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An Experimental Investigation into the Effect of Flap Angles for a Piezo-Driven Wing

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ABSTRACT
This article presents a comparison of results from six degree of freedom force and moment measurements and Particle Image Velocimetry (PIV) data taken on the Air Force Institute of Technology’s (AFIT) piezoelectrically actuated, biomimetically designed Hawkmoth, *Manduca Sexta*, class engineered wing, at varying amplitudes and flapping frequencies, for both trimmed and asymmetric flapping conditions to assess control moment changes. To preserve test specimen integrity, the wing was driven at a voltage amplitude 50% below the maximum necessary to achieve the maximal Hawkmoth total stroke angle. 86˚ and 65˚ stroke angles were achieved for the trimmed and asymmetric tests respectively. Flapping tests were performed at system structural resonance, and at ±10% off system resonance at a single amplitude, and PZT power consumption was calculated for each test condition. Two-dimensional PIV visualization measurements were taken transverse to the wing planform, recorded at the mid-span, for a single frequency and amplitude setting, for both trimmed and asymmetric flapping to correlate with the 6-DoF balance data. Linear velocity data was extracted from the 2-D PIV imagery at ± 1/2 and ±1 chord locations above and below the wing, and the mean velocities were calculated for four separate wing phases during the flap cycle. The mean forces developed during a flap cycle were approximated using a modification of the Rankine-Froude axial actuator disk model to calculate the transport of momentum flux as a measure of vertical thrust produced during a static hover flight condition. Values of vertical force calculated from the 2-D PIV measurements were within 20% of the 6-DOF force balance experiments. Power calculations confirmed flapping at system resonance required less power than at off resonance frequencies, which is a critical finding necessary for future vehicle design considerations.

NOMENCLATURE

\( n \) = natural flier wing beat
\( C_L \) = mean lift coefficient
\( c \) = wing mean chord
\( AR \) = wing aspect ratio
\( dA \) = differential area element
\( F_v \) = vertical force
\( F_x \) = balance force in x-direction
\( F_z \) = balance force in z-direction
\( M_y \) = balance moment in y-direction
\( I \) or \( V_{rms} \) = root mean square current/voltage
\( \bar{P} \) = mean power
\( u \) = x-direction velocity
\( w \) = z-direction velocity
$A_d$ = generalized actuator disk area
$\nu$ = induced disk velocity
$T_m$ = mean thrust
$\phi(t)$ = wing flap angle at time, t
$\phi_{max}$ = max wing flap angle (downstroke)
$\theta$ = wing elevation angle
$A$ = drive signal amplitude
$\alpha$ = wing angle of attack
$\vartheta$ = current or voltage phase angle
$A_{trim}$ = wing trim parameter
$r$ = wing radius
$C_D$ = mean drag coefficient
$R$ = wing length root-to-tip
$S$ = wing area
$Re$ = Reynolds number
$F_h$ = horizontal force
$F_y$ = balance force in y-direction
$M_x$ = balance moment in x-direction
$M_z$ = balance moment in z-direction
$\nu$ = kinematic viscosity
$x_{cp}$ = x-direction center of pressure
$v$ = y-direction velocity
$\rho$ = air density
$A_{arc}$ = area of wing swept arc
$U_u$ = mean wing tip velocity
$T_{max}$ = maximum thrust
$\Phi$ = total wing stroke angle
$\phi_{min}$ = min wing flap angle (upstroke)
$\omega$ = wing flapping frequency
$\eta$ = intra-stroke bias parameter
$\beta$ = wing stroke plane angle
$\delta$ = PZT tip displacement
$T$ = linkage transmission ratio

1. INTRODUCTION
The confluence between biologists and engineers over the past 10–20 years have produced considerable research into the aerodynamics and flight mechanisms responsible for insect flight. Research, mainly from biologists, has revealed these amazing fliers develop more lift than their wings alone can generate through a standard aerodynamic static, or quasi-static treatment; meaning the additional lift is generated through the complex interaction of the flapping motion of the wings and the surrounding fluid medium. It is the study of this aerodynamic phenomenon, its characterization, and its particular application to the AFIT Flapping Wing Micro Air Vehicle (FWMAV) program, which is the topic of this research effort.

With all the remarkable discoveries, and the litany of impressive military and civilian aircraft developed over the past 100 years, it was not until the last two decades that aerodynamicists have earnestly investigated the flight physics of nature’s smallest fliers—*insects*. To borrow an old colloquialism, *necessity is the mother of invention*, and heretofore, the civilian and military market demanded—*bigger, faster, farther*—from the aeronautics industry, not—*smaller, slower, lighter*. As with life, military objectives notwithstanding, all things change, and so has the demands on military strategic and tactical intelligence. The demands of surveillance necessary to minimize collateral damage have refocused the efforts of researchers to study the flight physics of birds and insects. Flapping flight provides a coupled relationship between the lifting surfaces, guidance and control, thrust generation, and power supply, in addition to providing the unique capability to ‘hide in plain sight’, a requirement to enable such vehicles to provide the level of intelligence fidelity necessary to meet an evolving military mission.
2. BACKGROUND

Natural fliers operate effectively in flow regimes where the propulsive efficiency of conventional systems decrease with size, and demonstrate flight maneuvers not currently capable by FWMAVs. One of the features differentiating small flapping wing fliers from engineered, fixed, and rotary wing systems is the use of unsteady aerodynamic phenomena to produce the aerodynamic forces necessary for sustained flight. By understanding how natural fliers utilize and employ their enhanced aerial abilities, engineers can design FWMAV systems that take advantage of, and hopefully emulate, similar unsteady aerodynamic mechanisms. Thus, bio-inspiration offers a means to enhance the performance of the next generation MAVs.

The goal of changing the wing flap symmetry through the bias parameter, \( \eta \), is to move the location of the \( x \)-direction center of pressure, \( x_{cp} \), to motivate a change in control moment. The Particle Image Velocimetry (PIV) measurements are utilized as an alternate measure of vertical force generation and \( x_{cp} \) location to corroborate the 6-DoF balance data, and identify if there are areas of the complex flow not completely captured by the balance data. A quantitative understanding of the differences between the aerodynamic forces and moments developed during symmetric and asymmetric flapping is crucial for design of control algorithms for future full-scale vehicle designs. In the present work, symmetric wing flapping is defined as a trimmed flapping condition where the up and downstroke angles are equivalent (\( \phi_{min} = \phi_{max} \)) measured from the zero motion point, perpendicular to the wing mount. An asymmetric wing flap angle is defined as an untrimmed flapping condition, characterized by an unequal up and downstroke angle, measured from rest, irrespective of the actual total flap angle achieved, \( \Phi \).

Figure 1 illustrates the qualitative difference at a specific driving voltage.

![Figure 1: Symmetric vs. asymmetric wing flap angle. The symmetric, trimmed wing angle is shown in black, and has a nearly equivalent up and downstroke angle, while the asymmetric, untrimmed wing flap angle is shown in red, which is predominated by the downstroke half of the flap cycle.](image)

It is imperative asymmetric flapping produces a comparable, but not necessarily equal amount of vertical force as symmetric flapping, because in a full flight vehicle, control about the pitch axis is exclusively achieved through modulation of the stroke angle between two wings. Pitch control is achieved by precisely controlling the asymmetry in the wing’s flap stroke; a larger downstroke angle during the flap cycle produces a nose down pitch, while a larger upstroke angle during the flap cycle yields a nose up pitch attitude. Roll is achieved by flapping one wing symmetrically at a higher amplitude, and the other wing flapping symmetrically at a lower amplitude.

The AFIT FWMAV wing design is wholly predicated on the size, shape, and structural response of the Hawkmoth, *Manduca Sexta*, and therefore, the aerodynamics used here to describe the research documented in the literature will naturally focus on studies of Hawkmoth flight characteristics, and bioinspired Hawkmoth mechanical flapping devices. For live Hawkmoth species, Weis-Fogh reported the mean lift coefficient, \( \bar{C}_L = 1.2 \); a Reynolds number (Re) of 5400; a wing beat frequency, \( n = 26Hz \); a stroke amplitude, \( \Phi = 110^\circ \); an aspect ratio, \( AR = 5.4 \); and a mean wing length, \( R = 48.5mm \), from averages of a number of live specimens [1]. Willmott and Ellington reported more modest mean lift coefficients in the range of \( 0.7 < \bar{C}_L < 0.95 \) over a range of Re = 4200 – 6500 [2, 3].
n = 25Hz, R = 50mm, S = 0.002m²; $\mathcal{A} \bar{I} = 5.0$, $\bar{c} = 20mm, \Phi = 110^\circ$, the AFIT FWMAV has a $Re = 5900$, and a $C_L = 1.1$.

3. AERODYNAMICS

Any discussion about the theoretical underpinnings of the mechanism of flight typically begins with the fundamental generalized Navier-Stokes (NS) fluid dynamics equations of motion, which are then transformed and applied to the specific flight conditions, and platform characteristics pertinent to the problem under investigation.

3.1 Flapping Flight Equations of Motion

The viscous, incompressible form of the NS equations in quasi-linear differential form, for a Newtonian fluid, subject to the no-slip boundary condition, are given below in equation 1 [4, 5, 6, 7].

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla}p + \mu \nabla^2 \vec{v}$$

Although there are no formal diffusion terms in the NS equations, the viscous stress tensor, $\mu \nabla^2 \vec{v}$, is a second order derivative, and diffusive effects are mathematically expressed as the negative gradient of concentration, where peaks in concentration tend towards uniformity, proportional to a diffusivity constant, $\kappa$—which is a second order phenomenon [5]. Therefore, viscosity tends to diffuse the concentration of momentum in a fluid, proportional to the kinematic viscosity, $\nu$, which is effectively a diffusion. The Reynolds number is used to compare and characterize various flow regimes, it is defined as the ratio of inertial to viscous forces, or in terms of the development of the NS equations above, it is the ratio between the convection and diffusion of momentum through the fluid. A low Re describes a laminar flow where the viscous forces, those close to an impenetrable boundary in the flow, dominate, inferring flow momentum is highly diffused as the flow matriculates through the control volume. A high Re describes a turbulent flow, dominated by the inertial energy of the flow itself, where the effects of viscosity are negligible, and momentum is transported (convected) by the bulk fluid motion, with little dissipation. The Re is given by equation 2 below

$$Re = \frac{V_\infty L}{\nu_\infty} = \frac{\rho_\infty V_\infty L}{\mu_\infty}$$

where $\infty$ denotes the free stream condition, and $L$ is the reference length scale of the flow. Non-dimensional forms of equations are scale-invariant, thereby making it possible to compare flight physics across a wide range of scales. The choice of parameters used to non-dimensionalize the applicable equations is somewhat arbitrary. The method used here, and most conventionally used by most insect and flapping wing flight analysis, is the one Ellington developed in his six manuscript series published in the 1984 Transactions of the Royal Society [1, 3, 8, 9, 10, 11]. Equation 3 gives the non-dimensional form of the viscous, incompressible NS equations, non-dimensionalized by the Reynolds number.

