Motion control of chaotic permanent-magnet synchronous motor servo system with neural network–based disturbance observer

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Abstract
The permanent-magnet synchronous motor system will display a variety of chaotic phenomenon when its parameters or external inputs satisfy certain condition, and thus its performance would be deteriorated. Therefore, chaos should be suppressed or eliminated. In this article, a practical method which combines adaptive robust control with a single-layer neural network–based disturbance observer is proposed for elimination of the chaos and high-performance motion control of permanent-magnet synchronous motor. The proposed controller not only accounts for the load torque disturbance but also takes the parametric uncertainties into account. A single-layer neural network–based disturbance observer is designed to estimate the disturbance while an adaptive control law is designed to estimate the parameters respectively. Then, all the estimated values are used in the feedforward cancelation item in the controller via a backstepping technique. Lyapunov’s method is used to prove the stability of the novel control scheme. Sufficient comparative simulation results are obtained to validate the effectiveness of the proposed control strategy.

Keywords
Adaptive robust control, chaos, neural network, observer, permanent-magnet synchronous motor

Introduction
Nowadays, permanent-magnet synchronous motors (PMSM) have been adopted as the primary actuating components in many industries due to its high power density, high efficiency, eco-friendly, low cost, and large torque-to-inertia ratio\(^1\)\(^-\)\(^3\) compared with its hydraulic part.\(^4\) However, it is still a challenge to realize a high-accuracy motion control of this kind of motor servo system since it is often a multi-variable nonlinear system with parametric uncertainty and external disturbances, which may lead to undesired behavior of PMSM servo system and even make the system unstable. Most studies just focus on modeling and controller designing, while ignoring some special nonlinear dynamic characteristics in the system, such as the irregular oscillation and low-frequency electro-magnetic noise. Recently, more and more research results show that the motion systems driven by PMSM can make chaotic movements when their parameters reach some certain values.\(^5\)\(^-\)\(^7\) These nonlinear chaotic behaviors may explain why the aforementioned irregular
oscillation and low-frequency electro-magnetic noise often occur in the motion systems driven by PMSM. This would deteriorate the performance of the systems and even make them unstable.\(^5\)\(^{-11}\)

The Ott–Grebović–Yorke (OGY) method is a classic method for chaos control, which is proposed by Ott et al.\(^12\) and could stabilize a sequence of Poincare section crossing points by adjusting a control parameter. However, it costs too much time to stabilize the system. Then, the time-delay feedback control (TDFC) was proposed by Zhang et al.\(^13\) It could make the system move along a desired periodic trajectory when the system moves close to the periodic trajectory. However, the desired trajectory must be one of the unstable periodic trajectories of the system or at the equilibrium, which could not satisfy the requirement in the reality. Feedback linearization technique was used for the chaos control in motor servo systems in the literature.\(^14\),\(^15\) It enabled the controlled system to evolve to any desired value according to the need in the reality. However, it is not easy to realize a high-performance motion control for motor servo systems with this method since it is based on the system’s mathematical model while we could not get accurate mathematical models for the systems in reality. Then, a kind of adaptive feedback control strategy was proposed by Hu et al.\(^16\) A parametric adaptive law was designed to estimate the parameters of PMSM servo systems which are unknown and then the estimated values are used in the feedforward cancelation item in the controller via a backstepping technique.\(^17\)\(^{-19}\) Thus, the control performance was improved dramatically. However, in Hu et al.\(^16\) the load torque was just regarded as a known variable, while in reality, the load torque usually varies with time and it is also unknown. Thus, the hypothesis in Hu et al.\(^16\) is not appropriate. In fact, the load torque could be regarded as a kind of disturbance. In this case, the adaptive control cannot improve the control performance of the system and may also make the system unstable when the disturbance becomes larger and larger. Then, in Hu et al.,\(^20\) a kind of adaptive robust control strategy was proposed to suppress the chaos and improve the control performance. A parametric adaption law and a high-gain robust feedback item were designed respectively to handle parameter uncertainties and time-varying disturbance together in one controller. However, the high gain often tends to make systems unstable. Thus, it is better to design an observer to observe the disturbance which can be canceled in the controller designed later. In this way, the robust gain could be reduced and thus the system could be stable.

