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Electric-Magnetic duality in (linearized) Hořava-Lifshitz gravity

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Abstract. We present an implementation of a duality symmetry in the minimal linearized Hořava-Lifshitz gravity. Some difficulties arise when an attempt is made to implement off shell duality symmetries in theories that are not invariant under full spacetime diffeomorphisms. To avoid some of them our approach will consist of two steps: a) Proposing an explicit formulation of a covariant model and applying standard rules to relate a theory with its dual, and b) Implementing a projection of this covariant model to Hořava-Lifshitz gravity using a gauge fixing condition. In this way we are able to present a dual action of the standard formulation of minimal linearized Hořava-Lifshitz gravity.

1. Introduction
Electric-magnetic duality has emerged as a very interesting symmetry of diverse physical theories of increasing complexity, from zero to higher spin theories. In all cases two key properties keep reappearing: (i) The symmetry is an invariance of the action, and not just of the equations of motion, and (ii) The symmetry can be made manifest, while keeping the formulation simple, only at the price of giving up manifest spacetime covariance. The relation between duality and space time covariance exhibits a fascinating complementarity that can be confirmed by the fact that duality imply Lorentz invariance, at least in the simple case of an abelian gauge field. At a more technical level two general features that also appear are: (iii) The need to reformulate the theory in terms of new variables (prepotentials), and (iv) A corresponding doubling of the gauge symmetry group.

In the case of gravitational theories, it is generally expected that gravitational duality could be fundamental in the understanding of hidden symmetries of supergravities and M-theory. While there is overwhelming evidence in favor of the presence of these symmetries, the results obtained so far remain incomplete because of the still somehow mysterious role played by the dual graviton. The clue for unlocking the present difficulties might lie in a better grasp of the relationship between the graviton and its dual.

The linearized Einstein equations in $D$ spacetime dimensions can be written as twisted self-duality relations revealing that the linearized curvature tensor of the graviton (rank-two symmetric tensor) is dual to the linearized curvature tensor of the dual graviton described by a tensor of $(D - 3, 1)$ Young symmetry type. In the case of 4 dimensions, both the graviton and
its dual are rank-two symmetric tensors (Young symmetry type (1,1)), while in the case of 11 space-time dimensions relevant to M-theory, the dual graviton is described by a tensor of (8, 1) Young symmetry type.

It is known that KK reduction of 11-dimensional supergravity gives rise to exceptional hidden symmetries. In this context a conjecture has emerged: infinite dimensional Kac-Moody algebra E11 is a symmetry of supergravity (M-theory)[1]. Level decomposition of E11 with respect to SL(D) subgroups reproduce the field content of maximal supergravity in D dimensions. This identification of the level decomposition of E11 and supergravity field content requires in the supergravity side that one add to each field its dual. E11 predicts not only the 3-form of 11 dimensional supergravity but also a 6-form and higher level fields in mixed Young Tableaux representations. The lowest one is the graviton and its dual, the dual graviton.

At linearized level we have two equivalent formulation of Einstein Gravity in D spacetime dimensions: based on the metric $h_{\mu\nu}$ or in terms of its dual mixed Young Tableaux field $C_{\mu_1...\mu D-3|\nu}$ whose covariant action was given by Curtright [2]. At nonlinear level there are obstructions to the implementation of duality in the Einstein Hilbert action. The Curtright action does not admit non-abelian deformations under the standard assumptions of spacetime covariance and locality.

Actually a main trend of current research is the study of deformations of gravity. The basic idea consist in the construction of a new gravity that bypass the basic problems associated with the standard formulation of gravity as a field theory. As is well know the theory is not renormalizable and the corresponding quantum theory is not known. Among the many directions in this trend (the consistent deformations and quantization of gravity) we can mention non commutative gravity, stringy corrections to the action of GR, new massive gravities in many dimensions, double field theory and Hořava-Lifshitz gravity [3, 4, 5, 6, 7]. This last particular deformation pretend to be a UV completion of gravity using an “heretic” proposal [8](breaking the spacetime paradigm), breaking full diff invariance of GR and leaving an invariance subgroup that is still manifest in ADM formalism. As a consequence different treatment of the kinetic and potential terms are necessary. The kinetic term is still quadratic in time derivatives but is deformed by a new parameter $\lambda$, and the potential has high order space derivatives of the basic field, the metric $g_{ij}$. The deformed theory is power counting renormalizable, at large distances higher derivative terms are suppressed and the theory runs to standard GR. The new parameter $\lambda$ could be considered as the coupling parameter which controls the contribution of the trace of extrinsic curvature and of some higher derivative terms as for example the term proportional to $(R^{(3)})^2$ (see [6]). For $\lambda = 1$ we have a fixed point in the IR, recovering GR. For generic values of $\lambda$ the theory breaks full diffeomorphism invariance (the theory becomes a non-relativistic theory).

The aim of this note is to implement (in the same way as in the parental action approach) the duality symmetry [9, 10]) in Hořava-Lifshitz theory. The interesting point about this duality is that Hořava-Lifshitz gravity is not invariant under the complete spacetime diffeomorphism but only under a subgroup of them. However this non full covariance is not an impediment to implement duality and is an example of the role played by the complementarity between covariance and duality. Another very interesting way to construct duality in linearized GR is the pre potentials approach of [11].

