Noncommutative geometry admitting conformal Killing vectors: stability problem and dimensional constraint

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We study different dimensional fluids inspired by noncommutative geometry which admit conformal Killing vectors. The solutions of the Einstein field equations examined specifically for five different set of spacetime. We calculate the active gravitational mass and impose stability conditions of the fluid sphere. The analysis thus carried out immediately indicates that at 4-dimension only one can get a stable configuration for any spherically symmetric stellar system and any other dimensions, lower or higher, becomes untenable as far as the stability of a system is concerned.

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I. INTRODUCTION

Higher dimensional spacetime configuration is particularly important in the studies of the early phase of the Universe. It plays a significant role to describe not only the universe in early stage of its evolution but also some unobservable phenomena in the physical universe.

It is interesting to point out that Barrow [1] first tried to investigate the role played by the dimensions of space and spacetime in determining the form of various physical laws and constants of Nature becomes inevitable to investigate [1]. Basically he [1] employed the concept of fractal dimension under Kaluza-Klein theories obtained by dimensional reduction from higher dimensional gravity or supergravity theories. It has therefore been argued that the presently observed 4D spacetime is the compactified form of manifold with higher dimensions and as such in the arena of grand unification theory and also in superstring theory this kind of self-compactification idea of multidimensions have been invoked by different workers [2, 3]. There are also several other research articles available in the literature in connection to higher spatial dimension some of which can be consulted in the following Refs. [4–8].

In cosmology, the Kaluza-Klein inflationary theory with higher dimensions, it is believed that extra dimensions are reducible specially to four dimension which was associated with some physical processes. Ishihara [2] and later on Gegenberg and Das [10] have shown that within the Kaluza-Klein inflationary scenario of higher dimension a contraction of the internal space causes the inflation of the usual space. It is argued by Ibanez and Verduguer [11] that there are cases in FRW cosmologies where the extra dimensions contract as a result of cosmological evolution.

It has been observed that symmetries of geometrical as well as physical relevant quantities of general relativity, known as collineations and conformal Killing vectors (CKV), are most useful to facilitate generation of exact solutions to the Einstein field equations [12]. Therefore, in the present investigation following Yavuz et al. [13], we have imposed the condition that the spacetime manifold admits a CKV and thus tried to tackle the anisotropic
field equations in a suitably better way.

In connection to the anisotropic field equations of general relativity the study of compact objects has been of ample interest for a long time. It was argued long ago by Bowers and Liang [14] that the effects of local anisotropy may have important role for relativistic fluid spheres to attain hydrostatic equilibrium in connection to maximum equilibrium mass and surface redshift. Ruder- man [15] showed that in the stellar interior the nuclear matter may have anisotropic features at least in certain very high density ranges (> $10^{15}$ gm/cc) and thus advocated to treat the nuclear interaction relativistically. Very recently, based on some observed compact stars Kalam et al. [16] made an extensive analysis to show the anisotropic behavior of the samples.

Therefore, under the above theoretical background our motivation is indeed to study stability problem and dimensional constraint in connection to solutions of higher dimensional spherically symmetric systems within the framework of noncommutative geometry. The present investigation thus based on the following scheme: After providing the basic field equations of Einstein in the Sec. II, we seek the solutions under CKV in Sec. III for various dimensions ranging from 3D to 11D spacetime. Sec. IV deals with active gravitational mass where we get indication for minimum potential energy at 4D spacetime. Therefore, in Sec. V we continue our investigation by imposing equilibrium conditions and arrive at the conclusion that higher dimensional spacetime is not tenable for stability of the system. Sec. VI offers some concluding remarks in favor of the results obtained.

II. THE BASIC FIELD EQUATIONS OF EINSTEIN

The spacetime metric describing a spherically symmetric system in higher dimension is taken as

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2d\Omega^2_n,$$  

where the line element $d\Omega^2_n$ on the unit n-sphere is given by

$$d\Omega^2_n = d\theta_1^2 + \sin^2\theta_1d\theta_2^2 + \sin^2\theta_1\sin^2\theta_2d\theta_3^2 + \cdots + \prod_{i=1}^{n-1} \sin^2\theta_i d\theta_i^2.$$  

The general energy momentum tensor which is compatible with static spherically symmetry as

$$T^\mu_\nu = (\rho + p_r)u^\mu u^\nu - (\rho + p_r)g^\mu_\nu + (p_t - p_r)\eta^\mu_\nu$$  

with $u^\mu u_\mu = \eta^\mu_\mu = 1$.

