Stability and correlations in dilute two-dimensional boson systems

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The hyperspherical adiabatic expansion method is used to describe correlations in a symmetric boson system rigorously confined to two spatial dimensions. The hyperangular eigenvalue equation turns out to be almost independent of the hyperradius, whereas the solutions are strongly varying with the strength of the attractive two-body potentials. Instability is encountered in hyperangular, hyperradial, and mean-field equations for almost identical strengths inversely proportional to the particle number. The derived conditions for stability are similar to mean-field conditions and closely related to the possible occurrence of the Thomas and Efimov effects. Renormalization in mean-field calculations for two spatial dimensions is probably not needed.

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Introduction. Lower dimensions than three are necessary in several branches of physics, for instance surface physics, semiconductor physics, artificial atoms, and quantum dots. The advanced tools of well-controlled external fields, used in atomic and molecular physics to manipulate the effective interactions, allow confinement of the systems to lower dimensions \[1, 2\]. The experimental investigations employ continuous variation of the dimension by use of tunable deformed external fields.

The basic properties vary dramatically with the dimensionality of the system, as highlighted in two dimensions \((2D)\) where the centrifugal s-wave barrier is negative for two particles and even an infinitesimally small attraction provides a bound state \[3, 4\]. The stability is strongly dependent on the deformation or the effective dimension of the confining potential \[3, 4, 5\]. The simplest \(N\)-body structures are the Bose-Einstein condensates for identical bosons. These systems are dilute, weakly interacting, and well described by mean-field models \[8, 9\] with an interaction strength adjusted to reproduce the low-energy scattering cross section in the Born approximation. The interaction in \(2D\) is then obtained from short- or zero-range three-dimensional \((3D)\) potentials restricted to the mean-field Hilbert space \[10, 11\].

Inclusion of correlations in the wave function prohibits this renormalization. Instead a finite short-range potential with the correct scattering length should be used. If the correlations are appropriately accounted for, the large-distance behavior must come out correctly with the realistic interaction \[12, 13\]. The consequences for lower dimensions are not yet investigated. The huge difference between two- and three-body properties in two and three dimensions is most likely more pronounced for \(N\)-body systems. Experimental results from varying dimensionality are easier to interpret if the limits are known.

The purpose of this letter is to (i) formulate a framework for investigations of correlated structures under strict \(2D\) confinement for \(N\) interacting identical bosons in an external harmonic field, (ii) derive stability conditions in terms of particle number and two-body interaction properties, (iii) extract the basic features of the solutions, and (iv) provide a conceptual link between the successfully renormalized mean-field models and the correlated solutions with the bare (effective) interaction.

Theoretical method. We shall briefly sketch the method to establish the notation and the pertinent formulae for two dimensions. We follow the derivation for three dimensions given in \[14\]. The \(N\) identical bosons have masses \(m\) and coordinates \(\vec{r}_i\). We use the hyperspherical adiabatic expansion method where the only length coordinate is the hyperradius \(\rho\) defined by

\[
\rho^2 \equiv \frac{1}{N} \sum_{i<j}^N \frac{r_{ij}^2}{\bar{R}} = \frac{N}{NR^2} \sum_{i=1}^N r_i^2 - NR^2 ,
\]

where \(r_{ij} = \vec{r}_i - \vec{r}_j\) and \(\bar{R} = \sum_i r_i/N\) is the center-of-mass coordinate. The remaining \(2N - 3\) relative coordinates in two dimensions are angles where we define \(\alpha_{ij}\) related to the size of \(r_{ij}\) by \(r_{ij} \equiv \sqrt{2}\rho \sin \alpha_{ij}\). If permitted in the context, we shall omit the indices \(ij\).

The center of mass separates out and we only need to deal with relative coordinates. The related volume element is \(\rho^{2N-3} d\rho \sin \alpha \cos^{3N-5} d\alpha d\theta d\Omega_{N-2}\), where \(\alpha = \alpha_{ij}\), \(\theta\) describes the direction of \(\vec{r}_{ij}\) and \(\Omega_{N-2}\) denotes the remaining angular part of the volume element corresponding to the last \(N-2\) relative vector coordinates.

