Semi-supervised Calibration of Risk with Noisy Event Times (SCORNET) Using Electronic Health Record Data

Yuri Ahuja, Liang Liang, Selena Huang, Tianxi Cai

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Abstract

Leveraging large-scale electronic health record (EHR) data to estimate survival curves for clinical events can enable more powerful risk estimation and comparative effectiveness research. However, use of EHR data is hindered by a lack of direct event times observations. Occurrence times of relevant diagnostic codes or target disease mentions in clinical notes are at best a good approximation of the true disease onset time. On the other hand, extracting precise information on the exact event time requires laborious manual chart review and is sometimes altogether infeasible due to a lack of detailed documentation. Current status labels – binary indicators of phenotype status during follow up – are significantly more efficient and feasible to compile, enabling more precise survival curve estimation given limited resources. Existing survival analysis methods using current status labels focus almost entirely on supervised estimation, and naive incorporation of unlabeled data into these methods may lead to biased results. In this paper we propose Semi-supervised Calibration of Risk with Noisy Event Times (SCORNET), which yields a consistent and efficient survival curve estimator by leveraging a small size of current status labels and a large size of imperfect surrogate features. In addition to providing theoretical justification of SCORNET, we demonstrate in both simulation and real-world EHR settings that SCORNET achieves efficiency akin to the parametric Weibull regression model, while also exhibiting non-parametric flexibility and relatively low empirical bias in a variety of generative settings.
1 Introduction

The Electronic Health Record (EHR) has in recent years become an increasingly available source of data for clinical and translational research (Kohane and others, 2012; Hripcsak and Albers, 2012; Miotto and others, 2016). Comprising heterogeneous clinical encounters including diagnostic and procedural billing codes, lab tests, prescriptions, and free text clinical notes for millions of patients, these rich data offer abundant opportunities for in silico epidemiological analysis. One application that has garnered recent interest is estimation of population disease risk within EHR patient cohorts, which can enable more powerful and precise estimation of real-world disease risks as well as comparative effectiveness analysis of alternative treatment strategies (Hodgkins and others, 2017; Dean and others, 2003; Liu and others, 2018; Panahiazar and others, 2015; Steele and others, 2018).

Several studies have had success estimating time to death within rule-defined disease cohorts (Panahiazar and others, 2015; Steele and others, 2018). However, estimating the temporal risk of developing a disease is a more challenging task due to EHR’s lack of direct observations of either disease status or the timing of disease onset. Convenient proxies of disease status or onset time based on readily available features such as International Classification of Disease (ICD) codes often exhibit low specificity and systematic temporal biases, potentially yielding highly biased disease risk estimators if used as event time labels (Cipparone and others, 2015; Uno and others, 2018). On the other hand, extracting precise information on disease outcomes requires labor-intensive manual chart review, which is particularly challenging for event times since the event may occur outside of the hospital system and only be mentioned during follow-up visits. It is thus only practically feasible to annotate the current status $\Delta = I(T \leq C)$ of the event time $T$, where $C$ is the follow up time.

In this paper, we consider the problem of estimating the disease risk $F(t) = P(T \leq t)$ when only a small number of labels on $\Delta$ and a large quantity of unlabeled EHR features $W$, including proxies of $T$, are available. Supervised survival curve estimation with current status data on $\{\Delta, C\}$ is well established in the statistical literature with several available parametric, semi-parametric and non-parametric procedures (Vardi, 1982; Huang,
1996; van der Laan and Robins, 1998; van der Laan and Jewell, 2003; Lin and others, 2019, e.g.). For example, van der Laan and Robins (1998) proposed a non-parametric, locally efficient estimator via inverse probability of censoring weighting (IPCW), assuming that (1) $T$ and $C$ are conditionally independent given some informative baseline covariates $Z_0 \subset W$ (e.g., age, sex, etc.) and (2) a consistent estimator for the conditional density of $C | Z_0$ is available. However, these existing estimators do not leverage unlabeled EHR feature information such as time to first surrogate ICD code, which may greatly improve risk estimation precision.

Since $W$ may be highly predictive of $T$, the estimation of $S(t)$ can potentially be improved via semi-supervised learning (SSL) leveraging both the small set of $\Delta$ observations in the labeled set and the EHR features $W$ in the unlabeled set. SSL has been shown to significantly mitigate bias and/or improve efficiency for various risk prediction applications (Chai and others, 2017; Liang and others, 2016; Bair and Tibshirani, 2004; Golub and others, 1999). For example, several studies employ semi-parametric models to impute event times in the unlabeled set for subsequent input into an outcome survival model alongside labeled data (Chai and others, 2017; Liang and others, 2016; Zhao and others, 2014; Uno and others, 2018; Hassett and others, 2017; Chubak and others, 2012; Choi and others, 2015; Kaji and others, 2019; Ruan and others, 2019; Ahuja and others, 2020a). While such imputation approaches may improve efficiency under correct specification of the imputation model, they are subject to significant bias if the imputation model is misspecified. In addition, these existing methods do not allow for use of current status labels for training. Other general augmented inverse probability weighting methods in the missing data literature (Seaman and White, 2013; Rotnitzky and Robins, 2014, e.g.) are not directly applicable here since the probabilities of labels being observed tend to zero in the SSL setting.

