Companion poles: from the $a_0(980)$ to the $X(3872)$

F. GIACOSA$^{a,b}$

$^a$Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland

$^b$Institut für Theoretische Physik, Johann Wolfgang Goethe- Universität, 60438 Frankfurt am Main, Germany

When an unstable ordinary quark-antiquark state couples strongly to other low-mass mesons (such as pions, kaons, $D$-mesons, etc.), the quantum fluctuations generated by the decay products dress the bare ‘seed’ $q\bar{q}$ state and modify its spectral functions. The state is associated to a pole on the complex plane. When the coupling to the decay products is sufficiently large, a remarkable and interesting phenomenon takes place: dynamically generated companion states (or poles) might emerge. Some resonances listed in the PDG, such as the $a_0(980)$, the $K_0^*(700)$, and the $X(3872)$, can be well understood by this mechanism, that we briefly review in these proceedings. On the other hand, we show that the $Y(4008)$ and $Y(4260)$ are not independent resonances (or poles), but manifestations of $\psi(4040)$ and $\psi(4160)$, respectively.

1. Introduction

The idea behind companion poles is quite simple: in the easiest scenario, one starts with a single bare field with quantum numbers $J^{PC}$ which corresponds to a well-defined $q\bar{q}$ state with a certain bare mass, typically very close to the predictions of the quark model [1]. Then, a Lagrangian in which this “seed” state couples strongly to some standard mesons (such as pions, kaons, $D$-mesons, $\rho$-mesons, ...) is written down. As a consequence, mesonic loops dress the original state. The original pole on the real axis moves down in the complex plane. Moreover, when the interaction is strong enough, other poles can appear: these are dynamically generated companion poles. In some cases, such poles can be interpreted as additional resonances and some of the supernumerary states listed in the PDG [2] can have such an origin. A general feature of companion poles is that they fade away in the large-$N_c$ limit [3]: in fact, the coupling of the original $q\bar{q}$ state to

(1)
other ordinary mesons scales as $1/\sqrt{N_c}$, therefore the quantum fluctuations become smaller for and additional poles do not emerge.

The mechanism outlined above was described in Refs. [4, 5] in the context of light scalar mesons. Later on, the concept of companion poles has been revisited in detail in Ref. [6], in which the $a_0(980)$ is described as a companion pole of a predominantly standard $\bar{q}q$ resonance $a_0(1450)$ and where a detailed comparison with the previous works of Ref. [4, 5] is presented. In the last four years, the approach has been studied for various states [7, 8, 9, 10, 11], as we shall discuss in more detail in the next section (see Table 1). For related ideas about the emergence of companion poles, we refer also to [12, 13, 14, 15] and refs. therein.

### 2. Companion poles: status

In this section, we describe the present status of the approach and the results obtained through its applications to resonances both in the light and in the charmonium sectors.

In order to explain the idea, we consider two explicit examples. In the first one, the seed state is the scalar state $K_0^*$, which decays to $K\pi$ and -after dressing- mainly corresponds to $K_0^*(1430)$; in the second case, the seed state is the $\bar{c}c$ bare field $\psi_\mu$, which decays into $DD$ and predominantly corresponds to $\psi(3770)$. The Lagrangians for these two systems are:

\begin{equation}
\mathcal{L}_{K_0^*} = a K_0^* \pi^0 K^+ + b K_0^* \partial_\mu \pi^0 \partial^\mu K^+ + \ldots, \quad (1)
\end{equation}

\begin{equation}
\mathcal{L}_\psi = ig \psi_\mu \left( \partial^\mu D^+ D^- - \partial^\mu D^- D^+ \right) + \ldots, \quad (2)
\end{equation}

where the dots refer to other isospin combinations, see details in [7, 8]. In both cases, the decay widths as function of the ‘running’ mass of the decaying particle can be expressed as:

\begin{equation}
\Gamma_{K_0^*}(m) = \Gamma_{K_0^*}^{\mathrm{tl}}(m) F_{\Lambda}(m) ; \Gamma_\psi(m) = \Gamma_\psi^{\mathrm{tl}}(m) F_{\Lambda}(m), \quad (3)
\end{equation}

where the tree-level part (tl) is obtained from the standard Feynman rules for the local Lagrangian in Eqs. (1)-(2), while the quantity $F_{\Lambda}(m)$ is a vertex function which takes into account the finite dimensions of the mesons. It could be formally introduced already at the Lagrangian level by rendering it nonlocal [15]. The function $F_{\Lambda}(m)$ should guarantee convergence of the loops, thus $F_{\Lambda}(m \to \infty) = 0$ sufficiently fast. A typical choice, valid in the reference frame of the decaying particle, is $F_{\Lambda}(m) = e^{-2k(m)}/\Lambda$, where $k(m)$ is the modulus of the three-momentum of one of the outgoing decay products and $\Lambda \simeq 0.5$ GeV is the typical energy scale for the overlap of extended mesons. Note, even if the vertex function cuts the three-momentum $k,$
Lorentz invariance is guaranteed \[17\].