An external harmonic potential \(m\omega^2 \sum_i r_i^2/2\) of angular frequency \(\omega\) is by use of eq. \[11\] divided into a center-of-mass part and a hyperradial part. The relative Hamiltonian is then separated into a hyperradial part and a
The corresponding radial equation is then

$$\hat{H} = \frac{\hbar^2}{2m} \left( -\frac{1}{\rho^{2N-4}} \frac{\partial}{\partial \rho} \rho^{2N-3} \frac{\partial}{\partial \rho} + \frac{\rho^2}{b_i^2} + \hat{h}_\Omega \right),$$

(2)

where all differential angular dependence, except $\alpha$, is collected in $D_{\text{angle}}$. The trap length $b_i$ is given by $b_i^2 \equiv \hbar/(m\omega)$. The two-body interaction $V_{ij}$ is of short range, e.g. a Gaussian $V_0 \exp(-r_{ij}^2/b^2)$ or a square well $V_0 \Theta(r_{ij} < b)$, where $\Theta$ is the truth function.

The reduced potentials are given by

$$V = \rho \left( \frac{N - 3}{2} \right)^2 \left( N - \frac{5}{2} \right) - \frac{\rho^2}{b_i^2}.$$  

(3)

$$\hat{h}_\Omega = \hat{P}^2 + D_{\text{angle}} + \frac{2m \rho^2}{\hbar^2} \sum_{i<j} N V(r_{ij}),$$

(4)

where $\hat{P}$ and $\Theta$ are the momentum operator and the angular average, consisting of three terms, i.e. the angular average of the interaction $V_{ij}$.

The radial wave function $\Psi(\rho, \Omega)$ obeys the Schrödinger equation

$$\hat{H} \Psi(\rho, \Omega) = E \Psi(\rho, \Omega),$$

(5)

where $E$ is the energy. We write $\Psi$ as an adiabatic expansion [3, 14] where the first term is

$$\Psi(\rho, \Omega) = \rho^{-\frac{(2N-3)/2}{2}} f(\rho) \Phi(\rho, \Omega),$$

(6)

with the hyperradial volume element explicitly extracted.

The angular wave function $\Phi(\rho, \Omega)$ is for fixed $\rho$ an eigenfunction of $\hat{h}_\Omega$ with the eigenvalue $\lambda(\rho)$, i.e.

$$\hat{h}_\Omega \Phi(\rho, \Omega) = \lambda(\rho) \Phi(\rho, \Omega).$$

(7)

The corresponding radial equation is then

$$-\frac{\hbar^2}{2m} \frac{d^2}{d\rho^2} + U(\rho) - E \right) f(\rho) = 0,$$

(8)

$$\frac{2mU(\rho)}{\hbar^2} = \lambda(\rho) \rho^2 + \frac{(2N - 3)(2N - 5)}{4\rho^2} + \frac{\rho^2}{b_i^2},$$

(9)

where the adiabatic potential $U$ is a function of the hyperradius consisting of three terms, i.e. the angular average $\lambda(\rho)$ of the interactions and kinetic energies, the generalized centrifugal barrier, and the external field.

For large particle distances only relative $s$ waves contribute. With a Faddeev decomposition of the angular function only the dependence on distance $\alpha$ is left, i.e.

$$\Phi(\rho, \Omega) = \sum_{i<j} N \phi(\rho, \alpha_{ij}) \equiv \sum_{i<j} N \phi(\rho, \alpha_{ij}) \sin^{2N-2} \alpha_{ij} \cos^{N-2} \alpha_{ij},$$

(10)

where we again explicitly extracted the square root of the volume element.

The integro-differential equation for $\phi(\rho, \alpha)$ is obtained from eq. (11) by integrating over all other angles than $\alpha_{12} = \alpha$, denoted by $\tau$, i.e.

$$\left(-\frac{\partial^2}{\partial \tau^2} + v_1 + v_2 - \lambda \right) \phi(\rho, \alpha) = \int d\tau G.$$

(11)

where $G = G(\tau, \alpha)$ is linear in both $\phi$ and $V$ [14], and the reduced potentials are given by

$$v_1 = \frac{(2N - 5)(2N - 7)}{4} \tan^2 \alpha - \frac{\cot^2 \alpha}{2} - \frac{4N - 9}{2},$$

(12)

$$v(\alpha) = 4a_B \left( \frac{\rho}{b^2} \right)^2 \exp(-2(\rho\sin\alpha/b^2)),$$

