RESISTIVE MAGNETIC FIELD GENERATION AT COSMIC DAWN

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Received 2010 June 23; accepted 2011 January 6; published 2011 February 10

ABSTRACT

Relativistic charged particles (CRs for cosmic rays) produced by supernova explosion of the first generation of massive stars that are responsible for the reionization of the universe escape into the intergalactic medium, carrying an electric current. Charge imbalance and induction give rise to a return current, \( j_r \), carried by the cold thermal plasma which tends to cancel the CR current. The electric field, \( E = \eta j_r \), required to draw the collisional return current opposes the outflow of low-energy CRs and ohmically heats the cold plasma. Owing to inhomogeneities in the resistivity, \( \eta(T) \), caused by a structure in the temperature, \( T \), of the intergalactic plasma, the electric field possesses a rotational component which sustains Faraday’s induction. It is found that a magnetic field is robustly generated throughout intergalactic space at a rate of \( 10^{-17} \) to \( 10^{-16} \) G Gyr\(^{-1} \), until the temperature of the intergalactic medium is raised by cosmic reionization. The magnetic field may seed the subsequent growth of magnetic fields in the intergalactic environment. The role of CR-driven instabilities is discussed, and nonlinear effects are briefly considered.

Key words: intergalactic medium – large-scale structure of universe – magnetic fields – methods: analytical – methods: numerical – plasmas

Online-only material: color figures

1. INTRODUCTION

Massive stars characterized by high emission of ionizing UV photons are the main contributors to the process of cosmic reionization (e.g., Ciardi & Ferrara 2005). At the end of their life they explode as supernovae (SNe) accelerating, large amounts of cosmic-ray (CR) protons (Krymsky 1977; Axford et al. 1977; Bell 1978; Blandford & Ostriker 1978). Since dynamo generation of magnetic fields operates very quickly within stars, stars can magnetize their surroundings through their own magnetized stellar winds and even a significant fraction of the parent galaxy itself (Rees 2006). Diffusive shock acceleration around SNe, therefore, can take place even without a pre-existing galactic magnetic field. Furthermore, only small magnetic fields are needed for supernova remnant (SNR) shocks to accelerate CRs to the GeV energies required for the process considered in the paper. Owing to their much higher energy and diffusive free path compared to thermal particles, CRs eventually escape from the parent galaxy into the intergalactic medium. Their escape may or may not be collimated by the galaxy disk (e.g., by the magnetic field in the wind around stars or a pre-existing galactic field if it exists). In any case, the CR protons carry a small but important electric current \( j_r \) which couples them to the thermal intergalactic medium and induce magnetic field generation there. The phenomena described in this paper depend on the electrical current carried by the CR. Most of the current is carried at GeV energies, so our discussion does not depend upon CR acceleration to TeV or PeV energies. Also, we neglect CR electrons as they are typically two orders of magnitude less numerous than protons (Schlickeiser 2002) and suffer much more severe energy losses (even more so at high redshift when the cosmic microwave background (CMB) energy density was much higher, e.g., by a factor of \( 10^4 \) at redshift 9), although the injection process is not completely understood and the CR proton to electron ratio could in principle change at high redshift.

The Larmor radius of a proton of momentum \( p \) and rest mass \( m \) in a magnetic field \( B \) is \( r_L = (p/mc)(B/10^{-15} \text{ G})^{-1} \) kpc, where \( c \) is the speed of light. In our discussion, we will consider fields up to \( 10^{-15} \) G and proton energies up to 1 GeV, so the magnetic field will place very little restriction on intergalactic CR propagation. Even a spatial diffusivity \( D \) unrealistically as small as the Bohm diffusivity \( D_B \sim c r_g/3 \), in which the mean free path is set equal to the Larmor radius \( r_g \), would allow GeV protons to diffuse several Mpc in a \( 10^{-15} \) G field after 1 Gyr. The magnetic field can only restrict CR propagation through the intergalactic medium if the field exceeds \( 10^{-15}(D/D_B)^{1/2} \) G. \( D \) is probably as small as \( D_B \) close to a shock during CR acceleration, but in other astrophysical circumstances we expect \( D \gg D_B \) (Dimitrakoudis et al. 2009; Dröge & Kartavykh 2009). Since we only calculate fields up to \( 10^{-16} \) G, we expect CR transport not to be inhibited by the magnetic field in the intergalactic medium. Whether in the freely streaming or mildly diffusive regime, though, the CR current remains basically unaltered and determined, on average, solely by the CR production rate inside the parent galaxy. If the CR propagation velocity is reduced by diffusion, the CR number density will correspondingly increase to maintain on average a constant CR flux and prevent CR accumulation in space. Interparticle collisions also place no restriction on propagation, since the Coulomb collision time for GeV protons in a characteristic plasma density \( \sim 10^{-4} \text{ cm}^{-3} \) is \( \sim 10^3 \) Gyr.

