Area, entanglement entropy and supertranslations at null infinity

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Abstract
The area of a cross-sectional cut $\Sigma$ of future null infinity ($I^+$) is infinite. We define a finite, renormalized area by subtracting the area of the same cut in any one of the infinite number of BMS-degenerate classical vacua. The renormalized area acquires an anomalous dependence on the choice of vacuum. We relate it to the modular energy, including a soft graviton contribution, of the region of $I^+$ to the future of $\Sigma$. Under supertranslations, the renormalized area shifts by the supertranslation charge of $\Sigma$. In quantum gravity, we conjecture a bound relating the renormalized area to the entanglement entropy across $\Sigma$ of the outgoing quantum state on $I^+$.

Keywords: BMS group, entanglement entropy, entropy bounds

1. Introduction

The Bekenstein–Hawking area-entropy law [1, 2]

$$S_{\text{BH}} = \frac{\text{Area}}{4\hbar G}$$

(1)

ascribes an entropy to a null surface proportional to its cross-sectional area in Planck units. This law has a number of fascinating generalizations [3–22], including the Bousso bound [23–29] which bounds the change in the area to the entropy flux through the null surface.

One of the most interesting null surfaces is future null infinity ($I^+$) which is a future boundary of asymptotically Minkowskian spaces. It is a universal observer horizon for all eternal observers which do not fall into black holes. It is natural to try to relate the change in the area of cross-sectional ‘cuts’ $\Sigma$ of $I^+$ to the energy or entropy flux across $I^+$. An immediate obstacle is that both the areas and area changes of such cuts are infinite. The Bousso bound is obeyed but in a trivial manner.

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In this paper we define a finite renormalized area of cuts of $I^+$ and conjecture a nontrivial bound relating it to the entropy radiated through $I^+$. A family of regulated null surfaces parametrized by $r$ which approach $I^+$ for $r \to \infty$ is introduced. For finite $r$ these have finite area for any cut $\Sigma$. We then define a subtracted area by subtracting the area of the same cut in a fiducial vacuum geometry. The gravitational vacuum has an infinite degeneracy labeled by an arbitrary function $C_0$ on the sphere at $I^+$. Under BMS supertranslations, also parameterized by an arbitrary function (denoted $f$) on the sphere, $C_0 \to C_0 + f$ and these vacua transform into one another. We show that the subtracted area, denoted $A_{\Sigma}^{\Sigma_0}$, is finite (and typically negative) in the $r \to \infty$ limit. However it retains ‘anomalous’ dependence on the choice of a fiducial $C_0$.

This renormalized area $A_{\Sigma}^{\Sigma_0}$ is found to have several interesting properties. When $C_0$ coincides with the physical vacuum at the location of the cut, $A_{\Sigma}^{\Sigma_0}$ is the negative of the so-called modular energy of the region $I^+_\Sigma$ lying to the future of $\Sigma$, including ‘soft graviton’ terms which are linear in the Bondi news. It tends to increase towards the far future, and asymptotically reaches zero from below when $C_0$ coincides with the asymptotic future vacuum. Moreover, under supertranslations it shifts by the supertranslation charge on $\Sigma$.

In quantum gravity, the outgoing quantum state is supported on $I^+$. The cut $\Sigma$ divides $I^+$ into two regions, and a quantum entanglement entropy $S_{\text{ent}}^{\Sigma_0}$ of the portions of the outgoing quantum state on opposing sides of the cut is expected. In principle, unlike the entanglement across generic fluctuating interior surfaces, $S_{\text{ent}}^{\Sigma_0}$ should be well defined because gravity is weakly coupled near the boundary. However it is beset by both ultraviolet (UV) divergences from short wavelength entanglements near the cut and infrared (IR) divergences from soft gravitons. A choice of vacuum is required for subtraction of UV divergences, so $S_{\text{ent}}^{\Sigma_0}$ will also acquire an ‘anomalous’ dependence on the fiducial $C_0$. We will not try to give a precise definition of $S_{\text{ent}}^{\Sigma_0}$ herein which would, among other things, require a decomposition of the soft graviton Hilbert space. Nevertheless we will motivate a conjecture that a suitably defined $S_{\text{ent}}^{\Sigma_0}$ obeys the bound

$$-\frac{A_0^{\Sigma}}{4\hbar G} \geq S_{\text{ent}}^{\Sigma_0}(\Sigma)$$

for any cut $\Sigma$ of $I^+$. Typically, at late times both sides of this equation are positive and decreasing. This relation incorporates the BMS structure at $I^+$ into the study of the relation between area and entanglement entropy.

