Co-evolutionary multi-task learning for dynamic time series prediction

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Abstract

Multi-task learning employs shared representation of knowledge for learning multiple instances from the same or related problems. Time series prediction consists of several instances that are defined by the way they are broken down into fixed windows known as embedding dimension. Finding the optimal values for embedding dimension is a computationally intensive task. Therefore, we introduce a new category of problem called dynamic time series prediction that requires a trained model to give prediction when presented with different values of the embedding dimension. This can be seen a new class of time series prediction where dynamic prediction is needed. In this paper, we propose a co-evolutionary multi-task learning method that provides a synergy between multi-task learning and coevolution. This enables neural networks to retain modularity during training for building blocks of knowledge for different instances of the problem. The effectiveness of the proposed method is demonstrated using one-step-ahead chaotic time series problems. The results show that the proposed method can effectively be used for different instances of the related time series problems while providing improved generalisation performance.

Keywords:
Coevolution; multi-task learning; modular neural networks; chaotic time series; and dynamic programming.

1. Introduction

Multi-task learning employs shared representation knowledge for learning multiple instances from the same problem with the goal to develop models with improved generalisation performance [1, 2, 3, 4]. On the other hand, multi-task evolutionary algorithms have been proposed for optimisation problems with the intention of exploring and exploiting the common knowledge between the tasks and enabling transfer of knowledge between them for optimisation [5, 6]. It has been shown that knowledge from related tasks can help in speeding up the optimisation process and obtain better quality solutions when compared to single-task approaches. Inspired by this new phenomenon, in this paper, we present a study on multi-task learning for time series prediction. Recently, evolutionary multi-tasking has been used for efficiently training feedforward neural networks for n-bit parity problem [7], where different tasks were implemented as different topologies given by the number of hidden neurons that obtained improved training performance.
Time series prediction using neural networks have been popular for applications that range from business [8] to weather, and climate prediction [9, 10]. In the past, different neural network architectures that include feedforward and recurrent neural networks have been used, with a wide range of algorithms that can be characterised as gradient based approaches [11, 12], neuro-evolution [13, 14, 15], hybrid algorithms and ensemble learning [16, 17, 18]. Time series prediction typically involves a preprocessing stage where the original time series is reconstructed into a state-space vector. This involves breaking the time series using overlapping windows known as embedding dimension taken at regular intervals or time lags [19]. The optimal values for embedding dimension and time lag are used to train the chosen model. These values vary on the type of problem and requires costly computational evaluation for model selection, hence, some effort has been made to address this issue. Multi-objective and competitive coevolution methods have been used to take advantage of different features from the embedding dimension during training [20, 21]. Moreover, neural network have been used for determining optimal embedding dimension of selected time series problems [22].

In time series for natural disasters such as cyclones [23, 10], it is important to develop models that can make predictions dynamically, i.e the model has the ability to make a prediction as soon as minimal data is available for the time series. The minimal value for the embedding dimension can have huge impact for the case of cyclones, where data is only available every 6 hours [24]. A way to address such categories of problems is to devise robust training algorithms and models that are capable of performing given different types of input or tasks. We define dynamic time series prediction as the need for a single model that can be used to make prediction for different values of the embedding dimension after training. It has been highlighted in recent work [24] that recurrent neural networks trained with a predefined embedding dimension can only generalise for the same embedding dimension which makes dynamic time series prediction a challenging problem. In this paper, we define tasks as different instances of the embedding dimension.

Furthermore, we note that different values in the embedding dimension can be used to generate several distinct datasets that have overlapping features which can be used to train modules for shared knowledge representation as needed for multi-task learning. Hence, it is important to ensure modularity is retained during learning. Modular neural networks are motivated from repeating structures in nature [25]. Modular networks were introduced for for visual recognition tasks that were trained by genetic algorithms and produced generalisation capability [25]. More recently, a modular neural network was presented where the performance and connection costs were optimised through neuro-evolution which achieved better performance when compared to fully connect neural network [26]. Modular neural networks have also been designed with the motivation to learn new tasks without forgetting old ones [27]. It was shown that modular networks learn new tasks faster from knowledge of previous tasks. Modular neural network architectures have been beneficial for hardware implementations [28]. Modular neural networks enable smaller networks to be used as building blocks for a larger network.

In dynamic programming, a large problem is broken down into sub-problems, from which at least one sub-problem is used as a building block for the optimisation problem. Although dynamic programming has been primarily used for optimisation problems, it has been briefly explored for data driven learning [29] [30]. The concepts in using sub-problems as building block in dynamic programming can be used in developing algorithms for multi-task learning. Cooperative coevolution (CC) is a divide and conquer approach that has been initially used for optimisation problems [31] and later been effective for neuro-evolution [32] and applied to time series problems [14, 15]. CC provides more diverse solutions through the subcomponents when compared to conventional single population-based evolutionary algorithms [32].
Time series prediction problems can be generally characterised into three major types of problems that include one-step prediction [16, 12, 14], multi-step-ahead prediction [33, 34, 35], and multi-variate time series prediction [36, 37, 38]. These problems at times may overlap with each other, for instance, a multi-step-ahead prediction can have a multi-variate component. Similarly, a one-step prediction can also have a multi-variate component, or a one-step ahead prediction can be used for multi-step prediction and vice-versa. In this paper, we identify a special class of problems that require dynamic prediction with the hope that the trained model can be useful for different instances of the problem. It would be able to feature different values of the embedding dimension or incorporate additional features in the case of multivariate problems.

