Effectively irreversible fringe contrast attenuation induced by one degree of freedom

A R Bosco de Magalhães¹ and Adélcio C Oliveira²

¹ Departamento de Física e Matemática, Centro Federal de Educação Tecnológica de Minas Gerais, Belo Horizonte, MG, 30510-000, Brazil
² Departamento de Física e Matemática, Universidade Federal de São João Del Rei, CP 131, Ouro Branco, MG, 36420-000, Brazil

E-mail: arthur.magalhaes@pq.cnpq.br

Received 26 January 2012
Accepted for publication 23 July 2012
Published 10 August 2012
Online at stacks.iop.org/PhysScr/86/035001

Abstract

The attenuation of fringe contrast in a Ramsey interferometer induced by atom–field interaction is analyzed. We show that short-time power series expansion is not a proper tool to find the relevant time scale for such a process. Analytical expressions for quantifying the relevant time scale for the fringe contrast decay and for characterizing the long-term effectiveness of this process are proposed. For the Pegg–Barnett phase state initial conditions, these expressions suggest that an increase of the energy of the field leads to slower vanishing of fringe contrast, and an increase of the field photon number variance leads to a more effective attenuation in the final regime. Numerical simulations with coherent and thermal initial states are in qualitative agreement with such results.

PACS numbers: 03.65.Yz, 03.65.Aa, 42.50.Pq

1. Introduction

The deleterious action of the environment over quantum coherences has been a fundamental ingredient in the study of the foundations of quantum mechanics, because it sheds light on the quantum to classical transition problem [1]. This process is called decoherence, and plays a central role in quantum information, as it is the main obstacle to quantum computation [2]. Progressive loss of coherence was observed in an important cavity quantum electrodynamics (QED) experiment [3]. At the theoretical level, decoherence is frequently studied by considering that the system of interest is coupled to another system, the environment [4]: if we take the trace over the environmental degrees of freedom, we access the statistics we are interested in. Other decoherence approaches are presented in [5], based on coarse-grained measurements, and in [6], where a fluctuation of some classical parameter is responsible for coherence loss.

Although the environment is usually modeled as a many degrees of freedom system [7–9], decoherence has also been investigated by coupling the system of interest to a few degrees of freedom whose classical analogues exhibit chaotic (or chaotic-like) behaviors [10–15]. In [16], we used a quartic oscillator coupled through cross-Kerr interaction to a variable number of bosons to show that effectively irreversible loss of quantum coherences may be induced even by a one degree of freedom environment without chaos. In the present paper, we continue the research on these lines by focusing on the effectively irreversible attenuation of fringe contrast in a two-way interferometer induced by one degree of freedom. The implementation of such an interferometer in the cavity QED context is described in [17]. That implementation is of particular interest for the present discussion, since the single-field mode that fulfills the function of a beam splitter also plays a role analogous to the role often played by infinity degrees of freedom reservoirs: due to the atom–field entanglement, it makes non-diagonal elements of the reduced atomic density operator decrease, leading to diminution of the fringe contrast. In order to study the interferometer in [17], we naturally employed the Jaynes–Cummings model [18] in the rotating wave approximation, as addressed in section 2. We wish to emphasize that although this model has been widely studied in recent decades, we believe that our results are new, since the Jaynes–Cummings model is used here as a means
of studying the induction of classicality by a few degrees of freedom, which has attracted much attention in recent times.

The characterization of relevant time scales for the fringe contrast decay (FCD) requires, in the present case, a different approach from that of [16], where decoherence time scales were defined by means of short-time power series expansions, as usual [19]. Thus, in section 3 we propose analytical expressions based on dephasing of complex terms to quantify FCD time scales. There, we also indicate a method for quantifying the effectiveness of the attenuation process for times much longer than the calculated time scales. These results, obtained for the Pegg–Barnett phase state [20] bosonic initial conditions, suggest that: (a) increasing the photon number variance of the field leads to a more effective attenuation of the fringe contrast in the final regime; (b) increasing the energy of the field leads to slower vanishing of such a contrast. In section 4, we focus on the Ramsey interferometer described in [17]. We show that for a large mean photon number the interference fringes would disappear if long atom–field interaction times were taken. This agrees qualitatively with the results in section 3. In section 5, we analyze the thermal initial bosonic state case by taking into account the parameters encountered in a recent experiment [21]. The conclusions are presented in section 6.

