Masses of tetraquarks with two heavy quarks in the relativistic quark model

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Masses of tetraquarks with two heavy quarks and open charm and bottom are calculated in the framework of the diquark-antidiquark picture in the relativistic quark model. All model parameters were regarded as fixed by previous considerations of various properties of mesons and baryons. The light quarks and diquarks are treated completely relativistically. The $c$ quark is assumed to be heavy enough to make the diquark configurations dominating. The diquarks are considered not to be point-like but to have an internal structure which is taken into account by the calculated diquark form factor entering the diquark-gluon interaction. It is found that all the $(cc)(\bar{q}q')$ tetraquarks have masses above the thresholds for decays into open charm mesons. Only the $I(J^P) = 0(1^+)$ state of $(bb)(\bar{u}\bar{d})$ lies below the $BB^*$ threshold and is predicted to be narrow.

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I. INTRODUCTION

Recent experimental studies of the heavy meson spectroscopy revealed several new states, such as $X(3872)$, $Y(4260)$, $D_{s0}^*(2317)$ etc., which cannot be simply accommodated in the quark-antiquark ($q\bar{q}$) picture [1]. These states can be considered as indications of the possible existence of exotic multiquark states which were proposed long ago, e.g. in [2]. The idea to revisit the multiquark picture using diquarks has been put forward by Jaffe and Wilczek [3]. At present, vast experimental and theoretical evidence of the important role played by diquark correlations in hadrons is collected [4].
The simplest multiquark system is a tetraquark, composed of two quarks and two antiquarks. Heavy tetraquarks are of particular interest, since the presence of a heavy quark increases the binding energy of the bound system and, as a result, the possibility that such tetraquarks will have masses below the thresholds for decays to mesons with open heavy flavour. If such strong decays are kinematically forbidden, then corresponding tetraquarks can decay only weakly or electromagnetically and thus they should have a small decay width. In this paper we consider tetraquarks with two heavy quarks as bound systems of a diquark and antidiquark. Therefore we assume that both $b$ and $c$ quarks are heavy enough to make the attractive $QQ$ interaction stronger than the $Q\bar{q}$ one. For tetraquarks containing $c$ quarks the obtained results crucially depend on this assumption. In particular, the doubly heavy $(QQ')(\bar{q}\bar{q}')$ tetraquark ($Q = b, c$ and $q = u, d, s$) is considered as the bound state of the heavy diquark $(QQ')$ and light antidiquark $(\bar{q}\bar{q}')$, while the $(cq)(b\bar{q}')$ tetraquark is the bound state of the heavy-light diquark $(cq)$ and antidiquark $(b\bar{q}')$. Masses of heavy tetraquarks with hidden charm $(cq)(\bar{c}\bar{q})$ and bottom $(bq)(\bar{b}\bar{q})$ were calculated in our previous paper [5]. There the dynamical analysis has shown that $X(3872)$ and $Y(4260)$ can be indeed the diquark-antidiquark tetraquarks with hidden charm. It was also argued that the corresponding ground-state tetraquarks with hidden bottom have masses below the open bottom threshold, and thus they should be narrow states.

It is important to investigate the possible stability of the $(QQ')(\bar{q}\bar{q}')$ tetraquarks since they are explicitly exotic states with the heavy flavour number equal to 2. Thus, their observation would be a direct proof of the existence of the multiquark states. Estimates of the production rates of such tetraquarks indicate that they could be produced and detected at present (SELEX, Tevatron, RHIC) and future facilities (LHC, LHCb, ALICE) [6].

To calculate the masses of heavy tetraquarks we use the relativistic quark model based on the quasipotential approach in quantum field theory. Previously we considered in our model the mass spectra of the ground-state and excited doubly-heavy $(QQq)$ [7] and heavy $(qqQ)$ [8, 9] baryons in the heavy-diquark–light-quark and light-diquark–heavy-quark approximations, respectively. The light quarks and light diquarks were treated completely relativistically.

The internal structure of the light and heavy diquarks was taken into account by calculating diquark-gluon form factors in terms of the obtained diquark wave functions. Such scheme proved to be very effective and successful in our calculation of the masses of heavy baryons in good agreement with experimental data [10]. The predicted masses of the $\Omega^{*}_c$, $\Sigma_b$, $\Sigma^*_b$ and $\Xi_b$ baryons proved to be very close to the recently measured ones [11, 12, 13]. Moreover, in Ref. [9] it was shown that currently available experimental data on excited charmed baryons can be accommodated in the picture treating a heavy baryon as the bound system of the light diquark and heavy quark, experiencing orbital and radial excitations. This gives us additional confidence in the reliability of the diquark approximation within our model and motivates the consideration of tetraquarks as diquark-antidiquark bound systems. It is important to note that all parameters of our model were fixed in the previous calculations of meson mass spectra and decays, and we will keep their values in the following analysis of heavy tetraquarks.

