Learning with Latent Structures in Natural Language Processing: A Survey

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Abstract

While end-to-end learning with fully differentiable models has enabled tremendous success in natural language process (NLP) and machine learning, there have been significant recent interests in learning with latent discrete structures to incorporate better inductive biases for improved end-task performance and better interpretability. This paradigm, however, is not straightforwardly amenable to the mainstream gradient-based optimization methods. This work surveys three main families of methods to learn such models: surrogate gradients, continuous relaxation, and marginal likelihood maximization via sampling. We conclude with a review of applications of these methods and an inspection of the learned latent structure that they induce.¹

1 Introduction

With recent advances in deep learning, end-to-end differentiable modules such as LSTM (Hochreiter and Schmidhuber, 1997) and Transformer (Vaswani et al., 2017), coupled with gradient-based optimization, have become the mainstream paradigm in NLP and have helped achieve state-of-the-art results in NLP without much task-specific, language-specific, or linguistically-inspired design (Peters et al., 2018; Devlin et al., 2019; Raffel et al., 2020, inter alia).

On the other hand, human languages possess inherent structures, such as syntactic, semantic, and discourse structures (Chomsky, 1965; Heim and Kratzer, 1998). Consequently, NLP systems have historically leveraged such structures to improve their quality. This was traditionally done with pipelined systems, where a separately trained parser is first used to decode an intermediate structure, which is then used as input for downstream tasks (Sennrich and Haddow, 2016; Eriguchi et al., 2016; Chen et al., 2017, inter alia). This paradigm, however, faces a few challenges. First, its disjoint modules tend to suffer from cascading errors. Second, such parsers are usually trained in a supervised fashion which, in order to ensure coverage for robustness, can require large amounts of annotated data. Such annotations often require domain expertise and, despite recent efforts such as the Universal Dependencies project (Zeman et al., 2021), remain difficult and expensive to obtain, especially for low-resource languages and domains. Even when sufficient data exist to train a high-quality formalism-specific parser, this specific structural formalism may not be the optimal one for the downstream task, and it is sometimes advantageous to allow the model to discover the best structure targeting the downstream task (Kim et al., 2017; Choi et al., 2018; Maillard et al., 2019, inter alia).

Therefore, previous studies have attempted to jointly train the parser and the downstream predictor and allow the parser to induce the best task-specific structure without the need for structural supervision, a

¹This work is inspired by Martins et al. (2019).
| Family | Method | Structure Discrete Inference Output Algo. |
|--------|--------|----------------------------------|
| Surrogate | STE \( (\text{Hinton, 2012}) \) | Both ✓ MAP |
| | SPIGOT \( (\text{Peng et al., 2018}) \) | ✓ ✓ MAP |
| Relaxation | softmax | ✓ ✓ |
| | sparsemax \( (\text{Martins and Astudillo, 2016}) \) +† | ✓ ✓ |
| | Part-Marginalization | ✓ ✓ Marg. |
| | SparseMAP \( (\text{Niculae et al., 2018a}) \) | ✓ ✓ MAP |
| Sampling | Score Function Estimator \( (\text{Williams, 1992}) \) | Both ✓ Sampling |
| | Rectified Distributions \( (\text{Louizos et al., 2018}) \) | ✓ ✓‡ |
| | Gumbel-Max \( (\text{Gumbel, 1954}) \) | ✓ ✓ |
| | Gumbel-softmax \( (\text{Jang et al., 2017; Maddison et al., 2017}) \) | ✓ ✓ |
| | Perturb-and-Parse \( (\text{Corro and Titov, 2019a}) \) | ✓ ✓ MAP |
| | Direct Loss Minimization \( (\text{McAllester et al., 2010}) \) | ✓ ✓ |

Table 1: A high-level summary of the family and the methods reviewed in this survey. † sparsemax+ refers to sparsemax and other related functions (see Section 5). We list if the methods are primarily used for inducing structured or unstructured objects, or both. Methods marked with ✓ may still be employed for structured induction when the parser decisions are local, such as transition-based parsers (see Section 7). We also mark if the methods yield discrete output, which can be desirable for, for example, interpretability, and sometimes enables specialized algorithms (e.g., Hu et al., 2021). ‡ Some rectified distributions still have a non-zero probability to output continuous solutions, though they may be discretizable. Finally, we indicate the structured inference algorithms required for all methods that are used for inducing structures: MAP, marginalization, or sampling. Depending on the problem, there may not exist an efficient version of a particular inference algorithm, affecting the choice of methods.

paradigm which we review in this survey. Such systems, however, suffer from one major difficulty. To induce an intermediate discrete structure, the \( \text{arg max} \) function is usually used to make decisions as a layer in the parser. However, the gradient of \( \text{arg max} \) is either zero (almost everywhere) or undefined, so it does not work well with gradient-based optimization methods, the dominant optimization scheme in NLP and deep learning.

In this work, we survey three main families of approaches that tackle the non-differentiability of \( \text{arg max} \). The first family uses biased estimators with some proxy to the true gradient. The second circumvents discreteness using continuous relaxation. The last family casts the optimization problem as marginal likelihood maximization and uses sampling to approximate the gradient. We summarize these methods and several of their key properties in Table 1. We hope this work could catalyze future studies in augmenting recent NLP models with structural inductive biases.

2 Background: Structure Prediction

The task of structure prediction aims to automatically extract structures that, in NLP, are often rooted in linguistic theories \( (\text{Smith, 2011}) \). In this survey, we loosely define “structure” and consider any collection of inter-dependent variables as a structure. Abstracting away from specific formalisms, we generically denote any structure with \( z \in \mathcal{Z} \) where \( \mathcal{Z} \) is the set of all possible structures and encodes any structural constraints enforced by a formalism.\(^2\) As an example, for many bilexical dependency formalisms, syntactic \( (\text{Zeman et al., 2021}) \) or semantic \( (\text{Ivanova et al., 2012}) \), \( z \) can be viewed as a collection of binary parts \( z = [z_1, \cdots, z_n] \in \{0, 1\}^n \) where each entry denotes the existence of an

\[^2\] \( \mathcal{Z} \) usually depends on \( x \), but we drop this dependency notationally for clarity.
edge between a pair of words in a sentence. Here, the structural constraint encoded by $Z$ could require, for instance, the arborescence structure for syntactic dependencies. As another example, in matching problems, the words in a source sentence with length $m$ are matched with the words in a target sentence with length $n$. Accounting for non-matched words by matching them with a special $\epsilon$ symbol, $z$ can again be viewed as consisting of binary parts, with length $(m + 1)(n + 1)$, where each entry denotes if two words are matched. These two types of structures are visualized in Figure 1, and more examples can be found in Section 7.

A parser finds the highest-scoring structure $\hat{z} = \arg \max_{z \in Z} S(z \mid x; \theta)$ for some input $x$ with some scoring function $S$ parameterized by $\theta$. We use $\pi(z \mid x; \theta)$ to denote a normalized scoring function which yields a probability distribution over all $z \in Z$. For example, for dependency parsing, a graph-based parser is commonly used to produce a score for every part (for the example in Figure 1a, this would be a score for every cell), and the scores that correspond to the activated parts in a structure $z$ are summed. Specifically, we can write the scoring function as $S(z \mid x; \theta) = \pi(z) f(x; \theta) = z^T s$ where $s := f(x; \theta)$ denotes the per-part score produced by some model $f$.

In this survey, for simplicity, we default to settings and terminologies for graph-based dependency parsing.

### 3 Latent Structure Prediction

As argued in Section 1, it is often desirable to model discrete structures latently as an intermediate representation in a system. Specifically, we still have $s = f(x; \theta); \hat{z} = \arg \max_{z \in Z} z^T s$ as the best structure. We then feed $\hat{z}$ to some downstream task-specific predictor $\hat{y} = g(\hat{z}; \phi)$ parameterized by $\phi$ and incur some loss $\ell = L(\hat{y}, y)$. For some applications, the original input $x$ is also used as an input to $g$ in addition to the intermediate structure $\hat{z}$, but we omit it for clarity.

g can be seen as a structure encoder. With linguistic structures, for example, it usually takes the form of graph neural networks which encodes the structure $\hat{z}$. TreeLSTMs (Tai et al., 2015) have been widely used (Maillard et al., 2019; Choi et al., 2018), including variants that use different composition functions (Hu et al., 2021). With the advent of graph convolutional networks (Kipf and Welling, 2017), more works have based themselves on this architecture and its variants (Corro and Titov, 2019b; Wu et al., 2021).

In practice, $g$ can be pretrained before serving as a frozen component in the final system (Wu et al., 2019; Marcheggiani and Titov, 2017; Wu et al., 2021), or jointly trained with $f$ with separate loss signals using multi-task learning (Bowman et al., 2016; Hashimoto et al., 2017). Nevertheless, driven by the desiderata outlined in Section 1, in this survey, we only consider the case where $f$ and $g$ are jointly trained using only the downstream loss.

