operators are included in
the EFT calculation, terms suppressed by the SUSY scale are missed (see the discussion
in \([64]\)). Therefore, the EFT calculation can become inaccurate for low SUSY scales.

In order to ensure a precise prediction for low, intermediary, and high SUSY scales,
the fixed-order approach and the EFT approach are combined in FeynHiggs \([54,62,65,69]\).
This is achieved by adding the resummed logarithms obtained in the EFT approach

\(^2\)The two-loop self-energy corrections are computed in the approximation of vanishing electroweak
gauge couplings and vanishing external momentum (see however \([50,60,85]\) for studies going beyond
this approximation).
to the self-energies obtained in the fixed-order approach and removing the double-counted logarithms by subtraction terms.

Finally, the renormalized self-energies, $\hat{\Sigma}$, supplemented by the resummed logarithms are used to obtain the pole masses of the Higgs bosons. For the neutral Higgs bosons this means that one has to find the poles of the propagator matrix, whose inverse is given by

$$\hat{\Gamma}_{hHA}(p^2) = \left[ p^2 1 - \begin{pmatrix} m_h^2 & 0 & 0 \\ 0 & m_H^2 & 0 \\ 0 & 0 & m_A^2 \end{pmatrix} \right] + \begin{pmatrix} \hat{\Sigma}_{hh}(p^2) + \Delta_{hh}^{\text{logs}} & \hat{\Sigma}_{hH}(p^2) + \Delta_{hH}^{\text{logs}} & \hat{\Sigma}_{hA}(p^2) \\ \hat{\Sigma}_{hH}(p^2) + \Delta_{hH}^{\text{logs}} & \hat{\Sigma}_{HH}(p^2) + \Delta_{HH}^{\text{logs}} & \hat{\Sigma}_{HA}(p^2) \\ \hat{\Sigma}_{hA}(p^2) & \hat{\Sigma}_{HA}(p^2) & \hat{\Sigma}_{AA}(p^2) \end{pmatrix}. \tag{1}$$

The mixing with the neutral Goldstone boson and the $Z$ boson yields subleading two-loop contributions to the mass predictions and is therefore neglected. The $\Delta$-terms contain the resummed logarithms, obtained in the EFT approach, as well as the corresponding subtraction terms. If all input parameters are real, $\hat{\Sigma}_{hA}$ and $\hat{\Sigma}_{HA}$ vanish, and the $(3 \times 3)$ mixing is reduced to a $(2 \times 2)$ mixing.

The real parts of the complex poles yield the physical Higgs-boson masses. The masses are conventionally labelled as $M_{h_i}$ ($i = 1, 2, 3$) in the case of $(3 \times 3)$ mixing, and as $M_h$, $M_H$ and $M_A$ in the case of $(2 \times 2)$ mixing.

In order to treat external Higgs bosons on-shell (e.g. in decay rates), the (non-unitary) $Z$-matrix is calculated. \cite{17,79,111–113} (see also \cite{114} and Sect. 5.3 of \cite{115}). It relates the tree-level mass eigenstates to the external physical states. Also an approximated form of the $Z$-matrix is given in the output, the $U$-matrix. It is by default defined as the unitary matrix diagonalizing the inverse propagator matrix, Eq. (1), in the approximation of vanishing momentum \cite{111,116} and is used to obtain effective couplings.

FeynHiggs furthermore provides an estimate of the remaining theoretical uncertainties from unknown higher-order corrections for all Higgs boson masses, for the $Z$-matrix, and for the $U$-matrix \cite{40}.

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The resummation of large logarithms is so far restricted to the $hh$, $hH$ and $HH$ self-energies. In the case of the SM as low-energy EFT, the resummation of logarithms in the $hH$ and $HH$ self-energies is approximated by assuming that the bulk of the correction originates from the top/stop sector. The coupling of the $H$ boson to top quarks is suppressed by $\tan \beta$ in the limit of large $M_A$. Therefore, the corrections to the $hH$ and $HH$ self-energies are obtained by dividing the correction to the $hh$ self-energy by $\tan \beta$ and $\tan^2 \beta$, respectively (see \cite{51,69} for more details). The region of high $\tan \beta$ and low $M_A$, where the accuracy of this approximation is questionable, is already tightly constrained by experimental searches for heavy Higgs bosons.
2.2. Other observables

In the following we list further (pseudo-)observables that are evaluated by FeynHiggs.

The calculated Higgs masses and the \( Z \)-matrix are used as input for the prediction of various other observables in the MSSM.

The implemented decay widths are summarized in Tab. [1]. The NLO QCD corrections to the decays to massless gauge bosons are implemented in the heavy (s)quark limit. For the decays into massive vector bosons, the phrase “reweighting of SM results” refers to rescaling the SM result with the relevant coupling of the considered MSSM Higgs boson.

Furthermore, approximations (for fast evaluation)—making use of tabulated SM results—of the main Higgs production cross-sections for given LHC energies and PDF sets are part of FeynHiggs, see Tab. [2]. In this Table, the phrase “reweighting of SM results” refers to taking the SM cross section (for the given value of the Higgs boson mass) and rescale it with the relevant coupling of the considered MSSM Higgs boson, see [121] for more details. Information about the “c-factor” of the \( gg \) production cross section can be found in [122] (and references therein). The “k-factor” method applies higher-order k-factors to the squared amplitude, taken from [123, 124]. “Reweighting of THDM results” refers to the application of the \( \Delta_b \) corrections to the bottom Yukawa coupling in the Two-Higgs-Doublet-Model cross section, which is given in type II as a function of \( M_{H^\pm} \) and \( \tan \beta \). More details about the various cross sections can be found in the references given in Tab. [2]. Moreover, the output contains a list of effective Higgs-boson couplings.

In order to test the parameter space we also evaluate several (pseudo-)observables that are connected to the Higgs-boson sector only via higher-order corrections. In Tab. [3] we list the included electroweak precision observables. The SUSY corrections to \( \Delta \rho \) include full one-loop corrections, two-loop SUSY-QCD corrections from gluons and gluinos as well as leading two-loop electroweak corrections. The leading two-loop SUSY corrections to \( \Delta r \) (and thus to \( M_W \)) and \( \sin \theta_W^{\text{eff,lept}} \) are incorporated via the \( \rho \)-parameter. \( M_W \) is calculated from \( \Delta r \) including full one-loop corrections and the SUSY two-loop corrections in terms of \( \Delta \rho \). For the calculation of \( \sin \theta_W^{\text{eff,lept}} \) only the one-loop SUSY corrections through \( \Delta r \) and the two-loop SUSY corrections through \( \Delta \rho \) are taken into account. From the SM side, the predictions for \( \Delta r \), \( M_W \) and \( \sin \theta_W^{\text{eff,lept}} \) contain all higher-order corrections currently known (for more details see [125]). “Partial 2L” in the \( (g - 2)_\mu \) and the EDMs predictions refers to the leading two-loop corrections. Details can be found in the given literature.

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4 The references focus on the corrections actually implemented into the code, but do not reflect the full status of the field of the corresponding available higher-order corrections. Reviews of Higgs boson production and decay, electroweak precision observables, EDM constraints and flavour constraints in the MSSM can be found in [117], [48], [118] and [119], respectively.

5 Various refinements to some of these decays, discussed in [120], will soon be implemented in FeynHiggs.
| decay width / branching ratio | precision level | references |
|-------------------------------|----------------|-----------|
| $h_i \rightarrow \gamma \gamma, gg$ | LO + NLO QCD | [126-129] |
| $h_i \rightarrow \gamma Z$ | LO | – |
| $h_i \rightarrow ZZ, W^± W^±$ | reweighting of SM result | [130-131] |
| $h_i \rightarrow f \bar{f}$ | NLO | [132] |
| $H^± \rightarrow f f'$ | LO + NLO QCD | [133-134] |
| $h_i \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0$ | LO | – |
| $h_i \rightarrow \tilde{\chi}_i^± \tilde{\chi}_j^±$ | LO | – |
| $H^± \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^±$ | LO | – |
| $h_i \rightarrow h_j Z$ | LO | – |
| $H^± \rightarrow h_j W^±$ | LO | [134] |
| $h_i \rightarrow h_j h_k$ | NLO + log resum. | [132-135] |
| $h_i \rightarrow f \bar{f}'$ | LO | – |
| $H^± \rightarrow \tilde{f}_u \tilde{f}_d'$ | LO | [134] |

Table 1: Higgs decay widths/branching ratios computed by FeynHiggs. For decays including (excluding) loop corrections the $Z$-matrix ($U$-matrix) is employed by default, which includes propagator-type corrections at the same level of accuracy as the mass predictions.

The flavor observables are given in Table 4. For many observables the corresponding SM predictions are given, in order to facilitate the comparison between MSSM and SM predictions. For the flavour observables, the recommendation is to use the values given in the output only to be added to the best available SM predictions (which are not provided by FeynHiggs), as in: $O_{\text{MSSM, best}} = O_{\text{SM, best}} + (O_{\text{MSSM, FH}} - O_{\text{SM, FH}})$. Correspondingly, the references refer to the SUSY contribution only.

We stress again, as already mentioned above, that the references listed in the Tables are not meant to provide a comprehensive literature list for the quoted observable. We list here only references containing corrections that are implemented into FeynHiggs.

2.3. Using FeynHiggs

FeynHiggs is mostly written in Fortran but can also be called from C/C++ and Mathematica, or accessed from a Web interface. In order to build FeynHiggs a Fortran and C compiler and, to build the FeynHiggs executables for Mathematica, a working Mathematica/MathLink installation are needed. The code has been thoroughly tested with gfortran, ifort, and pgf90 in several versions on several platforms.
production cross section | precision level | references
--- | --- | ---
$\bar{b}b \rightarrow h_i + X$ | reweighting of SM results | [88, 89]
$\bar{b}b \rightarrow h_i + X$ (one tagged $b$) | reweighting of SM results | [88, 89, 136]
$gg \rightarrow h_i + X$ (c-factor) | reweighting of SM results | [88, 89]
$gg \rightarrow h_i + X$ (k-factor) | reweighting of SM results | [88, 89]
$qq \rightarrow qqh + X$ | reweighting of SM results | [88, 89]
$qq, gg \rightarrow t\bar{t}h_i + X$ | reweighting of SM results | [88, 89]
$qq \rightarrow Wh_i + X$ | reweighting of SM results | [88, 89]
$qq \rightarrow Zh_i + X$ | reweighting of SM results | [88, 89]
$pp \rightarrow \tilde{t}_1\tilde{t}_1 h$ | LO | [137, 138]
$gb \rightarrow tH^-$ | reweighting of THDM results | [88, 89, 139–142]
$t \rightarrow H^+ b$ | LO + NLO QCD | [102, 143]

Table 2: Higgs production cross-sections computed by FeynHiggs.

| EWPO | precision level | references
--- | --- | ---
$\Delta \rho$ | 1L + 2L SUSY-QCD/EW | [48, 144, 145]
$\Delta r$ | 1L + 2L SUSY-QCD/EW (full SM) | [48, 144–146]
$M_W$ | 1L + 2L SUSY-QCD/EW (full SM) | [48, 144–146]
$\sin \theta_W^{\text{eff,lep}}$ | 1L + 2L SUSY-QCD/EW (full SM) | [48, 144–147]
$(g - 2)_{\mu}$ | 1L + partial 2L | [148, 149]
EDM of Th, n, and Hg | 1L + partial 2L | [150, 153]

Table 3: Electroweak precision observables computed by FeynHiggs. The abbreviation “full SM” is used to indicate that all known SM corrections are taken into account.

| flavour observable | precision level | references
--- | --- | ---
$B \rightarrow X_s \gamma$ | LO | [154]
$\Delta M_s$ | LO + NLO QCD | [155]
$B_s \rightarrow \mu^+ \mu^-$ | LO + NLO QCD | [156]

Table 4: flavour observables computed by FeynHiggs. All implemented corrections allow one to take non-minimal flavour violation into account.
After downloading the latest tar file from http://feynhiggs.de, the configuration and installation follow these steps:

```bash
tar xvfz FeynHiggs-2.14.x.tar.gz
cd FeynHiggs-2.14.x
./configure
make
make install
```

After building the code, FeynHiggs provides several ways to use it:

- The FeynHiggs Fortran library libFH.a can be linked to Fortran or C/C++ programs, where the latter include CFeynHiggs.h.

