In memoriam two distinguished participants of the Bregenz Symmetries in Science Symposia: Marcos Moshinsky and Yurii Fedorovich Smirnov

Maurice R Kibler$^{a,b,c}$

$^a$ Université de Lyon, F–69622, Lyon, France
$^b$ Université Claude Bernard Lyon 1, Villeurbanne, France
$^c$ CNRS/IN2P3, Institut de Physique Nucléaire de Lyon, France

E-mail: m.kibler@ipnl.in2p3.fr

Abstract. Some particular facets of the numerous works by Marcos Moshinsky and Yurii Fedorovich Smirnov are presented in these notes. The accent is put on some of the common interests of Yurii and Marcos in physics, theoretical chemistry, and mathematical physics. These notes also contain some more personal memories of Yurii Smirnov.

1. Introduction

Yurii Fedorovich Smirnov passed away in 2008 and Marcos Moshinsky in 2009. They were two famous physicists with common interests in nuclear physics, atomic and molecular physics, and mathematical physics. More generally, both of them were at the origin of significant achievements in symmetry methods in physics. They actively participated in several Symmetries in Science Symposia in Bregenz. These two giants had parallel centers of interest in the sense that they developed separately some complementary works in nuclear physics (that led in particular to the concept of Moshinsky-Smirnov coefficients), dealt with some related problems in atomic, molecular and mathematical physics, and, finally, combined efforts to produce a beautiful and very useful book [1] on the applications of the harmonic oscillator system to various areas of physics and chemistry.

It is not the purpose of these notes to extensively list and analyse the numerous papers by Marcos and Yurii. I shall focus on some particular facets of their works. I had the opportunity to meet Marcos and Yurii several times in Bregenz and on several other occasions, and to discuss with them about Wigner-Racah algebras for finite groups, Lie groups and quantum groups. I had also a chance to collaborate with Yurii Smirnov. Therefore, I shall devote the main part of these notes to some specific domains of importance to Yurii and Marcos and to some more personal reminiscences on Yurii.

2. Marcos Moshinsky

Marcos Moshinsky was a Mexican physicist. He was born in Kiev (Ukraine) in 1921. He arrived as a refugee in Mexico when he was three years old and obtained Mexican citizenship in 1942. He received a Bachelor’s degree in physics from the Universidad Nacional Autónoma de México (U.N.A.M.) and a Ph.D. degree in theoretical physics, under the guidance of Eugene P. Wigner,
from Princeton University. Marcos was also the recipient a post-doctoral fellowship at the Institut Henri Poincaré in Paris. Afterwards, he returned to Mexico and pursued a brilliant career at the U.N.A.M. in Mexico City.

Professor Marcos Moshinsky had important responsibilities as the President of the Sociedad Mexicana de Física, as a member of El Colegio Nacional, and as a member of the editorial board of several international scientific reviews. He produced and/or co-produced more than 200 scientific papers and four books among which the most well-known are the one written in collaboration with Thomas A. Brody on transformation brackets for nuclear shell-model calculations [2] and the one with Yurii F. Smirnov on the applications of the harmonic oscillator in various fields of physics and quantum chemistry [1]. He received several prizes, namely the Premio Nacional de Ciencias y Artes in 1968, Premio Luis Elizondo in 1971, Premio U.N.A.M. de Ciencias Exactas in 1985, Premio Príncipe de Asturias de Investigación Científica y Técnica in 1988, and the prestigious UNESCO Science Prize in 1997 for his work in nuclear physics. He also received the Wigner medal in 1998.

