Optimal deployment and principles of tracing for the distributing lines of pumping plants in the mountain terrain

E V Dmitriev¹, A V Kalach¹², V G Mokrozub² and E A Cherepanov³

¹Voronezh institute of the Russian Federal Penitentiary Service, Russia, Voronezh, Irkutskaya 1a
²Tambov State Technical University, Russia, Tambov, Leningradskaya, 1a
³Ural Institute of EMERCOM of Russia, Ekaterinburg, Mira, 22

E-mail: AVKalach@gmail.com

Abstract. In this paper, we propose new graphical algorithms for planning and optimal placement of pumps for outdoor fire extinguishing networks and propose mathematical optimization models for constructing various outdoor fire water supply networks for microdistricts located in mountainous areas (with significant elevation differences). The solution was carried out by constructing Fermat-Torricelli-Steiner points for the optimization problem of constructing a spatial dead-end network in three-dimensional space.

1. Introduction

It should be noted that in the applied construction and architectural tasks rather often [1-11] there appears a need to consider them in more generalized spaces than the Euclidian one.

For example, if on the plane (ground level) the optimal coverage area from the permanent center for the fixed time is a circle of a certain radius (Figure 1a), construction of communications and relocations in the mountains encounters with a problem when expenses in the vertical section of the plan upwards and downwards are of a certain value and along the terraces (to the right and to the left are the other ones. Note that the former ones are usually greater than the latter ones. An optimal zone in this case can be the interior of rhomb or ellipse in a dependence of saturation of their periphery on the saturation with the objects requiring a supply Figure 1, b,c). In fact, it means an appropriateness in considering optimization problem in the coordinate Banach spaces with the different but not the Euclidian metrics. This metrics is generated by a set of individual balls.
2. Optimization of a circular construction

Construction of the minimum in length circular communication of all the points corresponding to FH locations in the form of a closed graph, without circuits, where one graph edge enters and one graph edge exits each point of the graph is a well-known traveling salesman problem (TSP) [9]. This problem can be also solved in 3D space. Graph without circuits progressing once through all the points is named Hamilton’s graph. Determination of Hamilton’s graph minimal in length is just the traveling salesman problem. This problem can be solved by a computer searching and enumeration or with a help of a number of special known algorithms [9]. It is known that an exact solution of this problem for a number of points is a very complicated problem. Optimization definition of the problem is related to the class of NP-difficult tasks just as a lot of its special cases. Traveling salesman problem is related to the type of transcomputational ones: just for the relatively low number of the graph nodes (66 and more) it can not be solved by a simple enumeration of possibilities with any theoretically conceivable computers for the time no less than several billions years. However, an approximate solution of this problem is accessible to the computer technology. Here some results of solution for this problem are presented [10]. On March, 2005 the problem with 33 810 nodes was solved by the program Concord: the way with a length of 66 048 945 was calculated and the absence of more short ways was proved. In April, 2006 solution was found for the specimen with 85 900 nodes. Using decomposition techniques, it is possible to calculate solutions for the cases of problem with millions of nodes where the length is by less than 1% more than the optimal one. In practice, implying that in our case the number of points is proportional to the number of buildings in micro-region (Figure 2), i.e. it is about 20-30 pieces. So, the problem can be solved.

In our case the routing in micro-region comprising of the square buildings located rather closely to each other reduces to zero the possibility of projecting in Euclidian metrics (along the straight line).

In this case the traveling salesman problem should be solved not in the Euclidian metrics, but in the metrics of $l_1$ space. Note that the program of computer solution of the problem should be done using this metrics. The distance between points $M_k$ and $M_i$ in this program is calculated using formula

$$\rho(M_k, M_i) = |x_k - x_i| + |y_k - y_i| + |z_k - z_i|.$$  \hspace{1cm} (1)

while Euclidian metrics is written as

$$\rho(M_k, M_i) = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2 + (z_k - z_i)^2}.$$ \hspace{1cm} (2)

However, computer program can operate with any metrics, while calculation of the distance between the points in 3D space determines only the weight values in the traveling salesman problem and it completely does not have an impact on its complication or solution. Mathematical record of the problem can be expressed as: to determine Hamilton’s circuit with a minimal length or, in other words, to find $\sum \rho(M_k, M_i) \rightarrow \text{min}$, where points $M_k$ and $M_i$ form a certain Hamilton’s circuit.
3. Development of the dead-end networks of a special kind

Assume that we got a permission to construct pump station of the new kind. This is specially operated station which can separately supply FFW after receiving of a certain signal to the hydrants that are situated in an immediate vicinity from the hotbed of fire and does not supply water to all other fire hydrants thus saving all of the possible expenses. Thus there appears a problem – where should this station be situated in order the total length of the pipe sections is the minimal one (Figure 7). The problem of finding coordinates for this station is just the task of determination of the coordinates for the Fermat-Torichelli-Shteiner (GTSh) point. Just at this item the difference form the first part of work is comprised in following: planning will be made in the mountain terrain with rather great difference in height.

