Absence of Overscreened Kondo Effect in Ferromagnetic Host

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We study the low temperature behavior of a boundary magnetic impurity $S' = 1/2$ in an open ferromagnetic Takhatajjan-Babujian spin-$S$ chain. For antiferromagnetic Kondo coupling, it is shown via Bethe ansatz solution that the impurity spin is always locked into the critical behavior of the bulk. At low temperature, a local composite of spin-$S = 1/2$ forms near the impurity site and its contribution to specific heat is of simple power law $T^\Delta$. The absence of overscreened Kondo effect is due to the large correlation length of host spins which is divergent near the quantum critical point.

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Multichannel Kondo problem was originally proposed to study the magnetic impurity behavior in real metals. At low temperature for antiferromagnetic Kondo coupling, the impurity spin $S'$ is completely compensated by the surrounding $n$-channel electrons when $2S' = n$; while it is undercompensated for $2S' > n$ or overcompensated for $2S' < n$. The later situation (so-called overscreened Kondo effect) bears no resemblance to a non-interacting gas of electrons and provides us a possible interpretation of non-Fermi liquid behavior observed in certain uranium alloys as well as in certain tunneling problems in two-level systems. As this effect is immediately destroyed by weak anisotropy, it has been a challenge to deduce its signature relevant to other anomalous behavior in a variety of exotic metals, especially when the systems show reduced effective dimensionality, or undergo the proximity to quantum critical phase transitions. In this paper we will concentrate on multichannel Kondo problem in a typical quantum critical ferromagnet, with the number of channels $n$ being represented by the host spin, $n = 2S$. Contrary to the problem in antiferromagnetic host chain, where overscreened Kondo effect appears when $S > S'$, we will find that due to the divergent correlation length near the critical point, the overscreened Kondo effect does not exist in ferromagnetic host.

While the Kondo problems in paramagnetic or antiferromagnetic hosts have been intensively investigated over three decades, they attract little attentions in ferromagnetic hosts. The main reason is the difficulty in approaching this problem, i.e., usual perturbation techniques as well as bosonization fail in the critical region because of the absence of the “large energy scale”, i.e., the Fermi energy. Recent investigations have been shown that as the system closes to a quantum critical point, the impurity behavior depends strongly on the host properties and is non-universal. Another reason is due to the fact that the existence of Kondo effect in ferromagnetic host remains controversial. At a first glance, the traditional Kondo screening seems possible only at temperature higher than the Curie point where host magnetization vanishes. Otherwise, a local impurity could not change the total magnetization, opposite to what happened in dilute alloys. However, as far as the screening of a local moment is concerned, the issue should be how the impurity behavior changes as the temperature lowers down. The above picture does not necessary hold true even for the host goes away from the critical point where the long range order is destroyed by strong fluctuations. Yet there is no rigorous proof of the absence of overscreening in low temperature ferromagnet. In a earlier publication, Larkin and Mel’nikov first studied the Kondo effect in an almost ferromagnetic metal. With traditional perturbation theory they shown that the impurity susceptibility is almost Curie type with logarithmic corrections at intermediate low temperature and the usual Kondo screening is not effective. More recently, the authors studied a similar problem in 1D, i.e., a boundary Kondo impurity of spin $S'$ in $S = 1/2$ ferromagnet. The model provides us a first exact soluble example for the Kondo problem near a quantum critical point. Via Bethe ansatz solution it has been shown that at sufficient low temperature, the impurity spin is completely screened by $2S'$ host spins. Thus a local composite forms near the impurity site and shows a simple power law dependence of specific heat on temperature. Therefore, these interest results strongly motivate us to know what will happen for a magnetic impurity in a multichannel ferromagnetic host at low temperature and if there exhibits the conventional overscreened Kondo effect.

