Staggering of the Nuclear Charge Radii in a Superfluid Model with Good Particle Number

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Abstract

A simple superfluid model with an effective four body interaction of monopole pairing type is used to explain the staggering of the charge radii in the isotope chains. The contribution of deformation and of the particle number projection are analyzed for the Sn isotopes. Good results are obtained for the staggering parameters and neutron pairing energies.

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Detailed studies of the nuclear charge radii exhibit significant deviations from an average dependence of the mass number \([1]\). The systematic study of these deviations \([1]\) revealed two important effects (1) the change in slope of the variation of the charge radii versus the neutron number when crossing a magic number and (2) the staggering of the charge radii in isotope chains (odd - even effect). These effects are found over the whole table of nuclei and are confirmed with increasing accuracy \([2] - [9]\).

The odd-even effect exhibit some important characteristics \([4]\) (a) at the beginning of a neutron shell the amplitude of the effect is rising and it decrease to the end of the shell; (b) such variations in the amplitude are almost insensitive to the proton configuration; (c) at the end of a neutron shell an increasing of the amplitude for lighter elements is observed.

Many attempts to explain the odd-even effect have been done during the last 25 years but the output was often contradictory. Uher and Sorensen \([10]\) have extensively calculated the charge radii for many isotopes chains, in the framework of pairing plus quadrupole model, allowing for monopole and quadrupole core polarization but, they were not able to describe the odd even staggering. Reehal and Sorensen \([11]\) were able to find agreement with the data for some cases taking into account the influence of the neutron blocking on the ground state quadrupole vibrations in a mixed, microscopic and liquid drop model.

Calcium isotopes have been extensively studied \([2]\), \([3]\), \([12]\), the main mechanism involved being the core polarization, either in the Isospin Projected Hartree Fock (IPHF) framework or using a simple parameterization of the neutron proton interaction in a \(j^n\) configuration. The IPHF mechanism \([13]\) gives only a qualitative agreement with the data but, the simple parameterization of Talmi \([12]\) nicely describe the charge radii of calcium isotopes. However, the fitted parameters entering Talmi’s formula have not an additional physical meaning and the realistic calculation of the core polarization contribution give a much smaller effect \([2]\). The formal extension of the \(j^n\) formula to the case of Pb isotopes has also received some criticism \([3]\).

Some peculiar cases, connected with large deformation effects, received particular attention. The large amplitude of the staggering in neutron deficient Hg and Au isotopes has
been attributed to an oblate-prolate instability \cite{14}. For the transitional nuclei Nd, Sm and Gd, it is possible to obtain, in some cases, a staggering of correct magnitude by taking into account the zero point motion of the nuclear surface and the effect of the \( h_{11/2} \) neutron intruder state \cite{15}. Other sophisticated calculations like the large shell model configuration for light nuclei \cite{16} or the HFB model taking into account the continuum states \cite{17} for Sn isotopes, have used to describe the odd-even effect.

All the above mentioned models have not been able to put in evidence an underlying mechanism for this effect, observed with few exceptions on the whole table of atomic nuclei. If successful, they pointed out some specific contributions like the \( j^n \) configuration, the prolate oblate instability, the effect of zero point surface vibration, etc. The decrease of the staggering toward the closure of the neutron shell indicate that the primary mechanism which discriminate between even and odd neutron number is pairing. This observation made by Zawischa \cite{18} has been used to find a mechanism that strongly connect the neutron pairing properties with the proton properties. He looked for an universal mechanism which involves the collective properties of the neutrons and protons. The failure of the standard nuclear models to describe the staggering indicates that some small residual interaction can strongly contribute due to the collectivity. For the neutron system the collective quantity must be the pairing tensor, which have strong odd-even variation due to the blocking mechanism.

A first order coupling of the neutron pairing tensor to the proton field is not possible in a quasiparticle picture. The only possibility is to introduce effective many body forces. Zawischa and coworkers \cite{19}, \cite{20} have introduced a separable four body interaction between protons and neutrons within a HFB model and they were able to correctly describe the effect for a large range of elements. A four body interaction was indicated by the above mentioned requirements for the underlying staggering mechanism, and by the importance of four body correlations to the alpha clustering effects \cite{21}. Finally, they have shown \cite{20}, \cite{22} that a three body interaction is enough to explain the staggering amplitude.