Let us resemble the general state of FTSh problem: to determine coordinates of the point M in the coordinate Banach space where a sum of distances from this point to the fixed n points \((A_1, A_2, ..., A_n)\) is minimal in the metrics of this space.

This space can be of any dimensionality. It is also possible to consider different metrics for measurements of the distances.

FTSh-points (more correctly, their coordinates) in 3D space and the distances in Euclidian metrics were found with the use of iteration programs. The results were graphically illustrated in [12,13].

Let us present the schemes of algorithm for this case for a completeness of description.

Consider the following algorithm that does not depend on metrics where the measuring of the distance is performed, and then divide it into successive procedures.

4. Algorithm of numerical solution of the problem in space

Divide algorithm into successive procedures.

1. First procedure – scaling of the source data. Let \(P_k = (x_k, y_k, z_k)\) \(k = 1, 2, ..., n\) are the points in 3D space that represent source data. Define the boundaries of distribution for these points. Let all of the points are in the first quadrant, and

\[
\begin{align*}
\min x_k &= a, \min y_k = b, \min z_k = c; \\
\max (x_k - a) &= A, \max (y_k - b) = B, \max (z_k - c) = C. 
\end{align*}
\]

Then transformation of coordinates is presented as

\[
\begin{align*}
\xi &= \frac{x - a}{A}, \eta = \frac{y - b}{B}, \zeta = \frac{z - c}{C}.
\end{align*}
\]

moves all these points into a unit cube - \([0,1]^3\), where a guided search will be realized. Inverse transformation provides coordinates of the problem’s solution. Let us designate an exact solution of the problem as \(M = (x, y, z)\).

2. Second procedure – calculation of the values of objective function (in a dependence of metrics it is different) in the mesh points of cube splitting \([0,1]^3\). Each of the cube side one can split into two equal intervals with a length of \(|\Delta| = 0.5\), and in this case the cube itself will be divided into eight equal cubes. Centers of these eight cubes of the less size are named as mesh points of the splitting cube: \(m_{i,j,k}(1) = \left(\frac{1}{4} + \Delta \times i, \frac{1}{4} + \Delta \times j, \frac{1}{4} + \Delta \times k\right)\) where: \(i = 0, 1; j = 0, 1; k = 0, 1\). Then we successively calculate the values of objective function in these points \(\Phi(m_{i,j,k}(1))\) and determine the least value. At this item the first step is over.

If the precision of calculation while its value being defined in the area is enough then this point is acknowledged as solution of the problem. Second step starts from the third procedure.

3. Third procedure – search of the needed point now proceeds in that of the eight quadrats where objective function achieved its minimum just in its center. According to the formulas (2) and (3) we transfer to the new unit cube \([0,1]^3\) and repeat second procedure. The mesh points of the new cube splitting prove to be centers of the new cubes with a lesser size - \(m_{i,j,k}(2)\). Once again we successively calculate the values of objective function \(\Phi(m_{i,j,k}(2))\) in these points and find the least
value. If the precision is enough then the point is acknowledged as a solution. If not so, then all of the procedures are repeated.

Note. Accuracy of the approximate calculations of coordinates increases as a power function \( O \left( \frac{1}{2^m} \right) \).

For the analysis of possibility of the practical application for the algorithms developed in these works while routing the pipes in the mountain terrain let us consider one of the figures obtained in these works (Figure 2).

Assume that the served points are water supply objects (hydrants) situated near the buildings in the mountain terrain. Considered algorithm is not applicable since the figure demonstrates that the best deployment of the pump station (central point) can appear in the air just as the pipelines for water supply can completely or partially appear in the air. Thus, this algorithm for the use of the spatial FTSh points for optimization of the pump stations deployment in the mountain terrain proves to be not suitable in a general case. In the next item we shall change the algorithm in such a way that it can operate in the mountain terrain.

