Is the fair sampling assumption supported by EPR experiments?

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Abstract
We analyse optical EPR experimental data performed by Weihs et al in Innsbruck 1997–1998. We show that for some linear combinations of the raw coincidence rates, the experimental results display some anomalous behaviour that a more general source state (like non-maximally entangled state) cannot straightforwardly account for. We attempt to explain these anomalies by taking account of the relative efficiencies of the four channels. For this purpose, we use the fair sampling assumption, and assume explicitly that the detection efficiencies for the pairs of entangled photons can be written as a product of the two corresponding detection efficiencies for the single photons. We show that this explicit use of fair sampling cannot be maintained to be a reasonable assumption as it leads to an apparent violation of the no-signalling principle.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
The experimental violation of Bell inequalities [1–4] in optical EPR experiments [5, 6] can only be validated under the additional assumption of fair sampling [4, 7, 8]. This type of test is crucial for modern quantum communication research [9, 10]; it is unfortunate that the result of a test meant to disprove local realism would depend on such an additional assumption, since local realism is a priori no less plausible an assumption than fair sampling is.

In this paper we examine data from past experiments and look for possible traces of failure of the fair sampling assumption. For this purpose, we were recently given the chance to examine raw data from the long distance experiment with fast switching performed in Innsbruck in 1997–1998 by Gregor Weihs et al. We show that the near perfect agreement with the predictions of quantum mechanics obtained from experiments can be questioned when extracted data are looked at from a different perspective, that is, when comparing other
linear combinations of the normalized coincidence rates than the correlation function with the predictions of quantum mechanics.

2. Channel efficiencies

In a real experiment, not all photons are detected and one should take account of the efficiencies of each of the four channels involved. This problem is at the heart of the detection efficiency loophole, and can be used to design local realistic models that reproduce the main experimental features of optical EPR experiments [11].

We label $\eta_{A}^{+}(\alpha)$ and $\eta_{A}^{-}(\alpha)$ the single-count channel efficiencies of Alice’s plus and minus channels, respectively, and similarly $\eta_{B}^{+}(\beta)$ and $\eta_{B}^{-}(\beta)$ for Bob’s plus and minus channels; the parameter dependence in $\alpha$ and $\beta$ reflects the fact that these efficiencies may, in principle, depend on the local settings. It is important to stress that we can put in the concept of channel efficiencies all possible variations in the experimental conditions that are not directly related to the quantum state associated with the source (such as signal intensity variations, detector efficiencies, optical misalignments, collection efficiencies, etc). This idea is important since it is precisely the aim of our paper to analyse experimental quantities that can directly be compared with quantum predictions—under the assumption of fair sampling—independently of all these experimental contingencies encapsulated in the channel efficiencies.

We can expect the number of single counts to be proportional to the respective channel efficiencies:

$$N_{A}^{\varepsilon_{1}}(\alpha) \approx \eta_{A}^{\varepsilon_{1}}(\alpha)N_{A}/2$$

$$N_{B}^{\varepsilon_{2}}(\alpha) \approx \eta_{B}^{\varepsilon_{2}}(\beta)N_{B}/2,$$

where $N_{A}$ and $N_{B}$ are the (unknown) number of photons actually sent respectively to Alice and Bob, and where $\varepsilon_{1}$ and $\varepsilon_{2}$ can each be either + or −, to shorten the forthcoming equations.

