Dynamic quality prediction and control in rotary sponge iron kilns

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Abstract: The quality of sponge iron produced in the coal-fired rotary kilns at TATA Steel Long Products Limited (TSLPL) is permitted to vary in a small window of 80-83% Fe. In order to adhere to this window, the expected quality in the next hour is predicted, and appropriate actions are taken if changes or rates of change that take place are not in the right direction. Airflow rate through the primary air blower is controlled constantly. For quality prediction and control purposes, the One Step Ahead Adaptive Control and Prediction Algorithm (OSAA) and Grey (1,1) prediction algorithm have been tailored and tuned. The working principle of OSAA and Grey(1,1) are described, and their accuracy is compared. The results show that OSAA performs better than the Grey(1,1). The OSAA is therefore used to predict the changes required in volume of air blown through the primary air blower and also the amount of injection coal. An expert control system has been developed based on allowable windows of control variables.

1. INTRODUCTION

Rotary kiln (500 tons) is a long (80 m), and a slightly inclined rotating horizontal shaft in which iron ore and coal are fed from one end (feed end), and coal and primary air are injected from the other (discharge) end. Secondary Air Blowers (SAB) are installed along the length of kiln to supply additional air required for heating and combusting coal to produce a mixture of CO and CO₂. The reduction of iron oxides to iron is carried out primarily by CO gas. Some direct reduction by carbon also takes place. For a defined feed rate of ore, the main control parameters are: coal injection rate, rotation speed (RPM) of kiln, adjustment of primary air and secondary air flow rates and coal injection pressure; coal injection pressure (depending upon coal quality) is however allowed to vary only in a small range and is not frequently altered. Maintaining the quality in a specific window is crucial for the plant. For example, if the quality goes above the window (80-83% Fe), then more coal and/or air than needed has been used (higher cost) and if it goes below the window, then the product has to be rejected. The control parameters have to be changed in the proper direction and proper magnitude. In the present work, OSAA Model has been adapted and tuned to plant practice.

The feasibility of OSAA has been reported in the literature for several other processes as well. For example, OSAA has been used to control the gas turbine plant for a wide range of electrical loads [1]. The reliability of OSAA has been checked in the wind turbine system [2]. OSAA has been used for control of active rectifier in wild frequency applications [3]. All these systems are, however, fast...
response systems.

The use of autoregression in OSAA takes care of the nonlinearity in data, and the dynamic parameter update takes care of the dynamic behaviour of the plant. The same can be done by the popular Grey(1,1) model as well. The principle of both OSAA and Grey (1,1) models are described in sections 2 and 3. The performance of the OSAA and Grey (1,1) model for actual plant practice at TATA Steel Long Products is compared in Section 4.

The special aspect of the present work is that in rotary kilns, the response time of some variables is much longer (in hours) while some respond almost immediately (within a few seconds). For example, the kiln pressure and pressure differential (difference of inlet and outlet pressures) change almost immediately if moisture in coal changes, but the effect of kiln pressure on iron ore reduction rate (hence on the quality window) may precipitate after 1-6 hours of time. Hence it is important to consider the sponge iron rotary kiln system as a combination of fast and slow response but interacting variables. By analysis of plant data, it is necessary to find the optimum number of variables and data points (time scale of data) for good control of the system. A large amount of plant data has been analysed for this purpose in the present work before finalizing the optimal control strategy. It has been shown earlier that the sponge iron rotary kiln is a chaotic, dynamical (nonlinear, time-varying) system [4-8]. The present work is the first application of both OSAA and Grey (1,1) models to a chaotic dynamical system. The chaotic dynamical system can be controlled by breaking into small segments of data points (on time scale), and this works until a sharp change in trajectory takes place when the system moves away from chaotic attractor(s). Once the system trajectory changes sharply to a new direction, then new estimates are generated by giving proper weight to certain points through the Grey (1,1) model. In this way, system is stabilized successfully in most instances. Only in rare cases, however, the system does drift continuously and unpredictably for some time till it is brought back to control. This is unavoidable in a chaotic system.

