Investigation of the orientation of galaxies in clusters: the importance, methods and results of research

Paulina Pajowska, a Włodzimierz Godłowski, a Zong-Hong Zhu, b Joanna Popiela, a Elena Panko c and Piotr Flin d

aUniwersytet Opolski, Institute of Physics, ul. Oleska 48, 45-052 Opole, Poland
bDepartment of Astronomy, Beijing Normal University, Beijing 100875, China
cI.I. Mechnikov Odessa National University, Theoretical Physics and Astronomical Department, Park Shevchenko, Odessa, 65014, Ukraine
dJan Kochanowski University, Institute of Physics, ul. Swietokrzyska 15, 25-406 Kielce, Poland

E-mail: paoletta@interia.pl, godlowski@uni.opole.pl, zhuzh@bnu.edu.cn, joa_ols@wp.pl, panko.elena@gmail.com, sfflin@cyf-kr.edu.pl

Received August 10, 2018
Revised November 23, 2018
Accepted December 17, 2018
Published February 5, 2019

Abstract. Various models of structure formation can account for various aspects of the galaxy formation process on different scales, as well as for various observational features of structures. Thus, the investigation of galaxies orientation constitute a standard test of galaxies formation scenarios since observed variations in angular momentum represent fundamental constraints for any model of galaxy formation. We have improved the method of analysis of the alignment of galaxies in clusters. Now, the method allows to analyze both position angles of galaxy major axes and two angles describing the spatial orientation of galaxies. The distributions of analyzed angles were tested for isotropy by applying different statistical tests. For sample of analyzed clusters we have computed the mean values of analyzed statistics, checking whether they are the same as expected ones in the case of random distribution of analyzed angles. The detailed discussion of this method has been performed. We have shown how to proceed in many particular cases in order to improve the statistical reasoning when analyzing the distribution of the angles in the observational data. Separately, we have compared these new results with those obtained from numerical simulations. We show how powerful is our method on the example of galaxy orientation analysis in 247 Abell rich galaxy clusters. We
have found that the orientations of galaxies in analyzed clusters are not random. It means that we genuinely confirmed an existence of the alignment of galaxies in rich Abells’ galaxy clusters. This result is independent from the clusters of Bautz-Morgan types.

**Keywords:** galaxy clusters, galaxy formation

**ArXiv ePrint:** 1808.02573
1 Introduction

Solving the problem of the structures formation is one of the most significant issues of modern extragalactic astronomy. Many authors investigated the scenarios of structures formation since Peebles [1], Zeldovich [2]. New scenarios are mostly modifications and improvements of the older ones [3–16].

The final test of veracity in a given scenario is the convergence of its predictions with observations. One of the possibilities of such test is analysing the angular momenta of galaxies. Investigating the orientation of galaxy planes in space is of great importance since various scenarios of cosmic structures formation and evolution predict different distributions of galaxies angular momentum, [1, 8–10, 12, 15, 17–22], i.e. provide distinct predictions concerning the orientation of objects at different levels of structure — in particular clusters and superclusters of galaxies. Our model assumes that normals to the planes of galaxies are their rotational axes, which seems to be quite reasonable, at least for the spiral galaxies. Various models can account for various aspects of the galaxy formation process on different scales, as well as for various observational features of structures. This provides us with a method for testing scenarios of galaxy formation. In other words, the observed variations in angular momentum give us simple but fundamental test for different models of galaxy formation [23–25].

From the observational point of view it is not very difficult to investigate the distribution of the angular momenta for the luminous matter i.e. real galaxies and their structures. One should note however, that in real Universe, observed luminous matter of galaxies are surrounded by dark matter halos that are much more extended and massive. Direct observation of dark mater halos and theirs angular momentum is not so easy. Fortunately, there are the relation between luminous and dark mater (sub)structures. As a result we have a dependence between dark matter halos and luminous matter (real galaxies) orientation [13, 15, 16, 26–29]. Recently, the analysis of the Horizon-AGN simulation shows the similar dependence [30, 31]. It means that the analysis of angular momentum of luminous matter gives us also information about angular momentum of the total structure hence the analysis of the angular momentum of real (luminous) galaxies is still useful as a test of galaxy formation. The investigation of the galaxies orientation in clusters are also very important with regard to investigation of weak gravitational lensing For more detailed discussion of the
significance of this problem see Heavens et al. [32], Heymans et al. [33], Kiessling et al. [24], Stephanovich & Godlowski [34], Codis et al. [35].

Since the angular momenta of galaxies and also the directions of galaxy spin are usually unknown, the orientations of galaxies are investigated instead. In order to acquire this, either the distributions of galaxy position angles only [36] or the distributions of the angles giving the orientation of galaxy planes [37, 38] are examined. Many authors investigated the orientation of galaxies in different scales. The review of the observational results on the problem of galaxies orientation and structures formation was presented both theoretically [39] and observationally [25, 40].

One of the most meaningful aspects of the problem of the origin of galaxies involves the investigation of the orientation of galaxies in clusters. During the analysis of the angular momentum of a galaxy cluster, in principle we should take into account that total angular momentum of the cluster could come from both the angular momentum of each galaxy member and from the rotation of the cluster itself. However, one should note that there is no evidence for rotation of the groups and clusters of galaxies themselves. So, it is commonly agreed that such structures do not rotate (for example Regos & Geller [41], Diaferio & Geller [42], Diaferio [43], Rines et al. [44], Hwang & Lee [45], Tovmassian [46], see however Kalinkov et al. [47] for the opposite opinion). An especially important result is obtained by Hwang & Lee [45]. They have examined the dispersions and velocity gradients in 899 Abell clusters and have found a possible evidence for rotation in only six of them. This allowed us to conclude that any non-zero angular momentum in groups and clusters of galaxies should arise only from possible alignment of galaxy spins. Moreover, the stronger alignment means the larger angular momentum of such structures.

For many years, astrophysicists paid a lot of attention to the orientation of galaxies in clusters. It was investigated both theoretically (see for example [48, 49]) and observationally. Generally, summarizing the research results provided by various authors, it can be stated that we have no satisfactory evidence for the alignment of galaxies in groups and poor clusters of galaxies, while there is an ample evidence of this kind for rich clusters of galaxies [50] (see also Godlowski [40] for an improved analysis and Stephanovich & Godlowski [34] for review).

Thus, an interesting problem arises if there are any dependence on the alignment to the mass of the structure. Godlowski et al. [50] suggested that the alignment of galaxies in clusters should increase with the mass of the cluster. Thus, Godlowski et al. [50] hinted that the alignment should increase with the number of objects (richness) in a particular cluster, too. These suggestions were later confirmed by Aryal [51]. These authors analyzed a total of 32 clusters of different richness and BM types. They confirmed that the alignment is changing with the richness and moreover that they change with BM type of the clusters. However, one should note that both Godlowski et al. [50] and Aryal [51] investigations were qualitative only. The next step is to test this hypothesis also quantitatively.

This was the reason why Godlowski et al. [52] examined the orientation of galaxies in clusters both qualitatively and quantitatively. In this paper it was found that the alignment of galaxy orientation increased with numerousness of the cluster. However, the problem that we may obtain is whether we found a significant alignment in analyzed sample of 247 rich Abell clusters, or increasing alignment with cluster richness only. For this reason Godlowski [53] analyzed the distribution of position angles using $\chi^2$ test, Fourier test and autocorrelation test as well as Kolmogorov test, showing that it is not random.

In the present paper, following the ideas of Godlowski [53] and Panko et al. [54], we improved method allows us to analyze the distributions both of the position angles $p$ and
Figure 1. A schematic illustration of angles $\delta_D$ (the polar angle between the normal to the galaxy plane and the main plane of the coordinate system) and $\eta$ (the azimuth angle between the projection of this normal onto the main plane and the direction towards the zero initial meridian). $i$ is the inclination angle with respect to the observer’s line of sight, $P$ is the position angle in the reference system, $N_1$ and $N_2$ are possible positions of the normal to galaxy plane while $L$ and $B$ are the longitude and latitude of the reference coordinate system (for more details see Flin & Godlowski [37], Aryal et al. [147]).

distribution of two angles giving spatial orientation of galaxies. We denote $\delta_D$ angle (the polar angle between the normal to the galaxy plane and the main plane of the coordinate system) and the $\eta$ angle (the azimuth angle between the projection of this normal onto the main plane and the direction towards the zero initial meridian), see figure 1 for geometry of the angles. The main idea of our method is to analyze the distributions of these angles using statistical tests. We have analyzed in more details and improved the statistical tests used in [53] as well as introduce new statistical tests into the method.

We analyzed how the tests changes if expected values of galaxies in particular bins varies (as in the case of analysis of the $\delta_D$ angle). It slightly changes for autocorelation test but it is very important for Fourier test and Kolmogorov-Smirnov test. We also introduced to our improved method of investigation of galaxy alignment in clusters, the “control tests” that neglects a possible asymmetry of the distribution according to main coordinate plane. The idea of such tests is to analyze only the difference between more “parallel” or more “perpendicular” orientation according to the coordinate system main plane (or main direction towards the zero initial meridian in the case $\eta$ angle). We have checked how the Kolmogorov-Smirnov
test behaves in the investigation of the orientation of galaxies in clusters. We have introduced alternative tests, namely Crámer-von Mises and Watson that showed more explicitly that the allignment truly exists.

Usually the effect of allignment of galaxies in structures is not very strong and its analysis requires precise statistical considerations. In such a case it is very important to verify that no other observational systematics can affect. To avoid a problem with the possible impact on the obtained results by data systematics, we think it is necessary to test the method on a well-tested sample of galaxy clusters. We have decided to use a sample of the galaxy clusters selected on the basis of the PF catalog [55]. Hence, on the example of analysis of position angles in 247 rich Abell clusters we show how the method works in case of observational data. For our sample of 247 clusters, we computed the mean values of the analyzed statistics. Our null hypothesis $H_0$ is that the mean value of the analyzed statistics is as expected in the case of random distribution of analyzed angles. At first, we have compared the theoretical prediction with the results obtained from numerical simulations. Later, they are compared with the results obtained from the real sample of the 247 Abell clusters. Separately, we analyzed the sample when only galaxies brighter than $m_3 + 3^m$ were considered. Moreover, we decided to analyze if there are any differences in allignment of galaxies in the clusters belonging to different Bautz-Morgan (BM) types. In order to exclude the case that the obtained results comes from errors in observational measurements, we have used two separate methods. We have analyzed the sample assuming random errors in position angles and additionally we have used jackknife method especially to investigate the possible influence of background objects.

The novelty of our approach is to gather many methods of analysis of statistics of all angles $p$, $\delta_D$ and $\eta$ not only for some particular galaxy clusters but also for big samples of clusters. Unfortunately, such approach inevitably turns to the analysis on a case by case basis. That is why in each case, we point out possible difficulties and show which method has to be used. At first glance, most cases looks very similar, but one has to be careful not to omit the crucial differences. The advantage of the approach is that by analysing much more data at once, we are able to draw more general conclusions.

2 Observational data

In the present paper we have analyzed the sample of 247 rich Abell clusters containing at least 100 members galaxies each [52, 53]. The sample was selected on the basis of the PF catalogue [55]. The structures in the Panko & Flin [55] catalogue were extracted from the Muenster Red Sky Survey (MRSS hereafter) [56] using the 2D Voronoi tessellation technique [57], see Panko et al. [58] for details. Note that the confidence level for cluster search was 95% [57] and the list of clusters is reliable.

During analysis of the orientation of galaxies in structures the curcual point is to remove the non-galactic objects — mostly stars and artifacts. The advantage of MRSS list of galaxies is that the author of the catalogue very carefully analyzed the classification of all objects in the survey. Basic data for MRSS are 217 ESO Southern Sky Atlas R Schmidt plates covering an area of about 5000 square degrees in the southern hemisphere, with $b < -45^\circ$. All plates were digitized with the two PDS 2020GMPlus microdensitometers of the Astronomisches Institut Münster with a step width of 15 microns (1600 dpi), corresponding to 1.01 arcseconds per pixel. Objects search in digitized plates was made using the program SEARCH based on FOCAS algorithm [59], see also Ungruhe [60], MRSS [56] for detailed aplication to MRSS catalogue. The analysis of variations of background densities, in particular vignetting of the
telescope, and the influence of threshold of the background for the objects detection were made carefully. The influence of nearest objects was minimized due to final analysis inside the small frame including the object. Final galaxy search based on 6 parameters allows to select only galaxies. More than 2700000 uncertain objects were checked visually; they were faint objects mainly [56]. So, all selected objects are galaxies.

Resulting list of MRSS galaxies contains more than 5 millions ones till to $r_F = 21^m$ detected on the best plates. However, the limit of completeness of MRSS is $r_F = 18.3^m$ [56]. This short list contains 1200000 galaxies with reliable definitions parameters. The ellipticity and position angle for each galaxy were calculated using the covariance ellipse method [61]. The ellipticities and positional angles of galaxy images were calculated using both intensities and coordinates, so inside intensity distribution was accounted [56]. The problem of possible systematic effects was analyzed by Ungruhe [60], MRSS [56] while the detailed study of uncertainties on the position angles, including vary with galactocentric distance, was executed in Biernacka & Flin [62]. They confirmed the results of Nilson [63]. Following the results, we supposed the uncertainties on the position angles of galaxies on the level $2^\circ$ for galaxies elongated images of galaxies, in the worst case the uncertainties were on the level $5^\circ$. Obviously the uncertainties quickly increase for rounded images.

The PF Catalogue was created using only MRSS galaxies inside the completeness limit $r_F = 18.3^m$. The PF Catalogue defines a cluster as a structure which contains at least ten galaxies in the magnitude range between $m_3$ and $m_3 + 3^m$, where $m_3$ is the magnitude of the third brightest galaxy located in the considered structure region. The criterion of $m_3 + 3^m$ is a well known criterion to galaxy membership for the cluster if, as in the case MRSS [56] we have no information about radial velocities of particular galaxies. Panko et al. [58] checked the correctness of this limit using statistical completed sample contained 547 PF structures.

The full PF [55] catalogue includes 6188 galaxy clusters and groups and contains positions of the clusters, their radii, areas, the number of all galaxies in the field of structure, number of galaxies within the magnitude range $m_3$ and $m_3 + 3^m$, as well as an estimated number of background galaxies, ellipticity and position angles for each structure, magnitudes of the first, the third and the tenth galaxy in a structure (taken from the MRSS). The full PF catalogue contains not only the list of the clusters but also the lists of galaxies belonging to each structures, where the data for each galaxy member were taken from the MRSS. This data includes: the equatorial coordinates of galaxies ($\alpha$, $\delta$), the diameters of major and minor axes of the galaxy image ($a$ and $b$ respectively) and the position angle of the major axis $p$ (see also Godlowski [53]). Because the position angles in MRSS serve in clockwise system, we recomputed original position angles from MRSS clockwise system to standard counterclockwise system. We performed our computation in Equatorial and Supergalactic Coordinate System defined in Flin & Godlowski [37]. In the case of Supergalactic Coordinate System position angles $p$ were recomputed to supergalactic position angles $P$. The photometrical redshifts were calculated for each cluster using the relation $z(m_{10})$ [Biernacka et al. [64]) and rich PF clusters have redshifts $z < 0.12$ while median $z=0.08$. The positions of PF and APM [65] galaxy clusters are in good agreement [64].

In the present paper, as in Godlowski et al. [52] and Godlowski [53] we have analyzed the sample of rich clusters that have at least 100 members and belong to ACO clusters [66]. The advantage of such sample is that they have the Bautz-Morgan morphological types (BM types). There are 239 such objects in the PF catalogue. Moreover, 9 objects can be identified with two ACO clusters. We decide to include them in our investigation and increased our sample to 248 objects. We excluded the structure A3822, which potentially has
Therefore, finally our analyzed sample contains 247 clusters. Because all analyzed clusters have ACO identification, the distances can be found from literatures or extrapolated from 10th brightest galaxy in clusters [55, 64]. The numbers of clusters with particular BM types are from 35 (BM I) till 59 (BM II).

In our investigation we have decided to analyze two subsamples of data. The first one contains all galaxies lying in the region regarded as cluster. In the second one, for avoiding possible role of background object, only galaxies brighter than \( m_3 + 3 \) were taken into account.

3 The method of investigation

The analysis of the orientation of galaxies has usually been examined by two main methods. In the first one [36] the distribution of the position angles of the major axis of galaxies is performed. During the analysis of position angles, we exclude from examination all galaxies with axial ratio \( q = b/a > 0.75 \), because for the face-on galaxies position angles give only marginal information connected with the orientation of galaxy. In the second method based on the de-projection of the galaxy images, we have analyzed spatial orientation of galaxies. This idea was introduced by Oepik [69], applied by Jaaniste & Saar [38] and significantly modified by Flin & Godłowski [37], Godłowski [70, 71]. In this method, we take into account both galactic position angles \( p \) and another important parameter — the galaxy inclination with respect to the observers’ line of sight \( i \). Using these two angles we have determined two possible orientations of the galaxy plane in space, which gave two possible directions perpendicular to the galaxy plane. As was discussed in the introduction, it is expected that one of these normal corresponds to the direction of galactic rotation axis. One should note however, that de-projection of galaxy images on the celestial sphere gives four solutions for the angular momentum vector. Usually we consider only two distinguishable solutions since we do not know the direction of galaxy rotation.

The inclination angle has been computed from the galaxy image according to the formula:
\[
\cos^2 i = \frac{(q^2 - q_0^2)}{(1 - q_0^2)},
\]
where the observed axial ratio \( q = b/a \) and \( q_0 \) is “true” axial ratio. This formula is valid for oblate spheroids [72]. The value \( q_0 = 0.2 \) is used in the case when we have no information about morphological types of galaxies (as in MRSS catalog). For each galaxy we determined two angles \( \delta_D \) and \( \eta \). Following Godłowski [53] we performed our computation both in Equatorial and Supergalactic coordinate systems (Flin & Godłowski [37] based on Sandage & Tammann [73]). The relations between angles \( (\delta_D, \eta) \) and \( (i, P) \) in the Supergalactic coordinate system \((L, B)\) (figure 1) are the following ones (similar formulae may be obtained for Equatorial coordinate system)

\[
\sin \delta_D = - \cos i \sin B \pm \sin i \cos r \cos B, \tag{3.1}
\]
\[
\sin \eta = (\cos \delta_D)^{-1} [- \cos i \cos B \sin L + \sin i (\mp \cos r \sin B \sin L \pm \sin r \cos L)], \tag{3.2}
\]
\[
\cos \eta = (\cos \delta_D)^{-1} [- \cos i \cos B \cos L + \sin i (\mp \cos r \sin B \cos L \mp \sin r \sin L)], \tag{3.3}
\]

where \( r = P - \pi/2 \). As a result of the reduction of our analysis into two solutions only, it is necessary to consider the sign of the expression:
\[
S = - \cos i \cos B \mp \sin i \cos r \sin B
\]
and for \( S \geq 0 \) we should reverse the sign of \( \delta_D \) respectively (see Godłowski et al. [52]). Please note the usually the researchers use the simplified version of the method, taking into account only equations (3.1) and (3.2), which however caused serious problems in the interpretation of the results of the analysis of the spatial orientation of galaxies.
The essential progress of the investigation of galaxy alignment was made by Hawley & Peebles [36]. Their method of investigation of galaxies orientation is based on statistical analysis of the distribution of galaxies position angles. The essence of the method was to use three type of statistical tests: \(\chi^2\)-test, Fourier and First Autocorrelation. It was shown later that this methodology can also be used to study the spatial orientation of galaxies planes [37, 52, 54, 74–76].

The main idea of the paper is to show how to make the statistical methods more reliable and to interpret the obtained results. We show how the methods work in particular cases and apply it to the analysis of the distribution of position angles of observational sample of 247 rich Abell clusters. In particular we determine if the orientations of galaxies in clusters are isotropic or not.

