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Published in:
Optics Express

Publication date:
2022

Document Version
Early version, also known as pre-print

Citation (APA):
Karamehmedovic, M., Scheel, K., Pedersen, F. L-S., Villegas, A., & Hansen, P-E. (2022). Steerable photonic jet for super-resolution microscopy. Manuscript submitted for publication.
STEERABLE PHOTONIC JET FOR SUPER-RESOLUTION MICROSCOPY

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Abstract. A promising technique in optical super-resolution microscopy is the illumination of the sample by a highly localized beam, a photonic jet (also called photonic nanojet). We propose a method of computation of incident field amplitude and phase profiles that produce photonic jets at desired locations in the near field after interaction with a fixed micro-scale dielectric lens. We also describe a practical way of obtaining the incident field profiles using spatial light modulators. We expect our photonic jet design method to work for a wide range of lens shapes, and we demonstrate its application numerically using two-dimensional micro-lenses of circular and square cross-sections. We furthermore offer a theoretical analysis of the resolution of photonic jet design, predicting among other that a larger lens can produce a narrower photonic jet. Finally, we give both theoretical and numerical evidence that the waist width of the achieved designed jets is increasing linearly and slowly over a large interval of radial shifts. With uniform plane wave illumination, the circular two-dimensional micro-lens produces a similar-sized jet at a fixed radial shift, while the square lens does not form a jet at all. We expect our steerable optical photonic jet probe to enable highly localized adaptive real-time measurements and drive advances in super-resolution optical microscopy and scatterometry, as well as fluorescence and Raman microscopy. Our relatively weak peak jet intensity allows application in biology and health sciences, which require high resolution imaging without damaging the sample bio-molecules.

1. Introduction

Several far-field and near-field optical microscopy techniques overcome the Abbe diffraction limit [1] and achieve super-resolution imaging of particles and features. In a recent review article [2], Huszka and Gijs give a good introduction to the most common techniques, one of which is the use of photonic jets (also called photonic nanojets or PNJs [3, 4]; see Figure 1 for definitions). Here, laser or white light illumination of a dielectric micro-lens (cylindrical, hemispherical, spherical) produces a highly focused beam, a PNJ, in the shadow region of the near field exterior to the lens. PNJ formation was observed experimentally in 2006 [5], and it was since shown to depend on the micro-lens radius, shape, and refractive index relative to the surrounding medium, as well as on the illumination wavelength. [6, 3] Several works exploit this to create PNJs of various shapes and sizes using lens shaping and plane wave or gaussian illumination [7, 8]. The high field intensity and the small width of the PNJ focus, relative to the illuminating laser wavelength, allows highly local measurements, such as fluorescence [9] and Raman [10] microscopy, in addition to scatterometric measurements [11, 12] and optical imaging [13, 14, 15]. However, the highly localized nature of PNJs also makes it difficult to find the area of interest. It is therefore desirable to realize a steerable PNJ that can quickly scan the local
Figure 1. A photonic nanojet (PNJ), its radial shift, waist width, and decay length, as in [18]. The full width at half maximum (FWHM) amplitude contour is w.r.t. the amplitude $|E_{\text{tot}}|$ of the total electric field.

region without any need for lens adjustment or sample movement. Also, a precisely and quickly steerable PNJ could potentially allow rapid selective photo-switching of closely spaced sites along a biomolecule [9, 10], or unlock the full potential of the emerging label-free microsphere-assisted optical super-resolution microscopy; here, a sub-classical sample is placed near or inside a PNJ, enabling the localization, characterization (e.g., sizing) [5, 11, 12, 4] and even magnified optical image creation [13, 17, 14, 15, 16] of the sample.

We propose a method for fast and precise PNJ steering using a computed spatially inhomogeneous incident wave and a simple fixed micro-lens. The method enables rapid focus point movement without any adjustment or translation of the lens; it is not confined to the traditional spherical (3D) or circular (2D) micro-lenses; and it results in a consistently narrow PNJ and a large range of achievable PNJ positions. We describe the mathematical foundation of the method in Section 2, and propose a practical way to realize the computed incident fields in Section 3. We estimate theoretically the achievable PNJ waist width as a function of the radial shift in Section 4, and present numerical results for 2D circular and square cross section dielectric lenses in Section 5.

