Resonance Fluorescence in Transport through Quantum Dots: Noise Properties

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We study a two-level quantum dot embedded in a phonon bath and irradiated by a time-dependent ac field and develop a method that allows us to extract simultaneously the full counting statistics of the electronic tunneling and relaxation (by phononic emission) events as well as their correlation. We find that the quantum noise of both the transmitted electrons and the emitted phonons can be controlled by the manipulation of external parameters such as the driving field intensity or the bias voltage.

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The complete knowledge of the statistics and, in concrete, the properties of the fluctuations of the number of particles emitted from a quantum system has been a topic of intense studies in Quantum Optics [1, 2] and, in more recent years, in Quantum Transport [3, 4]. In particular, purely quantum features like an anti-bunching of photons emitted from a closed two-level atom under a resonant field [5], or a bunching of electrons tunneling through interacting two-levels quantum dots (QD) [6] have been reported. The electronic case receives special interest since it has been recently addressed for many different physical systems such as single electron tunneling devices [7-9, 10], molecules [11], charge shuttles [12], surface acoustic waves driven single electron pumps [13] or beam splitting configurations [14]. Here, we show that the combined statistics of Fermions and Bosons is a very sensitive tool for extracting information from time-dependent, driven systems. In particular, phonon emission has been measured by its influence on the electronic current in two-level systems [15]. We analyze the electron and phonon noises and find that they can be tuned back and forth between sub- and super-Poissonian character by using the strength of an ac driving field or the bias voltage. For this purpose, we develop a general method to simultaneously extract the full counting statistics of single electron tunneling and (phonon mediated) relaxation events.

Our system consists of a two level quantum dot (QD) connected to two fermionic leads by tunnel barriers cf. Fig. 1. The Coulomb repulsion inside the QD is assumed to be so large that only single occupation is allowed (Coulomb blockade regime). The lattice vibrations induce, at low temperatures, inelastic transitions from the upper to the lower state. In analogy to Resonance Fluorescence (RF) in quantum optics, a time-dependent ac field with a frequency \( \omega \) drives the transition between the two levels \( \varepsilon_1, \varepsilon_2 \) close to resonance, \( \Delta \varepsilon = \varepsilon_2 - \varepsilon_1 - \omega \approx 0 \), which allows us to assume the rotating wave approximation. Thus, the electron in the QD is coherently delocalized between both levels performing photon-assisted Rabi oscillations [16]. For simplicity, we consider spinless electrons.

\[ \Gamma_L \]

\[ \Gamma_R \]

\[ \gamma \]

\[ \Omega \]

\[ \mu \]

FIG. 1: A two-level QD coupled to two electronic leads. Electrons entering the QD from the left oscillate between both levels with Rabi frequency \( \Omega \), can relax with rate \( \gamma \), or tunnel out if its energy is greater than \( \mu \).

The components of the density matrix \( \rho(t) = \sum_{n_e,n_{ph}} \rho^{(n_e,n_{ph})}(t) \) give the probability that, during a certain time interval \( t \), \( n_e \) electrons have tunneled out of a given electron-phonon system and \( n_{ph} \) phonons have been emitted [17]. We define the generating function (GF) [20, 21]

\[
G(t, s_e, s_{ph}) = \sum_{n_e, n_{ph}} s_e^{n_e} s_{ph}^{n_{ph}} \rho^{(n_e,n_{ph})}(t),
\]

where \( s_{e,ph}(t) \) are the electron (phonon) counting variables whose derivatives give us the correlations:

\[
\frac{\partial^{n_e + q_{ph}}}{\partial s_e^n \partial s_{ph}^q} G(t, 1, 1) = \left( \prod_{i=1}^{n_e} \prod_{j=1}^{q_{ph}} (n_e - i + 1)(n_{ph} - j + 1) \right).
\]

Thus, we are able to obtain the mean number \( \langle n_{\alpha} \rangle \), the variance \( \sigma_{\alpha}^2 = \langle n_{\alpha}^2 \rangle - \langle n_{\alpha} \rangle^2 \) (which give the \( \alpha = e, ph \) current and noise, respectively), or define the correlation between the electron and phonon counts, \( \langle n_e n_{ph} \rangle \).

We integrate the equations of motion for the GF:

\[
G(t, s_e, s_{ph}) = M(s_e, s_{ph}) G(t, s_e, s_{ph}),
\]

that generalises the Master equation, \( \dot{\rho}(t) = M(1,1)\rho(t) \), by introducing the counting variables in those terms corresponding to the tunneling of an electron to the collector lead and the emission of a phonon.

