Relativistic nuclear energy density functional approach to magnetic-dipole excitation

Tomohiro Oishi\textsuperscript{1}, Goran Kružić\textsuperscript{2}, and Nils Paar\textsuperscript{1}

\textsuperscript{1}Department of Physics, Faculty of Science, University of Zagreb, Bijenička c. 32, 10000 Zagreb, Croatia
\textsuperscript{2}Research department, Ericsson - Nikola Tesla, Krapinska 45, 10000, Zagreb, Croatia
E-mail: toishi@phy.hr

Abstract. Magnetic-dipole (M1) excitations of $^{18}$O and $^{42}$Ca nuclei are investigated within a relativistic nuclear energy density functional framework. In our last work \cite{1}, these nuclei are found to have unique M1 excitation and its sum rule, because of their characteristic structure: the system consists of the shell-closure core plus two neutrons. For a more systematic investigation of the M1 mode, we have implemented a framework based on the relativistic nuclear energy density functional (RNEDF). For benchmark, we have performed the RNEDF calculations combined with the random-phase approximation (RPA). We evaluate the M1 excitation of $^{18}$O and $^{42}$Ca, whose sum-rule value (SRV) of the M1 transitions can be useful to test the computational implementation \cite{1}. We also apply this RNEDF method to $^{208}$Pb, whose M1 property has been precisely measured \cite{2, 3, 4, 5}. Up to the level of the M1 sum rule, our result is in agreement with the experiments, except the discrepancy related with the quenching factors for $g$ coefficients.

1. Introduction
The M1 excitation is one of the fundamental phenomena triggered by the electro-magnetic interactions with atomic nuclei. This is the leading mode to couple the unnatural-parity states, i.e. $J^P = 1^+$ states. One can expect that, from the form of the M1 operator, its resonance can be useful for investigation of spin-orbit level splitting, tensor-force effect, pairing correlations in medium \cite{1}, etc. Noticeable collective motions, including scissors mode in deformed nuclei \cite{6}, can be also activated by the M1 excitation. Also, the analogy between the M1 and Gamow-Teller (GT) modes has attracted a special interest in recent studies \cite{7}. Indeed, zero component of the GT transition is almost identical to the isovector spin-M1 excitation. The GT resonance is expected as the dominant ingredient in neutrino-nucleus reactions in the energy scale of supernova, which can be a key to explain the origin of several elements. For an accurate evaluation of neutrino-nucleus reactions, certain theoretical framework, which can predict the GT as well as M1 excitations throughout the nuclear chart, has been on a serious demand. See also Refs. \textsuperscript{6, 8, 9} for more details on the M1 phenomena.

For an evaluation of the M1 mode without limitations on mass numbers, the mean field calculation based on the relativistic nuclear energy density functional (RNEDF) theory can be a suitable option \textsuperscript{10, 11, 12, 13}. As an important feature of the M1 mode, its transition mainly occurs between the spin-orbit partners, e.g. $f_{7/2} \rightarrow f_{5/2}$ for Ca isotopes. From the RNEDF effective Lagrangian, this spin-orbit splitting can be naturally concluded \textsuperscript{12, 13}. On the other
hand, for computations including unnatural-parity states, one should be careful for the residual interactions in RNEDF. These interactions do not contribute in the ground state (GS) with \( J^P = 0^+ \), and thus, GS data cannot provide a reference to determine their model parameters. In order to optimize those parameters, one needs to shift focus to the measurable process, where the residual interaction plays an essential role. The M1 excitation is indeed a suitable reference for this purpose.

In this work, we develop a RNEDF-based framework to compute the collective M1 excitation. We adopt the CGS-Gauss system of units in this article.

2. Formalism

In the present study, the RNEDF framework has been employed to describe the nuclear ground state properties within the relativistic mean field model at the Hartree level, and nuclear excitations are described using the relativistic random phase approximation (RPA). The respective formalism is derived from an effective Lagrangian density with four fermion contact interaction terms including the isoscalar-scalar, isoscalar-vector, and isovector-vector channels \([12, 13]\). The effective Lagrangian contains the free-nucleon and density dependent interaction terms, coupling of protons to the electromagnetic field, and the derivative term accounting for the leading effects of finite-range interactions necessary for a quantitative description of nuclear density distribution and radii. Detailed formalism and overview of the model calculations are given in Refs. \([13, 15, 16]\). In this work, we employ the point-coupling interaction DD-PC1 \([15, 16]\). The respective set of parameters for the RNEDF has been utilized in several applications, resulting in successful agreement with the experimental data on nuclear ground state properties and excitations.