In order to study the multichannel Kondo problem in ferromagnet via exact solution, we consider a magnetic impurity $S' = 1/2$ coupled to an open spin-$S$ chain which is integrable by Bethe ansatz. A well known integrable extension of isotropic $S = 1/2$ spin chain to arbitrary spin $S$ is

$$H_S = J_0 \sum_{j=1}^{N-1} Q_{2S}(\vec{S}_j \cdot \vec{S}_{j+1}), \quad (1)$$

where $Q_{2S}(x)$ is a polynomial of degree $2S$ of $SU(2)$ invariant quantities $x = \vec{S}_j \cdot \vec{S}_{j+1}$. The general construction for $Q_S(x)$ is given in Ref.[10,11], one recovers the standard Heisenberg model and Takhatajjan-Babujian model...
for $S = 1/2$ and $S = 1$ respectively. Here we follow the definition of $H_S$ appeared in Ref. [11] so that the single-magnon dispersion coincides with that of the standard Heisenberg ferromagnet. Thus for $J_0 < 0$, the model is qualitatively equivalent to the standard spin-$S$ ferromagnetic Heisenberg chain. The boundary impurity spin is now coupled to the first host spin as

$$H_{imp} = J S_1 \cdot \vec{S}. \quad (2)$$

The novelty of this construction is that the total Hamiltonian $H_S + H_{imp}$ is again integrable for any $J_0$ and $J$. The result is compared with the problem of a single impurity in periodic spin chains \cite{14}, where the integrability requires a fixed Kondo coupling $J$ and a fine tuned impurity interaction instead of simple form \cite{15}. As long as spin dynamics is concerned, the integrable spin-$S$ chain represents the multichannel host, because Kondo screening of the impurity is equivalent to usual multichannel Kondo problem in antiferromagnetic case ($J_0 > 0$) \cite{16}; the impurity spin is completely screened for $S' = S$; partially screened for $S' > S$, with Schottky anomaly when an external magnetic field $H$ is applied; and overscreened for $S' < S$, giving rise to quantum critical behavior.

The model Hamiltonian is solved within the framework of Bethe ansatz approach to impurity problem \cite{14, 15}. Eigenstates of the model are parameterized by a set of rapidities $\{ \lambda_n \}_{n=1}^{M}$ ($M$ being the number of down spins), with total magnetization $S_z = NS + 1/2 - M$. Each rapidity represents a magnon of energy $-J_0S/\lambda^2 + S^2$, they are solutions of Bethe ansatz equations

$$e_{imp}(\lambda_n) e_{2S}^{1/2}(\lambda) = \prod_{\alpha} \prod_{\beta \neq \alpha} e_{2}(\lambda_n \pm \lambda_\beta) \quad (3)$$

where $e_\alpha(x) = \frac{x + \alpha}{x - \alpha}$, $e_{imp}(x) = e_{1+2c}(x)e_{1-2c}(x)$ with $c = \sqrt{(S + 1/2)^2 - J_0^2}$. In the following, we only consider the case $J_0 < 0$, $J > 0$, so $c \geq S + 1/2$. In the thermodynamics limit the solutions are classified by $n$-strings or $n$-magnons, they are associated with functions $\eta_n(\lambda)$ (with real rapidity $\lambda$), which are thermal equilibrium for finite temperature $T$ and field $H$ satisfy an infinite set of non-linearly coupled integral equations:

$$\ln \eta_n = \frac{\pi}{g} |J_0| \delta_{n,2S} + G \ln[(1 + \eta_{n-1})(1 + \eta_{n+1})], \quad (4)$$

with the asymptotic condition $\lim_{n \to \infty} \ln \eta_n(\lambda) = H/T = 2x_0$. Here, $G$ is an integral operator with kernel $g(\lambda) = 1/2 \cosh(\pi \lambda)$. Notice that the free energy, expressed in terms of $\eta_n(\lambda)$, consists of three parts $F = F_{bulk} + \frac{1}{N}(F_{imp} + F_{edge})$, bulk part is always the dominate one, impurity and edge parts are both of order of $1/N$, edge part is only due to the open boundary condition and is irrelevant to spin dynamics. We will discuss bulk and impurity parts in the following.

