Limits on the mass of the lightest Higgs in supersymmetric models

M. Masip\textsuperscript{(1)}, R. Muñoz-Tapia\textsuperscript{(1)} and A. Pomarol\textsuperscript{(2)}

\textsuperscript{(1)}Departamento de Física Teórica y del Cosmos, Universidad de Granada, 18071 Granada, Spain
\textsuperscript{(2)}Institut de Física d’Altes Energies, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain

Abstract

In supersymmetric models extended with a gauge singlet the mass of the lightest Higgs boson has contributions proportional to the adimensional coupling $\lambda$. In minimal scenarios, the requirement that this coupling remains perturbative up to the unification scale constrains $\lambda$ to be smaller than $\approx 0.7$. We study the maximum value of $\lambda$ consistent with a perturbative unification of the gauge couplings in models containing nonstandard fields at intermediate scales. These fields appear in scenarios with gauge mediation of supersymmetry breaking. We find that the presence of extra fields can raise the maximum value of $\lambda$ up to a 19\%, increasing the limits on the mass of the lightest Higgs from 135 GeV to 155 GeV.
The main motivation of supersymmetric (SUSY) extensions of the standard model is their stability against quantum corrections. SUSY models provide a framework to integrate large energy scales together with the observed low-energy physics. This generic motivation has been recently underlined by the celebrated perturbative unification of the three gauge couplings in the minimal extension (MSSM). Up to now, however, there is no observation in disagreement with the standard model predictions. Supersymmetry, although attractive from a theoretical point of view, is still lacking experimental confirmation.

SUSY models have been flexible enough to respect all experimental constraints, but this flexibility does not translate into a complete lack of low-energy predictivity. The most compelling prediction of SUSY models is probably the presence of a light Higgs field. In particular, the MSSM forces the CP-even scalar field $h^0$ to have a tree-level mass $m_h$ smaller than $M_Z$:

$$m_h^2 \leq M_Z^2 \cos^2 2\beta,$$

where $\tan \beta$ is the ratio of vacuum expectation values (VEVs) $v$ and $\bar{v}$ of the Higgs fields $H$ and $\bar{H}$ that give mass to the up and down type quarks, respectively (see [1] for a review). This tree-level bound is shared by any SUSY model with only doublets in the Higgs sector [2].

In models with gauge singlets, trilinear terms in the superpotential of the type

$$W \supset \lambda SH\bar{H}$$

introduce new quartic interactions for the scalar Higgs doublets. The tree-level bound becomes

$$m_h^2 \leq M_Z^2 \cos^2 2\beta + \lambda^2 \nu^2 \sin^2 2\beta,$$

with $\nu = \sqrt{v^2 + \bar{v}^2} = 174$ GeV. The impact of this new term, however, is limited by the following argument [3]. The $\beta$-function fixing the running of $\lambda$ is at one loop

$$\beta_\lambda = \frac{\lambda}{16\pi^2} (4\lambda^2 + 3h_t^2 + 3h_b^2 - g_1^2 - 3g_2^2),$$

where $h_t$ and $h_b$ are the top and bottom Yukawa couplings, and $g_1$ and $g_2$ are the $U(1)_Y$ and $SU(2)_L$ gauge couplings, respectively. The evolution of $\lambda$ will be dominated by $h_t$, which means that its value increases with the energy. As a consequence, the value of $\lambda$ at the weak scale must be small if we want to be in the perturbative regime up to the grand unification scale $M_X = 1.4 \times 10^{16}$ GeV. For the top quark observed at CDF [4], this argument implies that the low-energy value of $\lambda$ must be smaller than $\approx 0.7$ [5]. Moreover, any Yukawa coupling that can be added to the superpotential, like trilinears

$$W \supset -\frac{1}{3} k S^3,$$
gives a positive contribution to $\beta_\lambda$ and further decreases the maximum value of $\lambda$ that remains perturbative up to $M_X$.