We address this shortcoming by proposing Semi-supervised Calibration of Risk with Noisy Event Times (SCORNET) for estimation of $S(t)$. SCORNET utilizes current status labels while also employing a robust semi-supervised imputation approach on the extensive unlabeled set to maximize survival estimation efficiency. To mitigate imputation bias and maximize efficiency gain from the unlabeled data, SCORNET utilizes a highly flexi-
ble semi-non-parametric kernel regression model with EHR features as covariates, which ensures the validity of
the resulting risk estimator without requiring the imputation model to hold. In addition to providing theoret-
ical justifications for the SCORNET estimator, we illustrate via simulation studies that SCORNET substantially
outperforms existing methods with regards to the bias-variance tradeoff. The rest of the paper is organized
as follows. In Section 2, we detail the SCORNET procedure along with its associated inference procedures. In
Section 3, we report risk estimation performance relative to existing methods in diverse simulation studies. To
further illustrate the utility of SCORNET in clinical applications, we apply it to a real-world EHR study estimat-
ing the risk of heart failure among rheumatoid arthritis patients in Section 4. Finally, in Section 5 we briefly
discuss the strengths, weaknesses, and potential applications of SCORNET.

2 Methods

2.1 Setup

Let \( T \) denote the event time for which we are interested in estimating a cumulative distribution function
\( F(t) = P(T \leq t) \) and survival function \( S(t) = 1 - F(t) \). In the EHR study we do not observe \( T \) but rather \( \Delta = I(T \leq C) \)
for a small labeled subset, where \( C \) is the follow up time with finite support \([0, \tau_c]\). For all subjects, we also
observe a set of baseline covariates \( Z_0 \) and longitudinal EHR features \( Z \). Since codes used in the EHR are often
highly sensitive but not specific, there often exists some filter variable \( F \in \{0, 1\} \) such that \( \Delta_i | (F_i = 0, W_i) = 0 \)
almost surely, where \( W_i = (Z_{0,i}^T, Z_i^T)^T \). Moreover, we assume that \((T, Z)C | Z_0\). We assume that data for analysis
consist of a small set of \( n \) current-status-labeled observations randomly selected among those with \( F = 1 \) along
with a larger set of \( N \) unlabeled observations: \( \mathcal{D} = \{D_i = (C_i, V_i \Delta_i, W_i, V_i, F_i)^T, i = 1, ..., N\} = \mathcal{L} \cup \mathcal{U} \), where
\( \mathcal{L} = \{(C_i, \Delta_i, W_i, 1, 1)^T : F_i = 1, V_i = 1, i = 1, ..., n\} \) and \( \mathcal{U} = \{(C_i, 0, W_i, 0, F_i)^T : V_i = 0, i = n + 1, ..., N\} \) with
\( \log(N)/\log(n) \rightarrow \nu_0 > 3/2 \) as \( n \rightarrow \infty \).
Since the censoring \( C \) may depend on \( Z_0 \), we follow the IPCW strategy of van der Laan and Robins (1998) to weight observations by

\[
\omega_{t,b}(C \mid Z_0) = \frac{K_b(C-t)}{f_c(t \mid Z_0)}
\]

where \( K_b(s) = K(s/b)/b \), \( f_c(t \mid Z_0) = dF_c(t \mid Z_0)/dt \), \( F_c(t \mid Z_0) = P(C \leq t \mid Z_0) \), \( K(\cdot) \) is some symmetric density function, and \( 0 < b = O(n^{-\nu}) \) is the bandwidth thereof with \( \nu \in (1/5, 1/3] \). IPCW enables consistent estimation of functionals of \( T \leq t \) and \( W \) since for any reasonable choice of function \( q(\cdot) \) and \( a, d \in (0, 1) \),

\[
E \{ \Delta_i^d q(W_i)|^\pi \omega_{t,b}(C_i \mid Z_{0,i}) \} = E \{ I(T_i \leq t)^d q(W_i)|^\pi \} + O(b^2).
\]

The IPCW estimator for \( \pi(t) = P(T_i \leq t \mid F_i = 1) \) proposed by van der Laan and Robins (1998) essentially corresponds to

\[
\hat{\pi}(t) = \frac{\sum_{i=1}^n \bar{F}_i \Delta_i \omega_{t,b}(C_i \mid Z_{0,i})}{\sum_{i=1}^n \bar{F}_i \omega_{t,b}(C_i \mid Z_{0,i})}
\]

