Cooperative relaying with energy harvesting relays: Asymptotic analysis using extreme value theory for non-identically distributed RVs

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Abstract

This paper derives the limiting distribution of the maximum end-to-end signal to noise ratio (SNR) in a cooperative relaying (CR) scenario with multiple relays capable of both time splitting (TS) and power splitting (PS) based energy harvesting (EH). Considering an opportunistic relay selection scenario, we derive the limiting distribution of end-to-end SNR over independent and non-identical relay links between the source and the destination. Contrary to the majority of literature in communication which uses extreme value theory (EVT) to derive the statistics of extremes of sequences of independent and identically distributed (i.i.d.) random variables (RVs), we demonstrate how tools from EVT can be used to derive the limiting statistics of sequences of independent and non-identically distributed (i.n.i.d.) SNR RVs and hence derive the corresponding expressions for asymptotic ergodic and outage capacities. Finally, we present the utility of the asymptotic results for deciding the optimum TS and PS factors of the hybrid EH relays to minimize outage probability at the destination. Furthermore, we demonstrate how results from stochastic ordering can be utilized for simplifying the corresponding optimization problem.

Index Terms

cooperative relaying, energy harvesting, extreme value theory, non identical links, opportunistic scheduling
I. INTRODUCTION

Cooperative relaying (CR) has long been identified as one of the promising technologies capable of addressing issues like fading, poor coverage, increased power consumption etc. [1]. The benefits of CR come from the spatial diversity achieved by the virtual antenna array created by the cooperating nodes, which relay information between the source and destination. Unlike co-located antenna arrays, the components of the virtual array have a dynamic nature depending upon the state and availability of the cooperating nodes. The authors of [1]–[4] and their references therein analyze different aspects of CR schemes. These studies establish the merits offered by CR over other non-cooperative methods of communications.

Recently, CR schemes with energy harvesting (EH) nodes have gained significant attention [5], [6]. The idea of EH nodes in wireless communication brings out the possibility of self-sustaining nodes, harnessing energy from ambient energy sources in the environment as well as radio frequency (RF) EH from information signals (both from the desired transmitter and also from the interferers) [7], [8]. Amongst these alternatives, RF-EH is considered as a reliable and viable alternative to battery-powered wireless nodes and the corresponding theoretical and practical aspects have been extensively studied for several applications like cognitive radio, wireless sensor networks (WSN), personal area networks (PAN), device to device communication (D2D), CR etc. [3], [9]–[12]. Detailed surveys on simultaneous wireless information and power transfer (SWIPT) are available in works such as [13], [14]. Self-sustainable CR can be realised with EH in the source or relay nodes of the CR network. Here, the source (or a relay) which has continuous access to an energy source transfer RF energy to the energy-starved relay (or the source). The authors of [?], [15] analyses the performance of dual-hop CR models where the source is capable of harvesting energy from a single antenna relay node whereas [16] considers a multi-antenna EH relay node operating in mm wavebands. While [?] analyses the average symbol error rate at the destination, [16] studies the asymptotic (in terms of the number of relay antennas) energy harvested, spectral efficiency and system throughput at the destination.

Energy transfer in any SWIPT system follows either of the three EH protocols, namely, power splitting (PS) or time switching (TS) [17] or a hybrid of the two [18]. Energy received over a certain fraction of the time slot is used for EH and information processing (IP) is performed over the rest of the time slot in the TS protocol. Whereas in the PS protocol, fraction of the received energy over a time slot is divided and used for both EH and IP. The hybrid protocol performs
both TS and PS in all the time slots. The choice of EH protocol depends on the system hardware constraints and in turn decides the system performance. For example, in an EH system with TS protocol, the authors of [15], [19] discuss optimisation problems to choose the optimal time fraction for EH such that rate maximisation and outage minimisation are respectively achieved. The authors of [20], [21] study the performance of PS protocol in terms of outage probability, system throughput etc. Similarly, [22], [23] discusses the choice of optimal PS ratios in different relay systems. Both the TS and PS protocols can be derived as special cases of the hybrid protocol and hence the hybrid protocol is of particular interest from an analysis perspective. Hence, in this work we study the performance of an hybrid EH system. Since the energy transfer over the wireless medium suffers from large path loss, the amount and utility of energy harvested largely depends on the transmission scheme, channel conditions and the particular application considered. The authors in [24], [25] consider this harvested energy as a non-linear function of the received energy. Even though a non-linear model is more accurate than the linear model, a linear relationship between the received and harvested energy is more tractable for analysis and is considered in works like [3], [15], [19], [26], [27] and many more. Hence we also consider the linear EH model for simplicity.

Relay selection has been proven to be an effective method to enhance the end-to-end (e2e) performance of relaying systems [28], [29] while keeping the decoding complexity at the destination minimal. Hence, several works like [3], [27], [30], [31] study the performance of CR where the relay with the largest end-to-end signal to interference plus noise ratio (SINR)/signal to noise ratio (SNR) is chosen for information transmission to the destination. The asymptotic performance of the system when the number of relays grows to infinity facilitates easy comparison of the performance with respect to variations in other system parameters. For example, several works like [3], [27], [32], [33] rely on asymptotic analysis for understanding system performances for diverse applications. Hence, in this work, we study the asymptotic end-to-end SNR of an EH-CR system when a large number of EH relays are available between the source and destination node. Though the analysis is asymptotic, in Section V we show that the results hold fairly well for scenarios with moderate number of relays between the source and destination.

Extreme value theory (EVT) is the branch of statistics dealing with the asymptotics of extreme events (events with the extreme deviations from the median of probability distributions) [34]. Tools from EVT has been efficiently used for solving several problems in wireless communication
as well [3], [32], [33], [35]–[41]. Recently, [3] used EVT to analyze the asymptotic throughput of an opportunistic relay selection system when the relays are capable of harvesting energy from the desired signal as well as interferer signals. One important factor to notice in all these seminal works is that EVT has been used to derive the statistics of extremes over sequences of independent and identically distributed (i.i.d.) random variables (RVs). To the best of our knowledge, there is no previous work using EVT to derive the limiting statistics of extremes of a sequence of independent and non-identically distributed (i.n.i.d.) SNR RVs. While [41] derives the pdf of maxima of i.n.i.d. generalized-K variates using EVT, in it the pdf of each of the RV differ only by their mean values. Hence, the sequence of i.n.i.d. RVs could be easily transformed into a sequence of i.i.d. RVs by taking the difference of each RV with a common mean value and thus the analysis for i.i.d. RVs can be used for the analysis of the extreme values of the sequence. Several works like [3], [42]–[44] assume statistically identical source to relay and relay to destination links when analyzing opportunistic relay selection schemes in CR models. However, note that each of the relays is present at different location with respect to the source node. Thus the signal received at the different relays experiences independent channel fading and non-identical path loss effects owing to the differences in path lengths. Similarly, the channel gain over each of the relay to destination links will also be i.n.i.d.. Hence, in this work, we use EVT to derive the limiting distribution of end-to-end SNR in a dual-hop CR scenario with opportunistic relay selection and non-identical links over the EH relay nodes. We further highlight the need for a specific analysis of the statistics of the maximum of i.n.i.d. RVs using an example in Section.V.

