Prospects for constraining interacting dark energy cosmology with gravitational-wave bright sirens detected by future SKA-era pulsar timing arrays

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Pulsar timing arrays (PTAs) have the potential to detect Nanohertz gravitational waves (GWs) that are usually generated by the individual inspiraling supermassive black hole binaries (SMBHBs) in the galactic centers. The GW signals as cosmological standard sirens can provide the absolute cosmic distances, thereby can be used to constrain the cosmological parameters. In this paper, we analyze the ability of future SKA-era PTAs to detect the existing SMBHBs candidates assuming the root mean square of timing noise $\sigma_t = 20$ ns, and use the simulated PTA data to constrain the interacting dark energy (IDE) models with energy transfer rate $Q = \beta H \rho_c$. We find that, the future SKA-era PTAs will play an important role in constraining the IDE cosmology. Using only the mock PTA data consisting of 100 pulsars, we obtain $\sigma(H_0) = 0.239$ km s$^{-1}$Mpc$^{-1}$ and $\sigma(\Omega_m) = 0.0103$ in the I$\Lambda$CDM model, which are much better than the results from the Planck TT, TE, EE+lowE. However, the PTA data cannot provide a tight constraint on the coupling parameter $\beta$ compared with Planck, but the data combination of Planck+PTA can provide a rather tight constraint, i.e., $\sigma(\beta) = 0.00232$, since the PTA data could break the parameter degeneracies inherent in CMB. In the I$\omega$CDM model, we obtain $\sigma(\beta) = 0.00137$ and $\sigma(w) = 0.0492$ from the Planck+PTA data combination. In addition, we find that with the increase of the number of pulsars in PTA, the constraint results from the Planck+PTA will be further improved to some extent. We show that the observations of Nanohertz GWs with future SKA-era PTAs will provide a powerful tool for exploring the nature of dark energy and measuring the coupling between dark energy and dark matter.

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I. INTRODUCTION

Since the accelerating expansion of the universe was discovered by observations of Type Ia supernovae (SN Ia) [1, 2], and subsequently was confirmed by the observations of cosmic microwave background (CMB) and large scale structure [3–6], dark energy (DE) with negative pressure was first proposed to explain this counterintuitive phenomenon [7–15]. However, the nature of DE still remains a deep mystery. To describe the characteristics of DE, many DE theoretical models to date have been constructed. The simplest candidate for DE was the cosmological constant $\Lambda$ proposed by Einstein in 1917. The standard $\Lambda$ cold dark matter (ΛCDM) model is in excellent agreement with the mainstream cosmological observations, and the corresponding cosmological parameters have been determined to an impressive accuracy by the current observational data sets [16].

Accurate measurements of cosmological parameters are crucial for understanding the dynamic evolution of the universe and the nature of dark energy (DE) and dark matter (DM). Current primary electromagnetic (EM) observational datasets predominantly consist of cosmic microwave background (CMB) data [16], supernovae type Ia (SN Ia) data [17], and baryon acoustic oscillations (BAO) data [18–20]. Gravitational wave (GW) sources offer a unique opportunity as they allow the direct determination of luminosity distances from the waveform analysis, establishing them as vital standard sirens in cosmology. Particularly when GW events are accompanied by EM counterparts, known as bright sirens, their redshifts can be obtained. Binary neutron star (BNS) mergers are exemplary GW sources, typically associated with kilonovae and short gamma-ray bursts as EM counterparts [21–24], exemplified by the GW170817 event [25–27], which provided an initial measurement of the Hubble constant ($H_0$) with approximately 14% precision [28]. With continued observations from LIGO and Virgo, the precision of $H_0$ measurements is anticipated to reach 2% within five years [29]. Though stellar-mass binary black hole (SBBH) coalescences typically lack EM counterparts and are thus considered dark sirens, they still serve as valuable cosmological probes. For instance, the analysis of 47 gravitational-wave sources from the Third LIGO–Virgo–KAGRA Catalog has yielded an $H_0$ measurement with 17% precision [30]. Moreover, massive black hole binary (MBHB) mergers, which may emit EM counterparts [31–41], could further refine $H_0$ estimates. Specifically, utilizing MBHBs as both bright and dark sirens within the forthcoming Taiji–TianQin–LISA net-
work is projected to enhance the precision of $H_0$ measurements to 0.9\% \[42\].

