An Adaptive Threshold Image Reconstruction Algorithm of Oil-Water Two-Phase Flow in Electrical Capacitance Tomography System

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Abstract. The subject investigated in this paper is the ECT system of 8-electrode oil-water two-phase flow, and the measuring principle is analysed. In ART image-reconstruction algorithm, an adaptive threshold image reconstruction is presented to improve quality of image reconstruction and calculating accuracy of concentration, and generally the measurement error is about 1%. Such method can well solve many defects that other measurement methods may have, such as slow speed, high cost, and poor security and so on. Therefore, it offers a new method for the concentration measurement of oil-water two-phase flow.

1. Introduction to ECT
Electrical Capacitance Tomography (ECT) technique is a new computer tomography technique that was presented by UMIST in Britain between late 1980 and early 1990. By using with other measurement techniques or instruments can be applied to do a real-time measurement on total mass flow, split-phase mass flow and flow velocity of multiphase flows. Compared with other techniques, ECT technique has many characteristics, such as simple structure, non-intrusion, fast speed, low cost, good safety performance and so on. Therefore, it can offer an effective path to solve the measurement problem of two-phase concentration.

2. Systemic structure and principle of ECT
2.1. ECT system structure of oil-water two-phase flow
A typical ECT schematic diagram of 8-electrode oil-water two-phase flow is shown as figure 1. An ECT system is generally composed of three basic parts: EC sensors, an acquisition system for EC data, and a computer for image reconstruction. Its basic principle is that it can use different dielectric constants of each phase in multiphase media and get EC values between electrode couples by EC sensors which are evenly installed on the ectotheca of insulating pipeline. Since these EC values can reflect the distribution condition of dielectric constants in the pipeline, the computer can utilize the data to accomplish image reconstruction based on a certain algorithm. Then we can acquire phase-distribution charts on the cross section of pipeline, which means we can get concentration values of two-phase flow and visualized gain phase-distribution information of multiphase flow.

2.2. Measurement principle
The measurement principle for the ECT system of two-phase flow is described as: medium of each
phase in two-phase flow corresponds to different dielectric constant, therefore when the concentration of each-phase component and its distribution are changed, it can cause equivalent dielectric constant of two-phase mixed flow to be changed and EC measuring values can correspondingly be altered. Apparently, extent of EC values can reflect the concentration extent of various phases and the distribution in two-phase flow. By making use of multi-electrode array-type EC sensors and the intercombination between electrodes, we can get multiple EC measuring values that can reflect the distribution condition of each phase. By utilizing one proper image-reconstruction algorithm based on medical CT principle and such EC values, we can reconstruct the image that can reflect the distribution condition of each phase on a certain cross section of the pipeline or device, and then we can realize visual measurement of flow regime or parameter measurement of two-phase flow.

For a N-plate system, we can get the total number ‘M’ of independent electrode couples as follows:

\[
M = C_N^2 = \frac{N \cdot (N - 1)}{2}
\]  

(1)

The subject investigated of this paper is major in a typical 8-electrode EC sensor, and we can acquire 28 independent measuring values C1, C2, ..., C28.

An EC measuring value between plates is composed of two parts: the EC value formed by the dielectric media between plate couples or inside the pipeline, and the EC value formed by the dielectric media outside the pipeline or in the shielding layers between shielding covers. By a rational design for EC sensors, EC value of the second part can be greatly reduced, and we can ignore its influence on the total measuring value of EC. On the premise of this hypothesis, we study a kind of simple case, the continuous phase and discrete phase of two-phase flow are respectively defined as ‘\(\varepsilon_1\)’ and ‘\(\varepsilon_2\)’, and the discrete phase can be evenly distributed in the continuous phase. If the volumes of the two media are defined as ‘\(V_1\)’ and ‘\(V_2\)’, the equivalent dielectric constant ‘\(\varepsilon\)’ can be described as (2):

\[
\varepsilon = (V_1/V) \cdot \varepsilon_1 + (V_2/V) \cdot \varepsilon_2
\]  

(2)

