Abstract—Ensemble pruning, selecting a subset of individual learners from an original ensemble, alleviates the deficiencies of ensemble learning on the cost of time and space. Accuracy and diversity serve as two crucial factors while they usually conflict with each other. To balance both of them, we formalize the ensemble pruning problem as an objection maximization problem based on information entropy. Then we propose an ensemble pruning method including a centralized version and a distributed version, in which the latter is to speed up the former’s execution. At last, we extract a general distributed framework for ensemble pruning, which can be widely suitable for most of existing ensemble pruning methods and achieve less time consuming without much accuracy decline. Experimental results validate the efficiency of our framework and methods, particularly with regard to a remarkable improvement of the execution speed, accompanied by gratifying accuracy performance.

Index Terms—ensemble learning, ensemble pruning, diversity, composable core-sets

I. INTRODUCTION

Thanks to its remarkable potential, ensemble learning has attracted huge amount of interest in machine learning community [1] and has been applied widely in many real-world tasks such as object detection, object recognition, and object tracking [2]–[5]. As it is also known as committee-based learning, multiple classifier systems, or mixtures of experts [1], [6], [7], an ensemble is a set of learned models that make decisions collectively rather than relying on one single model. The variety of types of individual models categorizes an ensemble as heterogeneous ensembles and homogeneous ensembles. And most of ensemble methods concentrate on the latter such as bagging [8] and boosting [9], [10].

The success of ensemble methods is commonly attributable to two key issues: the accuracy of individual classifiers and the diversity of them [11]. For classification problems, one classifier is accurate if its error rate is better than random guessing on new instances; two classifiers are diverse if they make different errors on new instances. Unfortunately, researchers still have not reached a consensus yet on an official definition or measurement of diversity. Besides, the diversity among individual classifiers usually decreases when these individuals approach a higher level of accuracy. Thus how to handle the trade-off between the two criteria is an essential issue in ensemble learning.

Although ensemble methods are efficacious, one important drawback there is that both the required memory and processing time increase visibly with the number of individual models in the ensemble. To mitigate this shortcoming motivates ensemble pruning that aims to select a subset of individual models in an ensemble, called as ensemble selection or ensemble thinning as well [12]–[18]. It improves the generalization performance of an ensemble with a smaller size [19]. There has been a great amount of progression on ensemble pruning methods in the last two decades. Most of existing pruning methods, however, are centralized in which all individual classifiers have to be stored and processed on one single machine. As the scale of data and an ensemble itself enlarges rapidly in the context of big data, the performance of centralized methods is being the bottleneck in execution time which is why distributed approaches need to emerge.

To deal with ensemble pruning problems fast with balancing diversity and accuracy appropriately, we firstly treat ensemble pruning as an objection maximization problem using information entropy to reflect diversity and accuracy. The objection function we aim to maximize is a trade-off between diversity and accuracy from an information entropy perspective. Secondly, we transform this approach to one distributed version to speed up the execution, inspired by the emerging concept of “composable core-sets” in recent years. It adopts the same idea as a two-round divide and conquer strategy, which is particularly suitable for distributed setting. Thirdly, we extract a general distributed framework for ensemble pruning from our method’s distributed version. It could be widely applicable to various existing methods for ensemble pruning and achieve less time consuming without much accuracy decline.

Our contributions in this paper are four-fold:

- We formalize the ensemble pruning problem as an objection maximization problem based on information entropy, in order to balance diversity and accuracy.
- We propose an ensemble pruning method including a centralized version and a distributed version, utilizing accuracy and diversity concurrently.
- We propose a general distributed framework for ensemble pruning, which could be widely utilized and achieve less time consuming without much accuracy decline.
- We design detailed experiments to validate the effectiveness of our distributed framework and approaches.

II. RELATED WORK

In this section, we explain diversity—a key issue in ensemble learning—and existing research on it firstly. Then we describe the difficulty and existing methods in ensemble
pruning. Finally, we introduce a concept of “composable core-sets” and its development which sheds some light on our work.