2. ONE STEP AHEAD ADAPTIVE CONTROL AND PREDICTION ALGORITHM

This algorithm has two parts. The first part of the algorithm tracks the dynamical behaviour of the system by updating the chosen parameter of the system through the Recursive Least Square Algorithm [1]. Recursive Least Square (RLS) Algorithm estimates the system parameter at each time step by using input and output information [1]. The second part of the algorithm generates control information (required change in the control variable) for the system to get the desired output in the next time step. Thus, for the generation of control information, the algorithm requires the updated parameters of the system, the current output of the system, and the desired output.

In a nonlinear time-varying system, the output of the system at the current time depends on the system states and input to the system at the previous time [1]. In this work, as also described in [1], a linear model with time-varying parameters is used to approximate the system. This model is linear for each time step, but as a whole, it takes nonlinear time-varying behaviour in the form of a collection of linear behaviour segments. For this work, the well-known first-order ARMA (Auto-Regressive Moving Average model [1,9-10]) with continuous updating (one step ahead) has been considered. If \( A_{i-1} \) and \( B_{i-1} \) are parameter vectors of the model at \((i-1)t\) time step, \( X_i \) is the output vector at \( it \) timestep, \( U_{i-1} \) is the input vector at \((i-1)t\) time step then,

\[
X_i = A_{i-1}X_{i-1} + B_{i-1}U_{i-1}
\]

Equation(1) can also be written as follows.

\[
X_i = \theta_{i-1} \phi_{i-1}
\]

where

\[
\theta_{i-1} = [A_{i-1}B_{i-1}]
\]
Here $\theta_{t-1}$ and $\phi_{t-1}$ are known as the parameter matrix and information matrix, respectively.

Now $y_i$ is another vector such that

$$y_i = X_{i+1}$$  \hspace{1cm} (3)

From equation (1),

$$y_{t-1} = A_{t-1}X_{t-1} + B_{t-1}U_{t-1}$$ \hspace{1cm} (4)

Recursive Least Square Algorithm can be used to calculate $\theta_{t-1}$ when $y_{t-1} = X_i$ and $\phi_{t-1}$ are given.

RLS works as follows.

$$\theta_t = \theta_{t-1} + K_t (y_t - \phi_t^T \theta_{t-1})$$ \hspace{1cm} (5)

where

$$K_t = \frac{P_{t-1} \phi_t}{1 + \phi_t^T P_{t-1} \phi_t}$$ \hspace{1cm} (6)

$$P_{t-1} = P_{t-2} - \frac{P_{t-2} \phi_{t-1} \phi_{t-1}^T P_{t-2}}{1 + \phi_{t-1}^T P_{t-2} \phi_{t-1}}$$ \hspace{1cm} (7)

The initial value of $\theta$ and $P$ are required to use the RLS Algorithm. The following initial values are assumed:

$$\theta_0 = [0 \ 0]$$

$$P_0 = k I_{n \times n}$$

where $k$ is the sensitivity of the system; in present work, being a low sensitivity system, $k$ is equal to $10^6$[1]. $I_{n \times n}$ is the identity matrix where $n$ is the dimension of the information matrix. If data up to $ith$ time step is given then the quality at $(i + 1)th$ time step (denoted by $X_{i+1}$) can be predicted. Also, the required input for getting the desired quality at $(i + 1)th$ time step can be predicted. This required input should be given to the system at $ith$ time step. From Equation (1) the values of coefficient $A$ and $B$ at $(i - 1)th$ time step can be calculated if data up to $ith$ time step is given. Assuming the change of $A$ and $B$ to be small in one time step, $A_i$ and $B_i$ can be written as follows

$$A_i \cong A_{i-1}$$

$$B_i \cong B_{i-1}$$

Since the values of coefficients at $ith$ time step are found, the output(quality) at $(i + 1)th$ time step can be predicted from the following expression,

$$X_{i+1} = A_i X_i + B_i U_i$$ \hspace{1cm} (8)

This is the method of prediction of quality at one step ahead.

