Weak decays of doubly heavy baryons: the $1/2 \rightarrow 3/2$ case

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As a continuation of our previous works, we investigate the weak decays of doubly heavy baryons into a spin-$3/2$ singly or doubly heavy baryon. Light-front approach is adopted to handle the dynamics in the transitions, in which the two spectator quarks are approximated as a diquark. Results for form factors are then used to calculate decay widths of semi-leptonic and nonleptonic processes. The flavor SU(3) symmetry and symmetry breaking effects in semi-leptonic decays modes are explored, and we point out that in charm sector, there are sizable symmetry breaking effects. For nonleptonic decay modes, we study only the factorizable channels induced by the external W-emission. We find that branching fractions for most $1/2$ to $3/2$ transitions are approximately one order of magnitude smaller than the corresponding ones for the $1/2$ to $1/2$ transitions. Parametric uncertainties are also investigated in detail. This work, together with our previous works, are beneficial to the experimental studies of doubly heavy baryons at LHC and other experiments.

I. INTRODUCTION

Quite recently, LHCb collaboration reported the discovery of a doubly charmed baryon $\Xi^{++}_{cc}$ with the mass given as \cite{1}

\[ m_{\Xi^{++}_{cc}} = (3621.40 \pm 0.72 \pm 0.27 \pm 0.14)\text{MeV}. \] (1)

This discovery has already triggered great theoretical interests on the study of doubly heavy baryons from various aspects \cite{2,21]. Also after observing the first decay mode, $\Xi^{++}_{cc} \rightarrow \Lambda^{+}_c K^{-} \pi^{+} \pi^{+}$, LHCb collaboration is continuing the experimental analyses of doubly heavy baryon decays. This includes, but is not limited to, searches for the $\Xi^{+}_{cc}$ and charmed-beauty $\Xi_{bc}$ baryons \cite{22]. Comprehensive theoretical studies on weak decays must be performed and the golden modes for discoveries must be derived in order to optimize the experimental resources. In our previous work \cite{4}, we have presented the calculation of $1/2$ to $1/2$ weak decays. It is generally anticipated that the $1/2$ to $3/2$ processes will also be important. For instance, it is very likely that the $\Xi^{++}_{cc} \rightarrow \Lambda^{+}_c K^{-} \pi^{+} \pi^{+}$ comes from more than one intermediate $1/2 \rightarrow 3/2$ transitions.

A doubly heavy baryon is composed of two heavy quarks and one light quark. Light flavor SU(3) symmetry arranges the doubly heavy baryons into the presentation $\mathbf{3}$. For spin-$1/2$ doubly heavy baryons, we have $\Xi^{++}_{cc}$ and $\Omega^{+}_{cc}$ in the $cc$ sector, $\Xi^{0}_{bb}$ and $\Omega^{-}_{bb}$ in the $bb$ sector. There are two sets

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of baryons for $bc$ sector depending on the symmetric property under interchange of $b$ and $c$ quarks. If it is symmetric under interchange of $b$ and $c$ quarks, this set is denoted by $\Xi_{bc}^{+,0}$ and $\Omega_{bc}^0$, while for the asymmetric case, the corresponding set is denoted by $\Xi_{bc}^{+,0}$ and $\Omega_{bc}^0$. In reality these two sets probably mix with each other, which is not taken into account in this work. Spin-3/2 doubly heavy baryons have the same flavor wave functions with but different spin structures compared to the spin-1/2 counterparts. The quantum numbers of low-lying doubly heavy baryons can be found in Table I.

| Baryon | Quark Content | $S_h^q$ | $J^P$ | Baryon | Quark Content | $S_h^q$ | $J^P$ |
|--------|---------------|--------|-------|--------|---------------|--------|-------|
| $\Xi_{cc}$ | $(cc)q$ | $1^+$ | $3/2^+$ | $\Xi_{bb}$ | $(bb)q$ | $1^+$ | $1/2^+$ |
| $\Xi_{cc}^*$ | $(cc)q$ | $1^+$ | $1/2^+$ | $\Xi_{bb}^*$ | $(bb)q$ | $1^+$ | $3/2^+$ |
| $\Omega_{cc}$ | $(cc)s$ | $1^+$ | $1/2^+$ | $\Omega_{bb}$ | $(bb)s$ | $1^+$ | $1/2^+$ |
| $\Omega_{cc}^*$ | $(cc)s$ | $1^+$ | $3/2^+$ | $\Omega_{bb}^*$ | $(bb)s$ | $1^+$ | $3/2^+$ |
| $\Xi_{bc}$ | $(bc)q$ | $1^+$ | $3/2^+$ | $\Omega_{bc}$ | $(bc)s$ | $1^+$ | $1/2^+$ |
| $\Xi_{bc}^*$ | $(bc)q$ | $1^+$ | $1/2^+$ | $\Omega_{bc}^*$ | $(bc)s$ | $1^+$ | $3/2^+$ |

The decay final state of the $\Xi_{cc}$ and $\Omega_{cc}$ contains baryons with one charm quark and two light quarks. Light flavor SU(3) symmetry arranges them into the presentations $3 \otimes 3 = 6 \oplus \bar{3}$, as can be seen from Fig. I. The irreducible representation $3$ is composed of $\Lambda_c^+$ and $\Xi_c^{+,0}$ while the sextet is composed of $\Sigma_c^{++}, +, 0$, $\Xi_c^{+,0}$ and $\Omega_c^0$. They all have spin 1/2, while for spin-3/2 sextet, we will denote them by $\Sigma_c^{++}, +, s0$, $\Xi_c^{+,s0}$ and $\Omega_c^{00}$. The singly bottom baryons can be analyzed in a similar way.

To be explicit, we will investigate the following decay modes of doubly heavy baryons.

- **cc sector**

  \[
  \Xi_{cc}^{++}(ccu) \to \Sigma_c^{++}(dcu)/\Xi_c^{s0}(scu),
  \Xi_{cc}^+(ccd) \to \Sigma_c^{s0}(dcd)/\Xi_c^{s0}(scd),
  \Omega_{cc}^+(ccs) \to \Xi_c^{s0}(dcs)/\Omega_c^{s0}(scs),
  \]

- **bb sector**

  \[
  \Xi_{bb}^0(bbu) \to \Sigma_b^{s0}(ubu)/\Xi_b^{s0}(cbu),
  \Xi_{bb}^-(bbd) \to \Sigma_b^{s0}(ubd)/\Xi_b^{s0}(cbd),
  \Omega_{bb}^- (bbs) \to \Xi_b^{s0}(ubs)/\Omega_b^{s0}(cbs),
  \]

1 It should be noted that the convention here for $bc$ sector is the opposite of that in Ref. [23].
• *bc* sector with the *c* quark decay

\[
\Xi_{bc}^+(cbu)/\Xi_{bc}^+(cbu) \rightarrow \Sigma_{bc}^{0}(dbu)/\Xi_{bc}^{*0}(sbu), \\
\Xi_{bc}^0(cbd)/\Xi_{bc}^0(cbd) \rightarrow \Sigma_{bc}^{*-}(dbd)/\Xi_{bc}^{*-0}(sb), \\
\Omega_{bc}^0(cbs)/\Omega_{bc}^0(cbs) \rightarrow \Xi_{bc}^{*-}(dbs)/\Omega_{bc}^{*-0}(sbs),
\]

• *bc* sector with the *b* quark decay

\[
\Xi_{bc}^+(bcu)/\Xi_{bc}^+(bcu) \rightarrow \Sigma_{bc}^{++}(ucu)/\Xi_{bc}^{*++}(ccu), \\
\Xi_{bc}^0(bcd)/\Xi_{bc}^0(bcd) \rightarrow \Sigma_{bc}^{-}(ucd)/\Xi_{bc}^{++}(ccd), \\
\Omega_{bc}^0(bcs)/\Omega_{bc}^0(bcs) \rightarrow \Xi_{bc}^{*-}(ucs)/\Omega_{bc}^{*++}(ccs).
\]

In the above, the quark components have been explicitly shown in the brackets, in which the quarks that participate in weak decay are placed first.

To deal with the dynamics in the decay, we will adopt the light front approach, which has been widely used to study the properties of mesons [24–41]. Its application to the baryon sector can be found in Refs. [42–46], in which the two spectator quarks are viewed as a diquark. In this scheme, the role of the diquark system is similar to that of the antiquark in the meson case, as can be seen from Fig. 2. Following the same method, in this work we will study the 1/2 → 3/2 transition [46], where the spectator is a 1+ diquark system.

The authors of Ref. [7] have investigated the doubly heavy baryon decays with the help of flavor SU(3) symmetry. Based on the available data, a great number of decay modes ranging from semi-leptonic decays to multi-body nonleptonic decays can be predicted. However, in the *c* quark decay, SU(3) symmetry breaking effects may be sizable and cannot be omitted. A quantitative study of SU(3) symmetry breaking effects will be conducted within the light-front approach.

The rest of the paper is arranged as follows. In Sec. II, we will present briefly the framework of light-front approach under the diquark picture, and flavor-spin wave functions will also be discussed. Numerical results are shown in Sec. III, including the results for form factors, predictions on semi-leptonic and nonleptonic decay widths, detailed discussions on the SU(3) symmetry, the error estimates and a comparison with the previous 1/2 to 1/2 results. A brief summary and discussions on future improvements are given in the last section.

II. THEORETICAL FRAMEWORK

Theoretical framework for 1/2 → 3/2 transition will be briefly introduced in the first subsection, including the definitions of the states for spin-1/2 and spin-3/2 baryons, and the extraction of form factors. More details can be found in [42, 46]. Flavor-spin wave functions will be given in the second subsection.
FIG. 1: Anti-triplets (panel a) and sextets (panel b) of charmed baryons with one charm quark and two light quarks. These baryons are spin-1/2, while spin-3/2 baryons constitute another sextets.

