Limits on the ion temperature anisotropy in the turbulent intracluster medium

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1 INTRODUCTION

The intracluster medium (ICM) of galaxies comprises a weakly collisional and magnetized plasma, with turbulent motions at large scales. Cosmological mergers of subclusters are thought to be the major sources of this turbulence in the ICM. The turbulence timescale for the ICM is $\tau_{\text{turb}} \sim 10^{16} \text{s}$ (using $L_{\text{turb}} = 500 \text{kpc}$ and $U_{\text{turb}} = 10^3 \text{km s}^{-1}$ as the length-scale and velocity of the largest-scale turbulent motions), while the time-scale for the Coulomb collisions between ions is estimated as $\tau_{i} \sim 10^{15} \text{s}^1$ (mean free path of $\sim 30 \text{kpc}$ for ion–ion collisions), which requires a nearly collisionless approach.

The conservation of the first adiabatic invariant of the charged particles (magnetic moment) combined with the large-scale plasma motions that stretch/compress the magnetic fields in the ICM leads to the development of differences between the parallel (to the local field lines) and the gyro components of the thermal velocities of the ions. Therefore, because it is highly turbulent and weakly collisional, the ICM naturally develops anisotropies in the local distribution of the thermal velocities of the ions. This anisotropy is a source of free energy, which can trigger electromagnetic plasma instabilities (such as the ion-cyclotron, mirror and firehose instabilities; see for example Gary 1993), and thus plays a very important role regarding the dynamics of the system and the evolution of the turbulence itself (Schekochihin & Cowley 2006; Kowal, Falcke-Gonçalves & Lazarian 2011; Santos-Lima et al. 2014; Mogavero & Schekochihin 2014). This role, however, remains poorly understood.

An anisotropic magnetohydrodynamic (AMHD) approximation that assumes a bi-Maxwellian distribution of thermal velocities, namely one that takes into account two independent temperature components (one for the thermal velocity parallel to the local magnetic field and another for the gyro-motion of the particles), can be employed in this situation. The solutions of the AMHD equations reveal some linear instabilities (mirror and firehose), corresponding to the long-wavelength (fluid) limit of these plasma instabilities (see for example Hau & Wang 2007; Kowal et al. 2011).

The effects of anisotropy-driven instabilities at micro-scales are still a matter of debate. In one scenario, the plasma instabilities saturate the anisotropy at low levels, close to the instability

ABSTRACT

Turbulence in the weakly collisional intracluster medium (ICM) of galaxies is able to generate strong thermal velocity anisotropies in the ions (with respect to the local magnetic field direction), if the magnetic moment of the particles is conserved in the absence of Coulomb collisions. In this scenario, the anisotropic pressure magnetohydrodynamic (AMHD) turbulence shows a very different statistical behaviour from the standard MHD one and is unable to amplify seed magnetic fields. This is in contrast to previous cosmological MHD simulations that are successful in explaining the observed magnetic fields in the ICM. On the other hand, temperature anisotropies can also drive plasma instabilities that can relax the anisotropy. This work aims to compare the relaxation rate with the growth rate of the anisotropies driven by the turbulence. We employ quasi-linear theory to estimate the ion scattering rate resulting from the parallel firehose, mirror and ion-cyclotron instabilities, for a set of plasma parameters resulting from AMHD simulations of the turbulent ICM. We show that the ICM turbulence can sustain only anisotropy levels very close to the instability thresholds. We argue that the AMHD model that bounds the anisotropies at the marginal stability levels can describe the Alfvénic turbulence cascade in the ICM.

Key words: MHD – plasmas – turbulence – galaxies: clusters: intracluster medium.
thresholds (see e.g. Mogavero & Schekochihin 2014). In another scenario, if the anisotropy survives on dynamical time-scales and anisotropic thermal stresses dominate the dynamics of the system, there is a change in the traditional MHD turbulence picture with the presence of instabilities at fluid scales. Studies of the turbulence statistics and the magnetic field amplification when applying the latter scenario to galaxy clusters (Kowal et al. 2011; Santos-Lima et al. 2014; Falceta-Gonçalves & Kowal 2015) as well as to the Earth’s magnetosphere (Meng et al. 2012) revealed drastic differences compared with the isotropic MHD approach. However, in the anisotropic case the numerical description is incomplete, as the instabilities that should develop at subgrid scales may influence the large-scale anisotropy evolution (see Mogavero & Schekochihin 2014 and the discussion in Section 6.4).

All these collisionless effects possibly influence cosmic ray (CR) propagation and acceleration in the ICM. For instance, compressible modes rather than Alfvénic turbulence have been identified as the dominant agent for particle acceleration (Yan & Lazarian 2002). In the absence of the anomalous scattering of the ions produced by the kinetic instabilities, the large parallel viscosity of the ions will efficiently damp the compressible modes in the ICM. At the same time, if magnetic fluctuations caused by the temperature anisotropy are present in the large-scale ICM, they could have a direct impact on the propagation of CRs in the medium (e.g. Nakwacki & Peralta-Ramos 2013).

Obviously, the effects of the plasma instabilities at kinetic scales cannot be captured by any MHD model (see the discussion in Section 6.4 about the general effects of the subgrid phenomena). The impact of a fast thermal relaxation owing to particle scattering by kinetic instabilities on the turbulence cascade and on the magnetic field amplification was investigated in Santos-Lima et al. (2014), where the rate of this process was considered as a free parameter. The pitch-angle scattering rate caused by some of these instabilities has been investigated for the solar wind through two-dimensional particle-in-cell (PIC) and hybrid (PIC-MHD) simulations (Gary, Yin & Winske 2000), and also using a quasi-linear approach (Seough & Yoon 2012; Yoon & Seough 2012). The results point to a scattering time of the order of the linear growth rate of the instabilities (which can be of the order of ion kinetic times-scales). In fact, these studies only considered the evolution of the instabilities starting from an unstable anisotropy level (see Section 6.2). For cases of a very slow driving of the thermal anisotropy (compared with the ion thermal gyrofrequency), recent two-dimensional PIC simulations (Riquelme et al. 2012; Riquelme, Quataert & Verscharen 2015) and hybrid schemes (Kunz, Schekochihin & Stone 2014; Melville, Schekochihin & Kunz 2016) have demonstrated that the anisotropy relaxation arising from the instabilities does not necessarily result in instantaneous anomalous scattering of the ions during the time when the anisotropy is being driven by the turbulent motions (see Section 6.3).

A self-consistent treatment of the feedback of these instabilities connected to the turbulence cascade is, however, still lacking. A reference model was developed relating consistently both plasma instabilities induced by high-energy CRs (gyroresonance instability) and the turbulence in the interstellar and intergalactic media (Lazarian & Beresnyak 2006; Yan & Lazarian 2011).

The aim of this work is to evaluate the limits on the temperature anisotropy, particularly in the turbulent intergalactic or intracluster medium, taking into account the scattering produced by the electromagnetic instabilities triggered by temperature anisotropy in an approach similar to that in Yan & Lazarian (2011). In order to do this, we will compare the ion scattering rate obtained from quasi-linear theory with the anisotropy generation rate by turbulence obtained from AMHD simulations (Santos-Lima et al. 2014).

This paper is organized as follows: in Section 2 we review the observed relationship between the bounds on the temperature anisotropy in the solar wind and collisionless instabilities; in Section 3 we describe briefly the AMHD simulations used in this work; in Section 4 we present the quasi-linear equations employed for calculating the scattering rate of the ions; and in Section 5 we present the results. In Section 6 we discuss some limitations and consequences of our study and relate it to other works. Finally, in Section 7 we summarize and conclude our analysis.

2 EMPIRICAL BOUNDS ON THE TEMPERATURE ANISOTROPY

The distribution function of the thermal velocities of the species in the nearly collisionless plasmas of the Earth’s magnetosphere is accessible from direct measurements by spacecrafts. The data accumulated during the last decades have shown that the electrons and ions in the solar wind at a distance of ~1 au present a bi-Maxwellian distribution, with the maximum anisotropy in the temperatures anticorrelated with the local plasma $\beta$, which is the ratio between the thermal and magnetic energy densities (for further details see Marsch 2006 and references therein; Hellinger et al. 2006; Štverák et al. 2008). These limits on the magnitude of anisotropy are below the expected levels when one assumes adiabatic conservation of the magnetic momentum $v_\perp p_\perp / 2B$ (where $v_\perp$ and $p_\perp$ are the perpendicular velocity and momentum of the particle) during the expansion/compression of the solar wind (see for example Bale et al. 2009).