The essence of the method Godlowski [53] is to compute the mean values of analyzed statistics for the whole sample of analyzed cluster and compare them with that obtained from numerical simulations. Our null hypothesis \(H_0\) is that the mean value of the analyzed statistics is as expected in the case of random distribution of analyzed angles. In all tests, the range of the \(\theta\) angle (where for \(\theta\) one can put \(\delta_D + \pi/2\), \(\eta\), \(p\) (or \(P\)) respectively) is divided into \(n\) bins of equal width. We have used \(n = 36\) bins. As a check, we repeated the division for other values of \(n\), but generally we have not found any significant difference. There is one exception, namely is Kolmogorov-Smirnov test and we discuss this in detail in the section “Numerical Simulations and Results”.

In the whole paper we denote as \(N\) the total number of galaxies in analyzed clusters while \(N_k\) is the number of galaxies with orientations within the \(k\)-th angular bin and \(N_{0,k}\) is the expected number of galaxies in the \(k\)-th bin. In the case of the analysis of the position angles \(p\) or \(P\) and \(\eta\) angles all \(N_{0,k}\) are equal to \(N_0\), which is also the mean number of galaxies per bin. In the case of the analysis of the angles \(\delta_D\) of course \(N_{0,k}\) are not equal and are obtained from the cosine distribution. The case when not all \(N_{0,k}\) are equal was not analyzed in the paper of Godlowski [53], so adding such analysis significantly improves the method of analyzing the alignment of galaxies in clusters. This improvement means that the method is now valid also in the case when some (or even all) \(n\) bins have not equal width.

The first group of tests is based on the \(\chi^2\) test:

\[
\chi^2 = \sum_{k=1}^{n} \frac{(N_k - N \cdot p_k)^2}{N_p k} = \sum_{k=1}^{n} \frac{(N_k - N_{0,k})^2}{N_{0,k}},
\]

(3.4)

where \(p_k\) is the probability that a chosen galaxy falls into \(k\)-th bin. Because we have \(n\) bins, the number of degrees of freedom of the \(\chi^2\) test is \((n - 1)\). This causes that the mean value \(E(\chi^2) = n - 1\) and the variance \(\sigma^2(\chi^2) = 2(n - 1)\). For \(n = 36\) this leads to the values \(E(\chi^2) = 35\) and \(\sigma^2(\chi^2) = 70\). When we analyzed the sample of \(m\) clusters and computed the mean value of statistic, then \(E(\chi^2) = n - 1\), but \(\sigma^2(\chi^2)\) decreased by the factor \(m\) and equaled \(\frac{\sigma^2(\chi^2)}{m}\). For \(n = 36\) and \(m = 247\) this gives \(\sigma^2(\chi^2) = 0.2834\) and \(\sigma(\chi^2) = 0.5324\).

In the basic investigation the range of \(\delta_D\) angle is from \(-\pi/2\) to \(\pi/2\). The idea of the control test is to restrict our analysis only to the case of the absolute value of \(\delta_D\) angle [37, 51, 77–79]. Then, we neglected a possible asymmetry of the distribution according to main coordinate plane and analyzed only the differences between more “parallel” or more “perpendicular” orientation according to the coordinate system main plane. So the range of \(\delta_D\) angle is from 0 to \(\pi/2\) and we divided the entire range of a \(\theta\) angle into 18 instead of 36 bins. During the analysis of 247 clusters the mean value of statistic is of course \(E(\chi^2) = 17\).
while \( \sigma^2(\chi_2^2) = 0.1377 \) and \( \sigma(\chi_2^2) = 0.3710 \). Analogically, during analysis of the position and azimuthal \( p \) and \( \eta \) angles, we also reduced ranges of analyzed angles from 0 to \( \pi/2 \), so we divided the entire range of a \( \theta \) angle into 18 instead of 36 bins. Of course in the case of \( \eta \) angle, the idea of the control test is to analyze the asymmetry between more “parallel” or more “perpendicular” projection to the normal to the galaxy plane according to the main direction towards the zero initial meridian of the coordinate system.

The second group of our tests is based on the first auto-correlation test \cite{36}. Probably the best test for autocorrelation is the von-Neumann-Durbin-Watson test. However, in our paper we do not analyse full autocorrelation. Our idea is, as noted above, to obtain the average value for analysed statistic (and later to check if the mean value of the analysed statistics is as expected in the case of random distribution of analysed angles). Although, the Hawley & Peebles \cite{36} first autocorrelation test may not work as well as the von-Neumann-Durbin-Watson test, the idea presented there is also widely used (see for example Percival & Walden \cite{80}- especially chapter 6 of the book). So, it seems that the use of this test works well enough. However, we analyzed its properties in more detail than Hawley & Peebles \cite{36} before we used it.

The first auto-correlation test quantifies the correlations between galaxy numbers in neighboring angle bins. This correlation is measured by the statistic \( C \):

\[
C = \sum_{k=1}^{n} \frac{(N_k - N_{0,k})(N_{k+1} - N_{0,k+1})}{[N_{0,k}N_{0,k+1}]^{1/2}}
\]

(3.5)

where \( N_{n+1} = N_1 \). According to the original paper Hawley & Peebles \cite{36}, in the case of an isotropic distribution, the expected value of \( C \) is \( E(C) = 0 \) with the standard deviation \( \sigma(C) = n^{1/2} \).

In the paper Godlowski \cite{53}, it was shown that original Hawley & Peebles \cite{36} result was an approximation only that is not correct in our case, since they assumed that \( N_k \) are independent from each other. Therefore, in the formula for \( E(C) \) is present an additional term connected with the covariance between \( N_k \) and \( N_{k+1} \):

\[
E(C) = -\sum_{k=1}^{n} \frac{N_k p_k p_{k+1}}{\sqrt{N_{0,k}N_{0,k+1}}}
\]

(3.6)

When all \( p_k \) and hence \( N_{k,0} \) are equal (\( p_k = 1/n \)), as it was in the case of position angles, then \( E(C) = -1 \). Moreover, \( \sigma^2(C) \) contains a term which is the variance of the products of \( N_k \) and \( N_{k+1} \) which are not independent. As a result, the correct value of \( D^2(C) \) is only approximately equal to \( n \) and the correct value must be computed using numerical simulations. Moreover, for one cluster the difference between the results in expected values of \( C \) (0 or -1) is relatively small compared to \( \sigma(C) \approx \sqrt{n} \). However, when we analyse the sample of \( m \) clusters, the situation is different because the variance is decreased by the factor \( m \). As a result, in the case of sample 247 clusters, \( \sigma(C) \approx \sqrt{n/247} \approx 0.3818 \) is significantly smaller than 1 (which is the difference between expected values) \cite{53}.

In the case of analysis of the \( \delta_D \) angles the situation is more complicated because of course \( N_{0,k} \) and as results all \( p_k \) are not equal and they are obtained from cosine distribution. As a result in our case \( n = 36 \), \( E(C) = -\sum_{k=1}^{n} N_k p_k p_{k+1} = -0.9973 \).

Similarly as in the case of \( \chi^2 \) test we introduced the control first auto-correlation test. Also in this case, we restricted our analysis only to the case of the absolute value of \( \delta_D \) angle, so the range of \( \delta_D \) angle is from 0 to \( \pi/2 \). We divided entire range of a \( \theta \) angle into 18 instead
of 36 bins. $E(C_c) = -\sum_{k=1}^{n} N \ p_k p_{k+1} = -0.9701$. As in the case of the basic test we could approximate standard deviation of $C_c$, even if correct value must be obtained from numerical simulations. For the case of the position angles and 247 clusters $\sigma(C_c) \approx \sqrt{n/247} = 0.26995$ and is again significantly smaller than the difference in expected values (which is again equal to 1). Analogically as in the case of control $\chi^2$ test, during analysis the $p$ and $\eta$ angles, we also reduced ranges of analyzed angles from 0 to $\pi/2$, so we divided entire range of a $\theta$ angle into 18 instead of 36 bins.

The most popular test used for analysis of galaxy alignment is the Fourier Test [36] and its modifications, even if doubts are sometimes raised (we will discuss them separately below), as to the adequacy and the scope of applicability of this type of tests. The idea of this test is, that if the deviation from isotropy is a slow varying function of the angle $\theta$ then the expected number of galaxies with orientations within the $k$-th angular bin $N_k$ is in the most general form given by formulae Godłowski [71]:

$$N_k = N_{0,k}(1 + \Delta_{11} \cos 2\theta_k + \Delta_{21} \sin 2\theta_k + \Delta_{12} \cos 4\theta_k + \Delta_{22} \sin 4\theta_k + \ldots).$$

In Fourier test the crucial is the amplitude $\Delta = (\sum_i \sum_j \Delta_{ij})^{1/2}$ and probability that the amplitude $\Delta$ is greater than a fixed value. Using maximum-likelihood method we obtain the expressions for the $\Delta_{ij}$ coefficients. Usually only first or maximum two first modes are used in the investigation.

For that, equation (3.7) could be rewritten in the form:

$$\frac{N_k - N_{0,k}}{N_{0,k}} = \Delta_{11} \cos 2\theta_k + \Delta_{21} \sin 2\theta_k + \Delta_{12} \cos 4\theta_k + \Delta_{22} \sin 4\theta_k.$$  

If we define $I$ vector as:

$$I = \begin{pmatrix} \Delta_{11} \\ \Delta_{21} \\ \Delta_{12} \\ \Delta_{22} \end{pmatrix}$$

then the solution for $x \equiv I$ is given by Brandt [81] equation (9.2.26):

$$x = -(A^T G_y A)^{-1} A^T G_y c$$

where: $c$ is the vector of particular $y_i = \frac{N_k - N_{0,k}}{N_{0,k}}$:

$$c = \begin{pmatrix} N_{1} - N_{0,1} \\ N_{2} - N_{0,2} \\ \vdots \\ N_{n} - N_{0,n} \end{pmatrix}$$

$G_y$ is the inverse matrix to the covariance matrix of particular $y_i$ i.e weight matrix:

$$G_y = \begin{pmatrix} g_1 & \ldots & \cdot \\ \cdot & g_2 & \ldots \\ \cdot & \cdot & \ldots \\ \cdot & \cdot & \cdot & g_n \end{pmatrix} = \begin{pmatrix} N_{0,1} & \ldots & \cdot \\ \cdot & N_{0,2} & \ldots \\ \cdot & \cdot & \ldots \\ \cdot & \cdot & \cdot & N_{0,n} \end{pmatrix}$$

- 9 -
and matrix $\mathbf{A}$ of coefficients with particular $\Delta_{ij}$ has form:

$$
\mathbf{A} = - \begin{pmatrix}
\cos 2\theta_1 \sin 2\theta_1 \cos 4\theta_1 \sin 4\theta_1 \\
\cos 2\theta_2 \sin 2\theta_2 \cos 4\theta_2 \sin 4\theta_2 \\
\cos 2\theta_n \sin 2\theta_n \cos 4\theta_n \sin 4\theta_n \\
\end{pmatrix}
$$

(3.13)

while matrix $G(\mathbf{x}) = \mathbf{A}^T G_y \mathbf{A}$ (see [81] equation (9.2.27)) where covariance matrix $C_{\text{cov}}(\mathbf{x}) = G(\mathbf{x})^{-1}$. For detailed form of solutions for $\Delta_{ij}$ coefficients and their covariance matrix as well as the formulae for probability that the amplitude $\Delta$ is greater than a fixed value in the particular cases of the analysis see appendix.

During analysis of alignment of galaxies, it is important not only the power of the deviation from isotropy, but also its direction. The sign of the coefficient $\Delta_{11}$ gives us the information about direction of such departure from isotropy. If $\Delta_{11} < 0$, then the excess of the galaxies with $\theta$ angle near $90^\circ$ is observed, while $\Delta_{11} > 0$ means the excess for $\theta$ angle near $0^\circ$ (for detailed discussion see appendix).

In the paper Godłowski [53] it was discussed the properties of statistics $\Delta_{ij}/\sigma(\Delta_{ij})$, $\Delta_1/\sigma(\Delta_1)$ and the $\Delta/\sigma(\Delta)$, in the case of the distributions of the position angles. It was showed, that equation (A.22) (Hawley & Peebles [36] equation 26) is obtained as a result of the theorem of propagation of errors what is not good approximation because the theorem of propagation errors is obtained in the linear model whereas $\Delta_j = \left(\Delta_{ij}^2 + \Delta_{2j}^2\right)^{1/2}$ is strictly nonlinear (see equations (A.27) and (A.20)). Hence, the notation $\frac{\Delta_j^2}{\sigma^2(\Delta_j)}$ means only that elements of $\Delta^2$ should be divided by elements of covariance matrix $\Delta_{ij}$. Consequently, the notation $\Delta_j/\sigma(\Delta_j)$ does not mean that coefficient $\Delta_j$ is divided by its error. Such an interpretation is only a rough approximation based on linear model [53]. As a results, the correct values are the following: $E(\Delta_{ij}^2/\sigma^2(\Delta_1)) = 2$, $E(\Delta^2/\sigma^2(\Delta)) = 4$, $D(\Delta_{ij}^2/\sigma^2(\Delta_1)) = 4$ and $D(\Delta^2/\sigma^2(\Delta)) = 8$. Moreover, $D^2(\Delta_1/\sigma(\Delta_1)) = 1/2$, and $D^2(\Delta/\sigma(\Delta)) = 1/2$. The last results are obtained again from theorem of propagation errors, however in the paper Godłowski [53] it was showed that in presently analyzed case it works quite well, even if the correct values must be obtained from numerical simulations.

Because $E(X) = \sqrt{E(X^2) - D^2(X)}$, we get

$$
E \left( \frac{\Delta_1}{\sigma(\Delta_1)} \right) = \sqrt{E \left( \frac{\Delta_1^2}{\sigma^2(\Delta_1)} \right) - D^2 \left( \frac{\Delta_1}{\sigma(\Delta_1)} \right)} = \sqrt{2 - 0.5} = 1.2247
$$

(3.14)

and

$$
E \left( \frac{\Delta}{\sigma(\Delta)} \right) = \sqrt{E \left( \frac{\Delta^2}{\sigma^2(\Delta)} \right) - D^2 \left( \frac{\Delta}{\sigma(\Delta)} \right)} = \sqrt{4 - 0.5} = 1.8708
$$

(3.15)

One should note that our sample contain 247 clusters. So $\sigma^2(\Delta_1/\sigma(\Delta_1))$ and $\sigma^2(\Delta/\sigma(\Delta))$ are equal $\frac{1/2}{247} = 0.000204$ while standard deviations of $\sigma(\Delta_1/\sigma(\Delta_1))$ and $\sigma(\Delta/\sigma(\Delta))$ are equal $\sqrt{\frac{1/2}{247}} = 0.04499$.

The case of the analysis of the distribution of the $\eta$ angles is similar to the case of position angles. One should note however that the analysis of the distribution of the $\delta_D$ angles is more complicated. At first, it is because not all $N_{0,k}$ are equal. It is the reason that now $\sigma^2(\Delta_{11})$ and $\sigma^2(\Delta_{21})$ are not exactly but only approximately equal $N/2$ (see equations (A.17)
(A.18)) and what is crucial, when we analyze both $2\theta$ and $4\theta$ Fourier modes together, not all $\Delta_{ij}$ coefficients are independent of each other. Also the case of the control test for Fourier test is more complicated than in the case of $\chi^2$ and autocorrelation test.

The simplest situation is the statistics of $\Delta_{ij}/\sigma(\Delta_{ij})$. There are the cases of one dimensional (1D) Gaussian distribution. In these cases the situation is not changing with comparison of Godkowski [53] and it is very clear. Variables $\Delta_{11}/\sigma(\Delta_{11})$ and $\Delta_{21}/\sigma(\Delta_{21})$ are still normalized gaussian variables with expected value equal 0 and variance equal 1. Of course $\sigma^2(\Delta_{ij}/\sigma(\Delta_{ij})) = 1/247 = 0.00405$ and $\sigma(\Delta_{ij}/\sigma(\Delta_{ij})) = 0.06363$.

Unfortunately, when we consider $\Delta_{ij}$ is not the diagonal matrix (i.e. not all $\Delta_{ij}$ are independent of each other). In such a case, even if we take into account only first Fourier mode i.e coefficients $\Delta_{11}$ and $\Delta_{21}$ which are not independent to each other, then (see equation (A.9)) the auxiliary value $J$ has the form:

$$J = A\Delta_{11}^2 + 2Y\Delta_{11}\Delta_{21} + B\Delta_{21}^2 = \frac{\Delta_{11}^2}{1/A} + \frac{\Delta_{11}\Delta_{21}}{1/2Y} + \frac{\Delta_{21}^2}{1/B}$$  (3.16)

In the present case $\Delta_{11}$ and $\Delta_{21}$ are not independent but it could be very easy transformed to the form where they are independent [82]. In the present case the transformation: $\Delta'_{11} = \frac{\Delta_{11}}{\sigma(\Delta_{11})\sqrt{1-\rho^2}} - \frac{\rho\Delta_{21}}{\sigma(\Delta_{21})\sqrt{1-\rho^2}}$ and $\Delta'_{21} = \frac{\Delta_{21}}{\sigma(\Delta_{21})}$ (where $\sigma(\Delta_{11})$, $\sigma(\Delta_{21})$ and $\text{cov}(\Delta_{11}, \Delta_{21})$ are given by equation (A.10) and as a result the correlation ratio $\rho = \text{cov}(\Delta_{11}, \Delta_{21})/\sigma(\Delta_{11})\sigma(\Delta_{21}) = \frac{\sqrt{Y}}{\sqrt{AB}}$ gives the variables $\Delta'_{11}$ and $\Delta'_{21}$ independent to each other. It leads to situation when $(\Delta'_{11}, \Delta'_{21})$ has the standard bivariate normal distribution and consequently

$$J' = \frac{\Delta_{11}^2}{1} + \frac{\Delta_{21}^2}{1},  $$  (3.17)

where of course $J' \equiv J$. Because $J'$ is the sum of $\Delta_{11}^2/\sigma^2(\Delta_{ij})$ where $\Delta_{ij}^2$ are independent to each other, then it is $\chi^2$ distributed. As a result the value $J'$ (i.e. $\Delta_{ij}^2/\sigma^2(\Delta_{ij})$) in our “old” notation, where $\Delta'_{ij} = (\Delta_{ij}^2 + \Delta_{ij}^2)^{1/2}$ given by formulae:

$$J' = \frac{\Delta_{11}^2}{\sigma^2(\Delta_{11})} + \frac{\Delta_{21}^2}{\sigma^2(\Delta_{21})}$$  (3.18)

has $\chi^2$ distribution with 2 degrees of freedom. So again $E(\Delta_{ij}^2/\sigma^2(\Delta_{ij})) = 2$, $D^2(\Delta_{ij}^2/\sigma^2(\Delta_{ij})) = 4$ and $\sigma^2(\Delta_{ij}^2/\sigma(\Delta_{ij})) = 1/2$. Consequently, the result obtained in equation (3.14) is still valid and $E\left(\frac{\Delta_{ij}}{\sigma(\Delta_{ij})}\right) = 1.2247$, where using our “new” notation, $\frac{\Delta_{ij}}{\sigma(\Delta_{ij})}$ should be noted as $\sqrt{J'}$, which means $E(\sqrt{J'}) = 1.2247$. Because $J' \equiv J$ then above results are valid for original $J = \sum_i\sum_j I_i^T G_{ij} I_j$ and $\sqrt{J}$ so in analyzed case $E(\sqrt{J}) = 1.2247$. Of course the above approximation (for the sample 247 clusters) $\sigma^2(\Delta'/\sigma(\Delta'))$, written now as $\sigma^2(\sqrt{J}) \approx 1/247 = 0.002024$ and $\sigma(\Delta'/\sigma(\Delta'))$, now $\sigma(\sqrt{J}) \approx 1/\sqrt{247} = 0.04499$ are still valid.

