ABSTRACT

The star formation rate in galaxies should be related to the fraction of gas that can attain densities large enough for gravitational collapse. In galaxies with a turbulent interstellar medium, this fraction is controlled by the effective barotropic index \( \gamma = \frac{d \log P}{d \log \rho} \) which measures the turbulent compressibility. When the cooling timescale is smaller than the dynamical timescale, \( \gamma \) can be evaluated from the derivatives of cooling and heating functions, using the condition of thermal equilibrium. We present calculations of \( \gamma \) for protogalaxies in which the metal abundance is so small that \( \text{H}_2 \) and HD cooling dominates. For a heating rate independent of temperature and proportional to the first power of density, the turbulent gas is relatively “hard”, with \( \gamma > \sim 1 \), at large densities, but moderately “soft”, \( \gamma < \sim 0.8 \), at densities below around \( 10^5 \text{cm}^{-3} \). At low temperatures the density probability distribution should fall rapidly for densities larger than this value, which corresponds physically to the critical density at which collisional and radiative deexcitation rates of HD are equal. The densities attained in turbulent protogalaxies thus depend on the relatively large deuterium abundance in our universe. We expect the same physical effect to occur in higher metallicity gas with different coolants. The case in which adiabatic (compressional) heating due to cloud collapse dominates is also discussed, and suggests a criterion for the maximum mass of Population III stars.

Key words: molecular processes; turbulence; stars: formation; ISM: molecules; galaxies: ISM; galaxies: evolution

1 INTRODUCTION

A physical understanding of the star formation rate (SFR) in galaxies remains elusive. Probably the only consensus is that the SFR must be controlled by the rate of formation of substructure, or “clouds”, dense enough that they can undergo gravitational collapse and fragmentation. Whether the rate of formation of dense clouds is controlled by large-scale gravitational instabilities, thermal instability, turbulent compression, or some other process is currently unknown, although all these possibilities have been discussed in various degrees of detail.

Of particular interest is the SFR in high-redshift galaxies. The reionization of the intergalactic medium and the existence of heavy elements and dust in galaxies with redshifts up to \( z \sim 5 \) (see discussion in sec 4.2 below) suggest that the first luminous objects must have appeared at larger redshifts. A large number of papers have tried to estimate the SFR and the mass of these early objects, the latter based mostly on a criterion that the cooling time due to \( \text{H}_2 \) rovibrational transitions be smaller than the collapse time (e.g. Ostriker & Gnedin 1996, Tegmark et al. 1997, Abel et al. 1998, Nishi & Susa 1999, and many earlier references given in these papers). The problem is complicated because the \( \text{H}_2 \) abundance is generally not in equilibrium (Shapiro & Kang 1987, Galli & Palla 1998 and references therein) and because there is feedback between the UV background generated by the earliest stars and the \( \text{H}_2 \) abundance, a feedback which may be negative (e.g. Haiman, Abel & Rees 1999) or positive (Ferrara 1998). The chemistry, cooling, hydrodynamics, and radiative transfer are all complex (e.g. Abel et al. 2000 and references therein). Norman & Spaans (1997) and Spaans & Norman (1997) proposed, using a very detailed model for the chemistry, cooling, and radiative transfer (but not the SFR or hydrodynamics), a scenario in which the SFR is kept at low levels until fine structure cooling by metals allows a thermal instability, resulting in the enhanced formation of dense clouds.

What all these studies have in common is that they consider the gas to be quasi-static, except for gravitational collapse. On the other hand, there is overwhelming evidence that the Milky Way and nearby galaxies are turbulent in some sense, with supersonic motions occurring over a very large range of scales (see the papers in Franco & Caraminha 1999 for a survey). Even in protogalaxies for which there is no stellar input to drive the turbulence, it seems plausible that the gas will be turbulent. The Reynolds numbers are extremely large, so shear and compression can be
amplified and transferred to smaller or larger scales non-linearly by fluid advection, and there is probably no magnetic field (or at least only a weak one) which might suppress the motions. The gravitational potential of the inhomogeneous protogalaxy may be the original energy source. When one considers that it is not easy to separate monolithic collapse from the accretion of smaller, perhaps transient, gas clouds, the protogalactic “turbulence” might resemble smaller-scale versions of the dynamically and spatially irregular systems modeled by Haenelt et al. (1998). However, numerical simulations are incapable of determining whether small-scale turbulence will occur because of lack of spatial resolution. Some weak evidence that protogalaxies may be turbulent was also found in the simulations of Kulsrud et al. (1997). Turbulence in protogalaxies might also alleviate the long-standing “cooling problem” for protodisk systems (see Navarro & Steinmetz 1997 and references therein) by reducing the efficiency of star formation even in the presence of strong cooling.