FIG. 2: Feynman diagrams for baryon-baryon transitions in the diquark picture. $P^{(i)}$ is the momentum of the incoming (outgoing) baryon, $p_1^{(i)}$ is the initial (final) quark momentum, $p_2$ is the diquark momentum and the cross mark denotes the weak interaction.

A. Light-front approach

In the framework of light-front approach, the wave functions of $1/2^+$ baryon with an axial-vector diquark is expressed as

$$|B(P, S = 1/2, S_z)\rangle = \int \{d^3p_1\} \{d^3p_2\} 2(2\pi)^3 \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \times \sum_{\lambda_1, \lambda_2} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) |Q(p_1, \lambda_1)\{di\}(p_2, \lambda_2)\rangle.$$  \hspace{1cm} (2)

Here

$$\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{A}{\sqrt{2(p_1 \cdot P + m_1 M_0)}} \bar{u}(p_1, \lambda_1) \Gamma u(\tilde{P}, S_z) \phi(x, k_{\perp})$$ \hspace{1cm} (3)

with

$$\Gamma = -\frac{1}{\sqrt{3}} \gamma_5 \gamma^\lambda (p_2, \lambda_2), \quad m_1 = m_1, \quad \tilde{P} = p_1 + p_2,$$ \hspace{1cm} (4)

$$\phi = 4 \left(\frac{\pi}{\beta^2}\right)^{3/4} \sqrt{\frac{e_1 e_2}{x_1 x_2 M_0}} \exp\left(-\frac{\vec{k}^2}{2\beta^2}\right)$$ \hspace{1cm} (5)

and

$$A = \sqrt{\frac{-3m_1 M_0 + p_1 \cdot P}{3m_1 M_0 + p_1 \cdot P + 2(p_1 \cdot p_2)(p_2 \cdot P)/m_2^2}}.$$ \hspace{1cm} (6)
In analog to the $1/2^+$ baryon case, $3/2^+$ baryon state has a similar expression like Eq. (2) but with Eq. (3) being replaced by

$$\Psi^{SS_z}(\bar{p}_1, \bar{p}_2, \lambda_1, \lambda_2) = \frac{A'}{\sqrt{2(p_1 \cdot \bar{P} + m_1 M_0)}} \bar{u}(p_1, \lambda_1) \epsilon^{\alpha}(p_2, \lambda_2) u_\alpha(\bar{P}, S_z) \phi(x, k_\perp), \quad (7)$$

where

$$A' = \sqrt{\frac{3m_2^2 M_0^2}{2m_2^2 M_0^2 + (p_2 \cdot \bar{P})^2}}. \quad (8)$$

With the help of Eqs. (2), (3) and (7), the transition matrix element can be derived as

$$\langle B_f(P', S' = 3/2, S'_z)| \bar{q}_\gamma^\mu (1 - \gamma_5) Q| B_i(P, S = 1/2, S_z) \rangle = \int \{d^3 p_2\} \varphi'(x', k'_1) \varphi(x, k_\perp) \frac{\varphi(x', k'_1) \varphi(x, k_\perp)}{2\sqrt{p_1^+ p_1^{+*} (p_1 \cdot \bar{P} + m_1 M_0)(p'_1 \cdot \bar{P} + m'_1 M'_0)}} \times \sum_{\lambda_2} \bar{u}_\alpha(P', S'_z) \left[ \epsilon^{\alpha}(\lambda_2)(\bar{p}_1' + m_1') \gamma^\mu (1 - \gamma_5) (p_1 + m_1) \left( -\frac{1}{\sqrt{3}} \gamma_5 \bar{f}_2(\lambda_2) \right) \right] u(\bar{P}, S_z), \quad (9)$$

where

$$m_1 = m_Q, \quad m_1' = m_q, \quad m_2 = m_{di},$$

and $\varphi^{(i)} = A^{(i)} \phi^{(i)}$, $Q$ ($q$) represents the quark $b/c$ ($u/d/s/c$) in the initial (final) state, $p_1$ ($p_1'$) denotes its four-momentum, $P$ ($P'$) stands for the four-momentum of $B_i$ ($B_f$). The form factors for $1/2 \rightarrow 3/2$ transition are parameterized as

$$\langle B_f(P', S' = 3/2, S'_z)| \bar{q}_\gamma^\mu Q| B_i(P, S = 1/2, S_z) \rangle = \bar{u}_\alpha(P', S'_z) \left[ \gamma^\mu \rho_{\alpha \rho_{\beta}} f_1(q^2) \frac{1}{M} + f_2(q^2) \rho_{\alpha \rho_{\beta}} \right] \gamma_5 u(P, S_z), \quad (10)$$

$$\langle B_f(P', S' = 3/2, S'_z)| \bar{q}_\gamma^\mu \gamma_5 Q| B_i(P, S = 1/2, S_z) \rangle = \bar{u}_\alpha(P', S'_z) \left[ \gamma^\mu \rho_{\alpha \rho_{\beta}} g_{\alpha \beta} \right] \gamma_5 u(P, S_z). \quad (11)$$

Here $q = P - P'$, and $f_i$, $g_i$ are the form factors.

These form factors $f_i$ and $g_i$ can be extracted in the following way \[46\]. Multiplying Eq. (9) by $\bar{u}(P, S_z)(\Gamma^{\mu \beta}_5)_{i} u_\beta(P', S'_z)$ with $(\Gamma^{\mu \beta}_5)_{i} = \{ \gamma^\mu \rho_{\alpha \rho_{\beta}}, \rho_{\alpha \rho_{\beta}} \rho_{\alpha \rho_{\beta}}, \rho_{\alpha \rho_{\beta}} \}_{\gamma_5}$ respectively, and taking the approximation $P^{(i)} \rightarrow \bar{P}^{(i)}$ within the integral, and then summing over the polarizations in the initial and final states, one can arrive at

$$F_i = \int \{d^3 p_2\} \frac{\varphi'(x', k'_1) \varphi(x, k_\perp)}{2\sqrt{p_1^+ p_1^{+*} (p_1 \cdot \bar{P} + m_1 M_0)(p'_1 \cdot \bar{P} + m'_1 M'_0)}} \sum S'_z \sum \lambda_2 \left\{ \bar{u}_\beta(\bar{P}', S'_z) \bar{u}_\alpha(\bar{P}', S'_z) \right\}$$
\[ \times \epsilon^\alpha (\lambda_2)(\phi'_1 + m'_1) \gamma_\mu (\phi'_1 + m_1) \left( -\frac{1}{\sqrt{3}} \gamma_5 \epsilon^\sigma (\lambda_2) \right) u(\bar{P}, S_z) \bar{u}(\bar{P}, S_z)(\Gamma^\mu_5)_{i} \]  

(12)

with \( (\Gamma^\mu_5)_{i} = \{ \gamma^\mu \bar{P}^\beta, P^\mu \bar{P}^\beta, P^\mu \bar{P}^\beta, g^\mu \beta \} \gamma_5 \).

Multiplying the difference of Eq. (10) and Eq. (11) by the same factor \( \bar{u}(\bar{P}, S_z)(\Gamma^\mu_5)_{i} u_\beta (P', S'_z) \), and also summing over the polarizations in the initial and final states, one can arrive at

\[ F_i = \text{Tr} \left\{ u_\beta (P', S'_z) \bar{u}_\alpha (P', S'_z) \left[ \gamma^\mu P^\alpha \frac{f_1(q^2)}{M} + \frac{f_2(q^2)}{M^2} P^\alpha P^\mu + \frac{f_3(q^2)}{MM' M' P^\mu} + f_4(q^2) g^{\alpha \mu} \right] \right\} \gamma_5 \times u(P, S_z) \bar{u}(P, S_z)(\Gamma^\mu_5)_{i} \].  

(13)

The form factors \( f_i \) can then be extracted by equating Eqs. (12) and (13). With the same method, one can obtain the form factors \( g_i \).

### B. Flavor-spin wave functions

In subsection II A, the flavor-spin wave function was not taken into account. We consider first the initial state. For the doubly charmed baryons, the wave functions are given as

\[ \mathcal{B}_{cc} = \frac{1}{\sqrt{2}} \left[ \left( -\frac{\sqrt{3}}{2} c^l (c^2 q)_S + \frac{1}{2} c^l (c^2 q)_A \right) + (c^l \leftrightarrow c^2) \right], \]  

(14)

with \( q = u, d \) or \( s \) for \( \Xi^{++}_{cc}, \Xi^{+0}_{cc} \) or \( \Omega^{++}_{cc} \), respectively. It is similar for the doubly bottom baryons. For the bottom-charm baryons, there are two sets of states, with \( bc \) as a scalar or an axial-vector diquark. The wave functions of bottom-charm baryons with an axial-vector \( bc \) diquark are

\[ \mathcal{B}_{bc} = -\frac{\sqrt{3}}{2} b(cq)_S + \frac{1}{2} b(cq)_A = -\frac{\sqrt{3}}{2} c(bq)_S + \frac{1}{2} c(bq)_A, \]  

(15)

while those with a scalar \( bc \) diquark are given as

\[ \mathcal{B}_{bc}' = -\frac{1}{2} b(cq)_S - \frac{\sqrt{3}}{2} b(cq)_A = \frac{1}{2} c(bq)_S + \frac{\sqrt{3}}{2} c(bq)_A, \]  

(16)

with \( q = u, d \) or \( s \) for \( \Xi_{bc}^{(l)+}, \Xi_{bc}^{(l)0} \) or \( \Omega_{bc}^{(l)0} \), respectively. Note that the conventions for \( \mathcal{B}_{bc}^{(l)} \) in Ref. [23] are opposite to ours.

