Ten-Dimensional Super-Twistors and Super-Yang-Mills

Nathan Berkovits

Instituto de Física Teórica, UNESP-Universidade Estadual Paulista
R. Dr. Bento T. Ferraz 271 - Bl. II, 01140-070, São Paulo, SP, Brasil

Four-dimensional super-twistors provide a compact covariant description of on-shell \( \mathcal{N} = 4 \) \( d=4 \) super-Yang-Mills. In this paper, ten-dimensional super-twistors are introduced which similarly provide a compact covariant description of on-shell \( d=10 \) super-Yang-Mills. The super-twistor variables are \( Z = (\lambda^\alpha, \mu^\alpha, \Gamma^m) \) where \( \lambda^\alpha \) and \( \mu^\alpha \) are constrained bosonic \( d=10 \) spinors and \( \Gamma^m \) is a constrained fermionic \( d=10 \) vector. The Penrose map relates the twistor superfield \( \Phi(Z) \) with the \( d=10 \) super-Yang-Mills vertex operator \( \lambda^\alpha A_\alpha(x, \theta) \) which appears in the pure spinor formalism of the superstring, and the cubic super-Yang-Mills amplitude is proportional to the super-twistor integral \( \int dZ \Phi_1 \Phi_2 \Phi_3 \).
1. Introduction

In four dimensions, twistor variables were introduced in 1967 by Penrose as an alternative description of spacetime in which light-like lines replace points as the fundamental objects [1]. Instead of the usual spacetime vector variable \( x^\mu \) for \( \mu = 0 \) to \( 3 \), Penrose’s twistor variables consist of bosonic two-component spinors \( (\lambda^a, \mu_{\dot{a}}) \) where \( a, \dot{a} = 1 \) to \( 2 \). The relation between these two descriptions is given by

\[
\mu_{\dot{a}} = x_\mu \sigma^{\mu}_{a\dot{a}} \lambda^a, \quad P^\mu = \lambda^a \sigma^{\mu}_{a\dot{a}} \tilde{\lambda}^{\dot{a}}
\]

where \( P^\mu \) is the light-like momentum, \( \sigma^{\mu}_{a\dot{a}} \) are the d=4 Pauli matrices, and \( \tilde{\lambda}^{\dot{a}} \) is the canonical momentum variable to \( \mu_{\dot{a}} \). These d=4 twistor variables transform linearly under \( SO(4,2) \) conformal transformations and provide a compact description of massless states.

In 1978, Ferber [2] generalized Penrose’s twistors to four-dimensional super-twistors consisting of the bosonic spinor variables \( (\lambda^a, \mu_{\dot{a}}) \) as well as \( N \) fermionic scalar variables \( \eta^J \) for \( J = 1 \) to \( N \) where \( N \) is the number of supersymmetries. These super-twistor variables transform linearly under superconformal transformations and are related to the usual \( (x^\mu, \theta^{aJ}, \bar{\theta}^{\dot{a}J}) \) superspace variables by the map

\[
\mu_{\dot{a}} = x_\mu \sigma^{\mu}_{a\dot{a}} \lambda^a + \bar{\theta}^{\dot{a}J} \eta^J, \quad \eta^J = \lambda^a \theta^{aJ},
\]

\[
P^\mu = \lambda^a \sigma^{\mu}_{a\dot{a}} \tilde{\lambda}^{\dot{a}}, \quad q^a_J = \lambda^a \bar{\eta}^J, \quad \bar{q}^{\dot{a}}_J = \eta^J \tilde{\lambda}_{\dot{a}}
\]

where \( \tilde{\lambda}^{\dot{a}} \) is the canonical momentum variable to \( \mu_{\dot{a}} \) and \( \bar{\eta}^J \) is the canonical momentum variable to \( \eta^J \).

When \( N = 4 \), these super-twistors provide a compact covariant description of on-shell maximally supersymmetric \( d = 4 \) super-Yang-Mills. Expanding in powers of \( \eta^J \), a scalar twistor superfield \( \Phi(\lambda, \mu, \eta) \) of momentum \( P^\mu = \lambda^a \sigma^{\mu}_{a\dot{a}} \pi^{\dot{a}} \) (where \( \pi^{\dot{a}} \) is the eigenvalue of the operator \( \lambda^{\dot{a}} \)) has the expansion

\[
\Phi(\lambda, \mu, \eta) = e^{\mu_{\dot{a}} \pi^{\dot{a}}}(a_- + \eta^J \bar{s}_J + \eta^K \eta^3 \phi_{JK} + \eta^4 s^J + \eta^4 a_+)(1.3)
\]

where \( (a_-, a_+) \) are the \((-1, +1)\) helicities of the gluon, \( (\bar{s}_J, s^J) \) are the \((-\frac{1}{2}, +\frac{1}{2})\) helicities of the gluino, and \( \phi_{JK} \) are the six scalars. Recently, these super-twistors have played an important role in simplifying the computation of \( d=4 \) super-Yang-Mills scattering amplitudes. Starting from the supersymmetric expression for the MHV tree amplitude [3], Witten showed how to use super-twistors to compute non-MHV super-Yang-Mills tree
amplitudes [4]. These super-twistor methods were further developed in hundreds of papers and drastically simplify the conventional Feynman diagram techniques for computing $\mathcal{N} = 4$ $d=4$ super-Yang-Mills scattering amplitudes.

Despite this progress in computing $\mathcal{N} = 4$ $d=4$ super-Yang-Mills amplitudes using super-twistors, there has been very little discussion of super-twistors in higher dimensions. Although super-Yang-Mills is only conformally invariant in four dimensions, the most natural formulation of maximally supersymmetric Yang-Mills is in ten dimensions where the only physical fields are a gluon and gluino. Moreover, super-Yang-Mills in ten dimensions is the low energy limit of open superstring theory, and superstring theory and twistor theory have many similar features [5] [6]. For example, both superstring theory and twistor theory provide drastic simplifications to conventional Feynman diagram methods, at least when the external states are on-shell. This suggests that any $d=10$ super-twistor description of super-Yang-Mills might be related to superstring theory.

