Comment on Multiple Quantum Oscillations in the de Haas - van Alphen Spectra of the Underdoped High-Temperature Superconductor YBa$_2$Cu$_3$O$_{6.5}$.

Recent observations (1, 2 and references therein) of slow magneto-oscillations in some underdoped cuprate superconductors are one of the most striking discoveries in the field of superconductivity since many probes of underdoped cuprates clearly point to a non Fermi-liquid normal state. Here we argue that a large magnitude and strong magnetic-field dependence of the residual resistance invalidate the conventional interpretation of these magneto-oscillations as a normal-state dHvA effect.

The normal state in-plane resistivity, $\rho$, of underdoped samples with a doping close to YBa$_2$Cu$_3$O$_{6.5}$ remains almost flat as a function of temperature above $T_c$, with the magnitude c.a. $250 \div 300 \mu \Omega$cm. It remains flat or even increases at temperatures below $T_c$, when the transition is shifted by the magnetic field [3]. Indeed, using the resistance and the size of a sample, where the oscillations have been observed, (ref. [1], supplement), one obtains $\rho \approx 280 \mu\Omega$cm in the field $B = 60$ Tesla at $T=1.5K$. The lowest value of the Dingle temperature, $T_D = h/(2\pi k_BT_D)$, is $T_D \succeq \hbar e^2 k_F^2/(2\pi^2 m_k B)$, where $1/\tau$ is the scattering rate. This expression yields $T_D \succeq 25 K$ with the Fermi momentum $k_F \approx 1.3 \text{ nm}^{-1}$ and the cyclotron mass $m \approx 1.8 m_e$, found in the framework of the conventional description of oscillations [1, 2] ($c \approx 1.17 \text{ nm}$ is the c-axis lattice constant). Such a large Dingle temperature, several times higher than is found from the conventional analysis of oscillations [1, 2], should make any dHvA effect undetectable. To be compatible with the observed oscillation amplitudes the residual resistivity has to be below c.a. $70 \mu \Omega$cm, which is not the case in YBa$_2$Cu$_3$O$_{6.5}$ for any magnetic field above 20 Tesla [1], where the sample is allegedly in the normal state [2]. Adding more Fermi-surface pockets only increases the lowest value of $T_D$.

We suggest that the magnetic oscillations have been observed in the mixed superconducting state well below the upper critical field, rather than in the normal state. Our suggestion is supported by a large magnetoresistance, measured in all experimentally accessible magnetic fields [1], and by an estimate of the upper critical field, $H_{c2} = (\phi_0 k_B T_c m)^2/h^4 F \gtrsim 100 \text{ Tesla}$ (here $\phi_0$ is the flux quantum, $T_c = 50K$, $m = 2 m_e$, and the oscillation frequency $F = 500 \text{ Tesla}$). In fact this mean-field expression grossly underestimates $H_{c2}$, as follows from a number of measurements of underdoped samples with a sharp in-plane resistive transition in the magnetic field [2]. We propose that the oscillations are most likely caused by quantum interference of vortex and crystal lattice modulations of the superconducting order parameter [4]. An excellent fit of the oscillations in Fig.1 can be obtained with a simple theoretical expression describing an oscillating part of the magnetic response of the vortex lattice, $\Delta \chi(B) = B^{-2} \sum A_i e^{-b_i/B} \cos(\sqrt{B_i}/B)$. As shown by the residue (data minus fit, Fig.1 inset) the quality of our fit is comparable with the conventional fit [2] but the number of fitting parameters is several times smaller.

Unlike the conventional description [2], where four oscillation frequencies emerge from the blue, our characteristic fields fitting the experiment, $B_a = 1.09593 \times 10^6$ Tesla and $B_0 = 1.02776 \times 10^6$ Tesla, are remarkably close to the theoretical fields $B_a = 8 \pi^3 h/e a^2 \approx 1.118 \times 10^6$ Tesla and $B_0 = 8 \pi^3 h/e b^2 \approx 1.079 \times 10^6$ Tesla defined by the crystal lattice in-plane constants, $a \approx 0.382 \text{nm}$ and $b \approx 0.389\text{nm}$, respectively [4]. If the vortex lattice contains triangular and square domains, the number ($i$) of characteristic fields contributing to $\Delta \chi(B)$ could be larger than two. As a result we conclude that C. Proust and colleagues [1, 2] have observed a novel quantum interference phenomenon in the mixed state of underdoped cuprate superconductors proposed by one of us (ASA) [3], rather than the conventional dHvA effect.

We gratefully acknowledge valuable discussions with Nigel Hussey and Viktor Kabanov and financial support of one of us (IOT) from the SEPON project (EC VII FP, contract number ERC-2008-AdG-227457).

A. S. Alexandrov$^1$ and I. O. Thomas$^2$

$^1$Department of Physics, Loughborough University, Loughborough LE11 3TU, United Kingdom.

$^2$Istituto per i Processi Chimici Fisici, Consiglio Nazionale della Ricerche, Via G. Morruzzi 1, 156124 Pisa, Italy.

[1] N. Doiron-Leyraud et al., Nature 447, 565 (2007).
[2] A. Audouard et al., Phys. Rev. Lett. 103, 157003 (2009).
[3] V. F. Gantmakher et al., Zh. Eksp. Teor. Fiz. 115, 268 (1999) [JETP 88, 148 (1999)].
[4] A. S. Alexandrov, J. Phys.: Cond. Mat. 20, 192202 (2008).