It is therefore of interest to examine the ability of turbulent motions to generate large density contrast clouds in protogalaxies with very small metal abundances. Compressible turbulence causes the gas in galaxies to “slosh” around, giving rise to a spectrum of density fluctuations. Simulations and phenomenological studies (Scalo et al. 1998, Passot & Vazquez-Semadeni 1998) for turbulent galaxies have shown that the density probability distribution (density) function \( f(\rho) \) may be controlled by the turbulent compressibility or the effective barotropic index \( \gamma = d \log \gamma / d \log \rho \). When \( \gamma \) is small, the gas is “soft” with respect to compression by the flow, and large densities may be attained compared to gases with larger \( \gamma \). If \( \gamma \) is significantly smaller than unity, an approximately power-law \( f(\rho) \) is predicted, while for \( \gamma \) close to unity \( f(\rho) \) should have a nearly lognormal form (Scalo et al. 1998, Passot & Vazquez-Semadeni 1998, Nordlund & Padoan 1999). To the extent that the SFR is some increasing function of the integral of \( f(\rho) \) at large densities, we expect the SFR to be much larger in the power-law (small-\( \gamma \)) case compared to the lognormal (large-\( \gamma \)) case, due to the presence of a longer tail at large densities in the former case (cf. Fig. 10 of Scalo et. al. 1998). The situation is more complicated when the role of magnetic field fluctuations is considered (Vazquez-Semadeni & Passot 2000), but protogalaxies are expected to have negligible magnetic fields (Ruzmaikin et al. 1988) and hence we neglect this effect here.

The present paper tries to estimate the effective softness, as measured by the effective barotropic index \( \gamma \), for a turbulent protogalaxy whose cooling is dominated by \( \text{H}_2 \) (and HD at the lowest temperatures). Our approach does not include the details of \( \text{H}_2 + \text{HD} \) chemistry and UV feedback, but instead evaluates \( \gamma \) for a galaxy in which \( \text{H}_2 + \text{HD} \) cooling dominates, with the \( \text{H}_2 \) and HD fractions given as parameters. As will be seen, the behavior of \( \gamma \) is largely independent of the \( \text{H}_2 \) fraction as long as it is small. Our goal is only to map out the regions of the temperature-density space in which \( \text{H}_2 + \text{HD} \) cooling yields “soft” (\( \gamma < 1 \)) or “hard” (\( \gamma \gtrsim 1 \)) behavior, which suffices to estimate the relative fraction of the gas driven to large densities.

We emphasize that our view of the SFR is fundamentally different from the model proposed by Norman & Spaans (1997) and Spaans & Norman (1997); see also Spaans & Carollo (1997). These authors presented detailed comprehensive models for many of the physical processes as a function of redshift, but ignored the potential effects of turbulent compressions. For this reason they were led to assume that efficient star formation will only occur when sufficient metal production had occurred so that fine-structure cooling would induce a phase transition via thermal instability, at redshifts \( z \sim 1 - 2 \). Besides, in view of the possibility that turbulence effectively suppresses the thermal instability phase transition (Vazquez-Semadeni, Gazol & Scalo 2000), this picture neglects the possibility that high density condensations can be formed by turbulence even in a pure \( \text{H}_2 \) galaxy, if the value of \( \gamma \) is sufficiently less than unity. Our view of turbulent cloud and star formation is similar to the picture envisioned by Elmegreen (1993) and Padoan (1995), except for the sensitivity to \( \gamma \).

