Gradual Change Reliability Sensitivity Design for Spindle System on Fully Stochastic Process

Xin-gang WANG, Bao-yan WANG, Xiaohui CHEN, Dongxiao HOU
School of Mechanical Engineering and Automation, Northeastern University, Qinhuangdao 066004, P. R. China
xgwang@neuq.edu.cn; wangbaoyan2005@163.com;

Abstract. Gradual change reliability design of mechanical components is considered by semi-stochastic process model of loading with change of time and fully stochastic process model of known degradation function of strength at present, yet it can’t obtain reasonable reliability design by existing information of components for these models. Thus the influence of existing information and time-dependent parameters of components on reliability should be taken into account in order to access reliability correctly. By studying effect of loading action and time-dependent character of strength, strength of components is taken as a process of independent increments and autocorrelation coefficient of strength is also calculated, meanwhile, we present a method for computing time-dependent reliability. Combining the reliability design theory with sensitivity analysis method, a numerical method for time-dependent reliability sensitivity design of components based on fully stochastic process is proposed, the variation rules of reliability sensitivity of parameters are given at any moment and the effects of design parameters on reliability of components are also studied. The method presented in this paper provides the theoretical basis for structural design and life prediction of mechanical components.

1. Introduction
Most of the characteristics data of mechanical products vary with time gradually, making reliability of the product show the characteristic of gradual or gradual change change, for example, the decrease of mechanical strength caused by fatigue, wear and corrosion. The change of the product's characteristics is in fact a process of gradual change over time. We must have it processed as a stochastic process, of course, the product's reliability also inevitable is a function of time, and now its focus is transforming from the invariant-time reliability to the gradual change reliability of the whole life structure process composed by design, operation, maintenance and degradation. At present, a lot of research on mechanical gradual change reliability have been did by many scholars, yet most of which is to set up the mathematical model of pure theory, don't put the current state of components and the given gradual change information into the theoretical model, so certain error is brought into the reliability design. Although gradual change characteristics of load are now very in-depth research, gradual change characteristics of strength still need further research[1-3]. Mainly because there are ample causes for the strength degeneration of components, coupled with the change of physical properties of the material itself, it is hard to show the non-stationary stochastic process of strength degeneration by a certain mathematical model. In this paper, certain work was done about this problem, in which we fully used the measured value of components at the present moment to make correction about the hypothetical stochastic process model of strength[4]. Here, what is assumed is the model on stochastic process of...
strength when the parts to be failure, and then use the measured values to amend. Thus we can get the model on stochastic process of strength for existing components in the future using period.

2. Time-dependent performance of stress
For mechanical structure and components in existence, the fact that strength degenerate and random loads change due to the influence of external work environment and internal factors cannot be neglected, and strength and loads is related with time. Thus, it is necessary to simulate the structural strength and the effect of random loads during the course of the stochastic process. The expression of state function is given as,

\[ g(t) = r(t) - \sigma(t) \]  

where \( r(t) \) is the stochastic process of strength degeneration, and \( \sigma(t) \) is the stochastic process of load effect. Formula for the time-dependent reliability is represented as follows

\[ R(t) = \Phi(\beta(t)) = P[r(t) > \sigma(t)] \quad t \in [0, T] \]  

Because in the calculation of the structural reliability index, we adopted the first-order second-moment analysis method which considers probability distribution types of basic random variable, the stochastic process of load effect must be converted to random variable, which is maximum load, in the design service period or required service period in future. Although load changing with time randomly, its expected value of maximum distribution will not decrease with the passage of time. In order to identify the effect on components from the equivalent maximum load of this random load, the random load is required to be equally discreted as \( n \) parts from the moment \( t_i \) to a certain moment \( t_i + \Delta t \) in actual measurement. The components should not be failure, when components were not failure under the action of the maximum load \( S_{max} \) of \( n \) parts. It can be represented as follows,

\[ P(t) = P[r > \sigma(S_{max})] = P[r > \sigma(S_1), r > \sigma(S_2), \cdots, r > \sigma(S_n)] \]  

