Primary Feynman rules to calculate the $\epsilon$—dimensional integrand of any 1-loop amplitude

R. Pittau

*Departamento de Física Teórica y del Cosmos y CAFPE Universidad de Granada,*
E-18071 Granada, Spain.
E-mail: pittau@ugr.es

**Abstract:** When using dimensional regularization/reduction the $\epsilon$-dimensional numerator of the 1-loop Feynman diagrams gives rise to rational contributions. I list the set of fundamental rules that allow the extraction of such terms at the integrand level in any theory containing scalars, vectors and fermions, such as the electroweak standard model, QCD and SUSY.

**Keywords:** NLO, radiative corrections, electroweak model, QCD.
1. Introduction

New techniques [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] for computing 1-loop corrections led to a NLO revolution [15, 16], that, directly or indirectly, has allowed an impressive improvement in our ability to predict physical observables at the NLO accuracy [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37]. Basically all new methods need a special treatment of the contributions that are not proportional to the scalar 1-loop functions, the so called rational terms [38, 39, 40, 41]. That is achieved, in Unitarity and Generalized Unitarity methods, by computing the entire amplitude in different numbers of space-time dimensions [42], or via bootstrapping techniques [13, 14], or through $d$-dimensional cuts [45]. The Ossola-Papadopoulos-Pittau (OPP) approach of reference [4] requires, instead, the computation (once for all for the theory at hand) of a special set of tree level Feynman rules [46, 47, 48, 49, 50] up to 4-point interactions $^1$.

When using dimensional regularization/reduction, the origin of such terms lies in the $\epsilon$-dimensional numerator of the 1-loop Feynman diagrams $^2$. To be more specific, let us consider the general expression for the integrand of a generic $m$-point one-loop (sub-)amplitude

$$\tilde{A}(\bar{q}) = \frac{\tilde{N}(\bar{q})}{D_0D_1\cdots D_{m-1}}, \quad \tilde{D}_i = (\bar{q} + p_i)^2 - m_i^2, \quad (1.1)$$

$^1$The contributions to higher-point functions vanish because of UV finiteness.

$^2$Another contribution, which is however directly linkable to the cut-constructible part of the amplitude [51], is generated by the $\epsilon$-dimensional 1-loop denominators.
where $\vec{q}$ is the integration momentum and

$$\vec{q}^2 = q^2 + \vec{q}^2 = q^2 - \mu^2. \quad (1.2)$$

In the previous expressions and in all the following ones, a bar denotes objects living in $d = 4 + \epsilon$ dimensions, whereas a tilde represents $\epsilon$-dimensional quantities.

The numerator function $\tilde{N}(\vec{q})$ can be split into a 4-dimensional plus an $\epsilon$-dimensional part

$$\tilde{N}(\vec{q}) = N(q) + \tilde{N}(\mu^2, q, \epsilon). \quad (1.3)$$

$N(q)$ lives in 4 dimensions, while $\tilde{N}(\mu^2, q, \epsilon)$, which originates from the splitting of $d$-dimensional objects

$$\vec{q} = q + \vec{q},$$

$$\vec{\gamma}_\mu = \gamma_\mu + \vec{\gamma}_\mu, $$

$$\vec{g}^{\mu\nu} = g^{\mu\nu} + \vec{g}^{\mu\nu}, \quad (1.4)$$
gives rise to a rational piece called $R_2$ in the OPP language:

$$R_2\bigg|_{HV} = \frac{1}{(2\pi)^4} \int d^d \vec{q} \frac{\tilde{N}(\mu^2, q, \epsilon)}{D_0 D_1 \cdots D_{m-1}}, \quad (1.5)$$

with

$$\int d^d \vec{q} = \int d^4 q \int d^\epsilon \mu. \quad (1.6)$$

$R_2$ has a pure ultraviolet origin [38, 40], so that eq. (1.3) also holds when infrared/collinear divergences are present in the loop integrals. It can be shown [39, 42] that $\tilde{N}(\mu^2, q, \epsilon)$ is polynomial in $\mu^2$ and at most linear in $\epsilon$, and that the $\epsilon$ dependence can be reabsorbed in the regularization scheme [52]. Therefore, beside eq. (1.3), which defines $R_2$ in the 't Hooft-Veltman (HV) scheme, a Four Dimensional Helicity scheme (FDH) can be used in which the $\epsilon$ dependence in the numerator function is discarded before integration

$$R_2\bigg|_{FDH} = \frac{1}{(2\pi)^4} \int d^d \vec{q} \frac{\tilde{N}(\mu^2, q, \epsilon = 0)}{D_0 D_1 \cdots D_{m-1}}. \quad (1.7)$$

As for the virtual part of the NLO corrections, FDH is equivalent to Dimensional Reduction [53].

It is clear that explicitly using the rules in eq. (1.4) allows an analytic extraction, Feynman diagram by Feynman diagram, of the coefficients of the various powers of $\mu^2$ and $\epsilon$. For example, the GoSam [54, 55] approach achieves this on the fly, by linking to algebraical manipulation programs providing the necessary algebra when building the amplitude. The case of QCD is particularly simple, and computations based on a standard Passarino-Veltman [56] decomposition are relatively easy [57, 58]. In addition, for gluonic amplitudes, a super-symmetric decomposition relates the contribution of the rational terms
to a scalar massive gluon running in the loop \[59\], which can also be computed by using massive Cachazo-Svrček-Witten Feynman rules \[60, 61, 62\].

