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Estimating time constants for underfloor heating control

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Abstract. Time constants are used for tuning PID control parameters and should be correctly estimated to achieve a well-performing controller. However, the estimation of time constants for a very slowly reacting system such as underfloor heating is not easy in practice. In this work, different methods were used to estimate the time constants for underfloor heating. Graphical estimation and transfer function fitting were the core approaches. Measurement data and simulations on a calibrated model were used for the estimations. The resulting time constant values for the graphical method depended clearly on the setback duration and the values differed up to a factor of 100. For the transfer function method, time constant values did not vary significantly. For light construction, the time constant was around 10 hours and for massive, around 30 hours for the otherwise identical room.

1. Introduction
A well-tuned proportional-integral-derivative (PID) controller is the basis of many model predictive control (MPC) algorithms used in the current literature on heating control [1]. While MPC controls the setpoints, a room-level PID controller must complete the setpoint tracking [2]. In building heating control, a derivative part is often omitted, and proportional-integral (PI) controllers are used instead. In practice, thermostats (P-controllers) are used. There has been no shift towards PI controllers for hydraulic underfloor heating. One possible reason for this is that the control parameters need to be tuned in each room separately, while the parameters are not easily tuneable for systems with high inertia. Tuning is especially difficult for massive systems, as conventional tuning tests can take an unreasonably long time. For systems that are over-dimensionalized, for example for safety margin or heat-up possibility, tuning tests can cause temperatures that exceed the comfort limit.

Several PID tuning methods use either a time constant directly or a time constant-based model for calculating control parameters. However, estimating the time constant and simple transfer function models for underfloor heating can be challenging in practice. As the time constants can vary significantly, we test a range of methods and compare the measured time constant values. We aim to determine the difference between values achieved by tests with practical length and by ideal steps.

2. Theoretical background
2.1. Modelling approach
To track the temperature setpoint with PI control, we need to simplify several phenomena. First, we need a single-input-single-output (SISO) model of the system. Therefore, we omit all load disturbances,
such as solar irradiation and internal gains. Second, to describe the system with only one time constant (and a time lag if needed), we linearize all non-linear phenomena that occur in the floor and heat transfer from floor to room air. This time constant should not be confused with the cool-down time constant of the building. The time constant in this study describes first and foremost the heating system, though building construction and air exchange also have effects.

For model-based PI tuning, a transfer function model is the typical option. A second order transfer function is sometimes represented as a state-space model [3]. A transfer function has clearly defined time constants; there is one time constant if the model is first order. A first order process model with time delay (P1D) is shown in Equation 1 where \( r \) is the time constant, \( K_p \) is the process gain and \( L \) is time lag. Here and throughout, \( P \) is the process model and \( s \) is the complex frequency variable. The step response of this model in a time domain is shown in Equation 3. In our context, \( T \) is the room air temperature dependent on time \( (t) \). The second order transfer functions with time delay are shown in Equations 2 and 4 where the model in Equation 2 is the general case. Equation 4 represents an overdamped case (P2D) which can be separated as two first order systems, and so has two time constants, \( \tau_1 \) and \( \tau_2 \). The first of the two describes the same phenomena as P1D’s time constant. If the denominator does not have two real zeros, the system is underdamped (P2DU) and described by the general Equation 2. However, then the time constant no longer describes the same phenomena.

These models would need to be fitted from measured data. To fit the transfer function step response directly, an ideal step would be needed. In systems with high inertia, however, settling times are so long that it is impossible to apply this in practice. In practice, the same analysis is often used for steps that are not ideal; this results in shorter time constant values.

\[
P(s) = \frac{K_p}{1 + r \cdot s} e^{-sL}
\]

Equation 1

\[
P(s) = \frac{K_p}{1 + 2\lambda \omega + \omega^2 s^2} e^{-sL}
\]

Equation 2

\[
P(s) = \frac{K_p}{1 + \tau \cdot s + \tau^2} e^{-sL}
\]