2. TAILORING THE INCIDENT FIELD

The procedure for tailoring the incident field that produces the desired PNJ is based on the solution of a Lippmann-Schwinger equation, such as [19, Eq. (8.13), p. 309], which we here derive explicitly for completeness (Eq. (8) below). Figure 2 shows the setup of the problem. We consider the time-harmonic, two-dimensional transverse electric (2D TE) case with the electric field pointing out of the plane, and with the time-dependence factor $\exp(j\omega t)$ implied and suppressed; here $j$ is the imaginary unit. The operating free-space wavelength and wavenumber are $\lambda_0$ and $k_0 = 2\pi/\lambda_0$, respectively. A cylindrical dielectric lens with a constant refractive index $n_L > 1$ and a cross-section profile $L$ is immersed in vacuum and illuminated with an incident field $E_{\text{inc}}(x, y) = A(x, y) \exp(jP(x, y))$. Here a Cartesian coordinate system $(x, y, z)$ is introduced with the $z$-axis aligned with the lens axis. The
real-valued functions $A(x, y)$ and $P(x, y)$ give the amplitude and the phase profile of the incident field, respectively. Define the 'lens contrast' $\alpha = k_0^2(n_L^2 - 1)$, the characteristic function

$$\chi_L(x) = \begin{cases} 1, & x \in L, \\ 0, & \text{otherwise}, \end{cases}$$

and the piecewise constant wavenumber

$$k(x) = k_0[1 + \chi_L(x)(n_L - 1)], \quad x \in \mathbb{R}^2.$$  

We introduce the 'desired total field' $E_{\text{tot}}$ as the solution of the Helmholtz problem

$$\begin{cases} (\Delta + k(x)^2)E_{\text{tot}}(x) = 0, & x \in S, \\ E_{\text{tot}}(x) = \xi(x), & x \in C, \end{cases}$$

where $S$ is an adequately small open neighborhood of $L$, as explained shortly; the curve $C \subset S$ and the function $\xi$ together define the desired near-field pattern which we wish to achieve by a computed structured illumination of the lens. For example, $C$ can indicate where we want a high total field intensity, and $\xi$ can be a large real constant. We readily check that

$$\begin{cases} (\Delta + k_0^2)E_{\text{tot}}(x) = (k_0^2 - k(x)^2)E_{\text{tot}}(x) = -\alpha \chi_L(x)E_{\text{tot}}(x), & x \in S. \end{cases}$$

As usual, decompose the total field $E_{\text{tot}}$ outside $L$ into the sum $E_{\text{tot}} = E_{\text{inc}} + E_{\text{sca}}$ of an incident and a scattered field, where the incident field satisfies

$$\begin{cases} (\Delta + k_0^2)E_{\text{inc}}(x) = f(x), & x \in \mathbb{R}^2. \end{cases}$$

Here $f$ is the source of the incident field radiating in free space. Assume $f(x) = 0$ in $S$; this is what we mean by $S$ being 'adequately small.' (The set $\overline{L} = L \cup \partial L$ is the closure of $L$.) For $x \in S$, we now readily get

$$\begin{cases} (\Delta + k_0^2)E_{\text{sca}}(x) = (\Delta + k_0^2)E_{\text{sca}}(x) + f(x) = (\Delta + k_0^2)E_{\text{tot}}(x) = -\alpha \chi_L(x)E_{\text{tot}}(x). \end{cases}$$

Since the right-hand member of (6) is compactly supported, and since our scattered field must satisfy the Sommerfeld radiation condition in the plane, we thus have

$$E_{\text{sca}}(x) = -\alpha \Phi_0 * (\chi_L E_{\text{tot}})(x) = -\alpha \int_{y \in L} \Phi_0(x - y)E_{\text{tot}}(y)dy, \quad x \in S \setminus \overline{L},$$

Figure 2. Photonic jet design by amplitude- and phase-modulated illumination of a dielectric micro-lens.
and finally

\[ E^{inc}(x) = E^{tot}(x) - E^{sca}(x) = E^{tot}(x) + \alpha \int_{y \in L} \Phi_0(x - y)E^{tot}(y)dy, \quad x \in S \setminus L. \]

