Abstract

A lattice gauge theory with an action polynomial in independent field variables is considered. The link variables are described by unconstrained complex matrices instead of unitary ones. A mechanism which permits to switch off in the continuous limit the arising extra fields is discussed. The Euclidean version of the theory with an 4-dimensional lattice is described. The canonical form of this theory in Lorentz coordinates with continuous time is given. The canonical formulation in the light-front coordinates is investigated for the lattice in 2-dimensional transverse space and for the 3-dimensional lattice including one of the light-like coordinates. The light-front zero mode problem is considered in the framework of this canonical formulation.
1 Introduction

Ultraviolet regularization of nonabelian gauge theories via introduction of space-time lattice is widely used in nonperturbative considerations. It is usual to apply Wilson-Polyakov lattice, where gauge field is described by unitary matrices related to lattice links. This lattice is convenient for numerical calculations owing to compactness of the space of parameters defining the unitary matrices. However the action of the theory is nonpolynomial in these parameters which are independent dynamical variables. This complicates the analytic investigation of the theory. For example canonical formulation of lattice gauge theory on the Light Front (LF) encounters difficulties when Wilson-Polyakov action is applied. In this paper we consider the formulation of gauge invariant lattice theory with an action polynomial in independent variables. The idea is rather simple and originates from Bardeen-Pearson paper devoted to gauge theory on the LF with transverse space lattice. It can be explained as follows.

Let us introduce a cubic lattice in space-time and relate arbitrary complex $N \times N$ matrix $W$ to every link with positive direction respectively to corresponding coordinate axis. For the same link having opposite direction we use hermitian conjugated matrix $W^+$. With any closed directed loop on the lattice one relates the trace of the product of such matrices taking into account the direction of each link along the loop. Gauge transformations act on this matrices like in Wilson-Polyakov lattice theory, i.e. by $N \times N$ unitary matrices at the end points of corresponding links. The above mentioned traces belonging to closed loops remain invariant under this transformations. One can use linear combinations of such invariants to construct the action. In this theory the role of independent dynamical variables play the matrix elements of $W$ and $W^+$, and the action is polynomial in these variables. After transition to continuous space one gets a theory with usual gauge fields and with equal number of additional (extra) fields. This extra fields can be made to acquire infinite mass in the limit of zero lattice spacing and to switch off from the theory. Recently the analog of Bardeen-Pearson approach was applied to $(2 + 1)$-dimensional gauge theory on the LF. All extra fields was kept in the theory in order to describe phenomenologically the effective interaction at low energies. It was possible to reproduce the mass spectrum (known from Wilson-Polyakov lattice calculations) by fitting free parameters in effective interaction terms.

The objective of our paper is to develop this approach in a general form using appropriate choice of dynamical variables. This choice allows to control easily the naive continuous limit of the model and to illuminate the correspondence with usual continuous theory. We start with $U(N)$ gauge theory on 4-dimensional Euclidean lattice and introduce the gauge invariant "mass" term for extra fields. These fields decouple when the corresponding mass goes to infinity in continuous limit. To get $SU(N)$ gauge theory we add to the action another "mass" term which gives "infinite mass" to abelian part of the $U(N)$ field in the limit of zero lattice spacing.

Further we develop canonical formalism in Lorenz coordinates using 3-dimensional space lattice and in the LF coordinates using transverse space lattice (here we apply periodic boundary conditions for gauge fields defined on finite interval of light-like coordinate $x^- = \frac{1}{\sqrt{2}}(x^0 - x^3)$ with the $x^+ = \frac{1}{\sqrt{2}}(x^0 + x^3)$ being the "time"). We show the advantage of lattice LF formulation in solving canonical zero mode problem.

The layout of the rest of the paper is as follows. In Sec. 2 it is considered the lattice model in 4-dimensional Euclidean space with the action polynomial in independent field variables. Here the cubic lattice is introduced for all four dimensions. In Sec. 3 it is considered the gauge theory in canonical form in Lorentz coordinates with continu-
ous time and with 3-dimensional cubic lattice in the space coordinates. In Sec. 4 it is considered the gauge theory in the LF coordinates. The Sec. 4.1 is devoted to canonical formulation on the LF with continuous light-like coordinates and with a lattice in two transverse coordinates. In Sec. 4.2 it is considered briefly the possibility to introduce a lattice simultaneously in transverse and in light-like $x^\tau$-coordinates.

2 Gauge field theory on the lattice in 4-dimensional Euclidean space

We consider $U(N)$ gauge theory without scalar and spinor fields. We introduce 4-dimensional cubic lattice and denote by $e_\mu$ the vector connecting two neighbouring sites on the lattice and directed along positive $\mu$-coordinate axis. The $x^0, x^1, x^2, x^3$ denote the coordinates of the sites, and the $a$ is lattice spacing. We relate complex $N \times N$ matrix $W$ to a link directed from the site $x - e_\mu$ to the site $x$

\[ W(x, \mu) \]

an the $W^+$ to the same link in opposite direction

\[ W^+(x, \mu) \]

The elements of these matrices are considered as dynamical variables. We relate to any closed directed loop on the lattice the trace of the product of such matrices seating on the links directed along this loop. For example, the expression

\[ \text{Tr} \left\{ W(x, \nu)W(x - e_\nu, \mu)W^+(x - e_\mu, \nu)W^+(x, \mu) \right\}. \]

is related to the loop shown in fig. 1.