For the final state, the spin-3/2 baryon with quark contents of \( Qqq' \) has

\[ \mathcal{B}_{Qqq'}^{*} = q(Qq')_A = q'(Qq)_A, \]  

(17)

while for the \( Qqq \) baryon, an additional factor \( \sqrt{2} \) should be added. For the spin-3/2 baryon with quark contents of \( QQ'q \), we have

\[ \mathcal{B}_{QQ'q}^{*} = Q(Q'q)_A = Q'(Qq)_A, \]  

(18)
TABLE II: Flavor-spin space overlapping factors

| Transitions | Overlapping Factors | Transitions | Overlapping Factors |
|-------------|---------------------|-------------|---------------------|
| $\Xi_{cc}^{++}(ccu)\rightarrow \Sigma_{c}^{++}(dcu)/\Xi_{cc}^{++}(scu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ | $\Xi_{bb}^{0}(bbu)\rightarrow \Sigma_{b}^{0}(ubu)/\Xi_{bb}^{0}(cbu)$, | 1, $\frac{1}{\sqrt{2}}$ |
| $\Xi_{cc}^{0}(ccd)/\Xi_{cc}^{0}(scd)$, | 1, $\frac{1}{\sqrt{2}}$ | $\Xi_{bb}^{0}(ubd)\rightarrow \Sigma_{b}^{0}(ubd)/\Xi_{bb}^{0}(cbd)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ |
| $\Omega_{cc}^{+}(css)\rightarrow \Xi_{cc}^{0}(dcs)/\Omega_{cc}^{0}(scs)$, | $\frac{1}{\sqrt{2}}$, 1 | $\Omega_{bb}^{+}(bbu)\rightarrow \Sigma_{b}^{0}(ubs)/\Omega_{bb}^{0}(cbu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ |
| $\Xi_{bc}^{0}(cbu)\rightarrow \Sigma_{b}^{0}(dbu)/\Xi_{bc}^{0}(sbu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ | $\Xi_{bc}^{0}(bcd)\rightarrow \Sigma_{c}^{++}(ucu)/\Xi_{bc}^{++}(ccu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ |
| $\Xi_{bc}^{0}(cbu)\rightarrow \Sigma_{b}^{0}(dbu)/\Xi_{bc}^{0}(sbu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ | $\Xi_{bc}^{0}(bcd)\rightarrow \Sigma_{c}^{++}(ucu)/\Xi_{bc}^{++}(ccu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ |
| $\Omega_{bc}^{0}(bcs)\rightarrow \Xi_{b}^{+}(dbs)/\Omega_{b}^{+}(sbs)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ | $\Omega_{bc}^{0}(bcs)\rightarrow \Xi_{c}^{+}(ucu)/\Omega_{c}^{+}(ccu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ |
| $\Xi_{bc}^{+}(cbu)\rightarrow \Sigma_{b}^{0}(dbu)/\Xi_{bc}^{0}(sbu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ | $\Xi_{bc}^{0}(bcd)\rightarrow \Sigma_{c}^{++}(ucu)/\Xi_{bc}^{++}(ccu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ |
| $\Xi_{bc}^{0}(cbu)\rightarrow \Sigma_{b}^{0}(dbu)/\Xi_{bc}^{0}(sbu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ | $\Xi_{bc}^{0}(bcd)\rightarrow \Sigma_{c}^{++}(ucu)/\Xi_{bc}^{++}(ccu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ |
| $\Omega_{bc}^{0}(bcs)\rightarrow \Xi_{b}^{+}(dbs)/\Omega_{b}^{+}(sbs)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ | $\Omega_{bc}^{0}(bcs)\rightarrow \Xi_{c}^{+}(ucu)/\Omega_{c}^{+}(ccu)$, | $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$ |

while for the $QQq$ baryon, an additional factor $\sqrt{2}$ should be added. Here the asterisk denotes that the baryon is spin-3/2 and $q^{(i)} = u, d, s$.

Finally, the overlapping factors are determined by taking the inner product of the flavor-spin wave functions in the initial and final states. The corresponding results are collected in Table II.

### III. NUMERICAL RESULTS AND DISCUSSIONS

All the inputs will be given in the first subsection. Numerical results for form factors, semi-leptonic and nonleptonic decays will be shown subsequently. They are presented in this order: $cc$ sector, $bb$ sector, $bc$ sector with the $c$ quark decay, $bc$ sector with the $b$ quark decay, $bc'$ sector with the $c$ quark decay, $bc'$ sector with the $b$ quark decay. Some discussions will also be given.

The variable $\omega$ will be introduced

$$\omega \equiv v \cdot v' = \frac{P \cdot P'}{MM'},$$  \hspace{1cm} (19)

which can be easily changed to the squared momentum transfer $q^2$, and vice versa.

The vectorial spinor for spin-3/2 baryon is given as

$$u^\alpha = (\epsilon^\alpha - \frac{1}{3}(\gamma^\alpha + v^\alpha)\gamma)u$$  \hspace{1cm} (20)

with $v^\alpha = p^\alpha/m$, while its helicity eigenstate can be found in Eq. (20) of Ref. \[47\]

$$u^\alpha(p, \lambda) = \sum_{\lambda_1, \lambda_2} \left( \frac{1}{2}, \lambda_1, 1, \lambda_2 | \frac{3}{2}, \lambda \right) \times u(p, \lambda_1) e^\alpha(p, \lambda_2).$$  \hspace{1cm} (21)

#### A. Inputs

The constituent quark masses are given as (in units of GeV) \[33 \, 41\]

$$m_u = m_d = 0.25, \quad m_s = 0.37, \quad m_c = 1.4, \quad m_b = 4.8.$$  \hspace{1cm} (22)
The masses of the axial-vector diquarks are approximated by \( m_{(Qq)} = m_Q + m_q \). The shape parameters \( \beta \) in Eq. [28] are given as (in units of GeV) [28]

\[
\begin{align*}
\beta_{u(cq)} &= \beta_{d(cq)} = 0.470, & \beta_{s(cq)} &= 0.535, & \beta_{c(cq)} &= 0.753, & \beta_{b(cq)} &= 0.886, \\
\beta_{u(bq)} &= \beta_{d(bq)} = 0.562, & \beta_{s(bq)} &= 0.623, & \beta_{c(bq)} &= 0.886, & \beta_{b(bq)} &= 1.472,
\end{align*}
\]

where \( q = u, d, s \).

The masses and lifetimes of the parent baryons are collected in Table III [1, 3, 23, 48–50]. Note that, in the Table III, the masses and lifetimes of \( B_{bc} \) and \( B'_{bc} \) are taken the same. Also note that we have taken a new value for the lifetime of \( \Omega^+_c \). Compared with our previous work [4], because according to Ref. [48], lifetimes of doubly charmed baryons should satisfy the following pattern:

\[
\tau(\Xi^{++}_{cc}) \sim \tau(\Omega^+_{cc}) \ll \tau(\Xi^{++}_{cc}).
\]

\[\text{(24)}\]

- **TABLE III:** Masses (in units of GeV) and lifetimes (in units of fs) of doubly heavy baryons.

| baryons | \( \Xi^{++}_{cc} \) | \( \Xi^{0}_{cc} \) | \( \Omega^+_{cc} \) | \( \Xi^{(0)}_{bc} \) | \( \Xi^{(0)}_{hc} \) | \( \Omega^{(0)}_{bc} \) | \( \Xi^0_{bb} \) | \( \Xi^0_{bb} \) | \( \Omega^0_{bb} \) |
|--------|-----------------|-----------------|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| masses | 3.621 [1]       | 3.621 [1]       | 3.738 [23]       | 6.943 [23]      | 6.943 [23]      | 6.998 [23]      | 10.143 [23]     | 10.143 [23]     | 10.273 [23]     |
| lifetimes | 300 [3] | 100 [3] | 100 [48] | 244 [49] | 93 [49] | 220 [50] | 370 [49] | 370 [49] | 800 [50] |

The masses of the final state baryons are given in Table IV [23, 51]. Fermi constant and CKM matrix elements are given as [51]

\[
G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2},
\]

\[
|V_{ud}| = 0.974, \quad |V_{us}| = 0.225, \quad |V_{ub}| = 0.00357,
\]

\[
|V_{cd}| = 0.225, \quad |V_{cs}| = 0.974, \quad |V_{cb}| = 0.0411.
\]

\[\text{(25)}\]

In the calculation of nonleptonic decays, these mesons will present in the final states: \( \pi, \rho, a_1, K, K^*, D, D^*, D_s, D_s^* \). Their masses can be found in Ref. [51], while their decay constants are given as follows [28, 41, 52]:

\[
\begin{align*}
f_\pi &= 130.4 \text{MeV}, & f_\rho &= 216 \text{MeV}, & f_{a_1} &= 238 \text{MeV}, & f_K &= 160 \text{MeV}, & f_{K^*} &= 210 \text{MeV}, \\
f_D &= 207.4 \text{MeV}, & f_{D^*} &= 220 \text{MeV}, & f_{D_s} &= 247.2 \text{MeV}, & f_{D_s^*} &= 247.2 \text{MeV}.
\end{align*}
\]

\[\text{(26)}\]

Wilson coefficients \( a_1 = C_1(\mu_c) + C_2(\mu_c)/3 = 1.07 \) [53], will be used.

