Energy levels of an anharmonic oscillator in both weak and strong coupling limit using convergency of Morse-Feshbach non-linear perturbation series

Biswa Nath Rath\textsuperscript{1}, Pramoda Kumar Samal\textsuperscript{2}, Radhanath Mishra\textsuperscript{2} and Basudeb Sahu\textsuperscript{1}

\textsuperscript{1}Department of Physics, North Orissa University, Baripada, 757003, India
\textsuperscript{2}Department of Physics, Utkal University, Bhubaneswar, 751004, India

Abstract

We make an extensive rigorous study on convergent behaviour of Morse-Feshbach nonlinear perturbation series (MFNPS) to find out energy levels of the anharmonic oscillator (AHO) in both weak and strong coupling limit. We develop a new method of multi step optimal splitting in order to get convergency in MFNPS for ground state of AHO and found that two step optimal splitting is sufficient to provide convergency in MFNPS. Unlike the ground state the optimal splitting parameters for excited states is modified according to their dependency on state in order to achieve convergency in MFNPS.

Keywords: Morse-Feshbach, non-linear perturbation series, perturbation theory, anharmonic oscillator, energy level

1 Introduction

In quantum mechanics, there are only few problems whose Hamiltonians can be solved exactly. However for any arbitrary Hamiltonian one has to use some sort of approximations [1]. Perturbation theory is one of the few principal methods of approximation for solution to eigenvalue problems in quantum mechanics [1-10]. In perturbation theory one needs to solve the Hamiltonian of the type

\[ H = H_0 + \lambda H_1 \]  

(1)
where $H_0$ is the unperturbed Hamiltonian, which can be solved exactly and $\lambda H_1$ is the perturbation term. If $E$ corresponds to energy of the total Hamiltonian $H$ and $E_n^0$ corresponds to unperturbed Hamiltonian $H_0$, then one can obtain $E$ in two different ways as

$$E = f(E_0^0, \lambda)$$  \hspace{1cm} (2)

or

$$E = f(E, E_0^0, \lambda)$$  \hspace{1cm} (3)

Let us analyse the above two relations. From the relation in Eq (2) it is evident that R.H.S is a function of known energy $E_0^0$ and coupling constant. This method of calculation is known as constant function perturbation theory as seen from the literature [1-8]. Now analyse the the relation in Eq (3). In this case both L.H.S and R.H.S are the function of unknown energy $E$. This type of perturbation series is known as nonlinear series. The most simplest nonlinear perturbation series is the Morse-Feshbach nonlinear perturbation series (MFNPS) [9-10]. Further it is seen that almost all attemption of perturbation theory reflect Eq (2) with AHO as an example. So far there is very few work on nonlinear series reflecting Eq (3). Because of highly nonlinear in nature MFNPS is extremely difficult to solve analytically and needs high computational skill to solve it numerically in order to get higher order convergency. Even the most simplest form of nonlinear series is the MFNPS is in literature for more than fifty years in literature, only it has been applied only to the ground state of AHO [10]. The basic aim of this paper is to study the applicability of this series by calculating excited state energy considering AHO as an example. In a more explicit version of the aim is to address the followings:

- whether the convergency on ground state energy obtained previously [10] can be improved by introducing multi step optimal splitting parameter?
- whether the previous method applied for ground state will remain valid for the excited state energy calculation?
- whether for excited state energy calculation one has to make some modification?

To arrive at a suitable conclusion to above questions we have to make a systematic study in the following sections.
2 Morse-Feshbach nonlinear perturbation series (MFNPS)

In this section we write the MFNPS in a simplified language. Let us solve the unperturbed Hamiltonian

\[ H_0 \ket{n} = E_n^{(0)} \ket{n} \]  

where \( \ket{n} \) is the unperturbed eigenfunction corresponding to energy eigenvalue \( E_n^{(0)} \). According to MFNPS the energy \( E \) for the perturbed Hamiltonian \( H \) is given as

