Quadratic Partial Eigenvalue Assignment Problem with Time Delay for Active Vibration Control

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Abstract. Partial pole assignment in active vibration control refers to reassigning a small set of unwanted eigenvalues of the quadratic eigenvalue problem (QEP) associated with the second order system of a vibrating structure, by using feedback control force, to suitably chosen location without altering the remaining large number of eigenvalues and eigenvectors. There are several challenges of solving this quadratic partial eigenvalue assignment problem (QPEVAP) in a computational setting which the traditional pole-placement problems for first-order control systems do not have to deal with. In order to these challenges, there has been some work in recent years to solve QPEVAP in a computationally viable way. However, these works do not take into account of the practical phenomenon of the time-delay effect in the system. In this paper, a new “direct and partial modal” approach of the quadratic partial eigenvalue assignment problem with time-delay is proposed. The approach works directly in the quadratic system without requiring transformation to a standard state-space system and requires the knowledge of only a small number of eigenvalues and eigenvectors that can be computed or measured in practice. Two illustrative examples are presented in the context of active vibration control with constant time-delay to illustrate the success of our proposed approach. Future work includes generalization of this approach to a more practical complex time-delay system and extension of this work to the multi-input problem.

1. Introduction and Background

Control of undesirable vibrations to avoid failure in structure can be achieved by passive and active means. Passive control devices based upon the classical Frahm’s dynamic vibration absorber [1] are being utilized in automobile, aerospace and civil engineering applications [2-5]. These passive devices have several advantages due to simple design, reliability, low cost, no power requirements, good stability and larger broadband response. However, they are limited in their application to those systems only in which the dynamics of the structure do not change considerably or are limited within a small range of vibration amplitude and frequencies. Although passive devices are widely being used they are restricted in their application due to lack of adaptability with respect to the changes in dynamic environment, limited utilization for either storing or dissipating energy, its capability of producing only local controlling force, and its characteristics of altering the global dynamics of the system by introducing additional natural frequencies in the case of dynamic absorber. To overcome these limitations of the passive devices, active and/or semi-active control strategies are being...
developed and studied extensively for wide range of applications for vibration suppression [6-11]. Active Vibration Control (AVC) techniques are capable of supplying the desirable controlling forces in a dynamic environment (real time) by combining sensors (to sense the states), computers (to compute the controlling force in real time) and actuators (to supply the controlling force) in order to suppress the vibration corresponding to a wide range of dynamic operating conditions.

In vibration analysis the dynamics of the open loop multi-degree-of-freedom discrete system is governed by the following second order matrix differential equation,

$\mathbf{M}\ddot{\mathbf{v}}(t) + \mathbf{C}\dot{\mathbf{v}}(t) + \mathbf{K}\mathbf{v}(t) = \mathbf{0},$  \hspace{1cm} (1)

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices respectively of dimension $(n \times n)$. By using separation of variables of the form

$\mathbf{v}(t) = \mathbf{x}e^{\lambda t},$  \hspace{1cm} (2)

where $\mathbf{x}$ a constant vector, in (1) leads to the problem of finding the eigenvalues and eigenvectors of the quadratic pencil

$P(\lambda) = \lambda^2\mathbf{M} + \lambda\mathbf{C} + \mathbf{K}.$  \hspace{1cm} (3)

The characteristic roots of the quadratic polynomial (3) $\lambda_k$ are known as eigenvalues, and the corresponding vectors $\mathbf{x} \neq 0$ are corresponding eigenvectors which satisfy

$P(\lambda_k)\mathbf{x}_k = 0, \hspace{1cm} \text{for} \hspace{1cm} k = 1, 2, \cdots, n.$ \hspace{1cm} (4)

For AVC application the system modeled by (1) is modified by applying a controlling force $\mathbf{bu}(t)$,

$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{bu}(t),$ \hspace{1cm} (5)

with a constant vector $\mathbf{b} \in \mathbb{R}^n$, defining the location of control and associated control force $\mathbf{u}(t)$ as,

$\mathbf{u}(t) = f^{\top}\mathbf{x}(t) + g^{\top}\mathbf{x}(t), \hspace{1cm} (6)$

where, $f \in \mathbb{R}^{n \times 1}$ and $g \in \mathbb{R}^{n \times 1}$ are velocity and displacement feedback control gain vectors respectively. By substituting (2) in (5), we obtain the quadratic matrix pencil associated with the controlled system