A possible way to increase the value of $\lambda$ (and consequently $m_h$) is to introduce new matter fields at intermediate scales. The effect of these fields on $\lambda$ would be indirect, in the sense that they increase the evolution rate of the gauge couplings, which in turn decreases the evolution rate of $\lambda$. Note that $g_1^2$ and $g_2^2$ contribute negatively to $\beta_\lambda$; larger values of these couplings imply a slower running of $\lambda$ and then that larger initial values of this coupling would remain perturbative up to $M_X$. This argument was outlined by Kane and collaborators in Ref. [6]. They only introduced extra Higgs doublets because only $g_1^2$ and $g_2^2$ (and not $g_3^2$) appear in $\beta_\lambda$. They found that the effect on $\lambda$ is always small. A sizeable effect would require the inclusion of many doublets at low-energy scales, but then the gauge couplings become non-perturbative before $M_X$. In addition, the presence of Higgs doublets spoils the unification of the gauge couplings observed in the MSSM, which constitutes so far the only phenomenological motivation for supersymmetry.

The previous analysis, nevertheless, can be improved. First, it will be convenient to introduce extra quarks as well as extra leptons (or Higgs fields). The addition of matter with $SU(3)_C$ interactions will raise the intermediate values of $g_3$. Since the contribution of $g_3$ enters in $\beta_{h_t}$ with negative sign, larger values of $g_3$ will imply lower values of $h_t$, which would decrease the evolution rate of $\lambda$ (note that $h_t \approx g_3 \approx 1$ are the dominant couplings in the renormalization group equations). Second, there is a way to introduce extra matter fields that respects the perturbative unification of the three gauge couplings of the MSSM. If we add complete representations of a simple group [$SU(5), SO(10), E_6, ...$] that contains $SU(3)_C \times SU(2)_L \times U(1)_Y$ as a subgroup, then the (one-loop) effect on the running of the three gauge couplings is such that the couplings still meet at the same unification scale $M_X$ but with a higher final value. This fact, well known by the practitioners, allows larger intermediate values of $g_1$ and $g_2$ together with smaller values of $h_t$ (via larger $g_3$), and will define the scenario for the absolute perturbative bound on $\lambda$. In Fig. 1 we plot the running of the gauge couplings in the MSSM and in a model extended with four families of $5 + \overline{5}$ at 1 TeV.

The presence of matter in vectorlike representations of the standard model symmetry finds its primary motivation on models with gauge mediated supersymmetry breaking (GMSB) [7]. The extra fields, called messengers $\Phi - \bar{\Phi}$, have a mass $M$ that can vary from $\sim 30$ TeV to $M_X$, and couple directly to the fields that break supersymmetry (to the secluded sector). This coupling induces a scalar-fermion mass splitting $\sqrt{F}$ inside the messenger superfields that is transmitted, at the loop level, to the stan-
dard superfields. Actually, the minimal scenarios for GMSB could be closer to the singlet model than to the MSSM. The reason is that these models have serious difficulties to generate the $\mu$ term (the Higgsino mass) in the superpotential [8], and usually require the presence of non-standard fields and couplings. One simple possibility [9] is to introduce a singlet superfield with the coupling in Eq. (2) and generate $\mu = \lambda \langle S \rangle$ via VEVs. We will discuss later in some detail aspects of the singlet model which are specific to GMSB scenarios.

In this rapid communication we study the bounds on $\lambda$ in scenarios with vectorlike fields at intermediate scales. The couplings involved in our analysis are $g_1, g_2, g_3, h_t, h_b, h_\tau$, and $\lambda$. We take a top quark mass of 180 GeV (pole mass) and $\alpha_s(M_Z) = 0.118$ [4]. The extra matter present at a given scale is parametrized by the number $n_{5\bar{5}}$ of $5 + \bar{5}$ representations of $SU(5)$, which is the lowest dimensional vector representation of a simple group containing the standard model symmetry. In the appendix we include the two-loop renormalization group equations for these parameters. For different values of $\tan \beta$, we will look for the highest low-energy value of $\lambda$ consistent with a perturbative value of all the parameters up to the unification scale. Note that $\beta_\lambda$ includes a negative two-loop contribution $-10\lambda^4$. In consequence, at this order $\lambda$ does not have the ultraviolet Landau pole of non-SUSY $\phi^4$ theories. Here the large values of $\lambda$ will grow with the energy scale but only up to the point where the one-loop and the two-loop contributions to $\beta_\lambda$ cancel. This value will correspond approximately to $\lambda = \sqrt{4/10}$. We shall then consider that $\lambda$ is non-perturbative if at a scale below $M_X$ the running value is $\lambda > 0.3$. The same criterion will be used for the $h_t$ and $h_b$. The change from the perturbative to the non-perturbative regime is quite abrupt, and therefore the results do not depend on the actual maximum value for the running couplings at $M_X$ that we choose.

