On the Momentum Dependence of the Nucleon - Nucleus Optical Potential

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The momentum dependence of the mean-field contribution to the real part of the optical model potential is investigated employing realistic nucleon-nucleon interactions. Within a non-relativistic approach a momentum dependence originates from the non-locality of the Fock exchange term. Deducing the real part of the optical model from a relativistic Dirac Brueckner Hartree Fock approximation for the self-energy of the nucleons yields an additional momentum dependence originating from the non-relativistic reduction of the self-energy. It is demonstrated that large Fock terms are required in the non-relativistic approach to simulate these relativistic features. A comparison is made between a local density approximation for the optical model and a direct evaluation in finite nuclei.

I. INTRODUCTION

The nucleon - nucleus optical potential is a very important tool for the analysis of nuclear reaction data. Therefore a lot of empirical information on nucleon - nucleus scattering for a large spectrum of energies and various nuclei has been collected and the global fits to these optical potentials [1–3] provide a very condensed information on nuclear structure properties. It is obvious that theoreticians have devoted much interest to the study of this important quantity. Rather than trying to give a short overview on such investigations we refer to some recent review articles on this subject [4–7].

From a theoretical point of view the real part of the optical potential can be split into a mean-field contribution and a dispersive correction [8]. The name dispersive contribution originates from the fact that this real part is related to the imaginary part of the optical potential by a dispersion relation. This dispersion relation yields an explicit energy-dependence also for the real part. In principle, the dispersive contribution could be determined from the empirical data on the imaginary component. This would require the knowledge of the imaginary part at all energies. For large energies, however, the imaginary part gets contributions from nucleonic excitations, due to the strong short-range components of a realistic NN interaction [9], and the excitation of sub-nucleonic degrees of freedom, like particle production, which is beyond the scope of the present discussion. Therefore applications of such an dispersive optical-model analysis typically consider only the imaginary part at low energies [10] and use some kind of subtracted dispersion relation [8].

If one tries to describe the remaining mean-field contribution of the optical potential in terms of a local potential to be used in a Schroedinger equation, one observes a significant dependence of this mean-field part on the energy or asymptotic momentum of the scattered nucleon [9,11]. There are two possible sources for this momentum dependence. Within a non-relativistic approach one may attribute this momentum dependence to the non-locality of the exchange part in the Hartree-Fock (HF) or Brueckner-Hartree-Fock (BHF) approximation for the mean-field part of the optical potential. Transforming this non-local HF potential to an equivalent local potential leads to such a momentum dependence [9,12] (see also chapter 2).
Another source for the momentum-dependence of the mean-field part of the optical potential is of relativistic origin. Relativistic approaches have been very successful to describe bulk properties of nuclei. This is true for more phenomenological models, like the $\sigma - \omega$ model of Walecka and Serot [13], but also for microscopic Dirac-BHF calculations based on realistic nucleon-nucleon (NN) forces [14-17]. Characteristic for all these calculations is the relativistic structure of the nucleon self-energy, which contains a large Lorentz-scalar and a large time-like vector component. In a simple Dirac-Hartree calculation these components are local and energy independent. Reducing the two coupled Dirac equations for the upper and lower component to one Schroedinger equation, one derives from these scalar and vector components a central Schroedinger equivalent potential (SEP), which depends on the energy or momentum of the nucleon [18].

The scalar and vector components of the relativistic self-energy obtained in a Dirac-HF or Dirac-BHF calculation are non-local. Therefore both sources for the momentum dependence of the mean-field part of the optical model discussed above get relevant. It is one aim of the present investigation to explore the importance of these two effects and their interplay. This study is based on the self-consistent Dirac-BHF calculations, which were recently performed directly for finite nuclei [19,20].

There have been many very successful attempts to deduce the mean field part of the optical potential directly from the NN T-matrix or G-matrix [1] within a non-relativistic [21-24] and relativistic framework [25-27]. The impulse approximation which is used in most of these investigations is rather successful in particular for proton scattering at energies above 100 MeV. Medium effects are typically derived from nuclear matter and taken into account in a local density approximation [25].