- **TABLE IV:** The masses of baryons in the final states [23, 51].

| \( \Sigma^{++} \) | \( \Sigma^+ \) | \( \Sigma^{0} \) | \( \Xi^{0} \) | \( \Xi^{+0} \) | \( \Xi^{0+} \) | \( \Omega^{00} \) | \( \Omega^{++} \) | \( \Omega^{+} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 2.518           | 2.518           | 2.646           | 2.646           | 2.766           | 3.692           | 3.692           | 3.822           | 3.822           |
| 5.832           | 5.833           | 5.949           | 6.085           | 6.985           | 6.985           | 7.059           | 7.059           | 7.059           |
TABLE V: Values of form factors at $q^2 = 0$ for $cc$ sector. Single pole assumption in Eq. (27) will be adopted, and $m_{pole}$ is taken as 1.87 GeV.

| $F$ | $F(0)$ | $F$ | $F(0)$ |
|-----|--------|-----|--------|
| $f_1^{\Xi_c^0\to\Sigma_c^0}$ | $-1.121$ | $g_1^{\Xi_c^0\to\Sigma_c^0}$ | $-8.292$ |
| $f_2^{\Xi_c^0\to\Sigma_c^0}$ | $1.764$ | $g_2^{\Xi_c^0\to\Sigma_c^0}$ | $-0.156$ |
| $f_3^{\Xi_c^0\to\Sigma_c^0}$ | $-3.793$ | $g_3^{\Xi_c^0\to\Sigma_c^0}$ | $7.427$ |
| $f_4^{\Xi_c^0\to\Sigma_c^0}$ | $-1.827$ | $g_4^{\Xi_c^0\to\Sigma_c^0}$ | $0.295$ |
| $f_1^{\Xi_c^0\to\Xi_c^0}$ | $-1.318$ | $g_1^{\Xi_c^0\to\Xi_c^0}$ | $-14.180$ |
| $f_2^{\Xi_c^0\to\Xi_c^0}$ | $1.494$ | $g_2^{\Xi_c^0\to\Xi_c^0}$ | $-0.882$ |
| $f_3^{\Xi_c^0\to\Xi_c^0}$ | $-5.251$ | $g_3^{\Xi_c^0\to\Xi_c^0}$ | $13.600$ |
| $f_4^{\Xi_c^0\to\Xi_c^0}$ | $-2.147$ | $g_4^{\Xi_c^0\to\Xi_c^0}$ | $0.294$ |
| $f_1^{\Omega_c^0\to\Xi_c^0}$ | $-1.154$ | $g_1^{\Omega_c^0\to\Xi_c^0}$ | $-8.801$ |
| $f_2^{\Omega_c^0\to\Xi_c^0}$ | $2.227$ | $g_2^{\Omega_c^0\to\Xi_c^0}$ | $-0.118$ |
| $f_3^{\Omega_c^0\to\Xi_c^0}$ | $-4.444$ | $g_3^{\Omega_c^0\to\Xi_c^0}$ | $7.915$ |
| $f_4^{\Omega_c^0\to\Xi_c^0}$ | $-1.896$ | $g_4^{\Omega_c^0\to\Xi_c^0}$ | $0.298$ |
| $f_1^{\Omega_c^0\to\Omega_c^0}$ | $-1.339$ | $g_1^{\Omega_c^0\to\Omega_c^0}$ | $-14.470$ |
| $f_2^{\Omega_c^0\to\Omega_c^0}$ | $1.939$ | $g_2^{\Omega_c^0\to\Omega_c^0}$ | $-0.811$ |
| $f_3^{\Omega_c^0\to\Omega_c^0}$ | $-5.575$ | $g_3^{\Omega_c^0\to\Omega_c^0}$ | $13.850$ |
| $f_4^{\Omega_c^0\to\Omega_c^0}$ | $-2.204$ | $g_4^{\Omega_c^0\to\Omega_c^0}$ | $0.314$ |

B. Results for form factors

To access the $q^2$-distribution, the following single pole structure is assumed for form factors:

$$F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m_{pole}^2}},$$

(27)

$F(0)$ is the value of the form factors at $q^2 = 0$, the corresponding numerical results predicted by the light-front approach are collected in Tables VI to VIII. For $c \to d/s$ decays, $m_{pole}$ is taken as 1.87 GeV, while for $b \to u/c$ decays, $m_{pole}$ is taken as 5.28 GeV and 6.28 GeV, respectively. In practice, these quantities are taken as the masses of $D, B$ and $B_s$ mesons. The discussion for the validity of this assumption can be found in our previous work [41].

The physical form factor can be obtained by multiplying Eq. (27) by the corresponding overlapping factor.

C. Results for semi-leptonic decays

Helicity amplitudes are defined by

$$H_{\lambda,\lambda_W}^V \equiv \langle \mathcal{B}_j^i(\lambda')|\bar{q}\gamma^{\mu}Q|B_i(\lambda)\rangle \epsilon_{W\mu}(\lambda_W)$$ and

$$H_{\lambda',\lambda_W}^{A} \equiv \langle \mathcal{B}_j^i(\lambda')|\bar{q}\gamma^{\mu}\gamma_5 Q|B_i(\lambda)\rangle \epsilon_{W\mu}(\lambda_W)$$ respectively, where $\lambda = \lambda_W - \lambda'$ is understood. These helicity
TABLE VI: Values of form factors at $q^2 = 0$ for $bb$ sector. Single pole assumption in Eq. (27) will be adopted, and $m_{\text{pole}}$ is taken as follows: for $b \to q$ process, $m_{\text{pole}} = 5.28$ GeV while for $b \to c$ process, $m_{\text{pole}} = 6.28$ GeV.

| $F$ | $F(0)$ | $F$ | $F(0)$ |
|-----|--------|-----|--------|
| $f_1^{bb \to \Sigma_b}$ | -0.185 | $g_1^{bb \to \Sigma_b}$ | -0.219 |
| $f_2^{bb \to \Sigma_b}$ | 0.230 | $g_2^{bb \to \Sigma_b}$ | 0.089 |
| $f_3^{bb \to \Sigma_b}$ | -0.153 | $g_3^{bb \to \Sigma_b}$ | 0.118 |
| $f_4^{bb \to \Sigma_b}$ | -0.328 | $g_4^{bb \to \Sigma_b}$ | 0.087 |
| $f_1^{bb \to \Xi_b}$ | -0.791 | $g_1^{bb \to \Xi_b}$ | -3.044 |
| $f_2^{bb \to \Xi_b}$ | 1.284 | $g_2^{bb \to \Xi_b}$ | -0.441 |
| $f_3^{bb \to \Xi_b}$ | -1.287 | $g_3^{bb \to \Xi_b}$ | 3.170 |
| $f_4^{bb \to \Xi_b}$ | -1.439 | $g_4^{bb \to \Xi_b}$ | 0.342 |
| $f_1^{\Omega_b \to \Xi_b}$ | -0.178 | $g_1^{\Omega_b \to \Xi_b}$ | -0.207 |
| $f_2^{\Omega_b \to \Xi_b}$ | 0.225 | $g_2^{\Omega_b \to \Xi_b}$ | 0.092 |
| $f_3^{\Omega_b \to \Xi_b}$ | -0.148 | $g_3^{\Omega_b \to \Xi_b}$ | 0.104 |
| $f_4^{\Omega_b \to \Xi_b}$ | -0.321 | $g_4^{\Omega_b \to \Xi_b}$ | 0.086 |
| $f_1^{\Omega_b \to \Omega_b}$ | -1.759 | $g_1^{\Omega_b \to \Omega_b}$ | -2.755 |
| $f_2^{\Omega_b \to \Omega_b}$ | 1.122 | $g_2^{\Omega_b \to \Omega_b}$ | -0.272 |
| $f_3^{\Omega_b \to \Omega_b}$ | -1.089 | $g_3^{\Omega_b \to \Omega_b}$ | 2.520 |
| $f_4^{\Omega_b \to \Omega_b}$ | -1.390 | $g_4^{\Omega_b \to \Omega_b}$ | 0.366 |

Amplitudes are related to the form factors by the following expressions.

$$H_{3/2,1}^{V,A} = \pm i \sqrt{2} M M' (\omega \mp 1) f_4^{V,A},$$

$$H_{1/2,1}^{V,A} = i \sqrt{2} \frac{M M' (\omega \mp 1)}{3} \left[ f_4^{V,A} - 2 (\omega \pm 1) f_2^{V,A} \right],$$

$$H_{1/2,0}^{V,A} = \pm i \frac{1}{\sqrt{q^2}} \frac{2}{\sqrt{3}} \sqrt{M M' (\omega \pm 1)} \left[ (M \omega - M') f_4^{V,A} \mp (M \mp M') (\omega \pm 1) f_2^{V,A} + M (\omega^2 - 1) f_2^{V,A} + M (\omega^2 - 1) f_3^{V,A} \right],$$

where the upper (lower) sign corresponds to $V (A)$, $f_i^V = f_i^A = g_i$, $\omega$ is defined in Eq. (19), $M$ ($M'$) is the mass of the baryon in the initial (final) state. The remaining helicity amplitudes can be obtained by

$$H_{-\lambda',-\lambda W}^{V,A} = \mp H_{\lambda',\lambda W}^{V,A}.$$

Partial differential decay widths are obtained as

$$\frac{d\Gamma_T}{d\omega} = \frac{G_F^2}{(2\pi)^3} |V_{\text{CKM}}|^2 q^2 M^2 \sqrt{\omega^2 - 1} \frac{12 M}{12 M} \left[ |H_{1/2,1}|^2 + |H_{-1/2,1}|^2 + |H_{3/2,1}|^2 + |H_{-3/2,-1}|^2 \right],$$

$$\frac{d\Gamma_L}{d\omega} = \frac{G_F^2}{(2\pi)^3} |V_{\text{CKM}}|^2 q^2 M^2 \sqrt{\omega^2 - 1} \frac{12 M}{12 M} \left[ |H_{1/2,0}|^2 + |H_{-1/2,0}|^2 \right].$$
TABLE VII: Same as Table V but for $bc^{(i)}$ sector with the c quark decay.

| $F$ | $F(0)$ | $F$ | $F(0)$ |
|-----|--------|-----|--------|
| $f_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_c$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_c$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_c$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_c$ |
| $f_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_b$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_b$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_b$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_b$ |
| $f_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_c$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_c$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_c$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_c$ |
| $f_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_b$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_b$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_b$ | $g_{bc}^{(i)} \rightarrow \Sigma^{*(0)}_b$ |

Numerical results are collected in Tables IX to XIV. Some comments are given in subsection IIIF.