In a recent series of papers \cite{23}- \cite{27}, an effective four body interaction of monopole pairing type, between pairs of protons and neutrons, have been introduced and extensively
studied. The main consequences of this new interaction, when used in a variational approach with BCS wave functions, are (i) the mutual induction of the superfluidity from the neutron system to the proton one [28, 27]; (ii) the appearance of local minima in the functional energy versus the gap parameters [24, 25], which can be interpreted as metastable (superfluid isomer) states, possible identified with the $0^+_2$ state in $^{152}$Sm. The property (i) has been successfully used to described the main part of the staggering amplitude for the Pb isotopes [27]. This property represents, in fact, the necessarily strong collective coupling between the superfluid neutron system and the proton system which become superfluid. A variation in the neutron pairing tensor, due to the blocking mechanism in the odd system, induce a similar variation in the proton gap. The proton single particle distribution around the Fermi level follows this variation and the charge radii of the odd isotopes are lower than the average radii of the neighbor even isotopes. This mechanism is in agreement with the qualitative explanation given by Zawischa [18]. The HFB model include much more matrix elements which can connect the long range mean field with the pairing field; this is the reason why it includes different type of contributions to the staggering. For example, with three body forces only, the main contribution comes from the coupling of the neutron pairing tensor to the proton long range mean field [20] which lead to variations of the proton single particle wave functions, while with four body forces the main effect is given by the staggering of the proton pairing tensor induced by the neutron pairing tensor.

This debate was a stimulation for the present work. In a previous paper [27] the Pb isotope chain has been studied because for that case no large deformation effects are expected due to proton magic number. The staggering amplitude was fairly described for the $N \sim 126$ isotopes whose deformations are known to be small [28]. A partial agreement was obtained for the neutron deficient isotopes possible affected by the deformation [29]. In this work, the Sn isotopes are studied to further establish the contribution of the effective four body interaction to the staggering mechanism. The effect of the deformation is also taken into account. The Sn isotopes are almost spherical, but their $\beta_2$'s extracted from the BE2 values [28] are 3-4 times larger than the similar ones for Pb isotopes. However, their variation
with the neutron number is rather small and we expect no influence of the shape fluctuation to the staggering. It is also interesting to investigate the influence of the particle number violation (inherent in any quasiparticle description) to the odd-even staggering effect. To this goal we shall compare the results with and without the projection on good number of particles.

In our model, the protons and the neutrons are supposed to move in deformed single particle orbits described by a canonical mean field $H^{mf}$, and they are affected by a residual interaction of the usual pairing type $H^\text{pair}$ and a new four body interaction of monopole pairing type between pairs of protons and neutrons $H_4$

$$H = \sum_{i=p,n} (H_i^{mf} + H_i^{\text{pair}}) + H_4 ,$$

where

$$H_i^{mf} = \sum_{s_i\sigma_i} E_{s_i} a_{s_i\sigma_i}^+ a_{s_i\sigma_i} , \quad i = p, n ,$$

$$H_i^{\text{pair}} = -G_i P_i^+ P_i , \quad P_i = \sum_{s_i} a_{s_i} - a_{s_i}^+ ,$$

$$H_4 = -G_4 P_p^+ P_n^+ P_n P_p ,$$

$a_{s_i\sigma_i}^+$ is the creation operator of a nucleon in a single particle state with the sign of the angular momentum projection on the intrinsic symmetry axis $\sigma_i$, $s_i$ representing all the other single particle quantum numbers. The reason to preserve only this component from a full four body interaction is that all the other components seem to give only a renormalization effect to the two body strengths. This conclusion is based on the assumption that the two-body pairing strengths are phenomenological parameters, which can incorporate terms depending weakly on the extensive properties of the system, like $v^4$ (terms like $(G_p + G_{4p} \chi_p^2)$ still have small renormalization effects). As a consequence we keep only that part of the full four-body interaction which has a strong dependence on the product of the proton and neutron paring tensors. Another reason is its simplicity that permits to obtain complete solutions (in a BCS approximation), which exhibit very interesting physical properties.
The variational solutions of the proposed Hamiltonian in the space of BCS wave functions are given \[25\] by the single particle densities

\[ v_{s_i}^2(u_{s_i}^2) = \frac{1}{2}(1 - (\pm) \frac{E_{s_i} - \lambda_i}{\varepsilon_{s_i}}) , \] (5)

the quasiparticle energies

\[ \varepsilon_{s_i} = [(E_{s_i} - \lambda_i)^{1/2} + \Delta_i^{2}]^{1/2} , \] (6)

the constraints for good average number of particles

\[ N_i = \sum_{s_i} (1 - \frac{E_{s_i} - \lambda_i}{\varepsilon_{s_i}}) \] (7)

and the gap equations

\[ (G_p + G_4 \chi_n^2) \sum_{s_p} \frac{1}{\varepsilon_{s_p}} = 2 \] (8)

\[ (G_n + G_4 \chi_p^2) \sum_{s_n} \frac{1}{\varepsilon_{s_n}} = 2 \] (9)

