Novel Meta-heuristic Algorithm for Feature Selection, Unconstrained Functions and Engineering Problems

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ABSTRACT  This paper proposes a Sine Cosine hybrid optimization algorithm with Modified Whale Optimization Algorithm (SCMWOA). The goal is to leverage the strengths of WOA and SCA to solve problems with continuous and binary decision variables. The SCMWOA algorithm is first tested on nineteen datasets from the UCI Machine Learning Repository with different number attributes, instances, and classes for feature selection. It is then employed to solve several benchmark functions and classical engineering case studies. The SCMWOA algorithm is applied for solving constrained optimization problems. The two tested examples are the welded beam design and the tension/compression spring design. The results emphasize that the SCMWOA algorithm outperforms several comparative optimization algorithms and provides better accuracy compared to other algorithms. The statistical analysis tests, including one-way analysis of variance (ANOVA) and Wilcoxon’s rank-sum, confirm that the SCMWOA algorithm performs better.

INDEX TERMS  Artificial intelligence, Machine learning, Optimization, Sine Cosine algorithm, Modified whale optimization algorithm

I. INTRODUCTION

Stochastic algorithms are traditionally characterized as a heuristic, although current research often labels them metaheuristic. According to Glover’s example, all-natural algorithms are termed metaheuristic [1]. Generally speaking, heuristic refers to the process of finding or detecting through trial and error. Meta- shows a level achieved above and beyond the fundamental heuristic as they are not problem-specific. In his foundational work [1], Fred Glover coined the word “metaheuristics” as “a master technique that drives other heuristics towards the local optimism to generate answers that have to be produced differently” [2]. So, metaheuristics can be considered as randomized local searches. While quality solutions may be found to optimize problems within an acceptable amount of time, there is no guarantee that the optimal solution might be achieved. It is most probable that these techniques will succeed. Yet, this is impossible. For high probability global optimization, almost any metaheuristic technique may be used [3].

Meta-heuristics have two characteristics in terms of search behavior: intensification and diversification [4]. Diversifying entails developing various solutions that look at the search field across the globe and intensifying implies restricting the search field to a limited area with superior information. A proper balance between intensity and diversity should be maintained throughout the solution selection process to speed algorithm concordance. The solution is selected for optimum convergence while randomization enhances the search for the
The updating process of the agents’ positions in the search space between the sine cosine and the modified WOA algorithms during iterations to avoid the low convergence rate.

The SCMWOA algorithm evaluation in the experiments is divided into three scenarios. The first scenario is designed to test the ability of the SCMWOA algorithm in feature selection problems based on nineteen different tested datasets from the UCI public machine learning repository. The SCMWOA is compared to original Grey Wolf Optimizer (bGWO) [18], bPSO [19], Stochastic Fractal Search (bSFS) [20], Whale Optimization Algorithm (bWOA) [21], Multi-verse Optimization (bMVO) [22], Satin Bowerbird Optimizer (bSBO) [23], Firefly Algorithm (bFA) [24], bGA [25] algorithms, Modified GWO (bMGWO) [26], hybrid of Particle Swarm Optimization (PSO) and GWO (bGWO-PSO) [27], hybrid of Genetic Algorithm (GA) and GWO (bGWO-GA) [26], and hybrid of SC and PSO (bSCA-PSO) [28] in which \( b \) at the front of a name denotes the binary variant of the algorithm.

The next scenario examines the SCMWOA algorithm’s ability to solve benchmark functions divided into unimodal and multimodal functions. Twenty-three functions are employed in this scenario. The SCMWOA in the second scenario is compared to original GWO [18], PSO [19], WOA [21], Feedforward Error Propagation (FEP) algorithm [29], Gravitational Search Algorithm (GSA) [30], GA [25] algorithms, Enhanced Grey Wolf Optimizer (EGWO) [31], hybrid of Crow Search Algorithm (CSA) and GWO (GWO-CSA) [31], and hybrid of SC and PSO (bSCA-PSO) [28]. The third and last scenario is designed in this work for testing the ability of the algorithm for solving classical constrained optimization problems of tension/compression spring design (TCSD) [32] and welded beam design [33]. In addition, the SCMWOA algorithm results are compared in the third scenario with the original GWO [18], PSO [19], WOA [21], and GSA [34] algorithms’ results to get the minimum cost.

The main contribution of this work can be summarized as follows.

- A Sine Cosine Modified Whale Optimization Algorithm (SCMWOA) is presented.
- A binary SCMWOA is presented.
- Ability of the binary SCMWOA algorithm in feature selection problems is tested.
- Ability of the SCMWOA algorithm to solve twenty-three benchmark functions is tested.
- Ability of the SCMWOA algorithm for solving two constrained optimization problems of Tension/Compression Spring and Welded Beam designs is confirmed.

The following sections are organized as follows. The materials and methods of WOA, modified WOA, and SCA are discussed in Section II. Section III and IV present the proposed SCMWOA algorithm in continuous and discrete forms. Section V shows the results and discussion of the designed scenarios of feature selection, benchmark functions, and solving constrained optimization problems. Conclusion and future work are introduced in Section VI.
II. MATERIALS AND METHODS

In this section, the WOA, the modified WOA, and the SCA optimization algorithms are presented.

A. WHALE OPTIMIZATION ALGORITHM

This algorithm was first proposed in 2016 [21]. It mimics the bubble-net foraging strategy of humpback whales. In this algorithm, a number of \( n \) whales in the WOA algorithm can "swim" in an \( n \)-dimensional search space. To get the food (global solution), the position of each whale should be updated in the space search during iterations. To achieve this, the following equation was implemented in the WOA algorithm.

\[
X(t+1) = X^*(t) - A_i|C_iX^*(t) - X(t)| \tag{1}
\]

where the vector \( X(t) \) represents the \( t \)th iteration’s solution. The vector \( X^*(t) \) indicates the prey’s possible position. The "\( \cdot \)" symbol between vectors represents the pairwise multiplication. The \( A \) and \( C \) vectors are updated during iterations as \( A = 2a_i r_1 - a, C = 2r_2 - a \), where \( a \) is decreasing linearly from 2 to 0. \( r_1 \) and \( r_2 \) are selected randomly between \( [0, 1] \).

