PARTIAL SUPERSYMMETRY BREAKING AND AdS$_4$ SUPERMEMBRANE

F. Delduc$^{a,1}$, E. Ivanov$^{b,2}$, S. Krivonos$^{b,3}$

$^a$ Laboratoire de Physique, Groupe de Physique Théorique ENS Lyon, 46, allée d’Italie, F - 69364 - Lyon CEDEX 07

$^b$ Bogoliubov Laboratory of Theoretical Physics, JINR, Dubna, 141 980 Moscow region, Russia

Abstract

We consider partial spontaneous breaking of $N = 1$ AdS$_4$ supersymmetry $OSp(1|4)$ down to $N = 1, d = 3$ Poincaré supersymmetry in the nonlinear realizations framework. We construct the corresponding worldvolume Goldstone superfield action and show that it describes the $N = 1$ AdS$_4$ supermembrane. It enjoys $OSp(1|4)$ supersymmetry realized as a field-dependent modification of $N = 1, d = 3$ superconformal symmetry and goes into the superfield action of ordinary $N = 1, D = 4$ supermembrane in the flat limit. Its bosonic core is the Maldacena-type conformally invariant action of the AdS$_4$ membrane. We show how to reproduce the latter action within a nonlinear realization of the AdS$_4$ group $SO(2, 3)$. The same universal nonlinear realizations techniques can be used to construct conformally-invariant worldvolume actions for $(d - 2)$-branes in generic AdS$_d$ spaces.

E-Mail:
1) francois.delduc@ens-lyon.fr
2) eivanov@thsun1.jinr.ru
3) krivonos@thsun1.jinr.ru
1 Introduction

A view of superbranes as theories explicitly exhibiting the phenomenon of partial spontaneous breaking of global supersymmetry (PBGS) [1, 2] received a considerable attention (see, e.g., [3, 4] and references therein). In the approach with PBGS as the guiding principle, the manifestly worldvolume supersymmetric superbrane actions emerge as the Goldstone superfield actions associated with nonlinear realizations of some global space-time supersymmetry groups spontaneously broken down to smaller supersymmetries. The Goldstone superfields carry the superbrane physical worldvolume multiplets and are identified with coordinates of some superspace of the full supersymmetry. In this sense, superbranes in the PBGS approach bear a clear analogy with the standard nonlinear sigma models which are nonlinear realizations of spontaneously broken internal symmetries and describe Goldstone fields parametrizing the appropriate bosonic coset manifolds.

Until now, the PBGS approach was applied to spontaneously broken Poincaré supersymmetries in diverse dimensions, in general properly extended by some central-charge generators. All systems of this kind amount to $p$- or $Dp$- superbranes on flat Minkowski backgrounds. It is tempting to generalize the PBGS approach to the case of curved backgrounds. In most general setting, this problem amounts to coupling, in a proper way, the flat PBGS actions to the target and worldvolume supergravities, thus implying the passage from the PBGS concept to that of the partial breaking of local supersymmetry. As the first step toward this goal it is natural to adapt the PBGS approach for constructing worldvolume superfield actions of superbranes on some special homogeneous superbackgrounds. In view of the famous AdS/CFT correspondence [5, 6, 7], it seems of primary interest to look, from this point of view, at the superbranes living on the superextended $\text{AdS}_n \times S^m$-type backgrounds. The Green-Schwarz-type worldvolume actions for such superbranes were intensively discussed in literature (see., e.g., [8]-[11]). To the best of our knowledge, no relevant worldvolume superfield actions were explicitly constructed.

In this letter we present the PBGS action for a simple example of AdS superbranes, the AdS$_4$ supermembrane, and demonstrate that it goes into the known PBGS action of the ordinary $D = 4$ supermembrane [12] in the limit of the infinite AdS radius. The bosonic core of the action is a 3-dimensional analog of the scale invariant 4-dimensional Maldacena action [5]. We also show how this bosonic membrane action can be independently derived from the appropriate nonlinear realization of the AdS$_4$ group $SO(2, 3)$. The derivation can be directly extended to the generic case of $(d - 2)$-brane in AdS$_d$.

The AdS (super)groups are realized as (super)conformal groups on the (super)spaces the bosonic dimension of which is smaller by 1 compared to the bulk AdS (super)spaces. Thus, from the worldvolume perspective, the corresponding PBGS techniques should deal with the appropriate nonlinear realizations of superconformal groups in diverse dimensions. While nonlinear realizations of superconformal symmetries in superspace were already considered in the literature [13], we shall argue here on the example of the AdS$_4$ supermembrane that relevant to the PBGS approach is a rather special type of them. In the considered case the underlying supersymmetry is the $N = 1$ AdS$_4$ one which eventually takes the form of nonlinearly realized $N = 1, d = 3$ superconformal symmetry on the worldvolume superspace of the supermembrane.
2 AdS$_4$ membrane from the coset approach

To fix our basic assertions, we start with the bosonic case of AdS$_4$ membrane. Whereas it is known how to derive the Nambu-Goto action for the branes in the $d$-dimensional flat Minkowski background from the nonlinear realizations (coset) approach applied to relevant Poincaré group [14, 15], no such a self-contained construction was presented so far for AdS branes. Here we do this for AdS$_4$ membrane.

The algebra of the AdS$_4$ group SO(2,3) in the $d = 3$ spinor SL(2, R) notation and in the basis most appropriate for our purposes reads:

\[
[M_{ab}, M_{cd}] = \varepsilon_{ac}M_{bd} + \varepsilon_{ad}M_{bc} + \varepsilon_{bd}M_{ac} + \varepsilon_{bd}M_{cd} \equiv (M)_{ab,cd} = -(M)_{cd,ab},
\]
\[
[K_{ab}, K_{cd}] = -(M)_{ab,cd}, [M_{ab}, K_{cd}] = (K)_{ab,cd}, [M_{ab}, P_{cd}] = (P)_{ab,cd},
\]
\[
[K_{ab}, D] = -2P_{ab} + 2mK_{ab}, [P_{ab}, D] = -2mP_{ab}, [P_{ab}, P_{cd}] = 0,
\]
\[
[K_{ab}, P_{cd}] = -2(\varepsilon_{ac}\varepsilon_{bd} + \varepsilon_{bc}\varepsilon_{ad})D - m(M)_{ab,cd}.
\]

(2.1)

Here, $a, b = 1, 2$ and the contraction parameter $m$ is proportional to the inverse AdS$_4$ radius. We use the following conjugation rules

\[
P^\dagger_{ab} = P_{ab}, M^\dagger_{ab} = -M_{ab}, K^\dagger_{ab} = -K_{ab}, D^\dagger = D, m^\dagger = -m,
\]

(2.2)

which allow us to avoid the appearance of the imaginary unit in the commutation relations. The SO(1,2) generators $M_{ab}$ together with $K_{ab}$ form the algebra of $D = 4$ Lorentz group SO(1,3). In the $m = 0$ limit (2.1) goes over into $D = 4$ Poincaré algebra, with $P_{ab}, D$ forming the 4-momentum generator.