We now consider the number of coincidence counts experimentally registered. If we consider a pair of photons as a whole, the probability that it is detected in each specific combination of two channels should depend as well on a combined channel efficiency. We label these combined channel efficiencies as $\eta^{+\varepsilon_{1}}(\alpha, \beta)$, $\eta^{\varepsilon_{1}\varepsilon_{2}}(\alpha, \beta)$, $\eta^{\varepsilon_{1}+\varepsilon_{2}}(\alpha, \beta)$ and $\eta^{\varepsilon_{1}+\varepsilon_{2}}(\alpha, \beta)$. The number of coincidences in a pair of channels $(\varepsilon_{1}, \varepsilon_{2})$ should thus be proportional to the relevant combined efficiency $\eta^{+\varepsilon_{1}}(\alpha, \beta)$, the (unknown) number of pairs sent $N_{AB}$, and the relevant joint probability predicted by quantum mechanics $P_{QT}^{\varepsilon_{1}\varepsilon_{2}}(\alpha, \beta)$, that is,

$$N_{\exp}^{\varepsilon_{1}\varepsilon_{2}}(\alpha, \beta) \approx \eta^{\varepsilon_{1}\varepsilon_{2}}(\alpha, \beta)N_{AB}P_{QT}^{\varepsilon_{1}\varepsilon_{2}}(\alpha, \beta).$$

(1)

Here the angular dependence of the efficiencies is clearly unwanted since we are interested in an experimental test of the predictions of quantum mechanics, independently of the inaccuracies involved in any particular experimental setup. Our immediate purpose is therefore to get rid of these angular dependences due to the combined efficiencies.

In order to do so, we assume that the ensemble of detected pairs of photon provides a fair statistical sample of the ensemble of emitted pairs (fair sampling assumption). A consequence of this assumption is that the probabilities of non-detection for Alice and Bob should be independent of one another. Indeed, for fixed settings $(\alpha, \beta)$, the probability of a non-detection should be independent of the polarization state of the photon (otherwise the sampling would clearly be unfair), and should thus be independent of the fate of the distant correlated photon.

1 The use of the symbol $\approx$ is here to remind of the statistical variability in any experimental results that is naturally expected to induce small deviation from the predictions. The amplitude of these deviations are expected to decrease with the number of trials obtained experimentally.
Hence, the channel efficiencies should be the same for all photons going into a specific channel, independently of whether a photon happens to be single or to be paired with a distant detected photon. That is, the above combined efficiencies for pairs of particles should be equal to the product of the relevant channel efficiencies for the single counts:

$$\eta^{e_1,e_2}(\alpha, \beta) = \eta^{e_1}_A(\alpha)\eta^{e_2}_B(\beta).$$

With this factorization of the channel efficiencies, we can rewrite the predicted number of coincidence counts as

$$N^{e_1,e_2}_{\exp}(\alpha, \beta) \approx N_{AB}\eta^{e_1}_A(\alpha)\eta^{e_2}_B(\beta)P^{e_1,e_2}_{QT}(\alpha, \beta).$$

### 3. Normalizing using the single counts

The probabilities derived from the standard normalization procedure with coincidence counts still depend explicitly on the channel efficiencies, so that some properties of the photon pairs will remain out of reach of the experimenter.

True enough, in some specific cases the standard normalization procedure can make the channel efficiencies disappear from the final result. For instance, if on one side the channels are balanced, (e.g. $\eta_B^{e_2}(\beta) = \eta_B(\beta)$), it is straightforward to show that the standard normalization with the total sum of coincidences removes the channel efficiency dependence of the correlation function:

$$\frac{N^{++}_{\exp} + N^{--}_{\exp} - N^{+-}_{\exp} - N^{-+}_{\exp}}{\sum_{e_1,e_2} N^{e_1,e_2}_{\exp}} \approx P^{++}_{QT} + P^{--}_{QT} - P^{+-}_{QT} - P^{-+}_{QT} = E_{QT}. \tag{4}$$

However, for other linear combination of the coincidence counts, the normalization by the total sum of coincidences cannot remove the channel efficiency dependence, even if on one side the channels are balanced. For instance, if $\eta_B^{e_2}(\beta) = \eta_B(\beta)$, then

$$\frac{N^{++}_{\exp} + N^{--}_{\exp}}{\sum_{e_1,e_2} N^{e_1,e_2}_{\exp}} \approx \eta_A^{e_2} + \eta_A \left( P^{++}_{QT} + P^{--}_{QT} \right) \neq \left( P^{++}_{QT} + P^{--}_{QT} \right). \tag{5}$$

This problem makes it necessary to circumvent these dependences in all cases by means of the new normalization procedure. Our idea is that most of the counts registered in the channels are non-coincidence events, because the channel efficiencies are low. Since there are many times more non-coincident events than coincident ones, they provide a useful and accurate additional statistical information about the relative efficiency of the channels.