We start from the action

$$S_{HL} = 2 \int dt d^3x \sqrt{g} N \left( K_{ij} K^{ij} - \lambda K^2 + R^{(3)} \right)$$

in $d = 4$ dimensions and ADM separation, representing the minimal HL deviation from GR, we take the linearized part. Perturbation around flat spacetime $g_{ij} = \eta_{ij} + \epsilon h_{ij}$, $N = 1 + \epsilon n$ and
\[ N_i = \varepsilon n_i \text{ into } S_{\text{HL}} \text{ gives} \]
\[ L_{\text{HL}}^{(2)} = \varepsilon^2 \int d^3x \left[ \frac{1}{2} \dot{h}_{ij} \dot{h}^{ij} - \frac{\lambda}{2} h^2 + \partial_i n_j \partial^i n^j + (1 - 2\lambda)(\partial_i n^i)^2 - 2\partial_i n_j (\dot{h}^{ij} - \lambda n^i \dot{h}) \right. \]
\[ \left. - \frac{1}{2} \partial_k h_{ij} \partial^k h^{ij} + \frac{1}{2} \partial_i h \partial^i h + \partial^i h_{ij} \partial_k h^{kj} - \partial_t h^{ij} \partial_j h + 2n(\partial_i \partial_j h^{ij} - \Delta h) \right] \quad (1) \]

\((h \equiv n^{ij} h_{ij})\). From now on our focus will be the theory defined by this Lagrangian, so we drop the superscript \((2)\) of \(L_{\text{HL}}\). This Lagrangian deviates from FP theory in those terms proportional to \(\lambda\), to which this theory reduce in the IR limit given by \(\lambda \to 1\). Even thought the modifications due to a value of \(\lambda \neq 1\) are subtle, the structural consequences are considerable and have attracted much attention in recent research \(^1\).

In contrast to FP theory, the gauge transformations of this theory are given by the linearized foliation preserving diffeomorphisms \(\delta x^i = \xi^i(x, t)\) and \(\delta t = \xi^0(t)\), acting on the variables of the theory as
\[ \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i, \quad \delta n = -\xi_0. \quad (2) \]

This is a subset of the full group of diffeomorphisms \(\delta \xi h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu\). The latter do not leave the action defined by \((1)\) invariant but rather its variation gives
\[ \delta \xi S_{\text{HL}} = \int dt d^3x (\lambda - 1)(\dot{h} - 2\partial^i n_i) \Delta \xi_0. \quad (3) \]

Restricting the diffeomorphisms to those that satisfy \(\Delta \xi_0 = 0\) we get the isometries given by eq. \((2)\). The solution of this equation is \(\xi_0\) a constant in space\(^2\), i.e., \(\xi_0 = \xi_0(t)\).

2. Dual of linearized minimal HL gravity

The scheme depicted in fig. 1

![Diagram](image)

**Figure 1.** Role of the parental Lagrangian in \(d\) dimensions for FP theory. While \(h_{ab}\) are the first order perturbations of the metric, interpreted as the graviton field, the variables \(X^{abc}_{\mid d}\) are the components of the dual graviton. Both child theories enjoy arbitrary shifts of some variables as gauge symmetries; those pure gauge fields are marginalized through gauge fixing conditions.

The parental \(\mathcal{L}_{\text{FP}}[\epsilon, Y]\)

\[ L_{\text{FP}}[\epsilon_{ab}] \]

\[ L_{\text{FP}}[h_{ab}] \]

\[ L_{\text{FP}}[Y^{abc}_{\mid d}] \]

\[ L_{\text{FP}}[X^{abc}_{\mid d}] \]

\[ \text{gauge fixing } \epsilon_{[ab]} \]

\[ \text{gauge fixing } Z^{ab} \]

\[ \text{gauge fixing } e_{[ab]} \]

\[ \text{gauge fixing } Z^{ab} \]

\(\text{gives the general picture of the role of a parental action for the implementation of the duality symmetry. It is a first order action that delivers both the second order FP action in vielbein}\)

\(^1\) An incomplete list of references to this last topic is\([12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 8]\).

\(^2\) Equation \(\Delta \xi_0 = 0 \Rightarrow \partial_\xi \xi_0\) should be the transverse part of a vector, but 0 is the only vector whose transverse part is a gradient, so \(\partial_\xi \xi_0 = 0\) and this imply that \(\xi_0\) does not depend on the space coordinates.
formalism and the one of its dual theory as a subsequent action (see [10]). In order to apply this scheme for the case of linearized HL gravity we will construct an action in the \textit{vielbein} language as our starting point, in the same way as the one that should be obtained by a parental action (see left branch of fig. 2).

In the following subsection we will first construct an action of this sort, which results in an improved action in the sense that it is diffeomorphism invariant (but not local Lorentz invariant). This action will be the linearized HL theory through a gauge fixing procedure. From this point of view we can parallel the case of FP and construct the parental action from which we obtain a dual theory in section 2.2. We wonder if we can also impose on the latter a gauge fixing condition to define the dual of linearized HL theory, as suggested in the right branch of fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The Lagrangian density $\mathcal{L}_\lambda[e, Y]$ depends on the \textit{vielbeins} and it is parametrized by $\lambda$, the same parameter appearing in linearized HL gravity defined by Lagrangian (1). This theory is diffeomorphism invariant, so it is not the linearized HL gravity; the latter is obtained from $\mathcal{L}_\lambda[e_{ab}]$ by gauge fixing conditions that in fact eliminate antisymmetric components of $e_{ab}$. Analogously on the dual side $\mathcal{L}_\lambda[Y_{abc|d}]$ has some variables that can be eliminated through gauge fixing conditions in order to get a Lagrangian depending only on the dual graviton components.}
\end{figure}

\subsection{2.1. Linearized HL gravity in vierbein language}

Linearization of the Einstein-Hilbert theory, written in \textit{vielbein} variables, corresponds to the FP theory

$$
\mathcal{L}_{\text{FP}}[e_{ab}] = -\frac{1}{2} \left( \Omega^{abc} \Omega_{abc} + 2 \Omega^{abc} \Omega_{acb} - 4 \Omega^a \Omega_a \right),
$$

where $\Omega_{abc} = 2 \partial_{[a} e_{bc]}$ are the linearized anholonomy coefficients, and $\Omega_a \equiv \Omega_{ab}^b$. Two points are important here: tangent and world space indices are identified, and the coefficients $e_{ab}$ do not have a definite symmetry. This Lagrangian depends on $d^2$ fields in $d$ space-time dimensions, certainly more than the independent components of the metric. However local Lorentz invariance takes care of this excess of degrees of freedom.

