On Shadow of the Moon in Extensive Air Shower Data

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the date of receipt and acceptance should be inserted later

Abstract. A new technique has been devised for the analysis of extensive air shower data in observing the effect of the moon on this data. In this technique the number of EAS events with arrival directions falling in error circles centered about the moving moon is compared to the mean number of events falling in error circles with centers randomly chosen in the sky. For any assumed angular radius of the error circle the deficit in EAS event count in the direction of moon which is a moon-related effect is interpreted as the shadow of the moon. A simple theoretical model has been developed to relate $N_{\text{sky}}$ to the angular radius of the error circle and has been applied to the counts from the moon’s direction in order to extract the physical parameters of the shadow of the moon. The technique and the theoretical model has been used on $1.7 \times 10^5$ EAS events recorded at Alborz observatory.

Key words. Cosmic Ray, Extensive Air Shower, Angular resolution.

1. Introduction

In 1957, Clark (Clark 1975) recognized that the moon and the sun cast a shadow in the isotropic flux of cosmic-ray nuclei. As they pass overhead during a transit, the moon

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and the sun block cosmic rays, so their shadows in the cosmic ray flux should be visible to Extensive Air Shower (EAS) arrays with sufficiently good angular resolution. It was not until the early 1990’s that an extensive air shower array had the required angular resolution to observe this shadow. Since that time the shadow of the sun and the moon have been used to measure the angular resolution of extensive air shower arrays (Urban et al. 1990, Ambrosio et al. 2003, Amenomori et al. 2007). Observing the shadow of the moon in EAS experiments which usually might have a much larger error circle than the disk of the moon is a very difficult task and requires a careful scrutinization of the data. Normally study on the shadow of moon is needed as the first step to find angular resolution and systematic errors of any observational data which is to be used for any astronomical study in particular for the research of point sources in sky.

The object of this paper is to describe a new technique and procedure devised for the analysis of extensive air shower data in observing the effect of the moon on this data and also to present a theoretical model from which we can extract the relevant physical parameters of the shadow of moon. The technique has been previously briefly outlined on an investigation of energy spectrum of EGRET $\gamma$–ray point sources with EAS experiment (Khakian et al. 2005 a). In Sec.2 we will describe this new technique and its procedure for EAS data analysis. In Sec.3 we will present a theoretical model from which we can extract the relevant physical parameters, In Sec.4 the result of applying the procedure and the theoretical model to $1.7 \times 10^5$ EAS data of Alborz observatory is presented and the obtained results on shadow of the moon is also presented. Sec.5 is devoted to discussion and some concluding remarks.

2. Description of the New Technique and EAS data Analysis procedure

This technique and the corresponding EAS data analysis procedure for observing the effect of the moon on the data and determining the pertinent physical parameters is based on corrected measured EAS data (corrected for systematic errors). The data must contain the following information for each EAS event: The arrival time of each shower, $t_s$, and the coordinates of the arrival direction of the shower. The local coordinates of each shower arrival direction should be converted to declination and right ascension of each shower with arrived direction denoted here by $\delta_s$, $RA_s$ respectively. Here, we denote the declination and right ascension of the center of the moving moon by $\delta_m(t)$ and $RA_m(t)$ at the arrival time of each shower. The EAS experiments generally might have an experimental uncertainty which results in the experiment’s error circle to be larger than the disk of the moon. Furthermore, normally, realistic determination of the radius of the error circle is best obtained by observation of the shadow moon in EAS data. However, in this new proposed technique and the procedure described here, there is no need for the use of a predetermined radius of error circle or a fitted value for it, and, instead it is
based on the observation of any possible deficit in shower counts falling in the error circle centered about the moving location of moon as compared to the average shower counts falling in error circles centered at other positions in sky during the observation time for a wide range of assumed values for the radius of the error circle ranging from 0.2° to a maximum value relevant to the particular EAS data set under analysis. For any assumed radius of circle of error, or equivalently, its angular radius $\theta_{\text{err}}$, the number of showers falling in each circle is determined by calculating the angular separations $\theta_{sm}$ between the arrival direction of each shower event ($\delta_s, RA_s$) and the direction of the center of the moon at the time of recording of that event, using the following equation from spherical geometry:

$$\cos \theta_{sm} = \cos \delta_m \cos \delta_s + \sin \delta_m \sin \delta_s \cos (RA_m - RA_s).$$  \hspace{1cm} (1)

