\( X(1835), X(2120), X(2370) \) and \( \eta(1760) \) in chiral quark model

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**Abstract:** By analyzing the meson spectra obtained in the constituent quark model, we find that the pseudoscalar mesons \( \eta(2S_0) \), \( \eta'(2S_0) \), \( \eta(3S_0) \) and \( \eta'(3S_0) \) are the possible candidates of \( \eta(1760) \), \( X(1835) \), \( X(2120) \) and \( X(2370) \). The strong decay widths of these pseudoscalars to all the possible decay modes are calculated within the framework of the \(^3P_0\) model. Although the total width of \( \eta' \) is compatible with the experimental value of BES for \( \eta(1760) \), the partial decay width to \( \omega \omega \) is too small, which is not consistent with the result of BES. If the state \( X(1835) \) is interpreted as \( \eta(4S_0) \), the total decay width is compatible with the experimental data, and the main decay modes will be \( \pi a_0(980) \) and \( \pi a_0(1450) \), which needs to be checked. The assignment of the state \( X(2120) \) to \( \eta'(3S_0) \) and \( \eta'(4S_0) \) is also disfavored in the present calculation because of the incompatibility of the decay widths.

**Key words:** \( X(1835), X(2120), X(2370), \eta(1760) \), chiral quark model, pseudoscalar meson

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1 Introduction

In 2005, the BES Collaboration observed a narrow peak in the \( \eta' \) invariant mass spectrum in the process \( J/\psi \rightarrow \eta' + \pi^+ + \pi^- \) with a statistical significance of 7.7\( \sigma \). Fitting with Breit-Wigner function yields mass and width [1]

\[
M = 1833.7 \pm 6.1(stat) \pm 2.7(syst) \text{ MeV}/c^2 \\
\Gamma = 67.7 \pm 20.3(stat) \pm 7.7(syst) \text{ MeV}/c^2,
\]

and the product branching fraction

\[
B(J/\psi \rightarrow \gamma \eta X(1835))B(X(1835) \rightarrow \pi^+ + \pi^- \eta') = (2.2 \pm 0.4(stat) \pm 0.4(syst)) \times 10^{-4}.
\]

BES-III confirmed it in the same process with statistical significance larger than 20\( \sigma \). The fitted mass and width are \( M = 1836.5 \pm 3.0(stat) \pm 9(syst) \text{ MeV}/c^2 \), \( \Gamma = 190 \pm 10(stat) \pm 36(syst) \text{ MeV}/c^2 \). Meanwhile, another two new resonances, \( X(2120) \) and \( X(2370) \), are also observed in the same process with the statistical significance larger than 7.2\( \sigma \) and 6.4\( \sigma \), respectively. The fitted masses and widths are [2]

\[
M = 2122.4 \pm 6.7(stat) \pm 7(syst) \text{ MeV}/c^2, \\
M = 2376.3 \pm 8.7(stat) \pm 1.3(syst) \text{ MeV}/c^2,
\]

respectively. \( \eta(1760) \), which its nature is in controversial, was first reported by Mark III collaboration in the \( J/\psi \) radiative decays to \( \omega \omega \) [3] and \( \rho \rho \) [4]. And DM2 collaboration observed a large bump peaked at 1.77 GeV/c\(^2\) in \( \omega \omega \) invariant mass distribution in the process of \( J/\psi \rightarrow \gamma \omega \omega \) (\( \omega \rightarrow \pi^+ \pi^- \pi^0 \)) [5] and the study of the decays \( J/\psi \rightarrow \gamma \pi \pi \pi \rightarrow \pi^+ \pi^- \pi^0 \) and \( J/\psi \rightarrow \gamma \pi^+ \pi^- \pi^0 \) showed that both decays have a large \( \rho \rho \) dynamics [6]. The fitted mass and width are \( M = 1760 \pm 11 \text{ MeV} \), \( \Gamma = 60 \pm 16 \text{ MeV} \). Recently BES collaboration reported its results on the decays \( J/\psi \rightarrow \gamma \omega \omega \), \( \omega \rightarrow \pi^+ \pi^- \pi^0 \). The mass and width turn to be \( M = 1744 \pm 10(stat) \pm 15 \text{ MeV} \), \( \Gamma = 244^{+24}_{-21} \pm 25 \text{ MeV} \).

Many works have been devoted to the underlying structures of \( X(1835) \) and \( \eta(1760) \) [3]. For \( X(1835) \), Some interpret it as a \( p\bar{p} \) bound state [8,12]. By calculating the mesonic decays of a baryonium resonance, Ding et al. claimed that the \( p\bar{p} \) bound state favors the decay channel \( X \rightarrow \eta \pi \pi \) over \( X \rightarrow \eta 3\pi \) [3]. In fact, it is just this work that stimulates the obser-
The pseudoscalar meson spectrum is determined by the chiral quark model, the mixing angle between $X_n$ and $X_s$ is fixed through the system dynamics. Based on the mass spectrum, the possible candidates of $X(1835)$, $X(2120)$, $X(2370)$ and $\eta(1760)$ are assigned. Then the strong decay widths of the states are calculated in the framework of $^3P_0$ model, to see the assignment is reasonable or not. The paper is organized as follows: a brief review of $^3P_0$ model is given in section 2. The chiral quark model is introduced and meson spectrum and wave function scale parameter $\beta$ of the involved mesons are obtained in section 3. The numerical result of the strong decay are shown in section 4. The last section is a summary.

