Detection of Charged MSSM Higgs Bosons
at CERN LEP-II and NLC

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Abstract

We study the possibility of detecting the charged Higgs bosons predicted in the Minimal Supersymmetric Standard Model ($H^\pm$), with the reactions $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$, using the helicity formalism. We analyze the region of parameter space ($m_A^0 - \tan \beta$) where $H^\pm$ could be detected in the limit when $\tan \beta$ is large. The numerical computation is done for the energy which is expected to be available at LEP-II ($\sqrt{s} = 200 \text{ GeV}$) and for a possible Next Linear $e^+e^-$ Collider ($\sqrt{s} = 500 \text{ GeV}$).
Although the Standard Model (SM) \([1]\) provides a precise description of existing data on electroweak interactions, the Higgs boson \([2]\), an essential ingredient of the model, has not been observed. It is quite possible that the actual scalar sector in nature has more than one doublet of Higgs bosons or has Higgs bosons in other multiplets. This is expected in many theories that go beyond the SM. A discovery of charged Higgs bosons would be unambiguous evidence that the electroweak symmetry-breaking sector of the SM consists of at least two Higgs doubles. The theoretical framework of this paper is the Minimal Supersymmetric extension of the Standard Model (MSSM), which doubles the spectrum of particles of the SM and the new free parameters obey simple relations. The scalar sector of the MSSM \([3]\) requires two Higgs doublets, thus five physical Higgs bosons are predicted: two CP-even Higgs bosons, \(h\) and \(H^0\) with \(m_{h^0} < m_{H^0}\) a CP-odd Higgs boson, \(A^0\), and two charged Higgs bosons, \(H^\pm\), whose detection would be a clear signal of new physics. The sector of Higgs is specified at tree level by fixing two parameters, which can be chosen as the mass of the pseudoscalar \(m_A\) and the ratio of vacuum expectation values of the two doublets \(\tan \beta = \frac{v_2}{v_1}\), then the mass \(m_{h^0}, m_{A^0}\), and \(m_{H^\pm}\) and \(\alpha\), the mixing angle of the CP-even Higgs bosons can be fixed. However, since radiative corrections produce substantial effects on the predictions of the model \([4]\), it is necessary to specify also the squark masses, which are assumed to be degenerated. In this paper, we focus on the phenomenology of the charged Higgs bosons \((H^\pm)\). The cross section is given by \([5]\): \(\sigma(e^+e^- \to H^+H^-) \approx 0.31(1 - 4m^2_{H^\pm}/s)^{3/2} \text{[pb]}\), with \(m_A^2 = m_{H^\pm}^2 + m_{A^0}^2\).

The decay modes of the Higgs boson determine the signatures in the detector. Charged Higgs bosons are expected to decay predominantly into the heaviest kinematically accessible fermion pair. If the top quark were light than the charged Higgs boson, the reaction \(H^+ \to tb\) would be dominant, and thus the following reactions are most important:

\[
e^+e^- \to H^+H^- \to c\bar{s}s\bar{c}, t\bar{b}b, cs\tau^-\bar{\nu}_\tau, \tau^+\nu_\tau\tau^-\bar{\nu}_\tau.
\]

The resulting signatures are events with four jets, two jets, a \(\tau\) lepton and missing energy, and two \(\tau\) leptons with large missing energy. No signal has been observed.

The search for Higgs bosons of the MSSM at LEP-2 will be based mainly on the Higgs-strahlung process \(e^+e^- \to Z^0 + h^0(H^0)\) and the associated production \(e^+e^- \to A^0 + h^0\) \([6]\). These two mechanisms are somehow complementary. In fact, for small values of \(\tan \beta\) the first process dominates, whereas at large \(\tan \beta\) the second reaction becomes quantitatively more important \([7]\). The dominant decay modes of the neutral Higgs particles are in general \(b\bar{b}\) (\(\sim 90\%\)) and \(\tau^+\tau^-\) decays (\(\sim 10\%\)) which are easy to detect experimentally at \(e^+e^-\) colliders \([7][8]\). Charged Higgs particles decay predominantly into \(\tau\nu_\tau\) and \(tb\) pairs.