II. RELATIVISTIC QUARK MODEL

In the quasipotential approach and diquark-antidiquark picture of heavy tetraquarks the interaction of two quarks in a diquark and the diquark-antidiquark interaction in a tetraquark are described by the diquark wave function $(\Psi_d)$ of the bound quark-quark state.
and by the tetraquark wave function \( \Psi_{d,T} \) of the bound diquark-antidiquark state, respectively, which satisfy the quasipotential equation of the Schrödinger type \[14\]

\[
\left( \frac{b^2(M)}{2 \mu_R} - \frac{p^2}{2 \mu_R} \right) \Psi_{d,T}(p) = \int \frac{d^3 q}{(2\pi)^3} V(p, q; M) \Psi_{d,T}(q),
\]

where the relativistic reduced mass is

\[
\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m^2_1 - m^2_2)^2}{4M^3},
\]

and \( E_1, E_2 \) are given by

\[
E_1 = \frac{M^2 - m^2_2 + m^2_1}{2M}, \quad E_2 = \frac{M^2 - m^2_1 + m^2_2}{2M}.
\]

Here, \( M = E_1 + E_2 \) is the bound-state mass (diquark or tetraquark), \( m_{1,2} \) are the masses of quarks \( (q_1 \text{ and } q_2) \) which form the diquark or of the diquark \( (d) \) and antiquark \( (d') \) which form the heavy tetraquark \( (T) \), and \( p \) is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

\[
b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}.
\]

The kernel \( V(p, q; M) \) in Eq. (1) is the quasipotential operator of the quark-quark or diquark-antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. In the following analysis we closely follow the similar construction of the quark-antiquark interaction in mesons which were extensively studied in our relativistic quark model \[14, 15\]. For the quark-quark interaction in a diquark we use the relation \( V_{qq} = V_{q\bar{q}}/2 \) arising under the assumption of an octet structure of the interaction from the difference in the \( qq \) and \( q\bar{q} \) colour states.\(^1\) An important role in this construction is played by the Lorentz structure of the confining interaction. In our analysis of mesons, while constructing the quasipotential of the quark-antiquark interaction, we assumed that the effective interaction is the sum of the usual one-gluon exchange term and a mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli terms. We use the same conventions for the construction of the quark-quark and diquark-antidiquark interactions in the tetraquark. The quasipotential is then defined as follows \[14, 15\].

(a) For the quark-quark \( (qq') \), \( (Qq) \), \( (QQ') \) interactions, \( V(p, q; M) \) reads

\[
V(p, q; M) = \bar{u}_1(p) \bar{u}_2(-p) V(p, q; M) u_1(q) u_2(-q),
\]

with

\[
V(p, q; M) = \frac{1}{2} \left[ 4 \alpha_s D_{\mu\nu}(k) \gamma_1^{\mu} \gamma_2^{\nu} + V^{V}_{\text{conf}}(k) \Gamma_1^{\mu}(k) \Gamma_2_{\mu}(-k) + V^{S}_{\text{conf}}(k) \right],
\]

Here, \( \alpha_s \) is the QCD coupling constant; \( D_{\mu\nu} \) is the gluon propagator in the Coulomb gauge,

\[
D^{00}(k) = -\frac{4\pi}{k^2}, \quad D^{ij}(k) = -\frac{4\pi}{k^2} \left( s^{ij} - \frac{k^ik^j}{k^2} \right), \quad D^{0i} = D^{i0} = 0,
\]

\(^1\) Obviously, it is important to study diquark correlations in gauge-invariant color-singlet hadron states on the lattice.
where the four-vector \( q = p - q \); \( \gamma_\mu \) and \( u(p) \) are the Dirac matrices and spinors,

\[
u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \left( \frac{1}{\epsilon(p) + m} \right) \chi^\lambda,
\]

with \( \epsilon(p) = \sqrt{p^2 + m^2} \).

The effective long-range vector vertex of the quark is defined \([15]\) by

\[
\Gamma_\mu(k) = \gamma_\mu + \frac{i\kappa}{2m} \sigma_{\mu\nu} \tilde{k}^\nu, \quad \tilde{k} = (0, k),
\]

where \( \kappa \) is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In configuration space the vector and scalar confining potentials in the nonrelativistic limit reduce to

\[
V_{\text{conf}}^V(r) = (1 - \varepsilon)V_{\text{conf}}(r), \\
V_{\text{conf}}^S(r) = \varepsilon V_{\text{conf}}(r),
\]

with

\[
V_{\text{conf}}(r) = V_{\text{conf}}^S(r) + V_{\text{conf}}^V(r) = Ar + B,
\]

where \( \varepsilon \) is the mixing coefficient.

(b) For the diquark-antidiquark \((dd')\) interaction, \( V(p, q; M) \) is given by

\[
V(p, q; M) = \frac{\langle d(P)|J_\mu|d(Q')\rangle}{2\sqrt{E_d E_d'}} \left[ \frac{4}{3} \alpha_s D^{\mu\nu}(k) \frac{\langle d'(P')|J_{\nu'}|d'(Q')\rangle}{2\sqrt{E_d' E_d'}} \right] \\
+ \psi_{d'}^\ast(P) \psi_d^\ast(P') \left[ J_{d\mu} J_{d'}^\nu V_{\text{conf}}^V(k) + V_{\text{conf}}^S(k) \right] \psi_d(Q) \psi_d'(Q'),
\]

where \( \langle d(P)|J_\mu|d(Q')\rangle \) is the vertex of the diquark-ghon interaction which takes into account the finite size of the diquark and is discussed below \([11]\).