The supermassive binary black holes (SMBHBs), with masses $\geq 10^8 M_\odot$ residing in galactic centers, emit gravitational waves (GWs) in the nano-Hertz frequency range $(10^{-9} - 10^{-6}$ Hz). These frequencies are particularly within the sensitivity range of pulsar timing arrays (PTAs). Currently, there are three principal PTA initiatives globally: the Parkes Pulsar Timing Array \[43\], the European Pulsar Timing Array \[44\], and the North American Nanohertz Observatory for Gravitational Waves \[45\]. These projects collaborate under the umbrella of the International Pulsar Timing Array \[46\], aiming to boost detection sensitivities. While the primary focus has been on identifying the stochastic gravitational waves \[45\]. These projects collaborate under the umbrella of the International Pulsar Timing Array \[46\], aiming to boost detection sensitivities. While the primary focus has been on identifying the stochastic gravitational waves background (SGWB) \[47–51\], research also extends to the detection of individual SMBHBs \[52–59\]. These SMBHBs, when utilized as standard sirens, offer a novel avenue to constrain cosmological parameters.

In Ref. \[60\], the authors performed a preliminary investigation on constraining the equation-of-state (EoS) parameter of DE (with only EoS parameter $w$ set free, the other cosmological parameters all fixed) using the SMBHBs expected to be detected by SKA-era PTAs, and found that the EoS of DE can be constrained to an uncertainty of $\Delta w \sim 0.02 - 0.1$. Subsequently, in Ref. \[61\] the authors analyze the ability of SKA-era PTAs to detect existing SMBHB candidates in light of the simulation of timing residuals of pulsar signals, and use the mock data to constrain the cosmological parameters. They found that only 100 millisecond stable pulsars (MSPs) are needed to achieve precision cosmology if the root mean square (rms) of timing residuals could be reduced to 20 ns, and the SMBHB bright sirens could effectively break the cosmological parameter degeneracies inherent in the CMB, improving the constraint precision of the EoS of DE to 3.5\% level, which is comparable with the result of Planck 2018.

However, with regard to the interacting dark energy (IDE) cosmology, the study on the impacts of SMBHB bright siren observations on the possible coupling between DE and DM is still absent so far. Therefore, as a further step along this line, in the present work, we would analyze the ability of the PTA in detecting the GWs from SMBHBs, and obtain the luminosity distance information of SMBHBs through numerical simulation, then use the mock data, namely the distance-redshift relations of SMBHBs, to constrain the cosmological parameters in the IDE models. This work will make the analysis of the impacts of bright sirens from PTAs on improving cosmological parameter estimation more complete.

In the non-interaction models, the energy density of each fluid component is conserved separately, $\dot{\rho}_i + 3H(1 + \omega_i)\rho_i = 0$, where $H$ is the Hubble parameter, $\omega$ is the EoS parameter, and the subscript $i = (r, b, c, d, e)$ represents radiation, baryons, CDM and DE, respectively. In the IDE models, however, the energy continuity equations for DE and DM are of the following forms

$$\dot{\rho}_{de} + 3H(1 + \omega)\rho_{de} = -Q, \quad (1)$$
$$\dot{\rho}_c + 3H\rho_c = Q, \quad (2)$$

where $Q$ is the energy transfer rate between CMD and DE. Here, the case of $w = -1$ corresponds to the IACDM model, and the case of $w$ as a free parameter corresponds to the IwCDM model. There are numerous specific forms for the energy transfer rate $Q$ in the literature [62–100]. While in this paper, we only employ a general phenomenological form of $Q = \beta H\rho_c$, where $\beta$ denotes a dimensionless coupling parameter. From Eqs. (1) and (2), it can be seen that $\beta > 0$ indicates that CMD decays into DE, $\beta < 0$ indicates that DE decays into CDM, and $\beta = 0$ means that there is no interaction between DE and CDM.

The rest of this paper is organized as follows. In Sec. II, we introduce the methods including how to use the PTAs to obtain the information of SMBHBs, and use the mock data to constrain the cosmological parameters in IDE models. Then, we give the constraint results and analysis in Sec. III. Finally, the conclusion is given in Sec. IV.

II. METHODOLOGY

A. PTA and SMBHB

The effect of gravitational waves (GWs) on MSPs can be reflected in the residuals of time of arrivals (ToAs) by removing the model-predicted data from the observational ToAs, various noise model, and the timing model. The r.m.s. of these time residuals can be used to measure and constrain the GWs signal.

