On the local fractional metric dimension of corona product graphs

S Aisyah¹, M I Utoyò², L Susilowati²

¹Departement of Mathematics, Kaltara University, Indonesia
²Departemen of Matematics, Airlangga University, Indonesia

*E-mail: m.i.utoyo@fst.unair.ac.id; aisyah_rasyid84@yahoo.com

Abstract. A vertex in a connected graph is said to resolve a pair of vertices \{u, v\} if the distance from to is not equal to the distance from v to x. A set of vertices of is a resolving set for G if every pair of vertices is resolved by some vertices of S. The smallest cardinality of a resolving set for G is called the metric dimension of G, denoted by \dim(G). For the pair of two adjacent vertices \{u, v\} is called the local resolving neighbourhood and denoted by \mathcal{N}(u, v). A real valued function \(g: V(G) \rightarrow [0,1]\) is a local resolving function of G if for every two adjacent vertices \(u, v \in V(G)\). The local fractional metric dimension of G is defined as \(\dim_f(G) = \min \{|g|: g \text{ is local resolving function of } G\}\) where \(|g| = \sum_{v \in V} g(v)\). Let \(G = \mathcal{N}(1, 2)\) and be two graphs of order \(n_1\) and \(n_2\), respectively. The corona product \(G \boxdot H\) is defined as the graph obtained from G and H by taking one copy of G and \(n_1\) copies of H and joining by an edge each vertex from the \(i^{th}\) copy of H with the \(i^{th}\)-vertex of G. In this paper we study the problem of finding exact values for the fractional local metric dimension of corona product of graphs.

1. Introduction

Let \(G = (V(G), E(G))\) be a finite, simple, undirected, connected graph. For a graph G, the vertex set of G and \(E(G)\) is the edge set of G, respectively. For any two vertices \(x, y \in V(G)\), \(d_G(x, y)\) denotes the distance between \(x, y\) in G. A resolving set of G is a subset \(W \subseteq V(G)\) such that \(d_G(x, \emptyset) \neq d_G(y, \emptyset)\) for any two distinct vertices \(x, y \in V(G)\). The metric dimension of G, denoted by \(\dim(G)\), is the minimum cardinality of the resolving sets of G. Metric dimension was first introduced in the 1970s, independently by Harary and Melter [3] and by Slater [4].

Let \(g: V(G) \rightarrow [0,1]\) be a real valued function. For \(W \subseteq V(G)\), define \(g(W) = \sum_{v \in W} g(v)\). We call \(g\) is resolving function of G if \(g(R_G(x, y)) > 1\) for any two distinct vertices \(x, y \in V(G)\). The fractional metric dimension, denoted by \(\dim_f(G)\), is given by \(\dim_f(G) = \min \{|g|: g \text{ is resolving function of } G\}\), where \(|g| = g(V(G))\). Arumugam and Mathew [1] formally introduced the fractional metric dimension of graphs and obtained some basic results. For an ordered set \(X = \{x_1, x_2, ..., x_d\} \subseteq V(G)\) of vertices, we use the ordered \(k\)-tuple \(r(\{x_1, x_2, ..., x_d\}) = (d(x_1, x_2), d(x_1, x_3), ..., d(x_1, x_d))\) as the representation of \(x_1\) with respect to \(X\). If every two adjacent vertices of \(G\) has distinct representation with respect to \(X\) then \(X\) is called a local resolving set for \(G\). A minimum local resolving set is called local basis for \(G\). A local metric dimension for \(G\), denote by \(\dim_f(G)\), is the number of vertices in a local basis for \(G\) by Okamoto et al. [5]. From the idea of Arumugam and Mathew [1], Okamoto et al. [5] we define the local resolving neighbourhood as follow. For the pair \(\{u, v\}\) of two adjacent vertices of \(G\), we define the local resolving neighbourhood
is a connected graph of order $\mathcal{G}$. We refer to the vertices $v_i$ of $\mathcal{G}$ and $v_j$ of $\mathcal{H}$.

A graph $\mathcal{G}$ is defined as the copy of $\mathcal{G}$ for complete graphs, complete bipartite graphs, and for two isomorphic graphs $\mathcal{G}$ and $\mathcal{H}$, respectively. The corona product of graphs was studied in [8], besides the fractional metric dimension of corona product graphs. We begin by giving some basic concepts and notations. For two adjacent vertices $u$ and $v$, we use the notation $u \bowtie v$ to denote the join graph $\mathcal{G} \bowtie \mathcal{H}$. A graph $\mathcal{G}$ is isomorphic to the join graph $\mathcal{G} \bowtie \mathcal{H}$. We use the notation $u \bowtie v$ to denote the local resolving function $g_i$ of $\mathcal{G}$.

Figure 1 shows two examples of corona product graphs where the factors are $\mathcal{G}$ and $\mathcal{H}$. The local fractional metric dimension of $\mathcal{G}$ is defined as $\dim_{fr}(\mathcal{G}) = \min\{d(x, v) : v \in V(\mathcal{G})\}$, where $d(x, v)$ is the distance between the point $x$ and the vertex $v$ in the graph $\mathcal{G}$.

From the left, we show the corona graphs $\mathcal{G} \bowtie \mathcal{H}$ and $\mathcal{G} \bowtie \mathcal{H}$. The local fractional metric dimension of $\mathcal{G}$ is $\dim_{fr}(\mathcal{G}) = 1, 2, \ldots$, where $\dim_{fr}(\mathcal{G})$ is the order of the graph $\mathcal{G}$.