The diquark state in the confining part of the diquark-antidiquark quasipotential \([11]\) is described by the wave functions

\[
\psi_d(p) = \begin{cases} 
1 & \text{for a scalar diquark}, \\
\varepsilon_d(p) & \text{for an axial-vector diquark},
\end{cases}
\]

where the four-vector

\[
\varepsilon_d(p) = \left( \frac{\mathbf{e}_d \cdot \mathbf{p}}{M_d}, \mathbf{e}_d + \frac{(\mathbf{e}_d \cdot \mathbf{p}) \mathbf{p}}{M_d (E_d(p) + M_d)} \right), \\
\varepsilon_d^\mu(p) p_\mu = 0,
\]

is the polarization vector of the axial-vector diquark with momentum \( p \), \( E_d(p) = \sqrt{\mathbf{p}^2 + M_d^2} \), and \( \varepsilon_d(0) = (0, \mathbf{e}_d) \) is the polarization vector in the diquark rest frame. The effective long-range vector vertex of the diquark can be presented in the form

\[
J_{d\mu} = \begin{cases} 
\frac{(P + Q)_\mu}{2\sqrt{E_d E_d}} & \text{for a scalar diquark}, \\
- \frac{(P + Q)_\mu}{2\sqrt{E_d E_d}} + i \frac{\mu_d \Sigma_\nu \tilde{k}_\nu}{2M_d} & \text{for an axial-vector diquark},
\end{cases}
\]
TABLE I: Masses $M$ and form factor parameters (for definitions see Eq. (19)) of light diquarks. $S$ and $A$ denote scalar and axial-vector diquarks, antisymmetric $[q,q']$ and symmetric $\{q,q'\}$ in flavour, respectively.

| Quark content | Diquark type | $M$ (MeV) | $\xi$ (GeV) | $\zeta$ (GeV$^2$) |
|---------------|--------------|-----------|-------------|-------------------|
| $[u,d]$       | S            | 710       | 1.09        | 0.185             |
| $\{u,d\}$    | A            | 909       | 1.185       | 0.365             |
| $[u,s]$       | S            | 948       | 1.23        | 0.225             |
| $\{u,s\}$    | A            | 1069      | 1.15        | 0.325             |
| $\{s,s\}$    | A            | 1203      | 1.13        | 0.280             |

where $\tilde{k} = (0, k)$. Here, the antisymmetric tensor $\Sigma_{\mu}^\nu$ is defined by

$$
\left(\Sigma_{\rho\sigma}\right)^\nu_{\mu} = -i(g_{\mu\rho}\delta^\nu_{\sigma} - g_{\mu\sigma}\delta^\nu_{\rho}),
$$

and the axial-vector diquark spin $S_d$ is given by $(S_d)_{il} = -i\varepsilon_{kil}$; $\mu_d$ is the total chromomagnetic moment of the axial-vector diquark.

The constituent quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV and the parameters of the linear potential $A = 0.18$ GeV$^2$ and $B = -0.3$ GeV have the values typical in quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of charmonium radiative decays [14] and the heavy-quark expansion [16]. The universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia $^3P_J$ states [14]. In this case, the long-range chromomagnetic interaction of quarks vanishes in accordance with the flux-tube model.

### III. LIGHT, HEAVY-LIGHT AND HEAVY DIQUARKS

As the first step, we calculate the masses and form factors of the diquarks. As is well known, the light quarks are highly relativistic, which makes the $v/c$ expansion inapplicable and thus a completely relativistic treatment is required. To achieve this goal in describing light and heavy-light diquarks, we closely follow our recent consideration of the spectra of light mesons and adopt the same procedure to make the relativistic quark potential local by replacing $\varepsilon_{1,2}(p) \equiv \sqrt{m_{1,2}^2 + \mathbf{p}^2}$ by $E_{1,2}$ (see the discussion in Ref. [17]). The resulting quark–quark interaction potential is equal to 1/2 of the $q\bar{q}$ interaction in the meson. We solve numerically the quasipotential equation with this complete relativistic potential which depends on the diquark mass in a complicated, highly nonlinear way. The obtained ground-state masses of scalar and axial-vector light and heavy diquarks [5, 7, 8] are presented in Tables II and III.