The main emphasis of the present investigation is to study the differences and similarities in non-relativistic and relativistic approaches in particular at lower energies. These energies are important for the extrapolation of the single-particle potential at negative energies, connecting the optical potential to the binding properties. For that purpose we consider simple parameterizations of G-matrix approaches, derived from realistic NN forces. In the case of the non-relativistic approach this will be the so-called M3Y parameterization [29]. In the case of the relativistic treatment this will be the effective meson parameterization of [19,20]. This enables us to study and compare the origin of the momentum dependence obtained in the non-relativistic and relativistic approach in detail. Furthermore we would like to compare the results for nucleon-nucleus scattering obtained from a non-local Dirac self-energy which is calculated directly for the finite nuclei to results which are obtained in the frequently used local-density approximation.

After this introduction we will briefly recall the connection between the non-locality and the momentum dependence of an equivalent local mean field in section 2. In section 3 we will shortly describe the momentum dependence of the SEP derived from a local Dirac self-energy. There we will also describe an efficient method to calculate scattering phase shifts from a non-local relativistic self-energy. The results of our investigation are presented and discussed in section 4 and the conclusions are summarized in a final section 5.

**II. NON-LOCALITY AND EXCHANGE EFFECTS**

More than 15 years ago Bertsch et.al. determined a very efficient parameterization of an effective NN interaction from the G-matrix evaluated for realistic forces [29]. These so-called M3Y potentials are given in terms of local Yukawa potentials. The central part of the interaction can be written as a function of the distance $r$ between the interacting nucleons

$$V_{\text{M3Y}} = V_0(r) + V_\tau(r)\vec{\tau}_1 \cdot \vec{\tau}_2 + V_\sigma(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_{\tau\tau} \vec{\tau}_1 \cdot \vec{\tau}_2 \vec{\sigma}_1 \cdot \vec{\sigma}_2.$$  \hspace{1cm} (1)

The operators $\vec{\tau}_i$ and $\vec{\sigma}_i$ denote the Pauli matrices acting on the isospin and spin of nucleon $i$, respectively. The radial shapes of the functions $V_\alpha(r)$ are given in terms of 2 Yukawa potentials

$$V_\alpha(r) = \sum_{k=1}^{2} C^{(k)}_\alpha \frac{\exp (-\mu_k r)}{\mu_k r},$$  \hspace{1cm} (2)
only for the spin-isospin channel $V_{\sigma\tau}$ a third Yukawa term has been added, which represents the central component of the pion-exchange potential

$$\Delta V_{\sigma\tau} = \frac{1}{\lambda} \frac{g_\rho^2}{4\pi} \frac{m_\rho^2}{m_\pi^2} \frac{\exp(-\mu_\pi r)}{\mu_\pi r}$$

Here $\mu_\pi = m_\pi/(hc) = 0.7072$ fm$^{-1}$ and $C_\pi = 3.4877$ MeV, which corresponds to a pseudoscalar coupling constant for the pion of $g_\rho^2/(4\pi) \approx 13.9$. Assuming for the mass parameter $\mu_k$ in Eq.(3) values of 4 fm$^{-1}$ and 2.5 fm$^{-1}$, for $k = 1$ and 2, respectively, the 8 strength parameter $C_\alpha^{(k)}$ were adjusted to reproduce G-matrix elements of the Reid potential [32] and the Elliot potential [31] in an oscillator basis. The resulting parameters are listed in table 1.

For the mean-field contribution of the optical potential for scattering of nucleons on closed shell nuclei with N=Z only the spin- and isospin averaged part of the NN interaction is relevant. For the direct interaction leading to the Hartree part of the potential this is identical to the scalar isoscalar term in Eq.(1)

$$V_{\text{dir}} = V_0(r)$$

$$= 7999.0 \frac{\exp(-4r)}{4r} - 2134.25 \frac{\exp(-2.5r)}{2.5r}.$$  (4)

The resulting direct contribution to the optical potential is given by

$$U_{\text{dir}}(r) = \int d^3r' \rho_0(r') V_{\text{dir}}(|r - r'|)$$  (5)

with the nuclear density $\rho_0(r)$ = $\rho(r, r)$ obtained in the Hartree-Fock approximation

$$\rho(r, r') = \sum_{i=1}^{N} \Phi_i^*(r) \Phi_i(r').$$  (6)

Here $\Phi_i$ stands for the single-particle wavefunctions occupied in the Hartree-Fock approximation for the target nucleus.

