A Robust Iterative Learning Control for Continuous-Time Nonlinear Systems with Disturbances

MICHELE PIERALLINI¹,², (Student, IEEE), FRANCO ANGELINI¹,², (Member, IEEE), RICCARDO MENGACCI¹,², (Student, IEEE), ALESSANDRO PALLESCHI¹,², (Student, IEEE), ANTONIO BICCHI¹,²,³, (Fellow, IEEE), and MANOLO GARABINI¹,², (Member, IEEE)

¹Centro di Ricerca “Enrico Piaggio”, Università di Pisa, Largo Lucio Lazzarino 1, 56122 Pisa, Italy
²Dipartimento di Ingegneria dell’Informazione, Università di Pisa, Largo Lucio Lazzarino 1, 56122 Pisa, Italy
³Soft Robotics for Human Cooperation and Rehabilitation, Fondazione Istituto Italiano di Tecnologia, via Morego, 30, 16163 Genova, Italy

Corresponding author: Michele Pierallini (e-mail: michele.pierallini@gmail.com).

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ABSTRACT In this paper, we study the trajectory tracking problem using iterative learning control for continuous-time nonlinear systems with a generic fixed relative degree in the presence of disturbances. This class of controllers iteratively refine the control input relying on the tracking error of the previous trials and some properly tuned learning gains. Sufficient conditions on these gains guarantee the monotonic convergence of the iterative process. However, the choice of the gains is heuristically hand-tuned given an approximated system model and no information on the disturbances. Thus, in the cases of inaccurate knowledge of the model or iteration-varying measurement errors, external disturbances, and delays, the convergence condition is unlikely to be verified at every iteration. To overcome this issue, we propose a robust convergence condition, which ensures the applicability of the pure feedforward control even if other classical conditions are not fulfilled for some trials due to the presence of disturbances. Furthermore, we quantify the upper bound of the nonrepetitive disturbance that the iterative algorithm is able to handle. Finally, we validate the convergence condition simulating the dynamics of a two degrees of freedom underactuated arm with elastic joints, where one is active, and the other is passive, and a Franka Emika Panda manipulator.

INDEX TERMS Iterative Learning Control, Nonlinear control systems, Robustness, Robots

I. INTRODUCTION

STARTING from the 80s, a new control framework, namely iterative learning control (ILC), was introduced [1], [2]. The basic idea is to polish, iteratively, the current control input until the system is able to effectively perform the desired task. The iterative algorithm does not require any accurate description of the model, leading to good tracking performance without any substantial modification of the system dynamics, while incorporating persistent disturbances, e.g., gravity acceleration. Not surprisingly, ILC proved to be an excellent tool for repetitive tasks. Indeed, its field of applications are multiple, e.g., robotic manipulation [3], the wafer stage [4], manufacturing process [5], quadrotors [6], and soft robotics [7]–[10].

The iterative algorithm can be robust to disturbances [11] and can follow a switching policy between learning gains [12]. Additionally, the control law can involve a rectifying action for the initial state [13], can be combined with feedback control, e.g., proportional [14] or model predictive control [15], and can learn the desired trajectory even in the case of variations in the learning process [16].

The main problem when dealing with iterative processes is guaranteeing convergence. For continuous-time linear systems and discrete-time systems, it is possible to draw sufficient and necessary convergence conditions [17]–[19], while it is still an open problem for continuous-time nonlinear...
systems [19].

Disturbances such as model uncertainties [11], [20], error in the measurements, dynamic/external interactions [7], and actuation delays or faults [21] may cause a failure of the convergence condition.

Feedback controllers can mitigate the undesired effect of disturbances through the application of suitable high gains [22]. This leads to a profound alteration of the system dynamics, which is not acceptable in some applications like, for example, soft robotics [7]. In this case, the use of feedback control actions is strongly limit [23], and pure feedforward control action, e.g., ILC, is preferable. However, feedforward methods lack robustness in the case of disturbances. Thus, what happens in the case of disturbances? Which kind of disturbances can an iterative learning controller manage? What can we guarantee in terms of convergence?

The robustness of a pure feedforward iterative control law problem has already been widely investigated in the case of discrete-time systems [18], [20], [24], [25]. However, it is still under-studied for continuous-time systems. In [26], the sampled-data ILC algorithm for continuous-time systems can manage the time nonrepetitive disturbances, while in [27], the Authors tackle the same problem in the case of systems with a fixed relative degree equal to one, constant linear input and output fields, and saturated inputs.

In this paper, we design an iterative pure feedforward controller for multiple-input multiple-output (MIMO) continuous-time nonlinear systems with a generic fixed relative degree. We prove its convergence in the case of a great variety of disturbances. We distinguish disturbances on their dependency on the state or on time. Additionally, we classify them as repetitive or nonrepetitive depending on their occurrences w.r.t. the iteration domain. In [11], [18], [24], [25], the Authors already guarantee a bounded error in the presence of time-dependent nonrepetitive disturbance. We propose and prove a convergence condition (D-condition), which guarantees a robust convergence also in the presence of state-dependent nonrepetitive disturbances. Theoretically, we propose a necessary and sufficient converge condition for a restricted class of nonlinear systems. Then, we quantify the iteration-frequency and module of the nonrepetitive disturbances that the iterative algorithm can handle. Additionally, we prove that the D-condition does not modify the already known convergence results in [11], [18], [24], [25] dealing with time-dependent nonrepetitive disturbances.

Finally, we validate the D-condition on two simulated robotic systems varying disturbances types and output functions. The first robot is an underactuated compliant arm with two degrees of freedom (DoFs), in which the first elastic joint is active, while the other is passive. The second system is a Franka Emika Panda manipulator.

