Direct observation of the degree of quantum correlations using photon-number resolving detectors

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Optical parametric down-conversion is a common source for the generation of non-classical correlated photonic states. Using a parametric down-conversion source and photon-number resolving detectors, we measure the two-mode photon-number distribution of up to 10 photons. By changing the heralded collection efficiency, we control the level of correlations between the two modes. Clear evidence for photon-number correlations are presented despite detector imperfections such as low detection efficiency and other distorting effects. Two criteria, derived directly from the raw data, are shown to be good measures for the degree of correlation. Additionally, using a fitting technique, we find a connection between the measured photon-number distribution and the degree of correlation of the reconstructed original two-mode state. These observations are only possible as a result of the detection of high photon number events.

PACS numbers: 42.50.Ar, 42.50.Dv, 42.65.Lm

Non-classical states of light are an essential resource for novel protocols in quantum information and quantum metrology [1, 2]. The most common tool for producing such states is the nonlinear process of optical parametric down-conversion (PDC). In this process, a parent pump photon is split in a nonlinear material into two daughter down-converted photons, while conserving energy and momentum. As the down-converted photons originate from a single quantum system, they possess correlations in many degrees of freedom, such as their polarization, frequency and momentum. Because for any photon emitted into one optical mode there is a sister photon emitted into the other optical mode, there are also photon-number correlations between the two modes. These correlations have been used to produce heralded Fock states [3, 4] and enhance the precision of optical measurements [5, 6, 7].

The down-converted photons are distributed over a range of spatial and spectral modes. However, most experiments require that the photons occupy a single mode. A specific mode is then post-selected by spatial and spectral filtering of the photons. The collected modes must be carefully matched to obtain a high-quality produced state [10]. The collection of matching modes would result in a joint photon-number distribution of a non-classical highly-correlated state. On the other hand, collecting two unrelated modes would result in a classical joint photon-number distribution which is a product of the two individual states. Recent developments in photon-number resolving detectors allow the direct measurement of the joint photon-number distribution and photon-number correlations between two down-converted modes. However, imperfections in the detection process, such as collection losses and false detections, alter the measured photon statistics and reduce their correlations. Non-classical correlations of multimode distributions from PDC have been demonstrated indirectly using reconstruction techniques [11, 12] and directly with a system of relatively low loss [13, 14].

In this Letter, we use a novel photon-number resolving detection scheme in order to measure the joint photon-number distribution of a collinear type-II PDC process. These distributions were measured up to the 10 photon terms. The two down-converted photons, which have orthogonal polarizations, are each collected from a single spatial and spectral mode. The degree of correlation was controlled by varying the amount of overlap between the two collected modes. We directly observe the transition between a separable product state and a highly correlated state, despite the presence of low detection efficiency and other distorting effects. We introduce measures for the degree of correlations between the two modes and relate these measures to the degree of non-classicality of the collected state.

The PDC states are generated by a type-II collinear BaB₂O₄ nonlinear crystal. The crystal is pumped by amplified and frequency doubled Ti:Sapphire laser pulses at a repetition rate of 250 kHz. The orthogonally polarized down-converted photons at 780 nm are split using a polarizing beam-splitter and coupled into separate silicon photomultiplier (SiPM) photon-number resolving detectors (Hamamatsu Photonics, S10362-11-100U). Before coupling to the detectors, the down-converted photons are spatially filtered using single-mode fibers and spectrally filtered using 3 nm bandpass filters, to ensure the collection of a single spatio-temporal mode [15]. The degree of correlation g of the measured photon state is determined by the amount of spectral and spatial overlap between the two collected modes. The amount of overlap is tuned by translating the optical fibers in order to collect different spatial modes and by tilting the bandpass filter, in order to shift its spectral band. The amount of overlap is evaluated using the heralded efficiency γ, defined as the ratio between the coincidence
and the single count rates in the limit when the average number of photons approaches zero. The parameter $\gamma$ is linearly proportional to the degree of correlation between the modes $g$ and to the overall photon detection efficiency $\eta$. $\gamma$ was evaluated using standard photon-number non-discriminating detectors (Perkin Elmer SPCM-AQ4C).

The SiPMs are composed of a two-dimensional array of avalanche photo diodes (APD) operating in Geiger mode. These detectors are operable at room temperature and require a relatively low operating voltage. They generate a current which is proportional to the number of detected photons. The output signals are sampled simultaneously within a 1 ns sampling window, and analyzed in real-time using programmable electronics. A computer receives the results and continuously displays the joint photon-number distribution between the two polarization modes. The dark count and afterpulsing rates were minimized by synchronizing the sampling time of the SiPM analog output with the arrival time of the photons. Furthermore, the detectors are moderately cooled using a thermoelectric cooler to $\sim -10^\circ\text{C}$. The bias voltage is adjusted accordingly, so that the detection efficiency is not affected. As a result, we obtain a good photon-number resolution with an error of less than 1%. Figure 1(a) shows an example of the histogram of the output intensities, in which up to 14 photons can be resolved.

