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ABC: ATTENTION WITH BOUNDED-MEMORY CONTROL

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ABSTRACT

Transformer architectures have achieved state-of-the-art results on a variety of sequence modeling tasks. However, their attention mechanism comes with a quadratic complexity in sequence lengths, making the computational overhead prohibitive, especially for long sequences. Attention context can be seen as a random-access memory with each token taking a slot. Under this perspective, the memory size grows linearly with the sequence length, and so does the overhead of reading from it. One way to improve the efficiency is to bound the memory size. We show that disparate approaches can be subsumed into one abstraction, attention with bounded-memory control (ABC), and they vary in their organization of the memory. ABC reveals new, unexplored possibilities. First, it connects several efficient attention variants that would otherwise seem apart. Second, this abstraction gives new insights—an established approach (Wang et al., 2020b) previously thought to not be applicable in causal attention, actually is. Last, we present a new instance of ABC, which draws inspiration from existing ABC approaches, but replaces their heuristic memory-organizing functions with a learned, contextualized one. Our experiments on language modeling, machine translation, and masked language model finetuning show that our approach outperforms previous efficient attention models; compared to the strong transformer baselines, it significantly improves the inference time and space efficiency with no or negligible accuracy loss.

1 INTRODUCTION

Transformer architectures are now central to modeling in natural language processing (Vaswani et al., 2017), computer vision (Dosovitskiy et al., 2021), computational biology (Jumper et al., 2021), and other application areas. They rely on the attention mechanism (Bahdanau et al., 2015) to contextualize the input. The context can be seen as a random access memory whose size linearly grows with the sequence length; each query reads from the memory using a softmax-weighted linear combination, with an overhead linear in the memory size. This amounts to a quadratic complexity overall, making transformers’ computational overhead prohibitive, especially for long sequences.

One way to improve attention’s efficiency is to bound its memory size. Imposing a constant-sized constraint over the memory ensures that reading from it has constant time and space overhead, yielding a linear overall complexity in sequence lengths. This is in fact a common strategy adopted by several recent works. In this work, we show that some of these works are closely connected in ways that, to date, have gone unremarked. We propose attention with bounded-memory control (ABC), a unified abstraction over them. In ABC, constant-size memories are organized with various control strategies, e.g., induced from heuristic patterns (Beltagy et al., 2020; Zaheer et al., 2020; Ainslie et al., 2020; Rae et al., 2020, inter alia), locality assumptions (Parmar et al., 2018; Liu et al., 2018), or positions (Wang et al., 2020b).

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These strategies, by and large, are “context-agnostic.” In response to this, we propose ABC\textsubscript{MLP}, a particular instance of ABC that learns a contextualized control strategy from data. Specifically, ABC\textsubscript{MLP} uses a neural network to determine how to store each token into the memory (if at all). Compared to previous bounded-memory models, it strikes a better trade-off between accuracy and efficiency: controlling for the accuracy, ABC\textsubscript{MLP} can get away with much smaller memory sizes.

ABC models (including ABC\textsubscript{MLP}) come with a linear complexity in sequence lengths, and admit recurrent computation graphs in causal attention (self-attention over the prefix). Therefore they are appealing choices in a variety of applications, including text encoding, language modeling and text generation. This leads to a surprising finding. Linformer (Wang et al., 2020b), an established efficient attention method, was previously thought to be not applicable in causal attention (Jay et al., 2020) or autoregressive decoding. Through the ABC view, we show that it actually is, and achieves competitive performance in our machine translation experiments. This finding reveals new insights into the properties of the important Linformer model.

ABC connects existing models that would otherwise seem distinct, reveals new insights into established approaches, and inspires new efficient attention architectures. We explore its applications in transformers, as a drop-in substitute for the canonical softmax attention. This abstraction offers a novel lens that can help future research in the analysis of transformers, where the theoretical insights are still catching up with the empirical success. Experiments on language modeling, machine translation, and masked language model finetuning show that our ABC\textsubscript{MLP} model outperforms previous ABC approaches in accuracy with a much smaller memory size. Compared to the strong transformer baseline, ABC\textsubscript{MLP} achieves a significant speedup and memory savings at inference time, with no or negligible accuracy loss. Our analysis shows that the efficiency improvements are more prominent for long sequences, suggesting that the asymptotic savings are more appealing in application areas involving working with very long sequences. We will release our code upon publication.

2 AN OUTER-PRODUCT VIEW OF ATTENTION

This section reviews softmax attention and presents an outer-product memory perspective of it, which allows for a smooth transition to later discussion.

In attention, a sequence of query vectors \(\{q_i\}_{i=1}^N\) attend to a memory with \(N\) slots, each storing a key and value pair: \(K = [k_1, \ldots, k_N]^T\), \(V = [v_1, \ldots, v_N]^T \in \mathbb{R}^{N \times d}\). Reading from the memory with query \(q\) produces a \(d\)-dimensional output vector using a softmax-normalized linear combination: \(\text{att}(q, \{k_i\}_{i=1}^N, \{v_i\}_{i=1}^N) = V^T \text{softmax}(Kq) \in \mathbb{R}^d\). This takes \(O(N)\) time and space. When the attention with \(N\) queries can be parallelized (e.g., in text encoding), it takes linear time and quadratic space; when it cannot be (e.g., in decoding), it takes quadratic time and linear space.

The memory can be equivalently represented as sums of vector outer products:

\[
K = IK = \sum_{i=1}^N e_i \otimes k_i, \quad V = IV = \sum_{i=1}^N e_i \otimes v_i. \tag{1}
\]

\(I\) is the identity matrix, and \(\otimes\) denotes the outer product between vectors (i.e., \([x \otimes y]_{i,j} = x_i y_j\)). The \(N\)-dimensional \(e_i\) vectors form the standard basis, i.e., \(e_i\) has the \(i\)th element being one and others zeros. We can view \(e_i\) as control vectors that determine where to store \(k_i\) and \(v_i\):

\[
e_i \otimes k_i = \begin{bmatrix} 0 \ldots 0 & 1 & 0 \ldots 0 \end{bmatrix}^T \otimes k_i = \begin{bmatrix} 0 ; k_i ; 0 \end{bmatrix}_{d \times (i-1) \: d \times (N-i)} \in \mathbb{R}^{N \times d}. \tag{2}
\]

The \(N\)-by-\(d\) matrix on the right-hand side has its \(i\)th row being \(k_i^T\) and all others being zeros. In this sense, \(k_i\) is stored in the \(i\)th slot by \(e_i\), not affecting others.