D. Results for nonleptonic decays

For the nonleptonic processes, we are constrained to consider only those of a W boson emitting outward. For the process with a pseudoscalar meson in the final state, the decay width is obtained as

$$
\Gamma = |\lambda|^2 f^2 \frac{M}{6\pi} |F|^3 \left[ (\omega - 1) (B^2 - 2AB) + 2A^2 \omega \right],
$$

(34)
TABLE VIII: Same as Table VII but for the $bc$ sector with the $b$ quark decay.

|       |       |       |
|-------|-------|-------|
| $F$   | $F(0)$| $F$   | $F(0)$|
| $f_1^{\Xi_{bc}^{(0)} \to \Sigma_c^+}$ | $-0.114$ | $g_1^{\Xi_{bc}^{(0)} \to \Sigma_c^+}$ | $-0.024$|
| $f_2^{\Xi_{bc}^{(0)} \to \Sigma_c^+}$ | $0.040$ | $g_2^{\Xi_{bc}^{(0)} \to \Sigma_c^+}$ | $0.062$|
| $f_3^{\Xi_{bc}^{(0)} \to \Sigma_c^+}$ | $0.030$ | $g_3^{\Xi_{bc}^{(0)} \to \Sigma_c^+}$ | $-0.069$|
| $f_4^{\Xi_{bc}^{(0)} \to \Sigma_c^+}$ | $-0.239$ | $g_4^{\Xi_{bc}^{(0)} \to \Sigma_c^+}$ | $0.156$|
| $f_1^{\Omega_{bc}^{(0)} \to \Xi_{cc}^+}$ | $-0.497$ | $g_1^{\Omega_{bc}^{(0)} \to \Xi_{cc}^+}$ | $-0.446$|
| $f_2^{\Omega_{bc}^{(0)} \to \Xi_{cc}^+}$ | $0.265$ | $g_2^{\Omega_{bc}^{(0)} \to \Xi_{cc}^+}$ | $0.118$|
| $f_3^{\Omega_{bc}^{(0)} \to \Xi_{cc}^+}$ | $-0.011$ | $g_3^{\Omega_{bc}^{(0)} \to \Xi_{cc}^+}$ | $0.117$|
| $f_4^{\Omega_{bc}^{(0)} \to \Xi_{cc}^+}$ | $-0.969$ | $g_4^{\Omega_{bc}^{(0)} \to \Xi_{cc}^+}$ | $0.539$|

TABLE IX: Semi-leptonic decays for the $cc$ sector.

| channels | $\Gamma$ (GeV) | $B$ | $\Gamma_L/\Gamma_T$ |
|----------|----------------|-----|---------------------|
| $\Xi_{cc}^{++} \to \Sigma_c^{*+} e^+ \bar{\nu}_e$ | $1.26 \times 10^{-15}$ | $5.73 \times 10^{-4}$ | $0.85$ |
| $\Xi_{cc}^{+} \to \Sigma_c^{0} e^+ \nu_e$ | $2.51 \times 10^{-15}$ | $3.82 \times 10^{-4}$ | $0.85$ |
| $\Omega_{cc}^{+} \to \Xi_{cc}^{0} e^+ \nu_e$ | $1.19 \times 10^{-15}$ | $1.82 \times 10^{-4}$ | $0.87$ |
| $\Xi_{cc}^{++} \to \Xi_{cc}^{++} e^+ \nu_e$ | $1.61 \times 10^{-14}$ | $7.34 \times 10^{-3}$ | $0.99$ |
| $\Xi_{cc}^{+} \to \Xi_{cc}^{0} e^+ \nu_e$ | $1.61 \times 10^{-14}$ | $2.45 \times 10^{-3}$ | $0.99$ |
| $\Omega_{cc}^{+} \to \Omega_{cc}^{0} e^+ \nu_e$ | $3.20 \times 10^{-14}$ | $4.87 \times 10^{-3}$ | $0.99$ |

TABLE X: Semi-leptonic decays for the $bb$ sector.

| channels | $\Gamma$ (GeV) | $B$ | $\Gamma_L/\Gamma_T$ |
|----------|----------------|-----|---------------------|
| $\Xi_{bb}^{++} \to \Sigma_b^{*+} e^+ \bar{\nu}_b$ | $3.88 \times 10^{-17}$ | $2.18 \times 10^{-5}$ | $0.85$ |
| $\Xi_{bb}^{+} \to \Sigma_b^{0} e^+ \bar{\nu}_b$ | $1.94 \times 10^{-17}$ | $1.09 \times 10^{-5}$ | $0.85$ |
| $\Omega_{bb}^{0} \to \Xi_{bb}^{0} e^+ \bar{\nu}_b$ | $1.90 \times 10^{-17}$ | $2.32 \times 10^{-5}$ | $0.84$ |
| $\Xi_{bb}^{++} \to \Xi_{bc}^{++} e^- \bar{\nu}_e$ | $6.37 \times 10^{-15}$ | $3.58 \times 10^{-3}$ | $1.43$ |
| $\Xi_{bb}^{+} \to \Xi_{bc}^{0} e^- \bar{\nu}_e$ | $6.37 \times 10^{-15}$ | $3.58 \times 10^{-3}$ | $1.43$ |
| $\Omega_{bb}^{0} \to \Omega_{bc}^{0} e^- \bar{\nu}_e$ | $7.03 \times 10^{-15}$ | $8.55 \times 10^{-3}$ | $1.31$ |
TABLE XI: Semi-leptonic decays for the $bc$ sector with the $c$ quark decay.

| channels                                    | $\Gamma$/ GeV | $\mathcal{B}$ | $\Gamma_L/\Gamma_T$ |
|---------------------------------------------|---------------|--------------|------------------|
| $\Xi_{bc}^{1+} \rightarrow \Sigma_b^{*0} e^+ \nu_e$ | $1.10 \times 10^{-15}$ | $4.07 \times 10^{-4}$ | 0.69 |
| $\Xi_{bc}^{0} \rightarrow \Sigma_b^{*0} e^+ \nu_e$ | $2.17 \times 10^{-15}$ | $3.06 \times 10^{-4}$ | 0.69 |
| $\Omega_{bc}^{0} \rightarrow \Xi_b^{*0} e^+ \nu_e$ | $6.98 \times 10^{-16}$ | $2.33 \times 10^{-4}$ | 0.80 |
| $\Xi_{bc}^{0} \rightarrow \Xi_b^{*0} e^+ \nu_e$ | $1.33 \times 10^{-14}$ | $4.95 \times 10^{-3}$ | 0.77 |
| $\Xi_{bc}^{0} \rightarrow \Xi_b^{*0} e^+ \nu_e$ | $1.27 \times 10^{-14}$ | $1.80 \times 10^{-3}$ | 0.78 |
| $\Omega_{bc}^{0} \rightarrow \Omega_b^{*0} e^+ \nu_e$ | $1.47 \times 10^{-14}$ | $4.91 \times 10^{-3}$ | 0.97 |

TABLE XII: Semi-leptonic decays for the $bc$ sector with the $b$ quark decay.

| channels                                    | $\Gamma$/ GeV | $\mathcal{B}$ | $\Gamma_L/\Gamma_T$ |
|---------------------------------------------|---------------|--------------|------------------|
| $\Xi_{bc}^{1+} \rightarrow \Sigma_c^{*+} e^- \bar{\nu}_e$ | $3.32 \times 10^{-17}$ | $1.23 \times 10^{-5}$ | 0.81 |
| $\Xi_{bc}^{0} \rightarrow \Sigma_c^{*+} e^- \bar{\nu}_e$ | $1.66 \times 10^{-17}$ | $2.35 \times 10^{-6}$ | 0.81 |
| $\Omega_{bc}^{0} \rightarrow \Xi_c^{*+} e^- \bar{\nu}_e$ | $1.26 \times 10^{-17}$ | $4.23 \times 10^{-6}$ | 0.84 |
| $\Xi_{bc}^{+} \rightarrow \Xi_c^{*+} e^- \bar{\nu}_e$ | $8.96 \times 10^{-15}$ | $3.32 \times 10^{-3}$ | 1.18 |
| $\Xi_{bc}^{0} \rightarrow \Xi_c^{*+} e^- \bar{\nu}_e$ | $8.96 \times 10^{-15}$ | $1.27 \times 10^{-3}$ | 1.18 |
| $\Omega_{bc}^{0} \rightarrow \Omega_c^{*+} e^- \bar{\nu}_e$ | $7.53 \times 10^{-15}$ | $2.52 \times 10^{-3}$ | 1.28 |

with

$$\lambda \equiv \frac{G_F V_{cb} V^*_{q_1 q_2}}{\sqrt{2}} a_1,$$

$$A = (M - M') \frac{g_1}{M} + \frac{g_2}{M^2} (P \cdot q) + \frac{g_3}{MM'} (P' \cdot q) + g_4,$$

$$B = - (M + M') \frac{f_1}{M} + \frac{f_2}{M^2} (P \cdot q) + \frac{f_3}{MM'} (P' \cdot q) + f_4.$$

Here $a_1 \equiv C_1 + C_2/3$.