The exchange part of the NN interaction is evaluated as the scalar isoscalar part of the Fierz transformation [32] of Eq.(1) and is given as

$$V_{\text{exc}} = -\frac{1}{4} [V_0(r) + 3V_\sigma(r) + 3V_\rho(r) + 9V_{\sigma\tau}(r)]$$

$$= 4631.375 \frac{\exp(-4r)}{4r} - 1787.125 \frac{\exp(-2.5r)}{2.5r} - 7.8474 \frac{\exp(-0.7072r)}{0.7072r}.$$  (7)

This local parameterization of the G-matrix and slight modifications of it have been used for various nuclear studies including the evaluation of the optical potential for heavy-ion scattering [33]. The exchange part of the NN interaction yields the Fock contribution to the mean field which is non-local

$$U_{\text{exc}}(r, r') = \rho(r, r') V_{\text{exc}}(|r - r'|)$$  (8)

The local equivalent potential is defined by the requirement

$$U_{\text{exc}}^{\text{loc}}(r) \Psi_E(r) = \int d^3r' U_{\text{exc}}(r, r') \Psi_E(r') ,$$  (9)

where $\Psi_E(r)$ is the solution of the Schroedinger equation for a nucleon at energy $E$ moving in the single-particle potential $U = U_{\text{dir}} + U_{\text{exc}}$. Negele and Vautherin [34] demonstrated that Eq.(8) can well be approximated by

$$U_{\text{exc}}^{\text{loc}}(r) = \int d^3r' V_{\text{exc}}(|r - r'|) \rho(r, r') \frac{\sin k(r') |r - r'|}{k(r') |r - r'|} ,$$  (10)

with the local approximation to evaluate $k$.
\[ k^2(r) = \frac{2m}{\hbar^2} [E - U(r)] \]  

It is evident from Eq. (11) that the local equivalent potential depends on the energy \( E \) or the momentum \( k(\infty) \) of the scattered nucleon. In Eq. (11) this momentum-dependence is hidden in the local momentum \( k(r) \), which must be calculated in a self-consistent way from Eq. (11).

The mixed density \( \rho(r, r') \) entering the exchange part of the mean field potential is frequently approximated by the so-called Slater approximation obtained in nuclear matter

\[ \rho(r, r') = \rho_b \left( \frac{r + r'}{2} \right) \frac{3}{|\vec{r} - \vec{r}'|} |\vec{r} - \vec{r}'| k_F, \]

with the Spherical Bessel function \( j_1(x) \) and the local Fermi momentum, which is related to the baryon density \( \rho_b \) at the central point between \( \vec{r} \) and \( \vec{r}' \) by the nuclear matter relation

\[ k_F^3 = \frac{3\pi^2}{2} \rho_b \]  

III. DIRAC EQUATION AND SCHROEDINGER EQUIVALENT POTENTIAL

A simple parameterization of Dirac-Brueckner-Hartree-Fock (DBHF) calculations for nuclear matter employing realistic NN interactions has been presented in Ref. [19,20]. For each density \( \rho_b \) two coupling constants for a scalar \( (G_{\sigma}) \) and vector meson \( (G_{\omega}) \) exchange have been determined such that a simple Dirac Hartree calculation within the \( \sigma - \omega \) model reproduces the bulk properties of nuclear matter at this density obtained from DBHF. The density dependence of the coupling constants reflects the density dependence of the correlations taken into account in DBHF.

Using these density dependent coupling constants one can determine the nucleon self-energy in a Dirac-Hartree (DH) approximation

\[ \Sigma_{\text{DH}}^s = \Sigma_{\text{DH}} - \gamma^0 \Sigma_{\text{DH}}^0 \]  

with a scalar and vector contribution given by

\[ \Sigma_{\text{DH}}^s(r) = -G_{\sigma}(r)m_{\sigma} \int_0^\infty r'^2 dr' G_{\sigma}(r') \rho_s(r') I_0(m_{\sigma}r_<) \bar{K}_0(m_{\sigma}r_>), \]  

\[ \Sigma_{\text{DH}}^0(r) = -G_{\omega}(r)m_{\omega} \int_0^\infty r'^2 dr' G_{\omega}(r') \rho_b(r') I_0(m_{\omega}r_<) \bar{K}_0(m_{\omega}r_>). \]  

In these equations the coupling constants \( G_{\sigma}(r) \) and \( G_{\omega}(r) \) denote the density dependent coupling constants discussed above calculated at the nuclear density which is equal to the baryon density \( \rho_b \) of the nucleus at the position \( r \). The meson mass parameters were chosen to be \( m_{\sigma} = 550 \text{ MeV} \) and \( m_{\omega} = 783 \text{ MeV} \) in accordance with the realistic OBE potentials [14]. The functions \( I_L(x) \) and \( \bar{K}_L(x) \) arise from the multipole expansion of the meson propagator in coordinate space and are defined using the modified spherical Bessel functions \( I \) and \( K \) [35]:

\[ I_L(x) = \frac{I_{L+1/2}(x)}{\sqrt{x}} \quad \bar{K}_L(x) = K_{L+1/2}(x). \]  