If we take into account the $2\theta$ and $4\theta$ Fourier modes together, the situation are complicating further. When not all $N_{0,k}$ are equal as it is in the case of the analysis of the distribution of the $\delta_D$ angles, then even in the situation when theoretical distribution of $N_{0,k}$ are symmetric with respect to value $\delta_D = 0$ (i.e $N_{0,k} = N_{0,n-k}$) not all $\Delta_{ij}$ coefficients are independent (see equation (A.15)). Now, $J$ is given by equation (A.5) and $J = \sum_i\sum_j I_i^T G_{ij} I_j$.
has the form:

\[
J = A\Delta_{11}^2 + B\Delta_{21}^2 + C\Delta_{12}^2 + D\Delta_{22}^2 + 2U\Delta_{11}\Delta_{12} + 2W\Delta_{21}\Delta_{22}
\] (3.19)

Amplitude \(\Delta\) (see equation (A.6)) is described by 4D Gaussian distribution. Fortunately, also in this case we could transform the vector of variables \(\Delta_{ij}\) (i.e I vector described by formulae (A.5)) to the form in which variables \(\Delta'_{ij}\) give a vector of independent random variables each with standard normal distribution. Let \(I'\) denote vector constructed from the \(\Delta'_{ij}\)

\[
I' = \begin{pmatrix}
\Delta'_{11} \\
\Delta'_{21} \\
\Delta'_{12} \\
\Delta'_{22}
\end{pmatrix}
\] (3.20)

the transformation between \(I'\) and \(I\) has a form [82]:

\[
I' = L^{-1}(I - \mu)
\] (3.21)

where lower triangular matrix \(L\) is obtained from Choleski decomposition of the covariance matrix \(C = LL^T\) and \(\mu\) is a vector expected value of \(\Delta_{ij}\). In our case the covariance matrix \(C\) is given by equation (A.15) while \(\mu = 0\) because all expected values of \(\Delta_{ij}\) are equal 0. It means that equation (3.21) has in fact simple form \(I' = L^{-1}I\).

The above result means that again \(J'\) is the sum of standard normalized independent variables over theirs errors

\[
J' = \frac{\Delta'_{11}^2}{\sigma^2(\Delta_{11})} + \frac{\Delta'_{21}^2}{\sigma^2(\Delta_{21})} + \frac{\Delta'_{12}^2}{\sigma^2(\Delta_{12})} + \frac{\Delta'_{22}^2}{\sigma^2(\Delta_{22})}
\] (3.22)

(where now all \(\sigma(\Delta'_{ij}) = 1\) and has \(\chi^2\) distribution with 4 degrees of freedom. As a result \(E(\Delta'_{11}^2/\sigma^2(\Delta')) = 4, D^2(\Delta'_{21}^2/\sigma^2(\Delta')) = 8\) and \(\sigma^2(\Delta'_{12}/\sigma(\Delta')) = 1/2\). Consequently, the result obtained in equation (3.15) is still valid and \(E\left(\frac{\Delta'_{11}}{\sigma(\Delta_{11})}\right) = 1.8708\), where using our “new” notation, \(\frac{\Delta'_{11}}{\sigma(\Delta_{11})}\) should be noted as \(\sqrt{J'}\), hence \(E(\sqrt{J'}) = 1.8708\). Because \(J' \equiv J\) it leads to the conclusion that above results are valid for original \(J = \sum_i\sum_j I'^T G_{ij} I_j\) and \(\sqrt{J}\) so in the present case \(E(\sqrt{J}) = 1.8708\). Analogically as in the case of approximation \(\sigma^2(\Delta_{11}/\sigma(\Delta_{11}))\) the approximation for \(\sigma^2(\Delta_{21}/\sigma(\Delta))\) \(\approx 0.002024\) and \(\sigma(\Delta_{12}/\sigma(\Delta))\) \(\approx 0.04499\) are also still valid. Of course all above results will be valid also when the theoretical distribution is not symmetric according to the value of \(\delta_D = 0\) presented by formulae (A.4)–(A.5).

Similarly, as in the case of \(\chi^2\) and auto-correlation tests we introduce a control Fourier test. The Fourier test requires the range of the \(\theta\) angle \((0; \pi)\). Because the original idea of control test is to restrict our analysis only to the case of the absolute value of \(\delta_D\) angle then it is natural to do it in the following way. In the control test, the bins are equidistant located oppositely to the zero value of \(\delta_D\) angle. So \(N'_{k'} = (N_k + N_{37-k})/2\) (for \(k' = k \leq .36\)). Analogically, we repeat this procedure during analysis of the position \(p\) and azimuthal \(\eta\) angles. One should note, that in the case of the Fourier test the number of bins in the basic and the control test is the same and it is equal to \(n = 36\).

However, if we restrict our analysis only to the case of the absolute value of \(\delta_D\) angle, it is clear that we are able to neglect \(\Delta_{21}\) and \(\Delta_{22}\) coefficients, because they are equal to zero (see also [37, 51, 52, 77–79]). In that case \(\Delta_1\) is reduced to \(|\Delta_{11}|\), while \(\Delta\), now denoted as
\( \Delta_c \) is the function of coefficients \( \Delta_{11} \) and \( \Delta_{12} \) only [52]. The above-mentioned observation is still correct in the case of the analysis of the position \( p \) and azimuthal \( \eta \) angles.

Now, we compute the expected value of \( \left| \Delta_{11}/\sigma(\Delta_{11}) \right| \). Because the auxiliary variable \( z = \frac{\Delta_{11}}{\sigma(\Delta_{11})} \) has the standard normal distribution then the expected value of \( |z| = |\frac{\Delta_{11}}{\sigma(\Delta_{11})}| \) can be obtained from the following formulae:

\[
E(|z|) = \int_{-\infty}^{+\infty} |z| f(|z|) dz = \frac{2}{\sqrt{2\pi}} \int_{0}^{+\infty} z \exp\left(-\frac{z^2}{2}\right) dz = \sqrt{2/\pi} \approx 0.7973.
\]

(3.23)

Now, the variance of \( |z| \) is given by formulae: \( D^2(|z|) = E(|z|^2) - (E(|z|))^2 \). Because \( z \) has the standard normal distribution (i.e. \( E(z) = 0; D^2(z) = 1 \)) then \( E(z^2) = D^2(z) + (E(z))^2 = 1 \) what means that

\[
E(z^2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} z^2 \exp\left(-\frac{z^2}{2}\right) dz = 1.
\]

(3.24)

From the above formulae it is easy to see that the expected value of \( |z|^2 \) must be equal to the expected value of \( z^2 \) i.e. \( E(|z|^2) = E(z^2) = 1 \). So the variance of \( |z| = |\Delta_{11}/\sigma(\Delta_{11})| \) is equal

\[
D^2(|\Delta_{11}/\sigma(\Delta_{11})|) = 1 - (E(|\Delta_{11}/\sigma(\Delta_{11})|))^2 = 1 - 0.635742 = 0.364258
\]

(3.25)

So, the error of \( \sigma(|\Delta_{11}/\sigma(\Delta_{11})|) \) is 0.6035. Of course because our sample has 247 clusters then \( \sigma^2(|\Delta_{11}/\sigma(\Delta_{11})|) = \sigma^2(|\Delta_{11}/\sigma(\Delta_{11})|)/247 = 0.001474 \) and \( \sigma(|\Delta_{11}/\sigma(\Delta_{11})|) = 0.0384 \).

Fortunately, the case of \( \Delta_c \) is much easier to solve. It is because the coefficients \( \Delta_{21} \) and \( \Delta_{22} \) are equal to 0 so the equation (3.19) is reduced to the form

\[
J = A\Delta_{11}^2 + C\Delta_{12}^2 + 2U\Delta_{11}\Delta_{12}
\]

(3.26)

It is very easy to see that this equation is analogous to equation (3.16) with only differences that \( \Delta_{21} \) is substituted by \( \Delta_{12} \) and consequently (see equations (A.1) and (A.2)) in equation (3.26) instead of coefficients \( Y \) we have \( U \) and instead of \( B \) we have \( C \). It means that the reasoning carried for the cases when we take into account only first Fourier mode with coefficients \( \Delta_{11} \) and \( \Delta_{21} \) which are not independent to each other, (see equations from (3.16) to (3.18)) are still valid. As a result we obtain that

\[
J' = \Delta_{11}^2/\sigma^2(\Delta_{11}) + \Delta_{12}^2/\sigma^2(\Delta_{12})
\]

(3.27)

has \( \chi^2 \) distribution with 2 degrees of freedom and \( E(\Delta_{11}^2/\sigma^2(\Delta_{11})) = 2 \), \( D^2(\Delta_{12}^2/\sigma^2(\Delta_{12})) = 4 \) while \( \sigma^2(\Delta_{11}/\sigma(\Delta_{11})) = 1/2 \). Consequently \( E \left( \frac{\Delta_{11}}{\sigma(\Delta_{11})} \right) = 1.2247 \), (using our “new” notation, \( E(\sqrt{J}) = E(\sqrt{\lambda}) = 1.2247 \)) and (for the sample 247 clusters) again the approximation \( \sigma^2(\Delta_{11}/\sigma(\Delta_{11})) \), now \( \sigma^2(\sqrt{J}) \approx 0.002024 \) and \( \sigma(\Delta_{c}/\sigma(\Delta_{c})) \), now \( \sigma(\sqrt{J}) \approx 0.04499 \) are still valid.

As in Godlowski [53], we investigate the isotropy of the resultant distributions of \( \theta \) angles with the help of Kolmogorov-Smirnov test (K-S test). We assume that the theoretical random distribution contains the same number of objects as the observed one. In such studies the key is statistics \( \lambda \):

\[
\lambda = \sqrt{n_p} D_n
\]

(3.28)
which is given as the limit of the Kolmogorov distribution, where
\[
D_n = \sup |F(x) - S(x)|
\]

(3.29)

\(n_p\) is number of investigated points and \(F(x)\) and \(S(x)\) are theoretical and observational distributions of \(\theta\). Now, our interest is to compute the expected value and the standard deviation of the statistic for the real sample (247 rich Abell clusters). As in the previous case (especially \(\chi^2\) test) we introduce the K-S control test. Again, the range of \(\delta_D, p\) and \(\eta\) angles is from 0 to \(\pi/2\) and we divided the entire range of \(\delta_D, p\) and \(\eta\) angles into 18 instead of 36 bins. Because of the reason discussed in the next section, in all cases of the basic and control K-S tests, the expected values of \(\lambda\), as well as their standard deviations are obtained from numerical simulations.

4 Numerical simulations and results

In the beginning, we would like to check whether the statistical methods used in our investigation lead to reliable statistical tests that are suitable to solve investigated problems. Since the statistics described by equation (3.4) has only limit chi-squared distribution [83–86], the first question is whether the approximation of the resulting distribution by chi-squared distribution is acceptable. Secondly, does the \(C\) statistic described by equation (3.5) have normal distribution with standard deviation approximated by equation \(\sigma(C) = n^{1/2}\)? Thirdly, we want to check if the Fourier transform [36] works well, what in practice means to check if exponential formulae (A.28) are valid in the investigated case. This problem has been tested by 1000 simulations of the sample of 2227 galaxies in Godlowski [87] using build-in Fortran Lahey generator (the quality ot this generator was tested in Godlowski [53]), but the answers to above questions were never discussed in referred journals.

In Godlowski [87] it was found that the answers for all above questions are yes, although this thesis is available only in Polish and the answers are only quantitatively. In the present paper, in more detail, this is checked with the help of Kolmogorov- Smirnov test. We have checked if we could reject the hypothesis \(H_0\) that the distributions obtained from the simulations are the same as those approximated. They should be, in the case of statistics presented by equation (3.4), the \(\chi^2\) distribution (with 35 degrees of freedom), in the case \(C\) statistic described by equation (3.5), normal distribution with mean value \(E(C) = -1\) and standard deviation \(\sigma(c) = 6\), while in the case of \(\Delta_{11}\) (equation (A.16)) the normal distribution with mean value equal 0 and standard deviation given by formulae (A.18). The results of these tests for analysis \(\delta_D\) and \(\eta\) as well as for position angles \(P\) is presented in table 1. We present in the table 1 value of statistics \(\lambda\) (see equation (3.28)). At the significance level \(\alpha = 0.01\) the value \(\lambda_{cr} = 1.628\). In any case, the obtained value of statistic \(\lambda\) has not exceed the critical value. It means, that the result of Kolmogorov- Smirnov test has not excluded, on the assumed level of significance, the hypothesis, that analyzed distribution are as expected. Moreover, only in one case (\(C\) statistic for \(\delta_D\) angle) obtaining value of \(\lambda\) is grater than critical value \(\lambda_{cr} = 1.358\) at the significance level \(\alpha = 0.05\). Above results mean that our approximations work well in the case of analysis of the distribution of the \(\delta_D\) and \(\eta\) angles while in the case of analysis of position angles the approximations work perfectly well.

In our analysis we have 11 tests. We have analyzed \(\chi^2, \chi^2_c, \Delta_1/\sigma(\Delta_1), \Delta/\sigma(\Delta), \Delta_c/\sigma(\Delta_c), C, C_c, \lambda, \lambda_c, \Delta_{11}/\sigma(\Delta_{11})\) and \(|\Delta_{11}/\sigma(\Delta_{11})|\) statistics. For most of them we have theoretical prediction given in the previous section. The exception are variances of \(C\) and \(C_c\) statistics, where we only have approximation statistics and statistics \(\lambda\) and \(\lambda_c\)
Table 1. The result of Kolmogorov-Smirnov test for analysis of $\delta_D$, $\eta$ and $P$ angles. The distribution of tested statistics for 1000 simulations of the sample of 2227 galaxies.

| Test | $P$   | $\delta_D$ | $\eta$   |
|------|-------|------------|----------|
| $\chi^2$ | 0.885 | 1.170      | 1.202    |
| $C$  | 1.123 | 1.628      | 0.776    |
| $\Delta_{11}$ | 0.442 | 1.150      | 0.556    |

describing Kolmogorov-Smirnov test. Moreover, the standard deviation of the $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$ and $\Delta_c/\sigma(\Delta_c)$ statistics are obtained from theorem of propagation errors. As a result, (see equations (3.14) and (3.15)) theoretically we have good prediction for means of the $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$ and $\Delta_c/\sigma(\Delta_c)$ statistics, hence in reality we should obtain them also from numerical simulations. However, in all cases it is possible to perform the simulations and obtain Cumulative Distribution Function (CDF) and Probability Density Function (PDF).

The basic problem in numerical simulations is the choice of a random number generator. Unfortunately, many of the popular generators fail to give correct results in multidimensional simulations [88]. This problem, with respect to analysis of alignment of galaxies, was analyzed in detail in Godłowski [53]. In the paper it was shown that most suitable is RANLUX (level 4) generator [88–90] and this generator has been chosen as our base generator. The detailed discussion of different types of Random Generator showing the superiority of RANLUX was carried also for example by Shchur & Butera [91].

At first, using Monte-Carlo simulations, we simulated 247 fictitious clusters, each with 2360 random oriented members of galaxies. The details of our procedure are the following. For each galaxy we simulated the position angle (assuming uniform distribution) and inclination angle (cosine distribution). We have performed this procedure twice, first with galaxies in the clusters with coordinates distributed as in the real clusters and second independently for galaxies randomly distributed around the whole celestial sphere. Now, we obtain from equations (3.1), (3.2) and (3.3) the value of $\delta_D$ and $\eta$ angles. We performed 1000 simulations and on that basis we obtained PDF and CDF of analyzed angles.

Please note that instead of uniform and cosine distribution we could take any theoretically motivated distribution as for example von Mises circular distribution, which is useful during testing the effects of theoretical model of galactic formations [87, 92, 93]. Instead of simulating distribution of position $p$ and inclination angles $i$ and then computing the value of $\delta_D$ and $\eta$ angles, it is possible, if necessary, simulate directly $\delta_D$ and $\eta$ angles according distribution motivated theoretically, for example from Horizon-AGN simulation [30, 31].

On the basis of obtained PDF, now it is possible to compute the mean values of analyzed statistics and its standard deviations. We compute the standard deviation of the mean (denoted as $\sigma(\bar{x})$) and estimate $S^*$ denoted in the tables as $\sigma(x)$, that is the estimator of the standard deviation in the sample as well as the standard deviation of $S^*$ which is equal to $\sigma(S^*) = S^*/\sqrt{2(l-1)}$ [81]. We repeated these simulations again with 247 fictitious clusters each with number of member galaxies the same as in the real cluster. The reason of this is that the number of galaxies in our real clusters is small in some cases what could influence the results of statistical tests. In tables 2–4 we present mean values of the analyzed statistics, its standard deviation, standard deviation in the sample as well as its standard deviation for the sample of 247 clusters each with 2360 galaxies in the case of the analysis of the angles $p$, $\delta_D$ and $\eta$ respectively. First of all, we have analyzed how results of the simulation for $p$
obtained by theorem of propagation of errors. Moreover, the function 
prediction in relation to the squares of statistics only, and variances of analyzed statistics are 
their variance (see formulae (3.14), (3.15) and (3.27)). Unfortunately, we have the theoretical 
theoretical computations predicted by formulae (3.23), (3.25).

| Test          | $\bar{x}$ | $\sigma(x)$ | $\sigma(\bar{x})$ | $\sigma(\sigma(x))$ |
|---------------|-----------|-------------|-------------------|---------------------|
| $\chi^2$      | 34.9978   | 0.5442      | 0.0172             | 0.0128              |
| $\chi^2_0$    | 16.9984   | 0.3550      | 0.0112             | 0.0079              |
| $\Delta_1/\sigma(\Delta_1)$ | 1.2524 | 0.0424 | 0.0013 | 0.0009 |
| $\Delta/\sigma(\Delta)$   | 1.8794   | 0.0460      | 0.0014             | 0.0010              |
| $\Delta_c/\sigma(\Delta_c)$ | 1.2549 | 0.0419 | 0.0013 | 0.0009 |
| $C$           | -0.9917   | 0.3899      | 0.0123             | 0.0087              |
| $C_c$         | -0.9916   | 0.2509      | 0.0079             | 0.0056              |
| $\lambda$     | 0.7708    | 0.0166      | 0.0005             | 0.0004              |
| $\lambda_c$   | 0.7314    | 0.0166      | 0.0005             | 0.0004              |
| $\Delta_{11}/\sigma(\Delta_{11})$ | -0.0010 | 0.0643 | 0.0020 | 0.0014 |
| $|\Delta_{11}/\sigma(\Delta_{11})|$ | 0.7984 | 0.0393 | 0.0012 | 0.0009 |

Table 2. The result of numerical simulation — sample of 247 cluster each with 2360 galaxies.

angles in table 2 are in agreement with theoretical predictions. Our first test is the $\chi^2$ test. The theoretical value is $\chi^2 = n - 1 = 35$ while we have obtained the value 34.9978. It 
means that the difference is less than $1\sigma(\chi^2) = 0.0172$. Because we analyzed 247 clusters 
then for $n = 36$ the theoretical variance is equal $\sigma^2(\chi^2) = 0.2834$ and standard deviation 
equals 0.5324. The simulations have given 0.5442, so the differences are again less than the 
value of $\sigma(S)$. The similar situation is in the case of the $\chi^2$ control test, however differences 
between theoretical and simulated values of standard deviation is on the $2\sigma$ level. In the case of 
autocorrelation test, in both basic and control tests, the simulated and theoretical mean 
values of $C$ agree. The obtained value of standard deviation of $C$ also is not significantly 
deviated from value going from approximation $\sqrt{n/247} = 0.3818$. One should note however 
that in the case of control test the difference is bigger (more than $3\sigma$). When we analyzed the 
statistics $\Delta_{11}/\sigma(\Delta_{11})$ we have again obtained that simulations and theoretical mean (0.0000) 
and standard deviation (0.06363) values agree. Also in the case of analyzed $|\Delta_{11}/\sigma(\Delta_{11})|$ 
statistics we obtained a perfect agreement between values obtained from simulations and 
theoretical computations predicted by formulae (3.23), (3.25).