CR propagation within galaxies is not necessarily as clear cut, since it is possible that the magnetic field, although largely unknown, may conceivably be much larger than in the intergalactic medium as a result of processes associated with stars and SNe. However, CRs appear to diffuse freely around our own galaxy with present magnetic fields in the \( \mu \text{G} \) range. Even at energies in the GeV range, the presumption is that the galactic CR spectrum is determined by a balance between CR production by SNR and escape from the galaxy (Hillas 2005). Moreover, there is relatively direct evidence that even in the large
magnetic field close to the center of our galaxy CR diffusion, far exceeding Bohm diffusion, is relatively free (Dimitrakoudis et al. 2009). One other possible process which might inhibit CR propagation is the excitation of streaming instabilities by the CR. While this will be considered retrospectively toward the end of the paper in Section 7, after the physical conditions have been set out in intervening sections, we anticipate here that such instabilities are qualitatively not important for the process described in this paper.

In this paper, we show that the electric currents carried by CR protons produced in the first generation of galaxies that are also responsible for the reionization of the universe are sufficiently large to generate significant magnetic fields throughout intergalactic space. The magnetic field is generated by the curl of the electric field associated with the return currents induced by the CR particle propagation. This process operates most efficiently while the intergalactic medium is cold, and effectively shuts down once reionization raises the intergalactic gas temperature above $10^4$ K, i.e., when the universe was about 1 Gyr old (Ciardi & Ferrara 2005). The generated magnetic field is typically of order several $10^{-17}$ G at the end of reionization. This magnetic field may eventually be further enhanced by a turbulent dynamo both in the shocked intergalactic medium (Ryu et al. 2008) and in galaxies (Arshakian et al. 2009), although CR currents may play a further role in magnetic field amplification through Lorentz’s force (Bell 2004, 2005).

Magnetic fields are observed in most astrophysical bodies from planetary scales, to stars and galaxies, and up to the largest structures in the universe (Zel’dovich et al. 1983; Kulsrud & Zweibel 2008). Nearby galaxies have been known to be magnetized for some time, but recent studies show that they acquire their fields when the universe was less than half its present age (Bernet et al. 2008; Kronberg et al. 2008). Since the growth of a large-scale magnetic field in a dynamo model is exponential in time, this poses serious restrictions on the timescale on which galactic-dynamo must operate.

Evidence for intergalactic magnetic fields is provided by Faraday rotation measure and diffuse synchrotron radiation which reveal the existence of $\mu$G strong magnetic fields in the hot plasma of galaxy clusters (Clarke et al. 2001; Carilli & Taylor 2002). These probes are considerably less sensitive to fields in adjacent structures such as small groups and filaments of galaxies. Nevertheless, these fields are expected to be there and large efforts are being made in an attempt to measure them. In addition, intergalactic magnetic fields affect the propagation of ultra-high-energy CR particles, introducing potentially measurable effects on their energy spectrum, arrival direction, and composition (Sigl et al. 2004; Dolag et al. 2005; Hooper & Taylor 2010). Finally, the presence of magnetic fields in voids can affect the observed spectrum of extragalactic TeV $\gamma$-ray sources. Multi-TeV photons are absorbed by the diffuse extragalactic background light and converted into $e^\pm$ pairs which emit secondary cascade multi-GeV $\gamma$-rays by inverse Compton on the CMB. The observed flux of pair produced GeV photons can be suppressed if the pairs are deflected from the line of sight by a magnetic field within an energy loss distance (Aharonian et al. 1994; Nerlonov & Semikoz 2009). Based on these ideas, in very recent studies the $\gamma$-ray spectra of blazars were used to set lower limits to the value of magnetic fields in voids in the range $5 \times 10^{-15}$–$5 \times 10^{-17}$ G (Neronov & Vovk 2010; Tavecchio et al. 2010).  

The presence of magnetic fields in astrophysical plasma requires a mechanism for their generation because, given the high mobility of the charges, sustained electrostatic fields are non-trivial to set up. However, even a weak seed may suffice because magnetic field amplification can occur at the expense of the plasma motions through the induced electric field, $\vec{E} = -\vec{v} \times \vec{B}$. Specific lower limits on the required seed strength depend on the astrophysical systems.

There already exist various scenarios for the generation of magnetic fields each with its strengths and weaknesses; among others are Weibel’s instability at shocks (Medvedev et al. 2006; Schlickeiser & Shukla 2003; Medvedev 2007), battery effects, e.g., due to Compton drag (Harrison 1970; Ichiki et al. 2006) or Biermann’s mechanism either at cosmic shocks or ionization fronts (Subramanian et al. 1994; Kulsrud et al. 1997; Gnedin et al. 2000), galactic winds (Beck et al. 1996; Bertone et al. 2006; Donnert et al. 2009), and various processes in the early universe (Widrow 2002). In fact, the magnetization of cosmic space is complex and various processes are likely to contribute to it, though to a different extent in different environments. One of the strengths of the model we describe below is that it generates relatively strong macroscopic fields throughout cosmic space, while having a simple description based on well-understood physical processes.