In (2), ‘\(V\)’ is the total volume of two-phase flow, namely ‘\(V=V_1+V_2\)’. Then we can get (3):

\[
\varepsilon = [(V-V_2)/V] \cdot \varepsilon_1 + (V_2/V) \cdot \varepsilon_2 = \varepsilon_1 + (V_2/V) \cdot (\varepsilon_2 - \varepsilon_1)
\]  

(3)

And the discrete-phase concentration ‘\(\beta\)’ can be defined as follows:

\[
\beta = V_2/V
\]  

(4)

In this way we can draw a conclusion that EC measuring value ‘\(C\)’ is the function of equivalent dielectric constant ‘\(\varepsilon\)’, and it is expressed as follows:

\[
C = K \cdot \varepsilon = K \cdot g(V_2/V) = K \cdot g(\beta)
\]  

(5)

Thus it can be seen that the numerical range of ‘\(C\)’ can be looked on as the measuring basis of ‘\(\beta\)’.

According to electricity principle, we can ignore the distributing change of various split phase on the radial of pipeline and the influence on the shielding layer. For EC value ‘\(C_{ij}\)’ between Electrode ‘\(i\)’ and Electrode ‘\(j\)’ in a 8-electrode array-type EC sensor, we can approximatively express it as follows:

\[
C_{ij} = \iint_{D} \varepsilon(x,y) \cdot S_{ij}(x,y,\varepsilon(x,y)) \, dx \, dy
\]  

(6)

In (6): \(j=1, 2, \ldots, 28\); ‘\(D\)’ is the cross section of pipeline; ‘\(\varepsilon(x,y)\)’ is the distribution function of dielectric media on the cross section of the pipeline; ‘\(S_{ij}(x,y,\varepsilon(x,y))\)’ is distribution function of sensitivity for EC value ‘\(C_{ij}\)’, which means the sensitive extent forming from ‘\(C_{ij}\)’ to the medium on the spot ‘\((x,y)\)’.
We define gray scale of picture element for media distribution inside the pipeline as follows:

\[
f(x, y) = \frac{f(x, y) - \epsilon_1}{\epsilon_2 - \epsilon_1}, \quad f(x, y) \in [0, 1]
\]  

(7)

Well then the above definition can directly connect the media distribution with gray scale of the image, and it is more important that we can get (8) on the basis of (3) and (4) as follows:

\[
f(x, y) = \frac{f(x, y) - \epsilon_1}{\epsilon_2 - \epsilon_1} = \frac{V_1}{V} = \beta
\]  

(8)

In (8): ‘\(V_1\)’ and ‘\(V\)’ are respectively referred to volumes of discrete phase and multiphase flow at the spot ‘\((x, y)\)’. Such definition of gray scale ‘\(f(x, y)\)’ can reflect the content of discrete phase at the spot ‘\((x, y)\)’. Therefore, we can calculate the concentration of multiphase flow by reconstructing the cross-section image of multiphase-flow pipeline.

2.3. Mathematic principle of image reconstruction

According to infinitesimal theory of double integral, we can approximately calculate the distributing information of sensitivity field on the basis of higher accuracy and get the corresponding mathematical model as follows:

\[
P = WF
\]  

(9)

In (9): ‘\(F=[f_1, f_2, \ldots, f_m]^T\)’ is gray-scale vector, which shows gray scales of ‘\(m\)’ infinitesimal picture elements in the whole pipeline; ‘\(f_i \in [0, 1]\)’ is the gray scale of Infinitesimal ‘\(i\)’; ‘\(P=[p_1, p_2, \ldots, p_n]^T\)’ is projection vector, which represents ‘\(n\)’ projection values (in a 8-electrode system, \(n=28\)), and ‘\(p_j=C_{0j}\)’ (\(j=1, 2, \ldots, n\)); ‘\(W\)’ is the weight-coefficient matrix of gray-scale vector related to the distribution of sensitivity field in the pipeline, namely ‘\(W(j, i) = S_{0j}(i)\)’. We can expand the matrix equation of image-reconstruction model depending on (9) to the form of linear system of equations as follows:

\[
p_1 = w_{11}f_1 + w_{12}f_2 + \cdots + w_{1m}f_m
\]

\[
p_2 = w_{21}f_1 + w_{22}f_2 + \cdots + w_{2m}f_m
\]

\[
\vdots
\]

\[
p_n = w_{n1}f_1 + w_{n2}f_2 + \cdots + w_{nm}f_m
\]  

(10)

In (10): ‘\(N=28\)’ is the number of projection; ‘\(m=100\)’ is the number of picture element. According to (10), the problem of image reconstruction can correspondingly be boiled down to the problem of solving linear system of equations.

