PASSIVE CONTROL FOR A CLASS OF NONLINEAR SYSTEMS
BY USING THE TECHNIQUE OF ADDING A POWER INTEGRATOR

JINGLAI QIAO
School of Mathematics, Liaoning University, Shenyang, 110036, China
College of Science, Liaoning Shihua University, Fushun, 113001, China
LI YANG* AND JIAWEI YAO
School of Mathematics, Liaoning University, Shenyang, 110036, China
(Communicated by Wanquan Liu)

ABSTRACT. This paper studies the problem of passive control for a class of uncertain nonlinear lower-triangle systems. We extend the feedback designing tool named adding a power integrator. By using it repeatedly, the passive controller is given. Under this designing method, we don’t need the system to be feedback linearizable. Moreover, comparing with the backstepping technique, the coordinate in the controller designing process of this method does not need to be transformed.

1. Introduction. Passive control theory has been rapidly developed and widely applied in recent years. With the characteristics of passivity, passive control has been used to study the stabilization for some linear and nonlinear systems. To name a few, U. E. Kocamaz constructed a controller for the Rucklidge chaotic system [5]; Chen designed the passivity based control law for a class of switched control systems [3]; Liu established the coordinated passivity controller for SVC in [7]. In addition, Y. Liu studied the stabilization of switched nonlinear systems with passive and non-passive subsystems [9]. Due to the original meaning of passivity, passive control has been wildly used in physics, electrical engineering, network and so on. For example, the smearing of shocking wave [3], suppression of nonlinear vibration system [4], as well as the research for multimachine power system [15]. With the development of these theories, controller design has become an issue.

With the help of linear matrix inequalities (LMI) and Lyapnuov method, passive control for many linear systems has been solved [17]. H. Xiao made a sliding mode control design to study the robust passive control for uncertain switched time-delay systems [16]; Y. Li researched the problem for T-S fuzzy systems [8]; Z. Wu solved the problem for systems with time-varying delays [14]; L. Yang studied the issue about dissipative control for singular impulsive dynamical systems [18]. However, if
we focus on nonlinear systems, the application of LMI will be limited. In particular, passive control becomes more difficult in the case of high-order nonlinear systems, because the systems can not be linearized.

As an indispensable tool for stability research, the Lyapunov function is a positive definite function whose derivative is less than zero. The storage function is also positive defined, which plays an important role in the discuss of passivity. Its derivative is less than a function called supply rate. Thus, there are some functions which can be considered as Lyapunov functions and storage functions. As a consequence, some methods of solving the stabilization problem can be used to discuss the passive control problem after changing the control law. In [9] and [10], Wei Lin designed the tool called adding a power integrator to deal with the global robust stabilization problem and the adaptive regulation for high-order systems. This method also help us to stabilize high-order nonlinear systems[12], high-order time-delay nonlinear systems[2] and high-order stochastic nonlinear systems[13]. It has been also exploited to solve the problem of global finite-time stabilization for lower-triangular nonlinear systems.

Because of the complexity and diversity for nonlinear systems, there is no uniform approach to design passive control law. Considering that passive control has intuitive realistic significance, this study mainly propose the design of the passive controller for investigated nonlinear systems. Since there are close connections between passivity and stability, the passive control problem is solved by extending the designing tool in [9] and [10], which was applied to stabilize nonlinear systems. Besides, passive controller can also make the system stable if the external input is 0.

The rest of the paper is organized as follows. Section 2 gives the system description and the definition of passive. In Section 3, we introduce some inequalities and extend the design tool called adding a power integrator. In Section 4, we establish the design method of passive controller. The conclusion is given in Section 5.

