Acknowledgments

We acknowledge the support of the following agencies: CIEMAT, the Spanish Ministry of Science and Innovation, and the Spanish Government.

References

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CIEMAT Technical Report 1499 *

Abstract

We classify the possible deviations from the Standard Model in the QED-dominated $e^+e^- \rightarrow \gamma\gamma$ process under the assumption of a preserved $SU(2)_L \times U(1)_Y$ symmetry. We find that the only deviations really observable in practice correspond to a correction of the differential cross section by a factor $(1 + \frac{c_8}{\Lambda^4} \sin^2 \theta)$, where $\Lambda$ is the scale of new physics, $\theta$ is the polar angle of any of the final state photons and $c_8$ is a constant of order 1. We also provide sensitivity estimates for QED deviations at future $e^+e^-$ facilities. An $e^+e^-$ collider operating at $\sqrt{s} = 3$ TeV could provide sensitivity to $\Lambda$ scales as large as 15 TeV, provided that acceptances and efficiencies are controlled at the per mille level. Finally, we also discuss the possibility of a measurement of the luminosity at the FCC-ee with $\lesssim 10^{-4}$ precision, using analyses of the $e^+e^- \rightarrow \gamma\gamma$ process at $\sqrt{s} \approx m_Z$ energies.

1 Introduction

The successful confrontation of the Standard Model (SM) \[1,2,3\] to measurements at present collider experiments provides strong evidence for an underlying $SU(2)_L \times U(1)_Y$ symmetry in particle physics interactions. Despite the impressive level of agreement between SM predictions and experimental observations, there are known weak points in the scheme: gravitational interactions not considered, unnatural and unexplained origin of mass hierarchies, insufficient amount of CP violation, implications of the presence of massive neutrinos, unknown unification scheme of fundamental forces at high energy, ... The search for new physics beyond the SM is one of the main goals of present and future high-energy experiments and, within this context, classifying and constraining the most sensitive terms is an important exercise.

The $e^+e^- \rightarrow \gamma\gamma$ process has been used since long time as a golden channel to test the validity of QED [4, 5, 6]. Experimentally, its signature is clean and systematic uncertainties can be easily controlled [7, 8, 9, 10]. In these latest analyses, performed at LEP2 energies [11], deviations were quantified in terms of effective Lagrangians that respect the underlying QED $U(1)$ symmetry [12]. In this report we present instead a prospect study of the possible deviations under the assumption of a preserved $SU(2)_L \times U(1)_Y$ symmetry, as suggested by the overwhelming evidence for the validity of the SM up to the electroweak scale. As we will see, this scheme leads to a simpler scheme to study and isolate new physics effects.

The measurement of the $e^+e^- \rightarrow \gamma\gamma$ cross section has also been been suggested as reference for an ultra-precise determination of the luminosity at future Higgs factories running at the electroweak scale, owing to the fact that the QCD uncertainties in this process are expected to...
contribute at the $\lesssim 10^{-5}$ level [13]. This has to be compared with the current $\approx 10^{-4}$ relative uncertainties expected from Bhabha scattering at very low polar angles, which will likely require alignment precisions of the luminosity monitors at the micron level [13] [15]. The advantages and potential issues related with the use of the $e^+e^- \rightarrow \gamma\gamma$ process as luminometer reference is also addressed in this report.

In general, the study of neutral gauge boson pair production offers information complementary to the $e^+e^- \rightarrow f\bar{f}$ and $e^+e^- \rightarrow W^+W^-$ processes, and the observation of new physics effects in $e^+e^- \rightarrow \gamma\gamma$ will either confirm or rule out some of the possibilities. An interesting example is the possible existence of strong gravitational interactions at the TeV scale [16] [17], which should manifest simultaneously in all channels.

The classification of the relevant operators preserving the $SU(2)_L \times U(1)_Y$ symmetry is presented in the first sections of the report. We discuss which terms are acceptable, redundant or essentially excluded by present observations. The second part of the study is devoted to phenomenological and experimental studies assessing the new physics reach of the process at future $e^+e^-$ Higgs factories and the experimental uncertainties expected in the measurement of the $e^+e^- \rightarrow \gamma\gamma$ cross section at a Tera-Z facility like the one proposed at a future FCC-ee collider.