3 ATTENTION WITH BOUNDED-MEMORY CONTROL

A straightforward way to improve attention’s efficiency is to bound its memory size. Our outer-product view of attention (Eq.1) provides a straightforward way to devise this, by replacing the \(e_i\) vectors with control vectors that select \(n \ll N\) vectors to attend to. We denote this approach attention

\(\footnote{The number of queries and key-value pairs may differ, e.g., in the cross attention.} \)
with bounded-memory control (ABC). Concretely, let \( \tilde{K}, \tilde{V} \in \mathbb{R}^{n \times d} \) denote a constant-size memory with \( n \) slots, with \( n \) set a priori. \( \{ \phi_i \in \mathbb{R}^n \}^N_{i=1} \) denotes a sequence of control vectors:

\[
\tilde{K} = \sum_{i=1}^{N} \phi_i \odot k_i, \quad \tilde{V} = \sum_{i=1}^{N} \phi_i \odot v_i,
\]

with the output similarly calculated with a softmax-normalized linear sum, but over \( \tilde{K} \) and \( \tilde{V} \):

\[
\text{ABC} (\{ q_i \}^N_{i=1}, \{ k_i \}^N_{i=1}, \{ v_i \}^N_{i=1}, \{ \phi_i \}^N_{i=1}) = \tilde{V}^\top \text{softmax}(\tilde{K}q).
\]

Ways to construct \( \{ \phi_i \} \) vary, which we will soon discuss. Reading from the memory takes a constant \( O(n) \) time and space; therefore ABC’s overall complexity is \( O(Nn) \), linear in the sequence length.

Eq. [5] offers an equivalent recurrent computation, which is particularly useful in causal attention where only the prefix is looked at, as we will explore in our experiments.

\[
K_{t+1} = K_t + \phi_{t+1} \odot k_{t+1}, \quad V_{t+1} = V_t + \phi_{t+1} \odot v_{t+1}.
\]

\( K_t \) and \( V_t \) can be seen as the hidden state that accumulates the information of prefix \( x_{\leq t} \).

In what follows, we study several existing efficient attention approaches and show that they are in fact instances of the ABC abstraction.

### 3.1 Linformer

Linformer (Wang et al. [2020b]) is an established efficient transformer variant that proves successful in masked language modeling and text encoding. It assumes fixed-length inputs and learns a low-rank approximation of the attention weights. A learned \( n \)-by-\( N \) matrix \( W_{\text{LF}} \) down projects the \( N \)-by-\( d \) dimensional keys and values along the sequence length dimension to an \( n \)-by-\( d \) dimensional memory,

\[
\tilde{K}_{\text{LF}} = W_{\text{LF}} K, \quad \tilde{V}_{\text{LF}} = W_{\text{LF}} V,
\]

which are then used for attention computation with Eq. [4]. This yields a linear complexity in the sequence length. Linformer is an ABC instance where \( \phi_{\text{LF}}^i = W_{\text{LF}}^i \) (ith column), and in this sense, it learns a control vector for each position.

Previous works have noted that Linformer cannot be efficiently applied in causal attention (Table 1 of Tay et al. [2020]). Indeed, it is less straightforward to avoid mixing future with the past when projecting along the sequence length. The ABC lens reveals that, in fact, Linformer is applicable in causal attention. Like all ABC models, it admits the linear complexity recurrent computation as in Eq. [3] \( \tilde{K}_{\text{LF}}^{t+1} = K_t + \phi_{\text{LF}}^{t+1} \odot k_{t+1} \). This confirms ABC’s benefits: it reveals new insights about existing models and reassesses their applications and impact. Our experiments show that Linformer is indeed applicable to causal attention, with competitive performance in machine translation.

### 3.2 Clustering-Based Attention

Improving attention’s efficiency with clustering has received an increasing amount of interest (Kitaev et al. [2020], Roy et al. [2020], Wang et al. [2020a], inter alia). ABC bears interesting connections to clustering-based methods. Here we discuss an approach that closely follows Vyas et al. [2020], except that it clusters keys and values instead of queries. To reduce the effective context size, it clusters keys/values, and only attends to the centroids. Formally, \( \{ k_i \} \) are grouped into \( n < N \) clusters \( \{ k_j^\text{CL} \}_{j=1}^n \) and \( \{ v_i \} \) into \( \{ v_j^\text{CL} \}_{j=1}^n \). Let an \( N \)-by-\( N \) binary matrix \( M \) denote the cluster membership matrix, and it is shared between keys and values. \( M_{i,j} = 1 \) iff. \( k_i \) is assigned to cluster \( k_j^\text{CL} \) and \( v_i \) to \( v_j^\text{CL} \). The \( j \)th centroids for keys and values are respectively

\[
\tilde{K}_j^\text{CL} = \sum_{i=1}^{N} \frac{M_{i,j}}{\sum_{\ell=1}^{N} M_{\ell,j}} k_i, \quad \tilde{V}_j^\text{CL} = \sum_{i=1}^{N} \frac{M_{i,j}}{\sum_{\ell=1}^{N} M_{\ell,j}} v_i.
\]

\( ^{2} \)Using bounded memory distinguishes ABC models from the canonical softmax attention. If growing-size memory were allowed (\( n = N \)), an ABC with \( \phi_i = e_i \) would fall back to the softmax attention.

\( ^{3} \)We use \( k_j^\text{CL} \) to denote both the \( j \)th cluster for keys and its centroid. Likewise for the values.
Then the most recent token can be put into the slot freed up:

\[ \mathbf{K}_{t+1} = \mathbf{U} \mathbf{K}_t + \phi_{t+1} \otimes \mathbf{k}_{t+1} \]

Table 1: A comparison of memory-organization strategies of different ABC models. \( N \) denotes the sequence length, and \( n \) the memory size. \( \phi_t \) denotes the memory control vector for \( \mathbf{k}_t \) and \( \mathbf{v}_t \), and \( \text{unif} \) is the discrete uniform distribution.

