Coupling a single electron to a Bose–Einstein condensate

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The coupling of electrons to matter lies at the heart of our understanding of material properties such as electrical conductivity. Electron–phonon coupling can lead to the formation of a Cooper pair out of two repelling electrons, which forms the basis for Bardeen–Cooper–Schrieffer superconductivity. Here we study the interaction of a single localized electron with a Bose–Einstein condensate and show that the electron can excite phonons and eventually trigger a collective oscillation of the whole condensate. We find that the coupling is surprisingly strong compared to that of ionic impurities, owing to the more favourable mass ratio. The electron is held in place by a single charged ionic core, forming a Rydberg bound state. This Rydberg electron is described by a wavefunction extending to a size of up to eight micrometres, comparable to the dimensions of the condensate.

In such a state, corresponding to a principal quantum number of \( n = 202 \), the Rydberg electron is interacting with several tens of thousands of condensed atoms contained within its orbit. We observe surprisingly long lifetimes and finite size effects caused by the electron exploring the outer regions of the condensate. We anticipate future experiments on electron orbital imaging, the investigation of phonon-mediated coupling of single electrons, and applications in quantum optics.

Charged impurities were very successfully used as probes of elementary excitations in the early studies of superfluidity in liquid helium. These applications span the interaction of ion impurities with phonons and rotons, the creation and study of vortex lattices by impurities, and the coupling of electrons to surface ripplons. Additionally, they have been proposed for applications in quantum information. The emergence of Bose–Einstein condensation of alkali atoms has renewed interest in impurity physics. Positively charged impurities in a Bose–Einstein condensate (BEC) were first created by Penning ionization of metastable atoms and photoionization. Recently, single ion trapping has allowed the first study of the interaction between a single charged impurity and a BEC in a well controlled manner. In addition to bare losses by classical scattering, chemical reactions (both with the impurity and catalytic reactions mediated by the impurity) have been observed. Alternatively, the creation of neutral spin impurities in Fermi liquids offers tunable interaction strength with the bulk by means of a Feshbach resonance: the character of the interaction can be changed, from free impurities via a quasiparticle state (so-called Fermi polaron) to bound diatomic molecules to bound diatomic molecules.

For all such impurities, the interaction is inversely proportional to the reduced mass of the impurity and the bulk species. Light impurities such as electrons are therefore, in general, better suited to obtaining strong coupling. However, electrons require an appropriate trapping potential, because even the tiniest electric field will lead to a considerable acceleration. The simplest trap for an electron offered by nature is a positively charged Rydberg core \( \text{Rb}^+ \), forming a Rydberg bound state. This Rydberg electron is interacting with several tens of thousands of condensed atoms contained within its orbit. We observe surprisingly long lifetimes and finite size effects caused by the electron exploring the outer regions of the condensate. We anticipate future experiments on electron orbital imaging, the investigation of phonon-mediated coupling of single electrons, and applications in quantum optics.

\[ V_{\text{pseudo}}(r, R) = \frac{2\pi\hbar^2 a}{m_e} \delta(r - R) \]

\( a \) is the short-range interaction of the quasi-free Rydberg electron at position \( r \) with neutral ground-state atoms at \( R \).

Figure 1 Size comparison in the spatial and energy domains. a. Depending on the Rydberg state, the electron impurity is localized in different volumes in a BEC consisting of \( N \approx 5 \times 10^3 \) atoms. The sizes of the lowest (110S, blue) and highest Rydberg state (202S, red) under investigation are indicated. The sizes of the BEC and the surrounding thermal cloud are to scale. The lower bound of the blockade radius \( r_B \) for the 110S state is denoted as a blue circle in the projection. The blockade radii for the higher Rydberg states are off the scale. b. The corresponding interaction potentials \( V(r) \) (blue and red) from equation (2) are orders of magnitude stronger than the contribution of the positively charged Rydberg core \( \text{Rb}^+ \) (green) except for very small distances. The mean interaction strength (black dashed lines) can be set below and above the chemical potential \( \mu \) of the condensate (horizontal black line) by choosing the Rydberg state. The healing length \( \xi = 274 \text{ nm} \) of the condensate (vertical black line) is much smaller than the spatial extent of the electron wavefunction.