Here \( \Phi_0(x) = (j/4)H^{(2)}_0(k_0|x|) \) is the outgoing fundamental solution of the Helmholtz operator in the plane, and \( H^{(2)}_0 \) is the Hankel function of order zero and of the second kind. This suggests the following procedure for producing a PNJ at a desired location in the near field:

1. Define a neighborhood \( S \) of the lens \( L \) such that the desired PNJ is included in \( S \). Pick a curve \( C \subset S \) and a function \( \xi \) on \( C \) that represent the desired PNJ well via the second condition in (3). Also, pick a curve \( \Gamma \subset S \setminus L \) at which the tailored incident field is to be computed.

2. Solve the system in (3) (numerically) for the desired \( E^{tot} \) in \( S \). Prepare a program that approximates the numerical values of \( E^{tot} \) in \( L \) and at \( \Gamma \), for example by interpolation.

3. Compute the incident field \( E^{inc} \) at \( \Gamma \) using (8) and the program from the previous step.

4. Illuminate the micro-lens using a source \( f \) supported outside \( S \), and that radiates a field approximating \( E^{inc} \) at \( \Gamma \).

If the realized lens illumination produces an unsatisfactory approximation to the desired PNJ then one may repeat step 4 above using a better approximation to \( E^{inc} \) at \( \Gamma \), or one may start over and modify the curve \( C \) and function \( \xi \). For the numerical results presented in this paper, we always choose the curve \( C \) to be a short straight linear piece located at the desired PNJ position and pointing radially away from the origin. Our numerical experimentation has shown that more complicated choices of the geometry of \( C \), such as multi-component curves, closed curves, or non-radial curves, can result in involved interference patterns in the near field. While this should certainly be studied systematically and exploited for advanced near-field design, we feel it falls outside of the scope of this paper.

The curve \( \Gamma \) at which the incident field profile is tailored need not be a linear piece. However, for simplicity, in this paper we always choose \( \Gamma \) as illustrated in Figure 2, that is, a linear piece parallel with the \( x \)-axis and placed just above the micro-lens. We did not investigate the practical capabilities and performance of our PNJ design method for more complicated choices of \( \Gamma \).

3. REALIZING THE INCIDENT FIELD PROFILES USING SPATIAL LIGHT MODULATORS

The spatial profile of an optical beam as computed in Section 2 could be achieved in practice using diffractive optical elements such as phase filters or computer generated holograms (CGHs) implemented with digital micromirror devices (DMDs) or liquid crystal spatial light modulators (SLMs). These mechanisms allow to generate complex fields by effectively varying the diffraction efficiency with phase modulations [20, 21] or by transferring phase modulations onto amplitude modulations using polarizing optics [22]. We here propose a mechanism, illustrated in Figure 3, to generate the complex field required to probe the micro-lens and hence generate the nanojet based on the latter technique. We consider a linearly polarized light beam of spatial profile \( u_0(x) \) propagating along the \( z \) direction.
Figure 3. Amplitude and phase modulation of an optical beam with initial spatial profile \( u_0(x) \). Using two spatial light modulators (SLMs) and polarization optics we generate an arbitrary complex field \( u(x) = \Psi(x) \exp(i \Phi(x)) \) by imprinting the phases \( \varphi_1(x) \) and \( \varphi_2(x) \), respectively with the SLMs. The dimensions of the beam are adjusted with a demagnifying telescopic system.

Without losing generality, let us consider a Gaussian beam \( u_0(x) = A_0 \exp\left(-|x|^2/w_0^2\right) \) with horizontal polarization along the horizontal direction. With the use of a half-wave plate (HWP) the polarization of the beam is rotated 45º so that it has equal components of magnitude \( A_0/\sqrt{2} \) in the horizontal (\( \hat{H} \)) and vertical (\( \hat{V} \)) direction. The beam is reflected (or transmitted) by a first spatial light modulator (SLM1) imprinting a phase \( \varphi_1(x) \) to the horizontal component of the beam in the following way:

\[
u(x) = \frac{1}{\sqrt{2}} u_0(x) (e^{i\varphi_1(x)} \hat{H} + \hat{V}).\]

