Management of Municipal Public Transport Vehicle Journeys by Using the PERT Method

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Abstract: These days, seamless moving about a city is a determinant of the city’s competitiveness, and is decisive for the life quality in the city. Hence, taking care of appropriate traffic organisation is one of the major tasks of the city authorities. Development of optimal production and spatial interrelations, considering their costs, efficiency and scope of services rendered to individual entities, enables economic and ecological development of the region. Therefore, the issue of major importance for a city is implementing a transport policy that makes it possible to choose a specified method of development (transformation) of the existing transport system in such a way so that it is coherent with the adopted strategy for the city development. The purpose of a transport policy conducted by city authorities should be maintaining the functioning of the urban transport on at least a satisfactory level. The main purpose of the article is to present an innovative solution for optimising the journeys of public transport vehicles, using the PERT (Program Evaluation and Review Technique) method. The method was developed on the basis of the research carried out in the city of Opole, Poland. The article presents a Multicriteria Model of Controlling the vehicles of the Municipal Public Transport-CVMPT) and the method implementation algorithm along with assumptions. The model presented in this article shows the possible way of optimising public transport systems operated in cities, taking into account the travel time of any given bus service, based on normal distribution and the computed probability of such traffic. The application of the method has brought many positive effects through providing optimisation measures in the structure of the municipal public transport.

Keywords: city logistics; telematics; PERT method; optimization; public transport; security management; critical path; CVMPT

1. Introduction

Cities play a major role in economic and social development of any country, as they are the places where business operations—such as trade, management and transport—are concentrated. Cities attract people by providing possibilities of education, employment and entertainment. Currently, a significant part of the world’s population lives in cities, and this trend is forecast to continue [1].

Road traffic in cities contributes to numerous negative effects [2-4]:

• time wasting and hindered access to some destinations as a result of congestion,
• decreased road safety,
• increased air pollution, street noise, and global warming,
• vehicles parked in traffic lanes are often obstacles to pedestrians, cyclists and disabled persons.
Solving the above-mentioned problems is not an easy task due to limited space, especially in historical city centres. Construction of new roads in core city centres is impossible, while imposing entrance and parking restrictions may not affect the city centre inhabitants, therefore they are ineffective. Reducing the number of vehicles on city streets will have a positive result on fuel consumption, accident rate, road congestion, air pollution, and excessive noise emissions [5].

What else can be done in order to reduce the negative effects of urban road traffic? A major issue in this respect is changing people’s mobility-related behaviours and encouraging city users e.g., to travel with public transport, by foot or bicycle, or make use of car sharing and carpooling [6].

Currently it is hard to imagine functioning of any city without public transport. Public transport makes it possible for people to get from one place to another in a fast and relatively cheap manner. The structure and capacity of any urban transport network directly affect the level of accessibility or the ease of moving from one place to another within the city [7]. Decision-makers take numerous measures in order to convince the inhabitants and city users to give up individual transport and use collective transport instead. A key issue in this respect is proper planning and organisation of collective transport, which is an important factor affecting people’s life quality [8,9]. Short distances may be easily covered on foot, however, when the distance is considerable, people look for faster and more comfortable forms of moving around the city. In the latter case, we can choose between individual and collective transport. In the case of individual transport, we should take into account the cost of purchase, operation and maintenance (fuel, repairs, insurance, mandatory technical inspections), but also we cannot forget about the traffic congestion. When we compare the costs of having one’s own means of transport to the costs of using collective transport, choosing the latter form of transport appears to be more reasonable. Collective transport planning consists not only in specifying the routes, but also in coordinating the times of transfers at the nodes in such a way so that passengers can quickly reach their destinations. Proper organisation of collective transport not only raises the city’s attractiveness and competitiveness, but also it impacts the region’s sustainability level. The key determinants considered in collective transport planning are [10]:

- optimising the accessibility of individual parts of the city,
- minimising the travel time,
- reducing the negative environmental impacts.

An adequately designed and planned collective transport system has a considerable impact on regional development patterns, cost effectiveness, natural environment, and maintaining a socially acceptable level of life quality in urbanised areas. Planning of collective transport should also take into account destinations, time and spatial schedule of the journey as well as costs [10].

Making decisions in the area of designing and planning of a transport system is a complex task, as decision-makers have to consider a number of possible effects that may arise at the moment any given solution is implemented. At the same time, it is necessary to bear in mind that making decisions based exclusively on economic factors is inappropriate. In fact, such decisions should also take into account environmental and social considerations [11]. A decision-making process with regard to transport planning should be based on the stakeholders’ opinions [12], in this case these will be mainly people who use public transport services. It is important that any changes in organisation of transport services should be accepted by the urban community, as any such changes directly affect their lives.

