Multi-objective Lightweight Design of Bracket Based on Sensitivity Analysis

Guo Zhounan, Wang Bingzhe, Zhao Fei
The 713th Research Institute of China Shipbuilding Industry Corporation, Zhengzhou, 450015

Abstract: This paper takes a bracket as the research object. In order to realize the lightweight design on the basis of ensuring its static characteristics, use the sensitivity analysis method to analyze the magnitude of the impact of key dimensions on the design of the bracket structure; Combined with the multi-objective optimization design method, the lightest mass, maximum deformation and maximum stress as the optimization objectives, to obtain a set of optimal size values. Finite element analysis of the static characteristics of the structure before and after optimization. After optimization, the weight of the bracket decreased by 2.35%, and the performance was also improved. The results show that combining sensitivity analysis and multi-objective optimization is an effective way to design lightweight structures.

1. Introduction
The lightweight design of weapons and equipment has become the development trend of modern weapons. The bracket serves as the core support of a shipborne weapon, and its rigidity and strength directly affect the accuracy of firepower and the safety of equipment. On the premise of meeting the design requirements, the lightweight bracket can improve the mobility of the equipment and reduce energy consumption.

With the bracket as a support, designers often design the structure based on the overall structure layout and their own experience. With the development of technology, some designers began to use the finite element method to analyze and optimize the bracket structure, but the optimization often only considers the impact of a single parameter on a single performance. They did not carry out multi-objective and multi-parameter optimization design for the complicated working environment of the bracket. The structural safety margin of the final design is often large, which cannot meet the development needs of lightweight modern weapons.

This paper takes a bracket as the optimization object, adopts the sensitivity analysis method, reasonably selects the size parameter as the design variable, and analyzes the influence degree of the parameter change on the bracket deformation and weight. Then, it takes the maximum deformation and the lightest weight as the optimization goal, and carries out multi-objective and light-weight optimization design\[1,2\]. The results showed that the optimized bracket weight decreased by 2.35%, and the performance was also improved.

2. Static sensitivity analysis

2.1. Static characteristic finite element analysis
The finite element analysis of static characteristics is the basis of finite element analysis of structural design. It is used to analyze the structural response (such as displacement, stress, strain, etc.) when the
structure is subjected to a constant external force load (such as gravity, pressure, etc.). According to
the results of statics analysis, the maximum point of structural design deformation, stress, and strain
can be clarified; the static stiffness distribution of the structure can be obtained; the weakness of the
structural design can be found, and the structural improvement and optimization design can be guided.

When using finite element analysis to solve structural mechanical properties, there are three basic
methods including displacement method, stress method and hybrid method. The displacement method
is simple in calculation, strong in regularity and easy for computer programming, so it is commonly
used in the static analysis of the structure by the finite element method.

2.2. The basic principle and process of sensitivity analysis
Sensitivity analysis\cite{3-5} refers to the sensitivity of changes in the structural design objective function $f$
($x$) to changes in the structural design parameter $x$. Generally, it can also be described as: if the
function $f(x_i)$ is an objective function composed of parameters $x_1, x_2, ..., x_i, ..., x_n$, then the derivative or
partial derivative of the objective function $f(x_i)$ for the parameter $x_i$ is the sensitivity of the objective
function $f(x_i)$ to the change of the parameter $x_i$, which can be expressed as:

$$S(x) = \frac{df(x)}{dx}$$

or

$$S(x_i) = \frac{\partial f(x_1, x_2, ..., x_i, ..., x_n)}{\partial x_i}$$

The sensitivity value indicates how much the parameter changes affect the objective function. Using sensitivity analysis can guide designers to design and optimize key dimensions in structural
optimization design.

The general steps of sensitivity analysis are shown in Figure 1.

