Weyl conformal symmetry model of the dark galactic halo

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The postulate of universal conformal (local Weyl scaling) symmetry modifies both general relativity and the Higgs scalar field model. The conformal Higgs model (CHM) generates an effective cosmological constant that fits observed accelerating Hubble expansion for redshifts \( z \leq 1 \) (7.33 Gyr) accurately with only one free parameter. Growth of a galaxy is modeled by central accumulation of matter from an enclosing empty spherical halo whose radius expands with depletion. Details of this process account for the nonclassical radial centrifugal acceleration observed as excessive orbital velocities in galactic haloes. There is no need for dark matter.

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I. INTRODUCTION

Universal conformal symmetry, requiring local Weyl scaling covariance\(^{[1-4]}\) of all elementary physical fields\(^5\), offers a paradigm alternative to consensus \(\Lambda CDM\) scaling covariance\(^{[1-4]}\) of all elementary physical fields\(^5\), accurately for redshifts \( z \). The CHM determines centrifugal cosmic acceleration accurately for redshifts \( z \leq 1 \) (7.33 Gyr) \(^{[6, 7]}\). Conformal gravity (CG) replaces the Einstein Lagrangian density by a quadratic contraction of the conformal Weyl tensor \(^{[8, 10-14]}\). Substantial empirical support for this proposed break with convention is provided by applications of CG to anomalous galactic rotation velocities. CG has recently been fitted to rotation data for 138 galaxies\(^{[15-19]}\). The CHM precludes existence of a massive Higgs particle, but conformal theory is found to be compatible with a compound gauge diboson \( W_2 \) of mass 125 GeV\(^{[20]}\), consistent with the observed LHC resonance\(^{[21, 22]}\). Fits of CG and the CHM to observed galactic and cosmological data do not require dark matter\(^{[9]}\).

II. DARK MATTER

When it became possible to measure orbital velocities in the outer reaches of galaxies, they were found systematically to exceed the uniformly decreasing value implied by standard Einstein/Newton gravity. The general functional form of \( v(r) \) was observed to level off at a characteristic radial acceleration \( a_0 \approx 10^{-10} m/s^2 \). This led to the conjecture that standard gravity due to observed galactic mass was augmented by some additional gravitational source. Since this source was not directly observed it was called dark matter.

Alternatively, general relativity might be modified to account for this excess centripetal acceleration. The most successful model assumes modified Newtonian dynamics (MOND)\(^{[23, 24]}\). The basic postulate for radial acceleration \( a \), given Newtonian \( a_N \), is that \( a \rightarrow a_N a_0 \) for \( a_N \ll a_0 \).

When conformal gravity (CG) is applied to a Schwarzschild model (a central gravitational source with spherical symmetry) it has an exact solution in the form of Schwarzschild radial potential \( B(r) \)

\[
B(r) = -2\beta/r + \alpha + \gamma r - kr^2.
\]

Outside a source of finite radius\(^{[10]}\),

\[
B(r) = -2\beta/r + \alpha + \gamma r - kr^2.
\]

The depleted halo model\(^{[8]}\), described below, treats all matter outside a defined galactic radius as uniform and isotropic. Only spherical symmetry is considered. Following Mannheim and Kazanas\(^{[11]}\), galactic mass within this radius is treated essentially by classical gravitation, describing detailed nonspherical geometric structure. Dark matter is replaced by the anomalous acceleration parameter \( \gamma \)

An alternative to the multiplicative postulate of MOND is provided by additive acceleration parameter \( \gamma \), which has the conceptual advantage of arising from a well-defined variational field theory. Fits to anomalous galactic rotational velocities by CG and MOND are of comparable accuracy in the flat velocity range. However, the halo cutoff parameter \( \kappa \), unique to CG, is found to be relevant at very large galactic radii\(^{[16, 17, 25]}\).
III. DARK ENERGY AND HUBBLE EXPANSION