In the case of $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$ and $\Delta_c/\sigma(\Delta_c)$ statistics the situation is a little bit 
more complicated. It is because for obtaining the theoretical mean values we need to know 
their variance (see formulae (3.14), (3.15) and (3.27)). Unfortunately, we have the theoretical 
prediction in relation to the squares of statistics only, and variances of analyzed statistics are 
obtained by theorem of propagation of errors. Moreover, the function $y = \sqrt{x^2}$ is non linear so 
the results obtained by theorem of propagation of errors is only an approximation. According 
to this approximation $\sigma^2(\Delta_1/\sigma(\Delta_1)) = \sigma^2(\Delta/\sigma(\Delta)) = \sigma^2(\Delta_c/\sigma(\Delta_c)) = 1/2$. In our cases for 
247 clusters it leads to the value $\sigma(x) = 0.04499$. From the inspection of table 2 it is easy to 
see that only for statistics $\Delta/\sigma(\Delta)$ the differences between theoretical and observed values 
of $\sigma(x)$ is on the $1\sigma$ level, while for remaining two statistics the differences are a little bit 
more than $3\sigma$. Consequently the mean values of all three statistics varies from theoretical 
predictions and only for $\Delta/\sigma(\Delta)$ statistic the difference is not very high (1.8794 instead of 
1.8708 with $\sigma(\bar{x}) = 0.0014$). It clearly indicates that the correct values of $\Delta_1/\sigma(\Delta_1)$, $\Delta/\sigma(\Delta)$ 
and $\Delta_c/\sigma(\Delta_c)$ statistics must be obtained from numerical simulations.
Table 3. Results of numerical simulations — sample of 247 clusters each with 2360 galaxies.

| Test       | $\bar{x}$ | $\sigma(x)$ | $\sigma(\bar{x})$ | $\sigma(\sigma(x))$ |
|------------|-----------|-------------|-------------------|---------------------|
| angle $\delta_D$ |
| $\chi^2$   | 35.5837   | 0.5739      | 0.0181            | 0.0128              |
| $\chi_c^2$ | 16.8006   | 0.3882      | 0.0123            | 0.0087              |
| $\Delta_1/\sigma(\Delta_1)$ | 1.2570 | 0.0484 | 0.0015 | 0.0011 |
| $\Delta/\sigma(\Delta)$ | 1.8870 | 0.0494 | 0.0016 | 0.0011 |
| $\Delta_c/\sigma(\Delta_c)$ | 1.0254 | 0.0343 | 0.0012 | 0.0008 |
| $C$        | -0.6594   | 0.4100      | 0.0130            | 0.0092              |
| $C_c$      | -1.2522   | 0.2529      | 0.0080            | 0.0057              |
| $\lambda$  | 0.8164    | 0.0216      | 0.0009            | 0.0005              |
| $\lambda_c$| 0.6779    | 0.0151      | 0.0005            | 0.0003              |
| $\Delta_{11}/\sigma(\Delta_{11})$ | 0.0001 | 0.0475 | 0.0016 | 0.0011 |
| $|\Delta_{11}/\sigma(\Delta_{11})|$ | 0.6138 | 0.0284 | 0.0009 | 0.0006 |

Table 4. Results of numerical simulations — sample of 247 clusters each with 2360 galaxies.

| Test       | $\bar{x}$ | $\sigma(x)$ | $\sigma(\bar{x})$ | $\sigma(\sigma(x))$ |
|------------|-----------|-------------|-------------------|---------------------|
| angle $\eta$ |
| $\chi^2$   | 37.3439   | 0.5806      | 0.0184            | 0.0130              |
| $\chi_c^2$ | 18.1021   | 0.3955      | 0.0125            | 0.0088              |
| $\Delta_1/\sigma(\Delta_1)$ | 1.4786 | 0.0479 | 0.0015 | 0.0011 |
| $\Delta/\sigma(\Delta)$ | 2.1339 | 0.0473 | 0.0015 | 0.0011 |
| $\Delta_c/\sigma(\Delta_c)$ | 1.4009 | 0.0475 | 0.0015 | 0.0011 |
| $C$        | 0.4096    | 0.3793      | 0.0120            | 0.0085              |
| $C_c$      | -0.4442   | 0.2626      | 0.0083            | 0.0059              |
| $\lambda$  | 0.8630    | 0.0195      | 0.0006            | 0.0004              |
| $\lambda_c$| 0.7985    | 0.0192      | 0.0006            | 0.0004              |
| $\Delta_{11}/\sigma(\Delta_{11})$ | 0.0021 | 0.0718 | 0.0023 | 0.0016 |
| $|\Delta_{11}/\sigma(\Delta_{11})|$ | 0.9167 | 0.0448 | 0.0014 | 0.0010 |

The more difficult is the problem when we analyzed the distribution of $p$ angle (the easiest one), using Kolmogorov-Smirnov test. The investigated statistic are $\lambda$ and $\lambda_c$. The original Kolmogorov-Smirnov test is a nonparametric test of the equality of continuous, one-dimensional probability distributions. The test could also be adapted for discrete variables and also for the case when theoretical distribution depends on the estimated parameters. One should note that distribution of statistics $D_n$ and consequently $\lambda$ and $\lambda_c$ (see equation (3.28)) depend both on binning process as well as on theoretical distributions $F(x)$ and on true (but unknown) value of estimated parameters. The limiting form for the distribution function of Kolmogorov’s $D_n$ was analyzed by Wang et al. [94]. They showed that the mean and variance of $\lambda = \sqrt{n}D_n$ are $\mu = 0.868731$ and $\sigma^2 = 0.067773$ what led to the value $\sigma = 0.260333$. 


However, because the distribution depends on binning process the Monte Carlo or other methods of the simulations are required. In our cases we simulated the sample of 247 cluster each with 2360 galaxies and performed 1000 simulations. In the case of analysis of the position angle $p$, the range of analyzed angle is divided into $n$ bins of equal width. In the basic test the number of bins $n = 36$ while in the case of control test it is reduced to $n = 18$. In analyzed case we have obtained the expected value $\lambda = 0.7708$ and $\lambda_c = 0.7314$. Of course for different binning the value of galaxies in clusters, the number of clusters and number of simulations we will obtain different values. For example for 1000 simulations of the sample of 500 clusters each with 10000 galaxies having axial ratio $q = b/a > 0.75$, binned on $n = 100$, the expected value $\lambda = 0.8106$. For the case of simulated sample of 247 clusters each with 2360 galaxies and $n = 36$ bins the standard deviation of $\lambda$ and $\lambda_c$ equals 0.0166, what is in perfect agreement with the result of Wang et al. [94] i.e. $\sigma = 0.260333/\sqrt{247} = 0.01656$.

Unfortunately, during analysis of $\delta_D$ and $\eta$ angles the agreement is not so good (see tables 2–4). The above analysis clearly shows that expected value of $\lambda$ as well as their standard deviation, although valid in particular cases, must be obtained from numerical simulations.

For investigation of the uniformity on a circle, the alternative to the Kolmogorov-Smirnov test are Cramér-von Mises test [95, 96] and Watson test [84, 97] based on the statistics:

$$\omega^2 = \int_{-\infty}^{+\infty} (F(x) - S(x))^2 d(F(x)) \quad (4.1)$$

where again $F(x)$ is the theoretical distribution and $S(x)$ is the empirically observed distribution. The asymptotic form of above distribution was analyzed by Watson [98].

In Cramér-von Mises test one uses statistics:

$$W^2 = \sum_{i=1}^{n} \left( F(x_i) - \frac{2i - 1}{2n} \right)^2 + \frac{1}{12n} \quad (4.2)$$

while in the advanced modification, called Watson test [84, 97], one uses statistics:

$$U^2 = W^2 - n \left( \bar{F}(x) - \frac{1}{2} \right)^2 \quad (4.3)$$

where the average value $\bar{F}(x) = \frac{1}{n} \sum_{i=1}^{n} F(x_i)$.

However, because such tests are based on differences between observed and theoretical distribution like Kolmogorov-Smirnov test in power two, it should be checked the dependence of the distribution on binning process, as in the case of the Kolmogorov-Smirnov test. Again, as in the case Kolmogorov-Smirnov test, we simulated the sample of 247 cluster each with 2360 galaxies and performed 1000 simulations and compute the average values $\bar{W}^2$ and $\bar{U}^2$.

In the case of test with the number of bins $n = 36$ of equal width we have obtained the expected value $\bar{W}^2 = 0.01278$ and $\bar{U}^2 = 0.00401$ with standard deviation $\sigma(W^2) = 0.00049$ and $\sigma(U^2) = 0.000070$ respectively. Unfortunately, again for different binning of the value of galaxies in clusters, the number of clusters and number of simulations we have obtained different values. For example for 1000 simulations of the sample of 500 clusters each with 10000 galaxies with axial ratio $q = b/a > 0.75$, binned on $n = 100$, the expected value $\bar{W}^2 = 0.00501$ and $\bar{U}^2 = 0.00167$ with standard deviation $\sigma(W^2) = 0.00015$ and $\sigma(U^2) = 0.000024$.

The above analysis clearly shows that the expected value of statistics $W^2$ and $U^2$ as well as their standard deviations must be obtained from numerical simulations. Unfortunately, this
Figure 2. The Probability Density Function (PDF) (left panel) and Cumulative Distribution Function (CDF) (right panel) for analyzed statistics. The figure was obtained from 1000 simulations of samples of 247 clusters each with number of members galaxies the same as in the real clusters. From up to down we present statistics: $\chi^2, \chi^2_c, \Delta_1/\sigma(\Delta_1)$.

means that the use of Cramér-von Mises and Watson tests instead of the Kolmogorov-Smirnov test do not give significant progress in our method.

In contrary to analysis of the position angles $p$ where for each galaxy we have one solution, during the analysis of spatial orientation of galaxies we have two solutions for each galaxy i.e we have two possible values of angles $\delta_D$ and $\eta$ for each galaxy. It should be pointed out, that till now, nobody assumed that there could be any difference between the values of analyzed statistics in the case of the $p$, $\delta_D$ and $\eta$ angles. However the analysis of the tables 2–4 (as well as figures 2–5) indicates the presence of such difference.
The significance of the above observation can be investigated using Kolmogorov—Smirnov test. We have chosen for testing the statistics $\chi^2$ because we have a good theoretical predictions about both the mean values and variances of this statistics. The analysis of Godłowski [53], caried in the case of analysis of the position angles $p$, showed that the statistics $\chi^2$ were normal distributed (even though the $\chi^2$ statistics was not normal distributed) with the mean and standard deviation as expected from theoretical analysis i.e. $E(\chi^2) = 35$ and $\sigma^2(\chi^2) = 0.2834$. In order to reject the $H_0$ hypothesis that the distribution is Gaussian with the mean value and variance as assumed, the value of observed statistics $\lambda$ should be greater than $\lambda_{cr}$. At the significance level $\alpha = 0.05$ the value $\lambda_{cr} = 1.358$. In the case of analysis
Figure 4. The Probability Density Function (PDF) (left panel) and Cumulative Distribution Function (CDF) (right panel) for analyzed statistics. The figure was obtained from 1000 simulations of samples of 247 clusters each with number of members galaxies the same as in the real clusters. From up to down we present statistics: $C_c$, $\lambda$, $\lambda_c$.

of position angles the obtained values of $\lambda$ statistic was less than critical one, what means that we can not exclude the $H_0$ hypohthesis [53]. Now, the similar analysis for $\delta_D$ and $\eta$ angles shows the opposite result because the obtained values of $\lambda$ are greater than critical ones. As a result we were able to exclude the hypothesis that distributions are Gaussian with theoretical parameter as noted above.

The next step is to test the new hypothesis $H_0$ that the analyzed statistics are normally distributed with parameters as obtained from simulations. During such an investigation the problem that usually arise is, as shown by Massey [99] and Lilliefors [100], that the standard tables used for the Kolmogorov-Smirnov test are valid only in the case of analysis
Figure 5. The Probability Density Function (PDF) (left panel) and Cumulative Distribution Function (CDF) (right panel) for analyzed statistics. The figure was obtained from 1000 simulations of samples of 247 clusters each with number of members galaxies the same as in the real clusters. From up to down we present statistics: $\Delta_{11}/\sigma(\Delta_{11})$ and $|\Delta_{11}/\sigma(\Delta_{11})|$.

a completely specified continuous distribution. When we test if the distribution is normal, but parameters of the distribution are estimated from the sample, the modification of the classical Kolmogorov-Smirnov test, known as Kolmogorov — Lilliefors test, should be used instead [100]. The significance of this problem for investigation of the galaxy alignment was discussed in detail in Godlowski [53].

In the present analysis we conclude that in the case of all 11 analyzed statistics the values of $\lambda$ are significantly less than critical values $D_{cr}$ (for our case i.e. $n = 1000$ and the significance level $\alpha = 0.05$, $D_{cr} = 0.028$, Godlowski [53]) which means that we can not exclude our $H_0$ hypothesis. Summarizing, we conclude that the obtained results are not in conflict with our prediction that the statistics is normally distributed with parameters as obtained from simulations.

Because of relatively small numbers of galaxies in some clusters, we repeated our analysis with 1000 simulations of 247 fictitious clusters, each cluster with the number of member galaxies the same as in the real clusters (figures 2–5). It is easy to see the differences between distributions of analyzed statistics for $p$, $\delta_D$ and $\eta$ angles. One could observe that usually the analysis of the $\eta$ angles gives the higher values of observed statistics as in other cases. The exception is the analysis of $\Delta_{11}/\sigma(\Delta_{11})$ statistics, where for all analyzed angles PDF and CDF are very similar.

One should note that we have performed this procedure twice, first with galaxies in the clusters with coordinates distributed as in the real clusters and second independently for
we analyzed the sample of 2360 galaxies the difference between the case of galaxies in clusters and for galaxies randomly distributed around the whole celestial sphere. Firstly, we have compared the distribution of the position angles \( p \) and the results are presented in the table 5. When we analyzed the sample of 2360 galaxies the difference between the case of galaxies in clusters distributed as in the real clusters and the case of galaxies randomly distributed around the whole celestial sphere is in all cases less than 2 \( \sigma \) and usually is on 1 \( \sigma \) level. For the sample of clusters with real number of galaxies the situation is similar, but the differences are a little bit higher, up to 3 \( \sigma \). The exception is only for \( \Delta / \sigma (\Delta) \) and for \( \lambda_c \) statistics. The reason is (as it is noted above) that the variance of \( \Delta / \sigma (\Delta) \) is obtained from linear approximation hence the mean value of \( \Delta / \sigma (\Delta) \) is only approximated, while the simulated value of \( \lambda_c \) depends on the binning process. When we compared the results of the statistics obtained for cluster with the real number of galaxies with that obtained for fictitious cluster 2360 galaxies each, the differences between statistics are typically on the 2 \( \sigma \) level, but in any case are less than 3 \( \sigma \). The above results have clearly showed that during analysis of the alignment of galaxies in clusters, we had to compare the observational distribution of the analyzed angles with the results of numerical simulations based on catalogues that contain the clusters populated the same as real ones, but could not base only on pure theoretical predictions. Of course, a good simulated catalog should also have the same possible systematic effect as the real one.

Result of analogous analysis for angles giving the spatial orientation of galaxies i.e. angles \( \delta_p \) and \( \eta \) are presented in tables 6–7 and in figures 6–9. Again we have observed the differences between the cases when we analyzed a huge populated cluster and the cluster with the real number of galaxies in clusters as well as in the case of cluster with galaxies distributed around the whole celestial sphere and the case when the galaxies are distributed as in the real clusters only with the exception of \( \Delta_1 / \sigma (\Delta_1) \) statistic. The crucial observation is that the differences in the latter cases are much higher than in the former one. The presence of the above differences shows that the results of analysis of alignment in real clusters should be rather compared with the numerical simulations instead of pure theoretical predictions.

| Test   | 2360 Galaxies | 2360 coordinates | Real Number | Real coordinates |
|--------|---------------|------------------|-------------|------------------|
| \( \bar{x} \) | 34.9978 0.0172 | 35.0281 0.0166 | 34.9978 0.0170 | 35.0080 0.0168 |
| \( \sigma (\bar{x}) \) | 0.0172 | 0.0166 | 0.0170 | 0.0168 |

Table 5. Results of numerical simulations — sample of 247 clusters each with 2360 galaxies, with numbers of galaxies as in real clusters simulated both with coordinates distributed as in real clusters and for galaxies randomly distributed around the whole celestial sphere.
In our opinion the reason for such differences is mostly caused by the fact that during the process of deprojection of the spatial orientation of galaxies from its optical images we obtain two possible orientations — see equations (3.1)–(3.3). From analysis of these equations it is easy to see that solutions are not independent and as a result the distribution of analyzed statistics is modified and must be obtained from numerical simulations.

| Test  | 2360 Galaxies | 2360coordinates | Real Number | Realcoordinates |
|-------|---------------|-----------------|-------------|-----------------|
| Test  | $\bar{x}$  | $\sigma(\bar{x})$ | $\bar{x}$  | $\sigma(\bar{x})$ | $\bar{x}$  | $\sigma(\bar{x})$ |
| angle $\delta_D$ | | | | |
| $\chi^2$ | 35.5837 0.0181 | 35.5722 0.0172 | 35.5679 0.0186 | 35.8683 0.0179 |
| $\lambda_c^2$ | 16.8006 0.0123 | 17.7032 0.0127 | 16.7960 0.0122 | 17.7239 0.0128 |
| $\Delta_1/\sigma(\Delta_1)$ | 1.2570 0.0015 | 1.2440 0.0013 | 1.2490 0.0015 | 1.2427 0.0013 |
| $\Delta/\sigma(\Delta)$ | 1.8870 0.0016 | 1.8818 0.0014 | 1.8779 0.0016 | 1.8788 0.0014 |
| $\Delta_c/\sigma(\Delta_c)$ | 1.0254 0.0012 | 1.2275 0.0013 | 1.0772 0.0011 | 1.2250 0.0013 |
| $C$ | $-0.6594$ 0.0130 | $-0.4865$ 0.0125 | $-0.5077$ 0.0126 | $-0.5057$ 0.0123 |
| $C_c$ | $-1.2522$ 0.0080 | $-0.6211$ 0.0083 | $-1.1155$ 0.0081 | $-0.6405$ 0.0086 |
| $\lambda$ | 0.8164 0.0009 | 0.7790 0.0005 | 0.8089 0.0006 | 0.7781 0.0006 |
| $\lambda_c$ | 0.6779 0.0005 | 0.7524 0.0006 | 0.6870 0.0005 | 0.7521 0.0006 |
| $\Delta_{11}/\sigma(\Delta_{11})$ | 0.0001 0.0016 | $-0.0031$ 0.0020 | $-0.0009$ 0.0017 | 0.0006 0.0019 |
| $|\Delta_{11}/\sigma(\Delta_{11})|$ | 0.6138 0.0009 | 0.7626 0.0012 | 0.6366 0.0010 | 0.7618 0.0011 |

Table 6. Results of numerical simulations — sample of 247 clusters each with 2360 galaxies, with numbers of galaxies as in real clusters simulated both with coordinates distributed as in real clusters and for galaxies randomly distributed around the whole celestial sphere.

| Test  | 2360 Galaxies | 2360coordinates | Real Number | Realcoordinates |
|-------|---------------|-----------------|-------------|-----------------|
| Test  | $\bar{x}$  | $\sigma(\bar{x})$ | $\bar{x}$  | $\sigma(\bar{x})$ | $\bar{x}$  | $\sigma(\bar{x})$ |
| angle $\eta$ | | | | |
| $\chi^2$ | 37.3439 0.0184 | 36.2309 0.0181 | 36.3915 0.0179 | 36.2255 0.0176 |
| $\lambda_c^2$ | 18.1021 0.0125 | 17.6242 0.0124 | 17.7058 0.0124 | 17.6016 0.0119 |
| $\Delta_1/\sigma(\Delta_1)$ | 1.4786 0.0015 | 1.3619 0.0015 | 1.3761 0.0014 | 1.3640 0.0015 |
| $\Delta/\sigma(\Delta)$ | 2.1339 0.0015 | 2.0053 0.0015 | 2.0218 0.0015 | 2.0053 0.0015 |
| $\Delta_c/\sigma(\Delta_c)$ | 1.4009 0.0015 | 1.3369 0.0014 | 1.3498 0.0014 | 1.3363 0.0013 |
| $C$ | 0.4096 0.0120 | $-0.3124$ 0.0124 | $-0.1704$ 0.0125 | $-0.2839$ 0.0123 |
| $C_c$ | $-0.4442$ 0.0083 | $-0.7211$ 0.0084 | $-0.6574$ 0.0084 | $-0.7053$ 0.0085 |
| $\lambda$ | 0.8630 0.0006 | 0.8140 0.0006 | 0.8201 0.0006 | 0.8147 0.0006 |
| $\lambda_c$ | 0.7985 0.0006 | 0.7684 0.0006 | 0.7731 0.0006 | 0.7677 0.0006 |
| $\Delta_{11}/\sigma(\Delta_{11})$ | 0.0021 0.0023 | 0.0050 0.0022 | 0.0069 0.0022 | 0.0017 0.0021 |
| $|\Delta_{11}/\sigma(\Delta_{11})|$ | 0.9167 0.0014 | 0.8679 0.0014 | 0.8778 0.0013 | 0.8672 0.0013 |

Table 7. Results of numerical simulations — sample of 247 clusters each with 2360 galaxies, with numbers of galaxies as in real clusters simulated both with coordinates distributed as in real clusters and for galaxies randomly distributed around the whole celestial sphere.
Figure 6. The Cumulative Distribution Function (CDF) for $\delta_D$ (left panel) and $\eta$ (right panel) for analyzed statistics. The figure was obtained from 1000 simulations of samples of 247 clusters. Each simulation was done 4 times, with the number of members galaxies the same as in the real cluster, and with 2360 Galaxies. In both cases we used coordinates distributed as in the real clusters and independently coordinates of galaxies randomly distributed around the whole celestial sphere. From up to down we present statistics: $\chi^2, \chi^2_c, \Delta_1/\sigma(\Delta_1)$.