In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, it is necessary to calculate the corresponding matrix element of the quark current between diquark states. This (diagonal) matrix element can be parameterized by the following set of elastic form factors:

(a) scalar diquark ($S$)

$$
\langle S(P)\mid J_{\mu}\mid S(Q)\rangle = h_{+}(k^2)(P + Q)_{\mu},
$$

(16)
TABLE II: Masses $M$ and form factor parameters (for the definitions see Eq. (19)) of heavy-light and heavy diquarks. $S$ and $A$ denote scalar and axial-vector diquarks, antisymmetric $[Q,q]$ and symmetric $\{Q,q\}$ in flavour, respectively.

| Quark content | Diquark type | $Q = c$ | $Q = b$ |
|---------------|-------------|---------|---------|
|               |             | $M$ (MeV) | $\xi$ (GeV) | $\zeta$ (GeV$^2$) | $M$ (MeV) | $\xi$ (GeV) | $\zeta$ (GeV$^2$) |
| $[Q,u]$       | $S$         | 1973    | 2.55    | 0.63    | 5359    | 6.10    | 0.55    |
| $[Q,u]$       | $A$         | 2036    | 2.51    | 0.45    | 5381    | 6.05    | 0.35    |
| $[Q,s]$       | $S$         | 2091    | 2.15    | 1.05    | 5462    | 5.70    | 0.35    |
| $[Q,s]$       | $A$         | 2158    | 2.12    | 0.99    | 5482    | 5.65    | 0.27    |
| $[Q,c]$       | $S$         | 3226    | 1.30    | 0.42    | 6526    | 1.50    | 0.59    |
| $[Q,b]$       | $A$         | 6526    | 1.50    | 0.59    | 9778    | 1.30    | 1.60    |

(b) axial-vector diquark $(A)$

$$\langle A(P)|J_{\mu}|A(Q)\rangle = -[\varepsilon_d^*(P) \cdot \varepsilon_d(Q)]h_1(k^2)(P + Q)_\mu$$

$$+ h_2(k^2) \left\{ [\varepsilon_d^*(P) \cdot Q] \varepsilon_{d,\mu}(Q) + [\varepsilon_d(Q) \cdot P] \varepsilon_{d,\mu}(P) \right\}$$

$$+ h_3(k^2) \frac{1}{M_A} [\varepsilon_d^*(P) \cdot Q][\varepsilon_d(Q) \cdot P](P + Q)_\mu,$$

where $k = P - Q$, and $\varepsilon_d(P)$ is the polarization vector of the axial-vector diquark.

Using the quasipotential approach with the impulse approximation for the vertex function of the quark-gluon interaction, we find

$$h_+ (k^2) = h_1 (k^2) = h_2 (k^2) = F(k^2),$$

$$h_3 (k^2) = 0,$$

$$F(k^2) = \frac{\sqrt{E_d M_d}}{E_d + M_d} \int \frac{d^3p}{(2\pi)^3} \Psi_d \left( p + \frac{2\varepsilon_2(p)}{E_d + M_d} k \right) \left[ \frac{\varepsilon_1(p) + m_1}{2\sqrt{\varepsilon_1(p + k)}(\varepsilon_1(p) + m_1)} \right] \Psi_d(p) + (1 \leftrightarrow 2),$$

where $\Psi_d$ are the diquark wave functions. We calculated the corresponding form factors $F(r)/r$, which are the Fourier transforms of $F(k^2)/k^2$, using the diquark wave functions found by numerically solving the quasipotential equation. Our estimates show that this form factor can be approximated with high accuracy by the expression

$$F(r) = 1 - e^{-\xi r - \zeta r^2},$$

which agrees with previously used approximations. The values of the parameters $\xi$ and $\zeta$ for light, heavy-light and heavy scalar diquark $[q,q']$ and axial-vector diquark $\{q,q\}$ ground states are given in Tables and.
IV. MASSES OF HEAVY TETRAQUARKS

As the second step, we calculate the masses of heavy tetraquarks considered as bound states of diquark and antidiquark. For the potential of the diquark-antidiquark interaction \((\Pi)\) we get \(^2\)

\[
V(r) = \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}(r) + \frac{1}{2} \left\{ \left[ \frac{1}{E_1(E_1 + M_1)} + \frac{1}{E_2(E_2 + M_2)} \right] \hat{V}_{\text{Coul}}(r) - \left[ \frac{1}{M_1(E_1 + M_1)} + \frac{1}{M_2(E_2 + M_2)} \right] \right\} \frac{V'_{\text{conf}}(r)}{r} + \frac{1}{E_1E_2} \left\{ \left[ \frac{1}{E_1(E_1 + M_1)} - \frac{1}{E_2(E_2 + M_2)} \right] \hat{V}'_{\text{Coul}}(r) - \left[ \frac{1}{M_1(E_1 + M_1)} - \frac{1}{M_2(E_2 + M_2)} \right] \right\} \frac{V''_{\text{conf}}(r)}{r} \cdot \mathbf{L} \cdot (\mathbf{S}_1 + \mathbf{S}_2) + \frac{1}{r} \left[ \hat{V}'_{\text{Coul}}(r) + \frac{\mu_d}{4} \left( \frac{E_1}{M_1} + \frac{E_2}{M_2} \right) V''_{\text{conf}}(r) \right] \mathbf{L} \cdot (\mathbf{S}_1 + \mathbf{S}_2) + \frac{\mu_d}{4} \left( \frac{E_1}{M_1} - \frac{E_2}{M_2} \right) \frac{V''_{\text{conf}}(r)}{r} \mathbf{L} \cdot (\mathbf{S}_1 - \mathbf{S}_2) + \frac{1}{3} \left[ \hat{V}'_{\text{Coul}}(r) - \hat{V}''_{\text{Coul}}(r) + \frac{\mu_d^2}{4} \frac{E_1E_2}{M_1M_2} \left( \frac{1}{r} V'_{\text{conf}}(r) - V''_{\text{conf}}(r) \right) \right] \times \left[ \frac{3}{r^2} (\mathbf{S}_1 \cdot \mathbf{r}) (\mathbf{S}_2 \cdot \mathbf{r}) - \mathbf{S}_1 \cdot \mathbf{S}_2 \right] + \frac{2}{3} \left[ \Delta \hat{V}_{\text{Coul}}(r) + \frac{\mu_d^2}{4} \frac{E_1E_2}{M_1M_2} \Delta V_{\text{conf}}(r) \right] \mathbf{S}_1 \cdot \mathbf{S}_2 \right\},
\]

