Manifestation of the new laser-electron nuclear spectral effects in the thermalized plasma: QED theory of co-operative laser-electron-nuclear processes

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Abstract. A consistent QED perturbation theory approach is applied to calculation of the electron-nuclear \( \gamma \)-transition spectra of nucleus in the multicharged ion. The intensities of satellites are defined in the relativistic version of the energy approach (S-matrix formalism). As example, the nuclear transition in the isotope \( ^{57}_{26} \text{Fe} \) with energy 14.41 keV is considered. The results of the relativistic calculation for the electron-nuclear \( \gamma \)-transition spectra (set of electron satellites) of the nucleus in a multicharged atomic ion \( \text{Fe}^{XIX} \) are presented. The possible experiments for observation of the new effect in the thermalized plasma of O- like ions are discussed.

1. Introduction
In recent years a great number of papers have been devoted to the development of new theoretical methods for diagnostics of the elementary processes in high temperature multi-charged ions plasma (c.f. [1]). A great interest to this topic is also stimulated by importance of understanding the key physical processes in the multi-charged ion plasma in thermonuclear reactors, laser plasma etc. A development of methods of the laser spectroscopy [2] allowed observing and further using the little changes in structure of atomic and molecular spectra because of the corresponding alteration of the internal state of a nucleus (co-operative laser-electron-nuclear effects). Such effects as the isomer shift in the vibrational spectrum of a molecule because of the equivalent increasing of mass of the excited nucleus. It should be also mentioned a selective photoionization of atoms with the isomer nucleus and possibility of the quick physical separation of the isomer nuclei. This effect is of a great importance for the \( \gamma \)-laser problem (c.f. [1, 2]). It is of a great importance to observe new co-operative laser-electron-nuclear effects in the multicharged ions in corresponding plasma and use them as new basis for plasma parameters diagnostics.

Any alteration of the atomic, ionic or molecular state must be manifested in the quantum transitions, for example, in a spectrum of the \( \gamma \)-radiation of a nucleus. It is well known that it is possible the transfer of part of a nuclear energy to atom or molecule under radiating (absorption) the \( \gamma \) quanta by a nucleus (c.f. [3, 4, 15]). The first references to the neutral
recoil are due to Migdal and Levinger [4], who evaluated approximately the ionization of an atom undergoing sudden recoil in due to neutron impact and in a radioactive disintegration respectively. The neutral recoil situation differs radically from processes involving a charged particle for which the sudden recoil approximation is often invalid (c.f. [13, 14]). Simple, as a rule, non-relativistic quantum-mechanical models (c.f. [3–15] have been developed to evaluate the excitation or ionization of an atom, the electronic redistribution of an ion or an atom induced a sudden recoil of its nucleus occurring when a neutral particle is either emitted (γ-radioactivity) or captured (neutron capture for instance) [14–17]. The most consistent approach to considered problems must be based on the quantum electrodynamics (QED). The nuclear emission or absorption spectrum of an atom possesses a set of electron satellites, which are due to an alteration of the state of the electron shell [15–17]. The mechanism of formation of the satellites in neutral atoms and highly charged ion is different. In the first case (loose electron shell) a shaking of the shell resulting from the interaction between the nucleus and γ-quanta is predominant. In the second case (rigid electron shell) the mechanism involves a direct interaction between γ-quanta and electrons. The traditional selection rules and familiar intensity hierarchy with respect to electron transition multiplicity do not pertain to the second mechanism. Consequently, the satellite spectrum is much enriched and transitions between the fine and hyper fine structure components, 0-0 transitions and transitions, which do not involve a change in the electron configuration, can be considered. The intensive vibrational satellites can appear in a spectrum of the γ-radiation in molecule under radiating (absorption) the γ quanta by a nucleus. An appearance of these molecular nuclear lines is interesting as it opens a possibility of the changing the γ-radiation spectrum by means of the changing vibrational state of a molecule by coherent laser light [16]. Probability of the vibrational or rotational state changing (in difference from the atomic electrons state changing) is not small and must be taken into account even in the zeroth approximation. In any case there are a great number of different channels for the electron-nuclear processes in atoms, ions and molecules. Possibility of their interference makes the analysis more complicated. A consistent analysis of cited processes must be, as a rule, based on the quantum electrodynamical formalism.

