Influence of the gain saturation on the output performance of quantum-well heterostructures with modified distributed-feedback cavities

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Abstract. Influence of the gain saturation on the output performance of quantum-well heterostructures with modified distributed-feedback cavities is considered. Taking into account symmetrical boundary conditions, distribution of the electromagnetic field in the active region of the heterostructure laser diode with a phase-amplitude grating is determined. It is shown that both the output performance of quantum-well heterostructures and distribution of the electromagnetic field in the active region depend on the pump current and optical nonlinearity characteristics.

1. Introduction
At present, one of the actual problems is the rapid transmission of large volume of information. For solving this problem, the ultrafast fiber communication lines, which permit to transmit several tens of information channels in single-mode fibers, are attractive. Operation in such network lines is impossible without multi-channel transmission laser modules. Single-frequency laser diodes can to be the base of the required modules.

It is known that the spectral width of semiconductor lasers is quite large. For narrowing the laser spectral line, the distributed feedback (DFB) structures are used, the change in the period of which controls the tuning of the lasing frequency within the gain band of the active medium [1]. As known, the DFB lasers based on the phase grating have the lowest threshold gain for two modes simultaneously. Previously [2, 3], a method was suggested for obtaining the stable single-mode lasing in a DFB laser on the phase grating with a low-frequency sinusoidal modulation of the amplitude of the coupling coefficient of counter-propagating waves \(\alpha(z) = \alpha_0 \sin(Gz)\) (\(G\) is the inverse period of the amplitude modulation). The proposed DFB laser has technological advantages, because it may be fabricated on structures with semiconductor surfaces and in photosensitive optical fibers by the direct holographic writing.

2. Basic equations
One can represent the modulation of the refractive index \(n\) and gain coefficient \(k\) of an active medium as

\[
 n(z) = n_0 + 2\bar{n}\sin(Gz)\cos(gz), \quad k(z) = k_0 + 2\bar{k}\sin(Gz)\cos(gz),
\] (1)
where $g$ is the inverse period of the DFB structure, $n_0$ and $k_0$ are the non-perturbed values of the refractive index and gain coefficient, $\bar{n}$ and $\bar{k}$ determine the modulation amplitudes of the phase and amplitude gratings correspondingly. We assume that $|k| \ll n_0\kappa_0$ and $|\bar{n}| \ll n_0$. Here, $\kappa_0 = \omega/c$, $\omega$ is the lasing angular frequency, and $\sin(Gz)$ is a slowly changing function of the $z$-coordinate ($G \ll g$).

In the framework of the coupled-wave method [4], the solution of the wave equation

$$\frac{d^2 E}{dz^2} + \frac{\omega^2}{c^2} \left[ n(z) - i \frac{c}{\omega} k(z) \right]^2 E = 0 \quad (2)$$

is obtained in the form of two counter-propagating waves

$$E(z) = E_+(z)e^{i\frac{Gz}{2}} + E_-(z)e^{-i\frac{Gz}{2}}, \quad (3)$$

where $E_{\pm}(z)$ are the slowly changing amplitudes at the frequency $\omega \approx \omega_{Br} = gc/2n_0$.

Substitution of (3) into (2) leads to the reduced equation system [3]

$$\pm \frac{d E_\pm}{dz} - \left( \frac{k_0}{2} + \frac{i d}{2} \right) E_\pm = i\alpha(z) E_\pm(z) \quad (4)$$

that describes the linear regime of lasing of the DFB laser with sinusoidal modulation of the coupling coefficient. In this case, we have

$$\alpha(z) = \frac{\kappa_0}{2} \left( \bar{n} - \frac{i\bar{k}}{2\kappa_0} \right) \sin(Gz) = \alpha_0 \sin(Gz), \quad \alpha_0 = \Re \alpha_0 + \Im \alpha_0 = \alpha_0' + i\alpha_0'', \quad (5)$$

and the Bragg detuning $\delta = \kappa_0 n_0 - g/2$. The solution is

$$E_{\pm}(z) = \mathcal{E}_{\pm}(z)e^{i\psi_{\pm}(z)}. \quad (6)$$

Here, $\mathcal{E}_{\pm}(\mp L/2) = E_0$ , $\psi_{\pm}(\mp L/2) = 0$ are the boundary conditions, $L$ is the length of the cavity.

