A non-invasive stochastic-optical method (SOM) for estimating the volume fraction in granular flows: application on interrogation windows with different aspect ratios

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Abstract. Granular flows are involved in geophysical phenomena and industrial applications. The knowledge of the volume fraction is essential for better understanding their dynamics. Indeed, this quantity is highly coupled with the rheology of granular media. Here, we investigated the performance of the stochastic-optical method (SOM), proposed by [Sarno et al. Granular Matter (2016) 18: 80]. The method works thanks to highly-controlled illumination conditions, guaranteed by a flickering-free planar lamp, and uses a high-speed digital camera. Namely, the indirect estimation of the near-wall volume fraction $c_{3D}$ is made possible by the estimation of a quantity, called two-dimensional volume fraction $c_{2D}$, which is measurable through an opportune binarization of gray-scale images. With the purpose of assessing the performance of the SOM method on rectangular interrogation windows with different aspect ratios, we present a novel experimental campaign on dispersions of matte-white plastic beads immersed in a dense fluid, where the angle of incidence of light was 25°. Moreover, we explored various settings of the binarization algorithm, incorporated in the SOM method. The accuracy of the method is found to be reasonably high with a root-mean-square error on $c_{3D}$ lower than 0.03 for a wide range of settings and independently from the aspect ratio.

1. Introduction
Granular media are involved in several natural phenomena (e.g. debris flows, snow avalanches, pyroclastic flows) as well as in numerous industrial applications, where pellets or powders are handled and conveyed. The growing interest in understanding the dynamics of granular systems and in mathematically describing their propagation has strongly encouraged theoretical and experimental investigations [1-10].

The occurrence of various dissipation mechanisms and different flow regimes render the dynamics of granular media extremely complex and still far from being entirely understood (e.g., [11-14]). Among the various research approaches, the experimental investigation of granular materials at the laboratory scale represents an invaluable tool to better understand some aspects of their motion, which
can be hardly isolated at larger scales. In laboratory the main physical quantities, worth to be investigated, are the velocity and also the solid volume fraction, which is defined as the ratio between the volume occupied by the grains and the total volume of the mixture. The knowledge of this latter quantity is particularly useful to understand the granular dynamics. Indeed, the volume fraction is strongly coupled with the rheological behaviour of the granular medium, especially in the case of free-surface flows where a stress-free boundary condition occurs at the free surface. The necessity to keep the granular flow undisturbed led to the employment of several non-invasive techniques, exploiting different regions of the electromagnetic spectrum. Measurement techniques, like X-ray tomography, magnetic resonance imaging (MRI) and γ-radiation, allow measurements everywhere in the granular domain (e.g., [15-17]). Nevertheless, these techniques are rather expensive as regards to both installation and operating costs. Additionally, X-ray tomography generally does not allow high-frequency image acquisitions, while MRI only works with a limited number of materials. Conversely, the techniques that only use the visible region of the electromagnetic spectrum, often simply referred to as optical methods, are capable of taking measurements only at the boundaries of a non-transparent granular domain. Despite this disadvantage, they are very cost-effective and, thus, they become popular in standard laboratory investigations (e.g., [2,4,18,19,20]).

More recently, Sarno et al. [20] proposed a stochastic-optical method (SOM) that is capable of providing estimations of the near-wall volume fraction $c_{3D}$, by exploiting a controlled light source coming from a constant direction. The method indirectly estimates $c_{3D}$, by means of a measurable quantity, the two-dimensional volume fraction $c_{2D}$, which is accessible from greyscale images after an appropriate binarization. Namely, an extensive numerical investigation, based on Monte Carlo simulations, revealed that an exponential stochastic relationship exists between $c_{2D}$ and $c_{3D}$. Thanks to an ad-hoc local binarization algorithm to estimate $c_{2D}$ from gray-scale images, Sarno et al. [20] experimentally validated the SOM method on random dispersions of white plastic beads, where various light directions were investigated. However, in that validation only squared interrogation windows (IW) centred in the region of interest (ROI) were utilized.

