Simultaneous intraportation of many quantum states within the quantum computing network

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Abstract

A scheme is proposed for simultaneous intraportation of many unknown quantum states within a quantum computing network. It is shown that our scheme, much different from the teleportation in the strict sense, can be very similar to the original teleportation proposal[Phys.Rev.Lett.70 (1993)1895] and the efficiency of the scheme for quantum state transmission is very high. The possible applications of our scheme are also discussed.

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Quantum computing\textsuperscript{[1]} is an interesting and hot topic in the quantum theory, which can treat efficiently some nondeterministic polynomial-time problems inaccessible for the existing computer, such as factorization of large numbers\textsuperscript{[2]}, or solve some problems more rapidly, e.g., searching a certain item from a large disordered system\textsuperscript{[3]}, etc. It has been proven that any operation in the quantum computing can be decomposed into a series of two basic operations\textsuperscript{[4]}. One is controlled-NOT(CN) gate, defined as $|\epsilon_1\rangle>|\epsilon_2\rangle \rightarrow |\epsilon_1\rangle|\epsilon_1\oplus\epsilon_2\rangle$ with $\epsilon_{1,2}=0,1$, and the other is Hadamard gate $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ 1 & -1 \end{pmatrix}$, which transforms $|0\rangle$ and $|1\rangle$ to $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$, respectively. As Hadamard gate is a rotation on a single qubit, which is easily realized, the physical realization of the CN operation is the key to the achievement of an actual quantum computing. The experimental demonstration of the CN operation on a trapped ultracold $\text{Be}^+$ showed that the quantum computing can be actually carried out after some technical difficulties, such as the decoherence and ultracold cooling for large quantities of trapped ions, have been overcome\textsuperscript{[5,6]}.

The teleportation of an unknown quantum state over arbitrary distance is another interesting and hot topic in the quantum theory. It is also a striking demonstration of the nonlocal character of quantum states\textsuperscript{[7]}, which starts at a joint measurement on a particle(labelled as particle $a$) in an unknown state and a particle $b$ being one-half of a maximally entangled pair of particles($b$ and $c$), and ends at a suitable unitary rotation of particle $c$ to restore the state of particle $a$ with the help of two bits of classical message. It has been proven\textsuperscript{[8,9]} that the teleportation is a special reversible quantum operation, which hides the quantum information within the correlation between the system and the environment. Recently, the scheme has been extended to the teleportation of continued variables\textsuperscript{[10]}, and teleportation with GHZ state\textsuperscript{[11]}. Milburn et al\textsuperscript{[12]} demonstrated theoretically the teleportation via a two-mode squeezed vacuum state in light of the proposal in Ref.[13], and the teleportation was also proposed to construct a variety of quantum gates, associated with other operations\textsuperscript{[14]}. As far as we know, the teleportation has been demonstrated experimentally by using parametric down-conversion in interferometric Bell state analyzers and in k-vector entanglement\textsuperscript{[15,16]}, \textit{et al}.\textsuperscript{[17]}.
NMR method\cite{17} and continuous variables of electromagnetic field\cite{18}.

Since the essence of the teleportation is based on the quantum entanglement which is also the basis of the quantum parallelism in the quantum computing, a scheme\cite{19} for achieving teleportation via quantum computing was proposed recently. In that scheme, three quantum states are put into a circuit consisting of Hadamard operations and CN ones, in which one(input from channel \(a\)) is the unknown quantum state needed to be teleported and the other two are auxiliary ones (input from channels \(b\) and \(c\) respectively) to be entangled. After the entanglement of the states in channels \(a\) and \(b\)(one-half of the entanglement of \(b\) and \(c\)), one carries out a series of CN and Hadamard operations on the state in channel \(c\) with the help of the quantum information from channels \(a\) and \(b\). Finally, the quantum state put in the channel \(a\) reappears at the output of the channel \(c\). Obviously, this scheme is somewhat different from the original scheme proposed by Bennett et al\cite{7}. First, there is no need of joint measurement with Bell basis states. Although the authors made some discussions for the measurement on the channels \(a\) and \(b\) at the border line of the sender Alice and the receiver Bob, it is easily found that the final result is irrelative to these measurements. Secondly, Bob needs not only a unitary rotation to the quantum state in channel \(c\), but also CN operations to the quantum state with the help of the information from channels \(a\) and \(b\) being control bits. Particularly the CN operation between the channels \(a\)-\(c\) and \(b\)-\(c\) implies that \(a\) and \(b\) are not arbitrarily far away from \(c\). So this scheme is not actually referred to the teleportation, but, in some sense, the *intraportation* which transmits a state from a channel at the sender to another at the receiver within a network system.