In this paper, a new $d=10$ super-twistor description will be introduced which consists of the bosonic $d=10$ spinors $(\lambda^\alpha, \mu^\alpha)$ for $\alpha = 1$ to 16, and the fermionic $d=10$ vector $\Gamma^m$ for $m = 0$ to 9. These super-twistor variables will be constrained to satisfy

$$\lambda \gamma^m \lambda = 0, \quad \mu^\alpha \lambda^\alpha = 0, \quad \mu \gamma^m \lambda = 2 \Gamma^m \Gamma^n, \quad \Gamma^m (\gamma_m \lambda)_\alpha = 0 \quad (1.4)$$

where $\gamma^m_{\alpha\beta}$ are the $d=10$ Pauli matrices satisfying $\gamma^{(m}_{\alpha\beta} \gamma^n)_{\beta\gamma} = 2\eta^{mn} \delta^\gamma_\alpha$. The first constraint of (1.4) implies that $\lambda^\alpha$ is a $d=10$ pure spinor with 11 independent components, and the remaining constraints of (1.4) imply that $\mu^\alpha$ and $\Gamma^m$ each have 5 independent components. These $d=10$ super-twistor variables are related to the usual $d=10$ superspace variables $(x^m, \theta^\alpha)$ by the map

$$\mu^\alpha = x_m \gamma^m_{\alpha\beta} \lambda^\beta + \frac{1}{2} \Gamma^m (\gamma_m \theta)_\alpha, \quad \Gamma^m = \lambda \gamma^m \theta, \quad (1.5)$$

$$P^m = \lambda \gamma^m \bar{\lambda}, \quad q_\alpha = \bar{\Gamma}_m (\gamma^m \lambda)_\alpha - \Gamma^m (\gamma_m \bar{\lambda})_\alpha$$

where $\bar{\lambda}^\alpha$ is the canonical momentum variable to $\mu^\alpha$ and $\bar{\Gamma}_m$ is the canonical momentum variable to $\Gamma^m$.

The use of bosonic pure spinor variables $(\lambda^\alpha, \mu^\alpha)$ to describe higher-dimensional twistors has previously been discussed in [7] [8] [9], and the $d=10$ super-twistor variables of (1.4) can be understood as a “complexified” version of the real $d=10$ super-twistor variables introduced in [10]. Unlike in four dimensions where the super-twistor variables transform linearly under $d = 4$ superconformal transformations, the $d=10$ super-twistor
variables transform linearly only under $d=10$ super-Poincaré transformations. Note that $d=4$ super-twistor variables involving fermionic vectors have been discussed in \[11\], and $d=10$ super-twistor variables involving fermionic scalars have been discussed in \[12\].

To describe on-shell $d=10$ super-Yang-Mills, the twistor superfield $\Phi(\lambda, \mu, \Gamma)$ should satisfy the constraint $B\Phi = 0$ where

$$B = (\lambda \gamma^m \bar{\lambda}) \bar{\Gamma}_m - \frac{1}{2} (\bar{\lambda} \gamma^m \lambda) \Gamma_m$$

is a super-Poincaré covariant operator. The condition $B\Phi = 0$ implies that $\Phi$ depends on only 4 of the 5 independent $\Gamma$’s, so $\Phi$ has $2^4$ component fields as expected. Expanding in $\Gamma^m$, the component super-Yang-Mills fields appear in the twistor superfield as

$$\Phi(\lambda, \mu, \Gamma) = e^{\mu \lambda} \bar{\pi}^\alpha (s + \Gamma_m a_-^m + \Gamma_n s^{mn} + (\bar{\pi} \gamma_{mnqr} \bar{\pi}) \Gamma^m \Gamma^n \Gamma^p h^q a_+^r + (\bar{\pi} \gamma_{mnqr} \bar{\pi}) \Gamma^m \Gamma^n \Gamma^p \Gamma^q h^r s)$$

where $P^m = \lambda \gamma^m \bar{\pi}$, $h^m$ is any constant vector satisfying $h_m P^m = 1$, the $d=10$ gluon polarization has been split as $a^m = a_-^m + a_+^m$ with $a_-^m (\gamma_m \bar{\pi})_\alpha = a_+^m (\gamma_m \lambda)_\alpha = 0$, and the $d=10$ gluino polarization has been split as $\psi^\alpha = \bar{\pi}^\alpha s + (\gamma_{mn} \lambda)^\alpha s^{mn} + \lambda^\alpha s$ with $s^{mn} (\gamma_n \bar{\pi})_\alpha = 0$.

Using the relation of (1.5) to map super-twistor variables into superspace variables, one finds that $B$ maps into $\frac{1}{2} u_\alpha P^m (\gamma_m D)^\alpha$ where $D_\alpha$ is the $d=10$ supersymmetric derivative and $u_\alpha$ is any spinor satisfying $u_\alpha \lambda^\alpha = 1$. And $\Phi$ of (1.7) maps to $\lambda^\alpha A_\alpha(x, \theta)$ where $A_\alpha(x, \theta)$ is the super-Yang-Mills spinor gauge superfield in the gauge $P^m (\gamma_m D)\alpha A_\beta = 0$.

In the pure spinor formalism for the superstring \[13\], $V = \lambda^\alpha A_\alpha(x, \theta)$ is the unintegrated open string vertex operator for super-Yang-Mills and the composite $b$ ghost is $\frac{1}{2} u_\alpha P^m (\gamma_m D)\alpha + \ldots$ where $\ldots$ involves non-minimal variables. $N$-point super-Yang-Mills tree amplitudes are computed in this superstring formalism by evaluating the $\alpha' \to 0$ limit of the disk correlation function

$$A_N = \langle V_1(z_1) V_2(z_2) V_3(z_3) \prod_{r=4}^N \int dz U_r(z_r) \rangle$$

where $U_r(z_r) = b_{-1} V_r(z_r)$ is a dimension-one vertex operator and $b_{-1}$ denotes the single pole with $b$.

Since $\Phi$ maps to $V$ and $B$ maps to the $b$ ghost, it is natural to try to formulate a similar prescription for $d=10$ super-Yang-Mills tree amplitudes in terms of twistor superfields. As
in d=4, it will be shown that the cubic super-Yang-Mills amplitude in d=10 is proportional to the super-twistor integral

\[ \int d^{10} \lambda \lambda^5 \mu \mu^5 \Gamma \Phi_1 \Phi_2 \Phi_3. \] (1.9)

However, it is not yet understood how to obtain the correct proportionality factor coming from momentum conservation for this cubic d=10 amplitude. Furthermore, for higher-point amplitudes, the appropriate super-twistor prescription is not known and will require the construction of a worldsheet action for the super-twistor variables.