2 Review of $^3P_0$ model of meson decay

The $^3P_0$ model also known as the Quark-Pair Creation (QPC) model, applied to the decay of meson $A$ to meson $B + C$ was first proposed by Micu [23], and then developed by Le Yaouanc, Ackleh, Robert et al. [24-26]. The $^3P_0$ model assumes that there is a pair of quark and antiquark created in vacuum. The quantum number of the pair of quark and antiquark is $J^{pc}=0^{++}$. Since vacuum is colorless and flavorless, so color and flavor singlet should be satisfied. The created pair recombines with the quark-antiquark pair in initial meson and form two mesons in the final state in two possible ways, which is shown in Fig 1.

![Fig. 1. The two possible diagrams contributing to $A \rightarrow B + C$ in the $^3P_0$ model.](image)

In the non-relativistic limit, the transition operator $T$ takes form as

$$T = -3\gamma \sum_m \langle 1m1-m|00\rangle \int dp_3 dp_4 d^3(p_3+p_4) \times Y_1^m(\frac{P_3-P_4}{2})\chi_{1-m}^{34} \phi_0^{34} \omega_0^{34} b_3^d(p_3)d_4^f(p_4),$$ (1)

where $\gamma$, which is a dimensionless parameter, represents the strength of the quark-antiquark pair creation from the vacuum and can be obtained by fitting the experimental data. $p_3$ and $p_4$ denote the momenta of the created quark and antiquark respectively. $Y_1^m(p)$ is the $l$-th solid harmonic polynomial that gives the momentum-space distribution of the created quark-antiquark pair. $\chi_{1-m}^{34}$ reflects triplet state of spin. $\phi_0^{34} = (u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$ and $\omega_0^{34} = (r\bar{r}+g\bar{g}+b\bar{b})/\sqrt{3}$ correspond to flavor and color singlets, respectively. $b_3^d(p_3)d_4^f(p_4)$ are the creation operators of the quark and antiquark, respectively.
To depict the meson state, we define
\[ |A(n_A 2^{s_A+1} L_A J_A M_JA)(P_A)\rangle = \sqrt{2E_A} \sum_{M_{L_A} M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A}|J_A M_JA \rangle \]
\[ \int dp_A \chi_{S_A M_{S_A}}^{12}(\phi_A^{12} \omega_A^{12}) |q_1 (m_1/m_1 + m_2 P_A + p_A) \]
\[ \times q_2 (m_2/m_1 + m_2 P_A + p_A) \rangle, \]
(2)
The wave function is normalized to
\[ \langle A(n_A 2^{s_A+1} L_A J_A M_JA)(P_A)\rangle = 2E_A \delta(\mathbf{P}_A - \mathbf{P}_A'), \]
where \(\chi_{S_A M_{S_A}}^{12}(\phi_A^{12} \omega_A^{12})\) represent the spin, flavor and color wave function respectively; \(\mathbf{P}_A\) is the CM momentum of meson A, and \(\mathbf{p}_A = (m_2 \mathbf{p}_1 - m_1 \mathbf{p}_2)/(m_1 + m_2)\) is the relative momentum of \(q\bar{q}\) pair. \(n_A\) is the radial quantum number; \(\langle L_A M_{L_A}|S_A M_{S_A}|J_A M_JA \rangle\) are the quantum number of orbit angular momentum between \(q\bar{q}\) pair in meson A, the total spin of the pair and the total angular momentum, respectively; \(\langle L_A M_{L_A} S_A M_{S_A}|J_A M_JA \rangle\) denotes a Clebsch-Gordan coefficient, \(E_A\) is the total energy of the meson.

To describe a strong decay process of \(A \rightarrow B + C\), the S-matrix is written out as
\[ \langle BC|S|A\rangle = i - 2\pi i \delta(E_A - E_B - E_C) \langle BC|T|A\rangle, \]
and then
\[ \langle BC|T|A\rangle = \delta^3(\mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_C) \mathcal{M}^{M_{JA} M_{JB} M_{JC}}, \]
where \(\mathcal{M}^{M_{JA} M_{JB} M_{JC}}\) is the helicity amplitude of \(A \rightarrow B+C\). Taking the center of the mass frame of the meson A: \(\mathbf{P}_A = 0\). One can obtained \(\mathcal{M}^{M_{JA} M_{JB} M_{JC}}\) for decay process in terms of overlap integrals,