All the approaches mentioned so far have advantages and drawbacks. It is however beyond doubt that it would be desirable to have a way to compute \( R_2 \) independent of the theory at hand, four dimensional and not requiring the use of analytical manipulations. In this paper, I provide such a method in the form of primary Feynman rules, which reproduce the polynomial dependence on \( \mu^2 \) of the integrand in eq. (1.7) \(^3\) when working in the renormalizable gauge. As will be explained in the next section, such rules are uniquely determined by reading the original propagators and vertices of the theory, and solely depend on their Lorentz structure. They can therefore be used as any other Feynman rules by programs such as MadLoop [12], HELAC-NLO [13], FeynRules [63] or within a methods such as Open Loops [64] to automatically generate \( R_2 \).

This paper is organized as follows: in section 2.1 I give an introductory discussion and fix my conventions. Section 3 introduces vectors, scalars and their mutual interactions. The fermionic case is more delicate, and it is discussed in section 4. In section 5 the generic formulae are specialized to the QCD case and, in section 6, I draw my conclusions. Further details are reported in appendix A.

2. Preliminaries and conventions

I am interested in reconstructing the polynomial dependence on \( \mu^2 \) of the \( \epsilon \)-dimensional numerator in eq. (1.7). In a renormalizable gauge the relevant expansion is

\[
\tilde{N}(\mu^2, q, \epsilon = 0) = \sum_{j=1}^{2} (\mu^2)^j c_j(q).
\]

(2.1)

As already stated in section 1, an explicit dependence on \( \epsilon \), such as that implied by eq. (1.5), can always be reproduced by a change of regularization scheme. The translation rules relevant for QCD and for the electroweak standard model are given in appendix A.

By inserting eq. (2.1) into eq. (1.7) the relevant integrals are of the kind

\[
\frac{1}{(2\pi)^4} \int \frac{d^\epsilon q}{D_0 D_1 \cdots D_{m-1}} (\mu^2)^j q_{\mu_1} \cdots q_{\mu_r} \quad \text{with} \quad m \leq 4, \ 0 < j \leq 2, \ r \leq 2,
\]

(2.2)

which give a non-vanishing contribution, in the limit \( \epsilon \to 0 \), only when \( 4 + 2j + r - 2m \geq 0 \). They are computed in [51].

Powers of \( \mu^2 \) in eq. (2.1) are generated by the contraction of the integration momentum with itself only in the presence of vectors and fermions, which are indeed the only particles bringing the integration momentum in the numerator. For each vector field \( V_\alpha \), I therefore introduce a scalar field \( \tilde{V} \), whose propagator corresponds to the propagation of its \( \epsilon \)-dimensional components. In the same way, for any fermion \( F \), I introduce a fermionic field \( \tilde{F} \) and its corresponding propagator. In the following, \( \tilde{V} \) and \( \tilde{F} \) are called \( \epsilon \)-particles.

\(^3\)The conversion to the HV scheme of eq. (1.5) is presented in appendix A.
and their propagators are graphically represented by a dashed line and a dashed arrow, respectively.

$\epsilon$-particles interact among themselves and with the particles of the original theory, bringing an explicit dependence on $\mu$ and are only allowed to circulate in the loop. I call $\epsilon$-vertices the special vertices involving $\epsilon$-particles, and $\epsilon$-diagrams the Feynman diagrams that contain $\epsilon$-vertices. Given the fact that the only possible mechanisms for generating powers of $\mu^2$ are those illustrated in eq. (1.4), it is easy to determine the $\epsilon$-vertices by simply looking at the Feynman rules of the original theory. In particular, the Lorentz and color structures are completely dictated by the nature of the particles entering the $\epsilon$-vertices, so that, in general, only the relative phases of $\mu$ and/or $\sqrt{\mu}$ have to be determined. I fixed them by explicitly writing down all possible classes of 2, 3, and 4-point Feynman diagrams involving scalars, vectors and fermions and by requiring that the $\epsilon$-vertices reproduce the results obtained with an explicit calculation of $R_2$. It should be possible to determine the $\epsilon$-vertices directly from the original Lagrangian, by splitting it into 4 and $\epsilon$ dimensional parts, but I did not choose such an approach. In the following two sections, I list all possibilities involving vectors, scalars and fermions. I do it in a completely generic way, in the sense that any particular model can be implemented just by giving a specific value to the constants in front of the listed Lorentz structures. For example, the electroweak model is obtained by fixing them according to reference [65] and the minimal supersymmetric standard model by using the rules in [66].

3. Vectors, scalars and their interactions

The propagator of a scalar $\epsilon$-particle associated with a vector field is given in fig. 1. From the original three-vector vertex, two corresponding $\epsilon$-vertices are derived, as illustrated in fig. 2. To determine the sign of the first $\epsilon$-vertex, one should keep track of the flow of the loop momentum $q$. The original four-vector vertex gives rise to the two $\epsilon$-vertices of fig. 3 while two-vector-one-scalar, one-vector-two-scalar and two-vector-two-scalar vertices generate just one $\epsilon$-vertex each, as shown in figs. 4 to 6.