The remainder of this paper is organized as follows. In Section 2, we introduce the resistive mechanism for the generation of a magnetic field. In Section 3, we describe a cosmological simulation to determine some of the parameters describing the intergalactic medium that are required to quantify the generation of magnetic field. In Sections 4 and 5, we discuss the generation of magnetic field around individual galaxies using a numerical and slightly simplified analytical solutions, respectively. The solution is extended to the case of the intergalactic space in Section 6. In Section 7, we consider the effect of streaming instabilities. Finally, we briefly summarize the main findings of this paper in Section 8.

In the following, we use SI units and, unless explicitly stated, lengths are expressed in physical units, not comoving units.

2. RESISTIVE MECHANISM

Though as discussed in the previous section, CR propagation in intergalactic space is unaffected by Coulomb collision and magnetic fields deflections, CRs are not completely decoupled from intergalactic plasma. This is due to the small current, $\vec{j}$, carried by the CR flux, which is assumed mainly to consist of protons. In fact, to maintain quasi-neutrality the CR current must be balanced by a return current carried by the thermal plasma $\vec{j}_i$. While the CRs are collisionless with very long mean free paths, the thermal particles, due to their low energy, have mean free paths shorter than scale lengths of interest here. For example, at redshift $z \approx 10$, when the intergalactic plasma had a temperature $\sim 1$ K and density $\sim 10^{-4}$ cm$^{-3}$, thermal electrons had a Coulomb mean free path of only $10^3$ m$^{-1}$, where $\psi$ is the fractional ionization. Consequently, an electric field in the rest frame of the plasma, $E'$, is required to draw the return current, namely

$$E' = \eta \frac{\vec{j}_i}{},$$  \hspace{1cm} (1)

where $\eta$ is the plasma resistivity. Since Coulomb collisions dominate over charge–neutral collisions, the resistivity takes approximately the Spitzer value, $\eta = 65 T^{-3/2} \log \Lambda \Omega m$, where $T$ is the temperature of the ambient thermal plasma in K, and $\log \Lambda \simeq 20$ is the Coulomb logarithm. The Spitzer resistivity

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3 Incidentally, these results appeared after the first submission of this paper.
is independent of electron number density and consequently independent of the degree of ionization.

The electric field transfers energy from the CR flux to the thermal plasma through ohmic heating, described by the equation

\[ \frac{3}{2} n k_B \frac{dT}{dt} = \eta j_c^2, \]

(2)

where \( k_B \) is the Boltzmann constant. Since the charge–neutral collision time is much smaller than the expansion time of the universe, in Equation (2) \( n \) is the total number of particles in the plasma, to include heating of neutral as well as ionized particles. As a result of ohmic heating the temperature of the initially cold thermal plasma is raised.

The electric field also opposes the current carried by the CRs. In particular, CRs escape the parent galaxy to a distance \( R \) only if their kinetic energy at source exceeds the potential at \( R \), \( \phi = \int_0^R |E|dR \). This can determine the minimum energy of CR reaching \( R \), giving

\[ p_{\text{min}} = \frac{e\phi(R)}{c} \left( 1 + 2 \frac{mc^2}{e\phi(R)} \right)^{1/2}. \]

(3)

As will be seen in Figure 3, the electric potential is not sufficient to limit the escape of mildly relativistic protons from the galaxy for the cases considered here, largely because we assume that gas within 100 kpc of a galaxy is hot (see below), and therefore has a low resistivity which results in a low electric field even though the CR current is relatively large close to the galaxy.

In a uniform medium, the electric field drawing the return current \( j_c \) cannot have a curl and therefore there is no generation of magnetic field. If the medium is non-uniform on a given scale, the forward and return currents can become separated on that scale producing a current loop which in turn supports a magnetic field. However, the separation of forward and return currents, and the corresponding magnetic field, is opposed by inductive effects, and the actual growth of the magnetic field must be determined by a self-consistent solution of the Maxwell equations

\[ \nabla \times \vec{B} = \mu_0 (\vec{j}_c + \vec{j}_i), \]

(4)

\[ \frac{\partial \vec{B}}{\partial t} = - \nabla \times \vec{E}. \]

(5)

The displacement current can be neglected because timescales are much longer than the light transit time and the system evolves through a series of quasi-neutral steady states. Using Ampère's law (Equation (4)) to eliminate the thermal current we find that the electric field (Equation (1)) in the frame in which the plasma moves at velocity \( \vec{v} \) is \( \vec{E} = - \vec{v} \times \vec{B} + (\eta/\mu_0) \nabla \times \vec{B} - \eta \vec{j}_c \). The curl of this electric field then produces the growth of the magnetic field according to Faraday's law:

\[ \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \nabla \times \left( \frac{\eta}{\mu_0} \nabla \times \vec{B} \right) + \nabla \times (\eta \vec{j}_c). \]

