1. INTRODUCTION

The recent breakthrough in numerical relativity has allowed a direct computation of the linear momentum flux (“kick”) produced during the coalescence of a BH binary (Baker et al. 2006, 2007; Campanelli et al. 2007a, 2007b; González et al. 2007a, 2007b; Herrmann et al. 2007a, 2007b; Koppitz et al. 2007a, 2007b; Campanelli et al. 2007a, 2007b; González et al. 2007a). In the context of producing a BH binary (Baker et al. 2007b; González et al. 2007a). Recent literature has focused on the astrophysical implications of high-velocity kicks, which may displace or remove supermassive BHs from galactic centers (e.g., Merritt et al. 2004; Madau & Quataert 2004; Loeb 2007) and inhibit the growth of BHs at high redshifts ($z \gtrsim 6$), where the escape velocities from low-mass galactic halos are small ($\lesssim 100$ km s$^{-1}$; Haiman 2004; Yoo & Miralda-Escudé 2004).

Another implication of kicks is that they may help produce electromagnetic (EM) counterparts to gravitational wave (GW) sources detected by the Laser Interferometric Space Antenna (LISA) satellite. The discovery of such a counterpart would constitute a milestone for fundamental physics and astrophysics (e.g., Kocevski et al. 2007a). If the BH binary is surrounded by a circumbinary gas disk, the disk will respond promptly (on the local orbital timescale) to such a kick. If this results in warps or shocks, the disturbed disk could produce a transient EM signature (Milosavljević & Phinney 2005). The sky localization uncertainty from LISA is typically a few tenths of a degree, containing a large number of sources; monitoring this area for transient events may be the key to securely identifying counterparts (the monitoring can even begin in a few square degree area several weeks prior to coalescence; Kocevski et al. 2007a, 2007b; Lang & Hughes 2007).

The final recoil velocity of the BH depends on the masses, orbital parameters, and spins of the BHs, and in special configurations (i.e., with spins antialigned with each other), it can reach velocities as high as $\approx 4000$ km s$^{-1}$ (Campanelli et al. 2007a, 2007b; González et al. 2007a). In the context of producing a prompt EM counterpart, we expect that the direction, in addition to the magnitude of the kick, will be important. Naively, one expects that a kick within the plane of any circumbinary disk will be more likely to cause density enhancements and “light up” the disk than a kick perpendicular to it. In this Letter, we focus on the small scales and short timescales ($\lesssim$ years) that are relevant to prompt LISA counterparts. The kick direction, however, can be important on larger scales, as well. For example, the angle relative to a large-scale galactic disk can determine whether a kicked BH is ejected from the galaxy, and whether shocks are produced when a BH binary plunges into a large-scale disk.

Schnittman & Buonanno (2007, hereafter SB07) used the “effective one-body approach” and derived a scaling formula that yields the recoil velocity vector for arbitrary mass ratios and spin vectors. While their results have not been tested for generic spins, they agree well (to within 20%–30%) with numerical results in those special configurations where they were tested (including numerical calculations for configurations with the spins parallel or at intermediate angles with respect to the orbital plane; see Table 1 in SB07).

In this Letter, we investigate the response of a circumbinary to the kick, by following the perturbed, Keplerian orbits of collisionless massless test particles around the recoiling BH. Our goal is to demonstrate that prompt shocks or strong density enhancements arise when the kick is in the plane of the circumbinary disk, whereas they may be less likely for highly inclined kicks. We will then use the formulae of SB07 for the kick speed and direction to argue that a significant fraction ($\gtrsim$ a few %) of kicks may be both sufficiently large and well aligned with the orbital plane for strong shocks to be produced within a few weeks after coalescence.

2. CIRCUMBINARY DISKS

We begin with the assumption that the kicked BH is surrounded by a rotationally supported, geometrically thin gaseous
For a quantitative assessment of the disk’s response to the kick, we employ the following approximation: the disk particles are assumed to be massless, collisionless, and initially on coplanar, circular orbits. The kick simply adds the velocity $v_{\text{kick}}$ to the instantaneous orbital velocity of each particle (in the inertial frame centered on the BHB). We used $N = 10^6$ particles, distributed randomly and uniformly along the two-dimensional surface of the disk. The kick velocity was varied in the range 500 km s$^{-1} < v_{\text{kick}} < 4000$ km s$^{-1}$, and directed either perpendicular or parallel to the initial disk plane. Note that in both the perpendicular and parallel case, we start with a two-dimensional particle distribution (i.e., an infinitely thin disk), but in the perpendicular case, we then follow the orbits in 3D.