![Figure 1 General steps of sensitivity analysis](image)

3. Establishment of finite element model
Use Creo to build a three-dimensional model of the bracket as shown in Figure 2, ignoring its detailed
features. When establishing the model, according to the theoretical analysis and design experience, the
parameters related to the design purpose will be parameterized. When parameterizing, the prefix DS_
should be added before the parameter name, so that when using ANSYS Workbench finite element
simulation, the parameter variables imported into the 3D model can be identified. In this paper, the
parameter variables selected for optimal design are shown in Table 1. The bracket material is
aluminum alloy 7A09, and its material properties are: Strength limit $\sigma_s = 480MPa$; density
$\rho = 2850Kg/m^3$; Elastic Modulus $E = 71Gpa$; Poisson’s ratio $\mu = 0.33$; Shear modulus $G = 26.5Gpa$.
4. Multivariate sensitivity analysis

Using ANSYS Workbench to perform finite element simulation analysis on the key dimensions of the above structure, the distribution law of the maximum deformation, maximum stress and mass of the bracket with the change of parameter variables is obtained. Taking the maximum deformation of the structure as an example, the distribution of the maximum deformation of the bracket with the change of parameter variables is shown in Figure 3.

![Figure 3 Distribution law of the maximum deformation of the bracket with different parameters](image)

Through the numerical analysis method, the sensitivity of the parameter variable change to the maximum deformation of the bracket is further calculated, and the least square method is used to fit the objective function, the maximum deformation of the bracket. Some researchers have studied the
fitting curve of the objective function in the sensitivity analysis. The results show that the relationship between the maximum deformation of the bracket structure and the above parameters can be viewed as a linear relationship. Therefore, in this paper, the least square method\(^{6,7}\) is used to fit the relationship between the maximum deformation of the bracket and the above dimensional change, so as to obtain the sensitivity value of the response.

The relationship between the maximum deformation of the bracket and a certain parameter can be fitted into a linear equation:

\[ f(x_i) = \alpha x_i + \beta \]

In the formula, the absolute value of \( \alpha \) is the sensitivity value of the objective function \( f(x_i) \) to the parameter \( x_i \).

When the sum of the square of the difference (ie, the residual value) between the theoretically calculated value \( f(x) \) and the experimental value \( f(x_i) \) is the smallest, the fitting degree of the fitted curve is high. First of all, it is necessary to calculate the sum of squared differences, which is the minimum value of the calculation formula.

\[ SS = \sum_{i=1}^{n} (f(x_i) - f(x)) \]

Solve the joint formula and get

\[ \beta = \frac{\sum x_i f(x_i) - n \bar{x} f(x)}{\sum x_i^2 - n \bar{x}^2} \]

\[ \alpha = \frac{\sum x_i f(x_i) - n \bar{x} f(x)}{\sum x_i^2 - n \bar{x}^2} \]

Calculated

\[ f(x_{D1}) = -0.0672x_{D1} + 2.2249 \]
\[ f(x_{D2}) = -0.0042x_{D2} + 2.7296 \]
\[ f(x_H) = 0.0010x_H + 0.0513 \]
\[ f(x_L) = -0.0003x_L +1.5855 \]

Then the relationship between the maximum deformation of the bracket and the change of a certain parameter is:

\[ f(x) = -0.0672x_{D1} -0.0042x_{D2} -0.0003x_H +0.0010x_H +6.5912 \]

From the expression, the sensitivity of each parameter change to the maximum deformation of the bracket can be obtained, see Table 2.

| Parameter name | Sensitivity |
|----------------|-------------|
| L              | 0.0003      |
| H              | 0.0010      |
| D1             | 0.0672      |
| D2             | 0.0042      |

From the sensitivity calculation results, it can be seen that the change in parameter D1 has the largest sensitivity value to the maximum deformation of the bracket, and the change in parameter L has the smallest sensitivity value to the maximum deformation of the bracket. Therefore, the change of parameter D1 has the greatest influence on the maximum deformation of the bracket, and the change of parameter L has the smallest influence on the maximum deformation of the bracket. The sensitivity of each parameter changes to the maximum deformation of the bracket from large to small is: D1, D2, H, L. Similarly, the sensitivity of each parameter change to the maximum stress of the bracket and the mass of the bracket is calculated, as shown in Table 3.
Table 3 The sensitivity of each parameter change to the maximum stress and the mass

| Parameter name | Sensitivity Maximum stress | Sensitivity Mass |
|----------------|---------------------------|-----------------|
| L              | 0.0003                    | 0.0001          |
| H              | 0.0010                    | 0.00005         |
| D1             | 0.0672                    | 0.0174          |
| D2             | 0.0042                    | 0.0011          |

