The respond of the electric and magnetic fields of quark gluon plasma in heavy ion collisions

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Abstract

In this work, we investigate the electromagnetic respond of the quark gluon plasma during the motion of a resistive fluid in the presence of coupled transverse electric and magnetic fields. We compute the analytic formula for the electric and magnetic fields in the conducting fluid with the finite electric conductivity \( \sigma \). The exact algebraic expressions for both electric and magnetic fields are obtained. Moreover, the space-time profiles of electromagnetic fields and energy density are calculated based on analytic expressions. We find that the energy density evolution of the mentioned fluid depends on space time rapidity \( \eta \) as well as proper time \( \tau \). We also estimate the transverse momentum spectrum emerging from the magneto-hydrodynamic solutions.

Keyword: Magneto-hydrodynamic, Quark Gluon Plasma

1 Introduction

It has been known that in the relativistic heavy-ions collisions is expected to be formed a hot and dense fireball matter which is called the deconfined quark gluon plasma (QGP). Also during these collisions, extremely strong electromagnetic fields generate due to the relativistic motion of the colliding heavy ions carrying large positive electric charge. It has been shown that in a typical non-central collisions such as Pb-Pb at the center of mass energy \( \sqrt{s} = 2.76\text{TeV} \) and Au-Au at the center of mass energy \( \sqrt{s} = 200\text{GeV} \) an intense magnetic field is created of the order \( 10^{18} - 10^{19}\text{G} \), which is corresponds to \( |eB| \simeq 1\text{GeV}^2 \) and \( 10^{13} \) times larger than the strongest steady magnetic field which is find in the laboratory [1–6].

It has been argued that the existence of these strong fields is important for a wide variety of new phenomena such as Chiral Magnetic Effect (CME), Chiral Magnetic wave (CMW), Chiral Electric Separation Effect (CESE), Chiral Hall Separation Effect (CHSE), pressure anisotropy in QGP, influence on the direct and elliptic flow and shift of the critical temperature. Some references and review articles can be found in Refs.[7–16].

Studies have shown the electromagnetic fields are very strong almost at the time of collision and decrease fast in very short time after the collision [5, 17]. It has been claimed that the magnetic field drops by two to three orders of magnitude in about 0.5 fm/c from the collision time. The authors have shown the strong electric field
remains in about 0.25 fm/c after collisions and maybe acts as an accelerator on charge particles. Therefore, it should not have the sizeable effects on the final state hadrons in late time. However the medium effects have not be considered in these studies. The main respond of the plasma to the fields is the electrical conductivity $\sigma$. In fact Ohms currents will be induced in the plasma and slow down the decrease of the fields. Thus, the presence of a medium with finite electrical conductivity could substantially delay the decay of the electromagnetic field [3, 18].

There are some studies on the space-time evolution of electromagnetic fields created by the colliding charged beams moving at relativistic speed in z-direction, as a solution of the magneto-hydrodynamic equations [3, 5, 17–21]. Also a series of primary results is obtained by estimating the significance of strong EM fields on the QGP medium [22–25]. Almost in this works, the evolution of the EM fields and their influence on the flow coefficients have been studied by considering decoupled Maxwell equations from the time evolution of the QGP. Thus, it has been shown that the electromagnetic fields depend on the impact parameter of the colliding nucleons $b$, the center of mass energy $\sqrt{s}$ and the electric and chiral magnetic conductivities of the QGP; besides, its decrease with time is much slower than in vacuum. In addition, it turns out that the electromagnetic field has no influence on the evolution of the velocity of the fluid and vice versa. It is clear that a perfect description of the hot plasma in the presence of EM fields can be applied by complicated relativistic magneto-hydrodynamic equations. Thus, one needs a numerical code that solves the equations of (1+3) dimensional relativistic magneto-hydrodynamics (RMHD) [26–31].

In this work we investigate the respond of the QGP with the constant electrical conductivity to the electromagnetic fields in heavy ion collisions. We study the respond of resistive fluid with finite electrical conductivity $\sigma$ to the presence of coupled transverse electric and magnetic fields analytically. Here, we consider the combination of relativistic hydrodynamic equations with Maxwell equations and solve in (1+1) dimensions a set of coupled RMHD equations. Based on previous studies we assume a magnetic field produces in y-direction (y-axis is perpendicular to the reaction plane), and a electric field creates in reaction plane in x-direction. We consider QGP medium has been created a short time after collisions with the Bjorken longitudinal expansion.

Outlines of this paper are as follows: In the next section we introduce the resistive relativistic magneto-hydrodynamic (RRMHD) equations in their most general form, considering them in the case of a plasma with finite electrical conductivity. The results and the properties of EM fields are shown in Section 3. Finally, the last section is devoted to some concluding remarks.

