Presence versus absence of end-to-end nonlocal conductance correlations in Majorana nanowires: Majorana bound states versus Andreev bound states

Yi-Hua Lai, Jay D. Sau, and Sankar Das Sarma

1Department of Physics, Condensed Matter Theory Center and the Joint Quantum Institute, University of Maryland, College Park, MD 20742

By calculating the differential tunneling conductance spectra from the two ends of a Majorana nanowire (i.e., a semiconductor nanowire in the presence of proximity induced superconductivity, intrinsic Rashba spin-orbit coupling, and external magnetic field induced Zeeman spin splitting) with a quantum dot embedded at one end, we establish that a careful examination of the nonlocal correlations (or not) between the zero bias conductance peaks as measured separately through tunneling from the two ends of the wire can distinguish between topological Majorana bound states and trivial Andreev bound states. In particular, there will (not) be identical correlated zero bias peaks from both ends for Majorana (Andreev) bound states, and thus the presence or absence of correlated zero bias conductance in the measured tunneling spectra from the two wire ends could imply the presence or absence of topological Majorana zero modes in the system. We present detailed results for the calculated conductance, energy spectra, and wavefunctions for different chemical potentials at the same magnetic field values to motivate end-to-end conductance correlation measurements in Majorana nanowires.

I. INTRODUCTION

It was pointed out by Kitaev\textsuperscript{1} that isolated Majorana zero modes (i.e., topological bound states with precise zero energy) existing at the ends of a one dimensional (1D) spinless p-wave superconductor are effective non-Abelian anyons which could potentially be used for fault-tolerant topological quantum computation.\textsuperscript{2–4} It was also theoretically established around the time of Kitaev’s work that such Majorana zero modes in a topological p-wave superconductor can be detected by the usual normal metal-superconductor (NS) differential tunneling conductance measurements which would reflect a quantize zero bias conductance peak (ZBP or ZBCP) associated with the Majorana bound states (MBS).\textsuperscript{5} The fact that the presence or absence of a quantized ZBCP in the NS tunneling spectroscopy signals the presence or absence of MBS was later rediscovered and expanded on by several groups.\textsuperscript{6–9} The subject took on particular significance after it was shown theoretically that two-dimensional and 1D semiconductor structures could actually host MBS under well-defined experimentally achievable conditions,\textsuperscript{10–14} and soon after these concrete predictions for the possible existence of MBS in low-dimensional semiconductor structures, Mourik et al.\textsuperscript{15} reported the experimental observation of a ZBP in the tunneling conductance of 1D InSb nanowires (on NbTiN superconducting substrates) loosely consistent with the theoretical predictions. This started a deluge of theoretical and experimental activity, which continues unabated for the last 7 years, in semiconductor (InSb and InAs) nanowires (with NbTiN and Al as the parent superconductor) aimed at the observation and elucidation of ZBCPs which are considered to be the signatures for the putative MBS in these Majorana nanowires. The subject got particular impetus from the important backing of Microsoft Corporation, which started a large technological development effort in building a topological quantum computer based on these semiconductor Majorana nanowires.\textsuperscript{16}

In spite of enormous experimental progress\textsuperscript{17–26} in materials fabrication leading to the ubiquitous observation of impressive ZBCPs in Majorana nanowires in many laboratories far surpassing the quality of the ZBCPs originally reported by Mourik et al.,\textsuperscript{15} questions, however, linger on whether MBS (as opposed to mere ZBPs in the tunneling measurements) have actually been seen yet. In particular, Ref.\textsuperscript{27} forcefully raised the key question on whether many, if not all, of the experimentally observed ZBCPs in Majorana nanowires could have originated from accidental non-topological (often called ‘trivial’ in this context) Andreev bound states (ABS) which fortuitously happen to reside near zero energy inside the superconducting gap. These trivial almost-zero-energy midgap ABS could be producing the ZBPs in the experiments fooling everybody into thinking that MBS have been observed whereas in reality what have been observed are the ZBP associated with these non-topological ABS. Actually, the possibility that there could be generic low-lying in-gap fermionic bound states in Majorana nanowires arising from impurity disorder\textsuperscript{28–32} and/or inhomogeneous chemical potential\textsuperscript{27,33–37} was pointed out early in the literature, and the fact that well-defined ZBCPs could arise from low-lying ABS in confined semiconductor structures was also experimentally demonstrated.\textsuperscript{38} If topological MBS and trivial ABS could both produce tunneling ZBCP in Majorana nanowires, a serious problem arises in the interpretation of the experimental data. One cannot automatically assume the experimental observation of a ZBCP as evidence for the existence of an underlying isolated MBS. This is particularly true in light of the fact that low-energy midgap ABS seem to be generic in Majorana nanowires arising from chemical potential inhomogeneity or isolated impurities acting as quantum dots in the system. The interplay of spin-orbit coupling and Zeeman splitting generically allows Andreev bound states to reside...
close to zero energy over finite ranges of the external magnetic field. The ABS thus mimic the zero-energy MBS with the big difference that MBS arise as zero energy modes in the nanowire only in the topological regime (which translates to the induced Zeeman spin splitting being larger than the critical field necessary for the topological quantum phase transition, TQPT) whereas the ABS arise in the trivial regime for Zeeman field below the critical field. Experimentally, unfortunately, there is no independent way of precisely knowing the critical field, so one does not apriori know whether an observed ZBCP happens to be in the topological or trivial regime. Thus, although the existence of a ZBCP may be a necessary condition for the existence of underlying MBS, it is by no means sufficient since the almost zero-energy ABS produces similar ZBCP in the nontopological regime. The inability to decisively distinguish between ZBCPs arising from MBS and ABS has been the crucial stumbling block in further progress in the subject.

Given the great importance of the distinction between ABS and MBS to the subject of Majorana nanowires, it is understandable that many theoretical papers have appeared following Ref.\textsuperscript{27} with various proposals on how to discern MBS from ABS.\textsuperscript{35,36,39–42} The situation has, however, remained unclear, and even the unambiguous observation\textsuperscript{24} of the predicted quantized Majorana ZBCP can be interpreted in terms of underlying ABS accidentally localized at midgap.\textsuperscript{42} This is the context of the current work where we propose a definitive experiment which, in principle, can distinguish between ABS and MBS based on the extensively used conductance tunneling spectroscopy.

We show that a careful comparison between the conductance spectra obtained by carrying out tunneling measurements from the two ends of a Majorana nanowire should be able to decisively distinguish between ZBCPs arising from MBS and ABS. In particular, ZBCPs from MBS (ABS) would manifest highly correlated (uncorrelated) low-energy behavior. We provide extensive numerical simulations to demonstrate the importance of simultaneous tunneling measurements from both ends in the context of MBS versus ABS distinction. The specific system we consider is motivated by the experiment of Deng et al.,\textsuperscript{21} who carried out tunnel conductance measurements in a Majorana nanowire with an embedded quantum dot at the wire end where the NS tunnel barrier resides. They found well-defined conductance features away from zero bias at low magnetic field values, which merged at zero bias at higher magnetic field values producing sharp ZBCP. This was interpreted by the authors as evidence in favor of trivial ABS (at finite energy) transforming into zero-energy topological MBS as the magnetic field sweeps through the TQPT. The experiment of Deng et al. was critically reanalyzed by Liu et al.\textsuperscript{27} who showed that most likely the experiment is demonstrating the existence of low-energy midgap ABS, induced by the quantum dot, which is producing the ZBCP rather than the transformation of trivial ABS into topological MBS as envisioned in Ref.\textsuperscript{21} Of course, the possibility that some of the observed ZBCP in the experiment arise from MBS cannot be ruled out theoretically, but the real problem is that there is no way to know apriori which ZBCP arise from ABS and which ones from MBS.

