Coherent processing of a light pulse stored in a medium of four-level atoms

A. Raczyński\textsuperscript{1}, J. Zaremba\textsuperscript{1} and S. Zielińska-Kaniasty\textsuperscript{2}

\textsuperscript{1}Instytut Fizyki, Uniwersytet Mikołaja Kopernika, ul.Grudziądzka 5, 87-100 Toruń, Poland,
\textsuperscript{2}Instytut Matematyki i Fizyki, Akademia Techniczno-Rolnicza, Al. Prof. S. Kaliskiego 7, 85-796 Bydgoszcz, Poland.

Abstract

It is demonstrated that the properties of light stored in a four-level atomic system can be modified by an additional control interaction present during the storage stage. By choosing the pulse area of this interaction one can in particular continuously switch between two channels into which light is released.

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*email: raczyn@phys.uni.torun.pl
In recent years a number of nonlinear optical phenomena in weak fields have been intensively investigated. They are connected with a coherent excitation of an atomic medium, the optical properties of which can then be drastically modified. An important example is the electromagnetically induced transparency \[\text{[1]},\] possibly additionally controlled in time, which is manifested as light slow-down or even its storage and a controlled release \[\text{[2,3,4,5,6,7]}\]. The simplest realization of those effects occurs in atomic systems with three active states in a \(\Lambda\) configuration. Extending such a configuration by adding a coherently coupled fourth level opens new possibilities of an external control of such processes \[\text{[8,9,10,11]}\]. In our previous paper \[\text{[12]}\] we have shown that in a double \(\Lambda\) system it is possible to change the light frequency of the stored light or even to release two pulses of different frequencies by applying two control fields, properly chosen and delayed in time.

Light storing in the form of atomic coherences joins the advantages of the efficiency of light as an information carrier and of an atomic medium as an information store. Thus it might be a question of a practical importance how to modify in a controlled way the properties of the released light by processing the atomic medium during the storage stage. In the present work we investigate two possibilities of controlling the released pulse or pulses by modifying the atomic coherence due to the stopped light. In the case (a) the lower, initially empty state of a typical \(\Lambda\) system is additionally coupled to a fourth state by another laser. In the case (b) the initially occupied state of the \(\Lambda\) system is additionally coupled to a fourth state by another laser. We show that the Rabi oscillations due to the new interactions modify in a coherent way the properties of the released light.

We consider a quasi one-dimensional medium of four-level atoms with three lower metastable states \(|b\rangle\) and \(|c\rangle\) and \(|d\rangle\) and an upper state \(|a\rangle\) (Fig. 1). The position of an atom is described by the variable \(z\), which is considered continuous. The states \(b, a\) and \(c\) constitute a typical \(\Lambda\) system with the weak signal field 1 and a strong control field 2. In the case (a) the state \(c\) is additionally coupled with another state \(d\) by a laser field 4. In the case (b) the state \(a\) is coupled with a fourth state \(d\) by a weak signal field 3 while the states \(b\) and \(d\) are coupled by some effective coupling 4. The interaction Hamiltonian in the case (a) is \(V = -\hat{d}\sum_{j=1,2,4}\epsilon_j \cos \phi_j\), with \(\phi_j = \omega_j t - k_j z\), \(\epsilon_j = \epsilon_j(z,t)\) being slowly varying envelopes and \(\hat{d}\) - the dipole moment operator; we have assumed that all the fields have the same linear polarization. In the case (b) the Hamiltonian reads \(V = -\hat{d}\sum_{j=1,2,3}\epsilon_j \cos \phi_j + [(iU)|b>< d| - (iU)|d>< b|] \cos \phi_4\), where for numerical reasons we have made a physically insignificant assumption that the matrix element \(iU\) of the effective interaction 4 is imaginary. The matrix elements of the dipole moment \(d_1 = (\hat{d})_{ab}, d_2 = (\hat{d})_{ac}, d_3 = (\hat{d})_{ad}, d_4 = (\hat{d})_{cd}\) are taken real. Resonant conditions concerning all the couplings are assumed.

