Recovering 3D clustering information with angular correlations

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ABSTRACT

We study how to recover the full 3D clustering information of $P(\vec{k}, z)$, including redshift space distortions (RSD), from 2D tomography using the angular auto and cross spectra of different redshift bins $C_\ell(z, z')$. We focus on quasilinear scales where the minimum scale $\lambda_{\text{min}}$ or corresponding maximum wavenumber $k_{\text{max}} = 2\pi/\lambda_{\text{min}}$ is targeted to be between $k_{\text{max}} = \{0.05 - 0.2\} \ h\text{Mpc}^{-1}$. For spectroscopic surveys, we find that we can recover the full 3D clustering information when the redshift bin width $\Delta z$ used in the 2D tomography is similar to the targeted minimum scale, i.e. $\Delta z \approx \{0.6 - 0.8\} \lambda_{\text{min}}H(z)/c$ which corresponds to $\Delta z \approx 0.01 - 0.05$ for $z < 1$. This value of $\Delta z$ is optimal in the sense that larger values of $\Delta z$ lose information, while smaller values violate our minimum scale requirement. For a narrow-band photometric survey, with photo-z error $\sigma_z = 0.004$, we find almost identical results to the spectroscopic survey because the photo-z error is smaller than the optimal bin width $\sigma_z < \Delta z$. For a typical broad-band photometric survey with $\sigma_z = 0.1$, we have that $\sigma_z > \Delta z$ and most radial information is intrinsically lost. The remaining information can be recovered from the 2D tomography if we use $\Delta z \approx 2\sigma_z$. While 3D and 2D analysis are shown here to be equivalent, the advantage of using angular positions and redshifts is that we do not need a fiducial cosmology to convert to 3D coordinates. This avoids assumptions and marginalization over the fiducial model. In addition, it becomes straightforward to combine RSD, clustering and weak lensing in 2D space.

Key words: galaxy clustering; angular correlations; photometric redshift surveys

1 INTRODUCTION

In recent years, galaxy redshift surveys have provided new information about the cosmological model of our Universe, in pace with precision cosmology from other probes like CMB and type Ia Supernovae. We are now entering exciting times for cosmology, when surveys will go deeper and wider with increasing number of galaxy positions in each catalogue. With deep surveys we can use weak lensing information to improve constraints on cosmological parameters and also trace directly the dark matter distribution at large scales. Theoretical analysis of weak lensing (WL) is usually made through a 2D (angular) analysis of the measured galaxy shear maps. Future surveys will have less shot noise, allowing for more freedom in how we break the sample into multiple redshift shells, so that galaxy correlations can also be measured in and between shells. In doing angular correlations, we are projecting all the radial information within each redshift bin. But if we are able to use very thin radial shells, we can maybe recover the radial information using the angular cross-correlations between all the redshift bins (see Montanari & Durrer 2012 for a related idea). This is what we want to investigate in this paper.

This goal is also connected to recent studies of galaxy surveys using a combination of redshift space distortions (RSD) and WL galaxy-shear and shear-shear correlations. These allow measurements of galaxy bias and the breaking of degeneracies between growth history and cosmic history, as has been recently proposed (Gaztañaga et al. 2012; Cai & Bernstein 2012). RSD are usually studied in 3D, which complicates a joint analysis with WL which is usually 2D.
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(see Kitching et al. 2011 for a comparative analysis with 3D cosmic shear). If we could study RSD in 2D without loss of information, then it would be possible to do a joint analysis of both probes using only angular correlations with the corresponding simplification in the covariance analysis.

Observations directly probe redshifts and angular coordinates on the sky. Doing an angular analysis therefore does not require any prior knowledge of the cosmological model, while for doing 3D analysis we have to assume a fiducial cosmology to convert to comoving spatial coordinates. This then requires modelling the Alcock-Paczynski effect when fitting different models to our observables (Alcock C., Paczynski B. 1979). As the transformation is redshift dependent one has to make sure that this procedure is not biasing the parameter constraints. If the theoretical prediction for the correlations in angle and redshift can be calculated for each model, an angular analysis relating directly to the observables is much more direct.

The final goal of this paper is to analyze the bin optimization that allows us to recover the 3D constraints on clustering using a 2D tomographic approach. We have studied this in the framework of several idealized surveys: a spectroscopic survey in a redshift range similar to SDSS redshift range; a survey with photometric redshifts from a camera with narrow-band filters like the camera that Physics of the range; a survey with photometric redshifts from a camera tomographic survey in a redshift range similar to SDSS redshift clustering using a 2D tomographic approach. We have studied the Alcock-Paczynski effect when fitting different models to our observables (Alcock C., Paczynski B. 1979). As the transformation is redshift dependent one has to make sure that this procedure is not biasing the parameter constraints. If the theoretical prediction for the correlations in angle and redshift can be calculated for each model, an angular analysis relating directly to the observables is much more direct.

The goal of this paper is to show under which conditions, if any, one can recover the full 3D clustering information from a tomography study. By this we mean a combination of all the auto and cross angular spectra after the survey volume has been divided in a set of consecutive redshift bins. The angular spectra within each bin will include information mainly from transverse modes, while cross correlations between different shells account for radial modes with scales comparable to the bin separation.

We investigate this idea in the context of a spectroscopic survey as well as two photometric surveys with different accuracies in the redshift determination. In what follows we describe these “typical” surveys, the assumed galaxy samples, the observables considered and the figures of merit used to compare 3D and 2D tomography results.

