Optimal Power Dispatch of Dispersed Sources in Direct-Current Networks with Nonlinear Loads

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Abstract. The problem of the optimal power dispatch of dispersed generators in direct-current networks under the presence of nonlinear loads (constant power terminals) is addressed through a combinatorial optimization strategy by using a master-slave solution methodology. The optimal power generation in the dispersed is solved in the master optimization stage through the application of the vortex-search algorithm. Each combination of the power outputs at the dispersed generation sources is provided to a power flow methodology known as the hyperbolic power flow approach for direct current networks. The main advantage of the proposed optimization method corresponds to the possibility of solving a complex nonlinear programming problem via sequential quadratic programming, which can be easily implemented at any programming language with low computational effort and high-quality results. The computational tests of the master-slave optimization proposal are evaluated in a 21-bus system, and the numerical results are compared with the implementation of the exact nonlinear programming model in the General Algebraic Modeling System (i.e., GAMS). All the computational results are conducted through the MATLAB programming environment licensed by Universidad Tecnológica de Pereira for academic usage.

1. Introduction
Nowadays, direct current (DC) microgrids are a powerful electrical network tendency that becomes a possible economic and reliable alternative for providing electrical services to millions of users around the world [1]. The main advantage of using DC grids lie in the possibility of integrating a lot of dispersed energy resources, such as renewable generation [3] (solar, wind technologies, among others), and energy storage systems [2] (batteries, supercapacitors or fuel-cells) with DC-DC converters or ac-DC inverters, thus reducing the back-to-back topologies required to integrate them into conventional AC power grids [4]. Those reductions must be reflected in the total cost of the dispersed energy resources, as well as lower power loss and high electrical efficiency of DC networks in comparison with their AC counterparts [5]. Another important driver of the popularization of DC grids is that they are easier to analyze and operate because disappear frequency and reactive power control, key concepts of classical AC grids [5].

One of the most important problems in DC networks is the optimal power flow problem (OPF) because it represents an essential tool for planning, operating, and controlling DC networks [5–7]. The OPF problem is interesting for the research community because it is nonlinear and non-convex, and solving it requires of purposing new and efficient methodologies [8, 9].
In the case of Colombian electrical power system, regulatory entities have impulsed the massive integration of renewable energy sources and energy storage devices in all the voltage levels; in addition, the concept of microgrids for isolated applications has also impulsed by utilities for improving their grids in terms of voltage profiles and power loss. These characteristics have converted DC power systems into a powerful alternative for improving the efficiency and reliability of the power system in Colombia, and this paper tries to contribute with powerful tools for its analysis as is the case of the OPF solvers.

According to the review of the state-of-the-art above, only two combinatorial optimization methods have been applied to solve OPF problems in DC power grids, such as the continuous genetic algorithm proposed in [1] and the black hole optimizer presented in [10]. In that sense, this paper identifies a research gap in the field and offers a VSA approach [11]. The main advantage of the proposed approach in this work, in comparison with convex methods, lies in the fact that the number of variables of the power flow problem remains constant and no eigenvalue decomposition is required to recover power flow variables [8].

To complement the proposed VSA algorithm in the master stage, a Taylor-based method recently proposed in [12] for power solution at the slave stage. This combination produces a new hybrid optimization approach ahead named VSA-TBM. Additionally, this method does not require the usage of specialized software, and it can be implemented in any programming language since its solution methodology is pure-algorithmic [1, 10].

The remainder of this paper is organized as follows: Section 2 presents the conventional OPF problem for DC power grids with a focus on the possibility of including dispersed generation in the grid by controlling their percentages of penetration. Section 3 introduces the main characteristics of the VSA method as well as its evolution process and its application to the master problem; in addition, it shows the conventional TBM formulation for solving power flow problems in DC power grids with its use to the slave problem. Section 4 describes the test system, the numerical results, as well as the comparative methods. Finally, Section 5 draws the main conclusions and possible future research derived from this work, followed by acknowledgments and references.

2. Mathematical model
The OPF problem in electrical distribution networks operated with DC technology can be represented as a nonlinear non-convex optimization problem, where the most common objective function analyzed in literature is the minimization of the total grid power losses. This objective function is constrained with the power equilibrium at each node, the voltage regulation bounds, and the maximum capabilities of power generation, among others. The most complicate constraint is the hyperbolic relation among voltages and powers in the power balance constraints [8, 9]. The entire OPF model is listed below.

