Neutrinoless double $\beta$ decay, neutrino mass hierarchy, and neutrino dark matter

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Abstract

Recently the evidence of the neutrinoless double $\beta$ ($0\nu\beta\beta$) decay has been announced. This means that neutrinos are Majorana particles and their mass hierarchy is forced to $m_1 \sim m_2 \gg m_3$ (Type B) or degenerate mass, $m_1 \sim m_2 \sim m_3$ (Type C) patterns in the diagonal basis of charged lepton mass matrix, where $m_i$ is the absolute value of $i$-th generation neutrino mass. We analyze the magnitude of $0\nu\beta\beta$ decay in the degenerate neutrinos including the cosmological constraint of neutrino dark matter, since the Type B mass hierarchy pattern always satisfies the cosmological constraint. The upper bound of neutrino absolute mass is constrained by $0\nu\beta\beta$ experiment or cosmology depending on Majorana $CP$ phases of neutrinos and solar mixing angle.
Recently the evidences of neutrino oscillations are strongly supported by both of the atmospheric [1, 2] and the solar neutrino experiments [3, 4, 5, 6]. The former suggests an almost maximal lepton flavor mixing between the 2nd and the 3rd generations, while the favorable solution to the solar neutrino deficits is given by large mixing angle solution between the 1st and the 2nd generations (LMA, LOW) [7, 8]. Neutrino oscillation experiments indicate that the neutrinos have tiny but finite masses, with two mass squared differences \( \Delta m^2_\odot < \Delta m^2_{\text{atm}} \). However, we cannot know the absolute values of the neutrinos masses from the oscillation experiments.

Recently, a paper [9] announces the evidence of the neutrinoless double \( \beta \) (0\( \nu \beta \beta \)) decay. This paper suggests

\[
\langle m \rangle = (0.05 - 0.86)\text{eV} \quad \text{at } 97\% \text{ c.l.} \quad \text{(best value } 0.4 \text{ eV)}.
\]

This result is very exciting [10]. It is because 0\( \nu \beta \beta \) decay experiments could tell us about the absolute value of the neutrino masses, while neutrino oscillation experiments show only mass squared differences of neutrinos. Some papers [11] have studied from this date. The evidence for 0\( \nu \beta \beta \) decay also means neutrinos are the Majorana particles and the lepton number is violated, since 0\( \nu \beta \beta \) decay cannot occur in the case of Dirac neutrinos. This evidence is also closely related to the recent topics of the cosmology such as the dark matter candidate of universe [12].

The tiny neutrino masses and the lepton flavor mixing have been discussed in a lot of models beyond the Standard Model (SM). One of the most promising ideas is that light neutrinos are constructed as Majorana particles in the low energy, such as the see-saw mechanism [13]. Here we are concentrating on the light Majorana neutrinos which masses are induced by the dimension five operators in the low energy effective Yukawa interactions.

In the previous paper [14], we have estimated the magnitude of 0\( \nu \beta \beta \) decay in the classification of the neutrino mass hierarchy patterns as Type A, \( m_{1,2} \ll m_3 \), Type B, \( m_1 \sim m_2 \gg m_3 \), and Type C, \( m_1 \sim m_2 \sim m_3 \), where \( m_i \) is the absolute values of the \( i \)-th generation neutrino [15]. The magnitude of 0\( \nu \beta \beta \) decay strongly depends on the neutrino mass hierarchy. According to the analysis in Ref. [14], the results of 0\( \nu \beta \beta \) suggest that neutrino mass hierarchy is forced to Type B or Type C patterns in the diagonal basis of charged lepton mass matrix.

In this paper we will estimate the magnitude of 0\( \nu \beta \beta \) decay in the degenerate neutrinos including the cosmological constraint of neutrino dark matter. The relation of 0\( \nu \beta \beta \) decay and neutrino dark matter has been analyzed in Ref. [12]. We will analyze this relation including mass squared differences of the solar LMA solution and the value of \( U_{e3} \) in accordance with Majorana phases of neutrinos masses. Since the Type B mass hierarchy pattern always satisfies the cosmological constraint, \( \Sigma = m_1 + m_2 + m_3 \leq 4.4 \text{ eV} \) [16], we

* There are arguments for this result in Refs. [10].
will be concentrating on degenerate neutrino masses, Type C. We will see the upper bound of neutrino absolute mass is constrained by $0\nu\beta\beta$ experiment or cosmology depending on Majorana $CP$ phases of neutrinos and solar mixing angle.