$$\frac{\partial \vec{v}^*}{\partial t^*} + (\vec{v}^* \cdot \nabla^*) \vec{v}^* = -\nabla^* P^* + \frac{1}{Re} \nabla^2 \vec{v}^*$$

Ellington proposed a form of the Re number used in flapping flight analysis, adapted from equation 2, which is the convention both biologists and aerodynamicists have adopted in comparing flapping flight regimes and their associated flows, given below in equation 4

$$Re = \frac{\bar{c}U_t}{\nu} = \frac{4\Phi n R^2}{\nu \bar{R}}$$
where Ellington defined the mean maximum tip velocity, $U_t$, as $U_t = 2\Phi n R$, $\Phi$ is in radians, $n$ is in Hz, $R$ is the wing length (half span) and the $AR$ is defined below in equation 5. The Re for typical insect flight ranges from $10^2 – 10^4$, well within the laminar region of flow.

$$AR = \frac{4R^2}{S}$$  \hspace{1cm} (5)

The wing area, $S$, is given by equation 6 below [3].

$$S = 2 \int_{0}^{R} c(r) \, dr$$  \hspace{1cm} (6)

The NS equations are not complete without a statement of the conservation of energy equation; however, if the flow is incompressible and isothermal, then the energy equation is decoupled from the mass and momentum equations, and it can be left out of the aerodynamic equations of motion. Therefore, the continuity and momentum equations alone are sufficient to solve problems in flapping wing flight. These equations can be shown in short-hand vector form as

$$\frac{\partial U}{\partial t} + \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} = Q$$  \hspace{1cm} (7)

where the components of the flux tensor, $f, g, h$, are vector quantities given by

$$f = \begin{bmatrix} \rho u \\ \rho u^2 + p - \tau_{xx} \\ \rho uv - \tau_{xy} \\ \rho uw - \tau_{xz} \end{bmatrix}, \quad g = \begin{bmatrix} \rho v \\ \rho uv - \tau_{xy} \\ \rho vu - \tau_{yy} \\ \rho vw - \tau_{yz} \end{bmatrix}, \quad h = \begin{bmatrix} \rho w \\ \rho uw - \tau_{xz} \\ \rho vw - \tau_{yz} \\ \rho ww + p - \tau_{zz} \end{bmatrix}$$  \hspace{1cm} (8)

which are all 5x1 column vectors, the energy equation terms are not shown leaving a 4x1 column vector, and the source term vector, $Q$, is a 5x1 column vector, given by equation 9.

$$Q = \begin{bmatrix} 0 \\ \int_{S} c \, \hat{\mathbf{n}} \\ W_f + q_H \end{bmatrix}$$  \hspace{1cm} (9)

### 3.2 Lift & Drag

The lift and drag resultant forces exerted by the flow on a solid body are obtained from the integral form of the momentum equation given below in equation 10.

$$\frac{\partial}{\partial t} \int_{\Omega} \rho \mathbf{v} \, d\Omega + \int_{\Omega} \rho (\mathbf{v} \cdot \nabla) \mathbf{v} \, d\Omega = - \int_{\partial \Omega} \nabla P \, d\Omega + \int_{\partial \Omega} \Delta \mathbf{v} \, d\Omega + \int_{\partial \Omega} \rho f_s \, d\Omega$$  \hspace{1cm} (10)

If the control volume, $\Omega$, contains a solid body such as a wing, then an additional reaction force, $-\mathbf{R}$, must be added to the right-hand side of equation 10. The reaction forces, $-\mathbf{R}$, are comprised of a normal component, lift force, $\mathbf{L}$, and a parallel component, drag force, $\mathbf{D}$, in the direction opposite to the relative velocity. If the boundary of the control surface (wing), $S$, is coincident with the solid body...
surface of the wing, \( S_{\text{wing}} \), then equation 10 can be reduced to the incompressible, viscous NS equations, absent external forces, at the wing surface, shown in equation 11 \[5\].

\[
\vec{R} \equiv \vec{L} + \vec{B} = - \int_{\Omega_{\text{wing}}} \vec{\nabla} P \, d\Omega + \mu \int_{\Omega_{\text{wing}}} \Delta \vec{v} \, d\Omega
\]  

The solution to the NS equations yield information about the flow physics in the near-field control volume, and the resultant wake in terms of the velocity components and pressure gradients. The lift and drag forces acting on the wing surface can be evaluated from the pressure and stresses along its surface. The local forces acting perpendicular and parallel to the wing, lift and drag, are a summation of the viscous and inviscid fluxes along the wing, influenced by the pressure distribution above and below the surface, which is determined by the vortex wake in the near-field flow \[12\].

### 3.3 Insect & Flapping Wing Flight

The application of the NS equations developed in section 3.1 above to the analysis and design of FWMAV is a difficult problem. The flow fields surrounding flapping wings in general are characterized as unsteady; although portions of the flow are completely steady. The flow produced by a flapping wing can be broken down into periods of steady flow separated by unsteady transitions between phases. Flapping wings are characterized by a steady period of translation in the forward or downstroke where the wing translates with a constant velocity and feather angle, \( \alpha \), or angle of attack; a rapid deceleration to supination, which then is followed by a steady rotation to reverse the orientation of the wings, maintaining the leading edge into the direction of the relative wind; followed by a rapid acceleration that transitions the wing to another period of steady translation in the rearward, or upstroke, and the cycle is finalized by deceleration to pronation, and rotation of the wings back to the downstroke posture. Figure 2 shows the a sequential progression of a Hawkmoth’s flap cycle with reference to its body.

![Figure 2: A complete Manduca Sexta flap cycle from high-speed video capture sequence. (A-B) & (L-M) signify the mid-points of supination & pronation respectively. (K) & (R) represent the maximum up and downstrokes respectively. The white wing surfaces are the dorsal or top view, and the shaded areas are the ventral or bottom wing surfaces, which illustrate the amount of wing flip and flexion during a single cycle—reproduced with permission from JEB [13].](image-url)
Figure 3 depicts a graphic of the upstroke and the downstroke phases of insect flight with a horizontal stroke plane. Three Degrees of Freedom (DOF), given by three body centered cartesian angles: i) stroke plane angle, $\beta$, the angle between the mean plane of the wing’s $3/4$ chord point and the horizontal; ii) elevation angle, $\theta$, the angle between the $3/4$ chord point and $\beta$, when not in the stroke plane, $(+\theta)$ is above $\beta$ and $(-\theta)$ is below $\beta$; and iii) wing position angle, $\phi(t)$, the angle of the wing at a specific time in the stroke phase. In conjunction with a single wing rotation angle, the angle of attack, $\alpha$, the angle between the leading edge and the relative wind, completely describes the wing orientation in space, as well as the rotation of the wing about a longitudinal axis [1, 3, 8, 14].

Figure 3: Depiction of insect flight during a complete flap cycle. Left: downstroke. Right: upstroke. Blue line represents 2-D chord slice of the wing with the Leading Edge (LE) shown with a dot, and the Trailing Edge (TE) without. The red dot represents the $3/4$ chord point or Center of Mass (CoM) of the wing from which the cartesian angles are determined. The three kinematic cartesian angles shown completely describe the wing orientation during the stroke cycle. The bottom of each graphic shows a 2-D wing trace schematic of the wing and the orientation progression of the lift and drag forces during the flap cycle. Lastly, the black curved lines show the growth and development of Leading Edge Vortex (LEV) and Trailing Edge Vortex (TEV) sections at each stage in the flap cycle—modified from [15]

The unsteady aerodynamic phenomena driving small-scale flapping wings, which generate sufficient aerodynamic forces for lift and forward flight, are produced by high speed dynamic rotations of the wing. The bulk wing motion, comprised of a coupling of translation and rotation, are oscillatory in nature, and encompass a large variety of motion profiles and associated tip paths for different species of insects. The tip path associated with a specific winged flier depends on the morphology, the configuration of the wing and body structures and joints, and the physiology, which determines how the wing is actuated [1, 3, 16, 17]. Figure 4 shows a graphic depiction of a typical wing stroke tip trace with a slightly inclined stroke plane. Determination of the wing position with respect to its body axis coordinates will be essential later during analysis of the aerodynamic forces and moments created by the AFIT FWMAV.
Sane summarized in his article on the aerodynamics of insect flight that all of the effects on a flapping wing may be reduced to three major sources of aerodynamic phenomena: i) the creation and sustainment of a leading edge vortex; ii) the quasi steady-state aerodynamic forces on the wing observed during translation; and iii) the wing’s contact, and subsequent recapture, with its shed wake from previous strokes, which is a form of dynamic stall [19]. Of course, the mathematical rigor to fully characterize these seemingly innocuous flight mechanisms are time dependent and ephemeral. There is no clear delineation between the start of one mode and the end of the previous, and the prevalence of one varies as the size of the flight system changes. The size of flying insects ranges from about 20 µg to about 3 g; and as flight mass increases, there is a corresponding increase in wing area, and a decrease in wing beat frequency [20, 19, 14, 1, 3, 21]. For larger insects, the Reynolds number may be as high as 10^4, and for smaller insects, it may be as low as 10^2. This means that viscous effects are much more important to the more diminutive fliers, although the flow is still laminar, even up to the largest flapping wing fliers [19].

4. MECHANISM DESIGN

The AFIT FWMAV design is comprised of a flapping mechanism and transmission, a drive train, the AFIT engineered wing (designed from the Hawkmoth) mounting apparatus, and the associated electronics, connections, and sensing and diagnostic equipment. The following sub-sections detail the important aspects of the flapper anatomy and system design.

4.1 AFIT Engineered Wings

O’Hara designed a biomimetic wing model with material properties, and structural responses closely matching those of the Hawkmoth. They were designed through i) careful dissection of numerous biological specimens; ii) medical Computer Tomography (CT) scanning of the inner venation patterns to catalog their thickness and stiffness; iii) 3-D scanning laser tomography, with and without scales, to capture the precise wing planform shape; iv) 3-D finite element modeling to model the 1st and 2nd bending and torsion modes of the wing to aide in the design of a carbon fiber lay-up schematic so the man-made analog mimics the thickness, taper, camber and stiffness of the Hawkmoth, and accurately reproduces the same system structural dynamics; and v) an exhaustive precision laser CNC cutting technique developed to ensure accurate and repeatable manufacturing [22, 23, 24, 25, 26]. Figure 5 shows the finite element model of the engineered wing made from the material properties, modal testing, and 3-D rendering of the Manduca Sexta biological representative. Table 1 lists the material properties of the AFIT engineered wing, which were modeled from the properties calculated through the structural analysis of the Hawkmoth wings, which were determined from of finite element model developed using ABAQUS commercial modeling software [26]. Figure 6 shows a liberated Hawkmoth
wing (left), prototypes of the AFIT designed thin (center), and thick (right) ribbed engineered wing trials. Table 2 lists the 1st bending and 2nd torsion modes of all three wings depicted in Figure 6, evaluated in both air and in a vacuum, in the AFIT FWMAV lab [26, 24].