Our current work establishes that an experiment similar to that in Ref.\textsuperscript{21} with the tunneling spectroscopy carried out from both ends of the wire in the same sample (with a quantum dot only at one end) can distinguish between the ZBCPs arising from MBS and ABS through a simple examination of the correlations (or not) between two sets of tunneling data. The ZBCP arising from MBS (ABS) will be (un)correlated between the two ends—if the same ZBCP shows up in the tunneling from both ends it is likely to be associated with MBS whereas if the ZBCP exists only in the tunneling from one end (but not the other), then it is likely to be arising from ABS. Essentially, all one needs is redoing the Deng experiment by having NS tunneling from both ends of the wire. For completeness, we also briefly consider the effect of embedded end quantum dots on the cross-conductance, which can also be measured in the same set-up. Such measurements have recently been proposed as a way to distinguish ABSs versus MZMs by detecting the TQPT.\textsuperscript{43}

The rest of this paper is organized as follows. In Sec. II, we describe our model, theory, and calculations. In Sec. III, we present our numerical results for the calculated tunnel conductance from both ends of the wire and provide discussions on how such correlated tunneling spectroscopy could resolve the ABS versus MBS conundrum. We conclude in Sec. IV providing a summary and commenting on the experimental prospects.

\section{Model, Theory, and Calculations}

We analyze the conductance $G_\alpha = dI_\alpha/dV_\alpha$, where $\alpha = L, R$ corresponding to left or right lead of the Majorana nanowire device in a set-up shown in Fig. 1 and $I_\alpha$ and $V_\alpha$ denote the current and voltage in lead $\alpha$. We model the device within a minimal single-band model\textsuperscript{10,13,14} described schematically by the Bogoliubov-de Gennes (BdG) Hamiltonian\textsuperscript{10}:

\begin{equation}
\hat{H} = \frac{1}{2} \int dx \psi^\dagger(x) H_{NW} \psi(x),
\end{equation}

\begin{equation}
H_{NW} = \left( -\frac{\hbar^2}{2m} \nabla_x^2 - i\alpha_R \sigma_x \sigma_y - \mu \right) \tau_z
+ V_\sigma \sigma_x + \Delta(V_z) \tau_x - i\Gamma,
\end{equation}

where $\psi = (\psi_\uparrow, \psi_\downarrow, \psi_\uparrow^\dagger, \psi_\downarrow^\dagger)^T$ is the wave function in Nambu space, $\sigma_{x,y,z}$ (or $\tau_{x,y,z}$) are Pauli matrices in spin (particle-hole) space. The electrons are assumed to have an effective mass, $m^*$, Rashba spin-orbit coupling $\alpha_R$, and a magnetic field-induced Zeeman splitting $V_z$. The superconducting pairing potential, $\Delta(V_z)$, is assumed to be suppressed by Zeeman splitting $V_z$ as:

\begin{equation}
\Delta(V_z) = \Delta_0 \sqrt{1 - (V_z/V_c)^2},
\end{equation}

where $\Delta_0$ and $V_c$ are the superconducting gap and nonequilibrium critical magnetic field, respectively.
where $\Delta_0$ is the original induced superconducting gap without the magnetic field background, and $V_c$ is the field where the superconducting gap collapses. One may think of this collapse of the bulk gap as arising from the Clogston effect due to the bulk spin polarization in the parent superconductor (and other effects). Experimentally, such a bulk gap collapse is always present, and $V_c$ is a phenomenological parameter defining this field for the bulk gap collapse in the theory. A phenomenological dissipation parameter $\Gamma$ is introduced to account for possible anomalous broadening of the conductance, which is often observed experimentally.\cite{44,45} Finite temperature acts as an additional (thermal) broadening mechanism – the electron temperature may be well above the base temperature in experiments. We note that both $\Gamma$ and $V_c$ are nonessential aspects of our theory with respect to the ABS/MBS distinction – we have them in the theory to make the results realistic, not because they are necessary for the main point of left/right ZBP-correlations being made in this work.

The potential profile for the device in Fig. 1 is assumed to contain a quantum dot at the left end that can generate low energy subgap Andreev bound states even in the non-topological phase.\cite{27} We model the Hamiltonian of the quantum dot to be

$$H_{QD} = \left( -\frac{\hbar^2}{2m^*} \partial_x^2 - i\alpha_R \partial_x \sigma_y - \mu + V_{\text{dot}}(x) \right) \tau_z + V_x \sigma_x - i\Gamma,$$ \hspace{1cm} (3)

where $V_{\text{dot}}(x) = V_D \cos(3\pi x/2l_D)$ is the confinement potential in the quantum dot. The quantum dot length $l_D$ is only a fraction of the total nanowire length $L$. The precise form of the quantum dot potential is irrelevant for our consideration as no qualitative conclusion depends on these details. A more thorough discussion of the model can be found in Ref.\textsuperscript{27}

The leads in the set-up in Fig. 1 are described by the BdG Hamiltonians

$$H_{\text{lead}} = \left( -\frac{\hbar^2}{2m^*} \partial_x^2 - i\alpha_R \partial_x \sigma_y - \mu + E_{\text{lead}} \right) \tau_z + V_x \sigma_x - i\Gamma,$$ \hspace{1cm} (4)

where an additional on-site energy $E_{\text{lead}}$ is added as a gate voltage. Each lead induce a NS tunnel barrier at the junction connected to the nanowire. The tunnel barrier that controls the conductance into the Majorana nanowire is described by a BdG Hamiltonian:

$$H_{\text{barrier}} = \left( -\frac{\hbar^2}{2m^*} \partial_x^2 - i\alpha_R \partial_x \sigma_y - \mu + V_{\text{barrier}}(x) \right) \tau_z + V_x \sigma_x - i\Gamma,$$ \hspace{1cm} (5)

where $V_{\text{barrier}} = E_{\text{barrier}} \Pi_{\text{barrier}}(x)$ is a box-like potential with height $E_{\text{barrier}}$ and width $l_{\text{barrier}}$.

Note that the Hamiltonian in Eq. (1)-(5) do not overlap with each other in real space in our calculations. From the left most end, it starts with $H_{\text{lead}}$, and then $H_{\text{barrier}}, H_{QD}, H_{\text{NW}}, H_{\text{barrier}}$ and $H_{\text{lead}}$ to the right, as they are setup in Fig. 1. Other than the infinitesimal dissipation term $i\Gamma$, mostly from the vortices in the parent superconductor, that may decrease the conductance,\cite{44,45} we also take into account the effect of temperature, which also smears the conductance profiles. The finite-temperature conductance $G_T$ is then calculated from the zero-temperature conductance $G_0$ by the convolution of the derivative of the Fermi-Dirac distribution, i.e.,

$$G_T(V) = -\int_{-\infty}^{\infty} dE G_0(E) \frac{df(E-V)}{dE}$$ \hspace{1cm} (6)

We numerically calculate the zero-temperature conductance $G_0 = dI/dV$ by discretizing the Hamiltonian (Eq. (1)-Eq. (5)) into a lattice chain and obtaining the scattering matrix\textsuperscript{46} from the python package Kwant\textsuperscript{47} for quantum transport. The zero-temperature conductance (in unit of $e^2/h$) is computed using the following formula.

$$G_0 = 2 + \sum_{\sigma,\sigma'=\uparrow,\downarrow} \left( |r_{\sigma\sigma'}|^2 - |r_{\sigma'\sigma}|^2 \right),$$ \hspace{1cm} (7)

where $r_{\sigma\sigma'}$ and $r_{\sigma'\sigma}$ are the Andreev and normal reflection amplitudes, respectively. The factor of 2 in Eq. (7) is contributed by the two spin channels while we consider a one-subband system. The generalization to multisubband situations is straightforward, but adds no new element to the left/right tunneling correlations being discussed in this work.

