ON HYBRID DYNAMICS OF THE COPENHAGEN
DICHOTOMIC WORLD

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In the Copenhagen viewpoint, part of the world is quantized and the complementary part remains classical. From a formal dynamic aspect, standard theory is incomplete since it does never account for the so-called 'back-reaction' of quantized systems on classical systems except for the highly idealized system–detector interaction. To resolve this formal issue, a certain 'hybrid dynamics' can be constructed to account for the generic interaction between classical and quantized parts. Hybrid dynamics incorporates standard quantum theory, including collapse of the wave function during system–detector interaction. Measurable predictions are robust against shifting the classical–quantum boundary (von Neumann–cut).

1 Introduction

The standard quantum theory, as it is described e.g. in Ref. 1, can be analyzed from naive dynamic aspects. It turns out of course that the interaction of classical and quantum degrees of freedom are treated asymmetrically and also non-dynamically. This issue is exposed in the Sect. 2. While Copenhagen quantum mechanics is a correct theory as it stands, I propose a dynamical formulation which is thought to be equivalent to it. In Sec. 3, I present certain 'hybrid' differential equations of motion, developed in several early and recent works, profiting from related ideas as well. The Sec. 4 is devoted to a proof that hybrid dynamics in proper limit reproduces the system–detector interaction exactly in the form as it is postulated within the collapse theory[4] of standard quantum mechanics. In Sec. 5, I verify that, similarly to the standard theory, the hybrid dynamics experimental predictions are robust against the shift of the boundary (von Neumann cut) between classical and quantized systems. Finally, an outlook is given in Sec. 6.
2 Copenhagen Universe: \( C \times Q \)

The physical world of standard (Copenhagen) quantum mechanics is dichotomic. The Universe \( U \) consists of a classical and a quantized part:

\[ U = C \times Q. \]  

Macroscopic degrees of freedom, building up \( C \), are invariably described by classical canonical equations:

\[ \frac{dx}{dt} = \partial_p H_C(x, p), \quad \frac{dp}{dt} = -\partial_x H_C(x, p), \]  

while microscopic degrees of freedom (\( Q \)) are quantized, their quantum state \( \hat{\rho}_Q \) obeys quantum dynamics

\[ \frac{d\hat{\rho}_Q}{dt} = -i[\hat{H}_Q(x, p), \hat{\rho}_Q], \]  

where the Hamiltonian operator depends on the classical dynamic variables of \( C \) as external parameters. The macroscopic variables \( x, p \) are usually robust variables as compared to the quantized microscopic variables. Therefore, the back-reaction of the quantized system upon the classical one can and used to be ignored. We just keep going on with Eqs. (2).

There is a paradigmatic exception, however. There are macroscopic systems which are not robust against back-reactions from certain microsystems. Instead, these macrosystems are definitely sensitive and will magnify the tiniest back-reaction. As a rule, we use such macrosystems as detectors detecting the given quantized microsystem. The process of back-reaction is called the quantum measurement process. Standard Copenhagen theory does not intend to describe the details of such measurement process. It considers idealized cases of measurement where, instead of solving dynamic equations, only the random final states \( \hat{\rho}_f \) after the back-reaction (measurement) are specified in terms of the initial state \( \hat{\rho}_i \) (before measurement):

\[ \hat{\rho}_f = \begin{cases} \frac{1}{p_1} \hat{P}_1 \hat{\rho}_i \hat{P}_1 & \text{with probability } p_1 = tr[\hat{P}_1 \hat{\rho}_i] \\ \frac{1}{p_2} \hat{P}_2 \hat{\rho}_i \hat{P}_2 & \text{with probability } p_2 = tr[\hat{P}_2 \hat{\rho}_i] \\ \vdots & \vdots \\ \frac{1}{p_n} \hat{P}_n \hat{\rho}_i \hat{P}_n & \text{with probability } p_n = tr[\hat{P}_n \hat{\rho}_i] \end{cases} \]  

where the \( \hat{P}_n \)’s form a complete orthogonal set of Hermitian projectors.

