Energy losses in contractions for the ice slurry considered as the Bingham fluid

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Abstract. The paper presents the results of the experimental investigations on ice slurry energy losses and loss coefficients during its flow through contractions. Six contraction ratios were covered: 0.500, 0.615, 0.650, 0.769, 0.800 and 0.813. In the experimental research, the mass fraction of solid particles in the slurry ranged from 5% to 30% (5%, 10%, 15%, 20%, 25%, 30%). The average size of ice crystals (width/length) 0.1/0.15 mm was used in the experimental studies. The ice slurry is a non-Newtonian fluid. The presented experimental studies allow for determining the theoretical correlations for the ice slurry considered as a Bingham fluid in order to calculate the loss coefficients in contractions during laminar flow. The correlation for calculating the kinetic energy correction factor in the laminar range flow of liquid in the Bingham model was used in this work. The results of experimental research also confirmed that the loss coefficients in contractions in turbulent flow of the ice slurry are the same as in the case of Newtonian liquids.

1. Introduction

Ice slurry is a mixture of a base fluid, e.g. pure water or water with a freezing-point depressant, and small ice crystals, which are usually less than 0.5 mm in size. It is mainly used as a refrigerant in intermediate refrigeration systems. Local loss coefficients for ice slurry flowing through pipe contractions have been presented in [12]. The authors analyzed the flow of 10% and 30% ice slurry through a pipe contraction with a contraction ratio of \( \beta = 0.78 \). The local loss coefficient results presented in the paper depend on mean slurry velocity. It was noted that for the ice slurry with a 30% content of ice particles, the value of the coefficient increased as the velocity decreased. The 10% slurry showed constant loss coefficient values in the investigated velocity range. Sadly, the paper fails to propose a correlation for the calculation of the local loss coefficient for contractions. Therefore, in order to find an analogy in the flow of ice slurry and to use general equations for the calculation of local loss coefficients, other publications on various non-Newtonian fluids flowing through contractions were analyzed.

Papers [1,6,15] look at the flow of various slurries through contractions in the laminar flow range. For the turbulent flow range, the authors observed the same local loss coefficient values as for Newtonian fluids. Turian et al. [15] studied the flow of concentrated slurries through a contraction...
with the diameter ratio of $\beta = 0.508$. On the basis of their results, the authors proposed the following correlation for the laminar flow range to determine the local loss coefficients in the contraction [15]:

$$k = \frac{900}{Re}$$

(1)

The same type of correlation was proposed for the determination of local loss coefficients in [1]. The paper analyzed the flow of several slurries through contractions with diameter ratios between 0.85 and 0.22. The authors proposed that the constant in correlation (1) have a variable value of between 155 and 364, depending on the type of contraction.

Liu and Duan in [6] discuss the laminar flow of various slurries with a carbon particle content of 57%, 59% and 62% through contractions with diameter ratios of 0.386 and 0.500. The equation proposed in the paper is as follows [6]:

$$k = \frac{C}{Re^b}$$

(2)

where the constant C and exponent b have individual values for particular contractions, slurry types and mass fractions. Paper [9] builds on the idea proposed by the authors of paper [6] that C and b in equation (2) be functions of other elements, such as: the contraction ratio $\beta$, the ratio between the mean diameter of particles and the diameter of the pipe $Kn$ and a function of the mass fraction of ice in slurry $x_S$. The equations put forward in [9] are referred to the generalized flow of fluid through the Reynolds, Metzner and Reed numbers.

Paper [7] sets out the equations proposed by Edwards et al. and Kaye et al. to calculate the local loss coefficients of viscous fluids in contractions. The equation proposed by Edwards et al. was applied for diameter ratios ranging from 0.445 to 0.66. It is as follows [7]:

$$k = \left(0.307 + \frac{159}{Re}\right)(1 - \beta^4)$$

(3)

The equation proposed by Kaye et al. was applied for diameter ratios ranging from 0.161 to 0.797. It is as follows [7]:

$$k = \left(1.2 + \frac{160}{\beta Re}\right)(1 - \beta^4)$$

(4)

The vast majority of researchers treat ice slurries as a rheologically stable non-Newtonian fluid. However, different rheological models are adopted to describe them. The most frequent rheological models used to describe ice slurry flow are the following: Bingham’s model [2,9-11], Ostwald-de Waele’s power model [13], and Casson’s model [5]. In this paper, Bingham’s model will be used to describe the rheological properties of the ice slurry. This model has been selected as, taking into consideration an analysis of pipe flow curves, it best describes the rheological properties of ice slurry with an aquatic solution of ethyl alcohol as the base fluid [10,11].