2. Preliminaries. Consider the following nonlinear system

\[
\begin{align*}
\dot{z} &= f(z, x_1, d(t)), \\
\dot{x}_1 &= x_2^{p_1} + \phi_1(z, x_1, d(t)), \\
\dot{x}_2 &= x_3^{p_2} + \phi_2(z, x_1, x_2, d(t)), \\
&\vdots \\
\dot{x}_i &= x_{i+1}^{p_i} + \phi_i(z, x_1, \ldots, x_i, d(t)), \\
&\vdots \\
\dot{x}_r &= u^{p_r} + \phi_r(z, x_1, \ldots, x_r, d(t)),
\end{align*}
\]

(1)

where \( z \in \mathbb{R}^{n-r}, x = (x_1, x_2, \ldots, x_r)^T \) are the states, \( u \in \mathbb{R} \) is the control input, \( p_i \) are positive integers for \( i = 1, 2, \ldots, r \), \( d(t) \) is an unknown continuous disturbance or parameter in a known compact set. Functions \( \phi_i : \mathbb{R}^{n-r+i} \times \mathbb{R}^s \to \mathbb{R}^1 \) and \( f : \mathbb{R}^{n-r+i} \times \mathbb{R}^s \to \mathbb{R}^{n-r} \) are smooth functions of states satisfying \( \phi_i(0, d(t)) = 0, f(0, d(t)) = 0 \) for all \( d(t) \).
Definition 2.1. [11] A nonlinear system is given as
\[
\begin{align*}
\dot{x} &= f(x) + g(x)u, \\
y &= h(x),
\end{align*}
\] (2)
where \(x \in \mathbb{R}^n\) is state variable, \(u \in \mathbb{R}^m\) is the input, \(y \in \mathbb{R}^p\) is the output, \(f(\cdot), g(\cdot)\) are smooth vector fields, \(h(\cdot)\) is smooth mapping. Then system (2) is said to be passive if for all \(t \in [0, \infty)\) and \(u(\cdot), x(\cdot), y(\cdot)\) with \(x(0) = 0\), there holds
\[
\int_0^t u^T(\tau) y(\tau) d\tau \geq 0.
\]

Remark 1. A sufficient criterion of passive is the existence of a storage which is a function \(V : \mathbb{R}^m \to [0, \infty)\) with the properties \(V(0) = 0\) and \(V - u^T y \leq 0\).

Lemma 2.2. [9] (Modified Young’s inequality) For any positive integers \(m, n\) and any real-valued function \(\gamma(x, y) > 0\), the following inequality holds:
\[
|x|^m|y|^n \leq \frac{m}{m+n}\gamma(x, y)|x|^{m+n} + \frac{n}{m+n}\gamma^{-\frac{m}{n}}(x, y)|y|^{m+n}.
\] (3)
Proof. Let \(a = |x|^m\gamma^{\frac{m}{m+n}}\), \(b = |y|^n\gamma^{-\frac{m}{n}}\), \(p = \frac{m+n}{m}\), \(q = \frac{m+n}{n}\).

From Young’s inequality
\[
|ab| \leq \frac{|a|^p}{p} + \frac{|b|^q}{q},
\] (4)
the lemma is immediately proved. □

Lemma 2.3. For any given smooth function \(N(X, y), x \in \mathbb{R}^m, y \in \mathbb{R}\), supposing that there is a real-valued smooth function \(\gamma(X, y) \geq 0\) and exists a positive integer \(l\) such that
\[
|\frac{\partial N(X, y)}{\partial y}| \leq (\|X\|^l + |y|^l)\gamma(X, y),
\] (5)
then for any real-valued function \(W(X) > 0\) and a smooth function \(y^* = X^T \alpha(X)\), the following holds:
\[
|N(X, \xi + y^*) - N(X, y^*)| \leq \|X\|^{l+1}W(X) + \xi^{l+1}\rho(X, \xi),
\] (6)
where \(\rho(X, \xi) \geq 0\) and \(\rho(X, \xi) \in C^\infty\).