The conductance in the tunneling limit can be understood from the local density of states that can be
estimated from the energy spectrum $E$ and the wavefunction density $|\Psi(x)|^2$. To calculate these, we ignore the tunnel barrier effect from the leads, considering only the semiconductor nanowire coupled with the quantum dot. The Hamiltonian is a combination of Eq. (1) and Eq. (3).

$$H_{tot} = H_{QD} + H_{NW} + H_{t},$$
$$H_{t} = u + u^\dagger = \hat{f}_A \left(-t\delta_{\alpha\beta} + i\alpha R\sigma_{\alpha\beta}^R\right) \hat{c}_\beta + h.c., \tag{8}$$

where $H_{QD}$ is the isolated quantum dot, $H_{NW}$ is the semiconductor nanowire and $H_{t}$ is the coupling between them. $\hat{f}$ creates an electron at the end of the dot adjacent to the nanowire, while $\hat{c}$ annihilates an electron at the end of the nanowire connected to the dot. Then we diagonalize the total Hamiltonian in Eq. (8), obtaining the spectrum shown in Fig. 2-13.

Our goal in this paper is to see the (non)correlations of the ZBCPs probed from leads on both ends when the MBS (ABS) appear. We not only analyze the conductance, but also carefully look into the form of the lowest lying wave functions and the nanowire energy spectra. Thus, it is important for us to separate the eigenstates of Eq. (8) and combine them into the form of Majorana modes. Consider a low-energy eigenfunction $\phi_\epsilon$ of a positive energy $\epsilon \ll \Delta$ such that this eigenfunction at position $n$ is represented as $\phi_\epsilon(n) = (u_n, u_{n+1}, v_n, v_{n+1})^T$ in Nambu space. Particle-hole symmetry guarantees the existence of a eigenfunction of negative energy $-\epsilon$ described by $\phi_{-\epsilon}(n) = (v^*_n, v^*_{n+1}, u^*_n, u^*_{n+1})^T$. Combining these eigenstates, we construct the states of the form satisfying the Majorana conditions in Eq. (9).

$$\psi_A(n) = \frac{1}{\sqrt{2}} [\phi_\epsilon(n) + \phi_{-\epsilon}(n)] \tag{9a}$$
$$\psi_B(n) = -\frac{i}{\sqrt{2}} [\phi_\epsilon(n) - \phi_{-\epsilon}(n)] \tag{9b}$$

Here, $\psi_\alpha(n) = (u_{\alpha n\uparrow}, u^*_{\alpha n\downarrow}, v^*_{\alpha n\uparrow}, v_{\alpha n\downarrow})^T$ have a spinor structure, where $\alpha = A, B$. Besides, $u_{\alpha n\sigma} + v^*_{\alpha n\sigma}$ and $u_{B n\sigma} = -i(u_{A n\sigma} - v^*_{A n\sigma})$, which meet the Majorana condition. Generally, $\psi_A$ and $\psi_B$ are not eigenstates of the BdG Hamiltonian, except for $\epsilon = 0$, while the Majorana representation of the eigenstates of the BdG Hamiltonian, $\phi_{\pm \epsilon} = \frac{1}{\sqrt{2}}(\psi_A \pm i\psi_B)$ is generic.

In our calculations, we obtain the Majorana-form wave function probabilities $|\psi_A|^2$ and $|\psi_B|^2$ over the spatial space in the nanowire, by first summing over the inner degrees of freedom (spin, particle-hole spaces). If $|\psi_A|^2$ and $|\psi_B|^2$ are localized separately at two ends of nanowire, then they are an MBS pair, which means the ZBCP we see in the corresponding conductance plot results from Majorana zero modes. On the contrary, if we cannot separate them clearly by Eq. (9), then the ZBCP comes from ABS. By comparing the behavior of wavefunctions on both ends, we can distinguish the sources of ZBCP. We emphasize that the trivial almost-zero-energy ABS here are all composed of double MBS modes which overlap strongly spatially. When the two MBS are well-separated without overlap, being localized at the two ends of the wire, we have topological Majorana zero modes.

Considering that the theoretical methodology in our current work is standard (although the questions we ask and answers we provide are new) and has been widely studied in the literature,9,11,27,42,44–46,48 we do not provide any further details about the theory. Instead, we focus on the numerical results we compute based on above methods.

The parameters in all of our numerical results (Fig. 2-17) are chosen as follows (with the InSb nanowires in mind although we do not attempt any quantitative comparison with experiments because of the large number of unknown parameters in the semiconductor-superconductor hybrid system, e.g., the spin-orbit coupling, the effective mass, the effective g-factor, the lead-nanowire tunnel coupling, the superconductor-nanowire tunnel coupling, the chemical potential, the active wire length, the applicable coherence length in the nanowire, quantum dot confinement potential). The effective mass is $m^* = 0.015m_e$, nanowire length $L = 5\mu m$, induced superconducting gap $\Delta_0 = 0.9$ meV, spin-orbit coupling $\alpha_R = 0.5$ eVÅ. The gate voltage in the lead is $E_{lead} = -25$ meV, with the induced tunnel barrier height $E_{barrier} = 10$ meV, and the barrier length $l_{bar} = 20$ nm. The strength of the confinement potential in the quantum dot is $V_D = 4$ meV, with length $l_D = 0.3\mu m$. The temperature, which smears the conductance profile by thermally broadening all sharp features, is set at 0.02 meV. The phenomenological dissipation parameter is $\Gamma = 0.01$ meV. The above parameters will be fixed throughout for all the cases in our results, and other tuning parameters, including the chemical potential $\mu$, the Zeeman energy $V_z$, along with the superconducting gap collapsing point $V_c$, will be provided in the captions of the figures. The topological quantum phase transition (TQPT) field $V_{Zc}$, $\sqrt{\mu^2 + \Delta_0^2}$ is also provided in each case. We note that we only consider the case where $V_c > V_{Zc}$, so in principle, the topological regime exists for $V_z < V_c < V_z$ so that MBS induced ZBCPs can manifest itself. Experimentally, the situation $V_z < V_{Zc}$ is allowed, and in such a case, all ZBCP must arise from trivial MBS.

### III. RESULTS AND DISCUSSIONS

The key idea underlying the current work is simple: Topological Majorana modes have nonlocal correlations, and any zero bias peak associated with MBS must manifest itself in tunneling from each end of the wire since MBS must always exist in pairs at both ends, whereas by contrast ABS is a non-topological subgap fermionic bound state which will be randomly localized near one or the other end of the wire and as such will have no nonlocal correlations. The implication of this simple idea is that ABS induced ZBCP would arise only when tunneling connects to the relevant ABS which is necessarily at one end, thus ABS induced ZBCP are not correlated from both
ends whereas MBS induced ZBCP must necessarily be correlated. The details of this implication are, however, quite subtle (and depend crucially on system parameters, particularly, the chemical potential) as the results of our extensive numerical simulations of nanowire tunneling conductance show. We discuss these results below.