2 Resistive relativistic magneto-hydrodynamic

In order to describe the interaction of matter and electromagnetic fields in quark gluon plasma, we consider the relativistic magneto-hydrodynamic (RMHD) framework [26, 32, 33]. For the sake of simplicity, we assume an ideal relativistic plasma with massless particles and finite electrical conductivity ($\sigma$). In addition, the fluid is considered to be ultra-relativistic, thus implying that the rest mass contributions to the equation of state (EOS) have been neglected, and the pressure is simply proportional to the energy density: $P = \kappa \epsilon$ where $\kappa$ is constant. For an ideal fluid with finite electrical conductivity which is called resistive fluid, the equations of RMHD
can be written in the form of the covariant conservation laws:
\[ d_\mu T_{\mu \text{matter}}^{\nu} = -J_\lambda F^{\lambda \nu} \]  
\[ d_\mu F^{\mu \nu} = 0 \]  
\[ d_\mu F^{\nu \mu} = -J^{\nu}, \quad d_\mu F^{\mu} = 0 \]

where energy momentum tensor for the fluid is:
\[ T_{\mu \text{matter}}^{\nu} = (e + P)u^\mu u^\nu + Pg^{\mu \nu}, \quad p = \kappa \epsilon \]
\[ u^\mu, e \text{ and } P \text{ are fluid four velocity, energy density and pressure respectively.} \]

The tensors of electromagnetic field and current density are given by:
\[ F^{\mu \nu} = u^\mu b^\nu - u^\nu b^\mu + \epsilon^{\mu \nu \lambda \kappa} b_\lambda u_\kappa \]  
\[ F^{\ast \mu \nu} = u^\mu b^\nu - u^\nu b^\mu - \epsilon^{\mu \nu \lambda \kappa} b_\lambda u_\kappa \]  
\[ J^\mu = \sigma e^\mu \]

where \( d_\mu \) is covariant derivative and \( \epsilon^{\mu \nu \lambda \kappa} = -(-g)^{-1/2}[\mu \nu \lambda \kappa] \) is the space time Levi-Civita tensor density \( \epsilon^{\mu \nu \lambda \kappa} = (-g)^{-1/2}[\mu \nu \lambda \kappa] \) with \( g = \det(g_{\mu \nu}) \) and \([\mu \nu \lambda \kappa] \) is the alternating Levi-Civita symbol. Besides:
\[ e^\mu = F^{\ast \mu \nu} u_\nu, \quad b^\mu = F^{\mu \nu} u_\nu, \quad (e^\mu u_\mu = b^\mu u_\mu = 0) \]

\( e^\mu \) and \( b^\mu \) are electric and magnetic field four vector in the local rest frame of the fluid, which is related to the one measured in the lab frame. Moreover, the single fluid four velocity \( u_\mu (u_\mu u^\mu = -1) \) is given by:
\[ u^\mu = \gamma(1, \vec{v}) \gamma = \frac{1}{\sqrt{1 - v^2}} \]

In Eqs. [7] to [9] the covariant derivatives are given by:
\[ d_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\nu_{\mu \lambda} A^\lambda \]  
\[ d_\mu A^{\ast \mu \nu} = \partial_\mu A^{\ast \mu \nu} + \Gamma^\nu_{\mu \lambda} A^{\ast \mu \lambda} + \Gamma^\mu_{\mu \lambda} A^{\ast \mu \lambda} \]

where \( \Gamma^i_{jk} \) are the Christoffel symbols:
\[ \Gamma^i_{jk} = \frac{1}{2} g^{im} \left( \frac{\partial g_{mj}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^m} \right) \]

It is more convenient to work with Milne coordinates rather than the standard Cartesian coordinates for a longitudinally boost invariant flow:
\[ (\tau, x, y, \eta) = \left( \sqrt{\frac{1}{2} - z^2}, x, y, \frac{1}{2} \ln \frac{1 + z}{1 - z} \right) \]

Here, the metric is given by:
\[ g^{\mu \nu} = \text{diag}(-1, 1, 1, 1/\tau^2), \quad g_{\mu \nu} = \text{diag}(-1, 1, 1, \tau^2) \]

Working in Milne coordinates, one can easily obtain the Christoffel symbols: the only non-zero ones being:
\[ \Gamma^r_{\eta \eta} = \tau \text{ and } \Gamma^\eta_{\tau \eta} = 1/\tau. \]

By implementing the projection of \( d_\mu T_{\mu \text{matter}}^{\nu} = -J_\lambda F^{\lambda \nu} \) along the longitudinal and transverse direction with respect to \( u^\mu \), one can rewrite the conservation equations as:
\[ u_\nu (d_\mu T_{\mu \text{matter}}^{\nu}) = -J_\lambda F^{\lambda \nu} \rightarrow D_\epsilon + (\epsilon + P)\Theta = e^\lambda J_\lambda \]  
\[ \Delta_{\alpha \nu} (d_\mu T_{\mu \text{matter}}^{\nu}) = -J_\lambda F^{\lambda \nu} \rightarrow (\epsilon + P) D_{\alpha} u_\alpha + \nabla_{\alpha} P = g_{\alpha \nu} F^{\mu \nu} J_\lambda - u_\alpha e^\lambda J_\lambda \]
where:

\[ D = u^\mu d_\mu, \quad \Theta = d_\mu u^\mu, \quad \nabla^\mu = d^\mu + u^\mu D \quad \Delta^\nu_\mu = g^\nu_\nu + u^\nu u_\nu \quad (16) \]

