Two atoms in dissipative cavities in dispersive limit: entanglement sudden death and long-lived entanglement

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Abstract
We investigate the entanglement dynamics and coherence of two two-level atoms interacting with two coherent fields of two spatially separated and dissipative cavities. It is in particular shown that entanglement sudden death is obtained within a short interaction time. However, after a long interaction time a long-lived entanglement is shown; that is, the initial entanglement of two atoms could be partially preserved. In addition, the coherence of the two atoms will not be lost during the evolution.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Quantum entanglement is at the heart of quantum information processing and quantum computation [1–4]. In recent years, many efforts have been devoted to the study of the evolution of joint systems formed by two qubits [5–15]. In particular, Yu and Eberly [5, 6] have found out that the single-qubit dynamics and the global dynamics of an entangled two-qubit system subjected to independent environments are rather different. For a single-qubit system subjected to an environment, the local coherence decays asymptotically while the entanglement of an entangled two-qubit system may disappear for a finite time during the dynamics’ evolution. The nonsmooth finite-time disappearance of entanglement is called ‘entanglement sudden death’ (ESD). More recently, the ESD phenomenon has been observed in the laboratory by two groups [16, 17].

On the other hand, the long-lived entanglement in cavity QED or solid state systems was investigated by several authors [18–22]. In [18], the authors have discussed the effects of the classical noise on entanglement dynamics and pointed out that there is long-lived entanglement in the presence of global dephasing noise. The influence of white noise on the entanglement dynamics of two atoms within two cavities has been investigated by Xu and Li [19]. They found out that if only one atom is driven by the white-noise field, then there is long-lived entanglement. The entanglement dynamics in two effective two-level trapped ions interacting with a laser field has been studied in [20]. It has been shown by Dajka et al that there is long-lived entanglement of some composite systems even if coupled with a thermal bath [21]. It has also been pointed out that phase-damped cavity can lead to long-lived entanglement in a quantum system consisting of a single-Cooper-pair box irradiated by a quantized field [22].

In the present paper, we investigate the entanglement dynamics and coherence of a quantum system formed by two two-level atoms within two spatially separated and dissipative cavities in the dispersive limit by the employing concurrence and linear entropy, respectively. The two atoms are initially entangled and the cavities are initially prepared in coherent states. We show that the ESD phenomenon appears in the present system and the entanglement of two atoms decreases with the time in the short term. However, the long-term behaviour is very different since a long survival of the entanglement is shown in the system. In other words, there is long-lived entanglement (stationary-state entanglement) in the presence of the dissipation of cavities, implying that the initial entanglement of two atoms could be partially preserved even when they are put into dissipative cavities. We find that the long-lived entanglement depends on the initial state of the two atoms and the ratio of the decay rate of the cavities to the
atom–field coupling constant. Finally, we discuss the coherence of the two atoms using the linear entropy. Our results show that the linear entropy of each atom at any time is equal to the initial linear entropy, that is, the coherence of each atom is preserved. We find that the initial coherence (entanglement) of the two atoms could be preserved (partially preserved) in the dispersive limit even if they are put into two dissipative cavities.

The present paper is organized as follows. In section 2, we obtain an explicit analytical solution of one atom interacting with a dissipative cavity in the dispersive limit. In section 3, we consider a quantum system consisting of two atoms within two spatially separated cavities. In section 4, the entanglement dynamics and coherence of the two two-level atoms are investigated by employing the concurrence and linear entropy, respectively. Finally, we summarize our results in section 5.

2. One atom in a dissipative cavity

We first consider a quantum system consisting of a two-level atom interacting with a single-mode cavity. Under the electric dipole and rotating wave approximation, the Hamiltonian of the present system is [23, 24]

\[ H = \omega a^\dagger a + \frac{g}{2} \sigma_z + g(a^\dagger \sigma_- + a \sigma_+), \]

where \( g \) is the atom–field coupling constant, \( \sigma_z = |e\rangle\langle e| - |g\rangle\langle g| \) and \( \sigma_\pm \) are the atomic spin flip operators characterizing the effective two-level atom with frequency \( \omega_0 \). The symbols \( |e\rangle \) and \( |g\rangle \) refer to the excited and ground states for the two-level atom. Here, \( a^\dagger \) and \( a \) are the creation and annihilation operators of the field with frequency \( \omega_0 \), respectively. The dispersive limit is obtained when the condition |\Delta| = |\omega_0 - \Omega| \gg \sqrt{\pi T g} is satisfied for any relevant \( n \). Then the interaction Hamiltonian \( g(a^\dagger \sigma_- + a \sigma_+) \) can be regarded as a small perturbation and the effective Hamiltonian of the model can be recast as [25]

\[ H_e = \omega a^\dagger a + \frac{\omega_0}{2} \sigma_z + \Omega [ (a^\dagger a + 1) |e\rangle\langle e| - a^\dagger a |g\rangle\langle g| ] - \frac{g}{2} \sigma_z. \]