Our results are plausibly relevant to and were motivated by the black hole information paradox. A unitary resolution of this paradox would amount, roughly speaking, to showing that late and early time Hawking emissions are correlated in such a way that, for a pure incoming state, the full quantum state on $I^+$ is a pure state. However a more precise BMS-invariant statement is needed. One would like to compute the entanglement entropy $S_{\text{ent}}^{\Sigma_0}$ across any cut $\Sigma$. Naively, one expects that it approaches zero for all cuts in the far past and far future and has a maximum somewhere in the middle, possibly at the Page time [37, 38]. Given both the IR and UV subtractions needed to define $S_{\text{ent}}^{\Sigma_0}$, the resulting anomalies in supertranslation invariance and the discovery of soft hair [39, 40], it is not obvious to us what precisely the expectation following from unitarity should be. In particular, the requirement that $S_{\text{ent}}^{\Sigma_0}$ vanish in the far future is not fully supertranslation invariant. We do not attempt to resolve these issues herein. Rather we view the present effort as a first step in obtaining a precise statement of the black hole information paradox.

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2 Prescient early discussions of vacuum degeneracy are in [31–33].
3 A relevant discussion appears in [34–36].
This paper is organized as follows. Section 2 contains preliminaries and notation. In section 3 we define a renormalized area $A^F_{Σ}$ in which we subtract the area associated to the asymptotic future vacuum and relate it to the ’hard’ modular energy of the region to the future of the cut. In section 4 we show that $A^F_{Σ}$ varies under supertranslations into the hard part of the supertranslation charge. Section 5 introduces the more general renormalized $A^F_{Σ0}$ involving an arbitrary vacuum subtraction. Its variation under supertranslations is shown to involve the full modular energy including soft graviton contributions. In section 6 we motivate and conjecture a bound relating the renormalized area to the entanglement entropy which can be viewed as the second law of $I^+$.

Throughout this paper we assume for simplicity that the geometry reverts to a vacuum in the far future and that all flux though $I^+$ is gravitational. This highlights many of the salient features but a treatment of more general cases would be of interest.

2. Preliminaries

In retarded Bondi coordinates, asymptotically flat metrics [41–44] near $I^+$ take the form

$$ds^2 = -du^2 - 2du dr + 2r^2 \gamma_{\bar{z}z} dz d\bar{z} + \frac{2m_B}{r} du^2 + rC_{zz} dz^2 + rC_{\bar{z}\bar{z}} d\bar{z}^2 + D^z C_{zz} du dz + D^\bar{z} C_{\bar{z}\bar{z}} d\bar{u} d\bar{z} + \ldots$$

(3)

Here $\gamma_{\bar{z}z}$ is the round metric on the unit $S^2$ and $D^z$ is the associated covariant derivative. Defining

$$N_{zz} = \partial_u C_{zz},$$

$$T_{uu} = \frac{1}{2} N_{zz} N_{zz},$$

$$U_z = i D^z C_{zz}, \quad U = U_z dz + U_{\bar{z}} d\bar{z},$$

$$V_z = i D^\bar{z} N_{zz}, \quad V = V_z dz + V_{\bar{z}} d\bar{z},$$

$$\epsilon = i \gamma_{\bar{z}z} dz \wedge d\bar{z},$$

(4)

the leading order vacuum constraint equation reads

$$\partial_u m_B du \wedge \epsilon = -\frac{1}{2} T_{uu} du \wedge \epsilon - \frac{1}{4} du \wedge dV.$$  

(5)

We could easily add a matter contribution to $T_{uu}$ but we omit this for brevity. We assume that near the future boundary $I^+_{\perp}$ of $I^+$ the spacetime reverts to a vacuum so that

$$m_B|_{I^+_{\perp}} = 0, \quad C_{zz}|_{I^+_{\perp}} = -2D^2 \epsilon,$$

(6)

for some function $C_F(z, \bar{z})$. In the quantum theory we denote the corresponding vacuum state by $|F\rangle$. Given $C_F$ and the Bondi news tensor $N_{zz}$, the mass aspect $m_B$ is determined by integrating the constraint equation (5) backwards from $I^+_{\perp}$.

