BLACK HOLE SINGULARITY IN ADS/CFT

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We present a short review of hep-th/0306170. In the context of AdS/CFT correspondence, we explore what information from behind the horizon of the bulk black hole geometry can be found in boundary CFT correlators. In particular, we argue that the CFT correlators contain distinct, albeit subtle, signals of the black hole singularity.

It is a well-founded expectation that black holes should provide a key, or at least a window, into quantum gravity. Typically, black holes contain curvature singularities, where classical laws of general relativity break down, to be replaced by more fundamental quantum physics. Unfortunately, an asymptotic observer cannot see this quantum singularity resolution in action, because the near-singularity physics is cloaked by an event horizon, the defining feature of a black hole. The fact that black holes are causally nontrivial, signaled by an event horizon, has also associated with it long-standing puzzles, such as the information paradox.

To unravel the mysteries of black holes, we call to aid string theory, which has already proved very successful in resolving many timelike singularities. Spacelike singularities appear to be more subtle, and we expect that a fully nonperturbative formulation of string theory will be essential for this endeavor. While there are several remarkable formulations, the one best suited for our purposes is the well-known AdS/CFT correspondence. In particular, we will use the duality between 10-dimensional IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$, 4-dimensional Super Yang Mills gauge theory, living on the boundary of AdS.

More specifically, to study a singularity and an event horizon in the bulk gravity theory, we will consider the Schwarzschild-AdS geometry, describing a large black hole in AdS, whose gauge dual corresponds to an approximately thermal state. By analyzing the properties of this state,
such as the expectation values of various operators, we can hope to extract information about the black hole. Indeed, due to the powerful nature of the AdS/CFT duality, we expect to be able to decode the full quantum near-singularity bulk physics from the gauge dual.

However, since such decoding requires that the gauge theory encodes behind-the-horizon physics in the first place, we must first ask, how much of the bulk physics does the gauge dual encode? Naive considerations might lead one to think that the CFT encodes only the region of the bulk which is causally connected to the boundary, i.e. outside the event horizon. On the other hand, an event horizon is a global object (defined as the boundary of the past of the future infinity), which means that we cannot determine the presence or position of the event horizon without knowing the entire future evolution of the spacetime. Hence if the horizon were to bound what the CFT can encode, then the AdS/CFT correspondence would have to be very nonlocal in time.

We illustrate this point by the following gedanken-experiment\(^2\) (cf. Fig.1): Start with “empty” AdS space, and consider some event, labeled \(p\) in Fig.1a. As a boundary observer, one can obtain instantaneous information about the event \(p\), for instance by measuring appropriately decorated Wilson loop\(^3\) \(W\). After this measurement has been made, i.e. entirely in the future of \(W\), one can send in a shell \(s\) of radiation with sufficient energy

![Figure 1. Sketch of the proposed process of abstracting information from inside of the horizon: a) An event \(p\), in locally pure AdS space is measured by \(W\). b) After this measurement is performed, a shell \(s\) collapses, forming a black hole with a horizon \(H\) which encompasses \(p\).]
such that when it implodes in the center of AdS, it forms a large black hole, as sketched in Fig. 1b. The main observation is that the global event horizon for this spacetime, denoted by $H$ in Fig. 1b, can originate at the center of AdS prior to $p$. In particular, provided the shell is sent in within time of order the AdS radius after the measurement of $p$ is performed, one can always make the resulting black hole large enough for its horizon to encompass $p$. This implies that one has in fact succeeded in “measuring” an event, $p$, which is inside a black hole horizon.

While the above argument indicates that the CFT should encode at least some physics inside a horizon, it does not lend itself to detailed computational analysis. Instead, it is more fruitful to turn to a simpler set-up, namely that of the eternal black hole, which is static outside the horizon and has both future and past singularities, as well as two boundaries. This geometry was analyzed previously in 3 dimensions by a number of authors; here we study the higher-dimensional analog.

The real-time or thermofield formalism for thermal field theory is especially well-suited for this analysis. One copy of the CFT resides on each of the two asymptotic regions; the CFTs are noninteracting but entangled through the Hartle-Hawking state. In this approach, the one boundary thermal description is recovered by tracing over the Hilbert space of the other boundary CFT.