2 $e^+e^-\gamma\gamma$ contact interactions and $SU(2)_L \times U(1)_Y$ symmetry

We will assume that new physics: a) lies above the electroweak scale, b) respects the $SU(2)_L \times U(1)_Y$ symmetry, and c) decouples from the SM at low energy. The possible deviations are parametrized in terms of effective Lagrangians containing inverse powers of the scale of new physics, $\Lambda$:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{n=5}^{\infty} \frac{1}{\Lambda^{n-4}} \left( \sum_j c_{nj} \mathcal{O}_{nj} \right)$$

The index $n$ is the dimension of the $\mathcal{O}_{nj}$ operator and the sum on $j$ is extended to all possible terms respecting the symmetry. Provided that the $\mathcal{O}_{nj}$ operator is present and is hermitian, the Wilson coefficients $c_{nj}$ are expected be real and of order 1. In the limit in which new physics lies much above the electroweak scale and the characteristic scale of the collision, $\Lambda \gg v, \sqrt{s}$ ($v = 246$ GeV), terms of dimension $n$ are suppressed at least by factors of order $(\frac{\text{max}(v,\sqrt{s})}{\Lambda})^{n-4}$ and only the Lagrangians with the lowest dimension become relevant. The following standard notation is employed in the following:

- left-handed SM iso-doublet containing the electron: $e_L$,
- right-handed electron iso-singlet: $e_R$,
- gauge fields associated to $SU(2)_L$: $W^I_\mu$,
- field strengths associated to $SU(2)_L$: $W^I_{\mu\nu} \equiv \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g \epsilon_{IJK} W^J_\mu W^K_\nu$,
- gauge field associated to $U(1)_Y$: $B_\mu$,
- field strength associated to $U(1)_Y$: $B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$,
- photon field: $A_\mu \equiv \cos \theta_w B_\mu + \sin \theta_w W^3_\mu$,
- $Z$ field: $Z_\mu \equiv \cos \theta_w W^3_\mu - \sin \theta_w B_\mu$, 

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• Higgs field: $\Phi$. In the unitary gauge: $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$.

• covariant derivatives:
  - for fermions and the Higgs particle: $D_\mu \equiv \partial_\mu - ig\tau^I W^I_\mu - ig'Y B_\mu$,
  - for $SU(2)_L$ tensor strengths: $D_\mu W^I_{\nu\lambda} \equiv \partial_\mu W^I_{\nu\lambda} + g \epsilon_{IJK} W^J_\mu W^K_{\nu\lambda}$,
  - for the $U(1)_Y$ tensor strength: $D_\mu B_{\nu\lambda} \equiv \partial_\mu B_{\nu\lambda}$,
  - and dual tensors: $\tilde{W}^I_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} W^I_{\rho\lambda}$, $\tilde{B}_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} B_{\rho\lambda}$.

In the previous expressions $g$ and $g'$ are the $SU(2)_L$ and $U(1)_Y$ coupling constants, respectively, and $\theta_w$ is the Weinberg angle. The Pauli matrices are denoted by $\tau^I; I \in \{1, 2, 3\}$, the value of the hypercharge is $Y$ and the electromagnetic coupling constant satisfies: $e \equiv 4\pi\sqrt{\alpha} = g\sin\theta_w$. We will also assume a massless electron. This assumption leads to many simplifications, and it is certainly justified from the phenomenological and experimental points of view.

2.1 Effective Lagrangians of dimension six

No Lagrangians containing $e^+e^-\gamma\gamma$ couplings and satisfying the general conditions described above can be built at dimension five. At dimension six, it is well known that all potential $e^+e^-\gamma\gamma$ contact terms are redundant with operators of higher order. The reason is that they can be expressed via classical equations of motion (EOMs) as combinations of other operators that do not involve fields with two fermions and two bosons $[18, 19]$. The immediate consequence is that all new physical effects will be at least of order $\mathcal{O}((\text{energy})^2/\Lambda^4)$.