With regard to a single GW source coming from a direction $\Omega$, its induced pulsar timing residuals at time $t$ on the Earth can be written as \[101\]

$$s(t, \hat{\Omega}) = F_+ (\hat{\Omega}) A_+ (t) + F_\times (\hat{\Omega}) A_\times (t), \quad (3)$$

where $F_+ (\hat{\Omega})$ and $F_\times (\hat{\Omega})$ are antenna pattern functions and are defined by \[58, 102\]

$$F_+ (\hat{\Omega}) = \frac{1}{4(1 - \cos \theta)} \left\{ (1 + \sin^2 \delta) \cos^2 \delta_p \cos [2(\alpha - \alpha_p)] - \sin \delta \sin 2\delta_p \cos (\alpha - \alpha_p) + \cos^2 \delta (2 - 3 \cos^2 \delta_p) \right\};$$
$$F_\times (\hat{\Omega}) = \frac{1}{2(1 - \cos \theta)} \left\{ \cos \delta \sin 2\delta_p \sin (\alpha - \alpha_p) - \sin \delta \cos^2 \delta_p \sin [2(\alpha - \alpha_p)] \right\}. \quad (4)$$

Here, $(\alpha, \delta)$ and $(\alpha_p, \delta_p)$ are the right ascension and declination of the GW source and pulsar, respectively. $\theta$ is the angle between the GW source and pulsar with respect to the observer

$$\cos \theta = \cos \delta \cos \delta_p \cos (\alpha - \alpha_p) + \sin \delta \sin \delta_p. \quad (5)$$
the covariance matrix, which contains the error information of the parameters. In this work, the parameter set \{p\} includes eight GW source parameters, i.e., \{M_c, \alpha, \delta, \iota, \psi, \phi_0, f_0, d_L\}. Since the disk direction is randomly distributed in 4\pi solid angle, we need consider the prior of inclination angle \(P(\iota) \propto \sin \iota\).

![FIG. 1: The simulated precision of the luminosity distances (\(\Delta d_L/d_L\)) of SMBHB candidates with SNR>10 based on the Fisher matrix by using 100, 200, and 500 MSPs, respectively. Here, the luminosity distances \(d_L\) are calculated based on the Planck 2018 best-fit \(\Lambda\)CDM model, with the errors \(\Delta d_L\) being 1\(\sigma\) confidence level.](image)

For the GW sources, we use the current available 154 SMBHB candidates obtained from various characteristic signature. Most (149) of these samples are obtained via periodic variations in their light curves [106–108], and the others are Mrk 231 [109], NGC 5548 [110], OJ 287 [111], SDSS J0159+0105 [112], and Ark 120 [113]. Here, we assume \(m_1 = m_2, \iota = \pi/2, \psi = 0, \phi_0 = 0\) for all SMBHB samples. For the luminosity distance of SMBHB, based on the redshift of sample, we employ the Planck 2018 best-fit \(\Lambda\)CDM model as the fiducial cosmology [16], i.e.,

\[
h = 0.67, \Omega_\Lambda = 0.6847, \Omega_k = 0, \Omega_b = 0.049, \n
w = -1, n_s = 0.963, \sigma_8 = 0.811, N_{\text{eff}} = 3.046. \tag{13}
\]

The capability of PTA to detect GWs can be affected by the number of MSPs, the rms of timing noise, the cadence of monitoring ToAs, the observation time span, and other important factors. In this work, we construct an SKA-era PTA by using the Australia Telescope National Facility (ATNF) pulsar catalog [114]. Since the red noise can be removed by model building or using mathematical methods, and the neighboring MSPs may have a higher flux, the integrated pulse profile has a higher SNR and a more accurate timing, we select MSPs within 3 kpc from the Earth with the rms of timing noise \(\sigma_i = 20\) ns in this work. Following Ref. [60], we assume that the ToA data are obtained via monitoring the pulses from MSPs
with typical cadence of two weeks and the observation span of 10 years.

B. Cosmological Parameter Estimation

We first calculate the SNR of different GW sources according to Eq.(11), then we can naturally obtain the constraints on the luminosity distances of SMBBHs by using the Fisher matrix as described in the last subsection. We only select the SMBBHs with SNR greater than 10 as the useful SMBBH candidates in this work. We find that 11, 16, and 22 SMBBHs meet the requirement of PTA composed of 100, 200, and 500 MSPs, respectively. Our simulated accuracies for the luminosity distances \((\Delta d_L/d_L)\) of SMBBHs are shown in Fig.1. It can be obviously find that for the same SMBBH candidate, when the number of pulsars is larger, the constraint on the luminosity distance \(d_L\) is tighter.

