Top and Bottom Seesaw from Supersymmetric Strong Dynamics

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ABSTRACT: We propose a top and bottom seesaw model with partial composite top and bottom quarks. Such composite quarks and topcolor gauge bosons are bound states from supersymmetric strong dynamics by Seiberg duality. Supersymmetry breaking also induces the breaking of topcolor into the QCD gauge coupling. The low energy description of our model reduces to a complete non-minimal extension of the top seesaw model with bottom seesaw. The non-minimal nature is crucial for Higgs mixings and the appearance of light Higgs fields. The Higgs fields are bound states of partial composite particles with the lightest one compatible with a 125 GeV Higgs field which was discovered at the LHC.

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## Contents

1. Introduction ................................................. 2
2. Composite Particles from SUSY Strong Dynamics .......... 3
3. Top and Bottom Seesaw ..................................... 7
4. Conclusions ............................................... 21
1 Introduction

The Standard Model (SM) of electroweak interactions, based on the spontaneously $SU(2)_L \times U(1)_Y$ gauge symmetry breaking, has been extremely successful in describing phenomena below the electroweak scale. The most important problem in the SM is the source of the electroweak gauge symmetry breaking and the related problem of hierarchical flavor structure. It is well known that the top quark is very heavy comparing to the other SM fermions and its value is obtained in an ad hoc manner by adjusting the phenomenologically introduced Yukawa couplings. Besides, the top quark couples more strongly to electroweak symmetry breaking sector than the light quarks and it is possible that some of the electroweak symmetry breaking is due to top sector. The idea of top condensation [1] is an attractive approach to explain these problems.

However, the minimal top condensation framework predicts a too high top quark mass $m_t$ as well as a high Higgs mass, and then the extreme fine-tuning is needed to trigger the condensation. Also, the Nambu-Jona-Lasinio (NJL) model must be considered as an approximation of some new strong dynamics—the topcolor gauge interactions. One can combine topcolor with technicolor to get a TC2 model [2] in which the electroweak symmetry breaking gets contributions from both the top condensation and the technicolor sectors. The other very interesting scenario is the top seesaw model [3] which naturally predicts the acceptable top quark mass without the need of new electroweak symmetry breaking sector. The UV completion of topcolor needs more matter contents and certain interactions which are put in by hand. We would like to present a model which will give rise to these terms automatically.

It is well known that the SM requires the existence of Higgs fields to trigger electroweak gauge symmetry breaking. However, the quantum corrections to Higgs boson masses have quadratic divergences. Thus, the entire SM mass spectrum, which depends on the Vacuum Expectation Value (VEV) of Higgs field, is directly and indirectly sensitive to the cut-off scale of the theory like the Planck scale. This is the gauge hierarchy problem. One natural solution is supersymmetry (SUSY) by adding supersymmetric partners of the SM particles to cancel the quadratic divergences. However, the ATLAS and CMS Collaborations at the LHC have not found any signal of supersymmetric particles (sparticles) yet. Moreover, SUSY can provide a viable dark matter candidate, achieve the gauge coupling unification as well as be an essential ingredient to certain quantum gravity candidate. Thus, it is possible that our Universe could adopt supersymmetry at relatively high scale.

It had been conjectured long time ago that all the building blocks of the SM are composite particles instead of being fundamental particles. The existence of chiral symmetry is essential to guarantee the disappearance of the known fermion masses. However, ’t Hooft anomaly matching conditions [4] are very restrictive and hardly can one obtain the realistic composite models. A very interesting progress was achieved by Seiberg who discovered the duality [5] between different SUSY gauge theories. Seiberg duality is highly non-trivial and satisfies the ’t Hooft anomaly matching conditions and decoupling conditions as well as the other consistent checks. Besides, new emergent gauge groups and composite fermions appear in certain case of the dual description. We conjecture that the SM particles are com-
posite and such compositeness are the consequences of SUSY strong dynamics and SUSY breaking. The observed small mass terms of the SM fermions are the consequences of the strong dynamics arise from the emergent gauge interactions. Especially, the Higgs boson mass around 125 GeV, which was discovered at the LHC recently [6, 7], can be realized as well.

This paper is organized as follows. In Section 2, we discuss the emergent topcolor gauge group and matter contents from SUSY strong dynamics. SUSY is broken by rank conditions in our scenario, which results in the ISS-type metastable vacua [8]. In Section 3, we discuss the complete top and bottom seesaw sector. The composite matter content from Seiberg duality results in partial composite physical top and bottom quarks. Composite multiple Higgs doublets appear in our model at low energy and are fully responsible for electroweak gauge symmetry breaking. Section 4 contains our Conclusions.

2 Composite Particles from SUSY Strong Dynamics

Top quark, which couples more strongly to the electroweak symmetry breaking sector than other light quarks, could be responsible for electroweak symmetry breaking. The idea of top condensation is fairly attractive and gives an explanation on how top quark can participate in the electroweak symmetry breaking mechanism and obtain a dynamically-generated mass term. The UV completion of the top condensation idea suggests the existence of new topcolor gauge interactions. The complete topcolor sector requires new Higgs fields in (3, \bar{3}) representation to break the topcolor gauge symmetry down to SU(3)_C. Besides, top seesaw sector requires new vector-like particles. We want to obtain all the required ingredients from SUSY strong dynamics. The most simple setting is the vector-like supersymmetric QCD.

Let us consider SU(N_C) SUSY QCD which has the massive vector-like quarks Q_i and \tilde{Q}^i with i = 1, 2, ..., N_F, and the SU(N_C) singlet massive messenger fields \bar{f} and f for gauge mediation. The global flavor symmetry is SU(N_F)_1 \times SU(N_F)_2 \times U(1)_V. We adopt the following superpotential

\[ W = m^i_j Q_i \tilde{Q}^j + \kappa^{ij} \frac{Q_i \tilde{Q}^j \bar{f} f}{M_*} + M_0 \bar{f} f. \] (2.1)

where the following mass terms

\[ m^i_j = m_0 \delta^i_j, \] (2.2)

break the flavor symmetry down to SU(N_F)_V \times U(1)_V. This superpotential is of the simplified gauge mediation type proposed in Ref. [9].

According to the Seiberg duality [5], this theory is dual to an SU(N_F - N_C) gauge theory. We can identify the dual magnetic gauge group as the new topcolor-like SU(3)_1. Besides, we require that the dual magnetic gauge group be IR-free which sets \[ N_C + 1 < N_F < 3/2N_C. \] Thus, the only possible choice is \[ N_C \geq 6. \] We chose \[ N_C = 7 \] and \[ N_F = 10 \] in our scenario.
Similarly, the $Q$ and $U$ which lead to meta-stable vacua. In our scenario, we can see that the first three terms have 

is to explicitly break the R-symmetry by introducing small R-symmetry violation terms

exact R-symmetry forbids gaugino masses which is not acceptable. One possible solution

anomaly matching conditions. Components of the meson fields

the global symmetry $SU(6)$ theories with the respectively dynamical transmutation scales $\Lambda_e$ and $\Lambda_m$.

In general, the SUSY breaking requires the presence of R-symmetry [10]. However, an exact R-symmetry forbids gaugino masses which is not acceptable. One possible solution is to explicitly break the R-symmetry by introducing small R-symmetry violation terms which lead to meta-stable vacua. In our scenario, we can see that the first three terms have a $U(1)_R$ symmetry with $R(\bar{M}) = 2$ and $R(q) = R(\bar{q}) = R(\bar{f}) = R(f) = 0$. Such an exact $U(1)_R$ symmetry is obviously broken to an approximate one by the last term.

The electric theory is dual to a magnetic $SU(3)_1$ gauge theory with superpotential

$$W = hTr(\bar{q}Mq) + h\Lambda mTr(\bar{M}) + \frac{\Lambda}{M_*}Tr(\kappa\bar{M})\bar{f}f + M_0\bar{f}f,$$  

and the scale is defined as

$$(-1)^{N_f-N_e} \Lambda^b_e + b_m = \Lambda_e^{3N_e-N_f} \Lambda_m^{2N_f-3N_e},$$

where $b_e$ and $b_m$ are respectively the SUSY QCD beta functions of the electric and magnetic theories with the respectively dynamical transmutation scales $\Lambda_e$ and $\Lambda_m$.