Let us start analyzing the minimal model with no extra matter at intermediate scales. To obtain the bound on $\lambda$ we take $k = 0$. The lowest allowed value of $\tan \beta$ is 1.88, as for smaller values $h_t$ becomes non-perturbative [i.e., $\frac{h_t(M_X)}{4\pi} > 0.3$] even if $\lambda = 0$. For larger values of $\tan \beta$ the initial value of $\lambda$ is constrained by the simultaneous conditions that $h_t$ and $\lambda$ remain perturbative up to $M_X$. For $\tan \beta$ up to 2.31 the dominant condition is that $h_t$ remains perturbative, whereas for $2.31 < \tan \beta < 59.81$ $\lambda$ itself is the first coupling to become non-perturbative. For very large values of $\tan \beta$, from 59.81 to 61.23, the dominant condition is that $h_b$ remains perturbative up to $M_X$. We plot in Fig. (2) the maximum value of $\lambda$ for each $\tan \beta$. The absolute limit is $\lambda < 0.69$, which corresponds to $\tan \beta = 10$.

When extra matter is included the intermediate values of the gauge couplings
grow [see Fig. (1)] decreasing \( \beta \). To find the absolute limit on \( \lambda \), we add the maximum number \( n_{5\text{f}} \) of \( 5 + \mathbf{5} \) families consistent with a perturbative unification of the gauge couplings. It turns out that we can add four families at 250 GeV, or four families at one TeV plus another one at \( 10^{11} \) GeV, or five families at 60 TeV. We plot in Fig. (2) the first case, although the limits are similar in the other two cases. The maximum value \( \lambda = 0.82 \), obtained for \( \tan \beta = 8 \), is a 19\% higher than in the case with no vectorlike matter at intermediate scales. We also observe that lower values of \( \tan \beta \) are possible without going to non-perturbative values of \( h_t \); here the limit is \( \tan \beta = 1.19 \) versus \( \tan \beta = 1.88 \) in the MSSM. This fact is remarkable because the main contribution to \( m_h \) in the singlet model comes at low \( \tan \beta \).

It is now straightforward to translate this maximum values of \( \lambda \) into the limits on the mass of the lightest Higgs boson. In addition to the tree-level contributions in Eq. (3), we include top-quark radiative corrections. Following the procedure described in [5], one obtains that radiative corrections contributing to \( m_h^2 \) vary from \( (95 \text{ GeV})^2 \) at \( \tan \beta = 1.2 \) to \( (90 \text{ GeV})^2 \) at \( \tan \beta = 74 \). In Fig. (3) we plot the bound to \( m_h \) in the MSSM, in the singlet model with no matter, and in the singlet model with a maximum content of vectorlike matter. As it is apparent from this figure, the bounds on \( m_h \) are considerably relaxed; if the presence of a singlet takes the MSSM bound from \( m_h \leq 128 \) GeV to \( m_h \leq 135 \) GeV, the presence of matter at intermediate scales pushes this bound further up, to \( m_h \leq 155 \) GeV.