This can be seen by separating $e_{ab}$ in symmetric $h_{ab} \equiv 2 e_{(ab)}$ and antisymmetric $f_{ab} \equiv 2 e_{[ab]}$ parts. Direct computation shows that the Lagrangian $\mathcal{L}_{\text{FP}}[e_{ab}]$ depends on $h_{ab}$ and $f_{ab}$ in very different ways: while $h_{ab}$ is a dynamical field, $f_{ab}$ enters only through a total derivative term,

$$
\mathcal{M} \equiv \partial_a \left( -\frac{1}{2} h^c \partial_b f^{ab} \right).
$$

\footnote{In fact, they are the components in tangent space of the perturbation to the metric $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ around flat spacetime.}
The Lagrangian density (4) is invariant under local shifts of \( f_{ab} \) with parameters \( \omega_{ab}(x) = -\omega_a(x) \). The full gauge invariance is given by \( \delta e_{ab} = \partial_a \xi_b + \partial_b \xi_a + \omega_{ab} \), where \( \xi_a \) parametrize general coordinate transformations: \( \delta x^\mu = \xi^\mu(x) \). In other words, the FP theory is invariant under
\[
\delta \xi h_{ab} = \partial(a \xi_b) \quad \text{and} \quad \delta \omega f_{ab} = \frac{1}{2} \omega_{ab}.
\]
Using the gauge freedom for the \( f_{ab} \), they can be fixed away and then we can set them to zero. So they can be eliminated from the theory. As a consequence the Lagrangian (4) reduces to a theory for the symmetric fields \( h_{ab} \) only.

Linearized HL gravity is a deviation of FP theory parametrized by \( \lambda \), as in (1). To fix some ideas, we will consider the case of \( d = 4 \) and as our starting point we propose
\[
\mathcal{L}_\lambda[e_{ab}] = \mathcal{L}_{FP}[e_{ab}] + 2(1 - \lambda)(\Omega_0)^2,
\]
where \( \Omega_0 \) is the “0” component of the trace of \( \Omega_{abc} \). As in the case of FP Lagrangian (4) the above Lagrangian density depends on both the symmetric \( h_{ab} \) and antisymmetric \( f_{ab} \) components of the vierbein. As one can expect from general grounds the presence of the last term for \( \lambda \neq 1 \) breaks Lorentz invariance, so the dependence of the Lagrangian upon the fields \( f_{ab} \) are not of the form of a total derivative term. The antisymmetric part of the vierbein turns out to be a set of dynamical fields contributing to the total degrees of freedom of the theory. However, and this is the crux of our argument, we will be able to get rid of them by imposing a set of gauge fixing conditions. At the end we will have a theory with just the symmetric components \( h_{ab} \).

Let’s inspect the symmetries of the Lagrangian (7), where
\[
\Omega_0 = \frac{1}{2} [h - \partial^i(h_{0i} + f_{0i})], \quad h \equiv \eta^{ij} h_{ij}.
\]
For the following it is convenient to make use of the ADM variables, in which \( n_i = h_{0i} \). We also define the notation \( e_i \equiv f_{0i} \). The Lagrangian (7) does not share all the symmetries of \( \mathcal{L}_{FP} \) given in eq. (6).

The symmetry transformations acts on the term \((\Omega_0)^2\) as
\[
\delta \Omega_0^2 = \frac{1}{2} \Omega_0 \left( \partial^i \xi_i - \Delta \xi_0 - \partial^i \omega_{0i} \right).
\]
Then, this piece is invariant under global Lorentz transformations where \( \omega_{ab} \) is constant\(^4\). It is not invariant under local Lorentz transformations though. Instead, in order to have a the gauge symmetry that we need \( \partial^i (\xi_i - \partial_i \xi_0 - \omega_{0i}) = 0 \). This is an equation for \( \omega_{0i} \) so they can not be arbitrary but are determined by the equation of motion. The general solution for it is given by the sum of the solution to the homogeneous equation (any transverse –divergenceless– 3-vector \( \xi^i \)) and a particular solution to the inhomogeneous equation; that is
\[
\omega_{0i} = \xi_i + \dot{\xi}_i - \partial_i \xi_0, \quad \text{such that} \quad \partial^i \xi_i(x,t) = 0.
\]
The parameters \( \xi_i \), however, remain arbitrary, so the Lagrangian (7) is invariant under general coordinate transformations.

In conclusion, the deviation of FP theory given by eq. (7) have arbitrary shifts of \( f_{ij} \) with parameters \( \omega_{ij} \) among its gauge symmetries, as well as arbitrary shifts of the transverse part of \( e_i \) with parameters \( \zeta_i \)\(^5\). The transformations of the longitudinal part of \( e_i \) is linked with the

\(^4\) The anholonomy coefficients transform as \( \delta \Omega_{abc} = 2 \partial_a \partial_c \zeta_b + 2 \partial_a \omega_{bc} \), and so \( \delta \Omega_{abc} = 0 \) for global Lorentz transformations.

\(^5\) Actually, the parameters of the gauge transformations of that part would be \( \zeta_i \) plus the transverse part of \( (\xi_i - \partial_i \xi_0) \), still arbitrary.
where we kept the full total derivative and plug it into the above expression, we have
\[ \frac{\partial}{\partial t} \]
see that if we impose the condition
\[ \delta \xi h_{ij} = \delta_i (\xi_j), \quad \delta \xi n_i = \frac{1}{2} (\xi_i + \partial_i \xi_0), \quad \delta \xi n = -\frac{1}{2} \xi_0, \quad (10a) \]
\[ \delta \omega f_{ij} = \frac{1}{2} \omega_{ij}, \quad \delta \omega e_i = \frac{1}{2} (\xi_i + \dot{\xi}_i - \partial_i \xi_0). \quad (10b) \]

