Evaluation drag coefficients for circular patch vegetation with different riverbed roughness

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Abstract: The drag coefficient of a one-line circular patch of emergent sparse vegetation in an open-channel flow or riverbeds with varying bed roughness sizes was experimentally investigated. The total roughness of the channel was accounted for using the rate of energy dissipation that incorporated resistance by bed particles and cylinder obstacles. A total of 48 flume experiments were conducted by varying the flow rate, bed roughness, and water depth. The analysis of flow resistance due to the one-line patch of sparse vegetation and the variation in bed roughness was achieved through dimensional analysis. The results show that the drag coefficient $C_d$ is a function of stem Reynolds number, $R_d$, Froude number, $F_r$, and the ratio between stem diameter and flow depth ($d/H$). Experiments showed that the drag coefficient varied with the stem Reynolds number, Froude number, and bed roughness. The $C_d$ generally increased with decreasing stem Reynolds number and Froude number, and this increase in drag coefficient appeared significant for a rough bed of large particles. Results showed that Reynold numbers for different bed roughness had a minimal influence on the drag coefficients. The $F_r$ increases as bed materials are smoother. Statistical regression analysis was achieved to drive the relationship between drag coefficient versus $R_d$, $F_r$, and $d/H$ with a coefficient of determination of more than 0.97.

Subjects: Civil, Environmental and Geotechnical Engineering; Water Engineering; Hydraulic Engineering

Keywords: Vegetation; BED roughness; drag coefficient; stem Reynolds number

1. Introduction

Vegetation in natural rivers, floodplains, and irrigation canals exerts a crucial effect on the ecological environment in many respects. The growth of aquatic plants in rivers and natural streams causes an increase in water level and in the flow velocity around a vegetation patch, attenuates flood discharge, and controls the quantity of sediments due to blockage (Al-Husseini et al., 2020; Liu & Zeng, 2016; Mulahasan et al., 2021). Vegetation in aquatic ecosystems can be
found in different configurations and alignments (Cheng & Nguyen, 2011; Cornacchia et al., 2018; Coscorella et al., 2021; D’Ippolito et al., 2019; Gong et al., 2019; Mulahasan & Stoesser, 2016; Mulahasan et al., 2017; Tanino & Nepf, 2008; Wang et al., 2014).

A circular patch of vegetation may be found in natural rivers as an isolated patch (Chang & Constantinescu, 2015; Cheng et al., 2019; Takekuma & Tanaka, 2007; Tanaka & Yagisawa, 2010; Tanaka et al., 2007), neighboring patches (Cornacchia et al., 2018; Kitsikouds et al., 2020; De Lima et al., 2015) or may be found as a one-line circular patch of vegetation (Li et al., 2020; Yokojima et al., 2014). Flow and turbulence structures through an isolated circular patch of vegetation was numerically investigated by a number of researchers (Chang & Constantinescu, 2015, 2015; Kim et al., 2018; Yu et al., 2013, 2014, 2013; Zhang et al., 2019), and experimentally examined by (Cheng et al., 2019; Kitsikouds et al., 2020; Zong & Nepf, 2011, 2012). Flow and turbulence in the wake of neighboring rigid and emergent circular patches of vegetation were experimentally examined by Kitsikouds et al. (2020) and numerically by Lima et al. (De Lima et al., 2015) using Computational Fluid Dynamics (CFD) response. Cornacchia et al. (2018) determined that neighboring patches tend to grow in a slightly angled line, and an isolated patch was found to grow in V-like shapes to reduce drag forces. The disagreement between the numerical large eddy simulation model and the experimental data was attributed to the uniform distribution of the drag coefficient of a vegetation patch. Anjum and Tanaka (2019) experimentally investigated flow variation and backwater effects due to vegetation patches, changing the vegetation density and water depth. Zhou and Venayagamoorthy (2019) investigated wake dynamics of flow through a suspended cylindrical patch using large eddy simulation (LES) at deep flow conditions. Variable number of cylinders was used to study the effects of patch density on the wake dynamics.

The drag coefficient of an isolated vegetation patch was investigated by a number of academic research papers (Cheng et al., 2019; Taddei et al., 2016; Zhou & Venayagamoorthy, 2019). Generally, the drag coefficient of vegetation is influenced by the trunk and leaves due to viscosity and pressure (Albayrak et al., 2012). Flow resistance of two types of vegetation configuration (continuous or discontinuous patch of vegetation along the center-line of the flume) was investigated experimentally and numerically (Yokojima et al., 2014). Flow resistance from the combined effects of bed friction and vegetation elements was investigated (Dupuis et al., 2016). Impact of the porosity of an isolated circular vegetative patch on the drag coefficient was investigated (Cheng et al., 2019). Flow characteristics around a one-line patch of emerged vegetation located on one side of a channel was studied (Li et al., 2020). Flow resistance of floodplains vegetated by a mixture of flexible plants and grasses of low to high vegetation densities was studied by (Box et al., 2021). It was found that flow resistance by floodplain vegetation decreased by 35-90% when the submergence was increased from 1 to 2. Experimentally, the drag coefficient has been evaluated in consideration of changes in flow conditions. The analysis of the connection between vegetation and water flow is a complex issue due to the difference in physical mechanisms that influence the phenomenon and the biomechanical effects that describe different vegetation types (D’Ippolito et al., 2019).