**Notation**

Let \( I_m \in \mathbb{R}^{m \times m} \) be the identity matrix and \( 0_{n \times m} \in \mathbb{R}^{n \times m} \) be a zeros matrix. Let \( f(\cdot), g(\cdot) : x \in \mathbb{R}^n \to \mathbb{R}^n \) be two vector fields, \( L_t g(x) \) stands for the Lie derivative of \( g(x) \) along \( f(x) \), i.e.,

\[
L_t g(x) = \frac{dg(x)}{dx} f(x).
\]

For any vector \( v \in \mathbb{R}^n \), for any matrix \( A \in \mathbb{R}^{n \times m} \), we denote with \(|v|\) and \(||A||\) their infinity norm. Let \( \lambda \) be a positive constant, for any vector \( v \in \mathbb{R}^n \), we denote with \(|v|_{\lambda} = \sup \{ |v| e^{-\lambda t} \} \). Let \( y : t \in \mathbb{R} \to \mathbb{R}^n \) be a vector function, we denote with \( y'(t) \) its \( i\)-th time derivative. Let \( U \) be a set, we use the notation \#U to indicate its cardinality. Finally, all physical units may be assumed to be expressed in SI (MKS), and angles in radian.

**II. PROBLEM DEFINITION**

Let us consider an iterative process, where \( j \in U \) is the iteration index, and \( U \) is the iteration set. The class of continuous-time nonlinear systems under analysis can be written as

\[
\begin{align}
\dot{x}_j(t) &= f_j(x_j(t)) + g_j(x_j(t)) u_j(t) + d^{px}_j(x_j(t)) + d^{pl}_j(t) + d^{px}_j(x_j(t)) + d^{pl}_j(t), \\
y_j(t) &= h_j(x_j(t)) + d^{t^y}_j(t),
\end{align}
\]

with \( x_j(0) \) as initial condition, \( x_j(t) \in \mathbb{R}^n \) is the state vector, \( t \in [0, t_i] \) is the time variable, \( t_i \) is the termination time, \( u_j(t) \in \mathbb{R}^m \) is the control action, \( y_j(t) \in \mathbb{R}^n \) is the output, \( h_j : \mathbb{R}^n \times [0, t_i] \to \mathbb{R}^n \) is the output map, \( f_j : \mathbb{R}^n \times [0, t_i] \to \mathbb{R}^n \) and \( g_j : \mathbb{R}^n \times [0, t_i] \to \mathbb{R}^n \) are the drift and control vector field, respectively. Additionally, the system is affected by disturbances \( d^{px}_j(x_j(t)), d^{pl}_j(t), d^{px}_j(x_j(t)), d^{pl}_j(t) \in \mathbb{R}^n \), and \( d^{t^y}_j(t) \in \mathbb{R}^n \) which we classify in relation to their dependency on iteration, state and time domain. In particular, considering the iteration domain \( j \in U \), we distinguish between repetitive and nonrepetitive disturbances. Furthermore, we divide them into state disturbances and time disturbances, respectively. It is instrumental for the development of the method to introduce the following definitions.

**Definition 1.** A disturbance \( d^{px}(\cdot) : U \times [0, t_i] \times \mathbb{R}^n \to \mathbb{R}^n \), Lipschitz, and bounded is said to be state-repetitive (or state-persistent).

**Definition 2.** A disturbance \( d^{pl}(\cdot) : U \times [0, t_i] \to \mathbb{R}^n \), Lipschitz, and bounded is said to be time-repetitive (or time-persistent).

**Definition 3.** A disturbance \( d^{t^y}(\cdot) : T \subset U \times [0, t_i] \times \mathbb{R}^n \to \mathbb{R}^n \), Lipschitz and bounded, i.e., \( \max_j \| d^{t^y}(x_j(t)) \| = \bar{d}^{t^y} \), is said to be state-nonrepetitive.

**Definition 4.** A disturbance \( d^{t^y}(\cdot) : U \times [0, t_i] \to \mathbb{R}^n \), bounded, i.e., \( \max_j \| d^{t^y}(t) \| = \bar{d}^{t^y} \) and such that \( d^{t^y}(\cdot) \neq d^{t^y}(\cdot) \) and \( \forall i \neq j \), is said to be time-nonrepetitive.

It is worth highlighting that, nonrepetitive disturbances, i.e., Def. 4, have already been widely studied in the literature, e.g., [11], [20], [26], [28]. However, the other types of disturbances have not been properly analyzed yet. In the following remark, we present a few practical examples of these definitions.

**Remark 1.** State-repetitive disturbances (Def. 1) can represent an external force field, e.g., an unmodeled gravity vector...
in the dynamics of a robot. Time-repeatitive disturbances (Def. 3) can model additive uncertainties in the system nominal model. It is worth remarking that repetitive disturbances are present at each iteration of the whole iterative process.

State-nongenrepetitive disturbances (Def. 3) can derive from the interaction between a robot and the environment or actuators failure/delays. Time-nongenrepetitive disturbances (Def. 3) are disturbances with no relation with the state, e.g., measurements noise. It is worth remarking that nonrepetitive disturbances occur only for a few iterations (or change at each iteration) during the whole learning process.

Assumptions
We assume for the system (1)-(2) what follows:
A1) The system (1)-(2) is square, i.e., \( n_g = m \).
A2) The system (1)-(2) has a fixed relative degree (vector) \( r \), such as \( r = \{ r_1, \ldots, r_m \} \) (see, e.g., [29]):
- \( L_{r_1} L_{r_2} \cdots L_{r_m} b_i(\cdot) = 0 \), \( i, z \in [1, m], s \in \{ 0, r_i - 1 \} \).
- \( \operatorname{rank}(D(\cdot)) = m, \forall \epsilon \in \mathbb{R}^m \), such as \( D(\epsilon) \in \mathbb{R}^{m \times m} \) and \( D_{i,j}(\cdot) = L_{r_1} L_{r_2} \cdots L_{r_{i-1}} b_j(\cdot) \) with \( i, j \in [1, m] \).
A3) The initial condition \( x_j(0) \in \mathbb{R}^n \) is such that \( x_j(0) = x_d(0) \), \( \forall j \in U \).
A4) \( f_\epsilon(\cdot), g(\cdot), h(\cdot), L_{f_\epsilon} h(\cdot), s = 1, \ldots, r \), and \( D(\cdot) \) are globally Lipschitz with constants \( \bar{f}, \bar{g}, \bar{h}, \bar{F}, \bar{f}, \bar{g}, \bar{h} \), and \( \bar{f}, \bar{g}, \bar{h} \in \mathbb{R}^+ \), respectively, i.e., \( ||f_\epsilon(x) - f_\epsilon(x')|| \leq \bar{f} ||x - x'||, \forall x, x' \in \mathbb{R}^n \).
A5) The desired output trajectory \( y_d : [0, t_f] \rightarrow \mathbb{R}^n \) is feasible, continuous and differentiable for \( r \) times, \( \forall t \in [0, t_f] \).

