Simultaneous arrival of information in absorbing wave guides

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We demonstrate that the temporal peak generated by specific electromagnetic pulses may arrive at different positions simultaneously in an absorbing wave guide. The effect can be used for triggering several devices all at once at unknown distances from the sender or generally to transmit information so that it arrives at the same time to receivers at different, unknown locations. This simultaneity cannot be realized by the standard transmission methods.

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In a previous paper [1] we have described a surprising effect, namely, that the temporal peak of a quantum wave from a source with a sharp onset may arrive at different locations simultaneously in absorbing media. This “ubiquitous peak” (UP) effect was demonstrated for the Schrödinger equation, where, unlike the Hartman effect, it holds at arbitrarily large distances (see [1] for further differences with the Hartman effect); and also for a relativistic wave equation, but limited to distances where the time of arrival of the peak is larger than the time of the very first front [1]. In this letter we demonstrate that the effect is also found within a broad spatial range in an absorbing wave guide when the source emits (more realistic) smoothed pulses instead of a perfectly sharp step signal. The optimal carrier frequency is barely below the cut-off but, at variance with other “ultrafast” wave phenomena based on anomalous dispersion in absorbing media [2, 3, 4, 5], which depend on the dominance of light, infinite or negative group velocities, the ubiquitous peak (UP) effect was demonstrated for the quantum particle described by the Schrödinger equation, where, unlike the Hartman effect, it holds indefinitely for distances where

\[ \eta(\omega) = \sqrt{1 - \frac{\omega_0^2 - \omega^2 - 2i\delta\omega}{\omega_p^2(\omega^2 - \omega_0^2)^2 + 4\delta^2\omega^2}}, \]

and consider, for a frequency interval around \( \omega_0 \), \( |\omega - \omega_0| < \Delta \), the conditions \( \delta \gg |(\omega^2 - \omega_0^2)/(2\omega)| \) and \( \delta \gg \omega_p^2/(2\omega) \), so that

\[ \eta(\omega) \approx \sqrt{1 + i\frac{\omega_p^2}{2\delta\omega}} = 1 + i\frac{\omega_p^2}{4\delta\omega} = 1 + i\frac{1}{\omega}n_1. \]

(With this choice Eq. 1 is similar to the Klein-Gordon equation discussed in [3].)

We also assume “source” boundary conditions with the value of \( \phi(0, t) \equiv \phi_0(t) \) given for all \( t \), and require \( \phi(z, t) = 0 \) for \( z > 0, t < 0 \), which fixes a unique solution \( \phi(z, t) \). The UP effect was found first for the sharp-onset source function \( e^{-i\omega_0 t}\Theta(t) \) but we shall now examine smoother variants,

\[ \phi_{0m}(t) = \frac{i}{2\pi} \int_{-\infty}^{\infty} d\omega f_m(\omega - \omega_0) e^{-i\omega_0 t}/(\omega - \omega_0 + i0). \]

In the reference case \( f_m(\omega) = f_1(\omega) \equiv 1, \phi_{01}(t) = e^{-i\omega_0 t}\Theta(t) \). The solution of Eq. 1 with the boundary condition in Eq. 3 fulfilling the demand that \( \phi_{m}(z, t) = 0 \) for \( z > 0, t < 0 \) is

\[ \phi_{m}(z, t) = \frac{i}{2\pi} \int_{-\infty}^{\infty} d\omega f_m(\omega - \omega_0) e^{i\omega z - i\omega_0 t}/(\omega - \omega_0 + i0). \]

with the dispersion relation

\[ k(\omega) = \sqrt{\frac{\omega^2}{c^2} - \eta^2(\omega) - \gamma^2} = \frac{c}{1} \sqrt{(\omega + in_1)^2 - \omega_c^2}, \]

and \( \omega_c = c\gamma \) being the cut-off frequency. We may apply the saddle-pole approximation of the integral in Eq. 4 for the case \( f_m(\omega) = 1 \). The approximation \( \phi \approx \Theta(g)\phi_p + \phi_+ + \phi_- \) can be found by following
the steps described e.g. in \[54.\]. The saddle points are \( \omega_{s \pm} = \pm \beta - im_1 \) with \( \beta = \omega_c / \sqrt{1 - z^2 / (c^2 \tau^2)} \). The pole contribution, if \( 0 < g(z, t) = (\omega_0 - \beta)(\omega_0 \beta - \omega_c^2) + n_1 (\beta^2 - \omega_c^2)(-\omega_0^2 + 2\beta \omega_0 - \omega_c^2) \), is \( \phi_p(z, t) = -\exp(i\omega_0 t - i\omega_0 t) \), whereas the saddle contributions are

\[
\phi_{s \pm}(z, t) = \frac{i z_1^{1/2} \sqrt{1 - \frac{z^2}{c^2 \tau^2}}}{\omega_c + \mp \frac{1}{1 - \frac{z^2}{c^2 \tau^2}} (\omega_0 + im_1)} \exp\left(-t m_1 \mp it \omega_c \sqrt{1 - \frac{z^2}{c^2 \tau^2}}\right)
\]

