Transport through two interacting resonant levels connected by a Fermi sea

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We study transport at finite bias, i.e. beyond the linear regime, through two interacting resonant levels connected by a Fermi sea, by means of time-dependent density matrix renormalization group. We first consider methodological issues, like the protocol that leads to a current-carrying state and the characterization of the steady state. At finite sizes both the current and the occupations of the interacting levels oscillate as a function of time. We determine the amplitude and period of such oscillations as a function of bias and extension of the Fermi sea. In particular, the occupations on the two dots oscillate with a relative phase which depends on the distance between the impurities and on the Fermi momentum of the Fermi sea, as expected for RKKY interactions. Also the approximant to the steady-state current displays oscillations as a function of the distance between the impurities. Such a behavior can be explained by resonances in the free case. We discuss finally the incidence of interaction on such a behavior.

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I. INTRODUCTION

The study of quantum transport across nanostructures has been the subject of intense theoretical and experimental attention for decades. One of the most intensively studied systems is that of quantum dots, both because of their great experimental versatility and because they unveil an extremely rich physics, as exemplified by the Kondo effect1 in quantum dots2,3. When considering a system of two quantum dots, a further interesting phenomenon emerges, the Rudermann-Kittel-Kasuya-Yosida (RKKY) interaction4. It describes the indirect interaction between two magnetic impurities mediated by the electrons of the surrounding Fermi sea, and is characterized by oscillations related to the Fermi wavevector. The competition between the RKKY interaction and the Kondo effect was studied in the frame of numerical renormalization group5, and conformal field theory6,7. An experimental realization with two quantum dots coupled by a Fermi sea was meanwhile reported8.

Recently, a great deal of progress was achieved towards the theoretical determination of steady-state transport properties focusing on a quantum dot described by the interacting resonant level model (IRLM)9,10, that consists of spinless fermions with a nearest-neighbor repulsive interaction for the sites adjacent to the dot. This model was studied with several theoretical techniques, ranging from integrable field theories and Bethe Ansatz (see Boulat et al.9 and references therein), functional renormalization group10,11, real-time renormalization group12, to density-matrix renormalization group (DMRG) techniques13,14,15. These works provide the I-V characteristics out of equilibrium at finite bias and up to large values of the interaction16, and a detailed knowledge of the relaxation dynamics16,17 in the regime of small interaction, including also the incidence of finite temperatures17. The shot noise and the full counting statistics have been studied by means of exact diagonalization18 (in the free case), DMRG and thermodynamical Bethe Ansatz19,20.

Such an attention on a model that arguably cannot be experimentally realized in an electronic system is due to the fact that, in contrast to the Anderson impurity model, the important energy scales of the problem are accessible and controllable in numerical simulations, avoiding to deal with the Kondo scale, that requires high resolution in energy.

In contrast to the great attention devoted to the one impurity case, little is known about the case with more impurities21,22. In particular, to the best of our knowledge, the case of two IRLs separated by a Fermi sea under a finite bias awaits still a theoretical treatment. Here we consider two leads modeled as tight-binding chains with uniform hopping, coupled to two quantum dots interacting with their nearest-neighbor sites and a Fermi sea in between, focusing on the dynamics of the system when it is taken out of equilibrium with the application of a finite bias. The set-up is that of a quantum quench, where the initial state corresponds to the ground state of a Hamiltonian, but the time evolution is governed by a different (time independent) one. We considered two different protocols, where the bias is included either in the initial or in the final Hamiltonian. We discuss also the characterization of the steady-state and the incidence of finite-size effects.