The challenge associated with this paradigm lies in the gradient (or Jacobian, in the case of structured output) of $\arg \max$, which is either zero (almost everywhere) or undefined (Figure 2a). We hence do not
have a meaningful gradient with respect to $\theta$ for optimization. Below, we introduce three main families of methods that tackle this difficulty.

We note that the non-differentiability of \textit{arg max} poses an issue not only for latent structure prediction, but also for non-structural discrete latent decisions, such as in the context of quantization (Hubara et al., 2017). Therefore, for most methodological families that we review below, we start with the unstructured case. Nevertheless, structure prediction comes with additional difficulties. For example, the feasible set is usually exponential in the number of structural parts, effectively ruling out enumeration, and it also often comes with structural constraints. These methods, therefore, sometimes cannot be directly applied to the structural setting and need modifications to overcome these challenges.

### 4 Surrogate Gradients

One family of approaches directly optimize $L(g(\hat{z}; \phi), y)$ by introducing some gradient-like quantity for backpropagation. Backpropagating through the model described in Section 3 yields a well-defined $\nabla_{\hat{z}} \ell$. However, since \textit{arg max} is used to obtain $\hat{z}$ from $s$, $\nabla_{s} \ell = 0$, and hence $\nabla_{\theta} \ell = 0$ and the parameters of the parser will not be updated.

In straight-through estimator (STE; Hinton, 2012; Bengio et al., 2013), $\nabla_{s} \ell$ is manually overridden. For example, in the identity STE:

$$\nabla_{s} \ell = \nabla_{\hat{z}} \ell$$

as if the identity activation $\hat{z} = s$ were performed in the forward pass (cf. the actual $\hat{z} = \arg \max_{z \in Z} z^T s$). It effectively “skips” the \textit{arg max} operation during the backward pass, hence the name “straight-through.”

Alternatives exist that override the gradient with that of other activation functions including sigmoid (Bengio et al., 2013) and variants of the tanh (Hubara et al., 2016) and ReLU (Cai et al., 2017) functions. This procedure results in a biased estimator for the true gradient. Nevertheless, despite this biased nature and its simplicity, it empirically works well (Bengio et al., 2013; Chung et al., 2017). Much research in understanding and justifying why such a mismatch between the objective and the learning algorithm remains beneficial is still underway. For example, Yin et al. (2019) showed that the gradients provided by properly chosen STE variants correlate to the descent direction that minimizes the population loss.

Being mindful of the structural constraints for STE can help improve the learning process. To see this, we first examine an alternative formulation of STE as proposed by Mihaylova et al. (2020). Consider the

Figure 2: Plot of the (a) \textit{arg max} and (b) \textit{softmax} functions. Figure is taken from Corro and Titov (2019b).
hypothetical case where the optimal structure \( z^* \) is accessible. It can be used to incur some intermediate loss, such as the structured perceptron loss \( L_{\text{structure}}(\hat{z}, z^*) = s^\top \hat{z} - s^\top z^* \).

In practice, however, such an optimal structure may not always be accessible. Nevertheless, an approximation to the optimal structure can be induced using the downstream loss \( \ell \). One can view \( \hat{z} \) as an “updatable” quantity and use \( \tilde{z} = \hat{z} - \nabla_{\hat{z}} \ell \approx z^* \) for this approximation. Effectively, this procedure performs a one-step gradient descent to induce an approximation of the optimal structure. Mihaylova et al. (2020) noted that this formulation recovers the identity STE in Equation (1):

\[
\nabla_{s} \ell = \hat{z} - z^* \approx \hat{z} - \tilde{z} = \hat{z} - (\hat{z} - \nabla_{\hat{z}} \ell) = \nabla_{\hat{z}} \ell
\]

Peng et al. (2018) noted that this approximation \( \tilde{z} \) can be outside of the feasible set \( Z \) or its convex hull \( P = \text{conv}(Z) \). So \( \tilde{z} \) can be an invalid, let alone optimal, structure. They proposed the method SPIGOT that projects \( \tilde{z} \) back to \( P \) when it is outside of \( P \):

\[
\tilde{z}^{\text{SPIGOT}} = \text{proj}_P(\hat{z} - \nabla_{\hat{z}} \ell) \approx z^*
\]

Then the gradient with respect to \( s \) can be similarly calculated:

\[
\nabla_{s} \ell = \hat{z} - z^* \approx \hat{z} - \tilde{z}^{\text{SPIGOT}}.
\]

Compared to the one-step gradient descent in the case of STE, SPIGOT can be viewed as performing one step of projected gradient descent. This process is illustrated in Figure 3.

Inspired by SPIGOT, Mihaylova et al. (2020) explored more numbers of (projected) gradient update steps, an alternative loss based on the CRF loss (Lafferty et al., 2001) in the place of the structured perceptron loss, and exponentiated gradient updates (Kivinen and Warmuth, 1997). Nevertheless, they did not observe significant improvement in the structured case. They hypothesized that an optimal intermediate approximation may not overall help learning.

5 Continuous Differentiable Relaxation

Another family of approaches relax the nondifferentiable \( \arg \max \) function with some continuous and differentiable function. In general, rather than taking a single \( \hat{z} = \arg \max_{z \in Z} S(z \mid x; \theta) \), they consider \( \hat{z} = \mathbb{E}_{z \sim \pi(z \mid x; \theta)}[z] \) (or \( \hat{z}_i = \mathbb{E}_{z \sim \pi(z \mid x; \theta)}[z_i] \) for a specific part \( z_i \) in the structure), where \( \pi(z \mid x; \theta) \) represents the normalized scores. Writing this out:

\[
\hat{z} = \mathbb{E}_{z \sim \pi(z \mid x; \theta)}[z] = \sum_{z \in \mathcal{Z}} \pi(z \mid x; \theta)z
\]

We can see that, in Equation (4), rather than only considering one best \( z \), it encodes uncertainty in the latent space by taking a weighted average of all \( z \). The attention mechanism is a common example of this type of softening. In the hard case, a model aligns to the token with the highest compatibility score. This, however, is non-differentiable and restricts the focus to only one token. Hence, recent works obtain a

\[3\text{Note that the intermediate } L_{\text{structure}} \text{ is different from the downstream loss function } L.\]
distribution among all candidate tokens, a paradigm that has achieved tremendous success (Bahdanau et al., 2015; Vaswani et al., 2017, inter alia).

There can be many different choices of $\pi(\cdot)$. A straightforward and widely-used choice uses the $\text{softmax}$ function. Consider first some categorical and unstructured latent variable where $Z$ can be seen as the set of classes represented as one-hot vectors. Then:

$$
\pi(z | x; \theta) = z^\top \text{softmax}(s) \tag{5}
$$

Nevertheless, one downside of $\text{softmax}$ is that it is soft. Even when the temperature parameter (Hinton et al., 2015) is sometimes used to induce sparse probabilities (Jang et al., 2017; Zhou and Neubig, 2017; Maillard et al., 2019), it never yields completely discrete 0-1 solutions. On the other hand, discreteness is sometimes desirable for, for example, interpretability, so many works have sought sparser alternatives to $\text{softmax}$ that are still differentiable.

These alternatives are inspired by the equivalence of $\text{softmax}$ and $\text{arg max}$ coupled with a Shannon entropy term as penalty:

$$
\text{softmax}(s) = \arg\max_{z \in \Delta^{|Z|-1}} z^\top s + H(z) \tag{6}
$$

Substituting the entropy term $H(z)$ with other forms of penalty results in functions including $\text{sparsemax}$ (Martins and Astudillo, 2016), $\alpha$-entmax (Peters et al., 2019), $\text{fusedmax}$ (Niculae and Blondel, 2017), etc.

As derived in Niculae and Blondel (2017):

$$
\text{sparsemax}(s) = \arg\max_{z \in \Delta^{|Z|-1}} z^\top s - \frac{1}{2} \|z\|_2^2 \tag{7}
$$

$$
\alpha\text{-entmax}(s) = \arg\max_{z \in \Delta^{|Z|-1}} z^\top s - \frac{1}{\alpha(\alpha - 1)} \|z\|_\alpha^\alpha \tag{8}
$$

$$
\text{fusedmax}(s) = \arg\max_{z \in \Delta^{|Z|-1}} z^\top s - \frac{1}{2} \|z\|_2^2 - \lambda \sum_i |z_i - z_{i-1}| \tag{9}
$$

$\alpha$-entmax, of which $\text{sparsemax}$ is a special case, penalizes the norm of the structure, effectively encouraging sparser structures. $\text{fusedmax}$ builds on $\text{sparsemax}$ while including an additional penalty term that discourages changes in value between consecutive parts. See Figure 4 for a visualization.

Figure 5 (left) illustrates the relationship between $\text{arg max}$, $\text{softmax}$, and this family of approaches using $\text{sparsemax}$ as an example.