A study complementary to the present work has been given by Spaans & Silk (2000), who calculated \( \gamma \) for conditions expected in quasi-static self-gravitating molecular (i.e. shielded) clouds with metallicities \( Z > 0.01Z_\odot \). Their calculations include a detailed treatment of the chemistry and radiative transfer, various coolants not considered in the \( Z=0 \) cases treated here, the thermal coupling between gas and dust, and heating by the infrared Cosmic Background Radiation, which is important for redshifts between about 10 and 40 if the metallicity is large enough to allow significant dust abundances. They also discuss many implications of the resulting \( \gamma \) for stellar masses, high-redshift star formation, and starburst galaxies, all in the context of the ability of self-gravitating objects to collapse. In contrast, the present work is concerned with the \( Z < 0.01Z_\odot \) case, parameterizes the chemistry in terms of assumed \( \text{H}_2 \) and HD abundances, neglects dust grains (probably justified for these small metallicities), and is focused on the maximum densities, and hence star formation rates, that can be attained in a turbulent galaxy or cloud. Primarily because of the larger assumed metal abundances, the values of \( \gamma \) derived by Spaans and Silk and their dependence on density is somewhat different than found here for a pure \( \text{H}_2 + \text{HD} \) gas.

In section 2, we show explicitly how \( \gamma \) is determined by the logarithmic derivatives of the heating and cooling rates. Numerical evaluation of these derivatives and \( \gamma \) for a gas containing \( \text{H}_2 \) and HD are presented in section 3. Our interpretation of these results and their applicability to galaxies are discussed in section 4.

## 2 The Effective Barotropic Index

There are several physical processes by which turbulent interactions affect the density field, and these are mediated by \( \gamma \). A specific process for density amplification would be the supersonic interaction of two velocity streams or clouds, which will form a dense slab which may be subject to gravitational instability (Elmegreen 1993) and Vishniac’s (1994) nonlinear bending mode instability (Blondin et al. 1996, *It should be realized that even terrestrial turbulence is simply a ubiquitous empirical fact which numerical simulations attempt to model, but could not predict, at least until very recently.*
Klein & Woods 1998). Vazquez-Semadeni, Passot & Pouquet (1996) pointed out that, while the density jump behind a shock resulting from such an interaction is expected to be of order $M^2$ ($M$ = Mach number) for $\gamma = 1$ (isothermal flows), the density jump approaches $e^{M^2}$ as $\gamma = 0$. Also the minimum Mach number required to induce gravitational instability in a given region is a strong function of $(1 - \gamma)$, as shown in various contexts by Hunter & Fleck (1982), Tohline et al. (1987) among others. The importance of the effective barotropic index ($\gamma$) of the ISM of galaxies was explored by numerical simulations by Vazquez-Semadeni et al. (1996) in the context of supersonic turbulence.

For a large range of conditions relevant for the interstellar gas in galaxies it is true that, except for regions immediately behind shock fronts, the dynamical timescale of the gas motions in a region of size $L$ and characteristic velocity $v$, $L/v$, is much larger than the cooling time due to microscopic processes. This inequality holds for a large number of environments, including diffuse HI and molecular gas in our own and other galaxies, including protogalaxies (depending on the scale of interest). The inequality implies that the heating and cooling terms in the fluid energy equation will easily adjust to balance each other on the timescale of the dynamical flows, i.e. the gas can be considered to be in thermal equilibrium, as first emphasized by Vazquez-Semadeni et al. (1996). The velocity and density fields are controlled by the momentum and continuity equations, while the temperature (and hence pressure) are slaved to the relatively slowly-varying density field through the thermal equilibrium condition, which determines a unique temperature (and pressure) for any given density. In effect, the gas behaves as a barotropic fluid with pressure given by $P = \rho^\gamma$ where $\gamma$ is determined by the thermal equilibrium condition.

A general expression for $\gamma$ can be derived from the thermal equilibrium condition

$$\Lambda(\rho, T) = \Gamma(\rho, T)$$

where $\Lambda$ and $\Gamma$ are the cooling and heating rates per unit volume. The index $\gamma$ is defined by

$$\gamma \equiv \frac{d \log \rho}{d \log P} = 1 + \frac{d \log T}{d \log \rho}$$

where the perfect gas equation of state was used in the last step and we neglect variations in the mean molecular weight. Defining the function $F(\rho, T) = \Gamma(\rho, T) - \Lambda(\rho, T) = 0$ (by eq.1), we have after implicit differentiation,

$$\frac{d \log T}{d \log \rho} = \left( \frac{d \log T}{d \log \Lambda} \right) \left( \frac{d \log \Lambda}{d \log T} \right)$$

