Intelligent Warehouse Allocator for Optimal Regional Utilization

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Abstract
In this paper, we describe a novel solution to compute optimal warehouse allocations for fashion inventory. Procured inventory must be optimally allocated to warehouses in proportion to the regional demand around the warehouse. This will ensure that demand is fulfilled by the nearest warehouse thereby minimizing the delivery logistics cost and delivery times. These are key metrics to drive profitability and customer experience respectively. Warehouses have capacity constraints and allocations must minimize inter warehouse redistribution cost of the inventory. This leads to maximum Regional Utilization (RU). We use machine learning and optimization methods to build an efficient solution to this warehouse allocation problem. We use machine learning models to estimate the geographical split of the demand for every product. We use Integer Programming methods to compute the optimal feasible warehouse allocations considering the capacity constraints. We conduct a back-testing by using this solution and validate the efficiency of this model by demonstrating a significant uptick in two key metrics Regional Utilization (RU) and Percentage Two-day-delivery (2DD). We use this process to intelligently create purchase orders with warehouse assignments for Myntra, a leading online fashion retailer.

Keywords: Fashion E-commerce; Inventory Allocation; Regional Utilisation; Two-Day-Delivery; Warehouse Assignment; Stock Keeping Unit (SKU); Integer Programming

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1 Introduction
Buying and Replenishment is a major, throughout the year activity in large fashion e-commerce firms. Fast changing trends, seasonality, and just-in-time inventory models make inventory procurement an everyday activity. Hundreds of Purchase Orders (POs) are raised every day to procure styles of different article types from vendors. Once the buying quantity is fixed, typically using a demand forecasting model, the next important task is to distribute the procured inventory optimally between the principal warehouses. Logistics cost is a significant cost component in e-commerce operations and reduction in this by a few percentage points will result in cost savings of the order of millions of dollars. This will drive companies towards profitability. Optimal warehouse stocking leads to lower logistics costs and faster delivery of products and hence improved customer experience. An intelligent solution, which can optimally distribute the inventory between warehouses, is of great value.

The distribution must mirror the geographical demand split. The quantity of the product (SKU) that gets located in a warehouse must be in proportion to the regional demand of the product. This is the localised demand from the areas in the vicinity of the warehouse. Every postal pin-code is mapped to the nearest warehouse. Sets of pincodes mapped to the same warehouse partition the whole region into geographical clusters with an associated unique nearest warehouse. The quantity that gets located in a given warehouse must be in proportion to the demand of the associated cluster. The problem is to determine optimal stocking levels for these warehouses.

Warehouses have capacity constraints and old inventory which must be accounted while determining the new stocking quantities. Geographical distribution of the demand can be different for every SKU.

2 Related Work
Optimal allocation of resources is a well-studied problem across e-commerce and supply chain management. There is literature [5] for solving problems in production planning, distribution planning, and transport planning using optimization methods. Problems in logistics are modelled using costs such as inventory holding cost, transport cost, etc, and have realistic fixed limits to the variables. [1] have modelled
Supply Chain Networks using a general framework to support the operational decisions using a combination of an optimization model and discrete-event simulation.

Although some variables are linearly relaxed, the addition of integer constraints like knapsack constraints or capacity in volume or number makes the problem partly discrete and hence needs Mixed Integer Programming methods.

There has been work using Integer Programming methods to allocate resources optimally between Brick and Mortar and online stores [2], [4] discuss in detail about a production and distribution problem with boolean and general integer variables, formulated as an Integer Linear Program. [3] study an inventory planning problem where the procurement of items and their quantity is modelled using a mixed 0/1 integer program.

## 3 Problem Statement and Solution Outline

The objective is to determine the optimal stocking quantity of each SKU for individual warehouses which will maximise regional utilization. Regional Utilization (RU) is defined as the fraction of total demand which is fulfilled from the nearest warehouse. In cases when warehouses cannot inward the recommended quantities due to capacity constraints, the next nearest warehouse must be prioritised to minimise the cost of redistribution.

We formalise these requirements by defining a penalty matrix $L$ for redistribution costs and expressing the requirements as a constrained optimization problem. Table 1 lists down all the variables and notation used in the paper. We can formally state the problem as follows.