The exploitation phase of the WOA algorithm is based on a shrinking encircling mechanism that decreases with the value of \( a \), and a spiral updating and is calculated as the distance between whale’s location and location of the prey. The process of spiral is expressed as in the following equation.

\[
X(t+1) = |X^*(t) - X(t)| e^{bl} \cos(2\pi l) + X^*(t) \tag{2}
\]

where \( l \) is selected randomly between \([-1, 1] \). The spiral’s shape is represented by the constant \( b \). To simulate the process of prey’s encircling and spiral movement, this equation is applied.

\[
X(t+1) = \begin{cases}
X^*(t) - AD & \text{if } r_3 < 0.5 \\
D' e^{bl} \cos(2\pi l) + X^*(t) & \text{otherwise}
\end{cases} \tag{3}
\]

where \( r_3 \) is selected randomly in \([0, 1] \) to control switching between a spiral or circular movement.

On the other side, the exploration Phase (searching for a prey) is done based on the vector \( A \). By this process, the agent goes away from the leader. Thus, the agent position will be updated according to a random whale \( X_r \). This allows the optimizer a more global search. This can be achieved by the following equation.

\[
X(t+1) = X_r - A_i|C_iX_r - X| \tag{4}
\]

The \( A \) vector is used to control switching between exploration and exploitation. The termination criterion of the WOA algorithm will be due to the number of iterations. The pseudo-code of the original WOA algorithm is shown in Algorithm 1.

Algorithm 1: The WOA algorithm [21]

1. Initialize WOA population \( X_i(i = 1, 2, ..., n) \), size \( n \), maximum iterations \( Max_{iter} \), and objective function \( F_n \).
2. Initialize WOA parameters \( a, A, C, l, r_1, r_2, r_3 \).
3. Calculate objective function \( F_n \) for each \( X_i \).
4. Find best solution \( X^* \).
5. while \( t \leq Max_{iter} \) do
6. for \( (i = 1 : i < n + 1) \) do
7. if \( (r_3 < 0.5) \) then
8. if \( (|A| < 1) \) then
9. Update current agents’ positions by Eq. 1
10. else
11. Select a random agent \( X_r \)
12. Update current agents’ positions by Eq. 4
13. end if
14. else
15. Update current agents’ positions by Eq. 2
16. end if
17. end for
18. Update \( a, A, C, l, r_3 \)
19. Calculate objective function \( F_n \) for each \( X_i \)
20. Find best solution \( X^* \)
21. end while
22. Return \( X^* \)

B. MODIFIED WHALE OPTIMIZATION ALGORITHM

As presented in the original WOA algorithm in the previous section, the position of search agent is changed/upated based on only one random whale, named \( X_r \), that is determined from the population randomly to give the optimizer a more global search capability (exploration ability). By increasing the number of random agents in the modified WOA (MWOA), the global search can be more effective and be achieved. The following equation is applied to replace equation 4 of the original WOA algorithm for increasing the number of random agents up to three agents and give the algorithm more exploration ability.

\[
X(t+1) = w_1 * X_{\alpha} + \zeta * w_2 * (X_{\beta} - X_{\gamma}) + (1 - \zeta) * w_3 * (X(t) - X_{\alpha}) \tag{5}
\]

where the three random agents are indicated as \( X_{\alpha}, X_{\beta}, \) and \( X_{\gamma} \), which are employed in the MWOA algorithm instead of one random agent. The parameter of \( \zeta \) is computed as follows.

\[
\zeta = 1 - \left( \frac{t}{Max_{iter}} \right)^2 \tag{6}
\]

where \( Max_{iter} \) as maximum number of iterations during the execution process. The \( w_1, w_2, \) and \( w_3 \) parameters are selected randomly in \([0, 1] \).
Algorithm 2: The SCMWOA Meta-heuristic Algorithm

1. Initialize SCMWOA algorithm population $X_i(i = 1, 2, \ldots, n)$, size $n$, maximum iterations $Max_{iter}$, objective function $F_n$.
2. Initialize SCMWOA algorithm parameters $a, A, C, l, r_1, r_2, r_3, r_4, r_5, r_6, r_7, w_1, w_2, w_3, t = 1$
3. Calculate Objective function values $F_n$ for each agent $X_i$
4. Find best solution $X^*$ based on $F_n$
5. while $t \leq Max_{iter}$ do
6. for ($i = 1 : i < n + 1$) do
7. if ($t \% 2 == 0$) then
8. if ($r_3 < 0.5$) then
9. if ($|A| < 1$) then
10. Update current agents’ positions based on the following equation
11. $X(t + 1) = X^*(t) - A.[C.X^*(t) - X(t)]$
12. else
13. Select three different random agents $X_{\alpha}, X_{\beta}$, and $X_{\gamma}$ from the population
14. Update $\zeta$ by the following equation.
15. $\zeta = 1 - \left(\frac{t}{Max_{iter}}\right)^2$
16. Update current agents’ positions based on the following equation using random agents
17. $X(t + 1) = w_1 \times X_{\alpha} + \zeta \times w_2 \times (X_{\beta} - X_{\gamma}) + (1 - \zeta) \times w_3 \times (X(t) - X_{\alpha})$
18. end if
19. else
20. if ($r_7 < 0.5$) then
21. Update current agents’ positions based on the following equation
22. $X(t) + r_4 \times \sin(r_5) \times |r_6.X^*(t) - X(t)|$
23. else
24. Update current agents’ positions based on the following equation
25. $X(t) + r_4 \times \cos(r_5) \times |r_6.X^*(t) - X(t)|$
26. end if
27. end if
28. end for
29. Update parameters $a, A, C, l, r_3, r_7$
30. Calculate objective function $F_n$ for each agent $X_i$ and update old values
31. Find best solution $X^*$ based on $F_n$ and update old value
32. Set $t = t + 1$
33. end while
34. Return best solution $X^*$