and

$$\gamma \equiv 1 + \frac{d \log \rho}{d \log P}$$

γ then follows from eq. 2. Our goal is to evaluate the derivatives of the cooling function for cases in which the dominant coolants are H$_2$ and HD. In the present work we use the detailed calculations of H$_2$ cooling by LeBoule, Pineau des Forets, & Flower (1999) and Flower et al. (1999), which include collisions with H, H$_2$, and He, and use quantum mechanical cross-sections that supersede previous work. The disagreement in $\gamma$ that would be obtained using previous work can be seen by examining the slopes of the cooling curves presented in Figure A1 of Galli & Palla (1998).

For the heating rate we take the parameterized form $\Gamma \propto \rho^aT^b$. The simplest case is for optically thin heating by UV photons or heating by cosmic rays, in which case $a = 1$ and $b = 0$ to good approximation. Since in true primordial protogalaxies with no stars it is difficult to imagine such heating, we are assuming that some low level of star-forming activity has already occurred. It is instructive to notice that if the cooling rate is similarly parameterized as $\Lambda = \rho^cT^d$, then

$$\gamma = 1 + (1 - c)/d$$

If $c=1$ (for $a = 1$ and $b = 0$), this immediately shows that for densities large enough for collisional deexcitation to be important in controlling the level populations, so that $\Lambda \sim \rho$ (instead of $\rho^2$ at lower densities), the effective barotropic index will be close to unity (isothermal), and hence the turbulence will be relatively “hard.” The common assertion, supported marginally by observations, that dense molecular gas without internal star formation should be nearly isothermal is due to the fact that the densities in in these regions exceed the critical density for CO collisional deexcitation, not because the CO cooling is particularly efficient, as is often stated. In fact the range of densities for this condition to hold for CO is relatively small, as pointed out by Scalo et al. (1998). The same sort of effect will be seen below for H$_2$+HD.

We next examine the behavior of the logarithmic derivatives of the H$_2$ and HD cooling rates, and evaluate the turbulent compressibility for the case of a simple UV or cosmic ray dominated heating rate.

### 3 RESULTS

From the expression for $\gamma$ derived in the previous section we see that it is determined by the derivatives of the cooling function. Thus, in order to obtain the correct compressibility it is important to determine the proper cooling function (and its derivatives) for the primordial gas. Fortunately, making a theoretical estimate of the cooling function in the primordial universe is simpler than the corresponding problem in the present day universe, where there exists a large number of coolants.

Since Saslaw & Zipoy (1967) first discussed the importance of the H$_2$ molecule in cosmology it has been established that main coolant in the early universe below a temperature of 10$^4$ K is the H$_2$ molecule (see Galli & Palla 1998 for a review of astrochemistry of the early universe, and Stancil, Lepp and Dalgarno 1996 on the possible importances of the other molecules, such as HD and LiH). The calculation of the H$_2$ abundance is considerably complicated by the non-equilibrium nature of the chemical kinetics (Shapiro & Kang, 1987), which elevates the H$_2$ abundance compared to its equilibrium concentration. The detailed chemistry of H$_2$ formation is not treated here, and we take the H$_2$/H ratio as a parameter. Estimates of the H$_2$/H fraction by Tegmark et al. (1997) and Ferrara (1998) suggest an H$_2$ fraction of 10$^{-3}$ for protogalaxies. However a detailed consideration of the chemical pathways for H$_2$ formation by Galli & Palla (1998) gives for average cosmic gas density H$_2$/H ~ 10$^{-6}$ for redshifts less than about 100. At high redshifts ($z \geq 100$), H$_2$ formation is inhibited even in overdense regions because the required intermediaries H$_3^+$ and H$^+$ are dissociated by cosmic microwave background (CMB) photons. The H$_2$ abun-
dance can be as high as $10^{-3}$ inside collapsed clouds (Abel & Haiman 1997, Abel et al. 2000; see section 4.1 for discussion). We will show that the dependence of $\gamma$ on $H_2$ fraction is very small for $H_2$ fractions within three orders of magnitude of the adopted value of $10^{-6}$, because $\gamma$ only depends on the logarithmic derivatives of the cooling function with respect to temperature and density, not the absolute value of the cooling function itself. This result becomes invalid if the $H_2/H$ fraction is a strong function of temperature or density, which would need to be included in the logarithmic derivatives. We neglect this effect here, since they require detailed chemistry calculation. We have calculated $\gamma$ for $H_2$ abundances of $10^{-6}$ and $10^{-3}$ here.