In order to get rid of channel efficiencies in the above equations, we thus define the following experimental quantities, which are proportional to the ratio of the number of coincidence counts in a combined channel over the product of the two corresponding single counts:

$$f^{e_1,e_2}_{\exp}(\alpha, \beta) \equiv \frac{1}{4} \frac{N^{e_1,e_2}_{\exp}(\alpha, \beta)}{N^{e_1}_{\exp}(\alpha)N^{e_2}_{\exp}(\beta)}. \tag{6}$$

Replacing the single counts and coincidence counts by their expressions given respectively in (1) and (3), we obtain

$$f^{e_1,e_2}_{\exp}(\alpha, \beta) \approx \frac{N_{AB}}{N_AN_B} P^{e_1,e_2}_{QT}(\alpha, \beta) \tag{7}$$

for which the only angular dependence is the one due to the quantum mechanical term.
Since the four joint probabilities $P_{QT}^{++}(\alpha, \beta)$, $P_{QT}^{+-}(\alpha, \beta)$, $P_{QT}^{-+}(\alpha, \beta)$ and $P_{QT}^{--}(\alpha, \beta)$ add up to unity, summing these four equations together yields

$$\sum_{\varepsilon_1, \varepsilon_2} f^{\varepsilon_1, \varepsilon_2}_{\exp}(\alpha, \beta) \approx \frac{N_{AB}}{N_A N_B}$$

(8)

and finally,

$$P_{QT}^{\varepsilon_1, \varepsilon_2}(\alpha, \beta) \approx \frac{f^{\varepsilon_1, \varepsilon_2}_{\exp}(\alpha, \beta)}{\sum_{\varepsilon_1, \varepsilon_2} f^{\varepsilon_1, \varepsilon_2}_{\exp}(\alpha, \beta)}.$$  

(9)

Since the normalization procedure that we propose here is different to the standard one based on the coincidence counts alone, a word on the relevance of equation (9) to experiments might be necessary here. The validity of equation (9) as an approximation of quantum probabilities by experimental frequencies (based on the single counts) depends only on the validity of the fair sampling assumption, an assumption routinely made in EPR experiments exhibiting a violation of Bell inequalities\textsuperscript{2}. Indeed, equation (2) is but a consequence of this assumption, and leads straightforwardly to equations (6), (9) and (10).

Hence, by using the fair sampling assumption together with experimental statistics for the single counts, we are able to obtain experimental quantities that should coincide with the prediction given by quantum mechanics for the joint probabilities, independently of any channel efficiency imbalance. As far as we know, this procedure is new and offers new perspectives for comparing experimental result with quantum predictions.

In particular, this new normalization procedure allows us to obtain experimental quantities that should directly coincide with the marginal probabilities:

$$\frac{f^{++}_{\exp}(\alpha, \beta) + f^{+-}_{\exp}(\alpha, \beta)}{\sum_{\varepsilon_1, \varepsilon_2} f^{\varepsilon_1, \varepsilon_2}_{\exp}(\alpha, \beta)} \approx P_{QT}^{++}(\alpha, \beta) + P_{QT}^{+-}(\alpha, \beta)$$

$$\frac{f^{-+}_{\exp}(\alpha, \beta) + f^{--}_{\exp}(\alpha, \beta)}{\sum_{\varepsilon_1, \varepsilon_2} f^{\varepsilon_1, \varepsilon_2}_{\exp}(\alpha, \beta)} \approx P_{QT}^{-+}(\alpha, \beta) + P_{QT}^{--}(\alpha, \beta)$$