Although the $\epsilon$-vertices can be used in any stage of the calculation to compute $\epsilon$-diagrams reproducing the coefficients $c_j(q)$ of eq. (2.1), it may be useful and illuminating to consider them in strict connection with the original Feynman diagrams. With this kind of reasoning, one may associate to any 1-loop Feynman diagram contributing to the amplitude under study, a set of $\epsilon$-diagrams which fully reconstruct its $\mu^2$ dependence. In particular, when using the same rooting for the loop momentum in each of them, this reconstruction even holds at the integrand level. Furthermore, given the fact that the $\epsilon$-vertices bring known powers of $\mu$ into the $\epsilon$-diagrams, one can easily disentangle sub-sets generating specific powers of $\mu^2$ in eq. (2.1). A concrete example is given in fig. 7 for two particular diagrams contributing to a $VV \rightarrow VV$ scattering, where the sum of the two $\epsilon$-diagrams in the second line yields the term $\mu^4 c_2(q)$ corresponding to the original box diagram.
\[
V_\alpha \sim \frac{p}{p^2 - M_V^2} V_\beta = -i \frac{g_{\alpha\beta}}{p^2 - M_V^2}
\]

\[
\hat{V} \sim \frac{-1}{p^2 - M_V^2}
\]

**Figure 1:** Propagator of a vector particle (box on the top) and propagator of its corresponding scalar \(\epsilon\)-particle.

\[
V_1 \alpha \quad p_1 \quad V_2 \beta \quad p_2 \quad V_3 \gamma \quad p_3 = -ie C \left[ g_{\alpha\beta}(p_2 - p_1)\gamma + g_{\beta\gamma}(p_3 - p_2)\alpha + g_{\gamma\alpha}(p_1 - p_3)\beta \right]
\]

\[
\hat{V}_1 \quad \pm q \quad V_2 \beta \quad \pm q \quad \hat{V}_3 \beta \quad V_3 \gamma = -ie C (\pm i\mu) g_{\beta\gamma}
\]

**Figure 2:** Three-vector vertex (box on the top) and its corresponding \(\epsilon\)-vertices. \(q\) represents the flow of the loop momentum.

4. Fermions and their interactions with vectors and scalars

The study of the interactions involving fermions is complicated by the presence of \(\gamma_5\). It is convenient to start from the chiral fermions of the original theory and to split the fermion propagator in chirality flipping and chirality preserving parts, as illustrated in the top part of fig. 8. The \(\epsilon\)-propagator corresponding to the \(\epsilon\)-particle associated with a fermion is chirality flipping, being reminiscent of the presence of a \(\not{p}\) in the numerator, as shown in the bottom part of fig. 8. Fermions can interact with vectors and scalars with the standard vector-fermion-fermion and scalar-fermion-fermion vertices shown in the top parts of figs. 9 and 10, while the corresponding \(\epsilon\)-vertices are drawn in the bottom parts. Notice also that all vertices in fig. 8 are chirality flipping, due to the presence of \(\gamma_5\) in the original interaction, while those in fig. 10 are chirality preserving, because of their scalar nature. Particularly interesting is the vertex in fig. 8 (c), which represents a \(\hat{V} \hat{F} F\) interaction. Although no \(\gamma\)
matrix is present in that vertex, it should be considered as a chirality flipping one, because of its vectorial origin.

$\epsilon$-diagrams are built by using the rules of figs. 8 to 10 and reading, as usual, the fermionic line backward starting from the arrow. After the last vertex is encountered, a chirality projector $\omega^\pm = \frac{1}{2}(1 \pm \gamma_5)$ should be inserted, according to the chirality of
\[ V_\alpha S_1 p_1 S_2 = i e C (p_1 - p_2)_\alpha \]

\[ \hat{V} \quad \pm q \quad S_1 \quad = \quad \hat{V} \quad \pm q \quad S_2 \quad = \quad i e C (\pm i\mu) \]

**Figure 5:** One-vector-two-scalar vertex (box on the top) and its corresponding \( \epsilon \)-vertex. \( q \) represents the flow of the loop momentum.

\[ V_{1\alpha} S_1 = i e^2 C g_{\alpha\beta} \]

\[ V_{2\beta} S_2 = i e^2 C \]

\[ \hat{V}_1 S_1 = \]

\[ \hat{V}_2 S_2 = i e^2 C \]

**Figure 6:** Two-vector-two-scalar vertex (box on the top) and its corresponding \( \epsilon \)-vertex.

the fermion entering into it. For example, the diagram in fig. 11 generates the fermionic structure

\[ \bar{\nu}(1) \gamma_\beta \gamma_\alpha \gamma^\beta \omega^h u(2) . \]  

(4.1)

For fermionic loops, the starting point is arbitrary and should be kept fixed when summing over diagrams and families in order to preserve the right cancellations [47].

