Search for $CP$ violation in $\tau \to K\pi\nu_{\tau}$ decays.

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Abstract

We search and find no evidence for $CP$ violation in $\tau$ decays into the $K\pi\nu_{\tau}$ final state. We provide limits on the imaginary part of the coupling constant $\Lambda$ describing a relative contribution of the $CP$ violating processes with respect to the Standard Model to be $-0.172 < \Im(\Lambda) < 0.067$ at 90% C.L..
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The origin and source of CP violation in fundamental fermion interactions is a topic of great interest. CP violation has now been observed in the quark sector [1–4]. Increasing evidence for the existence of neutrino masses and their mixing opens the possibility of CP violation in the neutrino sector [5]. It would be odd if the mixing effects were limited to the quarks and neutrinos only and did not appear in the charged lepton sector. Such mixing could lead to CP violation. There are strict limits on the mixing among the charged leptons coming from the searches for lepton number violation [6]. Nevertheless, various extensions of the Standard Model allow for the existence of CP violation not only due to the mixing but also due to the interference between the W mediated and scalar boson mediated decays [7,8] of the τ to the same final state. This paper describes a search for CP violation in τ decays. The results of the search are interpreted within the context of a model where CP is violated due to interference of W-exchange and an exchange of a charged scalar with complex coupling constant Λ. We assume that CP symmetry is conserved at the τ pair production vertex. For this model, the matrix element for the τ− decay into Kπ−ντ final state is

\[
A(\tau^− \rightarrow K\pi^−\nu_\tau) \sim \bar{u}(\nu)\gamma_\mu(1-\gamma_5)u(\tau)f_V Q^\mu + \Lambda\bar{u}(\nu)(1+\gamma_5)u(\tau)f_SM, \]

where \(f_V\) and \(f_S\) are the vector and the scalar form factors chosen to be Breit-Wigner shapes for \(K^*(892)\) and \(K_0^*(1430)\) resonances, \(M = 1\) GeV/c^2 is a constant providing a normalization of the scalar term, and \(Q^\mu\) is

\[
Q^\mu = [(p_\pi - p_K)^\mu - \frac{m_\pi^2 - m_K^2}{(p_\pi + p_K)^2}(p_\pi + p_K)^\mu].
\]

Here, \(p_\pi, p_K, m_\pi, \) and \(m_K\) are the momenta and masses of the outgoing pion and kaon. The square of the matrix element is
\[ |A^2| \sim |f_V|^2(2(q \cdot Q)(Q \cdot k) - (q \cdot k)Q^2) + |\Lambda|^2|f_S|^2M^2(q \cdot k) + 2\Re(\Lambda)\Re(f_Sf_V^*Mm_\tau(Q \cdot k) \]
\[-2\Im(\Lambda)\Im(f_Sf_V^*Mm_\tau(Q \cdot k), \quad (4)\]

where \( q \) and \( k \) are the 4-vectors of the \( \tau \) lepton and of the neutrino, respectively, and \( m_\tau \) is the \( \tau \) lepton mass. The first three terms are \( CP \) even and the last, underlined term both violates SU(3) flavor symmetry and is \( CP \) odd.

To construct the optimal observable we need to express \( (q \cdot Q), (Q \cdot k), Q^2 \), and \( (q \cdot k) \) in terms of experimentally measured decay parameters. From the energy and momentum conservation law we get

\[ (q \cdot Q) = (Q \cdot k) = -2((m_\tau^2 + m_H^2)/2m_H^2 - m_\tau^2)[(m_\tau^4 + (m_\tau^2 - m_K^2)^2)/(4m_H^2)]^{1/2} \cos \alpha, \quad (5)\]

\[ Q^2 = 2m_\tau^2 + 2m_K^2 - [m_\tau^4 + (m_\tau^2 - m_K^2)^2]/m_H^2, \quad (6)\]

\[ (q \cdot k) = (m_\tau^2 - m_H^2)/2, \quad (7)\]