\[ \mathcal{M}^{M_{JA} M_{JB} M_{JC}} = \sum_{\{M\}} (L_A M_{L_A} S_A M_{S_A}|J_A M_JA \rangle \langle L_B M_{L_B} S_B M_{S_B}|J_B M_JB \rangle \langle L_C M_{L_C} S_C M_{S_C}|J_C M_JC \rangle \]
\[ \times (1m_1 - m_0)|L_{SB} M_{SB} \chi_{S_B M_{S_B}}^{32} L_{SC} M_{SC} \chi_{S_C M_{S_C}}^{34} |(\omega_B^{32} \omega_C^{34} |(\phi_B^{32} |(\phi_C^{34} \rangle \]
\[ \times I_{MLB,MLC}^{M_{LA}}(P, m_1, m_2, m_3) + (-1)^{1 + S_A + S_B + S_C} \langle L_{SB} M_{SB} \chi_{S_B M_{S_B}}^{32} L_{SC} M_{SC} \chi_{S_C M_{S_C}}^{34} |(\omega_B^{32} \omega_C^{34} |(\phi_B^{32} |(\phi_C^{34} \]
\[ \times I_{MLB,MLC}^{M_{LA}}(-P, m_2, m_1, m_3) \]
(6)
where \(\{M\} = M_{L_A}, M_{S_A}, M_{L_B}, M_{S_B}, M_{L_C}, M_{S_C}, m\), The momentum space integral \(I_{MLB,MLC}^{M_{LA}}(P, m_1, m_2, m_3)\) and \(I_{MLB,MLC}^{M_{LA}}(-P, m_2, m_1, m_3)\) are given by
\[ I_{MLB,MLC}^{M_{LA}}(P, m_1, m_2, m_3) = \sqrt{8E_A E_B E_C} \int dp \psi^*_n B L_{LB} M_{LB} \left( \frac{m_3}{m_1 + m_3} P + p \right) \psi^*_n C L_{LC} M_{LC} \left( \frac{m_3}{m_2 + m_3} P + p \right) \]
\[ \times \psi^*_n A L_{LA} M_{LA} (P + p) \mathcal{Y}^{mn}_{1}(p) \]
(7)
\[ I_{MLB,MLC}^{M_{LA}}(-P, m_2, m_1, m_3) = \sqrt{8E_A E_B E_C} \int dp \psi^*_n B L_{LB} M_{LB} \left( \frac{m_3}{m_1 + m_3} P + p \right) \psi^*_n C L_{LC} M_{LC} \left( \frac{m_3}{m_2 + m_3} P + p \right) \]
\[ \times \psi^*_n A L_{LA} M_{LA} (-P + p) \mathcal{Y}^{mn}_{1}(p) \]
(8)
where \(P_B = -P_C = P, p = p_a\), and \(m_3\) is the mass of the created quark. The spacial wavefunction one take is the simple harmonic oscillator (SHO) wavefunction. In momentum-space, the SHO wavefunction reads
\[ \Psi_{nLM_L}(p) = (-1)^n (-i)^L R^{L+\frac{1}{2}} \sqrt{\frac{2n!}{\Gamma(n + L + \frac{1}{2})}} \]
\[ \times \exp \left( -\frac{R^2 p^2}{2} \right) L^{L+\frac{1}{2}}_n \mathcal{Y}_{LM_L}(p), \]
(9)
here \(\mathcal{Y}_{LM_L}(p)\) is the solid harmonic polynomial; \(R\) is the parameter of SHO wavefunction; \(p\) is the relative momentum between \(q\bar{q}\) pair within one meson; \(\mathcal{M}^{JL}(A \rightarrow BC)\) is the Laguerre polynomial. The decay width can be written as follows
\[ \Gamma = \pi^2 \frac{|\mathcal{P}|}{M_A^2 (1 + \delta_{BC})} \sum_{JL} |\mathcal{M}^{JL}|^2, \]
(10)
where \(\mathcal{M}^{JL}\) is the partial wave amplitude, which is related to the helicity amplitude \(\mathcal{M}^{M_{JA} M_{JB} M_{JC}}\) via the Jacob-Wick formula [27]
\[ \mathcal{M}^{JL}(A \rightarrow BC) = \frac{2L + 1}{2J + 1} \sum_{M_{JA},M_{JB}} \langle L0J M_{JA}|J_M J_A \rangle \]
\[ \times \langle J_B M_{JB} J_C M_{JC}|J_M M_{JA}\rangle \mathcal{M}^{M_{JA} M_{JB} M_{JC}}(P), \]
(11)
QCD. The constituent quark mass originates from the spontaneous breaking of chiral symmetry and nonperturbative (color confinement and spontaneous breaking of chiral symmetry) properties of QCD. The model, constituent quark model, is used. The model Hamiltonian is

\[ H = m_1 + m_2 + \frac{p^2}{2\mu} + V^C + V^G + V^0 + V^s, \]  

(12)

where \( J = J_B + J_C, \) \( J_A = J_B + J_C + L. \) Then the decay width in terms of the partial wave amplitude is taken as, where \( |P| = |P_B| = |P_C|. \) According to the calculation of 2-body phase space, one can get

\[ |P| = \sqrt{\frac{M_A^2 - (M_B + M_C)^2}{2M_A^2}} \]

where \( M_A, M_B, \) and \( M_C \) are the masses of the meson \( A, B, \) and \( C, \) respectively.