Figure 1. Spectral-threshold characteristics of the DFB laser on (a) the phase and (b) amplitude gratings. (a) Spectrum of eigen modes $\omega_j$ at $\alpha_0' = 1 \text{ cm}^{-1}$ and $GL = \pi$, (b) dependence of the threshold gain coefficient $(k_0L)_{\text{th}}$ on the parameter $\alpha_0''$, curve 1 is the mode of the first order ($j = 1, \omega_{\pm1}$), curve 2 is the mode of the second order ($j = 2, \omega_{\pm2}$).
Figure 2. Change in the spectral dependence of amplitudes of output light waves of the DFB laser with the growth of the amplitude of the imaginary component of the coupling coefficient at $\alpha_0 = 10^{-2}$ cm$^{-1}$ and $GL = \pi$ ($m = 0$). (a) $k_0L = 14.4$, $\alpha_0' = 0$, (b) $k_0L = 13.6$, $\alpha_0'' = 10^{-2}$ cm$^{-1}$, (c) $k_0L = 10.8$, $\alpha_0'' = 5 \times 10^{-2}$ cm$^{-1}$. The bold curves represent the mode spectrum for direct propagating waves and the dash curves represent the mode spectrum for back propagating waves.

2.1. Phase ($\bar{k} = 0$) and amplitude ($\bar{n} = 0$) gratings

In consequence of numerical analysis of (4), we showed [5] that spectral-threshold characteristics of a DFB laser on the phase grating with sinusoidal modulation of the coupling coefficient at small values of the parameter $GL$ (for example, $GL = \pi$) are similar to the characteristics of the DFB laser on the amplitude grating at the constant coupling coefficient. The value of $(k_0L)_{th}$ increases with the growth of the mode number $j$, i.e., the discrimination of modes is observed and $k_0(\omega_j)_{th} = k_0(\omega_{-j})_{th}$ (Fig. 1 a.). At the same time, while increasing the amplitude of the coupling coefficient, the monotonous decrease in $(k_0L)_{th}$ occurs.

In the case of the amplitude grating, the Bragg mode of the frequency $\omega_0 = \omega_{Br}$ is forbidden. At the same time, two lowest modes of the first order with frequencies $\omega_{\pm 1}$ are symmetrically shifted relative to the correspondingly displaced Bragg frequency $\omega_{Br}$, and have the least (equal) threshold gain. The spectral-threshold characteristics of the DFB laser under consideration at the given frequency modulation coupling coefficient (Fig. 1 b) are similar to the characteristics of the DFB laser on the phase grating with a constant coupling coefficient [5].
2.2. Phase-amplitude grating
Evolution of spectral-threshold characteristics of the DFB laser with the growth of the amplitude component $k$ of the coupling coefficient is presented in Fig. 2. As seen, the modal set of the output field depends on the value of $k_0L$ [5]. One can see that the spectral picture of amplitudes of the output waves of the DFB laser on the phase grating (Fig. 2 a) monotonically transforms with increasing $k$ to the picture corresponding to the DFB laser on the amplitude grating (Figs. 2 b, c). In this case, the Bragg mode ($\omega = \omega_{B1}$) transforms to the minus first-order mode ($\omega_{-1}$) for the DFB laser on the amplitude grating. Correspondingly, the plus first-order mode ($\omega_{+1}$) shifts to the left from the point $\delta z = 0$. In this case, the threshold gains for these modes become identical.