2. The model

Let us consider a two-level system coupled to a resonant oscillator by the Hamiltonian

$$H = \frac{i}{\hbar} [a^\dagger \sigma_z + h a \sigma_x] + h g \left( \sigma_x a + \sigma_a a^\dagger \right),$$

where $a^\dagger$ and $a$ are the creation and annihilation bosonic operators, and $\sigma_z = |e\rangle \langle e| - |g\rangle \langle g|$, $\sigma_x = |e\rangle \langle g| + |g\rangle \langle e|$ are spin-$\frac{1}{2}$ operators. This Hamiltonian may concern a Rydberg atom coupled to one microwave mode in a lossless superconducting cavity [22]. Although analytically solvable, the Jaynes–Cummings model presents a rich dynamics. This is unveiled in seminal papers on the collapses and revivals of the Jaynes–Cummings model presents a rich dynamics. This permits us to obtain analytical results.

Figure 1. Time evolution of $|\rho_{eg}(t)|^2$ for initial state (2) with $c_x = c_g = 1/\sqrt{2}$.

The fringe contrast in the Ramsey interferometer of [17] is proportional to the absolute value of the non-diagonal elements of the reduced atomic density operator

$$\rho_{eg} = \rho_{ee} |e\rangle \langle e| + \rho_{gg} |g\rangle \langle g| + \rho_{eg} |g\rangle \langle e| + \rho_{ge} |e\rangle \langle g|$$

right after the resonant atom–field interaction. Let us first consider $c_x = c_g = 1/\sqrt{2}$, which leads to

$$\rho_{eg}(t) = \frac{1}{2(r+1)} \left\{ i e^{i\sqrt{g} \tau_c} \sin \left( g \sqrt{r+1} \right) \right.$$ 

$$+ \sum_{n=0}^{r} e^{-i\sqrt{g} \tau_c \sqrt{n+1} - \sqrt{n+1} \tau_c} \left\}\right.$$ 

This matrix element is calculated as the sum of complex terms with different phases. When $t = 0$, the phases are correlated (all the terms are real) and $|\rho_{eg}|$ assumes its maximum value. An FCD time scale may be defined as the time required for the vanishing of most of these correlations. Such a time will be assumed here as the time spent by the slowest term in equation (4) to do a complete oscillation:

$$\tau_d = \frac{2\pi}{g \left( \sqrt{r+1} - \sqrt{r} \right)},$$

which grows with $\sqrt{r}$ for large $r$ values. The evolutions of $|\rho_{eg}(\tau_d)|^2$ plotted in figure 1 corroborate the validity of equation (5). This behavior was observed for $r \leq 160$; larger $r$ could not be investigated due to numerical limitations.

When $t = \tau_d$, most of the phase correlations of the terms in equation (4) are lost: the low value of $|\rho_{eg}(\tau_d)|^2$ is a consequence of the mutual cancellation of the terms in the sum. Although revivals of $|\rho_{eg}|$ may occur, they will not be complete, since the frequencies of the periodic functions summed in equation (4) are not commensurable. As $r$ grows, each term in equation (4) decreases and mutual cancellation becomes more effective. Along these lines we propose a way to quantify such a process: assuming that for long times the complex phases (and also the argument in the sine function) behave as random variables, with no correlation among them, uniformly distributed between 0 and $2\pi$, the mean value and

3. Analytical results

We begin by analyzing the dynamics for the initial state

$$|\psi(0)\rangle = (c_x |e\rangle + c_g |g\rangle) \otimes \sum_{n=0}^{\infty} \frac{1}{\sqrt{r+1}} |n\rangle.$$  

The bosonic state is a particular approximate Pegg–Barnett phase state. This is not usually built in cavity fields, but it permits us to obtain analytical results.

$$\tau_d = \frac{2\pi}{g \left( \sqrt{r+1} - \sqrt{r} \right)},$$

which grows with $\sqrt{r}$ for large $r$ values. The evolutions of $|\rho_{eg}(\tau_d)|^2$ plotted in figure 1 corroborate the validity of equation (5). This behavior was observed for $r \leq 160$; larger $r$ could not be investigated due to numerical limitations.

