Research Article

Modeling of Solitary Wave Interaction with Curved Face Seawalls Using Numerical Method

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This paper considers the solitary wave interaction with seawalls of different curved geometries and sloped faces using numerical modeling. This interaction was simulated using the Finite Volume Method-Volume of Fraction (FVM-VOF) approach. To model the turbulent free-surface flow, coupled VOF and k-ε-RNG methods were used. To validate the model, the numerical results for a conventional sloping seawall were compared with the available experimental data. Then the interaction of solitary waves and seawalls with different sinusoidal, logarithmic, and parabolic functions front faces and linear slope was modeled. The results showed that for these seawalls in general by increasing the solitary wavelength, the wave reflection coefficient ($C_r$) increases. However, the wave run-up on seawalls demonstrates an oscillatory decrease. Furthermore, for parabolic walls in comparison to conventional linear sloping seawalls, the wave run-up and wave reflection increased by 4.1% and 4.7%. For sinusoidal walls, the wave run-up and wave reflection increased by 5% and 1.8%. For logarithmic walls, the wave run-up and wave reflection increased by 6.3% and decreased by 1.1%, respectively. This means that wave run-up on logarithmic walls is more than that of the sinusoidal, parabolic, and sloped walls. The simulation results revealed that normalized maximum run-up increases with an increase in normalized incident wave height for all types of curved walls.

1. Introduction

Seawalls are structures constructed almost parallel to the coastline to separate the land area from the sea region. They are used to protect the shoreline from erosion and reduce the risk of long-term effects of waves. The real effects of these structures on beaches can be estimated through the erosion process. After the waves break, a portion of the remaining energy will energize the bore that will run-up the face of the beach or sloped shore structure. Figure 1 depicts this process where run-up $R$ is the maximum height above the still water level to which water rises and $h_0$ is the water depth in front of the slope. These structures are only protecting their own vicinity and do not have any protective action on adjacent areas exposed to littoral drift. The front walls of these structures can be designed and constructed vertically or sloped in different shapes such as linear, stepped wise, or curved, although for performance purposes a combination of the above sections may be applied. Usually, the crest elevation is selected for these structures allowing a limited overtopping. It is possible to select this elevation so that no overtopping occurs even in storm conditions. However, such a high structure not only blocks the sea view but is also not economical.