Still, one can consider indirect ways of connecting $e^+e^-$ and $\gamma\gamma$ states via dimension-six operators. The exchange of neutral gauge bosons in the s-channel is excluded because no anomalous triple neutral gauge boson couplings of dimension six preserving $SU(2)_L \times U(1)_Y$ exist. The exchange of Higgs particles is another possibility, but a large anomalous coupling to electrons is in contradiction with the absence of observed effects in the $e^+e^- \rightarrow e^+e^-$ cross section for Higgs masses up to the weak scale. Finally, one expects contributions from operators of the electric or magnetic dipole type: $\sim (\epsilon_L \sigma^\mu\nu \Phi R) F_{\mu\nu}$. They provide effects of order $\mathcal{O}(v^2s/\Lambda^4)$ because they connect electron states with different chirality and therefore do not interfere with the SM amplitude. They are suppressed with respect to other physical effects at a similar scale because they must be created via loop level diagrams $[20]$, and present a dependence as a function of the polar angle equivalent to that of the dimension-eight operators that are discussed in the next section. Last but not least, they are extremely constrained by low-energy precision experiments $[21, 22]$.

2.2 Effective Lagrangians of dimension eight

No Lagrangians of dimension seven with $e^+e^-\gamma\gamma$ couplings and satisfying the condition of $SU(2)_L \times U(1)_Y$ invariance can be built $[24]$. At dimension eight, and despite the apparent increase in the number of possibilities, most of the potential terms can be effectively ignored, thus easing the classification task. CP-odd terms and all operators that connect electrons of different chirality via $\overline{e_L}\Phi e_R$, $\overline{e_L}\sigma^{\mu\nu}\Phi e_R$ components can be dropped. They give contributions of order $\mathcal{O}((\text{energy})^4/\Lambda^8)$ because they do not interfere with the SM process. The application of EOMs also helps in reducing the number of possible structures.

The starting point is a fermionic component of the type $\overline{e_L}\gamma^\mu e_R$. It can be complemented with covariant derivatives ($D^\mu$), gauge fields ($F^{\mu\nu}$, $\tilde{F}^{\mu\nu}$) and Higgs fields by pairs ($\Phi^I, \Phi$), but it
only terms with one covariant derivative and two gauge fields lead to genuine e^+e^−γγ coupling terms. Terms with five or three derivatives are equivalent to terms with more gauge fields and less derivatives. Using partial integration, the EOMs and the Bianchi identities we can also ignore in the classification terms containing derivatives of F_{\mu\nu} and \tilde{F}_{\mu\nu} fields. The following final set is obtained:

\[ O_{eR eR BB} = (i \bar{e}_R \gamma_\mu D_\nu e_R) B^{\mu\rho} B_\rho^\nu + h. c. \]  
\[ O_{eL eL BB} = (i \bar{e}_L \gamma_\mu D_\nu e_L) B^{\mu\rho} B_\rho^\nu + h. c. \]  
\[ O_{eR eR WW} = (i \bar{e}_R \gamma_\mu D_\nu e_R) W^{I \mu\rho} W^I_\rho^\nu + h. c. \]  
\[ O_{eL eL WW} = (i \bar{e}_L \gamma_\mu D_\nu e_L) W^{I \mu\rho} W^I_\rho^\nu + h. c. \]  
\[ O_{eL eL WB} = (i \bar{e}_L \tau^I \gamma_\mu D_\nu e_L) W^{I \mu\rho} B_\rho^\nu + h. c. \]  
\[ O_{eL eL BW} = (i \bar{e}_L \tau^I \gamma_\mu D_\nu e_L) B^{\mu\rho} W^I_\rho^\nu + h. c. \]

All operators are structurally similar. Consequently, only the following structure can contribute to the e^+e^- → γγ process at order \( O((\text{energy})^2/\Lambda^4) \) and at the same time respect the underlying SM symmetry:

\[ O_{ee γγ} \rightarrow \left[ i \bar{e}_\gamma_\mu \frac{1 \pm \gamma_5}{2} \partial_\nu e \right] A^{\mu\rho} A_\rho^\nu + h. c. \]  

2.3 Beyond dimension eight

The next level of deviations from the SM corresponds to operators of dimension ten. We may create new e^+e^-γγ couplings by combining operators of dimension eight with additional Higgs fields or covariant derivatives by pairs. The leading corrections in this case are of order \( O((\text{energy})^3/\Lambda^6) \), which can be absorbed via a redefinition of the dimension-eight couplings via form factors.