3. Model of the quantum-well heterostructure
For modeling optical transitions in quantum wells (QWs), we use the self-consistent analytical approximation of the two-band model [6, 7]. In the general case, the gain coefficient follows, in dependence on the radiation density flow in the QW, the law [7]

$$k_{QW} = \frac{k_0}{1 + \alpha S},$$

(7)

where $k_0$ is the initial gain coefficient (including the light confinement effects in the laser waveguide), $\alpha$ is the nonlinearity parameter. The nonlinearity parameter depends on the radiation frequency $\nu$, difference of the quasi-Fermi levels $\Delta F_0$ and surface (sheet) photon density $S$. The quantum yield $\eta_{sp}$ of luminescence (spontaneous radiative recombination) can be taken constant and equal $\approx 1$.

In the dependence $k_{QW}$ on the photon density $S$ there are exist three regions of the gain saturation. At low radiation fluxes $k_{QW} \approx k_0(1 - \alpha_0 S)$, where the initial nonlinearity parameter $\alpha_0$ is introduced. When the gain coefficient varies by a factor of two ($k_{QW} \approx k_0/2$), it is worthwhile to use an average value of the nonlinearity parameter $\alpha_{1/2}$. At high powerful radiation fluxes the gain drops as $k_{QW} \approx k_0/\alpha_{\infty} S$, where the nonlinearity parameter is equal to $\alpha_{\infty}$. Generally, the relation $\alpha_0 < \alpha_{1/2} < \alpha_{\infty}$ is fulfillment. For evaluations of the influence of the gain saturation on the dynamic processes in DFB laser structures it is preferable to apply the quantity $\alpha_{1/2}$ (or $\alpha_{\infty}$).

Taking into account transitions through high-lying excited subbands, we find

$$\alpha_{1/2} = \alpha_{norm} v \frac{1}{2} \left( \frac{e^{\Delta_{10} - \varepsilon_1} - 1}{e^{\Delta_{10} - \varepsilon_1} + 1} + \sum_i H_i(\varepsilon_i) \right) \left( \sum_i \ln \left( \frac{(1 + e^{\Delta_{10} - \delta_i})(e^{\Delta_{10} - \varepsilon_1} + 1)}{3e^{\Delta_{10} - \delta_i} + e^{\Delta_{10} - \varepsilon_1} + e^{\varepsilon_1 - \delta_i} + 1} \right) \frac{e^{\varepsilon_1 - \delta_i}(e^{\Delta_{10} - \varepsilon_1} - 1)(e^{\Delta_{10} - \varepsilon_1} + 1)}{(1 + e^{\Delta_{10} - \delta_i})(3e^{\Delta_{10} - \delta_i} + e^{\Delta_{10} - \varepsilon_1} + e^{\varepsilon_1 - \delta_i} + 1)} \right)^{-1}.$$  

(8)

Here, the normalization parameter $\alpha_{norm} = \eta_{sp} \kappa/2A_{ev}N_{c1}$ is determined by the Einstein coefficient $A_{ev}$, effective density of states $N_{c1}$, and maximum value of the gain coefficient in the QW equal $\kappa/2$. Other definitions are, as follows, $v$ is the group light velocity, the subband quantum number $i = 1, 2, ..., \varepsilon_i = (h\nu - h\nu_i)/2k_BT$, $\Delta_i = (\Delta F - h\nu_i)/2k_BT$, $k_BT$ is the thermal energy, $H_i(\varepsilon_i)$ is the Heaviside step function. Relative to the ground states we can determine $\varepsilon_i = \varepsilon_1 - \delta_i$, $\Delta_i = \Delta_1 - \delta_i$, $\delta_i = (h\nu_i - h\nu)/2k_BT$. The reduced quantity $\delta_i$ characterizes a set of optical resonances. Values $\varepsilon_1$ and $\Delta_1$ correspond to transitions through the ground subband states of electrons and holes $(i=1)$ where $\delta_1=0$. At the gain saturation $(0 < \varepsilon_1 < \Delta_{i0})$, the excitation level $\Delta_i$ of the QW changes from $\Delta_{i0}$ to $\varepsilon_i$, i.e., the quasi-Fermi level difference $\Delta F$ decreases from the initial value $\Delta F_0$ to the quantum energy $h\nu$. 
Detail analysis show that at a fixed value $\varepsilon$ (frequency) the nonlinearity parameter $(\varepsilon, \Delta_0)$ decreases with the growth of $\Delta_0$ (pump) (Fig. 3 a) and at a given $\Delta_0$ the quantity $\alpha(\varepsilon, \Delta_0)$, as a rule, increases versus $\varepsilon$. Normalization coefficient is can be presented as $\alpha_{\text{norm}} = \eta_{sp}/2k_B T \nu \rho d$, where $d$ is the QW width, $\rho$ is the electromagnetic mode density. Therefore the value $\alpha \sim \alpha_{\text{norm}}$ increases with lowering the temperature $T$ and narrowing the QW width $d$ and decreases inverse proportional to the squared energy band gap $E_g$ of the semiconductor ($h\nu \approx E_g$). Involving the excited subband states in the optical transitions results in a definite sharp increasing of $\alpha$ (Fig. 3 b).

