Multi-Criteria Knapsack Problem for Disease Selection in an Observation Ward

N Lurkittikul1 and O Kittitheepranochai1

1 Department of Industrial Engineering, Chulalongkorn University, Bangkok, Thailand 10330

E-mail: nopparuth.l.ty127@gmail.com

Abstract. The aging population and the introduction of Thailand universal healthcare have increased inpatients and outpatients to public hospitals, particularly to a hospital that provides special and comprehensive health services. Many inpatient wards have experienced large influx of inpatients as the hospitals have to admit all patients regardless their conditions. These overcrowding wards cause stress to medical staffs, block access between medical departments, hospital-acquired infections, and ineffective uses of resources. One way to manage such inundated inpatient is to select some patients whose conditions require less clinical attention or whose lengths of stay are predictable and short and, then, place them at an observation ward. This intermediate ward increases turnover of beds and reduces unnecessary paperwork as patients are considered to be outpatients. In this article, we studied inpatient data of a tertiary care hospital in which an observation ward was considered to alleviate the overcrowding problem at Internal Medicine Department. The analysis of data showed that the hospital can balance inpatient flow by managing a group of patients who is admitted because of treatments ordered by its special clinics. Having explored several alternatives, we suggested patient selection criteria and proposed a layout at an observation ward. The hospital should increase medical beds in a new building ward because the current observation ward can handle 27.3% of total short stay patients, while the observation ward is projected to handle 80% of total short stay patients.

1. Introduction

The aging population and the introduction of Thailand universal healthcare have increased numbers of inpatients and outpatients in public hospitals for the last decade. According to Bureau of Policy and Strategy Ministry of Health, the numbers of inpatients and outpatients have approximately doubled from five millions inpatients and 90 million outpatients in 2001 to twelve millions inpatients and 170 million outpatients in 2011, respectively. In particular, public tertiary-care hospitals that operate by the government and provide comprehensive health services have experienced large influx of inpatients as the hospitals are equipped with sufficient resources and have to admit all patients regardless their conditions. These overcrowding wards cause stress to medical staffs, access block between medical departments, hospital-acquired infections, and ineffective uses of resources. All of which undermines the public health quality in terms of prevention and treatment.

Many researchers and health offices have addressed this patient overcrowded problem and proposed recommendations, such as shortening patient’s length of stay by streamlining non-medical processes, reducing waiting time by scheduling and preparing key resources, and managing the flow of
patients by grouping inpatients by their conditions and/or symptoms. One way to manage such inundated inpatients while accomplishing these three recommendations is to select some patients whose conditions require less clinical attention or whose lengths of stay are predictable and short and, then, place them at a special area, called an observation ward. The observation ward can be viewed a buffer ward between Outpatient Department and Inpatient Department as targeted patients usually stay in the ward during 8 - 48 hours. This intermediate ward increases turnover of beds as it promotes the rapid flow of inpatients and reduces unnecessary paperwork as patients are considered to be outpatients. In some case, an observer ward could reduce the waiting time of patients and the cost of treatments.

The concept of an observation ward – providing specialized services to selected patients so that the patients receive better services in terms of treatment and prevention – is a rational one. In fact, many suggestions and case studies have been reported in the area of Emergency Medicine.

2. Literature review

The relevant researches to this article can be grouped into two categories: observation ward and analytic model in health service.

2.1. Observation ward

An observation ward – also referred to as an observation unit or an observation room – is a health facility located outside inpatient wards that consists of beds and medical equipment designed for providing a short term therapy and/or observing and evaluating symptoms of selected patients [2]. Establishing an observation ward benefits three stakeholders:

- **Patient aspect:** Recuperating at an observation ward improves the patient’s quality of life and satisfaction. Patients who recover in an observation ward usually stay in a hospital shorter than those who stay in a general ward because the patients experience less hospital-acquired infection and receive prompt treatments [2, 5]. Some researchers reported that an observation ward helps reduce the patient mortality [7].
  - **Medical staff aspect:** As patients who recover in an observation ward usually have mild symptoms or minor injuries, the comprehensive investigations of patients, e.g. full triage cross examination, and patient relative interview, are not required [2, 4, 5]. As a result, medical staffs, particularly nurses, spend more time tending patient’s needs that, in turn, helps detecting any complications.
- **Hospital aspect:** A hospital could reduce overall expenditure as patients recover faster with less infection [5, 6].