We performed our studies by means of a time-dependent DMRG (t-DMRG) simulation24,25. This method allows to study the time evolution of the system up to intermediate times (∼ 40ℏ/t0, where t0 is the nearest-neighbor hopping between the sites of the leads) in a nonperturbative way. The time evolution of the current on each link of the chain and of the particle-density on the dots exhibits oscillations that depend not only on the bias, as in the single dot case, but also on the extension of the Fermi sea. Furthermore, the occupations on the two dots oscillate with a relative phase which depends on the distance between the impurities and on the Fermi momentun of the Fermi sea. This can be explained in terms of the RKKY interaction. The currents in the sites
connecting the quantum dots to the leads show also oscilla-
tions but with a phase shift with respect to the density.
For the approximant to the steady-state current we find
that it oscillates as a function of the distance between the
impurities. In the free case the behavior of the current
can be understood in terms of resonances that appear
in the transmission coefficient of a single particle propa-
gating through the system. We finally show the effect
of interaction. While it suppresses the resonances found
in the free case, for strong interaction we find that large
oscillations of the current as a function of the inter-
impurity distance arise, with a periodicity which matches the
RKKY prediction.

This paper is organized as follows. Section II is devoted
to the discussion of methodological issues. In particular
in Sec. II A we define the model, the observables and the
numerical technique. We show the effect of different
quench schemes and motivate our choice in Sec. II B. In
Sec. II C we detail how the approximant of the steady-
state current is obtained and benchmark our DMRG re-
results for the one impurity system with those of Boulat
et al. Section III displays our results. In Sec. III A the
time evolution of the occupations and the currents is
shown and its relation with RKKY interaction is dis-
cussed. In Sec. III B we concentrate on the approximant
to the steady-state values of the current as a function of
the distance. We consider first the free case, for which we
establish a connection with the problem of transmission of
a single particle propagating in the system, and then
move to the interacting case. In Sec. IV we summarize
our results.

II. MODELS AND METHODS

A. Hamiltonian and observables

We study a system characterized by the presence of
two quantum dots at positions $d_1$ and $d_2$ separated by a
distance $R = d_2 - d_1$. The region in-between harbours a
Fermi sea. The Hamiltonian of the whole system is given by
\[ \hat{H}_\text{chain} = \hat{H}_D + \hat{H}_T + \hat{H}_F , \]
where
\begin{align}
\hat{H}_D &= -t_C (c_{d_1-1}^\dagger \hat{c}_{d_1} + c_{d_1}^\dagger \hat{c}_{d_1+1} + \text{H.c.}) \\
&- t_C (c_{d_1}^\dagger \hat{c}_{d_1+1} + c_{d_1+1}^\dagger \hat{c}_{d_1} + \text{H.c.}) \\
&+ U_C \sum_{\alpha = d_1, d_2} \sum_{r = \pm 1} \left( \hat{n}_\alpha - \frac{1}{2} \right) \left( \hat{n}_{\alpha + r} - \frac{1}{2} \right) ,
\end{align}
corresponds to the dots and their nearest-neighbors, where the interaction is present. The leads connecting
the quantum dot are described by the tight-binding
Hamiltonian $\hat{H}_T$,
\[ \hat{H}_T = -t_0 \sum_{j=1}^{d_2-2} c_{j+1}^\dagger \hat{c}_j + t_0 \sum_{j=d_1+1}^{L-1} c_{j+1}^\dagger \hat{c}_j + \text{H.c.} . \]
Furthermore, the Fermi sea is described by the Hamilton-
ian $\hat{H}_F$,
\[ \hat{H}_F = -t_0 \sum_{j=d_1+1}^{d_2-2} (\hat{c}_j^\dagger \hat{c}_{j+1} + \text{H.c.}) \]
\[ (4) \]
In what follows we call the sites at positions $c_1 \equiv d_1 - 1$ and $c_2 \equiv d_2 + 1$ contacts. The total number of sites of the system is given by $L$, which we take even. If $R$ is
odd, we choose the position of the dots such that the left
and the right leads have the same number of sites. If $R$
is even the position of the dots is given by $(L - R)/2 + 1$
and $(L + R)/2 + 1$, implying that the left lead has one
more site with respect to the right one. In Eqs. (2) - (4)
we have $\hat{n}_j = \hat{c}_j^\dagger \hat{c}_j$, where $\hat{c}_j^\dagger$ ($\hat{c}_j$) are creation (annihilation)
operators for spinless fermions, $U_C$ is the interaction
coupling the dots and their nearest neighbors, $t_C$ is the
hopping between the dot and its nearest-neighbors. The
hopping elements in the leads and in the Fermi sea are
all set to $t_0$. Energies are measured in units of $t_0$ and
time in units of $h/t_0$. The number of particles in the sys-


Figure 1. (Color online) Picture of the model Eq. 4 for a
system of $L = 14$ sites and $R = 5$. The shaded light blue
areas indicate the presence of the bias (Eq. 4).

