Mathematical Model of LoRaWAN Channel Access

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Abstract—While 3GPP has been developing NB-IoT, the market of Low Power Wide Area Networks has been mastered by cheap and simple Sigfox and LoRa/LoRaWAN technologies. Being positioned as having an open standard, LoRaWAN has attracted also much interest from the research community. Specifically, many papers address the efficiency of its PHY layer. However MAC is still underinvestigated. Existing studies of LoRaWAN do not take into account the acknowledgement and retransmission policy, which may lead to incorrect results. In this paper, we carefully take into account the peculiarities of LoRaWAN transmission retries and show that it is the weakest issue of this technology, which significantly increases failure probability for retries. The main contribution of the paper is a mathematical model which accurately estimates how packet error rate depends on the offered load. In contrast to other papers, which evaluate LoRaWAN capacity just as the maximal throughput, our model can be used to find the maximal load, which allows reliable packet delivery.

Index Terms—LoRa, LoRaWAN, LPWAN, Channel Access, Performance Evaluation, ALOHA

I. INTRODUCTION

LoRaWAN is a relatively new protocol designed to provide cheap and reliable wireless connectivity in various Internet of Things scenarios. Being a Low Power Wide Area Network technology operating in the ISM band, it rapidly got popularity in both industry and academic communities. Literature review shows that in spite of numerous studies of its PHY layer [1]–[3], the MAC layer got little attention, even though it has multiple issues [4], [5] that limit its performance. However, as LoRaWAN is designed to support networks of thousands of devices, it is crucial not only to consider the performance of this technology in point-to-point scenarios, but also to evaluate its applicability in case of highly-populated networks.

To calculate throughput of LoRaWAN networks, in existing studies of the MAC layer (e.g., see [6]), the authors typically use the classical approach for modeling ALOHA networks [7]. The papers (e.g. [8]) also limit the study to unacknowledged mode, which has no control acknowledgements (ACKs). Thus, with no control traffic the throughput increases. However the reliability of transmission decreases.

In this paper, we provide a mathematical model for a LoRaWAN network operating in the acknowledged mode. We explain why the usage of classical ALOHA-like approach underestimates the collision probability and develop an accurate mathematical model which takes into account LoRaWAN peculiarities related to retransmission policy.
III. PROBLEM STATEMENT

Consider a LoRaWAN network that consists of a GW and $N$ motes and operates in $F$ main channels and one downlink channel. The motes use data rates $0, 1, ..., R$, set by the GW. Let $p_i$ be the probability that a mote uses data rate $i$.

We consider that a frame collision occurs when two frames are transmitted in the same channel at the same data rate, and they intersect in time.

The motes generate frames according to a Poisson process with total intensity $\lambda$ (the network load). All motes transmit frames with 51-byte Frame Payload which corresponds to the lowest data rate. The frames are transmitted in the acknowledged mode, and ACKs carry no frame payload. We consider a situation, when motes have no queue, i.e., if two messages are generated, a mote transmits the most recent one.

For the described scenario, it is important not only to know the nominal channel capacity, but also to find the maximal load at which the network can provide reliable communications. In other words, we need to find the packet error rate (PER) as a function of network load $\lambda$.

IV. MATHEMATICAL MODEL

To solve the problem, we develop a mathematical model of the transmission process. As the first transmission attempts are described by the Poisson process, to find the PER in these assumptions, in Section IV-A we consider the approach used to evaluate ALOHA networks [7] and extended to take into account ACKs. This approach is however inapplicable for retransmissions, because they do not form a Poisson process, so in Section IV-B we propose another way to take them into account and thus to improve the accuracy of the model.

A. The First Transmission Attempt

The first transmission attempt is successful with probability

$$P_{S,1} = \sum_{i=0}^{R} p_i P_{Data} P_{Ack}^i,$$

where $P_{Data}^i$ is the probability that the data frame is transmitted without collision at data rate $i$ and $P_{Ack}^i$ is the probability that at least one ACK out of two is received by the mote, provided that the data frame is successful.

Since the packets transmitted in different channels and at different rates do not collide, we need to consider separately each combination of channel and data rate. Specifically for rate $i$ and one of $F$ channels, the load equals $r_i = \frac{p_i}{F}$.

