On the theory of three types of polaritons (phonon, exciton and plasmon polaritons)

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Abstract. We have investigated the similarities and difference between three well-known types of polaritons: phonon polariton, exciton polariton and surface plasmon polariton. For first two types (phonon polariton and exciton polariton) the interaction between photon and media can be expressed via a longitudinal-transversal splitting (LT-splitting), while for third type of polariton (surface plasmon polariton) via the boundary condition. Considering an analogy of these three types of polaritons, an effective LT-splitting was introduced for surface plasmon polariton. We discuss a possible existence of an evanescent state in the band gap of polaritons. Finally, the Nambu broken symmetry theory and Anderson-Higgs mechanism are discussed for lower branch of these polaritons.

1. Introduction

In recent years, the study of polariton has a great attention on both experimental and theoretical sides. The concept of polaritonics is recently introduced to the studies of polariton and its applications. The frequency spectra of polaritonics ranges from hundreds of gigahertz to several terahertz, which is the frequency gap between electronic and photonics. Therefore, polaritonics plays an important role in enabling advanced signal processing and spectroscopy application [1], allows observation of polariton condensation specially at room temperature [2-4], stimulates scattering [5], quantized vortices and super fluid behavior [6-7]. Besides that, polaritons can be applied in ultrafast optical switches, ultralow threshold polariton lasers and light emitting diodes [8-10].

Polariton can be simply defined as the strong coupling of photon with another quasiparticle. One can distribute polariton in different types: phonon polariton, exciton polariton and plasmon polariton.

The phonon polariton concept was introduced by Kun Huang (1951) when he studied the coupling of photon with phonon. In 1958, Hopfield studied the coupling of photon with exciton and suggested the concept of exciton polariton. Since then, the theoretical models of exciton polariton and phonon polariton have mainly focused on bulk polariton and neglected surface polariton lines which are in LT gap. In the case of plasmon polariton, most of works have focused
on surface plasmon polariton with frequency less than surface plasmon frequency. The range of frequency between bulk plasmon and surface plasmon is considered as the forbidden gap. Recently, the existence of novel evanescent state in the forbidden gap has shown in [12-15]. Topological insulator is an example for state in the forbidden band gap. The surface state of topological insulator has interesting properties that are protected by the time reversal symmetry and can not be destroyed by non-magnetic disorder. Because of these interesting properties, topological insulator is a promising material applied in nanotechnology [16-18]. In this work, in order to understand more clearly the nature of the surface plasmon polariton, we review the picture of three types of polaritons. We will analyze and compare three well-known types of polaritons to find out the similarities and differences between them. Both phonon polariton and exciton polariton have coupling constant described via longitudinal-transversal splitting. Considering a similarity of three types of polaritons as referred above, we propose a new model for bulk plasmon polariton by introducing an effective plasmon-photon coupling constant with an effective LT-splitting. Besides that, for both phonon polariton and exciton polariton there exist evanescent state in the forbidden band gap, so a question is if a similar state exists in the forbidden gap of surface plasmon polariton.

2. Phonon polariton
Phonon polariton is the result of the interaction of phonon with electromagnetic field ( photon). The interaction between electromagnetic wave and phonon can be described by Maxwell equations. In the case of plane wave with the wave vector \( \vec{k} \) and there is no charge in the medium, the Gauss equation is

\[
\nabla \vec{D} = 0, \tag{1}
\]

where \( \vec{D} \) is the electric displacement vector. This equation is equivalent to

\[
\varepsilon(\omega) (\vec{k} \cdot \vec{E}) = 0, \tag{2}
\]

which has two solutions \( \varepsilon(\omega) = 0 \) and \( \vec{k} \cdot \vec{E} = 0 \). The first solution corresponds to longitudinal mode with the longitudinal resonance frequency \( \omega = \omega_L \), the second solution corresponds to transversal mode with the transversal resonance frequency \( \omega = \omega_T \).

Using the Maxwell equations, the dispersion relation of phonon polaritons can be expressed as follows

\[
k^2 = \frac{\omega^2}{c^2} \varepsilon(\omega), \tag{3}
\]

where \( c \) is velocity of photon in vacuum, and \( \varepsilon(\omega) \) is the dielectric function of material,

\[
\varepsilon(\omega) = \varepsilon_{\infty} \frac{\omega_L^2 - \omega^2}{\omega_T^2 - \omega^2}. \tag{4}
\]

The solution of equation (3) is the dispersion relation of phonon polariton

\[
\omega^2 = \frac{1}{2\varepsilon_{\infty}} \left\{ c^2 k^2 + \varepsilon_{\infty} \omega_L^2 \pm \sqrt{(c^2 k^2 + \varepsilon_{\infty} \omega_L^2)^2 - 4\varepsilon_{\infty} c^2 k^2 \omega_T^2} \right\}. \tag{5}
\]

