Fate of $k_{\perp}$-factorization for hard processes in nuclear environment

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Large thickness of heavy nuclei brings in a new scale into the pQCD description of hard processes in nuclear environment. The familiar linear $k_{\perp}$-factorization breaks down and must be replaced by a new concept of the nonlinear $k_{\perp}$-factorization introduced in [1]. I demonstrate the salient features of nonlinear $k_{\perp}$-factorization on an example of hard dijet production in DIS off heavy nuclei. I also discuss briefly the non-linear BFKL evolution for gluon density of nuclei.

1. INTRODUCTION

The linear $k_{\perp}$-factorization is part and parcel of the pQCD description of high energy hard processes off free nucleons. A large thickness of a target nucleus introduces a new scale - the so-called saturation scale $Q_{s}^{2}$, which breaks the linear $k_{\perp}$-factorization theorems for hard scattering in nuclear environment. This property can be linked to the unitarity constraints for the colour dipole-nucleus interaction. In this talk I review the recent work by the ITEP-Jülich-Landau collaboration in which a new concept of the nonlinear $k_{\perp}$-factorization has been introduced [1234] and illustrate the major results on an example of dijet production in DIS off heavy nuclei.

2. $k_{\perp}$-FACTORIZATION FOR DIS OFF FREE NUCLEONS

The parton-fusion approach to shadowing introduced in 1975 [5] is equivalent to the unitarization on the colour dipole-nucleus interaction [9]. One starts with the colour-dipole factorization for DIS at small $x \lesssim x_{A} = 1/R_{A}m_{N}$, when the coherency over the thickness of the nucleus holds for the $q\bar{q}$ Fock states of the virtual photon:

$$\sigma_{T}(x,Q^{2}) = \langle \gamma^{*} | \sigma(x,r) | \gamma^{*} \rangle$$

$$= \int_{0}^{1} dz \int d^{2}r \Psi_{\gamma^{*}}(z,r)\sigma(x,r)\Psi_{\gamma^{*}}(z,r).$$

Here $z$ and $(1-z)$ is the energy partition between $q & \bar{q}$ and $r = \text{size of the colour dipole}$. There is an equivalence between colour dipole and $k_{\perp}$-factorization [678]:

$$\sigma(x,r) = \alpha_{s}(r) \int \frac{d^{2}\kappa 4\pi[1 - e^{iR\kappa}]}{N_{c}\kappa^{4}} \frac{\partial G_{N}}{\partial \log \kappa^{2}} .$$

$$f(x,\kappa) = \frac{4\pi}{N_{c}\sigma_{0}(x)} \frac{1}{\kappa^{2}} \frac{\partial G_{N}(x,\kappa)}{\partial \log \kappa^{2}} .$$

The $x$-dependence of $\sigma(x,r)$ is governed by the colour dipole BFKL equation [9]. The unintegrated gluon density $f(x,\kappa)$ furnishes a universal description of $F_{2p}(x,Q^{2})$ and of the final states. For instance, the linear $k_{\perp}$-factorization for forward dijet cross section reads

$$\frac{d\sigma_{N}}{dz d^{2}p_{+}d^{2}\Delta} = \frac{\alpha_{s}(p^{2})}{2(2\pi)^{2}} f(x,\Delta) \times |\Psi(z,p_{+}) - \Psi(z,p_{+} - \Delta)|^{2} ,$$

where $\Delta = p_{+} - p_{-}$ is the jet-jet decorrelation momentum.

3. COLLECTIVE NUCLEAR GLUE

The colour dipole-nucleus cross-section [6]

$$\sigma_{A}(r) = 2 \int d^{2}b [1 - \exp(-\frac{1}{2}\sigma(r)T(b))]$$

defines the collective nuclear glue per unit area in the impact parameter space, $\phi(b,\kappa)$ [1011]:

$$\Gamma_{A}(b,r) = [1 - \exp(-\frac{1}{2}\sigma(r)T(b))]$$

$$= \int d^{2}\kappa \phi(b,\kappa) \{1 - \exp[i\kappa r]\} .$$

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A useful expansion is

\[ \phi(b, \kappa) = \sum_{j=1}^{\infty} w_j(b) f^{(j)}(\kappa), \]

where \( \nu_A(b) = \frac{1}{2} \sigma_0(x) T(b) \) with \( \sigma_0(x) = \sigma(x, r) \big|_{r \to \infty} \) and \( T(b) \) being the optical the thickness of a nucleus measures the opacity of a nucleus to large dipoles, \( w_j \) is the probability to find \( j \) overlapping nucleons at impact parameter \( b \) in a Lorentz-contracted nucleus and \( f^{(j)} \) is a collective glue of \( j \) overlapping nucleons:

\[ f^{(j)}(\kappa) = \int \prod_{i} d^2 \kappa_i f(\kappa_i) \delta(\kappa - \sum_{i} \kappa_i). \]