With the development of artificial intelligence technology, the intelligent control underwent a rapid development. Neural networks have attracted much attention and have been applied in many ways.\(^21\),\(^22\) In Feng,\(^23\) the parametric uncertainty and uncertain nonlinearity of nonlinear systems are generally considered as a kind of system model uncertainty. Neural networks are used to approximate the model uncertainty of the whole system, and feedforward compensation technology is used to compensate the uncertainty to improve the control performance of the system. Although this method can realize the integrated design of nonlinear controller, it increases the modeling error and the approximation burden of neural network. When the system has only parametric uncertainty, this kind of controller design method is obviously inferior to the parameter adaptive control strategy based on the system model. Considering this point, we just design an intelligent compound controller which combines the advantage of neural network and the advantage of adaptive control, and it is different from the existing methods and novel. In virtue of the universal approximation of the neural network,\(^24\) we would like to use it as an observer of load torque disturbance to estimate the load torque. Adaptive control method is used to estimate the uncertain parameters. Then, both the parametric uncertainty and the disturbance (uncertain nonlinearity) could be compensated with feedforward cancelation technique using their estimation values. At this time, the residual model uncertainty is very small and a robust item with low gain could be designed to suppress it. Thus, the system could be more stable and more accurate. In a word, an integrated controller design idea of fusing parameter adaption with neural network–based load torque disturbance estimation is proposed, in order to suppress the chaos and improve the tracking performance of the PMSM system by combining the advantages of these two methods and overcome their drawbacks. A single-layer neural network is designed to approximate and compensate the load torque disturbance, and at the same time, a model-based parameter adaption control strategy is designed to estimate the uncertain parameter to reduce the parametric uncertainty. In addition, a low-gain robust item is also designed to suppress the observation error and other compensation errors. The extensive comparative simulation results are presented in the article and they prove the validity of this method.

This article is arranged as follows. The chaotic dynamics of the PMSM servo system is analyzed in section “Problem description,” and the proposed controller is designed in detail in section “Design of the controller with neural network–based disturbance observer.” The simulation results are illustrated in section “Results of the simulation,” and finally, the conclusion is given in section “Conclusion.”
Problem description

According to Li et al., the electromechanical behavior of PMSM servo system can be described by the following first-order differential equations

\[
\begin{align*}
\dot{x} &= \sigma(y - x) - \tilde{T}_L \\
\dot{y} &= (\mu - z)x - y + \tilde{u}_q \\
\dot{z} &= -z + xy + \tilde{u}_d
\end{align*}
\]

where \( x = \frac{R}{L_q} \hat{x} \), \( y = \frac{p_n L_q \psi_{fd}}{R B_{equ}} \hat{y} \), \( z = \frac{p_n L_q \psi_{fd}}{R B_{equ}} \hat{z} \), \( \hat{x} = \omega \), 
\( \hat{y} = i_q, \hat{z} = i_d \), \( \sigma = \frac{B_{equ} L_q}{J_{equ} R} \), \( \mu = \frac{p_n \psi_{fd}^2}{B_{equ} R} \), \( \tilde{u}_d = \frac{p_n L_q \psi_{fd}}{R^2 B_{equ}} u_d \),
\( \tilde{u}_q = \frac{p_n L_q \psi_{fd}}{R^2 B_{equ}} u_q \), and \( \tilde{T}_L = \frac{L_q^2}{R^2 J_{equ}} T_L \), where \( u_q \) and \( u_d \) represent the quadrature- and direct-axis voltage of the stator, respectively; \( i_q \) and \( i_d \) represent the quadrature- and direct-axis current of the stator, respectively; \( L_q \) and \( L_d \) represent the quadrature- and direct-axis inductance of the stator respectively; \( \psi_{fd} \) is the direct-axis flux; \( R \) is the resistance of the stator; \( p_n \) is the number of pole-pairs; \( \omega \) is the frequency of the motor angle; \( T_L \) is the load torque, \( B_{equ} \) is the equivalent friction coefficient; and \( J_{equ} \) is the equivalent polar moment of inertia.

