Intrinsic Parity of the $(j, 0) \oplus (0, j)$ Mesons

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Contrary to the usual belief, by carefully examining the operation of parity transformation on the $(1, 0) \oplus (0, 1)$ mesons in the generalized canonical representation, we establish that the $(j, 0) \oplus (0, j)$ meson-antimeson pair have opposite intrinsic parity. This opens up the possibility that while the particles without an internal structure may utilize one representation of the Lorentz group, phenomenologies of composite particles may exploit a different representation. As such (perhaps, only some of) the meson structures beyond the standard $q\overline{Q}$ may exploit the $(j, 0) \oplus (0, j)$ generalized canonical representation of the Lorentz group – this would result in a meson and the associated antimeson to manifest themselves in different partial waves.

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*This work was done under the auspices of the U. S. Department of Energy.
In this letter we argue that the relative intrinsic-parity of the particle-antiparticle pair emerges as a (Lorentz group) representation-dependent kinematical object, in apparent contradiction to the usual belief. Our motivation for this investigation is to continue our ab initio study, based on Refs. [1,3], to establish a relativistic phenomenology [4] for high spin hadronic resonances. These resonances will become increasingly more accessible at CEBAF, NIKHEF, RHIC, a possible upgrade of LAMPF, and other new medium energy nuclear physics facilities.

For concreteness we look at the relativistic phenomenology of the $j = 1$ mesons. Following conventions defined by Ryder [3] the $(1/2, 1/2)$ representation of the Lorentz group corresponds to the description of the $j = 1$ matter fields in terms of the vector field $A^{\mu}(x)$. The vector field $A^{\mu}(x)$ satisfies the Proca equation. Within this framework the relative intrinsic parity of the meson-antimeson pair is same. By considering the $(1,0) \oplus (0,1)$ representation in detail we will show that the relative intrinsic parity for the meson-antimeson, described by the generalized canonical [4a,b] representation $(1,0) \oplus (0,1)$ matter field, is opposite. The generalization of this result to the mesons with $j > 1$ will be seen as essentially obvious. As noted in the abstract, this result opens up the possibility that while structureless fundamental particles may utilize one representation of the Lorentz group, say $(1/2,1/2)$, the composite particles may find their phenomenological description in terms of other representations, such as $(1,0) \oplus (0,1)$.

We begin with the classical considerations similar to the ones found for the $(1/2,0) \oplus (0,1/2)$ Dirac field in the standard texts, such as Nachtmann’s treatment in Ref. [6, Sec. 4.5]. The $(1,0) \oplus (0,1)$ wave function satisfies the spin one Weinberg [1, 4h] equation

$$\left(\gamma_{\mu\nu} \partial^{\mu} \partial^{\nu} + m^2\right) \psi(t, \vec{x}) = 0 \quad .$$

\[\text{---}\]

2 There is a slight ambiguity even in the definition of relative intrinsic parity. For this ambiguity the curious reader is referred to footnote [11] on p. 570 of Ref. [7].

3 For example, see Ref. [1, footnote 13] and Ref. [2].
The $6 \times 6$ ten $\gamma_{\mu\nu}$ matrices in the generalized canonical representation [4h] are given by

$$
\gamma_{oo} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma_{o\ell} = \gamma_{\ell o} = \begin{pmatrix} 0 & -J_{\ell} \\ J_{\ell} & 0 \end{pmatrix}, \\
\gamma_{\ell j} = \gamma_{j\ell} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} g_{\ell j} + \begin{pmatrix} \{J_{\ell}, J_{j}\} & 0 \\ 0 & -\{J_{\ell}, J_{j}\} \end{pmatrix};
$$

(2)

where $\vec{J}$ are the $3 \times 3$ spin one matrices with $J_z$ diagonal; $I$ are the $3 \times 3$ identity matrices; $g_{\mu\nu}$ is the flat spacetime metric with $diag(1, -1, -1, -1)$; $\ell, j$ run over the spacial indices $1, 2, 3$; and $\{J_{\ell}, J_{j}\}$ is the anticommutator of $J_{\ell}$ and $J_{j}$. We seek the parity-transformed wave function

$$
\psi'(t', \vec{x}') = S(\Lambda_P) \psi(t, \vec{x}),
$$

(3)

such that Eq. (4) holds true for $\psi(t', \vec{x}')$

$$
(\gamma_{\mu\nu} \partial^\mu \partial^\nu + m^2) \psi(t', \vec{x}') = 0.
$$

(4)

