Sequential Quarkonium Suppression

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We use recent lattice data on the heavy quark potential in order to determine the dissociation temperatures of different quarkonium states in hot strongly interacting matter. Our analysis shows in particular that certain quarkonium states dissociate below the deconfinement point.

I. INTRODUCTION

The behavior of the heavy quarkonium states in hot strongly interacting matter was proposed as test of its confinement status, since a sufficiently hot deconfined medium will dissolve any binding between the quark-antiquark pair \[ \langle 0 \rangle \]. Another possibility of dissociation of certain quarkonium states (subthreshold states at \( T = 0 \)) is the decay into open charm (beauty) mesons due to in-medium modification of quarkonia and heavy-light meson masses \[ \langle 2 \rangle \].

The production of \( J/\psi \) and \( \Upsilon \) mesons in hadronic reactions occurs in part through production of higher excited \( c\bar{c} \) (or \( b\bar{b} \)) states and their decay into quarkonia ground state. Since the lifetime of different subthreshold quarkonium states is much larger than the typical life-time of the medium which may be produced in nucleus-nucleus collisions their decay occurs almost completely outside the produced medium. This means that the produced medium can be probed not only by the ground state quarkonium but also by different excited quarkonium states. Since different quarkonium states have different sizes (binding energies), one expects that higher excited states will dissolve at smaller temperature than the smaller and more tightly bound ground states. These facts may lead to a sequential suppression pattern in \( J/\psi \) and \( \Upsilon \) yield in nucleus-nucleus collision as the function of the energy density.

Here we will discuss heavy quarkonium dissociation below the deconfinement point where it is due to in-medium modification of the open charm (beauty) threshold \[ \langle 3 \rangle \] as well as above the deconfinement point due to the well known screening phenomenon \[ \langle 4 \rangle \].

II. QUARKONIUM PRODUCTION AND FEED-DOWN

It is well known that \( J/\psi \) production in hadron-hadron collision is to a considerable extent due to the production and subsequent decay of higher excited \( c\bar{c} \) states \[ \langle 5 \rangle \]. The feed-down from higher excited states was systematically studied in proton-nucleon and pion-nucleon interactions with 300GeV incident proton (pion) beams \[ \langle 6 \rangle \]. In these studies the cross sections for direct production of different charmonium states (excluding feed-down) were measured. Then making use of the known branching ratios \( B[\chi_1(1P) \to \psi(1S)] = 0.27 \pm 0.02, B[\chi_2(1P) \to \psi(1S)] = 0.14 \pm 0.01, \) and \( B[\psi(2S) \to \psi(1S)] = 0.55 \pm 0.05, \) one obtains the fractional feed-down contributions \( f_i \) of the different charmonium states to the observed \( J/\psi \) production; these are shown in the second and third columns of Tab. 1.

In the case of bottomonium the experiment provides only the inclusive (i.e. including also the feed-down from higher states) cross section for different \( (nS) \) states \[ \langle 7 \rangle \]. The feed-down from \( (nP) \) states is known only for transverse momenta \( p_T \geq 8 GeV/c \) \[ \langle 8 \rangle \]. To analyze the complete feed-down pattern, we thus have to find a way to extrapolate these data to \( p_T = 0 \) as well as to determine the direct cross section for different \( (nS) \) states. This can be done using the most simple and general model for quarkonium production, the color evaporation model \[ \langle 9 \rangle \]. In particular this model predicts that the ratios of cross sections for production of different quarkonium states are energy independent. This prediction was found to be true for a considerable range of energies \[ \langle 10 \rangle \]. The ratios between the different \( \chi_i(1P) \) states in this model are predicted to be governed essentially by the orbital angular momentum degeneracy \[ \langle 11 \rangle \]. we thus expect for the corresponding cross-sections

\[
\chi_0(1P) : \chi_1(1P) : \chi_2(1P) = 1 : 3 : 5.
\]

From Table 1 we have for \( \pi^- N \) collisions \( \chi_2(1P)/\chi_1(1P) \simeq 1.44 \pm 0.38 \) and thus reasonable agreement with the predicted ratio 1.67. Actually, for \( pN \) interactions, the experiment measures only the combined effect of \( \chi_1 \) and \( \chi_2 \) decay (30% of the overall \( J/\psi \) production); the listed values in Tab. 1 are obtained by distributing this in the ratio 3:5.