Although the classical Fisher–Tippett theorem in EVT was proposed in the year 1928, the first work discussing the order statistics over sequences of i.n.i.d. RVs was published only 40 years later by Mejzler [45]. The typical approach in identifying the limiting distribution of the maximum or the minimum over a sequence of i.i.d. RVs using EVT includes the test to identify the maximum domain of attraction (MDA)\(^1\) of the common distribution function and then finding the parameters of the limiting distribution (the normalizing constants) [46]. The choice of these normalizing constants is not unique and there are several common choices for

\(^1\)It is known that under certain conditions, the limiting distribution of the maximum or minimum RV of a sequence of i.i.d. RVs will only be one of the three extreme value distributions (EVD) (Frechet, Gumbel or Weibull). A distribution function \(F\) is said to belong to the MDA of an EVD \(G\) if \(F^n(a_n x + b_n) \rightarrow G(x)\) for some normalizing constants \(a_n\) and \(b_n\).
all possible limiting distributions available in literature [46], [47]. However, certain additional technical conditions are required to ensure the convergence of the distribution of the extreme statistic to a non-degenerate distribution function for the case of i.n.i.d. RVs. These conditions require the statistician to make appropriate choices for the normalizing constants of the limiting distribution. Although works like [45], [48], [49] presented conditions under which the limiting distribution of the maximum or the minimum of sequences of i.n.i.d. RVs exists, to the best of our knowledge, none of them provided general methods for identifying the specific normalizing constants of the corresponding limiting distributions. Hence, the key challenge in characterizing the distribution of the maximum end-to-end SNR over non-identical relay links is to identify the normalizing constants of the limiting distribution. In this work, we derive one choice of normalizing constants which enables us to characterize the limiting distribution of the maximum end-to-end SNR in a decode and forward (DF) CR system where the relays harvest energy from the source node via hybrid EH protocol.

A. Contributions and Outline

In this work, we derive the limiting distribution of the maximum end-to-end SNR with i.n.i.d. source to destination links over dual-hop EH relays. We present the system model in Section.II and the detailed derivation of the limiting distribution is discussed in Section.III. The limiting statistics of the end-to-end SNR is then used to derive the asymptotic ergodic and outage capacities at the destination in Section.III-A. Next, we use results from stochastic ordering to study the ordering of the end-to-end SNR RV with respect to various system parameters in Section.III-B. The asymptotic results are then used to identify the optimal TS and PS parameter at the relays to minimize the outage probability at the destination. Furthermore, in Section.IV we demonstrate the utility of the stochastic ordering results in simplifying and speeding up the solution for this optimization problem. The validity of the results presented throughout the paper are demonstrated through simulation results in Section.V and finally, we conclude the work in Section.VI.

II. System Model

We consider a dual-hop CR scenario where a source communicates with the destination via energy-constrained relays equipped with EH circuitry. The direct link between the source and destination is assumed to be in a permanent outage similar to [3], [50], [51]. L EH relays
present between the source and destination path harvest energy from the source to decode the data for the destination node. The relay which maximizes the end-to-end SNR is then chosen to forward the data to the destination. Furthermore, similar to [3], [15], [16] we assume that our relays use all the energy harvested for sending data to the destination. Fig.1 shows such a system model where $S$ represents the source node, $D$ the destination node and $\{R_1, \cdots, R_L\}$ are the $L$ EH relays. Here, $\{g_{1,\ell}; \ell = 1, \cdots, L\}$ and $\{g_{2,\ell}; \ell = 1, \cdots, L\}$ represents the small scale fading channel gains of the source to $\ell^{th}$ relay and the $\ell^{th}$ relay to the destination path respectively. Similarly, $\{d_{1,\ell}; \ell = 1, \cdots, L\}$ and $\{d_{2,\ell}; \ell = 1, \cdots, L\}$ represents the distances from the source to $\ell^{th}$ relay and the $\ell^{th}$ relay to destination respectively. Further, we assume that all the channels experience independent Rayleigh fading with $g_{i,\ell} \sim \mathcal{CN}(0,1); i \in \{1, 2\}$ and $\ell \in \{1, \cdots, L\}$. Also, the channel is assumed to remain constant during the transmission of one block of information and it varies independently from one block to another. Data transmission from the source to destination happens over three phases over a time slot of length $T$, as shown in Fig.2. In the first phase, over a duration of $\alpha T$, the source transmits data to the relays and the relays harvest this energy. Here, $\alpha$ is the time splitting (TS) factor. Then, the signal received at the $\ell^{th}$ relay during $\alpha T$ is given by

$$y_{1,\ell} = \sqrt{P_s d_{1,\ell}^{-\zeta}} g_{1,\ell}s + w_{\ell},$$  

(1)

where $P_s$ is the transmit power of the source node, $s$ is the signal transmitted and $w_{\ell}$ is the additive noise at the $\ell^{th}$ relay. Furthermore, similar to [22] we assume that the effect of the RF to DC conversion noise on the received signal is negligible. Over the second phase of duration $(1-\alpha)T/2$, the source continues transmission to the relays. $\lambda$ (power splitting (PS) factor) fraction of this energy is harvested and the rest is used for information decoding (ID). The signal for ID is hence given by

$$\tilde{y}_{1,\ell} = \sqrt{1-\lambda} \left( \sqrt{P_s d_{1,\ell}^{-\zeta}} g_{1,\ell}s + w_{\ell} \right).$$  

(2)

Thus, the total energy harvested at the $\ell^{th}$ relay over the two phases is given by,

$$E_{\ell} = \eta \left( P_s d_{1,\ell}^{-\zeta} |g_{1,\ell}|^2 \right) T \left( \alpha + \lambda \frac{1-\alpha}{2} \right),$$  

(3)

where $\eta$ is the efficiency of the EH circuit. In the third phase, the relay selected for transmission in the second hop uses all the energy harvested from the previous phase to send the decoded information to the destination. Similar to [3], [18], [26], we assume that the processing power
required at the relays is negligible as compared to the power used for signal transmission. Thus, the transmit power available at the $\ell^{th}$ relay is given by

$$P_\ell = \frac{2E_\ell}{(1 - \alpha)T}. \quad (4)$$

Then, the signal received at the destination from the $\ell^{th}$ relay can be written as

$$y_{2,\ell} = \sqrt{P_\ell d_{2,\ell}^{-\zeta} g_{2,\ell} s} + w_D. \quad (5)$$

The SNR over the first and second hops of the $\ell^{th}$ relay is thus given by

$$\gamma_{1,\ell} = \frac{(1 - \lambda)P s d_{1,\ell}^{-\zeta}|g_{1,\ell}|^2}{\sigma_\ell^2}$$
and

$$\gamma_{2,\ell} = \frac{P_\ell d_{2,\ell}^{-\zeta}|g_{2,\ell}|^2}{\sigma_D^2}, \quad (7)$$

respectively. Here, $\sigma_\ell^2$ and $\sigma_D^2$ are the noise powers at the $\ell^{th}$ relay and destination respectively.