A. Diversity in Ensemble Learning

Diversity, intuitively considered as the difference among individual learners in an ensemble, is a fundamental issue in ensemble methods [1], with several alternative names as dependence, orthogonality or complementarity of learners [6]. Practically, individual classifiers are usually trained on the subsets of the same training data, which drives them highly correlated, breaks the assumption—the independency of individual classifiers, and makes it hard to seek the diversity. Numerous ensemble methods attempt to encourage diversity implicitly or heuristically [20]. For instance, boosting and bagging promote diversity by re-weighting and sub-sampling existing training samples respectively [8-10, 21].

Unfortunately, researchers still have not reached a consensus yet on an official measurement of diversity. Several measures have been proposed to represent diversity, which could be divided into pairwise measures and non-pairwise measures [6], while no superior exists [7]. Besides, few researchers can tell how diversity works in ensemble methods exactly although its crucial role has been widely accepted.

In the last decade or so, Brown [22] claimed that from an information theoretic perspective, diversity within an ensemble did exist on numerous levels of interaction between the classifiers. This inspired Zhou and Li [23] to propose that the mutual information should be maximized to minimize the prediction error of an ensemble from the view of multi-information. Subsequently, Yu et al. [20] claimed that the diversity among individual learners in pairwise manner, used in their diversity regularized machine (DRM), could reduce the hypothesis space complexity, which implied that controlling diversity played the role of regularization in ensemble methods.

B. Ensemble Pruning

Ensemble pruning deals with the reduction of an ensemble while improving its efficiency and predictive performance [24]. Margineantu and Dietterich [25] showed the possibility to obtain nearly the same level of performance as the entire set by selecting a subset of learners from an ensemble in the first study on ensemble pruning. Zhou et al. [19] provided the bias-variance decomposition of error as the key factor of the success of their approach named Genetic Algorithm based Selective Ensemble (GASEN), and claimed that pruning could lead to smaller ensembles with better generalization performance. It is difficult, however, to select the sub-ensembles with the best generalization performance. One trouble is to estimate the generalization performance of a sub-ensemble, and the other is that finding the optimal subset is a combinatorial search problem with exponential computational complexity [26]. Note that selecting the best combination of classifiers from an ensemble is NP-complete hard and even intractable to approximate [27].

Numerous ensemble pruning methods have been proposed to overcome shortcomings of ensemble learning over the last two decades, which could be categorized into three general families: ranking-based, clustering-based, and optimization-based. Ranking-based pruning methods, the simplest conceptually, order the learners in the ensemble and select the first few of them according to different evaluation functions [24], including minimizing the error (e.g. Orientation Ordering [28]), maximizing the diversity (e.g. KL-divergence Pruning and Kappa Pruning [25]), or combining the both (e.g. Diversity Regularized Ensemble Pruning [26]). Clustering-based pruning methods employ a clustering algorithm to detect groups of learners which make similar predictions initially, and then prune each cluster separately to increase the overall diversity of the ensemble [24]. Notice that an intrinsic properties that those methods can be executed in a parallel manner is ignored frequently in the second phase. Optimization-based pruning methods pose ensemble pruning as an optimization problem which is to find the subset of the original ensemble that optimizes a measure indicative of its generalization performance. Searching exhaustively in the space of ensemble subsets is infeasible for a moderate ensemble size since this problem is NP-complete hard. Thus various techniques are utilized to alleviate this predicament including genetic algorithm [29], greedy algorithm [30], hill climbing [31] and bi-objective evolutionary optimization [32].

C. Composable Core-sets

Over the last few years, an effective technique, captured via the concept of “composable core-sets”, arises in order to solve optimization problems over massive data sets in distributed computing literature. Its effectiveness has been confirmed empirically for many machine learning applications, such as diverse nearest neighbor search [33], diversity maximization [34], and feature selection [35].

The notion of “composable core-sets” is introduced explicitly by Indyk et al. [33] for the very first time, while the notion of “core-sets” can be dated back to [36]. A core-set for an optimization problem, informally, is a subset (with a guaranteed approximation factor) of that data on which solving the underlying problem could yield an approximate solution for the original data. Composable core-sets are a collection of core-sets in which the union of them gives a core-set for the union of the original data subsets [33].