Most previous studies were based on laboratory flume tests or numerical simulations of vegetated streams with smooth beds. A variety of bed roughness can be found in rivers and streams, such as large pebbles and gravel, so the effect of surface roughness was considered together with the growth of apparent plants above the water surface. Low-density vegetation was chosen, distributed in the form of a circular vegetation patch at equal distances along the length of the laboratory flume. This experimental study aims to evaluate the combined effects of vegetation obstacles and bed roughness on drag coefficients. A series of circular patches of emergent, sparse vegetation located at the center line of a laboratory flume with changing flow rate, water depth, and bed material size is studied. In addition, the present study will provide hydraulic characteristics of flow past free surface piercing cylinders such as drag coefficient is of particular relevance to offshore structures. The following section focuses on the flow resistance coefficient by an array of vegetation elements.
2. Materials and methods

2.1. Drag Coefficient relationships
Emergent vegetation in natural streams is usually described by its diameter, height, and area of plants. In laboratory experiments, emergent vegetation is usually mimicked by rigid stems and may be distributed in a lined, staggered, or random arrangement. Its density ($\lambda$) can be expressed by solid volume fraction (SVF) and can be expressed as follows:

$$\lambda = \frac{md^2}{4}$$  \hspace{1cm} (1)

where, $m$ is number of stems per unit bed area, and $d$ is the stem diameter. In the vicinity of rigid vegetation in stream flows, the roughness coefficient is expressed as a function of drag force exerted by the flow on the vegetation elements, stem diameter, and velocity approach due to the different vegetation distributions as follows:

$$F_D = 0.5\rho C_d A_{veg}v_a^2$$ \hspace{1cm} (2a)

$$F_D = 0.5\rho C_d h_{veg}dv_a^2$$ \hspace{1cm} (2b)

where, $\rho$ is the fluid density, $C_d$ is the drag coefficient, $A_{veg} = h_{veg}d$ is the projected vegetation area, $v_a$ is the approach velocity, $h_v$ is the depth of immersed part of vegetation cylinder and $d$ is the vegetation diameter. The drag coefficient is a function of stem Reynolds number $Rd$, Froude number $Fr$, $d/H$ ratio, and vegetation density $\lambda$. The stem Reynolds number, $Rd$ for rigid emergent vegetation elements is defined as follows:

$$Rd = \frac{\rho Ud}{\mu} = \frac{Ud}{\nu}$$  \hspace{1cm} (3)

where $\rho$ is the fluid density, $d$ is the characteristic length and here is stem diameter, $U$ is the fluid velocity, $\mu$ is fluid dynamic viscosity and $\nu$ is the fluid kinematic viscosity.

A number of relationships between the drag coefficient $C_d$ and stem Reynolds number, $Rd$ was extracted experimentally by Schlichting (1982) as follows:

$$C_d = 3.07Rd^{-0.168}, \hspace{0.5cm} (Rd<800)$$  \hspace{1cm} (4a)

$$C_d = 1, \hspace{0.5cm} (800 \leq Rd<8000)$$  \hspace{1cm} (4b)

$$C_d = 1.2, \hspace{0.5cm} (8000<Rd<10^5)$$  \hspace{1cm} (4c)

Different expressions of the drag coefficients are found in the literature review of the topic, as reported in the following paragraphs:

Based on drag force measurements, the drag coefficient is induced in the following expressions by Ishikawa et al. (2000), Kothyari et al. (2009), D’Ippolito et al. (2019), and Ishikawa et al. (2000) conducted experimental work on the drag force of rigid vegetation arranged in an open-channel flow with varying Froude number ($Fr$) and an empirical expression was presented as follows:
\[ C_d = -0.32Fr + 1.24 \]  

where, \( Fr = \frac{v}{\sqrt{gh}} \), g is the acceleration due to gravity, and \( h \) is the flow depth.