Another basis in the algebra (2.1), which makes manifest its interpretation as $d = 3$ conformal algebra, is achieved by passing to the generators

\[
\tilde{K}_{ab} = \frac{1}{m}K_{ab} - \frac{1}{2m^2}P_{ab}, \tilde{D} = \frac{1}{m}D,
\]

(2.3)

which are the standard $d = 3$ special conformal and dilatation generators:

\[
[\tilde{K}_{ab}, \tilde{K}_{cd}] = 0, [M_{ab}, \tilde{K}_{cd}] = (\tilde{K})_{ab,cd}, [\tilde{K}_{ab}, \tilde{D}] = \tilde{D} = 2\tilde{K}_{ab},
\]
\[
[P_{ab}, \tilde{D}] = -2P_{ab}, [\tilde{K}_{ab}, P_{cd}] = -2(\varepsilon_{ac}\varepsilon_{bd} + \varepsilon_{bc}\varepsilon_{ad})\tilde{D} - (M)_{ab,cd}.
\]

(2.4)

In this conformal basis, any dependence on the dimensionful parameter $m$ disappears.

One more basis can be obtained by passing to the generator $\hat{P}_{ab} = P_{ab} - mK_{ab}$. It corresponds to the “old” standard form of the AdS$_4$ algebra (see, e.g., [16]), the basic commutation relation of which can be written as $[P_A, P_B] = m^2M_{AB}$, $A, B = 0, 1, 2, 3$, where $M_{AB} = (M_{ab}, K_{cd})$ are generators of the $D = 4$ Lorentz group SO(1,3) and $P_A \equiv (\hat{P}_{ab}, D)$ are generators of the curved SO(2,3)/SO(1,3) translations. In the “old” standard basis the AdS$_3$ subalgebra $so(2,2) \propto (\hat{P}_{ab}, M_{ab})$ of $so(2,3)$ is manifest. On the contrary, in our basis (2.1) the $d = 3$ Poincaré subalgebra $\propto (P_{ab}, M_{ab})$ is manifest (together with the manifest $so(1,3)$) basis The generators $(P_{ab}, D)$ form the maximal solvable subalgebra of $so(2,3)$. Any AdS$_d$ algebra $so(2, d - 1)$ can be written in the basis where the $(d - 1)$-dimensional Poincaré symmetry algebra is manifest, the $(d - 1)$-dimensional translation operator together with the dilatation generator form a solvable
subalgebra and also the $d$-dimensional Lorentz group algebra $so(1, d - 1)$ is manifest \[17\]. This basis, the particular case of which is just (2.1), is indispensable while considering AdS branes.

Now we consider the coset $SO(2, 3)/SO(1, 2)$ in the following parametrization:

$$g = e^{x^{ab} P_{ab}} e^{q(x) D} e^{\Lambda^{ab}(x) K_{ab}}.$$  
\hspace{10cm} (2.5)

The parameters $x^{ab} = -(x^{ab})^\dagger$ and $q(x) = -q^\dagger(x)$ provide a specific parametrization of the coset $SO(2, 3)/SO(1, 3) \sim$ AdS$_4$, just adapted to the above solvable-subgroup basis of the $so(2,3)$-algebra \[13\]. The extra vector parameter $\Lambda^{ab}(x) = (\Lambda^{ab}(x))^\dagger$ parametrizes the coset $SO(1, 3)/SO(1, 2)$. We shall see that its inclusion is imperative for deducing the AdS$_4$ membrane action from the coset approach (just like the case of membrane in a flat background, which corresponds to the nonlinear realization of the $D$-brane action from the coset approach (just like the case of membrane in a flat background, which corresponds to the nonlinear realization actually describes the spontaneous breaking of $SO(2, 3)$ down to its $d = 3$ Poincaré subgroup. The latter will finally be the only subgroup realized linearly on the AdS$_4$ membrane worldvolume $\{x^{ab}\}$.

The full set of the $SO(2, 3)$ transformations of the coset parameters in (2.5) can be found by acting on (2.5) from the left by various $SO(2, 3)$ group elements. For our purposes most interesting are the $d = 3$ conformal transformations of the AdS$_4$ coordinates $(x^{ab}, q(x))$. They are generated by the left shift with $g_0 = e^{\delta x^{ab} K_{ab}}$, where $K_{ab}$ was defined in (2.3):

$$\delta x^{ab} = 4 \left( x^{ab} b^{ab} - 2 x^{cd} h_{cd} x^{ab} \right) - \frac{1}{2m^2} e^{4m q} b^{ab}, \quad \delta q = - \frac{4}{m} x^{ab} b_{ab}. \hspace{10cm} (2.6)$$

It is important to realize that the algebra of these transformations still coincides with the $d = 3$ conformal group algebra, they simply provide a different realization of the latter, such that the Goldstone field $q(x)$ proves essentially involved. It is worth mentioning here that the same generators $P_{ab}, D$ can be regarded to span another coset of $SO(2,3)$, that over its subgroup $(K_{ab}, M_{ab})$. With this choice of the stability subgroup the same coordinates $x^{ab}, q$ carry the standard realization of $SO(2,3)$ as the $d = 3$ conformal group: in $\delta x^{ab}$ only the first piece remains, while $q$ still has the same transformation rule, being a dilaton. Thus different choices of the stability subgroups give rise to different realizations. From the perspective of the full coset $SO(2, 3)/SO(1, 2)$ (2.5), these two realizations are related by nonlinear redefinitions of the coset parameters, such that $x^{ab}, q(x)$ and $\Lambda^{ab}(x)$ are mixed up in a non-trivial way. It turns out that just the separation of the coset parameters into $d = 3$ space-time coordinates and Goldstone fields as in (2.5) immediately leads to the correct AdS$_4$ action. Correspondingly, it is just the field-dependent $d = 3$ conformal transformations (2.6) under which this action proves to be invariant.

