The Research on a parallel and synchronous encoding and decoding method of multi-level LDPC

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Abstract. Based on the research of Binary Low-Density Parity-Check (LDPC) code, this paper studies the Quaternary LDPC code. Based on LU decomposition and BP algorithm, a block and partition parallel mode is proposed to make the data receiving and code compiling process run at the same time, which not only improves the coding speed, but also greatly reduces the time delay caused by data receiving before coding, and the feasibility of the algorithm is verified by simulation. At the same time, synchronization technology is added to the system to minimize the error caused by parallel transmission and ensure the reliability of data reception. The simulation results show that the new algorithm can effectively reduce the time delay and the bit error rate to ensure the effectiveness and reliability of information transmission.

1. Introduction

5G as a new generation of mobile communication technology, its rapid development trend brings new direction for the application of wireless network. At the same time, the limited bandwidth resources of wireless network, the harsh environment of random time-varying, and the severe multipath fading all put forward higher requirements for the reliability and band utilization of signal transmission. In order to improve the wireless communication environment to the greatest extent and alleviate the serious interference caused by the channel to the signal, it is necessary to find a good channel coding method to realize any small error probability coding at a non-zero rate less than the channel capacity [1]. At this time, low density parity check (LDPC) code has attracted more and more scholars' attention because of its error correction performance approaching Shannon limit, simple decoding, error detection and other characteristics, and has become a research hotspot in the current academic and industrial circles.

LDPC code has high complexity in encoding. Although it can be encoded in linear time, its complexity is still too large compared with real-time encoding signals such as convolutional code. When the code length is very long, it must receive all the information bits before coding, which also brings a certain delay to the signal coding[2]. In order to solve this problem, this paper preprocesses the check matrix and coding signal, and divides them into blocks according to the specific way, so that the data receiving and coding process can be carried out at the same time. It can not only improve the coding rate, but also reduce the time delay caused by data reception before coding. At the same time, synchronization technology is added to reduce the bit error rate and ensure the reliability of data reception.
2. LDPC code

LDPC code is a kind of linear block code, which was first invented by Gallager in 1962 [3][4]. However, due to its high decoding complexity, it has not been widely spread and studied. It was not until Robert. J. MC Elice and others [5] elaborated the implementation of belief propagation algorithm and message passing algorithm that LDPC code was widely studied and became a milestone in the development history of LDPC code. In 2016, 3GPP organization took LDPC code as the data channel coding scheme of 5G eMBB scenario, which set off a research upsurge of LDPC code.

2.1. Check matrix

In LDPC coding, a check matrix H needs to be generated first, so as to obtain the generation matrix G, and then generate different code words. At present, the construction methods of check matrix generally include random construction and structural construction. In this paper, the Quasi-Cyclic (QC) code construction method is used.

The check matrix of QC-LDPC code is a cyclic structure, and also a grid matrix. Each block is a cyclic sub matrix. Assuming that the length of each segment is l, for a check matrix H with a size of M × n, mark \( m = m_b l \), \( n = n_b l \), then the classical expression of QC-LDPC code is:

\[
H_{m \times n} = \begin{bmatrix}
P_{0,0} & \cdots & P_{0,m_b-1} \\
\vdots & \ddots & \vdots \\
P_{m_b-1,0} & \cdots & P_{m_b-1,n_b-1}
\end{bmatrix}_{m_b \times n_b}
\]  

Each sub block represents a sub matrix of size L × L, and any sub matrix can be rotated to obtain a check matrix [6]. Therefore, in general, a basic matrix is constructed first, and then a complete check matrix H is obtained by cyclic shift expansion. However, when the basic matrix is constructed, it is easy to form a closed loop when the elements of the matrix are connected end to end. When a ring is a short ring, especially a "4-ring" with a length of 4, it will lead to cross correlation in LDPC code decoding. This cross-correlation will cause errors in signal transmission, slow the convergence speed of decoding, and fail to converge in serious cases, resulting in larger way to construct the check matrix to avoid the generation of 4-ring.

Based on AP sequence, this paper studies the construction method of QC-LDPC code check matrix without "4-ring" [7]. Firstly, the reference matrix \( H_b \) is constructed. By selecting the appropriate expansion factor K, a submatrix with dimension \( k \times K \) is constructed, so that the elements in each position of the reference matrix are modular K multiplication. Then, the part of \( H_b1 \) corresponding to the code word check bit is intercepted from the reference matrix \( H_b \), and then the \( H_b1 \) is extended to form the information bit \( H_1 \) of the code word. Then the check bit \( H_2 \) of the codeword is formed and combined with \( H_1 \) to complete the construction of the check matrix H.