Control of quality involves giving the desired input one time-step ahead. The desired quality at $(i + 1)th$ time step is denoted by $X_{i+1}^*$. On replacing $X_{i+1}$ in equation(8) by $X_{i+1}^*$, the required $U_i$ can be calculated.

$$X_{i+1}^* = A_i X_i + B_i U_i$$ \hspace{1cm} (9)

$$U_i = B_i^{-1} (X_{i+1}^* - A_i X_i)$$ \hspace{1cm} (10)

In the case of rotary kiln, if PAB, RPM, and quality data up to $ith$ time step are given, then quality at $(i + 1)th$ time step can be calculated using Equation (8), and then required input for desired quality one step ahead can be calculated using Equation (10). Step by step derivation of RLS Algorithm for
Parameter Estimation is explained in the Appendix[12,13].

3. GREY (1,1) MODEL

Grey (1,1) model [11] is also used for predicting the next value of a non-linear time series but having a small number of data points.

3.1. Algorithm for Implementation of Grey (1,1)

The steps are as follows:

Step 1: Suppose \( X^0 = \{x^0(1), x^0(2), x^0(3), ..., x^0(n)\} \) is a sequence of data points. \( X^1 = \{x^1(1), x^1(2), x^1(3), ..., x^1(n)\} \) is generated by accumulation generation operation on \( X \) by using equation (11).

\[
x^1(i) = \sum_{i=0}^{n} x^0(i)
\]

Step 2: The grey differential equation is given as follows.

\[
\frac{dx^1(k)}{dk} + ax^1(k) = b
\]

The solution of the differential equation can be obtained by the Least Square Method, which is given as follows.

\[
[a, b]^{-1} = (B^TB)^{-1}B^TY
\]

where

\[
Y = [x^0(2), x^0(3), x^0(4), ..., x^0(n)]^T
\]

\[
B = \begin{bmatrix}
\frac{-x^1(1) + x^1(2)}{2} & 1 \\
\frac{-x^1(2) + x^1(3)}{2} & 1 \\
\vdots & \vdots \\
\frac{-x^1(n-1) + x^1(n)}{2} & 1
\end{bmatrix}
\]

The solution of the differential equation(12) is as follows.

\[
x^1_p(k) = \left(x^0(1) - \frac{b}{a}\right)e^{a(k-1)} + \frac{b}{a}
\]

where \( x^1_p \) is predicted accumulated value.

Step 3: The actual predicted value is obtained by Inverse accumulation generation operation.

\[
x^0_p(k) = x^1_p(k) - x^1_p(k-1)
\]

where \( k = 2, 3, ..., n \) and \( x^0_p(k) \) is the kth predicted value.

\[
X_p = \{x^0_p(1), x^0_p(2), x^0_p(3), ..., x^0_p(k)\}
\]

Future value of time series can be predicted by taking \( k > n \).

4. DESCRIPTION OF SAMPLE DATA

The data pertaining to quality control at TSLPL includes the volume of air inflow through primary air blower (PAB), the volume of air inflow through secondary air blowers (SAB), coal and iron ore feed rates, the rotation speed of rotary kiln (RPM), temperature at the various positions of rotary kiln (eleven thermocouples placed along the entire length), and quality of sponge iron produced (percentage of iron content in reduced iron ore); a sample of data of quality, RPM and PAB is given in Table 1. In this work, the input parameters PAB and RPM (as a function of time) are found as the useful parameters for prediction of quality from regression and chaos analysis. It can be seen from
Table 1 that PAB data is collected every seven seconds, RPM data every five minutes, and quality data every hour. All information, however, needs to be at the same time scale in order to use them in a model. The average of twelve RPM data is taken as the RPM data in an hour. The average of 514 data of PAB is taken as the PAB data in an hour. Quality data is collected every hour. These data of PAB, RPM, and quality are used in One Step Ahead Prediction and Control Method. Grey(1,1) model, being a single parameter model, only uses the quality data to predict quality in the next hours.