**Ferromagnetic host.** The groundstate of ferromagnetic host has $M = 0$ with finite magnetization $S$ per site. The pure states are those with spin aligned along an arbitrary axis. With finite $T$ and $H$ the elementary excitations are $n$-magnons, bound states carrying $n$ quanta of reduction of magnetization along the ordered axis. Each $n$-magnon has energy $\epsilon_n = 2|J_0|n_{a,2S}(\lambda)$ with crystal momentum $K = 4 \sum_{p=1}^{n_{a,2S}} \arctan \frac{\pi(2S + 1 - 2p)}{2|J_0|}$. In open boundary problem, $K$ is positive. The energy spectrum $\epsilon_n(K)$ is a continuous function, increases monotonically as $K$ varies in $[0, \pi \min(n,S)]$. The long-wavelength limit, i.e., $K \to 0$, $\epsilon_n(K) \sim |J_0|S^2/n$, corresponds to $|\lambda| \sim \infty$. The absence of gap for all $n$-magnons at boundaries of Brillouin zones $\pi \min(n,S)$ indicates the exact cancellation of Umklapp processes \cite{17}. Correspondingly in thermodynamics limit, the driving term in Eq. (4) dominates when $T \to 0$, all $\eta_n(\lambda)$ functions are relevant. The situation is quite different to antiferromagnetic host ($J_0 > 0$), where only $\eta_{S}(\lambda)$ is relevant as $T \to 0$. So the solutions of Eq. (5) are more complicated for $J_0 < 0$. Here two limiting cases are important: (a) Weak coupling limit: $T \to \infty$. The driving term disappears, there is only one parameter in equations, $x_0 = H/T$, the solutions are

$$\eta_n = \frac{\sinh^2[(n + 1)x_0]}{\sinh^2 x_0} - 1 \quad (5)$$

for all $\lambda$ and $n$, this is the free spin limit. (b) Strong coupling limit: $T \to 0$. In this case the driving term diverges, $\ln(1 + \eta_n) \sim \ln \eta_n$, we obtain

$$\eta_n(\lambda) = \exp[nH + \pi a_{n,2S}(\lambda)|J_0|/T]. \quad (6)$$

At zero temperature, $\eta_n = \infty$ for all $n$, i.e., no magnons, the ferromagnetic groundstate with $S_z/N = S$ and $E/N = H$ is reproduced. At low but finite temperature, $\eta_n(\lambda)$ as function of $\lambda$ and $n$ show crossovers between the two asymptotic solutions (3) and (4). In principle, they can be solved numerically for fixed $T$ and $H$. To study the critical behavior of $S = 1/2$ ferromagnetic Heisenberg chain, Schlottmann \cite{18} proposed an analytic method to solve these equations, based on an elegant but simple correlation-length approximation, and the result coincides with the numerical one very well \cite{18}. His idea is now developed for arbitrary $S$ as follows.

Let us first notice that for sufficient large $|\lambda|$, the driving term becomes negligible and one reproduces the weak coupling solution Eq. (3) for all $n$, up to $O(e^{-\pi|\lambda|}|J_0|/T)$. Similarly, for sufficient large string index $n$, $\eta_n(\lambda)$ is again close to Eq. (4) because the effect of the driving term is also negligible. On the other hand, for small $|\lambda|$ and $n$, the driving term dominates, $\eta_n(\lambda)$ approach the strong coupling solution (5). For intermediate $\lambda$ and $n$ we have a crossover region. By equating two solutions (3, 4) for small $\lambda$ and $H$, we define a crossover scale $n_c(T)$ as $2S|J_0|n_c = T \ln[n_c(n_c + 1)]/2$, or solving it iteratively for $T << |J_0|$,

$$n_c(T) \approx \frac{2S|J_0|}{T} \left[ -\frac{1}{\ln(\frac{2S|J_0|}{T})} + \ln\frac{2S|J_0|}{T} \right] + \cdots. \quad (7)$$
The meaning of $n_c(T)$ is clear: for $n > n_c$, $\eta_n$ is close to weak coupling solution while for $n < n_c$ it is close to strong coupling one. Therefore $n_c(T)$ serves as the correlation length of host, because it is the average number of the correlated spins in thermal equilibrium. Similarly, for each $n$-magnon, we define a crossover scale for rapidity,

$$\lambda_c^n(T) \approx \sqrt{\frac{n}{\ln(n+1)}} \frac{2S|J_0|}{T}. \quad (8)$$

