Colour Octet Effects in Quarkonium Physics
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The importance of including colour octet contributions in describing decay and production of quarkonium states is briefly discussed for two cases: the radiative decays of the Υ and the production of J/ψ in the inclusive B decays. It is shown how information on the non-perturbative matrix elements can be obtained by comparing the theoretical expressions computed at next-to-leading order in $\alpha_s$ with the experimental data.

1. Introduction

It is now about four years that the non-relativistic QCD approach for the description of phenomena involving quarkonium production and decays, has been introduced [1]. Without any doubt, Bodwin, Braaten and Lepage have provided us with a powerful and rigorous framework where calculations can be undertaken both at perturbative and non-perturbative level. Within this approach, the factorization between short-distance and long-distance physics allows to express physical quantities as inclusive decay widths or cross sections as:

$$ Q(H) = \sum_n c_n \langle O_n(H) \rangle $$

where in $c_n$ are encoded the high-energy modes of the theory and so can be calculated perturbatively in QCD, while the matrix elements (MEs) are non-perturbative and can be either obtained by lattice simulation [2] or by comparison with experimental data. Moreover, by means of the power counting rules of NRQCD, it is possible to organize the MEs in a hierarchy based on the relative velocity $v \ll 1$ of the heavy-quarks in the bound state. In the end one is able to organize all these terms in a double expansion in $\alpha_s$ and in $v$ and to make predictions, in principle, at any given order of accuracy.

With NRQCD in hands, much of the effort of the last years (for very nice reviews on more recent developments see for example [3]) has been devoted mainly to improve at NLO in $\alpha_s$ the determination of the short distance coefficient $c_n$ for the most relevant terms in the $v$ expansion and use the data available on quarkonium production and decays to extract information on the non-perturbative MEs. The aim of the game has been to check the cornerstone of the predictive power of the theory: the MEs should be insensitive to the details of the hard processes and depend in a universal way only on the bound state physics. This allows, in principle, to extract the values of the non-perturbative parameters from the experimental data on one process and then to use them to make predictions for others.

In the following Sections I will discuss the necessity of including the colour octet contributions at NLO to correctly predict and/or explain experimental data for two cases of interest.

2. The photon spectrum in bottomonium decay

A consistent description of the photon energy spectrum in $\Upsilon \rightarrow \gamma + X$ decay requires the inclusion of the fragmentation components within the NRQCD factorization approach [4]. The differential photon decay can be expressed in terms of a convolution between partonic kernels $C_a$ and the fragmentation functions $D_{a\rightarrow\gamma}$:

$$ \frac{d\Gamma}{dz} = C_\gamma(z) + \sum_{a=q,g} \int_{z}^{1} \frac{dx}{x} C_a(x, \mu_F) D_{a\rightarrow\gamma}(\frac{z}{x}, \mu_F) $$

1Emission from final state light quarks has been surprisingly considered only recently by Catani and Hautmann in the CSM framework (and presented here in Montpellier in the QCD94 Conference ! [5]).
where $z = E_\gamma/m_Q$ is the rescaled energy of the photon ($m_Q$ is the heavy quark mass). The first term corresponds to what is usually called the ‘prompt’ or ‘direct’ photon production where the photon is produced directly in the hard interaction while the second one corresponds to the long distance fragmentation process where one of the partons fragments and transfers a fraction of its momentum to the photon. The NRQCD expansion for the coefficients $C_i(x)$ reads:

$$C_i = \sum_{Q} \hat{C}_i[Q](\alpha_s(m_Q)) \frac{\langle \Upsilon|O(Q)|\Upsilon \rangle}{m_Q},$$  \hspace{1cm} (3)$$

