Competitive balance theory: modeling the conflict of interest in a heterogeneous network

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ABSTRACT

The dynamics of networks on Heider's balance theory moves toward reducing the tension by constantly reevaluating the interactions to achieve a state of balance. Conflict of interest, however, is inherent in most complex systems; frequently, there are multiple ideals or states of balance and moving towards one could work against another. In this work, by introducing the competitive balance theory, we study the evolution of balance in the presence of conflicts of interest. In our model, the assumption is different states of balance compete in the evolution process to dominate the system. We ask, whether, through these interactions, different states of balance compete to prevail their own ideals or a set of co-existing ideals in a balanced condition is a possible outcome. The results show that although there is a symmetry in the type of balance, the system evolves either towards a symmetry breaking where one of the states of balance dominate the system, while the non-dominant part maintains a marginal presence. However, the tolerance of the dominant type with respect to the marginal type depends on the size of network.

Introduction

Conflict of interest is frequently observed in social networks, with conflicting sources of authority pushing the network towards their own ideals. As the system changes and evolves towards balance, these different ideals compete to dominate the network. In this paper we aim to extend the structural balance model, as theorized in Heider's pioneering work¹, to take this competition of ideals into account. A half-century of research has shown that balanced networks have been very well-theorized²–⁶ and are frequently observed empirically⁷–¹¹, first introduced by Heider and then modeled in signed graphs by Cartwright and Harary¹², is developed to explain the structure of conflicts and balance in a signed network; a network in which the connections are characterized as friendship and hostility, denoted by ±1. A triad is balanced when all three agents are friends, or when two friends have a common enemy; the cycle, otherwise, is tensioned. Over time, the system either reaches a global minima (either all links become positive or two hostile clusters emerge) or gets trapped in a local minimum (jammed state)¹³.

This original form of structural balance theory has been extended in several directions¹⁴–¹⁶. For example, Antal, Krapivsky, and Redner¹⁷ (2005) go beyond a static description of balanced relations, and investigate different dynamical rules¹⁸–²⁰ for achieving balance. Also, Marvel, Strogatz, and Kleinberg²¹ (2009), on the basis of social psychology theories, suggest that certain triads are more stable than others, and taking this statement as the driving force for change, they extract the energy landscape of the networks.

Heider’s balance theory, in its original form, states that the idea of balance holds when the system includes one type of link, but the conflict of interest here is defined by taking the existence of several
Figure 1. A network with two different conceptualizations of friendship and hostility. The solid and dashed lines show friendship and hostility, respectively, and the colors refer to the types of links (real or imaginary).

types, of $\pm$ links into account. In this work, we extend the original theory to propose the competitive balance theory, i.e. balance in the presence of conflict of interest. Conflict of interest is inherent in most complex systems; frequently, there are multiple ideals or states of balance and moving towards one could work against another. In this work, by introducing the competitive balance theory, we study the evolution of balance in the presence of several types of $\pm$ links.

We can employ the social scientific concept of "conflicting discourses" to clarify the idea of conflict of interest and different conceptualizations of friendship and hostility (i.e. different types of $\pm$ links). Discourse, as a social scientific term and as elaborated by Michel Foucault, in this context, refers to socially shared habits of thoughts, perceptions, and behaviors. The dominant discourse is attached to power, is continuously reaffirmed as the normative discourse, and it often involves ideas about a particular group of people (e.g. LGBTQ* community, Jews, or women). As the central point of commonality, the dominant discourse is the accepted way of perceiving the social reality, or behaving in social space. Alternative discourses, are usually that of the non-power holding others and are then shaped in response to the dominant discourse, frequently at the margins of systems, especially at the beginning of their emergence.