For the investigation of alignment we are able to analyze the distribution of the $p$, $\delta_D$ and $\eta$ angles. Unfortunately, if we want to analyze the distribution of the real values, the following problem of $\delta_D$ and $\eta$ angles arises. If we do not know the morphological types of galaxies, we have to assume the real axial ratio. This is usually done by assuming, during calculation the inclination angle, the average value $q_0 = 0.2$, but then the effect of deprojection masks any possible alignment as it is shown in Godłowski & Ostrowski [101], Godłowski [40, 102], Pajowska [103]. In the above papers it was shown that this problem can be solved when we
Figure 7. The Cumulative Distribution Function (CDF) for $\delta_D$ (left panel) and $\eta$ (right panel) for analyzed statistics. The figure was obtained from 1000 simulations of samples of 247 clusters. Each simulation was done 4 times, with the number of members galaxies the same as in the real cluster, and with 2360 Galaxies. In both cases we used coordinates distributed as in the real clusters and independently coordinates of galaxies randomly distributed around the whole celestial sphere. From up to down we present statistics: $\Delta/\sigma(\Delta), \Delta_c/\sigma(\Delta_c), C$.

know the morphological type of individual galaxies and use true values $q_0$ depending on the morphological type (according to [104] with the help of [105] corrections of $q$ to standard photometrical axial ratios) of axial ratio instead of the average value $q_0 = 0.2$. Unfortunately MRSS [56] does not provide information about morphological type of individual galaxies. Therefore in the present paper we have analyzed, like in Godłowski [53], only the distribution of the position angles $p$ in the sample (A) of 247 rich Abell clusters both in Equatorial and Supergalactic coordinate systems. Moreover we have analyzed the restricted sample (B) in
Figure 8. The Cumulative Distribution Function (CDF) for $\delta_D$ (left panel) and $\eta$ (right panel) for analyzed statistics. The figure was obtained from 1000 simulations of samples of 247 clusters. Each simulation was done 4 times, with the number of members galaxies the same as in the real cluster, and with 2360 Galaxies. In both cases we used coordinates distributed as in the real clusters and independently coordinates of galaxies randomly distributed around the whole celestial sphere. From up to down we present statistics: $C_c$, $\lambda$, $\lambda_c$.

which only galaxies brighter than $m_3 + 3^m$ are taken into account. The results are presented in table 8.

Our null hypothesis $H_0$ is that the mean value of the analyzed statistics is as expected in the cases of a random distribution of the position angles, against $H_1$ hypothesis that the analyzed values are different than predicted in the case of random distribution. For nearly all performed tests the result are significant on at least $3\sigma$ level. In all cases there are no significant differences when we analyzed the distribution of Equatorial position angles $p$ and
Supergalactic position angles $P$. One can see from PDF and CDF presented in the figures 2–5 that the probability that such results are coming from random distributions is less than 0.1%. The exception is only $\chi^2$ where the effect is on 2 $\sigma$ level and for $\Delta_{11}/\sigma(\Delta_{11})$ statistic where we have seen no effect.

Moreover, we have checked our result using Watson test. The theoretical simulations show that for the sample of 247 clusters each with numbers of galaxies as in real clusters the expected mean value of $U^2$ statistic is 0.0280 with standard deviation equal 0.0011. For the real sample of 247 cluster we have obtained the value $\bar{U}^2 = 0.0642$ with $\sigma(\bar{U}^2) = 0.0035$, hence this test rejects $H_0$ hypothesis on the 10 $\sigma$ level.

The statistics of $\Delta_{11}/\sigma(\Delta_{11})$ shows the direction of deviation from isotropy with respect to the assumed coordinate system main plane. Our results show that in the case of $\Delta_{11}/\sigma(\Delta_{11})$ test we can not exclude our null hypothesis $H_0$ that the mean value of statistic is the same as predicted for the case of random distribution. We have obtained the results for both Equatorial and Supergalactic coordinate systems. Of course, there should be no physical reason that the detected alignment could be connected with equatorial plane. Moreover, since we have analyzed the sample of clusters with redshift up to $z = 0.12$, which is much more distant than the Local Supercluster, there is also no reason to expect the special meaning of the Local Supercluster equator. Because in both cases the obtained values of
Table 8. The value of analyzed statistics for position angles $p$, the real sample of 247 Abell clusters.

| Sample | Test | Equatorial coordinates | Supergalactic coordinates |
|--------|------|-------------------------|----------------------------|
|        |      | $\bar{x}$ | $\sigma(\bar{x})$ | $\bar{x}$ | $\sigma(\bar{x})$ |
| A      | $\chi^2$ | 36.8591 | 0.5924 | 36.7899 | 0.6315 |
|        | $\chi^2_e$ | 17.7579 | 0.4030 | 18.0619 | 0.4355 |
|        | $\Delta_1/\sigma(\Delta_1)$ | 1.7046 | 0.0622 | 1.7021 | 0.0626 |
|        | $\Delta/\sigma(\Delta)$ | 2.2663 | 0.0594 | 2.2746 | 0.0591 |
|        | $\Delta_c/\sigma(\Delta_c)$ | 1.4619 | 0.0540 | 1.5682 | 0.0562 |
|        | $C$ | 1.1940 | 0.4530 | 1.1220 | 0.4237 |
|        | $C_c$ | -0.1030 | 0.3003 | 0.1904 | 0.2990 |
|        | $\lambda$ | 0.9177 | 0.0240 | 0.9138 | 0.0220 |
|        | $\lambda_c$ | 0.8365 | 0.0242 | 0.8561 | 0.0248 |
|        | $\Delta_{11}/\sigma(\Delta_{11})$ | -0.0005 | 0.0855 | 0.0940 | 0.0924 |
|        | $-\Delta_{11}/\sigma(\Delta_{11})$ | 1.0347 | 0.0543 | 1.1206 | 0.0588 |
| B      | $\chi^2$ | 36.4000 | 0.6072 | 36.2919 | 0.6124 |
|        | $\chi^2_e$ | 17.5943 | 0.3963 | 17.8530 | 0.4216 |
|        | $\Delta_1/\sigma(\Delta_1)$ | 1.6283 | 0.0577 | 1.6316 | 0.0578 |
|        | $\Delta/\sigma(\Delta)$ | 2.2055 | 0.0565 | 2.2199 | 0.0554 |
|        | $\Delta_c/\sigma(\Delta_c)$ | 1.4522 | 0.0521 | 1.5070 | 0.0525 |
|        | $C$ | 0.8843 | 0.4355 | 0.7863 | 0.4212 |
|        | $C_c$ | -0.1070 | 0.3012 | -0.0671 | 0.3063 |
|        | $\lambda$ | 0.8928 | 0.0224 | 0.8934 | 0.0210 |
|        | $\lambda_c$ | 0.8313 | 0.0228 | 0.8360 | 0.0228 |
|        | $\Delta_{11}/\sigma(\Delta_{11})$ | 0.0023 | 0.0826 | 0.0810 | 0.0866 |
|        | $-\Delta_{11}/\sigma(\Delta_{11})$ | 1.0079 | 0.0519 | 1.0700 | 0.0565 |

$\Delta_{11}/\sigma(\Delta_{11})$ statistic are close to zero, it increases the probability that the observed alignment is related not to a particular global plane, but with the alignment with respect to galaxy cluster or cluster’s parent supercluster planes. Therefore, our result confirms the prediction that the detected alignment is not connected with equatorial plane nor with Supergalactic plane. The final interpretation of this phenomenon, especially in the context of the evolution of galaxies and their structures needs a detailed future study.

We separately analyzed the sample B (only with galaxies brighter than $m_3 + 3^\text{m}$). Our investigation confirms the conclusion obtained by [53] that the observational alignment is weaker than in the case of sample A where all galaxy cluster members were analyzed, but still significant. As above, $\Delta_{11}/\sigma(\Delta_{11})$ is close to zero which is in agreement with the predictions of our null hypothesis $H_0$. For $\chi^2$ test the result is significant at 2 $\sigma$ level while for remaining tests the results are very significant i.e. on 3 $\sigma$ level. This result is important because, for avoiding possible role of background object, the restricted sample B took into account only galaxies brighter than $m_3 + 3^\text{m}$. This leads to the conclusion that the presence of background objects has no significant effect for all our results.
Table 9. The value of analyzed statistics for position angles $p$, the real sample of 247 Abell clusters. Random errors in position angles $p$ are included.

It is also necessary to investigate possible influence of errors in measurement of the position angles. For that, we repeat our analysis presented above in table 8 adding uncertainties in measurements of the position angles. We assumed standard error $\sigma(P) = 2^\circ$. It means that even on the $3\sigma$ level, the deviation will be $6^\circ$, what is more than in the worst case of the uncertainties ($5^\circ$). One notes that for real data, the uncertainties quickly increase for rounded images. However, this is not important in our analysis because face-on galaxies with axial ratio $q > 0.75$ are excluded from the analysis. From table 9 it is easy to conclude that uncertainty in determining position angles does not significantly affect the results we have received.

The alternative method to investigate the impact of errors and possible influence of background object is to use the jackknife method [106, 107]. The jackknife technique is based on drawing all possible samples of $N - 1$ values from the $N$ data points and repeating the test of $x$ statistic calculations on them, which allows us to calculate the standard deviations in the analyzed values of $x$, $\sigma_j(x)$. The best estimator for the standard errors in the value of $x$ is then just $\sqrt{N - 1}\sigma_j$ [107].
where we observe that again effects for sample B divided by the weighted average uncertainty. From table 10 it is easy to conclude that again errors presented in table 8. Only the H$_x$ for example Brandt \cite{81} i.e. $\bar{x}$ such a case average value of statistic should be obtained by weighted arithmetic mean (see presented in tables 8 and 9, statistics errors for individual clusters are not the same. In the errors are as predicted by the theory. Please note that now contrary to the analysis following statistics:

\begin{equation}
\text{Test} & \quad \bar{x} & \sigma_j(\bar{x}) & (\bar{x} - E(\bar{x}))/\sigma_j(\bar{x}) & \bar{x} & \sigma_j(\bar{x}) & (\bar{x} - E(\bar{x}))/\sigma_j(\bar{x}) \\
\chi^2 & 34.1996 & 0.7052 & <0. & 33.7071 & 0.6906 & <0. \\
\chi^2 & 15.8378 & 0.4903 & <0. & 15.8050 & 0.4785 & <0. \\
\Delta_1/\sigma(\Delta_1) & 1.7176 & 0.0621 & 7.44 & 1.6304 & 0.0617 & 6.09 \\
\Delta/\sigma(\Delta) & 2.2761 & 0.0621 & 6.40 & 2.2044 & 0.0610 & 5.34 \\
\Delta_c/\sigma(\Delta_c) & 1.4583 & 0.0517 & 3.30 & 1.4175 & 0.0595 & 2.76 \\
C & 0.2654 & 0.5168 & 2.49 & 0.0262 & 0.5080 & 2.05 \\
C_c & -0.4880 & 0.3447 & 1.51 & -0.4784 & 0.3406 & 1.56 \\
\lambda & 0.8670 & 0.0257 & 3.69 & 0.8390 & 0.0252 & 2.66 \\
\lambda_c & 0.7823 & 0.0263 & 1.99 & 0.7690 & 0.0254 & 1.53 \\
|\Delta_{11}/\sigma(\Delta_{11})| & 0.0024 & 0.0642 & 0.02 & 0.0060 & 0.0642 & 0.07 \\
|\Delta_{11}/\sigma(\Delta_{11})| & 1.0335 & 0.0642 & 3.67 & 1.0081 & 0.0642 & 3.26 \\
\end{equation}

\textbf{Table 10.} Jacknife analysis for position angles $p$, the real sample of 247 Abell clusters.

Now, we can use the jacknife technique for the analyzed galaxy clusters and see if the errors are as predicted by the theory. Please note that now contrary to the analysis presented in tables 8 and 9, statistics errors for individual clusters are not the same. In such a case average value of statistic should be obtained by weighted arithmetic mean \cite{81}. Respectively, the weighted average uncertainty is given by formulae: $\sigma(\bar{x}) = \left(\frac{1}{\sum_{i=1}^{n} \frac{1}{\sigma_i^2}}\right)^{-1}$. We present obtained results in table 10 where additionally we present differences between obtained weighted arithmetic mean and values expected from simulations (presented in the tables 5, column RealNumber, values $\bar{x}$) divided by the weighted average uncertainty. From table 10 it is easy to conclude that again $\Delta_{11}/\sigma(\Delta_{11})$ is close to zero which is in agreement with the predictions of our null hypothesis $H_0$. Generally, the received weighted average uncertainties are similar, but a little greater than errors presented in table 8. Only the $\chi^2$ test does not survive jacknife procedure while for other tests the results are still deviating from prediction of $H_0$ hypothesis. One could observe that again effects for sample B are weaker than in the case of sample A, but is still significant. These results confirm above conclusion that uncertainty in determining position angles does not significantly affect our results and moreover lead to conclusion that the influence of background objects is not significant for our results.

Finally we have analyzed the differences between clusters with different BM types (table 11). For this, we have used the means and standard deviations for all subsamples and compare the mean values of the statistic using the following well know statistical test. When comparing the mean values from subsamples with standard deviations known, we use the following statistics:

\begin{equation}
U = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \quad (4.4)
\end{equation}

where $\bar{X}_1$ and $\bar{X}_2$ are the mean values of samples (subsamples) and $n_1$ and $n_2$ are the samples.
where the variance estimators are:

\[ \chi^2 = \frac{S_1^2}{S_2^2} \]

(4.5)

where the variance estimators are: \( S_1^2(x) = \frac{n_1}{n_1 - 1} S_i^2(x) = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2 \). Under \( H_0 \) hypothesis that the analyzed variances are equal to each other, the \( F \) statistics has \( F \) Snedecor distribution with \( (n_1 - 1, n_2 - 1) \) degrees of freedom. In table 11 we presented the estimator of the standard deviation for the mean values, i.e. \( S_{Mi}(\bar{x}) = \sqrt{\sum_{i=1}^{n_i}(X_{ij} - \bar{X}_i)^2/[n_i(n_i - 1)]} \).

![Table 11](image)

| Members | BM I | BM I-II | BM II | BM II-III | BM III |
|---------|------|--------|-------|-----------|--------|
| Test    | \( \bar{x} \) | \( \sigma(\bar{x}) \) | \( \bar{x} \) | \( \sigma(\bar{x}) \) | \( \bar{x} \) | \( \sigma(\bar{x}) \) |
| \( \chi^2 \) | 36.603 | 1.546 | 36.975 | 1.286 | 36.232 | 1.134 | 37.888 | 1.177 | 36.544 | 1.609 |
| \( \chi^2_c \) | 17.837 | 1.024 | 16.992 | 0.930 | 18.035 | 0.696 | 18.596 | 0.915 | 17.268 | 1.003 |
| \( \Delta_1/\sigma(\Delta_1) \) | 1.658 | 0.154 | 1.593 | 0.114 | 1.539 | 0.113 | 1.920 | 0.154 | 1.827 | 0.160 |
| \( \Delta/\sigma(\Delta) \) | 2.328 | 0.150 | 2.252 | 0.109 | 2.060 | 0.111 | 2.492 | 0.143 | 2.239 | 0.152 |
| \( \Delta_c/\sigma(\Delta_c) \) | 1.393 | 0.150 | 1.446 | 0.091 | 1.413 | 0.102 | 1.689 | 0.138 | 1.333 | 0.128 |
| \( C \) | 1.486 | 1.162 | 0.778 | 0.974 | 0.279 | 0.888 | 2.875 | 1.011 | 0.696 | 1.075 |
| \( C_c \) | -0.969 | 0.815 | 0.093 | 0.483 | -0.215 | 0.525 | 1.170 | 0.792 | -0.974 | 0.750 |
| \( \lambda \) | 0.938 | 0.063 | 0.877 | 0.045 | 0.850 | 0.044 | 0.985 | 0.055 | 0.955 | 0.063 |
| \( \lambda_c \) | 0.784 | 0.059 | 0.814 | 0.044 | 0.819 | 0.044 | 0.934 | 0.063 | 0.809 | 0.058 |
| \( \Delta_{11}/\sigma(\Delta_{11}) \) | -0.287 | 0.205 | 0.120 | 0.163 | 0.125 | 0.164 | 0.203 | 0.219 | -0.311 | 0.196 |
| \( |\Delta_{11}/\sigma(\Delta_{11})| \) | 0.932 | 0.137 | 0.968 | 0.093 | 0.996 | 0.100 | 1.222 | 0.142 | 1.021 | 0.134 |

Table 11. The value of analyzed statistics position angles \( p \), the real sample of 247 Abell clusters, BM types, Equatorial coordinates, sample A.

size. Under the assumption of the new null hypothesis \( H_0 \) that the real mean values of \( \bar{X}_1 \) and of \( \bar{X}_2 \) are equal, the statistic \( U \) has standard normal distribution. One should note however, that in the real case the standard deviation is not a’priori known and is estimated from the samples. Hence the crucial check is if both standard deviations are equal to each other. We do it by using a very well known \( F \) Fisher test. In this test we make use of the statistic \( F \):

\[ F = \frac{S_1^2}{S_2^2} \]

(4.5)

where the variance estimators are: \( S_1^2(x) = \frac{n_1}{n_1 - 1} S_i^2(x) = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2 \). Under \( H_0 \) hypothesis that the analyzed variances are equal to each other, the \( F \) statistics has \( F \) Snedecor distribution with \( (n_1 - 1, n_2 - 1) \) degrees of freedom. In table 11 we presented the estimator of the standard deviation for the mean values, i.e. \( S_{Mi}(\bar{x}) = \sqrt{\sum_{i=1}^{n_i}(X_{ij} - \bar{X}_i)^2/[n_i(n_i - 1)]} \). Our analysis shows that in majority cases we can not exclude \( H_0 \) that \( \sigma_1^2 = \sigma_2^2 \).