(2)

With \( \Omega = g^2/\Delta \). In the interaction picture, the interaction Hamiltonian is

\[ V = \Omega [(a^\dagger a + 1) |e\rangle\langle e| - a^\dagger a |g\rangle\langle g| ] |. \]

(3)

We assume the two-level atom interacting with a coherent field in a dissipative environment. This interaction causes the losses in the cavity which is presented by the superoperator \( D = k(2a^\dagger a^\dagger - a^\dagger a - a^\dagger a) \), where \( k \) is the decay constant. For the sake of simplicity, we confine our consideration in the case of zero temperature cavity. Then the master equation that governs the dynamics of the system can be written as follows:

\[ \dot{\rho}' = -i[V, \rho'] + D\rho' = -i[V, \rho'] + k(2a^\dagger a^\dagger \rho' - a^\dagger a \rho' - \rho' a^\dagger a) = -i[V, \rho'] + k(2M\rho' - \mathcal{R}\rho' - \mathcal{L}\rho'). \]

(4)

where \( \rho' \) is the density matrix of the atom–field system, and \( \mathcal{M}, \mathcal{R} \) and \( \mathcal{L} \) are defined by

\[ \mathcal{M}\rho' = (a^\dagger a^\dagger)\rho' = a^\dagger a^\dagger \rho' = a^\dagger a \rho', \]

\[ \mathcal{R}\rho' = (a^\dagger a)\rho' = a^\dagger \rho' a, \]

\[ \mathcal{L}\rho' = (a^\dagger a^\dagger a^\dagger a)\rho' = a^\dagger a^\dagger a^\dagger a \rho'. \]

(5)

Here the superoperators \( a^\dagger, a, a^\dagger a, a^\dagger a^\dagger \) represent the action of creation and annihilation operators on an operator.

\[ (a^\dagger a)\rho = a^\dagger a \rho, \]

\[ (a^\dagger a^\dagger a^\dagger a)\rho = a^\dagger a^\dagger a^\dagger a \rho. \]

(6)

We can express the density matrix in the following form:

\[ \rho'(t) = \rho'_{ee}(t) \otimes |e\rangle\langle e| + \rho'_{eg}(t) \otimes |g\rangle\langle g| + \rho'_{ge}(t) \otimes |e\rangle\langle g| + \rho'_{gg}(t) \otimes |g\rangle\langle e|, \]

(7)

where \( \rho'_{ij} \)’s are defined as \( \rho'_{ij} = \langle i|\rho'(t)|j\rangle, \rho'_{ij} = \rho'_{ij}^*, i, j = e, g \). A straightforward calculation shows that

\[ \frac{d\rho'_{ee}(t)}{dt} = [-i\Omega(R - L) + k(2M - R - L)]\rho'_{ee}(t) = \mathcal{L}_{ee}\rho'_{ee}(t), \]

\[ \frac{d\rho'_{eg}(t)}{dt} = [i\Omega(R - L) + k(2M - R - L)]\rho'_{eg}(t) = \mathcal{L}_{eg}\rho'_{eg}(t), \]

\[ \frac{d\rho'_{ge}(t)}{dt} = [-i\Omega(R + L + 1) + k(2M - R - L)]\rho'_{ge}(t) = \mathcal{L}_{ge}\rho'_{ge}(t), \]

\[ \frac{d\rho'_{gg}(t)}{dt} = [-i\Omega(R + L + 1) + k(2M - R - L)]\rho'_{gg}(t) = \mathcal{L}_{gg}\rho'_{gg}(t). \]

(8)

It is easy to check that the superoperators \( \mathcal{M}, \mathcal{R} \) and \( \mathcal{L} \) satisfy the relations

\[ [\mathcal{R}, \mathcal{M}] = [\mathcal{L}, \mathcal{M}] = -\mathcal{M}, [\mathcal{R}, \mathcal{L}] = 0. \]

