(Non)-supersymmetric Marginal Deformations from Twistor String Theory

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Abstract

The tree-level amplitudes in $\beta$-deformed theory are studied from twistor string theory. We first show that a simple generalization of the proposal in hep-th/0410122 gives the correct results for all of the tree-level amplitudes to the first order of the deformation parameter $\beta$. Then we give a proposal to all orders of $\beta$ and show this matches the field theory. We also show the prescription using connected instantons and the prescription using disconnected instantons are equivalent in the deformed twistor string theory. The tree-level amplitudes in non-supersymmetric $\gamma$-deformed theory are also obtained.
in this framework. The tree-level purely gluonic amplitudes in theories with generic marginal deformations are also discussed in the twistor string theory side.

1 Introduction

One of the important issues in AdS/CFT correspondence [1, 2, 3] is to study the gravity dual of the gauge theories with less supersymmetries. In [4], the gravity dual of some gauge theories with less supersymmetries was studied using a solution generating transformation. One of the gauge theories studied in [4] is the $\beta$-deformed theory which has $\mathcal{N} = 1$ supersymmetry. Later this discussion was generalized to $\gamma$-deformed theory which has no supersymmetries [5, 6, 7]. These deformed SYM can be obtained from a certain kind of star product among the superfields. Using this star product, it is easy to see that the planar amplitudes in such theories are identical to those of the $\mathcal{N} = 4$ theory up to an overall phase factor [4, 17, 7].

On the other hand, Witten found a new relation between perturbative gauge theory and a topological string theory whose target space is the super-twistor space [18]. While the proposed AdS/CFT correspondence is a strong-weak duality, the new correspondence found by Witten is a perturbative one. The gauge theory amplitudes can then be computed using localization to either the connected instantons in twistor space [18, 19, 20, 21], or the completely disconnected instantons [22]. Later it was shown that in fact these two prescriptions are equivalent [23] and a family of intermediate prescriptions are also studied [23, 21]. Following Witten’s original paper [18], there appeared a lot of works on computing tree-level and/or one-loop amplitudes in four dimensional gauge theories, some of which are [25–41].

The twistor string theory corresponding to the gauge theories with less supersymmetries obtained by the Leigh-Strassler deformation [42] was proposed in [43], motivated by earlier work on deformations in the open-closed topological string theories [44, 45]. There it was suggested that the effect of the marginal deformations in field theory can be captured in twistor string theory by introducing a non-anticommutative star product among the fermionic coordinates in the super-twistor space. This star product can be shown heuristically to partially arise from a particular fermionic deformation in the

\footnote{The deformations in gauge theory were also studied in [8–15] and some generalizations of the work in [5] including a generalization of $\gamma$-deformation can be found in [16].}
closed string background. A prescription for calculating the tree-level amplitudes in the deformed theory using the connected instantons was also given in that paper to the first order of the deformation parameters. In this prescription, a non-anti-commutative star product among the wavefunctions of the external particles plays an important role. The authors of [43] computed examples of amplitudes corresponding to degree one curves in twistor space (these amplitudes are dubbed 'analytic amplitudes' in [46, 47]) and found that the results coincide with field theory results. Later they claimed one can generalize the prescription to amplitudes from higher degree curves [48]. The precise origin of this prescription is still not well understood. In particular, a prescription valid to all orders of the deformation parameter is absent until now. It is therefore interesting to make some progress in this direction.

Since the planar amplitudes in $\beta$-deformed and $\gamma$-deformed theories are rather simple, one may wonder whether they can be reproduced using the twistor string theory. Up to now it’s not clear how to calculate the amplitudes beyond tree-level from twistor string technique, we will from now on focus on tree-level amplitudes. We first show that the tree-level amplitudes in $\beta$-deformed theory can be obtained from the prescription in [43] to first order of the deformation parameter. Notice our result is not restricted to the degree one case. In fact, we study all of the tree-level amplitudes exploiting a rather simple generalization of [43] based on previous work for the $\mathcal{N} = 4$ theory [19, 20, 21]. We then propose an all-order exact prescription and show that this new prescription gives the right field theory results. The MHV diagrams and the equivalence between the prescriptions using connected instantons and completely disconnected instantons are also discussed in the case of the $\beta$-deformed theory. The $\gamma$-deformed theory is no longer supersymmetric, and doesn’t belong to the class of field theories studied in [43]. In this case, we also give an prescription to all orders of $\gamma$ to reproduce tree-level amplitudes from the twistor string theory.

Encouraged by these results, we proceed to find some further evidence for a possible all-degree generalization of [43] for theories with generic Leigh-Strassler deformations. We consider tree-level scattering amplitudes involving only particles in the $\mathcal{N} = 1$ vector supermultiplet. One can easily see that the particles in the the $\mathcal{N} = 1$ chiral supermultiplets cannot appear in the contributing tree-level Feynman diagrams. So these amplitudes in the theory with generic Leigh-Strassler deformations are the same as the ones in the undeformed $\mathcal{N} = 4$ theory. In [43], this result was shown to be valid for the analytic amplitudes to the linear order of the deformation parameters.
Here we improve on this result by showing that it is in fact true for all of the tree-level amplitudes involving only particles in $\mathcal{N} = 1$ vector supermultiplet.