Since one of the motivations for enlarging the MSSM with the singlet \( S \) and extra vectorlike fields arises from GMSB models, we would like to make a final remark on the viability of these theories. In minimal scenarios of GMSB one has that the trilinear soft masses are smaller than the other soft masses [7]. As a consequence, these models suffer from the presence of a too light scalar field [8, 10]. A possible solution to this problem is to introduce mixing between the messenger and the ordinary matter sectors [11, 12]. Then the scalar trilinears are induced at the one-loop level, as the other soft masses. There are different possibilities. One could introduce messenger-matter mixing from the couplings \( HQ\Phi, S\Phi\Phi, H\Phi\Phi \) or \( SH\Phi \) (and equivalently for \( H \to \bar{H} \)) [8, 12, 13]. Any of them induces at one loop scalar trilinears. For example, the coupling \( W \supset \lambda^\prime H\Phi\Phi \) gives to the trilinear and scalar mass the new contributions

\[ \delta A = -\frac{\lambda^2}{16\pi^2} \frac{F}{M}, \]

\[ \delta m_H^2 = -\frac{\lambda^2}{48\pi^2} \frac{F^4}{M^6} + \frac{\lambda^2}{256\pi^4} (4\lambda^2 + 3h_t^2 - \frac{3}{5}g_1^2 - 3g_2^2) \frac{F^2}{M^2}, \]

where we are assuming that \( F \) and \( M \) are the same for the two messenger fields coupled to \( H \) and \( F < M^2 \) (if this is not the case, see ref. [8]). The first term of eq. (7) enters
at the one-loop level but it is suppressed for $F < M^2$; the second term, calculated in Ref. [12], arises at the two-loop level. We have checked that, after including the above soft mass contributions, there are regions of the parameter space of the model that lead to phenomenologically viable scenarios of electroweak symmetry breaking. In particular, we find that the model proposed in ref. [11], with $S$ playing the role of sliding singlet [14], has a (at least local) minimum in which $\mu$ is of the order of the weak scale [4]. Of course, a certain degree of fine tuning of the parameters is required in order to obtain an acceptable minimum, but this is a generic problem of GMSB theories. Different conclusions were reached in ref. [10], but there the contribution (7) to the scalar soft masses was not included.

From the above we conclude that GMSB scenarios with a singlet require extra couplings of the Higgs with the intermediate matter (messengers). These couplings will modify the running of $\lambda$ discussed previously. For example, the inclusion of a new leptonic coupling $\lambda' = 0.5$ at 30 TeV will move the maximum low-energy value of $\lambda$ a 2% down. However, the presence of extra matter will increase the maximum values significantly also in this case. In particular, adding four $5 + \bar{5}$ families at the same energy scale would allow values of $\lambda$ a 10% larger, up to $\lambda = 0.74$.

In summary, we have analyzed how the presence of vectorlike fields relaxes the bounds on $m_h$ in extensions of the MSSM with a gauge singlet. These scenarios could be motivated by GMSB. The gauge unification of the minimal model is preserved and the range of $\tan \beta$ consistent with a perturbative value of $h_t$ and $h_b$ up to $M_X$ is extended. The mass of the lightest Higgs can be raised from 135 to 155 GeV. This is a correction that does not change the generic feature of SUSY models, the presence of a light neutral Higgs. However, since this corresponds to the range of energies to be explored in the near future, its phenomenological impact can be important.

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1. It is not clear if this model could survive after the recent LEP2 bounds on the superpartners masses.
APPENDIX

In this appendix we write the two-loop renormalization group equations for the couplings involved in our analysis. The parameter $n_{5\bar{5}}$ expresses the number of $5 + \bar{5}$ representations of $SU(5)$ present at a given scale. The equations have been deduced from the general expressions given in \[13\]. To obtain the initial values of $h_t$ and $h_b$ from the pole masses, see \[10\]. We denote with a tilde the parameter over $4\pi$: $\tilde{g} = g/4\pi$, and $t = \ln \mu$. For the three gauge couplings we have

$$\frac{d}{dt} \tilde{g}_1 = \left( \frac{33}{5} + n_{5\bar{5}} \right) \tilde{g}_1^3 + \tilde{g}_1^3 \left[ \left( \frac{199}{25} + \frac{7}{15} n_{5\bar{5}} \right) \tilde{g}_1^2 + \left( \frac{27}{5} + \frac{6}{5} n_{5\bar{5}} \right) \tilde{g}_2^2 + \left( \frac{88}{5} + \frac{32}{15} n_{5\bar{5}} \right) \tilde{g}_3^2 \right] - \frac{26}{3} \tilde{h}_t^2 - \frac{14}{3} \tilde{h}_b^2 - 6\tilde{h}_t^2 - 2\lambda^2$$