As it will be discussed in more detail in Sec. II B
we will follow the transport process in the frame of a
quantum quench, where a given initial state $|\Psi_0\rangle$ evolves
in time under the action of a given Hamiltonian, such
that the state of the system at a time $\tau$ is $|\Psi(\tau)\rangle = \exp(-i\hat{H}\tau)|\Psi_0\rangle$. Accordingly, the time-dependent occu-
pations on each site are given by
\[ n_j(\tau) = \langle \Psi(\tau)|\hat{n}_j|\Psi(\tau)\rangle . \]
The current on each bond connecting nearest-neighbor
sites can be obtained as:
\[ I_j = i\frac{e}{h} j(\Psi(\tau)|(\hat{c}_j^\dagger \hat{c}_{j+1} - \hat{c}_{j+1}^\dagger \hat{c}_j)|\Psi(\tau)) , \]
where $e$ is the electron charge, $j$ is the hopping on the
bond connecting sites $j$ and $j + 1$.

The results presented in this work are obtained with
the t-DMRG24-27. We typically simulate systems with
In order to implement the time evolution, we use the Trotter decomposition. Our code is adaptive, meaning that the number of states used at each time step changes dynamically keeping the discarded weight below a given threshold. The maximum number of states used in our computation is \(m \sim 1000\) and the discarded weight \(\varepsilon\) is kept below \(\sim 10^{-7}\). In the absence of interactions we employ also exact diagonalization.

### B. Quench schemes

In order to initiate transport processes in the system described by Eq. (1), a bias \(\Delta V\) has to be applied on the left and the right lead. It is described by:

\[
\hat{H}_B = \frac{\Delta V}{2} \left( \sum_{j=1}^{d_1-1} \hat{n}_j - \sum_{j=d_2+1}^{L} \hat{n}_j \right). \tag{7}
\]

As previously discussed for a single impurity, we can start with the ground state of \(\hat{H}_{\text{chain}}\) and follow the evolution of the system dictated by a Hamiltonian \(\hat{H}_{\text{chain}} + \hat{H}_B\). We denote such a procedure scheme (A). In such a scheme, however, the bounded nature of the spectrum of a lattice model becomes evident whenever the bias exceeds the bandwidth. In that case, there are no states available for transport through the system, as shown in Fig. 2 (the determination of the currents depicted will be discussed in detail in Sec. II C). It was suggested previously that in order to avoid such an artifact of a lattice model, the opposite scheme can be used, namely, the initial state is the ground state of \(\hat{H}_{\text{chain}} + \hat{H}_B\), and the evolution is studied switching off \(\hat{H}_B\). As shown in Fig. 2 such a quench scheme leads to a saturation of the attained current, with similar behavior for a single impurity or two of them with a Fermi sea inbetween. The current in scheme (B) saturates at large values of the bias, because of the finite bandwidth of the system.

For \(\Delta V\) smaller than half the bandwidth, both schemes lead to the same result. Moreover, for the whole range of biases studied in the one-impurity case, the I-V curves can be brought in this way to coincide with analytical results from conformal field theory, that correspond to the infinite bandwidth limit.

In scheme (B) the initial state is characterized by a particle imbalance between the left and right lead, due to the presence of the bias, and the distribution of particles in the central region is not uniform. However, we find \(\rho_c = \rho\) if the system is at half filling. In the other cases there is a discrepancy which can be controlled by performing a finite-size scaling.