Asymptotically flat spacetimes admit an infinite dimensional symmetry group, known as the Bondi–Metzner–Sachs (BMS) group [41–43]. The supertranslations are labeled by an arbitrary function $f(z, \bar{z})$ on the $S^2$ and are generated by the vector fields

$$\xi = f \partial_u + \frac{1}{r} (D^z f \partial_z + D^\bar{z} f \partial_{\bar{z}}) - \frac{1}{2} D^2 f \partial_u, \quad D^2 = 2D^z D_z.$$

(7)

Infinitesimal supertranslations act on the geometry as [45, 46]
\[ \delta J_{zz} = fN_{zz} - 2D_{z}f, \]
\[ \delta J_{CF} = f, \]
\[ \delta J_{Uz} = fV_z + i D_{z}fN_{zz} - i D_{z}^2 f, \]
\[ \delta J_{mB} = f \partial_{u} m_{B} + \frac{1}{4} D_{z}^2 fN_{zz} + \frac{1}{4} \bar{D}_{z}^2 fN_{zz} + \frac{i}{2} \partial_{z} fV_{z} - \frac{i}{2} \partial_{z} \bar{f}V_{z}, \]
\[ \delta J_{T_{uu}} = f \partial_{u} T_{uu}. \]

Equation (8)

These transformations are generated by the supertranslation charges

\[ Q[f] = \int \mathcal{I}^{+} f(z, \bar{z}) m_{B} \epsilon = \frac{1}{4} \int \mathcal{I}^{+} f(z, \bar{z}) \ du \wedge dV + \frac{1}{2} \int \mathcal{I}^{+} f(z, \bar{z}) T_{uu} \ du \wedge \epsilon. \]

Equation (9)

The first term, known as the soft charge, is linear in the gravitational field. It is written in terms of the zero-mode of the Bondi news, and creates soft-gravitons when acting on physical states. The second term, or hard charge, is quadratic in the matter and gravitational fields and characterizes the hard energy flux through \( \mathcal{I}^{+} \).

3. Renormalizing the area

We wish to study the area of a cut \( \Sigma \) of \( \mathcal{I}^{+} \) defined by

\[ u = u_{\Sigma}(z, \bar{z}) \]

in the geometry (3) and also to study its variation under supertranslations of \( \Sigma \)

\[ u_{\Sigma} \rightarrow u_{\Sigma} + f \]

with the geometry held fixed. Of course this area is infinite so we must introduce both a regulator and a subtraction. We regulate the area by the replacement of \( \mathcal{I}^{+} \) with the past lightcone of a point which approaches \( i^{+} \). For the flat Minkowski metric the null hypersurface

\[ r = -\frac{1}{2} (u - u_0) \]

approaches \( \mathcal{I}^{+} \) for \( u_0 \rightarrow \infty \) with \( u \) held fixed. More generally we solve the ODE

\[ \left( 1 - \frac{2m_{B}}{r} \right) du + 2dr = 0, \]

Equation (13)

which guarantees that the surface is null, and choose the integration constants at each \( (z, \bar{z}) \) so that the surface lies at large radius, approaching infinity, for finite \( u \). The null condition (13) has \( \frac{1}{r} \) corrections. For brevity such corrections are suppressed here and hereafter whenever they drop out of the large-\( r \) limit. Equation (13) can be rewritten as

\[ \frac{dr^2}{du} = 2m_{B} - r. \]

Equation (14)