2 METHODOLOGY

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Table 1. Comoving galaxy number densities at $z = 0.55$ assumed in this paper for the spectroscopic and narrow-band photometric surveys. Case 1 corresponds to a low shot noise level ($nP_{gal} = 2\%$, where $P_{gal}$ is the monopole of the galaxy spectrum at $z = 0.55$ and $k = 0.1 h \text{Mpc}^{-1}$) while Case 2 corresponds to a high shot noise level ($nP_{gal} = 10\%$).

| Case          | $n(r) (h^3 \text{Mpc}^{-3})$ |
|--------------|-------------------------------|
| Low Shot-Noise | $3.14 \times 10^{-3}$          |
| High Shot-Noise | $6.89 \times 10^{-4}$         |

Throughout the paper we used CAMB sources (Challinor & Lewis 2011) to compute the matter 3D power spectra as well as the angular power spectra, including cross correlations between radial bins.

2.1 Fiducial surveys and galaxy samples

In this section we describe our fiducial surveys and galaxy samples. We characterize them by a redshift range, a given accuracy of redshift measurements, a galaxy redshift distribution and bias.

In all cases we assume a full sky coverage. In ideal conditions this implies that the covariance matrix of observables such as $C_\ell$ is diagonal in $\ell$ (but notice that this assumption is not expected to change the conclusions of this paper). In all three surveys the overall redshift distribution of galaxies per degree$^2$ is taken as,

$$
\frac{dN}{dzd\Omega} = N_{gal} \left( \frac{z}{0.55} \right)^2 e^{-\left(\frac{z}{0.55}\right)^{1.5}}
$$

(1)

which is typical of a flux-limited sample with a magnitude cut at $i_{AB} < 24$. In Eq. $N_{gal}$ is a normalization related to the total number of galaxies per square degree under consideration.

2.1.1 Spectroscopic survey

Our benchmark spectroscopic survey has radial positions given by true redshifts (i.e. $\sigma_z = 0$ in the formulation below) and a redshift range $0.45 < z < 0.65$. Hence for the 2D tomography of this survey we use top hat bins to compute angular power spectra. In Table 2 we show the different bin configurations considered, characterized by the number of bins in which we divide the survey volume and their width. Provided with the narrow redshift range we can assume that the bias does not evolve, hence we take $b = 2$ throughout.

Lastly we discuss two cases for this survey, one where shot-noise is non-negligible and another where it is a subdominant source of error. These cases are detailed in Table 1 and for the redshift range under consideration imply 9M and 40M galaxies respectively (assuming full sky surveys).

1 www.pausurvey
2 www.darkenergysurvey.org
3 camb.info/sources
4 To satisfy differentiability requirements at the edges we use in practice $\phi(z) \propto \exp \left[-((z-\bar{z})/(\Delta z/2))^2\right]$, where $\bar{z}$ is the mean redshift of the bin and $\Delta z$ the full width.
Figure 1. Top panel shows the redshift distribution in the spectroscopic and narrow band photo-z survey (violet). For the narrow band case we show how the true redshift distributions given by Eq. (1) look like if we divide the volume in eight consecutive redshift bins. Bottom panel shows the same but for a broad band photometric survey divided in five bins.

2.1.2 Narrow Band Photometric survey

This case intends to be representative of a configuration such as the one proposed for the PAU survey where a set of narrow band filters is expected to deliver “low-resolution” spectra in a redshift range actually broader than the one considered here (Benítez et al. 2009; Gaztañaga et al. 2012). Hence our narrow-band photo-z survey has accurate photometric redshifts of \( \sigma_z = 0.004 \), in the same redshift range of the spectroscopic case (0.45 < z < 0.65). The bias \( b = 2 \) and the shot-noise cases considered match those of Sec. 2.1.1 (and are given in Table 1).

In turn the bin configurations assumed for the 2D tomography are also the same as for the spectroscopic survey given in Table 2 but with bin limits that now refer to photometric redshifts. Thus the true redshift distribution of galaxies in each bin is no longer a top hat, but rather has a small overlap with the nearest neighbouring bins due to the photo-z error, as described in Eq. (18) below. In the top panel of Fig. 1 we show this effect for the particular case of 8 bins.

2.1.3 Broad Band Photometric survey

On the other hand we consider a photometric survey that uses broad-band filters such as DES, Pan-Starrs or the future imaging component of Euclid. These surveys are expected to achieve photometric redshift estimates with accuracies \( \sigma_z \sim 5\%/(1 + z) \) (Banerji et al. 2008; Ross et al. 2011). In what follows we do not consider a possible redshift evolution of the photometric error but instead assume a conservative value of \( \sigma_z = 0.1 \).

Typically optical photo-z surveys are fainter and sample a much larger number of galaxies than spectroscopic ones, hence we assume a broader redshift range, 0.4 < z < 1.4, and only a low shot-noise case as given in Table 1. For the redshift range assumed this implies \( \sim 150 \times 10^6 \) galaxies. Table 3 show the bin configurations we have considered for this case. While in the previous cases we have assumed the bias is constant with redshift (because of the narrow redshift range), for the broadband photometric survey we introduce an evolution following (Fry 1996),

\[
b(z) = 1 + (b_\star - 1) \frac{D(z)}{D(z_\star)}
\]

where \( b_\star = 2 \) is the bias at \( z_\star = 1 \). In turn for the evolution of bias we have always assume the fiducial cosmology.

2.2 Spatial (3D) power spectrum

Since we are only interested in quasi-linear scales we assume the following simple model for the 3D galaxy power spectrum in redshift space,

\[
P_g(k, \mu, z) = \left(b + f \mu^2\right)^2 D^2(z) P_0(k) e^{-k^2 \sigma^2(z) \mu^2},
\]

where \(\sigma(z)\) is the peculiar velocity.