- The FeynHiggs executable FeynHiggs allows one to run FeynHiggs from the command-line.

- The MathLink executable MFeynHiggs allows one to call FeynHiggs from within a Mathematica session.

The Web interface at http://feynhiggs.de/fhucc allows one to run FeynHiggs without downloading it.

FeynHiggs receives its input parameters from an input file in either the SLHA [157, 158] or its native format, or directly through the API routines. The contents of the input file are read into a data structure called the FeynHiggs Record, which can be thought of as an SLHA superstructure that also encodes loops over parameters. Routines to read an input file into the Record and step through the loops in the Record are also available through the API. The SLHA carries mostly $\overline{\text{DR}}$ mass parameters, whereas FeynHiggs uses mostly OS masses internally. Care has to be taken if FeynHiggs is used as the starting point of an SLHA chain as FeynHiggs presently cannot convert its mass parameters to $\overline{\text{DR}}$ before writing them to the SLHA; this is indicated by a $\overline{\text{DR}}$ scale of 0 in the corresponding block.

For more details, we refer to the manual pages which are included in the tar file or are available at http://feynhiggs.de.

3. Improvements of the fixed-order calculation

In this section we describe improvements of the fixed-order calculation starting from FeynHiggs 2.13 (released in early 2017). The first improvement is the implementation of an optional $\overline{\text{DR}}$ renormalization of the stop sector. Second, we discuss an adaptation of the renormalization in the Higgs sector at the two-loop level.
3.1. Optional $\overline{\text{DR}}$ renormalization

FeynHiggs by default employs a mixed OS/$\overline{\text{DR}}$ renormalization scheme (see [79] for more details). In particular, the parameters of the stop/top sector are defined using OS renormalization conditions [80] (stop masses and stop mixing parameter $X_t$). FeynHiggs also offers the possibility to use $\overline{\text{DR}}$ input parameters, however. Before the release of FeynHiggs 2.14, these were converted to OS parameters at the one-loop level. The obtained OS parameters were then used as input for the rest of the calculation. This procedure has the advantage that a $\overline{\text{DR}}$ result and the default OS/$\overline{\text{DR}}$ result of FeynHiggs can easily be compared. If the calculation is performed identically except for the renormalization schemes, the difference between the two results can be interpreted as a part of the theoretical uncertainty.

As shown in [65], this procedure is, however, problematic if the fixed-order result is supplemented by a resummation of large logarithms obtained in an EFT approach. The parameter conversion induces additional logarithmic higher-order terms which can become large for large SUSY scales and therefore spoil the resummation. To circumvent this issue, an optional $\overline{\text{DR}}$ renormalization of the stop sector was employed in [65]. Here we describe the practical implementation of this optional renormalization scheme.

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The counterterm of $X_t$ is fixed by imposing a condition on the off-diagonal stop mass counterterm $\delta m_{\tilde{t}_1\tilde{t}_2}$ employing on-shell external momenta. See e.g. [80] for more details.
This scheme is implemented with the stop-mass scale $M_S = \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}}$ as $\overline{\text{DR}}$ scale. Inserting the relation\(^7\)

$$X_t^{\overline{\text{DR}}}(M_S) = X_t^{\text{OS}} + \delta^{\text{OS}} X_t(M_S)\bigg|_{\text{fin}}$$

and employing a Taylor expansion around $X_t^{\text{OS}}$ we obtain

$$\hat{\Sigma}(X_t^{\overline{\text{DR}}}(M_S)) = \hat{\Sigma}(X_t^{\text{OS}}) + \left( \frac{\partial}{\partial X_t} \hat{\Sigma} \right) \cdot \delta^{\text{OS}} X_t(M_S)\bigg|_{\text{fin}}, \quad (3)$$

where $\hat{\Sigma}$ is a generic renormalized self-energy, e.g. the $hh$ self-energy. The second term on the right-hand side corresponds to the subloop-renormalization diagrams involving $\delta X_t$ which are depicted in Fig. 1.

In this way, changing the renormalization scheme and scale of the stop sector becomes straightforward.\(^8\) It amounts to the calculation of all subloop-renormalization diagrams involving the stop mass or the stop mixing counterterms with the renormalization scale set equal to the stop mass scale $M_S$. It should be noted that due to the $SU(2)_L$ gauge symmetry also some sbottom counterterms depend on stop counterterms (see e.g. \([17]\)). Hence, also these contributions have to be taken into account. Adding the result to the existing self-energies with an OS renormalized stop sector, we have obtained the self-energies with a $\overline{\text{DR}}$-renormalized stop sector.

This calculation is automated (see Section 5) and also works for complex input parameters. In contrast, the explicit conversion to OS parameters had been implemented for real input parameters only and was in practice applied to the absolute value while the phase was left unchanged. The new procedure is presently used in the stop sector of the mass calculation; if the parameters of the sbottom sector are input in the $\overline{\text{DR}}$ scheme, FeynHiggs still uses the explicit $\overline{\text{DR}}$/OS conversion to obtain the parameters renormalized in the mixed OS/$\overline{\text{DR}}$ scheme which is employed for the sbottom sector \([37]\). The explicit conversion is likewise still used for the calculation of other observables (e.g. decay rates to scalar tops), with the exception of the $h_i \to h_j h_k$ modes. For the calculation of the latter, the $\overline{\text{DR}}$ parameters of the stop sector are used in order to consistently combine the NLO result \([132,135]\) (see also Tab. 1) with a resummation of large logarithms.

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\(^7\)The subscript “fin” indicates that only the finite part of the OS counterterm is taken into account. The UV-divergent part is cancelled by the corresponding $\overline{\text{DR}}$ counterterm.

\(^8\)Another approach would have been to replace the on-shell counterterms by $\overline{\text{DR}}$ counterterms taking into account the renormalization scale dependence.
3.2. Adapted renormalization of the Higgs sector

Another improvement concerns the renormalization of the Higgs sector at the two-loop level. If the mass of the $\mathcal{CP}$-odd Higgs boson $A$ is used as input mass (as done by default in the case of $(2 \times 2)$ mixing), the following OS renormalization conditions are employed:

\[ \delta^{(1)} m_A^2 = \text{Re} \left[ \Sigma_{AA}^{(1)}(m_A^2) \right], \]
\[ \delta^{(2)} m_A^2 = \text{Re} \left[ \Sigma_{AA}^{(2)}(m_A^2) \right] - \delta^{(1)} Z_{AA} \delta^{(1)} m_A^2 - \delta^{(1)} Z_{GA} \delta^{(1)} m_{AG}^2 + \text{Im} \left[ \Sigma_{AA}^{(1)'}(m_A^2) \right] \text{Im} \left[ \Sigma_{AA}^{(1)}(m_A^2) \right], \]

where the $\delta^{(1)} Z$-s are one-loop field renormalization constants (following the conventions of [82]).

The physical mass squared, $M_A^2$, is given by the real part of the corresponding propagator pole. In the absence of $\mathcal{CP}$-violation, i.e. if all input parameters are real, this pole is obtained by solving the equation

\[ p^2 - m_A^2 + \Sigma_{AA}(p^2) = 0. \]

Expanding up to the two-loop level yields

\[ M_A^2 = m_A^2 - \text{Re} \left[ \hat{\Sigma}_{AA}^{(1)}(m_A^2) \right] - \text{Re} \left[ \hat{\Sigma}_{AA}^{(2)}(m_A^2) \right] + \text{Re} \left[ \hat{\Sigma}_{AA}^{(1)'}(m_A^2) \hat{\Sigma}_{AA}^{(1)}(m_A^2) \right], \]

where the renormalized self-energies, marked by a hat, are given in terms of the unrenormalized self-energies containing the subloop renormalization and counterterms by

\[ \hat{\Sigma}_{AA}^{(1)}(m_A^2) = \Sigma_{AA}^{(1)}(m_A^2) - \delta^{(1)} m_A^2, \]
\[ \hat{\Sigma}_{AA}^{(2)}(m_A^2) = \Sigma_{AA}^{(2)}(m_A^2) - \delta^{(1)} Z_{AA} \delta^{(1)} m_A^2 - \delta^{(1)} Z_{AG} \delta^{(1)} m_{AG}^2 - \delta^{(2)} m_A^2. \]

The superscript marks the loop order, and the prime is used to denote a derivative with respect to $p^2$. Employing the conditions defined in Eqs. (4) and (5), we straightforwardly obtain

\[ M_A^2 = m_A^2, \]

meaning that the input mass $m_A$ is equivalent to the physical mass $M_A$. Before the release of FeynHiggs 2.14.0, the term in the last line of Eq. (5) had been omitted.

\[ ^9 \text{At the two-loop level, all self-energy contributions implemented in FeynHiggs are obtained by default in the limit of vanishing external momentum. Therefore, the counterterms are adapted accordingly if they appear at the two-loop level (see e.g. [82] for more details). An exception are the } \mathcal{O}(\alpha_t \alpha_s) \text{ corrections for which optionally the full momentum dependence can be taken into account [52,60]. Note that the new additional contribution to the two-loop counterterms } \delta^{(2)} m_A^2 \text{ and } \delta^{(2)} m_{H^\pm}^2, \text{ discussed in this section, is not of } \mathcal{O}(\alpha_t \alpha_s). \]
If the charged Higgs boson mass \( m_{H^\pm} \) is used as input parameter and renormalized on-shell (as done by default in the case of \((3 \times 3)\) mixing in the neutral Higgs sector), its two-loop counterterm is adapted accordingly,

\[
\delta^{(2)} m_{H^\pm}^2 = \text{Re} \left[ \Sigma^{(2)}_{H^\pm H^\pm}(m_{H^\pm}^2) \right] - \delta^{(1)} Z_{H^\pm H^\pm} \delta^{(1)} m_{H^\pm}^2 \\
- \frac{1}{2} \left( \delta^{(1)} Z_{G^\pm H^\pm} \delta^{(1)} m_{G^\pm H^\pm} + \delta^{(1)} Z_{G^\pm H^\pm}^{\ast} \delta^{(1)} m_{G^\pm H^\pm}^{\ast} \right) \\
+ \text{Im} \left[ \Sigma^{(1)}_{H^\pm H^\pm}(m_{H^\pm}^2) \right] \text{Im} \left[ \Sigma^{(1)}_{H^\pm H^\pm}(m_{H^\pm}^2) \right],
\]

(11)

whereas

\[
\delta^{(2)} m_A^2 = \delta^{(2)} m_{H^\pm}^2 - \delta^{(2)} M_W^2.
\]

(12)

In the approximation of vanishing electroweak gauge couplings, as employed for all two-loop corrections implemented in FeynHiggs, the two-loop counterterm of the \( W \) boson mass \( \delta^{(2)} M_W^2 \) is equal to zero.