Although, neuro-evolution has been successfully applied for training neural networks, multi-task learning for enhancing neuro-evolution has not been fully explored. There has not been any work that explores the embedding dimension of a time series as tasks for multi-task learning. This can be beneficial for dynamic time series prediction that requires a model to make robust prediction.

In this paper, we propose a co-evolutionary multi-tasking method that provides a synergy between multi-task learning and coevolution and enables neural networks to be trained with shared knowledge representation while retaining modularity. This enables the learning process to employ modules of knowledge from different but related tasks as building blocks of a single model. The proposed method is used for one-step-ahead chaotic time series problems using feedforward neural networks for seven benchmark problems.

The rest of the paper is organised as follows. Section 2 gives a background on multi-task learning, cooperative neuro-evolution and time series prediction. Section 3 gives details of the co-evolutionary multi-task learning method for dynamic time series prediction. Section 4 presents the results with discussion. Section 5 presents the conclusions and directions for future research.

2. Background and Related Work

2.1. Multi-task learning and applications

Multi-task learning employs a shared representation of knowledge for learning several different instance of the same or related problems [1]. A number of approaches have been presented that considers multi-task learning for different types of problems that include supervised and unsupervised learning [39, 40, 41, 42]. Negative transfer has been a major challenge for multi-task learning. The major approach to address it has been through task grouping where knowledge transfer is performed only within each group [43, 44]. Bakker et. al for instance, presented a Bayesian approach in which some of the model parameters were shared and others loosely connected through a joint prior distribution learnt from the data [44]. Zhang and Yeung presented a convex formulation for multi-task metric learning by modeling the task relationships in the form of a task covariance matrix [43]. Moreover, Zhong et. al presented flexible multi-task learning framework to identify latent grouping structures in order to restrict negative knowledge transfer [45].

Multi-task learning has recently contributed to a number of successful real-world applications that gained better performance by exploiting shared knowledge for multi-task formulation. Some of these applications include 1) multi-task approach for “retweet” prediction behaviour of individual users [46], 2) recognition of facial action units [38], 3) automated Human Epithelial Type 2 (HEp-2) cell classification [47], 4) kin-relationship verification using visual features [48] and 5) object tracking [49].
2.2. Cooperative Neuro-evolution

Neuro-evolution employs evolutionary algorithms for training neural networks [50]. Neuro-evolution can be classified into direct [50, 51], and indirect encoding strategies [52]. In direct encoding, every connection and neuron is specified directly and explicitly in the genotype [50, 51]. In indirect encoding, the genotype specifies rules or some other structure for generating the network [52]. Performance of direct and indirect encodings vary for specific problems. Indirect encodings seem very intuitive and have biological motivations, however, in several cases they have shown not to outperform direct encoding strategies [53, 54].

Cooperative coevolution for training neural networks is known as cooperative neuroevolution [32, 55]. Although cooperative coevolution faced challenges in problem decomposition, it showed promising features that included modularity and diversity [32]. Further challenges have been in area of credit assignment for subcomponents [32, 55], problem decomposition, and adaptation due to issues of separability [56]. In cooperative neuro-evolution, problem decomposition has a major effect in the training and generalisation performance. Although several decomposition strategies have been implemented that vary for different network architectures, the two established decomposition methods are those on the synapse level [53] and neuron level [57, 56, 58]. In synapse level, the network is decomposed to its lowest level where each weight connection (synapse) forms a subcomponent [53, 13]. In neuron level, the neurons in the network act as the reference point for the decomposition [59, 58]. They have shown good performance in pattern classification problems [60, 57, 58]. Synapse level decomposition has shown good performance in control and time series prediction problems [53, 13, 14], however, they gave poor performance for pattern classification problems [56].

