POSSIBLE SIGNATURES OF QUARK-HADRON PHASE TRANSITIONS INSIDE NEUTRON STARS

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The spin-down power of an isolated neutron star can drive its central density increase and overall structural changes, and trigger a quark–hadron phase transition. A series of observational signatures may be seen as a result of the phase transition, including pulsar spin-down and glitch behaviors, Soft Gamma Repeaters or Gamma-Ray Bursts.

1 Introduction

The sky has long been a successful test ground and discovery site for fundamental physics. Examples include the discovery of Helium from the solar spectrum in 1868, evidence for gravity waves from binary pulsars in 1978, and more recently in 1985, the discovery of Bucky Balls (C_{60}), though not directly in the sky, through experiments designed to simulate environments near stars in favor of forming long chain carbon molecules.

Quark–hadron phase transitions are being pursued in accelerators in the high temperature regime. Meanwhile, the densities inside neutron stars may be high enough to allow the existence of deconfined quark matter.

1.1 A Neutron Star with a Quark Core

The structure of a neutron star with a quark core can be solved using standard equations of stellar structure together with the equations of state (EOSs) of quark matter and normal neutron matter. The EOS of quark matter is much softer than that of neutron matter because of QCD asymptotic freedom. A neutron star containing a quark core is thus more compact than a normal neutron star. Consequently, the star has a larger maximum spin frequency. However, submillisecond pulsars have not been discovered and hence quark stars have not been identified this way.

1.2 Rotating Relativistic Stars

To solve the structure of a rotating relativistic star, Hartle developed a perturbation solution based on the Schwarzschild metric of a static, spherically
symmetric object. Rotation distorts the star away from spherical symmetry, and its perturbed metric has the form
\[
    ds^2 = -e^{2\nu} dt^2 + e^{2\lambda} dr^2 + e^{2\mu} d\theta^2 + e^{2\phi}(d\phi - \omega dt)^2 + O(\Omega^3).
\]
In this line element, \( \omega \) is the angular frequency of the star’s fluid in a local inertial frame and depends on the radial coordinate \( r \). It is related to \( \Omega \), the rotational frequency seen by a distant observer. In this way, the mass–central density relation can be solved for rotating compact stars. We note that, the mass “increase”, which is a main result in these works, represents the mass difference between two stars at different angular velocities with the same central density. Hence, these theoretical results are about star families relating to observations through pulsar statistics, and are difficult to be tested.

2 Predictions on Quark-Core Neutron Star Spin-Down Behavior

2.1 Another Way to Apply Hartle’s Perturbation Method

Instead of deriving mass–central density relations for a set of neutron stars, we have suggested to trace the evolution of an isolated neutron star during its spin-down process. Associated with the central density increase, neutron matter is continuously converted to quark matter. The overall structure and spin-down behaviors of the star are modified. In Hartle’s perturbation method, the central density is fixed as an input parameter, then stellar masses are solved as a function of angular frequency. In principle, we can derive a set of equations parallel to those of Hartle, with a fixed mass as an input and central density as a perturbation, and solve the central density and stellar structure as a function of angular velocities. In practice, it is more convenient to apply Hartle’s method in a different way to solve the change of central density of an isolated star.

An isolated neutron star has approximately constant gravitational mass during its spin–down process. The binding energy of a typical neutron star \( W \sim 10^{53} \) erg, and the ratio of rotational energy to gravitational energy is less than 0.1 for the most rapidly rotating neutron star. Since \( M \sim 10^{54} \) erg, the rotational energy is only 1% of the stellar mass. In calculating the stellar configurations at different angular velocities, we need first to plot the mass–central density relation at different angular velocities using Hartle’s method as in previous work. Under our approximation, we cross these curves with a line of constant mass, and the corresponding central densities are those of a rotating star at different angular velocities.
2.2 Pulsar Glitches

The quarks inside the star are charged particles and are coupled with the solid crust through magnetic fields. They rotate at a different velocity from superfluid neutrons. During the spin–down process, the central density and the quark core grow and so does the fractional moment of inertia of quark matter as compared to the whole star \( I_q / I \). In a simple starquake model, the post–glitch behavior of a pulsar can be described approximately as

\[
\Omega(t) = \Omega_0(t) + \Delta \Omega_0 [Q e^{-t/\tau} + 1 - Q],
\]

where \( Q \) is the healing parameter of the pulsar. \( Q = -\tau \Delta \Omega(t=0)/\Delta \Omega \), and \( \tau = -\Delta \Omega/\Delta \Omega(t=0) \). It has been shown that \( Q \propto I_n / I \) (ref. 3), where \( I = I_n + I_c \) is the total moment of inertia of the star, \( I_n \) is the moment of inertia carried by neutral particles, and \( I_c \) is that of charged particles. Hence, a decreasing \( Q \) over a long term may indicate continuous conversion from neutron matter to quark matter inside the pulsar.

2.3 Spin-Down History

The pulsar spin-down can be described as

\[
\dot{\Omega} = -k \Omega^n,
\]

and the braking index \( n \) increases as the moment of inertia decreases,

\[
\Delta n / n \propto -\Delta I / I.
\]

As a result, \(|\dot{\Omega}|\) is larger because the quark-core neutron star has a smaller \( I \) and thus is easier to brake. At the same time, the star tends to “spin–up” during the early stages of the phase transitions causing a higher \( \Omega \) than in the normal neutron star case. Consequently, the “characteristic age”

\[
\tau = \Omega / 2 |\dot{\Omega}| = k^{-1} \Omega^{1-n}
\]

may underestimate the true age of the neutron star with a quark core, while for a normal neutron star \( \tau \) is an upper limit.

