Probing the strange quark condensate by di-electrons from $\phi$ meson decays in heavy-ion collisions at SIS energies

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Abstract

QCD sum rules predict that the change of the strange quark condensate $\langle \bar{s}s \rangle$ in hadron matter at finite baryon density causes a shift of the peak position of the di-electron spectra from $\phi$ meson decays. Due to the expansion of hadron matter in heavy-ion collisions, the $\phi$ peak suffers a smearing governed by the interval of density in the expanding fireball, which appears as effective broadening of the di-electron spectrum in the $\phi$ region. The emerging broadening is sensitive to the in-medium change of $\langle \bar{s}s \rangle$. This allows to probe directly in-medium modifications of $\langle \bar{s}s \rangle$ via di-electron spectra in heavy-ion collisions at SIS energies with HADES.

Keywords: heavy-ion collisions, di-electrons, in-medium modifications of vector mesons
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I. INTRODUCTION

The starting heavy-ion collision experiments with the High Acceptance DiElectron Spectrometer (HADES) [1] at the heavy-ion synchrotron SIS at GSI/Darmstadt provide the rare opportunity to measure via the di-electron channel ($e^+e^-$) in-medium modifications of the light vector mesons with high accuracy. High accuracy is particularly important for the $\phi$ meson since the commonly expected change of the $\phi$ meson spectral function in a nuclear medium is not too strong.

From the theoretical side there are at least two motivations for studying in detail the in-medium modifications of the $\phi$ meson. First, as shown within various QCD sum rule approaches [2,3] the mass shift of the $\phi$ meson in nuclear matter is directly related to the in-medium change of the strange quark condensate $\langle \bar{s}s \rangle$ and, therefore, provides a direct access to the chiral symmetry restoration in the strange sector [4]. This is in contrast to the $\rho$ and $\omega$ mesons whose in-medium mass shifts are sensitive to both chiral condensate $\langle \bar{q}q \rangle$ and the poorly known four-quark condensate $\langle (\bar{q}q)^2 \rangle$, in particular for the $\omega$ meson [3]. (For the $\rho$ meson there is also a strong broadening effect expected [5,6], so that the $\rho$ meson can even disappear as a quasi-particle in a dense nucleon medium.) The second reason to focus on the $\phi$ meson in-medium effects is related to the great interest in measuring and
understanding the strangeness content of the nucleon and nuclear matter [7]. This issue has a lot of physical applications in various branches such as deep-inelastic lepton-nucleon scattering, production of strange particles in hadron and nuclear collisions, speculations on the nature of dark matter etc.

Within model approaches based on effective hadronic Lagrangians the $\phi$ meson mass in a nuclear medium is affected by the $\phi N$ scattering. Because of the small real part of the $\phi N$ scattering amplitude the corresponding change of the $\phi$ meson mass appears to be tiny in nuclear matter at saturation density. The $\phi$ meson mass shift remains still small in such approaches even if one takes into account various effects related to the $K\bar{K}$ in-medium loop in the $\phi$ meson self-energy [5,8]. This means that the main cause of a possible $\phi$ meson mass shift in a nuclear medium is connected with the change of the density dependent $\langle \bar{s}s \rangle$ condensate which actually acts here as a density dependent mean field [2,4,9]. The most consistent way to incorporate mean field effects is through the QCD sum rules, which match directly the in-medium spectral function with the density dependent strange quark condensate.

In the present work we employ the results [2,3] of the QCD sum rule evaluations of the $\phi$ meson mass shift at finite baryon density to find the possible modifications of the di-electron spectrum in the region of the $\phi$ peak in heavy-ion collisions at SIS energies. Due to the collective expansion of the matter formed in the course of heavy-ion collisions the baryon density $n$ depends on time so that a time average of the mass shift is expected. As a result, the mass shift is smeared out over an interval related to the temporal change of the density and looks like an "effective" broadening of the $\phi$ peak. It is remarkable that this effective $\phi$ broadening reflects essentially the in-medium behavior of the strange condensate $\langle \bar{s}s \rangle$ but is almost insensitive to the collision broadening. This offers a chance to probe the in-medium strange quark condensate via measuring the effective broadening of the $\phi$ peak in the $e^+e^-$ spectrum.

To get the di-electron spectrum from the $\phi$ meson decays we use a transparent hydrodynamical model for the space-time evolution of the matter including the obvious possibility that the $\phi$ meson does not chemically equilibrate. In spite of the schematic character of our dynamical model, it delivers results which are confirmed by the transport model calculations of BUU type [10].