Weber [1] proposed a conceptual design of a curved seawall with the combination of a parabolic and a circular arc which brings a smooth change in the direction of propagation from horizontal to vertical and vice versa to reduce the wave-induced pressures. Freeman and Le Mehaute [2] and Iwasaki and Togashi [3] improved the computational efficiency of this model. However, their predicted run-up values were not in good agreement with measured data. One of the leading studies in this area is the work of Synolakis [4], who provided analytical solutions and lab work in the field of wave run-up. Kobayashi et al. [5, 6] further estimated the run-up of broken periodic waves on rough slopes using friction coefficients of bedding. Kobayashi and Wurjanto [7] developed a numerical model...
to predict the decrease of wave reflection due to the increase
of wave overtopping on sloped coastal structures. Yeh et al.
[8] performed laboratory work on wave run-up on the
sloping bed and concluded that broken waves would move
faster on the dry surface. Zelt and Raichlen [9] found that
bed friction effects during the inundation phase were im-
portant for numerical simulations and experiments of soli-
tary wave run-up on a dry horizontal shore. Subramanya
and Grilli [10] proposed a very robust solution for the
calculation of run-up of solitary waves with an initial height
close to the limiting wave height. Murakami et al. [11]
proposed a new type of circular arc nonovertopping seawall
and measured the pressures and forces on the seawall due
to regular waves. Kano Lu and Synolakis [12] based on as-
ymptotic results concluded that solitary waves can interact
with piecewise linear topographies in a counterintuitive way.
It means that for a composite beach with the vertex below
the equilibrium surface, the run-up of solitary waves would
depend only on the slope closest to the shoreline. Kamikubo
et al. [13] investigated the characteristics of the curved
seawall and reproduced the fluid flow near the seawall
through numerical simulation using the finite volume
method (FVM). The run-up and run down of nonbreaking
and breaking solitary waves on a plane beach were inves-
tigated by solving the nonlinear shallow-water equations.
Numerical results of the free surface evolution, the shoreline
movement, the maximum run-up height, and the particle
velocity were compared with experimental data. Further-
more, variations of energy transformation and dissipation
with respect to the incident wave heights and slopes have
been discussed; see, e.g., Lin et al. [14] and Li and Raichlen
[15, 16]. Based on energy conservation, Li and Raichlen [17]
proposed an empirical formula to predict the maximum
run-up height of breaking solitary waves on a plane slope.
The reliability of the formula was confirmed by comparing
predictions with experimental data. Kobayashi and Karjadie
[18] developed numerical models for finite-amplitude,
shallow-water waves with arbitrary incident angles to predict
detailed wave motions on a slop in the vicinity of the still
waterline. Christakis et al. [19] developed and used a new
numerical model of wave dynamics based on volume of fluid
(VOF) techniques for wave impacts at coastal structures
during the critical period of impact and breaking. Hubbard
and Dodd [20] presented a 2D numerical model using an
upwind finite volume technique to simulate wave run-up
and overtopping. Allsop et al. [21] studied and validated
prediction methods for wave overtopping discharges and
velocities for steep, battered, composite, and vertical sea-
walls. Park et al. [22] developed a large eddy simulation
(LES) model by using a finite differential algorithm to in-
vestigate solitary wave run-up on a vertical wall, flow over a
broad-crested weir, and regular waves overtopping on a
sloping seawall. Nwogu and Demirbilek [23] presented the
results of a combined laboratory and numerical investiga-
tion into the role of infragravity motions in the wave run-up
process over fringing coral reefs. They developed a nu-
merical model based on the Boussinesq equations applied to
laboratory data and described the complex changes to the
wave spectrum over the reef flat due to nonlinear wave-wave
interactions and wave breaking as well as run-up at the
shoreline. Anada et al. [24] performed an experimental study
to measure the run-up and overtopping of three different
types of curved-front face seawall models. They also mea-
sured the wave dynamic pressures exerted on these seawalls.
Saelevik et al. [25] used particle image velocimetry (PIV) to
investigate the run-up of solitary waves at straight and
composite beaches with different inclinations. They com-
pared the results with numerical simulations using a Navier-
Stokes’ solver with zero viscosity. Didenkulova et al. [26]
studied the run-up of long solitary waves of different po-
larities on a beach in the case of composite bottom top-
graphy. They showed that nonlinear effects are more
strongly preferred for the run-up of a wave with negative
polarity (wave trough). Stockdon et al. [27] investigated the
run-up and its components using a parameterized model
developed by comparing run-up observations with offshore
wave height, wave period, and local beach slope. The results
indicated that the parameterized predictions of the setup
may need modification for extreme conditions, and nu-
merical simulations can be used to extend the validity of the
parameterized predictions of the infragravity swash. Reh-
man et al. [28] investigated the reduction in run-up heights
of solitary waves on steep beaches by installing parallel rows
of submerged breakwaters. They observed that three rows
of breakwaters six seawall heights apart resulted in a signi-
ficant decrease in wave run-ups as compared to two rows. Yao et al.
[29] studied the effects of reef morphologies (fore-reef slope,
back-reef slope, reef-flat width, and reef crest width) on
storm-like solitary wave transformation and run-up and pro-
posed an empirical formula based on experimental data.
Subramaniam et al. [30] studied the effects of dike curvature
on wave run-ups on regular wave attacks by employing a
numerical model. They studied various dike-opening angles
and compared them with physical model test results. In
addition to the mesh-based methods that need to readjust or
rezone the grid after each time step, smoothed particle
hydrodynamics (SPH) is a meshless particle method with
strong self-adaptability [31] which was initially proposed for
astrophysical problems by Gingold and Monaghan [32]. Lots
of work was conducted to increase the accuracy and stability
of the standard SPH scheme. Mahmoudi et al. [33, 34]
developed the periodic wave breaking process on a plane slope
using the weakly-compressible SPH (WCSPH) method.
Rostami and Ketabdar [35] proposed the WCSPH method
to solve the continuity and momentum equations with
laminar viscosity and the subparticle scale (SPS) turbulence

\[ \eta(x,t) \]

\[ H \]

\[ h_0 \]

\[ a \]

\[ R \]

