A unifying approach to left handed material design

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In this letter we show that equivalent circuits offer a qualitative and even quantitative simple explanation for the behavior of various types of left-handed (or negative index) meta-materials. This allows us to optimize design features and parameters, while avoiding trial and error simulations or fabrications. In particular we apply this unifying circuit approach in accounting for the features and in optimizing the structure employing parallel metallic bars on the two sides of a dielectric film.
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Left-handed materials exhibit a negative permeability, $\mu$, and permittivity, $\epsilon$, over a common frequency range $[1]$. Negative permeability is the result of an external magnetic field; negative permittivity can appear either by a plasmonic or a resonance response (or both) to an external electric field. Negative $\mu$ and negative $\epsilon$ lead to negative index of refraction, $n$, and to a left-handed triad of $\vec{k}, \vec{E}, \vec{H}$; hence, the names negative index materials (NIMs) or Left-handed Materials (LHMs). Pendry\cite{2,3} suggested a double metallic split-ring resonator (SRR) design for negative $\mu$ and a parallel metallic wire periodic structure for an adjustable plasmonic response. Several variation of the initial design have been studied; among them a single ring resonator with several cuts has been proved capable of reaching negative $\mu$ at higher frequency $[1]$; in Fig. 1(a) a two cut single ring is shown schematically. This, by a continuous transformation, can be reduced to a pair of carefully aligned metal bars separated by a dielectric spacer of thickness $t$.\cite{2,6}; in Figs 1(b) and 1(c) the view in the $(\vec{E}, \vec{k})$ and $(\vec{E}, \vec{H})$ planes of this structure is shown together with the directions of $\vec{k}, \vec{E}, \vec{H}$ of the incoming $EM$ field.

The design shown in Figs 1(b,c), besides its simplicity, has distinct advantages over conventional SRRs. The incident electromagnetic wave is normal to the structure as shown in Fig. 1(b), which enable us to build NIMs by only one layer of sample and achieve relatively strong response. Conventional SRRs, although they exhibit magnetic resonance which may produce negative $\mu$, they fail to give negative $\epsilon$ at the same frequency range and, hence, they are incapable by themselves to produce NIMs. An extra continuous wire is needed to obtain negative $\epsilon$ via plasmonic response.\cite{2,7}. In contrast, the pair of parallel metallic plates is expected to exhibit not only a magnetic resonance [Fig. 2(c), antisymmetric mode], but to show an electric resonance as well [symmetric mode] properly located in frequency by adjusting the length, $l$, of the pair.

The simulations were done with the CST Microwave Studio (Computer Simulation Technology GmbH, Darmstadt, Germany) using the lossy metal model for copper with a conductivity $\sigma = 5.8 \times 10^7$ for a single unit cell with periodic boundary in the $(E,H)$ plane, field distribution and scattering amplitudes have been calculated. The $\epsilon, \mu$ in Fig. 3 have been obtained by a retrieval procedure.\cite{5} At the magnetic resonance the two plates sustain anti-parallel currents producing a magnetic field $\vec{B}$ confined mainly in the space between the plates and directed opposite to that shown in Fig. 1(c); the electric field, because of the opposite charges accumulated at the ends of the two plates, is expected to be confined within the space between the plates and near the end points. Indeed, detailed simulations, shown in Fig. 2(c), confirm this picture.

At the electric resonance the currents at the two bars
are parallel (symmetric mode); the magnetic field lines go around both bars, while the electric field is mostly confined in the space between the nearest neighbor edges of the two pairs of bars belonging to consecutive unit cells.

The field and current configurations for both the antisymmetric and the symmetric mode can be accounted for by the equivalent circuit (b), which, since points 1 and 2 are equivalent because of the periodicity, reduces to circuit (c) and (d) for the magnetic (c) and electric (d) mode respectively.

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The magnetic resonance frequency, \( \omega_m \), is obtained by equating the impedance \( Z \) (of \( L_m \) and \( C_e \) in parallel) with minus the impedance \(-i/C_m\omega\) of the capacitance \( C_m \).

Since \( Z = iL_m\omega/(1 - L_mC_e\omega^2) \) we obtain

\[
\omega_m = \frac{1}{\sqrt{L_mC_m + C_e}} \approx \frac{1}{\sqrt{L_mC_m}}. 
\]