When $t = \tau_d$, most of the phase correlations of the terms in equation (4) are lost: the low value of $|\rho_{eg}(\tau_d)|^2$ is a consequence of the mutual cancellation of the terms in the sum. Although revivals of $|\rho_{eg}|$ may occur, they will not be complete, since the frequencies of the periodic functions summed in equation (4) are not commensurable. As $r$ grows, each term in equation (4) decreases and mutual cancellation becomes more effective. Along these lines we propose a way to quantify such a process: assuming that for long times the complex phases (and also the argument in the sine function) behave as random variables, with no correlation among them, uniformly distributed between 0 and $2\pi$, the mean value and
the standard deviation of $|\rho_{eg}|^2$ will be, respectively, given by

$$
M = \left( \frac{1}{2} (r+1) \right)^2 \left( r + \frac{1}{2} \right),
$$

$$
\sigma = \left( \frac{1}{2} (r+1) \right)^2 \sqrt{r^2 + \frac{1}{8}},
$$

decreasing with $1/r$ for large $r$. The long-time behavior of $|\rho_{eg}|^2$ is displayed in figure 2, where the effectiveness of equations (6) and (7) is exemplified in different orders of magnitude (note the different vertical axis scales).

Let us look at the relevance of calculating a FCD time scale through short-time power series expansions for the present case. The Taylor series of $|\rho_{eg}(t)|^2$ may be written as

$$
|\rho_{eg}(t)|^2 = |\rho_{eg}(0)|^2 + \left( \frac{d}{dt} \left( |\rho_{eg}(t)|^2 \right) \right)_{t=0} t + \left( \frac{1}{2} \frac{d^2}{dt^2} \left( |\rho_{eg}(t)|^2 \right) \right)_{t=0} t^2 + O(t^3),
$$

where $O(t^3)$ denotes the terms of third order in $t$. The first-order term in this expansion is zero. Accordingly,

$$
|\rho_{eg}(t)|^2 \approx |\rho_{eg}(0)|^2 + \left( \frac{1}{2} \frac{d^2}{dt^2} \left( |\rho_{eg}(t)|^2 \right) \right)_{t=0} t^2
$$

$$
= \left( |\rho_{eg}(t)|^2 \right)_{ap}
$$

for short times. The time scale for relevant changes in this approximation may be calculated as the time taken for the absolute value of the second-order term to reach 1, leading to the definition of the second-order FCD time scale:

$$
\tau_2 = \frac{1}{\sqrt{\frac{1}{2} \left( \frac{d}{dt} \left( |\rho_{eg}(t)|^2 \right) \right)_{t=0}}}
$$

$$
= \frac{2 \sqrt{r+1}}{g \sqrt{2 \sqrt{r(r+1)} + \sum_{n=0}^{\infty} \left( \sqrt{n+1} - \sqrt{n} \right)^2}}
$$

In figure 3, we show the ratio $\tau_2/\tau_d$ for $10 \leq r \leq 200$. It is clear, from the dynamics shown in figure 1, that the time scale for the decrease of $|\rho_{eg}|^2$ is really given by $\tau_d$, at least for $r$ around the values investigated. In view of the very low ratios displayed in figure 3, we conclude that $\tau_2$ does not give the relevant time scale for the vanishing of the fringe contrast. This may be understood with the help of figure 4, where we compare $|\rho_{eg}(t)|^2$ with its second-order approximation for $r = 40$. We see that although the approximation (9) is good for very short times, the higher-order terms become relevant long before the second order term reaches one. Namely, the second-order approximation is not good in the entire interval $0 \leq t \leq \tau_2$, which makes definition (10) uncorrelated with the decay of the fringe contrast.