Table 2. Bin configurations used for the 2D tomography in the case of the spectroscopic and the narrow band photometric survey in a redshift range of 0.45 < z < 0.65. We show the number of radial bins and their range of widths in redshift and comoving distance.

| Number of bins | \( \Delta z \) | \( \Delta r (h^{-1}\text{Mpc}) \) |
|---------------|---------------|------------------|
| 1             | 0.20          | 468              |
| 4             | 0.05          | 113 - 122        |
| 8             | 0.025         | 56 - 61          |
| 16            | 0.0125        | 28 - 31          |
| 20            | 0.010         | 22 - 25          |

5 www.darkenergysurvey.org
6 pan-starrs.ifa.hawaii.edu
7 www.euclid-imaging.net

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where \( P_0 \) is the linear spectrum at \( z = 0 \) (properly normalized), \( D(z) \) is the linear growth factor and the remaining amplitude depends on the bias \( b(z) \) and the linear growth rate \( f(z) \equiv d\ln D/d\ln a \). The Gaussian cut-off accounts for the fact that the radial information might be diluted due to photometric redshift errors \( \sigma_z \). In Eq. (3) this redshift error propagates to scales through \( \sigma_z(z) = c\sigma_z/H(z) \). Notice that \( \sigma_z \) depends also on the cosmic history. This should be taken into account when constraining relevant cosmological parameters (e.g. \( \Omega_m \)).

For a spatial analysis the measured 3D power spectrum depends on the cosmological model assumed to convert redshift and angles to distances. Hence for every model to be tested against the data one must perform a new measurement. This process is very costly. Instead one can choose a reference cosmological model where the measurement is done once, and then transform the model prediction to this reference frame (Alcock C., Paczynski B. 1979).

Let us call \( P^{\text{obs}}(k, \mu) \) the power spectrum measured in the reference cosmology and \( P^{\text{mod}}(\tilde{k}, \tilde{\mu}) \) the model prediction at the point in cosmological parameter space being tested. The transformation of distances and angles from the cosmological model being tested \( (\tilde{k}, \tilde{\mu}) \) to those in the reference model \((k, \mu)\) is done through the scaling factors

\[
\begin{align*}
c_{||} &= \frac{H(z)}{H^{\text{mod}}(z)}; \\
c_{\perp} &= \frac{d_A^{\text{mod}}(z)}{d_A(z)},
\end{align*}
\]

as \( \tilde{k}_|| = k_||/c_|| \) and \( \tilde{k}_\perp = k_\perp/c_\perp \), where || indicates modes parallel to line of sight and \( \perp \) perpendicular. The Hubble parameter and the angular diameter distances are given by

\[
H(z) = 100h\sqrt{\Omega_m(1+z)^3 + \Omega_D E(1+z)^{\gamma(1+w)}}
\]

\[
d_A(z) = \int_0^z \frac{c \, dz}{H(z)}
\]

From the above one trivially finds,

\[
\begin{align*}
\tilde{k} &= k\sqrt{(1 - \mu^2)c_\perp^2 + \mu^2c_||^2} \\
\tilde{\mu} &= \mu c_\perp^{-1}\sqrt{(1 - \mu^2)c_\perp^2 + \mu^2c_||^2}.
\end{align*}
\]

In addition the power spectrum is sensitive to the volume element. Thus we must re-scale \( P^{\text{mod}} \) by the differential volume element with respect to the reference cosmology : \( c_\perp^2 \). Lastly, following (Tegmark 1997) and (Seo et al. 2003) we construct the \( \chi^2 \) for each radial bin \( i \) as,

\[
\chi^2_{3D}(i) = \int_{r_{\min}^{\text{mod}(i)}}^{r_{\max}^{\text{mod}(i)}} \frac{d^2r}{4\pi^2} \left( n(r) - \bar{n}(r) \right)^2 \]

\[
- \frac{1}{c_{||}c_{\perp}^3} P^{\text{mod}}(\tilde{k}, \tilde{\mu}, z_i)
\]

where \( c_{||}c_{\perp}^3 \) is defined for every bin \( i \) according to,

\[
\text{Cov}^{-1}(k, \mu) = \int_{r_{\min}^{\text{mod}(i)}}^{r_{\max}^{\text{mod}(i)}} d^3r \left( \frac{n(r)}{1 + n(r)P^{\text{obs}}(k, \mu, z_i)} \right)^2.
\]

This is what the covariance of the power spectrum is accounted for, which we assume to be diagonal in \( k \). It has contributions from both sample variance and shot noise. In Eqs. (9)(10) \( P^{\text{obs}} \) is the measured spectra in the chosen reference cosmology which we take as our fiducial cosmological model introduced in Sec. (2.4).

For the spectroscopic survey we assume that bins are uncorrelated. Thus the total \( \chi^2 \) is given by,

\[
\chi^2_{3D} = \sum_i \chi^2_{3D}(i),
\]

where the sum runs over all the bins considered.

### 2.3 Angular (2D) power spectrum

In our 2D analysis we consider the exact computation of the angular power spectrum of projected overdensities in a radial shell,

\[
C^{\ell}_\ell = \frac{2}{\pi} \int dk \, k^2 P_0(k) \left( \Psi^{\ell}_\ell(k) + \Psi^{\ell\perp}_\ell(k) \right)^2
\]

where

\[
\Psi^{\ell}_\ell(k) = \int dz \, \phi(z) b(z) D(z) j_\ell(kr(z))
\]

is the kernel function in real space and

\[
\Psi^{\ell\perp}_\ell(k) = \int dz \, \phi(z) f(z) D(z) \frac{2\ell^2 + 2\ell - 1}{(2\ell + 3)(2\ell - 1)} j_\ell(kr) - \frac{\ell - 1}{(2\ell - 1)(2\ell + 1)} j_{\ell - 2}(kr) - \frac{(\ell + 1)(\ell + 2)}{2\ell + 1} j_{\ell + 2}(kr)
\]

should be added to \( \Psi^{\ell}_\ell \) if we also include the linear Kaiser effect (Fisher, Scharf & Lahav 1994; Padmanabhan et al. 2007). In turn, photo-z effects are included through the radial selection function \( \phi(z) \), see below. This model then has the same assumptions as the 3D spectrum from Eq. (4).