C. SINE COSINE ALGORITHM

The SCA algorithm was presented in [35] by switching between the sine and cosine based functions. To know the direction of the movement and how far the movement will be, SCA is based on a set of random variables. The following equation was used to update positions in this optimizer:

$$X(t + 1) = \begin{cases} 
X(t) + r_4 \times \sin(r_5) \\
|r_6.X^*(t) - X(t)| & r_7 < 0.5 \\
X(t) + r_4 \times \cos(r_5) \\
|r_6.X^*(t) - X(t)| & r_7 \geq 0.5 
\end{cases}$$

(7)

where the position of current solution id represented as $X(t)$, while the best solution is indicated as $X^*(t)$. The $r_5$, $r_6$, and $r_7$ parameters are selected randomly in $[0,1]$ during iterations. To make a balance between the process of exploration and the process of exploitation, $r_4$ is changed during iterations as follows.

$$r_4 = a - \frac{a \times t}{Max_{iter}}$$

(8)

where $t$ as current iteration, $a$ as constant, and $Max_{iter}$ represents the maximum number of iterations.
The presented Sine Cosine Modified Whale Optimization (SCMWOA) algorithm is explained in this section. The SCMWOA algorithm is shown in Algorithm 2. The presented algorithm is proposed by balancing the updating process of the agents’ positions between the sine cosine and the modified WOA algorithms during iterations.

The SCMWOA algorithm starts by initializing the population and algorithm parameters, then calculates all agents’ objective functions to get the initial best solution in steps from 1 to 4. For iterations $t \% 2 = 0$ of current iteration $t$, the modified WOA algorithm is applied in steps from 8 to 18. As presented in the modified WOA algorithm, the position of the search agent is changed and updated based on three different random whales, named $X_{a}$, $X_{b}$, and $X_{c}$, that can be determined from the population to give the algorithm more global search capability. While for the rest of the iterations, the cosine algorithm is applied in steps from 20 to 25. Steps from 28 to 30 are applied to update the parameters and find the current best solution $X^{*}$.

The SCMWOA algorithm’s computational complexity according to Algorithm 2 will be discussed. Let $n$ as number of population; $M_{t}$ as total number of iterations. For each part of the algorithm, the time complexity can be defined as:

- Objective function evaluation: $O (M_{t} \times n)$.
- Best individual update: $O (M_{t} \times n)$.
- Iteration counter increment: $O (M_{t})$.

The overall complexity of the proposed SCMWOA algorithm is $O (M_{t} \times n)$. Considering the number of variables as $m$, the final computational complexity of the algorithm will be $O (M_{t} \times n \times m)$.

### III. PROPOSED SCMWOA META-HEURISTIC ALGORITHM

#### IV. BINARY SCMWOA ALGORITHM

The SCMWOA algorithm has a binary version based on MWOA and SCA. To get a probability value of two discrete classes, an activation function, named Sigmoid, can be employed for binary classification [36]. The classification using this function gives output values between zero or one. The optimizer’s outputs are changed to binary values from the continuous ones by the following equation.

$$X_{d}^{(t+1)} = \begin{cases} 1 & \text{if } \text{Sigmoid}(X^{*}) \geq 0.5, \\ 0 & \text{otherwise} \end{cases}$$

(9)

where the best position is represented by $X^{*}$. The function $\text{Sigmoid}$ is mainly scales the continuous values to 0 or 1 values and is calculated as follows.

$$\text{Sigmoid}(X^{*}) = \frac{1}{1 + e^{-10(X^{*} - 0.5)}}$$

(10)

The proposed binary SCMWOA algorithm is discussed step by step in Algorithm 3. The binary SCMWOA algorithm starts by initializing the population and algorithm parameters in step 1. All the solutions are changed to binary ones in step 2. The algorithm calculates the agents’ objective function values to get the initial best solution in steps 3 and 4. The k-NN model training and the error are calculated to adjust the performance at step 5. The SCMWOA algorithm is then applied in step 7, and the output solutions are updated to get the optimal solution.
binary ones in step 8. Steps from 9 to 11 evaluate the objective function redetermining the best solution and update the algorithm parameters for the next iteration.

V. EXPERIMENTAL RESULTS AND DISCUSSION

The experimental results section is divided into three scenarios. The first scenario is designed to test the ability of the SCMWOA algorithm in feature selection problems based on nineteen different tested datasets from the UCI public machine learning repository. The next scenario examines the presented algorithm’s ability to solve twenty-three benchmark functions divided into unimodal and multimodal functions. The third and last scenario is designed in this work for testing the ability of the algorithm for solving two constrained optimization problems of Tension/Compression Spring and Welded Beam designs.

A. FEATURE SELECTION SCENARIO

Nineteen UCI repository datasets are tested in this work to analyze the ability of the proposed algorithm for feature selection problems. The nineteen datasets, shown in Table 1, are determined with various number of features/attributes, instances, and classed that the algorithms may be evaluated on working with various concerns. In this scenario, the presented algorithm of binary SCMWOA (bSCMWOA) is compared to the original bGWO [18], bPSO [19], bSFS [20], bWOA [21], bMVO [22], bSBO [23], bFA [24], bGA [25] algorithms, Modified GWO (bMGWO) [26], hybrid of PSO and GWO (bGWO-PSO) [27], hybrid of GA and GWO (bGWO-GA) [26], and hybrid of SCA and PSO (bSCA-PSO) [28] in which b denotes the binary variant of the algorithm. Configuration of the presented SCMWOA and compared algorithms during the experiments are discussed in Tables 2 and 3 with 100 iterations and 10 agents initiated at the start of each algorithm.

