Mechanically induced current and quantum evaporation from Luttinger liquids

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We investigate transport through a tunnelling junction between an uncorrelated metallic lead and a Luttinger liquid when the latter is subjected to a time dependent perturbation. The tunnelling current as well as the electron energy distribution function are found to be strongly affected by the perturbation due to generation of harmonics in the density oscillations. Using a semiconducting lead instead of a metallic one results in electrons being injected into the lead even without applied voltage. Some applications to carbon nanotubes are discussed.

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I. INTRODUCTION

The influence of static impurities on the transport properties of one-dimensional (1D) correlated electron systems is well understood [1, 2] (for a recent review on 1D physics see e. g. 3). On the contrary, the effect of dynamic time-dependent perturbations on transport from open Luttinger liquids (LLs) has been barely addressed in the literature. Yet, investigations into the role of dynamic sources in other correlated systems reveal interesting phenomena. So, it was seen in quantum evaporation (QE) experiments from superfluid helium 4 that such perturbations can significantly influence the structure of the complicated collective particle state. A phonon source (which is essentially a dynamic perturbation), imbedded into a superfluid is able to excite single particles above the energy needed to escape from the condensate. In this paper we address the question whether a correlated 1D electron system with open boundary shows similar physics. Research in this direction is particularly interesting because of a possibility to make related measurements on single-wall carbon nanotubes (SWNTs). Their electronic degrees of freedom are known to be described by the (four-channel) LL model [4].

An experimental realisation of a periodic perturbation can, for example, be achieved by laser radiation which causes localised oscillations of the lattice coupled to the electron density. Alternatively, it has been realised by the oscillation of the nanotube itself [5]. If the transport through the nanotube changes significantly in the presence of such perturbation, it could be used to gain information about the cause of the oscillation. This would add to already known applications of nanotubes as detector or sensor devices [6].

The system under investigation is a half-infinite LL with an open end. At the boundary it is contacted by an electrode which can be metallic, semiconducting or is even opened into vacuum (the latter case can be modelled by a metallic electrode with the chemical potential sent to infinity as in case of the field emission effect [7, 8]). We are interested in the current-voltage characteristics and in the electron energy distribution function (also called the total energy distribution function: TED) in the presence of a dynamic scatterer inside the LL. The TED is proportional to the energy resolved current in the case of emission into vacuum.

In the experimental setups similar to that of Ref. [9], the nanotube oscillates as a whole. Clearly such oscillations strongly affect the phase of the electron wave-function, but cause little elastic stress and hence only a weak back-scattering. It turns out, somewhat surprisingly, that a dynamic forward scattering perturbation non-trivially affects the electron transport. Contrary to a back-scattering perturbation, static forward scattering contributes neither to the TED nor to the current, so that the contribution of the dynamic part can be singled out. Because of these reasons, we concentrate here on the forward scattering model, which is very simple and exactly solvable and yet leads to interesting predictions for the electron transport.

The paper is organised as follows. In the next Section we discuss the TED of electrons in the vicinity of the open end in the presence of a time-dependent perturbation. We derive a general formula for the TED. In the subsequent Section we derive a similar formula for the tunnelling current across the interface to an additional electrode. Section [10] is then devoted to applications of the results to different situations and contains discussion of the main findings.

II. TIME-DEPENDENT PERTURBATION IN 1D

Let us begin by recollecting some known results for the TED of electrons in the presence of a time-dependent perturbation. Since a static forward scatterer only changes the phase of the wave function, it cannot influence the TED. As soon as the perturbation becomes time-dependent, the scattering process is not elastic any more and particles can acquire or lose energy. Hence the TED is changed. In the case of a periodic perturbation with the frequency $\Omega$, the electron can lose or gain the energy $n\Omega$ upon scattering, $n$ being an integer. Thereby an infinite number of equidistant sidebands emerge in the TED even in non-interacting systems [11, 12]. As a result, the TED is distorted in a staircase manner as shown in Fig. [13]. We stress that the forward scattering is sufficient...
to cause these effects.

