Constructing and Displaying the Trajectory Path on the Aircraft On-board Cockpit Multifunction Displays

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Abstract

Background/Objectives: Development of algorithms for flight plan data transformation into the projected trajectory of the aircraft based on the unification of used trajectories in the horizontal plane to reduce the consumed computing resources. Methods: In order to display the flight route, specialized software is used in the on-board systems enabling to convert a flight plan data set into a predictable trajectory of the aircraft path for its subsequent visualization. Findings: The authors have defined, developed and presented the principles and algorithms for computation of route trajectories ensuring the building of calculated flight path of the aircraft to be displayed on the on-board displays. The route displaying is carried out by consistent plotting of the typical trajectories that constitute the route path and in their turn consist of a set of graphic primitives such as line and arc. The database to be displayed is based on the flight plan, made in accordance with the “Jeppesen” database or similar, after bringing it to the formats of three typical defined trajectories. Applications/Improvements: The algorithms of the flight plan data conversion into a projected trajectory in the horizontal plane unify the process of information generation for displaying on the on-board multifunction displays and optimize a set of computation data to control the motion along a trajectory.

Keywords: Area Navigation, Aircraft Trajectory Path, Navigation

1. Introduction

In order to ensure the flight execution as per the route specified in the flight mission (flight plan), the route is displayed on the cockpit navigation displays.

Today’s requirements regarding the accuracy of en-route navigation, including the area navigation rules that require the track keeping to be 0.95 not more than ±1.85 km (RNP shall be not worse than RNP 1), determine also the increased requirements for the precision of the route imaging on the on-board displays, including the displaying of areas of transition between fixed route trajectories that must correspond at best to the actual implementation of these transitions that is ensured by the on-board control system¹–³. Implementation of these requirements is realized by using modern multi-functional displays and route description in the form of separate final trajectories.

In general, the representation of the route comprises a set of predefined WayPoints (WP) and a set of route forming trajectories determined in geodetic coordinate system. The route displaying on the display is made by the conversion of WP and trajectories from the geodesic to the display-oriented coordinate system with due account to the selected display window and image scale⁴.

Initial data for the route building include a database of navigation information as per specific flight mission (flight plan) and containing a list of sequential WP and description of flight paths between them. Depending on the assigned tasks, the description of WP and paths can be made in accordance with the ARINC-424 standards or internal database structure⁵–⁷.

The on-board electronic flight instrument system converts predetermined navigation data into a database forming a route of trajectories each of which is formed...
by the WP connected to each other. Coordinates of the trajectory WP and type of connecting line are determined by calculation based on the given parameters of the corresponding route leg. Conversion principles and algorithms are not regulated and are determined by intended purpose of the aircraft and the technical characteristics of the on-board electronic flight instrument system.

An example of display of flight trajectories on the on-board multifunction display is shown in Figure 1.

As a variant of the algorithms of conversion of navigation information determined by flight mission, a typical variant of database forming in order to display the route (route legs) on the on-board displays to ensure the flight execution as per the area navigation rules or similar has been presented.

2. Trajectory Database

A database of each trajectory includes a set of consistent graphic primitives such as arcs and line segments defined in geodetic coordinate system. In general, each trajectory consists of a set of arcs plotted following indicative fixes and connected by line segments.

The scheme of the typical trajectory building is shown in Figure 2. For its displaying, a number of the following calculated parameters forming two or three arc and line sections are used:

- geodetic coordinates of the current or initial position (B0, L0),
- turn radius (R);
- azimuth of the center of the turn on crosswind leg (A1);
- angle of the arc of the turn on crosswind leg (L1) (significant);
- geodetic coordinates of the final turn (BR, LR);
- turn radius (R);
- azimuth of the center of the final turn (A2);
- angle of the arc of the final turn (L2) (significant);
- geodetic coordinates of the termination point of the trajectory (BE, LE);
- radius of the arc (RF) (only for an arc flight);
- azimuth of the center of the arc (A3) (only for an arc flight) (significant);
- angle of the arc flight (L3) (only for an arc flight).

When building a route, all the route forming paths are displayed in sequence. Doing this, the end point of the trajectory (BE, LE, or BF, LF - for an arc flight) is taken as the point of the current position (B0, L0) for the next trajectory.