Le Bourlot et al. (1999) have recently calculated the cooling functions for a gas containing $H_2$ molecules, which are collisionally excited by $H$, $He$, and $H_2$. They have computed the $H_2$ cooling rates per molecule from a detailed computation of non-LTE level populations and quantum-mechanical collisional cross-sections, assuming the gas to be optically thin to these transitions. They have provided a dataset of calculated cooling values in a wide temperature ($10^2-10^4K$), density ($1-10^6cm^{-3}$), $n_{H}/n_{H_2}$ ($10^{-8}$ to $10^0$), and $n_{ortho}/n_{para}$ (0.1 to 3) range that we have adopted for calculating cooling values for our purpose. The results are insensitive to $n_{ortho}/n_{para}$ which we take as unity.

Although the HD abundance is much less than $H_2$, the presence of a non-zero permanent dipole moment and smaller rotational constant make it a potentially more important coolant at lower temperatures. The question of HD abundance in the post-recombination era is also well-studied (see Puy et al 1993, Tegmark et al. 1997 and references therein). We have adopted values from Galli & Palla (1998) which give this ratio to be $[HD/H_2] = 1.1 \times 10^{-3}$.

The HD cooling function is taken from Flower et al (1999), who provide a routine for calculating cooling values due to HD rotational transitions within the vibrational ground state of HD, collisionally excited by $H$, $H_2$, and $He$, again assuming optically thin transitions.

The $H_2$ and HD cooling rates have been combined to estimate the total cooling rates for a wide temperature ($10^2-10^4K$) and density ($1-10^6cm^{-3}$) range. These are shown in Figure 1. Since Le Bourlot et al. have provided cooling rates only for values of temperature and density which are too widely spaced for accurate calculation of derivatives by direct differentiation, we first needed to produce a local polynomial fit for the cooling function to avoid “jittery” derivatives that will result from their linear interpolation program. From the requirement of continuous second-order derivatives we used a cubic spline interpolation of the cooling function.

In the adopted parameterized heating rate $\Gamma \propto \rho^{\alpha}T^{\beta}$, the constant of proportionality is arbitrary. We do not adopt any particular value for this constant of proportionality as the cosmic ray and/or UV fluxes in protogalaxies are highly uncertain, depending upon (among other things) the unknown star formation rate. We incorporate this arbitrary scaling of the magnitude of the heating rate by letting the temperature be an independent variable, not coupled to the density by the thermal equilibrium condition (which we use only to calculate logarithmic derivatives). Thus the labelling of curves in our plots by temperature is essentially a labelling by the amplitude of the heating rate. The temperature range in actual protogalaxies may be inferred by comparison with the hydrodynamic simulations of Abel et al. (2000), which suggest that $T = 200-800K$ may be appropriate. A brief discussion of $\gamma$ in the case where adiabatic heating dominates is given in sec. 4.1 below.

Figures 2(a) and 2(b) show the logarithmic derivative of the $H_2+HD$ cooling function with respect to logarithmic density and temperature. The derivative with respect to logarithmic density has a value close to 2 at low densities and 1 at high densities as anticipated. The transition of cooling function from a quadratic dependence on density to a linear dependence takes place around the critical density above which collisional deexcitation dominates radiative decay as discussed, for example in Spitzer(1978). The derivatives with respect to temperature do not show appreciable variation with density at higher temperature and remain more or less constant at a value close to 3. This general behaviour can be shown using the analytic prescription for the $H_2$ cooling function given by Tegmark et al. (1997) based on Hollenbach and McKee (1979).

