Angular electromagnetic response of double-ring metamaterials for TE polarization

J.H. Shi1,*, N. Papasimakis2, V.A. Fedotov2, Z.P. Wang1 and N.I. Zheludev2
1College of Science, Harbin Engineering University, Harbin 150001, China
2Optoelectronics Research Center, University of Southampton, Southampton SO171BJ, UK

* E-mail: shijinhui@hrbeu.edu.cn

Abstract We investigate transmission characteristics in symmetric planar double-ring metamaterials at oblique angles of incidence theoretically. The double-ring microwave metamaterials are used to study resonant response to incident TE electromagnetic wave. The trapped-mode resonant feature results from the excitation of antisymmetric current mode and the quality factor is associated to the angle of incidence.

1. Introduction
Metamaterials have attracted a tremendous amount of attention in recent years since it was fabricated to exhibit simultaneously negative values of effective permeability $\mu_{\text{eff}}(\omega)$ and permittivity $\varepsilon_{\text{eff}}(\omega)$ at microwave frequencies $[1,2]$. And planar metamaterials or metafilms are very thin metal films that are patterned on a subwavelength scale. Initially used as microwave frequency selective surfaces (FSS) $[3,4]$, metafilms now attract attention due to their unusual functionalities ranging from invisibility $[5]$, magnetic mirror $[6]$ and optical magnetism $[7]$ to asymmetric transmission $[8]$ and optical activity $[9]$, and have become a subject of intense research in the terahertz domain demonstrating application potential in wave manipulation and sensing $[10-13]$. The response of metafilms can also be engineered to mimic electromagnetically induced transparency (EIT) of atomic systems $[14-19]$. The spectral response of most FSSs, however, depends strongly on the polarization and, more importantly, on the angle of incidence of incident electromagnetic waves. Limited attempts were made to reduce their angular sensitivity $[20-24]$, which led to a more complex structures exploiting bilayered compositions $[25]$ and, more recently, fractal $[26,27]$ and volume split-ring resonators $[28]$. Further, Metamaterials with polarization and direction insensitive resonant transmission response mimicked EIT $[29]$. However, the oblique incidence of electromagnetic wave provides some desirable properties of artificial composites. The oblique incidence resulted in optical activity of metamaterials without chirality $[30]$ and split resonance $[31]$. The oblique response of a double negative metamaterial slab was investigated to show that the negative index characteristics remain nearly the same up to incidence angle of 45° $[32]$. One fishnet metamaterial was demonstrated tuning of the transmission resonance when the oblique incidence occurred $[33]$. 

* To whom any correspondence should be addressed.
In this letter, Fano-type resonances in the planar double-ring metamaterials was reported for TE polarization incidence theoretically that insensitively depend on angles of incidence, not resulting from breaking the symmetry of the metamaterial structure.

2. Double-ring metamaterial structure
The double-ring metamaterial structure has a simple design ideally suited for the existing manufacturing planar micro- and nanotechnologies, which was used to mimic electromagnetically induced transparency experimentally and show resonant features that weakly depend on polarization and angles of incidence. Here, we provide theoretical support to the experimental results of double-ring trapped-mode metamaterial showing resonant transmission spectra. The metamaterial was formed by a planar array of concentric subwavelength ring resonators, which had ever been considered for achieving negative refraction in bulk composites. The double-ring pattern was etched from 35 µm copper cladding covering FR4 PCB substrate of 1.6 mm thickness. The unit cell of the pattern contains one double-ring structure and is a square having size of 15 × 15 mm². The metamaterial structure is illustrated in figure 1, showing geometrical parameters. The linewidth of metallic rings is 0.4 mm and the mid-radii of inner and outer ring are 4.5 mm and 5.45 mm, respectively. In the experiment, the sample size was only 220 µm × 220 µm that sufficiently generated strong electromagnetic response. Such a metamaterial did not diffract at frequencies below 10 GHz for any angle of incidence. However, for the simplification the overall size of the sample is infinite in the model, which enables us to conduct transmission for any incident angle ranging from 0° to 90°. The metal pattern was treated as a perfect conductor and the substrate was assumed to be a lossy dielectric with ε = 4.05 - i0.1.