Let’s take now a closer look into \( \mathcal{L}_\lambda[e_{ab}] \). Using the definition \( m_i \equiv n_i + e_i \) a direct computation gives
\[
\mathcal{L}_\lambda = \mathcal{L}_{\text{FP}} + \frac{(1 - \lambda)}{2} (\dot{h} - \partial^i m_i)^2 \\
= \frac{1}{2} \left( h_{ij} \dot{h}^{ij} - \lambda h^2 - 4 \partial_j n_i \dot{h}^{ij} + (4 \partial^i n_i - 2 \partial^i m_i + 2 \lambda \partial^i m_i) \dot{h}ight) \\
- 2 \partial_i n_j \partial^i n_j + (1 - \lambda) \partial^i m_i \partial^j m_j + 2 \partial_i n_j \partial^j n_j \\
- \partial_k h_{ij} \partial^k h^{ij} + \partial_j h^i \partial_i h_j + 2 \partial^i h_{ij} \partial_k h^{kj} - 2 \partial_j h^{ij} \partial_i h_j + 4n(h_{ij} \partial_j h^{ij} - \Delta h) \\
+ \mathcal{M}[h_{ij}, n_i, n; f_{ij}, e_i],
\]
where we kept the full total derivative \( \mathcal{M} \) for clarity of the argument. It is straightforward to see that if we impose the condition
\[ \rho \equiv \partial^i n_i - \partial^i e_i = 0 \quad (11) \]
and plug it into the above expression, we have \( \partial^i m_i = 2 \partial^i n_i \) and then
\[
\mathcal{L}_\lambda |_{\rho=0} = \frac{1}{2} \left( h_{ij} \dot{h}^{ij} - \lambda h^2 - 4 \partial_j n_i \dot{h}^{ij} + (2(1 - 2 \lambda) \partial^i n_i \partial^j n_j + 2 \partial_i n_j \partial^j n_j \\
- \partial_k h_{ij} \partial^k h^{ij} + \partial_j h^i \partial_i h_j + 2 \partial^i h_{ij} \partial_k h^{kj} - 2 \partial_j h^{ij} \partial_i h_j + 4n(h_{ij} \partial_j h^{ij} - \Delta h) \\
+ \mathcal{M} |_{\rho=0},
\]
\[ = \mathcal{L}_{\text{HL}} + \mathcal{M} |_{\rho=0}. \quad (12) \]
So we recover the Lagrangian of linearized HL theory from (7) by imposing eq. (11) and inside \( \mathcal{M} \) we have the longitudinal part of \( e_i \) in terms of \( n_i \). (This is the full content of \( \rho = 0 \), since this condition does not have any information about the transverse part.) Now we can go further and get rid of \( f_{ij} \) and the transverse part of \( e_i \) inside \( \mathcal{M} |_{\rho=0} \) by setting them to zero using the gauge symmetries (10b). In this way we get a Lagrangian for the symmetric \( h_{ab} \), which is the one of linearized HL gravity.

It can be seen that eq. (11) is actually a gauge fixing condition. The way to see this is by going into the Hamiltonian formalism corresponding to \( \mathcal{L}_\lambda \) and performing the Dirac consistency analysis. Among the resulting constraints we find first class constraints generating the gauge transformations
\[ \delta \xi h_{ij} = 0, \quad \delta \xi n_i = \frac{1}{2} \partial_i \xi_0 = -\delta \omega e_i, \quad \delta \xi n = -\frac{1}{2} \dot{\xi}_0, \]
i.e., those of eq. (10) for \( \xi^\mu = (\xi^0, 0, 0, 0) \) with \( \xi^0 = \xi^0(x, t) \) arbitrary. The condition (11) can be taken as a gauge fixing condition of one of those first class constraints.

Since we are dealing with a singular theory, it is pertinent to ask if plugging the projection \( \rho = 0 \) into the Lagrangian is consistent, i.e. if the dynamics of the linearized HL gravity as the above computation suggests is compatible with the corresponding variational principle. Comparison of the EoM for \( h_{ij}, n_i, n, e_i \) coming from \( \mathcal{L}_\lambda \) with the EoM for \( h_{ij}, n_i, n \) for
linearized HL gravity represented by Lagrangian (1) requires indeed the projection \( \rho = 0 \) to match both dynamics, but there is a subtlety that worths to be mentioned. The dynamical equations for \( h_{ij} \) coming from \( \mathcal{L}_\lambda \) are projected onto the corresponding equations of linearized HL gravity, and the projected constraint equations arising from variations of \( n_i, \ e_i \) combine to recover the constraint equation arising from variations of \( n_i \) in \( \mathcal{L}_\text{HL}^0 \). There remains, however, some constraint equations of the theory defined with that \( \mathcal{L}_\lambda \) that are seen only at the stages of secondary constraints found by the consistency analysis of linearized HL gravity.

In conclusion, by considering \( \mathcal{L}_\lambda \) simultaneously with \( \rho = 0 \) we have able to find a theory for the symmetric components \( h_{ab} \) of the vierbeins, whose dynamics is given by the EoM of the Lagrangian density \( \mathcal{L}_\text{HL} \). Moreover, since the gauge fixing condition given by eq. (11) transforms as

\[
\delta \rho = \partial^i (\delta \xi n_i) - \partial^i (\delta \omega e_i) = \frac{1}{2} \partial^i (\dot{\xi}_i + \partial_t \xi_0) - \frac{1}{2} \partial^i (\dot{\xi}_i - \partial_t \xi_0) = \Delta \xi_0
\]

under (10), then keeping it invariant restricts the gauge symmetries to \( \xi^a = (\xi_0, \vec{\xi}) \) with \( \xi^i = \xi^i(x, t) \) completely arbitrary but \( \xi^0 \) subject to equation \( \Delta \xi_0 = 0 \). As a consequence we have \( \xi^0 = \xi^0(t) \). These are precisely the foliation preserving diffeomorphisms of linearized HL gravity.

2.2. Parental action and dual theory

We will follow an approach that consist in taking the theory defined by \( \mathcal{L}_\lambda[e_{ab}] \) as a first order parental action from which we extract the second order dual theory for the corresponding dual graviton (see fig. 2 –in the following we use the same notation as in [10]–). The first order parental action from which both the “electric” and its “magnetic” dual can be obtained (in \( d \) dimensions) depends on the vielbein and some auxiliary fields. Neither of them belong to irreducible representations of the Lorentz group, but contain Young tableaux that does. In addition to the procedure to obtain the dual of \( \mathcal{L}_\lambda[e_{ab}] \), to get the dual of linearized HL gravity for the symmetric components of the vielbeins in \( d = 4 \) we have to impose a “magnetic” analogue of the “electric” condition given by eq. (11).