where \( m_H \) is an invariant mass of the \((\pi K)\) system and \( \alpha \) is an angle between the direction of a pion and the direction of a \( \tau \) lepton in a pion-kaon rest frame. The angle \( \alpha \) is not measurable due to the unknown direction of the \( \tau \). However, the cosine of this angle is statistically equal to the product of cosines of two other measurable angles in the pion-kaon rest frame: \(< \cos \alpha > = < \cos \beta \cos \psi >\). The brackets denote an averaging over the unobserved neutrino direction. The definitions of the angles \( \beta \) and \( \psi \) in terms of measurable quantities can be found in Ref. [11,12].

In this study we use Eqs. (5)-(7) to express the \( CP \)-odd and -even parts of the squared matrix element in Eq. (4). We use the \( CP \)-odd and -even parts of the squared matrix element in Eq. (4) to derive the optimal observable \( \xi \).

The data used in this analysis were collected with the CLEO detector at the Cornell Electron Storage Ring (CESR) operating on or near \( \Upsilon(4S) \) resonance. The data correspond to a total integrated luminosity of 13.3 \( \text{fb}^{-1} \) and contain 12.2 million \( \tau^+\tau^- \) pairs. Versions of the CLEO detector employed here are described in Refs. [13] and [14]. We estimate backgrounds by analyzing large samples of Monte Carlo events following the same procedures that are applied to the actual CLEO data. The generation of \( \tau \) pair production and decay is modeled by the KORALB event generator [17], suitably modified to include the charged scalar contribution to the \( \tau \to K\pi\nu_\tau \) decay. The detector response is simulated with a GEANT-based [18] Monte Carlo simulation.

Tau leptons are produced in pairs in \( e^+e^- \) collisions. Since the CLEO detector is more efficient for detecting \( K^0_S \to \pi^+\pi^- \) decays than for unambiguously identified kaons, we choose to make use of the \( \tau \to K_S^0\pi^+\nu_\tau \) decay, which has a 3-prong topology. We select the candidate events on the basis of the one-vs-three topology. The one-prong ‘tag’ is based on a \( \tau \) candidate decaying into an electron, muon or a single charged hadronic track, and no more than one additional \( \pi^0 \). If one prong is identified as a lepton we require the presence of no more than one photon candidate with energy smaller than 100 MeV. The other, ‘signal’
\( \tau \) is required to decay into a \( K_S^0 \), a charged pion, and a neutrino. We select events with four charged tracks and zero net charge. At CESR beam energies, the decay products of \( \tau^+ \) and \( \tau^- \) are well separated in the detector. Each event is divided into two hemispheres by requiring one charged track to be isolated by at least 90° from the other three tracks. Each track must have a momentum smaller than 0.85 \( E_\text{beam} \) to minimize background from Bhabha scattering and muon pair production. The momenta of all charged tracks are corrected for the energy loss in the beam pipe and tracking system. To ensure the existence of a \( K^0_S \) decay in the three-prong hemisphere, the separation of the tracks in \( z \) at the \( r - \phi \) intersection must be smaller than 12 mm, the radial decay length of the \( K^0_S \) candidate must be larger than 15 mm and the radial impact parameter of the \( K^0_S \) must be less than 1 mm.

Background from photon conversions is suppressed by requiring the cosine of the angle between two tracks to be smaller than 0.99. The invariant mass of tracks forming \( K_S^0 \) candidates must be within 12.5 MeV/\( c^2 \) of the known \( K_S^0 \) mass. To suppress background due to accidental combinations of the tracks we require the minimum impact parameter of the \( K^0_S \) daughter tracks to be greater than 500 \( \mu m \), i.e., two times larger than the typical position resolution in the detector. To suppress background from \( \tau \to K^0_S K \nu_\tau \) decay with the charged kaon misidentified as a pion we require \( dE/dx \) information for the charged track accompanying the \( K^0_S \) to be consistent with a pion.