This paper is organized as follows. In the next section we briefly review the method of computing field theory tree-level amplitudes from connected instantons in twistor string theory. In section 3, we study the amplitudes in $\beta$-deformed gauge theory from the twistor string theory. An exact prescription for the amplitudes in $\gamma$-deformed theory is given in section 4. In section 5, we discuss the tree-level amplitudes with only particles in the $\mathcal{N} = 1$ vector supermultiplet in the case of generic Leigh-Strassler deformations. The final section devoted to conclusions and discussions on further studies.

2 Review of tree-level Yang-Mills amplitudes from connected instantons in twistor string

2.1 Amplitudes of the $\mathcal{N} = 4$ theory from twistor string theory

We denote the tree-level partial amplitude of $\mathcal{N} = 4$ super Yang-Mills theory, which is the coefficient of $\text{Tr}(T^{a_1} \cdots T^{a_n})$, by

$$A_{n}^{\mathcal{N}=4}(\{\pi_i, \tilde{\pi}_i, h_i, t_i\}; \{\pi_n, \tilde{\pi}_n, h_n, t_n\}),$$

or simply by $A_{n}^{\mathcal{N}=4}(\{\pi_i, \tilde{\pi}_i, h_i, t_i\})$, here we decompose the momenta of the $i$-th massless external particles into two Weyl spinors, i.e., we define the 'bi-spinor' $p_{i\dot{a}}$ as $p_{i\dot{a}} \equiv p_{i\mu}^{\dot{a}} \sigma_{\mu\dot{a}}$ and choose a decomposition of this 'bi-spinor' as $p_{i\dot{a}} = \pi_{i\dot{a}} \tilde{\pi}_{i\dot{a}}$ (for more details, see, for example [18, 49]), and we use $h_i$ to denote the helicity of this external particle and $t_i$ its $SU(4)_R$ quantum numbers.

This amplitude can be computed in twistor string theory from localization on the connected instantons [18, 19, 20, 21] (some aspects of this approach were summarized in [49]). The main formula is:

$$A_{n}^{\mathcal{N}=4}(\{\pi_i, \tilde{\pi}_i, h_i, t_i\}) = \int dM_d \langle \int_C J_1 \Psi_1 \cdots \int_C J_n \Psi_n \rangle,$$

in the following we will explain the ingredients in this formula. First we note that in the supertwistor space $\mathbb{CP}^{3|4}$ with coordinates denoted by $Z^I = \ldots$
\( (\lambda^a, \mu^\alpha), \psi^A, (a, \hat{a} = 1, 2, I, A = 1, \ldots, 4) \), we have the following expansion \( \mathcal{A} \):

\[
\mathcal{A} = A + \psi^4 \chi_4 + \psi^I \chi_I + \psi^A \psi^I \phi_I + \frac{1}{2} \psi^I \psi^J \epsilon_{IJK} \tilde{\phi}^K + \frac{1}{2} \psi^4 \psi^I \psi^J \epsilon_{IJK} \chi^K + \frac{1}{3!} \psi^I \psi^J \psi^K \epsilon_{IJK} \chi^4 + \frac{1}{3!} \psi^4 \psi^I \psi^J \psi^K \epsilon_{IJK} G.
\]

As in [43], we split the four fermionic coordinates in \( \mathbb{C}P^3|4 \), \( \psi^A (A = 1, \ldots, 4) \), into \( \psi^I (I = 1, 2, 3) \) and \( \psi^4 \). For latter convenience, we write the above expansion as

\[
\mathcal{A} = \sum_{h=-1,-1/2,0,1/2,1} g_h^a(\psi^4, \psi^I) \Phi_{ha}(\lambda, \mu),
\]

where \( \Phi_{ha} = A, \chi_4, \chi_I, \phi_I, \tilde{\phi}^I, \chi^K, \chi^4, G \) respectively. \( \Psi_i(\pi_i, \tilde{\pi}_i, h_i, t_i) \) in eq. (3) is the wavefunction of the i-th particle in the supertwistor space, which is given by

\[
\Psi_i(\pi_i, \tilde{\pi}_i, h_i, t_i) = \bar{\delta}(\langle \lambda, \pi_i \rangle) \left( \frac{\lambda}{\pi_i} \right)^{2h_i-1} \exp \left(i[\tilde{\pi}_i, \mu](\pi_i/\lambda)\right) g_h^a(\psi^4, \psi^I),
\]

where the definition of the delta function \( \bar{\delta}(f) \) is

\[
\bar{\delta}(f) = \bar{\delta} f \delta^2(f)
\]

following [22]. In eq. (3), \( J_i \) is a holomorphic current made of free fermions in the wouldvolume theory of the D5-branes (the details can be found in [18, 49]). Since what we actually compute is the partial amplitude which is the coefficient of \( \text{Tr}(T^{a_1} \cdots T^{a_n}) \), in eq. (3) we only pick out the following coefficient of \( \text{Tr}(T^{a_1} \cdots T^{a_n}) \) in the correlation function \( \langle J_1(u_1) \cdots J_n(u_n) \rangle \):

\[
\frac{\prod_i \langle u_i, du_i \rangle}{\prod_k \langle u_k, u_{k+1} \rangle}.
\]

In the prescription using the connected \( D \)-instantons which are D-strings wrapped on algebraic curves, these amplitudes only receive contributions from the curves with genus zero and degree satisfying \( d = \frac{1}{2} \sum_{i=1}^{n} (1 - h_i) - 1 \).