\[
E_n = E_n^{(0)} + \lambda \bra{n} H_1 \ket{n} + \lambda^2 \sum_{m \neq n} \frac{\bra{n} H_1 \ket{m} \bra{m} H_1 \ket{n}}{(E_n - E_m^{(0)}) (E - E_k^{(0)})}
+ \lambda^3 \sum_{m, k \neq n} \frac{\bra{n} H_1 \ket{m} \bra{m} H_1 \ket{k} \bra{k} H_1 \ket{n}}{(E_n - E_m^{(0)}) (E_n - E_k^{(0)}) (E_n - E_p^{(0)})}
+ \lambda^4 \sum_{m, k, p \neq n} \frac{\bra{n} H_1 \ket{m} \bra{m} H_1 \ket{k} \bra{k} H_1 \ket{p} \bra{p} H_1 \ket{n}}{(E_n - E_m^{(0)}) (E_n - E_k^{(0)}) (E_n - E_p^{(0)}) (E_n - E_q^{(0)})}
\cdots + \lambda^K \sum_{m, \ldots, z \neq n} \frac{\bra{n} H_1 \ket{m} \cdots \bra{z} H_1 \ket{n}}{(E_n - E_m^{(0)}) \cdots (E_n - E_z^{(0)})}
\]  

Here \( K \) is the order of perturbation in MFNPS.

3 Higher order calculation on convergence test for groundstate using two step optimal splitting

In this section we test the convergence of the ground state energy of the AHO, whose Hamiltonian is given as

\[ H = \frac{p^2}{2} + \frac{x^2}{2} + \lambda x^4 \]  

using two step optimal splitting in higher order calculation. The coordinates \( x \) and momentum \( p \) satisfy the relation

\[ [x, p] = i\hbar \]
Now we rewrite the Hamiltonian in Eq (6) as

\[ H = \frac{p^2}{2} + w^2 \frac{x^2}{2} + \lambda x^4 - f(x) \hspace{1cm} (8) \]

where

\[ f(x) = \frac{6\lambda x^2}{2w} \hspace{1cm} (9) \]

and

\[ w^2 = 1 + \frac{6\lambda}{w} \hspace{1cm} (10) \]

Further we rewrite the Eq (8) as

\[ H = \frac{p^2}{2} + W_1^2 \frac{x^2}{2} + \lambda x^4 - f(x) - F_1(x) \hspace{1cm} (11) \]

where

\[ F_1(x) = \frac{6\lambda x^2}{2W_1} \hspace{1cm} (12) \]

and

\[ W_1^2 = w^2 + \frac{6\lambda}{W_1} \hspace{1cm} (13) \]

Now we rewrite the Hamiltonian as

\[ H = H_D + H_N \hspace{1cm} (14) \]

where \( H_D \) is the diagonal term and considered as unperturbed Hamiltonian. Similarly \( H_N \) is the non-diagonal term and considered as perturbation term.

The explicit expression for \( H_D \) in terms of creation operator \( a^\dagger \) and annihilation operator \( a \) is

\[ H_D = \frac{W_1 + \frac{1}{W_1}}{4} \left( 2a^\dagger a + 1 \right) + \lambda \frac{3 + 12a^\dagger a + 6(a^\dagger)^2 a^2}{4W_1^2} \hspace{1cm} (15) \]

Similarly the expression for \( H_N \) in terms of creation operator \( a^\dagger \) and annihilation operator \( a \) is

\[ H_N = \lambda \left[ \frac{a^4 + (a^\dagger)^4 + 4(a^\dagger)^3 a + 4a^\dagger a^3}{4W_1^2} - \frac{3(a^2 + (a^\dagger)^2)}{2wW_1} \right] \hspace{1cm} (16) \]

The non zero expectation values of the diagonal Hamiltonian \( H_D \) are given as
\[ \langle n|H_D|n \rangle = \left( \frac{2n + 1}{4} \right) \left( W_1 + \frac{1}{W_1} \right) + \frac{3\lambda}{4W_1^2}(2n^2 + 2n + 1) \quad (17) \]

The non zero expectation values of the non-diagonal Hamiltonian \( H_N \) are given as

\[ \langle n|H_N|n + 2 \rangle = \frac{\lambda \sqrt{(n+1)(n+2)}}{W_1} \left( \frac{n}{W_1} - \frac{3}{2w} \right) \quad (18) \]

\[ \langle n|H_N|n + 4 \rangle = \frac{\lambda \sqrt{(n+1)(n+2)(n+3)(n+4)}}{4W_1^2} \quad (19) \]

Now replacing \( \lambda H_1 = H_N \) and \( H_0 = H_D \) in Eq (14, 15) we calculate the groundstate energy of the AHO up to 14 order and the results are given in Table 1.

