Estimation of Stress Concentration Factor of Plate with Hole using Piezoelectric Actuator and Finite Element Method

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Abstract. In this paper, the effect of piezoelectric actuators placed above a circular hole of a rectangular plate subjected to uniform uniaxial tension is studied. The core idea is to investigate the stress (compression/tension) produced by the piezoelectric actuators on the stress distribution around the hole and along the width of the host plate. For this purpose, Finite Element Analysis (FEA) was carried out through parametric study in ANSYS software. The results demonstrated that the positive electric field would decrease and change the state of the stress distribution along the width of the host plate in contrast to the negative applied electric filed which increases the stress distribution smoothly without affecting its behaviour. The results also indicated that the reduction of the stress concentration factor increases with the decrease of the ratio (D/W) for the same applied positive electric field.

1. Introduction
Holes location in engineering structures will produce stress or strain concentrations and hence will decrease the mechanical properties and may cause a crack initiation. The amount of strains or stresses concentration due to the presence of the hole is known as the stress-concentration factor (SCF). The stress concentration factor is the ratio of the maximum stress at the edge of the hole to the nominal (reference) stress [1].

Recently, structures incorporating smart materials such as piezoelectric transducers and actuators are becoming attractive for many researchers. These smart structures could be applied in many engineering applications, such as aircraft structures, satellites, large space structures, etc. [2-5]. The piezoelectric materials are widely used in different engineering structures as actuators and sensor due to their coupling behavior. This coupling behavior is attributed to the interaction effects of the mechanical and electrical properties.

Many theoretical and experimental investigations show that the piezoelectric actuator affects the strain and stress in isotropic materials [6]. The crack tip stress concentration reduction by using piezoelectric actuator to generate counteracting bending moment has been investigated by Wang et al [7]. The piezoelectric actuator stress has been employed by Liu [8] to reduce the stress intensity factor of a crack beam using the fracture mechanics approach. The compression stress induced by the piezoelectric actuator near the crack tip has been used to reduce the fatigue damage propagation in thin aluminum panels [9]. The stress produced by the piezoelectric actuator was used to control the edge debonding in beams strengthened with composite materials [10]. Mode-I active repair using the piezoelectric behaviour restricted to the square actuator without accounting for the singularity of the
stress at the crack tip and the influence of adhesive properties in the repair process was carried out by Abuzaid et al. [11-12].

The control of the stress concentration for plate with a hole by using piezoelectrics actuator has been studied by Sensharma et al. [13]. Shah et al. [14] demonstrated that the stress concentration around the hole in a plate under tension can be reduced effectively by using the stress produced by the piezoelectric actuators. They investigated numerically two locations for the piezoelectric actuator around the hole. Their results show that the stress flow in the plate can reduce smoothly.

Lately, the effect of stiffness and thickness ratio of host plate and piezoelectric patches on reduction of the stress concentration factor has been investigated by Fesharaki et al. [15]. Their results indicated the importance of the stiffness ratio on the piezoelectric best placement for the SCF reduction. Fesharaki and Golabi [16] furthered the state of knowledge by specifying the actuators pattern recognition for the optimization of the stress concentration reduction.

Most of the previous works analyzed the piezoelectric actuator effects near the discourteous locations for the purpose of actively controlling the high stress concentration of the host structure. However, only limited numbers of studies have investigated the effects of the piezoelectric actuator when employed above the discontinuous locations. In this paper the effects of the piezoelectric actuators placed above the circular hole of a rectangular plate under uniform uniaxial tension were considered. The core idea is to investigate the stress (compression/tension) generated by the piezoelectric actuators on the stress distribution around the hole. For this purpose, the finite element analysis (FEA) was employed and two cases for the applied electric filed (negative/positive) were studied. The analysis of the two cases was carried out through parametric study in ANSYS finite element software.

2. Formulation of the problem

The rectangular plate containing a central hole with integrated piezoelectric actuator (placed above the hole) and loaded in a uniform uniaxial tension shown in figure 1 is considered. When the uniform tension is applied, the theoretical stress concentration factor $K_t$ in case without actuator is [1]:

$$K_t = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}}$$  \hspace{1cm} (1)

where $\sigma_{\text{max}}$ is the maximum stress and $\sigma_{\text{nom}}$ is the nominal stress. For the linear piezoelectric materials, the strain is linearly related to the electric field. Therefore, the displacement produced by an actuator can be easily controlled by changing the applied electric field. Applying electric field to the piezoelectric actuator would induce stress on the host structure which affects the host plate stress concentration factor and the stress distribution.

![Figure 1. Plate with circular hole and adhesively bonded piezoelectric actuator over the hole.](image)
Two different applied electric fields were considered in this study: the first case is a positive applied electric field (produces compression stress in the piezoelectric actuator) while the second case is a negative applied electric field (produces tension stress in the piezoelectric actuator).