(9)

It is worth noting that \([\mathcal{R} + \mathcal{M}], \mathcal{M} = 2\mathcal{M} \); the superoperators \( \mathcal{R} + \mathcal{M} \) and \( \mathcal{L} \) form a shift operator algebra. Thus, we can expand the exponential of a linear combination of \( \mathcal{R} + \mathcal{M} \) and \( \mathcal{L} \):

\[ e^{C_{\mathcal{R} + \mathcal{L}}} = e^{(e^{2\Omega t} - 1)\mathcal{M}e^{-i(\mathcal{R} + \mathcal{L})t}}Re^{i(\mathcal{R} + \mathcal{L})t}, \]

\[ e^{C_{\mathcal{R} + \mathcal{L}}} = e^{(e^{2\Omega t} - 1)\mathcal{M}e^{-i(\mathcal{R} + \mathcal{L})t}Re^{i(\mathcal{R} + \mathcal{L})t}C_{\mathcal{R} + \mathcal{L}}. \]

(10)

Suppose that the atom is initially prepared in the state \( \left( \frac{\xi_a}{\xi_b} \right) \) and the field is initially prepared in the coherent state \( |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^\infty \frac{a^n}{\sqrt{n!}} |n\rangle \) with \( \alpha \) being a complex number. Here, \( |n\rangle \) is the Fock state with \( a^\dagger a|n\rangle = n|n\rangle \). Therefore, the initial state of the atom–cavity system is

\[ \rho'(0) = \left( \frac{\xi_a}{\xi_b} \right) \otimes |\alpha\rangle\langle \alpha| = (\xi_a|e\rangle\langle e| + \xi_b|g\rangle\langle g| + \xi_e|e\rangle\langle g| + \xi_g|g\rangle\langle e|) \otimes |\alpha\rangle, \]

(11)

Comparing equation (7) with equation (11), one gets

\[ \rho'_{ee}(0) = \xi_a|\alpha\rangle\langle \alpha|, \]

\[ \rho'_{ge}(0) = \xi_g|\alpha\rangle\langle \alpha|, \]

\[ \rho'_{eg}(0) = \xi_e|\alpha\rangle\langle \alpha|, \]

\[ \rho'(0) = \rho'_{gg}(0). \]

(12)
In the basis obtained the reduced matrix density of two atoms conveniently.

The reduced density matrix of the atom is obtained by tracing out the variables of the field from the above density matrix. The EWL states belong to the class of the 'X' states. Explicitly, if the density matrix of a quantum state is of the form

\[ \rho_{ij}(t) = \langle \alpha_+ | \rho(t) | \alpha_- \rangle \]

then it belongs to the class of the X states.

The EWL states have the following advantages. First, we can easily find that if the initial state of two atoms is X structure, then the reduced density matrix at arbitrary time \( t \) is still X structure under the single atom evolution determined by the master equation (4) (in the basis \(| \alpha_1 \rangle \) and \(| \alpha_2 \rangle \). For the sake of simplicity, we assume \( \alpha_1 = \alpha_2 \), the decay rates of the two cavities are equal and the atom–field coupling constants are the same. The schematic picture of the present model is presented in figure 1.

3. Two atoms within two spatially separated cavities

In this section, we consider a quantum system consisting of two noninteracting atoms each locally interacts with its own single-mode cavity and interacts with its own cavity field locally. Note that there is no direct interaction between two atoms once they have been put into cavities.

Combining equations (10), (12) and the relation \( \rho_{ij}'(t) = e^{i\omega t} \rho_{ij}(0) \), we find that the matrix elements \( \rho_{ij}'(t) \) at time \( t \) are given by

\[
\rho_{ee}'(t) = \xi_0 | \alpha_+(t) \rangle \langle \alpha_+(t) |,
\rho_{gg}'(t) = \xi_0 | \alpha_-(t) \rangle \langle \alpha_-(t) |,
\rho_{eg}'(t) = \rho_{ge}'(t) = \rho_{ee} \times e^{-i\omega t}
\]

\[ f(t) = \exp \left( -i\omega t + |\alpha|^2 (e^{-2it} - 1) \right) \]

\[ \times \exp \left( -\frac{|\alpha|^2 k}{k + i\Omega} - e^{-2it+i\Omega} \right) \]

The reduced density matrix of the atom is obtained by tracing out the variables of the field from the above density matrix

\[ \rho_{\text{atom}}(t) = \xi_0 | \langle e \rangle + \xi_0 | \langle g \rangle + [\xi_e f(t) \lambda(t) | e \rangle | g \rangle + h.c. \rangle \]

\[ \lambda(t) = | \langle \alpha_- | \alpha_+ \rangle | \]

(14)

where \( h.c. \) denotes the Hermitian conjugate.