| Order | Convergent value \( E_0 \) with \( w = 2.0 \) |
|-------|-----------------------------------------------|
| K     | \( W_1 = 2.52510225481 \)                      |
| 0     | 0.847907429                                    |
| 1     | 0.847907429                                    |
| 2     | 0.8041081069829833                            |
| 3     | 0.8039091999664616                            |
| 4     | 0.8037978280048715                            |
| 5     | 0.8037792238820139                            |
| 6     | 0.8037726920274627                            |
| 7     | 0.8037713351909546                            |
| 8     | 0.8037708048619517                            |
| 9     | 0.8037707015253137                            |
| 10    | 0.8037706427848315                            |
| 11    | 0.8037706383463737                            |
| 12    | 0.8037706263550815                            |
| 13    | 0.8037706301691552                            |
| 14    | 0.8037706243946594                            |
| 15    | 0.8037706287404748                            |

Table 1: Convergent value for ground state energy \( E_0 \) of an AHO with \( \lambda = 1 \) in MFNPS using two-step optimal splitting parameters.
4 Higher order calculation on convergence test for ground state using multi step optimal splitting

| Order | $W_1 = 2.52510225481$ | $W_1 = 2.52510225481$ | $W_1 = 2.52510225481$ |
|-------|------------------------|------------------------|------------------------|
| $w = 2.0$ | $E_0 = 0.901242878$ | $E_0 = 0.901242878$ | $E_0 = 0.901242878$ |
| $K$ | $W_2 = 2.90538656129$ | $W_2 = 2.90538656129$ | $W_2 = 2.90538656129$ |
| $W_3 = 3.21090388686$ | $W_3 = 3.21090388686$ | $W_3 = 3.21090388686$ |
| $W_4 = 3.469745959594$ | $W_4 = 3.469745959594$ | $W_4 = 3.469745959594$ |
| $0.901242878$ | $0.901242878$ | $1.00178467$ |
| $0.953331243$ | $0.953331243$ | $1.00178467$ |
| $0.802222400795698$ | $0.8042036141630704$ | $0.809218969904901$ |
| $0.8054651813042190$ | $0.8096942202109880$ | $0.8159975210361090$ |
| $0.8037461158266676$ | $0.8035523554262308$ | $0.803652961629886$ |
| $0.8038036096317446$ | $0.8040771905224384$ | $0.8048413909232490$ |
| $0.8037708004664887$ | $0.8037491193175370$ | $0.8037061119915903$ |
| $0.8037713901307034$ | $0.8038780364646635$ | $0.8038694389755376$ |
| $0.8037706721375962$ | $0.8037690121024319$ | $0.8037599392046063$ |
| $0.8037706545007351$ | $0.8037715434308201$ | $0.8037800051832189$ |
| $0.8037706342530531$ | $0.8037705356047065$ | $0.8037692091978080$ |
| $0.8037706322885981$ | $0.8037706797772240$ | $0.8037715412805357$ |
| $0.8037706314198754$ | $0.8037706406762437$ | $0.8037704590622494$ |
| $0.8037706312900128$ | $0.8037706508130255$ | $0.8037707215178032$ |
| $0.8037706312192222$ | $0.8037706475443096$ | $0.8037706160697888$ |
| $0.8037706312169512$ | $0.8037706481678014$ | $0.8037706405540735$ |

Table 2: Convergent value for ground state energy $E_0$ of an AHO with $\lambda = 1$ in MFNPS using multi step optimal splitting parameters.

In this section we introduce a multi step optimal splitting approach on parameter calculation with a aim to improve the convergency. Following the previous procedure, we write the expression for $H_D$ as

$$H_D = \left( W_k + \frac{1}{W_k} \right) \left( 2a^1a + 1 \right) + \lambda \frac{3 + 12a^1a + 6(a^1)^2a^2}{4W_k^2}$$  \hspace{1cm} (20)
Similarly the expression for $H_N$ as

$$
H_N = \lambda \left[ \frac{a^4 + (a^\dagger)^4 + 4(a^\dagger)^3a + 4a^\dagger a^3}{4W_k^2} \right]
- \frac{3(a^2 + (a^\dagger)^2)}{2W_k} \left( \frac{1}{W_{k-1}} + \frac{1}{W_{k-2}} + \cdots + \frac{1}{W_1} + \frac{1}{w} \right)
$$