We start by considering the Lagrangian \( \mathcal{L}_\lambda[e_{ab}] \) given in eq. (7). It is not hard to see that the corresponding parental action can be defined with the Lagrangian density

\[
\mathcal{L}_\lambda[e, Y] = -2 \left( Y^{abc} \Omega_{abc} - Y^{abc} Y_{ac} \right) + \frac{1}{(d - 2)} Y^a Y_a + 2\frac{(\theta - 1)}{(d - 2)} (Y_0)^2,
\]

with \( \theta \) the function of \( \lambda \) given by

\[
\theta = \frac{(d - 2) \lambda}{(d - 1) \lambda - 1}.
\]

Here, besides the vielbein, we have some tensor variables \( Y^{ab|c} \) antisymmetric in \( ab \) indices but with no definite symmetry for \( c \) (we use the notation \( Y^a \equiv Y^{ab|b} \) for the trace). The above Lagrangian is assumed to be written in the same reference frame as (7).

We see from the EoM of \( Y^{abc} \) that these are auxiliary fields, solved in terms of \( e_{ab} \) and their derivatives as

\[
Y_{ab|c} = \frac{1}{2} \Omega_{abc} - \Omega_{c[a|b]} + 2\eta_{c[a} \delta_{b]} d \left( \Omega_d - (1 - \lambda) \eta_{db} \Omega_0 \right).
\]

Plugging eq. (15) into the first order Lagrangian (13) we get the second order “electric” Lagrangian (7).

Before going onto the dual theory, notice that the gauge symmetry transformations (10) of \( \mathcal{L}_\lambda[e_{ab}] \) are captured by gauge invariances of the parental Lagrangian (13). This is achieved

\[ \text{The constraints arising from variations of } n \text{ do not even need the projection } \rho = 0 \text{ to match.} \]
by extending the former with proper transformations of the auxiliary $Y^{ab|c}$. In fact, the restriction given in eq. (9) arise also here from general assumptions: using that $\delta e_{ab} = \partial_a \xi_b + \partial_b \xi_a + \omega_{ab} \Rightarrow \delta \Omega_{abc} = 2(\partial_c \partial_a \xi_b + \partial_b \partial_a \omega_{bc})$, we get from eq. (15)

$$\delta Y_{ab|c} = 2 \partial_a \partial_b \xi_c - \partial_c \omega_{ab} + 2 \eta_{|a}(\eta_{b|d} - (1 - \lambda)\eta_{b|0}\delta^{0|d})(\partial^d \partial^e \xi_e - \partial^e \partial_c \xi^d + \partial_c \omega^{ed}). \quad (16)$$

For the variation of the Lagrangian we get

$$\delta \mathcal{L} = -2\left(\delta Y^{ab|c}\Omega_{abc} + Y^{ab|c}(\delta \Omega_{abc} - 2 \delta Y_{ac|b} + \frac{2}{(d-2)} \eta_{bc} \delta Y_a) + 4 \frac{(\theta - 1)}{(d-2)} Y_0 \delta Y_0 \simeq 0. \right.$$

The first term in this equation involves the $e_{ab}$ variables. Requiring its invariance implies eq. (9), a restriction that eventually gives $\delta Y_0 = 0$ making the last term invariant on its own. The rest of the terms cancel out in account of the antisymmetry of $Y^{ab|c}$ in $ab$.

Now we compute the dual theory by taking in eq. (13) the $e_{ab}$ variables as Lagrange multipliers of the constraints $\partial_a Y^{ab|c} = 0$. These are solved by some potentials $Y^{abc|}_c$ completely antisymmetric in its indices $abc$ and such that $Y^{ab|}_c = \partial_c Y^{abc|}_c$. Reduction of Lagrangian (13) give us the dual theory:

$$\mathcal{L}_\lambda[Y^{abc|}_c] = 2\left(\frac{1}{(d-2)} Y_{ac|b} - \frac{1}{(d-2)} Y^{a} Y_a + 2 \frac{(\theta - 1)}{(d-2)} Y_0^2 \right) \quad (17)$$

(where the field strengths $Y^{ab|}_c$ are functions of the potentials $Y^{abc|}_c$). It is not difficult to see that the action defined by this Lagrangian is invariant under transformations (16) for the field strength, the restriction (9) coming here from the last term. However, these do not exhaust the gauge symmetries of the dual theory. There are more gauge symmetries coming from an ambiguity in the potentials solving $\partial_a Y^{ab|}_c = 0$. The redundancy is given by the gauge transformations

$$\delta_\phi Y^{abc|}_d = \partial_c \phi^{abc|}_d \quad (18)$$

with $\phi^{abc|}_d$ arbitrary and completely antisymmetric in indices $abc$. These transformations leave the field strengths $Y^{ab|}_d$ invariant and so the Lagrangian (17), since the latter is quadratic in those field strengths.

We can get a closer look to the symmetries of the dual “magnetic” theory if we perform the irreducible decomposition (see [10])

$$Y^{abc|}_d = X^{abc|}_d + \delta^{[a}_d Z^{bc]},$$

which consists of extracting from $Y^{abc|}_d$ its trace (the fields $X^{abc|}_d$ are completely antisymmetric in $abc$ indices and traceless). Plugging the above decomposition into Lagrangian (17) we end up with the dual theory written as

$$\mathcal{L}_\lambda[X, Z] = \mathcal{L}_{FP}[X, Z] + \frac{2(d-2)}{9} (\theta - 1) \partial^i Z^{i|0})^2. \quad (19)$$

The $\mathcal{L}_{FP}[X, Z]$ part is the dual of the FP Lagrangian in $d$ dimensions, where

$$\mathcal{L}_{FP}[X, Z] = 2 \partial_d X^{abc|}_d \partial^e X_{ace|b} + \mathcal{M}[X, Z],$$

7 With this restriction, using eq. (16) it can be seen that $\delta(\partial_a Y^{ab|}_c) = 0$, meaning that the constraint defining the dual potentials $Y^{abc|}_d$ is gauge invariant.
the last term being the total derivative
\[ M \equiv \frac{2}{3} \partial_e \left( 2 \partial_d X^{cda}[b] Z_{ab} + Z_a [b] \partial_b Z^a c \right) \] (20)
(and the only place where \( \mathcal{L}_{FP}[X, Z] \) depends on \( Z^{ab} \)).

