Diagnosis of deformation-derived ascending areas in a rainband
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\textbf{ABSTRACT}
This paper demonstrates that, for a moist baroclinic frontal system, the large-value deformation belt in the low-level atmosphere overlaps with precipitation. To precisely describe the relationship between deformation and heavy precipitation, deformation is introduced into the non-geostrophic $Q^2$-vector. $Q^2$ is then decomposed into three parts: the divergence-related term, the vorticity-related term, and the deformation-related term. By calculating the divergence of $Q^2$ and its components, it is found that in strong ascending areas within precipitation regions the non-geostrophic $Q^2$-vector divergence shows strong negative values. Its deformational component can contribute about 68\% to these negative values. This verifies that strong deformation in a precipitating atmosphere is favorable for the development of convection and precipitation. In addition, by calculating the correlation coefficients between the $Q^2$-vector (including its components) divergence and vertical motions, it is also found that the $Q^2$-vector divergence shows higher correlation with vertical motion within the precipitation belt and lower correlation in the non-precipitation areas, which indicates a larger contribution of $Q^2$ to vertical motion when precipitation occurs and implies an effect of $Q^2$ to the precipitation distribution or spatial variability. Among the three components of the $Q^2$-vector, the correlation coefficients between the deformational component and vertical motion are the most similar in pattern to that of the correlation coefficients between the $Q^2$-vector and vertical motion, which further reflects the important contribution of deformation to the large spatial variability of precipitation.

\section{1. Introduction}

Previous studies (e.g. Weldon 1979; Deng 1986; Jiang, Wang, and Mei et al. 2013) have researched the correlation between the low-level deformation field and strong precipitation, perhaps motivated by meteorologists noticing that deformation is closely related to cloud belts. From numerous satellite images, Weldon (1979) presented the high-level cloud-band distributions associated with deformation. He defined the cloud-stretching belt along the deformation dilation axis as the ‘deformation belt’ in satellite images and indicated that deformation can often influence deep layers of the atmosphere. A ‘deformation belt’ can not only occur along the front when two thermodynamically distinct flows encounter one another, but also in the vortex, high-level or low-level jet, and shear/convergence line, which may be invisible.

The relation between the deformation and cloud band implies its relation to precipitation. This was studied statistically by Deng (1986), who found that deformation can indicate heavy precipitation areas 12–24 h in advance. Recently, Jiang et al. (2013) also indicated that the strong deformation belt in a saddle-shaped field is aligned with the rainband. Several possible mechanisms may explain the collocation of large deformation belts and precipitation belts. The first is the deformational frontogenesis effect, which may cause dynamic and thermodynamic adjustment of the atmosphere, induce transverse frontal circulations, and thus trigger heavy precipitation. The second is by concentrating moisture. Through a simple numerical experiment, Gao et al. (2008) showed the process of deformation redistributing water vapor. The third may come from an impact of deformation on mesoscale disturbances or vortex evolutions, which is the direct...
producer of precipitation (Jiang 2011). These mechanisms give reasonable explanations and evidence regarding the correlation of deformation and precipitation, which to a certain extent indicates a possible application of deformation in precipitation diagnosis. However, an unsolved problem is that whilst these mechanisms explain how deformation may influence precipitation, they tell us little about how much they affect the evolution of precipitation.

Deformation, together with divergence and vorticity, is the basic characteristic of the wind field. The importance of divergence and vorticity to precipitation or the vertical motion associated with precipitation is relatively clear and can be diagnosed by the continuity equation and \( \omega \) equation (Bluestein 1992). A quantitative relation between deformation and vertical motion, however, has not been studied. Therefore, in this paper, to provide a quantitative examination of the correlation between deformation and the vertical motion associated with precipitation, a diagnostic method is utilized. The method is based on the non-geostrophic \( \mathbf{Q}^d \)-vector (Zhang 1999), which has been used extensively to diagnose the vertical motion associated with heavy precipitation (Yao, Yu, and Shou 2004; Yang, Gao, and Wang 2007; Cao and Gao 2007). Moreover, the relative importance of deformation, divergence and vorticity to vertical motion can also be diagnosed using this method.

The paper is structured as follows: Section 2 describes the formulas for building a relation between deformation and ascending motion based on the non-geostrophic \( \mathbf{Q}^d \)-vector and \( \omega \) equation. In Section 3, the applicability of the method in a front-related precipitation case is examined. Conclusions are drawn in Section 4.

