Mathematical modeling of economic processes in complex systems (on the example of Krasnoyarsk municipality)

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Abstract. The article is devoted to one of the topical issues of scientific research, mathematical modeling of economic systems. In the activities of the municipality mathematical modeling is of great importance, as it helps to make decisions in complex economic systems. In this article the role of mathematical modeling on the example of Krasnoyarsk municipality in the framework of socio-economic development forecasting is considered, the algorithm of mathematical modeling is presented. The purpose is to study the features of the use of mathematical modeling of complex economic systems in the activities of the municipality of Krasnoyarsk. Research methods: analysis and synthesis, methods of mathematical modeling. As a result, it is noted that this model is actually a tool for determining how the change in any indicator affects other parameters of system development.

1. Introduction

In the development of various fields of human activity mathematics has a significant impact. Its role was formed historically and depended on two factors: the degree of development of mathematical concepts and mathematical apparatus, as well as the degree of maturity of knowledge about research object. A more accurate mathematical description of processes and phenomena caused by the needs of modern science leads to the emergence of complex systems of integral, differential, integral, transcendental equations and inequalities that cannot be solved by analytical methods explicitly. To solve such problems it is necessary to resort to computational algorithms, to use any infinite processes converging to the final result. Approximate solution of the problem is obtained by performing a certain number of steps. In the activities of the municipality mathematical modeling is of great importance, as it helps to make decisions in complex economic systems.

Mathematical methods for solving economics and management problems are becoming more and more common. They are used not only for the description of general theories in these social sciences, but also for practical tasks of management and optimization. They are behind the information systems that drive the running not only the large manufacturing, commercial and transport companies, but also activities of municipality. Mathematical methods require the formulation of precise assumptions and the logical justification of each step of the process when solving problems. They provide a solid foundation for the further development and refinement of knowledge in economics or management. From another point of view, it can be stated that mathematical methods allow the emergence of models that can serve empirical testing and their adaptation to specific conditions [1].

Modern economics widely uses mathematical methods both for solving practical problems and for modeling socio-economic phenomena and processes. Mathematical models are the most important research and forecasting tool. They are the basis of computer modeling and information processing, give
a deeper understanding of the laws of economic processes, and contribute to the formation of thinking and analysis at a new, higher level. Today, in the context of globalization of the world economy and the formation of a new type of society (information society) mathematical model is becoming a powerful tool for forecasting the evolution of civilization, which allows determining the optimal line of economic development, especially in terms of human life. With the further development of society more and more important is to develop ways of improving economic relations from the standpoint of optimal use of natural, industrial, material and labor resources. Therefore, it is no accident that economists and mathematicians involved in the application of mathematics in the economy pay great attention to the development of mathematical methods for building optimal plans. So the purpose of this work is to study the features of the use of mathematical modeling of complex economic systems in the activities of the municipality of Krasnoyarsk.

2. Materials and methods

The municipality is a complex territorial socio-economic system with many heterogeneous elements. Management of such a system is a task that requires taking into account a complex of factors of internal and external environment. Thus, the task of forecasting becomes one of the primary ones in territorial management. However, to date, this procedure is associated with a number of difficulties, the main reasons for which are the following:

1. Inconsistency of objectives at different levels of management.
2. Functioning within the municipal system of multiple entities, with different and often opposite objectives.
3. Lack of systemic nature of economic policy.
4. Insufficient quality of initial information.
5. The openness of the system and the significant impact of external influences on the economy of the municipality. These problems are relevant for both the state and municipal levels of government.

However, they are most clearly manifested at the level of municipalities due to a number of features (relatively small space, proximity of government to the population and economic entities, low independence). At the same time, many problems of municipal development management are interrelated and interdependent [3, 4].

In this regard, a comprehensive approach to their elimination is required. This provision determines the relevance and importance of the choice and justification of tools used in the development of the forecast. In the literature, there are about 150 different methods of forecasting, but 20-30 are used in practice.

The most important type of formalized sign modeling is mathematical modeling, carried out by means of the language of mathematics and logic. For the study of any class of phenomena of the external world, its mathematical model is built, i.e. an approximate description of this class of phenomena, expressed with the help of mathematical symbols.