For the purpose of comparison, explicit relationship between the applied mechanical stress \( \sigma_{\text{app}} \) and the applied electric field \( E_0 \) was computed. The applied electric field \( E_0 \) was found such that it would produce equal stress in the infinite piezoelectric actuator to the applied mechanical stress \( \sigma_{\text{app}} \) [14-15].

### 3. Piezoelectric Constitute Equations

The constitutive equations governing the linear electro-mechanical piezoelectric effect are given as below [17].

\[
S_1 = s_{11}^E T_1 + d_{31} E_3 \quad (2)
\]

\[
D_3 = d_{31} T_1 + \varepsilon_{33}^T E_3 \quad (3)
\]

where \( S_1 \) is the mechanical strain, \( T_1 \) is the stress, \( D_3 \) is the electrical displacement, \( E_3 \) is the electric field, \( s_{11}^E \) is the mechanical compliance at zero field, \( \varepsilon_{33}^T \) is the dielectric constant at zero stress, \( d_{31} \) is the piezoelectric coefficient.

### 4. Finite Element Analysis

**Finite Element model**

In ANSYS, the structural element SOLID186 is used to model both the host structure and the bonding layer. This element is appropriate for the analysis of solid structures (3D) and recommended for linear application. Fine resolution is used to model the plate near the circular area. Piezoelectric actuator is molded with the structural element SOLID226.

The dimensions of the host plate considered in this work are \( W = 100 \text{ mm}, H = 200 \text{ mm} \), with a central hole of diameter \( D = 0.2W \) and thickness \( t \) of 1mm. The PIC151 piezoelectric actuator and the adhesive have the dimensions of \( h_p = h_{ad} = 1.5D, w_p = w_{ad} = 1.5D \), and thickness of \( t_p = 0.5t \), \( t_{ad} = 0.03 \text{ mm} \). The material used in this simulation for host structure has the parameters: density \( \rho = 2715 \text{ kg m}^{-3} \), Young’s modulus \( E = 200 \text{ Gpa} \) and Poisson ratio \( \nu = 0.3 \). For the piezoelectric actuator PIC151 was used. Some of the important material and electrical data for the piezoelectric actuator are: density \( \rho_p = 7800 \text{ kg m}^{-3} \), elastic compliance constant \( S_{11} = 19 \times 10^{-12} \text{ m}^2 \text{ N}^{-1} \), piezoelectric constant \( d_{31} = -2.10 \times 10^{-10} \text{ m/V} \) and electric permittivity coefficient \( \varepsilon_{33}^T = 2400 \).

**Validations of the finite element model**

In order to validate the finite element model, the plate with a hole and piezoelectric actuator placed on the top/bottom of the hole in figure 1 of Ref. [15] is considered. By comparing the results obtained by Ref. [15] and the present finite element results, there is a good agreement as shown in table 1.

### Table 1. Verification of the present finite element model.

| Condition       | Applied electric field | SCF Ref. [11] | SCF present work |
|-----------------|------------------------|---------------|------------------|
| Without actuator| -                      | 3.17          | 3.16             |
| With actuator   | -E0                    | 1.7           | 1.68             |

### 5. Results and Discussion

In this section, the FEA was employed to study the effect of the piezoelectric actuators placed above the circular hole of a rectangular plate under uniform uniaxial tension for the problem shown in figure 1. In this study the stress \( \sigma_y \) is normalized by the applied stress \( \sigma_{\text{app}} \).
The normalized stresses $\sigma_y$ around the circular hole in host plate with positive and negative applied electric field are depicted in figures 2 and 3, respectively. The results show that the normalized stress distribution $\sigma_y$ reduced with the application of the positive electric filed ($+E_0$) and this reduction increases with increasing the positive applied electric field (figure 2). Alternatively, the application of the negative electric field ($-E_0$) leads to increase of the SCF and at the same time the stress distribution (figure 3). This could be attributed to the strain produced by the piezoelectric actuator which is positive in the case of the ($+E_0$) and hence lead to reduction of the effective stress around the hole. It is noted from figure 2 that applying electric field of ($2E_0$) would change the state and the location of the maximum stress around the hole. It is also observed that at the compression zone of the stress around the hole ($\theta \approx 90$), the curves meet each other for both cases of the applied electric field (negative/positive).

The normalized stresses $\sigma_y$ along the width of the host plate with various applied electric fields (positive/negative) are illustrated in figures 4 and 5, respectively. The results show that the application of the positive electric filed decreases the SCF and the stress distribution $\sigma_y$ along the host plate width (figure 4). The results also show that there is a dip in the curves at the end location of the piezoelectric actuator.

**Figure 2.** The stress $\sigma_y$ around hole in host plate (Case 1).

**Figure 3.** The stress $\sigma_y$ around hole in host plate (Case 2).

**Figure 4.** The distributions of the stress $\sigma_y$ of the plate along the $x$ -axis (Case 1).

**Figure 5.** The distributions of the stress $\sigma_y$ of the plate along the $x$ -axis (Case 2).
It is also observed that the reduction of the stress distribution at far distance from the hole and the piezoelectric actuator edge are slightly less compared to stress reduction near the vicinity of the hole. In contrast, the negative applied electric field smoothly increases the SCF and also the stress distribution along the width of the host plate (figure 5).