Using consideration based on color evaporation model, in particular Eq. (1), the feed-down from higher excited \( b\bar{b} \) states to \( \Upsilon \) production can be predicted \[ \langle 12 \rangle \]; the feed-down fraction are summarized in Tab.1. Alternatively the feed-down fraction from higher excited \( b\bar{b} \) states can be predicted using NRQCD factorization formula \[ \langle 13 \rangle \]. The results of this analysis are summarized in the last column of Tab. 1.
Recent lattice calculations of the heavy quark potential show evidence for the string breaking at finite temperature $T$. On the lattice the potential is calculated from the Polyakov loop correlator, to which it is related by

$$V(r, T) = -\ln <L(r)L(0)^\dagger> + C,$$

where $L(r)$ is the Polyakov loop (see e.g. Ref. $\text{[3]}$ for definition). The normalization constant $C$ contains both the cut-off dependent self-energy and the entropy contributions $-TS$ (for $T \neq 0$ $-\ln <L(r)L(0)^\dagger>$ is actually the free energy of the static $Q\bar{Q}$ pair). For a properly chosen normalization constant $C$, $V(r, T)$ is the ground state energy of an infinitely heavy $Q\bar{Q}$ pair. In absence of dynamical quarks (quenched QCD) $V(r, T)$ is linearly rising with $r$ for large separations indicating the existence of a flux tube (string). If dynamical quarks are present the flux tube can decay (the string can break) by creating a pair of light quarks $q\bar{q}$ from the vacuum once $V(r, T)$ is larger than twice the binding energy of a heavy-light $Q\bar{q}$ meson $\frac{1}{2}$. Thus the potential at very large distances is constant $V_\infty(T)$ and is equal to twice the binding energy of a heavy-light meson.

It is expected that medium effects are not important at very short distances. Therefore at very short distances the potential $V(r, T)$ should be given by the Cornell potential $\text{[13]}$

$$V(r) = -\frac{e}{r} + \sigma r$$

We use this fact to determine the normalization constant $C$ and set the potential to be of the Cornell form at the smallest distance $rT = 0.25$ available in lattice studies of $\text{[3]}$, with $e = 0.4$ as expected for (2+1)-flavor QCD $\text{[12]}$. The resulting potential and $V_\infty(T)$ are shown in Fig. $\text{[3]}$. Note the strong temperature dependence of $V_\infty(T)$. Since for sufficiently heavy quarks ($m_Q \gg \Lambda_{QCD}$) it does not matter whether the quark is infinitely heavy or just merely heavy, the open charm (beauty) meson masses are approximately given by $2M_{D,B}(T) = 2m_{c,b} + V_\infty(T)$.

Now the temperature dependence of the different quarkonium states should be addressed. At zero temperature the heavy quark masses permit an application of potential theory for description quarkonium spectroscopy (see e.g. $\text{[11]}$). Furthermore it turns out that the time scale of gluodynamics relevant for quarkonium spectroscopy is smaller than $(m_Qv)^{-1}$ ($v$ being the heavy quark velocity) $\text{[14]}$. For sufficiently heavy quarks this time scale is much larger than the typical hadronic time scale $\Lambda_{QCD}^{-1} \sim 1 fm$. The decay of the flux-tube like all other hadronic decays has time scale of

\[ \text{Similar phenomenon occurs of course at } T = 0. \text{ However it is much more difficult to observe it on lattice (see e.g. $\text{[11]}$).} \]