(20)

where

\[
\hat{V}_{\text{Coul}}(r) = -\frac{4}{3} \frac{\alpha_s}{r} \frac{F_1(r)F_2(r)}{r}
\]

is the Coulomb-like one-gluon exchange potential which takes into account the finite sizes of the diquark and antidiquark through corresponding form factors \(F_{1,2}(r)\). Here, \(\mathbf{S}_{1,2}\) and \(\mathbf{L}\) are the spin operators of diquark and antidiquark and the operator of the relative orbital angular momentum. Since we limit our considerations to the ground states of heavy diquark-antidiquark bound systems \((\mathbf{L}^2) = 0\), the spin-orbit and tensor terms in the potential \((20)\) do not contribute in the further analysis. In the following we choose the total chromomagnetic moment of the axial-vector diquark \(\mu_d = 0\). Such a choice appears to be natural, since

\(^2\) In our paper [3] first two spin-orbit terms were missed. However they do not influence published numerical results, since masses mostly of ground states were calculated. Orbital excitations were considered only for the tetraquarks composed of the scalar diquark and scalar antidiquark for which the missed terms vanish.
the long-range chromomagnetic interaction of diquarks proportional to $\mu_d$ then also vanishes in accordance with the flux-tube model.

We substitute the quasipotential (20) in the quasipotential equation (11) and solve the resulting differential equation numerically. The calculated masses $M$ of tetraquarks with open charm and/or bottom composed from the heavy diquark, containing two heavy quarks ($QQ'$, $Q = b, c$), and the light antidiquark ($\bar{q}\bar{q}'$, $q = u, d, s$) are presented in Table III. In this table we give the values of the lowest thresholds $T$ for decays into two corresponding heavy-light mesons [(Q$q) = D^{(*)}, D_s^{(*)}, B^{(*)}, B_s^{(*)}]$, which were calculated using the measured masses of these mesons [10]. We also show values of the difference of the tetraquark and threshold masses $\Delta = M - T$. If this quantity is negative, then the tetraquark lies below the threshold of the decay into mesons with open flavour and thus should be a narrow state which can be detected experimentally. The states with small positive values of $\Delta$ could be also observed as resonances, since their decay rates will be suppressed by the phase space. All other states are expected to be very broad and thus unobservable. We find that the only tetraquark which lies considerably below threshold is the $0(1^{+})$ state of $(bb)(\bar{u}\bar{d})$. All other $(QQ')(\bar{q}\bar{q}')$ tetraquarks are predicted to lie either close to or significantly above corresponding thresholds. It is evident from the results presented in Table III that the heavy tetraquarks have increasing chances to be below the open flavour threshold and thus have a narrow width with the increase of the ratio of the heavy diquark mass to the light antidiquark mass.

| System | State | $I(J^P)$ | $Q = Q' = c$ | $Q = Q' = b$ | $Q = c, Q' = b$ |
|--------|-------|----------|--------------|--------------|----------------|
| $(QQ')(\bar{u}\bar{d})$ | 0(0+) | 7239 | 7144 | 95 | 7239 | 7144 | 95 |
| | 0(1+) | 3935 | 3871 | 64 | 10502 | 10604 | -102 | 7246 | 7190 | 56 |
| | 1(1+) | 4056 | 3729 | 327 | 10648 | 10558 | 90 | 7383 | 7144 | 239 |
| | 1(1+) | 4079 | 3871 | 208 | 10657 | 10604 | 53 | 7396 | 7190 | 206 |
| | 1(2+) | 4118 | 4014 | 104 | 10673 | 10650 | 23 | 7422 | 7332 | 90 |
| $(QQ')(\bar{u}\bar{s})$ | $\frac{1}{2}(0^+)$ | 7444 | 7232 | 212 | 7444 | 7232 | 212 |
| | $\frac{1}{2}(1^+)$ | 4143 | 3975 | 168 | 10706 | 10693 | 13 | 7451 | 7277 | 174 |
| | $\frac{1}{2}(1^+)$ | 4221 | 3833 | 388 | 10802 | 10649 | 153 | 7540 | 7232 | 308 |
| | $\frac{1}{2}(1^+)$ | 4239 | 3975 | 264 | 10809 | 10693 | 116 | 7552 | 7277 | 275 |
| | $\frac{1}{2}(2^+)$ | 4271 | 4119 | 152 | 10823 | 10742 | 81 | 7572 | 7420 | 152 |
| $(QQ')(\bar{s}\bar{s})$ | 0(1+) | 7684 | 7381 | 303 | 7684 | 7381 | 303 |
| | 0(0+) | 4359 | 3936 | 423 | 10932 | 10739 | 193 | 7673 | 7336 | 337 |
| | 0(1+) | 4375 | 4080 | 295 | 10939 | 10786 | 153 | 7683 | 7381 | 302 |
| | 0(2+) | 4402 | 4224 | 178 | 10950 | 10833 | 117 | 7701 | 7525 | 176 |
TABLE IV: Masses $M$ of diquark $(cq')$–antidiquark $(bq)$ states. $T$ is the lowest threshold for decays into two heavy-light $(Qq)$ mesons and $\Delta = M - T$; $T'$ is the threshold for decays into the $B_c^{(*)}$ and a light meson $(q'q)$, and $\Delta' = M - T'$. All values are given in MeV.

