ABSTRACT
State-of-the-art mechanisms for oblivious RAM (ORAM) suffer from significant bandwidth overheads (greater than 100x) that impact the throughput and latency of memory accesses. This renders their deployment in high-performance and bandwidth-constrained applications difficult, motivating the design of low-overhead approaches for memory access obfuscation.

In this work, we introduce and formalize the notion of a differentially private ORAM that provides statistical privacy guarantees, and which to the extent of our knowledge, is the first of its kind. The formalization of differentially private ORAM opens up a large design space of low-bandwidth ORAM protocols that can be deployed in bandwidth constrained applications.

We present Root ORAM, a family of practical ORAMs that provide a tunable, multi-dimensional trade-off between the desired bandwidth overhead, outsourcing ratio, and the system security, and that provide rigorous privacy guarantees of differentially private ORAMs. The Root ORAM protocols can be tuned to achieve application-specific bandwidth constraints, enabling practical deployment, at the cost of statistical privacy guarantees quantified under the differential privacy framework and lower outsourcing ratios.

We demonstrate the practicality of Root ORAM using theoretical analysis, simulations, as well as experiments on Amazon EC2. Our theoretical analysis rigorously quantifies the privacy offered by Root ORAM, and provably bounds the information leaked from observing memory access patterns. Our experimental analysis shows the feasibility of these protocols using realistic simulations. The simplest protocol in the Root ORAM family requires a bandwidth of mere 10 blocks, at the cost of a logarithmic overhead compared to conventional ORAMs. This is an order of magnitude improvement over the existing state-of-the-art ORAM protocols.

1. INTRODUCTION
Cloud storage and computing are important tools to outsource data but have given rise to significant privacy concerns due to the non-local nature of data storage. Though encryption goes a long way in assuring data confidentiality, recent work [6,12] has shown that encryption is not sufficient. Encryption does not hide memory access patterns; an untrusted storage server can thus perform traffic analysis of access patterns over encrypted emails [2]. Similarly, Dautrich et al. have shown that access patterns over database tuples can leak ordering information [5].

Oblivious RAM (ORAM), first introduced by Goldreich and Ostrovsky [10,11], is a cryptographic primitive which allows a client to protect its data access pattern from an untrusted server storing the data. Since its introduction, substantial progress has been made in developing novel and efficient ORAM schemes [4,9,16,17,20,22]. Recent work has also shown the promise of using ORAMs as a critical component in developing protocols for cryptographic primitives such as Secure Multi-Party Computation [9].

However, ORAM schemes incur a large overhead in terms of bandwidth that renders them impractical. For example, even the most efficient ORAM protocols [17,21,22] incur a logarithmic overhead compared to conventional RAMs (greater than 100x including constants). This significantly impacts the throughput and latency of memory accesses, and presents a bottleneck for real-world deployment of ORAMs in high-performance and bandwidth constrained applications. The lack of low-bandwidth ORAMs, despite considerable efforts from the security community, is an undeniable indicator for the need of a fundamentally new approach.

Hence, we propose a novel approach for developing practical ORAM protocols that can support even a constant bandwidth overhead compared to conventional RAMs. Our key approach is to provide statistical privacy guarantees and to proposes tunable protocol designs. We first formalize the notion of a differentially private ORAM that provides statistical privacy guarantees, and which to the extent of our knowledge, is the first of its kind. As the name suggests, we use the differential privacy framework developed by Dwork et al. [6] with its $(\epsilon,\delta)$-differential privacy modification [7]. In the current formulation of an ORAM, the output is computationally indistinguishable for any two input sequences.

In a differentially private ORAM, we characterize the effect of statistical privacy guarantees quantified under the differential privacy framework and lower outsourcing ratios.

1 Outsourcing ratio is defined as the amount of data that can be outsourced for a certain amount of local storage. Refer to Sec. 2 for more details.
of a small change in the ORAM input to the change in the probability distribution at the output.

We also present Root ORAM, a family of ORAM schemes that provide tunable ORAM protocols allowing variable bandwidth overheads, system security and outsourcing ratios and including a design point that supports constant bandwidth construction and provide rigorous privacy guarantees of differentially private ORAMs. The low bandwidth protocols, achieved at the cost of statistical privacy and lower outsourcing ratios, are an order of magnitude improvement over previous work in which the protocols still incur a logarithmic bandwidth [9,21,22].

The formalization of a differentially private ORAM opens up a large underlying design space currently not considered by the community. With rigorously quantified privacy guarantees, we propose Root ORAM as the first step in the direction of statistically private ORAMs.

1.1 Our Contributions

Root ORAM introduces a number of paradigm shifts in the design of ORAM protocols while at the same time building on the prevailing ideas of contemporary ORAM constructions. Our main contributions can be summarized as follows:

The notion of a differentially private ORAM: We formalize the notion of a differentially private ORAM, which to the extent of our knowledge is the first of its kind. Formally discussed in Section 3, a differentially private ORAM bounds the information leakage from memory access patterns of an ORAM protocol.