Kothyari et al. (2009) formulated the following equation for the drag coefficient \( C_d \) with Froude number \( Fr \) and vegetation density \( \lambda \) and stem Reynolds number \( Rd \) by measuring the fluid forces exerted on the cylinders by a load cell. This equation is applicable for both sub-critical and super critical flows.

\[ C_d = 1.8Rd^{0.5}[1 + 0.45\ln(1 + 100\lambda)](0.8 + 0.2Fr - 1.5Fr^2) \]  

For a linear arrangement of cylinders, (D'Ippolito et al., 2019) evaluated the drag coefficient in terms of vegetation density as follows:

\[ C_d = 0.211\ln(100\lambda) + 0.784 \quad 1000 < Rd < 10000 \]  

Based on Ergun's (1952) equation for both the laminar and turbulent components of the pressure loss across a packed bed, Tanino and Nepf (2008) proposed an expression for the drag coefficient of rigid circular cylinders as follows:

\[ C_d = 2\left[\frac{\alpha_0}{Rd} + \alpha_1\right] \]  

where, \( \alpha_0 = 5 + 313.17\lambda, \alpha_1 = 0.46 + 3.8\lambda, \lambda \) is the projected area occupied by the vegetation elements per unit bed area in an array of cylinders.

Tanaka and Yagisawa (2010) investigated the drag coefficient of clump-type emergent rigid vegetation with a changing number of cylinders inside the patch of vegetation and water depth. Cheng and Nguyen (2011) suggested an empirical relation between \( C_d \) and \( R_v \) through a series of experiments, as shown below:

\[ C_d = \frac{50}{R_v^{0.442}} + 0.7\left(1 - e^{-R_v}\right) \left( R_v = 5.2 - 5.6 \times 10^5 \right) \]  

where \( R_v = \frac{v_0}{v_c} \), \( r_c \) is the vegetation-related hydraulic radius defined as \( r_c = \pi d \left(\frac{1}{\lambda} - \frac{1}{\lambda_c}\right) \), and \( v_0 \) is the velocity approaching the vegetation.

Wang et al. (2014) performed many experiments to obtain a best-fit function explaining the relationship between \( C_d \) and \( R_v \) in a subcritical flow, as shown as follows:

\[ C_d = \frac{90}{Re_v^{0.6}} + 4.5 \frac{d}{H} - 0.303 \ln \lambda - 0.9 \]  

where \( Re_v \) is defined as \( Re_v = \frac{(1-\lambda)\rho v_c}{\mu} \), and \( \lambda = \frac{aN(\lambda)}{\lambda_c} \), where \( N \) is number of stems in an array of vegetation elements.

### 2.2. Experimental setup

The experiments were conducted in a hydraulic recirculating plastic walled flume of 4.58 m long and 0.3 m wide and depth. The flume was set to a bed slope of 0.0005. A sharp-crested weir was installed at the end of the flume’s working section to control the flow depth. Four different flow rates of 3.21, 4.47, 5.88, and 6.90 l/s were used. Discharge values (Q) were measured and determined using a gauge reading installed on the canal pipe entering the laboratory canal.
These values were checked after knowing and measuring the volume of water leaving the canal at downstream using a vessel of a known volume and a stopwatch for each laboratory experiment. This check is repeated three times for each discharge run.

The vegetation was mimicked using sets of rigid wooden circular cylindrical rods of 1 cm in diameter. A series of vegetation patches was installed along the centre line of the flume’s working section with a 20 cm center to center distance (figure 1). The vegetation patches were designed with a diameter D = 10 cm having five cylinders (d = 1 cm) arranged as shown in figure 2. Eleven patches of cylinders were installed at 0.2 m intervals along the flume’s center line, beginning 1.2 m from the flume’s inlet (figure 2a). A control volume was chosen to include one patch and half the area before and after the patch. So, the vegetation density would be the same for the entire channel section and each control volume (figure 2b). The drag coefficient was considered due to vegetation cylinders and bed roughness. Two types of bed roughness in comparison with a smooth bed were considered; coarse gravel (4.75–9.5 mm) mixed with fine gravel (2.36–4.75 mm) and fine gravel (2.36–4.75 mm) as shown in figure 3. The size of the gravel was estimated in the laboratory using the sieve analysis method. Water depths were measured under uniform flow conditions along the mid distance among patches.