Figure 1: Schematic of wave run-up on a sloped wall.
model. Fathi and Ketabdari [36] used the SPH method to examine the run-up and overtopping of solitary waves on semicircular breakwaters (SBW). He et al. [37] improved a meshless method and presented a numerical investigation of solitary wave breaking over a slope by using a finite particle method (FPM) and demonstrated that FPM performs better than SPH qualitatively and quantitatively.

Such broad literature shows that most studies were carried out on sloping beaches and seawalls with vertical faces restricted to the action of regular or random waves. Posttsunami conditions has added a new dimension to the problem of the response of such structures to shallow-water waves. Solitary waves represent closely the characteristics of a tsunami. This promoted the authors to undertake the present study. Therefore, this study uses the FVM-VOF method by FLOW-3D® software [38] to introduce a numerical model of solitary wave-seawall interaction and explain the effects of sinusoidal, logarithmic, and parabolic functions of seawall front faces on wave run-up and reflection.

2. Governing Equations and Numerical Solution

In this paper, a numerical simulation was undertaken using the Reynolds-averaged Navier-Stokes equations (RANS) with the k-ε-RNG renormalization group turbulent model. To model the complex geometric boundary by the fractional area/volumes obstacle representation FAVOR technique Hirt and Sicilian [39], the general continuity and momentum equations for incompressible turbulent flows are formulated with the area and volume fraction functions:

\[
\frac{\partial \langle u_x \rangle}{\partial t} + \langle u_x \rangle \frac{\partial \langle u_x \rangle}{\partial x_i} = \frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x_i} + \frac{1}{\rho V_F} \frac{\partial}{\partial x_i} \left[ A_i \left( \langle \tau_{ij} \rangle + R_{ij} \right) \right].
\]

(2)

In (2), \( \tau_{ij} = 2 \mu S_{ij} \) and \( S_{ij} \) is calculated from the following:

\[
S_{ij} = \frac{\left( \frac{\partial \langle u_j \rangle}{\partial x_i} + \frac{\partial \langle u_i \rangle}{\partial x_j} \right)}{2}.
\]

(3)

denotes the ensemble-averaged or so-called time-averaged properties, \( u \) is the velocity component in the subscript direction, the subscripts = 1, 2 represent x and z directions, respectively, \( p \) is the pressure intensity, \( \rho \) is the fluid density, \( g \) is the gravitational acceleration, and \( \mu \) is the absolute viscosity. \( V_F \) is the fractional volume open to the flow and \( A \) is the fractional area open to flow in the subscript direction. The above governing equations are reduced to standard RANS equations as both \( V_F \) and \( A \) are set to unity. The Reynolds stress term in (2) is expressed by

\[
R_{ij} = 2 \rho \gamma S_{ij} - \frac{2}{3} \rho k \delta_{ij},
\]

(4)

where \( \gamma \) is the eddy viscosity, \( k \) is the turbulent kinetic energy, and \( \delta_{ij} \) is the Kronecker delta function such that \( \delta_{ij} = 1 \) when \( i=j \); \( \delta_{ij} = 0 \), when \( i \neq j \).

