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The construction of partner potential from the general potential Rosen-Morse and Manning Rosen in 4 dimensional Schrodinger system

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Abstract. The solution of the Schrödinger equation with physical potential is the important part in quantum physics. Many methods have been developed to resolve the Schrödinger equation. The Nikiforov-Uvarov method and supersymmetric method are the most methods that interesting to be explored. The supersymmetric method not only used to solve the Schrödinger equation but also used to construct the partner potential from a general potential. In this study, the Nikiforov-Uvarov method was used to solve the Schrödinger equation while the supersymmetric method was used to construction partner potential. The study about the construction of the partner potential from general potential Rosen-Morse and Manning Rosen in D-dimensional Schrödinger system has been done. The partner potential was obtained are solvable. By using the Nikiforov-Uvarov method the eigenfunction of the Schrödinger equation in D-dimensional system with general potential Rosen-Morse and Manning Rosen and the Schrödinger equation in D-dimensional system with partner potential Rosen-Morse and Manning Rosen are determined. The eigenfunctions are different between the Schrödinger equation with general potential and the Schrödinger potential with the partner potential.

1. Introduction
The Schrödinger equation is the important part in quantum physics. The Schrödinger equation can describe the characteristics of particles in the quantum potential field. Many methods have been developed to solve the Schrödinger equation. For example the Nikiforov-Uvarov method [1-3], the supersymmetric method [4-6], the Ansatz wave function method [7-9], the asymptotic iteration method (AIM)[10-12], and other [13-14]. In this study, we used the Nikiforov-Uvarov method and the supersymmetric method. The Nikiforov-Uvarov method was used to solve the Schrödinger equation while the supersymmetric method was used to construction partner potential. The Nikiforov-Uvarov differential equation was the equation that formulated from the general hypergeometric differential equation.

Some study about construction partner potential has been done. For example the construction partner potential from the Hulthen potential [15] and the construction partner potential from the Hylleraas potential [16]. The main idea of the construction partner potential is using the supersymmetric operators, the ground state eigenfunction and the original potential. In this study, we choose potential Rosen-Morse and potential Manning Rosen as the original potential. The general form of Rosen-Morse and Manning Rosen potential are [5]
\[ V_i(\theta_i) = \frac{\hbar^2}{2m} \left( \frac{\nu(\nu+1)}{\cos^2 \theta_i} - 2q \tan \theta_i \right) \]

where \( \nu \) and \( q \) are potential parameters, \( m \) is mass and \( \hbar \) is reduced Planck constant.

The first section of this paper we will review about Nikiforov-Uvarov method and supersymmetric method. The next section we will explain about the Schrodinger equation in D-dimensional system. The third section we will present about the solution of D-dimensional Schrodinger equation with potential Rosen-Morse and Manning Rosen using Nikiforov-Uvarov method. The next section we will present about construction partner potential from origin potentials Rosen-Morse and Manning Rosen in D-dimensional Schrodinger system. The next section we will present about the solution of D-dimensional Schrodinger Equation with partner potential Rosen-Morse and Manning Rosen. In this study, we obtained the eigenfunction of Schrodinger equation with potential original and Schrodinger equation with the partner potential.

2. Method

2.1. Nikiforov-Uvarov method

The Schrodinger equation with physical potential field reduced become hypergeometric differential equation type Nikiforov-Uvarov by the variable substitution method. The general equation of Nikiforov-Uvarov differential equation as follows [17]

\[ \frac{\partial^2 \Psi(s)}{\partial s^2} + \frac{\bar{r}(s) \partial \Psi(s)}{\sigma(s)} - \frac{\bar{\sigma}(s)}{\sigma^2} \Psi(s) = 0 \]  

with \( \sigma(s) \) and \( \bar{\sigma}(s) \) are second order polynomials, \( \bar{r}(s) \) is first order polynomials. By set \( \Psi = \phi(s)y(s) \), Eq. (2) become

\[ \sigma \frac{\partial^2 y}{\partial s^2} + \tau(s) \frac{\partial y}{\partial s} + \lambda y(s) = 0 \]