In contrast with the energy momentum tensor \( T^{\mu \nu} \), the dual electromagnetic tensor \( F^{*\mu \nu} \) is antisymmetric; hence the homogeneous Maxwell equation, \( d_\mu F^{*\mu \nu} = 0 \), leads to the following equations:

\[ \partial_x F^{*xz} + \partial_y F^{*yz} + \partial_\eta F^{*\eta z} = 0 \quad (17) \]
\[ \partial_\tau F^{*\tau x} + \partial_y F^{*yx} + \partial_\eta F^{*\eta y} + \frac{1}{\tau} F_{\tau x} = 0 \quad (18) \]
\[ \partial_\tau F^{*\tau y} + \partial_x F^{*xy} + \partial_\eta F^{*\eta y} + \frac{1}{\tau} F_{\tau y} = 0 \quad (19) \]
\[ \partial_\tau F^{*\tau \eta} + \partial_x F^{*x\eta} + \partial_y F^{*y\eta} + \frac{1}{\tau} F_{\tau \eta} = 0 \quad (20) \]

In the same way, the inhomogeneous Maxwell equation \( d_\mu F^{\mu \nu} = -J^\nu \) are given by:

\[ \partial_x F^{xz} + \partial_y F^{yz} + \partial_\eta F^{\eta z} = -J^x \quad (21) \]
\[ \partial_\tau F^{\tau x} + \partial_y F^{yx} + \partial_\eta F^{\eta x} + \frac{1}{\tau} F_{\tau x} = -J^z \quad (22) \]
\[ \partial_\tau F^{\tau y} + \partial_x F^{xy} + \partial_\eta F^{\eta y} + \frac{1}{\tau} F_{\tau y} = -J^y \quad (23) \]
\[ \partial_\tau F^{\tau \eta} + \partial_x F^{x\eta} + \partial_y F^{y\eta} + \frac{1}{\tau} F_{\tau \eta} = -J^\eta \quad (24) \]

Indeed the non-central collisions can create strong out of reaction plane magnetic fields and in plane electric fields [34, 35]. The magnetic field in non-central collisions is dominated by \( y \) component which induces a Faraday current in \( xz \) plane; then, the Lorentz force is directed along \( x \) direction. In particular, we are here interested in obtaining analytic solutions representing the RMHD extension of one-dimensional Bjorken flow along the \( z \)-direction with velocity \( u^\mu = \gamma (1, 0, 0, z/t) \). We assume that electric field is oriented in \( x \) direction and the magnetic field is perpendicular to the reaction plane, pointing along the \( y \) direction in an inviscid fluid with finite electrical conductivity, following the Bjorken expansion along the \( z \) direction. Then we can make the following setup:

\[ u^\mu = (1, 0, 0, 0), \quad b^\mu = (0, 0, b_y, 0), \quad e^\mu = (0, e_x, 0, 0) \quad (25) \]

where \( u^\mu e_\mu = u^\mu b_\mu = 0, \quad u^\mu u_\mu = -1 \) are satisfied.

With the initial conditions in our hand, for conservations energy Eqs. [14]-[15] we can write:

\[ \partial_\tau e + \frac{(1 + \kappa) e}{\tau} = \sigma e_x^2 \quad (26) \]
\[ \partial^x P = 0 \quad (27) \]
\[ \partial^y P = -\frac{1}{\tau} \sigma e_x b_y \quad (28) \]

Using all the above assumptions the set of Eqs. [14]-[15] and Maxwell Eqs. [17]-
reduce to:

\[ \partial_x \epsilon + \frac{(1 + \kappa)\epsilon}{\tau} = \sigma \epsilon_x^2 \]  
(29)

\[ \frac{\kappa}{\tau^2} \partial_y \epsilon = -\frac{1}{\tau} \sigma \epsilon_x b_y \]  
(30)

\[ \partial_x P = 0 \]  
(31)

\[ \partial_y P = 0 \]  
(32)

\[ \partial_x e_x = \partial_y e_x = \partial_x b_y = \partial_y b_y = 0 \]  
(33)

\[ \partial_x b_y - \frac{1}{\tau} \partial_y e_x + \frac{b_y}{\tau} = 0 \]  
(34)

\[ \partial_x e_x - \frac{1}{\tau} \partial_y b_y + \frac{e_x}{\tau} + \sigma e_x = 0 \]  
(35)

The combination of the two last equations are given the following equations:

\[ -\partial_y^2 e_x + \tau^2 \partial_x^2 e_x + (3\tau + \sigma^2) \partial_x e_x + (1 + 2\sigma \tau)e_x = 0 \]  
(36)

\[ -\partial_y^2 b_y + \tau^2 \partial_x^2 b_y + (3\tau + \sigma^2) \partial_x b_y + (1 + \sigma \tau)b_y = 0 \]  
(37)