In the rest of the work we choose quench scheme (B) because it avoids the artifact introduced by a bounded spectrum.

### C. Time averages

As already discussed in the Refs. 16 and 30 in the case of a single quantum dot, the time evolution of a current in a finite system is affected in various ways. On the one hand, right after switching the bias on (or off), there is a transient time, where the current grows from zero to a quasi-stationary state. On the other hand, at long times, the current bounces back at the ends of the system. In the intermediate quasi-stationary state, periodic vari-

![Figure 2](image1.png)

Figure 2. (Color online) I-V characteristics for a system with one (a) and two impurities (b). Black empty circles and red full squares refer to quench schemes (A) and (B) respectively. All the curves are obtained with \(L = 100\), except for \(R = 7\), scheme (A), for which \(L = 300\) sites are used. Data for \(t_C = 0.8t_0\) and \(U_C = 0.0\). The current is shown in absolute value.

![Figure 3](image2.png)

Figure 3. (Color online) Finite size scaling of the oscillation amplitudes \(I_j\) from cosine fits as discussed in main text, extracted from the left-contact current \(I_{L1}\). Data refer to a system with \(t_C = 0.8t_0\), \(U_C = 0\), \(R = 0\) (empty symbols) and \(R = 7\) (full symbols).
\[ I_\alpha(\tau) = \tilde{I} + \tilde{I}_3 \cos(2\pi \tau/T_3 + \tilde{\phi}), \]
where \( \alpha = c_1 \) or \( c_2 \) denotes the left or right contact, and the free parameters of the fit are \( \tilde{I}, \tilde{I}_3 \) and \( \tilde{\phi} \).

In the case of two impurities without interaction we find the same time scales, with minor differences. In particular the transient time also depends on the distance between the two impurities, and the amplitude of the Josephson oscillations is also affected by \( R \). Nevertheless, as we show in Fig. 3, it is still possible to extract the approximant to the steady-state current by fitting the Josephson oscillations as mentioned above, obtaining an amplitude that also vanishes in the thermodynamic limit. In the presence of interaction, both for one and two impurities, additional frequencies emerge. In Fig. 4 we show the current on the left contact for \( U_C = 5t_0 \) as an example, where additional oscillations superimposed to the Josephson oscillations (they have in this case a period \( T_3 \sim 12.6h/t_0 \)) are clearly visible. In order to deal with the appearence of several frequencies, we perform a discrete Fourier transform (DFT) by first identifying an interval of time where the evolution is quasi-stationary, with a duration that is an integer number of Josephson periods \( T_3 \). Then we do a reconstruction of the current by picking up only the few most important frequencies from the DFT, which always include the zero frequency component (the approximant to the steady-state current), the Josephson frequency \( \nu_J \), and the one due to interaction with the highest Fourier weight \( \nu_I \), as displayed in Fig. 4, where the quality of such a reconstruction can be seen for two different numbers of frequencies considered. We associate to the approximant to the steady-state current the uncertainty:

\[ \Delta I \equiv \frac{1}{M} \sqrt{\sum_{i=1}^{M} \left( I(\tau_i) - \tilde{I}(\tau_i) \right)^2}, \]

where \( \tau_i \) with \( i = 1, M \), are the equally spaced times lying in the interval where the DFT is performed, \( I(\tau_i) \) is the current measured at \( \tau_i \) and \( \tilde{I} \) is the reconstructed current. The uncertainty \( \Delta I \) is typically within the size of the symbols in our plots.

By using the procedure described above we reproduce in Fig. 5 the I-V characteristics of a single impurity in the full range of interactions and biases with excellent agreement with the original work\(^9\).

\[ I_{\text{cl}}[e/h] \]

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(8)

where \( \tau_i \), with \( i = 1, M \), are the equally spaced times lying in the interval where the DFT is performed, \( I(\tau_i) \) is the current measured at \( \tau_i \) and \( \tilde{I} \) is the reconstructed current. The uncertainty \( \Delta I \) is typically within the size of the symbols in our plots.