For evaluating the feature selection ability of the proposed algorithm, the following metrics are employed in experiments. For \( M \) as the number repetitions, \( g_j \) represents the optimal solution, and \( N \) be the total number of points. The following equation can compute the Average Error for \( L \) as a class for point, \( C \) as classifier output for that point, and \( Match \) to represent the matching between the two inputs.

\[
AvgError = 1 - \frac{1}{M} \sum_{j=1}^{M} \frac{1}{N} \sum_{i=1}^{N} Match(C_i, L_i) \quad (11)
\]

The Average Fitness can be computed, for \( size(g_j^*) \) as the vector \( g_j^* \) size and \( D \) represents the size of dataset, as follows.

\[
AvgSelectSize = \frac{1}{M} \sum_{j=1}^{M} \frac{size(g_j^*)}{D} \quad (12)
\]

The Best Fitness and the Worst Fitness are computed as in the following equations.

\[
BestF_n = \min_{j=1}^{M} g_j^* \quad (13)
\]

\[
WorstF_n = \max_{j=1}^{M} g_j^* \quad (14)
\]

The Mean and the Standard Deviation (SD) are represented as in the following equations.

\[
SD = \sqrt{\frac{1}{M-1} \sum (g_j^* - Mean)^2} \quad (15)
\]
FIGURE 1: Feature selection average results acquired over all the datasets.

| Dataset    | bSCMWOA | bGWO | bGWO-PSO | bPSO | bSFS | bW AO | bMGWO | bMVO | bSBO | bGWO-GA | bFA | bGA | bSCA-PSO |
|------------|---------|------|----------|------|------|-------|-------|------|------|----------|-----|-----|---------|
| ISOLET     | 0.6472  | 0.8561 | 0.7878   | 0.9665 |      |       |       |      |      | 0.7228  |     |     | 0.4758  |
| Lymphography | 0.2244  | 0.39727| 0.246826 | 0.50894 | 0.54441|       |       |      |      | 0.6472  |     |     | 0.4758  |
| Average    | 0.3518  | 0.4361 | 0.3896   | 0.6750 | 0.4872| 0.3770| 0.4741| 0.4036| 0.3560| 0.4758  |     |     | 0.4758  |

TABLE 2: Proposed and compared algorithms’ average error.

| Dataset    | ISOLET | Lymphography | Ring | Diabetes | Parkinsons | ZCR | Titanic | Tiger | Tungstom | WaveformE&W | Tic-Tac-Toe | Murat | HA (Smartphones) | ISOLET | Lymphography | Breast Cancer | Parkinsons | Diabetes | Packet Size | Packet Error Rate |
|------------|--------|---------------|------|----------|-------------|-----|---------|-------|----------|-------------|-------------|-------|------------------|--------|---------------|----------------|-------------|----------|-------------|------------------|
| ISOLET     | 0.3560 | 0.6724         | 0.2737| 0.2967   | 0.3696      |     | 0.4758  | 0.4758| 0.4758   | 0.4758     | 0.4758     | 0.4758| 0.4758         | 0.4758| 0.4758       | 0.4758        | 0.4758     | 0.4758   | 0.4758     | 0.4758            |
| ISOLET     | 0.6472 | 0.8561         | 0.7878| 0.9665   | 0.7228      |     | 0.4758  | 0.4758| 0.4758   | 0.4758     | 0.4758     | 0.4758| 0.4758         | 0.4758| 0.4758       | 0.4758        | 0.4758     | 0.4758   | 0.4758     | 0.4758            |

TABLE 5: Average select size of the proposed and compared algorithms.

| Dataset    | ISOLET | Lymphography | Ring | Diabetes | Parkinsons | ZCR | Titanic | Tiger | Tungstom | WaveformE&W | Tic-Tac-Toe | Murat | HA (Smartphones) | ISOLET | Lymphography | Breast Cancer | Parkinsons | Diabetes | Packet Size | Packet Error Rate |
|------------|--------|---------------|------|----------|-------------|-----|---------|-------|----------|-------------|-------------|-------|------------------|--------|---------------|----------------|-------------|----------|-------------|------------------|
| ISOLET     | 0.3560 | 0.6724         | 0.2737| 0.2967   | 0.3696      |     | 0.4758  | 0.4758| 0.4758   | 0.4758     | 0.4758     | 0.4758| 0.4758         | 0.4758| 0.4758       | 0.4758        | 0.4758     | 0.4758   | 0.4758     | 0.4758            |
| ISOLET     | 0.6472 | 0.8561         | 0.7878| 0.9665   | 0.7228      |     | 0.4758  | 0.4758| 0.4758   | 0.4758     | 0.4758     | 0.4758| 0.4758         | 0.4758| 0.4758       | 0.4758        | 0.4758     | 0.4758   | 0.4758     | 0.4758            |
### Table 6: Average fitness of the proposed and compared algorithms.

| Dataset | ISOLET | WaveformEW | Titanic | Hepatitis | HAR (Smartphones) | iPhone | bSCMWOA | bGWO | bGWO-PSO | bPSO | bSFS | bW AO | bMGWO | bMVO | bSBO | bGWO-GA | bFA | bGA | bSCA-PSO |
|---------|--------|-------------|---------|-----------|------------------|-------|---------|------|----------|-----|------|------|-------|------|------|--------|-----|------|--------|
| Mean    | 0.6486 | 0.5437      | 0.4576  | 0.5453    | 0.513            | 0.685 | 0.773   | 0.739| 0.846    | 0.835| 0.854| 0.873| 0.854| 0.873| 0.846| 0.835  | 0.835| 0.854| 0.873  |
| Standard Deviation | 0.0894 | 0.0646      | 0.0764  | 0.0898    | 0.0824          | 0.049 | 0.0764  | 0.076| 0.0894   | 0.082| 0.0894| 0.089| 0.0894| 0.089| 0.0894| 0.0894 | 0.0894| 0.0894| 0.0894 |