4. Improvements of the EFT calculation

Apart from the fixed-order calculation, also the EFT calculation that is implemented in FeynHiggs has been improved.

The first advancement concerns the threshold corrections, which appear at each of the matching scales (the calculation can contain up to four different matching scales: the electroweak scale, the electroweakino scale, the gluino scale and the sfermion scale). Up to FeynHiggs 2.12.2, all threshold corrections were implemented in their degenerate form. This means that at a threshold all particles which are integrated out were assumed to have the same mass, which is moreover equal to the matching scale. As an example, in the EFT calculation it was assumed that the soft SUSY-breaking masses of the stop sector, \( M_{Q_3} \) and \( M_{U_3} \), are equal to each other. No such assumptions have been made in the fixed-order calculation, however. Therefore, the effect of non-degeneracy was captured in the fixed-order calculation at the full one-loop level and the two-loop level in the limit of vanishing electroweak gauge couplings.

In FeynHiggs 2.13.0, the full non-degenerate one-loop and the two-loop threshold corrections of \( \mathcal{O}(\alpha_t \alpha_s) \) [57] have been implemented. In FeynHiggs 2.14.1, also the non-degenerate two-loop threshold corrections of \( \mathcal{O}(\alpha_t^2) \) [64] have been included. This means that while e.g. the two stops are still integrated out at the same scale, their masses are not assumed to be equal anymore. While this prescription yields precise results for \( M_{Q_3} \sim M_{U_3} \), more than one matching scale would be required in case of a large hierarchy between the two soft SUSY-breaking masses.

This facilitated to lift also a further restriction. Before, the low-energy threshold of the gluino could only be taken into account in the case of LL and NLL resumma-
tion. If NNLL resummation was activated, the gluino mass $M_\tilde{g}$ was set equal to the SUSY scale $M_{\text{SUSY}}$ in the EFT calculation. The implementation of the non-degenerate threshold correction of $\mathcal{O}(\alpha_t \alpha_s)$ enables us to set $M_\tilde{g}$ independently of $M_{\text{SUSY}}$ in all relevant threshold corrections.\footnote{Note that formally we would have to expand the threshold corrections in $M_\tilde{g}/M_{\text{SUSY}}$ if the gluino remains in the EFT below the scale $M_{\text{SUSY}}$. However, from a practical point of view contributions of $\mathcal{O}(M_\tilde{g}/M_{\text{SUSY}})$ in the threshold corrections are negligible in scenarios in which the gluino threshold has a sizeable numerical impact (i.e. if $M_\tilde{g} \ll M_{\text{SUSY}}$). It should be noted that logarithms involving the SUSY scale and the gluino mass are not resummed for $M_\tilde{g} > M_{\text{SUSY}}$.} No two-loop threshold corrections need to be taken into account if the gluino is integrated out from the effective theory below the scale $M_{\text{SUSY}}$. This is due to the couplings of the gluino: It couples either through a quark–squark–gluino or a gluon–gluino–gluino vertex. Therefore, the gluino only contributes to the matching of the Higgs self-coupling in the effective theory below the scale $M_{\text{SUSY}}$—in which all squarks are integrated out—at the three-loop level and beyond. The RGEs of the EFT are modified already at the one-loop level, however. Corresponding one- and two-loop RGEs are listed in [62],[110] and have been crosschecked using SARAH, version 4 [159]. The modifications of the three-loop RGEs are unknown (except for the modification of the three-loop running of the strong gauge coupling [160]). The two-loop threshold corrections are also valid for low mass gluinos. Even so, the resum- mation of NNLL logarithms has to be considered as an approximation in this case due to the unknown three-loop RGEs. Based on the finding that in the SM the effects from three-loop running are negligible (see e.g. [61]), it is, however, conceivable that also the three-loop running in the SM plus gluino is negligible.\footnote{In addition, we checked that the implementation of the gluino contribution to the three-loop beta function of the strong gauge coupling [160] leads to a negligible shift in $M_h$ of $\mathcal{O}(1 \text{ MeV})$.} Taking into account the two-loop threshold correction also valid for low mass gluinos, however, turns out to be numerically relevant. Therefore, using this two-loop threshold correction together with the SM three-loop RGEs should be a good estimate of the correct result.

As second improvement, an interpolation of the EFT result was introduced for complex parameters in FeynHiggs 2.13.0. A pure EFT calculation taking into account phases in the threshold corrections has been performed in Ref. [161], however, for the hybrid approach no calculation for complex parameters is available at the moment. Therefore, we follow the approach that is employed in FeynHiggs for those fixed-order contributions which are only known for the case of real parameters and interpolate the EFT calculation in the case of complex parameters. The interpolation is carried out for the Higgsino mass parameter $\mu$, the trilinear coupling in the stop sector $A_t$, and the gluino mass parameter $M_\tilde{g}$, which are all allowed to take complex values. The interpolation is performed by evaluating the EFT result at $|P|$ and $-|P|$ ($P = \mu, A_t, M_\tilde{g}$) and afterwards linearly interpolating between the obtained values.

\begin{equation}
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\end{equation}
5. Improvements of code structure

The code of FeynHiggs is structured in three parts:

- code hand-written for FeynHiggs
  The ‘back bone’ of FeynHiggs is of course written by hand. Most code has been developed specifically for FeynHiggs, with some adaptations from external sources, e.g. LoopTools \[162\] or SLHALib \[163\]. Code falling into this category includes
  - all structural code: data structures, frontend, I/O, record handling, etc.
  - utility functions: matrix diagonalization, loop integrals, ordinary-differential-equation solver, etc.
  - contributions taken from the literature: the EDMs, some of the RGEs and threshold corrections in the EFT sector, higher-order SM parts of \( \Delta r \), etc., (for references see Sect. \[2\]).

- code generated from external expressions
  FeynHiggs includes several contributions which originated from independent projects and for which the original (typically large) expressions are available, usually in Mathematica format: several of the two-loop contributions to the Higgs self-energies, several ingredients of the EFT calculation, the muon \( g - 2 \), the two-loop parts of \( \Delta r \), etc.
  This is already more practical than hand-coded expressions since modifications (e.g. a change in conventions) can be done in Mathematica which is much easier and safer than search/replace in an editor. Also the code can be re-generated at any time and can be optimized, too. On the other hand, it is nearly impossible to extend or significantly change results implemented in this way.

- code generated from calculations done in/for FeynHiggs
  This mode is most convenient for perturbatively calculable quantities since it allows full control over model content, particle selection, resummations/K-factors, the renormalization prescription, etc. Calculations done in this way can usually be generalized to other models relatively straightforwardly. Note that we do not pursue a ‘generator generator’ approach as done in some other packages, i.e. even if our scripts ran (or were modified to run) with an ‘arbitrary’ model file, the produced code would still need to be embedded in and called from the main program, in which the inputs have to be properly adjusted.
  Calculations at this stage of automation can be found in the ‘gen’ subdirectory of FeynHiggs.
Currently, it includes (for references see Sect. 2)

- the entire set of renormalized one-loop Higgs self-energies (gen/oneloop),
- the $\mathcal{O}(\alpha^2)$ contributions to the two-loop Higgs self-energies (gen/tlsp),
- the shifts at two-loop order from $\overline{\text{DR}}$ input parameters (gen/drbar),
- the shifts at two-loop order from finite $Z$ factors (gen/dzhfin),
- the one-loop decay rates (gen/decays),
- the one-loop corrections to $\Delta_b$ (gen/db),
- several flavour observables at the one-loop order (gen/bsg,bsll,dms),
- the one-loop MSSM contribution to $\Delta\rho$ (gen/deltarho).

The code-generation scripts generally follow the approach of [98] and, in case of improvements or bugfixes, can be re-run with a few keystrokes.

(Another subdirectory, ‘gen/prod’, contains code for the empirical fitting of cross-sections from tabulated data. It falls somewhat outside the sort of code generation described here and shall not be discussed further.)

In the following we describe the main improvements in FeynHiggs version 2.14.

The unrenormalized one-loop Higgs self-energies have been generated with a high degree of automatization for all versions since 2.0. Before FeynHiggs 2.14, however, the entire renormalization was hard-coded. (At the time of the first implementation of the self-energies, a model file for the MSSM including the complete set of one-loop counter-terms [164] did not yet exist.) Old FeynHiggs versions actually encoded various options of renormalization schemes which were used for testing at that time. The only recommended scheme became the one used in the model file. The flags fieldren and $\tan\beta_{\text{ren}}$, which selected these schemes, were correspondingly dropped in 2.14.

The new procedure instead reads the renormalization (counter-terms plus renormalization constants) from the model file, making as few assumptions as possible. It needs to know the relevant flags governing the renormalization, of course, such as $\$\text{MHpInput}$, which selects whether $M_A$ or $M_{H^0}$ is the input mass for the Higgs sector, for which it generates the necessary if statements in the output. Diagram computation and code generation rely heavily on FeynArts [165] and FormCalc [162], and to achieve the level of automation we desired, we had to enhance and add several of FormCalc’s code-generation functions [166].

New in 2.14 are also the two-loop shifts induced by the use of $\overline{\text{DR}}$ input parameters and finite field-renormalization factors in the one-loop Higgs self-energies.

Even though FeynHiggs does not (yet) go beyond the MSSM in scope, there are three ‘models’ used internally: ‘mfv’ and ‘nmfv’, the MSSM with minimal and non-minimal flavour-violation, and ‘g1’, the gaugeless version used e.g. in the two-loop calculations. An important task was also to consolidate various sources of Feynman rules for the MSSM which had grown over the years.
All one-loop self-energies are automatically split into the parts corresponding to FeynHiggs’ \texttt{mssmpart} flag: \( t/\tilde{t}; t/\tilde{t} + b/\tilde{b}; f/\tilde{f} \); all, so that individual sectors of the MSSM can still be looked at even in the presence of a generated renormalization. Our code generation routines are generic enough to deal with things such as different renormalization schemes and simple extensions of the MSSM but are also to a certain extent model-aware, e.g. know how to simplify the \((2 \times 2)\) sfermion mixing matrices, and are hence not directly applicable to ‘arbitrary’ models. Planned directions in this programme are the implementation of recent two-loop results (e.g. \cite{84,85}) and the extension to the NMSSM based on \cite{114,120,167}.