These limits are interpreted as resulting from the non-linear saturation of the kinetic instabilities driven by the temperature anisotropy (Gary 1993). The linear dispersion of a plasma with one or more species having a bi-Maxwellian distribution presents a few instabilities resulting from the temperature anisotropy. The observed limits on the temperature anisotropy have been identified with the approximate thresholds for the firehose, mirror and ion-cyclotron instabilities (see for example Hellinger et al. 2006; Bale et al. 2009; Maruca, Kasper & Gary 2012).

The physical process limiting the temperature anisotropy depends on the specific instability and on the initial anisotropy level (see discussion in Sections 6.2 and 6.3). After the instabilities have reached growth saturation, this process is understood in terms of collisionless dissipation, with particles being scattered by the collective electromagnetic fluctuations caused by the instabilities (Kunz et al. 2014). These wave–particle interactions (quasi-collisions) diffuse the momentum of the particles and thus their pitch angle, relaxing the distribution function towards a Maxwellian one. This effect is observed not only in the solar wind, but also in laboratory plasmas (Keiter 1999) and in fully non-linear plasma simulations (Tajima, Mima & Dawson 1977; Tanaka 1993; Gary et al. 1997, 1998; Gary et al. 2000; Le et al. 2010; Nishimura, Gary & Li 2002; Riquelme et al. 2012; Kunz et al. 2014; Riquelme et al. 2015; Sironi & Narayan 2015; Sironi 2015).

3 TEMPERATURE ANISOTROPY DEVELOPMENT IN THE TURBULENT ICM: AMHD SIMULATIONS

In Santos-Lima et al. (2014, hereafter SL+14), a numerical study of the ICM turbulence was carried out by means of anisotropic
MHD simulations of forced turbulence in a periodic box. The temperature anisotropy evolution was modelled using the CGL closure (Chew, Goldberger & Low 1956), modified by the addition of a phenomenological pitch-angle scattering term:

$$\frac{\partial A}{\partial t} = \left( \frac{\partial A}{\partial t} \right)_{\text{CGL}} + \left( \frac{\partial A}{\partial t} \right)_{\text{scat}},$$

(1)

$$\frac{\partial A}{\partial t} = -\nabla \cdot (Au) + 3Ab \cdot [(b \cdot \nabla) u],$$

(2)

$$\frac{\partial A}{\partial t} = -v_S \left( 2A^2 - A - 1 \right),$$

(3)

where $A = T_i/T_\perp$ is the ratio between the temperature components; $u$ and $B$ are respectively the velocity and magnetic fields, with $b = B/B_0$; and $v_S$ is the pitch-angle scattering rate. The ions and electrons were considered to have identical temperature components, for simplicity. Also for simplicity, the cooling employed was considered not to affect the temperature anisotropy.

The effective scattering rate $v_S$ accounts for the effect of both the Coulomb collisions and the non-linear particle–plasma wave interactions. In SL+14, the Coulomb collisions were neglected and the scattering was attributed only to the action of the mirror and firehose instabilities whenever the anisotropy $A$ exceeded the threshold values for these instabilities. Different values were considered for $v_S$, ranging from the limit of no scattering ($v_S = 0$) to the extreme case in which the scattering time is very short or infinitely small compared with the resolved time-scales of the simulation ($v_S = \infty$).

Our aim here is (i) to provide an evaluation of the scattering rate $v_S$ resulting from the plasma instabilities, and (ii) to estimate the limits on the ion anisotropy $A_i$ in the ICM plasma by imposing statistical equilibrium between the terms $(\partial A_i/\partial t)_{\text{CGL}}$ and $(\partial A_i/\partial t)_{\text{scat}}$. In order to accomplish this, we will follow three steps: (1) from the MHD turbulence simulation, obtain the characteristic time for the anisotropy development $\tau_{\text{A}} = A_i(\partial A_i/\partial t)_{\text{CGL}}$ as a function of the ion plasma parameters $A_i$ and $\beta_{i\parallel}$; (2) estimate $v_S(A_i, \beta_{i\parallel})$ using quasi-linear theory and then calculate the characteristic time for the anisotropy relaxation $\tau_s = A_i(\partial A_i/\partial t)_{\text{scat}}^{-1}$; (3) find the values of $A_i(\beta_{i\parallel})$ for which $\tau_s = \tau_{\text{A}}$, in order to estimate the maximum anisotropy level that the turbulence can sustain in the presence of the instability scattering.²

The AMHD turbulence simulation we employ in step (1) has $v_S = 0$ (which is unrealistic, as will be seen). It corresponds to the model A2 presented in SL+14. The value of $v_S$ of the MHD simulation is of little importance at this stage, because it should not influence the evolution of $\tau_{\text{A}}$ (at least in order of magnitude), although it changes considerably the spreading of the probability distribution function (PDF) of the plasma parameters ($A_i, \beta_{i\parallel}$). In order to confirm this, we repeated our analysis using an AMHD model with a physically more plausible value of $v_S$ ($v_S \sim 10\tau_{\text{ turb}}$; see below). We considered a uniform magnetic field in the domain; the ratio between the unperturbed thermal pressure and the magnetic pressure of this uniform magnetic field has the value of $\beta_0 = 200$, which is representative of the ICM. Super-Alfvénic and subsonic turbulence (with Alfvénic Mach number $M_A \equiv \langle |u|/c_s \rangle \approx 1.2$ and sonic Mach number $M_S \equiv \langle |u|/c_s \rangle \approx 0.6$) is considered, with an injection scale $L_{\text{inj}} = 0.4L_0$, where $L_0$ is the computational box size. The employed resolution ($512^3$) allows for the solution of a modest inertial range covering the range of dimensionless wavenumbers $2.5 \lesssim kL_{\text{inj}} \lesssim 20$. Further details on the numerical setup, code and the turbulence statistics analysis can be found in SL+14.

We define the following physical dimensions for our simulations: $L_0 = 100$ kpc is the box size, $\rho_0 = 10^{-22}$ g cm$^{-3}$ is the mean density, and $c_{S0} = 10^8$ cm s$^{-1}$ is the unperturbed thermal speed (corresponding to the gas temperature $T_\parallel \approx 6 \times 10^6$ K). With this choice of units, $\tau_{\text{ turb}} = 40$ kpc, $u_{\text{rms}} \approx 7 \times 10^7$ cm s$^{-1}$, and $B_0 = 3$ μG is the intensity of the mean magnetic field, corresponding to the ion thermal gyroradius $\Omega_{\text{i}} = 3 \times 10^{-2}$ s$^{-1}$.

## 4 Quasilinear Evolution of the Kinetic Instabilities

The electromagnetic waves in the plasma can interact with the particles, exchanging energy and momentum. This process can be described statistically as a diffusion of the distribution function in the velocity space.

In a collisionless plasma composed of ions (protons) and electrons, the electromagnetic fluctuations driven by thermal anisotropy that are the most important for the scattering of the ions are generated by the firehose, mirror and ion-cyclotron instabilities (Gary 1993). The firehose instability can be excited when $T_i < T_\parallel$, and the mirror and ion-cyclotron instabilities can be excited in the opposite regime, namely $T_i > T_\parallel$. The resulting scattering from these instabilities decreases the temperature anisotropies and consequently regulates the growth of the instabilities themselves. The fastest growth modes for these instabilities occur for scales close to the ion Larmor radius, with growth rates that can be of the order of the ion Larmor frequency. The electron anisotropy is expected to be relaxed on faster time-scales (by the whistler and firehose modes; see Gary 1993; Nishimura et al. 2002; Šverák et al. 2008).

The non-linear development of the instabilities can be investigated analytically using quasi-linear theory, which assumes small perturbations of the distribution functions and of the electromagnetic fields (compared with the zeroth-order, background values). Quasi-linear theory also assumes the superposition of non-interacting plasma waves with random phases, which satisfy the linear dispersion relation of the plasma. The second-order effects of these waves on the particle distribution function give rise to a diffusion term in momentum space, which can be interpreted as resulting from effective collisions. In Section 6.2 we discuss the limitations of the quasi-linear approximation for examining the evolution of instabilities.