where $k$ is the order of multi step optimal splitting. The optimal splitting parameters $W_k$ are given by

$$
W_k^2 = W_{k-1}^2 + \frac{6\lambda}{W_k}
$$

with

$$
W_1^2 = w^2 + \frac{6\lambda}{W_1}
$$

The non zero expectation values of diagonal Hamiltonian $H_D$ are given as

$$
\langle n | H_D | n \rangle = \left( \frac{2n + 1}{4} \right) \left( W_k + \frac{1}{W_k} \right) + \frac{3\lambda}{4W_k^2} (2n^2 + 2n + 1)
$$

The non zero expectation values of non diagonal Hamiltonian $H_N$ are given as

$$
\langle n | H_N | n + 2 \rangle = \lambda \sqrt{(n+1)(n+2)} \left[ \frac{n}{W_k} - \frac{3}{2} \left( \frac{1}{W_{k-1}} + \frac{1}{W_{k-2}} + \cdots + \frac{1}{W_1} + \frac{1}{w} \right) \right]
$$

$$
\langle n | H_N | n + 4 \rangle = \frac{\lambda \sqrt{(n+1)(n+2)(n+3)(n+4)}}{4W_k^2}
$$

Here considering different values of $k$, we calculate the groundstate energy for $\lambda = 1$ and the results are given in Table 2. We notice that two step optimal splitting is sufficient to provide convergency in MFNPS for ground state of AHO.

Two step optimal splitting approach is also applied to the ground state of AHO for higher coupling parameter $\lambda$. Last three higher order convergent results for different coupling parameter $\lambda$ is given in Table 3.
Table 3: Convergent value for ground state $E_0$ of an AHO with different values of coupling parameter $\lambda$ in MFNPS using two-step optimal splitting.

| Order | Convergent value | Convergent value | Convergent value |
|-------|------------------|------------------|------------------|
| K     | $E_0$ for $\lambda = 0.01$ | $E_0$ for $\lambda = 10$ | $E_0$ for $\lambda = 100$ |
| 13    | 0.5072562106523008 | 1.5049724276222453 | 3.13184278221808 |
| 14    | 0.5072562106523008 | 1.504972406538491 | 3.13184221926994 |
| 15    | 0.5072562106523008 | 1.504972417516847 | 3.13184248591904 |

5 State-independent parameter calculation on excited state using two step optimal splitting

In this section we apply previously calculated optimal splitting parameters for ground state of AHO to the exited state of AHO. The results are given in Table 3. We observe that for $n \geq 5$ the MFNPS is not giving any convergent value and also for $n \leq 5$ the number of convergent digits decreases as the increase of state value $n$. Again the convergent results are not comparable for higher excited states with the results calculated previously by several authors [2-4]. This is expected as the excited state energy depends on the state value $n$. So the optimal splitting parameters has to be modified according to their state dependency.

NC-No Convergency

6 State-dependent parameter calculation on excited state using single step parameter

The state dependent parameter $W$ is determined in such a way that it will make $H_D$ and $H_N$ as small as possible simultaneously. One of the simplest ways is to determine $W$ from $H_D$ for the desired state, say $n$, using the variational principle

$$\frac{d\langle n|H_D|n\rangle}{dW} = 0 \quad (27)$$

$W$ is determined from the cubic equation

$$W^3 - W - \frac{6\lambda(2n^2 + 2n + 1)}{(2n + 1)} = 0 \quad (28)$$
For ground state $n = 0$ the above equation reduces to the state independent equation as discussed in Eq 10.

Now with this variational parameter the non zero expectation value of diagonal Hamiltonian $H_D$ is given as

$$
\langle n | H_D | n \rangle = \frac{2n + 1}{4} \left( W + \frac{1}{W} \right) + \frac{3\lambda}{4W^2} (2n^2 + 2n + 1)
$$

The non zero expectation value of non diagonal Hamiltonian $H_N$ is given as

$$
\langle n | H_N | n + 2 \rangle = \left[ \frac{1}{4} \left( -W + \frac{1}{W} \right) + \frac{\lambda}{W^2} \left( n + \frac{3}{2} \right) \right] \sqrt{(n+1)(n+2)}
$$

$$
\langle n | H_N | n + 4 \rangle = \frac{\lambda \sqrt{(n+1)(n+2)(n+3)(n+4)}}{4W^2}
$$

