Application of least squares fitting algorithm in transformer terminal unit electrical quantities acquisition

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Abstract. In order to accurately estimate the frequency and harmonics of voltage and current signals acquired in transformer terminal unit (TTU), the least squares fitting algorithm is applied to its harmonic analysis. The principle of the algorithm and its flowchart are presented, and the implementation of the algorithm in TTU is primarily discussed. The noisy voltage signals with harmonics and signal frequencies ranging from 49.8 Hz to 50.2 Hz are generated and the real ones are acquired. The harmonic analysis results of the least squares fitting algorithm and the Hanning window and interpolation algorithm reveal that the errors in the signal frequency, direct current (DC) component, amplitude and phase of fundamental and harmonics of the former are only about 1/3 of that of the latter.

1. Introduction
The development of industry imposes a very high requirement on power supply reliability and power quality. Distribution automation is an inevitable trend of power system. Distribution automation usually includes feeder terminal unit (FTU) [1] of circuit breaker, central control unit (CCU) [2-3] of substation, transformer terminal unit (TTU) [4] of distribution transformer, etc. TTU needs to acquire the three-phase voltage, three-phase current, and calculate the corresponding frequency, active power, reactive power and other parameters for the distribution transformer and at the same time, evaluate the power quality. Therefore, it is necessary to carry out harmonic analysis on the acquired voltage and current signals.

Typical power system harmonic analysis methods include the window and interpolation algorithm [5-6], modified ideal sampling frequency method [7], instantaneous reactive power algorithm [8-9], artificial neural network (ANN) method [10-11] and least squares fitting algorithm [12], etc. The window and interpolation algorithm can reduce spectral leakage and suppress harmonic interference, but it has relatively high requirement on sampling duration. The modified ideal sampling frequency method can also suppress the error caused by spectral leakage by using modified signal, but the correction process will introduce error. The instantaneous reactive power algorithm is proposed by H. Akagi, et al. It can accurately measure the harmonics of the symmetrical three-phase three-wire system. But this algorithm is particularly suitable for this type of system. Because of its strong approximation ability, ANN method has been widely used in harmonic analysis. The method has strong adaptive ability, and some ANN methods have high real-time performance, which is suitable for online harmonic analysis. The least squares fitting algorithm establishes the objective function according to the least squares method, and analyzes harmonic by minimizing the objective function. This method has the advantage of high accuracy, but the application of the algorithm in TTU electrical quantities acquisition has not been reported.
In order to fix this problem, based on the introduction of the principle of the least-squares fitting algorithm, the influence of number of iterations on the error is investigated. Then, for the simulated and real voltage signals, the Hanning window and interpolation algorithm [6] and the least-squares fitting algorithm are used for harmonic analysis. The errors of the two algorithms in signal frequency, direct current (DC) component, amplitude and phase of fundamental and harmonics are systematically compared, and the latter in TTU harmonic analysis is verified.

2. Harmonic analysis method based on least squares fitting

2.1. Algorithm principle

The sum of harmonics can be used to approximate the power system signal. Let \( s'(t) \) be the sum of the harmonics, which can be expressed by Equation (1).

\[
s'(t) = A_0 + \sum_{m=1}^{M} (A_{m1} \sin(2\pi f_m t) + A_{m2} \cos(2\pi f_m t))
\]

where \( t \) is time; \( A_0 \) is the DC component; \( A_{m1} \) and \( A_{m2} \) are the amplitudes of the sine and cosine components of the \( m \)th harmonic respectively; \( M \) is the maximum order of harmonics; \( f \) is the fundamental frequency.

The error of the \( n \)th sampling point is shown in Equation (2).

\[
e_n = s'((n+1)\Delta t) - s(n\Delta t), \quad n=0, 1, 2, \ldots, N-1
\]

where \( e_n \) is the error of the \( n \)th sampling point; \( s(n\Delta t) \) is the measured power system signal at the \( n \)th sampling point; \( \Delta t \) is the sampling interval; \( N \) is the number of sampling points.

The objective function based on the method of least squares is displayed in Equation (3).

\[
E = \sum_{n=0}^{N-1} e_n^2 = \sum_{n=0}^{N-1} (A_0 + \sum_{m=1}^{M} (A_{m1} \sin(2\pi f_m n\Delta t) + A_{m2} \cos(2\pi f_m n\Delta t)) - s(n\Delta t))^2
\]

where \( E \) is a nonlinear function taking the fundamental frequency, DC component, the amplitude of the sine and cosine components of fundamental and harmonic as variables. The key of the least squares fitting algorithm is to adjust \( f, A_0, A_{m1} \) and \( A_{m2}, m=1, 2, \ldots, M \) according to the optimization algorithm and making \( E \) tend to the minimum.

