Evaluation of Plain River Network Hydrologic Connectivity Based on Improved Graph Theory

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Abstract. Based on graph theory, the river network was generalized by a graph model. Considering the influence of water quantity on the hydrologic connectivity of river network, the connectivity factor reflecting water quantity index was taken as the weighted value, the quantitative evaluation model of hydrologic connectivity degree was set up. By calculating the river network weighted connectivity degree with MATLAB, the quantitative evaluation of river network hydrologic connectivity was realized. Taking the Qinhuai River Basin as an example, the HEC-HMS hydrological model was established for the simulation of short-term and long-term flood. The connectivity level of the river network in the basin under different scale floods was graded by the quantitative evaluation results of connectivity degree. The results showed that the constructed evaluation model has good applicability and reliability.

1. Introduction
Changes in hydrologic connectivity pattern will affect the water cycle path and regulatory capacity of the river, thus affect the play of hydrologic connectivity functions[1]. The evaluation method of hydrologic connectivity has undergone a process from qualitative analysis to quantitative analysis. More and more attention has been paid to the research of water system connectivity evaluation in our country with the increasing demand of flood control and disaster reduction, water resource allocation and water environment improvement.

However, the study in this aspect is still in its infancy. Guanglai Xu[2] constructs the degree of patency matrix by taking the reciprocal of the hydraulic resistance as the weight of the edge and took the average of the degree of patency at all the vertexes as the weighted connectivity degree of the river network; Xing Chen[3] applies the graph theory separately to quantitatively evaluating the structural connectivity and hydraulic connectivity of the Yanjingwei Plain River Network, in Changshu City before and after river system planning; Erős[4] searches key river sections for protection and environmental management based on network analysis function of the graph theory, and using graph theory to quantify the connectivity of freshwater ecosystem habitats further, proving that graph theory can be used for the protection of freshwater ecological resources; Segurado[5] uses the relevant theory of graph theory to determine the obstacles that affect the structural connectivity of the basin, and then judges which obstacles should be cleared firstly to improve the overall connectivity more effectively. In this paper, the connectivity degree reflects the degree of connectivity of plain river network based on the improved graph theory.

2. Establishment of Evaluation Model

2.1. Fundamentals
Based on the graph theory, the river network graph model $G(V, E)$, which can be represented by an adjacency matrix $A(a_{ij})_{non}$, $a_{ij}$ is the number of edges directly connected from $V_i$ to $V_j$. According to the judgement criteria, if all of the elements of the judgment matrix $S$ are not zero, the graph $G$ is a connected graph, otherwise it is a non-connected graph.

$$S(s_{ij})_{non} = \sum_{k=1}^{n-1} A^k(a_{ij}^{(k)})$$

\[ (1) \]

2.2. Establishment of the Evaluation Model

The existing graph theory can only reflect whether the river channels are connected with each other, it can not reflect the dynamic transfer capacity of water quantity among river networks, and can not analyze the impact of water quantity on hydrologic connectivity either. To make up the above deficiencies, the improved graph theory was proposed in the paper (Figure 1).

2.2.1. Selection of the water quantity index. There are many indexes to characterize the water quantity in a river channel, such as runoff $Q$, total runoff volume $W$, runoff modulus $M$, runoff depth $R$, runoff coefficient $\alpha$ and so on[6]. Runoff coefficient $\alpha$ can indicate the relative relationship between rainfall and runoff, and can comprehensively reflect the impact of various natural factors on the runoff formation. Therefore, it is more practical to select runoff coefficient $\alpha$ as the index to reflect water quantity in a river.

2.2.2. Weight of the edge. It can avoid the problem that the complicated calculation process and the long calculation time by using runoff coefficient $\alpha$, which can be used as the connectivity factor to characterize the weight of the river network model. Equation (3)~(5) are the calculation process of the connectivity factor $b_{ij}$.

$$W_i = \int_0^T Q \, dt$$

$$R_i = \frac{W_i}{A}$$

\[ (3) \]

\[ (4) \]
Where: $W_i$ is total runoff volume at the node of the river network, $m^3$; $\Delta t$ is the study period, h; $Q_i$ is the instantaneous runoff, $m^3 \cdot s^{-1}$; $R_i$ is runoff depth, mm; $A$ is the area of the study area, $km^2$; $P_i$ is rainfall depth over the period, mm. It is determined by Thiessen polygons method, which assumes that precipitation in each sub-basin is represented by the nearest rainfall station.

### 2.2.3 Calculation of weight connectivity degree.

The connectivity degree of each vertex is expressed by the average of all connected paths between $V_i$ and $V_j$, and the equation is:

$$b_{ij} = \frac{R_i}{P_i}$$

(5)

Where: $s_{ij}$ is the sum of the connected paths whose lengths are 1, 2, ... $n$-1 from $V_i$ to $V_j$; $t_{ij}$ is the sum of the connected factors whose lengths are 1, 2, ... $n$-1 from $V_i$ to $V_j$.

