Magnetic Field Induced Insulating Phases at Large $r_s$

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Exploring a backgated low density two-dimensional hole sample in the large $r_s$ regime we found a surprisingly rich phase diagram. At the highest densities, beside the $\nu = 1/3$, 2/3, and 2/5 fractional quantum Hall states, we observe both of the previously reported high field insulating and reentrant insulating phases. As the density is lowered, the reentrant insulating phase initially strengthens, then it unexpectedly starts weakening until it completely disappears. At the lowest densities the terminal quantum Hall state moves from $\nu = 1/3$ to $\nu = 1$. The intricate behavior of the insulating phases can be explained by a non-monotonic melting line in the $\nu$-$r_s$ phase space.

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A perpendicularly applied magnetic field ($B$) resolves the density of states of a two-dimensionally confined charged system into a set of discreet energies, called the Landau levels (LL). When all carriers fall into the lowest LL these systems have a variety of ground states. The most prominent of these are the fractional quantum Hall liquids (FQHL) which develop at certain fractional values of the LL filling factor $\nu$ along with the Wigner solid (WS) [1]. In the presence of disorder the condition for the development of the FQHL and the WS phases is thought to be the dominance of the carrier-carrier interaction over the carrier-disorder interaction.

The insulating phase at the highest $B$-fields found beyond $\nu = 1/5$ in early measurements of two-dimensional electron systems (2DES) has been interpreted as a magnetically induced WS [2]. Numerous properties of this high field insulating phase (HFIP) were found to be consistent with those of the WS [2]. Of these, perhaps the microwave resonances [4] constitute the most direct evidence that the HFIP is the long sought WS rather than a phase of singly localized particles. The insulating behavior at low frequencies can be understood as the pinning of the WS by the impurities present in the host crystal and the microwave resonances are the oscillation modes of the WS experiencing the pinning potential of these impurities [2]. The HFIP has also been found in two-dimensional hole systems (2DHS), but in the vicinity of the terminal filling $\nu = 1/3$ [2,4,5]. The difference between the 2DES and the 2DHS can be qualitatively understood in terms of the evolution of the boundary of the WS in the zero temperature limit, also referred to as the melting line, in the $\nu$-$r_s$ phase space. The $r_s$ parameter has been introduced in metal physics as a measure of the Coulomb interaction in units of the Fermi energy and for a perfectly 2D system it is expressed as $r_s = m^*e^2/(4\pi\hbar^2\sqrt{mp})$, with $m^*$ being the effective mass and $p$ the areal density of the charges. While to date the melting line lacks a full fledged theory, several of its properties are known. It is now well established that in the $r_s = 0$ limit the melting line starts at $\nu \simeq 1/6.5$ [8,10], it is believed that it monotonically moves towards higher $\nu$ as $r_s$ increases [10,11,12,13], and it extrapolates to $r_s \simeq 37$ in the $B = 0$ limit [14]. Due to the larger effective mass of holes in GaAs-AlGaAs structures at similar densities, $r_s$ is significantly larger than that of electrons and, from the behavior of the melting line, a larger terminal filling results.

Simultaneously with the observation of the HFIP, it was found that the insulating behavior persists even at $B$-fields below that of the terminal FQHL, in particular in the range of $1/5 < \nu < 2/9$ in 2DES [2] and $1/3 < \nu < 2/5$ in 2DHS [6]. This new insulating phase has been termed the reentrant insulating phase (RIP) and it has also been interpreted as being the WS [2,6]. Thus the terminal FQHL is sandwiched in between two WS phases, a property that is a result of the lowering of the energy of the terminal FQHL below that of the WS [2,6]. The RIP has been reported to strengthen with decreasing density in 2DHS [6]. This property is consistent with that of the WS since at lower densities the WS is expected to be more stable [10,11,12,13] resulting therefore in a stronger insulating behavior.

Several features of the $\nu$-$r_s$ phase diagram remain unelucidated. First, a recent study of a 2DHS having $r_s = 30$ hinted that there is no clear reentrant behavior at such large $r_s$ [12]. This result indicates a reversal of the earlier reported trend of strengthening of the WS with increasing $r_s$ [6] and it implies an intricate evolution of the RIP. Second, the shape of the melting line is unknown at very large $r_s$ where the terminal quantum Hall state is expected to move to a filling higher than $\nu = 1/3$. We note that these issues cannot be tackled by current theories. One of the reasons is that theories comparing the energies of the WS to that of FQHL [3,10] make predictions at fractional values of $\nu$ only, usually of the form of $1/m$ with $m$ an integer. In addition, the monotonic melting line obtained from a heuristic estimation [11] and from a simple Lindemann-type criterion [12,13], as well as
Each panel is labeled by the density in units of $10^{12}$ change of the vertical scale for the 4 largest densities.

