Abstract

A factorization theorem for $P$-wave quarkonium production, recently derived by Bodwin, Braaten, Yuan and Lepage, is applied to $\Upsilon \to \chi_{cJ} + X$, where $\chi_{cJ}$ labels the $^3P_J$ charmonium states. The widths for $\chi_{cJ}$ production through color-singlet $P$-wave and color-octet $S$-wave $c\bar{c}$ subprocesses are computed each to leading order in $\alpha_s$. Experimental data on $\Upsilon \to J/\psi + X$ is used to obtain an upper bound on a nonperturbative parameter (related to the probability for color-octet $S$-wave $c\bar{c}$ hadronization into $P$-wave charmonium) that enters into the factorization theorem. The bound obtained here adds to the limited information so far available on the color-octet mechanism for $P$-wave quarkonium production.
Factorization theorems play a basic role in perturbative QCD calculations of many hadronic processes. A well known factorization theorem for the decay and production of $S$-wave quarkonium follows from a nonrelativistic description of heavy quark-antiquark ($Q\bar{Q}$) binding \[1\]. Nonperturbative effects are factored into $R_s(0)$, the nonrelativistic wave function at the origin, leaving a hard $Q\bar{Q}$ subprocess matrix element that can be calculated in perturbation theory. This factorization is valid to all orders in the strong coupling $\alpha_s$, and to leading order in $v^2$, where $v$ is the typical center-of-mass velocity of the heavy quarks.

Remarkably, the correct factorization theorems for the decay \[2\] and production \[3\] of $P$-wave quarkonium have only recently been derived. These new theorems resolve a long standing problem regarding infrared divergences which appear in some cases to leading order in the rates for $P$-wave $Q\bar{Q}$ states \[4\]. In previous phenomenological calculations, the divergence was replaced by a logarithm of a soft binding scale, such as the binding energy or confinement radius \[1,1\]. However, a rigorous calculation requires that one consider additional components of the Fock space for $P$-wave quarkonium, such as $|Q\bar{Q}g\rangle$, where the $Q\bar{Q}$ pair is in a color-octet $S$-wave state, and $g$ is a soft gluon \[2,3\].

A renewed study of the decay and production of $P$-wave quarkonium is therefore of considerable interest, since one may gain new information on a nonperturbative sector of QCD that has largely been neglected in the quark model description of heavy quarkonium. This is also of practical consequence; for example, $J/\psi$ production provides a clean experimental signature for many important processes, and $P$-wave charmonium states have appreciable branching fractions to $J/\psi$.

In this paper the factorization theorem for $P$-wave quarkonium production is applied to $\Upsilon \to \chi_{cJ} + X$, where $\chi_{cJ}$ labels the $^3P_J$ charmonium states. The widths for $\chi_{cJ}$ production through color-singlet $P$-wave and color-octet $S$-wave $c\bar{c}$ subprocesses are computed each to leading order in $\alpha_s$. Experimental data on $\Upsilon \to J/\psi + X$ is used to obtain an upper bound on a nonperturbative parameter (related to the probability...
for color-octet $S$-wave $c\bar{c}$ hadronization into $P$-wave charmonium) that enters into the factorization theorem. The bound obtained here adds to the limited information so far available on the color-octet mechanism for $P$-wave quarkonium production. The color-octet component in $P$-wave decay was estimated in Ref. [2] from measured decay rates of the $\chi_{c1}$ and $\chi_{c2}$. A rough estimate of the color-octet component in $P$-wave charmonium production was obtained in Ref. [3] from data on $B$ meson decays; however, an accurate determination in that case requires a calculation of next-to-leading order QCD corrections to the color-singlet component of $B \to \chi_{cJ} + X$, which is so far unavailable [3].

The factorization theorem for $P$-wave quarkonium production has two terms, and in the case of $\Upsilon$ decay takes the form:

$$\Gamma(\Upsilon \to \chi_{cJ} + X) = H_1 \hat{\Gamma}_1(\Upsilon \to c\bar{c}(^3P_J) + X; \mu)$$

$$+ (2J + 1)H'_8(\mu)\hat{\Gamma}_8(\Upsilon \to c\bar{c}(^3S_1) + X).$$

(1)

$\hat{\Gamma}_1$ and $\hat{\Gamma}_8$ are hard subprocess rates for the production of a $c\bar{c}$ pair in color-singlet $P$-wave and color-octet $S$-wave states respectively. The quarks are taken to have vanishing relative momentum. The nonperturbative parameters $H_1$ and $H'_8$ are proportional to the probabilities for these $c\bar{c}$ configurations to hadronize into a color-singlet $P$-wave bound state. $H_1$, $H'_8$ and $\hat{\Gamma}_8$ are independent of the total angular momentum $J$.