### Table 7: Proposed and compared algorithms’ best fitness.

| Dataset | ISOLET | WaveformEW | Titanic | Hepatitis | HAR (Smartphones) | iPhone | bSCMWOA | bGWO | bGWO-PSO | bPSO | bSFS | bW AO | bMGWO | bMVO | bSBO | bGWO-GA | bFA | bGA | bSCA-PSO |
|---------|--------|-------------|---------|-----------|------------------|-------|---------|------|----------|-----|------|------|-------|------|------|--------|-----|------|--------|
| Mean    | 0.6486 | 0.5437      | 0.4576  | 0.5453    | 0.513            | 0.685 | 0.773   | 0.739| 0.846    | 0.835| 0.854| 0.873| 0.854| 0.873| 0.846| 0.835  | 0.835| 0.854| 0.873  |
| Standard Deviation | 0.0894 | 0.0646      | 0.0764  | 0.0898    | 0.0824          | 0.049 | 0.0764  | 0.076| 0.0894   | 0.082| 0.0894| 0.089| 0.0894| 0.089| 0.0894| 0.0894 | 0.0894| 0.0894| 0.0894 |

The results of the present and compared algorithms acquired over the nineteen datasets, summary results, to measure the performance of the bSCMWOA algorithm. The results respectively. The standard deviation fitness results of the tested algorithms are shown in Table 9. Tables 10 presented the p-values of the proposed and other tested algorithms for the nineteen datasets, which reflects the performance of the suggested algorithm with a p-value less than 0.005 for all datasets. Figure 1 shows the feature selection average results acquired over all the datasets, summary results, to measure the performance of the bSCMWOA algorithm. The results
and multimodal-based fixed dimension functions. This scenario tests the ability of the presented algorithm to get the best solution for the benchmark functions. Tables from Table 4 to Table 10 and Figure 1 shown in tables from Table 4 to Table 10 and Figure 1 confirm the performance of the binary SCWAOA algorithm for feature selection problem.

### B. BENCHMARK FUNCTIONS SCENARIO

This scenario tests the ability of the presented algorithm to get the best solution for the benchmark functions. Twenty-three functions, divided into seven unimodal, six multimodal, and ten multimodal-based fixed-dimension benchmark functions, are employed in this sub-section. Figure 21 describe the unimodal functions parameters and range. The multimodal and multimodal-based fixed dimension functions description of the range and minimum values are shown in Figures 22 and 23. The SCWAOA in the second scenario is compared to original GWO [18], PSO [19], WOA [21], FEP [29], GSA [30], GA [25] algorithms, EGWO [31], hybrid of CSA and GWO (GWO-CSA) [31], and hybrid of SCA and PSO (bSCA-PSO) [28].

The mean and standard deviation (StDev) results of the suggested and compared algorithms over the benchmark functions (f1 to f23) are shown in Table 11. This table shows that the proposed SCWAOA algorithm achieved zero values in Mean and StDev in some cases and better results than the compared single and hybrid algorithms in other cases.

Table 12 shows the ANOVA test for sample functions (f1, f2, f3, f9, f11, f23). The T-test analysis test for all benchmark functions (f1 to f23) using the suggested algorithm against the compared algorithms is presented in Table 13. The histogram interpolation of a sample function (f11) based on the SCWAOA algorithm and PSO, GWO, WOA, and GA algorithms is discussed in Table 14.

Box plot of the suggested and compared algorithms for benchmark function (f1 to f7) and (f8 to f19) and (f10 to f23) are shown in Figures 2, 3, and 4, respectively. The histogram of the suggested and compared algorithms for benchmark function (f1, f2, f3, f9, f11, and f23) and the convergence curves based on the benchmark functions (f1, f2, f3, f9, f11, and f23) are presented in Figures 6 and 7.

The results of the proposed continuous SCWAOA algorithm in this scenario, compared to the state-of-the-art algorithms, confirm the performance of the algorithm for the benchmark functions.
TABLE 11: Mean and standard deviation (StDev) of the suggested and compared algorithms over the benchmark functions (f1 to f23)

| Algorithm | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev |
|-----------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|-------|
| SCMWOA   | -6.66 -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 |
| PSO       | -6.66 -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 |
| WOA       | -6.66 -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 |
| GWO       | -6.66 -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 |
| FEP       | -6.66 -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 |
| GA        | -6.66 -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 |
| EGWO      | -6.66 -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 |
| GWO-CSA   | -6.66 -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 |
| SCA-PSO   | -6.66 -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 | -1.02 | -2.06 |

C. SOLVING CONSTRAINED OPTIMIZATION PROBLEMS SCENARIO

This section is designed to validate the SCMWOA algorithm to solve two constrained optimization example of tension/compression spring and welded beam designs. The two engineering problems are described mathematically in equations 17-23. In addition, the SCMWOA algorithm results are compared with GWO [18], WOA [21], GSA [34], and PSO [19] algorithms result to get the minimum cost.

1) Tension/Compression Spring design Problem

Figure 8 shows the schematic diagram of tension/compression spring design (TCSD) [32]. TCSD is considered as a constrained problem. The algorithm aims to minimize the volume of a coil spring under a constant tension/compression load. The TCSD has three design variables which are the number of spring’s active coils, \( L \), the diameter of the winding, \( d \), and the diameter of the wire, \( w \). The mathematical formulation of the TCSD can be described as follows:

\[
\text{Minimize } f(w, d, L) = (L + 2)w^2d
\]  

Subject to the following constraints

\[
g_1 = 1 - \frac{d^3 + L}{7178.5w^4} \leq 0
\]
\[
g_2 = \frac{1}{w^3(12566d - w)} + 1 \leq \frac{5108w^2 - 1}{2} \leq 0
\]  

(17)  

(18)
FIGURE 2: Box plot of the suggested and compared algorithms for benchmark function ($f_1$ to $f_7$).

where the three variables range are as follows:

$$0.05 \leq w \leq 2.0,$$
$$0.25 \leq d \leq 1.3,$$
$$2.0 \leq L \leq 15$$

(19)

The box plot results of Tension/Compression Spring design based on different algorithms are shown in Figure 9. The histogram results of Tension/Compression Spring design based on different algorithms are discussed in Figure 10. Table 17 shows the comparison of one sample t-test analysis of the tension/compression spring design among other algorithms.

