Analysis of ellipsometric parameters in the reflection of circularly polarized light from a thin anisotropic plate

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Abstract. The article presents the results of the analysis of the angular spectra of the ellipsometric parameters of the reflected wave when a circularly polarized light wave is incident on an anisotropic plate. The given dependences show a very high sensitivity of the ellipsometric parameters of the reflected light on the angle of incidence and the angle between the optical axis and the normal to the plate boundary. The energy reflection spectra themselves show much less variability when these parameters change. It should be especially emphasized the nature of the change in the ellipsometric angle \( \Delta \), which is responsible for the type of elliptical polarization - when \( \Delta > 0 \), the polarization is left-handed, and when \( \Delta < 0 \), it is right-handed. It is shown that a thin anisotropic plate at certain angles can serve as a polarization converter of the incident radiation. The ellipsometry parameter \( \rho \) characterizes the degree of compression of the ellipse - when \( \rho = 1 \), the ellipse is transformed into a circle, and the light is circularly polarized in this case. Thus, a thin anisotropic plate can not only convert left-handed polarization to right-handed, but it can also control the very shape of the polarization ellipse. Such a plate can be used in conjunction with a layered medium, for example, a one-dimensional photonic crystal, to control the polarization of the incident circularly polarized light.

1. Introduction
Until recently, in problems of spectroscopy of materials, as a rule, linearly polarized light was used. In this case, the reflected and transmitted light was analyzed for mutually perpendicular polarizations - s and p. The difference in the reflectance spectra for these polarizations provided additional information compared to the diagnostics of materials with unpolarized light. The most complete information on the properties of various materials is provided by spectroscopy using elliptically polarized light. When light is reflected from the sample under study, the nature of this polarization can change in a certain way, which can serve as an additional method for diagnosing the material. For example, in [1], circularly polarized light was used to analyze the optical properties of a quantum dot. The authors of the work emphasize that the emerging ellipticity of light when exposed to a quantum dot plays an essential role. Elliptically polarized light is used not only for diagnostic purposes, but also as a unique way to excite molecular systems. For example, in [2], a model of nonadiabatic rotational excitation of molecules was described. using short specific elliptically polarized laser pulses. It is shown in [3] that when using elliptically polarized light by changing the elliptic parameter, it is possible to change the linear or nonlinear absorption of the light wave. One of the most subtle optical methods of materials diagnostics is the ellipsometry method. An exact ellipsometric method for the analysis of coherent light with low ellipticity was proposed in [4]. Methods for transforming the polarization of light play a special role. Work [5] reports on a new method for converting linearly polarized light into elliptical light using a...
plane-parallel plate. In article [6], circularly polarized light is used to analyze the properties of a sheet. The author's work [7] analyzes the polarization of reflected light from various materials when circularly polarized light is incident on it. In this case, the boundary of only two environments was considered. In [8-10], with the participation of the author, the analysis of ellipsometric parameters for nanocomposite materials and the application of the surface plasmon resonance method for diagnostics of thin films were carried out. In this work, we analyze the reflection of circularly polarized light from thin anisotropic plate.

1.1. Formulation of the problem
Let us consider the incidence of a circularly polarized light wave on an anisotropic plate, the optical axis of which lies in the plane of incidence. The thickness of the plate is d. The angle of incidence is denoted by \( \theta \). It is necessary to analyze the angular spectra of reflection of the wave - the energy reflection coefficient, as well as its ellipsometric parameters. The problem is solved by the method of characteristic matrices ([9]). The calculation was carried out for the following parameter values: \( \varepsilon_0 = 2.1 \), \( \varepsilon_e = 2.5 \), \( \varepsilon_1 = 1 \), \( \varepsilon_3 = 4 \), plate thickness \( d = 2 \mu \), radiation wavelength \( \lambda = 0.64 \mu \).

![Figure 1. Geometry of the reflection of a light wave from an anisotropic plate.](image)

1.2. Theoretical consideration
To obtain the elements of the dielectric constant tensor, we pass from the main coordinate system \( OX'Z' \) to the laboratory \( OXZ \). Let us consider the case when in a uniaxial crystal the optical axis \( -OZ' \) is at an angle \( \varphi \) to the \( Z \) axis (figure 2). In the main coordinate system \( X'Y'Z' \), the electric induction vector has the following form:

\[
\begin{align*}
D_{x'} &= \varepsilon_{x'x'}E_{x'} \\
D_{y'} &= \varepsilon_{y'y'}E_{y'} \\
D_{z'} &= \varepsilon_{z'z'}E_{z'}
\end{align*}
\]