2.3. Flow resistance in vegetated streams
For steady uniform flow conditions where the water-level slope is parallel to the bed slope, the driving force of the flow equates with the bed resistance and form drag according to Mulahasan and Stoesser (2016) is expressed as:

\[
F_G = F_f + F_D
\]

(11)

where \(F_G = \rho gSH\left(1 - m\frac{dD}{dH}\right)\) is the gravitational force \(F_f = \tau_D\left(1 - m\frac{dD}{dH}\right)\), is the wall friction drag force, and \(F_D = \frac{1}{2}m\rho C_D H \frac{U^2}{dH}\) is the drag force exerted by the vegetation. From the definition of each term in Equation (11), the following results are obtained:
where \( \rho \) is the fluid density, \( g \) is the gravitational acceleration, \( S \) is the bed slope, \( H \) is the flow depth, \( \tau_b = \rho g S H \) is the bed shear stress, \( h, d \) is the stem diameter, \( R \) is the hydraulic radius = \( A/P \), \( A \) is the cross section area of non-vegetated channel, \( P \) is the wetted parameter for non-vegetated channel, \( L \) is the reach length, and \( U \) is the bulk velocity defined as

\[
U = \frac{QL}{(A_b - A_g)H} \tag{13}
\]

where \( A_b = WL \) is the total frictional bed area, \( W \) is the channel width, and \( A_g = N \pi \frac{d^2}{4} \) is the bed area occupied by stems. Equations 12 and 13 were used in calculating the drag coefficient after measuring and calculating the other variables from the practical results in the laboratory. It is noted that the values of \( S = 1/2000 \), \( N = 5 \). The properties of liquid used (water) are:
Figure 3. Bed roughness for: a) mixed sizes, and b) mono-size bed material.

Figure 4. Variations of water levels along the canal for different discharges.
\[ \rho = 1000 \frac{kg}{m^3}, \mu = 1.002 \times 10^{-3} \frac{N\cdot s}{m^2}, \eta = 1.002 \times 10^{-6} \frac{m^2}{s}, \] and the rest of the variables were measured in the laboratory. The only value of the drag coefficient was calculated for each laboratory experiment.

### 2.4. Dimensional analysis

The relationships between vegetation and resistance under different conditions, such as bed roughness, plant diameter, flow velocity, water depth, and fluid properties, were examined. We assumed that the drag coefficient depends on the variables of water density, water viscosity, gravitational acceleration, depth of water \( H \), flow velocity \( u \), and stem diameter \( d \), and it is expressed as follows:

\[ C_d = f(\rho, \mu, g, H, u, d) \tag{14} \]

The dimensionless numbers of parameters were determined through Buckingham's \( \pi \)-theorem dimensional analysis; \( C_d \) simulated using dimensionless parameters, was expressed as follows (see Appendix A):

\[ C_d = \frac{F\left(R_d, Fr, \frac{d}{H}\right)}{\eta^2} \tag{15} \]

Forty-eight series of experimental data were performed in a subcritical flow and used them to calculate \( C_d \) for patches of vegetation located along the centre line working section of the laboratory flume. Equations were formulated for different bed roughness using the Statistical Package for Social Science (SPSS) software programme (version 24). Equation 15 from the dimensional analysis was used to derive an equation for the drag coefficient through the experiments conducted.

### 3. Results

The results of laboratory experiments for various different flows (\( Q = 3.21, 4.47, 5.88, \) and 6.90 l/s) were as follows:

#### 3.1. Water surface waves

In consideration of free-surface waves, the changes in water levels in the part of the populated flume should be monitored using rods and the fluctuation in surface water around each single rod should be regulated. Flow depth was calculated as an average value through the flume reach using a point gauge with an accuracy of 0.10 mm. Figure 4 shows the measured water-level profile along the entire channel for four discharges at different gate openings of 10 mm, 12 mm, 15 mm, and 20 mm for each discharge of flow. From Figure 4, the water level in the channel increases with the increase in the water discharge at the given gate opening. That is evident for all other gate openings. For example, if the amount of flow discharge is 3.2 l/s and the gate opening is 10 mm, the water level in the channel is about 7 cm. When flow discharge is 4.4 l/s, the water level is about 12 cm. Increasing the flow discharge to 9.6 l/s produces the water level is about 17 cm for the same gate opening of 10 mm. Increasing flow discharge in the presence of vegetation leads to an increase in water level in the canal.

#### 3.2. Bed roughness

To demonstrate the effect of the bed roughness of the channel on the calculation of the drag coefficient, three types of beds were used in the channel, as follows:

- Smooth bed
- Mixed coarse (size 4.75–9.5 mm) and fine gravels (size 2.36–4.75 mm) bed, and
- Fine gravels (size 2.36–4.75 mm) bed.
3.3. Drag coefficient, $C_d$