Proof. Using the Taylor expansion formula with integration remainder, we have
\[
|N(X, \xi + y^*) - N(X, y^*)| = \left| \int_0^1 \frac{\partial N(X, y)}{\partial y}|_{y=y^*+\lambda \xi} d\lambda \right|
\leq |\lambda| \int_0^1 (\|X\|^l + |y^* + \lambda \xi|^l)\gamma(X, y^* + \lambda \xi)d\lambda.
\]
Based on \(\left(\frac{|a|+|b|}{2}\right)^n \leq \left(\frac{|a|+|b|}{2}\right)^n \leq \frac{|a|^n+|b|^n}{2^n}\) and Cauchy inequality, we obtain
\[
|N(X, \xi + y^*) - N(X, y^*)| \leq |\lambda| \int_0^1 (\|X\|^l + |\lambda \xi|^l)\hat{\gamma}(X, \lambda \xi)d\lambda
\leq |\lambda| (\|X\|^l + |\lambda|^l) \int_0^1 \hat{\gamma}(X, \lambda \xi)d\lambda,
\]
where \(\hat{\gamma}(X, \lambda \xi) \geq 0\) is a smooth function.

From Lemma 2.1, one can come to (6). □
3. Adding a power integrator. In the process of studying the passive control problem for the nonlinear system (1), the design tool called adding a power integrator is extended. Compared with adding a power integrator used in the robust stabilization problem, this tool will contribute to the construction of passive feedback control law, which makes the close-loop system to be passive. Similarly, the adding a power integrator tool is also based on the modification of the Young’s inequality.

In this section, we consider the nonlinear system:

\[ \dot{X} = f(X, z, d(t)), \]
\[ \dot{z} = u^{l_2} + \phi(X, z, d(t)), \]

where \( X \in \mathbb{R}^m \) and \( z \in \mathbb{R} \) are the states, \( u \in \mathbb{R} \) is the input, \( d(t) \) is an unknown continuous disturbance or parameter belonging to a known compact with \( f(0, 0, d(t)) = 0 \) and \( \phi(0, 0, d(t)) = 0 \). Assuming that there are two real-valued, nonnegative smooth functions \( \gamma_1(X, z) \) and \( \gamma_2(X, z) \), which satisfies

\[
(A1) \|f(X, z, d(t))\| \leq (\|X\|^{l_1} + |z|^{l_1})\gamma_1(X, z);
\]
\[
(A2) |\phi(X, z, d(t))| \leq (\|X\|^{l_2} + |z|^{l_2})\gamma_2(X, z);
\]

where \( l_0 \geq l_1 \geq l_2 \geq 1 \) are odd positive integers.

**Lemma 3.1.** (Adding one power integrator). For system (7), suppose that there exists a smooth function \( z = z^*(X) \) with \( z^*(0) = 0 \) and a smooth positive definite Lyapunov function \( V(X) \) such that

\[
(H1) (\partial V/\partial X)f(X, z^*(X), d(t)) \leq -\|X\|^{l_0 + 1}W(X);
\]
\[
(H2) |(\partial V/\partial X)\partial f(X, z, d(t))|/\partial z| \leq (\|X\|^{l_0} + |z|^{l_0})\gamma_3(X, z);
\]

where \( W(X) \) and \( \gamma_3(X, z) \) are in \( C^\infty \), \( W(X) > 0 \) and \( \gamma_3(X, z) \geq 0 \), then there is a smooth Lyapunov (storage) function \( U(X, z) \), which is positive definite and proper, and exists a smooth controller \( u^* = u^*(X, y) \) with \( u^*(0, 0) = 0 \), which makes system passive from the input to a constructed output.

**Proof.** For the sake of simplicity, we use the symbols as follows: \( f = f(X, z, d(t)) \), \( f^* = f(X, z^*(X), d(t)) \), \( \Delta = f - f^* \) and \( \tilde{z} = z - z^* \). System (7) can be written as:

\[
\dot{X} = f^* + \Delta
\]
\[
\dot{z} = u^{l_2} + \phi(X, z, d(t)).
\]