Plots of turbulent compressibility ($\gamma$) for various temperatures and $H$ number density for $[H_2/H] = 10^{-6}$ and $[HD/H_2] = 1.1 \times 10^{-3}$ are shown in Figures 3(a) and 3(b) respectively. From Figure 3(a) it is noticed that at the lowest temperatures $\gamma$ values shoot up near the critical density of HD (see section 4.3) as collisional deexcitation takes over from radiative decay. On the other hand, at high temperatures $\gamma$ varies quite smoothly with density. We attribute this to the fact that at higher temperatures the higher energy states are populated, and there is no single critical density characteristic of a two-level system, as pointed out by Le Bourlot et al. (1999) and others. Overall, between temperatures of 100 to 2000 K, above which $H_2$ molecules begin to dissociate, $\gamma$ remains within a range of 0.7 to 1. It takes a large value of 1 at large densities because cooling rates become proportional to number density. With decreasing
density, $\gamma$ decreases as the cooling function becomes proportional to $n^2$ in the low density limit. Fig. 3(b) shows that there is not much dependence of $\gamma$ on temperature for a given density.

In Figures 4(a) and 4(b) we show how $\gamma$ varies with density when the $H_2/H$ ratio is different ($10^{-3}$). The $HD/H_2$ ratio is assumed to be the same as earlier. A comparison of Figure 4(b) with Figure 3(a) shows there is little difference in $\gamma$ due to change in $H_2/H$ ratio within three orders of magnitude. Figure 4(a) shows how $\gamma$ varies with density when HD is absent. A comparison of Figure 4(a) with 4(b) shows that at small densities and low temperatures the gas is much harder in absence of HD. This implies that HD cooling is “softer” at low densities than $H_2$ cooling. At higher temperatures the two plots look almost identical because the contribution of HD to the total cooling function becomes insignificant in those temperatures.

4 DISCUSSION

4.1 Density Range

Because our derived values for the turbulent compressibility depend sensitively on whether the density is above or below the effective critical density for collisional deexcitation, it is important to understand the density ranges expected in protogalactic objects. Although no certain conclusion can be reached, suggestive results can be found in the study of Abel, Bryan, & Norman (2000). Abel
et al. found that primordial molecular clouds with masses \( \sim 10^7 \text{M}_\odot \) are formed at the intersections of filaments at redshifts around 40, with average \( \text{H}_2 \) fraction less than about \( 10^{-4} \) and particle densities much less than \( 1 \text{cm}^{-3} \). The density at the center of the most massive clump during the period \( Z = 35 \) to 23 is about \( 0.3 \) to \( 3 \text{ cm}^{-3} \), with \( T \) rising from \( \sim 200 \text{K} \) to \( \sim 800 \text{K} \) due to adiabatic heating, and \( f(\text{H}_2) \) increasing from about \( 10^{-5} \) to \( 10^{-4} \) during this period. After redshift \( \sim 23 \) the density increases rapidly, reaching \( 10^4 \text{cm}^{-3} \). In the densest core the cooling time is comparable to the free fall time, so our assumption of fast cooling is not valid.

For these reasons our calculations are applicable only to the GMC-like structures which have smaller densities. The largest densities are presumably due to the fact that the low-density gas is highly compressible, as we have explained in terms of the basic cooling physics here, while we expect the evolution of the dense cores, even when aided by self-gravity, to be relatively "hard", since at these large densities the \( \text{H}_2 \) is nearly in LTE and the value of \( \gamma \) should be large. The value of \( \gamma \) probably remains large during the contraction of the cores if they are able to gravitationally collapse (Spaans & Silk 2000).

This can be seen as follows. If adiabatic (compressional) heating at the free-fall rate dominates, then using \( \nabla \cdot \vec{u} = -d\log \rho / dt \sim 1/\tau_{ff} \) in the internal energy equation shows that the heating rate scales as \( \rho^{3/2} \) (using \( \tau_{ff} \sim \rho^{-1/2} \)). Using this heating rate and reading off the logarithmic derivative of the cooling rate from Fig. 2, equations 2 and 3 show that at densities \( n = (10^4, 10^5, 10^6), \gamma \approx (0.8, 1.0, 1.3) \). The latter value of \( \gamma \) holds approximately for all densities greater than \( 10^7 \text{cm}^{-3} \). Thus the gas becomes increasingly "hard" with increasing density. Only at the lowest densities is the gas moderately "soft". This is similar to what we found for diffuse heating, except that the values of \( \gamma \) are larger and the densities required for "soft" behavior are smaller in the compressional heating case. Notice that for \( n \gtrsim 10^6 \) the value of \( \gamma \) is sufficiently close to the critical value of \( 4/3 \) that classical fragmentation should be prevented, i.e. the thermal Jeans mass cannot decrease with increasing density. A similar result was found by Spaans & Silk (2000) who consider gas-grain heating rather than adiabatic compressional heating. This implies that the mass associated with densities around \( 10^6 \text{cm}^{-3} \) will be the maximum mass of Pop III objects. However the result is tentative, since our assumption of thermal balance may not be valid for collapsing objects.