The first part eigenfunction as follows

\[ \frac{\phi'}{\phi} = \frac{\pi}{\sigma} \]  

and we obtained the equation that will use to determined energy spectra and the second part eigenfunction \( y_n \) as follow

\[ \pi = \left( \frac{\sigma - \tau}{2} \right) \pm \sqrt{\left( \frac{\sigma - \tau}{2} \right)^2 - \sigma + k\sigma} \]

and

\[ \lambda = k + \pi', \quad \tau = \tau + 2\pi, \quad \lambda_n = -n\pi' - \frac{n(n-1)}{2} \sigma, n = 0, 1, 2, \ldots \]

The \( k \) in Eq. (5) obtained from the condition that the value under square-root of Eq. (5) was first order polynomials and perfect quadratic so the discriminate of the quadratic equation is zero. The energy in Eq. (4) obtained from the condition \( \pi' < 0 \). The second eigenfunction \( y_n(s) \) satisfy the Rodrigues relation, as follow

\[ y_n(s) = \frac{C_n}{\rho(s)} \frac{d^n}{ds^n} \left[ \sigma^n(s) \rho(s) \right] \]
where \( C_n \) is normalization constant, and \( \rho(s) \) satisfy the condition
\[
\frac{\partial(\sigma \rho)}{\partial s} = \tau(s) \rho(s) \tag{8}
\]

If the general hypergeometric equation that we get as [18]
\[
\frac{\partial^2 \psi(s)}{\partial s^2} + \left( c_1 - c_2 s \right) \frac{\partial \psi(s)}{\partial s} + \left( -\epsilon_1 s^2 + \epsilon_2 s - \epsilon_3 \right) \psi(s) = 0 \tag{9}
\]
by compared Eq. (9) with Eq. (2), we obtained the eigenvalue as
\[
c_2 n - (2n + 1) c_3 + (2n + 1) \left( \sqrt{c_9} + c_3 \sqrt{c_8} \right) + n(1 - c_3) c_3 + c_7 + 2c_3 c_8 + 2\sqrt{c_9 c_8} = 0 \tag{10}
\]
and the eigenfunction as follows
\[
\psi(s) = N_{np} s^{c_2/(1 - c_3 s)} - c_2 P_n^{c_0 - 1/(c_1/c_3) - c_0 - 1} (1 - 2c_3 s) \tag{11}
\]
with \( N_{np} \) is normalization constant \( P \) is Jacobi polynomials and
\[
c_4 = \frac{1}{2} (1 - c_1), c_5 = \frac{1}{2} (c_2 - 2c_3), c_6 = c_5^2 + \epsilon_1, c_7 = 2c_4 c_5 - \epsilon_2, c_8 = c_4^3 + \epsilon_3, \\
c_9 = c_3 c_7 + c_3^2 c_8 + c_6, c_{10} = c_1 + 2c_4 + 2\sqrt{c_9} c_{12} = c_4 + \sqrt{c_9}, \\
c_{11} = c_2 - 2c_3 + 2 \left( \sqrt{c_9} + c_3 \sqrt{c_8} \right), c_{13} = c_5 - \left( \sqrt{c_9} + c_3 \sqrt{c_8} \right), c_{14} = c_{12} + \frac{c_{11}}{c_3} \tag{12}
\]