The end-to-end SNR of the DF network when relay $\ell$ is transmitting in the second hop is defined as

$$\gamma_{e2e,\ell} = \min(\gamma_{1,\ell}, \gamma_{2,\ell}). \quad (8)$$

With opportunistic relay selection, the index of the relay transmitting in the second hop can be written as

$$\hat{\ell} = \arg\max_{\ell = 1, \ldots, L} \gamma_{e2e,\ell}. \quad (9)$$

and

$$\gamma_{e2e,max}^L = \max\{\gamma_{e2e,\ell}; \ell = 1, \cdots, L\} \quad (10)$$
is the corresponding end-to-end SNR. Note that owing to the path loss component $d_i^{-\zeta}$; $i \in \{1, 2\}$, $\ell \in \{1, \cdots, L\}$ the sequence of RVs $\{\gamma_{e2e,\ell}; \ell \in \{1, \cdots, L\}\}$ are all independent but not identically distributed.

III. ASYMPTOTIC DISTRIBUTION OF END-TO-END SNR

In this section, we derive the asymptotic distribution of the maximum end-to-end SNR $\gamma_{e2e,max}^L$, for large $L$ i.e we derive the distribution of $\gamma_{e2e,max} := \lim_{L \to \infty} \gamma_{e2e,max}^L$. Note that, here we need to evaluate the distribution of the maximum over a sequence of i.n.i.d. RVs $\{\gamma_{e2e,\ell}; \ell \in \{1, \cdots, L\}\}$. As mentioned in the introduction, EVT has been widely used for the performance analysis of communication systems, but for the analysis of extreme statistics over i.i.d. RVs. In the section,
we elaborate on how to identify one possible choice of normalizing constants to characterize the limiting distribution of $\gamma_{\text{Z2e, max}}^L$ using EVT. D.G.Mejzler in [45] studied the limiting distribution of the maximal term of a sequence of RVs when each of the elements of the sequence are i.n.i.d.. Barakat et al. in [52] extends this result to the case of i.n.i.d. random vectors and the corresponding result is reproduced here for the case of univariate RVs. We first begin with the necessary uniformity assumptions (UAs) and then present the key result we utilize in Theorem.1.

Let $Z_{\text{max}}^L = \max\{Z_1, Z_2, \cdots, Z_L\}$ where $Z_\ell \sim F_\ell(z)$ for $\ell = 1, \cdots, L$, then the distribution function (df) of $Z_{\text{max}}^L$ can be explicitly written as

$$H_{\text{max}}^L(z) = \mathbb{P}(Z_{\text{max}}^L \leq z) = \prod_{\ell=1}^L F_\ell(z).$$  \hspace{1cm} (11)

The sequence $\{F_\ell(z)\}$ of dfs and the sequences $a_L \geq 0$ and $b_L$ of normalizing constants is said to satisfy the UAs for maximum vector $Z_{\text{max}}^L$ if

$$\max\{1 - F_\ell(a_Lz + b_L); a_Lz + b_L > \alpha(F_\ell); 1 \leq \ell \leq L\} \to 0 \text{ as } L \to \infty,$$

where $\alpha(F_\ell) := \inf\{z : F_\ell(z) > 0\} > -\infty$ and $\omega(F_\ell) := \sup\{z : F_\ell(z) < 1\} < \infty$. Also, for any fixed number $0 < t \leq 1$ and each sequence of integers $\{m_L\}_L$ such that $m_L < L, m_L \to \infty$ and $\frac{m_L}{L} \to t$ as $L \to \infty$, we have that

$$u(t, z) = \lim_{L \to \infty} \sum_{\ell=1}^{m_L} (1 - F_\ell(a_Lz + b_L)))$$

exists and is finite for all $0 \leq t \leq 1$, whenever it is finite for $t = 1$. Under the UA $C_1$ and $C_2$, we have the following theorem [52]:

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Theorem 1. Under the UA $C_1$ and $C_2$, a non-degenerate df $H_{\text{max}}(z)$ is the limiting distribution of $\frac{Z_{\text{max}}^L - b_L}{a_L}$ i.e.

$$H_{\text{max}}^L(a_Lz + b_L) = \prod_{\ell=1}^{L} F_{\ell}(a_Lz + b_L) \xrightarrow{D} H_{\text{max}}(z) \text{ as } L \to \infty,$$

(12)

where $\xrightarrow{D}$ stands for convergence in distribution if and only if

$$u(z) = u(1, z) = \lim_{L \to \infty} \sum_{\ell=1}^{L} (1 - F_{\ell}(a_Lz + b_L)) < \infty.$$

(13)

Moreover, $H_{\text{max}}(z)$ should have the form $H_{\text{max}}(z) = e^{-u(z)}$ and either (i) $\log H_{\text{max}}(z)$ is concave or (ii) $\omega_{\text{max}} = \omega(H_{\text{max}}(z))$ is finite and $\log H_{\text{max}}(\omega_{\text{max}} - e^z)$ is concave or (iii) $\alpha_{\text{max}} = \alpha(H_{\text{max}}(z))$ is finite and $\log H_{\text{max}}(\alpha_{\text{max}} - e^z)$ is concave where $z > 0$ in (ii) and (iii).

Proof. Please refer [52] for the proof.

From the above theorem, it is clear that if we can find normalizing constants $a_L$ and $b_L$ satisfying (13) for $F_{\ell}(\gamma) = F_{\gamma_{e2,\ell}}(\gamma)$, then we can identify the form of $F_{\gamma_{e2,\text{max}}}(\gamma)$ which is the limiting distribution of the RV $\gamma_{e2,\text{max}}$. We begin by characterizing the distribution $F_{\gamma_{e2,\ell}}(\gamma)$. According to the system model discussed in Section II, $|g_{1,\ell}|^2$ will be exponentially distributed with unit mean. Then, by making use of the scaling properties of exponential random variables, we have $\gamma_{1,\ell} \sim \text{Exp}(\theta_{\ell})$, where $\text{Exp}(\theta)$ represents the exponential distribution with scale parameter $\theta$ and

$$\frac{1}{\theta_{\ell}} = \frac{(1 - \lambda)P_d d_{1,\ell}^{-\zeta}}{\sigma_{\ell}^2}.$$

(14)

Similarly, note that $\gamma_{2,\ell} = \gamma_{1,\ell} \times \varphi_{2,\ell}$, where

$$\varphi_{2,\ell} = \frac{\eta \sigma_{\ell}^2}{1 - \lambda} \left( \lambda + \frac{2\alpha}{1 - \alpha} \right) \frac{d_{2,\ell}^{-\zeta} |g_{2,\ell}|^2}{\sigma_D^2}.$$

(15)