In the above formulae the \( E_{s_i} \) are the renormalized single particle energies \[25\], \( \lambda_i \) is the Fermi level for the system \( i \),

\[ \chi_i = \sum_{s_i} u_{s_i} v_{s_i} \] (10)

is the pairing tensor for a system with an even number of particles or

\[ \chi_i = \sum_{s_i \neq s_0} u_{s_i} v_{s_i} \] (11)

for a system with an odd number of particles, when the \( s_0 \) level is blocked. The coupling constants \( G_p, G_n \) and \( G_4 \) can eventually be extracted from experimental binding energies combination \[25\] assuming that they have the following \( A \) dependence:

\[ G_p = \frac{C_p}{A} , \quad G_n = \frac{C_n}{A} , \quad G_4 = \frac{C_4}{A^2} \] (12)

The gap equations (8) - (9) are strongly coupled by the \( G_4 \) strength and the collective pairing tensor of the complementary system, due to the proposed four body interaction.
This coupling has two consequences (i) the proton system, which is in a normal phase for a magic number due to the Belyaev condition, can become superfluid due to the contribution of the $G_4\chi^2$ term; (ii) the staggering of the neutron pairing tensor induced by the blocking mechanism is fairly followed by the proton gap $\Delta_p$ (see Eq. (8)), the single particle densities $v_{sp}^2$, and finally by the charge radii

$$< r^2 > = \frac{2}{Z} \sum_{sp} < sp| r^2 | sp > v_{sp}^2 . \quad (13)$$

In particular, the charge radii are a little bit larger in an even system, due to the fact that a larger proton gap gives larger probabilities for protons to occupy higher single particle states with larger single particle radii.

This simple model has the advantage that it meets both the requirements of Zawischa and it can be extended to include quantum fluctuations. It is interesting to try to include a 3-body effective force in this type of dynamical calculations (not affecting the details of the single particle wave functions but only their superfluid properties). The most simplified form of a proton-neutron 3-body effective interaction

$$H_3 = -G_{pn} \sum_{sp,\sigma_p} a_{sp,\sigma_p}^+ a_{sp,\sigma_p} P_n^+ \{ P_n = -G_{np} \sum_{sn,\sigma_n} a_{sn,\sigma_n}^+ a_{sn,\sigma_n} P_p^+ \} ,$$

has only the effect of a renormalization of the pairing coupling constants $G_p(G_n)$ by the quantities $G_{pn}N_p(G_{np}N_n)$ proportional with the number of particles. This linear dependence of $G_p$ cannot explain any staggering. This represents a clear indication that the staggering induced by 3 body forces within the HFB model [22] originates mainly from the modification of the shape of the single particle wave functions due to the nonlinear coupling of the neutron pairing tensor to the long range mean field.

One of the drawbacks of the BCS wave functions consists in their nonconservation of the particle number. Fluctuations of the particle number give large contributions to quantities like the total energy but, they can be important also for such delicate observables like the small variation of the charge radii. In order to study such an effect, particle number projected BCS wave functions are used
\[ |\Psi| = P_A|BCS| = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{i\phi(\hat{N} - A)} \prod_{s_i} (u_{s_i} + v_{s_i} a_{s_i}^+ a_{s_i}^-)|-\rangle , \quad (14) \]

where \( \hat{N} \) is the particle number operator. The relevant matrix elements can be obtained

\[ <\Psi|\Psi| = R_0^0 , \quad (15) \]

\[ <\Psi|a_{s}^+ a_{s}|\Psi| = R_1^1(s) , \quad (16) \]

where \( R_m^m \) functions can be analytically written, but a recursive relation \[30] \[ R_m^m (s_1...s_m) = u_s^2 R_{m+1}^m(s_1...s_m) + v_s^2 R_{m+1}^m(s_1...s_m) \] is much more suitable for computer calculations. The charge radius is given by

\[ <r^2> = \frac{2}{Z R_0^0} \sum_{s_p} <s_p|r^2|s_p> v_{s_p}^2 R_1^1(s_p) . \quad (18) \]

The qualitative effect of the particle number projection on the BCS wave functions is that the single particle distribution around Fermi surface is squeezed.

