Properties of bound states containing fourth family quarks

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Abstract. The heavy fourth generation of quarks that have sufficiently small mixing with the three known SM families form hadrons. In the present work, we calculate the masses and decay constants of mesons containing either both quarks from the fourth generation or one from fourth family and the other from observed SM quarks, namely charm or bottom quark, in the framework of the QCD sum rules. In the calculations, the two gluon condensate diagrams as nonperturbative contributions are taken into account. The obtained numerical results are reduced to the known masses and decay constants of the \( \bar{b}b \) and \( \bar{c}c \) quarkonia, when the fourth family quark is replaced by the bottom or charm quark.

1. Introduction

In the standard model (SM), we have three generation of quarks experimentally observed. Among these quarks, the top (\( t \)) quark does not form bound states (hadrons) as a consequence of the high value of its mass. The top quark immediately decays to the bottom quark giving a \( W \) boson and this transition has full strength. The number of quark and lepton generations is one of the mysteries of nature and can not be addressed by the SM. There are flavor democracy arguments that predict the existence of the fourth generation of quarks \([1, 2, 3]\). It is expected that the masses of the fourth generation quarks be in the interval (300 – 700) \( GeV \) \([4]\). The last value coincides with upper limit following from partial-wave unitarity at high energies \([5]\). Within the flavor democracy approach, the Dirac masses of the fourth family fermions are almost equal, whereas masses of the first three family fermions, as well as CKM and PMNS mixings are obtained via small violations of democracy \([6, 7]\). For the recent status of the SM with four generations (SM\textsubscript{4}), see e.g. \([8, 9, 10]\) and references therein.

Although the masses of fourth generation quarks are larger than the top quark mass (the last analysis of the Tevatron data implies \( m_{d_4} > 372 \, GeV \) \([11]\) and \( m_{u_4} > 358 \, GeV \) \([12]\)), they can form bound states as a result of the smallness of mixing between these quarks and ordinary SM quarks \([13, 14, 15, 16, 17, 18, 19]\). As the mass difference between these two quarks is small, we will refer to both members of the fourth family by \( u_4 \). The condition for formation of new
hadrons containing ultra-heavy quarks \((Q)\) is given by \([20]\):

\[
|V_{Qq}| \leq \left( \frac{100 \text{ GeV}}{m_Q} \right)^{3/2}.
\]  

For t-quark with \(m_t = 172 \text{ GeV}\). Eq. (1) leads to \(V_{tq} < 0.44\), whereas single top production at the Tevatron gives \(V_{tb} > 0.74\) \([21]\). When the fourth family quarks have sufficiently small mixing with the ordinary quarks, the hadrons made up from these quarks can be long enough lived and the bound state \(\bar{u}_4u_4\) decays through its annihilation and not via \(u_4\) decays to a lower family quark plus a \(W\) boson \([19]\). Concerning flavor democracy approach, this situation is realized for parameterizations proposed in \([7]\) and \([22]\), whereas parameterization \([6]\) predicts \(V_{u4} \sim 0.2\) which does not allow formation of the fourth family quarkonia for \(m_{u4} > 300 \text{ GeV}\).

Considering the above discussions, the production of such bound states if exist will be possible at LHC. The conditions for observation of the fourth SM family quarks at the LHC has been discussed in \([13, 23, 24, 25, 26, 27, 28, 29, 30]\). As there is a possibility to observe the bound states containing fourth family quarks at the LHC, it is reasonable to investigate their properties, theoretically and phenomenologically.