Hence, $\gamma_{2,\ell}$ is the product of two exponential random variables with scale parameters $\theta_{\ell}$ and $\nu_{\ell}$ where

$$\frac{1}{\nu_{\ell}} = \frac{\eta \sigma_{\ell}^2}{1 - \lambda} \left( \lambda + \frac{2\alpha}{1 - \alpha} \right) \frac{d_{2,\ell}^{-\zeta}}{\sigma_D^2}.$$

(16)

Thus, we have the following theorem giving the distribution of $\gamma_{e2,\ell}$:

Theorem 2. The CDF of the RV $\gamma_{e2,\ell} = \min \{ \gamma_{1,\ell}, \gamma_{1,\ell} \times \varphi_{2,\ell} \}$ where $\gamma_{1,\ell} \sim \text{Exp}(\theta_{\ell})$ and $\varphi_{2,\ell} \sim \text{Exp}(\nu_{\ell})$ is given by

$$F_{\gamma_{e2,\ell}}(\gamma) = 1 - \theta_{\ell} \sum_{k=0}^{\infty} \frac{(-\nu_{\ell})^k}{k!} \gamma E_k(\theta_{\ell}\gamma),$$

(17)
where \( E_k(.) \) is the exponential integral function.

**Proof.** Please refer Appendix A for the proof.

Using the above CDF, we derive the asymptotic distribution of \( \gamma_{e2e,max}^L \) and the results are presented in the following theorem:

**Theorem 3.** The asymptotic distribution of \( \gamma_{e2e,max}^L \) is given by

\[
F_{\gamma_{e2e,max}}(\gamma) = \exp(-u(\gamma)) \quad \text{where}
\]

\[
u(\gamma) = \sum_{\ell=1}^L \exp(-\theta_\ell \gamma - \nu_\ell), \quad \theta_\ell = \frac{\sigma_\ell^2}{(1 - \lambda)P_\lambda d_{1,\ell}}, \quad \nu_\ell = \frac{(1 - \lambda)(1 - \alpha)\sigma_D^2}{\eta\sigma_D^2(2\alpha + \lambda(1 - \alpha))d_{2,\ell}}.
\]

**Proof.** Please refer to Appendix B for the proof.

Note that the asymptotic distribution of \( \gamma_{e2e,max}^L \) is far easier to evaluate than the exact distribution of the maximum which is given by the product of \( L \) distribution functions, each of the form of (17). Such a simplified expression proves to be propitious for analyzing system performance and resource planning. For the special case of i.i.d. links from the source to the relays and the relays to the destination with \( \theta_\ell = \theta \) and \( \nu_\ell = \nu \ \forall \ell \), the asymptotic distribution will be the Gumbel distribution with scale parameter \( \beta = \frac{1}{\theta} \) and location parameter \( \mu = \frac{\log(L) - \nu}{\theta} \).

### A. Asymptotic ergodic and outage capacity

Given that we have characterized the distribution of the asymptotic end-to-end SNR, we proceed to derive the asymptotic ergodic capacity defined as

\[
C_{e2e,max} = \lim_{L \to \infty} \frac{1}{2} \mathbb{E} \left[ \log_2 \left( 1 + \gamma_{e2e,max}^L \right) \right].
\]

Here, the factor \( \frac{1}{2} \) accounts for the transmission of information happening over two phases [53]. Since we have already proved that \( \gamma_{e2e,max}^L D_{\gamma_{e2e,max}} \), the asymptotic ergodic capacity can be evaluated as

\[
C_{e2e,max} = \frac{1}{2} \mathbb{E} \left[ \log_2 \left( 1 + \gamma_{e2e,max} \right) \right].
\]

This result can be easily derived by applying continuous mapping theorem and monotone convergence theorem to the expression in (20). The steps are very similar to the proof in Appendix...
B of [38] and hence we do not repeat them here. Now, the expression in (21) can be evaluated via numerical integration of the following expression:
\[
C_{e,\text{max}} = \frac{1}{2} \times \int_0^\infty \log_2 (1 + \gamma) f_{\gamma_{e,\text{max}}}(\gamma) \, d\gamma.
\] (22)

Here, \( f_{\gamma_{e,\text{max}}} \) is the pdf of \( \gamma_{e,\text{max}} \) and is given below:
\[
f_{\gamma_{e,\text{max}}} = \exp \left[- \sum_{\ell=1}^L e^{-(\theta_{\ell} \gamma + \nu_{\ell})} \right] \times \sum_{\ell=1}^L \theta_{\ell} e^{-(\theta_{\ell} \gamma + \nu_{\ell})}.
\] (23)

The effective information transmission time decides the achievable throughput which in this case is defined as follows [53]
\[
R_{e,\text{max}} = (1 - \alpha) C_{e,\text{max}}.
\] (24)

Similarly, we can characterize the asymptotic outage capacity of the system using the statistics of the maximum end-to-end SNR. Outage capacity is defined as the maximum constant rate that can be maintained over the fading blocks with a specified outage probability [53]. Here, the outage capacity is given by
\[
C_{e,\text{max}}^{\text{out}} = \frac{1}{2} \left[ 1 - P_{e,\text{max}}^{\text{out}}(\gamma_{\text{th}}) \right] \log_2 (1 + \gamma_{\text{th}}).
\] (25)

Here, \( P_{e,\text{max}}^{\text{out}}(\gamma_{\text{th}}) \) is the probability of outage for a threshold of \( \gamma_{\text{th}} \); i.e \( P_{e,\text{max}}^{\text{out}}(\gamma_{\text{th}}) = \mathbb{P}(\gamma_{e,\text{max}} \leq \gamma_{\text{th}}) \). Hence, the outage capacity can be easily derived from the asymptotic CDF of \( \gamma_{e,\text{max}} \) as is given by
\[
C_{e,\text{max}}^{\text{out}} = \frac{\log_2 (1 + \gamma_{\text{th}})}{2} F_{\gamma_{e,\text{max}}} (\gamma_{\text{th}}).
\] (26)

B. Ordering of asymptotic end-to-end SNR

Stochastic ordering allows ordering of RVs with respect to variations in their parameters. An RV \( X \) is said to be stochastically smaller than an RV \( Y \) if
\[
\mathbb{P}(X > z) \leq \mathbb{P}(Y > z), \ \forall z \in \mathbb{R},
\] (27)
and is written as \( X \leq_{st} Y \) [54]. Such an ordering of SNR RVs allows us to study the variations of SNR and hence functions of SNR with changes in different channel parameters. This will be

\[\text{Note that this is particularly useful for slowly varying channels, where the instantaneous SNR remains constant over a large number of symbols.}\]
highly useful for system planning and resource allocation without much computational burden every time a decision is to be made. Stochastic ordering has been effectively used for analysing the performance of various communication systems in works like [55]–[57]. Furthermore, we demonstrate one utility of these stochastic ordering results to simplify the solution of a resource allocation problem in the next section.