In (4), the eddy viscosity \( \gamma \) is related to the effect of the space and time distribution of the turbulent motion, which is solved here using the renormalization group method k-ε-RNG model. The k-ε-RNG turbulent model was proposed by Yakhot and Orszag [40] improving on the k-ε model. The transport equations of the k-ε-RNG model are expressed in a Cartesian coordinate system as

\[
\frac{\partial k}{\partial t} + \langle u_j \rangle \frac{\partial k}{\partial x_i} = \frac{1}{\rho} \frac{\partial}{\partial x_i} \left( \mu \frac{\partial k}{\partial x_i} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) - \epsilon, \quad (5)
\]

\[
\frac{\partial \epsilon}{\partial t} + \langle u_j \rangle \frac{\partial \epsilon}{\partial x_i} = \frac{1}{\rho} \frac{\partial}{\partial x_i} \left( \mu \frac{\partial \epsilon}{\partial x_i} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) + \frac{\partial}{\partial x_i} \left( \alpha_1 \gamma \frac{\partial \gamma}{\partial x_i} \right) - c_2 \epsilon^2 - R_0, \quad (6)
\]

where \( \epsilon \) is the turbulent dissipation rate, \( k \) is the turbulent kinetic energy, \( \mu \) is the dynamic viscosity, \( \alpha_1 \) is a constant, \( \gamma \) is the eddy viscosity, \( R_0 \) is the constant, \( c_2 \) is a constant, and \( \epsilon \) is the turbulent dissipation rate. The constants in the k-ε-RNG model are determined by the following equations:

\[
\eta = \sqrt{\frac{2k}{\epsilon}} \frac{\partial \langle u_j \rangle}{\partial x_i}, \quad (7)
\]

\[
\nu_{eff} = \nu \left[ 1 + \frac{c_k^2 k}{\mu \sqrt{\epsilon}} \right], \quad (8)
\]

\[
R_0 = \frac{c_p \eta (1 - (\eta/\eta_0))^2}{(1 + c_3 \eta/\eta_0)^2}, \quad (9)
\]

\[
\eta = \sqrt{\frac{2k}{\epsilon}} \frac{\partial \langle u_j \rangle}{\partial x_i}, \quad (10)
\]

The numerical solutions are implemented by FLOW-3D® [38], which utilizes the true VOF method of Hirt and Nichols [42] to accurately track the free water surface and models efficiently the solid geometries using the FAVOR technique. Several studies applied Flow-3D to successfully model the issues of the interaction of waves and structures, such as Choi et al. [43], Jin and Meng [44], and Dentale et al. [45]. Also, several studies on solitary wave characteristics, such as in Yang and Chen [46], Zhang et al. [47, 48], and Guo et al. [49], was investigated and in this paper, the solution of solitary wave derived from Boussinesq equations was used as the incident wave, which was expressed as

\[
\eta(x,t) = H \sec h \left[ \frac{3H}{4h_0} (x - ct) \right]. \quad (11)
\]
Table 1: Constants for $k$-$\varepsilon$-RNG turbulence model.

| Constants | Amount |
|-----------|--------|
| $\varepsilon_0$ | 0.0120 |
| $c_p$ | 0.0850 |
| $c_{1k}$ | 1.4200 |
| $c_{2k}$ | 1.6800 |
| $\alpha_k$ | 0.7194 |
| $\alpha_v$ | 0.7194 |
| $\eta_0$ | 4.3800 |

where $\eta$ is the free surface elevation, $h_0$ is the still water depth, $H$ is the wave height, and $c$ is the wave celerity expressed by

$$C = \sqrt{gh_0 \left( 1 + \frac{H}{2h_0} \right)}.$$  \hspace{1cm} (12)

3. Case Study

In this research, four seawalls with sinusoidal, logarithmic, parabolic, and sloping faces were studied. The values of wave height and depth of water used in modeling are presented in Table 2.

Based on Table 2, a solitary wave was modeled with five different heights: 15, 17.5, 20, 22.5, and 25 cm. Also, a constant water depth in front of the seawalls was modeled with five different heights: 35, 40, 45, 50, and 55 cm. The curvatures of sinusoidal, logarithmic, and parabolic walls were modeled with the powers of 2, 3, 5, and 8 and for the sloped seawall, the gradient was 1:1. In Figure 2, a 2D view of sloped and curved walls is presented. In FLOW-3D® software, in Cartesian coordinates, there are six distinct parameters has no numeric value.