In an earlier paper \([11]\) has been explored the possibility of finding one or more of the neutral Higgs bosons predicted by the MSSM in \(gg \to b\bar{b}h (h = h^0, H^0, A^0)\) followed by \(h \to b\bar{b}\), profiting from the very high \(b\)-tagging efficiencies. In other works \([12]\), the discovery reach of the Tevatron and the LHC for detecting a Higgs boson \((h)\) via the processes \(p\bar{p}/pp \to b\bar{b}h(\to b\bar{b}) + X\) has been studied and the possibility of detecting SUSY Higgs bosons at Fermilab and LHC if \(\tan \beta\) is large has been shown. In other paper, we study the detection of neutral MSSM Higgs bosons at \(e^+e^-\) colliders, including three-body process \([13]\).

In the case of the hadron colliders the three-body diagrams come from gluon fusion and this fact makes the contribution from these diagrams more important, due to the gluon...
abundance inside the hadrons. The advantage for the case of $e^+e^-$ colliders is that the signals of the processes are cleaner. The situation for the charged Higgs $H^\pm$ is apparently even less optimistic and extremely complicated. Indeed, the cross section for charged Higgs boson production via the channel $e^+e^- \rightarrow H^+H^-$ [13, 14] (although not particularly small at LEP-2) yields a signal which is very hard to extract, because of the huge irreducible background in $e^+e^- \rightarrow W^+W^-$ events. In fact, on the one hand, the MSSM mass relations tell us (at tree-level) that $m_{H^\pm}^2 = m_W^2 + m_A^2$ and, on the other hand, kinematic bounds dictated by the LEP-2 centre of mass energy imply that only $H^\pm$ scalars with mass $m_{H^\pm} \leq \sqrt{s}/2$ can be produced. The typical signature of a $H^\pm$ scalar would be most likely an excess of $\tau$ events with respect to the rates predicted by the SM, as the lepton-neutrino decay channel has the largest branching ratio.

For $m_{H^\pm} > m_t + m_b$, one consider searching for the decay of the charged Higgs to $t\bar{b}$. This signal is most promising when used in conjunction with the production processes $gg \rightarrow t\bar{b}H^-$, $b\bar{b}H^+$, and tagging several of the four $b$ jets in the final state [13]. For moderate tan $\beta$, the production cross section is suppressed such that the signal is not observable above the irreducible $t\bar{b}b\bar{b}$ background. The potential of this process is therefore limited to small and large values of tan $\beta$. With 200 $fb^{-1}$, a signal may be observable for $\tan \beta < 2$ and $m_{H^\pm} < 400$ GeV, and for $\tan \beta > 20$ and $m_{H^\pm} < 300$ GeV.

For $m_{A^0} \leq m_{Z^0}$, and if 50 events criterion are adequate, the $H^+H^-$ pair production will be kinematically allowed and easily observable [3, 13, 18]. For $m_{A^0} > 120$ GeV, $e^+e^- \rightarrow H^+H^-$ must be employed for detection of the three heavy Higgs bosons. Assuming that SUSY decays are not dominant, and using the 50 event criterion $H^+H^-$ can be detected up to $m_{H^\pm} = 230$ GeV [3, 13, 18], assuming $\sqrt{s} = 500$ GeV. The upper limits in the $H^+H^-$ mode are almost entirely a function of the machine energy (assuming an appropriately higher integrated luminosity is available at a higher $\sqrt{s}$). Two recentstudies [13, 20] show that at $\sqrt{s} = 1$ TeV, with an integrated luminosity of 200 $fb^{-1}$, $H^+H^-$ detection would extended to $m_{A^0} \sim m_{H^\pm} \sim 450$ GeV even if substantial SUSY decays of these heavier Higgs bosons are present.

In the present paper we study the production of charged SUSY Higgs bosons at $e^+e^-$ colliders. We are interested in finding regions that could allow the detection of the SUSY Higgs bosons for the set parameter space ($m_{A^0} - \tan \beta$). We shall discuss the charged Higgs bosons production $\tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$ in the energy range of LEP-II and NLC for large values of the parameter $\tan \beta$, where one expects to have high production. Since the couplings $\tau^-\bar{\nu}_\tau H^+$ ($\tau^+\nu_\tau H^-$) are directly proportional to $\tan \beta$, the cross section will receive a large enhancement factor when $\tan \beta$ is large. We consider the complete set of Feynman diagrams at the tree level and use the helicity formalism [21, 27] for the evaluation of the amplitudes. The results obtained for the three-body processes are compared with the dominant mode two-body reactions for the plane ($m_{A^0} - \tan \beta$). Succinctly, our aim in this work is to analyze how much the results of the mode two-body [Figs. 1.1, 1.4, 1.6 and 1.9] would be enhanced by the contribution from the diagrams depicted in Figs. 1.2, 1.3, 1.5, 1.7, 1.8, and 1.10, in the which the SUSY Higgs boson is radiated by a $\tau^-\bar{\nu}_\tau (\tau^+\nu_\tau)$ lepton.