Obviously $\lambda_c^n(T)$ is the correlation length in momentum space: for $|\lambda| > \lambda_c^n(T)$ (long wave-length or low energy), $\eta_n(\lambda)$ is close to weak coupling solution, while for $|\lambda| < \lambda_c^n(T)$ (and $n < n_c$) it is close to strong coupling one. Thus we adopt the following strategy to solve $\eta_n(\lambda)$, or more conveniently, to calculate $\xi_n(\lambda) = \ln[1 + \eta_n(\lambda)] - |\pi a_n| |J_0|/T$. (i) As a zero order approximation, we assume $\eta_n(\lambda) = \infty$ for $n < n_c$ and $|\lambda| < \lambda_n^c$, and elsewhere is given by Eq.(8). (ii) Substituting this approximation into the right hand side of Eq.(8), we get the first order approximation for $\xi_n(\lambda)$. The result is

$$\xi_n(\lambda) \approx \sum_{m=1}^{n_c} \left[ \ln[1 + \frac{1}{m(m+2)}] - \frac{2}{2m+1} \right] B_{mn}(\lambda_c^m - \lambda) \quad (9)$$

$$+ 2n \ln \frac{\sinh(1+n_c)x_0}{\sinh n_c x_0},$$

with $B_{mn}(\lambda) = \int_{-\lambda_n^c}^{\lambda_n^c} A_{mn}(\lambda - \lambda') d\lambda'$. The leading contribution to bulk free energy $F_{bulk} = -T \int d\lambda g(\lambda) \xi_2^S(\lambda)$ for small $T$ and $H/T$ is estimated as

$$F_{bulk} = -1.1 \left( \frac{2S}{|J_0|} \right)^{1/2} T^{3/2}$$

$$-0.42 |J_0| \left( \frac{2S|H|}{T^2} \right)^2 \left[ 1 - \frac{\ln(2S|H|)}{\ln(2S|J_0|)} \right] \frac{2S|J_0|}{T^2} + \cdots \quad (10)$$

The zero field dependence of the free energy, as given above, is due to the $n$-magnon contributions from those within the cut-off, $n_c$; while the contributions beyond the cut-off is of higher order, i.e., $\sim T^2$. So this part eventually approach the exact one as $n_c \to \infty$ or $T \to 0$. The finite-field contribution of the free energy, which comes from all kinds of $n$-magnons, is a result of competition of both parts, and it is approximately proportional to the correlation length $n_c$, giving rise logarithmic corrections to the bulk susceptibility. Notice that by rescaling $2S|J_0| \to J_0$, the bulk free energy is proportional to that of the standard spin–1/2 ferromagnetic Heisenberg chain, so one obtains $C_{bulk} = 2SC_{1/2} \sim T^{1/2}$ and $\chi_{bulk} = 2S\chi_{1/2} \sim T^{-2} \ln^{-1} T$.

**Boundary impurity.** When the impurity is coupled to the ferromagnetic host via antiferromagnetic $J > 0$, Eq.(6) has a pure imaginary mode $\lambda = i(e - 1/2)$. This mode contributes a negative energy $\varepsilon_{imp} = |J_0|/|S^2 - (e-1/2)^2| < 0$ to the system, indicating the formation of a bound state, i.e., a local composite made of the impurity and its neighboring spin $S$. One notices that $\lambda = i(e - 1/2)$ is the only possible boundary mode for $S' = 1/2$ and no boundary string is allowed because $\lambda = i(e + 1/2)$ is forbidden as shown in Eq.(3).

Therefore, by taking into account the pure imaginary mode, the BAE for the bulk rapidities read

$$e_{2S}^N(\lambda_n) e_{2S-3}(\lambda_n) \frac{e_{2S-1}(\lambda_n)}{e_{2S}} = \prod_{i=1}^{(M-1)} \prod_{\beta \neq \alpha} e_1(\lambda_n \pm \lambda_\beta). \quad (11)$$

When $2c$ is an integer, the impurity free energy is the difference of contribution from two effective ”ghost” spins $S_+ = c - 1/2$ and $S_- = c - 3/2$.

$$F_{imp} = -\frac{T}{2} \int d\lambda g(\lambda) [\xi_{2c-1}(\lambda) - \xi_{2c-3}(\lambda)]. \quad (12)$$