Besides, a composable core-set with $\alpha$ approximate factor yields a solution which is an approximation of the optimal solution for the original optimization problem, and the approximation is guaranteed by a factor $\alpha$, which is $1/12$ [34] and can be improved to $8/25$ [35].

III. Methodology

In this section, we firstly elaborate our objection maximization based on information entropy for ensemble pruning in a centralized way, then attain a distributed version by introducing the concept of composable core-sets, and finally extract a general distributed framework for ensemble pruning.
A. Objection Maximization Based on Information Entropy for Ensemble Pruning

Given a large data set \( D \) with the size \( d \) of labeled instances obtained gradually from stream data, and their labels represented by a \( d \)-dimensional vector \( c \). Consider a set of \( n \) trained individual classifiers \( \mathcal{H} = \{ h_i \}_{i=1}^n \) as the original ensemble, in which each one maps the feature space of instances to the label space. The classification result vector of any individual classifier \( h_i \) from the ensemble \( \mathcal{H} \) on the data set \( D \), similar to the class label vector \( c \), is represented by a \( d \)-dimensional vector \( \mathbf{h}_i \). The ensemble pruning task aims to find a compact subset of the original ensemble which will predict the labels with high accuracy. These chosen individual classifiers need to be diverse and accurate simultaneously to achieve this goal. To this end, we select some diversified individual classifiers from the original ensemble which are relevant to the vector of class labels. Hence we seek to define a metric distance between individual classifiers in view of diversity and accuracy concurrently inspired by [35], so that the ensemble pruning problem would be reduced to an objection maximization problem.

Given two discrete random variables \( X \) and \( Y \), Cover and Thomas [37] define the mutual information \( I(\cdot;\cdot) \) between them,

\[
I(X;Y) = H(X) - H(X|Y) = \sum_{x \in X, y \in Y} p(x, y) \log \frac{p(x, y)}{p(x)p(y)},
\]

then Zadeh et al. [35] define the normalized mutual information \( \text{MI}(\cdot,\cdot) \) and the normalized variation of information \( \text{VI}(\cdot,\cdot) \) of them,

\[
\text{MI}(X, Y) = \frac{I(X;Y)}{\sqrt{H(X)H(Y)}}, \quad \text{(2)}
\]

\[
\text{VI}(X, Y) = 1 - \frac{I(X;Y)}{H(X, Y)}, \quad \text{(3)}
\]

wherein \( p(\cdot,\cdot) \), \( H(\cdot) \), and \( H(\cdot,\cdot) \) are the joint probability, the entropy function, and the joint entropy function respectively.

Consider the class label vector \( c \) and two classification result vectors (\( \mathbf{h}_i \) and \( \mathbf{h}_j \)) generated from the data set \( D \) by any two individual classifiers (\( h_i \) and \( h_j \)). The normalized mutual information \( \text{MI}(h_i, c) \) reflects the relevance between this individual classifier \( h_i \) and the class label vector \( c \), implying the accuracy of this individual classifier on the training data set. The normalized variation of information \( \text{VI}(h_i, \mathbf{h}_j) \) reflects the redundancy between the two individual classifiers, implying the diversity between them. Since class labels have already been discrete values and these values are only relevant to the number of classes in those used data sets, we do not need to discretize continuous variables to calculate the probabilities used in MIs and VIs, while Zadeh et al. [35] have to deal with it.

In order to take both diversity and accuracy into consideration concurrently, the objective function between two individual classifiers (a tradeoff between diversity and accuracy of two individual classifiers, \( \text{TDAC} \)) is defined naturally as

\[
\text{TDAC}(h_i, h_j) = \begin{cases}
\lambda \text{VI}(h_i, h_j) + (1 - \lambda) \frac{\text{MI}(h_i, c) + \text{MI}(h_j, c)}{2}, & \text{if } i \neq j; \\
0, & \text{otherwise}
\end{cases}
\]

\( \text{(4)} \)

where a regularization factor \( \lambda \) is introduced to indicate the importance of each criterion implicitly. The regularization factor \( \lambda \) is also an equilibrium between two criteria in Eq. (4).

