Measurements of the exchange stiffness of YIG films using broadband ferromagnetic resonance techniques

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Abstract
Measurements of the exchange stiffness $D$ and the exchange constant $A$ of Yttrium Iron Garnet (YIG) films are presented. YIG films with thicknesses from 0.9 to 2.6 $\mu$m were investigated with a microwave setup in a wide frequency range from 5 to 40 GHz. The measurements were performed with the external static magnetic field applied in-plane and out-of-plane. The method of Schreiber and Frait (1996 Phys. Rev. B 54 6473), based on the analysis of the perpendicular standing spin wave mode frequency dependence on the applied out-of-plane magnetic field, was used to obtain the exchange stiffness $D$. This method was modified to avoid the influence of internal magnetic fields during the determination of the exchange stiffness. Furthermore, the method was also adapted for in-plane measurements. The results obtained using all methods are compared and values of $D$ between $(5.18 \pm 0.01) \cdot 10^{-17}$ T·m$^2$ and $(5.40 \pm 0.02) \cdot 10^{-17}$ T·m$^2$ were obtained for different thicknesses. From this, the exchange constant was calculated to be $A = (3.7 \pm 0.4)$ pJ·m$^{-1}$.

Keywords: anisotropy, perpendicular standing spin waves, exchange stiffness, exchange constant

(Some figures may appear in colour only in the online journal)

1. Introduction
In order to employ the degree of freedom of the spin in future information technology, materials with low Gilbert damping and long spin-wave propagation distances are needed. Yttrium Iron Garnet (YIG) is a material which fulfills the aforementioned requirements. New technologies employing YIG are being developed and new physical phenomena are being investigated. Logic operations with spin waves in YIG waveguides [2–4], data-buffering elements [5] and magnon transistors [6] are only a few examples of the latest technological progress. In particular, YIG films in the nanometer thickness range [7–12] are of large importance since they allow for the realization of nano- and microstructures [6, 8, 13, 14] and an enhancement of spin-transfer-torque related effects [12, 15]. In this context the precise determination of the material parameters of YIG are of crucial importance for its applications.

In a magnetic system, the exchange interaction contributes strongly to the energy of the system. From a classical point of view, this interaction is responsible for the parallel alignment of adjacent spins and, thus, it strongly influences the spin-wave characteristics. The strength of the exchange interaction is given by the exchange stiffness $D$. However, the existing approaches for its measurement are often influenced by internal magnetic fields which are consequently dependent on crystal anisotropies and the saturation magnetization. As a result, there is the...
need for methods to be utilized in the exact determination of the exchange stiffness without the uncertainties added by the aforementioned parameters. Here, such a method is presented and the results are compared to those obtained by commonly used data evaluation methods. Firstly, the classical approach of Schreiber and Frait [1] is used in the determination of the exchange stiffness when the external static magnetic field is applied out-of-plane. Secondly, the method is modified to avoid any influence of the anisotropy fields and the saturation magnetization in order to achieve precise values for D. Thirdly, the method of Schreiber and Frait is adapted and used for in-plane measurements. All values of D obtained by using the different methods are compared and, then, values of the exchange constant A are calculated for our YIG samples.

2. Theory

The precessional motion of the magnetization in an effective magnetic field is described by the Landau–Lifshitz and Gilbert equation [16]. The effective magnetic field includes the precessional motion of the magnetization in an effective field etc.). For this reason, the growth rate \( \nu \) was varied to obtain YIG films (0.005 \( \leq \nu \leq 0.015 \)) with reduced lattice misfits. Important material parameters of the samples are shown in Table 1. It can be seen that the lattice misfit increases with a decrease in the growth rate. The film thickness \( d \) was measured by a prism coupler technique [20] and the YIG/GGG lattice misfit values were determined by x-ray diffraction. The samples were then cut in sizes of 3 \( \times \) 3 mm\(^2\) for microwave studies.

For the measurements of the magnetization dynamics we use a broadband ferromagnetic resonance system based on a coplanar waveguide structure, for details see reference [21]. A microwave field with a power of 10 dBm is applied in a wide frequency range from 5 GHz to 40 GHz with a rotatable coplanar waveguide (CPW) such that the angle between the static field and the sample surface can be varied from 0° to 360°. For the in-plane measurements the external magnetic field is applied along the edges of the sample which is positioned in the center of the CPW.