This factorization theorem is valid to all orders in $\alpha_s$ and to leading order in $v^2$. The hard subprocess rates are free of infrared divergences. $\hat{\Gamma}_1$ and $H'_8$ depend on an arbitrary factorization scale $\mu$ in such a way that the physical decay rate is independent of $\mu$. In order to avoid large logarithms of $m_\Upsilon/\mu$ in $\hat{\Gamma}_1$, $\mu$ of $O(m_\Upsilon)$ should be used. The factorization theorem for $P$-wave charmonium decay contains a nonperturbative parameter $H_8$ analogous to $H'_8$. However a production process must be used to determine $H'_8$ phenomenologically [3].

In the usual nonrelativistic quark model, $H_1$ can be expressed in terms of the
$P$-wave color-singlet $c\bar{c}$ wave function:

$$H_1 \approx \frac{9}{2\pi} \frac{|R_P'(0)|^2}{m_c^2} \approx 15 \text{ MeV},$$  \hspace{1cm} (2)

where the numerical estimate was obtained in Ref. [2] from measured decay rates of the $\chi_{c1}$ and $\chi_{c2}$.

$H'_8$ cannot be rigorously expressed perturbatively in terms of $R_P$, since it accounts for radiation of a soft gluon by a color-octet $c\bar{c}$ pair. The scale dependence of $H'_8(\mu)$ is determined by the following renormalization-group equation (to leading order in $\alpha_s(\mu)$) \[2, 5\]:

$$\mu \frac{d}{d\mu} H'_8(\mu) \approx \frac{16}{27\pi} \alpha_s(\mu) H_1,$$  \hspace{1cm} (3)

which is readily integrated. For example \[3\]:

$$H'_8(m_b) = H'_8(\mu_0) + \left[ \frac{16}{27\beta_3} \ln \left( \frac{\alpha_s(\mu_0)}{\alpha_s(m_c)} \right) + \frac{16}{27\beta_4} \ln \left( \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right) \right] H_1$$  \hspace{1cm} (4)

(for $\mu_0 < m_c$), where $\beta_n = (33 - 2n)/6$. If $H'_8(\mu_0)$ is neglected in the limit of large $m_b$ one obtains $H'_8(m_b) \approx 3$ MeV, using $\alpha_s(\mu_0) \sim 1$ \[3\]. While one might not expect the physical value of $m_b$ to be large enough to neglect $H'_8(\mu_0)$, an estimate for $H'_8(m_b)$ obtained in Ref. \[3\] from experimental data on $B$ meson decays is consistent with the above result.

A calculation of $\hat{\Gamma}_1$ and $\hat{\Gamma}_8$ in Eq. (1) each to leading order in $\alpha_s$ can be obtained from a calculation of the infrared divergent width $\Gamma_{\text{div}}$ for $\Upsilon \to c\bar{c}(3P_J) + ggg$, where the $c\bar{c}$ pair is in a color-singlet $P$-wave state:

$$\Gamma_{\text{div}}(\Upsilon \to c\bar{c}(3P_J) + ggg; \mu_0) \equiv \frac{20\alpha_s^5 G_F^3}{3^7\pi^3 m_c} \left[ F_{1J}(\mu) + (2J + 1) \frac{16}{27\pi} \ln \left( \frac{\mu}{\mu_0} \right) F_8 \right] H_1.$$  \hspace{1cm} (5)

$F_{1J}$ and $F_8$ are dimensionless infrared-finite form factors. $\mu_0$ is an infrared cutoff on the energy of soft gluons, and $\mu$ is an arbitrary factorization scale [the $\mu$ dependence of $F_{1J}$ exactly cancels that of the explicit logarithm in Eq. (5)].

The constants in Eq. (5) include a color-factor of $5/216$ and phase space factors, including $1/3$ for $\Upsilon$ spin-averaging, and $1/3!$ for the phase space of the three...
indistinguishable gluons [cf. Eq. (13) below]. \( G_1^T \) is related to the usual \( S \)-wave \( b \bar{b} \) nonrelativistic wave function:

\[
G_1^T \approx \frac{3}{2\pi} \frac{|R_S^T(0)|^2}{m_b^2} \approx 108 \text{ MeV},
\]

where the numerical value is obtained from the electronic decay rate of the \( \Upsilon \) \[3\].