A measurement of the joint photon-number distribution between the horizontal and vertical polarization modes with maximal spatio-temporal overlap is shown in Fig. 1(b). We measured the full joint probability matrix up to 10-photon terms. Ideally, the photon-number distribution of the correlated state would be composed of only diagonal elements which correspond to the same number of photons in both modes. However, the distribution exhibits a large number of non-zero probability values for events which contain different photon numbers. Similar probability values were measured for events which involve the same number of photons, indicated by the different color groups in Fig. 1(b). Note however that the probability of events which contain no photons in either one of the modes are slightly higher than the remaining probabilities within the same group.

The distortion in the photon-number distribution is a result of the imperfect detection process in SiPM detectors [17]. Not every photon impinging on the detector creates a signal due to imperfect overall photon detection efficiency. Additionally, false signals can be generated by thermally-excited discharges (dark counts). Furthermore, when a detection element is triggered, it might trigger additional neighboring elements due to optical crosstalk [16], in which a spurious photon generated during a discharge in one APD element propagates and is detected by another element. These inherent effects alter the photon number distributions, reducing the photon-number correlations even between highly correlated modes.

In order to study the photon-number correlations in our measurements, we compare the measured correlated
distribution with its corresponding non-correlated product distribution, obtained by multiplying the photon-number probabilities \( P(n_h) \) and \( P(n_v) \) of the individual polarization modes. \( n_h \) (\( n_v \)) is the number of photons in the horizontal (vertical) polarization mode. This result is shown in Fig. 2(a). The distribution is displayed such that each curve connects events which contain the same total number of photons \( S = n_h + n_v \), and the event counts are presented as a function of the photon number difference \( D = n_h - n_v \). This representation is similar to that of the joint distribution in Fig. 1(b), if each color group were to be joined with a solid line. The correlated distribution shows a clear distinction from its corresponding product result. On the other hand, a similar measurement between two relatively uncorrelated modes, results in a distribution which is almost identical to its corresponding product result (see Fig. 2(b)).

We have recorded a series of joint probability distributions for different values of the heralded efficiency \( \gamma \). By maintaining a fixed value for the overall detection efficiency, the heralded efficiency can be used as a direct measure of the mode overlap. In order to quantify the deviation of the joint distribution from an uncorrelated product state, we define the ratio between the joint photon-number probabilities and the corresponding product of the probabilities of their individual polarization modes

\[
R(n_h, n_v) = \frac{P(n_h, n_v)}{P(n_h) \cdot P(n_v)}.
\]

As can be seen from Fig. 2(a), the values of \( R \) are fairly uniform for all probability values except for the extremes, when there are zero photons in one of the modes. The average values of \( R \) obtained for all probabilities \( R(n_h \neq 0, n_v \neq 0) \) for the series of distributions are presented in Fig. 3(a). These values approach \( R \approx 1 \) for the least correlated states and increase linearly up to \( R \approx 2 \) for the highly correlated state. Numerical calculations of the dependence of \( R \) on the degree of non-classicality in the original state confirmed the linear relation between the two parameters. The exact value of \( R \) depends on the specific values of the average number of photons in the original distribution, the detection probability, the dark count rate and the crosstalk probability. Thus, even though it may seem that the original correlations are completely washed out, the ratio between the joint and product probabilities can be used as a direct measure for the degree of correlation between the two modes.

A mathematical tool which provides a more universal quantitative measure for the similarity between a given two-mode distribution and its closest product state is the singular value decomposition [18]. The decomposition of an \( n \times n \) probability matrix results in \( n \) singular values \( s_i \), ordered as \( s_1 \geq s_2 \geq \cdots \geq s_n \geq 0 \) and normalized to \( \sum_i s_i^2 = 1 \). The decomposition of a separable product state results in a single non-zero value, \( s_1 \neq 0 \), whereas a maximally correlated state results in \( n \) values of \( s_i = 1/\sqrt{n} \). The normalized Euclidean distance between a measured matrix \( M \) and the closest product state \( M_{\text{prd}} \) can be expressed using the singular values of \( M \) and is given by |\( \Delta M \)| = \( \| M - M_{\text{prd}} \| = \sqrt{s_2^2 + s_3^2 + \cdots + s_n^2} \).

The Euclidean distances \( |\Delta M| \) for several heralded efficiency values are shown in Fig. 3(b). The errors in \( |\Delta M| \) are estimated using a bootstrapping procedure and assuming Poissonian noise. The distance exhibits a clear increase by an order of magnitude as the heralded efficiency is varied between its minimal and maximal values. Thus, the Euclidean distance \( |\Delta M| \) is a measure for the degree of correlation of the two-mode state. It is applied directly to the raw data, and can detect correlations despite large imperfections in the detection apparatus.