With the use of a linear polarizer (LP) at 45º followed by a HWP the amplitude of the beam becomes \( A_0/\sqrt{2} \) in the horizontal (\( \hat{H} \)) and vertical (\( \hat{V} \)) direction, then a second spatial light modulator (SLM2) imprints a phase \( \varphi_2(x) \), and the resulting beam is

\[
u(x) = u_0(x) \cos\left(\frac{\varphi_1(x)}{2}\right) \exp\left(i\frac{\varphi_1(x) + 2\varphi_2(x)}{2}\right).
\]

The amplitude and phase of the beam after the SLM2 can be engineered by setting the required phases \( \varphi_1 \) and \( \varphi_2 \). Note that an imaging system with unit magnification is required to prevent spurious phase delays due to propagation between the first and the second SLMs, for instance a 4\( f \) imaging system consisting of two lenses of equal focal lengths \( f \) located at a distance \( 2f \) between them in a way that the distance from the first (second) lens to the first (second) SLM is \( 3f \). To generate an arbitrary beam with electric field \( u(x) = \psi(x) \exp(i \phi(x)) \) one would need to induce \( \varphi_1(x) = 2 \arccos(\psi(x)/u_0(x)) \) to SLM1 and \( \varphi_2(x) = (2\phi(x) - \varphi_1(x))/2 \) to SLM2.

We next address the achievable resolution of the SLM-generated incident field profiles. In general, SLMs can generate pixelated 8-bit and 10-bit phase-maps (such as \( \varphi_1(x) \) and \( \varphi_2(x) \) above) with an arbitrary phase distribution of values discretized in steps of \( 2\pi \text{ rad}/255 \approx 24.6 \text{ mrad} \) \((\approx 1.4^\circ)\) and \( 2\pi \text{ rad}/1023 \approx 6.1 \text{ mrad} \) \((\approx 0.35^\circ)\) respectively. The actual performance depends on the model of the SLM and on the operating wavelength.

For the "SLM-generated" part of the numerical results in Section 5, we select the phase map resolution of 24.6 mrad, and Gaussian illumination sufficiently large to cover the area of interest of the SLM, but just enough not to generate diffraction.
effects due to the borders of the SLM. Since we choose to modulate the theoretical incident field profiles in Section 5 in 902 steps of 26.6 nm over a domain (the curve \( \Gamma \) in Figure 2) of length 24 \( \mu \)m, we here start with an illumination beam of diameter (FWHM) equal to 902 pixels of the SLM display. This way, each pixel of the SLM corresponds to a pixel in the domain \( \Gamma \). For an SLM with pixel size of 8 \( \mu \)m (such as Thorlabs EXULUS HD2), the required beam diameter is 7.2 mm and the beam waist 5.2 mm. After passing through the two SLMs, the beam will have the desired amplitude and phase profiles, however at a larger scale. To reduce the size of the beam to 24 \( \mu \)m we would require a 300X demagnifying imager, which can be realized in multiple steps, for instance, three telescopic systems: A, B) 10X \( \rightarrow f_1 = 1000 \) mm, \( f_2 = 100 \) mm, and C) 3X \( \rightarrow f_5 = 300 \) mm, \( f_6 = f_2 \). Putting these three systems (A, B and C) one after the other, the desired field is obtained at the final focal plane of the last system, which corresponds to the reference space domain \( \Gamma \) above the micro-lens. The imager requires ultra-precise alignment and position placement of the demagnifying lenses and of the micro-lens, which can be found by rigorous computation. We can compensate for small diffraction effects caused by misalignment, cross-talk and aberrations by introducing some feedback to the SLM phases \( \varphi_1(x) \) and \( \varphi_2(x) \).

4. Waist-width analysis of designed photonic jets

The reader interested only in the performance of the PNJ design algorithm proposed in sections 2 and 3 can skip the current section entirely. We here estimate the theoretically smallest achievable PNJ waist width as a function of the radial shift \( \varrho \geq 0 \) of the PNJ, using a micro-lens with a circular cross-section of radius \( R_L \).