The basic ingredients in constructing the action are left-invariant Cartan one-forms. In the obvious notation, they are defined by the standard relation

$$g^{-1} dq = \omega_P \cdot P + \omega_D D + \omega_K \cdot K + \omega_M \cdot M.$$  
\hspace{10cm} (2.7)

As usual, the forms associated with the coset generators are transformed homogeneously, while the $so(1, 2)$ Cartan form $\omega_M^\dagger$ has an inhomogeneous transformation rule. For our purposes it is
enough to know the explicit structure of the following two coset space Cartan forms

\[ \omega_P^{ab} = e^{-2mq} \left( dx^{ab} + \frac{4\lambda^{ab}_c dx^{cd}}{1 - 2\lambda^2} \right) + \frac{2\lambda^{ab} dq}{1 - 2\lambda^2} \equiv E^{cd}_{ab} dx^{cd}, \quad (2.8) \]

\[ \omega_D = \frac{1 + 2\lambda^2}{1 - 2\lambda^2} \left( dq + \frac{4e^{-2mq}\lambda^{ab} dx^{ab}}{1 + 2\lambda^2} \right), \quad (2.9) \]

where \( \lambda^{ab} \equiv \tanh \sqrt{2\Lambda^2} \Lambda^{ab}, \lambda^2 = \lambda^{ab} \lambda^{ab}. \quad (2.10) \)

From the structure of (2.9) it is clear that \( \lambda^{ab} \) can be covariantly expressed through \( q(x) \) by the inverse Higgs [13] constraint

\[ \omega_D = 0 \Rightarrow \frac{4\lambda^{ab}}{1 + 2\lambda^2} = -e^{2mq}\partial_{ab}q \Rightarrow \lambda^{ab} = -\frac{1}{2}e^{2mq} \frac{\partial_{ab}q}{1 + \sqrt{1 - \frac{1}{2}e^{4mq}(\partial q)^2}}. \quad (2.11) \]

Then the dreibein in (2.8) takes the simple form

\[ E^{ab}_{cd} = e^{-2mq} \delta^{(a}_{c} \delta^{b)}_{d}) - \frac{1}{2}e^{2mq} \frac{1}{1 + \sqrt{1 - \frac{1}{2}e^{4mq}(\partial q)^2}} \delta^{ab}_c \partial_{cd}q. \quad (2.12) \]

The simplest appropriate invariant is the covariant volume of the \( d = 3 \) space,

\[ \int d^3x \det E(q), \]

and the correct invariant action vanishing when \( q = 0 \) is given by (up to a normalization factor)

\[ S = \int d^3x \left[ e^{-6mq} - \det E(q) \right] = \int d^3x \ e^{-6mq} \left( 1 - \sqrt{1 - \frac{1}{2}e^{4mq}\partial_{ab}q\partial_{ab}q} \right). \quad (2.13) \]

By construction, it possesses all symmetries of the AdS4 space and in the limit \( m = 0 \) goes into the standard static-gauge form of the Nambu-Goto action for membrane in four-dimensional Minkowski space. Note that the term \( \sim \int d^3x e^{-6mq} \) is invariant under the nonlinear \( d = 3 \) conformal transformations (2.6) and dilatations on its own right.

Another, equivalent way to deduce the same action (2.13) is to start from the dreibein in (2.8) with \( q \) and \( \lambda^{ab} \) as independent fields. Then the invariant action takes the form

\[ S = \int d^3x \left[ e^{-6mq} - \det E(q, \lambda) \right] = -2 \int d^3x \ e^{-6mq} \left( \frac{2\lambda^2 + e^{2mq}\lambda^{ab} \partial_{ab}q}{1 - 2\lambda^2} \right). \quad (2.14) \]

Equation of motion for \( \lambda^{ab} \) yields just the inverse Higgs expression (2.11), and substituting it back into (2.14) brings the latter into the form (2.13).

To see that the action (2.13) indeed describes a membrane embedded into the AdS4 background, let us look at the induced distance defined as the square of \( \omega_P^{ab} \) with the dreibein (2.12)

\[ ds^2 = \omega_P^{ab} \omega_{Pab} = e^{-4mq} (dx^{ab} dx^{ab}) - \frac{1}{2} dq dq. \quad (2.15) \]
Introducing $U = e^{-2mq}$ and rescaling $x^{ab} = \frac{1}{2\sqrt{2m}}\tilde{x}^{ab}$, one can rewrite (2.15) and (2.13), up to some overall constant factors, in the form

$$ds^2 = U^2 (d\tilde{x}^{ab}d\tilde{x}_{ab}) - \left(\frac{dU}{U}\right)^2, \quad S = \int d^3\tilde{x} U^3 \left(1 - \sqrt{1 - \left(\frac{\partial U \cdot \partial U}{U^4}\right)}\right).$$

Thus $ds^2$ is recognized as the $d = 3$ pullback of the standard invariant interval on AdS$_4$ in the parametrization which is of common use in the AdS/CFT literature, while $S$ as the $d = 3$ analog of the Maldacena scale-invariant brane action on AdS$_5$ ([5] actually, of the scalar fields piece of his full D3-brane action). The derivation of this form of the AdS$_4$ interval from the coset $SO(2,3)/SO(1,3)$ parametrized by coordinates associated with the solvable subgroup generators (and a generalization to the generic case of AdS$_d$), as well as deducing the field-dependent conformal transformations (2.6), were given in [17] (see also [18]). To our knowledge, the explicit derivation of the AdS$_4$ membrane action from the coset approach we have given here is new. It can be straightforwardly extended to the case of $(d - 2)$-brane in AdS$_d$ in a static gauge for an arbitrary dimension $d$ [8, 11]. In the generic case, the only basic Goldstone field is also dilaton $q(x)$, while an analog of the extra 3-dimensional coset factor $SO(1,3)/SO(1,2)$ in (2.5) is the $(d - 1)$-dimensional coset $SO(1,d - 1)/SO(1,d - 2)$, with the parameters basically becoming $x$-derivatives of the dilaton after employing the inverse Higgs constraints. The conformally-invariant $(d - 2)$-brane action is an obvious modification of (2.13), (2.16) adapted to the $(d - 1)$-dimensional worldvolume.

