Critical behavior of YBa$_2$Cu$_3$O$_{7-\delta}$ and Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ from the dual Ginzburg-Landau model

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(Received November 23, 2018)

An unifying scenario for the critical behavior of YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) and Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCO) is proposed. It is shown that both critical crossovers observed in these materials follow by considering two different scalings in the dual Ginzburg-Landau model. The first scaling leads to the critical exponents $\nu \approx 2/3$, $\nu' \approx 1/3$ and $\alpha \approx 0$, with $\nu$ and $\alpha$ being respectively the correlation length and specific heat exponents while $\nu'$ is the magnetic field penetration depth exponent. These values for the critical exponents agree with the ones obtained experimentally for YBCO single crystals. For the second scaling we obtain $\nu = 1$ and $\alpha = -1$ which must be compared with the measured values $\nu \approx 1$ and $\alpha \approx -0.7$ for BSCO. For the penetration depth exponent it is predicted the value $\nu' = 1$.

Pacs: 74.20.-z, 05.10Cc, 11.10.-z

There is generally accepted that the experimentally accessible critical region of high temperature superconductor YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) lies in a $^4$He universality class \cite{3,1}. This means that the critical behavior is governed by the 3D XY ($X_Y$) model for short) nontrivial fixed point. The $X_Y$ critical behavior of YBCO has been probed both in zero and nonzero external field regimes \cite{2}. For instance, a quantity which can be accurately measured in zero field is the magnetic field penetration depth, $\lambda$, whose scaling behavior near $T_c$ is given by $\lambda \sim |t|^{\nu'}$, with $\nu' = 0.33 \pm 0.1$ \cite{2} and $t$ being the reduced temperature. This value of $\nu'$ is consistent with a critical regime described by a $X_Y$ universality class where we expect $\nu' = \nu/2$ with $\nu \approx 2/3$ \cite{1}. Also, specific heat measurements give $\alpha \approx -0.013$ \cite{2}, which is also consistent with the $X_Y$ universality class. These results hold for single crystals of optimally doped YBCO. However, this scenario may not hold for thin films of YBCO. For example, the value $\nu' = 1/2$ has been measured at zero field regime \cite{3,1} which may be associated either to a mean-field behavior \cite{2} or to some other critical crossover \cite{4}. It is worth to cite also the recent results of Charalambous et al. \cite{5} for bulk YBCO. They reported different values of $\alpha$ depending on whether $T_c$ is approached from below or from above.

The $X_Y$ universality class is not always shared by other cuprate superconductors. For instance, a different critical behavior is probed in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCO). It is obtained that $\alpha \approx -0.7$ and $\nu \approx 1$ \cite{6,7}. This is not very different from the critical exponents of the 3D $O(N)$ model in the large $N$ limit where we find exactly $\alpha = -1$ and $\nu = 1$ \cite{8}. In the context of superconductors, these exponents are obtained in a Hartree approximation in external field but with the gauge field fluctuations being neglected \cite{9}. Indeed, the Hartree approximation consists of a gap equation analogous to the one obtained in the large $N$ limit of the $O(N)$ model. The critical exponents $\alpha = -1$ and $\nu = 1$ can be obtained also through a weakly interacting Bose gas model \cite{10}. In this case the specific heat exhibits a triangular peak at zero field characteristic of a Bose-Einstein condensation (BEC).

In this paper we will show that the critical crossovers observed in both YBCO and BSCO are described by two different scalings in the dual version of the Ginzburg-Landau model \cite{7,11,12,13} (to be referred from now on as the dGL model). The description of the $X_Y$ scaling of YBCO by the dGL model is well known \cite{13,14}. However, we want to emphasize the fact that two different crossovers are obtained here from a same model. We provide in this way an unifying view for the YBCO and BSCO critical behaviors.

The dGL model was introduced in the literature as a continuum version of the lattice dual Ginzburg-Landau (GL) model \cite{15}. The dGL model is therefore a way towards a field theoretical description of the inverted $X_Y$ transition \cite{16}. The inverted $X_Y$ scenario must be valid at least for superconductors in the type II regime and ensures the existence of a charged infrared stable fixed point for the GL model \cite{17,18}. One important difference of the inverted $X_Y$ relative to the ordinary $X_Y$ universality class is the scaling $\lambda \sim \xi^{\nu'/\nu}$, where $\xi$ is the correlation length. This scaling implies that $\nu' = \nu$ instead of $\nu' = \nu/2$ as in the ordinary $X_Y$ fixed point. Recent high precision Montecarlo simulations give further support to the duality scenario and to the existence of the inverted $X_Y$ critical point \cite{19,20}. Unfortunately, the critical region corresponding to this nontrivial charged fixed point is still experimentally out of reach.