Next, we use the simulated data to constrain the cosmological parameters by performing a Markov Chain Monte Carlo (MCMC) analysis. In addition, we employ the cosmic microwave background (CMB) angular power spectra data of Planck 2018 TT,TE,EE,lowE [16] for the joint analysis and comparison. In this paper, in order to avoid the perturbation divergence problem in the IDE cosmology, we employ the ePPF framework to calculate the cosmological perturbations [78, 79]. We use the CosmoMC package [115] to perform the MCMC calculations and insert the ePPF code as a part of it to deal with the perturbation divergence problem in the global cosmological fit.

### III. RESULT

In this section, we shall report the constraint results on the models of IΛCDM and 1wCDM from the CMB data, simulated PTA data, and the data combinations of CMB and PTA. The constraint results are shown in Figs. 2–5, and summarized in Tables I–II. Note here that, we use “Planck” to denote the CMB data, use the abbreviations “PTA100”, “PTA200”, and “PTA500” to denote the PTA data sets consisting of 100, 200, and 500 MSPs, respectively. In the following, for a cosmological parameter \(\xi\), we use the \(\sigma(\xi)\) to represent its absolute errors.

At first glance in Fig. 2 and Fig. 4, we can find that the CMB data and the simulated PTA data have different degenerate orientations. Thus, combining the CMB and PTA data could break the parameter degeneracies in the IDE models, and the parameter constraints would be evidently improved compared with the cases of using either the CMB-alone data or PTA-alone data.

Fig. 2 shows the 1\(\sigma\) and 2\(\sigma\) posterior distribution contours for the parameters in the IΛCDM model from the Planck, Planck+PTA100, and PTA100 data sets. We find that the future GW bright sirens observation from PTA can give rather tight constraints on \(H_0\) and \(\Omega_m\). With the PTA100 data alone, we get \(\sigma(H_0) = 0.239\) km s\(^{-1}\)Mpc\(^{-1}\) and \(\sigma(\Omega_m) = 0.0103\), which are much better than the results of \(\sigma(H_0) = 1.760\) km s\(^{-1}\)Mpc\(^{-1}\) and \(\sigma(\Omega_m) = 0.0253\) from the Planck data alone. When combining the CMB data with the PTA100 data, we obtain \(\sigma(H_0) = 0.173\) km s\(^{-1}\)Mpc\(^{-1}\) and \(\sigma(\Omega_m) = 0.0027\). Compared with the case of Planck data alone, the data combination Planck+PTA100 can improve the constraint accuracies of \(H_0\) and \(\Omega_m\) by \((1.760 - 0.173)/1.760 = 90.2\%\) and \((0.253 - 0.0027)/0.253 = 89.3\%\) respectively.

For the coupling parameter \(\beta\) in IΛCDM, we find that the PTA-alone data can only give a relatively weak constraint, with \(\sigma(\beta) = 0.01481\). However, the Planck-alone data can give a much tighter constraint, with \(\sigma(\beta) = 0.00232\). Actually, in the IDE models with \(Q = \beta H\rho_c\), the CMB observation is usually more sensitive to the coupling parameter \(\beta\) than the other cosmological parameters. This is because in the early universe, both the Hubble parameter \(H\) and the cold dark matter density \(\rho_c\) take rather high values, and the energy transfer rate \(Q\) can take a moderate value even if the value of \(\beta\) is very small. Thus, the CMB data as an early-universe probe could provide a much tighter constraint on \(\beta\) compared with the PTA data as a late-universe probe. But for the other cosmological parameters, the PTA data could give much tighter constraints than the CMB data. Since the degeneracy orientations for \(\beta\) and other cosmological parameters in CMB and PTA are almost entirely different as shown in Fig. 2, we can obtain much tighter constraints on \(\beta\) when combining the CMB data with the PTA data, with the constraint results being \(\sigma(\beta) = 0.00081\), 0.00080, and 0.00080 from the data combinations of Planck+PTA100, Planck+PTA200 and
FIG. 2: Constraints on the cosmological parameters in the IΛCDM model from Planck, Planck+PTA100, and PTA100, respectively.