We present our main results in a set of twelve figures (each with five panels except for Figs. 2 and 3 consisting of four panels each—see below) in Figs. 2-13 with each figure corresponding to the chemical potential (ranging between 0 meV and 8 meV) increasing with the figure number. All system parameters other than the chemical potential are fixed in Figs. 2-13 in order to facilitate a one to one comparison among the results. Since the topological regime depends on the chemical potential with the TQPT field \( V_{Zc} \) increasing with increasing chemical potential, the applicable Zeeman field (along the x axis of the figures) also increases with increasing chemical potential. The five panels in each figure (except for Figs. 2 and 3 which contain only four panels each) describe the following calculated results: tunneling conductance (in color) from left (a) and right (b) wire ends as a function of Zeeman field along x-axis and bias voltage along y-axis; lowest wave functions at a low Zeeman field below TQPT (\( V_z < V_{Zc} \)) (c) and at a high Zeeman field above TQPT (\( V_z > V_{Zc} \)) (d); the low-lying energy spectra (e). Figures 2 and 3, at the lowest values of chemical potential (\( \mu = 0 \) meV, 1 meV, respectively in Figs. 2 and 3), contain 4 panels only with the wave function results shown in panel (c) only at one value of \( V_z > V_{Zc} \) and panel (d) showing the energy spectrum. The reason we do not show the wave function for \( V_z < V_{Zc} \) in Figs. 2 and 3 is that the system does not manifest any MBS-induced ZBCP for \( V_z < V_{Zc} \) for low values of chemical potential (as can be seen in panels a and b in Figs. 2 and 3 where no ZBCP manifests in the non-topological regime of \( V_z < V_{Zc} \)), and therefore, the wavefunctions for \( V_z < V_{Zc} \) become moot for Figs. 2 and 3.

In Figs. 2 and 3, the chemical potential being relatively small (\( \mu < \Delta_0 \) in Fig. 2 and \( \mu \sim \Delta_0 \) in Fig. 3), no zero energy ABS form, as is known from earlier theoretical work.\(^{27}\) In Fig. 2 (3), the prominent ZBCP for \( V_z > V_{Zc} = 0.9 \) (1.0) meV manifest for tunneling from both ends indicating that this ZBP indeed arises from MBS and is topological in nature. The MBS induced ZBCPs are correlated, and hence they must appear in tunneling from both ends in the same magnetic field range as can be seen in Figs. 2 and 3. Although the ZBCP from both ends manifest in the identical Zeeman field range in Figs. 2 and 3, their strengths or conductance values are, however, not identical since the magnitude of the conductance depends on many parameters, most importantly on the effective tunneling strength,\(^{40}\) which are different at the two ends because of the existence of the quantum dot at the left. Hence, in general, we expect the MBS-ZBCP from the left tunneling to be weaker at finite temperature than the MBS-ZBCP from the right tunneling. This is indeed the case in Figs. 2 and 3, but the occurrence of the two ZBCPs from the two ends must be correlated with respect to the magnetic field if they arise from an MBS origin. If at a particular Zeeman field range, ZBCPs exist for tunneling from both ends, we can be pretty sure that these ZBCP are arising from MBS. We emphasize that although the MBS-ZBCPs from the two ends are correlated with respect to their simultaneous occurrence in the same field range in the same sample, the conductance magnitudes for the left and right ZBCPs do not need to be the same (except that the conductance must not exceed \( 2e^2/h \)). They will have the same strength only at \( T = 0 \) independent of tunneling strength, given by the universal quantized value \( 2e^2/h \), but at any finite temperature, the MBS-ZBCPs from the two ends in general would have different strengths because of differing tunneling at the two ends of the sample.\(^{10}\) Both in Figs. 2 and 3, the left and right ZBCPs occur over the same Zeeman field ranges, but with differing strengths. We also show the wavefunctions for the low lying states for \( V_z > V_{Zc} \) in Figs. 2 and 3, which are clearly localized at the two wire ends, thus validating their MBS nature. The calculated energy spectra also show that the only prominent zero energy states in Figs. 2 and 3 are in the topological regime (i.e. ZBCPs occurring only for \( V_z > V_{Zc} \)). We mention that in Fig. 3 (in panels a and d) one can see the beginning of a weak formation of an ABS just below the TQPT (\( V_z \sim 1.35 \) meV), but this ABS quickly hybridizes with the MBS arising for \( V_z > 1.35 \) meV, and is not clearly distinguishable as a distinct ABS. We therefore conclude that the correlated ZBCPs showing up in Figs. 2 and 3 for \( V_z > V_{Zc} \) are all MBS-induced, and are therefore observable in the tunneling spectra from both ends.

In the ten figures, Figs. 4-13, we show contrasting ZBCPs arising from both ABS (left tunneling only) and MBS (both left and right tunneling) with the ABS-induced ZBCP being qualitatively similar to the MBS-induced ZBCP except that they only manifest for tunneling from the left end. By contrast, the MBS-induced ZBCP (for \( V_z < V_{Zc} \)) always manifest in a correlated manner in tunneling from both ends. The wavefunctions for MBS are individually localized at both ends of the wire implying topological nonlocality of isolated Majorana modes. By contrast, the ABS wavefunctions are localized at the left end (close to the quantum dot creating the ABS) and are composed of two strongly overlapping Majorana states. Because of the overlapping (isolated) nature of ABS (MBS) wavefunctions, the ZBCPs are unprotected (protected) for ABS (MBS), allowing the ABS to lead to ZBCP in the topologically trivial (\( V_z < V_{Zc} \)) phase. The calculated energy spectra show that the ABS (MBS) energy corresponds to \( V_z < (>) V_{Zc} \). The most important aspect of the results shown in Figs. 4-13 from our perspective is that the ZBCPs induced by MBS (ABS) manifest for tunneling from both ends (just the left end) of the wire. Thus, the existence of correlated ZBCPs in tunneling from both ends provides strong support for the existence of protected topological Majorana modes whereas the manifestation of ZBCP in tunneling just from
one end and not the other indicates the existence of non-topological Andreev bound states. The central message conveyed by Figs. 4-13 is precisely what is stated in the title of our paper: Presence versus absence of end-to-end nonlocal conductance correlations in nanowires indicates the existence (or not) of Majorana bound states versus Andreev bound states.