### 3 The masses of the mesons

To calculate the meson spectrum, a QCD-inspired model, constituent quark model, is used. The model incorporates the perturbative (one gluon exchange) and nonperturbative (color confinement and spontaneous breaking of chiral symmetry) properties of QCD. The constituent quark mass originates from the spontaneous breaking of chiral symmetry and consequently constituent quarks should interact through the exchange of Goldstone bosons [28], in addition to the one-gluon-exchange. To describe the hadron-hadron interaction, the chiral partner of pion, \( \sigma \)-meson, is also used. So the model Hamiltonian is

\[ V_{SO}^C = -\lambda_1^c \cdot \lambda_2^c \frac{a_\mu e^{-\mu x}}{4m_1^2 m_2^2 r} \left( [m_1^2 + m_2^2] (1 - 2a_\mu) + 4m_1 m_2 (1 - a_\mu) \right) S \cdot L \]

\[ V^G = V_B^C + V_{SO}^G + V_T^G \]

\[ V_{SO}^G = \frac{\alpha_s}{4} \lambda_1^c \cdot \lambda_2^c \left\{ \frac{1 - \sigma_1 \cdot \sigma_2}{6m_1 m_2 r r_0(\mu)} e^{-r/r_0(\mu)} \right\} \]

\[ V_{OGE}^T = -\frac{1}{16} \frac{\alpha_s}{m_1 m_2} \lambda_1^c \cdot \lambda_2^c \left\{ \frac{1}{r^3} e^{-r/r_0(\mu)} \right\} \left( \frac{1}{r^2} + \frac{1}{3r_0^2(\mu)} \right) \]

\[ V = (v_0^x + v_0^y) \sum_{a=1}^3 \lambda_a \cdot \lambda_a \]

\[ = \left( \sum_{a=1}^3 \lambda_a \cdot \lambda_a \right) + \left( \sum_{a=1}^3 \lambda_a \cdot \lambda_a \right) \]

\[ = \sum_{a=1}^3 \lambda_a \cdot \lambda_a \]

where \( r = |r_1 - r_2|, \) and \( p = (p_1 - p_2)/2, \) \( r_0(\mu) = \hat{r}_0/\mu, \)

\[ r_0(\mu) = \hat{r}_0/\mu. \]  

Other symbols have their usual meanings. The effective running coupling constant is given by

\[ \alpha_s(\mu) = \frac{\alpha_s}{\ln \left( \frac{\mu^2 + p_0^2}{\Lambda^2} \right)}. \]

(13)

where \( \mu \) is the reduced mass of the \( q\bar{q} \) system. The chiral coupling constant \( g_{ch} \) is determined from the \( \pi NN \) coupling constant through

\[ g_{ch}^2 = \frac{3}{5} \left( \frac{3}{4\pi} g_{\pi NN} m^2_{\pi NN} \right)^2 \]

(14)
The meson spectrum is obtained by solving the Schrödinger equation,

\[ H \Psi = E \Psi, \quad (15) \]

\[ \Psi = \left[ \psi_{nLM}(\chi_{SM}); J_{MF} \chi_c \chi_f \right], \quad (16) \]

Table 1. Model parameters. The masses of mesons $\pi, K, \eta$ take the experimental values.

| $m_{ud, d}$ | $m_s$ | $\alpha_c$ | $\mu_c$ | $\Delta$ | $\alpha_s$ |
|------------|-------|------------|--------|---------|----------|
| MeV        | MeV   | MeV fm^{-1} | MeV    |         |
| 313        | 555   | 430        | 0.7    | 181.10  | 0.777    |

\[ a_0 = \Lambda_0 \mu_0 \tau_0 \tau_g \quad \text{fm}^{-1} \text{MeV fm MeV fm} \]

\[ a_1 = \Lambda_{1/2} \Lambda_1 \Lambda_{3/2} \sigma_{LM}^2 / 4\pi \quad \theta^\rho \quad \text{fm}^{-1} \text{MeV fm MeV fm} \]

| 4.20 | 4.20 | 5.20 | 5.20 | 0.54 | -15 |

where $\chi_{SM}; \chi_c, \chi_f$ are spin, color and flavor wavefunctions of the meson, respectively and can be constructed through the symmetry. The spatial wavefunction $\psi_{nLM} = R_{nL}(r)Y_{LM}(\Omega)$ is obtained by solving the second-order differential equation. The efficient numerical method: Numerov method is used here. The model parameters, which are listed in Table 1 are fixed by fitting the experimental data of meson spectrum. Parts of the obtained meson spectrum are shown in Tables 2 and 3. The detailed results can be found in Ref. [20]. To calculate the strong decay of mesons analytically in $3P_0$ model, the obtained radial part of the spacial wavefunction $R_{nL}(r)$ is fitted by the simple harmonic oscillator (SHO),

\[ R_{nL}(r) = \frac{\beta^{L+\frac{1}{2}} \left( \frac{2n!}{\Gamma(n + L + \frac{1}{2})} \right) \exp \left( -\frac{\beta^2 r^2}{2} \right) \cdot r^n L_{n-\frac{1}{2}} \left( \beta^2 r^2 \right)}{r^n L_{n-\frac{1}{2}} \left( \beta^2 r^2 \right)}. \quad (17) \]

The fitted values of parameter $\beta$ are also listed in Table 3.