The evolution equation \(i\hbar \dot{\rho} = [H, \rho]\) for the density matrix \(\rho = \rho(z,t)\) for an atom at position \(z\), after making the rotating-wave approximation, transforming-off the rapidly oscillating factors: \(\rho_{ab} = \sigma_{ab} \exp(-i\phi_1), \rho_{ac} = \sigma_{ac} \exp(-i\phi_2), \rho_{bc} = \sigma_{bc} \exp[i(\phi_1 - \phi_2)], \rho_{db} = \sigma_{db} \exp(-i\phi_4), \rho_{dc} = \sigma_{dc} \exp[i(\phi_2 - \phi_3)], \rho_{ad} = \sigma_{ad} \exp[i(\phi_3)], \rho_{bd} = \sigma_{bd}\), and after adding relaxation terms describing the spontaneous emission within the system, takes the form in the more complicated case (b)

\[
i\hat{\sigma}_{aa} = -\frac{1}{2\hbar}\epsilon_1 d_1 (\sigma_{ba} - \sigma_{ab}) + \frac{1}{2}\Omega_2 (\sigma_{ca} - \sigma_{ac}) - \frac{1}{2\hbar}\epsilon_3 d_3 (\sigma_{da} - \sigma_{ad}) - i(\Gamma^a_b + \Gamma^a_c + \Gamma^a_d)\sigma_{aa},
\]
\[ i\dot{\sigma}_{bb} = -\frac{1}{2\hbar}\epsilon_1 d_1 (\sigma_{ab} - \sigma_{ba}) + \frac{1}{2\hbar} iU (\sigma_{bd} + \sigma_{db}) + i \Gamma_b^a \sigma_{aa}, \]
\[ i\dot{\sigma}_{cc} = -\frac{1}{2} \Omega_2 (\sigma_{ac} - \sigma_{ca}) + i \Gamma_c^a \sigma_{aa}, \]
\[ i\dot{\sigma}_{dd} = -\frac{1}{2\hbar} \epsilon_3 d_3 (\sigma_{ad} - \sigma_{da}) - \frac{1}{2\hbar} iU (\sigma_{bd} + \sigma_{db}) + i \Gamma_d^a \sigma_{aa}, \]
\[ i\dot{\sigma}_{ab} = -\frac{1}{2\hbar} \epsilon_1 d_1 (\sigma_{ab} - \sigma_{ba}) + \frac{1}{2} \Omega_2 \sigma_{cb} - \frac{1}{2\hbar} \epsilon_3 d_3 \sigma_{db} + \frac{1}{2\hbar} iU \sigma_{ad} - \frac{i}{2} (\Gamma_b^a + \Gamma_c^a + \Gamma_d^a) \sigma_{ab}, \]
\[ (1) \]
\[ i\dot{\sigma}_{ac} = -\frac{1}{2\hbar} \epsilon_1 d_1 \sigma_{ac} + \frac{1}{2} \Omega_2 (\sigma_{cc} - \sigma_{ac}) - \frac{1}{2\hbar} \epsilon_3 d_3 \sigma_{de} - \frac{i}{2} (\Gamma_b^a + \Gamma_c^a + \Gamma_d^a) \sigma_{ac}, \]
\[ i\dot{\sigma}_{ad} = -\frac{1}{2\hbar} \epsilon_1 d_1 \sigma_{bd} + \frac{1}{2} \Omega_2 \sigma_{cd} - \frac{1}{2\hbar} \epsilon_3 d_3 (\sigma_{dd} - \sigma_{aa}) - \frac{1}{2\hbar} iU \sigma_{ab} - \frac{i}{2} (\Gamma_b^a + \Gamma_c^a + \Gamma_d^a) \sigma_{ad}, \]
\[ i\dot{\sigma}_{ac} = -\frac{1}{2\hbar} \epsilon_1 d_1 \sigma_{ac} - \frac{1}{2} \Omega_2 \sigma_{cb} + \frac{1}{2\hbar} iU \sigma_{dc}, \]
\[ i\dot{\sigma}_{bd} = -\frac{1}{2\hbar} \epsilon_1 d_1 \sigma_{ad} + \frac{1}{2\hbar} \epsilon_3 d_3 \sigma_{ba} + \frac{1}{2\hbar} iU (\sigma_{dd} - \sigma_{bb}), \]
\[ i\dot{\sigma}_{cd} = \frac{1}{2} \Omega_2 \sigma_{aa} + \frac{1}{2\hbar} \epsilon_3 d_3 \sigma_{ca} - \frac{1}{2\hbar} iU \sigma_{cb}, \]

where \( \Gamma_b^a \) is the decay rate of the state \(|a >, b \rangle\), etc., and \( \Omega_2 = -\epsilon_2 d_2 / \hbar \) is the Rabi frequency corresponding to the driving field 2.