Table 1. Sample data of quality, PAB, and RPM

| Time   | Quality | Time   | RPM   | Time   | PAB   |
|--------|---------|--------|-------|--------|-------|
| 1:00:00| 84.5    | 01:00:00| 371.2 | 01:00:00| 39.5  |
| 2:00:00| 83.6    | 01:05:00| 368.8 | 01:00:07| 39.225|
| 3:00:00| 83      | 01:10:00| 377.8 | 01:00:14| 39.45 |
| 4:00:00| 82.4    | 01:15:00| 373.6 | 01:00:21| 39.425|
| 5:00:00| 82.2    | 01:20:00| 361.8 | 01:00:28| 39.5  |
| 6:00:00| 82.6    | 01:25:00| 374.4 | 01:00:35| 39.5  |
| 7:00:00| 83.2    | 01:30:00| 372.8 | 01:00:42| 39.4  |
| 8:00:00| 81      | 01:35:00| 365.6 | 01:00:49| 39.5  |

5. RESULTS AND DISCUSSION OF APPLICATION OF MODELS

The result section consists of two subsections. The first one describes the error of quality prediction by different methods and selection of the optimum length (time scale) of the dataset for training. The second subsection describes the prediction of the magnitude of control input for the desired output of quality and selection of the optimum length of the dataset for training.

Table 2. Error in quality prediction by different methods

| Test | Grey  | OSAA(R) | OSAA(P) | OSAA(R and P) |
|------|-------|---------|---------|---------------|
| 1    | 0.65% | 0.68%   | 0.72%   | 0.79%         |
| 2    | 0.85% | 1.30%   | 0.72%   | 3.29%         |
| 3    | 0.18% | 0.18%   | 0.28%   | 0.19%         |
| 4    | 0.08% | 0.73%   | 0.98%   | 0.17%         |
| 5    | 2.43% | 0.61%   | 1.40%   | 1.38%         |
| 6    | 1.83% | 0.08%   | 0.57%   | 2.83%         |
| 7    | 1.56% | 1.50%   | 0.99%   | 3.10%         |
|      | Average| 1.08%   | 0.73%   | 0.81%         | 1.68%   |
|      | Standard Deviation| 0.8816 | 0.5261 | 0.3569         | 1.3731524 |
|      | R-squared value | 0.0204 | 0.4702 | 0.2532         | 0.227   |

5.1. Comparison of Prediction Error in Quality

Table 2 shows the percentage of prediction error for the different methods. In Table 2, the number of data points taken for training is 5 for the first three tests and is ten for test number 4 to 7. Different methods in Table 2 are described as follows. Grey(1,1) model takes only the quality data and predicts the quality one step ahead. OSAA(R) method means the One Step Ahead Adaptive Prediction Method using RPM as input and quality of sponge iron as output. OSAA(P) method means the One Step Ahead Adaptive Prediction Method using PAB as input and quality of sponge iron as output. OSAA(R and P) means the One Step Ahead Adaptive Method using both RPM and PAB as input and quality of sponge iron as output. The data for training is converted into an hour time scale before training. It can be seen from Table 2 that OSAA(P) and OSAA(R) perform better than other methods.
as they show the lowest mean error and also the least standard deviation of error. Comparing the R-squared value in Table 2, it is clear that OSAA(R) is the best model for prediction of quality. Therefore, OSAA(R) is selected to predict the quality of sponge iron by taking 5 data points for training. Figure 1 to Figure 4 shows the plot of predicted quality versus actual quality for Grey (1,1), OSAA(R), OSAA(P), and OSAA(R and P) model, respectively. Figure 5 shows the percentage of prediction error for the different methods.

5.2. Error in Input Parameter Prediction

Prediction of PAB is made to get the desired quality at one step ahead by considering the OSAA(P) Control model. OSAA(P) Control technique is based on SISO (single input single output) system with PAB as input and quality as the output. Data up to a specific time step are taken to train the OSAA(P) control model. Then the quality value of the next time step is considered as the desired quality, and the required PAB value (control value) is predicted. The difference in predicted PAB and actual PAB is the error in PAB prediction. Table 3 shows details of PAB prediction where desired quality is taken as 83% Fe and actual PAB value corresponding to the desired quality was 64.43. Tests of determining error for PAB prediction has been done for different lengths of the training dataset. This experiment was done to see the performance of the OSAA(P) Control Method and also to determine the optimum length of the dataset for training. From figure 6, it is clear that minimum error in prediction is obtained when thirteen data points are taken to train the model.