Nevertheless, this scheme is an interesting extension for the original teleportation proposal, which enlightens us to make a more efficient transmission of the unknown states via quantum computing network. In this contribution, we will extend this idea to try to transmit simultaneously many unknown quantum states with the circuit similar to Ref.\cite{19}. It can be found that the pure state, instead of the entangled state required in Ref.\cite{7}, can be used as the auxiliary state for the state transmission, and fewer classical messages are
needed in our scheme than in Ref.\[7\]. However, the entanglement is still essential to our scheme. A comparison of our scheme with Ref.\[7\] will be made to show the advantages of our scheme. The possible applications of our scheme will be discussed in the secure transmission of quantum states as well as the preparation of nonclassical states.

Suppose first that two quantum states $a|1\rangle + b|0\rangle$ and $e|1\rangle + f|0\rangle$ put respectively in channels 1 and 3 will be intraported, and the other state $c|1\rangle + d|0\rangle$ in the channel 2 is auxiliary. (In this work, we set channels 1, 2 and 3 corresponding to the the channels from the top to the bottom.) Alice need not make any measurement at the position of the vertical dotted line denoting the border line of Alice and Bob, but informs Bob by the broadcast or telephone with one bits of message about the values of $c$ and $d$. If $c = d = \frac{1}{\sqrt{2}}$, Bob will carry out a series of corresponding operations, as shown in Fig.1 where and in the following figures the channels for transmitting the classical message are omitted. Then he will obtain $a|1\rangle + b|0\rangle$ and $e|1\rangle + f|0\rangle$ at the outputs of the channels 2 and 1 respectively. If $c = 0$ and $d = 1$, the operation by Bob is shown in Fig.2, where he obtains $e|1\rangle + f|0\rangle$ and $a|1\rangle + b|0\rangle$ at the outputs of the channels 1 and 3 respectively. Similarly, if $c = 1$ and $d = 0$, the operation by Bob is demonstrated in Fig.3. The later two procedures in fact result in the swapping of quantum states, or identity interchange of quantum states\[20\].

The scheme can be generalized to the intraportation of the unknown quantum states input respectively from channels 1 and 2, or channels 2 and 3, where the values of $e$ and $f$, or values of $a$ and $b$ should be informed to Bob. The circuit shown in Fig.4 is an example in these respects. From Figs. 1-3 we know that there are three kinds of possible outputs for a certain kind of input under the present operations. For example, for the input case in Fig.1, the three kinds of possible outputs are $(e|1\rangle + f|0\rangle)(a|1\rangle + b|0\rangle), (|1\rangle + |0\rangle)(a|1\rangle + b|0\rangle), (e|1\rangle + f|0\rangle)(a|1\rangle + b|0\rangle), (|1\rangle + |0\rangle)(a|1\rangle + b|0\rangle), (e|1\rangle + f|0\rangle)(a|1\rangle + b|0\rangle)$. So If the auxiliary state input by Alice is restricted to three cases, that is, $c = d = \frac{1}{\sqrt{2}}$, $c = 0$ and $d = 1$, and $c = 1$ and $d = 0$, then there are totally nine different cases for each intraportion, which in fact constitute the protocol between Bob and Alice for different operations performed by Bob corresponding to different inputs set by Alice.
Obviously, in the protocol, it is important to make clear from which channel the auxiliary state is input, since different situation for input channels of auxiliary states correspond to different operations Bob should perform.

As we know, three CN operation sequences will result in the swapping of two quantum states, i.e., \( CN_{12}CN_{21}CN_{12} |\Psi >_1 |\Phi >_2 = |\Phi >_1 |\Psi >_2. \) So with different group of these operations, three input quantum states can also reappear at the outputs of different channels via quantum computing, as demonstrated in Fig.5 where \( \Psi, \Phi, \theta \) are arbitrary quantum states. But this swapping operation is different from our intraportation scheme. The major difference is that, in our scheme of intraportation, the quantum information is divided into two parts: one is related to the entangled state, and the other is the purely classical information. Moreover, the three input quantum states are fully entangled before they are sent to the receiver in our scheme, instead of the swapping operation depending on the interaction between arbitrary two states of the three input states. However, by means of the swapping operations, we can simplify above protocol for intraportation, that is, Bob only needs to consider one kind of output case for each input case. For example, in Fig.1, Bob chooses the operations to produce \( (e|1 > + f|0 >)_{1}(a|1 > + b|0 >)_{2}(|1 > + |0 >)_{3}, \) and the other two kinds of outputs can be obtained via a group of the swapping operations. Therefore, the protocol for the present intraportation scheme can be reduced to including only three different cases, and the other different output cases are resorted to the post-intraportation treatment.