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Figure 4. (Color online) (a) Black continuous line: left-contact current \( I_{\text{cl}} \) for a system of \( L = 100 \) with two impurities at distance \( R = 7, t_C = 0.8t_0 \), \( \Delta V = 0.5t_0 \), and \( U_C = 5t_0 \). Horizontal continuous straight line: zero frequency component of the DFT in the interval [20, 45] (delimited by vertical dashed lines). (b) DFT of the black curve in (a). The red dotted curve in (a) corresponds to \( N_f = 2 \) frequencies: the zero frequency component and the Josephson frequency \( \nu_J \). The green dashed curve in (a) is found using also the frequencies framed by the dashed line in (b).

Figure 5. (Color online) I-V characteristics of a system at \( t_C = 0.5t_0 \) with quench scheme (B). The crosses are data from Ref. 9, the symbols are those obtained with our code (the parameters of our simulations are \( L = 96, m = 600 \) states, discarded weight \( \epsilon < 10^{-7} \)).

### III. RESULTS

#### A. Phase relations

As is well known, the RKKY interaction is an indirect exchange interaction between two localized spins mediated by the surrounding electrons of the Fermi sea. In the present case, since we are dealing with spinless fermions, only a coupling to the density will result. The RKKY interaction depends on the distance between the impurities \( R \) via \( 2k_F \) oscillations\(^4\) and is expected to induce correlations between the densities on the two dots and, consequently, on the currents in the contacts. We now show that the occupations on the dots fulfill the predictions of the RKKY interaction. The same correlations are also visible in the currents, but with a phase shift.

We take the system at half-filling, i.e. \( N/L = 0.5 \) in the free case and concentrate on the quasi-steady regime. In
the left panels of Fig. 6 we show the occupations on the two quantum dots. They oscillate with the Josephson

Figure 6. (Color online) Panels (a) and (c): time evolution of the number of particles on the left (blue continuous line) and right dot (red dashed line); panels (b) and (d): time evolution of the left-contact (blue continuous line) and right-contact (red dashed line) currents. Data for a system of $L = 100$, $t_C = 0.8t_0$, $\Delta V = 0.5t_0$, half-filling, quench scheme (B) and $U_C = 0$.

frequency $\nu_J$, which characterizes also the current (see Sec. II C). More interestingly we observe that when $R$ is odd the densities oscillate in opposition of phase, while if $R$ is even they oscillate in phase. This is a regular pattern which we find in all the range of $R$ considered. This behavior is compatible with the $2k_F$ oscillations of the RKKY interaction, as shown by Fig. 7. There it can be seen that the static susceptibility, that displays $2k_F$ oscillations as a function of $R$, is positive for $R$ odd and negative for $R$ even. Therefore, for $R$ odd the densities at the dots experience an effective repulsive interaction, while for $R$ even it is attractive.

If we now move to the right panels of Fig. 6 we find the opposite situation. When $R$ is odd the currents oscillate in phase (they are exactly equal in this case) and when $R$ is even they are in opposition of phase. In the latter case averaging the currents of the two contacts cancels out the oscillations. This effect is visible only in the quasistationary regime, as we can see from the left panels of Fig. 5.

The phase shift between densities and currents can be understood by noticing that when the mean density on a dot increases, transfer of a particle to (from) the dot is suppressed (enhanced) Then, for $R$ odd, while one dot has a higher density, the other has a lower one. Considering the current on the links to the left of $d_1$ and to the right of $d_2$, charge flow is enhanced on both links when $d_1$ has an increased density and $d_2$ a reduced one, while in the opposite case current is suppressed. On the other hand, when $R$ is even, both dots have an enhanced density or a suppressed one, such that when charge can be transferred on one link, the current is suppressed on the other.