2.2. Supersymmetric method

Definition of supersymmetric Hamiltonian \( H_{ss} \) partner for change operator that commute with \( H_{ss} \) by Witten as follow[19]
\[
H_{ss} = \begin{pmatrix}
\frac{d^2}{dx^2} + \frac{d \varphi(x)}{dx} + \varphi^2(x) & 0 \\
0 & -\frac{d^2}{dx^2} - \frac{d \varphi(x)}{dx} + \varphi^2(x)
\end{pmatrix} = \begin{pmatrix}
H_+ & 0 \\
0 & H_-
\end{pmatrix} \tag{13}
\]
The Hamiltonian partner \( H_+ = H_1 \) and \( H_- = H_2 \), and partner potential \( V_+ = V_1 \) and \( V_- = V_2 \) are
\[
V_-(x) = V_1 = \varphi^2(x) - \varphi'(x) \quad \text{and} \quad V_+(x) = V_2 = \varphi^2(x) + \varphi'(x) \tag{14}
\]
The relation between effective potential \( V_{eff} \) and first potential \( V_1 \) as follow
\[
V_{eff}(x) = V_-(x; a_0) + E_0 = V_1(x; a_0) \tag{15}
\]
with \( E_0 \) is ground state energy for effective potential. By setting the new supersymmetric operator are

the lowering operator \( A \) and the rising operator \( A^\dagger \) as
\[
A^\dagger = -\frac{d}{dx} + \varphi(x), \\
A = \frac{d}{dx} + \varphi(x) \tag{16}
\]
so the Hamiltonian partner in Eq. (13) can be rewritten as
\[
H_-(x) = H_1 = A^\dagger A, \\
H_+(x) = H_2 = AA^\dagger \tag{17}
\]
and also
\[ A\psi^{(-)}_0 = A\psi_0 = 0 \] (18)
The new partner potential \( V_2 \) from Eq. (15) and Eq. (18) we get the reaction [16]
\[ V_2(x) = V_1 - 2 \frac{d^2}{dx^2} \ln \psi_0 = V_{cf} - E_0 - 2 \frac{d^2}{dx^2} \ln \psi_0 \] (19)
This equation will be used to construct the partner potential from the original potential in Eq. (1).

3. Results and discussion

3.1. The Schrodinger equation in D-dimensional system

The general Schrodinger equation in D-dimensional system [20]
\[ -\frac{\hbar^2}{2m} \nabla^2 D \Psi(r,\Omega) + V(r,\Omega)\Psi(r,\Omega) = E\Psi(r,\Omega) \] (20)
with \( D \) is dimensional and the Laplacian operator as [21]
\[ \nabla^2 D = \frac{1}{r^{D-1}} \frac{\partial}{\partial r} \left( r^{D-1} \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \sum_{j=1}^{D-2} \frac{1}{\sin^2 \theta_j \sin^2 \theta_{j+1} \cdots \sin^2 \theta_{D-1}} \times \]
\[ \left\{ \frac{1}{\sin^2 \theta_j} \left( \frac{\partial}{\partial \theta_j} \sin^{j-1} \theta_j \frac{\partial}{\partial \theta_j} \right) \right\} + \frac{1}{r^2} \left\{ \frac{1}{\sin^2 \theta_{D-1}} \left( \frac{\partial}{\partial \theta_{D-1}} \sin^{D-2} \theta_{D-1} \frac{\partial}{\partial \theta_{D-1}} \right) \right\} \] (21)
By using variable separation method from Eq. (20) we get the D-dimensional Schrodinger equation radial part and polar part. The polar part D-dimensional Schrodinger equation for \( D = 4 \) as follow
\[ \frac{1}{\sin^2 \theta_1 \sin^2 \theta_2} \left( \frac{\partial^2}{\partial \theta_1^2} \right)^2 + \frac{1}{\sin^2 \theta_3} \frac{\partial}{\partial \theta_3} \left( \sin \theta_3 \frac{\partial}{\partial \theta_3} \right) \frac{V(\theta_1)}{\sin^2 \theta_2 \sin^2 \theta_3} + \frac{2m}{\hbar^2} \left( \frac{V(\theta_3)}{\sin^2 \theta_3} + V(\Omega_2) \right) = \lambda_1 Y'''(\Omega_2) \] (22)
To simplify calculation we set \( h = 1, 2m = 1 \). By setting \( Y'''(\Omega_2) = P_1(\theta_1), P_2(\theta_2), P_3(\theta_3) \), and \( V_1(\theta_1), V_2(\theta_2), V_3(\theta_3) \) from Eq. (1) and By using algebra method, from Eq. (22) we have the Schrodinger equation for \( \theta_1 \) as follow
\[ \frac{1}{R_1(\theta_1)} \left( \frac{\partial^2}{\partial \theta_1^2} R_1(\theta_1) \right) - V(\theta_1) + \lambda_1 = 0 \] (23)
the Schrodinger equation for \( \theta_2 \) as follow
\[ \frac{1}{R_2(\theta_2)} \left( \frac{\partial}{\partial \theta_2} \left( \sin \theta_2 \frac{\partial}{\partial \theta_2} \right) \right) - V(\theta_2) - \frac{\lambda_1}{\sin^2 \theta_2} = -\lambda_2, \] (24)
and the Schrodinger equation for \( \theta_3 \) as follow