The aim of this research study is to present a descriptive model of municipal transport fleet management (CVMP) and an algorithm for implementing the method, along with its assumptions. The method was verified on the example of the city of Opole,
Poland. In this article, the authors present the first phase of designing the model, and the following selected tools:

- application of the PERT (Program Evaluation and Review Technique) method that makes it possible to optimise the travel times (for the purposes of this study, the focus was only on the time from getting on a means of public transport to getting off),
- a survey carried out among users of the municipal collective transportation system, to find out whether it was easy for them to move between various parts of the city.

The PERT method may be applied in solving various strategic as well as operating problems faced by business units, in order to reduce costs or save time. Examples of the PERT method application may be:

- approaching analyses of fuzzy networks and projects [13],
- developing linear models for evaluation of contractor data, making it possible to estimate uncertainty of information regarding the contractor’s various criteria [14],
- calculating the statistical algorithms of time analysis, predicting the distribution of probability of a circuit delay, at the same time considering the effects of correlation of spatial changes in intragroup parameters [15],
- designing and perfecting various clinical processes [16],
- business management, project planning control, logistic support [17],
- logical design [18],
- optimisation of time and costs connected with production projects [19],
- modelling the renovation of various building structures [20].

The PERT method makes it possible to prepare a model that specifies distribution for probable performance of each action within a specified time span. The Authors decided to apply this method, as it enables development of a model that takes into account any uncertainty of evaluating the durations of individual actions, which depend on the amount of available information, and in the presented case it is limited. The model developed by means of the PERT method made it possible to present the selected urban reality (public transport vehicle traffic), which is connected with uncertainty of evaluating the durations of individual actions due to the lack of full information about any possible events that could delay the process. Thus, the developed model (that takes into account the limitations) formed the basis for further measures aimed at optimisation of the passenger flow process in the city. It should also be noted that the said method had not been applied before for similar purposes, which makes it an innovative approach. In connection with various above-mentioned possibilities of applying the PERT method, the authors also identified a certain dependency between project management and municipal public transport functioning. Namely, it is not possible to precisely specify how much time it will take for a bus to travel from one bus stop to another, as there are unpredictable factors such as congestion, road accidents or how the other road traffic participants are driving.

2. The PERT Method in Logistic Management of Collective Passenger Transport

The PERT method (Program Evaluation and Review Technique, or Program Evaluation Research Task) is a probabilistic method of planning and control, which makes use of network programming, usually applied in project management [21]. It was developed by the United States Department of Defence in the years 1956–1957, and applied by the Booz, Allen and Hamilton consulting company and the Missile Systems Division of Lockheed in connection with implementation of the Polaris missile programme for the US Navy. PERT graphs were used for preparing, at the right time and place and in appropriate amounts, the ingredients necessary for production of missiles [22]. Thanks to the method, the programme duration was shortened from five to three years.

Initially, the method was applied mainly to large, multi-year military programmes, but later on it was found useful also for civilian projects [23]. In accordance with the PERT method, a project is shown in the form of a network diagram, i.e., a directed graph where
vertices represent tasks being part of the project, while arrows represent directed ties between the tasks, with assigned durations of individual activities that need to be completed before moving on to the next task. The diagram is most often developed using the AoN (Activity on Nodes) notation, where tasks are presented in the form of boxes (nodes) connected with arrows. All the data are entered in the node, where also computations are made [24]. Alternatively, the AoA (Activity on Arrows) notation may also be used [25], however, currently it is rarely applied due to some of its features (e.g., the need to enter dummy activities) [26].

This study is aimed at presenting the possibilities of applying the PERT method in order to optimise the routes in logistic processes. The main task of the PERT method is to identify a sequence of activities that determine the duration of the whole process (i.e., the critical path in terms of time) [27]. Evaluation of duration of specific activities is usually predicated on human experience which is biased and diversified. Due to the diversity and subjectivity, it is hard to specify the durations in a definitively reliable manner, which is a fairly significant drawback of the method.

This study presents the application of the PERT method to determine the shortest travel time between bus terminals, and to compute precise travel times between intermediate bus stops. Table 1 presents the classical approach to application of the PERT method [28] in relation to logistic management of a collective urban transportation system.

| Description   | Application of Classical PERT | Application of PERT in Logistic Management of a Collective Urban Transportation System |
|---------------|--------------------------------|-------------------------------------------------------------------------------------|
| $t_a$ — optimistic time | the shortest possible duration of a given activity (it will occur in conditions that are particularly favourable for carrying out the task) | the shortest possible travel time for the given road section ($V = 50$ km/h) |
| $t_b$ — pessimistic time | the longest possible duration of a given activity (it will occur in the case of a number difficulties leading to a delay in carrying out the task) | the longest possible travel time for the given road section ($V = 1$ km/h) |
| $t_m$ — realistic time | the most probable (or realistic) duration of a given activity ($t_a < t_m < t_b$) | real averaged travel time for the given road section, resulting from the study |
| AoA notation node | A — event number, B — earliest possible moment of event, C — latest possible moment of event, D — differences between the latest and the earliest moments of the event (event slack). | A — number of a given intersection, B — earliest time of arrival at the intersection, C — latest time of arrival at the intersection, D — differences between the latest and the earliest arrival of the bus at the intersection (travel delay) |
| Connecting arrow | activity duration | Travel time from intersection to intersection |