Tunable/parametric protocol family: In bandwidth constraint applications, large bandwidth overhead (>100x) of conventional ORAM schemes is a significant bottleneck. We propose the tunable reduction of bandwidth with outsourcing ratio and a rigorously quantified privacy loss. We propose a new family of ORAM schemes called Root ORAM which can be tailored as per the needs and constraints of the application to achieve desired security and bandwidth. This serves as a key enabler for practical deployment and is discussed in more detail in Section 4.

Security: We analyze and provide theoretical guarantees for the security offered by Root ORAM schemes in the new differentially private ORAM framework. As a consequence of our analysis, we also give a new proof of the Path ORAM protocol security in the new framework. Our approach is general and will be useful for rigorously reasoning about the security of alternative statistically private ORAM schemes in the future. This is presented in Section 5.

Performance: Root ORAM introduces a new design point with constant bandwidth overhead. The simplest protocol of the family has bandwidth usage per access as low as a constant around 10 data blocks compared to 10 · log N blocks in the case of Path ORAM. But this comes at the cost of a rigorously quantified security loss as well as a reduction in outsourcing ratio. At the same time, the server-side storage efficiency can be as high as 1:2 i.e., one dummy block per real block outsourced (compared to Path ORAM which uses around 1:10). We implement Root ORAM and demonstrate its practicality using simulations as well as real world experiments on Amazon EC2. These results are covered in Section 6.

These contributions are order of magnitude improvement over state-of-the-art protocols, though we would like to remind the reader that these come at the cost of a rigorously quantified privacy loss and a reduction in outsourcing ratio. Finally, Root ORAM does not assume any server-side computation and uses low, practical amounts of client-side storage at the same time being extremely simple to implement at both the client and the server side.

2. PRELIMINARIES: STATISTICAL PRIVACY

![Figure 1: A representation of an ORAM. The protocol box translates an input access sequence into an output access sequence.](image)

The significant bandwidth overhead in conventional ORAM schemes despite considerable research efforts necessitates a paradigm shift in our approach to protocol design. To this extent, we formulate the concept of statistical privacy in ORAMs.

Here we present a brief overview of the notion of statistical privacy in the ORAM context. A perfect ORAM roughly leaks no information about the input access sequence. In other words, we can consider an ORAM to be a black-box with an input sequence as X and an output sequence as Y as shown in Fig. 1. An ORAM with perfect privacy would guarantee the independence of X and Y. A slightly stronger condition could be to say that the distribution of the output sequence is uniform over its space for any given input X.

The most natural way to extend the latter condition for designing statistically private ORAMs is to consider ORAM schemes that give non-uniform distributions of the output sequences Y for a given input X and use security metrics that quantify the “non-uniformity” of this distribution. This is graphically illustrated in Fig. 2 where an attacker aims to guess the original access pattern after observing the output access pattern o.

5We say roughly because ORAMs could leak information such as timing of accesses/access pattern size.

6This is stronger because conventional definitions of perfect ORAMs involve outputs being computationally indistinguishable which hides a small detail which we shall see in Sec. 6.
3. DIFFERENTIALLY PRIVATE ORAM

The notion of statistical privacy is certainly not new across security/privacy applications in current literature [1,13]; but it has never been previously explored in the context of ORAMs. We believe formulating such a framework would greatly expand the ability of the research community to develop novel ORAM protocols with low-bandwidth overhead, serving as an enabler for real-world deployment of this technology.

Over the years, a number of papers have been published in the ORAM domain which adopt quite a few different definitions to quantify ORAM bandwidth overhead. We will use the original and straightforward definition of bandwidth as the average number of blocks transferred for one access [4].

**Definition 1.** The bandwidth cost of a storage scheme is given by the average number of blocks transferred in order to read or write a single block.

Formally, an ORAM is defined as a mechanism (possibly randomized) which takes an input access sequence \( y \) as given below,

\[
\overrightarrow{y} = ((\text{op}_1, \text{addr}_1, \text{data}_1), ..., (\text{op}_i, \text{addr}_i, \text{data}_i))
\]

and outputs a resulting output sequence denoted by \( \text{ORAM}(\overrightarrow{y}) \). Here, \( M \) is the length of the access sequence, \( \text{op}_i \) denotes whether the \( i^{th} \) operation is a read or a write, \( \text{addr}_i \) denotes the address for that access, and \( \text{data}_i \) denotes the data (if \( \text{op}_i \) is a write). Denoting by \( |\overrightarrow{y}| \) the length of the access sequence \( \overrightarrow{y} \), the currently accepted security definition for ORAM security can be summarized as follows [22].