In this experimental study we further tested the robustness of the SOM method, by specifically investigating its performance on rectangular IWs ($h \times l$) with different aspect ratios, $h/l$. Moreover, a few settings of the binarization algorithm are explored. One major purpose of the present work is to quantitatively assess the accuracy attainable by the SOM method in cases of very elongated IWs (i.e. $h/l < 1$), which are particularly useful in applications on granular flows, where a higher spatial resolution is required in the transverse direction than in the stream-wise direction. It should be noted that the employment of very elongated IWs is a challenging application, considering that in laboratory the light source typically consists of a planar lamp of finite size that cannot guarantee a perfectly homogenous illumination far away from the central point. Nonetheless, it is worth anticipating that a reasonably good accuracy is observed for a wide range of aspect ratios (ranging from 1 to $\approx 0.06$), with the root-mean-square error (RMSE) on $c_{3D}$ generally lower than 0.03. Moreover, a higher accuracy is observed by increasing the area of the IW. These findings indicate that the SOM method is ready for advanced applications on granular flows: e.g. the estimation of the volume fraction profiles in chute flows or in rotating drum, where an elongated IW is preferable over a squared IW.

The paper is composed of the following sections. Sec. 2 briefly recalls the rationale and some details of the SOM method. The experimental arrangement is described in Sec. 3, while the results are reported and discussed in Sec. 4. The conclusions are finally summarized in Sec. 5.

2. The SOM method
In this section we briefly describe the SOM method, originally proposed in [20] for the estimation of the solid volume fraction. For further details we refer the reader to the main paper by Sarno et al. [20].
The SOM method works in the presence of a transparent wall delimiting the granular domain and utilizes a well controlled illumination system, where the angle of incidence of light $\zeta$ (lighting direction) is kept constant. The method also requires a high-speed camera, to be placed in front of the wall, so that several pictures of the granular medium can be captured from the perpendicular direction (i.e. the viewing direction). A sketch of the experimental setup required for the application of the SOM method is reported in figure 1(a). By exploiting information coming from both viewing and lighting directions, similarly to a triangulation approach, the SOM method allows the estimation of the near-wall volume fraction, $c_{3D}$ (i.e. the volume fraction related to a finite reference volume delimited by the wall). The accomplishment of this task relies on a measurable quantity, proposed by Sarno et al. [20] and named two-dimensional volume fraction, $c_{2D}$. With reference to a generic interrogation window (IW) located within the imaging plane, $c_{2D}$ is defined as the ratio between the projection onto the IW of all the visible and illuminated surface elements belonging to the grains and the total area of the IW. An extensive numerical investigation on random distributions of spheres, obtained by using the Monte Carlo (MC) method, was carried out by Sarno et al. [20] and revealed the existence of a stochastic transfer function

$$c_{3D} = a(\zeta) \exp(b(\zeta)c_{2D}),$$

where $a$ is proportional to $1/\cos\zeta$ and $b$ weakly depends on $\zeta$. Sarno et al. [20] found an almost constant value of $b \approx 5.5$ for perfectly spherical grains, while $b$ is expected to vary depending on the grain shape. The numerical investigation involved a large number ($\sim 10^5$) of virtual spheres of diameter $d$, randomly loaded into a virtual cubic box of side $10d$ (figure 1(b)), so that different values of $c_{3D}$ could be numerically investigated. In order to limit the effects of the box boundaries on the local volume fraction, an inner cube of side $6d$, frontally delimited by the transparent wall, was considered for calculations (figure 1(c)). Various values of $\zeta$ were studied and the best correlation between $c_{2D}$ and $c_{3D}$, in terms of $c_{2D}$-scatter and sensitivity of $c_{2D}$ on $c_{3D}$ ($d(c_{3D})/d(c_{2D})$), was found for $20^\circ < \zeta < 45^\circ$. A clear advantage of this purely numerical investigation was that both $c_{3D}$ and $c_{2D}$ could be exactly calculated for each distribution, since the spheres' locations in the box were known a priori.

After that the theoretical feasibility of the approach was verified, the practical use of the SOM method in real laboratory applications required the implementation of an automatic vision algorithm, capable of reliably estimating $c_{2D}$ from gray-scale digital images. A local binarization algorithm, which takes into account the local illumination conditions, was proposed in [20]. According to this approach and with reference to a generic IW, the binary image, $O_{binary}$, is calculated as

$$O_{binary}(i, j) = \left[ \frac{O_{i,j} - O_{min,N(i,j)}}{O_{max,N(i,j)} - O_{min,N(i,j)}} > s \right],$$