4
\[
\frac{1}{P_i \theta_i} \left( \frac{\partial}{\sin^2 \theta_i} \frac{\partial P_i}{\partial \theta_i} \right) - \frac{V(\theta_i) - \lambda_2}{\sin^2 \theta_i} + \lambda_3 = 0
\]  
(25)

where \( V(\theta) \) in Eq. (1). The eq. (23), (24) and (25) are the formula that we will solve at next section.

3.2. The Solution of 4 dimensional Schrödinger equation with potential Rosen-Morse and Manning Rosen

First calculation, the solution Schrödinger equation for \( \theta_i \) with Rosen-Morse potential by using the Nikiforov-Uvarov method. By substituting Eq. (1) into Eq. (23) and multiplied with \( P_i(\theta_i) \) we have

\[
\left( \frac{\partial^2 P_i(\theta_i)}{\partial \theta_i^2} \right) - \left( \frac{\nu(\nu + 1)}{\cos^2 \theta_i} - 2q \tan \theta_i \right) P_i(\theta_i) + \lambda_i P_i(\theta_i) = 0
\]

(26)

By set
\[ i(1 - 2s_i) = \tan \theta_i \]
(27)

from Eq. (26) we get

\[
\frac{\partial^2 P_i(s_i)}{\partial s_i^2} + \frac{(1 - 2s_i)}{s_i(1 - s_i)} \frac{\partial P_i(s_i)}{\partial s_i} P_i(s_i) - \frac{(\nu(\nu + 1))}{s_i^2(1 - s_i)^2} P_i(s_i)
\]

\[
+ \frac{(-[\nu(\nu + 1) + qi])}{s_i^2(1 - s_i)^2} P_i(s_i) - \frac{q^i \lambda_i}{s_i^2(1 - s_i)^2} P_i(s_i) = 0
\]

(28)

By compared Eq. (28) with Eq. (9) we get

\[ c_1 = 1, c_2 = 2, c_3 = 1, \varepsilon_i = (-\nu(\nu + 1)). \]

\[ \varepsilon_2 = (-[\nu(\nu + 1) + qi]), \varepsilon_3 = \left( -\frac{q^i \lambda_i}{4} \right) \]

(29)

By using the relation in Eq. (12) and Eq. (11) we get the eigenfunction as follow

\[
\psi(\theta_i) = \left[ \frac{N_{nl}}{2^{1/2} \sqrt{\frac{1 - \nu(\nu + 1)}{4}}} \right] \left( 1 + i \tan \theta_i \right) \sqrt{\frac{1 - \nu(\nu + 1)}{2}} \left( -i \tan \theta_i \right), \quad n = 0, 1, 2, 3, \ldots
\]