Tables 15 and 16 presents the best solution and the statistical results of proposed and compared algorithms for Tension/Compression Spring design Problem, respectively. The results of the proposed SCMWOA algorithm in this scenario compared to the state-of-the-art algorithms confirm the performance of the algorithm for solving the Tension/Compression Spring design.

2) Welded Beam design problem

The next constrained problem is the welded beam design [33]. The schematic diagram of the welded beam design is shown in Figure 11. It is considered as an important benchmark to test different optimization methods. The main objective is to minimize the fabricating cost of the welded beam which comprised of the setup, welding labor, and material costs. The properties constraints are on the shear stress, bending stress, buckling load, end deflection, and the side constraint. Four design variables of $w$, $L$, $d$, and $h$ are considered here. The mathematical formulation of the problem can be described as follows:

Minimize

$$f(w, L, d, h) = 1.10471w^2L + 0.04811dh(14.0 + L)$$

(20)
Subject to the following constraints

\begin{align}
g_1 &= w - h \leq 0 \\
g_2 &= \delta - 0.25 \leq 0 \\
g_3 &= \tau - 13,600 \leq 0 \\
g_4 &= \sigma - 30,000 \leq 0 \\
g_5 &= 0.125 - w \leq 0 \\
g_6 &= 6000 - P \leq 0 \\
g_7 &= 0.10471w^2 + 0.04811hd(14 + L) - 0.5 \leq 0
\end{align}

where

\begin{align}
\delta &= \frac{65856}{30000hD^3}, \quad \tau = \sqrt{\alpha^2 + \left(\frac{\alpha\beta L}{D}\right)^2} + \beta^2 \\
\alpha &= \frac{6000}{\sqrt{2}wL}, \quad \beta = \frac{QD}{J} \\
Q &= 6000 \left(14 + \frac{L}{2}\right), \quad D = \frac{1}{2} \sqrt{L^2 + (w + d)^2} \\
J &= \sqrt{2}wL \left[\frac{L^2}{6} + \frac{(w + d)^2}{2}\right] \\
\sigma &= \frac{504,000}{hd^2} \\
P &= 0.61432 \times 10^6 \frac{dh^3}{6} \left(1 - \frac{d\sqrt{30/48}}{28}\right)
\end{align}

FIGURE 3: Box plot of the suggested and compared algorithms for benchmark function \((f_8\ to\ f_{16})\).
TABLE 12: ANOVA test for sample functions ($f_1$, $f_2$, $f_3$, $f_9$, $f_{11}$, $f_{23}$).

| Function | SS (DF, DFd) | DF | MS | F (DFn, DFd) | P value |
|----------|--------------|----|----|--------------|---------|
| $f_1$    | 1.46E-07     | 29 | 5.03E-09    | F (29, 116) = 1.000 | P = 0.4765 |
| Row Factor | 2.42E-07     | 4  | 6.05E-08    | F (4, 116) = 12.04  | P < 0.0001 |
| Column Factor | 5.83E-07 | 116 | 5.03E-09    | -                    | -        |
| Residual   | 253          | 29 | 8.723       | F (29, 116) = 1.000 | P = 0.4765 |
| $f_2$    | 4.25E+10     | 4  | 1.06E+10    | F (4, 116) = 186.8  | P < 0.0001 |
| Row Factor | 6.59E+09     | 116| 56818063    | -                    | -        |
| Column Factor | 561.1     | 4  | 140.3       | F (4, 116) = 29.04  | P < 0.0001 |
| Residual   | 35.28        | 116| 4.831       | -                    | -        |

TABLE 13: T-test for the benchmark functions (from $f_1$ to $f_{23}$) based on the suggested SCMWOA algorithm against the compared algorithms.

| Function | PSO | GWO | WOA | GA |
|----------|-----|-----|-----|----|
| $f_1$    | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_2$    | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_3$    | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_4$    | <0.0001 | <0.0001 | <0.0001 | 0.0887 |
| $f_5$    | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_6$    | 0.0103 | 0.0003 | <0.0001 | <0.0001 |
| $f_7$    | 0.0003 | <0.0001 | <0.0001 | <0.0001 |
| $f_8$    | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_9$    | <0.0001 | <0.0001 | 0.3256 | 1    |
| $f_{10}$ | 0.1388 | <0.0001 | <0.0001 | <0.0001 |
| $f_{11}$ | <0.0001 | <0.0001 | <0.0001 | 0.0001 |
| $f_{12}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_{13}$ | <0.0001 | <0.0001 | <0.0001 | 0.0004 |
| $f_{14}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_{15}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_{16}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_{17}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_{18}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_{19}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_{20}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_{21}$ | 0.0007 | 0.0021 | 0.0008 | <0.0001 |
| $f_{22}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |
| $f_{23}$ | 0.1233 | 0.1233 | <0.0001 | <0.0001 |

where the four variables range are as follows:

\[ 0.1 \leq w, h \leq 2.0, \]
\[ 0.1 \leq L, d \leq 10 \]

The box plot results of the Welded Beam design problem based on different algorithms are shown in Figure 12. The histogram results of the Welded Beam design problem based...
TABLE 14: Interpolation of a Histogram of a sample function ($f_{11}$).