Consider the Lyapunov function

\[
U(X, z) = V(X) + \left( \frac{z - z^*(X)}{l_0 - l_2 + 2} \right)^{l_0 - l_2 + 2},
\]

which is positive definite and proper. By using (3), the derivate can be calculated as

\[
\dot{U}(X, z) = \dot{V}(X) + \tilde{z}^{l_0 - l_2 + 1}(u^{l_2} + \Phi(X, \tilde{z}, d(t)))
\]
\[
= \frac{\partial V}{\partial X}(f^* + \Delta) + \tilde{z}^{l_0 - l_2 + 1}(u^{l_2} + \Phi(X, \tilde{z}, d(t)))
\]
\[
\leq -\|X\|^{l_0 + 1}W(X) + \frac{\partial V(X)}{\partial X}\Delta + \tilde{z}^{l_0 - l_2 + 1}(u^{l_2} + \Phi(X, \tilde{z}, d(t))),
\]

where
where $\Phi(X, \bar{z}, d(t)) = \phi(X, \bar{z} + z^*(X), d(t)) - \frac{\partial z^*}{\partial X}f(X, \bar{z} + z^*(X), d(t))$. By (A1), (A2) and $l_1 \geq l_2$, we have

\[
\|\Phi(X, \bar{z}, d(t))\| \\
\leq (\|X\|^l_2 + |\bar{z} + z^*(X)|^l_2)\gamma_2(X, \bar{z} + z^*(X)) \\
+ |\frac{\partial z^*}{\partial X}||\|X\|^{l_1} + |\bar{z} + z^*(X)|^{l_1}|\gamma_1(X, \bar{z} + z^*(X)) \\
\leq (\|X\|^l_2 + |\bar{z} + z^*(X)|^l_2)(\gamma_2(\cdot) + \gamma_1(\cdot)|\|X\|^{l_1-l_2} + |\bar{z} + z^*(X)|^{l_1-l_2})) \\
\leq (\|X\|^l_2 + |\bar{z} + z^*(X)|^l_2)\tilde{\gamma}(X, \bar{z}),
\]

for $\tilde{\gamma}(X, \bar{z}) \geq 0$ in $C^\infty$. Then

\[
|\bar{z}^{l_0-l_2+1}\Phi(X, \bar{z}, d(t))| \leq |\bar{z}|^{l_0-l_2+1}\|X\|^l_2\tilde{\gamma}(X, \bar{z}) + \bar{z}^{l_0+1}\tilde{\gamma}(X, \bar{z}).
\]

Let $x = \|X\|$, $m = l_2$, $n = l_0 - l_2 + 1$ and $y = |\bar{z}|^{l_0-l_2+1}\tilde{\gamma}(X, \bar{z})$, according to Lemma 2.2,

\[
\bar{z}^{l_0-l_2+1}\Phi(X, \bar{z}, d(t)) \\
\leq \frac{l_2}{l_0 + 1}\|X\|^{l_0-1}\gamma(x, y) + \frac{l_0 - l_2 + 1}{l_0 + 1} \gamma \frac{l_2}{m-l_2+2-1}(x, y)(|\bar{z}|^{l_0-l_2+1}\tilde{\gamma}(X, \bar{z}))^{\frac{l_0+1}{m-l_2+2-1}} \\
+ \bar{z}^{l_0+1}\tilde{\gamma}(X, \bar{z}) \\
= \frac{\|X\|^{l_0+1}W(X)}{4} + \frac{l_0 - l_2 + 1}{l_0 + 1} \bar{z}^{l_0+1}\tilde{\gamma}^{\frac{l_0+1}{m-l_2+2-1}} \left(\frac{4l_2}{(l_0 + 1)W(X)}\right)^{\frac{l_0}{m-l_2+2-1}} + \bar{z}^{l_0+1}\tilde{\gamma}(X, \bar{z}) \\
\leq \frac{\|X\|^{l_0+1}}{4}W(X) + \bar{z}^{l_0+1}\rho_1(X, \bar{z}),
\]

(10)

for $\rho_1(X, \bar{z} \geq 0)$ in $C^\infty$.