This paper is organized as follows. We present in Sect. II the relevant details of the calculations. Sections III contains the results for the process $e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \tau^+\nu_\tau H^-$ at LEP-II and NLC. Finally, Sec. IV contains our conclusions.
II. HELICITY AMPLITUDE FOR CHARGED HIGGS BOSONS PRODUCTION

When the number of Feynman diagrams is increased, the calculation of the amplitude is a rather unpleasant task. Some algebraic forms [28] can be used in it to avoid manual calculation, but sometimes the lengthy printed output from the computer is overwhelming, and one can hardly find the required results from it. The CALKUL collaboration [29] suggested the Helicity Amplitude Method (HAM) which can simplify the calculation remarkably and hence make the manual calculation realistic.

In this section we discuss the evaluation of the amplitudes at the tree level for \( e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+ \), \( \tau^+\nu_\tau H^- \) using the HAM [21–27]. This method is a powerful technique for computing helicity amplitudes for multiparticle processes involving massless spin-1/2 and spin-1 particles. Generalization of this method that incorporates massive spin-1/2 and spin-1 particles, are given in Ref. [27]. This algebra is easy to program and more efficient than computing the Dirac algebra.

A charged Higgs boson \( H^\pm \) can be produced in scattering \( e^+e^- \) via the following processes:

\[
e^+e^- \rightarrow \tau^-\bar{\nu}_\tau H^+, \quad (1) \\
e^+e^- \rightarrow \tau^+\nu_\tau H^-. \quad (2)
\]

The diagrams of Feynman, which contribute at the tree-level to the different reaction mechanisms are depicted in Fig. 1. Using the Feynman rules given by the minimal supersymmetric standard model (MSSM), as are summarized in Ref. [30], we can write the amplitudes for these reactions. For the evaluation of the amplitudes we have used the spinor-helicity technique of Xu, Zhang, and Chang [22] (XZC) which is a modification of the technique developed by the CALKUL collaboration [29]. Following XZC, we introduce a very useful notation for the calculation of the processes (1) and (2).

A. Cases \( \tau^-\bar{\nu}_\tau H^+ \) and \( \tau^+\nu_\tau H^- \)

Let us consider the process

\[
e^- (p_1) + e^+ (p_2) \rightarrow \{ \tau^- (k_2) + \bar{\nu}_\tau (k_3) + H^+ (k_1), \tau^+ (k_2) + \nu_\tau (k_3) + H^- (k_1) \}, \quad (3)
\]

in which the helicity amplitude is denoted by \( \mathcal{M}[\lambda(e^-), \lambda(e^+), \lambda(\tau^\pm), \lambda(\nu_\tau)] \). The Feynman diagrams for this process are shown in Fig. 1. From this figure it follows that the amplitudes that correspond to each graph are

\[
\begin{align*}
\mathcal{M}_1 &= -iC_1 P_H^{-}(k_2 + k_3) P_Z(p_1 + p_2) T_1, \\
\mathcal{M}_2 &= iC_2 P_\tau(k_1 + k_3) P_Z(p_1 + p_2) T_2, \\
\mathcal{M}_3 &= -iC_3 P_\nu(k_1 + k_2) P_Z(p_1 + p_2) T_3, \\
\mathcal{M}_4 &= -iC_4 P_H^{-}(k_2 + k_3) P_\gamma(p_1 + p_2) T_4, \\
\mathcal{M}_5 &= iC_5 P_\tau(k_1 + k_3) P_\gamma(p_1 + p_2) T_5, \\
\mathcal{M}_6 &= -iC_1 P_H^+(k_2 + k_3) P_Z(p_1 + p_2) T_6, \\
\mathcal{M}_7 &= -iC_2 P_\tau(k_1 + k_3) P_Z(p_1 + p_2) T_7,
\end{align*}
\]
\[ \mathcal{M}_8 = iC_3 P_\nu(k_1 + k_2) P_Z(p_1 + p_2) T_8, \]
\[ \mathcal{M}_9 = -iC_4 P_{H^+}(k_2 + k_3) P_\gamma(p_1 + p_2) T_9, \]
\[ \mathcal{M}_{10} = -iC_5 P_\tau(k_1 + k_3) P_\gamma(p_1 + p_2) T_{10}, \]