According to equation (1), some conclusions can be reached that both Hopf bifurcation and chaos would appear when some parameters of the PMSM servo system drop into a certain range. For example, when the systematic parameters are as follows: \( \sigma = 10.5 \), \( \tilde{T}_L = 6 \), and the control values are set as \( \tilde{u}_q = -x \) and \( \tilde{u}_d = 0 \) which indicates linear feedback control, we could obtain the bifurcation figure with \( \mu = 24.8 \) changing from 1 to 500, which is shown in Figure 1. It can be seen in Figure 2 that the phase trajectory falls into the chaotic movement when \( \mu = 24.8 \).

In this article, our aim is to suppress the chaos since it would degrade the system’s performance and to make the system motion speed track a desired value or trajectory. Thus, we need to find some accessible manipulated variables first. From system dynamical equation (1), it can be seen that \( \tilde{u}_q, \tilde{u}_d \) could be regarded as two manipulated control inputs and thus could be properly designed to drive the systematic trajectory to a desired one. Now, the control inputs should be designed. The general feedback linearization control method could achieve the aforementioned goal effectively when there is no parametric uncertainty or external disturbance in the system. However, there always exist all these uncertainties in PMSM which would degrade the control effect of the general feedback linearization control method. To make up for its deficiency, a single-layer neural network–based disturbance observer and a parameter adaption law would be designed to estimate the disturbance and the parameters. All the estimated values could be used in the controller designed later.

To realize the aforementioned aim, the system state equation is built first as follows

\[
\begin{align*}
\dot{x}_1 &= \sigma(x_2 - x_1) - \tilde{T}_L \\
\dot{x}_2 &= -x_2 - x_1 x_3 + \mu x_1 + \tilde{u}_q \\
\dot{x}_3 &= -x_3 + x_1 x_2 + \tilde{u}_d
\end{align*}
\]

where \( x_1 = x, x_2 = y, \) and \( x_3 = z \) are the system states. The specific control strategy for system (2) is given in the next part.

Design of the controller with neural network–based disturbance observer

Parameter adaption

Denote \( \hat{\theta} \) as the estimate of \( \theta \) and \( \tilde{\theta} \) as the error of the estimation (i.e. \( \tilde{\theta} = \theta - \hat{\theta} \)). Define a discontinuous projection as
Pr ojₘ(•) = \begin{cases} 
0, & \hat{\theta}_i = \theta_{i\max} \text{ and } \hat{\theta}_i > 0 \\
0, & \hat{\theta}_i = \theta_{i\min} \text{ and } \hat{\theta}_i < 0 \\
\hat{\theta}_i, & \text{otherwise} 
\end{cases} \quad (3)

In equation (3), \dot{\theta}_i represents the \(i\)th component of the vector \(\theta\).

An adaptation law is given as follows

\[ \dot{\theta} = \text{Pr oj}_\theta(\Gamma \tau), \text{ with } \theta_{\min} \leq \dot{\theta} \leq \theta_{\max} \quad (4) \]

where \(\Gamma > 0\) is a diagonal matrix of adaptation rate and \(\tau\) is a parameter regression law to be deduced later. For any \(\tau\), the projection mapping used in equation (4) guarantees\(^1\)

\[ \text{(P1) } \hat{\theta} \in \Omega_\theta \equiv \left\{ \theta : \theta_{\min} \leq \hat{\theta} \leq \theta_{\max} \right\} \quad (5) \]

\[ \text{(P2) } \hat{\theta}[\Gamma^{-1} \text{Pr oj}_\theta(\Gamma \tau) - \tau] \leq 0 \quad \forall \tau \quad (6) \]

**Radial basis function–based load torque disturbance observer design**

**Step 1.** Design the structure of the radial basis function (RBF) network. RBF networks are universal approximators in that they can approximate any smooth nonlinear function within arbitrary accuracy, given a sufficient number of hidden layer neurons and input information. Here, an RBF neural network is trained online to approximate load torque disturbance \(\hat{T}_L\) (Figure 3).

The input–output map of RBF network can be expressed as

\[ f(x) = W^T h(x) + \epsilon_{\text{approx}} = \hat{T}_L \quad (7) \]

\[ h_j = \exp \left( \frac{||x - c_j||^2}{2h_j^2} \right) \quad (8) \]

where \(x\) is the input of the network, \(j\) is the \(j\)th node of the hidden layer, \(h = [h_j]^T\) is the output of network’s Guassian radial function, \(W^*\) is the ideal weight value of the network, and \(\epsilon_{\text{approx}}\) is the approximation error of the network and \(\epsilon_{\text{approx}} \equiv \epsilon_N\).