| Model          | Section | \( \phi_t \)                          | Mem. Control |
|----------------|---------|---------------------------------------|--------------|
| Sliding-window | §3.3    | \( \mathbf{e}_n \)                    | \( \mathbf{K}_{t+1} = \mathbf{U} \mathbf{K}_t + \phi_{t+1} \otimes \mathbf{k}_{t+1} \) |
| Linformer      | §3.1    | \( \mathbf{W}_{j}^{\text{LF}} \)     | \( \mathbf{K}_{t+1} = \mathbf{U} \mathbf{K}_t + \phi_{t+1} \otimes \mathbf{k}_{t+1} \) |
| L2G Pattern    | Appx. A.1 | \( \mathbf{e}_j \) if \( x_i \) is the \( i \)th token       | \( \mathbf{K}_{t+1} = \mathbf{U} \mathbf{K}_t + \phi_{t+1} \otimes \mathbf{k}_{t+1} \) |
| ABC\text{CD}   | Appx. A.2 | \( \mathbf{e}_{i_t} \), where \( i_t \sim \text{unif} \{1, n\} \) | \( \mathbf{K}_{t+1} = \mathbf{U} \mathbf{K}_t + \phi_{t+1} \otimes \mathbf{k}_{t+1} \) |
| Comp. Trans.   | Appx. A.3 | \( \mathbf{e}_{t, n/N} \) | \( \mathbf{K}_{t+1} = \mathbf{U} \mathbf{K}_t + \phi_{t+1} \otimes \mathbf{k}_{t+1} \) |
| Clustering     | §3.2    | \( \sum_{j=1}^{n} \left( M_{t,j} / \sum_{\ell=1}^{N} M_{t,\ell} \right) \mathbf{e}_j \) | \( \mathbf{K}_{t+1} = \mathbf{U} \mathbf{K}_t + \phi_{t+1} \otimes \mathbf{k}_{t+1} \) |
| ABC\text{MLP}  | §4      | \( \exp \left( \mathbf{W}_{\phi} \mathbf{x}_t \right) / \sum_{i=1}^{\ell} \exp \left( \mathbf{W}_{\phi} \mathbf{x}_t \right) \) | \( \mathbf{K}_{t+1} = \mathbf{U} \mathbf{K}_t + \phi_{t+1} \otimes \mathbf{k}_{t+1} \) |

Attention over the centroids then proceeds as Eq.\[\text{III}\] in the same way as the softmax attention, but over \( \{ \mathbf{K}_{j}^{\text{CL}} \}_{j=1}^{n} \) and \( \{ \mathbf{v}^{\text{CL}}_j \}_{j=1}^{n} = (\mathbf{K}_j^{\text{CL}})^{\top} \) \text{softmax}(\mathbf{K}_j^{\text{CL}} \mathbf{q}^\top), where \( \mathbf{K}^{\text{CL}} = [\mathbf{k}_1^{\text{CL}}, \ldots, \mathbf{k}_n^{\text{CL}}]^{\top} = \sum_{j=1}^{n} \mathbf{e}_j \otimes \mathbf{K}_j^{\text{CL}} = \sum_{j=1}^{n} \mathbf{e}_j \otimes \sum_{i=1}^{N} \sum_{\ell=1}^{\infty} M_{t,i} \mathbf{k}_i = \sum_{i=1}^{N} \left( \sum_{j=1}^{n} \mathbf{e}_j \otimes M_{t,i} \right) \otimes \mathbf{k}_i. \) \( \text{(8)} \)

By the last line, this model is an instance of ABC: \( \phi_t = \sum_{j=1}^{n} \left( M_{t,i} / \sum_{\ell=1}^{N} M_{t,\ell} \right) \mathbf{e}_j \). The stack of centroids can be seen as the constant-size memory. Putting aside the clustering overhead (i.e., constructing \( \mathbf{M} \) and computing centroids), it has a linear complexity in the sequence length.

### 3.3 Sliding-Window Attention

In some applications, being able to remove entries from the memory could be beneficial. For example, older context can be removed to clear up slots for more recent contexts, promoting a locality inductive bias. ABC offers the capability to do so, if augmented with an additional matrix multiplication. We use the sliding-window attention as an example. Attending to the most recent \( n \) input tokens (Parmar et al. 2018; Beltagy et al. 2020; Zaheer et al. 2020; inter alia) can be seen as a first-in-first-out queue that “pops” out the oldest token while “pushing” in the most recent one:

\[ \mathbf{K}_{t, n}^{\text{WD}} = [\mathbf{k}_{t-n+1}, \ldots, \mathbf{k}_t]^{\top}, \quad \mathbf{V}_{t, n}^{\text{WD}} = [\mathbf{v}_{t-n+1}, \ldots, \mathbf{v}_t]^{\top}. \] \( \text{(9)} \)

The pop operation can be achieved by multiplying an \( n \)-by-\( n \) upper shift matrix: \( U_{i,j} = \delta_{i+1,j} \), with \( \delta \) being the Kronecker delta (i.e., \( U \) has ones only on the superdiagonal and zeros elsewhere). Left-multiplying \( U \) against \( \mathbf{K}_{t, n}^{\text{WD}} \) shifts its rows one position up, with zeros appearing in the last:

\[ \mathbf{U} \mathbf{K}_{t, n}^{\text{WD}} = \mathbf{U} \begin{bmatrix} \mathbf{k}_{t-n+1}, \ldots, \mathbf{k}_t \end{bmatrix}^{\top} = \begin{bmatrix} \mathbf{k}_{t-n+2}, \ldots, \mathbf{k}_{t-1}, \mathbf{k}_t, \mathbf{0} \end{bmatrix}^{\top} \in \mathbb{R}^{n \times d}. \] \( \text{(10)} \)

Then the most recent token can be put into the slot freed up:

\[ \mathbf{K}_{t+1, n}^{\text{WD}} = \mathbf{U} \mathbf{K}_{t, n}^{\text{WD}} + \mathbf{e}_n \otimes \mathbf{k}_{t+1}. \] \( \text{(11)} \)

\( \mathbf{U} \) and \( n \)-dimensional \( \phi_t = \mathbf{e}_n \) ensure a first-in-first-out queue. Dilated and stride convolution patterns (Beltagy et al. 2020) can be similarly recovered, but with a double-ended queue (Appendix A.4).

Recurrently multiplying \( \mathbf{U} \) simulates the discrete pop operation (Grefenstette et al. 2015; Joulin & Mikolov 2015; Yoga 2018; Yogatama et al. 2018) in a differentiable way. This is reminiscent of recurrent neural networks, while in this case \( \mathbf{U} \) is fixed and never updated as parameters. It is exciting to explore learning such a recurrent matrix, but is beyond the scope of this work.

**Discussion.** In addition to the models discussed above, certain variants of Rae et al. (2020) and sparse attention patterns (local-to-global attention; Beltagy et al. 2020; Zaheer et al. 2020; Ainslie et al. 2020) can also be seen as instances of ABC (Appendix A).

ABC provides a unified perspective of these approaches, and at the same time points out their limitations: their control strategies are context-agnostic. In response to this, we propose to learn a contextualized strategy from data in \( \text{§4} \). Closing this section, Table 1 analyzes various ABC models.
4 LEARNED MEMORY CONTROL

The ABC abstraction connects several existing approaches that would otherwise seem distinct. This could inspire the design of new architectures. We hypothesize that learning a contextualized strategy could achieve better performance in practice. This section introduces \( \text{ABC}_{\text{MLP}} \), which parameterizes \( \phi \) with a single-layer multi-layer perceptron (MLP) that takes as input the token representation, and determines which slots to write it into and how much, if at all.

\[
\alpha_i = \exp \left( W_{\phi} x_i \right), \quad \phi_i = \frac{\alpha_i}{\sum_{j=1}^{N} \alpha_j}.
\]  

(12)

Matrix \( W_{\phi} \) is learned. \( \exp \) is an elementwise activation function. The motivation is to allow for storing a “fractional” (but never negative) amount of input into the memory.\(^1\) Using a non-negative activation, however, has a drawback: the scales of \( \sum_i \phi_i \otimes k_i \) and \( \sum_i \phi_i \otimes v_i \) would grow with the sequence lengths, making training less stable. To overcome this, we divide \( \alpha_i \) vectors by their sum. This functions as normalization and aims to offset the impact of varying sequence lengths.\(^1\) It admits the recurrent computation graph as in Eq.\(^5\) and has a linear complexity in the sequence length.