4. Model Validation

Synolakis [4] generated the solitary waves in a laboratory flume. He measured wave run-up $R$ at a plane beach of slope 1 in 19.85 for relative wave height $H/h_0$ ranging 0.005–0.633, in which $H$ is the offshore wave height and $h_0$ is the constant offshore depth beyond the plane beach. Based on his work, the recorded run-up values tended towards two distinct asymptotic forms depending upon whether the solitary wave had broken or not, given by

$$\frac{R}{h_0} = \alpha \left( \frac{H}{h_0} \right)^{\beta},$$  \hspace{1cm} (13)

where $\alpha$ and $\beta$ are empirical coefficients. The values of these coefficients are $\alpha = (11.0, 1.12)$ and $\beta = (1.22, 0.59)$ for the lower and upper asymptotes, respectively. In this paper, these experimental data were used to verify the numerical simulation of solitary waves. The experimental results from Synolakis’s study have been widely used in simulation verification, e.g., Lynett et al. [50]. The referenced experimental water depth ($h_0$) for the simulation was 0.21 m; the ratio of the wave height to water depth $H/h_0$ was 0.28, and the beach slope was 1/20. To verify the numerical model, in Figure 5 normalized values of 1:1 sloping seawall with 0.005 × 0.015 mesh sizes were compared with the laboratory work of Hall and Watts [51]. Lower and upper asymptotes modified by Borthwick et al. [52, 53] based on the outputs of Synolakis [4] were also plotted in relevant diagrams. The existence of a good agreement between numerical and experimental results is evident. The error in the same ratio of $H/d$ is in the range of 2% to 4%.

Before setting up the model runs, it is required to perform a sensitivity analysis to determine the optimum
Figure 2: Four 2D view model of sloping and curved seawalls.

Table 3: The boundary conditions and their symbols were used in this research.

| No. | Direction of the axes | Boundary condition | Symbol |
|-----|-----------------------|--------------------|--------|
| 1   | x-left                | Wave               | WV     |
| 2   | x-right               | Out flow           | O      |
| 3   | y-front               | Wall               | W      |
| 4   | y-back                | Wall               | W      |
| 5   | z-down                | Wall               | W      |
| 6   | z-up                  | Out flow           | O      |

Figure 3: (a) Boundary conditions on six face blocks and (b) model mesh size of (0.015 × 0.015) in this research.

The wave is assumed to come from a flat bottom reservoir, which is outside the computational domain.

Figure 4: Inputs required for solitary wave generation: wave height, mean water depth, and current velocity.
number of meshes to minimize the computational costs and time. Therefore, a concave seawall with a wave height of 0.30 m and a water depth of 0.35 m was investigated for different mesh sizes. The results are presented in Figure 6. It is evident from this figure that for mesh sizes less than 0.015 m × 0.015 m, the wave’s run-up on this seawall, remains nearly constant. Therefore, all the models in this study were analyzed with 0.015 m × 0.015 m mesh sizes.

Figures 7 show the simulation results of the wave run-up on Figure 8 slope. The circular points and solid lines in this figure represent the experimental data from Synolakis [4] and numerical simulation results respectively. A comparison among the waveform variations demonstrates that the numerical accuracy could be obtained by using the computational mesh of 0.015 m × 0.015 m. The existence of a good agreement between experimental results of Synolakis data and numerical modeling by using the FVM-VOF method is evident. The amount of error is in a range of 3% to 6%.

To verify the accuracy of the proposed numerical scheme, the computer code FLOW-3D® was applied to reproduce the proposed formula provided by Munk [54]. The propagation of a solitary wave with an incident wave height of $H = 0.25$ m on a plane beach model of concave seawall with a radius of 1 meter was investigated. The still water depth of the seawall was $d = 0.35$ m. The solitary wave profile generated by the FVM-VOF method agrees well with the theoretical solution. The effective wavelength of the
Figure 9: Comparison of numerical and theoretical results of free surface profile between 2 and 3 seconds of wave generation with \( H = 0.25 \) m and \( h_0 = 0.35 \) m.

Figure 10: Model results of reflected wave height for parabolic and sloped walls.

Figure 11: Model results of wave run-up for parabolic and sloped walls.