The hard subprocess rates of Eq. (1) are identified from \( \Gamma_{\text{div}} \) by using the perturbative expression for the infrared divergence in \( H_8' \), obtained from Eq. (3) by neglecting the running of the coupling \[3\]

\[
H_8' (\mu) \sim \frac{16}{27\pi} \alpha_s \ln \left( \frac{\mu}{\mu_0} \right) H_1.
\]

Thus:

\[
\hat{\Gamma}_1 (\Upsilon \to c \bar{c} (3P_J) + ggg; \mu) = \frac{20\alpha_s^5}{3^7\pi^3 m_\chi} \mathcal{F}_{1J}(\mu),
\]

and

\[
\hat{\Gamma}_8 (\Upsilon \to c \bar{c} (3S_1) + g) = \frac{20\alpha_s^4}{3^7\pi^3 m_\chi} \mathcal{F}_8.
\]

Note that \( \hat{\Gamma}_1 \) is suppressed by \( O(\alpha_s) \) compared to \( \hat{\Gamma}_8 \). However, the nonperturbative parameters \( H_1 \) and \( H_8' \) which accompany these subprocess rates in Eq. (4) are independent, hence \( \alpha_s H_1 \) need not be small compared to \( H_8' \) \[2,3\]. We therefore proceed to calculate \( \hat{\Gamma}_1 \) and \( \hat{\Gamma}_8 \) each to leading order; all further corrections to \( P \)-wave production are then guaranteed to be suppressed by at least one power of \( \alpha_s \) compared to what is included here.

In order to extract \( \mathcal{F}_{1J} \) and \( \mathcal{F}_8 \) individually, it is necessary to explicitly identify the infrared logarithm in the calculation of \( \Gamma_{\text{div}} \). This can be done analytically, as described in the following.

There are 36 \( O(\alpha_s^5) \) diagrams contributing to \( \Gamma_{\text{div}} \). One of these is shown in Fig. 1. All infrared divergences are associated with a gluon that is radiated from a charm quark line; in Fig. 1 this gluon carries four momentum \( k_1 \) and polarization \( \epsilon_1 \). Define the invariant amplitude \( \mathcal{M}_{1J}(2,3;1) \) corresponding to the sum of all Feynman diagrams where gluon “1” is radiated from the charm quark line. The amplitude
is readily computed using expressions for $S$- and $P$-wave $q\bar{q}$ currents given in Ref. [7].

$$\mathcal{M}_J(2, 3; 1) \equiv -\frac{m_{\chi} m_{\gamma} B_{\mu}(2, 3) C_{J}^{\mu}(1)}{[(k_2 + k_4) \cdot k_3][(k_3 + k_4) \cdot k_2][(k_2 + k_3) \cdot k_4] k_4^2 (k \cdot k_1)^2}, \quad (10)$$

where

$$\epsilon_4^\mu B_{\mu}(2, 3) = \{ \epsilon_4 \cdot \epsilon_2 [ -k_4 \cdot k_3 \epsilon_3 \cdot k_2 \epsilon_0 \cdot k_4 - k_2 \cdot k_3 \epsilon_2 \cdot k_0 \epsilon_3 \cdot k_4 - k_4 \cdot k_3 k_2 \cdot k_2 \epsilon_0 \cdot \epsilon_3 ]$$

$$+ \epsilon_0 \cdot \epsilon_3 [k_4 \cdot k_3 \epsilon_4 \cdot k_2 \epsilon_2 \cdot k_3 + k_2 \cdot k_4 \epsilon_3 \cdot k_4 \epsilon_2 \cdot k_3 - k_4 \cdot k_2 \epsilon_4 \cdot k_3 \epsilon_2 \cdot k_3 ] \} + \{ 2 \leftrightarrow 3 \} + \{ 3 \leftrightarrow 4 \} \quad (11)$$

