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Study of transmission dynamics of COVID-19 mathematical model under ABC fractional order derivative

Sabri T.M. Thabet a, Mohammed S. Abdo b, Kamal Shah c, Thabet Abdeljawad d,e,f,∗

a Department of Mathematics, University of Aden, Aden, Yemen
b Department of Mathematics, Hodeidah University, Al-Hodeidah, Yemen
c Department of Mathematics, University of Malakand Chakdara, Dir(L), Pakhtunkhwa, Pakistan
d Department of Mathematics and General Sciences, Prince Sultan University, Riyadh, Saudi Arabia
e Department of Medical Research, China Medical University, Taichung 40402, Taiwan
f Department of Computer Science and Information Engineering, Asia University, Taichung, Taiwan

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A B S T R A C T

The current research work is devoted to address some results related to the existence and stability as well as numerical finding of a novel Coronavirus disease (COVID-19) by using a mathematical model. By using fixed point results we establish existence results for the proposed model under Atangana–Baleanu–Caputo (ABC) derivative with fractional order. Further, using the famous numerical technique due to Adams Bashforth, we simulate the concerned results for two famous cities of China known as Wuhan and Huanggang which are interconnected cities. The graphical presentations are given to observe the transmission dynamics from February 1 a=2020 to April 20, 2020 through various fractional order. The concerned dynamics is global in nature due to the various values of fractional order.

1. Introduction

The dynamics mathematical models of the contagious diseases are now throughout. These models play a remarkable role in aid to realize strategies to control contagious diseases and alleviate their potential effects [1–4]. There are many wide studies of infectious diseases in the frame of mathematical models, we shall refer reader to [5] and references therein. The Coronavirus are a broad group of viruses that contain viruses which may occasion a group of diseases in humans, getting between from colds to the sharp respiratory syndrome. As well, viruses from this collection occasion many of animal diseases.

Latterly, the whole world suffers a hardship of a novel Corona virus (COVID-19). It is an infectious disease put forward by acute respiratory syndrome Coronavirus (CoV-2), that was claimed to stream have first prevalence at Wuhan city, China on November 28, 2019. It has since spread globally, this led to the outbreak of a pandemic in 2020. The COVID-19 pandemic is considered the world’s largest universal impedance such as economics and health system of each country in the world which has been pushed into a very critical placement. Furthermore, caused the millions of confirmed infections, hundreds of thousands of deaths accompany him worldwide. According to the WorldMeter website, the last statistics as of May 26, 2020, confirmed cases of COVID-19 have exceeded 6 millions worldwide, while the number of deaths has reached greater than 3 hundreds and the total of people who have recovered has increased to greater than 3 millions.

The application of a suitable isolationism against disease spread is another defiance. The mathematical modeling technique is one of the key tools for dealing with those challenges. Many of the diseases infectious models were developed in recent literature that allows us to better tracer the prevalence and control of such diseases. Most of those models are established on classical differential equations see [6–13].

In the 18th century many important results in classical calculus by some famous scientists, such as Liouville, Riemann, Euler, Fourier, and others, were registered in literature. Simultaneously and in that period, significant contributions were done in the field of fractional calculus (FC). In fact, several applications of FC in the area of mathematical modeling, for example where sundry inherited materials and memory measures cannot be clarified plainly by classical calculus, were investigated. Because FC, which incorporates classical calculus, has a more degree of freedom in their fractional differential operators as compared to classical differential operators which are local in nature. The important and remarkable applications of the mentioned FC can be found in [14–18]. Recently, new different types of nonlocal fractional
derivatives have been suggested in the existing literature to treat the reduction of derivative operators with power-law. For instance, a new fractional derivative has been introduced by Caputo–Fabrizio [19] based on the exponential kernel. However, this operator has some turmoil concerning the locality of the kernel. To eliminate Caputo–Fabrizio’s turmoil, Atangana and Baleanu in [20] have proposed a novel modified version of a fractional derivative with Mitrig-Leffler function as a non-local and non-singular kernel. The Atangana–Baleanu–Caputo (ABC) fractional derivative provides an accurate description of the memory. The important applications of the ABC operator can be found in [21–27].