In section 2 of this paper, the four-dimensional super-twistor description of \( \mathcal{N} = 4 \) d=4 super-Yang-Mills is reviewed. In section 3, the d=10 super-twistor variables are introduced, the twistor superfield for d=10 super-Yang-Mills is constructed, and the super-twistor prescription for super-Yang-Mills tree amplitudes is discussed.

2. Review of Four-Dimensional Super-Twistors

2.1. \( \mathcal{N} = 4 \) d=4 super-twistor variables

\( \mathcal{N} = 4 \) d=4 super-twistor variables consist of the bosonic Weyl and anti-Weyl spinors \( (\lambda^a, \mu_\dot{a}) \) for \( a, \dot{a} = 1 \) to 2 and the fermionic scalars \( \eta^J \) for \( J = 1 \) to 4. Note that unlike the usual superspace variables \( (x^\mu, \theta^{aJ}, \bar{\theta}_{\dot{a}J}) \) for \( \mu = 0 \) to 3, super-twistor variables carry the opposite statistics from those expected by the spin-statistics relation.

These variables transform linearly under \( PSU(2,2|4) \) superconformal transformations which are generated by

\[ P^\mu = \lambda^a \sigma^{\mu}_{a\dot{a}} \bar{\lambda}^{\dot{a}}, \quad q_{aJ} = \lambda_a \bar{\eta}_J, \quad \bar{q}_\dot{a}^J = \eta^J \bar{\lambda}_{\dot{a}}, \quad M^{\mu \nu} = \frac{1}{2} (\sigma^{\mu \nu})^b_a \lambda^a \bar{\mu}_b + \frac{1}{2} (\sigma^{\mu \nu})^b_{\dot{a}} \mu_{\dot{b}} \bar{\lambda}^\dot{a}, \quad (2.1) \]

\[ R^J_K = \eta^J \bar{\eta}_K, \quad K^\mu = \sigma^\mu_{a\dot{a}} \bar{\mu}_a \bar{\lambda}_{\dot{a}}, \quad D = \lambda^a \mu_a - \mu_{\dot{a}} \bar{\lambda}^\dot{a}, \quad s_a = \eta^J \bar{\mu}_a, \quad \bar{s}_{\dot{a}} = \mu_{\dot{a}} \bar{\eta}_J, \]

where \( (\bar{\mu}_a, \bar{\lambda}_{\dot{a}}, \bar{\eta}_J) \) are the canonical momenta variables to \( (\lambda^a, \mu_{\dot{a}}, \eta^J) \) and \( \sigma^\mu_{a\dot{a}} \) are the d=4 Pauli matrices. These superconformal generators all commute with the generator

\[ H = \lambda^a \mu_a + \mu_{\dot{a}} \bar{\lambda}^\dot{a} + \eta^J \bar{\eta}_J \] (2.2)

which defines the “projective weight”.

The above super-twistor variables are related to the usual \( \mathcal{N} = 4 \) d=4 superspace variables \( (x^\mu, \theta^{aJ}, \bar{\theta}_{\dot{a}J}) \) by the relation

\[ \mu_{\dot{a}} = x_\mu \sigma^{\mu}_{a\dot{a}} \lambda^a + \bar{\theta}_{\dot{a}J} \eta^J, \quad \eta^J = \lambda^a \theta_{aJ}. \] (2.3)

One can easily check that this map is invariant under the above \( PSU(2,2|4) \) superconformal transformations if one defines \( (x^\mu, \theta^{aJ}, \bar{\theta}_{\dot{a}J}) \) to transform in the standard manner.
2.2. Twistor superfield for $d=4$ super-Yang-Mills

Just as on-shell $\mathcal{N} = 4$ $d=4$ super-Yang-Mills can be described using gauge and field-strength superfields depending on $(x, \theta, \bar{\theta})$ superspace variables, it can also be described by a scalar twistor superfield $\Phi(\lambda, \mu, \eta)$. The map of (2.3) relates this twistor superfield with the spacetime field-strength superfield $F_{\bar{a}b}(x, \theta, \bar{\theta})$ whose $\theta = \bar{\theta} = 0$ component is the linearized self-dual field-strength $f_{\bar{a}b} = (\sigma^{\mu\nu})_{\bar{a}b} \partial_\mu a_\nu$.

To show this relation, first note that if $F_{\bar{a}b}$ is written in momentum space where the light-like momentum satisfies $P^\mu = \lambda^a \sigma^{\mu a} \bar{\pi}^a$, $F_{\bar{a}b}$ can be expressed as $F_{\bar{a}b} = \bar{\pi}_a \pi_b F(x, \theta, \bar{\theta})$ where the $\theta = \bar{\theta} = 0$ component of $F$ is the $-1$ helicity component of the gluon. In other words, if the gluon polarization $a^\mu$ is split as

$$a^\mu = (\epsilon^a \sigma^{\mu a} \bar{\pi}^a) a_- + (\epsilon^\dot{a} \sigma^{\mu \dot{a}} \lambda^a) a_+$$

(2.4)

where $(\epsilon^a, \bar{\epsilon}^\dot{a})$ are arbitrary spinors satisfying $\epsilon^a \lambda_a = \bar{\epsilon}^\dot{a} \pi_\dot{a} = 1$, $a_-$ is the $\theta = \bar{\theta} = 0$ component of $F$. Furthermore, the superspace constraints $\bar{D}^J F_{\bar{a}b} = 0$ and $\sigma_{\mu \nu} \partial_\mu D^\mu F_{\bar{a}b} = 0$ imply that $F$ satisfies

$$\bar{D}^J F = \lambda_a D^a F = 0$$

(2.5)

where $D_{aJ} = \frac{\partial}{\partial \theta^a} - \frac{1}{2} (\sigma^{\mu \nu}) a_{\mu J} \partial_\nu$ and $\bar{D}^J = \frac{\partial}{\partial \bar{\theta}^a} - \frac{1}{2} (\sigma^{\mu \bar{\nu}})^\dot{a} J \partial_\bar{\nu}$ are the $\mathcal{N} = 4$ $d=4$ supersymmetric derivatives.