Table 2. The mass of $I = 1, \frac{1}{2}$ mesons and the values of fitted $\beta$.

| $n^{2S+1}L_J$ states | Isospin | Mass (MeV) | $\beta$ (fm^{-1}) | $R$ (GeV^{-1}) |
|----------------------|---------|------------|---------------------|----------------|
| $1^{3}S_0$ $\pi$    | 1       | 139        | 2.308               | 2.196          |
| $2^{3}S_0$ $\pi(1300)$ | 1     | 1288       | 1.434               | 3.534          |
| $1^{3}P_1$ $\rho$   | 1       | 772        | 1.438               | 3.522          |
| $2^{3}S_1$ $\rho(1450)$ | 1   | 1478       | 1.096               | 4.624          |
| $1^{3}P_1$ $B_1(1235)$ | 1.234 | 1.234      | 4.077               |               |
| $1^{3}P_0$ $a_0(980)$ | 1     | 984        | 1.473               | 3.440          |
| $2^{3}P_0$ $a_0(1450)$ | 1     | 1587       | 1.125               | 4.505          |
| $1^{3}P_1$ $a_1(1260)$ | 1     | 1205       | 1.300               | 3.898          |
| $1^{3}P_2$ $a_2(1320)$ | 1     | 1327       | 1.106               | 4.582          |
| $1^{3}P_2$ $a_2(1700)$ | 1     | 1732       | 0.890               | 5.694          |

For $I = 0$ states, there are two types of them, one is composed of $u, d$-quark and $\bar{u}, \bar{d}$-antiquark, another is composed of $s$-quark and $\bar{s}$-antiquark. They are mixed in the flavor SU(3) symmetry to form flavor singlet and octet. However, flavor SU(3) is broken. In experiments, we have $\eta$ and $\eta'$ instead of $\eta_1$ and $\eta_8$ for pseudoscalar. In the present calculation, flavor SU(3) symmetry is not used, so we have flavor wavefunctions $X_n$ and $X_s$. As a consequence of $K$-meson exchange, they are mixed. To obtain the masses of $I = 0$ states, the following procedure is taken. First, solving the Schrödinger equation for $X_n$ and $X_s$ separately ($K$-meson exchange is not employed). Secondly, by using the wavefunctions $\Psi_n$ and $\Psi_s$ obtained in the first step and taking into account of $K$-meson exchange, the eigen-energies and eigen-states can be obtained by diagonalizing the Hamiltonian matrix

\[ \begin{pmatrix} H_{nn} & H_{ns} \\ H_{sn} & H_{ss} \end{pmatrix} \begin{pmatrix} C_n \\ C_s \end{pmatrix} = E \begin{pmatrix} C_n \\ C_s \end{pmatrix}. \quad (18) \]

where $H_{nn} = \langle \Psi_n | H | \Psi_n \rangle$, $H_{ns} = \langle \Psi_n | V_K | \Psi_s \rangle = H_{sn}$ and $H_{ss} = \langle \Psi_s | H | \Psi_s \rangle$. The eigen-state is $| \Psi \rangle = C_n | \Psi_n \rangle + C_s | \Psi_s \rangle$. The obtained eigen-energies and eigen-states are shown in Table 3. From Table 3 one finds that $\eta'(1760), X(1835), X(2120), X(2370)$ may be interpreted as $\eta'(2^1S_0), \eta'(4^1S_0), \eta'(3^1S_0)$ and $\eta'(4^3S_0)$ respectively by comparing the theoretical masses with the experimental data. To check these assignments, the decay properties of the states should be studied, which is discussed in the next section.
4 The strong decay of the candidates

for $\eta(1760), X(1835), X(2120), X(2370)$

$\eta, \eta'$ and their radial excitations have the same quantum numbers $IJ^{PC} = 0^+$. According to the $^3P_0$ model discussed above, the isospins of mesons $B$ and $C$ can takes the values $I = 0, 1/2, 1$ with the condition $I_B + I_C = I_A$. If not forbidden kinetically, the allowed decay modes of $\eta(\eta')$ family are listed in Table 4.

Table 4. Allowed decay modes and the amplitudes of the radial excited states of $\eta$ and $\eta'$. For $X_n$ decay, $\phi_f = \sqrt{\frac{2}{3}}, \sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}, 0$ for $I_B = I_C = 1$, $I_B = I_C = 1/2, 0(X_n), 0(X_s)$ and for $X_s$ decay, $\phi_f = 0, \sqrt{\frac{2}{3}}, 0, \sqrt{\frac{1}{3}}$ for $I_B = I_C = 1$, $I_B = I_C = 1/2, 0(X_n), 0(X_s)$.

| $(nL)^{PC}$ | states | Mass (MeV) | $C_\eta$ | $C_s$ | $\beta$ (fm$^{-1}$) | $R$(GeV$^{-1}$) |
|-------------|--------|------------|----------|-------|----------------|-------------|
| $1^1S_0$    | $\eta$ | 572        | $8.6564 \times 10^{-1}$ | -5.0066 $\times 10^{-1}$ | 1.732693 | 2.924 |
| $1^1S_0$    | $\eta'(958)$ | 956 | $5.0066 \times 10^{-1}$ | 8.6564 $\times 10^{-1}$ | 2.064307 | 2.455 |
| $2^1S_0$    | $\eta(1295)$ | 1290 | $9.6360 \times 10^{-1}$ | 2.67323 $\times 10^{-1}$ | 1.183-$1.666$ | 3.041-4.284 |
| $2^1S_0$    | $\eta'(1670)$ | 1795 | $2.6732 \times 10^{-1}$ | 9.6360 $\times 10^{-1}$ | 1.183-$1.666$ | 3.041-4.284 |
| $3^1S_0$    | $\eta(3S)$ | 1563 | $9.9350 \times 10^{-1}$ | -1.1380 $\times 10^{-1}$ | 0.929-$1.360$ | 3.726-5.455 |
| $3^1S_0$    | $\eta'(3S)$ | 2276 | $1.1380 \times 10^{-1}$ | 9.9350 $\times 10^{-1}$ | 0.929-$1.360$ | 3.726-5.455 |
| $4^1S_0$    | $\eta(4S)$ | 1807 | $9.9935 \times 10^{-1}$ | -3.5928 $\times 10^{-2}$ | 0.6725-$1.0995$ | 4.607-7.530 |
| $4^1S_0$    | $\eta'(4S)$ | 2390 | $3.5928 \times 10^{-2}$ | 9.9935 $\times 10^{-1}$ | 0.6725-$1.0995$ | 4.607-7.530 |