This paper is going on our studying the co-operative dynamical phenomena (c.f. [16, 17]) due the interaction between atoms, ions, molecule electron shells and nuclei nucleons. Earlier a consistent QED perturbation theory approach is developed and applied to calculation of the electron-nuclear γ-transition spectra of nucleus in the different atomic systems. In this paper a consistent QED perturbation theory approach is applied to calculation of the electron-nuclear γ-transition spectra of nucleus in the multicharged ion. The intensities of satellites are defined in the relativistic version of the energy approach (S-matrix formalism) [16]. Decay and excitation probabilities are linked with imaginary part of the energy of the 'nuclei nucleons-electron shells-field' system. For radiate decays it is manifested as effect of retarding in interaction and self-action and calculated within QED perturbation theory. As example, the nuclear transition in the isotope $^{57}_{26}Fe$ with energy 14.41 keV is considered. The results of the relativistic calculation for the electron-nuclear γ-transition spectra (set of electron satellites) of the nucleus in a multicharged atomic ion FeXIX are presented and compared with the corresponding non-relativistic estimates [5, 11]. The possible experiments for observation of the new effect in the thermalized plasma are discussed. It is considered a situation when electron satellites are not overlapped by the Doppler contour of the γ-line (plasma source).

2. Energy approach to QED theory of co-operative electron-nuclear processes

Following to refs. [16, 17], we consider the following model of atomic system: rigid nuclear core (c), above core proton (p) and electron (e). The masses of three particles are equal correspondingly: $\mu_c M$, $\mu_p M$, $\mu_e M$, where $M$ mass of all atom; $\mu_e + \mu_p + \mu_c = 1$; the space co-ordinates of the particles are denoted as $r_c$, $r_p$, $r_e$. The charge of nuclear core is $z$. Besides,
the value of $z^*$ denotes an effective charge for Coulomb field of the optically active electron in ion. Naturally, the majority of the excited states of nuclei have the multi-particle character [2]. As exclusion, one may consider the first excited states with one or two quasi-nucleons or quasi-vacancies above ‘even-even’ core. These states are more suitable for theoretical consideration as the one-particle model could be used. It is very important to underline that a generalization on the multi-particle case does not lead to qualitatively new results as the dynamical (radial) parts of the nuclear matrix elements do not enter into expressions for relative intensities of the electron satellites and ground line of the nuclear transition. QED is needed here as for obtaining the correct formula as carrying out the precise calculations.

Within the QED energy approach the main our purpose is calculating the imaginary part of energy of the excited state for atomic system. Detailed description of an approach was given earlier (c.f. [16, 17]). Here we consider only the key elements of the calculation procedure. Following the quasi-potential method, we introduce the bare interaction as follows:

$$V(r_e, r_p, r_e) = v(r_{pc}) - Z e^2/r_{ee} - e^2/r_{pe}. \quad (1)$$

Here $v(r_{pc})$ imitates the interaction of the proton with the core (nuclear and Coulomb); other interactions are obvious. Then imaginary part of the energy of the excited state for atomic system. Detailed description of an approach was given in the lowest QED perturbation theory order can be written as follows:

$$\text{Im}E = e^2 \text{Im}i \lim_{\gamma \to 0} \int \int d^4x_1d^4x_2 e^{\gamma(t_1+t_2)} \times$$

$$\times \langle \Psi | (j_{c}(x_1)j_{c}(x_2))|\Psi \rangle + D(r_{c1}, r_{c2}) \langle \Psi | (j_{p}(x_1)j_{p}(x_2))|\Psi \rangle +$$

$$+ D(r_{e1}, r_{e2}) \langle \Psi | (j_{e}(x_1)j_{e}(x_2))|\Psi \rangle. \quad (2)$$

Here $D(r_{c1}, r_{c2})$ is the photon propagator; $j_{c}, j_{p}, j_{e}$ are the four-dimensional components for the current operator for particles: core, protons, electrons; $x = (r_e, r_p, r_e, t)$ includes the space co-ordinates of three particles and time (equal for all particles); $\gamma$ is the adiabatic parameter. For the photon propagator the exact electrodynamical expression is used:

$$D(12) = -\frac{i}{8\pi^2} \int_{-\infty}^{\infty} d\omega e^{i\omega t_{12} + i\omega|r_{12}|}. \quad (3)$$