Similarly, the $(3,\bar{6})/(\bar{3},6)$ components of the dual quarks $(3,\bar{10})/(\bar{3},10)$ are transformed in terms of $SU(3)_1 \times SU(3)_2 \times SU(2)_L \times U(1)_Y$ as

$$q(3,\bar{6}) \sim (3,3,1)_0 \oplus (3,1,2)_{1/3} \ominus (3,1,1)_{-2/3},$$

$$\bar{q}(\bar{3},6) \sim (\bar{3},3,1)_0 \oplus (\bar{3},1,2)_{-1/3} \ominus (\bar{3},1,1)_{2/3}.$$
Thus, in our theory we can identify
\[
T_L \equiv \begin{pmatrix} t_L \\ b_L \end{pmatrix} \sim (3,1,2)_{1/3}, \quad X_L \equiv \begin{pmatrix} \chi_L \\ \omega_L \end{pmatrix} \sim (3,1,2)_{-1/3}, \quad X_L \equiv \begin{pmatrix} \chi_L \\ \omega_L \end{pmatrix} \sim (3,1,2)_{1/3} ,
\]
\[
P^c_L \equiv \begin{pmatrix} \rho^c_L \\ \sigma^c_L \end{pmatrix} \sim (1,\bar{3},2)_{-1/3}, \quad b^c_L \sim (1,\bar{3},1)_{2/3}, \quad \bar{\omega} L \sim (1,3,1)_{-2/3}, \quad \bar{\rho} L \sim (3,1,1)_{-2/3},
\]
\[
\bar{\omega} L \sim (3,1,2)_{2/3}, \quad \bar{\rho}^c_L \sim (1,\bar{3},1)_{2/3}, \quad H_1 \sim (1,1,2)_{-1}, \quad H_2 \sim (1,1,2)_{1},
\]
\[
\Phi_1 \sim (3,3,1)_0, \quad \Phi_2 \sim (3,3,1)_0, \quad S^a \sim (1,1,1)_0^a \quad (a = 1, 2). \quad (2.12)
\]

From the dynamical superpotential by Seiberg duality, we can identify the following interactions
\[
W \supset h \begin{pmatrix} \chi_L \\ \omega_L \end{pmatrix}^T \Phi_1 \begin{pmatrix} \chi^c_L \\ \omega^c_L \end{pmatrix} + h \begin{pmatrix} t_L \\ b_L \end{pmatrix}^T \Phi_2 \begin{pmatrix} \rho^c_L \\ \sigma^c_L \end{pmatrix} + h \begin{pmatrix} t_L \\ b_L \end{pmatrix}^T \begin{pmatrix} \chi^c_L \\ \omega^c_L \end{pmatrix} S_a
\]
\[
+ h \bar{\omega}^c L \Phi_1 \bar{\omega} L + h \bar{\rho}^c_L \Phi_2 \bar{\sigma} L + h \begin{pmatrix} t_L \\ b_L \end{pmatrix} H_1 \bar{\omega}^c L + h \begin{pmatrix} \chi^c_L \\ \omega^c_L \end{pmatrix} H_2 \bar{\sigma} L + h \bar{\omega} L \bar{\rho}^c L S_a . \quad (2.8)
\]

We also introduce the right-handed top quark chiral supermultiplets in terms of gauge group \( SU(3)_2 \times SU(2)_L \times U(1)_Y \times U(1)_1 \) quantum number
\[
t^c_L \sim (1,\bar{3},1)_{(-4/3,1)} , \quad P_L \equiv (\rho L, \sigma L) \sim (1,3,2)_{(1/3,1)} , \quad (2.9)
\]
and possible Higgs sector to completely break \( U(1)_1 \) at low energy. The necessity of chiral fermions is obvious. SUSY QCD is vector-like and the resulting dual gauge theory is still vector-like. In order to get the chiral fermions, we must introduce by hand at least one chiral component. This fact also appears in the (latticed) extra dimensional interpretation of top seesaw [11]. Localized heavy kink mass terms are necessary to get the localized chiral fermions.

Supersymmetry is broken by rank conditions [8]. Neglecting temporarily the contributions of the messenger fields, we can see from the rank conditions
\[
- F^{*}_{\Phi^j} = \lambda q^j q^j + \Delta_q^j m_0 , \quad (2.10)
\]
that supersymmetry is indeed broken. This is a typical in ISS-like models. The scalar potential is minimized along a classical pseudo-moduli space of vacua which is given by [8]
\[
M = \begin{pmatrix} 0 & 0 \\ 0 & \phi_0 \end{pmatrix}, \quad q = \begin{pmatrix} q_0 \\ 0 \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \tilde{q}_0 \\ 0 \end{pmatrix} , \quad (2.11)
\]
with
\[
q_0 \tilde{q}_0 = m_0 \Lambda , \quad (2.12)
\]
and arbitrary \( \phi_0 \). In our scenario, the \( q_0 \) and \( \tilde{q}_0 \) parts corresponding to the VEVs of \( \Phi_1 \) and \( \Phi_2 \) fields within the dual quark decomposition.

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- 5 -
Flat pseudo-moduli will in general be lifted by quantum effects. The one-loop stable minimum by Coleman-Weinberg potential \[8\] is

\[\phi_0 = 0_{N \times N}, \quad q_0 = M_1 1_{N \times N}, \quad \tilde{q}_0 = M_2 1_{N \times N}, \quad M_1 = M_2 = \sqrt{-m\Lambda}.\] (2.13)

The \(U(1)_R\) violation terms involving the messengers will shift the minimum of \(\tilde{M}\) through one-loop Coleman-Weinberg potential by an amount

\[< \phi_0 > = \Delta \phi_0 \equiv s_1 \sim \frac{\lambda^3 m\Lambda^4}{M\tilde{M}^2}.\] (2.14)

The lifetime of the metastable vacua requires

\[|\epsilon| \sim \sqrt{\frac{m}{\Lambda m}} \ll 1,\] (2.15)

with the tunneling probability \(e^S\) to exceed the lifetime of our universe \(e^{40}\) seconds

\[S \sim \epsilon^{-\frac{4(N_c - N_f)}{N_f - N_c}} > 40.\] (2.16)

There are large viable parameter spaces that can satisfy this requirement.

Possible new SUSY breaking minimum can arise through the combination of \(m_{ij}\) and \(\frac{\kappa_{ij}\Lambda}{M_pl}\tilde{f}f\). For example, a possible new minimum may be possible if \(\tilde{f}f = \frac{mM}{\kappa}.\) However, the lifetime (for tunneling to the new possible minimum) of the previous metastable vacuum can be long enough if we set

\[\frac{M^2 M_s}{\lambda} \gtrsim m\Lambda^2.\] (2.17)

The F-term of the meson fields induce the scalar mass terms for \(T_L, X^c_L, \tilde{\sigma}_L,\) and \(\tilde{\omega}_L\) from the induced superpotential \(\tilde{q}Mq.\) Other soft SUSY masses can be generated through the effective messenger fields

\[M_{mess} = M_0 + \frac{\lambda\Lambda}{M_pl} < \tilde{M} > \simeq M_0,\] (2.18)

with

\[F_{mess} = \frac{\kappa_{ij}\Lambda}{M_s} F_M = \frac{\kappa_{ii}m_0\Lambda^2}{M_s}.\] (2.19)

Thus, we obtain the gaugino masses \[12\]

\[M_a \simeq \frac{\alpha_a}{4\pi} \sum_I n_a(I) \frac{F_M}{M},\] (2.20)

and sfermion masses

\[m_{\phi_a}^2 \simeq 2 \left[ \sum_a \left( \frac{\alpha_a}{4\pi} \right)^2 C_a(i)n_a(I) \right] \left( \frac{F_M}{M} \right)^2.\] (2.21)

Below the scale \(\sqrt{F}\) which is the typical scalar masses for dual squarks, the SUSY QCD reduce to non-supersymmetric dynamics. The gaugino and remaining sfermions can acquire masses from gauge loops.

Thus, at the low energy we get an \(SU(3)_1 \times SU(3)_2 \times SU(2)_L \times U(1)_Y \times U(1)_1\) gauge theory with proper matter contents and interactions. The gauge group of the \(SU(3)_1\) is emergent and almost all the matter contents are composite particles.
3 Top and Bottom Seesaw

It is well known from the topcolor dynamics that the predicted top quark mass is too high if topcolor is responsible for full electroweak symmetry breaking. In order to get realistic top quark mass, top seesaw model was proposed by introducing additional vector-like particles besides the topcolor matter content. In our model, partial composite top and bottom quarks will naturally lead to top and bottom seesaw mechanism.

After $\Phi_1$ and $\Phi_2$ respectively acquire VEVs $M_1$ and $M_2$, the $SU(3)_1 \times SU(3)_2$ gauge symmetry is broken down to $SU(3)_C$. The theory has a set of massless gluons and massive octet colorons. The remaining QCD coupling is

$$\frac{1}{g_c^2} = \frac{1}{h_1^2} + \frac{1}{h_2^2}, \quad \cot \theta = \frac{h_1}{h_2}, \quad (3.1)$$

where $h_1$ and $h_2$ are the gauge couplings for $SU(3)_1$ and $SU(3)_2$, and the massive colorons acquire masses $M_B^2 = (h_1^2 + h_2^2)(M_1^2 + M_2^2)$.

After we integrate out the coloron fields, we obtain the effective four-fermion interaction

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} - (M_2 X_L^T C X_L^c + M_1 T_L^T C P_L^c + s_1 T_L^T C X_L^c) + \mathcal{L}_{\text{int}} \quad (3.2)$$

with

$$\mathcal{L}_{\text{int}} = -\frac{h_2^2}{M_B^2} \left( X_L^c \tilde{\sigma}^\mu \frac{M_A}{2} X_L \right) \left( t_L^c \tilde{\sigma}^\mu \frac{M_A}{2} (t_L^c)^\dagger \right) + (X_L \rightarrow P_L, t_L \rightarrow P_L^c) + \cdots \quad (3.3)$$

After performing the Fierz rearrangement and at the leading order in $1/N_c$, we have

$$\mathcal{L}_{\text{int}} = \frac{h_2^2}{M_B^2} \left[ (\bar{X_L} t_R)(\bar{t_R} X_L) + (\bar{X_L} P_R)(\bar{P_R} X_L) + (\bar{P_L} P_R)(\bar{P_R} P_L) + \cdots \right]. \quad (3.4)$$