The value of the radius of the turn used in the trajectory arcs building is determined by a given flight speed and a given bank angle, and is calculated by the formula:

$$ R = \frac{V_{ist}^2}{g \cdot \tan(\gamma)} $$

where $V_{ist}$ - true speed (pre-determined on this trajectory); $g$ - downward acceleration; $\gamma$ - value of a given bank angle on the current trajectory.

To determine the end of the trajectory having a linear turn lead, a value of calculated turning radius for a flight as per the next trajectory ($R_E$) is used, and it is defined by the above formula where the initial data are the given parameters related to the next trajectory. When building a route, all the route forming paths are displayed in sequence. Doing this, the end point of the trajectory (BE, LE, or BF, LF - for an arc flight) is taken as the point of the current position (B0, L0) for the next trajectory.

3. Trajectories of the Flight in the Horizontal Plane

Trajectories of the flight in the horizontal plane can be represented by three types of trajectories. Variants of executed typical trajectories in the horizontal plane, and the initial data for their calculation are presented in Table. 1.

When building a route which consists of a sequence of trajectories, a calculated azimuth of the end of the last trajectory is assumed to be the current course angle.

The standard types of trajectories, including some area navigation trajectories as typical ones are shown in Table 2. In the same table, the initial data conversion in order to calculate trajectories using the typical methods is presented as well.
Table 1. Typical trajectories in the horizontal plane

| Trajectory                                      | Schematic view | Initial data                                                                 |
|-------------------------------------------------|----------------|-------------------------------------------------------------------------------|
| Path line                                       |                | B0, L0 – current location coordinates. Ψ0 – current track angle. BF, LF –     |
|                                                 |                | coordinates of the executed navigation point. AF – final great-circle azimuth.|
| Direct to a fix (at a line tangent to a circle   |                | B0, L0 – current location coordinates. Ψ0 – current track angle. BF, LF –     |
| built in the intended direction)                |                | coordinates of the executed navigation point. AF – given direction of the     |
|                                                 |                | approach. RF – circle radius. TDF – turn direction.                           |
| Arc flight                                      |                | B0, L0 – current location coordinates. Ψ0 – current track angle. BF, LF –     |
|                                                 |                | arc center coordinates. AF – given azimuth of the executed navigation point.  |
|                                                 |                | RF – arc radius. TDF – turn direction.                                        |

Table 2. Presentation of the trajectories in a typical format

| Path and Terminator                           | Trajectory initial data                                                                 | Typical trajectory initial data |
|------------------------------------------------|----------------------------------------------------------------------------------------|--------------------------------|
| Track to a Fix                                 | B1, L1 – first fix coordinates; B2, L2 – second fix coordinates                       | Path line                       |
|                                                 |                                                                                       | BF, LF = B2, L2                |
|                                                 |                                                                                       | AF = \(F_{\text{br}}(B_2, L_2, B_1, L_1) + \pi\) |
| Course to a Fix                                | B, L – fix coordinates; ZPU – course                                                  | Path line                       |
|                                                 |                                                                                       | BF, LF = B, L                |
|                                                 |                                                                                       | AF = ZPU                       |
| Direct to a Fix                                | B, L – fix coordinates                                                                | Direct to a Fix                |
|                                                 |                                                                                       | BF, LF = B, L              |
|                                                 |                                                                                       | AF = 0, RF = 0               |
|                                                 |                                                                                       | TDF = 0                       |
| Course from a Fix to an altitude               | B, L – fix coordinates; ZPU – course; \(\Delta H\) – climb to necessary height       | Path line                       |
|                                                 |                                                                                       | BF, LF = \(F_{\text{pr}}(B, L, ZPU, \Delta H/\text{grad})\) |
|                                                 |                                                                                       | where \(\text{grad}\) – climb gradient |
|                                                 |                                                                                       | AF = ZPU                       |
Constructing and Displaying the Trajectory Path on the Aircraft On-board Cockpit Multifunction Displays

The following parameters are assumed to be the initial data: current course angle (Ψ0); final great circle azimuth (AFP).

The following parameters calculated by solving the geographical inverse on ellipsoid between the current location and the location of the executable navigation point, are assumed to be the measured parameters:

- Azimuth of the current location from an executable navigation point (AFP);
- Azimuth to an executable navigation point from the current location (D0F);
- Distance to an executable navigation point (Z).