At this point in the “magnetic” theory we are in a stage corresponding to the “electric” eq. (7). The FP Lagrangian \( \mathcal{L}_{FP} \) is invariant under the arbitrary shifts given by \( \delta_\omega Z^{ab} = \tilde{\omega}^{ab} = -\tilde{\omega}^{ba} \). These include the projection of the gauge symmetries given in eq. (18) for those variables. It can be seen that \( \delta_\phi (\partial_a Z^{ab}) = 0 \), and so it is straightforward that these transformations leave also invariant the last term in Lagrangian (19). This term is not invariant, however, under arbitrary shifts with parameters \( \omega_{ab} \). Instead, demanding invariance of this term will restrict them.

Identifying \( Z^{0i} \equiv Z^i \) with the components of a vector in \( d = 1 \) dimensions, we split them into transverse and longitudinal parts: \( Z^i = Z^i_\perp + Z^i_\parallel \), and doing the same with the parameters \( \tilde{\omega}^{0i} = \tilde{\xi}^i + \omega^i \), where the transverse (or divergenceless) \( \tilde{\xi}^i \) are involved in the local shifts of the transverse part \( Z^i_\perp \), the invariance of the last term in Lagrangian (19) requires
\[ \delta_\omega (\partial_i Z^i_\perp) = \delta_\omega (\partial_i Z^i_\parallel) = \partial_i \tilde{\omega}^{0i} = \partial_i \omega^i = 0. \]
This relation implies that \( Z^i_\perp \) are pure gauge (\( \delta_\omega Z^i_\perp = \tilde{\xi}^i \) with \( \tilde{\xi}^i \) arbitrary). On the other hand the parallel components \( \omega^i \) are restricted by the above equation to be constant along the spatial hypersurface, that is \( \delta_\omega Z^i_\parallel = \omega^i (t) \).

We have here a very similar breaking of local Lorentz invariance that we had in the “electric” theory, restricting the gauge invariances to those of eq. (10). In summary, the gauge symmetries for the dual theory are given by
\[ \delta_\phi X^{abc|d} = \partial_e \phi^{abc|d} + \frac{3}{(d - 2)} \delta^d [a] \partial_e \phi^{bc|e} , \] (21a)
\[ \delta_\omega Z^{ij} = \tilde{\omega}^{ij} , \quad \delta_\omega Z^i = \tilde{\xi}^i + \omega^i (t) \], such that \( \partial_i \tilde{\xi}^i = 0. \) (21b)

We have constructed the dual of the theory with Lagrangian (7) in \( d \) dimensions, and identified its gauge symmetries. We did it by providing the parental action defined with the Lagrangian (13), from which both “electric” and “magnetic” theories are obtained. In the case of the “electric” theory we managed to identify and characterize the way in which linearized HL gravity is obtained from Lagrangian (7), imposing the gauge fixing condition (11). We wonder now if there is an analogous “magnetic” projection for Lagrangian (19) in \( d = 4 \) dimensions that turns the reduced dual theory into the linearized HL gravity again.

To explore this idea we use the field redefinition
\[ T_{a_1...a_d-3|c} \equiv \frac{1}{3!} \epsilon_{a_1...a_d-3|e} g X^{e|f|g} c, \quad \text{such that} \quad T_{[a_1...a_d-3|c]} = 0. \] (22)
In \( d = 4 \) dimensions \( T_{ab} \) is a symmetric tensor. In these variables the action defined with the dual Lagrangian \( \mathcal{L}_{FP}[T, Z] \) is again the FP theory for the dual graviton. We also have that the symmetries (21a) can be written as \( \phi^{abcd|e} = \epsilon^{abcd|e} \chi_a \), with \( \chi_a \) arbitrary, giving \( \delta_\phi T_{ab} = -\partial_\chi (a \chi_b) \).

8 They separate as
\[ \delta_\phi X^{abc|d} = \partial_e \phi^{abc|d} \rightarrow \left\{ \begin{array}{l}
\delta_\phi Z^{ab} = \frac{3}{(d - 2)} \partial_e \phi^{abc}, \quad \phi^{abc} \equiv \phi^{abcd|d} \\
\delta_\phi X^{abc|d} = \partial_e \phi^{abc|d} + \frac{3}{(d - 2)} \delta^d [a] \partial_e \phi^{bc|e} .
\end{array} \right. \]
The \( \delta_\phi X^{abc|d} \) leave \( \mathcal{L}_{FP} \) invariant modulo a total derivative.
In these variables the action defined with the Lagrangian (19) for the dual theory is written as
\[ S_\lambda[T, Z] = \int d^4x \left( \mathcal{L}_{FP}[T, Z] + \frac{4}{9}(\theta - 1)(\partial_i Z_i)^2 \right) \]
\[ = 2 \int d^4x \left( \partial^a T^{bc} \partial_a T_{bc} - 2\partial_a T^{ac} \partial^b T_{bc} + 2\partial^b T^{a}_c \partial^c T_{bc} - \partial_a T_{bc} \partial^b T_{ce} \right. \]
\[ \left. + \mathcal{M}[T, Z] + \frac{2}{9}(\theta - 1)(\partial_i Z_i)^2 \right). \]

We can fix part of the second term (to zero) using the gauge invariance under arbitrary shifts of \( Z_{ij} \) and \( Z^j \), the variables that are pure gauge. But the \( Z^i \) survive on the boundary because the action does not exhibit arbitrary shifts of these variables as a gauge invariance. To describe the theory in terms of the fields \( T_{ab} \) we need to impose a “magnetic” analogue of the “electric” gauge fixing condition given in eq. (11). With such condition we would like to write \( Z^i \) (or more precisely, its divergence) in terms of some components of \( T_{ab} \).