The results of the drag coefficient were obtained for the bed of the canal and for the various depths of water and discharges under the conditions of the different gate openings as shown in the Table 1–3 in an Appendix using equation 10. Many laboratory experiments were conducted with consideration given to the effect of the drag coefficient in various conditions such as plant presence, flow velocity and flow depth in the canal. From the previously performed dimensional analysis (Equation 13), the first dimensional variable affecting the drag coefficient values is the stem Reynolds number, $Rd$. This non-dimensional variable was calculated through field experiments for different bed roughness in preparation for finding a relationship between this dimensionless variable and the drag coefficient, and this relation represented in figure 5 was drawn. Reynolds number, $Rd$ depends on the depth of flow instead of the diameter of the cylinder used as a plant and its effect on the calculation of the drag coefficient. figure 6 shows the variation of the Reynolds number, $Rd$ and the drag coefficient are given for the various roughness of the channel bed surface. Froude number is the second dimensional variable affecting the drag coefficient, according to the equation of equation 13. The $C_d$ was calculated as shown in Table 1–3, and the relationship between this number and the drag coefficient, represented in figure 7. The dimensional expression $\frac{d}{H}$ represents the last variable in the dimensional analysis affecting the values of the drag coefficient. figure 8 represents the relationship between this dimensionless expression and the drag coefficient for different bed roughness. To obtain a single relationship between drag coefficient, $C_d$ and $Rd, Fr, \frac{d}{H}$, for each type of channel bottom surface roughness, a statistical regression analysis method was used. Statistical regression analysis of the results in the form of equation (13) was performed using the SPSS software programme (version 24). The drag coefficient was calculated as in the following equations. For a smooth bed, the drag coefficient was defined as follows with $R^2$ of 0.977:

$$C_d = 11.457Rd^{0.072} + 0.482Fr^{-1.196} + 0.887\frac{d}{H} - 21.229$$ \hfill (16)

For mixed coarse and fine gravels, the drag coefficient was expressed as follows $R^2$ of 0.994:

$$C_d = 4.938 \times 10^6Rd^{-3.244} + 48.214Fr^{521.473} - 11.946\frac{d}{H} + 2.362$$ \hfill (17)

For mono-sized fine gravel materials, the drag coefficient was as follows $R^2$ of 0.997:

$$C_d = 4.3266 \times 10^6Rd^{-2.894} + 42.407Fr^{515.494} - 11.289\frac{d}{H} + 2.036$$ \hfill (18)

4. Discussion

The measurements were used in the water balance equation to evaluate the drag coefficient; $C_d$. Figure 4 shows the measured water-level profile along the entire channel for four discharges at different gate openings in the vicinity of a series of vegetated patches. The figure demonstrates that vegetation has a significant impact on water depth; that is, for a given discharge, the water depth increases in front of the vegetation elements. The presence of vegetation in the channel exerts an effect on the relationship of variations in water level with distance curve compared with the case without vegetation. Consequently, increases in water depths affect the drag coefficient. Forty-eighty runs of experiments were completed. The measurements of total drag coefficients for all cases are summarised in Table 1–3 with smooth bed, coarse and fine gravel bed; and fine gravel bed, respectively. The values of the drag coefficient $C_d$ were calculated in Table (1–3), depending on equation 10. These tables illustrate the use of four different water discharges, and for each amount, four openings for the gate were used at the downstream of the canal (10, 12, 15, and 20 cm) to demonstrate the effect of the different hydraulic conditions in the presence of the plant on the calculation of the drag coefficient.
Table 1. Results of computing drag coefficient and experimental measurements for smooth bed