This extended theory of structural balance can be used to model the dynamic behavior of the diverse, interacting, and possibly competing, discourses that coexist in a system. The competitive balance theory acknowledges that the meanings of hostility and friendship are localized in a social space, and different discourses, with potential or perceived conflicts of interest, compete to reach balance or dominance. Conflict of interest frequently arises between the dominant and alternative discourses. In the global environment, with several discourses operating at the same time, the meanings of friendship/hostility (links) are localized. Friendship and hostility are defined based on the discourse(s) in which the interactions are taking place, with each of active discourses having its own conceptualization of links. Discourses are also in dialogue all the time; they are rarely, if ever, independent from one another or entirely cut off from the global environment. Although each discourse has its own internal conflicts, conflicts of interest among discourses to dominate the system create a conflictual and contentious social space. In short, in our model

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there are various and potentially competing forms of links, and different desired balanced states.

Considering the inter- and intra-discourse interactions, the question is, whether, through these exchanges, the competition evolves towards a homogenization of a type of link (i.e. one discourse privileging its own set of meanings) or, alternatively, a set of co-existing discourses in a balanced condition (i.e. co-existing states of balance) is a possible outcome?

**Methods**

As explained in the introduction, in the real world, the meanings of hostility and friendship are localized in social space; in other words, people have different sets of reasons to be friend or resent one another. Taking this heterogeneity into account, the states of balance or unbalance in a triad is influenced by these different meanings of friendship and hostility.

Since in the original conceptualization of a balanced triad, the assumption is that there is a single meaning (or cause) of friendship or hostility, all triads are ‘pure’. In other words, going back to the concept of discourse, all dyadic relationship in a triad function are in the same discourse, or are derived from the same order of meanings. Extending the model to include diverse systems of meanings, in addition to the pure triads (getting their meanings from a single discourse), there will be ‘mixed’ ones with edges coming from different realms of meanings. In such mixed triads, the state of (un)balanced is influenced by the heterogeneity of the discourses involved.

For example, we know, from the original conceptualization of structural balance theory, that when two enemies have a common friend, the triad is unbalanced. If, however, the friendship and hostility in a triad are derived from different discourses (or, to put it simply, if the cause of friendship is different from that of hostility), in our system, the triad will be defined as balanced (see Fig. 2). Note however that unlike the edges in a multi-layered network, in the competitive balance model each two nodes can be connected to
each other with one, and only one, edge.

In this model, for simplicity, we use two different conceptualizations of friendship and hostility, i.e. four types of links altogether $\pm a, \pm b$. Fig 1 is a proper representation of such a network. The $\pm a$ edges aim to evolve towards the first desired balanced state, and the $\pm b$’s aim to evolve towards the second desired balanced state, with a conflict of interest occurring among $\pm a$’s and $\pm b$’s.

We, then, pursue to model the presence of two different types of friendship/hostility, satisfying the following conditions:

1. All the triads (pure and mixed) are homogeneously treated and their energies are equally weighted in our Hamiltonian function. The triads are homogeneous if, in moving towards lesser tension, the system does not prefer one balanced triad over another (i.e. the pure ones over the mixed ones, or the other way). In other words, the system moves towards the presence of more balanced triads, irrespective of the dyadic discursive relationships involved in each.

2. The definition (un)balanced pure triads follows the original conceptualization by Heider.

3. Mixed triads, the ones with dyadic relationships stemming from different discursive spaces, follow a symmetrical behavior according to their energies.

4. The edges and triads that are derived from different discursive spaces are always identifiable, and to this end we need to use a label to distinguish them.

To achieve these, we use the properties of complex numbers. Based on this choice, the edges of type $a$ (belonging to the first order of meanings) are denoted with real numbers $\pm 1$ and the edges of type $b$ (belonging to the second order of meanings), are denoted by the imaginary numbers $\pm i$. As $+1,-i$ indicate friendship and $-1,+i$ indicate hostility. Fortunately, by using complex numbers, we can distinguish between the two different types of friendship/hostility without disturbing the homogeneity of triads, i.e. using this method the energy of pure and mixed triads are treated equally in the Hamiltonian function. Translated into mathematical language of modeling:

\[
\forall i, j \in \{1, 2, \ldots, N\} : S_{ij} \in \{+1, -1, +i, -i\},
\]

thus the possible energies for triad $ijk$ is as below:

\[
E_{\triangle ijk} = -S_{ij}S_{jk}S_{ki} \Rightarrow E_{\triangle ijk} \in \{+1, -1, +i, -i\}.
\]