When scalars are present, the rules presented in figs. 8 to 10 define the diagrams up to a sign, which has to be included by hand. This is due to the anticommutation properties of \( \hat{q} \) with the four dimensional \( \gamma \) matrices. The rule for fixing the sign is as follows. For each pair of \( \epsilon \)-fermion lines present in a given diagram the result should be multiplied by \( (-)^{n_s + n_p} \), where \( n_s \) is the number of scalar vertices and \( n_p \) the number of chirality preserving propagators of the kind \( i \frac{m_p}{p^2 - m_p^2} \) between them. For example, a minus sign should
Figure 7: Examples of \( \epsilon \)-diagrams. In the two boxes I draw the original diagrams. The \( \epsilon \)-diagrams below each box reconstruct the complete \( \mu^2 \) dependence of the integrand of the corresponding diagram, provided the same rooting for the loop momentum is chosen in each of them.
\[
\begin{align*}
\frac{p}{F} \rightarrow \frac{\not{p}}{F} &= \frac{i}{\not{p} - m_F} = -\frac{h}{F} \rightarrow \frac{h}{F} + \frac{h}{F} \rightarrow \frac{h}{F} \\
-\frac{h}{F} \rightarrow \frac{p}{h} \rightarrow \frac{\not{h}}{F} &= \frac{h}{F} \rightarrow \frac{h}{F} = \frac{i}{p^2 - m_F^2}. \\
\frac{p}{\not{F}} \rightarrow \frac{\not{p}}{\not{F}} &= \frac{i}{p^2 - M_F^2} = -\frac{h}{\not{F}} \rightarrow \frac{h}{\not{F}}
\end{align*}
\]

Figure 8: In the box on the top a fermion propagator is split in chirality flipping and chirality preserving parts. \(h = \pm\) denotes right-handed or left-handed components and the dashed line on the bottom represents the propagator of the \(\epsilon\)-particle associated with a fermion.

to that one in fig. 12 (c).

A last subtlety concerns again the vertex in fig. 9 (c). The scalar \(\epsilon\)-dimensional degrees of freedom brought by the field \(\hat{\mathcal{V}}\) can only originate from one of the vertices introduced in section 3. In fact, \(\hat{\mathcal{V}}\) fields are only needed for non abelian theories because of the momentum dependent three-vector vertex of fig. 3 4. Therefore, diagrams where both ends of a \(\hat{\mathcal{V}}\) \(\epsilon\)-particle connect to a fermion line should be discarded, such as that one given in fig. 13.

Finally, in the case of Majorana fermions, such as neutralinos and gluinos in SUSY theories, the relative sign of interfering Feynman diagrams can be determined as described in [67, 68].

5. QCD

In the case of QCD no \(\gamma_5\) is present and splitting fermions into right-handed and left-handed components is no longer necessary. No additional sign needs to be inserted, since no scalar particles are involved in the \(\epsilon\)-fermionic vertices, although diagrams in which a scalar \(\epsilon\)-gluon connects 2 fermionic lines should be discarded, as explained in the previous section. I list the relevant QCD \(\epsilon\)-propagators and \(\epsilon\)-vertices in fig. 14. As can be seen, they have exactly the same general Lorentz structure described in sections 3 and 4, while terms of the original color structures split among the various contributions in a well defined way.

6. Conclusions

I presented the set of special Feynman rules allowing the reconstruction of the \(\epsilon\)-dimensional part of 1-loop amplitudes in theories with vectors, scalars and fermions. The rules are quite

\footnote{In pure QED, one just needs to introduce one \(\hat{F}\) field for each fermion family.}
\[ V_\alpha F_1 - h F_2 = ie\gamma_\alpha C^h \omega_h \]

(a) \[ V_\alpha - h \hat{F}_1 - h \hat{F}_2 = ie\gamma_\alpha C^h \mu \]

(b) \[ V_\alpha - h F_1 - h \hat{F}_2 = \mu \]

(c) \[ \hat{V} - h \hat{F}_1 - h \hat{F}_2 = ieC^h (\pm i \sqrt{\mu}) \]

**Figure 9:** Vector-fermion-fermion vertex (box on the top) and its corresponding \( \epsilon \)-vertices. \( h = \pm \) denotes right-handed or left-handed fermion components, \( \omega_h \) is a chirality projector and \( q \) represents the flow of the loop momentum.

\[ S F_1 - h F_2 = ieC^h \omega_h \]

\[ S \hat{F}_1 - h \hat{F}_2 = ieC^h \mu \]

\[ S \hat{F}_1 - h \hat{F}_2 = S \hat{F}_1 - h \hat{F}_2 = ieC^h \sqrt{\mu} \]

**Figure 10:** Scalar-fermion-fermion vertex (box on the top) and its corresponding \( \epsilon \)-vertices. \( h = \pm \) denotes right-handed or left-handed fermion components and \( \omega_h \) is a chirality projector.

simple, when assuming a renormalizable gauge, and easily derivable from the vertices of the original theory. They can be used to extract the \( \mu^2 \) dependence from the integrand of any contributing 1-loop Feynman diagram, namely the \( \epsilon \)-dimensional part generated by self contractions of the loop momentum.
The complete electroweak model can be studied by simply fixing the constants appearing in the vertices of figs. 1 to 6 and figs. 8 to 10 to their standard model values, while the interactions relevant for QCD are explicitly listed in fig. 14.

A four dimensional helicity scheme is used in this work, but simple translation rules to the ’t Hooft Veltman scheme are collected in an appendix.

SUSY and BSM theories sharing the same Lorentz structures studied in this paper can be treated in the same way.