To this end we use our results from [23] on the bandwidth of the singular spectrum of the forward operator \( F \) for the Helmholtz equation in the plane. The operator \( F \) maps any square-integrable (\( L^2 \)), time-harmonic, 2D TE electromagnetic source \( s \) supported in the disk of radius \( R_L \) and radiating in free space, to the trace of its radiated electric field at the boundary of the disk \( B_{R_L+\varrho} \). Indeed, in view of Eq. (7), we can express the field scattered by the micro-lens in terms of the field radiated in free space by a source \( s \) in \( B_{R_L} \). As in Section 2, we assume that the source of the incident field \( E_{\text{inc}} \) is supported outside some open neighborhood \( S \) of \( B_{R_L} \) and such that \( S \) includes the PNJ. Accordingly, given \( \varrho \), we

1. construct a desired PNJ profile \( E_{\text{PNJ}}^{\text{tot}} \) at \( \partial B_{R_L+\varrho} \)
2. interpret \( E_{\text{PNJ}}^{\text{tot}} \) as a field boundary datum at \( \partial B_{R_L+\varrho} \) for a single-frequency inverse source problem for the Helmholtz equation in free space, with the source supported in \( B_{R_L} \)
3. solve the inverse source problem and compute the optimal (in a precise sense given below) square-integrable source \( s \) supported in \( B_{R_L} \) that minimizes the \( L^2 \) norm of \( F(s) - E_{\text{PNJ}}^{\text{tot}} \) at \( \partial B_{R_L+\varrho} \)
4. compute the field \( E_{s}^{\text{sc}} = F(s) \) radiated by \( s \) at \( \partial B_{R_L+\varrho} \) and interpret \( E_{s}^{\text{sc}} \) as an approximation to the physically realizable field closest to the desired \( E_{\text{PNJ}}^{\text{tot}} \) at \( \partial B_{R_L+\varrho} \), and
5. find the FWHM of the field \( E_{s}^{\text{sc}} \) at \( \partial B_{R_L+\varrho} \) and interpret it as the actually achievable PNJ waist width, given the radial shift \( \varrho \) and the desired profile \( E_{\text{PNJ}}^{\text{tot}} \).
We estimate the smallest achievable FWHM using the above procedure, by letting the desired FWHM in $E_{\text{PNJ}}^{\text{tot}}$ approach zero and examining the corresponding asymptotic behavior of the realized FWHM in $E_{\text{sca}}$.

Let $R \geq R_L$ and assume a PNJ is centered at the intersection of the boundary $\partial B_R$ and the negative $y$-axis in Figure 2. The radial shift of this PNJ is $\varrho = R - R_L$.

In step 1 of the above procedure, we model the desired profile of the PNJ at $\partial B_R$ as a Gaussian,

$E_{\text{PNJ}}^{\text{tot}}(\theta) = \exp\left(-\frac{(\theta + \pi/2)^2}{d^2}\right),$

where $\theta$ is the angle coordinate in the plane. Given a desired FWHM of the PNJ profile in fractions of $\lambda_0$, we find the corresponding $d$ via

$d(\text{FWHM}, R) = \frac{\text{FWHM} \cdot \lambda_0}{2R\sqrt{\ln 2}}.$

Figure 4 shows four Gaussians, each modelling a desired PNJ profile with FWHM = $0.5\lambda_0$, for $R_L$ from $2\mu m$ to $8\mu m$. The radial shift of the PNJ is $2\lambda_0$ for all plots in this section. In step 3 of the above procedure, we solve the inverse source problem using a truncated singular value decomposition of the forward operator $F$. With $(\sigma_m, \psi_m, \phi_m)$ the singular system of $F$ described in [23, Section 2.1], we can write the action of $F$ on a $L^2(B_{R_L})$ (square integrable) source $s$ supported in $\overline{B_{R_L}}$ as