where $O_{i,j}$ is the brightness value of the generic pixel of coordinates $(i,j)$ in the gamma-decoded image, $O' = O^{1/\gamma}$ (for further details about gamma-decoding see [20]). In equation (2), $O'_{min,N(i,j)}$ is the minimum brightness value in a circular neighbourhood, $N(i,j)$, of the pixel $(i,j)$, $O'_{max,N(i,j)}$ is a maximum brightness value evaluated in the same neighbourhood and $s$ is a threshold to be experimentally calibrated (with values typically between 0.5-0.7). The size of $N(i,j)$ should be chosen large enough to encompass both illuminated and non-illuminated surface elements (i.e. the pixels in the digital image). Conversely, a too large $N(i,j)$ would significantly decrease the spatial resolution of the measurement. Hence, analogously to [20], a neighbourhood of size $2d$ is employed in the present experimental investigation. It should be noted that, different from $O'_{min,N(i,j)}$, $O'_{max,N(i,j)}$ is calculated not on $O'$ but on a pre-processed image that is preliminary obtained by applying an appropriate moving average filter (MAF) on $O'$.
Figure 1. (a) Geometrical set-up for the SOM method, (b) Example of a random dispersion of 400 spheres, generated by the MC method (the cubic reference volume of side 6d is highlighted in yellow), (c) frontal view of the same dispersion, (d) projection onto the IW of the visible and illuminated surface elements (represented in white), concurring to the calculation of $c_{2D}$.

As highlighted in [20], the employment of MAF for the identification of $O''_{\text{max},N(i,j)}$ is particularly useful to limit the detrimental effects of occasional glares on the grains, which could otherwise lead to inaccurate estimations of the $c_{2D}$. This glare effect is more noticeable if the interstitial fluid is highly transparent (e.g. air) and also if the granular material exhibits some direct reflectivity. Based on experimental investigations on matte white POM beads [18] immersed in water-sucrose solution and in air, Sarno et al. [20] observed that a small squared MAF of size $p=3\,\text{px}$ (corresponding to $\approx 0.2\,d$) is generally enough to attenuate these problems. The brightness value in $O_{\text{binary}}$ is 1, if the pixel is directly illuminated, otherwise it is 0. Hence, $c_{2D}$ can be straightforwardly calculated as follows

$$c_{2D} = \frac{\sum O_{\text{binary}}}{N}$$

(3)
where \( N \) is the total number of pixels in the IW. Due to the stochastic nature of the method, it is worth underlining that a reliable estimation of the volume fraction is only allowed if several estimations are averaged from different images. The SOM approach together with the aforementioned binarization strategy was experimentally validated by Sarno et al. [20] on random dispersions of matte-white POM grains by employing squared IWs of different sizes.

3. Experimental arrangement and image analysis

3.1. Experimental set-up

Motivated by the aim of investigating the performance of the SOM method on interrogation windows with different aspect ratios, here we present a new experimental campaign carried out on POM beads in water-sucrose solution. The experimental set-up is similar to that of [20]. Yet, different from [20], a unique lamp position corresponding to the effective angle of incidence of \( \zeta = 25^\circ \) was investigated: thus, the threshold \( s \) in (2) could be specifically calibrated for this angle. The granular material consists of matte-white acetal-polymeric (POM) spheroidal beads with mean diameter \( d \approx 3.3 \text{ mm} \) and density \( \rho_s = 1410 \text{ kg/m}^3 \) [18,20]. Before each experiment, a known amount of granular material was inserted into a cubic container, having a total volume of \( 1.6 \text{ dm}^3 \) and a transparent front face with size of \( 12 \text{ cm} \times 10 \text{ cm} \). A high-density sucrose-water solution (\( \approx 62\% \) in weight) was used as ambient fluid. The density and viscosity of the fluid allowed small sedimentation rates of the POM grains, so that an approximately random grain distribution could be obtained by manually shaking the box. As a consequence, several values of \( c_{id} \), even smaller than the volume fraction at deposit (\( \approx 0.6 \)), could be investigated, by inserting into the container different amounts of granular material. Specifically we experimentally investigated 7 values of \( c_{id} \): \{0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.62\}. For each \( c_{id} \), 50 different random dispersions of grains were obtained by manually shaking the container and were photographed for subsequent analysis.