Due to the fact that Sn has a proton magic number and in order to study the effects of different parameters, it is interesting to investigate this chain of isotopes in a spherical model. For the calculation of the single particle wave functions and energies, a program developed by Hird \[31\] has been used. It has the advantage that the eigenvectors are obtained in terms of the tridimensional harmonic oscillator wave function basis for which the matrix elements are analytically known.

The single particle potential is of the Woods Saxon type, allowing \( \beta_2 \) and \( \beta_4 \) deformations. The potential parameters are taken from Ref. \[32\], page 21 (quoted as Cheprunov in Table 1 of Ref. \[33\]). For computational reasons (to get double degenerate levels), the calculations have been performed with a very small deformation

\[ \beta_2 = 0.005 \quad , \quad \beta_4 = 0.0 \] .

The results are summarized in Table 1. The staggering is qualitatively obtained but the absolute results deviate from the experimental values. The main difficulty comes from the
fact that the slope of variation of the mean square radii versus the mass number A is not correctly reproduced. This difficulty has its roots in the shell model results (pairing residual interaction neglected) as can be extracted from the third column of Table 1. Possible explanations can be: no enough harmonic oscillator shells included, an inaccurate A and Z dependence of the Woods Saxon parameters, deformation effects not taken into account. The contribution from the higher harmonic oscillator shells has been carefully checked and found negligible. The parametrizations Blomqv - Wahlb, Rost and ”universal” from Table 1 in Ref. [33] have been checked but no qualitative improvements have been obtained. The effect of the deformation will be discussed in the next section.

The odd-even effect is obtained but the magnitude of the variation and the amplitude of the staggering are half the experimental one (see Table 1). In order to obtain this effect a fixed set of two and four body pairing strengths has been used

$$C_p = 33.5 \text{ MeV}, \quad C_n = 20. \text{ MeV}, \quad C_4 = 2. \text{ MeV}.$$  

These parameters can be extracted for every nucleus from the correlation energies $P_Z$, $P_N$ and $P_4$ (defined in Ref. [25]) with a relatively complicated procedure described in Ref. [25]. In this work we use the same strengths for the whole chain of isotopes and compare their values with the range given by the procedure described in Ref. [25]. The small slope of the radii versus the mass number A, force a relatively large proton pairing strength, $C_p$, and a small $C_4$ quantity (in Ref. [25] the experimentally extracted $C_4$ in the rare earth region are 10 times larger).

The staggering parameter [10]

$$\gamma_A = \frac{2[<r^2>_{A+1} - <r^2>_A]}{<r^2>_{A+2} - <r^2>_A},$$

represents an intrinsic measure of the effect. The calculated $\gamma_A$ (see Table 1) is in very good accord with the experiment, indicating that the proposed mechanism is adequate to describe the staggering.

One can try to improve the above results by including the deformation in the model. The static deformations for Sn isotopes are unknown but the $\beta_2$ values extracted from the
B(E2) values can give a fairly good approximation if the deformation is not too small. The $\beta_2$ values for the even isotopes can be extracted from the measured B(E2) values according to the following formula [34]:

$$\beta_2 \equiv \sqrt{\langle \beta_2^2 \rangle} = \sqrt{B(E2)} \frac{4\pi}{3ZR_0^2},$$  \hspace{1cm} (20)

where $R_0$ is usually taken as $1.4 A^{1/3}$ fm. The extracted $\beta_2$ values for the Sn isotopes are around 0.1 [28], 3-4 times larger than the similar values for the Pb isotopes, and their total variation is around 0.04 (see Table 2). In order to have an idea of the effect of this variation of deformation on the change in the mean square radii one can use the model of a uniformly charged deformed nucleus [4]

$$\delta < r^2 > = \delta < r^2 >_{sph} + < r^2 >_{sph} \frac{5}{4\pi} \delta < \beta^2 >. \hspace{1cm} (21)$$

According to this formula a deviation of 0.01 in $\beta_2$ can give, for nuclei with $A \sim 120$, a deviation of 0.01 in $\delta < r^2 >$. Such a magnitude is just the unit in the staggering. This classical result was checked also in a microscopic calculation.