(10)

$$\frac{f^{++}_{\exp}(\alpha, \beta) + f^{+-}_{\exp}(\alpha, \beta)}{\sum_{\varepsilon_1, \varepsilon_2} f^{\varepsilon_1, \varepsilon_2}_{\exp}(\alpha, \beta)} \approx P_{QT}^{++}(\alpha, \beta) + P_{QT}^{+-}(\alpha, \beta)$$

An important prediction of quantum mechanics for these marginal probabilities is the no-signalling principle. A correlation $P(\varepsilon_1, \varepsilon_2|\alpha, \beta)$ is non-signalling if and only if its marginal probabilities are independent of the other side input: $\sum_{\varepsilon_1} P(\varepsilon_1, \varepsilon_2|\alpha, \beta)$ is independent of $\beta$ and $\sum_{\varepsilon_2} P(\varepsilon_1, \varepsilon_2|\alpha, \beta)$ is independent of $\alpha$ [14].

This non-signalling property does not depend on the state of the source of photons. Whatever is the source sent to Alice and Bob, they cannot use it to communicate. Alice’s marginal probabilities, represented by the first two equations above, cannot depend on Bob’s measurement setting $\beta$. Similarly, Bob’s marginal probabilities, represented by the two last equations above, cannot depend on Alice’s measurement setting $\alpha$.

\textsuperscript{2} To some extent, one could argue that John Bell used an implicit fair sampling assumption in deriving his theorem, and that although he discarded the possibility that it would not be fulfilled as very unlikely, it remained however a necessary condition that had to always be assumed, but was never fully tested experimentally. In that sense, the fair sampling assumption is as essential to the whole framework of Bell’s theorem as are the locality assumption and the realism assumption.
Hence, if only one local parameter varies (say, $\alpha$), then only two out of these four quantities can vary accordingly, the other two remaining constant. It is precisely this no-signalling prediction conditioned on the validity of the fair sampling assumption that we want to check here.

### 4. Experimental results

#### 4.1. Long distance with fast switching

Out of all the various runs we had at hand, we chose first a set of files containing data from a long distance fast switching experiment, with a significant number of coincidences and with enough measurement angles. This run was performed on 1 May 1998 in Innsbruck. The files were referred to as `scanblue`, meaning that a scan varying ‘blue’ (for Alice) side modulator bias was performed, with both sides randomly fast switching between an equivalent $+0^\circ$ and $+45^\circ$ angle. Modulating the bias from $-100$ to $+100$ was linearly equivalent to rotating the corresponding polarizing beam splitter from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$.

We look at the data for the particular case when both switches were set to $+0^\circ$ angle. As can be seen in the coincidence rate figures (see figure 1(a)), the coincidence rates exhibit minima close to zero, and cosine-squared shape, as expected from the predictions of quantum mechanics. However, the maxima of the four coincidence curves differ significantly. In spite of this anomalous behaviour, the correlation function computed with the standard normalization (i.e., with the sum of all coincidence counts) coincides very well with the quantum mechanical predictions (see figure 1(b)).

It is interesting to note that possibly similar anomalies were observed first in the two-channel EPR experiments performed by Alain Aspect in Orsay in the early 1980s. The anomalies were reported in Aspect’s PhD Thesis [15] in the following words.

> For some measurements, we have observed abnormal differences between coincidence rates that were expected to be equal (for instance $N_{+-}$ and $N_{-+}$). It turns out however that even for these measurements the correlation coefficient $E(a,b)$ remains equal to the quantum predictions, with better than two standard deviations. We have no completely convincing explanation, either for these anomalies, or for their compensation.³

These anomalies could in principle be explained by a non-rotationally invariant source state, such as a non-maximally entangled state (see the appendix). The information available on these anomalies is reduced to the above quotation; it is however impossible to give a definite explanation for them. These anomalies are nevertheless interesting since they indicate that, assuming the state produced by the source was a singlet state and that the channel efficiencies where balanced on each side, the experimental data would not have coincided with the predictions of quantum mechanics for all possible linear combinations of the normalized coincidence counts, although the fit with the correlation function was excellent.