Figure 1 shows, as an example, a face-on view of the surface density of the disk 90 days after a kick with $v_{\text{kick}} = 500$ km s$^{-1}$ in the plane of the disk ($i_{\text{kick}} = 0^\circ$). The sharp, tightly wound spiral features clearly seen in the figure trace the locus of points where particles cross each other, corresponding formally to a density caustic. The spiral caustic first forms at $\sim 30$ days, and then propagates outward at a speed of $\approx 500$ km s$^{-1}$. This behavior can be roughly understood as follows: at a given radius $r$, the caustic forms at the time $t$, when the radial
epicyclic motions from two neighboring annuli, separated by the epicyclic amplitude \( r_c \sim (v_{kick}/h_{orbit})r_c \), overlap. This predicts that the shock at \( r_c \) forms at the time \( t_c \sim (r_c d\Omega/dr)^{-2} \), where \( \Omega(r) \propto r^{-3/2} \) is the epicyclic frequency (equal to the Kepler orbital frequency). Ignoring factors of order unity, this yields \( v_c \sim \nu_{kick} \). (Note that the epicyclic approximation requires \( v_{kick}/h_{orbit} \ll 1 \); since \( v_{orb} \approx 2100 \text{ km s}^{-1} \) at \( r_{orb} \), this condition is only marginally satisfied for \( v_{kick} = 500 \text{ km s}^{-1} \).) We also found that as \( t_{kick} \) is increased, the spiral caustics spread farther out. Eventually, for kicks strong enough so that a significant fraction of the disk is unbound, the caustics lose their coherent spiral patterns and develop complex two-dimensional shapes.

Figure 2 shows a side view of the 3D particle density one week after a kick with the velocity \( v_{kick} = 500 \text{ km s}^{-1} \) perpendicular to the disk (\( i_{kick} = 90^\circ \)). The density profile in this case remains azimuthally symmetric, but still develops concentric rings of density fluctuations. The important difference from the parallel kick case is that the density enhancements are much weaker (at the 10% level). By examining the time-evolving radial cross sections, we have verified that sharp density enhancements, i.e., true caustics caused by the orbit crossing of particles, first appear only after 1 yr, and involve a smaller fraction of the disk particles.

More generally, kicks can be expected to occur in a direction intermediate between the two extremes shown in Figures 1 and 2. Following a kick at an arbitrary inclination angle, each particle still remains on a Kepler orbit. Each new orbit will follow an ellipse in a plane that is tilted by an angle of order between \( \pm v_{kick}/h_{orbit} \) about the axis connecting the instantaneous position of the particle with the central BHB. Hence, after an orbital time, the particles of the disk will be smeared vertically, effectively thickening the original disk. The simulations we performed for initially infinitely thin, two-dimensional disks do not conclusively tell us a critical kick inclination angle for prompt density caustics to be produced—this would require three-dimensional simulations, following the orbits of particles in a 3D disk of finite thickness. However, since the prompt, strong density enhancements appear to develop in the plane of the kick, near the inner edge of the disk, one may conjecture that this critical inclination angle is given roughly by \( i_{kick} \leq \arctan(h/r) \) (with \( h/r = 0.46 \) evaluated at \( r_{avGV} \)).

In Figure 3 we show the distribution of kick angles, taken from SB07, assuming that both spins are randomly oriented. We require further that the component of \( v_{kick} \) within the orbital plane exceed 300–500 km s\(^{-1}\) (\( \sim 3–5 \times \) the sound speed; see justification in § 4 below). A strong kick within the orbital plane is produced if the spins are large and parallel to the angular momentum, but antialigned with each other.

The main conclusion to draw from Figure 3 is that caustic-producing kicks are not rare—in the cases shown in the panels, they occur at least a few percent of the cases. This conclusion remains true unless (1) the spins of the BHs is significantly below the maximal Kerr value (\( a \sim 0.9 \)), or (2) the spins are significantly aligned with each other. The latter may be expected if the spin of both BHs results from accretion from the same disk (Bogdanovic et al. 2007).

The criteria defined above for producing caustics may turn out very conservative. For example, prompt caustics could develop when particles from different layers of the disk cross along their orbits. A plausible weaker criterion for shocks is that the kick-induced tilts of the orbits do not thicken the disk beyond its original scale height. Apart from trigonometric factors of order unity that describe its azimuthal dependence, the tilt angle is given by \( i_{kick} \sim \nu_{kick}/h_{orbit} \). requiring only \( i_{kick} \leq \arctan(h/r) \), or \( i_{kick} \leq (v_{orb}/h_{kick}) \arctan(h/r) \). density caustics could arise even for nearly perpendicular kicks. In follow-up work, we will employ three-dimensional simulations, to clarify the relevant caustic formation criterion.

4. DISCUSSION

The main result of this Letter is that strong density enhancements can form promptly after a supersonic kick in the plane of the circumbinary disk, within a few weeks of the coalescence of a \( \sim 10^3 M_{\odot} \) BHB. Because the disk is cold, and caustics (Fig. 1) are formed when particles first cross each other along their orbits, this implies that corresponding shocks could occur in a gas disk. For hydrodynamical shocks to occur within a finite-pressure gas, the relative motions \( v_r \) between the neighboring particles that produce the caustic must exceed the sound speed. At the outermost radius where the disk is marginally bound to the BHB, one expects \( v_r \sim v_{kick} = \nu_{orb} \) relative motions will be slower further inside. The relative speed should roughly correspond to covering the epicyclic amplitude \( \sim (v_{kick}/h_{orbit})r_c \) in the caustic-formation time \( t_c \sim r_c h_{kick}^{-1} \), yield-
ing $v_c \sim v_{\text{kick}}/h_{\text{orb}}$. For $v_{\text{kick}} = 500$ km s$^{-1}$, this predicts $v_c \sim 25$ km s$^{-1}/(r/1000\text{yr})^{1/2}$; we have verified in our simulations that particles cross the caustics with speeds at about $\sim 30\%$ above this predicted value. Compared with the sound speed $c_s \approx 25$ km s$^{-1}/(r/1000\text{yr})^{-1/2}$, this suggests that the density waves produced by the kick in the gas beyond $\sim 700\text{yr}$ will indeed steepen into shocks.