Similarly, let $N(X, y) = \frac{\partial W}{\partial X}f$, together with Lemma 2.3 and (H2), we obtain

\[
|\frac{\partial W}{\partial X}\Delta| \leq \frac{\|X\|^{l_0+1}}{4}W(X) + \bar{z}^{l_0+1}\rho_2(X, \bar{z}),
\]

(11)

where $\rho_2(X, \bar{z} \geq 0)$ is in $C^\infty$.

Combining (9) (10) with (8), the derivate of $U$ can be calculated as

\[
\dot{U}(X, z) \leq -\frac{\|X\|^{l_0+1}}{2}W(X) + \bar{z}^{l_0-l_2+1}u^{l_2} + \bar{z}^{l_0+1}[\rho_1(X, \bar{z}) + \rho_2(X, \bar{z})].
\]

(12)

Then the state feedback control law can be designed as:

\[
u = -\bar{z}\left[-\frac{W(X)}{2} + \rho_1(X, \bar{z}) + \rho_2(X, \bar{z}) - \zeta(\bar{z})\bar{u}\right]^{\frac{1}{2}},
\]

(13)

where

\[
\zeta(\bar{z}) = \begin{cases} \\
\bar{z}^{-l_0}, & \bar{z} \neq 0, \\
0, & \bar{z} = 0.
\end{cases}
\]
Based on the equation (13),
\[ U(X, z) \]
\[ \leq -\left(\|X\|^{l_0+1} + \bar{z}^{l_0+1}\right) \frac{W(X)}{2} + \bar{z} \tilde{u} \]
\[ \leq -\left(\|X\|^{l_0+1} + \|X^T \alpha(X)\|^{l_0+1} \right) \frac{W(X)}{1 + \|\alpha(X)\|^{l_0+1}} + \bar{z} \tilde{u} \]
\[ \leq -\left(\|X\|^{l_0+1} + \|X^T \alpha(X)\|^{l_0+1} + \bar{z} - X^T \alpha(X)\right) \frac{W(X)}{2(1 + \|\alpha(X)\|^{l_0+1})} + \bar{z} \tilde{u}. \]
Because
\[ |X^T \alpha(X)|^{l_0+1} + \bar{z} - X^T \alpha(X)\] 
we can get
\[ U(X, z) \leq -\left(\frac{\|X\|^{l_0+1}}{2^{l_0}} + \frac{\|X\|^{l_0+1}}{2^{l_0}} \right) \frac{W(X)}{2 + 2\|\alpha(X)\|^{l_0+1}} + \bar{z} \tilde{u} \]
\[ \leq -\left(\|X\|^{l_0+1} + |z|^{l_0+1}\right) \tilde{W}(X, z) + \bar{z} \tilde{u}, \]
where
\[ \tilde{W}(X, z) = \frac{W(X)}{2^{l_0+1}(1 + \|\alpha(X)\|^{l_0+1})} > 0. \]
Therefore, the closed-loop system of (7) is passive from input \( \bar{u} \) to constructed output \( \bar{z} \).

4. Controller design. In this section, we will solve the passive control problem for system (1) with the help of the design tool in Lemma 3.1. The investigated system satisfies assumptions as the follows:

**Assumption 1.** For \( i = 1, 2, \ldots, r \) and any \( d(t) \),
\[ |\phi_i(x_0, x_1, x_2, \ldots, d(t))| \leq (\|x_0\|^{p_i} + |x_1|^{p_i} + |x_2|^{p_i} + \cdots + |x_i|^{p_i}) \rho_i(x_0, x_1, x_2, \ldots, x_i), \]
where \( \rho_i(x_0, x_1, x_2, \ldots, x_i) \) is a known nonnegative smooth function.

**Assumption 2.** \( p_1 \leq p_2 \leq \cdots \leq p_r \leq 1 \) are odd integers.

**Assumption 3.** Assume that there is a real-valued, nonnegative smooth function \( \gamma_1(x_0, x_1) \) such that
\[ \|f(x_0, x_1, d(t))\| \leq (\|x_0\|^{p_1} + |x_1|^{p_1}) \gamma_1(x_0, x_1). \]

Under these assumptions, the passive control problem can be solved by using adding a power integrator.