| System | State | $q' = u$ | $q' = s$ |
|-------|-------|---------|---------|
|       | $J^P$ | $M$  | $T$  | $\Delta$ | $T'$ | $\Delta'$ | $M$  | $T$  | $\Delta$ | $T'$ | $\Delta'$ |
| $(cq')(b\bar{u})$ | 0$^+$ | 7177 | 7144 | 33 | 6818 | 359 | 7294 | 7232 | 62 | 6768 | 526 |
|       | 1$^+$ | 7198 | 7190 | 8  | 6880 | 318 | 7317 | 7277 | 40 | 6820 | 497 |
|       | 1$^+$ | 7242 | 7190 | 52 | 6880 | 362 | 7362 | 7277 | 85 | 6820 | 542 |
|       | 0$^+$ | 7221 | 7144 | 77 | 6818 | 403 | 7343 | 7232 | 111 | 6768 | 575 |
|       | 1$^+$ | 7242 | 7190 | 52 | 6880 | 362 | 7364 | 7277 | 87 | 6820 | 544 |
|       | 2$^+$ | 7288 | 7332 | -44 | 7125 | 163 | 7406 | 7420 | -14 | 7228 | 178 |
| $(cq')\bar{b}s$ | 0$^+$ | 7282 | 7247 | 35 | 6768 | 514 | 7398 | 7336 | 62 | 6818 | 580 |
|       | 1$^+$ | 7302 | 7293 | 9  | 6820 | 482 | 7418 | 7381 | 37 | 6880 | 538 |
|       | 1$^+$ | 7346 | 7293 | 53 | 6820 | 526 | 7465 | 7381 | 84 | 6880 | 585 |
|       | 0$^+$ | 7325 | 7247 | 78 | 6768 | 557 | 7445 | 7336 | 109 | 6818 | 627 |
|       | 1$^+$ | 7345 | 7293 | 52 | 6820 | 525 | 7465 | 7381 | 84 | 6880 | 585 |
|       | 2$^+$ | 7389 | 7437 | -48 | 7228 | 161 | 7506 | 7525 | -19 | 7352 | 154 |

In Table IV the calculated masses $M$ of tetraquarks composed from a $(cq)$ diquark and a $(bq)$ antidiquark are listed. We also give the lowest thresholds $T$ for decays into heavy-light mesons as well as thresholds $T'$ for decays into the $B_c^{(*)}$ and light $(q'q)$ mesons and $\Delta' = M - T'$. We find that only 2$^+$ states of $(cq')(b\bar{u})$ have negative values of $\Delta$ and thus they should be stable with respect to decays into heavy-light $(B$ and $D)$ mesons. The predicted masses of lowest 1$^+$ states of $(cu)(b\bar{u})$ and $(cu)(b\bar{s})$ tetraquarks lie only slightly above the corresponding thresholds $T$. However, all $(cq')(bq)$ tetraquarks are found to be significantly above the thresholds $T'$ for decays into the $B_c^{(*)}$ and light $(q'q)$ mesons. Nevertheless, the wave function of the spatially extended $(cq')(b\bar{q})$ tetraquark would have little overlap with the wave function of the compact $B_c$ meson [1, 19], thus substantially suppressing the decay rate in this channel. Therefore the above-mentioned $(cq')(b\bar{q})$ tetraquark states which are below the $BD$ threshold have good chances to be rather narrow and could be detected experimentally.

It is important to note that the comparison of the masses of heavy tetraquarks given in Tables III and IV with our previous predictions [5] shows that the $(QQ')(\bar{q}q')$ states are, in general, heavier than the corresponding $(Qq)(\bar{Q}q')$ ones. This result has the following explanation. Although the relation [20] $M_{QQ} + M_{\bar{q}q}^S \leq 2M_{Qq}$ holds between diquark masses, the binding energy in the heavy-light diquark $(Qq)$– heavy-light antidiquark $(\bar{Q}q')$ bound

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3 Such $(cu)(b\bar{u})$ tetraquarks were recently argued [18] to be the best candidates for experimental detection.