The first criterion is to raise diversity by avoiding redundancy, and the second one is to promote accuracy by maximizing their relevance. Notice that VI is metric [38] and \( \text{MI}(\cdot,\cdot) \) is non-negative [35]. Consequently \( \text{TDAC} \) is metric as well, which means \( \text{TDAC}(h_i, h_j) + \text{TDAC}(h_j, h_k) \geq \text{TDAC}(h_i, h_k) \).

Subsequently, for an ensemble \( \mathcal{H} \) that is a set composed of \( n \) individual classifiers, the objection (a tradeoff between diversity and accuracy of a set of an ensemble, \( \text{TDAS} \)) is defined naturally as

\[
\text{TDAS}(\mathcal{H}) = \frac{1}{2} \sum_{h_i \in \mathcal{H}} \sum_{h_j \in \mathcal{H}} \text{TDAC}(h_i, h_j). \quad \text{(5)}
\]

Note that \( \text{TDAS}(\mathcal{H}) \) in Eq. (5) could be reformulated as

\[
\text{TDAS}(\mathcal{H}) = \frac{1}{2} \lambda \sum_{h_i \in \mathcal{H}} \sum_{h_j \in \mathcal{H}} \text{VI}(h_i, h_j) + \frac{n - 1}{2} (1 - \lambda) \sum_{h_i \in \mathcal{H}} \text{MI}(h_i, c), \quad \text{(6)}
\]

where VI of two similar individual classifiers will be near to zero. The first term in Eq. (6) prevents to select similar individual classifiers, and the second term ensures that those selected individual classifiers are relevant to the class labels. Hence the ensemble pruning task is formulated as an objective function maximization problem, which aims to find a subset \( P \subseteq \mathcal{H} \) with a specified condition \( |P| = k \) to restrict the size of the pruned sub-ensemble,

\[
\text{max}_{P \subseteq \mathcal{H}, |P| = k} \text{TDAS}(P) = \text{max}_{P \subseteq \mathcal{H}, |P| = k} \frac{1}{2} \sum_{h_i \in P} \sum_{h_j \in \mathcal{H} \setminus P} \text{TDAC}(h_i, h_j). \quad \text{(7)}
\]

Up to now, we have modeled the ensemble pruning task through an objective function maximization problem as shown in Eq. (7), which is enough to form a centralized algorithm to accomplish this goal of ensemble pruning. This centralized method, named as “Centralized Objection Maximization for Ensemble Pruning (COMEPP)”, selects greedily the current optimal classifier at each step, and could achieve a 1/2 approximation factor for objective function maximization problem according to [39].

Algorithm 1 COMEP

**Input:** Set of the original ensemble \( \mathcal{H} \), threshold \( k \) as the size of the pruned sub-ensemble.

**Output:** Set of the pruned sub-ensemble \( P \) meeting that \( P \subseteq \mathcal{H} \) and \( |P| \leq k \).

1. \( P \leftarrow \) an arbitrary individual classifier \( h_i \in \mathcal{H} \).
2. for \( 2 \leq i \leq k \) do
3. \( h^* \leftarrow \arg \max_{h_i \in \mathcal{H} \setminus P} \sum_{h_j \in P} \text{TDAC}(h_i, h_j) \).
4. Move \( h^* \) from \( \mathcal{H} \) to \( P \).
5. end for
B. Distributed Diversity Maximization for Ensemble Pruning

“Distributed Objection Maximization for Ensemble Pruning (DOMEP)”, the distributed version of COMEP, adopts a two-round divide and conquer strategy and composable core-sets \cite{40} as guidelines, which are particularly suitable for distributed setting. It partitions a set of individual classifiers of an ensemble into smaller pieces, solves the ensemble pruning problem on each piece, and eventually obtains a subset from the union of these representative subsets for all pieces.

Algorithm 2 DOMEP

\textbf{Input}: Set of the original ensemble \( \mathcal{H} \), threshold \( k \), number of machines \( m \).

\textbf{Output}: Set of the pruned sub-ensemble \( \mathcal{P} \) meeting that \( \mathcal{P} \subset \mathcal{H} \) and \( |\mathcal{P}| \leq k \).