**Theorem 4.1.** Consider the nonlinear system (1) which satisfies Assumptions 1-3. Suppose that there exists a function \( x_1^*(x_0) \) with \( x_1^*(0) = 0 \) and a smooth Lyapunov function \( V(x_0) \), which is positive definite and proper such that for any \( d(t) \),
\[ \frac{\partial V}{\partial z} f(x_0, x_1^*(x_0), d(t)) \leq -\|x_0\|^{p_0+1} W(x_0), \] \[ (14) \]
\[ \left| \frac{\partial V}{\partial x_0} f(x_0, x_1^*(x_0), d(t)) \right| \leq (\|x_0\|^{p_0} + |x_1|^{p_0}) \gamma_3(x_0, x_1), \] \[ (15) \]
where \( W(x_0) > 0 \) is in \( C^\infty \), \( p_0 \geq p_1 \) is an odd integer and \( \gamma_3(x_0, x_1) \) is a smooth function, then system (1) is passive from input to a constructed output under a control law \( u = u(x, v) \).
Proof. This proof is a process using Lemma 3.1 repeatedly.
Firstly, consider \( r = 1 \). Let
\[
X = x_0, \quad z = x_1, \quad u = x_2, \quad \phi(\cdot) = \phi_1(\cdot), \quad l_0 = p_0, \quad l_1 = l_2 = p_1,
\]
then the system is rewritten as
\[
\dot{X} = f(X, z, d(t)), \quad \dot{z} = u^{l_2} + \phi(X, z, d(t)), \quad y^*(X) = x_1^*(X) = X^T \tilde{\alpha}(X).
\]
It is easy to know that Assumptions 1 and 3, inequality (14) and (15) is equivalent to (A2) (A1) (H1) (H2) in Lemma 3.1 respectively. According to Lemma 3.1, there exists a smooth state feedback control law of the form (13)
\[
u = x_2^*(x_0, x_1, \tilde{u}_1), \tag{17}
\]
such that
\[
\dot{U}_1(x_0, x_1, \tilde{u}_1) \leq -\left(\|x_0\|_{p_0+1} + x_1^{l_0+1}\right)\tilde{W}(x_0, x_1) + \tilde{x}_0 \tilde{u}_1, \tag{18}
\]
where \( \tilde{W}(x_0, x_1) > 0 \) is in \( C^\infty \) and
\[
U_1(x_0, x_1) = V(x_0) + \frac{1}{p_0 - p_1 + 2} (x_1 - x_1^*(x_0))^p_0 - p_1 + 2.
\]
The closed-loop system of system (1) with \( r = 1 \) is passive.
Next, let \( \tilde{u}_0 = 0 \) and \( r = 2 \). We use the notations that \( X = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} \), \( z = x_2, \ u = x_3, l_0 = p_0, l_1 = p_1, l_2 = p_2 \) and
\[
\dot{f}(X, z, d(t)) = \begin{pmatrix} f(x_0, x_1, d(t)) \\ x_2^{l_2} + \phi_1(x_0, x_1, d(t)) \end{pmatrix} = \begin{pmatrix} f(X, d(t)) \\ z^{l_1} + \phi_1(X, d(t)) \end{pmatrix}.
\]
Using these notations, the system (1) with \( r = 2 \) can be rewritten as:
\[
\dot{X} = \dot{f}(X, z, d(t)), \quad \dot{z} = u^{l_2} + \phi_2(X, z, d(t)) = u^{l_2} + \phi(X, z, d(t)), \tag{19}
\]
\[
\dot{x}_2^*(X) = X^T \tilde{\alpha}(X).
\]
It is easy to know that (A1) and (A2) in Lemma 3.1 are satisfied. Inequality (18) implies that (H1) in Lemma 3.1 holds. According to \( U_1(x_0, x_1) \) and Lemma 2.2, it follows that
\[
\left| \frac{\partial U_1 \partial f(X, z, d(t))}{\partial X} \right| = \left| \frac{\partial U_1 \partial f(x_0, x_1, d(t))}{\partial X} \right| = \left| \frac{\partial U_1 \partial x_1^*(x_0)}{\partial X} \right| = \left| l_1 (x_1 - x_1^*(x_0))^{p_0 - l_1 + 1} z^{l_1 - 1} \right|
\]
\[
= \|X\|^{p_0 - l_1 + 1} z^{l_1 - 1} T X
\]
\[
\leq (\|X\|^{p_0} + |z|^m) \gamma_3(X, z),
\]
where \( \gamma_3(X, z) \leq 0 \) is in \( C^\infty \). Thus, (H2) is satisfied. From Lemma 3.1, there exist a controller
\[
u = x_3^*(x_0, x_1, x_2, \tilde{u})
\]
and a positive definite and proper Lyapunov function
\[
U_2(x_0, x_1, x_2) = U(x_0, x_1) + \frac{1}{p_0 - p_2 + 2} (x_2 - x_2^*(x_0, x_1, 0))^{p_0 - p_2 + 2},
\]
such that
\[ \dot{U}_2 \leq -(x_0^{l+1} + x_1^{l+1} + x_2^{l+1})\dot{W}(x_0, x_1, x_2) + \bar{x}_1 \bar{u}, \]
therefore the closed-loop system is passive from input \( \bar{u} \) to constructed output \( \bar{x}_1 \), which means theorem holds for \( r = 2 \).