Chandra et al. applied neural and synapse level decomposition for chaotic time series problems using recurrent neural networks [14]. Hence, it was established that synapse level encoding was more effective for time series and control problems [53, 14]. A competitive and collaborate method was proposed with very promising performance for chaotic time series problems [15].

| Alg. 1 Cooperative Coevolution |
|--------------------------------|
| **Step 1:** Decompose the problem (Neuron or Synapse level decomposition) |
| **Step 2:** Initialise and cooperatively evaluate each sub-population |
| for each cycle until termination do |
| for each Sub-population do |
| for n Generations do |
| i) Select and create new offspring |
| ii) Cooperatively evaluate the new offspring |
| iii) Update sub-population |
| end for |
| end for |
| end for |

In Algorithm 1, the network is decomposed according to the selected decomposition method. Neuron level decomposition is shown in Figure 1. Once the decomposition is done, the subcomponents that are implemented as sub-populations are initialized and evolved in a round-robin fashion, typically for a fixed depth of search given by generations. The evaluation of the fitness of each individual for a particular sub-population is done cooperatively by concatenating the current individual with the fittest individuals from the rest of the sub-populations [32]. The concatenated individual is then encoded into the neural network where its fitness is evaluated.
and returned. The fitness of the entire network is assigned to the particular individual of the sub-population, although it is a representative fitness. This is further illustrated in Figure 1.

Figure 1: Feedforward network with Neuron level decomposition. Note that 4 input neurons represent time series reconstruction with embedding dimension of 4.

2.3. Problems in time series prediction

We present details of the three major types of problems in time series that include one-step prediction, multi-step-ahead prediction, and multi-variate time series prediction. Another type of problem for time series prediction include applications that have missing data. Wu et. al approached the missing data problem in time series with non-linear filters and neural networks [61]. In their method, a sequence of independent Bernoulli random variables were used to model random interruptions which was later used to construct the state-space vector in preprocessing stage.

A number of methods have been used for one-step ahead time series prediction with promising results from neural networks with various architectures [12, 14] and algorithms that include gradient-based learning [62, 11], evolutionary techniques [13, 14, 15] and hybrid methods [16, 17, 12]. These methods can also be used for multi-step ahead and multivariate time series prediction.

Multi-step-ahead (MSA) prediction refers to the forecasting or prediction of a sequence of future values from observed trend in a time series [63]. It is challenging to develop models that produce low prediction error as the prediction horizon increases [33, 34, 35]. MSA prediction has been approached mostly with the recursive and direct strategies. In the recursive strategy, the prediction from a one-step-ahead prediction model is used as input for future prediction horizon [64, 65]. Although relatively new, a third strategy is a combination of these approaches [64, 66].

Multi-variate time series prediction typically involves the prediction of single or multiple values from multi-variate input that are typically interconnected through some event [36, 37, 38]. Examples of single value prediction are the prediction of flour prices of time series obtained from different cities [36] and traffic time series [67]. Note that the goal is to enhance the prediction performance from the additional features are in the input, although the problem can be solved in a univariate approach [67]. In the case of prediction of multiple values, the model needs to predict future values of the different features, for example, prediction of latitude and longitude that defines the movement of cyclones [68]. A recent study has shown that that multivariate
prediction would perform better than univariate for MSA as the prediction horizon becomes larger, multi-variate information becomes more important [69].

3. Co-evolutionary Multi-task Learning

3.1. Preliminaries: time series reconstruction

In state-space reconstruction, the original time series is divided using overlapping windows at regular intervals that can be used for one-step-ahead and MSA prediction. Taken’s theorem expresses that the vector series reproduces many important characteristics of the original time series [19]. Hence, given an observed time series \( x(t) \), an embedded phase space \( Y(t) = \{ x(t), x(t - T), ..., x(t(D - 1)T) \} \) can be generated, where, \( T \) is the time delay, \( D \) is the embedding dimension (window), \( t = 0, 1, 2, ..., N - DT - 1 \), and \( N \) is the length of the original time series. The optimal values for \( D \) and \( T \) must be chosen in order to efficiently apply Taken’s theorem [70]. Taken’s proved that if the original attractor is of dimension \( d \), then \( D = 2d + 1 \) will be sufficient to reconstruct the attractor [19]. In the case of using feedforward neural networks, \( D \) is the number of input neurons.

3.2. Problem: Dynamic time series prediction

Natural disasters such as torrential rainfall, cyclones, tornadoes, wave surges and droughts [71, 72, 9, 10] require dynamic and robust prediction models that can make a decision as soon as the event take place. Therefore, if the model is trained over specific months for rainy seasons, the system should be able to make a robust prediction from the beginning of the rainy season. We define the event length as the duration of an event which can be number of hours of a cyclone or number of days of drought or torrential rain.

As noted earlier, in a typical time series prediction problem, the original time series is reconstructed using Taken’s theorem [19, 70]. In the case of cyclones, it is important to measure the performance of the model when dynamic prediction is needed regarding track, wind or other characteristics of the cyclone [24]. Dynamic prediction can provide early warnings to the community at risk. For instance, data about tropical cyclone in the South Pacific is recorded at six hour intervals [73]. If the embedding dimension \( D = 6 \), the first prediction by the model at hand would come after 36 hours which could have devastating effects.