The mixed $\lambda, q$ representation for the membrane action (2.14) implies a new interesting (and rather strange) type of duality seemingly specific just for the AdS (super)branes. To reproduce the standard AdS$_4$ membrane action (2.13), we eliminated $\lambda_{ab}$ by its algebraic equation of motion. On the other hand, $q$ is also an unconstrained field and we can firstly vary (2.14) with respect to it, with the result

$$\partial_{ab} F^{ab} = 6m e^{-2mq} \left(1 - \sqrt{1 + 2F^2}\right), \quad F^{ab} \equiv \frac{2\lambda^{ab}}{1 - 2\lambda^2}.$$

In the flat limit $m = 0$ this equation becomes $\partial_{ab} F^{ab} = 0$ and can be interpreted as the Bianchi identity for a $d = 3$ Maxwell strength $F^{ab}$. After substitution of $F^{ab}$ expressed through the $d = 3$ gauge potential back into the $m = 0$ form of (2.14), the latter becomes just the $d = 3$ Born-Infeld action [13], thus displaying the well-known $d = 3$ duality between the membrane Nambu-Goto action in a static gauge and the $d = 3$ Born-Infeld action [20]. The situation radically changes in the $m \neq 0$ case: (2.17) does not longer impose any differential constraint on $F^{ab}$ and should be rather regarded as the equation expressing $q$ through $F^{ab}$:

$$e^{-2mq} = \frac{1}{6m} \frac{(\partial \cdot F)}{1 - \sqrt{1 + 2F^2}}.$$

In terms of the independent $d = 3$ vector field $F^{ab}(x)$, the action (2.14) takes the form

$$S = -\frac{1}{2} \left(\frac{1}{6m}\right)^3 \int d^3x \frac{(\partial \cdot F)^3}{\left(1 - \sqrt{1 + 2F^2}\right)^2}.$$

Thus, instead of the familiar NG - BI duality of the flat case, in the AdS$_4$ case the membrane with the scale-invariant action (2.13) proves to be dual (in the above sense) to some non-gauge
\( d = 3 \) vector field theory with the strange action \((2.19)\) which is singular in the limit \( m \to 0 \). For the time being, the meaning of such a theory is unclear for us. We hope to say more on it elsewhere.

3 AdS\(_4\) supermembrane

Our starting point will be the \( N = 1 \) AdS\(_4\) superalgebra \( osp(1|4) \), once again in the basis which is a natural generalization of \((2.3)\) and is most convenient for our purposes

\[
\begin{align*}
\{ Q_a, Q_b \} &= 2P_{ab} , \quad \{ S_a, S_b \} = 2P_{ab} - 4mK_{ab} , \quad \{ Q_a, S_b \} = 2\varepsilon_{ab}D - 2mM_{ab} , \\
[M_{ab}, Q_c] &= \varepsilon_{ac}Q_b + \varepsilon_{bc}Q_a \equiv (Q)_{ab,c} , \quad [M_{ab}, S_c] = (S)_{ab,c} , \\
[K_{ab}, Q_c] &= (S)_{ab,c} , \quad [K_{ab}, S_c] = - (Q)_{ab,c} , \\
[P_{ab}, Q_c] &= 0 , \quad [P_{ab}, S_c] = -2m(Q)_{ab,c} , \quad [D, Q_a] = mQ_a , \quad [D, S_a] = -mS_a .
\end{align*}
\]

The bosonic generators are the same as in the previous Section. The generators \( Q_a, P_{ab}, M_{ab} \) form \( N = 1, d = 3 \) super Poincaré algebra. The generators \( S_a \) in this basis have the same dimension as \( Q_a \) (\( Q_a = (Q_a)^\dagger \), \( S_a = (S_a)^\dagger \)). The passing to the conformal basis, besides the redefinitions \((2.3)\), implies the rescaling \( S_a = m\tilde{S}_a \), such that the generator \( \tilde{S}_a \) has the dimension opposite to \( Q_a \) and so has the natural meaning of the \( d = 3 \) conformal supersymmetry generator. The advantage of the basis \((3.1)\) is that it manifests the \( N = 1, d = 3 \) super Poincaré subalgebra of \( osp(1|4) \) and still yields the \( N = 1, D = 4 \) super Poincaré algebra (in the \( d = 3 \) notation) in the contraction limit \( m = 0 \). The \( N = 1, d = 3 \) Poincaré supertranslations subalgebra \( \propto (Q_a, P_{ab}) \) together with the generator \( D \) form the maximal solvable subalgebra of \( osp(1|4) \).

We wish to construct a \( OSp(1|4) \) extension of the AdS\(_4\) membrane action \((2.13)\), such that it possesses a manifest \( N = 1, d = 3 \) supersymmetry extending the manifest \( d = 3 \) Poincaré worldvolume invariance of \((2.13)\), and becomes the known action of the flat \( N = 1, D = 4 \) supermembrane \([12]\) in the contraction limit \( m = 0 \). Just like the latter action is the Goldstone superfield action for the 1/2 breaking of the \( N = 1, D = 4 \) Poincaré supersymmetry down to the \( N = 1, d = 3 \) one, the action we are seeking for is expected to be a Goldstone superfield action describing the 1/2 spontaneous breaking of the \( OSp(1|4) \) supersymmetry down to its \( N = 1, d = 3 \) super Poincaré subgroup.