3.1. Reduced density matrix of two atoms

Using the method introduced by Bellomo et al [13], we can obtain the reduced matrix density of two atoms conveniently. In the basis \(| 1 \rangle = | e e \rangle, | 2 \rangle = | e g \rangle, | 3 \rangle = | g e \rangle, | 4 \rangle = | g g \rangle \), and using equation (14), the reduced density matrix \( \rho(t) \) for the two-atom system is

\[
\rho_{11}(t) = \rho_{11}(0),
\rho_{12}(t) = f(t) \lambda(t) \rho_{12}(0),
\rho_{21}(t) = \rho_{21}(0),
\rho_{22}(t) = f(t) \lambda(t) \rho_{22}(0),
\rho_{33}(t) = f(t) \lambda(t) \rho_{33}(0),
\rho_{44}(t) = f(t) \lambda(t) \rho_{44}(0),
\rho_{13}(t) = f(t) \lambda(t) \rho_{13}(0),
\rho_{14}(t) = [f(t) \lambda(t)]^2 \rho_{14}(0),
\rho_{23}(t) = [f(t) \lambda(t)]^2 \rho_{23}(0),
\rho_{24}(t) = f(t) \lambda(t) \rho_{24}(0),
\rho_{34}(t) = f(t) \lambda(t) \rho_{34}(0),
\]

(15)

with \( \rho_{ij}(t) = \rho_{ij}^*(t) \) and \( i = 1, 2, 3, 4 \). We would like to point out that the above procedure allows us to obtain the reduced density matrix of the two-atom system for any initial state of the two atoms.

3.2. Extended Werner-like states

We assume that the initial states of the two-atom system are the extended Werner-like (EWL) states [13] defined by

\[ \rho_\Phi = p | \Phi \rangle \langle \Phi | + \frac{1 - p}{4} I, \]

\[ \rho_\Psi = p | \Psi \rangle \langle \Psi | + \frac{1 - p}{4} I, \]

\[ | \Phi \rangle = \mu | e e \rangle + \nu | g g \rangle, \]

\[ | \Psi \rangle = \mu | e e \rangle + \nu | g g \rangle, \]

(16)

where \( p \) is a real number which indicates the purity of initial states, \( I \) is a 4 × 4 identity matrix, \( \mu \) and \( \nu \) are complex numbers with \( |\mu|^2 + |\nu|^2 = 1 \). The parameter \( p \) is 1 for pure states and 0 for completely mixed states. It is worth noting that the EWL states belong to the class of the 'X' states. Explicitly, if the density matrix of a quantum state is of the form

\[ \rho = \begin{pmatrix} 0 & 0 & \rho_{14} \\ 0 & 0 & \rho_{24} \\ \rho_{14}^* & \rho_{24}^* & 0 \end{pmatrix}, \]

(17)

then it belongs to the class of the X states.

The EWL states have the following advantages. First, we can easily find that if the initial state of two atoms is X structure, then the reduced density matrix at arbitrary time \( t \) is still X structure under the single atom evolution determined by the master equation (4) (in the basis \(| \alpha_1 \rangle \) and \(| \alpha_2 \rangle \). Second, the EWL states allow us to clearly show the influence of the purity and the amount of entanglement of the initial states on the entanglement dynamics of two atoms simultaneously. The purity of the EWL states depends on the parameter \( p \) and the amount of the entanglement of the EWL states is related to \( \mu \) and \( \nu \). If \( p = 1 \), the EWL states reduce to the Bell-like states \(| \Phi \rangle \) and \(| \Psi \rangle \). In the case of \( p = 1, \mu = \nu = 1/\sqrt{2} \) the EWL states become the Bell states while in the case of \( p = 0 \) they are the maximally mixed states. Third, as we will see, it is easy for us to calculate the entanglement dynamics of two atoms which are initially prepared in EWL states.