Notice that in Eq. (12) we are only considering density and redshift space distortions terms. We are neglecting General Relativity (GR) effects as well as velocity and lensing terms, which are in our cases subdominant to the ones considered. Nonetheless the framework of angular auto and cross correlations could easily include these effects when required (Bonvin C., Durrer R. 2011; Challinor & Lewis 2011).
There are $N_z$ angular power spectrum, one per radial bin. But if we want to study all the clustering information we should add to our observables the $N_z(N_z - 1)/2$ cross-correlations between different redshift bins. These are given by
\[
C^{\ell^i}_{\ell^j} = \frac{2}{\pi} \int dk^2 P(k) \left( \Psi^i_{\ell^i}(k) + \Psi^{i*,\ell^i}(k) \right) \left( \Psi^j_{\ell^j}(k) + \Psi^{j*,\ell^j}(k) \right)
\]
(15)
Therefore, we are considering $N_z(N_z + 1)/2$ observable angular power spectra when reconstructing clustering information from tomography using $N_z$ bins.

2.3.2 Covariance matrix of angular power spectra

The covariance between angular spectra of redshift bins $ij$ and redshift bins $pq$ is given by
\[
\text{Cov}_{\ell^i,\ell^j}(p,q) = C^{\ell^i}_{\ell^j} \delta_{ij} \frac{1}{N_{\text{sub}}^2} \frac{1}{\Delta f_{\text{sky}}} \frac{1}{\Delta f_{\text{sky}}}
\]
(20)
where $N_{\text{sub}}$ is the number of galaxies per unit solid angle included in each radial bin. We define the observed power spectrum $C^{\ell^i}_{\ell^j}$ to account for the covariances and cross-covariances of auto and cross-correlations. In order to include observational noise we add to the auto-correlations in Eq. (15) a shot noise term
\[
\chi^2_{2D} = \sum_{\ell} \left( C^{\ell^i}_{\ell^j} - C^{\ell^i}_{\ell^j} \right) \delta_{ij} \frac{1}{N_{\text{sub}}^2} \frac{1}{\Delta f_{\text{sky}}} \frac{1}{\Delta f_{\text{sky}}}
\]
(21)
Notice that each term in the sum is the product of $N_z(N_z + 1)/2$-dimensional vectors $C^{ij}_{\ell}$ where $(ij)$ label all possible correlations of $N_z$ redshift bins, and a $N_z(N_z + 1)/2 \times N_z(N_z + 1)/2$ matrix corresponding to their (inverse) covariance.

Recall that we use the exact calculation of $C_\ell$ using CAMB sources, rather than the well-known Limber approximation (Limber 1954).

2.4 Nonlinear Scales

Both $\chi^2_{1D}$ and $\chi^2_{2D}$ depend sensibly on the maximum $k_{\text{max}}$ (or minimum scale) allowed in the analysis. In this paper, we chose to fix $k_{\text{max}}$ for all the bins and relate it to angular scales through $\ell_{\text{max}} = k_{\text{max}} r(z)$, where $\ell$ is the mean radial of the survey. In our fiducial cosmology we find $r(z) = 1471 h^{-1}$ Mpc in the redshift range $0.45 < z < 0.65$ and $r(z) = 2219 h^{-1}$ Mpc when $0.4 < z < 1.4$. In addition, we do not consider a dependence of $\ell_{\text{max}}$ with redshift (i.e. same $\ell_{\text{max}}$ for all redshift bins and their cross-correlation).

For the largest scale we use $k_{\text{min}} = 10^{-4} h^{-1}$ Mpc$^{-1}$ in the 3D analysis and $\ell_{\text{min}} = 2$ in the angular case. We have not found any significant dependence on $k_{\text{min}}$ or $\ell_{\text{min}}$.

2.5 Cosmological model and growth history

We assume the underlying cosmological model to be a flat ΛCDM universe with cosmological parameters $\omega = -1, h = 0.73, n_s = 0.95, \Omega_m = 0.24, \Omega_b = 0.042$ and $\sigma_8 = 0.755$. These parameters specify the cosmic history as well as the linear spectrum of fluctuations $P_{\theta}$. In turn, the growth rate can be well approximated by,
\[
f(z) \equiv \Omega_m(z)^{\gamma}
\]
(22)
and $\gamma = 0.545$ for ΛCDM. Consistently with this we obtain the growth history as
\[
D(z) \equiv \frac{\exp \left[ -\int_0^z \frac{f(z)}{1+z} dz \right]}{1+z}
\]
(23)
(where $D$ is normalized to unity today). The parameter $\gamma$ is usually employed as an effective way of characterizing modified gravity models that share the same cosmic history as GR but different growth history (Linder 2005). In part of
our analysis we focus in ΛCDM models and assume the GR value $\gamma = 0.545$. We deviate from this in Sec. 3.1.2 were we take $\gamma$ as a free parameter independent of redshift.

### 2.6 Likelihood analysis

In order to find constraints on cosmological models we integrate over the space of parameters defining the model, finding the value of the likelihood given by

$$-2 \log \mathcal{L} \propto \chi^2,$$

where we approximate the likelihood as Gaussian in the power spectra. Given the prior $\vartheta$ on the parameters one defines a probability for each sampled point $i$ in parameter space given by

$$P(i) \propto \mathcal{L}(i) \times \vartheta(i).$$

Finally, the mean and covariance matrix of the parameters is obtained from

$$\bar{p}_a = \sum_i P(i)p_a(i)$$

$$\Sigma(p_a,p_b) = \sum_i P(i)(p_a(i) - \bar{p}_a)(p_b(i) - \bar{p}_b).$$

where $p_a(i)$ is the value of the parameter $a$ in the grid point $i$, $\bar{p}_a$ is the mean value and $\Sigma(p_a,p_b)$ is the covariance between parameter $a$ and $b$. In Eqs. (26,27) $P(i)$ is normalized to unity over the grid. In addition we assume flat priors.