For that end we split the Lagrangian (19) in space and time. We have
\[ \mathcal{L}_\lambda[T, Z] = -2 \left( \dot{T}_{ij} \dot{T}^{ij} - \dot{T}^2 - 4\partial_i T_{0j}(\dot{T}^{ij} - \eta^{ij}\dot{T}) - 2\partial^j T_{0i}\partial^i T_{0j} - 2\partial_0 T_{0j}\partial^j T^{0j} + 2R_T \right) \]
\[ + \mathcal{M}[T, Z] + \frac{4}{9}(\theta - 1)(\partial_i Z_i)^2, \quad (23) \]
where
\[ R_T \equiv -\frac{1}{2}\partial_i T_{ij}\partial^k T^{ij} + \frac{1}{2}\partial_i T\partial^i T + \partial_i T^{ij}\partial^k T_{kj} - \partial_i T^{ij}\partial_j T - \partial_0(\partial_i \partial_j T^{ij} - \Delta T) \]
is nothing more but the potential in the Lagrangian (1) as a function of \( T_{ij} \) instead of \( h_{ij} \), and \( T \equiv \eta^{ij}T_{ij} \).

Taking the following ansatz\(^9\) for the “magnetic” projection condition
\[ \rho^* \equiv \partial_i Z^i - \varepsilon(\dot{T} - 2\partial^i T_{0i}) = 0, \quad \varepsilon \in \mathbb{R} - \{0\}. \quad (24) \]
Some arithmetics give
\[ \mathcal{L}_\lambda[T, Z] \big|_{\rho^*=0} = -2 \left( \dot{T}_{ij} \dot{T}^{ij} - \gamma \dot{T}^2 - 4\partial_i T_{0j}(\dot{T}^{ij} - \gamma \eta^{ij}\dot{T}) \right. \]
\[ \left. + 2(1 - 2\gamma)\partial^j T_{0i}\partial^i T_{0j} - 2\partial_0 T_{0j}\partial^j T^{0j} + 2R_T \right) \quad (25) \]
up to a total derivative, with the parameter \( \gamma \) given by
\[ \gamma \equiv 1 + \frac{2}{9} \left( \frac{1 - \lambda}{3\lambda - 1} \right) \varepsilon^2. \]
It is possible to see from the Hamiltonian Dirac analysis of the constraints that condition (24) is also a gauge fixing condition.

From eq. (23) to eq. (25), the latter a theory for \( T_{ab} \) only, we were able to get rid of \( \mathcal{M} \) because now this term is a total derivative that can be eliminated from the picture with boundary
\(^9\) The ansatz could be a general combination of \( \dot{T} \) and \( \partial^i T_{0i} \), which are the objects involved in the terms proportional to \( \lambda \) in the Lagrangian for linearized HL gravity. It can be seen that those terms should be in a ratio of \( \frac{\gamma}{\Delta T_{0i}} \sim \frac{1}{2} \).
conditions on $T_{ab}$. The resulting functional is the Lagrangian of linearized HL gravity but parametrized by $\gamma$ instead of $\lambda$. This parameter measures the deviation of the dual theory from the dual of FP action. Hence, the electric and magnetic theories are the same deviation of FP by terms measured with parameters belonging to “weak” and “strong” regimes respectively ($\lambda < 1 \Rightarrow \gamma > 1$, and vice versa –the case $\lambda = 1 = \gamma$ being FP theory–).

As in the “electric” theory, preservation of the condition given by eq. (24) restricts again the symmetries (21a), which are

$$\delta\phi T_{ab} = -\partial_i (\chi_i)$$

If we want to take the dual Lagrangian (19) together with such condition as the dual theory of linearized HL gravity, we would like to preserve the latter. Then, we have that

$$0 = \delta \rho^* = \delta\omega(\partial_i Z^i) - \varepsilon \left( \delta\phi(\dot{T}) - 2\delta\phi(\partial^i T_{0i}) \right)$$

$$= -\varepsilon \left( \eta^{ij} \partial_0 \delta\phi T_{ij} - 2\partial^i \delta\phi T_{0i} \right)$$

$$= -\varepsilon \left( \eta^{ij} \partial_0 (-\partial_i \chi_j) - 2\partial^i (-\partial_{(0} \chi_{i)}) \right)$$

$$= -\varepsilon \Delta \chi_0.$$  

This means that gauge symmetries are restricted to those with $\chi_0 = \chi_0(t)$, the foliation preserving diffeomorphisms of linearized HL gravity.

3. Conclusions

The construction of duality transformations in classical field theories using the powerful tools of covariant parent actions has been implemented in Hořava-Lifshitz gravity. For that end we proceeded in two steps: a) the construction of a covariant model that in an appropriate gauge reduce to Hořava-Lifshitz gravity. We have used a gauge is fixing procedure that it breaks the full diffeomosphism grip of General Realitivity to a subgroup that coincides with the symmetry group of Hořava-Lifshitz gravity., b) The implementation of parent action to construct the duality symmetry in a covariant model. In this way we were able to construct a dual action associated with the standard formulation of Hořava-Lifshitz gravity.

Starting from Lagrangian (7) we have constructed a dual action in $D$ dimensions (eq. (19)) and in particular for $D = 4$ our result is given in eq. (25). The details of our calculations will be published elsewhere. We have also work in progress along a generalisation of this duality implementation when the dual graviton is also present. This example shows that the Lorentz global invariance is not strictly necessary to implement duality symmetry at the variational level.

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Corrigendum: Electric-Magnetic duality in (linearized) Hořava-Lifshitz gravity

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We report a problem with the last version of our paper and the one that we sent for the final published version. The published version has an overlap with an already published work. The changes are only in the introduction. The correct introduction section appear below. The scientific content of our article and conclusions remains the same.

1. Introduction
Symmetries and variational principles play an essential role in the construction of interactions. The Noetherian symmetries are a central paradigm in this realm. In recent years the generalization of the concept of symmetry has been a subject of great interest.