4. Entanglement and coherence

In this section, we will analyse the entanglement dynamics and coherence of two atoms by employing the concurrence and linear entropy, respectively. In order to study the entanglement
of the above system described by the density matrix \( \rho \), we adopt the measure concurrence which is defined by [26]

\[
C = \max \{ 0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 \},
\]

where \( \lambda_i \) (\( i = 1, 2, 3, 4 \)) are the square roots of the eigenvalues in decreasing order of the magnitude of the 'spin-flipped' density matrix operator \( R = \rho (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y) \) and \( \sigma_y \) is the Pauli Y matrix, i.e. \( \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \). Particularly, for X structure states defined by equation (17), it is easy to find that the concurrence is

\[
C(t) = 2 \max \{ 0, |\rho_{23}(t)| - \sqrt{\rho_{11}(t) \rho_{22}(t)}, |\rho_{14}(t)| \} - \sqrt{\rho_{23}(t) \rho_{33}(t)}.
\]  

(19)

Combing equations (15), (16) and (19), we find that the concurrence of the two atoms is plotted as a function of the dimensionless scaled time \( \Omega t \) and the parameter \( \kappa / \Omega \) for different values of \( p \). From these two figures, one can easily find that the entanglement of the two atoms suddenly goes to zero and stays zero for a finite time, i.e. the ESD phenomenon appears in the present system. However, the entanglement cannot revive completely due to the dissipation of the cavity fields. The lower panels of figures 2 and 3 are the contour plots of the concurrence, where the severe shading areas indicate that the two atoms are completely disentangled. Comparing the upper panels of figures 2 and 3, one can clearly see that the entanglement dynamics relies heavily on the purity of the initial state which is represented by the parameter \( p \). For instance, the maximal value of the concurrence in figure 2 is about 0.7, while the maximal value of the concurrence in figure 3 is only about 0.4. The contour plots of these figures clearly show that the areas showing entanglement significantly decrease with the decrease of the purity of the initial state. To show this more intuitively, we plot the concurrence as a function of the dimensionless scaled time \( \Omega t \) for several values of \( p \) in figure 4. Also, the entanglement dynamics depends on the initial state of the field \( |\alpha| \) as one can see from the upper panel of figure 5. The concurrence decreases with the increase of the amplitude of \( \alpha \). For instance, in the case of \( k / \Omega = 0.01, \mu = v = 1/\sqrt{2}, p = 0.9 \) and \( \alpha = 0.5 \), there is no ESD. However, in the case of \( \alpha = 1 \) or \( \alpha = 2 \), the ESD phenomenon could appear.

In order to see the time limit (long-term) behaviour of the entanglement we plot the concurrence as a function of the dimensionless scaled time \( \Omega t \) for \( 0 \leq \Omega t \leq 500 \) for \( \alpha = 0.5 \) (solid line), \( \alpha = 1 \) (dashed line) and \( \alpha = 2 \) (dotted line) in the lower panel of figure 5. From this figure, we see that the entanglement of the two atoms survives for a long time if the mean photon number of the fields \( |\alpha|^2 \) has small values (see the solid and dashed lines of figure 5). The entanglement of the stationary state \( \rho(\infty) \) depends on the initial state of the two atoms and the ratio of the decay rate of two cavities to the
The concurrence of two atoms is plotted as a function of the dimensionless scaled time \( \Omega t \) with \( k/\Omega = 0.01 \), \( \alpha = 0.5 \), \( \mu = \nu = 1/\sqrt{2} \) for \( p = 0.8 \) (solid line), \( p = 0.6 \) (dashed line) and \( p = 0.5 \) (dotted line). Upper panel: the short-term behaviour of the entanglement between two atoms. Lower panel: the long-term behaviour of the entanglement between two atoms.

Figure 4. The concurrence of two atoms is plotted as a function of the dimensionless scaled time \( \Omega t \) with \( k/\Omega = 0.01 \), \( \alpha = 0.5 \), \( \mu = \nu = 1/\sqrt{2} \) for \( p = 0.8 \) (solid line), \( p = 0.6 \) (dashed line) and \( p = 0.5 \) (dotted line). Upper panel: the short-term behaviour of the entanglement between two atoms. Lower panel: the long-term behaviour of the entanglement between two atoms.