The plateau at small momenta of gluons,

\[ \phi(b, \kappa) \approx \frac{1}{\pi} \frac{Q_A^2(b)}{\kappa^2 + Q_A^2(b)^2}, \]

\[ Q_A^2(b, x) \approx \frac{4\pi^2}{N_c} \alpha_s(Q_A^2) G(x, Q_A^2) T(b), \]

is a signal of the saturation effect. The collective nuclear glue furnishes the linear \( k_t \)-factorization representation for DIS off nuclei,

\[ \sigma_{\gamma^* A} = \int d^2 b \langle \gamma^* | 2 \{ 1 - \exp[-\frac{1}{2} \sigma(r) T(b)] \} | \gamma^* \rangle \]

\[ = \int d^2 b \int \frac{d^2 p}{(2\pi)^2} \alpha_s(p^2) \]

\[ \times \int d^2 \kappa \phi(\kappa)(\Psi(z, p) - \Psi(z, p - \kappa))^2 \]

(4)

which is exactly the same as for the nucleon target, subject to \( f(\kappa) \leftrightarrow \phi(\kappa) \).

4. NON-ABELIAN INTRANUCLEAR EVOLUTION OF COLOUR DIPOLES

The two typical final states in DIS off heavy nucleus are shown in fig. 1. The coherent diffraction with large rapidity gap between the target nucleus in the ground state and diffractive hadronic debris of the photon makes \( \approx 50\% \) of the total cross section and gives exactly back-to-back correlated dijets. In the truly inelastic DIS with multiple colour excitation of the nucleus one encounters the non-Abelian intranuclear evolution of colour dipoles, the consistent description of which based on the ideas from [12,13] is found in [1]. Specifically, the ab initio calculation of the nuclear distortion of the two-parton density matrix the Fourier transform of which gives the spectrum of dijets, can be reduced, upon the closure over nuclear excitations, to the problem of intranuclear propagation of the colour-singlet 4-parton states as illustrated in fig. 2.

Figure 1. The typical unitarity cuts and dijet final states in DIS: (a), (b) - free-nucleon target, (c) - coherent diffractive DIS off a nucleus, (d) - truly inelastic DIS with multiple colour excitation of the nucleus.

Figure 2. Unitarity diagram for the dijet spectrum in terms of the 4-parton scattering amplitude.
\[
\frac{d\sigma_{\text{in}}}{dzd^2p_+d^2p_-} = \frac{1}{(2\pi)^4} \int d^2b_+ d^2b_-' d^2b_+ d^2b_- \times \exp[-ip_+(b_+ - b_') - ip_-(b_- - b_-')] \\
\times \left\{ S_{4A}(b_+', ..., b_-) - S_{4A}^D(b_+', ..., b_-) \right\} 
\]

where we subtracted the diffractive contribution.

To the standard dilute-gas nucleus approximation, the Glauber-Gribov theory gives

\[
S_{4A}(b_+', ..., b_-) = \exp\left\{ -\frac{1}{2} \sigma_4(b_+', ..., b_-) T(b) \right\} 
\]

where \( \sigma_4 \) is the coupled-channel operator in the space of singlet-singlet [11] or octet-octet [88] 4-body dipoles, see ref. [1] for more details.