Moreover, as demonstrated previously for 3 dimensions, one can probe physics behind the horizon by studying the correlator of two operators, one on each asymptotic boundary, each creating a large mass bulk particle. As the mass $m \to \infty$, the correlator can be evaluated in the semi-classical geodesic approximation and is given by $\exp(-mL)$, where $L$ is the (regularized) proper length of the spacelike geodesic joining the boundary points. Because the geodesic passes through spacetime regions inside the horizon, this boundary correlator reveals information about the geometry behind the horizon.

Unfortunately, the three dimensional case, which is simple enough to study analytically, is rather special. The geometry is locally that of pure $AdS_3$, and the black hole singularity is merely a result of the orbifold nature of this geometry. Consequently, the geodesics are not sensitive to the position of the singularity, and correspondingly the correlation function is relatively structureless.

On the other hand, this situation changes drastically in higher dimensions. In particular, the singularity in $d > 3$ dimensions approaches that of the $d$-dimensional Schwarzschild black hole singularity, which has a dra-
matic effect on the spacelike geodesics: pictorially, spacelike radial geodesics “bounce off” from the singularity. For sake of simplicity, we focus on $d = 5$ where the boundary CFT is four dimensional $\mathcal{N} = 4$ SYM; however, our results are qualitatively similar to those in other higher dimensions as well.

The first surprise, underlining the marked difference from the previously-studied 3-D case, is that the Penrose diagram\textsuperscript{*} of the Schw-AdS\textsubscript{5} spacetime is different. While the Penrose diagram for the Schw-AdS\textsubscript{3} geometry has the shape of a square, this can no longer be true in higher dimensions\textsuperscript{7}. One reason, apparent from Fig.2, is that outgoing radial null geodesics starting at the past singularity do not reach the boundary at a time-symmetric point. Apart from the differences in the Penrose diagrams, there are two important differences in the symmetric radial space-like geodesics: Unlike the 3-D case, in higher dimensions there exists a particular time $t_c$ beyond which there are no geodesics connecting the two boundaries, and the geodesics cross each other.

This has an important consequence for our present goal of seeing a signature of the singularity in the dual field theory. Consider the boundary to boundary correlator $\langle \phi \phi \rangle(t)$ between two high-dimension operators inserted in a symmetric fashion on the two boundaries at time $t$. Since this

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{penrose_diagrams.png}
\caption{Penrose diagrams of Schw-AdS in (a) 3 and (b) 5 dimensions. The top and bottom curves are the future and past singularities, the vertical lines are the boundaries, the dashed diagonal lines are the event horizons, and the remaining curves are spacelike radial geodesics.}
\end{figure}

\textsuperscript{*Penrose diagram is a conformal diagram of a spacetime including its boundary, used to describe its causal properties; time runs up and space runs sideways, such that null rays lie at 45 degree angles.
correlation function is related to the proper length of a spacelike geodesic connecting the two points, we expect that it reflects the special behaviour of the geodesics as \( t \to t_c \). In fact, direct evaluation suggests that we should see a pole in the correlation function:

\[
\langle \phi \phi \rangle (t) \sim e^{-mL(t)} \sim \frac{1}{(t - t_c)^{2m}} \quad \text{as} \quad t \to t_c.
\]  

(1)

This corresponds to a light-cone singularity in the field theory, since the geodesics are becoming almost null.

Had this been the full story, we would have a glaring signature of the black hole singularity in the gauge theory dual. However, general considerations of the boundary field theory rule this out; one can easily show that \( |\langle \phi \phi \rangle(t)| \leq |\langle \phi \phi \rangle(0)| < \infty \). What went wrong? In evaluating Eq.(1) we assumed that the correlator is dominated by a single geodesic, namely the real, “bounce” geodesic shown in Fig.2b. But, in fact, this geodesic does not dominate the correlator, for there are in general multiple geodesics that connect the two boundary points. One indicator of this fact is the intersection of nearby geodesics around \( t = 0 \); in a cut-off field theory there would therefore be three geodesics connecting the two points at (cut-off) boundaries. Alternately, in a Euclidean picture, one real and two purely imaginary geodesics contribute. At \( t = 0 \) their proper distances coincide, creating a branch point in the correlator which behaves as \( t^{4/3} \) for small \( t \).