This can be quantified for our system using the open boundary formalism of Ref. [13]. According to [13], in an open system the electrons can be regarded as chiral particles living in an infinite system. Thereby the operator \( \psi(x, t) \) on the negative half-axis describes the electrons moving towards the boundary (right-movers) and for positive \( x \) it corresponds to electrons moving in the opposite direction (left-movers). Since we are dealing with an explicitly time-dependent situation the TED \( n(\omega, t) \) at the boundary should be defined using the Wigner representation,

\[
n(\omega, t) = \int dt e^{i \omega t} \langle \psi^\dagger(0, t - \tau/2) \psi(0, t + \tau/2) \rangle . \tag{1}
\]

Because of the open boundary condition imposed on the electron wave functions the probability density to find any particles exactly at the boundary is, of course, zero. However, we assume the TED to be measured on the second last site of the underlying lattice model so that \( x \) in the last formula is actually not 0 but the lattice constant \( a_0 \).

Since we are dealing with a non-equilibrium situation, we resort to the Keldysh formalism [14, 15], in which the TED can be conveniently expressed through one of the off-diagonal Green’s functions (from now on we drop the spacial coordinate),

\[
n(\omega, t) = -i \int dt e^{i \omega t} g^{-}(t + \tau/2, t - \tau/2) , \tag{2}
\]

where (we give all Green’s functions for future reference),

\[
g^{-}(t, t') = i \langle \psi^\dagger(t') \psi(t) \rangle ,
\]

\[
g^{+}(t, t') = -i \langle \psi(t) \psi^\dagger(t') \rangle ,
\]

\[
g^{-}(t, t') = -i \langle \hat{T} \psi(t) \psi^\dagger(t') \rangle ,
\]

\[
g^{+}(t, t') = -i \langle \hat{T} \psi^\dagger(t) \psi(t') \rangle . \tag{3}
\]

Here \( T \) and \( \hat{T} \) denote the time- and anti-time-ordering operations, respectively. Notice that these functions are not translationally invariant in time because of the explicit time dependence of the perturbation \( U(t) \). All calculations are most transparent in the bosonized representation usually used in the theory of LLs [6]. The interacting electron field \( \psi(t) \) can then be expressed in terms of a free Bose field \( \hat{\phi}(t) \). The physical interpretation being that \( \hat{\phi} \) describes the collective low-energy plasmon excitations in our correlated electron system. At the system’s boundary we have [16, 17]:

\[
\psi(t) = \frac{1}{\sqrt{2 \pi a_0}} e^{i \hat{\phi}(t)/\sqrt{\pi}}, \tag{4}
\]

This equation contains the Luttinger liquid parameter \( g \), which is related to the interaction strength \( U_0 \) via \( g = (1 + 4U_0/\pi)^{-1/2} \) [18]. The mode expansion for the field \( \phi(x) \), in terms of boson creation \( b_q^\dagger \) and annihilation \( b_q \) operators, reads

\[
\phi(x) = \sum_{q > 0} \sqrt{\frac{\pi}{q L}} \left[ e^{i q x} b_q + e^{-i q x} b_q^\dagger \right] e^{-a_0 q^2/2}. \tag{5}
\]

We assume the system having length \( L \), so that the momentum \( q \) takes quantised values \( q = 2\pi n/L \), where \( n \) is an integer. At the end of all calculations we shall send \( L \) to infinity (any zero-modes omitted in Eq. (5) will drop out in this limit). In this representation, the (perturbed) LL Hamiltonian takes the following form,

\[
H = H_{LL}[\psi] + U(t) = v \sum_{q > 0} q b_q^\dagger b_q
+ \int dx [\Delta(x, t) + \Delta(-x, t)] \rho(x) , \tag{6}
\]

where \( \Delta(x, t) \) is the forward-scattering time-dependent perturbation and \( v = v_F/g \) is the renormalised sound velocity, \( v_F \) is the Fermi velocity of the non-interacting system and \( \rho(x) \) is the particle density operator. Notice that we are already working in the chiral representation therefore the last integral is performed over the whole real axis. For the same reason the scattering potential appears in the symmetrised form. In terms of the Bose field, the electron density \( \rho(x) \) reads

\[
\rho(x) = \psi^\dagger(x) \psi(x) = \frac{k_F}{\pi} + \frac{1}{2\pi\sqrt{g}} \partial_x \phi(x) . \tag{7}
\]