In this Letter we have systematically explored the evolution of the insulating phases, i.e. the HFIP and the RIP, with density in the large $r_s$ limit. To this end we performed standard low frequency magnetoresistance measurements on a low density 2DHS that has been backgated. The holes are confined to a 30 nm wide quantum well in a Si doped GaAs/AlGaAs heterostructure grown on a (311)A substrate. With the gate grounded the density is $p = 1.63 \times 10^{10} \text{cm}^{-2}$ and the mobility along the [233] axis at 33 mK is $\mu = 0.8 \times 10^6 \text{cm}^2/\text{Vs}$. The gate allowed a continuous tuning of the density from 0.64 to 2.85 $\times 10^{10} \text{cm}^{-2}$. Using the recently reported linealy increasing effective mass with density [16], the density range above corresponds to $r_s$ between 20 and 36.

In Fig.1 several traces of the diagonal resistivity $\rho_{xx}$ versus $B$ are shown for a set of representative densities.

![FIG. 1: The longitudinal resistivity $\rho_{xx}$ in units of the quantum resistance $h/e^2$ versus $B$ at 33 mK. Arrows mark $\nu = 1/3$. Each panel is labeled by the density in units of $10^{10} \text{cm}^{-2}$ (upper number) and the value of $r_s$ (lower number). Note the change of the vertical scale for the 4 largest densities.](image)

![FIG. 2: The dependence on density of the peak resistivity $\rho_{xx}^{\text{max}}$ of the RIP at 33, 42, and 60 mK (panel a.), the zero $B$-field mobility $\mu$ (panel b.), and the resistivity minima $\rho_{xx}^{\text{min}}$ at $\nu = 1/3$ and 2/3 (panel c.) at 33 mK. Lines are guides to the eye.](image)

serve clearly developed $\nu = 1$ and 2 integer quantum Hall states and, with the exception of the lowest densities, the $\nu = 1/3$ and 2/3 fractional quantum Hall states. The steeply increasing $\rho_{xx}$ at the highest $B$ values signals the HFIP. At the highest density of $2.85 \times 10^{10} \text{cm}^{-2}$, similarly to the earlier studied 2DHS with larger densities [16], our sample displays the reentrant insulating behavior. The peak in $\rho_{xx}$ at $B = 3.3$ T of peakheight $\rho_{xx}^{\text{max}} \simeq 9.5h/e^2$, a value that largely exceeds $h/e^2$, is due to this reentrant behavior. As the density is decreased to $2.45 \times 10^{10} \text{cm}^{-2}$, $\rho_{xx}^{\text{max}}$ increases. This property is similar to that of the earlier measured 2DHS in the density range from 4 to $12 \times 10^{10} \text{cm}^{-2}$ and it has been interpreted as the strengthening of the WS phase with decreasing density. With further lowering of the density, however, the reentrant insulating peak unexpectedly decreases and at $p = 1.50 \times 10^{10} \text{cm}^{-2}$ is not present any more. This non-monotonic dependence of $\rho_{xx}^{\text{max}}$ can better be seen in Fig.2a. We find that $\rho_{xx}^{\text{max}}$ at 33 mK reaches its largest value of $370 \Omega/\Box$ in the vicinity of $p = 2.2 \times 10^{10} \text{cm}^{-2}$ and it vanishes at $p = 1.54 \times 10^{10} \text{cm}^{-2}$. At the intermediate densities of 1.50, 1.16, and $1.03 \times 10^{10} \text{cm}^{-2}$, while the $\nu = 1/3$ and 2/3 FQHLs are well developed, the RIP is clearly missing. These two FQHLs at the lowest densities of 0.83, 0.71, and $0.64 \times 10^{10} \text{cm}^{-2}$ show gradual weakening until they get buried in a strong insulating background.