$P(\lambda) = \lambda^2\mathbf{M} + \lambda\left(\mathbf{C} - \mathbf{bf}^{\top}\right) + \left(\mathbf{K} - \mathbf{bg}^{\top}\right). \hspace{1cm} (7)$

Since vibration in a structure is caused by a few resonant or unstable eigenvalues, of practical importance here is to find the feedback vectors $f$ and $g$ such that these small numbers of unwanted eigenvalues are reassigned to suitably chosen numbers without altering the remaining large number of eigenvalues and eigenvectors of the pencil $P(\lambda)$. The last phenomenon is known as no spill-over and this problem is known as Quadratic Partial Eigenvalue Assignment Problem (QPEVAP). An obvious approach to solve this problem is to transform it to a standard first-order control problem and then use any of the several excellent methods available in the literature [12]. However, there are several practical computational challenges in solving QEVAP which the traditional pole placement problem does not have to deal with. These include (i) the problem must be solved using the knowledge of a small number of eigenvalues and eigenvectors of the quadratic pencil $P(\lambda)$ which are all that can be computed with the state-of-the-art computational technique, such as Jacobi-Davidson methods [12], (ii) transformation to a first-order control system or reduction of the order of the model must be avoided, since these processes may give rise to instability in computation and some practically inherent properties of the physical system, such as the symmetry, sparsity, etc., may be completely
destroyed, and above all (iii) the no spill-over must be guaranteed mathematically, since in practice it is impossible to verify them by recomputing them after application of the feedback. Meeting these challenges, a new approach has been developed in [13] for the QPEVAP, but the system considered there does not take into account of the time-delay. Implementation of AVC strategy, in real engineering applications using sensors and actuators in the feedback loop, is subjected to short time delays that deteriorates the control performance and may lead to instability. Suppose the controlled system (5) is modified by considering a constant time delay $\tau$ in the feedback loop as,

$$M\ddot{v}(t) + C\dot{v}(t) + Kv(t) = bu(t-\tau),$$

(8)

with the choice of the controlling force,

$$u(t-\tau) = f^T\dot{x}(t-\tau) + g^T\dot{x}(t-\tau),$$

(9)

where, the control gain vectors $f$ and $g$ depend upon the inherent time delay $\tau$ in the feedback loop. By substituting (2) in (8) we obtain the following characteristic quasi-polynomial,

$$P_2(\lambda) = M\lambda^2 + C\lambda + K - (bf^T\lambda + bg^T)e^{-\lambda\tau},$$

(10)

where the dynamics of the controlled system are characterized by the eigenvalues of the closed loop pencil (10). Numerous efforts have been made in the past to study the existence of the time delayed AVC strategies and its effect on system’s stability. The problem of delayed feedback control is summarized in the monographs [14-17] and the recent review papers [18, 19]. However, solution of time delayed AVC in a quadratic setting and effect of the time delay in partial pole assignment is almost non-existent in the literature. The partial pole assignment problem for the time delayed system (8) is to find the control gain vectors $f$ and $g$ such that

$$\det(M\dot{\lambda}^2 + C\dot{\lambda} + K - (bf^T\dot{\lambda} + bg^T)e^{-\lambda\tau}) = 0, \quad \text{for } k = 1, 2, \cdots, 2n,$$

(11)

has the desired poles $\{\lambda_i\}_{i=1}^m = \{\mu_1, \cdots, \mu_1, \lambda_{11}, \cdots, \lambda_{1m}\}$. That it contains $m$ numbers of newly assigned poles $(\mu_1, \cdots, \mu_m)$ without affecting the remaining poles $(\lambda_{11}, \cdots, \lambda_{1m})$ of the open loop system (1).

In section 2, by using an orthogonality relation from [13], we have derived an implicit solution to the QPEVAP with time delay. This method ensures no spillover will occur. We emphasize that our results are applicable in the general case and do not assume that $M, C, K$ are simultaneously diagonalizable. The method will be most advantageous in practical applications where $n$ is large and when it is required to assign only the eigenvalues $\lambda_{11}, \cdots, \lambda_{1m}$, where $m$ is much smaller than $2n$. This situation is typical in the vibration control and stabilization of flexible, large space structures [20, 21]. The method uses only the eigenvalues $\lambda_{11}, \cdots, \lambda_{1m}$ and their associated eigenvectors. These can be computed by Jacobi Davidson method (refer to the 2nd ed. of [12] to be published by SIAM, 2009), or by modal analysis measurements. Two illustrative examples are given in Section 3 validating the results for pole assignment without delay and analyzing the effect of time delay on the control gain vectors. The stability of the controlled system and its dependency on the time delay is shown in Section 4. Brief concluding remarks are presented in the final section.