If we choose homogeneous coordinates \( (u, v) \) on an abstract \( \mathbb{C}P^1 \), then the genus 0, degree \( d \) curve \( C \), which is a map from \( \mathbb{C}P^1 \) to \( \mathbb{C}P^{3|4} \), can be
parametrized as

\[ Z^I = P^I(u,v) = \sum_{\alpha=0}^{d} P^I_{\alpha} u^{\alpha} v^{d-\alpha}, \]
\[ \psi^A = Q^A(u,v) = \sum_{\alpha=0}^{d} Q^A_{\alpha} u^{\alpha} v^{d-\alpha}. \]  

(9)

Then the measure on the moduli space of the genus 0, degree \( d \) curves can be written as

\[ dM_d = \frac{\prod_{\alpha=1}^{d} \prod_{A=1}^{4} \prod_{I=1}^{4} dP^I_{\alpha} dQ^A_{\alpha}}{GL(2, \mathbb{C})}. \]  

(10)

Now we may write eq. (3) as

\[ A^{N=4}_n(\{\pi_i, \tilde{\pi}_i, h_i, t_i\}) = \int dM_d \prod_i \int_G \langle u_i, du_i \rangle \delta(\langle \lambda(u_i), \pi_i \rangle) \left( \frac{\lambda(u_i)}{\pi_i} \right)^{2h_i-1} \times \exp \left( i [\mu(u_i), \tilde{\pi}_i] (\pi_i/\lambda(u_i)) \right) g_{h_i}^{a_i}(\psi_i) \frac{1}{\prod_k \langle u_k, u_{k+1} \rangle}. \]  

(11)

### 2.2 Amplitudes in deformed SYM from twistor string theory

In [43], a proposition was proposed to calculate the amplitudes in deformed SYM from twistor string theory to the linear order of the deformation parameters.

The superpotential in the deformed theory considered in [43] can be written as

\[ W = W_{N=4} + \frac{1}{3!} h^{IJK} \text{Tr}(\Phi_I \Phi_J \Phi_K), \]  

(12)

to linear order of the deformation parameters \( h^{IJK} \), where \( h^{IJK} \) is totally symmetric. Here

\[ W_{N=4} = i \text{Tr}(\Phi_1[\Phi_2, \Phi_3]), \]  

(13)

is the superpotential of the \( N = 4 \) SYM and \( \Phi_i, i = 1, 2, 3 \) are three chiral superfields in the adjoint representation of the gauge group. We note that the deformed theory only has \( N = 1 \) supersymmetry.

\[ ^2 \text{As in [43], we pull the gauge coupling constant out of the superpotential. For ease of comparison, we also use the same normalization as [43].} \]
The authors of [43] used a star product among the $\psi$-dependent part of the wavefunctions (to the first order of the deformation parameters) as follows

$$f(\psi_1) \ast g(\psi_2) = f(\psi_1)g(\psi_2) - \frac{i}{4} V_{KL}^{IJ} \left( f(\psi_1) \frac{\partial}{\partial \psi_1^I} \psi_1^K \psi_2^L \left( \frac{\partial}{\partial \psi_2^J} g(\psi_2) \right) \right),$$

where the definition of $V_{KL}^{IJ}$ is

$$V_{KL}^{IJ} = h^{IQQ} \epsilon_{QKL} + \epsilon^{IQQ} \tilde{h}_{QKL},$$

then the amplitudes is given by

$$A_{\text{deformed}}^{\text{tree}}(\{\pi_i, \tilde{\pi}_i, h_i, t_i\}) = \int dM_d(\int_C J_1 \Psi_1 \ast \int_C J_2 \Psi_2 \ast \cdots \ast \int_C J_n \Psi_n),$$

to first order of the deformation parameters.\footnote{We note that the superscript of $\psi$ denotes its $SU(4)$ R-symmetry quantum number (the $SU(4)_R$ symmetry of the $N = 4$ SYM has been broken by the deformation) and the subscript of $\psi$ numbers the corresponding external particle.}

3 Tree-level amplitudes in $\beta$-deformed theory from twistor string theory

3.1 All tree-level amplitudes to linear order in $\beta$

The superpotential in the $\beta$-deformed SYM theory \cite{4, 17} is

$$W_{\beta-\text{deformed}} = i \text{Tr}(e^{i\pi \beta} \Phi_1 \Phi_2 \Phi_3 - e^{-i\pi \beta} \Phi_1 \Phi_3 \Phi_2).$$

Here $\beta$ is a real deformation parameter.