The dispersion curve of phonon polariton is plotted in figure 1 with two branches: upper and lower. One can see that the upper branch approaches to the constant value \( \omega_L \) when \( k \to 0 \), while the lower branch tends to \( \omega = ck/\sqrt{\varepsilon_{\infty}} \). On the other hand, as \( k \to \infty \) the upper branch approaches to \( \omega = ck/\sqrt{\varepsilon_{\infty}} \) and the lower branch approaches \( \omega_T \). In the figure 2, we plot \( \varepsilon(\omega) \) as a function of \( \omega \). It is easy to show that the dielectric function is positive when frequency is less than \( \omega_T \) or more than \( \omega_L \), so electromagnetic wave can propagate through the material.
For frequencies are between $\omega_T$ and $\omega_L$, the dielectric function is negative. In this range of frequency, electromagnetic wave does not propagate but decreases exponentially. This region is known as Reststrahlen band [19].

Phonon can interact with long-wavelength incident fields, creating a surface excitation mediated which known as surface phonon polariton. Surface phonon polariton can be excited in a polar dielectric crystal in the Reststrahlen band. When a plane wave which have frequency in the Reststrahlen band transmits to surface of a polar dielectric crystal, the positive ions move along the direction of the field and the negative ions move against it respectively. As a result the real part of permittivity is negative and an evanescent field occurs inside the polar dielectric crystal. This evanescent field is known as surface phonon polariton.

Using Maxwell equations, one can obtain the dispersion relation of surface phonon polariton as

$$ \varepsilon(\omega) \sqrt{k^2 - \frac{\omega^2}{c^2}} = -\sqrt{k^2 - \varepsilon(\omega) \frac{\omega^2}{c^2}}, \quad (6) $$

therefore

$$ \omega = ck \sqrt{\frac{1 + \varepsilon(\omega)}{\varepsilon(\omega)}}, \quad (7) $$

**Figure 1.** The dispersion curve of bulk phonon polariton in GaP (black solid line) with the values $\omega_L = 401.9 cm^{-1}$, $\omega_T = 366.3 cm^{-1}$, $\varepsilon_0 = 11$, $\varepsilon_{\infty} = 9.1$ are taken from [11].

**Figure 2.** The dielectric function of ion crystals.
We can see that the equation (6) can only be satisfied within the frequency range \( \omega_T \leq \omega < \omega_L \) because the dielectric function is negative in this region. The dispersion of surface phonon polariton starts at \( \omega = \omega_T, \, k = \omega_T/c \) and approaches to \( \omega_L \) when \( k \to \infty \) as presented in the figure 3.

\[
\omega = \omega_T, \, k = \omega_T/c
\]

**Figure 3.** The dispersion curve of surface phonon polariton in GaP (blue solid line). The LT-splitting of phonon polariton is quite small, so the surface phonon polariton is neglected.

### 3. Exciton polariton

Exciton is an electrically neutral quasiparticle which is formed by the electrostatic Coulomb interaction between an electron and a hole. Exciton can exist in insulators, semiconductors and in some liquids. An exciton is formed when semiconductor absorbs a photon, an electron can be excited into the conduction band and a hole with positive charged is left behind in the valence band. This electron-hole pair is known as an exciton.

Exciton polariton is defined as a quasiparticle which is a result of the coupling between a photon and an exciton. The interaction of photon with exciton can be also expressed by the Maxwell equations. One can find that the dispersion relation of exciton polariton has a form like equation (5). The dispersion curve is presented in the figure 4. There are two branches which are known as upper and lower branch. For the frequency in the region between \( \omega_T \) and \( \omega_L \), the electromagnetic wave decreases exponentially. The longitudinal-transversal splitting is

\[
\omega_{LT} = \omega_L - \omega_T.
\]

This splitting presents the coupling strength between the exciton and photon.

Surface exciton polariton can be referred as strong coupling of exciton with light which has a component of the wave-vector in plane of the surface. Solving the Maxwell equations with the continuity of the magnetic field at the surface, we obtain the relation between frequency and wave vector of surface exciton polariton as equations (6) and (7). One can see that, equations (6) and (7) equivalent to \( \varepsilon < -1 \). This condition is satisfied with frequencies in the range \( \omega_T < \omega < \omega_L \). The dispersion relation of surface exciton polariton in semiconductor is presented in the figure 5. By comparing phonon polariton and exciton polariton, we realize that both phonon polariton and
exciton polariton have longitudinal-transversal splitting that corresponds to the longitudinal-transversal symmetry breaking. Normally, the LT-splitting of exciton polariton is very small, about meV (for GaAs $\omega_{LT} = 0.08meV$ [20], ZnO $\omega_{LT} = 5.00meV$ [21]) and the dispersion curve of surface exciton polariton is nearly straight. This leads to the mass of surface exciton polarion approaches to infinity, so it can not move and the state in this band is also neglected.