For executing the control part of work on the shop floor, the PAB and coal injection rate are considered as usable inputs because there is an operational restriction on the frequent and significant variation of RPM (rotations per minute) in the plant. The change in RPM is directly related to the backflow of charge from the feed side. The plant operator first sees the present quality of sponge iron and then decides to change the specific plant parameter determined by OSAA. As mentioned earlier, kiln, being a chaotic dynamical system, this approach leads to an unpredictable change in sponge iron quality (sometimes only), even though proper action advised by the model to increase or decrease quality has been taken. The physical reason for some prediction failure cases can be identified. For example, in certain cases, the sudden dislodging of accretions inside the kiln leads to a sudden change in kiln pressure and in charge movement. To efficiently handle this situation, the operator compares the predicted quality trend by OSAA(R) and with the next quality step predicted by Grey(1,1) model. In case of step change (or difference between the two), the control actions may be held back for a few steps until the kiln begins to stabilize. On the whole, with the application of the present control model, the efficacy of adherence to the quality window has increased significantly as well as the number and duration of rejection periods have gone down. Kiln's availability has gone up because of enhanced campaign life.

| No of data points taken | Estimated PAB | Error   |
|-------------------------|---------------|---------|
| 5                       | 63.9658       | 0.5358  |
| 6                       | 63.9675       | 0.5375  |
| 7                       | 63.9249       | 0.4949  |
| 8                       | 64.8253       | 1.3953  |
| 9                       | 64.7781       | 1.3481  |
| 10                      | 64.6547       | 1.2247  |
| 11                      | 64.0535       | 0.6235  |
| 12                      | 64.0708       | 0.6408  |
| 13                      | 64.0041       | 0.5641  |
| 14                      | 65.5339       | 2.1039  |
| 15                      | 67.0777       | 3.6477  |
Figure 1. Plot of predicted quality by Grey (1,1) versus actual quality

Figure 2. Plot of predicted quality by OSAA(R) versus actual quality

Figure 3. Plot of predicted quality by OSAA(P) versus actual quality

Figure 4. Plot of predicted quality by OSAA(R and P) versus actual quality

Figure 5. Comparison of the percentage prediction error of quality by the different model in tests 1 to 7.

Figure 6. Estimation of error for prediction of required input PAB for desired quality for different length of data taken.

6. CONCLUSION AND FURTHER WORK

In this work, the fundamentals and the method of application of One Step Ahead Adaptive Control and Prediction Method (with full derivation in Appendix) and Grey(1,1) model have been explained. The reliability of the application of OSAA in the sponge iron plant has been demonstrated and proven in real-time application on shop floor for control and adherence of quality in the desired window. The drives for automatic airflow control of the SABs are required for further improvement (narrowing
down of quality window and further reduction of coal consumption), and once this task is completed, then SABs will also be included in the control action loop and decision logic table. At present, the air in SABs are changed only as per the accretion control model, manually, once in 8 hours [7]. The conflicts in control action (i.e., conflicts between quality control model (based on OSAA) and accretion control model described elsewhere [7]) are resolved through a two-level logistic decision support system; higher priority is given to quality control. This is a unique hybrid control system implemented for the first time for sponge iron rotary kilns.

APPENDIX

Step by Step Derivation of Recursive Least Square Algorithm for Parameter Estimation:
Consider a MISO (Multiple Input Single Output) System. Let \( \phi_1(t), \phi_2(t), ..., \phi_n(t) \) are \( n \) input to a system at time \( t \). The corresponding output of the system at time \( t \) is \( y(t) \). Then the linear regression model of the system can be written as follows.

\[
y(t) = \phi^T(t). \theta(t) + v(t)
\]

(15)

where

\[
\phi(t) = [\phi_1(t) \ \phi_2(t) \ ... \ \phi_n(t)]^T
\]

\[
\theta(t) = [\theta_1(t) \ \theta_2(t) \ ... \ \theta_n(t)]^T
\]

Here, \( \phi(t) \) is the input vector, \( \theta(t) \) is the parameter vector, and \( v(t) \) is the error term.