In expressions (2), (3) the summation of the directions of the photon polarization are fulfilled. Below we are limited by the lowest order of the QED perturbation theory, i.e. the next QED corrections to $\text{Im}E$ will not be considered. Note also that the expression (2) describes the one-photon processes. We need further relativistic solutions of the Dirac equation whose radial part is represented by

$$F' = -(\omega + |\omega|)F/r - \alpha(E + 2M\alpha^{-2})G - \alpha VG,$$

$$G' = (\omega - |\omega|)G/r + \alpha(E - V)F. \quad (4)$$

Here $\alpha$ is the fine structure constant; $\omega$ is the Dirac angular quantum number; $E$ is the state energy, $F, G$ being the large and small radial components correspondingly. In the non-relativistic limit the large radial component converts into the only component-solution of the non-relativistic radial Schrödinger equation. Substituting all expressions into (2), one may get the following general expression for imaginary part of the excited state energy of the three-quasi-particle system as a sum of the core proton and electron contributions:
\[ \text{Im} E = \text{Im} E_c + \text{Im} E_p + \text{Im} E_e, \]
\[ \text{Im} E_a = -Z_a^2/4\pi \int \int dr_{c1} dr_{c2} \int \int dr_{p0} dr_{p2} \int \int dr_{c1} dr_{c2} \Psi_e^*(1) \Psi_e^*(2) T_a(1,2) \Psi_F(1) \Psi_F(2), \quad (5) \]
\[ T_a(1,2) = (\sin(w_{IF} r_{a12})/r_{a12}) \left( 1/M \mu_a(\nabla r_{a1}, \nabla r_{a2}) + 1 \right). \]

Here \( r_{a12} = |r_{a1} - r_{a2}|; \) \( w_{IF} \) is the transition full energy, which includes changing of the kinetic energy of the ion, i.e. the recoil energy; \( \Psi_e, \Psi_p, \Psi_e \) are the second quantized field operators of the core particles, of the protons and of the electrons respectively. The sum according to \( F \) gives the summation of the final states of the system. In the second order of QED perturbation theory, the full width of a level is divided into the sum of the partial contributions, connected with the radiation decay into concrete final states of the system. These contributions are proportional to the probabilities of the corresponding transitions. The system of red (blue) satellites corresponds to the transitions with excitation (de-excitation) of the electron shell. The important quantity is the contribution of \( \text{Im} E_e \) to the relative intensity of satellite \( k = P(pe)/P(p) \) (here \( P(pe) \) is satellite intensity; \( P(p) \) is the intensity of the nuclear transition). The intensity of the line is linked with \( \text{Im} E \) (5) as:
\[ P = 2\text{Im} E/h. \quad (6) \]

A frequency of the \( \gamma \)-transition for nucleus with changing the electron state is defined by the following expression:
\[ h\omega_{\gamma f} = h\omega_{\gamma 0} \pm (\hbar \Delta \gamma + E_i - E_f). \quad (7) \]

Here \( \omega_{\gamma 0} \) is the frequency of the \( \gamma \)-transition without recoil; \( \hbar \Delta \gamma \) is the recoil energy; \( E_i \) and \( E_f \) are the initial and final energies of electron (sign ‘+’ is corresponding to absorption of the \( \gamma \)-quantum; sign ‘−’ is corresponding to emission of the \( \gamma \)-quantum). In fig.1 we present a schematic spectrum of the electron-nuclear lines of emission (lines, directed up) and absorption (lines, pointing down) of the \( \gamma \)-radiation for a nucleus in non-excited (a) and excited (b) neutral atom (left part of the figure). In the right part of the figure the corresponding quantum transitions in the system are presented.

Further it is convenient to separate the motion of the center of mass of the system, introducing the new variables:
\[ R = \mu_e r_e + \mu_p r_p + \mu_e r_e, \quad R_p = r_p - r_e, \quad R_e = r_e - r_c. \]

In the zeroth order perturbation theory approximation a dependence of \( \Psi_I, \Psi_F \) from the variables \( R, R_p, R_e \) is factorized:
\[ \Psi_A(R, R_p, R_e) = \Psi_A(R) \Psi_{Ap}(R_p) \Psi_{ Ae}(R_e). \quad (8) \]

Here \( \Psi_A \) is plane waves, \( \Psi_{Ap} \) is function of state of the proton in potential \( V(R_p) \), \( \Psi_{ Ae} \) is Coulomb relativistic function.