To estimate backgrounds coming from \( \tau \) decays other than signal we use a Monte Carlo sample containing 39.6 million \( \tau^+ \tau^- \) events in which all combinations of \( \tau^+ \) and \( \tau^- \) decay modes are present, except for our signal process. Non-\( \tau \) background processes include annihilation into multi-hadronic final states, namely \( e^+e^- \to q\bar{q} \) \((q = u, d, s, c \text{ quarks})\) and \( e^+e^- \to \Upsilon(4S) \to B\bar{B} \), as well as production of hadronic final states due to two-photon interactions. Backgrounds from the multi-hadronic physics are estimated using Monte Carlo samples which are slightly larger than the CLEO data and contain 42.6 million \( q\bar{q} \) and 17.3 million \( B\bar{B} \) events, respectively. The background due to two-photon processes is estimated from Monte Carlo simulation of 37 556 \( 2\gamma \to \tau^+\tau^- \) events, using the formalism of Budnev et al. [14]. To study the \( CP \)-violating effects, we use Monte Carlo samples generated with and without \( CP \) violation [17]. The Standard Model Monte Carlo sample contains 170 000 signal events which is four times that of the data, while \( CP \)-violating Monte Carlo samples generated with different values of the complex coupling \( \Lambda \) consist of 200 000 events each.

To suppress the background from the multi-hadronic events \((e^+e^- \to q\bar{q})\) we require the invariant mass of the signal hemisphere to be less than the \( m_\tau \). To suppress background from two-photon interactions we require the missing mass scaled with the center of mass energy to be less than 0.65 and the scaled transverse momentum to be greater than 0.02. We also require the cosine of the angle between the beam-pipe and the direction of the missing momentum to be less than 0.95. Here, missing mass is the invariant mass of the difference between the 4-vector of \( e^+e^- \) system and that for the total sum of all detected particles. Missing momentum is defined as a negative vector sum of all the momentum vectors of detected particles. The efficiency of the above selection criteria is \((11.3 \pm 0.1)\%\). A total of 11 970 events have been selected from the available CLEO data sample.

After applying the above selection criteria to the Monte Carlo simulation of \( e^+e^- \to BB \) and \( e^+e^- \to e^+e^- \gamma\gamma \) processes, we estimate that the remaining background from these sources contributes less than 0.2\% to the data sample. The background from \( e^+e^- \to q\bar{q} \) is estimated to be \((1.9 \pm 0.2)\%\). The dominant background is due to misidentified \( \tau \) decays.
with the largest contribution coming from the $\tau \rightarrow K K^0 \nu_\tau$ decay where we misidentify the charged kaon as a pion. We estimate this contribution to be $(15.2 \pm 1.7)\%$. The sources of background next in importance are due to $\tau$ decays with a $K^0_S$ in a final state with either a lost $\pi^0$, such as $\tau \rightarrow \pi \bar{K} \pi^0 \nu_\tau$ $[(9.5 \pm 1.0)\%]$ and $\tau \rightarrow K \bar{K} \pi^0 \nu_\tau$ $[(3.1 \pm 0.4)\%]$, or a lost $K^0_L$ in $\tau \rightarrow K^0 \pi K \nu_\tau$ $[(8.1 \pm 1.4)\%]$. All other $\tau$ decays contribute less than 1.0\% each. The total background from $\tau$ decays is estimated to be $(39.2 \pm 2.5)\%$, and from all sources, $(41.3 \pm 2.5)\%$. As a cross check of our signal selection procedure we calculate a branching fraction for $\tau \rightarrow (K\pi)_{f=1/2}\nu_\tau$ and obtain a value consistent with the Particle Data Group (PDG) $\xi$ within our statistical error.