**Definition 2. (Currently accepted ORAM Security):** Let \( y \) as given in Eq. [1] denote an input access sequence. Let \( \text{ORAM}(\overrightarrow{y}) \) be the resulting randomized data request sequence of an ORAM algorithm. The ORAM protocol guarantees that for any \( \overrightarrow{y} \) and \( \overrightarrow{y}' \), \( \text{ORAM}(\overrightarrow{y}) \) and \( \text{ORAM}(\overrightarrow{y}') \) are computationally indistinguishable if \( |\overrightarrow{y}| = |\overrightarrow{y}'| \), and also that for any \( \overrightarrow{y} \) the data returned to the client by ORAM is consistent with \( \overrightarrow{y} \) (i.e. the ORAM behaves like a valid RAM) with high probability.

This framework for ORAMs is constructed with complete security at its core [4,9,17,22] and there is no natural way to extend this to incorporate a statistical privacy notion.

### 3.1 Formalizing DP-ORAM

Hence, we introduce and formalize the following statistical notion of an ORAM: *differentially private ORAM*.

**Theorem 1 (Composability of DP-ORAM).** Given two access sequences \( s_1 \) and \( s_2 \) that differ in \( m \) accesses, a \((\epsilon, \delta)\)-differentially private ORAM mechanism guarantees,

\[
Pr[\text{ORAM}(s_1) \in S] \leq e^\epsilon Pr[\text{ORAM}(s_2) \in S] + m\delta
\]

The proof of the theorem directly follows from the composability property of the differential privacy mechanism [15]. In other words, Root ORAM guarantees can be extended to sequences which differ in multiple accesses and hence can be used to give rigorous guarantees for arbitrary access sequences.

4. ROOT ORAM OVERVIEW

In this section, we briefly describe our key design goals and give a high level overview of the Root ORAM protocol. The notation used is given in Table 1.

### 4.1 Design Goals

**Tunable ORAM scheme:** We target a tunable architecture with explicit low bandwidth regimes which can be used to design ORAM protocols for bandwidth constrained applications.

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 注: 有关DP-ORAM和ORAM的更详细信息，请参阅原始论文[1,13]。
process with parameter indistinguishable from normal requests, through a Poisson processes to keep a low stash value. The client machine inde-

recursion technique on the local stash. It also uses fake ac-

slightly more likely to be be the same as the old mapping

choose the distribution; for our purposes, we choose the

form among the leaves. There is tremendous flexibility in

written with the new data and a new mapping is generated.

the data element is mapped onto.

the client looks up the local mapping to find the leaf that

Access: The main invariant (same as Path ORAM) is

that any data block is along the path from the root to the

New Mapping: The relevant data block is then read or

written with the new data and a new mapping is generated.

It is important to note that this new mapping is not uni-

form among the leaves. There is tremendous flexibility in

choosing this distribution; for our purposes, we choose the
distribution to be given by Eq 4 i.e., the new mapping is
slightly more likely to be be the same as the old mapping
than any other random leaf.

Finally, new randomized encryptions are generated and all
the data is written back with elements being pushed down
further in the tree if possible (towards the leaf) and if new
elements can be written back to the tree.

Stash Recursion and Eviction: Root ORAM uses the
recursion technique on the local stash. It also uses fake ac-

cesses to keep a low stash value. The client machine inde-

pendently sends fake access queries to the server, completely
indistinguishable from normal requests, through a Poisson
process with parameter λ. The eviction process and the
recursion together ensure low stash size.

4.3 Comparison with Path ORAM

Root ORAM is inspired by the Path ORAM protocol and

we would like to give the authors all due credit. At the
same time, in this subsection, we would like to highlight the
critical differences between the two papers.

Differentially Private ORAM: Root ORAM introduces
a new rigorous metric to quantify ORAM security, which
extends current formalism to include the notion of a sta-
tistically private ORAM. We rigorously bound the privacy
offered by the Root ORAM (as well as Path ORAM) using
this metric.

Storage structure: Root ORAM uses a partial binary
tree as the storage structure at the server where the height
of the tree is a model parameter k. This is represented
in Fig 3. The parameter k governs the bandwidth of the
protocol. The Path ORAM protocol on the contrary has a
fixed height binary tree (complete binary tree).

Tunability: The ability to tune the protocol as per the
system constraints is a stark difference between Root ORAM
and Path ORAM. There is no way to optimize Path ORAM
when the bandwidth is constrained and statistical security
is acceptable. Root ORAM introduces the novel notion of
non-uniform mapping and gives rigorous statistical privacy
guarantees. Path ORAM’s update mapping scheme then
turns out to be a special case of this mapping.

Simply by tuning the parameters, Root ORAM matches
or exceeds the performance of Path ORAM. We provide the
ability to operate in the low bandwidth regime which Path
ORAM cannot support. The eviction scheme allows Root
ORAM to achieve perfect security (ε = 0) at even lower
bandwidth than the Path ORAM protocol.