**Objective function**

\[
\min p_{loss} = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} v_i v_j, \tag{1}
\]

where \(p_{loss}\) is the performance indicator (objective function) regarding the total power losses in all the distribution lines of the DC network (i.e., power losses in the resistive effect at each conductor); \(g_{ij}\) is the conductance component that connects nodes \(i\) and \(j\), which is obtained from the nodal conductance matrix; \(v_i\) and \(v_j\) are the values of the voltage at nodes \(i\) and \(j\), respectively. It is worth mentioning that \(n\) represent the total number of nodes of the electrical network under analysis.

**Set of constraints:**

\[
p_i^s = v_i \sum_{j=1}^{n} g_{ij} v_i v_j, \quad \forall i \in S, \tag{2}
\]

\[
p_i^d - p_i^d = v_i \sum_{j=1}^{n} g_{ij} v_i v_j, \quad \forall i \in N - S, \tag{3}
\]
\[ p^s_i \geq 0, \forall j \in S, \]  
\[ v_i = v_{is}, \forall i \in S, \]  
\[ 0 \leq p_{dg,i} \leq p_{dg,\text{max}}, \forall i \in N - S, \]  
\[ v_{i_{\text{min}}} \leq v_i \leq v_{i_{\text{max}}}, \forall i \in N - S, \]

where \( p^s_i \) is a vector that contains all the power injection in the slack sources; \( p_{dg,i} \) is a vector that contains all the power injections in the dispersed generators; \( p_{dg,\text{max}} \) represents the vector that contains all the power consumptions in the constant power nodes; \( p_{dg,\text{max}} \) is the maximum power generation bound in the dispersed source connected at node \( i \); \( v_{i_{\text{min}}} \) and \( v_{i_{\text{max}}} \) represent the minimum and maximum voltage limits associated at each node \( i \) contained in the distribution grid under study. Note that \( S \) is the set that contains all the slack sources.

The interpretation of the OPF model given from Equations (1) to (7) is the following: in Equation (1) is presented the performance indicator of the studied OPF problem which consists in the minimization of the total grid losses minimization; in Equations (2) and (3) are defined power equilibria constraints of the network, which are separated for the slack and demands nodes, respectively. Equations (4) and (5) define the nature of the decision variables, i.e., that the power generations must be positive definite and the constant magnitudes of the voltage profile at each slack node. On the other hand, Equations (6) and (7) define the positiveness characteristic of the power injected by dispersed sources as well as that the voltages must be contained between two positive limits known as the voltage regulation bounds.

Owing to the nonlinear non-convexity nature of the OPF problem under analysis, in the current literature can be found two main solution ways that allow reaching high-quality solutions with low computational effort. These alternatives are: i) use convex programming approximations [8, 13], or ii) apply combinatorial optimization methods based on metaheuristics [1, 10]. In this research, we adopt the second way to solve the OPF problem as will be detailed in the next section.

3. Strategy of solution
To address the OPF problem in electrical DC distribution networks here is proposed a combinatorial optimization approach based on master-slave strategy; where the master stage is guided by the vortex-search algorithm, and the slave strategy corresponds to a power flow based on the hyperbolic formulation adapted for DC grids.

3.1. Master stage
To solve the OPF problem in DC distribution networks, we proposed a master strategy based on VSA, which takes upon of defining the optimal power injections in all the dispersed generators. The general steps for implementing the VSA to solve the OPF problem are listed in Algorithm 1.

To know about all the mathematical elements and symbols presented in Algorithm 1, the following references are useful [11] and [14].

3.2. Slave stage
Power flow analysis for DC grids can be analyzed with different methodologies, including successive approximations, Gauss-Seidel, Newton-Raphson, and Taylor-based methods (TBMs), among others. Here, we select a TBM recently reported in [12]. This approach works by an iterative procedure based on a simplified Jacobian rule as presented in the recursive formula defined by Equation (8).