Figure 5: Finite element model of the Hawkmoth wing, which is used as the basis for the manufactured engineered wings used in this research [26, 24].

Table 1: Engineered Wing Structural Properties

| Mass Property     | x-axis (m) | y-axis (m) | z-axis (m) |
|-------------------|------------|------------|------------|
| Center of Area    | 22.1E-03   | 4.0E-03    | 0          |
| Crater of Mass    | 20.9E-03   | 4.7E-05    | 0          |
| Moment of Inertia |                         |            |            |
| Origin            | 7.69E-10   | 3.05E-08   | 3.13E-08   |
| Center of Mass    | 7.04E-10   | 8.58E-09   | 8.17E-09   |
| Product of Inertia|                         |            |            |
| Origin            | -1.13E-10  | -2.06E-12  | -1.37E-11  |
| Center of Mass    | -5.24E-11  | -2.08E-09  | -1.37E-11  |

Table 2: Biological and Engineered Wing Modal Analysis Results

| Wing        | Mass (mg) | 1st mode air (Hz) | 2nd mode air (Hz) | 1st mode vac (Hz) | 2nd mode vac (Hz) |
|-------------|-----------|-------------------|-------------------|-------------------|-------------------|
| Hawkmoth    | 55.8      | 46.8              | 71.8              | 87.5              | 152.5             |
| Thin vein   | 52.5      | 52.1              | 72.8              | 58.1              | 80.3              |
| Thick vein  | 61.7      | 68.4              | 77.8              | 68.4              | 84.6              |

4.2 Flapper & Drive Mechanism

To reliably measure the aerodynamic forces and moments of the wing apparatus, a drive mechanism had to be designed to mimic the flap and stroke cycle of the Hawkmoth as closely as possible. Extensive rod and gear driven mechanisms have been designed and tested in previous FWMAV
designs in the past, but they are limited in their variability to only being able to vary the flapping frequency without changing the drive linkages, or disassembling the mechanism and changing the rod lengths [28, 29, 30, 31]. Mechanical flappers are limited then to a fixed flapping waveform (the drive signal), drive bias (drive signal shape), and amplitude (wing tip angular arc), all governed by the mechanical components of the assembly mechanism itself [28, 29, 30, 31]. Further, as the entire scale of the flapping mechanism decreases, the ratio of inertial to structural loads decreases, making frictional losses critical to power and transmission efficiency when dealing with scales of milligrams of force, and driven by milliwatts of power [28]. A programmable drive system was desired to fine-tune the flap cycle without having to alter the mechanism itself.

4.2.1 Powerplant & Drive Train
Integral to the design of the AFIT FWMAV is the flapper mechanism. Anderson provided a thorough review of the typical mechanism designs presently being explored and implemented at the major MAV research centers [28]. A survey of the available power plant options for the myriad of design constraints the FWMAV designer must contend with, i.e. payload, vehicle size, flapping frequency, power requirements, control, electronics, almost exclusively implicates the PZT driven actuator as the most sensible and practical choice. Biologists have reached a consensus that insects generate sufficient aerodynamic forces to sustain controlled flight by flapping their wings at system resonance, through a series of ventral and dorsal muscular contractions originating in the mesothorax and metathoracic cavities; whereby the muscles contract linearly to produce an angular wing motion, which can be effectively modeled as a mechanical linear actuator. The bimorph cantilevered PZT driven actuator connected to a dual crank-slider transmission was selected as the most efficient adaptation of biological fliers. This complex biomechanical machine is most accurately simplified as a dual linear actuator model of the thoracic flight muscles, whose mechanical analog is a simple-crank slider mechanism [28, 32, 33, 30, 34, 29, 35]. Figure 7 illustrates a cross section cut-away of an indirect drive insect thorax, and how its biomechanical flapping mechanism is modeled as a simple mechanical model.

Figure 7: Insect flapper mechanics modeled as a mechanical 4-bar linkage.

The slider can be replaced with the addition of another pinned bar linkage, simplifying the model to a straight four-bar linkage mechanism. The free, unclamped tip of the PZT traverses more of an arc than straight translational motion; therefore, its motion can be used in place of the driving crank in the model.
flapping mechanism [29]. The AFIT FWMAV’s PZT and linkage drive train move in the same plane, aligned with the $z$-axis, which minimizes the number of moving joints, and provides for a robust testing architecture. Future flight vehicles, with miniaturized actuators, would require orienting the PZT 90° out-of-plane with the rest of the linkage assembly, in the longitudinal plane, along the vertical axis, which maximizes tip deflection, and therefore wing stroke angle, while minimizing the actuator tip strain, see [36, 37, 38].

The geometry of the flapping mechanism and the resulting rigid body kinematics are chosen based on the expected displacement of the drive actuator and the desired wing motion [29, 39, 31]. Figure 8 shows a diagram of the generalized AFIT 4-bar linkage flapper drive train. For illustration purposes, the PZT is replaced here by a rotating link, not a cantilevered beam.

![Figure 8: Schematic of the 4-bar linkage assembly used in the AFIT FWMAV drive transmission. Linkages, $L_i$, shown in red, displacement angles, $\theta_i$, shown in black, coordinates shown in blue [29, 39, 28].](image)

To define the linkage, the link lengths, $L_i$, and the relative location of the fixed rotation points, $(\Delta x, \Delta y)$ must be specified, which serve as the inertial, or global reference coordinates. For a given angular actuator deflection ($\theta_1$), $\delta$ defines the cartesian location of the linkage during the stroke. Given a specified linear actuator deflection, $\delta$, the location of point $(x_1, y_1)$ can be calculated. The distance from point $(x_2, y_2)$ to point $(x_i, y_i)$ is spanned by a two-link planar manipulator [28]. Equation 12 gives the trigonometric solution to the relative wing displacement angle, $\theta_3$ [29, 28].

$$\theta_3 = -\left[2 \arctan \sqrt{\frac{(L_2 + L_3)^2 - (x^2 + y^2)}{(x^2 + y^2) - (L_2 - L_3)^2}}\right]$$

where $x$ & $y$ are coordinate distances between the last two pivots of the linkage. For completeness, the intermediate angle, $\theta_2$ can be calculated as follows in equation 13 [29, 28]

$$\theta_2 = \arctan 2(y, x) + \arctan 2(L_3 \sin \theta_3, L_2 + L_3 \cos \theta_3)$$

where $\arctan 2$ is the four-quadrant arctangent function. The actual wing stroke angle, $\theta_4$, is $\theta_3$ plus its mounting position offset relative to $L_3$. Figure 8 shows an angle just past 90°. The position of the wing along $L_3$ is not fixed, but it is most beneficial to make $L_3$ as small as possible to maximize the amplification of the small amount of actuator displacement into a large amplitude wing stroke angle [28]. Flapping at resonance is critical to maximize the cantilever effect, and hence, generate the greatest translational displacement of the longitudinally mounted PZT, which is tantamount to maximizing the total wing stroke. The AFIT drive mechanism mimics this principle through the use of a linear actuator in the form of a cantilevered end PZT actuator, connected to a replicated robotic four
bar linkage. Equation 14 is the transmission ratio, $T$, of the replicated 4-bar linkage assembly, relating the wing flap angle to the linear PZT displacement, and is approximately equal to the inverse of the length of $L_3$ [29, 28].

$$
T \equiv \frac{\theta_{\text{wing}}}{\Delta} \approx \frac{1}{L_3}
$$

(14)

where $\Delta$ is the off-set in the vertical alignment between the flexures at $L_1/L_2$ and between $L_4$ and the mounting block, and $L_3$ is the length of the second crank. The length of the second crank determines the transmission ratio of the mechanism—how much the linear input motion is amplified to create angular output wing motion. For the largest amplitude, hence the greatest wing motion, the crank length, $L_3$, should be as small as possible. Figure 9 shows a close-up of the linkage assembly.

![Figure 9: Close-up of the linkage assembly with the parts labeled used in the design and assembly process. $L_1 = 2.96\,\text{mm}, L_2 = 2.36\,\text{mm}, L_3 = 1.25\,\text{mm}, L_4 = 2.50\,\text{mm}$.

Using the linkage lengths, $L_i$, provided in Figure 9, and the time history of the PZT displacement, $\delta$, from the displacement sensor, the theoretical wing angle, $\theta_{\text{wing}}$, can be calculated at any time during the stroke cycle using equation 15 [26, 39].

$$
\theta_{\text{wing}} = -\frac{\pi}{2} + \arccos \left\{ \frac{L_3^2 + (L_1 + L_2 - L_4 - \delta)^2 + L_4^2 - L_2^2}{2 \sqrt{L_3^2 + (L_2 - L_4)^2} \times \sqrt{L_3^2 (L_1 + L_2 - L_4 - \delta)^2 - L_3^2}} \right\} \\
\quad \times \left\{ 1 + \arctan \left( \frac{L_3}{L_1 + L_2 - L_4 - \delta} \right) + \arctan \left( \frac{L_2 - L_4}{L_3} \right) \right\}
$$

(15)

4.2.2 AFIT FWMAV Design

The flapping mechanism used in this research was designed to have a maximum positive stroke angle, $\phi_{\text{max}} = +55^\circ$, and a maximum negative stroke angle, $\phi_{\text{min}} = -55^\circ$, for a total stroke amplitude, $\Phi \approx 110^\circ$. The wing venation structure is manufactured of a high modulus lamina carbon fiber, YSH-70A, impregnated with RS-3C epoxy resin, multi-layered in a $0\,\text{°} - 90\,\text{°} - 0\,\text{°}$ sandwich, which was heated, pressed, and cured in a press (LPKF Multipress S) at $192^\circ\text{C}, 100\,\text{N/cm}^2$, for 120 minutes. The wing planform area is covered by a polyethylene terephthalate (PET) mylar film, with an elastic modulus of 3.7GPa, which is higher than its biological analog of 2.45GPa [24]. The linkages consist of 160$\mu$m carbon fiber with joint flexures created by sandwiching 12.5$\mu$m thick Kapton HN 50 between two pieces of carbon fiber, then cured at $192^\circ\text{C}, 30\,\text{N/cm}^2$, for four minutes. The cured carbon fiber components are cut on a precision laser-machining center (LPKF Protolaser U). Figure 10 shows the AFIT FWMAV components after the laminated carbon fiber sheets are cut on the laser and ready for assembly.
The PZT and linkage transmission are fixed to a rapid prototype mounting assembly. The wing is attached to $L3$ with crystal bond adhesive, and the rotation is controlled by a passive rotation stop made from $25\mu m$ thick Kapton HN 100, cut at 45°. The flapper mounting structure consists of a rapid prototype resin polymer structure, which is rigidly mounted to a Nano-17 Titanium force transducer with three $2M \times 10mm$ hex cap screws. Figure 11 shows the assembled flapper, and the flapper fastened to the mounting structure attached to the wired PZT.
The PZT used to drive the AFIT FWMAV in this research is a 50mm, 43mm effective length, cantilevered, Omega Piezo, OPT 60/20/0.6 strip actuator, with a maximum desired displacement of ±1.0mm of tip deflection. The piezo is driven in parallel (simultaneous) by a dual source with the crystals energized by a scaled DC bias \((A \cdot 200V)\) to maximize the drive potential of the actuator, which deflects more in the positive (elongation) direction. The drive signals originate from a desktop PC via Matlab©, to a data acquisition box (DAQ) (National Instruments USB-6229 ±10V), amplified \(30\times\), and then connected to the PZT actuator via a BNC interface card. The waveform shape of the drive signal can be altered in the Matlab© interface. The flapper drive signal sent to the DAQ box is generated in Matlab© using equation 16.