| Purpose of network solving | determining the shortest time of project completion, accounting for any constraints, logistic dependencies and estimated durations of individual activities | determining the optimal bus route between the specified points within the city, accounting for any dependencies and constraints |
|-----------------------------|-------------------------------------------------|----------------------------------------------------------|
| Critical path               | a sequence of activities (project subtasks), where a delay of any of them will delay the completion of the whole project. | specified route of a public bus service |
Combining the classical PERT method with its extension proposed by the authors will make it possible to implement the method in the aspect of logistic management of a collective urban transportation system.

3. Methodological Background

The procedure presented in the form of a diagram, connected with application of the PERT method in route planning in collective transport, is presented in Figure 1. The whole process was divided into 5 phases. For the purposes of this study, the authors focused on 4 phases in order to present the possibilities offered by the PERT method to optimise the bus routes in the collective urban transportation system. Phase 5 will constitute an element of further research studies which will include developing a procedure of implementing the PERT method in the structure of the collective transport system.
Figure 1. The diagram presenting the procedure for applying the PERT method in public transport route planning.
Phase I “City choice”—the model presented in the article—is universal, which allows it to be used in a city with a public transport system and an approximate fit, which was the city of Opole. Fitting to the model city consists in comparing the given cities in terms of the following categories: demographic data, commerce, culture/ recreation, education, financing, economy and infrastructure, followed by selecting with the expert method the cities that may apply the model. The comparative analysis of 15 Polish cities is presented in the article entitled “The Method to Compare Cities to Effective Management of Innovative Solutions” [29].

Phase II “City analysis”—focuses on collecting spatial and functional information about the city. At subsequent stages of the study, the obtained information will make it possible to determine Traffic Shaping Facilities, i.e., destinations which transport users want to reach. These are usually public utility, recreational and leisure facilities. At this stage, it is necessary to examine the city inhabitants’ and users’ levels of satisfaction with the currently functioning collective transportation system, and also to specify the significance level for the factors responsible for choosing public transport. Opinions of collective transport users may be obtained via direct standardised interviews, where respondents are asked relevant questions prepared in the form of a questionnaire.

At this point it is also necessary to conduct a statistical analysis of the city in question, which comprises computation of descriptive statistics including the mean, median, mode, mode amount, minimum, maximum, standard deviation, and coefficient of variation; examining whether the given feature in the population shows normal distribution; carrying out a Kruskal–Wallis H test (one-way ANOVA on ranks) in order to verify the hypothesis on irrelevance of differences between the medians of the examined variable in several populations; and conclusion drawing which consists in determining which data groups are closely related.

At this phase, the following information is obtained:

- routes of the individual bus lines, their starting and final points, intermediate points of bus lines, travel time between the initial and final bus terminals (data obtained on the basis of expertise and with application of data bases)
- changes depending on the day of the week (Monday, Tuesday, etc.), time of day (6:00, 8:00, etc.) (data obtained on the basis of expertise)
- preferences of city users with regard to the route, including bus stop locations (data obtained from surveys taken by municipal transport users)
- number of (all the data were obtained from the data bases managed by the city authorities):
  - car parks in the given city,
  - workplaces,
  - kinds of vehicles participating in the urban traffic,
  - shopping centres,
  - housing estates and city districts,
  - recreational facilities,
  - educational institutions,
  - municipal offices,
  - the city population (population density in individual districts).

Phase III “Traffic Flow Analysis” is the key stage that requires substantial precision and accuracy. Nowadays, there are numerous tools available that inform travellers about the current traffic situation: NaviExpert, Google Maps, targeo.pl, korkowo.pl, Tom Tom Traffic, Bing Maps, Waze. It is also possible to obtain any necessary data from traffic detectors. However, not all cities have such facilities at their disposal, due to considerable costs of purchasing the specialised equipment.

The analysis of the traffic flow is aimed at:

- indicating whether the travel direction is a significant element of the traffic intensity,
- indicating on which days the city traffic is similar,
- indicating the exact hours of intensified road traffic,
- identifying the roads being the so-called bottlenecks (which constrain the vehicle flow, thus constituting the capacity limit for the whole system).

Phases 4 and 5 are connected directly with application of the PERT method to establish new routes in the collective transport system, that are better adjusted to the needs of the present and potential passengers. Phase 4 was divided into two parts, the design (Figure 2) and the verification.

The design part was further divided into eight stages.