In the diagonal base of the charged lepton sector, the light neutrino mass matrix, $(M_\nu)_{ij}$, is diagonalized by $U_{ij}$ as

$$U^T M_\nu U \equiv M_\nu^{diag} = \text{diag}(m_1, m_2, m_3) .$$

(2)

Here the matrix $U$ is so-called MNS matrix\cite{17} denoted by

$$U = V \cdot P ,$$

(3)

where $V$ is the CKM-like matrix, which contains one $CP$-phase ($\delta$),

$$V = \begin{pmatrix}
    c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
    -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\
    s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} - s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} & c_{23}s_{13}
\end{pmatrix}$$

and $P$ contains two extra Majorana phases ($\phi_{1,2}$),

$$P = \text{diag.}(e^{-i\phi_1/2}, e^{-i\phi_2/2}, 1) .$$

For the suitable classification, we introduce the matrix

$$\tilde{M}_\nu \equiv P^* M_\nu^{diag} P^* = \text{diag.}(m_1 e^{i\phi_1}, m_2 e^{i\phi_2}, m_3) = \text{diag.}(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3) .$$

(4)

The results from the recent neutrino oscillation experiments\cite{1, 2, 3, 4, 5, 6, 7} indicate that the neutrinos have tiny but finite masses, with two mass squared differences $\Delta m^2_\odot < \Delta m^2_{\text{atm}}$. The naive explanation of the present neutrino oscillation experiments is that the solar neutrino anomaly is caused by the mixing of the 1st and the 2nd generations ($\theta_\odot \simeq \theta_{12}$, $\Delta m^2_\odot \simeq m_2^2 - m_1^2$), and atmospheric neutrino deficit is caused by the mixing of the 2nd and the 3rd generations ($\theta_{\text{atm}} \simeq \theta_{23}$, $\Delta m^2_{\text{atm}} \simeq m_3^2 - m_2^2$). We take the LMA solution for the solar neutrino solution, $\Delta m^2_\odot = (3 - 19) \times 10^{-5}$ eV$^2$ and $\tan^2 2\theta_\odot = (0.25 - 0.65)$, from the recent results including SK data\cite{3, 8}. Considering the results of the oscillation experiments, the hierarchical patterns of neutrino masses are classified by the following three types:

A : $m_3 \gg m_{1,2}$

B : $m_1 \sim m_2 \gg m_3$

C : $m_1 \sim m_2 \sim m_3$ .
Taking into account of the mass squared differences, $\Delta m^2_\odot$ and $\Delta m^2_{atm}$, the absolute masses of the neutrino in the leading are written by

| Type A          | Type B                                      |
|-----------------|---------------------------------------------|
| $m_1 : 0\sqrt{\Delta m^2_\odot}$ | $m_1 : \sqrt{\Delta m^2_{atm}}$             |
| $m_2 : \sqrt{\Delta m^2_\odot}$   | $m_2 : \sqrt{\Delta m^2_{atm}} + \frac{1}{2} \Delta m^2_{atm}$ |
| $m_3 : \sqrt{\Delta m^2_{atm}}$   | $m_3 : 0$                                    |

Type C
(1): normal
$m_1 : m_\nu$
$m_2 : m_\nu + \frac{1}{2} \Delta m^2_{atm}$
$m_3 : m_\nu + \frac{1}{2} \Delta m^2_{atm}$

(2): inverted
$m_1 : m_\nu + \frac{1}{2} \Delta m^2_\odot$
$m_2 : m_\nu + \frac{1}{2} \Delta m^2_{atm} + \Delta m^2_\odot$
$m_3 : m_\nu$

in each type, respectively. Where $m_\nu$ in Type C is the scale of the degenerated neutrino masses. The recent neutrino oscillation experiment shows