The Einstein equations (for the geometrized units $G = c = 1$) are

$$e^{-\lambda} \left( \frac{n(n-1)}{2r^2} + \frac{n\nu'}{2r} \right) - \frac{n(n-1)}{2r^2} = 8\pi\rho,$$  

$$e^{-\lambda} \left( \frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2} \lambda' \nu' + \frac{(n-1)}{r}(\nu' - \lambda') + \frac{(n-1)(n-2)}{2r^2} \right) = 8\pi p_t,$$  

where $\rho$, $p_r$, and $p_t$ are respectively the energy density, radial pressure and tangential pressure of the static fluid sphere. Here $\nu$ over $\nu$ and $\lambda$ denotes partial derivative w.r.t. radial coordinate $r$ only.

The energy density having a minimal spread Gaussian profile in higher dimension is taken as $m = \frac{m}{(4\pi\theta)^{(n+1)/2}} e^{\psi \left( \frac{r^2}{4\theta} \right)}$.

Here, $m$ is the total mass of the source which can be diffused throughout a region of linear dimension $\sqrt{\theta}$ due to the uncertainty and generally assumed to be closed to the Planck length scale.

III. THE SOLUTIONS UNDER CONFORMAL KILLING VECTORS

To find the exact solution, as is indicated in the introductory part, we use the well known inheritance symmetry of the spacetime as the symmetry under conformal Killing vectors [19, 20] given as:

$$L_\xi g_{ik} = \xi_{,ik} + \xi_{;i} = \psi g_{ik},$$  

where $\psi$ is an arbitrary function of $r$.

The above conformal Killing equations (5) provide the following set:

$$\xi^n + c = \text{constant},$$

$$\xi^1 \nu' = \psi;$$

$$\xi^1 \lambda' + 2\xi^1 = \psi,$$

where the subscript of comma denotes the partial derivative with respect to $r$.

These equations (9) - (12) imply

$$e^\nu = c_2^2 r^2;$$

$$e^\lambda = \left( \frac{c_1}{\psi} \right)^2;$$

$$\xi^1 = c_1 \delta^1_{n+2} + \left( \frac{\psi r}{2} \right) \delta^1_1.$$
where \( c_2 \) and \( c_3 \) are integration constants. The above equations (13) - (15) contain all the characteristic features derived from the existence of the conformal collineation.

Now, using the values of \( \nu \) and \( \lambda \) from equations (13) and (14), we can solve the field equations for the given energy density (7), i.e. we try to find out here the unknowns \( \psi, p_r, \) and \( p_t \) under different space-time with varying value of \( n, \) the index of dimension. As a sample study we only consider cases with \( n = 1, \) \( n = 2, \) \( n = 3, \) \( n = 8 \) and \( n = 9 \) representing \( 3D, \) \( 4D, \) \( 5D, \) \( 10D \) and \( 11D \) space-time respectively.

### A. 3D space-time (\( n = 1 \))

Starting this case of lower space-time with \( n = 1, \) the metric potential can be given by

\[
\lambda(r) = \ln \left( \frac{\theta}{4m(2\theta e^{-\frac{r^2}{4\theta}} - c_1)} \right). \tag{16}
\]

Here the pressures are taking the forms:

\[
p_r = \frac{m(2\theta e^{-\frac{r^2}{4\theta}} - c_1)}{2\pi r \theta^2}, \tag{17}
\]

\[
p_t = -\frac{me^{-\frac{r^2}{4\theta}}}{4\pi \theta}. \tag{18}
\]

In a similar way, by applying the boundary conditions at \( r = R, \) i.e. \( p_R = 0, \) at once we get

\[
c_1 = 2\theta e^{-\frac{R^2}{4\theta}}. \tag{19}
\]

1. **Matching conditions**

Our metric is

\[
ds^2 = c_2^2 r^2 dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Omega_{n=1}. \tag{20}
\]

The 3D metric is

\[
ds^2 = -(M_0 - \Lambda r^2) dt^2 + (M_0 - \Lambda r^2)^{-1} dr^2 + r^2 d\Omega_{n=1}. \tag{21}
\]

So the matching conditions yield the following results:

\[
c_1 = (M_0 + \Lambda R^2) \frac{\theta}{4m} + 2\theta e^{-\frac{R^2}{4\theta}}, \quad c_2 = \frac{1}{R} \sqrt{M_0 + \Lambda R^2}. \tag{22}
\]

### B. 4D space-time (\( n = 2 \))

This is the usual 4D space-time and in this case the metric potential can be given by

\[
\lambda(r) = \ln \left[ \frac{r \sqrt{\pi \theta}}{-\pi \sqrt{\pi \theta} \left( r + c_1 - 2m \text{erf} \left( \frac{r}{2\sqrt{\theta}} \right) \right) - 2m e^{-\frac{r^2}{4\theta}}} \right]. \tag{23}
\]

where \( \text{erf} \left( \frac{r}{2\sqrt{\theta}} \right) \) is the error function.

