Baryon asymmetry resulting from a quantum phase transition in the early universe

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Abstract – A novel mechanism for explaining the matter-antimatter asymmetry of the universe is considered. We assume that the universe starts from a completely symmetric state and then, as it cools down, it undergoes a quantum-phase transition which in turn causes an asymmetry between matter and antimatter. The mechanism does not require the baryon-number-violating interactions or CP violation at a microscopic level. Our analysis of the matter-antimatter asymmetry is in the context of conspicuous experimental results obtained in the condensed-matter physics.

One of the important and long-standing problems of modern cosmology and astrophysics is the matter-antimatter asymmetry: the observable part of the universe contains mostly baryons and antibaryons produced locally as the byproduct of nuclear reactions [1–4]. For the globally symmetric universe one can put a strong constraint on the size \( l \) of antimatter clusters [5], \( l > 1000 \text{ Mpc} \). This number may be compared with the visible size of the universe, 3000 Mpc.

A convenient dimensionless number which characterizes the magnitude of the baryon asymmetry is the ratio of the baryonic charge density \( n_B - n_{\bar{B}} \) to the density of the cosmic microwave background \( n_\gamma \) [6]

\[
\frac{n_B - n_{\bar{B}}}{n_\gamma} \lesssim 3 \cdot 10^{-10} \tag{1}
\]

Estimate (1), excluding antimatter on scales of order \( \sim 20 \text{ Mpc} \) [6], may be obtained assuming that at earlier times, at temperatures well above 100 MeV, the universe had one extra quark per about \( 10^{10} \) quark-antiquark pairs and this tiny excess is responsible for the entire baryonic matter in the present universe.

Although there is no logical contradiction to assume that an excess of quarks over antiquarks is built in as an initial condition, this \textit{ad hoc} hypothesis cannot be justified within the inflationary scenario which does not provide such an initial condition [1–4]. Thus, it becomes necessary to explain baryon asymmetry without introducing the minute particle excess at the initial stage of the big bang.

Sakharov [7] was the first to formulate the conditions necessary for generating the observed baryon asymmetry from an initially symmetric state. These are: the baryon number non-conservation, \( C \) and \( CP \) violation, and a departure from the thermal equilibrium. Numerous possible scenarios incorporating Sakharov’s conditions readily followed. The detailed discussion of these mechanisms such as electroweak baryogenesis, baryonic charge condensate, baryogenesis via leptogenesis, baryogenesis through evaporation of primordial black holes, out-of-equilibrium decays of massive particles, baryon number non-conservation caused by the triangle anomaly in baryonic current and baryogenesis in the presence of spontaneously broken Lorentz symmetry can be found in [1–4]. It is worth noting that all these scenarios require extension of the standard model of particle physics.

In this letter we propose a mechanism for explaining the matter-antimatter asymmetry of the universe, which does not require any extension of the standard model of particle physics or the standard model of cosmology. Our approach is based on the observation that the condensed-matter physics of strongly correlated Fermi systems and topologically protected gapless fermions without dispersion...
forming the flat bands may offer opening for designing such a mechanism [8–10]. We propose that the universe began from a completely symmetric state with the baryon number and CP conserved at the end of inflation when the particle production started. The observed asymmetry may be explained by suggesting that in the post inflation epoch, at baryogenesis, as the universe cooled down in approximately 10 orders of magnitude, it came near a quantum phase transition. At that time an excess of matter over the antimatter in the universe was generated. This phase transition could wash out the antibaryons from the universe ground-state wave function. Such a quantum phase transition can be represented by the fermion condensation quantum phase transition (FCQPT) that does not support quasiparticle-hole symmetry [11–14]. We note that flat bands related to FCQPT were observed in (2 + 1)-dimensional quantum field theory which is dual to a gravitational theory in the anti-de Sitter background [15]. For a detailed discussion of novel features exhibited by the strongly correlated Fermi systems see review [13].

Our suggestion is based on the results of theoretical [13,14,16] and experimental [17–19] studies of novel systems of condensed-matter physics—the strongly correlated Fermi systems and the quantum phase transitions within. These systems do not belong to the well-known class of Landau Fermi Liquid [20], and exhibit non-Fermi-Liquid behavior strikingly different from those of the Landau Fermi Liquid [21].