One should note that there are the combined electron-proton one-photon transitions already in the zeroth approximation. A contribution of the proton-electron interaction into satellite intensity is manifested only in the second order of the perturbation theory on this interaction. It has an additional order of smallness \( \sim 1/z^{\ast 2} \) (for coulomb part) and \( \mu^2 \) (for recoil interaction) in comparison with the main contribution. So, the main effect of causing the electron satellites for nuclear transitions has kinematics nature, which is in shifting center of mass of the system
Figure 1. Spectrum of the electron-nuclear lines of emission (lines, directed up) and absorption (lines, indicated down) of the $\gamma$-radiation for nucleus in non-excited (a) and excited (b) neutral atom (left part of the figure); in right part of the figure there are presented the corresponding quantum transitions in the system ($E_i$ and $E_f$ are the initial and final energies of electron, $E_{\gamma 0}$ is an energy of nuclear level).

under emission of $\gamma$-quanta relatively of the proton or electron orbital. In the concrete calculation one can use the standard expansion for operator $T$ on the spherical functions, which generates the multiple expansion for the decay probability. The details of the calculation procedure, the definition of all contributions and the corresponding matrix elements are described in refs. [16–21].

3. Results and discussion
It is very important to discuss the possible experimental observation of satellite effect. As indicated in ref. [15], in neutral atoms under standard experimental conditions the intensive satellites are overlapping by the Doppler contour of line of the $\gamma$-radiation. For their observation one should use the methods of inside Doppler spectroscopy [2]. In principle it is possible an observation of the satellites in the spectrum of emission or absorption without overlapping by the Doppler contour of the $\gamma$-line. Such a situation could be realized in plasma of multicharged ions [16]. The energy intervals between lowest electron levels may significantly exceed the Doppler shift of the $\gamma$ radiation. Let us suppose that the K shell is significantly destroyed. According to [15, 16], an average kinetic energy for ions in such plasma: $\sim E_i/10 \sim 1/20$ c.u. (Coulomb units are used), where $E_i$ is the ‘1s’ electron bond energy. The Doppler shift is as follows: $\delta\hbar\omega_D \approx \alpha\omega/(10M)^{1/2}$. The value $\alpha\omega$ is connected with the of $\gamma$ quantum by the following
The Doppler widths are shown qualitatively (disposition for some electron satellites in relation to the nuclear transition line is considered. - satellites are ones, which are corresponding to the transitions 2s−2p. In fig. 2 there are presented the lines which are accompanied by electron transitions: 1s−2s (monopole), 1s−2p/2 (dipole), 2p/2−2p/2 (quadruple). The detailed results of calculation for some of these transitions have been presented in Ref. [17], where are also indicated the corresponding non-relativistic data [15]. An account of the relativistic effects resulted in the shift of the curves to the region of the more large energies. The numerical difference of values for intensities of different satellites is connected with different values of the electron radial integrals, which are defined by the overlapping the wave functions. Intensity of satellite for transition to the 2p state is twice less than to the 2p state. The strongest satellites are ones, which are corresponding to the transitions 2s−2p. In fig. 2 a scheme of disposition for some electron satellites in relation to the nuclear transition line is considered. The Doppler widths are shown qualitatively (δhωD ≈ 5 eV).

The relative intensities for these satellites are equal ≈ 7·10−5. Satellites connected with the 1-2 transitions are separated from ωpe on value ≈ 6 keV, but their intensity is less. It easy to understand that naturally the relative electron satellite intensity values are sufficiently little because of the weak link between electron motion and motion of a nucleus under recoil. In fig. 2 there are presented the lines which are accompanied by electron transitions: 1s−2s2p41S0−2s2p53P1; 2−2s2p43P1−2s2p53P2; 2−2s2p43P2−2s2p53P1; 4−2s2p53P1−2p51S0. The relative intensities for these satellites are P(pee)/P(p) ≈ 7·10−5, the Doppler broadening is δhωD ≈ 5 eV (shown on figure 2 qualitatively). Thus, it is obvious that the electron-nuclear lines in spectra of emission or absorption can be experimentally observed in plasma of the O-and F-like multicharged ions and they are not overlapping by the Doppler broadening.