ART (Algebraic Reconstruction Technique,) algorithm [1] can realize image reconstruction not by directly solving matrix equations, but by iteratively solving the system of (10). In ART image-reconstruction algorithm, gray-scale vector ‘\(F=[f_1, f_2, \ldots, f_m]^T\)’ which is composed of ‘\(E\)’ picture elements can be looked at as one spot in a m-dimension space, and each projection equation in (10) can be looked at as a hyperplane. When the system of equations has a unique solution, all these hyperplanes can intersect at one spot. This spot is the solution to the system of equations, and the solution procedure can correspondingly be boiled down to iteratively solving the intersecting point of these hyperplanes. The iterative solution procedure of ART algorithm is accounted for as follows:

1. Assigning initial values: we can assign the initial estimated value of gray-scale vector for the reconstructed image to be ‘\(F^{[0]}\)’, namely ‘\(F^{[0]}=[f_1^{[0]}, f_2^{[0]}, \ldots, f_m^{[0]}]^T\)’.

2. ‘\(F^{[0]}\)’ is the vector of m-dimension space, and we can get ‘\(F^{[1]}\)’ formed by the projection from ‘\(F^{[0]}\)’ to the hyperplane determining by the first projection in (10). In this way, we can accomplish the iteration as follows:

\[
F^{[1]} = F^{[0]} - \frac{(w_i^T F^{[0]} - p_i)}{w_i} w_i
\]  

(11)

In (11): ‘\(w_i=[w_{i1}, w_{i2}, \ldots, w_{im}]^T\)’, and it is the weight coefficient of the first projection in (10).

3. Similarly, we can get ‘\(F^{[2]}\)’ by the projection from ‘\(F^{[1]}\)’ to the hyperplane determining by the second projection equation. Repeating the same process, we can get ‘\(F^{[j]}\)’ on Hyperplane ‘\(i\)’ from the projection of ‘\(F^{[j-1]}\)’ on Hyperplane ‘\(i-1\)’ as follows:
\[ F^{[i]} = F^{[i-1]} - \frac{\left(w_i F^{[i-1]} - p_i\right)}{w_i} \quad (12) \]

In (12): \( w_i = [w_{i1}, w_{i2}, \ldots, w_{im}]^T \). Finally we can get the projected result of the first rotation \( F^{[N]} \) from the projection of the hyperplane determined by the last projection in (10).

(4) We begin to get on the second rotation of projected iteration whose initial value is \( F^{[N]} \), which means by repeating iterative projected procedure on Step 3 we can get the projected result of the second rotation \( F^{[2N]} \), by the projection from \( F^{[N]} \) to the first hyperplane. In this way, we can get vector sequence \( F^{[N]}, F^{[2N]}, F^{[3N]}, \ldots \). If the unique solution exists, the sequence finally will be convergent to one point which is the gray-scale estimate of reconstructed image.

In the experiment, some gray scales of picture element acting as the iterated result perhaps exceed Range \([0, 1]\), so during the iterative procedure we can do a filtering adjustment as follows:

\[ f_i = \begin{cases} h_0, & \text{if } f_i > h_1 \\ f_i, & \text{if } h_0 \leq f_i \leq h_1 \\ h_1, & \text{if } f_i < h_0 \end{cases} \quad (13) \]

In (13): \( h_0 \) is the lower limit of gray scale, and its value is a little less than ‘0’, so in the experiment we choose the value to be ‘–0.1’; \( h_1 \) is the permissible upper limit of gray-scale whose value is a little greater than ‘1’, and in the experiment its value is often chosen to be ‘1.1’. During the imaging process we can set the threshold value to be ‘V’. If a certain gray scale of picture element is not less than ‘V’, we can set the value of the gray scale to be ‘1’ or else to be ‘0’ which can be described as (14), and in (14): ‘\( g_i \)’ is the gray scale of element ‘i’.