The baryon density \( \rho_b \) and the scalar density \( \rho_s \) are defined by

\[ \rho(r) = \frac{1}{4\pi} \sum_a (2j_a + 1) \left[ g_a^2(r) + f_a(r)^2 \right] \]

\[ \rho_s(r) = \frac{1}{4\pi} \sum_a (2j_a + 1) \left[ g_a^2(r) - f_a(r)^2 \right], \]  

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with a summation index \( a \) ranging over all occupied shells. The functions \( g_a \) and \( f_a \) refer to the small and large components of the Dirac spinor

\[
< r | \alpha > = \Psi_\alpha (\vec{r}) \equiv \left( \begin{array}{c} g_a (r) \\ -i f_a (r) \end{array} \right) Y_{l_a, m_a} (\Omega),
\]

(19)

using the notation of Ref. [20]. The Dirac spinors are obtained by solving the Dirac equation including the nucleon self-energy defined in Eq. (14). The definition of the self-energy and the solution of the Dirac equation are connected by a self-consistent requirement. Note, however, that in the present model, the baryon density \( \rho_b \) obtained from solving the Dirac equation enters the definition of the self-energy not only as displayed explicitly in Eqs. (15) and (16) but also via the density dependent coupling constants.

The contributions to the self-energy \( \Sigma^s_{DH} \) and \( \Sigma^0_{DH} \) are local in the Dirac Hartree approximation. The Dirac equation corresponds to two coupled equations for the large \( (g) \) and small \( (f) \) component of the resulting Dirac spinor. One can eliminate the small component in a standard way and obtain a Schroedinger equation for a wavefunction which at large distances agrees with the large component of the Dirac spinor [18,36]. The central part of the potential in this Schroedinger equation, called the central part of the Schroedinger Equivalent Potential (SEP) can be written

\[
U_{SEP}(r) = \Sigma^s(r) - \frac{\tilde{E}}{m} \Sigma^0 (r) + \frac{(\Sigma^s(r))^2 - (\Sigma^0 (r))^2}{2m} + U_{\text{Darwin}}(r)
\]

(20)

with

\[
U_{\text{Darwin}}(r) = \frac{3}{4} \left( \frac{1}{D(r)} \frac{d^2 D(r)}{dr^2} \right)^2 - \frac{1}{r D(r)} \frac{d D(r)}{dr} - \frac{1}{2 D(r)} \frac{d^2 D(r)}{dr^2},
\]

\[D(r) = m + E + \Sigma^s (r) + \Sigma^0 (r).\]

(21)

Therefore we see that this SEP depends on the energy of the scattered nucleon \( E = \tilde{E} - m \) or its asymptotic momentum. This is true already if we employ the Dirac - Hartree approximation, i.e. deal with local functions for the scalar and vector component of the nucleon self-energy.

If the self-energy is evaluated in the Dirac-Hartree-Fock approximation, the contributions are non-local. In order to evaluate the scattering observables obtained from a solution of the Dirac equation with such a non-local self-energy, we use the technique already described in Ref. [20]. In this method the radial functions \( g_a (r) \) and \( f_a (r) \), which define the stationary solutions of the Dirac equation (see Eq. (19)) are expanded in a discrete basis of spherical Bessel functions. The wave numbers for this basis are chosen such that this discrete basis is a complete orthonormal basis in a sphere of radius \( D \). This radius is chosen to be large enough that the results for the wavefunctions obtained at positive energies show the asymptotic form at \( r \approx D \). A typical value for \( D \) is \( D = 20 \) fm for the light nuclei being considered here. With this expansion for the wavefunctions the Dirac equation is rewritten in form of an eigenvalue problem and the eigenvalues \( (E_\nu) \) and eigenvectors (the expansion coefficients for \( g_a \) and \( f_a \)) are determined by a simple matrix diagonalisation. The expressions for the matrixelements of this matrix to be diagonalized are published elsewhere [20,37].