The problem arises when the gap between each data point in the times series is a day or number of hours. The problem with the existing models such as neural networks used for cyclones is the minimal embedding dimension \( D \) needed to make a prediction. It has been reported that recurrent neural networks trained with a given embedding dimension (Eg. \( D = 5 \), cannot make robust prediction for other embedding dimension (Eg. \( D = 7 \) or \( D = 3 \) ) [24]. Therefore, we introduce the problem of dynamic time series prediction (DTSP) that involves the minimum embedding dimension needed for a model to effectively reach a prediction for a given time-series. This enables different embedding dimension values to be used in a model for prediction, i.e. the model can provide a prediction irrespective of the embedding dimension.

3.3. Method

In the proposed method, a coevolution algorithm based on a dynamic programming strategy is proposed for multi-task learning. It features problem decomposition in a similar way as cooperative coevolution, however, the major difference lies in the way the solutions of the subcomponents are combined to build the final solution. Hence, the proposed co-evolutionary multi-task
The learning algorithm is inspired from the strategies used in dynamic programming where a subset of the solution is used as the main building block for the optimisation problem. In this case, the problem is learning the weights of a neural network and the base problem is the neural network with the smallest architecture and lowest number of input features. The weights in the base network are then mapped into larger network architectures that consist of more hidden neurons and input features. This can be viewed as modules of knowledge that are combined for larger tasks that use knowledge from smaller tasks as building blocks. The larger network architectures can also been seen as additional tasks, hence, we name the approach co-evolutionary multi-task learning (CMTL).

CMTL is used for training feedforward neural networks (FNNs) for dynamic time series prediction. It considers different tasks as neural network topologies defined by different number of input and hidden neurons. The different number of input neurons refer to additional features that are defined by the task. Let’s assume that different sets of features from the same problem make the different dataset with some overlapping component, i.e some of the features in these datasets are overlapping. Hence, multi-task learning can be used to represent the problem where there is some form of shared knowledge representation for the learning process which refers to the overlapped features from the different tasks. This means that the overlapping features can be grouped together as task = 1, while the remaining features as task = 2 in these types of problems.

In the CMTL algorithm, each sub-population is given as $S_1, S_2, ... S_N$, where $N$ is number of sub-populations. The sub-populations consist of matrix of variables that refer to the weights of the FNN that correspond to the different tasks $S = X_{i,j}$, $i$ is the number of variables and $j$ is the number of individuals in the respective sub-population. $S_{(task)}$ corresponds to a specific task where, task $= 1, 2, ..., N$, which corresponds to $Networ_k_{(task)}$, with data for each task with different embedding dimension of time series, $D_{(task)}$.

Suppose that $W_{(task)}$ is the set of input to hidden layer and hidden to output layer weights.

In our example, we limit number of output neurons as 1 for all the tasks. Therefore, we have the weights of a neural network topology for each task appended with the rest of the task knowledge which range from the smallest to the second largest task $x$, as shown in Equation 1.

$$x = [W_{(task=1)}, W_{(task=2)}, ..W_{(task=n)}]$$

$$Networ_k_{(task)} = [W_{(task),X}]$$

(1)

Algorithm 2 gives details for CMTL which begins by initialising all the components which include the sub-populations $S_{task}$ for co-evolutionary multitasking and the different neural network topologies defined by the tasks $Networ_k_{(task)}$ which feature the weights $W_{task}$ and respective task data $Data_{task}$. The $S_{task}$ are initialised with real values in a range $[-a, a]$ and also assigned arbitrary values for fitness $F_i$, for every individual $i$ in the sub-population.

Once the initialising phase has been completed, the algorithm moves into the evolution phase where each task is evolved for a fixed number for generations defined by depth of search, $depth$.

We then check if $task = 1$, then the $TaskSol_{task}$ returns the best solution $Sol_{best}$ from the sub-population $S_{task}$. Otherwise, the current task solution $Sol_{task}$ is appended with best solutions from previous tasks, $TaskSol_{task} = [Sol_{1}, Sol_{2}, ..Sol_{task}]$. Next, the task solution obtained is given as a parameter to Algorithm 3 along with the network topology $Networ_k_{(task)}$ of all the tasks in order to decode the task solution into the respective weights of the network. This procedure is done for every individual $i$ in the sub-population of the task, $S_{task}$. This procedure is repeated for every task for different phases until the termination condition is satisfied. The termination condition can be either the maximum number of function evaluations or a minimum fitness value.
Alg. 2 Co-evolutionary Multi-task Learning