4. Determination of exchange stiffness

4.1. Method of Schreiber and Frait

A typical dependence of the lock-in signal on the applied static field from the out-of-plane measurements is shown in Figure 1(a). The ferromagnetic resonance (\( n = 0 \)) can be found at the highest field values, whereas the thickness modes are located at lower field values. Resonances with even mode numbers are observed with a strong amplitude, whereas resonances with odd mode numbers have a significantly smaller amplitude. This effect can be understood based on the assumption of ‘perfect pinning’ since, in this case, only the even modes absorb energy from the homogeneous antenna field [22]. The experimental observation of odd PSSWs can be caused by small microwave inhomogeneities across the thickness of the film.

| Sample | Thickness \( d \) (\( \mu m \)) | Growth rate \( \nu \) (\( \mu m \ min^{-1} \)) | Lattice misfit \( \Delta a/a_{YIG} \) (10\(^{-4}\)) |
|--------|-------------------------------|--------------------------------|---------------------------------|
| E1     | 2.59 \( \pm \) 0.01           | 0.52                           | +5.33 \( \pm \) 0.07            |
| E2     | 1.59 \( \pm \) 0.02           | 0.32                           | +7.68 \( \pm \) 0.02            |
| E3     | 0.903 \( \pm \) 0.003         | 0.18                           | +8.72 \( \pm \) 0.03            |

(\( a_{YIG} = 12.376 \) Å) [18] the films exhibit a room temperature lattice misfit \( \Delta a/a = a_{GGG} - a_{YIG} \) which results in strained epitaxial films. This strain is one of the main factors defining the uniaxial anisotropy fields \( H_{u1} \) and \( H_{u2} \). For the case of LPE growth of garnet films the incorporation of lead ions from the PbO solvent plays an important role in adjusting the misfit in the films [19]. Therefore, the misfit essentially depends on the growth parameters (growth temperature, growth rate, etc.). For this reason, the growth rate \( \nu \) was varied to obtain YIG films (0.005 \( \leq \nu \leq 0.015 \)) with reduced lattice misfits. Important material parameters of the samples are shown in Table 1. It can be seen that the lattice misfit increases with a decrease in the growth rate. The film thickness \( d \) was measured by a prism coupler technique [20] and the YIG/GGG lattice misfit values were determined by x-ray diffraction. The samples were then cut in sizes of 3 \( \times \) 3 mm\(^2\) for microwave studies.

A microwave field with a power of 10 dBm is applied in a wide frequency range from 5 GHz to 40 GHz with a rotatable coplanar waveguide (CPW) such that the angle between the field and the sample surface can be varied from 0° to 360°. For the in-plane measurements the external magnetic field is applied along the edges of the sample which is positioned in the center of the CPW.

4 Pb contents in formular units were estimated from the lattice misfit by comparison with literature data [25].

Table 1. Parameters of the studied YIG samples. The average growth rate \( \nu \) was calculated from the thickness \( d \) and the deposition time which was 5 min for all YIG films.

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4 Pb contents in formular units were estimated from the lattice misfit by comparison with literature data [25].
In the classical approach of Schreiber and Frait the exchange stiffness is determined in the out-of-plane configuration using equation (2). Here, the anisotropy fields and the saturation magnetization are incorporated into the effective magnetization $M_{\text{eff},\perp}$. Then, the resonance field for a certain frequency is defined by:

$$H_n = M_{\text{eff},\perp} + \frac{\omega_1}{\gamma} - D \frac{\pi^2}{d^2} n^2,$$