Due to the choice of the basis for the expansion (Bessel functions, which vanish at \( r = D \): \( j_l(k_\nu D) = 0 \)) also the wavefunctions resulting from this diagonalisation must vanish at \( r = D \). This implies that we obtain results for the bound single-particle states and for those selected energies of scattering states \( E_\nu \) for which the wavefunction vanishes at \( r = D \). Assuming that the wavefunction has reached its asymptotic form, see discussion above, we can deduce the phase shifts \( \delta_l \) at these selected energies from the relation

\[
g_\nu (D) = \cos \delta_l j_l(k_\nu D) - \sin \delta_l y_l (k_\nu D) = 0,
\]

(22)

where \( y_l \) stands for the Neumann function and the asymptotic wavewumber \( k_\nu \) is related to the energy \( \tilde{E}_\nu \) by

\[
\tilde{E}_\nu = E_\nu + m = \sqrt{k_\nu^2 + m^2}.
\]

(23)
This method for the calculation of scattering phase shifts is a relativistic extension of the scheme employed in Refs. [38,39] for non-local potentials in a Schrödinger equation.

Also in the case of the Dirac-Hartree-Fock approximation one can try to find a local potential which used in a Schrödinger equation yields the same result for the scattering phase shifts as obtained from the Dirac-Hartree-Fock approximation. It is evident that this SEP will exhibit an energy dependence due to the Dirac effects but also due to the non-local Fock exchange terms as discussed in the previous section.

IV. RESULTS AND DISCUSSION

A parameterization of the Dirac-Brueckner-Hartree-Fock G-matrix calculated in nuclear matter for the version A of the Bonn OBE potential [14] has been presented in Ref. [20]. Employing this parameterization of the G-matrix in terms of a density-dependent $\sigma$ and $\omega$ meson exchange in Dirac-Hartree calculation of finite nuclei, results were obtained for the binding energy and radius of closed shell nuclei like $^{16}$O and $^{40}$Ca, which were in fair agreement with the experimental data [13,40]. The baryon densities and scalar densities obtained in such calculations for $^{16}$O and $^{40}$Ca are displayed in figures 1 and 2, respectively. These density distributions will be the starting point of our further investigations.

As a first approach we consider the scalar $\Sigma_{\text{DH}}$ and vector $\Sigma_{\text{DH}}^0$ contribution to the nucleon self-energy in the Dirac-Hartree approximation, which are related to these densities via the relations Eqs.(15) and (16). From these local scalar and vector contributions one can calculate a local SEP as discussed in the preceding section (see Eq.(20)). These local SEP are displayed in the lower parts of figures 1 and 2 for two different energies of the scattered nucleon. The attraction of the SEP for nucleons with an energy of 100 MeV is reduced by more than a factor 2 as compared to the SEP for nucleons at an energy of 5 MeV.

This energy dependence of the SEP, which is of course well known, is very important to obtain predictions for the elastic scattering which are in agreement with the experimental data. As an example we show in the upper parts of figures 3 and 4 results for the $l = 0$ phase shifts for elastic scattering of nucleons at various energies on $^{16}$O and $^{40}$Ca. As a bench-mark, representing the experimental data, we also show in these figures the phase-shifts obtained from a fit to the experimental data [3]. An acceptable agreement with these “experimental” values for the phase shifts is obtained, if we evaluate the phase shifts in the Dirac-Hartree approximation or, equivalently, consider the SEP for each energy consistently. If, however, the SEP calculated for a nucleon energy of 5 MeV would be used for the scattering of nucleons at higher energies as well, one obtains a substantial deviation (see dashed curves in the upper parts of figures 3 and 4).

As an alternative approach we now consider the non-relativistic parameterization of the G-matrix provided by the M3Y approach [29]. It should be mentioned, however, that other non-relativistic parameterizations show similar features. As we want to explore the difference between a relativistic and non-relativistic evaluation of the mean-field contribution to the optical potential, independent on the underlying baryon density, we use the same density distribution as in the Dirac-Hartree approach. For these densities we determine the scattering potential, including the exchange term in the local form of Eq. (10) with the Slater approximation of Eq.(12) for the mixed density $\rho(r,r')$.

The resulting local potentials are also displayed in figures 1 and 2 (dashed curves) again for nucleons with energies of 5 and 100 MeV. The shapes of the M3Y potentials deviate from those obtained in the Dirac-Hartree approach in a distinct manner. The M3Y potentials tend to be less attractive at the surface and more attractive close to the center. The energy dependence of these local potentials, however, seems to be quite similar to the one obtained in the Dirac-Hartree approach.