The corresponding equations for the case (a) are obtained from the above set (1) by interchanging the indices \( b \leftrightarrow c, 1 \leftrightarrow 2 \), by setting \( \epsilon_3 = 0, d_3 = 0, \Gamma_d^a = 0 \), by replacing the effective coupling \( iU \) by \( -\epsilon_4 d_4 \) (when multiplied by \( \sigma_{dc}, \sigma_{da}, \sigma_{ac}, \sigma_{db}, \sigma_{bc}, \sigma_{cc} \) and \( \sigma_{dd} \)) and by changing the sign of \( \omega_4 \).

The propagation equations for the signal field 1 (in the case (a)) and for both signal fields (1) and (3) (in the case (b)), in the slowly varying envelope approximation and in the conditions of the resonance read

\[ \frac{\partial \epsilon_1}{\partial z} + \frac{1}{c} \frac{\partial \epsilon_1}{\partial t} = -i N d_1 \omega_1 \epsilon_0 c \sigma_{ba}, \]
\[ \frac{\partial \epsilon_3}{\partial z} + \frac{1}{c} \frac{\partial \epsilon_3}{\partial t} = -i N d_3 \omega_3 \epsilon_0 c \sigma_{da}, \]

(2)

where \( N \) is the atom density and \( \epsilon_0 \) is the vacuum electric permittivity. Similarly as in earlier papers, we have neglected propagation effects for the driving fields, i.e. we take \( \epsilon_{2,4} = \epsilon_{2,4}(t) \).

Eqs (1) and (2) have been solved numerically in the moving window frame of reference: \( t' = t - z/c, z' = z \). Switching the driving field 2 on and/or off was modeled by a hyperbolic tangent, while the additional pulse 4 was taken rectangular. The initial probe pulse was taken as the sine square shape

\[ \epsilon_1(0, t) = \epsilon_0 \sin^2[\pi(t - \tau_1)/(\tau_2 - \tau_1)] \Theta(t - \tau_1) \Theta(\tau_2 - t), \]

while the initial condition for the atomic part was \( \sigma_{bb}(z, 0) = 1 \), with other matrix elements equal to zero.

We have performed model computations for data being of realistic orders of magnitude, however without making attempt to imitate any real atom. The atomic energies were \( E_a = -0.10 \) a.u., \( E_b = -0.20 \) a.u. \( E_c = -0.18 \) a.u. with \( E_d = -0.22 \) a.u (in the case (a)) and \( E_d = E_b + 10^{-7} \) a.u. in the case (b) (the latter value is of order of a magnetic energy splitting). The relaxation rates for the spontaneous emission from the level \( E_a \) to \( E_b, E_c \) and in the case
(b) also to $E_d$ were taken equal to $2.4 \times 10^{-9}$ a.u., from which the dipole moments have been calculated. The dipole moment for the electric transition $c \leftrightarrow d$ was taken $-2.74 \times 10^{-1}$ a.u., which corresponded to a negligible width of the level $E_c$. The length of the atomic sample was $2.5 \times 10^7$ a.u. (1.3 mm) in the case (a) and $3 \times 10^6$ a.u. (1.6 mm) in the case (b) and its density $3 \times 10^{-13}$ a.u. ($2 \times 10^{12}$ cm$^{-3}$). The initial signal pulse length was $10^{11}$ a.u. ($2.4 \mu$s) and $\epsilon_{10} = 10^{-10}$ a.u. (which corresponded to the power density of $3.5 \times 10^{-4}$ Wcm$^{-2}$); the maximum value of the amplitude of the control field $2$ was $1.2 \times 10^{-9}$ a.u. ($50$ mWcm$^{-2}$). The values of the $b - d$ effective coupling $U$ in the case (b) were of order of $10^{-10}$ a.u. while in the case (a) we took a coupling with $\epsilon_4 = 2 \times 10^{-9}$ a.u.