### 3.1 Problem Statement

Given a Purchase Order (PO) specification: $\{(sku_i, N_i) | 1 \leq i \leq M\}$ which is just a set of $(sku_i, quantity)$ pairs for $M$ unique SKUs and $K$ warehouses with capacities $[C_1, C_2, \ldots C_K]$ and existing inventory of the SKUs in warehouses $\{E_{ij} | 1 \leq i \leq M; 1 \leq j \leq K\}$, compute the optimal warehouse allocations $\{(sku_i, X_{ij}, \ldots X_{ik}, X_{ij}) | 1 \leq i \leq M\}$ where $X_{ij}$ is the number of $sku_i$ allocated to warehouse $j$. $X_{il}$ denotes the number of $sku_i$ not assigned to any warehouse. This is optionally allowed to handle cases where total SKU quantity is more than the combined capacity of all warehouses. Allocations must satisfy the following constraints.

$$\sum_{j=1}^{K} X_{ij} + X_{il} = N_i \quad 1 \leq i \leq M \quad (1)$$

$$\sum_{i=1}^{M} X_{ij} \leq C_j \quad 1 \leq j \leq K \quad (2)$$

### 3.2 Solution Outline

The solution comprises 2 steps.

**Table 1. Notation of Variables and Constants in the paper**

| PO          | Purchase Order Specification of size $M$  |
|-------------|------------------------------------------|
| PO$_i$      | A tuple of $(sku_i, N_i)$                |
| $N_i$       | Quantity of $sku_i$ to be inwarded       |
| $M$         | The total number of unique SKUs in the PO|
| $K$         | Number of Warehouses                     |
| $PO'$       | Exploded Purchase Order Specification of size $N$  |
| $E$         | Existing Inventory Matrix $\in \mathbb{Z}_+^{(M+K)}$ |
| $E_{ij}$    | The existing quantity of $sku_i$ in warehouse $j$ |
| $C$         | Warehouse Capacity Vector $\in \mathbb{Z}_+^K$ |
| $C_j$       | Capacity of warehouse $j$                |
| $X$         | Optimal Warehouse Allocation Matrix $\in \mathbb{Z}_+^{(M(K+1))}$ |
| $X_{ij}$    | The quantity of $sku_i$ to be inwarded to warehouse $j$ for $1 \leq j \leq K$ |
| $X_{il}$    | The quantity of $sku_i$ not assigned to any warehouse |
| $P$         | Ideal Split Probability Matrix $\in [0, 1]^{(M+K)}$ |
| $P_{ij}$    | The probability that the purchase event for $sku_i$ is located in geographic region for which nearest warehouse is $j$ |
| $I$         | Ideal Split Matrix $\in \mathbb{Z}_+^{(M+K)}$ |
| $I_{ij}$    | The ideal quantity of $sku_i$ to be inwarded to warehouse $j$ |
| $Y$         | Binary Decision Variable Matrix $\in \{0, 1\}^{(N+K)}$ (formulation 1) |
| $Y_{ij}$    | 1 if $item_i$ is inwarded to warehouse $j$ else 0 (formulation 1) |
| $L$         | Redistribution Cost Penalty Matrix $\in \mathbb{R}^{(K+K+1)}$ |
| $L_{ij}$    | The penalty for placing an item which is ideally assigned to warehouse $i$ in warehouse $j$ for $1 \leq i, j \leq K$ |
| $L'$        | Truncated Penalty Matrix $L' = L[1 : K][1 : K] \in \mathbb{R}^{(K+K)}$ |
| $1_m$       | A vector of 1's $\in \{1\}^{(m,1)}$     |

1. **Ideal Splits Computation**: Predicting the ideal split at the SKU level by a split prediction model and generating an ideal warehouse allocation given the order quantity from a demand model for every SKU assuming unlimited capacity in warehouses.

2. **Optimal Feasible Allocations**: Finding optimal feasible allocations considering warehouse capacity constraints by defining an allocation penalty matrix and solving the constrained optimization problem.

In the following sections, we will describe the above 2 steps and results in detail.
4 Ideal Splits Computation

Given a purchase order specification: \{\{\text{sku}_i, N_i\}|1 \leq i \leq M\} and \(K\) warehouses, we compute ideal splits \(\{I_{ij}|1 \leq j \leq K\}\) among \(K\) warehouses for each \(\text{sku}_i\):

\[
\sum_{j=1}^{K} I_{ij} = N_i \quad 1 \leq i \leq M \quad (3)
\]

We assume unlimited capacity in warehouses. Ideal splits \(I_{ij}\) are computed by estimating the split probabilities \(\{P_{ij}|1 \leq j \leq K\}\) for each \(\text{sku}_i\) defined as follows.

\[
P_{ij} = \text{Probability} (\text{Purchase event for } \text{sku}_i \text{ is located in geographical cluster with associated nearest warehouse being warehouse } j)
\]

\[
\sum_{j=1}^{K} P_{ij} = 1.0 \quad (4)
\]

We estimate these probabilities by learning a classifier that predicts the warehouse probabilities using predictors like attributes of the style like colour, brand, fabric, size, age group, price, sleeve length, neck-type, etc. We use around 10 - 12 relevant attributes for each article type and gender group.