2.2. Multi-level LDPC code

Based on the binary LDPC code, the multi-level LDPC code is obtained by extending it to the finite field \( GF(q) \), so it can also be represented by the check matrix. The elements in the matrix come from the elements \( \{0,1,2,...,q-1\} \) in the finite field \( GF(q) \), and the symbol in the corresponding multi-level LDPC code represents \( \log_2q \) information bits. Common coding algorithms include Gauss elimination, system coding, LU decomposition coding and so on [7]. The decoding algorithm has BP algorithm and so on. The encoding and decoding methods used in this paper are improved LU decomposition coding and BP decoding algorithm. The encoding and decoding process is shown in Figure 1.
After receiving the information, the receiver uses the channel likelihood information $f_n(a)$ to initialize the variable message. Define variable node message as $q_{nm}^{(0)}(a) = f_n(a)$, the information is displaced as it passes through the node [8]. The replaced message is shown in formula (2).

$$q_{nm}^{(1)}(a) = q_{nm}^{(0)} \left( \frac{a}{h_{nm}} \right)$$  \hspace{1cm} (2)

In (2), $n$ is the variable node, $m$ is the check node, $h_{nm}$ is the element of row $m$ and column $n$ in the check matrix $H_{M \times N}$, division is carried out in the finite field $GF(q)$. Then, the message of the verification node is expanded by using the total probability formula:

$$r_{nm}^{(0)}(a) = P(c_m = 0 | c_n = a) = \sum_{c_n = a} P(c_m = \sum_{n' \in E(n)} h_{mn'} \cdot c_{n'}) \prod_{n' \in E(n) \setminus n} q_{mn'}^{(0)}(a)$$  \hspace{1cm} (3)

In (3), $c_m$ represents the $m$-th verification relation, and $V$ represents all $c_m$ vector sets satisfying the verification relation.

When the verification node passes the message to the variable node, it also needs to pass through the replacement node, the replaced message is as shown in formula (4):

$$r_{nm}^{(1)}(a) = r_{nm}^{(0)}(a \cdot h_{mn})$$  \hspace{1cm} (4)

In (4), $h_{mn}$ is the element of row $m$ and column $n$ in the check matrix $H_{M \times N}$, and the multiplication is carried out in the finite field $GF(q)$.

Then update the variable node according to formula (5), calculate $Q_n(a)$ in formula (6) and find out the maximum value. Finally, according to formula (7) to determine the decoding stop, if the decoded codeword meets formula (7) in (a) or the number of iterations reaches the maximum, the decoding stops, otherwise, the decoding continues after returning to initialization.

$$q_{nm}^{(1)}(a) = a_{mn} \cdot f_n(a) \prod_{m' \in E(n) \setminus m} r_{m'n}^{(1)}(a)$$  \hspace{1cm} (5)

$$Q_n(a) = a_n \cdot f_n(a) \prod_{m' \in E(n)} r_{m'n}^{(1)}(a), \hat{c}_n = \arg \max_a Q_n(a)$$  \hspace{1cm} (6)

$$H_{M \times N} \cdot \hat{c}^T = 0 \hspace{1cm} (a)$$

$$l = l_{\text{max}} \hspace{1cm} (b)$$  \hspace{1cm} (7)

3. LDPC codec system with synchronization code

Although the multi-band LDPC code can effectively reduce the cross-talk caused by the multi-path channel, when the code length is long, it will cause additional delay due to the problem that all
information bits must be received before coding, which will reduce the transmission efficiency, especially when the information transmitted is large, this disadvantage is more significant. Therefore, this paper selects the appropriate code length after the test, and blocks the transmitted information according to the code length, so that the data can be transmitted in parallel and the transmission rate can be shortened. At the same time, the synchronization code is added to restore the original signal, reduce the bit error rate, and ensure the effectiveness of information transmission. In this paper, we use the central insertion code in group synchronization. This method uses a special group synchronization code group, which is inserted in front of the information code group, so that the receiver can easily capture the signal[8]. In order to identify the synchronization code easily by the sharp single peak autocorrelation, a 13 bit Barker code is used as the synchronization code group in group synchronization.