In order to solve the above equations, one could consider the electric and magnetic fields as:

\[ e_x(\tau, \eta) = g(\eta)f(\tau) \]  
(38)

\[ b_y(\tau, \eta) = s(\eta)h(\tau) \]  
(39)

Then, one gets:

\[ \tau^2 \frac{d^2 f(\tau)}{d\tau^2} + (3\tau + \sigma^2) \frac{df(\tau)}{d\tau} + (-m^2 + 1 + 2\sigma \tau)f(\tau) = 0 \]  
(40)

\[ \frac{d^2 g(\eta)}{d\eta^2} = m^2 g(\eta) \]  
(41)

\[ \tau^2 \frac{d^2 h(\tau)}{d\tau^2} + (3\tau + \sigma^2) \frac{dh(\tau)}{d\tau} + (-n^2 + 1 + \sigma \tau)h(\tau) = 0 \]  
(42)

\[ \frac{d^2 s(\eta)}{d\eta^2} = n^2 s(\eta) \]  
(43)

First of all, we will discuss about the behavior of \( \eta \) dependent functions \( g \) and \( s \). If we substitute \( e_x = g(\eta)f(\tau) \) and \( b_y = s(\eta)h(\tau) \) into Eq. (34), we have:

\[ \tau s(\eta)h'(\tau) - g'(\eta)f(\tau) + s(\eta)h(\tau) = 0 \]  
(44)

Multiplying by \( \frac{1}{s(\eta)f(\tau)} \) we get:

\[ \tau \frac{h'(\tau)}{f(\tau)} + \frac{h(\tau)}{f(\tau)} = \frac{g'(\eta)}{s(\eta)} = C \]  
(45)

which implies that:

\[ \tau h'(\tau) + h(\tau) = Cf(\tau) \]  
(46)

\[ g'(\eta) = Cg(\eta) \]  
(47)

We next substitute \( e_x = g(\eta)f(\tau) \) and \( b_y = s(\eta)h(\tau) \) into Eq. (35):

\[ \tau f'(\tau) + (1 + \sigma \tau)f(\tau) - C' h(\tau) = 0 \]  
(48)

\[ s'(\eta) - C' g(\eta) = 0 \]  
(49)
Finally one obtain:

\[ g'(\eta) = C s'(\eta) = CC'g(\eta), \quad (50) \]

\[ s''(\eta) = C'g'(\eta) = C'Cs(\eta) \quad (51) \]

Thus, separation constants are \( CC' = m^2 = n^2 \). For further application, we choose \( m^2 = l(l+2) + 1 \).

Then, the solution of (50) is given by:

\[ g_m(\eta) = A \cosh(m\eta) + B \sinh(m\eta) \quad (52) \]

and \( s(\eta) \) is:

\[ s_m(\eta) = \frac{Am}{C} \sinh(m\eta) + \frac{Bm}{C} \cosh(m\eta) = \frac{AC'}{m} \sinh(m\eta) + \frac{BC'}{m} \cosh(m\eta) \quad (53) \]

From the symmetry, it realizes that electric field in our setup to be rapidity-odd and in the large rapidity the electric field goes to zero. Besides, Considering \( b_y \) generated by two nuclei passing each other in heavy ion collisions, which should be an even function in \( \eta \) and most dominant at large rapidity after collisions [31], we may write:

\[ s(\eta) = b_m \cosh(m\eta) \rightarrow g(\eta) = e_m \sinh(m\eta) \quad (54) \]

Let us to solve Eq. (40). The equation is given by:

\[ \tau^2 \frac{d^2 f(\tau)}{d\tau^2} + (3\tau + \sigma\tau^2) \frac{df(\tau)}{d\tau} + (-l(l+2) + 2\sigma\tau)f(\tau) = 0 \quad (55) \]

Then, we consider \( f(\tau) = \tau^l G(\tau) = \tau^l \sum_{n=0}^{\infty} a_n \tau^n \) is power series function. When it is substituted into above differential equation, we find \( G(\tau) \) obeys by:

\[ \frac{d^2 G(\tau)}{d\tau^2} + \left( \sigma + \frac{2l+3}{\tau} \right) \frac{dG(\tau)}{d\tau} + \sigma \frac{(l+2)}{\tau} G(\tau) = 0 \quad (56) \]

When \( G(\tau) = \sum a_n \tau^n \) is substituted into the differential equation, one obtains a relation between various coefficients. It is easy to get the recursion relation:

\[ \frac{a_{n+1}}{a_n} = -\sigma \frac{n + l + 2}{n(n+1) + (2l+3)(n+1)} \quad (57) \]

We should assume the series terminates because a solution \( f(\tau) \) should be well behaved at infinity. This means for some \( n \) we must have:

\[ n + l + 2 = 0 \rightarrow n = -l - 2 \quad \text{for} \quad n = 0, 1, 2, ... \rightarrow l = -2, -3, ... \quad (58) \]

Then, we have chosen \( m^2 = l(l+2) + 1 \), so for \( l = -2, -3, ... m^2 = 1, 2, 3, ... \) This means that \( m \) is a integer (we consider \( m \) be positive).