The above review of papers by other authors focusing on ice slurry flow mainly suggests a broad diversity of the shapes and dimensions of the ice particles used in the studies and the diversity of the base liquids used in the slurries. Few research papers have been published to address the behaviour of ice slurry during its flow through pipeline fittings.
2. Experimental studies

The studies conducted in this project covered the flow of ice slurry with an ice content of between 5% and 30%, flowing (in the laminar range) through six investigated contractions (\( \beta = 0.500, 0.615, 0.650, 0.769, 0.800, \) and \( 0.813 \)). The mean size of ice particles in the ice slurry amounts to 0.125 mm. In order to measure the flow resistance of the ice slurry in the contraction, an experimental stand was used which enabled the flow resistance of the mixture to be investigated in various pipeline fittings (Figure 1). Ice slurry flows through the system thanks to the pumps installed in the pipeline (5). The ice slurry is generated in the ice slurry generator (2). It is collected in the storage tank (4), from where it is pumped towards the investigated contraction (1). The tank is fitted with mixers (3), which are supposed to maintain a homogenous composition of the mixture. Before the ice slurry is generated, it is necessary to prepare the essential exchangeable elements of the test stand, such as the investigated contraction (1), straight pipe sections (11) and the necessary measurement equipment.

Among the measurement devices used in the experimental setup are PT100 temperature sensors (6) used to measure the temperature of the ice slurry flowing through the system. The PT100 sensors were placed in thermometric sleeves and information about the measurements taken by the sensors was sent to computer memory (8). Other measurement devices used at the stand were Fuji Electric pressure difference sensors (9) with measurement ranges of 0÷1 kPa, 0÷6 kPa and 0÷32 kPa. 4 mm pressure impulse nozzles were used in the studies due to the presence of ice particles. Non-insulated, transparent impulse ducts (12) were used for measurements using the nozzles. The measurement of the mass stream of the ice slurry and its density was possible thanks to the use of the MASSFLO MASS 6000 mass flow-meter (7). The ice fraction was measured through random calorimetric measurements (10).

![Figure 1. Schematic diagram of an experimental test stand: 1 – contraction, 2 – ice slurry generator, 3 – mixer, 4 – storage tank, 5 – main pump, 6 – measurement of temperature and pressure, 7 – mass flow-meter, 8 – data collection point, 9 – multi-point measurement of pressure losses, 10 – measurement and control of ice volume fraction, 11 – straight pipe sections, 12 – impulse ducts](image-url)
The ice slurry studied in the paper was generated from a 10.6% aquatic solution of ethyl alcohol. One of several methods can be used to determine the ice fraction in the slurry. These methods include: calorimetric measurements, the use of the freezing curve of the aquatic solution of ethyl alcohol [8] and measurements of the density of the slurry using a mass flow-meter [10,11]. All three methods were used in this paper. The freezing curve and measurements of slurry density were used for continuous measurements, whereas the more accurate but more problematic calorimetric method was used to verify the results of the continuous measurements.

The problem of maintaining a homogenous composition of the ice slurry also applies to distribution pipelines. It is necessary to ensure that the flow velocity is never lower than the velocity necessary to ensure the homogeneity of slurry composition. The results presented in [3] suggest that regardless of the mass fraction of ice, below the velocity of 0.15 m/s, the flow of the ice slurry will correspond to the structure of moving sludge and will not be a homogenous structure.

The energy losses lead to the pressure drops in sudden contractions. In order to avoid possible disturbances of the measured values, measurements of pressure drops in contractions were made by measuring at the same time the total pressure drop in the contractions themselves and the friction resistances in the inlet and outlet sections. The friction resistances in the inlet and outlet pipes, measured along 1 m measurement sections, were then subtracted from the obtained measurement result.

The local loss coefficient for the contraction was calculated using the determined pressure drop \( Dp_{CON} \) in the contraction and equation (5) containing Coriolis coefficients \( \alpha \):

\[
k_{CON} = \frac{2Dp_{CON}}{\rho V^2} - \alpha_2 + \alpha_1 \frac{V^2_1}{V^2_2}
\]

where: \( \rho \) - ice slurry density [kg/m³], \( V \) – ice slurry velocity [m/s], and subscripts 1 and 2 represent the measurement location upstream and downstream of the investigated contraction, respectively.