Eviction scheme: The eviction schemes of the two pro-
tocols have subtle differences. Path ORAM relies on a suf-
ciently large bucket size to achieve its goals. In contrast,
Root ORAM uses an eviction scheme of fake accesses. Root
ORAM parameters can be tuned to achieve Path ORAM pro-
tocol, but the latter is not the lowest bandwidth full security
(ε = 0) protocol in the Root ORAM family.

Multi-dimensional design space: Root ORAM can be
tuned as per the user’s requirements in terms of the security
desired, the bandwidth available and local storage required.
Thus, Root ORAM offers attractive design points that can
support a wide range of operating conditions just by tuning
its parameters.

5. ROOT ORAM DETAILS

In this section, we provide the details of Root ORAM. We
begin by describing the basics of the protocol. The required
notation is tabulated in Table 1.

| Symbol | Description |
|--------|-------------|
| N = 2^k | Number of real data blocks outsourced |
| k ≥ 1 | Model parameter (to tune bandwidth) |
| p ∈ (0, 1 − 1/N] | Model parameter (to tune security) |
| Z | Number of blocks in each bucket |
| B | Size of each block (in bits) |
| P(x) | Path from leaf x to the root |
| P(x, i) | Node at level i in P(x) |
| x := position[a] | Data block a is currently mapped to leaf x i.e. a resides in some bucket in P(x) |

Table 1: Notation for Root ORAM

Secure framework: We target protocols that provide
rigorous privacy guarantees viz. that of differentially private
ORAMs formalized in Section 3.

Low Storage and Computation: The design should
use as low storage as possible both on the client as well
as the server side. Server side computation is not always
practical and hence we would like to avoid assuming any
such capability.

4.2 Approach Overview

Root ORAM protocol can be split into three components,
the access, the new mapping and the eviction. These are
briefly described below. As Path ORAM is an instantiation
of Root ORAM, the protocols are very similar in their
structure but have important subtle differences.

Since we demonstrate the feasibility of having a tunable
ORAM architecture, we just need to show the existence of
such a trade-off. In particular, we do not claim this con-
struction to be the optimal way of achieving trade-offs. We
choose the server-side storage to be stored in a partial bi-
ar tree where each node is a bucket which can hold up to
Z data blocks. A stash at the client is used to store a small
amount of data. Data elements are mapped to leaves and
this mapping is stored locally.

Access: The main invariant (same as Path ORAM) is
that any data block is along the path from the root to the
leaf it is mapped or is in the stash. To access a data element,
the client looks up the local mapping to find the leaf that
the data element is mapped onto.

New Mapping: The relevant data block is then read or
written with the new data and a new mapping is generated.
It is important to note that this new mapping is not uni-
form among the leaves. There is tremendous flexibility in
choosing this distribution; for our purposes, we choose the
distribution to be given by Eq 4 i.e., the new mapping is
slightly more likely to be be the same as the old mapping
than any other random leaf.

Finally, new randomized encryptions are generated and all
the data is written back with elements being pushed down
further in the tree if possible (towards the leaf) and if new
elements can be written back to the tree.

For ε = 0, Path ORAM uses a bandwidth of ~ 10 log N
data blocks per access whereas Root ORAM can perform
the same with around ~ 8 log N for the following choice
of parameters: Z = 2, λ = 1, p = 1 − 2^−k.

8A bucket contains multiple blocks of data storage which
can be either real or dummy.
Figure 3: **Root ORAM server storage**: The figure illustrates the server side storage. The level 0 to \( k - 1 \) form a binary tree and the last level of the tree contains \( N = 2^L \) leaves evenly distributed over the binary tree leaves.

**Bucket structure**: Each node is a bucket consisting of \( Z \) blocks, each block can either be real or dummy (encryptions of 0).

**Path structure**: The leaves are numbered in the set \( \{0, 1, ..., 2^L - 1\} \). \( P(x) \) denotes the path (set of buckets along the way) from leaf \( x \) to the root and \( P(x, i) \) denotes the bucket in \( P(x) \) at level \( i \). It is important to emphasize here that the path length in Root ORAM is \( (k + 1) \) blocks compared to the \( (\log N) + 1 \) in Path ORAM.

**Dummy blocks and randomized encryption**: We use the standard padding technique (fill buckets with dummy blocks when needed) along with randomized encryption to ensure indistinguishability of real and dummy blocks.

### 5.2 Invariants of the scheme

**Main Invariant (same as Path ORAM)**: The main invariant in Root ORAM is that each real data block \( a \) is mapped to a leaf \( x := \text{position}[a], x \in \{0, 1, 2, ..., 2^L - 1\} \) and at any point in the execution of the ORAM, the real block will be somewhere in a bucket \( \in P(x) \) or in the local Stash.