The lifetime behaviour is extracted from the pole near \( z = 0 \) in the Laplace transform of the GF, \( \tilde{G}(z, s_e, s_{ph}) = (z - M)^{-1} \rho(0) \). From the Taylor expansion of the
pole \( z_0 = \sum_{m,n>0} c_{mn}(s_e - 1)^m(s_{ph} - 1)^n \), we obtain
\[ G(t, s_e, s_{ph}) \sim g(s_e, s_{ph}) e^{i\omega t} \] and the central moments:
\[ \langle n_{e(ph)} \rangle = \frac{\partial g(1,1)}{\partial s_{e(ph)}} + c_{10(01)} t \] (3a)
\[ \sigma_{e(ph)}^2 = \frac{\partial^2 g(1,1)}{\partial s_{e(ph)}^2} \left( \frac{\partial g(1,1)}{\partial s_{e(ph)}} \right)^2 + (c_{10(01)} + 2c_{20(02)}) t \] (3b)
or the electron-phonon correlation (not discussed here), given by:
\[ \langle n_e n_{ph} \rangle - \langle n_e \rangle \langle n_{ph} \rangle = \frac{\partial^2 g(1,1)}{\partial s_e \partial s_{ph}} + c_{11} t. \] (4)
Higher moments can be straightforwardly obtained by

\[
M(s_e, s_{ph}) = \begin{pmatrix}
-2\Gamma_L - (\chi_1 + \chi_2)\Gamma_R & s_e \chi_1 \Gamma_R \\
\Gamma_L + s_e^{-1} \chi_1 \Gamma_R & -\chi_1 \Gamma_R \\
0 & i\frac{\Omega}{2} \\
0 & i\frac{\Omega}{2} \\
\Gamma_L + s_e^{-1} \chi_2 \Gamma_R & 0 \\
0 & i\frac{\Omega}{2} \\
0 & -i\frac{\Omega}{2} \\
0 & -i\frac{\Omega}{2} \\
\end{pmatrix}
\]
where \( \gamma = 2\pi|\lambda_{e_2-e_1}|^2 \) is the spontaneous phonon emission rate, \( \Gamma_\alpha = 2\pi|V_\alpha|^2 \) is the tunneling rate through the contact \( \alpha \), \( \Omega \) is the Rabi frequency, which is proportional to the intensity of the ac field, and \( \chi_i = f(\epsilon_i - \mu) = (1 + e^{(\epsilon_i - \mu)\beta})^{-1} \) and \( \bar{\chi}_i = 1 - \chi_i \) weight the tunneling of electrons between the right lead (with a chemical potential \( \mu \)) and the state \( i \) in the QD. The Fermi energy of the left lead is considered infinite, so no electrons can tunnel from the QD to the left lead. All the parameters in these equations, except the sample-depending coupling to the phonon bath, can be externally manipulated. In the limit \( \Gamma_L(R) \to 0 \) we recover the pure RF case for the statistics of the emitted phonons [20],
\[ F_{ph}(\Gamma_i = 0) = 1 - \frac{2\Omega^2(3\gamma^2 - 4\Delta_w^2)}{(\gamma^2 + 2\Omega^2 + 4\Delta_w^2)^2}, \] (6)
yielding the famous sub-Poissonian noise result at resonance (\( \Delta_w = 0 \)). In the following, we will restrict ourselves to the resonant case.

**Electron noise**– By tuning \( \mu \), we control the tunneling of electrons from the two leads. Let us first consider the non-driven case (\( \Omega = 0 \)). If \( \mu < \epsilon_2 \) (i.e., \( \chi_2 \approx 0 \)), the Fano factor becomes:

\[ F_e = 1 + \frac{2\Gamma_L \Gamma_R \left(-2(\gamma + \Gamma_R)^2 + \Gamma_L \Gamma_R \chi_1^2 + (2\gamma \Gamma_L + (\gamma + \Gamma_R)(2\gamma + 3\Gamma_R)) \chi_1 \right)}{(\gamma + \Gamma_R)(2\Gamma_L + \Gamma_R) - \Gamma_L \Gamma_R \chi_1^2} \] (7)

leading to super-Poissonian noise [6]:
\[ F_e = 1 + \frac{2\Gamma_L \Gamma_R}{\Gamma_R (\gamma + \Gamma_R) + \Gamma_L (2\gamma + \Gamma_R)} \] (8)
which has a finite well-defined value in spite of the strong current suppression. In contrast, the large bias case, \( \mu \ll \frac{\omega}{\Delta_w} \),

The dynamical channel blockade case is of special interest, as for \( \epsilon_1 < \mu < \epsilon_2 \) (\( \chi_1 \approx 1 \)) the occupation of the lower level blocks the transport through the upper level, this formalism. In the large time asymptotic limit, all the information is included in the coefficients \( c_{mn} \). Then, the Fano factor is \( F_e(\mu) = 1 + 2c_{20(02)}/c_{10(01)} \) so the sign of the second term in the right hand side defines the sub-\( (F < 1) \) or super-\( (F > 1) \) Poissonian character of the noise.