The input of the network is taken as \(x = [x_1, x_2]^T\), and the actual output of the network is

\[ f(x) = \hat{W}^T h(x) = \hat{T}_L \quad (9) \]

where \(\hat{W}\) is the estimation of \(W^*\). Then, the estimation error of \(\hat{T}_L\) can be expressed as

\[ \tilde{f}(x) = \hat{T}_L - T_L = \hat{W}^T h(x) - (W^T h(x) + \epsilon_{\text{approx}}) \]

\[ = - \hat{W}^T h(x) - \epsilon_{\text{approx}} \quad (10) \]

where \(\hat{W} = W^* - \hat{W}\) is the estimation error of the ideal weight value.

**Step 2.** In this step, we need to design the weights adaption law. Let \(x_{1d}\) be the desired velocity and denote \(z_1 = x_1 - x_{1d}\). Then, we could design the weights adaption law as follows

\[ \dot{\hat{W}} = \text{Pr oj}_\hat{W}(\Gamma_1 h(x)z_1) \quad (11) \]

where \(\Gamma_1\) is the weights adaption rate matrix.

**Adaptive robust backstepping controller design**

**Step 1.** In this step, \(x_2\) is regarded as a virtual manipulated control variable. Then, a control function \(x_{2eq}\) needs to be designed for the virtual control variable \(x_2\) so that the output tracking performance can be guaranteed.

According to the definition of \(z_1\), we have

\[ \dot{z}_1 = \dot{x}_1 - \dot{x}_{1d} = \alpha(x_2 - x_1) - \hat{T}_L - \dot{x}_{1d} \quad (12) \]

Denote \(z_2 = x_2 - x_{2eq}\) as the input error, and then we rearrange equation (12) as follows

\[ \dot{z}_1 = \alpha z_2 + \alpha x_{2eq} - \alpha x_1 - \hat{T}_L - \dot{x}_{1d} \quad (13) \]

According to equations (9) and (13), the resulting virtual control variable \(x_{2eq}\) can be given as follows

\[ x_{2eq} = x_{2eq1} + x_{2eq2} \]

\[ x_{2eq1} = \frac{1}{\alpha}[\sigma x_1 + \dot{x}_1^L + \dot{x}_{1d}], x_{2eq2} = u_{s1} + u_{s2} \quad (14) \]

\[ u_{s1} = -k_1 z_1, u_{s2} = -\frac{\sigma}{4\epsilon} z_1 \]
where $k_1 > 0$ is a feedback gain, and $\varepsilon$ can be regarded as a kind of control accuracy, which is bounded and positive.

In equation (14), $x_{2eq1}$ functions as a model-based compensation law through online disturbance estimation, and $x_{2eq2}$ is composed of $u_{\tau}$ and $u_{\tau2}$. $u_{\tau2}$ is a linear feedback stabilizing item which is used to stabilize the PMSM system. $u_{\tau2}$ is a nonlinear robust item which can be used to cancel the approximation error of the neural network and it satisfies the following conditions

1. $z_1[u_{\tau2} - e_{\text{approx}}] \leq \varepsilon$
2. $z_1u_{\tau2} \leq 0$

Substituting equation (14) into equation (13), we have

$$\dot{z}_1 = \sigma z_2 - k_1 z_1 + \frac{\lambda}{J_{\text{equ}}} - T_L + u_{\tau2} = \sigma z_2 - k_1 z_1 + u_{\tau2} - \tilde{W}^T h(x) - e_{\text{approx}}$$

(16)

**Step 2.** In step 1, a robust virtual control variable $x_{2eq}$ has been designed. In this step, an actual control law for $u_q$ needs to be designed. According to equation (2), the time derivative of $z_2$ can be obtained as

$$\dot{z}_2 = \dot{x}_2 - \dot{x}_{2eq} = -x_2 - x_{11} + \mu x_1 + \tilde{u}_q - \dot{x}_{2eq}$$

(17)