Figure 12: The incident wave and relevant reflected waves from sloped and parabolic walls (\( H/h_0 : 0.7 \)).

The solitary wave is about 6 m when evaluated by (11). Figure 9 shows a comparison of the numerical results by using the computational mesh of 0.015 m \( \times \) 0.015 m and the theoretical data of the free surface profile at 1.5 seconds after solitary wave generation. Solid lines are numerical results (FVM method), and symbols are theoretical data [54]. The existence of a good agreement between results is evident as the error for the peak of the solitary wave is in a range of 5% to 7%.

5. Results

The number of models analyzed in this study for each type of wall was 16. So, for four seawalls a total number of 208 models were investigated based on Table 2. In theory, a solitary wavelength \( L \) is infinite. However, in practice, a limited \( L \) is needed. For measurement of solitary wave \( L \) in a range of 3 to 8 m is about 85% wavelength. It is calculated as follows in which \( H \) and \( h_0 \) are the wave height and water depth respectively:

\[
L = 2\pi h_0 \sqrt{\frac{4h_0}{3H}}
\]

(14)

It can be seen that for this type of...
wall, with increasing \((H/h_0)\), \(C_r\) decreases. The average \(C_r\) on parabolic walls is about 18.4% higher than that of the sloping walls and can be expressed as a power function by \(Y = AX^B\). The coefficients range of this function for walls with all powers are \(A = 0.47 \sim 0.61\) and \(B = −0.37 \sim −0.46\) and the \(R^2\) coefficients for these functions are 0.94. However, \(R\) increases by increasing \((H/h_0)\). The average \(R\) on parabolic walls is about 4.5% higher than that of the sloping walls and can be expressed as a power function by \(Y = AX^B\). The coefficients range of this function for walls with all powers are \(A = 3.54 \sim 3.84\) and \(B = 0.60 \sim 0.69\) and the \(R^2\) coefficients for these functions are 0.98. For parabolic function front faces wall, the average amount of \(C_r\) and \(R\) to the power of 2 and 3 is the lowest and highest respectively and by increasing the power, these amounts are increased.

The incident and corresponding reflected wave profiles from parabolic and sloped walls for \((H/h_0)\) in the range of 0.27 to 0.71 were investigated. The results show that this kind of wall with the powers of 2, 3, 5, and 8 reflected the wave as much as 34%, 18%, 26%, and 32% less than the incident wave, respectively. Also, the comparison of this type of wall to the powers of 2, 3, 5, and 8 with sloping walls was reduced by 7%, 12%, 15%, and 9%, respectively. In Figure 12 for \(H/h_0 = 0.71\), the incident wave and the corresponding reflected waves are shown.

Figures 13 shows these parameters on the sinusoidal function front faces and sloped walls. It can be seen that for this type of wall, with increasing \((H/h_0)\), \(C_r\) decreases. The average \(C_r\) on sinusoidal walls is about 7.7% higher than that of the sloping walls and it can be expressed as a power function by \(Y = AX^B\). The coefficients range of this function for walls with all powers are \(A = 0.39 \sim 0.47\) and \(B = 0.60 \sim 0.69\) and the \(R^2\) coefficients for these functions are 0.98. For sinusoidal function front faces wall, the average amount of \(C_r\) and \(R\) to the power of 2 and 3 is the lowest and highest respectively and by increasing the power, these amounts are increased.
\begin{align*}
B &= -0.46 \sim -0.54 \text{ and the } R^2 \text{ coefficients for these functions are } 0.94. \text{ However, } R \text{ increases by increasing } (H/h_0). \text{ The average } R \text{ on sinusoidal walls is about 5.1\% higher than that of the sloping walls and can be expressed as a power function by } Y = AX^B. \text{ The coefficients range of this function for walls with all powers are } A = 3.52 \sim 3.78 \text{ and } B = 0.58 \sim 0.65 \text{ and the } R^2 \text{ coefficients for these functions are 0.98. For sinusoidal function front faces wall, the average amount of } C_r \text{ and } R \text{ to the power of } 2 \text{ and } 3 \text{ is the lowest and highest respectively and by increasing the power, these amounts are increased.}

