Bootstrap equations for effective theories and the calculation of the $G_T/G_V$ ratio

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Abstract

A method is described for dealing with effective theories of hadron scattering. It allows one to reduce the number of independent renormalization prescriptions in those theories and gives a possibility to make numerical predictions. As an illustration, we show the results of comparison with the known data on $\pi\pi$, $\pi K$ and $\pi N$ elastic scattering. This work presents a generalization and the further development of our results first discussed at the MENU’99 Symposium [1].

1 Preliminary notes

It is widely believed that to construct the complete theory of strong processes one needs to make two steps:

1. With the help of QCD (which is supposed to be the fundamental theory of strong forces) find the hadronic spectrum (poles of the Green functions) and construct the complete set of asymptotic states.

2. Construct the scattering theory for those (composite) states.

It is not yet clear how to solve the first problem. Is it possible to say anything about the scattering theory of hadrons, having no information on their inner structure? That is how the effective theory concept naturally comes to mind.

When constructing a theory of hadron scattering, we are forced to rely upon the Dyson series for the $S$-matrix, because this is the only known perturbative approach guaranteeing unitarity, causality and Lorentz-invariance of the results (see, e.g., [3]). In the case of effective theory this series can always be presented in the form

$$S_{fi} = \langle f | T_W \exp \left\{ -i \int H_{\text{int}} dx \right\} | i \rangle,$$

where $T_W$ stands for Wick’s (explicitly covariant) $T$-product and the Hamiltonian density (in the interaction picture) $H_{\text{int}}$ does not contain any noncovariant terms. Imposing any algebraic symmetry requirements on a theory based on the form (1) looks not more difficult

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1 Here this term is understood precisely in the same sense as in [3].
2 Linearly realized, for example isotopic $SU_2$ (or $SU_3$).
than if the Lagrangian picture is used: the $S$-matrix is covariant and satisfies the symmetry requirements, if $H_{\text{int}}$ does.

In contrast, the problem of dynamical symmetries looks much more transparent in the Lagrangian picture. And the transition from the Lagrangian picture to the Hamiltonian one looks almost hopeless when one deals with an effective theory. Indeed, in this case one needs to solve an infinite system of constraints arising, in particular, due to the presence of higher powers of time derivatives. However, as shown in \[2\], one needs only a finite number of Lagrangian terms when working to a given order in a small momentum. Thus, in the last case one can construct the corresponding Hamiltonian and, hence, avoid problems with unitarity. This program (first realized in \[4\]) gives us the natural way ("matching") to take into account the dynamical symmetry requirements in the effective theory based on the Hamiltonian. Namely, one needs to compute the amplitude of the process in question, expand it in powers of a small momentum (of the Goldstone particle) up to a given order, and, finally, compare the result with that following from the canonical approach (based on the invariant Lagrangian of the same order) and equate the corresponding constants.

Which fields should be included in the Hamiltonian? To be able to work not only in the low energy region, we include the resonance fields (like the $\rho$-meson) as well as the fields of the true asymptotic states (stable with respect to the strong forces, like the $\pi$-meson).\[3\] To avoid model dependence of the results, we reserve the possibility to work with an arbitrary (possibly infinite) number of resonances with arbitrarily high values of spin $J$ and mass $M$. The only limitation is suggested by experiment: we imply that there is a finite number of resonances with the same mass (though the mass spectrum may be unbounded). To put it another way, we imply the existence of a leading Regge trajectory (in the real plane of Re$M$ and $J$) which, however, is not necessarily linear. According to the phenomenology of strong interactions we do not deal with massless spin $J > \frac{1}{2}$ particles. Also, we assume that the maximal isospin value is $I = 1$ for mesons and $I = \frac{3}{2}$ for baryons.

Thus, we consider the effective hadron scattering theory based on Dyson's series \([1]\) with the Hamiltonian written in the form of an infinite sum of Lorentz-invariant (and SU(2)-invariant) local terms, each one constructed from the fields (and all powers of their derivatives of arbitrary high order) of pions, $K$-mesons, nucleons and all possible resonances.