In this work, we consider the collective M1 excitation of the \( ^{A}Z \) nucleus up to the one-body-operator level\(^{1}\)

\[
\hat{Q}_\mu(M1) \equiv \sum_{k \in A=N+Z} \hat{P}_\mu(k),
\]

where \( \hat{P}_\mu(k) \) with \( \mu = 0, \pm 1 \) is the single-particle M1 operator for the \( k \)th nucleon. That is \([17]\),

\[
\hat{P}_0 = \mu_N \sqrt{\frac{3}{4\pi}} \left( g_l \hat{l}_0 + g_s \hat{s}_0 \right), \quad \hat{P}_\pm = (\mp) \mu_N \sqrt{\frac{3}{4\pi}} \left( g_l \hat{l}_\pm + g_s \hat{s}_\pm \right),
\]

where \( \mu_N \) is the nuclear magneton, \( \hat{l}_0 = \hat{l}_z, \hat{l}_\pm = (\hat{l}_x + i \hat{l}_y)/\sqrt{2}, \hat{l}_- = \hat{l}_+ \), and similarly defined for spin operators. Considering the different \( g \) coefficients for protons and neutrons, \( \hat{Q}_\mu(M1) \) reads

\[
\hat{Q}_\mu(M1) = \mu_N \sqrt{\frac{3}{4\pi}} \left[ \sum_{i \in Z} \left( g_s^{(p)} \hat{s}_\mu(i) + g_l^{(p)} \hat{l}_\mu(i) \right) + \sum_{j \in N} \left( g_s^{(n)} \hat{s}_\mu(j) + g_l^{(n)} \hat{l}_\mu(j) \right) \right].
\]

Here \( g_l = 1 \) (0) and \( g_s = 5.586 (-3.826) \) for the proton (neutron) \([17, 18]\). Note that, utilizing the isospin \( \tau_0(k) = 2\tau_0(k) = \pm 1 \) for the \( k \)th proton (neutron), one can separate the collective M1 operator into the isoscalar (IS) and isovector (IV) terms. That is,

\[
\hat{Q}_\mu(M1) = \hat{Q}^{IS}_\mu(M1) + \hat{Q}^{IV}_\mu(M1) = \mu_N \sqrt{\frac{3}{4\pi}} \sum_k \left[ \left( g_s^{IS} \hat{s}_\mu(k) + g_l^{IS} \hat{l}_\mu(k) \right) + \tau_0(k) \left( g_s^{IV} \hat{s}_\mu(k) + g_l^{IV} \hat{l}_\mu(k) \right) \right],
\]

\(^1\) We neglect the meson-exchange-current effect, for which one needs to consider the relevant multi-body terms.
Table 1. Sum-rule values of M1 ($S_{M1}$) obtained in this work. The unit is $\mu_2^2$. The corresponding analytic result in Ref. [1] with the three-body model (3BM) is also shown for comparison.

| Method            | This work | Ref. [1] |
|-------------------|-----------|----------|
| $S_{M1}$ for $^{18}\text{O}$ | 2.73      | 2.79     |
| $S_{M1}$ for $^{42}\text{Ca}$ | 2.91      | 2.99     |
| $S_{M1}$ for $^{208}\text{Pb}$ | 52.86     | -        |

where $g_{l}^{IS} = g_{l}^{IV} = \frac{1}{2}$, $g_{s}^{IS} = \frac{g_{s}^{(p)} + g_{s}^{(n)}}{2} = 0.880$, and $g_{s}^{IV} = \frac{g_{s}^{(p)} - g_{s}^{(n)}}{2} = 4.706$. Notice that the IS spin-M1 response is often minor because of the cancellation of the $g$ coefficients.

For the $g$ coefficients, so-called quenching factors have been utilized in M1 calculations [19, 20, 21]: $g_{l,s}^{IS,IV} \rightarrow \zeta g_{l,s}^{IS,IV}$, where $\zeta$ often should be less than one for the agreement with experimental M1 data. This quenching effect is mainly from the second-order configuration mixing, or equivalently, coupling with two-particle-two-hole states [22]. In this article, however, we fix $\zeta = 1$, except the case with special mentioning.