(6)

The first term on the right-hand side transports the frozen-in field with the plasma and can stretch and amplify an already existing magnetic field. The second term represents resistive diffusion and can easily be verified to be insignificant for kpc distances and Gyr timescales. Crucially, in contrast, the final resistive term of the equation produces the magnetic field in a previously unmagnetized plasma. The magnetic field grows wherever the resistivity varies perpendicularly to the CR current. A larger electric field is needed to draw the return current where the resistivity is higher, so the electric field has a curl and magnetic field grows. These equations are well known in laser-produced plasmas where the current is carried by energetic laser-produced electrons in place of SNe-produced CR protons (Bell & Kingham 2003).

Temperature inhomogeneities, resulting in variations in conductivity, are naturally present throughout intergalactic plasma due to the growth of cosmological structure. In addition, because the volume heat capacity is proportional to density, where ohmic heating is significant, inhomogeneous temperature enhancements will arise from density inhomogeneities. On the other hand, the CR current is expected to be approximately uniform because as discussed in previous sections, CRs are relatively undeflected by collisions or magnetic fields in the intergalactic medium. Thus, the magnetic field may be generated by the resistive term at a rate

\[ \dot{B} = [\vec{j}_c \times \vec{V}_i] \sim \frac{j_c \eta}{L_T}. \]

(7)

where \( L_T \equiv T/|\nabla T| \) is the characteristic temperature scale. Here, we are concerned only to estimate the magnitude of the field produced by a locally uniform CR current \( \vec{j}_c \) without considering the feedback of the field on the CR flux. In this approximation, we assume that the relative orientation of \( \vec{j}_c \) and \( \nabla \eta \) is uncorrelated.

It is clear from the above expression that the resistive process depends sensitively on the plasma temperature through \( \eta \). In particular, it operates efficiently while the intergalactic medium is cold, and it effectively shuts down once reionization raises the intergalactic gas temperature above \( 10^4 \) K. For a universe that is reionized at redshift \( z \geq 6 \) (Ciardi & Ferrara 2005), the available time is of order a Gyr. Note that close to the star-forming galaxies the gas temperature is \( \geq 10^4 \) K due to both photoheating from the ionizing flux escaping the galaxies and the ohmic heating. For the above choice of the reionization epoch, the typical size of H\textsc{ii} regions at \( z \simeq 10 \) is about 100 kpc (or 1 comoving Mpc; Zahn et al. 2007).

In order to quantify the importance of the resistive process in the following, we first use a numerical simulation of structure formation to compute the scale of the temperature variations in the intergalactic medium. We then estimate the generation of magnetic field around individual star-forming galaxies at the epoch of reionization, including self-consistently the effect of the induced electric fields on the escaping CR particles and ohmic heating. Finally, we estimate the distribution of magnetic fields produced in intergalactic space.

3. COSMOLOGICAL SIMULATION

We extract characteristic scale length \( L_T \) and the characteristic range of values taken by \( L_T \) from a cosmological simulation of structure formation which includes hydrodynamics, dark matter, and gravity. The simulation does not include the CR effects discussed in this paper. The simulation uses a directionally unsplit higher order Godunov’s method for the hydrodynamics, a time-centered-modified symplectic scheme for the collisionless dark
important ionization radiation flux prior to reionization, collisions provide the most equilibrium. Note that, lacking any ionizing background computed by solving the system of ionization equations assuming equilibrium. These include colli-
sional excitation/ionization and recombination cooling from the various ionization species of H and He, bremsstrahlung and inverse Compton on the CMB. We describe these processes with the rates provided in Hui & Gnedin (1997). The abundance of the various ionization species (H$_i$, H$_{ii}$, He$_i$, He$_{ii}$, He$_{iii}$) is computed by solving the system of ionization equations assum-
ing equilibrium. Note that, lacking any ionizing background radiation flux prior to reionization, collisions provide the most important ionization/excitation process.

We adopt a flat ΛCDM universe with the following parameters normalized to the critical value for closure ($\rho_c = 3H_0^2/8\pi G$, with $H_0$ being the Hubble constant and $G$ Newton’s constant): total mass density, $\Omega_m = 0.2792$, baryonic mass density, $\Omega_b = 0.0462$, and vacuum energy density, $\Omega_{\Lambda} = 1 - \Omega_m = 0.7208$ (Komatsu et al. 2009). In addition, the normalized Hubble parameter is $h = H_0/100$ km s$^{-1}$ Mpc$^{-1} = 0.701$, the spectral index of primordial perturbation is $n_s = 0.96$, and the rms linear density fluctuation within a sphere with a comoving radius of 8 h$^{-1}$ Mpc is $\sigma_8 = 0.817$ (Komatsu et al. 2009). We generate the initial conditions with the grafic2 package (Bertschinger 2001). We use a computational box of comoving size $L = 1$ h$^{-1}$ Mpc, discretized with 512$^3$ comoving computational cells, providing a nominal spatial resolution of 2 h$^{-1}$ comoving kpc for the field components, and 512$^3$ particles with mass $4.8 \times 10^2$ h$^{-1}$ $M_\odot$ for the collisionless dark matter component. At redshift $z = 6$, the box size in physical units is $\simeq 143$ h$^{-1}$kpc and, likewise, the nominal spatial resolution is $\simeq 285$ h$^{-1}$pc. Thus, structures with spatial scales between 1 and 10 kpc should be adequately resolved for our purposes.