with \( f_c(t \mid Z_0) \) in \( \omega_{t,b}(C_i \mid Z_{0,i}) \) replaced by a consistent estimator that converges faster than \( n^{-1/4} \), which is not difficult to achieve under reasonable modeling assumptions since \( C_i \mid Z_{0,i} \) can be estimated using the full data \( D \). To this end, we propose to derive an estimator for the conditional density \( f_c(t \mid Z_0) = \lambda_c(t \mid Z_{0,i})S_c(t \mid Z_{0,i}) \) by imposing a semi-parametric model for \( C \mid Z_0 \). Although many commonly employed models can be used since once again \( C \) is fully observed for all patients, we illustrate our proposal by focusing on the Cox proportional hazards model (Cox, 1972) under which

\[
\lambda_c(t \mid Z_{0,i}) = \lambda_{0c}(t)e^{\gamma^T Z_{0,i}} \quad \text{and} \quad S_c(t \mid Z_{0,i}) \equiv 1 - F_c(t \mid Z_{0,i}) = \exp \left\{ -\lambda_{0c}(t)e^{\gamma^T Z_{0,i}} \right\},
\]

where \( \lambda_c(t \mid Z_{0,i}) \) is the conditional hazard function of \( C_i \mid Z_{0,i}, \lambda_{0c}(t) \) is the unknown baseline hazard function, \( \lambda_{0c}(t) = \int_0^t \lambda_{0c}(s)ds \), and \( \gamma \) is the vector of unknown covariate effects.

### 2.2 SCORNET Estimation

As outlined in Figure 1, SCORNET consists of three steps: (1) estimating the conditional censoring distribution \( h(t \mid Z_0) \) using \( D \); (2) fitting an imputation working model for \( \pi(t \mid W) \equiv P(T \leq t \mid W, F = 1) \) using \( L \), denoting...
the estimate of $\pi(t \mid W)$ as $(t \mid W)$; and (3) estimating $S(t)$ by marginalizing $(t \mid W)^{\mathbb{F}} + \Delta (1 - \mathbb{F}) = (t \mid W)^{\mathbb{F}}$ via IPCW.

![Figure 1: Schematic of the SCORNET algorithm.](image)

### 2.2.1 Step 1: Estimate $f_c(t \mid Z_0)$ Under the Cox Model for $C \mid Z_0$

To estimate $f_c(t \mid Z_0)$, we fit the Cox model (2) to the full data $\mathbb{D}$ to obtain the partial likelihood estimator $\hat{\gamma}$ for $\gamma$. We subsequently estimate $\Lambda_0(t)$ and $\Lambda_{0c}(t)$ respectively as the standard Breslow estimator $\hat{\Lambda}_0(t)$ and the kernel-smoothed Breslow estimator $\hat{\Lambda}_{0c}(t)$ (Basha and Hoxha, 2019), where

$$
\hat{\Lambda}_{0c}(t) = \sum_{j=1}^{N} \frac{I(C_j \leq t)}{\sum_{i=1}^{N} I(C_i \geq C_j) \exp(\hat{\gamma}^T Z_{0,i})}, \quad \hat{\Lambda}_0(t) = \sum_{j=1}^{N} \frac{K_{a_N}(C_j - t)}{\sum_{i=1}^{N} I(C_i \geq C_j) \exp(\hat{\gamma}^T Z_{0,i})},
$$

and $a_N = O(N^{-\nu_c})$ with $\nu_c \in (1/5, 1/3]$. We then obtain

$$
\hat{\lambda}_c(t \mid Z_0) = \hat{\Lambda}_{0c}(t) \exp(\hat{\gamma}^T Z_{0,i}) \quad \text{and} \quad \hat{S}_c(t \mid Z_0) = \exp\left\{ -\hat{\Lambda}_{0c}(t) \exp(\hat{\gamma}^T Z_{0,i}) \right\},
$$

and we estimate $f_c(t \mid Z_{0,i})$ as $\hat{f}_c(t \mid Z_{0,i}) = \hat{\lambda}_c(t \mid Z_{0,i}) \hat{S}_c(t \mid Z_{0,i})$. Following standard asymptotic results for non-parametric kernel regression (Pagan and Ullah, 1999), it is not difficult to show that $\sup_{Z_{0,i}} |\hat{\lambda}_c(t \mid Z_{0,i}) - \lambda_c(t \mid Z_{0,i})| = o_p(1)$ as $N \rightarrow \infty$. 

Figure 1: Schematic of the SCORNET algorithm.
\[ Z_0(t) - f_c(t \mid Z_0) = O_p\{\log(N)^{1/2}(N\alpha_N)^{-1/2}\} = o_p(). \]

We denote the resulting estimate for the censoring weight as \( \hat{\omega}_{b_n}(t \mid Z_{0,i}) = K_{b_n}(C_i - t)/\hat{f}_c(t \mid Z_{0,i}). \)