For the process with a vector meson in the final state, the decay width is obtained as

$$\Gamma = |\lambda|^2 f^2 \frac{m^2}{16\pi M^2} \left[ |H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + |H_{3/2,1}|^2 + |H_{-3/2,-1}|^2 + |H_{1/2,0}|^2 + |H_{-1/2,0}|^2 \right].$$

Note that in the above equations, $q^2 = m^2$ is understood, where $m$ is the mass of the meson.

TABLE XIII: Semi-leptonic decays for the $bc'$ sector with the $c$ quark decay.

| channels                                    | $\Gamma$/ GeV | $\mathcal{B}$ | $\Gamma_L/\Gamma_T$ |
|---------------------------------------------|---------------|--------------|------------------|
| $\Xi_{bc}^{1+} \rightarrow \Sigma_b^{*0} e^+ \nu_e$ | $3.30 \times 10^{-15}$ | $1.22 \times 10^{-3}$ | 0.69 |
| $\Xi_{bc}^{0} \rightarrow \Sigma_b^{*0} e^+ \nu_e$ | $6.50 \times 10^{-15}$ | $9.18 \times 10^{-4}$ | 0.69 |
| $\Omega_{bc}^{0} \rightarrow \Xi_b^{*0} e^+ \nu_e$ | $2.09 \times 10^{-15}$ | $7.00 \times 10^{-4}$ | 0.80 |
| $\Xi_{bc}^{+} \rightarrow \Xi_b^{*0} e^+ \nu_e$ | $4.00 \times 10^{-14}$ | $1.48 \times 10^{-2}$ | 0.77 |
| $\Xi_{bc}^{0} \rightarrow \Xi_b^{*0} e^+ \nu_e$ | $3.82 \times 10^{-14}$ | $5.40 \times 10^{-3}$ | 0.78 |
| $\Omega_{bc}^{0} \rightarrow \Omega_b^{*0} e^+ \nu_e$ | $4.40 \times 10^{-14}$ | $1.47 \times 10^{-2}$ | 0.97 |
TABLE XIV: Semi-leptonic decays for the $be'$ sector with the $b$ quark decay.

| channels                   | $\Gamma$/ GeV | $B$ | $\Gamma_L/\Gamma_T$ |
|---------------------------|----------------|-----|---------------------|
| $\Xi^{++}_{bc} \rightarrow \Sigma^{++}_{c} l^- \bar{\nu}_e$ | $9.97 \times 10^{-17}$ | $3.70 \times 10^{-5}$ | 0.81 |
| $\Xi^{0}_{bc} \rightarrow \Sigma^{+}_{c} l^- \bar{\nu}_e$ | $4.99 \times 10^{-17}$ | $7.05 \times 10^{-6}$ | 0.81 |
| $\Omega^{0}_{bc} \rightarrow \Xi^{++}_{cc} e^- \bar{\nu}_e$ | $3.79 \times 10^{-17}$ | $1.27 \times 10^{-5}$ | 0.84 |
| $\Xi^{+}_{bc} \rightarrow \Xi^{+}_{cc} e^- \bar{\nu}_e$ | $2.69 \times 10^{-14}$ | $9.97 \times 10^{-3}$ | 1.18 |
| $\Xi^{0}_{bc} \rightarrow \Xi^{+}_{cc} e^- \bar{\nu}_e$ | $2.69 \times 10^{-14}$ | $3.80 \times 10^{-3}$ | 1.18 |
| $\Omega^{+}_{bc} \rightarrow \Omega^{++}_{cc} e^- \bar{\nu}_e$ | $2.26 \times 10^{-14}$ | $7.55 \times 10^{-3}$ | 1.28 |

TABLE XV: Nonleptonic decays for $cc$ sector.

| channels       | $\Gamma$/ GeV | $B$ | $\Gamma$/ GeV | $B$ |
|----------------|---------------|-----|---------------|-----|
| $\Xi^{++}_{cc} \rightarrow \Sigma^{++}_{c} \pi^+$ | $1.16 \times 10^{-15}$ | 1.73 \times 10^{-3} | $3.78 \times 10^{-15}$ | 2.27 \times 10^{-5} |
| $\Xi^{0+}_{cc} \rightarrow \Sigma^{0+}_{c} K^+$ | $1.60 \times 10^{-16}$ | 1.15 \times 10^{-3} | $4.97 \times 10^{-17}$ | 3.21 \times 10^{-4} |
| $\Xi^{++}_{cc} \rightarrow \Xi^{++}_{cc} \pi^+$ | $2.24 \times 10^{-14}$ | 1.21 \times 10^{-5} | $4.85 \times 10^{-14}$ | 7.37 \times 10^{-3} |
| $\Xi^{++}_{cc} \rightarrow \Xi^{++}_{cc} K^+$ | $1.32 \times 10^{-15}$ | 1.60 \times 10^{-6} | $7.05 \times 10^{-16}$ | 1.07 \times 10^{-4} |
| $\Xi^{++}_{cc} \rightarrow \Sigma^{+}_{c} K^+$ | $2.32 \times 10^{-15}$ | 1.68 \times 10^{-4} | $5.34 \times 10^{-15}$ | 5.53 \times 10^{-4} |
| $\Xi^{++}_{cc} \rightarrow \Sigma^{+}_{c} K^+$ | $3.21 \times 10^{-16}$ | 4.87 \times 10^{-5} | $7.05 \times 10^{-16}$ | 7.20 \times 10^{-6} |
| $\Xi^{++}_{cc} \rightarrow \Xi^{++}_{cc} \pi^+$ | $2.24 \times 10^{-14}$ | 4.85 \times 10^{-14} | $7.37 \times 10^{-3}$ | 7.37 \times 10^{-3} |
| $\Xi^{++}_{cc} \rightarrow \Xi^{++}_{cc} K^+$ | $1.32 \times 10^{-15}$ | 1.60 \times 10^{-6} | $7.20 \times 10^{-6}$ | 1.07 \times 10^{-4} |
| $\Omega^{+}_{cc} \rightarrow \Xi^{++}_{cc} \pi^+$ | $1.10 \times 10^{-15}$ | 1.60 \times 10^{-4} | $3.64 \times 10^{-15}$ | 5.53 \times 10^{-4} |
| $\Omega^{+}_{cc} \rightarrow \Xi^{++}_{cc} K^+$ | $1.53 \times 10^{-16}$ | 2.32 \times 10^{-5} | $4.74 \times 10^{-17}$ | 7.20 \times 10^{-6} |
| $\Omega^{+}_{cc} \rightarrow \Sigma^{+}_{c} K^+$ | $4.26 \times 10^{-14}$ | 6.47 \times 10^{-3} | $9.88 \times 10^{-14}$ | 1.50 \times 10^{-2} |
| $\Omega^{+}_{cc} \rightarrow \Sigma^{+}_{c} K^+$ | $2.79 \times 10^{-15}$ | 4.24 \times 10^{-4} | $1.38 \times 10^{-15}$ | 2.10 \times 10^{-4} |

All the corresponding results are collected in Tables XV to XX. Some comments are given in subsection III.

E. SU(3) symmetry for semi-leptonic decays

According to the flavor SU(3) symmetry, there exist the following relations among these semileptonic decay widths [7], which can also be readily rederived using the overlapping factors given in Table II.

- $cc$ sector

\[
\Gamma(\Xi^{++}_{cc} \rightarrow \Sigma^{++}_{c} l+\nu) = \frac{\Gamma(\Xi^{++}_{cc} \rightarrow \Sigma^{0+}_{c} l+\nu)}{|V_{cd}|^2} = \frac{\Gamma(\Omega^{++}_{cc} \rightarrow \Xi^{++}_{cc} l+\nu)}{|V_{cd}|^2} = \frac{\Gamma(\Omega^{++}_{cc} \rightarrow \Omega^{++}_{cc} l+\nu)}{2|V_{cd}|^2}.
\]

- $bb$ sector

\[
\Gamma(\Xi^{0}_{bb} \rightarrow \Sigma^{0+}_{b} l^- \bar{\nu}) = 2\Gamma(\Xi^{0+}_{bb} \rightarrow \Sigma^{0+}_{b} l^- \bar{\nu}) = 2\Gamma(\Omega^{0}_{bb} \rightarrow \Xi^{0}_{bb} l^- \bar{\nu}),
\]
TABLE XVI: Nonleptonic decays for $bb$ sector.

| channels      | $\Gamma$ / GeV | $B$  | channels      | $\Gamma$ / GeV | $B$  |
|---------------|----------------|------|---------------|----------------|------|
| $\Xi_{bc}^0 \to \Sigma_{bc}^{*+} \pi^-$ | $6.34 \times 10^{-18}$ | $3.57 \times 10^{-6}$ | $\Xi_{bc}^0 \to \Sigma_{bc}^{*+} \rho^-$ | $2.19 \times 10^{-15}$ | $1.23 \times 10^{-3}$ |
| $\Xi_{bb} \to \Sigma_{bb}^{*+} a_1^+$ | $4.38 \times 10^{-18}$ | $2.47 \times 10^{-6}$ | $\Xi_{bb} \to \Sigma_{bb}^{*+} K^-$ | $9.99 \times 10^{-20}$ | $5.62 \times 10^{-8}$ |
| $\Xi_{bb} \to \Sigma_{bb}^{*+} K^{*-}$ | $1.67 \times 10^{-19}$ | $9.41 \times 10^{-8}$ | $\Xi_{bb} \to \Sigma_{bb}^{*+} D^-$ | $1.44 \times 10^{-19}$ | $8.12 \times 10^{-8}$ |
| $\Xi_{bb} \to \Sigma_{bb}^{*+} D^{*-}$ | $2.58 \times 10^{-19}$ | $1.45 \times 10^{-7}$ | $\Xi_{bb} \to \Sigma_{bb}^{*+} D_s^-$ | $3.77 \times 10^{-18}$ | $2.12 \times 10^{-6}$ |

\[
\Gamma(\Xi_{bb}^0 \to \Xi_{bb}^{*+} l^- \bar{\nu}) = \Gamma(\Xi_{bb}^0 \to \Xi_{bc}^{0*} l^- \bar{\nu}) = \Gamma(\Omega_{bb}^- \to \Omega_{bc}^{0*} l^- \bar{\nu}), \quad (40)
\]