We can transform the interaction eigenstates to the partial mass eigenstates by

$$t_L^c \rightarrow t_L' \equiv t_L^c, \quad X_L^c \rightarrow X_L'^c = X_L^c \cos \beta + P_L^c \sin \beta, \quad P_L^c \rightarrow P_L'^c = -X_L^c \sin \beta + P_L^c \cos \beta, \quad (3.5)$$

where

$$\tan(2\beta) = \frac{2s_1 M_1}{s_1^2 + M_2^2 - M_1^2}. \quad (3.6)$$

We define the unitary mixing matrix $T_i \equiv N_i^{-1} T_j$ with

$$\tilde{T}_1 \equiv T_L, \quad \tilde{T}_2 \equiv X_L, \quad \tilde{T}_3 \equiv P_L, \quad T_1 \equiv T_L, \quad T_2 \equiv X_L, \quad T_3 \equiv P_L. \quad (3.7)$$
In this basis, the NJL model takes the form

\[ \mathcal{L} = \mathcal{L}_{\text{kinetic}} - (M_1 X'_L X'_R + M_2 P'_L P'_R) \]

\[ + \frac{h^2}{M_B} \left\{ \left[ \sum_{i=1}^{N_2 T_i} t'_R \right] \left[ \sum_{i=1}^{N_2 T_i} t'_R \right] \right\} \]

\[ + \left[ \sum_{i=1}^{3} \sum_{j=2}^{3} N_{ji} T_i \right] \left( -X'_R \sin \beta + P'_R \cos \beta \right) \]

\[ \times \left[ \left( -X'_R \sin \beta + P'_R \cos \beta \right) \left( \sum_{i=1}^{3} \sum_{j=2}^{3} N_{ji} T_i \right) \right] \] (3.8)

with \( M_1 \) and \( M_2 \) the eigenvalues of the matrix

\[ \begin{pmatrix} s_1^2 + M_2^2 & s_1 M_1 \\ s_1 M_1 & M_1^2 \end{pmatrix}. \] (3.9)

Assume the gauge couplings for \( SU(3)_2 \) and \( U(1)_1 \) get strong quickly towards IR and trigger the fermion condensation. The vacuum is tilted by the \( U(1)_1 \) interactions so that condensation between \( \rho_L \) and \( t'_L \) is disallowed by the repulsive forces of \( U(1)_1 \).

From the expansion, we can see that possible types of dynamical condensations for \( X_L t_R \) are

\[ < t'_L t'_R >, \quad < \chi'L'_L >, \quad < \chi L'_R >, \quad < \rho L'_L >, \] (3.10)

with corresponding mass gap

\[ - \mu_{tt} t'_L t'_R - \mu_{t\chi} \chi L'_L t'_R - \mu_{t\rho} \rho L'_L t'_R. \] (3.11)

And they have the following relations

\[ \mu_{tt} = \mu N_{21}, \quad \mu_{t\chi} = \mu N_{22}, \quad \mu_{t\rho} = \mu N_{23}, \] (3.12)

so that they are not independent. Just as the case for ordinary topcolor, the dynamical mass terms \( \mu \) can be calculated through the gap equations. Tidious calculations give the
Figure 1. The gap equations for quark condensations in the top sector. The red, blue and green crosses denote the $\bar{t}'\chi'$, $\bar{t}'t'$ and $\bar{t}'\rho'$ condensations, respectively. While black crosses denote all the previous three condensations.

expression for $\bar{\chi}^Lt^R$ condensation

$$
\mu N_{22}^{-1} \approx \frac{\hbar^2 N_c}{4\pi^2 M_B^2} \left\{ \frac{N_{22}^3}{22} \mu + \left( \frac{M^2}{M_1^2} + N_{21} \mu M_1 \right) \ln \left( \frac{M_1 + M^2}{M_1} \right) \right.
$$

$$
- \left( \frac{N_{21}^2 + N_{22}^2 + N_{23}^2}{22} \mu^2 \log \left( \frac{M^2 + M_1^2}{M_2^2} \right) \right)
+ \frac{N_{22}^2 \mu^2 M_1^2}{M_2^2} \log \left( \frac{(M^2 + M_1^2) M_1^2}{(M^2 + M_2^2) M_2^2} \right)
$$

$$
+ \left( \frac{N_{21}^2 + N_{22}^2 + N_{23}^2}{22} \mu^2 \log \left( \frac{M^2 + M_2^2}{M_1^2} \right) \right)
+ \frac{N_{22}^2 \mu^2 M_1^2}{M_2^2} \log \left( \frac{(M^2 + M_2^2) M_2^2}{(M^2 + M_1^2) M_1^2} \right)
$$

$$
+ \left( \frac{N_{21}^2 + N_{22}^2 + N_{23}^2}{22} \mu^2 \log \left( \frac{M^2 + M_1^2}{M_2^2} \right) \right)
- \left( \frac{N_{21}^2 + N_{22}^2 + N_{23}^2}{22} \mu^2 \log \left( \frac{M^2 + M_2^2}{M_1^2} \right) \right)
$$

$$
= \mu N_{22}^{-1} \approx \frac{\hbar^2 N_c}{4\pi^2 M_B^2} \left\{ \frac{N_{22}^3}{22} \mu + \left( \frac{M^2}{M_1^2} + N_{21} \mu M_1 \right) \ln \left( \frac{M_1 + M^2}{M_1} \right) \right. 
$$

$$
- \left( \frac{N_{21}^2 + N_{22}^2 + N_{23}^2}{22} \mu^2 \log \left( \frac{M^2 + M_1^2}{M_2^2} \right) \right)
+ \frac{N_{22}^2 \mu^2 M_1^2}{M_2^2} \log \left( \frac{(M^2 + M_1^2) M_1^2}{(M^2 + M_2^2) M_2^2} \right)
$$

$$
+ \left( \frac{N_{21}^2 + N_{22}^2 + N_{23}^2}{22} \mu^2 \log \left( \frac{M^2 + M_2^2}{M_1^2} \right) \right)
+ \frac{N_{22}^2 \mu^2 M_1^2}{M_2^2} \log \left( \frac{(M^2 + M_2^2) M_2^2}{(M^2 + M_1^2) M_1^2} \right)
$$

$$
+ \left( \frac{N_{21}^2 + N_{22}^2 + N_{23}^2}{22} \mu^2 \log \left( \frac{M^2 + M_1^2}{M_2^2} \right) \right)
- \left( \frac{N_{21}^2 + N_{22}^2 + N_{23}^2}{22} \mu^2 \log \left( \frac{M^2 + M_2^2}{M_1^2} \right) \right) \right\}.
$$

From the gap equation, we can get the analytical expressions for the condensation scale $\mu$ and the effective critical coupling. This approach with mass insertion is an approximation at large $N_c$ expansion. We will deduce more precise forms of the condensation in symmetry broken phase.
Similarly, we can get the other condensations

\[ < \bar{t}_L \rho_R' >, < \bar{\chi}_L \rho_R' >, < \bar{\rho}_L' \rho_R' >, \cdots \]  

(3.14)

to give \( < \bar{X}_LP_L > \) and \( < \bar{P}_LP_L > \). After all condensation occurs, we get the most general possible mass matrix for top sector

\[
(t_L, \chi_L, \rho_L) \begin{pmatrix}
0 & s_a & M_1 \\
\mu & M_2 & \mu_1 \\
0 & 0 & \mu_2
\end{pmatrix} \begin{pmatrix} t_L^c \\ \chi_L^c \\ \rho_L^c \end{pmatrix}.
\]  

(3.15)

The mass eigenvalues and eigenstates can be obtained by the following unitary transformations

\[
\mathcal{M} = U^\dagger_L \mathcal{M}_{\text{diag}} U_R.
\]  

(3.16)

The analytical expressions are very complicate. Careful analysis indicates that the three mass eigenvalues are of order

\[
(m_\chi)_{\text{phy}}^2 \equiv \lambda_2^2 \sim M_1^2, \quad (m_\rho)_{\text{phy}}^2 \equiv \lambda_3^2 \sim M_2^2, \quad (m_t)_{\text{phy}}^2 \equiv \lambda_1^2 \approx \frac{s_a^2 \mu^2 \mu_2^2}{M_1^2 M_2^2},
\]  

(3.17)
in case \( M_2 = M_1 \gtrsim s_a \gg \mu \). We will not give the explicit expression of the mass eigenstates for the top quark sector. We just parameterize them as

\[
(t_L^m, \chi_L^m, \rho_L^m)^T = U^L_{ij} (t_L, \chi_L, \rho_L)^T, \quad (t_R^m, \chi_R^m, \rho_R^m)^T = U^R_{ij} (t_L, \chi_L, \rho_L)^T,
\]  

(3.18)

with the lowest mass eigenstates \( t_{L,R}^m \) corresponding to the physical top quark. One Higgs doublet field in the multiple-Higgs-doublets are the condensations

\[ H_1 \sim (\bar{\chi}_L t_R, \bar{\omega}_L t_R) = \left( (h_0 + \pi_1^0 + v_0)/\sqrt{2}, \pi^+ \right), \]  

(3.19)

and additional two singlets (and triplets) are from the condensations

\[ H_2 \sim \bar{X}_L \otimes P_R = \Delta_2(3) \oplus S_2(1), \quad H_3 \sim \bar{P}_L \otimes P_R = \Delta_3(3) \oplus S_3(1). \]  

(3.20)