### 4.2 Metallicity and Redshift

Our results are only relevant for protogalaxies whose metallicity is so small that \( \text{H}_2 \) and HD cooling dominates. Based on the detailed calculations of Norman & Spaans (1997) and Spaans & Norman (1997), the transition metallicity is \( Z \sim 0.01Z_\odot \). At larger \( Z \) cooling by fine structure atomic and ionic lines dominates for the densities and temperatures of interest. We adopt this as the critical metallicity here.

The question of the redshift at which this critical \( Z \) is reached in a cosmic-averaged sense cannot be answered at present. Metals and dust have been observed in galaxies with redshifts of at least 5 (Armus et al. 1998). Observations of Fe and, to a lesser extent, Zn in high-redshift Ly \( \alpha \) clouds (Prochaska & Wolfe 2000) indicates that the column density-weighted Fe abundance relative to the solar system is about 0.025 for redshifts between 1 and 4, and is remarkably constant. There is a large range in Fe abundance at given redshift, but only a small fraction of Ly \( \alpha \) clouds have Fe abundances as small as the estimated critical value, suggesting that the H\(_2\) cooling-dominated phase terminates at redshifts less than at least 4 on average. Pettini et al. (1997) estimate the average metallicity in damped Ly \( \alpha \) systems to be \( \sim 0.05Z_\odot \) at \( Z=3 \). For the lower column density Ly \( \alpha \) forest clouds, carbon abundances (Tytler et al. 1995, Songaila & Cowie 1996 and references therein) are approximately 0.01 times solar, but the overall metallicity (dominated by oxygen) may be larger. An extreme lower limit for the metallicity of the Ly \( \alpha \) forest at redshift 3 is

![Figure 4](image-url)
0.001Z⊙ (Songaila 1997). Observations of both emission and absorption lines intrinsic to QSOs give solar or higher metallicities out to redshifts of at least 4 (see Hamann 1998 and references therein). How these results relate to metallicities in the bulk of galactic gas is currently unknown, but do show that the present results are only applicable to galactic nuclei at much larger redshifts. Based on studies of the intergalactic medium and elliptical galaxies in clusters, Renzini (1998) has argued that the mean metallicity of the universe at z ∼ 3 is about 0.1Z⊙.

Theoretically, the problem is extremely complicated, since the metal enrichment rate depends on both the star formation rate, which is essentially a parameter in all models and is only very weakly constrained by observations, the IMF, which is unknown, the ejection of metals from galaxies by supernova-energized outflows, and other factors. The redshift corresponding to Z= 0.01Z⊙ in the Norman & Spaans (1997) models was about 3. A recent calculation of the evolution of the cosmic metallicity by Pei, Fall, & Hauser (1999), assuming an IMF lower mass limit of 0.1M⊙, shows that metallicities as small as 0.01Z⊙ occur only for redshifts greater than 5, consistent with the limit from the Ly α observations. As pointed out by Padoan et al. (1999) and others, if the IMF is strongly weighted toward high-mass stars, metal enrichment might only take about 10^7 yr because the metal yield (mass of metals produced relative to total mass incorporated into a generation of stars) will be large. A plot of yields as a function of IMF lower mass cutoff is given in Scalo (1990). Another way to look at the problem is to assume a redshift for the formation of the first star-forming objects (1990). Another way to look at the problem is to assume a redshift for the formation of the first star-forming objects and timescale for metal enrichment. For example, the simulations of Abel et al. (2000) produce GMC-mass clouds at redshift Z ∼ 40, in rough agreement with some previous non-simulation estimates (e.g. Tegmark et al. 1997). If star formation proceeds with a “normal” IMF, the duration of metal enrichment may be about 10^8 years, corresponding to a critical metallicity of 0.01Z⊙ at redshift 15 (assuming A = 0, q = 1/2, H$_2$ = 75 cosmology). If the protogalaxy IMF is extremely top-heavy, then the enrichment may only take about 10^7 yr, corresponding to a redshift of about 34 for the same cosmology.