In the following subsections, we establish the stochastic ordering of $\gamma_{e2e,max}$ with respect to variations in the source transmit power $P_s$, noise variance $\sigma^2$, the TS parameter $\alpha$ and the PS parameter $\lambda$. For all further analysis we make the assumption that $\sigma^2_D = \sigma^2_\ell = \sigma^2 \forall \ell \in \{1, \cdots, L\}$.

1) Ordering with respect to $P_s$: Let $X_1$ and $X_2$ be the RVs representing the asymptotic maximum end-to-end SNR with transmit power $P_1$ and $P_2$ respectively. We further assume that $P_1 > P_2$ and the rest of the parameters are considered to be the same for both the RVs. Hence, we have $X_1$ with parameters $\{\theta^{(1)}_\ell, \nu_\ell; \ell = 1, \cdots, L\}$ and $X_2$ with parameters $\{\theta^{(2)}_\ell, \nu_\ell; \ell = 1, \cdots, L\}$ where $\theta^{(i)}_\ell = \frac{\sigma^2_\ell}{(1-\lambda)P_i d_{i,\ell}}$ for $i \in \{1, 2\}$. $X_2 \leq_{st} X_1$ if the following is true:

$$\exp \left( - \sum_{\ell=1}^{L} \exp \left( -\theta^{(2)}_\ell z - \nu_\ell \right) \right) \geq \exp \left( - \sum_{\ell=1}^{L} \exp \left( -\theta^{(1)}_\ell z - \nu_\ell \right) \right)$$

i.e.

$$\sum_{\ell=1}^{L} \exp \left( -\frac{\tilde{\theta}_\ell z}{P_2} - \nu_\ell \right) \leq \sum_{\ell=1}^{L} \exp \left( -\frac{\tilde{\theta}_\ell z}{P_1} - \nu_\ell \right),$$

where

$$\theta^{(i)}_\ell = \frac{\tilde{\theta}_\ell}{P_i}.$$  

Upon further rearrangement, (29) can be re-written as follows

$$\sum_{\ell=1}^{L} \exp(-\nu_\ell) \left\{ \exp \left( -\frac{\tilde{\theta}_\ell z}{P_2} \right) - \exp \left( -\frac{\tilde{\theta}_\ell z}{P_1} \right) \right\} \leq 0.$$  

(31)

Note that for $P_1 > P_2$ and $z > 0$ term 1 will be negative for all values of $\ell$ and hence the inequality in (31) holds for all choices of $\tilde{\theta}_\ell, z$ and $\nu_\ell$. Thus, we conclude that $X_2$ is stochastically smaller than $X_1$ when $P_2 < P_1$. Note that this observation is intuitive since the end-to-end SNR is expected to increase with increase in transmit power. Nevertheless, this results reaffirm the

Note that the RVs $X_1$ and $X_2$ will have the same set of parameters $\{\nu_\ell; \ell = 1, \cdots, L\}$ since they are independent of the source transmit power.
utility of our asymptotic results in deriving meaningful inferences about the system performance with respect to various system parameters.

2) Ordering with respect to $\alpha$: Let $X_1$ and $X_2$ be the RVs representing the asymptotic maximum end-to-end SNR with TS parameter $\alpha_1$ and $\alpha_2$ respectively. We further assume that $\alpha_1 > \alpha_2$, RV $X_1$ has parameters $\{\theta_\ell, \nu^{(1)}_\ell ; \ell = 1, \cdots, L\}$ and $X_2$ has parameters $\{\theta_\ell, \nu^{(2)}_\ell ; \ell = 1, \cdots, L\}$ where $\nu^{(i)}_\ell = \frac{(1-\lambda)(1-\alpha_i)}{\eta d_{2,\ell}^2}$ for $i \in \{1, 2\}$. Here, $X_2 \preceq_{st} X_1$ if the following is true:

$$\sum_{\ell=1}^{L} \exp (-\theta_\ell z - \tilde{\nu}_\ell \tilde{\alpha}_2) \leq \sum_{\ell=1}^{L} \exp (-\theta_\ell z - \tilde{\nu}_\ell \tilde{\alpha}_1),$$

where

$$\nu^{(i)}_\ell = \tilde{\nu}_\ell \tilde{\alpha}_i, \quad \tilde{\nu}_\ell = \frac{(1-\lambda)}{\eta d_{2,\ell}^2} \text{ and } \tilde{\alpha}_i = \frac{1-\alpha_i}{2\alpha_i + \lambda(1-\alpha_i)}.$$ (33)

Upon further rearrangement, (32) can be re-written as follows

$$\sum_{\ell=1}^{L} \exp(-\theta_\ell) \left\{ \exp(-\tilde{\nu}_\ell \tilde{\alpha}_2) - \exp(-\tilde{\nu}_\ell \tilde{\alpha}_1) \right\} \leq 0.$$ (34)

Further analysis shows that for, $\alpha_2 < \alpha_1$, $\tilde{\alpha}_2 > \tilde{\alpha}_1$. Hence, term 2 will be negative for all values of $\ell$ and hence the inequality in (34) holds for all choices of $\tilde{\nu}_\ell$ and $\theta_\ell$. Thus, we conclude that $X_2$ is stochastically smaller than $X_1$ when $\alpha_2 < \alpha_1$. This means that the maximum end-to-end SNR increases with increase in TS factor i.e with increase in time over which energy is harvested. However note that we cannot choose $\alpha = 1$ since this would mean that the whole time slot is utilized for energy harvesting and no time is allocated for information transfer. Hence, practically we are constrained to choose a maximum value of $\alpha$ that still reserves time for information transmission both from the source to the relay and from the relay to the destination.

3) Ordering with respect to $\sigma^2$: Let $X_1$ and $X_2$ be the RVs representing the asymptotic maximum end-to-end SNR with noise power $\sigma_1^2$ and $\sigma_2^2$ respectively. We further assume that $\sigma_1^2 < \sigma_2^2$, RV $X_1$ has parameters $\{\theta^{(1)}_\ell, \nu_\ell; \ell = 1, \cdots, L\}$ and $X_2$ has parameters $\{\theta^{(2)}_\ell, \nu_\ell; \ell = 1, \cdots, L\}$ where $\theta^{(i)}_\ell = \frac{\sigma_\ell^2}{(1-\lambda)P_s d_{1,\ell}^2}$ for $i \in \{1, 2\}$. Following the analysis similar to the case of $P_s$ we can infer that $X_2$ is stochastically smaller than $X_1$ in this case.