Data: Requires data for different tasks $D_{\text{task}}$

Result: Weights as model parameters for FNN $\text{Network}_{\text{task}}$

Initialization

for each task do

1. Define different tasks using data $D_{\text{task}}$ that corresponds to neural network $\text{Network}_{\text{task}}$ given by different number of neurons: (input $i$, hidden $j$ and output $k$)
2. Define the weight space for the different tasks $W_{\text{task}}$
3. Initialise individuals of the sub-populations $S_{\text{task}}$, within the unified search space
4. Assign arbitrary fitness values for fitness $F_i$ of individuals in each sub-population $S_{\text{task}}$
5. Assign depth of search, eg. depth = 5 that defines the number of generation for each sub-population $S_{\text{task}}$

end

while each phase until termination do

for each task do

for each generation until depth do

- Get best Solution $Sol$ from $S_{\text{task}}$

if task == 1 then

- Get the current solution and assign TaskSol$_{\text{task}}$ = $Sol_1$

else

- Append the current task solution $Sol_{\text{task}}$ with best solutions from previous tasks, TaskSol$_{\text{task}}$ = [$Sol_1, Sol_2, ..., Sol_{\text{task}}$

end

for each Individual $j$ in S do

- Call Algorithm 3: This will encode the TaskSol$_{\text{task}}$ into the Network$_{\text{task}}$
- Load data $D_{\text{task}}$ for the task and evaluate the Network$_{\text{task}}$ for fitness $F$ given by RMSE

end

for each Individual $j$ in S do

- Select and create new offspring via evolutionary operators such selection, crossover and mutation

end

- Update $S$
- Update number of FE

end

end

- Test the obtained solution

for each task do

1. Load best solution $S_{\text{best}}$ from $S_{\text{task}}$
2. Map into the weight space for the task $W_{\text{task}}$
3. Load test data for TestData$_{\text{task}}$ and test the Network$_{\text{task}}$
4. Report RMSE

end

from the training or validation dataset. Figure 2 shows an exploded view of the neural network topologies associated with the respective tasks, however, they are all part of the same network as later shown in Figure 3. The way the task solution is decomposed and mapped into the network
Figure 2: Problem decomposition as tasks in co-evolutionary multi-task learning. Note that the colours associated with the synapses in the network are linked to their encoding that are given as different tasks. Task 1 employs a network topology with 2 hidden neurons while the rest of the tasks add extra input and hidden neurons. The exploded view shows that the different neural network topologies are assigned to the different tasks, however, they are all part of the same network as shown in Figure 3.

is given in Figure 3 and discussed detail in the next section.

Note that the major way CMTL differs from cooperative neuro-evolution (CNE) given in Algorithm 1 is by the way the problem is decomposed and the way the fitness for each individual is calculated. In CMTL, the fitness of an individual from a sub-population $S_{task}$ depends on the previous tasks if the task is greater than 1. This is different for the case of CNE as the fitness of an individual is calculated when it is concatenated with the best individuals from all the respective sub-populations. This is a major difference in the approach which makes CMTL useful for tasks where the problem changes or features increase with the task, whereas CNE can only be used for single-tasking. In CMTL, the number of input features related to the task data does not matter as long as it increases with the task. CMTL can be easily modified to be applicable for the tasks that have the same number of input features, and with same or different number of instances in the respective task datasets.

Finally, when the termination criteria has been met, the algorithm moves into the testing phase where the best solutions from all the different tasks are saved and encoding into their
respective network topologies. Once this is done, the respective task test data is loaded and the network is used to make a decision that results in a certain measure of error which can be given by the RMS $E$; however, any other measure can also be implemented.

Hence, we have highlighted the association of every individual in the respective sub-populations with different tasks in the multi-task learning environment. There is transfer of knowledge in terms of weights from smaller to bigger networks as defined by the task with its data which is covered in detail in next section.

3.4. Transfer of Knowledge from Tasks

One challenging aspect of the Algorithm 2 is the transfer of knowledge represented by the weights of the respective neural networks that is learnt by the different tasks in CMTL. We assume that the topology in terms of the number of input, hidden and output neurons increase with the tasks. Algorithm 3 is able to handle any number of increase in the respective neurons of the different tasks.

The purpose of Algorithm 3 is to transfer neural network weights that are mapped from different sub-populations defined by the tasks. Therefore, it is used for transfer for different number of tasks. The algorithm is given input parameters which are:

1. The current task ($task = 1, 2, \ldots, N$), where, $N$ is the number of tasks and each task corresponds to the respective sub-population and data with input and target instances;
2. The current task solution ($TaskSol_{task} = [Sol_{1}, Sol_{task-2}, \ldots + Sol_{task}]$). The solutions are appended with solutions of previous tasks in cases when $task > 1$;
3. The topology of the respective neural networks for the different tasks in terms of number of input, hidden and output neurons.

We describe the algorithm with reference to Figure 3 which shows a case where the network for $task = 3$ goes through the transfer where $task = 1$ and $task = 2$ are used as building blocks of knowledge given in the weights. Therefore, we use examples for the network topology as highlighted below.

1. Input is vector of number of input neurons for the respective tasks, eg. $Input = [2, 3, 4]$;
2. Hidden is vector of number of input neurons for the respective tasks, eg. $Hidden = [2, 3, 4]$;
3. Output: is vector of output neurons for the respective tasks, eg. $Output = [1, 1, 1]$. Note that since our application is limited for one step ahead time series prediction, we only consider 1 output neuron for all the tasks.