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Here the interaction strength is fully characterized by the scattering length $a$; $\hbar$ is Planck's constant divided by $2\pi$, $m_e$ is the mass of the electron and $\delta$ the Dirac delta function. Given the Rydberg electron wavefunction $\Psi(r)$, the pseudopotential leads to a mean field potential:

$$V(R) = \frac{2\pi \hbar^2}{m_c} |\Psi(R)|^2$$

(2)

The interaction is restricted to a range given by the size of the Rydberg atom, which is proportional to $n^2a_0$, where $a_0$ denotes the Bohr radius. If this size is comparable to the mean interparticle distance, the interaction can lead to a bound state in the case of negative scattering lengths. Diatomic and triatomic molecules of this class have been experimentally observed for $^{87}$Rb atoms at principal quantum numbers around $n = 40$ and densities of the order of $10^{12}$ cm$^{-3}$. At higher densities, however, the lifetime of these bound states was found to decrease significantly.

The experiments described here are performed in a completely different regime. We combine much higher Rydberg states at principal quantum numbers $n = 110–202$ with high densities of up to $10^{14}$ cm$^{-3}$ in a BEC. In this regime, a significant fraction of the BEC is inside the electron orbit. Consequently, the single electron impurity has an impact on the BEC wavefunction as a whole. We vary the absolute number of atoms inside the electron orbit from 700 to up to 30,000 by changing the principal quantum number, which increases the spatial extent of the electron wavefunction up to the edges of the condensate (see Fig. 1).

We start with a magnetically trapped BEC of $^{87}$Rb atoms in the $5S_{1/2}, m_F = 2$ state. We illuminate the BEC with light from two lasers coupling the ground state to the $|nS_{1/2}, m_S = 1/2\rangle$ state ($n = 110–202$) via the intermediate $5P_{3/2}$ state (see Methods). The strong van der Waals interaction between Rydberg atoms in such high states prevents the simultaneous excitation of two Rydberg atoms within a certain distance $r_p$. For Rydberg states with $n > 100$, this blockade effect allows only a single Rydberg excitation in the whole BEC, because the blockade radius here is significantly larger than the Thomas–Fermi radii of the condensate (see Fig. 1a and Supplementary Information). This enables us to create one single electron impurity in the condensate. After a defined interaction time of the order of the Rydberg state lifetime, we apply an electric field pulse to remove the ionic core as well as the electron. We repeat this sequence several hundred times in a single condensate, before releasing the BEC and taking an absorption image after a time of flight of 50 ms. The long expansion time isolates the low-momentum components of the trapped atom cloud, as fast particles have already travelled out of the region of interest. From these time of flight images, we extract the change in atom number and aspect ratio of the BEC at different detunings of the excitation lasers relative to the single atom Rydberg level, and at different principal quantum numbers (see Fig. 2a, b).

We observe the impact of the electron impurity as a loss of atoms from the BEC after the time of flight. Compared to the Rydberg resonances measured in a thermal cloud at low densities the observed lines in the BEC are shifted to the red, with the magnitude of the shift being inversely proportional to the principal quantum number of the Rydberg atom. This shift is caused by the low-energy scattering of the single Rydberg electron from the BEC atoms. The contribution of the positively charged Rydberg core can be neglected in our case, because the range of this polarization potential is very small compared to the electron wavefunction (see Fig. 1b). Whereas the mean depth of the interaction

![Figure 2](image-url)
potential decreases from 12 kHz to 290 Hz with increasing principal quantum number, the number of atoms inside the electron wavefunction increases. Integrating the mean field potential $V(R)$ over all atoms inside the Rydberg electron wavefunction we obtain the total shift, which depends to lowest order\(^{14,15}\) only on the mean atomic density $\bar{n}$:

$$
\Delta E = \int d^3 R V(R) n(R) = \frac{2\pi\hbar^2}{m_e} \bar{n}
$$

(3)