Another effect of the deformation is that it gives a better slope of variation of the mean square radii versus the mass number $A$. The results are presented in Table 2 and Fig. 1. The deformations used in the calculations for the even nuclei have been taken from Ref. [28]. For the odd nuclei the interpolated values have been used. Figure 1 indicates that the discrepancy in slope has been reduced but the experimental results are not completely reproduced. The remaining difference can come from a better $A$ dependence of the $R_0$ Woods Saxon parameter.

The two and four body pairing strengths used in the calculation are

$$C_p = 30 \text{ MeV } , \hspace{1cm} C_n = 18 \text{ MeV } , \hspace{1cm} C_4 = 10 \text{ MeV } .$$

The proton pairing strength is not yet decreased very much but, the four body strength is near the right magnitude [23], [27]. The magnitude of the variation of the of the radii and the staggering amplitude are now closer to the experimental ones (see Fig. 2). The
staggering parameter is smaller than the experimental one (see Fig. 3), indicating that the effect is underestimated.

The effect of particle number projection is shown in Fig. 3 where the staggering parameters with and without projection are compared. The results consist in an increase of $\gamma_A$ for the projected case in the neutron rich region. This behavior is due to the fact that the amplitude of the staggering comes from the amplitude of variation in the proton gap; for the neutron rich isotopes the gap is already small in the BCS approximation but, the projection shrinks further the proton density around the Fermi level, making the staggering amplitude smaller.

It is interesting to see if the quoted pairing constants are able to reproduce the experimental pairing energies,

\[
P_Z = \frac{1}{2} [2\mathcal{E}(Z - 1, N) - \mathcal{E}(Z, N) - \mathcal{E}(Z - 2, N)] \quad (22)
\]

\[
P_N = \frac{1}{2} [2\mathcal{E}(Z, N - 1) - \mathcal{E}(Z, N) - \mathcal{E}(Z, N - 2)] \quad , (23)
\]

where the $\mathcal{E} = B$ is the binding energy. These quantities are compared to the experiment in Fig. 4. The neutron pairing energy is fairly reproduced. The shell effect around $A = 116$, connected with a shell closure for $N = 64$ is also obtained. The experimental values for the proton pairing energies, $P_Z$, are larger than 1 MeV. This give some indication that the proton system is superfluid to some extent; this effect can be explained by the 4-body neutron-proton coupling. The theoretical description of the proton pairing energies is not very accurate, especially in the neutron deficient region. This could rise some doubts about the agreement obtained for the amplitude of the staggering. The origin of this discrepancy is due to a larger $C_p$ values necessary to assure proton superfluid properties to all even isotopes in the chain. However, if one wants to describe only the neutron deficient part of the isotope chain with a fixed set of $C_i$ strengths, one could do this with a lower value of $C_p$ and, as a consequence, with a better description of the proton pairing energies. A more accurate description of the neutron number dependence for the slope of the radii and for the $G_p, G_n, G_4$ strengths (see Eq. (12)) could solve this discrepancy.
Finally, it is interesting to compare the absolute rms radii with the values experimentally extracted [4]. The results are given in the last column of Table 2. They are in good agreement with the experiment for the neutron deficient isotopes (compare to the data in the prelast column of the table). Similar discrepancy have been obtained in the more sophisticated HF calculations for Te isotopes (see Table XIV in Ref. [4]).

In conclusion, the staggering of the nuclear charge radii of the Sn isotopes has been investigated in a simple superfluid model with an effective four body interaction of monopole pairing type included. The intrinsic effect has been obtained in a simple quasispherical approximation but, the absolute results deviate from the experimental values due to the weaker dependence of the slope of the charge radii versus the neutron number. The contribution of the particle number projection and of the deformation have been studied, their effects leading to better absolute results. This simple model has two interesting features: (i) it fulfill the qualitative physical requirements of the staggering mechanism discussed by Zawischa; (ii) it can be extended to include quantum fluctuations like particle number projection, RPA pairing and quadrupole vibrations in the ground state or fluctuations in the gauge space connected with the gaps. Its parameters have known physical meaning and its ability to describe the staggering data, make it a good candidate model to check the contributions of different mechanisms to the odd-even staggering of the charge radii.

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Table Captions

Table 1  Relative and the variation of the mean squared charge radii (in $fm^2$) for Sn isotopes. Experimental values are taken from Ref. [7]. Theoretical values obtained in the quasispherical model. Last two columns present results for the staggering parameter, Eq. (19).