Of course, these $\text{arg max}$-based formulations cannot be naïvely applied since, again, $\text{arg max}$ is not amenable to gradient-based optimization. Nevertheless, non-zero Jacobians specific to each of these functions have been derived. We refer readers to the original papers for details.

\footnote{For a proof, see the appendix of Niculae et al. (2018b).}
Figure 5: Illustration of the relationship between \( \text{arg max} \), \( \text{softmax} \), and \( \text{sparse max} \) in the probability simplex (left), as well as their structured counterpart in the convex hull of all possible structures (right). Figure is taken from Niculae et al. (2018a).

When \( z \) is a structure, \( \mathbb{E}_{z \sim \pi(z|x; \theta)}[z] \) can be intractable to obtain, and people have instead considered the part-marginal \( \mathbb{E}_{z \sim \pi(z|x; \theta)}[z_i] \) by marginalizing over exponentially many \( z \). Specifically:

\[
\pi(Z_i = z_i | x; \theta) = \mathbb{E}_{z \sim \pi(z|x; \theta)}[z_i] = \sum_{z \in Z} \pi(z | x; \theta) I(z_i = z_i)
\]

(10)

where \( I(\cdot) \) is the indicator function. Of course, directly calculating the right-hand side of Equation (10) is still in most cases intractable. Nevertheless, for specific cases, efficient algorithms exist, typically variants of the sum-product algorithm, such as the forward-backward algorithm (Rabiner, 1989) and the inside-outside algorithm (Baker, 1979).

Analogous to \( \text{sparse max} \) and related functions in the unstructured case, sparser alternatives to Equation (10) have been pursued. While the right-hand side of Equation (10) cannot be directly evaluated, with a sparse distribution over structures \( \pi(\cdot) \), we can only enumerate over structures with a nonzero probability which allows tractable inference. This provides an alternative to using sum-product algorithms which in some cases are unavailable and, even when available, only provide local features (e.g., the probability of an arc between a pair of words), incapable of modeling more global structures which can be beneficial (Corro and Titov, 2019b). Niculae et al. (2018a,b) developed the SparseMAP inference strategy which produces a combination of only a small number of structures. Figure 5 illustrates its correspondence with the unstructured setting. In addition to outperforming the \( \text{softmax} \) baseline on a variety of tasks, this sparse distribution over structures allows for improved interpretability: they discovered that it represents legitimate linguistic ambiguity. Niculae and Martins (2020) proposed LP-SparseMAP that builds on SparseMAP while allowing approximate inference.

### 6 Marginal Likelihood Maximization via Sampling

We now reconsider our objective probabilistically as a conditional marginal likelihood model. We want to maximize the likelihood of the label \( y \) given the input \( x \), \( p(y | x) \). Introducing and marginalizing over the latent variable \( z \), this can be written as

\[
p(y | x; \theta, \phi) = \sum_{z \in Z} p(y | x, z; \phi)p(z | x; \theta)
\]

(11)

This perspective in fact relates to using continuous relaxation where we considered \( p(y | x, \mathbb{E}_{z \sim \pi(z|x; \theta)}[z]; \phi) \). When the function that models the outer probability distribution is convex, though it often is not as parameterized by neural networks, by Jensen’s inequality, the continuous relaxation perspective is a lower bound of this quantity. Hence, optimizing Equation (11) provides an alternative to continuous relaxation.
Nevertheless, the challenge of optimizing Equation (11), or back to our formulation, \( E_{z \sim \pi (x; \theta)} [L(g(z; \phi), y)] \), with respect to \( \theta \) lies in the fact that the expectation is taken over a distribution parameterized by \( \theta \). This makes \( \nabla_\theta E_{z \sim \pi (x; \theta)} [L(g(z; \phi), y)] \) difficult to calculate. Sometimes this gradient can be directly evaluated with special structures and algorithms (e.g., Wiseman et al. (2018)). In this survey, we focus on the more general setting where this might be intractable and review two methods that rewrite this quantity in forms that allow its approximation via sampling: score function estimator and reparameterization.\(^5\)

### 6.1 Score Function Estimator

One class of such methods uses score function estimators, also known as the \textsc{REINFORCE} algorithm (Williams, 1992), to compute this gradient. They are based on the identity

\[
\nabla_\theta \pi(z \mid x; \theta) = \pi(z \mid x; \theta) \nabla_\theta \log \pi(z \mid x; \theta)
\]

which leads to

\[
\nabla_\theta E_{z \sim \pi (x; \theta)} [L(g(z))] = \nabla_\theta \sum_{z \in \mathcal{Z}} L(g(z)) \pi(z \mid x; \theta)
\]

\[
= \sum_{z \in \mathcal{Z}} L(g(z)) \nabla_\theta \pi(z \mid x; \theta)
\]

\[
= \sum_{z \in \mathcal{Z}} L(g(z)) \pi(z \mid x; \theta) \nabla_\theta \log \pi(z \mid x; \theta)
\]

\[
= E_{z \sim \pi (x; \theta)} [L(g(z)) \nabla_\theta \log \pi(z \mid x; \theta)]
\]

This formulation allows approximating the gradient with Monte-Carlo samples.

The score function estimator can suffer from high variance due to its sampling nature considering the exponential number of possible trees and the lack of intermediate rewards. For example, Nangia and Bowman (2018) showed that the trees induced by Yogatama et al. (2017), a method based on score function estimators, has a self-F1, which is a measure of variance, that is similar to random trees. Hence, variance reduction techniques are often employed in practice. For example, a control variate \( b(x) \) (Paisley et al., 2012) can be subtracted from \( L \):

\[
\nabla_\theta E_{z \sim \pi (x; \theta)} [L(g(z))] = E_{z \sim \pi (x; \theta)} [(L(g(z)) - b(x)) \nabla_\theta \pi(z \mid x; \theta)]
\]

### 6.2 Reparameterization

An alternative to score function estimators is the reparameterization trick (Kingma and Welling, 2014). Suppose we can sample from \( \pi(\cdot \mid x; \theta) \) by first sampling a noise \( \gamma \) from a simpler base distribution \( \mathcal{B} \) independent of \( \theta \) and then apply a transformation \( h(\gamma; \theta) \). Then:

\[
\nabla_\theta E_{z \sim \pi (x; \theta)} [L(g(z))] = \nabla_\theta \sum_{z \in \mathcal{Z}} L(g(z)) \pi(z \mid x; \theta)
\]

\[
= \nabla_\theta \sum_{\gamma \in \text{supp}(\mathcal{B})} L(g(h(\gamma; \theta))) \mathcal{B}(\gamma)
\]

\[
= \sum_{\gamma \in \text{supp}(\mathcal{B})} \mathcal{B}(\gamma) \nabla_\theta L(g(h(\gamma; \theta)))
\]

\[
= E_{\gamma \sim \mathcal{B}} [\nabla_\theta L(g(h(\gamma; \theta)))]
\]

\(^5\)Technically, this formulation does not apply directly in an autoencoding setup where \( y = x \), because we cannot use two separate networks to model \( p(x \mid z) \) and \( p(z \mid x) \). Directly maximizing the probability is often intractable (Ammar et al. (2014) being a notable exception), and requires optimizing the evidence lower bound, or \textsc{ELBO}. We omit the technical details for brevity, though most methods below also apply to \textsc{ELBO} optimization. See Rush et al. (2018) for a review.
This allows, once again, Monte-Carlo methods to approximate the original gradient.

We can compare the score function estimator and reparameterization-based methods by contrasting the final line in Equations (13) and (15). In reparameterization, we differentiate through the loss function, whereas in score function estimator, we only consider the gradient of the log probability of the structure and use the loss, or “reward” in reinforcement learning terminologies, as a black-box to drive the optimization. The former hence is more informative and empirically yields lower variance (Rush et al., 2018). Nevertheless, not all probability distributions are reparameterizable.

The reparameterization trick was originally introduced to train variational auto-encoders (Kingma and Welling, 2014). Thanks to the linearity of the normal distribution, sampling \( X \sim \mathcal{N}(\mu, \sigma^2) \) can be equivalently reparameterized as \( X = h(\gamma; \theta) = \mu + \sigma \gamma \) where \( \gamma \sim \mathcal{N}(0, 1) \), allowing gradients to be taken with respect to \( \mu \) and \( \sigma \) so that they can be learned. Nevertheless, it can be more difficult to reparameterize a discrete variable. We review a few methods to do this below.