We conclude that if the IMF in galaxies is not extremely top-heavy, then our calculations of γ for a gas dominated by H$_2$ + HD cooling may be relevant, in a cosmic-averaged sense, from the epoch of galaxy formation to redshift about 4-5, with great uncertainty. In the “worst case” scenario in which star formation was rapid and the IMF was heavily weighted to massive stars, the duration of metal enrichment ∼ 10^7 yr implies that the H$_2$-dominated phase covered the narrow redshift range from around 40 to 30. However the cosmic average may not be very relevant. One general conclusion that follows from the observational considerations above is that the universe was chemically very inhomogeneous at redshifts up to ∼ 5.

### 4.3 The critical density

To use these results to predict SFR in protogalaxies we need to have predictions about the critical value of γ below which the density pdf develops a power-law tail. This tail behavior occurs at γ values as large as 0.7 to 0.8 in simulations by Passot and Vazquez-Semadeni (1998) and Scalo et al. (1998).

So depending upon the critical value of γ, densities from several hundreds to several thousands should be commonly attained by turbulent compressions. If gravitational collapse can commence at such densities, then the SFR in protogalaxies should be appreciably larger than that has been previously speculated (e.g. Spaans & Norman 1997, Norman & Spaans 1997). On the other hand, γ approaches a value of unity above densities ∼ 10^3. So if such densities or higher are required for commencement of gravitational collapse then the SFR should be low in protogalaxies. In any case we note that the H$_2$+ HD gas can only attain a lowest value of γ of 0.7, which is still considerably “harder” than for fine structure cooling of atomic gas. Thus the conclusion of Norman & Spaans (1997) and Spaans & Norman (1997) that the SFR should peak when the fine structure cooling commences may be valid, but because of the differences in turbulent compressibility, not because of thermal instability.

The most interesting result is that at low temperatures the gas becomes abruptly hard above the critical density for collisional de-excitation for HD, implying that structure formation at those temperatures is unlikely to occur above the critical density. We conclude that the structures that form by turbulent interactions at temperatures below a few hundred degrees can be characterized by an upper density limit which is the critical density for the equality of collisional and radiative de-excitation, since the value of γ approaches unity for larger densities. This result is similar to the situation for CO-dominated cooling in contemporary molecular clouds where γ is small below the critical density for collisional de-excitation (Scalo et al. 1998). This result implies that realistic density distributions for protogalactic or present day large metallicity galaxies can only be obtained from simulations if the details of cooling function are accounted for; isothermal simulations will clearly underestimate the densities of clouds formed by turbulence.

We predict that the distribution of gas densities formed...
by turbulent interactions should have a peak corresponding to the critical density for HD collisional deexcitation, which is \( \approx 10^4 \text{cm}^{-3} \). It is important to note that if HD cooling is absent the critical density would be determined by \( \text{H}_2 \), which has a critical density of \( \approx 10^2 \), as seen from Fig. 5.

It is interesting that densities larger by a factor of hundred can be attained by turbulence in a universe like ours, with relatively large D abundance, compared to a situation in which D abundance is much smaller.

Galaxies with larger metal abundances should have a peak at densities corresponding either to fine structure cooling (if molecules cannot form) or carbon monoxide (dominant coolant for molecular gas at moderate densities) cooling (if molecules cannot form) or carbon monoxide (dominant coolant and peak at densities corresponding either to fine structure cooling or carbon monoxide (dominant coolant) cooling (if molecules cannot form) or carbon monoxide (dominant coolant) cooling (if molecules cannot form) or carbon monoxide (dominant coolant).

The critical density when CO is the dominant coolant is roughly \( 10^3 \text{cm}^{-3} \) depending on the temperature (see Goldsmith & Langer 1978). If the medium is optically thick to its own radiation, then radiative trapping can also reduce the density dependence of the cooling rate; however a proper treatment must include radiative line transfer, and there is currently no viable model for line transfer in a turbulent medium. Existing calculations that use escape probabilities show the sensitivity of the radiative trapping critical density to the adopted velocity gradient parameter (Goldsmith & Langer 1978, Qaiyum & Ansari 1987) for CO.

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