The area of a cut \( \Sigma \) of \( \mathcal{I}^{+} \) defined by \( u = u_{\Sigma}(z, \bar{z}) \) then follows from the metric induced from (3) and is given by

\[ \text{The generic such surface will terminate at a cusp rather than a point, but this will not matter as the quantities considered below do not have contributions from the endpoint of the surface.} \]
\[ A(\Sigma, N_{zz}, C_F) = \int_{\Sigma} d^2z \sqrt{\det g} = \int_{\Sigma} \left( r^2 \epsilon - \frac{1}{2} du \Sigma \wedge U \right). \] (15)

Both the area (15) as well as its variation with respect to retarded time are divergent in the large-\(r\) limit of interest. A subtraction is necessary to obtain a finite result. We define a fiducial ‘\(C_F\)-vacuum’ in which the news \(N_{zz}\) vanishes and \(C_{zz} = -2D_z^2C_F\) on all of \(\mathcal{I}^+\). This flat geometry coincides with (3) at late times. A fiducial null hypersurface in this fiducial space-time solving (14) (with \(m_B = 0\)) can then be found which coincides exactly with the solution of (13) in (3) at late times. A subtracted area, with a finite large-\(r\) limit, may then be obtained by subtracting the area of the fiducial hypersurface:

\[
A^\Sigma_F = A(\Sigma, N_{zz}, C_F) - A(\Sigma, 0, C_F) = \int_{\Sigma} \left[ (r^2 - r_0^2) \epsilon + \frac{1}{2} du \Sigma \wedge \Delta U \right].
\] (16)

Here \(\Delta U = U_F - U_\Sigma\) is the change in \(U\) and \(r_0\) is the radius in the fiducial vacuum. Using (14), we have

\[
\frac{d}{du} (r^2 - r_0^2) = 2m_B.
\] (17)

Integrating this equation from \(\mathcal{I}_+^0\) to \(u_\Sigma\), one finds

\[
r^2(u_\Sigma, z, \bar{z}) - r_0^2(u_\Sigma, z, \bar{z}) = -\int_{u_\Sigma}^{\infty} du 2m_B(u, z, \bar{z}).
\] (18)

The finite renormalized area is then given by

\[
A^\Sigma_F = -\int_{\mathcal{I}_+^0} du \wedge \left[ 2m_B \epsilon - \frac{1}{2} du \Sigma \wedge V \right].
\] (19)

where \(\mathcal{I}_+^0\) is the three-dimensional region of \(\mathcal{I}^+\) lying to the future of the cut \(\Sigma\). Using the constraint equation and the identity

\[
\int_{\mathcal{I}_+^0} du \wedge du \Sigma \wedge V = \int_{\mathcal{I}_+^0} (u - u_\Sigma) du \wedge dV = -\int_{\Sigma} u_\Sigma du \Sigma \Delta U,
\] (20)

the renormalized area can be rewritten

\[
A^\Sigma_F = -\int_{\mathcal{I}_+^0} (u - u_\Sigma) T_{uu} du \wedge \epsilon = -\int_{\Sigma} d^2z \gamma_{\Sigma} \int_{u_\Sigma}^{\infty} (u - u_\Sigma) T_{uu} du.
\] (21)

This expression could equivalently be derived through integration of the Raychaudhuri equation, and matches familiar expressions for the modular Hamiltonians of lightsheets [27, 47, 48]. We refer to this as the (negative of the) hard modular energy of the region \(\mathcal{I}_+^0\). We note that \(A^\Sigma_F\) is typically negative and increases to zero in the far future due to the subtraction scheme.

### 4. Supertranslations

\(A^\Sigma_F\) is strictly invariant under coordinate transformations which both move the cut and transform the physical and subtraction geometries. In particular, \(A^\Sigma_F\) is invariant if we simultaneously shift the cut \(u_\Sigma \to u_\Sigma + f\) and supertranslate the geometry by the inverse transformation.
However, one can consider evaluating the subtracted area on a supertranslated cut, sending $u_{\Sigma} \rightarrow u_{\Sigma} + f$ while keeping the geometry fixed. Starting from either (19) or (21) one easily finds

$$\delta_{f}A_{\Sigma}^{F} = \int_{\Sigma} f \left[ 2m_{B}\epsilon - \frac{1}{2} dU \right] = \int_{\mathcal{I}_{+}^{\Sigma}} f T_{\mu\nu} \, du \wedge \epsilon. \quad (22)$$

The right hand side is the hard part of the supertranslation charge on $\mathcal{I}_{+}^{\Sigma}$. Alternately, (22) may be derived by infinitesimally supertranslating the geometry according to (8).