As for the back-reaction of the quantized system on the detector’s classical variables, it is tacitly understood that one of them, ideally the position (say
of the detector’s pointer, taking a neutral value $x^i$ before, will become completely correlated with the random set (4) of final quantum states $\hat{\rho}^f$:

$$x^f \approx g n, \quad \text{with probability } p_n, \quad n = 1, 2, \ldots$$

(5)

The detection process (4,5) is usually considered as the measurement of the Hermitian observable $g A \equiv g \sum_n n P_n$, the random outcomes (5) are the eigenvalues of $g A$.

The interaction between 'detectors' and quantum systems, as specified by the standard Eqs. (4,5), differs from the Eq. (3) which is valid otherwise. But this is just another appearance of the basic dichotomy, without endangering logical consistency of the standard theory. Nonetheless, we present a certain 'hybrid' dynamics which treats all interactions by the same differential equations.

The consistency of the standard theory relies heavily upon the robustness of its measurable predictions against shifting the boundary between the classical and the quantized parts of the Universe:

$$C \times Q \rightarrow C' \times Q'$$

(6)

I mention, however, the notorious extreme case $U = Q$ where the standard quantum theory becomes paralyzed by the impossibility of detection due to the complete lack of classical systems. Still, pushing the boundary toward more and more quantized degrees of freedom is necessary since we have no evidence of an upper limit for the size of the quantized system $Q$. The back-reaction of the enlarged quantum system may grow significant. This would also motivate the next Section.

3 Hybrid dynamics for $C \times Q$

Constructing unified dynamics of hybrid systems requires compromises, c.f. Salcedo’s no-go proofs. Sharp values of the classical variables $x, p$, as well as sharply given phase space trajectories must be given up if the classical system $C$ is brought into interaction with the quantized system $Q$. Classical states will be described by phase space distributions $\rho_C(x, p)$ which should never be narrower than a single Planck cell neither should it bear particular structures within single Planck cells.

If the classical and quantized systems are uncorrelated then it is straightforward to construct the hybrid state

$$\hat{\rho}(x, p) = \hat{\rho}_Q \rho_C(x, p)$$

(7)
for the composite system $C \times Q$. In general, we represent the state of the hybrid system by a hybrid 'density' $\hat{\rho}(x,p)$ which is a phase space dependent non-negative operator. Its trace is the phase space distribution $\rho_C(x,p)$ of $C$ while its phase space integral yields the density operator $\hat{\rho}_Q$ of $Q$. When $\hat{\rho}(x,p)$ is not factorable the unconditional quantum state $\hat{\rho}_Q$ must be distinguished from the conditional quantum states:

$$\hat{\rho}_{xp} = \frac{\hat{\rho}(x,p)}{\rho_C(x,p)}$$

depending on the classical coordinates $x,p$ as conditions. The generic hybrid observables are Hermitian operators $\hat{F}(x,p)$ depending on the classical variables as well. Their expectation values are defined as follows:

$$\langle \hat{F}(x,p) \rangle = tr \int \hat{F}(x,p)\hat{\rho}(x,p) dx dp. \quad (9)$$

The Hamiltonian of the hybrid system takes this form:

$$\hat{H}(x,p) = H_C(x,p) + \hat{H}_Q(x,p). \quad (10)$$

One can construct the following canonical hybrid equation of motion for the hybrid state $\hat{\rho}(x,p)$:

$$\frac{\hat{\rho}(x,p)}{dt} = -i \hat{H} \left( x + \frac{\partial_x + i \partial_p}{2}, p + \frac{\partial_p - i \partial_x}{2} \right) \hat{\rho}(x,p) + H.C. \quad (11)$$

where : . . . : mean that all differentiations must be done first. There is an additional analytic condition for $\hat{\rho}(x,p)$ which assures the mathematical consistency of the hybrid equation.