Beyond dimension ten, one should consider not only dimension-twelve Lagrangians, but also the dimension-eight terms with contributions of order \( O((\text{energy})^4/\Lambda^8) \). Although highly suppressed, some of these operators (e.g., \( \tau e_R A^{\mu\nu} A_{\mu\nu} \)), provide differential cross sections that are experimentally distinguishable from the one predicted by the \( O_{ee γγ} \) Lagrangian.

3 A simple behavior for the leading deviations in the e^+e^- → γγ process

According to the previous discussion, the lowest-order deviations in the SM e^+e^- → γγ process allowed by \( SU(2)_L \times U(1)_Y \) invariance can be quantified using the following Lagrangian:

\[ O_8 = \frac{1}{\Lambda^4} \left[ \bar{e}_\gamma_\mu \left( f_{SL} \frac{1 - \gamma_5}{2} + f_{SR} \frac{1 + \gamma_5}{2} \right) i \partial_\nu e \right] A^{\mu\rho} A_\rho^\nu + h. c. \]  

where \( f_{SL}, f_{SR} \) are constants of order unity and \( \Lambda \) is the possible scale of new physics. The resulting differential cross section, including SM and new physics contributions, is:

\[ \frac{d\sigma}{d\Omega} = \frac{1}{2} \left( \frac{d\sigma}{d\Omega} \right)_{SM} \left[ \left( 1 + \frac{f_{SL} s^2}{16\pi\alpha\Lambda^4} \sin^2 \theta \right)^2 + \left( 1 + \frac{f_{SR} s^2}{16\pi\alpha\Lambda^4} \sin^2 \theta \right)^2 \right] \]  

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where $\theta$ is the polar angle between the photon and any of the beam particles, and $\Omega$ the solid angle, with $\cos \theta$ defined in the $[0,1)$ range. The term $(d\sigma/d\Omega)_{SM}$ is the Born-level QED differential cross section:

$$
\frac{(d\sigma)}{(d\Omega)}_{SM} = \frac{\alpha^2}{s} \frac{1 + \cos^2 \theta}{\sin^2 \theta}
$$

As expected, the effect is similar to equivalent dimension-8 $e^+e^-\gamma\gamma$ contact terms that were previously proposed \cite{24,25,26} and, at first order of deviation $(\propto s^2/\Lambda^4)$, to the one predicted by the inclusion of QED cutoff parameters $\Lambda_{\pm}$ in the electron propagator \cite{4}. Indeed, the $\Lambda$ scale coincides numerically with $\Lambda_{\pm}$ for new interactions with vector-like couplings of electromagnetic size:

$$
c_8 = f_{8L} = f_{8R} = \pm e^2 \Rightarrow \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{SM} \left( 1 \pm \frac{s^2}{2\Lambda_{\pm}^4} \sin^2 \theta + \frac{s^4}{16\Lambda_{\pm}^8} \sin^4 \theta \right)
$$

Note that the leading $O(s^2/\Lambda_{\pm}^4)$ deviations are either positive or negative depending on the sign of $c_8$, but the total cross section always stays positive.

Reducing the study of all dominant QED deviation effects to the study of only one operator ($O_8$) has strong implications. First, any conceivable effect on the $e^+e^- \rightarrow \gamma\gamma$ process from decoupled new physics must be of order $O(s^2/\Lambda^4)$ or smaller. Second, the only allowed deviation at this order is a relative increase or decrease of the differential cross section in the central region of detectors, precisely following a $\sin^2 \theta$ behaviour. Examples of new physics searches in this channel are excited electrons \cite{5,6,27} or the possible manifestation of extra-dimensional gravity effects at the TeV scale \cite{16,17,28}. Both types of signals indeed follow the required dependence at order $O(s^2/\Lambda^4)$, despite their totally different physics origin (magnetic-like coupling in t-channel versus spin-2 exchange in s-channel).