Simulation calculations show the sensitivity of each parameter change to each objective function, as shown in Figure 4. The height of the histogram represents the size of the sensitivity. Similarly, the positive and negative of the histogram does not represent the positive or negative of the sensitivity value. We still compare the absolute value. Comparing the simulation results with the sensitivity calculated by numerical analysis, it is found that the sensitivity values obtained by the two methods are consistent, which proves the correctness of the method of fitting the sensitivity using the linear function. In addition, the change of the parameter D1 has a great sensitivity to the mass of the bracket. In the lightweight design, the size of the parameter can be reduced to achieve the goal of rapid weight loss. However, the sensitivity of the change of the parameter D1 to the maximum deformation of the bracket is also great, so in the optimization design, the parameter D1 should be focused on, so as to meet the requirements of various design indicators.

![Figure 4 Histogram of the sensitivity of each parameter change to each objective function](image)

5. Lightweight design of bracket

5.1. Mathematical model
Building a mathematical model is an important work in structural optimization design. Three parts are included in the building process: parameter variables, constraints, and objective function. The simulation software solves the optimal solution of the objective function according to the constraint conditions through iterative calculation, so as to obtain the optimal design result. The correctness of the mathematical model directly affects the calculation speed, so it is important to pay attention to the mathematical model in the structural optimization design.

Parameter variables: According to the design structure space size, combined with design experience, the size parameters in Table 1 are optimized for design.

Constraint: The optimization range of parameter variables is shown in Table 4.
Table 4 Statistical Table of Variation Range of Parameter Variables

| Parameter name | Variation range (mm) |
|----------------|----------------------|
| L              | 1400–2000            |
| H              | 800–1200             |
| D1             | 10–25                |
| D2             | 340–450              |

Objective function: (1) Minimum mass; (2) Structural response deformation is less than or equal to 0.8mm; (3) Structural response maximum stress is less than or equal to 40Mpa.

5.2. Optimization and result analysis

The software obtains the convergence process of each objective function calculation through iterative calculation, function convergence, and optimization calculation, as shown in Figure 5. The cloud diagram of the finite element analysis of the bracket before and after optimization is shown in Figure 6. The parameters and performance indicators are shown in Table 5. After optimization, it was found that the weight of the bracket reduced by 2.35%; the maximum deformation of the bracket reduced by 6.54%, and the maximum stress of the bracket increased by 7.88%, but less than 40Mpa to meet the performance index requirements. The results show that the multi-objective optimization analysis method can achieve the goal of lightweight design of the bracket structure under the premise of ensuring the performance index requirements.

(a) Maximum deformation (b) Maximum stress (c) Mass

Figure 5 Schematic diagram of the convergence process of each function

(a) Deformation cloud diagram before optimization (b) Stress cloud diagram before optimization

(c) Optimized deformation cloud diagram (d) Optimized stress cloud diagram

Figure 6 Finite element analysis cloud diagram before and after optimization
| Parameter / performance index | Before optimization | After optimization | Rate of change |
|-------------------------------|---------------------|--------------------|----------------|
| Bracket bottom width          | 1600                | 1573               | -1.69%         |
| Bracket shoulder height       | 1050                | 858                | -18.29%        |
| Bracket plate thickness       | 20                  | 18.5               | -7.5%          |
| Bracket thickness             | 400                 | 431.5              | 7.88%          |
| Maximum deformation (mm)      | 0.8562              | 0.8010             | -6.45%         |
| Maximum stress (Mpa)          | 29.663              | 33.235             | 12.04%         |
| Mass (t)                      | 0.5104              | 0.4984             | -2.35%         |

### 6. Conclusion

Sensitivity analysis method is used to find and optimize the parameters that are sensitive to the design goals in the structural design variables, which is an accurate and efficient optimization method in the design of the bracket structure. Combined with the multi-objective optimization method, the lightweight design of the bracket can be achieved while ensuring its rigidity and strength performance. The results show that, using the above two methods, the optimized weight of the bracket reduced by 2.35%, and the performance is also improved. Therefore, the method used in this paper has its own engineering practical significance and reference value in the optimization design of mechanical structure.

### References

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