| $(nL)^{PC}$ | states | Mass (MeV) | $C_\eta$ | $C_s$ | $\beta$ (fm$^{-1}$) | $R$(GeV$^{-1}$) |
|-------------|--------|------------|----------|-------|----------------|-------------|
| $1^3P_1$    | $\omega(782)$ | 691 | $9.9499 \times 10^{-1}$ | 9.9967 $\times 10^{-2}$ | 1.547 | 3.276 |
| $1^3P_1$    | $\phi(1020)$ | 1020 | $-9.9967 \times 10^{-2}$ | 9.9499 $\times 10^{-1}$ | 1.918 | 2.642 |
| $2^3P_1$    | $\omega(1420)$ | 1444 | $9.9852 \times 10^{-1}$ | 5.4331 $\times 10^{-2}$ | 1.163 | 4.357 |
| $2^3P_1$    | $\phi(1680)$ | 1726 | $-5.4331 \times 10^{-2}$ | 9.9852 $\times 10^{-1}$ | 1.506 | 3.365 |
| $1^3P_1$    | $h_1(1170)$ | 1257 | 1.0 | 0 | 1.202 | 4.216 |
| $1^3P_1$    | $h'_1(1170)$ | 1511 | 1.0 | 1.0 | 1.581 | 3.205 |
| $3^3P_2$    | $f_2(1270)$ | 1311 | 1.0 | 0 | 1.112 | 4.557 |
| $3^3P_2$    | $f'_2(1525)$ | 1556 | 1.0 | 1.0 | 1.496 | 3.387 |
| Reaction | Mass Range | Expression |
|----------|------------|------------|
| $X \rightarrow J/\psi + P_h$ | $\pi a_0(980), \pi a_0(1450), \pi(1300)a_0(980)$ | $M^{JL} = M^{00} = M^{000}$ |
| $X \rightarrow S_0 + P_h$ | $KK^*_0(1430), K^0\eta(1450)$ | $M^{000} = \sqrt{11 m_{P_h}^2 + \Delta m_{P_h}^2}$ |
| $X \rightarrow S_0 + S_1$ | $KK^*, K^0\eta(1450), K^0(1460)K^*$ | $M^{JL} = M^{11} = -M^{000}$ |
| $X \rightarrow S_0 + D_1$ | $KK^*(1680)$ | $M^{000} = -\sqrt{11 m_{D_1}^2 + \Delta m_{D_1}^2}$ |
| $X \rightarrow S_1 + P_h$ | $\rho a_1(1260)$, $\rho a_1(1460)$, $K^0K^*_1(1273)$, $\omega f_1(1285)$ | $M^{JL} = M^{00} + M^{22}$ |
| $X \rightarrow S_1 + S_1$ | $\rho\rho(1450)$, $\omega\omega(1420)$, $K^*K^*$, $K^*K^*(1410)$, $\phi\phi$ | $M^{001} = -\sqrt{\frac{1}{3} m_{S_1}^2 + \Delta m_{S_1}^2}$ |
| $X \rightarrow S_1 + P_1$ | $\rho b_1(1235)$, $K^0K^*_1(1400)$, $\omega h_1(1170)$ | $M^{011} = -\sqrt{\frac{1}{3} m_{S_1}^2 + \Delta m_{S_1}^2}$ |
| $X \rightarrow S_1 + P_2$ | $\rho a_2(1320)$, $K^*K^*_2(1430)$ | $M^{011} = -\sqrt{\frac{1}{3} m_{S_1}^2 + \Delta m_{S_1}^2}$ |

All the possible decay modes of $\eta(\eta')$ family are shown in Table 4. To calculate the strong decay widths of mesons, the strength of the quark pair creation from the vacuum, $\gamma$, has to be fixed. It is obtained by fitting the experimental values of the strong decay widths of light and charged mesons, charmonium and baryons. In the present work, $\gamma = 6.95$, which is adopted by many researchers, is taken for the non-strange quark pair creation, and the strength of $s\bar{s}$ creation satisfies $\gamma_s = \gamma/3$.

### 4.1 $\eta'(21S_0)$

The experimental evidence for $\eta(1760)$ is controversial. There are large differences between the observations of MARK III, DM2 and BES collaborations. In our calculation, the mass of $\eta'(21S_0)$ is 1795 MeV, which is close to the experimental mass of $\eta(1760)$. So we take it as the candidate of $\eta(1760)$. In Fig 2 we show the dependence of the partial widths of the strong decay of the $\eta'(21S_0)$ on the $R_A$. Taking $R_A = 3.0 - 4.3$ GeV$^{-1}$ discussed above, the total width ranges from 256 to 404 MeV, which is much larger than the results given by the Mark III and DM2 collaboration, but falls in the range of the BES experimental data. In this range, $\eta'(21S_0)$ have a sizable branching ratio into $\pi a_0(980)$, $\pi a_0(1320)$, $\rho\rho$, and $KK^*$. But the partial width to $\omega\omega$ is rather small. If the BES results are reliable, the assignment of $\eta(1760)$ to $\eta'(21S_0)$ is disfavored in the present calculation. In Ref. [19], $\eta(1760)$ is taken as $\eta(3S)$, the total decay width is between 60-100 MeV, which falls in the range of DM2’s results, but is far below BES’s results.