Finally, the adherence to the FORTRAN 77 standard, kept mainly because of g77 (for many years the only free Fortran compiler), was dropped with version 2.14. Even though outwardly the code retains its fixed-format ‘F77’ look, it uses many F90 idioms, in particular vector syntax.

While the numerical stability of the code is generally satisfactory, some sections, for example the non-degenerate two-loop threshold corrections of the EFT results, can be affected by numerical artefacts even in not-too-extreme corners of the parameter space. A quadruple-precision version of FeynHiggs has been available for some time (\texttt{/configure --quad}) but this naturally runs vastly slower. In 2.14.3 we reorganized many of the internal utility functions, in particular the loop integrals, so that they compile to either a double- or a quadruple-precision object depending on the setting of a flag, and can now adjust higher precision for just the neuralgic parts, which improves overall precision appreciably and makes the slowdown hardly noticeable. Quadruple precision (\texttt{REAL*16, COMPLEX*32}) is currently available with gfortran and ifort. With gfortran, the alternate extended-precision type \texttt{REAL*10} can also be targeted, which is realized in hardware on Intel x86 chips, either overall (\texttt{/configure --quad --real10}) or just for the parts in need of extra precision (\texttt{/configure --real10}).

6. Numerical results

In this Section, we present some exemplary results highlighting various aspects of the improvements discussed above. Other examples of the improved Higgs-boson mass calculation are given in \cite{65,69,168,169}.

6.1. Improvements of the fixed-order calculation

First, we look at the improvements of the fixed-order calculation as discussed in Section 3: the numerical impact of the new optional DR renormalization on \( M_h \) obtained as a result of the hybrid approach has already been presented in \cite{65}, and we do not repeat this discussion here. We will, however, investigate scenarios with complex DR input parameters in Section 6.2.

The numerical effect of the adapted renormalization of the Higgs sector, see Section 3, namely of the additional term \( \text{Im}[\Sigma^{(1)/}] \) \( \text{Im}[\Sigma^{(1)}] \) in the two-loop counterterm of the input mass in Eq. (5) or Eq. (11) is shown in the left plot of Fig. 2 for a scenario
Figure 2: Left: Masses of the non-SM-like Higgs bosons as a function of $\tan \beta$. The results employing the adapted renormalization of the Higgs sector (solid) are compared to the results employing the old renormalization (dashed). Right: $M_h$ as a function of $X_{t}^{\text{DR}}/\sqrt{M_{Q,3}M_{U,3}}$. The results obtained using the non-degenerate and the degenerate form of the threshold correction of $O(\alpha_t^2)$ are compared. (See text for the values of the parameters.)

with the input values $M_{\text{SUSY}} = 1$ TeV (common mass scale of squarks and sleptons), $X_t^{\text{DS}}/M_{\text{SUSY}} = 2$, $m_A = 500$ GeV, and $\mu = -500$ GeV. The gaugino masses are set to $M_1 = M_2 = 500$ GeV, and $M_3 = 2.5$ TeV. All trilinear soft-breaking couplings apart from $A_t$ are set to zero.

Due to the chosen mass pattern, the additional term $\text{Im}[\Sigma_{AA}(m_{A}^{2})] \text{Im}[\Sigma_{AA}(m_{A}^{2})]$ only receives contributions from SM particles. One observes that the term is negligible in the range $2 \lesssim \tan \beta \lesssim 25$. For $\tan \beta \sim 1$, where the coupling of the heavy Higgs bosons to top quarks is not suppressed, a small upward shift of all three non-SM-like Higgs-boson masses is visible. Similarly, one finds a slightly larger upward shift for $\tan \beta \gtrsim 25$, where the coupling of the heavy Higgs bosons to bottom quarks becomes large. One also observes that with the adapted renormalization scheme the physical mass of the $A$-boson is, as expected, always equal to the input mass $m_A$.

6.2. Improvements of the EFT calculation

Next, we discuss the numerical impact of the improvements of the EFT calculation. We first consider the effect of the non-degenerate threshold corrections. Since, as already mentioned, the effect of non-degenerate particle masses was captured exactly up to the level of two-loop corrections via the fixed-order calculation before, the numerical impact of those for scenarios with SUSY masses around the TeV scale is quite small ($\lesssim O(100 \text{ MeV})$).

For multi-TeV SUSY masses larger effects can be observed, however. As an example, we investigate a scenario in which all soft-breaking masses, the mass of the $\mathcal{CP}$-odd Higgs boson, $m_A$, and the Higgsino mass parameter $\mu$ are set equal to $M_{\text{SUSY}} = 5$ TeV. Only the soft-breaking mass $M_{U,3}$ in the stop sector is chosen differently, $M_{U,3} = M_{\text{SUSY}}/4$. 
Figure 3: Comparison of results with and without interpolation of the EFT result for complex parameters. The input parameters $M_{\text{SUSY}} = 2$ TeV, $\tan \beta = 10$ and $\frac{\lambda_{\text{DR}}}{M_{\text{SUSY}}} = \sqrt{6}$ are chosen. Left: $M_h$ as a function of $\phi_{A_t}$. Right: $M_h$ as a function of $\phi_{M_3}$.

to generate a large non-degeneracy in the stop sector. $\tan \beta$ is set equal to 10. In the right plot of Fig. 2, we show $M_h$ as a function of $X_{\text{DR}}^{\text{t}}/\sqrt{M_{Q_3}M_{U_3}}$, comparing the results obtained with the degenerate and the non-degenerate threshold corrections of $O(\alpha_t^2)$. Due to the multi-TeV SUSY scale we observe a downwards shift of $\sim 1$ GeV for vanishing stop mixing. Moreover, we see that the values of $X_{\text{DR}}^{\text{t}}$ maximizing $M_h$ are shifted away from the expected value of $|X_{\text{DR}}^{\text{t}}/\sqrt{M_{Q_3}M_{U_3}}| \sim \sqrt{6}$ if the degenerate threshold correction of $O(\alpha_t^2)$ is used. This effect was especially relevant for the studies conducted in [169].

For a further example showing the impact of the non-degenerate threshold corrections of $O(\alpha_t^2)$, we refer to [69] where scenarios with low $m_A$ are investigated and shifts of up to 6 GeV in the prediction of $M_h$ have been found between results obtained using the degenerate and non-degenerate threshold corrections of $O(\alpha_t^2)$.

As second improvement we investigate the interpolation of the EFT result for the case of complex input parameters. We compare three methods to handle complex parameters in the EFT calculation: using the real part of the complex parameter as input, using its absolute value as input, and the interpolation method described in Section 4. For the investigation, we use a scenario like the one in the right plot of Fig. 2 but with $M_{U_3} = M_{\text{SUSY}} = 2$ TeV. In addition, we allow for nonzero phases of $A_t$ and $M_3$.

In the left panel of Fig. 3 we vary the phase of $A_t$ between $-\pi$ and $\pi$ and observe shifts in $M_h$ of up to 3 GeV for $\phi_{A_t} \sim \pm \frac{\pi}{4}$. Cutting off the imaginary part of $A_t$ leads to values of $M_h$ which are similar to those obtained from the interpolation in $\phi_{A_t}$ only close to $\phi_{A_t} = 0; \pm \pi$ where the imaginary part of $A_t$ is small. For phases in between, the predicted values of $M_h$ are smaller compared to those obtained from the interpolation. Using the absolute value conversely works better for $|\phi_{A_t}| \lesssim 0.7$ but is worse, as expected, for $\phi_{A_t} \sim \pm \pi$. Since the one-loop threshold correction involves only
even powers of $X_t$, and in the investigated scenario $A_t$ is similar in size to $X_t$ due to the relatively high value of $\tan \beta$, the dominant contribution causing these shifts is the threshold correction of $\mathcal{O}(\alpha_t \alpha_s)$.

This is confirmed by the right plot of Fig. 3, showing a variation of the gluino phase $\phi_{M_3}$. The threshold correction of $\mathcal{O}(\alpha_t \alpha_s)$ is a function of $X_t/M_3$. Therefore, a variation of $\phi_{M_3}$ is comparable to a variation of $\phi_{A_t}$, as observable in the plots. Cutting off the imaginary part of $M_3$ is not a good approximation here since $M_3$ appears in the denominator and its real part approaches zero for $\phi_{M_3} \sim \pm \frac{\pi}{4}$.

The different treatment of the phases is formally of NNLL order, since at the one- and two-loop level in the fixed-order calculation the phase dependence is taken into account without approximation. The remarkably large size of the effect is compatible with the shifts caused by overall NNLL resummation found in [62]. In contrast, a variation of $\phi_\mu$ leads only to very small shifts well below 1 GeV.

The results show that an interpolation of the EFT result yields more reliable results than just using the real part or absolute value of the complex parameters. Nevertheless, the displayed results motivate an improved EFT calculation taking the phases fully into account. We leave that for future work.

The plots shown in Fig. 3 are also examples of scenarios with complex $\overline{\text{DR}}$ input parameters. The conversion between the $\overline{\text{DR}}$ input parameters and the internally used OS parameters, as employed in earlier FeynHiggs versions, was in contrast not applicable to the case of complex parameters (i.e. the phases were not converted to the OS scheme).

7. Conclusions

After presenting a short overview over the Fortran code FeynHiggs (available at \url{http://feynhiggs.de}), whose main purpose is to provide precise numerical predictions for observables in the Higgs sector of the MSSM, we discussed various improvements in the calculation of the Higgs spectrum. For the prediction of the Higgs-boson masses, a diagrammatic fixed-order calculation—accurate for low SUSY scales—is combined with an EFT calculation—accurate for high SUSY scales—in order to provide an accurate result also for intermediate scales.

We first discussed improvements of the fixed-order calculation. We explained the implementation of an alternative $\overline{\text{DR}}$ renormalization of the stop sector (allowing one to input also complex $\overline{\text{DR}}$ parameters). Moreover, we showed how the two-loop renormalization of the Higgs sector is adapted in order to ensure that the input Higgs mass is equal to the corresponding physical mass. Numerically, this change of the renormalization scheme has been relevant in the considered scenario only for very low or very high values of $\tan \beta$.

\footnote{The uncertainty in the prediction of $M_h$ will be discussed in a future publication which will compare the different approaches in detail.}
Then, we addressed the improvements of the EFT calculation. We described the implementation of threshold corrections valid for arbitrary masses of the decoupled particles and showed that this can lead to sizeable numerical effects as compared to the result using degenerate threshold corrections e.g. in the case of a large separation between the two stop masses. Moreover, we explained how the EFT calculation is interpolated in the case of complex input parameters. The numerical effects of the phase variations can be important if the imaginary parts of the respective parameters are sufficiently large.