We calculate the excited state energies of an AHO up to 14 order and 15 order terms in MFNPS using single step variational parameter and the results are given in Table 5 and Table 6 respectively. Comparing results from Table 5 and Table 6 we achieve a convergency up to a minimum of 7 digit.

| State | $E_n$ for Coupling parameter $\lambda = 0.1$ | $E_n$ for Coupling parameter $\lambda = 1$ | $E_n$ for Coupling parameter $\lambda = 100$ |
|-------|----------------------------------------|----------------------------------------|----------------------------------------|
| 1     | 1.769502633601580                     | 2.737893473247960                     | 11.18727013754662                     |
| 2     | 3.138624640483020                     | 5.179368610682413                     | 21.90792389514790                     |
| 3     | 4.628893580258386                     | 7.944276200342911                     | 34.20660264641097                     |
| 4     | 6.220490587163873                     | 10.98830903940391                     | 48.00716948459711                     |
| 5     | 7.901913609979696                     | 14.41671095662173                    | 65.17919098201246                     |
| 6     | 9.674274270406038                     | NC                                    | NC                                    |
| 7     | 11.58029965657092                    | NC                                    | NC                                    |
| 8     | NC                                    | NC                                    | NC                                    |
| 9     | NC                                    | NC                                    | NC                                    |
| 10    | NC                                    | NC                                    | NC                                    |

Table 4: Convergent value for excited state energy $E_n$ of an AHO using 15 order of terms of the MFNPS
7 State-dependent parameter calculation on excited state using two step parameter

In this section we introduce state dependent parameter using two step optimal splitting procedure for the convergency test on excited states. The two optimal splitting parameters $W$ and $w$ are calculated from the cubic equation given by

$$W^3 - w^2W - \frac{6\lambda(2n^2 + 2n + 1)}{(2n + 1)} = 0$$  \hspace{0.5cm} (32)

where

$$w^2 = 1 + \frac{6\lambda}{w}$$  \hspace{0.5cm} (33)

The non zero expectation value diagonal Hamiltonian $H_D$ is given as

$$\langle n | H_D | n \rangle = \left( \frac{2n + 1}{4} \right) \left( W + \frac{1}{W} \right) + \frac{3\lambda}{4W^2}(2n^2 + 2n + 1)$$  \hspace{0.5cm} (34)

$$\langle n | H_N | n + 2 \rangle = \sqrt{(n + 1)(n + 2)} \left[ \frac{-W}{4} + \frac{1}{4W} + \frac{\lambda}{W^2} \left( n + \frac{3}{2} \right) \right]$$  \hspace{0.5cm} (35)

The non zero expectation value non diagonal Hamiltonian $H_N$ is given as
The excited energy levels calculated up to 14 and 15 order of terms in MFNPS using two step optimal splitting variational parameters and the results are given in Table 7 and Table 8.

| State | Coupling parameter $\lambda$ | Coupling parameter $\lambda$ | Coupling parameter $\lambda$ |
|-------|-------------------------------|-------------------------------|-------------------------------|
| $E_n$ for $\lambda = 0.1$ | $E_n$ for $\lambda = 1$ | $E_n$ for $\lambda = 100$ |
| 1 | 1.769502734911583 | 2.737955961049832 | 11.1886588385119 |
| 2 | 3.138624351464486 | 5.179303188325927 | 21.90710180727473 |
| 3 | 4.628882837847637 | 7.942406660726320 | 34.18256466416197 |
| 4 | 6.220300917364425 | 10.96358385150704 | 47.70721635883445 |
| 5 | 7.89767264110872 | 14.20313941758235 | 62.2812490910177 |
| 6 | 9.657840029418940 | 17.6340492493832 | 77.77077204538597 |
| 7 | 11.48731562514444 | 21.2364355766123 | 94.07804949179396 |
| 8 | 13.3824748917164 | 24.99493657142581 | 111.1279607854576 |
| 9 | 15.33864207925654 | 28.89725105542083 | 128.8606292267516 |
| 10 | 17.3519076749986 | 32.93326304139077 | 147.2269943696901 |

Table 6: Energy levels $E_n$ of the AHO using $K = 15$ order of terms in MFNPS using single step variational parameters.

\[
\langle n|H_N|n+4 \rangle = \frac{\lambda \sqrt{(n+1)(n+2)(n+3)(n+4)}}{4W^2}
\]  

(36)

The excited energy levels calculated up to 14 and 15 order of terms in MFNPS using two step optimal splitting variational parameters and the results are given in Table 7 and Table 8.