The equation proposed by Strzłecka and Jeżowiecka-Kabsch [14] was used to determine the value of the Coriolis coefficient for Reynolds numbers within the range of Re = 2800-100000:

\[
\alpha = 1 + 105 \left( \frac{10}{\ln^2 Re} \right)^3 - 11.88 \left( \frac{10}{\ln^2 Re} \right)^2 + 1.208 \left( \frac{10}{\ln^2 Re} \right)
\]

Equation [9] was used to determine the value of the Coriolis coefficient in the laminar area for ice slurry described with the Bingham model:

\[
\alpha = \frac{54(47\varepsilon^2 + 58\varepsilon + 35)}{35(3\varepsilon^2 + 2\varepsilon + 3)}
\]

where: \( \varepsilon = \tau_p/\tau_w \)

Shear stress at pipe wall (with \( \varepsilon < 0.5 \)) for the ice slurry in the Bingham model can be represented with equation (8) [10,11]:

\[
\tau_w = \frac{4}{3} \tau_p + \mu_p \frac{8V}{D}
\]

Critical (plastic) shear stress included in equation (8) for the ice slurry prepared on the basis of a 10.6% solution of ethyl alcohol and mid-sized 0.125mm particles is represented with equation (9) [11]:
where \( x_S \) represents the content of ice particles in the slurry [-].

Reynolds number for Bingham fluids was used in graphs to present the results of experimental studies and of the calculations of local loss coefficients for the ice slurry in contractions. It was represented with equation:

\[
Re_B = \frac{\rho z V D}{\mu_p}
\] (10)

The dynamic coefficient of plastic viscosity in equation (10) for the ice slurry prepared on the basis of a 10.6% solution of ethyl alcohol and mid-sized 0.125mm particles was determined experimentally on the basis of pipe flow curves and is represented with equation (11) [10,11]:

\[
\mu_p = 0.0035 + 0.0644(x_S) - 0.7394(x_S)^2 + 5.6963(x_S)^3 - 19.759(x_S)^4 + 26.732(x_S)^5
\] (11)

The results of experimental studies of ice slurry flow resistances in sudden contractions and the calculated values of the Coriolis coefficient were used to determine the local loss coefficients in these fittings.

3. Results of experimental studies

Figure 2 shows the values of local loss coefficients as a function of the Reynolds number (for a Bingham fluid) for all six investigated contractions and ice particle content in the slurry amounting to between 5% and 30%. It is difficult to directly and comprehensively compare the obtained values of local loss coefficients with the research conducted by other authors, mainly because few such studies have been published, and also because those that have been released were mainly presented as a function of the Reynolds, Metzner and Reed numbers. On the basis of the variation of local loss coefficients presented in Figure 2, for ice slurry with ice particle content of 5-30%, flowing through all of the investigated contractions, these coefficients were confirmed as consistent in the turbulent area with theoretical values given for a Newtonian fluid. This consistency is also reflected by the results of research by other authors investigating local resistance in pipeline fittings during the flow of non-Newtonian fluids [1,15]. In the laminar range, there is a clear impact of the ice particle content on the variation in local loss coefficients as a function of the Reynolds number (the higher the share of ice particles, the higher the value of the local resistance coefficient). The value of the local loss coefficient is also significantly impacted by \( \beta \), i.e. the ratio of the outer pipe diameters upstream and downstream of the contraction, and the value of the Reynolds number.

In principle, these observations overlap with the conclusions of other authors who focused on the determination of local loss coefficients both for ice slurries and other non-Newtonian fluids [1,11,12]. Another variable which should be taken into consideration while addressing the possible impacts on the value of local loss coefficients, is the ratio between the mean diameter of ice particles and the inner pipe diameter (\( Kn \)). The impact of this variable on the values of local loss coefficients has also been suggested by the authors of [4]. Paper [9] proposes equations for the calculation of the local loss coefficient of ice slurry flowing through contractions, but only for an ice particle content of 20-30%. In addition, the flow of ice slurry is treated as generalized flow of a non-Newtonian fluid with the use of the Reynolds, Metzner and Reed numbers. It was also proposed in [9] that the constants in correlation (2) be treated as functions of other values (\( x_S, \beta, Kn \)), which have an impact on the value of local loss coefficients.
Figure 2. Local loss coefficient of the ice slurry (5%-30%) as a function of the Reynolds number for a Bingham fluid for the contraction: a) 28/15 ($\beta=0.500$), b) 28/18 ($\beta=0.615$), c) 22/15 ($\beta=0.650$), d) 28/22 ($\beta=0.769$), e) 22/18 ($\beta=0.800$) and f) 18/15 ($\beta=0.813$)