In Figs. 14-16, we show the line cuts of the tunneling conductance from both ends (as a function of bias voltage) at fixed Zeeman splitting values for various chemical potentials, both below (i.e. the trivial regime, $V_z < V_{Z,c}$) and above (i.e. the topological regime, $V_z > V_{Z,c}$) TQPT so that the tunneling physics of both ABS and MBS are manifest, clearly demonstrating the role of end-to-end correlations in determining the identity of Majorana zero modes. In Fig. 14, we show right and left tunneling conductance comparisons for low values of chemical potential, $\mu = 0$ and 1 meV corresponding to Figs. 2 and 3 respectively. Here, the ZBCPs are all induced by MBS, occurring at $V_z > V_{Z,c}$, and as such, produce correlated conductance peaks at the same $V_z$ from both ends although the peak conductances are very different for tunneling from left and right ends for the chosen parameter values. This difference in the conductance strength arises from the presence of the quantum dot at the left end, and at $T = 0$ both right and left peaks will have the appropriate Majorana quantized conductance of $2e^2/h$ although at finite temperature, the actual conductance depends on all the system parameters as discussed already in the literature. It is therefore important to emphasize that end-to-end ZBCP correlations for MBS implies the existence of the ZBCP from both ends for the same values of $V_z$, but not that the tunnel conductances from two ends must have the same magnitude. In general, their magnitudes will be different at finite temperatures under realistic conditions since the tunnel couplings are in general different at the two wire ends. We see in Fig. 14 that the conductance magnitudes from two ends for MBS-ZBCPs could be very different while being perfectly correlated in terms of the Zeeman field values for their occurrence. In contrast to Fig. 14 which presents conductance line cuts for low values of chemical potential, we show in Fig. 15 comparative left and right conductance line cuts for large chemical potentials ($\mu = 5, 6, 7, 8$ meV all much larger than the induced superconducting gap 1 meV) at low values of $V_z(< V_{Z,c})$ in the trivial regime below TQPT. In each panel of Fig. 15, there is a prominent ABS-induced ZBCP in the left conductance, but not in the right, clearly establishing that the very prominent left ZBCP in these results must necessarily be ABS-induced features since these left ZBP do not correlate with any corresponding right ZBCP at the same Zeeman field. We note that the ABS-induced ZBCPs from the left in Fig. 15 (blue solid curves in Fig. 15) look essentially identical to the MBS-induced ZBCPs from the right in Fig. 14 (orange dashed curves in Fig. 14), the only difference between the two situations being that in Fig. 15 there is no corresponding ZBCP from the right (i.e. the orange dashed curves in Fig. 15 show basically zero conductance) whereas the corresponding left conductance in Fig. 14 do manifest ZBPs (i.e. the blue solid curves in Fig. 14 have small discernible peaks correlated with the prominent ZBCPs from the right). Thus, end-to-end ZBP correlation is the key to distinguishing between ABS and MBS—if the two ends do not exhibit simultaneous zero bias peaks, a ZBCP arising just from one end (and not from the other) is in all likelihood a trivial ABS peak, no matter how impressive the peak may be. We note that the ABS-induced ZBPs in Fig. 15 could be close to or above (below) the quantized value of $2e^2/h$, depending on the details of temperature, tunneling, and other parameters as already discussed extensively in the literature. The precise value of the ABS-induced ZBCP depends on the details of the tunneling matrix elements near the quantum dot, and could easily be exactly $2e^2/h$ since the tunnel probe may connect predominantly with one and not the other Majorana mode combining to form the ABS. In the idealized situation of $T = 0$ and strong tunneling, the ABS and MBS induced ZBCPs should have values of $4e^2/h$ and $2e^2/h$, respectively, but this ideal situation is rarely achieved at finite temperatures. We emphasize again that the conductance peak values from the two ends do not need to be identical in the realistic experimental situation for the ZBCP to arise from MBS, the necessary correlations are in the simultaneous existence of ZBCPs over the same range of magnetic field values in tunneling from both ends. In Fig. 16, we show extensive comparisons between right and left conductance at fixed magnetic field values for different chemical potentials ($\mu = 2$ to 8 meV). In each case, we show calculated examples of both trivial ZBCPs caused by ABS existing (below TQPT) only for tunneling from left (and not from right) as well as correlated ZBCPs caused by MBS in the topological regime (above TQPT) existing in tunneling from both ends. The presence (absence) of end-to-end conductance peak correlations manifesting in MBS (ABS) is obvious in Fig. 16, clearly bringing out the key message of our work. For MBS-induced ZBCP, the conductance magnitudes from right and left tunneling should become increasingly equal as the magnetic field increases, particularly for large values of chemical potential, as can be seen in Fig. 16.

In Fig. 17, we show the calculated comparative conductance plots from two wire ends (similar to Fig. 2-13) for the strong-coupling superconductor-semiconductor proximity model, where the tunnel coupling between the parent superconductor and the nanowire is large so that the induced superconducting gap in the nanowire is now limited by the parent gap and not by the tunneling at the interface (as it is in the weak coupling results presented so far in Figs. 2-16 where the proximity gap in the nanowire is smaller than the parent gap). The strong coupling model must necessarily incorporate the dynamical self-energy effects, and the detailed theory has been already provided in the literature, which we do not reproduce here. The calculated tunneling conductance results in Fig. 17 for various chemical potentials show the same qualitative
FIG. 2. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions above \( V_{Zc} = 0.90 \) meV (yellow dashed line in (a) and (b)), for the case of chemical potential \( \mu = 0 \) meV. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. (c): lowest lying wave functions in the nanowire at \( V_z = 1.5 \) meV (purple dashed line in (a) and (b)), above \( V_{Zc} \), which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where \( V_{Zc} \) marks. The superconducting collapsing field is at \( V_c = 3.2 \) meV.

FIG. 3. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions above \( V_{Zc} = 1.35 \) meV (yellow dashed line in (a) and (b)), for the case of chemical potential \( \mu = 1.0 \) meV. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. (c): lowest lying wave functions in the nanowire at \( V_z = 2.0 \) meV (purple dashed line in (a) and (b)), above \( V_{Zc} \), which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where \( V_{Zc} \) marks. The superconducting collapsing field is at \( V_c = 3.2 \) meV.

features as what is described above for the weak-coupling results presented in Figs. 2-16. In particular, the MBS-induced ZBCPs for \( V_z > V_{Zc} \) always manifest together in a correlated manner from both ends of the wire whereas the ABS-induced ZBCPs for \( V_z > V_{Zc} \) manifest only for tunneling from the left end without manifesting any correlations. Thus, our conclusion about the presence (absence) of end-to-end correlations in the ZBCP implying the presence of topological MBS (trivial ABS) in the system remains valid in the presence of strong-coupling proximity effect. Note that there are quantitative differences between the two approximations in the details of conductance magnitudes and TQPT points and so on as one would expect, but the key point of ZBCP correlations from the two ends being a decisive signature for MBS is equally applicable to both models.

In addition to conductance correlation between the two ends, the three terminal set-up in Fig. 1 allows access to two cross conductances \( G_{LR} = dI_L/dV_R \) and \( G_{RL} = dI_R/dV_L \). Such a non-local conductance has been proposed as a way to distinguish bulk states from potential inhomogeneity-induced sub-gap states. The appearance of bulk states at the gap closure of the Majorana nanowire is a hallmark of the TQPT at which Majorana modes are supposed to appear. Such a TQPT is thus characterized by a signature associated with bulk gap closure as seen in the cross-conductance \( G_{LR} \) versus voltage (Fig. 18(a)). While, as a matter of principle, the cross-conductance reveals the bulk gap closure, the cross-conductance is constrained to vanish at zero-voltage by particle-hole symmetry, which has motivated other non-local signatures such as heat transport and non-linear conductivity. Because of this, the gap-closure at the critical Zeeman field in Fig. 18(a) appears more as a soft-gap where the conductance continues to vanish at zero voltage. In Fig. 18(b), we plot the cross-conductance as a function of voltage for the case with a quantum dot on the left. We find that the quantum dot strongly suppresses the bulk gap closing signature near the critical Zeeman field by reducing the conductance scale further near zero voltage. Therefore, the observation of gap closing in cross-conductance might require appropriate tuning/engineering of the end potential to eliminate the effect of the quantum dots on the cross-conductance. This can probably be done with additional gate voltages to tune system parameters appropriately.

IV. CONCLUSION

We have shown by extensive numerical simulations that the zero bias tunneling conductance peaks arising from trivial ABS can be distinguished from those arising from topological MBS by comparing separate tunneling measurements carried out simultaneously from the two ends of the wire. The fact that the MBS-induced ZBCPs are
FIG. 4. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 1.75 \text{ meV}$ (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 1.5 \text{ meV}$. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V_z = 1.5 \text{ meV}$, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V_z = 1.65 \text{ meV}$ (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V_z = 2.5 \text{ meV}$ (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 3.7 \text{ meV}$.

FIG. 5. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 2.19 \text{ meV}$ (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 2.0 \text{ meV}$. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V_z = 1.75 \text{ meV}$, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V_z = 2.0 \text{ meV}$ (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V_z = 3.0 \text{ meV}$ (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 4.7 \text{ meV}$.