Hellinger et al. (2013) provide general quasi-linear expressions for the evolution of the mean velocity and temperature energy components of a general drifting bi-Maxwellian plasma composed of protons and electrons. Here we will use the simpler expressions derived in Yoon & Seough (2012) and Seough & Yoon (2012), namely a bi-Maxwellian distribution function for the ions and an isotropic distribution for the electrons, for the evolution of the temperature components arising from the parallel firehose, mirror and ion-cyclotron instabilities. We reproduce these expressions below.

### 4.1 Parallel firehose modes

The linear dispersion relation for firehose modes (modes with right-hand circular polarization) propagating parallel to the mean...
magnetic field is given by
\[ 0 = \frac{c^2 k^2}{\omega p} - \frac{\omega k}{\Omega_i} + \left( 1 - \frac{T_{i\perp}}{T_{i\parallel}} \right) \frac{\Omega_i}{k v_{i\parallel}} - \frac{T_{i\perp}}{T_{i\parallel}} \frac{1}{k v_{i\parallel}} Z \left( \frac{\omega k + \Omega_i}{k v_{i\parallel}} \right) \]
where \( \omega k = \omega(k) \) is the wave frequency complex for the wavevector \( k = k_\|, k_\perp = \sqrt{4 \pi \sigma_e c^2 / m_i} \), and \( \Omega_i = e B_0 / m_i c \) are respectively the plasma frequency and Larmor frequency for the ions, \( v_{i\parallel} = \sqrt{T_{i\parallel}/m_i} \) is the parallel thermal speed of the ions, \( Z(\xi) \) is the plasma function, and \( n_i, e, B_i \) and \( m_i \) are the ion density, elementary charge, background magnetic field intensity and ion mass, respectively. Terms of order \( (\omega k / \Omega_i)^2 \) and \( \omega k / \Omega_i \) were neglected in the above dispersion relation.

The evolution equations for the ion kinetic energies (second-order moments of the distribution function) resulting from the interaction with the parallel firehose modes are given by
\[ n_i \frac{d T_{i\perp}}{dr} = 8 \int_0^k dk \beta_i \frac{T_{i\perp}}{8 \pi} \left[ \frac{\Omega_i}{k v_{i\perp}} - 1 \right], \]
\[ \frac{n_i \frac{dT_{i\parallel}}{dr}}{2} = -4 \int_0^k dk \beta_i \frac{T_{i\parallel}}{8 \pi} \left[ \frac{\Omega_i}{k v_{i\parallel}} - 1 \right], \]
where \( v_{i\perp} = B_0 / \sqrt{4 \pi \sigma_e m_i} \) is the Alfvén velocity, \( \beta_i \) and \( \gamma_k \) are the real and imaginary parts of \( \omega_k \), respectively, and \( |\delta B_k|^2 / 8 \pi \) is the spectral energy density of the magnetic fluctuations, which evolves accordingly to the wave kinetic equation
\[ \frac{\partial |\delta B_k|^2}{\partial t} = 2 \gamma_k |\delta B_k|^2. \]

We refer the reader to Seough & Yoon (2012) for more details on the deduction of the above equations.

### 4.2 Ion-cyclotron and mirror modes

The linear dispersion relation for the ion-cyclotron modes (with left-hand polarization) propagating in an arbitrary oblique direction to the mean magnetic field is given by
\[ 0 = \frac{c^2 k^2}{\omega p} + \frac{\omega k}{\Omega_i} + \frac{\Omega_i}{k v_{i\parallel}} \left[ \frac{T_{i\perp}}{T_{i\parallel}} - 1 \right] \left( \frac{\Omega_i}{k v_{i\parallel}} - 1 \right) \frac{\zeta(\xi IC)}{2}, \]
where \( \omega_k = \omega IC(k) \) is the complex wave frequency for the two-dimensional wavevector \( k = (k_\|, k_\perp) \), \( \xi IC = \omega IC / k_\perp v_{i\perp} \), \( \xi IC = (\omega IC^2 - \Omega_i^2) / k_\perp v_{i\perp} \), \( \lambda = k_\perp v_{i\perp} / 2 \Omega_i \), \( \nu_{i\perp} = \sqrt{2 T_{i\perp}} / m_i \), and \( \nu_{i\perp} = \sqrt{2 T_{i\perp}} / m_i \). The modified Bessel function of the first kind of order \( j \), and \( \zeta(\xi) \) is the derivative of the plasma function \( Z(\xi) \).

The dispersion relation for the non-propagating mirror modes is in turn given by
\[ 0 = \frac{c^2 k^2}{\omega p} + 2 \lambda \left[ I_0(\lambda) - I_1(\lambda) \right] \exp(-\lambda) \left[ 1 + \frac{T_{i\perp}}{T_{i\parallel}} \frac{\zeta(\xi M)}{2} \right], \]
where \( \xi = i \xi M / k_\perp v_{i\perp} \). Similar to the case for the firehose instability, terms of order \( (\omega_k / \omega_0)^2 \) and \( \omega_k / \Omega_i \) are neglected in the dispersion relation for the ion-cyclotron and mirror modes.

The equations describing the evolution of the ion kinetic energy components are given by
\[ n_i \frac{dT_{i\perp}}{dr} = -16 \pi \int_0^k dk \beta_i \frac{T_{i\perp}}{8 \pi} \left[ 1 + \left( \frac{\zeta(\xi IC)}{\Omega_i} - 1 \right) \frac{\Omega_i^2}{k^2 v_{i\perp}^2} \right], \]
\[ \frac{n_i \frac{dT_{i\parallel}}{dr}}{2} = 8 \pi \int_0^k dk \beta_i \frac{T_{i\parallel}}{8 \pi} \left[ 1 + \lambda \left( \frac{\zeta(\xi IC)}{\Omega_i} - 1 \right) \frac{\Omega_i^2}{k^2 v_{i\parallel}^2} \right], \]
where we used the definition \( \Omega_i = I_1(\lambda) \exp(-\lambda) \). The kinetic wave equations for the ion-cyclotron and mirror modes are
\[ \frac{\partial |\delta B_k|^2}{\partial t} = 2 \gamma_k |\delta B_k|^2, \]
\[ \frac{\partial |\delta B_k|^2}{\partial t} = 2 \gamma_k |\delta B_k|^2. \]

The derivation of the above equations can be found in Yoon & Seough (2012).

### 4.3 Numerical methods

The quasi-linear equations for the evolution of the ion temperature components and of the magnetic energy modes were integrated using the LSODE solver from the numerical library ODEPACK (Hindmarsh 1983; Radhakrishnan & Hindmarsh 1993). At each iteration, the linear dispersion equation for each instability is solved numerically inside a discrete domain \( k_\perp(i), k_\parallel(j) \) defined by \( k_{i\perp}(i) = (i - 0.5) k_{\max} / N_1 \leq i \leq N_2 \), where \( 0 < k_{\max} < r_i \) is the thermal ion Larmor radius, and \( N_1 = 256 \). For the firehose modes, only the unidimensional grid \( k_\parallel(i) \) was used. For all the calculations presented, a flat spectrum of magnetic fluctuations \( |\delta B_k|^2 / B_0^2 = 10^{-7} \) is imposed at the beginning of the simulation.

### 5 RESULTS

The top panel of Fig. 1 shows the PDF of the macroscopic dimensionless plasma parameters \( \beta_i = 8 \pi n_i T_{i\perp} / B^2 \) and \( A_i = T_{i\perp} / T_{i\parallel} \) for the CGL-MHD numerical simulation of forced turbulence described in Section 3 (i.e. model A2 of SL+14 with null scattering rate \( n_s = 0 \)). Most of the plasma volume has the parameters \( (\beta_i, A_i) \) inside the unstable zones. The thresholds for the mirror and firehose instabilities are represented in Fig. 1 by the continuous grey lines, and the threshold for the parallel ion-cyclotron (IC) instability is represented by the dashed grey line. Note that the threshold for the ion-cyclotron instability is more constraining than that for the mirror instability in the regime \( \beta_i < 1 \).