In the present work, we calculate the masses and decay constants of the bound state mesons containing two heavy quarks with either both quarks from the SM or one from the heavy fourth family and the other from ordinary heavy \(b\) or \(c\) quark. Here, we consider ground state mesons with different quantum numbers, namely scalar (\(\bar{u}_4u_4, \bar{u}_4b\) and \(\bar{u}_4c\)), pseudoscalar (\(\bar{u}_4\gamma_5u_4, \bar{u}_4\gamma_5b\) and \(\bar{u}_4\gamma_5c\)), vector (\(\bar{u}_4\gamma_{\mu}u_4, \bar{u}_4\gamma_{\mu}b\) and \(\bar{u}_4\gamma_{\mu}c\)) and axial vector (\(\bar{u}_4\gamma_{\mu}\gamma_5u_4, \bar{u}_4\gamma_{\mu}\gamma_5b\) and \(\bar{u}_4\gamma_{\mu}\gamma_5c\)) mesons. These mesons, similar to the ordinary hadrons, are formed in a region of energy very far from the asymptotic region. Hence, perturbation theory can not be used in this region since the coupling constant between quarks and gluons is large. Therefore, to calculate the hadronic parameters such as the mass and leptonic decay constant, we need to consult to some nonperturbative approaches. Among the nonperturbative methods, the QCD sum rules \([31]\), which is based on QCD Lagrangian and is free from the model dependent parameter, is one of the most applicable and predictive approaches to hadron physics. This method has been successfully used to calculate the masses and decay constants of mesons both in vacuum and at finite temperature (see for instance \([32, 33, 34, 35, 36, 37, 38, 39, 40, 41]\)). Now, we extend the application of this method to calculate the masses and decay constants of the considered mesons containing fourth family quarkonia. For details see the original work \([42]\).

2. QCD sum rules for the mass and decay constant

We start this section considering the sufficient correlation functions responsible for calculation of the masses and decay constants of the bound states containing heavy fourth generation quarks in the framework of the QCD sum rules. The two point correlation function corresponding to the scalar (S) and pseudoscalar (PS) cases can be written as:

\[
\Pi^{S(PS)} = i \int d^4 x e^{ip \cdot x} \left< 0 \mid T \left( J^{S(PS)}(x) J^{S(PS)}(0) \right) \right> 0,
\]

where \(T\) is the time ordering product and \(J^{S}(x) = \pi_4(x)q(x)\) and \(J^{PS}(x) = \pi_4(x)\gamma_5q(x)\) are the interpolating currents of the heavy scalar and pseudoscalar bound states, respectively. Here, we consider the \(q\) to be either fourth family \(u_4\) quark or ordinary heavy \(b\) or \(c\) quark. Similarly for the vector (V) and axial vector (AV), the correlation function can be written as:

\[
\Pi^{V(AV)}_{\mu} = i \int d^4 x e^{ip \cdot x} \left< 0 \mid T \left( J^{V(AV)}_{\mu}(x) J^{V(AV)}_{\mu}(0) \right) \right> 0,
\]
where, the currents $J^V_\mu = \overline{q}(x)\gamma_\mu q(x)$ and $J^{AV}_\mu = \overline{q}(x)\gamma_\mu\gamma_5 q(x)$ are responsible for creating the vector and axial vector quarkonia, respectively from the vacuum with the same quantum numbers as the interpolating currents.

From the general philosophy of the QCD sum rules, we calculate the aforesaid correlation functions in two alternative ways. From the physical or phenomenological side, we calculate them in terms of hadronic parameters such as masses and decay constants. In QCD or theoretical side, they are calculated in terms of QCD degrees of freedom such as quark masses and gluon condensates by the help of operator product expansion (OPE) in deep Euclidean region. Equating these two representations of the correlation functions through dispersion relations, we acquire the QCD sum rules for the masses and decay constants. These sum rules relate the hadronic parameters to the fundamental QCD parameters. To suppress the contribution of the higher states and continuum, the Borel transformation with respect to the momentum squared is applied to both sides of the correlation functions.

First, to calculate the phenomenological part, we insert a complete set of intermediate states having the same quantum numbers as the interpolating currents. Performing the integral over $x$ and isolating the ground state, we obtain

$$
\Pi^{S(PS)} = \frac{\langle 0 \mid J^{S(PS)}(0) \mid S(PS) \rangle \langle S(PS) \mid J^{S(PS)}(0) \mid 0 \rangle}{m^2_{S(PS)} - p^2} + \cdots,
$$

where $\cdots$ represents the contributions of the higher states and continuum and $m_{S(PS)}$ is mass of the heavy scalar (pseudoscalar) meson. Similarly, for the vector (axial vector) case, we obtain

$$
\Pi^{V(\mu \nu)} = \frac{\langle 0 \mid J^{V(\mu \nu)}(0) \mid V(\mu \nu) \rangle \langle V(\mu \nu) \mid J^{V(\mu \nu)}(0) \mid 0 \rangle}{m^2_{V(\mu \nu)} - p^2} + \cdots.
$$