$\Delta m^2_{atm} = 3.2 \times 10^{-3} \text{ eV}^2$, \hspace{1cm} $\Delta m^2_\odot = 6.9 \times 10^{-5} \text{ eV}^2$, \hspace{1cm} $\sin^2 2\theta_{atm} = 1.0$, \hspace{1cm} $\tan^2 \theta_\odot = 3.6 \times 10^{-1}$, \hspace{1cm} \(\sin^2 2\theta_{13} < 0.1\),

and

In the zeroth order approximations, $\cos \theta_{12} = \cos \theta_{23} = 1/\sqrt{2}$ and $\sin \theta_{13} = 0$, we can obtain the zeroth order form of the MNS matrix as,

$$V^{(0)} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix}$$

The neutrino mass matrix $M_\nu$ is determined by $U$ and $M_\nu^{diag}$ from Eqs. (2)~(4). The zeroth order form of the neutrino mass matrix is determined by the approximated MNS matrix, $V^{(0)}$, according to the patterns of neutrino mass hierarchy, Types A~C. In Ref.[15], the zeroth order forms of neutrino mass matrices are shown when Majorana CP phases are 0 or $\pi$, which are shown in Table 1. These mass matrices are useful for the zeroth order approximations of estimating the probability of $0\nu\beta\beta$ as follows.
The effective neutrino mass $\langle m \rangle$, which shows the magnitude of 0νββ decay \[19\] in Eq.(1) is defined by

$$\langle m \rangle = |\sum_{i=1}^{3} U_{ei}^2 m_i| = |\sum_{i=1}^{3} U_{ei} m_i U_{ie}^T| = |V_{e1}^2 m_1 e^{-i\phi_1} + V_{e2}^2 m_2 e^{-i\phi_2} + V_{e3}^2 m_3|,$$  \(7\)

where $i$ denotes the label of the mass eigenstate ($i = 1, 2, 3$). The value of $\langle m \rangle$ is equal to the absolute value of $(1,1)$ component of $M_\nu$. Thus, Table 1 seems to suggest that the forms of the neutrino mass matrix should be B2 or C0 or C3, in order to obtain the suitable large magnitude of $(1,1)$ component. However, it is too naive estimation. The previous paper Ref.[14] has shown that C1 and C2 can also induce sizable magnitude of $\langle m \rangle$ by increasing the value of $m_\nu$. How can the value of $m_\nu$ be large? In fact there is the cosmological upper bound for $m_\nu$, $\Sigma = m_1 + m_2 + m_3 \leq 4.4$ eV \[16\]. This hot (neutrino) dark matter constraint comes from the CMB measurements and galaxy cluster constructions.

Now let us estimate the magnitude of 0νββ decay in the degenerate neutrinos including the cosmological constraint of neutrino dark matter\[12\]. We will analyze this relation including mass squared differences of solar LMA solution and the value of $U_{e3}$ in accordance with Majorana CP phases. At first, we show the case of Type B2, where the sign of $m_1$ is the same as that of $m_2$. The approximation in Table 1 shows $\langle m \rangle = O(\sqrt{\Delta m^2_{atm}}) \sim 0.057$ eV. We can see the value of $\langle m \rangle$ cannot be larger than 0.06 eV even if we change the parameters of $\phi_{1,2}$ and $U_{e3}$ in Type B \[14\]. The region where $\langle m \rangle > 0.05$ eV only exists just around B2 in the parameter space of $\phi_{1,2}$. This magnitude of $\langle m \rangle$ is the edge of the allowed region of experimental value of 97% c.l. in Eq.(1), and the value of $\Sigma$ is of order $2 \times \sqrt{\Delta m^2_{atm}}$, which is much smaller than the cosmological constraint, 4.4 eV. Thus, Type B neutrino mass pattern automatically satisfies the cosmological constraint. We would like to be concentrating on the degenerate neutrino mass patterns (Type C) from now on.