where

\[ C_1 = -\frac{g^3 m_\tau}{16\sqrt{2} m_W} \tan \beta \cos \theta_W \frac{\cos^2 \theta_W}{\cos^2 \theta_W} \]
\[ C_2 = \frac{g^3 m_\tau}{32\sqrt{2} m_W} \tan \beta \frac{1}{\cos^2 \theta_W} \]
\[ C_3 = \frac{g^3 m_\tau}{32\sqrt{2} m_W} \tan \beta \frac{1}{\cos^2 \theta_W} \]
\[ C_4 = \frac{g^3 m_\tau}{2\sqrt{2} m_W} \tan \beta \sin^2 \theta_W \]
\[ C_5 = \frac{g^3 m_\tau}{2\sqrt{2} m_W} \tan \beta \sin^2 \theta_W \]

while that the propagators are

\[ P_Z(p_1 + p_2) = \frac{(s - m_Z^2)}{(s - m_Z^2)^2 + (m_Z \Gamma_Z)^2}, \]
\[ P_{H^+}(k_2 + k_3) = \frac{(2k_2 \cdot k_3 - m_{H^+}^2)}{(2k_2 \cdot k_3 - m_{H^+}^2)^2 + (m_{H^\pm} \Gamma_{H^\pm})^2}, \]
\[ P_\tau(k_1 + k_3) = \frac{1}{m_{H^\pm}^2 + 2k_1 \cdot k_3}, \]
\[ P_\nu(k_1 + k_2) = \frac{1}{m_{H^\pm}^2 + 2k_1 \cdot k_2}, \]
\[ P_\gamma(p_1 + p_2) = \frac{1}{s}, \]

where \( s = (p_1 + p_2)^2 \) and the corresponding tensors are

\[ T_1^{\mu} = \bar{u}(k_2)(1 - \gamma_5) v(k_3) \bar{v}(p_2) (k_1 - k_2 - k_3)(v_\mu^* - a_\mu^* \gamma_5) u(p_1), \]
\[ T_2^{\mu} = \bar{u}(k_2) \gamma^\mu (v_\mu^* - a_\mu^* \gamma_5)(k_1 + k_3)(1 - \gamma_5) v(k_3) \bar{v}(p_2) \gamma_\mu(v_\mu^* - a_\mu^* \gamma_5) u(p_1), \]
\[ T_3^{\mu} = \bar{u}(k_2)(1 - \gamma_5)(k_1 + k_2) \gamma_\mu (v_\mu^* - a_\mu^* \gamma_5) v(k_3) \bar{v}(p_2) \gamma_\mu(v_\mu^* - a_\mu^* \gamma_5) u(p_1), \]
\[ T_4^{\mu} = \bar{u}(k_2)(1 - 5)(v(k_3) \bar{v}(p_2)(k_1 - k_2 - k_3) u(p_1), \]
\[ T_5^{\mu} = \bar{u}(k_2) \gamma_\mu (k_1 + k_3)(1 - \gamma_5) v(k_3) \bar{v}(p_2) \gamma_\mu u(p_1), \]
\[ T_6^{\mu} = \bar{u}(k_3)(1 + \gamma_5) v(k_2) \bar{v}(p_2)(k_2 + k_3 - k_1)(v_\mu^* - a_\mu^* \gamma_5) u(p_1), \]
\[ T_7^{\mu} = \bar{u}(k_3)(1 + 5)(k_1 + k_3) \gamma_\mu (v_\mu^* - a_\mu^* \gamma_5) v(k_2) \bar{v}(p_2) \gamma_\mu(v_\mu^* - a_\mu^* \gamma_5) u(p_1), \]
\[ T_8^{\mu} = \bar{u}(k_3) \gamma_\mu (v_\mu^* - a_\mu^* \gamma_5)(k_1 + k_3)(1 + \gamma_5) v(k_2) \bar{v}(p_2) \gamma_\mu(v_\mu^* - a_\mu^* \gamma_5) u(p_1), \]
\[ T_9^{\mu} = \bar{u}(k_3)(1 + 5)(k_1 + k_3) \gamma_\mu v(k_2) \bar{v}(p_2)(k_2 + k_3 - k_1) u(p_1), \]
\[ T_{10}^{\mu} = \bar{u}(k_3)(1 + 5)(k_1 + k_3) \gamma_\mu v(k_2) \bar{v}(p_2) \gamma_\mu u(p_1). \]
In fact, we rearrange the tensors $T$’s in such a way that they become appropriate to a computer program. Then, following the rules from helicity calculus formalism [21–27] and using identities of the type