The M1 excitation strength is evaluated as

$$B_{M1}(E_\gamma) = \sum_{\mu=0,\pm 1} \left| \left\langle f \mid \hat{Q}_\mu(\text{M1}) \mid i \right\rangle \right|^2,$$

(5)

where $E_\gamma = E_f - E_i$ is the excitation energy. For this evaluation, we utilize the random-phase approximation (RPA). Namely, the same procedure has been employed as in Refs. [14, 23], but in the present analysis the relativistic point coupling interaction is used, with the parameterization DD-PC1. In the transition matrix elements, the magnetic operator $\hat{Q}_\mu(\text{M1})$ is used. Then $B_{M1}(E_\gamma)$ is evaluated for the excitation from the $0^+$ ground state (GS), $|i\rangle$, to the $1^+$ excited state, $|f\rangle$. We assume the spherical symmetry in this work.

3. Result and Discussion

3.1. No-pairing sum rule in $^{18}\text{O}$ and $^{42}\text{Ca}$

In order to check the numerical implementation, sum rules of the excitation strength often provide a useful guidance. For the M1 mode, we can refer to one case-limited but available version of its sum rule in Ref. [1]. There, the non-energy-weighted sum-rule value (SRV) of the M1 excitation was evaluated for some specific systems, which consist of the shell-closure core and two valence neutrons or protons. Those systems include, e.g. $^{18}\text{O}$ and $^{42}\text{Ca}$. An advantage of that sum rule is that, when the pairing correlation between the valence nucleons is neglected, the SRV is determined analytically for the corresponding system of interest.

The numerical SRV is determined as

$$S_{M1} \equiv \int dE_\gamma B_{M1}(E_\gamma),$$

(6)

where $E_\gamma$ is the excitation energy. Our results for the M1 SRV are shown in Table 1 and in Figure 1 for $^{18}\text{O}$ and $^{42}\text{Ca}$. In our RNEDF calculations, the pairing energy is neglected, in order to keep consistency between the no-pairing result in Ref. [1]. For comparison, the results of the three-body model are also shown. We take strength values up to 60 MeV into account for this SRV. The actual $B_{M1}(E_\gamma)$ distribution is plotted in Figure 1. There is one significant peak each
in $^{18}$O and $^{42}$Ca. From the configuration results in RNEDF plus RPA calculations, it is found that these peaks are attributable to the neutron transitions of $d_{5/2} \rightarrow d_{3/2}$ and $f_{7/2} \rightarrow f_{5/2}$ in $^{18}$O and $^{42}$Ca, respectively. Namely, only the two valence neutrons are active for M1 transitions in these systems.

As displayed in Table 1, our SRV is obtained consistently to that in Ref. [1]. Thus, our implementation can be valid in the level of no-pairing sum rule. Note that, for $^{18}$O or $^{42}$Ca, the accurate measurement of M1 strength has not been achieved yet.

The excitation energy (position of the peak) of $B_{M1}(E_\gamma)$ shows an unnegligible difference between Ref. [1] and this work. This problem can be independent of the no-pairing SRV, but should be related with the spin-orbit splitting energies from the two models. We would like to remind that, in Ref. [1], the $B_{M1}(E_\gamma)$ distribution is shown to be sensitive to the choice of the pairing model. Also, in the RNEDF side, the M1-excitation energy is expected to depend on the residual interactions, especially pseudo-vector interaction. For more precise discussions, one needs to optimize these model parameters with respect to the experimental data. More details will be given in forthcoming publication [24].

3.2. Result in $^{208}$Pb
Following the consistency test of no-pairing SRV, we next apply our RNEDF framework to the $^{208}$Pb nucleus. This nucleus is one of the most precisely measured systems with respect to the M1 excitations. Its mean-excitation energy is measured as $E_\gamma = 7.3$ MeV [2,3,4,5]. The total
SRV of M1 is also evaluated as $S_{M1} = 15 - 20 \mu_N^2$ \[4, 5\].

In Figure 2, the calculated $B_{M1}$ distribution is plotted. The mean-excitation energy is in a good agreement with experiments. As a remarkable difference from $^{18}$O or $^{42}$Ca, the M1 strength shows two peaks. This result is consistent to the two-peak structure in the experimental data \[4, 5\]. The origin of this two-peak distribution is simple: in $^{208}$Pb, both the valence protons in the $0h_{11}/2$ orbit and neutrons in the $0i_{13}/2$ orbit can be available for the M1 transition. Note also that the other spin-orbit-partner levels are fully occupied, and thus, cannot be active.

The SRV is obtained as $S_{M1} = 52.86 \mu_N^2$ from our calculation. This value indeed overshoots the experimental result \[4, 5\]. Here we should mention that, in our calculations, the quenching factor $\zeta$ on the $g$ coefficient has been fixed as one. In some literature \[19, 20, 21\], however, it has been suggested that $\zeta \approx 0.6 - 0.7$ is necessary for consistency with experimental data. Notice that $S_{M1}$ as well as $B_{M1}(E_\gamma)$ should be reduced by $\zeta^2$. This procedure then concludes the result $S_{M1} \approx 20 \mu_N^2$, which is in a reasonable agreement with the experimental data. Note that the quenching factor does not change the excitation energy $E_\gamma$, which may be shifted only by changing the RNEDF parameters.