Every purchase event on the platform is associated with a postal pin code that is mapped to the nearest warehouse. This is used to create labelled training records on which the classifier is trained. We tried different classifiers like logistic regression, tree-based classifiers and feed-forward neural network classifiers. Table 2 compares the performance of these classifiers. The neural network model outperforms logistic regression, tree-based classifiers and feed-forward neural network classifiers. Table 2 lists the other classifiers and we use a 3-layer MLP for each article type, gender combination in our application. Table 3 lists the log loss for the three layer multi layer perceptron for various article types and genders.

5 Optimal Feasible Allocations

In this step, we optimally impose warehouse capacity constraints on ideal splits computed in the previous section.
Optimality is with regard to the inter warehouse redistribution task. We formalise this by defining a redistribution cost penalty matrix task. Let \( L \in \mathbb{R}^{K \times (K+1)} \) where \( L_{ij} \) (\( j \neq K + 1 \)) is the penalty for placing an item which is ideally assigned to warehouse \( i \) to warehouse \( j \). We set these penalties to mirror logistics cost involved in fulfilling an order from warehouse \( j \) when the nearest warehouse is warehouse \( i \).

Non-assignment is allowed to handle cases where total number of items in the PO is greater than the combined capacity of all warehouses. The \((K+1)\text{th} \) column of \( L \) represents the non-assignment penalty (Eqn. 9).

\[
L_{i,K+1} = \lambda_{NA} \quad 1 \leq i \leq K
\]  

However, we make sure that assignments are always preferred over non-assignments by setting \( \lambda_{NA} >> L_{ij}, 1 \leq i, j \leq K \).

Optimization algorithms will always choose assignment over non-assignment because the non-assignment penalty is greater than any assignment penalty.

The key idea behind these optimization formulations is the following. Warehouse allocation implied by ideal splits need not be always feasible because of warehouse capacity constraints. By defining a redistribution penalty matrix, we can compute an optimal re-arrangement of ideal allocations which will be capacity feasible. We can think of the final allocation as minimum distortion from the ideal splits allocation. The key idea behind these optimization formulations is minimum distortion from the ideal splits allocation.

\[
Z = 1_N - Y1_K
\]  

We determine the optimal feasible assignment \( Y_{\text{opt}} \) by solving the following optimization problem:

\[
\arg \min_Y \begin{align*} 
& \text{trace}(YL'W^T) + \lambda_{NA} \sum_{i=1}^N Z_i \\
& \text{subject to} \\
& \sum_{j=1}^K Y_{ij} \leq 1 \quad 1 \leq i \leq N, \\
& \sum_{i=1}^N Y_{ij} \leq C_j \quad 1 \leq j \leq K 
\end{align*}
\]  

Optimization objective (Eqn. 12a) represents the total redistribution cost for all SKUs given an ideal allocation \( W \) and final assignment \( Y \). \((YL'W^T)_{ij}\) represents the redistribution cost for \( \text{item}_i \). There are two constraints. Eqn 12b constrains that we choose at most 1 warehouse per item. Eqn. 12c constrains that total items assigned to warehouse \( j \) is not more than the capacity of the warehouse \( j \).

Using optimal feasible assignments \( Y_{\text{opt}} \), we compute optimal feasible warehouse splits for SKUs as follows.
The ideal assignments for \(sku_i\) is \(Y(sku_i)\) given by:

\[
Y(sku_i) = [Y_{opt}[j] | 1 \leq j \leq N \text{ s.t. } item_j = sku_i] \quad (13)
\]

\[
Y(sku_i) \in \{0, 1\}^{(N \times K)}
\]

\[
X_{ij} = \sum_{m=1}^{N_i} Y(sku_i)_{mj} \quad 1 \leq j \leq K \quad (14)
\]

\[
X_{il} = N_i - \sum_{j=1}^{K} X_{ij}
\]

\[
\{(sku_i, [X_{i1}, ..., X_{iK}, X_{il}]) | 1 \leq i \leq M\} \text{ are the desired optimal feasible warehouse allocations.}
\]

Binary Integer Programming formulation works well for small size orders. The number of decision variables is \(O(N \times K)\) where \(N\) is the total quantity of items and \(K\) is the total number of warehouses. For large sized orders, the number of decision variables will become too large and this will be very cumbersome to solve. In the next subsection, we describe an alternative formulation which can efficiently solve large orders.