$$\{\bar{u}_\lambda(p_1)\gamma^\mu u_\lambda(p_2)\}\gamma^\mu = 2u_\lambda(p_2)\bar{u}_\lambda(p_1) + 2u_{-\lambda}(p_1)\bar{u}_{-\lambda}(p_2), \tag{8}$$

which is in fact the so called Chisholm identity, and

$$\dot{p} = u_\lambda(p)\bar{u}_\lambda(p) + u_{-\lambda}(p)\bar{u}_{-\lambda}(p), \tag{9}$$

defined as a sum of the two projections $u_\lambda(p)\bar{u}_\lambda(p)$ and $u_{-\lambda}(p)\bar{u}_{-\lambda}(p)$.

The spinor products are given by

$$s(p_i, p_j) \equiv \bar{u}_+(p_i)u_-(p_j) = -s(p_j, p_i),$$

$$t(p_i, p_j) \equiv \bar{u}_-(p_i)u_+(p_j) = [s(p_j, p_i)]^\pm. \tag{10}$$

Using Eqs. (8)-(10), which are proved in Ref. [27], we can reduce many amplitudes to expressions involving only spinor products.

Evaluating the tensors of Eq. (7) for each combination of $(\lambda, \lambda')$ with $\lambda, \lambda' = \pm 1$ one obtains the following expressions:

$$\mathcal{M}_1(\pm, +) = F_1 f_1^{\pm, +} s(k_2, k_3)[s(p_2, k_1)t(k_1, p_1) - s(p_2, k_2)t(k_2, p_1) - s(p_2, k_3)t(k_3, p_1)],$$

$$\mathcal{M}_1(-, +) = F_1 f_1^{-, +} s(k_2, k_3)[t(p_2, k_1)s(k_1, p_1) - t(p_2, k_2)s(k_2, p_1) - t(p_2, k_3)s(k_3, p_1)], \tag{11}$$

$$\mathcal{M}_2(\pm, +) = F_2 f_2^{\pm, +} s(k_2, p_2)t(p_1, k_1)s(k_1, k_3),$$

$$\mathcal{M}_2(-, +) = F_2 f_2^{-, +} s(k_2, p_1)t(p_2, k_1)s(k_1, k_3), \tag{12}$$

$$\mathcal{M}_3(\pm, +) = F_3 f_3^{\pm, +} s(k_2, k_1)t(k_1, p_1)s(p_2, k_3),$$

$$\mathcal{M}_3(-, +) = F_3 f_3^{-, +} s(k_2, k_1)t(k_1, p_1)s(k_1, k_3), \tag{13}$$

$$\mathcal{M}_4(\pm, +) = F_4 s(k_2, k_3)[s(p_2, k_1)t(k_1, p_1) - s(p_2, k_2)t(k_2, p_1) - s(p_2, k_3)t(k_3, p_1)],$$

$$\mathcal{M}_4(-, +) = F_4 s(k_2, k_3)[t(p_2, k_1)s(k_1, p_1) - t(p_2, k_2)s(k_2, p_1) - t(p_2, k_3)s(k_3, p_1)], \tag{14}$$

$$\mathcal{M}_5(\pm, +) = F_5 s(k_2, p_2)t(p_1, k_1)s(k_1, k_3),$$

$$\mathcal{M}_5(-, +) = F_5 s(k_2, p_1)t(p_2, k_1)s(k_1, k_3), \tag{15}$$

$$\mathcal{M}_6(\pm, +) = -F_1 f_6^{\pm, +} t(k_3, k_2)[s(p_2, k_1)t(k_1, p_1) - s(p_2, k_2)t(k_2, p_1) - s(p_2, k_3)t(k_3, p_1)],$$

$$\mathcal{M}_6(-, +) = -F_1 f_6^{-, +} t(k_3, k_2)[t(p_2, k_1)s(k_1, p_1) - t(p_2, k_2)s(k_2, p_1) - t(p_2, k_3)s(k_3, p_1)], \tag{16}$$