In the framework of the study, the economic and mathematical model of the municipality was developed and tested, which based on the following principles: consistency, complexity of the description of the socio-economic system, balance, adaptability, and theoretical validity.

The algorithm for constructing an economic and mathematical model includes the following steps:

1) construction of a logical model (study of the statistical base and identification of the main indicators, as well as their interconnection, the choice of indicators);
2) mathematical formalization of relationships between the elements of the forecasting object, external parameters and the resulting indicators;
3) scenario modelling (scenarios of development);
4) filling the model.

The conceptual scheme of the proposed economic and mathematical model of the municipality is shown in Figure 1.

With the advent of computers method of mathematical modeling took a leading place among other research methods. This method plays a particularly important role in modern economic science. The
study and forecasting of any economic phenomenon by the method of mathematical modeling allows designing new technical means, predicting the impact of certain factors on this phenomenon, planning these phenomena even in the presence of an unstable economic situation.

As noted earlier, the high degree of openness of the municipal system does not allow considering it in isolation from the macroeconomic situation and trends in the development of the external environment.

**Figure 1.** System of development of economic and mathematical model of municipal formation

In particular, such parameters as consumer price index and the growth rate of the gross regional product of the constituent entity of the Russian Federation in which the municipality is located, the standard of living in neighboring municipalities and the region as a whole, the parameters of the budget and tax system, etc. should be taken into account. In turn, to describe the processes occurring within the municipality, it is required to develop a set of balance and regression equations that provide logical relationship indicators. Taking into account the target orientation of the development of this model as a management tool in solving the problems facing the administration of the municipality as a key performer of the territory development strategy, it seems appropriate to choose the managed parameters from the position of municipal management [5].

3. **Study of the mathematical modeling of economic processes in complex systems**

The proposed approach to the development of the forecast of socio-economic development of the municipality was tested on the data of the city district of Krasnoyarsk. The choice of modeling indicators is due to the need to comply with the principles of complexity and consistency of the study of such a complex object of modeling as the municipality. At the same time, the selection of indicators is influenced by the purpose of the study, determined by the tasks of developing a forecast of socio-economic development of the city in the medium term, as well as the features of the development of the city, the structure of its economy, the main problems and risks of further development.

Economic development of the city is characterized by such indicators, as volume of shipped goods of own production, works and services, the retail trade turnover, the volume of investments in fixed capital, the presence and input of fixed assets of organizations, average number of employees of enterprises and organizations, etc. The block of the municipal Finance presented the revenue and
expenditure parts of the budget of the urban district of the city of Salavat. In the revenue part of the budget the key components are allocated – tax, non-tax revenues and inter-budget transfers. The main directions of expenditure of budgetary funds identified national issues, the national economy, social policy and support for housing and communal services of the city. As a balancing indicator is the value of the surplus (deficit) of the budget. In addition, the model takes into account the impact of the external macroeconomic environment, including by taking into account the dynamics of the gross regional product of the Republic of Bashkortostan and the consumer price index. Thus, in total the developed economic and mathematical model of the city district of Krasnoyarsk includes about 50 indicators. At the next stage, the analysis of the relationship between these indicators and formed a logical model. The next stage of development of the model involves the definition of its managed parameters and a list of indicators to judge the overall development of the socio-economic system.

Taking into account the orientation of the developed model to solve the problems of municipal government, it seems appropriate as controlled parameters in this case to determine the following:

– the structure of the budget expenditures of the city Krasnoyarsk;
– the growth rate of investment in fixed capital;
– the growth rate of the average monthly nominal wage.

As indicators of the model, 7 quantitatively measurable indicators characterizing the effectiveness of the implementation of measures of socio-economic policy, including:

– population;
– the average monthly nominal wage;
– the volume of shipped products for all activities;
– the volume of investment in fixed assets;
– the amount of cash income of the population;
– budgetary security;
– the share of tax and non-tax revenues in budget revenues.