II. DI-ELECTRON SPECTRA FROM IN-MEDIUM $\phi$ DECAYS

Within the linear density approximation and for not too high temperatures $T < 100$ MeV the strange quark condensate in hadronic matter can be written as

$$\langle \bar{s}s \rangle_{\text{matter}} = \langle \bar{s}s \rangle_0 + \sum_h \frac{\langle h|\bar{s}s|h \rangle}{2M_h} n_h,$$

where $\langle \bar{s}s \rangle_0$ is the vacuum condensate, $\langle h|\bar{s}s|h \rangle$ denotes the matrix element corresponding a one-hadron state (we employ the normalization $\langle h|h \rangle = 2E_h$ with $E_h \approx M_h$ with $E_h$ and $M_h$ as respective energy and mass of the hadron species $h$), $n_h$ stands for the hadron density, and the sum runs over all hadrons in the medium, i.e., $h = N, \Delta, \Sigma, K, \cdots$. For the
nucleon matrix element $\langle N|\bar{s}s|N\rangle$ the dimensionless parameter $y$ is widely used to specify the strangeness content in the nucleon via

$$\frac{\langle N|\bar{s}s|N\rangle}{2M_N} = y \frac{\sigma_N}{2m_q},$$

where $\sigma_N$ is the nucleon sigma term and $m_q$ the light quark mass $m_q = \frac{1}{2}(m_u + m_d)$. Since the strangeness content of the nucleon is a yet poorly known quantity and matter of debate so far (see, for instance, the QCD lattice calculations in [11,12] and the phenomenological study in [13]) we shall vary below $y$ in the interval $0 \cdots 0.2$ to get the corresponding modifications of the di-electron spectra. We also simplify eq. (1) by the replacement

$$\sum_h \langle h|\bar{s}s|h\rangle^2 \frac{1}{2M_h} n_h \to y \frac{\sigma_N}{2m_q} n_N$$

keeping in mind that the presence of other hadron states in the sum can only increase the value of $y$ [9], so that our choice reflects a lower limit of the strangeness content of hadronic matter.

Basing on the above parameterization of the strange quark condensate in hadron matter one can perform the QCD sum rule evaluations which give for the $\phi$ meson mass the density dependence in leading order [3]

$$m_\phi = m_\phi^{\text{vac}} (1 - 0.14y \frac{n_N}{n_0}),$$

where $m_\phi^{\text{vac}}$ denotes the vacuum value of the $\phi$ mass, and the nuclear matter saturation density is $n_0 = 0.15 \text{ fm}^{-3}$.

The di-electron rate from the in-medium $\phi$ meson decays in ideal gas approximation at temperature $T$ is given by

$$\frac{dN}{d^4x d^4Q} = \frac{6}{(2\pi)^3} \exp \left( -\frac{u \cdot Q}{T} \right) M \Gamma_{\phi \to e^+ e^-} A(M^2, m_\phi, \Gamma_{\phi}^{\text{tot}})$$

where $\Gamma_{\phi \to e^+ e^-}$ and $\Gamma_{\phi}^{\text{tot}}$ are the di-electron and total decay widths of the $\phi$ meson, $Q_\mu$ denotes the four-momentum of the di-electron with invariant mass $M^2 = Q^2$, and $u_\mu$ is the four-velocity of the emitting medium. We use the Breit-Wigner parameterization of the spectral function

$$A(m, m_\phi, \Gamma_{\phi}^{\text{tot}}) = \frac{1}{\pi} \frac{m_\phi \Gamma_{\phi}^{\text{tot}}}{(m^2 - m_\phi^2)^2 + (m_\phi \Gamma_{\phi}^{\text{tot}})^2},$$

having in mind that the $\phi$ meson may survive as single-peaked quasi-particle excitation [5], while in-medium the width is expected to be noticeably enlarged, at least by collision broadening to $\Gamma_{\phi}^{\text{tot}} = 20 \cdots 30 \text{ MeV}$. Due to four-momentum conservation in the direct decay $\phi \to e^+ e^-$ one also gets the relation $m^2 = M^2$ interrelating eqs. (4, 5).