The Higgs scalar field\[26, 27\] is an essential element of electroweak physics. It has a spontaneously generated finite amplitude, constant in spacetime, responsible for finite mass of gauge bosons and fermions. Retaining Higgs $V(\Phi, \bar{\Phi}) = -\left(w^2 - \lambda \Phi^2 \bar{\Phi}^2\right)$, which depends on two assumed constants $w^2$ and $\lambda$, the postulate of universal conformal symmetry requires the CHM Higgs Lagrangian density to acquire a gravitational term, $-\frac{1}{16\pi G} R \Phi \bar{\Phi}$, where $R = g_{\mu\nu} R^{\mu\nu}$, trace of the Ricci tensor. The variation of Ricci on a cosmic time scale implies a very small but universal source density for the $Z_\mu$ neutral gauge field. Dressing of the scalar field by $Z_\mu$ determines Higgs parameter $w^2$ and dressing by diboson $W_2$ determines $\lambda$. These two parameters and Ricci scalar $R$ imply finite $\Phi$ amplitude and broken gauge and conformal symmetry.

In the uniform, isotropic cosmic geometry assumed for cosmology, the CHM implies a Friedmann cosmic evolution equation\[5, 7\] with parameters determined by the scalar Higgs field. This modified Friedmann equation contains an effective cosmological constant, defining dark energy density. The integrated luminosity distance, computed as a function of redshift, fits observed data back to cosmological acceleration data, with centrifugal acceleration, is accurate to redshift $z = 1/(7.33\text{Gyr})$\[7\].

IV. DEPLETED HALO MODEL

CG and the CHM are consistent but interdependent\[9\] in the context of a depleted dark halo model\[8\] for an isolated galaxy. A galaxy of mass $M$ is modeled by spherically averaged mass density $\rho_G/c^2$ within an effective galactic radius $r_G$, formed by condensation of primordial uniform, isotropic matter of uniform mass density $\rho_m/c^2$. A model valid for nonclassical gravitation can take advantage of spherical symmetry at large galactic radii, assuming classical gravitation within $r_G$. Nonspherical gravitation is neglected outside $r_G$. The dark halo inferred from gravitational lensing and centripetal acceleration is identified with the resulting depleted sphere of large radius $r_H$\[8\].

CG determines source-free Schwarzschild potential $B(r)$ as Eq.\[1\] outside galactic radius $r_G$\[8, 10\]. As shown in detail below, the physically relevant particular solution for $B(r)$\[9\] incorporates nonclassical radial acceleration $\gamma$ as a free parameter. Its value is determined by the halo model. Gravitational lensing by a spherical halo is observed as centripetal deflection of a photon geodesic passing from the external intergalactic space with postulated universal isotropic mass-energy density $\rho_m$ into the empty halo sphere. The conformal Friedmann cosmic evolution equation implies dimensionless cosmic acceleration parameters $\Omega_\gamma(\rho)$\[8\] which are locally constant but differ across the halo boundary $r_H$.

Smooth evolution of the cosmos implies observable centripetal particle acceleration $\gamma$ within $r_H$ proportional to $\Omega_\gamma(\rho) - \Omega_\gamma(\rho_m)$. Uniform cosmological $\rho_m$ implies constant $\gamma$ for $r \leq r_H$, independent of galactic mass\[9\]. This surprising result is consistent with recent observations of galactic rotational velocities for galaxies with directly measured mass\[23, 25\], implying that radial acceleration $a$ observed as orbital velocity is a function of Newtonian $a_N$ independently of orbital radius and galactic mass.

In the CHM, observed nonclassical gravitational acceleration $\frac{1}{2} \gamma c^2$ in the halo is proportional to $\bar{\Delta} \Omega_\gamma = \Omega_\gamma(0) - \Omega_\gamma(\rho_m) = \Omega_\rho(\rho_m)$, where, given $\rho_m$ and $H_0$, $\Omega_\rho(\rho_m) = \frac{2}{3} \frac{\tau c^2 \rho_m}{H_0^3}$\[6\], for Hubble constant $H_0$, $\Omega_m(\rho_m) \leq \frac{3}{4} \frac{\tau c^2 \rho_m}{H_0^3}$, for Hubble constant $H_0$ and $\tau < 0$. Thus the halo model determines constant $\gamma$ from universal universal cosmic baryonic mass density $\rho_m/c^2$, which includes radiation energy density here.