2. Deformation and vertical motion

Hoskins, Dagbici, and Darics (1978) introduced the quasi-geostrophic \( \mathbf{Q} \)-vector and derived the quasi-geostrophic \( \omega \) equation, which uses the divergence of the quasi-geostrophic \( \mathbf{Q} \)-vector as the only forcing term. Since then, the \( \mathbf{Q} \)-vector has been used extensively in vertical motion calculations and the diagnosis of strong convection weather (Dunn 1991; Keyser, Schmidt, and Duffy 1992; Xu 1992; Yue 1999). Limited by the quasi-geostrophic approximation, the \( \mathbf{Q} \)-vector derived by Hoskins, Dagbici, and Darics (1978) does not perform well at low latitudes and in some sub-synoptic-scale weather systems. Therefore, Zhang (1999) derived a non-geostrophic \( \mathbf{Q}^d \)-vector in the \( p \)-coordinate and a corresponding \( \omega \) equation, which has performed better in heavy rainfall diagnosis, especially at low latitudes.

Therefore, in this paper, we conduct derivations based on the non-geostrophic \( \mathbf{Q}^d \)-vector. According to Zhang (1999), the non-geostrophic \( \mathbf{Q}^d \)-vector can be written as

\[
\mathbf{Q}^d = \left( Q^d_x, Q^d_y \right),
\]

\[
Q^d_x = \frac{1}{2} \left[ f \left( \frac{\partial v}{\partial p} - \frac{\partial u}{\partial p} \right) - h \frac{\partial v_h}{\partial x} \right],
\]

\[
Q^d_y = \frac{1}{2} \left[ f \left( \frac{\partial u}{\partial p} - \frac{\partial v}{\partial p} \right) - h \frac{\partial v_h}{\partial y} \right],
\]

where \( \mathbf{v}_h = (u, v) \) is the horizontal wind vector, \( f \) is the Coriolis parameter, \( h = \frac{p}{\rho}/C_p \), \( R \) is the dry air gas constant, \( p \) is pressure, \( p_r \) is the reference constant, \( C_p \) the specific heat at constant pressure, and \( \theta \) is potential temperature. The relation between the non-geostrophic \( \mathbf{Q}^d \)-vector and vertical motion can be represented by the non-geostrophic \( \omega \) equation,

\[
\nabla^2(\sigma\omega) + \rho f \frac{\partial^2 \omega}{\partial p^2} = -2\nabla \cdot \mathbf{Q}^d.
\]

When the \( \omega \) field has a wavy pattern, \( \nabla \cdot \mathbf{Q}^d \propto \omega \). Therefore, when \( \nabla \cdot \mathbf{Q}^d < 0, \omega < 0 \) and there is ascending flow. On the other hand, when \( \nabla \cdot \mathbf{Q}^d > 0, \omega > 0 \) and there is descending flow.

Introducing vertical vorticity \( \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \), horizontal divergence \( D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \), shearing deformation \( E_{sh} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \) and stretching deformation \( E_{st} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \) into Equations (2) and (3) gives

\[
Q^d_x = \frac{1}{4} \left( f \left( \frac{\partial v}{\partial p} - h \frac{\partial \sigma}{\partial p} \right) - D \right),
\]

\[
Q^d_y = \frac{1}{4} \left( f \left( \frac{\partial u}{\partial p} - h \frac{\partial \sigma}{\partial p} \right) - D \right) + \frac{1}{4} \left( f \left( \frac{\partial u}{\partial p} E_{sh} + h E_{st} \right) - h \frac{\partial \sigma}{\partial p} \right)
\]

\[
= Q_{xd} + Q_{yd} + Q_{xe},
\]

Equations (5) and (6) show that the non-geostrophic \( \mathbf{Q}^d \)-vector can be decomposed into three parts: a divergence-related term \( \mathbf{Q}^d_d = \left( Q^d_{xd}, Q^d_{yd} \right) \), a vorticity-related term \( \mathbf{Q}^d_v = \left( Q^d_{vx}, Q^d_{vy} \right) \), and a deformation-related term \( \mathbf{Q}^d_e = \left( Q^d_{xe}, Q^d_{ye} \right) \). Combining Equations (5) and (6) with (4), the non-geostrophic equation is thus written as

\[
\nabla^2(\sigma\omega) + \rho f \frac{\partial^2 \omega}{\partial p^2} = -2\nabla \cdot \mathbf{Q}_d - 2\nabla \cdot \mathbf{Q}_v - 2\nabla \cdot \mathbf{Q}_e.
\]
As is shown, corresponding to the divergence-related part, the vorticity-related part, and the deformation-related part in $Q^0$, vertical motion also includes the divergence part, vorticity part, and deformation part, which can be used to diagnose the contribution of deformation to ascending flow compared with the other two parts.