We furthermore highlighted several improvements of the code structure, which are not directly visible for the user but should allow for an easier development and extension of FeynHiggs in the future.

**Acknowledgments**

We thank E. Bagnaschi, P. Slavich, and I. Sobolev for useful discussions and relentless testing. We thank E. Bagnaschi, P. Slavich, D. Stöckinger, K. Williams, and L. Zeune for contributions to the code. The work of S.H. is supported in part by the MEINCOP Spain under contract FPA2016-78022-P, in part by the “Spanish Agencia Estatal de Investigación” (AEI) and the EU “Fondo Europeo de Desarrollo Regional” (FEDER) through the project FPA2016-78022-P, in part by the “Spanish Red Consolider MultiDark” FPA2017-90566-REDC, and in part by the AEI through the grant IFT Centro de Excelencia Severo Ochoa SEV-2016-0597. G.W. acknowledges support by the DFG through the SFB 676 “Particles, Strings and the Early Universe”. The work of S.P. is supported by the ANR grant “HiggsAutomator” (ANR-15-CE31-0002). HR’s work is partially funded by the Danish National Research Foundation, grant number DNRF90. The authors would like to express special thanks to the Mainz Institute for Theoretical Physics (MITP) for its hospitality and support.
References

[1] G. Aad, et al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B716 (2012) 1–29. arXiv:1207.7214 doi:10.1016/j.physletb.2012.08.020

[2] S. Chatrchyan, et al., Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B716 (2012) 30–61. arXiv:1207.7235 doi:10.1016/j.physletb.2012.08.021

[3] G. Aad, et al., Combined Measurement of the Higgs Boson Mass in pp Collisions at \( \sqrt{s} = 7 \) and 8 TeV with the ATLAS and CMS Experiments, Phys. Rev. Lett. 114 (2015) 191801. arXiv:1503.07589 doi:10.1103/PhysRevLett.114.191801

[4] G. Aad, et al., Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at \( \sqrt{s} = 7 \) and 8 TeV, JHEP 08 (2016) 045. arXiv:1606.02266 doi:10.1007/JHEP08(2016)045

[5] P. Fayet, Supergauge Invariant Extension of the Higgs Mechanism and a Model for the electron and Its Neutrino, Nucl. Phys. B90 (1975) 104–124. doi:10.1016/0550-3213(75)90636-7

[6] P. Fayet, Spontaneously Broken Supersymmetric Theories of Weak, Electromagnetic and Strong Interactions, Phys. Lett. 69B (1977) 489. doi:10.1016/0370-2693(77)90852-8

[7] H. P. Nilles, Supersymmetry, Supergravity and Particle Physics, Phys. Rept. 110 (1984) 1–162. doi:10.1016/0370-1573(84)90008-5

[8] H. E. Haber, G. L. Kane, The Search for Supersymmetry: Probing Physics Beyond the Standard Model, Phys. Rept. 117 (1985) 75–263. doi:10.1016/0370-1573(85)90051-1

[9] J. R. Ellis, G. Ridolfi, F. Zwirner, Radiative corrections to the masses of supersymmetric Higgs bosons, Phys. Lett. B257 (1991) 83–91. doi:10.1016/0370-2693(91)90863-L

[10] Y. Okada, M. Yamaguchi, T. Yanagida, Upper bound of the lightest Higgs boson mass in the minimal supersymmetric standard model, Prog. Theor. Phys. 85 (1991) 1–6. doi:10.1143/ptp/85.1.1

[11] Y. Okada, M. Yamaguchi, T. Yanagida, Renormalization group analysis on the Higgs mass in the softly broken supersymmetric standard model, Phys. Lett. B262 (1991) 54–58. doi:10.1016/0370-2693(91)90642-4

[12] H. E. Haber, R. Hempfling, Can the mass of the lightest Higgs boson of the minimal supersymmetric model be larger than \( m(Z) \)?, Phys. Rev. Lett. 66 (1991) 1815–1818. doi:10.1103/PhysRevLett.66.1815

[13] J. R. Ellis, G. Ridolfi, F. Zwirner, On radiative corrections to supersymmetric Higgs boson masses and their implications for LEP searches, Phys. Lett. B262 (1991) 477–484. doi:10.1016/0370-2693(91)90626-2

[14] K. Sasaki, M. Carena, C. E. M. Wagner, Renormalization group analysis of the Higgs sector in the minimal supersymmetric standard model, Nucl. Phys. B381 (1992) 66–86. doi:10.1016/0550-3213(92)90051-1

[15] P. H. Chankowski, S. Pokorski, J. Rosiek, Charged and neutral supersymmetric Higgs boson masses: Complete one loop analysis, Phys. Lett. B274 (1992) 191–198. doi:10.1016/0370-2693(92)90552-6

[16] A. Brignole, Radiative corrections to the supersymmetric neutral Higgs boson masses, Phys. Lett. B281 (1992) 284–294. doi:10.1016/0370-2693(92)91142-V

[17] P. H. Chankowski, S. Pokorski, J. Rosiek, Complete on-shell renormalization scheme for the minimal supersymmetric Higgs sector, Nucl. Phys. B423 (1994) 437–496. arXiv:hep-ph/9303309 doi:10.1016/0550-3213(94)90141-4

[18] R. Hempfling, A. H. Hoang, Two loop radiative corrections to the upper limit of the lightest Higgs boson mass in the minimal supersymmetric model, Phys. Lett. B331 (1994) 99–106. arXiv:hep-ph/9401219 doi:10.1016/0370-2693(94)90948-2

[19] J. A. Casas, J. R. Espinosa, M. Quiros, A. Riotto, The lightest Higgs boson mass in the minimal supersymmetric standard model, Nucl. Phys. B436 (1995) 3–29, [Erratum: Nucl.
[20] A. Dabelstein, The one loop renormalization of the MSSM Higgs sector and its application to the neutral scalar Higgs masses, Z. Phys. C67 (1995) 495–512. arXiv:hep-ph/9409375 \[\text{doi:10.1016/0550-3213(95)00057-Y}]\[\text{[p5]}

[21] M. Carena, J. R. Espinosa, M. Quiros, C. E. M. Wagner, Analytical expressions for radiatively corrected Higgs masses and couplings in the MSSM, Phys. Lett. B355 (1995) 209–211. arXiv:hep-ph/9409375 \[\text{doi:10.1016/0370-2693(95)00694-G}]\[\text{[pp5]}

[22] M. Carena, M. Quiros, C. E. M. Wagner, Effective potential methods and the Higgs mass spectrum in the MSSM, Nucl. Phys. B461 (1996) 407–436. arXiv:hep-ph/9508343 \[\text{doi:10.1007/BF01624592}]\[\text{[p5]}

[23] D. M. Pierce, J. A. Bagger, K. T. Matchev, R.-J. Zhang, Precision corrections in the minimal supersymmetric standard model, Phys. Rev. D58 (1998) 091701. arXiv:hep-ph/9606211 \[\text{doi:10.1103/PhysRevD.58.091701}]\[\text{[pp4,5]}

[24] H. E. Haber, R. Hempfling, A. H. Hoang, Approximating the radiatively corrected Higgs mass in the minimal supersymmetric model, Z. Phys. C75 (1997) 539–554. arXiv:hep-ph/9609331 \[\text{doi:10.1007/s002880050498}]\[\text{[pp4,5]}

[25] S. Heinemeyer, W. Hollik, G. Weiglein, QCD corrections to the masses of the neutral CP-even Higgs bosons in the MSSM, Phys. Rev. D58 (1998) 091701. arXiv:hep-ph/9803277 \[\text{doi:10.1103/PhysRevD.58.091701}]\[\text{[pp4,5]}

[26] S. Heinemeyer, W. Hollik, G. Weiglein, Precise prediction for the mass of the lightest Higgs boson in the MSSM, Phys. Lett. B447 (1999) 89–97. arXiv:hep-ph/9808299 \[\text{doi:10.1016/S0370-2693(98)01575-5}]\[\text{[pp4,5]}

[27] R.-J. Zhang, Two loop effective potential calculation of the lightest CP even Higgs boson mass in the MSSM, Phys. Lett. B447 (1999) 89–97. arXiv:hep-ph/9808299 \[\text{doi:10.1016/S0370-2693(98)01116-2}]\[\text{[pp4,5]}

[28] S. Heinemeyer, W. Hollik, G. Weiglein, The masses of the neutral CP-even Higgs bosons in the MSSM: Accurate analysis at the two loop level, Eur. Phys. J. C9 (1999) 343–366. arXiv:hep-ph/9812472 \[\text{doi:10.1007/s100529900006,10.1007/s100520050537}]\[\text{[pp4,5]}

[29] S. Heinemeyer, W. Hollik, G. Weiglein, The mass of the lightest MSSM Higgs boson: A compact analytical expression at the two loop level, Phys. Lett. B491 (1997) 3–67. arXiv:hep-ph/9606211 \[\text{doi:10.1016/S0550-3213(96)00683-9}]\[\text{[pp4,5]}

[30] J. R. Espinosa, R.-J. Zhang, MSSM lightest CP even Higgs boson mass to $O(\alpha_s^2\alpha_t)$: The effective potential approach, JHEP 03 (2000) 026. arXiv:hep-ph/9912236 \[\text{doi:10.1088/1126-6708/2000/03/026}]\[\text{[pp4,5]}

[31] M. Carena, H. E. Haber, S. Heinemeyer, W. Hollik, C. E. M. Wagner, G. Weiglein, Reconciling the two loop diagrammatic and effective field theory computations of the mass of the lightest CP - even Higgs boson in the MSSM, Nucl. Phys. B580 (2000) 29–57. arXiv:hep-ph/0001002 \[\text{doi:10.1016/S0550-3213(00)00212-1}]\[\text{[pp4,5]}

[32] J. R. Espinosa, R.-J. Zhang, Complete two loop dominant corrections to the mass of the lightest CP even Higgs boson in the minimal supersymmetric standard model, Nucl. Phys. B586 (2000) 3–38. arXiv:hep-ph/0003246 \[\text{doi:10.1016/S0550-3213(00)00421-1}]\[\text{[pp4,5]}

[33] J. R. Espinosa, I. Navarro, Radiative corrections to the Higgs boson mass for a hierarchical stop spectrum, Nucl. Phys. B615 (2001) 82–116. arXiv:hep-ph/0104047 \[\text{doi:10.1016/S0550-3213(01)00429-1}]\[\text{[pp4,5]}

[34] G. Degrassi, P. Slavich, F. Zwirner, On the neutral Higgs boson masses in the MSSM for arbitrary stop mixing, Nucl. Phys. B611 (2001) 403–422. arXiv:hep-ph/0010096 \[\text{doi:10.1016/S0550-3213(01)00343-1}]\[\text{[pp4,5]}

[35] S. P. Martin, Two loop effective potential for a general renormalizable theory and softly broken supersymmetry, Phys. Rev. D65 (2002) 116003. arXiv:hep-ph/0111209 \[\text{doi:10.1103/PhysRevD.65.116003}]\[\text{[pp4,5]}