Figures 6 and 7 show the normalized stress distribution $\sigma_{\text{ypiezoe}}$ in piezoelectric patch with positive and negative applied electric field, respectively. It is clear from the results that increasing the applied negative electric field increases the tension stress (figure 6) while increasing the positive applied electric field increases the compression stress (figure 7).

The passive effects of the piezoelectric actuator with different actuator thicknesses on the normalized stress distribution and the stress $\sigma_y$ around the hole in host plate are given in figures 8 and 9, respectively. The results show that increasing the actuator thickness increases the reduction of the stress distribution. In fact, this is due to the increase of the integrated structure stiffness.

For the purpose of observing the effect of the piezoelectric actuator thickness on the plate stress distribution, the normalized stress $\sigma_y$ around the circular hole in host plate with negative/positive applied electric field is depicted in figure 10. From the figure, it is clear that thicker actuator with positive electric field relatively gives higher decrease of the SCF and the stress distribution compared...
to thin actuators. It is important to mention that the increase of the actuator thickness will not affect the location of the maximum stress around the hole of the host plate with application of the electric field.

Figure 11 demonstrates the effect of the piezoelectric actuator thickness on normalized stress $\sigma_y$ along the width of the host plate with various applied electric fields. The figure shows that applying electric field of $(E_0)$ would change the state of the stress distribution along the width of the host plate in contrast to the negative applied electric field which increases the stress distribution smoothly without affecting the behaviour of the stress distribution.

The effects of the ratio (D/W) with fixed piezoelectric actuator size on the normalized stress $\sigma_y$ along the width of the host plate with negative and positive applied electric fields are shown in figures 12 and 13, respectively. The results in figure 12 show that the reduction of the stress concentration factor increases with decrease of the ratio (D/W) with the application of the same positive electric filed $(E_0)$. It is also observed that the stress drop in the curves at the end location of the piezoelectric actuator increases with decrease of the ratio (D/W). It is clear from figure 13 that the reduction of the stress distribution at far distance from the hole and the piezoelectric actuator edge are slightly less compared to stress reduction near the vicinity of the hole.

Figure 10. The Stress $\sigma_y$ around hole with different piezoelectric actuator thickness and applied electric filed.

Figure 11. The stress distributions $\sigma_y$ of the plate along the $x$−axis with different piezoelectric actuator thickness.

Figure 12. The stress distributions $\sigma_y$ of the plate along the $x$−axis with different D/W ratio (Case 1).

Figure 13. The stress distributions $\sigma_y$ of the plate along the $x$−axis with different D/W ratio (Case 2).
6. Conclusion
This paper investigated the effect of piezoelectric actuators placed above a circular hole of a rectangular plate under uniform uniaxial tension. The methodology of this work is based on the investigation of the electromechanical behaviour of the piezoelectric actuator on the stress distribution around the hole and along the width of the host plate. The results showed that the normalized stress distribution reduces with the application of positive electric field and this reduction increases with increasing the positive applied electric field. Alternatively, the application of a negative electric field would lead to increase of the SCF and at the same time the stress distribution. The results also indicated that the ratio (D/W) affects the reduction of the stress concentration factor and the stress distribution at the vicinity of the hole.

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References
[1] Walter D F P and Pilkey D 2008 Stress Concentration Factors (ed: John Wiley & Sons), p. 560.
[2] Crawley E F and De Luis J 1987 AIAA J. 25 1373
[3] Srinivasan A V and McFarland D M 2001 Smart structures: analysis and design (Cambridge University press).
[4] Frecker M I 2003 J. Intel. Mat. Syst. Str. 14, 207
[5] Chopra I 2002 AIAA J. 40 2145
[6] Abuzaid A, Hrairi M and Dawood M S I S 2015 Actuators 4 77
[7] Wang Q, Quek S T and Liew K M 2002 Smart Mater. Struct. 11 404
[8] Liu T J C 2007 Theor. Appl. Fract. Mech. 47 120
[9] Platz R, Stapp C, and Hanselka H 2010 Smart Mater. Struct. 20 085009
[10] Rabinovitch O 2007 J. Compos. Mater. 41 525
[11] Abuzaid A, Hrairi M, and Dawood M S I 2015 Adv. Mater. Res. 1115 517
[12] Abuzaid A, Dawood M S I, and Hrairi M 2015 Appl. Mech. Mater. 799-800 788
[13] Sensharma P K, Palantera M J, and Haftka R T 1993 J. Intel. Mater. Syst. Str. 4 509
[14] Shah D, Joshi S, and Chan W 1994 Smart Mater. Struct. 3 302
[15] Fesharaki J J, Madani S G 2016 Int. J. Adv. Struct. Eng. 8 229
[16] Fesharaki J J and Golabi S I 2016 Smart Struct. Syst. 17 725
[17] IEEE 1988 IEEE Standard on Piezoelectricity ANSI/IEEE Std 176-1987.