Are $e\mu$ colliders interesting?

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Abstract

We show that current experimental constraints already severely restrict what might be observable at $e\mu$ colliders. We identify some cases where it may be possible to probe physics beyond what might be possible at other facilities and make some remarks about physics capability of high energy $e\mu$ colliders.
Recently, Hou [1] and Choi et al. [2] have suggested the intriguing possibility of searching for lepton flavor violating (LFV) couplings at $e\mu$ colliders. They propose to search for $e$ and $\mu$ number violating couplings of hypothetical heavy particles $X$ that may be produced as resonances in $e\mu$ collisions. The implementation of this idea is hampered by our complete ignorance of the mass $M_X$, i.e. of not knowing the energy at which to operate the collider.

In this note, we first explore the more practical idea of looking for LFV interactions by searching for resonance production of known particles, which has the obvious advantage that we know the exact center of mass (CM) energies at which the collider should be operated. The other very important advantage of this idea is that we would not require a very high energy muon beam since most of the known resonances are lighter than about 10 GeV. We were especially motivated to examine this since it may well be that the development of cold high intensity muon beams with $E \sim 1$ GeV could be a needed first step for the development of a high energy muon collider. Unfortunately, we find that current limits on LFV interactions of known particles already put severe limits on the cross sections for producing these via $e\mu$ collisions, so that with the preliminary estimates [3] for the luminosity

$$L \sim 2 \times 10^{32} \left( \frac{E}{100 \text{ GeV}} \right)^{4/3} \text{cm}^{-2} \text{s}^{-1}$$

the expected rates, with 1–10 GeV $e\mu$ colliding beams, are generally well below the level of observability. Following Choi et al. [2] we next consider high energy $e\mu$ colliders designed to operate at the resonance $X$. We show that rates for LFV processes will be strongly limited if $X$ also couples to hadrons. For completeness, we point out exceptional scenarios where there could be observable rates at $e\mu$ colliders, but which cannot be probed at high energy $e^+e^-$ (and even $\mu^+\mu^-$) or hadron colliders.

**Resonance Production of Known Particles**

The best limits on $e\mu$ flavor violation came from the non-observation of the reaction $\mu Ti \rightarrow e Ti$, or the decays $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$. The branching fraction for these decays are smaller [4] than $5 \times 10^{-11}$ and $10^{-12}$, respectively. These bounds strongly limit the LFV $e\mu$ couplings of flavor-neutral vector mesons such as $\rho$, $\psi$, $\Upsilon$ (and their excitations)
which directly couple to single photons. It is easy to check, for example, that the limit on \( B(\mu \rightarrow e\gamma) \) leads to the remarkably strong bound \( \frac{f_{\mu e}^2}{4\pi} < 10^{-26} \), to be compared with \( \frac{f_{\rho\pi\pi}^2}{4\pi} < 10^{-21} \).

Spin-zero mesons cannot directly couple to the photon but can couple to \( e^+e^- \) pairs, and lead to \( \mu \rightarrow 3e \) decays if these mesons have LFV \( \mu e \) interactions. This decay cannot proceed via 1-photon exchange (even for the \( 0^+ \) state) since the electromagnetic current is exactly conserved, but occurs via multi-photon exchange or via the \( Z^0 \) exchange, and is further suppressed by the chirality factor \( (m_e/m_X)^2 \). The best limit on LFV interactions of spin-zero mesons appears to come from the non-observation of the decay \( \mu \rightarrow e\gamma\gamma \), the branching fraction for which is smaller \[4\] than \( 7 \times 10^{-11} \). For example, for \( X = \eta \), using \( \Gamma(\eta \rightarrow \gamma\gamma) \simeq 0.5 \) keV, we obtain \( \frac{f_{\eta\gamma\gamma}^2}{4\pi} \lesssim 10^{-17} \). A similar bound should apply on LFV couplings of other spin-zero mesons, assuming only that \( f_{X\gamma\gamma} \equiv f_{\eta\gamma\gamma} \).

There are direct limits \[4\] on \( \mu e \) LFV decays of flavored mesons \( K^0_L, D^0 \) and \( B^0 \). The strongest of these limits is on the branching fraction for \( K^0_L \rightarrow \mu e \) decay which is smaller than \( 3 \times 10^{11} \). The corresponding bounds for \( \mu e \) decay of \( D^0 \) (\( B^0 \)) mesons are \( \sim \) few \( 10^{-5} \) (few \( 10^{-6} \)). It is straightforward to check that the direct limit on \( B(K^0_L \rightarrow \mu^\pm e^\mp) \) is much more restrictive than the limit that would be obtained from \( \mu \rightarrow e\gamma\gamma \) decay using the observed decay rate for \( K^0_L \rightarrow \gamma\gamma \).

