Reflection of sound pulses from an inhomogeneous bubble medium

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Abstract. The effect of changes in the volume concentration of bubbles in the boundary zone of the bubble medium on the nature of reflection and radiation of the excited bubble medium is studied. The spectral characteristics of the radiation of a bubble medium are obtained at the initial stage of transition radiation and at large times when the radiation is stationary. It is shown that in the initial phase the emission spectrum is broadband and is located in the absorption band of the bubble medium, and at large times the emission spectrum is located outside this band.

1. Introduction
Bubble regions in natural conditions and technological processes are always inhomogeneities in the distribution of bubbles in space. In the marine environment, there are various small-scale inhomogeneities, among which gas bubbles make a significant contribution to the nonlinear acoustic characteristics of the marine environment. One of the important acoustic parameters are sound backscattering coefficients associated with sound absorption and acoustic nonlinearity of the marine environment [1]. Bubbles have the greatest effect on the scattering and attenuation of sound in the frequency range in the vicinity of the resonant frequency. The basis for theoretical studies are reliable experimental studies in the resonance region [2, 3]. In work [4] experimental and theoretical studies of the influence of resonant bubbles, located in a single layer in a polymer gel, on the transmittance at low ultrasonic frequencies are presented. In experiments, the effect of 2.7 times shifting the resonant frequency towards high frequencies is revealed. Bubble radiation is used to search for violations in underwater gas pipelines [5]. The effects of the interaction of sound with an inhomogeneous bubble medium are used to develop diagnostic systems for two-phase flow by acoustic sounding [6].

The aim of the work is to determine the spectral characteristics of sound reflected and emitted by an inhomogeneous bubble medium.

2. Physical statement of the problem
Part of the fluid unlimited in space is filled with identical bubbles. In the border region of the bubble medium, the volume concentration of bubbles $\alpha$ increases from 0 to a constant value according to the formula

$$\alpha = a \exp(-b(x-c)^2) (x-c)^2,$$

the coefficients $a$, $b$, $c$ determine the change in $\alpha$ in a given region. The objective of the study is the radiation of an excited bubble medium into a pure liquid.
3. Nonlinear wave system of equations

To study the radiation characteristics of a bubble medium, we use the wave model [6] in a one-dimensional formulation. The wave system of equations has the form:

\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = - \frac{1}{c^2} \frac{\partial}{\partial t} \left( \frac{p}{\rho} \ln(1-\alpha) \right)
\]

\[
R_k \frac{d^2 R_k}{dt^2} + \frac{3}{2} \left( \frac{dR_k}{dt} \right)^2 + 4\mu \frac{dR_k}{dt} + 2\sigma \frac{R_k}{\rho_0 R_k} = \frac{1}{\rho_0} P_0 - \frac{2\sigma}{R_k} \left( \frac{R_{k0}}{R_k} \right)^3 - \frac{P(x, t)}{\rho_0} + \frac{p(x, t)}{\rho_0}
\]

\[
\alpha(x, t) = \sum_k \nu_k(t) \cdot \delta(x - x_k(t))
\]

\[
\nu_k(t) = \frac{4}{3} \pi R_k^3(t)
\]

$k = 1,..., N_k$ defines the ordinal number of the bubble in the study area. $\delta(x-x_0)$ characterizes the position of the $k$th bubble in space, $P(x,t)$ is the pressure in the wave, $P_0$ is the initial pressure in the medium, $c$ is the speed of sound in water, $\alpha$ is the volume concentration of the bubbles, $R_k$ is the radius of the $k$th bubble, $\nu_k$ is the volume of the $k$th bubble, $\rho$ is the density of the liquid, $\sigma$ is the surface tension of the liquid, $\mu$ is the viscosity of the liquid, $t$ is the time, and $x$ is the spatial coordinate. The gas in the bubbles obeys the adiabatic law, and the pressure and density of water are related by expression $P(x, t) = \rho(x, t)c^2$.

For numerical solutions, system (2-5) is reduced to a dimensionless form using the relations

\[
\delta R_k = R_k / R_{k0}, \quad \delta P = P / P_w, \quad \delta t = t / (R_0 \sqrt{\gamma (\rho_0 / P_w)}), \quad \delta x = x / (cR_0 \sqrt{\gamma (\rho_0 / P_w)}),
\]

index $0$ denotes the initial state of the parameter. The studies are performed for two values of $P_w$ (the amplitude of the exciting pulse) of 10 Pa and 0.5 MPa. The duration of the exciting pulse is $\tau = 30 \cdot 10^6$ s. The initial pressure in the medium is $P_0=0.1$ MPa, the radius of the bubbles is $R_0=0.25 \cdot 10^{-3}$ m, the volume concentration of the bubbles is $\alpha=10^{-3}$, the gas in the bubble obeys the adiabatic law with the exponent $\gamma=1.4$, the density of water is $\rho_0=1000$ kg/m$^3$, and the speed of sound in a fluid is $c_0=1500$ m/s.