The fact that the equations of motion can have more symmetries than the corresponding variational principle ruled out some important symmetries that can be very useful in the analysis and understanding of physical theories. Symmetries like non-Noetherian symmetries (or s-equivalent symmetries) and Dualities are powerful tools in diverse areas like condensed matter and string theory. In particular Dualities are very interesting symmetries that relate different physical theories and/or the same theory at different couplings.

Among duality symmetries the electric-magnetic duality is perhaps the best well known of them. Recently the authors of [1, 2] proposed a variational formulation of this duality, that works not only at the level of the equations of motion. Even though this formulation is constructed giving up manifest spacetime covariance of the original Maxwell action the authors of [3] showed that in the case of linearized gravity it is possible to implement the electric-magnetic duality analogue at the level of an action. This implementation also breaks manifest spacetime covariance in order to have duality invariance manifest.

Motivated by these works we are wondering if Hořava-Lifshitz gravity, a completion of general relativity in the high energy regime that actually breaks spacetime covariance, admits also a duality implementation. The aim of this work is to present in detail such implementation, following the example of the Fierz-Pauli case. We will proceed, however, from another point of view, namely from a parent-action for the covariant Fierz-Pauli action and its dual theory.
Actually a main trend of current research is the study of deformations of gravity. The basic idea consist in the construction of a new gravity that bypass the basic problems associated with the standard formulation of gravity as a field theory. As is well know the theory is not renormalizable and the corresponding quantum theory is not known. Among the many directions in this trend (the consistent deformations and quantization of gravity) we can mention non commutative gravity, stringy corrections to the action of GR, new massive gravities in many dimensions, double field theory and Hořava-Lifshitz gravity [4, 5, 6, 7]. This last particular deformation pretends to be a UV completion of gravity using an “heretic” proposal [9](breaking the spacetime paradigm), breaking full diff invariance of GR and leaving an invariance subgroup that is still manifest in ADM formalism. As a consequence different treatment of the kinetic and potential terms are necessary. The kinetic term is still quadratic in time derivatives but is deformed by a new parameter $\lambda$, and the potential has high order space derivatives of the basic field, the metric $g_{ij}$. The deformed theory is power counting renormalizable, at large distances higher derivative terms are suppressed and the theory runs to standard GR. The new parameter $\lambda$ could be considered as the coupling parameter which controls the contribution of the trace of extrinsic curvature and of some higher derivative terms as for example the term proportional to $(R^{(3)})^2$ (see [7]). For $\lambda = 1$ we have a fixed point in the IR, recovering GR. For generic values of $\lambda$ the theory breaks full diffeomorphism invariance (the theory becomes a non-relativistic theory).

The aim of this note is to implement (in the same way as in the parental action approach) the duality symmetry [10, 11]) in Hořava-Lifshitz theory. The interesting point about this duality is that Hořava-Lifshitz gravity is not invariant under the complete spacetime diffeomorphisms but only under a subgroup of them. However this non full covariance is not an impediment to implement duality and is an example of the role played by the complementarity between covariance and duality. Another very interesting way to construct duality in linearized GR is the pre potentials approach of [3].

We start from the action

$$S_{HL} = 2 \int d^4 x \sqrt{g} N \left(K_{ij} R^{ij} - \lambda K^2 + R^{(3)} \right)$$

in $d = 4$ dimensions and ADM separation, representing the minimal HL deviation from GR, we take the linearized part. Perturbation around flat spacetime $g_{ij} = \eta_{ij} + \varepsilon h_{ij}$, $N = 1 + \varepsilon n$ and $N_i = \varepsilon n_i$ into $S_{HL}$ gives

$$L_{(2)}^{HL} = \varepsilon^2 \int d^3 x \left[ \frac{1}{2} h_{ij} \dot{h}^{ij} - \frac{\lambda}{2} \dot{\lambda}^2 + \partial_i n_j \partial^i n^j + (1 - 2\lambda)(\partial^i n^i)^2 - 2 \partial_i n_j (\dot{h}^{ij} - \lambda n^{ij} \dot{h}) 
- \frac{1}{2} \partial_k h_{ij} \partial^k h^{ij} + \frac{1}{2} \partial_i h \partial^i h + \partial^i h_{ij} \partial_k h^{kj} - \partial_i h^{ij} \partial_j h + 2n(\partial_i \partial_j h^{ij} - \Delta h) \right]$$  \hspace{1cm} (1)

($h = n^{ij} h_{ij}$). From now on our focus will be the theory defined by this Lagrangian, so we drop the superscript $^{(2)}$ of $L_{HL}$. This Lagrangian deviates from FP theory in those terms proportional to $\lambda$, to which this theory reduce in the IR limit given by $\lambda \to 1$. Even thought the modifications due to a value of $\lambda \neq 1$ are subtle, the structural consequences are considerable and have attracted much attention in recent research.

In contrast to FP theory, the gauge transformations of this theory are given by the linearized foliation preserving diffeomorphisms $\delta x^i = \xi^i(x,t)$ and $\delta t = \xi^0(t)$, acting on the variables of the theory as

$$\delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i, \hspace{0.5cm} \delta n_i = \xi_i, \hspace{0.5cm} \delta n = -\xi_0.$$  \hspace{1cm} (2)

1 An incomplete list of references to this last topic is [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 9].
This is a subset of the full group of diffeomorphisms $\delta \xi h_{\mu \nu} = \partial_\nu \xi_\mu + \partial_\mu \xi_\nu$. The latter do not leave the action defined by (1) invariant but rather its variation gives

$$\delta \xi S_{HL} = \int dt d^3x 2(\lambda - 1)(\dot{h} - 2\dot{\partial}_i n_i) \Delta \xi_0. \quad (3)$$

Restricting the diffeomorphisms to those that satisfy $\Delta \xi_0 = 0$ we get the isometries given by eq. (2). The solution of this equation is $\xi_0$ a constant in space, i.e., $\xi_0 = \xi_0(t)$. 

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\textsuperscript{2} Equation $\Delta \xi_0 = 0 \Rightarrow \partial_i \xi_0$ should be the transverse part of a vector, but 0 is the only vector whose transverse part is a gradient, so $\partial_i \xi_0 = 0$ and this imply that $\xi_0$ does not depend on the space coordinates.