$$\frac{d}{dt} \tilde{g}_2 = (1 + n_{5\bar{5}}) \tilde{g}_2^3 + \tilde{g}_2^3 \left[ \left( \frac{9}{5} + \frac{3}{5} n_{5\bar{5}} \right) \tilde{g}_1^2 + (25 + 3 n_{5\bar{5}}) \tilde{g}_2^2 + 24 \tilde{g}_3^2 - 6\tilde{h}_t^2 - 6\tilde{h}_b^2 - 2\lambda^2 \right]$$

$$\frac{d}{dt} \tilde{g}_3 = (-3 + n_{5\bar{5}}) \tilde{g}_3^3 + \tilde{g}_3^3 \left[ \left( \frac{11}{5} + \frac{8}{5} n_{5\bar{5}} \right) \tilde{g}_1^2 + 9 \tilde{g}_2^2 + (14 + \frac{16}{3} n_{5\bar{5}}) \tilde{g}_3^2 - 4\tilde{h}_t^2 - 4\tilde{h}_b^2 \right].$$

The equations for $h_t$, $h_b$, $h_r$, $\lambda$, and $k$ are

$$\frac{d}{dt} \tilde{h}_t = \tilde{h}_t \left( 6\tilde{h}_t^2 + \tilde{h}_b^2 + \tilde{h}_r^2 - \frac{13}{15} \tilde{g}_1^2 - 3\tilde{g}_2^2 - \frac{16}{3} \tilde{g}_3^2 \right) + \tilde{h}_t \left[ \frac{2743}{450} \tilde{g}_1^4 + \frac{15}{2} \tilde{g}_2^4 - \frac{16}{9} \tilde{g}_3^4 \right] + \tilde{h}_t \left[ \frac{136}{45} \tilde{g}_1^2 \tilde{g}_2^2 + 8 \tilde{g}_2^3 \tilde{g}_3 + \tilde{h}_t^2 \left( \frac{6}{5} \tilde{g}_1^2 + 6 \tilde{g}_2^2 + 16 \tilde{g}_3^2 \right) + \frac{2}{5} \tilde{g}_1^2 \tilde{h}_b^2 - 22 \tilde{h}_t^2 - 5 \tilde{h}_t \tilde{h}_b^2 - 5 \tilde{h}_b^2 - \tilde{h}_t \tilde{h}_r^2 - \lambda^2 \right)$$

$$\frac{d}{dt} \tilde{h}_b = \tilde{h}_b \left( 6\tilde{h}_b^2 + \tilde{h}_t^2 + \tilde{h}_r^2 - \frac{7}{15} \tilde{g}_1^2 - 3\tilde{g}_2^2 - \frac{16}{3} \tilde{g}_3^2 \right) + \tilde{h}_b \left[ \frac{287}{50} \tilde{g}_1^4 + \frac{15}{2} \tilde{g}_2^4 - \frac{16}{9} \tilde{g}_3^4 \right] + \tilde{h}_b \left[ \frac{16}{9} \tilde{g}_1^2 \tilde{g}_2^2 + 8 \tilde{g}_2^3 \tilde{g}_3 + 8 \tilde{g}_3^2 \tilde{g}_3 + \tilde{h}_b^2 \left( \frac{2}{5} \tilde{g}_1^2 + 6 \tilde{g}_2^2 + 16 \tilde{g}_3^2 \right) + \frac{4}{5} \tilde{g}_1^2 \tilde{h}_t^2 + \frac{6}{5} \tilde{h}_t^2 \tilde{h}_b^2 - 22 \tilde{h}_t^2 - 5 \tilde{h}_t \tilde{h}_b^2 - 5 \tilde{h}_b^2 - 3 \tilde{h}_b^2 \tilde{h}_r^2 - 3 \tilde{h}_b^4 - \tilde{h}_b^2 \tilde{h}_r^2 - 2 \tilde{h}_t \left( 3 \tilde{h}_b^2 + 4 \tilde{h}_b^2 + 2 \tilde{h}_t^2 + 3 \lambda^2 \right) \right]$$