Related results are obtained for $c_x = 1$ and $c_x = 0$. This is the phase state case corresponding to the coherent state case analyzed in [17]. Now, $|\rho_{eg}|^2$ starts at zero and performs damped slow oscillations with the characteristic time scales calculated as above (see figure 5). The evolution of $\rho_{eg}$ may
be displayed in the form

$$\rho_{ee}(t) = \frac{1}{2(r+1)} \sum_{n=1}^{r} \left[ \sin \left( g \left( \sqrt{n} - \sqrt{n+1} \right) t \right) + \sin \left( g \left( \sqrt{n} + \sqrt{n+1} \right) t \right) \right],$$

and the time taken for the slowest term in equation (11) to perform a complete oscillation defines the characteristic time scale, which is also given by equation (5) (the validity of this time scale is exemplified in figure 5). Under the random variables assumption for the arguments of the sine functions in equation (11), the mean value and the standard deviation of $|\rho_{eg}|^2$ are given by

$$M = \left( \frac{1}{2(r+1)} \right)^2 r,$$

$$\sigma = \left( \frac{1}{2(r+1)} \right)^2 \sqrt{2r^2 - \frac{15}{4} r + 3}.$$

The effectiveness of equations (12) and (13) is exemplified for different orders of magnitude in figure 6, where the plots were constructed with distinct vertical axes. These plots also constructed with distinct horizontal axes, in order to display the relevant scale of the dynamics in the long-time regime, when the slow oscillations that decay with $\tau_d$ are no longer appreciable. For each plot of figure 6, the faster terms of equation (11) perform a number of complete oscillations between 60 and 70 in the period shown.

In our numerical investigations we did not find any relevant revival where the behavior departs considerably from that shown in figures 2 and 6. This suggests that we approach towards irreversibility as $r$ increases.

4. The Ramsey interferometer

In [17] is reported a complementarity experiment exploring interesting features of a Ramsey interferometer. A Rydberg atom with relevant levels $e$ and $g$ is sent through a microwave cavity. The atom is initially in the excited level $e$ and the field is prepared in the coherent state $|\alpha\rangle$. The atom and the field interact resonantly during a period $t_\alpha$ defined by $\rho_{ee}(t_\alpha) = 1/2$. Then, by applying an electric field across the cavity mirrors, the relative phase $\phi$ of the probability amplitudes related to levels $e$ and $g$ is shifted by a variable amount. Finally, the atom crosses a Ramsey zone, and the transition probability between levels $e$ and $g$ ends with the value

$$P_g(\phi) = \frac{1}{2} \left[ 1 + \text{Re} \left( 2\rho_{ge}(t_\alpha) \exp(i\phi) \right) \right].$$

4
The contrast of the fringes depends on the atom–field entanglement [30], which is related to which-path information and is responsible for diminishing $|\rho_{ge}(t_0)|$.

The left plots of figure 7 show the values for $|\rho_{ge}(t_0)|$ concerning the first time that $\rho_{ee}(t)$ reaches $1/2$. As in the experimental situation in [17], such values increase with $|\alpha|$: the more the coherent field approaches a classical regime, the less which-path information will be available on it. However, if a much longer $t_0$ (also satisfying $\rho_{ee}(t_0) = 1/2$) had been chosen, an opposite behavior would be observed: $|\rho_{ge}(t_0)|$ would tend to be small for high values of $|\alpha|$, as is exemplified in the right plots of figure 7. As $|\alpha|$ increases, the field takes a longer time to store which-path information, but the flow back of this information becomes less significant.

Similarities to the results in section 3 become clear with the help of figure 8, which must be compared to figure 1. Enhancing $|\alpha|$ or $r$ corresponds to enhancing the mean photon number and the photon number variance. This leads, in both cases, to more effective, but retarded, FCD. For the phase state case, such a retardation seems to be mainly related to the energy increasing, since the slowest terms in expressions (4) and (11), which determine the FCD time scale, correspond to the highest occupied bosonic energy levels. The maintenance of the fringe contrast is related to the separability of the atom–field state. Thus, this slowdown of the contrast decreasing resembles the results found in [28], where it is shown, for specific atomic initial states and the field starting in a coherent state in the limit of large average photon number $\bar{n} \rightarrow \infty$, that the atom and the field remain separately in a pure state, for finite time and even for infinite times provided that the time $t$ goes to infinity slowly enough so that $t/\bar{n} \rightarrow 0$. On the other hand, also for the phase state case, the long-term effectiveness of the FCD is mainly related to the photon number variance: as this variance increases, each term in expressions (4) and (11) decreases, leading to a more effective mutual cancellation due to the pseudo-randomization of the relative phases. This is reflected in the factors $1/(r + 1)$ in equations (12) and (13). Although all energy levels are present in the coherent state, relevant coefficients are found in a finite range, and a similar mechanism for the FCD process may exist.