Obviously, if $\theta_{sm} < \theta_{\text{err}}$ that shower is counted as falling in the moon’s error circle. In order to compare the obtained result with random sampling and scrutinize the difference for each assumed value of $\theta_{\text{err}}$, some random locations in the sky denoted by celestial coordinates ($\delta_r, RA_r$) are chosen and the number of showers falling in the error circles centered about each of the random locations is determined similarly by calculating the angular separation $\theta_{sr}$ of each shower arrival direction ($\delta_s, RA_s$) with the direction of the center of the randomly chosen error circle denoted by ($\delta_r, RA_r$) from above equation with ($\delta_m, RA_m$) replaced by ($\delta_r, RA_r$). If for any shower event $\theta_{sr} < \theta_{\text{err}}$ that shower is counted as falling in the error circle of that random position. For any assumed angular error radius, $\theta_{\text{err}}$, some error circles are chosen in the sky centered about truly random locations. The number of random circles for every angular error radius should be limited such that it ensures that no overlap occurs between two or more random circles, so the number of random centers are varied from at least 1000 (for small error circles) to 77 a smaller number which depends on the data set under analysis (for larger error circles). Thus, for each assumed radius for the error circle, the mean of the shower counts falling in random circles could safely be used as the expected mean number of EAS events falling in the error circle in any direction in the moonless sky against which the number of EAS falling in the circle centered about the moving moon could be safely compared, and, the variance of its distribution could safely be used as the statistical error of the mean number. Obviously if the deficit in the number of showers falling in each error circle centered about the moving moon from the mean number exceeds the statistical uncertainly in the mean , then the deficit could only be attributed to the moon’s effect. This effect is the shadow of moon in the EAS data, and, as explained in Sec.3, from a quantitative analysis and comparison with the expected mean number of EAS showers falling in the randomly centered error circle with that falling in the circles centered about the moving moon, the physical parameters of the moon’s shadow could be extracted.
3. A theoretical model for Moon’s shadow in EAS data

Following the procedure described in above technique, one now has the expected mean number of showers falling in randomly chosen error circles in monless sky as well as that in the moon-centered error circle for a set of assumed radii of the error circles, \( \rho_{\text{err}} \), (or equivalently angular radii, \( \theta_{\text{err}} \)). Here we propose a simple physical model to obtain this expected mean number as a function of the angular radius of the error circle, as a function of \( \theta_{\text{err}} \). We now derive the expected mean number of EAS events from each random direction in the moonless sky as a function of the assumed radius for the error circle, \( N_{\text{sky}}(\rho_{\text{err}}) \). The derivation is based on the single assumption of the model, that is, the assumption of uniform intensity, \( I \), of EAS producing radiation everywhere in the \( 4\pi \) steradian for each area of the random moonless sky and thus for each element of the error circle. For the contribution from each element of area of the error circle \( 2\pi \rho d\rho \), we should take into account only a fraction of radiations coming at such an angle to be able to reach the point of observation, that is a fraction equal to \( \frac{Id}{4\pi} \). Where \( d\Omega \) is the solid angle subtended by the element of area element from the observation point, which is the projection of the area element divided by the square of its distance from observation point, that is, \( d\Omega = 2\pi \rho d\rho \left(\frac{d}{R}\right)^2 \), with \( R = \sqrt{\rho^2 + d^2} \) and \( d \) is the distance from point of observation to the center of the error circle, and it is merely a multiplicative constant factor relating the radius of the error circle to its angular radius \( \rho_{\text{err}} = d\tan\theta_{\text{err}} \). The integration is trivial. Thus we have:

\[
N_{\text{sky}}(\rho_{\text{err}}) = \int_{\rho_0}^{\rho_{\text{err}}} \frac{Id}{4\pi} \frac{2\pi \rho d\rho}{\left(\rho^2 + d^2\right)^{\frac{3}{2}}} = -\frac{Id}{2} \left[ \rho^2 + d^2 \right]^{-\frac{1}{2}} \big|_{\rho_0}^{\rho_{\text{err}}} = -\frac{Id}{2} \left( \frac{1}{\sqrt{\rho_{\text{err}}^2 + d^2}} - \frac{1}{d} \right) \quad (2)
\]

for the number of showers falling in the error circle centered about moon, \( N_{\text{moon}}(\rho_{\text{err}}) \) the integration has to be split into two or three parts involving the physical parameters of shadow of moon in EAS data. Here, we define the following three physical parameters used in this model:

a) \( r_m \equiv \) radius of umbra of shadow that is, from \( \rho = 0 \) to \( \rho = r_m \) the EAS producing radiation are assumed to be fully absorbed (totally blocked) and could not contribute to \( N_{\text{moon}}(\rho_{\text{err}}) \).