In agreement with the general set-up of figure 1(a), the measuring apparatus is merely composed of a digital camera and a planar LED lamp. A high-speed digital camera, model AOS S-PRI, equipped with a 25mm-lens (model Pentax B2514D), was employed to record grey-scale images of the granular dispersion. The camera was placed perpendicular to the transparent wall at a distance \( d_W = 40 \text{ cm} \), so that the centre of the ROI coincided with the centre of the wall and the length scale was \( 1 \text{ px} \approx 2 \times 10^{-4} \text{ m} \) (i.e. \( 1d \approx 16 \text{ px} \)). The illumination system consisted of a high-intensity flickering-free LED lamp, model MultiLED-LT (by Photo-Sonics Corp.), which was placed at a distance of \( d_L = 32 \text{ cm} \) from the wall and was pointed at the centre of the ROI (figure 2). The LED array is planar and forms a rectangle of size \( 10 \text{ cm} \times 6.5 \text{ cm} \). The lamp was mounted on a geared head (model Manfrotto Junior 410) that allowed its positioning with a millimetric accuracy. In order to obtain the desired angle of incidence \( \zeta = 25^\circ \), at the stage of lamp positioning we had to take into account the refractive index of the water-sucrose solution, which was measured equal to 1.446 by means of an Abbe refractometer (model ABBE-REF 1 by PCE instruments). Therefore, considering Snell's law, the horizontal angle, \( \zeta_{\text{lamp}} \), between the normal to the LED lamp and the normal to the transparent wall was set to 37.7\(^\circ\) (cf. figure 2). The camera was operated with the following settings: lens aperture of \( f/4 \) and shutter time of 50 \( \mu \text{s} \). We preliminarily checked that the environmental light was much dimmer than the LED lamp (\( \approx 7000 \text{ lumens} \)) and, thus, was irrelevant to the photographic exposure.
3.2. Image analysis

Different from [20], in which only squared IWs were investigated, here the SOM method is applied to rectangular IWs with various aspect ratios $n=h/l$, where $l$ and $h$ are the width (horizontal) and the height (vertical) of the IW, respectively (figure 3). Specifically, we explored two series of IWs with areas $A=16d^2$ and $A=36d^2$, and, for each area, we systematically investigated 11 different aspect ratios ranging from 0.0625 to 1 (i.e. the squared case). As already mentioned in Sec. 2, the employment of an elongated aspect ratio ($l>>h$) is particularly useful for estimating the volume fraction profiles in chute granular flows. In fact, in order to achieve a high accuracy (increasing with $A$) and also a high spatial resolution along the flow depth (decreasing with $h$), the optimal IW should extent as little as possible along the vertical direction, $z$, and, accordingly, it should have a much larger extension in the horizontal direction, $x$. Table 1 reports the two dimensions of the rectangular IW, $l$ and $h$, corresponding to the investigated aspect ratios.

| Investigated aspect ratios, $n=h/l$ | 1.0 | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 | 0.2 | 0.1 | 0.0625 |
|-----------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|--------|
| $A=16d^2$                         |     |     |     |     |     |     |     |     |     |     |        |
| $l/d$                             | 4.00| 4.22| 4.47| 4.78| 5.16| 5.66| 6.32| 7.30| 8.94| 12.65| 16.00  |
| $h/d$                             | 4.00| 3.79| 3.58| 3.35| 3.10| 2.83| 2.53| 2.19| 1.79| 1.26  | 1.00   |
| $A=36d^2$                         |     |     |     |     |     |     |     |     |     |     |        |
| $l/d$                             | 6.00| 6.32| 6.71| 7.17| 7.75| 8.49| 9.49|10.95|13.42|18.97 | 24.00  |
| $h/d$                             | 6.00| 5.69| 5.37| 5.02| 4.65| 4.24| 3.79| 3.29| 2.68| 1.90 | 1.50   |

Additionally, we carried out a sensitivity analysis on the binarization algorithm (2), by varying the size $p$ of the MAF, employed for estimating $O''_{\max,N(i,j)}$, between $0.2d$ and $2d$. Analogous to [20], the threshold $s$ in equation (2) was experimentally calibrated by minimizing the root mean square error (RMSE) on $c_{3D}$ (calculated on all 7 investigated values of $c_{3D}$). We found that $s$ weakly depends on the aspect ratio $n$, while it critically depends on the optical properties of the investigated system and also on the settings of the binarization formula. Therefore, $s$ was calibrated on the extreme case of $n=1$ (corresponding to a squared IW) and, then, was kept constant for all other investigated aspect ratios. Conversely, $s$ had to be calibrated for each investigated value of $p$. 

![Figure 2. Sketch of the experimental arrangement, where the S-PRI camera and the LED lamp are reported ($d_c=40cm$, $d_l=32cm$, $\zeta_{\text{lamp}}=37.7^\circ$).](image-url)
4. Results and discussion

In this section we present and discuss the experimental results.