Indeed, based on these two types and two signs of edges, twenty different kinds of triads can be constructed as shown in Fig. 2. These triads are divided into four categories according to their energy. A triad is balanced if its energy is negative and unbalance otherwise. On the other hand, a triad is classified in the real (imaginary) group if its energy is a real (an imaginary) number.

\[
\begin{align*}
\triangle_{re}, & \quad \text{if } E_{\triangle ijk} = \begin{cases} 
-1, & \text{balanced} \\
+1, & \text{unbalanced}
\end{cases} \\
\triangle_{im}, & \quad \text{if } E_{\triangle ijk} = \begin{cases} 
-i, & \text{balanced} \\
+i, & \text{unbalanced}
\end{cases}
\end{align*}
\]
As mentioned before, in each group, we name a triad pure if its sides are all of the same type, either real or imaginary and mixed if its sides are a combination of both types.

In the initial condition of simulation, we assume that all four types of edges are randomly distributed with an equal probability in a fully connected network. Since a random configuration is obviously not balanced, the network begins to evolve. As Antal has mentioned the natural tendency of members in social network is to reduce the unbalanced triads\textsuperscript{17} which is equal to tension reduction. So, in order to investigate the final states of the system, we define a Hamiltonian function which satisfies the tension reduction, as follows:

\[ E(t) = E_{re}(t) + E_{im}(t), \]  
\[ E_{re}(t) = \frac{1}{N_{\Delta}} \sum_{\Delta \in \Delta_{re}} E_{\Delta}(t) \Rightarrow -1 \leq E_{re}(t) \leq 1, \]  
\[ E_{im}(t) = \frac{1}{iN_{\Delta}} \sum_{\Delta \in \Delta_{im}} E_{\Delta}(t) \Rightarrow -1 \leq E_{im}(t) \leq 1. \]

Although, the share of \( E_{re} \) and \( E_{im} \) in total energy varies in time but the total energy of network is always in the range of -1 to +1:

\[ -1 \leq E(t) \leq 1 \]

This form of Hamiltonian simply apply Constrained Triad Dynamics (CTD)\textsuperscript{17} which corresponds to the reduction of the number of unbalanced triads. In other words, the priority is to maximize the balanced triads, irrespective of their types. In each update step, we choose an edge at random to flip to one of the three other relations with equal probability. The flip will be accepted if the total number of balanced triads\textsuperscript{28} increases.
In this way, the system dynamics moving toward structural balance, or lesser tension, can be studied when there are two interacting discourses at play. In the next section, we ask how does a system evolve in the presence of two different, but structurally equal, types of friendship/hostility, when the model does not prefer one type over another? What is the share of each, $E_{re}$ and $E_{im}$, in the total energy during the evolution? Whether, as a result of the competition, one of the orders of meanings dominates the system or they both co-exist in a balanced state?

Results

Fig. 3a represents the share of energy of the real and imaginary triads in total energy over time for 10,000 realizations. Note that the illustrated results are of a typical network of size $N=20$, at the end of this section we investigate the effect of size. At the initial state there is no preference for different types of edges, so the density of the four groups of triads (shown in Fig. 2) are equal; hence, $E_{re}$ and $E_{im}$ have roughly the same share. The fact that at the beginning, the number of balanced and unbalanced triads are equal implies that the starting point of the dynamic of each sample is around the origin of the coordinate. As time passes, different kinds of triads transform to each other and at a specific point there appears to be a symmetry breaking. From this point forward, as shown in Fig. 3a, the energy of network falls into either the blue branch, where the share of real energy is dominant, or the red branch, where the share of imaginary energy is dominant. Of course, in rare cases, the system stops in the middle of the path (green branch).