It is proposed in [11] that when determining a correlation for the local loss coefficient for bends and elbows, the ice particle content in the slurry be represented by the Hedström number $He$. Therefore, a decision was taken to account for the impact of four variables $k=f(Re_B, He, \beta, Kn)$ when determining the equation for the local loss coefficient during ice slurry flow through contractions. It was also noted that the variation of the local loss coefficient as a function of the Reynolds number for a Bingham fluid (for the laminar range) in contraction has a power course. It was assumed that the
sought form of the function, just as in the case of other non-Newtonian fluids [1,6,15], is as in equation (2), where: \( C \) and \( b \) are functions of: \( He \), \( \beta \) and \( Kn \). The following observations were made during the analysis of studies and calculation results concerning flow resistances in contractions:

- the impact of the Hedström number and of the ratio between the ice particle and pipe diameters on the determined values of the exponent in equation (2) is negligible,
- Hedström number values are similar for ice content of 20% and 30% and therefore it may not represent the ice fraction in a correlation specifying the local loss coefficients due to significant differences in the values of these coefficients for the above ice fractions,
- for an ice content of 5% to 15%, the value of the Hedström number changes significantly and therefore it may very well represent the ice content values in the equation used to calculate the local loss coefficient in contractions,
- it is very difficult to find universal correlations describing local loss coefficients in the entire range of variability of the ice fraction in the ice slurry which would be accurate enough in representing the values of this coefficient obtained in experimental studies.

Considering the above observations, the decision was taken to separately prepare equations for the calculation of the local loss coefficient in contractions for an ice content of 20% to 30%, and separately for a smaller ice content. For an ice content of between 5% and 15% in ice slurry flowing (in the laminar range) through the six investigated contractions (\( \beta = 0.500, 0.615, 0.650, 0.769, 0.800, \) and 0.813), equations (12) and (13) were obtained to supplement correlation (2) with parameters \( C \) and \( b \) [10]:

\[
C = \frac{1000}{-1.135 + 19.24 \beta^{2.5} - 1.48 \cdot 10^{-2} (0.5 Kn He)^{2}}
\]  
(12)

\[
b = \frac{1}{2893.6 - 319.3 \beta - 3357.4 \beta^{1.5} + 789.2 \beta^{2.5} + 3438.6 \beta^{0.5} \ln \beta}
\]  
(13)

For the ice content of 20% to 30% in the ice slurry flowing in the laminar range through the six investigated pipe contractions, equations (14) and (15) were obtained to calculate the local loss coefficients as a supplement to correlation (2) [10]:

\[
\ln C = 160.42 + 346.35 \beta \ln \beta - 146.85 \beta^{2.5} + 5.78 \cdot 10^{-3} (100 Kn x)\]  
(14)

\[
b = \frac{3.14 - 7.961 \beta + 5.169 \beta^{2}}{\left[1 - 1.352 \beta - 1.363 \beta^{2} + 1.935 \beta^{3}\right]}
\]  
(15)

A comparison of calculated and measured values of local loss coefficients for ice slurry flowing through six contractions has been presented in Figure 3. Figure 3a applies to an ice content of 5% to 15% and refers to equations (2), (12) and (13), whereas Figure 3b presents a comparison of calculations performed using equations (2), (14) and (15) and refers to ice slurry with the ice particle content of 20% to 30%.
4. Summary

The paper presents the results of experimental studies of flow resistance of ice slurry in pipe contractions. The experimental results made it possible to obtain the values of local loss coefficients for this slurry in pipe contractions ($\beta=0.500, 0.615, 0.650, 0.769, 0.800, \text{ and } 0.813$). A direct comparison of the values of local loss coefficients of the ice slurry in contractions with research conducted by other authors is difficult as very few results of such studies have been published and they are incomplete. The proposed correlations for the calculation of local loss coefficient values in the laminar range take into account the variation of several parameters which have an impact on the value of those coefficients. In the study, we did not find universal correlations describing local loss coefficients of ice slurry flowing through pipe contractions in the entire range of variability of ice content in the ice slurry which would be accurate enough in representing the experimentally obtained values of this coefficient. In light of the above, the experimental results were divided into two parts, depending on the content of ice particles in the slurry, and the correlations for the calculation of the local loss coefficient values in pipe contractions were determined separately for each group. For the ice particle content of 5-15% in the slurry, the local loss coefficients are a function: $k=f(Re, He, \beta, Kn)$, and for the ice particle content in the slurry of 20-30% the local loss coefficients are a function: $k=f(Re, xs, \beta, Kn)$. In the turbulent range, it was possible to obtain consistent values of local loss coefficients for ice slurry flowing through pipe contractions and those given for Newtonian fluids.

The paper also includes a comparison of the calculated and measured local loss coefficient values for ice slurry flowing through six pipe contractions. The results of the measurements and calculations of the local loss coefficient values for ice slurry flowing through contractions were quite consistent.

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