We investigate the mass spectrum of $1^{++}$ exotic mesons with quark content $(cq\bar{c}\bar{q})$, using molecular and diquark-antidiquark operators, in quenched lattice QCD with exact chiral symmetry. For the molecular operator $\{(q\gamma_i c)(\bar{c}\gamma_5 q) - (\bar{c}\gamma_i q)(q\gamma_5 c)\}$ and the diquark-antidiquark operator $\{(q^T c\gamma_i c)(\bar{q}C\gamma_5 \bar{c}^T) - (\bar{q}C\gamma_i \bar{c}^T)(q^T c\gamma_5 c)\}$, both detect a resonance with mass around $3890 \pm 30$ MeV in the limit $m_q \rightarrow m_u$, which is naturally identified with $X(3872)$. Further, heavier exotic meson resonance with $J^{PC} = 1^{++}$ is also detected, with quark content $(c\bar{s}c\bar{s})$ around $4100 \pm 50$ MeV.

PACS numbers: 11.15.Ha, 11.30.Rd, 12.38.Gc, 14.40.Lb, 14.40.Gx

Keywords: Lattice QCD, Exact Chiral Symmetry, Exotic mesons, Charm Mesons
1 Introduction

Since the discovery of $D_s(2317)$ by BaBar in April 2003, a series of new heavy mesons with open-charm and closed-charm have been observed by Belle, CDF, CLEO, BaBar, and BES. Among these new heavy mesons, the narrow charmonium-like state $X(3872)$ (with width < 2.3 MeV) first observed by Belle [5] in the exclusive decay $B^\pm \to K^\pm X \to K^\pm\pi^+\pi^-J/\psi$ seems to be the most remarkable. The evidence of $X(3872)$ has been confirmed by three experiments in three decay and two production channels [6, 7, 8]. Recently, the decay $X(3872) \to \gamma J/\psi$ has been observed by Belle [9], which implies that the charge conjugation of $X(3872)$ is positive. Further, $X(3872)$ is unlikely a vector $1^{--}$ state, since the bound $\Gamma(e^+e^-)Br(X \to \pi^+\pi^-J/\Psi) < 10eV$ at 90% C.L. has been obtained [10], with the data collected by BES at $\sqrt{s} = 4.03$ GeV. Now, the quantum numbers $0^{++}$ and $0^{+-}$ are ruled out based on the angular correlations in the $\pi^+\pi^-J/\psi$ system [11], also $2^{--}$ and $1^{--}$ are strongly disfavored according to the dipion mass distribution [11]. Thus it is likely that $X(3872)$ possesses $J^{PC} = 1^{++}$.

Theoretically, one can hardly interpret $X(3872)$ as $2P$ or $1D$ states in the charmonium spectrum, in view of its extremely narrow width. Thus it is most likely an exotic (non-$q\bar{q}$) meson (e.g., molecule, diquark-antidiquark, and hybrid meson). Now the central question is whether the spectrum of QCD possesses a resonance around 3872 MeV with $J^{PC} = 1^{++}$.

In this paper, we investigate the mass spectra of molecular and diquark-antidiquark interpolating operators whose lowest-lying states having $J^{PC} = 1^{++}$, in lattice QCD with exact chiral symmetry [12, 13, 14, 15, 16]. This study follows our recent investigation on the mass spectrum of exotic mesons with $J^{PC} = 1^{--}$ [17], which suggests that $Y(4260)$ [18] is in the spectrum of QCD, with quark content (cucu).

For two lattice volumes $24^3 \times 48$ and $20^3 \times 40$, each of 100 gauge configurations generated with single-plaquette action at $\beta = 6.1$, we compute point-to-point quark propagators for 30 quark masses in the range $0.03 \leq m_q a \leq 0.80$, and measure the time-correlation functions of the exotic meson operators which can overlap with $X(3872)$. The inverse lattice spacing $a^{-1}$ is determined with the experimental input of $f_\pi$, while the strange quark bare mass $m_s a = 0.08$, and the charm quark bare mass $m_c a = 0.80$ are fixed such that the masses of the corresponding vector mesons are in good agreement with $\phi(1020)$ and $J/\psi(3097)$ respectively [19].