Therefore, the radial and tangential pressure parameters, \( p_r \) and \( p_t, \) now are respectively taking the following forms:

\[
p_r = -\frac{-2r + 6m \text{erf} \left( \frac{r}{2\sqrt{\theta}} \right) - 3c_1 - 6m r e^{-\frac{r^2}{4\theta}}}{8r^3 \pi}, \tag{24}
\]

\[
p_t = \frac{-(-\sqrt{\pi \theta}^2 + m r^2 \sqrt{\theta} e^{-\frac{r^2}{4\theta}})}{8(r^2)^2 (\pi \theta)^{3/2}}. \tag{25}
\]

At the boundary surface \( (r = R) \) pressure should be considered as of vanishing order \( (p_{r=R} = 0) \). Thus we get the value of the constant \( c_1 \) as

\[
c_1 = \frac{2}{3} \left[ -R - 3 \frac{mR}{\sqrt{\pi \theta}} e^{-\frac{R^2}{4\theta}} + 3m \text{erf} \left( \frac{R}{2\sqrt{2}} \right) \right]. \tag{26}
\]

1. **Matching conditions**

In the present case our metric:

\[
ds^2 = c_2^2 r^2 dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Omega_{n=1}. \tag{27}
\]

On the other hand, 4D metric is Schwarzschild metric and can be supplied as

\[
ds^2 = -(1 - \frac{2M}{R}) dt^2 + \left( 1 - \frac{2M}{R} \right)^{-1} dr^2 + r^2 d\Omega_2. \tag{28}
\]

So matching conditions provide us the following expressions for the constant quantities:

\[
c_1 = -2R + 2M + 2m \text{erf} \left( \frac{R}{2\sqrt{2}} \right) - \frac{2mR}{\sqrt{\pi \theta}} e^{-\frac{R^2}{4\theta}}, \quad c_2 = \frac{1}{R} \sqrt{\frac{2M}{R} - 1}. \tag{29}
\]

### C. 5D space-time (\( n = 3 \))

Let us now move towards higher dimension by choosing the value of \( n > 2 \). The metric potential, the radial and tangential pressures for this 5D space-time can respectively be given by

\[
\lambda(r) = \ln \left[ \frac{3r^3 \pi^2 \theta}{\pi^2 \theta (3r^2 + 2c_1) + 2\pi m (r^2 + 4\theta) e^{-\frac{r^2}{4\theta}}} \right], \tag{30}
\]

\[
p_r = \frac{\pi \theta (9r^2 + 4c_1) + 4m (r^2 + 4\theta) e^{-\frac{r^2}{4\theta}}}{8\pi r^3 \theta}, \tag{31}
\]

\[
p_t = \frac{-4\pi \theta^2 + m r^2 e^{-\frac{r^2}{4\theta}}}{16\pi r^3 \theta^2}. \tag{32}
\]

At \( r = R, \) \( p_{r=R} = 0 \) and that gives \( c_1 = -\frac{4}{\pi^2} \left( R^2 + 4\theta \right) \text{exp}(-\frac{R^2}{4\theta}). \)
Our metric in this case is
\[ ds^2 = c^2_2 r^2 dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Omega_{n=3}. \] (33)

Again, the 5D Schwarzschild metric is
\[ ds^2 = \left( 1 - \frac{8GM}{3\pi R^2} \right) dt^2 + \left( 1 - \frac{8GM}{3\pi R^2} \right)^{-1} dr^2 + r^2 d\Omega_3. \] (34)

So matching conditions on the boundary immediately give us:
\[ c_1 = \frac{3}{2}(R^3 - R^2) - \frac{4GM R}{\pi} \frac{m}{4\theta} (R^2 + 4\theta e^{-R^2/4\theta}), \]
\[ c_2 = \frac{1}{R} \sqrt{1 - \frac{8GM}{3\pi R^2}}. \] (35)