Figure 1 shows the distribution of the baryonic gas density at $z \simeq 10$ on a two-dimensional slice across the simulation box. One can recognize a few high density collapsed structures where gas is rapidly cooling. These are the sites where stars and CRs are eventually produced. However, most of the gas ($\sim 97\%$ by mass) is still in the diffuse phase, with a density within a factor a few of the mean value. Figure 2 shows the occurrence of spatial scale lengths in the range 1–10 kpc in the low density, low temperature gas which occupies most of the intergalactic space. The most important mechanism determining the temperature gradients relevant for the resistive process is the growth of structure due to gravitational instability and the induced adiabatic compression of the gas sitting in the potential well of those structures. Larger gradients may be present on smaller scales where structures have collapsed and shock heating and radiative cooling are effective. However, these concern only a small fraction of the baryonic gas and cosmic space and are, therefore, do not characterize the resistive generation of intergalactic magnetic fields.

Note that the residual fraction of free electrons in the intergalactic medium after recombination keeps the gas temperature $T$ locked to the CMB temperature, $T_{\text{CMB}} = 2.725(1+z)$ K, until redshift $1 + z = 142(\Omega_b h^2/0.024)^{2/3} \simeq 140$ (Peebles 1993).
CRs with momentum greater than $p_{min}$ reach a radius $R$, the current carried by CRs at that radius is approximately given by $j_c = 5 \times 10^{-10} Q_c (p_{min}/m_p c)^{-0.3}/(p_{min}/m_p c + 1) \text{ A m}^{-2}$, where $m_p$ is the proton mass. Taking the efficiency of CR production to be fixed at $\epsilon_c = 0.3$:

$$j_c \approx 5.3 \times 10^{-14} \left( \frac{L}{L_\odot} \right) \left( \frac{R}{\text{kpc}} \right)^{-2} \left( \frac{p_{min}}{m_p c} \right)^{-0.3} \times \left( 1 + \frac{p_{min}}{m_p c} \right)^{-1} \text{ A m}^{-2}. \tag{8}$$

This CR current must be balanced by an equal but opposite return current drawn by an electric field $E = \eta j_c$, giving

$$E \approx 6.9 \times 10^{-11} \left( \frac{L}{L_\odot} \right) \left( \frac{R}{\text{kpc}} \right)^{-2} \left( \frac{p_{min}}{m_p c} \right)^{-0.3} \times \left( 1 + \frac{p_{min}}{m_p c} \right)^{-1} \left( \frac{T}{K} \right)^{-3/2} \text{ V m}^{-1}. \tag{9}$$

So, we can now solve numerically the coupled system of Equations (2), (3), (8), and (9) to determine the propagation of CRs through the intergalactic medium surrounding a star-forming galaxy, the associated electric field, and the ensuing generation of magnetic field. Our "standard" calculation is as follows. We treat the galaxy as a spherically symmetric source of CR which is constant in time. We set an inner radial boundary to the computational grid at $r_{min} = 10 \text{ kpc}$ which corresponds to the notional radius of the galaxy. We assume that all CRs produced by the galaxy escape through this boundary for the reasons given in Section 1. As will become clear below, changing $r_{min}$ has very little effect on the results for the intergalactic medium on Mpc scales. The CR flux is specified at the inner boundary as a power law ($\propto p^{-2.3}$) with a minimum momentum of $0.1 m_p c$ which corresponds to a proton velocity a few times the shock velocity of an expanding young SNR. We set the initial intergalactic temperature to $2 \times 10^5$ K within a radius of 100 kpc to represent heating due to ionization by the galaxy. Outside this radius we set the initial temperature to 1 K, with the temperature changing between the two regions over a distance of 10 kpc. We set the intergalactic density to $10^{-25}$ cm$^{-3}$.

