On the Intrinsic Bias in Detecting Caustic Crossings between Galactic Halo and Self-lensing Events in the Magellanic Clouds

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ABSTRACT
In this paper, we investigate the intrinsic bias in detecting caustic crossings between Galactic halo and self-lensing events in the Magellanic Clouds. For this, we determine the region for optimal caustic-crossing detection in the parameter space of the physical binary separations, $\ell$, and the total binary lens mass, $M$, and find that the optimal regions for both populations of events are similar to each other. In particular, if the Galactic halo is composed of lenses with the claimed average mass of $\langle M \rangle \sim 0.5\,M_\odot$, the optimal binary separation range of Galactic halo events of $3.5\,\text{AU} \sim \ell \sim 14\,\text{AU}$ matches well with that of a Magellanic Cloud self-lensing event caused by a binary lens with a total mass $M \sim 1\,M_\odot$; well within the mass range of the most probable lens population of stars in the Magellanic Clouds. Therefore, our computation implies that if the binary fractions and the distributions of binary separations of the two populations of lenses are not significantly different from each other, there is no strong detection bias against Galactic halo caustic-crossing events.

Key words: binaries: general – Galaxy: halo – gravitational lensing – dark matter

1 INTRODUCTION
From years of monitoring millions of stars located in the Large and Small Magellanic Clouds (LMC and SMC), the MACHO (Alcock et al. 1997) and EROS (Aubourg et al. 1993) collaborations have detected ~20 microlensing events. Among these events, two are caustic-crossing binary-lens events (see §2 for more details about the caustic-crossing binary-lens events): one toward the LMC (Bennett et al. 1996) and the other toward the SMC (Alcock et al. 1996; Afonso et al. 1998; Albrow et al. 1999; Alcock et al. 1998; Udalski et al. 1998). For a caustic-crossing event one can determine the lens proper motion, $\mu$, from which one can strongly constrain the lens location (Gould 1994; Nemiroff & Wickramasinghe 1994; Witt & Mao 1994; Peng 1997). The measured lens proper motions for the individual detected binary-lens events are $\mu \sim 20\,\text{km\,s}^{-1}$ and $\sim 80\,\text{km\,s}^{-1}$. Due to their small values of $\mu$, both lenses are suspected of being located within the Magellanic Clouds themselves. Furthermore, it is often hypothesized that stars within the Magellanic Clouds play a dominant role as gravitational lenses for events detected toward the Magellanic Clouds (Sahu & Sahu 1998).

The probability of detecting a caustic-crossing binary-lens event is strongly dependent on the binary separation in units of the combined Einstein ring radius ($b = \ell/\rho_E$; normalized binary separation). The combined Einstein ring radius is related to the total mass of the binary $M$ and its location on the line of sight toward the source star by

$$\rho_E = \left( \frac{4GM_{\odot} D_{sl}D_{ls}}{c^2 D_{os}} \right)^{1/2},$$

where $D_{sl}$, $D_{ls}$, and $D_{os}$ are the separations between the observer, lens, and source star. Therefore, the normalized binary separation is also related to the total mass and the location of the binary lens. The dominant lens populations (and thus the lens mass) and the locations of lenses for Galactic halo and Magellanic Cloud self-lensing events are different. Therefore, even if the binary fractions and the distribution of the physical binary separations $f(\ell)$ are similar each other, the distributions of the normalized binary separations $f(b)$ of the two populations of events might be significantly different from each other. If so, the probability of detecting Galactic halo caustic-crossing events will be systematically different from that of self-lensing events in the Magellanic Clouds, leading to a detection bias. Therefore, unless it is shown that this type of bias is not important, one cannot conclude that MACHOs in the Galactic halo are unlikely to...
be responsible for the events detected toward the Magellanic Clouds.

In this paper, we investigate the intrinsic bias in detecting caustic crossings between Galactic halo and self-lensing events in the Magellanic Clouds. For this investigation, we determine the optimal ranges of the physical binary separations for caustic crossings. If the determined ranges for the two populations of events are similar each other, there will be no strong detection bias against Galactic halo caustic-crossing events and this therefore will support the hypothesis that an important fraction of microlensing events detected toward the Magellanic Clouds are indeed caused by lenses in the Magellanic Clouds.

2 OPTIMAL BINARY SEPARATION FOR CAUSTIC CROSSING

When the lengths are normalized to the combined Einstein ring radius, the lens equation in complex notation for a binary-lens system is represented by
\[
\zeta = z + \frac{m_1}{z_1 - \bar{z}} + \frac{m_2}{z_2 - \bar{z}},
\]
where \(m_1\) and \(m_2\) are the mass fractions of individual lenses (and thus \(m_1 + m_2 = 1\); \(z_1\) and \(z_2\) are the positions of the lenses, \(\zeta = \xi + i\eta\) and \(z = x + iy\) are the positions of the source and images, and \(\bar{z}\) denotes the complex conjugate of \(z\) (Witt 1990). The amplification of the binary-lens event is given by the sum of the amplifications of individual images, \(A_i\), which are given by the Jacobian of the transformation \(2\) evaluated at the image position, i.e.
\[
A_i = \left(\frac{1}{|\det J|}\right)_{z=z_i} \quad \text{det } J = 1 - \frac{\partial \zeta}{\partial \xi},
\]
The source positions with infinite amplifications, i.e. \(\text{det } J = 0\), form closed curves called caustics. Whenever a source star crosses a caustic, an extra pair of source star images appear (or disappear), producing a sharp spike in the light curve (mao & Paczyński 1991).

In Figure 1, we present caustics of binary-lens events with various values of \(b\) and \(q\). In the figure, the mass positions \(z_1\) and \(z_2\) are chosen so that the center of mass is at the origin, both lenses are on the \(\xi\)-axis, and the heavier lens is to the right. The dotted circle represents the combined Einstein ring.