$F(s) \approx \sum_{|m| \leq B} \sigma_m(s, \psi_m) L^2(B_{R_L}) \phi_m,$
where \( B \) is the stable spectral bandwidth of \( F \), estimated in 2D in [23] in terms of
\[
B_- \leq B \leq B_+ \quad \text{with}
\]
\[
B_- = \arg\min_{m \geq 0} \{ j_{m,1} \geq k_0 R_L \},
\]
(12)
\[
B_+ = \arg\min_{m \geq 0} \{ y_{m,1} \geq k_0 R_L \}.
\]
(13)
Here \( j_{m,1} \) and \( y_{m,1} \) are the first positive zeros of the Bessel function \( J_m(x) \) and the Neumann function (Bessel function of the second kind) \( Y_m(x) \), respectively. Thus, we solve \( F(s) = E_{\text{tot}}^{\text{PNJ}} \) in terms of
\[
s_{\text{TSVD}} = \sum_{m \in \mathbb{Z} \atop |m| \leq B} \sigma_m^{-1} \langle E_{\text{PNJ}}^{\text{tot}}, \phi_m \rangle_{L^2(\partial B_R)} \psi_m,
\]
(14)
and in step 4 of the above procedure we find \( E_s^{\text{ca}} \) via
\[
E_s^{\text{ca}} = F(s_{\text{TSVD}}) \approx \sum_{m \in \mathbb{Z} \atop |m| \leq B} \sigma_m (s_{\text{TSVD}} \psi_m)_{L^2(\partial B_R)} \phi_m
\]
\[
= \sum_{m \in \mathbb{Z} \atop |m| \leq B} \langle E_{\text{PNJ}}^{\text{tot}}, \phi_m \rangle_{L^2(\partial B_R)} \phi_m.
\]
(15)
We here use the fact that the right singular vectors \( \psi_m \) of \( F \) constitute an orthonormal system. The computed \( s_{\text{TSVD}} \) is optimal in the sense that it is the fullest projection of the true solution \( s \) onto the domain of \( F \) (without the nullspace of \( F \)) that is based on the stable (noise-robust) component in the data \( E_{\text{tot}}^{\text{PNJ}} \); see [23] for more details. Since each left singular vector \( \phi_m(\theta) \) is proportional to \( \exp(j m \theta) \) [23, Lemma 1], the computation in (15) amounts to a band-limited projection of the desired profile \( E_{\text{PNJ}}^{\text{tot}} \) onto the Fourier basis at the unit circle. It is the sharp spectral cut-off exhibited by the map \( F \) at the bandwidth index \( |m| = B \), in conjunction with the lens radius \( R_L \) and the radial shift \( R - R_L \), that ultimately determine the smallest achievable PNJ waist width. From (12) and (13) it is evident that the bandwidth \( B \) increases with increasing lens radius \( R_L \). This is because \( j_{m,1} \) and \( y_{m,1} \) are monotonically increasing with \( |m| \). Thus a larger lens will be able to produce a narrower PNJ, since the corresponding \( F \) will have a larger bandwidth, and higher spatial frequency components will be present in the projection (15). The analysis in this section is directly extendible to 3D with the help of our singular spectrum results for \( F \) from [24].

We now fix the operating wavelength at \( \lambda_0 = 532 \) nm (common green laser). Figure 5 shows the physically viable field profiles produced using the 5-step procedure outlined above, taking \( B = B_- \) and starting with the desired profiles from Figure 4. We see that the achieved FWHM of the PNJ profile gets slightly closer to the desired FWHM of \( 0.5 \lambda_0 \) as we increase the lens radius \( R_L \) and thereby the bandwidth \( B \). Figure 6 shows more desired and achieved FWHM for the four lens radii \( R_L \). We again use \( B = B_- \), in order to arrive at an upper bound for the achievable FWHM. The curves representing larger lens radii, and therefore larger bandwidths, are closer to the desired FWHM. The plot further suggests that the bandwidth of the forward operator \( F \) is indeed the limiting factor for how narrow a PNJ can get. Figure 7 shows our lower and upper bounds on the achievable FWHM (based on \( B = B_+ \) and \( B = B_- \) respectively) for the four examined lens radii. These
Figure 5. Resulting physically viable PNJ profiles \(|E_{sca}^\text{es}(\theta)|\) at \(\partial B_{RL+\phi} \times 10^5 \text{ V/m}\) for different lens radii \(R_L\).

Figure 6. Waist-width analysis of PNJ profile for four different lens radii with a radial shift of \(2\lambda_0\). The black dots are the results from Figure 5.

bounds are the asymptotic values of the achieved FWHM as the desired FWHM approaches zero. We finally examine the achievable FWHM as function of the desired radial shift, again using the 5-step procedure outlined above. Figure 8 shows
Figure 7. Angular PNJ resolution analysis using the projection from (15) with both bandwidth estimates $B_{\pm}$ from [23].