To obtain the di-electron spectrum from the rate (4) one needs to specify the space-time evolution of the matter. According our experience [14] the detailed knowledge of the space-time evolution is not necessary to describe the experimental di-electron spectra in relativistic heavy-ion collisions in the low-mass region [15]. In this line, and also to avoid too many parameters, we employ here a variant of the blast wave model (cf. [16]) with a constant radial expansion velocity $v_r = 0.3$. We take for the initial baryon density $n_N(t = 0) = 3n_0$,
as suggested by transport code simulations for the maximum density at SIS-18 (cf. [18]),
and \( n_N(t_{f.o.}) = 0.3n_0 \) for the freeze-out density. The evolution of the matter, i.e., \( n(t) \), is
defined by the baryon conservation within the fireball volume; we use as baryon participant
number \( N_B = 330 \), and \( T(t = 0) = 90 \) MeV as initial temperature of the systems. These
numbers are to be understood as spatial averages, i.e., gradients are not explicitly taken
into account. The freeze-out temperature \( T_{f.o.} = 50 \) MeV, deduced in [21] from hadro-
chemistry, and the initial temperature are linked by entropy conservation. A convenient
parameterization for the relevant section of the isentrope with specific entropy 5 per baryon
is, e.g., \( T(t) = (a + bn(t))^{1/6} \) with \( a \) and \( b \) given by the initial and freeze-out conditions.

Within the described fireball dynamics we consider two cases for the evolution of the \( \phi \)
multiplicity:
(i) the number of \( \phi \) mesons is assumed to be governed by chemical equilibrium, so that it is
proportional to \( \exp(-m_\phi/T) \), and
(ii) the \( \phi \) mesons are still in thermal equilibrium with the bulk of matter but do not maintain
chemical equilibrium; to be specific, we take the number of as constant thus assuming a weak
\( \phi \) inelasticity. This assumption is supported by BUU transport calculations [18].

After freeze-out, the \( \phi \) meson decays in vacuum contribute to the di-electron spectrum
according to
\[
\frac{dN}{dM^2} = Br_{\phi \rightarrow e^+e^-} N_\phi(T_{f.o.}, M^2) A(M^2, m_\phi^{\text{vac}}, \Gamma_{\phi}^{\text{tot,vac}}),
\]
where \( Br_{\phi \rightarrow e^+e^-} \) is the branching ratio of the \( \phi \) decay \( \phi \rightarrow e^+e^- \) in vacuum, and \( N_\phi(T_{f.o.}, m_\phi) \)
is the number of of \( \phi \) mesons at freeze-out. This number is given in the spherical expansion
model for case (i) by
\[
N_\phi(T_{f.o.}, m_\phi) = \frac{1}{(2\pi)^3} \frac{4\pi R_{f.o.}^3}{\gamma} \int dp_{\perp}^2 \sqrt{2\pi T_{f.o.}/\gamma m_\perp} \exp \left( -\frac{\gamma m_\perp}{T_{f.o.}} \right) \times
\left[ \frac{\sinh a_{f.o.}(\gamma m_\perp + T_{f.o.})}{a_{f.o.}} - T_{f.o.} \cosh a_{f.o.} \right]
\]
where \( m_\perp = \sqrt{m_\phi^2 + p_{\perp}^2} \), \( a_{f.o.} = \gamma v_r p_{\perp}/T_{f.o.} \) and \( p_{\perp} \) is the transverse momentum of the \( \phi \)
meson; \( \gamma \) is the Lorentz factor related to the transverse expansion velocity \( v_r \).

The number of \( \phi \) mesons in case (ii) can be parameterized by including in eq. (7) an
off-equilibrium chemical potential which is governed by the condition \( N_\phi = \text{const} \) during the
expansion.

Note that, at present, the understanding of the \( \phi \) dynamics in heavy-ion collisions at
threshold energies is still in the infancy. The data base is poor [19], and the interpretation
by transport models meets uncertainties [18,20]. While the multiplicities of other hadrons,
after freeze-out, can fairly well be described by a statistical (equilibrium) model [21], the
situation for the \( \phi \) mesons remains unsettled. We are, therefore, left with the two above
extreme scenarios.
III. RESULTS