In figure 4 the RMSE on $c_{3D}$, obtained by different SOM analyses, is plotted against the dimensionless size of the MAF, $p/d$. Each curve, reported in figure 4, corresponds to the application of the SOM method on a IW with a specific aspect ratio, $n$ (cf. table 1). All the curves show approximately the same trend, for both investigated IW areas: $A=16d^2$ (figure 4(a)) and $A=36d^2$ (figure 4(b)). The RMSE weakly varies for $0<p<1d$, while its minimum is located at $p\approx1.2d$. Conversely the RMSE rapidly increases for $p>1.3d$, up to values larger than 0.035. These findings reveal that the location of the optimal value for the size of the MAF, $p$, is not sensitive with respect to the size and shape of the IW. Moreover, the curves are very close to each other, suggesting that the shape of the IW has a weak effect in the estimation of the volume fraction. These results indicate that the employment of elongated IWs, down to $n=0.06$, is feasible in the investigated geometrical setup.

Figure 4. Performance (in terms of RMSE on $c_{3D}$) of different SOM analyses, obtained by varying the aspect ratio, $n$, of the rectangular IW and the size of the MAF, $p$, in the binarization algorithm (2). Two sizes of the IW are investigated: (a) $A=16d^2$, (b) $A=36d^2$.

In figure 5, the RMSE on $c_{3D}$, resulting from different SOM analyses, is plotted as function of the aspect ratio $n$. The results with $A=16d^2$ are plotted with continuous lines, while those ones corresponding to $A=36d^2$ are plotted with dotted lines. Only two sizes of $p$ are selected for this comparison: $p=3px=0.2d$ (i.e. the same size used in [20]) and $p=15px=1d$, which is closer to the optimal point (cf. figure(4)). As one can see from figure 5, the performance of the SOM method does not show any clear trend with $n$: namely the accuracy in the estimation of the $c_{3D}$ does not significantly
change with the shape of the investigated IW. This basically confirms what has been already observed in figure 4. As well, figure 5 indicates that, for both cases of $A=16d^2$ and $A=36d^2$, the employment of $p=1d$ yields a generally higher accuracy than $p=0.2d$. However, it is interesting to note that, while this improvement is apparent for $n>0.3$, for smaller values of $n$ the RMSE is much less sensitive to the specific choice of $p$.

As explained in Sec. 2, some scatter of $c_{2D}$ is intrinsic to the stochastic nature of the SOM method. In order to investigate the performance of the method related to this aspect and highlight the differences between experimental and numerical investigations, figure 6 reports the behaviour of the standard deviations, $\sigma_{c_{2D}}$, of $c_{2D}$ estimations. In particular, the standard deviations obtained by various experimental SOM analyses (i.e. by choosing different values of $n$ and $p$) are compared with the theoretical ones, obtained from numerical MC generations on IWs of areas $16d^2$ (in figures 6(a)-(b)) and $36d^2$ (in figures 6(c)-(d)). While Sarno et al. [20] only reported the numerical analysis on a $6d \times 6d$ IW, here we also carried out numerical calculations by using a smaller $4d \times 4d$ IW on the same numerical dataset of random dispersions of spheres. Before comparing experimental and numerical data, it should be noted that, as expected, $\sigma_{c_{2D}}$ noticeably decreases with increasing IW area (cf. figures 6(a)-(b) and 6(c)-(d)). In fact, the larger is the IW the higher is the spatial average, which acts as a low-pass filter with respect to $c_{2D}$ fluctuations at the cost of a lower spatial resolution. From figure 6 it can be noted a strong overlap of all experimental curves, obtained by using different $n$. This indicates that the shape of the IW weakly influences $\sigma_{c_{2D}}$. Moreover, for all the SOM analyses the experimental behaviour of $\sigma_{c_{2D}}$ exhibits a decreasing trend with $c_{3D}$, similar to the theoretical curves. Yet, the experimental values of $\sigma_{c_{2D}}$ are almost everywhere larger than the numerical ones, especially for intermediate values of $c_{1D}$. The reason of this discrepancy could be due to the fact that the laboratory experimental setup slightly differs from that of the numerical simulations: e.g. the investigated plastic beads are spheroids instead of perfect spheres, the optical response of the grains’ surface could be inhomogeneous etc... Moreover, it should be considered that the experimental estimations of $c_{2D}$ incorporate additional variance due to inaccuracies of the binarization algorithm, which, conversely, is not employed in the purely numerical investigation. The differences between theoretical and experimental curves are only mildly reduced by using a larger MAF (cf. figures 6(a) vs. 6(b), and 6(c) vs. 6(d)). In the light of these observations, a further investigation on the capabilities of the binarization algorithm might be advisable. Nonetheless, it is worth underlining that the binarization algorithm remains quite accurate in estimating the averaged $c_{2D}$ and, consequently, the averaged values of $c_{3D}$, as the very small values of the RMSE indicate (cf. figure 5 and also figure 8, hereafter reported).
Figure 6. Standard deviations, $\sigma_{c2D}$, of individual $c_{2D}$ estimations plotted against $c_{3D}$. For comparison also the theoretical standard deviations, numerically obtained from MC distributions of perfect spheres, are reported. (a) IW of area $A=16d^2$ and MAF of size, $p=3px\approx0.2d$, (b) $A=16d^2$ and $p=15px\approx1d$, (c) $A=36d^2$ and $p=3px\approx0.2d$; (d) $A=36d^2$ and $p=15px\approx1d$.