A data frame transmission is successful if it intersects with no transmission of another frame or an ACK sent by the GW as a response to previous frame. Let $T_{i,Data}$ and $T_{i,Ack}$ be the durations of a data frame and an ACK, respectively, at rate $i$. Intersection with a frame does not occur if no frames are generated in the interval $[-T_{i,Data}, T_{i,Data}]$, relative to the beginning of the considered frame. For a Poisson process of frame generation, such an event happens with probability $e^{-2r_i T_{i,Data}}$. We consider that the GW cancels ACK transmission if it is receiving a data frame, so a collision can happen only if the ACK is generated in the interval $[-T_{i,Ack}, 0]$. The rate of ACK generation is $P_{Data}^i$, so the probability to avoid collision with an ACK is $e^{-r_i P_{Data} P_{Ack}}$. Finally, $P_{Data}$ can be found from the following equation:

$$P_{Data} = e^{-(2P_{Data} + P_{Data} P_{Ack}) r_i} e^{-T_{i,Data}}.$$

As for ACKs, the probability that at least one ACK arrives is calculated according to the inclusion-exclusion principle:

$$P_{Ack} = P_{Ack1} + P_{Ack2} - P_{Ack1} P_{Ack2},$$

where $P_{Ack1}$ and $P_{Ack2}$ are the probabilities that the first and the second ACK, respectively, is transmitted successfully, provided that data was transmitted at rate $i$. The first ACK is transmitted successfully if no data frame intersects it:

$$P_{Ack1} = e^{-\text{min}(T_{i,Data} + T_{i,Ack}) r_i}.$$

Here we take the minimum of $T_{i,Data}$ and $T_{i,Ack}$, because if a frame exceeds $T_{i,Data}$, it breaks the acknowledged frame, but such an event is already taken into account by $P_{Data}$. The second ACK is transmitted successfully if no data frame is successful in any other channel or at any other data rate, such that its second ACK would intersect the considered one:

$$P_{Ack2} = e^{-T_{i,Data}} \lambda (1 - \frac{p_i}{F}) \sum_{i=0}^{R} P_{Data} P_{Ack}^i.$$

B. Retransmissions

Consider a case, when two motes transmit frames with collision, as shown in Fig. 2. Let $0$ be the time when the frame of mote A begins, and $x$ be the offset for frame of mote B. Motes choose a channel for retransmission randomly. If they choose different channels, the collision is resolved. Otherwise, with probability $\frac{1}{2}$, they choose the same channel. In this case, let $y$ and $z$ be the times when motes A and B start their retransmission, respectively. The value of $y$ is distributed uniformly in the interval $[\tau, \tau + W]$, where $\tau$ is the frame duration $T$ plus the timeout for the ACK. The value of $z$ is distributed uniformly in the interval $[\tau + x, \tau + x + W]$. The retransmission results in a new collision, if $[z, z + T]$ intersects with $[y, y + T]$, which happens with the probability

$$P_x = \int_0^T \int_0^W \int_1^W x \int_0^x 1(y \leq z \leq y + T) + 1(z \leq y \leq z + T) dz dy dx = \int_0^T \int_0^W \int_0^W 1(y \leq z) dz dy dx = \frac{W^2}{2} \left(2W - \frac{3T}{2} - \frac{2}{T r_i^2} + \frac{1}{r_i \tanh(\frac{2T}{r_i})} \right),$$

where $1(\text{condition})$ is the indicator function which equals 1 if condition is true and 0 otherwise.

Motes have the same probability of being the first and the second one, so the probability that there is no collision equals