Note that we are working in the quenched approximation which in principle is unphysical. However, our previous results on charmed baryon masses [19], and also charmed meson masses and decay constants (theoretical predictions) [20] turn out to be in good agreement with the experimental values. This

---

1For recent reviews of these new heavy mesons, see, for example, Refs. [1, 2, 3, 4], and references therein.
suggests that it is plausible to use the quenched lattice QCD with exact chiral symmetry to investigate the mass spectra of the exotic meson operators constructed in this paper, as first steps toward the unquenched calculations. The systematic error due to quenching can be determined only after we can repeat the same calculation with unquenched gauge configurations. However, the Monte Carlo simulation of unquenched gauge configurations for lattice QCD with exact chiral symmetry, on the lattices $20^3 \times 40$ and $24^3 \times 48$ at $\beta = 6.1$, still remains a challenge to the lattice community. Thus, in this paper, we proceed with the quenched approximation, assuming that the quenching error does not change our conclusions dramatically, in view of the good agreement between our previous quenched mass spectra of charmed hadrons [19, 20] and the experimental values.

2 The Molecular Operators

In this section, we construct four molecular operators with quark content $(c\bar{q}c\bar{q})$ such that the lowest-lying state of each operator has $J^{PC} = 1^{++}$. Then we compute the time correlation function of each operator, and extract the mass of its lowest-lying state. Explicitly, these molecular operators are:

\[
M_1 = \frac{1}{\sqrt{2}} \left\{ (\bar{q} \gamma_i c)(\bar{c} \gamma_5 q) - (\bar{c} \gamma_i q)(\bar{q} \gamma_5 c) \right\} \\
M_2 = (\bar{q} \gamma_5 \gamma_i q)(\bar{c} c) \\
M_3 = \frac{1}{\sqrt{2}} \left\{ (\bar{q} \gamma_5 \gamma_i c)(\bar{c} q) + (\bar{c} \gamma_5 \gamma_i q)(\bar{q} c) \right\} \\
M_4 = (\bar{c} \gamma_5 \gamma_i c)(\bar{q} q)
\]

The time-correlation function\(^2\)

\[
C_M(t) = \sum_{\vec{x}} \langle M(\vec{x}, t) M^\dagger(\vec{0}, 0) \rangle
\]

is measured for each gauge configuration, and its average over all gauge configurations is fitted to the usual formula

\[
\frac{Z}{2ma} \left[ e^{-ma t} + e^{-ma (T - t)} \right]
\]

to extract the mass $ma$ of the lowest-lying state and its spectral weight

\[
W = \frac{Z}{2ma}
\]

\(^2\)Here we have neglected the $c\bar{c}$ and $q\bar{q}$ annihilation diagrams such that $C(t)$ does not overlap with any conventional meson ($c\bar{c}$ or $q\bar{q}$) states. Also, $C(t)$ has been averaged over $C_i$ (with $\gamma_i$ for $i = 1, 2, 3$, where in each case, the “forward-propagator” $C_i(t)$ and “backward-propagator” $C_i(T - t)$ are averaged to increase the statistics. The same strategy is applied to all time-correlation functions in this paper.
Theoretically, if this state is a genuine resonance, then its mass $m_a$ and spectral weight $W$ should be almost constant for any lattices with the same lattice spacing. On the other hand, if it is a 2-particle scattering state, then its mass $m_a$ is sensitive to the lattice volume, and its spectral weight is inversely proportional to the spatial volume for lattices with the same lattice spacing. In the following, we shall use the ratio of the spectral weights on two spatial volumes $20^3$ and $24^3$ with the same lattice spacing ($\beta = 6.1$) to discriminate whether any hadronic state under investigation is a resonance or not.