Lateral deviation from the great circle (Z):

\[ Z = D0F \cdot \sin(AF - AFP). \]

Great circle azimuth (ΨF):

\[ \Psi F = AF + \pi + (AFP - AF0), \text{ in the range } [-\pi; \pi]. \]

Turns on the trajectory capture leg are carried out with a specific radius (R).

4. Turn Direction

The direction of a turn on crosswind leg is determined by the condition for a smaller angle of turn needed to capture the path of approaching to the great circle.

The trajectory of approaching to the great circle is a straight line located at a predetermined angle of approaching to the great circle, passing into an arc of a circle tangent to the great circle, with a radius equal to the radius of the turn. The predetermined angle of approaching is assumed to be equal to 45° (π/4). In the case of another angle of approaching, the π/4 value in the following formulas shall be replaced by the corresponding value.

Turn direction (TD):

\[ \text{with } |Z| \geq R \cdot (1 - \cos(\pi/4)): \]

\[ TD = \text{sign}(\sin(\text{sign}(Z) \cdot \pi/4 - (\Psi 0 - \Psi F))). \]

With \( |Z| < R \cdot (1 - \cos(\pi/4))): \]

\[ TD = \text{sign}(\sin(\text{sign}(Z) \cdot \arccos(1 - |Z|/R) - (\Psi 0 - \Psi F))). \]

- with \( TD = 1 \) clockwise turn, with \( TD = -1 \) counterclockwise turn.

4.2 Angle of Approaching, Course

The angle of approaching is an angle of capture of the arc of a circle tangent to the great circle with a radius equal to the radius of the turn, and calculated from the direction of the great circle. The angle of approaching is determined by the lateral deviation from the great circle and the turn direction.

The angle of approaching (F):

\[ \text{With } TD \cdot Z > R \cdot (1 + \cos(\Psi 0 - \Psi F) - 2 \cos(\pi/4)): F = TD \cdot \pi/4. \]

With \( TD \cdot Z \geq R \cdot (\cos(\Psi 0 - \Psi F) - 1) \) and

\[ TD \cdot Z \leq R \cdot (1 + \cos(\Psi 0 - \Psi F) - 2 \cos(\pi/4)): F = \arccos(0.5 \cdot (1 + \cos(\Psi 0 - \Psi F) - TD \cdot Z/R)). \]

With \( TD \cdot Z < R \cdot (\cos(\Psi 0 - \Psi F) - 1) \):

\[ F = -TD \cdot \pi/4. \]

The course angle of approaching (PsID)

\[ \Psi D = \Psi F + F, \text{ in the range } [-\pi; \pi]. \]

4.3 Parameters of Trajectory Capture

The leg of capture of the trajectory following the track line - great circle, includes the arc of the turn on crosswind leg from the current location until the angle of approach and the arc of the final turn entering the great circle from the start point of the final turn, connected by a straight line. The radii of the arcs are equal to the specific turn radius.

Parameters of the track line capture are shown in Figure 3.
The coordinates of the start point of the final turn in geodetic coordinate system ($BR, LR$) are determined by solving the common survey computation in the space relative to the current location with the calculated azimuth and distance of the start point of the final turn from the current location.

The azimuth and distance of the start point of the final turn from the current location ($AR, DR$):

$$AR = \Psi F + \arctg \left( \frac{(Z - ZR)}{SR} \right);$$
$$DR = \sqrt{((Z - ZR)^2 + SR^2)},$$

where ($SR, ZR$) – coordinates of the start point of the final turn in the orthodromic coordinate system with the center displaced along orthodromy up to the current location abeam:

$$ZR = \text{sign}(F) \cdot R \cdot \left(1 - \cos(F)\right);$$
$$SR = TD \cdot R / \sin(F) \cdot (1 - \cos(\Psi_0 - FP - F)) + (Z - ZR) / \tan(F);$$

with $F = 0$: $SR = 0$

Azimuth of the center of the turn on crosswind leg ($A1$): $A1 = \Psi_0 + \pi/2 \cdot TD$.