Let say input and output data from \( t = 0 \) to \( t = N \) are given. A linear model is to be fitted at each time step by changing the parameters such that the sum of the square of error is least. The sum of the square of error is as follows.

\[
V_N(\theta(t)) = \sum_{k=1}^{t} [y(k) - \phi^T(k). \theta(t)]^2
\]

(16)

Equation(16) can be written as

\[
V_N(\theta(t)) = [Y - \Phi^T \theta(t)]^T[Y - \Phi^T \theta(t)]
\]

(17)

where

\[
Y = [y(1) \ y(2) \ ... \ y(N)]^T
\]

\[
\Phi^T = \begin{bmatrix}
\phi_1(1) & \phi_2(1) & \cdots & \phi_n(1) \\
\phi_1(2) & \phi_2(2) & \cdots & \phi_n(2) \\
\vdots & \vdots & \ddots & \vdots \\
\phi_1(N) & \phi_2(N) & \cdots & \phi_n(N)
\end{bmatrix}
\]

\[
\theta(t) = [\theta_1(t) \ \theta_2(t) \ ... \ \theta_n(t)]^T
\]

(18)

(19)

(20)

RHS of equation(17) can be written as follows

\[
[Y - \Phi^T \theta(t)]^T[Y - \Phi^T \theta(t)] = [Y^T - \theta^T(t) \Phi^T] [Y - \Phi^T \theta(t)]
\]

\[
= Y^TY - Y^T \Phi^T \theta(t) - \theta^T(t) \Phi^T Y + \theta^T(t) \Phi^T \Phi^T \theta(t)
\]

\[
= Y^TY - 2 \theta^T(t) \Phi Y + \theta^T(t) \Phi^T \Phi^T \theta(t)
\]

\[
= \left( [\theta^T(t) \Phi Y]^T = Y^T \Phi^T \theta(t) \right)
\]

\[
= 2 \left( \frac{1}{2} \theta^T(t) \Phi \Phi^T \theta(t) - \theta^T(t) \Phi Y \right) + Y^TY
\]

Now equation(17) can be written as follows

\[
V_N(\theta(t)) = 2 \left( \frac{1}{2} \theta^T(t) A \theta(t) - \theta^T(t) b \right) + Y^TY
\]

Taking \( A = \Phi \Phi^T \) and \( b = \Phi Y \)
Now differentiating $V_N(\theta(t))$ and equating to zero, following expression is obtained

$$\frac{\partial V_N(\theta(t))}{\partial \theta(t)} = 0$$

$$\Rightarrow 2 \frac{\partial \left(\frac{1}{2} \theta^T(t) A \theta(t) - \theta^T(t) b\right)}{\partial \theta(t)} + \frac{\partial (Y^T Y)}{\partial \theta(t)} = 0$$

$(We know \frac{\partial (Y^T Y)}{\partial \theta(t)} = 0)$

$$\Rightarrow \frac{\partial \left(\frac{1}{2} \theta^T(t) A \theta(t) - \theta^T(t) b\right)}{\partial \theta(t)} = 0$$

$$\Rightarrow A \theta(t) - b = 0$$

Lemma 1:

$$\frac{\partial \left(\frac{1}{2} \theta^T(t) A \theta(t) - \theta^T(t) b\right)}{\partial \theta(t)} = A \theta(t) - b$$

Solving equation(23), the value $\theta(t)$ is obtained for which the sum of the square of error value is minimum. This value of $\theta(t)$ is denoted by $\theta_{ls}(t)$.

$$\theta_{ls}(t) = A^{-1} b$$

$(\Phi \Phi^T)^{-1}$ can be written as follows.

$$(\Phi \Phi^T)^{-1} = \left(\sum_{i=1}^{t} \phi^T(i) \phi(i)\right)^{-1}$$

$$= \left(\sum_{i=1}^{t-1} \phi^T(i) \phi(i) + \phi^T(t) \phi(t)\right)^{-1}$$

$$= (P(t-1)^{-1} + \phi^T(t) \phi(t))^{-1} = P(t)$$

$$As P(t) = \left(\sum_{i=1}^{t} \phi^T(i) \phi(i)\right)^{-1}$$

$\Phi \cdot Y$ can be written as follows.