A key design choice of \( \text{ABC}_{\text{MLP}} \) is that its \( \phi_i \) depends only on current input \( x_i \). This helps (1) keep the recurrent computation efficient in practice (Lei et al., 2018), and (2) make it applicable in not only encoder self-attention and cross attention, but also causal attention. Goyal et al. (2021) and Ma et al. (2021), concurrently to this work, also learn contextualized control. They compute \( \phi_i \) from previous layer’s memory, revealing the full sequence to the control vectors. As a result, these two approaches are unsuitable for causal attention.\(^2\)

\( \text{ABC}_{\text{MLP}} \), as other ABC models, can be used as a drop-in replacement for the canonical softmax attention, and we apply its multihead variant in transformers. With proper parameter sharing, the amount of additional parameters \( \text{ABC}_{\text{MLP}} \) incurs is small: we tie \( \phi \)-MLP’s parameters across different layers, which adds less than 1% parameters to the models. This sharing is inspired by the both best-performing and most parameter-efficient configuration by Wang et al. (2020b).

\( \text{ABC}_{\text{MLP}} \): context-agnostic then context-dependent attention. We now dissect \( \text{ABC}_{\text{MLP}} \) and show that it can be seen as a cascade of two attention mechanisms: one with a learned context-agnostic query followed by one with a context-dependent query. Our analysis starts with a one-dimensional example; the conclusion generalizes to higher-dimensional cases.

**Example 1.** Consider \( \text{ABC}_{\text{MLP}} \) with a single memory slot \( (n = 1) \). It is parameterized with a learned vector \( w_{\phi} \), and \( \phi_i = \exp(w_{\phi} \cdot x_i) / \sum_{j=1}^{N} \exp(w_{\phi} \cdot x_j) \). Since \( \phi_i \) is a scalar here, \( \phi_i \otimes k_i = \phi_i k_i^{\top} \).

\[
\bar{K}^{\top} = \sum_{i=1}^{N} (\phi_i \otimes k_i)^{\top} = \sum_{i=1}^{N} \frac{\exp(w_{\phi} \cdot x_i)}{\sum_{j=1}^{N} \exp(w_{\phi} \cdot x_j)} k_i = \text{attn}(w_{\phi}, \{x_i\}_{i=1}^{N}, \{k_i\}_{i=1}^{N}).
\]  

(13)

In other words, \( \bar{K} \) uses \( w_{\phi} \) as a “pseudo-query” to attend to \( \{x_i\} \) and \( \{k_i\} \). Likewise, \( \bar{V}^{\top} = \text{attn}(w_{\phi}, \{x_i\}_{i=1}^{N}, \{v_i\}_{i=1}^{N}) \).

Despite its similarity to the standard softmax attention, Example 1 has a more efficient linear complexity in sequence lengths. \( w_{\phi} \)’s being context-independent is the key to the savings. Appendix B.3 compares its complexity to softmax attention’s.

Example 1’s conclusion generalizes to higher-dimensional cases: the \( j \)th dimension of \( \{\phi_i\} \) attends to \( \{x_i\} \) and \( \{k_i\} \) using the \( j \)th row of \( W_{\phi} \) as the context-independent pseudo-query; \( n \) such attention mechanisms run in parallel, stacking the results into \( n \)-by-\( d \) memory \( K \) and \( V \). Intuitively, it is the “real queries” \( \{q_i\} \) that encode “what information is useful for the prediction task.” Without access to them, \( \text{ABC}_{\text{MLP}} \) summarizes the input for \( n \) times using different pseudo-queries, aiming to preserve

\(^1\)We also experiment with other non-negative activation functions: ReLU and sigmoid (Appendix C.2).

\(^2\)Here encoder self-attention or cross attention is assumed, and the normalization sums over the entire sequence. Causal attention is slightly different, normalizing by the sum over the prefix instead: \( \phi_i = \alpha_i / \sum_{j=1}^{i} \alpha_j \). This does not require access to future tokens. Appendix B.1 details a linear complexity computation graph of causal \( \phi \).

\(^3\)Both are instances of ABC. See Appendix A.5 for a detailed discussion. Ma et al. (2021) resorts to a variant of Katharopoulos et al. (2020) for causal attention.
enough information in the memory for onward computation. The attention output is calculated with the context-dependent real queries using Eq. 4. Appendix B.2 presents a detailed derivation.

**Connections to other prior works.** Although starting from distinct motivations, ABCMLP closely relates to hierarchical attention (HA; Yang et al., 2016). HA summarizes the context into higher-level representations with a cascade of attention mechanisms, e.g., words to sentences, and then to documents. ABCMLP applies two types of attention. The first learns context-agnostic pseudo-queries and attend to the same sequence for \( n \) times in parallel, while the second retrieves from the memory with real queries. HA, in contrast, summarizes non-overlapping segments at each level.

The learned pseudo-queries closely relate to the inducing point method in set attention (ISA; Lee et al., 2019). ISA applies a non-linear feedforward network between a cascade of two attention modules. This precludes the outer-product memory computation and efficient recurrences in ABCMLP.

Another line of works “linearizes” attention through kernel tricks and also applies bounded memory: their feature map dimensions are analogous to memory sizes. They substitute the softmax with approximations (Peng et al., 2021; Choromanski et al., 2021), heuristically designed (Katharopoulos et al., 2020; Schlag et al., 2021), or learned (Kasai et al., 2021b) functions. ABCMLP keeps the softmax, but over a smaller constant-sized context. This can be useful in practice: (1) ABC provides a unified perspective of several efficient attention methods, allowing for borrowing from existing wisdom to design new architectures; (2) it draws a close analogy to the canonical softmax attention, and is better-suited as its drop-in substitute in various application settings, as we will show in the experiments; (3) empirically, we find that ABCMLP can get away with a much smaller memory size to retain the accuracy. Peng et al. (2021) and Schlag et al. (2021) use gating to promote recency bias. The same technique is equally applicable in ABC models.