The construction of the AdS\(_4\) superbrane action is not so straightforward as in the bosonic case. Already in the case of flat \( N = 1, D = 4 \) supermembrane \([12]\) the correct action is by no means a covariant supervolume of the \( N = 1, d = 3 \) superspace. The corresponding superfield Lagrangian density is not a tensor, but it is rather of Chern-Simons or WZW type, since it is shifted by a full derivative under the nonlinearly realized half of full supersymmetry. At present, the only known way of constructing such Goldstone superfield actions is to start from a linear realization of the partially broken supersymmetry in some appropriate superspace. The nonlinear realization is then recovered by imposing proper covariant constraints on the corresponding superfields (see, e.g., \([21, 22]\)). The correct Goldstone superfield actions arise from some simple superfield invariants of the initial linear realization after enforcing these constraints. There is a systematic way of searching for such covariant constraints \([23, 24]\). It is a generalization of the analogous approach worked out for the case of the totally broken supersymmetry in \([25]\). Now we are going to show that these techniques, applied earlier in PBGS systems with rigid Poincaré supersymmetries, work fairly well also in the curved case at hand and give rise to the sought PBGS action of the AdS\(_4\) supermembrane.
As a first step we need to define the appropriate analog of the aforementioned linear realization. It turns out that in the AdS case it is already a sort of nonlinear realization, but with weaker nonlinearities compared to the final nonlinear realization which underlies the AdS supermembrane action.

As a natural superextension of the bosonic coset element \((2.3)\) we choose the following one:

\[
g = e^{e^{ab} P_{ab}} e^{\theta^a Q_a} e^{\psi^a S_a} e^{u(z) D} e^{\Lambda^{ab} (z) K_{ab}}. \tag{3.2}
\]

Here, the parameters \(z \equiv (x^{ab}, \theta^a, \psi^a)\) are \(N = 2, d = 3\) superspace coordinates, while \(u = u(z)\) and \(\Lambda^{ab}(z)\) are Goldstone superfields given on this superspace. The subspace spanned by the coordinate set \(\zeta \equiv (x^{ab}, \theta^a)\) is the standard flat \(N = 1, d = 3\) superspace in which \(N = 1, d = 3\) Poincaré supertranslations \(\propto (Q_a, P_{ab})\) are realized in a standard way:

\[
\delta x^{ab} = a^{ab} - \frac{1}{2} (\epsilon^a \theta^b + \epsilon^b \theta^a), \quad \delta \theta^a = e^a. \tag{3.3}
\]

These transformation laws are obtained by acting on \((3.2)\) from the left by the group element \(g_0 = e^{a^{ab} P_{ab}} e^{\eta^a S_a}\).

The rest of the \(OSp(1|4)\) transformations except for the \(SO(1,2)\) rotations is nonlinearly realized on the coset coordinates, mixing the \(N = 2\) superspace coordinates with the Goldstone superfield \(u(z)\). Acting on \((3.2)\) from the left by different supergroup elements, it is easy to find the explicit form of these transformations.

**Broken supersymmetry:** \(g_0 = e^{\eta^a S_a},\)

\[
\delta x^{ab} = 2m \left( \theta^a x^{bc} + \theta^b x^{ac} \right) \eta_c + \frac{1}{2} e^{4m u} \left( \psi^a \eta^b + \psi^b \eta^a \right) + \frac{3}{2} me^{4m u} \psi^2 \left( \theta^a \theta^b + \theta^b \theta^a \right),
\]

\[
\delta \theta^a = 4m x^{ac} \eta_c + m \theta^2 \eta^a - 3me^{4m u} \psi^2 \eta^a, \quad \delta u = 2\theta^a \eta_a,
\]

\[
\delta \psi^a = \eta^a - 2m \left( \theta^b \theta^a \psi_b - \eta^a \theta^b \psi_b - \eta^b \theta^a \psi_b \right). \tag{3.4}
\]

**Dilatations:** \(g_0 = e^{\alpha D},\)

\[
\delta x^{ab} = 2\alpha m x^{ab}, \quad \delta \theta^a = \alpha m \theta^a, \quad \delta \psi^a = -\alpha m \psi^a, \quad \delta u = \alpha. \tag{3.5}
\]

As follows from \((3.1)\), all bosonic transformations (including the dilatations) are actually contained in the closure of the supersymmetry transformations, so it is not necessary to explicitly quote them here.

Covariant derivatives of the Goldstone superfield \(u(z)\) can be constructed by the supercoset element \((3.2)\) following the generic guidelines of the nonlinear realizations method. Of actual need for us will be spinor covariant derivatives. Without entering into details, these are as follows

\[
\tilde{\nabla}^Q_a u = \frac{1}{\sqrt{1 - 2\lambda^2}} \left( \nabla^Q_a - 2\lambda^b \nabla^S_b \right) u, \quad \tilde{\nabla}^S_a u = \frac{1}{\sqrt{1 - 2\lambda^2}} \left( \nabla^S_a + 2\lambda^b \nabla^Q_b \right) u, \tag{3.6}
\]

\[
\nabla^Q_a u = e^{e^{mu}} \left( D_a - m \psi^2 \frac{\partial}{\partial \psi^a} \right) u - 2e^{e^{mu}} \psi_a \equiv \tilde{\nabla}^Q_a u - 2e^{e^{mu}} \psi_a,
\]

\[
\nabla^S_a u = e^{-e^{mu}} \left[ \frac{\partial}{\partial \psi^a} + e^{e^{mu}} \left( \psi^b \partial_{ab} + 3m \psi^2 D_a \right) \right] u. \tag{3.7}
\]
is the standard covariant spinor derivative of \( N = 1, d = 3 \) Poincaré supersymmetry. The Goldstone superfield \( \lambda^{ab} \) is related to \( \Lambda^{ab} \) like in the bosonic case. Actually, in what follows we shall need only the “semi-covariant” derivatives \((3.7)\), and the superfield \( \lambda^{ab} \) will never appear in further consideration.

What we have at this stage, is a nonlinear realization of the \( N = 1 \) AdS\(_4\) supergroup on the \( N = 2, d = 3 \) Goldstone superfield \( u(x, \theta, \psi) \):

\[
\delta^* u(x, \theta, \psi) = -\left( \delta x^{ab}_a \partial_{ab} + \delta \theta^a \partial^b_a + \delta \psi^a \partial^\psi_a \right) u(x, \theta, \psi) + 2\theta^a \eta_a ,
\]

(3.9)

where \( \delta^* \) means the “active” form of the infinitesimal transformations. The first component in the \( \theta, \psi \) expansion of \( u \) can be regarded as the Goldstone dilaton field discussed in the previous Section. The spinor derivative \( D_a u \) is shifted by \( \eta_a \) under the \( S - \)supersymmetry, suggesting that we actually face the 1/2 spontaneous breaking of the AdS\(_4\) supersymmetry, with \( D_a u|_{\psi=0} \) as the corresponding Goldstone fermionic \( N = 1 \) superfield. However, \( u \) contains quite a few extra component fields having no immediate Goldstone interpretation.