If all classical variables were robust then Eq. (11) would reduce asymptotically to the standard Eqs. (2,3). In the next Section, however, we consider the paradigmatic opposite case: system–detector interaction.

4 Measurement interaction with hybrid dynamics

We can describe the schematic interaction (4,5) between the classical detector and the quantized system as a hybrid dynamic process. Let the detector's pointer be a harmonic oscillator with Hamiltonian $\frac{1}{2}(x^2 + p^2)$, shortly but strongly coupled to the observed quantum variable $\hat{gA} = g \sum_n n \hat{P}_n$ by the following interaction Hamiltonian:

$$\hat{H}_Q(x,p) = \delta(t)pg \hat{A}.$$ 

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Since $1/g$ will be the relative precision of the measurement we assume $g \gg 1$. We are interested in the states just before and, respectively, after the measurement hence the interaction Hamiltonian (12) dominates and the hybrid equation (11) will take this form:

$$\frac{d\hat{\rho}}{dt} = -ig\delta(t)[\hat{A}, \hat{\rho}] - \frac{1}{2}g\delta(t)[\hat{A}, \partial_x \hat{\rho}] + \frac{i}{2}g\delta(t)[\hat{A}, \partial_p \hat{\rho}].$$ \hspace{1cm} (13)

As it follows from this dynamics, $x$ will play the role of the pointer variable to indicate the value $g, 2g, \ldots$ of the operator $g\hat{A}$ after the measurement. We assume the following factorized initial state for the hybrid system:

$$\rho^i \exp\left[-\frac{i}{2}(x^2 + p^2)\right] = \hat{\rho}_C^i (x, p),$$ \hspace{1cm} (14)

where $\rho_C^i (x, p)$ corresponds to the pointer’s initial position $x^i = 0 \pm 1$. The evolution (13) acts on block-matrix elements of the initial state (14) as follows:

$$\exp\left(-igp[\hat{A}, .] - \frac{1}{2}g[\hat{A} \partial_x , .] + \frac{i}{2}g[\hat{A} \partial_p , .]\right) \hat{P}_n \hat{\rho}_n^i \hat{P}_n \rho_C^i (x, p) =$$

$$\exp\left(-\frac{(n-m)^2}{2g^2} + \frac{n-m}{2i}gp\right) \hat{P}_n \hat{\rho}_n^i \hat{P}_n \rho_C^i \left(x - \frac{n+m}{2g}, p\right).$$ \hspace{1cm} (15)

Since $g \gg 1$ the off–diagonal terms are heavily damped, so the initial hybrid state (14) tends into the diagonal final state:

$$\sum_n \hat{P}_n \hat{\rho}_n^i \hat{P}_n \rho_C^i (x - ng, p).$$ \hspace{1cm} (16)

This result clearly shows that the pointer’s coordinate shifts to the statistically well–separated positions $x^f = gn \pm 1, (n = 1, 2, \ldots)$ with probability $p_n = \text{tr}[\hat{P}_n \hat{\rho}]$. The quantized system’s conditional state (8) is $\hat{P}_n \hat{\rho}_n^i \hat{P}_n / p_n$, respectively. This scheme of the final quantum state $\hat{\rho}_f^i$ and classical pointer position $x^f$ is identical to the result (4,5) of the standard theory.

5 Shifting the quantum-classical boundary

I am going to prove the consistency of hybrid dynamics against the shift (6) of quantum vs. classical boundary. The new hybrid states, Hamiltonian, and observables will be denoted by $\hat{\rho}'(x', p'), \hat{H}'(x', p')$ and $\hat{F}'(x', p')$, respectively. First, we have to establish their correspondence with the old ones. Without restricting generality, one can consider a simplest shift in favor of $Q$ on $C$’s account. We quantize $x_1, p_1$, the other ones $(x_2, p_2, x_3, p_3, \ldots) \equiv (x', p')$ will remain classical. Then the new hybrid Hamiltonian should be chosen as