D. 10D space-time \((n = 8)\)

For the arbitrary choice of \(n = 8\) for higher dimensional case, we get the following results:
\[ \lambda(r) = \ln(128r^7\pi^5\theta^4) - \ln \left[ \frac{32\theta^2\pi^5(4r^7 + c_1) - 840\theta^4\pi^2 m \text{ erf} \left( \frac{r}{2\sqrt{\theta}} \right)}{256 \theta^5 \pi^6 (-9\pi r^7 - 126\theta r^5 - 126\theta^2 r^3 - 7560\theta^3 r)} \right] \] (36)

\[ p_r = -\frac{1}{256r^9\pi^6} (\pi^5 (-2048r^4 - 288c_1) + 7560\pi^2 m \text{ erf} \left( \frac{r}{2\sqrt{\theta}} \right) - \frac{\sqrt{\pi} \theta m e^{-\frac{r^2}{4\theta}}}{256 r^9 \theta^5} (-9\pi r^7 - 126\theta r^5 - 126\theta^2 r^3 - 7560\theta^3 r)), \] (37)

\[ p_t = -\frac{448\pi^4\theta^4 + \sqrt{\pi} \theta m e^{-\frac{r^2}{4\theta}}}{512 r^9 \pi^5 \theta^5 \theta}. \] (38)

At \(r = R\), \(p_r = 0\) and it provides
\[ c_1 = -\frac{2048R^7}{288} + \frac{7560}{288\pi^3} m \text{ erf} \left( \frac{R}{2\sqrt{\theta}} \right) - \frac{\sqrt{\pi} \theta m e^{-\frac{R^2}{4\theta}}}{288 \pi^3 \theta^4} (9R^7 + 126\theta R^5 + 1260\theta^2 R^3 + 7560\theta^3 R). \] (39)

E. 11D space-time \((n = 9)\)

We would like to study one more higher dimensional case with \(n = 9\). In this chosen case of 11D space-time, the metric potential and pressure parameters take the forms:
\[ \lambda(r) = -\ln \left[ \frac{64c_1 + 288r^8}{288r^8} \right] \] (40)

\[ p_r = \frac{320c_1 + 2592r^8}{256 \pi r^{10}} \] (41)

At \(r = R\), \(p_r = 0\) and we get
\[ c_1 = -\frac{2592R^8}{320} \frac{me^{-\frac{R^2}{4\theta}}}{320\pi^4 \theta^4} \times (5R^8 + 80 R^6 \theta + 960 R^4 \theta^2 + 7680 R^2 \theta^4 + 30720 \theta^6). \] (43)

IV. ACTIVE GRAVITATIONAL MASS IN VARIOUS DIMENSIONS

We apply the following relation to calculate active gravitational mass in various dimensions [21]
\[ M(R) = \int_0^R \left[ \frac{2\pi m}{r^{(n+1)/2}} \right] r^n \rho(r) dr. \] (44)
to be extracted from this study for our understanding of possible properties of different static and spherically symmetric fluid distribution. This obviously can be done through a systematic survey of the effect of higher dimensional spacetime for the different parameter set that fits the observed stars. We have collected data for the masses of some compact objects, e.g. PSR J1614-2230 [1.97 ± 0.08 $M_\odot$ [24]], PSR J1903+327 [1.667 ± 0.02 $M_\odot$ [23], Vela X-1 [1.77 ± 0.08 $M_\odot$ [24]], SMC X-1 [1.29 ± 0.05 $M_\odot$ [24] and Cen X-3 [1.29 ± 0.08 $M_\odot$ [24]]. The corresponding radii have been calculated by Takisa et al. [25] which respectively are as follows: 10.30 km, 9.82 km, 9.99 km, 9.13 km and 9.51 km (also see the Refs. 16, 26 for other data set for different compact stars).

Keeping in mind the above range of masses and radii, let us then realistically consider the following specific parameters set for different compact stars.

\[ M_\odot = 1.4 M_\odot, \quad R = 10 \text{ km and } \theta = 0.002, \quad \text{from a straightforward calculation, we get } M(R) = 39.38883833, 55.70423008, 1.979898987, 111.4084602, 222.8169203, 630.2214212, 891.2676813, 2520.885686 \text{ for the dimensions } 2, 3, 4, 5, 7, 10, 11 \text{ and } 14 \text{ respectively. It is very interesting to note from the Fig. 1 that the function } M(R) \text{ increases in a blowing up manner as the number of dimensions increases but it has only one minimum at the } 4D \text{ spacetime. There is also a small hump visible at } 3D \text{ spacetime. This means that noncommutative geometry admitting conformal Killing vectors with anisotropic fluid sphere permit only the } 4\text{-dimensional spacetime to make the spherically symmetric matter distribution in stable equilibrium.}