To construct the minimal Goldstone multiplet, we shall generalize the method which was applied in \[24\] to \( d = 2 \) PBGS systems and, in \[4\], to the case of flat-space \( N = 1, D = 4 \) supermembrane. Following the reasonings of \[4\] and keeping in mind that the minimal scalar multiplets of \( N = 1 \) AdS\(_4\) supergroup are represented by chiral \( N = 1, D = 4 \) or \( N = 2, d = 3 \) superfields, \[4\] we shall regard the Goldstone superfield \( u(z) \) to be complex and subject it to the covariant chirality constraint

\[
\left( \nabla^Q_a + i \nabla^S_a \right) u = 0 .
\]

(3.10)

This condition is equivalent to the similar one in terms of the full covariant derivatives \((3.6)\) in view of the relation

\[
\nabla^Q_a \pm i \nabla^S_a = \frac{\delta^b_a \pm i \lambda^b_a}{\sqrt{1 - \mu^2}} \left( \tilde{\nabla}^Q_b \pm i \tilde{\nabla}^S_b \right) ,
\]

(3.11)

and so it is covariant with respect to the whole \( OSp(1|4) \). Note that the transformation law \((3.9)\) of the complex superfield \( u \) cannot be rewritten in an equivalent passive form, because \( \delta x^{ab}, \delta \theta^a, \delta \psi^a \) defined by eqs. \((3.4)\) are essentially complex due to the presence of \( u(z) \). Nevertheless, the transformations \((3.9)\) are still self-consistent just because of complexity of \( u \) and can be checked to have the correct closure.

Using the explicit form of the covariant derivatives \((3.7)\), it is straightforward to check that \((3.10)\) is self-consistent, in the sense that the appropriate integrability condition is satisfied

\[
\{ \nabla^Q_a - i \nabla^S_a, \tilde{\nabla}^Q_b - i \tilde{\nabla}^S_b \} u - 2 \left( \nabla^Q_a + i \nabla^S_a \right) e^{mu} \psi_b - 2 \left( \tilde{\nabla}^Q_b - i \tilde{\nabla}^S_b \right) e^{mu} \psi_a = T^c_{ab} \left( \nabla^Q_c - i \nabla^S_c \right) u .
\]

(3.12)

\(^1\)From the structure relations \((3.1)\) it is seen that the complex combinations of spinor generators \( Q_a + i S_a \) or \( Q_a - i S_a \) form closed subgroups together with the bosonic \( SO(1,3) \) generators \( K_{ab}, M_{cd} \), thus showing the existence of two conjugated chiral subspaces in the \( OSp(1|4)/SO(1,3) \) superspace \((x^{ab}, \theta^a, \psi^a, q) \[16\].

\(^2\)Such a geometric equivalent form of this transformation is expected to exist in a complex chiral basis where \((3.10)\) become Grassmann Cauchy-Riemann condition. Here we shall not elaborate on this point.
The constraint (3.10) can be solved by expanding \( u \) in powers of \( \psi^a \). It is easy to check that (3.10) expresses all terms in this expansion in terms of \( u_0 \equiv u|_{\psi^a=0} \) and \( D_a \) derivatives of \( u_0 \). In particular,

\[
\frac{\partial u}{\partial \psi^a}|_{\psi=0} = -ie^{2mu}D_a u|_{\psi=0} .
\]

(3.13)

Thus the complex \( N = 1, d = 3 \) superfield

\[
u_0(x, \theta) \equiv q(x, \theta) + i\Phi(x, \theta) , \quad q^\dagger = -q , \quad \Phi^\dagger = -\Phi ,
\]

(3.14)

incorporates the full irreducible field content of the \( N = 2, d = 3 \) Goldstone chiral superfield \( u(x, \theta, \psi) \). Its \( S \)-supersymmetry transformation reads

\[
\delta u_0 = Lu_0 + 2\eta^a \theta_a + ie^{2mu_0} \eta^a D_a u_0 ,
\]

(3.15)

where

\[
L \equiv -m \left( \theta^2 \eta^a + 4x^{ab} \eta_b \right) \frac{\partial}{\partial \theta^a} + 4m \eta_c \theta^b \partial^{ac} \partial_{ab} .
\]

(3.16)

For the imaginary and real parts of \( u_0 \) eq. (3.15) implies the following transformation rules

\[
\delta q = Lq - e^{2mq} \eta^a \left[ \sin(2m\Phi) D_a q + \cos(2m\Phi) D_a \Phi \right] + 2\eta^a \theta_a ,
\]

\[
\delta \Phi = L\Phi + e^{2mq} \eta^a \left[ \cos(2m\Phi) D_a q - \sin(2m\Phi) D_a \Phi \right] .
\]

(3.17)

The nonlinear realization we have at this step is still non-minimal in the following sense. Besides the \( N = 1 \) superfield \( q(x, \theta) \) which contains all Goldstone fields required by the 1/2 breaking of \( OSp(1|4) \) down to its \( N = 1, d = 3 \) Poincaré subgroup \( (q)|_{\theta=0} \) for the dilatations, \( (D_a q)|_{\theta=0} \) for the broken \( S \)-transformations and \( \partial_a q|_{\theta=0} \) for the broken \( SO(1, 3)/SO(1, 2) \) transformations), there is an extra non-Goldstone \( N = 1, d = 3 \) superfield \( \Phi(x, \theta) \). The last step is to eliminate the latter in terms of \( q \) and its derivatives by imposing some nonlinear covariant constraint on \( u_0(x, \theta) \), analogous to the constraints imposed in the flat case [12]. The precise form of such a constraint can be found by applying the general procedure of refs. [24, 23, 24] to the given case. It goes straightforwardly, but it is rather technical in view of more nonlinearities involved as compared to the flat case. So we skip details and present the final form of the constraint

\[
\Phi = \frac{e^{2mq} D^a q D_a q}{4 + e^{2mq} D^2 \Phi} .
\]

(3.18)

It can be directly checked to be covariant with respect to (3.17).