(13)

$$v_1(\alpha) = 2a_B \left( \frac{\rho}{b^2} \right)^2 (N - 3)^2(N - 2)$$

$$\times \int dx \exp [2(x - 1)(\rho/b \cos \alpha)^2] \approx \frac{(N - 3)^2(N - 2)a_B}{\cos^2 \alpha},$$

(14)

$$v_2(\alpha) \approx \frac{4}{3} v_1(\alpha)(1 - \frac{1}{3} \tan^2 \alpha)N^{-4} \Theta(\alpha < \pi/3),$$

(15)

$$a_B = \frac{m}{2\pi \hbar^2} \int V(r) d\theta dr dr = \frac{mV_0 b^2}{2\hbar^2}.$$  

(16)

The last expression for $a_B$ is valid both for Gaussian and square-well potentials $V$. The approximations for $v_1$ and $v_2$ are very accurate for $\rho \cos \alpha \gg b$. For $\alpha = \pi/2$ we get exactly $v_1(\pi/2) = 2(N - 3)(N - 2)a_B(\rho/b^2)^2$, and $v_2$ is very small.

For a short-range interaction the right-hand side of eq. (11) is independent of $\rho$ as well as $v_1$ and $v_2$ when $\alpha$ is not too close to $\pi/2$. The only $\rho$ dependence is then through $\phi$, which approaches a zero-range interaction in $\alpha$ as $\rho$ increases. The eigenvalue $\lambda(\rho)$ is therefore expected to be constant in large ranges of $\rho$. These features are unique for two dimensions.

**Stability conditions.** For comparison we first consider the mean-field approximation in two dimensions. This was investigated in details in [12] with a potential derived from a three-dimensional zero-range potential by [16]. Neglecting the logarithmic energy dependent term in [12, 16] only a two-dimensional zero-range potential remains precisely as the delta-function limit of our Gaussian short-range potential. In general, for an interaction of short range, i.e. small $b$, the differential equation is in dimensionless quantities given by

$$\left(-\frac{\partial^2}{\partial \tau^2} - \frac{1}{x} \frac{\partial}{\partial x} + \left( \frac{b}{b_i} \right)^4 x^2 + 2a_B(N - 1)|f_m(x)|^2 - \epsilon \right) f_m(x) = 0,$$

(18)

where $x \equiv r/b$ and $\epsilon \equiv 2mE_m b^2/\hbar^2$ are measures of the single-particle mean-field coordinate $r$ and energy $E_m$. The radial wave function $f_m$ is approximated by a Gaussian, i.e. $f_m = \exp[-x^2/(2b^2)]/(\sqrt{\pi}d)$ renormalized as $\int d\theta dx |f_m(x)|^2 = 1$ as in [12]. The corresponding energy per particle $\epsilon$ is then as a function of $d$ given by

$$\epsilon = \left( \frac{b}{b_i} \right)^4 d^2 + \frac{1}{d^2} \left[ 1 + a_B(N - 1) \right],$$

(19)

which only has a minimum when

$$a_B(N - 1) > -1.$$  

(20)
Then the energy and the Gaussian width are
\[ d = \frac{b_t}{b} \left[ 1 + (N - 1)a_B \right]^{1/4}, \quad (21) \]
\[ \epsilon = 2 \left( \frac{b}{b_t} \right)^2 \sqrt{1 + (N - 1)a_B}. \quad (22) \]

These results coincide for \( N - 1 \approx N \) and \( a_B = \bar{g} \) with those derived in [14]. If the interaction strength in an experiment suddenly is changed from repulsive to an attractive value \( a_B \), the new state of the \( N \)-body system can only be stable for particle numbers smaller than the critical value \( N_c = 1 - 1/a_B \). For \( N > N_c \) the motion is towards larger densities and either a total collapse or a reduction of particles in the gas caused by molecular recombination.

Thus, only repulsive or very weakly attractive potentials provide stable mean-field solutions. The basic reason is that even infinitesimally small attractions bind two particles in two dimensions. The corresponding two-body Schrödinger equation with wave function \( \psi \) is
\[ \left( -\frac{\partial^2}{\partial x^2} - \frac{1}{x} \frac{\partial}{\partial x} + 2a_B \left( \frac{V(x) - E_2}{V_0} \right) \right) f_2(x) = 0, \quad (23) \]
which for a weakly attractive interaction has the bound-state energy \( E_2 = -4\hbar^2/(mb^2) \exp(2/a_B - 2\gamma) \) and the mean-square radius \( \langle r^2 \rangle = 2\hbar^2/(3mE_2) \), where \( \gamma \) is Euler’s constant, see [3].