We propose that the universe exhibits non-Fermi-Liquid behavior and therefore shares the features of non-Fermi Liquid, in particular the spontaneous breakdown of quasiparticle-hole symmetry. The quasiparticle-hole asymmetry manifests itself on a macroscopic scale as an asymmetric conductivity [17–19]. This phenomenon of quasiparticle-hole asymmetry serves as the guiding principle in our suggestion of explaining matter-antimatter asymmetry from the results of condensed-matter physics of strongly correlated Fermi systems. Below we briefly discuss the quasiparticle and hole properties in Landau Fermi Liquid and non-Fermi Liquid which are necessary to illustrate our suggestion and provide the basis for interpreting the observed baryon asymmetry as resulting from a quantum phase transition. Throughout holes will represent the matter (baryons) and quasiparticles will serve as an analogy of antimatter (antibaryons).

As is well known, the basic thermodynamic and transport properties of Landau Fermi Liquid are described in terms of quasiparticles—the weakly excited states over the Fermi sea (or the Fermi level $E_F$) [20]. Landau Fermi Liquid is symmetric with regard to quasiparticles and holes. The latter are the “mirror images” of quasiparticles with the same mass but opposite charge; in particle physics terminology the quasiparticles above and the holes below the Fermi sea are fermions above and antifermions below the Dirac sea. The microscopic Hamiltonian describing Landau Fermi Liquid is fully symmetric with regards to holes and quasiparticles, and this “matter-antimatter symmetry” holds on a macroscopic scale as well [20].

The theory of Landau Fermi Liquid is based on a representation of the system as a gas of interacting quasiparticles, the number of which is equal to the number $N$ of particles [20]. The ground-state energy $E$ of a uniform Fermi system is treated as a functional of the quasiparticle distribution $n(p)$. Under arbitrary variation of $n(p)$, conserving the particle number density $x$, the energy $E$ is changed according to the formula [11,12]

$$\delta E = \int (\varepsilon(p) - \mu) \delta n(p) \frac{dp}{(2\pi)^3}. \quad (2)$$

Here $\varepsilon(p) = \delta E/\delta n(p)$ is the energy of a quasiparticle and $\mu$ is the chemical potential. The distribution $n(p)$ at $T = 0$ is a Fermi step $n(p) = \theta(p - p_F)$, $p_F$ is the Fermi momentum, and $x = \frac{p_F^3}{3\pi^2}$. At finite temperatures $n(p)$ is given by the Fermi-Dirac distribution

$$n(p,T) = \frac{1}{1 + \exp[(\varepsilon(p,T) - \mu)/T]}^{-1}. \quad (3)$$

A necessary stability condition for Landau Fermi Liquid is that the group velocity of quasiparticles be non-negative at the Fermi surface: $v_g(p) = \partial \varepsilon(p)/\partial p \geq 0$. In that case $\delta E > 0$ and we obtain from eq. (2) that $\varepsilon(p) = \mu$. This condition can be reformulated as an equation of the minimum [11,12]

$$\frac{\delta E}{\delta n(p)} = \varepsilon(p) = \mu; \quad p_i \leq p \leq p_f. \quad (4)$$

The distribution function found from eq. (4) differs from the step function in the interval from $p_i$ to $p_f$; inside this interval $0 < n(p) < 1$. Outside this interval $n(p)$ coincides with the step function. When the density $x \rightarrow x_{FC}$, where $x_{FC}$ is the quantum critical point (QCP) of FCQPT, a non-trivial solution of eq. (4) emerges. At finite $T$, a solution $n(p)$ of eq. (4) and the corresponding single-particle spectrum $\varepsilon(p)$ are depicted in fig. 1 [16]. At the Fermi level $\varepsilon(p,T) = \mu$, then from eq. (3) the distribution function $n(p,T) = 1/2$. The vertical dashed and solid lines in fig. 1 crossing the distribution function at the Fermi level illustrate the asymmetry of the corresponding distribution functions with respect to the Fermi level at $T = 0.01$ and $T = 0.0001$, respectively. As $T \rightarrow 0$ as seen from fig. 1, the number density of holes $H = \sum_{\varepsilon(p) < E_F} (1 - n(p))$ is finite, while the number density of quasiparticles $P = \sum_{\varepsilon(p) > E_F} n(p)$ vanishes. Clearly the solutions of eq. (4) strongly violate the particle-hole symmetry, and the asymmetry $R_A = (H-P)/(H+P)$ increases becoming more pronounced at diminishing temperatures.