**Table 1.** Energies of the L-levels for ion of FeXIX, counted from the ground level 2s22p3P2

| Configuration | 2s22p4 | 2s22p5 | 2p6 |
|---------------|--------|--------|------|
| State         | 3P0    | 3P1    | 1P0  |
| E, eV         | 9.4    | 11.1   | 20.9 |
|               | 20.9   | 40.3   | 114.4|
|               | 114.4  | 122.0  | 127.7|
|               | 127.7  | 157.1  | 264.6|

formulae: \( E_\gamma [keV] \approx 4Z(\alpha \omega) \). If, say, \( \alpha \omega = 1 \), then \( \delta h \omega_D \approx 1/200(Z)^{1/2} \) c.u. \( \approx 0.15(Z)^{1/2} \) eV. For comparison let us give the values of the 1s, 2s, 2p-2p electron transitions for one-electron ions with \( Z=10–50: E(1s–2p_{3/2}) = 1.3 \cdot 10^3 \) eV, \( E(2s–2p_{3/2}) = 0.1 \) eV.

One can see that the transition energies have an order of the Doppler shift value. A little value of the splitting in the one-electron ions is entirely provided by relativistic corrections. In a multi-electron system a situation is more favourable. Let us consider a case of the O-like and F-like multicharged ions. An additional splitting is defined by inter electron interaction. In table 1 we present the energies of levels for L shell of the oxygen-like ion FeXIX (\( Z=26 \)), counted from the ground level 2s22p3P2 [16].

The lines of big number of the electron satellites, connected with 2-2 transitions are sufficiently far from the Doppler contour. We consider the nuclear transition in the isotope \( ^{57}_{26}Fe \) with the quantum energy 14.41 keV. The period of the half decay of state \( T(1/2) = 9.77 \cdot 10^{-8} \) sec., the recoil energy \( 1.96 \cdot 10^{-6} \) keV. The parameter \( \alpha \omega = 0.27 \). We consider the following transitions: 1s−2s (monopole), 1s−2p/2, 2s−2p/2 (dipole), 2p/2−2p/2 (quadruple). The detailed results of calculation for some of these transitions have been presented in Ref. [17], where are also indicated the corresponding non-relativistic data [15]. An account of the relativistic effects resulted in the shift of the curves to the region of the more large energies. The numerical difference of values for intensities of different satellites is connected with different values of the electron radial integrals, which are defined by the overlapping the wave functions. Intensity of satellite for transition to the 2p/2 state is twice less than to the 2p/2 state. The strongest satellites are ones, which are corresponding to the transitions 2s−2p. In fig. 2 a scheme of disposition for some electron satellites in relation to the nuclear transition line is considered. The Doppler widths are shown qualitatively (\( \delta h \omega_D \approx 5 \) eV).

The relative intensities for these satellites are equal \( \approx 7 \cdot 10^{-5} \). Satellites connected with the 1-2 transitions are separated from \( \omega_{pe} \) on value \( \approx 6 \) keV, but their intensity is less. It easy to understand that naturally the relative electron satellite intensity values are sufficiently little because of the weak link between electron motion and motion of a nucleus under recoil. In fig. 2 there are presented the lines which are accompanied by electron transitions: 1s−2s2p41S0−2s2p53P1; 2−2s2p43P1−2s2p53P2; 3−2s2p43P2−2s2p53P1; 4−2s2p53P1−2p51S0. The relative intensities for these satellites are \( P(pee)/P(p) \approx 7 \cdot 10^{-5} \), the Doppler broadening is \( \delta h \omega_D \approx 5 \) eV (shown on figure 2 qualitatively). Thus, it is obvious that the electron-nuclear lines in spectra of emission or absorption can be experimentally observed in plasma of the O-and F-like multicharged ions and they are not overlapping by the Doppler broadening.
Figure 2. The positions of emission and absorption lines electron satellites (in a positive and negative direction of abscissa axe correspondingly) for ions FeXIX, FeXVIII in lowest states of the ground configurations $2s^22^1$, $2s^22^2$ (a) and states of the excited configuration $2s^22^5$, $2s^22^5nl$ (b) relatively the nuclear $\gamma$-transition in isotope of $^{57}$Fe with energy $\hbar\omega_{\gamma} = 14.41$ keV; $P(pe)/P(p)$ is relation of the satellite intensity to the nuclear transition line intensity.
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