[36] A. Brignole, G. Degrassi, P. Slavich, F. Zwirner, On the $O(\alpha_t^2)$ two loop corrections to the neutral
Higgs boson masses in the MSSM, Nucl. Phys. B631 (2002) 195–218. arXiv:hep-ph/0112177 doi:10.1016/S0550-3213(02)00184-0

[37] A. Brignole, G. Degrassi, P. Slavich, F. Zwirner, On the two loop sbottom corrections to the neutral Higgs boson masses in the MSSM, Nucl. Phys. B643 (2002) 79–92. arXiv:hep-ph/0206101 doi:10.1016/S0550-3213(02)00748-4

[38] S. P. Martin, Two loop effective potential for the minimal supersymmetric standard model, Phys. Rev. D66 (2002) 096001. arXiv:hep-ph/0206136 doi:10.1103/PhysRevD.66.096001

[39] S. P. Martin, Complete two loop effective potential approximation to the lightest Higgs scalar boson mass in supersymmetry, Phys. Rev. D67 (2003) 095012. arXiv:hep-ph/0211366 doi:10.1103/PhysRevD.67.095012

[40] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. Weiglein, Towards high precision predictions for the MSSM Higgs sector, Eur. Phys. J. C28 (2003) 133–143. arXiv:hep-ph/0212020 doi:10.1140/epjc/s2003-01152-2

[41] A. Dedes, P. Slavich, Two loop corrections to radiative electroweak symmetry breaking in the MSSM, Nucl. Phys. B657 (2003) 333–354. arXiv:hep-ph/0212132 doi:10.1016/S0550-3213(03)00173-1

[42] A. Dedes, G. Degrassi, P. Slavich, On the two loop Yukawa corrections to the MSSM Higgs boson masses at large \( \tan \beta \), Nucl. Phys. B672 (2003) 144–162. arXiv:hep-ph/0305127 doi:10.1016/j.nuclphysb.2003.08.033

[43] S. P. Martin, Evaluation of two loop selfenergy basis integrals using differential equations, Phys. Rev. D68 (2003) 075002. arXiv:hep-ph/0307101 doi:10.1103/PhysRevD.68.075002

[44] S. P. Martin, Two loop scalar self energies in a general renormalizable theory at leading order in gauge couplings, Phys. Rev. D70 (2004) 016005. arXiv:hep-ph/0312092 doi:10.1103/PhysRevD.70.016005

[45] S. P. Martin, Strong and Yukawa two-loop contributions to Higgs scalar boson self-energies and pole masses in supersymmetry, Phys. Rev. D71 (2005) 016012. arXiv:hep-ph/0405022 doi:10.1103/PhysRevD.71.016012

[46] B. C. Allanach, A. Djouadi, J. L. Kneur, W. Porod, P. Slavich, Precise determination of the neutral Higgs boson masses in the MSSM, JHEP 09 (2004) 044. arXiv:hep-ph/0406166 doi:10.1088/1126-6708/2004/09/044

[47] S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein, High-precision predictions for the MSSM Higgs sector at \( O(\alpha_s^3) \), Eur. Phys. J. C39 (2005) 465–481. arXiv:hep-ph/0411114 doi:10.1140/epjc/s2005-02112-6

[48] S. Heinemeyer, W. Hollik, G. Weiglein, Electroweak precision observables in the minimal supersymmetric standard model, Phys. Rept. 425 (2006) 265–368. arXiv:hep-ph/0412214 doi:10.1016/j.physrep.2006.04.002

[49] S. P. Martin, D. G. Robertson, TSIL: A Program for the calculation of two-loop self-energy integrals, Comput. Phys. Commun. 174 (2006) 133–151. arXiv:hep-ph/0501132 doi:10.1016/j.cpc.2005.08.005

[50] S. P. Martin, Two-loop scalar self-energies and pole masses in a general renormalizable theory with massless gauge bosons, Phys. Rev. D71 (2005) 116004. arXiv:hep-ph/0502168 doi:10.1103/PhysRevD.71.116004

[51] S. P. Martin, Three-loop corrections to the lightest Higgs scalar boson mass in supersymmetry, Phys. Rev. D75 (2007) 055005. arXiv:hep-ph/0701051 doi:10.1103/PhysRevD.75.055005

[52] R. V. Harlander, P. Kant, L. Mihaila, M. Steinhauser, Higgs boson mass in supersymmetry to three loops, Phys. Rev. Lett. 100 (2008) 191602. [Phys. Rev. Lett.101,039901(2008)]. arXiv:0803.0672 doi:10.1103/PhysRevLett.101.039901,10.1103/PhysRevLett.100.191602

[53] P. Kant, R. V. Harlander, L. Mihaila, M. Steinhauser, Light MSSM Higgs boson mass to three-loop accuracy, JHEP 08 (2010) 104. arXiv:1005.5709 doi:10.1007/JHEP08(2010)104

[54] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein, High-precision predictions for...
the light CP-even Higgs boson mass of the Minimal Supersymmetric Standard Model, Phys. Rev. Lett. 112 (14) (2014) 141801.  
[arXiv:1312.4937 doi:10.1103/PhysRevLett.112.141801]

[55] P. Draper, G. Lee, C. E. M. Wagner. Precise estimates of the Higgs mass in heavy supersymmetry, Phys. Rev. D89 (5) (2014) 055023.  
[arXiv:1312.5743 doi:10.1103/PhysRevD.89.055023]

[56] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich, W. Hollik. Momentum-dependent two-loop QCD corrections to the neutral Higgs-boson masses in the MSSM, Eur. Phys. J. C74 (8) (2014) 2994.  
[arXiv:1404.7074 doi:10.1140/epjc/s10052-014-2994-0]

[57] E. Bagnaschi, G. F. Giudice, P. Slavich, A. Strumia. Higgs mass and unnatural supersymmetry, JHEP 09 (2014) 092.  
[arXiv:1407.4081 doi:10.1007/JHEP09(2014)092]

[58] G. Degrassi, S. Di Vita, P. Slavich. Two-loop QCD corrections to the MSSM Higgs masses beyond the effective-potential approximation, Eur. Phys. J. C75 (8) (2015) 2994.  
[arXiv:1410.3432 doi:10.1140/epjc/s10052-015-3280-5]

[59] J. P. Vega, G. Villadoro. SusyHD: Higgs mass determination in supersymmetry, JHEP 07 (2015) 159.  
[arXiv:1504.05200 doi:10.1007/JHEP07(2015)159]

[60] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich, W. Hollik. Renormalization scheme dependence of the two-loop QCD corrections to the neutral Higgs-boson masses in the MSSM, Eur. Phys. J. C75 (9) (2015) 424.  
[arXiv:1505.03133 doi:10.1140/epjc/s10052-015-3648-6]

[61] G. Lee, C. E. M. Wagner. Higgs bosons in heavy supersymmetry with an intermediate m_A, Phys. Rev. D92 (7) (2015) 075032.  
[arXiv:1508.00576 doi:10.1103/PhysRevD.92.075032]

[62] H. Bahl, W. Hollik. Precise prediction for the light MSSM Higgs boson mass combining effective field theory and fixed-order calculations, Eur. Phys. J. C76 (9) (2016) 499.  
[arXiv:1608.01880 doi:10.1140/epjc/s10052-016-3648-6]

[63] E. Bagnaschi, J. Pardo Vega, P. Slavich. Improved determination of the Higgs mass in the MSSM with heavy superpartners, Eur. Phys. J. C77 (5) (2017) 334.  
[arXiv:1703.08166 doi:10.1140/epjc/s10052-017-4885-7]

[64] H. Bahl, S. Heinemeyer, W. Hollik, G. Weiglein. Reconciling EFT and hybrid calculations of the light MSSM Higgs-boson mass, Eur. Phys. J. C78 (1) (2018) 57.  
[arXiv:1706.00346 doi:10.1140/epjc/s10052-018-5544-3]

[65] R. V. Harlander, J. Klappert, A. Voigt. Higgs mass prediction in the MSSM at three-loop level in a pure DR context, Eur. Phys. J. C77 (12) (2017) 814.  
[arXiv:1708.05720 doi:10.1140/epjc/s10052-017-5368-6]

[66] B. C. Allanach, T. Kwasnitza, J.-H. Park, A. Voigt, D. Stöckinger, J. Ziebell. FlexibleSUSY 2.0: Extensions to investigate the phenomenology of SUSY and non-SUSY models, Comput. Phys. Commun. 230 (2018) 145–217.  
[arXiv:1710.03760 doi:10.1016/j.cpc.2018.04.016]

[67] A. Pilaftsis. CP odd tadpole renormalization of Higgs scalar - pseudoscalar mixing, Phys. Rev. D58 (1998) 096010.  
[arXiv:hep-ph/9803297 doi:10.1103/PhysRevD.58.096010]

[68] D. A. Demir. Effects of the supersymmetric phases on the neutral Higgs sector, Phys. Rev. D60
A. Pilaftsis, C. E. M. Wagner, Higgs bosons in the minimal supersymmetric standard model with explicit CP violation, Nucl. Phys. B553 (1999) 3–42. arXiv:hep-ph/9902371, doi:10.1016/S0550-3213(99)00385-8

S. Y. Choi, M. Drees, J. S. Lee, Loop corrections to the neutral Higgs boson sector of the MSSM with explicit CP violation, Phys. Lett. B481 (2000) 57–66. arXiv:hep-ph/0002287, doi:10.1016/S0370-2693(00)00421-4

T. Ibrahim, P. Nath, Corrections to the Higgs boson masses and mixings from chargino, W and charged Higgs exchange loops and large CP phases, Phys. Rev. D63 (2001) 035009. arXiv:hep-ph/0008237, doi:10.1103/PhysRevD.63.035009

S. Heinemeyer, The Higgs boson sector of the complex MSSM in the Feynman diagrammatic approach, Eur. Phys. J. C22 (2001) 521–534. arXiv:hep-ph/0108059, doi:10.1007/s100520100819

T. Ibrahim, P. Nath, Neutralino exchange corrections to the Higgs boson mixings with explicit CP violation, Phys. Rev. D66 (2002) 015005. arXiv:hep-ph/0204092, doi:10.1103/PhysRevD.66.015005

M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein, The Higgs boson masses and mixings of the complex MSSM in the Feynman-diagrammatic approach, JHEP 02 (2007) 047. arXiv:hep-ph/0611326, doi:10.1088/1126-6708/2007/02/047

S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein, The Higgs sector of the complex MSSM at two-loop order: QCD contributions, Phys. Lett. B652 (2007) 300–309. arXiv:0705.0746, doi:10.1016/j.physletb.2007.07.030

W. Hollik, S. Pašehr, Two-loop top-Yukawa-coupling corrections to the Higgs boson masses in the complex MSSM, Phys. Lett. B733 (2014) 144–150. arXiv:1401.8275, doi:10.1016/j.physletb.2014.04.026