Table 2  Charge radii results with deformations included. First three columns give the relative mean square charge radii in $fm^2$. $\beta_2$ values are taken from Ref. [28]. Last two columns show the absolute charge radii in $fm$; experimental values from Ref. [5].
Figure Captions

Figure 1  Relative mean squared charge radii for Sn isotopes. Experimental values (squares and solid line) taken from Ref. [7]. Theoretical values (circles and solid lines) are calculated with four body forces and deformation included. Theoretical values calculated with projected wave functions are denoted by pluses and are connected by dashed lines. All lines are drawn to guide the eye.

Figure 2  Staggering of the mean squared charge radii. The significance of signs and lines is the same as in the caption to Fig. 1.

Figure 3  Theoretical staggering parameter compared with the experimental one. The significance of signs and lines is the same as in the caption to Fig. 1.

Figure 4  Neutrons (left) and protons (right) pairing energies given by Eqs. (23) and (22). Experimental values marked by squares and solid line. Theoretical values (circles and dashed lines) are calculated with four body forces and deformation included. All lines are drawn to guide the eye.
| A  | Exp. | Shell model | 4N model | $\delta < r^2 >_{A,A-1}$ | $(\gamma A)_{exp}$ | $(\gamma A)_{4N}$ |
|----|------|-------------|----------|--------------------------|-------------------|-------------------|
| 110| -0.638 | -0.439 | -0.360 | | 0.736 | 0.876 |
| 111| -0.586 | -0.388 | -0.323 | 0.037 | | |
| 112| -0.497 | -0.339 | -0.275 | 0.048 | 0.908 | 0.806 |
| 113| -0.438 | -0.293 | -0.244 | 0.031 | | |
| 114| -0.367 | -0.246 | -0.198 | 0.046 | 0.687 | 0.706 |
| 115| -0.322 | -0.202 | -0.173 | 0.025 | | |
| 116| -0.236 | -0.159 | -0.127 | 0.046 | 0.758 | 0.718 |
| 117| -0.189 | -0.118 | -0.103 | 0.024 | | |
| 118| -0.112 | -0.077 | -0.060 | 0.043 | 0.750 | 0.707 |
| 119| -0.070 | -0.038 | -0.039 | 0.021 | | |
| 120| 0.0 | 0.0 | 0.0 | 0.039 | 0.832 | 0.732 |
| 121| 0.042 | 0.038 | 0.019 | 0.019 | | |
| 122| 0.101 | 0.074 | 0.052 | 0.033 | 0.725 | 0.723 |
| 123| 0.134 | 0.109 | 0.069 | 0.017 | | |
| 124| 0.192 | 0.144 | 0.099 | 0.030 | | |
| 125| 0.225 | 0.178 | 0.112 | 0.013 | | |
| A  | Exp.  | 4N model | Projected | $\beta_2$ | rms exp. | rms 4N model |
|----|-------|----------|-----------|----------|----------|--------------|
| 110| -0.638| -0.504   | -0.512    | 0.126    | 4.5820   |              |
| 111| -0.586| -0.462   | -0.467    | 0.1245   | 4.5866   |              |
| 112| -0.497| -0.386   | -0.390    | 0.1227   | 4.5958   | 4.5949       |
| 113| -0.438| -0.329   | -0.328    | 0.1208   | 4.6011   |              |
| 114| -0.367| -0.257   | -0.259    | 0.119    | 4.6103   | 4.6089       |
| 115| -0.322| -0.230   | -0.229    | 0.1155   | 4.6118   |              |
| 116| -0.236| -0.151   | -0.159    | 0.1118   | 4.6261   | 4.6203       |
| 117| -0.189| -0.135   | -0.135    | 0.111    | 4.6320   | 4.6220       |
| 118| -0.112| -0.074   | -0.077    | 0.1106   | 4.6395   | 4.6287       |
| 119| -0.070| -0.057   | -0.054    | 0.109    | 4.6448   | 4.6306       |
| 120| 0.0   | 0.0      | 0.0       | 0.1075   | 4.6522   | 4.6367       |
| 121| 0.042 | 0.015    | 0.024     | 0.1055   | 4.6383   |              |
| 122| 0.101 | 0.070    | 0.072     | 0.1036   | 4.6633   | 4.6442       |
| 123| 0.134 | 0.086    | 0.106     | 0.100    | 4.6460   |              |
| 124| 0.192 | 0.135    | 0.150     | 0.0953   | 4.6736   | 4.6512       |
| 125| 0.225 | 0.165    | 0.191     | 0.092    | 4.6545   |              |
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Figure 4