It is a straightforward exercise to find that $S(\Lambda_P)$ must simultaneously satisfy the following requirements

$$
S^{-1}(\Lambda_P) \gamma_{oo} S(\Lambda_P) = \gamma_{oo}, \quad S^{-1}(\Lambda_P) \gamma_{o\ell} S(\Lambda_P) = -\gamma_{\ell o}, \\
S^{-1}(\Lambda_P) \gamma_{jo} S(\Lambda_P) = -\gamma_{j o}, \quad S^{-1}(\Lambda_P) \gamma_{\ell j} S(\Lambda_P) = \gamma_{\ell j}.
$$

(5)

Referring to Eqs. (2), we now note that while $\gamma_{oo}$ commutes with $\gamma_{\ell j}$ it anticommutes with $\gamma_{o\ell}$

$$
[\gamma_{oo}, \gamma_{\ell j}] = [\gamma_{oo}, \gamma_{j \ell}] = 0, \quad \{\gamma_{oo}, \gamma_{o\ell}\} = \{\gamma_{oo}, \gamma_{j o}\} = 0.
$$

(6)

As a result, confining to the norm preserving transformations (and ignoring a possible global phase factor), we identify $S(\Lambda_P)$ with $\gamma_{oo}$, yielding

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4The operation of parity is defined via $x'^\mu = \Lambda^\mu_\nu x^\nu$, with $\Lambda_P \equiv [\Lambda^\mu_\nu] = diag(1, -1, -1, -1)$. All conventions, unless indicated otherwise, are that of Ref. [3].
\[ \psi'(t', \vec{x}') = \gamma_{oo} \psi(t, \vec{x}) \iff \psi'(t', \vec{x}') = \gamma_{oo} \psi(t', -\vec{x}'), \quad (7) \]

This prepares us to proceed to the field theoretic considerations. The \((1,0) \oplus (0,1)\) matter field operator may be defined as follows [4a]

\[
\Psi(x) = \sum_{\sigma = 0, \pm 1} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2 \omega_{\vec{p}}} \times \left[ u_\sigma(p) a_\sigma(\vec{p}) \exp(-ip \cdot x) + v_\sigma(p) b_\sigma(\vec{p}) \exp(+ip \cdot x) \right], \quad (8)
\]

with \(\omega_{\vec{p}} = \sqrt{m^2 + \vec{p}^2}\). The explicit general-canonical-representation expressions [4h,b,f] for the \((1,0) \oplus (0,1)\) spinors \(u_\sigma(\vec{p})\) and \(v_\sigma(\vec{p})\), which appear in Eq. (8), are

\[
u_{+\sigma}(p) = \begin{pmatrix} m + [(2p_z^2 + p_+ p_-)/2(E + m)] \\ p_z p_+ / \sqrt{2}(E + m) \\ p_z^2 / 2(E + m) \\ -p_+ / \sqrt{2} \end{pmatrix}, \quad u_{\sigma}(p) = \begin{pmatrix} m + [p_+ p_-/(E + m)] \\ p_z / \sqrt{2} \\ -p_{\pm} / \sqrt{2}(E + m) \\ 0 \end{pmatrix}.
\]
\[
u_{-i}(p) = \begin{pmatrix}
p^2/2(E + m) \\
-p_z p_-/\sqrt{2}(E + m) \\
m + [(2p_z^2 + p_+ p_-)/2(E + m)] \\
0 \\
p_-/\sqrt{2} \\
-p_z
\end{pmatrix}
\]

\[
v_\sigma(p) = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} u_\sigma(p).
\]

(9)