\[ v_d^{t+1} = \left[ G_{dd} + \text{diag}^{-2}(v_d^t) \text{diag} \left( p_{dg} - p^d \right) \right]^{-1} \left[ 2\text{diag}^{-1}(v_d^t) \left( p_{dg} - p^d \right) - G_{ds}v_s \right], \]  

where \( p_{dg} \) is a vector that contains all the power inputs in the dispersed sources, \( p^d \) is the vector that contains all the powers absorbed in the constant power terminals; \( v_d \) is a vector that includes all the
Data: Select the DC grid topology and define all the parameters of the VSA
Define the starting center of the hypersphere $\mu_m$;
Define the initial radius of the hypersphere $r_m$;
Generate the initial population $S^m$;
Verify the lower and upper bounds of all the individuals;
Evaluate each $s^m_i$ and find $s^m_{best}$;
for $m = 1 : m_{max}$ do
    Update the center of the hypersphere $\mu_{m+1}$;
    Calculate the new radius of the hypersphere $r_{m+1}$;
    Calculate the new population $S^{m+1}$;
    Verify the feasibility of all the solution individuals;
    Verify the lower and upper bounds of all the individuals;
    Evaluate all the individuals $s^{m+1}_i$ and find $s^{m+1}_{best}$;
    Revise the non-consecutive improvements of the objective function;
    if $k \geq k_{max}$ then
        Select as solution of the problem $\mu_{m+1}$;
        Return the solution of the OPF problem;
        break;
    end
end
Result: Provide the solution of the OPF problem
Algorithm 1: General application of the VSA to solve the OPF problem in DC networks

Voltage variables in the constant power terminals; $v_s$ represents a vector that includes all the voltage in the slack sources (i.e., a vector of known variables); and $G_{ds}$ and $G_{dd}$ are components of the conductance matrix that connects slack and demands nodes among them. In addition, $t$ is incremented until $t_{max}$ (maximum number of iterations) and the search stop when $\max \left\{ |v_d^{i+1} - v_d^i| \right\} \leq \epsilon$. It is important to mention that $\epsilon$ represents the maximum convergence error allowed between two consecutive iterations. In specialized literature, this parameter is typically selected as $1 \times 10^{-10}$ [12].

4. Numerical validation
The computational validation of the proposed master-slave optimization method for OPF solution in DC distribution networks is made in the 21-node test feeder [8]. This test system consists of 21 nodes and 20 lines with multiple constant power terminals. In addition, said system has a unique voltage controlled node (i.e., substation bus at node 1). All of the information of this test system can be consulted in [8]. This study analyzes the possibility of installing three dispersed generators considering penetration percentages from 20% to 60% of the total power consumption. Note that, via heuristic search methods, nodes 9, 12 and 16 were selected for DGs’ location [10].

Table 1 shows the generation at each DG considering different percentages of power penetration, total power generation, and objective function values that are total power losses in the network DC. Note that, in each simulation case, the maximum power injection allowed into the DC grid reaches 1.0954 p.u, 2.2160 p.u and 3.3240 p.u for dispersed generator penetration scenarios of 20%, 40% and 60%, respectively. In that sense, from the fifth column in Table 1, it can be noticed that the proposed VSA-TBM method as well as the GAMS optimizer use more than 99% of the maximum generation allowed to reduce total power losses in the DC network. In terms of objective function minimization, note that the method proposed in this work presents an estimation error lower than 0.30% (compared to the SCIP solver) for each simulation case. Therefore, such hybrid VSA-TBM method offers adequate numerical convergence compared to large-scale nonlinear packages.

It is important to stand out that the power delivered by each dispersed generator exhibits small
Table 1: Numerical comparison between the proposed approach and the GAMS package [15]

| Penetration [%] | DG 9 [p.u] | DG 12 [p.u] | DG 16 [p.u] | Total Gen [p.u] | Losses [p.u] | Reduction [%] |
|----------------|------------|-------------|-------------|----------------|--------------|--------------|
| 20%            | 0.0168     | 0.0405      | 1.0381      | 1.0954         | 0.1382       | 49.91        |
| 40%            | 0.2472     | 0.6981      | 1.2707      | 2.2160         | 0.0661       | 76.04        |
| 60%            | 0.8451     | 1.0294      | 1.4495      | 3.3234         | 0.0306       | 88.90        |

Solutions provided by the GAMS and the SCIP solver

| Penetration [%] | Losses [p.u] | Reduction [%] |
|----------------|--------------|---------------|
| 20%            | 0.1426       | 50.50         |
| 40%            | 0.2464       | 76.00         |
| 60%            | 0.8442       | 88.87         |

variations compared to GAMS optimizer results in the solutions obtained by means of the proposed approach. These differences can be attributed to the nonlinear continuous nature of the OPF problem and to the use of metaheuristic method, which implies that multiple dispersed generator power combinations with identical objective functions could exist. The proposed VSA-TBM method and the comparative method via GAMS implementation exhibit similar performance in terms of power losses reduction, which clearly validates the proposed approach in terms of numerical convergence. In addition, when the reduction of the final power loss respect to the case without dispersed generation are compared as can be seen in the seventh column of Table 1, we can be sure that our proposed approach is efficient and reliable for OPF analysis in relation to well-known commercial solvers.