The algorithm begins by assigning $BaseTask = 1$, as base case is applied irrespective of the number of tasks. In Step 1, the Transfer for Input-Hidden layer weights from the $TaskSol$ is done in a straight simple manner as shown by weights (1-4) in Figure 3. Step 2 executes the transfer for Hidden-Output layer weights from the $TaskSol$ as shown by weights (5-6) in Figure 3.

Note that Step 1 and 2 are applied for all the cases given by the number of tasks. Once this is done, the algorithm terminates if $task = 1$ or proceeds if $task >= 2$. Moving on, in Step 3, the situation is more complex as we consider $task >= 2$. In this case, Step 1 and 2 are executed before moving to Step 3 where $TaskSol$ contains the appended solution sets from previous task, $TaskSol_{task} = [Sol_{1}, Sol_{task-2}]$. If we consider the position $t$ of $TaskSol(t)$, by the time the
algorithm reaches Step 3, it in principle points to the beginning of the solution given by sub-population for task = 2. Here, the transfer for Input-Hidden layer weights (7-9) is executed from the TaskSol for Task = 2, which is marked by position $i$ that increments itself during the transfer. Note that in this case, we begin with the weights with reference to the number of hidden neurons from previous task $j = Hidden_{(task-1)} + 1$, and move to the number of hidden neurons of the current task $j = Hidden_{task}$ in order to transfer the weights to all the input neurons. This refers to weights (7-9) in Figure 3. Before reaching to the transfer for task = 3, task = 1 and task = 2 transfer would have already taken place and hence the weights (13-16) would be transferred as shown in the same figure.

Moving on to Step 4, we first consider the transfer for Input-Hidden layer weights for task = 2 through the transfer of weights from beginning of previous task input, $i = Input_{(task-1)} + 1$ to current task input connected with all hidden neurons. This is given by weights (10-11) in Figure 3. For the case of task = 3, this would refer to weights (17-19) in the same figure.

Finally, in Step 5, the algorithm executes the transfer for Hidden-Output layer weights based on the hidden neurons from previous task to the current task that are linked to the output neuron. In case of task = 2, this results in transferring weight (12) and for task = 3, the transfer is weight (20) in Figure 3, respectively.

Note that the algorithm can transfer any number of input and hidden neurons as the number of tasks increase. It can also consider whether to transfer when either all the input or hidden neurons are of the same size for the different tasks.

4. Simulation and Analysis

This section presents an experimental study that compares the performance of CMTL with single task learning methods such as cooperative neuro-evolution and evolutionary algorithm for dynamic time series prediction. Note that single task learning methods only provide a comparison while they cannot be applied to dynamic time series as they cannot handle different values of embedding dimension while training a single model. In the beginning, seven chaotic benchmark time series problems are employed and compared with EA and cooperative neuro-evolution (CNE).

4.1. Benchmark Chaotic Time Series Problems

In the benchmark chaotic time series problems, the Mackey-Glass, Lorenz, Henon and Rossler are the four simulated time series problems. The experiments use the chaotic time series with length of 1000 generated by the respective chaotic attractor. The first 500 samples are used for training and the remaining 500 is used for testing.

In all cases, the phase space of the original time series is reconstructed with the embedding dimensions for 3 datasets for the respective tasks with embedding dimension $D = 3, 5, 7$ and time lag $T = 2$. All the simulated and real-world time series were scaled in the range $[0,1]$. Further details of each of the time series problem is given as follows.

The Mackey-Glass time series [74] has been used in literature as a benchmark problem due to its chaotic nature. The Lorenz time series was introduced by Edward Lorenz who has extensively contributed to the establishment of Chaos theory [75]. The Henon time series is generated with a Henon map which is a discrete-time dynamical system that exhibit chaotic behavior [76] and the Rossler time series is generated using the attractor for the Rossler system, a system of three non-linear ordinary differential equations as given in [77].

The real-world problem are the Sunspot, ACI finance and Lazer time series. The Sunspot time series is a good indication of the solar activities for solar cycles which impacts Earth’s climate,
Figure 3: Transfer of knowledge from tasks encoding as sub-populations in co-evolutionary multi-task learning algorithm. This diagram shows transfer of knowledge from Task 1 to Task 2 and then finally to Task 3. Note that Task 2 utilises the knowledge of Task 1. The same concept is used for Task 3 which utilizes knowledge of the previous tasks. Once the previous tasks knowledge is transferred into Task 3, the network loads the Task 3 data of (4 features in this example) for further evolution of the sub-population that links with Task 3.
Alg. 3 Transfer of knowledge from previous tasks

Input Parameters: Task, TaskSol, Input, Hidden and Output

BaseTask = 1

Step 1:
for each $j = 1$ to Hidden(BaseTask) do
  for each $i = 1$ to Input(BaseTask) do
    $W_{(i,j)} = TaskSol(t)$
    $t = t + 1$
  end
end