In Fig. 1, the ratio $(R = W_{20}/W_{24})$ of spectral weights of the lowest-lying state extracted from the time-correlation function of $M_1$ on the $20^3 \times 40$ and $24^3 \times 48$ lattices is plotted versus the quark mass $m_qa \in [0.03, 0.80]$. (Here the quark fields $q$ and $\bar{q}$ are always taken to be different from $c$ and $\bar{c}$, even in the limit $m_q \to m_c$.) Evidently, $R \simeq 1.0$ for the entire range of quark masses, which implies that there exist $J^{PC} = 1^{++}$ resonances, with quark content $(cs \bar{c} \bar{s})$, and $(cu \bar{c} \bar{u})$ respectively.

In Fig. 2, the mass of the lowest-lying state extracted from the molecular
operator $M_1$ is plotted versus $m_q a$. In the limit $m_q \to m_u \simeq 0.00265 a^{-1}$ (corresponding to $m_\pi = 135$ MeV), it gives $m = 3895(27)$ MeV, which is in good agreement with the mass of $X(3872)$.

For $m_q = m_s = 0.08 a^{-1}$, the time-correlation function and effective mass of $M_1$ are plotted in Fig. 3a and Fig. 3b respectively. With single exponential fit, it gives $m[(\bar{s}\gamma_i c)(\bar{c}\gamma_5 s) - (\bar{c}\gamma_i s)(\bar{s}\gamma_5 c)] = 4109(21)$ MeV.

Next we turn to the molecular operators $M_2$, $M_3$, and $M_4$. In each of these cases, the ratio of spectral weights ($R = W_{20}/W_{24}$) behaves as $R \simeq 1.0$ for $m_q a > 0.05$, but deviates from 1.0 with large errors as $m_q \to m_u$. This seems to suggest that $M_2$, $M_3$ and $M_4$ have little overlap with the resonance detected by $M_1$ as $m_q \to m_u$. We suspect that this is due to quenching artifacts as $m_q \to m_u$, mostly coming from the scalar meson $(\bar{q}q)$, or $(\bar{c}q)$, or $(\bar{q}c)$, as well as the pseudovector $(\bar{q}\gamma_i \gamma_5 q)$. However, all molecular operators give compatible masses as $m_q \to m_u$. This is also the case for the resonance at $m_q = m_s$. 

Figure 2: The mass of the lowest-lying state of $M_1$ versus the quark mass $m_q a$, on the $24^3 \times 48$ lattice at $\beta = 6.1$. The solid line is the linear fit.
3 The Diquark-Antidiquark Operator

We construct the diquark-antidiquark operator with $J^{PC} = 1^{++}$ as

$$
X_4(x) = \frac{1}{\sqrt{2}} \left\{ (q^T C \gamma_i c)_{xa} (q C \gamma_5 \bar{c} T)_{xa} - (q C \gamma_i \bar{c} T)_{xa} (q^T C \gamma_5 c)_{xa} \right\} \quad (5)
$$

where $C$ is the charge conjugation operator satisfying $C \gamma_\mu C^{-1} = -\gamma_\mu^T$ and $(C \gamma_5)^T = -C \gamma_5$. Here the “diquark” operator $(q^T \Gamma Q)_{xa}$ for any Dirac matrix $\Gamma$ is defined as

$$
(q^T \Gamma Q)_{xa} \equiv \epsilon_{abc} q_{x\alpha a} \Gamma_{\alpha\beta} Q_{x\beta c} \quad (6)
$$