$CP$ can be violated as a result of an interference between a vector [dominated by the $K^*(892)$] and a scalar [e.g., the $K^*_0(1430)$] resonance in the final state. To look for evidence of higher mass resonances we plot in Fig. 1 the invariant mass of the $(K^0_S \pi)$ system for the data, signal Monte Carlo and backgrounds. We see no evidence for the $K^*_0(1430)$ resonance. It can be seen in Fig. 1 that the $K^*$ mass peak in the data is shifted by approximately $4.7 \pm 0.9$ MeV/$c^2$ with respect to the Monte Carlo simulation (which is based on the PDG mass of the $K^{*+}$). This is under study, but it does not affect the results presented in this paper.

Another check for the $CP$-violating scalar component in the $\tau$ decay is to look at the average value of the optimal observable as a function of the $(K^0_S \pi)$ invariant mass. We expect the $CP$-violating effects to be maximal in the invariant mass range laying between the resonances, i.e., between 0.9 and 1.4 GeV/$c^2$. In Fig. 2 we plot $<\xi>$ separately for $\tau^-$ and $\tau^+$ as a function of the $(K^0_S \pi)$ invariant mass for the data and for the Monte Carlo with maximum $CP$ violation. A difference between the $<\xi>$ distributions for $\tau^-$ and $\tau^+$ would indicate $CP$ violation. We observe no difference in the $<\xi>$ distributions for the data and, therefore, no $CP$ violation.

To calculate the limit on the $CP$ violation parameter $\Lambda$, we obtain the $\xi$ distribution for the data, for the Standard Model Monte Carlo simulation, and for the background Monte Carlo predictions. In Fig. 3 we plot the $\xi$ distribution for both the full data sample and for the restricted region of the $(K\pi)$ invariant mass $0.85$ GeV/$c^2 < M(K\pi) < 1.45$ GeV/$c^2$ where the sensitivity to $CP$ violation is maximal. Here, we change the sign of $\xi$ distribution for the $\tau^+$ decays to add $\tau^-$ and $\tau^+$ samples together. The corresponding average values of $<\xi>$ are listed in Table I.

### TABLE I

Average value of the optimal observable in data, Standard Model and background Monte Carlo samples.

| Sample                              | $<\xi>$, $10^{-3}$ |
|-------------------------------------|---------------------|
|                                     | Full sample         | $0.85$ GeV/$c^2 < M(K\pi) < 1.45$ GeV/$c^2$ |
| data                                | $-1.5 \pm 1.5$      | $-1.7 \pm 1.7$ |
| signal Monte Carlo                  | $0.4 \pm 1.0$       | $0.5 \pm 1.1$ |
| $\tau$ background Monte Carlo      | $0.6 \pm 1.6$       | $0.7 \pm 2.3$ |
| $qq$ background Monte Carlo         | $-18.1 \pm 14.7$    | $-23.1 \pm 19.1$ |
| data (background subtracted)       | $-2.0 \pm 1.8$      | $-2.3 \pm 1.9$ |
FIG. 1. The \((K^0_S\pi)\) invariant mass for data (squares), signal Monte Carlo prediction (solid line) and background (hatched histogram).
FIG. 2. Average value of the optimal observable as a function of the $(K^0_S\pi)$ invariant mass for (a) data and (b) Monte Carlo with maximum $CP$ violation $\Im(\Lambda) = 1$.

FIG. 3. The distribution of $\xi$ for the data (squares) compared to the sum of the background (hatched histogram) and a Standard Model Monte Carlo prediction for the (a) whole data sample and (b) for the events with the mass of the $(K\pi)$ system ranging between 0.85 and 1.45 GeV/$c^2$.
To relate the observed mean value of the optimal observable $<\xi>$ to the $CP$-violating imaginary part of the coupling constant $\Lambda$, the $\Im(\Lambda)$ dependence of $<\xi>$ must be known. The optimal observable is a pure $CP$-odd quantity, therefore, its average value can be expanded in odd powers of the $CP$-odd part of $\Im(\Lambda)$. We have analyzed a Standard Model Monte Carlo sample with twice as many events as the data, finding an average value of $\xi$ consistent with zero (Table I). Therefore, the selection criteria do not introduce artificial $CP$ violating asymmetry. A study of the $CP$-violating Monte Carlo sample showed that at small values of $\Im(\Lambda)$ only the few first terms in the expansion contribute to the $<\xi>$

$$<\xi> \simeq c_1 \Im(\Lambda) + c_3 \Im(\Lambda)^3.$$  

(8)