The incident and corresponding reflected wave profiles from sinusoidal and sloped walls for \((H/h_0)\) in the range from 0.27 to 0.71 were investigated. The results show that this kind of wall to the powers of 2, 3, 5, and 8 reflected the wave as much as 40\%, 28\%, 36\%, and 40\% less than the incident wave, respectively. Also, the comparison of this type of wall to the powers of 2, 3, 5, and 8 with sloping walls was reduced by 1\%, 13\%, 5\%, and 1\%, respectively. In Figure 15 for \(H/h_0 = 0.71\), the incident wave and the corresponding reflected waves are shown.

Figures 16 show these parameters on the logarithmic Figure 17 function front faces and sloped walls. It can be seen that for this type of wall, with increasing \((H/h_0)\), \(C_r\) decreases. The average \(C_r\) on logarithmic walls is about 5.1\% lower than that of the sloping walls and can be expressed as a power function by \(Y = AX^B\). The coefficients range of this function for walls with all powers are \(A = 0.25 \sim 0.44\) and \(B = -0.51 \sim -0.84\) and the \(R^2\) coefficients for these functions are 0.94. However, \(R\) increases by increasing \((H/h_0)\). The average \(R\) on logarithmic walls is about 6.4\% higher than that of the sloping walls and can be expressed as a power function by \(Y = AX^B\). The coefficients range of this function for walls with all powers are \(A = 3.51 \sim 3.87\) and \(B = 0.56 \sim 0.65\) and the \(R^2\) coefficients for these functions are 0.99. For the logarithmic function front faces wall, the average amount of \(C_r\) and \(R\) to the power of 2 and 3 is the lowest and highest respectively and by increasing the power, these amounts are increased.

The incident and corresponding reflected wave profiles from logarithmic and sloped walls for \((H/h_0)\) in the range from 0.27 to 0.71 were investigated. The results show that this kind of wall to the powers of 2, 3, 5, and 8 reflected the wave as much as 40\%, 28\%, 36\%, and 40\% less than the incident wave, respectively. Also, the comparison of this type of wall to the powers of 2, 3, 5, and 8 with sloping walls was reduced by 1\%, 13\%, 5\%, and 1\%, respectively. In Figure 15 for \(H/h_0 = 0.71\), the incident wave and the corresponding reflected waves are shown.

Figures 16 show these parameters on the logarithmic Figure 17 function front faces and sloped walls. It can be seen that for this type of wall, with increasing \((H/h_0)\), \(C_r\) decreases. The average \(C_r\) on logarithmic walls is about 5.1\% lower than that of the sloping walls and can be expressed as a power function by \(Y = AX^B\). The coefficients range of this function for walls with all powers are \(A = 0.25 \sim 0.44\) and \(B = -0.51 \sim -0.84\) and the \(R^2\) coefficients for these functions are 0.94. However, \(R\) increases by increasing \((H/h_0)\). The average \(R\) on logarithmic walls is about 6.4\% higher than that of the sloping walls and can be expressed as a power function by \(Y = AX^B\). The coefficients range of this function for walls with all powers are \(A = 3.51 \sim 3.87\) and \(B = 0.56 \sim 0.65\) and the \(R^2\) coefficients for these functions are 0.99. For the logarithmic function front faces wall, the average amount of \(C_r\) and \(R\) to the power of 2 and 3 is the lowest and highest respectively and by increasing the power, these amounts are increased.