### 5.2 Integer Programming Formulation

From Step 1 (Sec. 4), we have computed ideal splits: \(\{(sku_i, [I_{i1}, I_{i2}, ..., I_{ik}]) | 1 \leq i \leq M\}\). This gives us the ideal split matrix: \(I \in Z_{+}^{(M \times K)}\) where \(I_{ij}\) represents the quantity of \(sku_i\) assigned to warehouse \(j\). Penalty matrix \(L \in R^{(K+1) \times (K+1)}\) is the same as before.

We define a final split Decision Variable Tensor \(Y \in Z_{+}^{(M \times K+1)}\) as follows:

\[
Y_{iuv} = \text{number of } sku_i \text{ ideally allocated to warehouse } u \text{ which is finally placed in warehouse } v, 1 \leq i \leq M; 1 \leq u \leq K; 1 \leq v \leq K+1(v = K+1 \text{ implies not assigned to any warehouse})
\]

We determine optimal final split tensor \(Y_{opt}\) by solving the following integer programming problem.

\[
\begin{align}
\text{arg min}_Y & \quad \sum_{i=1}^{M} \text{trace}(Y_i \ast L^T) \\
\text{subject to} & \quad \sum_{m=1}^{K+1} Y_{ijm} = I_{ij} \quad 1 \leq i \leq M, 1 \leq j \leq K, \\
& \quad \sum_{i=1}^{M} \sum_{j=1}^{K} Y_{ijm} \leq C_m \quad 1 \leq m \leq K
\end{align}
\]

Optimization objective (16a) represents total redistribution cost for \(\{sku_i | 1 \leq i \leq M\}\) given an ideal split and a final split. The term \(\text{trace}(Y_i \ast L^T)\) is the redistribution cost for \(sku_i\) as depicted in Figure 1. Eqn 16b and 16c are optimization constraints for SKU order quantities and warehouse capacities respectively.

Once we have \(Y_{opt}\) by solving the optimization problem, we compute final optimal feasible allocations as follows:

\[
X_{ij} = \sum_{m=1}^{K} (Y_{opt})_{imj} \quad 1 \leq i \leq M, 1 \leq j \leq K
\]

\[
X_{il} = \sum_{m=1}^{K} (Y_{opt})_{im,K+1} \quad 1 \leq i \leq M
\]

\[
\{(sku_i, [X_{i1}, ..., X_{iK}, X_{il}]) | 1 \leq i \leq M\} \text{ are the desired optimal feasible warehouse allocations.}
\]

The second formulation (Sec. 5.2) is computationally more efficient. The number of decision variables is \(O(M \times K^2)\) which is usually much smaller than binary integer programming case when \(K\) is not very large. With this we can efficiently solve the optimization problem with total SKU quantity ranging in millions, number of SKUs ranging in few thousands and number of warehouses is less than 100 using open source solvers like coin-or/Cbc.

Note that for this problem, we can always supply an initial feasible solution to warm start the optimization. One trivial feasible solution is the one which does not assign warehouses to any \(sku\). This also has the maximum objective value. Using ideal splits and some heuristics, we can supply much better initial feasible solutions to speed up the optimization.

We use this second formulation for our back-testing and for optimization, we used the coin-or/Cbc solver at https://github.com/coin-or/Cbc.

### 6 Evaluation and Results

In this section, we describe the back-testing set up and results. We back-tested the model on purchase orders created during an entire month at Myntra. We tested it on major business
units like Apparel, Footwear and Personal Care, covering 23 different Article types. Around 8,000 purchase orders were created for 43,000 SKUs and a total quantity of 3.4 million.

We used two warehouse constraint scenarios (Table 4 and Table 5) provided by the Supply-Chain-Management team.

We solved the optimization problem using constraints depicted in tables 4 and 5. After every purchase order is created, we deduct the total quantity of the order from the capacities and update the constraints.