It is worth noting that, thanks to assumptions [A3] and [A5], there exist bounded \( u_d, x_d \), and \( y_d \), which are the desired control input, state and output, respectively, such that \( x_d(t) = f_{\epsilon_d}(x_d(t)) + g(x_d(t))u_d(t) \) and \( y_d(t) = h(x_d(t)) \).

Goals
Considering the disturbed system (1)-(2), given the desired trajectory \( y_d(\cdot) : [0, t_f] \rightarrow \mathbb{R}^n \), and assumptions [A1]-[A5]. The main purpose of this paper is to investigate the robustness of the iterative feedforward control law \( u_j(\cdot) \) in the presence of disturbances \( \delta^\epsilon(x_j(\cdot)) \), \( \delta^\rho(x_j(\cdot)) \), \( \delta^\nu(x_j(\cdot)) \), \( \delta^\lambda(x_j(\cdot)) \), and \( \delta^\nu(\cdot) \). In particular, we summarize the goals of this work as follows.
G1) Design an iterative feedforward control law \( u_j(\cdot) : [0, t_f] \rightarrow \mathbb{R}^m \) able to follow \( y_d(\cdot) \) \( \forall t \in [0, t_f] \), i.e., \( \lim_{j \rightarrow +\infty} \| y_d(t) - y_j(\cdot) \|_\lambda = 0 \).
G2) Propose a robust convergence condition, namely D-condition, which guarantees (G1) even in the presence of state-nongenrepetitive disturbances \( \delta^\epsilon(x_j(\cdot)) \).
G3) Find an upper-bound of the state-nongenrepetitive disturbances \( \delta^\epsilon(x_j(\cdot)) \), which can be dealt with by the convergence condition proposed in (G2).
G4) Prove that the D-condition in (G2) handles the presence of time-repeatitive \( \delta^\epsilon(t) \) and time-nongenrepetitive \( \delta^\nu(t) \) disturbances, guaranteeing \( \lim_{j \rightarrow +\infty} \| u_d(t) - u_j(t) \|_\lambda \leq b_u \) with \( b_u > 0 \).

III. SOLUTION
This section is divided into four parts. Firstly, we present the employed control law. Secondly, we report well-known results for this iterative control. Third, we propose the main result of this paper, i.e., a robust convergence condition for the control law (3). This convergence condition is able to cope with state repetitive and nonrepetitive disturbances, i.e., Def. [1] and [3]. Finally, the fourth section extends the main result considering also the presence of time repetitive and nonrepetitive disturbances, i.e., Def. [2] and [4].

A. ITERATIVE CONTROL LAW
In this paper we employ an ILC control law, which is purely feedforward. This has already been widely used in literature, for example in [9] and [30], achieving (G1). Recalling the system (1)-(2) and the assumptions [A1]-[A5], we employed control law is

\[
\begin{align*}
u_{j+1}(t) &= u_j(t) + \Gamma_j(\cdot) e_j(t), \\
\end{align*}
\]

where \( \Gamma_j(\cdot) \in \mathbb{R}^{m \times m} \) is the time and iteration varying learning gain and the error signal \( e_j(\cdot) \in \mathbb{R}^m \) is defined as

\[
\begin{align*}e_j(t) &\equiv \sum_{i=0}^r \Gamma_i \left( y_d^{(i)}(t) - y_j^{(i)}(t) \right) \\
&= \sum_{i=0}^r \Gamma_i \left( L_{f_\epsilon} h(x_d) - L_{f_\epsilon} h(x_j) \right) \\
&+ \Phi(x_d, x_j) \\
&+ \Theta_i \left( D(x_d) u_d - D(x_j) u_j \right),
\end{align*}
\]

where \( \Theta_i \in \mathbb{R}^{m \times m}, \Theta_i \succ 0, \forall i = 0, \ldots, r \) are tunable control gain matrices, which affect the convergence velocity [9]. The initial guess \( u_0(\cdot) \in \mathbb{R}^m \) of the iterative control law (3) can be arbitrarily chosen.

Assuming that the measurements of \( y_j^{(i)}(t) \) for \( i = 0, \ldots, r \) can be easily obtained through sensors, for each iteration \( j \) and time instant \( t \in [0, t_f] \), the control law (3) requires \( (r+1)(m^2 + m) \) operations. In the case that the derivative measurements are not available the method complexity increases depending on the adopted algorithm.

It is instrumental for the design of the method to introduce the following definition.

Definition 5. If for any initial guess \( u_0(\cdot) : [0, t_f] \rightarrow \mathbb{R}^m \), the iterative control law (3) converges to \( u_d(\cdot) : [0, t_f] \rightarrow \mathbb{R}^m \) in such a way \( \| u_d(t) - u_j(t) \|_\lambda = 0 \) when \( j \rightarrow +\infty \), then (3) is said to be convergent.

Lemma 1. If the control law is convergent (Def. 5), then the error (4) is such that \( \| e_j(\cdot) \|_\lambda \rightarrow 0 \) when \( j \rightarrow +\infty \).