To relate $F(x, \theta, \bar{\theta})$ to $\Phi(\lambda, \mu, \eta)$, define the scalar twistor superfield with momentum $P^\mu = \lambda^a \sigma^{\mu a} \bar{\pi}^a$ as

$$\Phi(\lambda, \mu, \eta) = e^{\mu a} \pi^a f(\eta^J)$$

(2.6)

where $f(\eta^J)$ is an arbitrary function of $\eta^J$. The map relating $F(x, \theta, \bar{\theta})$ and $\Phi(\lambda, \mu, \eta)$ is defined by

$$F(x, \theta, \bar{\theta}) = \bar{\Phi}(\lambda, x, \theta, \bar{\theta})$$

(2.7)

where $F(x, \theta, \bar{\theta})$ has momentum $P^\mu = \lambda \sigma^{\mu a} \bar{\pi}^a$ and $\bar{\Phi}(\lambda, x, \theta, \bar{\theta})$ is obtained from $\Phi(\lambda, \mu, \eta)$ by setting $\mu_a = x^{\mu a} \lambda^a + \bar{\theta}_{aJ} \eta^J$ and $\eta^J = \lambda^a \theta^a J$ as in (2.3). It is easy to use (2.3) to verify that $\bar{D}^J \Phi = \lambda^a D_{aJ} \bar{\Phi} = 0$, so (2.3) is satisfied. Using the identification of (2.7) with $F(x, \theta, \bar{\theta})$, one learns that

$$f(\eta^J) = a_- + \eta^J \bar{s}_J + \eta^J \eta^K \phi_{JK} + (\eta^J)^3 s^J + (\eta^4)^a a_+$$

(2.8)
where \( a_\pm \) are defined in (2.4), \( s^J \) and \( \tilde{s}_J \) are the gluinos of \( \pm \frac{1}{2} \) helicity defined by \( \psi^J_\alpha = \pi_\alpha s^J \) and \( \tilde{\psi}^J_\alpha = \tilde{\pi}_\alpha \tilde{s}_J \), and \( \phi_{JK} \) are the six scalars. Note that \( f(\eta^J) \) has +2 projective weight since under \( \lambda^a \to c\lambda^a \) and \( \bar{\pi}^a \to c^{-1}\bar{\pi}^a \),

\[
(a_-, \tilde{s}_J, \phi_{JK}, s^J, a_+) \to (c^2 a_-, c\tilde{s}_J, \phi_{JK}, c^{-1}s^J, c^{-2}a_+) .
\]  

(2.9)

2.3. \( d=4 \) super-Yang-Mills tree amplitudes

Four-dimensional super-twistors have recently been used to compute super-Yang-Mills tree amplitudes where tree amplitudes of different helicity violation involve curves in twistor space of different degree [4] [14]. Only the degree zero curve where \( \lambda^a \) is constant will be discussed here, which is non-vanishing for cubic “self-dual” amplitudes, i.e. the supersymmetric completion of amplitudes involving two gluons of \(-1\) helicity and one gluon of \(+1\) helicity. Note that although the cubic amplitude vanishes for real momentum in signature \((d-1,1)\), it is non-vanishing for real momentum in signature \((\frac{d}{2}, \frac{d}{2})\).

In signature \((2,2)\), there are two possible ways for the momentum conservation condition \( \sum_{r=1}^{3} P^\mu_{(r)} = \sum_{r=1}^{3} (\lambda^a_{(r)} \sigma^\mu \bar{\pi}^a_{(r)}) = 0 \) to be satisfied. Either \( \lambda^a_{(1)} = \lambda^a_{(2)} = \lambda^a_{(3)} \) and \( \sum_{r=1}^{3} \bar{\pi}^a_{(r)} = 0 \), or \( \bar{\pi}^a_{(1)} = \bar{\pi}^a_{(2)} = \bar{\pi}^a_{(3)} \) and \( \sum_{r=1}^{3} \lambda^a_{(r)} = 0 \). The first solution corresponds to the “self-dual” amplitude involving a degree zero curve where \( \lambda^a \) is constant, whereas the second solution corresponds to the “anti-self-dual” amplitude involving a degree one curve where \( \lambda^a \) is non-constant.

Suppose one uses projective invariance to scale \( \lambda^1_{(1)} = \lambda^1_{(2)} = \lambda^1_{(3)} = 1 \). Then if one defines

\[
\Phi_{(r)}(\lambda, \mu, \eta) = \delta(\lambda^2 - \lambda^2_{(r)}) e^{\mu a} \pi^a_{(r)} f_{(r)}(\eta)
\]

(2.10)

where \( f_{(r)}(\eta) \) is defined in (2.8), the cubic self-dual super-Yang-Mills amplitude can be expressed as the super-twistor integral

\[
\mathcal{A} = \int d\lambda^2 \int d^2\mu \int d^4\eta Tr([\Phi_{(1)}, \Phi_{(2)}] \Phi_{(3)})
\]

(2.11)

where the trace is over the color indices of \( \Phi \). The integral over \( \int d\lambda^2 \int d^2\mu \int d^4\eta \) is easily performed and gives

\[
\mathcal{A} = \delta(\lambda^2_{(3)} - \lambda^2_{(1)}) \delta(\lambda^2_{(3)} - \lambda^2_{(2)}) \delta^2(\sum_r \pi_{(r)}) Tr([a^{(1)}_-, a^{(2)}_-] a^{(3)}_+) + ... 
\]

(2.12)

\[
= (\pi^a_{(1)} \pi^a_{(2)} \pi^a_{(3)}) \delta^2(\sum_r \lambda^2_{(r)} \pi_{(r)}) \delta^2(\sum_r \pi_{(r)}) Tr([a^{(1)}_-, a^{(2)}_-] a^{(3)}_+) + ... 
\]

\[
= (\pi^a_{(1)} \pi^a_{(2)} \pi^a_{(3)}) \delta^4(\sum_r P_{(r)}) Tr([a^{(1)}_-, a^{(2)}_-] a^{(3)}_+) + ... ,
\]

which is the correct expression for the self-dual super-Yang-Mills amplitude where ... is the supersymmetric completion of the self-dual gluon amplitude.
3. Ten-Dimensional Super-Twistors