3. Case study

The relations between deformation and heavy rainfall are illustrated with the aid of a rainfall case in North China on 21 July 2012. In this case, extreme rainfall in Beijing on 21 July was induced, which caused heavy casualties and huge economic losses, thus sparking considerable attention throughout China. The data for the analysis are from the Global Forecasting System on a $0.5^\circ \times 0.5^\circ$ latitude–longitude grid.

Figure 1 shows two different times during the rainfall case on 21 July to observe the wind stream pattern and precipitation distribution. These two times were chosen because they correspond to the occurrence of heavy precipitation in the Beijing area (approximately 41°N, 116.5°E). At 0600 UTC, as in Figure 1(a), precipitation presented a northeast–southwest-oriented belt shape, extending southwesterly from the middle part of Inner Mongolia to the combined area of Gansu, Ningxia, and Sichuan provinces. Two strong precipitation centers were present therein. One center was located in the combined area of Inner Mongolia, Shanxi, and Shannxi provinces (39°N, 111°E), while the other was in the combined area of Gansu and Sichuan provinces (33°N, 106°E). From Figure 1, it is clear that the wind streams associated with the precipitation belt have evident features of deformation. Within the precipitation belt, northwesterly flows from high latitude encountered southeasterly flows from low latitude, forming a deformation pattern. As can also be seen, from the black curved arrows, which illustrate the flow pattern, the contraction axis of the deformation zone (axis along which air converges) was perpendicular to the rain band, while the dilatation axis (axis along which air stretches) was along the rain band. The relative orientation of the dilatation axis and rain band was favorable for air convergence of the rain belt. Apart from the flow pattern, we can also see that large values of deformation correspond well to the precipitation belt. As in Figure 1(a), the total deformation (red solid lines) presents two centers corresponding to the precipitation centers, also with a northeast–southwest-oriented belt shape. At 1200 UTC, when Beijing was experiencing the strongest precipitation (Figure 1(b); ~90 mm/6 h), the flow pattern was similar to that at 0600 UTC. However, the high-value deformation belt moved eastward, corresponding to the precipitation belt, with the center over Beijing and Hebei. This shows that, in this ‘Beijing extreme rainfall’ case, deformation had a close relation with the strong precipitation. This relation can also be seen in the distributions of deformation tick marks at 0600 UTC and 1200 UTC 21 July 2012. Deformation tick marks are a series of solid lines with their directions parallel to the dilation axis of deformation, and their lengths are equal to the magnitude of deformation. The dilation axis is an axis along which the deformational flow stretches air parcels. As shown in Figure 2(a) and (b), the lengths of deformation tick marks (short red lines) over precipitation regions are basically much longer than the surrounding non-precipitation areas. In addition, Figure 2 also shows the distributions of $\theta_E$ contours denoting the location of the cold front, which was a key system that influenced the occurrence of this extreme rainfall in Beijing. As in Figure 2(a), dense $\theta_E$ contours are apparent over the precipitation region, which are basically southwest–northeast-oriented and curved to the south at the northeast edge of the precipitation region. Within the precipitation area (west of 112°E), the deformation tick marks are quasi-east–west-oriented
and show evident angles with the $\theta_e$ contours. However, east of 112°E, in the dashed areas, the deformation tick marks are mainly distributed along the $\theta_e$ contours, and veer with contour curvature. At 1200 UTC (Figure 2(b)), the precipitation center entered Beijing and strengthened. Compared to Figure 2(a), the lengths of the deformation tick marks in the dashed areas are enlarged, while their directions retain small angles to the $\theta_e$ contours in Figure 2(b). According to deformational frontogenesis theory, the small angles (<45°) between the $\theta_e$ contours and the deformation tick marks in the dashed areas will induce frontogenesis, which is a triggering mechanism for strong precipitation due to secondary frontal circulation.