Stage I—at this point, the mean value is computed on the basis of the analysis of traffic intensity, carried out by means of the surveying tool, taking into account the following criteria:

- dividing the road infrastructure into sections in accordance with the following guidelines: the sections take into account the travel times at least between two intersections, even though there may be shorter sections; the sections account for all changes in the traffic intensity shown on google.maps, any sections considered to be bottlenecks as a result of the traffic flow analysis have been rejected, any sections that are unsuitable for bus travel due to traffic impediments have been rejected, therefore, the permissible speed of 50 km/h was adopted for vehicles driving within the city,
- computation of travel time using the length of a section and velocity at which the vehicle may be driven along the section, by means of Formula (1).

\[ t = \frac{s}{v} \]  

(1)

where: \( s \)—the road section length calculated on the basis of http://mapy.geoportal.gov.pl/ (accessed on 1 March 2021), \( v \)—specified only according to the colour shown at http://google.maps (accessed on 1 March 2021) (for confirmation, the speed was assumed on the basis of the tests conducted in real traffic conditions).
Figure 2. The multi-criteria model CVMPT—Part 1—the model design.
• green (1)—50 km/h,
• yellow (2)—25 km/h,
• orange (3)—10 km/h,
• red (4)—1 km/h,
• repeating the steps enumerated above for, respectively: day of the week—Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday, and hours: 6:00, 8:00, 10:00, 12:00, 14:00, 16:00, 18:00, 20:00, 22:00,
• determination of the most probable time lengths via averaging all values for each section, taking into account the day of the week and the hour (Figure 3).

Figure 3. Stage 1—Allocation of average travel time to road sections. Source: Google Maps.

This stage is the most time-consuming one due to the fact that the data to be used in further analyses should be detailed enough to specify all the possible travel variants through the analysed centre, as well as to enable specification of concrete sections. The \( s \) and \( V \) values are estimated manually, using appropriate websites. The Authors have been working on software aimed at automation of this part of the model.

Stage II—creating a radial city layout for the analysed city. This means performing two activities (Figure 4):
• specifying the Interchange Point in the city centre (taking the location in the city centre or in the vicinity of the Main Station as the criterion), which should serve as the distribution point for passengers using the collective urban transportation system,
• indicating on the city outskirts all the Extreme points that can be reached by any road (the criterion is met when the shortest (in terms of distance) routes from the Extreme points leading to the Interchange Point will run through all districts in the city).
Figure 4. Stage 2—Creation of a radial city layout.

At this stage, it is necessary to identify all points that are Extreme points within the analysed city.

Stage III—determining the shortest Connecting Routes \( n \), where \( n \) means the number of identified Extreme Points from Stage II between the Interchange Point and the Extreme Point. The shortest distance between the two points may be determined via any optimisation method (Figure 5).

Figure 5. Stage 3—Determining potential shortest routes from the city’s Extreme Points to the Interchange Points.
In order to compute the shortest distance at this stage, it is best to use the node method that identifies the shortest possible route between two points.

Stage IV consists in identifying, for each bus line separately, the points located in the direct vicinity of the Connecting Routes, regarding the Traffic Shaping Facilities in the City (Figure 6).

![Figure 6. Stage 4—Identifying the closest locations of Traffic Shaping Facilities in the City.](image)

Stage 4 requires the greatest concentration, as in the case of the presented model an important issue is specification of all Traffic Shaping Facilities in the City.

Stage V is a graphic visualisation of the PERT method in the AoA notation of all the bus lines, accounting for: Node as an intersection (selection of Nodes should be determined by the nearest vicinity connected with the Traffic Shaping Facilities in the City, running as close as possible to the Connecting Route), the Linking Arrow as the most probable travel time (tm) from one Node (intersection) to another (Figure 7).

At this stage, the shortest route calculated at Stage III is assigned to the points which are located in the direct vicinity of the route, which have a significant impact on the inhabitants’ mobility, and which passengers would like to reach seamlessly.

Stage VI is closely connected with Stage V, and it includes making basic computations of all created Nodes, in accordance with the principles of the classic approach to the PERT method and with the “forward” and “backward” principle. Moreover, the method assumes indicating the Critical Path, which in the model shows a new route in the public transport system for the bus traffic for the nth line, which passes through the Traffic Shaping Facilities in the City, which are relevant for this route. Indication of the Critical Path is based on connecting the Nodes with the zero (0) Travel Delay (Figure 7).
Determining the critical path consists in traditional computing of the route travel time in accordance with the PERT method. Figure 8 shows that it is the longest possible route that may be covered by a bus between those points. Specifying a critical path makes it possible to determine routes that are shorter in terms of travel time. While making such changes, it is necessary to remember about the Traffic Shaping Facilities in the City. On that basis it is possible to design routes that save time, nevertheless, it should be borne in mind that the indicated alternative route should run through the Traffic Shaping Facilities in the City. In the future, in addition to the time criterion, it would be advisable to consider introduction of a criterion corresponding to e.g., capacity of concrete places in the city or bus stops.