3.1. \( d = 10 \) super-twistor variables

As discussed in [7, 8, 9], the natural generalization of four-dimensional twistor variables \((\lambda^a, \mu_a)\) to higher dimensions is \((\lambda^\alpha, \mu^\alpha)\) where \(\lambda^\alpha\) is a pure spinor and \(\mu^\alpha\) is related to the spacetime variables \(x^m\) by the map \(\mu_\alpha = x^m (\gamma^m \lambda_\alpha)\). In ten dimensions, a Weyl spinor has 16 components (i.e. \(\alpha = 1\) to 16), and a pure spinor must satisfy \(\lambda \gamma^m \lambda = 0\) which implies that \(\lambda^\alpha\) has only 11 independent components. Furthermore, \(\mu_\alpha = x^m (\gamma^m \lambda_\alpha)\) implies that \(\mu_\alpha \lambda^\alpha = \mu \gamma^{mn} \lambda = 0\), which implies that \(\mu_\alpha\) has only 5 independent components. Note that in spacetime with signature (9,1), \((\lambda^\alpha, \mu_\alpha)\) are complex variables. But just as four-dimensional twistors are real variables in signature (2,2), ten-dimensional twistors are real variables in signature (5,5). In this paper, we shall choose the signature (5,5) so that \(\lambda^\alpha\) and \(\bar{\lambda}^\alpha\) are independent real variables.

As in four dimensions, \(\mu_\alpha\) can be interpreted as the canonical momentum variable to \(\bar{\lambda}^\alpha\) where the light-like spacetime momentum is \(P^m = \lambda^\alpha \gamma^m_{\alpha \beta} \bar{\lambda}^\beta\). Note that \(\bar{\lambda}^\alpha\) is not required to be a pure spinor, and \(P^m P_m = 0\) follows from the \(d=10\) gamma-matrix identity \(\gamma_{m \alpha (\beta} \gamma^m_{\delta)} = 0\) together with \(\lambda \gamma^m \lambda = 0\).

To generalize to ten-dimensional super-twistors, one introduces a fermionic vector \(\Gamma^m\) which is constrained to satisfy \(\Gamma^m (\gamma_m \lambda)_\alpha = 0\). So \(\Gamma^m\) has 5 independent components. As in four dimensions, the statistics of the fermionic twistor variable is opposite from the statistics one would expect from its Lorentz representation. One also modifies the constraints on \(\mu_\alpha\) to \(\mu_\alpha \lambda^\alpha = \mu \gamma^{mn} \lambda - 2 \Gamma^m \Gamma^m = 0\).

So the ten-dimensional super-twistor space is defined by the variables \((\lambda^\alpha, \mu_\alpha, \Gamma^m)\) which are constrained to satisfy

\[
\lambda \gamma^m \lambda = 0, \quad \mu_\alpha \lambda^\alpha = 0, \quad \mu \gamma^{mn} \lambda - 2 \Gamma^m \Gamma^m = 0, \quad \Gamma^m (\gamma_m \lambda)_\alpha = 0. \quad (3.1)
\]

These constraints imply that \(\mu_\alpha\) and \(\Gamma^m\) can be expressed in terms of \(d=10\) superspace variables \((x^m, \theta^\alpha)\) as

\[
\mu_\alpha = x^m (\gamma^m \lambda)_\alpha + \frac{1}{2} \Gamma^m (\gamma_m \theta)_\alpha, \quad \Gamma^m = \lambda \gamma^m \theta, \quad (3.2)
\]

which closely resembles the four-dimensional relation of (2.3). Furthermore, these super-twistor variables transform linearly under \(d=10\) super-Poincaré transformations which are generated by

\[
P^m = \lambda \gamma^m \bar{\lambda}, \quad q_\alpha = (\gamma^m \lambda)_\alpha \bar{\Gamma}_m - \Gamma^m (\gamma_m \bar{\lambda})_\alpha, \quad M^{mn} = \frac{1}{2} \lambda \gamma^{mn} \bar{\mu} + \frac{1}{2} \mu \gamma^{mn} \bar{\lambda} + \Gamma^[m \bar{\Gamma}^n], \quad (3.3)
\]
where \((\bar{\mu}_\alpha, \bar{\lambda}^\alpha, \bar{\Gamma}_m)\) are canonical momentum variables for \((\lambda^\alpha, \mu_\alpha, \Gamma^m)\).

It is easy to verify that the generators of (3.3) commute with the constraints of (3.1) and form a d=10 super-Poincaré algebra. As in four dimensions, the operator

\[
H = \lambda^\alpha \bar{\mu}_\alpha + \mu_\alpha \bar{\lambda}^\alpha + \Gamma^m \bar{\Gamma}_m
\]  

(3.4)
defines the projective weight and commutes with the super-Poincaré generators. Finally, it will be useful to define the fermionic operator

\[
B = (\lambda \gamma^m \bar{\lambda}) \bar{\Gamma}_m - \frac{1}{2} \Gamma^m (\bar{\lambda}_m \bar{\lambda})
\]  

(3.5)
which commutes with both the constraints of (3.1) and with the super-Poincaré generators of (3.3).

If one sets the fermionic variables \(\Gamma^m\) and \(\bar{\Gamma}_m\) to zero, the d=10 Poincaré algebra can be extended to a conformal algebra by including the generators \(K^m = \mu_\alpha \gamma^m \bar{\mu}\) and \(D = \mu \bar{\lambda} - \lambda \bar{\mu}\). However, after including \(\Gamma^m\) and \(\bar{\Gamma}_m\), there is no obvious way to extend the d=10 super-Poincaré algebra to a superconformal algebra. This is of course not surprising since d=10 super-Yang-Mills is not superconformally invariant.

3.2. d=10 twistor superfield

In this section, it will be shown that on-shell d=10 super-Yang-Mills is described by a scalar twistor superfield \(\Phi(\lambda, \mu, \Gamma)\) of +1 projective weight which is annihilated by the \(B\) operator of (3.3). This twistor superfield can be mapped to the spacetime superfield \(V = \lambda^\alpha A_\alpha(x, \theta)\) which appears in the pure spinor formalism, and the condition of +1 projective weight is related to the +1 ghost-number of \(V\). The condition that \(B\Phi = 0\) comes from a gauge-fixing condition on \(A_\alpha\) and implies that \(\Phi\) depends on only 4 of the 5 \(\Gamma\)'s, which is the same number of fermionic variables as in the four-dimensional super-twistor.