V. CONSISTENCY OF CG AND CHM

CG and CHM must be consistent for an isolated galaxy and its dark halo, observed by gravitational lensing. CG is valid for anomalous outer galactic rotation velocities in the static spherical Schwarzschild metric, solving a differential equation for Schwarzschild gravitational potential $B(r)$\[3, 10\]. The CHM is valid for cosmological Hubble expansion in the uniform, isotropic FLRW metric, solving a differential equation for Friedmann scale factor $a(t)$\[6\]. Concurrent validity is achieved by introducing a common hybrid metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -B(r)dt^2 + a^2(t) \left( \frac{dr^2}{B(r)} + r^2 d\omega^2 \right).$$

Metric tensor $g_{\mu\nu}$ is determined by conformal field equations derived from $L_\gamma + L_\Phi$\[8\], driven by energy-momentum tensor $\Theta_{\mu\nu}$, where subscript $m$ refers to conventional matter and radiation. The gravitational field equation within halo radius $r_H$ is

$$X_{\mu\nu}^\gamma + X_{\Phi}^{\mu\nu} = \frac{1}{2} \Theta_{\mu\nu}^m,$$

where $X_{\mu\nu}^\gamma$ is a metric functional derivative\[3, 9\]. The gravitational equations are decoupled by separating mass/energy source density $\rho$ into uniform isotropic mean density $\bar{\rho}$ and residual $\rho = \rho - \bar{\rho}$, which extends only to galactic radius $r_G$ and integrates to zero over the defining volume. Defining mean density $\bar{\rho}_G$ and residual density $\rho_G = \rho_G - \bar{\rho}_G$, and assuming $\Theta_{\mu\nu}^m(\rho) \equiv \Theta_{\mu\nu}^m(\bar{\rho}) + \Theta_{\mu\nu}^m(\rho)$, for $r \leq r_G$ of the two equations

$$X_{\gamma}^{\mu\nu} = \frac{1}{2} \Theta_{\mu\nu}^m(\bar{\rho}_G), X_{\Phi}^{\mu\nu} = \frac{1}{2} \Theta_{\mu\nu}^m(\bar{\rho}_G)$$

decouple, implying a solution of the full equation.
VI. COMPUTED PARAMETERS OF SCHWARZSCHILD POTENTIAL B(R)

Given mass/energy source density \( f(r) \) enclosed within \( \bar{r} \), the Schwarzschild field equation is [10, 13]

\[
\partial_i^2 (rB(r)) = rf(r),
\]

for \( f(r) \sim (\Theta^0_0 - \Theta^r_r)_m \) determined by source energy-momentum tensor \( \Theta^\mu_\nu \) [3].

Derivative functions \( y_i(r) = \partial_i^k (rB(r)) \) for \( 0 \leq i \leq 3 \) satisfy differential equations [3, 9]

\[
\partial_r y_i = y_{i+1}, \quad 0 \leq i \leq 2,
\]

\[
\partial_r y_3 = rf(r).
\]

The general solution, for independent constants \( c_i = y_i(0) \), determines coefficients \( \beta, \alpha, \gamma, \kappa \) such that at endpoint \( \bar{r} \)

\[
y_0(\bar{r}) = -2\beta + \alpha\bar{r} + \gamma\bar{r}^2 - \kappa\bar{r}^3,
\]

\[
y_1(\bar{r}) = \alpha + 2\gamma\bar{r} - 3\kappa\bar{r}^2,
\]

\[
y_2(\bar{r}) = 2\gamma - 6\kappa\bar{r},
\]

\[
y_3(\bar{r}) = -6\kappa.
\]

Gravitational potential \( B(r) \) is required to be differentiable and free of singularities. \( c_0 = 0 \) prevents a singularity at the origin. Specific values of \( \gamma \) and \( \kappa \), consistent with Hubble expansion and the observed galactic dark halo [6, 8], can be fitted by adjusting \( c_1, c_2, c_3 \), subject to \( c_0 = 0, \alpha^2 = 1 - 6\beta \gamma \). [13].

A particular solution for \( B(r) \) [10, 13], assumed by subsequent authors, derives an integral for \( \gamma \) that vanishes for residual source density \( \bar{\rho} \). This is replaced here by an alternative solution for which \( \gamma \) is a free parameter [9]. Because the Weyl tensor vanishes identically in uniform geometry, CG applies only to residual density \( \bar{\rho} \).