Besides, many researchers have started to focus strikingly on studying the FC. Indeed, a fractional derivative is a specific integral which geometrically interprets the piling up of the whole function or the entire range which globalizes it. In an examination of differential equations for numerical and analytical studies, noteworthy contributions have been made by analysts and researchers, we allude to a few [28–34].

Recently, it has been observed that fractional differential equations can be employed to modeling phenomena of worldwide more accurately. The global problem of the spread of the disease attracted the attention of researchers from various fields, which led to the emergence of a number of proposals to analyze and anticipate the development of the epidemic, for more details see [35–42]. Some recent papers dealt with different models of the COVID-19 based on ABC operator, see [43–47].

Very recently, authors in the work [48] discussed the dynamic behavior of COVID-19, that was by analyzed a fractional-order model of COVID-19 with inter-city networked coupling effects based on the real-data from January 23 to March 18, 2020.

In this paper, our major contribution relates to the consideration of the most popular category which is now appears in medical journals [49,50]. This new work includes theoretical, practical analyzes and numerical simulations on studying the dynamics behavior of the speed of spreading Coronavirus (COVID-19) infection and how to reduce the spread of infection in society. Motivated by the work [48] and with inspiration from [41], we will study the following SEIHDR model in sense of ABC fractional derivative:

\[
\begin{align*}
&ABC^D S_k(t) = -\sum_{i=1}^n \beta_{k,i} \left( \frac{S_k(t)I_i(t)}{N_k} + \frac{S_k(t)E_i(t)}{N_k} \right), \\
&ABC^D E_k(t) = \sum_{i=1}^n \beta_{k,i} \left( \frac{S_k(t)I_i(t)}{N_k} + \frac{S_k(t)E_i(t)}{N_k} \right) - \mu_{k} E_k(t) - r_k E_k(t), \\
&ABC^D I_k(t) = r_k E_k(t) - \delta_k I_k(t) - \mu_k I_k(t), \\
&ABC^D H_k(t) = \delta_k I_k(t) - \lambda_k H_k(t) - K_k H_k(t), \\
&ABC^D R_k(t) = \lambda_k H_k(t), \\
&ABC^D D_k(t) = \mu_k E_k(t) + \mu_{2k} I_k(t) + K_k H_k(t),
\end{align*}
\]

(1)

with the initial conditions
\[
\begin{align*}
S_1(0) &= S_0(0) \geq 0, \\
E_1(0) &= E_0(0) \geq 0, \\
I_1(0) &= I_0(0) \geq 0, \\
H_1(0) &= H_0(0) \geq 0, \\
R_1(0) &= R_0(0) \geq 0, \\
D_1(0) &= D_0(0) \geq 0,
\end{align*}
\]

(2)

where \(0 \leq t < T < \infty\) and \(ABC^D\) denotes the ABC fractional derivative of order \(0 < q \leq 1\). The parameters details of given model are describing as follows:

- \(N_k\) is denote the total population of city \(k\) which classified into six sets are \(S_k\), \(E_k\), \(I_k\), \(H_k\), \(R_k\), and \(D_k\) for \(k = 1, \ldots, n\), representing the number of susceptible (uninfected), exposed (without clinical symptoms), infectious (with obvious clinical symptoms and not hospitalized), hospitalization, recovered and deaths individuals at time \(t\) in city \(k\), respectively.
- The interaction of susceptible people \(S_k\) with \(E\) and \(I\) implies to transition of the infectiousness to them which given by \(\sum_{i=1}^n \beta_{k,i} \left( \frac{S_k(t)I_i(t)}{N_k} + \frac{S_k(t)E_i(t)}{N_k} \right)\), where \(\beta_{k,i}, (i = 1, \ldots, n)\) is the disease transmission coefficient.
- The death people \(D_k\) are the death during hospitalization \(K_k(t)H_k\), infective \(\mu_{2k} I_k\) and exposure \(\mu_k E_k\), where \(\mu_k(j = 1, 2)\) and \(K_k(t)\) represent the disease-related death rate.
- The parameters: \(r_k\) is the transit rate of the exposed individuals \(E_k\), \(\delta_k\) be hospitalization rate of the infectious individuals, \(\lambda_k(t)\) denote the recovery rate into hospitalization. Furthermore, \(\lambda_k, K_k, \beta_k, \mu_k, \delta_k\) and \(r_k(j = 1, 2)\) are positive constants.