(30)

with \( N_{nl} \) is normalization constant and \( P \) is Jacobi polynomials. By using Eq. (10) we get

\[
\lambda_i = \frac{4q^2 - 1}{4 \left( 2n + 1 \pm \sqrt{1 - 4\nu(\nu + 1)} \right)^2}, \quad n = 0, 1, 2, 3, \ldots
\]

(31)

Second calculation, the solution of Schrödinger equation for \( \theta_i \) with Manning Rosen potential using Nikiforov-Uvarov method. Substituting q. (1) into Eq. (24) and then multiplied by \( Y_2(\theta_i) \) and by setting \( Y_2(\theta_i) = P_2(\theta_i) \sin^{-1/2} \theta_i \) we have
by setting \(i(1-2s_2) = \cot \theta_2\), and by using the steps used to solve the first part of angular Schrodinger equation in Eq (26), we obtain the the solution of Eq. (34) as follow

\[
\psi(\theta_2) = \left[ \frac{N_{nl}}{2} \left( \frac{1+i \cot \theta_2}{\sin^2 \theta_2} \right)^{\frac{1}{2}} \left( \frac{1-i \cot \theta_2}{2} \right)^{\frac{1}{2}} \right] P_n(\cot \theta_2), \quad n = 0,1,2,3,\ldots
\]

(33)

and also

\[
\lambda_2 = \frac{-4q^2 + \frac{1}{4} \left\{ (2n+1) \pm 2 \sqrt{\nu(\nu-1)+\lambda_2} \right\}^4}{\left\{ (2n+1) \pm 2 \sqrt{\nu(\nu-1)+\lambda_2} \right\}^2} - \frac{1}{2}, \quad n = 0,1,2,3,\ldots
\]

(34)

with \(N_{nl}\) is normalization constant, \(P\) is Jacobi polynomials and \(\lambda_i\) as shown in Eq. (31)

Third calculation, the solution of Schrodinger equation for \(\theta_3\) with Manning Rosen potential using Nikiforov-Uvarov method. Substituting Eq. (1) into Eq. (25) and by setting \(Y_3(\theta_3) = P_3(\theta_3) \sin^{-1} \theta_3\) lead us to

\[
\left( \frac{\partial^2}{\partial \theta_3^2} P_3(\theta_3) \right) - \left( \frac{\nu(\nu-1) + \lambda_2}{\sin^2 \theta_3} - 2q \cot \theta_3 \right) P_3(\theta_3) + \left( 1 + \lambda_3 \right) P_3(\theta_3) = 0
\]

(35)

In the similar way, we obtain the solution of third part of angular Schrodinger we have the solution of Eq. (35) as follow

\[
\psi(\theta_3) = \left[ \frac{N_{nl}}{2} \left( \frac{1+i \cot \theta_3}{\sin^2 \theta_3} \right)^{\frac{1}{2}} \left( \frac{1-i \cot \theta_3}{2} \right)^{\frac{1}{2}} \right] P_n(\cot \theta_3), \quad n = 0,1,2,3,\ldots
\]

(36)

and we get

\[
\lambda_3 = \frac{-4q^2 + \frac{1}{4} \left\{ (2n+1) \pm \sqrt{1+4(\nu(\nu-1)+\lambda_2)} \right\}^4}{\left\{ (2n+1) \pm \sqrt{1+4(\nu(\nu-1)+\lambda_2)} \right\}^2} - 1, \quad n = 0,1,2,3,\ldots
\]

(37)

with \(N_{nl}\) is normalization constant, \(P\) is Jacobi polynomials. The eigenfunction that we obtained will be used to construct the new partner potential of the potential Rosen-Morse and Manning Rosen.