In Fig. 2 we show the released part of the pulse 1 as a function of the local time $t'$ for different values of the area of the pulse 4 (case (a)). The pulse can be lowered, completely damped or its sign reversed depending on the final phase of the Rabi oscillations between the levels c and d. Of course the final results do not depend on particular time instants of switching the interaction 4 on and off, provided that the pulse arrived after the signal pulse 1 has been stored and before the release stage has started. The presence of the Rabi oscillations becomes clearly visible in the situation in which the control pulses 2 and 4 partially overlap. In this case, with the latter pulse being now by an order of magnitude stronger than before, the restored pulse is constructed of parts freed in those intervals of the Rabi period in which the coherence $\sigma_{bc}$ differs significantly from zero. The Rabi oscillations between the levels c and d are thus imposed on the leaving signal pulse (see Fig. 3).

The Rabi oscillations due to the additional control field may be used not only to destroy in a reversible way the atomic coherence $\sigma_{ab}$ necessary to release the pulse 1 (case (a)) but also to create a new coherence $\sigma_{ad}$ which can be converted into a new pulse 3 (case (b)). In Fig. 4 we show the shapes of the two signal pulses 1 and 3 for different values of the area of the control pulse 4, switched on and off in the storage stage. If the area is a multiple of $\pi$ only the pulse 1 is released, with its sign being changed in the case of an odd multiple. If the area is an odd multiple of $\pi/2$ only the pulse 3 appears, alternatively with a changed sign. For pulse areas being not a particular multiple of $\pi/2$ both pulses 1 and 3 are released, their heights being under control.

As in the previous papers the problem can be analyzed in terms of dark state polaritons. Such an analysis allows one to describe the whole process of light storing in a single Λ system in terms of a shape preserving solution of the Maxwell-Bloch equations (Eqs (1,2)), the components of which, i.e. the signal field and the atomic coherence, adiabatically turn one into another. In the case of a four-level system the evolution could not in general be fully adiabatic, which means that bright-state polaritons must appear at some stage of the process and are later damped [12]. Thus the dark-state polaritons at the initial and final stages are not identical.

The approach of Ref. [12] generalized in our case (b) leads to the following results. One can attempt to solve Eqs (1) and (2) perturbatively (as concerns signal fields), in an adiabatic and relaxationless approximation. The stage of light stopping occurs as in the case of a three-level system: the polariton solution

$$\Psi = \frac{\Omega_2 \epsilon_1 + \frac{2 \omega_1 N d_1}{\epsilon_0 \sigma_{bc}} \sigma_{bc}}{\sqrt{\Omega_2^2 + \frac{2 \omega_1 N d_1^2}{\epsilon_0 \sigma_{bc}}} \left| d_2 \right|}$$

(3)

describes an adiabatic conversion of the pulse 1 into the coherence $\sigma_{bc}$ and the sign correction
guarantees that for large $|\Omega|$ we get $\psi = \epsilon_1$ (we assume that $\epsilon_2 > 0$). After pulse stopping, say at time instant $t_1$, the control pulse 4 is switched on and is present up to the time instant $t_2$. As a consequence the density matrix evolves and at $t = t_2$ we obtain in the case (b) \[ \sigma_{bb} = \cos^2 \theta, \quad \sigma_{dd} = \sin^2 \theta, \quad \sigma_{bd} = -\sin \theta \cos \theta, \quad \sigma_{bc} = \sigma_{bc}(t_1) \cos \theta, \quad \sigma_{dc} = -\sigma_{bc}(t_1) \sin \theta, \]
where \[ \theta = \frac{U(t_2 - t_1)}{2\hbar} \] is the pulse area.

At time instant $t_3$ ($t_3 > t_2$) the control field 2 is switched on in order to release the trapped pulse. In the assumed approximations $\sigma_{bb}, \sigma_{dd}$ and $\sigma_{bd}$ do not change any more. The pulses 1 and 3 satisfy the equations

\[ \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \right) \epsilon_j = \frac{1}{\Omega_2} \frac{\partial}{\partial \Omega_2} \sum_k M_{jk} \epsilon_k, \]

where $j, k = 1, 3$ and \[ M_{11} = -\frac{2N\omega_1^2}{\epsilon_0 \hbar} \cos^2 \theta, \quad M_{13} = \frac{2N\omega_1 \omega_2 d_3}{\epsilon_0 \hbar} \sin \theta \cos \theta, \quad M_{33} = -\frac{2N\omega_2 \omega_3 d_1}{\epsilon_0 \hbar} \sin^2 \theta. \]