**Secondary Invariant**: We maintain the secondary invariant that after each access to an element, its new mapping is governed by a constant but non-uniform distribution \( D \). There is tremendous flexibility in choosing this distribution; for our purposes, we consider the distribution \( D \) given by the following equation and shown graphically in Fig. 4.

\[
P_{z,x} = p_2 + (p_1 - p_2)\delta_{zx}
\]

Where \( P_{z,x} \) is the probability that an element accessed from leaf \( x \) is mapped to a leaf \( z \), \( \delta_{ij} \) is the Kronecker\(^{11}\) delta, \( p_1 = (1 - p) \) and \( p_2 = p/(N-1) \) where \( p \) is the model parameter as defined in Table. 1.

### 5.3 Client Storage

**Position Map**: The client side stores a position map which maps real data blocks to leaves of the server tree.

\(^{10}\)It is important to note that the invariant does not say that the position each data block is uniform over the set of leaves, as shall be clarified by the second invariant.

\(^{11}\)Kronecker delta is defined as

\[
\delta_{ij} = \begin{cases} 
0 & \text{if } i \neq j \\
1 & \text{if } i = j 
\end{cases}
\]

### 5.4 Main idea

The main idea of the protocol is very simple, we read data along a path, try to write data back to the same path (with some modifications and new encryptions) and if there is insufficient storage, we retain those overflown data elements back in the local Stash.

Along with this, there is an independent access process of fake accesses\(^{12}\). These accesses are made by the user to the server and are indistinguishable from real accesses. Fake accesses are drawn from a Poisson process with a parameter \( \lambda \). It is important to note that in Root ORAM, the real

\(^{12}\)This is the similar to the eviction scheme described in \[19\] with the crucial difference that our fake accesses are completely indistinguishable from real accesses.
access by the user and the fake accesses by the client machine are exactly the same and hence are indistinguishable from the server’s perspective.

5.5 Details of the protocol

An access is defined as a 3-tuple

\[ \text{Access}_i = (\text{data}_i, \text{element}_i, \text{operation}_i) \]

For a real access, given a particular access 3-tuple, the user finds the mapping of the data block needed using his local position map. He then requests the whole path of that leaf from the server tree. After processing the data and generating new randomized encryptions, the user writes the data back to the tree with the element that was accessed at a new location along the path. But the key idea here is that the element that was accessed has a non-uniform distribution of it being mapped to other leaves. It is more likely to be mapped to the same leaf than to others and the probabilities involved are decided by the security parameter \( p \).

The broader picture of the protocol is as follows. The client systems makes real as well as fake accesses to the server. The real access is as described in the previous paragraph. There is a parameter \( \lambda \) which controls the amount of fake accesses. One way of implementing the protocol is in the following way\(^\text{[13]}\)

**normal_access(a):** A normal access consists of the following functions in order: read(a), push_down(position[a]), update_mapping(a) and finally a write(a).

\[
\text{Access}(\text{op}, a, \text{data}^*) = \begin{cases} 
\text{while ORAM is under use do} & \\
\alpha \leftarrow \text{Poisson}(\lambda) & \\
\text{for } i = 1 : \alpha \text{ do} & \\
\text{normal_access(a)} & \\
\text{end for} & \\
\text{fake_access()} & \\
\text{end while} & \\
\end{cases}
\]

**read(a):** The reading phase is the same as that in Path ORAM; Using the local client side mapping, the client finds out the leaf to which the data element a is currently mapped i.e find \( x \) such that \( x = \text{position}[a] \). We then request all the data blocks in the buckets along path \( P(x) \). The invariant ensures that the client can retrieve its data element, from these. This completes the reading phase of the protocol.

```
read(a):
1: x ← position[a]
2: for i ∈ {0, 1, ..., k} do
3: S ← S ∪ ReadBucket(P(x, i))
4: end for
```

**update_mapping(a):** After reading a data block, we modify its mapping using the distribution mentioned in Eq\([3]\) i.e update_mapping keeps the mapping same with probability \((1 - p)\) and with the remaining probability changes it to a uniformly random leaf among the remaining leaves.

\[^{13}\]The distribution becomes uniform if \( p = 1 - 2^{-L} = 1 - 1/N \).

\[^{14}\]It should be noted that the code has been structured in the following way for clarity of understanding and hence can be optimized in a number of ways.

**push_down(position[a]):** When any path is accessed, this function tries to place any data blocks along the path \( P(\text{position}[a]) \) or in the Stash to lower positions on the same path if possible.

```
push_down(position[a]) = \begin{cases} 
1: x ← position[a] & \\
2: \text{if } \text{Bernoulli}(p) = 0 \text{ then} & \\
3: \text{return } x & \\
4: \text{else} & \\
5: \text{return } \text{UniformRandom}(\{0, 1, 2, \ldots 2^L - 1\} \setminus \{x\}) & \\
6: \text{end if} & \\
\end{cases}
```

**write(a):** Once the mapping is updated, say initially \( x = \text{position}[a] \) and after updating the mapping \( z = \text{position}[a] \), we try to write the data block back into the bucket which is the lowest intersection of the two paths in consideration i.e. lowest bucket in \( P(x) \cap P(z) \) (with the convention that bucket with the highest level number is the root at level 0) which has an empty/dummy block.