- $bc$ sector with the $c$ quark decay

\[
\frac{\Gamma(\Xi_{bc}^+ \to \Sigma_{bc}^0 l^+ \nu)}{|V_{cd}|^2} = \frac{\Gamma(\Xi_{bc}^0 \to \Sigma_{bc}^0 l^- \nu)}{|V_{cs}|^2} = \frac{\Gamma(\Omega_{bc}^0 \to \Omega_{bc}^{0*} l^- \nu)}{|V_{cd}|^2} = \frac{\Gamma(\Omega_{bc}^0 \to \Omega_{bc}^{0*} l^- \nu)}{|V_{cs}|^2}, \quad (41)
\]
Also, we have compared the predictions of the light-front approach with those of SU(3) symmetry

Some comments are given in order.

- **bc sector with the b quark decay**

\[
\Gamma(\Xi_{bc}^{+} \rightarrow \Sigma_{bc}^{*+} l^- \bar{\nu}) = 2\Gamma(\Xi_{bc}^{+} \rightarrow \Sigma_{bc}^{*+} l^- \bar{\nu}) = 2\Gamma(\Omega_{bc}^{0} \rightarrow \Xi_{bc}^{*+} l^- \bar{\nu}),
\]

\[
\Gamma(\Xi_{bc}^{0} \rightarrow \Xi_{bc}^{*+} l^- \bar{\nu}) = \Gamma(\Xi_{bc}^{0} \rightarrow \Xi_{bc}^{*+} l^- \bar{\nu}) = \Gamma(\Omega_{bc}^{0} \rightarrow \Omega_{bc}^{*+} l^- \bar{\nu}),
\]

(42)

- **bc' sector with the c quark decay**

\[
\Gamma(\Xi_{bc}^{+} \rightarrow \Sigma_{bc}^{*0} l^+ \bar{\nu}) = \frac{\Gamma(\Xi_{bc}^{0} \rightarrow \Sigma_{bc}^{*0} l^+ \bar{\nu})}{|V_{cd}|^2} = \frac{\Gamma(\Omega_{bc}^{0} \rightarrow \Xi_{bc}^{*0} l^+ \bar{\nu})}{|V_{cd}|^2}
\]

\[
\Gamma(\Xi_{bc}^{0} \rightarrow \Xi_{bc}^{*0} l^+ \bar{\nu}) = \Gamma(\Xi_{bc}^{0} \rightarrow \Xi_{bc}^{*0} l^+ \bar{\nu}) = \Gamma(\Omega_{bc}^{0} \rightarrow \Omega_{bc}^{*0} l^+ \bar{\nu}),
\]

(43)

- **bc' sector with the b quark decay**

\[
\Gamma(\Xi_{bc}^{+} \rightarrow \Sigma_{bc}^{*+} l^- \bar{\nu}) = 2\Gamma(\Xi_{bc}^{0} \rightarrow \Xi_{bc}^{*+} l^- \bar{\nu}) = 2\Gamma(\Omega_{bc}^{0} \rightarrow \Xi_{bc}^{*+} l^- \bar{\nu}),
\]

\[
\Gamma(\Xi_{bc}^{0} \rightarrow \Xi_{bc}^{*+} l^- \bar{\nu}) = \Gamma(\Xi_{bc}^{0} \rightarrow \Xi_{bc}^{*+} l^- \bar{\nu}) = \Gamma(\Omega_{bc}^{0} \rightarrow \Omega_{bc}^{*+} l^- \bar{\nu}).
\]

(44)

Also, we have compared the predictions of the light-front approach with those of SU(3) symmetry method taking cc and bb sectors as examples, which can be seen in Tables XXI and XXII.

Some comments are given in order.

- Note that, 1/2 to 3/2 process has completely the same SU(3) relations as the corresponding 1/2 to 1/2 case. This can be expected, because spin-3/2 baryon shares the same flavor wave function as the corresponding spin-1/2 baryon.

- SU(3) predictions for the corresponding two channels in bc and bc' sectors have completely the same form, as can be explained by the facts that they have the same final states and the formally fixed initial states as in Eqs. (15) and (16).
The small deviation of 6% in Table XXI and 2% in Table XXII can be explained by the fact that we have taken a larger value for the mass of quark and diquark, where $Q = c/b$. And also, note that, SU(3) symmetry breaking in c quark decay is usually larger than that in b quark decay.

- We can see from Table XXI that, sizable SU(3) symmetry breaking takes place between the $c \to d$ and $c \to s$ processes. Of course, this can be attributed to the model parameter: we have taken different quark mass for $d$ quark and $s$ quark.

- The small deviation of 6% in Table XXI and 2% in Table XXII can be explained by the fact that we have taken a larger value for the mass of Qs diquark than that of Qu or Qd diquark, where $Q = c/b$.
TABLE XIX: Nonleptonic decays for \(bc\) sector with the \(c\) quark decay.

| channels \(\Xi_{bc}^i \rightarrow \Sigma_{bc}^{j0}\pi^+\) | \(\Gamma/\text{GeV}\) | \(\mathcal{B}\) | channels \(\Xi_{bc}^i \rightarrow \Sigma_{bc}^{j0}K^{++}\) | \(\Gamma/\text{GeV}\) | \(\mathcal{B}\) |
|-------------------------------------------------|----------------|-------|-------------------------------------------------|----------------|-------|
| \(\Xi_{bc}^i \rightarrow \Sigma_{bc}^{j0}\pi^+\) | \(1.14 \times 10^{-15}\) | \(4.22 \times 10^{-4}\) | \(\Xi_{bc}^i \rightarrow \Sigma_{bc}^{j0}\rho^+\) | \(9.73 \times 10^{-15}\) | \(3.61 \times 10^{-3}\) |
| \(\Xi_{bc}^i \rightarrow \Sigma_{bc}^{j0}K^{++}\) | \(5.32 \times 10^{-16}\) | \(1.97 \times 10^{-4}\) | \(\Xi_{bc}^i \rightarrow \Sigma_{bc}^{j0}\rho^+\) | \(6.95 \times 10^{-17}\) | \(2.58 \times 10^{-5}\) |
| \(\Xi_{bc}^i \rightarrow \Xi_{bc}^{j0}\pi^+\) | \(2.29 \times 10^{-14}\) | \(8.49 \times 10^{-3}\) | \(\Xi_{bc}^i \rightarrow \Xi_{bc}^{j0}\rho^+\) | \(1.52 \times 10^{-13}\) | \(5.63 \times 10^{-2}\) |
| \(\Xi_{bc}^i \rightarrow \Xi_{bc}^{j0}K^{++}\) | \(6.61 \times 10^{-15}\) | \(2.45 \times 10^{-4}\) | \(\Xi_{bc}^i \rightarrow \Xi_{bc}^{j0}\rho^+\) | \(1.11 \times 10^{-15}\) | \(4.10 \times 10^{-4}\) |

F. Comparison

For a comparison, we also list the results in Ref. [41]. Some comments will be given on the results of semi-leptonic and nonleptonic decays.

\[ \mathcal{B}(B_c \rightarrow B_s \bar{\nu}) = 1.51 \times 10^{-2}, \]
\[ \mathcal{B}(B_c \rightarrow B_s^* \bar{\nu}) = 1.96 \times 10^{-2}, \]
\[ \mathcal{B}(B_c \rightarrow B \bar{\nu}) = 1.04 \times 10^{-3}, \]
\[ \mathcal{B}(B_c \rightarrow B^* \bar{\nu}) = 1.34 \times 10^{-3}, \]
\[ \mathcal{B}(B_c \rightarrow B_s \pi) = 4.1 \times 10^{-2}, \]
\[ \mathcal{B}(B_c \rightarrow B_s^* \pi) = 2.0 \times 10^{-2}. \] (45)

- Since there exist large uncertainties in the lifetimes, we have also presented the results for decay widths.
- We find that the result for 1/2 to 3/2 process is roughly one order of magnitude smaller than the corresponding 1/2 to 1/2 case except for the \(B_{bc}'\) decays. Both the \(c\) quark and \(b\) quark decays of \(B_{bc}'\) baryons are comparable to the corresponding 1/2 to 1/2 cases.
- \(\mathcal{B}(H_{bc} \rightarrow H_{bc} \bar{\nu}) \sim 10^{-2}\) holds for the corresponding results in Refs. [4] and [41], while in this work, it ranges from \(10^{-3}\) to \(10^{-2}\). \(\mathcal{B}(H_{bc} \rightarrow H_{bc} \bar{\nu}) \sim 10^{-3}\) holds for the corresponding results in Refs. [4] and [41], while in this work, it ranges from \(10^{-4}\) to \(10^{-3}\). Here \(H_{bc}\) stands for the \(B_c\) meson or the \(B_{bc}'\) baryon.
- \(\mathcal{B}(H_{bc} \rightarrow H_{bc} \bar{\pi}) \sim 10^{-2}\) holds for the corresponding results in Refs. [4] and [41], while in
### TABLE XX: Nonleptonic decays for be' sector with the b quark decay.