To illustrate this point with our particular set of data, we computed the four possible sum of even normalized coincidence counts with odd normalized coincidence counts (see figure 2). In an ideal setup with balanced channels, these four curves should coincide with Alice’s and Bob’s marginal probabilities (that is, 1/2 for a singlet state). The result displayed in figure 2 shows clearly that other linear combination of the normalized coincidence counts

³ Our translation. The original text is as follows: "Pour quelques mesures on a observé des différences anormales entre taux de coincidences que l’on attendait égaux (par exemple $N_{+-}$ et $N_{-+}$). Mais il se trouve que pour ces mesures, le coefficient de corrélation $E(a,b)$ reste égal à la prévision quantique, à mieux que deux écarts-types près. Nous n’avons aucune explication complètement convaincante, ni pour ces anomalies, ni pour leur compensation."
Figure 1. (a) Coincidence counts for a long distance Bell test with fast measurement switching (Innsbruck 1998, source file: scanblue, with switches on position 00), (b) correlation function.

than the correlation function leads to incorrect results. For this purpose we extract first the single counts. The result is displayed in figure 3.

As can be seen from the figure, some slight variations of Alice’s single-count rates can be observed as she varies her bias. It is nevertheless of small amplitude, and remains in any case always local, in the sense that only Alice’s single counts vary. More important is the fact that the efficiency of + and − channels differs significantly for Alice.

As can be seen from figure 4, the four experimental quantities expressed by (10) vary with Alice’s measurement angle α.

For the first two, corresponding to Alice’s marginal probabilities, this dependence can be accounted for by a non-rotationally invariant source state, such as a non-maximally entangled state (see the appendix). However, for the remaining two, there should be no angle dependence
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Figure 2. The four possible curves with $(N_{\text{exp}}^{\text{even}} + N_{\text{exp}}^{\text{odd}}) / \sum_{\epsilon_1, \epsilon_2} N_{\text{exp}}^{\epsilon_1, \epsilon_2}$, showing that the standard normalization with coincidence counts leads to incorrect results.

Figure 3. Single counts (switches 00, scanblue).

on $\alpha$ at all, since they should depend only on $\beta$ which remains fixed. The angular dependence of the experimental quantities that should coincide with Bob’s marginal probabilities is admittedly weak, but statistical analysis show that a linear fit provides a much poorer fit than a nonlinear fit for these plots.

Nevertheless, since we had other sets of data available, we decided to investigate whether this anomalous behaviour could be exhibited in other experimental setup that would not necessarily be meant to close the locality loophole, but would have clearer results.
4.2. Short distance without fast switching

We thus chose a set of data that have a relatively high maximum number of coincidences and low dark rates, as well as enough different measurement settings, in order to better exhibit the rather tenuous anomaly we observed in all the runs. This run was performed in 1997 in Innsbruck. The files were referred to as bluesine, meaning that only the ‘blue’ side (that is Alice) varied the measurement setting $\alpha$, while the ‘red’ side (that is, Bob) kept the same
setting. It had no fast switching and was performed at short distance, but since our goal was not to focus on the locality loophole but rather on detection and sampling issues, this would have been irrelevant anyway.

As can be seen in the coincidence rate figures (see figure 5(a)), the coincidence rates exhibit minima close to zero, and cosine-squared shape, as expected from the predictions of quantum mechanics. However, just like in the long distance experimental setup, the maxima of the four coincidence curves differ significantly.

If we now extract the single counts, in order to use our normalization procedure, we can see that the efficiency of + and − channels differ significantly for Bob (see figure 5(b)).