### 5. The Fate of \( k_\perp \)-Factorization for Nuclear Targets

The single-quark spectrum in DIS exhibits the Abelianization property. Namely, the truly inelastic,

\[
\frac{d\sigma_{\text{in}}}{d^2b_+d^2p_+dz} = \frac{1}{(2\pi)^2} \times \left\{ \int d^2\kappa \phi(\kappa)^2 |\Psi(z, p) - \Psi(z, p - \kappa)|^2 \\
- \left| \int d^2\kappa \phi(\kappa)(\Psi(z, p) - \Psi(z, p - \kappa)) \right|^2 \right\} 
\]

and coherent diffractive

\[
\frac{d\sigma_D}{d^2b_+d^2p_+dz} = \frac{1}{(2\pi)^2} \times \left| \int d^2\kappa \phi(\kappa)(\Psi(z, p) - \Psi(z, p - \kappa)) \right|^2 
\]

spectra add to precisely eq. [4]. I.e., the linear \( k_\perp \)-factorization in terms of the collective nuclear glue holds, which is a feature of DIS where the photon is a colour singlet projectile. The same is not true for other projectiles, see Schäfer’s talk at this conference [14].

The nuclear dijet spectrum takes a simple form in the large-\( N_c \) approximation:

\[
\frac{d\sigma_{\text{in}}}{d^2b_+d^2p_+dz} = \frac{1}{2(2\pi)^2} \alpha_S \sigma_0 T(b) \int_0^1 d\beta \\
\times \Phi(2\beta \lambda A_{\nu A}(b), \Delta - \kappa) \frac{d\sigma_N}{d^2b_+d^2p_+dz}. 
\]

Figure 3. The color excitation of the dipole in the large-\( N_c \) approximation.
where 
\[ \Phi(\nu_A(b), \kappa) = \exp(-\nu_A(b))\delta^{(2)}(\kappa) + \phi(\nu_A(b), \kappa) \]
and \( \lambda_e \equiv C_A/2C_F \). The convolution form \( \Phi \) makes the nuclear enhancement of decorrelation obvious one. Semihard dijets, \( |p_\perp|^2 \ll Q_A^2 \), are completely decorrelated.

The work on applications to the centrality dependence of single-jet spectra to dijet decorrelations at RHIC [15] is in progress.

6. SMALL-\( x \) EVOLUTION OF COLLECTIVE NUCLEAR GLUE.

Despite the manifest breaking of the linear \( k_\perp \)-factorization, the collective nuclear glue remains a useful concept. For a free nucleon target the effect of the \( q\bar{q} \) and higher Fock states in the photon is reabsorbed in the linear BFKL evolution for the dipole cross section with the photon treated as the \( q\bar{q} \) state. One possible definition of the nonlinear BFKL evolution for nuclear glue is to insist on the same representation for nuclear cross section. It is indeed possible although without a closed-form evolution equation.

We comment here on first correction \( \propto \log \frac{1}{x} \) to the nuclear profile function and nuclear collective glue. The correction \( \delta \Gamma_A(x, b, r) \) to the colour dipole-nucleus profile function for the \( q\bar{q} \) Fock state in the photon equals

\[
\left. \int q^2 b \frac{\partial \delta \Gamma_A(x, b, r)}{\partial \log \frac{1}{x}} \right|_x = K_0 \int d^2 \rho \frac{r^2}{\rho^2 (\rho - r)^2} \int d^2 b [\Gamma_{3A}(b, \rho, r) - \Gamma_A(b, r)]
\]

\[ \Gamma_{3A}(b, \rho, r) = 1 - S_{3A}(b, \rho, r) = 1 - \exp\left[-\frac{1}{2} \sigma_3(\rho, r) T(b)\right] \]

where \( \sigma_3(\rho, r) \) is the 3-parton cross section [13].

A simplified Glauber-Gribov formula holds at large-\( N_c \), \( S_{3A}(b, \rho, r) = S_{2A}(b, \rho - r)S_{2A}(b, \rho) \). Here \( \partial \delta \Gamma_A(x, b, r)/\partial \log \frac{1}{x} \) is a nonlinear functional of \( \Gamma_{2A} \), the identification of \( \Gamma_A(x, b, r) \) with \( \Gamma_{2A}(x, b, r) \), and the extension of the first iteration to what has become known as the closed-form Balitsky-Kovchegov nonlinear equation [17] is unwarranted. In terms of the nuclear transparency for large dipoles, \( S_A(b, \sigma_0) = \exp(-\frac{1}{2} \sigma_0 T(b)) \), the first correction to the unintegrated nuclear glue takes the form

\[
\frac{\partial \delta \phi_A(x, b, \Delta)}{\partial \log \frac{1}{x}} = S_A(b, \sigma_0) K_{BFKL} \otimes \phi(b, \Delta)
\]

\[
+ K_0 \int d^2 p d^2 k \phi(b, k) \left\{ K(\Delta + p, \Delta + k) \phi(b, p) - K(p, p + \Delta + k) \phi(b, \Delta) \right\}
\]

\[ = S_A(b, \sigma_0) K_{BFKL} \otimes \phi(b, \Delta) + + K_{NonLin} [\phi(b, \Delta)] \]

where \( K(p, k) = (p - k)^2/p^2 k^2 \). It contains an absorption suppressed linear BFKL term with the familiar kernel \( K_{BFKL} \) [18]. For central DIS off heavy nuclei \( S_A \to 0 \) and evolution is entirely driven by the nonlinear term quadratic in \( \phi(k) \).