[Fig. 2. The possible strong decay of the $\eta'(21S_0)$]
4.2 \( \eta(4^1S_0) \)

\( X(1835) \) was first observed by BESII in the \( \pi^+\pi^-\eta' \) invariant-mass spectrum in the decay channel \( J/\psi \to \gamma \pi^+\pi^-\eta' \) with a statistical significance of 7.7 \( \sigma \)[1]. BESIII confirmed it in the same process with statistical significance larger than 20\( \sigma \)[2]. In the present calculation, the mass of \( \eta(4^1S_0) \) = 1807 MeV is close to the mass of \( X(1835) \), so the assignment of \( X(1835) \) to \( \eta(4^1S_0) \) is possible, which is different from the assignment of Ref. [12], \( \eta'(3S) \). In Fig. 2 the dependence of the partial widths of the strong decay of the \( \eta(4^1S_0) \) on the \( R_A \) is shown. From the mass calculation, \( R_A \) = 4.6-7.5 GeV\(^{-1} \) is obtained. In this range, the total width ranges from 54 to 692 MeV, which falls in the range of the BES experimental data, and the main decay modes are \( \pi a_0(980) \) and \( \pi a_0(1450) \).

We suggest experimental search for \( X(1835) \) in these modes to make sure whether it is \( \eta(4^1S_0) \) assignment.

4.3 \( \eta'(3^1S_0) \) and \( \eta'(4^1S_0) \)

Besides confirmed the existence of \( X(1835) \) in the \( \pi^+\pi^-\eta' \) invariant-mass spectrum in the process \( J/\psi \to \gamma \pi^+\pi^- \), other two states \( X(2120) \) and \( X(2370) \) are observed by BESIII with statistical significance larger than 7.2\( \sigma \) and 6.4 \( \sigma \), respectively. By comparing the masses of the \( \eta(\eta') \) family, it is possible to take \( \eta'(3^1S_0) \) and \( \eta'(4^1S_0) \) as the candidates of \( X(2120) \) and \( X(2370) \). Because of their large masses, many strong decays modes are allowed. In Figs. 4 and 5 the partial widths of their strong decays are shown. For \( \eta'(3^1S_0) \) with \( R_A = 3.7-5.6 \) GeV\(^{-1} \) and for \( \eta'(4^1S_0) \) with \( R_A = 4.6-7.5 \) GeV\(^{-1} \), the decay widths are much higher than the experimental data of BESIII. Because both \( X_\pi \) and \( X_s \) have contributions to
the state $n\bar{s}s\bar{u}$, the partial width of the strong decay to two isospin $I=\frac{1}{2}$ mesons is generally much larger than to two isospin $I=1$ or 0 mesons. $\eta' (3^1 S_0)$ have large partial to $KK^*$ and $KK^*$. And the main decay modes of $\eta' (4^1 S_0)$ are $KK^*$, $KK_1(1352)$, $KK_0^*(1430)$, $KK^*_0(1950)$.

If we describe $X(2120)$ and $X(2370)$ as $\eta'(3^1 S_0)$ and $\eta'(4^1 S_0)$ respectively with parameters in this work, it is not appropriate obviously.

![Graph of possible strong decay of the $\eta' (4^1 S_0)$](image)

**Fig. 5. The possible strong decay of the $\eta' (4^1 S_0)$**

### 5 summary and discussions

By using chiral quark model, the mass spectrum of $\eta(\eta')$ family are calculated, where the mixing between $(u\bar{u}+d\bar{d})/\sqrt{2}$ and $\bar{s}s$ is determined by system dynamics, $K$-meson exchange. Based on the mass spectrum, the possible candidates of four $J^{PC} I^G = 0^- 0^+$ mesons, $\eta(1600)$, $X(1835)$, $X(2120)$ and $X(2370)$ are assigned to $\eta'(2^1 S_0)$, $\eta'(4^1 S_0)$, $\eta'(3^1 S_0)$, $\eta'(4^1 S_0)$. Furthermore, all kinematically allowed two-body strong decays of them can calculated in the framework of the $J^P_C$ model. The wavefunctions needed in the calculation are obtained from the mass calculation. To simplify the calculation, the SHO wavefunctions are used to mimic the real wavefunctions.

The decay widths turn out to be strongly dependent on the SHO wave function scale parameter $\beta$. For $\eta(1600)$, the width is larger than the result of [6] and is compatible with the results of BES observation [1] in the $R_A$ range. However the partial width to $\omega\omega$ is too small, which is incompatible with experimental date [1] [3] [4] [5]. So the assignment of $\eta(1760)$ to $\eta(2S)$ is disfavored in the present calculation. For the state $X(1835)$, the calculated decay width is consistent with experiment data, and $\pi\eta_0(980)$ and $\pi\eta_0(1450)$ are the main decay modes. To justify this assignment, the experimental investigation of the $\pi\eta_0(980)$ and $\pi\eta_0(1450)$ decay modes of $X(1835)$ is needed. Since $X(1835)$ is around the threshold of $pp$, it may be the mixture of $q\bar{q}$ and baryonium. Further study of the state $X(1835)$ by taking into account of the mixture is essential to understand the nature of the state.