We obtain the precise gap equation of this theory in the broken phase [13] at the cut-off
\[
\mathcal{L}_\Lambda = \left( \bar{t}_L , \chi_L , \bar{\rho}_L \right) \begin{pmatrix}
0 & s_a & M_1 \\
\mu & M_2 & \mu \mu_1 \\
0 & 0 & \mu_2
\end{pmatrix} \begin{pmatrix}
t_R \\
\chi_R \\
\bar{\rho}_R
\end{pmatrix} - \frac{h_2 M}{\sqrt{2} M_B} \bar{\chi} t (h_0 + v_{h_0}) - \frac{i h_2 M}{\sqrt{2} M_B} \bar{\chi} \gamma^5 t \pi_i^0 \\
- \frac{h_2 M}{M_B} \bar{\omega}_L t_R \pi_i^- - \frac{h_2 M}{\sqrt{2} M_B} \bar{X}_L P_R (S_2 + \Delta_2 + v_{S_i}) - \frac{h_2 M}{\sqrt{2} M_B} \bar{\rho}_L P_R (S_3 + \Delta_3 + v_{S_i}) \\
- \frac{1}{2} \frac{M^2}{2} \left( \bar{h}_0^2 + \sum_{i=0}^{3} (S_i^2 + 2 |\Delta_i|^2) \right) - \frac{M^2}{2} \left( \bar{v}_{h_0} \bar{h}_0 + \sum_{i} v_{S_i} S_i \right) \\
\geq - \frac{1}{2} \frac{M^2}{2} \left( \bar{h}_0^2 + \sum_{i=0}^{3} (S_i^2 + 2 |\Delta_i|^2) \right) - \frac{M^2}{2} \left( \bar{v}_{h_0} \bar{h}_0 + \sum_{i} v_{S_i} S_i \right) \\
- \frac{h_2 M}{\sqrt{2} M_B} \left( U_{L21}^{-1} \bar{t}_L + U_{L22}^{-1} \bar{\chi}_L + U_{L23}^{-1} \bar{\rho}_L \right) \left( U_{R11}^{-1} t_R + U_{R12}^{-1} \chi_R + U_{R13}^{-1} \bar{\rho}_R \right) h_0 \\
- \frac{h_2 M}{\sqrt{2} M_B} \left( U_{L31}^{-1} \bar{t}_L + U_{L32}^{-1} \bar{\chi}_L + U_{L33}^{-1} \bar{\rho}_L \right) \left( U_{R31}^{-1} t_R + U_{R32}^{-1} \chi_R + U_{R33}^{-1} \bar{\rho}_R \right) S_2 \\
- \frac{h_2 M}{\sqrt{2} M_B} \left( U_{L31}^{-1} \bar{t}_L + U_{L32}^{-1} \bar{\chi}_L + U_{L33}^{-1} \bar{\rho}_L \right) \left( U_{R31}^{-1} t_R + U_{R32}^{-1} \chi_R + U_{R33}^{-1} \bar{\rho}_R \right) S_3. \\
(3.21)
\]

After we integrate out the heavy fields \( \chi^m, \rho^m \), we obtain the low energy effective theory

\[
\mathcal{L}_\mu = - \lambda_1 \bar{t}_L t_R^m + \frac{h_2 M}{\sqrt{2} M_B} U_{L21}^{-1} \bar{t}_L U_{R11}^{-1} t_R^m h_0 + \frac{h_2 M}{\sqrt{2} M_B} U_{L21}^{-1} \bar{t}_L U_{R31}^{-1} t_R S_2 \\
+ \frac{h_2 M}{\sqrt{2} M_B} U_{L31}^{-1} \bar{t}_L U_{R31}^{-1} t_R S_3 + \frac{1}{2} Z_{h_0} (\partial_\mu h_0)^2 + \frac{1}{2} \sum_i Z_{S_i} (\partial_\mu S_i)^2 + \sum_i Z_{\Delta_i} |\partial_\mu \Delta_i|^2 \\
- \frac{1}{2} \left( \bar{M}_{h_0}^2 \bar{h}_0^2 + \sum_i M_{\Delta_i}^2 |\Delta_i|^2 \right) - \sum_i M_{\Delta_i}^2 |\Delta_i|^2 - \sum_i M_{\Delta_i} S_i h_0 \\
- \bar{M}_{23} S_2 S_3 - M_{23} \Delta_2 \Delta_3 - V(h_0, S_i, \Delta_i) - \Delta T_{h_0} h_0 - \sum_i \Delta T_{S_i} S_i, \\
(3.22)
\]

The tadpole cancellation condition is

\[
Z_{h_0}^{1/2} \Delta T_{h_0} = Z_{h_0}^{-1/2} v_{h_0} M^2 + \delta \tilde{T}_{h_0} = 0, \\
(3.23)
\]

with \( \delta \tilde{T} \) the one-loop tadpole contributions. Through the tadpole cancellation condition we can obtain the exact gap equation

\[
\mu = \frac{h_2^2 M^2 N_c}{8 \pi^2 M_B^3} \left[ \sum_{i=1}^{3} \Re \left( U_{L2i} U_{L2i}^{-1} \right) \left( \lambda_i - \frac{\lambda_3^3}{M^2} \ln \left( \frac{M^2 + \lambda_3^2}{\lambda_i^2} \right) \right) \right], \\
(3.24)
\]

with the fact that \( \mu = Z_{h_0}^{-1/2} h_2 M v_{h_0} / \sqrt{M_B} \). Such form is consistent with the previous large \( N_c \) expansion approach with mass insertion.
From the wave function renormalization of the composite Higgs fields, we can get the precise form of the Pagels-Stokar formula

\[ v_{h_0}^2 = \frac{\mu^2 M^2 N_c}{8\pi^2 M_B^2} \left[ \sum_{i=1}^3 2 \left| U_{L2i}^{-1} U_{R1i}^{-1} \right|^2 \log \left( \frac{M^2 + \lambda_i^2}{\lambda_i^2} \right) \right. \]

\[ + \sum_{i=1}^3 \sum_{j=1;i<j}^3 \left( \left| U_{L2i}^{-1} U_{R1j}^{-1} \right|^2 + \left| U_{L3j}^{-1} U_{R1i}^{-1} \right|^2 \right) \log \left( \frac{M^2 + \lambda_j^2}{\lambda_j^2} \right) \right], \tag{3.25} \]

with other possible Higgs VEVs from bottom sector \( v_{h_i}^2 (i \neq 0) \)

\[ \sum_i v_{h_i}^2 = v_{EW}^2. \tag{3.26} \]

The VEV of \( S_3 \) breaks the \( U(1)_1 \) gauge symmetry completely due to its non-vanishing \( U(1)_1 \) quantum number. The expression of \( S_3 \) will be given in subsequent Section. Due to the mixing in the Higgs sector, the physical Higgs fields can be obtained by diagonalizing the relevant mass matrix. We will discuss the complete Higgs sector after we include the bottom-type quarks.

Similar setting can be seen for the bottom quark. We rewrite the relevant terms for bottom quarks

\[
\begin{align*}
\bar{b}_L^c & \sim (1,3,1)_{(-2/3,0)}, \quad \tilde{\omega}_L \sim (1,3,1)_{(0,-2/3)}, \quad \tilde{\sigma}_L \sim (3,1,1)_{(-1/3,-1/3)}, \\
\bar{\omega}_L^c & \sim (3,1,1)_{(1/3,1/3)}, \quad \bar{\sigma}_L^c \sim (1,3,1)_{(0,2/3)}, \quad H_1 \sim (1,1,2)_{(-1,0)}, \\
H_2 & \sim (1,1,2)_{(1,0)},
\end{align*}
\tag{3.27}
\]

and the induced interactions

\[ W \supset h \tilde{\omega}_L^c \Phi_1 \tilde{\omega}_L + h \tilde{\sigma}_L^c \Phi_2 \bar{\sigma}_L + h \left( \begin{array}{c} t_L \\ b_L \end{array} \right) H_1 \tilde{\omega}_L^c + h \left( \begin{array}{c} \lambda_L^c \\ \omega_L^c \end{array} \right) H_2 \tilde{\sigma}_L + h \tilde{\omega}_L \tilde{\sigma}_L^c S. \tag{3.28} \]

We can see from the identification that the most general bottom quark mass matrix is

\[
(b_L, \omega_L, \sigma_L, \tilde{\omega}_L, \tilde{\sigma}_L) \left( \begin{array}{ccccc}
0 & s_1 & M_1 & 0 & 0 \\
\bar{\mu} & M_2 & \mu_1 & 0 & \mu_3 \\
0 & 0 & \mu_2 & 0 & \mu_4 \\
\mu_5 & 0 & \mu_6 & M_1 & \mu_7 + s_\alpha \\
0 & 0 & 0 & 0 & M_2
\end{array} \right) \left( \begin{array}{c}
b_L^c \\
\tilde{\omega}_L^c \\
\tilde{\sigma}_L^c \\
\omega_L^c \\
\sigma_L^c
\end{array} \right). \tag{3.29} \]

Similarly, we can diagonalize the mass matrix and obtain the relevant eigenvalues. We note that the determinant of the mass matrix is

\[ \det M_b = s_1 \mu_2 M_1 M_2, \tag{3.30} \]

which is important to determine the lightest bottom-type quark masses. For \( M_2 = M_1 \gtrsim s_1 \gg \mu_i \), we have the eigenvalues of various mass eigenstate in order

\[
\tilde{\lambda}_2^2 \equiv \tilde{m}^2_{\tilde{\omega}_m} \sim M_1^2, \quad \tilde{\lambda}_3^2 \equiv \tilde{m}^2_{\tilde{\sigma}_m} \sim 2M_1^2, \quad \tilde{\lambda}_4 \equiv \tilde{m}^2_{\tilde{\omega}_m} \sim M_2^2/2,
\]

\[
\lambda_5^2 \equiv \tilde{m}^2_{\tilde{\sigma}_m} \sim M_2^2/2, \quad \lambda_6^2 \equiv \tilde{m}^2_{\tilde{\omega}_m} \sim s_1^2 \mu_2^2 M_1^2 M_2^2. \tag{3.31} \]
Here the expression for the lightest bottom-type quark mass is not precise. This formula is used to determine the order of the physical bottom mass. We can also parameterize the mixings in the bottom sector as

\[
(b_m^L, \omega_m^L, \sigma_m^L, \tilde{\omega}_m^L) = Z_{ij}^L(b_L, \omega_L, \sigma_L, \tilde{\omega}_L),
\]

\[
(b_m^R, \omega_m^R, \sigma_m^R, \tilde{\omega}_m^R) = Z_{ij}^R(b_R, \omega_R, \sigma_R, \tilde{\omega}_R).
\]