($\epsilon_0$ is the polarization of the $\Upsilon$), and

$$\epsilon_{4\mu} C_{J=0}^{\mu}(1) = \frac{\sqrt{1}}{6} [\epsilon_1 \cdot \epsilon_4 k_1 \cdot k_4 - \epsilon_1 \cdot k_4 \epsilon_4 \cdot k_1] \left( m_{\chi}^2 + k \cdot k_4 - k_4^2 \right)$$

$$\epsilon_{4\mu} C_{J=1}^{\mu}(1) = \frac{1}{2} m_{\chi} k_4^2 \varepsilon_{\alpha \beta \gamma \delta} \epsilon_4^\gamma \epsilon_1^\delta \epsilon_1^\delta,$$

$$\epsilon_{4\mu} C_{J=2}^{\mu}(1) = \frac{\sqrt{1}}{2} m_{\chi}^2 \left( k_1 \cdot k_4 \epsilon_1^\alpha \epsilon_4^\beta + k_4^\alpha k_1^\beta \epsilon_1 \cdot \epsilon_4 - k_1^\alpha \epsilon_4^\beta \epsilon_1 \cdot k_4 - k_4^\alpha \epsilon_1^\beta \epsilon_4 \cdot k_1 \right) e^{\alpha \beta} \quad (12)$$

e$^\alpha$ is a spin-1 polarization vector and $e^{\alpha \beta}$ is a spin-2 polarization tensor.

For convenience the virtual gluon is labeled in Eqs. (10)-(12) by polarization $\epsilon_4$ and momentum $k_4$ ($k_4 = P - k_2 - k_3 = k + k_1$). Terms which vanish due to the on-shell conditions $\epsilon_i \cdot k_i = 0$ ($i = 1, 2, 3$) and $\epsilon_0 \cdot P = 0$ have been dropped. The charm quark current $C_{J}^{\mu}(1)$ is symmetric under interchange of labels 1 and 4 (up to terms which vanish due to the on-shell conditions). The bottom quark current $B_{\mu}(2, 3)$ is explicitly symmetric under interchange of labels 2, 3 and 4.

The overall factors in Eq. (5) are such that:

$$\mathcal{F}_{1J}(\mu) + (2J + 1) \frac{16}{27\pi} \ln \left( \frac{\mu}{\mu_0} \right) \mathcal{F}_{8} \equiv 3 \int d[\Phi_4] \sum_{\text{spins}} \left[ \mathcal{M}_J^2(2, 3; 1) + 2 \mathcal{M}_J(2, 3; 1) \mathcal{M}_J(1, 3; 2) \right], \quad (13)$$

$^1$Overall factors in the quark currents including couplings, color amplitudes, and wave functions have been accounted for in Eq. (5).
where $\Phi_n$ denotes (infrared-cutoff) $n$-body phase space, normalized according to

$$\Phi_n[P \to p_1, \ldots, p_n] \equiv \int \prod_{i=1}^{n} \frac{d^3p_i}{2E_i} \delta^4(P - \sum_i p_i). \quad (14)$$

The factor of 3 on the right hand side of Eq. (13) accounts for symmetrization of $\mathcal{M}_J(2,3;1)$ under gluon label interchanges $1 \leftrightarrow 2$ and $1 \leftrightarrow 3$, taking account of the symmetry in the three gluon phase space.

The infrared divergence comes entirely from the first term in square brackets in Eq. (13), and is due to the $P$-wave charm quark propagator $1/(k \cdot k_1)^2$ in Eq. (10). It is therefore advantageous to organize the four-body phase space integral in Eq. (13) by taking the invariant mass of the $\chi_c J$ and gluon “1” as one integration variable [8]

$$\int d[\Phi_4] = \int_0^{(m_\Upsilon - m_\chi)^2} d(k_{23}^2) \int_{m_\chi^2 + 2\mu_0 m_\chi}^{(m_\Upsilon - m_23)^2} d(k_{1X}^2)$$

$$\times \Phi_2[P \to k_{23}, k_{1X}] \Phi_2[k_{23} \to k_2, k_3] \Phi_2[k_{1X} \to k_1, k], \quad (15)$$

where $m_{23}^2 \equiv k_{23}^2$. Note the infrared cutoff $\mu_0$ on the energy of gluon “1” in the rest frame of the $\chi_c J$.