5. A general subtraction

The subtraction used in (16) to obtain a finite area change has a teleological nature: we must know which vacuum the geometry settles into in the far future in order to define $A_{\Sigma}^{F}$. In this section we consider a more general, non-teleological subtraction of the area of $\Sigma$ at $u = u_{\Sigma}$ in the null hypersurface defined by solving (14) in an arbitrary vacuum characterized by the arbitrary function $C_{0}$ with $C_{0z} = -2D_{z}C_{0}$. Unlike the case in (16), the subtracted geometry is not identical to the physical one at late times, and so the late-time contributions are not manifestly finite or well-defined. To characterize the resulting ambiguity we introduce a late-time cutoff by terminating both surfaces at a final cut $\Sigma_{F}$ at $u = u_{F}(z, \bar{z})$, in the late-time vacuum region with $m_{B} = 0$. One finds

$$A_{0}^{\Sigma} = A_{F}^{\Sigma} + \frac{1}{2} \int_{\Sigma} d(u_{\Sigma} - u_{F}) \wedge (U_{0} - U_{F}). \quad (23)$$

where $U_{0}$ and $U_{F}$ are constructed from $C_{0}$ and $C_{F}$ according to (4). As may be easily verified, this expression is invariant if we supertranslate the physical geometry, the fiducial vacuum $C_{0}$ and both cuts at $\Sigma$ and $\Sigma_{F}$. We now restrict consideration to the case $u_{F} = \text{constant}$, in which case this expression reduces to

$$A_{0}^{\Sigma} = - \int_{\mathcal{I}_{+}^{\Sigma}} du \wedge (u - u_{\Sigma})(\epsilon T_{\mu\nu} + \frac{1}{2} dV) + \frac{1}{2} \int_{\Sigma} du_{\Sigma} \wedge (U_{0} - U). \quad (24)$$

Fixing the geometry and varying $u_{\Sigma} \rightarrow u_{\Sigma} + f$, we find

$$\delta_{f}A_{0}^{\Sigma} = \int_{\Sigma} f \left[ 2m_{B}\epsilon - \frac{1}{2} d(U_{0} - U) \right]. \quad (25)$$

The right hand side is the full supertranslation charge in the special case $U = U_{0}$ on $\Sigma$.

6. An area-entropy bound conjecture

Given a cut $\Sigma$ and a vacuum state $|C_{0}\rangle$ on all of $\mathcal{I}^{+}$ we may define a density matrix on the region $\mathcal{I}_{+}^{\Sigma}$ to the future of $\Sigma$ by

$$\sigma_{0} = \text{tr}_{<}|C_{0}\rangle \langle C_{0}|, \quad (26)$$

where the trace is over the region prior to $\Sigma$ and the dependence on the choice of cut is suppressed. Similarly for an excited state $|\Psi\rangle$ we define the density matrix on $\mathcal{I}_{+}^{\Sigma}$

$$\rho = \text{tr}_{<}|\Psi\rangle \langle \Psi|, \quad (27)$$

It would be interesting to analyze the more generic case of the area change over a more general finite interval.

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5 It would be interesting to analyze the more generic case of the area change over a more general finite interval.
We normalize so that $\text{tr}\rho = \text{tr}\sigma_0 = 1$. The modular Hamiltonian which measures local Rindler energies relative to $|C_0\rangle$ is

$$-\ln\sigma_0.$$ (28)