3. Marcos and Yurii

A first seminal paper by Marcos concerned the transient dynamics of particle wavefunctions, a phenomenon that gives rise to diffraction in time [3]. However, most of his scientific work dealt with collective models of the nucleus, canonical transformations in quantum mechanics, and group theoretical methods in physics, with a special emphasis on symplectic symmetry in nuclear, atomic and molecular physics. These themes were of interest to Yurii too. Following the pioneering work of Talmi (who prepared his Ms.S. thesis with Guilio Racah, his Doctorate thesis with Wolfgang Pauli and who was a post-doctoral fellow with Eugene P. Wigner) [4], both Marcos and Yurii were interested in the description of pairs of nucleons in a harmonic-oscillator potential. In 1959, Moshinsky developed a formalism to connect wavefunctions in two different coordinate systems for two particles (with identical masses) in a harmonic-oscillator potential [5]. In this formalism, any two-particle wavefunction $|n_1 \ell_1, n_2 \ell_2, \lambda \mu\rangle$, expressed in coordinates with respect to the origin of the harmonic-oscillator potential, is a linear combination of wavefunctions $|n \ell, NL, \lambda \mu\rangle$, expressed in relative and centre-of-mass coordinates of the two particles. The so-called transformation brackets $\langle n \ell, NL, \lambda | n_1 \ell_1, n_2 \ell_2, \lambda \rangle$ make it possible to pass from one coordinate system to the other. Moshinsky gave an explicit expression of these coefficients in the case $n_1 = n_2 = 0$ and derived recurrence relations that can be used to obtain the coefficients for $n_1 \neq 0$ and $n_2 \neq 0$ from those for $n_1 = n_2 = 0$ [6, 7]. At the end of the fifties, Smirnov worked out a similar problem, viz. the calculation of the Talmi coefficients for unequal mass nucleons, and gave solution for the case $n_1 \neq 0$ and $n_2 \neq 0$ [6, 7]. (Indeed, the transformation brackets and the Talmi coefficients are connected via a double Clebsch-Gordan transformation.) The coefficients $(n \ell, NL, \lambda | n_1 \ell_1, n_2 \ell_2, \lambda\rangle$, called transformation brackets by Moshinsky and total Talmi coefficients by Smirnov, are now referred to as Moshinsky-Smirnov coefficients. Both the Moshinsky-Smirnov coefficients and the Talmi coefficients were revisited at the end of the seventies in terms of generating functions in the framework of the approaches of Julian S. Schwinger and Valentine Bargmann to the harmonic-oscillator bases [8]. (The work by Mehdi Hage Hassan [8], who prepared his State Doctorate thesis at the Institut de Physique Nucléaire de Lyon and conducted his career in Beyrouth, constitutes a very deep and original approach to the Talmi coefficients and Moshinsky-Smirnov coefficients.) It should be noted that the transformation brackets or Moshinsky-Smirnov coefficients are also of importance for atoms and molecules as shown by Marcos and Yurii in their book [1] written during the time Yurii was a visiting professor at the Instituto de Física of the Universidad Nacional Autónoma de México.

A second area of common interest to both Marcos and Yurii concerns the many-body problem considered from the point of view of unitary and symplectic groups and the use of...
nonlinear and nonbijective canonical transformations. In this vein, Moshinsky and some of
his collaborators introduced the concept of an ambiguity group, a group required for taking
into account the nonbijectivity of certain canonical transformations [9]. Indeed, this concept
is closely related to the one of Lie algebra under constraint [10], which in turn is connected to
nonbijective transformations like Hopf fibrations on spheres and Hopf fibrations on hyperboloids
[11]. Among the nonbijective transformations, one may mention the $\mathbb{R}^4 \rightarrow \mathbb{R}^3$
Kustaanheimo-Stiefel transformation and the $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ Levi-Civita transformation as well as their various extensions [11, 12]. In particular, the Kustaanheimo-Stiefel transformation allows one to pass
from a four-dimensional harmonic oscillator subjected to a constraint to the three-dimensional
hydrogen atom (see for instance [13]). This subject was of interest to Yurii and he revisited the
hydrogen-oscillator connection with Corrado Campigotto [14].

The harmonic oscillator is a central ingredient in numerous studies by Smirnov and
Moshinsky. Many applications of the nonrelativistic and relativistic harmonic oscillators to
modern physics (from molecules, atoms and nuclei to quarks) were pedagogically exposed in
the book by Marcos and Yurii [1] with a special attention paid to the n-body problem (in the
Hartree-Fock approximation), the nuclear collective motion and the interacting boson model.

But their common interests were not limited to transformation brackets and harmonic
oscillators. Let us briefly mention that both of them were also interested in group theoretical
methods and symmetry methods in physics and also contributed to several fields of mathematical
physics including, for instance, the state labelling problem, special functions, and generating
functions (see, for example, [15, 16]).

4. Yurii Fedorovich Smirnov
Yurii Fedorovich Smirnov was a Russian physicist. He was born in the city of Il’inskoe (Yaroslavl’
region, Russia) in 1935. He graduated from Moscow State University. Subsequently, he
completed his Doctorate thesis at the same university under the guidance of Yurii M. Shirokov
and benefited from fruitful contacts with other distinguished physicists including Yakov A.
Smorodinsky. He pursued his career in the (Skobeltsyn) Institute of Nuclear Physics and in
the Physics Department of (Lomonosov) Moscow State University with many stays abroad.
The last fifteen years of his life were shared between Moscow and Mexico City where he was
a visiting professor and later a professor at the Instituto de Física and at the Instituto de
Ciencias Nucleares of the U.N.A.M. (he spent almost 11 years in Mexico). He received prestigious
awards: the K.D. Sinel’nikov Prize of the Ukrainian Academy of Sciences in 1982 and the M.V.
Lomonosov Prize in 2002. He was also a member of the Academy of Sciences of Mexico.