For a grid of values \( (\beta_i, A_i) \) where the PDF of the CGL-MHD simulation is above an arbitrary cutoff of \( 10^{-7} \) (lighter grey dots in
Figure 1. (Top) Probability distribution function for the macroscopic plasma parameters $\beta_{i||} = 8\pi n_i T_{i||}/B^2$ and $A_i = T_{i\perp}/T_{i||}$ obtained from the statistically stationary state of forced turbulence of the simulation using the CGL-MHD approximation by Santos-Lima et al. (2014, model A2 there). (Bottom) Initial values of $\beta_{i||}$ and $A_i$ from the quasi-linear calculations (lighter grey dots) and values of the same parameters after the time interval $\Omega t = 500$ (red dots). The successively darker grey dots represent the system at the times $t = 10, 20$ and $40 \Omega^{-1}$. The grey solid lines represent the thresholds for the mirror, $A_i = 1 + 0.87\beta_{i\perp}^{-0.56}$ ($A_i > 1$), and parallel firehose, $A_i = 1 - 0.61\beta_{i\perp}^{-1.03}$ ($A_i < 1$), instabilities; the grey dashed line represents the threshold for the ion-cyclotron (IC) instability, $A_i = 1 + 0.43\beta_{i\perp}^{-0.42}$ (all the thresholds are obtained from linear theory; see Seough & Yoon 2012 and references therein).

Figure 2. (Top) Ion scattering rate averaged over time $\langle \nu_S \rangle$ (normalized by the Larmor frequency $\Omega_i$) for each initial state $(\beta_{i||}, A_i)$ of the quasi-linear evolution. The average in $\nu_S$ only considers times for which $\nu_S \geq 0.6\nu_{max}$, where $\nu_{max}$ is the maximum scattering rate during the system evolution. (Bottom) Maximum magnetic energy density in the ion-cyclotron + mirror ($A_i > 1$) and firehose ($A_i < 1$) modes during the quasi-linear evolution of each initial state $(\beta_{i||}, A_i)$. The grey lines have the same meaning as in Fig. 1.

The bottom panel of Fig. 1, we calculated the quasi-linear evolution of the ion temperatures $T_{i\perp}$ and $T_{i||}$ resulting from the wave-particle scattering of the ions by the parallel firehose, mirror and ion-cyclotron modes. The evolved values of $(\beta_{i||}, A_i)$ after a time interval $\Omega t = 500$ are shown as red dots in the bottom panel of Fig. 1. For each initial condition, the plasma parameters evolved to values close to the marginal equilibrium state.

The top panel of Fig. 2 depicts the ion scattering rates $\langle \nu_S \rangle$ (normalized by the ion Larmor frequency $\Omega_i$) resulting from the quasi-linear evolution, as a function of the initial states $(\beta_{i||}, A_i)$. These scattering rates were obtained from a temporal average of the instantaneous scattering rates, taking into account only values of $\nu_S \geq 0.6\nu_{max}$, where $\nu_{max}$ is the maximum scattering rate obtained during the time evolution. The values of $\langle \nu_S \rangle/\Omega_i$ are mostly in the interval $10^{-2}-10^{-1}$, but inside the stable region they drop quickly to zero (this cannot be visualized in the figure, as the values of $\langle \nu_S \rangle$ fall below the colour scale range in the region near $A_i \sim 1$). We further note that the values of $\langle \nu_S \rangle/\Omega_i$ increase with $\beta_{i||}$.

The bottom panel of Fig. 2 shows, as a function of the initial states $(\beta_{i||}, A_i)$, the maximum magnetic energy density in the modes (ion-cyclotron, mirror and firehose) during the quasi-linear evolution, normalized by the energy density of the background magnetic field $B_0$. For most of the initial conditions, this quantity is below unity and does not break the assumption of small perturbations of the Larmor orbit. However, for initial conditions far from the thresholds (especially in the high-$\beta_{i||}$ region for $A_i < 1$), the normalized magnetic energy density achieves values of the order of or larger than 1. For this region, the values of $\langle \nu_S \rangle$ shown in the top panel of Fig. 2 must be taken with caution (see Section 6.2). These initial conditions are not, however, expected to be accessible by the ICM plasma if the wave–particle scattering is taken into account during the CGL-MHD evolution (see below).

The top panel of Fig. 3 shows the characteristic rate of the anisotropy relaxation caused by the scattering owing to the instabilities $\Gamma_v = |(\partial A_i/\partial t)|A_i^{-1}$ (normalized by the ion Larmor frequency $\Omega_i$) as a function of the initial plasma parameters $(\beta_{i||}, A_i)$, according to equation (3) and $\langle \nu_S \rangle$ from the quasi-linear calculations. The characteristic rate at which the anisotropy changes in the CGL-MHD turbulence simulation described above, $\Gamma_A = |(\partial A_i/\partial t)|_{CGL}A_i^{-1}$ (normalized by the ion Larmor frequency $\Omega_0$ in the uniform magnetic field $B_0$; see equation 2), is shown in the bottom panel of Fig. 3. The average in $(\partial A_i/\partial t)|_{CGL}$ considers only the plasma volume of the simulation with $(\partial A_i/\partial t)|_{CGL} > 0$ when $A_i > 1$ and $(\partial A_i/\partial t)|_{CGL} < 0$ when $A_i < 1$, in order to capture the rate at which the anisotropy is driven in zones other than the stable zone. It is evident that $\Gamma_v \gg \Gamma_A$ throughout all of the unstable region.\footnote{However, the values of $\Gamma_A$ from the simulations are expected to increase with resolution; the average value obtained in the simulation presented here could be up to 4 orders of magnitude below the real one (see the discussion}
values of $A/\Gamma_1$ turbulence can sustain are limited by the temperature anisotropy rates. The average was performed using only the plasma volume where the anisotropy $A_i$ was increasing for $A_i > 1$ and decreasing for $A_i < 1$.

It is clear that the maximum and minimum values of $A_i$ that the turbulence can sustain are limited by the temperature anisotropy relaxation rates resulting from the instabilities. By comparing $\Gamma_A$ and $\Gamma_\nu$, we can find for each value of $\beta_\parallel$ the maximum/minimum values of $A_i (A_i^\pm)$ from balancing $\Gamma_A(A_i^\pm) = \Gamma_\nu(A_i^\pm)$. Fig. 4 shows the ratio $\Gamma_A/\Gamma_\nu$ between the rates presented in Fig. 3. The separation of $A_i^\pm$ from the mirror and firehose thresholds cannot be resolved for the grid in the $(\beta_\parallel, A_1)$-plane used in our calculations. However, it is evident that this separation is $\ll 1$. This unresolved separation shows that the turbulence can only sustain values of the temperature anisotropy $A_i$ that are extremely close to the instability thresholds. Therefore, the anisotropy levels featuring in the CGL-MHD simulation for the ICM turbulence are far from realistic.

We repeated the above analysis, but replacing the CGL-MHD simulated model used so far (model A2 of SL+14 with null $v_S$) for a simulated AMHD model in which a non-null constant value of $v_S$ was employed (model A3 of SL+14, with $v_S \sim 10\kms/turb$). Fig. 5 presents the evolution of $(\beta_\parallel, A_i)$ (top panel), and the ratio between the anisotropy change rate $\Gamma_A = |(\partial A_1/\partial t)_{CGL}|A_i^{-1}$ and the characteristic rate of anisotropy relaxation $\Gamma_\nu = |(\partial A_1/\partial t)_{CGL}|A_i^{-1}$. The results are similar to the previous ones, with the equality between rates $\Gamma_A$ and $\Gamma_\nu$ very close to the thresholds for the instabilities.

6 DISCUSSION

6.1 Limitations of the CGL-MHD model in the description of compressible modes

The CGL closure provides the simplest fluid model for a collisionless plasma, and assumes no heat flux. In particular, the linear dispersion of the CGL-MHD equations is known to deviate from

![Figure 3](image1.png)

**Figure 3.** (Top) Characteristic rate of anisotropy relaxation (normalized by the proton Larmor frequency $\Omega_L$) arising from the instability scattering $\Gamma_\nu = |(\partial A_1/\partial t)_s|A_i^{-1}$ calculated using the average quasi-linear scattering rates ($v_S$) (see equation 3). (Bottom) Characteristic rate of anisotropy increase (for $A_i > 1$) or decrease (for $A_i < 1$) obtained from the CGL-MHD turbulence simulation of Fig. 1 (top). $\Gamma_A = |(\partial A_1/\partial t)_{CGL}|A_i^{-1}$ normalized by the proton Larmor frequency $\Omega_L$ of the mean magnetic field $B_0$ (see equation 2). The average was performed using only the plasma volume where the anisotropy $A_i$ was increasing for $A_i > 1$ and decreasing for $A_i < 1$.