Then, the following control law $u_q$ is proposed

$$\tilde{u}_q = \dot{x}_{2eq} + x_2 - x_{11} - \mu x_1 - k_2 z_2$$

(18)

where $k_2 > 0$ is a feedback gain, and $\mu$ is the estimation of $\mu$. Since $\mu$ is a critical value which determines the chaotic movement of PMSM and we could not know its true value, we need to estimate its value. Substituting equation (18) into equation (17), we have

$$\dot{z}_2 = -k_2 z_2 - (\mu - \mu) x_1 = -k_2 z_2 - \mu x_1$$

(19)

**Step 3.** In this step, the parametric adaption law is designed for $\mu$ as follows

$$\dot{\mu} = \text{Proj}_\mu \{ x_1 z_2 \}$$

(20)

where $\gamma$ is the parameter adaption rate.

**Step 4.** In this step, an actual control law should be designed for $u_{\tau d}$. Similarly, let $x_{3d}$ denote the desired output $d$-axis current and $z_3 = x_3 - x_{3d}$, then a feedback control law is designed as $\tilde{u}_d = x_3 - x_1 x_2 - k_3 z_3 + x_{3d}$. where $k_3 > 0$ is a feedback gain. When the control law is substituted into the third equation of system (2), the error dynamical equation could be obtained as follows

$$\dot{z}_3 = \dot{x}_3 - \dot{x}_{3d} = -k_3 z_3$$

(21)

which ensures that the tracking error dynamics of is globally uniformly stable.

**Main results**

**Theorem 1.** With neural network-based control laws (9), (14), and (18) and parameter adaptive laws (11) and (20), the output tracking error of the system is bounded and it converges to $\sqrt{\frac{e}{k_1}}$ exponentially with a converging rate larger than $k_1$, which indicates that it is guaranteed that the tracking has a prescribed transient and steady-state performance.

**Proof.** For proof, see Appendix 1.

**Remark.** Theorem 1 indicates that the proposed controller could not only guarantee a prescribed transient output tracking performance but also guarantee the final tracking accuracy. After a finite time, the tracking error can be suppressed to a given range by adjusting the parameter $e_{\text{approx}}$. Thus, it can be made arbitrarily small in theory. In addition, a neural network is used to estimate the load torque and an adaptive control method is used to estimate the uncertain parameters. Both the parametric uncertainty and the disturbance are compensated with feedforward cancelation technique using their estimation values. Thus, the residual model uncertainty is very small and a robust item with low gain could be designed to suppress it. Thus, the robustness of the system is strong.

**Results of the simulation**

The parameters of PMSM used in the simulation are chosen as follows: the number of pole-pairs $p_n = 4$, inductance $L_d = L_q = 0.10\ \text{H}$, stator resistance $R = 2.5\ \Omega$, rotor flux linkage amplitude $\psi_f = 0.45\ \text{Wb}$, equivalent viscous friction coefficient $B_{\text{equ}} = 1.27 \times 10^{-2}\ \text{Ns}$, equivalent inertia $J_{\text{equ}} = 2.512 \times 10^{-5}\ \text{kg} \cdot \text{m}^2$. After a series of calculation, we have $\sigma = 10.5$ and $\mu = 25$, which means the system is in the chaotic movement state. The control objective of $x_1$ is set to be $x_{1d} = \sin(t)$ and the control objective of $x_2$ is still set to be $x_{2d} = 0$.

The following three controllers are compared:

1. **Neural network observer-based adaptive robust nonlinear feedback controller (NNARC).** This is the control strategy proposed in this article, which combines adaptive robust controllers (14) and (18) with adaptive laws (11) and (19) and neural network-based disturbance observer (9).
First, we need to choose the number of nodes of hidden layer. Generally speaking, the more the number of hidden layer nodes, the higher the approximation accuracy of the neural network. However, with the increase in the number of nodes, the computational burden will increase correspondingly and the control cycle would be prolonged which would lead to a decreased control performance. Thus, it is necessary to select a suitable number of nodes. There are several empirical formulas of how to choose the number of nodes such as \( m = \log_2 n \), in which \( m \) means the number of nodes of hidden layer and \( n \) means the number of the nodes of input layer. First, we choose the number of nodes of hidden layer according to this formula. Then, we rectify it through multiple simulation tests. Finally, we decide to choose five nodes for hidden layer.