Our manuscript is arranged as follows. Section 1 dedicated to the introduction. Some fundamental concepts results are given in Section 2. In Section 3, we give the theoretical results. Numerical results are provided in 4. In the last, a brief conclusion is given in Section 5.

2. Preliminaries

In this section, we will present some important preliminaries related to the fractional calculus and nonlinear analysis [20,51–57].

**Definition 2.1.** The left-sided ABC fractional derivative with the lower limit zero of order \(a \in (0, 1]\) for a function \(\phi \in H^l(0, T)\) is defined by

\[
ABC^D \phi(t) = \frac{N'(a)}{1-a} \int_0^t \frac{\phi'(\theta)}{\Gamma(a)} d\theta, \quad t > 0,
\]

where \(N'(a)\) is the normalization function which is defined as \(N'(0) = N'(1) = 1\), and \(E_a\) is called the Mittag-Leffler function defined by the series

\[
E_a(z) = \sum_{k=0}^\infty \frac{z^k}{\Gamma(ak+1)},
\]

and \(Re(a) > 0\) and \(\Gamma(.)\) is a gamma function.

**Definition 2.2.** The left-sided ABC fractional integral with the lower limit zero of order \(a \in (0, 1]\) for a function \(\phi \in L^1(0, T)\) is defined by

\[
ABC^I \phi(t) = \frac{1}{N(a)} \phi(t) + \frac{1}{N(a)} \int_0^t (t-\theta)^{-a-1} \phi(\theta) d\theta, \quad t > 0.
\]

**Lemma 2.2.1 (See Proposition 3 in [55]).** The solution of the following problem for \(a \in (0, 1]\)

\[
ABC^D \phi(t) = \psi(t), \quad \phi(0) = \phi_0,
\]

is given by

\[
\phi(t) = \phi_0 + \frac{1}{N(a)} \psi(t) + \frac{1}{N(a)} \int_0^t (t-\theta)^{-a-1} \psi(\theta) d\theta.
\]

**Theorem 2.3.** Assume \(\mathcal{Y}\) be a Banach space and \(D \subset \mathcal{Y}\) be a convex, closed and bounded set. If \(\Psi: D \longrightarrow D\) is a continuous operator such that \(\Psi D \subset \mathcal{Y}\) and \(\Psi D\) is relatively compact, then \(\Psi\) has at least one fixed point in \(D\).

3. Theoretical approach

At the beginning, in order to study the existence, uniqueness and stability of solutions for the model (1)–(2), we need to reformulation the such model to the following appropriate form:

\[
\begin{align*}
ABC^D S_k(t) &= G_k(t, S_k(t), E_k(t), I_k(t), H_k(t), R_k(t)), \\
ABC^D E_k(t) &= G_k(t, S_k(t), E_k(t), I_k(t), H_k(t), R_k(t)), \\
ABC^D I_k(t) &= G_k(t, S_k(t), E_k(t), I_k(t), H_k(t), R_k(t)), \\
ABC^D H_k(t) &= G_k(t, S_k(t), E_k(t), I_k(t), H_k(t), R_k(t)), \\
ABC^D R_k(t) &= G_k(t, S_k(t), E_k(t), I_k(t), H_k(t), R_k(t)), \\
ABC^D D_k(t) &= G_0(t, S_k(t), E_k(t), I_k(t), H_k(t), R_k(t)),
\end{align*}
\]
For the qualitative analysis, let us present the Banach space \( \mathcal{N} \). We can write the model (1)–(2) as follows:

\[
\begin{align*}
\mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) &= -\sum_{i=1}^{n} \beta_k \left( S_i(t) I_i(t) + S_i(t) E_i(t) \right), \\
\mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) &= -\mu_k (E_k(t) - r_k E_k(t)), \\
\mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) &= r_k E_k(t) - \delta_k I_k(t) - \mu_k I_k(t), \\
\mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) &= \delta_k I_k(t) - \lambda_k I_k(t) H_k(t) - K_k I_k(t), \\
\mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) &= \mu_k I_k(t) H_k(t), \\
\mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) &= \mu_k (E_k(t) + r_k I_k(t)) + K_k I_k(t) H_k(t).
\end{align*}
\]