The constant contribution (average density) gives rise to an overall energy shift and will therefore be dropped in what follows. Any additional terms containing back-scattering processes are dropped as well. Combining this relation with Eq. (6) we rewrite the perturbation in terms of the Bose operators,

\[
U(t) = i \frac{1}{2} \sum_{q > 0} \sqrt{\frac{q}{\pi g L}} [\Delta_q(t) + \Delta_{-q}(t)] (b_q - b_q^\dagger) e^{-a_0 q^2/2} , \tag{8}
\]

where \( \Delta_q(t) \) is the Fourier transform of the perturbation amplitude. \( \Delta_q(t) = \int dx e^{i q x} \Delta(x, t) \). To evaluate the Green’s functions all we need is the time dependence of the Bose operators \( b_q \). In our simple model, the latter can easily be obtained as a solution of the corresponding equation of motion, which has the following form,

\[
i \frac{\partial}{\partial t} b_q^\dagger(t) = [b_q^\dagger, H] =
= -v q b_q^\dagger(t) - i \frac{1}{2} \sqrt{\frac{q}{\pi g L}} [\Delta_q(t) + \Delta_{-q}(t)] b_q e^{-a_0 q^2/2} .
\]

The solution of this equation is

\[
b_q^\dagger(t) = (b_q^\dagger + f(t)) e^{i v q t} , \tag{9}
\]

where

\[
f(t) = - \frac{1}{2} \sqrt{\frac{q}{\pi g L}} \int_{-\infty}^{t} d\tau [\Delta_q(\tau) + \Delta_{-q}(\tau)] e^{-i v q (\tau - t)} e^{-a_0 q^2/2} .
\]
Using this solution we can write down the Keldysh Green’s function with indices \( (ij) \) as products of the bare Green’s function \( g_0^ij(t-t') \) (i.e. without the perturbation \( U(t) \)) and multipliers, which are responsible for the breaking of the translational symmetry in time,

\[
g^{ij}(t, t') = g_0^{ij}(t-t') e^{i(\chi(t) - \chi(t'))},
\]

where the phase factors are

\[
\chi(t) = -\frac{1}{2g} \int_0^\infty d\tau \left[ \Delta(-v\tau, t - \tau) + \Delta(v\tau, t - \tau) \right].
\]

Remarkably, factorisation (10) holds for interacting systems; it is a consequence of bosonization.

We are now able to establish the connection between the TED in the presence of a dynamic scatterer and that of the unperturbed system \( n_0(\omega) \). In order to accomplish this we combine Eqs. (3), (10) and (14). As a result we obtain

\[
n(\omega, t) = \int \frac{d\omega'}{2\pi} n_0(\omega') \int d\tau e^{i(\omega - \omega')\tau} \\
\times e^{i(\chi(t + \tau/2) - \chi(t - \tau/2))}.
\]

Let us from now on concentrate on a separable harmonic perturbation, adequate for describing the setup of Ref. [9].

\[
\Delta(x, t) = \sin[\Omega t] \eta(x).
\]

We define the function \( F(\tau) \) responsible for the geometry of the scatterer,

\[
F(\tau) = -\frac{1}{2g} [\eta(-v\tau) + \eta(v\tau)].
\]

After simple algebra we obtain

\[
n(\omega, t) = \int \frac{d\omega'}{2\pi} n_0(\omega') \int d\tau e^{i(\omega - \omega')\tau} \\
\times \exp \left( - i \frac{2}{\Omega} \sin[\Omega \tau/2] |z| \sin[\Omega t + \Phi] \right),
\]

where the constant \( z \) and the phase delay \( \Phi \) between the TED oscillations and those of the perturbation are

\[
z = \int_0^\infty d\tau e^{i\Omega \tau} F(\tau),
\]

\[
\Phi = \arcsin[\text{Re}[z]/|z|].
\]

In realistic systems such as SWNTs the frequency \( \Omega \) is expected to be quite high (in the far MHz range) so that it is unlikely that the explicit time resolution of the TED or other physical properties of the system would be experimentally accessible. Therefore it is instructive to investigate time-averaged quantities (i.e. averaged over the period of the perturbation). We define the averaged TED as

\[
\bar{n}(\omega) = \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} dt n(\omega, t).
\]