Therefore, the reliability after the effect of \( n \) times of random load is equivalent to the corresponding reliability of maximum load value of \( n \) load samples. The maximum load in the load samples can be defined as the equivalent load of \( n \) times’ load effect. According to order statistic theory, the maximum load \( S_{max} \) actually is the maximum order statistic variable \( S_n \) which is defined by the load sample \( S_1, S_2, \ldots, S_n \).

\[ F_i(x) = P(\max S_i \leq x) = P(S_1 \leq x)P(S_2 \leq x) \cdots P(S_n \leq x) = \prod_{i=1}^{n} P(S_i \leq x) = \left[F_i(x)\right]^n \]  

Thus the probability model of stochastic process of load is set up.

The formula of reliability under the action of the stochastic process of load is given as

\[ R(t) = P[r > \max \sigma(t)] \quad t \in [t_i, t_i + \Delta t] \]  

3. Time-dependent performance of stress
At present, there are two kinds of gradual change model studying strength degeneration of mechanical components basically, the first model, directly converting strength to random variable of each period, substituting section random variable at one moment for stochastic process, which cannot reflect the correlation and other random characteristics among every random variable of strength at each moment; the second model, to show the strength degeneration process by some certain function, to transform the non-stationary stochastic process to stationary stochastic process. Due to the extreme complexity of the strength degeneration process, it is difficult to use certain mathematical model to express its degeneration function, and some information of the given components can’t be used fully.

Because the great uncertainty of correlation among each strength at any time exists, in this paper, using independent increment principle of stochastic process, we view correlation among each strength as independent increment of stochastic process, and make corrections to the original stochastic process model of strength by the measured value at current time, thus the stochastic process model of strength degeneration for components in future using time is formed. Then suppose stochastic process of strength degeneration to be \( \{ r(t), t \in [t_i, t_i + \Delta t] \} \), the strength \( r(t_i) \) is a random variable at the present
moment \( t_i \), its mean value is \( E[r(t_i)] \), variance is \( D[r(t_i)] \). The stochastic process of strength degeneration in design is \( \{r(t_i), t \in [t_i, t_i + \Delta t]\} \), and its mean value function is \( E[r_0(t)] \), variance function is \( D[r_0(t)] \). \( E[r(t_i)] \) is equal to \( E[r_0(t_i)] \), \( D[r(t_i)] \) is equal to \( D[r_0(t_i)] \) at current moment \( t_i \) in the meaning of statistics and theory. Then the stochastic process of strength is defined as
\[
r(t) = r(t_i) + [r_0(t_i) - r_0(t_i) = 0]
\]
(6)

The mean value function and the variance function are given as follows
\[
E[r(t)] = E[r(t_i)] + E[r_0(t)] - E[r_0(t_i)]
\]
(7)
\[
D[r(t)] = D[r(t_i)] + D[r_0(t_i)] - 2 \text{cov}[r_0(t), r_0(t_i)]
\]
(8)

where \( \text{cov}[r_0(t), r_0(t_i)] \) is the covariance function of strength degeneration. If \( \{r(t), t \in [t_i, t_i + \Delta t]\} \) is independent increment process, the autocorrelation coefficient of strength is
\[
\rho_r[\{r(t_i), r(t_i + \Delta t)\}] = \frac{\text{cov}[r(t_i), r(t_i + \Delta t)]}{\sqrt{D[r(t_i)]D[r(t_i + \Delta t)]}}
\]
(9)

According to formula (2), the structural reliability of components is expressed by function equation as follows,
\[
R(t) = P[g(t) > 0, t \in [0, T)]
\]
(12)

The formula (11) is to show that when \( r(t) \) of every time \( t \) in design service period is greater than \( \sigma(t) \), the component structure is in the reliable state. The probability of failure is
\[
F(t) = 1 - R(t) = P[g(t) < \sigma(t), t \in [0, T)]
\]
(13)