![Figure 4](image2.png)

**Figure 4.** Ratio between the characteristic rate of anisotropy change obtained from the CGL-MHD turbulence simulation, $\Gamma_A = |(\partial A_1/\partial t)_{CGL}|A_i^{-1}$, and the characteristic rate of anisotropy relaxation calculated from quasi-linear theory, $\Gamma_\nu = |(\partial A_1/\partial t)_s|A_i^{-1}$ (both presented in Fig. 3).

![Figure 5](image3.png)

**Figure 5.** (Top) As Fig. 1 (bottom panel), but using the initial values of $(\beta_\parallel, A_i)$ obtained from a turbulence AMHD simulation that employed a non-null constant rate $v_S$ in the equation of anisotropy evolution (model A3 of SL+14). Here the grey dots correspond to the times $t = 0, 20$ and $40 \Omega_0^{-1}$ (from the lighter to the darker dots). (Bottom) As Fig. 4, but for the model above.
the long-wavelength limit of kinetic theory for compressible modes, resulting in a different threshold for the mirror instability (which is over-stable compared with the threshold obtained from kinetic theory). In addition, for simplicity we considered a CGL-MHD model with the same anisotropy in temperature and total thermal energy for both the ions (protons) and electrons (see the discussion in SL+14 and below).

Another serious limitation of the CGL-MHD model is that it does not capture the collisionless damping effects of the compressible modes (see for example Yan & Lazarian 2004). Alternative higher-order closures exist that can mimic the Landau damping of thecompressible modes, at least for a narrow range of wavelengths (see for example Snyder, Hammett & Dorland 1997; Sharma et al. 2006). In view of this, we should be cautious with regard to compressible modes cascade (and shocks) in CGL-MHD-based models.

The spatial scale at which the collisionless damping may be dominant in the collisionless ICM is \( \approx \text{0.1–1 kpc} \) (Brunetti & Lazarian 2007), which is well below the approximate inertial range of the turbulent models discussed here (between 5 and 40 kpc). Thus a potential influence of Landau damping in the models discussed here would be only in shock regions formed by the turbulence.

On the other hand, if a considerable reduction of the parallel ions mean free path is assumed to occur continuously in time in most of the plasma volume – through the scattering or magnetic trapping of the ions by the plasma instabilities (see the following sections), this problem could be solved, at least in part, because the large-scale turbulence in the ICM would become effectively ‘collisional’. However, knowledge of the spatial/temporal statistics of the parallel ions mean free path in the turbulent ICM is non-trivial, because the state of the micro-physical instabilities depends not only on the instantaneous properties of the flow and the macroscopic variables, but also on their evolutionary history (Melville et al. 2016; see also the discussion in the following sections).

### 6.2 Limitations of the quasi-linear theory applied to initially unstable plasma configurations

The quasi-linear theory used here to calculate the evolution of the plasma instabilities arising from an initially unstable configuration has, of course, limitations, which are (at least in part) related to: (i) the linear approximations assumed; (ii) the assumption that the distribution function is bi-Maxwellian all the time; (iii) the neglect of non-linear interaction between waves; and, in particular, (iv) the assumption of an homogeneous final state of plasma equilibrium.

Considering the limitation imposed by (i), it should be pointed out that, although the quasi-linear approximation is formally only applicable for very small perturbations, the thermal ions are not sensitive to perturbations much larger than their gyro-radius, which are also generated by the instabilities. Thus, the condition \( \delta B^2 / B_0^2 \ll 1 \) can be slightly relaxed, considering the magnetic energy of the fluctuations \( \propto \delta B^2 \) integrated over all the spectrum.

Recently, Seough, Yoon & Hwang (2014) performed one-dimensional PIC simulations of the ion-cyclotron instability for a limited set of initial conditions (with a fixed anisotropy \( T_{ei}/T_{ii} = 4 \) and various values of \( \beta_{i0} \)). They compared the evolution of the thermal energy components and of the total magnetic energy in the instabilities with the quasi-linear predictions, finding good agreement for the moderate- and high-beta regimes (\( \beta_{i0} = 1 \) and 10), for which the linear assumption \( \delta B^2 / B_0^2 \ll 1 \) is maintained all the time. In the low-beta regime (\( \beta_{ii} = 0.1 \)), however, the exponential growth of the instability ceased soon after the wave energy reached the background magnetic energy level (at \( t \approx 50 \Omega_i^{-1} \)), being replaced by a nearly linear growth until saturation. Nonetheless, the quasi-linear predictions still provided a reasonable approximation to the PIC experiment in this case for the evolution of the thermal anisotropy. The authors also observed that the ion distribution function deviates from a bi-Maxwellian during the early stages of the instability evolution, but this deviation vanishes at late times when the system achieves the stationary, saturated state (after \( \sim 100 \Omega_i^{-1} \)).

We also carried out comparisons of the evolution of the instabilities obtained from two-dimensional hybrid simulations by Gary et al. (2000) for a plasma with a dominant perpendicular temperature using quasi-linear calculations, taking into account both the oblique ion-cyclotron and mirror modes (see Appendix A). These results show good agreement (within an order of magnitude) between the scattering rates, especially for large values of the initial ion-cyclotron growth rate. For the smallest values, the quasi-linear scattering rates seem to overestimate those from the simulations.

On the other hand, in two-dimensional PIC and hybrid simulations the dominance of the mirror modes (which are oblique to the background magnetic field) over the ion-cyclotron modes has been verified for regimes with \( \beta_{ii} \gtrsim 1 \), even when the ion-cyclotron modes have growth rates comparable to those of the mirror modes (Kunz et al. 2014; Riquelme et al. 2015). These last numerical experiments focused on the situation in which the thermal energy is initially isotropic and in which one component of the external magnetic field has its intensity changed at a constant rate (in a shear box configuration, representing the magnetic field shearing caused by the large-scale MHD turbulence motions), in this way driving the increase of the perpendicular temperature (see Section 6.3). Also employing two-dimensional PIC simulations, Sironi & Narayan (2015) showed that the relative roles of the mirror and ion-cyclotron instabilities are dependent on the electron to ion temperature ratio \( T_e/T_i \), with the ion-cyclotron instability dominant only when \( T_e/T_i \lesssim 0.2 \) for high-beta plasmas (for the studied range \( \beta_i \sim 5 – 30 \)). Even in this situation, the mirror modes can dominate after one time-scale associated with the anisotropies generation rate. In the turbulent ICM, only a detailed modelling of the thermodynamical evolution of the species (taking into account electron–ion anomalous collisional processes) could provide information on the local deviations from the thermal equilibrium between electrons and ions (see the discussion in Section 6.4). With respect to the global ICM properties, Takizawa (1998, 1999) showed that during the merger of subclusters of galaxies, the electron temperature can be half that of the ions in the post-shock ICM gas in the outskirts of the cluster (where the electron–ion collision time is longer because of the lower density). However, these studies considered the thermal coupling between ions and electrons mediated by Coulomb collisions only, and did not include any magnetic fields.

A detailed study comparing fully non-linear plasma simulations with a quasi-linear approximation is still lacking for the mirror instability. However, the stabilization mechanism of the mirror instability can be very different depending on the initial conditions of the temperature anisotropy. Very large anisotropies could produce modes with wavelengths close to the ion Larmor radius, in the case when irreversible ion scattering is likely to drive the system to marginal stability. However, these required levels of anisotropy can be artificially high, similar to those generated by the CGL-MHD turbulence presented in this paper. In this scenario, the quasi-linear
scattering rates calculated here can be considered as a ‘zeroth-order’ approximation.

For moderate values of the anisotropy beyond the threshold, the saturated state of the mirror instability can be achieved by a highly inhomogeneous and stable configuration of the plasma and magnetic field (Kivelson & Southwood 1996), without breaking the magnetic momentum of the ions through anomalous scattering. Total trapped equilibrium can be achieved by betatron cooling of the trapped protons only (Pantellini 1998).