Then we need to choose the parameters of the controller. We mainly need to tune the parameter \( k_1 \) in equation (14) and \( k_2 \) in equation (18). We just simply choose a large constant to implement the robust control law (14) without the control item of model feed-forward compensation \( x_{2eq} \) since it not only reduces the online computation time significantly but also facilitates the gain tuning process in the implementation. After we have chosen an appropriate value for \( k_1 \), we add \( x_{2eq} \) in the control law. It is the same process for \( k_2 \). Finally, the control gains are chosen as \( k_1 = 1.0 \) and \( k_2 = 0.2 \). The bounds of the parameters are given by \( \mu_{\min} = 1.0 \) and \( \mu_{\max} = 30.0 \). The control accuracy \( e_s = 0.3 \). The initial value of \( \mu \)'s estimation is chosen as 1.0. The adaptation rate matrix is set as \( \Gamma = \text{diag} \{ 0.05, 0.05, 0.01, 0.01, 0.01 \} \) and \( \gamma = 0.15 \). Parameters of the neural network are chosen as \( c_i = 0.5 \times [ 2 \ 1 \ 0 \ 1 \ 2 ]^T \) and \( b_i = 5.0 \) according to the range of neural network's inputs \( x_1 \) and \( x_2 \).

2. General nonlinear feedback controller (GNFC).
This is the controller discussed in the study of Xu and Yao\(^1\) which includes \( x_{2eq} \) in equations (14) and (18), but just uses the nominal value of \( \bar{T}_L = 5.0 \) in \( x_{2eq} \) which is different from the practical one. The controller's parameters are the same as those of NNARC.

3. Robust nonlinear feedback controller (RNFC).
This is the controller discussed in the study of Xu and Yao\(^1\) which includes equations (14) and (18), but there is no estimation of \( T_L \) in \( x_{2eq} \). The controller's parameters are the same as those of NNARC.

Two cases are tested for these three controllers.

Case 1. Load torque disturbance is a constant value
In this case, a constant load torque disturbance is implemented by applying \( \bar{T}_L = 6.0 \) to the physical motion system. The simulation results are obtained for tracking the desired output trajectory. The working time of the controller is set as \( t = 15 \) s to show the control effect clearly. The three systematic state variables are shown in Figure 4, which shows that all the systematic states have been driven to the desired motion trajectory with NNARC. The control inputs are shown in Figure 5. Figure 6 shows the compare curves of three controllers. As it can be seen, NNARC and RNFC controllers are better than GNFC controller in terms of both the transient performance and the final tracking accuracy since both the controllers employed robust item to cancel the influence of uncertain load torque disturbance. This confirms that the proposed robust control law is effective. In addition, the maximal steady tracking error of NNARC is about 0.02\(^\circ\), which is slightly better than that of RNFC, since the former controller could estimate the load torque disturbance and cancel it.
Case 2. Load torque disturbance is a time-varying value

In this case, a time-varying load torque disturbance $\dot{T}_L = 0.5x_1 + x_2$ is applied to the motion system. The systematic dynamics is almost totally changed with this time-varying disturbance and the simulation results can be regarded as the most exhaustive verification. Figure 7 shows the compare curves of the three controllers. As it can be seen, all these three controllers can eliminate the chaos when the motion system tracks a sinusoidal desired output. However, NNARC and RNFC controllers are better than GNFC controller in terms of both the transient performance and the final tracking errors since both the controllers employed robust item to cancel the influence of uncertain load torque disturbance. This confirms that the proposed robust control law is effective further. However, the maximal steady tracking error of NNARC controller is about 0.01°, which means it is the best one in all the three controllers. The reason is that the NNARC controller could estimate and cancel the time-varying load torque disturbance effectively using neural network and the residual uncertainty which is very small and handled by the robust item which leads to a better control performance, meanwhile the NNARC could compensate the parametric uncertainty with adaption law (the adaptive parameters and the disturbance estimation are shown in Figures 8 and 9, respectively). Thus, the overall uncertainties are greatly reduced and the tracking performance has been greatly improved.