Figure 8. Waist-width analysis with varying radial shift of a single PNJ with desired FWHM of $0.5\lambda_0$, for four different lens radii $R_L$. The black dots are the results from figures 4 and 5.

Our numerical results; here we try to produce a PNJ with a FWHM waist-width of $0.5\lambda_0$ (as in Figure 5). The achieved FWHM waist-width of the PNJ seems to be approximately linearly increasing with increasing radial shift. The rate of increase is lower for larger lenses, which implies that, theoretically, we can achieve higher angular resolution at large distances from the lens by increasing the lens radius.
5. Achieved PNJ design

We demonstrate our method of PNJ design numerically using a 2D circular cross-section lens of radius 4\(\mu\)m and a 2D square cross-section lens of side length 8\(\mu\)m. Both lenses have refractive index \(n_L = 1.4607\) (SiO\(_2\) at 532 nm), and the operating wavelength is \(\lambda_0 = 532\) nm (common green laser). The illumination is time-harmonic and 2D TE, that is, the electric field vector points out of the lens cross-section plane, and the coordinate system origin is at the center of the lens cross-section.

The curve \(C\), defining the desired location of the PNJ in (3), is a straight, 10 nm-linear piece pointing radially away from the origin, and we choose \(\xi(x) = 1\) for \(x \in C\). The curve \(\Gamma\) where the incident field is tailored is the straight linear piece defined by \(x \in [-12\mu\text{m}, 12\mu\text{m}], y = 4\mu\text{m}\). Thus \(\Gamma\) is always 'just above' the micro-lens, and parallel with the \(x\)-axis. We solve (3) for \(E^{\text{tot}}\) using COMSOL, although other near-field numerical methods are also applicable [25]. Furthermore, we solve (8) for \(E^{\text{inc}}\) at \(\Gamma\) using numerical quadrature and interpolation in a Matlab script. Finally, we input the computed \(E^{\text{inc}}\) at \(\Gamma\) to a second COMSOL model to find the resulting total field.

The left-most and right-most columns in Figure 9 demonstrate the realized PNJ design using the theoretical (Section 2) and the simulated SLM-produced (Section 3) incident fields, respectively. For practical purposes, there seems little difference between the theoretical and the SLM-produced near fields. The SLM angles \(\phi_1(x)\) and \(\phi_2(x)\) are here discretized with the resolution of 24.6 mrad. We conjecture that the lack of symmetry in some of the profiles of the tailored incident field in the first four cases in Figure 9 is a numerical artifact due to the finite field sampling in the steps 2 and 3 described in Section 2. Evidently, the solution compensates for the asymmetry, and correct symmetric PNJs are ultimately achieved as desired in these cases.

The numerical results indicate that our method can design the PNJ position well and over a large domain in the near field. Specifically, Figure 10 shows the achieved radial shifts for the two lens geometries over large ranges of desired radial shifts. The relative error in the achieved radial shifts in that figure is on average 9.9\% in the circular lens case (for both the theoretical and the SLM-produced incident fields), and on average 1.5\% (theoretical) and 2.7\% (SLM) in the square lens case.

In Figure 11 we observe that the waist width and the decay length of the realized PNJs are increasing with increasing radial shift, and for both lens geometries. The linear nature of the waist width increase was predicted in Section 4, but overestimated, especially for large radial shifts. We stress that the analysis in Section 4 uses a simple Gaussian profile to model the field traces. In conclusion, both the radial and the lateral dimension of our near-field PNJ design increase with increasing distance from the lens, with the lateral dimension growing much slower and to the point of being practically constant over large intervals of radial shifts.