### 2.2.2 Step 2: Estimate an Imputation Model \( \pi(t \mid W_i) \equiv P(T_i \leq t \mid W_i, \bar{F}_i = 1) \)

To leverage the unlabeled data, we fit a flexible imputation working model

\[
\pi(t \mid W_i) = g\{\alpha(t) + \beta_0(t)^T\hat{Z}_{0,i} + \beta(t)^TZ_i\} = g\{\theta(t)^T\hat{W}_i\}
\]

where \( Z_i \) denotes the EHR surrogate features, \( \theta(t) = (\alpha(t), \beta_0(t)^T, \beta(t)^T)^T \), and \( \hat{W}_i = (1, \hat{Z}_{0,i}, Z_i)^T \). Under (3), \( P(T_i \leq t \mid W_i, \bar{F}_i = 1) = g\{\theta(t)^T\hat{W}_i\} \), and hence we may estimate \( \theta(t) \) as \( \hat{\theta}(t) = (\hat{\alpha}(t), \hat{\beta}_0(t)^T, \hat{\beta}(t)^T)^T \), the solution to the IPCW estimating equation evaluated with \( L_n \)

\[
\sum_{i=1}^{n} \hat{\omega}_{i,b_n}(C_i \mid Z_{0,i})\bar{F}_i\hat{W}_i \left\{ \Delta_i - g\left(\hat{\theta}^T\hat{W}_i\right) \right\} = 0,
\]

where \( b_n = O(n^{-\nu}) \) with \( \nu \in (1/5, 1/2) \). In practice, \( b_n \) can be chosen via either standard cross-validation or heuristic plug-in values. For a future observation with filter status \( \bar{F}_i = 1 \) and covariates \( W_i \), we impute \( I(T_i \leq t) \) as the conditional risk \( \hat{\pi}(t \mid W_i) = g\{\hat{\theta}(t)^T\hat{W}_i\} \).

It is not difficult to show that \( \hat{\theta}(t) \) converges in probability to \( \hat{\theta}(t) \), the solution to the limiting estimating equation

\[
E\left[\hat{W}_i \left\{ I(T_i \leq t) - g(\theta^T\hat{W}_i) \right\} \mid \bar{F}_i = 1\right] = 0,
\]

which ensures that

\[
E\{\hat{\pi}(t \mid W_i) \mid \bar{F}_i = 1\} = P(T_i \leq t \mid \bar{F}_i = 1), \quad \text{where} \quad \hat{\pi}(t \mid W_i) = g\{\hat{\theta}(t)^T\hat{W}_i\}.
\]

regardless of the adequacy of the imputation model (3).

### 2.2.3 Step 3: Estimate \( F(t) \) by Marginalizing Imputed Risks

Finally, we marginalize the imputed values \( \hat{\pi}_{il} = \hat{\pi}(t \mid W_i) \forall \bar{F}_i = 1 \) and \( \Delta_i = 0 \forall \bar{F}_i = 0 \) to estimate \( F(t) \). Since \( \bar{F}_i \) depends on \( C_i \), we again employ IPCW to marginalize \( \hat{\pi}_{il}\bar{F}_i + \Delta_i(1 - \bar{F}_i) = \hat{\pi}_{il}\bar{F}_i \) and thereby construct our
final estimator for $F(t)$:

$$
\hat{F}(t) = \frac{\sum_{i=1}^{N} \{ \hat{\Delta}_i (1 - F_i) \} \hat{\omega}_{t,a_N} (C_i \mid Z_{0,i})}{\sum_{i=1}^{N} \hat{\omega}_{t,a_N} (C_i \mid Z_{0,i})} = \frac{\sum_{i=1}^{N} \hat{\Delta}_i \hat{\omega}_{t,a_N} (C_i \mid Z_{0,i})}{\sum_{i=1}^{N} \hat{\omega}_{t,a_N} (C_i \mid Z_{0,i})}.
$$

### 2.3 Inference for $\hat{F}(t)$

Following standard theory for non-parametric kernel regression (Pagan and Ullah, 1999), we show in the Supplementary Materials that $\hat{F}(t) \rightarrow P(T_i \leq t, F_i = 1) + P(T_i \leq t, F_i = 0) = P(T_i \leq t) = F(t)$ in probability under mild regularity conditions and correct specification of the censoring model regardless of the adequacy of the imputation model. Here, we note that for any $C \in \mathbb{R}$, under mild regularity conditions and correct specification of the censoring model regardless of the adequacy of the imputation model. Here, we note that for any $t \in [0, \tau_c]$, $0 = P(\Delta_i = 0 \mid F_i = 0, C_i = t, W_i)$ implies that $P(T_i \leq t \mid F_i = 0) = 0$. Furthermore,

$$
(nb_n)^{1/2} \left\{ \hat{F}(t) - F(t) \right\} = \left( \frac{b_n}{n} \right)^{1/2} \sum_{i=1}^{n} \hat{\omega}_{t,b_n} (C_i \mid Z_{0,i}) F_i \left\{ \Delta_i - \bar{\pi}(t \mid W_i) \right\} + o_P(1)
$$

and

$$
\sup_t |\hat{f}_c(t \mid Z_0) - f_c(t \mid Z_0)| = o_P().
$$

It follows that $(nb_n)^{1/2} \left\{ \hat{F}(t) - F(t) \right\}$ is asymptotically normal with mean 0 and variance