Considering measured value of components at current time \( t_i \), reliability in the mechanical parts' future service period \( [t_i, t_i + \Delta t] \) can be given as
\[
R(t) = P[g(t_i) > \sigma(t), t_i \in [t_i, t_i + \Delta t)]
\]
(14)

where \( g(t_i) \) is the strength of any time \( t_i \) for components, \( \Delta t \) is the effect of random load of any time \( t_i \). According to formula (14), a component is equal to a series system made up by \( n \) samples in the whole design service period. From the reliability definition of the series system, if to make the whole system effective, every subsystem is effective too. And the time-dependent reliability is
\[
R(t) = P\left\{ \bigcap_{i=1}^n g(t_i) > \sigma(t), t_i \in [t_i, t_i + \Delta t]\right\}
\]
(15)

On the basis of the independent increment process, correlation coefficient of the state function is
\[
\rho_x[g(t_i), g(t_j)] = \frac{\text{cov}[g(t_i), g(t_j)]}{\sqrt{D[g(t_i)]D[g(t_j)]}} = \frac{\rho_r[r(t_i), r(t_j)]}{\sqrt{D[r(t_i)]D[r(t_j)]}} = \frac{\rho_r[r(t_i), r(t_j)]}{\sqrt{D[r(t_i)]D[r(t_j)]} + D[\sigma(t_i)] + D[\sigma(t_j)]}
\]
(16)

4. Design on gradual change reliability sensitivity

4.1. Mechanical Model of a Land Axle

Spindle force distributing diagrammatic is shown in figure 1.

**Figure 1.** Force analysis of the spindle.

After the analysis of the spindle, it can be known that the closer spindle to the symmetric line, the greater bending moment produced by gravity, and as a result
\[ G_1 = G_2 = \frac{G}{2}, \quad N_1 = N_2 = \frac{G}{2} \]  

(17) \[ M_{\text{max}} = M_a = M_h = G_1L \]  

(18)

4.2. Computation of the model

After being detected a spindle of a certain type of spindle system, which have worked 500 hours, the mean value function and variance function of its strength are respectively \( E[r(t)] \) and \( D[r(t)] \). \( E[r(t)]=2.6e^{-0.00003t}, D[r(t)]=5.3\exp(-2\times10^{-9}t^2) \). The first two moments of diameters are \( d_1 \) and \( d_2 \) of the stepped shaft, \( d_1=(10.24, 0.087)\text{mm}, \quad d_2=(7.92, 0.06)\text{mm} \) and \( L=(128, 0.18)\text{mm} \). The measured load \( G \) with the first two moments at the current moment is \( G=(310.88\times10^3 \text{N}, 3.5\times10^3 \text{N})\text{Nmm} \), and the maximum equivalent load effect of load stochastic process obeys extremum distribution. Suppose the strength of the land axle obeys exponential distribution at the moment \( t_i \), to calculate the reliability and gradual change reliability sensitivity of \( d_1, d_2 \) and \( L \) within next 5000 hours.

From figure 2, it is accord with the actual working condition that structural reliability \( R \) of the spindle system is gradually reduced with the increase of service time. Figure 3 to 5 show changes of sensitivity of parameters \( d_1, d_2 \) and \( L \), it can be seen that reliability \( R \) with respect to sensitivity of \( d_1 \) is larger than that of \( d_2 \) and \( L \) at any time, thus \( d_1 \) is the most sensitive. Mean value of \( d_1 \) and \( d_2 \) is the larger, and the spindle system is more reliable. Sensitivity of \( L \) is negative value, which shows the spindle system is unreliable, that is improving failure rate, with the increase of mean value of \( L \).
5. Conclusions
(1) Based on practical engineering and the actual effect of the load, derived from the theory of the stress-strength interference and independent increment process model of strength, the computing method of gradual change reliability for mechanical components which is on the basis of the fully stochastic process is established, and the change rule of reliability with time is also given in this paper.
(2) On the basis of gradual change reliability consideration for a spindle system, a numerical method of gradual change reliability sensitivity is proposed. It reflects the sensitive degree of each structural parameter to reliability of the whole mechanical structure within different use period.
(3) Owing to the consideration of measured information and time response of strength and load for components, it is closer to the reliability problem in practical engineering, meanwhile, the complexity for computing higher order integrals of series system is avoided.

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