Step 2:
for each $k = 1$ to Output(BaseTask) do
  for each $j = 1$ to Hidden(BaseTask) do
    $W_{(j,k)} = TaskSol(t)$
    $t = t + 1$
  end
end

if task $>= 2$ then
  Step 3
  for each $j = Hidden(task-1) + 1$ to Hidden(task) do
    for each $i = 1$ to Input(task) do
      $W_{(i,j)} = TaskSol(t)$
      $t = t + 1$
    end
  end

  Step 4
  for each $j = 1$ to Hidden(task) - 1 do
    for each $i = Input(task-1) + 1$ to Input(task) do
      $W_{(i,j)} = TaskSol(t)$
      $t = t + 1$
    end
  end

  Step 5
  for each $k = 1$ to Output(task) do
    for each $j = Hidden(task-1) + 1$ to Hidden(task) do
      $W_{(j,k)} = TaskSol(t)$
      $t = t + 1$
    end
  end
end

weather patterns, satellite and space missions [78]. The Sunspot time series from November 1834 to June 2001 is selected which consists of 2000 points.

The ACI financial time series is obtained from the ACI Worldwide Inc. time series, which is one of the companies listed on the NASDAQ stock exchange. The data set contains closing stock prices from December 2006 to February 2010, which is equivalent to approximately 800 data points. The closing stock prices were normalized between 0 and 1. The data set features the recession that hit the U.S. market in 2008 [79]
the *Lazer time series* is time series measured in a physics laboratory experiment that were used in the *Santa Fe Competition* [80]. All the real world time series used the first 50 percent samples for training and remaining for testing.

4.2. Experimental Design

In the case of cooperative neuro-evolution, neuron level problem decomposition is applied for training feedforward networks [56] on the given problems. We employ covariance matric adaptation evolution strategies (CMAES) [81] as the evolutionary algorithm in sub-populations of CMTL, CNE and the population of EA. The training and generalisation performances are reported for each case given by the different tasks in the respective time series problems. Note that only CMTL can be used to approach dynamic time series problem with the power of multi-task learning, however, we provide results for single tasking approaches (CNE and EA) in order to provide baseline performance.

The respective neural networks used both sigmoid units in the hidden and output layer for all the different problems. The RMSE is given in Equation 2 and used as the main performance measure. Each neural network architecture was tested with different numbers of hidden neurons.

We employ fixed depth = 5 generation in the sub-populations of CMTL as the depth of search as it has given optimal performance in trial runs. CNE also employs the same value. Note that all the sub-populations evolve for the same depth of search. The population size of CMAES in all the respective methods is given by $P = 4 + \text{floor}(3 \times \log(W))$, where $W$ is the total number of weights that is encoded into the sub-population (for case of CNE and CMTL) or the population (for case of EA).

The termination condition is fixed at 30 000 function evaluations for each task, hence, CMTL employs 120 000 function evaluations while single tasking approaches use 30 000 for each of the respective tasks for all the problems. Note that since there is a fixed training time, there was no validation set used to stop training.

The root mean squared error (RMSE) is used to measure the prediction performance as given in Equation 2.

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}$$ (2)

where $y_i, \hat{y}_i$ are the observed and predicted data, respectively. $N$ is the length of the observed data. These two performance measures are used in order to compare the results with the literature.

4.3. Results for Benchmark Problems

The results for the 7 benchmark chaotic time series problems are given in 4 to 10 which highlight the training (Train) and generalisation performance (Test) given by the respective single task learning methods (EA and CNE) and CMTL. We limit our discussion to the generalisation performance, although the training performance is also shown.

Figure 4 shows that CMTL generalisation performance is better than EA and CNE for timespan $D = 3, 5, 7$. CMTL and EA beat the performance of CNE in all the cases of the respective timespan. The same trend is shown in general for Lorenz and Henon time series as shown in Figure 5 and Figure 6, respectively. There is one exception, $D = 5$ for Henon time series where CME gives better performance than EA, however, worse than CMTL. Figure 7 shows the results for the Rossler time series which follows a similar trend when compared to the previous
problems. Hence, we can conclude here that CMTL generalisation performance is the best when compared to the single-tasking methods (CNE and EA) for the 4 simulated time series problems which have little or no external noise present.

Moving on to the real-world chaotic time series, Figure 8 for the Sunspot problem shows that CMTL provides the best generalisation performance when compared to EA and CNE for all the cases (timespan). The same is given for first two timespan cases for ACI-Finance problem as shown in Figure 9, except for one case, \( D = 7 \), where EA and CMTL gives the same performance. In the case of the Lazer time series in Figure 10, which is known as one of the most chaotic time series problems, CMTL beats CNE and EA, except for one case, \( D = 7 \). Therefore, at this stage, we can conclude that CMTL gives the best performance for most of the cases in the real-world time series problems.