Finally, just like in the long distance setup, the four experimental quantities that should coincide with the marginal probabilities (see figure 6) vary with Alice’s measurement angle α.

This time, the experimental quantities that should coincide with Bob’s marginal clearly depend on Alice’s measurement setting α. It should be noted that our normalization procedure based on the fair sampling assumption reduces the magnitude of this behaviour, but without removing it completely. If the standard normalization based only on the coincidence counts are used (by computing quantities like $N^{++} + N^{+-}$ divided by the total sum of coincides), the visibility of these anomalies are even greater.

5. Conclusion

Our result, obtained under the assumption of fair sampling, shows that Bob’s marginal probabilities clearly vary with Alice’s setting α, in violation of the no-signalling principle. It logically means that these anomalies imply either the violation of the fair sampling assumption, or the violation of the no-signalling principle, or the violation of both.

It should be noted that these anomalies cannot be the result of variations or discrepancies in the single-count channel efficiencies. The main feature of our normalization procedure is indeed that—once fair sampling has been explicitly assumed—the dependence on the channel efficiencies disappear from the final expressions relating quantum predictions to directly
measurable experimental quantities. This is an improvement on the standard normalization procedure, which in general leads to results depending not only on the predictions for the considered quantum state but also on the channel efficiencies.

The no-signalling principle being a fundamental feature of quantum mechanics, as well as of local realism, the most reasonable interpretation of these anomalies is that the fair sampling assumption should be rejected\(^4\), as it is the only extra assumption that we used to observe them.

In other words, we do not believe that our investigation supports the idea of faster than light signalling, although this possibility cannot be logically excluded. Once the fair sampling assumption is rejected, there is no evidence of violation of the no-signalling principle, and therefore no evidence whatsoever of a possible superluminal communication. It should nevertheless be stressed that if one prefers to maintain the fair sampling assumption at all cost, the most striking result that can be derived under this assumption is not so much the violation of Bell inequalities, but rather the violation of no-signalling principle.

In any case, our results show that more experiments are definitely required to understand whether this feature is unique or not in EPR experiments.

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Appendix. Predictions of quantum mechanics for a non-maximally entangled state

As an example of the non-signalling principle, we can consider the source of pairs of photons to be represented by a non-maximally entangled state of the form

\[
|\psi\rangle = \frac{1}{\sqrt{1 + p^2}}[|H\rangle_1 \otimes |V\rangle_2 - p|V\rangle_1 \otimes |H\rangle_2].
\]  

With this state, it is straightforward to obtain the joint probability of detecting a ++ event when polarizers 1 and 2 are oriented at \(\alpha\) and \(\beta\) respectively. The result is given in quantum mechanics by the Born rule as

\[
P_{\text{QT}}^{++}(\alpha, \beta) = |\langle (+_\alpha | \otimes (+_\beta |)\psi\rangle|^2.
\]  

\(^4\) Note that this can be interpreted in the framework of contextual probabilities [12, 13]. By choosing a pair of settings \(\Pi = (\alpha, \beta)\), we choose a selection procedure for pairs of particles. Each selection procedure produces its own ensemble of pairs \(S_{\Pi}\), and the statistical properties of \(S_{\Pi}\) does depend on \(\Pi = (\alpha, \beta)\). It therefore depends explicitly on an experimental context \(C\Pi\) that can relate to Bohr’s reply to Einstein in 1935. It is not clear how Bohr would have reacted to such an explicit contextuality, but since he would probably have rejected a remote contextuality based on an explicit nonlocality [16], we believe that our interpretation of contextuality based on selection procedure offers a link between Einstein’s ensemble interpretation of quantum mechanics and Bohr’s contextual viewpoint. The point with an explicit contextuality as this one is that Bell’s derivation of his famous inequality becomes impossible as it is derived from a fixed probability measure \(\mu\). This problem was discussed in [17], p 95, where generalizations of Bell inequalities were obtained within the framework of contextuality. Such generalized Bell inequalities are not violated by experimental data.
The joint probability $P_{QT}^{\varepsilon_1,\varepsilon_2}(\alpha, \beta)$ to get the results $\varepsilon_1$ and $\varepsilon_2$, given the measurement settings $\alpha$ and $\beta$, can be derived explicitly as