A schematic phase diagram of the system which is driven to FCQPT by variation of $x$ is reported in fig. 2. Upon approaching QCP of FCQPT at $x = x_{FC}$, the system remains in the Landau Fermi-Liquid region at sufficiently low temperatures as is shown by the shadow area. The temperature range of the shadow area shrinks as the system approaches QCP. At $x_{FC}$ shown by the
Fig. 1: (Colour on-line) The single-particle energy \( \epsilon(k,T) \) (a) and the distribution function \( n(k,T) \) (b) at finite temperatures as functions of the dimensionless variable \( k = p/p_F \). Temperature is measured in units of \( E_F \). At \( T = 0.01 \) and \( T = 0.0001 \), the vertical dashed and solid lines, respectively, show the position of the Fermi level \( E_F \) at which \( n(k,T) = 0.5 \) as depicted by the horizontal line. As \( T \to 0 \) and consistent with eq. (2), the single-particle energy \( \epsilon(k,T) \) becomes more flat in the region \( (p_f - p_i) \) and the distribution function \( n(k,T) \) in this region becomes more asymmetric with respect to the Fermi level \( E_F \) producing the quasiparticle-hole asymmetry related to the non-Fermi-Liquid behavior.

Fig. 2: (Colour on-line) Schematic phase diagram of a system with FCQPT. The number density \( x \) is taken as the control parameter and depicted as \( x/x_{FC} \). The quantum critical point at \( x/x_{FC} = 1 \) of FCQPT is denoted by the arrow. At \( x/x_{FC} > 1 \) and sufficiently low temperatures, the system is in the Landau Fermi-Liquid state as shown by the shadow area. At finite temperatures and beyond the critical point, \( x/x_{FC} < 1 \), the system is above the quantum critical line depicted by the dashed line and shown by the vertical arrow. The location of the system in the NFL region is characterized by the quasiparticle-hole asymmetry related to the non-Fermi-Liquid behavior.

Fig. 3: (Colour on-line) \( R_A \) as a function of the dimensionless temperature measured in \( E_F \). Our calculations are shown by squares. The solid curve represents the fit \( R_A \approx a_0 + a_1 \sqrt{T} \) (shown by the vertical arrow and the dashed line in fig. 2) rather than a quantum critical point. It is seen from fig. 2 that at finite temperatures there is no boundary (or phase transition) between the states of systems located before or beyond QCP shown by the arrow. Therefore, at elevated temperatures the properties of systems with \( x/x_{FC} < 1 \) or with \( x/x_{FC} > 1 \) become indistinguishable, while the particle-hole symmetry is restored [13]. In fig. 3 we present the asymmetry \( R_A \) vs. dimensionless temperature. At low temperatures \( T/E_F \leq 1 \) the asymmetry \( R_A \approx a_0 + a_1 \sqrt{T} \) and the symmetry is restored at \( T \approx E_F \) since \( R_A \approx 0 \).

The function \( R_A \) is of universal form and the values of \( a_0 \) and \( a_1 \) are determined by the location of the system at the quantum critical line shown in fig. 2. We note that \( a_0 \) is given by the temperature-independent part \( S_0 \) of the entropy, \( S(T \to 0) \to S_0 \) [13,14,16].

One of the manifestations of the quasiparticle-hole symmetry on a macroscopic scale is the symmetric electric conductivity. It is straightforward to demonstrate that in Landau Fermi Liquid the differential conductivity \( \sigma_d \) appearing in Ohm’s law

\[
dI = \sigma_d dV
\]  

is a symmetric function of the voltage \( V \), i.e. \( \sigma_d \) may depend on the absolute value of \( V \), but not on its sign. From eq. (5) it follows that when the conductivity is a symmetric function of voltage, reversing the sign of \( V \) results in \( I \to -I \); the electric current maintains its magnitude and flows in the opposite direction. This reasonable feature may be readily derived in the framework of the Landau Fermi-Liquid theory. Let us recall that the electric current can be expressed in terms of the distribution function given by eq. (3) as [17]

\[
I(V) = \text{const} \int d\epsilon [n(\epsilon - V) - n(\epsilon)].
\]  

Now, from eqs. (5) and (6) it immediately follows that \( \sigma_d \) is a symmetric function of voltage \( V \). This result is a direct consequence of the quasiparticle-hole symmetry which is
an inherent feature of the Landau Fermi-Liquid theory. The symmetric conductivity has been observed so many times that it lead to a perception that the conductivity cannot be asymmetric.

This perception has been invalidated by recent experimental observations where it was shown that at low temperatures the electric conductivity in high-$T_c$ superconductors [17] as well as in some heavy-fermion metals, such as CeCoIn$_5$ and YbCu$_{5-x}$Al$_x$ [18,19] is clearly asymmetric, this asymmetry vanishing as the temperature increases. Evidently, the asymmetry cannot be explained in the framework of the Landau Fermi-Liquid theory since the quasiparticle-hole symmetry unavoidably leads to the step function for the fermion distribution function in the low-temperature regime which, in turn results in a symmetric conductivity. Therefore, in order to explain the asymmetric conductivity, it is necessary to consider fermion systems more general than Landau Fermi Liquid.