Applying this prescription to Eq. (15) one finds that

\[
\bar{n}(\omega) = \int \frac{d\omega'}{2\pi} n_0(\omega') \int d\tau e^{i(\omega - \omega')\tau} \\
\times J_0[2|z| \sin[\Omega \tau/2]],
\]

where \( J_0 \) is the Bessel function of the first kind. The \( \tau \) integral can be performed by using the formula

\[
\int dt e^{ixt} J_0[y \sin|x\tau/2|] = 2\pi \sum_{n=-\infty}^\infty \delta \left( \frac{\omega}{\Omega} - n \right) J_n^2(y/2).
\]

As a result one obtains the following expression:

\[
\bar{n}(\omega) = \sum_{n=-\infty}^\infty n_0(\omega - n\Omega) J_n^2(|z|),
\]

where \( z \) is the only free parameter in this formula. It contains all essential information about the scatterer properties. Hence, the TED of a system with a harmonic perturbation is a weighted superposition of infinite number of original unperturbed TEDs centred at energies \( n\Omega \).

This result is in accordance with previous findings for non-interacting systems [3, 4]. The important fact is that factorisation (10) continues to hold even for interacting systems so that formula (21) still applies.

Due to the presence of the oscillating perturbation the actual TED is distorted even for non-interacting systems, see Fig. 1. For a weak scatterer \( |z| \ll 1 \) (in the figure \( |z| \) is set to 0.5-1.0 in order to make the details of the plot clearer) in a non-interacting system, where \( n_0(\omega) \) is just a step function, the particles can barely receive energy larger than \( \Omega \) upon scattering, so that the TED \( \bar{n}(\omega) \) only acquires additional step of the width \( \Omega \) above the Fermi energy \( E_F \) and a dip of the same width just below \( E_F \). At higher \( |z| \) additional sidebands emerge and at sufficiently large \( |z| \) the TED is bounded from above.
only by the actual bandwidth of the host material. The latter can be quite high and exceed the work function so that particles can leave the system, or to evaporate from it in analogy to QE.

The TED $n_0(\omega)$ of any unperturbed system is a product of the Fermi distribution function and the energy-dependent local density of states (LDOS). Therefore the TED $\bar{n}(\omega)$ for an LL differs from that for the non-interacting system only by the energy-dependent details in vicinity of each step. For example, in Fig. 1 we plotted the TED for an LL with interaction parameter $g$, where the LDOS is given by

$$\nu_\psi(\epsilon) = C_\psi |\epsilon|^{1/\gamma - 1}.$$  \hfill (21)

$C_\psi$ is a constant, which for a non-interacting system coincides with the energy independent LDOS. The above discussion applies at zero-temperature. It is not difficult to generalise the formulas to finite temperatures. This is not of immediate interest though as clearly the time-dependent perturbation is harmonic, Eq. (13), is not really restrictive. In such situation the field emission (Auger) singularity to emerge at every $\nu$, however, they would be weaker than in the case without the time-dependent perturbation.

III. GENERAL RESULTS FOR THE TUNNELLING CURRENT

We now turn to the calculation of the current-voltage characteristics of the system which is contacted by an additional electrode at $x = 0$. The corresponding Hamiltonian,

$$H = H_{LL}[\psi] + H_0[c] + \gamma [\psi^\dagger(0)c(0) + c^\dagger(0)\psi(0)] + U(t),$$  \hfill (22)

contains a contribution $H_0[c]$ describing the electronic degrees of freedom in the uncorrelated lead (on the right of the tunnelling junction). Here $\gamma$ is the tunnelling amplitude between the leads. The current operator can be derived in the usual way calculating the time derivative of the particle number operator on the left or, equivalently, on the right side of the junction. Its average should be calculated by means of the Keldysh technique because of the explicit time dependence of $U(t)$ and the finite voltage $V$ which is applied to the junction,

$$j(t) = i\gamma \langle [c^\dagger(0,t)\psi(0,t) - \psi^\dagger(0,t)c(0,t)]S_C \rangle_0,$$  \hfill (23)

where $S_C$ denotes the non-equilibrium $S$-matrix defined on the Keldysh contour $C$,

$$S_C = T_C \exp \left\{-i\gamma \int_C dt' [\psi^\dagger(0,t')c(0,t')] + c^\dagger(0,t')\psi(0,t') \right\}.$$  \hfill (24)