$$
\sigma^2(t) = R(K)E \left\{ V(t \mid Z_{0,i}) / f_c(t \mid Z_{0,i}) \right\},
$$

where

$$
V(t \mid Z_{0,i}) = E \left[ F_i \{ I(T_i \leq t) - \bar{\pi}(t \mid W_i) \}^2 \mid Z_{0,i} \right]
$$

and $R(K) = \int K(x)^2 dx$. Our derivation for the asymptotic distribution of $\hat{F}(t)$ can effectively ignore the variability associated with the estimation of censoring weights, which simplifies the asymptotic variance $\sigma^2(t)$. Importantly, $\sigma^2(t)$ decreases as the imputation model approximates $\pi(t \mid W_i)$ better since $V(t \mid Z_{0,i}) = E \left[ F_i \{ I(T_i \leq t) - \pi(t \mid W_i) \}^2 \mid Z_{0,i} \right] + E \left[ F_i \{ \pi(t \mid W_i) - \bar{\pi}(t \mid W_i) \}^2 \mid Z_{0,i} \right]$ decreases. To estimate $\sigma^2(t)$ in practice, one may construct a plug-in estimator,

$$
\hat{\sigma}^2(t) = \frac{b_n}{n} \sum_{i=1}^{n} \hat{\omega}_{t,b_n} (C_i \mid Z_{0,i}) \{ \Delta_i - g \left( \hat{\theta}^{(t)\top} \hat{W}_i \right) \}^2.
$$
By contrast, the supervised IPCW estimator that incorporates filter negative patients takes the form
\[
\hat{P}(t) = \frac{\sum_{i=1}^{N} \hat{\lambda}(t) \hat{F}_i + \Delta_i (1 - \hat{F}_i) \hat{\omega}_{t,a_N}(C_i \mid Z_{0,i})}{\sum_{i=1}^{N} \hat{\omega}_{t,a_N}(C_i \mid Z_{0,i})} = \frac{\sum_{i=1}^{N} \hat{\lambda}(t) \hat{F}_i \hat{\omega}_{t,a_N}(C_i \mid Z_{0,i})}{\sum_{i=1}^{N} \hat{\omega}_{t,a_N}(C_i \mid Z_{0,i})}.
\]

The asymptotic variance of \((nb_n)^{1/2}(\hat{P}(t) - F(t))\) is then \(\sigma^2(t) = R(K)E\left[\mathbb{V}(t \mid Z_{0,i})/f_c(t \mid Z_{0,i})\right]\), where
\[
\mathbb{V}(t \mid Z_{0,i}) = E \left[\mathbb{V}_i\{I(T_i \leq t) - \pi(t)\}^2 \mid Z_{0,i}\right].
\]
The variance \(\sigma^2(t)\) is equivalent to that of SCORNET if and only if the feature set \(W\) is uninformative for \(T\) (i.e. \(TW\)). Supervised IPCW is otherwise less efficient, with relative efficiency controlled by the relative magnitudes of the marginal error \(E\left[\{I(T_i \leq t) - F(t)\}^2 \mid F_i = 1, Z_{0,i}\right]\) and the conditional error \(E\left[\{I(T_i \leq t) - \hat{\pi}(t \mid W_i)\}^2 \mid F_i = 1, Z_{0,i}\right]\).

3 Simulation Study

We conduct extensive simulation experimentation to evaluate the finite sample performance of the proposed SCORNET estimator in realistic settings with \(n \in \{100, 200\}\) observed labels within the set of filter-positive patients, defining the filter to have 99% sensitivity and 88% specificity for \(\Delta\). We compare SCORNET to three existing survival function estimators with current status data: 1) parametric Weibull Accelerated Failure Time (AFT) regression with interval event times (Lin and others, 2019), 2) semi-parametric Cox Proportional Hazards regression with interval event times and Breslow baseline hazard estimation (Huang, 1996; Cox, 1972; Breslow, 1972), and 3) non-parametric IPCW estimation (van der Laan and Robins, 1998). We incorporate the filter in the Weibull and Cox models by setting \(\Delta_i = 0\) and weighting the \(n\) labeled filter-positive patients by \(\frac{1}{n} \sum_{i=1}^{N} F_i\). Weibull and Cox are implemented using the icenReg package in R, while IPCW is implemented per the algorithm detailed in van der Laan and Robins (1998), estimating \(C \mid Z_0\) using \(D\) under the Cox model. We note that estimating the censoring distribution using \(L\) yields similar asymptotic performance to using \(D\), but in finite sample settings the latter offers higher efficiency.
Table 1: Generative parameters employed in our simulation study.