The decays \( K_L \rightarrow \mu^\pm e^\mp \) probe different sources of LFV than do the corresponding decays of \( K^* \) on which the bounds are considerably weaker. Nevertheless, from the limit \[4\] \( B(K^+ \rightarrow \pi^+ \mu^+ e^-) \lesssim 2 \times 10^{-10} \) we may infer bounds such as \( B(K^* \rightarrow \mu^+ e^-) \lesssim 10^{-21} \), since otherwise the rate for the decay \( K^+ \rightarrow \pi^+ \mu^+ e^- \) mediated by virtual \( K^* \) would exceed its experimental bound. It should be amply clear that a similar bound applies to all strange resonance \( K_X \) with \( J^P = 0^+, 1^-, 2^+, \ldots \) since the \( K K_X \pi \) vertex is allowed by strong interactions. We thus conclude that the \( K_X \mu e \) coupling can only be significant for strange mesons with \( J^P = 0^-, 1^+, 2^- \) etc. Within the quark model framework, however, this possibility also appears to be excluded since any quark bilinear \( s\Gamma d \) with non-vanishing matrix elements between these \( J^P = 0^-, 1^+, 2^- \) etc. states and the vacuum would also result in \( K^0_L \rightarrow \mu e \).
decays, unless various contributions cancel to a very high precision, or form factors become
tiny for no apparent reason. We thus conclude that $\mu e$ LFV couplings of strange mesons
with all $J^P$ quantum numbers consistent with the quark model are very small.

We now turn to the examination of what might be possible at $e\mu$ colliders. The peak
cross section for resonance production of a particle $X^0$ with spin $S = (0 \text{ or } 1)$ is given by

$$\sigma = \frac{4\pi}{M_X^2} (2S + 1) B_{e\mu}, \quad (2)$$

where $B_{e\mu}$ is the branching fraction for the decay $X^0 \rightarrow e^-\mu^+$ (or $e^+\mu^-$), depending on the
initial beams. Eq. (2) presumes that the spread in energy ($\Delta$) is much smaller than the
width $\Gamma$ of the resonance. In the case that $\Delta \gg \Gamma$, the effective luminosity, and hence the
event rate at the peak, is reduced by a factor $\sim \Gamma/\Delta$, the exact number depending on the
beam profile. In what follows, we take $\Delta \sim 10$ MeV, which for $\sqrt{s} = 1–10$ GeV corresponds
to a beam resolution of (0.1–1)% , to be compared with the the projected [5] beam resolution
of better than 0.1% for muon beams and typical resolutions of a few $\times 10^{-4}$ at existing $e^+e^-$
colliders. In order to maximize the event rate, it is clear that we should focus on particles
with widths larger than 10 MeV.

The most obvious $X$ candidate is $Z^0$ which, however, is excluded for reasons that we
have already mentioned in another context: the experimental bound $B(\mu \rightarrow 3e) < 10^{-12}$, in
turn, limits $B(Z^0 \rightarrow \mu^+e^- + e^-\mu^+) < 6 \times 10^{-13}$. From Eq. (2) it is then straightforward to
check that even with an integrated luminosity of 100 fb$^{-1}$, we would expect $\lesssim 0.1$ event at
an $e\mu$ collider operating on $Z^0$.

We are thus led to examine the possibility of producing known hadrons in $\mu e$ collisions.
For a CM energy $E \sim 1$ GeV, Eq. (1) yields an integrated luminosity of $\sim 4$ pb$^{-1}$, assuming
collider operation for $10^7$ s. Using Eq. (2), we see that for resonances with $\Gamma \gtrsim \Delta$, we may
expect about