Planck+PTA500, respectively. Comparing with the results of \( \sigma(\beta) = 0.00232 \) from the Planck-alone data, we can see that the future GW observations from PTA will exhibit a powerful capability in detecting the possible interaction between dark sectors.

Fig. 3 shows 1σ and 2σ measurement error contours for the parameters in IΛCDM from the Planck CMB data combined with the PTA data sets comprised of different numbers of MSPs. It can be clearly find that with the increase of the number of MSPs in PTA, the constraint results will become tighter little by little. We get \( \sigma(H_0) = 0.119 \) km s\(^{-1}\)Mpc\(^{-1}\) for Planck+PTA200 and \( \sigma(H_0) = 0.077 \) km s\(^{-1}\)Mpc\(^{-1}\) for Planck+PTA500, which can be improved by \((0.173 - 0.119)/0.173 = 31.2\%\) and \((0.173 - 0.077)/0.173 = 55.5\%\), respectively, compared with the case of Planck+PTA100. For the parameter \( \Omega_m \), we get \( \sigma(\Omega_m) = 0.0021 \) for Planck+PTA200 and \( \sigma(\Omega_m) = 0.0017 \) for Planck+PTA500, which can be improved by \((0.0027 - 0.0021)/0.0027 = 22.2\%\) and \((0.0027 - 0.0017)/0.0027 = 37.0\%\), respectively, compared with the case of Planck+PTA100.

The constraint results on the IwCDM model are listed in Table II. In Figs. 4, we show the posterior distribution contours for the parameters of \( \beta \) and \( w \) we are most interested in. We can see that the Planck data alone cannot give a good constraint on the DE EoS parameter \( w \), i.e., \( \sigma(w) = 0.3003 \). This is because the CMB observation is originated from the early universe (\( z \approx 1100 \)), but DE mainly dominates the evolution of late time universe. Therefore, the CMB observation is not sensitive to the DE EoS parameter \( w \), and it is necessary to employ the late-universe probes to break the parameters degeneracies inherent in the CMB. As a late-universe probe, the PTA data can provide a rather tight constraint on \( w \), i.e., \( \sigma(w) = 0.1495 \) from PTA100, which is much better than the result given by the Planck-alone data. With regard to the coupling parameter \( \beta \), as discussed above, using only the Planck data can give a tight constraint on \( \beta \), i.e., \( \sigma(\beta) = 0.00163 \), which is much better than the result from the PTA100 data, \( \sigma(\beta) = 0.01903 \).

Since the Planck data can tightly constrain \( \beta \) and the PTA data can tightly constrain \( w \), their degeneracy orientations are completely different, as shown in Fig. 4. Nevertheless, combining the Planck data with the PTA data could break the parameter degeneracies between \( \beta \) and \( w \), greatly improving the correspond-
The GW bright sirens from SMBHBs that can be detected by PTA in the nanohertz band can serve as a late-universe cosmological probe with great potential to independently measure the absolute distances of the sources. In this work, we investigate the capabilities of future bright siren observations from SKA-era PTAs on constraining the cosmological parameters in the IDE models. We select some existing SMBHB candidates, and use the Fisher information matrix to forecast their accuracies of luminosity distances based on the simulation of timing residuals of pulsar signals. Then, we use the MCMC method to show how the future GW bright sirens from SKA-era PTAs can determine the cosmological parameters and play a key role in breaking the parameter degeneracies inherent in the CMB data.

Taking the future SKA experiment as the standard, we simulate three PTA data sets separately consisting of 100, 200, and 500 pulsars with white noise $\sigma_i = 20$ ns. The observational time spans for all pulsars are 10 years with evenly spaced 2 weeks ToA measurements. In our simu-
We select the SMBHB candidates with SNR > 10, and find that with the increase of the number of pulsars in PTA, the number of SMBHB candidates and the accuracies of their luminosity distances can be largely improved.

We also find that the future SMBHB bright sirens from PTAs could effectively break the parameter degeneracies inherent in the CMB. In the case of PTA consisting of 100 pulsars, the combination of the CMB and PTA can improve the accuracy of $\beta$ in $\Lambda$CDM by 65.1%, and can improve the accuracy of $\beta$ and $w$ in $I_{w}$CDM by 16.0% and 83.6%, respectively, compared with the case of the CMB-alone data. Besides, with the increase of pulsars in PTA, the constraints on the cosmological parameters can also be improved to some extent.

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