Resonance in Bose-Einstein condensate oscillation from a periodic variation in scattering length

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Abstract.
Using the explicit numerical solution of the axially-symmetric Gross-Pitaevskii equation we study the oscillation of the Bose-Einstein condensate induced by a periodic variation in the atomic scattering length $a$. When the frequency of oscillation of $a$ is an even multiple of the radial or axial trap frequency, respectively, the radial or axial oscillation of the condensate exhibits resonance with novel feature. In this nonlinear problem without damping, at resonance in the steady state the amplitude of oscillation passes through maximum and minimum. Such growth and decay cycle of the amplitude may keep on repeating. Similar behavior is also observed in a rotating Bose-Einstein condensate.

PACS numbers: 03.75.-b, 03.75.Kk

Submitted to: J. Phys. B: At. Mol. Opt. Phys.
1. Introduction

The successful detection \[1,2\] of Bose-Einstein condensates (BEC) in dilute weakly-interacting bosonic atoms employing magnetic trap at ultra-low temperature stimulated intense theoretical studies on different aspects of the condensate \[3,4,5\]. The experimental magnetic trap could be either spherically symmetric or axially symmetric. The properties of an ideal condensate at zero temperature are usually described by the time-dependent, nonlinear, mean-field Gross-Pitaevskii (GP) equation \[6\] which incorporates appropriately the trap potential as well as the interaction between the atoms forming the condensate.

One of the problems of interest in Bose-Einstein condensation is the study of the oscillation in the condensate induced by a sudden change in the interaction acting on it. There have been both experimental \[7\] and theoretical \[8\] studies of the motion of the condensate under a sudden or continuous variation of the trapping frequencies. There has also been similar studies induced by a sudden variation of the atomic scattering length. In the latter class there have been experimental studies of explosive emission of atoms from a condensate when the scattering length is suddenly turned negative (attractive) from positive (repulsive), thus leading to collapse and explosion of the condensate \[9\]. This could also lead to the observation of a soliton train \[10\]. There have been theoretical attempts to explain the dynamics of the condensate for collapse and explosion \[11\] as well as for soliton train \[12\] using the GP equation.

Resonance is an interesting feature of any oscillation under the action of an external periodic force manifesting in a very large amplitude. Usually, a resonance appears when the frequency of the external periodic force becomes equal to that of the natural oscillation of the system \[13\]. Such a resonance appears in many areas of physics. There have been studies of resonance in different problems of linear dynamics and the appearance of resonance is well understood in these problems both analytically and numerically. The investigation of resonance in nonlinear dynamics is far more interesting and nontrivial. Consequently, resonances in nonlinear problems are not so well understood. This makes the study of resonance in the dynamics generated by the nonlinear GP equation of general interest. Apart from mere theoretical interest, such resonance in the oscillation of the BEC governed by the GP equation can also be studied experimentally so that one can compare theoretical prediction with experiment.

The possibility of generating a resonance in the oscillation of a BEC subject to a varying trapping potential has been investigated theoretically \[8\]. The variation of trapping potential in the GP equation corresponds to a linear external perturbation. Here we investigate the resonance dynamics of the BEC subject to a periodic variation of the scattering length, which should be considered as a nonlinear periodic force acting in the GP equation. Such a variation of the scattering length is possible near a Feshbach resonance by varying an external magnetic field \[14\]. For experimentalists a periodic variation of the external magnetic field is easier to implement and the magnetic field is nonlinearly related to the scattering length. However, a simple periodic variation of the
scattering length is a cleaner theoretical possibility and we shall apply such a variation in this study. The appearance of a resonance in the oscillation of a BEC due to a periodic variation of the scattering length has been postulated recently in a spherically symmetric trap [15]. Here we present a complete study of the resonance dynamics in this case. In addition, we extend this investigation to the more realistic and complicated case of an axially symmetric trap as well as to vortex states in such a trap.