The above problem belongs to the nonlinear least squares problem, and the Levenberg-Marquardt algorithm is particularly suitable for solving this problem. In this algorithm, the Hessian matrix is approximated by Equation (4).

\[
H = J^T J + \lambda I
\]

where \( H \) is the Hessian matrix; \( J \) is the Jacobian matrix; \( \lambda \) is a variable which prevents the Hessian matrix calculated by Equation (4) from being significantly different from the actual Hessian matrix; \( I \) is a \( 2M+2 \times 2M+2 \) identity matrix.

Assume that \( e = [e_0, e_1, \ldots, e_{2N-1}]^T \) is the error vector. Then the variables can be adjusted according to Equation (5).

\[
W(k+1) = W(k) - (J(k)^T J(k) + \lambda I)^{-1} J(k)^T e(k)
\]

where \( W = [A_0, A_0, A_{m1}, A_{m2}, \ldots, A_{m1}, A_{m2}] \); \( k \) is the number of iterations.

If \( E(k+1) \leq E(k) \), then \( \lambda = \lambda / 10 \). Otherwise, if \( E(k+1) > E(k) \), then \( \lambda = 10 \lambda \) and the adjustment about the variables is discarded. The algorithm flowchart is shown in Figure 1.
2.2. Discussion of algorithm implementation in TTU

Considering that the least squares fitting algorithm needs several iterations to minimize the objective function, a microcontroller unit (MCU) with relatively fast computing speed should be chosen. The STM32F4 family is a high-performance ARM® Cortex®-M4 MCU developed by STMicroelectronics. The MCU features a floating point unit (FPU) single precision. It can operate at a frequency of up to 168 MHz. The processor can operate at speeds up to 210 million instructions per second (DMIPS). In addition, it offers three 12-bit analog-to-digital converters (ADCs) with a minimum sampling time of 0.41 μs and a maximum sampling rate of 2.4 million samples per second (MSPS) and even 7.2 MSPS in triple interleaved mode. The above algorithm can be programmed with C language, compiled and linked into executable files. And it can be imported into the read-only memory (ROM) of STM32F4. By this way, the TTU has harmonic analysis capability.

3. Validation of proposed algorithm

3.1. Simulated voltage signal

The following signal is used to simulate the voltage signal acquired in TTU and validate the proposed algorithm.

\[ u(t)=0.01B+Bs\sin(2\pi ft-\pi/3)+0.015Bs\sin(6\pi ft-\pi/2)+0.01Bs\sin(10\pi ft-\pi/5)+0.01Brandn \]  

(6)

where \(u\) is voltage in V; \(t\) is time in s; \(B\) is the fundamental amplitude which is equal to 311.13; \(f\) is the fundamental frequency in Hz which varies in the range of 49.8~50.2 and the step size is 0.02; \(randn\) produces a standard normally distributed random variable. The sampling frequency and sampling duration are 40 kHz and 0.05 s, respectively. Change of \(E\) and error in frequency with number of
iterations is displayed in Figure 2. According to Figure 2, if the number of iterations is larger than 2, $E$ and error in frequency are essentially constant as number of iterations increases. Therefore, the number of iterations is set to 5 in the algorithm.

![Image](a) Sum of squared errors  
![Image](b) Error in signal frequency estimation

**Figure 2.** Change of sum of squared errors and signal frequency error with number of iterations.

In order to suppress the interference caused by noise, 1000 signals are randomly generated based on Equation (6) for each parameter combination. The errors in signal frequency, DC component, amplitude and phase of fundamental and harmonics are calculated by the Hanning window and interpolation algorithm and the least squares fitting algorithm, as shown in Figure 3. Note that the error in Figure 3 is the mean value of 1000 error amplitudes.

![Image](a) Signal frequency  
![Image](b) DC component  
![Image](c) Fundamental amplitude  
![Image](d) Fundamental phase  
![Image](e) Amplitude of the third harmonic  
![Image](f) Phase of the third harmonic

**Figure 3.** Changes of errors in signal frequency, DC component, amplitude and phase of fundamental and harmonics with frequency.
As can be seen from Figure 3, the errors of the least squares fitting algorithm are significantly smaller than those of the Hanning window and interpolation algorithm. Especially for the errors of signal frequency, fundamental amplitude, phase of fundamental and harmonics, even the error of the former is only one percent of that of the latter. This is because when the signal frequency is not 50 Hz, it is in the asynchronous sampling state. The Hanning window and interpolation algorithm has spectral leakage and picket-fence effect, and the error may be quite significant. By minimizing the objective function, the least squares fitting algorithm can make the error tend to 0 when the signal is noise-free.