Nevertheless, an observation ward has drawbacks such as increasing of re-admission rate [6, 7] and demanding additional space and staffs if it is poorly managed. Researchers reported the abuses of an observation ward on difference purposes, such as to prolong length of stay of inpatients, to bill emergency visits, and to handle post-operation patients. Therefore, it is important to establish the clear goals. According to Brillman et al. [1], the goals of observation ward are usually intertwined with a medical department that is directly responsible for an observation ward and can be grouped into three types as follows:

- **Emergency Department:** Majority of observation wards are located in Emergency Department and led by emergency physicians to assist patients who require further treatments or evaluations. The main objectives are to improve quality of treatments and to assess conditions of patients. The efficient treatment and effective assessment should reduce admitted patients, waiting times, and cost of treatments.
- **Outpatient Department:** Some observation ward aims at patients who wait for admission or transferring. This ward is generally operated by non-emergency physicians. Located in
Outpatient Department, its main goal is to manage patients who are medically stable or ready for disposition.

- Inpatient Department: Aimed at cost reduction and alleviated crowded admitted patients, the observation ward located in Inpatient Department targets short stay patients. A group of inpatient physicians and emergency physician are usually in charges of these units to ensure uninterrupted flows of patients.

In some cases, an observation ward could use to observe and to diagnose patients whose causes of disease are unknown or as a temporary ward for treating short stay patients or evaluating the patient’s psychosocial needs.

Beside the direct responsible department that usually contributes to the location of an observation ward, the other important parts of establishing the clear goals are the targeted patients and the appropriated time limit. The targeted patients should exhibit clinical conditions easy to observe and treat [1-3] e.g., patient’s consciousness, strong vital signs, exact goal of treatment, and mild symptom. An observation ward has to operate within an appropriate time limit to ensure the constant and uninterrupted flow of patients. As a result, many studies suggested different durations in which patients should stay in an observation ward ranging from 8 - 48 hours and depending on the goals. Nevertheless, the previous studies agreed that patients who stay more than 72 hours should be admitted into an inpatient ward [5] because their presence increases the workload and diminishes the effectiveness of an observation ward. The effectiveness of an observation ward depends largely on sufficient resources, such as basic equipment and versatile medical staffs, as patients share similar non-chronic symptoms and an observation ward has to handle patients as a one-stop service.

2.2. Analytic model in health service

A health system is a dedicated and complex system that usually involves the life of human beings. As a result, any initiative faces challenges and securities from stakeholders resulting strong resistance. One way to understand the ramifications of an initiative before the actual implementation is to use an analytic model, such as simulation models and mathematical models.

Simulation models have been widely applied to the health system for different purposes. To narrow the scope of this review, we focus on the simulation models to determine numbers of resources and allocation policy. One of the most competing resources in hospital is an operation room as a surgery may be critical to a patient’s life and requires many resources, for example surgeons, anesthesiologists, scrub nurses, and instrument. As a result, the maximum numbers of patients who can be prepped for surgeries was proposed by Ballard et al [9] to improve the patient flow while maintain the hospital standards. The simulation leaded to significant numbers of patients who receive surgeries before the original planned. Besides the resources, the workload balancing and bottleneck elimination are important issues in an operation room. Therefore, Zheng et al [13] proposed a simulation model to improve process and utilization of key resources, including manpower and facilities. All these studies discover that a simulation model is fundamental for re-engineering and understanding what-if scenario.

Since the simulation model requires enormous data and statistics, one of sensible approaches is a mathematical model embedded with historical data. The inspiration of this approach derives from engineering disciplines, particularly factory and plant design. For example, Caputo et al [10] studied the environmental situation at an industrial plant that emits many hazard materials and the decisions by a safety manager to reduce overall hazard released in order to comply with environmental regulations. Therefore, the portfolio of safety measures with budget constraints is modeled as multi-criteria knapsack model. The sensitivity analysis of the model with respected to budgets reveals the useful information for further analytic decision. Another example is the study of foundry industry by
Camargo et al [11]. As one of the important Brazilian industries, the key to sustain growth and to maintain competitiveness is the efficient production planning as production sequence and quantities highly affect operation costs. To solve this generalized lot-sizing problem, the two-step heuristic method that combines a genetic algorithm and a knapsack algorithm is proposed and compared with the optimal solution. The result is applicable to the industry as the quality of solution and the computation time are both within reasonable range.

This article aims to illustrate the analysis and procedures necessary for design an observation ward. Having addressed the contribution, the remaining sections are organized as follows. Section 3 overviews the background of the case study hospital along with the analysis of data that lead to the hospital decision to consider an observation ward. Section 4 describes the mathematical model that enables the selection of diseases for an observation ward and the design of an observation room based on available space. Section 5 discusses the conclusion and guidelines to the future research.