The special vertices presented here may also be considered as a practical tool to determine the counter-terms needed to restore gauge invariance in calculations where the numerator function of the 1-loop Feynman diagrams is computed in four dimensions. This
\[
\begin{align*}
p_{\alpha_1, \alpha_2 \beta_1, \beta_2} &= -i \frac{g_{\alpha_1 \beta_1}}{p^2} \delta_{\alpha_2 \beta_2}, \quad \frac{p}{l} = i \delta_{kl}, \quad \frac{p}{k} - m_q, \quad \frac{k}{\alpha_1, \alpha_2} = -ig^{\alpha_1 \alpha_2 \gamma} \alpha_1, \\
\frac{p_1}{p_2} = g f^{abc} \left[ g_{\alpha_1 \beta_1} (p_2 - p_1) + g_{\beta_2 \gamma} (p_3 - p_2) + g_{\gamma_1 \alpha} (p_1 - p_3) \right], \\
\frac{p_3}{\gamma, \gamma_1} &= -ig^2 \left[ f^{fbc} f^{eda} (g_{\beta_2 \gamma} g_{\alpha_1 \gamma_1} - g_{\alpha_1 \beta_1} g_{\gamma_1 \gamma}) + f^{ebd} f^{ecb} (g_{\alpha_1 \beta_1} g_{\gamma_1 \gamma} - g_{\beta_2 \gamma} g_{\alpha_1 \gamma_1}) \right].
\end{align*}
\]

**Figure 14:** QCD Feynman rules (box on the top) and corresponding $\epsilon$-propagators and $\epsilon$-vertices. $q$ represents the flow of the loop momentum.
possibility is particularly appealing in conjunction with schemes such as dimensional redu-
tion, where the use of particular classes of four-dimensional identities involving $\gamma_5$ is
forbidden.

It would also be interesting to generalize this approach to the Unitary gauge and beyond 1-loop. I leave these two issues to future investigations.

Acknowledgments

I would like to thank Fabio Maltoni for useful discussions. This research is supported by
the MEC project FPA2008-02984.

A. From the FDH scheme to the HV scheme

In this appendix, I give the translation rules from the FDH scheme (or dimensional re-
duction) reproduced by the Feynman rules given in sections 3 to 5 and the HV scheme
of eq. (1.5). For QCD, once a 1-loop amplitude $A^{(1)}$ has been computed in FDH, the
corresponding HV result can be obtained with the help of the formula

$$A^{(1)}_{HV} = A^{(1)}_{FDH} + A^{(0)} g^2 \frac{N_c}{16\pi^2} \left[ \frac{N_c(n_q + n_Q - 2)}{6} - \frac{n_q}{2} \frac{N_c^2 - 1}{2N_c} \right],$$

(A.1)

where $A^{(0)}$ is the tree level result, $N_c$ the number of colors, $n_q$ the number of massless quarks
and $n_Q$ the number of massive quarks. For the electroweak model, once the renormalized
1-loop amplitude has been determined in FDH, since the terms proportional to $\epsilon$ in eq. (1.5)
are separately gauge invariant, their total contribution can be completely reabsorbed by
shifts of the renormalization constants. The HV result can then be obtained through the

replacements\(^5\)

\[
\begin{align*}
\delta t &\rightarrow \delta t - \frac{e}{8\pi^2 s} M_W^3 \left(1 + \frac{1}{2c^4}\right) \\
\delta M_H^2 &\rightarrow \delta M_H^2 + 3 \frac{e^2}{16\pi^2 s^2} M_W^2 \left(1 + \frac{1}{2c^4}\right) \\
\delta Z_H &\rightarrow \delta Z_H \\
\delta M_W^2 &\rightarrow \delta M_W^2 + \frac{e^2}{24\pi^2 s^2} M_W^2 \\
\delta Z_W &\rightarrow \delta Z_W - \frac{e^2}{24\pi^2 s^2} \\
\delta M_Z^2 &\rightarrow \delta M_Z^2 + \frac{e^2c^2}{24\pi^2 s^2} M_Z^2 \\
\delta Z_{ZZ} &\rightarrow \delta Z_{ZZ} - \frac{e^2c^2}{24\pi^2 s^2} \\
\delta Z_{AZ} &\rightarrow \delta Z_{AZ} + \frac{e^2c}{12\pi^2 s} \\
\delta Z_{ZA} &\rightarrow \delta Z_{ZA} \\
\delta Z_{AA} &\rightarrow \delta Z_{AA} - \frac{e^2}{24\pi^2} \\
\delta m_{f,i} &\rightarrow \delta m_{f,i} - \frac{m_{f,i}}{2} \frac{e^2}{16\pi^2} \left(\frac{1}{4s^2c^2} - 6 \frac{Q_{f,f_W}^3}{c^2} + 6 \frac{Q_f^2}{c^2} + \frac{1}{2s^2}\right) \\
\delta Z_{f,L}^{i,i} &\rightarrow \delta Z_{f,L}^{i,i} + \frac{e^2}{16\pi^2} \left(\frac{1}{4s^2c^2} - 2 \frac{Q_{f,f_W}^3}{c^2} + \frac{Q_f^2}{c^2} + \frac{1}{2s^2}\right) \\
\delta Z_{f,R}^{i,i} &\rightarrow \delta Z_{f,R}^{i,i} + \frac{e^2}{16\pi^2} \frac{Q_f^2}{c^2}.
\end{align*}
\]

(A.2)

From eq. (A.2), the necessary shifts in the charge renormalization constant and in the sine and cosine of the weak mixing angle (relevant when using the on-shell scheme) can be determined from the equations

\[
\begin{align*}
\delta Z_e &= -\frac{1}{2} \left(\delta Z_{AA} + \frac{8}{c} \delta Z_{ZA}\right) \\
\frac{\delta c}{c} &= \frac{1}{2} \left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) \\
\frac{\delta s}{s} &= -\frac{e^2}{s^2} \frac{\delta c}{c}.
\end{align*}
\]

(A.3)

The rules in eq. (A.2) are easily derived from the explicit knowledge of the part proportional to \(\lambda_{HV}\) in the 2-point functions listed in \[47\].