We also found that the inclination of the kick may be important in determining the strength and timing of such shocks—perpendicular kicks would only produce weaker density enhancements, at least until a delay of about a year. This delay is, however, shorter for larger kicks; we find that the first caustics occur at approximately 90, 30, and 10 days, for perpendicular kicks with $v_{\text{kick}} = 1000$, 2000, and 3000 km s$^{-1}$, respectively. A nonnegligible fraction ($\approx$ several %) of kicks could, however, be sufficiently large and well aligned with the orbital plane for shocks to be produced within a few weeks after coalescence.

The nature of the emission resulting from the shocks or density enhancements will have to be addressed in future work, by computing the heating rate at the spiral shocks and modeling the overall disk structure and vertical radiation transport. However, for reference, it is useful to express the disk luminosity resulting from the kick, $L_{\text{kick}}$, as a fraction of the BBH’s Eddington luminosity. We imagine that the shock propagates through the entire disk; i.e., the entire disk mass interior to $r$ participates in a shock at some time prior to $t$. For the fiducial case discussed above, the disk mass enclosed within $r < (10^{10} - 10^4) r_0$ is $\approx 50 - 1200 M_\odot$. If the shocked gas is heated to temperatures corresponding to $T_{\text{shock}}$, and $t_{\text{shock}}$ is the timescale on which the corresponding thermal energy is converted to photons, then $L_{\text{kick}} \approx (1/2) M_{\text{shock}} v_{\text{shock}}^2 / t_{\text{shock}}$. Taking $v_{\text{shock}} \approx v_c \approx 25 - 80$ km s$^{-1}$ and $t_{\text{shock}} = t_{\text{orb}} \approx 70$ days to 1 yr, we find $L_{\text{kick}} / L_{\text{Edd}} \approx 0.3 \times 10^{-4}$ to $1.6 \times 10^{-2}$. These numbers estimate the average luminosity prior to a given time between 2 months and 2 years after coalescence. The instantaneous luminosity may be a factor of 29/10 $\approx$ 3 higher if it evolves with time as steeply as $L_{\text{kick}} \propto t^{-9/10}$ (see next paragraph). We conclude that the luminosity may be a small but nonnegligible fraction, 0.2% to 5% of the Eddington luminosity. This suggests that the afterglows may be detectable, at least for nearby BHs and/or for the most massive BHs in LISA’s range (which extends up to $M_\odot \approx 10^3 M_\odot$).

We may also speculate on the spectral evolution of our fiducial “kick afterglow.” Assuming $M_{\text{shock}} \propto \Sigma r dr \propto r^{11/2}$ (with $\Sigma \propto r^{-1/2}$ and $dr \propto r^{-1/2}$, the epicyclic amplitude), and $v_{\text{shock}} \approx v_c$, we find $L_{\text{shock}} \propto r^{21/2}$. This suggests that the luminosity may be dominated by the outermost shocked shells, with the spectrum peaking at the characteristic photon energy corresponding to $kT_{\text{shock}} \propto v_c^2 \propto v_{\text{orb}}^2 \propto r$. The shocks could therefore result in an afterglow, starting from $700 v_c / h_{\text{kick}} \approx 50$ days, first peaking in the UV band ($\sim 3$ eV or $\sim 0.3\mu$m, corresponding to $\sim 25$ km s$^{-1}$), and then hardening to the soft X-ray ($\sim 50$ eV) range after $\sim 2$ yr. The detection of such an afterglow would help identify EM counterparts to GW sources discovered by LISA. Unfortunately, atmospheric and interstellar attenuation is significant at these photon energies. For reference, we note that interstellar extinction at $\approx 3$ eV is about 5 magnitudes; and even in the direction of the Lockman Hole, which has the lowest neutral hydrogen column density ($N_{\text{HI}} \approx 4.5 \times 10^{19}$ cm$^{-2}$), photoelectric absorption will be significant in the 13.6–100 eV range. This will make the detection of the afterglow difficult or impossible, unless the spectrum extends to either lower or higher photon energies by a factor of $\sim 2$.

Our results need to be verified in simulations that follow the orbits within a disk that has a finite initial thickness. A realistic disk model, incorporating gas dynamics, is needed to study the correspondence between collisionless caustics and gaseous shocks. Finally, the SB07 formula needs to be confirmed for generic spin configurations. Nevertheless, our results do suggest that kicks due to gravitational waves may produce a prompt EM signal.

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