This is supported by the inspection of the scattering phase shifts in the lower parts of figures 3 and 4. Also for the M3Y potentials we obtain an acceptable agreement with the “experimental” data if the energy dependence of the local potential is taken into account. If, however, a scattering potential determined for an energy of 5 MeV is used for the scattering of nucleons with higher energies as well, the calculated phase shifts are far above the experimental at energies around 100 MeV.
The results obtained for the phase shifts display a remarkable similarity comparing the Dirac-Hartree and the M3Y approach. Similar results are also obtained for other angular momenta and the total cross section. For angular momenta \( l \) different from zero, and consequently also for the total cross section, the Dirac-Hartree approach tends to give slightly larger results. This can be related to the larger radii of the Dirac-Hartree potentials.

At this stage one may expect that a combination of the energy dependence in the local potential due to the Dirac effects and due to the Fock exchange effects, as considered in a Dirac-Hartree-Fock calculation should lead to an energy dependence of the calculated phase shifts, which is larger than in the Dirac-Hartree or in the non-relativistic Hartree-Fock approximation and therefore exceeds the energy dependence of the empirical data.

This, however, is not the case. To demonstrate this, we use the Dirac-Hartree-Fock parameterization of the G-matrix evaluated in nuclear matter again for version \( A \) of the OBEP (including \( \pi \) and density dependent \( \sigma \) and \( \omega \) exchange). We take the same density distributions for \(^{16}\text{O}\) and \(^{40}\text{Ca}\) as discussed above. (The self-consistent Dirac-Hartree-Fock densities deviate from the Dirac-Hartree densities used here and are not considered as we would like to study the differences in the self-energies which are not due to different densities.) For these ingredients one can calculate the non-local Dirac-Hartree-Fock self-energy and evaluate the phase using the technique discussed in section 3.

Results for the \( l = 0 \) phase shifts are displayed in figure 5 for the case of the nucleus \(^{40}\text{Ca}\). These phase-shifts are rather close to those obtained in the Dirac-Hartree approximation. In fact the energy dependence obtained in the Dirac-Hartree-Fock approximation is even slightly weaker than the one obtained in the Dirac-Hartree calculation.

Figure 5 also shows results for the scattering phase shifts calculated in a local density approximation (LDA) directly from the nucleon self-energy in nuclear matter \(^{28}\). This means that one determines the different components of the self-energy for various densities of nuclear matter and identifies the local self-energy at position \( \vec{r} \) with the corresponding nuclear matter result calculated for the baryon density at this position. For the scalar part of the self-energy this means

\[
\Sigma_{\text{LDA}}^s(r, E) = \Sigma_{\text{Nuc.Mat.}}^s(\rho(r), k) \tag{24}
\]

where \( \Sigma_{\text{Nuc.Mat.}}^s(\rho, k) \) is the scalar part of the nucleon self-energy calculated in nuclear matter of density \( \rho \) for a nucleon with momentum \( k \), which is equal to the asymptotic momentum of the scattered nucleon. Equivalent expressions define the other components of the self-energy. Inserting this expression into Eq. \( \tag{20} \) one obtains the SEP in this local density approximation. The phase shifts can be calculated either directly from the solution of the Dirac equation with a self-energy according to Eq. \( \tag{24} \) or by solving a Schroedinger equation for the SEP.

From figure 5 we see that the LDA yields results which are in good agreement with the phase shifts calculated in Dirac-Hartree and Dirac-Hartree-Fock approximation. Very similar agreement is also obtained in the case of the nucleus \(^{16}\text{O}\), which is not presented here. In particular the energy or momentum dependence of the SEP seems to be contained already in the nuclear matter approximation. This can also be seen from the plot of the SEP in figure 2. The LDA yields predictions for the local equivalent potential, which are in very good agreement with the Dirac-Hartree results. In particular the energy dependence is well reproduced. The radial shapes deviate in a characteristic way. The LDA reflects the fluctuations in the baryon density more drastically and exhibits a smaller radius than the Dirac-Hartree approximation. This is due to the suppression of finite range effects in the nucleon-nucleon interaction, which is characteristic for local density approximations \(^{28}\).