We can write the model (1)–(2) as follows:

\[
\begin{align*}
\mathcal{X}_0 &= S_0(t, \mathcal{X}(t)) = \langle \mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) \rangle, \\
\mathcal{X}_0 &= S_0(t, \mathcal{X}(t)) = \langle \mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) \rangle, \\
\mathcal{X}_0 &= S_0(t, \mathcal{X}(t)) = \langle \mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) \rangle, \\
\mathcal{X}_0 &= S_0(t, \mathcal{X}(t)) = \langle \mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) \rangle, \\
\mathcal{X}_0 &= S_0(t, \mathcal{X}(t)) = \langle \mathcal{G}_0(t, S_k, E_k, I_k, H_k, R_k, D_k) \rangle.
\end{align*}
\]

Here, the symbol \( (\cdot) \) denotes the transpose vector. In Lemma 2.2.1, the system (6) is equivalent to the following fractional integral equation:

\[
\mathcal{X}(t) = \mathcal{X}_0 + \frac{1}{N(q)} \mathcal{G}(t, \mathcal{X}(t)) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau.
\]

For the qualitative analysis, let us present the Banach space \( \mathcal{II} \) endowed with the norm \( ||\mathcal{X}|| := \sup_{t \in [0,T]} ||\mathcal{X}(t)|| \).

We define the bounded closed convex ball \( B_p = \{ \mathcal{X} \in \mathcal{I} : ||\mathcal{X}|| \leq \rho, \rho > 0 \} \)

\[
\mathcal{X}_0 \leq \mathcal{X}_0 + \frac{1}{N(q)} \mathcal{G}(t, \mathcal{X}(t)) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau.
\]

We need to show that \( (YB_p) \subset B_p \). Then for all \( t \in [0,T] \), we have

\[
||\mathcal{X}(t)|| \leq \mathcal{X}_0 + \frac{1}{N(q)} \mathcal{G}(t, \mathcal{X}(t)) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau.
\]

For \( \mathcal{X} \in B_p \), we get

\[
||\mathcal{X}(t)|| \leq \mathcal{X}_0 + \frac{1}{N(q)} (1 + ||\mathcal{X}||) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau
\]

\[
\leq \mathcal{X}_0 + \frac{1}{N(q)} (1 + ||\mathcal{X}||) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau.
\]

This is \( (YB_p) \subset B_p \).

Next, we will prove that the operator \( Y \) is continuous. For that, we assume \( \{ \mathcal{X}_n \} \) is a sequence such that \( \mathcal{X}_n \to \mathcal{X} \) in \( B_p \) as \( n \to \infty \). Then for each \( t \in [0,T] \), we get

\[
\mathcal{X}_n(t) - \mathcal{X}(t) \to 0 \text{ at } n \to \infty.
\]

i.e. \( Y \) is continuous on \( B_p \).

Finally, we are going to show that \( (YB_p) \) is relatively compact operator. Since \( (YB_p) \subset B_p \) hence \( (YB_p) \) is uniformly bounded.

In order to show that \( Y \) is equicontinuous on \( B_p \), let \( \mathcal{X} \in B_p \) and \( t_1, t_2 \in [0,T] \) with \( t_1 < t_2 \), then we have

\[
||\mathcal{X}(t_2) - \mathcal{X}(t_1)|| \leq \frac{1}{N(q)} \left| \mathcal{G}(t_2, \mathcal{X}(t_2)) - \mathcal{G}(t_1, \mathcal{X}(t_1)) \right| + \frac{q}{N(q)} \frac{1}{T(q)} \left( \int_0^{t_2} (t_2 - \tau)^{q-1} - \int_0^{t_1} (t_1 - \tau)^{q-1} \right) ||\mathcal{G}(\tau, \mathcal{X}(\tau))||
\]

\[
\leq \frac{1}{N(q)} \left| \mathcal{G}(t_2, \mathcal{X}(t_2)) - \mathcal{G}(t_1, \mathcal{X}(t_1)) \right| + \frac{q}{N(q)} \frac{1}{T(q)} \left( \int_0^{t_2} (t_2 - \tau)^{q-1} - \int_0^{t_1} (t_1 - \tau)^{q-1} \right) ||\mathcal{G}(\tau, \mathcal{X}(\tau))||
\]