Yurii Smirnov authored and/or co-authored eleven books and more than 250 scientific articles.
He also translated several scientific books into Russian. He translated, for example, a book on
the harmonic oscillator written by Marcos Moshinsky in 1969, precisely the book that was a
starting point for their common book on the same subject, published in 1996 [1]. He was
a member of the editorial board of several journals and a councillor of the scientific councils
of the Skobeltsyn Institute of Nuclear Physics and of the Chemistry Department of Moscow
State University, as well as of the Institute for Theoretical and Experimental Physics (ITEP) in
Moscow.

5. Some personal reminiscences on Yurii
My first contact with the work of Yurii Smirnov dates back to 1978 when my colleague J. Patera
showed me, on the occasion of a NATO Advanced Study Institute organised in Canada by J.C.
Donini, a beautiful book written by D.T. Sviridov and Yu.F. Smirnov [17]. This book dealt with
the spectroscopy of $d^N$ ions in inhomogeneous electric fields (part of a disciplinary domain known
as crystal- and ligand-field theory in condensed matter physics and explored via the theory of
level splitting from a theoretical point of view). In 1979, B.I. Zhilinskiï, while visiting Dijon and
Lyon in France in the context of an exchange programme between the USSR and France, gave me another interesting book, on \( f^N \) ions in crystalline fields, written by D.T. Sviridov, Yu.F. Smirnov and V.N. Tolstoy [18]. At that time, the references for mathematical aspects of crystal- and ligand-field theory were based on works by Y. Tanabe, S. Sugano and H. Kamimura from Japan [19], J.S. Griffith from England [20], and Tang Au-chin and his collaborators from China [21] (see also some contributions by the present author [22]). The two above-mentioned books by Smirnov and his colleagues shed some new light on the mathematical analysis of spectroscopic and magnetic properties of partly filled shell ions in molecular and crystal surroundings. In particular, special emphasis was put on the derivation of the Wigner-Racah algebra of a finite group of molecular and crystallographic interest from that of the group \( SO(3) \sim SU(2)/Z_2 \).

My second (indirect) contact with Yurii is related to an invitation to participate in the fifth workshop on \textit{Symmetry Methods in Physics} in Obninsk in July 1991. Unfortunately, I did not get my visa on time reducing my participation to a paper in the proceedings of the workshop edited by Yu.F. Smirnov and R.M. Asherova [23].

In the beginning of the 1990’s, I had a chance to discover another facet of Yurii’s work. In 1989, a Russian speaking student from Switzerland, C. Campigotto, spent one year in the group of Prof. Smirnov. He started working on the Kustanheimo-Stiefel transformation, an \( \mathbb{R}^4 \rightarrow \mathbb{R}^3 \) transformation associated with the Hopf fibration \( S^3 \rightarrow S^2 \) with compact fiber \( S^1 \). (Such a transformation makes it possible to connect the Kepler-Coulomb system in \( \mathbb{R}^3 \) to the isotropic harmonic oscillator in \( \mathbb{R}^4 \).) Then, Campigotto (well-prepared by Smirnov and his team, especially Andrey M. Shirokov and Valeriy N. Tolstoy) came to Lyon to prepare a Doctorate thesis (partly published in [24]). He defended his thesis in 1993 with George S. Pogosyan (representing Yu.F. Smirnov) as a member of the jury.