$T_C$ is the contour ordering operation. Performing the second order perturbative expansion in the tunnelling amplitude one obtains the usual expression for the current $j(t)$

$$j(t) = \frac{\gamma^2}{2} \int dt' \left[ G_{0}^- (t' - t)g_0^- (t,t') - G_{0}^- (t - t')g_0^- (t',t) \right. $$

$$+ \left. G_{0}^+ (t' - t)g_0^+ (t,t') - G_{0}^+ (t - t')g_0^+ (t',t) \right].$$  \hfill (25)

We are again using the local Keldysh Green’s functions defined as in Eq. (2). The Green’s functions on the opposite side of the junction, denoted by $G_{ij}^\pm (t)$, are defined in a similar way with the change $\psi \rightarrow c$ and depend, contrary to the situation on the left side of the junction, only on the time differences. Taking this and the factorisation relation (26) into account we obtain an expression for the current similar to Eq. (25).

$$j(t) = \gamma^2 \int dt' G(t') \exp \left(-i2\sin[\Omega t']/2,\sin[\Omega t + \Phi] \right),$$  \hfill (26)

the function $G(t)$ being

$$G(t) = G_{0}^- (-t)g_0^- (-t) - G_{0}^- (-t)g_0^- (-t) + G_{0}^- (-t)g_0^+ (-t) - G_{0}^+ (t)g_0^+ (-t).$$  \hfill (27)
It can be shown, e. g. by doing a perturbative expansion in $|z|$, that the time-ordered part of this function does not contribute to the tunnelling current and can be dropped.

Again, we expect that it is hardly possible to measure the time evolution of $j(t)$ directly. A more easily accessible quantity is the current averaged over one (or more) period of the oscillating perturbation. Performing the average in the same way as for the TED we obtain the following general result,

$$j = \frac{\gamma^2}{2\pi} \sum_n J_n^2 ||z|| \int d\omega (G_0^+(\omega)g_0^{-}(\omega + \Omega n) - G_0^-(\omega + \Omega n)g_0^+(\omega)) .$$

(28)

We stress again that we have chosen the specific time-dependence of the perturbation, (13), for the sake of clarity. A similar formula will hold for any periodic perturbation (with Bessel functions substituted by Fourier coefficients relevant for the given perturbation.)

IV. APPLICATIONS AND CONCLUSIONS

The junction is biased by a finite voltage $V$. Setting the chemical potential in the host to zero and the chemical potential of the lead to $-V$, we can immediately write down the Green’s functions entering Eq. (28),

$$G_0^-(\omega) = -i \Theta(\omega + V)\nu_c ,$$

(29)

$$G_0^+(\omega) = i \Theta(-\omega - V)\nu_c ,$$

$$g_0^+(\omega) = -i \Theta(\omega)\nu_\psi(\omega) ,$$

$$g_0^-(\omega) = i \Theta(-\omega)\nu_\psi(\omega) ,$$

where $\nu_c$ is a constant density of states on the right electrode and $\nu_\psi(\omega)$ is the LDOS in the host which, in general, does depend on the energy $\omega$ in the relevant energy range (set by $V$ and $\Omega$).

Let us first neglect this energy dependence, i. e. take $\nu_\psi(\omega) = \nu_\psi^{(0)}$. Then substituting the above Green’s functions into (28) we find that because of the sum rule,

$$\sum_{n=-\infty}^{+\infty} J_n^2 ||z|| = 1 ,$$

(30)

the usual linear current-voltage characteristics [19] is restored:

$$j = \gamma^2 \nu_c \nu_\psi^{(0)} V/2\pi$$

(31)

It is remarkable that while the dynamic scatterer profoundly affects the TED, all these contributions completely cancel out (for a constant LDOS) in the total current. (A sum rule similar to (30) and hence the result (31) can be shown to hold for arbitrary periodic perturbation.) Thus, as long as the energy dependence of the LDOS can cancel out (for a constant LDOS) in the total current. (A sum rule similar to (30) and hence the result (31) can

Therefore, as soon as the particle-hole symmetry is violated, the current becomes strongly influenced by the source of oscillations. Notice that it is essential to have a finite voltage applied across the junction. As is the case for the TED, Eq. (33) can be thought of as a sum of contributions from different harmonics, that are generated by the perturbation $U(t)$. The prefactors $J_n^2(||z||)$ govern the amplitude of $n$-th harmonic. The argument $|z|$ of the Bessel functions sets a characteristic energy scale and effectively cuts off the sum over $n$ because $J_n(||z||)$ as a function of
n decreases exponentially for \( n > |z| \). An important consequence is that for \( |z| < 1 \) the perturbation is not strong enough to produce higher harmonics and according to Eq. (23) the linear behaviour of \( j(V) \) is restored with an \( \Omega \) dependent conductance.