(3.32)

We can introduce auxiliary fields in symmetry breaking phase to obtain the precise gap equations

\[
\mathcal{L}_\lambda^b = -\tilde{\lambda}_1 b_m^L b_m^R - \tilde{\lambda}_2 \tilde{\omega}_m^L \tilde{\omega}_m^R - \tilde{\lambda}_3 \tilde{\sigma}_m^L \sigma_m^R - \tilde{\lambda}_4 \tilde{\omega}_m^R \omega_m^R - \tilde{\lambda}_5 \tilde{\sigma}_m^R \sigma_m^R - \frac{h_2M}{M_B} \tilde{\omega}_L b_R \tilde{H}_1
\]

\[
- \frac{h_2M}{\sqrt{2M_B}} \tilde{X}_L P_R(\Delta_2 + S_2 + v_{S_2}) - \frac{h_2M}{\sqrt{2M_B}} \tilde{P}_L \bar{P}_R(\Delta_3 + S_3 + v_{S_3}) - \frac{h_2M}{M_B} \tilde{\omega}_L b_R \tilde{S}_4
\]

\[
- \frac{h_2M}{M_B} \tilde{\omega}_L P_R \tilde{H}_2 - \frac{h_2M}{M_B} \tilde{X}_L \tilde{\sigma}_R \tilde{H}_3 - \frac{h_2M}{M_B} \tilde{P}_L \tilde{\sigma}_R \tilde{H}_4 - \frac{h_2M}{\sqrt{2M_B}} \bar{\tilde{\sigma}}_L \tilde{\sigma}_R \tilde{S}_5
\]

\[
- \frac{h_2M}{M_B} \tilde{\sigma}_L \tilde{\sigma}_R \tilde{S}_6 - M^2 \left( \sum_{i=1}^{4} |\tilde{H}_i|^2 + \sum_{i=2}^{6} |\tilde{S}_i|^2 + \sum_{i=2}^{3} |\Delta_i|^2 \right).
\]

(3.33)

Again we can integrate out the heavy modes and obtain the low energy effective interactions

\[
\mathcal{L}_\mu^b = -\tilde{\lambda}_1 b_m^L b_m^R - \frac{h_2M}{\sqrt{2M_B}} Z_{L21}^{-1} Z_{R11}^{-1} b_m^L b_m^R (h_1 + v_{h_1}) - \frac{h_2M}{\sqrt{2M_B}} \left[ Z_{L21}^{-1} Z_{R31}^{-1} b_m^L b_m^R (\Delta_{2,0} + S_2 + v_{S_2}) \right]
\]

\[
+ Z_{L21}^{-1} Z_{R31}^{-1} b_m^L b_m^R (\Delta_{3,0} + S_3 + v_{S_3}) - \frac{h_2M}{\sqrt{2M_B}} \left\{ Z_{L41}^{-1} Z_{R41}^{-1} \left[ b_m^m(b_4 + v_{h_2}) + i b_m^m \gamma \tilde{b}_4^m \tilde{a}_4^m \right] + Z_{L21}^{-1} Z_{R51}^{-1} \left[ b_m^m(h_3 + v_{h_3}) + i b_m^m \gamma \tilde{b}_4^m \tilde{a}_4^m \right] + Z_{L41}^{-1} Z_{R51}^{-1} \left[ b_m^m(S_5 + v_{S_5}) + i b_m^m \gamma \tilde{b}_4^m \tilde{a}_4^m \right] \right\}
\]

\[
- \frac{1}{2} \left( \sum_{i=1}^{4} Z_{S_i} (\partial_{\mu} S_i)^2 + \frac{1}{2} \sum_{i=2}^{6} Z_{S_i} (\partial_{\mu} S_i)^2 + \frac{1}{2} \sum_{i=2}^{3} Z_{S_i} (\partial_{\mu} S_i)^2 + \sum_{i=2}^{3} Z_{S_i} (\partial_{\mu} S_i)^2 \right)
\]

\[
- Z_{\Delta_i} (\partial_{\mu} \Delta_i)^2 + \frac{1}{2} \sum_{i=1}^{4} Z_{S_i} (\partial_{\mu} S_i)^2 - \frac{1}{2} \sum_{i=2}^{6} Z_{S_i} (\partial_{\mu} S_i)^2 + \frac{1}{2} \sum_{i=2}^{3} Z_{S_i} (\partial_{\mu} S_i)^2 + \sum_{i=2}^{3} Z_{S_i} (\partial_{\mu} S_i)^2
\]

\[
- \frac{1}{2} \left( \sum_{i=1}^{4} M_{h_i}^2 h_i^2 + \frac{1}{2} \sum_{i=2}^{6} M_{S_i}^2 S_i^2 \right) + \sum_{i=2}^{3} M_{S_i}^2 |\Delta_i|^2 - \frac{1}{2} \sum_{i=2}^{4} \sum_{j=1}^{6} M_{S_i}^2 S_i S_j - \frac{1}{2} \sum_{i=2}^{6} \sum_{j=2}^{3} M_{S_i}^2 S_i S_j
\]

\[
- \frac{1}{2} \sum_{i=1}^{4} \sum_{j=1}^{3} M_{h_i} h_i h_j - M_{32} \Delta_2 \Delta_3 - V(h_i, S_i, \Delta_i) - \sum_{i=1}^{4} \Delta_{T_i} h_i - \sum_{i=2}^{6} \Delta_{T_i} S_i.
\]

(3.34)

where we use the parameterization

\[
\tilde{H}_i(i = 1, 2, 3, 4) \sim \left( \frac{n_i^+ + n_i^0}{\sqrt{2}}(h_i + n_i^0) + v_{h_i} \right), \quad \tilde{S}_i(i = 4, 5, 6) \sim \frac{1}{\sqrt{2}}(S_i + \tilde{n}_i^0 + v_{S_i}).
\]

(3.35)

The tadpole cancelation conditions

\[
Z_{h_i}^{1/2} \Delta_{T_h} = Z_{h_i}^{-1/2} v_{h_i} M^2 + \delta T_{h_i} = 0,
\]

(3.36)

\[
Z_{S_i}^{1/2} \Delta_{T_S} = Z_{S_i}^{-1/2} v_{S_i} M^2 + \delta T_{S_i} = 0.
\]

(3.37)
determine the exact gap equations

\[ \mu_{H_1} = G_{21}^B, \mu_{H_2} = G_{43}^B, \mu_{H_3} = G_{25}^B, \mu_{H_4} = G_{35}^B, \mu_{S_3} = G_{44}^B, \mu_{S_5} = G_{45}^B, \mu_{S_6} = G_{55}^B \]  

(3.38)

while \( \mu_{S_2} \) and \( \mu_{S_3} \) are

\[ \mu_{S_2} = G_{23}^B + G_{25}^T, \mu_{S_3} = G_{23}^B + G_{33}^T. \]  

(3.39)

Here we use the notation

\[ \mu_{H_1} \equiv \tilde{\mu}, \mu_{S_2} \equiv \mu_1, \mu_{S_3} \equiv \mu_2, \mu_{H_3} \equiv \mu_3, \mu_{H_4} \equiv \mu_4, \mu_{S_4} \equiv \mu_5, \mu_{H_6} \equiv \mu_6, \mu_{S_5} \equiv \mu_7, \]

and define

\[
G_{ab}^B = \frac{h_2^2 M^2 N_c}{8\pi^2 M_B^2} \left[ \sum_{i=1}^{5} 2\Re (Z_{Lai}^{-1} Z_{Rbi}^{-1}) \left( \frac{\lambda_i - \frac{\lambda_i^2}{M^2} \ln \left( \frac{M^2 + \lambda_i^2}{\lambda_i^2} \right) \right) \right], \tag{3.40} 
\]

\[
G_{ab}^T = \frac{h_2^2 M^2 N_c}{8\pi^2 M_B^2} \left[ \sum_{i=1}^{3} 2\Re (U_{Lai}^{-1} U_{Rbi}^{-1}) \left( \lambda_i - \frac{\lambda_i^2}{M^2} \ln \left( \frac{M^2 + \lambda_i^2}{\lambda_i^2} \right) \right) \right]. \tag{3.41} 
\]

From the wave function normalization, we can get the Pagels-Stokar formula in the bottom sector

\[ v_{h_i}^2 = \mu_{H_i} P_{21}^B, \quad v_{h_2}^2 = \mu_{H_2} P_{43}^B, \quad v_{h_3}^2 = \mu_{H_3} P_{25}^B, \quad v_{h_4}^2 = \mu_{H_4} P_{35}^B, \quad v_{h_5}^2 = \mu_{H_5} P_{35}^B, \quad v_{h_6}^2 = \mu_{H_6} P_{45}^B, \quad v_{h_7}^2 = \mu_{H_7} P_{55}^B, \]