According to the Eq. (56), the analytical solution of \( G(\tau) \) is given by:

\[ G(\tau) = C_1 e^{-\frac{1}{2} \sigma \tau - l - \frac{1}{2} \tau^2} \left( \sigma \tau I_{\frac{1}{2}+l}(\frac{1}{2}\sigma \tau) + (\sigma \tau + 4l+2)I_{\frac{1}{2}+l}(\frac{1}{2}\sigma \tau) \right) + 
C_2 e^{-\frac{1}{2} \sigma \tau - l - \frac{1}{2} \tau^2} \left( -\sigma \tau K_{\frac{1}{2}+l}(\frac{1}{2}\sigma \tau) + (\sigma \tau + 4l+2)K_{\frac{1}{2}+l}(\frac{1}{2}\sigma \tau) \right) \quad (59) \]
where \( I_\alpha \) and \( K_\alpha \) are the modified Bessel functions. The constant coefficients \( C_1 \) and \( C_2 \) can be determined by assuming a proper profile for \( G(\tau) \) and applying the orthogonal properties of Bessel functions. Equivalently, the solution of \( G(\tau) \) for each \( l \) can be written as following:

\[
\begin{align*}
l & = -2 \quad \rightarrow \quad G_1(\tau) = C_{11} - C_{12}e^{-\sigma\tau}(\sigma\tau + 1) \\
l & = -3 \quad \rightarrow \quad G_2(\tau) = C_{21}(3 - \sigma\tau) + C_{22}e^{-\sigma\tau}(\sigma^2\tau^2 + 4\sigma\tau + 6) \\
l & = -4 \quad \rightarrow \quad G_3(\tau) = C_{31}(20 - 8\sigma\tau - 8\sigma^2\tau^2) - C_{32}e^{-\sigma\tau}(\sigma^3\tau^3 + 9\sigma^2\tau^2 + 36\sigma\tau + 60) \\
, & \quad \text{...}
\end{align*}
\]

Then, \( f_m(\tau) \) is given by:

\[
\begin{align*}
f_1(\tau) &= \frac{G_1(\tau)}{\tau^2} = C_{11} \frac{1}{\tau^2} - C_{12} \frac{e^{-\sigma\tau}(\sigma\tau + 1)}{\tau^2} \\
f_2(\tau) &= \frac{G_2(\tau)}{\tau^3} = C_{21} \frac{3 - \sigma\tau}{\tau^3} + C_{22} \frac{e^{-\sigma\tau}(\sigma^2\tau^2 + 4\sigma\tau + 6)}{\tau^3} \\
f_3(\tau) &= \frac{G_3(\tau)}{\tau^4} = C_{31} \frac{(20 - 8\sigma\tau - 8\sigma^2\tau^2)}{\tau^4} - C_{32} \frac{e^{-\sigma\tau}(\sigma^3\tau^3 + 9\sigma^2\tau^2 + 36\sigma\tau + 60)}{\tau^4} \\
, & \quad \text{...}
\end{align*}
\]

Besides, we know if \( \sigma \to \infty \), then \( e_2 \) should go to zero. Therefore, we have to set \( C_{1i} = 0, i = 1, 2, 3, \ldots \). Finally, the most general solution for the electric field is:

\[
\begin{align*}
e_x(\tau, \eta) &= e_1 \sinh(\eta) \frac{e^{-\sigma\tau}(1 + \sigma\tau)}{\tau^2} + e_2 \sinh(2\eta) \frac{e^{-\sigma\tau}(6 + 4\sigma\tau + \sigma^2\tau^2)}{\tau^3} + \\
e_3 \sinh(3\eta) \frac{e^{-\sigma\tau}(60 + 36\sigma\tau + 9\sigma^2\tau^2 + \sigma^3\tau^3)}{\tau^4} + \ldots
\end{align*}
\]

where \( e_m \) are constants and should be determined.

To find the time dependence of the magnetic field, one can substitute the \( e_x(\tau, \eta) \) from the above equation in to Eq. (35) or solve Eq. (37). Then, the magnetic field is given by:

\[
\begin{align*}
b_y(\tau, \eta) &= F_1(\tau) + e_1 \cosh(\eta) \frac{e^{-\sigma\tau}}{\tau^2} + e_2 \cosh(2\eta) \frac{e^{-\sigma\tau}(6 + 2\sigma\tau)}{\tau^3} + \\
& \quad e_3 \cosh(3\eta) \frac{e^{-\sigma\tau}(60 + 24\sigma\tau + 36\sigma^2\tau^2)}{\tau^4} + \ldots
\end{align*}
\]