In summary, we can conclude that the SOM method is almost insensitive to the employment of elongated IWs, since the IW aspect factor cause no significant variation in the estimations of the averaged near-wall volume fraction and of the $c_{2D}$-scatter. This represents an important result, as it allows the employment of the SOM method to elongated IWs, e.g. for the estimation of the volume fraction profiles in chute granular flows or in a rotating drum with a reasonably high spatial resolution.

In order to further describe the effects of the MAF on the binarization task, in figure 7 example binary images, obtained by employing different $p$, namely $p=0.2d$ (figure 7(b)) and $p=1.0d$ (figure 7(c)) are reported. As one can see from figure 7, with both settings the binarization algorithm is generally capable of distinguishing the illuminated and visible surface elements (white pixels), namely the pixels concurring to the $c_{2D}$ estimation, from the non-illuminated ones (black). However, minor improvements, especially at the borders of the grains, can be observed in the case of the larger MAF.

Finally, in figure 8 the experimental data ($c_{2D}$, $c_{3D}$), obtained from averaging 50 different estimations of $c_{2D}$, are reported together with their standard deviations (horizontal bars) and are compared with the numerical data, obtained from MC generations by using the same angle of incidence of the light ($\zeta=25^\circ$). Two comparisons are reported by using the two investigated interrogation areas: $A=16d^2$ (figure 8(a)) and $A=36d^2$ (figure 8(b)), while $p=1d$ and $n=0.5$ are employed in both series of experimental data. Conversely, the numerical data are obtained on squared IW of sizes $4d \times 4d$ (figure 8(a)) and $6d \times 6d$ (figure 8(b)), so that the interrogation areas are equal to the experimental ones.
Figure 7. Example of binarized images, obtained by using different sizes of the moving average filter: (a) original picture ($c_{3D}=0.15$), (b) binarized image obtained with $p=0.2d$, (c) binarized image obtained with $p=1d$.

Figure 8. Comparison between the experimental data ($c_{2D}, c_{3D}$), from pictures of POM grains, and the numerical data, from MC generations. The experimental data are obtained by using a rectangular IW with aspect ratio $n=0.5$, while the numerical data are obtained on a squared IW with the same area: (a) $A=16d^2$ (RMSE on $c_{3D}$ of 0.0253), (b) $A=36d^2$ (RMSE on $c_{3D}$ of 0.0215).

The agreement between the averaged experimental estimations and the best-fitting exponential transfer function, obtained from numerical data, is excellent. Indeed, the RMSE on $c_{3D}$ is equal to 0.0253 and 0.0215 for $A=16d^2$ and $A=36d^2$, respectively. In agreement with that already observed in figure 6, the scatter of $c_{2D}$ decreases with increasing interrogation area both in numerical and experimental investigations. In figure 8(a), a slight underestimation of the volume fraction can be observed for $c_{3D}=0.55$. The employment of a larger $A$ (figure 8(b)) yields a slightly better agreement, especially for the upper end of $c_{3D}$ values (i.e. $c_{3D}=0.55$ and $c_{3D}=0.62$).
5. Conclusions

The SOM method, originally proposed by Sarno et al. [20], allows the non-invasive estimation of the near-wall volume fraction, \( c_{3D} \), in granular media. This estimation is made possible by a correlated quantity, named two-dimensional volume fraction, \( c_{2D} \), which is accessible from a transparent wall delimiting the granular domain. The method works thanks to well controlled illumination conditions, where the angle of incidence of light is kept constant within the region of interest. Extensive numerical investigations permitted the identification of a strong stochastic relationship between \( c_{2D} \) and \( c_{3D} \), whose parameters depend on the angle of incidence of the light. For the practical use in real laboratory applications, the SOM approach needs a local binarization algorithm for estimating \( c_{2D} \) from gray-scale images captured by a high-speed digital camera.