It should be noticed that the trace related to closed loop consisting of the same links passed in both directions is not identically unity because the matrices $W$ are not unitary. (See, for example, fig. 2.) The unitary matrices $U(x)$ of gauge transformations act on the $W$ and $W^+$ in the following way:

\[ W(x, \mu) \to W'(x, \mu) = U(x)W(x, \mu)U^+(x - e_\mu), \] (1)

\[ W^+(x, \mu) \to W'^+(x, \mu) = U(x - e_\mu)W(x, \mu)U^+(x). \] (2)

Quantities related to closed loops on the lattice as described are invariant under this transformations. In order to construct gauge invariant action having correct naive limit at $a \to 0$ we use the analogy with formulation in continuous space. We write

\[ W(x, \mu) = I - gaV_\mu(x), \] (3)
\[ W^+(x, \mu) = I - gaV^+_\mu(x), \]  

where the \( g \) is the coupling constant. Further, we define in fundamental representation the generators \( T^a \) of \( SU(N) \)-group and the generator \( T^0 \) of \( U(1) \) group. The indeces like \( a, b, \ldots \) are related only to \( SU(N) \) generators and the indeces like \( \tilde{a}, \tilde{b}, \ldots \) run over all set of \( U(N) \)-generators. We define them as follows:

\[ T^{\tilde{a}} = T^{\tilde{a}+}, \]  

\[ \text{Tr} \left( T^{\tilde{a}}T^{\tilde{b}} \right) = \delta^{\tilde{a}\tilde{b}}, \]  

\[ [T^a, T^b] = it^{abc}T^c, \]  

\[ \text{Tr} T^a = 0, \quad \text{Tr} T^0 = \sqrt{N}. \]  

We decompose the matrices \( V_\mu(x) \) and \( V^+_\mu(x) \) putting

\[ V_\mu(x) = B_\mu(x) + iA_\mu(x), \]  

\[ V^+_\mu(x) = B_\mu(x) - iA_\mu(x), \]  

where \( B^+_\mu = B_\mu, A^+_\mu = A_\mu \), and separate the abelian parts:

\[ B_\mu(x) = T^0b_\mu(x) + \bar{B}_\mu(x), \]
\[ A_\mu(x) = T^0 a_\mu(x) + \tilde{A}_\mu(x), \] (12)

\[ \tilde{B}_\mu(x) = T^a B^a_\mu(x), \quad \tilde{A}_\mu(x) = T^a A^a_\mu(x), \quad \text{Tr} \tilde{B}_\mu = 0, \quad \text{Tr} \tilde{A}_\mu = 0. \]

In naive limit \( a \to 0 \) the \( \tilde{A}_\mu \) coincides with usual nonabelian \( SU(N) \) gauge field, the \( a_\mu \) becomes abelian gauge field, and \( b_\mu, \tilde{B}_\mu \) become extra fields which should be switched off.

We can define an analog of covariant derivative. For any field \( \varphi \) localized at the sites of the lattice it is

\[ \tilde{D}\varphi(x) = \frac{1}{a} (\varphi(x) - W(x, \mu)\varphi(x - e_\mu)) = \tilde{\partial}\varphi(x) + gV_\mu(x)\varphi(x - e_\mu), \] (13)

where

\[ \tilde{\partial}\varphi(x) = \frac{1}{a} (\varphi(x) - \varphi(x - e_\mu)) \] (14)

is the analog of usual derivative. It follows that under gauge transformation

\[ \tilde{D}\varphi(x) \to \tilde{D}'\varphi'(x) = U(x)\tilde{D}\varphi(x), \] (15)

where \( \varphi(x) \to \varphi'(x) = u(x)\varphi(x) \).

Hence,

\[ (\tilde{D}'_\mu \tilde{D}'_\nu - \tilde{D}'_\nu \tilde{D}'_\mu) \varphi'(x) = U(x) (\tilde{D}_\mu \tilde{D}_\nu - \tilde{D}_\nu \tilde{D}_\mu) \varphi(x - e_\mu - e_\nu), \] (16)

and one can define the analog \( G_{\mu\nu}(x) \) of the usual tensor field via the relation

\[ (\tilde{D}'_\mu \tilde{D}'_\nu - \tilde{D}'_\nu \tilde{D}'_\mu) \varphi'(x) = gG_{\mu\nu}(x)\varphi(x - e_\mu - e_\nu), \] (17)

where

\[
G_{\mu\nu}(x) = \tilde{\partial}_\mu V_\nu(x) - \tilde{\partial}_\nu V_\mu(x) + g (V_\mu(x)V_\nu(x - e_\mu) - V_\nu(x)V_\mu(x - e_\nu)) = \\
= \frac{1}{a^2 g} (W(x, \mu)W(x - e_\mu, \nu) - W(x, \nu)W(x - e_\nu, \mu)).
\] (18)

It can be represented in the form

\[ G_{\mu\nu}(x) = \frac{1}{a^2 g} \left( \begin{array}{c} x - e_\mu - e_\nu \\ x - e_\mu - e_\nu \end{array} \right). \] (19)

This field transforms under gauge transformations as follows:

\[ G_{\mu\nu}(x) \to G'_{\mu\nu}(x) = U^+(x)G_{\mu\nu}(x)U(x - e_\mu - e_\nu). \] (20)

Therefore the action

\[ S_1 = \frac{a^4}{4} \sum_{x, \mu, \nu} \text{Tr} \left( G^+_\mu(x)G_{\mu\nu}(x) \right) \] (21)
is $U(N)$ gauge invariant. It can be represented as follows:

$$S_1 = \frac{1}{4g^2} \sum_{x, \mu, \nu} \text{Tr} \left\{ \left( \begin{array}{ccc} x - \epsilon_\mu & x & x \\ x - \epsilon_\mu - e_\nu & x & x \\ x - \epsilon_\mu - e_\nu & x & x \end{array} \right) \right\} =$$

$$= \frac{1}{4g^2} \sum_{x, \mu, \nu} \text{Tr} \left( \begin{array}{ccc} x - \epsilon_\mu & x & x \\ x - \epsilon_\mu - e_\nu & x & x \\ x - \epsilon_\mu - e_\nu & x & x \end{array} \right) (22)$$

One can see that this action is real and nonnegative.

In the limit $a \to 0$ the quantities $\tilde{D}_\mu \varphi$, $G_{\mu \nu}$, and $S_1$ become

$$\tilde{D}_\mu \varphi(x) \rightarrow D_\mu \varphi(x) + gB_\mu(x)\varphi(x), \quad (23)$$

$$G_{\mu \nu}(x) \rightarrow iF_{\mu \nu}(x) = D_\mu B_\nu - D_\nu B_\mu + g[B_\mu, B_\nu], \quad (24)$$

$$S_1 \rightarrow \frac{1}{4} \int d^4x \sum_{\mu, \nu} \text{Tr} \left\{ F_{\mu \nu}^2 + (D_\mu B_\nu - D_\nu B_\mu)^2 - g^2 \text{Tr} ([B_\mu, B_\nu]^2) - 4iF_{\mu \nu} [B_\mu, B_\nu] \right\}, \quad (25)$$

where

$$F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig [A_\mu, A_\nu], \quad (26)$$

$$D_\mu B_\nu = \partial_\mu B_\nu + ig [A_\mu, B_\nu]. \quad (27)$$

Here we denote by $D_\mu$ the usual $U(N)$ covariant derivative. The form of gauge transformation in the limit $a \to 0$ becomes

$$iA'_\mu(x) = u(x)iA_\mu(x)u^+(x) + \frac{1}{g}u(x)\partial_\mu u^+(x), \quad (28)$$

$$B'_\mu(x) = u(x)B_\mu(x)u^+(x). \quad (29)$$

Thus if we switch off the extra field $B_\mu = \frac{1}{\sqrt{N}}B_\mu + \tilde{B}_\mu$ in the limit $a \to 0$ we obtain usual continuous $U(N)$ gauge theory for the field $A_\mu$.