The relation between deformation and precipitation is diagnosed by showing its relation to vertical motion with the derived formula above. Figure 3 presents the horizontal distributions of vertical motion, $Q^e$-vector divergence and its three components at 1200 UTC 21 July 2012 around the Beijing area. As shown, at 1200 UTC, when Beijing encountered the strongest precipitation on 21 July, a strong ascending center appears near the precipitation center, with the largest vertical velocity being $-0.8$ pa s$^{-1}$. Corresponding to the ascending areas in Figure 3(a), the divergence of the total $Q^e$-vector (Figure 3(b)) shows strong negative values in the precipitation areas. The mean value of $\nabla \cdot Q^e$ over the precipitation center (black box in Figure 3(b); (39.5°–42.5°N, 115°–118°E)) is $-1.8 \times 10^{-17}$ pa$^{-1}$ s$^{-3}$. Comparing Figure 3(c–e) to (b), it can be seen that the divergence of the deformation-related $Q^e$-vector (Figure 3(e)) over the precipitation areas of Beijing is much larger than the divergence of the vorticity-related $Q^e$-vector (Figure 3(c)) and the divergence-related $Q^e$ (Figure 3(d)). The mean values of $\nabla \cdot Q^e$, $\nabla \cdot Q^u$ and $\nabla \cdot Q^v$ over the precipitation center (black boxes) are $-0.41 \times 10^{-17}$ pa$^{-1}$ s$^{-3}$, $-0.16 \times 10^{-17}$ pa$^{-1}$ s$^{-3}$ and $-1.23 \times 10^{-17}$ pa$^{-1}$ s$^{-3}$, respectively, which shows that the deformation part contributed about 68% of the total $Q$. In order to show that the deformation component not only plays a dominant role in terms of mean values, but also contributes to the spatial variability of vertical motion or precipitation, Figure 4 illustrates the spatial distribution of the correlation coefficients between $\nabla \cdot Q^e$ (including its components) and vertical motion during the whole process of the Beijing heavy rainfall event during 20–21 July 2012. In Figure 4(a), which presents the average rainfall of the heavy rainfall case, there is a strong precipitation center (black rectangle) embedded within a northeast–southwest-oriented rain belt, implying a large spatial variability of precipitation. Ascending areas (also averaged during 20–21 July 2012) basically correspond to the rain belt, with strong ascending centers near the precipitation center (black rectangle). From Figure 4(b) it is interesting to see that the correlation coefficient between $\nabla \cdot Q^e$ and vertical motion (denoted as $C_{Q^v}$) also shows large spatial variability, with higher correlation within the precipitation belt (black lines) and lower correlation in the non-precipitation areas. The largest $C_{Q^v}$ is approximately 0.8, right near the precipitation center in the black rectangle. This implies a contribution of $Q^e$ to precipitation distributions by acting on vertical motion. Comparing Figure 4(b) to (c–e), all three components of $Q^e$ show large correlation coefficients to vertical motion near the precipitation center within their respective black rectangles. However, the correlation between $\nabla \cdot Q^e$ and vertical motion (denoted as $C_{Q^v}$) shares a pattern that is more similar to $C_{Q^v}$. This is not only consistent with the above result regarding the
significant contribution of the deformational component to $Q^d$, but also reflects the important contribution of deformation to the large spatial variability of precipitation by acting on vertical motion.

4. Conclusion

Based on the non-geostrophic $Q^d$-vector and $\omega$ equation, this paper uses vorticity, divergence and deformation to substitute the total horizontal motion in the $Q^d$-vector and thus introduces deformation into $\omega$ equation. In the new $\omega$ equation (Equation (7)), the effect of deformation on vertical motion compared to divergence and vorticity can be evaluated quantitatively, which means the role of deformation during heavy precipitation can be diagnosed. By calculating the divergence of the non-geostrophic $Q^d$-vector and its three components during a front-related heavy precipitation case, it is shown that in the strong precipitation area, which features strong vertical motion, the divergence of the non-geostrophic $Q^d$-vector presents strong negative values. Mean-value analysis shows that the deformation-related part plays the most significant role in these negative values among the three components of the non-geostrophic $Q^d$-vector, which implies the importance of strong flow deformation on the convection and precipitation. In addition, the correlation between $\nabla \cdot Q^d$ (including its components) and vertical motion is analyzed to examine the role of deformation in the spatial distribution of precipitation. The results show that the correlation coefficients between $\nabla \cdot Q^d$ and vertical motion (denoted as $C_{Q^d}$) are higher within the precipitation belt and lower in non-precipitation areas, which indicates a larger contribution of $Q^d$ to vertical motion when precipitation occurs. This reflects an effect of $Q^d$ on the precipitation distribution or spatial variation.
variability. Among the three components of the $Q^*$-vector and vertical motion, the correlation coefficients between the deformational component of $Q^*$ and vertical motion share a pattern that is more similar to $C_{Q^*}$, which then implies a contribution of deformation to the large spatial variability of precipitation.

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