$$\Phi \cdot Y = \sum_{i=1}^{t} \phi(i) y(i)$$

$$= \sum_{i=1}^{t-1} \phi(i) y(i) + \phi(t) y(t)$$

Now from equation(25),(26) and (28), $\theta_{ls}(t - 1)$ can be estimated as follows.

$$\theta_{ls}(t - 1) = \left(\sum_{i=1}^{t-1} \phi^T(i) \phi(i)\right)^{-1} \sum_{i=1}^{t-1} \phi(i) y(i)$$

$$= P(t - 1) \sum_{i=1}^{t-1} \phi(i) y(i)$$
\[ \Rightarrow \sum_{i=1}^{t-1} \phi(i)y(i) = P(t-1)^{-1}\theta_{ls}(t-1) \]

So, \( \Phi Y \) can be written as follows.

\[ \Phi Y = P(t-1)^{-1}\theta_{ls}(t-1) + \phi(t)y(t) \]

Now from equation (25),

\[ \theta_{ls}(t) = P(t). \left( P(t-1)^{-1}\theta_{ls}(t-1) + \phi(t)y(t) \right) \]

From equation (27),

\[ P(t)^{-1} = P(t-1)^{-1} + \phi^T(t)\phi(t) \]

From equation (29) and (30),

\[ \theta_{ls}(t) = P(t). \left( \left[ P(t)^{-1} - \phi^T(t)\phi(t) \right] \theta_{ls}(t-1) + \phi(t)y(t) \right) \]

\[ \Rightarrow \theta_{ls}(t) = \theta_{ls}(t-1) - P(t)\phi^T(t)\phi(t)\theta_{ls}(t-1) + P(t)\phi(t)y(t) \]

\[ \Rightarrow \theta_{ls}(t) = \theta_{ls}(t-1) + P(t)\phi(t)(y(t) - \phi^T(t)\theta(t-1)) \]

\[ \Rightarrow \theta_{ls}(t) = \theta_{ls}(t-1) + K(t)(y(t) - \phi^T(t)\theta(t-1)) \]

where

\[ K(t) = P(t)\phi(t) \]

Lemma 2:

\[ (A + BD)^{-1} = A^{-1} - A^{-1}B(I_m + DA^{-1}B)^{-1}DA^{-1} \]

Where \( A, B, C, D \) and \( I_m \) are of appropriate dimension.

From equation (23) and Lemma 2,

\[ P(t) = P(t-1) - P(t-1)\phi(t)(1 + \phi^T(t)P(t-1)\phi(t))^{-1}\phi^T(t)P(t-1) \]

\[ \Rightarrow P(t) = P(t-1) - \frac{P(t-1)\phi(t)\phi^T(t)P(t-1)}{1 + \phi^T(t)P(t-1)\phi(t)} \]

(32)

Now \( K(t) \) can be written as follows using equation (32).

\[ K(t) = P(t)\phi(t) \]

\[ = P(t-1)\phi(t) \left[ 1 - \frac{\phi^T(t)P(t-1)\phi(t)}{1 + \phi^T(t)P(t-1)\phi(t)} \right] \]

\[ \Rightarrow K(t) = \frac{P(t-1)\phi(t)}{1 + \phi^T(t)P(t-1)\phi(t)} \]

(33)

Now, from equation (31), (32) and (33), the expressions for Recursive Least Square Algorithm is obtained.

\[ \theta_{ls}(t) = \theta_{ls}(t-1) + K(t)(y(t) - \phi^T(t)\theta_{ls}(t-1)) \]

\[ K(t) = \frac{P(t-1)\phi(t)}{1 + \phi^T(t)P(t-1)\phi(t)} \]
\[ P(t) = P(t - 1) - \frac{P(t - 1)\phi(t)\phi^T(t)P(t - 1)}{1 + \phi^T(t)P(t - 1)\phi(t)} \]

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