$$\mathcal{M}_7(\pm, +) = -F_2 f_7^{\pm, +} t(k_3, k_1)s(k_1, p_2)t(p_1, k_2),$$

$$\mathcal{M}_7(-, +) = -F_2 f_7^{-, +} t(k_3, k_1)s(k_1, p_2)t(p_1, k_2), \tag{17}$$

$$\mathcal{M}_8(\pm, +) = -F_3 f_8^{\pm, +} t(k_3, k_1)[s(p_2, k_1)t(k_1, k_3) + s(p_2, k_2)t(k_2, k_3)],$$

$$\mathcal{M}_8(-, +) = -F_3 f_8^{-, +} t(k_3, k_2)[t(p_1, k_1)s(k_1, k_3) + t(p_1, k_2)s(k_2, k_3)], \tag{18}$$

$$\mathcal{M}_9(\pm, +) = -F_4 t(k_3, k_2)[s(p_2, k_1)t(k_1, p_1) - s(p_2, k_2)t(k_2, p_1) - s(p_2, k_3)t(k_3, p_1)],$$

$$\mathcal{M}_9(-, +) = -F_4 t(k_3, k_2)[t(p_2, k_1)s(k_1, p_1) - t(p_2, k_2)s(k_2, p_1) - t(p_2, k_3)s(k_3, p_1)], \tag{19}$$

$$\mathcal{M}_{10}(\pm, +) = -F_5 t(k_3, k_1)s(k_1, p_2)t(p_1, k_2),$$

$$\mathcal{M}_{10}(-, +) = -F_5 t(k_3, k_1)s(k_1, p_1)t(p_2, k_2), \tag{20}$$

where
we present our results. We utilize the helicity formalism for the evaluation of their amplitudes. In the next subsection, respectively. We consider the complete set of Feynman diagrams (Fig. 1) at the tree level and including three-body mode diagrams [Figs. 1.1, 1.4, 1.6, and 1.9] assuming an integrated luminosity of \( L = 500 \, \text{pb}^{-1} \) and \( L = 10 \, \text{fb}^{-1} \) at \( \sqrt{s} = 200 \, \text{GeV} \) and 500 \( \text{GeV} \) for LEP-II and NLC, respectively. In other works \([9,32–34]\), the possibility of detecting SUSY Higgs bosons at gg predicted by the MSSM in \( e^+e^- \rightarrow \tau^-\nu_\tau H^+, \tau^+\nu_\tau H^- \).

### III. DETECTION OF CHARGED HIGGS BOSONS AT LEP-II AND NLC ENERGIES

In an earlier paper \([15]\) has been explored the possibility of finding charged Higgs bosons predicted by the MSSM in \( gg \rightarrow b\bar{b}H^+, \bar{t}bH^- \), and tagging several of the four \( b \) jets in the final state. In other works \([7,22,44]\), the possibility of detecting SUSY Higgs bosons at LEP-II and NLC if \( \tan \beta \) is large has been shown.

In this paper, we study the detection of charged MSSM Higgs bosons at \( e^+e^- \) colliders, including three-body mode diagrams [Figs. 1.2, 1.3, 1.5, 1.7, 1.8, and 1.10] besides the dominant mode diagrams [Figs. 1.1, 1.4, 1.6, and 1.9] assuming an integrated luminosity of \( \mathcal{L} = 500 \, \text{pb}^{-1} \) and \( \mathcal{L} = 10 \, \text{fb}^{-1} \) at \( \sqrt{s} = 200 \, \text{GeV} \) and 500 \( \text{GeV} \) for LEP-II and NLC, respectively. We consider the complete set of Feynman diagrams (Fig. 1) at the tree level and utilize the helicity formalism for the evaluation of their amplitudes. In the next subsection, we present our results.