4) Ordering with respect to $\lambda$: Let $X_1$ and $X_2$ be the RVs representing the asymptotic maximum end-to-end SNR with PS parameter $\lambda_1$ and $\lambda_2$ respectively. We further assume that $\lambda_1 < \lambda_2$,
RV $X_1$ has parameters $\{\theta^{(1)}_\ell, \nu^{(1)}_\ell; \ell = 1, \cdots, L\}$ and $X_2$ has parameters $\{\theta^{(2)}_\ell, \nu^{(2)}_\ell; \ell = 1, \cdots, L\}$ where

$$\theta^{(i)}_\ell = \frac{\sigma^2_\ell}{(1 - \lambda_i) P_s d_{1,\ell}^c} \quad \text{and} \quad \nu^{(i)}_\ell = \frac{(1 - \lambda_i)(1 - \alpha)\sigma^2_D}{\eta \sigma^2_\ell (2\alpha + \lambda_i (1 - \alpha)) d_{2,\ell}^c}. \quad (35)$$

Furthermore, we define

$$\tilde{\theta}_\ell = \frac{\tilde{\theta}_\ell}{1 - \lambda_i}, \quad \tilde{\nu}_\ell = \tilde{\nu}_\ell \tilde{\lambda}_i, \quad \tilde{\nu}_\ell = \frac{d_{2,\ell}^c (1 - \alpha)}{\eta} \quad \text{and} \quad \tilde{\lambda}_i = \frac{1 - \lambda_i}{2\alpha + \lambda_i (1 - \alpha)} \quad \text{for } i \in \{1, 2\}. \quad (36)$$

Here, $X_2 \leq_{st} X_1$ if the following is true:

$$\sum_{\ell=1}^{L} \underbrace{\exp(-\tilde{\theta}_\ell z)}_{\text{Term 3}} \underbrace{\exp(-\tilde{\nu}_\ell \tilde{\lambda}_2)}_{\text{Term 4}} - \underbrace{\exp(-\tilde{\theta}_\ell z)}_{\text{Term 5}} \underbrace{\exp(-\tilde{\nu}_\ell \tilde{\lambda}_1)}_{\text{Term 6}} \geq 0. \quad (38)$$

Unlike the case of TS factor $\alpha$, the ordering with respect to $\lambda$ does not remain the same for all values of $\theta_\ell$ and $\nu_\ell$, $\ell = 1, \cdots, L$. Here, note that for $\lambda_1, \lambda_2 \in (0, 1)$ and $\lambda_1 < \lambda_2$, term 3 is smaller than term 5 and term 4 is larger than term 6. Hence the sign of left-hand-side (LHS) of (38) will depend on whether Term 3 (or Term 5) dominates Term 4 (or Term 6) or otherwise in each of the $\ell$ sum terms. Note that in the high SNR scenario $\tilde{\theta}_\ell$ will be small since $\tilde{\theta}_\ell$ is inversely proportional to $\gamma_s = \frac{P_s}{\sigma^2}$. Now if $\lambda_i$ is not very close to 1, the product terms in (38) will be dominated by term 4 and term 6. If $\lambda_i$ is close to 1, the denominator of the exponent of terms 3 and 5 tends to zero and hence term 3 and 5 approaches zero. Similarly, for the low SNR regime $\tilde{\theta}_\ell$ will be large and hence smaller values of $\lambda_i$ will increase the value of the product terms. However, for all the cases in between the high and low SNR values we cannot have a general conclusion about the ordering of $\gamma_{e2e,\text{max}}$ with respect to the variations in the PS factor $\lambda$.

Given that we have established the ordering of $\gamma_{e2e,\text{max}}$ with respect to variations in $P_s, \alpha$ and $\sigma^2$ we can extend this to the case of asymptotic ergodic capacity by making use of the following result from the theory of stochastic ordering.

**Lemma 1.** RV $X$ is stochastically less than or equal to RV $Y$ if and only if the following holds for all increasing functions $\phi(.)$ for which the expectations exist:

$$\mathbb{E}[\phi(X)] \leq \mathbb{E}[\phi(Y)]. \quad (39)$$
The above lemma is discussed in detail in chapter 1 of [54]. Using Lemma (1) we can easily extend the ordering results in section III-B1-III-B4 to the ordering of asymptotic ergodic capacity $C_{e2e,max}$. Thus, this lemma allows us to make inferences about the resultant ergodic capacity with respect to variations in the system parameters easily. Note that such observations are otherwise difficult to be derived directly from the integral expressions for ergodic capacity as given in (22).

IV. OPTIMAL TS AND PS PARAMETER

Note that the statistics of the end-to-end SNR depends on the choice of TS and PS parameters. In this section, we discuss one possible method to choose the optimal TS and PS parameter for minimizing outage probability. More precisely, we look at the following optimization problem:

$$\min_{\alpha, \lambda} \exp\left(-\sum_{\ell=1}^{L} \exp\left(-\theta_{\ell} \gamma_{\ell} - \nu_{\ell}\right)\right)$$

subject to

$$0 \leq \alpha \leq \alpha^{\text{max}},$$

$$0 \leq \lambda \leq \lambda^{\text{max}},$$

where $\alpha^{\text{max}}$ and $\lambda^{\text{max}}$ are the maximum value of $\alpha$ and $\lambda$ feasible within the hardware constraints of the system. Note that $\alpha = 1$ or $\lambda = 1$ would mean the relays only harvest energy but does not transmit information and hence cannot be the optimal choice of the TS parameter. The above minimization problem is equivalent to solving the following maximization problem:

$$\max_{\alpha, \lambda} \sum_{\ell=1}^{L} \exp\left(-\theta_{\ell} \gamma_{\ell} - \nu_{\ell}\right)$$

subject to

$$0 \leq \alpha \leq \alpha^{\text{max}},$$

$$0 \leq \lambda \leq \lambda^{\text{max}}.$$
function evaluated for $\alpha = \alpha^\ast$. There are a number of algorithms for finding the optima of non convex, nonlinear constrained/unconstrained optimization problems. Here, we propose to use the method of sequential quadratic programming (sqp) to find the optimum solution [58], which under specific conditions is proven to demonstrate faster convergence as compared to algorithms like the interior point method. Furthermore, we can make use of the stochastic ordering results in high and low SNR regimes to do clever initialization of the sqp algorithm and thus accelerate the convergence of the algorithm. More details regarding the initialization of the algorithm are presented along with the simulation results.

V. SIMULATION RESULTS

Before we begin with the detailed discussion of the simulation results for the i.n.i.d. scenario, we further motivate the importance of the analysis through an example. In most of the practical scenarios, we are interested in analyzing the maximum or minimum statistics over a sequence of i.n.i.d. RVs. However, several works in the literature assume them to be i.i.d. RVs and the right method for approximating a sequence of i.n.i.d. RVs with a sequence of i.i.d. RVs is an interesting problem in itself. In the following two experiments, we consider $L$ i.n.i.d. RVs of the form $\gamma_{e,\ell}$ each with CDF given by Theorem.2. We choose the parameters as $\nu_\ell = \nu = 0.2 \forall \ell \in \{1, \cdots, L\}$ and $\theta_\ell$ to be non-identical across the RVs with values between 1 and 3. Here, the solid curve in Fig.3 and Fig.4 shows the empirical CDF of the maximum of such RVs over sequences of length $L = 64$. Now, if we approximate this sequence of RVs with an sequence of i.i.d. RVs each with $\theta_\ell = \theta$ chosen to be the mean of the non-identical parameters, the limiting distribution of the maximum will be a Gumbel distribution with location and scale parameters as discussed.
in Section III. This approximate CDF generated using the theoretical distribution function of the Gumbel RV is plotted as the red dashed curve in the figures. Here we notice that this approximation (of the sequence of i.n.i.d. RVs with the sequence of i.i.d. RVs by choosing the parameter to be the mean of the non-identical parameters) is a good choice for the first case whereas, in the second case, the theoretical approximation and the true simulated CDF are not close. In other words for i.n.i.d. RVs an approximation which works well for a set of values may be poor for another set of values. Hence, one should directly derive the limiting distribution of the maximum of the sequence of i.n.i.d. RVs to derive more accurate inferences about system performance and utility.