Equation 3

\[
P(s) = \frac{K_p}{(1 + \tau_1 \cdot s)(1 + \tau_2 \cdot s)} e^{-sL}
\]

Equation 4

2.2. Graphical approach

A practical approach is to run setbacks and calculate the time constant from the rising temperature curve. For some hours, heating is turned off, and then turned on again to heat the room to the initial setpoint. For the heating-up period, minimum and maximum temperature levels are determined. Time at 63.2% of the temperature rise is regarded as the time constant. If there is a clear time lag between changing the setpoint and the start of a temperature change, this time lag is deducted from the time constant. Often, however, a time lag is not clear, and the time delay remains in the time constant value. Therefore, during setbacks, an ideal step response cannot be achieved, as the higher temperature level is limited and the lower level is not stable. The non-ideal step and the time delay in the time constant both preclude the result from being the theoretically defined time constant.

3. Methods

We applied both the modelling and the graphical approach described above, and compared the resulting time constants. With the aim of creating close-to-ideal steps, we measured air temperatures during two long setbacks in a test house located at TalTech campus, described in [4]. We observed mainly one room (R5) with a net floor area of 10.4 m² and two exterior walls each with a 4 m² window, one facing north and the other west. The underfloor heating with wet installation had a design power of 68 W/m². During calibration, it was reduced to 61 W/m². The floor construction is depicted in Figure 2. The setbacks were induced by changing the room thermostat setpoints from 21 °C to 18 °C and back. The first setback was 2 days and the second 3 days long. The measurement time step was 10 minutes. During the heating-up periods, the outdoor temperature stayed between -5 °C and +5 °C. Based on these measurements, we improved the cool-down calibration of the pre-calibrated model of the test building in IDA-ICE [5]. The calibration results are shown in Section 4.2. The heating curve was fitted from previous measurements to provide the supply temperature for the model. The resulting line is characterized in Equation 5, where \( T_{\text{supply}} \) is the flow temperature and \( T_{\text{out}} \) the outdoor temperature.
We conducted 2 types of simulation tests on the calibrated model:

1. Setbacks for 7 days, where the length of the setback is varied between 1, 3, 6, 12 and 24 hours
2. Close-to-ideal step change: two months of cooling the room (no heating, balance temperature develops as other rooms in the house are heated) and one month of heating

All tests were carried out in two outdoor conditions and for two construction types. Constant outdoor conditions with no solar irradiation were defined with outdoor temperatures at 0°C and -15°C, characterizing average and cold winter conditions in Estonia. The thermal mass was varied between light and massive versions by changing the exterior wall construction. The inner layer was 13 mm of gypsum board followed by an insulated wood frame construction in the case of light construction. For the massive case, there was concrete behind the gypsum board. The exact layering of the constructions is shown in Table 4 in the Appendix. For all the cases, doors between rooms were closed and no internal gains were included.

For measurements and ideal simulation cases, first and second order transfer function models according to the formats in Equations 1 and 3 were fit to the data. Fitting was done in a MATLAB system identification toolbox [6]. Additionally, the time constant was calculated as the time at 63.2% of the temperature increase for all cases.

### 4. Results

#### 4.1. Calculations based on measurement results

Measurement results and the setpoint are shown in Figure 3. As the actual average temperatures were shifted from the setpoint, but not equally on upper and lower levels, a range of possible start and end points could be used for the time constant calculation. At both ends, either the setpoint level or the actual average temperature level can be used. Correspondingly, the temperature difference $\Delta T$ between the minimum and maximum levels as well as the temperature at 63.2% between them ($\tau_r$) vary. Calculation results for all these cases are shown in Table 1. During both heating-up periods, the lowest temperature value occurred at the second time step after the setpoint rise to 21°C. Therefore, time delays of 10 minutes were detected. Resulting time constant values range from 9.5 to 12.2 hours.