Figure 12 shows, for the theoretical incident field profiles, the maximal PNJ electric field intensity relative to the maximal tailored incident electric field intensity, as function of the achieved radial shift in the PNJ. For the circular lens, this quantity fluctuates about 30\% to 35\%, while for the square lens it decays significantly with increasing radial shift, and seems to stabilize at about 40\%. The fact that we achieve PNJs with maximum amplitudes comparable to those of the tailored
Figure 9. Left column: PNJ scanning achieved at the single optical wavelength $\lambda_0 = 532$ nm (common green laser). A 2D SiO$_2$ micro-lens with a circular cross-section of radius 4 $\mu$m, or a square cross-section of side length 8 $\mu$m, is illuminated along the negative $y$-axis by a computed structured incident field (see Section 2). The plots show the amplitude (in V/m) of the resulting total near field, normalized to maximum intensity of 1. Middle column: The computed amplitude and phase profiles of the incident field that produce the desired total near field. The desired PNJ locations in $\mu$m are, from top to bottom: $(x, y) = (0, -4.532)$, $(x, y) = (0, -9.32)$, $(x, y) = (0, -4.532)$, $(x, y) = (0, -9.32)$ (radial shifts 1 $\lambda_0$ or 10 $\lambda_0$), and $(x, y) = (3, -9)$. Right column: PNJ scanning achieved with the incident field profiles simulated as being produced by an SLM with phase steps of 24.6 mrad in the angles $\phi_1(x)$ and $\phi_2(x)$. See Section 3 for details.
incident fields suggests that the phenomenon underlying our PNJ formation is not primarily focusing, but rather an interference effect.

Table 1 shows mean values of achieved PNJ parameters, for the realizations of the theoretical incident field profiles treated in figures 10–12. For comparison,

|                | circular lens | square lens |
|----------------|---------------|-------------|
| mean waist width | $0.88\lambda_0$ | $0.70\lambda_0$ |
| mean decay length | $2.22\lambda_0$ | $1.58\lambda_0$ |
| mean $\max |E^{PNJ}|/\max |E^{inc}|$ | 0.32 | 0.71 |

Table 1. Mean values of the achieved PNJ parameters in figures 10–12, for theoretical incident field profiles (Section 2).

The simulated SLM-produced incident field profiles yield a mean waist width of $0.88\lambda_0$ (circular lens) and $0.70\lambda_0$ (square lens), as well as a mean decay length of $2.24\lambda_0$ (circular lens) and $1.60\lambda_0$ (square lens). For the theoretical incident field profiles, the predicted FWHM of the achieved PNJs in the example in Figure 8 is between approximately $0.75\lambda_0$ and $1.00\lambda_0$ for $R_L = 4\mu m$ and for radial shifts between approximately zero and $3\lambda_0$. This corresponds well with the numerically obtained waist widths in Figure 11 for the circular lens and for achieved radial shifts from approximately zero to approximately $3\lambda_0$. (We stress that our PNJ design procedure from Section 2 does not include specifying a desired PNJ waist width.) We next note that the PNJs produced using a square lens seem not only more precisely steerable, but also smaller than those produced using a circular lens. Their peak intensity relative to the incident field maximum intensity is also on average greater than in the circular-lens case, albeit this intensity ratio seems to drop quickly with the radial shift of the PNJ.

Finally, for reference, we investigate the near-field localization occurring when a simple $-\hat{y}$-directed uniform plane wave at wavelength $\lambda_0 = 532$ nm illuminates the
Figure 11. Achieved waist width and decay length vs. achieved radial shift. a) circular lens, b) square lens.

The circular cross-section lens produces a fixed PNJ with radial shift $2.22\lambda_0$, waist width $0.84\lambda_0$, decay length $3.21\lambda_0$, and maximum PNJ field intensity (relative to maximum incident electric field intensity) 18.92. The waist width is $0.04\lambda_0 \approx 21$ nm smaller than its mean counterpart in our steerable PNJs in the circular-lens case (but larger than that for our steerable PNJs in the square-lens case), while the decay length is almost one full wavelength greater than the mean decay length in the corresponding steerable PNJs. The main difference is the fact that the steerable PNJs exhibit 60 times smaller peak intensity relative to the maximum incident field
intensity, and hence probably need a more powerful illumination in order to achieve comparable maximum field strengths. Alternatively, the relatively weak PNJs are well-suited as steerable optical probes in the investigation of bio-molecules and other sensitive samples.

The near-field structure in Figure 13 of the square lens illuminated by a uniform plane wave seems too far off an admissible PNJ structure for analysis.

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