We also know if \( \sigma \to \infty \) then \( b_y \) should be go to \( \frac{1}{\tau^3} \). Therefore, we have \( F_1(\tau) = b_0 \frac{\tau^3}{\tau^3} \). On the other hand, it is expected in vacuum \(( \sigma \to 0 \) the magnetic field decays as \( \frac{1}{\tau^4} \):

\[
b_y(\tau, 0) \approx b_0 \frac{\tau^3}{\tau^3}
\]

By using the above equation, one can deduce that all coefficients of \( e_1 \) should be zero except \( e_2 \). Consequently, the electric field and magnetic field are given by:

\[
\begin{align*}
e_x(\tau, \eta) &= b_1 \frac{\tau^3}{\tau^3} e^{-\sigma\tau} \sinh(2\eta)(6 + 4\sigma\tau + \sigma^2\tau^2) \\
b_y(\tau, \eta) &= b_0 \frac{\tau^3}{\tau^3} \left(1 - e^{-\sigma\tau}\right) + b_1 \frac{\tau^3}{\tau^3} e^{-\sigma\tau} \cosh(2\eta)(6 + 2\sigma\tau)
\end{align*}
\]
In order to obtain the energy density $\epsilon$, one should solve the Eqs. (29) and (30) in consistent way. Using Eq. (29) one can find:

$$\epsilon(\tau, \eta) = \frac{K(\eta)}{r_{1+\kappa}} + \frac{\sigma}{r_{1+\kappa}} \int_{0}^{\tau} d\tau' \, \tau'^{1+\kappa} \, e_{x}^{2}(\tau', \eta)$$  \hspace{1cm} (66)

Besides, using Eq. (30) one obtains:

$$\epsilon(\tau, \eta) = -\frac{\sigma}{k} \int_{-\eta}^{\eta} d\eta' \, e_{x}(\tau, \eta') b_{y}(\tau, \eta') + L(\tau)$$  \hspace{1cm} (67)

The first term in the r.h.s of the Eq. (68) is canceled because the integrated function is a rapidity-odd function. In order to determine the function $K(\eta)$ one has to substitute the Eq. (66) in (67) then:

$$K(\eta) = \tau_{0}^{1+\kappa} L(\tau_{0})$$  \hspace{1cm} (69)

Finally, $\epsilon$ is:

$$\epsilon(\tau, \eta) = \epsilon_{0}(\frac{\tau_{0}}{\tau})^{1+\kappa} + \frac{\sigma}{r_{1+\kappa}} \left(b_{1} \tau_{0}^{3} \sinh(2\eta)\right)^{2} \int_{\tau_{0}}^{\tau} d\tau' \, \tau'^{\kappa-5} e^{-2\sigma\tau'}(6 + 4\sigma\tau' + \sigma^{2}\tau'^{2})^{2}$$  \hspace{1cm} (70)

In order to evaluate our results, we consider the ideal RMHD, i.e. electrical conductivity is infinite ($\sigma \to \infty$). In this case, we have:

$$\epsilon(\tau) = \epsilon_{0}(\frac{\tau_{0}}{\tau})^{1+\kappa}, \quad b_{y}(\tau) = b_{0}(\frac{\tau_{0}}{\tau}), \quad e_{x} = 0.$$  \hspace{1cm} (71)

which is consistent with previous studies.

3 Results and discussions

We shall now investigate the space-time evolution of EM fields which are given by Eq. (65) and energy density Eq. (70). Besides, the transverse momentum spectrum and velocity of fluid can be studied and results can be compared with experimental data. We estimate the typical magnetic field produced in heavy ion collisions about $|eB| \approx 1$GeV$^{2}$ at a proper time $\tau_{0} = 0.5 fm$ after collision and suppose there is a simple relation between the fixed coefficients $b_{0}, b_{1}$ as ratio $b_{1} b_{0} = \alpha$ and this constant can be discussed in different ranges.

Because, the magnetic field is an even function and the electric field is an odd function in $\eta$, the electric field is zero in the plane of $\eta = 0$. We only consider the evolution of magnetic field at $\eta = 0$. Fig. 1 shows the time evolution of magnetic field for different values of $\sigma$ at $\eta = 0$. The value of $eb_{y}$ is plotted as a function of $\tau$ for different value of $\alpha$ at $\eta = 0$. Figures indicate the presence of a conducting medium delays the decay of the magnetic field; however, the $\alpha$ parameter which comes from initial conditions has big effects on the delay of $b_{y}$.