| Setting | $Z_0 \sim$ | $C \mid Z_0 \sim$ | $T \mid Z_0 \sim$ | $Z \sim$ |
|---------|-------------|-----------------|-----------------|---------|
| 1       | Unif($-1, 1$) | Weibull ($10e^{-0.5Z_0}, \frac{3}{2}$) | Weibull ($15e^{-0.3Z_0}, \frac{5}{2}$) | Normal$\{T + 1, \sigma(C)/2\}$ |
| 2       | Unif($-1, 1$) | Weibull ($10e^{-0.5Z_0}, \frac{3}{2}$) | Weibull ($15, \frac{5}{2}$) | Normal$\{T + 1, \sigma(C)/2\}$ |
| 3       | Unif($-1, 1$) | Weibull ($10e^{-0.5Z_0}, \frac{3}{2}$) | Weibull ($15e^{-0.3Z_0}, \frac{5}{2}$) | Normal$\{T + 1, \sigma(C)/2\}$ |
| 4       | Unif($-1, 1$) | Weibull ($10e^{-0.5Z_0}, \frac{3}{2}$) | Logistic $\{15 - 4Z_0, 3\}$ | Normal$\{T + 1, \sigma(C)/2\}$ |
| 5       | Unif($-1, 1$) | Weibull ($10, \frac{3}{2}$) | Weibull ($15e^{-0.3Z_0}, \frac{5}{2}$) | Normal$\{T + 1, \sigma(C)/2\}$ |
| 6       | Unif($-1, 1$) | Weibull ($10e^{-0.5Z_0^2}, \frac{3}{2}$) | Weibull ($15e^{-0.3Z_0}, \frac{5}{2}$) | Normal$\{T + 1, \sigma(C)/2\}$ |

We consider 6 diverse generative mechanisms as detailed in Table 3, including cases where Weibull-distributed accelerated failure time of $T \mid Z_0$, proportional hazards of $T \mid Z_0$, and proportional hazards of $C \mid Z_0$ are respectively violated, as well as cases where SCORNET’s imputation model is and is not misspecified. In settings 1, 2, and 5, we consider various cases where SCORNET and all comparator methods are correctly specified. In setting 1 we consider the specific case where $C$ and $T$ both depend on $Z_0$, and both $C \mid Z_0$ and $T \mid Z_0$ are Weibull-distributed satisfying accelerated failure time and proportional hazards. In setting 2, by contrast, we consider a case where $TZ_0$ to assess robustness to over-parametrization of this relationship, and in setting 5 we consider a case where $CZ_0$ to evaluate robustness to over-parametrization thereof. In settings 3 and 4 we assess the benefit of SCORNET and IPCW’s robustness to the distribution of $T \mid Z_0$ when this distribution satisfies neither Weibull accelerated failure time nor proportional hazards. We evaluate SCORNET’s sensitivity to misspecification of the imputation model in settings 1, 3, and 5, as compared to correct specification thereof in settings 2 and 4. Finally, in setting 6 we assess the sensitivity of SCORNET and IPCW to misspecification of the conditional censoring model $C \mid Z_0$. For each given configuration, we compute the empirical bias, standard error, and root mean squared error (RMSE) of all estimators for $F(t)$ based on their average performance on
500 simulated datasets evaluated at 100 equally-spaced time points $t \in [Q_C(0.1) + b_n, Q_C(0.9) - b_n]$, where $Q$ denotes the quantile function of $C$ under the configuration. We used plug-in bandwidths of $b_n = \hat{s}(C) n^{-1/4}$ and $\alpha_n = \hat{s}(C) N^{-1/4}$ for the imputation (Step 2) and marginalization (Step 3) steps of SCORNET respectively, where $\hat{s}$ is the empirical standard deviation of observed $C$. We present the performance of the estimators averaged over the selected time points using $n = 200$ labels in Figure 2. The performance at each time point can be found in Supplementary Figure 1, and time-averaged performance using $n = 100$ labels can be found in Supplementary Table 1 of the Supplementary Materials.

![Figure 2](https://example.com/figure2.png)

**Figure 2:** Time-averaged empirical absolute biases (left), standard errors (second from left), root relative efficiencies (second from right), and relative RMSEs (right) of the Weibull Accelerated Failure Time (red), Cox Proportional Hazards w/ Breslow baseline (blue), supervised IPCW (green), and SCORNET estimators using weakly informative (purple) and strongly informative (orange) surrogates, in various simulated settings with $n = 200$ observed current status labels.
As Figure 2 demonstrates, imputing using a strongly informative feature \( Z \) (SCORNET-Strong) results in consistently higher efficiency than just using the weakly informative baseline \( Z_0 \) (SCORNET-Weak), which in turn is markedly more efficient than not leveraging the unlabeled set at all (IPCW). SCORNET makes minimal assumptions regarding the distribution of \( T \mid Z_0 \), settling for non-parametric efficiency in exchange for enhanced flexibility. By contrast, the Weibull regression model fully parametrizes \( T \mid Z_0 \), and the Cox model assumes proportional hazards thereof, increasing efficiency at the expense of bias in the case of misspecification. As expected, Weibull consistently achieves higher empirical efficiency than Cox, which in turn is more efficient than IPCW across settings. Notably, SCORNET consistently achieves empirical efficiency comparable to Weibull and significantly higher than Cox despite being far more flexible than both, again highlighting the efficiency gained by leveraging auxiliary information to impute unobserved risks. At the same time, SCORNET is much less susceptible to model misspecification bias than Weibull, as demonstrated by the latter’s significantly higher bias and RMSE in Setting 4. Indeed, SCORNET achieves relatively low mean absolute bias across settings, with MSE apparently dominated by variance rather than bias in the setting of 100-200 labels. Consistent with the theory, SCORNET is robust to misspecification of the imputation model in settings 1, 3, and 5, achieving equivalently insignificant bias as in setting 2 and marginally but not meaningfully higher bias than in setting 4. That said, correctness of the imputation model in settings 2 and 4 does not yield any meaningful change in relative efficiency, likely because inherent variability functionally dominates imputation model bias given so few labels. Reassuringly, SCORNET (and IPCW) appear insensitive to misspecification of \( C \mid Z_0 \) in setting 6, achieving functionally equivalent bias to the correctly-specified Weibull and Cox models. Altogether, these results corroborate the assertion that SCORNET’s semi-supervised utilization of informative feature data to impute risks in the unlabeled set improves estimation efficiency without introducing bias regardless of the validity of the imputation model. Moreover, they suggest that SCORNET is particularly valuable in settings where (1) flex-
ibility is desired with regard to the distribution of $T \mid Z_0$, and (2) there exists a large set of unlabeled patients with associated EHR data – both commonplace in retrospective observational studies.