FIG. 6. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 2.66 \text{ meV}$ (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 2.5 \text{ meV}$. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V_z = 1.45 \text{ meV}$, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V_z = 2.5 \text{ meV}$ (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V_z = 3.0 \text{ meV}$ (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 5.7 \text{ meV}$.
nonlocally correlated in tunneling measurements from the two ends was already emphasized in Ref., but the current work incorporates the complications arising from the existence of nontopological ABS in the wire, a situation not addressed in Ref.. The basic idea behind our work is simple: Topological MBS are intrinsic nonlocal objects with their wavefunctions existing at both ends of the wire whereas the nontopological ABS can only exist near one or the other end depending on the extrinsic details giving rise to the ABS. Thus, an NS tunneling measurement from a particular wire end can only probe an ABS if it exists near that end whereas the MBS, if it exists, must be equally accessible from both ends because of its nonlocal nature. This simple physics is reflected in our simulations and should be observable experimentally. There are complications, of course, as the ABS may interfere with the ZBCP produced by the MBS in the topological regime. Such ABS-MBS interplay may manifest itself as small kinks or anti-crossing features in the zero bias tunnel conductance more prominently from one end than the other (and has in fact been proposed as evidence for MBS by itself in recent publications) as the level repulsion between the ABS and the MBS is likely to be stronger near one end because of the wavefunction overlap effect. We clearly see such anti-crossing features in several of our simulations, where one can see extrinsic ABS coming down and anticrossing with MBS features near zero bias, but these anticrossings affect only very small regions of the Zeeman field whereas the MBS-ZBCP conductance lasts over a large Zeeman field regime. These small level repulsion effects, therefore, do not detract from our key finding that an ABS-induced ZBCP would only show up in the tunnel conductance from one end and not the other whereas an MBS-induced ZBCP will show up in the tunneling spectra from both ends. A comparison between the tunnel conductance from the two ends as a function of the same range of values of the Zeeman field should show robust correlated ZBCPs from both ends if they arise from MBS and uncorrelated ZBCPs from the two ends at different values of the magnetic field if they arise from ABS. It should therefore be possible to validate the MBS existence from the ZBCP provided the same ZBCP shows up in the tunnel conductance spectra from both wire ends in exactly the same Zeeman field range. We emphasize, however, that at finite temperatures the correlated MBS-induced ZBCPs from the two ends will exist robustly at the same range of values of the magnetic field, but their conductance values may differ in magnitudes since the existence of a quantum dot (or some other potential fluctuation in the wire) will modify the tunneling strengths at the two ends in general. Thus, the nonlocal correlations (or not) are only in the precise agreement in the magnetic field range where the ZBCPs show up—for MBS the two magnetic field ranges from the two ends coincide whereas for ABS they do not. Of course, as is the case in our simulations in the ABS case, MBSs can ultimately appear at large Zeeman fields even when the ZBCPs do not correlate at the ends. However, it is rather difficult to ascertain MBSs in this case. The conductance values from the two ends for the MBS-induced ZBCPs should come closer together as temperature decreases and/or chemical potential increases.

One possible complication in the context of end-to-end tunneling conductance correlations is the finite (and often rather short) length of the Majorana nanowire used in the experiments. In general, the wire length should be much larger than the superconducting coherence length so that the wavefunction overlap between the two MBS at the two ends is negligible, leading to the concept of isolated MBS. Only such isolated MBS are topological anyons, and, as such, distinguishable from nontopological ABS. If the two MBS at the two ends overlap considerably because of the short wire length, then the MBS should be considered equivalent to an ABS since an ABS in a Majorana nanowire is basically composed of two nearby or strongly overlapping MBSs. Overlapping MBS from the two wire ends typically produce “Majorana oscillations” as a function of increasing Zeeman field (which suppresses the superconducting gap leading to an enhancement of the coherence length, thus reducing the dimensionless wire length) if the wire is short. In the effective topological regime, these Majorana oscillations in the conductance should be quantitatively correlated in the tunneling spectra from the two ends. By contrast, ABS manifest fairly random structures around zero bias as a function of the Zeeman field, and hence the ABS features in the tunneling spectra from the two wire ends would be generically uncorrelated. A significant difference between ABS-and MBS-induced field-dependent structures around zero bias is that for MBS-induced Majorana oscillations, the conductance oscillation amplitude grows monotonically with increasing magnetic field in a predictable manner whereas the ABS features are random, which may increase or decrease or even disappear with changing magnetic field.

It is interesting to note that the Majorana nanowire experiments manifesting ZBCPs typically show little signatures of Majorana oscillations. Nothing mimicking the theoretical predictions of Majorana oscillations has ever been reported in any existing nanowire experiments although a few experiments see some structures sometimes around zero bias which could be construed rather arguably as one or two oscillations. The lack of Majorana oscillations in nanowire experiments remains a mystery, and may be a reflection of strong dissipation or effectively high electron temperature or bad energy resolution around zero bias suppressing any oscillatory structure or simply a long effective wire length (i.e. equivalently a short effective coherence length). One experiment with a putative observation of Majorana oscillations manifests oscillation amplitudes decreasing with increasing magnetic field directly contradicting the theoretical predictions and has been explained on the basis of the existence of ABS rather than MBS in the system. Since ABS induced ZBCPs are never exactly at zero energy, they may reflect “oscillatory” structures around zero bias as a function of magnetic field, but the amplitudes of this oscillatory
FIG. 7. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 3.13$ meV (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 3.0$ meV. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V_z = 1.3$ meV, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V_z = 2.8$ meV (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V_z = 4.0$ meV (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 6.7$ meV.

FIG. 8. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 3.91$ meV (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 3.8$ meV. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V_z = 1.5$ meV, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V_z = 2.5$ meV (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V_z = 4.5$ meV (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 8.2$ meV.

FIG. 9. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 4.59$ meV (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 4.5$ meV. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V_z = 1.7$ meV, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V_z = 2.0$ meV (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V_z = 5.0$ meV (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 9.4$ meV.
structure could increase or decrease with increasing magnetic field in a random manner. These oscillations in the ZBCPs should be correlated (uncorrelated) depending on whether they arise from MBS (ABS) as our work shows clearly. One unfortunate logical possibility underlying the lack of any experimentally observed Majorana oscillations is that all observed ZBCPs arise from ABS, hence no MBS oscillations. Another logical possibility for the lack of experimental Majorana oscillations is that perhaps the experimental coherence length is much shorter than the theoretical estimate, which is equivalent to the experimental wire length being effectively very large. We note that our chosen wire length is longer than the experimental wire lengths in order to avoid complications arising in the simulations from Majorana oscillations which are invariably present in any theoretical MBS simulations for shorter wires.

One experimental (or materials) issue may severely compromise the observation (and verification) of MBS in realistic nanowires which we now discuss. This is the issue of the experimentally observed collapse of the parent bulk gap at some magnetic field (≈ 1T) universally happening in all the Majorana nanowire experiments. The reason for this gap collapse is unclear, and one possibility is a strong magnetic field induced orbital effect in the parent superconductor. It could also be arising simply from the parent SC reaching the Clogston limit with the Zeeman spin splitting becoming equal to the bulk SC gap. Denoting the corresponding Zeeman splitting for this gap collapse by $V_c$ (to be contrasted with the critical TQPT field $V_z$, defining the topological regime $V_c > V_z$), it is obvious that if $V_c < V_z$, MBS cannot appear in the system as an isolated anyon. Unfortunately, $V_z$ is unknown in experiments, only $V_c$ is known. Typically, nothing happens experimentally for $V_z > V_c$ with all conductance signals in the nanowire basically vanishing above this parent SC gap collapse point. If $V_z > V_c$ generically (e.g. perhaps because the chemical potential is always rather large, making $V_z$ also large), then all observed ZBCPs arise from ABS and MBS are simply inaccessible until $V_z < V_c$ can be achieved by tuning system parameters. Obviously, any topological MBS can exist only in the Zeeman field regime $V_z < V_c < V_z$. Therefore, if $V_c < V_Z$, with the SC bulk gap collapsing before $V_z$ reaches the TQPT point), our proposed nonlocal correlation experiments would not work, and all observed ZBCPs would manifest no correlations in the tunneling data sets from the two wire ends, since they all must be ABS in such an unfortunate scenario. Making $V_z > V_z$ is an all-important materials challenge deserving serious experimental efforts for achieving further progress in the search for topological Majorana modes in superconductor-semiconductor hybrid systems.