In real systems, such as SWNTs, the main reason for the energy dependence of the LDOS is, of course, correlations. In LLs, the LDOS is strongly energy-dependent, see Eq. (21). Combining Eqs. (28), (29) and (21) we obtain the expression for the current between the dynamically perturbed LL and a non-interacting lead:

\[
 j = g \frac{\gamma^2}{2\pi} \nu_c C_{\psi} \left\{ J_0^2 ||z|| \text{sgn}(V) |V|^{1/g} + \sum_{n=1}^{\infty} J_n^2 ||z|| \left( \text{sgn}(V - \Omega n) |V - \Omega n|^{1/g} + \text{sgn}(V + \Omega n) |V + \Omega n|^{1/g} \right) \right\}. \tag{34}
\]

Formula (34) is the main result of this paper. As expected from the previous discussion, turning off the interactions, \( g = 1 \), restores the linear current-voltage characteristics expected for a tunnelling junction between two uncorrelated metallic leads. For the interacting case we predict frequency and amplitude dependent current across the junction. Since the SWNTs oscillate at very high frequencies (several \( 10^{10} \) Hz which corresponds to \( \sim 10^{-4} \) eV, see e.g. Ref. [9]) we would expect that in the transport experiments on oscillating SWNTs even the lowest harmonics should be observable. The most interesting features arise in the voltage dependence of the differential conductance \( dj/dV \), see Fig. 2, where it is plotted for three different perturbation strengths. At the onset of every harmonic, \( V/\Omega = \text{integer} \) there are pronounced dips at the positions of additional sidebands in the TED.

Another important setup is when a LL is coupled via a tunnelling junction to a semiconducting electrode (without the bias voltage). In this case one of the Green’s function is identically zero, \( G_{0+}^+(\omega) = 0 \) as the conductance band of the semiconductor (SC) is nearly empty. The other off-diagonal Green’s function contains the SC gap \( \Delta \), and a constant \( \nu_s \) directly connected with the LDOS in the SC in vicinity of the conductance band bottom \( [20] \),

\[
 G_{0+}^+(\omega) = -i \nu_s \Theta(\omega - \Delta). \tag{35}
\]

Substituting this into Eq. (28) results in

\[
 j = g \frac{\gamma^2}{2\pi} \nu_s C_{\psi} \sum_{n>\Delta/\Omega} \infty J_n^2 ||z|| (\Omega n - \Delta)^{1/g}. \tag{36}
\]

Contrary to a metallic junction, \( j \) is nonzero even in the non-interacting case. As in the previous setups, the current is constituted by contributions from different harmonics. However, since the energy of a given harmonic has to overcome the SC gap to contribute to the current, the sum starts at \( n = \Delta/\Omega \) (we assume \( \Omega \) to be positive). The tunnelling current shows a very sharp threshold at \( |z| = \Delta/\Omega \) because for \( |z| < \Delta/\Omega \) the contribution of the sum on the right-hand-side of Eq. (36) is exponentially small. Of course, Eq. (36) gives the current immediately after the perturbation was switched on since the charging effects across the junction would tend to suppress it. Such process gives rise to a finite potential difference across the junction which can be detected. As it is related to the frequency \( \Omega \) and amplitude of the oscillations, such hetero-junction might be employed as a detector.
In conclusion, we analysed the influence of a time-dependent forward scattering perturbation on the transport properties of correlated one-dimensional electron systems. We derived the exact Green’s functions of the system in the presence of the dynamic perturbation. This knowledge allowed us to establish the relation between the total TED of the electrons in the case with and without the perturbation. In the case of a periodic perturbation with the frequency $\Omega$ the TED develops sidebands at energies $E_F + n\Omega$, $n$ being an integer. The precise energy dependence of the sidebands coincides with the behaviour of the local density of states in the particular interacting system without the perturbation. We further developed this theory to calculate the current-voltage characteristics of a tunnelling junction between an interacting electron system and a metallic or semiconducting electrode and found that the transport is strongly affected by the additional dynamic perturbation.

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