By construction the likelihood peaks at the fiducial value considered in the analysis. In all our studies we have chosen wide prior limits and therefore have found no dependence with these limits, and find the mean agrees with the fiducial value and the posteriors are quite Gaussian. Then in the case of only one nuisance parameter $p$, solving $\chi^2(p) - 1 = 0$ gives the same variance as likelihood sampling which allows us to speed up constraints considerably.

### 2.7 Figures of Merit

We consider two different analyses in order to compare 3D clustering with 2D tomography including all the auto and cross-correlations between redshift bins.

On the one hand, a bias fixed case, in which we only vary $\Omega_m$ (which affects both the shape and the amplitude of the power spectrum, and can be constrained as if we had a good knowledge of the bias prior to the analysis).

On the other hand we consider a bias free case, in which only $b$ and $\gamma$ (hence $f$ through Eq. (22)) are allowed to vary. This changes the (anisotropic) amplitude of the power spectrum, but not the underlying shape. This case is virtually the same as the standard analysis of redshift space distortions (White et al. 2009; Ross et al. 2011). For this case we had to adapt CAMB sources slightly, see the discussion in Appendix A.

To make the comparison quantitative we define a figure of merit (FoM) based on the covariance matrix $\Sigma$,

$$\text{FoM}_S = \frac{1}{\sqrt{\text{det}[\Sigma] S}},$$

where $S$ is the subspace of parameters we are interested in. If this subspace correspond to only one parameter, then the FoM is the inverse of the square root of the variance of the corresponding parameter. Thus we have the following cases,

- **FoM$_{\Omega_m}$**: Constraints on $\Omega_m$, with other parameters fixed at fiducial values.
- **FoM$_b$ and FoM$_\gamma$**: bias and $\gamma$ constraints when marginalized over $\gamma$ and bias, respectively. Other parameters are fixed at their fiducial values.
- **FoM$_{b\gamma}$**: Joint constraint on bias and $\gamma$, with other parameters fixed at fiducial values.

### 3 RESULTS

In this section we present the forecasts on $\Omega_m$ (bias fixed) and $b$ and $\gamma$ (bias free) from the measurement of either spatial or angular power spectra in the spectroscopic survey described in Sec. 2.1.1. Next we perform the bias fixed analysis in the narrow-band photometric survey with accurate photo-z discussed in Sec. 2.1.2 and the broad-band photometric survey defined in Sec. 2.1.3. Notice that despite photometric redshift errors large scale redshift space distortions can be measured in photometric surveys for binned data (Nock, Percival, & Ross 2010; Crocce, Cabré, & Gaztañaga 2011; Crocce et al. 2011), albeit with possible large error bars. Nonetheless for photometric surveys we concentrate the bias fixed case only.

All the analyses introduced above have been done for three different $k_{\text{max}} = \{0.05, 0.1, 0.2\}$ h Mpc$^{-1}$ (with corresponding $\ell_{\text{max}}$ as detailed in Sec. 2.1), and several bin configurations (see Table 2 and 3). We then study for which redshift bin width the information obtained using angular power spectra (quantified by the FoM of Sec. 2.1) are similar to those derived from the 3D power spectra.

#### 3.1 Spectroscopic redshifts

##### 3.1.1 Bias fixed case

Top panels of Fig. 2 show the FoM on $\Omega_m$ for different $k_{\text{max}}$ and $\ell_{\text{max}} = r(z)k_{\text{max}}$ as a function of the number of redshift bins $N_z$ in which we divide the full survey volume (see Table 2). Here dashed lines are results from fitting the 3D power spectrum according to Eqs. (9,11), while solid are from the 2D tomography including all the auto and cross correlations of bins, as in Eq. (21). The left (right) panel corresponds to the low (high) shot noise case as defined in Table 1.

As expected we find that the FoM increases for increasing $k_{\text{max}}$, $\ell_{\text{max}}$. Including more modes to the $\chi^2$ adds more information to our analysis and therefore results in better constraints. We also see that FoM$_{\Omega_m}$ from the 3D analysis only show a marginal dependence on the bin configuration. This is because the $\chi^2$ per redshift bin is roughly proportional to the volume of the redshift shell, see Eq. (21). Thus, increasing the number of bins at the expense of decreasing their volume keeps the FoM$_{\Omega_m}$ unchanged. We obtained the
same result for all the cases studied in this paper, as long as $P_g$ does not change abruptly with redshift. Thus from now on we will only refer to the 3D results in the whole survey.

This picture changes for the 2D tomography. Here the transverse information is fixed once $\ell_{\text{max}}$ is set ($2\ell + 1$ modes per $\ell$ value up to $\ell_{\text{max}}$). As we increase the number of narrower bins $N_z$ (with fixed total redshift range) we have several effects:

(i) Decreasing the number of galaxies per bin increases the shot noise per bin.

(ii) Increasing the number of bins so that they are thinner proportionally increases the signal auto power spectrum in each bin (there is less signal power suppression due to averaging along the radial direction).

(iii) When we split a wide redshift bin in two, we double the number of angular auto power spectra (transverse modes). This results in a larger FoM because the signal to noise in each bin remains nearly constant (the shot noise and signal in each bin both increase proportionately). This gain is illustrated by the dotted line in Fig. 2 which corresponds to the FoM produced by just using auto-correlations. For even narrower redshift bins the bins will become correlated and the gain will saturate, but this is not yet the case in our results as the redshift bins are still large compared to the clustering correlation length. In the limit in which all modes of interest are very small compared to shell thickness and they are statistically equivalent, for a single power spectrum amplitude parameter one expects

Figure 2. Spectroscopic survey & bias fixed. Top panels show $\text{FoM}_{\Omega_m}(2D)$ and $\text{FoM}_{\Omega_m}(3D)$ as a function of the number of bins in which we divide the survey for the analysis (left panel for a low shot-noise survey and right to a high shot noise). Dashed line corresponds to the 3D analysis, dotted to the 2D tomography using only auto-correlations and solid to auto plus cross correlations. Different colors correspond to different minimum scales, as detailed in the bottom panel inset labels. Bottom panels show the ratio of $\text{FoM}_{\Omega_m}(2D)$ (auto plus cross) and $\text{FoM}_{\Omega_m}(3D)$ as a function of the bin width $\Delta r$ normalized by the minimum scale assumed in the 3D analysis. Remarkably the recovered constraints from full tomography match the 3D ones for $\Delta r \sim \lambda_{3D}^{\text{min}}$ for all $\lambda_{3D}^{\text{min}}$. We note that different lines in the bottom panels are truncated differently merely because we have done the three $k_{\text{max}}$ cases down to the same minimum $\Delta r$. 