![Graph](image)

**FIG. 1:** The centrifugal terms \( v_c \) and \( v_1 \) from eqs. (12) and (14) as function of \( \alpha \). The thick lines show \( v_c + v_1 \) for \( N = 20 \), various values of \( a_B \), and \( \rho/b = 10^3 \) unless otherwise indicated. The thin, solid lines show the contributions from \( v_1 \) and \( v_c \) for \( a_B = -0.02 \).

The radial potential in eq. (9) depends crucially on \( \lambda \) determined from eq. (11) which in turn is dominated by the terms \( v_c \) and \( v_1 \) shown in fig. 1. When \( \alpha \) approaches \( \pi/2 \) the approximation in eq. (15) and \( \epsilon \), eq. (14) both diverge as \( \cos^{-2}\alpha \approx (\pi/2 - \alpha)^{-2} \). Thus, when \( a_B < -(5/2)(N - 7/2)/(N - 2)(N - 3)^2 \approx -1/(N - 2) \) an attractive pocket inevitably appears for large \( \rho \). This divergence at the point \( \alpha = \pi/2 \) disappears when the exact expression in eq. (14) is used for \( v_1 \). Then the lowest eigenvalue \( \lambda \) would be finite and proportional to \( \rho^{-2} \). These solutions correspond to many particles close together which probably violates our assumption of s-wave dominance in the wave function. To avoid this divergence at large \( \rho \) the strength of the attraction must be limited by \( a_B(N - 2) > -1 \), which is almost identical to eq. (20). Thus, remarkably enough the mean-field stability condition is precisely also obtained from the angular potential when \( N - 2 \approx N \).

For small \( \alpha \) the angular centrifugal term \( v_c \) from eq. (12) diverges as \( -\alpha^2/4 \). This is the limit rigorously separating attractions leading to either no bound states or infinitely many bound states of Thomas or Efimov character [5]. Thus, a small two-body attraction \( v \) is for \( \rho \) sufficient to bind a state in the pocket at small \( \alpha \). This is seen by substituting \( x = \sqrt{2\rho \alpha}/b \) in eq. (11) when only the potential \( v \) and \( v_c \) are included.

The energy and the mean-square radius of the solution becomes \( \lambda(\rho) \rightarrow 2m\rho^2E_2/b^2 \) and \( \langle \alpha^2 \rangle \rightarrow 2/(3|\lambda|) = 1/12(b/\rho)^2 \exp(2\gamma - 2/a_B) \). If the size in \( \alpha \) space has to be smaller than unity, \( \rho/b \) must exceed \( \exp(-1/a_B) \) which is huge when \(-1/N < a_B < 0\), i.e. the interaction is attractive but allows physical solutions in agreement with eq. (20). Thus, the diverging \( \lambda \) corresponding to the bound two-body state is never encountered because either the interaction is too attractive leading to solutions in the pocket at large \( \alpha \), or the interaction is too weak to bind at small \( \alpha \) for \( \rho \) values less than the trap length. Therefore diatomic recombination is unlikely.

In any case the angular potential provides two types of minima at small and large \( \alpha \)-values, respectively. The large \( \alpha \)-behavior corresponds to the stability condition obtained by mean-field calculations. The structure attempts to maximize the two-body attractions by a rather similar distance between all particles confined to the volume allowed by the given hyperradius, i.e. two particles are far apart but all others are correspondingly close.

The small \( \alpha \)-behavior turns out to be unimportant for stability, because the extremely weak binding requires a spatially extended wave function attempting to reach beyond the confining boundaries of the trap. A huge trap would in principle allow binding by this pocket, which corresponds to a structure with one two-body bound state and consequently this does not resemble a condensed state. Such a structure would not be found by mean-field calculations although the two-body bound state always is present for overall attraction in two dimensions.

The deeper-lying reason for not populating this two-body bound state seems to be related to the choice of strength for a two-dimensional zero-range interaction.
The 2D scaling property of a finite range (Gaussian) interaction matches with maintaining fixed values of both the scattering length and the two-body bound state energy. Thus, both mean-field zero-range and finite-range correlation calculations can use the strength determined to reproduce a given value of the scattering length in the Born approximation. This is not possible in three dimensions, see the renormalization paragraph below.