The learned contextualized memory control is reminiscent of the content-based addressing in neural Turing machines (NTM; Graves et al., 2014). ABCMLP computes the control vectors \( \phi_i \) as a function of the input, but not of the memory as in NTM. This ensures that the control vectors at different timesteps can be computed in parallel, improving the time efficiency in practice (Lei et al., 2018; Peng et al., 2018). Analogies between memory and neural architectures are also made by other previous works (Hochreiter & Schmidhuber, 1997; Weston et al., 2015; Le et al., 2020, inter alia).

5 Experiments

We evaluate ABC models on language modeling (§5.1), sentence-level and document-level machine translation (§5.2), and masked language model finetuning (§5.3). Dataset statistics and implementation details are summarized in Appendix C.

5.1 Language Modeling

**Setting.** We experiment with WikiText-103, sampled text from English Wikipedia (Merity et al., 2017). Our BASE model that uses the standard softmax attention is the strong transformer-based language model by Baevski & Auli (2019). In addition, we compare the following ABC variants, which build on BASE, but replace the softmax attention with linear-complexity bounded-memory attention alternatives while keeping other components the same.

- ABCMLP as described in §4, learns a contextualized exp-MLP as the \( \phi \) function.
- Linformer (Wang et al., 2020b), as described in §3.1
- ABCRD stores each token in a randomly-selected memory slot with \( \phi_t = e_{i_t} \), \( i_t \) is uniformly drawn from \( \{1, \ldots, n\} \) at each time step. This helps us quantify the differences between random and learned bounded memory control.

We consider two model size settings:

- The first one follows Baevski & Auli (2019). It compares models with 16 layers and around 242M parameters. They train with 512-token segments; at evaluation time, the context size is 0 or 480, i.e., a 0 or 480 length prefix is attached to each evaluation segment.
- The second one follows Kasai et al. (2021b), aiming to compare ABCMLP head-to-head to several recently-proposed kernel-based efficient attention variants: ELU (Katharopoulos et al., 2020), RFA (Peng et al., 2021), and T2R (Kasai et al., 2021b). This setup uses 32 layers, applies layer
Table 2: WikiText-103 language modeling perplexity (lower is better). \( n \) denotes the memory size. Bold numbers perform the best among linear-complexity models.

| Model  | Cross \( n \) | Causal \( n \) | BLEU |
|--------|---------------|---------------|------|
| BASE   | -             | -             | 27.2 |
| ABC\_RD | 32           | 32           | 25.7 |
| ABC\_RD | 64           | 64           | 26.2 |
| Linformer | 32          | 32           | 26.6 |
| Linformer | 64           | 64           | 26.7 |
| ABC\_MLP | 32          | 8            | 27.1 |
| ABC\_MLP | 32           | 32           | 27.3 |

(a) Bolded number outperforms BASE.

(b) Kasai et al. (2021b)’s 32-layer setting. A 256-length context is used at evaluation time. † numbers are due to Kasai et al. (2021b).

Table 3: Machine translation test SacreBLEU. Left: sentence-level translation with WMT14 EN-DE; Right: document-level translation with IWSLT14 ES-EN.

| Model  | Cross \( n \) | Causal \( n \) | BLEU |
|--------|---------------|---------------|------|
| BASE   | -             | -             | 39.9 |
| Linformer | 128          | 64           | -    |
| ABC\_RD | 128          | 64           | 38.6 |
| ABC\_MLP | 128         | 64           | 39.7 |

(b) Linformer fails to converge even with multiple random seeds. Bold number performs the best among ABC models.

5.2 Machine Translation

**Datasets.** To assess their performance over various sequence lengths, we compare ABC models on both sentence-level and document-level machine translation.

- Sentence-level translation with WMT14 EN-DE (Bojar et al., 2014). The preprocessing and data splits follow Vaswani et al. (2017).
- Document-level translation with IWSLT14 ES-EN (Cettolo et al., 2014). We use Miculicich et al. (2018)’s data splits and preprocessing. Following standard practice (Voita et al., 2019), a 4-sentence sliding window is used to create the dataset, i.e., each instance has 4 sentences.

**Setting.** Here we compare the ABC variants described in \( \S \)5.1 Appendix C.2 further compares to the clustering-based (\( \S \)5.2) and sliding-window (\( \S \)5.3) variants of ABC. The BASE model that they are
All linear-complexity models achieve consistent decoding speed for different lengths (Figure 1), with multiple random seeds, suggesting the limitations of its purely position-based strategy in tasks with RealNews, and drop Stories (Trinh & Le, 2018), whose public access is broken at the time of this work. T2R needs to be finetuned from a pretrained transformer to match its performance, while others don’t. The trend is similar on document-level translation with IWSLT14 ES-EN (Table 3b), except that a sequence-to-sequence generation experiment. Three linear-complexity models are compared: A scaling linearly in the sequence length, A decoding efficiency varying sequence lengths. We follow Peng et al. (2021)’s setting, and consider efficiency improvements could be more prominent for longer sequences. We study A underperforms A, possibly be due to overfitting, or a discrepancy between our data and RoBERTa’s. Linformer achieves inference time speed gains and memory savings. Linformer is effective despite its bounded memory size. Linformer fails to converge even with multiple random seeds, suggesting the limitations of its purely position-based strategy in tasks involving varying-length sequences, especially long text decoding.

5.3 Masked Language Model Finetuning

Setting. We compare the ABC variants described in §5.1. It is interesting to pretrain from scratch, but we lack the resources to do so. Instead, we warm start from a pretrained RoBERTa-base (Liu et al., 2019) trained with the softmax transformer, and continue pretraining with the masked language modeling (MLM) objective on a concatenation of BookCorpus (Zhu et al., 2015), English Wikipedia, OpenWebText (Tokaslan & Cohen, 2019), and RealNews (Zellers et al., 2019). Then the models are finetuned and evaluated on downstream classification datasets from the GLUE benchmark (Wang et al., 2018). This is an appealing setting: being able to convert a pretrained transformer into its efficient alternatives could avoid the majority of the expensive pretraining cost. In preliminary experiments, all ABC models fail on downstream datasets without continued pretraining.

Results. Table 4 compares ABC models against RoBERTa on downstream text classification accuracy. Following standard practice (Liu et al., 2019), we report results on development data. Continued training on our data, RoBERTa-Ours slightly underperforms RoBERTa. This could possibly be due to overfitting, or a discrepancy between our data and RoBERTa’s. Linformer achieves competitive accuracy, aligned with Wang et al.’s (2020b) results. ABCMLP outperforms Linformer, and performs on par with or better than RoBERTa-Ours, affirming the benefits of using contextualized memory organization in masked language modeling. With sequence length 512 and batch size 16, both ABCMLP and Linformer achieve inference time speed gains and memory savings. Linformer is faster, since its linear projection is cheaper to compute than ABCMLP’s MLP. Their memory overhead is similar. ABCRD fails to converge in this experiment.