Proof. Recalling assumptions [A3] and [A5], i.e., no shift in the initial condition and the feasibility of the desired trajectory, the proof is trivial. □
B. STATE OF THE ART

A sufficient convergence condition \[\text{[50]}\] for the controller \[\text{[3]}\], which we call not-disturbed (ND) convergence condition, is

\[
||I_m - \Gamma_\tau(j)D(x_j)|| < 1, \forall j \in U, \forall x \in \mathbb{R}^a, \forall t \in [0,t] . \tag{5}
\]

If \[\text{(5)}\] is verified, then the iterative process is guaranteed to converge. This occurs also in the presence of state-repetitive disturbances (Def.\[\text{[1]}\]), see, e.g., [7], [31]. Indeed, considering the system \[\text{(1)}\], the state-persistent disturbances \[d^p(x_j(t))\] can be included in the vector \[f_o(x_j(t))\], which is still Lipschitz. For this reason, in the following, without loss of generality, we can directly consider the disturbed drift vector

\[
f(x_j(t)) \doteq d^p(x_j(t)) + f_o(x_j(t)) . \tag{6}
\]

It is worth noting that, the Lipschitz constant of \[f(x_j(t))\] is still \[\bar{f} \in \mathbb{R}\], i.e., assumption \[\text{[A4]}\].

Additionally, \[\text{(5)}\] can also deal with both time-nonrepétitive and time-repetitive disturbances (Def.\[\text{[2]}\] and \[\text{[3]}\]). However, in this case the iterative process will not have a perfect convergence as in Def.\[\text{[5]}\] but it will be bounded, i.e., \[||u_0(t) - u_j(t)|| < b_u\] with \[b_u > 0\] finite, see, e.g., [11].

On the other hand, \[\text{(5)}\] does not guarantee the so-called control contraction when state-nonrepétitive disturbances (Def.\[\text{[3]}\]) occur. Therefore, the main contribution of this work is to propose a robust convergence condition (D), which extends \[\text{(5)}\]. This is presented in the following section.

C. MAIN RESULT: STATE-NONREPÉTITIVE DISTURBANCES

For the sake of clarity, let us define what follows.

Definition 6. Let \[U \equiv \mathbb{N}\] be the iteration set. \[U\] is such that \[U = T \cup V\], where \[V\] contains all that iteration \[j\] such that \[\text{(5)}\] is not fulfilled, while \[T = U - V\].

The following Theorem represents the main result of this paper. It enables the controller \[\text{[3]}\] to cope with state-nonrepétitive disturbance such as in Def.\[\text{[3]}\] achieving \[\text{[32]}\].

Theorem 1. Let us consider the system in the form \[\text{(1)}\] with \[d^p(t) \equiv 0_{nx1}, d^f_j(t) \equiv 0_{nx1}, d^f_0(t) \equiv 0_{mx1}\] and let \[y_0(t) \in \mathbb{R}^m\] be the desired output trajectory. Let \[N \in \mathbb{N}\] be a finite constant. Under assumptions \[\text{[A4]} \text{[A5]}\] if the learning gain \[\Gamma_j(t) \in \mathbb{R}^{mxm}\] satisfies

\[
\prod_{i=j}^{j+N-1} ||I_m - \Gamma_i \tau(j)Y,D(x_i)|| \leq \prod_{i=j}^{j+N-1} \rho_i < 1 , \tag{7}
\]

\[\forall j \in \mathbb{N}, s \in \mathbb{N}, \forall t \in [0,t_j]\] then, the control law \[\text{[3]}\] is convergent (Def.\[\text{[5]}\]), i.e., \[\|e_j(t)\|_\lambda \rightarrow 0\] when \[j \rightarrow +\infty\].

Proof. For the sake of clarity, we omit the time dependency.

Given the control laws \[\text{[3]}\] and \[\text{[4]}\], we have

\[
u_d - u_{j+1} = (I_m - \Gamma_j \tau(j)D(x_j)) (u_d - u_j) - \Gamma_j \Phi(x_j,x_d) + \Gamma_j Y_D(D(x_j) - D(x_d)) u_d . \tag{8}
\]

Defining \[\delta u_j \doteq u_d - u_j\] and \[\delta x_j \doteq x_d - x_j\], we can write

\[
||\delta u_{j+1}|| \leq ||I_m - \Gamma_j \tau(j)Y,D(x_j)|| ||\delta u_j|| + ||\Gamma_j|| ||\Phi(x_j,x_d)|| + ||\Gamma_j|| ||Y_D|| ||D(x_j) - D(x_d)|| ||u_d|| . \tag{9}
\]

Given \[\text{[4]}\] and \[\text{[A4]}\] we compute

\[
||\Phi(x_j,x_d)|| \leq \sum_{i=0}^{j} ||\gamma_i \Phi|| ||\delta x_j|| \leq (r+1) ||\Phi|| ||\delta x_j|| , \tag{10}
\]

with \[\Phi \in \mathbb{R}\] such that \[\Phi > \max_{r=0,...,\tau} \{||\gamma_i \Phi||,||\Phi||\}\].

Recalling \[\text{[A4]}\] let \[\tau_j\] be such that \[||I_m - \Gamma_j Y_D(x_j)|| \leq \tau_j\], defining \[\mu \doteq \sup_{\tau_j} \{||\gamma_i \Phi||,||\Phi||,||\gamma_i \Phi|| + (r+1) \Phi||,||\delta x_j||\}, one has

\[
||\delta u_{j+1}|| \leq \tau_j ||\delta u_j|| + \mu ||\delta x_j|| . \tag{11}
\]

Using again assumption \[\text{[A4]}\] we can write the following inequality for the system \[\text{(1)}\]

\[
||\delta x_j|| \leq \int_0^t \left(\bar{f} + g \|u_d(\tau)\|\right)||\delta x_j(\tau)|| + \left(g(x_j(\tau))\right)||\delta u_j(\tau)|| d\tau . \tag{12}
\]

Applying the Gronwall’s Lemma to \[\text{(12)}\], leads to

\[
||\delta x_j|| \leq \int_0^t c_1 ||\delta u_j(\tau)|| e^{c_2(t-\tau)} d\tau , \tag{13}
\]

where \[c_1 \doteq \sup_{\tau_j} \{||g(x_j(\tau))||\}\] and \[c_2 \doteq \sup_{\tau_j} \{\bar{f} + g \|u_d\|\}\].