In the simplest case of a damped classical oscillator under a periodic external force with the natural frequency of oscillation, the resonance appears due a constant phase difference between the oscillation and the external force [13]. This leads to a gradual increase in kinetic energy over each cycle of oscillation during an interval of time and hence in the amplitude of oscillation in this interval. Eventually, the rate of loss of energy due to damping is exactly compensated for by the increase in energy from the external force and the oscillator oscillates with a large constant amplitude at large times at resonance. The constant phase difference between the external force and the oscillation is possible only at resonance when the two frequencies match and is necessary for resonance.

The situation is much more complicated in the case of a trapped BEC subject to a nonlinear external force due to a periodic variation in the scattering length. When subject to a sudden perturbation, the natural frequency of oscillation of a trapped spherical BEC governed by the GP equation with nearly zero nonlinearity is $2\omega$ where $\omega$ is the frequency of the existing trap [9,16]. This result is exact in the linear case $n = 0$. However, for the nonlinear problem ($n \neq 0$), this natural frequency may change. In this paper we shall only be considering small nonlinearity where the natural frequencies can be taken approximately as $2\omega$. In physical analogy with the classical oscillator, a resonance is expected in the BEC oscillation when the scattering length oscillates with frequency $2\omega$ or its multiples. In our study we find that this is indeed the case. However, a much richer dynamics emerges in this case compared to the case of linear systems. We find that due to nonlinearity, the phase difference $\delta$ between the BEC oscillation and the external nonlinear force varies with time. Consequently, during a certain interval of time the BEC may steadily gain energy from the external force in all cycles and the amplitude of oscillation may grow leading to a resonance. In the case of BEC there is no damping and for a constant phase difference $\delta$ the amplitude is expected to grow beyond limit. However, as a result of the variation of the phase $\delta$ with time due the nonlinearity, after some time the system starts to lose energy due to the action of the external force. Eventually, the oscillation of the the system can virtually stop. By that time the modified phase difference favors the increase of energy and the amplitude starts to grow again. Consequently, the resonance of the system has a peculiar nature; the amplitude of oscillation passes through pronounced maxima and minima. In a single classical damped oscillator this is not possible under the action of a periodic force [13]. However, this is possible in case of a coupled system with exchange of energy between the two oscillators.

We also investigate the resonant oscillation of an axially symmetric BEC due to
a periodic variation of the scattering length. When the frequency of oscillation of the scattering length equals the natural frequency of the radial or axial oscillation of the BEC, we find resonance in the radial or axial oscillation, respectively.

In section 2 we describe briefly the time-dependent GP equation with spherical and axial traps. The essential details of the split-step Crank-Nicholson method used for numerical solution is also described in this section. In section 3 we report the numerical results of the present investigation of resonance for the spherically symmetric case. In section 4 we present the same for the axially symmetric case for condensates with zero angular momentum as well as for vortex states with nonzero angular momentum. Finally, in section 5 we present a discussion and summary of our study.

2. Nonlinear Gross-Pitaevskii Equation

At zero temperature, the time-dependent Bose-Einstein condensate wave function $\Psi(r; \tau)$ at position $r$ and time $\tau$ may be described by the following mean-field nonlinear GP equation \[3,10\]

\[
-\frac{\hbar^2}{2m} \nabla^2 + V(r) + gN|\Psi(r; \tau)|^2 - i\hbar \frac{\partial}{\partial\tau} \Psi(r; \tau) = 0.
\] (2.1)

Here $m$ is the mass and $N$ the number of atoms in the condensate, $g = 4\pi\hbar^2a/m$ the strength of interatomic interaction, with $a$ the atomic scattering length. The normalization condition of the wave function is $\int dr |\Psi(r; \tau)|^2 = 1$.