$$
P_{QT}^{++}(\alpha, \beta) = \frac{(p \sin \alpha \cos \beta - \cos \alpha \sin \beta)^2}{1 + p^2},$$

$$
P_{QT}^{+-}(\alpha, \beta) = \frac{(\cos \alpha \cos \beta + p \sin \alpha \sin \beta)^2}{1 + p^2},$$

$$
P_{QT}^{-+}(\alpha, \beta) = \frac{(p \cos \alpha \cos \beta + \sin \alpha \sin \beta)^2}{1 + p^2},$$

$$
P_{QT}^{--}(\alpha, \beta) = \frac{(\sin \alpha \cos \beta - p \cos \alpha \sin \beta)^2}{1 + p^2}.
$$

(A.3)

It is straightforward to show that the marginal probabilities for this particular state are

$$
P_{QT}^{++}(\alpha, \beta) + P_{QT}^{+-}(\alpha, \beta) = \frac{\cos^2 \alpha + p^2 \sin^2 \alpha}{1 + p^2},$$

$$
P_{QT}^{-+}(\alpha, \beta) + P_{QT}^{--}(\alpha, \beta) = \frac{p^2 \cos^2 \alpha + \sin^2 \alpha}{1 + p^2},$$

$$
P_{QT}^{++}(\alpha, \beta) + P_{QT}^{+-}(\alpha, \beta) = \frac{p^2 \cos^2 \beta + \sin^2 \beta}{1 + p^2},$$

$$
P_{QT}^{-+}(\alpha, \beta) + P_{QT}^{--}(\alpha, \beta) = \frac{\cos^2 \beta + p^2 \sin^2 \beta}{1 + p^2}.
$$

(A.4)

which fulfil the non-signalling principle, as Alice’s marginal probabilities depend on $\alpha$ alone, while Bob’s marginal probabilities depend on $\beta$ alone.

References

[1] Einstein A, Podolsky B and Rosen N 1935 Phys. Rev. 47 777
[2] Bohm D J 1951 Quantum Theory (Englewood Cliffs: Prentice-Hall)
[3] Bell J S 1964 Physics 1 195
[4] Clauser J F, Horne M A, Shimony A and Holt R A 1969 Phys. Rev. Lett. 23 880
[5] Aspect A, Dalibard J and Roger G 1982 Phys. Rev. Lett. 49 1804–7
[6] Weihs G, Jennewein T, Simon C, Weinfurter H and Zeilinger A 1998 Phys. Rev. Lett. 81 5039–43
[7] Clauser J F and Shimony A 1978 Rep. Prog. Phys. 41 1881–927
[8] Adenier G and Khrennikov A Yu 2003 Proc. Conf. Quantum Theory: Reconsideration of Foundations-2 (Växjö, Sweden: Växjö University Press)
[9] Tittel W and Weihs G 2001 Photonic entanglement for fundamental tests and quantum communication Quantum Inf. Comput. 1 3-56
[10] Sergienko A V and Jaeger G S 2003 Contemp. Phys. 44 341
[11] Larsson J-Å 1998 Phys. Rev. A 57 3304–8
[12] Khrennikov A Yu 2001 J. Phys. A: Math. Gen. 34 9965–81
[13] Khrennikov A Yu 2004 Found. Phys. Lett. 17 691
[14] Gisin N 2005 Preprint quant-ph/0512168
[15] Aspect A 1983 PhD Thesis 2674 (Orsay)
[16] Wiseman H M 2005 Preprint quant-ph/0509061
[17] Khrennikov A Yu 2003 Interpretations of Probability (Utrecht-Boston: VSP)