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Figure 3. Spectroscopic survey & bias free. Top panels show the combined $b - \gamma$ constraint resulting from 3D clustering (dashed lines) or 2D tomography considering as observables only auto correlations in redshift bins (dotted lines), or adding to this the cross-correlations (solid lines). The $x$-axis corresponds to the number of radial bins considered in the analysis. Different colors label different minimum scales assumed (same values and labels as in Fig. 2). Middle and bottom correspond to individual $b$ or $\gamma$ constraints after marginalization over $\gamma$ or $b$ respectively. As for the bias fixed we find that 3D information can be recovered but now the role of radial modes is much more important because RSD (our bias free case) relies on the relative clustering amplitude of radial and transverse mode.

$\text{FoM} = 1/\sigma \propto \sqrt{N_z}$, as obtained in Fig. 2 for low $N_z$.

(iv) When we increase the number of narrower bins, we also include information of radial modes by adding the cross-correlation between different redshift bins (illustrated by the solid line in Fig. 2 that corresponds to the total FoM from auto plus cross-correlations). Note how adding the cross-correlations to the autocorrelations (solid lines in Fig. 2) only increases the FoM moderately as compared to the autocorrelation result (dotted line). This reflects the fact that there are fewer radial modes than transverse ones, while much of the $\Omega_m$ constraint comes from the shape of $P(k)$ that is isotropic.

(v) As shown in Fig. 2 the 2D FoM can exceed the 3D FoM. This happens because the 3D analysis is limited by construction to a maximum number of modes, given by $k_{max}$, while in 2D we only limit the analysis to $l < l_{max}$ and we can formally exceed the maximum number of narrow redshift bins, as explained in point (ii) and (iii) above. But in reality, these additional modes are not necessarily independent and they could well be in the non-linear regime, so it is not clear to what extent we can use them to increase the FoM. As we want to restrict our analysis to $k < k_{max}$ we should not use redshift bins that are smaller than $\lambda_{3D}^{min}$.

The bottom panels of Fig. 2 show the ratio of the 2D and 3D FoM's against the bin width (instead of $N_z$), now normalized by the minimum scale used in the 3D analysis $\lambda_{3D}^{min} = 2\pi/k_{max}$ (for three different $k_{max}$ as before). We find $\text{FoM}(2D) \sim \text{FoM}(3D)$ when $\lambda_{3D}^{min} \sim \Delta r$ for all $\lambda_{3D}^{min}$. More

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9 A similar effect can be seen on Figs. 8 and 9 of (Ross et al. 2011) in the context of RSD constraints in a broad band photometric survey. In their Fig. 8 the constraint in $f\sigma_8$ saturates when they consider only one redshift bin. However the error on $f\sigma_8$ from the combined measurements on several bins does not saturate (Fig 9).
Figure 4. Spectroscopic survey & bias free. Top panels show the ratio between combined FoM_{b\gamma} (2D) (auto plus cross correlations) and FoM_{b\gamma} (3D) with respect to $\lambda_{\text{3D}}^{\text{min}} = 2\pi/k_{\text{max}}$, normalized by the mean width of the redshift bins $\Delta r$ in the analysis. Middle and bottom panels show the same but for ratios of FoM_b and FoM_{\gamma}, respectively. We show results for Low Shot Noise and High Shot Noise in left and right panels, respectively. To reconstruct RSD information in practice, one needs bins slightly smaller than $\lambda_{\text{3D}}^{\text{min}}$. Precisely:

$$\Delta r = c\Delta z/H(z) \approx 0.8 \lambda_{\text{3D}}^{\text{min}}.$$  \hspace{1cm} (29)

Basically this means that the 3D clustering information is recovered once the binning is such that the radial bin width equals the minimum scale probed in the 3D analysis. In this case one is able to constrain the parameters without loss of information compared to a three dimensional analysis, though the actual range of scales around $k_{\text{max}}$ that are used in the 2D analysis may be slightly different from the ones used in the 3D analysis.

3.1.2 Bias free case

We now turn to the bias free case where we assume we know perfectly the shape of the power spectrum so that all the parameters are fixed at their fiducial values except the bias $b$ and the growth index $\gamma$.

In Fig. 4 we plot the combined FoM obtained for bias $b$ and growth index $\gamma$, and the FoM of each of these 2 parameters marginalized over the other, as a function of the number

$\lambda_{\text{3D}}^{\text{min}}/\Delta r$. 

Note that as mentioned in point (v) above, we can only really trust our results for the 2D FoM up to the limit in which they are equal or smaller than the 3D FoM, i.e., in the range in which the width of redshift bins is greater or similar than $\lambda_{\text{3D}}^{\text{min}}$. To use the smaller scales we first need to explore to what extent we can model the non-linear 2D clustering to improve the FoM. We are currently investigating this issue (Asorey et al., in prep.)

Lastly, note that including shot noise does degrade the FoM as shown in the right panel of Fig. 2. However this does not change the conclusions above.
of redshift bins considered in the analysis (for a fixed survey redshift range $0.45 < z < 0.65$). As in Fig. 3 dashed line corresponds to the 3D analysis, dotted line to the 2D tomography using only auto-correlations\(^{10}\) and solid line to the full 2D case where we add auto and cross angular correlations.