$X(2120)$ and $X(2370)$ are assigned to $\eta'(3^1 S_0)$ and $\eta'(4^1 S_0)$ respectively. Since they have larger masses, many strong decays modes are allowed and have a large phase space to some modes. The total decay widths are much higher than the experimental val-
ues. The large decay width may be due to the overestimated value of $\gamma$. To exclude the impact of parameters, the branching ratio is better to justify the assignment. More experimental data are needed. Since the lattice QCD predicts the $0^{-+}$ glueball is about 2.3–2.6 GeV, which is around the masses of $X(2120)$ and $X(2370)$, the study with the mixture of $q\bar{q}$, glueball and other configurations are necessary to understand the nature of $X(2120)$ and $X(2370)$ states.

References
1. Ablikim M et al. [BES Collaboration], Phys. Rev. Lett., 2005, 95, 262001:1-5
2. Ablikim M et al. [BES Collaboration], Phys. Rev. Lett., 2011, 106, 072002:1-5
3. Baltrusaitis R M et al. [MARKIII Collaboration], Phys. Rev. Lett., 1985, 55, 1723-1726
4. Baltrusaitis R M et al. [MARKIII Collaboration], Phys. Rev. D, 1986, 33, 1222-1232
5. Bisello D et al. [DM2 Collaboration], Phys. Lett. B, 1987, 192, 239-244
6. Bisello D et al. [DM2 Collaboration], Phys. Rev. D, 1989, 39, 701-712
7. Ablikim M et al. [BES Collaboration], Phys. Rev. D, 2006, 73, 112007:1-9
8. Klempt E and Zaitsev A, Phys. Rep., 2007, 454, 1-202 and references therein.
9. Ding G J and Yan M L, Phy Rev. C, 2005, 72, 015208:1-8; Ding G J and Yan M L, Eur. Phys. J. A, 2006, 28, 351-360
10. Dedonder J P, Loiseau B, El-Bennich B and Wycech S, Phys. Rev. C, 2009, 80, 045207:1-6
11. Liu C, Eur. Phys. J. C, 2008, 53, 413-419
12. Wang Z G and Wan S L, J. Phys. G, 2007, 34, 505-512
13. Kocheliev N and Min D P, Phys. Lett. B, 2006, 633, 283-288; Phys. Rev. D, 2005, 72, 097502:1-3
14. He X G, Li X Q, Liu X and Ma J P, Eur. Phys. J. C, 2006, 49, 731-736
15. Li B A, Phys. Rev. D, 2006, 74, 034019:1-8
16. Hao G, Qiao C F and Zhang A L, Phys. Lett. B, 2006 642, 53-61
17. Huang T and Zhu S L, Phys. Rev. D, 2006, 73, 014023:1-4
18. D. M. Li and B. Ma, Phys. Rev. D 77, 074004:1-7 (2008).
19. Yu J S, Sun Z F, Liu X and Zhao Q, Phys. Rev. D, 2011, 83, 114007:1-15
20. Vijaide J, Fernández F and Valcarce A, J. Phys. G, 2005, 31, 481-506
21. Li X Q and Page P R, Eur. Phys. J. C, 1998, 1, 579-583
22. Wu N, Yuan T N and Zheng Z P, Chin. Phys. C, 2001, 10, 611-612
23. Micu L, Nucl. Phys. B, 1969, 10, 521-526
24. Yaouanc A, Le, Oliver L, Pene O and Raynal J C, Phys. Rev. D, 1973, 8, 2223-2234; 1974, 9, 1415-1419; 1975, 11, 1272-1286
25. Roberts W and Silvestr-Brac B, Few-Body Syst., 1992, 11, 171-193
26. Ackleh E S, Barnes T and Swanson E S, Phys. Rev. D, 1996, 54, 6811-6829
27. Jacob M and Wick G C, Ann. Phys. (N.Y.), 1959, 7, 404-428
28. Manohar A and Georgi H, Nucl. Phys. B, 1984, 234, 189-212
29. Koonin S E and Meredith D C. Computational Physics. New York: Addison-Wesley, 1990. 56—57
30. Lu J, Deng W Z, Chen X L and Zhu S L, Phys. Rev. D, 2006, 73, 054012:1-6; Zhang B, Liu X, Deng W Z and Zhu S L, Eur. Phys. J. C, 2007, 50, 617-628; Chen C, Chen X L, Liu X, Deng W Z and Zhu S L, Phys. Rev. D, 2007, 75, 094017:1-13
31. Blundell H G and Godfrey S, Phys. Rev. D, 1996, 53, 3700-3711; Phys. Rev. D, 1996, 53, 3712-3722
32. Yang Y C, Xia Z R and Ping J L, Phys. Rev. D, 2010, 81, 094003:1-9
33. Yaouanc A Le, Oliver L, Pene O and Raynal J C, Phys. Lett. B, 1977, 72, 57-61