Stage VII—at this point, probability of covering the planned route within the stipulated time is specified. This stage consists of 8 activities:

- computations for the sections forming the critical path, pessimistic time \((t_b)\) for which the assumed speed is 10 km/h and optimistic time \((t_a)\) for which the assumed speed is 50 km/h, using the data from the traffic flow analysis carried out at Stage 1, item 1.
- drawing up a table showing the entire route made using the PERT method, additionally considering, for all sections in the analysed PERT network, the following values:
  - most probable travel time \((t_m)\):
    \[
    t_m = \frac{t_a + 4t_m + t_b}{6} \tag{2}
    \]
  - standard deviation \((\sigma)\):
\[ \sigma = \frac{t_b - t_a}{6} \]  
(3)

- variance \((\sigma^2)\):

\[ \sigma^2 = \frac{(t_b - t_a)^2}{6} \]  
(4)

- calculation of the sum of the Most Probable Times \((\sum t_m)\) on the Critical Path,
- calculation of the sum of Variations \((\sum \sigma^2)\) of all times on the Critical Path,
- indication of the Potential Travel Time \((D)\) as the time we want to check as likely for the journey,
- calculation of the Normal Distribution Function (variable \(Z\)), where:

\[ Z = \frac{D - \Sigma t_m}{\sqrt{\Sigma \sigma^2}} \]  
(5)

- reading from the Normal Distribution tables the probability that the bus covers the route within time \(D\), where \(D < \sum t_m\) or where \(D > \sum t_m\).
- indicating travel time \(D\), for which probability is >80%.

Formulas (2)–(5) were selected in accordance with the PERT methodology, based on the literature review [30].

Calculating the probability of covering the specified route within specified time facilitates effective preparation of precise timetables (Table 2).

Table 2. Calculation of variances and standard deviations based on the critical path.

| s/n | R    | U       | tm   | ta   | ts   | tv   | tα  | σ   | σ²   |
|-----|------|---------|------|------|------|------|-----|-----|------|
| 1   | 1–2  | STRZ2   | 61.77| 57.24| 286.2| 98.42| 38.16| 1456.19|
| 2   | 2–3  | Mor     | 141.11| 11.38| 551.88| 187.95| 90.08| 8115.01|
| 3   | 3–4  | Gru-tys | 159.06| 123.19| 615.96| 229.23| 82.13| 6745.06|
| 4   | 4–6  | Oz3     | 61.05| 37.08| 185.4| 77.78| 24.72| 611.08|
| 5   | 3–5  | Wsch2   | 143.54|      |       |      | 95.69| 0.00| 0.00|
| 6   | 2–7  | Strz3-kow | 193.7|      |       |      | 129.13| 0.00| 0.00|
| 7   | 6–5  | Glo2    | 61.77|      |       |      | 41.18| 0.00| 0.00|
| 8   | 6–8  | Oz 5, 6,7, 8 | 112.55| 79.63| 398.16| 154.67| 53.09| 2818.37|
| 9   | 8–9  | Oz 8, 9 | 63.03|      |       |      | 42.02| 0.00| 0.00|
| 10  | 8–10 | Rej2    | 44.9 | 33.34| 166.68| 63.27| 22.22| 493.88|
| 11  | 10–11| Rej1    | 63.52| 59.26| 296.28| 101.60| 39.50| 1560.51|
| 12  | 5–11 | Wsch1   | 56.73|      |       |      | 37.82| 0.00| 0.00|
| 13  | 11–7 | MI-1    | 38.04| 27.36| 136.8| 52.72| 18.24| 332.70|
| 14  | 10–13| 1Maj 1,2 | 71.59|      |       |      | 47.73| 0.00| 0.00|
| 15  | 7–12 | Ak1     | 95.05| 71.71| 358.56| 135.08| 47.81| 2285.64|
| 16  | 9–13 | Ple     | 44.37|      |       |      | 29.58| 0.00| 0.00|
| 17  | 13–12| Fab     | 46    |      |       |      | 30.67| 0.00| 0.00|
| 18  | 9–14 | Oz 10, 11 | 142.67|      |       |      | 95.11| 0.00| 0.00|
| 19  | 13–15| 1Maj 3, 4, 5 | 106.23|      |       |      | 70.82| 0.00| 0.00|
| 20  | 12–15| Ak2, 3, 4 | 112.38| 90.65| 453.24| 165.57| 60.43| 3651.99|
Table 3 presents a tool prepared in a worksheet, used for computing the probability of covering the route in the stipulated time.