2.1. Spherically Symmetric Case

In this case the trap potential is given by $V(r) = \frac{1}{2}m\omega^2r^2$, where $\omega$ is the angular frequency and $r$ the radial distance. The wave function can be written as $\Psi(r; \tau) = \psi(r, \tau)$. After a transformation of variables to dimensionless quantities defined by $x = \sqrt{2r/l}$, $t = \tau\omega$, $l \equiv \sqrt{\hbar/m\omega}$ and $\varphi(x, t) = x\psi(r, \tau)(4\pi l^3/\sqrt{8})^{1/2}$, the GP equation in this case becomes

\[
-\frac{\partial^2}{\partial x^2} + \frac{x^2}{4} + 2\sqrt{2}n \left| \frac{\varphi(x, t)}{x} \right|^2 - i\frac{\partial}{\partial t} \varphi(x, t) = 0,
\] (2.2)

where $n = Na/l$. The normalization condition for the wave function is $\int_0^\infty dx |\varphi(x, t)|^2 = 1$.

2.2. Axially Symmetric Case

The trap potential is given by $V(r) = \frac{1}{2}m\omega^2(\rho^2 + \lambda^2z^2)$ where $\omega$ is the angular frequency in the radial direction $\rho$ and $\lambda\omega$ that in the axial direction $z$. We are using the cylindrical coordinate system $r \equiv (\rho, \theta, z)$ with $\theta$ the azimuthal angle. In this case one can have quantized vortex states with rotational motion around the $z$ axis \[4\]. In such a vortex the atoms flow with tangential velocity $\hbar L/(mr)$ such that each atom has
quantized angular momentum $\hbar L$ along $z$ axis. The wave function can then be written as: $\Psi(r, \tau) = \psi(r, z; \tau) \exp(iL\theta)$, with $L = 0, \pm 1, \pm 2, \ldots$.

Using the above angular distribution of the wave function in (2.1), in terms of dimensionless variables $x = \sqrt{2}r/l$, $y = \sqrt{2}z/l$, $t = \tau \omega$, $l \equiv \sqrt{\hbar/(m\omega)}$, and $\varphi(x, y; t) = x \sqrt{4\pi l^3/\sqrt{8}}\psi(r, z; \tau)$, we get

\[
\left[ -\frac{\partial^2}{\partial x^2} + \frac{1}{x} \frac{\partial}{\partial x} - \frac{\partial^2}{\partial y^2} + \frac{L^2}{x^2} + \frac{1}{4} \left( x^2 + \lambda^2 y^2 - \frac{4}{x^2} \right) \right]
+ 2\sqrt{2}n \left| \frac{\varphi(x, y; t)}{x} \right|^2 - i\frac{\partial}{\partial t} \varphi(x, y; t) = 0. \tag{2.4}
\]

The normalization condition of the wave function is

\[
\int_0^\infty dx \int_0^\infty dy |\varphi(x, y; t)|^2 x^{-1} = 1. \tag{2.5}
\]

2.3. Numerical Detail

We solve the GP equation numerically employing a split-step time-iteration method using the Crank-Nicholson discretization scheme described elsewhere [17]. We discretize the GP equation spanning $x$ from 0 to 25 and $y$ from $-30$ to 30 in the axially symmetric case. In the spherically symmetric case the range of $x$ integration was taken from 0 to 30. This was enough for having a negligible value of the wave function at the boundaries. To calculate the solution for a specific nonlinearity $n$ the time iteration is started with the known harmonic oscillator solution of the GP equation for nonlinearity $n = 0$. These initial normalized solutions are

\[
\varphi(x) = (2/\pi)^{1/4}xe^{-x^2/4} \tag{2.6}
\]

\[
\varphi(x, y) = \left[ \frac{2\lambda}{\pi 2^{|L|}(|L|!)^2} \right]^{1/4} x^{1+|L|}e^{-(x^2+\lambda y^2)/4} \tag{2.7}
\]

for (2.2) and (2.4), respectively. The nonlinearity is then slowly changed in steps of 0.0001 during each time iteration until the desired value of $n$ is attained. As a result in the course of time iteration the initial $n = 0$ solution slowly converges towards the final solution with the desired $n$.