| Design Variables | Asymmetric Sigmoidal, SPL, X is log(concentration) | SCMWOA | PSO | GWO | WOA | GA |
|------------------|-----------------------------------------------|--------|-----|-----|-----|----|
| Best-fit values  | LogEC50                                       | -3.4   | -3.4| -3.4| -3.4| -3.4|
|                  | HillSlope                                      | -4.564 | -4.564| -4.564| -4.564| -4.564|
|                  | S                                              | -1.815 | -1.815| -1.815| -1.815| -1.815|
|                  | Top                                            | 115.5  | 115.5| 115.5| 115.5| 115.5|
|                  | Bottom                                         | 0.5455 | 0.5455| 0.5455| 0.5455| 0.5455|
|                  | EC50                                           | 0.000398| 0.000398| 0.000398| 0.000398| 0.000398|
| 95% CI (asymptotic)| Bottom                                        | -0.5931 to 1.684| -0.5931 to 1.684| -0.5931 to 1.684| -0.5931 to 1.684| -0.5931 to 1.684|
| Goodness of Fit  | Degrees of Freedom                             | 50     | 50  | 50  | 50  | 50  |
|                  | R squared                                      | 0      | 0   | 0   | 0   | 0   |
|                  | Adjusted R squared                             | -0.08  | -0.08| -0.08| -0.08| -0.08|
|                  | Sum of Squares                                 | 883.6  | 883.6| 883.6| 883.6| 883.6|
| Runs test        | Points above curve                             | 1      | 1   | 1   | 1   | 1   |
|                  | Points below curve                             | 54     | 54  | 54  | 54  | 54  |
|                  | Number of runs                                 | 2      | 2   | 2   | 2   | 2   |
|                  | P value (runs test)                            | 0.0364 | 0.0364| 0.0364| 0.0364| 0.0364|
| Deviation from Model | Significant                                      | Significant| Significant| Significant| Significant| Significant|
| Number of points | # of X values                                  | 55     | 55  | 55  | 55  | 55  |
|                  | # Y values analyzed                            | 55     | 55  | 55  | 55  | 55  |

TABLE 15: Best solution of proposed and compared algorithms for Tension/Compression Spring design Problem

| Algorithm   | Design Variables | Optimal Cost |
|-------------|------------------|--------------|
| PSO         | 0.051728         | 0.357644     |
| GSA         | 0.050276         | 0.323680     |
| WOA         | 0.051207         | 0.345215     |
| SCMWOA      | 0.051232         | 0.345805     |

TABLE 16: Statistical results of proposed and compared algorithms for Tension/Compression Spring design Problem

| Algorithm   | Optimal Cost | Average | Standard Deviation | Function Evaluations |
|-------------|--------------|---------|---------------------|----------------------|
| PSO         | 0.0126747    | 0.0139  | 0.0033              | 5460                 |
| GSA         | 0.0127022    | 0.0136  | 0.0026              | 4980                 |
| WOA         | 0.0126763    | 0.0135  | 0.0024              | 4820                 |
| SCMWOA      | 0.0126696    | 0.0134  | 0.0013              | 2460                 |

TABLE 17: One sample t-test analysis of the Tension/Compression Spring design problem based on different algorithms.

| Algorithm   | SCWMOA | PSO   | GSA   | GWO   |
|-------------|--------|-------|-------|-------|
| Theoretical mean | 0      | 0     | 0     | 0     |
| Actual mean   | 0.01267| 0.0128| 0.01277| 0.01282|
| Number of values | 19     | 19    | 19    | 19    |
| P value (two tailed) | **    | ****  | ****  | ****  |
| P value summary | Yes    | Yes   | Yes   | Yes   |
| Significant (alpha=0.05)? | Yes    | Yes   | Yes   | Yes   |

on different algorithms are shown in Figure 13. Table 20 shows the comparison of the one sample t-test analysis of the welded beam design problem among other algorithms.

Tables 18 and 19 presents the best solution and the sta-
Figure 4: Box plot of the suggested and compared algorithms for benchmark function (f₁₇ to f₂₃).

Statistical results of proposed and compared algorithms for Welded Beam design problem, respectively. The results of the proposed SCMWOA algorithm in this scenario compared to the state-of-the-art algorithms confirm the performance of the algorithm for solving the Welded Beam design.

VI. CONCLUSION

This paper proposed an optimization algorithm called Sine Cosine hybrid with Modified Whale Optimization Algorithm (SCMWOA). The SCMWOA algorithm is tested using nineteen datasets, from the UCI Machine Learning Repository, with different number attributes, instances, and classes for feature selection. The SCMWOA algorithm is also tested for twenty-three benchmark functions. The functions include seven unimodal, six multimodal, and ten multi modal based fixed-dimension functions. The two tested engineering problems are the tension/compression spring design and the welded beam design. The results emphasize that the SCMWOA algorithm outperforms several comparative optimization algorithms and provides high accuracy. Statistical analysis tests, including one-way analysis of variance (ANOVA) and Wilcoxon’s rank-sum, confirm that the SCMWOA algorithm has better performance. The SCMWOA algorithm will be tested for more classical engineering design problems in future work since the algorithm perform well only in the two mentioned problems in this paper. Other benchmark functions, such as CEC 2015 and CEC 2017, will also be considered in future work.
FIGURE 5: Histogram of the suggested and compared algorithms for benchmark function ($f_1$, $f_2$, $f_3$, $f_9$, $f_{11}$, and $f_{23}$).

FIGURE 6: QQ plot of the suggested and compared algorithms for benchmark function ($f_1$, $f_2$, $f_3$, $f_9$, $f_{11}$, and $f_{23}$).
FIGURE 7: Convergence curves of the suggested and compared algorithms based on the benchmark functions ($f_1$, $f_2$, $f_3$, $f_9$, $f_{11}$, and $f_{23}$).

TABLE 18: Best solution of proposed and compared algorithms for Welded beam design problem

| Algorithm | Optimal Cost | $w$   | $L$ | $d$ | $h$  | $c$  |
|-----------|--------------|-------|-----|-----|-----|-----|
| PSO       | 1.728024     | 0.202369 | 3.544214 | 9.048210 | 0.205723 | 1.728024 |
| GSA       | 1.879952     | 0.182129 | 3.856979 | 10.00000 | 0.203760 | 1.879952 |
| WOA       | 1.730499     | 0.205396 | 3.484293 | 9.037426 | 0.206276 | 1.730499 |
| SCMWOA    | 1.726738     | 0.205644 | 3.479712 | 9.041001 | 0.205739 | 1.726738 |

TABLE 19: Statistical results of proposed and compared algorithms for Welded beam design problem

| Algorithm | Average | Standard Deviation | Function Evaluations |
|-----------|---------|--------------------|----------------------|
| PSO       | 1.728024 | 0.01275             | 13770 |
| GSA       | 1.879952 | 1.28740             | 10750 |
| WOA       | 1.730499 | 0.02260             | 9900 |
| SCMWOA    | 1.726738 | 0.10162             | 8740 |

APPENDIX

This appendix includes three tables of benchmark functions. Table 21 shows the description of the unimodal benchmark functions. Table 22 shows the description of the multimodal benchmark functions. Table 23 shows the description of the multimodal based fixed-dimension benchmark functions.