We estimate $c_1$ and $c_3$ from the Monte Carlo generated with different values of $\Im(\Lambda)$ by fitting the observed average values of $\xi$ as a function of $\Im(\Lambda)$ to a cubic polynomial. The obtained coefficients $c_1$ and $c_3$ of Eq. (8) are given in Table II.

| Coefficient | Full sample | 0.85 GeV/$c^2$ < $M(K\pi)$ < 1.45 GeV/$c^2$ |
|-------------|-------------|------------------------------------------|
| $c_1$       | 0.0368 ± 0.0018 | 0.0410 ± 0.0020 |
| $c_3$       | −0.0135 ± 0.0019 | −0.0127 ± 0.0022 |

We use these empirically determined coefficients to estimate the value of $\Im(\Lambda)$. In Table II we list the results for both the full sample and the events with restricted $(K\pi)$ invariant mass range.

| Results     | Full sample | 0.85 GeV/$c^2$ < $M(K\pi)$ < 1.45 GeV/$c^2$ |
|-------------|-------------|------------------------------------------|
| $\Im(\Lambda)$ | −0.054 ± 0.049 | −0.046 ± 0.044 |
| 90% confidence limits | (-0.134, 0.027) | (-0.119, 0.027) |

To estimate the upper limit on the $CP$ violating parameter $\Im(\Lambda)$ we first must estimate systematic errors. There are several possible sources of systematic errors that can contribute to this analysis. We treat these errors to be multiplicative if the source can modify the value of $c_1$ and to be additive if the source can bias the central value of $<\xi>$.

To construct the optimal observable we parameterize the scalar hadronic current as a product of a Breit-Wigner shape of the poorly known scalar resonance $K_0^*(1430)$ and a normalization constant $M = 1$ GeV/$c^2$ [Eq. (2)]. This assumption is a source of a systematic error on the calibration coefficient $c_1$. We perform the following studies on the Monte Carlo simulation to estimate this error. We vary the width of the $K_0^*(1430)$ resonance by $\simeq 5\sigma$
and re-calculate the coefficient $c_1$. This results in a change of the value of $c_1$ of $\pm 4.4\%$. Similarly, we change the mass of the $K_0^*(1430)$ resonance from 1.35 GeV/$c^2$ to 1.45 GeV/$c^2$ and obtain a variation of the value of $c_1$ of $\pm 11\%$. Thus, the overall conservative estimate of the systematic error due to the uncertainty in the mass and the width of $K_0^*(1430)$ is $\pm 12\%$.

The choice of $M = 1$ GeV/$c^2$ is simple but not unique. Any Lorentz invariant quantity which has units of mass can be used as a normalization parameter. Another choice for $M$ is the invariant mass of the $(K\pi)$ system. We generate a Monte Carlo sample with the $(K\pi)$ mass as a normalization parameter and re-estimate the value of $c_1$ to be $(0.0326 \pm 0.0003)$. The resulting value of $c_1$ differs from its nominal value by 2%. The overall multiplicative error on $c_1$ due to a choice of the scalar current parameterization is $\pm 12\%$.

In Eq. (2) we define a Standard Model $W$ exchange to have only a pure transverse vector current. However, chiral perturbation theory demands a scalar component \cite{10}. It is assumed to be small and is neglected in TAUOLA. To estimate the systematic error due to the modeling of the $W$ current we modify TAUOLA to include a scalar part. We use a Breit-Wigner shape for the $K_0^*(1430)$ to describe the scalar component. The new value differs by $\simeq 3\%$ from the nominal value. We take this as an estimate of the multiplicative systematic error due to the parameterization of the vector current.