\[
\text{Drive Signal} = \frac{(A \cdot V_{\text{gain}}) \left( \frac{1}{2} + \sin(\omega t + \eta + A_{\text{trim}}) \right)}{30}
\]  

(16)

where \(A\) is the signal amplitude expressed as a fraction of the max voltage (–150V to +300), which can be varied from 0 \(\rightarrow\) 1, \(V_{\text{gain}} = 200V\). \(\eta\) is the stroke bias parameter, and \(A_{\text{trim}}\) is a wing trim parameter used to obtain symmetry between the up and downstroke \((\phi_{\text{min}} = \phi_{\text{max}})\) from an unperturbed, fixed wing position. \(A_{\text{trim}}\) is an artifact of the manual manufacturing and assembly process, and is dependent on the wing and assembly structure. It is calculated through an automated auto-tuning procedure, and must be determined for each individual flapper system under test, see [41] for details on the trim procedure. The signal can be further biased toward either the up or downstroke with the bias parameter \((\eta)\) while maintaining period invariance. The factor of \((\frac{1}{2})\) is added to the equation to oscillate the PZT more in the \((+)\) direction, where it has greater displacement potential. A factor of \((1)\) would cause the PZT to exceed its maximum voltage of \(V = +300V\) at the higher amplitude settings. Figure 12 illustrates the difference in both the drive signal and the resulting wing flap angle, \(\phi(t)\), between the symmetric (trimmed) and asymmetric (untrimmed) flap cases, where \(\phi(t)\) was calculated with equation 15, using the values for \(\delta\) from the PZT displacement sensor data.

![Sample Drive Signal. Amplitude = 0.3, \(\omega = 25Hz\)](image1)

(a) Drive signal

![Sample Drive Signal. Amplitude = 0.3, \(\omega = 25Hz\)](image2)

(b) Flap angle

Figure 12: Example of two cycles of wing flap angle at \(A = 0.3\) and \(\omega = 25Hz\) for a trimmed and untrimmed drive signal, where \(A_{\text{trim}} = -0.35\) for the symmetric example. 

Without a traditional bar, gear, and linkage drive design, the wing by virtue of its attachment, rotates around the spanwise axis independently during the stroke, and its motion cannot be prescribed. Therefore, to ensure the wing rotates without needless oscillation, and recaptures inertial losses at the end of the stroke, a passive rotation joint was added to the wing-linkage attachment to ensure the leading edge was ahead of the trailing edge during both the up and downstroke phases [28]. Figure 13 depicts the AFIT wing/flapper mechanism with its 45° passive rotation joint, and the reference coordinate system used by the Nano-17 Titanium force balance.
5. EXPERIMENTAL METHODOLOGY

Figure 14 shows a labeled front and back view of the FWMAV experimental test chamber used in the testing detailed below. Figure 15 shows the data flow between the commanded input signal to the flapper and the collected aerodynamic data using the experimental setup shown in Figure 14.

Figure 14: AFIT MAV Lab setup. Left: front view of the MAV Lab, not pictured, the vibrometer sensor head is located just off the right side of the frame; Right: rear view of the MAV test chamber, stainless steel braided hose connected to the top is the vacuum hose.
5.1. Preparing the Flapper for Testing

The following list enumerates the procedures developed to execute full-scale flapping tests and analysis using the AFIT designed and fabricated AFIT FWMAV:

I) A Matlab® script was written to predict total wing tip travel in degrees, using equations 12—15, given a PZT cantilever length, and transmission linkage lengths, with the linkages and wing modeled as linear beams. Although it is possible to design the linkages to achieve a prescribed wing travel with equations 12–15, the practical implementation of the fabrication and assembly of such diminutive designs rarely translates into the expected performance. It is desired to be able to track the motion of the PZT, and therefore the wing, reliably and accurately to determine flapping symmetry, without altering the system dynamics or changing the flapping configuration of the wing-flapper system. A Micro-Epsilon optoNCDT 1700 laser displacement sensor, with a 2.5kHz sampling rate, 0.1µm resolution, and 2.0µm maximum linearity was used to capture the PZT tip displacement. A rapid prototype mounting structure was designed and fabricated to mount the sensor in close proximity to the flapper mechanism without interfering with the flapper itself, or the surrounding aerodynamic forces created during wing motion. Figure 16 shows the displacement sensor in the test chamber.
A calibration procedure was developed to determine the conversion ratio from volts to millimeters. A sensitive calibration tool, comprised of a rotating metal drum, sub-scale accurate to 0.0025 in, full-scale accurate to 0.025 in, per rotation, was used to calibrate the displacement sensor. The sensor was connected to an oscilloscope, and voltage readings were taken at five complete rotations of the calibration tool from 0.10 in to 0.20 in. A straight difference was taken between voltage readings, and an average slope was calculated by equation 17.

\[ S = \frac{1}{S_n} \sum_{i=1}^{n} s_i \]  

(17)

The average slope was found to be 12.53 \( v/in \). Given the slope and the input from the sensor, the standard form for an equation of a line was used to calculate the PZT tip displacement (in inches) as follows in equation 18.

\[ x_{disp} = \frac{y_{volts} - b}{12.53 \ v/in} \]  

(18)

II) To maximize force production, the resonant flapping frequency, \( \omega \), has to be determined. Flapping at resonance is necessary to optimize the greatest amount of displacement of the PZT, and hence maximize the total wing stroke, while minimizing power consumption. A Matlab\textsuperscript{®} routine was incorporated in the flapper drive menus to drive the flapper with a linear frequency sweep utilizing a built-in signal processing chirp function that excites a user specified frequency range and amplitude. The function subsamples the frequency response data from the force, velocity, and distance sensors, depicted in Figure 15, to eliminate the starting and stopping transient responses. All of the individual frequency test data is averaged, yielding a single multi-run frequency response data set. From this, the frequency response function (FRF) is used to determine the 1st resonant mode of the wing-flapper system, which is the flapping frequency, \( \omega \), used in equation 16 in subsequent force and PIV testing. Figure 17 shows the coherence of the velocity and displacement signals, and their associated normalized amplitude system FRFs for the PZT/flapper/wing system under test used in the experiments detailed below. See [41] for specifics of the derivations used to calculate the FRF of the complete AFIT wing and flapper system. There were two clearly identifiable resonant frequencies at \( \approx 25\text{Hz} \) and \( \approx 50\text{Hz} \).[41]

III) With the resonant frequency of the wing and flapper system identified, a Matlab\textsuperscript{®} routine was developed to perform an automated, real-time adaptive tuning of a specific PZT, flapper, and wing assembly by using the displacement sensor data in a closed-loop feedback program, which proportionally adjusts the stroke trim parameter, \( A_{trim} \), until symmetric PZT displacement, and hence symmetric wing flapping (equal up and downstroke angles), is achieved at various amplitude settings. See [41] for more details on the specifics of the auto-tuning procedure, and results from an example flapper and wing assembly.

IV) The position of an untwisted wing tip in relation to a fixed point, such as the wing mount spar, is necessary to calculate the wing’s angular position, \( \phi \), in the wing stroke plane, \( \beta \). An improved Gaussian Mixture Model (GMM), originally developed by Zivkovic, was adopted and implemented to analyze the furthest wing tip pixel of a 1sec video clip of the wing flapping from a top mounted, downward focused, high-speed camera taken at 1000Hz. The GMM mode was implemented in a C++ program, and a 1000 frames of wing flapping video, from a single camera were used to track a single point on the wing tip. Since a single camera was used to track a single wing point, the resulting pixel coordinates can only be used to describe the wing tip’s 2-D arc, which can only be used to calculate the stroke angle, \( \phi \). See [41] for the application of the GMM model to the high speed video frames used to get the initial (\( x, y \)) coordinate locations of the wing tip. Figure 18 shows the actual angular wing tip trace results from the novel optical tacking algorithm employed on a sample set of video data.
Figure 17: Normalized velocity & displacement % coherence and system FRFs: $H_1$ (blue) is the cross correlation, $S_{xy}$, over the auto correlation of $x$ with itself, $S_{xx}$. $H_2$ (red) is the auto correlation of $y$ with itself, $S_{yy}$, over the cross correlation, $S_{yx}$ [41].

5.2 Force Balance
Insects generate very small, nearly imperceivable forces and moments, and likewise, so do aerodynamically scaled FWMAVs, and are therefore difficult to accurately measure. Specially constructed test equipment designed to capture minute forces is required to reliably measure the aerodynamic output of the AFIT FWMAV. An ATI Nano-17 Titanium Force Torque (F/T) sensor, coupled with a Netbox data communications interface was used to collect force and moment data on all three axis during the flapping testing. The Nano-17 Titanium has a published sensitivity rating below the threshold of the expected AFIT FWMAV forces. According to the manufacturer’s specifications, the balance has a resolution of 0.149gF in all three Cartesian coordinates.
Figure 18: Time trace plot of $\phi(t)$, using the results of the refined LSR optical tracking coordinates at $A = 0.35$. In this plot, $\phi_{\text{max}} = 42.2^\circ$, $\phi_{\text{min}} = -43.4^\circ$, and $\Phi \approx 86^\circ$.

Figure 19: Diagram of the Nano-17 Titanium F/T transducer and orientation of principal axes with the PZT, mounting block, and wing provided for perspective on location of the sensors with respect to the actual wing motion. Note in the orientation used in the AFIT MAV lab, the vertical force vector (lift), is aligned along the balance $x$-axis, the side force (spanwise) down the wing is aligned along the $y$-axis, and the axial force (thrust) is aligned out the front of the balance along the $z$-axis—modified from [42].