Angle of the arc of the turn on crosswind leg ($L1$): $L1 = TD \cdot L10$, where $L10 = TD \cdot (\Psi_0 - \Psi_0) \text{ in the range } [0; 2\pi]$.

Azimuth of the center of the final turn ($A2$): $A2 = \Psi ID - \pi/2 \cdot \text{sign}(F)$.

Angle of the arc of the final turn ($L2$): $L2 = (-FI) \cdot R$.

The total length of the orthodromic straight-line segment ($S0$):

$$S0 = -D0F \cdot \cos(AF0 - AF) - (SR + \text{sign}(F) \cdot R \cdot \sin(F)).$$

4.4 Trajectory Termination

End of the flight path following the track line is determined by the sign of the passage of an executable navigation point, which can be: passage of an executable point; achievement of turn lead.

Turn lead involves displacement of the termination point of the current trajectory from the executable navigation point to the linear turn lead ($SE$) in order to capture the next trajectory of the flight following the track line or the single turn arc.

The parameters of the next trajectory are assumed to be the following:

If the next trajectory is a track line:
- Azimuth to a navigation point of the next trajectory from the executable point ($AN$).
- Distance to the center of the next arc from the executable navigation point ($DN$);
- Next arc radius ($RN$);
- Direction of the turn following the next arc ($TDN$).

Turns in the leg of the next trajectory capturing are made with the specific turn radius ($RE$).

The parameters of the linear turn lead are presented in Figure 4.

If the next path is the track line, linear turn lead ($SE$) is the following:

$$SE = RE \cdot \frac{\sin((AF - AN)/2)}{1};$$

If the next trajectory is a circle flying, a linear turn lead is the following ($SE_E$):

$$SE = TD \cdot \sqrt{(BR + TDR \cdot RE)^2 - (DN \cdot RE - TDR \cdot DN \cdot \sin(AF - AN))^2} - DN \cdot \cos(AF - AN).$$

where, $TDR = \text{sign} \left( \cos(AF - AN) \right)$

In default of this condition or when $SE < 0$: $SE = 0$.

In order to avoid incorrect transition between adjacent paths, the value of the linear turn lead is limited to a certain specified value. As specified limit value, a turn radius is usually used: When $SE > RE$: $SE = RE$.

When passing an executed point (if there no sign of the use of linear turn lead), the linear turn lead is assumed to be equal to zero: $SE = 0$.

The coordinates of the termination point of the trajectory in geodetic coordinate system ($BO, LE$) are...
determined by solving common survey computation in space relative to the executable navigation point.

Azimuth and distance of the termination point of the trajectory from the executable navigation point \((AE, DE)\): 
\[
AE = AF + \pi; \ DE = SE
\]

Azimuth of the trajectory termination \((\Psi E)\):

\[
\Psi E = AF \ in \ the \ range \ [-\pi; \pi].
\]

4.5 Trajectory interruption

The trajectory interruption is determined by the condition: \(S0 - SE < 0\).

This condition means the inability to capture the trajectory of the path line until the attainment of the termination point of the trajectory. In this case, this trajectory is disregarded and is not computed.

5. Direct to a Fix Trajectory

The current course is assumed to be the initial data \((\Psi 0)\).

The following parameters calculated by solving the geographical inverse on ellipsoid between the current location and the location of the executable navigation point, are assumed to be the measured parameters:

– Azimuth of the current location from an executable navigation point \((AF0)\);
– Azimuth to an executable navigation point from the current location \((A0F)\);
– Distance to an executable navigation point \((DOF)\).

For combined method path section or in order to capture a circle tangent point in the given direction, the following parameters are additionally specified:

– Given direction of the approach \((AF)\);
– Circle radius \((RF)\);
– Turn along a given circle \((TDF)\) – if necessary.

If the flight trajectory is not a combined method leg or exit to a circle tangent point in the given direction, the \(AF\) and \(RF\) values are assumed to be zero.

Turns in the trajectory capture leg are executed by means of a calculation radius \((R)\).

5.1 Final Turn Direction

The circle of the final turn with a radius equal to the radius of the turn is built for the path leg of combined method flight along the trajectory or to capture the circle tangent point in the desired direction, and ensures the capture of the executable navigation point with a specified approach direction.