5. Algorithm for the optimal deployment of the pump stations in the mountain terrain
All of the works known to the authors, concerning optimal deployment of FTSh point as a space for search used either all of the space or all space of some cube in the real space. A practical directivity of this work advances obligatory claims: 1) the station must be deployed on the surface of the served area; 2) water pipelines should be laid under the surface of the area. These limitations define the changes in the algorithm proposed above.

Thus, a common application of the known programs for determination of the deployment of FTSh point in 3D space (Figure 2) is simply not suitable for solution of our problem.

In order to develop new algorithm, we shall proceed from the obligatory requirement – pump station is deployed on the surface of the area. In this item we propose to develop iteration algorithm where the space of search – not all of the points in space but only those ones associated with the square mesh depicted on the plan of the investigated terrain (Figure 3). Such assumption guarantees location of the pump station on the surface of the ground.
Figure 3. The point radiating the arrow lines is \( M^1(k,j) \) point. The points where the arrows enter are the points of water consumers \( P_i \).

Let us consider successive procedures of the algorithm:

1) Using satellite photos of the area let us place the photo of the studied region in the square \( A \) (Fig. 4). Within this square the points \( M^{1k} = (x_k, y_k, z_k) \) \( k = 1, 2, ..., n \) representing intersections of the uniform mesh with rather big size are marked (Figure 3). All these points were found at the photo of a district. These points are associated with the real coordinates in 3D space corresponding to the locations in the mountain terrain.

   We designate these points as \( M^1(k,j) \). We can introduce two coordinates in the mesh corresponding to the lines in different directions. For each of these points we calculate the value of objective function (it is, in fact, a sum of distances from \( M^1(k,j) \) to the points of water consumers near the buildings). Let us designate the points of water consumers as \( P_i \); assume that their number is \( N \), and \( i = 1, 2, ..., N \). They also have the real mountain coordinates from the Figure 3. For the fixed parameters of \( (k,j) \) objective function is expressed as:

   \[
   \Phi^1_{k,j} = \sum_{m=1}^{N} \rho(M^1(k,j), P_m).
   \] (5)

   In (3) we shall consider metrics (2).

2) Now we choose the minimal value from this finite sequence. Suppose that it is realized for the indexes \( (k_1, j_1) \). This point is designated by a circle in Figure 4, in the center of the square \( B \).

3) Using the mesh, we choose the least square with its center in the point with the indexes \( (k,j) \). Square \( B \) can be transformed into the square with the size \( A \) by linear transformation. Inside this square once again a similar rectangular mesh is built up with the points. Once again, all the values of objective function are calculated at the points \( M^2(k,j) \). Then we choose the minimal value from this finite sequence. Suppose that it is realized for the indexes \( (k_2, j_2) \). And so on until achieving of the rational value from the viewpoint of practice. Since the sequence of \( M^n(k_n, j_n) \) is non-ascending and it is limited and bound below then it has a limit.
We do not need to have this limit itself; reasonable approximation here is quite sufficient so that the sequence changes insignificantly.

This algorithm makes it possible to determine the place for optimal deployment of the pumping station. Further we represent block diagram of the algorithm, it differs from those ones presented in [11,12].

6. Block diagram of the algorithm for a search of the optimal deployment of the pumping station in the mountain terrain
Consider the difficulties appearing during routing of the pipeline in the mountain terrain:
1) Simulation of the mountain region is a complicated and unreasonable problem in such a situation;
2) an ordinary design and projecting onto terrain of the waterway tracks obtained as a result of the application of the described algorithm may not account as for the composite relief as for the possibility of passing the waterway track through the buildings.
Input data \( P_i \) and accuracy \( \varepsilon \).

Defining of the min. area including points, mesh, projection onto the square, preserving mountain coordinates.

Calculation of obj. function in the mesh points. Determination of its min.

Yes

Center of opt. pt.

side of square < \( \varepsilon \)

No

Separate min. square involving the point with min obj. func., expand it to the initial size, then plot the mesh with real horizontal coordinates in the mesh points.

**Figure 5.** Block diagram of the algorithm for a search of the optimal deployment of the pumping station in the mountain terrain.

However, the distance to water consumers was measured along straight lines in Euclidean metrics and so the pipeline can partially or even completely be suspended in the air, similar to the cableway in the mountains (Figure 4).

**Figure 6.** Dashed lines show the lines obtained with the use of algorithm without the account of relief features. Solid line marks trace routing made by designer in the manual or computer mode in \( l_1 \) metrics.