Strongly correlated fermion systems may serve as one of the examples of such systems. These systems have many novel phenomena which have been observed experimentally [13,21]; in our case the most attractive is the low-temperature asymmetric conductivity [13,17−19]. This macroscopic effect finds its explanation in the quasiparticle-hole asymmetry in theory of strongly correlated fermion systems. Fundamental microscopic interaction in this theory is fully symmetric with regard to quasiparticles and holes but FCQPT at low temperatures causes the spontaneous symmetry breakdown [13,14]. Asymmetry is caused by the simple fact that in strongly correlated fermion systems, in contrast to Landau Fermi Liquid, the single-particle energy $\varepsilon(p)$ is T dependent. Thus $n(p,T)$ given by eq. (3) does not reproduce the step function in the low-temperature limit [11,12]. The asymmetric part of the conductivity,

$$\Delta \sigma_d(V) \equiv \frac{\sigma_d(V) - \sigma_d(-V)}{2},$$

(7)

can be calculated [14] and comparison with experiments [18,19] leads to good agreement. The estimate is

$$\Delta \sigma_d(V) \sim \frac{V}{2T} \frac{p_f - p_i}{p_f}. \tag{8}$$

From eq. (8) it follows that a fairly large asymmetry is obtained when $(p_f - p_i)/p_f \approx 1$. When a magnetic field $B$ is applied the Landau Fermi-Liquid behavior is restored, particle-hole asymmetry is eliminated, and therefore the asymmetric part of the differential conductivity disappears [13,14,16]. In other words, the particle-hole symmetry is macroscopically broken, or $CP$ is violated, in the absence of applied magnetic fields. Conversely, the application of a magnetic field restores both the particle-hole symmetry and the Landau Fermi-Liquid state. This agrees with the experimental facts collected in measurements on YbCu$_{5-x}$Al$_x$ [13,19].

Now we are in a position to formulate our approach to the baryon asymmetry of the universe. As it cools down, the universe behaves as a non-Fermi Liquid; one of the manifestations of such systems is a strongly correlated Fermi system exhibiting FCQPT at $T=0$ as is shown in fig. 2. We suggest that this model describes the particle-antiparticle content of the universe. As seen from fig. 3, at finite temperatures baryon-antibaryon asymmetry emerges as an inherent property of the system located above the quantum critical line. The asymmetry results from the distortion of the Fermi surface, in other words from the deviation of the distribution function $n(p)$ from the step function at low temperature. At lowering temperature, the system approaches the quantum critical line and, correspondingly, the asymmetry increases. Details of this increase depend on the model chosen; the very existence and the universal qualitative behavior of the asymmetry is of great significance.

The picture for explaining baryon asymmetry emerging from the above is as follows. The initial excited state corresponding to the big bang with extremely high temperature possesses matter-antimatter symmetry. At the end of the inflation stage the dark matter emerges producing baryons (holes) and antibaryons (quasiparticles). At this stage the temperature is high and the chemical potentials of baryons and antibaryons are zero, so that the asymmetry is also zero. As the temperature drops the state with the baryon (hole) asymmetry is formed. As a result, the universe approaches the quantum critical line which corresponds to the state with the maximum baryon (hole) asymmetry as it is seen from fig. 3. This state is the eigenstate of the fully symmetric microscopic Hamiltonian with the eigenvalue lower than the one for the state with matter-antimatter symmetry and contains the visible matter—baryons (holes), and the dark matter in the vicinity of the visible matter. In our approach the visible matter is represented by the excitations (holes) of the dark matter. As there is almost no interaction between holes and the Fermi sea, we conclude that there is no direct interaction between dark matter and visible matter, that is the interaction if it exists is very weak. Then, proceeding along the universe–non-Fermi-Liquid analogy, since it takes about ten particles to create one quasiparticle [20], we estimate the ratio of the mass of visible matter to the mass of dark matter to be of order of ten which is close to the observed value $\Omega_{DM}/\Omega_b \approx 5$ [6,22]. The ground state of the universe which we identify with the dark energy is interpreted in our approach as the vacuum.