The final turn direction \((TDF)\) shall be calculated in the absence of its precise targeting. The final turn direction is determined by the hemisphere relative to the equator passing through executable navigational point with an azimuth equal to the specified approach direction \((AF)\), except for the supposed case of crossing of the equator during the turn. Selection of the direction of the final turn is shown in Figure 5.

\[
TDF = \text{sign}(R \cdot \text{sign}(\sin(\Psi 0 - AF)) \cdot (1 + \cos(AF0 - AF + \Psi 0)) + DOF \cdot \sin(AF0 - AF)).
\]

5.2 Turn Direction

The direction of a turn on crosswind leg is determined by the condition for a smaller turn angle needed to enter the azimuth of approaching to a predetermined navigation point or a circle tangent point.

The turn direction \((TD)\) is determined by the formula:

\[
TD = \text{sign} \left( \sin \left( \arcsin \left( \frac{RF}{DR} \right) \right) - \arcsin \left( \frac{RF}{DOF} \right) \right)
\]

where, the distance to the center of the final turn circle \((DR)\):

\[
DR = \sqrt{DOF^2 + RF^2 + 2 \cdot TDF \cdot DOF \cdot RF \cdot \sin(AF0)}
\]

The azimuth of the center of the final turn circle \((AR)\):

\[
AR = AF0 - TDF \cdot \arcsin(RF \cdot \cos(AF0)) / DR
\]

With \(TD = 1\) – clockwise turn, with \(TD = -1\) – counterclockwise turn.

5.3 Angle of Approach, Course

Angle of approach, which is also a course \((\Psi D)\) is a course to the exit to an executable navigation point, or for the path leg of combined method flight along trajectory or in order to capture the circle tangent point, course to the exit to the tangent line to the circle of the final turn.
Angle of approach ($\Psi D$) is:

$$\Psi D = AC + \arcsin\left(\frac{T \cdot R - T \cdot D \cdot RF}{DC}\right) \text{ in the range } [-\pi; \pi].$$

where, the distance between the centers of the circles of the turn on crosswind leg and the final turn (DC) is:

$$DC = \sqrt{DR^2 + R^2 + T \cdot 2 \cdot DR \cdot R \cdot \sin(\Psi 0 - AR)}.$$ 

The angle between the centers of the circles of the turn on crosswind leg and the final turn:

$$AC = AR - T \cdot D \cdot \arcsin\left(\frac{R}{DC} \cdot \cos(\Psi 0 - AR)\right).$$

When $|TD \cdot R - TDF \cdot RF| > DC$ – trajectory shall not be executed (see Trajectory interruption).

5.4 Parameters of the Trajectory Capturing

The leg of capture of the trajectory includes the arc of the turn on crosswind leg from the current location until the angle of approach with a radius equal to the radius of the turn.

Parameters of the trajectory capturing are presented in Figure 6.

Parameters of the trajectory capturing are calculated by the following formulas.

Azimuth of the center of the turn on crosswind leg ($A_1$):

$$A_1 = \Psi 0 + \pi/2 \cdot TD.$$

Arc of the angle of the turn on crosswind leg ($L_1$):

$$L_1 = T \cdot D \cdot L10,$$

where

$$L10 = T \cdot D \cdot (\Psi D - \Psi 0) \text{ in the range } [0; 2\pi].$$

The overall length of the straight section of trajectory ($S_0$):

$$S_0 = \sqrt{DC^2 - TDF \cdot (R - T \cdot D \cdot TDF \cdot RF)^2}.$$ 

5.5 Trajectory Termination

If the flight trajectory is not a combined method leg or exit to a circle tangent point in the given direction, the termination point of the trajectory (the coordinates of the trajectory termination fix) is determined in accordance with the termination of the trajectory of the path line and a predetermined sign of the passage of an executable navigation point, where:

$$AF = \Psi D + A0F - AF0 + \pi.$$ 

In the case of the leg of combined method flight along trajectory, or exit to a tangent point of given circle, the trajectory ends in a point of tangency of the final turn to the executable point. The coordinates of the termination point of the trajectory in geodetic coordinate system ($BE$, $LE$) are determined by a common survey computation in space relative to the executable navigation point.