3 Onset of the Phase Transition: Catastrophic or Continuous?

3.1 Event Rate

In the previous section we considered the neutron star with a developed quark core in it. What if the central density of a normal neutron star happens to
increase from sub-critical density to critical density? Apparently the chance for this to happen is low, and we can estimate the event rate as

\[
R \simeq 10^{-5} \left( \frac{P_i}{20 \text{ ms}} \right)^{-2} \left( \frac{R_{\text{NS}}}{10^{-2}} \right) \text{yr}^{-1} \text{galaxy}^{-1},
\]

where \( R_{\text{NS}} \) is the average neutron star birth rate in units of per year per galaxy and \( P_i \) is the initial spin period. \( R \) does not strongly depend on the exact critical density. The event rate is low because the central density increase is small if the initial spin is slow. However, equation (6) suggest that these types of events happen at a finite rate, especially when we have millions of galaxies in our view. While the accretion power in astrophysics has been stressed in many astronomical environments, equation (6) suggest that the “spin-down power” is not negligible. The spin-down power can apply to any rotating systems with a critical point, not limited to quark-hadron phase transitions, but include Kion or Pion condensations inside neutron stars. Even for a rotating white dwarf the spin-down power alone may trigger a type Ia supernova as the Chandrasekhar mass limit depends on the rotational velocity.

### 3.2 Catastrophic

If the phase transition is catastrophic, it may happen on a time scale of seconds. In such a short time the star contracts from a radius of \( \sim 15 \text{ km} \) to \( \sim 10 \text{ km} \), associated with a sudden spin up. Most importantly, the gravitational energy released

\[
E \sim \frac{GM^2}{R} \left( \frac{\Delta R}{R} \right) \simeq 10^{53} \left( \frac{\Delta R}{R} \right) \text{ergs}
\]

may be large enough to power a cosmological gamma-ray burst (GRB).

### 3.3 Non-Catastrophic

Even though quark-hadron phase transition is likely to be first order, it is different from water–vapor phase transition in that the former has an additional freedom of whether a proton deconfines to \( uud \) quarks or a neutron becomes \( udd \) quarks, which does not conserve electric charges in the two phases although the overall charge neutrality is conserved with the help from leptons. Hence, the two phases are not necessarily separated by gravity. In this case, the phase transition may happen slowly on a time scale of \( 10^5 \) years and the energy is released at an average rate of \( 10^{40} \text{ ergs s}^{-1} \). The majority of this energy is released via neutrino emission, and only a tiny fraction is used to heat the star up to a surface temperature of \( 3 \times 10^6 \text{ K} \), yielding a soft X-ray
luminosity of $\sim 10^{35}$ ergs s$^{-1}$. Note that it is 25 times more luminous than the sun, and it is not necessarily in a binary system. While the fluid core of the star is contracting, the solid crust may have stress built up in it, and the cracking of the crust can release bursts of energy.

4 Observations

In this section we confront theoretical predictions with observations. Radio pulsars constitute only a small fraction of all neutron stars because of the beaming nature of their radio emissions, and hence are not the best place to look for the signatures of the phase transition. So far only 3 pulsars have reliably measured $n$. Five glitches for Crab pulsar, and 7 for Vela are recorded with measured $Q$. Many more are needed to see the continuous phase transition in radio pulsars.

4.1 Soft Gamma Repeaters

Soft Gamma Repeaters (SGRs) are X-ray transient sources (with $\sim 20$ keV photons) associated with young ($10^4$ yr) supernova remnants (SNRs). They are usually also quiescent X-ray emitters (with $kT \sim 1$ keV, $L_X \sim 10^{35}$ ergs s$^{-1}$). So far 4 SGRs have been discovered in the Galaxy, and 1 in the Large Magellanic Cloud. Two SGRs show $\Omega/2|\dot{\Omega}| (\sim 10^3$ yrs) $< \text{SNR age} (\sim 10^4$ yrs).

Since many more Anomalous X-ray Pulsars (AXPs) share some properties of SGRs, it seems that our prediction in previous section that we should be able to see one at a time in our Galaxy is an underestimate. Note however that if the initial spin of a neutron star is 2 ms rather than 20 ms (that of the Crab pulsar), we get 100 times more event rate of phase transition. Fast initial spin has indeed been suggested to produce strong magnetic fields and slow down neutron stars much faster than Crab-type pulsars. Radio astronomers may have been biased toward Crab-type pulsars when picking up neutron stars.

4.2 Gamma-Ray Bursts

GRBs are energetic explosions with total energy $10^{51} - 10^{54}$ erg (if isotropic), most of which is released with 100 keV to a few MeV photons. The time scales range from milliseconds to minutes. The event rate is estimated to be $10^{-5}$ yr$^{-1}$ galaxy$^{-1}$ if the emission is isotropic, and can be much higher if the emission is highly beamed. We note that the event rate in equation (3) can still account for the GRBs even if they are beamed, and the faster initial spins of the neutron stars naturally offer large angular momentum for the sake of
beaming the electromagnetic radiations.

Gravity wave detectors such as LIGO may be able to observe gravity waves associated GRBs, and tell which is the correct model. However, the gravity wave is emitted more or less isotropically while the electromagnetic radiation is likely to be highly beamed (with a beaming factor 100–1000) in the angular momentum direction. Hence, LIGO may need to accumulate 100–1000 events before seeing a gravity wave signal at the same time of a GRB.

5 Conclusions

The accretion power in astrophysics has been stressed over decades, while the “spin-down power” is often neglected. Here we show that, in any astrophysical systems with angular momentum and a critical point, the spin-down power can drive the system to the critical point at a small but finite probability. If the transition to critical state is energetic enough to be seen from distant galaxies, we can observe these events regularly, as in the case for the signatures of quark deconfinement phase transitions in radio pulsars, SGRs, and GRBs.

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