For homogeneous densities the shift is therefore independent of the actual Rydberg state. This is consistent with our measurements at smaller principal quantum numbers where the local density approximation is fulfilled. For higher principal quantum numbers, however, we find a significant deviation. When the spatial extent of the Rydberg atom becomes comparable to the radial size of the BEC (see Fig. 1a), the electron is exploring mainly the low-density regions of the condensate. We can account for this by averaging the Thomas–Fermi density distribution over the size of the Rydberg atom. We assume the Rydberg atom is being excited in the centre of the condensate and use the average value of the peak density $\bar{n} = 8.6 \times 10^{13}$ cm\(^{-3}\) over one sequence. In Fig. 3a the calculated shift is plotted in comparison to the line shifts extracted from Fig. 2a. In addition to the values for a constant scattering length\(^{13}\) of $a = -16.1\ a_0$, a calculation taking higher-order scattering theory into account is shown (see also Supplementary Information). This simple model without any fitting parameters agrees quantitatively with our data.

The density-dependent shift of the Rydberg line allows us to control the position of the electron impurity inside the BEC with a precision much better than our optical resolution simply by changing the frequency of the excitation lasers. This is confirmed by the measured deformation of the condensate (Fig. 2b, c). The interaction of the electron impurity with the BEC excites a quadrupole shape oscillation (Fig. 2d and Supplementary Information) that leads to a change in the aspect ratio after the time of flight. In particular, we observe the strongest deformation on the red side of the BEC loss feature, that is, in the region of largest density-induced shift. Here the electron impurity is localized at the centre of the condensate and strong deformation of the condensate can be expected, whereas on the blue side and at high principal quantum numbers the mechanical effect averages to zero for repeated excitations. Quantitative modelling of this effect is, however, more involved and will be the subject of future studies.

To determine the actual interaction time of an individual impurity with the BEC, we measure the lifetime of the Rydberg atoms (see Fig. 3b and Supplementary Information). From empirical scaling laws\(^{22}\), we would expect the Rydberg lifetimes $\tau$ to increase with principal quantum number as $n^5$, here from 1.7 ms (at $n = 110$) to 10.8 ms (at $n = 202$). However, we observe a reduced lifetime of around 780 $\mu$s at densities around $10^{12}$ cm\(^{-3}\) in the thermal cloud for all Rydberg states investigated between $n = 110$ and 202. In the condensate, at peak densities just below $10^{14}$ cm\(^{-3}\), we find the lifetime to be further reduced by about two orders of magnitude. In this high-density regime we observe a lifetime increasing with principal quantum number. These two observations suggest that there is a dominant decay mechanism which is mainly dependent on the density of the gas. Further discussion of this effect can be found in Supplementary Information.

We now turn to the quantitative analysis of the fraction of atoms removed from the condensate owing to the electron impurity. To fully model this process, we have to take into account the coherent properties of the condensate and its excitations. We use perturbation theory to calculate the number of excitations in the condensate. Expressing the interaction potential $V(r)$ from equation (2) in terms of Fourier components $\rho_q = \int d^3 r e^{-iqr} \langle \Psi(r) \rangle^*$ of the electron density, the relevant part of the interaction reads (see Supplementary Information):

$$
\hat{H}_{\text{int}} = \frac{2\pi\hbar^2}{m_e} \bar{n} \sum_{q \neq 0} \rho_q (u_q - v_q) \left( b_q^+ + b_{-q}^+ \right)
$$

(4)

where $u_q$ and $v_q$ denote the Bogoliubov factors and $b_q^+$ is the creation operator of a collective excitation with quasi-momentum $q$. The finite lifetime $\tau$ of the Rydberg state leads to a time dependence of the perturbation, resulting in a finite Fourier width of the excitation. The shape of the potential $V(r)$ from equation (2) then determines which modes are actually populated within the allowed energy range. Because the outer edge of the electron density is located approximately at the Bohr orbit, which scales as $n^2 a_0$ (see Fig. 1b), the Fourier components $\rho_q$ show a maximum at low momenta, clearly in the regime of phonon type excitations. However we find that the excitations at higher momenta within the Fourier width also lead to a significant contribution (see Supplementary Information). After the time of flight, both phonon and free-particle excitations can be detected as atom losses. Taking this conversion into account, we are able to reproduce our experimental results without any free parameters. Figure 3c shows the maximum atom loss extracted from the data in Fig. 2a, divided by the square of the lifetimes $\tau$ measured for each Rydberg state. This way we remove the main dependence on $\tau$ (see Supplementary Information). The solid line in Fig. 3c shows the atom loss predicted by our Bogoliubov calculation, assuming a constant lifetime of $\tau = 10\ \mu$s (blue line) are shown. Error bars: 68% confidence bounds of Gaussian fits from Fig. 2a (a), exponential fits to the measured decay (b) and a combination of both (c).