And satellite map does not consider bends and pits which can possibly be present at this section of the region (Figure 4).

Considering the enumerated complications, we can suggest the following design activities.
Since the point of location (terrain coordinates) of the pumping station is already determined and coordinates of water consumers (points $P_i$) are also known, a designer of the pipelines plots the tracks connecting PS and the points $P_i$ in the interactive mode (either manually or using personal computer). In a dependence on the housing density of the area he has the right to use as metrics (1) as metrics (2). In addition, he can and ought to employ specific features of the terrain (Figure 7).

![Figure 7](image-url)

**Figure 7.** Dashed line designates the line obtained by visualization of the algorithm, solid line is obtained by designer in the interactive mode with the account of the buildings location and the features of relief. Algorithm is finished.

### 7. Conclusion

The main issue noted in the work is comprised in the fact that the current layout of the building’s location in the modern micro-districts and the size of buildings make impossible the application of the current legal regulations basing on the radial approach for designing of facilities for the external firefighting water supply. Deployment of the firefighting hydrant (FH) is realized without the matching with setting-out of a building. Measuring of distances along a straight line considerably complicates the practical actions during organization of the firefighting. There is no any problem definition concerning optimization in the normative requirements.

(In the first part of the work ring and dead-end water-supply networks EFFWS are considered for designing on the plane (level ground)). Route tracking is proceeded in the metrics of $l_1$ space, which takes into account mainly square shape of buildings. In the second part of the work ring and dead-end water-supply networks EFFWS are considered in order to make a design in the mountain terrain.

In this work similar operations were done for the planning and routing of lines in the mountain terrain. The features of this part of work are distinguished and its difference from the way of tracking on the plane (ground level).

### References

[1] Gurov A V and Gridnev E Y 2012 On the issue of creating a fire-fighting water supply *Bulletin of the Voronezh Institute of GPS of the Ministry of Emergency Situations of Russia* 1 pp 49-51

[2] Maltsev E D et al 1976 Hydraulics and fire water supply (Moscow)

[3] Abrosimov Yu G et al 2003 Hydraulics and fire-fighting water supply (Moscow: Academy of GPS of the Ministry of Emergency Situations of Russia)
[4] Ivanov E N 1986 Fire-fighting water supply (Moscow: Stroyizdat)
[5] Polyakov D V 2017 100 questions and answers on the design of AUVPT 20 pp 96-100
[6] Rodin A V, Kalach A Yu, Akulov E A and Cherepanov E F 2014 Algorithms for optimal location of hydrants for outdoor fire-fighting water supply
[7] Pivovarov N Yu and Tarantsev A A 2014 Modeling of water output of ring networks of external fire-fighting water supply system *Fire and Explosion Safety* 23(12) pp 69-75
[8] Vinogradov I N 1977 Mathematical Encyclopedia (Moscow) 1
[9] Menshikh V V 2012 Discrete mathematics. Textbook Voronezh Institute of the Ministry of Internal Affairs of Russia
[10] Ulanov E A and Uteshev A Y 2011 Analytical solution of the generalized Fermat-Torricelli-Steiner problem *Management Processes and Sustainability* ed A S Eremin (St. Petersburg: Izdat. House of St. Petersburg State University) pp 201–6
[11] Rodin V A and Rodina E V 2016 On Fermat-Steiner points in Banach spaces with different metrics *Syst. Management and Inform. Technologies* 1(63) pp 17-20
[12] Bondarenko E S, Grechany S A and Rodin V A 2017 Numerical simulation of problems of optimal placement of a service object using analogs of Fermat-Steiner points (Voronezh: VI Bulletin of the Ministry of internal Affairs of Russia) 2 pp 154-61
[13] Grishanov M V and Rodin V A 2018 Numerical modeling of optimal placement problems using analogs of Fermat-Steiner points *Physics and Mathematics Model Systems* pp 37-39
[14] Numerical Algorithm Framework URL: https://ilnumerics.net/index.html. (date of access: 15.05.2020)
[15] Andreev D G and Rodin V A 2020 Collection of student scientific papers of the Faculty of Computer Science of Voronezh State University. Algorithms for Numerical Determination of Coordinates of Analogs of the Fermat-Steiner Point pp 3-8
[16] Numerics I L High Performance Math Library for C# and .NET [Electronic resource]: URL: https://www.kdnuggets.com/2013/07/ilnumerics-high-performance-math-library-csharp-net.html (date of access: 15.05.2020)