A fourth opportunity to work with Yurii stemmed from our mutual interest in quantum groups and in nuclear and atomic spectroscopy. I meet him for the first time in Dubna in 1992. We then started a collaboration (partly with R.M. Asherova) on \( q \)- and \( qp \)-boson calculus in the framework of Hopf algebras associated with the Lie algebras \( su(2) \) and \( su(1,1) \) [25]. In addition, we pursued a group-theoretical study of the Coulomb energy averaged over the \( n\ell^N \)–atomic states with a definite spin [26]. We also had fruitful exchanges in nuclear physics. Indeed, Prof. Smirnov and his colleagues D. Bonatsos (from Greece), S.B. Drenska, P.P. Raychev and R.P. Roussev (all from Bulgaria) developed a model based on a one-parameter deformation of \( SU(2) \) for dealing with rotational bands of deformed nuclei and rotational spectra of molecules [27] (see also [28]). Along the same line, a student of mine, R. Barbier, developed in his thesis a two-parameter deformation of \( SU(2) \) with application to superdeformed nuclei in mass region \( A \sim 130 – 150 \) and \( A \sim 190 \) (partly published in [29]). It was a real pleasure to receive Yurii in Lyon on the occasion of the defence of the Barbier thesis in 1995. Indeed, from 1992 to 1995, Yurii made four stays in Lyon (one with his wife Rita and one with his daughter Tatjana) and we jointly participated in several meetings, one in Clausthal in Germany (organised by H.-D. Doebner, V.K. Dobrev and A.G. Ushveridze) and two in Bregenz in Austria (organised by B. Gruber and M. Ramek).

I cannot do justice to all of the fields in which Yurii was recognized as a superb researcher. It is enough to say that he contributed to many domains of mathematical physics (e.g., finite groups embedded in compact or locally compact groups, Lie groups and Lie algebras, quantum groups, special functions) and theoretical physics (e.g., nuclear, atomic and molecular physics, crystal- and ligand-field theory). Let me mention, among other fields, that he achieved alone and with collaborators significant advances in the theory of clustering of shell-model (nuclear) systems [30], in projection operator techniques for simple Lie groups [31], in the theory of heavy ion collisions [32], and in the so-called \( J \)-matrix formalism for quantum scattering theory (see [33] and references therein). The \( J \)-matrix formalism requires the solution of three-term recurrence relations (or second-order difference equations); along this line, Yurii and some of
his collaborators published several works (see for instance [34]). As another major contribution, at the end of the sixties he proposed in collaboration with Vladimir G. Neudatchin a method, the so-called (e,2e) method (an analog of the (p,2p) method used in nuclear physics), for the experimental investigation of the electronic structure of atoms, molecules and solids; this method was successfully tested in many laboratories around the world (see [35] and references therein).

Yurii was also an exceptional teacher. It was very pleasant, profitable and inspiring to be taught by him. I personally greatly benefited from discussions with Yurii Smirnov.

Closing

Yurii and Marcos had many students who are now famous physicists. They interacted with many collaborators in their countries and abroad, and had an influence on many scientists. Marcos Moshinsky and Yurii Fedorovich Smirnov will remain examples for many of us. We shall not forget them.