The synchronization technology is added to the encoding and decoding system of multi base LDPC code to study the feasibility and effectiveness of the hybrid algorithm. The system block diagram is shown in Figure 2.

![Figure 2. Encoding and Decoding System of Multi-level LDPC code.](image)

4. Simulation analysis

In this paper, MATLAB software is used to test the performance of the codec system. The message sent is a 64 * 64 gray-scale picture, code length 1000, code rate 1 / 2 of the Quaternary LDPC code, QPSK modulation. Then the modulated signal passes through the wireless channel with multipath effect, and Gaussian white noise is added. The signal-to-noise ratio is 0 ~ 60dB, which is decoded by block-BP algorithm. The simulation results are as follows:

![Figure 3. Send picture.](image)

![Figure 4. Constellation of Transmitting Signal.](image)

Figure 3 is the gray image sent by the source. Figure 4 is the constellation of the transmitted signal, (a) is the constellation of the picture information after QPSK modulation, (b) is the constellation of the channel plus noise. Through the comparison of the two images, it can be found that the signal received serious interference after passing through the multipath channel.
Figure 5 is the recovery diagram of each stage of sending picture, (a) is the received picture without the Quaternary LDPC coding system; (b) and (c) is the received picture after the Quaternary LDPC coding system, wherein (b) is a received picture without a synchronization code, (c) is a received picture with a synchronization code. From the (a) diagram, it is obvious that the signal is seriously interfered after passing through the multipath channel, and it is impossible to extract any useful information from the signal when it is output directly. Through the comparison of (a) and (b) diagrams, it can be seen that the signal has been greatly improved after being coded by multi-level LDPC, and useful signals can be roughly inferred, which effectively improves the impact of multipath effect and fully proves the effectiveness of the system. Through (c) diagram, it can be clearly found that the performance of the system has been greatly improved after adding synchronization code, and the sending information has been accurately recovered at the receiving end, which fully proves the reliability of the system.

Figure 6 is the error rate comparison curve, in which the red curve is the system error rate with synchronization code added and the green curve is the system error rate without synchronization code added. Through comparison, it can be clearly found that the error rate of the system with synchronization code decreases rapidly and tends to be stable with the increase of SNR, which shows that the system with synchronization code is more reliable.

| SNR(dB) | Add sync code | Sync code not added |
|---------|---------------|---------------------|
| 0       | $1.951621\times10^{-3}$ | $4.236189\times10^{-3}$ |
| Code Length | Error Rate (x10^-3) | Bit Error Rate (x10^-3) |
|------------|---------------------|------------------------|
| 5          | 1.265955 × 10^-3    | 4.239273 × 10^-3       |
| 10         | 7.753939 × 10^-4    | 4.231795 × 10^-3       |
| 15         | 3.771364 × 10^-4    | 4.232765 × 10^-3       |
| 20         | 1.030455 × 10^-4    | 4.235295 × 10^-3       |
| 25         | 1.026515 × 10^-5    | 4.239576 × 10^-3       |
| 30         | 7.803030 × 10^-6    | 4.238333 × 10^-3       |
| 35         | 7.803030 × 10^-6    | 4.237318 × 10^-3       |
| 40         | 7.803030 × 10^-6    | 4.238455 × 10^-3       |
| 45         | 7.803030 × 10^-6    | 4.237788 × 10^-3       |
| 50         | 7.803030 × 10^-6    | 4.237553 × 10^-3       |
| 55         | 7.803030 × 10^-6    | 4.239258 × 10^-3       |
| 60         | 7.803030 × 10^-6    | 4.239576 × 10^-3       |

Table 1 further explains the reliability of the synchronous code system through specific data. It is found that under the condition of certain code length and code rate, the bit error rate decreases gradually with the increase of signal-to-noise ratio, and tends to be stable when it reaches 30dB.

5. Conclusion
This paper mainly studies the coding and decoding method of LDPC code. On the basis of conventional LDPC encoding and decoding, a block and partition mode is proposed, which makes the process of data receiving and encoding at the same time. It not only improves the encoding and decoding speed, but also greatly reduces the delay of data receiving before encoding. At the same time, synchronization technology is added to minimize the bit error rate and ensure the reliability of data reception. Simulation results show that the algorithm can achieve convergence faster and ensure the correctness of information transmission.

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