4. Summary and Outlook

We have developed the RNEDF framework using the RPA in order to investigate the properties of M1 excitations in nuclei. In the benchmark test for $^{18}$O and $^{42}$Ca, our result consistently reproduces the no-pairing SRV \[1\]. We have also investigated the $^{208}$Pb, and found that, except for the quenching effect, our results are consistent to the experimental energy and the SRV. We note that in the forthcoming study the role of the pseudo-vector interaction terms in the residual RPA interaction will be studied in more details. In addition, the pairing effects on M1 transitions will be studied using the relativistic quasiparticle random phase approximation.

Acknowledgments

We sincerely thank A. Tamii for suggestions from the experimental side. This work is supported by the QuantIXLie Centre of Excellence, a project co financed by the Croatian Government and European Union through the European Regional Development Fund, the Competitiveness and Cohesion Operational Programme (KK.01.1.1.01).

References

[1] Oishi T and Paar N 2019 Phys. Rev. C 100(2) 024308
[2] Holt R J, Jackson H E, Laszowski R M and Specht J R 1979 Phys. Rev. C 20(1) 93–114
[3] Köhler R, Wartena J A, Weigmann H, Mewissen L, Poortmans F, Theobald J P and Raman S 1987 Phys. Rev. C 35(5) 1646–1660
[4] Laszewski R M, Alarcon R, Dale D S and Hoblit S D 1988 Phys. Rev. Lett. 61(15) 1710–1712
[5] Birkhan J, Matsubara H, von Neumann-Cosel P, Pietralla N, Ponomarev V Y, Richter A, Tamii A and Wambach J 2016 Phys. Rev. C 93(4) 041302
[6] Heyde K, von Neumann-Cosel P and Richter A 2010 Rev. Mod. Phys. 82(3) 2365–2419 and references therein.
[7] Langanke K, Martinez-Pinedo G, von Neumann-Cosel P and Richter A 2004 Phys. Rev. Lett. 93(20) 202501 and references therein.
[8] Kneissl U, Pitz H and Zilges A 1996 Progress in Particle and Nuclear Physics 37 349 – 433 ISSN 0146-6410
[9] Pietralla N, von Brentano P and Lisetskiy A 2008 Progress in Particle and Nuclear Physics 60 225 – 282 ISSN 0146-6410
[10] Walecka J D 1974 Annals of Physics 83 491
[11] Boguta J and Bodmer A R 1977 Nuclear Physics A 292 413
[12] Reinhard P G 1989 Reports on Progress in Physics 52 439
[13] Vretenar D, Afanasjev A V, Lalazissis G A and Ring P 2005 Physics Report 409 101–259 and references therein.
[14] Paar N, Ring P, Nikšić T and Vretenar D 2003 Phys. Rev. C 67(3) 034312
[15] Nikšić T, Vretenar D and Ring P 2008 Phys. Rev. C 78(3) 034318
[16] Nikšić T, Paar N, Vretenar D and Ring P 2014 Computer Physics Communications 185 1808 – 1821 ISSN 0010-4655
[17] Ring P and Schuck P 1980 The Nuclear Many-Body Problems (Berlin and Heidelberg, Germany: Springer-Verlag)
[18] Eisenber J and Greiner W 1970 Nuclear Theory Volume 2: Excitation Mechanisms of the Nucleus (Amsterdam: North-Holland Publishing Company)
[19] von Neumann-Cosel P, Poves A, Retamosa J and Richter A 1998 Physics Letters B 443 1 – 6 ISSN 0370-2693
[20] Vesely P, Kvasil J, Nesterenko V O, Kleinig W, Reinhard P G and Ponomarev V Y 2009 Phys. Rev. C 80(3) 031302
[21] Nesterenko V O, Kvasil J, Vesely P, Kleinig W, Reinhard P G and Ponomarev V Y 2010 Journal of Physics G: Nuclear and Particle Physics 37 064034
[22] Takayanagi K, Shimizu K and Arima A 1988 Nuclear Physics A 481 313 – 332 ISSN 0375-9474
[23] Nikšić T, Marketin T, Vretenar D, Paar N and Ring P 2005 Phys. Rev. C 71(1) 014308
[24] Kružić G, Oishi T and Paar N in forthcoming publication.