There is another way to construct the action with the same properties in the limit $a \to 0$. Let us define the quantities $H_{\mu \nu}(x)$ as

$$H_{\mu \nu}(x) = \frac{1}{a^2g} \left( \begin{array}{ccc} x - \epsilon_\mu & x & x \\ x - \epsilon_\mu - e_\nu & x & x \\ x - \epsilon_\mu - e_\nu & x & x \end{array} \right), \quad \text{at } \mu \neq \nu,$$

$$H_{\mu \mu}(x) = \frac{1}{a^2g} \left\{ \left( \begin{array}{ccc} x - \epsilon_\mu & x \\ x - \epsilon_\mu - e_\nu & x \end{array} \right) - \left( \begin{array}{ccc} x - \epsilon_\mu & x \\ x - 2e_\mu & x \end{array} \right) \right\}. \quad (30)$$
It means that

$$H_{\mu\nu}(x) = \frac{1}{a^2 g} \left( W(x-e_\mu, \nu)W^+(x-e_\nu, \mu) - W^+(x, \mu)W(x, \nu) \right) =$$

$$= \bar{\delta}_\mu V_\nu + \bar{\delta}_\nu V^+_{\mu} - g \left( V^+_{\mu}(x)V_\nu(x) - V_\nu(x-e_\mu)V^+_{\mu}(x-e_\nu) \right), \quad (31)$$

and $H_{\mu\nu}(x) = H_{\nu\mu}(x)$. Under $U(N)$ gauge transformations we get

$$H'_{\mu\nu}(x) = U(x-e_\mu)H_{\mu\nu}(x)U^+(x-e_\nu). \quad (32)$$

Hence, the quantity

$$S_2 = \frac{1}{4g^2} \sum_{x,\mu,\nu} \text{Tr} \left( H^+_{\mu\nu}(x)H_{\mu\nu}(x) \right) \quad (33)$$

is lattice gauge invariant and nonnegative. Any linear combination of quantities $S_1$ and $S_2$ can be used. In particular, we can find in the limit $a \to 0$ that

$$\frac{1}{2}(S_1|_{a \to 0} + S_2|_{a \to 0}) = \int d^4x \sum_{\mu,\nu} \text{Tr} \left\{ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2}(D_\mu B_\nu)^2 - \frac{1}{4} g^2 ([B_\mu, B_\nu])^2 \right\}. \quad (34)$$

In order to switch off the field $B_\mu$ in the limit $a \to 0$ one can add to the action $U(N)$ the gauge invariant quantity

$$S_m = \frac{m^2 a^2}{8g^2} \sum_{x,\mu} \text{Tr} \left( \left( x-e_\mu, x \right) - I \right)^2. \quad (35)$$

It can be written explicitly as follows:

$$S_m = \frac{m^2 a^2}{8g^2} \sum_{x,\mu} \text{Tr} \left\{ (W^+(x, \mu)W(x, \mu) - I)^2 \right\} =$$

$$= \frac{m^2 a^2}{8g^2} \sum_{x,\mu} \text{Tr} \left\{ (V^+_{\mu}(x) + V_\mu(x))^2 - 2ag(V^+_{\mu}(x) + V_\mu(x))V^+_{\mu}(x)V_\mu(x) +$$

$$+ a^2 g^2(V^+_{\mu}(x)V_\mu(x))^2 \right\}, \quad (36)$$

where the $m$ is some mass parameter.

In the limit $a \to 0$ the $S_m$ becomes:

$$S_m \xrightarrow{a \to 0} \frac{m^2}{2} \int d^4x \sum_{\mu} \text{Tr} \left( B^2_{\mu} \right) = \frac{m^2}{2} \int d^4x \sum_{\mu} \left( b^2_{\mu} + \text{Tr} \left( \bar{B}^2_{\mu} \right) \right). \quad (37)$$

Othertise the quantity

$$S_{mb} = \frac{m^2 a^2}{8Ng^2} \sum_{x,\mu} \left( N - \text{Tr} \left( x-e_\mu, x \right) \right)^2 =$$

$$= \frac{m^2 a^4}{8N} \sum_{x,\mu} \{ \text{Tr} \left( V^+_{\mu}(x) + V_\mu(x) \right) - ag \text{Tr} \left( V^+_{\mu}(x)V_\mu(x) \right) \}^2 \quad (38)$$

is gauge invariant and

$$S_{mb} \xrightarrow{a \to 0} \frac{m^2}{2} \int d^4x \sum_{\mu} b^2_{\mu}, \quad (39)$$
where the $m_b$ is another mass parameter. One can choose this mass parameters to depend on $a$ so that they go to infinity when $a \to 0$ giving infinite masses to extra fields.

This leads us to following form of the action on the lattice:

$$S = cS_1 + (1 - c)S_2 + S_m + S_{mb}, \quad (40)$$

where the $c$ is arbitrary number in the interval $0 \leq c \leq 1$. For the concretness we consider further only the action of the form

$$S = S_1 + S_m, \quad (41)$$

In order to get $SU(N)$ theory it is necessary to add to the action a term keeping only $SU(N)$ gauge invariance and switching off the abelian field $a_\mu$ in the limit $a \to 0$. An example of such term is

$$S_{det} = \frac{m^2_{det} a^4}{2Ng^2} \sum_{x,\mu} \left\{ \left( \det W^+(x, \mu) - 1 \right) \left( \det W(x, \mu) - 1 \right) \right\}. \quad (42)$$

In the limit $a \to 0$ we obtain

$$S_{det} \to \frac{m^2_{det}}{2} \int d^4x \sum_\mu \left( a_\mu^2(x) + b_\mu^2(x) \right). \quad (43)$$

We assume that $m_{det} \to \infty$ at $a \to 0$ to switch off the field $a_\mu$ in the limit. For $SU(3)$ theory this term is of sixth order in the fields $A_\mu$ and $B_\mu$ so that we get rather complicated theory.