References

[1] Moshinsky M and Smirnov Yu F 1996 The Harmonic Oscillator in Modern Physics (Amsterdam: Harwood Academic Publishers)
[2] Brody T A and Moshinsky M 1960 Tables of Transformation Brackets for Nuclear Shell-Model Calculations (Mexico City: Universidad Nacional Autónoma de México)
[3] Moshinsky M 1952 Phys. Rev. 88 625
[4] Talmi I 1952 Helv. Phys. Acta 25 185
[5] Moshinsky M 1959 Nucl. Phys. 13 104
[6] Smirnov Yu F 1961 Nucl. Phys. 27 177
[7] Smirnov Yu F 1962 Nucl. Phys. 39 346
[8] Hage Hassan M 1980 J. Phys. A: Math. Gen. 13 1903
[9] Moshinsky M and Seligman T H 1978 Ann. Phys., NY 114 243
Moshinsky M and Seligman T H 1979 Ann. Phys., NY 120 402
[10] Deenen J, Moshinsky M and Seligman T H 1980 Ann. Phys., NY 127 458
[11] Lambert D and Kibler M 1987 Levi-Civita, Kustaanheimo-Stiefel and other transformations Group Theoretical Methods in Physics ed R Gilmore (Singapore: World Scientific)
Lambert D and Kibler M 1988 J. Phys. A: Math. Gen. 21 307
Kibler M 1988 Lecture Notes in Physics 313 238
Kibler M and Labastie P 1989 On some transformations generalizing the Levi-Civita, Kustaanheimo-Stiefel, and Fock transformations Group Theoretical Methods in Physics eds Y Saint-Aubin and L Vinet (Singapore: World Scientific)
[12] Hage Hassan M and Kibler M 1990 Non-bijective quadratic transformations and theory of angular momentum Selected Topics in Statistical Mechanics eds A A Logunov, N N Bogolubov, Jr, V G Kadyshevsky and A S Shumovsky (Singapore: World Scientific)
Hage Hassan M and Kibler M 1991 On Hurwitz Transformations Le Problème de Factorisation de Hurwitz : Approche Historique, Solutions, Applications en Physique eds A Rouvax and D Lambert (Namur: FUNDP) (Preprint hep-th/9409051)
[13] Kibler M and Négadi T 1983 Lett. Nuovo Cimento 37 225
Kibler M and Négadi T 1983 J. Phys. A: Math. Gen. 16 4265
Kibler M and Négadi T 1984 Phys. Rev. A 29 2891
[14] Campigotto C and Smirnov Yu F 1991 Helv. Phys. Acta 64 48
[15] Moshinsky M, Patera J, Sharp R T and Winternitz P 1975 Ann. Phys., NY 95 139
[16] Raychev P P, Roussev R P and Smirnov Y F 1991 J. Phys. A: Math. Gen. 24 2943
[17] Sviridov D T and Smirnov Yu F 1977 Teorija Opticheskikh Spektrov Perekhodnykh Metallov (Moscow: Izd. Nauka)
Sviridov D T and Smirnov Yu F 1968 Soviet. Phys. Doklady 13 565
Vonsovski C V, Grimalov C V, Tcherepanov V I, Meng A N, Sviridov D T, Smirnov Yu F and Nikiforov A E 1969 Crystal-Field Theory and Optical Spectra of Partly Filled d Shell Transition Ions (Moscow: Nauka)
Sviridov D T, Sviridova R K and Smirnov Yu F 1976 Optical Spectra of Transition-Metal Ions in Crystal (Moscow: Nauka)
[18] Sviridov D T, Smirnov Yu F and Tolstoy V N 1975 Spektroskopiya Kristallov (Moscow: Akad. Nauk SSSR)
[19] Tanabe Y and Sugano S 1954 J. Phys. Soc. Japan 9 753
Tanabe Y and Sugano S 1954 J. Phys. Soc. Japan 9 766
Tanabe Y and Sugano S 1956 J. Phys. Soc. Japan 11 864
Tanabe Y and Kamimura H 1958 J. Phys. Soc. Japan 13 394
Sugano S and Tanabe Y 1958 J. Phys. Soc. Japan 13 880
Sugano S, Tanabe Y and Kamimura H 1970 Multiplets of Transition-Metal Ions in Crystals (New York: Academic Press)
[20] Griffith J S 1960 Molec. Phys. 3 79
Griffith J S 1960 Molec. Phys. 3 285
Griffith J S 1960 Molec. Phys. 3 457
Griffith J S 1960 Molec. Phys. 3 477
Griffith J S 1961 The Theory of Transition-Metal Ions (Cambridge: Cambridge Univ. Press)
Griffith J S 1962 The Irreducible Tensor Method for Molecular Symmetry Groups (Englewood Cliffs: Prentice-Hall)
[21] Tang An-chin, Sun Chia-chung, Kiang Yuan-sun, Deng Zung-hau, Liu Jo-chuang, Chang Chain-er, Yan Go-sen, Goo Zien and Tai Shu-shan 1966 Sci. Sinica (Peking) 15 610
Tang An-chin, Sun Chia-chung, Kiang Yuan-sun, Deng Zung-hau, Liu Jo-chuang, Chang Chain-er, Yan Gosen, Goo Zien and Tai Shu-shan 1979 Theoretical Method of the Ligand Field Theory (Peking: Science Press)
[22] Kibler M 1968 J. Molec. Spectrosc. 26 111
Kibler M 1969 Int. J. Quantum Chem. 3 795
Kibler M 1969 C.R. Acad. Sci. (Paris), Ser. B 268 1221
Kibler M R 1977 Group theory around ligand field theory Group Theoretical Methods in Physics eds R T Sharp and B Kolman (New York: Academic Press)
Kibler M R 1979 Finite symmetry adaptation in spectroscopy Recent Advances in Group Theory and Their Application to Spectroscopy ed J C Donini (New York: Plenum Press)
Kibler M and Daoud M 1992 Symmetry adaptation and two-photon spectroscopy of ions in molecular or solid-state finite symmetry Symmetry Methods in Physics eds Yu F Smirnov and R M Asherova (Obninsk: Russian Federation Ministry of Atomic Energy – Institute of Physics and Power Engineering)
[24] Campigotto C and Smirnov Yu F 1991 Helv. Phys. Acta 64 48
Drăganescu Gh E, Campigotto C and Kibler M 1992 Phys. Lett. A 170 339
Kibler M and Campigotto C 1993 Int. J. Quantum Chem. 45 209
Kibler M and Campigotto C 1993 Phys. Lett. A 181 1
Campigotto C, Smirnov Yu F and Enikeev S G 1995 J. Comp. Appl. Math. 57 87
Smirnov Yu F and Campigotto C 2004 J. Comp. Appl. Math. 164–165 643
[25] Smirnov Yu F and Kibler M R 1993 Some aspects of q-boson calculus Symmetries in Science VI: From the Rotation Group to Quantum Algebras ed B Gruber (New York: Plenum Press)
Kibler M, Campigotto C and Smirnov Yu F 1994 Recursion relations for Clebsch-Gordan coefficients of $U_q (su(2))$ and $U_q (su(3)_1)$ Symmetry Methods in Physics eds A N Sissakian, G S Pogosyan and S I Vinitsky (Dubna: Joint Institute for Nuclear Research)
Kibler M R, Asherova R M and Smirnov Yu F 1995 Some aspects of q- and qp-boson calculus Symmetries in Science VIII ed B Gruber (New York: Plenum Press)
[26] Kibler M and Smirnov Yu F 1995 Int. J. Quantum Chem. 53 495
[27] Bonatsos D, Drenskas S B, Raychev P P, Roussev R P and Smirnov Yu F 1991 J. Phys. G: Nucl. Part. Phys. 17 L67
Raychev P P, Roussev R P and Smirnov Yu F 1990 J. Phys. G: Nucl. Part. Phys. 16 L137
Bonatsos D, Raychev P P, Roussev R P and Smirnov Yu F 1990 Chem. Phys. Lett. 175 300
Zhilinskii B I and Smirnov Yu F 1991 Sov. J. Nucl. Phys. 54 10
[28] Georgieva A and Dankova Ts 1994 J. Phys. A: Math. Gen. 27 1251
Georgieva A I, Goleminov I D, Ivanov M I and Geyer H B 1999 J. Phys. A: Math. Gen. 32 2403
[29] Barbier R, Meyer J and Kibler M 1994 J. Phys. G: Nucl. Part. Phys. 20 L13
Barbier R, Meyer J and Kibler M 1995 Int. J. Mod. Phys. E 4 385
Barbier R and Kibler M 1995 A system of interest in spectroscopy: The qp-rotor system Finite Dimensional Integrable Systems eds A N Sissakian and G S Pogosyan (Dubna: Joint Institute for Nuclear Research)
Barbier R and Kibler M 1995 On the use of quantum algebras in rotation-vibration spectroscopy Modern Group Theoretical Methods in Physics eds J Bertrand, M Flato, J-P Gazeau, D Sternheimer and M Iracl-Astaad (Dordrecht: Kluwer)
Barbier R and Kibler M 1996 Rept. Math. Phys. 38 221
[30] Neudatchin V G and Smirnov Yu F 1969 Nucleon Clusters in Light Nuclei (Moscow, Nauka)
Nemets O F, Neudatchin V G, Rudchik A T, Smirnov Yu F and Tchuvil’sky Yu M 1988 *Nucleon Clusters in Atomic Nuclei and Many-Nucleon Transfer Reactions* (Kiev: Naukova Dumka)