We find a similar trend for the evolution of the different FoM of the $\gamma$ and $b$ parameters (either combined or marginalized) than when varying $\Omega_m$. Constraints given by spatial power spectrum are stable, while constraints from projected power spectrum in the bins increases with the number of bins in which we divide the survey. However there is a substantial difference in regards to the contribution of radial modes. Now the contribution of cross-correlations is very large (compare solid to dotted lines in the left panel of Fig. 19). In fact, without cross-correlations we do not recover all the 3D information. This is because redshift space distortion information (i.e. our bias free case) is based in the relative clustering amplitude of modes parallel and transverse to the line of sight. The contribution from radial modes is much more evident for the $\gamma$ constraint (FoM$\gamma$ and then FoM$\gamma b$) because $\gamma$ is basically what quantifies this relative clustering amplitude (in addition $f \equiv \Omega(z)$ depends on redshift while we assume bias does not).

As we have done with FoM$\Omega_m$, we show in Fig. 4 the dependence of the ratios between 2D and 3D FoM with respect to $\lambda^{3D}_{\min}/\Delta r$. We find that both analyses produce the same constraints when the mean redshift bin width is slightly smaller than $\lambda^{3D}_{\min}$ (and we use auto and cross 2D correlations). Comparing these results with the bias fixed case, it seems that for the RSD probe we need to extract more radial information. In this case:

$$\Delta r = c\Delta z / H(z) \simeq 0.6 \lambda^{3D}_{\min}$$

\(^{10}\) We note that we refer here to observables. The covariance of the auto-correlations does include cross-correlations of redshift bins, see Eq. (19).
as compared to 0.8 in Eq. (29). This means that we have to include more radial bins when developing the fit to angular correlations than when only fitting $\Omega_m$ if we want to match the constraints from 3D clustering. This in practice corresponds to using slightly narrower redshift bins. This may also result in more information being included from radial modes with $k > k_{max}$, though a detailed analysis of the implications of this is beyond the scope of the current paper.

### 3.2 Photometric redshifts

In this section we show how the results found in the previous section extend to the photometric surveys detailed in Sec. 2.1.2 and 2.1.3. For concreteness we will only consider the bias fixed study where all cosmological parameters are fixed at their fiducial values except for $\Omega_m$.

#### 3.2.1 Narrow-band photometric survey (PAU-like)

In top panels of Fig. 5 we show the $\Omega_m$ constraints (bias fixed case) from 3D and 2D analysis (dashed and solid lines respectively) in a narrow band photometric survey with $\sigma_z = 0.004$. In bottom panels we show how the ratio between 2D and 3D FoM depends on the ratio between the minimum scale of the radial shells and the mean comoving width of radial shells.

We find basically the same result as in the spectroscopic survey. Constraints from a projected or unprojected analysis are equivalent when the mean width of the radial shells (set by our binning strategy) is equal to the minimum scale considered in 3D analysis $\lambda_{\min}^{3D}$. The absolute value of each FoM is degraded with respect the FoM reached with an spectroscopic survey because photo-z errors dilute clustering in the radial direction. This broadens the selection function $s$ in the 2D analysis and introduces a cutoff already at quasi-linear scales in the 3D $P(k)$. In both cases the consequence is that signal to noise reduces and thus errors of observables degrade. But if we compare Fig. 2 and Fig. 5 we see that the spectroscopic survey and a photometric one with very accurate redshifts are almost indistinguishable in terms of bin width optimization.

#### 3.2.2 Broad-band photometric survey (DES-like)

We now consider a deep survey ($i_{AB} < 24$) with redshifts estimated by photometry with broadband filters ($\sigma_z = 0.1$), and use the full catalogue with $0.4 < z < 1.4$. We obtain the FoM for $\Omega_m$ shown in the top left panel of Fig. 6.

Now the large photo-z error removes most of the radial information, thus all FoM$_{\Omega_m}$ are degraded with respect to spectroscopic and narrow-band photometric surveys. In addition, we find that FoM$_{\Omega_m}$ saturates with the number of redshift bins included in the survey for every $k_{max}$. This ef-
fect is produced by the overlapping between true galaxy distributions at different bins induced by photo-z transitions.

We also find that the configuration in which spatial and projected analysis constrain \( \Omega_m \) equally corresponds to the same number of bins for all the \( k_{\text{max}} \) considered. Therefore, as we can see in bottom left panel of Fig. 5, the scale given by \( \lambda_{\text{min}}^D \) is not ruling the dependencies. Instead it is the scale of the photometric redshifts which is affecting both clustering analyses. This is shown in the right panel of Fig. 5 where we plot the ratio of figures of merit (2D vs. 3D) against a new scaling: \( \sigma_z/\Delta \sigma \). We find that for a DES-like case, with the assumption of \( \sigma_z = 0.1 \), one needs roughly 5 bins for the 2D tomography to optimally recover the 3D clustering information. This corresponds to:

\[
\Delta z \approx 2\sigma_z.
\]  
(31)

With a lower \( \sigma_z \), the number of bins will increase.

4 CONCLUSIONS

In this paper we have studied the redshift bin width that allows us to recover the full 3D clustering constraints from tomography of angular clustering (i.e. the combination of all the auto and cross correlations of redshift bins). We explore three surveys with different properties: a spectroscopic and a narrow band photometric survey in a redshift range 0.45 < \( z < 0.65 \), and a deeper broadband photometric survey that covers redshifts in the range 0.4 < \( z < 1.4 \). We have considered how well we can recover the shape of the power spectrum by allowing \( \Omega_m \) to be free and fixing the amplitude of clustering, including bias. We call this the bias fixed case. We have also explored how to recover the information from redshift space distortions (RSD), by measuring the anisotropic amplitude of the power spectrum allowing for both a free bias and a free growth index. This is the bias free case. We restrict our study to quasi-linear scales and we only consider scales above some minimum scale \( \lambda_{\text{min}}^D = 2\pi/k_{\text{max}} \), where \( k < k_{\text{max}} \) and \( k_{\text{max}} \) is either 0.05, 0.1 or 0.2 \( h \) Mpc\(^{-1}\). In angular space this corresponds to \( l < l_{\text{max}} \approx k_{\text{max}}r(z) \), where \( r(z) \) is the radial distance to the mean redshift bin.