Finally, we discuss possible false signals in our proposed end-to-end NS tunneling conductance correlation measurements. In particular, we address the important question: Is it possible for the ABS-induced ZBCPs to manifest apparent end-to-end correlations which the MBS-induced ZBCPs are generically guaranteed to manifest? The answer is that although it is, in principle, possible for ABS-induced ZBCPs from the two ends to manifest an apparent correlated behavior occasionally, it is highly unlikely and such a behavior is completely non-generic in contrast to the MBS situation. End-to-end correlations in ABS-induced ZBCPs necessitate an extremely fine-tuned system where there are two accidental, but effectively identical, quantum dots in the nanowire near the two ends, i.e., the nanowire has an exact reflection symmetry purely fortuitously even in the presence of extrinsic potential fluctuations! In such a highly unlikely (but not entirely impossible) scenario, there could be accidental equivalent ABS-formation at both wire ends, giving rise to an apparent correlated behavior in the ABS-induced ZBCP. It should, however, be relatively easy to rule out such an accidental end-to-end apparent correlation in the ABS-induced ZBCPs by tuning the gate voltage near the wire ends which should modify the extrinsic quantum dots, suppressing the non-generic accidental end-to-end ABS correlations. By contrast, the end-to-end MBS correlations are generic arising from the nonlocal topological nature of the Majorana zero modes, and their end-to-end correlations should remain robust to any tuning of gate voltages near the wire ends. It is possible for ABSs to manifest spurious end-to-end correlations in the very short wire situation, where the ABS formed at one end of the wire may still have overlap with the tunneling lead at the other end because the wire length is shorter than the superconducting coherence length. This is also the same situation where the MBSs manifest strong end-to-end correlated Majorana oscillations because of the short wire length. Therefore, such a spurious end-to-end correlations in ABS conductance could be easily checked by increasing the magnetic field to see if correlated Majorana oscillations manifest from both ends or not. In such a short wire (with wire length < coherence length) situation, neither the ABS nor the MBS are topologically protected, and should be avoided by making the wire longer. Since the existing semiconductor nanowire experiments do not manifest any Majorana oscillations at any magnetic field, we have not considered in the current work the situation with very short wires. We do note an extremely unfortunate scenario, where, if $V_c < V_z$, then it is possible in short wires for the ABS to manifest spurious end-to-end ZBCP correlations without the system manifesting any Majorana oscillations at higher fields, since the system enters the hopelessly regime of $V_z < V_c$ before it can enter the topological regime of $V_z > V_c$. In such a situation, topologically protected MBS simply cannot exist in the system by virtue of $V_z < V_c$, and the issue of end-to-end ZBCP correlations becomes moot since all ZBCPs in such a case arise from ABS. Our current work assumes that the experimental systems do not belong to this unfortunate scenario.

We believe that the end-to-end tunneling correlation measurements can be carried out in the laboratory right now, and are encouraged by the fact that several groups
FIG. 10. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 5.08$ meV (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 5.0$ meV. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V_z = 1.5$ meV, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V_z = 1.9$ meV (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V_z = 6.0$ meV (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 10.4$ meV.

FIG. 11. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 6.07$ meV (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 6.0$ meV. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V_z = 1.7$ meV, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V_z = 2.0$ meV (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V_z = 7.0$ meV (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 12.4$ meV.

FIG. 12. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 7.06$ meV (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 7.0$ meV. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V_z = 2.05$ meV, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V_z = 2.5$ meV (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V_z = 8.0$ meV (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 14.4$ meV.
FIG. 13. Differential conductance measured from both ends, energy spectrum, and the lowest lying wave functions below and above $V_{Zc} = 8.05$ meV (yellow dashed line in (a) and (b)), for the case of chemical potential $\mu = 8.0$ meV. (a): differential conductance measured from the left lead. (b): differential conductance measured from the right lead. The cyan dashed line in (a) and (b) is at $V = 1.7$ meV, where the ZBCP first starts. (c): lowest lying wave functions in the nanowire at $V = 2.5$ meV (first purple dashed line in (a) and (b)), below $V_{Zc}$, which cannot be separated into Majorana wave functions on both ends of nanowire. (d): lowest lying wave functions in the nanowire at $V = 8.5$ meV (second purple dashed line in (a) and (b)), above $V_{Zc}$, which are separated into Majorana wave functions on both ends of nanowire. (d): energy spectrum. The black dashed line is where $V_{Zc}$ marks. The superconducting collapsing field is at $V_c = 16.4$ meV.

FIG. 14. Line cuts of the tunneling conductance as a function of bias voltage from both ends (The blue solid line is from the left end, while the orange dashed line is from the right end.), for low values of chemical potential $\mu$ and $V_z \geq V_{Zc}$. (a): $\mu = 0$ meV and $V_z = 1.5$ meV, with $V_{Zc} = 0.90$ meV. (b): $\mu = 0$ meV and $V_z = 2.5$ meV, with $V_{Zc} = 0.90$ meV. (c): $\mu = 1.0$ meV and $V_z = 1.35$ meV, with $V_{Zc} = 1.35$ meV. (d): $\mu = 1.0$ meV and $V_z = 2.0$ meV, with $V_{Zc} = 1.35$ meV. Note that the superconducting collapsing field is fixed at $V_c = 20$ meV.

FIG. 15. Line cuts of the tunneling conductance as a function of bias voltage from both ends (The blue solid line is from the left end, while the orange dashed line is from the right end), for high values of chemical potential $\mu$ and low values of magnetic field $V_z (< V_{Zc})$. (a): $\mu = 5.0$ meV and $V_z = 5.08$ meV, with $V_{Zc} = 5.08$ meV. (b): $\mu = 6.0$ meV and $V_z = 2.5$ meV, with $V_{Zc} = 6.07$ meV. (c): $\mu = 7.0$ meV and $V_z = 2.5$ meV, with $V_{Zc} = 7.06$ meV. (d): $\mu = 8.0$ meV and $V_z = 3.0$ meV, with $V_{Zc} = 8.05$ meV. Note that the superconducting collapsing field is fixed at $V_c = 20$ meV.

are currently trying to implement such multiprobe tunneling measurements where NS tunneling probes are used in several contacts along the wire. In particular, Grivnin et al. has already shown the way by carrying out a pioneering multiprobe tunneling measurement searching for the simultaneous closing and opening of a bulk gap along with the appearance of a zero bias peak. It should be straightforward to adapt this multiprobe setup to carry out simultaneous NS tunneling measurements from both ends of the wire to look for conductance correlations as proposed in our work. One interesting conclusion of Grivnin et al. is that there are different kinds of conductance zero bias peaks in the nanowires, and not all zero bias peaks are similar. This is of course the precise conclusion of the current work also (ABS-induced and MBS-induced zero bias peaks are fundamentally different with respect to nonlocal correlations), but much more work along the line of correlated tunneling spectroscopy from both ends is necessary before any firm conclusion is possible. In particular, tunnel probes themselves may introduce ABS and hence ABS-induced ZBCPs, complicating the experimental situation, but our conclusion about nonlocal correlations in the MBS-induced ZBCP in contrast to lack of correlations in ABS-induced ZBCPs would still apply.