To proceed, we need to know the matrix elements of the interpolating currents between the vacuum and mesonic states. These matrix elements are parametrized in terms of the leptonic decay constants as:

$$
\langle 0 \mid J(0) \mid S \rangle = f_S m_S,
\langle 0 \mid J(0) \mid PS \rangle = f_{PS} \frac{m^2_{PS}}{m_{PS}^2 + m_q^2},
\langle 0 \mid J(0) \mid V(\mu \nu) \rangle = f_{V(\mu \nu)} m_{V(\mu \nu)} \epsilon_{\mu \nu},
$$

where $f_i$ are the leptonic decay constants of the considered bound state mesons. Using the summation over the polarization vectors in the $V(\mu \nu)$ case via

$$
\epsilon_{\mu \nu} = -g_{\mu \nu} + \frac{p_\mu p_\nu}{m^2_{V(\mu \nu)}},
$$

we get, the final expressions of the physical sides of the correlation functions as:

$$
\Pi^S = \frac{f_S^2 m^2_S}{m^2_S - p^2} + \cdots,
\Pi^{PS} = \frac{f^2_{PS} \left( \frac{m^2_{PS}}{m_{PS}^2 + m_q^2} \right)}{m^2_{PS} - p^2} + \cdots,
\Pi^{V(\mu \nu)} = \frac{f^2_{V(\mu \nu)} m^2_{V(\mu \nu)}}{m^2_{V(\mu \nu)} - p^2} \left[ -g_{\mu \nu} + \frac{p_\mu p_\nu}{m^2_{V(\mu \nu)}} \right] + \cdots.
$$
where to calculate the mass and decay constant in the $V(AV)$ case, we choose the structure $g_{\mu\nu}$.

In QCD side, the correlation functions are calculated in deep Euclidean region, $p^2 \ll -\Lambda_{QCD}^2$ via OPE where the short or perturbative and long distance or non-perturbative effects are separated. For each correlation function in $S(PS)$ case and coefficient of the selected structure in $V(AV)$ case, we write
\[
\Pi^{QCD} = \Pi^{pert} + \Pi^{nonpert}. \tag{9}
\]

The short distance contribution (bare loop diagram in figure (1) part (a)) in each case is calculated using the perturbation theory, whereas the long distance contributions (diagrams shown in figure (1) part (b)) are parameterized in terms of gluon condensates. To proceed, we write the perturbative part in terms of a dispersion integral,
\[
\Pi^{QCD} = \int \frac{d\rho(s)}{s - p^2} + \Pi^{nonpert}, \tag{10}
\]

where, $\rho(s)$ is called the spectral density. To calculate the spectral density, we calculate the Feynman amplitude of the bare loop diagram by the help of the Cutkosky rules, where the quark propagators are replaced by Dirac delta function, i.e., \[\frac{1}{p^2 - m^2} \rightarrow (-2\pi i)\delta(p^2 - m^2).\] As a result, the spectral density is obtained as follows:
\[
\rho(s) = \frac{3s}{8\pi^2} (1 - \frac{(m_1 \pm m_2)^2}{s}) \sqrt{1 - \frac{2m_1^2 + m_2^2}{s} + \frac{(m_1^2 - m_2^2)^2}{s^2}}. \tag{11}
\]

where + sign in $(m_1 \pm m_2)$ is chosen for scalar and axial vector cases and − sign is for pseudoscalar and vector channels. Here, $m_1 = m_{u_4}$ and $m_2$ is either $m_{u_4}$ or $m_{c(b)}$.

To obtain the non-perturbative part, we calculate the gluon condensate diagrams represented in part (b) of figure (1). For this aim, we use Fock-Schwinger gauge, $x^\mu A_\mu^a(x) = 0$. In momentum space, the vacuum gluon field is expressed as:
\[
A_\mu^a(k') = -\frac{i}{2}(2\pi)^4 G^a_{\mu\rho}(0) \frac{\partial}{\partial k'_{\rho}} \delta^{(4)}(k'), \tag{12}
\]

where $k'$ is the gluon momentum and in calculations, we use the quark-gluon-quark vertex as:
\[
\Gamma^a_{ij\mu} = ig\gamma_\mu \left(\frac{\lambda^a}{2}\right)_{ij}, \tag{13}
\]