$\sigma_0$ has contributions from the entanglement of both hard and soft modes across the surface $\Sigma$. Hard mode entanglements contribute [47, 48]

$$-\ln\sigma_0|_{\text{hard}} = \frac{1}{4\hbar G} \int_{I^+} du \wedge (u - u_{\Sigma}) (e^{\hat{T}_{uu}} + \text{constant}) = -\frac{\hat{A}_\Sigma}{4\hbar G} + \text{constant},$$ (29)

where here $\hat{T}_{uu}$ and $\hat{A}$ are both operators and the constants depend on the normal ordering prescription. It would be extremely interesting, but beyond the scope of this paper, to regulate, define and compute the soft contributions to $\sigma_0$. The precise form of $\sigma_0$ may well depend on the renormalization scheme. Here we simply conjecture, motivated by the structures encountered in the previous section, that these contributions can be defined in such a way that

$$-\ln\sigma_0 = -\frac{\hat{A}_\Sigma}{4\hbar G} + \text{constant},$$ (30)

where the operator-valued area appearing here is

$$\hat{A}_\Sigma^\Sigma = -\int_{I^+} du \wedge (u - u_{\Sigma}) (\epsilon\hat{T}_{uu} + \frac{1}{2} d\hat{V}) + \frac{1}{2} \int_{\Sigma} du u_{\Sigma} \wedge (U_0 - \hat{U}).$$ (31)

We interpret the first term as the full modular Hamiltonian, including soft terms. The last is a soft term which vanishes when $U_0 = \hat{U}$ on the cut $\Sigma$. The $C_0$-vacuum subtracted modular energy of the state $|\Psi\rangle$ restricted to $I^+_+$ is

$$K_0 = -\text{tr}\rho \ln\sigma_0 + \text{tr}\sigma_0 \ln\sigma_0.$$ (32)

This expression vanishes for $\rho = \sigma_0$, as does $A_0^\Sigma$ when the physical geometry is the $C_0$ vacuum. Hence the constant is fixed so that

$$K_0 = -\frac{A_0^\Sigma}{4\hbar G}.$$ (33)

We further define the regulated entanglement entropy

$$S^\text{ent}_{0} = -\text{tr}\rho \ln\rho + \text{tr}\sigma_0 \ln\sigma_0$$ (34)

and the relative entropy

$$S(\rho|\sigma_0) = \text{tr}\rho \ln\rho - \text{tr}\rho \ln\sigma_0.$$ (35)

Evidently

$$S(\rho|\sigma_0) = K_0 - S^\text{ent}_{0}.$$ (36)

Positivity of relative entropy and the conjecture (30) then implies the bound

$$-\frac{A_0^\Sigma}{4\hbar G} \geq S^\text{ent}_{0}.$$ (37)

We note that the renormalized area $A_0^\Sigma$ is typically negative while the entanglement entropy is typically positive. If the renormalized area and entanglement entropy both tend to zero when

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6 Note that our normalization of the stress energy tensor as defined in (5) differs by a factor of $8\pi G$ from some other references.
the cut $\Sigma$ is taken to $\mathcal{I}^+$, then it follows from (37) that the change (final minus initial) $\Delta A$ in the renormalized area and the change $\Delta S^\text{ent}$ in the entanglement entropy obey the ‘second law of $\mathcal{I}^+$’,

$$\frac{\Delta A}{4\hbar G} + \Delta S^\text{ent} \geq 0.$$  

(38)

In this inequality, $\Delta A$ is typically positive while $\Delta S$ is typically negative, reflecting the fact that the outgoing flux after the cut $\Sigma$ is correlated with the flux prior to $\Sigma$ if it is to restore quantum purity.

7. Future directions

Our work leaves open several important questions meriting further investigation. It seems imperative to define, regulate, and compute the soft contributions to the modular Hamiltonian and the entanglement entropy at $\mathcal{I}^+$. This question seems related to the careful treatment of soft quanta required at the horizons of black holes [39]. A more pressing question regards the apparent ambiguity in subtraction scheme introduced in section 5. Our vacuum subtraction prescription is reminiscent of the subtraction procedure employed in the Euclidean approach to black hole thermodynamics in asymptotically flat spacetimes. There the boundary term in the on-shell action has a large-radius divergence which must be regulated with a vacuum subtraction. In that case the subtraction scheme is essentially fixed by requiring agreement with already known results calculable by other means. In the present context, there is no known answer to be reproduced. However, one might hope that a unique subtraction scheme could be singled out by other means, or that a covariant scheme exists, perhaps with a simple counter-term prescription. We leave this interesting question for future investigation.

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