\(^5\)I use the same notations and conventions of \[65\] and assume a unit CKM matrix.
References

[1] Z. Bern, L. J. Dixon, D. C. Dunbar, and D. A. Kosower, One loop n point gauge theory amplitudes, unitarity and collinear limits, Nucl.Phys. B425 (1994) 217–260, [hep-ph/9403226].

[2] Z. Bern, L. J. Dixon, D. C. Dunbar, and D. A. Kosower, Fusing gauge theory tree amplitudes into loop amplitudes, Nucl.Phys. B435 (1995) 59–101, [hep-ph/9409265].

[3] R. Britto, F. Cachazo, and B. Feng, Generalized unitarity and one-loop amplitudes in N=4 super-Yang-Mills, Nucl.Phys. B725 (2005) 275–305, [hep-th/0412103].

[4] G. Ossola, C. G. Papadopoulos, and R. Pittau, Reducing full one-loop amplitudes to scalar integrals at the integrand level, Nucl.Phys. B763 (2007) 147–169, [hep-ph/0609007].

[5] D. Forde, Direct extraction of one-loop integral coefficients, Phys.Rev. D75 (2007) 125019, [arXiv:0704.1835].

[6] G. Ossola, C. G. Papadopoulos, and R. Pittau, CutTools: A Program implementing the OPP reduction method to compute one-loop amplitudes, JHEP 0803 (2008) 042, [arXiv:0711.3596].

[7] C. Berger, Z. Bern, L. Dixon, F. Febres Cordero, D. Forde, et. al., An Automated Implementation of On-Shell Methods for One-Loop Amplitudes, Phys.Rev. D78 (2008) 036003, [arXiv:0803.4186].

[8] W. Giele and G. Zanderighi, On the Numerical Evaluation of One-Loop Amplitudes: The Gluonic Case, JHEP 0806 (2008) 038, [arXiv:0805.2152].

[9] R. Ellis, K. Melnikov, and G. Zanderighi, Generalized unitarity at work: first NLO QCD results for hadronic W+3-jet production, JHEP 0904 (2009) 077, [arXiv:0901.4101].

[10] A. van Hameren, C. Papadopoulos, and R. Pittau, Automated one-loop calculations: A Proof of concept, JHEP 0909 (2009) 106, [arXiv:0903.4665].

[11] C. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, et. al., Next-to-Leading Order QCD Predictions for W+3-Jet Distributions at Hadron Colliders, Phys.Rev. D80 (2009) 074030, [arXiv:0907.1984].

[12] V. Hirschi, R. Frederix, S. Frixione, M. V. Garzelli, F. Maltoni, and R. Pittau, Automation of one-loop QCD corrections, JHEP 1105 (2011) 044, [arXiv:1103.0622].

[13] G. Bevilacqua, M. Czakon, M. Garzelli, A. van Hameren, A. Kardos, et. al., HELAC-NLO, [arXiv:1110.1499].

[14] V. Hirschi, New developments in MadLoop, [arXiv:1111.2708].

[15] G. P. Salam, Perturbative QCD for the LHC, PoS ICHEP2010 (2010) 556, [arXiv:1103.1318].

[16] R. Ellis, Z. Kunszt, K. Melnikov, and G. Zanderighi, One-loop calculations in quantum field theory: from Feynman diagrams to unitarity cuts, [arXiv:1105.4319].

[17] T. Binoth, G. Ossola, C. Papadopoulos, and R. Pittau, NLO QCD corrections to tri-boson production, JHEP 0806 (2008) 082, [arXiv:0804.0350].

[18] R. Ellis, W. Giele, Z. Kunszt, K. Melnikov, and G. Zanderighi, One-loop amplitudes for W+3 jet production in hadron collisions, JHEP 0901 (2009) 012, [arXiv:0810.2762].
[19] C. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, et. al., Precise Predictions for $W + 3$ Jet Production at Hadron Colliders, Phys.Rev.Lett. 102 (2009) 222001, arXiv:0902.2760.

[20] A. Bredenstein, A. Denner, S. Dittmaier, and S. Pozzorini, NLO QCD corrections to $pp \rightarrow t \bar{t} b \bar{b} + X$ at the LHC, Phys.Rev.Lett. 103 (2009) 012002, arXiv:0905.0110.

[21] G. Bevilacqua, M. Czakon, C. Papadopoulos, R. Pittau, and M. Worek, Assault on the NLO Wishlist: $pp \rightarrow t \bar{t} b \bar{b}$, JHEP 0909 (2009) 109, arXiv:0907.4723.

[22] A. Bredenstein, A. Denner, S. Dittmaier, and S. Pozzorini, NLO QCD Corrections to Top Anti-Top Bottom Anti-Bottom Production at the LHC: 2. full hadronic results, JHEP 1003 (2010) 021, arXiv:1001.4006.

[23] G. Bevilacqua, M. Czakon, C. Papadopoulos, and M. Worek, Dominant QCD Backgrounds in Higgs Boson Analyses at the LHC: A Study of $pp \rightarrow t \bar{t} + 2$ jets at Next-To-Leading Order, Phys.Rev.Lett. 104 (2010) 162002, arXiv:1002.4009.