### 4.1. CR Propagation and Magnetic Field Generation

Figure 3(a) shows the results for our standard calculation, as defined above, for CR propagation and plasma heating as a function of radius for a galaxy with typical luminosity $L = L_\odot$ at time $t = 1 \text{ Gyr}$. The horizontal scale is logarithmic and stretches from the edge of the galaxy at 10 kpc to a distance of 10 Mpc which is the characteristic of the distance between bright galaxies at this epoch. Ohmic heating is effective out to a radius of $\sim 3$ Mpc where the electric and magnetic fields reach their maximum values. The magnetic field is less within this radius because ohmic heating reduces the resistivity and the growth of magnetic field. Within 100 kpc the magnetic field is much smaller because the temperature is initialized to a much larger value to represent ionization by the galaxy. The magnetic field is largest at $\sim 3$ Mpc, but the maximum is broad, and fields above $10^{-7}$ G extend from 100 kpc to 10 Mpc. Figure 3(b) displays the profiles at the earlier time of 0.1 Gyr for the same parameters. The fields are slightly reduced but still reach $2.5 \times 10^{-7}$ G, once again with a broad maximum. This shows that most of the field generation occurs quickly before ohmic heating reduces.
the resistivity. Fields in the range $10^{-17}$ to $10^{-16}$ G are produced even if suitable conditions for growth apply for much less than 1 Gyr.

We take Figure 3(a) as the standard calculation and then vary individual parameters in Figures 3(c)–(f). Fields of a similar magnitude are produced if the initial temperature is raised to 10 K (Figure 3(c)), the CR luminosity of the galaxy is reduced tenfold (Figure 3(d)), the density is increased to $10^{-3}$ cm$^{-3}$ (Figure 3(e)), or the minimum CR energy is increased to $p_{\text{min}} = m_p c$ (Figure 3(f)). Clearly, the production of fields in the range $10^{-17}$ to $10^{-16}$ G is robust. The results indicate that fields as large as $\sim 10^{-20}$ T ($10^{-16}$ G) are produced in a few to several $\sim$ Mpc$^3$ volume surrounding galaxies if the temperature or density varies on kpc scales.

The plots of the electric potential $\phi$ in Figure 3 show that the electric field does not inhibit the escape of CRs from the galaxy.

5. ANALYTIC SOLUTION FOR A FIXED CURRENT

For the cases considered above, the electric potential does not inhibit CR transport. If the galaxy can be assumed to produce CRs at a constant rate, the CR current is constant at any point outside the galaxy. The evolution of the temperature and magnetic field at that point is then determined by the equations

$$\frac{d\phi}{dt} = \frac{\eta_j j}{L_T}; \quad \frac{dT}{dt} = \frac{2n j^2}{3nk_B}.$$  \hspace{1cm} (10)

The solution is

$$B = \frac{3}{2} B_1 \frac{j_1}{j_c} \left( 1 + \frac{5}{3} \frac{j_c^2}{j_1^2} \right)^{2/5}.$$  \hspace{1cm} (11)

where $T_1$ is the initial temperature in Kelvin, $B_1 = (\eta_j n k_B T_1 / L_T^2)^{1/2}$, $j_1 = (nk_B T_1 / \eta_j t)^{1/2}$, and $\eta_j$ is the resistivity at temperature $T_1$. The magnetic field is largest when $j_c = 5.15 j_1$. When $j_c$ exceeds 5.15 $j_1$, ohmic heating increases the temperature, and reduces the resistivity, which in turn reduces the growth of magnetic field. Ohmic heating is marginally important when $j_c = 5.15 j_1$. At this CR current the magnetic field achieves a maximum value, $B_{\text{max}} = 1.046 B_1$. Expressed in more meaningful terms, after a time $t$ in Gyr, the maximum field is

$$B_{\text{max}} = 8.2 \times 10^{-17} \left( \frac{L_T}{\text{kpc}} \right)^{-1} \left( \frac{T_1}{\text{K}} \right)^{-1/4} \times \left( \frac{n}{10^{-4} \text{cm}^{-3}} \right)^{1/2} \left( \frac{t}{\text{Gyr}} \right)^{1/2} \text{G.} \hspace{1cm} (12)$$

The maximum field occurs when the CR current ($j_c = 5.15 j_1$) is

$$j_{\text{max}} = 3 \times 10^{-20} \left( \frac{n}{10^{-4} \text{cm}^{-3}} \right)^{1/2} \left( \frac{T_1}{\text{K}} \right)^{5/4} \times \left( \frac{t}{\text{Gyr}} \right)^{-1/2} \text{Am}^{-2} \hspace{1cm} (13)$$

and the maximum field occurs at a distance $R_{\text{max}}$ from a galaxy with luminosity $L$, where

$$R_{\text{max}} = 1.9 \left( \frac{n}{10^{-4} \text{cm}^{-3}} \right)^{-1/4} \left( \frac{T_1}{\text{K}} \right)^{-5/8} \left( \frac{L}{L_*} \right)^{1/2} \times \left( \frac{p_{\text{min}}}{0.1 m_p c} \right)^{-0.15} \left( 1 + \frac{p_{\text{min}}}{0.1 m_p c} \right)^{-1/2} \left( \frac{t}{\text{Gyr}} \right)^{1/4} \text{Mpc,} \hspace{1cm} (14)$$
in a medium with a proton density \( n \). With the proviso that \( R_{\text{max}} \) lies in a realistic range, \( R_{\text{max}} \) is independent of the emissivity of the CR source, the CR momentum \( p_{\text{min}} \), and the fraction of CR escaping the galaxy. \( B_{\text{max}} \) is proportional to the inverse of the temperature scale length, but all other dependences are relatively weak. The weak dependence on the initial temperature implies that a significant magnetic field is generated even if the initial temperature is greater than 1 K. Hence, the characteristic magnetic field is robustly of the order of \( 10^{-17} - 10^{-16} \) G when the effect of Ohmic heating is included in the calculation. The maximum magnetic fields and their spatial locations when the effect of Ohmic heating is included in the calculation. The maximum magnetic fields and their spatial locations