Now let us focus our attention on the plasma regime in which the parallel temperature is dominant, and therefore where the firehose instability is present. Seough, Yoon & Hwang (2015) compared directly the quasi-linear evolution of the parallel firehose instability with one-dimensional PIC simulations with fixed initial anisotropy $T_{\parallel}/T_{\perp} = 0.1$ and various values of the plasma beta parameter: $\beta_{\parallel} = 2.5, 5,$ and 10. Similar to in the ion-cyclotron study (Seough et al. 2014), the quasi-linear predictions provide a better agreement for the highest values of $\beta_{\parallel}$. After a short initial phase of exponential growth when the quasi-linear calculations are almost identical to the simulations, however, the saturation values of the magnetic energy modes predicted by the quasi-linear calculations are found to be larger than the values obtained from the plasma simulations. For the lowest value of $\beta_{\parallel}$ tested ($\beta_{\parallel} = 2.5$), the agreement is the poorest and the final saturated value of the anisotropy is far from the threshold of the firehose instability. These authors also observed that the deviation from the initial bi-Maxwellian velocity distribution is larger for smaller $\beta_{\parallel}$. They suggest that the existence of strong wave–wave interactions could be responsible for the deviation from the quasi-linear calculations.

The quasi-linear calculations presented in this study consider only the evolution of the plasma instabilities from a set of initially unstable plasma configurations taken from the statistics of numerical simulations of CGL-MHD turbulence that do not consider self-consistently the feedback of the small-scale (subgrid) plasma instabilities. If our quasi-linear rates of anisotropy relaxation owing to the ion scattering are valid at least to an order of magnitude, the straightforward conclusion that can be drawn is that there is an obvious physical inconsistency in neglecting the micro-instability effects on the evolution of the temperature anisotropy in AMHD simulations of turbulence, at least for the observed conditions of the ICM. Even for an AMHD model with an imposed anisotropy relaxation rate of $\tau_{\text{rel}} \approx 10^{-3} \tau_{\text{turn}}$ (where $\tau_{\text{turn}}$ is the turbulence turn-over time) over all the firehose and mirror unstable volume, the levels of temperature anisotropy achieved would generate micro-instabilities so strong in a real plasma that they would bring the anisotropy to the (near) marginal state almost ‘instantaneously’ (parallel to the plasma regime in which the temperature anisotropy is tightest).

The most obvious complication of this description is related to the evolution of the macroscopic pressure components relative to the magnetic field amplification in the ICM, and also analysed the decay/evolution of the instabilities when the driving of anisotropy ceases or is reversed.

These studies clearly show that the temperature anisotropy is tightly limited by the firehose and mirror marginal stability thresholds in the asymptotic limit $S \ll \Omega$. For the firehose instability, in the regime $\abs{S\delta/\Omega} \ll 1$ (relevant for the ICM parameters), the anomalous scattering is set by the macroscopic anisotropy generated rate $S$ after a time delay $\delta t \ll S^{-1}$, while in the regime where $\abs{S\delta/\Omega} \gtrsim 1$ (relevant for the early scenario of magnetic field amplification in the ICM), the time interval $\delta t$ for the development of the magnetic fluctuations able to scatter the ions at a rate that equilibrates the anisotropy generation is $\delta t \gtrsim S^{-1}$ (Kunz et al. 2014; Melville et al. 2016). In both cases, the firehose fluctuations decay exponentially at a rate $\sim \Omega_i/\beta$ after the shutdown of the anisotropy generation (Melville et al. 2016). In contrast, for the mirror instability the magnetic fluctuations keep increasing continuously during the shear time $S^{-1}$, with the maintenance of the marginal stability condition resulting from the increasing fraction of ions trapped in regions where the increase of the magnetic field is compensated by the magnetic fluctuations (the trapped particles do not feel the increase of the mean magnetic field and are not subject to betatron acceleration; see Kunz et al. 2014; Riquelme et al. 2015; Rincon, Schekochihin & Cowley 2015). These magnetic structures have $\delta B_\perp \gg \delta B_\parallel$ and are elongated in the direction of the local magnetic field. In situations for which the generation of anisotropy is removed at $S \approx 1$, the mirror fluctuations decay at a rate $\sim \Omega_i/\beta$, although slower than exponential (Melville et al. 2016).

Melville et al. (2016) also analysed the situation in which the direction of the anisotropy generation is reversed after a time $S^{-1}$. The firehose development on the top of the residual mirror modes proceeds very similarly to its development from the homogeneous and isotropic initial conditions. In the case when the driving of an excess of parallel pressure is inverted to drive an excess of perpendicular pressure, the plasma only develops enough anisotropy to trigger mirror modes after a substantial decay of the firehose modes.

In the next section we further discuss our results in the light of those above, putting our work in a broader context.

### 6.4 Applicability of the bounded anisotropy model to turbulence simulations of the ICM

The physical fields evolved in our AMHD simulations of the ICM are in fact mean fields, in the sense that they represent macroscopic averages in space and time, over scales much larger than those related to the firehose and mirror modes that are expected to develop there. This macroscopic description therefore filters the ‘microscale’ magnetic fluctuations that can achieve intensities comparable to the macroscopic magnetic field (for example, the firehose modes in the ‘ultra-high’-beta regime” described in Melville et al. 2016).

The most obvious complication of this description is related to the evolution of the macroscopic pressure components relative to the magnetic field amplitude in the ICM,

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4 The local shear rate $S \sim \delta v_\perp/l$, produced by turbulence on a scale $l$ is expected to be coherent during the cascading time of these scales $\sim l/\delta v_\perp \sim S^{-1}$.

5 The ‘moderate’ and ‘ultra-high’-beta regimes (respectively $\beta \ll \Omega_i/S$ and $\beta \gtrsim \Omega_i/S$) are defined in Melville et al. (2016), where the estimates for the critical $\beta$ in the ICM turbulence are provided: $\beta_c \sim 10^{-9}$, corresponding to magnetic field intensities $\sim 10^{-9} - 10^{-8}$ G.
direction of the macroscopic magnetic field. For example, the development of a microscale transverse magnetic field component does not change the direction (or intensity) of the mean magnetic field. However, it modifies the direction of the magnetic field on small scales, and at least part of the parallel pressure (with respect to the microscopic local magnetic field) should contribute to increasing the macroscopic perpendicular pressure. Furthermore, the changes in the magnetic field intensity owing to the microcomponent should produce changes in the macroscopic pressure anisotropy. In other words, the macroscopic thermal anisotropy is influenced by the development of microscale magnetic field fluctuations, even assuming perfect conservation of the magnetic moment of the particles and excluding any kinetic effect. In summary, in the presence of micro-instabilities, the CGL closure for the mean, large-scale fields is at least incomplete, as the microscopic effects eventually modify macroscopic thermal anisotropy evolution. Therefore, the inclusion of a ‘subgrid’ model for the evolution of anisotropy in the AMHD description of the ICM turbulence is needed, even in the absence of any irreversible scattering of the particles.

Another complication is that the rate at which anisotropy is generated by turbulence increases in a way that is inversely proportional to the scale of the motions: $\Gamma_A \sim (0A/\partial t)_{\text{CGL}} \sim -d\ln B/\partial t \sim l_{\perp}^{-5/3}$ or $l_{\parallel}^{-1/3}$ (considering the fast or Alfvénic/slow scaling for the velocity gradients; see Yan & Lazarian 2011). This means that the statistics of the driving rate of anisotropy are strongly dependent on the inertial range of the simulation, and, therefore, on the numerical resolution. In this way, considering the dominant scale for the anisotropy generation rate as the lowest scale of the inertial range of our simulations ($\sim 10^{22}$ cm), and using the power law corresponding to the Alfvénic/slow velocity gradients in order to extend it to ion kinetic scales $\sim 10^3$ km, the average value of $\Gamma_A$ from our simulations could increase by a factor of at most $\sim 10^4$.