Obviously that the right-hand side of the above inequality tends to zero as \( t_2 \to t_1 \). In view of well known Arzelà–Ascoli theorem, we have \( (YB_p) \) is relatively compact, and hence \( Y \) is completely continuous. As a consequence of Theorem 2.3, we conclude that the model (1)–(2) has at least one solution. The proof is finished. \( \square \)

In next theorem, we will introduce the stability result of solution of the model (1)–(2) according to caused a small change in initial conditions.

\[
\begin{align*}
\mathcal{X}(t) &= \mathcal{X}_0 + \frac{1}{N(q)} \mathcal{G}(t, \mathcal{X}(t)) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau, \\
\mathcal{X}(t) &= \mathcal{X}_0 + \frac{1}{N(q)} \mathcal{G}(t, \mathcal{X}(t)) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau.
\end{align*}
\]

\[
\begin{align*}
\mathcal{X}_0 &= \mathcal{X}_0 + \frac{1}{N(q)} \mathcal{G}(t, \mathcal{X}(t)) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau, \\
\mathcal{X}_0 &= \mathcal{X}_0 + \frac{1}{N(q)} \mathcal{G}(t, \mathcal{X}(t)) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau.
\end{align*}
\]

For \( \mathcal{X} \in B_p \), we get

\[
||\mathcal{X}(t)|| \leq \mathcal{X}_0 + \frac{1}{N(q)} (1 + ||\mathcal{X}||) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau.
\]

\[
\leq \mathcal{X}_0 + \frac{1}{N(q)} (1 + ||\mathcal{X}||) + \frac{q}{N(q)} \frac{1}{T(q)} \int_0^t (t - \tau)^{q-1} \mathcal{G}(\tau, \mathcal{X}(\tau)) d\tau.
\]
Proof. The solutions of the models (6) and (12) are equivalent to integral Eqs. (8) and
\[ \dot{X}(t) = X(t) + \frac{1 - q}{N(q)} f(t, X(t)) + \frac{q}{N(q)} \int_0^t (t - \tau)^{q-1} f(t, X(\tau)) d\tau. \] (14)
respectively. Then, for each \( t \in [0, T] \), we get
\[ \|X(t) - \hat{X}(t)\| \leq |e| + \frac{1 - q}{N(q)} \|f(t, X(t)) - f(t, \hat{X}(t))\| + \frac{q}{N(q)} \int_0^t (t - \tau)^{q-1} \|f(\tau, X(\tau)) - f(\tau, \hat{X}(\tau))\| d\tau. \]
Thus, we have
\[ \|X(t) - \hat{X}(t)\| \leq |e| + \frac{1 - q}{N(q)} \int_0^t (t - \tau)^{q-1} f(\tau, X(\tau)) d\tau. \]
Hence,
\[ \|X(t) - \hat{X}(t)\| \leq \left( 1 - \frac{(1 - q)T^q}{N(q)T^q} \right) |e|, \]
this proves the theorem. \( \Box \)

Remark 1. If \( e = 0 \), in the above theorem, we conclude that the solution of the model (1)–(2) is unique.

4. Numerical approach

Here, we supply the numerical solutions of the considered model (1)–(2). Then the numerical results are gained through the proposed scheme. To this purpose, we utilize the fractional Adams Bashforth method [24] to approximate the AB fractional integral.