\[ g_i = \begin{cases} 1, & \text{if } f_i \geq V \\ 0, & \text{if } f_i < V \end{cases} \quad (14) \]

3. Measurement principle for the concentration of oil-water two-phase flow based on ECT

3.1. Conventional method

In conventional measuring methods, the dielectric constant of water is greater than that of oil; therefore the gray scale of picture element is the water cut of picture element. The water cut of two-phase flow in the extent of pipeline corresponded to the plate length of sensors can be calculated by the equations below [2][3][4]:

\[ \beta = \frac{\text{sum}(f(i) \cdot (f(i) > V))}{100} \quad (15) \]

\[ \beta = \frac{1}{28} \sum_{i=1}^{28} p_i \quad (16) \]

\[ \beta = \frac{1}{20} \sum_{i=1}^{20} p_i, \quad p_i \in \{20 \text{ measuring values of EC between non-adjacent plates}\} \quad (17) \]

From the above three calculation methods, the accuracy of (15) is the highest. If a bigger oil drop exists in the pipeline, there can be a great difference between projected data (measured EC value) when the oil drop locates near the polar plate and at the center of the pipeline. The value of the latter is obviously greater than that of the former, therefore according to (16) and (17) we cannot assure the accuracy of water cut. Because the sensitivity field of EC between adjacent plates is excessively centralized (only in a small area between two plates), it is of disadvantage to reflect split-phase content in the whole pipeline. Therefore, in (17) we need not calculate these eight EC values, and the calculating accuracy of content is better than that of (16).

According to the above method we can get on reconstructing image and calculating content, but the quality of image and the accuracy of content are not ideal. Especially the error of content in oil-water two-phase flow is greater which can come to 13.5\%, thus we need to study an improved method.

3.2. Improved method

When the contents are different by experimental observation, the imaging threshold values should follow up the change in order to get higher quality of image reconstruction and lower calculating error
of content. Therefore we choose 16 representative samples that oil cut are respectively 0, 5, 10, 15, 25, 30, 40, 50, 55, 70, 75, 85, 90, 95, 97.5, 100. Then we detect their suitable imaging threshold values, and fit these 16 data by a curve. At last we get a relation curve between content and threshold value shown as figure 2, and the computing method for content is also corrected as follows:

$$\beta = \frac{1}{100} \sum_{i=1}^{100} g_i$$

(18)

In the course of image reconstruction, the determination of threshold values and the method of image reconstruction are described as follows:

1. According to ART algorithm, we can calculate the gray scale of each picture element and then calculate split-phase content ‘h0’ based on (15).

2. According to ‘h0’, we can find a new corresponding threshold value ‘V’ on the curve.

3. According to (14) we calculate the imaging gray scale of ‘g1’, and then we can calculate new content ‘h1’ based on (18).

4. If ‘|h1−h0|>t’ and ‘t’ is a preinstalled constant, we will get ‘h0=h1’ and then turn to Step 2.

5. According to ‘V’, we can reconstruct the image and then calculate the content.

Calculated capacity from Step 2 to Step 4 is very small and the required time related to ART iteration can be ignored. Therefore, it cannot influence the real-time reconstruction of image.

4. Experimental result and discussion

We perform an experiment on an 8-electrode system and the subject investigated is the oil-water two-phase flow. The cross section of pipeline can be divided by a mesh of 20×20 when imaging, and we can totally get 316 effective picture elements which can be divided into 25 blocks when reconstructing image. At the same time we experiment on some typical flow regimes, such as stratiform flow, core flow and so on, and we apply to statistical filtering threshold value when imaging. In the experiment we choose the back-projection image-reconstruction algorithm (BP) in order to make a comparison, and the experimental result is shown as figure 3. From the experimental results, we can get better effect in calculating the concentration of oil-water two-phase flow on the basis of improved ART algorithm.

**Figure 2.** Relation curve between content rate and threshold value.

**Figure 3.** Image reconstruction on the cross section in oil-water two-phase flow and measuring results of concentration.

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