If we let the system evolve long enough, it is finally trapped in a minimum which may be local or global. Global minima are absorbing states of the system in which $E = E_{re} + E_{im} = -1$. If the share of $E_{re}(E_{im})$ in the total energy is significantly higher than the share of $E_{im}(E_{re})$, then the system is said to be in the real (imaginary) domain. It is worth noting that the symmetry is spontaneously broken, in the sense that without the intervention of an external force, in global minima the major share of energy belongs to either the real or the imaginary part. Also, less frequently, the system is trapped in local minima, called jammed states$^{17}$, and, as illustrated, unlike global minima, in jammed states $E_{re}$ and $E_{im}$ have close shares. If the system does not get stuck in jammed states, the energy path is finally absorbed into one of the real or imaginary domains. We averaged over the energy paths of different realizations that led to either local
Figure 5. 3D plot, energy vs. time for a system that falls in the (a) real domain or (b) imaginary domain. The energy fluctuates between fixed points with the same magnitude of energy.

minima (i.e. jammed states), or global minima (i.e. real and imaginary domains, separately), as shown in Fig. 3b. It is clearly evident that there is a symmetry on the path to both real and imaginary domains.

Fig. 4a illustrates the final energy of 10,000 realizations with the same initial condition after $N^4$ Monte-Carlo steps. As shown, all paths are absorbed into a limited number of discrete fixed points, categorized into three sets: jammed states, real and imaginary domains. As the density of occurrence shows in Fig. 4b, it is apparent that some absorbing points are more preferable than the others. Also, as shown, reaching a real or an imaginary domain is significantly more probable than getting stuck in the jammed states.

Fig. 5 is a 3D representation which shows the share of $E_{re}$ and $E_{im}$ in total energy versus time for two arbitrary configurations. As the figure shows, the energy path quickly reaches a balanced state in one domain, then starts to fluctuate between fixed points with the same magnitude of energy in that domain.

A macro-level analysis showed us that the global behavior of the network leads to the formation of three possible domains, based on the final energy levels. We now turn to a micro-level analysis, study the details of network structure and detect the kinds of triads that will remain in each domain. Here, we leave aside the jammed states and follow the density of triads over time for the paths which finally settle to the global minima of energy.

As explained in Fig. 2, the triads are classified into balanced and unbalanced, with real and imaginary energies. Since the initial density of all types of edges are equal, as it is shown in Fig. 6a, all four groups of triads have the same density at the beginning. Over the early steps of evolution the number of balanced triads dominates unbalanced ones resulting in division of branches in the figure. As time goes by, at a point, symmetry between the number of imaginary and real triads is spontaneously broken and one of the balanced groups predominates, while the density of the other balanced group decreases. After saturation the dominant balanced group occupies more than 80 percent of all triads leaving less than 20 percent for defeated balanced group and nothing for unbalanced groups.

Another question is the share of different possible balanced triads in the final states. Surprisingly, we observed that despite the fact that constraints concerning energy allow existence of 10 forms of balanced triads but only subgroups of them coexist in the final global states. For example if the system falls into
Figure 6. (a) The changes in the density of the dominant and defeated balanced and unbalanced groups across time. The unbalanced groups start to disappear from the beginning, and either the real or the imaginary balanced group dominates the network. (b) The density of different balanced triads across time. The legend shows the triads in the real or imaginary domains, in two separate columns. The circled triads in the legend are the ones that remain in the system, in each specific domain.

the real domain, out of 10 balanced triads only five of them remains: two pure real balanced triads and three mixed imaginary ones. They have been specified by circles in the left column in Fig. 6b. This form of selection, indicates that it is more favorable to the system to go to the state with most real links. In the same way if the system gets into the imaginary domain, the most dense triads are respectively pure imaginary and mixed real ones which have been identified in the right column. As a result, the steady state of a competitive system is where real or imaginary links overcome the other one. Indeed, in real (imaginary) domain, real (imaginary) links predominate in the network and a few imaginary (real) links remain sporadic across them.