b) \( r_p \equiv \) radius of penumbra of shadow; that is from \( \rho = r_m \) to \( r = r_p \) only a fraction \( (f) \) of EAS producing radiation penetrate the penumbra and contribute to \( N_{\text{moon}}(\rho_{\text{err}}) \).

c) \( f \equiv \) the fraction of EAS producing radiation which penetrate the moon’s penumbra. Obviously, if \( \rho_{\text{err}} \leq r_p \) the integration will only be split into two parts, that is, \( \int_0^{\rho_{\text{err}}} \rightarrow 0 \times f \int_0^{r_m} + f \int_{r_m}^{r_p} \).

For the case of \( \rho_{\text{err}} > r_p \), the integral will be split into three parts:

\[
\int_0^{\rho_{\text{err}}} \rightarrow 0 \times f \int_0^{r_m} + f \int_{r_m}^{r_p} + 1 \times f \int_{r_p}^{r_{\text{err}}}.
\]

The result of these integrations giving \( N_{\text{moon}}(\rho_{\text{err}}) \) in terms of physical parameters of the moon’s shadow \( (r_m, r_p, f) \) is given in Appendix. It should be emphasized here that the strict explicit assumption of uniform flux of EAS producing radiation used in this
model requires that when the EAS data is used to extract the parameters of the moon’s shadow from this model one has to make sure that the data may only have statistical error (as this has also been implicitly assumed as outlined in the procedure for obtaining shower counts in the error circles), that is, the data should have been corrected for any systematic errors such as those related to the site of observation and non-uniformities in the exposure time in various directions in sky.

4. Application of the Technique to ALBORZ EAS data

The technique described in Sec.2 for determination of the moon shadow has been applied to $1.7 \times 10^5$ EAS data collected in 280 hours of observations in April-June 2002, with the small EAS array of the prototype of ALBORZ Observatory of Sharif University located in Tehran, Iran ($51^\circ 20' E$ and $35^\circ 43' N$, elevation 1200 m $\equiv 890$ g cm$^{-2}$). For details of array and data, see [Bahmanabadi et al. 2003]. As explained in our previous report [Khakian 2005 b], the data has been corrected (scaled for uniform exposure) for site-dependent factors effecting shower counts from different directions in sky. The information on the celestial coordinates of the moon during the observation time of the collected data has been obtained from the internet site [http://aa.usno.navy.mil]. The moon’s data has been obtained for time increments of one minute, and the location of moon in Right ascension and declination coordinate at the recorded time of arrival of each EAS event has been calculated and used.

For an assumed set of radii for error circles ranging from $0^\circ$ to $6^\circ$ (increments of $0.2^\circ$) we have calculated the number of corrected shower counts falling in each error circle. For every assumed radius, some random moon-like locations passing through the paths like as moon’s path in sky was chosen according to the radius of error circles and the number of corrected EAS counts falling in each error circle was found according to the procedure described in Sec.2. In table 1 the number of random circles and the mean of counts for various assumed radii of error circles are given in second and third columns, also in this table (4th column) the number of corrected shower counts from the error circle centered about the moving moon is shown. In column 5 and 6 of the table the deficit of counts from moon’s error circle from the mean count of the random sky error circle and the statistical error of deficits are given. The last column of the table gives the statistical significance of these deficits calculated with Li&Ma method [Li & Ma 1983]. Fig.1 shows the variation of the mean number of events for moon-like error circles with random centers as a function of the chosen radius of the error circles. The smooth curve shown that calculated according to our theoretical model of Sec.3 and it fits the mean count from random sky with a regression of 0.996. Error bars are taken from 4th column of table 1. The good fit of random sky counts with model shows that we can safely use these mean number of events to compare with that falling in the error circles centered about the
moving moon and rule out the possibility of the deficit in the number of events falling in the moon centered circle as due to statistical fluctuations. In Fig. 2 we have shown the variation of events falling in each circle centered about the moon and the mean number of events falling in the error circles centered about random moving moon-like locations as a function of radius of the error circles. As seen in Fig. 2 moon counts are less than mean counts from random moon-like centers for all error circles that we considered. In Fig. 3 we have shown the number of deficit events for each radius of the error circles. We have fitted the deficit counts falling in the error circle centered about the moving moon from that for moon-like circles with random centers to our theoretical model (sec. 3 and Eq. A2 in Appendix) and have obtained the following results:

\[ \theta_m = 0.5^\circ, \theta_p = 4.5^\circ, f = 0.80. \]