In the experiment, the heralded efficiency \( \gamma \) was first optimized such that the largest amount of coincident events were observed. This condition is fulfilled when the two collected polarization modes maximally overlap, both spatially and spectrally. Then, by misaligning one of the collected modes, the overlap between the two modes was reduced, as well as the photon-number correlations. Thus, the joint probability of the two down-converted modes is a linear combination of the probability \( P_{\text{pdc}} \) of a correlated PDC distribution and that of an uncorrelated product distribution \( P_{\text{prd}} \)

\[
P(n_h, n_v, g) = g \cdot P_{\text{pdc}}(n_h, n_v) + (1-g) \cdot P_{\text{prd}}(n_h, n_v),
\]

where the degree of correlation \( g \) is the amount of overlap between the two collected modes. For \( g = 1 \), the
distribution is that of a collinear type-II PDC

\[ P_{\text{pdc}}(n_h, n_v) = \begin{cases} 0 & \text{if } n_h \neq n_v \\ \frac{1}{(n+1)} \left( \frac{\langle n \rangle^2}{\langle n \rangle + 1} \right)^n & \text{if } n_h = n_v = n, \end{cases} \]

where the parameter \( \langle n \rangle \) is the average number of photons in each mode. For \( g = 0 \), the distribution is that of a product state of two thermal modes

\[ P_{\text{prod}}(n_h, n_v) = \left( \frac{1}{(n+1)^2} \left( \frac{\langle n \rangle^2}{\langle n \rangle + 1} \right) \right)^{n_h + n_v}. \]

By modelling the effects of loss, dark counts and crosstalk in the SiPM detectors, we reconstructed the original two-mode photon-number distribution using a fitting method, similar to that presented in Ref. [17]. The measured distribution \( P_m \) is related to the original distribution \( P \) as

\[ \bar{P}_m = M_{\text{ct}} \cdot M_{\text{dk}} \cdot M_{\text{loss}} \bar{P}, \]

where \( \bar{P}_m \) and \( \bar{P} \) are vector representations of the measured and the original two-mode probability matrices, respectively. The matrices \( M_{\text{loss}} \), \( M_{\text{dk}} \), and \( M_{\text{ct}} \) represent respectively the effects of loss, dark counts and crosstalk according to the detector model of Ref. [17]. The fitting procedure is performed in two stages. First, we perform a least squares fit to the computed product state and obtain the average number of photons per mode \( \langle n \rangle \), the overall detection probability \( \eta \), the average number of dark counts, and the crosstalk probability for each of the two modes. Then, the measured data is fitted to Eq. 3 with the degree of correlation \( g \) as the free parameter.

Some example results of the fitting process are shown in Fig. 4. Three distributions taken for different values of \( g \) are presented. The photon-number probabilities in both polarization modes were kept constant for all measurements, thus maintaining the same product state (black lines). Transition from a highly correlated state to a product state is observed as the value of the heralded efficiency is decreased. The solid lines in Fig. 4 are fits to Eqs. 4. All fits result in similar values for the dark counts, detection efficiency, crosstalk probability, and \( \langle n \rangle \) up to the experimental error. The fits clearly differ in their values for the degree of correlation \( g \).

Figure 5 shows the reconstructed distributions from the data of Fig. 4. For the highest correlated state we observe strong photon-number correlations, which gradually disappear as the value of the heralded efficiency \( \gamma \) is decreased. Even for the lowest measured value of \( \gamma \), photon-number correlations are still evident.

The degree of correlation \( g \) for additional values of \( \gamma \) are shown in Fig. 5(c). As expected, the degree of correlation \( g \) depends linearly on the heralded efficiency. The parameter \( g \) is also the degree of non-classicality of the state. Applying the non-classicality criterion for the photon statistics of two-mode radiation of Lee [19] on Eq. 4 shows that a necessary (but not sufficient) condition for non-classicality is that \( g > 0 \). We have tested all of the reconstructed distributions against Lee’s criterion and found that all states with \( g > 0 \) satisfy it.

In conclusion, we have measured the two-mode photon-number distribution of a collinear type-II PDC process for different degrees of correlation. Clear evidence for photon-number correlations are presented despite the
low detection efficiency, the dark counts and the optical crosstalk effects of SiPM number-resolving detectors, which highly distort the number-correlations. These observations are only possible as a result of the detection of high photon number events. Both the ratio between the measured and the product matrix probabilities, and the singular value decomposition of the measured probability matrices, are shown to be good measures for the degree of correlation. These two criteria are derived directly from the raw data. Additionally, using a fitting technique, we found a connection between the measured photon-number statistics and the degree of correlation of the reconstructed original two-mode state.

The authors thank O. Gat for fruitful discussions.

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