We will assume that we know \( \varepsilon_0 \) and \( \varepsilon_e \), i.e.:

\[
\begin{align*}
\varepsilon_{x'x'} &= \varepsilon_{y'y'} = \varepsilon_0 \\
\varepsilon_{z'z'}(\neq \varepsilon_{x'x'}) &= \varepsilon_e
\end{align*}
\]
System (1) is transformed as follows:

\[
\begin{align*}
D'_{x'} &= \varepsilon_o E_{x'} \\
D'_{y'} &= \varepsilon_o E_{y'} \\
D'_{z'} &= \varepsilon_e E_{z'}
\end{align*}
\]  

(3)

In the laboratory coordinate system XYZ, the dielectric constant tensor will have the form:

\[
\hat{\varepsilon} = \begin{pmatrix}
\varepsilon_{xx} & 0 & \varepsilon_{xz} \\
0 & \varepsilon_{yy} & 0 \\
\varepsilon_{xz} & 0 & \varepsilon_{zz}
\end{pmatrix}
\]

(4)

For the values of the elements of the dielectric constant tensor, we obtain the following result:

\[
\begin{align*}
\varepsilon_{xx} &= \varepsilon_o \cos^2 \phi + \varepsilon_e \sin^2 \phi \\
\varepsilon_{yy} &= \varepsilon_o \\
\varepsilon_{zz} &= \varepsilon_e \cos^2 \phi + \varepsilon_o \sin^2 \phi \\
\varepsilon_{xz} &= (\varepsilon_e - \varepsilon_o) \cos \phi \sin \phi
\end{align*}
\]  

(5)

2. Calculation results

The results of calculating the angular spectra for different angles \( \phi \) between the optical axis \( (z' \text{ axis}) \) and the normal to the plate boundary \( (z \text{ axis}) \) are shown in figures 3-8. Figure 3 shows the angular spectrum of the ellipsometric parameter \( \rho_R \) for reflected radiation at different values of the angle \( \phi \).

2.1. Modulus of the ellipsometric parameter

Figure 3 shows the angular spectrum of the ellipsometric parameter \( \rho_R \) for reflected radiation at different values of the angle \( \phi \).
Figure 3. Angular spectra of the modulus of the ellipsometric parameter of the reflected wave \( \rho_R \) at different values of the angle \( \phi \): 1 °, 45 °, 60 °.

2.2. Angular spectrum of the ellipse parameter \( \Delta \)

Figure 4 shows the angular spectrum of the ellipsometric parameter \( \Delta R \) for reflected radiation at different values of the angle \( \phi \).

Figure 4. Angular spectra of the ellipsometric parameters \( \Delta R \) for reflected radiation at different values of the angle \( \phi \): 1 °, 45 °, 60 °.

2.3. Character of polarization changes

Let us consider how the character of polarization changes at different angles of incidence \( \theta \), as well as angles \( \phi \). Figure 5 is a view of the incident circular polarization curve. Further figures 6-8 show the change in the shape of the original circle when the above angles change.
2.4. Position of the minimum angle in the angular spectrum of the ellipsometry parameter module

Figure 5 shows the dependence of the position of the minimum angle in the angular spectrum of the ellipsometry parameter module on the refractive index of the layer under study. It follows from this figure that the method is very sensitive to a change in the refractive index and can be used for precision measurements of this parameter for a wide class of substances.

**Figure 5.** The shape of the curve, which is described by the end of the vector of the electric field strength of the incident radiation — left-hand circular polarization.

**Figure 6.** Reflected light. Incidence angle $\theta = 30^\circ$, $\phi = 1^\circ$: right-hand elliptical polarization.
3. Conclusion
The given dependences show a very high sensitivity of the ellipsometric parameters of the reflected light on the angle of incidence $\theta$ and the angle $\phi$ between the optical axis and the normal to the plate boundary. At the same time, the energy reflection spectra themselves show much less variability when these parameters change. It should be especially emphasized the nature of the change in the ellipsometric angle $\Delta$, which is responsible for the type of elliptical polarization - when $\Delta > 0$, the polarization is left-handed, and when $\Delta < 0$, it is right-handed. The incident radiation has left-hand circular polarization. Consequently, a thin anisotropic plate at certain angles can serve as a polarization converter of the incident radiation. The ellipsometry parameter $\rho$ characterizes the degree of compression of the ellipse - when $\rho = 1$, the ellipse is transformed into a circle, and the light is circularly polarized in this case. The smaller this parameter, the more compressed the ellipse will be - you can compare figures 5 and 8. Thus, a thin anisotropic plate can not only transform left-handed polarization into right-handed, but also with its help you can control the very shape of the polarization ellipse. Such a plate can be used in conjunction
with a layered medium, for example, a one-dimensional photonic crystal, to control the polarization of the incident circularly polarized light.

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