Making use of an explicit form of \( K(p, k) \), one can recast [11] for the leading conformal twist nuclear glue in an alternative form

\[
\frac{\partial \delta \phi_A(x, b, \Delta)}{\partial \log(1/x)} = K_{BFKL} \otimes \phi(b, \Delta) + Q[\phi](b, \Delta). \tag{11}
\]

Here the linear term evolves with the conventional BFKL kernel, whereas the nonlinear term takes a particularly simple form

\[
Q[\phi|(b, \Delta)] = -2 K_0 \frac{\Delta^2}{\Delta^2} \left[ \int \Delta^2 d^2 q \phi(b, q) \right]^2
\]

\[ -2 K_0 \phi(b, \Delta) \int \Delta^2 d^2 p \int \Delta^2 d^2 q \phi(b, q). \]

For hard gluons, \( \Delta^2 > Q_A^2 \), one can use an approximation \( \phi(q) \sim \phi(\Delta) (\Delta^2/q^2)^2 \) with the result

\[ Q[\phi|(\Delta; b)] \approx -4 K_0 \cdot \Delta^2 \phi^2(\Delta) \propto \frac{\phi(\Delta)}{\Delta^2}. \]

The nonlinear component in [11] gives a pure higher twist contribution. It doesn’t exhaust the nuclear higher twist terms, though, because the one is contained also in \( \phi(\nu, \Delta; b) \), see the discussion in [11]. The character of nonlinearity in terms of \( G_A(b, x, Q) \) is instructive:

\[
\frac{\partial^2 \delta G_A(b, x, Q)}{\partial \log(1/x) \partial \log Q^2} = K_{BFKL} \otimes \frac{\partial G_A(b, x, Q)}{\partial \log Q^2}
\]
\[ -\frac{4\alpha_s(Q^2)T(b)}{Q^2} \left( \frac{\partial G_A(b, x, Q)}{\partial \log Q^2} \right)^2 \]

Now have a look at the plateau region of soft gluons, \( \Delta^2 \ll Q_A^2 \). Here eq. (11) takes the form

\[ \frac{\partial \delta \phi_A(x, b, \Delta)}{\partial \log \frac{1}{x}} = -2C\pi K_0 \phi(x, b, 0) \] (12)

where the constant factor, \( C \sim 1 \), depends on the form of the collective nuclear glue. If we recall that \( \phi(x, b, 0) \sim \pi Q_A^2 b \), then (12) entails an expansion of the plateau width with the decrease of \( x \):

\[ Q_A^2(b) \Rightarrow Q_A^2(b) \left[ 1 + 2C\pi K_0 \log \frac{1}{x} \right] \]

The full-fledged nonlinear evolution will be in effect for soft-to-hard gluon momenta \( \Delta^2 \ll Q_A^2 \).

7. CONCLUSIONS

Nuclear saturation is a straightforward consequence of opacity of heavy nuclei to large colour dipoles. The imposition of unitarity constraints within the colour-dipole approach leads to a unique definition and expansion of nuclear unintegrated glue in terms of the collective glue of overlapping nucleons. The problem of a nonabelian intranuclear evolution of colour dipoles has been solved and consistent description of single-jet and dijet production in DIS off nuclei has been developed. We have proven the breaking \( k_\perp \)-factorization and instead formulated the nonlinear \( k_\perp \)-factorization for forward dijet production in DIS. The formalism is readily extendible to proton-nucleus collision in the kinematic conditions of the RHIC experiments, the corresponding work is in progress. We applied our technique to the nonlinear BFKL evolution of collective nuclear glue and explored the twist properties of the nonlinear component of this equation. The work an applications to jet production in \( pA \) collisions at RHIC is in progress.

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