![Figure 3: Empirical coverage probabilities averaged over time (left) and plotted over time (right) of SCORNET-Strong’s 95% confidence intervals constructed with the bootstrap (red) and plug-in (blue) standard error estimators in various simulated settings with $n = 200$ observed current status labels. See Table 1 for details of the generative mechanism employed in each setting.](image)

To assess the finite sample performance of the proposed interval estimation procedures, we obtain standard error estimates both using the proposed plug-in estimator $\hat{\sigma}(t)$ and via bootstrap with 500 replicates. In Figure 3 we demonstrate empirical coverage probabilities of SCORNET’s 95% Wald confidence intervals constructed using each standard error estimator, both averaged over the selected timepoints (left) and at each timepoint (right). Reassuringly, we find that the 95% confidence intervals using both plug-in and bootstrap estimators achieve nearly 95% mean coverage across settings. Coverage only drops below 90% at the tails of the event time
support due to moderately increased bias from kernel smoothing thereabout. The plug-in estimator achieves marginally lower coverage than the bootstrap estimator at the right tail due to underestimation of the true standard error, likely because of overfitting of the imputation model given low local censoring density (and thus low effective $N$). Notably, we do not observe this trend in setting 4, wherein correct specification of the imputation model obviates overfitting. Thus, we posit that the plug-in estimator can be reliably used for finite sample problems with $n \in [100, 200]$ labels as long as the conditional censoring density $f_c(C \mid Z_0)$ is sufficiently high and the timepoints evaluated are sufficiently far from the tails of the event time support.

4 Application to Assessing Heart Failure Risk Among Rheumatoid Arthritis Patients

Rheumatoid arthritis (RA), a chronic inflammatory disease that affects approximately 1% of the general population, is associated with dramatically increased risk of heart failure (HF) morbidity and mortality (Kaplan, 2010; Nicola and others, 2005, 2006; Ahlers and others, 2020). One study estimated that RA patients have a 1.9-fold lifetime risk of developing HF compared to matched RA-negative controls (Nicola and others, 2005), while another estimated that HF accounts for 13% of excess mortality among RA patients (Nicola and others, 2006). Ongoing interest lies in estimating the risk of developing HF subtypes in RA cohorts and quantifying the risk modifying effect of various RA treatments (Ahlers and others, 2020). Due to the increased availability of electronic health record (EHR) data, it is now possible to assess HF risk for a broader RA population using these data. For example, at Mass General Brigham we previously established an EHR cohort of $N_0 = 16,358$ RA patients (Huang and others, 2020). This large RA cohort can potentially be used to study the longitudinal risk of HF among RA patients.

