The Tsallis Distribution at Large Transverse Momenta

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Abstract

Fits to the transverse momentum distributions of charged particles produced in \( p - p \) collisions at LHC energies based on the Tsallis distribution have been shown to work over 14 orders of magnitude. Two versions of the Tsallis distribution are compared and discussed, the thermodynamically consistent version leads to a temperature of \( T = 74 \pm 13 \text{ MeV} \) at the highest beam energy, a result which is in agreement with previous analyses done with lower transverse momentum data.

It is well known that the Tsallis distribution fits excellently to the transverse momentum \( (p_T) \) distributions observed at RHIC [1–3] and at the LHC [4–8] with only three parameters, \( T, q \) and \( dN/dy \) (or, alternatively, a volume \( V \) [10]). It was recently shown that these fits extend to values [9] of \( p_T \) up to 200 GeV [11–13], this means that the Tsallis description can be used over 14 orders of magnitude. This is unexpected because in this kinematic range hard scattering processes dominate the scattering process and extrapolations from low (soft) \( p_T \) to very high (hard) \( p_T \) are unexpected [14]. In this paper we revisit the results obtained in [11–14] where very good fits to transverse momentum distributions were obtained and show that the parameters needed are the same as those obtained at lower transverse momenta. We confirm the quality of the fits but obtain different values albeit using a different version of the Tsallis model.

The transverse momentum distributions of charged particles produced in \( p - p \) collisions at LHC energies are fitted using a sum of three Tsallis distributions. These consist of fits for \( \pi^+ \)'s, \( K^+ \)'s and protons, \( p \). The following expression, at mid-rapidity and \( \mu = 0 \), was used to fit the distributions obtained in various experiments [15–17]:

\[
\left. \frac{d^2N_{ch}}{dp_T \, dy} \right|_{y=0} = 2p_T \frac{V}{(2\pi)^2} \sum_{i=1}^{3} g_i m_{T,i} \left[ 1 + \left( q - 1 \right) \frac{m_{T,i}}{T} \right] \frac{e^{-q}}{(q-1)}
\]

where \( i = \pi^+, K^+, p \). The relative weights between particles were determined by the corresponding degeneracy factors and given by \( g_{\pi^+} = g_{K^+} = 1 \) and \( g_p = 2 \). The factor 2 on the right hand side takes into account the contributions from
antiparticles, $\pi^-, K^-$ and $\bar{p}$.

The parameter $T$ in Eq. 1 can be regarded as a temperature as it is related to the entropy via the standard thermodynamic relation [16, 18–20]

$$T = \frac{\partial E}{\partial S} \bigg|_N,$$  

where $S$ refers to the Tsallis entropy. This is to be contrasted to the formula used in [11–14]

$$\left. \frac{d^2N_{ch}}{dp_T dy} \right|_{y=0} = A \left[ 1 + (q - 1) \frac{m_T}{T} \right]^{\frac{1}{q-1}}$$

in this formula $T$ is a parameter used to fit the transverse momentum distribution and is not related to a temperature in the thermodynamic sense as in Eq. 2. It is remarkable that this formula works over 14 orders of magnitude [11–13]. Using the formula Eq. 3 a value $T \approx 130$ MeV, which is considerably larger than the value extracted in other approaches [16–19, 21].

We will show below that the use of Eq. 1 also leads to a very good fit over 14 orders of magnitude but with a value of $T$ (as used in Eq. 1) which is in line with previous analyses (we will use the same symbols in Eq. 1 and 3, the text will make clear which one is being used). The results are summarized in Table 1 and show that the values obtained for the temperature $T$ and parameter $q$ as defined in Eq. 1 are compatible with those obtained in previous works [16–19, 21].

The fits to the transverse momentum distribution at 7 TeV are shown in Fig. 1. The ratios of the experimental data over the fit values are shown in Fig. 2. An intriguing log-periodic oscillation scillation as a function of the transverse momentum has been observed. This was first noticed and discussed in [22, 23]. It has to be remarked that our value of the temperature $T$ is about half of the value obtained in [11–13] but, of course, the underlying formulas are quite different.

| Experiment | $\sqrt{s}$ (TeV) | $q$         | $T$ (MeV)      | $\chi^2$/NDF |
|------------|-----------------|-------------|---------------|--------------|
| CMS [9]    | 7               | 1.153 ± 0.003 | 74.17 ± 13.5 | 0.51 / 24    |
| ATLAS [7]  | 7               | 1.148 ± 0.005 | 83.25 ± 8.78 | 0.52 / 33    |

Table 1: Values of the $q$ and $T$ parameters and $\chi^2$/NDF obtained using Eq. 1 to fit the data.

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Figure 1: Transverse momentum distributions of charged particles as measured by the CMS [9] and ATLAS [7] collaborations in $p - p$ collisions at $\sqrt{s} = 7$ TeV fitted with Eq. 1.
Figure 2: Ratio of the transverse momenta of charged particles as measured by the CMS [9] and ATLAS [7] collaborations in $p - p$ collisions at $\sqrt{s} = 7$ TeV and the fit values obtained using Eq. 1.
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