After straightforward but lengthy calculations, the non-perturbative part for each case in momentum space is obtained as:
\[ \Pi^i_{\text{nonpert}} = \int_0^1 (\alpha_s G^2) \Theta^i \Theta^i (m_1 \leftrightarrow m_2) \frac{96 \pi (m_2^2 + m_1^2 x - m_2^2 x - p^2 x + p^2 x^2)^2}{dx} \]  

(14)

where \( \Theta^i (m_1 \leftrightarrow m_2) \) means that in \( \Theta^i \), we exchange \( m_1 \) and \( m_2 \). The explicit expressions for \( \Theta^i \) are given in [42].

The next step is to match the phenomenological and QCD sides of the correlation functions to get sum rules for the masses and decay constants of the bound states. To suppress contribution of the higher states and continuum, Borel transformation over \( p^2 \) as well as continuum subtraction are performed. As a result of this procedure, we obtain the following sum rules:

\[ m_{S(V)(AV)}^2 f_{S(V)(AV)}^2 \frac{-m_{S(V)(AV)}^2}{M^2} = \int_{m_{u_4} + m_{q_4}}^{m_0} ds \rho_{S(V)(AV)}(s) e^{-\frac{M^2 s}{s_0}} + B\Pi_{\text{nonpert}}^{S(V)(AV)}, \]

\[ \frac{m_{PS}^2 f_{PS}^2}{(m_{u_4} + m_{q_4})^2} \frac{m_{PS}^2}{M^2} = \int_{m_{u_4} + m_{q_4}}^{m_0} ds \rho_{PS}(s) e^{-\frac{M^2 s}{s_0}} + B\Pi_{\text{nonpert}}^{PS}, \]

(15)

where \( M^2 \) is the Borel mass parameter and \( s_0 \) is the continuum threshold. The sum rules for the masses are obtained applying derivative with respect to \( -\frac{1}{M^2} \) to both sides of the above sum rules and dividing by themselves, i.e.,

\[ m_{S(V)(AV)}^2 = \frac{-d}{d(\frac{1}{M^2})} \left[ \int_{m_{u_4} + m_{q_4}}^{m_0} ds \rho_{S(V)(AV)}(s) e^{-\frac{M^2 s}{s_0}} + B\Pi_{\text{nonpert}}^{S(V)(AV)} \right], \]

(16)

where

\[ B\Pi_{\text{nonpert}}^i = \int_0^1 e^{-\frac{m_2^2 + x (m_1^2 - m_2^2)}{M^2 x (x - 1)}} \frac{\Delta^i (m_1 \leftrightarrow m_2)}{\pi 96 M^6 (x - 1)^4 x^4} (\alpha_s G^2) dx, \]

(17)

and explicit expressions for \( \Delta^i \) are given in [42].

### 3. Numerical Results

To obtain numerical values for the decay constants and masses of the considered bound states containing heavy fourth family from the obtained QCD sum rules, we take the mass of the \( u_4 \) in the interval \( m_{u_4} = (450 - 550) \text{ GeV} \), \( m_b = 4.8 \text{ GeV} \), \( m_c = 1.3 \text{ GeV} \) and \( \langle 0 | \frac{1}{2} \alpha_s G^2 | 0 \rangle = 0.012 \text{ GeV}^4 \). The sum rules for the masses and decay constants also contain two auxiliary parameters, namely Borel mass parameter \( M^2 \) and continuum threshold \( s_0 \). The standard criteria in QCD sum rules is that the physical quantities should be independent of the auxiliary parameters. Therefore, we should look for working regions of these parameters such that our results be approximately insensitive to their variations. The working regions for the Borel mass parameter and the continuum threshold are found in [42].