Table 1 shows the mean of RMSE and confidence interval across the 3 embedding dimension. Here, we find that the CMTL performs better than EA and CNE for almost all the problems. The
Lazer problem is the only exception where the EA is slightly better than CMTL.

Table 1: Performance across the 3 embedding dimensions

| Problem       | EA         | CNE        | CMTL        |
|---------------|------------|------------|-------------|
| Mackey-Glass  | 0.0564 ±0.0081 | 0.0859 ±0.0147 | 0.0472 ±0.0054 |
| Lorenz        | 0.0444 ±0.0067 | 0.0650 ± 0.0127 | 0.0353 ± 0.0049 |
| Henon         | 0.1612 ± 0.0120 | 0.1721 ± 0.0128 | 0.1267 0.0127 |
| Rossler       | 0.0617± 0.0091 | 0.0903± 0.0138 | 0.0489 ±0.0054 |
| Sunspot       | 0.0529 ±0.0062 | 0.0773 ±0.0137 | 0.0399 ±0.0052 |
| Lazer         | 0.0917 ±0.0056 | 0.1093 ±0.0099 | 0.0936 ±0.0077 |
| ACI-finance   | 0.0565 ±0.0091 | 0.0866 ±0.0159 | 0.0471 ±0.0087 |
4.4. Discussion

The goal of the experiments were to evaluate if the proposed CMTL method can deliver similar results when compared to single task learning approaches for the introduced dynamic time series problems. Therefore, the comparison was to ensure that the approach does not lose quality in terms of generalisation performance when compare to single tasking approaches. The results have shown that CMTL not only addresses the problem of minimal timespan in dynamic time series, but is a way to improve the performance if each of the multi-task learning cases were alienated and approached as single tasks.

It is important to understand why CMTL has shown better results for almost all the cases when compared to single-tasking approaches with same the neural network topology and data for respective tasks. Note that the CMTL is an incremental evolutionary learning approach as the algorithm employs the consecutive evolution of each task for a small depth of search in terms of number of generations. Therefore, this can be viewed as incremental knowledge-based learning where the bigger tasks take advantage of knowledge gained from learning smaller tasks. This is done through collaborative fitness evaluation, where in the bigger tasks, the best solution from the
smaller task is combined. However, when it comes to the smaller tasks, they do not combine with bigger task solutions as would have been in conventional cooperative coevolution. Through such concatenation of knowledge, there is diversity in incremental knowledge development from the base task which seems to be beneficial for future tasks. However, the reason why the base task produces better results when compared to similar experimental design in a single-task learning approach is of good interest that can be explored with further analysis during the learning process. Note that the features in the bigger tasks contain all the overlapping features from the base tasks - hence bigger tasks can be seen as those that have additional features that guide the bigger network(s) with more hidden neurons during training.

This type of incremental learning not only improved the learning, but also enables modularity that is used for dynamic time series prediction. Modularity for knowledge is essential for dynamic problems, where groups of knowledge can be combined as the nature or complexity of the problem increases. Modularity is important for design of neural network in hardware [28] as disruptions in certain synapse(s) can result in problems with the whole network which can be eliminated by persevering knowledge as modules [26].

Due to being evolutionary in nature, CMTL can be seen as a flexible method that can be used for multiple sets of data that have different features of which some are overlapping and distinctly contribute to the problem. The common features can be captured as a task. Through multi-task learning, the overlapping features can be used as building blocks to learn the nature of the problem through the model at hand. Although feedforward neural networks have been used in CMTL, other other neural network architectures and learning models can be used depending on the nature of the tasks.

In case of computer vision applications such as face recognition, the different tasks can be different number of features, i.e. the algorithm can execute face recognition based on either $D = 10$, $D = 15$ or $D = 20$ features that link with different features from the same problem.

The major limitation of the method is the training time as CMTL is evolutionary in nature. However, with help of gradient based local search methods, hybrid instances of CMTL can be developed, either as a two stage evolutionary global-local search method or as memetic algorithms where local refinement occurs during the evolution [82].

Figure 10: Performance given by EA, CNE, CMTL for Lazer time series
5. Conclusions and Future Work

We presented a novel algorithm that provides a synergy between coevolution and multitasking for training neural networks for dynamic time series problems. The results show that the proposed algorithm not only addresses the problem of minimal timespan in dynamic time series problems, but also provides better performance in most cases when compared to single-tasking approaches. Each point in a given timespan represents a number of hours and the proposed algorithm can be used to train a model that can work with multiple timespan values which makes the prediction dynamic and robust.

In future work, the proposed approach can be used for other time series problems that can be broken into multiple tasks, such as multiple step ahead time series prediction. The proposed method can also be extended for transfer learning problems that can include both heterogeneous and homogeneous domain adaptation cases. In case of tropical cyclones, which is multi-variate time series problems, the different tasks can be seen as features that include cyclone tracks, sea surface temperature and humidity.

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