We see for the small value of $\alpha$ the magnetic field decays slowly specially for $0 < \sigma < 1$ (Fig. 1(a)). The Fig. 1(c) shows when $\alpha \simeq 1$ the delay of the magnetic field
\( \alpha = 0.01 \)
\( \sigma = 0 \text{ fm}^{-1} \)
\( \sigma = 1 \text{ fm}^{-1} \)
\( \sigma = 5 \text{ fm}^{-1} \)
\( \sigma = 10 \text{ fm}^{-1} \)
\( 0.5 \ 1.0 \ 1.5 \ 2.0 \ 2.5 \ 3.0 \)
\( 0.0 \)
\( 0.2 \)
\( 0.4 \)
\( 0.6 \)
\( 0.8 \)
\( 1.0 \)
\( \tau \text{ [fm]} \)

by \((\tau,0)\text{ [GeV}^2/e]\)

\( \alpha = 0.1 \)
\( \sigma = 0 \text{ fm}^{-1} \)
\( \sigma = 1 \text{ fm}^{-1} \)
\( \sigma = 5 \text{ fm}^{-1} \)
\( \sigma = 10 \text{ fm}^{-1} \)
\( 0.5 \ 1.0 \ 1.5 \ 2.0 \ 2.5 \ 3.0 \)
\( 0.0 \)
\( 0.1 \)
\( 0.2 \)
\( 0.3 \)
\( 0.4 \)
\( \tau \text{ [fm]} \)

by \((\tau,0)\text{ [GeV}^2/e]\)

\( \alpha = 2 \)
\( \sigma = 0 \text{ fm}^{-1} \)
\( \sigma = 1 \text{ fm}^{-1} \)
\( \sigma = 5 \text{ fm}^{-1} \)
\( \sigma = 10 \text{ fm}^{-1} \)
\( 0.5 \ 1.0 \ 1.5 \ 2.0 \ 2.5 \ 3.0 \)
\( 0.0 \)
\( 0.1 \)
\( 0.2 \)
\( 0.3 \)
\( 0.4 \)
\( \tau \text{ [fm]} \)

by \((\tau,0)\text{ [GeV}^2/e]\)

\( \alpha = 10 \)
\( \sigma = 0 \text{ fm}^{-1} \)
\( \sigma = 1 \text{ fm}^{-1} \)
\( \sigma = 5 \text{ fm}^{-1} \)
\( \sigma = 10 \text{ fm}^{-1} \)
\( 0.5 \ 1.0 \ 1.5 \ 2.0 \ 2.5 \ 3.0 \)
\( 0.00 \)
\( 0.05 \)
\( 0.10 \)
\( 0.15 \)
\( 0.20 \)
\( \tau \text{ [fm]} \)

by \((\tau,0)\text{ [GeV}^2/e]\)

Figure 1: The magnetic field versus \( \tau \) by considering different constant \( \alpha \) at \( \eta = 0 \) with different values of \( \sigma \).

happens for the larger value of \( \sigma \). On the other hand, for \( \alpha \simeq 10 \), one can ignore the first term of Eq. (65), and the magnetic field decays like \( \frac{1}{\tau^3} \) (Fig. 1(d)).

In order to investigate the effects of the medium on the evaluation of the electric and magnetic fields, the energy density one should consider \( \eta \neq 0 \). In Fig. 2, the profiles of the magnetic field versus \( \eta \) is illustrated by considering the different values of \( \sigma \) at initial proper time \( \tau = 0.5 \text{ fm} \) and \( \tau = 1.2 \text{ fm} \). We see that the magnetic field evaluates like \( \frac{1}{2} \) for the large value of the electrical conductivity and it dose only depends on the proper time (Fig 2(b)).

Figure 2: The profiles of magnetic field at (a) \( \tau = 0.5 \text{ fm} \) and (b) \( \tau = 1.2 \text{ fm} \) with different values of \( \sigma \).
In Fig. 3, the profile of electric field is plotted by considering the different values of \( \sigma \) at \( \eta = 1 \). As seen, the electric field decreases rapidly for the large value of \( \sigma \). The Fig 4. shows the evolution of the electric and the magnetic fields at \( \sigma = 1 \text{fm}^{-1} \).

Figure 3: The dependence of electric field on electrical conductivity \( \sigma \) at \( \eta = 1 \).

Figure 4: The space-time evolution of (a) electric field and (b) magnetic field at \( \sigma = 1 \text{fm}^{-1} \) for \( \alpha = 2 \).

The correction of energy density as a function of \((\tau, \eta)\) is given by Eq. 70. The evolution of energy density for different values of \( \sigma \) is shown in Fig. 5 at \( \eta = 1 \). The results show a finite electrical conductivity will lead to a slow decay of the fluid energy density and increase the magnitude of energy density. Although, when the electrical conductivity tend to infinity the effect of second term in Eq. (70) becomes negligible and the magnitude of energy density reach to approximately to \( \sigma = 0 \) (Fig. 5(b)).
\[ \sigma = 0 \text{ fm}^{-1} \]
\[ \sigma = 1 \text{ fm}^{-1} \]
\[ \sigma = 3 \text{ fm}^{-1} \]

\[ \tau \text{[fm]} \]
\[ \epsilon(\tau, 1) \]

Figure 5: The evolution of energy density for different values of \( \sigma \) at \( \eta = 1 \) for \( \alpha = 2 \).

3.1 The transverse velocity and transverse momentum spectrum in the presence of electromagnetic fields

In the previous sections we have obtained analytical solutions of the electric and magnetic fields and energy density. Now we can use these results to estimate the transverse momentum spectrum emerging from the magneto-hydrodynamic solutions.