As an example, let us consider the case of the bound state \( \bar{u}_4 u_4 \). The dependence of the masses of scalar \( \bar{u}_4 u_4 \), pseudoscalar \( \bar{u}_4 \gamma_5 u_4 \), vector \( \bar{u}_4 \gamma_\mu u_4 \) and axial vector \( \bar{u}_4 \gamma_5 \gamma_\mu u_4 \) are presented in figures (2-5) at three different fixed values from the considered working region for the continuum threshold. From these figures, we see a good stability of the masses with respect to the Borel mass parameter \( M^2 \). From these figures, it is also clear that the results do not depend on the continuum threshold in its working region. The dependence of the decay constants of the scalar \( \bar{u}_4 u_4 \), pseudoscalar \( \bar{u}_4 \gamma_5 u_4 \), vector \( \bar{u}_4 \gamma_\mu u_4 \) and axial vector \( \bar{u}_4 \gamma_5 \gamma_\mu u_4 \) are presented in figures (6-9) also at three different fixed values of the continuum threshold. These figures also
Figure 2. Dependence of mass of the scalar $\bar{u}_4u_4$ on the Borel parameter, $M^2$ at three fixed values of the continuum threshold. The upper, middle and lower lines belong to the values $s_0 = (m_1 + m_2 + 3.7)^2 \text{GeV}^2$, $s_0 = (m_1 + m_2 + 3.5)^2 \text{GeV}^2$ and $s_0 = (m_1 + m_2 + 3.3)^2 \text{GeV}^2$, respectively.

Figure 3. The same as Fig. 2 but for pseudoscalar $\bar{u}_4\gamma_5u_4$.

Figure 4. The same as Fig. 2 but for vector $\bar{u}_4\gamma_{\mu}u_4$.

depict approximately insensitivity of the results under variation of the Borel mass parameter in its working region. The results of decay constants also show very weak dependency on the continuum threshold in its working region. From the similar way we analyze the mass and decay constants of the cases when one of the quarks belong to the heavy fourth generation and the other is ordinary bottom or charm quark. The numerical results deduced from the figures are collected in Tables I-VI for three different values of the $m_{u_4}$, namely $m_{u_4} = 450 \text{ GeV}$, $m_{u_4} = 500 \text{ GeV}$ and $m_{u_4} = 550 \text{ GeV}$. The errors presented in these tables are only due to the uncertainties coming from the determination of the working regions for the auxiliary parameters. Here, we should
stress that the obtained results in Tables I-VI are within QCD and do not include contributions coming from the Higgs couplings to the ultra heavy quarks. Such contributions to the binding energy have been calculated in [19], where it is shown that these contributions are more than several GeV in the case when both quarks belong to the fourth family. The Higgs contribution calculated in [19] is proportional to the product of two quark masses. When we replace one of the ultra heavy quarks by \( b \) or \( c \) quark, the binding energy obtained in [19] reduces to a value which is less than the QCD sum rules predictions in the present work. However, when both quarks belong to the fourth family, the binding energy obtained in the present work is very small comparing to the Higgs corrections in [19].
Figure 8. The same as Fig. 6 but for the decay constant of vector $\bar{u}_4\gamma_\mu u_4$.

Figure 9. The same as Fig. 6 but for the decay constant of axial vector $\bar{u}_4\gamma_5\gamma_\mu u_4$.

Table 1. The values of masses of different bound states obtained using $m_{u_4} = 450$ GeV.

| mass (GeV) | $u_4\bar{c}$       | $u_4\bar{b}$       | $u_4\bar{u}_4$       |
|-----------|---------------------|---------------------|-----------------------|
| Scalar    | 453.01 ± 0.25       | 456.45 ± 0.25       | 901.68 ± 0.50         |
| Pseudoscalar | 452.62 ± 0.15     | 455.95 ± 0.15       | 901.12 ± 0.30         |
| axial vector | 453.00 ± 0.25       | 456.44 ± 0.25       | 901.70 ± 0.50         |
| vector    | 452.62 ± 0.15       | 455.94 ± 0.15       | 901.13 ± 0.30         |

At the end of this part, we would like to mention that the obtained QCD sum rules in the present work reproduce the masses and decay constants of the ordinary $\bar{b}b(\bar{c}c)$ states when we set $u_4 \rightarrow b(c)$. The obtained numerical values in this limit are in a good consistency with the existing experimental data [43] and QCD sum rules predictions [40, 41].