To exclude reflections from the boundaries, the width of the computational domain is 3 m. The width of the bubble region $h=1.5$m. The study of the interaction of short pulses with the bubble layer lies in the time range of $2 \cdot 10^{-3}$ s.

A description of the algorithm for numerical solution of the system of equations (2) - (5) and verification of solutions for compliance with experimental data is given in [6].

4. Discussion of the results

The formation of sound fields is strongly affected by the intensity of the exciting wave. Depending on the amount of energy in the exciting pulse, sound fields of various configurations are formed. The energy of a weak wave with an amplitude of 10 Pa is $E_w=5.71 \cdot 10^{10}$ J. The energy of a strong wave with an amplitude of 1 MPa is $E_w=5.76$ J. The duration, shape and structure of inhomogeneous bubble medium did not change in the calculations. The distance at which the volume concentration of bubbles in the bubble medium changes according to formula (1) from 0 to $10^{-3}$ is 0.5 m. This distance is ten times the spatial size of the pulse, which is $4.5 \cdot 10^{-3}$ m. The time scale of the problem, in accordance with (6), for weak wave is $1.22 \cdot 10^{-3}$ s, and for strong wave it is $3.68 \cdot 10^{-6}$ s.

Figure 1 shows the spatial structure of the sound field in the boundary regions of a pure liquid and a bubble medium for a weak exciting pulse with an amplitude of 10 Pa. In the region of low excitation
energies, three characteristic types of waves are formed. The main wave structure is a standing wave, which is formed in the zone of variable $\alpha$ when $\alpha$ reaches a constant value. The energy density in this region is an order of magnitude higher than in other regions and in the steady state it is this region that is the source of radiation. In Figure 1, this zone is designated as (b). The energy contained in the zone of the standing wave $E=1.78\times10^{10}$ J. The energy in inhomogeneous bubble medium in a standing wave is evenly distributed between the bubbles and the wave. In the sound component of the standing wave field, $E_s=8.68\times10^{11}$ J, and the energy $E_b=9.1\times10^{11}$ J is concentrated in the bubbles. These values for the averaged energy are obtained from 20 realizations. In the sound precursor moving towards the bubble medium, the energy is two orders of magnitude smaller and equal to $E=5.68\times10^{12}$ J. The wave component $E_w=3.29\times10^{12}$ J is always greater than the energy in the bubbles $E_s=2.39\times10^{12}$ J. About the same amount of energy is emitted into the water $E_r=4.64\times10^{12}$ J. The energy in the radiation into the liquid and in the sound precursor is calculated on the same spatial interval as the spatial size of the standing.

At high intensities of the exciting pulse, the structure of the wave field changes significantly.

**Figure 1.** The spatial structure of the wave field. Weak wave.

**Figure 2.** The spatial structure of the wave field. Strong wave.

Figure 2 shows the spatial structure of the wave field after exposure to a strong wave at the moment when all elements of the field are formed. The solution of system (2) - (5) shows that a typical wave structure is formed in an inhomogeneous bubble medium as well as in a homogeneous medium for given parameters [7]. While maintaining the general structure of the wave field, the parameters of its components have changed significantly. In a homogeneous medium, a resonant soliton is completely formed directly at the boundary in a time equal to its own duration and at a distance from the boundary equal to its own length [7]. In the case of an inhomogeneous medium, the transformation of the exciting pulse into a resonant soliton takes much longer. The formation of the soliton occurs when the volume concentration of bubbles reaches a constant value. From this moment on, the formed soliton moves through a homogeneous medium without changing its shape and retains its energy. The main property of a resonant soliton, as in a homogeneous medium, is that the bubbles and pressure field in the soliton constitute a single stable dynamic system. The pressure field in the resonant soliton and the bubbles are in dynamic equilibrium. The pressure change in the soliton is such that the bubbles inside the soliton are compressed to a minimum size, then they smoothly return to the equilibrium state and do not pulsate afterwards. The second feature of the interaction of pulses with an inhomogeneous bubble medium is a change in the nature of radiation in the direction of a pure liquid. The radiation amplitude of an inhomogeneous medium is much larger with respect to the reflected wave than their ratio in a homogeneous medium. This is because the reflection is formed at the boundary, where the volume concentration of the bubbles is still very small, and the radiation is formed inside the bubble medium, where the volume concentration of bubbles has already grown significantly. Under the
accepted conditions of the problem, the ratio of the amplitude of the radiation wave in the initial phase to the amplitude of the reflection wave $P_{\text{red}}/P_{\text{ref}}=21.2$. For a homogeneous medium with the same parameter values, the $P_{\text{red}}/P_{\text{ref}}$ ratio is close to 1. An additional condition that determines stronger radiation in the direction of a pure liquid is associated with a decrease in acoustic resistance in this direction due to a decrease in the difference in the equilibrium speed of sound in a bubble medium and the speed of sound in a pure liquid.