**fake_access():** A fake access is issued to push back elements from the stash to the tree. One data block, say \( a' \), is chosen at random from the stash\(^\text{[15]}\) and a normal access is performed on \( a' \), i.e., \( \text{read}(a') \) followed by \( \text{push_down}(\text{position}[a']) \) followed by an \( \text{update_mapping}(a') \) followed by a \( \text{write}(a') \).

6. THEORETICAL EVALUATION

In this section, we shall state our main theorems, their proofs and a few interesting special cases.

**Theorem 2.** (Main theorem). Given a stash size \( C \), the Root ORAM protocol with parameters \( k, p, Z \) and \( \lambda \) is \((\epsilon, \delta)\)-differentially private for \( \epsilon = 2 \log \left( \frac{(N-1)p}{p \Delta} \right) \)

and \( \delta = (1-p)^{M_k} \) where \( M_k = (C + Z(k+1) + 1) \)

**Proof:** First, we remark that due to the conservative security analysis (Refer Sec. 6.1), \( \lambda \) does not appear in the above expressions. The theorem has two parts, the \( \epsilon \) bound and the \( \delta \) bound. Firstly, we give a brief insight into the two security parameters \( \epsilon \) and \( \delta \). The proof is then structured as follows:

**The \( \epsilon \) bound:**
- Set up the differential privacy framework for ORAM.
- Then we set up the probability evaluation model to find the probability of that a particular real sequence leads to a particular output sequence by the ORAM.
- Then we compute the maximum change that one access in the input sequence can have on the probability of the output sequence (over all output sequences).
- Finally we complete the \( \epsilon \) bound.

**The \( \delta \) bound:**
- We first show the need for \( \delta \) in the security.
- We then conservatively evaluate a bound on \( \delta \).

\( \epsilon, \delta \) interpretation: Given an ORAM scheme with an unbounded amount of local stash, we show that such a scheme is \( \epsilon \)-differentially private. But the moment we introduce a\(^\text{[15]}\) the security analysis takes care of the correlation of the fake access with stash elements by a worst case analysis.
Table 2: Additional notation for security analysis

| Symbol | Description |
|--------|-------------|
| $M$    | Access pattern size |
| $C$    | Stash size |
| $p_1$  | $(1 - p)$ |
| $p_2$  | $p/(N - 1)$ |
| $M_k$  | $M_k = Z(k + 1) + C$ |

The $\epsilon$ bound

6.1 Framework set-up

The additional notation used is specified in Table 2. We conservatively assume that the adversary knows the real sequences (in other words we assume $\lambda = \infty$). Essentially, among the $M$ access of which some are real and some are fake, we conservatively assume that these can be distinguished. In practice, the security offered by our approach is higher since the untrusted server storage cannot differentiate fake accesses from real accesses in practice.

More formally, let $f_i$ denote the set of fake accesses and $r_i$ denotes the real set of accesses made by the ORAM. We denote by $R_i$, the complete set of accesses made ($r_i$ along with $f_i$). Thus we have that:

$$\max_{|r_1 - r_2| = 1} \frac{\Pr[\text{ORAM}(r_1) = o]}{\Pr[\text{ORAM}(r_2) = o]} \leq \max_{|R_1 - R_2| = 1} \frac{\Pr[\text{ORAM}(R_1) = o]}{\Pr[\text{ORAM}(R_2) = o]}$$

(5)

where $\text{ORAM}(R_i)$ denotes the ORAM protocol output on sequence $R_i$ without any additional fake accesses. But to prove the bounds of differential privacy in the theorem, we need to bound the following term:

$$\max_{|r_1 - r_2| = 1} \frac{\Pr[\text{ORAM}(r_1) = o]}{\Pr[\text{ORAM}(r_2) = o]} \leq \epsilon^\delta$$

Hence, it suffices to bound the latter term in Eq. 5 by $\epsilon^\delta$.

6.2 Probability model

Next, we evaluate the ratio of the probabilities by invoking the secondary invariant. Recall that our secondary invariant is: after each access(real/fake) for an element, the position map of that element (and none other) changes randomly according to the distribution $D$ given by Eq. 4.