2. Formulation for Partial Pole Assignment with Delayed Feedback Control Gains

Consider the following orthogonality relation proved in [13]. Suppose $M, C, K \in \mathbb{R}^{n \times n}$ are symmetric and $M$ is positive definite. Let the $n \times 2n$ system of equations

$$MX\dot{\lambda}^2 + CX\lambda + KX = 0,$$

(12)
where \( X \in \mathbb{C}^{n \times 2n} \) and \( \Lambda = \text{diag} \{ \lambda_1, ..., \lambda_{2n} \} \in \mathbb{C}^{2n \times 2n} \) with distinct \( \lambda_i \) be an eigen-decomposition of the quadratic open loop pencil (3). Then

\[
\Lambda X^T M X - X^T K X = D_1
\]

where \( D_1 \) is a diagonal matrix. Using this orthogonality relation, we now present a solution to the following partial eigenvalue assignment problem for the pencil (3).

Given \( m \) complex numbers \( \mu_1, ..., \mu_m \), \( m \leq n \), and a vector \( b \in \mathbb{R}^{m \times 1} \), we are required to find \( f, g \in \mathbb{R}^{m \times 1} \) which are such that the closed loop pencil (10) has the spectrum \( \{ \mu_1, ..., \mu_m, \lambda_{m+1}, ..., \lambda_{2n} \} \). We will now use the following partitions of eigenvalue and eigenvector matrices,

\[
X = \begin{pmatrix} X_1 & X_2 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \Lambda_1 & \Lambda_2 \\ \Lambda_2 & \Lambda_1 \end{pmatrix},
\]

where \( X_i \) and \( \Lambda_i \) are pair wise self-conjugate. The eigenvalue matrix \( \Lambda_i \) consist of \( m \) eigenvalues \( \Lambda_i \) is the eigenvectors corresponding to these eigenvalues.

**THEOREM 2.1**: Let

\[
f = MX \Lambda \beta, \quad g = -KX \beta, \quad \beta \in \mathbb{C}^{m \times 1},
\]

then for any choice of \( \beta \)

\[
MX_2 \Lambda_2 + CX_2 \Lambda_2 + KX_2 - (bf^T X_1 \Lambda_2 + bg^T X_2) e^{-\Lambda r} = 0. \quad \text{(no spillover)}
\]

**Proof**: Note that the left hand side of (16) equals

\[
-(bf^T X_1 \Lambda_2 + bg^T X_2) e^{-\Lambda r},
\]

since \( \Lambda_2 \), \( X_2 \) are eigenmatrix pairs of the open loop pencil. Furthermore, by substituting (15) in (17) and rearranging we have,

\[
-b(\beta^T e^{-\Lambda r} \Lambda_1 X_1^T M X_2 \Lambda_2 - \beta^T e^{-\Lambda r} X_1^T K X_2 - (bf^T X_1 \Lambda_2 + bg^T X_2) \Lambda 1 X_1^T M X_2 \Lambda_2 - X_1^T K X_2) = 0,
\]

since from [22],

\[
\Lambda_1 X_1^T M X_2 \Lambda_2 - X_1^T K X_2 = 0,
\]

and hence from (12) the relationship (16) can be proved.

In order to use this theorem to solve the partial pole assignment problem, we need to choose \( \beta \) such that it will move \( \{ \lambda_j \}_{j=1}^n \) of the pencil (3) to \( \{ \mu_j \}_{j=1}^m \) in (10). If there is such a vector \( \beta \) then there exists an eigenvector matrix \( Y \in \mathbb{C}^{m \times m} \),

\[
Y = (y_{1}, ..., y_{m}), y_j \neq 0, j = 1, ..., m,
\]

such that

\[
MYD^2 + CYD + KY - (bf^T Y D + bg^T Y) e^{-Dr} = 0.
\]
where, $D = \text{diag}\{\mu_1, \ldots, \mu_m\}$, and $e^{-br} = \text{diag}\{e^{-r}, \ldots, e^{-r}\}$. Now by substituting for $f$ and $g$ from (15) in (21) and after rearranging we obtain,