To sum up, unlike the top quark, the heavy fourth generation of quarks that have sufficiently small mixing with the three known family SM quarks form hadrons. Considering the arguments mentioned in the text, the production of such bound states will be possible at LHC. Hoping this possibility, we calculated the masses and decay constants of the bound state objects containing two quarks with either both quarks from the SM$_4$ or one from heavy fourth generation and the other from observed SM bottom or charm quarks in the framework of the QCD sum rules. The obtained numerical results approach to the known masses and decay constants of the $\bar{b}b$ and $\bar{c}c$ heavy quarkonia, when the fourth family quark is replaced by the bottom or charm quark.
Table 2. The values of masses of different bound states obtained using $m_{a_4} = 500 \, \text{GeV}$.

| mass (GeV) | $u_4 \bar{c}$ | $u_4 b$ | $u_4 \bar{u}_4$ |
|-----------|----------------|---------|----------------|
| Scalar    | 502.91 ± 0.28  | 506.36 ± 0.28 | 1001.61 ± 0.55 |
| Pseudoscalar | 502.52 ± 0.17  | 505.86 ± 0.17 | 1001.04 ± 0.33 |
| Axial Vector | 502.91 ± 0.28  | 506.35 ± 0.28 | 1001.60 ± 0.55 |
| Vector    | 502.57 ± 0.17  | 505.85 ± 0.17 | 1001.04 ± 0.33 |

Table 3. The values of masses of different bound states obtained using $m_{a_4} = 550 \, \text{GeV}$.

| mass (GeV) | $u_4 \bar{c}$ | $u_4 b$ | $u_4 \bar{u}_4$ |
|-----------|----------------|---------|----------------|
| Scalar    | 552.82 ± 0.31  | 556.27 ± 0.31 | 1101.67 ± 0.60 |
| Pseudoscalar | 552.43 ± 0.18  | 555.78 ± 0.18 | 1101.11 ± 0.36 |
| Axial Vector | 552.81 ± 0.31  | 556.25 ± 0.31 | 1101.68 ± 0.60 |
| Vector    | 552.42 ± 0.18  | 555.77 ± 0.18 | 1101.12 ± 0.36 |

Table 4. The values of decay constants of different bound states obtained using $m_{a_4} = 450 \, \text{GeV}$.

| Leptonic decay constant $f$ (GeV) | $u_4 \bar{c}$ | $u_4 b$ | $u_4 \bar{u}_4$ |
|----------------------------------|----------------|---------|----------------|
| Scalar                           | 0.12 ± 0.01    | 0.15 ± 0.02 | 0.28 ± 0.03    |
| Pseudoscalar                      | 0.17 ± 0.01    | 0.34 ± 0.02 | 4.01 ± 0.20    |
| Axial Vector                      | 0.12 ± 0.01    | 0.15 ± 0.02 | 0.28 ± 0.03    |
| Vector                           | 0.17 ± 0.01    | 0.34 ± 0.02 | 4.01 ± 0.20    |

Table 5. The values of decay constants of different bound states obtained using $m_{a_4} = 500 \, \text{GeV}$.

| Leptonic decay constant $f$ (GeV) | $u_4 \bar{c}$ | $u_4 b$ | $u_4 \bar{u}_4$ |
|----------------------------------|----------------|---------|----------------|
| Scalar                           | 0.11 ± 0.01    | 0.13 ± 0.01 | 0.26 ± 0.03    |
| Pseudoscalar                      | 0.15 ± 0.01    | 0.30 ± 0.02 | 3.91 ± 0.19    |
| Axial Vector                      | 0.11 ± 0.01    | 0.13 ± 0.01 | 0.26 ± 0.03    |
| Vector                           | 0.15 ± 0.01    | 0.29 ± 0.02 | 3.91 ± 0.19    |

Table 6. The values of decay constants of different bound states obtained using $m_{a_4} = 550 \, \text{GeV}$.

| Leptonic decay constant $f$ (GeV) | $u_4 \bar{c}$ | $u_4 b$ | $u_4 \bar{u}_4$ |
|----------------------------------|----------------|---------|----------------|
| Scalar                           | 0.10 ± 0.01    | 0.12 ± 0.01 | 0.26 ± 0.03    |
| Pseudoscalar                      | 0.14 ± 0.01    | 0.27 ± 0.01 | 4.19 ± 0.20    |
| Axial Vector                      | 0.10 ± 0.01    | 0.12 ± 0.01 | 0.26 ± 0.03    |
| Vector                           | 0.14 ± 0.01    | 0.27 ± 0.01 | 4.18 ± 0.20    |
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