4 For the non-strange $(cq')(bq)$ tetraquarks we give thresholds $T'$ for decays of the $I = 0$ states into $B_c^{(*)}$ and $\eta$ or $\omega$. These states should be more stable than the $I = 1$ ones, since their decays to $B_c^{(*)}$ and $\pi$ violate isospin.
TABLE V: Comparison of different theoretical predictions for the masses of heavy \((QQ')(\bar{q}q')\) tetraquarks (in MeV).

| System | \(I(J^P)\) | this work | Ref. [21] | Ref. [22] | Ref. [23] | Ref. [24] | Ref. [25] | Ref. [26] | Ref. [27] |
|--------|------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| \((cc)(\bar{u}d)\) | 0\((1^+)\) | 3935 | 3931 | 3876 | 3764 | 3927 | 3905 | 4000 ± 200 |
| | 1\((0^+)\) | 4056 | | | | | | | 4150 |
| | 1\((1^+)\) | 4079 | | | | | | | 4186 |
| | 1\((2^+)\) | 4118 | | | | | | | 4211 |
| \((bb)(\bar{u}d)\) | 0\((1^+)\) | 10502 | 10525 | 10504 | 10558 | 10261 | 10426 | 10200 ± 300 |
| | 1\((0^+)\) | 10648 | 10587 | 10766 | 10690 | | | | |
| | 1\((1^+)\) | 10657 | 10712 | 10644 | 10774 | 10698 | | | |
| | 1\((2^+)\) | 10673 | 10735 | 10790 | 10707 | | | | |
| \((bb)(\bar{u}s)\) | 1\(\frac{1}{2}(1^+)\) | 10706 | | 10680 | | | | | |
| | 1\(\frac{1}{2}(2^+)\) | 10823 | | 10816 | | | | | |
| \((bc)(\bar{u}d)\) | 0\((0^+)\) | 7239 | | 7206 | | | | | |
| | 0\((1^+)\) | 7246 | | 7244 | | | | | |
| | 1\((2^+)\) | 7422 | | 7422 | | | | | |
| \((bc)(\bar{u}s)\) | 1\(\frac{1}{2}(2^+)\) | 7572 | | 7496 | | | | | |

System is significantly larger than in the corresponding heavy diquark \((QQ')\)–light antiquark \((\bar{q}q)\) one. This fact is well known from the meson spectroscopy, where heavy quarkonia \(QQ\) are more tightly bound than heavy-light mesons \(Q\bar{q}\). For instance, we found that some of the \((cu)(\bar{c}u)\) tetraquarks lie below open charm thresholds while all ground-state \((cc)(\bar{u}d)\) tetraquarks are found to be above such thresholds.

In Table V we confront our results for masses of heavy \((QQ')(\bar{q}q')\) tetraquarks with other theoretical predictions \[21, 22, 23, 24, 25, 26, 27\]. In Ref. \[21\] the authors solve the four-body problem using the expansion in the harmonic-oscillator basis in the framework of the nonrelativistic quark model with a phenomenological potential. The same model with a different expansion basis, which can accommodate asymptotic states of two free mesons, is applied for the calculation of heavy-tetraquark properties in Ref. \[22\]. The stability of tetraquarks with heavy flavours is studied by using a variational approach and a nonrelativistic potential model in Ref. \[23\]. In Refs. \[24, 25\] tetraquarks are analyzed in the chiral constituent quark model using a variational approach. The potential of this model includes one-gluon, confinement and meson-exchange interactions. The existence of a virtual tetraquark state \(cc\bar{u}\bar{d}\) is discussed in Ref. \[26\] on the basis of semi-empirical mass relations. QCD sum rules are applied for the \(QQ\bar{u}\bar{d}\) tetraquarks in Ref. \[27\]. The main difference between our approach and the above quoted papers consists in that, from the very beginning, we explicitly reduce the relativistic four-body problem to the subsequent solution of two relativistic two-body problems assuming the diquark-antidiquark structure.
of the \((QQ')(\bar{q}q')\) tetraquarks. From Table \(V\) we see that most of the presented approaches predict that only the \(1^+\) state of the \((bb)(\bar{u}d)\) tetraquark lies below the open-bottom threshold (see also \(28\)). In Ref. \(22\) it is claimed that also the \(1^+\) state of the \((cc)(\bar{u}d)\) tetraquark is weakly bound against the \(DD^*\) threshold, if it has a molecular structure. Note that such structures are absent in our approach. A large binding energy in the \(1^+\) state of the \((cc)(\bar{u}d)\) tetraquark is found only in Ref. \(24\) and it is claimed to originate from the meson-exchange part of the quark interaction potential. The recent QCD sum rule analysis \(27\) finds that only the \((bb)(\bar{u}d)\) tetraquark is expected to be a narrow state.