$$\frac{d}{dt} \tilde{h}_r = \tilde{h}_r \left( 4\tilde{h}_r^2 + 3 \tilde{h}_b^2 + \tilde{g}_t^2 - \frac{9}{5} \tilde{g}_1^2 - 3 \tilde{g}_2^2 \right) + \tilde{h}_r \left[ \frac{243}{50} \tilde{g}_1^4 + \frac{15}{2} \tilde{g}_2^4 + \frac{9}{5} \tilde{g}_3^2 \tilde{g}_2 + \tilde{h}_r \left( \frac{6}{5} \tilde{g}_1^2 + 6 \tilde{g}_3^2 \right) + \tilde{h}_b^2 \left( \frac{2}{5} \tilde{g}_1^2 + 16 \tilde{g}_3^2 \right) - 10 \tilde{h}_t^4 - 9 \tilde{h}_b^2 \tilde{h}_r^2 - 9 \tilde{h}_b^4 - 3 \tilde{h}_b \tilde{h}_r^2 - \tilde{h}_r^2 \tilde{h}_b^2 \right) - \frac{2}{5} \tilde{h}_r^2 + 2 \tilde{h}_b^2 + 2 \lambda^2 \right) \right]$$

$$\frac{d}{dt} \tilde{k} = \tilde{k} \left( 6\tilde{\lambda}^2 + 6 \tilde{k}^2 \right) + \tilde{k} \left[ \frac{18}{5} \tilde{g}_1^2 \tilde{\lambda}^2 + 18 \tilde{g}_2^2 \tilde{\lambda}^2 - \tilde{\lambda}^2 \left( 12 \tilde{\lambda}^2 - 18 \tilde{\lambda}^2 - 18 \tilde{\lambda}^2 - 2 \tilde{\lambda}^2 - 2 \tilde{\lambda}^2 \right) \right]$$

$$\frac{d}{dt} \tilde{\lambda} = \tilde{\lambda} \left( 3 \tilde{\lambda}^2 + 3 \tilde{\lambda}^2 + 4 \tilde{\lambda}^2 + 2 \tilde{k}^2 - \frac{3}{5} \tilde{g}_1^2 - 3 \tilde{g}_2^2 \right) + \tilde{\lambda} \left[ \frac{207}{50} \tilde{g}_1^4 + \frac{15}{2} \tilde{g}_2^4 + \frac{9}{5} \tilde{g}_1^2 \tilde{g}_2^2 + \frac{6}{5} \tilde{g}_2^2 \tilde{\lambda}^2 + \frac{6}{5} \tilde{g}_2^2 \tilde{\lambda}^2 + \frac{4}{5} \tilde{g}_1^2 \tilde{h}_t^2 - \frac{2}{5} \tilde{g}_1^2 \tilde{h}_b^2 + 6 \tilde{g}_2^2 \tilde{\lambda}^2 + 16 \tilde{g}_3 \tilde{\lambda}^2 + 16 \tilde{g}_3 \tilde{\lambda}^2 - 9 \tilde{h}_t^4 - 6 \tilde{h}_b^2 \tilde{h}_r^2 - 9 \tilde{h}_b^4 - 3 \tilde{\lambda}^4 - 8 \tilde{\lambda}^4 - 8 \tilde{\lambda}^4 - 8 \tilde{\lambda}^4 - 8 \tilde{\lambda}^4 \right]$$

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Figure 1: Evolution of the three gauge couplings in the MSSM (solid) and in the model extended with four complete $5 + \overline{5}$ representations of $SU(5)$ at 1 TeV (dashes). The scale $\mu$ is given in GeV units.
Figure 2: Limits on the value of $\lambda$ at the weak scale. We plot the singlet model in the cases with a maximal matter content at intermediate scales (upper) and without extra matter (lower).
Figure 3: Limit on $m_h$ (in GeV) in the MSSM, in the singlet model with no intermediate matter, and in the singlet model with a maximal matter content at intermediate scales. We have included top radiative corrections with $(m_t^2 + m_{\tilde{t}}^2)/2 = 1 \text{ TeV}^2$. 