The infrared logarithm on the right-hand side of Eq. (13) can now be identified analytically by observing that $B_\mu(2,3) C^\mu_J(1)$ in Eq. (10) is given by a sum of terms each containing exactly one factor of $k_1$, if $k_4 = P - k_2 - k_3$ is used to eliminate the virtual gluon momentum. With this convention, one has

$$\sum_{\text{spins}} \mathcal{M}^2_J(2,3;1) = \frac{\gamma_J(k_1; P, k, k_2, k_3)}{(k \cdot k_1)^2}, \quad (16)$$

where $k_1$ appears explicitly in the function $\gamma_J(k_1; P, k, k_2, k_3)$ only in the combination $k_1/k \cdot k_1$.

$\mathcal{F}_8$ is then given in terms of a manifestly infrared-finite three-body phase space integral, taking account of the fact that $\Phi_2(k_{1X} \to k_1, k) = \frac{1}{4} k \cdot k_1/k_{1X}^2 \int d\Omega_{1X}^*, \Omega_{1X}^*$ is the center-of-mass solid angle of the two body system:

$$\mathcal{F}_8 = \frac{27\pi}{32m_\chi^2} \int_{0}^{(m_\Upsilon - m_\chi)^2} d(k_{23}^2) \Phi_2[P \to k_{23}, k]$$

$$\times \Phi_2[k_{23} \to k_2, k_3] \int d\Omega_{1X}^* \gamma_J(k_1; P, k, k_2, k_3), \quad (17)$$
where
\[ \tilde{k}_1 \equiv \lim_{k \cdot k_1 \to 0} \frac{k_1}{k \cdot k_1}. \] (18)

The finite four-vector \( \tilde{k}_1 \) is readily expressed directly in terms of \( k_2^2 \) and \( \Omega_{1\chi}^* \). An expression for \( F_{1J} \) can be obtained from Eqs. (13) and (17) by analogy with the identity \( \int dx f(x)/x = f(0) \ln x + \int dx [f(x) - f(0)]/x \).

The contraction of currents and sum over polarizations in Eqs. (10) and (13) were performed symbolically using REDUCE [9] (leading to lengthy expressions, particularly for \( J = 2 \)). The \( \chi_{cJ} \) spin sums were done using (see e.g. Ref. [7]):

\[
\begin{align*}
\sum_e e\mu e\nu &\equiv -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_\chi^2} \equiv P_{\mu\nu}, \\
\sum_e e\mu e\nu \epsilon_{\alpha\beta} &\equiv \frac{1}{2} \left[ P_{\mu\alpha} P_{\nu\beta} + P_{\mu\beta} P_{\nu\alpha} \right] - \frac{1}{3} P_{\mu\nu} P_{\alpha\beta}.
\end{align*}
\] (19)

The phase space integrals were evaluated numerically using VEGAS [10]; modest integration grids are found to give very good convergence.

The fact that \( F_8 \) should be independent of \( J \) provides a stringent check of these calculations, given that the three currents \( C_{J}^{\mu} \) have very different structures [cf. Eq. (12)]. This was verified explicitly in numerical calculations of Eq. (17), to better than a few tenths of a percent for all \( m_\chi/m_\Upsilon \) on a modest integration grid.

Figure 2 shows the numerical results for \( F_8 \) over a range of hypothetical meson masses. In Fig. 3 results for \( F_{1J}(\mu) \) are shown using a factorization scale \( \mu = m_\Upsilon \).

The available experimental data on charmonium production in \( \Upsilon \) decay is for the \( J/\psi \):

\[ B_{\exp}(\Upsilon \to J/\psi + X) \begin{cases} 
= (1.1 \pm 0.4) \times 10^{-3} & \text{CLEO [11]}, \\
< 1.7 \times 10^{-3} & \text{Crystal Ball [12]}, \\
< 0.68 \times 10^{-3} & \text{ARGUS [13]}. 
\end{cases} \] (20)

An upper bound on \( H_8^\prime \) can be extracted from this data by computing the “indirect” production of \( J/\psi \) due to the \( \chi_{cJ} \) states. Assuming that radiative cascades from \( \chi_{c1} \) and \( \chi_{c2} \) dominate, with branching fractions \( B_{\exp}(\chi_{c1} \to \gamma J/\psi) \approx 27\% \) and \( B_{\exp}(\chi_{c2} \to \)
\(\gamma J/\psi \approx 13\%\) \(^3\), the results presented here give:

\[
H'_8(m_\Upsilon) \approx \left\{ \sum B(\Upsilon \to \chi_{cJ} + X' \to J/\psi + X) \frac{1}{2.9 \times 10^{-5}} + 1.4 \right\} \text{MeV.} \quad (21)
\]

The first number in brackets above comes from the color-octet subprocess rate \(\hat{\Gamma}_8\), and the second number from the color-singlet rate \(\hat{\Gamma}_1\). The experimental value for the total width \(\Gamma_{\text{tot}}(\Upsilon) \approx 52\text{ keV}\) \(^3\) was used, along with \(\alpha_s(m_\Upsilon) \approx 0.179\) \(^1\), and the values of \(H_1\) and \(G_\Upsilon^T\) given in Eqs. \((2)\) and \((6)\).