The 3D analysis has almost no dependence on the number of redshift bins because radial modes are already included in each bin. In contrast the 2D tomographic analysis depends strongly on the number of bins (or equivalently on redshift bin widths), since broad bins average down transverse power on scales smaller than the bin width, and it is only by using multiple thin shells that radial modes are included.

For the bias fixed case in the spectroscopic survey we have found that we recover all the information with 2D tomography when the width of the redshift bins that we use to do the tomography is similar to the minimum scale used in the 3D observables, \( \lambda_{\text{min}}^D \). More precisely we find that the optimal bin width is (see Fig. 2 and Eq. (29)): \( \Delta \sigma = c\Delta z/H(z) \approx 0.8 \lambda_{\text{min}}^D \). In addition most of the 2D constraints come from autocorrelations. When studying RSD, i.e. in the bias free case, we see that if we want to recover the 3D constraints we need radial shells which are slightly smaller, i.e. \( \Delta \sigma \approx 0.6 \lambda_{\text{min}}^D \) (see Fig. 4), which means that we would need more bins than in the case in which we just want to measure the shape of \( P(k) \). In addition we find necessary to include in the observables the cross correlation between redshift bins. This is expected because in the RSD case we are comparing the clustering in radial and transverse direction to the light of sight: information from radial modes should be more important than in the case in which we just study information in the isotropic shape of the power spectrum. Also note how we can not recover the 3D information from RSD when we just use autocorrelations (see dotted line in Fig. 4).

We found that in the bias fixed case, the narrow-band photometric survey is almost equivalent to an spectroscopic survey, and we therefore reach the same conclusions with respect to the optimal bin width for the tomography of galaxy counts. In the case of a deeper broadband photometric survey we find that the typical uncertainty in photometric redshifts \( \sigma_z \) severely limits the accuracy of the radial information for both 3D and 2D cases. In this case the information recovery does not depend strongly on \( \lambda_{\text{min}}^D \), because this is smaller than the scale corresponding to the photometric redshift accuracy, i.e. \( c\sigma_z/H(z) > \lambda_{\text{min}}^D \). The optimal redshift bin width in this case is simply given by \( \Delta z \approx 2\sigma_z \).

For a redshift range 0.4 < \( z < 1.4 \) and \( \sigma_z = 0.1 \) (DES-like survey) we find that we will need only 5 redshift bins to constrain \( \Omega_m \) using tomography with the similar precision than a full 3D analysis of the survey. In comparison, for a PAU-like survey with \( \sigma_z \approx 0.004 \) and \( k_{\text{max}} = 1 \) we need about 44 redshift bins of width \( \Delta z \approx 0.023 \) each.

We conclude from our analysis that it seems possible to recover the full 3D clustering information, including RSD information, from 2D tomography. This has the disadvantage of needing a potentially large number of redshift bins, and correspondingly large covariance matrices between observables. But it has the great advantage of simplifying the combination with WL and of just using observed quantities, i.e. angles and redshifts, avoiding the use of a fiducial cosmology to convert angles and redshifts into 3D comoving coordinates. In practice, probably both types of analysis should be used to seek for consistency.

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and cross-correlation is given by:

\[
C_{ij}^{\ell} = b_i b_j C_{ij}^{(0)} + b_i f_i C_{ij}^{(2)} + f_i f_j C_{ij}^{(4)} 
\]

(A1)

\[
C_{ij}^{\ell} = b_i b_j C_{ij}^{(0)} + b_i f_i C_{ij}^{(2)} + b_j f_j C_{ij}^{(2)} + f_i f_j C_{ij}^{(4)} 
\]

(A2)

where \(b_i\) is the bias of the bin \(i\) and \(f_i\) is the growth rate given by Eq. (22), evaluated at the mean redshift of the bin \(i\). This factorization assumes \(f(z)\) does not vary much within the redshift range of the bin (neither \(b\)). We have tested this assumption using the exact CAMB sources evaluation or the reconstruction of Eqs. (A1,A2) and found an excellent match for the bin widths considered in this paper.

Using the observed \(C_{ij}^{\ell}\) and solving a linear set of equations using different values for \(b_i\) we can store the value of \(C_{ij}^{(2)}, C_{ij}^{(2)}(2), C_{ij}^{(4)}(2)\) and \(C_{ij}^{(4)}\). The values of \(C_{ij}^{(0)}(0)\) and \(C_{ij}^{(4)}(0)\) are obtaining by excluding RSD in \(C_{ij}^{\ell}\). Then, we sample \(b\) and \(\gamma\) space using these factors and the reconstruction given by Eqs. (A1) and (A2) obtaining \(C_{ij}^{\ell}\) in parameter space.

In the reconstruction we assume the underlying value of \(\Omega_m = 0.24\) given by our reference cosmology while the growth factor \(D(z)\) is included in the integrals that are contained in the cosmic history dependent factors \(C_{ij}^{(n)}\).

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APPENDIX A: MODIFYING CAMB SOURCES TO SAMPLE GROWTH RATE AND BIAS

In order to consider the bias free case we had to modify CAMB sources to accept as (independent) inputs bias and growth rate (parameterized through \(\gamma\) as in Eq (22)). In addition this case does not involve changes in the shape of the real space spectrum, thus one should be able to sample parameter space without the need to compute the transfer functions at each point of parameter space.

To fulfil these needs we have factorized the terms in our observables that depend on the cosmic history (for our reference cosmology) from those that depend on the bias \(b\) and growth index \(\gamma\). The factorization in the case of auto