The teleportation of the entangled state is also an interesting topic for the quantum communication. With the original teleportation proposal\(^{[7]}\), it is easily proven that an unknown entangled state can be teleported via a known entangled state as well as two bits of classical message. Our scheme simultaneously transmitting two pure quantum states, in some sense, also means that it can intraport entangled states. As shown in Fig.6, two quantum states \( c|1 > + d|0 > \) and \( e|1 > + f|0 > \) are entangled, after the Hadamard gate and CN gate, to be \( \Psi = \frac{1}{\sqrt{2}}(c + d)(e|01 > + f|00 >) + \frac{1}{\sqrt{2}}(d - c)(e|10 > + f|11 >). \) With the help of the auxiliary state \( |0 > \) and the corresponding operations by Bob, \( \Psi \) reappears at the outputs.
of channels 1 and 2. In this procedure, the intraportion only needs a pure state to be auxiliary, and one bits of classical message about the auxiliary state from Alice to Bob, which is more efficient than that with the original teleportation proposal. Obviously, before the intraportion, the protocol should be made between Alice and Bob for Bob's operation corresponding to the different auxiliary state input by Alice.

The present scheme can be readily extended to intraporting simultaneously many unknown quantum states and the intraportion of many-particle entangled states, with the increase of the channels transmitting the quantum states, and a little bit modification of the circuit. For example, for the case of four channels, we can intraport simultaneously at most three unknown quantum states via one bit of classical message after the protocol has been made by Alice and Bob, as shown in Figs.7, 8 and 9 where the auxiliary states were input from the lowest channel. Comparing these figures with Figs.1-4, and by direct deductions for other many-channel situations, we find that, for the case of N channels, no matter from which channel the auxiliary state is input, the operations performed by Alice can be $H_{N-1}CN_{N-1}N_{N-2}CN_{N-2}N_{N-3} \cdots CN_{23}CN_{12}H_1$, and the first five operations by Bob are $CN_{N-1}NHNCN_{1N}H_1H_N$. The other operations by Bob should be performed in terms of the specific input situation. These characteristics are useful for the production of the protocol for intraportion. According to above discussion, we know that, even for the many-channel situation, there are still three different cases in the protocol if we use the swapping operations for the post-intraportion treatment.

As the quantum states are fully entangled before they are reconstructed at the output locations, one possible application of our scheme is the secure transmission of the everyday quantum messages within a quantum computing network. Suppose an eavesdropper wants to get messages from the channels between the sender Alice and the distant receiver Bob. The eavesdropper will not succeed even if he eavesdrops the quantum information simultaneously from all channels, and meanwhile receives the classical message send by Alice, as long as he do not know the protocol made by Alice and Bob. As the eavesdropper is not clear from which channel the auxiliary state is input, the probability for his successful eavesdropping
decreases as the increase of the number of the channels, and the action of the eavesdropper can be detected by Bob from the comparison of the output results with the expected results given by the protocol. Although it also needs the judgement for whether there exists the eavesdropping, however, different from the standard cryptographic scheme\cite{21}, our scheme does not involve the quantum key distribution process because we have supposed that Alice and Bob are not much far away from each other. The protocol in our scheme can be made by Alice and Bob meeting at a common place. One may ask: why does not Alice send her quantum message to Bob directly when they meet each other? The key point in our scheme is that, once the protocol has been made, Alice can send her quantum messages to Bob efficiently and safely at any time when required. They may meet each other once a month or longer to change their protocol for keeping secure transmission of the quantum information in the next several days. Strictly speaking, our scheme is less practical for two much distant users than the standard quantum cryptography, whereas it might be more convenient and practical for the everyday transmission of the quantum information within a limited quantum computing network in future. Moreover, a by-product of this application is the preparation of different quantum states at the output location. By suitably choosing the operations as well as the coefficients of the input states, some useful quantum states, such as Bell states\cite{7}, Greenberger-Horne-Zeilinger states\cite{22} and so on, can be obtained from the output states. For example, in Fig.1, if Bob does not perform the last CN$_{23}$, but make a measurement at the channel 3, then he will obtain $ea|11> + fb|00> + eb|10> + fa|01>$ from channels 1 and 2 by projecting the output state of the channel 3 on $|0>$ or $|1>$. 

A shortcoming of our scheme is the increase of the quantum gates with the increase of the number of the channels. In practical application, we may restrict the number of the channels in terms of the quantity of the quantum message to be transmitted. We can also try to integrate some quantum gates, according to the regularity of operations referred to above, to reduce the number of the operation.