5. Taking into account recent experimental parameters

In this section, we assume that the field mode was previously thermalized:

$$\rho (0) = (c_e |e\rangle + c_g |g\rangle) (c_e^* \langle e\rangle + c_g^* \langle g\rangle)$$

$$\times \otimes (1 - e^{-\frac{\hbar \omega}{k_B T}}) \sum_{n=0}^{\infty} e^{-\frac{\hbar \omega}{k_B T}} |n\rangle \langle n|, \quad (15)$$

where $k_B$ is Boltzmann’s constant and $T$ is the absolute temperature [31]. In figure 9, we show the evolution of $|\rho_{ge}|^2$ taking into account the same parameters as in the experiment described in [21]. We see that the higher the temperature of the initial state, the more the bosonic system
effective in destroying $|\rho_{eg}|$ in the long-time regime. However, higher temperatures also retard this regime. These results are analogous to those found in sections 3 and 4. In the actual experimental setup, the field decays according to a cavity damping time $T_c = 130 \text{ ms}$ (which corresponds to $gt = 4.08 \times 10^4$). Since $T_c$ is much longer than the time spent to achieve the final regime observed, the imperfections of the cavity were disregarded. For the actual temperature of the experimental setup 0.8 K, the field is near the vacuum and there occurs almost complete cyclical recurrence of $|\rho_{eg}|$. Nevertheless, for the other temperatures in figure 9, interference fringes will be hardly observed if the atom and the field had interacted for a long time.

6. Conclusions

Short-time power series expansions are usually employed to define the characteristic time scales for different physical quantities. We show that such a time scale is not a proper tool to analyze the FCD due to long atom–field interaction times in a Ramsey interferometer such as the one described in [17]. This can be understood by observing that, for this model, higher-order terms become relevant very early in the scale defined by the second-order term. For the present case, a suitable time scale is defined through the analysis of dephasing of complex phases. Of course, time scales defined by means of short-time power series expansions are applicable in several situations, such as, for example, the case described in [16], where analytical and numerical investigations show the relevance of the second-order time scale to the decoherence process.

According to our analytical and numerical investigations, an oscillator performing a dynamics with no chaotic (or chaotic-like) behavior is capable of destroying interference fringes in an effectively irreversible fashion analogously to a reservoir. However, important differences between the case studied here and other situations involving actual reservoirs still exist. The single oscillator cannot produce the total dissipation of the atomic energy as some infinite degrees of freedom reservoirs do. Also, the bosonic system will not behave as a reservoir in the sense that it relaxes to a unique thermal equilibrium state. Another relevant difference: as is shown in [32], the dynamics of the spin–boson system can be reversed by applying a transformation on the atomic part. It is also important to stress that a complete disappearance of interference fringes is not expected, but only attenuation. As in the system studied in [33], quantum phenomena disappear only when we consider a finite experimental resolution.

In the Ramsey interferometer of [17], the coherent field acts as a beam splitter. Since it is part of a measuring device, it must act classically [34]. This classicality is achieved when the energy of the field is increased, which leads to slower atom–field entanglement. The quantum system (atom) and the classical system (field) must interact for the shortest time that produces the atom’s state splitting required for the interferometry. If this interaction time is long, quantum and classical systems get entangled. Then, another kind of classical limit may be envisaged: the field acts as the environment, destroying quantum coherences of the atom. The crucial factor seems to be not the energy of the field, but the photon number variance.