$$MYD^T + CYD + KY = b(f^TYD + g^TY)e^{-br} = b\beta^T(\Lambda_iX_i'MYD - X_i'KY)e^{-br} = b\beta^TZ^T = be^T,$$

where,

$$Z = e^{-br}(DY^TM\Lambda_i - Y^TKX_i),$$

and,

$$c = Z\beta,$$

is a vector that will depend on the scaling chosen for the eigenvectors in $Y$. In order to obtain $Y$, we can solve for each of the eigenvectors $y_i$ using the equations

$$(\mu_j^2M + \mu_jC + K)y_j = b, \ j = 1, \ldots, m.$$

This corresponds to choosing the vector $c = (1, \ldots, 1)^T$, so, having computed the eigenvectors, we could solve the $m$-square system

$$Z\beta = (1, \ldots, 1)^T,$$

for $\beta$, and hence determine the control gain vectors $f$ and $g$.

### 3. Examples

In this section we present two illustrative examples associated with single degree and two degree of freedom non-conservative system for which control gains assuring the partial pole assignment with time delay is considered.

#### 3.1 Example 1 (Pole assignment in Single Degree of Freedom System)

Let, $M = 1$, $C = 0.01$, $K = 5$, $b = 1$, $\tau = 0.1$ and with these values the open loop system eigenvalues and eigenvectors satisfying (4) are,

$$\lambda_{1,2} = -0.005 \pm 2.236i$$

and $x_1 = -0.0009 + 0.4082i$ $x_2 = -0.0009 - 0.4082i$. (27)

We want to calculate the control gains such that the closed loop system will replace the poles (27) to their new values: $\mu_1 = -1$, $\mu_2 = -3$. First we find $Y$ from (25) by using the assigned eigenvalues $\mu_{1,2}$ as: $Y = (0.1669, 0.0005)$. By denoting,

$$D = \begin{pmatrix} -1 & 0 \\ 0 & -3 \end{pmatrix}$$

and

$$e^{-br} = \begin{pmatrix} e^{0.1} & 0 \\ 0 & e^{0.3} \end{pmatrix},$$

we compute from (23) and (24),

$$Z = \begin{pmatrix} -0.1676 + 0.3766i & -0.1676 - 0.3766i \\ -2.1309 + 0.1014i & -2.1309 - 0.1014i \end{pmatrix}$$

and $\beta = \begin{pmatrix} -1.3499 - 0.7269i \\ -1.3499 + 0.7269i \end{pmatrix}$ (29)

with $c = (1, 1)^T$. The values of control gain now can be estimated from (15) as,

$$f = -2.4646$$

and

$$g = 2.9553,$$
which is the same value as obtained in [23] using the receptance method. The control gains obtained in (30) satisfy (11), ensuring the determinant to be smaller than \(10^{-13}\) with \(\mu_1, \mu_2\). Now by varying the constant time delay from \(\tau = 0\) (case of no delay in the feedback loop) to \(\tau = 2\) seconds, the steps (27) to (29) are repeated and control gains for each \(\tau\) are computed and plotted in the Figure 1. It is clear from Figure 1 that the requirements on the control gain vectors changes substantially as the time delay in the feedback loop changes.

3.2 Example 2 (Partial Pole assignment in Two Degree of Freedom System)

Let, 
\[
M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0.1 & -0.1 \\ -0.1 & 0.1 \end{pmatrix}, \quad K = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \tau = 0.01
\]

For the above system, the open loop eigenvalues are:
\[
\lambda_{1,2} = -0.0053 \pm 0.6183i, \quad \lambda_{3,4} = -0.0947 \pm 1.6145i.
\]

We wish to replace the eigenvalues \(\lambda_{1,2}\) with \(\mu_{1,2} = -1 \pm i\) without altering \(\lambda_{3,4} = -0.0947 \pm 1.6145i\).