We estimate the values of the coefficients $c_1$ and $c_3$ in Table [1] from the Monte Carlo simulation. Therefore, the quality of the simulation may affect the result. We study the momentum distributions of the pion and of the reconstructed $K_0^*$ candidate in the signal $\tau$ decay, both in data and in the Monte Carlo. We estimate a systematic error on the result by re-calculating the coefficients $c_1$ and $c_3$ for the deviations between data and Monte Carlo parameterized as a slope of the ratio of the momenta distributions for the real and generated data. The multiplicative systematic error due to imperfect simulation of the data is estimated to be 9.3%.

We study the systematic effects due to a possible difference of the track reconstruction efficiency for $\pi^+$ and $\pi^-$ as a function of the pion momentum. To estimate the size of this effect, we study the momentum distribution for charged pions in $\tau^\pm \rightarrow K_0^*\pi^\pm\nu_\tau$ decay. The ratio of these distributions for $\tau^+$ and $\tau^-$ decays is consistent with 1, and the maximum deviation characterized by a slope is fitted to be $0.01 \pm 0.04$. The introduction of such a slope to the data sample changes the value of $\Im(\Lambda)$ by $\pm 0.009$. We take this as a measure of an additive systematic error.

A possible source of a bias is an asymmetry of the $\xi$ distribution induced by the remaining background. If we denote the number of signal and background events by $S$ and $B$, then the contribution to the optimal observable due to the background is

$$\Delta <\xi> = <\xi>_B B/(S+B).$$

Here, $<\xi>_B$ is the value of the optimal observable in the background. We can estimate $<\xi>_B$ from the $\tau$ generic and multi-hadronic Monte Carlo simulations. For the $\tau$ background $B/S$ is $0.413 \pm 0.025$ and $<\xi>_B$ is estimated to be $(0.6 \pm 1.6) \times 10^{-3}$. Therefore, $\Delta <\xi>$ is equal to $(0.2 \pm 0.5) \times 10^{-3}$. Such a change will modify the value of $\Im(\Lambda)$ by $\pm 0.014$. Similarly, the background contribution from the multi-hadronic processes is estimated to be $\pm 0.009$. Therefore, the overall background contribution can modify the central value of $\Im(\Lambda)$ by $\pm 0.017$. 

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The overall multiplicative error is estimated to be ±15%, and the overall additive error on \(\Im(\Lambda)\) is ±0.019. Within our experimental precision we observe no significant asymmetry of the optimal observable and, therefore, no \(CP\) violation in \(\tau \to K\pi\nu_{\tau}\) decay. For a restricted range of the \((K\pi)\) mass (between 0.85 and 1.45 GeV/c\(^2\)) we obtain a value of the imaginary part of the scalar component in the \(\tau\) decays as

\[
\Im(\Lambda) = (-0.046 \pm 0.044 \pm 0.019)(1 \pm 0.15).
\]  

The first error is statistical and the second is additive systematic. The overall expression is multiplied by the multiplicative systematic error. The corresponding limits are

\[-0.172 < \Im(\Lambda) < 0.067,\]  at 90% C.L.

This limit is an order of magnitude more restrictive than that obtained in the previous search \cite{12} for \(CP\) violation in \(\tau \to K\pi\nu_{\tau}\) decays. These results constrain the value of \(\Im(\Lambda)\) at a comparable level to those from our study of \(\tau^{-}\tau^{+} \to (\pi^{-}\pi^{0}\nu_{\tau})(\pi^{-}\pi^{0}\bar{\nu}_{\tau})\) \cite{9}. However, the current result is about a factor of 10 more restrictive on the \(CP\)-violating parameters of Multi-Higgs-Doublet Models \cite{7}.

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