**Table 3. Stage 7—Computing the probability of covering the route in the stipulated time.**

| s/n | R     | U  | t_m | t_s | t_b | t_v | σ  | σ^2 |
|-----|-------|----|-----|-----|-----|-----|----|-----|
| 21  | 14–15 | Kol 1, 2 | 55.98 | 37.32 | 0.00 | 0.00 |     |     |
|     | Total: |     | 889.43 | 590.84 | 3449.16 | 1923.36 | 476.39 | 28070.42 |

| Normal Distribution |
|----------------------|
| Checked value (D)    |
| 1035                 |
| Probability          |
| 0.87                 |
| Value read in the tables |
| 80.78%               |

There is a 80.78% chance that the bus will cover the route running along the critical path within 1035 s, i.e., 17 min

Stage VIII consists in repeating stages IV–VII for all the determined Connecting Routes.

**Figure 8.** Modelled route of the current bus line no. 3.

**4. Evaluation**

**Use Case Description**

Opole is a medium-size city located in the south-west of Poland, in the Silesian Lowland. Since 1950 it has been the capital city of the Opole Voivodeship. At the end of 2017, its population was 128,224, out of which 52.8% were women, and 47.2% were men. As from 1 January 2017, the city boundaries were extended to include the adjacent villages, now it’s the 27th biggest city in Poland in terms of population size. The city of Opole takes up the area of 148.9 km², with the population density of 861.14 persons/km².

Opole is divided into 29 districts, which are administrative units of the municipality. After 2018, following the accession of some districts, the city became a larger and more
significant entity in Poland. Housing estates have auxiliary administrative functions and are managed separately by Community Councils elected by the residents, which are the decision-making bodies with regard to local development. The largest district in terms of area is Czarnówšy (14.1 km²), and the smallest Borki (0.7 km²).

The population density coefficient, which shows the number of people per m² of the area, is as follows: Armii Krajowej—12.69 persons/m², which is almost twice as high as in the case of Malina street with just 6.76 persons/m². The authors relied on the above-mentioned coefficient when specifying the frequency of bus runs in selected areas. In view of the above, the authors adopted the following categories of bus service frequency: from 0 to 0.5—seldom (1 run per hour), from 0.51 to 1—occasional (3 runs per hour), from 1.01 to 10—frequent (4 runs per hour) and above 10.01—very frequent (6 runs per hour).

Nowadays, city development relies on numerous factors affecting economic, social or political aspects. Based on the PESTEL analysis completed for the city of Opole [31], it can be concluded that the most advantageous factor is the social and cultural sphere, which is described by such aspects as: satisfaction, mentality or development trends of the city inhabitants. Political aspects have the least influence on the city development, as they play a minor role in creating a metropolis.

Public transport in Opole is mainly based on road transport. The entity responsible for passenger transport is MZK Opole, which currently operates 18 day-time and 5 night-time bus lines. The fleet includes 92 buses that are used to transport passengers within the city limits. Timetables differ in terms of departure times: there are different timetables for weekdays, Saturdays, Sundays and public holidays. The most frequently running bus lines can be found in the city centre. An average waiting time at a bus stop is from 10 to 15 min. The municipality has ensured that inhabitants can use bus services to get to any district of Opole. However, in order to get to the city suburbs, it may be necessary to wait from 30 to 60 min. Telematics solutions functioning so far in public transport in Opole are:

- vehicle tracking system—GPS (aimed at tracking buses to find out their exact location at the moment, and to specify an approximate time of arrival at the next bus stop),
- variable message boards (the latest telematics solution aimed at notifying passengers waiting at a bus stop, at what time any given bus is scheduled to arrive),
- dynamic passenger information system—informing passengers about time of arrival of the means of transport. The main advantage of this system is that it provides information in real time, telling when a bus of any given bus line is going to arrive at the next bus stop. Unfortunately, the variable message boards and the dynamic passenger information system have not been combined yet, and the boards display the scheduled bus arrival times.

In order to find out about the attitudes of public transportation users in the analysed city, direct standardised interviews were held with randomly chosen passengers of the selected bus line. The results have shown that 80% of the respondents were open to changes in the bus routes in Opole [32], especially when the changed plans would take into account suggestions provided by the respondents with regard to the location of bus stops. Summing up, the respondents voiced suggestions regarding the timing of bus services and—as mentioned above—the locations of bus stops. For the purposes of the research study, bus line no. 3 was selected (Figure 8).

In order to simplify the procedure of further research work, the following assumptions were made [32]:

- for the purposes of the study, the direction of vehicle movement in the city is unimportant (the statement was based on the research results in the analysed city (the results were shown in: [32], even though it is probable that the direction of vehicle movement is important in other cities, as one-way streets and time of day may have a significant impact on bus travel time lengths),
the analysis results have shown that week days from Monday through Friday are similar, so they form one category, whereas the other category comprises Saturday and Sunday,

- the analysis has shown that similarities in terms of particular hours of traffic were diverse and they can be divided into 5 groups: Group 1: 6:00, 20:00, Group 2: 8:00, 14:00, Group 3: 10:00, 12:00, 18:00, Group 4: 16:00, Group 5: 22:00,
- traffic intensity for the above groups, starting from the least congested streets: Group 4, Group 2, Group 3, Group 1, Group 5,
- the most congested streets in Opole are: the Opole ring-road (road no. 46), ul. Bohaterów Monte Casino, ul. Niemodlińska, ul. Wrocławska, ul. Piastowska.