$$\hat{H}'(x', p') = \hat{H}(x_1, p_1, x', p');$$ \hspace{1cm} (17)
where \( : \ldots : \) means normal ordering for \( \hat{x}_1, \hat{p}_1 \). The new hybrid state must be defined by the implicit relation

\[
\langle x_1, p_1 | \hat{\rho}'(x', p') | x_1, p_1 \rangle = \hat{\rho}(x, p)
\]  

where \( | x_1, p_1 \rangle \) are normalized coherent states, i.e. eigenstates of \( \hat{x}_1 + i \hat{p}_1 \). This relation defines \( \hat{\rho}'(x', p') \) uniquely since, as we mentioned in Sec. 3, a certain analyticity condition has been imposed on \( \hat{\rho}(x, p) \). Now assume that \( \hat{\rho}(x, p) \) satisfies the hybrid Eq. (11). Then it follows from the construction of the hybrid dynamics that the shifted hybrid state (18) will satisfy the hybrid equation (11) with the shifted Hamiltonian (17).

We have to prove that the physical predictions are robust against the shift. Let us construct the shifted observable \( \hat{F}'(x', p') \) from the original one in the following way:

\[
\hat{F}'(x', p') = \{ \hat{F}(\hat{x}_1, \hat{p}_1, x', p') \}_{\text{sym}}
\]  

where a Wigner-Weyl symmetrization for \( \hat{x}_1, \hat{p}_1 \) is understood on the r.h.s. If \( \hat{F}(x, p) \) is a smooth enough function of \( x_1, p_1 \) on the scale of Planck cell then the expectation value of \( \hat{F}'(x', p') \) in state \( \hat{\rho}'(x', p') \) tends to be identical to the expectation value of \( \hat{F}(x, p) \) in \( \hat{\rho}(x, p) \). (The exact relation between the two expectation values is given in Refs. 4, 5.) This assures the robustness of physical predictions provided the observables are smooth functions vs. the Planck cell.

We see that the shift of von Neumann’s cut in one direction means quantization of a classical canonical system. In the opposite direction it means de-quantization of a quantized system. These two procedures are exact inverses of each other. As it is well-known in standard quantum theory, canonical quantization (hence de-quantization, too) can be defined in many ways, depending e.g. on the operator ordering of the Hamiltonian. The concrete meaning of the shift of von Neumann’s cut depends on conventions of quantization/de-quantization. Hay and Peres have recently applied Wigner–functions as the phase–space distribution of de-quantized systems. In our hybrid dynamics the Husimi–function (18) plays the same role. Hay and Peres suggest the invariance of cascaded standard measurements against the shift of von Neumann’s cut provided the quantized pointer variables are Wigner–Weyl–ordered Hermitian operators. In hybrid dynamics, however, these pointer variables should be the normal–ordered ones (17). The discrepancy follows from the different conventions of quantization.

\footnote{Such invariance has been pointed out within a phenomenological theory of open systems, intrinsically related to hybrid dynamic theory.}
In fact, in hybrid dynamics we departed from the more popular convention of classical–quantum correspondence and we have replaced Wigner–functions by Husimi–functions. This allows de-quantizing any quantum states including those with non-positive Wigner–functions. The price we pay is the loss of reversibility of hybrid dynamics. On the other hand, it can thus describe the standard Copenhagen system–detector interaction which is irreversible.

6 Outlook

The introduction of hybrid dynamics is not a necessity, it is a possibility. It does not alter the physical predictions of standard quantum theory. It is thought to reproduce them. Hybrid dynamics turns out to be surprisingly effective in cases of ‘continuous’ observation or massive continuous ‘back-reaction’ of quantized systems where standard detector concept requires cumbersome considerations (though they work, finally) or gross simplifications (which are always restrictive). These positive features of hybrid dynamic approach have recently been emphasized by the present author, Gisin, and Strunz. In particular, I do expect relevant applications of hybrid dynamics in quantum cosmology where massive ‘back-reaction’ is being currently treated by a number of semi-phenomenological models.

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