8 Conclusion

In this paper we have taken a very fruitful approach to get convergency in MFNPS in order to get energy levels of an AHO. The problem of energy levels of AHO has been analysed from wide angles to achieve convergency in MFNPS. MFNPS seems to be one of most simplest non linear series to be used for energy levels of AHO. Optimal splitting method is found to be very efficient technique to achieve convergency in MFNPS.
Table 7: Energy levels $E_n$ of the AHO using $K = 14$ order of terms in the MFNPS perturbation series using two step optimal splitting variational parameters.

| State | Coupling parameter $\lambda = 0.1$ | Coupling parameter $\lambda = 1$ | Coupling parameter $\lambda = 100$ |
|-------|-----------------------------------|----------------------------------|----------------------------------|
| 1     | 1.769502595495307                 | 2.737892280858456                | 11.18829020411844                |
| 2     | 3.138624197794214                 | 5.179291722504740                | 21.90689767968495                |
| 3     | 4.628882511628982                 | 7.942403919498243                | 34.18252348780879                |
| 4     | 6.220300863131970                 | 10.96358293850466                | 47.70720519550517                |
| 5     | 7.899767113294260                 | 14.20313925236565                | 62.28123822043900                |
| 6     | 9.657840059798382                 | 17.63404889881106                | 77.77077201647030                |
| 7     | 11.48731530776505                | 21.23643596676358                | 94.07805770355252                |
| 8     | 13.38247452708933                | 24.99493915652584                | 111.127999324749                 |
| 9     | 15.33864161377721                | 28.89726023457015                | 128.8606564368714                |
| 10    | 17.35190770595816                | 32.93326418012689                | 147.2269846271021                |

References

[1] C. M. Bender and T. T. Wu Phys. Rev 184, 1234 (1965).

[2] W. E. Caswell Ann. Phys (N.Y) 123, 153 (1979).

[3] E. J. Weniger Phys. Rev. Lett 77, 2862 (1996).

[4] W. Janke and H. Kleinert Phys. Rev. Lett 75, 2787 (1995).

[5] F. M. Fernandez, A. M. Meson and E. A. Castro J. Phys A 18 , 1398 (1985).

[6] B. Rath Eur. J. Phys 11 , 184 (1990).

[7] I. G. Halliday and P. Suranyi Phys. Rev D 21, 1529 (1980).

[8] B. Rath J. Phys. Soc (Jpn), 67 (9), 3044 (1998).

[9] B. Rath Int. J. Mod. Phys A 14 (13), 2103 (1999).
| State | Coupling parameter $\lambda = 0.1$ | Coupling parameter $\lambda = 1$ | Coupling parameter $\lambda = 100$ |
|-------|-----------------------------------|----------------------------------|----------------------------------|
| 1     | 1.769502595720013                 | 2.737892290808430                | 11.18725175268262                |
| 2     | 3.138624197891925                 | 5.179291724439308                | 21.90689768817313                |
| 3     | 4.62888251166889                  | 7.942403920161726                | 34.18252348715269                |
| 4     | 6.220300863145563                 | 10.96358293822675                | 47.70720518995203                |
| 5     | 7.899767113290646                 | 14.20313925025467                | 62.28123817953492                |
| 6     | 9.657840059675530                 | 17.63404886347468                | 77.77077131768290                |
| 7     | 11.48731530570292                | 21.23643553577421                | 94.07805012397570                |
| 8     | 13.38247450558015                | 24.99493599000598                | 111.1279501536521                |
| 9     | 15.33864146634275                | 28.89724877539235                | 128.8606557529643                |
| 10    | 17.35190696572501                | 32.93326503128077                | 147.2269836271021                |

Table 8: Energy levels $E_n$ of the AHO using $K = 15$ order of terms in the MFNPS perturbation series using two step optimal splitting variational parameters.

[10] P. M. Morse, H. Feshbach Method of Theoretical Physics Part-II (Mc Graw Hill, New York 1963)