Substituting \[\text{(13)}\] in \[\text{(11)}\], leads to

\[
||\delta u_{j+1}|| \leq \tau_j ||\delta u_{j+1}|| + \mu c_1 \int_0^t ||\delta u_j(\tau)|| e^{c_2(t-\tau)} d\tau . \tag{14}
\]

Computing the \(\lambda\)-norm of \[\text{(14)}\], we obtain

\[
||\delta u_{j+1}||_\lambda \leq \tau_j ||\delta u_j||_\lambda + \mu c_1 \int_0^t e^{c_2(t-\tau)} ||\delta u_j||_\lambda d\tau . \tag{15}
\]

Grouping for \[||\delta u_{j+1}||_\lambda\] and solving the integral, leads to

\[
||\delta u_{j+1}||_\lambda \leq \left(\tau_j + \mu c_1 \int_0^t \frac{e^{c_2(t-\tau)}}{\lambda} d\tau\right) ||\delta u_j||_\lambda , \tag{16}
\]

which can be rewritten as

\[
||\delta u_{j+1}||_\lambda \leq (\tau_j + \nu_j(\lambda)) ||\delta u_j||_\lambda \leq \rho_j ||\delta u_j||_\lambda . \tag{17}
\]

Considering \[\tau_j < 1\], then \(\forall c_2 \geq 0, \exists \lambda \geq 0\) such that \[\tau_j + \nu_j(\lambda) < 1, \forall j \in U\]. It is worth mentioning that this proves the ND-condition \[\text{(5)}\].

On the other hand, the presence of state-nonrepétitive disturbances \[d^p(x_j)\] (Def.\[\text{[4]}\]) affects the constant \[c_2\] in \[\text{(13)}\], leading to \[c_2^j \doteq c_2 + d^p_j\]. This may lead to a failure in the convergence condition \[\text{(5)}\]. Indeed, \(\forall c_2, \lambda\) (already selected), \(\exists d^p_j : \tau_j + \nu_j(\lambda, d^p_j) > 1, \forall j \in V\) in \[\text{(17)}\].
Without loss of generality, we can group the windows of $N$ trials, which contains iterations belonging to both $V$ and $T$. This leads to

$$
\|\delta u_{j+N}\|_\lambda \leq \sum_{i=j}^{j+N-1} \rho_i \|\delta u_j\|_\lambda \triangleq P_j \|\delta u_j\|_\lambda ,
$$

(18)

which is a control contraction for hypothesis, i.e., $P_j < 1$.

We substitute all the iterations of the iterative process, and we compute the limit for $j \to +\infty$

$$
\lim_{j \to +\infty} \|\delta u_{j+1}\|_\lambda \leq \lim_{j \to +\infty} \prod_{j=0}^{\infty} P_j \|\delta u_0\|_\lambda = 0 .
$$

(19)

The right-hand side of (19) is an infinite product of factors $P_j$ such that $0 \leq P_j < 1$. This implies that $\prod_{j=0}^{\infty} P_j = 0$. Recalling Lemma 1, we state that $\|e_j(t)\|_\lambda \to 0, j \to +\infty$. Thus, the proof is completed.

Note that, if we choose $N = 1$, convergence condition (7) (D) shrinks into (5) (ND). Conversely, choosing $1 < N < +\infty$, leads to a convergence condition, which is more robust than (5). Indeed, (7) guarantees the convergence even if (5) is not fulfilled for some iterations.

A necessary and sufficient convergence condition for the controller (3) and nonlinear system (1)- (2) is still an open problem. However, restricting the class of nonlinear systems under study, it is possible to obtain the necessary and sufficient convergence condition for the controller (3), as proven in the following Theorem.

**Theorem 2.** Under the same assumption of Theorem 1 let $D(x)$ be the decoupling matrix, such that $D(x) = D$, with constant matrix such that $\eta = \|D\| \in \mathbb{R}$. Then, (7) is the necessary and sufficient convergence condition for the control law (3).

**Proof.** Sufficiency. We refer to Theorem 1.

Necessity. By contradiction, let us assume that $\|e_j(t)\|_\lambda \to b \geq 0$, when $j \to +\infty$.

Recalling (4), and (A4) leads to

$$
\|e_{j+N}\| \leq (r + 1)\Phi \|\delta x_{j+N}\| + \eta \|\delta u_{j+N}\|.
$$

(20)

Defining $\Phi \triangleq (r + 1)\Phi$, and substituting (12) into (20), one has

$$
\|e_{j+N}\|_\lambda \leq (\Phi + v_{j+N} + \eta) \|\delta u_{j+N}\|_\lambda \leq (\Phi + v_{j+N} + \eta) P_j \|\delta u_j\|_\lambda .
$$

(21)

Since $\|e_j\|_\lambda \to b \in \mathbb{R}^+, j \to +\infty$, then $P_j \geq 1$ for some $j$, which is absurd ($P_j < 1 \forall j \in U$). Thus $\|e_j(t)\|_\lambda \to 0$, and the proof is completed.

Since the windows $N$ is not known a priori, (7) results not trivial for a practical interpretation. To have a trivial comparison with a classic convergence condition (ND), i.e., (5), we state what follows.