Figure 19 shows a drawing of the Nano-17 Titanium and the reference coordinate directions used by the balance. The balance coordinate system differs from traditional aerodynamic balances where the body axes relating the normal, side, and axial forces are $x_b$, $y_b$, $z_b$ respectively. The moments are measured about the balance center ($M_b$), and are transferred to the pivot point ($M_p$), see figure 9, of the linkage/mounting block interface through equation 19

$$F_p = F_b$$
$$M_p = M_b - r_{b-p} \times F_b$$

(19)

where $r_{b-p}$ is the vector coordinate, $[\Delta x_b, \Delta y_b, \Delta z_b]$, from the balance to the pivot point, given here as $r_{b-p} = [0, -10\, \text{mm}, 62.25\, \text{mm}]$. Equation 20 gives the matrix equation used to transfer the moments from the balance center to the pivot point.
5.3 PIV

The AFIT 2-D PIV system was used to collect two dimensional, \(u\) and \(v\), velocity field data with AFIT’s functional FWMAV model, operating in a closed cell seeded particulate environment. The PIV system used in these tests was a Dantec Dynamics all-in-one 2-D PIV and data processing system. The PIV system laser is a New Wave Research, Solo 120 Neodymium-doped Yttrium Aluminum Garnet (Nd:YAG) laser. An armature with straightening mirrors directs the energy emitted by the lasing cavity from the laser housing to the laser head optics, which is used to generate the planar laser sheet and adjust its thickness. The laser sheet was adjusted to a thickness of \(\approx 2\) mm to accommodate the highly three dimensional nature of wake vortex structures emanating from flapping wings to illuminate particles traveling longitudinally in and out of the 2-D planar sheet. Figure 20 depicts the PIV equipment setup, and the actual test equipment used in these experiments.

![Figure 20: PIV equipment setup.](image)

A Kodak Megaplus ES 4.0/E camera was used to capture the illuminated image pairs. The camera is a high speed, black and white, asynchronous still-frame camera with a 2048 \(\times\) 2048 pixel array Charged Coupled Device (CCD), equipped with electronics to accommodate rapid inter-frame acquisition of image pair sequences. When the Q-switch receives an external, or internal laser fire command, the camera aperture opens and the CCD chip is exposed to scattered light from the first pulse of the laser sheet, and a 2048 \(\times\) 2048 pixel image is acquired. The first image is immediately stored in a temporary buffer, the CCD array is electronically wiped clean, the aperture remains open and then registers light from the second pulse of the laser sheet, and the second 2048 \(\times\) 2048 pixel image is acquired. Both images are transferred to the FlowMap processor through a BNC connection. The time dependent unsteady nature of flapping wing aerodynamics makes it critical to specify exactly when the laser fires to capture flow phenomenon at specific instances in the flight profile—phase locked or phase averaged. The Dantec FlowMap PIV system is equipped with an advanced automatic synchronization mode that has a short activation delay of 20 \(\mu\)s, which permits the ability to externally trigger the laser sheet with minimal delay between fire command and laser execution, which is critical to capture and freeze wing motions on the order of \(\approx 25\)Hz. See [41] for more details on the PIV setup.

The Flow Manager processing software transforms the time stamped pixel data to the frequency domain by performing an FFT on each user specified integration window to correlate velocity vectors. To mitigate the effect of aliasing, windowing was used to manipulate the image grey-scale by multiplying the registered pixel intensity by a variable gain between 0 and 1, which depends on the
pixel position within the interrogation area [44]. This eliminates false correlations at the edge of the interrogation area, and prevents correlating edge artifacts with actual flow particles. To mitigate data loss at the window peripheries, a 50% window overlap is chosen to increase the pixel count in each window, ensuring pixels across integration boundaries are not capriciously eliminated.

The size of the interrogation window is dependent on the maximum expected particle speed in the flow region, the amount and distribution of seeding particles in the flow, the size of the camera lens, the f-stop setting, and the distance the camera is from the test section. Sample image pairs were processed using varying size interrogation windows. The most realistic and repeatable results were obtained from an interrogation area of 64×64 pixels, with 50% region overlap, and a Gaussian window. The cross correlation of each image pair were further refined by applying velocity peak and range validation filters. The peak-to-peak height ratio relative to an adjacent peak was set to 1.1—meaning a peak that is greater than 10% higher than a neighboring peak, both spatially and temporally, is eliminated—a higher ratio admits more vectors in the cross correlation matrix, but increases the probability of spurious data being included in the velocity field. The range validation filter was set to 10 \( \text{m/s} \), which eliminates any velocity vector greater than the maximum value set in the filter. The range validation filter value was based on the maximum expected particle velocity, \( U_T \). Several researchers utilizing several flow visualization methods on live Hawkmoths, and scaled models, confirmed the maximum velocity occurs at the wing tip [45, 46, 47, 48, 49, 50, 51]. The average tip velocity for the symmetric and asymmetric tests was \( \approx 3.75 \text{ m/s} \) and \( \approx 2.8 \text{ m/s} \), respectively, and were calculated with equation 21 [9, 20]

\[
U_t = 2\Phi \omega R
\]  

where \( \Phi \) is the value of the actual total stroke angle, \( = 86^\circ \) for symmetric, and \( = 65^\circ \) in the asymmetric tests, and was calculated using the optical tracking algorithm identified above, \( \omega \) is the flapping frequency (previous insect studies adapted the used of \( n \) to denote the wing beat frequency), 25Hz, and \( R \) is the wing length from root to tip, 50mm. For these experiments, the range filter was set 2× higher than the calculated mean tip velocity to ensure no valid peaks were eliminated. As a result of expanding the filter, some spurious vectors may have been admitted in the solution.

A subset of the preliminary force balance tests, representing the highest recorded vertical force, \( F_{x'} \), were selected to perform PIV analysis on. Four phases, with 60 image pairs collected per phase, for both symmetric and asymmetric wing flapping, defined earlier as equal up and downstroke angles, at the mid-span location only, were recorded at each test condition. The four phases captured during each of the flapping tests in these experiments was i) max downstroke; ii) max upstroke; iii) mid-downstroke; and iv) mid-upstroke. Table 3 lists the flapping test points along with the camera and laser settings used in the PIV tests.

| Amplitude | Frequency (Hz) | \( A_{trim} \) | Stroke Phase | Span Location (%R) | Test Time (s) | Pulse \( \Delta t \) (\( \mu s \)) | Pulse Duration (\( \mu s \)) | # Bursts |
|-----------|---------------|--------------|--------------|---------------------|-------------|-------------------|-------------------|----------|
| 0.3       | 25            | 0            | Max Up       | 50%                 | 33          | 500               | 0.01              | 60       |
| 0.3       | 25            | 0            | Mid-Down     | 50%                 | 33          | 500               | 0.01              | 60       |
| 0.3       | 25            | 0            | Max Down     | 50%                 | 33          | 500               | 0.01              | 60       |
| 0.3       | 25            | -0.35        | Mid-Up       | 50%                 | 33          | 500               | 0.01              | 60       |
| 0.3       | 25            | -0.35        | Max Down     | 50%                 | 33          | 500               | 0.01              | 60       |
| 0.3       | 25            | -0.35        | Mid-Up       | 50%                 | 33          | 500               | 0.01              | 60       |

A Matlab\textsuperscript{®} script was written to synchronize the laser pulses and image pair capture from the PIV system, with the flapping period of the wing, and sample rate; irrespective of the flapping frequency, amplitude, \( A_{trim} \), or intra-period offset (\( \eta \)), enabling all the image pairs to be captured at the same phase...
in the flap cycle—phase locked. The laser TTL signal was written as a carrier signal, packaged with the flapper drive signal, sent to the data acquisition box. Figure 21 shows the flapper drive signal for both symmetric and asymmetric flapping with the four phases marked on the signal wave, and an example of a mid-downstroke 5V TTL signal overlaid with the drive signal.

The TTL signal was designed such that for a flapping frequency of 25Hz, four seconds of test time should emit 100 laser triggers, and record 100 PIV image pairs. However, the laser Q-switch and flash lamp repetition rate is not fast enough to fire the laser at every \(2\pi\) interval of wing phase, meaning the laser did not fire every time the wing passed through the specific phase under consideration. Six laser pulses were fired for every three seconds of flap time at a 5000Hz sample rate. About 33s of flap time was required to collect 60 image pairs.

6. RESULTS & DISCUSSION
Preliminary experiments were conducted on the standard AFIT FWMAV as proof of concept of the efficacy of the force balance, test equipment, flapper drive system, and user software to ensure the system is successfully integrated, and able to collect and resolve force and moment data with sufficient accuracy to be used to make aerodynamic assessments of the wing and flapper system. The following flapping flight tests were performed on the complete AFIT FWMAV test specimen, listed in Table 4 below. The cycle averaged flap angle, \(\Phi\), was computed with equation 15 to calculate the time history of the flap angle, \(\phi(t)\), using the PZT displacement measurement data, sampling the middle 70% of the data set to eliminate the transient wing motions at the beginning and end of the test, and the total cycle average flap angle was computed as the average of the sum of the absolute value of the local maximum and minimum values over the sample range.

6.1 Varying Amplitude (A)
The next series of plots show graphical results of the flapping force and moment data collected for the tests listed in Table 4. The first two plots, Figures 22 and 23, show cycle averaged force and moment results of symmetric vs. asymmetric wing flap angles (\(A_{\text{trim}} = \text{value from auto-tune to generate}\)
symmetric PZT motion, and $A_{trim} = 0$, which is asymmetric) at the resonant flapping frequency, $\omega = 25\text{Hz}$, as a function of both changing amplitude and corresponding PZT tip displacement. Multiple second tests at each test condition were executed and averaged together to get a single raw time history data set. The time history was subsampled to retain the middle 70% of the flapped data time, then the mean value was taken of the resampled raw data to calculate the cycle averaged forces and moments.

Figure 24 shows an example of the raw time history data, and a 0.05s 3-cycle average of the time history for different amplitudes to illustrate the growth in vertical force, $F_x$, and pitching moment, $M_y$, for symmetric and asymmetric flapping respectively. Overall, Figures 22-24 demonstrate expected trends in force and moment development. An increase in flapping amplitude; and therefore, the total stroke angle, $\Phi$, corresponds to an increase in vertical force, as well as an increase in pitching moment about the wing center of pressure. Symmetric flapping produces more usable lifting force than asymmetric flapping given the same drive signal amplitude. See [41] for the symmetric and asymmetric error bar plots, and a statistical development of a 95% confidence interval developed from the time averaged force data. Table 4 shows at an amplitude of $A = 0.3$, the symmetric flapping generated more vertical force, $F_x$, due to the signal drive bias, pre-polarizing the PZT crystal along its principal axis, permitting more total displacement. Symmetric flapping consumed slightly more power than the asymmetric case at the same amplitude because of the extra voltage term, $(A \cdot A_{trim} \cdot A_{DC})$, in equation 16, which causes the PZT to flap against the pre-strained axis from the DC bias. Because of the higher forces generated, symmetric flapping also developed a higher pitching moment, $M_y$, than the asymmetric case.