The Google Maps service was used to estimate the traffic intensity in the analysed case. The tool made it possible to visualise the road traffic in the selected sections that are relevant for the research study procedure:

In order to optimise the current way of running the bus service, a PERT-based model was applied to the current bus line no. 3 (Figure 8). Following the analysis of places that could be of vital significance for the route of the bus service, it was possible to suggest some optimising solutions.

The table presents three sections that are important from the point of view of the Opole public transport users. They are not covered by the current route of bus line no. 3, but when they are included in the route, this will make it possible to optimise the travel time. All the presented sections have been included in the procedure of further research work (Table 4).

Table 4. Points of optimising bus service no. 3.

| 1. Sosnkowskiego–Mikołajczyka–Okulickiego | 2. Oleska–Chabrów–Batalionów Chłopskich | 3. Ozimska–Reymonta–1 Maja–Plebiscytowa |
|------------------------------------------|------------------------------------------|------------------------------------------|
| ![Map of Sosnkowskiego–Mikołajczyka–Okulickiego](image1) | ![Map of Oleska–Chabrów–Batalionów Chłopskich](image2) | ![Map of Ozimska–Reymonta–1 Maja–Plebiscytowa](image3) |

In cases no. 1 and 3, the currently designed network is a solution that takes less time, resulting from the model-based computations, whereas in case no. 2 there is a possibility that taking another, alternative route between the two points may be a faster solution than in the case of the planned new bus line, even though the route runs through more Traffic Shaping Facilities in the City.

Another step in the proposed model is making a critical path which will indicate a bus route that best covers the Traffic Shaping Facilities in the City, but shows the longest travel time. Figure 9 presents making a critical path for this case.
The critical path in the pivotal points shows that there are identical solutions of the current first two cases. Selection of the more advantageous route must be predicated on some assumptions: through which intermediate points the given bus service should run, and what will be the time difference between the two transport variants. For that purpose, it is necessary to compute the probability at which a given bus is able to cover the new route made by the critical path, in accordance with the time schedule. Thus, Table 5 presents the travel times through all the points in the network. Only the points on the critical path have been provided with optimistic and pessimistic times.

Table 5. Computation of variance and standard deviation for the current bus line no. 3.

| l.p | R     | U      | t   | t   | Tp  | Tnp | O   | W       |
|-----|-------|--------|-----|-----|-----|-----|-----|---------|
| 1   | 20–19 | Puż1   | 131.37 | 101.45 | 507.24 | 246.69 | 189.03 | 67.63 | 4574.04 |
| 2   | 19–18 | Sos7   | 72.14  | 47.74  | 238.68 | 119.52 | 95.83  | 31.82  | 1012.72 |
| 3   | 18–17 | Mik    | 36.97  | 20.52  | 102.6  | 53.36  | 45.17  | 13.68  | 187.14  |
| 4   | 17–15 | Ole4   | 63.69  | 47.52  | 237.6  | 116.27 | 89.98  | 31.68  | 1003.62 |
| 5   | 18–16 | Sos6   | 50.74  | 50.74  | 33.83  | 0.00   | 0.00   | 0.00   | 0.00   |
| 6   | 16–15 | Oku    | 31.89  | 31.89  | 21.26  | 0.00   | 0.00   | 0.00   | 0.00   |
| 7   | 15–14 | Ole5,6 | 84.67  | 84.67  | 56.45  | 0.00   | 0.00   | 0.00   | 0.00   |
| 8   | 14–12 | BCH    | 50.84  | 50.84  | 33.89  | 0.00   | 0.00   | 0.00   | 0.00   |
| 9   | 15–13 | Cha    | 96.94  | 63.94  | 319.68 | 160.19 | 128.56 | 42.62  | 1816.75 |
| 10  | 13–12 | Lub5,6 | 142.1  | 104.33 | 521.64 | 256.02 | 199.06 | 69.55  | 4837.43 |
| 11  | 12–11 | NH,2KsO3 | 49.7 | 37.58 | 187.92 | 91.73 | 70.72 | 25.06 | 627.84 |
| 12  | 11–10 | Sad, Sie2,1 | 59.29 | 44.28 | 221.4 | 108.32 | 83.81 | 29.52 | 871.43 |
| 13  | 10–9 | Ole9 | 14.48  | 9.86   | 49.32  | 24.55  | 19.52  | 6.58   | 43.25  |
| 14  | 9–8 | Gr2Żer1Rey6-Pk | 74.2 | 32.9 | 164.52 | 90.54 | 82.37 | 21.94 | 481.22 |
| 15  | 8–7 | Oz12,11,10 | 124.02 | 68.94 | 344.52 | 179.16 | 151.59 | 45.93 | 2109.56 |
| 16  | 7–5 | Ple   | 44.37  | 32.47  | 162.36 | 79.73  | 62.05  | 21.65  | 468.65 |
The considerable sum of variances indicates considerable data dispersion in relation to their mean. In this case it is a normal situation caused by very diversified types of traffic and sections considered in the analysis. The result of applying the PERT method in the infrastructure of the bus service run so far is an answer within what time and at what probability the bus will cover the route. Table 6 presents the probability at which the given bus service will cover the route in 23 min.