Next, we present results of simulation experiments to validate the results derived in the previous sections. Here, we have chosen the noise power to be identical at all the relays as well as the destination and we define $\gamma_s := \frac{P_s}{\sigma^2}$ where $\sigma^2$ is the noise power. Throughout the simulations, we have chosen $\gamma_s = 25$ dBm, $\eta = 0.9$, $L = 20$, $\alpha = 0.3$, $\lambda = 0.4$ and $\gamma_{th} = 1$ dB unless stated otherwise. Furthermore, we assume that the straight line distance between source and destination is normalized to unity. Hence, the distance from the source to relays and relays to destination are uniformly chosen within intervals $(0.5, 0.8)$ and $(0.5, 0.7)$ respectively. Here, Fig.5 and Fig.6 show the simulated and theoretical CDF of $\gamma_{e2e,\text{max}}$ for different values of $L$ and $\gamma_s$. From the figures, we see that the asymptotics hold good even when the maximum SNR is evaluated over a smaller number of relays, $L$. Furthermore, we can see that the convergence of the exact distribution of the minimum to the asymptotic distribution improves with an increase in $L$. Fig.7 shows the theoretical value of outage capacity $C_{e2e,\text{max}}^{\text{out}}$ with respect to variations in the TS and
PS parameters. Here we notice that the outage capacity decreases significantly with the increase in the PS parameter $\lambda$.

Next, we present simulation results to validate the convergence of the ergodic capacity to the proposed value of asymptotic ergodic capacity. Fig.8 shows the simulated and theoretical values of achievable throughput for different values of $\gamma_s$ and $L$. We see that the simulated and theoretical values are in good agreement for all values of $L > 10$. This validates the utility of the asymptotic results in many system planning problems where the achievable throughput is an important factor. The variation in the theoretical values of achievable throughput for different values of TS and PS parameters are presented in Fig.9. It is to be noted that the achievable throughput does not show the same trend as the outage capacity but decreases with increase in $\alpha$ beyond a certain value.

The ordering results in Section III-B are verified in Fig.10 for $L = 15$. For clarity in presentation, we have plotted only the theoretical curves of CDF in Fig.10(b). Next, in Fig.11 and 12 we present the solutions for the optimization problem to choose the optimal TS and PS factors. Here the optimal solutions ($\alpha^*, \lambda^*$) is shown using a red star in the figure. In Fig.11 we show the log of outage probability and the corresponding choice of optimal $\alpha$ and $\lambda$ for $\gamma_s = 4$ dBm and a threshold of $\gamma_{th} = 15$ dBm. As discussed in Section IV for a low SNR scenario, only one factor of each of the product terms in (38) dominates and smaller values of $\lambda$ will increase the objective and hence decrease the outage. Hence we propose that $\lambda = 0$ will be a
good initialization. In fact, for the very low SNR scenario, we observe that the optimal choice corresponds to the TS relaying protocol. Also, it was observed that in these cases initializing $\lambda = 0$ for the sqp algorithm reduces the number of iterations by half when compared to the number of iterations required for convergence when the initialization is $\lambda = 1$. This emphasizes the utility of our ordering results and further reiterates the fact that the right initialization can ensure faster convergence.
Next, in Fig. 12 we show another example of the optimization problem for $\gamma_s = 40$ dBm and the same threshold of $\gamma_{th} = 15$ dBm. From the stochastic ordering results, we know that here optimal $\lambda$ can be away from zero but not equal to one as well. Hence, we decide that $\lambda = 0.5$ can be a good initialization. Here also we observe slightly faster convergence with this initialization as compared to other initializations away from the optimal solution. For any other SNR scenario we propose to use $\lambda = 0.5$ as the initialization since the solution has to be between the above two cases.

For the outage-optimal TS and PS factors we compare the outage probability and the rate achieved $R_{e2e,max}$ (computed using (24)) for the three EH protocols in Fig.13 and Fig.14 respectively. From Fig.13, we observe that the hybrid protocol and the TS protocol achieves identical performance in terms of outage probability. However from Fig.14 we understand that both the hybrid as well as TS protocol achieves lower values of outage probability by sacrificing the rate $R_{e2e,max}$. Also, observe that for the low SNR scenario, the rate performance of hybrid EH protocol is more identical to the PS protocol whereas as the SNR increases, the hybrid protocol achieves the same rate as achieved from the TS protocol. Note that for each SNR value the optimal $\alpha^*$ and $\lambda^*$ that minimizes outage probability is used to compute the outage probability and rate in each of these figures. Furthermore, note that this trend is observed for an outage threshold of $\gamma_{th} = 1$ dB and the SNR range for which the hybrid protocol performs similar to TS and PS strategies depends upon the outage threshold to be maintained.
VI. Conclusions

This paper derives the asymptotic distribution of the maximum end-to-end SNR in an CR scenario with opportunistic selection of EH relays. We demonstrate the viability of a particular choice of normalizing constants to characterize this limiting distribution of the maximum over a sequence of i.n.i.d. RVs using EVT. Furthermore, we show the utility of these results in deciding the optimum TS and PS factors for minimizing the outage probability at the destination. The solution for this optimization problem is further simplified using results from stochastic ordering.

APPENDIX A

Proof for Theorem 2

Note that \( \gamma_{e2e,\ell} = \gamma_{1,\ell} \min(1, \varphi_{2,\ell}) \). Now, let \( Y = \min(1, \varphi_{2,\ell}) \). Then, the CDF of \( Y \) is given by

\[
F_Y(y) = \begin{cases} 
1, & y \geq 1, \\
1 - \exp(-y \nu_{\ell}), & 0 \leq y \leq 1, \\
0, & \text{o.w.}
\end{cases}
\]

(42)

Hence, the CDF of \( \gamma_{e2e,\ell} \) is given by \( F_{\gamma_{e2e,\ell}}(\gamma) = P(\gamma_{1,\ell} Y \leq \gamma) \). Thus,

\[
F_{\gamma_{e2e,\ell}}(\gamma) = \int_{0}^{\gamma} F_{Y} \left( \frac{\gamma}{x_{1}} \right) f_{\gamma_{1,\ell}}(x_{1}) \, dx_{1}.
\]

(43)