3. Case Study Hospital
The case study hospital is the 855-bed tertiary care hospital located in the western region of Thailand. The hospital covers 14 medical specialties for outpatients and 10 medical specialties for inpatients and is capable to handle 600,000 outpatients and 44,000 inpatients annually as shown in figure 1.

![Figure 1](image-url) Numbers of patients admitted into specialty wards at the case study hospital in 2011 and 2012.

The figure shows numbers of inpatients in each specialty ward in 2011 and 2012. With exceptions of Pediatrics wards, numbers of patients admitted into major specialty wards increase in 2012. This aligns with the high aging population and low birthrate in the western region. Figure 1 also suggests that 30% of total inpatients are admitted into Internal Medicine Department. This department can be further divided into nine wards as shown in figure 2.
Figure 2. Number of hospital beds at each ward is responsible by Internal Medicine Department.

Figure 2 depicts numbers of registered beds at each ward responsible by Internal Medicine Department which can be classified by expenditure into two types: private bed and general bed. Private beds are dedicated beds located in separated rooms designed to serve patients who prioritize privacy and capable to afford additional expense, whereas general beds are economic beds located in common area and easy to access by nurses. Wards 1, 2, 3 and 4 are main wards of the department as their targeted patients are those who recuperate from incentive care units or have chronic diseases, such as Cancel, Pneumonia, and Leukemia. As the tertiary health facility and the central hospital in the region, the hospital has to admit all patients regardless their conditions, including a large numbers of non-chronic patients. As a result, the numbers of inpatients in these main wards exceed the numbers of registered beds, and the hospital has to use stretchers as temporary medical beds and place non-chronic patients alongside corridors. To investigate this observation, we analyzed the length of stay of inpatient in Internal Medical Wards 1, 2, 3, and 4 in 2011 and 2012, as shown in figure 3.

Figure 3. Length of stay of inpatient in Internal Medical Wards 1, 2, 3, and 4 in 2011 and 2012.

Figure 3 confirms our observation. That is, a large portion of inpatients stays in the wards less than two days or 48 hours. In addition, numbers of inpatients who recuperate more than two day gradually decrease. Evidently, these medical wards serve more non-chronic patients than their originally
designed, and reducing number of short-stay inpatients should alleviate the overcrowded wards and improve the quality of treatments. However, it is worth to understand arrival patterns of these short-stay inpatients before suggest any solutions, as shown in figure 4.

![Figure 4](image.png)

**Figure 4.** Arrival patterns of inpatients distributions of short stay patient demand on each day of week.

Figure 4 illustrates the arrival patterns and frequencies of the short-stay inpatients on each day of the week. Despite relatively high variations, numbers of the patients who arrive on weekend are less than those who arrive on weekday. The figure shows that the inpatients tend to arrive on Monday or with average of 16.6 and standard deviation of 6.5 patients. The similar pattern can be observed on Wednesday and Friday. The further investigation and the interview with stakeholders reveal that the pattern is influence by special clinics that require specific treatments such as blood transfusion and bone marrow extractions. Moreover, these patients visit the hospital at fixed and predictable time intervals. Having contemplated these evidences and organized series of meetings, the hospital suggests that establishing an observation ward should help to alleviate numbers of short-stay patients in these wards and to streamline medical documents required for these inpatients.

4. Designing an observation ward

After reviewing the purpose and resources required, an observation ward is decided to be located in Emergency Department as shown in figure 5.

![Figure 5](image.png)

**Figure 5.** Layout and numbers of medical beds of the proposed observation ward in Emergency Department.
Because of the limited space in Emergency Department, this observation ward contains only three medical beds and serves as a temporary faculty until a new building is constructed and fully functioned. During the meetings, some topics such as availability of nurses, information flows, and supporting activities are raised. Nevertheless, the two biggest concerns are: utilization of medical beds in an observation ward and selection of patients. The answers to these concerns are important and paramount to the success of an observation ward.

Before applying an analytical model to answer these concerns, it is important to understand the nature of length of stay. Unlike the processing time in manufacturing machines, two identical patients who are admitted and diagnosed with the same disease may have different length of stay as illustrated in figure 6.

Figure 6. Length of stay of Essential hypertension disease (ICD10:I10).