\[
F_1 = -2iC_1 P_{H^\pm}(k_2 + k_3) P_Z(p_1 + p_2),
\]
\[
F_2 = 4iC_2 P_{\tau}(k_1 + k_3) P_Z(p_1 + p_2),
\]
\[
F_3 = -4iC_3 P_{\nu}(k_1 + k_2) P_Z(p_1 + p_2),
\]
\[
F_4 = -2iC_4 P_{H^\pm}(k_2 + k_3) P_{\tau}(p_1 + p_2),
\]
\[
F_5 = 4iC_5 P_{\tau}(k_1 + k_3) P_{\tau}(p_1 + p_2),
\]

and

\[
f_1^{+,+} = f_6^{+,+} = (v_\nu^z - a_\nu^z),
\]
\[
f_2^{+,+} = f_7^{+,+} = (v_\nu^z + a_\nu^z),
\]
\[
f_2^{+,+} = f_7^{+,+} = (v_\nu^z - a_\nu^z)^2,
\]
\[
f_3^{+,+} = f_8^{+,+} = ((v_\nu^z)^2 - (a_\nu^z)^2),
\]
\[
f_3^{+,+} = f_8^{+,+} = (v_\nu^z + a_\nu^z)(v_\nu^z - a_\nu^z),
\]
\[
f_3^{+,+} = f_8^{+,+} = (v_\nu^z + a_\nu^z)(v_\nu^z + a_\nu^z).
\]

Here, \( v_\nu^z = -1 + 4 \sin^2 \theta_W, a_\nu^z = -1, v_\nu^z = 1 \) and \( a_\nu^z = 1 \), according to the experimental data \([31]\).

After the evaluation of the amplitudes of the corresponding diagrams, we obtain the cross sections of the analyzed processes for each point of the phase space using Eqs. (11)-(20) by a computer program, which makes use of the subroutine RAMBO (Random Momenta Beautifully Organized). The advantages of this procedure in comparison to the traditional “trace technique” are discussed in Refs. \([21,27]\).

We use the Breit-Wigner propagators for the \( Z^0 \) and \( H^\pm \) bosons. The mass \( (M_{Z^0} = 91.2 \, \text{GeV}) \) and width \( (\Gamma_Z = 2.4974 \, \text{GeV}) \) of \( Z^0 \) have been taken as inputs; the width of \( H^\pm \) are calculated from the formulas given in Ref. \([30]\). In the next section we present the numerical computation of the processes \( e^+e^- \rightarrow \tau^-\nu_\tau H^+, \tau^+\nu_\tau H^- \).
A. Detection of $H^\pm$

In order to illustrate our results on the detection of the $H^\pm$ Higgs boson, we present graphs in the parameters space region $(m_{A^0} - \tan \beta)$, assuming $m_t = 175$ GeV, $M_\tau = 500$ GeV and $\tan \beta > 1$ for LEP-II and NLC. Our results are displayed in Fig. 2-5, for $e^+e^- \rightarrow H^+H^-$ dominant mode and for the processes at three-body $e^+e^- \rightarrow \tau^\mp \bar{\nu}_\tau H^\pm, \tau^\pm \nu_\tau H^\mp$.

The total cross section for the reaction $e^+e^- \rightarrow H^+H^-$ at LEP-II are show in Fig. 2 for each contour with 0.01, 0.001, and 0.0001 pb, which gives 5 events, 0.5 events, and 0.05 events, respectively.

For the case of the processes at three-body $e^+e^- \rightarrow \tau^\mp \bar{\nu}_\tau H^\pm, \tau^\pm \nu_\tau H^\mp$, the results on the detection of the $H^\pm$ are show in Fig. 3. The total cross section for each contour is 0.01, 0.001, and 0.0001 pb; this give 5 events, 0.5 events, and 0.05 events. We can see from this figure, that the effect of the reactions $\tau^- \nu_\tau H^+$ and $\tau^+ \nu_\tau H^-$ is lightly more important that $H^+H^-$, for most of the $(m_{A^0} - \tan \beta)$ parameters space regions. Nevertheless, there are substantial portions of parameters space in which the discovery of the $H^\pm$ is not possible using either $H^+H^-$ or $\tau^- \nu_\tau H^+$ and $\tau^+ \nu_\tau H^-$. On the other hand, if we focus the detection of the $H^\pm$ at Next Linear $e^+e^-$ Collider with $\sqrt{s} = 500$ GeV and $\mathcal{L} = 10$ fb$^{-1}$, the panorama for its detection is more extensive. Figure 4, show the contours lines in the plane $(m_{A^0} - \tan \beta)$, to the cross section of $H^+H^-$. The contours for this cross section correspond to 100, 10, and 1 events.

In the case of the processes at three-body $e^+e^- \rightarrow \tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$ Fig. 5, the cross section is 0.01, 0.001, and 0.0001 pb, which gives 100 events, 10 events, and 1 events, respectively.