Some of the new Lagrangians experimentally exploited at LEP \cite{11}, denoted by $\Lambda_6, \Lambda_7$ and $\Lambda_8$, were proposed in Reference \cite{12}. By construction they were not expected to respect the $SU(2)_L \times U(1)_Y$ symmetry, but just the $U(1)$ QED symmetry. However, it is illustrative to understand why they do not appear to be relevant in our classification:

- $i(\bar{e}_\mu \gamma^\nu D_\nu e)(g_6 F^{\mu\nu} + \tilde{g}_6 F^{\mu\nu})$: this Lagrangian is redundant at order $O(s/\Lambda^2)$ simply because it has dimension six \cite{19}. This is confirmed by the leading $\Lambda^{-4}$ dependence of the deviations and its angular dependence, which is exactly reproduced by the dimension-eight operator $O_8$ introduced above.

- $\frac{1}{3!}(g_7^S F^{\mu\nu} + ig_7^A \gamma_5 F^{\mu\nu})e F_{\mu\nu}$: this dimension-seven Lagrangian is not $SU(2)_L$-invariant, unless we add a Higgs field connecting the fermionic fields. After that modification it becomes a Lagrangian of dimension eight. The predicted deviations are suppressed by an additional factor $\frac{\alpha}{\Lambda^2}$, due not only to the inclusion of the Higgs field, but also to the lack of interference with the Standard Model process (it connects electron states of different chirality). The combined effect is an operator with a leading contribution of order $O(s^4/\Lambda^8)$, which was neglected in our classification of leading deviation terms.

- $\frac{1}{2} \epsilon^{\mu\nu\lambda\rho}(g_8^V - g_8^A \gamma_5)(\partial_\mu F^{\alpha\beta}) F_{\alpha\beta}$: this dimension-eight Lagrangian is insensitive to new physics from a practical point of view. It is related via the classical equations of motion with a Lagrangian proportional to the electron mass. This explains the almost negligible contribution obtained in Reference \cite{12}, the poor scale limits obtained at LEP and why it was discarded a priori in our approach.
4 Expectations at future $e^+e^-$ colliders

We will first evaluate the statistical sensitivity at different energies and luminosities assuming an extended likelihood fit to the differential distribution:

$$\frac{d\sigma}{d\cos \theta} = \frac{2\pi\alpha^2}{s} \frac{1 + \cos^2 \theta}{\sin^2 \theta} \left[ 1 + \lambda \frac{s^2}{2}\sin^2 \theta \right]$$

(13)

where $\cos \theta$ is defined in the $[0, 1)$ range and the parameter $\lambda$ provides a direct connection with the electromagnetic cut-off parameter limits obtained in past experiments: $\lambda \equiv \pm 1/\Lambda^4 \equiv \pm f/(e^2\Lambda^4)$.

In a real high-precision analysis, one will take into the exact shape of the differential distribution including higher order corrections (see also discussion in the next section) and adopt an appropriate definition of the event polar angle to account for cases where additional hard photons are emitted. At LEP [11] it was found that a simple definition that implicitly assumes additional beam-collinear radiation was adequate: $\cos \theta \equiv \tanh(|y_1 - y_2|/2)$, where $y_1$ and $y_2$ are the rapidities of the two high-energy selected photons. This should probably be complemented in the future with an additional boost of the system in the transverse direction to account for cases with additional non-collinear photons, similarly to the Collins-Soper treatment typically used at hadron colliders [29].