1: Partition \( \mathcal{H} \) into \( \{\mathcal{H}_i\}_{i=1}^m \) randomly.
2: for \( 1 \leq i \leq m \) do
3: \( \mathcal{P}_i \leftarrow \text{COMEP}(\mathcal{H}_i, k) \).
4: end for
5: \( \mathcal{P} \leftarrow \text{COMEP}(\bigcup_{i=1}^m \mathcal{P}_i, k) \).
6: \( \mathcal{P} \leftarrow \arg \max_{\mathcal{P}} \text{TDAS}(\mathcal{P}) \).

Consider a set of \( n \) trained individual classifiers \( \mathcal{H} = \{h_i\}_{i=1}^n \) as the original ensemble. In the first phase, a primary machine partitions all individual classifiers in the original ensemble into \( m \) groups \( \{\mathcal{H}_i\}_{i=1}^m \) and allocates them to different machines. Notice that \( m \) is the number of machines and \( \bigcup_{i=1}^m \mathcal{H}_i = \mathcal{H} \). For each \( i \) \((1 \leq i \leq m)\), machine \( i \) runs \text{COMEP} on its allocated set \( \mathcal{H}_i \) independently and selects a subset \( \mathcal{P}_i \) from it in parallel. In the second phase, the primary machine gathers all subsets, runs \text{COMEP} on their union \( \bigcup_{i=1}^m \mathcal{P}_i \) to produce a subset \( \mathcal{P} \), and eventually outputs the best one of them by comparing \( \mathcal{P} \) with \( \mathcal{P}_i \) \((1 \leq i \leq m)\) according to Eq. (7). It suffices to output the satisfactory set \( \mathcal{P} \) after these two phases (Lines [11][13] in Algorithm 2) in practice, and the extra comparison purposes to get a higher approximation factor which is 1/4 theoretically and could even be 8/25 under some extra conditions [35].

C. A General Distributed Framework for Ensemble Pruning

A general distributed framework is extracted from DOMEP, named as “Ensemble Pruning Framework in a Distributed Setting (EPFD)”, which likewise adopts the two-round divide and conquer strategy and composable core-sets \cite{40}. It enables the ensemble pruning problem to be solved fast in a distributed way. Ensemble pruning is usually described as a process to acquire the optimum subset from the original ensemble. Denote \( \text{ALG} \) as an arbitrary algorithm to perform this task and \( \mathcal{H} \) as the original ensemble. \( \text{EPFD} \) consists of two main phases just like \( \text{DOMEP} \) that can be regarded as a special case of \( \text{EPFD} \) choosing \text{COME}P as a pruning method (Line [35]). Another key difference is that the criterion (Line [3]) here is not limited to Eq. (7). For instance, it can use accuracy or other measures corresponding to data to compare different subsets. \( \text{EPFD} \) is a simple yet powerful framework to accelerate the original methods for ensemble pruning without much performance decline, which is elaborated in Section IV-C.

IV. EXPERIMENTS

In order to evaluate our algorithms, in this section, we elaborate our experiments on 17 binary and 12 multiclass data sets including an image dataset with 12,500 pictures (Dogs vs Cats)\footnote{http://www.kaggle.com/c/dogs-vs-cats} and 28 data sets from UCI repository \cite{41}. 10-fold cross-validation is used in these experiments, except the way we split every data sets is quite unusual: each one is split into three parts, with 10\% as the training set, 10\% as the validation set, and 80\% as the test set. We did this for two reasons: One of them was that our methods actually do not need such a big scale of the training set to achieve the same level of accuracy as other algorithms; The other was that the accuracy results of ten ensemble pruning methods would be close to or even reach 100\% if we used the standard cross-validation, which would be indistinguishable from the accuracy perspective. Therefore, we maintained deliberately a small proportion as the training set and kept a larger part for testing. We construct homogeneous ensembles using Bagging or AdaBoost on various kinds of classifiers including naive bayesian (NB) classifiers, K-nearest neighbor (KNN) classifiers, linear model (LM) classifiers and linear SVMs (LSVM). An ensemble is trained on the training set, then pruned by a pruning method on the validation set, and finally tested on the test set. The baselines we considered are a variety of ranking-based methods, namely KL-divergence Pruning (KL), Kappa Pruning (KP)\footnote{25}, Orientation Ordering Pruning (OO)\footnote{28}, Reduce-Error Pruning (RE)\footnote{42}, Diversity Regularized Ensemble Pruning (DREP)\footnote{26}, and Ordering-based Ensemble Pruning (OEP) as well as optimization-based methods, namely Single-objective Ensemble Pruning (SEP), and Pareto Ensemble Pruning (PEP)\footnote{32}. Notice that several methods cannot fix the number of learners after ensemble pruning (such as OO, DREP, SEP, OEP and PEP), while others could fix it by giving a pruning rate which is the up limit of the percentage of those discarded individual classifiers in the original ensemble. Those methods that cannot fix the size may lead to oversize or undersize sub-ensembles and affect their space cost. Due to space constraints, we only list the comparisons of the time cost and the test accuracy hereinafter.