Then, for testing the significance of the differences of the mean values in subsamples we should use the well know Student test. The statistics \( t \)

\[ t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{n_1 S_1^2 + n_2 S_2^2/n_1 + n_2}} \]

(4.6)

has Student distribution with \( n_1 + n_2 - 2 \) degrees of freedom under the assumption of \( H_0 \) hypothesis that the real mean values of \( \bar{X}_1 \) and of \( \bar{X}_2 \) are equal. Please note that this test is valid only in the case when standard deviations are equal.

In the few cases, when standard deviations in subsamples do not fulfill this condition, as for the comparison BM II-III subsample with B II and BI-II subsamples in the case of control tests (with exception of \( \chi^2_c \) test) for the comparison of the mean values of the statistics we
have to use the full Cochrane-Cox test \[85, 108–110\]. The statistics is given by the formula:

\[
CC = \frac{X_1 - X_2}{\sqrt{S_1^2/n_1-1 + S_2^2/n_2-1}}
\]  

(4.7)

with approximated critical value

\[
c(p, n1, n2) \geq \frac{S_1^2 t(p,n1-1)}{n_1-1} + \frac{S_2^2 t(p,n2-1)}{n_2-1} \approx \frac{S_1^2}{n_1-1} + \frac{S_2^2}{n_2-1}
\]  

(4.8)

where \(t(p,n_i-1)\) are critical values (quantiles) of Student test.

Our analysis shows that in majority cases we cannot exclude the \(H_0\) hypothesis that mean values of analyzed statistics for different BM types are equal. One should note that only for cluster type BM II-III the mean deviates from the case when cluster have other morphological type. The value of statistics for this type is higher than for other ones. In some cases these differences are significant, but only when we compare BM II and BM II-III statistics more then a half (i.e. 6) tests show that the differences is significant.

Finally, we have been able to check if the value of statistics for subsamples of cluster belonging to particular BM types deviates from the mean values obtained for the whole population. The possible difference is observed only in the case BM II-III type, while for other subsamples at most one test shows possible differences.

Similarly as in the case of the whole sample, also in the case of clusters with different BM types, we have investigated possible influence of errors in measurement of the position angles. The result are presented in table 12. One should note that also in this case, the uncertainty in determining position angles does not significantly affect the results we have received.

| Members | BM I | BM I-II | BM II | BM II-III | BM III |
|---------|------|--------|-------|-----------|--------|
| Test    | \(\chi^2\) | \(\chi^2_c\) | \(\Delta_1/\sigma(\Delta_1)\) | \(\Delta/\sigma(\Delta)\) | \(\Delta_c/\sigma(\Delta_c)\) | \(C\) | \(C_c\) | \(\lambda\) | \(\lambda_c\) | \(\Delta_{11}/\sigma(\Delta_{11})\) | \(|\Delta_{11}/\sigma(\Delta_{11})|\) |
| \(\bar{x}\) | 36.909 | 1.676 | 36.488 | 1.038 | 35.523 | 1.129 | 38.434 | 1.275 | 36.279 | 1.584 |
| \(\sigma(\bar{x})\) | 1.676 | 36.488 | 1.038 | 35.523 | 1.129 | 38.434 | 1.275 | 36.279 | 1.584 |
| \(\bar{x}\) | 16.594 | 1.034 | 18.130 | 0.790 | 17.878 | 0.668 | 18.709 | 0.944 | 17.383 | 1.083 |
| \(\sigma(\bar{x})\) | 1.034 | 18.130 | 0.790 | 17.878 | 0.668 | 18.709 | 0.944 | 17.383 | 1.083 |
| \(\Delta_1/\sigma(\Delta_1)\) | 1.621 | 0.155 | 1.592 | 0.116 | 1.545 | 0.113 | 1.928 | 0.154 | 1.825 | 0.160 |
| \(\Delta/\sigma(\Delta)\) | 2.325 | 0.150 | 2.233 | 0.110 | 2.054 | 0.109 | 2.507 | 0.143 | 2.275 | 0.150 |
| \(\Delta_c/\sigma(\Delta_c)\) | 1.367 | 0.145 | 1.400 | 0.094 | 1.430 | 0.102 | 1.703 | 0.143 | 1.341 | 0.126 |
| \(C\) | 0.767 | 1.188 | -0.057 | 1.001 | 0.425 | 0.890 | 2.028 | 1.143 | 0.383 | 1.005 |
| \(C_c\) | -0.523 | 0.622 | -0.826 | 0.648 | -0.300 | 0.589 | 1.220 | 0.883 | -0.985 | 0.683 |
| \(\lambda\) | 0.930 | 0.061 | 0.859 | 0.043 | 0.872 | 0.042 | 1.004 | 0.054 | 0.957 | 0.064 |
| \(\lambda_c\) | 0.763 | 0.058 | 0.808 | 0.045 | 0.820 | 0.041 | 0.943 | 0.064 | 0.781 | 0.060 |
| \(\Delta_{11}/\sigma(\Delta_{11})\) | -0.255 | 0.199 | 0.128 | 0.164 | 0.125 | 0.167 | 0.192 | 0.225 | -0.336 | 0.192 |
| \(|\Delta_{11}/\sigma(\Delta_{11})|\) | 0.909 | 0.131 | 0.958 | 0.098 | 1.005 | 0.103 | 1.249 | 0.145 | 1.004 | 0.132 |

Table 12. The value of analyzed statistics position angles \(p\), the real sample of 247 Abell clusters, BM types, Equatorial coordinates, sample A. Random errors in position angles \(p\) are included.
5 Discussion

Any statistical study involving orientations must take into account that directional variables are cyclic [111]; i.e. 359° is close to 1° or if the analyzed range is 180° (as in the case of positional angles) that 179° is close to 1°. It is one of the reason to include the procedure considered with the sign of the expression for $S$ (equations (3.1), (3.2), (3.3) and the paragraph below them). Fourier transform (see equation (3.7)) and Peeble’s first auto-correlation test (equation (3.5)) take it into account (the exception is the auto-correlation control test but the correct values and Cumulative Distribution Functions are obtained from simulations.

Another aspect of the problem is that when we study the distribution of cyclic variables, then it becomes important to determine the analogy of the Gaussian distribution. Usually, such a role is fulfilled by the von Mises circular distribution because it is a close approximation to the wrapped normal distribution (i.e. a wrapped probability distribution that results from the “wrapping” of the normal distribution around the unit circle) which is the circular analogue of the normal distribution [112]. In the context of research of the galaxy orientation, von Mises circular distribution was used for example during testing the effects of theoretical model of galactic formations [87, 92, 93].

The most important claim raised against the use of the Fourier test [36] is that from theoretical point of view the Fourier transform does not lead to reliable statistics tests because the conditions under which the exponential formulae (eq. (A.28)) are rarely valid in practice (see Chpt 5 of Percival & Walden [80]). This problem is well known and has been discussed many times in the literature in context of usability of power spectrum analysis (PSA) [107, 113–118], together with the Rayleigh test [119, 120]. For a given frequency, the Rayleigh power spectrum corresponds to the Fourier power spectrum. Newman et al. [116] pointed out that Yu & Peebles [118] version of Power Spectrum Analysis can be applied correctly only when a uniform distribution function is tested. In other cases, test statistics is significantly modified [80, 116] and must be obtained from simulations (see also Godlowksi et al. [113]). Fortunately, in case of position angles the theoretical distribution is uniform, hence the Fourier transform [36] should work well in this case and therefore the exponential formulae (equation (A.28)) are valid in practice.

In the papers [40, 52, 121] it was shown the alignment of galaxies in clusters increases with their richness. It allowed to conclude that the angular momentum of the cluster increases with the numbers of galaxies in clusters i.e. with the mass of the structure. With such dependency it is expected that in the sample of rich clusters we should observe the significant alignment. The result of the present paper confirmed such predictions. One should note that the increase of alignment of bright galaxy members (central galaxies) with the clusters richness has recently been found by Huang et al. [122]. They state that alignment of central galaxies may originate from the filamentary accretion processes, but also possibly affected by the tidal field.

If the alignment is increasing with the richness of the cluster the question which arises is what is its shape and reason. The relation between the angular momentum and the mass of the structure has been discussed since a long time and usually is presented as $J \sim M^{5/3}$ [123–126]. The explanation of this phenomenon in the light of the structure formation scenarios is not clear but it could be explained by the Li model [20, 50, 127] in which galaxies form in the rotating universe or by tidal torque scenario in hierarchical clustering model as suggested by Heavens & Peacock [128] and Catelan & Theuns [17] (see also Noh & Lee [129, 130]). For that, extending the idea of Heavens & Peacock [128] and Catelan & Theuns [17] we use a novel
theoretical approach Stephanovich & Godlowski [34], Stephanovich & Godlowski [131] in which the distribution function of dynamic characteristics of galaxies ensembles is calculated via tidal (shape-distorting) quadrupolar (and also higher multipolar) interaction between the galaxies. This function, among other things, may be used to a better statistical treatment of observational data, which permits to discriminate observationally the relevance of available theories of galaxies formation. The calculation of the average galaxies angular momenta with the help of the above distribution function permits to study theoretically their orientations. In the papers Stephanovich & Godlowski [34], Stephanovich & Godlowski [131] it was shown that with a reasonable assumption given, the angular momentum of galaxy structures increases with their richness, however the final form of the dependence (not necessary $J \sim M^{5/3}$) depends on the assumption about cluster morphology. The preliminary results given in Stephanovich & Godlowski [131] shows that the present data does not allow to discriminate between different dependence between angular momentum and mass. The results of this paper are in agreement with such theoretical predictions. In particular, this shows that the relation between alignment of galaxies in clusters and their mass is more complicated than simple increasing according to formulae $J \sim M^{5/3}$.

We also should note that during the studies of the angular momentum of galaxy cluster we have some complications that make analysis not so easy. Recall that generally clusters do not rotate [45], so the angular momentum of such structures is connected with galaxy members alignment, but there are small number of clusters with intrinsic rotation. A sample of six Hwang & Lee [45] rotating clusters was analyzed by Aryal et al. [132] who did not found any alignment for that cluster, so the angular momentum of such structures is coming from orbital movement, not from alignment. Finally we should note that recently Yadav et al. [133] analyzed the sample of dynamically unstable Abell clusters founding a random orientation of galaxies inside these clusters. One should note that recently was found that the alignment of galaxies evolves in time [34, 131, 134]. In the papers Stephanovich & Godlowski [34], Stephanovich & Godlowski [131] it was shown that angular momentum of galaxies in cluster increases with time. This could explain the result of Hao et al. [135] who found that alignment of Brightest Cluster Galaxies decreases with redshift. This could also potentially explain Yadav et al. [133] result because it seems to be reasonable to assume that the dynamically unstable Yadav et al. [133] clusters are young, hence their angular momenta are still small. Such predictions could also explain the results of [136] paper, who found that the alignment profile of cluster galaxies drops faster at higher redshifts. Our results shows that although the exceptions exist, they do not significantly influence the statistics of the whole sample analyzed in this paper.

During the investigation of alignment in clusters, the important problem is the influence of environmental effects to the origin of galaxy angular momenta. Godlowski et al. [137] shows possible impact of the membership clusters on the superclusters. Also Huang et al. [122], Huang et al. [138], Wang et al. [139] indicates possible role of environmental effects on central galaxy and radial alignments. Moreover, Huang et al. [122], Huang et al. [138] pointed out the role of central dominating galaxies in cluster and merger process events which tend to destroy alignment.

In particular, the discussion shows that even though the tidal torque theory is at the moment supported by observations, it is still a significant simplification. There exist rich clusters that undergone many mergers and accretion events in history and cannot be well modeled by this theory. In this way, numerical simulations of structure formation which capture some of the complexity may be more compelling.
One should note however, that even now, the investigation of the spatial orientation of galaxies in clusters would be possible with the use of information about frequency of galaxy occurrence in clusters with particular morphological types, since galaxy proportions with different spectral types can be estimated on the basis of density profiles in cluster, even if we do not have the information about morphological types for each particular galaxy [140–143]. In clusters, we may be able to estimate the fraction of galaxies having the particular morphological type. In numerical simulations, it will be taken into consideration the information about the frequency of occurrence of galaxies with particular morphological types in each cluster. Galaxy proportions with different spectral types in various cluster areas would be estimated on the basis of density profiles in cluster Dressler [142], Calvi et al. [140], Coenda et al. [141], Hoyle et al. [143]. Subsequently, from the formula: \( \cos^2 i = \left( q^2 - q_0^2 \right) / \left( 1 - q_0^2 \right) \) the observed value of \( q = a/b \) could be calculated for each galaxy. Using this value generated on the assumption of isotropy and \( q_0 = 0.2 \), the new values \( \cos^2 i \), as well as \( \delta_D \) and \( \eta \) angles would be enumerated. In such way we will obtain a new theoretical isotropic distributions for \( \delta_D \) and \( \eta \) angles in which the information about frequency of appearing galaxies with individual morphological types in clusters will be already included. Only with these corrected theoretical isotropic distributions we will be able to apply for testing the isotropy of galaxy orientations hypothesis when morphological types of particular galaxies are unknown. In this way we will compare obtained “theoretical isotropic distribution”, which will take into consideration both the information about galaxy proportions of occurrence of different morphological types in cluster and the value of average galactic axial ratio \( q_0 \) with the observational distributions obtained as well with the assumption \( q_0 = 0.2 \).

Although such alternative solution exists, it requires precise and complicated numerical simulations and has never been used in practice before. For the above reason we decide to postpone it to future investigation and in the present paper we have analyzed, like in Godłowski [53], only the distribution of the position angles \( p \) in the sample of 247 rich Abell clusters both in Equatorial and Supergalactic coordinate systems.

6 Conclusions

The motivating theoretical goal of the project has been to give an improvement in the discrimination among different models of galaxy formation. A general idea has been to analyze the angular momentum of galaxies in clusters and check if the results agree with scenarios predictions. That is why, in this paper we have focused on how we perform the analysis of the alignment of galaxies in clusters. In the original method presented in Godłowski [53] the distributions of the position angles for galaxies in each cluster were analyzed using statistical tests: \( \chi^2 \) test, Fourier tests, Autocorrelation test and Kolmogorov test. The mean value of the analyzed statistics was compared with theoretical predictions as well as with results obtained from numerical simulations. The method allows to check if the mean value of analyzed statistics is the same as expected in the case of random distribution of the position angles of galaxies.

In the present paper we have analyzed this method in detail, giving proposal of some significant improvements and introducing new statistical tests into the method. We have considered how the tests changes if we assume various expected values of galaxies in bins. In particular, in the autocorrelation test, the values of statistics slightly changes. However, in the Fourier test, not only the formulas for coefficients changes but also the coefficients need not be independent and we consider this in the analysis. We have also analyzed the properties
of the Kolmogorov-Smirnov test applied to the analysis of the alignment of galaxies in clusters and finally we introduced control tests to all considered tests. In all cases the theoretical predictions have been compared with the numerical simulations.

The second major advantage of the present paper in comparison to the previous investigation is that our analysis allowed us to expand the investigation of alignment of galaxies in cluster from the analysis of position angles only to the angles giving spatial orientation of galaxy planes, that has never been done before. The main difference is that the analysis of the position angles gives the information about orientation of galaxies only for edge-on galaxies. The analysis of the spatial orientation has allowed us to include all galaxies especially face-on galaxies. The difficulty that arises is that during the process of deprojection of the spatial orientation of galaxies from its optical images we obtain two possible orientations, and because we are able to find which solution is correct only in the small number of galaxies, both solutions must be taken into account during further analysis.

Another crucial problem during analysis of the angles giving spatial orientation of galaxies is that if for any reason we exclude from analysis any type of galaxies (for example face-on galaxies), then the theoretical distribution of analyzed angles will be modified, even in the case when the distribution of galaxy planes is random and isotropic. In this case, a random distribution of analyzed angles which is the base of comparison with the real one must be, in practice, obtained from numerical simulations. This problem was analyzed for example by Godłowski(1993a) [70], Aryal & Saurer [74] (for modern analysis see for example Flin et al. [121]). However, we have noticed that nobody took care of the fact that both obtained solutions for orientations are not independent of each other (only Panko et al. [54] made a remark that such problem could arise). Consequently, nobody analyzed if statistical test gives, even for “random” distributions, the same values of statistics as it is predicted in the case of the position angles. Our results have clearly showed that the expected mean values of the statistics for $\delta_D$ and $\eta$ angles varied from that obtained during analysis of the position angles. It means that “theoretical random distribution” must be modified this time. This is very easy to be observed in the example of nearly face-on galaxies, when both possible orientations are similar, which leads to the situation that both obtained values for $\delta_D$ and $\eta$ angles are similar. As a result in this case the first solution strongly affects the second. This phenomenon is also responsible for the results that during analysis of the spatial orientation of galaxies, we found the significant difference between the case when we assumed the real coordinates for galaxies in clusters and the case of the analysis of the fictitious clusters with coordinates distributed around the whole celestial sphere. In the paper we have analyzed this problem theoretically as well as show using numerical simulations how it affects the real data.

In this paper we have analyzed the sample of 247 rich Abell clusters containing at least 100 members using a significantly improved method of the investigation of the orientation of galaxies in clusters. We found that the mean values of tested statistics, obtained on the base of the analyzed sample, significantly deviated from the expected in the case of the random distributions. As a result, we could conclude that the orientations of galaxies in analyzed clusters are not random. It means that we genuinely confirmed an existence of the alignment of galaxies in rich Abells’ galaxy clusters, suggested by [53], especially by results of Crâmer-von Mises and Watson tests. Moreover, we have shown that the above results are not due to errors in measurement of position angles nor influence of background objects (also done by jackknife method).

The results that the alignment is increasing with richness and is observed in rich clusters supports the scenarios that predict such a thing (Li model, tidal torque scenario in the
hierarchical clustering model). The other scenarios like Zeldovich pancakes [2] and primordial turbulence [19] cannot explain such alignment and hence are not supported by our results.

It was natural to expect that observed alignment could not be connected with equatorial plane. Indeed, the obtained values of $\Delta_{11}/\sigma(\Delta_{11})$ statistics does not show any deviation from zero, as predicted in such cases. This result was obtained both in the case of the analysis in Equatorial and Supergalactic coordinate systems, what means that observed alignment is also not connected with Local Supercluster plane.

This result is generally independent from the clusters Bautz-Morgan types. Only cluster type BM II-III shows possible deviation from results obtained for other morphological types especially if we compare BM II-III with BM II type clusters. Our result clearly confirmed Godlowski et al. [52] opinion that, contrary to the suggestions of Aryal [77], Aryal & Saurer (2005b) [144], Aryal & Saurer (2005c) [145], Aryal [79], Aryal [51], the alignment of the orientation of galaxies is only weakly correlated with their morphological types according to the classification of Bautz-Morgan (BM). One should note that in the paper Biernacka et al. [146], during the analysis of the Binggeli effect for sample of 6188 galaxy clusters also selected from Panko & Flin [55] catalogue, the differences was found with the Binggeli effect for BM type II clusters. It sugests that both of this observations could be connected with different morphological populations of the clusters i.e. the late type clusters (BM II-III and BM III) are spiral-rich clusters.

It is important to note that the present observational results obtained for the sample of rich Abell clusters is based only on the analysis of the positions angles. It is due to the fact that we have no information connected with morphological type of members galaxies, hence the process of deprojection of the spatial orientation of galaxies from its optical images is a source of errors that are difficult to be controlled. Therefore, during the comparison of the real data with theoretical predictions and numerical simulations, we have concentrated on the analysis of the position angles only and postponed the spatial analysis of the real clusters to future studies. We should point out that our method of analysing the spatial angles is now well developed theoretically.

In the future studies, we will investigate the real samples also with the analysis of the distribution of the angles $\delta_D$ and $\eta$ giving spatial orientation of galaxies. The future investigation will be possible with the use of information about frequency of galaxy occurrence in clusters with particular morphological types, since galaxy proportions with different spectral types can be estimated on the basis of density profiles in cluster Dressler [142], Calvi et al. [140], Coenda et al. [141], Hoyle et al. [143]. We are also planning to extend our research to fewer galaxy clusters.

Finally, we would like to conclude that now we have a well-tested method of studying the orientation of galaxies in clusters that can be used for research on other data sets, such as these from the new Kilo-Degree Survey.