Figure 3. Dependencies (a) $\alpha(\Delta_0)$ at the given $\varepsilon = 0$ and (b) $\alpha(\varepsilon)$ at the given $\Delta_0=4.8$. Curve 1 corresponds to $\alpha_0/v\alpha_{\text{norm}}$, curve 2 corresponds to $\alpha_\infty/v\alpha_{\text{norm}}$, $d=10\text{nm}$, $T=300\text{ K}$, $\delta_2=2.82$ (GaAs).

For the GaInAs–GaInAsP laser system, the DFB structure parameters are fitted in such a way that in the gain spectrum of the QW structure, only one mode satisfying the Bragg condition has minimum losses. The amplification of this system is determined by difference between the gain coefficient of the QW structure $k_{\text{QW}}$ and losses introduced by the DFB cavity.

For the laser diode with the DFB structure, we have found a distribution of the electromagnetic field over the length of the active region. Fig. 4 illustrates the distribution of the amplitudes of counter-propagating waves and of the gain coefficient through the cavity.

At the small intensities of the input signal the maximum power density of light field is observed at the edges of the active region due to the amplification of the input waves over all the cavity length (Fig. 4 a). At increasing the input intensity, the gain coefficient falls at the edges of the active region due to the effect of the inversion burning. The light field concentrates in the center of the active region, where the main radiation amplification just occurs (Fig. 4 b). At further increasing the input radiation intensity, the gain coefficient decreases over all the length of the active region and radiation passes through the structure practically without amplification. When the pump current increases the effects associated with the inversion burning decrease.

The considered GaInAs–GaInAsP QW laser system with the incorporated DFB structure is promising for applications as a source for multi-channel transmitting networks. The control of the DFB structure parameters provides the required set of lasing carrier frequencies.

4. Conclusion
Introducing the modified DFB structure in the active region of laser diodes gives new possibilities to control their spectral and power performance characteristics. In particular, embedded in the
Figure 4. Distribution of (a, b) the amplitudes of the counter-propagating waves, (c, d) the power density of light, and (e, f) gain coefficient over the length of the active region at (a, c, e) small and (b, d, f) large intensities of the input signal. The results are represented for the pump level (1) $\Delta_0 = 1$, (2) 2, and (3) 3. The figures with prime at curves a, b correspond to the direct propagating waves.

active region the phase-amplitude grating provides, in dependence on the amplitude of the complex coupling coefficient components, efficient selection of the lasing modes at single Bragg resonance or two neighboring adjacent resonance frequencies. Distribution of the electromagnetic
field in the DFB cavity is essentially affected by the gain saturation. For QW laser diodes, the nonlinearity parameter has specific spectral dispersion that is related with the energy level structure in the subbands. Due to effects of the gain saturation the spatial distribution of the field in the amplifying DFB heterostructures is markedly transformed with the increasing of the pump current and especially of the input signal intensity.

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