**Corollary 1.** Under the same assumptions of Theorem 1 let $U = V \cup T$ be the iteration set such that $\#T = \infty$ while $\#V < \infty$. A sufficient condition for the convergence of (3) is

$$
\|I_m - \Gamma_j(t)Y_D(x_j)\| \leq \rho_j < 1 \quad \forall j \in T, \forall t \in [0, t_f].
$$

(22)

**Proof.** We here report only a sketch of it. Recalling (17), we substitute all the previous trials, we split the products, and we compute the limit

$$
\lim_{j \to +\infty} \|\delta u_{j+1}\|_\lambda \leq \lim_{j \to +\infty} \prod_{j \in V} (\chi_j + \nu_j) \prod_{j \in T} (\chi_j + \nu_j) \|\delta u_0\|_\lambda ,
$$

(23)

in which $\prod_{j \in T} (\chi_j + \nu_j) = 0$ and $\prod_{j \in V} (\chi_j + \nu_j) = \nu_\star \in \mathbb{R}^+ \setminus \{+\infty\}$. The proof is completed.

We tackle the goal (23) with the following Proposition.

**Proposition 1.** Under the same assumptions of Theorem 1 and given a window $N$ of iterations, let $N_V$ and $N_T$ be two sets such that $N = \#N_V + \#N_T$. The two sets $N_V$ and $N_T$ include the iteration indexes $j$ where a state-nonrepetitive disturbance occurs or not, respectively.

Let be $A \triangleq 1/\prod_{j \in N_T} \rho_j, 1 \leq A < +\infty$ and let the learning gain $\Gamma_j(t)$ equal to

$$
\Gamma_j(t) = \varepsilon \Gamma_j(1)^{-1}D^{-1}(x_j), \forall t \in [0, t_f], \varepsilon \in (0, 1], \forall j \in U .
$$

(24)

For any iteration window $N$, the D-condition (7) holds if the nonrepetitive disturbances are such that

$$
d_{xj}^\star = \max_{j \in N_V} \{d_{xj}^\star\} \leq \lambda - c_2
$$

$$
- W \left( \frac{t_c c_1 \mu}{\mu V / A} \exp \left( \frac{\varepsilon}{c_1 \mu} + 1 \right) \frac{t_c c_1 \mu}{\mu V / A} \right) \frac{1}{t_c} - \frac{\varepsilon}{c_1 \mu} + 1 ,
$$

(25)

where $W$ is the Lambert function [32], $\lambda \in \mathbb{R}^+ \setminus \{+\infty\}$, and $\mu, c_1, c_2$ are respectively defined in (11) and (13).

**Proof.** Since we assumed that (7) holds true, recalling (16) and Def. 6 we can write

$$
\prod_{j \in N_T} \rho_j \prod_{j \in N_T} \rho_j = A^{-1} \prod_{j \in N_T} \rho_j < 1 ,
$$

(26)

where $A^{-1} < 1$ and $\prod_{j \in N_T} \rho_j \in [1, +\infty)$. Substituting (24) into (26), and computing $d_{x}^\star = \max_{j \in N_V} \{d_{xj}^\star\}$, yield to

$$
\prod_{j \in N_V} \rho_j = \left( \frac{\mu c_1 \left( 1 - e^{c_2 \lambda - c_2 - d_{x}^\star} \right)}{\lambda - c_2 - d_{x}^\star} \right) ^{\#N_V} < A .
$$

(27)

Note that, we are looking for $d_{x}^\star \in \mathbb{R} \setminus \{\infty\}$, which satisfies (27). Then, after mathematical manipulation, and defining $\xi \triangleq \lambda - c_2 - d_{x}^\star$, one has $e^{-\xi} c_1 \mu > - \mu V / \xi - \varepsilon + c_1 \mu$, whose solution is (25).

In practice, (25) is difficult to apply, but it guarantees an upper bound w.r.t. the iteration frequency for any state-nonrepetitive disturbances.

\[\text{VOLUME 4, 2016}\]
Remark 2. The control law (3) depends on the control gains \( \Gamma_i \in \mathbb{R}^{m \times m} \) for \( i = 0, \ldots, r \). These directly multiply the derivative of the output. Large values could speed up the convergence of the method. However, the magnitude of the gains should be proportional to the reliability of the measurements, i.e., inaccurate measurements should be multiplied by low gains. Moreover, in practical applications, the control action could exceed the actuators physical limits and, eventually, damage the system.

D. OTHER RESULTS: ALL DISTURBANCES

In this section, we analyze the presence of also the time-nonrepetitive and repetitive disturbances (Def. [4] and [5]), achieving (G4). As discussed in Sec. [III-B], these disturbances do not affect (2), although, they lead to a bounded error (28) and (18). The following Theorem extends Theorem [1] w.r.t. all disturbances under analysis, relaxing also [A3].

Theorem 3. Let us consider the system in the form (1)–(2), and let \( y_d(t) \in \mathbb{R}^m \) be desired output trajectory.

Under assumptions (A1)–(A5), let us consider the initial condition such that \( x_j(0) = x_{0j}(0) + l_j, \forall j \in U \), with \( \sup_j ||x_j(0)|| \leq b_0 < +\infty \). Let us assume that the time-nonrepetitive disturbances \( d^\text{tv}(t) \in \mathbb{R}^m \) (Def. [4]) is time differentiable for \( r \) times with bounded derivatives, namely \( \bar{d}_0^\text{tv}, \ldots, \bar{d}_r^\text{tv} \in \mathbb{R}^{\{+\infty\}} \).

If \( \Gamma_j(t) \in \mathbb{R}^{m \times m} \) satisfies (4), then the controller (3) is such that \( ||u_j(t) - u_j(t)||_\lambda \leq b_0, \) when \( j \to +\infty \) with \( b_0 \in \mathbb{R}^{+\infty} \).

Proof. The presence of time-repetitive and nonrepetitive disturbances modify (11) such as

\[
||\bar{\delta} u_{j+1}|| \leq \bar{\delta} x_j ||\bar{\delta} u_j|| + \mu ||\bar{\delta} x_j|| + ||\Gamma_j|| ||\bar{d}^\text{tv}||, \tag{28}
\]

with \( \bar{d}^\text{tv} = (r+1) \max_j (||Y_0|| \, \bar{d}_0^\text{tv}, \ldots, ||Y_r|| \, \bar{d}_r^\text{tv}) \).