Acknowledgments

The authors thanks anonymous referee for detailed remarks which helped to improve the original manuscript.
A Some details of statistical tests

If we denote:

\[ A = \sum_{k=1}^{n} N_{0,k} \cos^2 2\theta_k, \quad B = \sum_{k=1}^{n} N_{0,k} \sin^2 2\theta_k, \]
\[ C = \sum_{k=1}^{n} N_{0,k} \cos^2 4\theta_k, \quad D = \sum_{k=1}^{n} N_{0,k} \sin^2 4\theta_k, \]
\[ U = \sum_{k=1}^{n} N_{0,k} \cos 2\theta_k \cos 4\theta_k, \quad W = \sum_{k=1}^{n} N_{0,k} \sin 2\theta_k \sin 4\theta_k, \]
\[ K = \sum_{k=1}^{n} (N_k - N_{0,k}) \cos 2\theta_k, \quad L = \sum_{k=1}^{n} (N_k - N_{0,k}) \sin 2\theta_k, \]
\[ M = \sum_{k=1}^{n} (N_k - N_{0,k}) \cos 4\theta_k, \quad N = \sum_{k=1}^{n} (N_k - N_{0,k}) \sin 4\theta_k \] (A.1)

and moreover

\[ Y = \sum_{k=1}^{n} N_{0,k} \cos 2\theta_k \sin 2\theta_k, \quad Z = \sum_{k=1}^{n} N_{0,k} \cos 4\theta_k \sin 4\theta_k, \]
\[ V = \sum_{k=1}^{n} N_{0,k} \cos 2\theta_k \sin 4\theta_k, \quad X = \sum_{k=1}^{n} N_{0,k} \sin 2\theta_k \cos 4\theta_k \] (A.2)

then, in the most general case, we obtain the following solution of equation (3.10). The inverse matrix to the covariance matrix of \( x \) (i.e. coefficients \( \Delta_{ij} \)) has a form:

\[ G = \begin{pmatrix}
A & Y & U & V \\
Y & B & X & W \\
U & X & C & Z \\
V & W & Z & D
\end{pmatrix} \] (A.3)

While we introduce auxiliary vector \( H \):

\[ H = \begin{pmatrix}
K \\
L \\
M \\
N
\end{pmatrix} \] (A.4)

then the resulting vector \( I \) is equal:

\[ I = \begin{pmatrix}
\Delta_{11} \\
\Delta_{21} \\
\Delta_{12} \\
\Delta_{22}
\end{pmatrix} = G^{-1} \cdot H \] (A.5)

The amplitude \( \Delta \equiv J = \sum_i \sum_j I_i^T G_{ij} I_j \) is described by 4D Gaussian distribution and expression for the required probability is the following:

\[ P(> \Delta) = (1 + J/2) \exp(-J/2). \] (A.6)
Even if we, like [36] take into account only first Fourier mode (i.e. only coefficients affiliated with $\cos 2\theta$ and $\sin 2\theta$) situation is not simple. In such a case the resulting vector $I$ (equation (3.9)) is reduced to the form:

$$I = \begin{pmatrix} \Delta_{11} \\ \Delta_{21} \end{pmatrix} \quad (A.7)$$

while auxiliary vector $H$ for:

$$H = \begin{pmatrix} K \\ L \end{pmatrix} \quad (A.8)$$

In this case $G$ matrix has form:

$$G = \begin{pmatrix} A & Y \\ Y & B \end{pmatrix} \quad (A.9)$$

while responsible covariance matrix $C = G^{-1}$ is equal:

$$C = G^{-1} = \begin{pmatrix} B / AB - Y^2 & -Y / AB - Y^2 \\ -Y / AB - Y^2 & -Y / AB - Y^2 \end{pmatrix} \quad (A.10)$$

and we obtain the following expression for the $\Delta_{i1}$ coefficients:

$$\Delta_{11} = \frac{BK - YL}{AB - Y^2} \quad (A.11)$$

$$\Delta_{21} = \frac{AL - YK}{AB - Y^2} \quad (A.12)$$

One should remember that then we take into account only first Fourier mode, then during computation of the probability, we have again 2D not 4D Gaussian distribution. So now the expression for the required probability is:

$$P(\Delta) = \exp (-J/2). \quad (A.13)$$

Such general situation as discussed above is rather unusual in practical applications and corresponds to the theoretical situation when theoretical distribution of $N_{0,k}$ is not symmetric i.e $N_{0,k} \neq N_{0,n-k}$. In the case of the analysis of the $\delta_D$ angle ($\theta = \delta_D + \pi/2$) it answers the situation when theoretical distribution is not symmetric according to the value of $\delta_D = 0$. Such theoretical models seem a bit strange, but an example could be a model with angular momentum pointed out directly to the Local Supercluster Center (Virgo Cluster center) i.e. hedgehog model. During analysis of such a model, we could take into account the fact that our Galaxy is not directly lying in the Local Supercluster plane and as a result the coordinates of the Virgo Cluster center in this supergalactic coordinate system are not $L = 0, B = 0$ but $L = 0, B = -3.19^\circ$ [37].

The case that $N_{0,k}$ are symmetric i.e $N_{0,k} = N_{0,n-k}$ (what means that for $\delta_D$ angle theoretical distribution is symmetric according value of $\delta_D = 0$) was analysed in details by Godlowski(1994a) [71]. In such a case the most important simplifications is that all auxiliary

\[1\] However please note that there are printed errors in Godlowski (1994). Most important is that eq. 18 should have form $P(\Delta) = (1 + J/2) \exp (-J/2)$. 

\[\]
values given in formulae (A.2) (i.e. $Y$, $V$, $X$, $Z$) are equal zero. As a result, if we analyze first and second Fourier mods together, solutions for coefficients $\Delta_{ij}$ have a form:

$$\Delta_{11} = CK - UM \frac{AC - U^2}{AC - U^2},$$

$$\Delta_{21} = DL - WN \frac{BD - W^2}{BD - W^2},$$

$$\Delta_{12} = -UK + AM \frac{AC - U^2}{AC - U^2},$$

$$\Delta_{22} = -WL + BN \frac{BD - W^2}{BD - W^2}$$

(A.14)

with covariance matrix $C_{\text{cov}}(x)$:

$$C_{\text{cov}}(x) = \begin{pmatrix}
\frac{C}{(AC-U^2)} & \frac{-U}{(AC-U^2)} & \frac{0}{(BD-W^2)} \\
\frac{D}{(BD-W^2)} & \frac{0}{(AC-U^2)} & \frac{-W}{(BD-W^2)} \\
\frac{0}{(AC-U^2)} & \frac{W}{(BD-W^2)} & \frac{B}{(BD-W^2)}
\end{pmatrix}$$

(A.15)

Please note when we analyzed first and second Fourier modes separately solutions are reduced to the explicit form Godlowski(1994a) [71]:

$$\Delta_{1j} = \frac{\sum_{k=1}^{n} (N_k - N_{0,k}) \cos 2J\theta_k}{\sum_{k=1}^{n} N_{0,k} \cos^2 2J\theta_k},$$

(A.16)

and

$$\Delta_{2j} = \frac{\sum_{k=1}^{n} (N_k - N_{0,k}) \sin 2J\theta_k}{\sum_{k=1}^{n} N_{0,k} \sin^2 2J\theta_k},$$

(A.17)

with the standard deviation

$$\sigma(\Delta_{1j}) = \left( \sum_{k=1}^{n} N_{0,k} \cos^2 2J\theta_k \right)^{-1/2} \approx \left( \frac{2}{nN_{0,k}} \right)^{1/2},$$

(A.18)

and

$$\sigma(\Delta_{2j}) = \left( \sum_{k=1}^{n} N_{0,k} \sin^2 2J\theta_k \right)^{-1/2} \approx \left( \frac{2}{nN_{0,k}} \right)^{1/2},$$

(A.19)

while the probability that the amplitude

$$\Delta_J = (\Delta_{1j}^2 + \Delta_{2j}^2)^{1/2}$$

(A.20)

is greater than a fixed value is now given by the formula:

$$P(> \Delta_J) = \exp \left( -\frac{1}{2} \left( \frac{\Delta_{1j}^2}{\sigma(\Delta_{1j})} + \frac{\Delta_{2j}^2}{\sigma(\Delta_{2j})} \right) \right) \approx \exp \left( -\frac{n}{4} N_0 \Delta_J^2 \right)$$

(A.21)

with standard deviation being approximately:

$$\sigma(\Delta_J) \approx \left( \frac{2}{nN_0} \right)^{1/2}$$

(A.22)
Finally the case then all $N_{0,k} = N_0$ are equal was analyzed in details by Godłowski [53]. Please note that in such a case formulae for $\Delta_{ij}$ coefficients are reduced to the explicit form

$$\Delta_{1j} = \frac{\sum_{k=1}^{n} N_k \cos 2J\theta_k}{\sum_{k=1}^{n} N_0 \cos^2 2J\theta_k},$$  \hspace{1cm} (A.23)

and

$$\Delta_{2j} = \frac{\sum_{k=1}^{n} N_k \sin 2J\theta_k}{\sum_{k=1}^{n} N_0 \sin^2 2J\theta_k},$$  \hspace{1cm} (A.24)

while formulae for probability (A.6) are reduced to the explicit form

$$P(\Delta > \Delta) = \left(1 + \frac{n}{4} N_0 \Delta^2\right) \exp\left(-\frac{n}{4} N_0 \Delta^2\right), \hspace{1cm} (A.25)$$

where amplitude $\Delta$:

$$\Delta = \left(\Delta_{11}^2 + \Delta_{21}^2 + \Delta_{12}^2 + \Delta_{22}^2\right)^{1/2}$$  \hspace{1cm} (A.26)

One should note that in the case when we analysed only first Fourier mode [148, 149] the formulae (A.23) and (A.24) for $\Delta_{11}$ are exactly the same as originaly obtained by Hawley & Peebles [36]. Also the formulae for probability that the amplitude $\Delta_{11}$ is greater than a fixed value:

$$P(\Delta_1 > \Delta) = \exp\left(-\frac{n}{4} N_0 \Delta_1^2\right)$$  \hspace{1cm} (A.27)

are exactly the same as obtained by Hawley & Peebles [36].

During analysis of the distribution of position angles ($\theta \equiv \rho$) $\Delta_{11} < 0$ means an excess of galaxies with position angles near $90^\circ$ - parallel to main plane of the coordinate system (equatorial or supergalactic in our case). It indicates the rotation axis tends to be perpendicular to the main plane. If $\Delta_{11} > 0$, then the excess of objects with position angles perpendicular to the main plane of the coordinate system is observed. Therefore, for $\Delta_{11} > 0$ the rotation axis tends to be parallel to the main plane.

We could do similar analysis for the angles giving information on the spatial orientation of galaxies. For $\theta \equiv \eta$, the positive sign $\Delta_{11}$ ($\Delta_{11} > 0$) means that projection of rotation axis for that plane tends to be directed toward $\eta = 0$. Therefore, for $\Delta_{11} < 0$ the projection of rotation axes tends to be perpendicular to zero point, respectively.

One can also deduce the direction of the departure from isotropy from the sign of $\Delta_{11}$ for distribution of $\delta_D$ angle ($\theta \equiv \delta_D + \pi/2$). If $\Delta_{11} < 0$, then an excess of galaxies with rotation axes parallel to the coordinate system main plane is observed, while for $\Delta_{11} > 0$ rotation axes tend to be perpendicular to the coordinate system main plane.

One should note that alternatively the direction of deviation from isotropy could be also obtained by computation of Directional Mean and Rayleigh’s Z statistics [119, 120]. It was proposed to use it for analysis of the orientation by Kindl [75] who proposed to use phase angle for any preferred orientation given by formulae: $\Theta_J = (2J)^{-1} \cdot \arctan(\Delta_{2J}/\Delta_{1J})$. Although the parameter is statistically interesting, it is not used in practice in the investigation of alignment of galaxies.
References

[1] P.J.E. Peebles, *Origin of the Angular Momentum of Galaxies*, *Astrophys. J.* 155 (1969) 393 [arXiv:0802.1051] [SPIRE].

[2] Ya. B. Zeldovich, *Gravitational instability: An approximate theory for large density perturbations*, *Astron. Astrophys.* 5 (1970) 84 [SPIRE].

[3] J. Blazek, Z. Vlah and U. Seljak, *Tidal alignment of galaxies*, *JCAP* 08 (2015) 15.

[4] R.G. Bower et al., *The broken hierarchy of galaxy formation*, *Mon. Not. Roy. Astron. Soc.* 370 (2006) 645 [arXiv:0511338] [SPIRE].

[5] C.B. Brook et al., *The Formation of Polar Disk Galaxies*, *Astrophys. J.* 689 (2008) 678 [arXiv:0802.1051] [SPIRE].

[6] S. Codis et al., *Connecting the cosmic web to the spin of dark halos: implications for galaxy formation*, *Mon. Not. Roy. Astron. Soc.* 427 (2012) 3320 [arXiv:1201.5794] [SPIRE].

[7] A. Giahi-Saravani and B.M. Schäfer, *Weak gravitational lensing of intrinsically aligned galaxies*, *Mon. Not. Roy. Astron. Soc.* 437 (2014) 1847 [arXiv:1302.2607] [SPIRE].

[8] J. Lee and U.-L. Pen, *Cosmic shear from galaxy spins*, *Astrophys. J.* 532 (2000) L5 [astro-ph/9911328] [SPIRE].

[9] J. Lee and U.-L. Pen, *Galaxy spin statistics and spin-density correlation*, *Astrophys. J.* 555 (2001) 106 [astro-ph/0008135] [SPIRE].

[10] J. Lee and U. Pen, *Detection of Galaxy Spin Alignments in the Point Source Catalog Redshift Survey Shear Field*, *Astrophys. J.* 567 (2002) L111.

[11] H.J. Mo, X.-H. Yang, F.C. van den Bosch and N. Katz, *Preheating by previrialization and its impact on galaxy formation*, *Mon. Not. Roy. Astron. Soc.* 363 (2005) 1155 [astro-ph/0506516] [SPIRE].

[12] J.F. Navarro, M.G. Abadi and M. Steinmetz, *Tidal torques and the orientation of nearby disk galaxies*, *Astrophys. J.* 613 (2004) L41 [astro-ph/0405429] [SPIRE].

[13] D. Paz, F. Stasyszyn and N. Padilla, *Angular momentum-Large-scale structure alignments in LCDM models and the SDSS*, *Mon. Not. Roy. Astron. Soc.* 389 (2008) 1127 [arXiv:0804.4477] [SPIRE].

[14] S.F. Shandarin, S. Habib and K. Heitmann, *Cosmic web, multistream flows, and tessellations*, *Phys. Rev. D* 85 (2012) 3005.

[15] I. Trujillo, C. Carretero and S.G. Patiri, *Detection of the effect of cosmological large-scale structure on the orientation of galaxies*, *Astrophys. J.* 640 (2006) L111 [astro-ph/0511680] [SPIRE].

[16] J. Varela, J. Betancort-Rijo, I. Trujillo and E. Ricciardelli, *The orientation of disk galaxies around large cosmic voids*, *Astrophys. J.* 744 (2012) 82 [arXiv:1109.2056] [SPIRE].

[17] P. Catelan and T. Theuns, *Evolution of the angular momentum of protogalaxies from tidal torques: Zel’dovich approximation*, *Mon. Not. Roy. Astron. Soc.* 282 (1996) 436 [astro-ph/9604077] [SPIRE].

[18] A.G. Doroshkevich, *The Origin of Rotation of Galaxies*, *Astroph. Lett.* 14 (1973) 11.

[19] G.A. Efstathiou and J. Silk, *The Formation of Galaxies*, *Fund. Cosmic Phys.* 9 (1983) 1.

[20] L.-X. Li, *Effect of the global rotation of the universe on the formation of galaxies*, *Gen. Rel. Grav.* 30 (1998) 497 [astro-ph/9703082] [SPIRE].
[23] B. Joachimi et al., *Galaxy alignments: An overview*, *Space Sci. Rev.* **193** (2015) 1 [arXiv:1504.05456] [inSPIRE].

[24] A. Kiessling et al., *Galaxy Alignments: Theory, Modelling & Simulations*, *Space Sci. Rev.* **193** (2015) 67 [arXiv:1504.05456] [inSPIRE].

[25] A.J. Romanowsky and S.M. Fall, *Angular Momentum and Galaxy Formation Revisited*, *Astrophys. J. Suppl.* **203** (2012) 17.

[26] P. Bett, V. Eke, C.S. Frenk, A. Jenkins and T. Okamoto, *The angular momentum of cold dark matter haloes with and without baryons*, *Mon. Not. Roy. Astron. Soc.* **404** (2010) 1137 [arXiv:0906.2785] [inSPIRE].

[27] T. Kimm, J. Devriendt, A. Slyz, C. Pichon, S.A. Kassin and Y. Dubois, *The angular momentum of baryons and dark matter halos revisited*, arXiv:1106.0538 [inSPIRE].

[28] D.J. Paz, M.A. Sgro, M. Merchan and N. Padilla, *Alignments of Galaxy Group Shapes with Large Scale Structure*, *Mon. Not. Roy. Astron. Soc.* **414** (2011) 2029 [arXiv:1102.2229] [inSPIRE].

[29] M.J. Pereira, G.L. Bryan and S.P.D. Gill, *Radial Alignment in Simulated Clusters*, *Astrophys. J.* **672** (2008) 825 [arXiv:0707.1702] [inSPIRE].

[30] S. Codis et al., *Galaxy orientation with the cosmic web across cosmic time*, *Mon. Not. Roy. Astron. Soc.* **481** (2018) 4753.

[31] T. Okabe et al., *Projected alignment of non-sphericities of stellar, gas and dark matter distributions in galaxy clusters: analysis of the Horizon-AGN simulation*, *Mon. Not. Roy. Astron. Soc.* **478** (2018) 1141 [arXiv:1804.08843] [inSPIRE].

[32] A. Heavens, A. Refregier and C. Heymans, *Intrinsic correlation of galaxy shapes: implications for weak lensing measurements*, *Mon. Not. Roy. Astron. Soc.* **319** (2000) 649.

[33] C. Heymans, M. Brown, A. Heavens, K. Meisenheimer, A. Taylor and C. Wolf, *Weak lensing with combo-17: estimation and removal of intrinsic alignments*, *Mon. Not. Roy. Astron. Soc.* **347** (2004) 895 [astro-ph/0310174] [inSPIRE].

[34] V.A. Stephanovich and W. Godlowski, *The Distribution of Galaxies’ Gravitational Field Stemming from Their Tidal Interaction*, *Astrophys. J.* **810** (2015) 167.

[35] S. Codis, Y. Dubois, C. Pichon, J. Devriendt and A. Slyz, *How the cosmic web induces intrinsic alignments of galaxies*, in *The Zeldovich Universe: Genesis and Growth of the Cosmic Web*, R. van de Weygaert, S. Shandarin, E. Saar and J. Einasto eds., proceedings of the International Astronomical Union, IAU Symposium, Volume 308, (2016) p. 437.

[36] D.I. Hawley and P.J.E. Peebles, *Distribution of observed orientations of galaxies*, *Astron. J.* **80** (1975) 477.

[37] P. Flin and W. Godlowski, *The orientation of galaxies in the Local Supercluster*, *Mon. Not. Roy. Astron. Soc.* **222** (1986) 525.

[38] J. Jaaniste and E. Saar, *Orientation of Spiral Galaxies as a Test of Theories of Galaxy Formation*, in *The large scale structures of the Universe*, M.S. Longair, J. Einasto and D. Reidel eds., Dordrecht (IAU Symp. 79), (1978), p. 488.

[39] B.M. Schaefer, *Galactic Angular Momenta and Angular Momentum Correlations in the Cosmological Large-Scale Structure*, *Int. J. Mod. Phys.* **18** (2009) 173.

[40] W. Godlowski, *Global and Local Effects of Rotation: Observational Aspects*, *Int. J. Mod. Phys. D* **20** (2011) 1643 [arXiv:1103.5786] [inSPIRE].