6.2.1 Rectified Distributions

Rectifying continuous distributions can ensure a non-trivial probability for discrete solutions. For example, Bastings et al. (2019) used a stretch-and-rectify procedure, originally proposed in Louizos et al. (2018), to design the continuous HardKumaraswamy distribution (solid curve in Figure 6) that is based on the Kumaraswamy distribution (dashed curve in Figure 6; Kumaraswamy, 1980), which is reparameterizable. Specifically, after a base noise is drawn \( \gamma \sim \text{unif}(0, 1) \), it is fed through the inverse CDF of the Kumaraswamy distribution \( k = F_{K}^{-1}(\gamma; a, b) \in (0, 1) \), stretched \( t = l + (r - l)k \), and collapsed \( z = \min(1, \max(0, t)) \). Intuitively, while the original Kumaraswamy distribution does not have 0 and 1 in its support, we can stretch it from its original support \((0, 1)\) to \([l, r]\) and collapse the probability mass beyond 0 and 1 to the endpoints, allowing a non-trivial probability at these two points. This stretch-and-rectify procedure is visualized in Figure 6, and we refer readers to Bastings et al. (2019) for more details.

6.2.2 Gumbel-Max

The Gumbel-Max trick (Gumbel, 1954; Maddison et al., 2014) can also be used to reparameterize the sampling of discrete variables. To sample from the unnormalized scores \( s \), the Gumbel-Max trick perturbs the scores with an additive noise \( \gamma \sim \text{Gumbel}(0, 1) \), or equivalently, \( U \sim \text{unif}(0, 1), \gamma = \ldots \).
\[- \log(- \log(U)). \] It can be shown that taking
\[ \hat{z} = h(\gamma; \theta) = \arg \max_{z \in Z} z^\top s + \gamma = \arg \max_{z \in Z} S(z | x; \theta) + \gamma \tag{16} \]
where \( Z \) consists of one-hot vectors, is equivalent to sampling from the softmax-normalized \( s \).\(^6\)

However, even with the Gumbel-Max trick, the non-differentiability still exists in Equation (16). Therefore, methods introduced in Section 4 or 5 may still be required. With surrogate gradients, it becomes Gumbel STE, while with continuous relaxation, it yields Gumbel-softmax (Jang et al., 2017; Maddison et al., 2017). The two can be combined, where the forward pass uses a discrete \( \arg \max \) like in STE, while the softmax-ed distribution is used in the backward pass (vs. the identity function in identity STE). This is sometimes called straight-through Gumbel-softmax (Jang et al., 2017).

In the structured case, it is intractable to perturb every possible \( z \in Z \) separately. Corro and Titov (2019a,b) proposed differentiable Perturb-and-Parse by perturbing scores of each part. Specifically:
\[ \hat{z} = \arg \max_{z \in Z} z^\top (s + \gamma) \tag{17} \]

This \( \arg \max \) can be solved with max-product algorithms, though these algorithms usually have \( \arg \max \) operations within them that break backpropagation (Mensch and Blondel, 2018). Therefore, Corro and Titov (2019a,b) relaxed these operations with softmax that preserves the gradient flow. Nevertheless, unlike in the unstructured case, sampling using this method is not exact because it perturbs local parts rather than the global structure. This is apparent by contrasting Equations (16) and (17).

### 6.2.3 Direct Loss Minimization

As an alternative to circumventing the non-differentiability introduced by Gumbel-Max using STE or Gumbel-softmax, Lorberbom et al. (2019) proposed to adopt direct loss minimization, originally introduced by McAllester et al. (2010). Direct loss minimization was developed to optimize arbitrary, not necessarily differentiable, loss functions under mild conditions. We state this formally in Theorem 1, taken from Song et al. (2016), which is a generalization of the original theorem proposed in McAllester et al. (2010):

**Theorem 1.** When given a finite set \( \mathcal{Y} \), a scoring function \( F(x, y, w) \), a data distribution, as well as a task-loss \( L(y, \hat{y}) \), then, under some mild regularity conditions (see the supplementary material of Song et al. (2016) for details), the direct loss gradient has the following form:
\[ \nabla_w E[L(y, y_w)] = \pm \lim_{\epsilon \to 0} \frac{1}{\epsilon} E[\nabla_w F(x, y_{direct}, w) - \nabla_w F(x, y_w, w)] \tag{18} \]

with
\[ y_w = \arg \max_{\hat{y} \in \mathcal{Y}} F(x, \hat{y}, w) \tag{19} \]
\[ y_{direct} = \arg \max_{\hat{y} \in \mathcal{Y}} F(x, \hat{y}, w) \pm \epsilon L(y, \hat{y}) \tag{20} \]

**Proof.** We refer readers to the supplementary material of Song et al. (2016). \( \square \)

In Theorem 1, in addition to executing the regular inference procedure to compute \( y_w \), one also needs an additional loss-adjusted inference for \( y_{direct} \).

Lorberbom et al. (2019) adapted Theorem 1 to work with the Gumbel-Max reparameterization in variational auto-encoders (VAEs) that allows further expansion of Equation (15) by plugging in Equation (16). This is formalized in Theorem 2, taken from their work and adapted to our notation:

\(^6\)See the appendix of Maddison et al. (2014) for a proof.
Theorem 2. Assume that $S(z \mid x; \theta)$ is a smooth function of $\theta$. Then
\begin{equation}
E_{\gamma \sim B} [\nabla_\theta L(g(h(\gamma; \theta)))] = E_{\gamma \sim B} [\nabla_\theta L(g(\hat{z}))] \\
= \nabla_\theta E_{\gamma \sim B} [L(g(\hat{z}))] \\
= \lim_{\epsilon \to 0} \frac{1}{\epsilon} E_{\gamma \sim B} [\nabla_\theta S(\hat{z}(\epsilon) \mid x; \theta) - \nabla_\theta S(\hat{z} \mid x; \theta)]
\end{equation}
with
\begin{align*}
\hat{z} &= \arg \max_{\hat{z} \in \hat{z}} S(\hat{z} \mid x; \theta) + \gamma \\
\hat{z}(\epsilon) &= \arg \max_{\hat{z} \in \hat{z}} S(\hat{z} \mid x; \theta) + \gamma + \epsilon L(g(z))
\end{align*}

Proof. We refer readers to the supplementary material of Lorberbom et al. (2019).

The expansion in Equation (21) circumvents the need to take the gradient through the $\arg \max$ that is introduced by Gumbel-Max in Equation (15). Intuitively, the gradient is estimated by slightly perturbing $\hat{z}$.

While Equation (21) holds theoretically, one cannot implement the limit easily in a machine learning system. Lorberbom et al. (2019) hence treated $\epsilon$ as a fixed hyperparameter that controls the bias-variance trade-off. Since too small an $\epsilon$ could lead to very large gradients, they only considered $\epsilon \geq 0.1$. This, therefore, leads to a biased estimator for the true gradient. Nevertheless, they compared their method with Gumbel-softmax and showed that $\epsilon$ better controls the bias-variance trade-off than the temperature parameter in Gumbel-softmax.

In the structured case, direct loss minimization suffers from the same intractability as Gumbel-Max. Lorberbom et al. (2019) therefore only considered pairwise structures. We believe attempting to combine direct loss minimization and methods such as Perturb-and-Parse would be an interesting future study.

7 Applications

The methods introduced above have been widely adopted for optimizing models with latent discrete decisions. We review some applications below. We first discuss two settings where a downstream task is used as the training signal for latent structure induction, where the latent $z$ can either be a linguistic structure or an extractive summary of the input sentence. We also review when the target output is the same as the input, an auto-encoding setup, that allows unsupervised structure induction.

7.1 Linguistic Structures

As argued in Section 1, latently inducing a linguistically-inspired structure that aids the downstream prediction can provide a useful inductive bias. Typically, a latent tree is induced from text, and some structure-encoding model such as TreeLSTM (Tai et al., 2015) or GCN (Kipf and Welling, 2017) is used on top of this structure to compose the word-level representations into a sentence representation for the final prediction for various end tasks.

We presented graph-based parsing for bilexical dependencies in Section 2. Transition-based parsers, on the other hand, uses the scoring function $S(z \mid x; \theta) = \sum_{\{a : \text{yield}(a) = z\}} \prod_{i=1}^{|a|} f(a_i \mid x, a_{<i}; \theta)$, where $\text{yield}(\cdot)$ is the mapping from a sequence of actions $a = [a_1, \cdots, a_m]$, such as SHIFT and REDUCE, to the deterministic structure produced by these actions. For constituency parsing, the default choice is the CKY algorithm (Cocke, 1969; Kasami, 1965; Younger, 1967), a special case of chart parsing, which determines the optimal parse structure with dynamic programming by finding the optimal split point for spans.

\footnote{Again ignoring the caveat in Footnote 5.}
Transition-based parsing and the CKY algorithm are similar in that, in most settings, every decision sequence yields a valid structure. Therefore, we can simply backpropagate through the sequence of discrete decisions using the methods presented above. In this manner: Maillard and Clark (2018) trained a latent shift-reduce parser with STE. Maillard et al. (2019) induced a latent constituency structure with continuous relaxation using a softmax to normalize over all possible split points. Bogin et al. (2021) used a similar architecture to improve compositional generalization for grounded question answering. Yogatama et al. (2017) learned a policy network for a latent dependency parser. Kim et al. (2019) trained an unsupervised recurrent neural network grammar (Dyer et al., 2016) that resembles a shift-reduce parser using reinforcement learning. Choi et al. (2018) latently induced a constituency tree by greedily choosing a pair of neighboring text spans to merge at each layer using straight-through Gumbel-softmax at each layer for sampling. Hu et al. (2021) used this method to design a heuristic pruning procedure that improves the $O(n^3)$ complexity of the CKY algorithm to $O(n)$. This efficiency also allowed them to pretrain this model with a language modeling objective.