### Table 4: Complete system (PZT+linkage+wing) flapping tests

| $\Phi$ (deg) | $A$ | Drive Voltage (V) | Max Voltage (V) | Min Voltage (V) | $\omega$ (Hz) | $A_{trim}$ | $\delta$ (mm) | $P_x$ (mgF) | $P$ (mW) | $M_{xy}$ (gF-mm) | $\chi_{zy}$ (mm) |
|-------------|----|------------------|-----------------|-----------------|---------------|-------------|--------------|-------------|---------|-----------------|------------|
| 12.1        | 0.05 | 10              | 15              | -5              | 25            | 0           | 0.1131       | 3.2         | 0.63    | 0.14            | 43.2       |
| 19.1        | 0.1  | 20              | 30              | -10             | 25            | 0           | 0.1785       | 28.8        | 2.96    | -0.74           | -25.9      |
| 35.2        | 0.2  | 40              | 60              | -20             | 25            | 0           | 0.33         | 66.7        | 10.9    | -0.77           | -11.5      |
| 62.7        | 0.3  | 60              | 90              | -30             | 25            | 0           | 0.5666       | 161.8       | 26.8    | -3.1            | -19.4      |
| 10.1        | 0.05 | 10              | 11.5            | -8.5            | 25            | -0.35       | 0.233        | 42.7        | 1.4     | -2.0            | -47.1      |
| 17.4        | 0.1  | 20              | 23              | -17             | 25            | -0.35       | 0.3921       | 68.1        | 6.1     | -1.6            | -23.3      |
| 25.4        | 0.2  | 40              | 46              | -34             | 25            | -0.35       | 0.5629       | 140.8       | 23.9    | -1.4            | -18.3      |
| 36.2        | 0.3  | 60              | 69              | -51             | 25            | -0.35       | 0.8052       | 196.0       | 77.2    | 3.5             | -9.9       |
| 54.8        | 0.35 | 70              | 80.5            | -59             | 25            | -0.35       | 1.1774       | 331.3       | 45.1    | 2.0             | -6.0       |
| 88.6        | 0.36 | 72              | 82.8            | -61.2           | 25            | -0.39       | 1.7767       | 560.7       | 109.3   | 4.5             | -8.1       |
| 91.7        | 0.37 | 74              | 85.1            | -62.9           | 25            | -0.39       | 1.8322       | 584.2       | 124.2   | 3.7             | -6.3       |
| 95.2        | 0.38 | 76              | 87.4            | -64.6           | 25            | -0.40       | 1.8884       | 649.8       | 128.8   | 5.3             | -8.2       |
| 99.1        | 0.39 | 78              | 89.7            | -66.3           | 25            | -0.50       | 1.9377       | 665.1       | 129.6   | 4.1             | -6.2       |
| 102.4       | 0.40 | 80              | 92              | -68             | 25            | -0.38       | 1.9923       | 683.1       | 131.3   | -2.1            | -3.1       |
| 105.7       | 0.41 | 82              | 94.3            | -69.7           | 25            | -0.38       | 2.033        | 733.0       | 135.8   | -2.8            | -3.9       |
| 107.5       | 0.42 | 84              | 96.6            | -71.4           | 25            | -0.38       | 2.058        | 764.9       | 137.9   | -2.2            | -2.9       |

| 54.8        | 0.35 | 70              | 80.5            | -59.5           | 22.5          | -0.35       | 1.0471       | 253.6       | 58.0    | —               | —          |
| 48.5        | 0.35 | 70              | 78.8            | -61.3           | 25            | -0.35       | 1.438        | 245.8       | 54.1    | —               | —          |

| 54.3        | 0.35 | 70              | 80.5            | -59.5           | 27.5          | -0.35       | 1.1643       | 412.5       | 61.8    | —               | —          |
| 54.3        | 0.35 | 70              | 78.8            | -61.3           | 27.5          | -0.35       | 1.165        | 431.9       | 58.6    | —               | —          |
| 54.5        | 0.35 | 70              | 77.7            | -62.3           | 27.5          | -0.39       | 1.7491       | 646.8       | 72.6    | —               | —          |
Figure 22: 3-axis forces for symmetric vs. asymmetric flapping at $\omega = 25\text{Hz}$ vs. stroke angle, $\Phi$, and PZT displacement, $\delta$.

Figure 23: 3-axis moments for symmetric vs. asymmetric flapping at $\omega = 25\text{Hz}$ vs. stroke angle, $\Phi$, and PZT displacement, $\delta$. Moments are transferred from the balance center to the pivot point, located $-10\text{mm}$ in the $y$-direction, and $62.25\text{mm}$ in the $z$-direction.
Figure 24: Symmetric and asymmetric flapping cases at $A = 0.3$, $\omega = 25$ Hz, $A_{\text{trim}} = \{0, -0.35\}$. (Left-Top:) 3-cycle time history ($F_x$); (Left-Bottom:) 3-cycle time history ($M_y$); (Right-Top:) Cycle averaged vertical force ($F_x$); (Right-Bottom:) Cycle averaged pitch moment at pivot point ($M_{yp}$).

Figure 25 shows the symmetric vs. asymmetric center of pressure location, $x_{cp}$ (in mm), as a function of drive signal amplitude, where the location is calculated by dividing the pitching moment at the pivot point, $M_{yp}$, by the vertical force, $F_x$. As expected, although there is some oscillation, especially at the lowest amplitude settings, where the forces and moments are smallest, and register near the balance’s lower threshold, the center of pressure is nearly constant as amplitude increases. The mean location of

Figure 25: Symmetric and asymmetric locations of center of pressure, $x_{cp} = M_{yp}/F_x$ as a function of flap angle, $\Phi$. 

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for all flap angles is $\approx -5.6\, \text{mm}$ for the symmetric tests, which agrees well with the expected location. The pivot point is $\approx -2\, \text{mm}$ behind the vertical force locus (toward the balance), and the result of $F_r / M_{\infty}$ should be located a constant distance of $-2\, \text{mm}$ behind the pivot point, but is slightly rear of that (toward the balance) due to the influence of the data at the lower amplitude settings, which recorded forces near the lower bound of the balance threshold. If the data below $A = 0.35$, $\Phi = 86^\circ$ is neglected, the value of $\bar{x}_{cp}$ decreases to $-4.5\, \text{mm}$. Although, not conclusive, the mean values of $\bar{x}_{cp}$ shows the movement of the asymmetric center of pressure slightly in the direction of the asymmetry in the flap motion, which decreases the overall moment lever length, reducing the distance along the $z$-axis between the balance center and the locus of force. The mean location of $\bar{x}_{cp}$ is $\approx -19\, \text{mm}$ for the asymmetric tests, which is a shift in the center of pressure location of $\approx -14\, \text{mm}$ closer to the balance.

### 6.2 ACTUATOR POWER

The next set of force plots show the average power, $\bar{P}$, consumed by the PZT actuator as a function of both flapping frequency, $\omega$, and drive signal amplitude, $A$. Since the drive signal is a time varying complex waveform, the standard electrical resistive relations, $V = IR$ and $P = I^2 R = IV$, cannot be used to calculate the average PZT consumed power. The equation for PZT mean power estimation used in this research was developed by NASA for PZT-based structural health monitoring types of applications, and is given in equation 22 below [52]

$$\bar{P} = \frac{1}{T} \sum_{t=T} V_{rms} (\omega, t) I_{rms} (\omega, t) \cos \theta_t \Delta t \quad (22)$$

where $V_{rms}$ and $I_{rms}$ are the root-mean-square of the drive voltage and current respectively, $T$ is the subsampled flapping period, $\theta_t$ is the phase angle difference between the instantaneous voltage and current, and $\Delta t$ is the sample interval, calculated by taking the inverse of the sample rate. The voltage and current are recorded as inputs by the data acquisition box. The $rms$ values are found by subsampling the instantaneous current and voltage data with the same algorithm used to subsample the force and moment data above, and then an ensemble average of the square root of the mean squared of the voltage and current was taken to build an array of $rms$ voltage and current values. The same signal processing techniques, used to calculate the system FRF, of windowing, zero padding the data set to the closest power of 2, taking the FFT of the subsampled data set, and then removing data above $1/2$ the sampling frequency, were used here to transform the voltage and current into its complex components.

The complex components, $x + yi$, are used to calculate the voltage and current phase angle, $\theta_t$, using equation 23. The phase difference is computed by taking the difference between their respective angles at each $\Delta t$.

$$\theta_t = \tan^{-1} \left( \frac{y}{x} \right) \quad (23)$$

Figure 26, shows the average power consumed, $\bar{P}$, for the variable amplitude symmetric and asymmetric flapping tests at $\omega = 25\, \text{Hz}$. Figure 27 shows the variable frequency tests at $A = 0.3$ and $A_{trim} = -0.35$. The results shown here are reasonable, and follow expected trends. Figure 26 shows for both the trimmed and untrimmed flapping conditions, the required power increased as the amplitude, (% max input voltage), was increased. Figure 27 shows the minimum consumed power, $P_{min}$, occurred at resonance as expected, and increased at both $\pm 10\%$ off-resonance, with $+10\%$ off-resonance requiring slightly more power than $-10\%$ off-resonance.
Hollenbeck et al. evaluated the thoracic flight muscle system of the Hawkmoth, Manduca Sexta. Their experiments focused on the tergum Dorso Ventral Muscles (DVM) and the phragma Dorso Longitudinal Muscles (DLM), which are the Hawkmoth’s primary flight actuation muscles. The experimental investigation assessed the static and dynamic loading on several fresh, and experimentally controlled desiccated specimens [53, 54]. Calculations were performed of the average power required to manually elevate the Hawkmoth’s wing in a flight attitude by integrating the load cell force necessary to move the tergum plate to actuate the DVM muscles from rest, to a representative flight stroke angle, divided by the total time [53]. Power consumption of the AFIT FWMAV flapped at resonance at various driving voltages and trim settings are cataloged in Table 4, and compare favorably to the method Hollenbeck et al. used to calculate the power needed to actuate Hawkmoth flight muscles and wings.

Figure 26: Cycle average vertical force ($F_x$) and PZT actuator average power ($P$) vs. varying drive amplitude at $\omega = 25$Hz

Figure 27: Test Conditions: $A = 0.3$ & $A_{trim} = -0.35$ (Symmetric), $A_{trim} = 0$ (Asymmetric)

6.3 PIV Results

The intent of the PIV data was to correlate vertical force and $x_{cp}$ location with balance data, and identify important aspects of the flow not possible through force and moment measurements alone to elucidate further aerodynamic insights for improved designs. The cross correlated raw PIV data was imported into Matlab®, and a mass batch processing routine was developed to further process and refine the data, plot the raw image files, calculate the velocity vector fields, and extract horizontal and vertical $u$ and $v$ velocity profile data at salient coordinate positions in the integration field of view. Figure 28 shows a
comparison between the symmetric ($A_{trim} = -0.35$), and asymmetric ($A_{trim} = 0$) velocity vector overlays at the maximum upstroke. The time-averaged, four-phase asymmetric PIV $u - v$ velocity vector plots are presented in Figure 29. Refer to [41] for the symmetric velocity vector plots.