With this invariant, we can compute the probability of a particular real sequence $R$ leading to a particular observed sequence $o$. For our computation, we write the real sequence (including fake accesses) below the observed sequence and calculate the probabilities according to the following rules:

- The first time a data block is accessed, its location is random. Hence, we write a $1/N$ below this access.
- When an element that was accessed before is accessed, we write a $p_1$ or $p_2$ in the probability calculation depending on whether the observed locations were same or different respectively.
- A background check is maintained, if at any time there are more than $(k + 1) \times Z + C$ data blocks mapped to the same location, the probability becomes 0, where the symbols can be found from the notations in Table 1.
- Finally, we multiply all the written probabilities to get the probability $\Pr[\text{ORAM}(R) = o]$.

This is demonstrated in Table 3.

Table 3: An example of writing probabilities given the real and observed access patterns $r$ and $o$. Different symbols are used for real and observed access patterns merely for the clarity of the demonstration. $p_1$ and $p_2$ are as defined in Eq. 4 and Table 2.

| Observed seq. | a b a c a a b d |
|---------------|-----------------|
| Real seq.     | x y z y y y y z x |
| Probabilities | $\frac{1}{N}$  $\frac{1}{N}$ $p_1$ $\frac{1}{N}$ $p_2$ $p_1$ $p_2$ $p_2$ |

Table 4: Only the blue symbols affect the probability that will be written under the data element shown by an enclosing box. The red elements show the previous and next access of the boxed data element.

| Observed seq. | a b a c a a b d |
|---------------|-----------------|
| Real seq.     | x y z y y y y z x |
| Probabilities | $\frac{1}{N}$  $\frac{1}{N}$ $p_1$ $\frac{1}{N}$ $p_2$ $p_1$ $p_2$ $p_2$ |

6.3 Maximum change

Next, we find the maximum change in the probabilities that can occur as a result of changing one access.

First we note that in the probability model, the probability written under each data element depends only its current observed location and its previous observed location and nothing else (and is governed by the distribution $D$ given by Eq. 4). Hence, if one data access is changed, the maximum change that can occur in the probability is at most in two places viz, the location which was modified and the next accessed location of that data element. With this, we can enumerate all the possible cases that can occur and find the maximum change in probabilities. To do this efficiently, we develop some more notation.

Let the accessed data element be changed from a to b. Let the previous location of access of data element a be $l_{pa}$ (leaf $pa$) and the next location be $l_{na}$. Similarly, the previous location of access of b is $l_{pb}$ and the next location as $l_{nb}$. If any of these 4 do not exist i.e the symbol was never accessed before or was never accessed afterwards, we define that leaf to be 0 for simplification of the equations. Let $l$ be the location of the access in consideration i.e the location of data access which was changed in $r_1$ and $r_2$. Note that in the DP-ORAM calculations, we have the same observed sequence for both the sequences $r_1$ and $r_2$, the location of access $l$ is the same in both the sequences. This is shown in the Fig. 5.

10 In other words, if data element a was never accessed after the location of access change, then $l_{na} = 0$. 
Now, the probabilities can differ in at most 3 places viz., the largest and the smallest probabilities. The same proof goes through for other probability distributions leading to an ε-differential privacy framework for ORAMs.

Let $M_k$ denote the number $(C + Z(k + 1) + 1)$. It is easy to see that there is a sudden jump in the probability from 0 to a non-zero value when the real access is changed at one location when we look at any such sequence. In particular we choose the following two sequences:

$$r_1 = (1, 2, 3, ..., M_k)$$
$$r_2 = (1, 2, 3, ..., M_k - 1, 1)$$

If $Pr[\text{ORAM}(r_i) = o] > 0$ for $i = 1, 2$, then we have already shown the ε bound and hence $\delta = 0$. So it remains to find the smallest probabilities. The same proof goes through for other probability distributions leading to $\epsilon = 2 \log \left( \frac{p_{\max}}{p_{\min}} \right)$. 

For that matter so can any sequence $a, a, ..., a$ for any $a \in \{1, 2, 3, ..., N\}$

The δ bound

$$\delta = \frac{1}{\log N}$$

Figure 5: The sequences $r_1$ and $r_2$ differ by one element (boxed). The previous accessed location and the next accessed location are as shown, dots are irrelevant accesses (not a or b). The observed sequence o is the same for both (DP-ORAM condition).

6.4 The need for δ

In this subsection, we show the need for δ in quantifying the security. We use the Path ORAM paper to demonstrate this shortcoming. We assume that the Stash size is bounded by $C$ and let $M_L$ denote $Z \log N + C + 1$. For demonstration purpose, we construct a minimal working example. Let:

$$\gamma = ((r, 1, \cdot), (r, 1, \cdot), ..., (r, l, \cdot))$$
$$\gamma' = ((r, 1, \cdot), (r, 2, \cdot), ..., (r, M_L, \cdot))$$

where $r$ denotes the read operation and $\cdot$ denotes data which is not important for the demonstration. In words, one access sequence consists of $M_L$ accesses to the same element and the second access sequence consists of $M_L$ different accesses to elements 1, 2, ..., $M_L$.