Now, let us recall (15), which becomes

\[
||\bar{\delta} x_j|| \leq b_0 e^{\kappa t} + \int_0^t \sum_j \left( c_1 ||\bar{\delta} u_j(\tau)|| + \bar{d}_x^\text{tv} e^{\kappa(t-\tau)} d\tau, \tag{29}
\]

where \( \bar{d}_x^\text{tv} = \max_j (d^\text{tv}_x(j)) \). Then, with analogous calculation from (14)–(18), we derive

\[
||\bar{\delta} u_{j+N}||_\lambda \leq \sum_{j=1}^{j+N-1} P_j ||\bar{\delta} u_j|| \sum_{k=1}^{N-1-j} \rho_d \bar{d} + \bar{d}_{j+N}, \tag{30}
\]

with \( \bar{d} \triangleq \sup_j (\bar{d}_j) \) is \( \sup_j \{|\Gamma_j||\bar{d}^\text{tv}|| + \mu b_1 + \mu \bar{d}_x^\text{tv}(\lambda)\} < +\infty \), and \( \bar{d}_{j+N} \triangleq \sup_j (\bar{d}_{j+N}) = \sup_j \{|\Gamma_j ||\bar{d}^\text{tv}|| + \mu b_1 + \mu \bar{d}_x^\text{tv}(\lambda)\} < +\infty \). Computing the limit for \( j \to +\infty \), using (7), and rearranging (30) by splitting into \( N \) iteration products, lead to

\[
\lim_{j \to +\infty} ||\bar{\delta} u_{j+N}||_\lambda \leq \sum_{j=1}^{+\infty} P_j ||\bar{\delta} u_0||_\lambda + \sum_{j=1}^{+\infty} P_j \bar{d} + \bar{d}_N, \tag{31}
\]

where \( \bar{d}_N \) is bounded because it is a finite sum of \( N \) bounded variables.

Recalling (7), and defining \( \bar{P} \triangleq \sup_j \max_i P_j \), one has

\[
\lim_{j \to +\infty} ||\bar{\delta} u_{j+N}||_\lambda \leq \frac{1}{1-P_d \bar{P} + \bar{d}_N} \triangleq b_u < +\infty. \tag{32}
\]

The proof is completed.

IV. VALIDATION

We validate the effectiveness of the D-condition through simulations, using MATLAB. Firstly, we simulate a 2 DoFs underactuated compliant robot, namely \( \mathcal{R} \), composed of two elastic joints, where only the first one is actuated. Secondly, we test the method on a Franka Emika Panda robot equipped with elastic joints.

The dynamic model is used for simulating the system and for tuning the gain \( \Gamma_j(t) \) of the controller (3). The gains \( \Gamma \) are chosen depending on the system, while \( \varepsilon = 0.9 \). The initial guess \( u_0 \) is chosen equal to the constant torque able to maintain the robot in the starting position of the trajectory \( y_d(0), i.e., \text{solving } f_u(x_0(t)) + g(x_0(t)) u_0(t) = 0 \).

To quantify the tracking performance, we use as a metric the root mean square (RMS) of the norm of each component of the output error, showing that the D-condition (7) extends the ND-condition (5). The learning is executed until the RMS error reaches a value of 0.001rad.

A. TWO DOFS UNDERACTUATED ROBOT: \( \mathcal{R} \)

We simulate the dynamics of a two DoFs underactuated arm with elastic joints. We refer to [2] for a more exhaustive treatment of the system dynamics. Let \( m = 0.55 \) kg, \( J = 0.001 \) kmgm², \( l = 0.085 \) m, \( a = 0.089 \) m, and \( d_v = 0.3 \) Nms/rd be the mass, inertia, length, center of mass distance, and damping of each link, respectively. The stiffness of each link is tested in two configurations: Soft, i.e., \( k = 1 \) Nm/rd, and Stiff, i.e., \( k = 3 \) Nm/rd. For the sake of clarity, let us recall that the state \( x \in \mathbb{R}^4 \) of the robot is \( x = [\, x_1, x_2, x_3, x_4, \,] ^T \), where \( x_1 \) and \( x_2 \) are the joint positions, while \( x_3 \) and \( x_4 \) are the joint angular velocities.

To test the robustness of the method, we design the learning gain \( \Gamma_j \) using a model whose parameters are different from the nominal one. In particular, the second link parameters \( \tilde{m}_2, \tilde{J}_2, \tilde{I}_2, \) and \( \tilde{a}_2 \) are decreased by 50%, this is a state-nonrepetitive disturbance \( d^\text{nx} \) in (1). Additionally, we test the control algorithm simulating measurement noise \( d^\text{tv} \), external disturbances, and delays in the controller \( u_j(t) \), which can be both modeled as state-nonrepetitive disturbances \( d^\text{nx} (x_j(t)) \) in (1).

The chosen output function \( h(x) \in \mathbb{R} \) is the absolute angle of the robot tip i.e., \( y = x_1 + x_2 \), which leads to a relative degree \( r = 2 \) iff (9)

\[
D(x) = \lambda_0 L_{f_h} h(x) = \frac{-b_2 \cos(x_2)}{\det M(x)} \neq 0, \tag{33}
\]

where \( b_1 = m_2 a_2^2 + m_1 I_2^2 + J_1, \) \( b_2 = m_2 a_2^2 + J_2, \) \( b_3 = a_1 I_2 m_2 \) and \( \det M(x) = b_1 b_2 + b_3 \cos x_2 \neq 0, \forall x \in \mathbb{R}^n \).
Finally, in the D - Model scenario, we have \(d^p(t) \equiv d^p_j(x_j(t)) \equiv 0_{2n \times 1} \) in (38).

Fig. 2 reports the simulation results. Fig. 2(a) depicts the tracking performance at the last iteration, while Fig. 2(b) and Fig. 2(c) show the error evolution over iterations.