5. Concluding Remarks

It is worth remarking that the application of the proposed technique to ALBORZ EAS data has yielded good agreement between the mean number of counts from error circles with centers chosen randomly in sky with no moon in the line of sight and the expected number according to our proposed theoretical model. This good agreement is very encouraging and prompted us to extract the physical parameters of the moon shadow (defined in Sec. 3) from this data. It should also be remarked that the data used for calculating shower counts in each error circle was the corrected counts scaled in order to obtain a uniform exposure of sky. The correction accounted for site-dependent systematic errors arising from uneven number of EAS events in various directions in sky due to two main factors: (1) varying amount of air mass which produces the EAS event as a function of zenith angle and depends on the elevation of the site (Khakian 2005 b), and (2) geomagnetic effect which depends on the components of magnetic field at the site’s location. Our attempt to extract the physical parameters of moon’s shadow from this data has been fully successful as can be seen from the reported result in Sec. 4. That is in fitting the corrected data to our theoretical model we are able to obtain a value for the radius of shadow’s umbra \( \theta_m = 0.5^\circ. \) However, according to statistical significance shown in the last column of table 1 we didn’t see the umbra with good significance but the obtained results show that in spite of low-statistics EAS data base this method is powerful to find shadow of moon. One may suggest that the value of \( \theta_p = 4.5^\circ \) we obtained is just the umbra’s radius rather than penumbra’s and resulted from low angular resolution of our array. This could be right since the angular resolution of our array which was reported before (Khakian et al. 2005 a) is about 4.3\(^\circ\) close to 4.5\(^\circ\) which we find here as the radius of penumbra. We believe that the main uncertainty in extracting results from Alborz data could be due to the following two reasons, both of which will be improved upon in future with much higher number of events and larger
1. Inaccurate and low-statistics EAS data base

Since the umbra’s radius is in the order of 0.5° it is hard to expect to extract it from inaccurate EAS data. The local coordinates associated with each EAS event in ALBORZ EAS data has been obtained from an array with a very small number of detectors. EAS data from observatories with large arrays, once corrected for the systematic site-dependent errors may be more accurate to yield better results for the physical parameters of moon’s shadow according to the technique presented here.

2. Incomplete data on Moon

As explained in Sec. 3 the information about the celestial coordinates of the moon was obtained from the internet site using time increments of one minute. The time of arrival of EAS events had been recorded with an uncertainty of 0.07 seconds. In the computations of shower counts in the error circles of various radii centered about the moving moon which are given in Table 1 to check whether a given EAS event falls in the error circle centered about the moon or falls outside it, we have used the coordinates of the moon at the one of the minute steps which is closest to the arrival time of the given EAS event. Obviously, this may have caused an extra inaccuracy in the counts given in column 5 of Table 1. In future application of this technique the interpolated or exact location of the moon at the instant of recording of each EAS event must be used and the variable earth-moon distance should also be taken into account. However, the study of the moon’s motion has been beyond the scope of the present work.

6. Appendix

For calculating the count in the error circle centered about the moon, we split Eqn.2 in three parts. The result of integrations for two regions are shown in following equations:

\[ N_{moon} = -\frac{I}{2} \left[ \cos \theta_{err} - \cos \theta_p \right] + f \left( \cos(\theta_p - \cos \theta_m) \right), \quad \theta_{err} > \theta_p \]

\[ N_{moon} = -\frac{I}{2} f \left( \cos \theta_{err} - \cos \theta_m \right), \quad \theta_m < \theta_{err} < \theta_p \]

\[ N_{back} - N_{moon} = -\frac{I}{2} \left[ (1 - f) \cos \theta_{err} - 1 + f \cos \theta_m \right], \quad \theta_m < \theta_{err} < \theta_p \] \quad (A.2)

\[ N_{back} - N_{moon} = -\frac{I}{2} \left[ (1 - f) \cos \theta_p - 1 + f \cos \theta_m \right], \quad \theta_p < \theta_{err} \]