In order to examine the effects of electromagnetic fields on the transverse momentum spectrum we need to obtain the velocity of charge particles in the spectrum such as proton and pion. We can use a proper approximation for transverse velocity as \([24]\):

\[ q \vec{v} \times \vec{B} + q\vec{E} - \mu m \vec{v} = 0, \]

where we solve the equation of motion for a charged fluid element with mass \( m \) in the local fluid rest frame. Here, magnetic and electric fields are given by the Eq. \((65)\), and the last term defines the drag force on a fluid element. For the purpose of simplicity we consider \( \mu m \) as a constant. It is given by \([24]\):

\[ \mu m = \frac{\pi\sqrt{6\pi}}{2} (1.5T_c)^2, \]

with \( T_c \approx 170 \text{MeV} \). Therefore, the components of the transverse velocity as a function of \((\tau, \eta)\) are given by:

\[ v_x(\tau, \eta) = \frac{q\mu m c_x(\tau, \eta)}{\mu m^2 + q^2 b_y(\tau, \eta)^2}, \quad v_y(\tau, \eta) = 0 \]

In Fig. 6 we plot the transverse velocity \( v_x \) in the \((\tau, \eta)\) plane for different values of the electric conductivity \( \sigma \). We observe that by increasing the electrical conductivity velocity decreases in an excessive range.

The transverse spectrum is calculated by the Cooper-Frye formula at the freeze out surface \([24, 36]\).

\[ S = \frac{q_s}{2\pi} \int_0^{x_f} m_T x_\perp \tau_f(x_\perp) K_1 \left( \frac{m_T u_x}{T_f} \right) I_0 \left( \frac{m_T u_x}{T_f} \right) dx_\perp \]

where \( u_x = 1 \), \( u_{x_\perp} = \gamma v_x = v_x \) and \( \tau_f(x_\perp) \) is the solution of the \( T(\tau_f, x_\perp) = T_f \). \( T_f \) is the temperature at the freeze out surface. It is the isothermal surface in spacetime at which the temperature of inviscid fluid is related to the energy density as
Figure 6: The transverse velocity $v_x$ in the $(\tau, \eta)$ plane for different values of $\sigma$. 
$T \propto \epsilon^{1/4}$ and for the energy density the Eq. (70) is applied. Also, $p_T$ is the detected transverse momentum, $m_T = \sqrt{m^2 + p_T^2}$ the corresponding transverse mass and $g_i = 2$ is degeneracy factor for the proton or pion.

The spectrum Eq. (75) is illustrated in Fig. 7 and Fig. 8 for three different values of electrical conductivity and compared with experimental results obtained at PHENIX [37] in non-central collisions. Our spectra appear to underestimate the experimental data, but their behavior with $p_T$ has the correct trend of a monotonically decrease. Although the conductivity has a significant effect on electric field, magnetic field and energy density, the final spectrum is not sensitive to $\sigma$ parameter.

4 Conclusion and outlook

The magnitudes and the evolution of electromagnetic fields play an important role in estimations of possible observable effects of the de-confinement and chiral phase transitions in heavy-ion collisions. There have been a lot works which have investi-
gated the electromagnetic field strength and their evolutions. It has been known in the initial stage the magnitude of the magnetic field falls rapidly with time \(|B_y| \sim \frac{1}{\tau^3}\); however, the presence of the hot quark-gluon plasma (QGP) may increase the lifetime of the strong magnetic field.

In the present work the effect of QGP with finite conductivity on the evolution of electromagnetic fields has been estimated. Making use of Milne coordinates, in our setup the medium is boost-invariant along the \(z\) direction. The magnetic field pointed in \(y\) and the electric field pointed in \(x\) directions are functions of \(\tau\) and \(\eta\). We have studied analytically (1+1) dimensional, longitudinally boost invariant motion of a fluid with the electrical conductivity \(\sigma\) in magneto-hydrodynamic framework. The energy and Euler equations together with Maxwell’s equations reduced to two coupled differential equations, which we solved analytically. We have computed analytic formula for electric and magnetic fields in the conducting fluid with the electric conductivity \(\sigma\). The exact algebraic expressions for both electric and magnetic fields have been obtained. Moreover, the space-time profiles of electromagnetic fields and energy density have been calculated based on analytic expressions. We have showed the time evolution of the magnetic and electric fields and the energy density dependence on \(\sigma\). Besides, we find that energy density evolution of the mentioned fluid depends on space-time rapidity \(\eta\) as well as proper time \(\tau\). The analytical results are in a satisfactory agreement with the previous ones where the ideal RMHD, i.e., electrical conductivity is infinite, are considered.

Finally we use the solutions for the transverse velocity and energy density in the presence of electromagnetic fields, to estimate the transverse momentum spectrum emerging from the magneto-hydrodynamic solutions.

Our analytic solutions are worth in order to acquire deeper understandings for more realistic numerical results.

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