We can easily generalize this approach to include "matter" fields localized at the sites of the lattice (for example the fermion fields). To do this we can use for the part of action containing matter fields in the same form as one on Wilson-Polyakov lattice with the substitution of variables $W$ and $W^+$ for the corresponding unitary matrix variables. All complications connected with fermions on the lattice remain in this approach. In this paper we do not discuss more detaily the theory with fermion fields.

Due to compactness of $U(N)$ and $SU(N)$ gauge groups it is possible to use lattice theory in nonperturbative calculations without any gauge fixing. This is true despite the noncompactness of the space of our dynamical variables. Nevertheless one can fix the gauge. It is not difficult to prove that by gauge transformation one can reduce the field $V_\mu(x) = B_\mu(x) + iA_\mu(x)$ to a form where

$$A_0(x) = 0, \quad \forall x \quad (44)$$

(or $A_1(x) = 0, \text{ or } A_2(x) = 0, \text{ or } A_3(x) = 0$). The part $B_0(x)$ of the field cannot be made equal to zero simultanously with the $A_0(x)$ by unitary gauge transformation. Only in the limit $a \to 0$ when extra fields switch off one gets usual theory in the gauge $A_0 = 0$.

The considered lattice model can be used also for invariant ultraviolet regularization of perturbation theory. As usual one can consider the corresponding functional integral and separate the quadratic part of the action. This "free" part of the action is invariant with respect to the abelian analog of gauge transformations (1), (2). Such abelian group is noncompact. Therefore it is necessary to fix the gauge (for example, like in eq. (44)) when perturbation theory is applied. This perturbation theory uses the propagators of the fields $A_\mu$ and $B_\mu$ and the vertices contained in the nonquadratic part of the action. The number of vertices is finite and does not grows when the order of perturbation theory increases in contrast with perturbation theory based on Wilson-Polyakov lattice.
Fourier transforms of the fields on the lattice are periodic functions of momenta. It is possible to get different but equivalent forms of perturbation theory in momentum space using different localizations of fields on the lattice. In particular we related the field \( V_\mu(x) \) on the link connecting the points \( x - e_\mu \) and \( x \) to the point \( x \). In the construction of perturbation theory it can be more convenient to localize this field in the point \( x - (1/2)e_\mu \). This leads to the modification of the form of propagators and vertices in momentum space although both variants of perturbation theory are equivalent.

3 The Hamiltonian formulation in Lorentz coordinates with a lattice in 3-dimensional space

Here we consider the lattice only in 3-dimensional space. The time coordinate \( x_0 \) remains continuous. Starting with the \( U(N) \) theory we use as before the matrices \( W(x, i), V_i(x), G_{ik}(x), (i = 1, 2, 3) \). The time component \( A_0(x) = A_0^a(x)T_a, (a = 0, 1, \ldots, N^2 - 1) \) is taken as in usual \( U(N) \) theory, i.e. without the \( B_0(x) \) complement. This field is localized in the sites of 3-dimensional lattice.

We define the covariant derivative \( D_0 \) as follows:

\[
D_0 \varphi(x) = (\partial_0 + igA_0(x))\varphi(x)
\]

and derive the components \( G_{0i}(x) \) of tensor fields from the equality

\[
(D_0 \tilde{D}_i - \tilde{D}_iD_0)\varphi(x) = gG_{0i}(x)\varphi(x - e_i).
\]

We get

\[
G_{0i}(x) = \partial_0 V_i(x) - i\tilde{D}_i A_0(x) + ig(A_0(x)V_i(x) - V_i(x)A_0(x - e_i)),
\]

\[
G_{i0}(x) = -G_{0i}(x).
\]

The analog of the action \( S_1 \) considered before is

\[
S_1 = \frac{a^3}{4} \sum_i \int d^3 \varphi \text{Tr} \left( 2 \sum_i G_{0i}^+(x)G_{0i}(x) - \sum_{i,k} G_{0i}^+(x)G_{ik}(x) \right) \equiv \int d^0 L_1,
\]

where \( x = x^1, x^2, x^3 \) and the minus before the second term is connected with the transition to pseudoeuclidean space. In order to switch off the extra fields \( B_i(x) \) in the limit \( a \to 0 \) we add to the \( S_1 \) the analog of the "mass" term \( S_m \) introduced before:

\[
S_m = -\frac{m^2 a^3}{8} \sum_{x, i=1}^3 \int d^0 \text{Tr} \left\{ (V_i^+(x) + V_i(x))^2 + 
-2ag \left( V_i^+(x) + V_i(x) \right) V_i^+(x)V_i(x) + a^2 g^2 \left( V_i^+(x)V_i(x) \right)^2 \right\} \equiv \int d^0 L_m.
\]

The action now is

\[
S = S_1 + S_m \equiv \int d^0 L,
\]

where \( L = L_1 + L_m \). The mass parameter \( m \) should be infinitely increased when \( a \to 0 \) in order to switch off the fields \( B_i(x) \).
For the $SU(N)$ theory we add to the action the analog of the "mass" term $S_{det}$ introduced before:

$$S_{det} = - \frac{m_{det}^2 a^3}{2N g^2} \sum_{x,i} \int dx^0 \left\{ (\det W^+(x,i) - 1) (\det W(x,i) - 1) \right\} = \int dx^0 L_{det}, \quad (51)$$

The parameter $m_{det}$ also tends to infinity when $a \to 0$.