where $x$, $\{a, b, c\}$ and $\{\alpha, \beta\}$ denote the lattice site, color, and Dirac indices respectively, and $\epsilon_{abc}$ is the completely antisymmetric tensor. Thus the diquark (6) transforms like color anti-triplet. For $\Gamma = C \gamma_5$, it transforms like $J^P = 0^+$, while for $\Gamma = C \gamma_i$ ($i = 1, 2, 3$), it transforms like $1^+$. In Fig. 4, the ratio ($R = W_{20}/W_{24}$) of spectral weights of the lowest-lying state extracted from the time-correlation function of $X_4$ on the $20^3 \times 40$ and
Figure 4: The ratio of spectral weights of the lowest-lying state of diquark-antidiquark operator $X_4$, for $20^3 \times 40$ and $24^3 \times 48$ lattices at $\beta = 6.1$. The upper-horizontal line $R = (24/20)^3 = 1.728$, is the signature of 2-particle scattering state, while the lower-horizontal line $R = 1.0$ is the signature of a resonance.

$24^3 \times 48$ lattices is plotted versus the quark mass $m_q a \in [0.03, 0.8]$. Evidently, $R \simeq 1.0$ for the entire range of quark masses, which implies that there exist $J^{PC} = 1^{++}$ resonances, with quark content ($c\bar{s}\bar{s}c$), and ($c\bar{c}\bar{u}u$) respectively.

In Fig. 5, the mass of the lowest-lying state of the diquark-antidiquark operator $X_4$ is plotted versus $m_q a$. In the limit $m_q \to m_u$, it gives $m = 3891(17)$ MeV, which is in good agreement with the mass of $X(3872)$.

For $m_q = m_u = 0.08 a^{-1}$, the time-correlation function and effective mass of the diquark-antidiquark operator are plotted in Fig. 6. The mass of the lowest-lying state is $4134(19)$ MeV.

4 Summary and Discussions

In this paper, we have investigated the mass spectra of several interpolating operators (i.e., the molecular operators $M_1$, $M_2$, $M_3$, and $M_4$, and the diquark-antidiquark operator $X_4$) with the lowest-lying $J^{PC} = 1^{++}$, in quenched lattice
Figure 5: The mass of the lowest-lying state of the diquark-antidiquark operator $X_4$ versus the quark mass $m_q a$, on the $24^3 \times 48$ lattice at $\beta = 6.1$. The solid line is the linear fit.

QCD with exact chiral symmetry. Our results for $M_1$ and $X_4$ are summarized in Table 1, where in each case, the first error is statistical, and the second one is our estimate of combined systematic uncertainty including those coming from: (i) possible plateaus (fit ranges) with $\chi^2/d.o.f. < 1$; (ii) the uncertainties in the strange quark mass and the charm quark mass; (iii) chiral extrapolation (for the entries containing u/d quarks); and (iv) finite size effects (by comparing results of two lattice sizes). Note that we cannot estimate the discretization error since we have been working with one lattice spacing. Even though lattice QCD with exact chiral symmetry does not have $O(a)$ and $O(ma)$ lattice artifacts, the $O(m^2 a^2)$ effect might turn out to be not negligible for $m_c a = 0.8$.

Evidently, both the molecular operator $M_1$ and the diquark-antidiquark operator $X_4$ detect a $1^{++}$ resonance around $3890 \pm 30$ MeV in the limit $m_q \to m_u$, which is naturally identified with $X(3872)$. This suggests that $X(3872)$ is indeed in the spectrum of QCD, with quark content ($c\bar{u}c\bar{u}$), and $J^{PC} = 1^{++}$.

Now, in the quenched approximation, our results suggest that $X(3872)$ has good overlap with the molecular operator $M_1$ as well as the diquark-antidiquark operator $X_4$. This is in contrast to the case of $Y(4260)$ in our recent study [17],
in which $Y(4260)$ seems to have better overlap with the molecular operator
\[
\{(q\gamma_5\gamma_i c)(\bar{c}\gamma_5 q) - (c\gamma_5\gamma_i q)(\bar{q}\gamma_5 c)\}
\] than any diquark-antidiquark operators. This seems to suggest that $X(3872)$ is more tightly bound than $Y(4260)$. It would be interesting to see whether this picture persists even for unquenched QCD.