The effect of incorporate $\tau^- \bar{\nu}_\tau H^+$ and $\tau^+ \nu_\tau H^-$ in the detection of the Higgs boson $H^\pm$ is more important than the case of two-body mode $H^+H^-$, because $\tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$ cover a major region in the parameters space $(m_{A^0} - \tan \beta)$. The most important conclusion from this figure is that detection of the charged Higgs bosons will be possible at $\sqrt{s} = 500$ GeV.

IV. CONCLUSIONS

In this paper, we have calculated the production of the charged Higgs bosons in association with $\tau^- \bar{\nu}_\tau$ and $\tau^+ \nu_\tau$ via the processes $e^+e^- \rightarrow \tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$, and using the helicity formalism. We find that this processes could help to detect a possible charged Higgs boson at LEP-II and NLC energies when $\tan \beta$ is large.

The detection of $H^\pm$ through of the reactions $e^+e^- \rightarrow \tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$, compete favorable with the mode dominant $e^+e^- \rightarrow H^+H^-$. The processes at three-body $\tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$ cover lightly a major portion of the parameter space $(m_{A^0} - \tan \beta)$ as is shown in Fig. 3 and the corresponding cross section for each contour is of $\sigma = 0.01, 0.001, 0.0001$ pb for LEP-II. For NLC energies we have that $\sigma = 0.01, 0.001, 0.0001$ pb and the corresponding contours are shown in Fig. 5. We can conclude that there is a region where the Higgs bosons $H^\pm$ could be detected at the next high-energy machines (NLC).

In summary, we conclude that the possibilities of detecting or excluding the charged Higgs bosons of the minimal supersymmetric standard model ($H^+H^-$) in the processes $e^+e^- \rightarrow \tau^- \bar{\nu}_\tau H^+, \tau^+ \nu_\tau H^-$ are important and in some cases are compared favorably with
the mode dominant $e^+e^- \rightarrow H^+H^-$ in the region of parameter space $(m_{A^0} - \tan \beta)$ with large $\tan \beta$. The detection of the charged Higgs bosons will require the combined use of a future high energy machine such as LEP-II and the Next Linear $e^+e^-$ Collider.

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Fig. 1 Feynman diagrams at tree-level for $e^+e^- \rightarrow \tau^\pm \nu_{\tau}H^\pm, \tau^\mp \nu_{\tau}H^-$.  

Fig. 2 Total cross sections contours in $(m_{A^0} - \tan \beta)$ parameter space for $e^+e^- \rightarrow H^+H^-$ at LEP-II with $\sqrt{s} = 200 \ GeV$ and an integrated luminosity of $\mathcal{L} = 500 \ pb^{-1}$. We have taken $m_t = 175 \ GeV$ and $M_{\widetilde{t}} = 500 \ GeV$ and neglected squark mixing.  

Fig. 3 Same as in Fig. 2 but for $e^+e^- \rightarrow \tau^- \nu_{\tau}H^+, \tau^+ \nu_{\tau}H^-$.  

Fig. 4 Total cross sections contours for an NLC with $\sqrt{s} = 500 \ GeV$ and $\mathcal{L} = 10 \ fb^{-1}$. We have taken $m_t = 175 \ GeV$, $M_{\widetilde{t}} = 500 \ GeV$ and neglected squark mixing. We display contours for $e^+e^- \rightarrow H^+H^-$, in the parameters space $(m_{A^0} - \tan \beta)$.  

Fig. 5 Same as in Fig. 4 but for $e^+e^- \rightarrow \tau^- \nu_{\tau}H^+, \tau^+ \nu_{\tau}H^-$.  

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Fig. 1
Fig. 2

\[ \tan(\beta) \]

\[ 0.0001 \]

\[ 0.001 \]

\[ 0.01 \]

\[ m_A (\text{GeV}) \]

\[ H^+ H^- \]
Fig. 3

\[ \tau \bar{\nu}_\tau H^+ + \bar{\tau} \nu_\tau H^- \]
Fig. 4

\( \tan(\beta) \)

\( H^+ H^- \)

\( 0.01 \)

\( 0.001 \)

\( 0.0001 \)

\( m_A (\text{GeV}) \)
Fig. 5

$\tan(\beta)$

$\tau \bar{\nu}_{\tau} H^+ + \bar{\tau} \nu_{\tau} H^-$

$m_A (\text{GeV})$