A. Comparison of COMEP and DOMEP to the State-of-the-art Ensemble Pruning Methods

In this subsection, we compare the quality of various ensemble pruning methods (the original centralized version)
including KL, KP, OO, RE, DREP, SEP, OEP, and PEP with our proposed centralized (COMEP) and distributed (DOMEP) methods. Experimental results reported in Table I contain the average test accuracy of each method and the corresponding standard deviation under 10-fold cross validation on each dataset. Each row in Table I compares the classification accuracy using bagging with the same type of individual classifiers. The results with higher accuracy and lower standard deviation are indicated with bold fonts for each dataset (row). Besides, we examine the significance of the difference in the accuracy performance between two methods by two-tailed paired t-test at 5% significance level to tell if two ensemble pruning methods have significant different results. Two methods end in a tie if there is no significant statistical difference; otherwise, one with higher values of accuracy will win. In the last row of Table I the performance of each method is compared with DOMEP in terms of the number of data sets that DOMEP has won, tied, or lost, respectively. It can be inferred that DOMEP does not underperform centralized methods in many datasets it only utilizes local information not like others, which confirms the reasonableness of DOMEP (and COMEP) utilizing accuracy and diversity simultaneously. Despite slightly lower values of accuracy in some cases, DOMEP still remains acceptable results. Similar results are reported in Table II and Table III using different individual classifiers. Figure 1 reports the comparison of the state-of-the-art methods with COMEP and DOMEP on the test accuracy using statistical test methods [32, 43]. Figure 1(a) shows COMEP has significant superiority over other compared centralized methods. Figure 1(b) presents the aggregated rank for each method.

B. DOMEP vs. COMEP

In this experiment, we employ a larger numbers of machines in DOMEP in order to test its speed-up in comparison with COMEP. Under the ideal conditions, Zadeh et al. [35] point that the speed-up between the distributed and centralized version is almost linear in terms of the number of used machines, since there is no overhead of information-sharing between those machines. Constrained by the machine capability we test, several ensemble pruning problems conducted on two or three machines are used as a typical example to present the performance of DOMEP. The results using various settings of this experiment are summarized in Figure 2, which indicates

### Table I
Comparison of the state-of-the-art methods with COMEP and DOMEP using Bagging to produce an ensemble with NBs as individual classifiers.

|                  | KL  | KP  | RE  | OO  | DREP | SEP  | OEP  | COMEP | DOMEP |
|------------------|-----|-----|-----|-----|------|------|------|-------|-------|
| images           | 66.30 ± 3.69 | 62.80 ± 3.52 | 66.10 ± 3.45 | 65.10 ± 3.41 | 63.10 ± 3.79 | 65.30 ± 3.98 | 64.20 ± 4.44 | 65.40 ± 2.69 | 66.90 ± 4.11 | 65.60 ± 4.36 |
| heart            | 80.00 ± 7.80 | 80.37 ± 7.99 | 79.63 ± 8.96 | 78.40 ± 7.31 | 79.63 ± 9.97 | 80.74 ± 7.73 | 80.37 ± 7.04 | 80.37 ± 7.04 | 82.22 ± 5.19 |
| liver            | 58.86 ± 9.86 | 57.00 ± 11.85 | 63.71 ± 12.06 | 57.14 ± 10.84 | 58.29 ± 14.75 | 63.14 ± 14.93 | 54.86 ± 12.57 | 57.14 ± 11.36 | 65.14 ± 9.83 | 64.00 ± 10.63 |
| sensor readings  | 93.51 ± 0.83 | 93.51 ± 0.83 | 93.51 ± 0.83 | 92.83 ± 1.86 | 93.51 ± 0.83 | 93.51 ± 0.83 | 95.71 ± 0.57 | 73.44 ± 28.95 | 93.51 ± 0.83 | 93.51 ± 0.83 |
| W/T/L            | 0/50 | 0/50 | 0/50 | 0/50 | 0/50 | 0/50 | 0/50 | 1/40 | 0/50 | 0/50 | 0/50 | 0/50 |