W. Hollik, S. Pašehr, Higgs boson masses and mixings in the complex MSSM with two-loop top-Yukawa-coupling corrections, JHEP 10 (2014) 171. arXiv:1409.1687, doi:10.1007/JHEP10(2014)171

M. D.Goodsell, F. Staub, The Higgs mass in the CP violating MSSM, NMSSM, and beyond, Eur. Phys. J. C77 (1) (2017) 46. arXiv:1604.05335, doi:10.1140/epjc/s10052-016-4495-9

S. Pašehr, G. Weiglein, Two-loop top and bottom Yukawa corrections to the Higgs boson masses in the complex MSSM, Eur. Phys. J. C78 (3) (2018) 222. arXiv:1705.07909, doi:10.1140/epjc/s10052-018-5655-8

S. Borowka, S. Pašehr, G. Weiglein, Complete two-loop QCD contributions to the lightest Higgs-boson mass in the MSSM with complex parameters, Eur. Phys. J. C78 (7) (2018) 576. arXiv:1802.09886, doi:10.1140/epjc/s10052-018-6055-y
W. Porod, SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at e+ e- colliders, Comput. Phys. Commun. 153 (2003) 275–315. arXiv:hep-ph/0301101 doi:10.1016/S0010-4655(03)00222-4

P. Athron, J. Park, D. Stöckinger, A. Voigt, FlexibleSUSY – A spectrum generator generator for supersymmetric models, Comput. Phys. Commun. 190 (2015) 139–172. arXiv:1406.2319 doi:10.1016/j.cpc.2014.12.020

A. Djouadi, J. Kalinowski, M. Spira, HDECAY: A Program for Higgs boson decays in the standard model and its supersymmetric extension, Comput. Phys. Commun. 108 (1998) 56–74. arXiv:hep-ph/9704448 doi:10.1016/S0010-4655(97)00123-9

R.V.Harlander, S.Liebler, H.Mantler, SusHi: A program for the calculation of Higgs production in gluon fusion and bottom-quark annihilation in the Standard Model and the MSSM, Comput. Phys. Commun. 184 (2013) 1605–1617. arXiv:1212.3249 doi:10.1016/j.cpc.2013.02.006

A. Djouadi, J. Kalinowski, M. Spira, HDECAY: A Program for Higgs boson decays in the standard model and its supersymmetric extension, Comput. Phys. Commun. 108 (1998) 56–74. arXiv:hep-ph/9704448 doi:10.1016/S0010-4655(97)00123-9

S. Liebler, S. Patel, G. Weiglein, Phenomenology of on-shell Higgs production in the MSSM with complex parameters, Eur. Phys. J. C77 (5) (2017) 305. arXiv:1611.09308 doi:10.1140/epjc/s10052-017-4849-y

H. Bahl, Pole mass determination in presence of heavy particles, JHEP 02 (2019) 121. arXiv:1812.06452 doi:10.1007/JHEP02(2019)121

W. Hollik, S. Paßeehr, Two-loop top-Yukawa-coupling corrections to the charged Higgs-boson mass in the MSSM, Eur. Phys. J. C75 (7) (2015) 336. arXiv:1502.02394 doi:10.1140/epjc/s10052-015-3558-7

R. Hempfling, Yukawa coupling unification with supersymmetric threshold corrections, Phys. Rev. D49 (1994) 6168–6172. doi:10.1103/PhysRevD.49.6168

L. J. Hall, R. Rattazzi, U. Sarid, The Top quark mass in supersymmetric SO(10) unification, Phys. Rev. D50 (1994) 7048–7065. arXiv:hep-ph/9306309 doi:10.1103/PhysRevD.50.7048

M. Carena, M. Olechowski, S. Pokorski, C. E. M. Wagner, Electroweak symmetry breaking and bottom - top Yukawa unification, Nucl. Phys. B426 (1994) 269–300. arXiv:hep-ph/9402253 doi:10.1016/0550-3213(94)90313-1

M. Carena, D. Garcia, U. Nierste, C. E. M. Wagner, Effective Lagrangian for the $\bar{t}bH^+$ interaction in the MSSM and charged Higgs phenomenology, Nucl. Phys. B577 (2000) 88–120. arXiv:hep-ph/9912515 doi:10.1016/S0550-3213(00)00146-2

J. Guasch, P. Hafliger, M. Spira, MSSM Higgs decays to bottom quark pairs revisited, Phys. Rev. D68 (2003) 115001. arXiv:hep-ph/0305101 doi:10.1103/PhysRevD.68.115001

D. Noth, M. Spira, Higgs Boson Couplings to Bottom Quarks: Two-Loop Supersymmetry-QCD Corrections, Phys. Rev. Lett. 101 (2008) 181801. arXiv:0808.0087 doi:10.1103/PhysRevLett.101.181801

D. Noth, M. Spira, Supersymmetric Higgs Yukawa Couplings to Bottom Quarks at next-to-next-to-leading Order, JHEP 06 (2011) 084. arXiv:1001.1935 doi:10.1007/JHEP06(2011)084

M. Arana-Catania, S. Heinemeyer, M. J. Herrero, S. Penaranda, Higgs Boson masses and B-Physics Constraints in Non-Minimal Flavor Violating SUSY scenarios, JHEP 05 (2012) 015. arXiv:1109.6232 doi:10.1007/JHEP05(2012)015

M. Arana-Catania, S. Heinemeyer, M. J. Herrero, S. Penaranda, Higgs Boson masses and B-Physics Constraints in Non-Minimal Flavor Violating SUSY scenarios, JHEP 05 (2012) 015. arXiv:1109.6232 doi:10.1007/JHEP05(2012)015

S. Heinemeyer, W. Hollik, F. Merz, S. Penaranda, Electroweak precision observables in the MSSM with nonminimal flavor violation, Eur. Phys. J. C37 (2004) 481–493. arXiv:hep-ph/0403228 doi:10.1140/epjc/s2004-02006-1

M. E. Gómez, T. Hahn, S. Heinemeyer, M. Rehman, Higgs masses and Electroweak Precision
[110] G. F. Giudice, A. Strumia, Probing high-scale and split supersymmetry with Higgs mass measurements, Nucl. Phys. B858 (2012) 63–83. arXiv:1108.6077 doi:10.1016/j.nuclphysb.2012.01.001

[111] S. Heinemeyer, W. Hollik, J. Rosiek, G. Weiglein, Neutral MSSM Higgs boson production at $e^+ e^-$ colliders in the Feynman diagrammatic approach, Eur. Phys. J. C19 (2001) 355–546. arXiv:hep-ph/0102081 doi:10.1007/s100520100631

[112] E. Fuchs, G. Weiglein, Breit-Wigner approximation for propagators of mixed unstable states, JHEP 09 (2017) 079. arXiv:1610.06193 doi:10.1007/JHEP09(2017)079

[113] E. Fuchs, G. Weiglein, Impact of CP-violating interference effects on MSSM Higgs searches, Eur. Phys. J. C77 (8) (2017) 562. arXiv:1706.00437 doi:10.1140/epjc/s10052-017-5104-2

[114] F. Domingo, P. Drechsel, S. Paßehr, On-Shell neutral Higgs bosons in the NMSSM with complex parameters, Eur. Phys. J. C77 (8) (2017) 562. arXiv:1705.05757 doi:10.1140/epjc/s10052-017-5104-2

[115] E. Fuchs, Interference effects in new physics processes at the LHC, Ph.D. thesis, U. Hamburg, Dept. Phys., Hamburg (2015). doi:10.3204/DESY-THESIS-2015-037

[116] S. Heinemeyer, W. Hollik, G. Weiglein, Decay widths of the neutral CP even MSSM Higgs bosons in the Feynman diagrammatic approach, Eur. Phys. J. C16 (2000) 139–153. arXiv:hep-ph/0003022 doi:10.1007/s100520050010

[117] M. Spira, Higgs Boson Production and Decay at Hadron Colliders, Prog. Part. Nucl. Phys. 95 (2017) 98–159. arXiv:1705.05757 doi:10.1140/epjc/s10052-018-5543-4

[118] M. Pospelov, A. Ritz, Electric dipole moments as probes of new physics, Annals Phys. 318 (2005) 119–169. arXiv:hep-ph/0504231 doi:10.1016/j.aop.2005.04.002

[119] G. Buchalla, et al., $B$, $D$ and $K$ decays, Eur. Phys. J. C57 (2008) 309–492. arXiv:0801.1833 doi:10.1140/epjc/s10052-008-0716-1

[120] F. Domingo, S. Heinemeyer, S. Paßehr, G. Weiglein, Decays of the neutral Higgs bosons into SM fermions and gauge bosons in the CP-violating NMSSM, Eur. Phys. J. C78 (11) (2018) 942. arXiv:1807.06322 doi:10.1140/epjc/s10052-018-6400-1

[121] T. Hahn, S. Heinemeyer, F. Maltoni, G. Weiglein, S. Willenbrock, SM and MSSM Higgs boson production cross-sections at the Tevatron and the LHC, in: TEV4LHC Workshop: 3rd Meeting Geneva, Switzerland, April 28–30, 2005, 2006. arXiv:hep-ph/0607308.

[122] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein, FeynHiggs 2.7, Nucl. Phys. Proc. Suppl. 205-206 (2010) 152–157. arXiv:1007.0956 doi:10.1016/j.nuclphysbps.2010.08.035

[123] M. Spira, HIGLU: A program for the calculation of the total Higgs production cross-section at hadron colliders via gluon fusion including QCD corrections. arXiv:hep-ph/9510347

[124] S. Catani, M. Grazzini, Higgs Boson Production at Hadron Colliders: Hard-Collinear Coefficients at the NNLO, Eur. Phys. J. C72 (2012) 2013, [Erratum: Eur. Phys. J.C72,2132(2012)]. arXiv:1106.4652 doi:10.1140/epjc/s10052-012-2132-9

[125] L. Zeune, Constraining supersymmetric models using Higgs physics, precision observables and direct searches, Ph.D. thesis, U. Hamburg, Dept. Phys., Cham (2014). doi:10.1007/978-3-319-22228-8

[126] M. Spira, A. Djouadi, D. Graudenzi, P. M. Zerwas, Higgs boson production at the LHC, Nucl. Phys. B453 (1995) 17–82. arXiv:hep-ph/9504378 doi:10.1016/0550-3213(95)00379-7

[127] M. Spira, QCD effects in Higgs physics, Fortsch. Phys. 46 (1998) 203–284. arXiv:hep-ph/9705337 doi:10.1002/(SICI)1521-3978(199804)46:3<203::AID-PROP203>3.0.CO;2-4
[128] U. Aglietti, R. Bonciani, G. Degrassi, A. Vicini, Analytic Results for Virtual QCD Corrections to Higgs Production and Decay, JHEP 01 (2007) 021. arXiv:hep-ph/0612266 doi:10.1088/1126-6708/2007/01/021

[129] R. Benbrik, M. Gomez Bock, S. Heinemeyer, O. Stal, G. Weiglein, L. Zeune, Confronting the MSSM and the NMSSM with the Discovery of a Signal in the two Photon Channel at the LHC, Eur. Phys. J. C72 (2012) 2171. arXiv:1207.1096 doi:10.1140/epjc/s10052-012-2171-2