However, such analysis is not straightforward as HF status is not readily available within the RA cohort. We propose to estimate HF risk among RA patients by leveraging (1) $n$ current status labels on HF status obtained
via manual chart review, and (2) informative yet unlabeled EHR data, including time to first ICD code for HF, as surrogate variables $Z$. We estimate both the age-specific HF risk, $F_{\text{age}}(\cdot)$, and the risk of developing HF after the patient’s incident ICD code for RA (714), $F_{\text{RA+}}(\cdot)$, among patients with at least 6 months of follow up whose incident RA codes occur after the age of 16 to select for adult-onset as opposed to juvenile RA. Among filter-positive patients, defined as having at least 1 ICD code for HF, we have $n = 300$ labels on censoring time HF status $\Delta$ for age-specific HF risk, and we have $n = 126$ for post-RA HF risk. We let the baseline covariates $Z_0$ include sex and decade of first EHR event for $F_{\text{age}}(\cdot)$, and sex, decade of first RA code, and age at first RA code for $F_{\text{RA+}}(\cdot)$. We obtain HF risk estimators based on SCORNET as well as the aforementioned comparator estimators. For the imputation model in Step 2 of SCORNET, we consider three EHR-derived surrogate risk predictors for $Z$: (1) the predicted $\Delta$ based on the unsupervised Multimodal Automated Phenotyping (MAP) algorithm, which uses the total counts of HF ICD codes and mentions of HF in clinical notes, as well as the total count of all ICD codes as a healthcare utilization measure (Liao and others, 2019), (2) the predicted $\Delta$ based on the unsupervised Surrogate-guided Ensemble Latent Dirichlet Allocation (sureLDA) algorithm, which leverages the features used in MAP as well as 121 additional manually-selected EHR features including counts of relevant medications, ICD codes, and concept unique identifiers (CUIs) in clinical notes (Ahuja and others, 2020b); and (3) the time to first HF ICD code. As in our simulation, we select plug-in bandwidths of $b_n = \tilde{s}(C)n^{-1/4}$ and $a_N = \tilde{s}(C)N^{-1/4}$ for the imputation and marginalization steps of SCORNET respectively, and we evaluate risk at 100 timepoints $t \in [Q_C(0.1) + b_n, Q_C(0.9) - b_n]$. We again compare the performance of SCORNET to that of Weibull, Cox, and IPCW, incorporating the filter in the Weibull and Cox models by propensity weighting as we do in the simulation study.

In Figure 4, we show the estimated HF risk curves along with their standard errors. Reassuringly, all methods appear to agree rather closely for estimation of both age-specific HF risk and HF risk after RA diagnosis. For the latter quantity, however, Weibull and Cox appear to underfit while IPCW appears to overfit relative to
Figure 4: Estimated age-specific and post-RA cumulative risks of heart failure (top) and bootstrap standard errors thereof (bottom) over time of the Weibull Accelerated Failure Time (red, short-long-dashed), Cox Proportional Hazards w/ Breslow baseline (blue, dot-dashed), supervised IPCW (green, dashed), and SCORNET (purple, solid) estimators.

the SCORNET estimator, which appears to achieve a reasonable middle ground. Moreover, SCORNET once again attains standard errors comparable to those of the Weibull estimator and meaningfully lower than those of the Cox and IPCW estimators. This suggests that while the Weibull and Cox models potentially fail to capture the complexity of the post-RA HF risk function, and IPCW is too unstable for a limited labeled set of size $n = 126$, SCORNET offers an attractive balance of efficiency and flexibility and is thus well conditioned for such a scenario. As shown in Figure 5, averaged over the selected timepoints, the root relative efficiency of SCORNET is 1.11, 2.55, and 3.31 compared to the Weibull, Cox, and IPCW estimators respectively for estimation of age-specific risk, and 1.34, 2.32, and 3.85 respectively for estimation of HF risk after RA diagnosis. Once again, the fact that SCORNET achieves efficiency moderately higher than the relatively inflexible Weibull model and sig-
nificantly higher than the Cox and IPCW estimators reflects the value of leveraging available information from the EHR to bolster risk estimation efficiency.

Figure 5: Time-averaged bootstrap standard errors (left) and empirical root relative efficiencies (right) of the Weibull Accelerated Failure Time (red), Cox Proportional Hazards w/ Breslow baseline (blue), IPCW (green), and SCORNET (purple) estimators for estimation of (1) age-specific HF risk (left), and (2) HF risk after RA diagnosis (right), among RA patients in the Partners EHR database.

5 Discussion

By leveraging a sizeable unlabeled data set containing imperfect surrogates of the true event times and a small set with observed current status labels, the SCORNET estimator serves as a robust and efficient alternative to existing model-free survival estimators with current status data. The semi-supervised nature of SCORNET makes it well-conditioned to EHR-based survival estimation in settings where only a limited number of labels are available or readily obtainable. Moreover, by only requiring current status labels rather than the precise
timing of event onset, SCORNET greatly reduces the burden of chart review and increases the feasibility of studying disease risk using EHR data.

To allow for covariate-dependent censoring, which is frequently present in observational settings, SCORNET requires additional assumptions on the distribution of $C \mid Z_0$. Although we choose the proportional hazards model for illustration, any standard semi-parametric model will yield similar properties for the resulting estimator. Since $\{C, Z_0\}$ are observed for all subjects, one can potentially allow for more flexible (i.e. non-parametric) censoring models. That said, our simulation results suggest that SCORNET is relatively insensitive to misspecification of the model for $C \mid Z_0$. Even under mild misspecification, it achieves consistently lower mean squared errors than existing estimators.

When interest lies in assessing how risk differs across different patient sub-populations, it is straightforward to extend SCORNET to estimate subgroup-specific risks for a small number of subgroups. However, future research is warranted to estimate covariate-specific risks for a general set of covariates.

6 Software

An R package, including a sample use case and complete documentation, is available at https://cran.r-project.org/web/packages/SCORNET/index.html. Source code can be found at https://github.com/celehs/SCORNET.

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