The incident and corresponding reflected wave profiles from logarithmic and sloped walls for \((H/h_0)\) in the range from 0.27 to 0.71 were investigated. The results show that this kind of wall to the powers of 2, 3, 5, and 8 reflected the wave as much as 40\%, 28\%, 36\%, and 40\% less than the incident wave, respectively. Also, the comparison of this type of wall to the powers of 2, 3, 5, and 8 with sloping walls was reduced by 1\%, 13\%, 5\%, and 1\%, respectively. In Figure 15 for \(H/h_0 = 0.71\), the incident wave and the corresponding reflected waves are shown.
walls is shown in Figure 19. It was found that with increasing the ratio Figure 18 of $H/h_0$, $R/h_0$ also increases. These values can be represented by a power function of $Y = AX^B$. The values of the coefficients for parabolic, sinusoidal, and logarithmic walls Figure 20 for this function Figure 21 are $(A = 3.54 \sim 3.84, B = 0.60 \sim 0.69)$, $(A = 3.52 \sim 3.79, B = 0.58 \sim 0.65)$, and $(A = 3.51 \sim 3.88, B = 0.57 \sim 0.65)$, respectively. The average value $R/h_0$ against $H/h_0$ for the parabolic walls with the powers of 2, 3, 5, and 8 was about 3%, 4%, 5%, and 5% higher than the sloped wall, respectively, for sinusoidal wall with the powers of 2, 3, 5, and 8 respectively was about 4%, 4%, 5%, and 7%, and for logarithmic wall with the powers of 2, 3, 5, and 8 respectively was about 5%, 6%, 7%, and 9% higher than that of the sloped wall.

6. Conclusion

In the present study, a numerical FVM-VOF model was developed to study the interaction of solitary waves with curved and sloped seawalls. The RANS model was used to consider turbulent flow. 208 models of seawalls with parabolic, sinusoidal, logarithmic, and sloped faces were analyzed. These analyses were examined for the scaled model solitary wavelength in the range from 3 to 8 m.

In light of the most important results of this research, the following conclusions can be drawn:

(i) As the wavelength increases, the wave reflection is increased with oscillation nature for curved faces and sloped walls as a power function.

(ii) Among all models, parabolic walls with the power of 3 had the highest amount of wave reflection and logarithmic walls with the power of 2 had the lowest one.

(iii) As the wavelength increases, the wave run-up is decreased as a power function.

(iv) Among all models, logarithmic walls with the power of 8 had the highest wave run-up and parabolic walls with the power of 2 had the lowest one.

(v) As normalized wave height ($H/h_0$) increases, normalized run-up ($R/h_0$) also increases as a power function for curved face walls and sloped ones.

(vi) Numerical results show that the parabolic seawalls have the highest amount of wave reflection and turbulence on the seaside.

List of Notations

- $a$: Seawall slope
- $a$: Fractional area open to the flow
- $\alpha$: Empirical coefficients
- $C$: Wave celerity
- $C_r$: Reflection coefficient
- $C_3$: Turbulence constant (dimensionless)
- $C_{\mu}$: Turbulence constant (dimensionless)
- $C_{\alpha1}$: Turbulence constant (dimensionless)
- $C_{\alpha2}$: Turbulence constant (dimensionless)
- $\alpha_4$: Turbulence constant (dimensionless)
- $\alpha_5$: Turbulence constant (dimensionless)
- $\eta_0$: Turbulence constant (dimensionless)
- $g$: Gravitational acceleration
- $H$: Wave height
- $h_0$: Water depth
- $k$: Turbulent kinetic energy
- $L$: Wavelength
- $p$: Pressure intensity
- $R$: Wave run-up
- $R_s$: Coefficient of determination
- $S_{ij}$: Strain tensor
- $R_i$: Reynolds stress term
- $u_i$: Instantaneous velocity in direction $x$
- $u_j$: Instantaneous velocity in direction $z$
- $V_F$: Fractional volume open to the flow


\( \nu \): Absolute viscosity

\( \nu_{\text{eff}} \): Effective viscous vorticity

\( \rho \): Fluid density

\( \nu \): Kinematic (laminar) viscosity

\( \nu_{\text{eff}} \): Effective viscosity

\( \nu_i \): Turbulent viscosity

\( \eta \): Free surface elevation

\( \delta_{ij} \): Kronecker delta function

\( \tau_{ij} \): Turbulence Reynold's stress

\( \varepsilon \): Turbulent energy dissipation

\( \mu_{\text{eff}} \): Ensemble-averaged or time-averaged properties.

**Data Availability**

The data used and/or analyzed are not available.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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