Motivated by the interest of evaluating the applicability of the SOM method to granular chute flows, where reasonable accuracy and good spatial resolution in the transverse direction should be kept, we experimentally investigated the performance of the SOM method on rectangular interrogation windows with different aspect ratios. Indeed, in these specific applications the employment of elongated interrogation windows along the stream-wise direction is preferable over a squared shape. An experimental campaign, involving a liquid-granular mixture, composed of matte-white POM beads and a high-density water-sucrose solution, is carried out. In this campaign the effective angle of incidence of light of \( 25^\circ \) is employed. Several aspect ratios of the interrogation window (ranging from 1 to 0.06), as well as two interrogation areas (i.e. \( 16d^2, 36d^2 \)), are investigated.

As expected, the agreement between experimental and numerical data increases with increasing interrogation area. However, this improvement comes at the cost of a lower spatial resolution of the measurements. Furthermore, independently from the aspect ratio of the rectangular interrogation window, no significant variation either in the estimation of the volume fraction or in the scatter of \( c_{2D} \) measurements is observed. In the light of these findings and, considering the very low RMSE values (<0.03) observed in this study, the usage of the SOM method on elongated interrogation windows with horizontal lengths up to \( \approx 24d = 8.3 \) cm appears totally feasible in the present experimental set-up (i.e. with a LED lamp of size \( 10 \times 6.5 \) cm, located at a distance of 32 cm from the wall). Moreover, we believe that the employment of a larger lamp, located at a greater distance from the wall, could further extend the applicability of the method to even larger interrogation windows, thanks to a more uniform illumination.

A sensitivity analysis on the moving average filter, involved in the binarization task, is also reported. Interestingly, from this brief analysis it emerged that a moving average filter of size \( \approx 1d-1.2d \) slightly improves the accuracy of the SOM method, compared to the smaller moving average filter of size \( \approx 0.2d \) previously suggested in [20]. Nonetheless, it has been observed that the scatter of the experimental estimations of \( c_{2D} \) is systematically larger than the theoretical one, independent from the size of the moving average filter. These discrepancies indicate that the binarization algorithm introduces some non-negligible random errors to \( c_{2D} \) estimations, which add up to the intrinsic fluctuations of \( c_{2D} \). A further improvement of the binarization algorithm may overcome these limitations, which, however, do not significantly influence the overall accuracy of the SOM method in estimating the averaged values of \( c_{3D} \).