Graph-based parsing usually needs to be mindful of structural constraints. Peng et al. (2018) used SPIGOT to train latent syntactic and semantics parsers. Kim et al. (2017) considered the expected local decisions by marginalizing over all possible structures, latently inducing a projective syntactic dependency tree structure (and also a linear sequential structure), which they termed “structured attention.” For example, if $Z_{pq}$ represents the existence of a syntactic edge between words $p$ and $q$ with $q$ being the parent, then one can obtain a context vector of $q$, $c_q$, by performing attention to the syntactic parent of $q$ with $c_q \propto \sum_p \pi(Z_{pq} = 1 \mid x; \theta)x_p$ where $x_p$ denotes the representation of the word $p$. Liu et al. (2018) leveraged structured attention networks to induce latent constituency trees for richer inter-sentence attention (Parikh et al., 2016). Structured attention networks, however, can be expensive during the marginalization step. For example, marginalizing over dependency trees using the inside-outside algorithm can result in a more than 10 times slowdown (Liu and Lapata, 2018) due to its lack of parallelizability. Additionally, it is inherently unable to consider non-projective dependency trees. To address these two issues, Liu and Lapata (2018) proposed a generalization of structured attention networks by marginalizing using the Matrix-Tree Theorem (Tutte, 1984) that allows the induction of non-projective dependency trees (Koo et al., 2007; Smith and Smith, 2007; McDonald and Satta, 2007) and with better parallelizability, achieving similar speed as their simple attention baseline. Bisk and Tran (2018) extended this method to machine translation and showed that it outperforms using simple attention as well as STE. Niculae et al. (2018b) used SparseMAP to induce sparse distributions over latent dependency parses for sentence classification, natural language inference, and reverse dictionary lookup. Zhou et al. (2020) used a rectified distribution to induce latent dependency trees for syntactic-then-semantic AMR (Banarescu et al., 2013). Corro and Titov (2019b) used Perturb-and-Parse to obtain a latent dependency parser.

7.2 Rationale Extraction

One line of work concerns the interpretability of neural models and aims to extract the words and phrases in a span of text that are responsible for some task prediction, usually termed “rationale extraction.” Lei et al. (2016) first introduced this task and considered the intermediate structure as a collection of Bernoulli variables, each corresponding to a word in the original text, with some independence assumption among them. A condensed summary is formed by collecting the words corresponding to the activated Bernoulli variables. Then, a predictor learns a mapping from the summary to the final prediction, which is expected to stay close to the gold label. The summary usually needs to be latently induced due to a lack of summary annotation.

Lei et al. (2016) proposed to jointly train a rationale extractor and a downstream task predictor using score function estimators. They experimented with both an unstructured intermediate rationale where each word is independently selected as a Bernoulli random variable, as well a structured rationale.
where the selection of each word recurrently depends on previous selections. Their method achieved similar performance to using the full text while in most cases including only up to 30% of the text. Bastings et al. (2019) used reparameterization with a rectified distribution and demonstrated improved performance.

7.3 Auto-Encoder

Another important body of work uses an autoencoding objective where the target $y$ is the same as the input $x$. A reconstruction loss is typically either maximized directly (Ammar et al., 2014) or, when it is intractable as is common with neural network parameterized models, using variational autoencoders (VAE; Kingma and Welling, 2014). An encoder, or inference network, $f$ that models $p(z \mid x)$ and a reconstruction network $g$ that models $p(x \mid z)$ are jointly learned, after which we can take only the inference network to predict a structure from text.

With the output the same as the input, this allows completely unsupervised learning. Sometimes some amount of intermediate supervision is also added to ensure that the intermediate structure resembles some target formalism.

Linguistic Structures When $z$ is a linguistic structure, this method enables unsupervised or semi-supervised parsing. Drozdov et al. (2019, 2020) and Xu et al. (2021) used a continuous relaxation based approach for unsupervised and distantly-supervised parsing. Shen et al. (2018) softly and latently attends to a word’s syntactic siblings using proximity values induced by a “syntactic distance” for language modeling and unsupervised constituency parsing. Yin et al. (2018) used reinforcement learning to train a semi-supervised semantic parser. Corro and Titov (2019a) used Perturb-and-Parse for semi-supervised parsing.

Others The methods reviewed in this survey have also been used in VAEs when $z$ is not a linguistic structure. For example, Miao and Blunsom (2016) considered $z$ as a compressed version of the input sentence and trained a semi-supervised sentence compression model. Using Gumbel-softmax, Zhou and Neubig (2017) sampled independent morphological label as $z$ for morphological re-inflection, and Rezaee and Ferraro (2021) treated semantic frames as $z$ and trained a model for semi-supervised event modeling.

8 What is Latently Learned?

We now inspect the structure that is latently learned. We focus on latent syntax-inspired trees as they are easy to compare to human’s syntactic judgment and there are treebanks, or silver parser-generated trees, for reference.

Perhaps surprisingly, despite the strong downstream performance of these latent structure models, the trees that many such models learn for the most part do not correspond to traditional linguistic formalisms and at best capture only shallow syntactic units. Williams et al. (2018) examined the reinforcement learning based model of Yogatama et al. (2017) (RL-SPINN) and the straight-through Gumbel based model of Choi et al. (2018) (ST-Gumbel). They took these models that are trained on NLI datasets and evaluated them on the Penn Treebank (PTB; Marcus et al., 1993). RL-SPINN achieved similar parsing performance as random trees, while ST-Gumbel was even worse. In particular, RL-SPINN’s induced trees are highly similar to a purely left-branching structure with an F1 of > 99%. In contrast to this deepest possible strategy, the trees of ST-Gumbel are very shallow with an average depth of 4.2, close to 3.9 for balanced trees, and much shallower than silver parses with a 5.7 average depth. Maillard and Clark (2018) confirmed that the models of themselves and of Maillard et al. (2019) do not induce trees that resemble those produced by the Stanford Parser. Bisk and Tran (2018) similarly showed that their method achieved an attachment score similar to or even worse than flat baselines.
Nangia and Bowman (2018) introduced the ListOps dataset for synthetic math expression evaluation which is easy to solve with a parse structure but difficult otherwise. Both RL-SPINN and ST-Gumbel even underperform a simple LSTM model that has no structured modeling, suggesting that these models, with the datasets and training algorithms they used, cannot learn the structure required for this task. Nevertheless, Havrylov et al. (2019), which improved over RL-SPINN as introduced in §6.1, achieved near-perfect performance on this dataset by reducing the gradient estimation variance and reducing the coadaptation issue of the parser and the downstream predictor. Overall, it seems that for the latently induced structure to bear a close resemblance to existing syntactic formalisms is not a necessary condition for strong downstream task performance. In fact, many studies showed that, in latent structure learning, using parses from a pretrained parser deteriorates the performance (Choi et al., 2018; Maillard et al., 2019), pretraining on these parses does not help (Kim et al., 2017), and neither does including an auxiliary parsing loss (Yogatama et al., 2017). Some studies qualitatively inspected the latent structure and discovered deviation from existing formalisms that may benefit composition (Bisk and Tran, 2018; Niculae et al., 2018b). For example, in coordination constructions, Universal Dependencies dictate the leftmost conjunct as the head (Zeman et al., 2021), while Niculae et al. (2018b) argued that a symmetric treatment may provide a better composition order. On the other hand, Shi et al. (2018) noted that trivial trees (i.e., balanced or linear trees) achieve performance similar to or better than latently induced trees or external parse trees on a variety of classification and generation tasks, casting doubt on the real source of performance improvement. More future work is needed to understand the role syntax plays in this picture and how it might vary across different tasks.

The story is different for methods for unsupervised parsing which can induce familiar linguistic formalisms with decent accuracy (though see Li and Risteski (2021) for a theoretical analysis of their limitations). These methods usually use a language modeling or autoencoding objective rather than receiving supervision from a natural language understanding task (Shen et al., 2018; Kim et al., 2019; Hu et al., 2021, inter alia). Why these objectives are more conducive to the induction of a latent structure that better resembles existing linguistic formalisms would be an interesting future research question.

9 Conclusion

In this work, we surveyed three families of methods that tackle the non-differentiability of discrete decisions caused by the $\arg \max$ function and introduced how they respectively handle the unique challenges of structure prediction. These methods and some of their key properties are summarized in Table 1. We also reviewed past work that studied the induced latent structures and highlighted that they mostly do not closely resemble existing linguistic formalisms. We hope this survey can serve to catalyze future research in this area.