ACKNOWLEDGMENT

The authors thank Taif University Accessibility Center for the study participants. We deeply acknowledge Taif Univer-
TABLE 20: One sample t-test analysis of the Welded Beam design problem based on different algorithms.

| Function | D | Range | SCMWOA | PSO | GSA | WOA |
|----------|---|-------|--------|-----|-----|-----|
| Theoretical mean | 0 | 0 | 0 | 0 | 0 | 0 |
| Actual mean | 1.727 | 1.73 | 1.893 | 1.734 |
| Number of values | 19 | 19 | 19 | 19 |
| One sample t test | t=1346, df=18 | t=1508, df=18 | t=238.2, df=18 | t=921.5, df=18 |
| P value (two tailed) | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| P value summary | **** | **** | **** | **** |
| Significant (alpha=0.05)? | Yes | Yes | Yes | Yes |

How big is the discrepancy?

- Discrepancy: 1.727, 1.73, 1.893, 1.734
- SD of discrepancy: 0.00591, 0.005, 0.03464, 0.008201
- SEM of discrepancy: 0.001283, 0.001147, 0.007948, 0.001881
- 95% confidence interval: 1.724 to 1.729, 1.727 to 1.732, 1.876 to 1.910, 1.730 to 1.738
- R squared (partial eta squared): 0.9997, 1, 0.9978, 1

TABLE 21: Description of the multimodal benchmark functions

| Function | D | Range | \(f_{\text{min}}\) |
|----------|---|-------|------------------|
| \(f_8(w) = \sum_{i=1}^{D} w_i \sin(\sqrt{|w_i|})\) | 30 | [-500, 500] | -12569.487 |
| \(f_9(w) = \sum_{i=1}^{D} w_i \sin(\sqrt{|w_i|})\) | 30 | [-512, 512] | 0 |
| \(f_{10}(w) = -20 \exp(-0.2 \sqrt{\sum_{i=1}^{D} w_i^2}) - \exp(\sum_{i=1}^{D} \cos(w_i)) + 20 + \eta\) | 30 | [-32, 32] | 0 |
| \(f_{11}(w) = \frac{1}{4000} \sum_{i=1}^{D} w_i^2 - \prod_{i=1}^{D} \cos(\frac{\pi y_i}{b_i}) + 1\) | 30 | [-600, 600] | 0 |
| \(f_{12}(w) = \frac{1}{\pi}(30 \sin(y_1 + 1) - 20)\) | 30 | [-50, 50] | 0 |
| \(f_{13}(w) = 0.1 \{10 \sin^2(3\pi y_1) + \sum_{i=1}^{D} (w_i - 1)^2[1 + 10 \sin^2(2\pi y_1)] + 20 \}\) | 30 | [-50, 50] | 0 |

TABLE 22: Description of multimodal based fixed-dimension benchmark functions

| Function | D | Range | \(f_{\text{min}}\) |
|----------|---|-------|------------------|
| \(f_{14}(w) = \left( \frac{1}{30} + \sum_{j=1}^{25} \frac{1}{\sum_{j=1}^{25} (w_i - h_{ij})^2} \right)^{-1}\) | 2 | [-65, 65] | 1 |
| \(f_{15}(w) = \sum_{i=1}^{11} h_i - \frac{w_i (2b_i + w_i) + w_i}{b_i + w_i + w_i} \) | 4 | [-5, 5] | 0.00030 |
| \(f_{16}(w) = 4w_1^2 - 2.1w_1^4 + \frac{1}{3} w_1^6 + w_1 w_2 - 4w_2^2 + 4w_2^4\) | 2 | [-5, 5] | -1.0316 |
| \(f_{17}(w) = \left( \frac{w_1}{w_1 + w_2 + 1} \right)^2 + 10 \left( 1 - \frac{w_1}{w_1 + w_2 + 1} \right) \cos w_1 + 10\) | 2 | [-5, 5] | 0.398 |
| \(f_{18}(w) = \left( \frac{w_1 + w_2 + 1}{w_1 + w_2 + 1} \right)^2 - 14w_1 - 4w_1^2 + 4w_2^4 + 2w_1 w_2 + 4w_1^2 + 2w_2^2 \) | 2 | [-2, 2] | 3 |
| \(f_{19}(w) = \sum_{i=1}^{4} b_i \exp(\sum_{j=1}^{10} h_j (w_j - p_{ij}))\) | 3 | [1, 3] | -3.86 |
| \(f_{20}(w) = \sum_{i=1}^{4} b_i \exp(\sum_{j=1}^{10} h_j (w_j - p_{ij}))\) | 6 | [0, 1] | -3.32 |
| \(f_{21}(w) = \sum_{i=1}^{4} (w_i - h_i) (w_i - h_i)^2 + b_i\) | 4 | [0, 1] | -10.1532 |
| \(f_{22}(w) = \sum_{i=1}^{4} (w_i - h_i) (w_i - h_i)^2 + b_i\) | 4 | [0, 1] | -10.4028 |
| \(f_{23}(w) = \sum_{i=1}^{4} (w_i - h_i) (w_i - h_i)^2 + b_i\) | 4 | [0, 1] | -10.5363 |
FIGURE 8: Tension/Compression spring design problem [32].

FIGURE 9: Box plot results of Tension/Compression Spring design based on different algorithms.

FIGURE 10: Histogram results of Tension/Compression Spring design based on different algorithms.

FIGURE 11: Welded beam design problem [33].

FIGURE 12: Box plot results of Welded Beam design problem based on different algorithms.

FIGURE 13: Histogram results of Welded Beam design problem based on different algorithms.

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