$$\mathcal{N} \simeq (2S + 1) B_{e\mu}(2 \times 10^{10})/M^{2/3} \quad (3)$$

events (here $M$ is in GeV units) during this period of operation, so that we can at best
probe LFV decays with a branching fraction $\lesssim 10^{-10}$ for $M \sim 1–5$ GeV. But our previous
discussion shows that current experimental bounds already essentially exclude this range for most known resonances. LFV decays of strongly decaying flavor-neutral mesons, we saw, were constrained to have branching fractions $\lesssim 10^{-26}$, while the bounds on their pseudoscalar counterparts were $\sim 10^{-17}$. LFV couplings of $1^+$ scalar bosons would have similar bounds on their couplings as $0^-$ bosons. We have also seen that the limits on LFV decays $K^0_L \to \mu e$ and $K^0_L \to \pi \mu e$ respectively limit the LFV branching fractions of $0^-, 1^+, 2^- \ldots (0^+, 1^-, 2^+ \ldots)$ mesons to be smaller than $3 \times 10^{-11}$ ($\sim 10^{-21}$), which again would be below the level of observability given in Eq. (3), unless luminosities significantly higher than those given by (2) are achieved.

Despite the fact that we have arrived at a generally negative assessment regarding the feasibility of observing known resonances in $e\mu$ collisions, there is one loophole in our arguments up to now. Recall that LFV interactions of unflavored $1^+$ mesons were constrained only by upper limits on the $e\mu$ decay rate of the corresponding pseudoscalar state. There are, however, no such limits on $\eta_c$ and $\eta_b$ decays. This leads us to suggest that at $e\mu$ colliders it may be possible to probe LFV couplings of $\chi_{c1}$ ($J^{PC} = 1^{++}$) which has a width $\sim 0.9$ MeV. Taking into account the suppression from the factor $\Gamma/\Delta$, we see from (3) that optimistically it should be possible to probe $B(\chi_{c1} \to e\mu)$ down to about $10^{-10}$ since there is no physics background to the signal $e\mu \to \chi_{c1} \to$ anything, where the invariant mass of the final state reconstructs to $M(\chi_{c1})$. Electron contamination from the decays of muons in the beam would lead to hadronic signals with smaller invariant mass. A $\sim 4\pi$ detector would thus be necessary to convincingly study the signal. We have checked that the current bounds on $B(\pi^0 \to \mu e)$ do not constrain the hypothetical $\chi_{c1} \to \mu e$ couplings. We note that $\chi_{b1}$ states are expected to be somewhat narrower so that the range of branching fractions that may be probed via $e\mu$ collisions is smaller by a factor of 5–10.

Finally, we turn to bare charm and bottom mesons. Current limits (4) on the decays $B^0 \to \mu e, D^0 \to \mu e$ are in the vicinity of few $\times [10^{-6}–10^{-5}]$, while limits on $B^+ \to D^+ \to \pi e\mu$ are $\sim$ few $\times 10^{-3}$. As we discussed for the kaon system, any significant LFV $\mu e$ couplings of $0^+, 1^-, 2^+ \ldots$ states of this system results in branching fractions for $D \to \pi e\mu$
decays in excess of experimental bounds, just because the $D$ states are so narrow. The same considerations hold for $B$ mesons. We thus focus on LFV couplings of $0^-, 1^+, 2^- \ldots D$ and $B$ mesons, which are small enough to escape the direct bounds, and whose widths are larger than $\sim 10$ MeV. The only established state that we could find was $D_1(2420) \ [J^P = 1^+]$, whose width is 19 MeV. Using (3), we see that it should be possible to probe a branching fraction of $\sim 10^{-10}$ after a year of $e\mu$ collider operation, to be compared with branching fraction limits of $\sim 10^{-5}$ available today, and at best of $\sim 10^{-8}$ that may be possible \[6\] at the Tevatron or charm meson factories. There is no suitable $B$ meson state that has been clearly identified, though it is quite possible that such a state may be discovered at $B$-factories. We also note that in principle $\mu^+e^-$ and $\mu^-e^+$ collisions probe LFV couplings that are a priori independent.

$e\mu$ Colliders at High Energy

Although this is not the main focus of this present study, we make a few remarks on $e\mu$ collider operation at very high energy where the elementary carrier of LFV interactions can be resonantly produced. For an integrated luminosity of 100 fb$^{-1}$, the number of events in the final state $f$ (assuming the collider is operated at the resonance peak) is given by

$$N = \frac{0.5(2S + 1) \times 10^9 B_{\mu e} B_f}{M_X (\text{TeV})^2},$$

where $B_{\mu e}$ and $B_f$ are the branching fractions for the decays $X \to \mu e$ and $X \to f$, respectively \[4\]. We see from (4) that the event rate can be $\sim 10^8/100$ fb$^{-1}$ even for $M_X \sim \text{TeV}$. This is not necessarily in conflict with limits on the branching fraction for $\mu \to 3e$ decay, which is given by