Due to the small volume concentration of bubbles in the excitation zone, the medium accumulates a smaller amount of energy. Therefore, energy is emitted much faster. Radiation of more than 90% of energy occurs during 20 intrinsic durations of the exciting wave, which corresponds to a time of $\sim 6\times10^{-5}$ s. This is an order of magnitude faster than for sharp boundaries between the bubble medium and liquid and a constant volume concentration of bubbles in the medium [8]. The second phase of the radiation has a significantly lower intensity and many times longer duration. Changes in the structure of wave fields entail changes in the spectral characteristics of radiation. Figure 3 presents the dimensionless energy spectrum of the initial phase of radiation of the bubble medium for a weak excitation wave with an amplitude of 10 Pa. The spectrum is a continuous strip with sharp boundaries. The lower cutoff frequency $f_{\text{cutoff}} = 0.162$ or in dimensional form 13.280 kHz, is close to the Minnaert frequency [9], which is equal to the resonant frequency of a pulsating bubble in an unbounded fluid regardless of surface tension and viscosity. For the bubble size adopted in this study, the Minnaert frequency is 13.53 kHz. The upper boundary dimensionless frequency is 0.651 or 53.37 kHz. This value is 4 times greater than the lower frequency of the emission band. The power in the spectral band decreases linearly from the lower frequency to the upper $(\delta P)^2 = -736\delta f + 49$.

For a strong wave, the emission spectrum shown in figure 4 has the same character, with the exception of quite strong radiation above the upper frequency of the spectrum of the main radiation. The quantitative characteristics of the spectra differ significantly. The lower dimensionless limit frequency of the continuous part of the spectrum is 0.159, which corresponds to a frequency of 43.3 kHz, and the upper limit frequency of the continuous part of the spectrum is 0.656 or 178.4 kHz. The spectrum between the boundary frequencies is described by the equation $(\delta P)^2 = -121\delta f + 110$. The additional high-frequency component in the spectrum under strong excitation has a maximum at a frequency of 0.686, and a dimensional frequency of 186 kHz. The results obtained are consistent with the data of [4]. In this work, the 2.7 times shift of the resonance frequency to the high-frequency region was measured experimentally when studying the transmission of a monodisperse layer into one bubble at the action of sound signals with the frequency from 30 to 250 kHz. The paper describes this effect using a self-consistent scattering model.

![Figure 3](image1.png)  
**Figure 3.** The spectrum of the initial phase of the radiation of the bubble region. Weak wave.

![Figure 4](image2.png)  
**Figure 4.** The spectrum of the initial phase of the radiation of the bubble region. Strong wave.

After the medium emits the main energy in both cases, the emission spectra at large times are similar in structure. Spectra consist of three components. The first two components are the boundary frequencies of the absorption band, and the third lies immediately with the upper line of the main
spectrum. The first two components are the boundary frequencies of the absorption band, and the third lies immediately with the upper line of the main spectrum. The spectrum consists of a line in the low-frequency region and a band in the high-frequency region. The low-frequency line has a dimensionless frequency of $0.158$ or $12.95 \text{ Hz}$. This frequency is lower than the Minnaert frequency. The upper frequency of the continuous absorption band is $0.657$ or $53.86 \text{ kHz}$. In the high-frequency region, due to radiation from the zone of the standing wave, the band has pronounced boundary frequencies.

The energy spectrum of a strong wave is shown in figure 6. Maintaining the same structure as a whole, the entire spectrum is substantially shifted to the high-frequency region and is much wider than the spectrum of weak excitation. The frequency range of the spectrum for the amplitude of the exciting wave $\delta P_w = 10 \text{ Pa}$ lies within $12.54 - 54.19 \text{ kHz}$, and for a wave with amplitude $\delta P_w = 1 \text{ MPa}$ within $42.41 - 178.62 \text{ kHz}$.

The frequency of the low-frequency boundary of the strong excitation spectrum is $3.27$ times greater than the frequency of the low-frequency boundary of the weak excitation, and the width of the strong excitation spectrum is $3.38$ times greater.

**Conclusions**

Studies have shown that when a short broadband sound pulse acts on a bubble medium with an inhomogeneous distribution of bubbles in the border zone, medium radiation consisting of two phases is formed. In the first short phase, more than $90\%$ of the energy introduced by the wave into the bubble medium is emitted. The emission spectrum is continuous and lies in the absorption band of the bubble medium with a linear power drop from low to high frequencies. The second phase of the radiation lasts much longer and its spectrum lies outside the boundaries of the absorption band. The structure of the spectrum weakly depends on the power of the exciting wave for a given type of heterogeneity, but with a strong wave the spectrum shifts significantly to the high-frequency region.

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