Modifying the CGL closure by imposing ‘hard wall’ limits on the pressure anisotropy (Sharma et al. 2006) in the AMHD description of the ICM is equivalent to assuming that the relaxation of the macroscopic anisotropy to the instability threshold occurs on a timescale that is negligible compared with the macroscopic time-scales. This assumption is well justified for both the firehose and mirror instabilities, whenever the rate of anisotropy generation $\dot{S}$ is much smaller than the ion Larmor frequency $\Omega_i$ (see discussions in Sections 6.2 and 6.3), independent of the development of pitch-angle scattering of the ions. However, the assumption of instantaneous relaxing also implies that the free energy released by the instabilities, which (at least in part) would be stored in microscopic magnetic fluctuations, is directly converted into internal energy irreversibly. First, in the case for which the instabilities scatter the ions almost ‘instantaneously’ during the anisotropy generation period, not necessarily all of the free energy of the instability is transferred to the ions (or is equally distributed between ions and electrons), as some of the electromagnetic energy cascades to scales below the ion Larmor radius (see Kunz et al. 2014) and ends up being transferred to the electrons. In any case, in the absence of a detailed description of the microturbulence cascading and of the full thermodynamics, including ion–electron collision rates, emission/cooling processes for each species, electron acceleration etc., it is not meaningful to pursue such a detailed energy distribution in AMHD simulations of the ICM (in the SL+14 simulations, thermal equipartition is assumed between the ions and electrons). Secondly, ‘removing’ instantaneously the energy from the microscale magnetic fluctuations causes the magnetic energy pressure to be underestimated. However, the largest values of the relative magnetic energy of the fluctuations $\delta B^2/B^2$ are of order unity (Kunz et al. 2014; Melville et al. 2016), and as the thermal $\beta$ values relevant for the ICM are high, the magnetic field pressure is secondary (compared with the thermal pressure) and also dynamically unimportant at large scales (especially in the dynamo context). This underestimation of the magnetic energy density could also affect, the detailed energy distribution between the species if radiative emission were taken into account. After the driving of anisotropy ceases, the microscopic magnetic fluctuations decay at a rate regulated by the scattering of the ions (Melville et al. 2016). The magnetic energy of the fluctuations gradually released is not converted again to free energy of the thermal anisotropy, because of the irreversible scattering of the ions. As discussed in the previous sections, the mirror and firehose magnetic fluctuations decay on a time-scale that is relatively short after the anisotropy generation has stopped, for moderate values of $\beta$. In the ‘ultra-high’-beta regime, however, these magnetic fluctuations persist over dynamical time-scales. This means that the bounded anisotropy model also cannot describe correctly the entropy evolution of the plasma.

To what extent could the ion scattering rate (and consequently the ion parallel mean free path) be derived from AMHD simulations of the turbulent ICM? Let us neglect for a moment the complexity arising from the fact that the statistics (spatial and temporal) of the turbulent shearing/compression rates may depend on the micro-instability state (and on the resolution of the simulation, as discussed above), and assume that the statistics of the shearing/compression are known. As discussed above, for values of beta representative of the ICM, the firehose fluctuations instantaneously spark ion scattering at a rate needed to keep the macroscopic anisotropy at the marginal threshold level, making it possible to derive the statistics of the scattering. For the mirror modes (and also for the firehose modes in the regime of ‘ultra-high’ beta), however, the determination of the scattering rate depends on knowledge of the microscopic magnetic fluctuation level that develops over the macroscopic time-scales of the shear/compressions. This means that the macroscopic fields of the plasma cannot be used to determine the local scattering rate at a given time.

Now consider again the influence of the ion scattering rate on the AMHD turbulence evolution itself. In the absence of a significant decrease of the parallel mean free path of the ions, a strong collisionless damping of the compressible modes propagating parallel to the local field can be expected. However, the shear velocities of the Alfvén modes, transverse to the local magnetic field, are not expected to be affected by the ion parallel mean free path. That is, an MHD-like Alfvénic cascade is expected to develop independently of the ion parallel viscosity (see Schekochihin et al. 2005). The linear Alfvén modes are expected to be affected only by the shear viscosity component. Neither the Braginskii shear viscosity $\sim r_i^2 v_i$ (where $r_i$ is the thermal ion Larmor radius and $v_i$ the ion–ion collision rate) nor the Landau damping can set a viscous scale for the Alfvénic strong cascade above the ion Larmor radius in the ICM regime of a high-beta-plasma subsonic turbulence (for a detailed discussion on this subject see Borovsky & Gary 2009). On the other hand, the compressible cascade will be damped on much larger scales. This damping is of kinetic origin, and its physics cannot be captured in AMHD simulations (see Section 6.1).

If we assume that the coupling between the compressible and Alfvénic modes in the anisotropic MHD is similar to that in isotropic pressure MHD (Cho & Lazarian 2003; Kowal & Lazarian 2010), the Alfvénic cascade must be energetically more important than and little affected by the compressible cascade in the ICM. Therefore, the macroscopic turbulence statistics of the ICM should be well represented by the bounded anisotropy
AMHD simulations if the precise thermodynamic description is not important.

However, in the absence of significant anomalous scattering of the ions (as is expected in the ‘ultra-high’-beta regime; see Melville et al. 2016), the micro-instability mechanism that will keep the pressure anisotropy limited is the suppression of the stretching rate of the magnetic field (Mogavero & Schekochihin 2014). This means that the velocity and magnetic fields from the micro-instabilities can affect the global stretching rate of the magnetic field, and, therefore, the small-scale dynamo evolution. In this scenario, the mirror instabilities, for example, could slow down considerably the turbulent amplification of the large-scale magnetic field; however, the situation is more complex because the persistent firehose fluctuations can suppress the development of the mirror modes (Melville et al. 2016). Such a contribution from the microscales to the induction equation cannot be included easily in fluid simulations, and hence the results of the magnetic field amplification obtained previously from both isotropic pressure MHD and bounded anisotropy AMHD simulations (e.g. SL+14) should be treated with caution, at least in regimes of very high beta (for further details on this subject, see Mogavero & Schekochihin 2014; Melville et al. 2016).

It is worthwhile to emphasize that several aspects of the ICM thermodynamics – such as entropy generation (Lyutikov 2007), ion heat conduction (Kunz et al. 2011), the physics of the compressible modes (see Section 6.1), which can have crucial importance for the ICM structuring and dynamics – cannot be self-consistently approached by the bounded anisotropy AMHD model without a detailed modelling of the micro-instability evolution (Melville et al. 2016).

6.5 Implications for particle acceleration in the ICM turbulence

As discussed in Section 6.4, a modification of the CGL-MHD equations is required in order to account for the limits on the temperature anisotropy imposed by the thresholds of the firehose and mirror instabilities and possibly of the ion-cyclotron instability (in regions of low $\beta$) in studies of the turbulent ICM. In SL+14, such limits (mirror and firehose) were shown to make the turbulence statistics similar to those of the collisional MHD counterpart, and the turbulent dynamo was also found to amplify the magnetic fields at rates comparable with those of collisional MHD (neglecting the effects of the suppression of the stretching rate of the magnetic field by the micro-instabilities, see Section 6.4).

If the scattering and trapping of the ions by the instabilities makes the effective collisional scale of the thermal particles much smaller than the Coulomb ion–ion parallel collision scale over a significant fraction of the plasma, there could be a drastic reduction of the collisionless damping of the compressible modes by the thermal plasma. Invoking such a scenario, Brunetti & Lazarian (2011) showed that the compressible modes can channel energy to re-accelerate efficiently relativistic particles in the ICM. In the absence of such anomalous scattering, only 10 per cent of the energy in the fast modes is available to accelerate the particles (Petrosian, Yan & Lazarian 2006; Brunetti & Lazarian 2007).

As discussed in Section 6.3, however, a knowledge of the magnetic fluctuation level of the micro-scale mirror modes (and also of the firehose modes in the ‘ultra-high’-beta regime; see Footnote 3) is required in order to make an estimate of the trapped fraction and scattering rate of the ions. In fact, as the evolution of the mirror modes occurs on macroscopic time-scales, the ion parallel mean free path will reduce gradually over time in the spatial location where the turbulence drives the anisotropy generation. A localized (in space and time) reduction of the ion mean free path in the turbulent plasma should be expected, but detailed statistics (spatial/temporal distribution) in connection with the turbulence cascade are necessary in order to understand the impact of the reduced ion mean-free-path on the damping of the compressible modes. This question deserves further extensive investigation.

7 SUMMARY AND CONCLUSIONS

Previous numerical simulations of forced turbulence representing the intracluster medium regime showed that the turbulence can produce very high levels of anisotropy in the temperature when the plasma instability feedback is neglected (SL+14). This anisotropy has an important impact on the turbulence statistics, producing significant modifications compared with one-temperature collisional MHD turbulence (see also Nakwacki et al. 2016 for a study of the impact of the pressure anisotropy on Faraday rotation maps of the ICM). High levels of anisotropy also prevents the turbulent amplification of the magnetic fields, which is believed to be responsible for sustaining the observed intensities and coherence lengths of the magnetic fields present in the ICM (Kotarba et al. 2011; Egan et al. 2016).