Figure 28: Test Conditions: $A = 0.3$, $\omega = 25$Hz. Symmetric vs. asymmetric velocity vector fields overlayed on a false color image of the wing and flapper. Top: maximum upstroke. Bottom: maximum downstroke. $u$-velocity is along the horizontal dimension, and $v$-velocity is along the vertical dimension of the page.

6.4 Forces from Mean Velocity

Figure 30 shows $v$-velocity profiles for all four wing phases at $-1/2$ chord below the wing for the symmetric and asymmetric test cases along with a built-in Matlab® ‘Lowess’ linear smoothing algorithm applied to the data to remove some of the jaggedness, which helps to visually discern the downward trend in velocity in the vicinity of the wing. The red line in each plot is the $v$-velocity at the maximum upstroke, which corresponds to the 2-D velocity vector image overlays shown in the top pane of Figure 28; while the green line corresponds to the maximum downstroke, shown in the bottom pane of Figure 28. The portion of the data outside the region of interest were excluded from all the four-phase velocity plots. The image area of the PIV camera CCD is $2048 \times 2048$ pixels, while the wing spans $1000$ pixels horizontally from maximum downstroke to maximum upstroke in the image plane. Including data from the entire width of the focus area biases the magnitude of the velocity line data by including regions of zero flow. The velocity profile data outside $\pm 10\%$ of the projected wing swept width of $\approx 1000$ pixels were not included in the velocity profile and thrust calculations.

The mean velocities above and below the wing are used to calculate the induced velocity, $v_i$, which is assumed equal to the velocity the volume of air reaches in a given stream tube as it is accelerated through a hypothetical actuator propeller disk. The mean forces developed during a flap cycle are approximated using a modification of the Rankine-Froude axial actuator disk model, more commonly known as disk momentum theory. The model stipulates purely axial flow through a fixed rotating disk, assuming an ideal, incompressible fluid operating medium. The rotating disk imparts a velocity to the fluid, and a resulting pressure differential ensues between the upper and lower surfaces of the actuator disk [55]. Figure 31 shows a graphic of an idealized actuator disk.
Figure 29: Asymmetric four phase velocity vector fields overlayed on a false color image of the wing and flapper mechanism.

Figure 30: Smoothed $v$-velocity profiles for all four wing phases (max, mid-down, min, mid-up) at $-\frac{1}{2}$ chord from 14mm < r < 86mm.
Figure 3: Graphical example of a Rankine-Froude pulsed actuator disk model used in axial force momentum calculations. (P) is the location of the the far-field in-flow. (Q) is the near-field location in vicinity of the actuator disk. (R) is the location of the far-field out-flow. (ν) is the induced flow velocity at the disk. (ν_{far}) is the induced velocity in the far-field—adapted with permission from JEB [46, 47, 55].

The pressure inside the arc of the pulsed actuator disk is greater than the pressure above the disk, leaving a discontinuity in the pressure field. Unlike solving for velocity and pressure through a pitot static tube, where the functions are continuous, and Bernoulli’s equations for an incompressible, inviscid flow, can be used to solve for the velocity and pressure conditions downstream of the inlet flow conditions, the discontinuity in the actuator disk field requires independent application of Bernoulli’s equations above and below the disk [55]. The total hydraulic head, \( H_{dis} \), the sum of the elevation head and static pressure head, for the upstream section, is equivalent to equation 24 [55, 56]

\[
H_{us} = P_o + \frac{1}{2} \rho V_c^2 = P + \frac{1}{2} \rho (V_c + \nu_i)^2
\]  

(24)

where \( V_c \) is the outside free stream velocity, \( \rho \) is the air density, \( \nu_i \) is the averaged induced velocity over the entire disk, \( P \) is the mean pressure, and \( P_o \) is the total pressure (far-field) and is equal to the sum of the static and dynamic pressures. The downstream total head can then be represented similarly by equation 25 [55, 56]

\[
H_{ds} = P_o + \frac{1}{2} \rho (V_c + \nu_i)^2 = P + \Delta P + \frac{1}{2} \rho (V_c + \nu_i)^2
\]  

(25)

where the only difference is the term \( P + \Delta P \), which is the pressure differential below the disk as a result of the force imparted to the fluid by the actuator disk. Manipulating the above expressions for the upstream and downstream head, and equating like terms, leaves a single expression in terms of the pressure differential, \( \Delta P \), shown by equation 26 [55, 56].

\[
\Delta P = \rho (V_c + \frac{1}{2} \nu_i) \nu_i
\]  

(26)

Equating the flow at the far field and at the disk:
where $mg$ is opposed by the mean vertical force (or thrust), $F_v$, and $\nu_{far}$ is:

$$\nu_{far} = \sqrt{\frac{2mg}{\rho A_d}}$$

arranging the weight terms on the left and the momentum flux terms on the right:

$$mg = \frac{1}{2} \rho \nu_{far}^2 A_d$$

rearranging and substituting $\nu_{far} = 2\nu_i$:

$$mg = 2 \rho \nu_i^2 A_d$$

Preserving continuity, as the area entrained by the net momentum flux, stream tube of flow, narrows in the far-field, the area necessarily varies inversely with the induced velocity, and the far-field area is thus, $A_{far} = \frac{A_d}{2}$. This is the standard result shown by [10, 46, 55]. A straightforward application of Newton’s 2nd law leaves a force balance expression in terms of the total average thrust, $\bar{T}$, produced as a function of the actuator disk radius, $R_d$, given in equation 31 [55, 56].

$$F_v \equiv -\bar{T} = mg = \pi R_d^3 \rho (V_c + \nu_i)^2 = 2 A_d \rho (V_c + \nu_i) \nu_i$$

where $A_d$ is the generalized actuator disk area; and for flapping wing applications, $A_d$ is twice the swept area of a single wing. The resultant direction of $\bar{T}$ is opposite the direction of $\nu_i$; and in a hover condition, $\nu_i$ is assumed vertically down (towards the earth), and $\bar{T}$ is pointing vertically up (towards the sky). In hover, or any static condition where the free stream velocity is zero, $V_c = 0$, and equation 31 is reduced to the form shown in equation 32 below [10, 55, 56].

$$F_v \equiv \bar{T} = 2 A_d \rho \nu_i^2$$

Equation 32 can be applied to the AFIT FWMAV to calculate the peak thrust at the wing tip for a specified set of flapping parameters. The equation requires slight modification, where $\nu_i$ is replaced by the average tip velocity, $V_t$, given in equation 21, and the actuator blade disk area, $A_d = \pi r^2$, is replaced by the swept area of a single AFIT flapper wing, which is approximated by the area of a circular arc, $A_{arc} = \frac{1}{2} r^2 \Phi$, where the radius of the arc segment, $r$, is equal to the wing length from tip to root, $R$, and $\Phi$ is the actual swept angle from the optical tracking results in Figure 18. Substituting the above modifications into equation 31 results in an expression for the maximum thrust, $T_{max}$, applied to a flapping wing specimen, given below in equation 33. After substituting the actual values of the wing and the test conditions used in these experiments into equation 33, the theoretical maximum thrust, $T_{max}$ is 649 mgF and 280 mgF for the symmetric ($A_{trim} = -0.35$, $\Phi = 86^\circ$), and asymmetric ($A_{trim} = 0$, $\Phi = 65^\circ$) tests respectively.

$$F_{v_{max}} \equiv T_{max} = 4 \rho \nu_i^2 \Phi \pi r^4$$

This derivation assumes an idealized scenario, where the assumption of an ideal, incompressible, inviscid fluid does not accurately represent the actual operating conditions; therefore, any periodicity in the pulsed momentum flux as a result of cyclic wing flapping, versus uniform rotor rotation, is ameliorated by the presence of viscous forces in the far field, meaning $\nu_{far}$ will vary only slightly from
the strictly uniform rotor blade disk analysis. Ellington stipulated in his treatise on the development of a comprehensive vortex theory of animal flight that although accounting for the periodicity of far field wake is a more satisfying physical description of flapping flight phenomenon, his experimental findings of insects with a horizontal stroke plane, i.e. the Manduca Sexta, resulted in less than a 20% difference in the induced power and induced velocity, as compared to those calculated from an application of the exceedingly more simplistic Rankine-Froude momentum disk theory [10].

The average 4-phase induced velocities derived from the PIV results shown in Figure 30, can be substituted in equation 32 to calculate the average sectional thrust, vertical force, produced by the symmetric and asymmetric flapping PIV experiments. The mean thrust, calculated from the $z^{1/2}$ chord velocity profiles, is substantially lower than both the maximum expected thrust at the wing tip, and the average vertical force recorded by the Nano-17. The velocity profiles are measured across a small differential area, $dA$, of the wing, spanned by the 2mm laser sheet.

In the limit, as the number of cross sectional measurements taken approach infinity, the three dimensional nature of the flow would become more evident—the same u-shaped velocity profile will grow in depth as the laser sheet progresses from the wing base to the wing tip. To calculate an estimate of the expected thrust from the induced velocity across the entire span, as an infinite number of PIV measurements were actually taken along the span, a coarse function of $\nu_i$ in terms of the wing span, $R$ was developed to integrate across the span. A first order, linear equation was fit to the line plot of the the induced velocity, $\nu_i = (0, -1, -3.75) \, m/s$ and $\nu_i = (0, -1, -2.8) \, m/s$ for the trimmed and untrimmed cases respectively, at several wingspan locations, $R = (0, 25, 50) \, mm$. The value of $\nu_i = 0$, at $R = 0$, is assumed because of the no-slip condition, and zero motion was assumed at the wing root/linkage connection point, the value of $\nu_i$ at $R = 25 \, mm$ is the 4-phase mean value from the PIV measurements, and finally, the average tip velocity, $U_t$, was chosen for the value of $\nu_i$ at $R = 50 \, mm$. The linear fit to the plot of $\nu_i$ vs. $R$ is given in equation 34.

$$\nu_i(r) = -0.0908r + 0.422 \quad \text{(symmetric)}$$
$$\nu_i(r) = -0.0959r + 0.736 \quad \text{(asymmetric)}$$

where the wing location, $r$, is in $mm$. The function for $\nu_i$ was integrated across the area of a circular arc, which is invariant, given by equation 35 below.

$$F_x = \bar{T} = \int_0^R \int_0^{\Phi} 2\rho \nu_i^2 \, dA$$

where:
$$dA = r \, d\Phi \, dr$$
and:
$$\nu_i(r) = -Ar + B \quad \text{(with } r \text{ in } mm)$$

substituting:
$$\bar{T} = (2\rho \Phi) \int_0^R r (-Ar + B)^2 \, dr$$

where $A$ and $B$ are the coefficients of the first order fit of the velocity vs. wing radial position line plot. The resultant expression for $\bar{T}$ is a function of the spanwise location, $r$, along the wing. Figure 32 shows a 3-D illustration of the differential area element, $r \, dr \, d\phi$, with respect to the total angle subtended by the flap arc, $\Phi$.