One finds that the leading contributions from the impurity is the same as that of $1/2$–spin (rescaling of $J_0$ is used), but is always negative, i.e., $C_{imp} = -C_{1/2} \sim T^{1/2}$, $\chi_{imp} = -\chi_{1/2} \sim (T^2 \ln T)^{-1}$, indicating the freezing of some degrees of freedom of the bulk. Notice that $c$-dependence arises only in the next orders. When $2c$ is not an integer, the situation is somewhat more complicated. In this case, there are additional contributions coming from the non-zero residue $(2c) = 2c - [2c]$ ($2c$ is the integer part of $2c$), which renormalize the effective Kondo temperature $T_K \sim 1/\cos(\pi(2c)/2)$ as well as the physical quantities. Explicit calculation shows that they only change the subleading behavior.

The effect of the local composite can be analyzed in the same way. This local composite is made of the impurity spin and the neighboring host spin. In contrast to the single channel problem where the local composite is always a spin singlet, the present one is not a spin singlet, but with a residual spin $S - 1/2$. Its feature is encoded in BAE as follows

$$e_S^{2(\xi_1^S)}(\lambda) = \exp\{i\phi(\lambda)\} \prod_{\beta \neq \alpha} e_1(\lambda_n \pm \lambda_\beta). \quad (13)$$

where $\phi(\lambda) = -i \ln[e_{2S}^N(\lambda_n) e_{2S-3}(\lambda_n)]$ represents the phase shift of a bulk spin wave scattering off the local composite. In the limit $J \to \infty$, one finds a momentum dependent phase shift $\phi(\lambda) \neq 0$ indicating no complete compensation of the impurity spin in multichannel ferromagnetic host. (except for $S = 1/2$ where $\phi(\lambda) = 0$). Similarly, For $2c =$integer, we estimate the local composite contribution to the free energy

$$F_{loc} = \frac{T}{2} \int d\lambda g(\lambda) \left[ \xi_{2c} + \xi_{2c-3} - \xi_{2c-1} \right]. \quad (14)$$

One immediately obtains $C_{loc} = (S - 1/2)C_{1/2} \sim T^{1/2}$, $\chi_{loc} = (S - 1/2)\chi_{1/2} \sim (T^2 \ln T)^{-1}$. So the leading order behavior of the local composite is similar to that
of \((S - 1/2)\)-spin, indicating that the impurity is neither completely screened as in one channel problem nor overscreened as in antiferromagnetic host, but is always locked into the critical behavior of the bulk.

Discussions and conclusions. In this paper, we studied a spin-1/2 boundary impurity coupled to the ferromagnetic spin-\(S\) chain. The results were obtained with the assumption that the crossover from strong coupling to weak one is abrupt. A smooth crossover region should not modify the low-\(T\) and small \(H/T\) dependences of the free energies obtained. Moreover, if the correlation lengths \(\kappa_c\) and \(\kappa'_c\) are scaled, the low-\(T\) dependence of our results remain unchanged, only the amplitudes are rescaled. The leading zero-field dependence of the bulk free energy is a rigorous result, even the amplitude agrees well with those obtained numerically for \(S \rightarrow 1/2\) or analytically for \(S > 1/2\) via spin-wave theory. However, since all \(n\)-strings contribute, the situation is more complicated for the susceptibility, logarithmic corrections in susceptibility arise even without Kondo impurity, indicating the existence of a marginal variable which does not exist in classical spin wave solution.

The situation for multichannel ferromagnetic is dramatically changed. Due to the presence of antiferromagnetic coupled impurity, there is always a pure imaginary mode in addition to the bulk \(n\)-magnons. The magnetization is suppressed due to the local composite which is a spin \(S - 1/2\) object. But the impurity spin is not overscreened, in contrast to that in antiferromagnetic host. Rather, it is locked into the critical behavior of the bulk, with impurity specific heat showing a simple power law \(C_{\text{imp}} \sim T^\lambda\). The local composite now effectively coupled \textit{ferromagnetically} to the other host spins via quantum fluctuation, showing the same power law specific heat \(C_{\text{loc}} \sim T^\lambda\). The absence of multichannel Kondo effect is due to the large correlation length \(\kappa_c\), which is divergent when \(T \rightarrow 0\). Though these results are limited to 1D, similar phenomena may exist in higher dimensions. In multichannel ferromagnet, the antiferromagnetic Kondo coupling always leads to a local composite of a smaller spin, and leads to ferrimagnetic state when there is a finite density distribution of the impurities.