It is worth mentioning that, taking \(N = 5\), \(\lambda = 1.8\), \(A = 1.52\), \(\mu = 3e - 4\), \(c_2 = \pi/4\) and \(c_1 = 26\), (25) holds. In particular, we have that in the D - Force scenario \(d^p_* = 0.5\), while in the D - Delay scenario \(d^p_* = 0.4\).

B. SERIAL MANIPULATOR

We simulate a 7-DoFs Franka Emika Panda\(^1\) manipulator adding a joint stiffness matrix \(K = \text{diag}\{5, 5, 5, 3, 3, 3, 3\}\) 1e2 and a joint damping matrix \(F = \text{diag}\{10, 10, 10, 5, 5, 5, 5\}\). Additional details on the dynamics model of the robot can be found in [33].

We design a Cartesian trajectory \((X - X_0)^2 + (Y - Y_0)^2 + Z_0^2 = R^2\)\(^2\), where \([X_0, Y_0, Z_0]^\top \in \mathbb{R}^3\) is the Cartesian starting position of the robot and \(R = 0.1\) m is the radius of the circumference. Solving the inverse kinematic leads to the desired time evolution of the joints, i.e., \(y_d(t) = \{\text{ones}(1, 7), \text{zeros}(1, 7)\} x_d\), in such a way that the relative degree \(r = 2\) in (34). We indicate the nominal inertia matrix of the robot as \(M(q)\) and its model with \(\hat{M}(q) = 0.9 M(q)\). This is a state-repetitive disturbance \(d^p_q\) in (1). Note that both \(M(q), \hat{M}(q) > 0\). The control gains are \(Y_0 = M^{-1}(q) \text{diag}\{5, 5, 3, 3, 7, 7, 10\}\) 1e1, \(\Gamma_1 = M^{-1}(q) \text{diag}\{3, 3, 3, 5, 5, 5, 5\}\), \(Y_2 = 0.1 \text{diag}\{\text{ones}(1, 7)\}\), where \(\hat{M}\) is either the nominal or the perturbed inertia matrix depending on the case under study.

We test the same task in three different conditions:

- **ND**: we use (24), where \(D(x_j) = M^{-1}(q)\).
- **D - Data Loss**: we design the learning gain such as \(\Gamma_j(t) = \epsilon \hat{M}(q)\), which is a model uncertainty, namely \(d^p_q\). Additionally, at trials \(j = 4, 6\), we simulate a complete loss of joint position data, i.e., \(\Gamma_{j+1} = \epsilon \hat{M}(q)\) leading to a failure of (3). The loss of data can be modeled as a state-nonrepetitive disturbance \(d^p_q(x_j(t))\).
- **D - Delay**: in addition to designing the learning gain such as in the D - Data Loss case, we simulate the presence of a delay of 0.8 s in the control action of

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\(^1\)https://www.franka.de/
the joints 1, 3, 6, 7 at the trails $j = 8, 12$. This leads to a failure of (5). The delay can be seen as state-nonnrepetitive disturbance $d_{rx}^j(x_j(t))$.

Note that the learning gain $\Gamma_j(t) \in \mathbb{R}^{m \times m}$ is nonlinear.

Thus, in the $D$ - Delay and Data Loss scenarios, recalling (1) and (6), the simulated system can be written as

$$\dot{x}_j(t) = f(x_j(t)) + g(x_j(t))u_j(t) + d_{rx}^j(x_j(t)) + d_{pt}(t).$$

Finally, in the $ND$ scenario, we have $d_{pt}(t) \equiv d_{rx}^j(x_j(t)) \equiv 0_{2n \times 1}$ in (38).

V. DISCUSSION

Results show that the proposed method improves the tracking error between the first and the last iteration $G1$, reaching the desired tracking error value (0.001rad) in presence of state-repetitive and state-nonnrepetitive disturbances (Theorem 1, goal $G2$) both in case of underactuated (Fig. 1(b)-2(b)) and MIMO systems (Fig. 3). Quantification of the robustness of the method is also presented (Proposition 1, goal $G3$). As expected, the error convergence is not achieved in the case of time-repetitive and nonrepetitive disturbances (Fig. 1(c)), where a bounded error is obtained (Theorem 3, goal $G4$).

If the employed model is exact, and there are no distur-
ierarchical, the converge is smooth, fast, and exponential Fig. 2(b)-Fig. 3. On the other hand, as expected, the presence of state-nonrepetitive disturbances leads to an increment of the error for some iterations, Fig. 1(b)-Fig. 2(b) Fig. 3 leading to a non-monotonic convergence. However, thanks to the fulfillment of the D-condition we proposed, the controller is able to achieve the same tracking performance (goal G2) Fig. 2(a) Fig. 4(a). This proves that the D-condition is more robust w.r.t. the original ND one. Indeed, the D-condition obtains the minimization of the error while dealing with the incorrect contribution added to the control input, achieving the same tracking performance as the ND-condition.

VI. CONCLUSION AND FUTURE WORK

In this paper, we tackled the problem of trajectory tracking for continuous-time nonlinear systems affected by disturbances. We define different classes of disturbances. The goal was to obtain a controller able to achieve good tracking performance even in presence of state-nonrepetitive disturbances. We proposed and proved a convergence condition for a class of iterative learning controllers. The algorithm is purely feedforward, and it copes with nonlinear systems with a generic and fixed relative degree. The proposed method is robust both to repetitive and nonrepetitive disturbances. Additionally, we presented an upper bound of the disturbance amplitude that can be dealt with. Finally, we validated the proposed method through simulations using an underactuated compliant arm and Franka Emika Panda robot, both subjected to different types of disturbances.

Future work will investigate the robustness of the iterative framework from both a theoretical and an experimental point of view. We will combine feedforward and feedback terms and design switching policies depending on the system relative degree. Additionally, the employed control law is based only on the output measurements. Future work will rely on state-observers to design a control law employing the knowledge of the whole state. Finally, from a more experimental point of view, we will implement the algorithm on a real soft continuum prototype and medical image encryption both with disturbances.

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