By the initial conditions and the operator \( {^{AB}T^q_{0+}} \), we turn fractional model (1) into the fractional integral equations
\[
\begin{align*}
S_1(t) - S_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_1(t, S_0(\tau)) d\tau, \\
E_1(t) - E_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_2(t, E_0(\tau)) d\tau, \\
I_1(t) - I_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_3(t, I_0(\tau)) d\tau, \\
H_1(t) - H_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_4(t, H_0(\tau)) d\tau, \\
R_1(t) - R_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_5(t, R_0(\tau)) d\tau, \\
D_1(t) - D_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_6(t, D_0(\tau)) d\tau,
\end{align*}
\] (15)
which gives
\[
\begin{align*}
S_1(t) - S_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_1(t, S_0(\tau)) d\tau, \\
E_1(t) - E_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_2(t, E_0(\tau)) d\tau, \\
I_1(t) - I_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_3(t, I_0(\tau)) d\tau, \\
H_1(t) - H_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_4(t, H_0(\tau)) d\tau, \\
R_1(t) - R_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_5(t, R_0(\tau)) d\tau, \\
D_1(t) - D_0(t) &= \frac{1}{\Gamma(q)} \int_0^t (t - \tau)^{q-1} H_6(t, D_0(\tau)) d\tau.
\end{align*}
\] (16)

To procure an iterative scheme, setting \( t = t_{i+1} \), for \( s = 0, 1, 2, \ldots \), in the above system leads to the model down
\[
\begin{align*}
S_{i+1} - S_i &= \frac{1}{\Gamma(q)} \int_t^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_1(t, S_i(\tau)) d\tau, \\
E_{i+1} - E_i &= \frac{1}{\Gamma(q)} \int_t^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_2(t, E_i(\tau)) d\tau, \\
I_{i+1} - I_i &= \frac{1}{\Gamma(q)} \int_t^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_3(t, I_i(\tau)) d\tau, \\
H_{i+1} - H_i &= \frac{1}{\Gamma(q)} \int_t^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_4(t, H_i(\tau)) d\tau, \\
R_{i+1} - R_i &= \frac{1}{\Gamma(q)} \int_t^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_5(t, R_i(\tau)) d\tau, \\
D_{i+1} - D_i &= \frac{1}{\Gamma(q)} \int_t^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_6(t, D_i(\tau)) d\tau.
\end{align*}
\] (17)

Utilize the two points interpolation polynomial for approximate the functions
\[
\begin{align*}
H_1(t, S_i) &= H_1(t, S_i) \equiv \frac{h}{\Gamma(q)} \int_0^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_1(t, S_i(\tau)) d\tau, \\
H_2(t, E_i) &= H_2(t, E_i) \equiv \frac{h}{\Gamma(q)} \int_0^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_2(t, E_i(\tau)) d\tau, \\
H_3(t, I_i) &= H_3(t, I_i) \equiv \frac{h}{\Gamma(q)} \int_0^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_3(t, I_i(\tau)) d\tau, \\
H_4(t, H_i) &= H_4(t, H_i) \equiv \frac{h}{\Gamma(q)} \int_0^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_4(t, H_i(\tau)) d\tau, \\
H_5(t, R_i) &= H_5(t, R_i) \equiv \frac{h}{\Gamma(q)} \int_0^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_5(t, R_i(\tau)) d\tau, \\
H_6(t, D_i) &= H_6(t, D_i) \equiv \frac{h}{\Gamma(q)} \int_0^{t_{i+1}} (t_{i+1} - \tau)^{q-1} H_6(t, D_i(\tau)) d\tau.
\end{align*}
\] (18)

where
\[
\begin{align*}
I_{i+1} &= \int_t^{t_{i+1}} (t_{i+1} - \tau)^{q-1} d\tau, \\
I_{i+1} &= \int_t^{t_{i+1}} (t_{i+1} - \tau)^{q-1} d\tau.
\end{align*}
\] (19)

By simple calculations of the integrals \( I_{i-1} \) and \( I_q \), we get
\[
\begin{align*}
I_{i-1} &= \frac{1}{q+1} \left( t_{i-1} - t \right)^{q+1} - \frac{1}{q+1} \left( t_{i-1} - t \right)^{q+1}, \\
I_q &= \frac{1}{q+1} \left[ (t_{i+1} - t) \left( t_{i+1} - t \right)^q \right] - \frac{1}{q+1} \left[ (t_{i-1} - t) \left( t_{i-1} - t \right)^q \right].
\end{align*}
\] (20)
and
\[ I_{i,t} = \frac{1}{q} \left( (t_{i+1} - t_i)(t_{i+1} - t_{i+1}^*) \right) \]
\[ = -\frac{1}{q} \left[ \left( t_{i+1} - t_i + t_{i+1}^* - t_{i+1}^* \right) \right]. \]