From our superfield \( u_0 \) we can construct the following two simplest invariants (up to normalization factors)

\[
S_1 = \frac{1}{2} \int d^3 x d^2 \theta \left( e^{-4mu_0} + e^{4mu_0^\dagger} \right) = \int d^3 x d^2 \theta e^{-4mq} \cos(4m\Phi) ,
\]

(3.19)

and

\[
S_2 = -\frac{1}{2im} \int d^3 x d^2 \theta \left( e^{-4mu_0} - e^{4mu_0^\dagger} \right) = \frac{1}{m} \int d^3 x d^2 \theta e^{-4mq} \sin(4m\Phi) .
\]

(3.20)

After solving the constraint (3.18)

\[
\Phi = \frac{e^{2mq} D^a q D_a q}{2 + \sqrt{4 + e^{4mq} D^2 (D^b q D_b q)}} ,
\]

(3.21)
in the Lagrangians in $S_1$ and $S_2$ only lowest terms in $\Phi$ survive in view of the nilpotency of $\Phi$ in (3.21). So the actions take the form

$$S_1 \sim \int d^3x d^2 \theta e^{-4mq},$$

(3.22)

$$S_2 \sim \int d^3x d^2 \theta \frac{e^{-2mq} D^a q D_a q}{2 + \sqrt{4 + e^{4mq} D^2 (D^b q D_b q)}},$$

(3.23)

Relevant for our purposes is just $S_2$, because it contains the kinetic term of the Goldstone superfield $q(\zeta)$. Its bosonic part, with the fermions omitted and the auxiliary field $B = D^2 q|_{\theta = 0}$ eliminated by its equation of motion ($B = 0$ in the vanishing fermions limit), is recognized as the AdS$_4$ membrane action (2.13).

We come to the conclusion that the Goldstone superfield action (3.23) is the natural superextension of the conformally-invariant AdS$_4$ membrane action. Besides being manifestly invariant under $N = 1, d = 3$ Poincaré supersymmetry, it is invariant under the nonlinearly realized part of $N = 1$ AdS$_4$ supersymmetry $OSp(1|4)$ which acts on the $N = 1, d = 3$ worldvolume as the Goldstone superfield-modified $d = 3$ superconformal transformations (eqs. (3.17) with (3.21) standing for $\Phi$). Thus it is a PBGS superfield form of the worldvolume action of $N = 1$ AdS$_4$ supermembrane. In the limit $m \to 0$, it reproduces the known PBGS action of $N = 1, D = 4$ supermembrane in the flat Minkowski background [12].

Finally, let us comment on the effect of adding extra invariant (3.22). The bosonic part of the modified action $S_2 + \alpha m^{-1} S_1, \alpha^\dagger = \alpha$, reads

$$S_2^{bos'} \sim \int d^3 x \left[ e^{-6mq} \left( 1 - \sqrt{1 - \frac{1}{4} e^{4mq} (2\partial q \cdot \partial q + B^2)} + 2\alpha e^{-4mq} B \right) \right].$$

(3.24)

After elimination of $B$ by its algebraic equation of motion, the bosonic action, again up to an overall renormalization factor, becomes

$$S_2^{bos'} \sim \int d^3 x e^{-6mq} \left( \frac{1}{\sqrt{1 + 16\alpha^2}} - \sqrt{1 - \frac{1}{2} e^{4mq} \partial q \cdot \partial q} \right).$$

(3.25)

Thus, the effect of non-zero parameter $\alpha$ amounts to the appearance of a sort of “cosmological” term on the worldvolume of the AdS$_4$ membrane (the structure of fermionic terms is also changed). The possibility of adding such an invariant term already exists in the flat case [12, 15], where $\alpha m^{-1} S_1 \to -\alpha \int d^3 x d^2 \theta q(\zeta)$.

4 Conclusions

In this paper, proceeding from a 1/2 partial breaking of the $N = 1$ AdS$_4$ supersymmetry in the nonlinear realizations description, we have constructed the worldvolume superfield action (3.23) for AdS$_4$ supermembrane. It is the first example of the complete PBGS Goldstone superfield action for superbranes on curved superbackgrounds and, in particular, for AdS superbranes. Its main characteristic feature is that the spontaneously broken part of $OSp(1|4)$ is realized as a Goldstone-superfield modified $d = 3$ superconformal symmetry. Like in the case of flat supermembrane [12], the superfield Lagrangian density of the AdS$_4$ supermembrane PBGS
action is not of a tensor form, it is shifted by a full derivative under the broken part of $OSp(1|4)$ transformations. In this sense, it resembles WZW or Chern-Simons terms.

Our consideration here can be regarded as a first step towards constructing analogous world-volume superfield actions for more interesting examples of branes on the superbackgrounds with the $AdS_n \times S^m$ bosonic part, including the appropriate $D_p$-branes. It still remains to be examined how such actions are related to the more familiar Green-Schwarz type ones. Usually, the component on-shell form of PBGS actions coincides with a static-gauge form of the appropriate G-S actions, with $\kappa$-supersymmetry also being properly fixed [26]. Their full off-shell superfield form can be recovered from the superembedding approach [27]. It would be of interest to establish similar relationships for PBGS actions of AdS branes, in particular, for the action constructed here. Note that there is a problem of the most convenient choice of the $\kappa$-symmetry gauge-fixing in the worldvolume actions of AdS superbranes (see, e.g., [18, 10, 29]). The PBGS approach yields the superbrane actions at once in terms of the physical worldvolume degrees of freedom (after elimination of the auxiliary fields, if they are present), therefore no problems related to the non-uniqueness of the $\kappa$-gauge fixing can arise in this approach.

Finally, let us shortly comment on some related works.

The partial breaking of $D = 4$ superconformal symmetries down to the corresponding Poincaré supersymmetries in the nonlinear realizations superspace framework was considered in [13]. In these studies, the superconformal symmetries are not regarded as AdS ones in higher dimensions and their realization on the Poincaré superspace coordinates has the standard form [30] involving no nonlinear Goldstone superfield terms (appearing in our AdS realization). Respectively, the Goldstone superfield actions of [13] admit no superbrane interpretation.