6 Analysis

Decoding efficiency varying sequence lengths. Scaling linearly in the sequence length, ABC’s efficiency improvements could be more prominent for longer sequences. We study ABCMLP’s decoding overhead varying sequence lengths. We follow Peng et al.’s (2021)’s setting, and consider a sequence-to-sequence generation experiment. Three linear-complexity models are compared: RFA (with cross-causal memory sizes of 256-128; Peng et al., 2021), T2R (32-4; Kasai et al., 2021b), and ABCMLP (32-8). These memory sizes are chosen to maximize efficiency without accuracy drop. T2R needs to be finetuned from a pretrained transformer to match its performance, while others don’t.

All linear-complexity models achieve consistent decoding speed for different lengths (Figure 1), substantially outpacing the softmax attention baseline, especially for long sequences. In particular, ABCMLP decodes around 1.25 times faster than RFA, another competitive model that can match

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1. Our data differs from RoBERTa’s, which we do not have access to. We replace CC-News (Nagel, 2016) with RealNews, and drop Stories (Trinh & Le, 2018), whose public access is broken at the time of this work.

2. Inference speed is measured on the same V100 GPU. Wang et al. (2020b) use batch sizes as large as the hardware allows, while we use the same 16 batch size for all models.
Table 4: Text classification development set accuracy. RoBERTa is the base-sized RoBERTa model, and its numbers are due to Liu et al. (2019). The second block continues pretraining RoBERTa based on our data with the MLM objective. Bold numbers perform the best among ABC models, and underlined ones perform on par with or better than RoBERTa-Ours. Inference speed (higher is better) and memory consumption (lower is better) are relative to RoBERTa’s.

Table 5: ABCMLP’s SacreBLEU on WMT14 EN-DE development data with varying memory sizes of cross and causal attention. All models apply greedy decoding, without checkpoint averaging.

Memory size’s impact on accuracy. Practically, one may want to minimize memory size to improve efficiency. We use the WMT14 EN-DE experiment to investigate how memory size affects accuracy. Using the §5.2’s setup, we vary ABCMLP’s cross and causal attention memory sizes and compare their translation quality on the development data. They are selected from \{8, 16, 32, 64\}, with cross attention’s equal to or larger than causal’s: as previous evidence shows, cross attention is more important than causal attention in machine translation (Michel et al., 2019; You et al., 2020). Our results (Table 5) align with this observation: when cross attention memory is large enough, reducing causal attention memory size from 64 to 8 has a minor 0.3 BLEU drop. Surprisingly, ABCMLP with 8-8 sized cross-causal memory is only 1.1 BLEU behind the best-performing configuration.

7 Conclusion

We presented attention with bounded-memory control (ABC). It provides a unified perspective of several recently-proposed models, and shows that they vary in the organization of the bounded memory. ABC reveals new insights into established methods: we showed that Linformer is efficiently applicable in causal attention, opposite to what was previously believed. ABC also has practical benefits and inspires new architectures. We proposed ABCMLP, a particular instance of ABC that
learns a contextualized memory control. On language modeling, machine translation, and masked language model finetuning, ABCMLP outperforms previous ABC models. Compared to the strong transformer baseline, ABCMLP achieves substantial efficiency improvements with no or negligible accuracy loss.

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Appendices

A OTHER ABC MODELS

A.1 SPARSE LOCAL-TO-GLOBAL ATTENTION

It sparsifies attention pattern to reduce the number of tokens that are attended to (Beltagy et al., 2020; Zaheer et al., 2020, inter alia). All queries attend to a subset of \( n < N \) “global tokens,” while ignoring others. Therefore the effective context size is reduced to \( n \). The global tokens are usually pre-selected by positions according to using some heuristics. Local-to-global attention is an instance of ABC: it can be recovered by letting \( \phi_t = e_i \) if \( x_t \) is the \( i \)th global token \((i = 1, \ldots, n)\), and the zero vectors for others.

A.2 RANDOM MEMORY CONTROL

As a baseline, \( \text{ABC}_{\text{RD}} \) stores each token in a randomly-selected memory slot. This is achieved by letting \( \phi_t = e_{i_t} \), where \( i_t \) is uniformly drawn from \( \{1, \ldots, n\} \) for each \( t \). It is designed as a baseline to \( \text{ABC}_{\text{MLP}} \) and Linformer to quantify the differences between random and learned bounded memory control.

Random sparse attention patterns are explored by Zaheer et al. (2020), where a subset of \( n < N \) tokens are randomly selected to be attended to by all. \( \text{ABC}_{\text{RD}} \) is different, and it attends to all tokens, but “squash” them into an \( n \)-slot memory.

A.3 COMPRESSION Transformer WITH MEAN Pooling

The compressive transformer (Rae et al., 2020) explores various ways to “squash” long context into smaller and more compact representations. It achieves state-of-the-art performance on several language modeling benchmarks. We show that at least the mean-pooling variant of the compressive transformer can be seen as an ABC instance.

The mean-pooling variant of the compressive transformer compresses the context by

\[
\text{K} = [k_1, \ldots, k_N]^{\top} \in \mathbb{R}^{N \times d}
\]

\[
\rightarrow \tilde{\text{K}} = \left[ \frac{k_1 + \cdots + k_c}{c}, \frac{(k_{c+1} + \cdots + k_{2c})}{c}, \ldots, \frac{(k_{N-c+1} + \cdots + k_N)}{c} \right]^{\top} \in \mathbb{R}^{n \times d}.
\]

where \( c = N/n \) is the compression ratio. Here \( N \mod n = 0 \) is assumed, since otherwise the sequence can be padded to.

The above model is an ABC instance by letting

\[
\phi_t = e_{\left[(i-1)/c\right]+1}/c. \tag{14}
\]

A.4 DILATED CONVOLUTION Attention PATTERNS

The dilated attention pattern is similar to the sliding window attention and only considers the context within a predefined window. It differs in that it attends to every other token:

\[
\tilde{\text{K}}_t = [k_{t-2n+2}, k_{t-2n+4}, \ldots, k_{t-2}, k_t]^{\top}. \tag{15}
\]

It can be simulated with two separate queues \( \tilde{\text{K}}^{\text{odd}} \) and \( \tilde{\text{K}}^{\text{even}} \):

\[
\tilde{\text{K}}_t^{\text{odd}} = \begin{cases} U\tilde{\text{K}}^{\text{odd}}_{t-1} + e_n \otimes k_t, & \text{if } t \text{ is odd} \\ \tilde{\text{K}}^{\text{odd}}_{t-1}, & \text{otherwise} \end{cases}
\]

\[
\tilde{\text{K}}_t^{\text{even}} = \begin{cases} U\tilde{\text{K}}^{\text{even}}_{t-1} + e_n \otimes k_t, & \text{if } t \text{ is even} \\ \tilde{\text{K}}^{\text{even}}_{t-1}, & \text{otherwise} \end{cases}
\]

Likewise for the values. Depending on \( t \), the query attends to one of the two queues:

\[
\text{output} = \begin{cases} (\tilde{\text{V}}^{\text{odd}})^{\top} \text{softmax}(\tilde{\text{K}}^{\text{odd}} q_t), & \text{if } t \text{ is odd} \\ (\tilde{\text{V}}^{\text{even}})^{\top} \text{softmax}(\tilde{\text{K}}^{\text{even}} q_t), & \text{otherwise} \end{cases}
\]
The above implementation could incur considerable amount of overhead and may be actually more expensive than the original dilated window formulation. Therefore it has more conceptual value than practical one.