Take \( t_i = \Delta t \), we can easily find that
\[ I_{i-1,q} = \frac{t_q}{q} \left[ (s + 1 - i)^9 (s - i + 2 + q) - (s - i)^9 (s - i + 2 + 2q) \right]. \]

Replacing (19) and (20) into (18), we attain
\[ S(t_{i+1}) = S(t_0) + \frac{1}{Y(q)} H(t_i, S(t_i)) + \frac{q}{Y(q)} \sum_{i=0}^{s} \frac{t}{q} \]
\[ = \left( \frac{H(t_i, S(t_i))}{I(q + 2)} \right) \left( (s + 1 - i)^9 (s - i + 2 + q) \right) \]
\[ - (s - i)^9 (s - i + 2 + 2q) \]
\[ - \frac{H(t_i, S(t_i))}{I(q + 2)} \left( (s + 1 - i)^9 (s - i + 1 + q) \right) \]
\[ E(t_{i+1}) = E(t_0) + \frac{1}{Y(q)} H(t_i, E(t_i)) + \frac{q}{Y(q)} \sum_{i=0}^{s} \frac{t}{q} \]
\[ = \left( \frac{H(t_i, E(t_i))}{I(q + 2)} \right) \left( (s + 1 - i)^9 (s - i + 2 + q) \right) \]
\[ - (s - i)^9 (s - i + 2 + 2q) \]
\[ - \frac{H(t_i, E(t_i))}{I(q + 2)} \left( (s + 1 - i)^9 (s - i + 1 + q) \right) \]
\[ I(t_{i+1}) = I(t_0) + \frac{1}{Y(q)} H(t_i, I(t_i)) + \frac{q}{Y(q)} \sum_{i=0}^{s} \frac{t}{q} \]
\[ = \left( \frac{H(t_i, I(t_i))}{I(q + 2)} \right) \left( (s + 1 - i)^9 (s - i + 2 + q) \right) \]
\[ - (s - i)^9 (s - i + 2 + 2q) \]
\[ - \frac{H(t_i, I(t_i))}{I(q + 2)} \left( (s + 1 - i)^9 (s - i + 1 + q) \right) \]
\[ \frac{1}{Y(q)} \left( \frac{h}{q} \right) \left( (s + 1 - i)^9 (s - i + 2 + q) \right) \]
\[ - (s - i)^9 (s - i + 2 + 2q) \]
\[ - \frac{1}{Y(q)} \left( \frac{h}{q} \right) \left( (s + 1 - i)^9 (s - i + 1 + q) \right) \]
\[ R(t_{i+1}) = R(t_0) + \frac{1}{Y(q)} H(t_i, R(t_i)) + \frac{q}{Y(q)} \sum_{i=0}^{s} \frac{t}{q} \]
\[ = \left( \frac{H(t_i, R(t_i))}{I(q + 2)} \right) \left( (s + 1 - i)^9 (s - i + 2 + q) \right) \]
\[ - (s - i)^9 (s - i + 2 + 2q) \]
\[ - \frac{1}{Y(q)} \left( \frac{h}{q} \right) \left( (s + 1 - i)^9 (s - i + 1 + q) \right) \]

4.1. Numerical simulations and discussion

Now to give the numerical simulation of the proposed model (1)–(2) that contains the ABC fractional operator, we will use the iterative solution given in (21)–(26). Here the time as days. The values of the parameters utilized in the simulation are designated in Table 1 for two cities \( k = 1 \) : Huanggang and \( k = 2 \) : Wuhan, which are interconnected cities. The graphical representations of the numerical solution of species \( S, E, I, H, R, D_k, (k = 1, 2) \) by a various fractional values \( q = 0.75, 0.85, 0.95, 1.0 \) from the proposed model (1)–(2) are shown in Figs. 1–12, respectively, and we consider the initial values as a proportion of the total population in millions from February 1 to April 20, 2020 is as follows:

For Huanggang city, \( S_0 = 7.4000959 \) million, \( E_0 = 0, I_0 = 0.002141, H_0 = 0.002141 \) million, \( R_0 = 0, D_1 = 0, D_2 = 0, \) and for Wuhan city, \( S_0 = 11 \) million, \( E_0 = 0, I_0 = 0.084, H_0 = 0.084 \) million, \( R_0 = 0, D_1 = 0, D_2 = 0. \)

In Figs. 1–6 and 7–12, Here we assumed that each infected is hospitalized. A global dynamics of each compartment in the proposed model (1) have presented by applying the numerical values in Table 1 contra different fractional order.