Eqs (4) can be decoupled by a linear transformation. One of the solutions can be shown, similarly as in previous papers, to be a shape-preserving solution traveling with a time-dependent velocity

\[ v(t) = \frac{1}{1 + \frac{1}{2N(d_1^2 \omega_1 \cos^2 \theta + d_3^2 \omega_3 \sin^2 \theta)}}. \]

(The other solution is zero due to the initial conditions at $t = t_2$.) The polariton, being a combination of two fields and two coherences, has the form

\[ \Psi = \frac{d_1}{\sqrt{d_1^2 \omega_1 \cos^2 \theta + d_3^2 \omega_3 \sin^2 \theta}} \frac{e^{i\Omega_1 t}}{\sqrt{1 + \frac{2N}{\epsilon_0 \hbar \Pi_2^2}(d_1^2 \omega_1 \cos^2 \theta + d_3^2 \omega_3 \sin^2 \theta)}} |d_1| \times \left[ \frac{d_1 \epsilon_1 \cos \theta - d_3 \epsilon_3 \sin \theta}{d_1^2 \omega_1 \cos^2 \theta + d_3^2 \omega_3 \sin^2 \theta} + \frac{2N}{\epsilon_0 \Omega_2} (\sigma_{bc} \cos \theta - \sigma_{dc} \sin \theta) \right]. \]

The solution (6) has been normalized so that it is equal to the solution (3) at $t = t_2$, i.e. $\Psi(t_2) = \sqrt{2N/\epsilon_0} [\sigma_{bc}(t_2) \cos \theta - \sigma_{dc}(t_2) \sin \theta]$. However, the final form of the polariton is, again for large $\epsilon_2$, a combination of the signal fields with the coefficients different from $\cos \theta$ and $\sin \theta$, except in the case of $d_1 = d_3$ and $\omega_1 = \omega_3$. In particular for $\theta = -\frac{\pi}{2}$ one finds that $\Psi \rightarrow \sqrt{2N/\epsilon_3} \epsilon_3 d_3/d_1 d_3$ instead of $\epsilon_j \frac{d_j d_3}{|d_1 d_3|}$. As described in detail in our previous paper [12], this means that the evolution cannot be fully adiabatic and bright state polaritons (which are later damped) must be invoked. In the case (a) the nonadiabatic element of the evolution is even more conspicuous: only the part of the transformed coherence, namely that proportional to $\cos \theta$ is active in the release stage and turns adiabatically, being a shape-preserving solution, into $\cos \theta \epsilon_1$. The other part of the excitation, namely that proportional to $\sin \theta$, survives the release stage inside the medium unless relaxations in the states $b$ and $d$ are taken into account.

In summary, we have demonstrated that a modification of the atomic coherence due to a stopped light pulse can be used as a new way of changing the properties of the released light. One can in particular release two pulses of different frequencies or polarizations, with their envelopes being regulated in a continuous way. This may serve as a kind of a switch which allows one e.g., to continuously steer the information by sending it to particular channels or to temporarily hide it.
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FIGURES

FIG. 1. Level and coupling schemes; the indices 1 and 3 refer to signal fields and 2 and 4 - to control fields.

FIG. 2. The field amplitude of the released pulse 1 as a function of the local time $t'$ for different pulse areas of the control field 4 in the case (a): curve 1: 0, curve 2: $\frac{\pi}{6}$, curve 3: $\frac{\pi}{4}$, curve 4: $\frac{\pi}{3}$, curve 5: $\frac{\pi}{2}$, curve 6: $\frac{2\pi}{3}$, curve 7: $\pi$.

FIG. 3. The field amplitude of the released pulse 1 as a function of the local time $t'$ in the case (a) for overlapping pulses 2 and 4: curve 1: part of the pulse transmitted before light storing, curve 1a: the released pulse in the absence of the additional coupling field, curve 1b: the released pulse in the presence of the additional coupling field, curve 2: the control field 2, curve 4: the additional coupling field 4. The values of the fields 2 and 4 have been reduced by the factors of 40 and 20, respectively.

FIG. 4. The amplitudes (in $10^{-11}$ a.u.) of the signal fields 1: full line, and 3: dashed line, as functions of the local time $t'$ (in $10^{11}$ a.u.) in the case (b) for different values of the pulse area of the interaction 4.
field amplitude (10^{-11} a.u.)