Now we compute the tree-level amplitudes in this theory using the all-degree generalized prescription we give above. First, we expand $W_{\beta-\text{deformed}}$ to first order of $\beta$:

$$W_{\beta-\text{deformed}} = W_{N=4} - \pi \beta \text{Tr}(\Phi_1 \{\Phi_2, \Phi_3\}) + O(\beta^2).$$

\footnote{As we have mentioned in the introduction, in \cite{43} only the analytic amplitudes were discussed, the formula here is valid for all tree-level amplitudes.
Then one can easily read off in this case the $h^{IJK}$'s in eq. (12) are simply $-2\pi\beta |\epsilon_{IJK}|$, so we find

$$V_{KL}^{IJ} = h_{KL}^{IJQ} \epsilon_{QKL} + \epsilon_{QKL} h_{KL}^{IJ} = -4\pi\beta \delta_{K}^{I} \delta_{L}^{J} \epsilon_{IJK} \alpha_{Q}$$

where the definition of $\alpha_{I}$'s is $\alpha_{1} = \alpha_{2} = \alpha_{3} = 1$.

Using this, it can be easily found that for arbitrary wavefunction $f$ and $g$ we have

$$f(\psi_{1}) * g(\psi_{2}) = f(\psi_{1})g(\psi_{2}) + i\pi\beta f(\psi_{1}) \left( \sum_{I,J,K} \epsilon_{IJK} \left( \frac{\partial}{\partial \psi_{1}} \psi_{1}^{I} \alpha_{K} \psi_{2}^{J} \frac{\partial}{\partial \psi_{2}} \right) \right) g(\psi_{2})$$

$$+ O(\beta^{2}).$$

Because the wavefunctions are all monomials in fermionic directions, we further have

$$f(\psi_{1}) \frac{\partial}{\partial \psi_{1}} \psi_{1}^{I} = \begin{cases} f(\psi_{1}) & \text{if } I \in f, \\ 0 & \text{otherwise.} \end{cases}$$

were by $I \in f$, we simply meant $\psi_{1}^{I}$ is a factor of $f(\psi_{1})$. In other words,

$$f(\psi_{1}) \frac{\partial}{\partial \psi_{1}} \psi_{1}^{I} = N_{f}^{I} f(\psi_{1}),$$

where $N_{f}^{I}$ is the number of $\psi_{1}^{I}$ in $f(\psi_{1})$ ($N_{f}^{I} \leq 1$ since $\psi_{1}^{I}$ is fermionic). Similarly,

$$\psi_{2}^{J} \frac{\partial}{\partial \psi_{2}} g(\psi_{2}) = N_{g}^{J} g(\psi_{2}).$$

So eq. (20) can also be re-written as

$$f(\psi_{1}) * g(\psi_{2}) = (1 + i\pi\beta \sum_{I,J,K} \epsilon_{IJK} N_{f}^{I} N_{g}^{J} \alpha_{K}) f(\psi_{1})g(\psi_{2}) + O(\beta^{2})$$

$$= (1 + i\pi\beta \sum_{I \in f, J \in g} \epsilon_{IJK} \alpha_{K}) f(\psi_{1})g(\psi_{2}) + O(\beta^{2}),$$

We can slightly rephrase the above results by considering the following $U(1) \times U(1)$ symmetry acting on the fermionic part of the supertwistor space.
\[ \mathbb{CP}^{3|4}, \text{under which } (\psi^1, \psi^2, \psi^3, \psi^4) \text{ are assigned the following charges} \]

\[
\begin{array}{cccc}
\bar{Q}_1 & \psi^1 & 0 & -1 \\
\bar{Q}_2 & \psi^2 & 1 & 0 \\
\end{array}
\]

\[
\begin{array}{cccc}
\bar{Q}_3 & \psi^3 & 1 & -1 \\
\bar{Q}_4 & \psi^4 & 0 & 0 \\
\end{array}
\]

(25)

from which we have \( \epsilon_{ijk} \alpha_k = \bar{Q}_1 \bar{Q}_2 - \bar{Q}_3 \bar{Q}_4 \). Using the identity

\[
\bar{Q}(\psi^1 \cdots \psi^n) = \bar{Q}^{\psi^1} + \cdots + \bar{Q}^{\psi^n},
\]

(26)

we get

\[
\Psi_1 \star \Psi_2 \star \cdots \star \Psi_n = \left( 1 + i\pi \beta \sum_{i < j} (\bar{Q}_1^i \bar{Q}_2^j - \bar{Q}_3^i \bar{Q}_4^j) \right) \Psi_1 \Psi_2 \cdots \Psi_n + O(\beta^2),
\]

(27)

Here \( \bar{Q}_1^i \) and \( \bar{Q}_2^i \) are the charges of the i-th wavefunction. We notice that there is the following relation between the \( \bar{Q}_i \), defined here and the \( Q_i \) defined in\cite{4, 17},

\[
Q_i(\Phi_{ha}) + \bar{Q}_i(g^a(\psi^4, \psi^I)) = 0.
\]

(28)

Replacing the charges using this identification, we find

\[
\mathcal{A}_n^{\beta-deformed} = \left( 1 + i\pi \beta \sum_{i < j} (Q_1^i Q_2^j - Q_3^i Q_4^j) \right) \mathcal{A}_n^{\mathcal{N}=4} + O(\beta^2).
\]

(29)

Here \( Q_1^i \) and \( Q_2^i \) are the charges of the wavefunction of i-th external particle. This result coincides with the results in \cite{17} to first order of \( \beta \).