LEP studies also showed that an acollinearity cut between the two selected photons provided a better agreement between the true distribution and a pure Born-level treatment, mostly at high polar angles [11]. These details should be of course polished in the future, using more precise theoretical calculations - beyond the permille level of accuracy - once they become available. In our study, and given the fact that we are only interested in the level of precision that can be be reached, a pure Born-level description will be assumed. A likelihood fit to the previous angular distribution of Equation 13 in the absence of signal ($\lambda = 0$) leads to the following uncertainty on $\lambda$:

$$\Delta \lambda = \frac{2}{s^2\sqrt{<\sin^4 \theta>}} \frac{1}{\sqrt{N_{ev}}}$$

(14)

where $N_{ev}$ is the number of selected $\gamma\gamma$ events and $<>$ denotes an average value over this sample. The statistical uncertainty $\Delta \lambda$ was determined assuming a parabolic behavior around the minimum of the fit. This is justified because $\Delta \lambda$ is expected to be rather small for the integrated luminosities of future colliders. Assuming a large constant acceptance ($\approx 100\%$) and a measurement in the region $\cos \theta < c_0$, an approximate estimate of $N_{ev}$ and $<\sin^4 \theta>$ is given by:

$$N_{ev} \approx L \frac{2\pi\alpha^2}{s} \left( \log\left(\frac{1 + c_0}{1 - c_0}\right) - c_0 \right),$$

(15)

$$<\sin^4 \theta> \approx \frac{(c_0 - c_0^2/3)}{\log\left(\frac{1 + c_0}{1 - c_0}\right) - c_0}$$

(16)

where $L$ is the integrated luminosity. The results of the fit for different $e^+e^-$ colliders, energies and proposed luminosities is collected in Table 1 for a cut $c_0 = 0.95$.

As expected, the sensitivity to new physics increases dramatically with the collision energy, due to the $s^2/\Lambda^4$ dependence. A CLIC collider at $\sqrt{s} = 3$ TeV will be sensitive to scales as large
Table 1: Achievable statistical uncertainties on $\lambda$, 68% CL relative cross section variations due to potential new physics effects, $\Delta\sigma_{NP}/\sigma_{SM}$, and 95% confidence level on the scales $\Lambda_\pm, \Lambda$ in the absence of QED deviations. The values are determined for different $e^+e^-$ collider options, energies and luminosities. We assume an angular fiducial cut of $\cos\theta < 0.95$ and an acceptance approaching 100%. The present combined LEP results [11] are also shown for comparison.

| Collider option | $\sqrt{s}$ [TeV] | $L$ [ab$^{-1}$] | $\Delta\lambda$ [TeV$^{-4}$] | $\Delta\sigma_{NP}/\sigma_{SM}$ | $\Lambda_\pm$ limit [TeV] | $\Lambda$ limit [TeV] |
|-----------------|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| FCC-ee          | 0.09             | 150.0           | $6.7 \times 10^{-4}$ | $1.1 \times 10^{-3}$ | 0.9             | 1.7             |
| FCC-ee          | 0.16             | 10.0            | $4.8 \times 10^{-1}$ | $7.2 \times 10^{-5}$ | 1.1             | 1.8             |
| FCC-ee          | 0.24             | 5.0             | $2.0 \times 10^{-1}$ | $1.5 \times 10^{-4}$ | 1.3             | 2.3             |
| FCC-ee          | 0.35             | 1.5             | $1.2 \times 10^{-1}$ | $4.0 \times 10^{-4}$ | 1.4             | 2.6             |
| CEPC            | 0.09             | 16.0            | $2.0 \times 10^{-1}$ | $3.2 \times 10^{-5}$ | 0.7             | 1.3             |
| CEPC            | 0.16             | 2.6             | $9.4 \times 10^{-1}$ | $1.4 \times 10^{-4}$ | 0.9             | 1.6             |
| CEPC            | 0.24             | 5.6             | $1.9 \times 10^{-1}$ | $1.4 \times 10^{-4}$ | 1.3             | 2.3             |
| ILC             | 0.25             | 2.0             | $2.8 \times 10^{-1}$ | $2.5 \times 10^{-4}$ | 1.2             | 2.1             |
| ILC             | 0.50             | 4.0             | $2.5 \times 10^{-2}$ | $3.5 \times 10^{-4}$ | 2.1             | 3.8             |
| CLIC            | 0.38             | 1.0             | $1.1 \times 10^{-1}$ | $5.4 \times 10^{-4}$ | 1.4             | 2.6             |
| CLIC            | 1.50             | 1.5             | $1.5 \times 10^{-3}$ | $1.7 \times 10^{-3}$ | 4.3             | 7.8             |
| CLIC            | 3.00             | 5.0             | $1.0 \times 10^{-4}$ | $1.9 \times 10^{-3}$ | 8.3             | 15.2            |