This list of indicators aggregates the development of the city for all three previously selected blocks of the model. Having thus formed a logical model of municipal education, it is necessary to proceed to its mathematical formalization. The analysis of available approaches in the field of economic and mathematical modeling has allowed drawing a conclusion that for the solution of an objective the most expedient is application of econometric methods. To determine the parameters of the regression and balance equations of the model, specialized software was used to assess the quality of the equations and integrate the individual equations into a single computational scheme. At the same time, preference was given to linear models, as the most clearly interpreted from a logical and economic point of view.

As an example, the authors consider the formation of cash income of the population, including income from wages, income from property, social transfers, cash income from business and other income. Each of the selected sources is calculated in the model according to a strictly defined formula and depends on certain factors (Table 1).

To take into account the instability and probabilistic nature of changes in the parameters of the economic system, as well as other risks, in the formation of the forecast of socio-economic development of the city of Krasnoyarsk, was considered three scenario options: conservative, basic and target. The choice of scenarios was carried out based on the recommendations of the Ministry of economic development to the forecast in the strategy of socio-economic development of the territory. Both external and internal parameters were the factors of scenario formation. External factors include the parameters of socio-economic development of the Russian Federation and Krasnoyarsk. The key internal factors in the formation of the scenarios are the dynamics of investments in fixed capital, the rate of wage growth, as well as the parameters of budget policy.
Table 1. Mathematical formalization of the economic model of Krasnoyarsk (fragment)

| Model parameter                  | Regression equation of the model                   | Equation quality parameter |
|----------------------------------|---------------------------------------------------|----------------------------|
| Salary ($X_s$)                   | $X'_s = 5693.19 + 7.66 \cdot N_{av} \cdot R_{AS}$ | $R^2 = 0.82$               |
|                                  | where $N_{av}$ – average annual number of         |                            |
|                                  | employees; $R_{AS}$ – average monthly              | $F = 23.45$                |
|                                  | accrued salary                                    | $t(a) = 3.08$              |
|                                  |                                                   | $t(b) = 4.84$              |
| Property income ($X_{pi}$)       | $X'_{pi} = 4599.47 + 0.31 \cdot X'_{t-1_{pi}}$   | $R^2 = 0.75$               |
|                                  |                                                   | $F = 48.64$                |
|                                  |                                                   | $t(a) = 5.68$              |
|                                  |                                                   | $t(b) = 2.21$              |
| Social transfers ($X_{st}$)      | $X'_{st} = -409.19 + 15.39 \cdot N_{pens} \cdot R_{pens}$ | $R^2 = 0.99$               |
|                                  | where $N_{pens}$ – number of pensioners; $R_{pens}$ –    |                            |
|                                  | the average size of calculated pension            | $F = 991.48$               |
|                                  |                                                   | $t(a) = 2.01$              |
|                                  |                                                   | $t(b) = 3.149$             |
| Cash income from business activities ($X_{ci}$) | $X'_{t_{ci}} = 12005.72 + 0.253 \cdot Q'_{ret}$ | $R^2 = 0.76$               |
|                                  | $Q'_{ret}$ – retail trade turnover                 | $F = 35.71$                |
|                                  |                                                   | $t(a) = 17.33$             |
|                                  |                                                   | $t(b) = 1.89$              |
| Cash income of the population ($Y$) | $Y_{t} = 375.79 + 1.02 \cdot (X'_{s} + X'_{pi} + X'_{st} + X'_{t_{ci}})$ | $R^2 = 0.99$               |
|                                  |                                                   | $F = 541.17$               |
|                                  |                                                   | $t(a) = 2.18$              |
|                                  |                                                   | $t(b) = 23.26$             |

4. Conclusion
Economics and management are dynamic areas that require the constant adaptation of knowledge to new conditions. Social developments bring about new phenomena that require new theories or new solutions. Mathematical methods are rich enough to provide the basis for this dynamic knowledge. This model is actually a tool for determining how the change of any indicator will affect other parameters of the system development.

Thus, the model will assess the effectiveness of: resource allocation;
- managerial decision-making;
- of budget spending;
- investment projects;
- adopted target programs for the development urban district and so on.

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