From the definition in (14) we have,
\[
X = \begin{pmatrix} -0.4474 + 0.0123i & -0.4474 - 0.0123i \\ -0.7233 & -0.7233 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} -0.0053 - 0.6183i & 0 \\ 0 & -0.0053 + 0.6183i \end{pmatrix}.
\]

By following the procedure in the previous example we compute
\[
Z = \begin{pmatrix} 0.2002 - 0.1283i & -0.1513 + 0.3960i \\ -0.1513 - 0.3960i & 0.2002 + 0.1283i \end{pmatrix}, \quad \beta = \begin{pmatrix} -2.8539 + 2.1735i \\ -2.8539 - 2.1735i \end{pmatrix},
\]

with \(c = (1 \ 1)^T\). Now the values of control gain vectors now can be estimated from (15) as,
\[
f = \begin{pmatrix} -1.2594 \\ -1.9659 \end{pmatrix}, \quad g = \begin{pmatrix} -0.8718 \\ -1.6281 \end{pmatrix}
\]

Once the control gain vectors are obtained, by computing
\[
D_k = \det \left( \hat{\lambda}_{k}^2 M + \hat{\lambda}_{k} C + K - b \left( \hat{\lambda}_{k} f^T + g^T \right) e^{-\hat{\lambda}_{k} \tau} \right), \quad \text{for} \ k = 1, 2, 3, 4
\]

where \(\hat{\lambda}_{1,2} = -1 \pm i, \hat{\lambda}_{3,4} = -0.0947 \pm 1.6145i\), it is verified that \(|D_k| < 10^{-11}\) for \(k = 1, 2, 3, 4\), validating the desired partial pole assignment.

4. A Note on Stability

It is important to note that the characteristic quadratic pencil associated with the controlled system without time delay in the feedback loop (7) is a polynomial; hence the \(n\) dimensional system will have \(2n\) roots. The location of these roots in the complex plane defines the stability of the system. Hence if the assigned poles have negative real part then the controlled system must be stable. However the characteristic equation of a controlled system with delay (10) is a quasi-polynomial and has an infinite number of roots in the complex plane satisfying (35). Hence assigning the poles with negative real part may not necessarily make the system stable because there may be one or more unstable roots satisfying (35). This point is highlighted in the recent publication by Ram et al. [23] and he has shown that for pole assignment in delayed system a posteriori analysis has to be done to identify and assign the primary eigenvalues ensuring that such assignment will avoid instability.
For example, reconsider example 2 in the previous section and assign partial poles $\lambda_1, \lambda_2$ to new stable real and negative poles, which are further from imaginary axis, $\mu_1 = -1$ and $\mu_2 = -50$. The pole assignment is achieved and the associated control gains satisfy (35). However, when plotting determinant (35) for a range of real valued $\hat{\lambda}$, as shown in figure (2), it is clear that there exist at least one positive real root which can bring the system to instability even if the assigned poles are stable. Hence, by following the posteriori analysis based on second order Taylor series expansion in [23] we obtain the following eigensystem,

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
1 & -\hat{\lambda} & 1 \\
\end{bmatrix}
\begin{bmatrix}
z \\
\lambda z \\
\end{bmatrix} = 0.
\]  

(36)

The primary eigenvalues of the system (36) are $\hat{\lambda}_1 = 0.5614$, and $\hat{\lambda}_2 = -0.9999$, which is consistent with the real roots shown in figure 1. Consider another case for the same example 2 and now we assign partial poles $\lambda_1, \lambda_2$ to $\mu_1 = -1$ and $\mu_2 = -3$. The characteristic real roots for this case are negative and therefore the system is stable. Moreover, the primary eigenvalues for such assignment from (36) are also found to be stable: $\hat{\lambda}_1 = -2.9731$, and $\hat{\lambda}_2 = -1.0002$. Hence, if the primary poles are assigned during partial pole assignment the system will remain stable.

5. Conclusions

This paper proposes a new approach for partial eigenvalue assignment in active vibration control for a second-order model with time-delay. This new approach has several desirable practical features, most important of which is that the approach is implementable with the knowledge of only a few eigenvalues and eigenvectors of the associate quadratic eigenvalue problem which are all that are computable using the state-of-the-art computational technique or can be measured in vibration laboratory. Furthermore, the no spill-over property of the closed-loop system is guaranteed with a mathematically proved result. Thus, the approach is applicable to control dangerous vibration and other forms of instability in large real-life structures. Such practically implementable methods for vibration control of large real-life structures are almost nonexistent. The research on extending the approach to partial eigenstructure assignment which involves reassigning not only a few unwanted eigenvalues but the associated eigenvectors as well, and to the multi-input time-delay feedback design problem, are currently underway. It is expected that by the time of the conference, validation studies of this technique with the lab scale experiment will be available.
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