[31] Asherova R M, Smirnov Yu F and Tolstoy V N 1973 *Theoret. Math. Phys.* 15 107

Tolstoy V N 2006 *Phys. Atom. Nuclei* 69 1058

[32] Smirnov Yu F and Tchuvil’sky Yu M 1984 *Phys. Lett.* B 134 25

[33] Smirnov Yu F, Shirokov A M, Lurie Yu A and Zaytsev S A 1995 Harmonic oscillator representation in the theory of scattering and nuclear reactions *Harmonic Oscillators* eds D Han and K B Wolf (Washington: NASA)

Bang J M, Mazur A I, Shirokov A M, Smirnov Yu F and Zaytsev S A 2000 *Ann. Phys.*, NY 280 299

[34] Smirnov Yu F, Suslov S K and Shirokov A M 1984 *J. Phys. A: Math. Gen.* 17 2157

nonum Braun P A, Shirokov A M and Smirnov Yu F 1985 *Molec. Phys.* 56 573

nonum Shirokov A M and Smirnov Yu F 1991 *J. Phys. A: Math. Gen.* 24 2961

nonum Zaitsev S A, Smirnov Y F and Shirokov A M 1988 *Teoret. Mat. Fiz.* 117 1291

[35] Neudatchin V G, Popov Yu V and Smirnov Yu F 1999 *Uspekhi Fiz. Nauk* 169 1111