| Channels   | $\Gamma$ / GeV | $B$  | Channels   | $\Gamma$ / GeV | $B$  |
|------------|----------------|------|------------|----------------|------|
| $\Xi_{bc}^{++} \to \Sigma_{c}^{++} \pi^-$ | $2.46 \times 10^{-18}$ | | $\Xi_{bc}^{++} \to \Sigma_{c}^{++} \rho^-$ | $5.55 \times 10^{-18}$ | $2.06 \times 10^{-6}$ |
| $\Xi_{bc}^{++} \to \Sigma_{c}^{++} e^- e^+$ | $7.90 \times 10^{-18}$ | $2.90 \times 10^{-9}$ | $\Xi_{bc}^{++} \to \Sigma_{c}^{++} K^-$ | $1.98 \times 10^{-19}$ | $7.34 \times 10^{-8}$ |
| $\Xi_{bc}^{++} \to \Sigma_{c}^{++} D^+$ | $5.11 \times 10^{-19}$ | $1.27 \times 10^{-17}$ | $\Xi_{bc}^{++} \to \Sigma_{c}^{++} D_s^+$ | $9.10 \times 10^{-18}$ | $3.37 \times 10^{-6}$ |
| $\Xi_{bc}^{++} \to \Sigma_{c}^{++} D_s^-$ | $2.53 \times 10^{-15}$ | $9.39 \times 10^{-4}$ | $\Xi_{bc}^{++} \to \Xi_{cc}^{++} \pi^-$ | $6.63 \times 10^{-15}$ | $2.46 \times 10^{-3}$ |
| $\Xi_{bc}^{++} \to \Xi_{cc}^{++} a_1^-$ | $1.23 \times 10^{-18}$ | $1.74 \times 10^{-7}$ | $\Xi_{bc}^{++} \to \Xi_{cc}^{++} \rho^-$ | $2.78 \times 10^{-18}$ | $3.93 \times 10^{-7}$ |
| $\Xi_{bc}^{++} \to \Xi_{cc}^{++} K^-$ | $3.43 \times 10^{-16}$ | $1.27 \times 10^{-4}$ | $\Xi_{bc}^{++} \to \Xi_{cc}^{++} K^-$ | $9.90 \times 10^{-20}$ | $1.40 \times 10^{-8}$ |
| $\Xi_{bc}^{++} \to \Xi_{cc}^{++} D^+$ | $4.84 \times 10^{-16}$ | $1.79 \times 10^{-4}$ | $\Xi_{bc}^{++} \to \Xi_{cc}^{++} D^-$ | $1.71 \times 10^{-19}$ | $2.42 \times 10^{-8}$ |
| $\Xi_{bc}^{++} \to \Xi_{cc}^{++} D_s^-$ | $6.35 \times 10^{-18}$ | $8.98 \times 10^{-7}$ | $\Xi_{bc}^{++} \to \Xi_{cc}^{++} D_s^+$ | $4.55 \times 10^{-18}$ | $6.43 \times 10^{-7}$ |

### TABLE XXI: Quantitative predictions of SU(3) breaking for semi-leptonic decays: cc sector.

| Channels   | $\Gamma/\text{GeV (LFQM)}$ | $\Gamma/\text{GeV (SU(3))}$ | $|\text{LFQM} - \text{SU(3)}|/\text{SU(3)}$ |
|------------|-----------------------------|-----------------------------|--------------------------------|
| $\Xi_{cc}^{++} \to \Sigma_{c}^{++} e^+ e^-$ | $1.26 \times 10^{-15}$ | $1.26 \times 10^{-15}$ | - |
TABLE XXII: Quantitative predictions of SU(3) breaking for semi-leptonic decays: $bb$ sector.

| channels                  | $\Gamma$/GeV (LFQM) | $\Gamma$/GeV (SU(3)) | $|\text{LFQM} - \text{SU(3)}|$/SU(3) |
|---------------------------|----------------------|-----------------------|----------------------|
| $\Xi_{bb}^0 \to \Sigma_b^+ e^- \bar{\nu}_e$ | $3.88 \times 10^{-17}$ | $3.88 \times 10^{-17}$ | - - |
| $\Xi_{bb}^- \to \Sigma_b^0 e^- \bar{\nu}_e$ | $1.94 \times 10^{-17}$ | $1.94 \times 10^{-17}$ | 0% |
| $\Omega_{bb}^- \to \Xi_b^{0*} e^- \bar{\nu}_e$ | $1.90 \times 10^{-17}$ | $1.94 \times 10^{-17}$ | 2% |
| $\Xi_{bb}^0 \to \Xi_b^{+*} e^- \bar{\nu}_e$ | $6.37 \times 10^{-15}$ | $6.37 \times 10^{-15}$ | - - |
| $\Xi_{bb}^- \to \Xi_b^{0*} e^- \bar{\nu}_e$ | $6.37 \times 10^{-15}$ | $6.37 \times 10^{-15}$ | 0% |
| $\Omega_{bb}^- \to \Omega_b^{0*} e^- \bar{\nu}_e$ | $7.03 \times 10^{-15}$ | $6.37 \times 10^{-15}$ | 10% |

this work, it is roughly $10^{-3}$. $\mathcal{B}(H_{bc} \to H_{bd}\pi) \sim 10^{-3}$ holds for the corresponding results in Refs. 4 and 41, while in this work, it is of order $10^{-4}$.

G. Uncertainties

We will also investigate the dependence of the decay widths on the model parameters. Take $\Xi^{++} \to \Sigma^+_c$ transition as an example. Varying the model parameters $m_{(di)}$, $\beta_i$, $\beta_f$ and $m_{\text{pole}}$ by 10% respectively, the corresponding error estimates are listed as follows

$$
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{++}_c e^+ \nu_e) = (1.26 \pm 0.26 \pm 0.21 \pm 0.25 \pm 0.12) \times 10^{-15} \text{ GeV}, \\
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{++}_c \pi^+) = (1.16 \pm 0.08 \pm 0.32 \pm 0.04 \pm 0.00) \times 10^{-15} \text{ GeV}, \\
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{++}_c \rho^+) = (3.78 \pm 0.81 \pm 0.63 \pm 0.79 \pm 0.40) \times 10^{-15} \text{ GeV}, \\
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{++}_c K^+) = (4.97 \pm 0.61 \pm 0.17 \pm 0.26 \pm 0.18) \times 10^{-17} \text{ GeV}, \\
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{++}_c K^{**}) = (1.60 \pm 0.44 \pm 0.59 \pm 0.49 \pm 0.25) \times 10^{-16} \text{ GeV}. \quad (46)
$$

For $\Xi^{++}_{cc} \to \Sigma^{+}_c$, the corresponding results are listed as follows

$$
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{+}_c e^+ \nu_e) = (1.04 \pm 0.04 \pm 0.02 \pm 0.11 \pm 0.15) \times 10^{-14} \text{ GeV}, \\
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{+}_c \pi^+) = (5.75 \pm 0.19 \pm 0.35 \pm 0.88 \pm 0.02) \times 10^{-15} \text{ GeV}, \\
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{+}_c \rho^+) = (2.61 \pm 0.08 \pm 0.08 \pm 0.29 \pm 0.27) \times 10^{-14} \text{ GeV}, \\
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{+}_c K^+) = (4.28 \pm 0.15 \pm 0.25 \pm 0.66 \pm 0.16) \times 10^{-16} \text{ GeV}, \\
\mathcal{B}(\Xi^{++}_{cc} \to \Sigma^{+}_c K^{**}) = (1.39 \pm 0.05 \pm 0.02 \pm 0.14 \pm 0.22) \times 10^{-15} \text{ GeV}. \quad (47)
$$

Some comments are given in order.

- Eq. (27) is also adopted for $\Xi^{++}_{cc} \to \Sigma^+_c$ transitions for this time. In our previous work Ref. 4, the following fit formulas were adopted

$$
F(q^2) = \frac{F(0)}{1 + \frac{q^2}{m_{\text{fit}}^2} + \delta \left(\frac{q^2}{m_{\text{fit}}^2}\right)^2}. \quad (48)
$$
However, only a few percent is changed in Eqs. (47) compared with our previous results in Ref. [4].

- It can be seen that, the variation in these parameters may cause a sizable change in the decay width, but the order of magnitude will not change.

IV. CONCLUSIONS

In our previous work, we have performed the calculation of doubly heavy baryon weak decays for 1/2 to 1/2 case. As a continuation, we investigate the 1/2 to 3/2 case in this work. Light-front approach under the diquark picture is once again adopted to extract the form factors. In Ref. [5], the same method was used to study the bottom and charm baryon decays and reasonable results were obtained. The extracted form factors are then applied to predict the decay widths of semi-leptonic and nonleptonic decays. We find that the result for 1/2 to 3/2 case is roughly one order of magnitude smaller than the corresponding 1/2 to 1/2 case except for the $B_{bc}'$ decays. For $B_{bc}'$ baryons, both the $c$ quark and $b$ quark decays are comparable to the corresponding 1/2 to 1/2 cases. SU(3) symmetry and sources of SU(3) symmetry breaking for semi-leptonic decays are discussed. The error estimates are also performed.

It should be noted that the decay branching ratio is proportional to the lifetime of the initial baryon. However, as we have pointed out in Ref. [4], there exist large uncertainties in the lifetimes of these doubly heavy baryons. Our future work will aim to fix this problem.

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