In the pure spinor formalism for the superparticle or superstring, linearized on-shell d=10 super-Yang-Mills is described by the vertex operator \(V = \lambda^\alpha A_\alpha(x, \theta)\) satisfying \(QV = 0\) where \(Q = \lambda^\alpha D_\alpha, D_\alpha = \frac{\partial}{\partial \theta^\alpha} - \frac{1}{2} (\gamma^m \theta)_{\alpha \beta} \frac{\partial}{\partial x^m}\), \(\lambda^\alpha\) is a d=10 pure spinor, \(A_\alpha(x, \theta)\) is the spinor gauge superfield satisfying \(D_{(\alpha A_\beta)} = \gamma^m_{\alpha \beta} A_m\), and \(A_m(x, \theta)\) is the vector gauge superfield [15]. The gauge superfields \(A_\alpha\) and \(A_m\) are defined up to the linearized gauge transformations \(\delta A_\alpha = D_\alpha \Omega\) and \(\delta A_m = \partial_m \Omega\).
A convenient gauge-fixing condition for $A_\alpha$ is $\partial_m (\gamma^m D)_\alpha A_\beta(x, \theta) = 0$. This gauge-fixing condition implies that $\partial^m A_m = 0$ and can be solved in a plane-wave basis with momentum $P_m$ by

$$A_\alpha = h^m (\gamma_m W)_\alpha, \quad A_n = h^m F_{mn}$$

where $h^m$ is any constant vector satisfying $h^m P_m = 1$, $W^\alpha = \frac{1}{10} \gamma^{m\alpha\beta}(D_\beta A_m - \partial_m A_\beta)$ is the superfield-strength whose $\theta = 0$ component is the gluino, and $F_{mn} = \partial_{[m} A_{n]}$ is the superfield-strength whose $\theta = 0$ component is the gluon field strength.

To relate $V = \lambda^\alpha A_\alpha$ with a twistor superfield, suppose that the momentum $P^m$ satisfies $P^m = \lambda^\alpha \gamma^m \bar{\pi}^\beta$ for some $\bar{\pi}^\beta$. Then the superfield identity $D_\alpha W^\beta = -\frac{1}{4} (\gamma^{mn})_\alpha^\beta F_{mn}$ implies that $V = (\lambda^\gamma W^\gamma) h_m$ satisfies

$$(\lambda^\alpha D_\alpha) V = (\lambda^\gamma mn D) V = (\bar{\pi}^\alpha D_\alpha) V = 0.$$  

(3.7)

Since the momentum $P^m = \lambda^\gamma m \bar{\pi}$ is invariant under the transformation $\delta \bar{\pi}^\alpha = (\gamma^mn\lambda)^\alpha \Omega_{mn}$ for arbitrary $\Omega_{mn}$, one can choose $\bar{\pi}^\alpha$ so that it is a pure spinor satisfying $\bar{\pi}^\gamma m \bar{\pi} = 0$. The d=10 twistor superfield will then be defined in analogy with (2.6) as

$$\Phi(\lambda, \mu, \Gamma) = e^{\mu_0 \bar{\pi}^\alpha} f(\Gamma^m).$$

(3.8)

To satisfy $B\Phi = 0$ where $B$ is defined in (3.2), $f(\Gamma^m)$ must satisfy $(\lambda^\gamma m \bar{\pi}) \frac{\partial}{\partial \Gamma^m} f = 0$ which implies that $f(\Gamma^m)$ depends on only four of the five independent $\Gamma$’s.

The map relating $V = \lambda^\alpha A_\alpha$ with $\Phi(\lambda, \mu, \Gamma)$ will be defined as in (2.7) by

$$V(\lambda, x, \theta, \phi) = \tilde{\Phi}(\lambda, x, \theta)$$

(3.9)

where $V(\lambda, x, \theta)$ has momentum $P^m = \lambda^\gamma m \bar{\pi}$ and $\tilde{\Phi}(\lambda, x, \theta)$ is obtained from $\Phi(\lambda, \mu, \Gamma)$ by setting $\mu_\alpha = x_m (\gamma^m \lambda)_\alpha + \frac{1}{2} \Gamma^m (\gamma_m \theta)_\alpha$ and $\Gamma^m = \lambda^\gamma m \theta$ as in (3.2). It is easy to use (3.2) to verify that $(\lambda^\alpha D_\alpha) \tilde{\Phi} = (\lambda^\gamma mn D) \tilde{\Phi} = (\bar{\pi}^\alpha D_\alpha) \tilde{\Phi} = 0$, so (3.7) is satisfied. Using the identification of (3.9) with $V(\lambda, x, \theta)$, one learns (up to constant coefficients) that

$$f(\Gamma^m) = \bar{s} + \Gamma_m a_-^m + \Gamma_m \Gamma_n s^{mn} + (\bar{\pi} \gamma_{mnpqr} \bar{\pi}) \Gamma^m \Gamma^n \Gamma^p h^q a_+^r + (\bar{\pi} \gamma_{mnpqr} \bar{\pi}) \Gamma^m \Gamma^n \Gamma^p h^q s$$

(3.10)

where $P^m = \lambda^\gamma m \bar{\pi}$, $h^m$ is any constant vector satisfying $h_m P^m = 1$, the d=10 gluon polarization has been split as $a^m = a_-^m + a_+^m$ with $a_-^m (\gamma_m \bar{\pi})_\alpha = a_+^m (\gamma_m \lambda)_\alpha = 0$, and the d=10 gluino polarization has been split as $\psi^\alpha = \bar{\pi}^\alpha \bar{s} + (\gamma_{mn} \lambda)^\alpha s^{mn} + \lambda^\alpha s$ with $s^{mn} (\gamma_n \bar{\pi})_\alpha = \psi^\alpha$. 

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0. Note that \( f(\Gamma^m) \) is annihilated by \( P^m \frac{\partial}{\partial \Gamma^m} \) and has +1 projective weight since under \( \lambda^a \rightarrow c\lambda^a \) and \( \bar{\pi}^\alpha \rightarrow c^{-1}\bar{\pi}^\alpha \),

\[
(a_-^m, a_+^m) \rightarrow (a_-^m, a_+^m) \quad \text{and} \quad (s, s^{mn}, s) \rightarrow (cs, c^{-1}s^{mn}, c^{-1}s).
\] (3.11)

Furthermore, \( \bar{\pi}^m \gamma^m \bar{\pi} = 0 \) implies that \( f(\Gamma^m) \) is independent of the explicit choice of \( h^m \).