The transition to the Hamiltonian formulation can be carried out as usual. We introduce at $x^0 = const$ the "momenta"

$$\Pi_i^\alpha(x) \equiv \frac{\partial L}{\partial (\partial_0 V_i^\alpha(x))} = \frac{a^3}{2} G_{0i}^{\alpha 

conjugated to the $V_i^\alpha(x)$, and the "momenta"

$$\Pi_i^\dagger(x) \equiv \frac{\partial L}{\partial (\partial_0 V_i^{\dagger \alpha}(x))} = \frac{a^3}{2} G_{0i}^{\dagger \alpha}, \quad (53)$$

conjugated to the $V_i^{\dagger \alpha}$. Besides, we get

$$\pi_{0i}^\dagger(x) \equiv \frac{\partial L}{\partial (\partial_0 A_0^i(x))} = 0. \quad (54)$$

The generalized Hamiltonian is

$$\tilde{H} = \sum_{x,i} \text{Tr} \left\{ \Pi_i^+(x) \partial_0 V_i(x) + \Pi_i(x) \partial_0 V_i^+(x) \right\} - L = H - \sum_x (A_0(x) \phi(x)), \quad (55)$$

where

$$H = \sum_x \text{Tr} \left\{ \frac{2}{a^3} \sum_i \Pi_i^+(x) \Pi_i(x) + \frac{a^3}{4} G_{ik}^{\alpha \dagger}(x) G_{ik}(x) \right\} - L_m, \quad (56)$$

$$\phi(x) = \sum_i \left\{ i \left( \partial_i \Pi_i^+(x + \epsilon_i) - \partial_i \Pi_i(x + \epsilon_i) \right) + i g \left( V_i^+(x + \epsilon_i) \Pi_i(x + \epsilon_i) - V_i^+(x + \epsilon_i) \Pi_i(x + \epsilon_i) - \Pi_i(x) V_i^+(x + \epsilon_i) + V_i(x) \Pi_i^+(x + \epsilon_i) \right) \right\}. \quad (57)$$

The $\phi(x)$ is the 1st class constraint. The Lagrangian can be written in Hamiltonian form

$$L^{(1)} = \sum_{x,i} \text{Tr} \left\{ \Pi_i^+(x) \partial_0 V_i(x) + \Pi_i(x) \partial_0 V_i^+(x) \right\} - H + \sum_x \text{Tr} \left( \tilde{A}_0(x) \phi(x) \right). \quad (58)$$

For the $SU(N)$ theory the term $(-L_{det})$ is to be added to the $H$. Separating hermitian and antihermitian parts of the fields

$$V_i = B_i + i A_i, \quad \Pi_i = \frac{1}{2} (P_i + i \pi_i), \quad (59)$$

where $B_i = B_i$, $A_i = A_i$, $P_i = P_i$, $\pi_i = \pi_i$, we get the pairs of real canonical variables $(A_i^\dagger, \pi_i^\dagger)$ and $(B_i^\dagger, P_i^\dagger)$.

The transition to quantum theory can be realized by two different ways. One of them is to fix the gauge by the relation $A_0 = 0$, limit the physical subspace of states by the condition

$$\phi(x) |\Psi_{ph}\rangle = 0, \quad (60)$$
and to solve the Schroedinger equation

\[(H - E) |\Psi_{ph}\rangle = 0\]  

(61)

in the physical subspace. Another way is to introduce the gauge \(A_3 = 0\), to solve explicitly the constraint with respect to \(\pi_3\) and to substitute it for the \(\pi_3\) in the Hamiltonian. However the presence of nonzero \(B_3\) when \(A_3 = 0\) makes the problem of finding the \(\pi_3\) from the constraint equation practically nonsolvable. Nevertheless one can slightly modify the model under consideration to get \(A_3 = B_3 = 0\) without destroying the gauge invariance. To do this we relate to the links which are directed along the \(x^3\) axis the unitary matrices \(U(x, 3) = \exp(-i\sigma \tilde{A}_3(x))\) instead of \(W(x, 3)\) and the \(U^+(x, 3) = \exp(i\sigma \tilde{A}_3(x))\) instead of \(W^+(x, 3)\). Other link variables remain unchanged. The gauge invariant action can be obtained from previous one by the following changings:

\[V_3(x) \rightarrow U(x, 3) - I, \quad V_3^+(x) \rightarrow U^+(x, 3) - I.\]  

(62)

In this modified theory it is possible to fix the gauge by the relation

\[U(x, 3) = I,\]  

(63)

i.e. \(A_3(x) = 0\). After this gauge fixing the modified action coincides with the action considered before with \(A_3 = B_3 = 0\). It follows that the condition \(A_3 = B_3 = 0\) in the unmodified model agrees in fact with gauge invariance. Using this condition and looking at the Hamiltonian formalism we see that

\[P_3 = 0, \quad \Pi_3 = i\pi_3.\]  

(64)

The Hamiltonian coincides with the eq. (56) at \(A_3 = B_3 = 0\) and \(\Pi_3 = i\pi_3\). The constraint (57) can be now easily resolved with respect to the \(\pi_3\) and the Hamiltonian can be written in terms of remaining independent variables. It may be noticed that one could consider the model in the gauge \(A_3 = 0\) with continuous \(x^0\) and \(x^3\) and with the lattice in the \(x^1\), \(x^2\). Then it would be possible to put \(B_0 = B_3 = 0\) immediately. However the ultraviolet regularization in such model is not complete.

4 Canonical formulation of gauge theory on the LF

4.1 Transverse space lattice with continuous \(x^+\) and \(x^-\)

As before we use the following denotations for the LF coordinates:

\[x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3), \quad x^\perp \equiv (x^1, x^2) \equiv x^k, \quad k = 1, 2,\]  

(65)

where the \(x^+\) plays the role of time \([7]\). Transverse coordinates \(x^k\) correspond to the sites of the transverse lattice.

Our canonical formulation of gauge theory on the LF is similar to Bardeen- Pearson one \([4]\) but with using of other independent variables and with more detailed taking into account of the \(x^-\) zero mode problem \([8, 9]\). This problem can be formulated canonically on the interval \(-L \leq x^- \leq L\) using the assumption of periodic boundary conditions in the \(x^-\) for gauge fields. Such regularization preserves translation and gauge invariance \([8, 9]\).
In continuous space this formulation deals with very complicated 2nd class constraints containing zero modes of gauge fields, and it is not clear how to treat them in terms of quantum operator variables. We show that the introduction of transverse lattice in the framework of such formulation allows to avoid this 2nd class constraints at all. Furthermore we present the consideration of the LF 1st class constraints [3].