Although the evolution of the current is more involved in the presence of interaction due to the appearance of additional oscillations, the same qualitative considerations hold also for $U_C \neq 0$. As an example, in Fig. 8 we show the currents and the densities in the presence of interaction, namely at $U_C = 5t_0$. The behavior of the densities is very clear and analogous to the free case. However, the interaction enhances the amplitude of the oscillations as can be seen comparing Figs. 6 and 8. In spite of the interaction, it is clearly visible that for the odd-$R$ case the currents are exactly equal and for even $R$ an opposition in phase is evident.

The phases characterizing the time evolution of the density and of the current described above reveal the effect of the RKKY interaction on the slow dynamics of the Josephson oscillations, giving rise to sustained and con-
trollable oscillations of the densities and of the currents in finite size systems. This fact may turn out to be observable in experiments focused on quantum dots set-ups in mesoscopic systems. Indeed there have been proposals of simulating quantum impurity systems and transport properties in cold atom systems. The first experimental progress done so far in this direction is the realization of a mesoscopic conducting channel in a cloud of Lithium atoms, performed by Brantut and collaborators.

B. Average current as a function of the distance

1. Free case

The results shown above indicate that the dynamics of the current and the density is regulated by 2k_F oscillations due to the RKKY interaction. We now investigate how the behavior of the steady-state current is affected by the distance between the impurities and the Fermi momentum. In Fig. 9 we show the approximant to the steady state current in absence of interaction as a function of the distance between the impurities. The simplest case is t_C = t_0, for which the data of Fig. 9 show very small variations as a function of R. Indeed these variations are only a finite size effect: as we show in Fig. 11 the current for R ≠ 0 converges in the thermodynamic limit to the value corresponding to R = 0. On the other side, from Fig. 9 we see that the curves with t_C = 0.5t_0 are the most sensitive to R, showing pronounced fluctuations. Furthermore, for t_C = 0.5t_0, there is a range of R where the oscillations have the largest amplitude. This range changes with ∆V. As an example, for ∆V = 0.5t_0 the range is given approximately by R ~ 7 / 19, while for ∆V = 0.75t_0 by R ~ 4 / 13. Moreover, the period of these oscillations is typically R = 2.

Following the Landauer-Büttiker approach, we now show that the patterns of the current of Fig. 9 can be understood in terms of the transmission properties for a single particle. Indeed, the physical mechanism at the root of the flow of current is that the dot, characterized by t_C ≠ t_0, is an effective tunnel barrier with an energy-dependent transmission probability p_s(ε) (where the subscript s stands for single). The presence of two dots requires the combination of the transmission probabilities in order to compute the total probability p_d(ε) (the subscript d standing for double). The transmission probability through a single dot is given by 18:

\[ p_s(\varepsilon) = \frac{1 - e^{2/(4t_0^2)} - 2(1 - p_s) \cos(2k(\varepsilon)R - 2\phi)}{1 + e^{2(t_d^2 - 2t_0^2)}/(4t_d^4)}. \]  

The total transmission probability can be obtained using the transfer matrix approach 34, 36 and gives:

\[ p_d = \frac{p_s^2}{1 + (1 - p_s)^2 - 2(1 - p_s) \cos(2k(\varepsilon)R - 2\phi)}, \]  

where \( \phi = kb \) and b is the size of the single tunnel barrier. In our case we have that t_C is present on three sites (the dot and its nearest-neighbors), so b = 3. The expression for the combined probability eq. (10) is valid provided R ≥ 3. Indeed, for R = 1, 2 one has to consider a single barrier of size b = 4, 5 respectively. In order to obtain the average current, one has to integrate the transmission probability over the energies of current-carrying states. This yields 34:

\[ I(\Delta V) = \int_{\epsilon_F - \Delta V/2}^{\epsilon_F + \Delta V/2} de \ p_d(\varepsilon). \]  

Figure 10. (Color online) Approximant to the steady state current for a system with t_C = t_0, ∆V = 0.25t_0 and scheme (B) for different sizes L, as obtained from time averaging in the interval \( \tau = [15 / 41] \). The dashed line is the current for R = 0.
Our results for $t_C = 0.5t_0$ are the blue dashed lines of Fig. [9]. We observe that there is a very good agreement with the current obtained by doing the time average.