### 3.2 The star product to all orders of \( \beta \)

Based on the calculation in the previous subsection, we propose the following star product among the wave functions

\[
f(\psi_1) \star g(\psi_2) = f(\psi_1) \exp \left( i\pi \beta \sum_{I,J,K} \frac{\partial}{\partial \psi_1^I} \psi_1^I \alpha_K \frac{\partial}{\partial \psi_2^J} g(\psi_2) \right)
\]

\[
\text{[50] In [50], similar star product was introduced in } \mathcal{N} = 4 \text{ light-cone superspace in stead of supertwistor space. Recently, a possible new relation between } \beta\text{-deformation and non-commutative field theories was discussed in [51].}
\]

9
As in the previous subsection, we can now re-write the latter one. This can be verified explicitly as follows:

\[
\sum_{n=0}^{\infty} \frac{(i \pi \beta)^n}{n!} \sum_{I_1, J_1, K_1} \cdots \sum_{I_n, J_n, K_n} f(\psi_1)(\epsilon_{I_1, J_1, K_1, L_1, M_1} \times g(\psi_2)) = \exp \left( i \pi \beta \sum_{I,J} \epsilon_{IJK} \right) f(\psi_1)g(\psi_2)
\]


(30)

will reproduce all tree-level field theory amplitudes to all orders of \( \beta \).

Since \( \frac{\partial}{\partial \psi_1^I} \) commutes with \( \psi_2^m \), we have

\[
f(\psi_1) * g(\psi_2) = \sum_{n=0}^{\infty} \frac{(i \pi \beta)^n}{n!} \sum_{I_1, J_1, K_1} \cdots \sum_{I_n, J_n, K_n} \epsilon_{I_1, J_1, K_1} \cdots \epsilon_{I_n, J_n, K_n} \times g(\psi_2)
\]

\[
= \sum_{n=0}^{\infty} \frac{(i \pi \beta)^n}{n!} \sum_{I_1, J_1, K_1} \cdots \sum_{I_n, J_n, K_n} \epsilon_{I_1, J_1, K_1} \cdots \epsilon_{I_n, J_n, K_n} \times N^f_{I_1} \cdots N^f_{I_n} N^g_{J_1} \cdots N^g_{J_n} f(\psi_1)g(\psi_2)
\]

\[
= \sum_{n=0}^{\infty} \frac{(i \pi \beta)^n}{n!} \left( \sum_{IJK} \epsilon_{IJK} N^f_{I} N^g_{J} \right)^n f(\psi_1)g(\psi_2)
\]

\[
= \sum_{n=0}^{\infty} \frac{(i \pi \beta)^n}{n!} \left( \sum_{I,J} \epsilon_{IJK} \right)^n f(\psi_1)g(\psi_2)
\]

\[
= \exp \left( i \pi \beta \sum_{I,J} \epsilon_{IJK} \right) f(\psi_1)g(\psi_2).
\]

(31)

As in the previous subsection, we can now re-write

\[
f(\psi_1) * g(\psi_2) = \exp \left( i \pi \beta (\bar{Q}_1^f \bar{Q}_2^g - \bar{Q}_2^f \bar{Q}_1^g) \right) f(\psi_1)g(\psi_2).
\]

(32)

From eq. (28), we know that the star product among the wavefunctions defined here is isomorphic to the one among the fields defined in [4,17], and hence the former star product is also associative due to the associativity of the latter one. This can be verified explicitly as follows:

\[
(f * g) * h = \exp \left( i \pi \beta (\bar{Q}_1^f \bar{Q}_2^g - \bar{Q}_2^f \bar{Q}_1^g) \right) (fg) * h
\]

\[
= \exp \left( i \pi \beta (\bar{Q}_1^f \bar{Q}_2^g - \bar{Q}_2^f \bar{Q}_1^g) + (i \pi \beta (\bar{Q}_1^f \bar{Q}_2^g - \bar{Q}_2^f \bar{Q}_1^g)) \right) fgh
\]
\[ \exp \left( i\pi \beta \left( \tilde{Q}_h^f \tilde{Q}_2^g - \tilde{Q}_2^f \tilde{Q}_1^g \right) \right) f g h \\
= \exp \left( i\pi \beta \left( \tilde{Q}_h^f \tilde{Q}_2^g - \tilde{Q}_2^f \tilde{Q}_1^g \right) \right) f * (g h) \\
= f * (g * h), \quad (33) \]

where \( \tilde{Q}_i^f = \tilde{Q}_i^f + \tilde{Q}_i^g (i = 1, 2) \) is used.

Similar to what we have done in the previous subsection, one can easily verify that

\[ A_{n}^{\beta\text{-deformed}} = \exp \left( i\pi \beta \sum_{i<j} (Q_i^1 Q_j^2 - Q_j^1 Q_i^2) \right) A_{n}^{N=4}. \quad (34) \]

This result is indeed the same as the one obtained in field theory computations [14, 17].

### 3.3 The MHV diagrams and the equivalence between different twistor space prescriptions

In this subsection we will discuss the MHV diagrams and the equivalence between the prescription using connected instantons and the prescription using completely disconnected instantons for the \( \beta \)-deformed theory [6].