For completeness, we have also studied the cross-
FIG. 16. Line cuts of the tunneling conductance as a function of bias voltage from both ends. (The blue solid line is from the left end, while the orange dashed line is from the right end) The magnetic fields $V_z$ for the first and the third columns are set below TQPT point ($V_z < V_{Zc}$), while $V_z$ for the second and the fourth columns are set above TQPT point ($V_z > V_{Zc}$). (a): $\mu = 2.0$ meV and $V_z = 1.75$ meV, with $V_{Zc} = 2.19$ meV. (b): $\mu = 2.0$ meV and $V_z = 3.0$ meV, with $V_{Zc} = 2.19$ meV. (c): $\mu = 2.5$ meV and $V_z = 1.45$ meV, with $V_{Zc} = 2.66$ meV. (d): $\mu = 2.5$ meV and $V_z = 4.0$ meV, with $V_{Zc} = 2.66$ meV. (e): $\mu = 3.0$ meV and $V_z = 1.3$ meV, with $V_{Zc} = 3.13$ meV. (f): $\mu = 3.0$ meV and $V_z = 4.0$ meV, with $V_{Zc} = 3.13$ meV. (g): $\mu = 5.0$ meV and $V_z = 1.9$ meV, with $V_{Zc} = 5.08$ meV. (h): $\mu = 5.0$ meV and $V_z = 6.0$ meV, with $V_{Zc} = 5.08$ meV. (i): $\mu = 6.0$ meV and $V_z = 2.5$ meV, with $V_{Zc} = 6.07$ meV. (j): $\mu = 6.0$ meV and $V_z = 8.0$ meV, with $V_{Zc} = 6.07$ meV. (k): $\mu = 8.0$ meV and $V_z = 6.0$ meV, with $V_{Zc} = 8.05$ meV. (l): $\mu = 8.0$ meV and $V_z = 10.0$ meV, with $V_{Zc} = 8.05$ meV. Note that the superconducting collapsing field is fixed at $V_c = 20$ meV.

Conductance that can be measured in the same multi-terminal set-up as the conductance correlation. Such measurements have been proposed as a way to detect the TQPT, which might help separate ABSs and MBSs. However, similar to conductance correlations, the presence of end quantum dot induced ABSs might obscure the signature of the TQPT. Therefore, the failure to observe the TQPT (and the bulk gap closure) in the cross-correlation would not imply the lack of MBSs. This is reminiscent of our discussion of the absence of conductance correlations not ruling out MBSs. At the same time, one could likely combine information from cross-conductance and conductance correlations to find MBSs where both tests give a weak signal. Similarly, the vanishing of cross-conductance in the Majorana wire away from zero energy can be used to estimate the coherence length relative to the length of the wire. This can eliminate the false positive from ABSs in short wires discussed earlier in this section. One serious problem is that current experiments provide no information on either the magnitude of the applicable coherence length or the location of the TQPT (i.e. the value of $V_{Zc}$) since the bulk gap closing feature has not yet been directly observed experimentally.

In conclusion, we have shown that the presence (absence) of correlated zero bias conductance from the two ends as a function of the applied magnetic field could indicate the presence (absence) of topological Majorana (trivial Andreev) bound states in nanowires.

Acknowledgement: This work is supported by Microsoft and Laboratory for Physical Sciences. Yi-Hua Lai thanks Chunxiao Liu, Haining Pan, and Yingyi Huang for helpful discussions and suggestions, particularly on the use of KWANT in the numerical simulations. The authors acknowledge the support of the University of Maryland High Performance Computing Center for the use of Deep Through II cluster for carrying out the numerical work.

1 A. Y. Kitaev, Phys.-Usp. 44, 131 (2001), http://stacks.iop.org/1063-7869/44/i=10S/a=S29.
2 C. Nayak, S. H. Simon, A. Stern, M. Freedman, and
FIG. 17. Differential conductance measured from both ends, including the self-energy. The self-energy coupling constant is $\lambda = 1.5$ meV, which renormalized the induced superconducting gap at $\omega = 0$ to be $\Delta_0 = 1.5$ meV. Note that the superconducting collapsing field is fixed at $V_c = 20$ meV. The first and third columns are measured from the left lead, while the second and fourth columns are measured from the right lead. The yellow dashed line is where $V_{Z_c}$ marks. (a)-(b): $\mu = 0$ meV and $V_{Z_c} = 1.50$ meV. (c)-(d): $\mu = 1.0$ meV and $V_{Z_c} = 1.80$ meV. (e)-(f): $\mu = 1.5$ meV and $V_{Z_c} = 2.12$ meV. (g)-(h): $\mu = 2.0$ meV and $V_{Z_c} = 2.50$ meV. (i)-(j): $\mu = 2.5$ meV and $V_{Z_c} = 2.92$ meV. (k)-(l): $\mu = 3.0$ meV and $V_{Z_c} = 3.35$ meV. (m)-(n): $\mu = 3.8$ meV and $V_{Z_c} = 4.09$ meV. (o)-(p): $\mu = 4.5$ meV and $V_{Z_c} = 4.74$ meV. (q)-(r): $\mu = 5.0$ meV and $V_{Z_c} = 5.22$ meV. (s)-(t): $\mu = 6.0$ meV and $V_{Z_c} = 6.18$ meV. (u)-(v): $\mu = 7.0$ meV and $V_{Z_c} = 7.16$ meV. (w)-(x): $\mu = 8.0$ meV and $V_{Z_c} = 8.14$ meV.
The bulk gap in (a) appears to close and reopen as $V_1$ voltage $V_1$. Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. 151001 (2015), https://www.nature.com/articles/nphys2015.1.  

$\Gamma_{\text{cross}}$ vanishes below the bulk gap. Majorana wire and one with a quantum dot on the left respectively. The cross-conductance vanishes below the bulk gap. Note that the induced tunnel barrier height is $E_{\text{barrier}} = 1$ meV.  

FIG. 18. Cross-conductance $G_{LR}$ as a function of applied voltage $V_2$ for different Zeeman splittings $V_1$ applied to the Majorana wire. Panels (a) and (b) show $G_{LR}$ for an ideal Majorana wire and one with a quantum dot on the left respectively. The cross-conductance vanishes below the bulk gap. The bulk gap in (a) appears to close and reopen as $V_2$ crosses the critical point. Such a gap closing is difficult to see in (b). Note that the induced tunnel barrier height is $E_{\text{barrier}} = 1$ meV.

S. Das Sarma, Rev. Mod. Phys. 80, 1083 (2008), https://link.aps.org/doi/10.1103/RevModPhys.80.1083.  

---

D. Castelvecchi, Nature News 541, 9 (2017), http://www.nature.com/news/quantum-computers-ready-to-leap-out-of-the-lab-in-2017-1.21239.

---

A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, Nature Physics 8, 887 (2012), https://www.nature.com/articles/nphys2479.

---

M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, Nano Lett. 12, 6414 (2012), https://doi.org/10.1021/nl303758v.

---

H. O. H. Churchill, V. Fatemi, K. Grove-Rasmussen, M. T. Deng, P. Caroff, H. Q. Xu, and C. M. Marcus, Phys. Rev. B 87, 241401 (2013), https://link.aps.org/doi/10.1103/PhysRevB.87.241401.

---

A. D. K. Finck, D. J. Van Harlingen, P. K. Mohseni, K. Jung, and X. Li, Phys. Rev. Lett. 110, 126406 (2013), https://link.aps.org/doi/10.1103/PhysRevLett.110.126406.

---

M. T. Deng, S. Vaitiekėnas, E. B. Hansen, J. Danon, M. Leijnse, K. Flensberg, J. Nygård, P. Kroogstrup, and C. M. Marcus, Science 354, 1557 (2016), http://science.sciencemag.org/content/354/6319/1557.