From this comparison one can see that the discussion of nuclear matter results using a local density approximation yields valuable information on general features of finite nuclei, like the momentum dependence of the optical model. It must be kept in mind, however, that the local-density approximation in general is not very reliable in predicting specific observables with high accuracy \(^{22}\). As an example we would like to mention that the LDA overestimates the calculated binding energy per nucleon by 3.2 MeV and 2.3 MeV for \(^{16}\text{O}\) and \(^{40}\text{Ca}\), respectively. Furthermore it yields smaller radii for the ground states of these nuclei than the corresponding direct calculation for the finite nuclei (see also figure 2 and discussion of the SEP above).

In our investigation we find that the energy- or momentum-dependence of the optical potential can be described within a non-relativistic as well as in a relativistic framework. This is the same situation as in the empirical approaches to describe nucleon-nucleus scattering \(^{1}\). The physical origin of the momentum-dependence of the local equivalent
potentials displayed in figures 1 and 2 is quite different in the two approaches. Within the non-relativistic description, the momentum dependence is determined by the strong Fock exchange term in the M3Y parameterization. If one calculates the binding of nuclear matter with this effective interaction (see Eq.(1)), one finds that the dominating contribution to the potential energy arises from the Fock exchange terms. This is displayed in figure 6, where the direct Hartree contribution to the potential energy of nuclear matter is compared to the total Hartree-Fock result for various densities. One finds that the Hartree term only yields about 25 percent of the total attraction.

These strong exchange terms are required to provide a sufficient momentum dependence for the optical potential. In order to identify the most important contribution to this momentum dependence figure 7 shows the momentum dependence of the single-particle potential in nuclear matter including the various components of the M3Y interaction. As our discussion of the local density approximation above demonstrated that the main ingredients of the energy-dependence are already obtained for nuclear matter, we will concentrate our discussion on nuclear matter. From the analysis displayed in figure 7 it is obvious that the momentum dependence is dominated by the components of medium range in the exchange part of the M3Y interaction. The pion-exchange contribution is much weaker and the repulsive short-range components increase the attraction for larger momenta. This medium range component in Eq.(6) is mainly due to $V_\tau$ and $V_{\sigma\tau}$. Translated into the language of mesons this means that it is due to the exchange of an isovector meson with a mass of around 500 MeV. As such a meson does not exist, we must consider this as an effective meson to parameterize the effects of correlations or other features contained in the G-matrix.

It is very difficult to decide whether such strong exchange effects required in the M3Y or other non-relativistic parameterizations are realistic or just an artefact of this parameterization. The only information available from NN scattering data and microscopic calculations of NN interactions is the information about the sum of direct and exchange term and therefore the separation into a direct and an exchange term is to some extent model-dependent. There are, however, some indications that such strong exchange effects do not show up within the relativistic framework.

It is worth noting that in relativistic parameterizations of the G-matrix, the self-energy is dominated by the direct Hartree-contributions. This is valid for the Dirac-Hartree-Fock parameterization of [20], which we have discussed so far. The same is also true for the more complete parameterization of the G-matrix derived from the Bonn potential of [41] and for the independent analysis of the group in Groningen [17,42]. The Fock- or exchange-terms yield corrections to the Hartree terms, which are as large as 30 percent of the Hartree term. However, one does not meet a situation that the exchange terms are more important as it is the case in the non-relativistic framework discussed above.

This is also reflected by the fact that the momentum or energy-dependence of the SEP derived from the Dirac-Brueckner-Hartree-Fock self-energy in nuclear matter is dominated by the energy dependence displayed directly in Eq.(20). This is reflected in figure 8, where the momentum dependence of the SEP in nuclear matter is displayed with (solid lines) and without (dashed lines) taking into account the momentum dependence of the Dirac self-energy. Both, for the Bonn potential and for the independent analysis of the Groningen group, one observes that the momentum dependence of the Dirac self-energy, which reflects the Fock exchange effects, is a small correction and tends to reduce the momentum dependence of the SEP.

V. CONCLUSIONS

Results for the real part of the optical potential for nucleon - nucleus scattering at low energies ($E < 100$ MeV) are discussed within a non-relativistic framework and the relativistic Dirac-Hartree-Fock approximations. The investigations are based on relativistic and non-relativistic parameterizations of a G-matrix derived from realistic NN interactions. Both approaches lead to local energy-dependent potentials, which, employed in a Schroedinger equation, reproduce the bulk features of the energy dependence of the empirical data for nucleon scattering on $^{16}O$ and $^{40}Ca$. However, for the same density distribution of the target nucleons different local equivalent potentials are obtained.