3.3. The construction of partner potential from origin potential Rosen Morse and Manning Rosen in Schrodinger 4 dimensional system

To construction the partner potential we use Eq. (19). So we obtained the new partner potential as follow
\begin{align}
V_2 (\theta_1) &= \left[ \frac{\nu (\nu + 1)}{\cos^2 \theta_1} - 2q \tan \theta_1 \right] - E_{01} \\
&= -2 \left[ \frac{1}{2} qi - \frac{1}{4} \lambda_i \right] i \sec^2 \theta_1 \tan \theta_1 \left( \frac{1 + i \tan \theta_1}{2} \right)^{-1} + \left( \frac{\sec^2 \theta_1}{2} \right)^2 \left( \frac{1 + i \tan \theta_1}{2} \right)^{-2}, \\
&+ 2 \left[ \frac{1}{2} qi - \frac{1}{4} \lambda_i \right] i \sec^2 \theta_1 \tan \theta_1 \left( 1 - i \tan \theta_1 \right)^{-1} - \left( \frac{\sec^2 \theta_1}{2} \right)^2 \left( 1 - i \tan \theta_1 \right)^{-2}, \\
&= \left[ \frac{\nu (\nu + 1) + \lambda_i - \frac{1}{4}}{\sin^2 \theta_2} - 2q \cot \theta_2 \right] - E_{02} \\
V_2 (\theta_2) &= -2 \left[ \frac{1}{8} \lambda_2 + \frac{1}{2} qi \right] i \sec^2 \theta_2 \cot \theta_2 \left( 1 + i \cot \theta_2 \right)^{-1} + \left( \frac{\sec^2 \theta_2}{2} \right)^2 \left( 1 + i \cot \theta_2 \right)^{-2}, \\
&+ 2 \left[ \frac{1}{8} \lambda_2 + \frac{1}{2} qi \right] i \sec^2 \theta_2 \cot \theta_2 \left( 1 - i \cot \theta_2 \right)^{-1} + \left( \frac{\sec^2 \theta_2}{2} \right)^2 \left( 1 - i \cot \theta_2 \right)^{-2}, \\
&= \left[ \frac{\nu (\nu + 1) - \lambda_2 - 2q \cot \theta_3}{{\sin^2 \theta_3}} \right] - E_{03} \\
V_2 (\theta_3) &= -2 \left[ \frac{1}{2} qi + \frac{1}{4} (1 + \lambda_3) \right] i \sec^2 \theta_3 \cot \theta_3 \left( 1 + i \cot \theta_3 \right)^{-1} + \left( \frac{\sec^2 \theta_3}{2} \right)^2 \left( 1 + i \cot \theta_3 \right)^{-2}, \\
&+ 2 \left[ \frac{1}{2} qi + \frac{1}{4} (1 + \lambda_3) \right] i \sec^2 \theta_3 \cot \theta_3 \left( 1 - i \cot \theta_3 \right)^{-1} - \left( \frac{\sec^2 \theta_3}{2} \right)^2 \left( 1 - i \cot \theta_3 \right)^{-2} \\
\text{where } E_{01} &= \left[ \frac{(\nu + 1)^2}{{\cos^2 \theta_1}} - \frac{q^2}{{(\nu + 1)^2}} \right] \text{ dan } E_{02} = E_{03} = \left[ \frac{(\nu - 1)^2}{{(\nu + 1)^2}} \right].
\end{align}

We use the new partner potential in Eq. (38) as a new potential. Then, the new partner potential solved in Schrodinger equation sistem by using the same steps that we used to solved the angular part Schrodinger equation with the original potential. The solution of the angular part Schrodinger equation with partner potential are