where

$$M_{\text{eff},\perp} = M_S - H_{\text{il}} + \frac{4}{3} H_e - H_{\text{e2}}$$

If the resonance field is plotted versus the square of the mode number $n^2$, the exchange stiffness can be extracted from the slope of a linear fit function, where the $y$-intercept provides information about the effective magnetization. The presence of resonances with odd mode numbers introduces some ambiguity regarding the identification of the modes. However, the mode intensity together with the $n^2$-dependence of the resonance field shift enables a consistent identification, as can be seen in figure 1(b). Here, the resonance fields of sample E2 are shown for different frequencies. The slopes of the linear functions are the same for all measurements and the $y$-intercepts are different due to the use of different excitation frequencies. In the performed measurements no deviations from the linear functions were detected which supports our perfect pinning assumption. In the case of mixed pinning, deviations from the linear relationship would occur for small $n$ due to the influence of the surface anisotropy [1, 23]. The exchange stiffness values for all samples presented in the left column of table 2 were extracted from the slopes of the graphs shown in figure 1. The average value for samples E1–E3 is $D = (5.32 \pm 0.07) \times 10^{-17}$ T·m².

Table 2. Results for the YIG samples with different thicknesses. The shown errors are the statistical fitting errors.

| Sample | Out-of-plane<sup>a</sup> | Out-of-plane<sup>b</sup> | In-plane |
|--------|--------------------------|--------------------------|----------|
| E1     | 5.33 ± 0.09              | 5.18 ± 0.01              | 5.29 ± 0.04 |
| E2     | 5.32 ± 0.09              | 5.34 ± 0.02              | 5.30 ± 0.02 |
| E3     | 5.29 ± 0.05              | 5.31 ± 0.02              | 5.40 ± 0.02 |

<sup>a</sup> The values in the column with the out-of-plane measurement are obtained using the original method of Schreiber and Frait.

<sup>b</sup> The values in the column with the out-of-plane measurements are obtained based on the difference between the resonance field of higher modes and the ferromagnetic resonance field.

This effect is clearly visible in the size of the error bars if compared with the modified method presented in the next section.

4.2. Modified method of Schreiber and Frait

As shown before, the method of Schreiber and Frait requires several parameters to be taken into account in order to obtain the exchange stiffness. Here, a method which is independent of assumptions regarding the anisotropy fields and the saturation magnetization is presented. For this, the ferromagnetic resonance field $H_{\text{res}}^{(0)}(0)$ is subtracted from the resonance fields of the higher modes $H_{\text{res}}^{(n)}(n \neq 0)$ in order to determine the exchange field $H_{\text{ex}}$ of the thickness modes. Since the resonance field of the ferromagnetic resonance contains all information about the anisotropy fields and $M_S$, as can be seen in equation (4), the exchange field only depends on $D$:

$$H_{\text{ex}} = \mu_0 H_{\text{res}}^{(n)} - \mu_0 H_{\text{res}}^{(0)} = D \frac{\pi^2}{d^2} n^2.$$
microwave frequency. Furthermore, all data points lie on a linear function with $H_{\text{ex}}(0) = 0$. These data can now be analyzed using a simple linear fit with no offset, i.e. only one fitting parameter is used. Thus, any mutual influence of parameters is avoided which is the reason for the significantly reduced statistical error. The results are shown in the center column of table 2. All values are in the same range as those obtained with Schreiber and Frait’s method. However, it is visible that the exchange stiffness of sample E1 is significantly smaller than that of the others. In comparison to the method of Schreiber and Frait, the statistical error is decreased by a factor of up to 9 because the influence of the effective magnetization during the data evaluation is avoided. This allows for the determination of the exchange stiffness with a high accuracy.

4.3. Method for in-plane measurements

Classically, the method of Schreiber and Frait is used for the determination of the exchange stiffness in the out-of-plane configuration. Here, the method is adapted for use in the in-plane configuration. A sample spectrum of the in-plane measurements is shown in figure 3(a). It is slightly modified in comparison to the out-of-plane spectrum. The resonances are shifted to smaller field values due to decreased demagnetizing effects. In the in-plane case, the former methods cannot be used for data evaluation since $\omega_\parallel$ is not linearly dependent on the static field in equation (1). However, the former procedure can be applied to the pure exchange field of the PSSWs. For this, we propose the following steps.