[24] J. Andersen et. al., The SM and NLO Multi-leg Working Group: Summary report, arXiv:1003.1241.

[25] K. Melnikov and M. Schulze, NLO QCD corrections to top quark pair production in association with one hard jet at hadron colliders, Nucl.Phys. B840 (2010) 129–159, arXiv:1004.3284.

[26] C. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, et. al., Precise Predictions for $W + 4$ Jet Production at the Large Hadron Collider, Phys.Rev.Lett. 106 (2011) 092001, arXiv:1009.2338.

[27] A. Denner, S. Dittmaier, S. Kallweit, and S. Pozzorini, NLO QCD corrections to $W W b b$ production at hadron colliders, Phys.Rev.Lett. 106 (2011) 052001, arXiv:1012.3975.

[28] G. Bevilacqua, M. Czakon, A. van Hameren, C. G. Papadopoulos, and M. Worek, Complete off-shell effects in top quark pair hadroproduction with leptonic decay at next-to-leading order, JHEP 1102 (2011) 083, arXiv:1012.4230.

[29] T. Melia, K. Melnikov, R. Rontsch, and G. Zanderighi, NLO QCD corrections for $W^+ W^-$ pair production in association with two jets at hadron colliders, Phys.Rev. D83 (2011) 114043, arXiv:1104.2327.

[30] R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, R. Pittau, et. al., Scalar and pseudoscalar Higgs production in association with a top-antitop pair, Phys.Lett. B701 (2011) 427–433, arXiv:1104.5613.

[31] F. Campanario, C. Englert, M. Rauch, and D. Zeppenfeld, Precise predictions for $W \gamma \gamma +$ jet production at hadron colliders, Phys.Lett. B704 (2011) 515–519, arXiv:1106.4009.

[32] R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, R. Pittau, et. al., $W$ and $Z/\gamma *$ boson production in association with a bottom-antibottom pair, JHEP 1109 (2011) 061, arXiv:1106.6019.

[33] K. Arnold, J. Bellm, G. Bozzi, M. Brieg, F. Campanario, et. al., VBFNLO: A parton level Monte Carlo for processes with electroweak bosons – Manual for Version 2.5.0, arXiv:1107.4033.

[34] H. Ita, Z. Bern, L. Dixon, F. Cordero, D. Kosower, et. al., Precise Predictions for $Z + 4$ Jets at Hadron Colliders, arXiv:1108.2229.
[35] G. Bevilacqua, M. Czakon, C. Papadopoulos, and M. Worek, *Hadronic top-quark pair production in association with two jets at Next-to-Leading Order QCD*, [arXiv:1108.2851](http://arxiv.org/abs/1108.2851).

[36] R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, R. Pittau, et. al., *Four-lepton production at hadron colliders: aMC@NLO predictions with theoretical uncertainties*, [arXiv:1110.4738](http://arxiv.org/abs/1110.4738).

[37] R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, R. Pittau, et. al., *aMC@NLO predictions for Wjj production at the Tevatron*, [arXiv:1110.5502](http://arxiv.org/abs/1110.5502).

[38] T. Binoth, J. Guillet, and G. Heinrich, *Algebraic evaluation of rational polynomials in one-loop amplitudes*, *JHEP* **0702** (2007) 013, [hep-ph/0609054](http://arxiv.org/abs/hep-ph/0609054).

[39] G. Ossola, C. G. Papadopoulos, and R. Pittau, *On the Rational Terms of the one-loop amplitudes*, *JHEP* **0805** (2008) 004, [arXiv:0802.1876](http://arxiv.org/abs/0802.1876).

[40] A. Bredenstein, A. Denner, S. Dittmaier, and S. Pozzorini, *NLO QCD corrections to t anti-t b anti-b production at the LHC: 1. Quark-antiquark annihilation*, *JHEP* **0808** (2008) 108, [arXiv:0807.1248](http://arxiv.org/abs/0807.1248).

[41] F. Campanario, *Towards pp → VVjj at NLO QCD: Bosonic contributions to triple vector boson production plus jet*, *JHEP* **1110** (2011) 070, [arXiv:1105.0920](http://arxiv.org/abs/1105.0920).

[42] W. T. Giele, Z. Kunszt, and K. Melnikov, *Full one-loop amplitudes from tree amplitudes*, *JHEP* **0804** (2008) 049, [arXiv:0801.2237](http://arxiv.org/abs/0801.2237).

[43] Z. Bern, L. J. Dixon, and D. A. Kosower, *Bootstrapping multi-parton loop amplitudes in QCD*, *Phys.Rev.* **D73** (2006) 065013, [hep-ph/0507005](http://arxiv.org/abs/hep-ph/0507005).

[44] S. Badger, E. Glover, and K. Risager, *One-loop phi-MHV amplitudes using the unitarity bootstrap*, *JHEP* **0707** (2007) 066, [arXiv:0704.3914](http://arxiv.org/abs/0704.3914).

[45] S. Badger, *Direct Extraction Of One Loop Rational Terms*, *JHEP* **0901** (2009) 049, [arXiv:0806.4600](http://arxiv.org/abs/0806.4600).

[46] P. Draggiotis, M. Garzelli, C. Papadopoulos, and R. Pittau, *Feynman Rules for the Rational Part of the QCD 1-loop amplitudes*, *JHEP* **0904** (2009) 072, [arXiv:0903.0353](http://arxiv.org/abs/0903.0353).