Let us start with the \( U(N) \) theory of pure gauge fields. The components of fields corresponding to continuous coordinates \( x^+, x^- \) can be taken in a form

\[
V_\pm = iA_\pm, \quad A^\pm_\pm = A_\pm
\]

because this simplification is allowed by a form of gauge transformations. Then we get in the same way as the eqv. (47) was derived the following relations:

\[
G_{\pm}(x) = iF_{\pm}(x), \\
G_{\pm k}(x) = \partial_\pm V_k(x) - i\tilde{\partial}_k A_\pm(x) + ig(A_+(x)V_k(x) - V_k(x)A_+(x - e_k)) \quad (67)
\]

To write the action in canonical (Hamiltonian) form we fix the gauge appropriately to periodic boundary conditions in the \( x^- \):

\[
\partial_- A_- = 0, \quad A^B_\pm(x) = \delta^B_\pm(x^+, x^-).
\]

The \( i, j \) are the \( U(N) \) matrix indexes \( i, j = 1, 2, \ldots, N \). We obtain

\[
S = \frac{a^2}{2} \sum_{x^-, x^+} \int_{-L}^{L} dx^+ \int_{-L}^{L} dx^- \text{Tr} \left( G_{\pm}^{-+}G_{\pm}^{+-} + \sum_{k=1,2} \left( G_{\pm k}^{+}G_{-k}^{-} + G_{-k}^{+}G_{+k}^{-} \right) - G_{12}^{-}G_{12}^{+} \right) + S_{m, \perp} =
\]

\[
= \frac{a^2}{2} \sum_{x^-, x^+} \int_{-L}^{L} dx^+ \int_{-L}^{L} dx^- \left\{ \sum_{i=1}^{N} 2L \left[ (F_{i-})_x + i\tilde{\partial}_k(V_k - V_k^+)(x + e_k) \right]_{(0)} \partial_+ V_i(x) + \right.
\]

\[
+ \sum_{i,j=1}^{N} \left[ \left( (\partial_- - igv_i(x) + igv_j(x - e_k))^+V_{k}^{+ij}(x) \right) \partial_+ V_k^{ij}(x) + h.c. \right] +
\]

\[
+ \sum_{i,j=1}^{N} A^B_\pm(x)Q^{ij}(x) - H(x) \right\},
\]

where the denotation \( ()_x \) means that all quantities inside the brackets are to be taken at the point \( x \); the \( S_{m, \perp} \) is the ”mass” term given by eq. (30) (with opposite sign) where one takes into account the transition to the transverse lattice (then it does depend only on transverse components of the fields); furthermore

\[
f_{(0)} \equiv \frac{1}{2L} \int_{-L}^{L} dx^- f(x^-), \quad (70)
\]

and the \( H(x) \) is the Hamiltonian density.

The generators of gauge transformations \( Q^{ij}(x) \) are defined by the following expression

\[
Q(x) = 2(D_-F_{i-})_x + i\tilde{\partial}_k(G_{i-}^{-}G_{-k}^{+} - G_{-k}^{+}G_{i-}^{-})(x + e_k) +
\]

\[
+ ig(V_kG_{i-}^{-}G_{-k}^{+} - G_{-k}^{+}V_k^+)(x) - ig(G_{i-}^{-}G_{-k}^{+}V_k^+ - V_k^+G_{-k}^{+})(x + e_k).
\]

(71)
The gauge constraints

\[ Q_{ij}^{ij}(x) = 0 \]  

(72)
can be resolved explicitly by expressing the quantities \( F_{ij}^{ij} \) in terms of other variables except of zero mode components \( F_{ii}^{ii}(0) \) which can not be found from this constraint equation. So the corresponding zero mode \( Q_{ii}^{ii}(x^+, x^+) \) of the constraint remains unresolved explicitly and it is considered as the condition on the physical quantum states:

\[ Q_{ii}^{ii}(x^+, x^+) |\Psi_{phys}\rangle = 0. \]  

(73)

In order to complete the derivation of the action in canonical form and to extract all independent canonical variables we make the Fourier transformation in the \( x^- \) of the transverse field components \( V_{ij}^{ij}(x) \) as follows:

\[ V_{ij}^{ij}(x) = \sum_{n=-\infty}^{\infty} \{ \Theta \left( p_n - gv^i(x) + gv^j(x) - e_k \right) \} V_{nk}^{ij}(x^+, x^+) + \Theta \left( -p_n + gv^i(x) - gv^j(x) - e_k \right) \} V_{nk}^{ij}(x^+, x^+) \times \left( 4L \left| p_n - gv^i(x) + gv^j(x) - e_k \right| \right)^{-1/2} e^{-ip_n x^-}, \]  

(74)

where \( p_n = \pi n/L, n \in \mathbb{Z} \).

Then the action is

\[ S = \frac{a^2}{2} \sum_{x^+} \int dx^+ \left\{ \sum_{i=1}^{N} 2L \left[ 2(F_{+}^-)_x + i \tilde{\partial}_k(V_k - V_k^+)_{x+e_k}^{ii} \right] \partial_+ v^i(x) + i \sum_{n=-\infty}^{\infty} \sum_{i,j=1}^{N} \left( V_{nk}^{ij+} \partial_+ V_{nk}^{ij} \right)_x + 2L \sum_{i=1}^{N} \left( A_{+}^{ii}(0) Q_{ii}^{ii}(0) \right)_x - \bar{\tilde{H}}(x) \right\}, \]  

(75)

where the quantities \( (V_k)_{(0)}^{ii} \) are expressed in terms of variables \( V_{0k}^{ii} \) according to eqs. (70), (74) as follows:

\[ (V_k)_{(0)}^{ii} = \frac{V_{0k}^{ii}}{\sqrt{4La|\partial v^i|}} \]  

(76)

and the \( \bar{\tilde{H}} \) is obtained from the \( H \) via substitution of the expressions for the \( F_{+}^{ij} \) found from the constraints (72).

We have the following set of canonically conjugated pairs of independent variables:

\[ \left\{ v^i(x), \Pi_i(x) = La^2 \left[ 2(F_{+}^-)_x + i \tilde{\partial}_k(V_k - V_k^+)_{x+e_k}^{ii} \right] \right\}, \]

\[ \left\{ V_{nk}^{ij}(x), \frac{i\alpha^2}{2} V_{nk}^{ij+}(x) \right\}. \]  

(77)

In quantum theory this variables become operators which satisfy usual canonical commutation relations.