If there is no pole contribution, \( \phi \approx \phi_{s+} + \phi_{s-} \), one may easily obtain lower and upper envelopes for the oscillating signal, \( I_- \leq |\phi_{s+} + \phi_{s-}| \leq I_+ \). Let us now examine the exact solutions for other “window functions” \( f_m \) with a central plateau (see Fig. \[1.\] for examples),

\[
f_2(\omega) = \begin{cases} 
1 & : 0 \leq |\omega| < \Delta \omega \\
1 - \frac{|\omega| - \Delta \omega}{\alpha} & : \Delta \omega \leq |\omega| < \Delta \omega + \alpha \\
0 & : \Delta \omega + \alpha < |\omega|
\end{cases}
\]

\[
f_3(\omega) = \begin{cases} 
1 & : 0 \leq |\omega| < \Delta \omega \\
\exp\left(-\frac{(|\omega| - \Delta \omega)^2}{\alpha^2 - (|\omega| - \Delta \omega)^2}\right) & : \Delta \omega \leq |\omega| < \Delta \omega + \alpha \\
0 & : \Delta \omega + \alpha < |\omega|
\end{cases}
\]

Since \( f_2 \) and \( f_3 \) are non-zero only in a range around 0 it is enough that the form of the refraction index, Eq. \( \ segueq .\), is fulfilled in an interval \( 2(\Delta \omega + \alpha) \) around \( \omega_0 \).

Figure \[2.a.\] shows \( |\phi_m(0, t)|^2 \) for different \( m \) and Fig. \[2.b.\] shows \( |\phi_m(z, t)|^2 \) for \( z = 150 \text{ m} \). A consequence of the smoothing of the signal onset at the source is the cancellation of the oscillations between the two envelopes, i.e., a much simpler signal structure. Also, the very first sharp causal front for \( f_1 \) is substituted by a smooth increase for the window functions \( f_2, f_3 \), but the maximum around \( t = 2 \mu s \) remains. We are interested in the time \( \tau_T \) of this maximum for \( m = 2, 3 \). The times \( \tau_T \) are depicted in Fig. \[3.\] versus \( z \) for different \( m \) and \( \alpha \): \( \tau_T \) for \( m = 2, 3 \) is nearly independent of \( z \) for intermediate values of \( z \), see also Fig. \[4.\] whereas at large \( z \) the maximum behaves “normally” and grows linearly with \( z \).
UP effect, it follows that the peak is predominantly composed by frequencies above the cut-off $\omega_c$. The group velocity calculated at the positive saddle $\Phi$ is always smaller than $c$, but it is unrelated here to the peak’s behaviour.

$\tau_{Ts+}(z)$ may be used to estimate a lower value for the start of the effect: $z_{\text{min}}$ is defined as the smallest $z$ where $|\Phi(z,\tau)| < 1/c$, so that the temporal maximum “moves” beyond $z_{\text{min}}$ faster than $c$ without violating causality in any way. Assuming $n_1 < \omega_0$ and $1 < \xi \equiv \omega_c/\omega_0 < 3/2^{3/2}$ we can find an analytical formula for the maximal value of $\tau_{Ts+}$,

$$\tau_M = \frac{1}{2n_1} \frac{3 - 2\xi - 3 \xi^2}{\xi^2} \approx \frac{1}{2n_1},$$

which gives the arrival time of the peak in the region where the UP effect holds. The values $\tau_M$ are shown in Fig. 3 and Fig. 4 with empty symbols.

The approximation $\phi \approx \phi_+$ breaks down when the saddle point reaches the edge of the window function, i.e. $\text{Re}(\omega_+)$ in $\omega_+ + \Delta \omega$: then the UP effect also breaks down and $\tau_T$ grows linearly with $z$. From the condition $\text{Re}(\omega_+) = \omega_0 + \Delta \omega$ the upper boundary for the effect is given by $z_{\text{max}} = cT_s \sqrt{1 - \omega_0^2/(\omega_0 + \Delta \omega)^2}$, which increases with $\Delta \omega$. This value of $z_{\text{max}}$ coincides with the penetration length of $\omega = \omega_0 + \Delta \omega$ defined by $l = 1/(2\mu|k(\omega)|)$. However, $z_{\text{max}}$ cannot be arbitrarily large since the maximum of the saddle eventually vanishes. (The value $z_M$ where this occurs is given by a lengthy expression. In the range of parameters considered in the examples $z_{\text{max}} < z_M$ so that $z_{\text{max}}$ is the true upper bound.)