The proposed particular solution, given \( \gamma, \kappa \), is

\[
rB(r) = y_0(r) = -\frac{1}{6} \int_0^r q^4f dq + \alpha r - \frac{1}{2} \int_r^\bar{r} q^3f dq + \gamma r^2 + \frac{1}{2} \int_0^{\bar{r}} q^2f dq - \kappa r^3 - \frac{1}{6} \int_r^\bar{r} qf dq.
\]

Integrated parameters \( c_i = y_i(0) \) are \( c_1 = \alpha - \frac{1}{2} \int_0^r q^3f dq, \)

\( c_2 = 2\gamma + \int_0^r q^2f dq, \)

\( c_3 = -6\kappa - \int_0^r qf dq, \) and at \( r = \bar{r}, \)

\( 2\beta = \frac{1}{3} \int_0^\bar{r} q^4f dq. \) Term \( \gamma r^2 + \frac{1}{2} \gamma r^2 \int_0^{\bar{r}} q^2f dq \) in this solution differs from prior reference [10]. Here \( \gamma \) is a free parameter that determines generally nonzero \( c_2 \).

For an isolated single spherical solar mass in a galactic halo, mean internal mass density \( \bar{\rho}_0 \) within \( r_0 \) determines an exact solution of the conformal Higgs gravitational equation, giving internal acceleration \( \Omega_q(\bar{\rho}_0) \). Given \( \gamma \) outside \( r_0 \), continuous acceleration across boundary \( r_0 \),

\[
\frac{1}{2} \gamma c^2 - cH_0\Omega_q(\bar{\rho}_0) = \frac{1}{2} \gamma c^2 - cH_0\Omega_q(0),
\]

determines constant \( \gamma_{0,1n} \) valid inside \( r_0 \). \( \gamma_{0,1n} \) is determined by local mean source density \( \bar{\rho}_0 \). \( \gamma \) in the halo is not changed. Its value is a constant of integration that cannot vary in the source-free halo [8, 9]. Hence there is no way to determine a mass-dependent increment to \( \gamma \). This replaces the usually assumed \( \gamma = \gamma_0 + N^* \gamma^* \) by \( \gamma = \gamma_H \), determined at halo boundary \( r_H \).

VII. IMPLICATIONS FOR COSMOLOGY

The common assumption for galactic growth is that a primordially accumulated dark matter halo subsequently attracts baryonic matter to form an observable galaxy. Conformal theory as well as MOND reverse this sequence, while eliminating the need for dark matter. The nonclassical CG gravitational acceleration is a byproduct of the gravitational accumulation of baryonic matter attracted to a growing galaxy. The CHM generates a uniform constant nonclassical centripetal acceleration within a halo of large expanding radius. The rate of galactic growth must depend on the net incoming flux of matter diffusing across the halo boundary, where the net gravitational radial acceleration vanishes.

Galactic collision is initiated by halo contact. Because halo volume is determined by galactic mass, it must remain constant, implying distortion of colliding halo boundaries analogous to collision of two spherical balloons. A dynamical model must consider diffusion of matter across a changing halo boundary. A new observable phenomenon affects neighboring galaxies in a galactic cluster. Once halos are in contact, the accessible source of primordial matter is restricted, hence reducing the rate of growth of both colliding galaxies. This should be observed as cessation of growth from the primordial background in the extreme case of a galaxy completely surrounded by a cluster of contiguous halos.

The empirical correlation relation of McGaugh et al. [28] establishes total radial acceleration as a function of its baryonic Newtonian value. This implies CG \( \gamma \) independent of galactic mass [28, 24], which places a strong constraint on galactic rotation curves. The selected particular solution, Eq. (8) for \( B(R) \), which differs from [10], depends on independently determined \( \gamma \). Resulting nonzero \( c_2 \) implies singular Ricci scalar at the galactic center [9], relevant to formation of a supermassive black hole.

Because the coefficient of the source term in the conformal Friedmann equation is negative, primordial energy density must cause centrifugal acceleration. This may create a dynamical Big Bang in the CHM without requiring a separate field. The relevancy of CHM should be explored. Weak time-dependence of scalar field \( \Phi \) is implied [6, 29]. For large redshifts, the Friedmann equation for scale factor \( a(t) \) and the CHM equation for \( \Phi(t) \) must be integrated together. A changed Higgs amplitude \( \phi_0 \) affects initial atomic abundances.
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