Figure 4 gives the distribution functions for the current and magnetic field for galaxies randomly distributed in space according to the Schechter function. Apart from the luminosity, the Monte Carlo calculation adopts the standard parameters used in Figure 3. The distribution functions are defined as the number of current and magnetic field for the standard parameters assumed in Figure 3. The maximum magnetic fields and their spatial locations in Figure 3 obey these equations. The weak dependences on various parameters result in the broad extent of the maxima in Figure 3.

6. SOLUTION FOR THE INTERGALACTIC MEDIUM

The number density of star-forming galaxies at redshift \( z \geq 6 \), with luminosity, \( L \), and per luminosity interval, \( dL/L_\ast \), is well described by a Schechter function

\[
\Phi(L) = \Phi_\ast \left( \frac{L}{L_\ast} \right)^{\alpha} \exp \left( -\frac{L}{L_\ast} \right)
\]

with the following parameters: \( L_\ast \simeq 5.2 \times 10^{28} \) erg s\(^{-1}\) Hz\(^{-1}\), \( \Phi_\ast \simeq 10^{-3} \) Mpc\(^{-3}\), and \( \alpha \simeq 1.77 \) (Bouwens et al. 2007; Oesch et al. 2009; Bouwens et al. 2010). \( L_\ast \) is the typical luminosity of a bright galaxy, and \( \Phi_\ast \) roughly corresponds to their number density. \( \alpha \) is the slope of the distribution at the faint luminosity end.

The electric current at any given point in intergalactic space is contributed by galaxies within the current horizon, \( R_{\text{jh}} \). As long as the diffusion coefficient remains much larger than Bohm’s value, as observed in the much more turbulent interstellar medium of the Galaxy (Dröge & Kartavykh 2009), even for \( B \sim 10^{-16} \) G, \( R_{\text{jh}} \) is of order a few tens of Mpc, i.e., larger than the average distance between bright galaxies. Magnetic field generation is thus expected to be dominated by the nearest luminous galaxy. This is the case even if the currents are beamed with an opening angle \( \theta \sim 0.5 \) rad. In this case, the number of CR current sources visible from any given point is

\[
N_c = \Phi_\ast \Gamma(2 - \alpha) \frac{3}{4\pi} R_{\text{jh}}^3 \sim 1,
\]

for \( R_{\text{jh}} \gg \) a few \( \times 10 \) Mpc. Therefore, each point in space is exposed to the CR current from about one galaxy. Note that because the faint-end slope of the above Schechter’s function is steep, i.e., \( \alpha \lesssim 2 \), the CR output per luminosity log interval, \( \epsilon_c L_\ast (L/L_\ast)^{\alpha} \Phi(L) \approx \epsilon_c L_\ast (L/L_\ast)^{2-\alpha} \Phi_\ast \), is approximately constant. However, the average distance between faint galaxies of luminosity \( L \) is

\[
\langle d_L \rangle = \langle L \Phi(L) \rangle^{-1/3} \propto L^{(\alpha-1)/3},
\]

so that their CR current at this distance scales as

\[
j_c(L, d_L) \propto \epsilon_c L/(d_L)^2 \propto L^{1/2},
\]

i.e., fainter galaxies are slightly less efficient at magnetizing their surroundings.

We now make a more quantitative estimate through a Monte Carlo simulation in which galaxies are distributed randomly in space according to the Schechter function as given in Equation (15). The currents from each galaxy are added vectorially and the calculation repeated many times to build up a distribution function for the CR currents found at any random point in space. The magnetic field is then calculated using Equation (11). Figure 4(a) gives the distribution functions for the current and the magnetic field for the standard parameters assumed in Figure 3(a). The distribution functions are defined as the number of logarithmic interval in \( j \) or \( B \). The magnetic field calculation assumes that the initial temperature is 1 K everywhere. It neglects heating during ionization, which was included in Figure 3 within a radius of 100 kpc, but this neglect has little effect on the distribution function, first because the ionized volume is relatively small compared with the characteristic distance of 10 Mpc between luminous galaxies, and second because, even starting from a low temperature, ohmic heating reduces the resistivity, and therefore the magnetic field, in regions close to galaxies. For fixed parameters such as the temperature scale length \( L_T \) and the plasma density \( n \), the magnetic field cannot...
exceed a maximum value as given in Equation (12). This produces an abrupt cutoff to the distribution function. The spike in the distribution function at the cutoff in Figure 4(a) reflects the broad spatial extent of the maxima in Figure 3(a). The magnetic field is close to the maximum value over a wide range of distances from individual galaxies. In reality, as shown in Figure 2, the temperature scale length $L_T$ takes a range of local values, and this will smear the cutoff. The effect of decreasing (increasing) the temperature scale length $L_T$ by a factor $\lambda$ would be to move the distribution function for $B$ a corresponding factor $\lambda$ to the right (left) in Figure 4(a), while leaving the distribution function for $j$ unchanged.