2.4. Resonance Formation

We study the formation of resonance in the oscillation of the condensate as the scattering length is varied periodically. In the scaled GP equations (2.2) and (2.4) the scattering length appears only through the nonlinearity parameter $n = Na/l$. In particular we consider the following variation of the nonlinearity parameter

\[
n = b + c \sin(\Omega t), \tag{2.8}
\]

where $b$ and $c$ are two constants and $\Omega$ is the frequency of variation of the nonlinear parameter.
In the GP equation (2.2), the scaled harmonic oscillator trap frequency is unity; the radial and axial trap frequencies of (2.4) are 1 and $\lambda$, respectively. For zero and small values of nonlinearity the natural frequency of oscillation of (2.2) is twice the scaled trap frequency, which is 2. The natural frequencies of oscillation of (2.4) in radial and transverse directions are twice the scaled trap frequencies, which are 2 and $2\lambda$, respectively. These natural frequencies of oscillation have been verified experimentally [9], as well as in theoretical calculations [16]. The perturbation (2.8) when applied to the GP equation behaves like a nonlinear external force acting in (2.2) and (2.4). When the frequency $\Omega$ of the perturbation coincides with the natural frequencies of oscillation of the GP equation, resonant behaviors are expected.

The classic example of resonance appears when the parameter $b$ in (2.8) is zero. In that case resonances are expected in the spherically symmetric case for $\Omega = 2$. In the axially symmetric case they are expected for $\Omega = 2$ and $\Omega = 2\lambda$ in the radial and axial oscillations, respectively. For $b \neq 0$, resonances also appear provided that the constant $c$ is not very small. We study these possibilities in the next sections.

3. Result for Spherical Trap

First we consider the case with $b = 0$ in (2.8). In this case the maximum permitted value of $c$ is 0.575. For $c > 0.575$, the minimum of nonlinearity $n$ could be less than $-0.575$. It is known that no stable solution of the spherically symmetric GP equation could be obtained for $n < -0.575$ [2, 3, 18]. The domain of values $n < -0.575$ corresponds to attractive interaction leading to collapse of the condensate.

In this case the natural frequency of oscillation of the condensate is 2 and resonances are expected for values of $\Omega$ equaling multiples of 2. We shall mostly concentrate here to the case $\Omega = 2$. In figures 1 (a), (b) and (c) we plot the root mean square (rms) radii $\langle x \rangle$ vs. time for $c = 0.1, 0.3$ and 0.5 and for $\Omega = 2$. We find that as $c$ increases the rms radii can grow to a large value signaling a resonance. One interesting feature of these resonances is that the oscillation of the rms radius can increase and subsequently decrease and such a cycle can repeat many times. A larger number of such growth and decay cycles can be accommodated in a given interval of time as $c$ increases.

In figure 1 (d) we plot the results for $c = 0.5$ and $\Omega = 4$. We find that again one can have large amplitudes of oscillation. Similar resonances appear for $\Omega = 6, 8, ...$ etc. However, in this work we shall only study the resonance of lowest order for $\Omega = 2$. Finally, in figure 1 (e) we present a off-resonance result for $c = 0.5$ and $\Omega = 2.2$. Compared to figure 1 (c) with same values of $b$ and $c$, we find oscillation with highly reduced amplitude in figure 1 (e) due to a variation of $\Omega$ to an off-resonance value. If $\Omega$ is taken further off-resonance the oscillations disappear quickly.