### A.5 Shared Workspace and Linear Unified Nested Attention

Concurrently to this work, shared workspace (SW; Goyal et al., 2021) and linear unified nested attention (LUNA; Ma et al., 2021) also learn contextualized memory control strategies. Both can be seen as instances of ABC. At layer $\ell$, their $\phi^\ell_i$ is a function of previous layer’s memory $\tilde{X}^{\ell-1} \in \mathbb{R}^{n \times d}$ and current layer’s input $X^\ell \in \mathbb{R}^{N \times d}$:

$$ \phi_i = \left[ \text{softmax} \left( \tilde{X}^{\ell-1} X^\ell_T \right) \right]_{:,i}, \quad (16) $$

where $[\cdot]_{:,i}$ denotes the $i$th column of a matrix. Query, key, and value projections are suppressed for notation clarity.

SW and LUNA reveal the entire sequence to the control vectors, by constructing $\phi$ as a function of previous layer’s memory. Although both admit the recurrent computation as all ABC models do, they are ill-suited for causal attention and autoregressive decoding, since future information is “leaked” to $\phi_i$ from the previous layer. LUNA resorts to a variant of Katharopoulos et al. (2020) in causal attention (Ma et al., 2021). In contrast, $A^{\text{BC-MLP}}$ never depends $\phi_i$ on previous layer’s memory, but only on current layer’s input.

### B More Details about ABC-MLP

#### B.1 Normalization in Causal Attention

An equivalent implementation to Eq. 12 is to normalize $\tilde{K}$ and $\tilde{V}$ instead of $\phi_i$ vectors:

$$ \alpha_i = \exp \left( W_\phi x_i \right), \quad \phi_i = \alpha_i, $$

$$ \tilde{K} = \tilde{K} \bigg/ \sum_{j=1}^N \alpha_j, \quad \tilde{V} = \tilde{V} \bigg/ \sum_{j=1}^N \alpha_j. $$

$$ \text{output} = \tilde{V}^T \text{softmax}(\tilde{K} q). $$

$M/z$ divides the $\ell$th row of matrix $M$ by vector $z$’s $\ell$th dimension. This admits a linear complexity computation graph for the causal variant of ABC-MLP.

#### B.2 Higher-dimensional Case of Example 1

This section generalizes Example 1 to higher dimensional cases. Assume that the constant-sized memory has $n$ slots. $\phi_i$ is calculated as in Eq. 12. Then $K = \sum_{i=1}^N \phi_i \otimes k_i \in \mathbb{R}^{n \times d}$. Each row of $K$ can be seen as a separate attention mechanism with a pseudo query. Let $[\cdot]_\ell$ denote the $\ell$th row/dimension of a matrix/vector. Then for any $\ell = 1, \ldots, n$

$$ [\tilde{K}]_\ell = \sum_{i=1}^N (\phi_i)_\ell \otimes k_i = \sum_{i=1}^N \frac{\exp([W_\phi]_\ell \cdot x_i)}{\sum_{j=1}^N \exp([W_\phi]_\ell \cdot x_j)} k_i^T $$

$$ = \text{attn} \left( [W_\phi]_\ell, \{x_i\}_{i=1}^N, \{k_i\}_{i=1}^N \right)^T \in \mathbb{R}^{1 \times d}. $$

In other words, there are $n$ attention mechanisms in total, each with a separately-parameterized pseudo-query $[W_\phi]_\ell$. They summarize the context for $n$ times in parallel, each producing a $d$-dimensional vectors. These output vectors are then stacked into $n$-by-$d$ memory $\tilde{K}$. $\tilde{V}$ is likewise.

#### B.3 Complexity Analysis for ABC

Table 6 compares ABC’s time and space complexity against softmax attention’s. Its savings come from a per-query complexity reduction from $O(N)$ to $O(n)$, for both time and space. Since the memory $\tilde{K}$ and $\tilde{V}$ is shared across different queries, ABC comes with a linear complexity in sequence lengths. Therefore significant efficiency improvements can be achieved when $n \ll N$. 

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Table 6: ABC’s time and space complexity in sequence length against the softmax attention’s. “Mem.” indicates the time and space needed for calculating and storing memory $\tilde{K}, \tilde{V}$. $N$ denotes the sequence length, and $n$ the memory size. The time complexity analysis assumes that the softmax attention cannot be parallelized across the queries. In practice, this is common in, e.g., autoregressive decoding, or for long sequences where the accelerators (e.g., GPUs) do not have enough threads to fully parallelize softmax attention’s computation across different queries.

The same analysis applies to ABC\textsubscript{MLP}.

C EXPERIMENTAL DETAILS

C.1 LANGUAGE MODELING

We closely build on Baevski & Auli (2019) and Kasai et al. (2021b). The hyperparameters are summarized in Table 9. All models are trained on 4 A100 GPUs.

C.2 MACHINE TRANSLATION

We experiment with a sentence-level (WMT14 EN-DE, Bojar et al., 2014) and a document-level benchmark (IWSLT14 ES-EN, Cettolo et al., 2014) to assess model performance over various sequence lengths. The preprocessing and data splits of WMT14 EN-DE follow Vaswani et al. (2017). A 32,768 byte pair encoding (BPE; Sennrich et al., 2016) vocabulary is shared between source and target languages. For IWSLT14, we follow Miculicich et al. (2018) and use the dev2010 subset for development and tst2010-2012 for testing. The tokenization is also the same as Miculicich et al. (2018): we tokenize and truecase Spanish and English with Moses (Koehn et al., 2007) and run byte-pair encoding with 30k splits, shared between the two languages. The final dataset contains 1421, 8, and 42 documents for training, development, and testing. On average, each document contains 126.7 sentences and each sentence contains 21.7(ES)/22.5(EN) BPE. We use a sliding window with length-4 and stride-one to generate our dataset. During inference, we use predicted context at the target side.