It might seem surprising that unlike the \( \mathcal{N} = 4 \) d=4 twistor superfield which is bosonic, the d=10 twistor superfield of (3.8) is fermionic. This is related to the fact that upon dimensional reduction to d=4, \( \Gamma^m = \lambda^\gamma^m \theta \) involves the chiral d=4 \( \theta^J_a \)'s for \( J = 1 \) to \( 3 \) in the linear combinations \( \eta^J = \lambda^a \theta^J_a \), but also involves the antichiral d=4 \( \bar{\theta}^a_4 \)'s in the combinations \( \lambda \sigma^4 \bar{\theta}_4 \). So the dimensional reduction of the d=10 twistor superfield is not the d=4 twistor superfield of (2.6).

3.3. d=10 super-Yang-Mills tree amplitudes

Since the d=10 super-twistors closely resemble the \( \mathcal{N} = 4 \) d=4 super-twistors, it is natural to try to generalize the super-twistor prescription for computing \( \mathcal{N} = 4 \) d=4 super-Yang-Mills tree amplitudes to ten dimensions. Although cubic super-Yang-Mills amplitudes vanish for real momenta in spacetime signature (9,1), they are non-vanishing in signature (5,5) where pure spinors have 11 real components. To analyze the kinematics in this signature, it is convenient to break manifest SO(5,5) Lorentz invariance to an \( SL(5) \) subgroup such that a spinor \( \lambda^a \) decomposes into \( (1,10,5) \) representations which will be denoted as \( (\lambda^+, \lambda^j, \lambda^j) \) for \( j = 1 \) to \( 5 \). When \( \lambda^a \) is a pure spinor, the constraint \( \lambda \gamma^m \lambda = 0 \) can be solved by setting \( \lambda^j = \frac{1}{8}(\lambda^+)^{-1}\epsilon^{jklmn}\lambda_{kl}\lambda_{mn} \), where it is assumed that the component \( \lambda^+ \) is non-zero.

If the momenta of the three external states are \( P^m_{(r)} = \lambda_{(r)}^a \gamma^m \bar{\pi}^a_{(r)} \), one can use the invariance \( \delta \bar{\pi}^a_{(r)} = (\gamma^{mn} \lambda_{(r)})^a \Omega_{(r)mn} \) (and the condition that \( \lambda^+_a \neq 0 \)) to fix \( \bar{\pi}^+_a_{(r)} = \bar{\pi}^a_{(r)jk} = 0 \). So the only non-zero components of \( \bar{\pi}^a_{(r)} \) are in the 5 representation, and the ten components of \( P^m_{(r)} \) decompose under \( SL(5) \) into

\[
P^j_{(r)} = \lambda^+_j \bar{\pi}^j_{(r)}, \quad P_{(r)j} = \lambda_{(r)jk} \bar{\pi}^k_{(r)}.
\] (3.12)

Now if one uses projective invariance to scale \( \lambda^+_1 = \lambda^+_2 = \lambda^+_3 = 1 \), momentum conservation implies as in d=4 that \( \sum_{r=1}^3 \bar{\pi}^a_{(r)} = 0 \). However, unlike in d=4, momentum conservation does not imply that \( \lambda^a_{(1)} = \lambda^a_{(2)} = \lambda^a_{(3)} \), and this will lead to a missing proportionality factor in the d=10 super-twistor prescription.
The natural d=10 generalization of the d=4 super-twistor prescription of (2.11) is

\[ A = \int d^{10} \lambda \int d^{5} \mu \int d^{5} \Gamma \; Tr([\Phi_{(1)}, \Phi_{(2)}] \Phi_{(3)}) \]  

(3.13)

where \( \Phi_{(r)} \) is defined as

\[ \Phi_{(r)}(\lambda, \mu, \Gamma) = \delta^{10}(\lambda_{jk} - \lambda_{(r)jk}) e^{\mu j \bar{\pi}_{(r)} f_{(r)}(\Gamma)} \]  

(3.14)

and \( f_{(r)}(\Gamma) \) is defined in (3.10). The integral \( \int d^{5} \Gamma \) will be defined as

\[ \int d^{5} \Gamma \; F(\Gamma) = \frac{1}{5!} (\lambda \gamma^{mnpqr} \lambda) \frac{\partial}{\partial \Gamma^{m}} \frac{\partial}{\partial \Gamma^{n}} \frac{\partial}{\partial \Gamma^{p}} \frac{\partial}{\partial \Gamma^{q}} F(\Gamma), \]  

(3.15)

which is consistent with the constraint \( \Gamma^{m}(\gamma_{m} \lambda)_{\alpha} = 0 \) since \( (\lambda \gamma^{mnpqr} \lambda) \frac{\partial}{\partial \Gamma^{m}} \; \Gamma^{s}(\gamma_{s} \lambda)_{\alpha} = 0. \)

Performing the integration \( \int d^{10} \lambda \int d^{5} \mu \int d^{5} \Gamma \), one finds that

\[ A = \delta^{10}(\lambda_{(3)jk} - \lambda_{(1)jk}) \delta^{10}(\lambda_{(2)jk} - \lambda_{(3)jk}) \delta^{5}(\sum_{r=1}^{3} \bar{\pi}^{j}_{(r)}) \]  

(3.16)

\[ Tr([a_{(1)}, a_{(2)}]P_{(3)m}a_{(3)n} + \psi_{(1)} \gamma^{m} \psi_{(2)} a_{(3)m} + \ \text{permutations of (1, 2, 3) } ). \]

This would be the correct cubic super-Yang-Mills amplitude if

\[ \delta^{10}(\lambda_{(3)jk} - \lambda_{(1)jk}) \delta^{10}(\lambda_{(2)jk} - \lambda_{(3)jk}) \delta^{5}(\sum_{r=1}^{3} \bar{\pi}^{j}_{(r)}) \]

were equal to

\[ \delta^{10}(\sum_{r=1}^{3} P_{(r)m}) = \delta^{5}(\sum_{r=1}^{3} \lambda_{(r)jk} \bar{\pi}^{k}_{(r)}) \delta^{5}(\sum_{r=1}^{3} \bar{\pi}^{j}_{(r)}). \]  

(3.17)

However, as remarked earlier, \( \sum_{r} P_{(r)m} = 0 \) does not imply \( \lambda_{(1)jk} = \lambda_{(2)jk} = \lambda_{(3)jk} \), so the first line of (3.16) is too restrictive.