In summary, we pointed out that a former teleportation scheme via quantum computing network is actually an intraportation within a quantum computing network, and along this
idea, we studied how to intraport simultaneously many unknown quantum states. As the classical message from Alice to Bob is necessary in the scheme, our scheme is more in tune with the original teleportation proposal than Ref.[19]. Moreover our scheme is also more efficient than the transmission of unknown quantum states with the original teleportation proposal. Different from Ref.[7], however, the present scheme do not obviously depend on the entanglement characteristic of the auxiliary state itself since the pure state can also act as the auxiliary state, whereas the entanglement is still the heart of our scheme. So from this viewpoint, the present scheme does not follow closely the original teleportation proposal. On the other hand, for more coincidence with the original teleportation proposal, we can use two auxiliary states for the many-channel cases. For example, in Fig.7, we can set $e = 0$ and $f = 1$, thus the Hadamard gate and the following CN operation make the two auxiliary states entangled. Then one part of the entanglement is correlated with the two unknown quantum states, and the other one is sent directly to Bob. As a result, Bob can reproduce in his side the two unknown quantum states by two bits of classical messages sent from Alice and some corresponding operations performed according to the protocol he made with Alice. Obviously, with this change, our scheme is well consistent with the proposal in Ref.[7], whereas our scheme is still more efficient as many unknown quantum states(e.g. two unknown quantum states in Fig.7) are transmitted simultaneously by means of two bits of classical message. From above analysis, we know that, the realization of the present scheme at least requires two prerequisites. One is the quantum channels between Alice and Bob, whereas with what material to construct these quantum channels is still an open question. The other is the experimental achievement of the quantum computing with many-qubit. As the seven-qubit quantum computing has been carried out via NMR[23], and many-qubit quantum computing via trapped ions will be achieved in the near future[24,25], we believe that our scheme will be helpful for the exploration of efficient transmission of quantum information in future actual quantum computing network.

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Note added: After finishing this work, the author was told that a work\textsuperscript{[26]} extending Ref.[19] along another direction had been carried out in the field of condensed matter physics.
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Captions of the figures

Fig.1 Intraportation of two quantum states, where the auxiliary state is $|0\rangle + |1\rangle$ and input from the middle channel.

Fig.2 The same as Fig.1, but the auxiliary state is $|0\rangle$.

Fig.3 The same as Fig.1, but the auxiliary state is $|1\rangle$.

Fig.4 The same as Fig.1, but the auxiliary state $|0\rangle$ is input from the lowest channel.

Fig.5 Quantum swapping of three quantum states.

Fig.6 Intraportation of an entangled state.

Fig.7 Intraportation of three quantum states, where the auxiliary state is $|0\rangle + |1\rangle$ and input from the lowest channel.

Fig.8 The same as Fig.7, but the auxiliary state is $|0\rangle$.

Fig.9 The same as Fig.7, but the auxiliary state is $|1\rangle$. 
\[
a \{1 \} > +b\{0 \} > \\
|1 \} > +\{0 \} > \\
e \{1 \} > +f\{0 \} > \\
\text{Alice} \\
\end{array} \quad \begin{array}{c}
H \\
H \\
\text{Bob} \\
(\{1 \} > +\{0 \} >)/\sqrt{2}
\end{array}
\]

Fig. 1

\[
a \{1 \} > +b\{0 \} > \\
|0 \} > \\
e \{1 \} > +f\{0 \} > \\
\text{Alice} \\
\end{array} \quad \begin{array}{c}
H \\
H \\
\text{Bob} \\
(\{1 \} > +\{0 \} >)/\sqrt{2}
\end{array}
\]

Fig. 2

\[
a \{1 \} > +b\{0 \} > \\
|1 \} > \\
e \{1 \} > +f\{0 \} > \\
\text{Alice} \\
\end{array} \quad \begin{array}{c}
H \\
H \\
\text{Bob} \\
(\{1 \} > -\{0 \} >)/\sqrt{2}
\end{array}
\]

Fig. 3

\[
a \{1 \} > +b\{0 \} > \\
c \{1 \} > +d\{0 \} > \\
|0 \} > \\
\text{Alice} \\
\end{array} \quad \begin{array}{c}
H \\
H \\
\text{Bob} \\
\text{c} \{1 \} > +d\{0 \} >
\end{array}
\]

Fig. 4

\[
\Psi \\
\Phi \\
\theta \\
\Psi
\]

Fig. 5

\[
|0 \} > \\
c \{1 \} > +d\{0 \} > \\
e \{1 \} > +f\{0 \} > \\
\text{Alice} \\
\end{array} \quad \begin{array}{c}
H \\
H \\
\text{Bob} \\
} \Psi
\end{array}
\]

Fig. 6
Fig. 7

Fig. 8

Fig. 9