2. Interacting case

We start considering the effect of a small interaction, namely $U_C = 1.0t_0$, and we choose $t_C = 0.5t_0$ in order to probe if and how the interaction affects the resonances (Fig. [11]). From the comparison with the free case we can see first that the current is enhanced, an effect that becomes stronger at larger values of the bias at larger values of $R$. The enhancement of the current by interaction is also observed in the one impurity case (see Fig. [9]) for small $U_C \lesssim t_0$ and $\Delta V \lesssim 2t_0$ Furthermore, it can be seen that the resonances observed in the free case are suppressed.

In Fig. 12 we show our results for the approximant to the steady-state current as a function of the distance between the impurities $R$ for a system of $L = 100$ sites with $U_C = 1.0t_0$ for different values of the bias and quench scheme (B). The currents in the non-interacting case (dotted, dash-dotted and dashed lines show the current and correspond to $\Delta V = 0.5, 0.8, 1.2t_0$ respectively) are shown as reference. The uncertainty on the value of the average current $\Delta I$ as discussed in Sec. II C is within the size of the symbols in the plot. (The current is plotted in absolute value.)

In Fig. 12 we show our results for the approximant to the steady-state current with increasing values of the interaction. While the current does not vary significantly for values of $U_C$ lower than the band-width, a qualitatively different behavior appears when $U_C$ is larger than $4t_0$. For $U_C = 6, 10t_0$ and $R \gtrsim 6$ we interestingly find that the current oscillates as a function of the distance with periodicity two, with a rather large amplitude, which is typical of RKKY oscillations at half-filling. The same behavior is also confirmed if we change the contact hopping, for example with $t_C = 0.8t_0$. In Fig. [9] we saw that without interaction the current is in this case almost independent on $R$, because the single tunnel barrier has a transmission coefficient close to unity (see Eq. [9]). Comparing Figs. 12 and 13 we find that the approximant to the steady-state current shows the same pattern if the interaction is large enough, i.e. $U_C \gtrsim 5.0t_0$, hinting at a signature of the RKKY interaction. It is also remarkable that the maxima of the current are of the same order as for the single-impurity case. This behavior and the features of the time evolution of the currents discussed in Sec. II A make the large interaction regime physically very interesting. By putting a Fermi sea between two
interacting levels, one finds that the current can be of the same order of the one impurity case, but with the advantage of controlling the phase relations between the currents and the occupations.

IV. SUMMARY

By studying the time dependence of the current and the density in a one-dimensional chain in the presence of two interacting resonant levels, we observed the interplay of the RKKY interaction and the characteristics of the quantum dots, concerning the dynamical behavior in a finite system as well as the approximant of the steady-state current.

Focusing on the time evolution, we found that, at finite size, the evolution of the current in the contacts and the occupations of the dots are characterized by oscillations, whose period depends on the applied bias as in the single dot case,[16] but interrelated in a way that depends on the size of the Fermi sea. In fact, we show that the densities on the dots oscillate with a relative phase which depends on the Fermi momentum of the Fermi sea and on the distance between the impurities, as expected for the RKKY interaction. An analogous behavior is found for the time evolution of the currents in the contacts, which are related to those of the density, but phase shifted with respect to them. Such a behavior could be exploited in experimental measurements in mesoscopic systems, giving thus a direct access to the RKKY interaction. As mentioned before, experimental investigation of transport in cold atomic systems,[32,33,37,38] would be an interesting set-up, where the variations of the density in the quantum dots could be accessed directly.

We have also studied the approximant to the steady state current, and its oscillations as a function of the distance between the dots. In the free case we identified resonances that can be traced back to the resonances affecting the transmission coefficients of a single particle propagating freely in the system. Turning interactions on the resonances are suppressed. However, for large values of the interaction we observe rather large oscillations of the current as a function of the distance with periodicity two. This corresponds to $2k_F$ oscillations, as expected for the RKKY interaction.

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