The origin of the narrow resonance can be traced to the excitation of the so-called trapped mode, a high-Q mode formed by counter-propagating current. Such a mode normally inaccessible but can be excited if, for example, the metamaterial symmetry is appropriately broken. As a result of the interference with continuum-like spectrum of the excitation the mode acquires an asymmetrically shaped resonance line characteristic to the well-known Fano resonance. The trapped mode nature of the observed pass-band was confirmed by numerical simulations based on method of moments.

![Figure 1](image)

Figure 1 Double-ring metamaterial structure. Dashed box indicates its elementary unit cell.

3. Simulation results
The double-ring metamaterial transmission was measured in an anechoic chamber in the frequency range of 4-10 GHz using broadband horn antennas (Schwarzbeck BBHA9120D) equipped with dielectric lens concentrators and a vector network analyzer (Agilent E8364B). Here, we present the theoretical transmission spectrum of double-ring metamaterial at angles of incidence ranging from 0°
to $80^\circ$ for TE polarization. Figure 2 shows a typical response of the planar metamaterial to incident linearly polarized waves, which was recorded in transmission.

![Graph showing transmission spectrum of double-ring metamaterial at angle of incidence for TE polarization.](image_url)

**Figure 2** Transmission spectrum of double-ring metamaterial at angle of incidence for TE polarization. The red dashed line indicates zero dB level.

It should be noted that although the data are presented for TE polarization, the metamaterial also reveals weakly changed angular response for TM polarization (not shown here). The simulated spectra of double-ring metamaterial illustrated in figure 2 show a good agreement with the measured one in Ref. 29. For the metamaterial, transmission pass-bands always happen at any angle of incidence and the electromagnetic response weakly depends on the angle of incidence. The peak transmission at the trapped-mode resonance gradually decreases with the increasing angle of incidence, but the transmission peaks are kept unmoved. Evidently, the spectrum of double-ring metamaterial reveals a sharp resonance near 6.2 GHz (marked as II), which is accompanied by two much weaker resonances (marked as I and III) corresponding to transmission dips. For the normal incidence, the sharp spectral response has the width of 0.8 GHz as measured at 3 dB below the maximum and the quality factor $Q$ of such response is about 8. While the sharp spectral response has the width of 0.15 GHz as measured at 3 dB below the maximum and the quality factor $Q$ of such response is up to 40 for the glazing incidence though a large reflection loss exists. The relation between the quality factor and the angle of incidence is shown in figure 3. Only large angle of incidence, even to the glazing incidence, leads to the dramatic change of $Q$, where the metamaterial shows strong coupling with the magnetic component of incident electromagnetic.

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antisymmetric current configuration of double metallic rings results in the electromagnetic energy confined to the metamaterial surface, revealing weak coupling to the free space and sharp resonance.

![Figure 3](image1.png)

**Figure 3** Angular response of the quality factor of double-ring metamaterial.

![Figure 4](image2.png)

**Figure 4** Absolute surface current distribution of double-ring metamaterial corresponding to the resonances.

4. Conclusions
In summary, the planar double-ring metamaterial exhibiting Fano-type resonance at oblique angles of incidence was demonstrated for TE polarization theoretically. The transmission resonances insensitively depend on the angles of incidence. The transmission response of double-ring metamaterials can show its unique functionality as wide-angular applications. Achieving the trapped-mode resonances will be especially important for low-loss metamaterials.

