An example illustrating the imprecision of the efficient approach for diagnosis of Petri nets via integer linear programming

Alban Grastien*

September 18, 2012

Abstract

This document demonstrates that the efficient approach for diagnosis of Petri nets via integer linear programming may be unable to detect a fault even if the system is diagnosable.

1 Introduction

The efficient approach for diagnosis of Petri nets via integer linear programming was proposed in 2009 by Basile et al. [BCD09]. Their work represents a novel approach to diagnose discrete event systems [CL99] modeled as a Petri net.

The original work contains two typos [BCD12]. In particular, the efficient approach comes at a cost, which is that a fault might come undetected even if the system is diagnosable. I illustrate this imprecision by presenting a diagnosable net system where the fault cannot be diagnosed using Basile et al.’s approach.

It might be argued that the community was already aware that this approach is not precise. Dotoli et al. [DFMU09] presented a similar yet precise approach, by removing the largest aspect of the efficiency in Basile et al. It seemed to me however that it was necessary to illustrate this point.

2 Efficient Diagnosis of Petri Net

We assume that the reader is familiar with the Petri net formalism, and provide only a short definition. Formal definitions and further references can be found in Basile et al.’s paper [BCD09].

A Petri net (cf. Fig. 1) consists in a set of places (represented by circles) and a set of transitions (represented by rectangles). A place contains a number of tokens (represented by dots) A number of edges connect places to transitions and transitions to places. A transition may trigger if all preceding places have a token; this consumes one token in each of these places and adds a token in each place that is a successor of the transition. A net system is a Petri net with a set of tokens.

Some transitions are observable: their firing is known. Observable transitions are filled in black; unobservable transitions are empty. It is assumed that the sub-network without the observable transitions is cycle-free. One transition is faulty. It is unobservable.

The purpose of diagnosis is the following. The system takes a sequence of transitions $s$ (under the liveness property, we assume that the system will never stop running, which means that $s$ can be assumed arbitrary long). The observation $o$ on the system is the projection of $s$ on the observable transitions (i.e., we remove from $s$ the unobservable transitions). Diagnosis consists in determining whether the faulty transition certainly/possibly occurred.

* Alban Grastien is with the Optimisation Research Group of NICTA and the Artificial Intelligence Group of ANU, Canberra, Australia. E-mail: alban.grastien@nicta.com.au.
I will not present diagnosis of Petri nets via integer linear programming as this is irrelevant for this discussion; I only need to say that this approach computes all the sequences $s'$ that can generate the observation $o$ and determine if all/some of these sequences contain the faulty transition. I will however discuss the efficient approach presented by Basile et al.

There are two important aspects in this approach:

1. Given an observation $o = [t_1, \ldots, t_n]$, the efficient approach computes the diagnosis of $o_1 = [t_1]$, of $o_2 = [t_1, t_2]$, \ldots, and of $o_n = [t_1, \ldots, t_n]$. If a fault is detected for some $o_i$, it is detected for $o$.

2. A second important point about the efficient algorithm is that the order of the observed transitions is lost when the diagnosis of $o_k$ is performed. For instance, if $o_2 = [A, B]$, then the sequences $s'$ of transitions that are generated are those that generate $[A, B]$ or $[B, A]$.

3 Example

![Figure 1: A diagnosable net system.](image)

I now present the example that shows that the efficient approach is not precise. The net system is presented on Fig. 1. The observable transitions are $A$, $B$, $C$, $D$, and $E$; the unobservable transitions are $f$, $u_1$, and $u_2$, where $f$ is the faulty transition.

There are three possible sequences of transitions:

- $u = [f, A, B, D, E, E, \ldots]$,
- $v = [u_1, B, A, D, E, E, \ldots]$, and
- $w = [u_2, A, B, C, E, E, \ldots]$.

Clearly, the system is diagnosable\(^1\), since a fault occurred iff $A$ is observed before $B$, and $D$ is observed.

\(^1\) Diagnosability means that a fault can always be detected.
Assume now that \( u \) takes place and \( o = [A, B, D, E, E, \ldots] \) is observed. I provide the list of explanations \( s' \) associated with each observation \( o_i \) (remember that is order between the observed transitions is dropped when the diagnosis of \( o_i \) is performed):

- \( o_0 = [] \): \( s' = [] \)
- \( o_1 = [A] \): \( s'_1 = [f, A] \), \( s'_2 = [u_2, A] \)
- \( o_2 = [A, B] \): \( s'_1 = [f, A, B] \), \( s'_2 = [u_2, A, B] \), \( s'_3 = [u_1, B, A] \)
- \( o_3 = [A, B, D] \): \( s'_1 = [f, A, B, D] \), \( s'_2 = [u_1, B, A, D] \)
- \( o_4 = [A, B, D, E] \): \( s'_1 = [f, A, B, D, E] \), \( s'_2 = [u_1, B, A, D, E] \)
- etc.

Clearly, the fault is never diagnosed, because there is always another explanation that includes no fault.

**Conclusion:** This demonstrates that the efficiency of Basile et al.’s approach comes at the cost of precision. Despite this negative result, I see this approach as very interesting, in particular in connection with Petri net specific techniques (i.e., the reduction to integer linear programming).

I think an interesting follow-up of this result would be to devise a precision testing that could decide whether a specific net system can be precisely diagnosed with the efficient approach. Even better would be to find sufficient restrictions on the Petri net (similar to the Petri net safety).

**Acknowledgments**

NICTA is funded by the Australian Government as represented by the Department of Broadband, Communications and the Digital Economy and the Australian Research Council through the ICT Centre of Excellence program.

**References**

[BCD09] Fr. Basile, P. Chiacchio, and Gi. De Tommasi. An efficient approach for online diagnosis of discrete event systems. *IEEE Transactions on Automatic Control (TAC)*, 54(4):748–759, 2009.

[BCD12] Fr. Basile, P. Chiacchio, and Gi. De Tommasi. Erratum to ”an efficient approach for online diagnosis of discrete event systems”, 2012. http://wpage.unina.it/detommas/publications/Erratum.pdf.

[CL99] Chr. Cassandras and St. Lafortune. *Introduction to discrete event systems*. Kluwer Academic Publishers, 1999.

[DFMU09] M. Dotoli, M. P. Fanti, A. M. Mangini, and W. Ukovich. On-line detection in discrete event systems by Petri nets and integer linear programming. *Automatica (Automatica)*, 45:2665–2672, 2009.