The results of our calculations of the di-electron invariant mass spectrum from \( \phi \) decays are exhibited in fig. 1 for the case (i), i.e., where the \( \phi \) meson is assumed to be in chemical equilibrium. As seen here the in-medium shift of the strange quark condensate, specified by the parameter \( y \), causes a noticeable shift of the peak position due to contributions of \( \phi \) decays inside the medium. One can even expect a double peak structure [22] which emerges from the superposition of vacuum and in-medium contributions. Due to the smearing of the \( \phi \) meson mass shift over an interval of density of the expanding fireball, one can actually observe an effective broadening of the second "shifted" peak. Such an effective broadening increases almost linearly with the parameter \( y \) independently of the value of the \( \phi \) meson width \( \Gamma_{\phi}^{\text{tot}} \): As displayed in fig. 2, the linear dependence of the effective broadening on the strange quark condensate still holds if one varies the width \( \Gamma_{\phi}^{\text{tot}} \) from the vacuum value up to the sizeable value of 20 MeV caused by \( \phi N \) collisions and predicted by effective hadronic interaction Lagrangians [5]. This offers a chance to probe the strange quark condensate in hadronic matter (and thus the parameter \( y \)) via measuring the effective broadening of the second "shifted" peak in the \( \phi \) meson region of the di-electron spectrum.

Due to the assumed chemical equilibrium, the number of \( \phi \) mesons drops rapidly with decreasing temperature. Therefore, the di-electron signal from \( \phi \) mesons decaying after freeze-out appears rather small, in contrast to naive expectations assuming a "life time of 45 fm/c". The peak height of the vacuum decay contribution varies strongly with changing values of \( T_{f.o.} \).

In fig. 3 we show the in-medium modifications of the di-electron spectrum from \( \phi \) mesons in the case of chemical off-equilibrium with fixed number of \( \phi \) mesons during the evolution. Here the in-medium modification of the strange quark condensate is reflected by the appearance of the characteristic l.h.s. shoulder additionally to the vacuum \( \phi \) peak. The effective broadening of the shoulder still increases linearly with the parameter \( y \) and is insensitive to variations of the width \( \Gamma_{\phi}^{\text{tot}} \) within the given range. In contrast to the above assumed chemical equilibrium situation, the vacuum decay contribution is here much stronger. This is since the \( \phi \) multiplicity is kept constant, according to our scenario (ii) and in line with [18]. Correspondingly, also the in-medium decay is somewhat stronger than in case (i). Nevertheless, the separation of this in-medium \( \phi \) signal will be an experimental challenge.

We have here displayed only the di-electron spectra from exclusive decays \( \phi \to e^+e^- \). Actually, there will be contributions from \( \phi \) Dalitz decays which give additional strength, sufficiently far at the low-mass side of the direct \( \phi \) signal. Both contributions sit on a steeply falling background which emerges essentially from the high-mass \( \rho \) decays, bremsstrahlung etc. As shown in [16], the \( \phi \) signal may be clearly separated from this background. For further studies of the feasibility of identifying the \( \phi \) signal within the HADES acceptance we refer the interested reader to [23].

IV. SUMMARY

In summary, the expansion of nuclear matter after the maximum compression stage in the course of heavy-ion collisions is expected to cause a broadening of the di-electron
spectrum in the $\phi$ meson region. The amount of the broadening appears to be almost insensitive to the collision broadening in the nuclear medium. Rather, the broadening is directly related to the dependence of the strange quark condensate $\langle \bar{s}s \rangle$ on the nucleon density which, in turn, is related to the strangeness content of the nucleon. This gives a good opportunity to probe the corresponding in-medium modification of $\langle \bar{s}s \rangle$ with HADES in heavy-ion collisions. The impact parameter dependence, which could be used to study details of the density dependence of $\langle \bar{s}s \rangle$, deserves further investigations. Supplementary information can be gained from the di-electron spectrum in the $\phi$ region in reactions of elementary projectiles with nuclei.

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FIG. 1. Di-electron spectra from $\phi$ meson decays as a function of invariant mass for case (i), i.e., assumed chemical equilibrium. "vac" labels the decay contribution after freeze-out, while the curves labeled by $y = 0$, 0.1 and 0.2 depict the in-medium decay contributions for various values of the strangeness content of the nucleon. The total width of the $\phi$ meson is assumed to be $\Gamma^{\text{tot}} = 20$ MeV.

FIG. 2. The full width half maximum of the in-medium $\phi$ peak in the $e^+e^-$ mass spectrum as a function of the strangeness content $y$ of nuclear matter for two values of the total $\phi$ meson decay width $\Gamma^{\text{tot}}$.

FIG. 3. As in fig. 1 but for case (ii), i.e., chemical off-equilibrium of $\phi$ mesons.