We proceed now to the extraction of the phase diagram in the $\nu$-$r_s$ phase space. First we need to establish the
values of the $B$-field at which the insulating phases lie. One way to achieve this is to determine the crossing of the $\rho_{xx}$ versus $B$ traces taken at different $T$ \cite{17}. Our traces, however, do not have a clear crossing in the $T$-range accessed (not shown). In the absence of a critical $B$-field separating the insulating and FQHL phases, we use the $\rho_{xx} \geq h/e^2$ condition to delimit the insulating phases. Such a definition is consistent with the $T$-dependence of the traces we measure. Furthermore we note that a small error in determining the exact location of the onset of the insulating behavior has only a minor effect: we expect that the qualitative features of the phase diagram to be insensitive to the relaxation of the condition above. By performing the described procedure, we obtain the phase diagram shown in Fig.3. In addition to our data (circles), Fig.3 is completed at $r_s < 20$ with data from an earlier measurement of ours of a 2DHS with $r_s = 18.5$, from Ref.\cite{6} (squares), and from Ref.\cite{7} (triangles).

The obtained phase diagram is unexpectedly rich. Besides the FQHL at $\nu = 1/3$ and 2/5, the phase diagram is dominated by a tortuous boundary of the HFIP and RIP. For $r_s < 24$ both the RIP and the HFIP are present and in between them lies the $\nu = 1/3$ FQHL. The $\nu = 2/5$ FQHL extending up to $r_s \approx 25$ appears to be interrupted by the RIP. In the 20 < $r_s < 22$ range the boundary of the RIP on the higher $\nu$ side is described by an increasing $r_s$ with increasing $\nu$. This trend is consistent with the increase of the terminal filling with $r_s$ from 1/5 in 2DES to 1/3 in 2DHS \cite{2, 6}, with the earlier reported density dependence of the RIP \cite{6}, and with theoretical expectations. Indeed, predictions of the melting line using heuristic arguments \cite{11} from a simple Lindemann-type of melting criterion \cite{12, 13}, as well as from a more sophisticated incorporation of the mixing of the LL \cite{10, 12} yield an increasing $r_s$ with $\nu$. We note that the lowest $r_s$ values of Fig.3 the RIP is not present \cite{6, 7}. For $r_s$ between 22 and 24 we find a decreasing $r_s$ as a function of increasing $\nu$ and the RIP surprisingly disappears for $r_s > 24$. The collapse of the RIP correlates with the vanishing $\rho_{xx}^{\text{max}}$ of Fig.2a. This collapse is anomalous as it cannot be accounted for by existing theories. We note that the disappearance of the RIP close to $r_s = 24$ is not a finite $T$ effect. Indeed, Fig.2a shows that the density or the $r_s$ of this collapse is independent of $T$ and, furthermore, for $r_s > 24$ we observe neither the RIP nor any precursor of it.

In the 24 < $r_s < 32$ region of the phase diagram, while the RIP is absent, the HFIP and the $\nu = 1/3$ FQHL are well developed. At even larger $r_s$, there is a sliver of the phase diagram ranging from $r_s \approx 32$ to 33 with quite interesting properties. Here the $\nu = 1/3$ FQHL, which is expected to fully develop in the $T = 0$ limit, is intercalated between two insulating phases. The insulating phase at the higher filling factor, however, does not resemble the RIP present at $r_s < 24$. The difference is that this new insulating phase extends to much larger $\nu$, closer to $\nu = 1$, and the height and the $T$-dependence of the resistivity peak are both smaller than that of the RIP. At $r_s > 33$, the $\nu = 1/3$ FQHL cannot be supported any more and the terminal quantum Hall state moves to the integer value of $\nu = 1$. Such a $\rho_{xx}$ versus $B$ trace is shown in the first panel of Fig.1 where $p = 0.64 \times 10^{10}$ cm$^{-2}$.

From the phase diagram in the $\nu$-$r_s$ plane it seems that the RIP forms an island that is topologically disconnected from the HFIP. However, at a more careful inspection it becomes apparent that the phase boundary of the RIP that is not parallel to the $\nu = 1/3$ vertical line is a prolongation of the boundary of the HFIP. In other words, one can understand the phase diagram as originating from a single insulating phase that is bound by a continuous and non-monotonic S-shaped melting line spanning the entire $r_s$ range. This insulating phase is interrupted at $\nu = 1/3$ by the FQHL. In this interpretation, the RIP and the HFIP have the same origin. The melting of the WS at the S-shaped boundary is attributed to quantum fluctuations caused by the zero point motion \cite{3, 11, 13} while melting at the two boundaries parallel to the $\nu = 1/3$ vertical line, i.e. the phase boundary of the HFIP and that of the RIP on the lower $\nu$ side, is due to Laughlin correlations \cite{13}. We note that small variations of the phase diagram are expected to occur in samples of various disorder and finite layer thickness \cite{6} without a change in the topology of the phase diagram.