In a problem of resonance of an uncoupled oscillator driven by a periodic external force the oscillator oscillates in phase with the external force. Hence it gains energy during every cycle of oscillation and the amplitude of oscillation keeps on increasing. So a pertinent question to ask is how can the amplitude of oscillation decrease in the present
Figure 1. The rms radii of the condensate $\langle x \rangle$ vs. time $t$ for $b = 0$ and (a) $c = 0.1$, $\Omega = 2$, (b) $c = 0.3$, $\Omega = 2$, (c) $c = 0.5$, $\Omega = 2$, (d) $c = 0.5$, $\Omega = 4$, (e) $c = 0.5$, $\Omega = 2.2$.

problem. In this nonlinear problem the phase between the oscillation at resonance and the external force varies with time. Hence during a certain interval of time the two are in phase and the amplitude of oscillation grows. After the amplitude attains a certain value, the external force and the oscillation become out of phase and the system loses energy in each cycle. Eventually, the amplitude of oscillation reduces to a small value and this growth and decay cycle may repeat for a long time. Due to the nonlinearity in the equation the system moves in and out of resonance when a constant frequency drive is applied. This leads to oscillations of varying time-dependent amplitude in the
Next we study the resonance in the situation with $b \neq 0$ and $\Omega = 2$. For a nonzero $b$, pronounced resonances appear as $c$ is increased from 0. In figures 2 (a), (b), and (c) we plot $\langle x \rangle$ vs. $t$ for $b = 1$, and $c = 0.3, 0.4$ and 0.5, respectively. The amplitude of oscillation increases as $c$ is increased. We again see the growth and decay cycles for small values of $c$. However, such periodic growth and decay cycles are lost with the increase in $c$ as one can find in figure 2 (c) for $c = 0.5$.

In figures 2 (d) and (e) we show the results for $b = 5$ and $c = 2$ and 2.4, respectively.
With the increase of $c$, the amplitude of oscillation has increased from figure 2 (d) to 2 (e). However, the periodic growth and decay cycles have been lost in this transition.

There are two time scales in each of the figures. The first one is the driving frequency and the second one is related to the nonlinear term in the GP equation. For the linear problem only the first time scale appears and the second frequency is absent. At resonance the second frequency should increase with the effective nonlinearity of the system although a simple relation between the two cannot be obtained. For $b = 0$ the effective nonlinearity increases with increasing $c$ and at resonance the second frequency is expected to increase as $c$ increases. However, for a positive $b$ the effective nonlinearity is large when $c$ is large and at resonance second frequency is expected to decrease as $c$ increases. These interesting features of the growth and decay cycles emerges from figures 1 and 2. At resonance for $b = 0$ more growth and decay cycles appear for large values of $c$. However, quite expectedly for $b \neq 0$ more growth and decay cycles appear for small values of $c$.

4. Result for Axial Trap

Next we continue our discussion of resonance to axially symmetric traps. We find that when the external frequency $\Omega$ coincides with the natural frequency of radial ($\omega = 2$) or axial ($\omega = 2\lambda$) oscillation, there is resonance in radial or axial oscillation, respectively. The general nature of this oscillation is similar to that in the spherically symmetric case. We studied the resonance dynamics for different sets of parameters. However, in the following we present results only for $b = 0$ and $c = 0.5$ for angular momenta $L = 0$ and 1. We consider both pancake ($\lambda > 1$) and cigar-type ($\lambda < 1$) deformation of the condensate in our study. We recall that $\lambda = 1$ corresponds to the spherical case considered in the last section.

4.1. States with zero angular momentum

Here, for angular momentum $L = 0$, we consider the following two values of the axial parameter $\lambda = \sqrt{8}$ and $1/\sqrt{8}$ as some of the early experiments \cite{1} were performed with these anisotropies. Also, these values of $\lambda$ correspond to substantial contraction and elongation along the axial direction and hence to a substantial deviation from the spherical symmetry. We present results for $\Omega = 2$ and $2\lambda$ in each of these cases.

In figures 3 (a), (b), (c), and (d) we plot the rms radial and axial sizes $\langle x \rangle$ and $\langle y \rangle$ for $\lambda = \sqrt{8}$ and $1/\sqrt{8}$ and for $\Omega = 2$ and $2\lambda$. In figures 3 (a) and (b) resonance in radial oscillation is achieved for $\Omega = 2$. This results in the large and rapidly varying rms radii $\langle x \rangle$ at resonance and a small and slowly varying rms size $\langle y \rangle$ off the resonance. In figures 3 (a) and (b) there is one complete growth and decay cycle for $\langle x \rangle$. Then the resonance is manifested by a different type of oscillation in $\langle x \rangle$. Similar behavior was also found in the spherically symmetric case, e.g., in figure 2 (e).