**TAB 1:** Feed-down fractions for from higher excited states to the $J/\psi$ and $\Upsilon$ states.

| state  | $f_i(\pi^-N)i$ [%] | $f_i(p\bar{N})$ [%] | state  | $f_i(p\bar{p})$ [%] | $f_i^{NRQCD}(p\bar{p})$ [%] |
|--------|-------------------|-------------------|--------|-------------------|---------------------------|
| $J/\psi(1S)$ | 57 ± 3 | 62 ± 4 | $\Upsilon(1S)$ | 52 ± 9 | 52 ± 34 |
| $\chi_1(1P)$ | 20 ± 5 | 16 ± 4 | $\chi_0(1P)$ | 26 ± 7 | 24 ± 8 |
| $\chi_2(1P)$ | 15 ± 4 | 14 ± 4 | $\Upsilon(2S)$ | 10 ± 3 | 8 ± 7 |
| $\psi(2S)$ | 8 ± 2 | 8 ± 2 | $\chi_2(2P)$ | 10 ± 7 | 14 ± 4 |
| $\Upsilon(3S)$ | 2 ± 0.5 | 2 ± 2 |
order 1 fm. Therefore in the potential theory the potential must always be of Cornell form (i.e. linearly rising at large distances). These considerations have direct phenomenological support. Namely, simple potential models with linearly rising potential can describe reasonably well also the quarkonium states above the open charm (beauty) threshold. Many of this higher excited states have effective radius of order or even larger than 1 fm [12,13]. Contrary to this situation in the case of the potential becoming flat around 1 fm (the expected radius of string breaking at $T=0$) the higher excited states above the open charm (beauty) threshold simply do not exist. Therefore we have determined the temperature dependent heavy quarkonium masses from the Schrödinger equation with the temperature dependent string potential [2].

![Graph](image)

**FIG. 1.** The heavy quark potential and its asymptotic value below deconfinement at different temperatures. The line on the left figure is a fit to the data points.

$$V_{\text{string}}(r,T) = -(e - \frac{1}{6} \arctan(2rT)) \frac{1}{r}$$

$$= (\sigma(T) - \frac{\pi T^2}{3} - \frac{2T^2}{3} \arctan\frac{1}{rT}) r + \frac{1}{2} \ln(1 + 4r^2T^2).$$ (4)

This form of the potential describes quite well the temperature dependence of the heavy quark potential in quenched QCD for appropriately chosen $\sigma(T)$ [13]. In order to make contact to real QCD we set $\alpha_s = 0.4$ and use $T_c/\sqrt{\sigma} = 0.425$ from [3] for the deconfinement temperature (by $\sigma$ we always denote the string tension at zero temperature). Furthermore we use the following values of the heavy quark masses, $m_c = 1.3 GeV$ and $m_b = 4.72 GeV$ as well as $\sqrt{\sigma} = 0.44 GeV$ for the zero temperature string tension. This values of parameters give a fairly good description of the observed quarkonium spectrum at zero temperature. The temperature dependence of the string tension was taken from [13]. The resulting quarkonia masses are shown in Fig 2. Since the smallest distance available on lattice is only 0.25 fm $^{-1}$ one may worry about possible medium effects at this distance and their role in determination of $V_{\infty}(T)$. Normalizing the Polyakov loop correlator (3) at $r = 1/(4T)$ with Eq. (4), we thus obtain what might be a more reliable estimate of the plateau $V_{\infty}(T)$ than with the $T = 0$ form (3). It turns out, however, that the two forms of short distance behavior resulting from the zero temperature Cornell potential (3) and (4) are practically identical, so that the normalization is in fact not affected by the in-medium modifications at larger distances. To consider further possible uncertainties of the normalization procedure, we have also normalized the Polyakov loop correlator at the next smallest distance $r = \sqrt{2}/(4T)$. The resulting two forms of $V_{\infty}(T)$ are shown in both Fig. 2. The difference between the two curves of $V_{\infty}(T)$ provides an estimate of the normalization error. Except for the region very near $T = T_c$, the uncertainty is seen to be quite small.

From Fig. 2 one can see that $\psi'$ and $\chi_c$ states become an open charm states well below $T_c$ and can dissociate by decaying into $DD$. The situation is similar for $\Upsilon(3S)$ and $\chi_c(2P)$ states which can decay into $BB$ below $T_c$. For $J/\psi$, $\chi_b(1P)$ and $\Upsilon(2S)$ it is not possible to say whether they will dissociate above $T_c$ or just below $T_c$. Finally, the $\Upsilon(1S)$ state will definitely dissociate above the deconfinement.