[130] A. Bredenstein, A. Denner, S. Dittmaier, M. M. Weber, Radiative corrections to the semileptonic and hadronic Higgs-boson decays $H \rightarrow WW/ZZ \rightarrow 4$ fermions, JHEP 02 (2007) 080. arXiv:hep-ph/0611234 doi:10.1088/1126-6708/2007/02/080

[131] A. Bredenstein, A. Denner, S. Dittmaier, M. M. Weber, Precise predictions for the Higgs-boson decay $H \rightarrow WW/ZZ \rightarrow 4$ leptons, Phys. Rev. D74 (2006) 013004. arXiv:hep-ph/0604011 doi:10.1103/PhysRevD.74.013004

[132] K. E. Williams, H. Rzehak, G. Weiglein, Higher order corrections to Higgs boson decays in the MSSM with complex parameters, Eur. Phys. J. C71 (2011) 1669. arXiv:1103.1335 doi:10.1140/epjc/s10052-011-1669-3

[133] A. Djouadi, P. Gambino, QCD corrections to Higgs boson selfenergies and fermionic decay widths, Phys. Rev. D5 (1995) 218–228, [Erratum: Phys. Rev.D53,4111(1996)]. arXiv:hep-ph/9406431 doi:10.1103/PhysRevD.51.218,10.1103/PhysRevD.53.4111.2

[134] A. Djouadi, J. Kalinowski, P. M. Zerwas, Two and three-body decay modes of SUSY Higgs particles, Z. Phys. C70 (1996) 435–448. arXiv:hep-ph/9511342 doi:10.1007/s002880050121

[135] K. E. Williams, G. Weiglein, Precise predictions for $h_a \rightarrow h_b h_c$ decays in the complex MSSM, Phys. Lett. B660 (2008) 217–227. arXiv:0710.5320 doi:10.1016/j.physletb.2007.12.049

[136] R. V. Harlander, W. B. Kilgore, Higgs boson production in bottom quark fusion at next-to-next-to leading order, Phys. Rev. D68 (2003) 013001. arXiv:hep-ph/0304035 doi:10.1103/PhysRevD.68.013001

[137] A. Djouadi, J. L. Kneur, G. Moultaka, Higgs boson production in association with scalar top quarks at proton colliders, Phys. Rev. Lett. 80 (1998) 1830–1833. arXiv:hep-ph/9711244 doi:10.1103/PhysRevLett.80.1830

[138] S. Dittmaier, M. Krämer, M. Spira, M. Walser, Charged-Higgs-boson production at the LHC: NLO supersymmetric QCD corrections, Phys. Rev. D83 (2011) 055005. arXiv:0906.2648 doi:10.1103/PhysRevD.83.055005

[139] M. Flechl, R. Klees, M. Krämer, M. Spira, M. Ubiali, Improved cross-section predictions for heavy charged Higgs boson production at the LHC, Phys. Rev. D91 (7) (2015) 075015. arXiv:1409.5615 doi:10.1103/PhysRevD.91.075015

[140] C. Degrande, M. Ubiali, M. Wiesemann, M. Zaro, Heavy charged Higgs boson production at the LHC, JHEP 10 (2015) 145. arXiv:1507.02549 doi:10.1007/JHEP10(2015)145

[141] J. G. Körner, M. C. Mauser, $O(\alpha_s)$ radiative corrections to polarized top decay into a charged Higgs, $t^\pm \rightarrow H^\pm + b$, Eur. Phys. J. C54 (2008) 175–185. arXiv:0806.2648 doi:10.1140/epjc/s10052-007-0510-5

[142] S. Heinemeyer, W. Hollik, D. Stöckinger, A. M. Weber, G. Weiglein, Precise prediction for $M(W)$ in the MSSM, JHEP 08 (2006) 052. arXiv:hep-ph/0604147 doi:10.1088/1126-6708/2006/08/052

[143] S. Heinemeyer, W. Hollik, G. Weiglein, L. Zeune, Implications of LHC search results on the W boson mass prediction in the MSSM, JHEP 12 (2013) 084. arXiv:1311.1663 doi:10.1007/
JHEP12(2013)084

[146] O. Stål, G. Weiglein, L. Zeune, Improved prediction for the mass of the W boson in the NMSSM, JHEP 09 (2015) 158. arXiv:1506.07465 doi:10.1007/JHEP09(2015)158

[147] S. Heinemeyer, W. Hollik, A. M. Weber, G. Weiglein, Z Pole Observables in the MSSM, JHEP 04 (2008) 039. arXiv:0710.2972 doi:10.1088/1126-6708/2008/04/039

[148] G. Degrassi, G. F. Giudice, QED logarithms in the electroweak corrections to the muon anomalous magnetic moment, Phys. Rev. D58 (1998) 053007. arXiv:hep-ph/9803384 doi:10.1103/PhysRevD.58.053007

[149] S. Heinemeyer, D. Stöckinger, G. Weiglein, Two loop SUSY corrections to the anomalous magnetic moment of the muon, Nucl. Phys. B690 (2004) 62–80. arXiv:hep-ph/0312264 doi:10.1016/j.nuclphysb.2004.04.017

[150] T. Ibrahim, P. Nath, The Neutron and the electron electric dipole moment in N=1 supergravity unification, Phys. Rev. D57 (1998) 478–488, [Erratum: Phys. Rev.D60,119901(1999)]. arXiv:hep-ph/9708456 doi:10.1103/PhysRevD.58.019901,10.1103/PhysRevD.60.079903,10.1103/PhysRevD.57.478

[151] D. A. Demir, O. Lebedev, K. A. Olive, M. Pospelov, A. Ritz, Electric dipole moments in the MSSM at large tan β, Nucl. Phys. B680 (2004) 339–374. arXiv:hep-ph/0311314 doi:10.1016/j.nuclphysb.2003.12.026

[152] D. Chang, W. Keung, A. Pilaftsis, New two loop contribution to electric dipole moment in supersymmetric theories, Phys. Rev. Lett. 82 (1999) 900–903, [Erratum: Phys. Rev.Lett.83,3972(1999)]. arXiv:hep-ph/9811202 doi:10.1103/PhysRevLett.83.3972,10.1103/PhysRevLett.82.900

[153] K. A. Olive, M. Pospelov, A. Ritz, Y. Santoso, CP-odd phase correlations and electric dipole moments, Phys. Rev. D72 (2005) 075001. arXiv:hep-ph/0506106 doi:10.1103/PhysRevD.72.075001

[154] T. Hahn, W. Hollik, J. I. Illana, S. Penaranda, Interplay between H → b anti-s and b → s gamma in the MSSM with non-minimal flavor violation. arXiv:hep-ph/0512315

[155] A. J. Buras, S. Jäger, J. Urban, Master formulae for Delta F=2 NLO QCD factors in the standard model and beyond, Nucl. Phys. B605 (2001) 600–624. arXiv:hep-ph/0102316 doi:10.1016/S0550-3213(01)00207-3

[156] C. Bobeth, A. J. Buras, F. Kruger, J. Urban, QCD corrections to $\bar{B} \rightarrow X_{d,s}\nu\bar{\nu}$, $\bar{B}_{d,s} \rightarrow \ell^+\ell^-$, $K \rightarrow \pi\nu\bar{\nu}$ and $K_L \rightarrow \mu^+\mu^-$ in the MSSM, Nucl. Phys. B630 (2002) 87–131. arXiv:hep-ph/0112305 doi:10.1016/S0550-3213(02)00141-4

[157] P. Z. Skands, et al., SUSY Les Houches accord: Interfacing SUSY spectrum calculators, decay packages, and event generators, JHEP 07 (2004) 036. arXiv:hep-ph/0311123 doi:10.1088/1126-6708/2004/07/036

[158] B. C. Allanach, et al., SUSY Les Houches Accord 2, Comput. Phys. Commun. 180 (2009) 8–25. arXiv:0801.0045 doi:10.1016/j.cpc.2008.08.004

[159] F. Staub, SARAH 4: A tool for (not only SUSY) model builders, Comput. Phys. Commun. 185 (2014) 1773–1790. arXiv:1309.7223 doi:10.1016/j.cpc.2014.02.018

[160] L. Clavelli, P. W. Coulter, L. R. Surguladze, Gluino contribution to the three loop beta function in the minimal supersymmetric standard model, Phys. Rev. D55 (1997) 4268–4272. arXiv:hep-ph/9611355 doi:10.1103/PhysRevD.55.4268

[161] M. Carena, J. Ellis, J. S. Lee, A. Pilaftsis, C. E. M. Wagner, CP Violation in Heavy MSSM Higgs Scenarios, JHEP 02 (2002) 123. arXiv:1512.00437 doi:10.1007/JHEP02(2002)123

[162] T. Hahn, M. Perez-Victoria, Automated one loop calculations in four-dimensions and D-dimensions, Comput. Phys. Commun. 118 (1999) 153–165. arXiv:hep-ph/9807565 doi:10.1016/S0010-4655(98)00173-8

[163] T. Hahn, SUSY Les Houches Accord 2 I/O made easy, Comput. Phys. Commun. 180 (2009) 1681–1693. arXiv:hep-ph/0605049 doi:10.1016/j.cpc.2009.03.012
[164] T. Fritzsche, T. Hahn, S. Heinemeyer, F. von der Pahlen, H. Rzehak, C. Schappacher, The Implementation of the Renormalized Complex MSSM in FeynArts and FormCalc. Comput. Phys. Commun. 185 (2014) 1529–1545. arXiv:1309.1692 doi:10.1016/j.cpc.2014.02.005 [p17]

[165] T. Hahn, Generating Feynman diagrams and amplitudes with FeynArts 3. Comput. Phys. Commun. 140 (2001) 418–431. arXiv:hep-ph/0012260 doi:10.1016/S0010-4655(01)00290-9 [p17]

[166] T. Hahn, New Features in FeynArts & Friends, and how they got used in FeynHiggs. arXiv:1906.02119 [p17]

[167] P. Drechsel, L. Galeta, S. Heinemeyer, G. Weiglein, Precise Predictions for the Higgs-Boson Masses in the NMSSM, Eur. Phys. J. C77 (1) (2017) 42. arXiv:1601.08100 doi:10.1140/epjc/s10052-017-4595-1 [p18]

[168] H. Bahl, E. Fuchs, T. Hahn, S. Heinemeyer, S. Liebler, S. Patel, P. Slavich, T. Stefaniak, C. E. M. Wagner, G. Weiglein, MSSM Higgs Boson Searches at the LHC: Benchmark Scenarios for Run 2 and Beyond. arXiv:1808.07542 [p18]

[169] E. Bagnaschi, et al., Supersymmetric Models in Light of Improved Higgs Mass Calculations. Eur. Phys. J. C79 (2) (2019) 149. arXiv:1810.10905 doi:10.1140/epjc/s10052-019-6658-y [pp18-20]