**Radial solutions.** To solve the hyperradial eq. (9) we need $\lambda$. For $\rho = 0$ only $v$ is present in eq. (11). The free angular solutions are

$$\tilde{\phi}_\nu = P_{\nu}^{(0,N-3)}(\cos 2\alpha), \quad \lambda_\nu^{(f)} = 4\nu(\nu + N - 2),$$

(24)

where $P_{\nu}^{(0,N-3)}$ is the Jacobi function and $\nu = 0, 1, 2, 3, \ldots$ is a non-negative integer. For very small $\rho$ perturbation then gives $\lambda \approx \lambda_\nu^{(f)} + 2a_B N(N - 1)(\rho/b)^2$.

Increasing $\rho$ for a repulsive or weakly attractive potential leaves approximately the free wave function, but the energies are shifted from the contribution of the interaction. In first-order perturbation with the free wave functions the eigenvalues are denoted $\lambda_\nu^{(d)}$, where the lowest for $\rho \gg b$ is found to be

$$\lambda_{\nu=0} = a_B N(N - 1)(N - 2).$$

(25)

When the attraction is stronger, both wave function and energy change. However, the only $\rho$ dependence in the angular equation is in $v$ and in $v_1$ when $\alpha \approx \pi/2$. Still, the eigenvalues $\lambda$ are essentially $\rho$ independent when $b \ll \rho \ll \exp(-1/a_B)$, but perhaps lower than $\lambda_\nu$. The eigenvalues must be obtained by numerical calculations.

The behavior of the angular eigenvalues in the interesting range of $\rho$ values are illustrated in fig. 2. They can be parametrized rather well by

$$\lambda(\rho) \approx \frac{\lambda_\nu^{(f)} - \lambda_\nu^{(d)}}{1 + (\rho/\rho_0)^2} + \lambda_\nu^{(d)},$$

(26)

$$\left(\frac{\rho_0}{b}\right)^2 = -\frac{2a_B N(N - 1)}{\lambda_\nu^{(f)} - \lambda_\nu^{(d)}} = \frac{2}{N - 2},$$

(27)

where the last equality only holds for $\nu = 0$. Thus, $\rho_0$ is independent of $a_B$ and inversely proportional to $N$. In these expressions we must for somewhat stronger attraction replace $\lambda_\nu^{(d)}$ by the solution obtained numerically. The curve for $a_B = -0.0559$ in fig. 2 suddenly decreases dramatically when $\rho/b \approx 0.03$. This reflects the pronounced effect of the attractive pocket for large $\alpha$ appearing in fig. 2 for large $\rho$.

The radial potential in eq. (10) has the same simple structure as the mean-field energy in eq. (11). Analogously stable solutions only exist for constant $\lambda$ when $\lambda \gg -(N - 3/2)(N - 5/2)$. Using $\lambda_\nu^{(d)}$ this implies collapse when $a_B < -(N - 3/2)(N - 5/2)/[N(N - 1)(N - 2)] \approx -1/(N + 1)$. This condition is again very similar to the mean-field condition in eq. (20), but now less surprising since the hyperradial and the mean-field radial equations both describe the same overall size dependence.

In fact a constant $\tilde{\phi}$ in eq. (10) corresponding to $\nu = 0$ in eq. (24) leaves the total wave function as a function of only the hyperradius $\rho$. For Gaussian single-particle wave functions this is precisely identical to a Gaussian $\rho$ dependence of $f(\rho)$. For all other than Gaussian radial shapes of single-particle and hyperradial wave functions the connection cannot be made explicit. In the present formulation correlations, and thereby clear-cut deviations from the mean-field approximation, are included through the angular dependence of $\Phi$. These effects beyond the mean-field are perhaps most striking by the ability to describe simultaneously both states resembling a condensate and two-body bound states within the $N$-body system. This only requires two adiabatic potentials.