The bare free energy density for the dGL model is given by

$$F = \frac{1}{2} \left( (\nabla \times h_0)^2 + M_0^2 h_0^2 \right) + |(\nabla - ie_0 d h_0) \psi_0|^2$$

$$+ \frac{\hbar_q^2}{2} |\psi_0|^2 + \frac{\hbar q^2}{2} |\psi_0|^4,$$

where the constraint $\nabla \cdot h_0 = 0$ should be understood. The bare dual charge $e_{0,d} = 2\pi M_0/\hbar q$ where $q_0 = 2e_0$ is the Cooper pair charge. Also, $M_0^2 = q_0^2 |\psi_0|^2$ where $\phi_0$
is the bare order parameter field of the GL model. Thus, $M_0$ is the photon mass generated in the Meissner phase of the GL model. The Meissner phase of the GL model is described as a symmetric regime in the dGL model. In this sense, the dGL model is a disorder field theory.

Due to the presence of two masses in the problem, we have more possibilities of scalings in the dGL model than in the GL model. A class of such scalings was discussed recently by us [3].

The scaling corresponding to the $XY_3$ behavior of YBCO is obtained by looking to the flow of the renormalized dimensionless couplings $f = e_d^2/\mu$ and $g = u/\mu$ with respect to $m$ at $M_0$, $u_0$ and $c_{0,d}$ fixed. The renormalized parameters are defined in a usual way [12]. They are given by $m^2 = Z_m^{-1}Z_\psi m_0^2$, $M^2 = Z_h M_0^2$, $e_d^2 = Z_h e_{0,d}^2$ and $u = Z_u^2 u_0/Z_u$. Such a scaling has been considered before in Refs. [15, 27] and for this reason we will omit the details. In this scaling the infrared stable fixed point corresponds to a nonzero $f$. As a consequence, the anomalous dimension $\eta_m = m_0 \ln Z_h/\partial m$ is given at the nontrivial fixed point by $\eta_m = 1$ exactly [15, 27]. Since $M_0$ is fixed it follows the scaling $M \sim m^{1/2}$ near the critical point. Thus, it follows that $\nu = \nu/2$ and the ratio $m/M \to 0$ as $m \to 0$. From this last observation we obtain that the gauge degrees of freedom decouple and the corresponding fixed point is $XY_3$. We have therefore that $\nu \approx 2/3$, $\nu' \approx 1/3$ and $\alpha \approx 0$, which are just the measured values in the bulk YBCO with optimal doping [15, 27].

For the scaling corresponding to the BSCCO critical behavior it is more convenient to choose $M$ as the running scaling variable. The BSCCO scaling is defined by demanding that the ratio $m_0/M_0$ is kept fixed together with $e_0$ and $u_0$ [28]. We assume therefore that both $m_0^2$ and $M_0^2$ are proportional to the reduced temperature $t$. In this scaling the renormalized quantities are defined as before and the corresponding renormalization constants are denoted by $Z_\psi$, $Z_h$, $Z_m$ and $Z_u$. The dimensionless couplings are now defined by $f = Z_\psi e_{0,d}^2/M$ and $g = Z_u^{-1} Z_\psi u_0/M$. It is convenient to define also the dimensionless parameter $\kappa_d = m/M$ which plays a role analogous to the Ginzburg parameter. Note that fixing $e_{0,d}$ is equivalent to consider that the amplitude fluctuations are frozen in the original GL model. We define the renormalization group (RG) functions:

$$\tilde{\eta}_\psi = M \frac{\partial \ln Z_\psi}{\partial M},$$

(2)

$$\tilde{\eta}_h = M \frac{\partial \ln Z_h}{\partial M},$$

(3)

$$\tilde{\eta}_m = M \frac{\partial \ln Z_m}{\partial M}.$$  

(4)