In the following we discuss four possibilities for the collapse of the RIP with decreasing density. The first

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{The boundary of the insulating phases (full symbols) and the $\nu = 1/3$ and 2/5 FQHL (open symbols) in the $\nu$-$r_s$ phase space. Continuous lines are guides to the eye. Dotted lines are loci of points obeying $E_c = \lambda \cdot h \omega_c$ (see text).}
\end{figure}
such possibility is related to the degradation of the sample quality as the density is lowered. Indeed, as shown in Fig.2b, the mobility $\mu$ of the 2DHS decreases with decreasing density. In an extreme case it is possible that the WS cannot be supported any more and a new insulating phase sets in. This new insulating phase could be another collective insulator or, more likely, a phase of singly localized particles. At the collapse of the RIP in the vicinity of $p = 1.54 \times 10^{10}$cm$^{-2}$, however, the mobility has a very gentle variation and $\rho_{xx}^{\text{max}} \ll \hbar/e^2$. We therefore think that such a scenario is unlikely. A second possibility is a periodic modulation of the melting line imposed by LL mixing. Such a modulation could arise every time the Coulomb energy $E_c$ encompasses an integer number of Landau levels $\hbar \omega_c$, i.e. $E_c = \lambda \cdot \hbar \omega_c$. [10]. The loci of points obeying the condition above are plotted on the phase diagram for several values of $\lambda$. Since the anomalous behavior of the melting line does not possess any periodicity as $\lambda$ is incremented, we conclude that the effect described above is not a good explanation either.

The third scenario involves a commensuration effect that is similar to the registry of Helium atoms with a graphite substrate [20]. If the potential the holes experience has a periodic spatial modulation of areal density of the minima of 2.2 $\times$ $10^{10}$cm$^{-2}$, this potential will favor the WS of the same density and the melting line will protrude to higher $\nu$. Since we cannot identify any periodicity of such a lengthscale in the host GaAs crystal, we discard this possibility as well. Finally, the fourth scenario relates to the composite-fermion WS interpretation of the insulating phases [21]. The Laughlin-Jastrow correlated trial wavefunction used is built on the single particle wavefunctions of the lowest LL. We note, however, that the RIP is beyond the lowest LL of the composite-fermions. If the wavefunction of the second LL is included with a weight, it could become energetically more favorable for the width of the wavefunction to spread out with decreasing density. We propose that the collapse of the RIP could be due to such a relaxation of the single particle wavefunction. Due to the spreading of the wavefunction there is less distance between the lattice sites of WS. This can lead to an increased zero point motion that melts the WS.

It is noteworthy to point out the behavior of the $\nu = 1/3$ FQHL near the collapse of the RIP at $p = 1.54 \times 10^{10}$cm$^{-2}$. As the the density is increased, at constant $T$ it is expected that FQHLs develop deeper resistivity minima $\rho_{xx}^{\text{min}}$. While $\rho_{xx}^{\text{min}}$ at $\nu = 2/3$ does have such a behavior, at $\nu = 1/3 \rho_{xx}^{\text{min}}$ has an unexpected increase between 1.47 and 1.62 $\times 10^{10}$cm$^{-2}$ (shown in Fig.2c). Since this anomalous behavior develops at the crossing of the S-shaped melting line described earlier and the $\nu = 1/3$ line, we think that it is a signature of the quantum fluctuations of the zero-point motion affecting the FQHL.

Our data implies, that the $r_s \simeq 33$ and $\nu = 1/3$ point is special. This point, denoted by $C$, is the endpoint of the $\nu = 1/3$ FQHL line and it is on the melting line of the solid as well. Thus, in this point there is an intriguing possibility of coexistence of the FQHL and the WS phases. This region of the phase diagram has recently been accessed in a slightly lower quality 2DHS [13] and the observed periodic modulation of $\rho_{xx}$ has been attributed to such a two-phase coexistence.

In conclusion, we have presented a thorough exploration of the phase diagram in the $\nu$-$r_s$ plane of two-dimensional holes in the lowest LL. We think that this diagram can be understood as originating from a monolithic insulating phase that is cut by the $\nu = 1/3$ FQHL into two disjoint phases: the HFIF and the RIP. The RIP appears to form an isolated island because of the non-monotonic shape of the melting line.

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