A local density approximation (LDA) contains the main features of the energy dependence of the optical potential. The LDA fails to reproduce the precise shape of the potentials leading to smaller radii. Also it does not provide accurate results for the binding energy.
Both kinds of approaches yield a similar energy- or momentum dependence. Therefore, analogous to the empirical studies of nucleon nucleus scattering, also the present investigation cannot give a definite answer to the question whether relativistic features are needed to describe these processes. The origin of the energy dependence is quite different in the two approaches. Within the non-relativistic scheme strong Fock exchange terms are required to obtain an energy dependence of the optical model as required from the empirical data. In the relativistic framework the energy dependence is due to the reduction of the Dirac equation to a Schroedinger equation and Fock exchange terms play a minor role. Therefore one may understand the strong exchange effects of the non-relativistic approach as a tool to incorporate the relativistic features.

It should be useful to study the optical potential for the elastic scattering of heavy ions \[33\] to investigate whether the empirical data for these systems allow to distinguish between the relativistic and the non-relativistic approach.

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TABLE I. Parameterization of the different central terms of the M3Y potential (see Eq.(1)), using the parameterization of Eq.(2). Note that for the $\sigma\tau$ channel the pion contribution of Eq.(3) has to be added.

| $k$ | $\mu_k$ [fm$^{-1}$] | $C_{0}^{(k)}$ [MeV] | $C_{\tau}^{(k)}$ [MeV] | $C_{\sigma}^{(k)}$ [MeV] | $C_{\sigma\tau}^{(k)}$ [MeV] |
|-----|---------------------|---------------------|---------------------|---------------------|---------------------|
| 1   | 4.0                 | 7999.00             | -4885.50            | -2692.25            | -421.25             |
| 2   | 2.5                 | -2134.25            | 1175.50             | 478.75              | 480.00              |
FIG. 1. Density distribution and optical potential for $^{16}O$. The density distribution displayed in the upper part of the figure has been obtained from the Dirac Hartree calculation of Ref.[20]. For this distribution the local equivalent optical potential of nucleon scattering have been calculated for energies of 5 MeV and 100 MeV, using the Dirac Hartree (solid line) and the non-relativistic folding potential calculated for the M3Y potential (dashed curve).

FIG. 2. Density distribution and optical potential for $^{40}Ca$. The lower part of the figure also displays the local density approximation to the optical potential defined in Eqs.(24) and (20). Further details see figure 1.
FIG. 3. Phase shifts for \( l = 0 \) scattering on \(^{16}\text{O}\) at various energies of the scattered nucleon. Results of the Dirac Hartree calculation are displayed in the upper part while those obtained from the non-relativistic M3Y potential are given in the lower part of the figure. Beside the results of the consistent calculations (solid lines) also phase-shifts obtained for the equivalent local potential at a fixed energy of 5 MeV (dashed line) are presented. For comparison also the phase-shifts obtained from the empirical fit of Ref.[3] are displayed as “experimental” data.

FIG. 4. Phase shifts for \( l = 0 \) scattering on \(^{40}\text{Ca}\) at various energies. Further details see figure 3.
FIG. 5. Calculated $l = 0$ phase shifts for scattering on $^{40}Ca$. Results are displayed for the Dirac-Hartree, the Dirac-Hartree-Fock and the local density approximation for the Dirac-Hartree approach.

FIG. 6. Contributions to the binding energy of nuclear matter at various densities calculated from the direct part of the M3Y interaction (Hartree) and from the direct plus exchange part (Hartree-Fock). The kinetic energy is not included.
FIG. 7. Momentum dependence of the single-particle potential in nuclear matter at saturation density \((k_F = 1.36 \text{ fm}^{-1})\) calculated for the M3Y potential. Results are displayed including the total exchange part of the interaction (solid line), the medium and short-range part (dashed line) and the short-range part only (line with circle labels). Displayed is the energy difference to the potential energy at momentum \(k = 0\) in each case.

FIG. 8. Momentum dependence of the Schroedinger equivalent potential (SEP) calculated according to Eq.(20) in nuclear matter at saturation density \((k_F = 1.36 \text{ fm}^{-1})\). Results are displayed which account for the momentum dependence of the scalar and vector terms in the self-energy (solid lines) and those for which the scalar and vector terms calculated at momentum \(k = k_F\) were taken for all momenta (dashed lines). Results derived from the Bonn A potential (Ref.[20]) are compared to those of the Groningen group (Ref.[17], lines with circle labels). Displayed is the energy difference to the potential energy at momentum \(k = 0\) in each case.
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