Results of fitting transfer functions on the measured heating-up periods are summarized in Table 2. Fitting precisions varied between 70 and 78%. It is clear that there was an effect from the setback length (i.e., the duration of the lowered set temperature), as the time constant from the first two models, which were fitted on the first heating-up, was about 9.3 hours, and from the second two, 11.5 hours. Time delay
values varied, ranging from 12 to 31 minutes. The second order time constant from models 2 and 4 was about 2.6 minutes. 

During the heating-up after the 3-day setback, a clear solar peak emerged; therefore, the fit percentages to the estimation data were lower for this heat-up and the time constants vary a bit more. Nevertheless, all models fit similarly to both heating-up periods. All fitted models are shown on the first measured heating-up period in Figure 4, regardless of which heating-up was used to fit them.

Figure 3. Measured air temperature in the room (blue) and the current setpoint (light blue), red lines indicate the average temperature when the lower setpoint is reached and green lines the upper setpoint period average

Table 1. Time constant and time delay values calculated by 63.2% rule from measurement data dependent on where to read the beginning and ending of the temperature increase; the highest value in a column is marked in pink and the lowest in blue

| Setback length | Begin | End | ΔT (°C) | T1 (°C) | τ1 + L (min) | L (min) | τ (min) |
|---------------|-------|-----|---------|---------|--------------|---------|--------|
| 2 days        | Setpoint | Setpoint | 3 | 19.9 | 9 h 40 min | 10 min | 9 h 30 min |
| 2 days        | Setpoint | Level | 3.1 | 19.95 | 9 h 50 min | 10 min | 9 h 40 min |
| 2 days        | Level | Setpoint | 2.8 | 19.95 | 9 h 50 min | 10 min | 9 h 40 min |
| 2 days        | Level | level | 2.9 | 20.05 | 10 h 20 min | 10 min | 9 h 10 min |
| 3 days        | Setpoint | Setpoint | 3 | 19.9 | 10 h 40 min | 10 min | 10 h 30 min |
| 3 days        | Setpoint | Level | 3.15 | 20 | 11 h 30 min | 10 min | 11 h 20 min |
| 3 days        | Level | Setpoint | 2.7 | 20 | 11 h 30 min | 10 min | 11 h 20 min |
| 3 days        | Level | Level | 2.85 | 20.1 | 12 h 20 min | 10 min | 12 h 10 min |

Table 2. Time constant and time delay values retrieved from transfer function model identifications; models with best fit are presented

| Setback length | Model type | Fit % | Kp (°C) | τ1 (min) | L (min) | τ2 (min) |
|---------------|------------|-------|---------|----------|---------|----------|
| 1             | 2 days     | P1D   | 78      | 21.256   | 9 h 22 min | 12       |
| 2             | 2 days     | P2D   | 78      | 21.254   | 9 h 18 min | 23       | 0 h 2 min 32 s |
| 3             | 3 days     | P1D   | 70      | 21.472   | 11 h 37 min | 13       |
| 4             | 3 days     | P2D   | 70      | 21.466   | 11 h 28 min | 31       | 0 h 2 min 45 s |

Figure 4. Fitted models adjusted to the measured air temperature during the first heating-up test by initial state estimation

4.2. Model calibration

The measured setback data was used to improve a previously calibrated model of the test house. The heating power was reduced to 90% in all tested rooms and the supply temperature was adapted to the heating curve given by Equation 5. The calibration results for the room R5 are shown in Figure 5.
4.3. Simulation results

The 63.2% time constants calculated from the setbacks are shown in Figure 6. The time constants depend mostly on the setback length varying from 0.5 to 6.5 hours and, additionally, on the thermal capacity of walls at longer setback lengths. The outdoor temperature has little impact on the time constant; there is a slight variation due to different outdoor temperatures only for the shortest setback length.