The neutrino masses are degenerate in Type C, and we set the value of the degenerate mass as $m_\nu$. In Ref.[13], Type C mass hierarchy is classified to four subgroups, C0, C1, C2 and C3, by relative signs of $m_1$, $m_2$ and $m_3$.

$$\begin{align*}
(m_1, m_2, m_3) = \begin{cases} 
&m_\nu(1, 1, 1) & (\phi_1, \phi_2) = (0, 0) & \text{(Type C0)} \\
&m_\nu(-1, 1, 1) & (\phi_1, \phi_2) = (0, \pi) & \text{(Type C1)} \\
&m_\nu(1, -1, 1) & (\phi_1, \phi_2) = (\pi, 0) & \text{(Type C2)} \\
&m_\nu(-1, -1, 1) & (\phi_1, \phi_2) = (\pi, \pi) & \text{(Type C3)}
\end{cases}
\end{align*}$$

Seeing the zeroth order neutrino mass matrices in Table 1, we suppose, naively, only Type C0 and C3 might explain 0νββ decay experiments because the (1,1) element of mass matrix
is of $O(m_{\nu})$. However Type C1 and C2 cases can also explain Eq. (11) in the suitable large value of $m_{\nu}$ [14].

We show the value of $\langle m \rangle$ in cases of normal and inverted hierarchies of C0 $\sim$ C3 including $\Delta m_{\odot}^2$ and $\Delta m_{\text{atm}}^2$. In Type C0 and C3, it is given by

$$\langle m \rangle = | V_{e1}^2 m_{\nu} + V_{e3}^2 (m_{\nu} + \frac{\Delta m_{\odot}^2}{2m_{\nu}}) \pm V_{e3}^2 (m_{\nu} + \frac{\Delta m_{\text{atm}}^2}{2m_{\nu}}) |, \quad \text{(normal)}$$

$$\langle m \rangle = | V_{e1}^2 (m_{\nu} + \frac{\Delta m_{\odot}^2}{2m_{\nu}}) + V_{e2}^2 (m_{\nu} + \frac{\Delta m_{\odot}^2 + \Delta m_{\text{atm}}^2}{2m_{\nu}}) \pm V_{e3}^2 m_{\nu} |. \quad \text{(inverted)}$$

By using the experimental values, $|V_{e3}| \leq 0.1$, $0.48 \leq |V_{e2}| \leq 0.63$, $\Delta m_{\odot} = 6.9 \times 10^{-5} \text{eV}^2$ and $\Delta m_{\text{atm}}^2 = 3.2 \times 10^{-3} \text{eV}^2$, we can see that the difference of normal and inverted hierarchies is significant in the range,

$$m_{\nu} \leq \frac{\Delta m_{\text{atm}}^2}{2m_{\nu}}.$$ 

It means that we can neglect the difference of the normal and inverted hierarchies in the region of $m_{\nu}^2 \gg 10^{-3} \text{eV}^2$.

On the other hand, in Type C1 and C2, the values of $\langle m \rangle$ are given by

$$\langle m \rangle = | (V_{e1}^2 - V_{e2}^2) m_{\nu} - V_{e2}^2 \frac{\Delta m_{\odot}^2}{2m_{\nu}} \pm V_{e3}^2 (m_{\nu} + \frac{\Delta m_{\text{atm}}^2}{2m_{\nu}}) |, \quad \text{(normal)}$$

$$\langle m \rangle = | (V_{e1}^2 - V_{e2}^2) m_{\nu} - V_{e2}^2 \frac{\Delta m_{\odot}^2}{2m_{\nu}} \pm V_{e3}^2 m_{\nu} - \frac{\Delta m_{\text{atm}}^2}{2m_{\nu}} (V_{e2}^2 - V_{e1}^2) |. \quad \text{(inverted)}$$

They suggest the difference of normal and inverted hierarchies is significant in the range of

$$| (V_{e1}^2 - V_{e2}^2) m_{\nu} - V_{e2}^2 \frac{\Delta m_{\odot}^2}{2m_{\nu}} \pm V_{e3}^2 m_{\nu} | \leq | V_{e3}^2 \frac{\Delta m_{\text{atm}}^2}{2m_{\nu}} |,$$

$$| (V_{e1}^2 - V_{e2}^2) m_{\nu} - V_{e2}^2 \frac{\Delta m_{\odot}^2}{2m_{\nu}} \pm V_{e3}^2 m_{\nu} | \leq \frac{\Delta m_{\text{atm}}^2}{2m_{\nu}} (V_{e2}^2 - V_{e1}^2) |.$$