Figure 5. The concurrence of two atoms is plotted as a function of the dimensionless scaled time \( \Omega t \) with \( k/\Omega = 0.01 \), \( p = 0.9 \), \( \alpha = 1 \) (dashed line) and \( \alpha = 2 \) (dotted line). Upper panel: the short-term behaviour of the entanglement between two atoms. Lower panel: the long-term behaviour of the entanglement between two atoms. Comparing the upper panel with the lower panel, one can observe that, in the short term, there is ESD and the maximal values of entanglement decrease with time. However, in the long term, there is long-lived entanglement (see the solid and dashed lines of the lower panel).

Comparing the upper panel with the lower panel, one can observe that, in the short term, there is ESD and the maximal values of entanglement decrease with time. However, in the long term, there is long-lived entanglement (see the solid and dashed lines of the lower panel).

atom–field coupling constant. If the initial state is completely mixed \( (p = 0) \), that is, the two atoms are disentangled initially, then no entanglement will be generated in the system.

In figure 6, we have presented the time evolution of entanglement as a function of \( kt \) and \( \Omega/k \). One can see clearly that the entanglement decreases gradually as \( kt \) increases for a given time. In principle, by using the explicit expression of the concurrence, one can find exactly the threshold value after which a long-lived entanglement can be obtained. It is easy to see that the oscillations of the concurrence gradually disappears as the time \( kt \) increases. Actually, from equation (21) one can see that \( C(t) = c \) (c is a constant) leading to the following condition: \( kt > kt_c \), which can be obtained from the following equation (we set \( \mu = \nu = 1/\sqrt{2} \)):

\[
|f(t)| = \frac{1}{|\langle \alpha_-|t\rangle| \langle \alpha_+|t\rangle|} \sqrt{\frac{2c + 1 - p}{2p}},
\]

(22)

then the entanglement has no chance to be oscillating when \( kt \) exceeds a threshold value \( kt_c \) (from figure 6, \( kt_c \approx 7 \)). Here we should point out that the effective threshold value of the decay time corresponding to the transition of the entanglement from the oscillating regime to a fully constant value is independent of the interaction time. From the above discussions, one can observe that the initial entanglement of the two atoms could be partially preserved in the present system which is useful for quantum information processing and quantum memory.

In order to show the dependence of the long-time behaviour of the entanglement of the atoms on the parameter \( \alpha \) and the purity of the initial state \( p \), we plot the concurrence as a function of \( \alpha \) and \( p \) with \( t = 500 \) in figure 7. From figure 7, we observe that the stationary-state entanglement of the atoms decreases with the increase of the parameter \( \alpha \) while increases with the purity of the initial state \( p \). This observation is consistent with the previous discussions. We
parameter $\alpha$ stationarystate entanglement of the atoms decreases with the time $t$ and the purity of the initial state $p$. From this figure, we see that the coherence of the two atoms can be preserved for a long time.

5. Conclusions

In the present paper, we have investigated the entanglement dynamics and coherence of a quantum system formed by two two-level atoms interacting with two spatially separated and dissipative cavities in the dispersive limit. With the help of the superoperator method, we obtained an explicit expression of the reduced density matrix of the two atoms and calculated the entanglement and coherence of them by employing the concurrence and linear entropy, respectively. In a short-term regime, the ESD phenomenon could be obtained, and the entanglement of the two atoms decreases with the time development. However, in the long term, the entanglement of the two atoms tends to a fixed value. This value depends only on the initial state of the two atoms, the atom–field coupling constant, and the decay rate of fields. Particularly, we found that there is long-lived entanglement (or stationary-state entanglement) in the presence of the dissipation of the fields. In other words, in the dispersive limit, the initial entanglement of the two atoms within two dissipative cavities could be partially preserved for a long time.

Finally, we calculated the coherence of the two atoms using the linear entropy. Our results show that the coherence of each atom can be preserved. Thus, the subspace of each atom is a decoherence-free subspace. In a word, the initial coherence (entanglement) of the two atoms could be preserved (partially preserved) in the dispersive limit even if they are put into two dissipative cavities. This feature is useful for quantum information processing and quantum memory. It is interesting to extend the present system to a many-qubit system. Note that equation (15) is not suitable for the many-qubit system. However, the method introduced by Bellomo et al [13] is still applicable. Thus, one can investigate whether the multipartite entanglement of the many-qubit system can be preserved.

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\[ S(t) = 1 - Tr[\rho^2(t)]. \] (23)
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