4. Plasmon polariton
Bulk plasmon polariton is quasiparticle which is a result of the coupling between plasmon and photon. This concept is similar to exciton polariton and phonon polariton. Using the Drude model for metal, we have metal dielectric constant

$$\varepsilon_m(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\gamma\omega}, \quad (9)$$

where $\gamma$ is the collision rate, $\omega_0$ is the frequency of restoring force, $\omega_p$ is the bulk plasma frequency, $\omega_p = \sqrt{ne^2/\varepsilon_0 m}$ in which $n$ is the electron density, $\varepsilon_0$ is the vacuum permittivity and $m$ is effective electron mass. We consider a simple case of no restoring ($\omega_0 = 0$) and small
collision rate \((\gamma = 0)\), then the metal dielectric function is

\[
\varepsilon_m(\omega) = 1 - \frac{\omega_P^2}{\omega^2}.
\]  

(10)

\[
\varepsilon_m(\omega) = 1 - \frac{\omega_P^2}{\omega^2}.
\]

Figure 6. The dielectric function of metal (blue solid line).

Solving Maxwell equations, we obtain the dispersion relation of bulk plasmon polariton which is also represented as equation (3). The dispersion relation of bulk plasmon polariton can be rewritten in an explicit form

\[
\omega^2 = \omega_P^2 + c^2 k^2.
\]  

(11)

The dispersion relation implies that bulk plasmon polariton frequency is always higher than bulk plasma frequency, that means no transverse electromagnetic wave with frequency smaller than the bulk plasma frequency can travel through medium. The dispersion curve of bulk plasmon polariton is plotted in the figure 7.

Considering the similarity of three types of polaritons, we propose a new model for bulk plasmon polariton with the transversal resonance frequency is \(\omega_{TB P} = 0\) and the longitudinal resonance frequency is \(\omega_{LBP} = \omega_P\), then bulk plasmon polariton also has an effective bulk plasmon polariton splitting

\[
\omega_{LT}^* = \omega_{LBP} - \omega_{TB P} = \omega_P.
\]  

(12)

The effective LT-splitting of bulk plasmon polariton equals to bulk plasmon energy about ten eV. This effective splitting also describes the effective coupling between bulk plasmon and photon similar to the case of phonon polariton and exciton polariton.

In the classical theory, surface plasmon polariton is electromagnetic wave bound to an interface between two media. Surface plasmon polariton can also be defined as a collective oscillation of charge carriers propagating on both side of the interface. We consider a simple model in which the dielectric medium is vacuum (the dielectric constants \(\varepsilon_d = 1\)). Solving the Maxwell equations at the metal/vacuum planar interface with continuity conditions of electric field, one can obtain the dispersion relation for surface plasmon as

\[
k = \omega \sqrt{\frac{\varepsilon_m(\omega)}{1 + \varepsilon_m(\omega)}}.
\]  

(13)

In the Drude model for metal, the dispersion relation of surface plasmon polariton can be written in the following form

\[
\omega = \sqrt{c^2 k^2 + \omega_{SP}^2} - \sqrt{c^4 k^4 + \omega_{SP}^4}.
\]  

(14)
in which $\omega_{SP} = \omega_{SP}/\sqrt{2}$. The dispersion is presented in the figure 8. This curve starts at $\omega = \omega_{TBP} = 0 \ (k = 0)$ and approaches to $\omega = \omega_{SP}$. The dispersion curve of surface plasmon polariton is in the band between $\omega_{TBP}$ and $\omega_{SP}$ similar to the case of surface phonon polariton and surface exciton polariton. There is no electromagnetic wave propagating in the band gap between $\omega_{SP}$ and $\omega_{LBP} = \omega_{P}$. This band gap can be called a forbidden gap.

Figure 7. The dispersion curve of bulk plasmon (blue solid line).

Figure 8. The dispersion curve of surface plasmon polariton (blue solid line).

In the case of phonon polariton and exciton polariton, there exists evanescent states in the band gap. The question is if a similar states in the forbidden gap of surface plasmon polariton? This problem will be investigated in our next work.

5. Conclusion
In summary, we have investigated the similarities and differences between three well-known types of polaritons: phonon polariton, exciton polariton and surface plasmon polariton. Both phonon polariton and exciton polariton have coupling constant which is described via longitudinal-transversal splitting, while this coupling absents for the case of surface plasmon polariton because surface plasmon is longitudinal and photon is transversal polarization.
Assuming that three types of polaritons are similar, we have considered a new model for bulk plasmon polariton by introducing an effective plasmon-photon coupling constant which is described via effective LT-splitting. The longitudinal-transversal splitting is equivalent to the longitudinal-transversal broken Galilean symmetry, so the Nambu broken symmetry theory and Anderson-Higgs mechanism could be applied to these types of polaritons. The existence of surface modes in the LT energy gap for all three types is noted. We calculated and showed that the surface dispersion curves of three types of polaritons are similar, so a common theory of surface polaritons is needed.

There is a wide forbidden gap occurs between $\omega_{SP}$ and $\omega_{LBP} = \omega_{P}$. The possible existence of new mode in the energy gap of surface plasmon polariton will be the subject of our next work.

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