Now we investigate the effect of size on our results. We performed simulation for a wide range of sizes in ensembles of 1,000 iterations. Fig. 7a illustrates the final energies for a selection of sizes, and Fig. 7b shows the probability density for the spectrums. A few interesting results were deduced from the size analysis. Size analysis revealed that jammed states can occur in any size. Although in Heider’s balance model, networks of certain smaller sizes (i.e. \(N < 9 \& N = 10\)) have no jammed states\(^1\), this is not the case in our model. Additionally, as the spectrum of final states in Fig. 7 shows, as the size grows the probable global minima move towards homogeneity. In other words final global minima get closer to the real or imaginary axes. This means that for the global minima falling in the real domain, the share of imaginary triads shrink as size grows. The same story happens if the system fall in the imaginary domain. In society this implies that in fully connected networks, for large sizes we approach a more homogeneous world where one of the discourses almost eliminates the other.

Now, another concern rises. What will be the density of the most probable triad for such uniform world? In size 20 we observed that out of 10 possible triads only five survived evolution of the system. As well we noticed that one forms of triads outnumbered the others by the rate of 60 percent, graphed in Fig. 6b. One might concern at what rate the most popular triad dominates the system in the thermodynamic limit.
Figure 7. (a) The fixed points of different domains, and (b) the probability of occurrence of fixed points for networks with sizes $N=8, 16, 32, 64$. 
Fig. 8 graphs the evolution of the share of the most popular triads over time for various sizes. It seems that as the size grows the share of the most probable triad grows as well. However, an upper bound appears in the graph and is revealed by a rough calculation. In a rather homogenized world, observed in larger sizes, with one of the discourses dominating the system, the configuration becomes either paradise or bipolar. Entropy, however, suggests the bipolar state, in which there are two strong subsets having intra-group friendship and inter-group hostile relationships. The number of possible choices in a bipolar network as such is maximized if each group has almost half the population. In this system two kinds of triads are formed: about $N^3/24$ triads with edges $(1,1,1)$ and $N^3/8$ triads with edges $(1,1,-1)$. So, referring back to Fig. 8, the $(1,1,-1)$ triad leads the density with an optimum share of 0.75.

Conclusion

Heterogeneity and conflict of interest is inherent in most real-life social networks. While a typical property of most social networks is their strive towards balance, as theorized by Heider pioneering work, we discuss that different desired states of balance may coexist in a network, with one state working against others. In this paper, we introduce the competitive balance theory by redressing the original balance function and modifying the model to take the conflict of interest into account. The competitive balance theory studies the evolution of networks towards different states of balance, in the presence of heterogeneous and potentially competing types of links.

In this work we assumed that the system evolves towards two competing states of balance, and comprises two different types of friendship and hostility. We used complex plane to distinguish between the two different types without disturbing the homogeneity of triads in the Hamiltonian function. This way, while proposing a new model, we follow the original conceptualization of (un)balanced triads by Heider.

In our simulation, we randomly distribute four different edges; two real and two imaginary, to represent different types of friendship and hostility. As the network evolves, although at first there is no preference for the share of real or imaginary parts in energy, at a specific point there appears to be a symmetry breaking. The system, typically, moves towards the domination of the real or imaginary domain, while the non-dominant type of energy maintains a marginal presence. For a better comprehension of this state,
imagine a normative discourse (e.g. heteronormative) staying- or an alternative discourse (e.g. LGBTQ inclusive) becoming- the dominant discourse, while the other maintains a marginal existence. Size analysis shows that in smaller sizes the network demonstrates higher levels of tolerance for the non-dominant type, though, as the network grows so does its intolerance for the marginal presence of the other discourse, while in the thermodynamic limit the non-dominant type of links ceases to exist.

Less frequently, the system gets trapped in unstable local minima, with high levels of heterogeneity and conflicts. Using the same analogy, this happens when different discourses of sexual orientation coexist in a conflictual environment, with none of them playing the role of dominant discourse. Note that, in this paper we take two different conflictual states of balance into account; this modeling choice, depending on the research question or the case under study, can be extended to more than two conflicting sets of ideals.

The empirical testing is an important next step in formal modeling. It mostly refers to the use of large N data to test the predictions and the processes highlighted by a model, taking the assumptions into account. Today, data creation, occurring at a record rate, offers new opportunities to empirically test formal models. There are several domains in which the conflicting nature of multiple states of balance in a system can be studied and modeled empirically; conflicting discourses of family and sexuality and conflicting political rhetoric as described in Supplementary Information, are cases in point.