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References

Waleed Ammar, Chris Dyer, and Noah A Smith. 2014. Conditional random field autoencoders for unsupervised structured prediction. In Advances in Neural Information Processing Systems, volume 27. Curran Associates, Inc.
Dzmitry Bahdanau, Kyung Hyun Cho, and Yoshua Bengio. 2015. Neural machine translation by jointly learning to align and translate. In Proceedings of the 3rd International Conference on Learning Representations.

J. K. Baker. 1979. Trainable grammars for speech recognition. The Journal of the Acoustical Society of America, 65(S1):S132–S132.

Laura Banarescu, Claire Bonial, Shu Cai, Madalina Georgescu, Kira Griffitt, Ulf Hermjakob, Kevin Knight, Philipp Koehn, Martha Palmer, and Nathan Schneider. 2013. Abstract Meaning Representation for sembanking. In Proceedings of the 7th Linguistic Annotation Workshop and Interoperability with Discourse, pages 178–186, Sofia, Bulgaria. Association for Computational Linguistics.

Jasmijn Bastings, Wilker Aziz, and Ivan Titov. 2019. Interpretable neural predictions with differentiable binary variables. In Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics, pages 2963–2977, Florence, Italy. Association for Computational Linguistics.

Yoshua Bengio, Nicholas Léonard, and Aaron Courville. 2013. Estimating or propagating gradients through stochastic neurons for conditional computation.

Yonatan Bisk and Ke Tran. 2018. Inducing grammars with and for neural machine translation. In Proceedings of the 2nd Workshop on Neural Machine Translation and Generation, pages 25–35, Melbourne, Australia. Association for Computational Linguistics.

Ben Bogin, Sanjay Subramanian, Matt Gardner, and Jonathan Berant. 2021. Latent Compositional Representations Improve Systematic Generalization in Grounded Question Answering. Transactions of the Association for Computational Linguistics, 9:195–210.

Samuel R. Bowman, Jon Gauthier, Abhinav Rastogi, Raghav Gupta, Christopher D. Manning, and Christopher Potts. 2016. A fast unified model for parsing and sentence understanding. In Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 1466–1477, Berlin, Germany. Association for Computational Linguistics.

Zhaowei Cai, Xiaodong He, Jian Sun, and Nuno Vasconcelos. 2017. Deep learning with low precision by half-wave gaussian quantization. In CVPR.

Qian Chen, Xiaodan Zhu, Zhen-Hua Ling, Si Wei, Hui Jiang, and Diana Inkpen. 2017. Enhanced LSTM for natural language inference. In Proceedings of the 55th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 1657–1668, Vancouver, Canada. Association for Computational Linguistics.

Jihun Choi, Kang Min Yoo, and Sanggoo Lee. 2018. Learning to compose task-specific tree structures. In Proceedings of the AAAI Conference on Artificial Intelligence.

Noam Chomsky. 1965. Aspects of the Theory of Syntax, 50 edition. The MIT Press.

Junyoung Chung, Sungjin Ahn, and Yoshua Bengio. 2017. Hierarchical multiscale recurrent neural networks. In International Conference on Learning Representations.

John Cocke. 1969. Programming Languages and Their Compilers: Preliminary Notes. New York University, USA.

Caio Corro and Ivan Titov. 2019a. Differentiable perturb-and-parse: Semi-supervised parsing with a structured variational autoencoder. In International Conference on Learning Representations.

Caio Corro and Ivan Titov. 2019b. Learning latent trees with stochastic perturbations and differentiable dynamic programming. In Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics, pages 5508–5521, Florence, Italy. Association for Computational Linguistics.
Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2019. BERT: Pre-training of deep bidirectional transformers for language understanding. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 4171–4186, Minneapolis, Minnesota. Association for Computational Linguistics.

Andrew Drozdov, Subendhu Rongali, Yi-Pei Chen, Tim O’Gorman, Mohit Iyyer, and Andrew McCallum. 2020. Unsupervised parsing with S-DIORA: Single tree encoding for deep inside-outside recursive autoencoders. In Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing (EMNLP), pages 4832–4845, Online. Association for Computational Linguistics.

Andrew Drozdov, Patrick Verga, Mohit Yadav, Mohit Iyyer, and Andrew McCallum. 2019. Unsupervised latent tree induction with deep inside-outside recursive auto-encoders. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 1129–1141, Minneapolis, Minnesota. Association for Computational Linguistics.

Chris Dyer, Adhiguna Kuncoro, Miguel Ballesteros, and Noah A. Smith. 2016. Recurrent neural network grammars. In Proceedings of the 2016 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, pages 199–209, San Diego, California. Association for Computational Linguistics.

Akiko Eriguchi, Kazuma Hashimoto, and Yoshimasa Tsuruoka. 2016. Tree-to-sequence attentional neural machine translation. In Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 823–833, Berlin, Germany. Association for Computational Linguistics.

Emil Julius Gumbel. 1954. Statistical theory of extreme values and some practical applications; a series of lectures. Applied mathematics series ; 33. U.S. Govt. Print. Office, Washington.

Kazuma Hashimoto, Caiming Xiong, Yoshimasa Tsuruoka, and Richard Socher. 2017. A joint many-task model: Growing a neural network for multiple NLP tasks. In Proceedings of the 2017 Conference on Empirical Methods in Natural Language Processing, pages 1923–1933, Copenhagen, Denmark. Association for Computational Linguistics.

Serhii Havrylov, Germán Kruszewski, and Armand Joulin. 2019. Cooperative learning of disjoint syntax and semantics. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 1118–1128, Minneapolis, Minnesota. Association for Computational Linguistics.

Irene Heim and Angelika Kratzer. 1998. Semantics in Generative Grammar. Blackwell Textbooks in Linguistics. Wiley.

Geoffrey Hinton. 2012. Neural networks for machine learning, coursera.

Geoffrey Hinton, Oriol Vinyals, and Jeffrey Dean. 2015. Distilling the knowledge in a neural network. In NIPS Deep Learning and Representation Learning Workshop.

Sepp Hochreiter and Jürgen Schmidhuber. 1997. Long short-term memory. Neural Computation, 9(8):1735–1780.

Xiang Hu, Haitao Mi, Zujie Wen, Yafang Wang, Yi Su, Jing Zheng, and Gerard de Melo. 2021. R2D2: Recursive transformer based on differentiable tree for interpretable hierarchical language modeling. In Proceedings of the 59th Annual Meeting of the Association for Computational Linguistics and the 11th International Joint Conference on Natural Language Processing (Volume 1: Long Papers), pages 4897–4908, Online. Association for Computational Linguistics.
Itay Hubara, Matthieu Courbariaux, Daniel Soudry, Ran El-Yaniv, and Yoshua Bengio. 2016. Binarized neural networks. In Advances in Neural Information Processing Systems, volume 29. Curran Associates, Inc.

Itay Hubara, Matthieu Courbariaux, Daniel Soudry, Ran El-Yaniv, and Yoshua Bengio. 2017. Quantized neural networks: Training neural networks with low precision weights and activations. J. Mach. Learn. Res., 18(1):6869–6898.

Angelina Ivanova, Stephan Oepen, Lilja Øvrelid, and Dan Flickinger. 2012. Who did what to whom? a contrastive study of syntacto-semantic dependencies. In Proceedings of the Sixth Linguistic Annotation Workshop, pages 2–11, Jeju, Republic of Korea. Association for Computational Linguistics.

Eric Jang, Shixiang Gu, and Ben Poole. 2017. Categorical reparametrization with gumbel-softmax. In Proceedings International Conference on Learning Representations 2017. OpenReviews.net.

T. Kasami. 1965. An efficient recognition and syntax analysis algorithm for context-free languages. Technical Report AFCRL-65-758, Air Force Cambridge Research Laboratory, Bedford, MA.

Yoon Kim, Carl Denton, Luong Hoang, and Alexander M. Rush. 2017. Structured attention networks.

Yoon Kim, Alexander Rush, Lei Yu, Adhiguna Kuncoro, Chris Dyer, and Gábor Melis. 2019. Unsupervised recurrent neural network grammars. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers), pages 1105–1117, Minneapolis, Minnesota. Association for Computational Linguistics.

Diederik P. Kingma and Max Welling. 2014. Auto-Encoding Variational Bayes. In 2nd International Conference on Learning Representations, ICLR 2014, Banff, AB, Canada, April 14-16, 2014, Conference Track Proceedings.

Thomas N. Kipf and Max Welling. 2017. Semi-supervised classification with graph convolutional networks. In International Conference on Learning Representations (ICLR).

Jyrki Kivinen and Manfred K. Warmuth. 1997. Exponentiated gradient versus gradient descent for linear predictors. Information and Computation, 132(1):1 – 63.