One consider the propagator of the seed state dressed by loops of the decay products. Its scalar part is

\[
\Delta_j^{-1}(m) = m^2 - M_0^2 + \Pi_j(m^2), \quad j = K^0, \psi ,
\]

where \(\Pi_j(m^2)\) is the loop function such that \(\text{Im}\,\Pi_j(m^2) = m\Gamma_j(m)\). Since the imaginary part is known, the loop function \(\Pi_j(m^2)\) can be obtained by dispersion relations. In the first Riemann sheet (IRS), \(\Pi_j(m^2)\) is regular everywhere, a part from a cut along the real axis. When the coupling constant(s) is (are) sent to zero, the so-called seed pole of \(\Delta_j(m)\) is \(m_{\text{seed}} = M_0 - i\varepsilon\). For nonzero couplings, the seed pole \(m_{\text{seed}}\) moves down in the IIRS: the pole mass is \(\text{Re}[m_{\text{seed}}]\) (usually, not far from \(M_0\)) and the decay width is \(-2\text{Im}[m_{\text{seed}}]\). But for coupling constant large enough, there can be a second, dynamically generated companion pole:

\[
m_{\text{comp}} \text{ such that } \Delta_j^{-1}(m_{\text{comp}})_{\text{IIRS}} = 0 \text{ and } m_{\text{comp}} \neq m_{\text{seed}} .
\]

The companion poles has a completely different ‘movement’ on the complex plane: for small coupling, it lies very far from the real axis and then it approaches the real axis from below when the coupling increases. Eventually, for very large coupling, it can be even closer to the real axis than the original seed pole. (This is not the case for the two examples above, but it applies for the \(a_0\)-system, see below).

Next, one defines the spectral function as \[18\] \[19\]

\[
d_j(m) = \frac{2m}{\pi} \text{Im}[\Delta_j(m)] \to \int_0^\infty d_j(m)dm = 1 .
\]

The normalization is a crucial property, since it allows to interpret \(d_j(m)\) as a mass probability density (for a detailed proof, see Refs. \[20\]). Note, even when the companion pole is present, strictly speaking there is only one ‘state’ properly normalized to unity. Typically, the companion pole generates an enhancement of the spectral function at low energies (or even a second peak as for \(a_0(980)\) and \(X(3872)\)). Note, the here outlined approach is valid at the (resummed) one-loop level. Fortunately, it seems to be a good approximation in hadron physics \[21\].

In Table 1 we report the present status of some resonances: for given quantum numbers, the bare fields with the spectroscopic notation and \(\bar{q}q\) content, the resulting predominantly \(q\bar{q}\) resonances, and the companion poles are listed. Then, below the Table we briefly discuss each case separately.
Table 1: Summary of light and heavy systems in which companion poles have been investigated.

| J^P_C | Bare field n^2S+1L_J | Main decays | Predom. \( \bar{q}q \) pole (GeV) | Companion pole (GeV) | Ref. |
|-------|-----------------|-------------|-------------------------------|----------------------|-----|
| 0^{++} | 0_0 \quad 1^3 P_0 \quad u \bar{d}, ... | KK \pi \eta, \pi \eta' | \( a_0(1450) \) 1.456, \(-i0.134 \) | \( a_0(980) \) 0.970, \(-i0.045 \) | [6] |
| 0^{++} | 0_0 \quad 1^3 P_0 \quad u \bar{s}, ... | K\pi | \( K_0^*(1430) \) 1.413, \(-i0.127 \) | \( K_0^*(700) \) 0.746, \(-i0.262 \) | [7] |
| 1^{−−} | 1^3 D_1 \quad c \bar{c} | DD | \( \psi(3770) \) 3.777, \(-i0.0123 \) | \( \lambda(800) \) 3.741, \(-i0.0018 \) | [8] |
| 1^{−−} | 3^3 S_1 \quad c \bar{c} | DD, DD^* \quad D^* D^*, D_s D_s | \( \psi(4040) \) 4.053, \(-i0.039 \) | \( \lambda(800) \) 3.934, \(-i0.030 \) | [9] |
| 1^{−−} | 2^3 D_1 \quad c \bar{c} | DD, DD^* \quad D^* D^*, D_s D_s | \( \psi(4160) \) 4.199, \(-i0.033 \) | \( \lambda(800) \) 3.87164, \(-i\varepsilon \) (virtual) | [10] |
| 1^{++} | \chi_{c,1}(2P) \quad c \bar{c} | DD^* | \( \chi_{c,1}(2P) \) 3.995, \(-i0.036 \) | \( X(3872) \) 3.87164, \(-i\varepsilon \) (virtual) | [11] |