as 15 TeV if acceptances and efficiencies can be controlled at the per mille level ($\Delta\sigma_{NP}/\sigma_{SM}$ column in Table 1). At collision energies below 1 TeV systematic effects will have to be controlled at the $10^{-4}$ level or so in order to fully profit from the available statistics. For FCC-ee measurements at the Z pole and at the WW thresholds, the luminosity uncertainty, $\Delta L \approx 10^{-4}$, is therefore a limiting factor.

One could also search for new physics effects using exclusively the shape of the differential distribution, in order to be independent of luminosity uncertainties. The likelihood function is in this case not extended, and the corresponding expression for $\Delta\lambda$ is:

$$\Delta\lambda = \frac{2}{s^2 \sqrt{<\sin^4\theta> - <\sin^2\theta>^2}} \frac{1}{\sqrt{N_{ev}}}$$

with:

$$<\sin^2\theta> \approx \frac{\left(c_0 + \frac{c_1^2}{3}\right)}{\left(\log\left(\frac{1+c_0}{1-c_0}\right) - c_0\right)}$$

The results of this alternative fit are reported in Table 2. While the sensitivity to new physics effects is reduced by almost a factor of two as expressed in terms of the $\lambda$ parameter, there is just a mild $\approx 15\%$ degradation in the $\Lambda_\pm$ limit.

5 Measuring the luminosity using $e^+e^- \rightarrow \gamma\gamma$ events

The $e^+e^- \rightarrow \gamma\gamma$ process has been proposed as alternative reference reaction for a precise determination of the luminosity at the FCC-ee, given its minimal dependence on hadronic corrections.
According to the results of Table 2, this uncertainty can be reduced at the $\Lambda_{\text{NP}}$ scale. Using a simple Born-level estimate and assuming a total integrated luminosity of 150 ab$^{-1}$ at the Z pole we obtain a statistical precision of $1/\sqrt{N} = 1.3 \times 10^{-5}$ for $|\cos \theta| < 0.95$, i.e. in a region fully covered by the tracker system, and $1/\sqrt{N} = 2.0 \times 10^{-5}$ for $|\cos \theta| < 0.7$, a typical region spanned by both barrel tracker and barrel electromagnetic calorimeters. This implies a statistical precision well below $10^{-4}$ per year of running at the Z pole is a realistic target.

Nevertheless, a luminosity measurement using this channel will be a sensible option only if “new physics” effects can be excluded or at least circumvented. At the Z pole, and assuming that scales $\Lambda \lesssim 0.7$ TeV are already excluded by LEP2, an analysis similar to the one employed to obtain Table 2 would provide uncertainties due to unknown new physics effects never larger than $\Delta \sigma/\sigma_{\text{SM}} \approx 4 \times 10^{-4}$. This implies that luminosity uncertainties below the permille level are a very realistic target. A way to further reduce this “new physics” uncertainty would be to perform a simultaneous fit to both the integrated luminosity and the size of QED deviations, following the strategy of decoupling normalization and shape-only effects described in the previous section. According to the results of Table 2 this uncertainty can be reduced at the $\Delta \sigma/\sigma_{\text{SM}} \lesssim 2.0 \times 10^{-5}$ level for the full sample, and certainly below the $10^{-4}$ level for yearly running periods. This decoupling strategy would also be more consistent with current proposals that ask for an integrated treatment of NLO corrections and theoretical uncertainties within an SMEFT context \[30\]. For instance, we note that the SM NLO weak corrections for this channel have the same angular behaviour as genuine QED deviations, and should therefore be disentangled carefully. These NLO effects contribute at the permille level at the Z peak, and at the percent level at WW, HZ or $t\bar{t}$ thresholds \[13\].