The last relation follows because, for the values we have used (\( l = 7\text{mm}, w = 1\text{mm}, t_s = 0.254\text{mm}, t_m = 10\mu\text{m} \) and \( b = 0.3\text{mm} \)), \( C_e \approx 0.1C_m \). Combining the Eq(1) and Eq(2) we find that

\[
f_m = \frac{\omega_m}{2\pi} = \frac{1}{2\pi l\sqrt{\mu\epsilon_s/2}} = \frac{1}{2\pi\sqrt{\epsilon_r\epsilon_s/2}l} \left( \frac{c}{l} \right) 
\]

where \( \epsilon_r = 2.53 \) is the reduced dielectric constant of the dielectric, \( \epsilon_r = \epsilon/\epsilon_0 \). In Fig(4) we compare our result of Eq(5) which shows that \( f_m \) is a linear function of only \( 1/l \), with detailed simulations results. Fig. 4 shows the dependence of the magnetic resonance frequency as obtained from the retrieved resonant effective \( \mu \) on the inverse length of the parallel metallic bars (Fig. 1(b)), for different widths \( w \) (separation between parallel bars \( t_s = 0.254\text{mm} \) fixed) and two different separations \( t_s \) (width of bars \( w = 1\text{mm} \) fixed). Complete quantitative agreement is obtained if \( c_1 = 0.22 \). Notice the independence of the simulation results on the width \( w \) and the dielectric thickness \( t_s \). It is worthwhile to point out that the result \( f_m \sim 1/l \) is robust over a wide range of parameters even if Eqs. 1 and 2 are not valid. To see this point, consider the extreme case of a pair of thin wires (as opposed to a pair of bars) of length \( l \), cross-section radius \( r \), at a distance \( d \) apart such that \( r \ll d \ll l \). For such a system \( L = (\mu/4\pi)[1 + 4\ln(d/r)] \approx (\mu l/\pi)\ln(d/r) \) and \( C \approx \epsilon\pi l/\ln(d/r) \). Thus again \( f_m \sim 1/\sqrt{\epsilon l} \).
the small distance b, it is in agreement with the simple formula (5). (t_s = 0.254mm for triangular, cross, circle; w = 1mm for diamond; and for all cases, b = 0.5 ~ 5.5mm, a_x = 20mm).

For frequencies near the electric resonance, because of mirror symmetry in Fig. 3(d), there is no current passing through the capacitances C_m. As a result the electric resonance frequency f_e is given by 

\[ f_e = \frac{1}{2\pi \sqrt{C_e L_e}} \]

where L_e is expected to be of the form \((\mu/\pi)g(w/l)\) where \(g(x)\) is a function which for \(x \to 0\) behaves as \(-\ln(x)\).

We point out that \(f_e\) is a rather sensitive function of \(e\) depends on \(b\), while \(L_e\) is practically independent on \(b\). Indeed the ratios \(f_e(2b)/f_e(b)\) and \(f_e(3b)/f_e(b)\) for \(b = 0.1mm\) according to the equation of \(C_e\) and equation of \(f_e\) are respectively 1.14 and 1.215 in good agreement with the simulation results in Fig. 5 (1.13 and 1.21 respectively); the dependence of both \(f_m\) and \(f_e\) on \(a_y/l = 1 + b/l\) is shown in Fig. 5.

Finally the width of the bars, \(w\), can increase until the bars join the ”infinite” wires producing thus a continuous connected network which can be constructed by opening periodically placed rectangular holes on uniform metallic films covering both sides of a dielectric sheet [10, 11, 12].

In this letter we have shown that \(L, C\) equivalent circuits can account for the \(EM\) properties of various negative index artificial meta-materials (NIMs), even at a quantitative level; furthermore, this simple unifying circuit approach offers a clear guidance in adjusting the design and optimizing the parameters for existing and possibly, future NIMs.

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FIG. 4: (Color online) Linear dependence of the magnetic resonance frequency, \(f_m\), as obtained by simulation, on the inverse length \(l\); this result as well as its independence on \(w\) and \(t_s\) is in agreement with the simple formula (5). (\(t_s = 0.254mm\) for triangular, cross, circle; \(w = 1mm\) for diamond; and for all cases, \(b = 0.5 \sim 5.5mm\), \(a_x = 20mm\)).

FIG. 5: (Color online) Magnetic resonant frequency \(f_m\) cross over with electrical resonant frequency \(f_e\) as \(a_y/l = 1 + b/l\) varies between 7.1mm and 7.3mm; \(a_x = 20mm\).

FIG. 6: (Color online) Retrieved \(\epsilon_{\text{eff}}\) (solid lines) and \(\mu_{\text{eff}}\) (dotted lines) for two cut wires. (a) and (b) correspond to points a \((a_y = 7.3mm, a_x = 20mm\)) and b \((a_y = 7.1mm, a_x = 20mm\)) in Fig. 5. Notice that both the response are Lorentz like.

This can be achieved by increasing \(C_e\) either by decreasing \(b\) or by increasing at the ends of each bar the width \(w\) choosing a double T shape for each bar.

Still another possibility to make the negative \(\epsilon\) region wider (and more negative) is to add continuous metallic wires as in Fig. 4(d) which produce a plasmonic response [5]. By adjusting the width of these wires their effective plasma frequency \(f_p\) can be made larger than the frequency, \(f_1\), at which the continuous curve in Fig. 6(b) crosses the axis \((f_1 \approx 16GHz)\).

We point out that \(f_e\) is a rather sensitive function of \(b\) due to the plasma frequency \(f_p\) can be made larger than the frequency, \(f_1\), at which the continuous curve in Fig. 6(b) crosses the axis \((f_1 \approx 16GHz)\).
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