| Run | Gate opening, mm | H, m  | R, m  | Q, l/s | U, m/s | Cd  | Fr  | Rd  | R_H |
|-----|------------------|-------|-------|--------|--------|------|-----|-----|------|
| 1   | 10               | 0.077 | 0.0298| 3.21   | 0.289  | 1.438| 0.332| 2887| 22,234|
| 2   | 12               | 0.055 | 0.0288| 3.21   | 0.303  | 1.026| 0.410| 3025| 16,731|
| 3   | 15               | 0.045 | 0.0282| 3.21   | 0.310  | 0.760| 0.467| 3104| 13,968|
| 4   | 20               | 0.039 | 0.0279| 3.21   | 0.314  | 0.568| 0.507| 3140| 12,278|
| 5   | 10               | 0.12  | 0.213 | 4.47   | 0.213  | 2.599| 0.196| 2127| 25,529|
| 6   | 12               | 0.102 | 0.250 | 4.47   | 0.250  | 1.814| 0.250| 2504| 25,539|
| 7   | 15               | 0.085 | 0.309 | 4.47   | 0.309  | 1.165| 0.339| 3092| 26,285|
| 8   | 20               | 0.063 | 0.366 | 4.47   | 0.366  | 0.718| 0.465| 3658| 23,119|
| 9   | 10               | 0.112 | 0.062 | 5.88   | 0.187  | 2.502| 0.178| 1868| 20,887|
| 10  | 12               | 0.095 | 0.059 | 5.88   | 0.202  | 1.812| 0.209| 2019| 19,183|
| 11  | 15               | 0.083 | 0.052 | 5.88   | 0.250  | 1.189| 0.277| 2501| 20,829|
| 12  | 20               | 0.061 | 0.044 | 5.88   | 0.313  | 0.545| 0.404| 3127| 19,073|
| 13  | 10               | 0.17  | 0.078 | 6.9    | 0.143  | 5.203| 0.111| 1428| 24,280|
| 14  | 12               | 0.154 | 0.074 | 6.9    | 0.157  | 4.121| 0.128| 1567| 24,139|
| 15  | 15               | 0.125 | 0.068 | 6.9    | 0.188  | 2.540| 0.170| 1882| 23,528|
| 16  | 20               | 0.1   | 0.058 | 6.9    | 0.245  | 1.366| 0.248| 2450| 24,425|
| Run | Gate opening, mm | H, m  | R, m  | Q, l/s | U, m/s | C\textsubscript{d} | F\textsubscript{r} | R\textsubscript{d} | R\textsubscript{H} |
|-----|------------------|-------|-------|--------|--------|--------------|-------------|-------------|-------------|
| 17  | 10               | 0.072 | 0.048 | 3.21   | 0.146  | 3.056        | 0.174       | 1459        | 10,482      |
| 18  | 12               | 0.064 | 0.042 | 3.21   | 0.176  | 2.140        | 0.223       | 1761        | 11,204      |
| 19  | 15               | 0.058 | 0.041 | 3.21   | 0.181  | 1.756        | 0.239       | 1811        | 10,578      |
| 20  | 20               | 0.058 | 0.041 | 3.21   | 0.184  | 1.732        | 0.244       | 1839        | 10,702      |
| 21  | 10               | 0.128 | 0.069 | 4.47   | 0.122  | 6.102        | 0.109       | 1217        | 15,559      |
| 22  | 12               | 0.104 | 0.061 | 4.47   | 0.150  | 3.609        | 0.148       | 1499        | 15,663      |
| 23  | 15               | 0.084 | 0.053 | 4.47   | 0.191  | 2.025        | 0.210       | 1907        | 16,106      |
| 24  | 20               | 0.070 | 0.047 | 4.47   | 0.227  | 1.253        | 0.274       | 2270        | 15,887      |
| 25  | 10               | 0.144 | 0.074 | 5.88   | 0.112  | 7.491        | 0.095       | 1125        | 16,166      |
| 26  | 12               | 0.115 | 0.065 | 5.88   | 0.143  | 4.176        | 0.134       | 1425        | 16,423      |
| 27  | 15               | 0.093 | 0.055 | 5.88   | 0.187  | 2.265        | 0.196       | 1873        | 17,481      |
| 28  | 20               | 0.077 | 0.049 | 5.88   | 0.229  | 1.386        | 0.263       | 2289        | 17,712      |
| 29  | 10               | 0.176 | 0.081 | 6.9    | 0.108  | 9.145        | 0.082       | 1076        | 18,950      |
| 30  | 12               | 0.141 | 0.071 | 6.9    | 0.139  | 5.006        | 0.118       | 1391        | 19,621      |
| 31  | 15               | 0.116 | 0.064 | 6.9    | 0.171  | 3.005        | 0.161       | 1713        | 19,803      |
| 32  | 20               | 0.086 | 0.053 | 6.9    | 0.231  | 1.409        | 0.251       | 2306        | 19,795      |

Table 2. Results of computing drag coefficient and experimental measurements for bed of mixed coarse and fine gravels

Gubash et al., Cogent Engineering (2022), 9: 2044574
https://doi.org/10.1080/23311916.2022.2044574
| Run | Gate opening, mm | H, m  | R, m  | Q, l/s | U, m/s | C_d  | F_r  | R_d  | R_m |
|-----|-----------------|-------|-------|--------|--------|------|------|------|------|
| 33  | 10              | 0.069 | 0.047 | 3.21   | 0.152  | 2.693| 0.185| 1525 | 10,554|
| 34  | 12              | 0.061 | 0.042 | 3.21   | 0.180  | 1.904| 0.232| 1798 | 10,992|
| 35  | 15              | 0.059 | 0.041 | 3.21   | 0.188  | 1.723| 0.247| 1879 | 11,070|
| 36  | 20              | 0.058 | 0.040 | 3.21   | 0.189  | 1.672| 0.251| 1894 | 11,023|
| 37  | 10              | 0.131 | 0.070 | 4.47   | 0.117  | 6.686| 0.103| 1166 | 15,248|
| 38  | 12              | 0.108 | 0.061 | 4.47   | 0.151  | 3.755| 0.147| 1509 | 16,221|
| 39  | 15              | 0.087 | 0.052 | 4.47   | 0.195  | 2.087| 0.211| 1946 | 16,851|
| 40  | 20              | 0.071 | 0.046 | 4.47   | 0.234  | 1.292| 0.279| 2337 | 16,667|
| 41  | 10              | 0.155 | 0.076 | 5.88   | 0.109  | 8.373| 0.089| 1092 | 16,949|
| 42  | 12              | 0.127 | 0.068 | 5.88   | 0.136  | 4.928| 0.122| 1358 | 17,210|
| 43  | 15              | 0.100 | 0.060 | 5.88   | 0.168  | 2.766| 0.170| 1684 | 16,874|
| 44  | 20              | 0.080 | 0.051 | 5.88   | 0.217  | 1.480| 0.246| 2173 | 17,279|
| 45  | 10              | 0.185 | 0.085 | 6.9    | 0.096  | 11.620| 0.071| 957  | 17,740|
| 46  | 12              | 0.151 | 0.075 | 6.9    | 0.123  | 6.529| 0.101| 1228 | 18,596|
| 47  | 15              | 0.126 | 0.068 | 6.9    | 0.150  | 4.024| 0.135| 1498 | 18,856|
| 48  | 20              | 0.091 | 0.056 | 6.9    | 0.205  | 1.763| 0.217| 2052 | 18,631|