Appendix:

Suggestion for empirical testing

The competitions among different perceptions of family, mostly as a result of the growing LGBTQ discourses and movements on a global scale, provides a relevant historical example. Evidence indicates that, in the past few decades, the discourses around the issue of family have been competing for dominance, and the conflict of interest in the system has resulted in a general shift towards a greater acceptance of LGBTQ-inclusive discourse of family (as opposed to the once dominant discourse of nuclear/heterosexual family)\textsuperscript{29}. Both discourses, those who see families as a heterosexual unit, and those who have a more inclusive idea of family, have their own internal conflicts (over issues such as children, families of origin, and finances), on the one hand, and are in conflict with each other on the other hand. Agents, have complex social identity, and often draw on multiple, at times competing, discourses to make sense of the reality. Depending on the cycle (local environment/ social space) in which one is interacting, s/he may interact based on heterosexual or LGBTQ-inclusive notions of family. In the process of reconsidering one’s perception about the notion of family, a single agent goes back and forth between multiple discourses, as see fit in their local environments and constantly reorganize a sense of who they are. Agents with multiple identities (i.e. using multiple discourses) negotiate their identities over time toward reaching stability, and that is when the system reaches a balance.

In political domain, conflicting political rhetorics is another case in point. Political communication strategists, while still predominantly using traditional channels such as TV debates or door knocking, since the 2008 Obama election, were fascinated with the potential of online social media platforms\textsuperscript{30}. Many researchers believe that in the age of Twitter political rhetoric has changed, and a new amateurish (yet authentic) discourse has emerged, that co-exists with the traditional professional discourse in social media\textsuperscript{31}. There are, for example, several case studies on Donald Trump’s rhetoric, described as authoritarian or populist, and fundamentally different from the established partisan rhetoric in the US\textsuperscript{33}. Professional rhetoric uses the established political language to promote a cause and uses research-based evidence to mobilise the users. The counter-trend moves towards deprofessionalism, using controversial language, and utilizing the dual role of celebrity-politician, and at times more likely to be circulated compared to messages using professional stylistic standards and established political language. This
counter-rhetoric corresponds to an image of authentic ideas and a genuine outsider positioning, using form and content to show spontaneity and sincerity. The conflict between these two political rhetorics in an election season, for example that of Donald Trump and Hillary Clinton, and the attempt to dominate the system can be modeled through the competitive balance theory.

The empirical testing of our model is not limited to the social or political domains, but can be extended to other areas such as such as human genomics, healthcare, oil and gas, search behaviour, surveillance, and finance.