Terry Koo, Amir Globerson, Xavier Carreras, and Michael Collins. 2007. Structured prediction models via the matrix-tree theorem. In Proceedings of the 2007 Joint Conference on Empirical Methods in Natural Language Processing and Computational Natural Language Learning (EMNLP-CoNLL), pages 141–150, Prague, Czech Republic. Association for Computational Linguistics.

P. Kumaraswamy. 1980. A generalized probability density function for double-bounded random processes. Journal of Hydrology, 46(1):79–88.

John D. Lafferty, Andrew McCallum, and Fernando C. N. Pereira. 2001. Conditional random fields: Probabilistic models for segmenting and labeling sequence data. In Proceedings of the Eighteenth International Conference on Machine Learning, ICML ’01, page 282–289, San Francisco, CA, USA. Morgan Kaufmann Publishers Inc.

Tao Lei, Regina Barzilay, and Tommi Jaakkola. 2016. Rationalizing neural predictions. In Proceedings of the 2016 Conference on Empirical Methods in Natural Language Processing, pages 107–117, Austin, Texas. Association for Computational Linguistics.

Yuchen Li and Andrej Risteski. 2021. The limitations of limited context for constituency parsing. In Proceedings of the 59th Annual Meeting of the Association for Computational Linguistics and the 11th International Joint Conference on Natural Language Processing (Volume 1: Long Papers), pages 2675–2687, Online. Association for Computational Linguistics.
Yang Liu, Matt Gardner, and Mirella Lapata. 2018. Structured alignment networks for matching sentences. In Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing, pages 1554–1564, Brussels, Belgium. Association for Computational Linguistics.

Yang Liu and Mirella Lapata. 2018. Learning structured text representations. Transactions of the Association for Computational Linguistics, 6:63–75.

Guy Lorberbom, Andreea Gane, Tommi Jaakkola, and Tamir Hazan. 2019. Direct optimization through \( \arg \max \) for discrete variational auto-encoder. In Advances in Neural Information Processing Systems, volume 32. Curran Associates, Inc.

Christos Louizos, Max Welling, and Diederik P. Kingma. 2018. Learning sparse neural networks through \( L_0 \) regularization. In International Conference on Learning Representations.

Chris J. Maddison, Andriy Mnih, and Yee Whye Teh. 2017. The Concrete Distribution: A Continuous Relaxation of Discrete Random Variables. In International Conference on Learning Representations.

Chris J Maddison, Daniel Tarlow, and Tom Minka. 2014. A* sampling. In Advances in Neural Information Processing Systems, volume 27. Curran Associates, Inc.

Jean Maillard and Stephen Clark. 2018. Latent tree learning with differentiable parsers: Shift-reduce parsing and chart parsing. In Proceedings of the Workshop on the Relevance of Linguistic Structure in Neural Architectures for NLP, pages 13–18, Melbourne, Australia. Association for Computational Linguistics.

Jean Maillard, Stephen Clark, and Dani Yogatama. 2019. Jointly learning sentence embeddings and syntax with unsupervised Tree-LSTMs. Natural Language Engineering, 25(4):433–449.

Diego Marcheggiani and Ivan Titov. 2017. Encoding sentences with graph convolutional networks for semantic role labeling. In Proceedings of the 2017 Conference on Empirical Methods in Natural Language Processing, pages 1506–1515, Copenhagen, Denmark. Association for Computational Linguistics.

Mitchell P. Marcus, Beatrice Santorini, and Mary Ann Marcinkiewicz. 1993. Building a large annotated corpus of English: The Penn Treebank. Computational Linguistics, 19(2):313–330.

André F. T. Martins and Ramón F. Astudillo. 2016. From softmax to sparsemax: A sparse model of attention and multi-label classification. In Proceedings of the 33rd International Conference on International Conference on Machine Learning - Volume 48, ICML’16, page 1614–1623. JMLR.org.

André F. T. Martins, Tsvetomila Mihaylova, Nikita Nangia, and Vlad Niculae. 2019. Latent structure models for natural language processing. In Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics: Tutorial Abstracts, pages 1–5, Florence, Italy. Association for Computational Linguistics.

David McAllester, Tamir Hazan, and Joseph Keshet. 2010. Direct loss minimization for structured prediction. In Advances in Neural Information Processing Systems, volume 23. Curran Associates, Inc.

Ryan McDonald and Giorgio Satta. 2007. On the complexity of non-projective data-driven dependency parsing. In Proceedings of the Tenth International Conference on Parsing Technologies, pages 121–132, Prague, Czech Republic. Association for Computational Linguistics.

Arthur Mensch and Mathieu Blondel. 2018. Differentiable dynamic programming for structured prediction and attention. In Proceedings of the 35th International Conference on Machine Learning, volume 80 of Proceedings of Machine Learning Research, pages 3462–3471. PMLR.
Yishu Miao and Phil Blunsom. 2016. Language as a latent variable: Discrete generative models for sentence compression. In *Proceedings of the 2016 Conference on Empirical Methods in Natural Language Processing*, pages 319–328, Austin, Texas. Association for Computational Linguistics.

Tsvetomila Mihaylova, Vlad Niculae, and André F. T. Martins. 2020. Understanding the mechanics of SPIGOT: Surrogate gradients for latent structure learning. In *Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing (EMNLP)*, pages 2186–2202, Online. Association for Computational Linguistics.

Nikita Nangia and Samuel Bowman. 2018. ListOps: A diagnostic dataset for latent tree learning. In *Proceedings of the 2018 Conference of the North American Chapter of the Association for Computational Linguistics: Student Research Workshop*, pages 92–99, New Orleans, Louisiana, USA. Association for Computational Linguistics.

Vlad Niculae and Mathieu Blondel. 2017. A regularized framework for sparse and structured neural attention. In *Proceedings of the 31st International Conference on Neural Information Processing Systems*, NIPS’17, page 3340–3350, Red Hook, NY, USA. Curran Associates Inc.

Vlad Niculae and Andre Martins. 2020. LP-SparseMAP: Differentiable relaxed optimization for sparse structured prediction. In *Proceedings of the 37th International Conference on Machine Learning*, volume 119 of *Proceedings of Machine Learning Research*, pages 7348–7359. PMLR.

Vlad Niculae, Andre Martins, Mathieu Blondel, and Claire Cardie. 2018a. SparseMAP: Differentiable sparse structured inference. In *Proceedings of the 35th International Conference on Machine Learning*, volume 80 of *Proceedings of Machine Learning Research*, pages 3799–3808. PMLR.

Vlad Niculae, André F. T. Martins, and Claire Cardie. 2018b. Towards dynamic computation graphs via sparse latent structure. In *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing*, pages 905–911, Brussels, Belgium. Association for Computational Linguistics.

John Paisley, David M. Blei, and Michael I. Jordan. 2012. Variational bayesian inference with stochastic search. In *Proceedings of the 29th International Conference on International Conference on Machine Learning*, ICML’12, page 1363–1370, Madison, WI, USA. Omnipress.

Ankur Parikh, Oscar Täckström, Dipanjan Das, and Jakob Uszkoreit. 2016. A decomposable attention model for natural language inference. In *Proceedings of the 2016 Conference on Empirical Methods in Natural Language Processing*, pages 2249–2255, Austin, Texas. Association for Computational Linguistics.

Hao Peng, Sam Thomson, and Noah A. Smith. 2018. Backpropagating through structured argmax using a SPIGOT. In *Proceedings of the 56th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 1863–1873, Melbourne, Australia. Association for Computational Linguistics.

Ben Peters, Vlad Niculae, and André F. T. Martins. 2019. Sparse sequence-to-sequence models. In *Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics*, pages 1504–1519, Florence, Italy. Association for Computational Linguistics.

Matthew E. Peters, Mark Neumann, Mohit Iyyer, Matt Gardner, Christopher Clark, Kenton Lee, and Luke Zettlemoyer. 2018. Deep contextualized word representations. In *Proceedings of the 2018 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long Papers)*, pages 2227–2237, New Orleans, Louisiana. Association for Computational Linguistics.

L.R. Rabiner. 1989. A tutorial on hidden markov models and selected applications in speech recognition. *Proceedings of the IEEE, 77(2):257–286.*
Colin Raffel, Noam Shazeer, Adam Roberts, Katherine Lee, Sharan Narang, Michael Matena, Yanqi Zhou, Wei Li, and Peter J. Liu. 2020. Exploring the limits of transfer learning with a unified text-to-text transformer. *Journal of Machine Learning Research*, 21(140):1–67.

Mehdi Rezaee and Francis Ferraro. 2021. Event representation with sequential, semi-supervised discrete variables. In *Proceedings of the 2021 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*, pages 4701–4716, Online. Association for Computational Linguistics.

Alexander Rush, Yoon Kim, and Sam Wiseman. 2018. Deep latent variable models of natural language. In *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing: Tutorial Abstracts*, Melbourne, Australia. Association for Computational Linguistics.