Gubash et al., *Cogent Engineering* (2022), 9:2044574 https://doi.org/10.1080/23311916.2022.2044574
Figure 5 depicts the behaviour of $C_d$ with stem Reynolds number, $R_d$ for different bed roughness of a one-line patch of vegetation. The figure indicates that the highest drag coefficient, $C_d$, of coarse and fine gravel bed materials shows a minimum stem Reynolds number, $R_d$ and that $C_d$ decreases with increasing $R_d$ for all experiments runs. Figure 5 also shows a match between two curves of mixed fine gravels with coarse gravels and mono-sized fine gravels. This is one of the results we have obtained, as shown in equations 17 and 18, the two cases are similar for the presence of gravel with sand in both cases, with the different nature of the roughness of the materials. At high values of $R_d$, the drag coefficient, $C_d$ tends to become constant, and this result is similar to those of D’Ippolito et al. (2019; Kothyari et al. (2009). Figure 6 presents the variation in drag coefficient, $C_d$ with Reynolds number based on water depth; $R_d$ ranges between 10,500 and 26,300. The figure shows that the drag coefficient $C_d$ increases with roughness, remaining constant with $R_d$. The same results were also obtained by D’Ippolito et al. (2019). Figure 7 shows the variation in drag coefficient with the Froude number $Fr$, which indicates that $C_d$ decreases with increasing $Fr$, similar to the variation in Reynolds number, $R_d$ versus drag coefficient. The second term in the water balance equation (equation 11) is the bed shear stress that plays a significant role in flow resistance. It can be said that the coarse and fine gravel bed surface and fine gravel bed surface are more valuable for the drag coefficient due to the presence of roughness compared with the smooth surface bed as shown in Figures 5 and 7.

The dimensional analysis indicates in Equation (15) that the drag coefficient, $C_d$ is a function of the ratio $d/H$. Figure 8 depicts the relationship between the dimensionless $d/H$ and drag coefficient, $C_d$. The trend shows that $C_d$ decreases with increasing $d/H$, as approximated by Morinaqa et al. (2012) for one-line trees. This relationship is illustrated in figure 8 and Equation 16. The drag coefficient decreases with an increase in the ratio $d/H$. As the depth of the water decreases with the stability of the cylinder diameter, this ratio increases, resulting in a lower drag coefficient. Figure 8 shows that the higher roughness gives a greater drag coefficient than the smooth surface bed for the same ratio $d/H$. In general, vegetation drag coefficients with a rough bed illustrate higher values than those with a smooth bed. Statistical regression analysis was used to examine the effect of various hydraulic conditions such as flow velocity, flow depth, as well as the presence of plants within the channel during the flow of water. Mathematical relationships were derived to represent the roughness of the channel bed surface and its effect under these different hydraulic conditions. The equations (16–18) show that the coefficient of determination ($R^2$) is higher than 90%, which indicates the strength of the correlation of the drag coefficient with the measured dimensionless variables ($R_d, Fr, \frac{d}{H}$). It was noticed from Equations 15 and 16 that the variable $R_d$ has
a greater effect than the other non-dimensional variables. This indicates that the presence of vegetation is more influential in calculating the drag coefficient than the Fr number and the ratio d/H. The Froude number (Fr) has the least effect on the drag coefficient in the mixed coarse and fine gravels bed and mono-sized fine gravel material bed, while its effect is visible on the smooth ground bed. Equation (16) shows that there is a clear influence of the three dimensionless variables (Rd; Fr; d/H) for a smooth bed.