Distance and azimuth of the trajectory termination point, calculated measured from the executable navigation point ($DE, AE$) are:

$$DE = RF \cdot \sqrt{2 \cdot \left(1 + \cos(A0F - AF0 + \Psi D - AF)\right)};$$

$$AE = \arccos\left(\frac{DE}{RF/2}\right).$$

Trajectory termination azimuth ($\Psi E$):

$$\Psi E = \Psi D + A0F - AF0 + \pi \text{ in the range } [-\pi; \pi].$$

5.6 Trajectory Interruption

The trajectory interruption is determined by any of the conditions:

$$|TD \cdot R - TDF \cdot RF| > DC \text{ or } S0 - SE < 0.$$ 

Presence of one of such condition means the inability to capture the trajectory of the path line until the attainment of the termination point of the trajectory. In this case, this trajectory is disregarded and is not computed.

6. Arc Flight Trajectory

The following parameters are assumed to be the initial data: current course angle ($\Psi 0$); arc radius ($RF$); predetermined azimuth of the executable navigation point from the center of the arc (AF); turn direction ($TDF$).

The following parameters calculated by solving the geographical inverse on ellipsoid between the current location and the location of the arc center, are assumed to be the measured parameters: azimuth of the current location from an executable navigation point ($A0F$); azimuth

![Figure 6. Parameters of the Direct to the Fix trajectory.](image)
of the center of an executable arc from the current location ($A0F$); distance to an executable navigation point ($D0F$).

Turns on the trajectory capture leg are carried out with a specific radius ($R$).

6.1 Turn Direction

The direction of a turn on crosswind leg is determined by the condition for a smaller angle of turn needed to capture the trajectory of approaching to the arc.

The trajectory of approaching to the arc is a straight line directed to or from the center of the arc passing into the arc of a circle with a radius equal to the turn and tangent to the given arc in the predetermined direction (TDF).

Sectors of turn direction selection (TD) depending on the distance from the center of the arc, the direction of the arc turn and the direction of the current course are shown in Figure 7.

Turn direction (TD):

When $D0F^2 \geq (R^2 + 2 \cdot RF \cdot R)$:

$$TD = \text{sign} (\sin (A0F - \Psi0))$$

When $D0F \geq RF$ and $D0F^2 < (R^2 + 2 \cdot RF \cdot R)$:

$$TD = \text{sign}(TDF \cdot \cos(A0F - \Psi0) - TDF \cdot \arccos((D0F^2 - RF^2)/2R/(R - D0F/R)))$$

When $D0F < RF$ and $D0F^2 > (R^2 - 2 \cdot RF \cdot R)$ and $D0F > (2 \cdot R - RF)$:

$$TD = -\text{sign}(TDF \cdot \cos(A0F - \Psi0) - TDF \cdot \arccos((D0F^2 - RF^2)/2R/(R - D0F/R)))$$

When $D0F^2 \leq (R^2 - 2 \cdot RF \cdot R)$ and $D0F > (2 \cdot R - RF)$:

$$TD = -\text{sign} (\sin (A0F - \Psi0))$$

With $TD = 1$ – clockwise turn is obtained, with $TD = -1$ – counterclockwise turn.

6.2 Angle of Approach, Course

Angle of approach, which is also a course ($\Psi D$) is a course to the exit to an arc of a circle with a radius equal to the turn radius and tangent to a specified executable arc.

Angle of approach ($\Psi D$) in the range $[-\pi; \pi]$:

When $DC \geq (RF + R)$:

$$\Psi D = AC - TD \cdot \arcsin(R/DC) + \pi$$

where, the distance from the center of the turn on crosswind leg to the predetermined arc center ($DC$) is:

$$DC = \sqrt{D0F^2 + R^2 + 2 \cdot TD \cdot D0F \cdot R \cdot \sin(\Psi0 - A0F)}$$

Azimuth from the center of the turn on crosswind leg to the predetermined arc center ($AC$) is:

When $D0F^2 > R^2 + DC^2$:

$$AC = \Psi0 + TD \cdot (\pi/2 - \arcsin(D0F \cdot \cos(\Psi0 - A0F))$$

When $D0F^2 \leq R^2 + DC^2$:

$$AC = \Psi0 - TD \cdot (\pi/2 - \arcsin(D0F \cdot \cos(\Psi0 - A0F)/DC))$$

6.3 Parameters of the Trajectory Capturing

The leg of exit to the arc flight trajectory includes the arc of the turn on crosswind leg from the current location to the angle of approach and the arc of final turn of the exit to a given executed arc from the start point of the final turn, connected by a straight line. The radii of the arcs during the turns are equal to turn radius. The parameters of the trajectory of the arc capture are shown in Figure 7.