The overall hydrologic connectivity degree is calculated as the mean of the average connectivity degree of all the vertices. The equation is:

$$Z = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} D_{ij}}{n^2}$$

(7)

### 3. Case Study

#### 3.1. Study Area

![River distribution of Qinhuai River Basin](image)

Figure 2. River distribution of Qinhuai River Basin

Qinhuai River Basin is located in the downstream of the Yangtze River in Jiangsu Province with an area of 2631 km². There are two sources of Qinhuai River, north source is the Jurong River, and south source originates the Lishui River in the Dong Lu mountain. The two rivers converge in the northwestern village of Jiangning District and become the main stream of the Qinhuai River. When flowing to Dongshan Town, they flow into two streams. The north branch flows along the main river
channel in Nanjing, outflowing from Wuding Gate. The west branch is Qinhuai New River, who outflows from the Qinhuai New River Gate. Both of them flow into Yangtze River[7] (Figure 2).

3.2. Data Processing
The water system was extracted and the study area was divided into 18 sub-basins based on GIS. The Thiessen polygon was established by the distribution of rainfall stations in the basin (Figure 3). Due to the deviation between the water system extracted based on GIS and the actual river network, the extracted river network was imported into Google Earth for correction, the digital water system graph was obtained and further generalized to 25 main points river network topology graph (Figure 4).

3.3. Flood Process Simulation
HEC-HMS hydrological model can be used to simulate the short-term and long-term flood processes in the Qinhuai River basin. The relative error of peak discharges and total runoff volumes were both less than 20%, the Nash coefficient and correlation coefficient were both greater than 0.8, and the Nash coefficient of long-term flood processes were all above 0.7, so this model can meet accuracy requirements[8]. The comparison between simulated values and measured values of some flood processes were shown in Figure 5.
Figure 5. Comparisons between simulated and measured values of some flood processes

Based on the improved graph theory method, the overall connectivity degree of the basin during some rainfall periods are as shown in Table 1.

Table 1. The overall connectivity degree of the basin during some rainfall periods

| Flood Processes | Simulated Values of Peak Discharge (m$^3$·s$^{-1}$) | Simulated Values of Total Runoff Volume (mm) | Nash | Degree |
|-----------------|---------------------------------|---------------------------------|------|--------|
| Short-term Floods | 199603 169.4 | 19.15 | 0.758 | 0.0013 |
| 198908 782.7 | 138.04 | 0.888 | 0.0066 |
| 199106 1336.9 | 400.53 | 0.751 | 0.0083 |
| 198707 910.2 | 564.72 | 0.705 | 0.0195 |
| Long-term Floods | 200306 1376 | 803.46 | 0.784 | 0.0327 |
| 199105 2058.7 | 1314.92 | 0.753 | 0.0545 |
| 1991 2140.6 | 1827.39 | 0.702 | 0.0617 |

4. Results and Discussions

4.1. Relationship between Total Runoff Volume and the Connectivity Degree

As shown from Table 1, the connectivity degree tends to increase with the increase of volume, and a quantitative relationship in line with the two can be studied further. Taking total runoff volume as independent variable, connectivity degree as dependent variable, they were basically in line with the linear relationship. If confidence level $\alpha$ is 0.05 and degree of freedom is 5, the critical value $r_{\text{critical}} = 0.754$, $r > r_{\text{critical}}$, indicating that there is a significant linear relationship between the two variables[9].

4.2. Classification of Hydrologic Connectivity

The level of hydrologic connectivity is divided into four classes, which are better, good, general and bad respectively in this paper. The better class indicates the level of hydrologic connectivity is high, which can meet demands of industry, agriculture and ecology. Other classes' level of connectivity are low relatively.

During the study period, a flood disaster broke out in 1991, taking the connectivity degree as the highest and using it as the standard value[10]. The evaluation criteria corresponding to different classes are shown in Table 2.

Table 2. The evaluation criteria corresponding to different classes

| Classes         | Evaluation Criteria                                                                 |
|-----------------|--------------------------------------------------------------------------------------|
| better          | the standard value×70%≤the overall connectivity degree≤the standard value              |
| good            | the standard value×50%≤the overall connectivity degree < the standard                |
Yu Huaizhi. Characteristics in the basin, which therefore, the evaluation results are reasonable.

4.3. The Reliability Proven of the Constructed Evaluation Model

According to the evaluation criteria proposed in this paper, the classification of simulated short-term floods are bad. The reason may be that although peak discharge is large, the water quantity is small due to the short period of time, which can not meet the water needs of industry and agriculture. Therefore, the evaluation results are reasonable.

5. Conclusions

Based on the evaluation model proposed in this paper, it proved that volume was positively correlated with the connectivity degree. Under the premise of meeting the needs of urban water supply, agricultural irrigation and ecological replenishment, the river channel can ensure the smooth flow if it can storage water to an extreme. Therefore, this model was of great guiding significance for the optimal dispatch of water in a basin.

Based on the corresponding evaluation criteria, it was of great significance to popularize the model to evaluate hydrologic connectivity of different scales floods in other basins. The corresponding standard values can be determined according to the hydrological characteristics in the basin, which was convenient and applicable.

The standard value of connectivity was determined at the annual scale in this paper. So, we can only evaluate the hydrologic connectivity under the annual scale of the river basin. The connectivity degree and evaluation of hydrologic connectivity under the longer time scale needs to be further studied.

6. References

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