[41] E. Regos and M.J. Geller, *Infall patterns around rich clusters of galaxies*, *Astron. J.* **98** (1989) 755.

[42] A. Diaferio and M.J. Geller, *Infall regions of galaxy clusters*, *Astrophys. J.* **481** (1997) 633 [astro-ph/9701034] [inSPIRE].
[43] A. Diaferio, Mass estimation in the outer regions of galaxy clusters, Mon. Not. Roy. Astron. Soc. 309 (1999) 610 [astro-ph/9906331] [INSPIRE].

[44] K. Rines, M.J. Geller, M.J. Kurtz and A. Diaferio, CAIRNS: The Cluster And Infall Region Nearby Survey 1. Redshifts and mass profiles, Astron. J. 126 (2003) 2152 [astro-ph/0306538] [INSPiRE].

[45] H.S. Hwang and M.G. Lee, Searching for rotating galaxy clusters in SDSS and 2dFGRS, Astrophys. J. 662 (2007) 236 [astro-ph/0702184] [INSPiRE].

[46] H.M. Tovmassian, The Rotation of Groups of Galaxies, Astrophysics 58 (2015) 471.

[47] M. Kalinkov, T. Valchanov, I. Valtchanov, I. Kuneva and M. Dissanska, Rotation of the cluster of galaxies A2107, Mon. Not. Roy. Astron. Soc. 359 (2005) 1491 [astro-ph/0505091] [INSPiRE].

[48] L. Ciotti and S.N. Dutta, Alignment and morphology of elliptical galaxies: the influence of the cluster tidal field, Mon. Not. Roy. Astron. Soc. 270 (1994) 390 [astro-ph/9404059] [INSPiRE].

[49] L. Ciotti and G. Giampieri, Motion of a Rigid Body in a Tidal Field: an application to elliptical galaxies in clusters, Cel. Mec. 68 (1997) 313.

[50] W. Godłowski, M. Szydłowski and P. Flin, Some remarks on the angular momenta of galaxies, their clusters and superclusters, Gen. Rel. Grav. 37 (2005) 615 [astro-ph/0502381] [INSPiRE].

[51] E. Panko, T. Juszczyk and P. Flin, Orientation of Brighter Galaxies in Nearby Galaxy Clusters, Astron. J. 138 (2009) 1709.

[52] J.F. Jarvis and J.A. Tyson, FOCA$S$ — Faint Object Classification and Analysis System, Astron. J. 86 (1981) 476.

[53] R. Ungruhe, The Munster Red Sky Survey — Large Scale Structures in the Universe, Astron. Nachr. 329 (2008) 1 ed. Plionis, M., O. Lopez-Cruz, Hughes D. Springer: Dordrecht, 335 eds. Plionis, M., O. Lopez-Cruz, Hughes D. Springer: Dordrecht, 409.

[54] M. Ramella, W. Boschin, D. Fadda and M. Nonino, Finding galaxy clusters using voronoi tessellations, Astron. Astrophys. 368 (2001) 776 [astro-ph/010411] [INSPiRE].

[55] E. Panko, T. Juszczyk and P. Flin, Orientation of Brighter Galaxies in Nearby Galaxy Clusters, Astron. J. 138 (2009) 1709.

[56] J.F. Jarvis and J.A. Tyson, FOCA$S$ — Faint Object Classification and Analysis System, Astron. J. 86 (1981) 476.

[57] R. Ungruhe, The Munster Red Sky Survey — Large Scale Structures in the Universe, Ph.D. Thesis, Astronomisches Institut der Universität Munster, Munster, Germany, (1999).

[58] D. Carter and N. Metcalf, The morphology of clusters of galaxies, Mon. Not. Roy. Astron. Soc. 191 (1980) 325.

[59] M. Biernacka and P. Flin, Dynamic evolution of nearby galaxy clusters, Astron. Nachr. 332 (2011) 537.

[60] M. Biernacka and P. Flin, Dynamic evolution of nearby galaxy clusters, Astron. Nachr. 332 (2011) 537.

[61] M. Biernacka and P. Flin, Dynamic evolution of nearby galaxy clusters, Astron. Nachr. 332 (2011) 537.

[62] M. Biernacka and P. Flin, Dynamic evolution of nearby galaxy clusters, Astron. Nachr. 332 (2011) 537.
[65] G. Dalton, S. Maddox, W. Sutherland and G. Efstathiou, The apm galaxy survey — V. Catalogues of galaxy clusters, Mon. Not. Roy. Astron. Soc. 289 (1997) 263 [astro-ph/9701180] [inSPIRE].

[66] G.O. Abell, H.G. Corwin Jr. and R.P. Olowin, A catalog of rich clusters of galaxies, Astrophys. J. Suppl. 70 (1989) 1 [inSPIRE].

[67] A. Biviano et al., The eso nearby abell cluster survey. 3. Distribution and kinematics of emission-line galaxies, Astron. Astrophys. 321 (1997) 84 [astro-ph/9610168] [inSPIRE].

[68] A. Biviano, P. Katgert, T. Thomas and C. Adami, The ESO Nearby Abell Cluster Survey. XI. Segregation of cluster galaxies and subclustering, Astron. Astrophys. 387 (2002) 8 [astro-ph/0201540] [inSPIRE].

[69] E.J. Oepik, Preferential Orientation of Galaxies: on the Possibility of Detection, Irish Astron. J. 9 (1970) 211.

[70] W. Godlowski, Galactic Orientation Within the Local Supercluster, Mon. Not. Roy. Astron. Soc. 265 (1993) 874.

[71] W. Godlowski, Some aspects of the galactic orientation within the Local Supercluster, Mon. Not. Roy. Astron. Soc. 271 (1994) 19.

[72] E. Holmberg, On the Apparent Diameters and the Orientation in Space of Extragalactic Nebulae, Medd. Lund. Astron. Obs. Ser. VI (1946) Nr. 117.

[73] A. Sandage and G.A. Tammann, The Virgo cluster. I — The equality of mean redshifts of E and S galaxies near the cluster center, Astrophys. J. 207 (1976) L1.

[74] B. Aryal and W. Saurer, Comments on the expected isotropic distribution curves in galaxy orientation studies, Astron. Astrophys. 364 (2000) L97.

[75] E. Kindl, Observations and models of galaxy orientations, Astron. J. 93 (1987) 1024.

[76] S.N. Yadav, B. Aryal and W. Saurer, Spatial Orientation of Spin Vectors of Blue-shifted Galaxies, arXiv:1606.02881.

[77] B. Aryal and W. Saurer, Spin vector orientations of galaxies in eight Abell clusters of BM type I, Astron. Astrophys. 425 (2004) 871.

[78] B. Aryal and W. Saurer, Morphological dependence in the spatial orientations of Local Supercluster galaxies, Astron. Astrophys. 432 (2005) 431.

[79] B. Aryal and W. Saurer, Spatial orientations of galaxies in 10 Abell clusters of BM type II–III, Mon. Not. Roy. Astron. Soc. 336 (2006) 438.

[80] B.D. Percival and A.T. Walden, Spectral Analysis for Physical Applications, Cambridge University Press, Cambridge, (1993).

[81] S. Brandt, Statistical and Computational Methods in Data Analysis, third edition, Springer Verlag, New York, U.S.A., (1997).

[82] R. Johnson, D. Wichern, Applied Multivariate Statistical Analysis, third edition, Prentice Hall, (1992).

[83] H. Chernoff and E.L. Lehmann, The Use of Maximum Likelihood Estimates in $\chi^2$ Tests for Goodness of Fit, Annals Math. Statist. 25 (1954) 579.

[84] C. Domanski, Statistical nonparametric tests, Statystyczne testy nieparametryczne (in Polish), PWE, Warszawa, Poland (1979).

[85] W. Krysicki, J. Bartos, W. Dyczka, K. Królikowska and M. Wasilewski, The probability calculus and mathematical statistics in exercises (in Polish), Rachunek Prawdopodobieństwa i Statystyka Matematyczna w Zadaniach II, PWN, Warszawa, Poland (1998).

[86] G.W. Snedecor and W.G. Cochran, Statistical Methods, Iowa University Press, U.S.A., (1967).

[87] W. Godlowski, Pewne cechy strukturalne Lokalnej Supergromady (in Polish), Ph.D. Thesis, Uniwersytet Mikołaja Kopernika, Toruń, Poland, (1992).
[88] M. Lüscher, A portable high quality random number generator for lattice field theory simulations, *Comput. Phys. Commun.* **79** (1994) 100 [hep-lat/9309020] [inSPIRE].

[89] F. James, RANLUX: A FORTRAN implementation of the high quality pseudorandom number generator of Lüscher, *Comput. Phys. Commun.* **79** (1994) 111 [Erratum ibid. **97** (1996) 357] [inSPIRE].

[90] M. Luescher, [http://luscher.web.cern.ch/luscher/ranlux/](http://luscher.web.cern.ch/luscher/ranlux/), (2010).

[91] L.N. Shchur and P. Butera, The RANLUX generator: Resonances in a random walk test, *Int. J. Mod. Phys. C* **9** (1998) 607 [hep-lat/9805017] [inSPIRE].

[92] W. Godłowski, Angular momentum of galaxies in the LSC, in Proceedings of the 3rd DAEC Workshop, D. Alloin and G. Stasińska eds., Publications de l’Observatoire de Paris, (1993), p. 350.

[93] W. Godłowski, Implications of galaxy alignment for the galaxy formation problem, in 1993 Violent Star Formation - From 30 Doradus to QSO, G. Tenorio-Tagle ed., Cambridge University Press (1994), p. 274.

[94] J. Wang, W.W. Tsang and G. Marsaglia, Evaluating Kolmogorov’s Distribution, *J. Stat. Softw.* **8** (2003) 1.

[95] H. Cramér, On the composition of elementary errors First paper: Mathematical deductions, *Scand. Actuarial J.* (1928) 13.

[96] R.E. von Mises, Wahrscheinlichkeit, Statistik und Wahrheit, Julius Springer, (1928).

[97] G.S. Watson, Goodness-of-fit tests on a circle. II, *Biometrika* **49** (1962) 57.

[98] G.S. Watson, Goodness-of-fit tests on a circle, *Biometrika* **48** (1961) 109.

[99] F.J. Massey, The Kolmogorov-Smirnov test for goodness of fit, *J. Am. Statist. Assoc.* **46** (1951) 68.

[100] H.W. Lilliefors, On the Kolmogorov-Smirnov test for normality with mean and variance unknown, *J. Am. Statist. Assoc.* **62** (1967) 399.

[101] W. Godłowski and M. Ostrowski, Investigation of galactic alignment in LSC galaxy clusters, *Mon. Not. Roy. Astron. Soc.* **303** (1999) 50 [astro-ph/9901172] [inSPIRE].

[102] W. Godłowski, Some observational aspects of the orientation of galaxies, *Acta Phys. Polon. B* **42** (2011) 2323 [arXiv:1111.1777] [inSPIRE].

[103] P. Pajowska, W. Godłowski, E. Panko and P. Flin, Some aspects of the orientation of galaxies in clusters, *J. Phys. Stud.* **16** (2012) 4901 [arXiv:1301.5294] [inSPIRE].

[104] J. Heidmann, N. Heidmann and G. de Vaucouleurs, Inclination and absorption effects on the apparent diameters, optical luminosities and neutral hydrogen radiation of galaxies — I. Optical and 21-cm line data, *Mon. Not. Roy. Astron. Soc.* **75** (1972) 85.

[105] P. Fouque and G.G. Paturel, Standard photometric diameters of galaxies. II — Reduction of the ESO, UGC, MCG catalogues, *Astron. Astrophys.* **150** (1985) 192.

[106] B. Efron, Bootstrap Methods: Another Look at the Jackknife, *Ann. Stat.* **7** (1979) 1.

[107] E. Hawkins, S.J. Maddox and M.R. Merrifield, No periodicities in 2df redshift survey data, *Mon. Not. Roy. Astron. Soc.* **336** (2002) L13 [astro-ph/0208117] [inSPIRE].

[108] W.G. Cochran and G.M. Cox, *Experimental designs*, second ed., John Wiley and Sons, New York, U.S.A., (1957).

[109] F.E. Satterthwaite, An approximate distribution of estimates of variance components, *Biometrics Bulletin* **2** (1946) 110.

[110] H. Toutenburg, *Experimental Design and Model Choice: The Planning and Analysis of Experiments with Continuous or Categorical Response*, Springer Verlag, Berlin Heidelberg GmbH, (1995).
[111] E.D. Feigelson and G.J. Babu, *Modern Statistical Methods for Astronomy with R Applications*, Cambridge University Press, (2012).

[112] N.I. Fisher, *Statistical Analysis of Circular Data*, Cambridge University Press, Cambridge, (1993).

[113] W. Godlowski, K. Bajan and P. Flin, *Weak redshift discretisation in the Local Group of galaxies?, Astron. Nachr.* 327 (2006) 103.

[114] B.N.G. Guthrie and V.M. Napier, *The Virgo cluster as a test for quantization of extragalactic redshifts*, Mon. Not. Roy. Astron. Soc. 243 (1990) 431.

[115] R.G. Lake and R.C. Roeder, *An Analysis of the Distribution of Redshifts of Quasars and Emission-Line Objects*, Jour. RASC 66 (1972) 111.

[116] W.I. Newman, M.P. Haynes and Y. Terzian, *Redshift data and statistical inference*, Astrophys. J. 431 (1994) 147.

[117] A. Webster, *The clustering of radio sources. I — The theory of power-spectrum analysis. II — The 4C, GB and MCI surveys*, Mon. Not. Roy. Astron. Soc. 175 (1976) 61.

[118] J.T. Yu and P.J.E. Peebles, *Superclusters of Galaxies?, Astrophys. J.* 158 (1969) 103.

[119] E. Batschelet, *Circular Statistics in Biology*, Academic Press, London, U.K., (1981).

[120] K.V. Mardia, *Statistics of Directional Data*, Academic Press, London, U.K., (1972).

[121] P. Flin, M. Biernacka, W. Godlowski, E. Panko and P. Piwowarska, *Some properties of galaxy structures*, Baltic Astron. 20 (2011) 251 [inSPIRE].

[122] H.-J. Huang et al., *Intrinsic alignments in redMaPPer clusters — I. Central galaxy alignments and angular segregation of satellites*, Mon. Not. Roy. Astron. Soc. 463 (2016) 222 [arXiv:1605.01066] [inSPIRE].

[123] P. Brosche, *Angular Momentum Versus Mass*, Comm. Astroph. 11 (1986) 213.

[124] L. Carrasco, M. Roth and A. Serrano, *Density scaling of the angular momentum versus mass universal relationship*, Astron. Astrophys. 106 (1982) 89.

[125] P.S. Wesson, *Self-similarity and the angular momenta of astronomical systems — A basic rule in astronomy*, Astron. Astrophys. 80 (1979) 296.

[126] P.S. Wesson, *Clarification of the angular momentum/mass relation (J = pM^2) for astronomical objects*, Astron. Astrophys. 119 (1983) 313.

[127] W. Godlowski, M. Szydlowski, P. Flin and M. Biernacka, *Rotation of the universe and the angular momenta of celestial bodies*, Gen. Rel. Grav. 35 (2003) 907 [astro-ph/0404329] [inSPIRE].

[128] A. Heavens and J. Peacock, *Tidal torques and local density maxima*, Mon. Not. Roy. Astron. Soc. 232 (1988) 339.

[129] Y. Noh and J. Lee, *The alignments of disk galaxies with the local pancakes*, astro-ph/0602575 [inSPIRE].

[130] Y. Noh and J. Lee, *The Anisotropy in the Galaxy Velocity Field Originated from the Gravitational Pancaking Effect*, Astrophys. J. Lett. 652 (2006) 71.

[131] V.A. Stephanovich and W. Godlowski, *The non-Gaussian distribution of galaxy gravitational fields*, Res. Astron. Astrophys. 17 (2017) 119.

[132] B. Aryal, H. Bhattacharai, S. Dhakal, C. Rajabahak and W. Saurer, *Spatial orientation of angular momentum vectors of galaxies in six rotating clusters*, Mon. Not. Roy. Astron. Soc. 434 (2013) 1939.

[133] S.N. Yadav, B. Aryal and W. Saurer, *Preferred alignments of angular momentum vectors of galaxies in six dynamically unstable Abell clusters*, Res. Astron. Astrophys. 17 (2017) 44.

[134] D.M. Schmitz, C.M. Hirata, J. Blazek and E. Krause, *Time evolution of intrinsic alignments of galaxies*, JCAP 07 (2018) 030 [arXiv:1805.02649] [inSPIRE].
[135] J. Hao et al., *Intrinsic Alignment of Cluster Galaxies: the Redshift Evolution*, *Astrophys. J.* **740** (2011) 39 [arXiv:1103.3500] [SPIRE].

[136] H. Song and J. Lee, *Modeling the Alignment Profile of Satellite Galaxies in Clusters*, *Astrophys. J.* **748** (2012) 98 [arXiv:1106.5104] [SPIRE].

[137] W. Godłowski, E. Panko and P. Flin, *The environmental effects in the origin of angular momenta of galaxies*, *Acta Phys. Polon. B* **42** (2011) 2313 [arXiv:1111.1776] [SPIRE].

[138] H.-J. Huang, R. Mandelbaum, P.E. Freeman, Y.-C. Chen, E. Rozo and E. Rykoff, *Intrinsic alignment in redMaPPer clusters — II. Radial alignment of satellites towards cluster centres*, *Mon. Not. Roy. Astron. Soc.* **474** (2018) 4772 [arXiv:1704.06273] [SPIRE].

[139] P. Wang et al., *Alignment between satellite and central galaxies in the SDSS DR7: dependence on large-scale environment*, *Astrophys. J.* **859** (2018) 115 [arXiv:1802.10105] [SPIRE].

[140] R. Calvi, B.M. Poggianti, G. Fasano and B. Vulcani, *The distribution of galaxy morphological types and the morphology-mass relation in different environments at low redshift*, *Mon. Not. Roy. Astron. Soc.* **419** (2012) L14.

[141] V. Coenda, H. Muriel and H.J. Martinez, *Comparing galaxy populations in compact and loose groups of galaxies*, *Astron. Astrophys.* **543** (2012) 119.

[142] A. Dressler, *A catalog of morphological types in 55 rich clusters of galaxies*, *Astrophys. J. Suppl.* **42** (1980) 565.

[143] B. Hoyle, K. Masters, R.C. Nichol and S.P. Bamford, *The fraction of early-type galaxies in low redshift groups and clusters of galaxies*, *Mon. Not. Roy. Astron. Soc.* **423** (2012) 3478 [arXiv:1110.6320] [SPIRE].

[144] B. Aryal and W. Saurer, *Spin vector orientations of galaxies in seven Abell clusters of BM type III*, *Astron. Astrophys.* **432** (2005) 841.

[145] B. Aryal and W. Saurer, *Spin vector orientation of galaxies in the region 15h48m ≤ α(2000) ≤ 19h28m, −68° ≤ δ(2000) ≤ −62°*, *Mon. Not. Roy. Astron. Soc.* **360** (2005) L25.

[146] M. Biernacka, E. Panko, K. Bajan, W. Godłowski and P. Flin, *The Alignment of Galaxy Structures*, *Astrophys. J.* **813** (2015) 20.

[147] B. Aryal, P.R. Kafle and W. Saurer, *Radial velocity dependence in the spatial orientations of galaxies in and around the local supercluster*, *Mon. Not. Roy. Astron. Soc.* **389** (2008) 741.

[148] W. Godłowski, F.W. Baier and H.T. MacGillivray, *Substructures and galaxy orientations in clusters. I. The cluster Abell 754*, *Astron. Astrophys.* **339** (1998) 709.

[149] F.W. Baier, W. Godłowski and H.T. MacGillivray, *Substructures and galaxy orientations in clusters. II. Cluster Abell 14*, *Astron. Astrophys.* **403** (2003) 847.