We derived the ideal time constant from a full step change of 1 month. In all cases, all models perform very similarly with a maximum of 1.5% difference. Therefore, only P1D results are shown in Table 3. Time constant values from P1D and P2D were same for the light cases and for the massive case at -15℃. At 0℃, the time constant for P2D and massive construction was 31.4 hours. This is 6 hours more than P1D predicted, but agrees well with the value obtained at -15℃. The 63.2% method results on the ideal steps are shown in the last column. These time constants probably describe a different effect.

| Tout (℃) | Construction | Fit % | L (min) | τ (hrs) | τ at 63.2% (hrs) |
|----------|--------------|-------|---------|---------|-----------------|
| 0        | light        | 95    | 7.9     | 7.7     | 84              |
| 0        | massive      | 79    | 1.0     | 25.3    | 165             |
| -15      | light        | 89    | 6.6     | 8.5     | 87              |
| -15      | massive      | 79    | 3.0     | 31.3    | 179             |

5. Discussion

Time constant values obtained for the light construction case from different methods are compared in Figure 7. The values vary from 0.5 to 87 hours. This range would result in very different control parameters and control accuracies; an analysis of these effects should be carried out in a follow-up work.

Both the highest and lowest time constant values have been achieved by the 63.2% graphical method. It is clear, then, that when using this method, we cannot substitute a long test with shorter ones, unless we know the relationship between the setback length and the time constant value for the particular room and heating system. It remains to be determined which setback length produces a time constant that results in the best-performing PI parameters.

The time constants from estimating transfer functions are here always in the order of 10 hours. However, only longer setbacks are tested with this method. The short setbacks are more difficult to model as small changes in temperature for a short time do not provide enough data to get reliable results due to measurement noise. In future work, a transfer function model could be estimated from simulations of profiles with different setpoint temperatures.

Time constant values for the massive construction could be estimated only from simulations and with the 63.2% model. The short setback end is similar to the light construction case, whereas the time constants from ideal step are 2 times higher. Therefore, on a graph similar to that of Figure 7 the crosses would not be very well aligned.
Figure 7. Time constant values dependent on setback length obtained from different methods. Crosses used for the 63.2% method and diamonds for modelling methods; blue colours indicate simulation data as source, orange ones are derived from measurements; both axes are logarithmic.

6. Conclusion
We tested different time constant calculation methods on measured and simulated data. For the widely used method of reading the time constant at 63.2% of the temperature increase, a very clear dependence on the performed setback length was found. The resulting time constants ranged from 0.5 to 87 hours for light construction and up to 179 hours for massive construction. Therefore, using the time constant of this method for PI parameter tuning could result in suboptimal control performance.

Time constants from the first and second order transfer functions that were fit in this work did not show a dependence on setback length. A light construction time constant obtained with this method ranged from 7.7 to 11.5 hours. For massive construction, only an ideal-like step was used to model the time constants; the values ranged from 25.3 to 31.1 hours. Using either measured or simulated data for the estimations, the time constant values agreed as the simulation model was calibrated. Future work should clarify which of these time constants would perform best for PI parameter tuning.

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Appendix
Table 4. Exterior wall constructions for light and massive variants

| Layers from inside to outside | Thickness (mm) | Thermal conductivity (W/(m K)) | Specific heat capacity (J/(kg K)) | Density (kg/m³) |
|------------------------------|----------------|-------------------------------|----------------------------------|-----------------|
| Massive / light              |                |                               |                                  |                 |
| Gypsum board                 | 13             | 0.22                          | 1090                             | 970             |
| Concrete / insulated frames  | 200 / 45       | 1.7 / 0.044                   | 880 / 1720                       | 2300 / 56       |
| EPS Silver / chip board      | 216 / 10       | 0.033 / 0.13                  | 750 / 1300                       | 20 / 1000       |
| - / insulated frames         | - / 245        | - / 0.044                     | - / 1720                         | - / 56          |
| Wind protection board        | 30             | 0.037                         | 750                              | 20              |
| Air gap                      | 25             | 0.11                          | 1006                             | 1.2             |
| Wooden panels                | 22             | 0.12                          | 2300                             | 500             |