Conclusion

In this article, an adaptive robust control with a neural network–based load torque disturbance observer has been proposed to suppress the chaos in PMSM and realize the high-accuracy motion control of PMSM system, which not only considers the load torque disturbance but also takes the systematic parameter uncertainties into account. Lyapunov’s method is used to prove the stability of the novel control strategy, which shows that the controller could control the chaos effectively and a prescribed tracking performance could be guaranteed theoretically in the presence of various uncertainties. Sufficient comparative simulation has been done and the results show that the proposed strategy is effective, that is, a wonderful tracking accuracy can be achieved via introducing neural network–based disturbance observer in adaptive robust control compared to the other nonlinear feedback control methods. The proposed scheme can suppress the chaos and improve the tracking accuracy of PMSM system by combining the advantages of the two approaches fundamentally.
Declaration of conflicting interests

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Appendix I

Proof of Theorem 1

A positive definite function is chosen as

\[ V = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} \tilde{\mu}^2 + \frac{1}{2} \Gamma^{-1} \tilde{W}^T \tilde{W} \]  

(22)

The time derivative of function (22) is

\[ \dot{V} = z_1 \dot{z}_1 + z_2 \dot{z}_2 + \tilde{\mu} \dot{\tilde{\mu}} + \Gamma^{-1} \tilde{W}^T \dot{\tilde{W}} \]

\[ = z_1 (\sigma z_2 - k_1 z_1 + u_{e_2} - \tilde{W}^T h(x) - e_{\text{approx}}) \]

\[ + z_2 (-k_2 z_2 - \tilde{\mu} x_1) + \tilde{\mu} \dot{\tilde{\mu}} + \Gamma^{-1} \tilde{W}^T \dot{\tilde{W}} \]

\[ = -k_1 z_1^2 + \sigma z_1 z_2 - k_2 z_2^2 + z_1 (u_{e_2} - e_{\text{approx}}) \]

\[ + \tilde{\mu} (-\tilde{\mu} - x_1 z_2) + \tilde{W}^T (\Gamma^{-1} \dot{\tilde{W}} - h(x) z_1) \]

\[ \leq -k_1 z_1^2 + \sigma z_1 z_2 - k_2 z_2^2 + \epsilon \]

\[ = -\frac{1}{2} k_1 z_1^2 + \sigma z_1 z_2 - \left(k_2 - \frac{1}{2} k_1\right) z_2^2 \]

\[ + \epsilon - k_1 z_1^2 - \frac{1}{2} k_1 z_2^2 \]

Choose to make \( \sigma = \sqrt{2k_1 k_2 - \frac{k_1^2}{2}} \), then we could obtain

\[ \dot{V} \leq -\left(\sqrt{\frac{1}{2} k_1 z_1} - \sqrt{k_2 - \frac{1}{2} k_1 z_2}\right)^2 + \epsilon - \frac{1}{2} k_1 z_1^2 - \frac{1}{2} k_1 z_2^2 \]

\[ \leq -\frac{1}{2} k_1 z_1^2 - \frac{1}{2} k_1 z_2^2 - \frac{k_1}{2} \tilde{\mu}^2 - \frac{k_1}{2} \Gamma^{-1} \tilde{W}^T \tilde{W} + \frac{k_1}{2} \tilde{\mu}^2 \]

\[ + \frac{k_1}{2} \Gamma^{-1} \tilde{W}^T \tilde{W} + \epsilon \leq -k_1 V + e \]

(24)

where \( \frac{k_1}{2} \tilde{\mu}^2 \leq e_1, \frac{k_1}{2} \Gamma^{-1} \tilde{W}^T \tilde{W} \leq e_2, e_1 = \epsilon + e_1 + e_2. \tilde{\mu} \) is bounded since \( \mu \) and \( \tilde{\mu} \) are bounded, and \( \tilde{W} \) is also bounded with the same reason.

Then we would have \( z_1^2 \leq |z_1(0)|^2 \exp(-k_1 t) + \frac{e}{k_1} [1 - \exp(-k_1 t)] \), from which we could get the conclusion that the output tracking is guaranteed to have a prescribed transient and steady-state performance in the sense that the tracking error bounded above by a known function exponentially converges to \( \sqrt{\frac{e_2}{k_1}} \) with a converging rate not less than \( k_1 \).