The parameter \( I (=497270) \) is determined by fitting Eqn.2 to the data of column 3 in table 1. By knowing \( I \) and fitting the data of deficit events (column 6 of table 1) with above equations (A.2), we obtained \( \theta_m, \theta_p, \) and \( f \).
| radii of error circle(°) | random moon-like locations | Circle centered about the moon | Deficit of counts from the moon | error in deficit (counts) | Statistical Significant |
|-------------------------|---------------------------|-------------------------------|-------------------------------|--------------------------|------------------------|
|                         | #random circles | mean count | counts | counts | in counts | Li&Ma Method |
| 0.1                     | 1000          | 7           | 5      | 2      | 2.24      | 0.89       |
| 0.3                     | 1000          | 15          | 6      | 9      | 2.46      | 3.67       |
| 0.5                     | 1000          | 26          | 17     | 9      | 4.13      | 2.18       |
| 0.7                     | 1000          | 40          | 22     | 18     | 4.70      | 3.83       |
| 0.9                     | 1000          | 55          | 42     | 13     | 6.50      | 2.0        |
| 1.1                     | 1000          | 74          | 52     | 22     | 7.23      | 3.05       |
| 1.3                     | 1000          | 97          | 73     | 24     | 8.56      | 2.81       |
| 1.5                     | 1000          | 125         | 93     | 32     | 9.67      | 3.31       |
| 1.7                     | 991           | 155         | 105    | 50     | 10.29     | 4.87       |
| 1.9                     | 793           | 187         | 134    | 53     | 11.63     | 4.57       |
| 2.1                     | 649           | 223         | 168    | 55     | 13.03     | 4.24       |
| 2.3                     | 541           | 261         | 204    | 57     | 14.38     | 3.98       |
| 2.5                     | 458           | 303         | 246    | 57     | 15.82     | 3.63       |
| 2.7                     | 393           | 348         | 287    | 61     | 17.09     | 3.59       |
| 2.9                     | 340           | 396         | 317    | 79     | 17.99     | 4.43       |
| 3.1                     | 298           | 446         | 365    | 81     | 19.36     | 4.23       |
| 3.3                     | 263           | 497         | 422    | 75     | 20.81     | 3.64       |
| 3.5                     | 233           | 553         | 454    | 99     | 21.67     | 4.63       |
| 3.7                     | 209           | 610         | 487    | 123    | 22.56     | 5.56       |
| 3.9                     | 188           | 669         | 518    | 151    | 23.39     | 6.61       |
| 4.1                     | 170           | 727         | 579    | 148    | 24.77     | 6.13       |
| 4.3                     | 155           | 791         | 636    | 155    | 26.03     | 6.12       |
| 4.5                     | 141           | 858         | 709    | 149    | 27.52     | 5.57       |
| 4.7                     | 129           | 924         | 780    | 144    | 29.03     | 5.13       |
| 4.9                     | 119           | 994         | 840    | 154    | 30.31     | 5.29       |
| 5.1                     | 110           | 1068        | 924    | 144    | 31.88     | 4.71       |
| 5.3                     | 102           | 1140        | 972    | 168    | 32.84     | 5.36       |
| 5.5                     | 94            | 1210        | 1048   | 162    | 34.32     | 4.97       |
| 5.7                     | 88            | 1286        | 1132   | 154    | 35.93     | 4.55       |
| 5.9                     | 82            | 1364        | 1205   | 159    | 37.31     | 4.55       |
| 6.1                     | 77            | 1434        | 1272   | 162    | 38.78     | 4.51       |

Table 1. Number of EAS events obtained in various error circles with random centers and moon center, of the low-statistics EAS data of Alborz observatory.

**Acknowledgement**

This research was supported by a grant No. NRCI 1853 from the national research council of Islamic republic of Iran for basic sciences.

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Fig. 1. Variation of the mean number of EAS events falling in the random error circles as a function of the angular radius of the error circle, $\theta_{err}$. The smooth curve is the result of computations according to our theoretical model (Sec.2).

Fig. 2. Variation of events falling in the random circles (•) and moving moon (△) as a function of the angular radius of the error circle, the smooth curve is the same as Fig.1.
Fig. 3. Variation of deficit events falling in moving moon circles from that in the random circles as a function of the angular radius of the error circle, the smooth two parts curve is the result of fitting data with the theoretical model equations see in appendix.