Let us examine what happens by changing $\omega_0$ or $n_1$. According to Fig. 4b, the effect exists also for carrier frequencies above the cut-off, $\omega_0 > \omega_c$, but in that case a much greater peak appears at small $z$ which travels with finite velocity, and the simultaneous arrival effect is only seen at larger distances from the source when the main forerunner has not yet arrived and a much smaller peak is formed first. For the applications described below (simultaneous triggering or sending information that arrives simultaneously to different receivers) it is more convenient to use energies just below the cut-off, because the traveling forerunner does not exist, the attenuation is minimal, and the spatial range in which the effect holds becomes maximal. In Fig. 4b we can see that a stronger absorption leads to faster arrival of the peak. The upper and lower limits of the effect diminish when $n_1$ is increased, but detection becomes more difficult because of the attenuation of the signal.

In the following we shall illustrate how this effect can be used to send a triggering signal or information to receivers at unknown distances in such a way that the information arrives at all receivers at nearly the same time.

\begin{align*}
\Phi_0(t) &= b_1\phi_{03}(t) + b_2\phi_{03}(t-t_2) + b_3\phi_{03}(t-t_3) + b_4\phi_{03}(t-t_4).
\end{align*}

As Eq. 11 is linear, $\Phi(z, t)$ can be found by adding the solutions corresponding to each term separately. An example of $|\Phi(z, t)|^2$ is plotted for different $z$ in Fig. 5 where the times of the maxima are nearly independent of $z$. Suppose that the receivers are located at unknown distances from the source and that they do not have synchronized clocks (only their unit time intervals are equal). Each receiver gets $|\Phi(z, t)|^2$ and may use the following operational procedure to find the peaks: $\tau_p$ is the time of finding a peak (the peak is at time $\tau_p - \Delta t$ if $|\Phi(z, \tau_p - \Delta t)|^2 > |\Phi(z, \tau_p)|^2$ for $\tau_p - \Delta t > t \geq \tau_p$). Moreover there should be a noise level $l_0$ so that $|\Phi(z, t)|^2$ is assumed as zero if $|\Phi(z, t)|^2 < l_0$. After finding a peak the search for the next peak is started if $|\Phi(z, t)|^2 < l_0$. The noise level may establish an upper limit $z_{\text{noise}}$ for the effect more strict than $z_{\text{max}}$, beyond which the attenuation makes impossible in practice to distinguish the peak.

The resulting times $\tau_p$ of the different peaks are plotted in Fig. 6. Clearly the receivers find the peaks at nearly the same times $\tau_p$. For comparison, lines with
and partly controllable domain. The task of sending in-wave guide arrive simultaneously at receivers in a broad ima generated by specific wave pulses in an absorbing
necessary that the peaks are sent at equal time intervals. for example, would represent a sequence 1010. It is not following peaks carry the message. The signal in Fig. 5, a logical 1 and the lower peak a logical 0, whereas the used for calibration, e.g. the higher peak may represent information coded in the value of triggering signal. Moreover it is possible to send bits of known distances if we use only the first maximum as the signal to them when it finds the peak. This shows that the peak earlier than if the nearest receiver sends a light slope 1
τ
[μs]
×
P
[10]
FIG. 6: Times of finding the peaks τp versus z (dots); l0 = 0.5 × 10−5, Δt = 0.1μs; the lines have gradient 1/c; for the other parameters see Fig. 5 and text for details.
slope 1/c are also plotted. In all cases the receivers get the peak earlier than if the nearest receiver sends a light signal to them when it finds the peak. This shows that the effect can be used to trigger receivers at different unknown distances if we use only the first maximum as the triggering signal. Moreover it is possible to send bits of information coded in the value of |Φ(z, t)|2 at the time of the maxima. The height of the first two peaks may be used for calibration, e.g. the higher peak may represent a logical 1 and the lower peak a logical 0, whereas the following peaks carry the message. The signal in Fig. 5, for example, would represent a sequence 1010. It is not necessary that the peaks are sent at equal time intervals.

Summarizing, we have shown that the temporal maxima generated by specific wave pulses in an absorbing wave guide arrive simultaneously at receivers in a broad and partly controllable domain. The task of sending in-
formation to arrive at different receivers simultaneously is different from the question of superluminal velocities because the information always arrives subluminally and it will be possible in principle to send information faster to a single fixed receiver than with the present effect.

The ubiquitous peak is dominated by above-cut-off frequencies so it is of a fundamentally different origin from effects based on superluminal tunneling and on negative and/or superluminal group velocities. In the case described by the Schrödinger equation it is closer in nature to the over-the-barrier, saddle dominated peak that arrives at the Büttiker-Landauer time in a non-absorbing waveguide, and in fact tends to it continuously when the absorption vanishes. However, that peak “moves” with a semiclassical tunneling velocity whereas in the absorbing medium it appears everywhere simultaneously within the domain of the effect.

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