### Table II
Comparison of the state-of-the-art methods with COMEP and DOMEP using Bagging to produce an ensemble with DTs as individual classifiers.

|                  | KL  | KP  | RE  | OO  | DREP | SEP  | OEP  | COMEP | DOMEP |
|------------------|-----|-----|-----|-----|------|------|------|-------|-------|
| iono             | 97.22 ± 2.48 | 96.39 ± 4.13 | 96.94 ± 2.90 | 96.94 ± 2.90 | 93.33 ± 0.90 | 92.22 ± 5.39 | 97.22 ± 2.78 | 96.94 ± 3.15 | 97.50 ± 2.90 | 96.11 ± 5.56 |
| wisconsin        | 99.57 ± 0.66 | 99.71 ± 0.58 | 99.86 ± 0.43 | 100.00 ± 0.00 | 99.71 ± 0.58 | 98.70 ± 1.01 | 100.00 ± 0.00 | 99.86 ± 0.43 | 100.00 ± 0.00 | 99.86 ± 0.43 |
| W/T/L            | 0/20 | 0/20 | 0/20 | 0/20 | 0/20 | 1/10 | 0/20 | 0/20 | 0/20 |

### Table III
Comparison of the state-of-the-art methods with COMEP and DOMEP using Bagging to produce an ensemble with LSVMs as individual classifiers.

|                  | KL  | KP  | RE  | OO  | DREP | SEP  | OEP  | COMEP | DOMEP |
|------------------|-----|-----|-----|-----|------|------|------|-------|-------|
| images           | 74.10 ± 5.47 | 67.90 ± 6.17 | 78.60 ± 2.84 | 79.70 ± 3.23 | 78.60 ± 3.56 | 80.60 ± 3.44 | 79.40 ± 4.10 | 81.10 ± 3.94 | 81.30 ± 3.61 | 80.50 ± 3.26 |
| sensor readings  | 86.31 ± 1.45 | 86.45 ± 1.44 | 86.45 ± 1.44 | 86.20 ± 1.47 | 86.38 ± 1.44 | 86.45 ± 1.44 | 86.09 ± 1.36 | 85.98 ± 1.43 | 86.45 ± 1.44 |
| gmm (3d)         | 97.26 ± 0.40 | 97.21 ± 0.40 | 97.21 ± 0.40 | 97.21 ± 0.60 | 97.21 ± 0.60 | 97.16 ± 0.59 | 97.06 ± 0.61 | 97.16 ± 0.59 | 97.36 ± 0.45 | 97.11 ± 0.43 |
| iono             | 92.78 ± 4.84 | 92.50 ± 6.70 | 92.50 ± 6.66 | 92.22 ± 7.93 | 90.83 ± 6.70 | 91.39 ± 6.27 | 91.67 ± 7.14 | 92.50 ± 6.76 | 94.44 ± 5.93 | 92.22 ± 5.53 |
| ames             | 63.00 ± 4.84 | 64.29 ± 6.97 | 66.14 ± 5.11 | 65.86 ± 3.86 | 67.00 ± 7.38 | 68.00 ± 5.72 | 66.86 ± 4.90 | 69.57 ± 4.70 | 72.29 ± 6.36 | 69.00 ± 7.26 |
| W/T/L            | 0/40 | 0/40 | 0/50 | 0/50 | 0/50 | 0/50 | 0/50 | 0/50 | 0/50 |

Fig. 1. Comparison of the state-of-the-art methods with COMEP and DOMEP on the test accuracy. (a) Friedman test chart (non-overlapping means significant difference) [43]. (b) The aggregated rank for each method (the smaller the better) [32].
**DOME P** runs faster than **COME P** even reaching superlinear speedup on a tiny minority of experiments.