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substituting:
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where $A$ and $B$ are the coefficients of the first order fit of the velocity vs. wing radial position line plot. The resultant expression for $\bar{T}$ is a function of the spanwise location, $r$, along the wing. Figure 32 shows a 3-D illustration of the differential area element, $r \, dr \, d\phi$, with respect to the total angle subtended by the flap arc, $\Phi$. 

Figure 32: Illustration of the differential arc area used to calculate the projected thrust along the entire span from the single PIV data location at mid-span.
A linearly space vector between 0mm and 50mm was used to plot the resulting mean sectional thrust as a function of span location. Figure 33 shows the 4-phase symmetric and asymmetric sectional thrust calculated at $-\frac{1}{2}$ chord below the wing, along with the mean 4-phase thrust plotted against the Nano-17 average vertical force, $F_x$, and the mean spanwise integrated thrust. Recognizing these are broad estimates, and actual measurements at many locations along the span would provide greater fidelity to the estimate of induced velocity as a function of span location, the symmetric and asymmetric approximations proved a good first order estimate of the vertical force. Table 5 shows the symmetric and asymmetric thrust approximations compared to their respective force balance results.

![Figure 33: 4-Phase thrust calculated at $-\frac{1}{2}$ chord from $14mm \leq r \leq 86mm$. The dotted line represents the mean sectional thrust calculated from the four individual phase mean velocities. The curve is a graphic of the integrated sectional thrust projected across the entire span, $R$. The solid line is the mean of the integrated sectional thrust. The dash-dot line is the average vertical force, $F_x$, measured by the Nano-17 at the same test conditions.](image-url)

| Measurement & calculation method | Mean Thrust ($\mathcal{F}$) (mgF) |
|----------------------------------|----------------------------------|
| Maximum expected thrust using max wing tip velocity, $U_t \approx \nu_t$ | 649 |
| Mean 6-DoF vertical force ($F_x$) (symmetric) | 196 |
| Mean integrated sectional thrust using linear fit (symmetric) | 166 |
| Maximum expected thrust using max wing tip velocity, $U_t \approx \nu_t$ | 280 |
| Mean 6-DoF vertical force ($F_x$) (asymmetric) | 161 |
| Mean integrated sectional thrust using linear fit (asymmetric) | 131 |

The mean integrated asymmetric thrust values are less than the symmetric values because the total flap angle, $\Phi$, subtended by the wing during testing was smaller in the asymmetric tests than the symmetric tests. Per equations 33-35, this difference in flap angle substantially effects the measured vertical velocity component, $\nu_z$, and therefore the predicted thrust. The angular difference between the symmetric and asymmetric testing was approximately $21^\circ$; $86^\circ$ symmetric vs. $65^\circ$ asymmetric. An approximate calculation of the asymmetric stroke angle was derived from the empirical results obtained from the symmetric flapping tests. The optical tracking procedure revealed the symmetric flapping tests, at an amplitude of $A = 0.35$, $A_{trim} = -0.35$, resulted in a total stroke angle of $\Phi = 86^\circ$, and the Optol7 laser distance sensor recorded an ensemble average PZT tip displacement of 1.1774mm. Applying the law of cosines and assuming small angles, for 1.1774mm of linear PZT tip displacement, the angular sweep of the PZT between the unperturbed position at the clamped end, and the fully deflected tip, was calculated to be $\approx 1.5^\circ$. Given 2.0mm of total PZT travel, the
mechanism was designed to yield a total wing stroke angle of $\Phi = 110^\circ$. Therefore, the testing revealed $\approx 60\%$ of the maximum PZT travel resulted in $\approx 78\%$ of the designed wing arc. The asymmetric flapping test point of, $A = 0.3, A_{trim} = 0$, produced 0.5866mm of total PZT travel, resulting in $\approx 0.75^\circ$ of PZT angular displacement. Using the ratio of known PZT angular displacement to the known wing swept angle, $1.5/86^\circ$, in the symmetric test, the expected asymmetric wing angle can be solved for given a PZT tip displacement. The intent of the auto-tuning procedure of the PZT and wing together is to generate symmetric PZT tip motion, thereby maximizing the blocking force, and ensuring a symmetric up-to-downstroke flap cycle, which generates the highest vertical force. The differences between the symmetric and asymmetric test cases at $A = 0.3$ can be summarized in Table 6.

As expected, at a given flapping amplitude, the symmetric case used more power than the asymmetric case because extra energy is required to change the trim value in the drive signal, which is working against the DC bias, forcing the PZT to flap more in a specified direction (depending on the sense of $A_{trim}$) to achieve the desired up-to-downstroke ratio. Although the symmetric flapping cases produced more vertical force than the corresponding asymmetric cases, it is imperative the asymmetric cases produce sufficient vertical force for flight as well as for gust avoidance and control.

### Table 6: Comparison of symmetric vs. asymmetric flapping cases at $A = 0.3$

|       | $\delta$ (mm) | $F_a$ (mgF) | $M_{xy}$ (gF-mm) | $x_{cf}$ (mm) | $\bar{F}$ (mW) | $T_{win,y}$ (mgF) | $\bar{T}$ (max-up) | $\bar{T}$ (mid-dwn) | $\bar{T}$ (max-dwn) | $\bar{T}$ (mid-up) |
|-------|---------------|-------------|------------------|--------------|---------------|------------------|------------------|------------------|------------------|------------------|
| Symmetric | 0.81         | 196         | -3.5             | -5.6         | 27            | 161              | 43.7             | 27.2             | 30.5             | 35.7             |
| Asymmetric | 0.59         | 161         | -3.1             | -19.6        | 26            | 131              | 8.8              | 27.2             | 27.9             | 11.5             |

Referencing the 4-phase $v$-velocity line plots in Figure 30, in conjunction with the representative sample max up and downstroke PIV velocity vector plots in Figure 28, the shift in the location of the center of the concentration of the vertical downwash from left to right ($= 20$ pixels) between the symmetric and asymmetric test cases is evident. This changes the wing center of pressure (CP), which also changes the mean aerodynamic center ($M_{ac}$). A deeper analysis of the change in horizontal $u$-velocity components between the symmetric and asymmetric flapping cases is beyond the scope of this analysis, but preliminary results indicate a slight positive $u$-velocity ($= 0.5 m/s$) in asymmetric flapping, which agrees with the small change in the axial force, $F_z$, between the two, shown in Figure 22.

### 7. CONCLUSION

Testing was conducted on AFIT’s piezo-driven, biomimetically designed Hawkmoth, class engineered wing with the intent to quantify the differences between trimmed and untrimmed flight conditions on aerodynamic performance to determine the change in pitch moment and the corresponding location of $x_{cp}$. The effect of setting a trim condition served to achieve symmetric up and downstroke wing flap angles, while the untrimmed tests, represented asymmetric flapping angles. Six degree of freedom force and moment measurements at varying amplitudes and flapping frequencies were presented. To preserve test specimen integrity, the wing was driven at a voltage amplitude 50% below the maximum necessary to achieve the maximal Hawkmoth total stroke angle. 86° and 65° stroke angles were achieved for the symmetric and asymmetric tests respectively. Additionally, two-dimensional PIV visualization measurements were taken transverse to the wing planform, recorded at the mid-span, for a single frequency and amplitude setting, for both symmetric and asymmetric flapping to complement the balance data with the goal to gain a more complete understanding of the complex flow physics on the AFIT FWMAV.

The force balance results showed the asymmetric vertical force developed was 18% less than the symmetric force. These values are less than the symmetric forces, mainly because the symmetric PZT displacement was $= 27\%$ greater than the asymmetric PZT displacement, resulting in a flap angle approximately 25% larger than the asymmetric flap angle. The symmetric $x$-direction center of pressure location remained nearly constant, $-6mm \leq x_{cp} \leq -2.5mm$, with changing flap angle, while the asymmetric center of pressure was $-25mm \leq x_{cp} \leq -20mm$, which is closer to the balance than the
symmetric $x_{cp}$, indicating asymmetrically flapping the wing by varying the bias off-set parameter, $\eta$, can produce desired control changes in pitch attitude.

Phase-averaged PIV data showed the downward $v$-velocity component imparted to the fluid at four phases during the flap cycle. Velocity data was extracted from the 2-D PIV imagery at $\pm \frac{1}{2}$ and $\pm 1$ chord locations above and below the wing, and the mean velocities were calculated at the mid-span for four separate wing phases during the flap cycle. A modified application of an axial actuator disk model was adopted to estimate the vertical momentum transfer, perpendicular to the stroke plane, from the phase averaged velocity data, to approximate the vertical force developed, and estimate the location of the $x_{cp}$ to compare with the 6-DoF balance data. The force balance data showed an 18% difference in vertical force between symmetric and asymmetric tests and correlated very well with the approximations calculated from the vertical velocity data, which showed a 19% difference between the two. Values of vertical thrust calculated from the 2-D PIV velocity measurements were within 20% of the 6-DOF force balance experiments in both symmetric and asymmetric testing at flap angles of 86° and 65° respectively.

Flapping tests were performed at system structural resonance and at $\pm 10\%$ off system resonance at a single amplitude. Drive system power consumption was calculated for each test condition. Power calculations confirmed flapping at system resonance required less power than at off resonance frequencies. At $-10\%$ off resonance (22.5Hz), power consumption was 128% higher than at resonance, and at $+10\%$ off resonance (27.5Hz), power consumption was 138% higher than at resonance. The PZT actuator power consumed for symmetric flapping at $\Phi = 86^\circ$, was about 50% of what Hollenbeck et al. calculated from manual deflections of a Hawkmoth’s thoracic flight muscles using a mechanical load cell.

8. FUTURE WORK

Future work on the FWMAV project will involve expanding the 2-D PIV testing to 3-D PIV to capture the out-of-plane $w$ – velocity component, which will enable us to better characterize the fully 3-D nature of flapping wing flows. Force and visualization experiments involving passive rotation stop angles different than 45°, and varying the wing stiffness and camber, by modifying the carbon fiber lay-up angles and the wing planform molds to assess the change in the unsteady aerodynamics during the transition from pronation to supination are forthcoming. Finally, incorporating the PIV and the force balance experiments into the same test chamber is underway, enabling simultaneous force and PIV experiments in subsequent testing.

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