Rico Sennrich and Barry Haddow. 2016. Linguistic input features improve neural machine translation. In *Proceedings of the First Conference on Machine Translation: Volume 1, Research Papers*, pages 83–91, Berlin, Germany. Association for Computational Linguistics.

Yikang Shen, Zhouhan Lin, Chin wei Huang, and Aaron Courville. 2018. Neural language modeling by jointly learning syntax and lexicon. In *International Conference on Learning Representations*.

Haoyue Shi, Hao Zhou, Jiaze Chen, and Lei Li. 2018. On tree-based neural sentence modeling. In *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing*, pages 4631–4641, Brussels, Belgium. Association for Computational Linguistics.

David A. Smith and Noah A. Smith. 2007. Probabilistic models of nonprojective dependency trees. In *Proceedings of the 2007 Joint Conference on Empirical Methods in Natural Language Processing and Computational Natural Language Learning (EMNLP-CoNLL)*, pages 132–140, Prague, Czech Republic. Association for Computational Linguistics.

Noah A. Smith. 2011. *Linguistic Structure Prediction*. Synthesis Lectures on Human Language Technologies. Morgan and Claypool.

Yang Song, Alexander Schwing, Zemel Richard, and Raquel Urtasun. 2016. Training deep neural networks via direct loss minimization. In *Proceedings of The 33rd International Conference on Machine Learning*, volume 48 of *Proceedings of Machine Learning Research*, pages 2169–2177, New York, New York, USA. PMLR.

Kai Sheng Tai, Richard Socher, and Christopher D. Manning. 2015. Improved semantic representations from tree-structured long short-term memory networks. In *Proceedings of the 53rd Annual Meeting of the Association for Computational Linguistics and the 7th International Joint Conference on Natural Language Processing (Volume 1: Long Papers)*, pages 1556–1566, Beijing, China. Association for Computational Linguistics.

W.T. Tutte. 1984. *Graph Theory*. Encyclopedia of mathematics and its applications. Cambridge University Press.

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Ł ukasz Kaiser, and Illia Polosukhin. 2017. Attention is all you need. In *Advances in Neural Information Processing Systems*, volume 30. Curran Associates, Inc.

Adina Williams, Andrew Drozdov, and Samuel R. Bowman. 2018. Do latent tree learning models identify meaningful structure in sentences? *Transactions of the Association for Computational Linguistics*, 6:253–267.

Ronald J. Williams. 1992. Simple statistical gradient-following algorithms for connectionist reinforcement learning. *Mach. Learn.*, 8(3–4):229–256.
Sam Wiseman, Stuart Shieber, and Alexander Rush. 2018. Learning neural templates for text generation. In *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing*, pages 3174–3187, Brussels, Belgium. Association for Computational Linguistics.

Zhaofeng Wu, Hao Peng, and Noah A. Smith. 2021. Infusing Finetuning with Semantic Dependencies. *Transactions of the Association for Computational Linguistics*, 9:226–242.

Zhaofeng Wu, Yan Song, Sicong Huang, Yuanhe Tian, and Fei Xia. 2019. WTMED at MEDIQA 2019: A hybrid approach to biomedical natural language inference. In *Proceedings of the 18th BioNLP Workshop and Shared Task*, pages 415–426, Florence, Italy. Association for Computational Linguistics.

Zhiyang Xu, Andrew Drozdov, Jay Yoon Lee, Tim O’Gorman, Subendhu Rongali, Dylan Finkbeiner, Shilpa Suresh, Mohit Iyyer, and Andrew McCallum. 2021. Improved latent tree induction with distant supervision via span constraints. In *Proceedings of the 2021 Conference on Empirical Methods in Natural Language Processing*, pages 4818–4831, Online and Punta Cana, Dominican Republic. Association for Computational Linguistics.

Pengcheng Yin, Chunting Zhou, Junxian He, and Graham Neubig. 2018. StructVAE: Tree-structured latent variable models for semi-supervised semantic parsing. In *Proceedings of the 56th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 754–765, Melbourne, Australia. Association for Computational Linguistics.

Daniel H. Younger. 1967. Recognition and parsing of context-free languages in time $n^3$. *Information and Control*, 10(2):189–208.

Daniel Zeman, Joakim Nivre, Mitchell Abrams, Elia Ackermann, Noëmi Aepli, Hamid Aghaei, Željko Agić, Amir Ahmad, Lars Ahrenberg, Chika Kennedy Ajede, Gabriéle Aleksandravičiūtė, Ika Alfin, Lene Antonsen, Katya Aplonova, Angelina Aquino, Carolina Aragon, Maria Jesus Aranzabe, Bilge Nas Arıcan, Hórunn Arnardóttir, Gashaw Arutie, Jessica Naraiswari Arwidarasti, Masayuki Asahara, Deniz Baran Aslan, Luma Ateyah, Furkan Atmaca, Mohammed Attia, Atziber Atutxa, Liesbeth Augustinus, Elena Badmaeva, Keerthana Balasubramani, Miguel Ballesteros, Esha Banerjee, Sebastian Bank, Verginica Barbu Mititelu, Starkaður Barkarson, Victoria Basmov, Colin Batchelor, John Bauer, Seyyit Talha Bedir, Kepa Bengoetxea, Gözde Berk, Yevgeni Berzak, Irshad Ahmad Bhat, Riyaz Ahmad Bhat, Erica Biagini, Eckhard Bick, Agné Bielinskienė, Kristín Bjarnadóttir, Rogier Blokland, Victoria Bobicev, Loïc Boizou, Emanuel Borges Völker, Carl Börstl, Cristina Bosco, Gosse Bouma, Sam Bowman, Adrian Boyd, Anouck Braggard, Kristina Brokaitė, Aljoscha Burcardt, Marie Candido, Bernard Caron, Gauthier Caron, Lauren Cassidy, Tatiana Cavalcanti, Gülşen Cebiroğlu Eryiğit, Flavio Massimiliano Cecchini, Giuseppe G. A. Celano, Slavomir Čeplň, Neslihan Cesur, Savas Cetin, Özlem Çetinoğlu, Fabricio Chalub, Shweta Chauhan, Ethan Chi, Taishi Chika, Yongseok Cho, Jinho Choi, Juyeon Chun, Alessandra T. Cignarella, Silvie Cinková, Aurélie Collomb, Çağrı Çöltekin, Miriam Connor, Marine Courtin, Mihaela Cristescu, Philemon. Daniel, Elizabeth Davidson, Marie-Catherine de Marnelle, Valeria de Paiva, Mehmet Oğuz Derin, Elvis de Souza, Arantza Diaz de Irarraz, Carly Dickerson, Arawinda Dinakaramani, Elisa Di Nuovo, Bamba Dione, Peter Dirix, Kaja Dobrovolec, Timothy Dozat, Kira Drogoanova, Puneet Dwivedi, Hanne Eckhoff, Sandra Eiche, Marhaba Eli, Ali Elkahky, Binyam Ephrem, Olga Erina, Tomaž Erjavec, Aline Etienne,
Tanaka, Samson Tella, Isabelle Tellier, Marinella Testori, Guillaume Thomas, Liisi Torga, Marsida Toska, Trond Trosterud, Anna Trukhina, Reut Tsarfaty, Utku Türk, Francis Tyers, Sumire Uematsu, Roman Untilov, Zdeňka Urešová, Larraitz Uria, Hans Uszkoreit, Andrius Utka, Sowmya Vajjala, Rob van der Goot, Martine Vanhove, Daniel van Niekerk, Gertjan van Noord, Viktor Varga, Eric Villemonte de la Clergerie, Veronika Vincze, Natalia Vlasova, Aya Wakasa, Joel C. Wallenberg, Lars Wallin, Abigail Walsh, Jing Xian Wang, Jonathan North Washington, Maximilan Wendt, Paul Widmer, Seyi Williams, Mats Wirén, Christian Wittern, Tsegay Woldemariam, Tak-sum Wong, Alina Wróblewska, Mary Yako, Kayo Yamashita, Naoki Yamazaki, Chunxiao Yan, Koichi Yasuoka, Marat M. Yavrumyan, Arife Betül Yenice, Olcay Taner Yıldız, Zhuoran Yu, Zdeněk Žabokrtský, Shorouq Zahra, Amir Zeldes, Hanzhi Zhu, Anna Zhuravleva, and Rayan Ziane. 2021. Universal dependencies 2.8.1. LINDAT/CLARIAH-CZ digital library at the Institute of Formal and Applied Linguistics (ÚFAL), Faculty of Mathematics and Physics, Charles University.

Chunting Zhou and Graham Neubig. 2017. Multi-space variational encoder-decoders for semi-supervised labeled sequence transduction. In Proceedings of the 55th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 310–320, Vancouver, Canada. Association for Computational Linguistics.

Qiji Zhou, Yue Zhang, Donghong Ji, and Hao Tang. 2020. AMR parsing with latent structural information. In Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics, pages 4306–4319, Online. Association for Computational Linguistics.