where $i = \gamma, q, \bar{q}, g$ and $\hat{C}_i[Q](x, \alpha_s(m_Q))$ are the perturbative coefficients. The NRQCD sum is performed over all the relevant $Q$ states that contribute at a desired order $v$. The perturbative coefficients for the colour octet states are now known at NLO $^3$. We can then investigate the phenomenological applications of colour octet states, if information on the NRQCD MEs can be obtained. To this aim we proceed in two steps: first we obtain estimates by solving the renormalization group (RG) equations $^3$ and then we test the reliability of these estimates analyzing their impact on the observable $R_\mu(\Upsilon) = \Gamma(\Upsilon \rightarrow \text{had})/\Gamma(\Upsilon \rightarrow \mu^+\mu^-)$, whose value is well-known both experimentally $^9$ and theoretically at NLO (both for singlet and octet contributions $^{10}$). As a result the RG estimates turn out to be smaller that one could expect form the naive velocity scaling rules and so legitimate doubts on their reliability rise. Nevertheless the above mentioned analysis shows that on one hand RG estimates are to be thought as lower limit while on the other, consistency between theory and experiment in total decay rates, strongly disfavor much larger colour octet MEs. I remark here that these conclusions strongly rely on our present knowledge of the theoretical expression which is only at NLO. As a recent calculation shows $^{11}$, NNLO corrections can be very large. In fig. 1 are shown the various contributions to the spectrum. At low values of $z$ the fragmentation from octets is of the same order of magnitude of the LO colour singlet one. Contrary to LO expectations in the framework of CSM $^6$, we conclude that the decay of $\Upsilon$ into a photon would not be useful for estimating photon fragmentation functions. For values of $z$ near the end-point, breaking of the fixed-order calculation is manifest and the resummation of both short-distance coefficient in $\alpha_s$ and the non-perturbative MEs in $v$, is called for. Finally we notice that the overall effect of octet states is at its minimum in the central region of the spectrum (exactly where the singlet LO direct contribution dominates) and so it should be used to make comparison with experimental data. Finally it has to be stressed that relativistic and higher order strong corrections to the singlet should be included to have a consistent theoretical picture at NLO $^2$.}

\[ ^2 \text{The calculation of NLO corrections to the decay } ^3S_1^{(1)} \rightarrow gg\gamma \text{ are in progress}^{14}. \]
3. The inclusive decay $B \to J/\psi + X$

In the case of a inclusive B decay into a charmonium state the following factorization formula holds \[1\]:

$$\Gamma(B \to H + X) = \sum_n C(b \to c\bar{c}[n] + x) \langle O^H[n] \rangle,$$  \hspace{1cm} (4)

which is valid up to power corrections of order $\Lambda_{QCD}/m_{b,c}$. (To this accuracy it is justified to treat the $B$ meson as a free $b$ quark.) The parameters $\langle O^H[n] \rangle$, defined in \[3\], describe the hadronization of a couple of heavy quarks into a charmonium state while the coefficient functions $C(b \to c\bar{c}[n] + x)$ describe the production of a $c\bar{c}$ configuration $n$ at short distances and can be expanded in the strong coupling $\alpha_s(\mu)$ at a scale $\mu$ of order $2m_c$. The terms of interest in the $\Delta B = 1$ effective weak Hamiltonian \[9\]

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=s,d} V_{cb}^* V_{cq} \left[ \frac{1}{3} C_{[1]} O_1 + C_{[8]} O_8 \right]$$  \hspace{1cm} (5)

contain the ‘current-current’ operators

$$O_1 = \left[ \bar{c} \gamma_{\mu}(1 - \gamma_5)c \right] \left[ \bar{b} \gamma^\mu(1 - \gamma_5)b \right]$$  \hspace{1cm} (6)

$$O_8 = \left[ \bar{c} T^A \gamma_{\mu}(1 - \gamma_5)c \right] \left[ \bar{b} T^A \gamma^\mu(1 - \gamma_5)b \right].$$  \hspace{1cm} (7)