(The 3 in the denominator of the exponent, implying a 3-sheeted Riemann surface, corresponds to the 3 geodesics.)

By studying various resolutions of this branch point, we show that as \( t \) increases from 0, the correlator defined by the boundary CFT is given by a symmetric sum of the two complex branches of this expression. Each of these can be attributed to a complex geodesic in complexified spacetime. But the correlator is an analytic function of \( t \) and can be continued onto the real sheet. This is the essential feature of the CFT which enables us to extract information which is not directly measurable. On the real sheet, the “light cone singularity” does appear. So the boundary correlator does contain information about the singularity, albeit in a subtle way.

\footnote{The full analytic structure of the correlator is given by the 3-sheeted Riemann surface of \( L(t) \), which can be derived implicitly in terms of “energy” \( E \) of the real geodesics;

\[
t = \frac{1}{2} \ln \left( \frac{\frac{1}{2}E^2 - E + 1}{\sqrt{1 + \frac{4}{3}E^4}} \right) - \frac{1}{2} i \ln \left( \frac{-\frac{1}{2}E^2 + iE + 1}{\sqrt{1 + \frac{4}{3}E^4}} \right), \quad L = \ln \left( \frac{2}{\sqrt{1 + \frac{4}{3}E^4}} \right)
\]}


So far, our discussion dealt with infinitely massive field $\phi$, as well as classical spacetime with vanishing string length $l_s$ and coupling $g_s$. We can extract further information about the singularity by deviating away from this limit. Although the branch point at $t = 0$ is smoothed out for any finite $m$, preventing analytic continuation to the bouncing geodesic, at each order in the $1/m$ expansion the branch point persists and one can follow this geodesic and the accompanying fluctuation corrections into the bouncing domain. Moreover, since the $1/m$ corrections are largest near the singularity, we can effectively isolate the near-singularity behaviour.

One may similarly make use of the $l_s$ and $g_s$ corrections, thereby extracting stringy and quantum near-singularity behaviour. The main corrections at finite but small $l_s$ (with $g_s$ kept equal to zero) can be expressed as modifications of the supergravity field equations, which does not change the basic picture outlined above: There still exists a branch point, allowing analytic continuation onto the real sheet. The $g_s$ expansion is more analogous to the $1/m$ expansion; finite $g_s$ smooths out the singularity. But by taking appropriate $g_s \to 0$ limits we can extract behavior around the singularity to all orders in $g_s$, as well as study certain leading nonperturbative effects.

In summary, we have seen that the CFT encodes physics behind the horizon, including the near-singularity region. Unlike the previously-studied three dimensional case, in higher dimensions there is a genuine curvature singularity which is bowed in on the Penrose diagram, leading to a special behaviour of the geodesics as a critical time $t_c$ is approached. Despite subtleties related to the nontrivial analytic structure of $\mathcal{L}(t)$, the CFT correlators reveal distinct signals of the black hole singularity. In fact, given the CFT data $\langle \phi \phi \rangle$, the properties of the singularity are computationally accessible\(^7\). Since the “$t_c$” singularity persists to all orders in $1/m$ and $g_s$, as well as for small $l_s$, we can also extract the stringy and quantum behaviour near the black hole singularity. Thus, using analyticity, we have demonstrated that a significant amount of information from behind the horizon, and in particular from near the singularity, is encoded in boundary theory correlators. Since analyticity was essential for our arguments, it would be worthwhile to understand its implications in the context of AdS/CFT at a much deeper level.

\(^7\)Of course, it is not presently feasible to obtain the CFT “data” $\langle \phi \phi \rangle$ in the relevant regime by a direct computation: the CFT is strongly coupled, and $m \to \infty$ implies infinitely large dimension operators. Nevertheless, it is still of use to ask the matter-of-principle question: with given information about the CFT, what can one learn about the bulk?
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Due to space restrictions, only a very limited set of references is listed here. Please refer to Ref.[7] for a more complete list.