Using the optimal splits computed using our optimization algorithm and the purchase data for these skus in subsequent months, we estimated Regional Utilization (RU) which is the fraction of orders which are fulfilled by the nearest warehouse. We also estimated percentage Two-Day-Delivery which is the fraction of orders which are fulfilled within two days of placing the order. Every warehouse has a set of pin-codes for which delivery can be done within 2 days. Using this, we can estimate the percentage Two-Day-Delivery. We observed a substantial uptick in these two key metrics — RU and 2DD over a heuristics based approach previously followed. We compared the estimates with the numbers produced using simple business heuristics. Tables 6, 7 and 8 depict the results.

- RU (ideal splits): RU estimate using ideal splits with unlimited warehouse capacities.
- RU (constrained splits): RU estimate using optimal feasible splits considering capacity constraints.
- RU (Heuristics based): Actual realised RU using a heuristics based warehouse allocation policy.

- 2DD (ideal splits): Percentage Two-Day-Delivery estimate using ideal splits with unlimited warehouse capacities.
- 2DD (constrained splits): Percentage Two-Day-Delivery estimate using optimal feasible splits considering capacity constraints.
- 2DD (Heuristics based): Actual realised Percentage Two-Day-Delivery using a heuristics based warehouse allocation policy.

Results clearly show a significant improvement in both RU and 2DD. RU improves by around 27% over the naive heuristic method and 2DD improves by around 20%. The performance trend is consistent across all the 3 categories and the 2 constraint scenarios.

7 Conclusion

We have developed an efficient solution to the problem of optimal warehouse allocations from a Regional Utilization

Table 4. Warehouse capacity constraint - Scenario 1

| Month   | Type     | C1       | C2       | C3       | C4       |
|---------|----------|----------|----------|----------|----------|
| April   | Apparel  | 1005714  | 502857   | 377143   | 754286   |
|         | Footwear | 91429    | 45714    | 34286    | 68571    |
|         | Personal | 102857   | -        | 17143    | -        |
| May     | Apparel  | 435810   | 217905   | 163429   | 326857   |
|         | Footwear | 76191    | 38095    | 28571    | 57143    |
|         | Personal | 85714    | -        | 14286    | -        |

Table 5. Warehouse capacity constraint - Scenario 2

| Month   | Type     | C1       | C2       | C3       | C4       |
|---------|----------|----------|----------|----------|----------|
| April   | Apparel  | 550979   | 721502   | 844438   | 546241   |
|         | Footwear | 50089    | 65591    | 76768    | 49658    |
|         | Personal | 56350    | -        | 38384    | -        |
| May     | Apparel  | 238758   | 312650   | 365923   | 236705   |
|         | Footwear | 21705    | 28423    | 33265    | 21518    |
|         | Personal | 24418    | -        | 16634    | -        |

Table 6. Apparel RU and 2DD Estimates

| Metric                      | Scenario 1 | Scenario 2 |
|-----------------------------|------------|------------|
| RU (ideal splits)           | 0.91       | 0.91       |
| RU (constrained splits)     | 0.82       | 0.84       |
| RU (Heuristics based)       | 0.64       | 0.64       |
| 2DD (ideal splits)          | 0.64       | 0.64       |
| 2DD (constrained splits)    | 0.58       | 0.61       |
| 2DD (Heuristics based)      | 0.48       | 0.48       |

Table 7. Footwear RU and 2DD Estimates

| Metric                      | Scenario 1 | Scenario 2 |
|-----------------------------|------------|------------|
| RU (ideal splits)           | 0.9        | 0.9        |
| RU (constrained splits)     | 0.87       | 0.9        |
| RU (Heuristics based)       | 0.38       | 0.38       |
| 2DD (ideal splits)          | 0.69       | 0.69       |
| 2DD (constrained splits)    | 0.66       | 0.69       |
| 2DD (Heuristics based)      | 0.34       | 0.35       |

Table 8. Personal Care RU and 2DD Estimates

| Metric                      | Scenario 1 | Scenario 2 |
|-----------------------------|------------|------------|
| RU (ideal splits)           | 0.93       | 0.93       |
| RU (constrained splits)     | 0.7        | 0.7        |
| RU (Heuristics based)       | 0.5        | 0.5        |
| 2DD (ideal splits)          | 0.72       | 0.72       |
| 2DD (constrained splits)    | 0.53       | 0.53       |
| 2DD (Heuristics based)      | 0.35       | 0.35       |
perspective using machine learning and optimization methods. We have computed ideal allocations using predictive ML models. We have formalised the requirements of optimal warehouse allocation problem using 2 novel optimization formulations. We have demonstrated the efficacy of the solution with an elaborate back-testing. We have demonstrated a substantial uptick in key business metrics like Regional Utilization and Two-Day-Delivery over a heuristics based approach.

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