Firstly, equation (1) must be rewritten in a way which is convenient for the fitting process:

$$\omega_\parallel = \gamma \mu_0 \sqrt{(H_0 + H_{\text{ex}})(H_0 + H_{\text{ex}} + M_{\text{eff},\parallel})}.$$  \hspace{1cm} (6)

Here, the different field contributions, including the saturation magnetization, are summarized in $M_{\text{eff},\parallel} = M_S - H_{\text{ex}} - H_c$. Now the effective magnetization is obtained by fitting the $n = 0$-mode (FMR mode).

Secondly, the same equation is used to obtain the exchange field $H_{\text{ex}}$ of the higher PSSW modes ($n \neq 0$). For this, the obtained value of $M_{\text{eff},\parallel}$ is used as a constant during the data evaluation process. The resulting exchange fields $H_{\text{ex}}$ of the PSSW modes depend on the square of a mode number which is unknown at first. As in the case of the out-of-plane measurements there is an ambiguity regarding the identification of the mode number $n$ for each observed mode. The identification procedure of the mode numbers is shown next.

Thirdly, the exchange fields of the modes are varied manually over the presumed mode numbers (see figure 3(b)). As a first indicator, the peak height of the resonance can be used to determine whether a mode is even or not. To prove the mode numbering a graphical feedback can be obtained by plotting $\mu_0 H_{\text{ex}}(n)$ over $n^2$, where an incorrect mode numbering would be directly visible. If the exchange field follows a linear function, as shown in figure 3(b), the exchange stiffness is given by the slope of this function.

For all three samples the values of the exchange stiffnesses are shown in the right column of table 2. They are in the same range as with the other methods which supports the validity of this method. However, the data evaluation is much more complicated and the systematic uncertainties are larger than for the other methods. Note that the error margins in table 2 only include statistical errors but not possible systematic errors.

5. Determination of the exchange constant

For the determination of the exchange constant $A = D M_S^2$ of our YIG samples, the saturation magnetization $M_S$ must be known. Vibrating sample magnetometry (VSM) was used to determine $M_S$ and values of 141 kA m$^{-1}$, 136 kA m$^{-1}$ and 137 kA m$^{-1}$ for samples E1, E2 and E3, respectively, are found with an accuracy of 10%. The large error margins of the values for the saturation magnetization are in part caused by the uncertainty of the volume determination of the YIG films. The exchange constant $A$ of the YIG films is determined to be $(3.7 \pm 0.4) \text{pJ m}^{-1}$ which is the average value of all measurements. The result is in good agreement with the values obtained by other groups [24].

6. Conclusion

Different methods were developed and compared in order to estimate the exchange stiffness $D$ from the microwave absorption spectra. Firstly, the method of Schreiber and Frait [1] was used to estimate $D$ for the out-of-plane magnetized samples. The method was shown to be influenced by the simultaneous fit of the exchange stiffness and the effective magnetization, which includes the saturation magnetization and anisotropy fields. Therefore, the exchange stiffnesses $D$ were accompanied with considerable errors. This problem is solved with the proposed method by avoiding additional fit parameters including anisotropy fields by extraction of the pure exchange field contributions (see equation (5)). With the use of this method it was demonstrated that more accurate results for $D$ can be obtained. The modified method is recommended for
Figure 3. (a) Example spectrum for the YIG sample E2 in the in-plane configuration at 20 GHz. The first two resonances overlap in such a way that a multiple resonance fit had to be used in order to extract position of both resonances. The spectrum on the left hand side of the dashed lines is magnified by a factor of 130. (b) The exchange fields of sample E3 follow a linear function of the square of the mode number. The slope of the function is proportional to the exchange constant. Each point in the graph stands for the exchange field of one resonance. Even modes are marked with vertical dashed lines.

the determination of the exchange stiffness in general. Finally, the former method was adapted for the in-plane configuration. The in-plane estimates of the exchange stiffness $D$ were found to agree well with those obtained in the out-of-plane configuration. However, because of the nonlinear dependence of the PSSW mode frequency versus $n^2$, an evaluation of the in-plane measurement data was more complicated and resulted in a similar spread of the exchange stiffness $D$ as in the original method of Schreiber and Frait [1].

Finally, an exchange constant of $A = (3.7 \pm 0.4) \text{ pJ} \cdot \text{m}^{-1}$ was extracted for thin YIG films.

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