Acknowledgments

We are grateful to M C Nemes, J G Peixoto de Faria and R Rossi Jr for fruitful discussions. ARBM acknowledges FAPEMIG, CNPq and CEFET-MG for partial financial support. ACO acknowledges FAPESB for partial financial support.

References

[1] Giuliani D, Joos E, Kiefer C, Kupsch J, Stamatescu I-O and Zeh H D 1996 Decoherence and the Appearance of a Classical World in Quantum Theory (Berlin: Springer)
[2] Bouwmeester D, Ekert A and Zeilinger A (ed) 2000 The Physics of Quantum Information (Berlin: Springer)
[3] Brune M, Hagley E, Dreyer J, Maitre X, Maali A, Wunderlich C, Raimond J M and Haroche S 1996 Phys. Rev. Lett. 77 4887
[4] Senitzky I R 1960 Phys. Rev. 119 670
[5] Kofler J and Brukner C 2007 Phys. Rev. Lett. 99 180403
[6] Bonifacio R, Olivares S, Tombesi P and Vitali D 2000 Phys. Rev. A 61 053802
[7] Ford G W, Kac M and Mazur P 1965 J. Math. Phys. 6 504
[8] Ford G W, Lewis J T and O’Connell R F 1988 Phys. Rev. A 37 4419
[9] Caldeira A O and Leggett A J 1983 Ann. Phys. 149 374
[10] Furuya K, Nemes M C and Pellegrino G Q 1998 Phys. Rev. Lett. 80 5524
[11] Blume-Kohout R and Zurek W H 2003 Phys. Rev. A 68 032104
[12] Rossini D, Benenti G and Casati G 2006 Phys. Rev. E 74 036209
[13] Casati G and Rossini D 2007 Prog. Theor. Phys. 166 70
[14] Bandyopadhyay J N 2009 Europhys. Lett. 85 50006
[15] Gabriela Barreto Lemos and Fabrizio Toccano 2011 Phys. Rev. E 84 016220
[16] Oliveira A C, and Bosco de Magalhães A R 2009 Phys. Rev. E 80 026204
[17] Bertet P, Osnagui S, Rauschenbeutel A, Nogues G, Auffeves A, Brune M, Raimond J M and Haroche S 2001 Nature 416 166
[18] Cummings F W 1965 Phys. Rev. 140 A1051
[19] Kim J, Nemes M C, de Toledo Piza A F R and Borges H E 1996 Phys. Rev. Lett. 77 207
[20] Pegg D T and Barnett S M 1997 J. Mod. Opt. 44 225
[21] Gleyzes S, Kuhr S, Guerlin C, Bernu J, Deleglise S, Hoff U B, Brune M, Raimond J M and Haroche S 2007 Nature 446 297
[22] Raimond J M, Brune M and Haroche S 2001 Rev. Mod. Phys. 73 565
[23] Eberly J H, Narozhny N B and Sanchez-Mondragon J J 1980 Phys. Rev. Lett. 44 1323
[24] Narozhny N B, Sanchez-Mondragon J J and Eberly J H 1981 Phys. Rev. A 23 236
[25] Yoo H Y, Sanchez-Mondragon J J and Eberly J H 1981 J. Phys. A: Math. Gen. 14 1383
[26] Phoenix S J D and Knight P L 1988 Ann. Phys., NY 186 381
[27] Geo-Banachcho J 1990 Phys. Rev. Lett. 65 3385
[28] Geo-Banachcho J 1991 Phys. Rev. A 44 5913
[29] Phoenix S J D and Knight P L 1991 Phys. Rev. A 44 6023
[30] Terra Cunha M O and Nemes M C 2002 Phys. Lett. A 305 313
[31] Kubo R, Toda M and Hashitsumi N 1978 Statistical Physics II—Nonequilibrium Statistical Physics (Springer Series in Solid-State Sciences vol 31) (Berlin: Springer)
[32] Morigi G, Solano E, Englert B-G and Walther H 2002 Phys. Rev. A 65 040102
[33] Oliveira A C, Peixoto de Faria J G and Nemes M C 2006 Phys. Rev. E 73 046207
[34] Bohr N 1935 Phys. Rev. 48 696