**C. Comparison Between the State-of-the-art Ensemble Pruning Methods and Their Distributed Versions Generated with EPFD**

In this subsection, we compare the quality of various centralized ensemble pruning methods with their respective distributed versions generated with **EPFD** in terms of accuracy and time cost. To test the quality of the selected sub-ensembles of each method, we control them under the same conditions (including employed ensemble methods or types of individual classifiers) in each experiment. Figure 3 shows the comparison results when individual classifiers are designated as LMs and assembled by Bagging. It can be inferred that, for each pruning method (each group on the horizontal axis), the accuracy of the distributed version is superior or equal to that of its corresponding centralized version. In consideration of the less time cost it takes, we have reasons to believe that the distributed version of each method outperforms its centralized version. Moreover, we can tell that the effectiveness of **EPFD** is extremely evident on **PEP**, a complicated method utilizing an evolutionary Pareto optimization combined with a local search subroutine. In addition, Figure 4 reports the results of the same experiment with **LSVMs** as individual classifiers for binary classification.

**D. Validating the Objective Function**

Regarding a new objective function, its relation with the classification accuracy is one of the major questions. We select two small-sized ensembles (small in the number of individual classifiers) and evaluate all possible combinations of these individual classifiers in order to test this issue. In this experiment, we compare the classification accuracy for all the 3-combinations and 4-combinations of individual classifiers in the original ensemble against their corresponding objective value with the \( \lambda \) parameter equal to 0.1. Each small black dot in Figure 5 represents the classification accuracy on a 3-combination or 4-combination of the individual classifiers with the size 8 of an ensemble in the **Iono** dataset, and the line is the regression line. We observed that the objective value and the classification are highly correlated from Figure 5 which means maximizing this objective function leads to our target—the highly accurate sub-ensembles.

**E. Effect of \( \lambda \) Value**

Crucial as other issues, the relation of two criteria needs to be investigated in the defined objective function. To reveal how the classification results are effected with the regularization factor \( \lambda \), different \( \lambda \) values (from 0 to 1 with 0.05 steps) are tested in the experiments of this part. Figure 6 exemplifies the effect of \( \lambda \) on the **Ringnorm** dataset. Figure 6(a) illustrates that the linear combination performs better than each MI term \( (\lambda = 0) \) or VI term \( (\lambda = 1) \) in Eq. 4 individually, although finding the optimal value of the \( \lambda \) is another challenge. Figure 6(b) presents that a global maximum around the optimal \( \lambda \) exists regardless of the size of the pruned sub-ensemble, which suggests that it may be related to the intrinsic properties of the dataset. Although proper results for all datasets have been brought with \( \lambda \) being set to 0.5 (in Table 1) for convenience,
Fig. 6. Effect of $\lambda$ value on the classification accuracy in the Ringnorm dataset using AdaBoost with Decision Trees as individual classifiers. (a) Accuracy of each criterion individually. (b) Slight differences of $\lambda$ value while selecting different size of the pruned sub-ensemble (7, 10, 14, 19).

**DOMEP** would achieve a better performance in practice when the $\lambda$ is adjusted for each dataset separately.

**V. Conclusion**

In this work, we have formalized ensemble pruning problem as an objection maximization problem based on information entropy to consider diversity and accuracy simultaneously. And we have proposed an ensemble pruning method according to this objection maximization problem (including two versions, **COME** and **DOMEP**) for ensemble pruning. We also presented that our methods (**COME** and **DOMEP**) were consistently competitive with various existing methods for ensemble pruning, which could handle large-scale ensembles fast yet efficiently through handling the accuracy and diversity of the ensembles in a proper way. At last, we have proposed a general distributed framework (**EPFD**) for ensemble pruning, which could be widely applied to various existing methods for ensemble pruning, to achieve less time consuming without much accuracy decline. The remarkable effectiveness of **EPFD** is definitely valuable for enormous data in the real world. For future work, it seems like a promising direction to explore the deeper theoretical basis and to try other objective functions to achieve better performance.

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