In the above expression we have defined \(p_\pm = p_x \pm ip_y\). The transformation properties of the “particle” [“antiparticle”] creation operators \(a_\sigma^\dagger(\vec{p})\) \([b_\sigma^\dagger(\vec{p})]\) are obtained from the condition

\[
U(\Lambda_P) \Psi(t', \vec{x}') U^{-1}(\Lambda_P) = \gamma_{oo} \Psi(t', -\vec{x}') ,
\]

(10)

where \(U(\Lambda_P)\) represents a unitary operator which governs the operation of parity in the Hilbert space of the single particle-antiparticle states. Using the definition of \(\gamma_{oo}\), Eqs. (2), and the explicit expressions for the \((1,0) \oplus (0,1)\) spinors \(u_\sigma(\vec{p})\) and \(v_\sigma(\vec{p})\) given by Eqs. (9), we find

\[
\gamma_{oo} u_\sigma(p') = + u_\sigma(p) ,
\]

\[
\gamma_{oo} v_\sigma(p') = - v_\sigma(p) ,
\]

(11)

with \(p'\) the parity-transformed \(p\) — i.e. for \(p^\mu = (p^o, \vec{p}), p'^\mu = (p^o, -\vec{p})\). The observation (11) when coupled with the requirement (10) immediately yields the transformation properties of the particle-antiparticle creation operators

\[
U(\Lambda_P) a_\sigma^\dagger(\vec{p}) U^{-1}(\Lambda_P) = + a_\sigma^\dagger(-\vec{p})
\]

\[
U(\Lambda_P) b_\sigma^\dagger(\vec{p}) U^{-1}(\Lambda_P) = - b_\sigma^\dagger(-\vec{p}) .
\]

(12)
Under the assumption that the vacuum is invariant under the parity transformation, 
\[ U(\Lambda_P) | \kappa \rangle = | \kappa \rangle, \]
we arrive at the result that the “particles” (described by the \( u \)-spinors) and “antiparticles” (described by the \( v \)-spinors) have opposite relative intrinsic parities

\[
U(\Lambda_P) | \vec{p}, \sigma \rangle^u = + | \vec{p}, \sigma \rangle^u, \\
U(\Lambda_P) | \vec{p}, \sigma \rangle^v = - | \vec{p}, \sigma \rangle^v.
\] (13)

The results (12) and (13) are precisely what we set out to prove. While the particle-antiparticle pairs in the \((1/2, 1/2)\) representation of the Lorentz group have \textit{same} relative intrinsic-parity, the particle-antiparticle pairs in the \((1, 0) \oplus (0, 1)\) representation have \textit{opposite} intrinsic parity. Because of the general structure of the Weinberg’s equations and the \((j, 0) \oplus (0, j)\) spinors, we assert that this result is true for \textit{all} spins. Whether (at least some of the) mesons beyond the standard \( q\overline{Q} \) structures exploit the \((j, 0) \oplus (0, j)\) representation of the Lorentz group remains an open experimental question. \textit{To sum up, the analysis of this work establishes that the relative intrinsic parity of a particle-antiparticle pair is a Lorentz-representation-dependent kinematical object. Which of the various representations, for a given spin, is actually realized in nature, such as in constructing the phenomenologies of composite particles, can be (contrary to the usual belief — for example, see Ref. [2]) experimentally determined.}

ACKNOWLEDGMENTS

An anonymous referee (of a related work) is to be thanked for bringing to my attention footnote 13 of Ref. [1]. Mikkel Johnson and Mikolaj Sawicki are to be thanked for being \textit{insistent} that we understand intrinsic-parity at a deeper level within the context of our other collaborative efforts — in addition they kindly read the rough draft of this work and provided

\[\text{ACKNOWLEDGMENTS}\]

\[\text{The reader may wish to note that all \((j, 0) \oplus (0, j)\) spinors satisfy relations very similar to Eq. (11); etc.}\]
comments and suggestions. It is also my pleasure to extend thanks to Dick Arnowitt, Terry Goldman and Barry Holstein for conversations on the subject matter of this work. Finally, I thankfully acknowledge financial support via a postdoctoral fellowship by the Los Alamos National Laboratory.
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