Thus, we can neglect the difference between the normal and inverted hierarchies in $m_{\nu} \gg 10^{-3} \text{eV}^2$. Above discussions mean the value of $\langle m \rangle$ does not depend on whether neutrino mass is normal or inverted hierarchies in the range of $m_{\nu}^2 \geq 10^{-2} \text{eV}^2$. We can see it explicitly in Fig.1 (normal hierarchy case) and Fig.2 (inverted hierarchy case).
We stress here that the maximal (minimum) value of $\langle m \rangle$ is given by C0 (C1) line independently of Majorana phases. It is because the relation of $|V_{e1}^2 m_1 - V_{e2}^2 m_2| > |V_{e3}^2 m_3|$ is always satisfied in Eq.(7) when $m_\nu > 2.8 \times 10^{-2}$eV, $|V_{e3}| \leq 0.1$ and $0.30 \leq |V_{e2}| \leq 0.58$. Let us show the normal hierarchy case at first. Figures (1.a)∼(1.c) show the values of $\langle m \rangle$ and $\Sigma$ for the value of $m_\nu$ with $U_{e3} = 0$. When $U_{e3} = 0$, C3 (C2) line falls on C0 (C1) line. Then, the maximal (minimum) value of $\langle m \rangle$ is induced by the cases C0, C3 (C1, C2) in the given value of $m_\nu$. In Fig.(1.a) we take the center value of LMA solution as $\tan^2 \theta_\odot = 0.36$. 

We can see that $0\nu\beta\beta$ decay constraint (Eq.(1)) is severer than the cosmological constraint, $\Sigma \leq 4.4$ eV, in Type C0 and C3. On the other hand, the cosmological constraint is severer than $0\nu\beta\beta$ result in cases of C1 and C2. Figure (1.b) ((1.c)) shows the case of $\tan^2 \theta_\odot = 0.2$ (0.65). In this case the line of minimum value $\langle m \rangle$, C1, C2, is lowered (lifted) since the cancellation between $m_1$ and $m_2$ in Eq.(7) is (not) enhanced. Other lines, C0, C3, and $\Sigma$, are not changed from Fig.(1.a). The case of $\tan^2 \theta_\odot = 0.65$ shows $0\nu\beta\beta$ result is severer than the cosmological constraint as shown in Fig.(1.c). Figure (1.d) show the case of $\tan^2 \theta_\odot = 0.27$ and $U_{e3} = 0$. In this case, the cosmological constraint is the same as $0\nu\beta\beta$ decay constraint in Type C1 and C2. However, the cosmological constraint is severer than $0\nu\beta\beta$ decay constraint in Type C0 and C3. Figure (1.e) shows the case of $U_{e3} = 0.1$ with LMA center value. This case split C0 from C3, and C1 from C2. However, this effect is not so large. All figures show that tritium $\beta$ decay constraint, $m_\nu < 3$ eV, is less severer constraint.

Figure 2 show the inverted hierarchy cases. We finds almost same results as the normal hierarchy case in the range of Eq.(1), since where $m_\nu^2 \geq 10^{-2}$ eV$^2$.

A recent paper[9] announces the evidence of $0\nu\beta\beta$ decay, and the value of $\langle m \rangle$ is large as Eq.(1). This means that neutrinos are Majorana particles and their mass hierarchy is forced to Type B or Type C patterns in the diagonal basis of charged lepton mass matrix. In this paper we have estimated the magnitude of $0\nu\beta\beta$ decay in the degenerate neutrinos including the cosmological constraint of neutrino dark matter, since the Type B mass hierarchy pattern always satisfies the cosmological constraint. The absolute value of neutrino mass is constrained by $0\nu\beta\beta$ result or cosmology depending on Majorana $CP$ phases of neutrino masses and solar mixing angle. C0 and C3 cases are constrained from $0\nu\beta\beta$ result. On the other hand, the constraint of neutrino absolute mass with $\Delta m^2_{\text{atm}} = 3.2 \times 10^{-3}$ and $\Delta m^2_{\odot} = 6.9 \times 10^{-5}$ is gived by the following.
| solar mixing angle | constraint          |
|--------------------|---------------------|
| $0.27 < \tan^2 \theta_\odot \leq 0.65$ | cosmology          |
| $\tan^2 \theta_\odot = 0.27$ | cosmology and $0\nu\beta\beta$ decay |
| $0.2 \leq \tan^2 \theta_\odot < 0.27$ | $0\nu\beta\beta$ decay |

The results of MAP satellite will make $\Sigma < 0.5$ eV, which will suggest more severer bound from the cosmology [20].