Figure 9 shows the comparison in the values of the drag coefficient of the various equations for the previous studies with the equation derived in the current study of the state of the smooth bed roughness by statistical regression analysis. It is evident from the figure that there is a good convergence in the values of the drag coefficient of the current study with the previous studies at high Reynolds numbers. It can also be noted that the Kothyari et al. (2009) equation is the closest
5. Conclusions
The drag coefficient in the vicinity of rigid emergent vegetation with different bed roughness was studied experimentally. Eleven patches of rigid circular cylinders and three types of bed roughness (smooth, coarse and fine, and fine) were used. The main result of this study showed that within the experimental range conducted, the drag coefficient for the coarse and fine beds was greater than that for other bed roughnesses at a minimum stem Reynolds number.
number, \( R_d \). All experimental runs indicated that the drag coefficient, \( C_d \), decreased with increasing stem Reynolds number, \( R_d \), and remained constant at high \( R_d \) for all bed roughness. The drag coefficients were investigated independently of the Reynolds number; when Reynolds number varied from 10,500 to 26,300. The results show that the drag coefficient \( C_d \) of the coarse and fine gravel bed and other surface fine gravel bed is more valuable than that of the smooth bed. The statistical program (SPSS) was used, and nonlinear equations were found through the method of regression analysis, and the correlation coefficient in these equations was above 90%, which indicates the strength of the correlation to find the drag coefficient with the non-dimensional variables. These non-dimensional terms have a clear effect on the channel’s smooth bed, and the effect of the Froude number is small compared to the stem Reynolds number for the vegetative patch at the bottom of the channels of mixed coarse and fine gravel and fine gravel materials. When comparing the statistical regression analysis (equations 16–18) that were derived, it becomes clear that the effect of the Froude number for the case of a smooth bed (equation 16) is greater than the Froude number for the case of the other two surfaces (equations 17 and 18).

Acknowledgements
The authors are thankful to Mustansiriyah University (www.uomustansiriyah.edu.iq) and the staff of the Hydraulic Laboratory- College of Engineering for supporting this research.

Funding
The authors received no direct funding for this research.

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Disclosure statement
No potential conflict of interest was reported by the author(s).

Citation information
Cite this article as: Evaluation drag coefficients for circular patch vegetation with different riverbed roughness, Karim R. Gubashi, Saad Mulahasan, Mohammed Abid Jameel & Abdul-Sahib T. Al-Madhhachi, Cogent Engineering (2022), 9: 2044574.

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Appendix A
Details of Dimensional Analysis

The drag coefficient is function of water density, \( \rho \), water viscosity, \( \mu \), gravitational acceleration, \( g \), depth of water \( H \), flow velocity \( u \), and stem diameter, \( d \), and it is expressed as follows:

\[
C_d = f(\rho, \mu, g, H, u, d)
\]  \hspace{1cm} (A1)

The dimensionless numbers of parameters were determined through Buckingham’s \( \pi \)-theorem dimensional analysis and assumed that repeating variables are \( \rho, U, H \). Therefore, the \( \pi \)-theorem is expressed as:

\[
\pi_1 = C_d
\]  \hspace{1cm} (A2)

\[
\pi_2 = \rho^a \cdot U^b \cdot H^c \cdot \mu
\]  \hspace{1cm} (A3)

\[
\pi_3 = \rho^a \cdot U^b \cdot H^c \cdot g
\]  \hspace{1cm} (A4)

\[
\pi_4 = \rho^a \cdot U^b \cdot H^c \cdot d
\]  \hspace{1cm} (A4)

This research assumed that the dimension variables are mass, \( M \), length, \( L \), and time, \( T \):

\[
M_0^a L_0^b T_0^c = \left( \frac{M}{L^3} \right)^a \left( \frac{L}{T} \right)^b \left( \frac{L}{T} \right)^c \hspace{1cm} (A5)
\]

\[
M : 0 = a + 1
\]  \hspace{1cm} (A6a)

\[
L : 0 = -3a + b + c - 1
\]  \hspace{1cm} (A6b)
\( T : 0 = -b - 1 \) \hspace{1cm} (A6c)

\( a = -1, b = -1, c = -1 \)

\[
\pi_2 = \frac{\rho^{-1} U^{-1} g^{-1}}{\mu} \rightarrow \mu \frac{\rho U H}{\mu} = \text{Re}or\text{Re}_d \hspace{1cm} (A7)
\]

Similarly, the other two terms are expressed as follows:

\[
\pi_3 = U^{-2} H g = \frac{g H}{U^2} or \frac{U^2}{g H} = \frac{U}{\sqrt{g H}} = Fr \hspace{1cm} (A8)
\]

Therefore, the drag coefficient is expressed as:

\[
C_d = F \left( Rd, Fr, d \frac{d}{H} \right) \hspace{1cm} (A9)
\]