Another result which comes as a bonus of our universe–non-Fermi-Liquid analogy is the high entropy of the universe. As is seen from fig. 1, $n(k,T)$ of holes (the visible matter) even at $T \to 0$ is non-integer, $0 < n(k,T) < 1$. The entropy $S(n(p,T))$ is given by the well-known expression [20]

$$S(n(p,T)) = -2 \int [n(p,T) \ln(n(p,T)) + (1 - n(p,T))] \frac{dp}{(2\pi)^3}. \tag{9}$$

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It follows from fig. 1 and eq. (9) that the entropy of the system is finite at \( T \to 0 \): \( S(T \to 0) \to S_0 \).

Let us introduce \( S_B \), entropy per baryon, as \( S/x \) where \( S \) is given by eq. (9) and \( x \) is the number density of baryons. Then from eq. (9) it follows that \( S_B \sim 1 \). This observation immediately explains the high entropy of the visible matter [23]. We also conjecture that the observed violation of CP-symmetry, leading to the violation of \( T \)-symmetry and making the finite term of the entropy, \( S(T \to 0) = S_0 \), may resolve the well-known problem of the time arrow.

It might seem that the presence of the Fermi level contradicts the relativistic invariance and the CPT theorem since the Fermi level and the Fermi sphere related to it are not transformed according to the invariance. As we shall see, at high energies the degrees of freedom related to the Fermi sphere become irrelevant and the relativistic symmetry is preserved in our approach. Let us assume that there are substantial gaps in the energy scales separating different states of the evolution of the Universe. One of these gaps separates the state behind FCQPT which produces \( CP \) violation and the symmetrical state lying above FCQPT. Consider the linear response function \( \chi(q, \omega) \) [see, e.g., (24)]

\[
\chi(q, \omega) = \sum_n \left| \rho(q)_{nn}\right|^2 \left( \frac{2\omega_n}{(\omega + i\gamma)^2 - \omega_n^2} \right),
\]

(10)

Here \( \rho(q) \) describes the fluctuations of the quasiparticle density, \( \omega_n = E_n - E_0 \) is the difference between the ground-state and the excited-state energies. In our case \( E_n \)'s are the energy of the excited states behind FCQPT. If \( \omega \) is sufficiently high so that \( \omega \gg (E_n - E_0) \), then eq. (10) defines the linear response function of non-interacting particles [24]

\[
\chi(q, \omega \to \infty) = \frac{xq^2}{m_{\omega'}^2},
\]

(11)

where \( m \) is the mass of quasiparticle at \( \omega \gg \omega_n \). In our case, it means that at these high energies the system is located in the region of its phase diagram located well above the critical line shown in fig. 2 where the \( CP \) symmetry is restored. Relativistic invariance is not yet restored since the very existence of quasiparticles ensures that the Fermi sphere still remains a relevant surface in phase space. Now we take into account the fact that the linear response function of this system is again given by eq. (10) with new \( \omega_n' \). Again at \( \omega \gg \omega'_{n} \) the response function is given by eq. (11) formed by particles the existence of that is not related to any Fermi sphere. Going along this way, we ascend to a level at which the inflation takes place and the relevant degrees of freedom are now not quasiparticles but particles of the Standard Model. As a result, we conclude that at elevated energies \( \omega \) the irrelevant degrees of freedom vanish and both the relativistic invariance and the \( CPT \) symmetry emerge. The details of the reappearance of relativistic invariance are defined by the concrete model chosen to describe baryogenesis.

The attractive feature of the scenario discussed above is in its “conservative” character—in order to introduce the matter-antimatter asymmetry it may suffice to suggest a mechanism based on the analysis of quantum phase transitions in Fermi systems. There is no need to introduce baryon number non-conservation or \( CP \)-violating interactions, as well as to invoke any extension of the standard cosmological model—ordinary quantum mechanics and quantum statistics applied to a multifermion system guarantee that the system starts from the symmetric state and at decreasing temperatures arrives at a maximally asymmetric state. This universal feature is present in strongly correlated Fermi systems with fermion condensation quantum phase transitions; the details depend on the concrete microscopic Hamiltonian postulated. We note that typical current-current interactions lead to the formation of flat bands [25]. These located at the Fermi surface, being topologically protected from interaction and other disturbances, lead to the robustness of the generic properties of the quantum vacuum generated by their existence [8–10]. Because of the universal features of our model, we have not concentrated on a particular picture that follows from some concrete dynamics, nor pursued the best quantitative description as the goal by itself. Rather, we demonstrate an opportunity to explain baryon asymmetry from the very general physics principles. Let us stress that the quasiparticle-hole asymmetry which manifests itself at the macroscopic scale is observed in experiments on heavy-fermion metals and analyzed theoretically is one of the few analogies of particle-antiparticle asymmetry observed in the universe, and thus deserves attention.

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