5. Conclusion and future work

This paper presented a combination of the vortex search algorithm approach in conjunction with the Taylor series-based method for solving the OPF problem in DC power grids. The VSA method is a soft variant of the widely-known random search and pattern search algorithms, which offer an adequate trade-off between exploration and exploitation of the solution space for continuous optimization models as the case of the OPF problem. To confirm this advantage, the large-scale nonlinear optimization package SCIP solver (available for GAMS) was used as comparative method, which allowed observing the excellent numerical convergence of the proposed hybrid VSA-TBM method for OPF analysis.

As future works, the VSA-TBM method presented in this paper could be used for solving the problem of optimal location and sizing of dispersed generation in DC power grids in conjunction with binary metaheuristic techniques or use it within an economic dispatch strategy of DC networks.

References

[1] Montoya O D, Gil-González W, and Grisales-Noreña L F 2018 Optimal Power Dispatch of DGs in DC Power Grids: a Hybrid Gauss-Seidel-Genetic-Algorithm Methodology for Solving the OPF Problem. WSEAS Transactions on Power Systems, 13(13) pp 335–346.
[2] Patterson M, Macia N F, and Kannan A M 2015 Hybrid microgrid model based on solar photovoltaic battery fuel cell system for intermittent load applications. IEEE Transactions on Energy Conversion, 30(1) pp 359–366.
[3] Ellabban O, Abu-Rub H, and Blaabjerg F 2014 Renewable energy resources: Current status, future prospects and their enabling technology. Renewable Sustainable Energy Rev., 39 pp 748–764.
[4] Parhizi S, Lotfi H, Khodaei A, and Bahramirad S 2015 State of the art in research on microgrids: A review. IEEE Access, 3 pp 890–925.
[5] Garces A 2017 Uniqueness of the power flow solutions in low voltage direct current grids. Electric Power Syst. Res., 151(Supplement C) pp 149–153.
[6] Li J, Liu F, Wang Z, Low S H, and Mei S 2018 Optimal Power Flow in Stand-Alone DC Microgrids. IEEE Trans. Power Syst., 33(5) pp 5496–5506.
[7] Garces A 2018 On Convergence of Newtons Method in Power Flow Study for DC Microgrids. IEEE Trans. Power Syst., 33(5) pp 5770–5777.
[8] Montoya O D, Gil-González W, and Garces A 2019 Optimal Power Flow on DC Microgrids: A Quadratic Convex Approximation. IEEE Trans. Circuits Syst. II, 66(6) pp 1018–1022
[9] Simpson-Porco J W, Dorfler F, and Bullo F 2015 On resistive networks of constant-power devices. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 62(8) pp 811–815.

[10] Velasquez O S, Montoya O D, Garrido-Arevalo V M, and Grisales-Noreña L F 2019 Optimal Power Flow in Direct-Current Power Grids via Black Hole Optimization. *Advances in Electrical and Electronic Engineering*, 17(1) pp 24–32.

[11] Aydin O, Tezcan S S, Eke I, and Taplamacioglu M C 2017 Solving the Optimal Power Flow Quadratic Cost Functions using Vortex Search Algorithm. *IFAC-PapersOnLine*, 50(1) pp 239–244 20th IFAC World Congress.

[12] Montoya O D, Garrido V M, Gil-González W, and Grisales-Noreña L F 2019 Power Flow Analysis in DC Grids: Two Alternative Numerical Methods. *IEEE Trans. Circuits Syst. II*, pp 1–5.

[13] Montoya O D, Gil-González W, and Garces A 2019 Sequential quadratic programming models for solving the OPF problem in DC grids. *Electr. Power Syst. Res.*, 169 pp 18–23.

[14] Dogan B and Olmez T 2015 Vortex search algorithm for the analog active filter component selection problem. *AEU - International Journal of Electronics and Communications*, 69(9) pp 1243–1253.

[15] GAMS Development Corp. GAMS free demo version [Online:] https://www.gams.com/.