We describe our system by the Hamiltonian:
\[ \dot{H}(t) = \sum \epsilon_i \hat{d}_i^\dagger \hat{d}_i + \frac{\Omega}{2} \left( \hat{d}_1^\dagger \hat{d}_1 + \hat{h.c.} \right) + \sum \lambda_i \hat{a}_i \hat{a}_Q + \sum \lambda_i \hat{v}_i \hat{a}_Q + \sum \lambda_i \hat{d}_i \hat{a}_Q + \hat{h.c.} \], where \( \hat{a}_Q, \hat{c}_k, \hat{\alpha} \) are annihilation operators of phonons and electrons in the leads and in the QD, respectively. Writing the density matrix as a vector, \( \rho = (\rho_{00}, \rho_{01}, \rho_{12}, \rho_{21}, \rho_{22})^T \), the equation of motion of the GF [2] is described in the Born-Markov approximation, by the matrix:

\[
\begin{pmatrix}
0 & \bar{\chi}_n \Gamma_R & \Gamma_R & 0 & s_e \chi_2 \Gamma_R \\
\Gamma_R & 0 & -i\frac{\Omega}{2} & i\frac{\Omega}{2} & -\bar{\chi}_n \gamma \Gamma_R \\
0 & i\frac{\Omega}{2} & 0 & -i\frac{\Omega}{2} & s_{ph} \chi_1 \Gamma_R \\
\chi_1 \Gamma_R & -\bar{\chi}_n \Gamma_R & i\frac{\Omega}{2} & 0 & 0 \\
\end{pmatrix}
\]
Suppressed for higher 

increases the noise. In the opposite limiting case, \( \epsilon \mu > \epsilon \)

\( F_e = 1 - \frac{4\Gamma_L\Gamma_R}{(2\Gamma_L + \Gamma_R)^2} \),

independent of the coupling to the phonon bath. Note that by comparing with the single resonant level case, \( \mu > \epsilon_2 \) the inclusion of an additional level in the bias window increases the noise. In the opposite limiting case, \( \mu > \epsilon_2 \) the transport is still possible due to the thermal smearing of the Fermi function, the noise tends to be Poissonian before the current is completely suppressed for higher \( \mu \) (cf. Fig. 2). The phonon bath mainly affects the current noise through the non-driven device in the region \( \epsilon_1 \mu > \epsilon_2 \) by reducing it as \( \gamma \)

For \( \mu \ll \epsilon_1 \), the Fano factor does not depend on the ac field and expression (9) is recovered.

**Phonon noise** – We now turn to the phonon statistics, cf. Fig. 3. In the non-driven case, a phonon can be emitted only when an electron enters the upper level with the lower level being already empty by an electron having tunneled out into the right contact. Therefore, the emission of phonons is anti-bunched and suppressed in the region where the transport of electrons is blocked by the occupation of the lower level (i.e., when \( \mu > \epsilon_1 \)):

\[ F_{ph} = 1 - \frac{2\gamma \Gamma_L \Gamma_R (\gamma + 2\Gamma_L + 2\Gamma_R) (1 - \epsilon_1)}{(\Gamma_L (2\gamma + \Gamma_R (2 - \epsilon_1)) + \Gamma_R (\gamma + \Gamma_R))^2} \]

However, the driving field gives an additional way of populating the higher level, increasing the noise and leading, for high enough field intensities, to the bunching of phonons (see Fig. 3). A crossover from this pure transport infinite bias regime (\( \mu \ll \epsilon_1 \)), where the phonon noise increases with the Rabi frequency, to the strictly sub-Poissonian noise where the electron transport is suppressed (and the RF limit is recovered) can be observed by increasing \( \mu \) and successively blocking the transport through the lower (for \( \mu > \epsilon_1 \)) and upper level (for \( \mu > \epsilon_2 \)), cf. Fig. 3. It can be easily seen that the noise suppression for this last case is largest when \( \Omega = \gamma / \sqrt{2} \). Thus, in the more interesting intermediate regime, \( \epsilon_1 < \mu < \epsilon_2 \), as in the RF case, a minimum at low intensities is observed which is shifted and modified by the electronic transport, and also the super-Poissonian noise characteristic of the large bias regime for \( \Omega > (4 (3\gamma + 2\Gamma_R) \Gamma_L / \Gamma_R + (13\gamma + 11\Gamma_R) \Gamma_L + 3\Gamma_R (\gamma + \Gamma_R)^{1/2} \) (see Fig. 3).