In Figs. 1–6, we have presented the transmission dynamics of Coronavirus-19 for Huanggang for the eighty days during the out break through the considered model. We have used fractional order derivatives of ABC-type. Here we state that in Figs. 6–12, we have shown the aforementioned dynamics for Wuhan city for the eighty days. Initially the decline in susceptible population is very fast and then as the time passed the concerned decline became slow until stable. Similarly the exposed population initially increase up with fast speed corresponding to small fractional order then became slow. The infection and hospitalization were decreasing with different rate and nearly in eighty days that up to April 20, 2020 the situation in China has reached to satisfactory position. During this time the recovery and death population also achieve their maximum peak as shown in Figs. Here we remark that the concerned derivative generates global dynamics of the suggested model, where the smaller order goes to stable rapidly as compared to integer order.

5. Conclusion

In this study by interconnected model for the current COVID-19 we have produced some results for the two cities of China one is Wuhan and other Huanggang. By using ABC derivative of fractional order and Adam Bashforth method, we have simulated the results. The concerned graphs have been presented for different fractional order. A global transmission dynamics has been received for the eighty days that from February 1 to April 20, 2020. Also, some results for the existence and stability of solution to the proposed model have been verified. On
Table 1
The physical interpretation of the parameters and numerical values.

| Parameters | Physical description | Numer. val. of Huanggang [58] | Numer. val. of Wuhan [58] |
|------------|----------------------|------------------------------|---------------------------|
| $N_k$      | The total population of city $k$ | 7.4031 millions | 11.084 millions |
| $\beta_k$  | The disease transmission coefficient | 0.3265 | 0.4012 |
| $K_k$      | The death rate during hospitalization $H_k$ | 0.0000019 | 0.00003 |
| $\mu_k$    | The death rate during exposure $E_k$ | 0.000005 | 0.00005 |
| $\lambda_k$| The death rate during infectious $I_k$ | 0.0012 | 0.009871 |
| $r_k$      | The transit rate of the exposed individuals $E_k$ | 0.1818 | 0.1818 |
| $\delta_k$ | Hospitalization rate of the infectious individuals $I_k$ | 0.0752 | 0.1370 |

Fig. 1. Dynamics of susceptible class for Huanggang during last eighty days at various fractional order.

Fig. 2. Dynamics of exposed class for Huanggang during last eighty days at various fractional order.

Fig. 3. Dynamics of infected class for Huanggang during last eighty days at various fractional order.
the light of Theorem 3.1 in [56], we would like to mention that the solutions obtained for the system (1-2) can verify the initial data when the right hand side of the system vanishes at 0. That is the derived solutions satisfy the system for $t > 0$ and verify the initial data (when $t = 0$) only at specific populations agree with the vanishing condition at 0.

CRediT authorship contribution statement

Sabri T.M. Thabet: Prepared the draft. Mohammed S. Abdo: Performed the analysis. Kamal Shah: Developed the concept, Methodology. Thabet Abdeljawad: The revision and viewing the paper finally.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Fig. 7. Dynamics of susceptible class for Wuhan during last eighty days at various fractional order.

Fig. 8. Dynamics of exposed class for Wuhan during last eighty days at various fractional order.

Fig. 9. Dynamics of hospitalized class for Wuhan during last eighty days at various fractional order.

Fig. 10. Dynamics of infected class for Wuhan during last eighty days at various fractional order.
Fig. 11. Dynamics of recovered class for Wuhan during last eighty days at various fractional order.

Fig. 12. Dynamics of death class for Wuhan during last eighty days at various fractional order.

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