\section{Conclusions}

In this paper we have calculated the masses of the ground states of tetraquarks with two heavy quarks assuming the diquark–antidiquark structure. Such approximation allowed us to reduce the very complicated relativistic four-body problem to the solution of two — significantly more simple — relativistic two-body problems. All considerations were done in the framework of the relativistic quark model which proved to be successful in describing numerous properties of mesons and baryons \(3, 5, 7, 8, 9, 14, 15, 16, 17\). The parameters of the model were fixed previously from the meson sector and are kept unchanged in the present analysis. The diquarks were treated dynamically. Their masses and form factors, which take into account the diquark structure, were calculated on the basis of a numerical solution of the corresponding relativistic quasipotential equation. Note that they are the same as in our previous studies of light and heavy diquarks in heavy \(8\) and doubly-heavy baryons \(11\), respectively. Light quarks and diquarks were treated completely relativistically without applying the \(v/c\) expansion. It was found that both the relativistic dynamics of light diquarks as well as their internal structure play an important role in the description of diquark-antidiquark bound states. The binding of a heavy diquark and a light antidiquark turned out to be weaker than the binding of a corresponding heavy-light diquark and heavy-light antidiquark. Thus, in contrast to the \((cq)(\bar{c}\bar{q})\) tetraquarks, which were discussed previously \(5\), all the \((cc)(\bar{q}q')\) tetraquarks are predicted to be above the decay threshold into the open charm mesons. Only the \(I(J^P) = 0(1^+)\) state of \((bb)(\bar{u}d)\) was found to lie below the \(BB^*\) threshold. As a result, this state can decay only weakly and thus it should be narrow. The strange partner of this state \((bb)(\bar{u}s)\) is predicted to lie slightly (13 MeV) above the \(B^*B_s\) threshold and, in principle, could be observed as a not too broad resonance decaying mainly into \(B^*B_s\).

The investigation of the decay widths of heavy tetraquarks which are predicted to lie below the threshold of the open flavours represents another very important and interesting problem. It could be considered by means of the relativistic generalization of the analysis performed in Ref. \(29\). However, this problem is beyond the scope of the present paper and will be considered elsewhere.

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[1] For a recent review, see E. S. Swanson, Phys. Rep. 429, 243 (2006), and references therein.
[2] R. L. Jaffe, Phys. Rev. D 15, 267 (1977); Phys. Rev. Lett. 38, 195 (1977); V. A. Matveev and P. Sorba, Lett. Nuovo Cim. 20, 443 (1977).
[3] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003).
[4] For a recent review, see R. L. Jaffe, Phys. Rep. 409, 1 (2005), and references therein.
[5] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Lett. B 634, 214 (2006).
[6] A. Del Fabbro, D. Janc, M. Rosina and D. Treleani, Phys. Rev. D 71, 014008 (2005).
[7] D. Ebert, R. N. Faustov, V. O. Galkin and A. P. Martynenko, Phys. Rev. D 66, 014008 (2002).
[8] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 72, 034026 (2005).
[9] D. Ebert, R. N. Faustov and V. O. Galkin, arXiv:0705.2957 [hep-ph].
[10] Particle Data Group, W.-M. Yao et al., J. Phys. G 33, 1 (2006).
[11] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 97, 232001 (2006).
[12] I. V. Gorelov [CDF Collaboration], arXiv:hep-ex/0701056.
[13] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 99, 052001 (2007); T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 99, 052002 (2007).
[14] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 67, 014027 (2003).
[15] D. Ebert, V. O. Galkin and R. N. Faustov, Phys. Rev. D 57, 5663 (1998); 59, 019902(E) (1999).
[16] R. N. Faustov and V. O. Galkin, Z. Phys. C 66, 119 (1995); D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 73, 094002 (2006).
[17] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 47, 745 (2006); Mod. Phys. Lett. A 20, 1887 (2005).
[18] H. J. Lipkin, arXiv:hep-ph/0703190.
[19] N. A. Törnqvist, Nuovo Cim. A 107, 2471 (1994).
[20] S. Nussinov and M. A. Lampert, Phys. Rep. 362, 193 (2002).
[21] B. Silvestre-Brac and C. Semay, Z. Phys. C 57, 273 (1993).
[22] D. Janc and M. Rosina, Few-Body Syst. 35, 175 (2004).
[23] D. M. Brink and Fl. Stancu, Phys. Rev. D 57, 6778 (1998).
[24] J. Vijande, F. Fernández, A. Valcarce and B. Silvestre-Brac, Eur. Phys. J. A 19, 383 (2004).
[25] J. Vijande, A. Valcarce and K. Tsushima, Phys. Rev. D 74, 054018 (2006).
[26] B. A. Gelman and S. Nussinov, Phys. Lett. B 551, 296 (2003).
[27] F. S. Navarra, M. Nielsen and S. H. Lee, Phys. Lett. B 649, 166 (2007).
[28] H. J. Lipkin, Phys. Lett. B 172, 242 (1986); Phys. Lett. B 580, 50 (2004); M. Karliner and H. J. Lipkin, Phys. Lett. B 638, 221 (2006).
[29] D. Melikhov and B. Stech, Phys. Rev. D 74, 034022 (2006).