## 2 Essential parametres, self-consistency and the bootstrap equations

It is possible to show that the essential parametres of the effective scattering theory are masses (real parts of pole positions) and those (and only those) combinations of coupling constants which are needed to fix the on-shell kinematic structure of tree-level vertices. When computing the $S$-matrix elements one does not need to impose a renormalization condition on each coupling constant appearing in the Hamiltonian: only the essential parametres require fixing of their finite parts.

The central idea of our work is that one cannot take independent renormalization prescriptions even for the essential parametres of the effective scattering theory: certain natural

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\[3\] No problem with unitarity occurs in spite of the fact that Dyson’s series is based on the Hamiltonian depending on resonance fields (see, e.g. \[3\]). In fact, this approach is used in the Standard Model of electroweak interactions: for instance, the $W$-boson is not an asymptotic state.

\[4\] These are the only parametres that appear in the $S$-matrix elements, see \[3\], Chapter 7.
self-consistency requirements impose an infinite number of constraints (bootstrap conditions) on the allowed physical values of the essential parameters. Namely, to make it possible to construct the one-loop approximation for the amplitude of a given scattering process (here we only discuss $2 \rightarrow 2$ processes), the corresponding tree-amplitude $A(s, t, u)$ must satisfy the following two requirements:

1. It must be a meromorphic function of the Mandelstam variables $s, t, u$, with poles and residues fixed by the Feynman rules.

2. This amplitude must be polynomially bounded in each independent energy-like variable at zero value of the corresponding momentum transfer.

As explained in [7], these two requirements turn out to be sufficient to derive the exact form of the tree-amplitude. At the same time they lead to an infinite set of equations connecting the tree-amplitude parameters among themselves (bootstrap equations). And if we write the Hamiltonian in terms of physical parameters (plus the necessary counterterms — what we can always do), then the tree-amplitude is automatically written in terms of physical (experimentally measurable) parameters. All this means that the bootstrap equations are not affected by the renormalization procedure and can be tested experimentally.

3 Comparison with experiment

In the cases of $\pi\pi$ and $\pi K$ elastic scattering (see [7, 8]) it has been found that the resulting equations strongly contradict the known data unless two light scalar resonances are taken into account. These are the $\sigma (0^+0^+)$ and $\kappa (0^+1^+)$ mesons with the following parameters estimated from the bootstrap equations: $m_\sigma \sim 500$ MeV; $\Gamma_\sigma \sim 300$ MeV; $m_\kappa \sim 1$ GeV; $\Gamma_\kappa \sim 500$ MeV.

These parameters are strongly supported by modern data, see, e.g. [7, 8]. It is interesting to note, that, as was shown in [10] (see also [9]), to preserve the unitarity bound for the $\pi\pi$ and $\pi K$ amplitudes one must take into account both the resonance and the (automatically implied in our approach) background interaction terms.

Perhaps, the most interesting result has been obtained from the analysis of the bootstrap equations for the $\pi N$ scattering amplitude parameters. It was possible to make the accurate estimate of the ratio $G_{NN\rho}^T / G_{NN\rho}^V = 6 (\pm 20\%)$, of tensor/vector $NN\rho$ coupling constants. This value turned out to be in nice agreement with experimental data. As far as we know, such a relation has never been explained in terms of model-independent theoretical arguments.

Besides, with the help of the bootstrap equations we have estimated the values of 40 coefficients in the expansion of the $\pi N$ amplitude around the crossing symmetry point, first introduced in [12]. The detailed analysis will be published elsewhere.

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5 Both of them are trivial if there is a finite number of terms in the Hamiltonian (Lagrangian).

6 No singularities except poles.

7 Polynomial boundedness of the meromorphic functions is understood as in complex analysis, see e.g. [6].

8 For the corresponding experimental data and notations see [1].
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