For $\rho$ values confined by the external field, we can insert the constant $\lambda_\nu^{(d)}$ given by eq. (25) in eq. (11) which then is the radial harmonic-oscillator equation for an effective angular-momentum quantum number number $l^*$ defined by $l^*(l^* + 1) = \lambda + (N - 3/2)(N - 5/2)$ or equivalently $l^* = -0.5 + [\lambda + (N - 3/2)(N - 5/2) + 1/4]^{1/2}$. When $l^*(l^* + 1) > -1/4$, the solutions are well defined and characterized by the corresponding real values of $l^* > -1/2$. The eigenfunctions and eigenvalues are $(\rho/b_i)^2 L_{l^*}^{(l^* - 1/2)}[\rho/b_i^2] \exp[-0.5(\rho/b_i)^2]$ and $\hbar \omega(2n + l^* + 3/2)$, where $L_{l^*}$ are generalized Laguerre polynomials and $n$ is a non-negative integer. The ground-state energy is then $\hbar \omega(l^* + 3/2)$ and the level spacing is precisely $2\hbar \omega$ as for harmonic-oscillator excitations maintaining parity.
Renormalization of the interaction strength. In mean-field 3D zero-range calculations the employed interaction is renormalized as $4\pi a_s^3 \delta(\vec{r}) \hbar^2 / m$, where $a_s$ is the true s-wave scattering length derived from the low-energy two-body scattering properties. A short-range Gaussian representation of the delta function with a strength adjusted to reproduce the same scattering length $a_s$ leads to totally different mean-field results. In order to get the desired results in the mean-field approximation with a Gaussian interaction, it is necessary to renormalize the Gaussian strength such that the Born approximation to the scattering length is reproduced instead of the true scattering length $\lambda_D$. Thus the interaction in 3D mean-field calculations must reproduce the correct scattering length by using the Born approximation.

This renormalization procedure guarantees three-dimensional mean-field results in agreement with the use of the above renormalization of the zero-range interaction. In the hyperspherical adiabatic method the large-distance asymptotic behavior of the angular eigenvalue should approach the expectation value for the mean-field zero-range interaction using the free wave function, i.e. the proper renormalization is seen to be equivalent to $\lambda_D \to \lambda_D^{(3)}$ for large $\rho$. Thus, to the extent the large-$\rho$ behavior of $\lambda_D$ is well described by $\lambda_D^{(3)}$, we conjecture that this renormalization is not needed in 2D calculations.

Let us now average a 3D Gaussian interaction over the $z$-coordinate. Then we find that the Born approximation to the scattering length of the new 2D-interaction is $a_B \propto a_B^{(3)} / b_z$, where $b_z$ is the trap length in the $z$-direction and $b \ll b_z$. Considering refs. [18] and neglecting the logarithmic terms in their results, the present result is precisely the same for zero-range interactions when the true scattering length is substituted by the Born approximation. Thus, in 3D mean-field calculations the Born approximation to the scattering length of the interaction potential, e.g. a Gaussian, is replaced by the true scattering length, whereas no replacement is needed in two dimensions.

The stability condition, $Na_B > -1$, is equivalent to $Na_B^{(3)}/b_z > -1$, which coincides with the condition obtained by variation of the Gaussian length parameters for a deformed trap for a zero-range interaction where the 3D scattering length is substituted by its Born approximation [17]. Again this indicates that renormalization is not needed for 2D-calculations.

When the 3D scattering length is small compared to $b_z$, the 2D and 3D coupling strengths for a zero-range interaction are related by $g_{2D} = g_{3D} / (b_z \sqrt{2\pi})$. When the axial confinement is strong, i.e. $b_z \ll a_s$, $g_{2D}$ becomes density dependent [16].

Treating correlations with the hyperspherical adiabatic method seems to require renormalization only in three dimensions where the mean-field and correlated results only coincide for differently renormalized interactions. For 2D-calculations the same interaction seems to produce the same large-distance mean-field and correlated results. Thus, no renormalization seems to be required in 2D.

Conclusions. We investigated symmetric $N$-boson systems in the rigorous two-dimensional limit for attractive two-body interactions. We derive the stability condition where the product of particle number, strength, mass, and square of the range has to be sufficiently small. The condition is the same for both mean-field and correlated solutions and independent of the external field. Diatomic recombination into two-body bound states is not likely, since the two-body bound state would extend beyond the trap length for potentials sufficiently weak to allow stable solutions. Thus, these weakly attractive potentials, inevitably binding two particles, are able to support stable condensates. These features are completely different from three-dimensional properties. These rigorous two-dimensional results cannot be compared to other calculations where the effective interactions are repulsive.

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