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References

[1] Bonamy D, Daviaud F, Laurent L 2002 Experimental study of granular surface flows via a fast camera: A continuous description Phys. Fluids 14(5) 1666-1673
[2] GDR Midi 2004 On dense granular flows Eur. Phys. J. E Soft Matter 14(4) 341-65
[3] da Cruz F, Emam S, Prochnow M, Roux J N, Chevoir F 2005 Rheophysics of dense granular materials: Discrete simulation of plane shear flows Phys. Rev. E 72(2) 021309
[4] Pudasaini S P, Hutter K, Hsiau S-S, Tai S-C, Wang Y, Katzenbach R 2007 Rapid flow of dry granular materials down inclined chutes impinging on rigid walls Phys. Fluids 19(5) 053302
[5] Sarno L, Martino R, Papa M N 2011 Discussion of “Uniform Flow of Modified Bingham Fluids in Narrow Cross Sections” by Alessandro Cantelli J. Hydraul. Eng. 137(5) 621–621
[6] Sarno L, Carravetta A, Martino R, Tai Y-C 2013 Pressure coefficient in dam-break flows of dry granular matter J. Hydraul. Eng. 139(11) 1126–1133
[7] Sarno L, Carravetta A, Martino R, Tai Y-C 2014 A two-layer depth-averaged approach to describe the regime stratification in collapses of dry granular columns Phys. Fluids 26(10) 103303
[8] Sarno L, Carravetta A, Martino R, Papa M N, Tai Y-C 2017 Some considerations on numerical schemes for treating hyperbolicity issues in two-layer models Adv. Water Resour. 100 183-198
[9] Papa M, Sarno L, Vitiello F, Medina V 2018 Application of the 2D depth-averaged model, FLATModel, to pumiceous debris flows in the Amalfi Coast Water (Switzerland) 10(9) 1159
[10] Rendina I, Viccione G, Cascini L 2019 Kinematics of flow mass movements on inclined surfaces. Theor. Comp. Fluid Dyn. https://doi.org/10.1007/s00162-019-00486-y
[11] Jenkins J T, Savage S B 1983 A theory for the rapid flow of identical, smooth, nearly elastic, spherical particles J. Fluid Mech. 130 187-202
[12] Savage S B and Hutter K 1989 The motion of a finite mass of granular material down a rough incline J. Fluid Mech. 199 177-215
[13] Gray J M N T, Wieland M, Hutter K. 1999 Gravity-driven free surface flow of granular avalanches over complex basal topography Proc. R. Soc. Lond. A 455(1985) 1841-1874
[14] Delannay R, Valance A, Mangeney A, Roche O, Richard P 2017 Granular and particle-laden flows: from laboratory experiments to field observations J. Phys. D Appl. Phys. 50(5) 053001
[15] Ancey C 2001 Dry granular flows down an inclined channel: Experimental investigations on the frictional-collisional regime Phys. Rev. E 65(1) 011304
[16] Nakagawa M, Altobelli S A, Caprihan A, Fukushima E, Jeong E-K 1993 Non-invasive measurements of granular flows by magnetic resonance imaging Exp. Fluids 16(1) 54-60
[17] Grudzień K, Niedostatkiewicz M, Adrien J, Tejchman J, Maire E 2011 Quantitative estimation of volume changes of granular materials during silo flow using X-ray tomography Chem. Eng. Process. 50(1) 59-67
[18] Sarno L, Papa M N, Martino R 2011 Dam-break flows of dry granular material on gentle slopes 5th Int. Conf. on Debris-Flow Hazards Mitigation: Mechanics, Prediction and Assessment ed R Genevois et al (Rome: Casa Editrice, Università La Sapienza) pp 503–512
[19] Sheng L-T, Kuo C-Y, Tai Y-C, Hsiau S-S 2011 Indirect measurements of streamwise solid fraction variations of granular flows accelerating down a smooth rectangular chute Exp. Fluids 51(5) 1329-1342
[20] Sarno L, Papa M N, Villani P, Tai Y-C 2016 An optical method for measuring the near-wall volume fraction in granular dispersions Granul. Matter 18(4) 80
[21] Eckart W, Gray J M N T, Hutter K 2003 Particle Image Velocimetry (PIV) for granular avalanches on inclined planes. Dynamic response of granular and porous materials under large catastrophic deformations - Lecture notes in applied & computational mechanics (Vol. 11) ed K Hutter and N Kirchner (Berlin: Springer) pp 195-218
[22] Jesuthasan N, Baliga B R, Savage S B 2006 Use of particle tracking velocimetry for measurements of granular flows: review and application KONA Powder Part. J. 24 15-26
[23] Sarno L, Carleo L, Papa M N, Villani P 2018 Experimental Investigation on the Effects of the Fixed Boundaries in Channelized Dry Granular Flows Rock. Mech. Rock Eng. 51 203-225
[24] Sarno L, Carravetta A, Tai Y-C, Martino R, Papa M N, Kuo C-Y 2018 Measuring the velocity fields of granular flows—Employment of a multi-pass two-dimensional particle image velocimetry (2D-PIV) approach Adv. Powder Tech. 29(12) 3107-3123
[25] Sarno L, Tai Y-C, Carravetta A, Martino R, Papa M N, Kuo C-Y 2019 Challenges and improvements in applying a particle image velocimetry (PIV) approach to granular flows XXVI A.I.VE.LA. Annual Meeting - J. Phys. Conf. Ser. 1-13

[26] Drake T G 1990 Structural features in granular flows J. Geophys. Res. B: Solid Earth 95(B6) 8681-8696

[27] Ahn H, Brennen C E, Sabersky R H 1991 Measurements of velocity, velocity fluctuation, density, and stresses in chute flows of granular materials J. Appl. Mech. 58(3) 792-803

[28] Capart H, Young D L, Zech Y 2002 Voronoï imaging methods for the measurements of granular flows Exp. Fluids 32(1) 121-135

[29] Spinewine B, Capart H, Larcher M, Zech Y 2003 Three-dimensional Voronoï imaging methods for the measurement of near-wall particulate flows Exp. Fluids 34(2) 227-241

[30] Barbolini M, Biancardi A, Natale L, Pagliardi M 2005 A low cost system for the estimation of concentration and velocity profiles in rapid dry granular flows Cold Reg. Sci. Technol. 43(1-2) 1-9

[31] Spinewine B, Capart H, Fraccarollo L, Larcher M 2011 Laser stripe measurements of near-wall solid fraction in channel flows of liquid-granular mixtures Exp. Fluids 50(6) 1507-1525