Finally, for $m_q = m_s$, heavier exotic meson resonance with $J^{PC} = 1^{++}$ is also detected, with quark content $(cs\bar{s}\bar{c})$ around $4100 \pm 50$ MeV, which serves as a prediction from lattice QCD with exact chiral symmetry.

**Acknowledgement**

This work was supported in part by the National Science Council, Republic of China, under the Grant No. NSC94-2112-M002-016 (T.W.C.), and Grant No. NSC94-2119-M239-001 (T.H.H.), and by the National Center for High Performance Computation at Hsinchu, and the Computer Center at National Taiwan University.
| Operator                                                                 | Mass (MeV)   | R/S |
|------------------------------------------------------------------------|--------------|-----|
| $\frac{1}{\sqrt{2}}[(\bar{u}\gamma_i c)(\bar{c}\gamma_5 u) - (\bar{c}\gamma_i u)(\bar{u}\gamma_5 c)]$ | 3895(27)(35) | R   |
| $\frac{1}{\sqrt{2}}[(\bar{s}\gamma_i c)(\bar{c}\gamma_5 s) - (\bar{c}\gamma_i s)(\bar{s}\gamma_5 c)]$ | 4109(21)(32) | R   |
| $\frac{1}{\sqrt{2}}\{(u^T C\gamma_i c)(\bar{u} C\gamma_5 e^T) - (u^T C\gamma_5 e)(\bar{u} C\gamma_i e^T)\}$ | 3891(17)(21) | R   |
| $\frac{1}{\sqrt{2}}\{(s^T C\gamma_i c)(\bar{s} C\gamma_5 e^T) - (s^T C\gamma_5 e)(\bar{s} C\gamma_i e^T)\}$ | 4134(19)(25) | R   |

Table 1: Mass spectra of the molecular operator $M_1$ and the diquark-antidiquark operator $X_4$ with $J^{PC} = 1^{++}$. The last column R/S denotes resonance (R) or scattering (S) state.

References

[1] E. S. Swanson, Phys. Rept. 429, 243 (2006)
[2] T. Barnes, AIP Conf. Proc. 814, 735 (2006)
[3] C. Quigg, PoS HEP2005, 400 (2006)
[4] J. L. Rosner, AIP Conf. Proc. 815, 218 (2006)
[5] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003)
[6] D. Acosta et al. [CDF II Collaboration], Phys. Rev. Lett. 93, 072001 (2004)
[7] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 93, 162002 (2004)
[8] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 71, 071103 (2005)
[9] K. Abe et al., hep-ex/0505037.
[10] C. Z. Yuan, X. H. Mo and P. Wang, Phys. Lett. B 579, 74 (2004)
[11] K. Abe et al., hep-ex/0505038.
[12] D. B. Kaplan, Phys. Lett. B 288, 342 (1992); Nucl. Phys. Proc. Suppl. 30, 597 (1993).
[13] R. Narayanan and H. Neuberger, Nucl. Phys. B 443, 305 (1995)
[14] H. Neuberger, Phys. Lett. B 417, 141 (1998)
[15] P. H. Ginsparg and K. G. Wilson, Phys. Rev. D 25, 2649 (1982)
[16] T. W. Chiu, Phys. Rev. Lett. 90, 071601 (2003); Phys. Lett. B 552, 97 (2003); hep-lat/0303008; Nucl. Phys. Proc. Suppl. 129, 135 (2004).
[17] T. W. Chiu and T. H. Hsieh [TWQCD Collaboration], Phys. Rev. D 73, 094510 (2006)

[18] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 95, 142001 (2005)

[19] T. W. Chiu and T. H. Hsieh, Nucl. Phys. A 755, 471 (2005)

[20] T. W. Chiu, T. H. Hsieh, J. Y. Lee, P. H. Liu and H. J. Chang, Phys. Lett. B 624, 31 (2005)