Parameters of the trajectory of the arc capture are computed by the following formulas. The coordinates of the start point of the final turn in geodetic coordinate system ($BR$, $LR$) are determined by solving the common survey computation in the space relative to the center of the executed arc with the calculated azimuth and distance of the start point of the final turn from the center of the arc.

Distance and azimuth of the start point of the final turn from the center of the arc ($DR$, $AR$):

When $DC \geq (RF + R)$:

$$DR = \sqrt{RF^2 + 2 \cdot RF \cdot R}; AR = A0F - A0F + \Psi D$$

When $DC < (RF + R)$ and $DC > (RF - R)$:

$$\Psi D = AC + TD \cdot \arcsin((DC^2 + 4R^2 - (RF + TDF + TD + R)) / 4/D0F/R)$$

When $DC \leq (RF - R)$:

Figure 7. Parameters of the trajectory of the arc capture.
Azimuth of the center of the turn on crosswind leg (A1): 
\[ A_1 = \psi_0 + \pi/2 \cdot TD. \]

The angle of the arc of the turn on crosswind leg (L1): 
\[ L_1 = TD \cdot L_{10}, \]
where \( L_{10} = TD \cdot (\psi_D - \psi_0) \) in the range \([0;2\pi]\).

The angle of the arc of the final turn (L2): 
\[ \text{When } DR > R_F: \]
\[ L_2 = -TDF \cdot \arccos ((R^2 + (RF + R)^2 - DR^2)/2R)/(RF + R). \]

\[ \text{When } DR < RF: \]
\[ L_2 = TDF \cdot \arccos ((R^2 + (RF - R)^2 - DR^2)/2R)/(RF - R). \]

Azimuth of the center of the final turn (A2): 
\[ A_2 = \psi_D + \pi/2 \cdot \text{sign}(L2) \]

### 6.4 Trajectory Termination
The trajectory of the arc is built from the point of exit to the arc to the executable navigation point. The coordinates of the point of exit to the arc (BE, LE) in a geodetic coordinate system are defined by solving the common survey computation in the space relative to the center of the executed arc with the calculated azimuth of the point of exit to the arc from the arc center and at the distance equal to the arc radius.

Azimuth of the point of exit to the arc from the arc center (A0): 
\[ A_0 = \psi_D + L_2 - \pi/2 \cdot TDF. \]

Azimuth of the center of turn along a given arc to the executable point (A3): 
\[ A_3 = A_0 + \pi. \]

The angle of the turn along a given arc (L3): 
\[ L_3 = TDF \cdot L_{30}, \] where \( L_{30} = TDF \cdot (AF - A_0) \) in the range \([0;2\pi]\).

Azimuth of the trajectory termination (\( \psi_E \)): 
\[ \psi_E = AF + \frac{\pi}{2} \cdot TDF \] in the range \([-\pi;\pi]\).

### 6.5 Trajectory Interruption
 Interruption of the trajectory is determined by limiting the magnitude of the angle of the turn along a given arc. In the case of the absolute magnitude of the angle of turn (L3) in the predetermined direction exceeding \((2\pi - \pi / L3)\), this trajectory is disregarded and its computing is not carried out. That is, the angle of flight along the arc is less than 345°.

### 7. Conclusion
Building of the flight route of the aircraft in the horizontal plane for displaying on cockpit displays is performed by conversion of specified trajectories that constitute the route defined by the flight mission (flight plan) into a fixed database of trajectories. The database contains a particular set of fixes in a geodetic coordinate system connected to each other either by an arc or a straight line.

It should be noted that any desired flight path of the aircraft (including the area navigation trajectory) can be represented in the form of one or more specific typical trajectories: path line, direct to a fix, or arc flight. This representation unifies the information generating algorithms for displaying on the navigation indicators and optimizes a set of calculated data to control the flight along the trajectory in the horizontal plane.

### 8. References
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