References

1. Heider F. Attitudes and cognitive organization. journal of Psychology. 1946;21:107.
2. Park J, Newman M. Solution for the properties of a clustered network. Physical Review E. 2005;72(2):026136
3. Saeedian M, Azimi-Tafreshi N, Jafari G, Kertesz J. Epidemic spreading on evolving signed networks. Physical Review E. 2017;95(2):022314.
4. Kirkley A, Cantwell GT, Newman MEJ. Balance in signed networks. Physical Review E. 2018;99(1):012320.
5. Rabbani F, Shirazi AH, Jafari GR. Mean-field solution of structural balance dynamics in nonzero temperature. Physical Review E. 2019;99(6):062302
6. Parravano A, Andina-Diaz A, Melendez-Jimenez M. Bounded confidence under preferential flip: A coupled dynamics of structural balance and opinions. PloS one. 2016;11(10):e0164323
7. Hart J. Symmetry and polarization in the European international system, 1870-1879: a methodological study. Journal of Peace Research. 1974;11(3):229–244.
8. Hummon N, Doreian P. Some dynamics of social balance processes: bringing heider back into balance theory. Social Networks. 2003;25(1):17–49.
9. Lerner J. Structural balance in signed networks: separating the probability to interact from the tendency to fight. Social Networks. 2016;40(6):733-741.
10. Leskovec J, Huttenlocher D, Kleinberg J. Predicting positive and negative links in online social networks. Proceedings of the 19th international conference on World wide web. 2010;641-650.
11. Doreian P, Mrvar A. Structural balance and signed international relations. J. Soc. Struct. 2015;16(1).
12. Cartwright D, Harary F. Structural balance: A generalization of heider’s theory. Psych. Rev. 1956;63:277.
13. Hedayatifar, L. and Hassanibesheli, F. and Shirazi, A.H. and Vasheghani Farahani, S. and Jafari, G.R. Pseudo paths towards minimum energy states in network dynamics. Physica A: Statistical Mechanics and its Applications. 2017;483:109 - 116.
14. Belaza AM, Hoefman K, Ryckebusch J , Bramson A , Heuvel M , Schoors K. Statistical physics of balance theory. PLoS one. 2017;12(8):e0183696.
15. Du H, He X, Wang J, Feldman, MW. Reversing structural balance in signed networks Physica A: Statistical Mechanics and its Applications. 2018;503:780–792.
16. Belaza AM, Ryckebusch J, Bramson A, Casert C, Hoefman K Schoors K, Heuvel M, VanDerMarliere B. Social stability and extended social balance—quantifying the role of inactive links in social networks. Physica A: Statistical Mechanics and its Applications. 2019;518:270–284.
17. Antal T, Krapivsky PL, Redner S. Dynamics of social balance on networks. Phys. Rev. E. 2005;72:036121.
18. Kulakowski K, Gawronski P, Gronek P. The heider balance a continuous approach. Int J Mod Phys C. 2005;16:707–716
19. Marvel SA, Kleinberg J, Kleinberg RD, Strogatz SH. Continuous-time model of structural balance. PNAS. 2011;108(5):771–776.
20. Abell P, Ludwig M. Structural balance: A dynamic perspective. The Journal of Mathematical Sociology. 2009;33(2):129–155.
21. Marvel SA, Strogatz SH, Kleinberg JM. Energy landscape of social balance. Phys. Rev. Lett. 2009;103:198701.
22. Gorski PJ, Kulakowski K, Gawronski P, Holyst J. Destructive influence of interlayer coupling on heider balance in bilayer networks. Scientific Reports. 2017;7(1):16047.
23. Hassanibesheli F, Hedayatifar L, Safdari H, Ausloos M, Jafari GR. Glassy states of aging social networks. Entropy. 2017;19(6):247.
24. Sheykhaligheh S. and Darooneh, A.H. and Jafari, G.R. Partial balance in social networks with stubborn links. Physica A: Statistical Mechanics and its Applications. 2019;123882.
25. Foucault M, Power/knowledge: Selected interviews and other writings, 1972-1977. Vintage. 1980.
26. Hall S. Foucault: Power, knowledge and discourse. discourse theory and practice: A reader. Sage in association with The Open University. 2001;72–81.
27. Spargo T. Foucault and queer theory. Cambridge: Icon books. 1999.
28. Terzi E, Winkler M. A spectral algorithm for computing social balance. WAW 2011, LNCS 6732. 2011;1–13.
29. Copley A. Sexual moralities in france, 1780-1980: new ideas on the family, divorce, and homosexuality: an essay on moral change. Routledge. 2019.
30. Cogburn DL, Espinoza-Vasquez FK. From networked nominee to networked nation: Examining the impact of web 2.0 and social media on political participation and civic engagement in the 2008 obama campaign. Journal of political marketing. 2011;10(1-2):189–213.
31. Engesser S, Ernst N, Esser F, Büchel F. Populism and social media: How politicians spread a fragmented ideology. Information communication and society. 2017;20(8):1109–1126.
32. Valentino NA, Neuner FG, Vandenbroek LM. The changing norms of racial political rhetoric and the end of racial priming. The Journal of Politics. 2018;80(3):757–771.
33. Enli G, Twitter as arena for the authentic outsider: Exploring the social media campaigns of trump and clinton in the 2016 us presidential election. European journal of communication. 2017;32(1):50–61.
34. Ott BL. The age of twitter: Donald j. trump and the politics of debasement. Critical studies in media communication. 2017;34(1):59–68.