Figure 6 depicts hours in which patients who are admitted with Essential (primary) hypertension disease spend in the wards. The figure suggests that the length of stay depends on an individual patient. To narrow diseases before using an analytical model, we applied Pareto principle or 80/20 rule to the 2011-2012 medical records. Among 793 diseases that patient spend less than 48 hours in the wards, 114 diseases account for 80% of total short stay patients. These diseases become candidates to an observation ward, and theirs data are embedded in the following mathematical model.

4.1. Mathematical Model
\[ \mathcal{D} = \text{set of diseases} \]
\[ \mathcal{W} = \text{set of day in week} \]
\[ p_d = \text{impact in disease } d \]
\[ l_d = \text{average length of stay of disease } d \]
\[ n_{dw} = \text{average numbers of patient of disease } d \text{ on day } w \]
\[ H_w = \text{available hours of each medical bed on day } w \]
\[ b = \text{number of medical beds in an observation ward} \]
\[ x_d = \begin{cases} 1, & \text{if disease } d \text{ is selected in the observation ward} \\ 0, & \text{otherwise.} \end{cases} \]
Expression 1 is the objective function that maximizes the total impacts of the selected diseases. The hospital is interested in impacts as many criteria such as numbers of patient in each disease, possibility to develop serious symptom, nature of treatment, available equipment and skilled of staff. These criteria are different from one hospital to others and require serious discussion among physicals and medical staffs. The constraints of this Integer Programming are listed in Expressions 2 and 3. Expression 2 ensures that the average duration required by patients who are served by an observation ward on each day of the week does not exceed the medical-bed capacities. This expression reflects the fact that medical beds are allocated directly to each department and cannot be easily borrowed or transferred. Expression 3 assures that the decision variable or the selection of disease is binary as an initial triage unable to predict the length of stay of each patient. Mathematically, Expressions 1 – 3 can be viewed as the multi-criteria knapsack problem – a generalization of classical knapsack problem in which a person must select a set of items to maximize total benefits while satisfy the limitations of all resources. Using the analogy of the multi-criteria knapsack problem, parameters $p_d$, $l_d$, $n_{dw}$, and $b \cdot H_w$ can be viewed as the benefit of each item, resources required of each item, and total limited resources, respectively.

4.2. Numerical Experience

We embedded the data of the case study hospital into our model and solved it with Microsoft Excel / Solver [12]. Since the available hours of each medical bed is a difficult parameter to be estimated, we varied the parameter from 14 hours to 24 hours with two-hour interval, i.e., \{14, 16, 18, 20, 22, 24\} and reported the result and numerical experience as shown in table 1.

| $H_w$ | Numbers of Diseases | % of Served Patients | Daily Average Utilization (%) | Daily Standard Deviation of Utilization (%) |
|-------|----------------------|----------------------|-------------------------------|------------------------------------------|
| 24    | 42                   | 27.3                 | 93.1                          | 10.4                                     |
| 22    | 39                   | 25.1                 | 93.4                          | 9.1                                      |
| 20    | 36                   | 23.3                 | 93.8                          | 9.1                                      |
| 18    | 29                   | 21.2                 | 93.9                          | 8.9                                      |
| 16    | 31                   | 19.2                 | 93.5                          | 10.0                                     |
| 14    | 28                   | 17.1                 | 94.6                          | 8.0                                      |

* percentage form 80% of total short stay patient (targeted patient)

As the available hours of each medical bed decrease, table 1 suggests that number of patients who are served by an observation ward gradually decreases. Decreasing $H_w$ results in changing the set of diseases. Interestingly, the number of diseases at $H_w = 16$ is 31 diseases which is larger than $H_w = 18$ or 29 diseases because many diseases are introduced into an observation ward at $H_w = 16$. In particular, the model selects diseases to maintain high utilization of an observation
ward. It is worth to mention that the model fails to reach 100% utilization as average numbers of patients on Saturday and Sunday are significant less than that on weekday, resulting to 93% utilization of medial beds.

4.3. Greenfield Design
Because of the limited space, the proposed observation ward can handle 17.1-27.3% of potential inpatients. This raises a question: how many medical beds does the hospital require if there is no spatial constraint? As a result, we modified Expression 2 by assuming $x_d = 1 \quad \forall d \in D$ to determine the number of medical beds required with Expression 4.

$$b = \max_{w \in W} \left\{ \frac{\sum_{d \in D} l_d \cdot n_{dw}}{H_w} \right\}$$

Number of beds in an observation ward ($b$) is the maximum round-up value of total hours all short-stay patients spend in an observation ward each day over available hours of bed. The result of this calculation, average utilization, and standard deviation are shown in table 2.