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1. P. Nozieres and A. Blandin, J. Phys. (Paris) \textbf{41}, 193(1980).
2. N. Andrei and C. Destri, Phys. Rev. Lett. \textbf{52}, 364(1984);
   P.B. Wiegmann and A.M. Tsvelik, Z. Phys. \textbf{54}, 201(1985); I. Affleck and A.W.W. Ludwig, Nucl. Phys. \textbf{B352}, 849(1991);
   \textbf{B360}, 641(1991); V.J. Emery and S. Kivelson, Phys. Rev. B \textbf{46}, 10812(1992);
3. D.L. Cox, Phys. Rev. Lett. \textbf{59}, 1240(1987); M.B. Maple et al., J. Phys.: Condens. Matter \textbf{8}, 9773(1996).
4. D.C. Ralph et al., Phys. Rev. Lett. \textbf{72}, 1064(1994).
5. B. Andraka and A.M. Tsvelik, Phys. Rev. Lett. \textbf{67}, 2886(1991); J.A. Hertz, Phys. Rev. B \textbf{14}, 1165(1976); A.J. Millis, Phys. Rev. B \textbf{48}, 7183(1993).
6. C.R. Cassanelllo and E. Fradkin, Phys. Rev. B \textbf{53}, 15079;
   \textbf{56}, 11246(1997); C. Gonzalez-Buxton and K. Ingersent, Phys. Rev. B \textbf{57}, 14254(1998).
7. A.I. Larkin and V.I. Mel'nikov, JETP \textbf{34}, 656(1972).
8. Y. Wang and J.H. Dai, Phys. Rev. B\textbf{59}, 13561(1999).
9. P.P. Kulish, N. Yu Reshetikhin and E.K. Sklyanin, Lett. Math. Phys. \textbf{A 5}, 393(1981).
10. L. Takhtajan, Phys. Lett. \textbf{87A}, 479(1982); H.M. Babujian, Nucl.Phys. B \textbf{215}, 317(1983);
   H.M. Babujian and T.M. Tsvelik, Nucl. Phys. B \textbf{265}, 24(1986).
11. F.D.M. Haldane, J. Phys. C: Solid State Phys. \textbf{15}, L1309(1982).
12. Y. Wang, Phys. Rev. B\textbf{56}, 14045(1997);
   H. Frahm and A. Zyuzin, J. Phys.: Cond. Matt. \textbf{9}, 9039(1997).
13. J.H. Dai and Y. Wang, Phys. Rev. B \textbf{60}, 6594(1999).
14. N. Andrei and H. Johannesson, Phys. Lett. \textbf{100 A}, 108(1981);
   K.-J.-B. Lee and P. Schlottmann, Phys. Rev. B \textbf{37}, 379(1987);
   P. Schlottmann, J. Phys.: Cond. Matt. \textbf{3}, 6619(1991).
15. N. Andrei, in \textit{Low-Dimensional Quantum Field Theories for Condensed Matter Physics}, edited by S. Lundqvist, G. Morandi and Yu Lu (World Scientific, 1995).
16. More precisely, \(\lambda\) in \(\eta_\lambda(\lambda)\) or \(\zeta_\lambda(\lambda)\) defined later could be complex, but limited to the condition \(|\text{Im}\lambda| < 1/2\).
17. P. Schlottmann, Phys. Rev. B \textbf{33}, 4880(1986).
18. P. Schlottmann, Phys. Rev. Lett. \textbf{54}, 2131(1985); M. Taka-
   hashi and M. Yamada, J. Phys. Soc. Japan \textbf{54}, 2808(1985).
19. Numerical bulk results for \(S = 1\) are obtained by many people,
   see References in Ref.[18]. For ferrimagnetic chain,
   the results are expected to be valid for \(S > 1/2\), up to a
   numerical factor dependent on \(S\).
20. M.E. Fisher, Am.J. Phys. \textbf{32}, 343(1964).