We have the following exact flow equations:

$$M \frac{\partial f}{\partial M} = (\tilde{\eta}_\psi - 1) f,$$

(5)

$$M \frac{\partial m^2}{\partial M} = (2 + \tilde{\eta}_\psi - \tilde{\eta}_m - \tilde{\eta}_h)m^2,$$

(6)

$$M \frac{\partial M_0^2}{\partial M} = (2 - \tilde{\eta}_h)M_0^2.$$  

(7)

A nontrivial fixed point with $\tilde{f}$, $\tilde{g}$ and $\kappa_d$ nonzero must verify the equations $\tilde{\eta}_h = 1$, $\tilde{\eta}_\psi = \tilde{\eta}_m = 1$ and $M \partial \tilde{g}/\partial M = 0$. If such a fixed point exists, we obtain from Eqs. (7) and (8) that $\nu' = \nu = 1$ and $\alpha = -1$ from the scaling relation $3\nu = 2 - \alpha$, which are the required exponents. Note that by assuming the existence of the nontrivial fixed point we have that the values of the exponents are exact and are the same as in the BEC transition. The existence of this nontrivial fixed point can be verified approximately through a simple 1-loop example. We have at 1-loop order,

$$\tilde{\eta}_\psi = -\frac{2}{3\pi} 1 + \kappa_d,$$

(9)

$$\tilde{\eta}_h = \frac{\tilde{f}}{24\pi \kappa_d},$$

(10)

$$\tilde{\eta}_m = -\frac{\tilde{g}}{4\pi \kappa_d},$$

(11)

$$M \frac{\partial \tilde{g}}{\partial M} = (2\tilde{\eta}_\psi - 1)\tilde{g} + \frac{\tilde{g}^2}{8\pi} + \frac{\tilde{f}^2}{2\pi},$$

(12)

where in neglecting higher order terms we have assumed $\kappa_{d,0} = m_0/M_0 \approx 1$. This assumption does not affect the generality of the problem though higher values of $\kappa_{d,0}$ lead to a worse result from the point of view of convergence. The numerical nontrivial fixed point values are $\kappa_d^* \approx 0.34$ and $\tilde{g}^* \approx 21.4$. The numerical flow diagram in the $\tilde{g}\kappa_d$-plane is shown in Fig. 1. This result can be improved further by computing higher order terms and using resummation methods. Although we have shown the validity of the proposed scaling through perturbative methods, we claim that this scenario holds also beyond perturbation theory. As mentioned earlier, the present scaling with $e_{0,d}$ and $m_0/M_0$ fixed corresponds to freeze the amplitude fluctuations in the original GL model. Numerical simulations with frozen amplitudes but taking into account the phase fluctuations can be used as a final test of the above picture.

Note that the scaling we have just discussed predict the value $\nu' = 1$ for the penetration depth exponent. This prediction implies the scaling $\lambda \sim \xi$ which is verified also in the vicinity of the inverted $XY_3$ critical point.
It would be very interesting to verify this prediction experimentally and also by numerical means.

Summarizing, we have considered two different scalings in the dGL model, each one corresponding to a different critical crossover. The first one gives the $XY_3$ scaling behavior which is probed experimentally in single crystals of YBCO while the second one reproduces approximately the scaling behavior of BSCCO. Moreover, we predict the value $\nu' = 1$ for the BSCCO penetration depth exponent. The results of this paper show the power of the dual approach. Indeed, all the possible crossovers obtained within the dGL model may be very difficult to obtain in the original GL model. The point is that a weak coupling regime in the dGL model corresponds to a strong coupling regime in the GL model. In this case nonperturbative effects become important.

Among the further directions is the inclusion of anisotropy and more general disorder parameters with higher symmetries like $d + is$ wave and $SO(5)$ symmetries. It would be also interesting to study other novel situations which occur in external field like the $\Phi$-transition of Téšanović. Finally, we hope that the present contribution will stimulate some experimental efforts towards the verification of the prediction $\nu' = 1$.

We would like to thank Z. Téšanović for bringing Ref. 3 to our attention and A. Sudbø for sending us Ref. 26 prior publication. We would like to thank specially Prof. A. Junod for his valuable comments on the experimental results in YBCO and BSCCO.

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FIG. 1. Flow diagram in the $\tilde{g}r_d$-plane.