(3.42)

with \( v_{h_i}^2 \) the \( U(1)_2 \) breaking scale and the fact \( \mu_{S_3} = Z_{S_3}^{-1/2} h_2 M v_{S_3}/\sqrt{\Sigma M_B} \). Here, we define

\[
P_{ab}^B = \frac{M^2 N_c}{8\pi^2 M_B^2} \left[ \sum_{i=1}^{5} 2 \left| Z_{Lai}^{-1} Z_{Rbi}^{-1} \right|^2 \log \left( \frac{M^2 + \lambda_i^2}{\lambda_i^2} \right) \right], \tag{3.40} 
\]

\[
P_{ab}^T = \frac{M^2 N_c}{8\pi^2 M_B^2} \left[ \sum_{j=1}^{3} 2 \left| U_{Lai}^{-1} U_{Rbi}^{-1} \right|^2 \log \left( \frac{M^2 + \lambda_j^2}{\lambda_j^2} \right) \right]. \tag{3.41} 
\]

The physical Higgs fields can be obtained by diagonalizing the \( 10 \times 10 \) mixing mass matrix between \( h_i (i = 0, \cdots, 4) \) and \( S_j (j = 2, \cdots, 6) \). Each entry can be calculated by the one-loop diagrams in the large \( N_c \) fermion bubble approximation. For example, the diagonal
entry can be calculated to be

$$
\begin{align*}
    m^2_{h_0} &= \frac{M_T^{21}}{Z_{h_0}}, \quad m^2_{h_1} = \frac{M_B^{21}}{Z_{h_1}}, \quad m^2_{h_2} = \frac{M_R^{21}}{Z_{h_2}}, \quad m^2_{h_3} = \frac{M_R^{23}}{Z_{h_3}}, \quad m^2_{h_4} = \frac{M_R^{25}}{Z_{h_4}}; \\
    m^2_{S_2} &= \frac{M_T^{23} + M_B^{23} - M^2}{Z_{S_2}}, \quad m^2_{S_3} = \frac{M_T^{33} + M_B^{33} - M^2}{Z_{S_3}}, \\
    m^2_{S_4} &= \frac{M_B^{41}}{Z_{S_4}}, m^2_{S_5} = \frac{M_B^{43}}{Z_{S_5}}, m^2_{S_6} = \frac{M_B^{55}}{Z_{S_6}},
\end{align*}
$$

(3.45)

where the wave function renormalizations are

$$
\begin{align*}
    Z_{h_0} &= \frac{h_2^2}{2}P_{21}^T, \quad Z_{h_1} = \frac{h_2^2}{2}P_{21}^B, \quad Z_{h_2} = \frac{h_2^2}{2}P_{43}^B, \quad Z_{h_3} = \frac{h_2^2}{2}P_{25}^B, \quad Z_{h_4} = \frac{h_2^2}{2}P_{35}^B, \\
    Z_{S_2} &= \frac{h_2^2}{2}(P_{23}^B + P_{23}^T), \quad Z_{S_3} = \frac{h_2^2}{2}(P_{33}^B + P_{33}^T), \\
    Z_{S_4} &= \frac{h_2^2}{2}P_{41}^B, \quad Z_{S_5} = \frac{h_2^2}{2}P_{45}^B, \quad Z_{S_6} = \frac{h_2^2}{2}P_{53}^B,
\end{align*}
$$

(3.46)

and the definitions for $M_{ab}^T$ and $M_{ab}^B$ are

\begin{align*}
    M_{ab}^T &\equiv \left\{ \left( 1 - \frac{h_2^2M^2}{8\pi^2M_B^2} \right) M^2 + \frac{h_2^2M^2}{8\pi^2M_B^2} \left[ 4 \sum_{i=1}^{3} \Re(U_{La}^{-1}sU_{Rb})^2 \lambda_i^2 \ln \left( \frac{M^2}{\lambda_i^2} \right) \right] \\
    &+ \sum_{i,j=1,i>j}^{3} 2\Re[(U_{La}^{-1}sU_{Rb})(U_{Laj}^{-1}sU_{Rbi})] \frac{\lambda_i\lambda_j}{(\lambda_i^2 - \lambda_j^2)} [\lambda_i^2 \ln \left( \frac{M^2}{\lambda_i^2} \right) - \lambda_j^2 \ln \left( \frac{M^2}{\lambda_j^2} \right)] \right\}, \\
    M_{ab}^B &\equiv \left\{ \left( 1 - \frac{h_2^2M^2}{8\pi^2M_B^2} \right) M^2 + \frac{h_2^2M^2}{8\pi^2M_B^2} \left[ 4 \sum_{i=1}^{3} \Re(Z_{La}^{-1}sZ_{Rb})^2 \tilde{\lambda}_i^2 \ln \left( \frac{M^2}{\tilde{\lambda}_i^2} \right) \right] \\
    &+ \sum_{i,j=1,i>j}^{5} 2\Re[(Z_{La}^{-1}sZ_{Rb})(Z_{Laj}^{-1}sZ_{Rbi})] \frac{\tilde{\lambda}_i\tilde{\lambda}_j}{(\lambda_i^2 - \lambda_j^2)} [\lambda_i^2 \ln \left( \frac{M^2}{\lambda_i^2} \right) - \lambda_j^2 \ln \left( \frac{M^2}{\lambda_j^2} \right)] \right\}.
\end{align*}

(3.47)

The mixings between the Higgs fields can be calculated accordingly. For simplicity, we
can define

\[
F_{a\beta,cd}^{BB} = -\frac{h_i^2 M_B^2 N_c}{8\pi^2 M_B^2} M^2 + \frac{h_j^2 M_B^2 N_c}{8\pi^2 M_B^2} \left[ \sum_{i=1}^{5} 4 \Re (Z_{La_i}^{-1} Z_{Rb_i}^{-1}) \Re (Z_{La_1}^{-1} Z_{Rd_1}^{-1}) \tilde{\lambda}_i^2 \ln \left( \frac{M^2}{\lambda_i^2} \right) \right] \\
+ \sum_{i,j=1; i > j}^{5} [(Z_{La_i}^{-1} Z_{Rb_i}^{-1})(Z_{La_j}^{-1} Z_{Rb_j}^{-1}) + (Z_{La_1}^{-1} Z_{Rb_1}^{-1})(Z_{La_1}^{-1} Z_{Rd_1}^{-1})] \\
\times \left( \tilde{\lambda}_i \tilde{\lambda}_j \left[ \tilde{\lambda}_i^2 \ln \left( \frac{M^2}{\lambda_i^2} \right) - \tilde{\lambda}_j^2 \ln \left( \frac{M^2}{\lambda_j^2} \right) \right] + \sum_{i,j=1; i > j}^{5} \left( (Z_{La_i}^{-1} Z_{Rb_i}^{-1})(Z_{La_j}^{-1} Z_{Rd_j}^{-1}) \right) \right) + (Z_{La_j}^{-1} Z_{Rb_j}^{-1})(Z_{La_i}^{-1} Z_{Rd_i}^{-1}) \left[ \tilde{\lambda}_i^2 \ln \left( \frac{M^2}{\lambda_i^2} \right) - \tilde{\lambda}_j^2 \ln \left( \frac{M^2}{\lambda_j^2} \right) \right], 
\tag{3.48}
\]

\[
F_{a\beta,cd}^{TT} = -\frac{h_i^2 M_B^2 N_c}{8\pi^2 M_B^2} M^2 + \frac{h_j^2 M_B^2 N_c}{8\pi^2 M_B^2} \left[ \sum_{i=1}^{3} 4 \Re (U_{La_i}^{-1} U_{Rb_i}^{-1}) \Re (U_{La_1}^{-1} U_{Rd_1}^{-1}) \lambda_i^2 \ln \left( \frac{M^2}{\lambda_i^2} \right) \right] \\
+ \sum_{i,j=1; i > j}^{3} [(U_{La_i}^{-1} U_{Rb_i}^{-1})(U_{La_j}^{-1} U_{Rb_j}^{-1}) + (U_{La_1}^{-1} U_{Rb_1}^{-1})(U_{La_1}^{-1} U_{Rd_1}^{-1})] \\
\times \left( \frac{\lambda_i \lambda_j}{(\lambda_i^2 - \lambda_j^2)} \left[ \lambda_i^2 \ln \left( \frac{M^2}{\lambda_i^2} \right) - \lambda_j^2 \ln \left( \frac{M^2}{\lambda_j^2} \right) \right] + \sum_{i,j=1; i > j}^{3} \left( (U_{La_i}^{-1} U_{Rb_i}^{-1}) \right) \right) + (U_{La_j}^{-1} U_{Rb_j}^{-1})(U_{La_i}^{-1} U_{Rd_i}^{-1}) \left[ \lambda_i^2 \ln \left( \frac{M^2}{\lambda_i^2} \right) - \lambda_j^2 \ln \left( \frac{M^2}{\lambda_j^2} \right) \right], 
\tag{3.49}
\]

where the mixing terms between \( h_i \) and \( h_j \) are

\[
M_{12}^{hh} = \frac{F_{21.43}^{BB}}{\sqrt{Z_{h_1} Z_{h_2}}}, M_{13}^{hh} = \frac{F_{21.25}^{BB}}{\sqrt{Z_{h_1} Z_{h_3}}}, M_{14}^{hh} = \frac{F_{21.35}^{BB}}{\sqrt{Z_{h_1} Z_{h_4}}}, \\
M_{23}^{hh} = \frac{F_{43.25}^{BB}}{\sqrt{Z_{h_2} Z_{h_3}}}, M_{24}^{hh} = \frac{F_{43.35}^{BB}}{\sqrt{Z_{h_2} Z_{h_4}}}, M_{34}^{hh} = \frac{F_{25.35}^{BB}}{\sqrt{Z_{h_3} Z_{h_4}}}, M_{0h}^{hh} = 0, 
\tag{3.50}
\]