- \( a_0(980) \) and \( a_0(1450) \) [6]. One starts with a unique field \( a_0 \) with a bare mass of about 1.2 GeV coupled to light mesons according to the constrains of chiral symmetry [22]. Then, upon including the loops, \( a_0(1450) \) is predominantly \( \bar{q}q \), and the resonance \( a_0(980) \) is a dynamically generated companion pole (for a detailed discussion of light scalar mesons, see [15]). Quite remarkably, the loops are so strong that the corresponding spectral function \( d_{a_0}(m) \) contains two peaks.

- \( K_0^*(700) \) and \( K_0^*(1430) \) [7]: the seed state \( K_0^* \) lies well above 1 GeV. \( K_0^*(1430) \) corresponds to the (dressed) \( \bar{q}q \) state, while \( K_0^*(700) \) is dynamically generated. In the spectral function there is no peak for this state, but a slight low-energy enhancement. Recently, the existence of this non-conventional meson has been in the centre of many investigations, e.g. Ref. [23]. The PDG2018 has re-named this state as \( K_0^*(700) \) (previously, \( K_0^*(800) \)) and included it in the summary table. Our study clearly confirms the existence of this state and provides a clear physical interpretation of its nature.
• $\psi(3770)$ [8]: the non-Breit-Wigner form of the spectral function is caused by the loops. Also in this case two poles appear. Yet, the dynamically generate pole is quite close to the seed one, hence no new name for an independent state is assigned.

• $\psi(4040)$ [9]: the bare $\bar{c}c$ state couples strongly to various $D$ mesons. The spectral function is strongly distorted and two poles are generated, just as for $\psi(3770)$. The dynamically generated pole does not correspond to the enhancement $Y(4008)$ [24]. A broad distorted resonance-like structure may emerge in the $j/\psi\pi\pi$ channel through the process $\psi(4040) \rightarrow DD^* \rightarrow j/\psi\pi\pi$ (because the real part of the loop $DD^*$ is peaked at the $DD^*$ threshold). Hence, $Y(4008)$ is not an independent state, but a $DD^*$-loop manifestation of $\psi(4040)$.

• $\psi(4160)$ [10] (actual mass: 4.191 GeV [2]): in this case there is a unique (relevant) pole. As before, the chain $\psi(4160) \rightarrow D_s^*D_s^* \rightarrow j/\psi\pi\pi$ generates a resonance-like structure peaked at about 4.222 GeV (this is the $D_s^*D_s^*$ threshold where again the real part of the loop is enhanced). This signal can be assigned to the $Y(4260)$ (also called $\psi(4260)$ in the PDG [2]). Then, $Y(4260)$ is not an independent resonance, but a loop manifestation of the state $\psi(4160)$ (and of its pole) shifted of about 40 MeV in mass.

• $X(3872)$ and $\chi_{c1}(2P)$ [11]: a bare seed state $\chi_{c1}(2P)$ gets dressed by $DD^*$ loops. At the lowest $D_0D_0^*$ threshold the spectral function develops a very high and narrow peak: the $X(3872)$. In the complex plane, there is a virtual pole just below $D_0D_0^*$. The state $\chi_{c1}(2P)$ has a well-defined pole, but the corresponding peak can fade away, explaining the difficulty to measure it in experiments.

3. Conclusions

In this work we have briefly reviewed the concept dynamical generation of companion poles and the status of some resonances on the basis of this idea. Other resonances could also emerge as companion poles, as for instance the state $D_s(2317)$. Moreover, as for the $Y(4008)$ and $Y(4260)$, other enigmatic $Y$ states (see [25] for a review) could be not real, but ‘loop’ manifestation of conventional $\bar{c}c$ states.

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