In figures 3 (c) and (d) resonance in axial oscillation is achieved for $\Omega = 2\lambda$. In
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Figure 3. The radial and axial rms sizes of the condensate $\langle x \rangle$ and $\langle y \rangle$, respectively, vs. time $t$ for $b = 0, c = 0.3, L = 0$ and for (a) $\lambda = \sqrt{8}, \Omega = 2$, (b) $\lambda = 1/\sqrt{8}, \Omega = 2$, (c) $\lambda = \sqrt{\pi}, \Omega = 2\lambda$, (d) $\lambda = 1/\sqrt{8}, \Omega = 2\lambda$.

In these cases the axial size $\langle y \rangle$ executes resonance oscillation, whereas the rms radius $\langle x \rangle$ increases in size representing a swelling. In figure 3 (c) the axial trapping frequency $\lambda = \sqrt{\pi}$ is large, which makes the axial oscillation less likely than in the case presented in figure 3 (d), where the small axial trapping frequency $\lambda = 1/\sqrt{8}$ makes the axial oscillation more favorable. Consequently, the amplitude at the resonance in axial oscillation in figure 3 (d) is about four times more than that in figure 3 (c).

From figures 3 (a) and (b) we find that at the resonance of radial oscillation the axial rms size $\langle y \rangle$ remains quite constant. We find in figures 3 (c) and (d) that at the resonance of axial oscillation the radial rms radius $\langle x \rangle$ varies more with time. One interesting feature has appeared in figures 3 (c) and (d), where for $\lambda = 1/\sqrt{8}$ the radial rms radii $\langle x \rangle$ vary with time corresponding to a slow radial swelling accompanied by small oscillation and not an oscillation with large amplitude. We verified, as in the spherically symmetric case, that a smaller value of the constant $c$ leads to many growth
and decay cycles of the amplitude at resonance in the axially symmetric case (however, not shown explicitly here).

4.2. Vortex States

Here for angular momentum \( L = 1 \) we consider the effect of a periodic variation of the scattering length via (2.8) on the quantized vortex states. Such states are of great importance to the study of the BEC, as they are intrinsically related to the existence of superfluidity [19]. The experimental detection of vortex states in BEC [20] makes the resonant oscillation of the vortex states under a periodic variation of scattering length experimentally observable.

To illustrate the resonance we consider the lowest angular excitation of the BEC rotating round the axial direction with each atom possessing angular momentum \( \bar{\hbar} \) corresponding to \( L = 1 \). Again we consider the anisotropy parameters \( \lambda = \sqrt{8} \) and

![Figure 4](image-url)

**Figure 4.** The radial and axial rms radii of the condensate \( \langle x \rangle \) and \( \langle y \rangle \), respectively, vs. time \( t \) for \( b = 0, c = 0.3, L = 1 \) and for (a) \( \lambda = \sqrt{8}, \Omega = 2 \), (b) \( \lambda = 1/\sqrt{8}, \Omega = 2 \), (c) \( \lambda = \sqrt{8}, \Omega = 2\lambda \), (d) \( \lambda = 1/\sqrt{8}, \Omega = 2\lambda \).
$1/\sqrt{8}$ and the external frequencies $\Omega = 2$ and $2\lambda$. In figures 4 (a), (b), (c), and (d) we plot the rms radial and axial sizes $\langle x \rangle$ and $\langle y \rangle$ in these cases. These figures should be compared with the respective zero angular momentum cases shown in figures 3 (a), (b), (c), and (d).