In [31], in a similar nonlinear realizations setting, a partial breaking of $N = 2$ AdS supersymmetries in $D = 3, 2$ to their $N = 1$ AdS subgroups was considered. The basic Goldstone superfield was found to be associated with an internal $U(1)$ symmetry generator. The corresponding invariant Goldstone superfield action (yet to be constructed) also seems to bear no direct links to AdS superbranes. Besides, it should be manifestly invariant under $N = 1$ AdS$_{3, 2}$ supersymmetries and so explicitly include the superspace coordinates, along the line of ref. [16]. This is in contrast with our action (3.23) which reveals manifest $N = 1, d = 3$ Poincaré supersymmetry.

In a recent preprint [32], a nonlinear realization of $N = 1, D = 4$ superconformal symmetry $SU(2, 2|1)$ treated as $N = 1$ AdS$_5$ supersymmetry was constructed. Conceptually, the approach of [32] is close to ours. However, the invariant $N = 1, d = 4$ Goldstone superfield action suggested there seems not to be the appropriate one to describe AdS$_5$ super 3-brane. Its Lagrangian (covariantized $N = 1, d = 4$ supervolume) behaves as a density under the broken transformations, while the correct minimal PBGS superbrane action is expected to be of non-tensor type, like the action constructed here.

**Acknowledgements**

We are grateful to D. Sorokin, M. Tonin and B. Zupnik for useful discussions. The work of E.I. and S.K. was supported in part by the grants RFBR-CNRS 98-02-22034, RFBR-DFG-99-02-04022, RFBR 99-02-18417, INTAS-00-00254, NATO Grant PST.CLG 974874 and PICS Project No. 593. E.I. thanks the Directorate of ENS-Lyon for the hospitality extended to him during the course of this work.
References

[1] J. Bagger, J. Wess, Phys. Lett. B 138 (1984) 105.

[2] J. Hughes, J. Liu, J. Polchinski, Phys. Lett. B 180 (1986) 370; J. Hughes, J. Polchinski, Nucl. Phys. B 278 (1986) 147.

[3] S. Bellucci, E. Ivanov, S. Krivonos, Nucl. Phys. Proc. Suppl. 102 (2001) 26.

[4] E. Ivanov, “Superbranes and Super Born-Infeld Theories as Nonlinear Realizations”, hep-th/0105210.

[5] J. Maldacena, Adv. Theor. Mah. Phys. 2 (1998) 231.

[6] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys. Lett. B 428 (1998) 105.

[7] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253.

[8] P. Claus, R. Kallosh, J. Kumar, P.K. Townsend, A. Van Proeyen, JHEP 9806 (1998) 004.

[9] R.R. Metsaev, A.A. Tseytlin, Nucl. Phys. B 533 (1998) 109; Phys. Lett. B 436 (1998) 281.

[10] R. Kallosh, J. Rahmfeld, Phys. Lett. B 443 (1998) 143; R. Kallosh, J. Rahmfeld, A. Rajaraman, JHEP 9809 (1998) 002.

[11] R. Kallosh, J. Kumar, A. Rajaraman, Phys. Rev. D 57 (1998) 6452.

[12] E. Ivanov, S. Krivonos, Phys. Lett. B 453 (1999) 237.

[13] K. Kobayashi, T. Uematsu, Nucl. Phys. B 263 (1986) 309; K. Kobayashi, K.-H. Lee, T. Uematsu, Nucl. Phys. B 309 (1988) 669; Y. Gotoh, T. Uematsu, Phys. Lett. B 420 (1998) 69.

[14] P. West, “Automorphisms, Non-linear Realizations and Branes”, hep-th/0001216.

[15] E. Ivanov, “Diverse PBGS patterns and Superbranes”, hep-th/0002204.

[16] E.A. Ivanov, A.S. Sorin, J. Phys. A13 (1980) 1159.

[17] L. Castellani, A. Ceresole, R. D’Auria, S. Ferrara, P. Fre, M. Trigiante, Nucl. Phys. B 527 (1998) 142.

[18] P. Pasti, D. Sorokin, M. Tonin, Phys. Lett. B 447 (1999) 251; “Branes in super-AdS background and superconformal theories”, hep-th/9912076.

[19] E.A. Ivanov, V.I. Ogievetsky, Teor. Mat. Fiz. 25 (1975) 164.

[20] A.A. Tseytlin, Nucl. Phys. B 469 (1996) 51.

[21] J. Bagger, A. Galperin, Phys. Rev. D 55 (1997) 1091; Phys. Lett. B 412 (1997) 296.

[22] M. Roček, A. Tseytlin, Phys. Rev. D 59 (1999) 106001.
[23] F. Delduc, E. Ivanov, S. Krivonos, Nucl. Phys. B 576 (2000) 196.

[24] E. Ivanov, S. Krivonos, O. Lechtenfeld, B. Zupnik, Nucl. Phys. B 600 (2001) 235.

[25] E. Ivanov, A. Kapustnikov, J. Phys. A 11 (1978) 2375; J. Phys. G 8 (1982) 167.

[26] A. Achucarro, J. Gauntlett, K. Itoh, P.K. Townsend, Nucl. Phys. B 314 (1989) 129.

[27] P. Pasti, D. Sorokin, M. Tonin, Nucl. Phys. B 591 (2000) 109; “Geometrical aspects of superbranes dynamics”, hep-th/0011020; I. Bandos, P. Pasti, A. Pokotilov, D. Sorokin, M. Tonin, “Space Filling Dirichlet 3-Brane in N = 2, D = 4 Superspace” hep-th/0103152.

[28] J.M. Drummond, P.S. Howe, “Codimension zero superembeddings”, hep-th/0103191.

[29] R. Kallosh, A.A. Tseytlin, JHEP 9810 (1998) 016.

[30] A. Ferber, Nucl. Phys. B 132 (1978) 55.

[31] M. Sano, T. Uematsu, Phys. Lett. B 503 (2001) 413.

[32] S.M. Kuzenko, I.N. McArthur, “Goldstone multiplet for partially broken superconformal symmetry”, hep-th/0109183.