Figure 3 | Energy shift and lifetime reduction of the Rydberg state in a condensate and Rydberg electron induced loss of BEC atoms. a. Theory curves with constant s-wave scattering length (blue) and taking higher-order scattering theory into account (red). The corresponding values neglecting the Thomas–Fermi (TF) density distribution are indicated as dotted lines. b. Power-law fits (dashed lines) to lifetime measurements in the condensate (circles) and the thermal cloud (squares). c. Plots of the atom loss per pulse from Fig. 2a divided by the square of the Rydberg lifetime $\tau$ in the condensate. Theory values taking the measured lifetime into account (red squares) and assuming a constant lifetime of 10 $\mu$s (blue line) are shown. Error bars: 68% confidence bounds of Gaussian fits from Fig. 2a (a), exponential fits to the measured decay (b) and a combination of both (c).
of a whole condensate by an impurity. A repulsive interaction between electron and BEC could be achieved by changing the spin state of the electron\(^2\). Tiny electric fields are sufficient to deform and manipulate the Rydberg electron wavefunction, offering further ways to control the coupling. The interaction of the impurity with excitations already present in the condensate could provide a model system for phonon-mediated coupling of electrons. From the perspective of the single Rydberg atom, the strong interaction of the excited electron with the trapped atoms will influence its quantum mechanical state as well as its motional degrees of freedom. These effects enable intriguing quantum optics applications. For example, the scattering from the cold gas could serve as a source of dephasing, forming a crucial part of the proposal for a single photon absorber\(^2\). Last, the BEC provides a sensitive probe for applications. For example, the scattering from the cold gas could serve as a source of dephasing, forming a crucial part of the proposal for a single photon absorber\(^2\).

**METHODS SUMMARY**

We start with a condensate of \(N = 8 \times 10^4\) atoms of \(^{87}\)Rb in the |S\(_{1/2}\), m\(_s\) = 2\rangle state in a cloverleaf-type magnetic trap at a high magnetic offset field of 1.355 mT (radial and axial trap frequencies are respectively \(\omega_j = 2 \pi \times 81.7\) Hz and \(\omega_z = 2 \pi \times 22.4\) Hz). We excite Rydberg states |nS\(_{1/2}\), m\(_s\) = 1/2\rangle (with \(n = 110\)–202) via a two-photon transition detuned by 500 MHz from the intermediate S\(_{1/2}\), m\(_s\) = 1/2\rangle state using continuous wave diode lasers at wavelengths of 780 nm and 480 nm. The blue laser beam has a power of 100 mW and is focused down to a size of 120 \(\mu\)m (1/e\(^2\) diameter). We choose the power of the red beam to be typically in the range of 3 \(\mu\)W at a 1/e\(^2\) diameter of 1 mm. The two laser beams are counter-propagating along the magnetic field axis of the trap. We address the desired transition by choosing \(\sigma^-\) and \(\sigma^+\) polarization for the 780-nm and 480-nm laser, respectively.

In each condensate, a sequence consisting of a 1-\(\mu\)s light pulse for Rydberg excitation and a 2-\(\mu\)s electric field pulse for removal of any remaining Rydberg atoms or ions, separated by 10-\(\mu\)s delay time, is repeated 200 (Fig. 2d), 300 (110S and 125S states) or 500 (other states) times at a rate of 62.5 kHz. The clearance field is set to 5.7 V cm\(^{-1}\), well above the ionization threshold of all Rydberg states under investigation. The magnetic trap is switched off immediately after the Rydberg sequence. After a time of flight of 50 ms we take an absorption image of the condensate. We determine the relative change in atom number and aspect ratio by comparing each measurement with a consecutive reference image in which the blue Rydberg laser is detuned by more than 40 MHz.

Further details about the set-up and the data analysis can be found in ref. 25 and in Supplementary Information.

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