The analytic amplitude in \( \beta \)-deformed theory is independent of \( \tilde{\lambda} \), since this amplitude equals the product of the corresponding analytic amplitude in \( N = 4 \) theory and a phase factor [7], both of which are independent of \( \tilde{\lambda} \). Then in the \( \beta \)-deformed theory, we can use the same off-shell continuation as in [22, 47] to define the analytic vertex. We can compute all of the tree-level amplitudes in \( \beta \)-deformed theory from the MHV diagrams obtained by connecting the analytic vertices with propagators as in [47]. Similar to the proof using Feynman rules in [17], we can prove that the amplitudes from the MHV diagrams are exactly the same as expected.

As in the \( N = 4 \) theory [22], the extended-CSW rules in the \( \beta \)-deformed theory mentioned above can be obtained from the prescription for calculating the tree-level amplitudes using completely disconnected instantons in the twistor string theory. Consider an expression in the completely disconnected prescription corresponding to a given MHV diagram. Notice that the twistor

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6In this subsection, we will only give some brief discussions, the details are omitted since they are similar to the discussions in the \( N = 4 \) case.

7Notice that the phase factor also depends on the internal particles.
space propagator $D$ proportional to $\delta^4(\psi - \psi')$ and the latter is just $(\psi - \psi')^4$ (in [23, 49] ($\psi$ and $\psi'$ are fermionic coordinates of the two ends of the twistor space propagator). In a given MHV diagram, each end of a propagator has a fixed helicity and a fixed $SU(4)_R$ quantum number. According to this helicity and quantum number, only one term in the expansion of $(\psi - \psi')^4$ will be picked out. This term is a product of two factors, one factor is a product of $\psi$'s the other of $\psi'$'s. So for every analytic vertex and a propagator connected to this vertex, there is an end of the propagator corresponding to the vertex. Then there is a corresponding product of the fermionic coordinates. In the expression corresponding to the MHV diagram, for every vertex, these factors from the propagators connected to it and the wavefunctions of the external particles connected to it should be multiplied using the star product in supertwistor space to give the amplitudes in $\beta$-deformed theory. Then, similar to the proof using Feynman rules in [17], we will finally get a star product among all of the external wavefunctions multiplied by ordinary multiplication with the ordinary product of the twistor space propagators. The associativity of the all-order star product defined in the last subsection is essential for these arguments.

Following the proof in [23] for the $\mathcal{N} = 4$ case, one can prove that the prescription using the connected instantons and the prescription using the completely disconnected instantons will give the same result. One need only notice that in the proof in [23], the poles of the integrands which contribute to the residues never came from the wave-functions themselves, nor from the case when two points where the wavefunctions are inserted come close to each other.

4 Tree-level amplitudes in $\gamma$-deformed theory from twistor string theory

A generalization of the $\beta$-deformation is the $\gamma$-deformation [5, 6, 7]. It is obtained by introducing the following star product

$$f \ast g = \exp \left( - i\pi Q^i_1 Q^j_2 \epsilon_{ijk} \gamma_k \right) f g$$

among the component fields in the Lagrangian of $\mathcal{N} = 4$ theory. Here $\gamma_i, i = 1, 2, 3$ are three real deformation parameters and $Q_i, i = 1, 2, 3$ are the charge of three $U(1)$ symmetries of the theory. The charges of the component fields
are the following [7]:

\[
\begin{array}{ccccccccc}
A_\mu & \phi_1 & \phi_2 & \phi_3 & \chi_1 & \chi_2 & \chi_3 & \chi_4 \\
Q_1 & 0 & 1 & 0 & 0 & 1/2 & -1/2 & -1/2 & 1/2 \\
Q_2 & 0 & 0 & 1 & 0 & -1/2 & 1/2 & -1/2 & 1/2 \\
Q_3 & 0 & 0 & 0 & 1 & -1/2 & -1/2 & 1/2 & 1/2 \\
\end{array}
\]

(36)

and for every component field \( \phi \), we have \( Q_i(\phi^\dagger) = -Q_i(\phi) \).

The \( \gamma \)-deformed theory is non-supersymmetric and the component Lagrangian of this theory can be found for example in [7]. To reproduce the tree-level amplitudes in the \( \gamma \)-deformed theory [5, 6, 7] from connected instantons in twistor string theory, we consider the following global \( U(1)^3 \) symmetry acting on the fermionic coordinates of the supertwistor space, under which \((\psi^1, \psi^2, \psi^3, \psi^4)\) have the following charge assignment

\[
\begin{array}{cccc}
\psi^1 & \psi^2 & \psi^3 & \psi^4 \\
\tilde{Q}_1 & -1/2 & 1/2 & 1/2 & -1/2 \\
\tilde{Q}_2 & 1/2 & -1/2 & 1/2 & -1/2 \\
\tilde{Q}_3 & 1/2 & 1/2 & -1/2 & -1/2 \\
\end{array}
\]