Despite this incorrect proportionality factor, it is remarkable that

\[ \int d^{5} \Gamma \; f_{(1)}(\Gamma)f_{(2)}(\Gamma)f_{(3)}(\Gamma) \]

correctly reproduces the polarization dependence of the cubic super-Yang-Mills amplitude in the second line of (3.16). This is related to the fact that \( f_{(r)}(\Gamma) \) is mapped to \( V_{(r)} = \lambda^{\alpha} A_{(r)\alpha}(x, \theta) \), and the cubic super-Yang-Mills amplitude prescription in the pure spinor formalism is \( A = \langle V_{(1)}V_{(2)}V_{(3)} \rangle \) where the normalization of \( \langle \rangle \) is defined by

\[ \langle (\lambda \gamma^{m}\theta)(\lambda \gamma^{n}\theta)(\lambda \gamma^{p}\theta)(\theta \gamma_{mnp}\theta) \rangle = 1. \]  

(3.18)
Note that $\Gamma^m = \lambda \gamma^m \theta$ implies that $\Gamma^m \Gamma^n \Gamma^p \Gamma^q \Gamma^r$ is proportional to

$$(\lambda \gamma^{mpqr}) \lambda (\lambda \gamma^s \theta) (\lambda \gamma^t \theta) (\lambda \gamma^u \theta) (\theta \gamma_{stu} \theta).$$  \hfill (3.19)

So (3.13) implies that $\int d^5 \Gamma \ F(\Gamma)$ is proportional to $\langle \tilde{F}(\lambda, \theta) \rangle$ where $\tilde{F}(\lambda, \theta)$ is obtained from $F(\Gamma)$ by setting $\Gamma^m = \lambda \gamma^m \theta$.

Because of its close relationship with the pure spinor formalism, it might be possible to get intuition about a super-twistor prescription for $N$-point tree amplitudes from the pure spinor formalism. The $N$-point tree amplitude prescription using this superstring formalism is

$$\mathcal{A} = \langle V(1)(z_1)V(2)(z_2)V(3)(z_3) \prod_{r=4}^{N} \int dz_r U(r)(z_r) \rangle$$  \hfill (3.20)

where $V = \lambda^\alpha A_\alpha$ is the dimension zero unintegrated vertex operator, $U = \partial \theta^\alpha A_\alpha + (\partial x^m - \frac{1}{2} \theta \gamma^m \partial \theta) A_m + \ldots$ is the dimension one vertex operator satisfying $QU = \partial V$, and $Q = \int dz \lambda^\alpha d_\alpha$ is the BRST operator.

In this formalism, the composite operator

$$b = \frac{1}{2} (\partial x^m - \frac{1}{2} \theta \gamma^m \partial \theta) (u \gamma_m d) + \ldots$$  \hfill (3.21)

satisfies $\{Q, b\} = T$ where $u_\alpha \lambda^\alpha = 1$, $T$ is the stress tensor, $d_\alpha$ is the worldsheet variable corresponding to $D_\alpha$, and ... involves non-minimal variables. After including dependence on the non-minimal variables, one can choose Siegel gauge for $V$ \cite{16} in which $b_0 V = 0$ where $b_n V$ denotes the pole of order $(2 + n)$ in the OPE of $b$ and $V$. In this gauge, the integrated vertex operator can be expressed as $\int dz U = \int dz b_{-1} V$ and the tree amplitude prescription of (3.20) can be expressed in terms of $V$ and $b$ as

$$\mathcal{A} = \langle V(1)(z_1)V(2)(z_2)V(3)(z_3) \prod_{r=4}^{N} \int dz_r b_{-1} V(r)(z_r) \rangle.$$  \hfill (3.22)

If $d_\alpha$ is defined as $d_\alpha = q_\alpha - P^m (\gamma_m \theta)_\alpha$ where $q_\alpha = (\gamma^m \lambda)_\alpha \Gamma_m - \Gamma^m (\gamma_m \lambda)_\alpha$ as in (3.3), one finds that

$$\frac{1}{2} P_m (u \gamma^m d) = \frac{1}{2} P_m (u \gamma^m q) = (u_\alpha \lambda^\alpha) [P^m \Gamma_m - \frac{1}{2} \Gamma_m (\bar{\lambda} \gamma^m \lambda)] = B$$  \hfill (3.23)

where the identity $P_m (\gamma^m \lambda)_\alpha = -\frac{1}{2} (\bar{\lambda} \gamma^m \lambda) (\gamma_m \lambda)_\alpha$ has been used. So the super-Poincaré invariant operator $B$ of (3.3) is related to the composite $b$ ghost. This suggests defining the $N$-point tree amplitude super-twistor prescription as

$$\mathcal{A} = \int d^{10} \lambda \int d^5 \mu \int d^5 \Gamma \Phi(1)(z_1) \Phi(2)(z_2) \Phi(3)(z_3) \prod_{r=4}^{N} \int dz_r B_{-1} \Phi(r)(z_r).$$  \hfill (3.24)
However, in order to evaluate this amplitude prescription, one first needs to define a worldsheet action for the super-twistor variables and compute their OPE’s. If this can be done, it seems reasonable to conjecture that the resulting twistor string theory will be related to the usual d=10 superstring theory by a field redefinition. It is intriguing that the bosonic super-twistor variables $\lambda^{\alpha}$ resemble the the bosonic ghost variables in the pure spinor formalism, whereas the fermionic super-twistor variables $\Gamma^m$ resemble the fermionic matter variables in the RNS formalism. In fact, a recent proposal in [17] for relating the pure spinor and RNS worldsheet variables defined fermionic variables $\Gamma^m = \lambda \gamma^m \theta$ and $\bar{\Gamma}_m = u \gamma_m d$ which were related to the RNS variables by twisting [18] as

$$\psi^m = \gamma^{-1} \Gamma^m + \gamma \bar{\Gamma}^m$$

(3.25)

where $\psi^m$ is the RNS fermionic matter variable and $\gamma$ is the RNS bosonic ghost variable.

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