the mixing terms within \( S_i \) are

\[
M_{23}^{SS} = \frac{F_{23.33}^{TT} + F_{23.33}^{BB}}{\sqrt{Z_{S_2} Z_{S_3}}}, M_{24}^{SS} = \frac{F_{23.41}^{BB}}{\sqrt{Z_{S_2} Z_{S_4}}}, M_{25}^{SS} = \frac{F_{23.45}^{BB}}{\sqrt{Z_{S_2} Z_{S_5}}}, \\
M_{34}^{SS} = \frac{F_{43.35}^{BB}}{\sqrt{Z_{S_3} Z_{S_4}}}, M_{35}^{SS} = \frac{F_{43.45}^{BB}}{\sqrt{Z_{S_3} Z_{S_5}}}, \\
M_{45}^{SS} = \frac{F_{41.45}^{BB}}{\sqrt{Z_{S_4} Z_{S_5}}}, M_{46}^{SS} = \frac{F_{41.55}^{BB}}{\sqrt{Z_{S_4} Z_{S_6}}}, 
\tag{3.51}
\]

\[ -16 - \]
the $h_i$ and $S_j$ mixing terms are

\[ M^{hS}_{02} = \frac{F_{TT}^{21,23}}{\sqrt{h_i Z_j Z_2}} \, , \quad M^{hS}_{03} = \frac{F_{TT}^{21,33}}{\sqrt{h_i Z_j Z_3}} \, , \quad M^{hS}_{04} = M_05 = M_06 = 0 \, , \quad M^{hS}_{12} = \frac{F_{BB}^{21,23}}{\sqrt{Z_{h_1} Z_{S_2}}} , \]

\[ M^{hS}_{13} = \frac{F_{BB}^{21,33}}{\sqrt{h_i Z_j Z_3}} \, , \quad M^{hS}_{14} = \frac{F_{BB}^{21,41}}{\sqrt{Z_{h_1} Z_{S_4}}} \, , \quad M^{hS}_{15} = \frac{F_{BB}^{21,45}}{\sqrt{Z_{h_1} Z_{S_5}}} \, , \quad M^{hS}_{16} = \frac{F_{BB}^{21,55}}{\sqrt{Z_{h_1} Z_{S_6}}} , \]

\[ M^{hS}_{22} = \frac{F_{BB}^{43,23}}{\sqrt{h_i Z_j Z_2}} \, , \quad M^{hS}_{23} = \frac{F_{BB}^{43,33}}{\sqrt{Z_{h_2} Z_{S_2}}} \, , \quad M^{hS}_{24} = \frac{F_{BB}^{43,41}}{\sqrt{Z_{h_2} Z_{S_3}}} \, , \quad M^{hS}_{25} = \frac{F_{BB}^{43,45}}{\sqrt{Z_{h_2} Z_{S_4}}} , \]

\[ M^{hS}_{26} = \frac{F_{BB}^{43,55}}{\sqrt{Z_{h_3} Z_{S_6}}} \, , \quad M^{hS}_{32} = \frac{F_{BB}^{25,23}}{\sqrt{Z_{h_2} Z_{S_2}}} \, , \quad M^{hS}_{33} = \frac{F_{BB}^{25,33}}{\sqrt{Z_{h_2} Z_{S_3}}} \, , \quad M^{hS}_{34} = \frac{F_{BB}^{25,41}}{\sqrt{Z_{h_2} Z_{S_4}}} , \]

\[ M^{hS}_{35} = \frac{F_{BB}^{25,55}}{\sqrt{Z_{h_3} Z_{S_6}}} \, , \quad M^{hS}_{36} = \frac{F_{BB}^{35,23}}{\sqrt{Z_{h_3} Z_{S_2}}} \, , \quad M^{hS}_{42} = \frac{F_{BB}^{35,41}}{\sqrt{Z_{h_4} Z_{S_2}}} \, , \quad M^{hS}_{43} = \frac{F_{BB}^{35,43}}{\sqrt{Z_{h_4} Z_{S_3}}} , \]

\[ M^{hS}_{44} = \frac{F_{BB}^{35,45}}{\sqrt{Z_{h_4} Z_{S_4}}} \, , \quad M^{hS}_{45} = \frac{F_{BB}^{35,55}}{\sqrt{Z_{h_4} Z_{S_6}}} \, , \quad M^{hS}_{46} = \frac{F_{BB}^{35,55}}{\sqrt{Z_{h_4} Z_{S_6}}} \, . \] (3.52)

and the relations are

\[ M^{hS}_{ij} = \frac{F_{TT,BB}^{ab,cd}}{\sqrt{Z_{h_i} Z_{S_j}}} \, , \quad M^{hS}_{ji} = \frac{F_{TT,BB}^{cd,ab}}{\sqrt{Z_{S_i} Z_{h_j}}} . \] (3.53)

One combination of $\pi_t^{0,\pm}$ and $\pi_{bi}^{0,\pm}$ will act as "would be" Goldstone bosons which will be eaten by $W^\pm$ and $Z_0$. The remaining $\pi_t^{+}$ and $\pi_t^{-}$ will combine into multiple charged Higgs fields $H_t^\pm$ while the other combinations of $\pi_{bi}^{0}$ and $\pi_t^{0}$ will be the CP-odd Higgs fields $A_0^i$. The mixings between triplet Higgs fields will give two mass eigenstates $\bar{\Delta}_2$ and $\bar{\Delta}_3$. There is enough parameter space to tune the lightest Higgs field to be at 125 GeV. We note that the non-minimal nature is crucial for Higgs mixing and the appearance of light Higgs field.

Quarks of the first two generation transform as $SU(3)_2$ fundamental representations and also carry $U(1)_1$ charges. As $SU(3)_2$ will become strongly coupled, additional $U(1)_1$ interactions can prevent the condensation between the first two generations. This is similar to that of the flavor-universal topcolor model [14].

The most important electroweak precision constraints on top seesaw comes from the electroweak oblique parameters $S$ and $T$ [15]. Minimal Top seesaw model can non-trivially satisfy the $S - T$ bounds. We know that the oblique parameter $S$ can be thought of as the measure of the total size of the new sector while $T$ is the measure of the weak-isospin breaking induced by it. Just as ordinary extended Top Seesaw model with bottom seesaw, the contributions to the oblique parameters are rather complicate. New contributions to the $T$ parameter

\[ T = \frac{4\pi}{\sin^2 \theta \cos^2 \theta M_Z^2} \left[ \Pi_{11} \big|_{\varphi^2=0} - \Pi_{33} \big|_{\varphi^2=0} \right] , \] (3.54)
from the quark sector are

\[
\delta T = \frac{4\pi}{s_W^2 c_W m_Z^2} \frac{4 N_c}{16 \pi^2} \left\{ \sum_{a=1}^{3} \sum_{b=1}^{5} \left[ \left( \frac{3}{i=1} U_{Lia}^* Z_{Lia}^{-1} \right) \left( \frac{3}{i=2} U_{Ria} Z_{Ria}^{-1} \right) \right] \right\} K(\lambda_a, \bar{\lambda}_b) \\
+ \sum_{a=1}^{3} \sum_{b=1}^{5} 2 \Re \left[ \left( \frac{3}{i=1} U_{Lia}^* U_{Lia}^{-1} \right) \left( \frac{3}{i=2} U_{Ria} U_{Ria}^{-1} \right) \right] L(\lambda_a, \bar{\lambda}_b) \\
- \sum_{a=1}^{3} \sum_{b=1; a \neq b}^{5} \left[ \left( \frac{3}{i=1} Z_{Lia}^* Z_{Lia}^{-1} \right) \left( \frac{3}{i=2} Z_{Ria} Z_{Ria}^{-1} \right) \right] K(\lambda_a, \bar{\lambda}_b) \\
- \sum_{a=1}^{3} \sum_{b=1; a \neq b}^{5} \left[ \left( \frac{3}{i=1} Z_{Lia}^* Z_{Lia}^{-1} \right) \left( \frac{3}{i=2} Z_{Ria} Z_{Ria}^{-1} \right) \right] L(\lambda_a, \bar{\lambda}_b) \\
- \frac{3}{a^2 - b^2} \left[ \left( \frac{3}{i=1} Z_{Lia}^* Z_{Lia}^{-1} \right) \left( \frac{3}{i=2} Z_{Ria} Z_{Ria}^{-1} \right) \right] \left( \lambda_a - \frac{1}{4} \lambda_1 \right), \tag{3.55}
\]

here we define

\[
K(a, b) \equiv \frac{1}{a^2 - b^2} \left[ \left( \frac{3}{i=1} U_{Lia}^* U_{Lia}^{-1} \right) \left( \frac{3}{i=2} U_{Ria} U_{Ria}^{-1} \right) \right] \\
L(a, b) \equiv \frac{ab}{a^2 - b^2} \left[ \left( \frac{3}{i=1} Z_{Lia}^* Z_{Lia}^{-1} \right) \left( \frac{3}{i=2} Z_{Ria} Z_{Ria}^{-1} \right) \right]. \tag{3.56}
\]