The probability of this event is 80.23%, which in accordance with the model criteria meets the adopted assumptions. Therefore, the last element to evaluate the applied model is checking whether or not the computed travel time is better after the model application. Table 7 presents a comparison between the real travel time and its modelled version.

**Table 6.** Probability of covering the current bus line route in scheduled time.

| Lp | R     | U       | t  | t  | t  | Tp | Tnp | O  | W           |
|----|-------|---------|----|----|----|----|-----|----|--------------|
| 17 | 8–6   | Rey 3,4,5 | 64.31 | 64.31 | 42.87 | 0.00 | 0.00  |
| 18 | 6–5   | 1Maj 4,5 | 86.78 | 86.78 | 57.85 | 0.00 | 0.00  |
| 19 | 5–4   | 1Maj2,1  | 41.59 | 38.52 | 247.68 | 109.26 | 75.43 | 3.86 | 1215.22     |
| 20 | 4–3   | Rey1     | 63.52 | 59.26 | 296.28 | 139.69 | 101.60 | 39.50 | 1560.51     |
| 21 | 3–2   | MI1      | 38.04 | 27.36 | 136.8  | 67.40  | 52.72  | 18.24 | 332.70      |
| 22 | 2–1   | AP2-JAG  | 157.76 | 126.5 | 632.52 | 305.59 | 231.68 | 84.34 | 7112.67     |
|    |       |          | 1210.18 | 863.17 | 4370.76 | 2517.27 | 1925.26 | 584.60 | 28254.77   |

There is a 80.23% chance that the bus will cover the route running along the critical path within 1353 s, i.e., 23 min.

**Table 7.** Comparison of mean bus travel times before and after applying the model.

| Mean Actual Bus Travel Time before the Optimisation (min) | Mean Optimised Bus Travel Time (min) | Passenger Car Travel Time (at 13.30, Tuesday) (min) |
|----------------------------------------------------------|--------------------------------------|-----------------------------------------------|
| Bus travel time (including the time of stopping at the bus stops) | 36                                   | 25                                    |
| Number of bus stops                                      | 23                                   | 23                                    |
| Time of stopping at the bus stops                        | 11.5                                 | 21                                    |
| Bus driving time                                         | 25                                   | 23                                    |

In conclusion, the suggested route makes it possible to save 1.5 min. The standard deviation of the time saved is 0.5 min, where it is possible to conclude that the time saved will fluctuate within the interval from 1 to 2 min. This result may be deemed very good in view of the fact that the study concerned changes related to only one run of one bus line. Consequently, over the whole day of that bus service runs, given that there are 47 bus runs a day, the total time savings amount to 70.5 min. Further research works will be
connected with designing the whole public transportation system in Opole, which will make it possible to obtain a better picture of the impact of the changes on streamlining the passenger transport.

5. Conclusions

The presented model makes it possible to optimise the public transport solutions used so far in cities, at the same time estimating the exact travel time for the specific bus line, based on normal distribution and computed probability of such traffic. The research study has shown that following the application of the PERT method to planning the travel times in relation to one bus line, nearly 1.5+/− 0.5 min could be saved on the selected route (as there are 47 bus runs a day, the total savings amount to 1 h and 10.5 min), which is a very good result, especially in view of the fact that travelling the same route in a passenger car takes merely 2 min less.

It should also be noted that determining the full impact on the city’s public transportation system will be possible upon designing the whole public transport system for Opole, using the proposed solutions. Summing up, the presented method of planning the public transport system routes makes it possible to achieve measurable benefits such as: shortening the travel times, designing the routes based on actual needs of public transport users with regard to locations of bus stops. This way of planning on one hand may result in increased user satisfaction, thus providing a viable alternative to using individual means of transport in the city, and on the other hand it has a positive impact on decreasing the operational load of the means of transport.

In view of the above, the authors see a considerable potential in the presented model which in the future may become an additional tool to support the operations of municipal public transport operators. The most important area of further research related to the proposed model of controlling the collective transportation fleet is to specify whether or not application of the suggested solutions in the real-life transportation system will bring measurable benefits. Therefore, measures should be taken to implement the model in the living organism of the city. At this point, it is important to develop a detailed implementation plan for incorporating model-related elements in the actual infrastructure. The measures will constitute a starting point for further research work.

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