\begin{align}
\psi_r (\theta_i) &= \left[ \frac{N_{\alpha l}}{2} \left( \frac{1}{2} \frac{2^\nu \nu_{\alpha l} \lambda_i}{2^\nu \nu_{\alpha l} \lambda_i} \right) \left( \frac{\nu + 1}{2} \nu_{\alpha l} \lambda_i \right) \right] \\
&\times \left[ \left( \frac{1}{2} \frac{2^\nu \nu_{\alpha l} \lambda_i}{2^\nu \nu_{\alpha l} \lambda_i} \right) \left( \frac{i + \nu_{\alpha l} \lambda_i}{2} \right) \right] (i \tan \theta_i),
\end{align}
\[
\psi'(\theta_2) = \begin{bmatrix}
N_n \left( \frac{1 + i \cot \theta_2}{2} \right)^{1/2} \left( \frac{1 + i \cot \theta_2}{2} \right)^{-1/2} \\
N_n \left( \frac{1 + i \cot \theta_2}{2} \right)^{1/2} \left( \frac{1 + i \cot \theta_2}{2} \right)^{-1/2}
\end{bmatrix}
\]

with the separated variable constant are

\[
\lambda'_2 = \frac{4q^2}{\left( (2n+1) + \sqrt{4n(\nu+1) + 8(c_{121} + c_{142})} \right)^2} - 2E_0,
\]

\[
\lambda'_3 = \frac{4q^2}{\left( (2n+1) + \sqrt{4n(\nu+1) + 8(c_{121} + c_{142})} \right)^2} - (E_0 + 1)
\]

where \( \lambda'_2 \) in Eq. (31), \( \lambda'_3 \) in Eq. (34), and

\[
c_{121} = \sqrt{\left( -\frac{1}{2} q \right)^2 - \frac{1}{4} \lambda_1},
\]

\[
c_{141} = \sqrt{\left( -\frac{1}{2} q \right)^2 - \frac{1}{4} \lambda_1},
\]

\[
c_{122} = \sqrt{\left( \frac{1}{2} + \frac{1}{4} \lambda_2 \right)^2 + \frac{1}{4} q^2},
\]

\[
c_{142} = \sqrt{\left( \frac{1}{2} + \frac{1}{4} \lambda_2 \right)^2 - \frac{1}{4} q^2},
\]

\[
c_{123} = \sqrt{\left( \frac{1}{2} q \right)^2 - \frac{1}{4} \lambda_3},
\]

\[
c_{143} = \sqrt{\left( \frac{1}{2} q \right)^2 - \frac{1}{4} \lambda_3}.
\]

And then, to see the form of eigenfunction angular part of Schrodinger equation with original potential and Schrodiger equation with new partner potential we make visualization by using matlab program, for example we present the eigenfunction of the angular part \( \theta_1 \) in Table 1.
Table 1. The eigenfunction of angular part $\theta_3$ Schrodinger equation for potential Manning Rosen and partner potential Manning Rosen

| $n_l$ | Original Potential Manning Rosen | Partner Potential Manning Rosen |
|-------|---------------------------------|---------------------------------|
| 0     | ![Original Potential](image)     | ![Partner Potential](image)     |
| 1     | ![Original Potential](image)     | ![Partner Potential](image)     |
| 2     | ![Original Potential](image)     | ![Partner Potential](image)     |

In Table 1 we can see that the eigenfunction of the original potential Manning Rosen and the partner potentials Manning Rosen have the same shape. The higher the number of angular quantum the more waves are formed. The significant difference in their amplitude. The amplitude of the eigenfunction of partner potential bigger than original potential.

From all of the calculations that we have done above, we can see that from the solution of quantum potential we can construct the new potentials partner. The new partner potentials are solvable so can also be solved in quantum mechanics system. The eigenfunction of original potential is different compared with the new partner potential.

4. Conclusion

We have constructed the partner potential from original potential Rosen-Morse and Manning Rosen in Schrodinger 4 dimensional system using the supersymmetric method. The partner potentials that we obtained are solvable. By using the Nikiforov-Uvarov method we obtained the eigenfunction of angular part of Schrodinger equation with partner potential Rosen-Morse and Manning Rosen. The eigenfunctions of Schrodinger equation with the potential original and the partner potential have same shape, but have the difference amplitude. The higher the number of angular quantum the more waves are formed and the amplitude of the eigenfunction of partner potential bigger than original potential.

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