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Figure 1: The values of $\langle m \rangle$ and $\Sigma$ with $\tan^2 \theta_\odot = 0.34$ (the LMA center value) in (1.a), $\tan^2 \theta_\odot = 0.1$ in (1.b), and $\tan^2 \theta_\odot = 0.5$ in (1.c). Other values are taken as the zeroth order values as $U_e3 = 0$ and $\theta_{atm} = \pi/4$ in (1.a)~(1.c) We show the values of $\langle m \rangle$ and $\Sigma$ with $\tan^2 \theta_\odot = 0.27$, $U_e3 = 0$ in (1.d) and $\tan^2 \theta_\odot = 0.34$ (the LMA center values), $U_e3=0.1$ in (1.e). The allowed region are $0.05 \text{ eV} < \langle m \rangle < 0.86 \text{ eV}$ from $0\nu\beta\beta$ and $\Sigma \leq 4.4 \text{ eV}$ from cosmology.
Figure 2: The values of $\langle m \rangle$ and $\Sigma$ with $\tan^2 \theta_\odot = 0.36$ (the LMA center value) in (2.a), $\tan^2 \theta_\odot = 0.2$ in (2.b), and $\tan^2 \theta_\odot = 0.65$ in (2.c). Other values are taken as the zeroth order values as $U_{e3} = 0$ and $\theta_{atm} = \pi/4$ in (2.a)∼(2.c) We show the values of $\langle m \rangle$ and $\Sigma$ with $\tan^2 \theta_\odot = 0.27$, $U_{e3} = 0$ in (2.d) and $\tan^2 \theta_\odot = 0.36$ (the LMA center value), $U_{e3}=0.1$ in (2.e). The allowed region are $0.05 \text{ eV} < \langle m \rangle < 0.86 \text{ eV}$ from $0\nu\beta\beta$ and $\Sigma \leq 4.4 \text{ eV}$ from cosmology.
| \( \tilde{M}_\nu \) | Neutrino mass matrix | \( C_0 \) \( \text{diag.}(1,1,1) \) | \( C_1 \) \( \text{diag.}(-1,1,1) \) | \( C_2 \) \( \text{diag.}(1,-1,1) \) | \( C_3 \) \( \text{diag.}(-1,-1,1) \) |
|---|---|---|---|---|---|
| B1 \( \text{diag.}(1,-1,0) \) | \[
\begin{bmatrix}
0 & -1/\sqrt{2} & 1/\sqrt{2} \\
-1/\sqrt{2} & 0 & 0 \\
1/\sqrt{2} & 0 & 0
\end{bmatrix}
\] | \[
\begin{bmatrix}
0 & 1/\sqrt{2} & -1/\sqrt{2} \\
1/\sqrt{2} & 1/2 & 1/2 \\
-1/\sqrt{2} & 1/2 & -1/2
\end{bmatrix}
\] | \[
\begin{bmatrix}
0 & -1/\sqrt{2} & 1/\sqrt{2} \\
-1/\sqrt{2} & 1/2 & 1/2 \\
1/\sqrt{2} & 1/2 & 1/2
\end{bmatrix}
\] | \[
\begin{bmatrix}
-1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix}
\] |
| A \( \text{diag.}(0,0,1) \) | \[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 1/2 & 1/2 \\
0 & 1/2 & 1/2
\end{bmatrix}
\] | | | | |
| B2 \( \text{diag.}(1,1,0) \) | \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1/2 & -1/2 \\
0 & -1/2 & 1/2
\end{bmatrix}
\] | | | | |

Table 1: The zeroth order neutrino mass matrices. In Type A and B, the eigenvalues of \( \tilde{M}_\nu \) and the neutrino mass matrices are normalized by \( \sqrt{\Delta m^2_{\text{atm}}} \). In Type C, they are normalized by \( m_\nu \).