| $H_w$ | number of beds ($b$) | % of served patients a | Daily average utilization (%) | Daily standard deviation of utilization (%) |
|-------|----------------------|------------------------|-----------------------------|-------------------------------------------|
| 24    | 17                   | 100                    | 69.4                        | 18.4                                      |
| 22    | 18                   | 100                    | 71.5                        | 18.9                                      |
| 20    | 20                   | 100                    | 70.8                        | 18.7                                      |
| 18    | 22                   | 100                    | 71.5                        | 18.9                                      |
| 16    | 25                   | 100                    | 70.8                        | 18.7                                      |
| 14    | 28                   | 100                    | 72.3                        | 19.1                                      |

a percentage form 80% of total short stay patients

As the available hours decrease, table 2 shows that the numbers of beds gradually increase. The result reveals that the utilization of beds depends on number of patients on Monday. This reflects the fact that majority of patients show up on Monday resulting to the under-utilization in other days. If numbers of inpatients were similar to other days, the observation ward would require only 15 beds instead of 20 beds in case of $H_w = 20$ or 25% less capacity. This observation leads to the discussion on patient flows. The imbalance of patient flows affects to utilization of beds. The average of utilization in Greenfield Design obviously lower than Numerical Experience because the model in Numerical Experience attempt to maximum impact and it affects high utilization simultaneously.

5. Conclusion and future works

We analyzed the data of a Thailand tertiary care hospital and found that the hospital has faced the overcrowded inpatient in the major wards responsible by Internal Medicine Department as the inpatients spends less than 48 hours in the wards. The additional investigation revealed that some of these short-stay patients are caused by special treatments. As a result, the hospital considers an observation ward to alleviate this problem and to streamline procedures. Furthermore, the meeting between Internal Medicine Department and Emergency Department concluded a temporary site of an observation ward. Because of a limited space, we proposed a mathematical model to select the set of
diseases and determined numbers of medical beds if there is no spatial constraint. The model suggests that three medical beds available at Emergency Department can ideally handle approximately 27.3% of targeted patients that consists of 42 diseases. Depending on the medical bed turnover, 17-28 medical beds are recommended if the hospital plans to setup a new observation ward in a new building. The result also implies that Internal Medicine Department should review special clinics schedule to balance the patient flows and to reduce a number of beds required in an observation ward.

The proposed mathematical model serves as a primary design tool as the deterministic arrival time and recuperated time are assumed. The further studies and comprehensive design using a discrete event simulation that accounts for the stochastic events are required. In addition, the design of an observation ward should incorporate with the procedure and information flow to ensure smooth and successful implementation.

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References
[1] Brillman J et al 1995 Management of observation units Annals of Emergency Medicine 25 823-830.
[2] Brand C et al 2004 Short Stay and Observation Units, Medical Assessment and Planning Units and Emergency Medical Units. Clinical Epidemiology and Health Service Evaluation Unit (Australia: Melbourne Health), Retrived from http://www.mh.org.au/.
[3] Dallos V and Mouzas GL 1981 An evaluation of the functions of the short-stay observation ward in the accident and emergency department British Medical Journal 282 37.
[4] Jones A, O'Driscoll K and Luke LC 1995 Head injuries and the observation ward Journal of Accident and Emergency Medicine 12 160.
[5] Cooke MW, Higgins J and Kidd P 2003 Use of emergency observation and assessment wards: a systematic literature review Emerg Med J 20 138-142.
[6] Daly S, Campbell DA and Cameron PA 2003 Short-stay units and observation medicine: a systematic review The Medical Journal of Australia (MJA) 178 559-563.
[7] Juan A, Salazar A, Alvarez A, Perez JR, Garcia L and Corbella X 2006 Effectiveness and safety of an emergency department shortstay unit as an alternative to standard inpatient hospitalization Emerg Med J 23 833-837.
[8] Ballard SM and Kuhl ME 2006 Winter Simulation Conference (WSC) .
[9] Zheng Q, Shen J, Liu Z, Fang K and Xiang W 2011 International Conference on Industrial Engineering and Engineering Management (IE&EM).
[10] Caputo AC, Pelagagge PM and Salini P 2013 A multicriteria knapsack approach to economic optimization of industrial safety measures Safety Science 51 534-360
[11] Camargo VCB, Mattiolli L and Toledo F 2012 A knapsack problem as a tool to solve the production planning problem in small foundries Computers and Operations Research 39 86-92
[12] Fylstra D, Lasdon L, Watson J and Waren A 1998 Design and use of the microsoft excel solver Interfaces 28 29-55.