One can also investigate whether equivalent or better limits have already been obtained with dedicated studies at the LHC. Elastic proton-proton scattering (interpreted as a $\gamma^*\gamma^* \rightarrow e^+e^-$ reaction) is one of the possibilities. Current results are based on rather limited statistics and have a non-negligible systematic contribution from single proton dissociation events \[31\].
expectations from inelastic collisions for $e^+e^-\gamma\gamma$ contact terms via photonic or leptonic PDFs seem equally limited \cite{32,33}. If we assume fermion universality for the $f\bar{f}\gamma\gamma$ contact terms the situation improves. For instance, a reinterpretation of the GRW \cite{28} CMS search for large extra-dimensions \cite{34} provides a direct limit on $\Lambda$. CMS sets a limit on the extra-dimension GRW scale of $M_S > 7.8$ TeV, which translates into a limit of $\Lambda = M_S/\sqrt{2} > 5.5$ TeV, well above the expected FCC-ee reach.

Experimental uncertainties and backgrounds must be understood to a similar level of precision. An almost background-free analysis can be performed by selecting events with zero tracking activity whatsoever around the two most energetic selected photons. The small remaining background from Bhabha events (despite their large cross section) can be quantified using a control Bhabha sample where one of the final electrons is selected with extremely tight criteria and the second electron is misidentified as a photon, due to either “internal” bremsstrahlung or real bremsstrahlung in the detector. Diphoton events lost due to photon conversions on both sides of the event can be estimated using a control sample of events where the conversion affects only one of the two photons, as previously done at LEP. While this control sample and the previous Bhabha control sample present an overlap, they can easily be disentangled by analyzing the energy spectrum of the charged electron (smaller on average in the case of conversions).

Last but not least, a precise knowledge of the detector acceptance requires a precise position of the edges of the measurement, at the level of $10^{-100}$ $\mu$m over distances of a meter, depending on the luminosity precision target ($10^{-5} - 10^{-4}$). Those precisions can only be obtained by a continuous monitoring of the electromagnetic calorimeter positions using charged tracks from Bhabha events. Note that such a procedure also monitors the size and density of collisions in the beam interaction region, which also affects the acceptance. All these experimental details should be investigated in more detail using realistic full simulations of the FCC-ee detectors, in order to finally establish whether a luminosity measurement at the $10^{-4}$ or better is feasible or not.

6 Conclusions

Under the assumption of a preserved $SU(2)_L \times U(1)_Y$ symmetry, and in the massless electron limit, any possible deviation from the Standard Model in the $e^+e^-\gamma\gamma$ process is suppressed at least by factors of order $O(s^2/\Lambda^4)$, where $\Lambda$ represents the characteristic scale of new physics. The differential cross section as a function of the photon polar angle at this order must follow the dependence:

\[
\left( \frac{d\sigma}{d\cos\theta} \right)_{SM+new} = \left( \frac{d\sigma}{d\cos\theta} \right)_{SM} \left[ 1 + \frac{c_8 \alpha^2}{8\pi^2 \alpha^2 s^2} \sin^2\theta \right]
\]

where $c_8$ is a constant of order 1. We find that any different behaviour of the differential cross section will be hardly observable, since it is suppressed by additional $(\text{energy}/\Lambda)^4$ factors.

We have provided sensitivity estimates for QED deviations at future $e^+e^-$ facilities. A CLIC collider operating at $\sqrt{s} = 3$ TeV could provide sensitivity to $\Lambda$ scales as large as 15 TeV, provided that acceptances and efficiencies are controlled at the per mille level. Finally, we have discussed in some detail the interplay between potential new physics effects and a possible measurement of the luminosity at the FCC-ee with $< 10^{-4}$ precision, using analyses of the $e^+e^-\rightarrow\gamma\gamma$ process at the Z pole.
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