In conclusion, we propose a solid-state, transport version of resonance fluorescence (driven two-level quantum dot with phonon emission), where the noise for both the transferred electronic current and the emitted phonons can be experimentally controlled by tuning the transport (chemical potential) and optics (ac field intensity) parameters. We found various regimes showing different combinations of sub and super-Poissonian electron and phonon Fano factors. Although not discussed here, our technique can also be applied to driven electron-phonon systems with higher complexity such as double quantum dots, and to obtain higher moments and correlations between electron and phonon distributions, which will be the scope of a future work.

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FIG. 2: Electron Fano factor as a function of (a) $\tilde{\gamma} = \gamma/T$ for $\tilde{\Omega} = \Omega/T = 0$ ($\Gamma = \Gamma_{ph}/T$) and (b) $\tilde{\Omega}$ for $\tilde{\gamma} = 0.1$ for different chemical potentials: $\mu < \varepsilon_1$ (solid), $\mu = \varepsilon_1$ (dashed), $\varepsilon_1 < \mu < \varepsilon_2$ (dash-dotted), $\mu = \varepsilon_2$ (dash-dot-dotted) and $\mu \gtrsim \varepsilon_2$ (dotted). Note that, in the non-driven case (a), the phonon bath does not affect the sub or super-Poissonian character of the electron noise which can be manipulated introducing the resonant driving field (b). Insets: (a) $F_e$ as a function of $\mu$ for $\Omega = 0$ and different phonon emission rates: $\tilde{\gamma} = 0.01$ (solid), $\tilde{\gamma} = 0.1$ (dashed), $\tilde{\gamma} = 1$ (dash-dotted) and $\tilde{\gamma} = 10$ (dotted) and (b) for $\tilde{\gamma} = 0.1$ and different driving intensities: $\tilde{\Omega} = 0$ (solid), $\tilde{\Omega} = 0.15$ (dashed), $\tilde{\Omega} = 0.3$ (dash-dotted), $\tilde{\Omega} = 0.492$ (dash-dot-dotted), $\tilde{\Omega} = 1$ (dotted). Parameters (holding for all figures, in meV): $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.8$, $\kappa_B T = 2 \times 10^{-3}$.

FIG. 3: $F_{ph}$ as a function of $\tilde{\Omega}$ for $\tilde{\gamma} = 1$ and different chemical potentials: $\mu < \varepsilon_1$ (solid), $\varepsilon_1 < \mu < \varepsilon_2$ (dashed) and $\mu > \varepsilon_2$ (dotted). In the large bias case, the noise always increases with $\tilde{\Omega}$ and becomes super-Poissonian (in this concrete configuration) for $\tilde{\Omega} > 4.046$. In the case $\mu > \varepsilon_2$, there is a pronounced minimum at $\tilde{\Omega} = 1/\sqrt{2}$ typical of the RF. The case $\varepsilon_1 < \mu < \varepsilon_2$ follows an intermediate behaviour showing the RF-like minimum and the super-Poissonian noise (typical of the large-bias case) for $\tilde{\Omega} > 5\sqrt{2}$ (see text). Inset: $F_{ph}$ as a function of $\mu$ for $\tilde{\gamma} = 10$ and different Rabi frequencies: $\tilde{\Omega} = 0.1$ (solid), $\tilde{\Omega} = 2$ (dashed), $\tilde{\Omega} = 6$ (dash-dotted), $\tilde{\Omega} = 10$ (dotted). For high $\tilde{\Omega}$, $\mu$ changes the character of the noise.

FIG. 4: Colour plot showing the different regions where one can find $F_e, F_{ph} \geq 1$ (white, appearing only for $\tilde{\Omega} = 0$), $F_e < 1$, $F_{ph} \geq 1$ (light grey), $F_e \geq 1, F_{ph} < 1$ (dark grey) and $F_e, F_{ph} < 1$ (black) by tuning $\mu$ and $\tilde{\Omega}$, for $\tilde{\gamma} = 1$.

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