(37)

and we define the star product among the wave functions as

\[
f(\psi_1) * g(\psi_2) = f(\psi_1) \exp \left( -i\pi \sum_{A,B} \frac{\partial}{\partial \psi_1} \tilde{Q}_i^A \tilde{Q}_j^B \epsilon_{ijk} \gamma_k \tilde{Q}_j^A \tilde{Q}_1^B \frac{\partial}{\partial \psi_2} \right) g(\psi_2),
\]

which is equivalent to

\[
f(\psi_1) * g(\psi_2) = \exp \left( -i\pi \sum_{A\in f, B\in g} \tilde{Q}_i^A \tilde{Q}_j^B \epsilon_{ijk} \gamma_k \right) f(\psi_1)g(\psi_2).
\]

(38)

This star product is also associative and it gives rise to the following tree-level amplitudes:

\[
A_{n-\text{deformed}}^\gamma = \exp \left( -i\pi \sum_{a<b} \tilde{Q}_i^a \tilde{Q}_j^b \epsilon_{ijk} \gamma_k \right) A_{n=4}^N
\]

(40)
again coinciding with field theory results [7] to all orders of $\gamma$. Similarly to what we have done in the previous section, we can show that the prescription using the connected instantons and the one using the completely disconnected instantons are equivalent which relies crucially on the associativity of the star product just defined.

5 Tree-level purely gluonic amplitudes in the general deformed theory

The tree-level purely gluonic amplitudes in theories with generic Leigh-Strassler deformations are the same as in the $\mathcal{N} = 4$ theory [43]. This is a trivial result in the field theory side since if the external particles are all gluons, the internal particles in the tree-level Feynman diagrams can only be gluons too. In the twistor string theory side, this was proved for the analytic amplitudes to first order of the deformation parameters in [43]. Here we will show that it is also true for all of the tree-level amplitudes obtained from twistor string theory. We take this as another consistent check of the all-degree generalization of the their prescription, i.e., eq. (16). We only consider the amplitudes $A_n(1^-, 2^-, 3^-, 4^+, \ldots, n^+)$ as an example. The demonstration of this result for other purely gluonic amplitudes is identical in spirit.

The star product one needs to compute is

$$\left(\epsilon_{I_1J_2K_3}\psi_{I_1}^{J_2} \psi_{K_3}\right) \ast \left(\epsilon_{J_1L_3M_4}\psi_{J_2}^{K_3} \psi_{M_4}\right) \ast \left(\epsilon_{M_1N_2O_3}\psi_{N_1}^{J_2} \psi_{O_3}\right)$$

$$= \epsilon_{I_1J_2K_3}\epsilon_{J_1L_3M_4}\epsilon_{M_1N_2O_3} \left(\psi_{I_1}^{J_2} \psi_{J_1}^{K_3} \psi_{L_3}^{M_4} \psi_{M_1}^{J_2} \psi_{N_1}^{K_3} \psi_{O_3}^{M_4} \right)$$

$$- \frac{i}{4} \left(3^2 \psi_{I_1}^{J_1} \psi_{J_1}^{K_1} \psi_{K_1}^{M_1} \psi_{M_1}^{J_1} \psi_{J_1}^{K_1} \psi_{K_1}^{M_1} \right) \psi_{I_1}^{J_1} \psi_{J_1}^{K_1} \psi_{K_1}^{M_1} \psi_{M_1}^{J_1} \psi_{J_1}^{K_1} \psi_{K_1}^{M_1} \right)$$

$$+ 3^2 \psi_{I_1}^{J_2} \psi_{J_2}^{K_3} \psi_{K_3}^{M_4} \psi_{M_4}^{J_2} \psi_{J_2}^{K_3} \psi_{K_3}^{M_4} \right) \psi_{I_1}^{J_2} \psi_{J_2}^{K_3} \psi_{K_3}^{M_4} \psi_{M_4}^{J_2} \psi_{J_2}^{K_3} \psi_{K_3}^{M_4} \right)$$

$$+ 3^2 \psi_{I_1}^{J_3} \psi_{J_3}^{K_1} \psi_{K_1}^{M_2} \psi_{M_2}^{J_3} \psi_{J_3}^{K_1} \psi_{K_1}^{M_2} \right) \psi_{I_1}^{J_3} \psi_{J_3}^{K_1} \psi_{K_1}^{M_2} \psi_{M_2}^{J_3} \psi_{J_3}^{K_1} \psi_{K_1}^{M_2} \right),$$

(41)

to the linear order of deformation.

The first term in the right hand side of the above equation gives the amplitudes in $\mathcal{N} = 4$ theory [18, 19, 20, 21, 23]. Now we will show that the second term gives no contributions. Since there are 3 external gluons with negative helicity, the contributing algebraic curves in supertwistor space should be the
ones with genus zero and degree 2. Such curves can be parametrized as

\[ Z^I = \sum_{\alpha=0}^{2} P^I_{\alpha} u^\alpha v^{2-\alpha}, \]  
\[ \psi^A = \sum_{\alpha=0}^{2} Q^A_{\alpha} u^\alpha v^{2-\alpha}, \]  

Now we define

\[ F^{I_1 I_2 I_3 J_1 J_2 J_3 K_1 K_2 K_3} \equiv \int \prod_{I=1}^{3} \left( \prod_{\alpha=0}^{2} dQ^I_{\alpha} \right) \psi^I_{I_1} \psi^I_{I_2} \psi^I_{I_3} \psi^J_{J_1} \psi^J_{J_2} \psi^J_{J_3} \psi^K_{K_1} \psi^K_{K_2} \psi^K_{K_3}. \]  

It is not hard to see that

\[ F^{I_1 I_2 I_3 J_1 J_2 J_3 K_1 K_2 K_3} \propto \epsilon^{I_1 I_2 I_3} \epsilon^{J_1 J_2 J_3} \epsilon^{K_1 K_2 K_3}, \]  

here the proportional coefficient is independent of the indices \( I_i, J_i, K_i \) (we will not need the concrete value of this coefficient in the following).