In the obtained formulation there are no 2nd class constraints for zero modes of the transverse field components. If one goes to the limit \( a \to 0 \) this constraints reappear in a form which contains quantum operators in definite order. This order was not clear earlier.
One can easily construct canonical operator of translations in the $x^-$:

$$P_{\text{can}}^i = \frac{a^2}{2} \sum_{x^\perp} \sum_{k=1,2} \sum_{n} \sum_{i,j=1}^{N} p_n \varepsilon \left(p_n - g v^i(x) + g v^j(x - e_k)\right) \left(V_{nk}^{ij} + V_{nk}^{ji}\right)_x. \quad (78)$$

This expression differs from the physical gauge invariant momentum operator $P_-$ by a term proportional to the constraint. The operator $P_-$ is

$$P_- = a^2 \sum_{x^\perp} \sum_{k=1,2} \int_{x^- L} dx^- \text{Tr} \left(G^+_k G^-_k\right)_x = P_{\text{can}}^+ + L a^2 \sum_{x^\perp} \sum_{i=1}^{N} \left(v^i Q_{(0)}^i\right)_x =$$

$$= \frac{a^2}{2} \sum_{x^\perp} \sum_{k=1,2} \left[ ga \sum_{i=1}^{N} \left(V_{0k}^{ii} - \sqrt{\frac{4L}{ga}} \hat{\partial}_k v^i\right)_x^+ + \left(V_{0k}^{ii} - \sqrt{\frac{4L}{ga}} \hat{\partial}_k v^i\right)_x^+ \right] +$$

$$+ \sum_{n,i,j}^\prime \left(p_n - g v^i(x) + g v^j(x - e_k)\right) \left(V_{nk}^{ij} + V_{nk}^{ji}\right)_x \quad (79)$$

where the $\sum'$ denotes the sum over all $n,i,j$ except of $i = j$ at $n = 0$.

The operators $Q_{(0)}^i(x^\perp, x^+)$ have the following form in terms of canonical variables:

$$2L Q_{(0)}^{ii}(x) = \sum_{k=1,2} \left\{ - g a \hat{\partial}_k \left( \varepsilon(\hat{\partial}_k v^i) \left( V_{0k}^{ii} - \sqrt{\frac{4L}{ga}} |\hat{\partial}_k v^i|\right)_x^+ + \left(V_{0k}^{ii} - \sqrt{\frac{4L}{ga}} |\hat{\partial}_k v^i|\right)_x^+ \right) + \right.$$

$$+ g \sum_{n,i,j}^\prime \left[\varepsilon(p_n - g v^i(x + e_k) - g v^j(x)) \left(V_{nk}^{ij} + V_{nk}^{ji}\right)_x^+ - \right.$$

$$\left. - \varepsilon(p_n - g v^i(x) + g v^j(x - e_k)) \left(V_{nk}^{ij} + V_{nk}^{ji}\right)_x^+ \right]. \quad (80)$$

In order to find general form of physical states satisfying the condition (78) it is convenient to introduce a basis consisting of following state vectors:

$$\prod_{x^\perp} \prod_{k=1,2} \prod_{n} \prod_{i,j=1}^{N} \left(V_{nk}^{ij}(x)\right)^{m^i_{nk}(x)} \left(R_k^i(x)\right)^{m^i_k(x)} |\bar{v}\rangle \quad (81)$$

where we use ”creation and annihilation” operators, $R_k^{i+}(x)$ and $R_k^i(x)$, defined as follows:

$$R_k^i(x) \equiv \left(V_{0k}^{ii} - \sqrt{\frac{4L}{ga}} |\hat{\partial}_k v^i|\right)_x \quad (82)$$

The $R_k^i(x)$ act on the vector $|\bar{v}\rangle$ like annihilation operators:

$$R_k^i(x) |\bar{v}\rangle = 0 \quad (83)$$

The vector $|\bar{v}\rangle$ is connected with the standard vector $|v\rangle$ satisfying the conditions

$$\hat{v}^i(x) |v\rangle = v^i(x) |v\rangle , \quad (84)$$

$$V_{nk}^{ij}(x) |v\rangle = 0, \quad (85)$$
by the following transformation:

\[ |\bar{v}\rangle = \exp \left\{ \frac{a^2}{2} \sum_{x^\perp} \sum_{k=1,2} \sum_{i=1}^{N} \left( \frac{4L}{ga} |\partial_k v^i| \right)^{1/2} V_{0k}^{ij}(x) \right\} |v\rangle . \]  

(86)

A set of nonnegative integer numbers \( m_{nk}^{ij}(x) \) is defined so that

\[ m_{0k}^{ii}(x) \equiv 0. \]

(87)

In this basis the conditions (73) take the form of following relations between the numbers \( m_{nk}^{ij}(x), m_k^i(x) \) and the \( v^i(x) \):

\[ \sum_{k=1,2} \left\{ -a\partial_k \left( \varepsilon(\partial_k v^i) m_k^i \right)_{x+e_k} + \sum_{n} \sum_{j=1}^{N} \left[ \varepsilon(p_n - gv^j(x) + e_k) m_{nk}^{ij}(x + e_k) - \varepsilon(p_n - gv^j(x) + e_k) m_{nk}^{ij}(x) \right] \right\} = 0 \]

One can find the eigenvalue \( p_- \) of the momentum \( P_- \) for such basis state:

\[ p_- = \sum_{x^\perp} \sum_{k=1,2} \left[ ga \sum_{i=1}^{N} \left( \partial_k v^i |m_k^i\rangle \right)_{x} + \sum_{n} \sum_{i,j=1}^{N} |p_n - gv^j(x) + e_k| m_{nk}^{ij}(x) \right] \]

(89)

and to require that this value be finite. The detailed analysis of this problem will be given in future publication.

Let us discuss the transition to the SU(\( N \)) gauge theory. In order to avoid abelian part of the field in the limit \( a \to 0 \) we add to the action the additional ”mass term” which is obtained from the expression (12) with opposite sign by transition to transverse lattice. Moreover we can restrict the general form of the \( A_+, A_- \) by the condition

\[ \text{Tr} A_+ = \text{Tr} A_- = 0. \]

(90)

We get in this way the theory with the same canonical structure in transverse variables as before. According to eq. (21) one has to choose the independent components among \( v^i \) and \( \Pi_i \), and to modify the expression for the constraint operator \( Q^{ij}(x) \) by subtraction of the \( (N^{-1})\text{Tr} Q(x) \). Analogous modification has to be done in the formulation of the problem (73).