**IV. QUARKONIUM DISSOCIATION BY COLOR SCREENING**

In the deconfined phase it is customary to choose the constant $C$ in (3) to be the value of the correlator at infinite separation $C = T \ln \langle L(r)L^\dagger(0) \rangle \equiv | < L > |^2$. The resulting connected correlator defines the so-called color averaged potential [16]

$$V(r, T) = -T \ln \langle L(r)L^\dagger(0) \rangle \frac{1}{|< L >|^2}$$ (5)

3
The color average potential can be written as the thermal average of the potentials in color singlet $V_1(T, r)$ and color octet $V_8(T, r)$ states:

$$\exp(-V(r, T)/T) = \frac{1}{9}\exp(-V_1(r, T)/T) + \frac{8}{9}\exp(-V_8(r, T)/T)$$

(6)

In potential models it is assumed that quarkonium is dominantly a singlet $Q\bar{Q}$ state. Furthermore the octet channel is repulsive (at least in perturbation theory) and therefore only a singlet $Q\bar{Q}$ pair can be bound in the deconfined phase. Thus we need to know the singlet potential. The lattice data in the relevant case of 3 flavor QCD exist only for the averaged potential $\langle V \rangle$. The averaged potential in 3 flavor QCD are shown in Fig. 3 for three representative temperatures. Note that within present accuracy of the lattice calculations the potential vanishes beyond some distance $r_0(T)$ denoted by vertical arrows in Fig. 3. In perturbation theory, the leading terms for both are at high temperature and small $r$ ($r << T^{-1}$) of Coulombic form,

$$V_1(T, r) = -\frac{4}{3}\frac{\alpha(T)}{r}, \quad V_8(T, r) = \frac{+1}{6}\frac{\alpha(T)}{r},$$

(7)

with $\alpha(T)$ for the temperature-dependent running coupling. In the region just above the deconfinement point $T = T_c$, there will certainly be significant non-perturbative effects of unknown form. We therefore first consider the high temperature regime, which we somewhat arbitrarily define as $T \geq 1.45T_c$. In this region, we attempt to parameterize the existing non-perturbative effects through a conventional screening form, replacing Eq. (7) by

$$-\frac{3}{4}V_1(T, r) = 6V_8(T, r) = \frac{\alpha(T)}{r}\exp\{-\mu(T)r\},$$

(8)

where $\mu(T)$ denotes the effective screening mass in the deconfined medium. We fit the lattice data by assuming $\alpha(T)$ and $\mu(T)$ to be unknown functions of $T$. Such a fit can describe the lattice data for $T > 1.45T_c$ very well. Furthermore the temperature dependence of $\alpha(T)$ can be well described by 1-loop formula for the coupling in QCD with $\Lambda_{QCD} = (0.34 \pm 0.01)T_c$ and the screening mass $\mu(T)$ turns out to be constant in units of the temperature $\mu(T) = (1.15 \pm 0.02)/T$. A similar behavior of the screening mass was found in pure SU(2) and SU(3) gauge theory \[8\] - \[21\]. We shall now assume that the above form of the screening mass continues to remain valid as we lower the temperature to $T_c$. Such a constant screening mass down to $T_c$ is again expected from studies of pure gauge theory \[8\]. On the other hand, quenched QCD (pure SU(3) gauge theory) studies indicate that when $T$ is lowered to $T_c$, the perturbative ratio $V_1/V_8 = -8$ will increase in favor of the singlet potential \[21\]. We therefore try to describe the behavior just above $T_c$ by a potential of the form (8), in which the color octet potential is given by

$$V_8(T, r) = \frac{c(T)}{6}\frac{\alpha(T)}{r}\exp\{-\mu(T)r\}$$

(9)

instead of Eq. (8); the factor $c(T) \leq 1$ accounts for the expected reduction of octet interactions as $T \to T_c$. In the interval $T_c < T < 1.45T_c$ we thus fit the lattice results for $V(T, r)$ in terms of the parameter $c(T)$ with $\alpha(T)$ and $\mu(T)$ given above. In this way we get again quite a good fit of the lattice data Refs. \[8\] - \[21\] at temperatures $T < 1.45T_c$. 