$$B(\mu \to 3e) = K \left( \frac{g_{\mu e} g_{ee}}{M_X^2} \right)^2 \frac{1}{G_F^2},$$

where the constant $K$, which is $\mathcal{O}(1)$ depends on the spin and assumed spacetime structure of the interactions of $X$ with electrons and muons. If $g_{\mu e} \sim g_{ee} \lesssim 10^{-3}$, $B(\mu \to 3e)$ is quite compatible with experimental bounds, and we can have a large event rate in (4) if $B_{\mu e} \approx B_{\mu \mu} \approx 1/2$. While couplings $\sim 10^{-3}$ may appear small for gauge interactions, it
is worth keeping in mind that $X$ could be a new spin-zero boson with modest couplings to leptons. Our message is that the physics of LFV interactions, should these exist, is completely unknown.

We also remark that $e\mu$ LFV interactions of $X$, are stringently constrained if $X$ also couples to hadrons. In this case $X$ will induce current-current LFV interactions, which as we saw in the previous section, are strongly constrained by experiment. For example, the vector component of the hadronic current that couples to $X$ leads to $\mu \rightarrow e\gamma$ if it is flavor-neutral, or to $K \rightarrow \pi\mu e$ decays if it is $\bar{d}\gamma_s$, etc. Similarly, there should be strong bounds on LFV couplings of sneutrinos if these also couple to hadrons in SUSY models where $R$-parity is not conserved.

Finally, we remark on whether it would be possible to study the resonance production of $X$ at other colliders. This, of course, depends on what $X$ couples to. If it couples to $e^+e^-$ and $e\mu$ pairs, the cross section for $e^+e^- \rightarrow X \rightarrow \mu e$ and $\mu e \rightarrow X \rightarrow e^+e^-$ should be equal. Since both processes are free from physics backgrounds we see no particular advantage of $e\mu$ colliders. If instead $X$ couples to $\mu^+\mu^-$ and $e\mu$ pairs, it can be searched for at $\mu^+\mu^-$ colliders. If, however, it only couples to $e\mu$ and $\tau^+\tau^-$ pairs, $e\mu$ colliders appear to provide the only way of directly searching for it. These colliders are also the unique facility to study the direct production of a hypothetical particle responsible for muonium–antimuonium oscillations, as long as it does not also couple to other channels.

What if $X$ couples to hadrons? One would then think that it would be simple to search for it at the LHC via its $\mu e$ decays. Such a search would be akin to the search for $Z'$ bosons except that the final state would be even more characteristic. We note, however, that at the LHC it is possible to search for a $Z'$ of mass 1 TeV only if $(\sigma \cdot B)_{Z'} \gtrsim (2 \times 10^{-3})(\sigma \cdot B)_Z$, assuming an integrated luminosity of 100 fb$^{-1}$. We thus warn the reader that if the couplings of $X$ to first generation quarks $\lesssim$ few $\times$ 10$^{-3}$ it could escape detection at the LHC, but, depending on its decay patterns may be observable at an $e\mu$ collider.
Summary

We have shown that existing constraints on LFV interactions severely restrict what might be observable at $e\mu$ colliders. Our arguments leave some loopholes for LFV interactions with special form of flavor and spacetime structure. In these cases, a low energy $\mu e$ collider operating at the resonance energy of $\chi_{c1}(3150)$, $\chi_{b1}(9892)$, or $D_1(2420)$ may be able to probe LFV $\mu e$ couplings beyond current bounds, and likely beyond what might be possible at other facilities. Moreover, our arguments do not apply to mesons with quantum numbers that do not correspond to those of quark-antiquark bound states as given by the quark model \[10\]. While we regard the theoretical case for such LFV interactions as far from compelling, our analysis suggest that it may be possible to use a relatively low energy muon beam that may be available during the first stages of muon collider construction to probe physics that may not be accessible elsewhere. We believe that the case for $e\mu$ colliders at high energy is less compelling. It appears that only for some special flavor structure of LFV couplings (or when these couplings are $\ll 1$) are $e\mu$ colliders a unique facility for discovering these new interactions.

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[6] See e.g. T. Liu and S. Pakvasa, in Proceedings of the Workshop on Heavy Quark Physics at C0, UH-511-867-97 (1997).

[7] Equation \textsuperscript{[4]} also shows that the event rate is not increased even if the resonance happens to be wide because it can decay via many different channels, contrary to the statement in Ref. \textsuperscript{[2]}.

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