**4.2 Lattice in coordinates \( x^1, x^2 \) and \( x^- \)**

The formulation of gauge theory on the LF with transverse lattice and continuous coordinates \( x^+, x^- \) gives only partial ultraviolet regularization. To complete the regularization we can introduce the lattice also along the \( x^- \). Let us discuss this possibility. We consider for simplicity the U(\( N \)) theory in the space with unbounded coordinate \( x^- \). We introduce different lattice spacing parameters in the transverse and in the \( x^- \) coordinates. Let us denote the transverse one as before by \( a \) and the one along the \( x^- \) direction by \( b \). The \( x^+ \) coordinate remains continuous, and corresponding field component is taken in the simplest form \( V_+ = iA_+ \), i.e. \( B_+ = 0 \). We relate the matrix \( W(x,-) \) to the link directed to positive side of the \( x^- \) axis, and the matrix \( W^+(x,-) \) to the link directed to the opposite side. The matrix \( W(x,-) \) is related to the link connecting the point \( x - e_- \) with the point \( x \). We put

\[ W(x,-) = I - gbV_-(x), \quad V_-(x) = B_-(x) + iA_-(x). \]

(91)
The component $G_{+-}$ is defined as follows

$$G_{+-}(x) = \partial_+ V_-(x) - i \tilde{\partial}_- A_+(x) + ig (A_+(x) V_-(x) - V_-(x) A_+(x - e_-)).$$

(92)

Let us take into account that the components $G_{+k}(x)$, $(k = 1, 2)$ change under gauge transformation as follows

$$G_{+k}'(x) = u(x) G_{+k}(x) u^+(x - e_k),$$

(93)

and choose the definition of the $G_{-k}(x)$ so that the quantity

$$\text{Tr} \left( G^+_+ (x) G^-_k (x) \right),$$

(94)

entering into the action remains gauge invariant. Such definition is

$$G_{-k}(x) = \frac{1}{2gab} \left( \begin{array}{ccc} x - e_k & x & \vspace{0.2cm} \\ x & x - e_k & x - e_k \\ x - e_k & x & x - e_k \end{array} \right) =$$

$$= \frac{1}{2gab} \left\{ 2W(x, -) W(x - e_-, k) W^+(x - e_k, -) - W(x, -) W^+(x, -) W(x - e_k, -) - W(x, k) W(x - e_k, -) W^+(x - e_k, -) \right\}. 

(95)

This expression agrees in the continuous limit with the usual tensor field if the extra fields $B_k, B_-$ are switched off in this limit. Now the action can be written in the form

$$S_1 = \frac{a^2 b}{2} \sum_{x^-, x^1, x^2} \int dx^+ \text{Tr} \left\{ G^+_+ (x) G^-_+ (x) + G^+_+ (x) G^-_- (x) + G^+_+ (x) G^-_{12} (x) \right\}. 

(96)

We add also necessary "mass" terms in order to switch off the extra fields in the continuous limit.

In order to get the canonical formalism we have to fix the gauge $A_- = 0$. However the $B_-$ part cannot be simultaneously set equal to zero by gauge transformation. This leads to the difficulty with solving of the canonical constraints

$$\pi^+_k = G^-_{-k},$$

(97)

where $\pi^+_k$ is the momentum conjugate to $A_k$. To pass over this difficulty we again apply the modified model in which the $W(x, -)$ is changed by unitary matrix $U(x, -) = \exp (-igb A_- (x))$. The components $G_{12}, G_{+k}$ remain unchanged. The quantities $G_{-k}$ become as follows:

$$G_{-k}(x) = \frac{1}{gab} \left( \begin{array}{ccc} x - e_k & x & \vspace{0.2cm} \\ x & x - e_k & x - e_k \\ x - e_k & x & x - e_k \end{array} \right) =$$

$$= \frac{1}{gab} \left( U(x, -) W(x - e_-, k) U^+(x - e_k, -) - W(x, k) \right),$$

$$G_{+-}(x) = \partial_+ \left( \frac{U(x, -) I}{(-ag)} - i \tilde{\partial}_- A_+(x) +$$

$$+ ig \left( A_+(x) \left( \frac{U(x, -) I}{(-ag)} - \left( \frac{U(x, -) I}{(-ag)} \right) A_+(x - e_-) \right) \right).$$

(98)
Further we substitute this expression for the $G_{\mu\nu}$ in the action

$$S = S_1 + S_m$$  \hspace{1cm} (99)$$
where $S_1$ is defined by eq. (96) and $S_m$ is the same as in eq. (95). We get again $U(N)$ gauge invariant action which has correct naive continuous limit if $m \to \infty$ when $a \to 0$, $b \to 0$. In the modified theory one can use the gauge

$$A_- = 0,$$  \hspace{1cm} (100)
i.e.

$$U(x, -) = I.$$  \hspace{1cm} (101)$$
In this gauge one has

$$G_{+}(x) = -i \tilde{\partial} A_{+}(x), \quad G_{-k}(x) = \tilde{\partial} V_k(x),$$  \hspace{1cm} (102)$$
and there remain previous expressions for the $G_{+k}, G_{12}$. Now there is no difficulty with solving the canonical constraints, and all steps necessary for the construction of Hamiltonian formalism can be done in standard way.

Let us discuss the question connected with the regularization of the "infrared" LF divergences at $p_- = 0$. To achieve this regularization one can as usual take the lattice with finite number of sites in the $x^-$ direction and choose periodic boundary conditions in the $x^-$. However this requires gauge fixing in the form (as in the section 4.1)

$$A_-(x) = v(x^+, x^1, x^2),$$  \hspace{1cm} (103)$$
where the $v$ is a diagonal matrix. In this gauge the unitary matrices $U(x, -)$ become diagonal but the action remains nonpolynomial in variables $v$. More detailed investigation of this question is under consideration.

Acknowledgements

This work was in part supported by the DFG Grant 436 RUS 113/205/1 on Russian-German cooperation. We thank B. Van de Sande for interesting discussions.

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