We average the checkpoints from the last five epochs to obtain the final model Vaswani et al. (2017). In inference, we apply beam search with size 5 and length penalty 0.6. Other hyperparameters are summarized in Table 10. All models are trained on 4 RTX 2080 Ti GPUs.

**Additional machine translation results.** In addition to the results presented in §5.2, Table 7 further compares, on the WMT14 EN-DE dataset, the clustering-based (§3.2) and sliding-window (§3.3) models of ABC, as well as ReLU and sigmoid variants of ABC\textsubscript{MLP}. Clustering and sliding-window ABC variants underperform ABC\textsubscript{MLP} with the same memory sizes by more than 0.5 BLEU. Both ReLU and sigmoid underperform their exp counterpart. We observe that ABC\textsubscript{MLP}-ReLU suffers a severe “the rich gets richer” issue: all tokens are stored in a handful of slots, no matter how large the memory size is. This could be the reason for its suboptimal accuracy. Further investigations are deferred to future work.

MLP-exp-all replaces the encoder’s softmax attention modules with ABC, in addition to the decoder’s. It underperforms ABC\textsubscript{MLP} by 0.3 BLEU.

Figure 2 compares ABC\textsubscript{MLP}’s (32-8 memory sizes) attention memory overhead with softmax attention’s. Following Kasai et al. (2021b), we consider a synthetic sequence-to-sequence generation task with varying sequence lengths. A batch size of 16 and greedy decoding is used. The models are of the same size as those in §5.2.
| Model   | $\phi$ | Cross $n$ | Causal $n$ | Encoder $n$ | BLEU |
|---------|--------|-----------|------------|-------------|------|
| BASE    | -      | -         | -          | -           | 27.2 |
| Window  |        | 32        | 32         | -           | 26.3 |
| Cluster |        | 32        | 32         | -           | 26.8 |
| MLP-ReLU|        | 32        | 8          | -           | 26.4 |
| MLP-ReLU|        | 32        | 32         | -           | 26.8 |
| MLP-sigmoid |    | 32    | 8          | -           | 27.0 |
| MLP-sigmoid |     | 32    | 32         | -           | 27.0 |
| MLP-exp |        | 32        | 8          | -           | 27.1 |
| MLP-exp |        | 32        | 32         | -           | **27.3** |
| MLP-exp-all |   | 32    | 32         | 32          | 27.0 |

Table 7: ABC variants’ SacreBLEU on WMT14 EN-DE sentence-level machine translation test set. MLP-ReLU with 32/8 memory sizes fails to converge. MLP-exp-all applies ABC in both the encoder and the decoder, while others only in the decoders.

Figure 2: Machine translation decoding memory overhead of the attention computation. The setting follows [Kasai et al., 2021b], with greedy decoding and batch size 16.

C.3 Masked Language Model Finetuning

Our data for continued pretraining is a concatenation of BookCorpus [Zhu et al., 2015], English Wikipedia, OpenWebText [Gokaslan & Cohen, 2019], and RealNews [Zellers et al., 2019]. Our data differs from RoBERTa’s pretraining data, which we do not have access to. We replace their CC-News [Nagel, 2016] with RealNews, and drop Stories [Trinh & Le, 2018]. At the time of this project, the public access to the Stories dataset is broken. Our machine does not have a large enough memory to load all the data. We therefore split the training data into 20 shards, after shuffling. Other preprocessing is the same as [Liu et al., 2019]. The hyperparameters for continued pretraining follows base-sized RoBERTa, part of which summarized in Table 11. All models are trained on a single TPU v3 accelerator.

For downstream task finetuning, we use the same hyperparameters as [Liu et al., 2019] Table 12 briefly introduces the tasks. The readers are referred to [Wang et al., 2018] for further details.

https://console.cloud.google.com/storage/browser/commonsense-reasoning/reproduce/stories_corpus?pli=1
https://github.com/pytorch/fairseq/blob/master/examples/roberta/README.pretraining.md
https://github.com/pytorch/fairseq/blob/master/examples/roberta/README.glue.md
Under review as a conference paper at ICLR 2022

| Data             | Train | Dev. | Test  | Vocab. | Sent./doc |
|------------------|-------|------|-------|--------|-----------|
| WikiText-103     | 103M  | 218K | 246K  | 268K   | -         |
| WMT14 EN-DE      | 4.5M  | 3K   | 3K    | 32K    | -         |
| IWSLT14 ES-EN    | 1713  | 8    | 56    | 30K    | 121.5     |

Table 8: Statistics for the datasets. WikiText-103 split sizes are in number of tokens, WMT14 in number of sentences, and IWSLT14 in number of documents.

| Hyperprams.       | Baevski & Auli (2019) | Kasai et al. (2021b) |
|-------------------|-----------------------|----------------------|
| # Layers          | 16                    | 32                   |
| # Heads           | 8                     | 8                    |
| Embedding Size    | 1024                  | 1024                 |
| Head Size         | 128                   | 128                  |
| FFN Size          | 4096                  | 4096                 |
| Batch Size        | 64                    | 64                   |
| Learning Rate     | 1.0                   | 1.0                  |
| Dropout           | 0.3                   | 0.3                  |
| Layer Dropout     | -                     | 0.2                  |
| Memory size       | [32, 64]              | 64                   |

Table 9: Hyperparameters used in the language modeling experiments.

| Hyperprams.       | WMT14 | IWSLT14 |
|-------------------|-------|---------|
| # Layers          | 6     | 6       |
| # Heads           | 8     | 8       |
| Embedding Size    | 512   | 512     |
| Head Size         | 64    | 64      |
| FFN Size          | 2048  | 1024    |
| Warmup Steps      | 6000  | 4000    |
| Dropout           | 0.1   | 0.3     |
| Cross Attention Memory Size | 32 | 128 |
| Causal Attention Memory Size | 8 | 64 |

Table 10: Hyperparameters used in the machine translation experiments.

| Hyperprams.       | Values             |
|-------------------|--------------------|
| # Layers          | 12                 |
| # Heads           | 12                 |
| Embedding Size    | 768                |
| Head Size         | 64                 |
| FFN Size          | 3072               |
| Dropout           | 0.1                |
| Memory Size       | [64, 128]          |

Table 11: Hyperparameters for continued pre-training in the masked language model fine-tuning experiments.

| Data             | Task         | Train  | Dev.  |
|------------------|--------------|--------|-------|
| MNLI             | Entailment   | 392K   | 9.8K  |
| QNLI             | Entailment   | 105K   | 5.5K  |
| QQP              | Paraphrase   | 363K   | 40K   |
| SST-2            | Sentiment    | 67K    | 873   |

Table 12: GLUE datasets and statistics. MNLI: Williams et al. (2018); QNLI is compiled by GLUE’s authors using Rajpurkar et al. (2016); QQP: Csernai (2017, accessed September 1, 2020); SST-2: Socher et al. (2013).