[47] M. Garzelli, I. Malamos, and R. Pittau, *Feynman rules for the rational part of the Electroweak 1-loop amplitudes*, *JHEP* **1001** (2010) 040, [arXiv:0910.3130](http://arxiv.org/abs/0910.3130).

[48] M. Garzelli, I. Malamos, and R. Pittau, *Feynman rules for the rational part of the Electroweak 1-loop amplitudes in the R$_x$ gauge and in the Unitary gauge*, *JHEP* **1101** (2011) 029, [arXiv:1009.4302](http://arxiv.org/abs/1009.4302).

[49] M. Garzelli and I. Malamos, *R2SM: A Package for the analytic computation of the R$_2$ Rational terms in the Standard Model of the Electroweak interactions*, *Eur.Phys.J.* **C71** (2011) 1605, [arXiv:1010.1248](http://arxiv.org/abs/1010.1248).

[50] H.-S. Shao, Y.-J. Zhang, and K.-T. Chao, *Feynman Rules for the Rational Part of the Standard Model One-loop Amplitudes in the ’t Hooft-Veltman $\gamma_5$ Scheme*, *JHEP* **1109** (2011) 048, [arXiv:1106.5030](http://arxiv.org/abs/1106.5030).

[51] G. Ossola, C. G. Papadopoulos, and R. Pittau, *Numerical evaluation of six-photon amplitudes*, *JHEP* **0707** (2007) 085, [arXiv:0704.1271](http://arxiv.org/abs/0704.1271).

[52] A. Signer and D. Stockinger, *Using Dimensional Reduction for Hadronic Collisions*, *Nucl.Phys.* **B808** (2009) 88–120, [arXiv:0807.4424](http://arxiv.org/abs/0807.4424).
[53] D. Stockinger, *Regularization by dimensional reduction: consistency, quantum action principle, and supersymmetry*, JHEP 0503 (2005) 076, [hep-ph/0503123](http://arxiv.org/abs/hep-ph/0503123).

[54] P. Mastrolia, G. Ossola, T. Reiter, and F. Tramontano, *Scattering AMplitudes from Unitarity-based Reduction Algorithm at the Integrand-level*, JHEP 1008 (2010) 080, [arXiv:1006.0710](http://arxiv.org/abs/1006.0710).

[55] G. Cullen, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, *et. al.*, *Automation of One-Loop Calculations with GoSam: Present Status and Future Outlook*, arXiv:1111.3339.

[56] G. Passarino and M. Veltman, *One Loop Corrections for e+ e- Annihilation Into mu+ mu- in the Weinberg Model*, Nucl.Phys. B160 (1979) 151.

[57] Z. Xiao, G. Yang, and C.-J. Zhu, *The Rational Part of QCD Amplitudes. I. The General Formalism*, Nucl.Phys. B758 (2006) 1–34, [hep-ph/0607015](http://arxiv.org/abs/hep-ph/0607015).

[58] S. Badger, R. Sattler, and V. Yundin, *One-Loop Helicity Amplitudes for t\bar{t} Production at Hadron Colliders*, Phys.Rev. D83 (2011) 074020, [arXiv:1101.5947](http://arxiv.org/abs/1101.5947).

[59] S. Badger, B. Biedermann, and P. Uwer, *NGluon: A Package to Calculate One-loop Multi-gluon Amplitudes*, Comput.Phys.Commun. 182 (2011) 1674–1692, [arXiv:1011.2900](http://arxiv.org/abs/1011.2900).

[60] R. Boels and C. Schwinn, *CSW rules for a massive scalar*, Phys.Lett. B662 (2008) 80–86, [arXiv:0712.3409](http://arxiv.org/abs/0712.3409).

[61] E. Nigel Glover and C. Williams, *One-Loop Gluonic Amplitudes from Single Unitarity Cuts*, JHEP 0812 (2008) 067, [arXiv:0810.2964](http://arxiv.org/abs/0810.2964).

[62] H. Elvang, D. Z. Freedman, and M. Kiermaier, *Integrands for QCD rational terms and N=4 SYM from massive CSW rules*, arXiv:1111.0635.

[63] N. D. Christensen and C. Duhr, *FeynRules - Feynman rules made easy*, Comput.Phys.Commun. 180 (2009) 1614–1641, [arXiv:0806.4194](http://arxiv.org/abs/0806.4194).

[64] F. Cascioli, P. Maierhofer, and S. Pozzorini, *Scattering Amplitudes with Open Loops*, arXiv:1111.5206.

[65] A. Denner, *Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200*, Fortsch.Phys. 41 (1993) 307–420, [arXiv:0709.1075](http://arxiv.org/abs/0709.1075).

[66] J. Rosiek, *Complete set of Feynman rules for the MSSM: Erratum*, hep-ph/9511250.

[67] A. Denner, H. Eck, O. Hahn, and J. Kublbeck, *Feynman rules for fermion number violating interactions*, Nucl.Phys. B387 (1992) 467–484.

[68] A. Denner, H. Eck, O. Hahn, and J. Kublbeck, *Compact Feynman rules for Majorana fermions*, Phys.Lett. B291 (1992) 278–280.

[69] A. Signer, *Helicity method for next-to-leading order corrections in QCD*, Ph.D. Thesis.