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Reference
[1] Veselago V G 1968 *Sov. Phys. Usp.* **10** 509
[2] Shelby R A, Smith D R, and Schultz S 2001 *Science* **292** 77
[3] Vardaxoglou J 1997 *Frequency Selective Surfaces* (Research Studies, England).
[4] Munk B A 2000 *Frequency Selective Surfaces: Theory and Design* (Wiley, New York)
[5] Fedotov V A, Mladenov P L, Prosvirnin S L, and Zheludev N I 2005 *Phys. Rev. E* **72** 056613
[6] Sievenpiper D, Zhang L, Broas R, Alexopolous N, and Yablonovitch E 1999 *IEEE Trans. Microwave Theory Tech.* **47** 2059
[7] Linden S, Enkrich C, Dolling G, Klein M W, Zhou J, Koschny T, Soukoulis C M, Burger S, Schmidt F, and Wegener M 2006 E J. Sel. Top. Quantum Electron. 12 1097
[8] Fedotov V A, Mladyonov P L, Prosvirnin S L, Rogacheva A V, Chen Y, and Zheludev N I 2006 Phys. Rev. Lett. 97 167401
[9] Plum E, Fedotov V A, and Zheludev N I 2008 Appl. Phys. Lett. 93 191911
[10] Chen H T, Padilla W J, Zide J M O, Gossard A C, Taylor A J, and Averitt R D 2006 Nature 444 597
[11] Singh R, Smirnova E, Taylor A J, O’Hara J F, and Zhang W 2008 Opt. Express 16 6537
[12] Singh R, Azad A K, O’Hara J F, Taylor A J, and Zhang W 2008 Opt. Lett. 33 1506
[13] O’Hara J F, Singh R, Brener I, Smirnova E, Han J, Taylor A J, and Zhang W 2008 Opt. Express 16 1786
[14] Fedotov V A, Rose M, Prosvirnin S L, Papasimakis N, and Zheludev N I 2007 Phys. Rev. Lett. 99 147401
[15] Papasimakis N, Fedotov V A, Zheludev N I, and Prosvirnin S L 2008 Phys. Rev. Lett. 101 253903
[16] Zhang S, Genov D A, Wang Y, Liu M, and Zhang X 2008 Phys. Rev. Lett. 101 047401
[17] Liu N, Kaiser S, and Giessen H 2008 Adv. Mater. 20 4521
[18] Tassin P, Zhang L, Koschny T, Economou E N, and Soukoulis C M 2009 Phys. Rev. Lett. 102 053901
[19] Singh R, Rockstuhl C, Lederer F, and Zhang W 2009 Phys. Rev. B 79 085111
[20] Roberts A and McPhedran R C 1988 IEEE Trans. Antennas Propag. 36 607
[21] Shaker J and Shafai L 1993 Electron. Lett. 29 1655
[22] Huang J, Wu T, and Lee S 1994 IEEE Trans. Antennas Propag. 42 166
[23] Wu T K and Lee S W 1994 IEEE Trans. Antennas Propag. 42 1484
[24] Van Labeke D, Gerard D, Guizal B, Baïda F I, and Li L 2006 Opt. Express 14 11945
[25] Shaker J and Shafai L 1995 IEEE Microw. Guid. Wave Lett. 5 324
[26] Zhou L, Chan C T, and Sheng P 2004 J. Phys. D 37 368
[27] Wen W, Zhou L, Hou B, Chan C T, and Sheng P 2005 Phys. Rev. B 72 153406
[28] Baena J D, Jelinek L, Marques R, Mock J J, Gollub J, and Smith D R 2007 Appl. Phys. Lett. 91 191105
[29] Papasimakis N, Fu Y H, Fedotov V A, Prosvirnin S L, Tsai D P, and Zheludev N I 2009 Appl. Phys. Lett. 94 211902
[30] Plum E, Liu X X, Fedotov V A, Chen Y, Tsai D P, and Zheludev N I 2009 Phys. Rev. Lett. 102 113902
[31] Enkrich C, Wegener M, Linden S, Burger S, Zschie Fridrich L, Schmidt F, Zhou J F, Koschny T, and Soukoulis C M 2005 Phys. Rev. Lett. 95 203901
[32] Alici K B and Ozbay E 2009 Opt. Lett. 34 2294
[33] Minovich A, Neshev D N, Powell D A, Shadrivov I V, Lapine M, McKerracher I, Hattori H T, Tan H H, Jagadish C, and Kivshar Y S 2010 Phys. Rev. B 81 15109
[34] Fano U 1961 Phys. Rev. 124 1866
[35] Zheludex N, Prosvirnin S L, Papasimakis N, and Fedotov V A 2008 Nat. Photonics 2 351