To determine the optimal binary separation for caustic crossing, we first compute the caustic-crossing probability map for caustic-crossing in the parameter space of selected binary axis and the impact parameter (normalized by \(r_E\)), \(\beta\), are randomly chosen in the ranges \(0 \leq \theta \leq 2\pi\) and \(0 \leq \beta \leq 1\). We assume that caustic crossings can be detected as long as the source star trajectory crosses any part of the caustics. The upper panel of Figure 2 shows the iso-probability map for caustic-crossing in the parameter space of \(b\) and \(q\). In the map, contours are drawn starting at 10% and in 10% intervals. One finds that caustic crossings can happen with an important probability only for some optimal values of \(b\), and the probability decreases rapidly for binaries with separations that are too small or too large. In addition, the probability depends weakly on the mass ratio.

Once \(P_{cc}(b, q)\) is computed, the caustic-crossing probability as a function of the normalized binary separation is determined by
\[
P_{cc}(b) = \int_0^1 P_{cc}(b, q) f(q) dq,
\]
where \(f(q)\) is the probability density function of the mass ratio. We use a uniform distribution for \(q\) in the range [0, 1].

Figure 1. Caustics (thick solid lines) of gravitational microlensing events caused by binary lenses for various normalized binary separations, \(b\), and mass ratios, \(q\). The positions of lenses are chosen so that their center of mass is at the origin. Both lenses are on the \(\xi\)-axis and the heavier lens is to the right. The dotted circle represents the combined Einstein ring.
Figure 2. Upper panel: contours of caustic-crossing probability as a function of the normalized binary separation and mass ratio, $P_{cc}(b, q)$. Contours are drawn at levels starting at 10% and increasing in steps of 10%. Lower panel: Caustic crossing probability as a function of only the normalized binary separation, $P_{cc}(b)$. One finds that caustic crossings occur with a probability of $P_{cc}(b) \geq 20\%$ when the normalized binary separation is in the range $0.6 \leq b \leq 2.3$.

where $f(q)$ is the distribution of binary mass ratios. To determine $P_{cc}(b)$, we assume that $f(q)$ is uniformly distributed. Partially, this is because $f(q)$ is poorly known, though more importantly because the probability $P_{cc}(b, q)$ is weakly dependent on the mass ratio. In the lower panel of Figure 2, we present $P_{cc}(b)$. One finds that a caustic crossing occurs with a probability of $P_{cc}(b) \geq 20\%$ when the normalized binary separation is in the range $0.6 \leq b \leq 2.3$.

3 BIAS IN DETECTING CAUSTIC CROSSING EVENTS

With the optimal range in $b$ we have determined, we then calculate the optimal region for caustic crossing in the parameter space of $\ell$ and $M$ and illustrate the result in Figure 3. In the figure, the shaded region (enclosed by solid lines) represents the optimal detection region for Magellanic Cloud self-lensing events and the unshaded region (enclosed by dashed lines) is for Galactic halo events. The optimal regions for the two populations of events overlap. For the self-lensing events in the Magellanic Clouds, we adopt the average lens-source separation of $\langle D_\ell \rangle = 5$ kpc, which is roughly half of the line-of-sight physical depth of the SMC (Mathewson, Ford, & Visvanathan 1986; Martin, Maurice, & Lequeux 1989; Hatzidimitrion et al. 1997).

Since the Galactic halo’s optical depth peaks at $D_\odot \sim 10$ kpc (Kerins & Evans 1998), we adopt $x = 0.2$ for this population of events. For the self-lensing events in the Magellanic Clouds, we adopt the average lens-source separation of $\langle D_\ell \rangle = 5$ kpc, which is roughly half of the line-of-sight physical depth of the SMC (Mathewson, Ford, & Visvanathan 1986; Martin, Maurice, & Lequeux 1989; Hatzidimitrion et al. 1997).

From the figure, one finds that a large portion of the optimal regions for the two populations of events overlap. For a given lens mass, the mean value of the optimal binary separation for Galactic halo events is systematically larger than that for Magellanic Cloud self-lensing events. However, considering the uncertainties in lens location $x$, this difference is not important. Particularly, if the Galactic halo is composed of lenses with the claimed mass of $\langle M \rangle \sim 0.5 M_\odot$, the optimal range of $\ell$ for the Galactic halo events (represented by the lower arrow) agrees well with that of a Magellanic Cloud self-lensing event produced by a binary lens with a mass of $M \sim 1 M_\odot$ (represented by the upper arrow).

$$b(M, \ell, x) = \left[ \frac{c^2 \ell^2}{4GM_\odot x (1-x)} \right]^{1/2}; \quad x = \frac{D_\odot}{D_\odot}$$

Figure 3. The region of optimal caustic crossing in the parameter space of the physical separations, $\ell$, and total masses of the binaries, $M$. The shaded region (enclosed by solid lines) represents the optimal region for Magellanic Cloud self-lensing events, and the unshaded region (enclosed by dashed lines) is for Galactic halo events. The regions are determined so that the normalized binary separation with given values of $\ell$ and $M$ lies in the determined optimal range of $0.6 \leq b \leq 2.3$. One finds that if the Galactic halo is composed of lenses with an average mass of $\langle M \rangle \sim 0.5 M_\odot$, the optimal range of $\ell$ for the Galactic halo events (represented by the lower arrow) agrees well with that of a Magellanic Cloud self-lensing event produced by a binary lens with a mass of $M \sim 1 M_\odot$ (represented by the upper arrow).
significant fraction of events detected toward the Magellanic Clouds are caused by lenses in the Magellanic Clouds themselves.

ACKNOWLEDGMENTS

We would like to thank P. Martini for carefully reading the manuscript.

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