New contributions to the oblique S parameter

\[
S = 16\pi \left[ \frac{\partial}{\partial q^2} \Pi_{33} \bigg|_{q^2=0} - \frac{\partial}{\partial q^2} \Pi_{33} \bigg|_{q^2=0} \right]. \tag{3.57}
\]
from the quark sector are

\[
-\frac{4\pi S}{N_c} = \frac{1}{3} \sum_{a=1}^{3} \left[ \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} - \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} - 4U_{Ria}^{-1} U_{Ria} \right) \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} - \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} \right) \right] \\
+ \frac{2}{9} \sum_{a=1}^{3} \ln(\lambda_{a}^2) \left[ \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} - \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} - 4U_{Ria}^{-1} U_{Ria} \right) \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} + \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} + 4U_{Ria}^{-1} U_{Ria} \right) \right] \\
+ \frac{1}{3} \sum_{a=1}^{3} \left[ \left( \sum_{i=1}^{3} Z_{Lia}^{-1} Z_{Lia} - \sum_{i=2}^{3} Z_{Ria}^{-1} Z_{Ria} \right) \\
\times \left( \sum_{i=1}^{3} Z_{Lia}^{-1} Z_{Lia} - 2 \sum_{i=4}^{5} Z_{Lia}^{-1} Z_{Lia} - \sum_{i=2}^{3} Z_{Ria}^{-1} Z_{Ria} + 2 \sum_{i=1,4,5} Z_{Ria}^{-1} Z_{Ria} \right) \right] \\
+ \frac{2}{9} \sum_{a=1}^{3} \ln(\lambda_{a}^2) \left[ \left( \sum_{i=1}^{3} Z_{Lia}^{-1} Z_{Lia} - \sum_{i=2}^{3} Z_{Ria}^{-1} Z_{Ria} \right) \left( \sum_{i=1}^{3} Z_{Ria}^{-1} Z_{Ria} - 2 \sum_{i=4}^{5} Z_{Ria}^{-1} Z_{Ria} \right) \right] \\
+ \sum_{a=1}^{3} \sum_{b=1}^{3} \left[ \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} - \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} \right) \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} + \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} + 4U_{Ria}^{-1} U_{Ria} \right) \right] \left( \frac{1}{3} - 4P(\lambda_{a}, \lambda_{b}) \right) \\
+ \sum_{a=1}^{3} \sum_{b=1}^{3} \left[ \left( \sum_{i=1}^{3} Z_{Lia}^{-1} Z_{Lia} - \sum_{i=2}^{3} Z_{Ria}^{-1} Z_{Ria} \right) \left( \sum_{i=1}^{3} Z_{Ria}^{-1} Z_{Ria} - 2 \sum_{i=4}^{5} Z_{Ria}^{-1} Z_{Ria} \right) \right] \left( \frac{1}{3} - 4P(\lambda_{a}, \lambda_{b}) \right) \\
+ \sum_{a=1}^{3} \sum_{b=1}^{3} \left[ \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} - \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} \right) \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} + \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} + 4U_{Ria}^{-1} U_{Ria} \right) \right] 2\lambda_{a}\lambda_{b} Q(\lambda_{a}, \lambda_{b}) \\
+ \sum_{a=1}^{3} \sum_{b=1}^{3} \left[ \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} - \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} \right) \left( \sum_{i=1}^{3} U_{Lia}^{-1} U_{Lia} + \sum_{i=2}^{3} U_{Ria}^{-1} U_{Ria} + 4U_{Ria}^{-1} U_{Ria} \right) \right] 2\lambda_{a}\lambda_{b} Q(\lambda_{a}, \lambda_{b}) .
\]

with the definition

\[
P(a, b) = \int_{0}^{1} x(x - 1) \ln[(a - b)x + b]dx ,
\]

\[
Q(a, b) = \int_{0}^{1} \frac{x(x - 1)}{(a - b)x + b} dx ,
\]

which we will not give their tedious analytic expressions. Although the precise values need
the detailed numerical studies, we note that the contributions to the $S$ parameter should
be very similar to that of the minimal top seesaw model because most new particle contents
are vector-like.
The contributions from the multiple Higgs doublets needs the Higgs spectrum as well as the knowledge of the mixings among different Higgs doublets. In general, they should drive the \( T \) parameter negative which however being compensated by isospin violating quark sector contributions. We will left the detailed numerical results to subsequence studies. We just anticipate that there are enough parameter space to make our model compatible with \( S-T \) bounds.

There are additional constraints from \( Z-b_L-b_L \) coupling. The mixing within the bottom seesaw sector change the vertex by

\[
\delta g_L^b = \frac{e}{2 \sin \theta \cos \theta} \left( \sum_{i=4}^{5} |Z_{1j}|^2 \right). \tag{3.60}
\]

We can see that \( \Gamma(Z \rightarrow \bar{b}b) \) will decrease with respect to the SM predictions. The updated data on \( R_b \) will constrain the mixings within the bottom sector.

We can properly choose the parameter \( M_1 = M_2 = 20 \text{ TeV} \) so that the physical top quark mass is given by

\[
\lambda^2 \approx \frac{\mu^2 \mu_2^2 s_1^2}{M_1^2 M_2^2} \approx (170 \text{GeV})^2. \tag{3.61}
\]

The gap equation depends implicitly on \( \mu \) and \( \mu_2 \) on the r.h.s and we checked that the following parameters

\[
s_1 \simeq 18 \text{ TeV}, \mu_2 \simeq 5.02 \text{ TeV}, \mu \simeq 0.76 \text{ TeV} \tag{3.62}
\]

can satisfy approximately the gap equation

\[
\frac{\mu}{\mu_2} \approx \frac{U_{L33}^{-1} U_{R33}^{-1}}{U_{L23}^{-1} U_{R13}^{-1}}. \tag{3.63}
\]

The mixing matrices can be obtained by diagonalizing the mass matrices

\[
\begin{pmatrix}
    t_L^m \\
    \chi_L^m \\
    \rho_L^m
\end{pmatrix} =
\begin{pmatrix}
    -0.2475 & 0.4940 & -0.8335 \\
    0.2225 & -0.8083 & -0.5451 \\
    0.9430 & 0.3204 & -0.0902
\end{pmatrix}
\begin{pmatrix}
    t_L \\
    \chi_L \\
    \rho_L
\end{pmatrix}, \tag{3.64}
\]

\[
\begin{pmatrix}
    t_R^m \\
    \chi_R^m \\
    \rho_R^m
\end{pmatrix} =
\begin{pmatrix}
    0.9988 & -0.0475 & -0.0132 \\
    -0.03766 & -0.5623 & -0.8261 \\
    0.03180 & 0.8256 & -0.5634
\end{pmatrix}
\begin{pmatrix}
    t_R \\
    \chi_R \\
    \rho_R
\end{pmatrix}, \tag{3.65}
\]

with the eigenvalues

\[
\lambda_1 \simeq 0.172 \text{ TeV}, \lambda_2 \simeq 13 \text{ TeV}, \lambda_3 \simeq 31.36 \text{ TeV}. \tag{3.66}
\]

The unitary nature of the mixing matrix indicates that the inverse mixing matrix is the form

\[
U_L^{-1} = \begin{pmatrix}
-0.2475 & 0.2225 & 0.9430 \\
0.4940 & -0.8083 & 0.3204 \\
-0.8335 & -0.5451 & -0.0902
\end{pmatrix}, U_R^{-1} = \begin{pmatrix}
0.9988 & -0.03766 & 0.03180 \\
-0.0475 & -0.5623 & 0.8256 \\
-0.0132 & -0.8261 & -0.5634
\end{pmatrix} \tag{3.67}
\]
From the Pagels-Stokar formula and setting \( v_t^2 \simeq (200\text{GeV})^2 \) and \( N_c = 3 \), we obtain the cut-off scale

\[
M \simeq 40 \text{ TeV}.
\]  

(3.68)

The coloron mass scale is \( M_B \simeq \sqrt{2}M_1 \simeq 30 \text{ TeV} \) if we assume \( h_2 \sim \mathcal{O}(1) \). The bottom quark sector can be similarly obtained. The lightest bottom-type quark mass is given approximately by

\[
\tilde{\lambda}_1 \approx \frac{\tilde{\mu}_1^2 \mu_2}{M_1 M_2} = 4.2 \text{ GeV},
\]  

(3.69)

which is related to the top quark sector through the relation \( \mu/\tilde{\mu} \equiv \tan \beta_1 \approx 40 \).

In general, the scalar type bound state of the NJL-type condensation has a mass of order \( 2\mu \) with \( \mu \) the corresponding dynamical mass in the gap equation. In our scenario, the lightest scalar states are mixing between various condensation bound states with the lightest bound state as light as \( 2\tilde{\mu} \sim \mathcal{O}(10 \text{ GeV}) \), and then can be as light as \( \mathcal{O}(100 \text{ GeV}) \).

4 Conclusions

The recent discovery of a 125 GeV Higgs-like particle at the LHC pushes us to ask the interesting question whether such scalar is composite or fundamental. On the other hand, top quark, which is much heavier than all the other SM fermions, indicates that it couples more strongly to electroweak symmetry breaking sector. Thus, it is possible that the top sector plays a key role in electroweak symmetry breaking mechanism and related intimately to the intrinsic nature of the Higgs field. Ordinary top seesaw model predicts too heavy Higgs mass and requires new matter contents and interactions that are put in by hand. We propose a typical non-minimal extended top seesaw model (with also bottom seesaw) and accommodate a light composite Higgs field. The non-minimal nature is crucial for Higgs mixing and the appearance of light Higgs field. Besides, supersymmetric strong dynamics can lead to almost composite top and bottom quark as well as new emergent topcolor gauge interactions. At the same time, supersymmetry breaking condition also leads to topcolor breaking as well. The low energy QCD coupling is partially emergent. This theory also act as an AdS/CFT dual to a Randall-Sundrum \[16\] type model which will be given in subsequent studies.

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