\section{V. EQUILIBRIUM: TOV EQUATION FOR 4D}

In view of the above result and discussion we are then in demand of verifying the equilibrium features of the spherically symmetric matter distribution. The active gravitational mass, for the present case, can be given by

\[ -M_g \frac{p_r + \rho}{r^2} e^{\frac{\lambda - \nu}{r}} - \frac{dp_r}{dr} + \frac{2}{r} (p_t - p_r) = 0, \quad (45) \]

where

\[ M_g = M_g(r) = \frac{1}{2} r^2 e^{\frac{\lambda - \nu}{r}} \frac{d\nu}{dr}. \quad (46) \]

Now equilibrium of the spherical symmetric system requires the following condition:

\[ F_g + F_h + F_a = 0, \quad (47) \]

where $F_g$, $F_h$ and $F_a$ are respectively the gravitational, hydrostatic and anisotropic forces.

Let us then provide the forces in action, i.e. gravitational, hydrostatic and anisotropic, respectively as follows:

\[ F_g = -M_g \frac{p_r + \rho}{r^2} e^{\frac{\lambda - \nu}{r}} \]

\[ = -\frac{1}{r} \left[ -2r + 6m \frac{erf\left(\frac{r}{2\sqrt{\theta}}\right)}{2\sqrt{\theta}} - 3c_1 - \frac{6mr}{\sqrt{\pi}e^{-\frac{r^2}{4\theta}}} \right] + \frac{m}{(4\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}}, \quad (48) \]

\[ F_h = \frac{dp_r}{dr} \]

\[ = \frac{d}{dr} \left[ -2r + 6m \frac{erf\left(\frac{r}{2\sqrt{\theta}}\right)}{2\sqrt{\theta}} - 3c_1 - \frac{6mr}{\sqrt{\pi}e^{-\frac{r^2}{4\theta}}} \right], \quad (49) \]

\[ F_a = \frac{2}{r} (p_t - p_r) \]

\[ = \frac{2}{r} \left[ -\sqrt{\pi} \frac{\theta^2}{2} + 6m r^2 \sqrt{\pi} e^{-\frac{r^2}{4\theta}} \right] + \left[ -2r + 6m \frac{erf\left(\frac{r}{2\sqrt{\theta}}\right)}{2\sqrt{\theta}} - 3c_1 - \frac{6mr}{\sqrt{\pi}e^{-\frac{r^2}{4\theta}}} \right]. \quad (50) \]

Even though the differential Eq. (49) for $F_h$ has not been worked out, one can draw information from Eq. (47) to have a primary conclusion. By using Maple we actually are able to plot without writing the derivative in analytical form. Thus, as far as the Fig. 2 is concerned, we observe that $F_a$ is the most dominant factor whereas the least one is $F_g$. Moreover, at lower dimension the joint action of $F_g$ and $F_a$ is much more than $F_h$ so that the system becomes unstable. On the other hand, as we approach towards $4D$ they balance each other and thus make a stable configuration.
VI. CONCLUDING REMARKS

The analysis done in the foregoing section immediately indicates that at 4-dimension only one can get a stable configuration for any spherically symmetric stellar system as such higher dimension becomes untenable as far as the stability of a system is concerned. In a study on higher dimensional framework of noncommutative geometry Farook et al. [29], replace pointlike structures with smeared objects and have found that wormhole solutions exist in the usual four, as well as in five dimensions also (only in a very restricted region), but they do not exist in higher-dimensional spacetime. However, it is now an open question whether the above remark is true for a noncommutative geometry admitting conformal Killing vectors or in any geometry this becomes feasible. In this connection we note that Farook et al. [7] studied a generalized Schwarzschild spacetime with higher dimensions and performed a survey whether higher dimensional Schwarzschild spacetime is compatible with some of the solar system phenomena. As a sample test they examined four well known solar system effects, viz., (1) Perihelion shift, (2) Bending of light, (3) Gravitational redshift, and (4) Gravitational time delay. It has been shown by them that under a N-dimensional solutions of Schwarzschild type very narrow class of metrics the results related to all these physical phenomena are mostly incompatible with the higher dimensional version of general relativity.

As a final comment we would like to add here that a much more deep and comprehensive studies are required before offering a concluding remark in connection to stability of a spherically symmetric fluid distribution at 4D only.

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