Using a grid of different initial conditions ($\beta_{ij}, A_i$) taken from a production distribution by a CGL-MHD simulation, we calculated the non-linear evolution of the ion temperature components owing to the pitch-angle scattering caused by the plasma mirror, ion-cyclotron, and parallel firehose instabilities, using quasi-linear theory in which we assumed an isotropic distribution for the electron temperature. The quasi-linear evolution brings the values of ($\beta_{ij}, A_i$) close to the limits of marginal stability after a few hundred ion thermal periods.

At the same time, we computed the average rate at which the simulated CGL-MHD turbulence pushes the temperature anisotropy in the direction of unstable values. We showed that this rate is several orders of magnitude smaller than the rate at which the pitch-angle scattering by the instabilities drives the temperature anisotropy towards stable values, even when starting with small deviations from the instability thresholds. The quasi-linear evolution of the ion temperature anisotropy used here was also compared with that obtained from two-dimensional hybrid simulations for a small set of unstable initial conditions in which the plasma develops ion-cyclotron modes. This comparison shows good accordance (within an order of magnitude) for the rate of pitch-angle scattering (see Appendix A).

Our quasi-linear analysis demonstrates clearly that, in the turbulent ICM, the fast scattering of ions by the instabilities rules out the presence of temperature anisotropy levels exceeding substantially the thresholds for the instabilities. When the anisotropy level is very close to the threshold of the instabilities, the slow instability growth favours adiabatic evolution to saturation, and then the quasi-linear scattering rates may become less representative (see Sections 6.2 and 6.3). This last observation is particularly relevant in the case of the mirror instability.

In addition, the development of the instabilities should take into account the continuous generation of anisotropy over macroscopic turbulence time-scales. Such a problem was recently addressed in studies by Kunz et al. (2014), Riquelme et al. (2015) and Melville et al. (2016), with the conclusion that the instabilities induced by the ICM turbulence do indeed keep the thermal anisotropy bounded by the instability thresholds on all the relevant macroscopic time-scales. The scattering rate of the ions is set ‘instantaneously’ (for the macroscopic time-scales) by the firehose instability, and it is responsible for the anisotropy relaxation that keeps the anisotropy...
limited. This is not the case regarding the mirror instability, however, for which the ion scattering increases gradually over the time-scale of the anisotropy generation by the turbulence shear/compression. In this case, the anisotropy is limited to the thresholds by processes that are essentially adiabatic (at least initially). In conclusion, all these results and considerations justify the modification of the CGL-MHD equations to include bounds of the anisotropy at the instability thresholds, which is appropriate for the description of the ICM turbulence. As the anisotropy relaxation rate derived from these bounds does not necessarily reflect the instantaneous scattering rate of the ions by the instabilities (at least for the mirror modes), such bounded anisotropy models cannot represent properly the thermodynamical evolution of the gas nor the damping of the compressible modes, which depend on the parallel ion mean free path. Even considering these limitations (see Section 6.4), the bounded anisotropy model should represent well the Alfvénic cascade of the turbulence, assuming that thermodynamical details play no major role in the turbulence dynamics. It was shown in the earlier study of SL+14 that, if the temperature anisotropy is restricted to stable values (considering only the mirror and firehose instabilities), the turbulence statistics and the magnetic field amplification arising from the small-scale dynamo have results indistinguishable from the one-temperature MHD description commonly used in studies of the ICM. We note, however, that this last study did not take into account the potential effects of the microscale instabilities on the stretching rate of the magnetic field during the turbulent amplification, which can be important, at least in very-high-beta regimes (Section 6.4).

Finally, it should be emphasized that any phenomena depending on the ion thermal parallel mean free path that can affect the ICM structuring (e.g. via heat conduction, thermal instabilities, cooling), and also cosmic ray acceleration (see Section 6.5) deserve further investigation, considering the complex interplay between the macroscopic turbulence and the detailed evolution of the microscopic instabilities.

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APPENDIX A: QUASI-LINEAR EVOLUTION OF THE ION-CYCLOTRON INSTABILITY COMPARED WITH PLASMA SIMULATIONS

We present here a comparison between the evolution of the temperature anisotropy obtained from two-dimensional hybrid simulations presented in Gary et al. (2000, hereafter G+00) and our quasi-linear approach described in Section 4.

We evolved the ion temperature components and the magnetic field wave energy using equations (10) to (13) for a set of initial values of $\beta_i^\parallel$ and $A_i = T_i^\perp/T_i^\parallel$, similar to those employed in G+00. We considered an initial flat spectrum for the magnetic waves with $|\delta B_k|^2/B_0^2 = 10^{-7}$.

Fig. A1 shows the evolution of the energy density of the magnetic fluctuations $W_B = \int \delta B_k^2/B_0^2$ (top panel), and the evolution of the ion temperature anisotropy (bottom panel) for one particular initial condition: $\beta_i^\parallel = 0.05$ and $A_i = 6.5$. This evolution is qualitatively similar to that obtained from the two-dimensional hybrid simulation of G+00 (see their fig. 1). After a short time interval, $W_B$ increases exponentially. The particle scattering resulting from these magnetic fluctuations modifies the ion thermal velocity distribution and leads to a fast decrease in the anisotropy at the same time at which the magnetic fluctuations grow very rapidly (bottom panel of Fig. A1). The anisotropy in temperature decreases exponentially during this time interval. After this phase, it continues to decrease but at a slower rate.

Fig. A2 shows for the complete set of the quasi-linear calculations the maximum values of the energy density of the magnetic fluctuations (top panel), the initial (blue triangles) and final (red squares) temperature anisotropy; the grey lines represent the thresholds (see Fig. 1) for the ion-cyclotron (dashed) and mirror (continuous) instabilities. (Bottom) Average scattering rate of the ions normalized by the ion Larmor frequency.

![Figure A1](https://example.com/figureA1.png)

**Figure A1.** Temporal evolution of the magnetic energy density perturbations (top) and temperature anisotropy (bottom). The initial conditions are $\beta_i^\parallel = 0.05$ and $T_i^\perp/T_i^\parallel = 6.5$. The dashed grey lines represent the fitting presented in Gary et al. (2000) for the two-dimensional hybrid simulations with similar initial conditions. The continuous grey lines represent the curves that are the best fit to the present quasi-linear results (see text).

![Figure A2](https://example.com/figureA2.png)

**Figure A2.** Results from the quasi-linear calculations using a set of different initial conditions ($\beta_i^\parallel, T_i^\perp/T_i^\parallel$) similar to those employed in the two-dimensional hybrid simulations by Gary et al. (2000). (Top) Maximum magnetic energy density in the ion-cyclotron modes during the evolution. (Middle) Initial (blue triangles) and final (red squares) temperature anisotropy; the grey lines represent the thresholds (see Fig. 1) for the ion-cyclotron (dashed) and mirror (continuous) instabilities. (Bottom) Average scattering rate of the ions normalized by the ion Larmor frequency.
Figure A3. Average ion scattering rate $\langle \nu S \rangle$ during the system evolution versus the maximum growth rate $\gamma_{\text{IC max}}$ of the unstable ion-cyclotron modes. The dashed line represents the fitted curve in Gary et al. (2000).

The average scattering rate $\langle \nu S \rangle$ for each simulation (bottom panel), as functions of the initial value of $\beta_{\|}$. We note that the scattering rate $\nu_S$ at each time-step was not calculated using equation (3). Instead, in order to make a more straightforward comparison with G+00, we used

$$\nu_S^* = -\frac{1}{A_i} \left( \frac{\partial A_i}{\partial t} \right)_{\text{scatt}},$$

(A1)

and the time-average accounted only for values of $\nu_S^* \geq 0.6\nu_{\text{max}}$, where $\nu_{\text{max}}$ is the maximum instantaneous scattering rate during the time-evolution.

The results from Fig. A2 are similar to those presented in fig. 2 in G+00, with differences no larger than one order of magnitude.

Fig. A3 shows the relation between the average scattering rate $\langle \nu S \rangle$ and the (initial) maximum growth rate of the unstable ion-cyclotron modes $\gamma_{\text{IC max}}$, for the same set of simulations. For comparison, also shown is the (dashed line) fitting for this relation obtained by G+00. The quasi-linear scattering rates agree, at least to an order of magnitude, with the totally non-linear calculation, and the similarity is closer for the highest initial values of the ion-cyclotron growth rate.

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