From this result, we can prove that

\[ \epsilon^{I_1 I_2 I_3} \epsilon^{J_1 J_2 J_3} \epsilon^{K_1 K_2 K_3} F^{I_1 I_2 I_3 J_1 J_2 J_3 K_1 K_2 K_3} V^{I_3 J_1} = 0. \]  

Using eq. (45), we know that the left hand side of the above equation is proportional to

\[ \epsilon^{I_1 I_2 I_3} \epsilon^{J_1 J_2 J_3} \epsilon^{K_1 K_2 K_3} \epsilon^{I_1 I_2 I_3} \epsilon^{J_1 J_2 J_3} \epsilon^{K_1 K_2 K_3} (\epsilon^{I_3 J_1 Q} \bar{h}^{Q I_3 J_1} + \epsilon^{I_3 J_1 Q} \epsilon^{Q I_3 J_1}) \]
\[ \propto \delta^{I_3}_{I_1} \delta^{I_3}_{J_1} (\epsilon^{I_5 J_1 Q} \bar{h}^{Q I_3 J_1} + \epsilon^{I_5 J_1 Q} \epsilon^{Q I_3 J_1}) = \epsilon^{I_5 J_1 Q} \bar{h}^{Q I_3 J_1} + \epsilon^{I_5 J_1 Q} \epsilon^{Q I_3 J_1} \]
\[ = 0. \]  

So the second term in eq. (41) vanishes after integral over the Grassmann odd coordinates of the moduli space of these curves. By the same calculation we find that neither the third term nor the forth term gives contributions. This completes our proof.

Using the same method, we can show that all amplitudes with only gluons and/or gluinos in the \( N = 1 \) vector supermultiplet (ie., \( \lambda_4 \)) in the deformed theory are the same as in the \( N = 4 \) theory to first order of deformations considered here.
6 Conclusion and discussions

In this paper, we studied the particle scattering amplitudes in four dimensional gauge theories with marginal deformations using the recently proposed twistor string theory, especially in the supersymmetric $\beta$-deformed theory and the non-supersymmetric $\gamma$-deformed theory.

In the $\beta$-deformed theory, we first re-investigate the amplitudes to linear order in $\beta$ using a generalization of the prescription in [43]. We find that the corresponding star product among the wavefunctions in this special case is drastically simplified compared to theories with general deformations. The special combinations $\frac{\partial}{\partial \psi^I_1} \psi^J_2$ and $\frac{\partial}{\partial \psi^I_2} \psi^J_1$ that appear in our expressions make it easy to convert our star-product into the form found in [4, 17]. These special operators merely count the number of $\psi^I_1$ and $\psi^I_2$ and this fact lead us to make a conjecture for the all-order star product and show that this conjecture is indeed correct. In other words, this identifies the exact star product we need to use to multiply two general functions $f(\psi_1)$ and $g(\psi_2)$ in the corresponding twistor string theory. In sharp contrast to our success in the real $\beta$ case, it turned out much harder to find the exact star product in other more general cases at this moment. We hope to return to this problem in the near future.

In the case of real $\gamma$ deformed theory, we were also able to find an all-order description which leads correctly to the field theory amplitudes. This is done essentially by by guesswork and we know even less about aspects of the B-model description in this case. It is an interesting open problem to find the correct closed string background which would give the self-dual part of the Lagrangian of this theory.

Finally we showed that the amplitudes involving only the fields in the $\mathcal{N} = 1$ vector supermultiplet obtained from the prescription is the same as the one expected from the field theory. Due to the difficulty alluded to above in generalizing our results in the special cases, we do this to linear order in the deformation parameters. This general result may be taken as yet another evidence of the possibility of completely understanding the closed B-model string background for the general marginal deformation of four dimensional gauge theory.

Despite the relative ease with which we were able to reproduce correct field theory amplitudes, we still lack a proper understanding of the precise origin of the prescription proposed in [43]. Notice the difficulty here is closely
related to the fact that the scattering amplitudes involve products among different wavefunctions while the holomorphic Chern-Simons theory concerns only the wavefunction of the five-brane. This means necessarily that modifying the product in the holomorphic Chern-Simons theory won’t give rise to the full star product required to reproduce four dimensional field theory results. We hope that our exact results in the above special cases may shed some more light on this issue.

Another interesting problem is to prove that the tree-level amplitudes obtained from twistor string theory satisfy the constraints from the parity invariance of the gauge theory. Since in twistor theory, the gluons with opposite helicities are treated in different manners, the parity invariance is not manifest at all. It has been proven that the amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory obtained from twistor string theory satisfy the constraints from parity invariance \cite{21,52}. It is quite interesting to generalize this proof in the theories with deformations, first at the linear order of the deformation parameters.

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