Now from (42), \( F_{Y} \left( \frac{\gamma}{x_{1}} \right) \) will be unity for all values of \( x_{1} > \gamma \) and hence we can rewrite the previous integral as follows:

\[
F_{\gamma_{e2e,\ell}}(\gamma) = \int_{0}^{\gamma} f_{\gamma_{1,\ell}}(x_{1}) \, dx_{1} + \int_{\gamma}^{\infty} (1 - \exp(-\nu_{\ell} \gamma/x_{1})) f_{\gamma_{1,\ell}}(x_{1}) \, dx_{1}
\]

(44)

\[
= \int_{0}^{\gamma} f_{\gamma_{1,\ell}}(x_{1}) \, dx_{1} + \int_{\gamma}^{\infty} \exp(-\nu_{\ell} \gamma/x_{1}) f_{\gamma_{1,\ell}}(x_{1}) \, dx_{1}.
\]

(45)

Since \( \gamma_{1,\ell} \sim \text{Exp}(\theta_{\ell}) \), \( F_{\gamma_{e2e,\ell}}(\gamma) \) can now be written as,

\[
F_{\gamma_{e2e,\ell}}(\gamma) = 1 - \theta_{\ell} \int_{\gamma}^{\infty} \exp \left( -\theta_{\ell} x_{1} - \frac{\nu_{\ell} \gamma}{x_{1}} \right) \, dx_{1}.
\]

(46)

Now, expanding the second exponential, we have
\[ F_{\gamma e_{2\ell}}(\gamma) = 1 - \theta \ell \int_{\gamma}^{\infty} \exp(-\theta \ell x_1) \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left( \frac{\nu \ell}{x_1} \right)^k k \, dx_1 \]  

(47)

By Fubini’s theorem, we can exchange the integral and the summation in the previous expression we have,

\[ F_{\gamma e_{2\ell}}(\gamma) = 1 - \theta \ell \left\{ \sum_{k=0}^{\infty} \frac{(-\nu \ell \gamma)^k}{k!} \int_{\gamma}^{\infty} \exp(-\theta \ell x_1) x_1^{-k} \, dx_1 \right\}. \]

(48)

Next, by applying the transformation \( y = \frac{\gamma}{\ell} \), (49) can be rewritten as

\[ F_{\gamma e_{2\ell}}(\gamma) = 1 - \theta \ell \left\{ \sum_{k=0}^{\infty} \frac{(-\nu \ell \gamma)^k}{k!} \int_{1}^{\infty} \gamma \exp(-\theta \ell y \gamma) (y \gamma)^{-k} \, dy \right\}. \]

(49)

Thus, we have,

\[ F_{\gamma e_{2\ell}}(\gamma) = 1 - \theta \ell \sum_{k=0}^{\infty} \frac{(-\nu \ell)^k}{k!} \gamma E_k(\theta \ell \gamma), \]

(50)

where \( E_n(x) \) is the exponential integral function given by

\[ E_n(x) = \int_{1}^{\infty} \exp(-xt)t^{-n} \, dt. \]

(51)

**APPENDIX B**

**PROOF FOR THEOREM 3**

Recall from Theorem.1 that if we can identify normalizing constants \( a_L \geq 0 \) and \( b_L \) such that

\[ u(\gamma) = \lim_{L \to \infty} \sum_{\ell=1}^{L} (1 - F_{\ell}(a_L \gamma + b_L)) < \infty, \]

(52)

then we can identify the form of the limiting distribution of \( \gamma_{\ell e_{2\ell, max}}^{L} \). Hence we begin by evaluating \( u(\gamma) \) for \( F_{\ell}(\gamma) = F_{\gamma e_{2\ell}}(\gamma) \). Note that the choice of normalizing constants is not unique and here we choose \( b_L = 0 \) and then identify the corresponding choice of \( a_L \) so that (52) is satisfied. Hence we have,

\[ u(\gamma) = \lim_{L \to \infty} \sum_{\ell=1}^{\infty} \theta \ell \sum_{k=0}^{\infty} \frac{(-\nu \ell)^k}{k!} (a_L \gamma) E_k(\theta \ell (a_L \gamma)) \]

(53)

Since we are interested in finding the limiting distribution of the maximum RV, we further simplify the function \( u(\gamma) \) by considering the approximation of the function for large values of \( \gamma \). The authors of [59] uses a similar approach where they expand the distribution function of an
exponential random variable using the corresponding Maclaurin series\(^4\) and hence approximate the infinite sum of such distribution functions to identify the limiting distribution of the minimum.

Now, the exponential integral function in (53) has the following asymptotic expansion:

\[
E_n(x) = \frac{\exp(-x)}{x} \left\{ 1 - \frac{n}{x} + \frac{n(n+1)}{x^2} - \cdots \right\}
\]

(54)

Using the above expansion, we can rewrite \(u(\gamma)\) as

\[
u(\gamma) = \lim_{L \to \infty} \sum_{\ell=1}^{L} \sum_{k=0}^{\infty} \frac{(-\nu_{\ell})^k}{k!} \exp(-\theta_{\ell} a_L \gamma) \left\{ 1 - \frac{k}{\theta_{\ell} a_L \gamma} + \frac{k(k+1)}{(\theta_{\ell} a_L \gamma)^2} - \cdots \right\}
\]

(55)

\[
u(\gamma) = \lim_{L \to \infty} \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{\ell=1}^{L} (-\nu_{\ell}) \exp(-\theta_{\ell} a_L \gamma) \left\{ 1 - \frac{k}{\theta_{\ell} a_L \gamma} + \frac{k(k+1)}{(\theta_{\ell} a_L \gamma)^2} - \cdots \right\}.
\]

(56)

Now, let us analyze \(g(k)\) for different values of \(k\). For \(k = 0\), we have \(g(0) = \sum_{\ell=1}^{L} e^{-\theta_{\ell} a_L \gamma}\).

Similarly, for \(k = 1\), we have

\[
\sum_{\ell=1}^{L} (-\nu_{\ell}) e^{-\theta_{\ell} a_L \gamma} \left\{ 1 - \frac{1}{\theta_{\ell} a_L \gamma} + \frac{1 \times (2)}{(\theta_{\ell} a_L \gamma)^2} - \cdots \right\}.
\]

(57)

Looking at the pattern we observe that if we choose \(a_L = 1\), \(u(z)\) will be finite. This is because, with \(a_L = 1\), terms of the form \(\frac{1}{(\theta_{\ell} a_L \gamma)^k}\) will approach zero for all \(k > 0\) and \(\gamma\) around infinity.

Here, \(\theta_{\ell}\) will be the power of the \(\ell\)-th source to relay link and hence will be finite. Thus we conclude that \(a_L = 1\) is a valid choice of normalizing constant and hence we have

\[
u(\gamma) = \sum_{\ell=1}^{L} e^{-(\theta_{\ell} \gamma + \nu_{\ell})}
\]

(58)

Thus, the limiting distribution of the maximum of end-to-end SNR is given by \(F_{\gamma_{e2e,max}}(\gamma) = \exp(-u(\gamma))\) for \(u(\gamma)\) as given in (58).

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\(^4\)There the interest was in deriving the limiting distribution of the minimum over a sequence of non negative random variables and hence the series expansion around zero was considered.
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