In Figure 3, it can be seen that the value of the maximum magnetic field is insensitive to changes in various parameters. However, the position of the maximum moves in radius. For example, the maximum magnetic field occurs at 1 Mpc when the CR luminosity of all galaxies is reduced by a factor of 10 (see Figure 3(d) where $L = 0.1 L_c$). The effect of this on the distribution function for $B$ can be seen in Figure 4(b). When the luminosity is reduced, there are still regions of space in which the magnetic field reaches the maximum value of $8 \times 10^{-17} \text{G}$, but the field is characteristically $10^{-17} \text{G}$ throughout most of the volume between galaxies. Similarly, the effect on the distribution function of changing other parameters can be deduced from Figure 3 in conjunction with Equation (12). Equation (12) gives a good estimate of the maximum magnetic field, while Figure 3 indicates the volume of space that is filled by fields close to the maximum value.

7. STREAMING AND CURRENT-DRIVEN INSTABILITIES

Here we consider the possible excitation of streaming and current-driven plasma instabilities caused by the relative drifts of CR, thermal electrons, and thermal ions. Instabilities can be categorized as (1) short-wavelength two-stream instabilities such as the Langmuir, Buneman, and ion- acoustic instabilities and (2) long-wavelength MHD instabilities.

Short-wavelength two-stream instabilities driven by the return current are probably inactive because the electron drift velocity is smaller than the electron thermal velocity, the ion thermal velocity, and the ion acoustic speed at all distances from the galaxy for conditions set out in Figure 3(a). Similarly, CRs propagate diffusively and do not directly excite instabilities as a cold well-collimated beam, except perhaps at microscales, with the wavenumber $k \approx \omega_e/c$, where $\omega_e$ is the electron plasma frequency, with the corresponding wavelength $\lambda = 2\pi/k \approx 10^{12} \text{cm}$. However, even if a two-stream or high-$k$ electromagnetic instability were excited on such small scales, it would saturate quickly, and the combination of short wavelength and small characteristic electric potential would leave CR trajectories unaffected.

Furthermore, if a two-stream instability were active, its dominant effect would be to produce an anomalous resistivity which might inhibit the return current. The large-scale electric field required to draw the return current would then be increased and resistive magnetic field generation would be enhanced. If the large-scale electric field were strong enough to inhibit the propagation of low-energy CRs, higher energy CRs would still propagate as required to drive the enhanced electric field.

Long-wavelength MHD instabilities are more likely to be important. Estimates of the growth rate, $\gamma = (kB_j/\rho)^{1/2}$, of the non-resonant CR-driven MHD instability (Bell 2004) in a magnetic field of $10^{-10} \text{G}$ indicate that this might grow on a kpc scale in a Gyr. A scenario can be envisaged in which resistive field generation produces the magnetic field of the order of $10^{-16} \text{G}$ which is then further amplified by the non-resonant instability as well as by turbulent processes. This will be the subject of future work.

8. SUMMARY

In conclusion, we have shown that CRs produced by SNe at the epoch of reionization when the first stars and galaxies form can be expected to have a substantial impact on the intergalactic medium out to distances of a several Mpc from the galaxy. Through the return electric current carried by the thermal plasma, the medium is heated ohmically and the magnetic field is generated in the range $10^{-17}$ to $10^{-16} \text{G}$. Because of self-compensation through ohmic heating, the maximum magnetic fields are relatively insensitive to parameters such as the galaxy luminosity, the efficiency of CR production, and the ambient intergalactic temperature and density. The magnitude of the magnetic field is however sensitive to the scale length $L_T$ of temperature variations in the intergalactic medium. Cosmological simulations support an assumption of scale lengths in the range 1–10 kpc. Self-consistent cosmological simulations including CR effects are a necessary next step in developing this theory, but our argument extends the possibility that resistive magnetic field generation, driven by CR streaming, may generate seed magnetic fields which might subsequently be turbulently amplified to form the intergalactic magnetic field at the present epoch. The effect of CR-driven instabilities may also deserve further work.

F.M. acknowledges several useful discussions with Dr. P. Oesch. The research leading to these results has received funding from the European Research Council under the European Community’s Seventh Framework Programme (FP7/2007-2013)/ERC grant agreement No. 247039.

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