FIG. 2. The masses of different quarkonia states and the open charm (beauty) threshold as function of the temperature. Shown are the charmonia masses and open charm threshold (left) and bottomonia masses and open beauty threshold (right) as function of the temperature. The thick solid line is the open charm (beauty) threshold obtained from normalization at $r = 1/(4T)$. The thin solid line in the open charm (beauty) threshold obtained from normalization at $r = \sqrt{2}/(4T)$ (see text).
Now we are in a position to discuss quarkonium dissociation due to color screening. It is natural to assume that the heavy $Q\bar{Q}$ pair cannot exist as a bound state if its effective binding radius (the mean distance between $Q$ and $\bar{Q}$) is larger than the screening radius of the medium. The effective radii for different bound states are calculated from the Schrödinger equation with the singlet potential described above.

$$\left[2m_a + \frac{1}{m_a} \nabla^2 + V_1(r)\right] \Phi_i^a = M_i^a \Phi_i^a, \quad (10)$$

The screening radius of the medium can be identified with $1/\mu(T)$. However, the value of $\mu(T)$ strongly depends on assumption we have made to determine it. A more model independent and more conservative approach would be to identify the screening radius with $r_0(T)$ defined above. We use the latter approach. In Fig. 3 we show the effective radius of $J/\psi$ and $\Upsilon$ states and the screening radius as function of the temperature. The intersection of these curves defines the dissociation temperature of $J/\psi$ and $\Upsilon$ states. Similar analysis was done for excited states which may survive above $T_c$.

V. SUMMARY AND CONCLUSIONS

We have considered quarkonium dissociation in hot strongly interacting matter below as well as above the deconfinement. In a confined medium dissociation of certain quarkonium states occur due to in-medium modification of the open charm (beauty) threshold as well as the quarkonia masses. In the deconfined medium quarkonium dissociation is due to color screening. We summarize the dissociation temperature of different quarkonium states in Tab. 2. Combining these dissociation temperature with the feed-down fractions determined in section II we can predict the sequential suppression pattern of $J/\psi$ and $\Upsilon$ states as function of the temperature. These are summarized in Fig. 4. For a more accurate determination of the quarkonium suppression patterns, it would be desirable to carry out direct lattice studies of the color singlet potential and of its quark mass dependence, which may become important near the critical temperature. Furthermore, to make contact with nuclear collision experiments, a more precise determination of the energy density via lattice simulations is clearly needed, as is a clarification of the role of a finite baryochemical potential. For the latter problem, lattice studies are so far very difficult; nevertheless, a recent new approach could make such studies feasible.

The methods used in the present study of in-medium modification of hadrons containing heavy quarks is of limited validity. A more accurate study of hadron properties in medium should be based on determination of hadron spectral function and their temperature dependence. Recent lattice studies indicate that a determination of hadron spectral function is possible at least within quenched approximation with the present days computer resources. Although at present the hadronic spectral function can be determined only in quenched approximation they may provide valuable test for the approach described here. In principle such studies will also include the dissociation of quarkonium states due to interaction with partonic constituents (thermal activation) in terms of width of the peaks in the calculated spectral function which is not accessible within present approach.

| $T/T_c$ | $q\bar{q}$ | $J/\psi$ | $\chi_c$ | $\chi_b(1S)$ | $\chi_b(1P)$ | $\chi_b(2S)$ | $\chi_b(2P)$ | $\chi_b(3S)$ |
|---------|----------|----------|--------|-------------|-------------|-------------|-------------|-------------|
| 1.10    | 0.74     | 0.2      | 2.31   | 1.13        | 1.10        | 0.83        | 0.75        |

TAB 2: Dissociation temperatures of different quarkonium states.
FIG. 4. The suppression pattern of $J/\psi$ (left) and $\Upsilon$ yield (right) as function of the temperature.

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