Equation \((21)\) yields the bound \(H'_8(m_\Upsilon) \lesssim 25\) MeV using the ARGUS upper limit, which is consistent with the other measurements. This bound is considerably larger than an estimate \(H'_8(m_b) \approx 3\) MeV based on \(B\) meson decays \(^3\), although a calculation of next-to-leading order QCD corrections to the color-singlet component of \(B \to \chi_{cJ} + X\) is required before an accurate determination of \(H'_8\) can be made in that case \(^3\).

This raises the possibility of significant direct production of \(J/\psi\) in the decay of the \(\Upsilon\), unless the branching fraction turns out to be considerably smaller than the ARGUS bound. Mechanisms for direct \(\Upsilon \to J/\psi + X\) in perturbative QCD were first discussed in Refs. \(^{14}\) and \(^{15}\). The direct production rate is suppressed by \(O(\alpha_s^2)\) compared to the \(P\)-wave color-octet production mechanism considered here. However, the nonperturbative matrix elements which enter into \(P\)-wave production are of \(O(v^2)\) relative to the corresponding parameter for \(S\)-wave production, where \(v\) is a typical relative velocity of the quarks. Moreover, there are many channels which contribute to direct production.

The full \(O(\alpha_s^6)\) perturbative QCD amplitude for direct \(\Upsilon \to J/\psi + X\) was recently evaluated in Ref. \(^{16}\), corresponding to one-loop diagrams for \(\Upsilon \to J/\psi + gg\), and tree diagrams for \(\Upsilon \to J/\psi + gggg\). The \(O(\alpha_s^2\alpha_s^2)\) electromagnetic amplitude for the two gluon decay mode was also evaluated. Unfortunately, only a crude estimate of the required phase space integrations was made in Ref. \(^{16}\) (there is a costly convolution

\(^2\)From Eq. \((3)\), \(H'_8(\mu)\) increases by only \(\approx 0.3\) MeV in the evolution from \(\mu = m_b\) to \(\mu = m_\Upsilon\).
with a numerical calculation of the loop integrals for $\Upsilon \to J/\psi + gg)$. Nevertheless, the calculation of Ref. [16] suggests a branching fraction for direct production of a few $\times 10^{-4}$. This would lead to a considerable reduction in the bound on $H'_8$ extracted from Eqs. (20) and (21).

To summarize, a complete calculation was made of the leading order rates for $\Upsilon \to \chi_{cJ} + X$, through both color-singlet $P$-wave and color-octet $S$-wave $c\bar{c}$ subprocesses. Experimental data on $J/\psi$ production was used to obtain an upper bound on the nonperturbative parameter $H'_8$, related to the probability for color-octet $S$-wave $c\bar{c}$ hadronization into $P$-wave charmonium. This work adds to the limited information so far available on the color-octet mechanism for $P$-wave quarkonium decay and production [2, 3]. These investigations provide new information on a nonperturbative sector of QCD that has largely been neglected in previous studies of heavy quarkonium. A quantitative estimate of $H'_8$ is phenomenologically important since this parameter is required as input for the calculation of a variety of processes. Improved experimental data, and a definitive calculation of the direct $J/\psi$ production rate along the lines of Ref. [16], would allow for an accurate determination of $H'_8$ from the results presented here.

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Figure 1: One of the 36 $O(\alpha_s^5)$ diagrams contributing to $\Upsilon \rightarrow c\bar{c}(3P_J) + ggg$.

Figure 2: Color-octet form factor $F_8$ as a function of $m_\chi/m_\Upsilon$.

Figure 3: Color-singlet form factors $F_{1J}$ as functions of $m_\chi/m_\Upsilon$: $J = 0$ (short-dashed line), $J = 1$ (long-dashed line), $J = 2$ (solid line). The form-factors were evaluated using a factorization scale $\mu = m_\Upsilon$. 