$$P_{i,Re} = 1 - 2P_x / F.$$

The average probability of a successful transmission $P_S$ is
PER 10
10 10 10
-4
-2
-1
0
radius of
LoRaWAN
network. As in [6],
we consider a scenario,
distributed as follows:
EU 863-880 MHz ISM
band. In this case, the
data rates are
estimate
of the model. From Fig. 3 we also see that we
correctly
transmissions, we have
significantly improved the
accuracy
50% greater
than
PER.
 With those obtained with the
developed
mathematical model. The
results are shown in Fig. 3. Because
packet error
transmission attempt
probability that a new frame
does not arrive during the
transmission
is the first one (not
a retry).
Thus, by taking into account
retransmission
process significantly over-
estimates efficiency of a
LoRaWAN network. In contrast,
our model takes into account
peculiarities of the
retransmission
process and correctly estimates
packet error rate when the
load is lower than some threshold
\( \lambda^* \), which is found in
the paper. However the area with
the higher loads is not
interesting from a practical point of
view. Indeed, after the
load exceeds the described threshold,
PER rapidly grows to
1 because retransmissions form an
“avalanche”. Thus in this
area LoRaWAN cannot provide
reliable communications.

\[
P_{S} = P_{1}P_{S,1} + (1 - P_{1})P_{S,Re},
\]

where \( P_{S,Re} \) is the probability of a successful
retransmission, calculated as in eq. (1), using
\( P_{i,Re}^{Data} \) instead of \( P_{i,Data} \), and
\( P_{1} \) is the probability that the transmission is the
first one (not a retry).
\( P_{1} \) is reverse to the average number of
transmission attempts per a frame:
\[
P_{1} = \left( 1 + (1 - P_{S,1}) \sum_{r=0}^{RL} (1 - P_{S,Re})^r P_{N}^{r+1} \right)^{-1},
\]
where \( P_{N} = \sum_{i=0}^{R} p_{i} e^{-\frac{R}{2}(D_{Data} + T_{2} + T_{Ack} + T_{Wait})} \) is the
probability that a new frame does not arrive during the
transmission and \( T_{Wait} = 1 + W/2 \) is the average interval
that a mote waits before a retransmission. The
Packet error
rate is calculated as \( PER = 1 - P_{S} \).

The model estimates PER correctly up to such network load,
that new frames arrive at the motes as quickly as the motes
drop the frames due to inability to resolve collisions after
\( RL \) retransmission attempts. It means that the load equals
\[
\lambda^* = F \left( \sum_{i=0}^{R} p_{i} (D_{Data} + T_{2} + T_{Ack} + T_{Wait}) RL \right)^{-1}.
\]

V. Numerical Results
Let us use the developed model to evaluate performance of a
LoRaWAN network. As in [6], we consider a scenario, when
the motes are distributed uniformly in a circular area with
radius of 1 km around the GW, and the path-loss is described
by Okumura-Hata model for urban environment. We consider
EU 863-880 MHz ISM band. In this case, the data rates
are distributed as follows: \( p_{0} = 0.28, p_{1} = 0.2, p_{2} = 0.14, p_{3} = 0.1, p_{4} = 0.08, p_{5} = 0.2. \) We simulate a network with 1000
motes and compare the average PER and PER1 for the first
transmission attempt with those obtained with the
developed mathematical model. The results are shown in Fig. 3. Because
of inefficient retransmission parameters the real PER is by
50% greater than PER1. Thus, by taking into account
retransmissions, we have significantly improved the
accuracy
of the model. From Fig. 3 we also see that we
correctly estimate \( \lambda^* \) which is the highest load when we can neglect
high-order collisions and the “avalanche effect” inherent to
the default retransmission parameters. Non-adaptive and small
retransmission window does not allow to resolve collisions
with high number of packets, and involving new motes in
collisions is faster than packet dropping or collision resolution.
This significantly limits the capacity of a LoRaWAN network.
While the network can transmit several packets per second,
because of a poor retransmission policy the PER rapidly tends
to 1, when the load exceeds \( 10^{-1} \) packets per second.

VI. Conclusion
In the paper, we develop the first accurate mathematical
model of acknowledged uplink transmissions in LoRaWAN
networks with class A devices. We have shown that leaving
out of consideration retransmission process significantly
overestimates efficiency of a LoRaWAN network. In contrast,
our model takes into account peculiarities of the retransmission
process and correctly estimates packet error rate when the
load is lower than some threshold \( \lambda^* \), which is found in
the paper. However the area with the higher loads is not
interesting from a practical point of view. Indeed, after the
load exceeds the described threshold, PER rapidly grows to
1 because retransmissions form an “avalanche”. Thus in this
area LoRaWAN cannot provide reliable communications.

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