The join graph $\mathcal{G} \bowtie \mathcal{H}$ is defined as $\mathcal{G} \bowtie \mathcal{H} = \mathcal{G} \cup \mathcal{H} \cup \{e : v \in V(\mathcal{G}) \rightarrow u \in V(\mathcal{H})\}$. Furthermore, we determine the exact value of the metric dimension of corona product of graphs for any graph $\mathcal{G}$.

Also, $\dim_{fr}(\mathcal{G}) = 1, 2, \ldots$, where $\dim_{fr}(\mathcal{G})$ is the order of the graph $\mathcal{G}$.
2. Main Result
In this section, we investigate the value of $\dim_f(G \ominus H)$ when $G$ and $H$ is a complete graph. We first recall some of the bounds on the local fractional metric dimension of graphs [13].

Theorem 1. For any graph $G$ of order $n$,

a. For the path graph $(P_n)$, $\dim_f(P_n) = 1$

b. For $K_n$, $\dim_f(K_n) = \frac{n}{2}$

c. For the cycle graph $(C_n)$ on $n \geq 3$ vertices, we have
\[ \dim_f(C_n) = \frac{n-1}{2}, \text{ if } n \text{ is odd} \]
\[ \dim_f(C_n) = 1, \text{ if } n \text{ is even} \]
d. For the star graph $(S_n)$, $\dim_f(S_n) = 1$

e. For the bipartite graph $(K_{n,m})$, $\dim_f(K_{n,m}) = 1$

To begin with, consider some straightforward cases. If $H$ is an empty graph, then $K_1 \ominus H$ is a star graph and $\dim_f(K_1 \ominus H) = 1$. Moreover, if $H$ is a complete graph of order $n$, then $K_n \ominus H$ is a complete graph of order $n + 1$ and $\dim_f(K_n \ominus H) = \frac{n+1}{2}$.

Theorem 2. Let $G$ be a connected non-trivial graph. For any empty graph $H$,
\[ \dim_f(G \ominus H) = \dim_f(G) \]

Proof: Take any of the resolving function $f: V(G \ominus H) \to [0,1]$. Take any two adjacent vertices $x, y \in V(G \ominus H)$ then there are two possibilities there are (1). $x, y \in V(G)$ and (2). $x \in V(G)$ and $y \in H(G)$. If

a. $(x, y) \in V(G)$ then there exist $U_{x_i} = \{x_{1}, \ldots, x_{n}\}$ with $i = 1, 2, \ldots, n$. Based on the definition of the resolving set, is obtained $U_{x_i} \subseteq R(x, y)$.

b. $x \in U_{x_i}$ and $y \in U_{x_j}$ with $j = 1, 2, \ldots, m$. Based on the definition of the resolving set, is obtained $R(x, y) = V(G \ominus H)$.

As a result, we can conclude that $\dim_f(G \ominus H) = \dim_f(G)$.

Theorem 3. Let $G$ be a connected graph and $H$ be a graph. Then $\dim_f(G \ominus H) = |V(G)| \dim_f(H)$

Theorem 4. Let $H$ be a non-empty graph. The following assertions hold.

i. If the vertex of $K_1$ does not belong to any local metric basis for $K_1 + H$, then for any connected graph $G$ of order $n$,
\[ \dim_f(G \ominus H) = n \cdot \dim_f(K_1 + H) \]

ii. If the vertex of $K_1$ belongs to a local metric basis for $K_1 + H$, then for any connected Graph $G$ of order $n \geq 2$,
\[ \dim_f(G \ominus H) = n \cdot (\dim_f(K_1 + H) - 1) \]

3. Conclusion
In this paper, we get the value of $\dim_f(G \ominus H)$ where $G$ and $H$ are a special graph, i.e complete graphs, complete bipartite graphs, cycle graphs, star graphs, and path graphs.

Open Problem 3.1. Determine value $\dim_f(G \ominus H)$ where $G$ and $H$ are general graph.

Open Problem 3.2. To further be able to do research local fractional metrics dimension of comb product graph.
Acknowledgement
We gratefully acknowledge the support from DIKTI Indonesia trough the “Penelitian Disertasi Doktor” RISTEKDIKTI 2018 research project.

References

[1] Arumugam S, and Mathew V 2012 *Discrete Mathematics* 312 1584–1590

[2] Arumugam S, Mathew V, and Shen J 2013 *Discrete Mathematics Algorithms Appl.* 5 (2) 1350037 8

[3] Harary F, Melter R A 1977 *Ars Combin.* 2 191–195 4 318

[4] Slater P J 1975 *Congr. Number.* 14 549–559

[5] Okamoto F, Phinezy B and Zhang P 2010 *Mathematica Bohemica* 135 no. 3 239-255

[6] Feng M, Wang B Lv K 2014 *Discrete Appl. Math* 170 55–63

[7] Frucht R, Harary F 1970 *Aequationes Mathematicae* 4 (3) 322–325

[8] Feng M, Wang B Lv K On the fractional metric dimension of corona product graphs and lexicographic product graphs arXiv:1206.1906v1.

[9] Kang C X The fractional strong metric dimension of graphs *Lecture Notes in Comput. Sci.* 8287 84–95

[10] Kang C X, Yero I G, and Yi E The fractional strong metric dimension in three graph products arXiv:1608.0549v1.

[11] Yi E 2015 *Acta Mathematica Sinica* Vol .31 No. 3 367-382

[12] Aisyah S, Utoyo M I, and Susilowati L Local Fractional Metric Dimension of Graphs *Discrete Mathematics* (On Submitted).

[13] Iswadi H, Baskoro E T, Simanjuntak R 2010 *Far East Journal of Mathematical Sciences* 52 155–170