In the case of vortex states very orderly (periodic) growth and decay cycles of the resonating amplitudes appear for $b = 0$ and $c = 0.3$ in (2.8). The periodic growth and decay cycles of the resonating amplitudes are replaced by irregular ones as in figures 3 (a), (b), (c), and (d) as the value of the constant $c$ is increased (not shown explicitly in this paper). The off-the-resonance amplitudes $-\langle y \rangle$ in figures 4 (a) and (b) and $\langle x \rangle$ in figures 4 (c) and (d) — remain constant without variation in all cases. However, the period of growth and decay cycles of the resonating amplitude varies from one case to another. In all four cases we have plotted the amplitudes up to 600 dimensionless time units. This accommodates about 2.3 growth and decay cycles of the resonating amplitude in figure 4 (a), 1.5 cycles in figure 4 (b), 5 cycles in figure 4 (c), and 0.5 cycles in figure 4 (d).

Comparing the situation of figure 4 (c) with that in figure 4 (d) we see that there is a reduction in the axial trapping frequency as we pass from figure 4 (c) to figure 4 (d). This reduction in the axial trapping frequency should favor the axial oscillation at resonance in figure 4 (d). Consequently, we find that the amplitude of axial oscillation at resonance is much larger in figure 4 (d) compared to that in figure 4 (c). Similar effect was also observed in figures 3 (c) and (d) in the $L = 0$ case.

5. Summary

We have studied the occurrence of resonant oscillation in a BEC due to a periodic variation in the atomic scattering length $a$. A variation in the atomic scattering length near a Feshbach resonance can be and has been achieved experimentally [14] by varying an external magnetic field. This has been executed experimentally in producing a sudden jump in the scattering length from positive (repulsive) to negative (attractive) which has led to collapse and explosion in the condensate [9] and several novel phenomena [10]. Here we demonstrate that interesting resonating oscillation can be generated in the condensate if the scattering length is varied periodically with a given frequency. This oscillation may be verified experimentally and novel and interesting phenomena may emerge as a consequence of such verification.

Mathematically, the periodically oscillating scattering length in the GP equation can be considered as an external nonlinear periodic driving force in the linear Schrödinger equation, which is an interesting problem of quantum mechanics and deserves special attention. Here we have considered a full numerical approach to its solution and have revealed interesting results. One could also employ an analytic perturbative procedure for its solution when the coefficient of the external force is small. This would be a problem of future interest.

We found that resonances with novel feature can appear in the oscillation of the
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BEC from a periodic variation of the scattering length when the frequency of the oscillation of scattering length $\Omega$ coincides with a multiple of the natural frequency of oscillation of the harmonically trapped BEC. For a spherically symmetric condensate, the natural frequency of oscillation of the BEC is twice the trapping frequency ($= 2 \omega$). Hence resonances in the BEC oscillation appear for $\Omega = 2\omega, 4\omega, 6\omega, \ldots$ etc. For an axially symmetric BEC, the natural frequency of oscillation of the BEC in radial and axial directions are twice the trapping frequencies in these directions, which are $= 2\omega$ and $= 2\lambda\omega$, respectively. Consequently, resonances in the BEC oscillation in the radial and axial directions appear for $\Omega = 2\omega, 4\omega, 6\omega, \ldots$ and $\Omega = 2\lambda\omega, 4\lambda\omega, 6\lambda\omega, \ldots$, respectively. The interesting feature of these resonances is that even in the absence of any damping (dissipative force), the amplitude of oscillation at resonance can grow and reduce with time. Such growth and decay cycle of the amplitude may repeat several times. The period of such growth and decay cycles is $\sim 100$ units of dimensionless time, which under usual experimental condition with $\omega \approx 500$ Hz/s corresponds to about 0.2 s – a period that can be precisely measured and compared with theoretical prediction to test the mean-field theory. We hope that the present investigation may stimulate further theoretical and experimental studies on this topic.

Acknowledgments

The work is supported in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico and Fundação de Amparo à Pesquisa do Estado de São Paulo of Brazil.

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