A next-to-leading order calculation has been recently completed for all $S$ and $P$ states \[3\] but I will focus here only on the decay into a $J/\psi$. At leading order in the velocity expansion the spin-triplet $S$-wave charmonium states are produced directly from a $c\bar{c}$ pair with the same quantum numbers, i.e. $n = 3S_1^{(1)}$. At order $v^4$ relative to this colour singlet contribution, a $\psi$ can materialize through the colour octet $c\bar{c}$ states $n = 3S_1^{(8)}$, $1S_0^{(8)}$, $3P_J^{(8)}$, where the subscript ‘$J$’ implies a sum over $J = 0, 1, 2$. Note that even if the colour octet contributions are suppressed by $v^4$ they must be included because the weak effective Hamiltonian favours the production of colour octet $c\bar{c}$ pairs. The approximate relation holds

$$C_{[1]}^2 \sim v^4 C_{[8]}^2,$$  \hspace{1cm} (8)

which shows how the NRQCD suppression is compensated by the structure of the weak Hamiltonian. In order to compare the final results with experimental data, we need some input information on MEs. We use $\langle O_1^{\psi}(\bar{c}S_1) \rangle = 1.16$ GeV$^3$ and the determination of $\langle O_8^{\psi}(\bar{c}S_1) \rangle = 1.06 \cdot 10^{-2}$ GeV$^3$ from direct $J/\psi$ production at large transverse momentum in $p\bar{p}$ collisions \[8\]. The other two colour octet MEs are not yet well determined separately. Defining

$$M_k^{\psi}(1S_0^{(8)}, 3P_J^{(8)}) = \langle O_8^{\psi}(\bar{c}S_0) \rangle + \frac{k}{m_c} \langle O_8^{\psi}(3P) \rangle,$$  \hspace{1cm} (9)

we can reproduce the CLEO data $\text{Br}(B \to J/\psi + X) = (0.80 \pm 0.08)\%$, with:

$$M_{3,1}^{\psi}(1S_0^{(8)}, 3P_J^{(8)}) = 1.5^{+0.8}_{-1.1} \cdot 10^{-2} \text{ GeV}^3$$  \hspace{1cm} (10)

where a conservative choice for the errors has been made. It is interesting to compare the central values in \[10\] and the upper limits with other determinations of the parameter $M_k^{\psi}(1S_0^{(8)}, 3P_J^{(8)})$. This is summarized in Tab. 1.

| Process                | $\langle O_8^{\psi}(\bar{c}S_0) \rangle$ | $k$ | $M_k$ |
|------------------------|------------------------------------------|-----|-------|
| Tevatron \[14\]        | 1.06                                     | 3.5 | 4.40  |
| f-t hadropr. \[16\]    | (0.66)                                   | 7.0 | 3.0   |
| Tevatron \[15\]        | 0.3                                      | 3.0 | 1.2   |
| Photoprod. \[17\]      | –                                        | 7.0 | 2.0   |
| This work              | (1.06)                                   | 3.1 | 1.5   |

The central value in the first line is about a factor 3 larger than the central values obtained in \[10\]. As emphasized in Ref. \[14\] the Tevatron extraction is very sensitive to various effects that affect the transverse momentum distribution. Indeed, Refs. \[15\] quote smaller values compatible with, or smaller than the central values above. The total production cross section in fixed target collisions probes $M_7^{\psi}(1S_0^{(8)}, 3P_J^{(8)})$ (assuming the validity of NRQCD factorization, which may be controversial). Given that a different combination of MEs enters, the values obtained in Ref. \[16\] are certainly consistent with the above central value. Considering the uncertainties involved in charmonium production in hadron collisions, the above upper limit on $M_{3,1}$ is the most stringent one existing at present.

\[3\] For simplicity I do not discuss here penguin contributions, although they have been included in the final results.
4. Conclusions

We have shown, by discussing both a decay and a production process, that the effects of including colour octet states contributions are relevant. In particular we have given examples on how information on non-perturbative MEs can be obtained by the comparison between theoretical quantities (where short distance coefficients have been calculated at NLO) and available experimental data. This procedure has allowed us (a) to make predictions for photon energy spectrum in the $\Upsilon$ decays gaining information from the total and leptonic decay rates; (b) to give an estimate for some of colour octet MEs relevant for the $J/\psi$ production in $B$ decays and so to check the “universality” by comparison with the values extracted from other processes. As a final comment I would like to note that in both the above analysis we have found values of the MEs smaller than one could expect from the naïve power counting of NRQCD. Whether this is a general feature of NLO calculations (as a preliminary analysis for fixed-target hadroproduction and photoproduction indeed shows) and how this can be reconciled with the scaling rules is under study.

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