With the help of inductive method, the theorem can be proved by using Lemma 3.1 \( r \) times.

5. Example. Consider the following system
\[
\begin{align*}
\dot{z}_1 &= (x_1 + x_1^2)z_2^2 - z_2, \\
\dot{z}_2 &= z_1^3 + z_2^3 + 2x_1^3, \\
\dot{x}_1 &= u^3 + (z_1^2 + \ln(1 + z_2^2))x_1^2.
\end{align*}
\]
For the first two subsystems, choose a positive define Lyapunov function
\[ V(z) = \frac{1}{4}(5z_1^4 + z_2^4) + (z_1 - z_2)^4. \]
Let \( x_1^* = -z_2 \), then
\[ \dot{V}(z)|_{x=x^*} = -z_1^6 - z_2^6 - 3z_1^4(z_1 - 2z_2)^2 \leq -z_1^6 - z_2^6. \]
Let \( p_0 = 5 \) and \( p_2 = p_3 = 3 \).
For the whole system, choose Lyapunov function:
\[ U = V + \frac{(x_1 - x_1^*)^4}{4}. \]
From the relationship
\[ |(z_1^2 + \ln(1 + z_2^2))x_2^2| \leq (|z_1|^3 + |z_2|^3 + |x_1|^3)(1 + x_1^2), \]
we can get
\[ \dot{U} \leq -z_1^6 - z_2^6 + x_1^3(u^3 + (|z_1|^3 + |z_2|^3 + |x_1|^3)(1 + x_1^2) - z_1^3 - z_2^3 - 2x_1^3). \]
Then, under the controller
\[ u = -\bar{x}_1(1 - (|z_1|^3 + |z_2|^3 + |x_1|^3)(1 + x_1^2) + z_1^3 + z_2^3 + 2x_1^4 - \zeta(\bar{x}_1)v^3, \]
we have
\[ \dot{U} \leq -z_1^6 - z_2^6 - x_1^6 + \bar{x}_1v. \]
Therefore, the closed-loop system is passive from input \( v \) to constructed output \( \bar{x}_1 \).

6. Conclusion. In this paper, we discuss the passive control problem for high-order lower-triangular nonlinear systems. The problem is solved by using iterative process and a design tool called adding a power integrator. The tool is extended to design the passive controller, which was used to stabilize high-order nonlinear systems. The method of controller design is given in an iterative process.
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Received May 2019; 1st revision September 2019; Final revision October 2019.

E-mail address: 925351034@qq.com
E-mail address: yangli2923@163.com
E-mail address: 594225264@qq.com