The noise in Equation (6) leads to a certain error. However, it is much smaller than the error caused by spectral leakage and picket-fence effect.

The statistical results of harmonic analysis errors by the two algorithms are shown in Table 1. \( E_m \) and \( E_M \) represent the mean and maximum error amplitudes, respectively. Similar to the results in Figure 3, the errors of the least squares fitting algorithm are significantly smaller than those of the Hanning window and interpolation algorithm. In addition, according to Table 1, the results show that the errors of signal frequency, DC component, amplitude and phase of fundamental and harmonics of the least squares fitting algorithm are only 5.29\times 10^{-3}~0.75 of that of the Hanning window and interpolation algorithm, and the mean value is only 0.30. In other words, the error of the former is about 1/3 of that of the latter.

Table 1. Statistics results of harmonic analysis error on the two algorithms, simulated voltage signal.

| Frequency/Hz | DC/V | Fundamental Amplitude/V | Phase/rad | Third Amplitude/V | Phase/rad | Fifth Amplitude/V | Phase/rad |
|--------------|------|--------------------------|-----------|-------------------|-----------|------------------|-----------|
| Hanning      | \( E_m \) 4.86\times 10^{-2} | 5.73\times 10^{-2} | 8.07\times 10^{-4} | 4.92\times 10^{-4} | 8.01\times 10^{-2} | 1.73\times 10^{-2} | 7.92\times 10^{-2} |
|              | \( E_M \) -7.49\times 10^{-2} | -4.26        | 0.68       | 1.30\times 10^{-2} | 1.07      | 3.41             | 0.56      |
| Least squares | \( E_m \) 2.83\times 10^{-3} | 0.29        | -0.41     | -2.31\times 10^{-3} | -0.45   | -9.33\times 10^{-2} | 0.4       |
|              | \( E_M \) 1.39\times 10^{-2} | 0.11        | 0.11      | 6.34\times 10^{-2} |

3.2. Real voltage signal

The real voltage signal is a sinusoidal waveform with an amplitude of 2 V, and the signal frequency varies in the range of 49.8~50.2 Hz. It is generated by a signal generator (Model DG1022U, RIGOL Technologies Co., Ltd., China). The signals are acquired by an oscilloscope (Model MSO5204, RIGOL Technologies Co., Ltd., China) with sampling frequency of 40 kHz and number of samples of 2000. It is only a fundamental wave with different frequencies and the harmonic content is the natural distortion of the generator. The errors of harmonic analysis using the Hanning window and interpolation algorithm and the least squares fitting algorithm are shown in Figure 4. The statistics results of error are presented in Table 2. The phase of fundamental and harmonic varies greatly with signal frequency. Therefore, the phase errors are not presented. According to Figure 4, the frequency error of the least squares fitting algorithm is significantly smaller than that of the Hanning window and interpolation algorithm. As for the harmonic amplitude error, the former is also slightly smaller than that of the latter. This is due to the fact that the voltage signal contains harmonics. The results in Table
2 are consistent with Figure 4. In addition, according to Table 2, the errors of signal frequency, DC component and amplitude of fundamental and harmonics of the least squares fitting algorithm are only $1.25 \times 10^{-2} \sim 1.67$ of the Hanning window and interpolation algorithm, and the mean value is only 0.38. In other words, the error of the former is about 1/3 of that of the latter.

![Graphs showing errors in signal frequency, DC component, fundamental amplitude, and third harmonic amplitude.]

Table 2. Statistics results of harmonic analysis error on the two algorithms, real voltage signal.

|                | Frequency/mHz | DC/mV  | Fundamental, amplitude/mV | Third, amplitude/mV | Fifth, amplitude/mV |
|----------------|---------------|--------|---------------------------|---------------------|---------------------|
| FFT            | $E_m$         | 58.29  | 75.78                     | 1.97                | 4.2                 | 1.18                |
|                | $E_M$         | 83.79  | -82.45                    | -5.05               | 9.69                | 2.7                 |
| Least squares  | $E_m$         | 2.46   | 0.95                      | 0.99                | 1.08                | 0.7                 |
|                | $E_M$         | -6.21  | 2.97                      | 1.97                | 3.01                | -1.51               |

4. Conclusions
In this work, the problem of harmonic analysis in TTU electrical quantities acquisition is investigated, and the least squares fitting algorithm is used to the harmonic analysis for simulated and real voltage signals. The results are compared with the Hanning window and interpolation algorithm. The results reveal that the errors of the signal frequency, DC component, amplitude and phase of fundamental and harmonics of the least squares fitting algorithm are only about 1/3 of those of the Hanning window and interpolation algorithm.
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