Now, of all the possible sequences ORAM($\gamma$) can produce, we can see that the sequence 1, 1, ..., 1 can be one of them. But, its not hard to see that the same sequence 1, 1, ..., 1 can never occur as ORAM($\gamma'$). The reason for this is simply because we cannot ever map more than $M_L$ elements to the same path (else the Path ORAM invariant is broken i.e. stash overflows) and hence the $M_L$ accesses to the same location cannot all be different elements.

We demonstrate this as an attack on the Path ORAM protocol. Consider a situation where a program is using the Path ORAM protocol to hide its access pattern and we know that the program has the following traits,

$$\text{Access Pattern} = \begin{cases} 1, 1, ..., 1 & \text{if Secret = 1} \\ 1, 2, 3, ..., M & \text{if Secret = 0} \end{cases}$$

If y is the real access pattern, if ever we see a sequence of $M_L$ or more access made to the same location in ORAM($\gamma$), we can immediately infer that Secret = 1.
the maximum $\delta$ when one of these terms is 0. WLOG, $Pr[\text{ORAM}(r_1) = 0] = 0$. Hence $\delta$ is the maximum value of $Pr[\text{ORAM}(r_2) = 0]$. Now, one simple upper bound on $\delta$ can be found by noting the following: Since the probabilities used to compute for each access are at most $p_1$ (they are either $p_1, p_2$ or $1/N$ and $p_1$ is the largest), we can get a quick upper bound on $\delta$ as

$$\delta \leq p_1^{M_k} = (1 - p)^{M_k}$$  \hspace{1cm} (8)

Where $M_k = (C + Z(k + 1) + 1)$. This completes the $\delta$ bound.

### 6.6 Bandwidth

**Theorem 3.** The bandwidth of the Root ORAM protocol with parameters $k, p, Z$ and $\lambda$ is $2 \times Z(k+1) \times (1+1/\lambda)$ per real access.

**Proof:** The number of blocks in any path of the tree is equal to $Z(k+1)$ and hence twice the number of blocks are transferred per read and write. Also, they way the parameter $\lambda$ is set (i.e. the way the fake accesses are programmed), we perform on an average $\lambda$ real accesses per fake access. (the average of a Poisson process with parameter $\lambda$). Hence, the bandwidth gets an addition factor of $(1+1/\lambda)$ per real access.

### 7. SYSTEMS EVALUATION

In the previous sections, we have established the design space made possible by formalizing DP-ORAM using theoretical analysis. We also saw Root ORAM as a way to achieve some interesting points in the design space. In this section, we analyze the stash size usage, in other words, the outsourcing ratio. We define **outsourced ratio** as the ratio of the total data outsourced to the amount of local storage required.

We resort to simulations to demonstrate the stash size required. We simulate Root ORAM for various values of the parameter to understand the impact of design parameters on the stash size. The parameter choices have been tabulated in Table 5. We begin by giving the details of our implementations.

### 7.1 Details of the implementation

We implemented the complete functionality of Root ORAM in C++. We plan to make our implementation publicly available as an open source software. We performed all experiments on a 1.4 GHZ Intel processor. The Amazon EC2 experiments were performed using a TCP connection for reliable data downloads.

### 7.2 Evaluation results

**Max stash usage vs $N$:** In light of the recent paper by Bindschaedler et al. [2], we base our experimental evaluation by giving due importance to the constants involved. Fig. 6 shows the dependence of the maximum stash used on $N$, the number of outsourced blocks. Different lines correspond to different values of $\lambda$. As can be seen, the effect of $\lambda$ goes down for relatively large values of $N$.

To put numbers in perspective, the worst case stash size for standard 4 KB blocks for 10 GB of data outsourced is roughly 40 MB (using a 4 KB block size).

| Symbol | Description |
|--------|-------------|
| $p$    | $1 - 2^{-i}$, $i = \{1..L\}$ |
| $L$    | From 10 to 21 |
| $k$    | Runs from 1 to $L$ |
| $Z$    | $Z \in \{2,3,4,5\}$ |
| $\lambda$ | $\in \{0.25,0.5,0.75,1,2,\infty\}$ |

Table 5: Simulations limits

Figure 6: This figure illustrates the maximum stash usage as a function of $N$. Different lines correspond to different values of $\lambda$. To put this in perspective, the maximum stash usage for 10 GB of outsourced data is roughly 40 MB (using a 4 KB block size).

We use random access patterns for the simulations and the maximum stash size is calculated excluding the transient storage for one path. Unlike current work, we independently study the effect of increasing the number of accesses ($M$) on the max stash size. This is equivalent to giving bounds on the failure probability of the stash. Next we briefly describe the aims of our evaluations before showing its results.

We study the effect of various parameters on the performance. In particular, we study the effect of $N$ on the maximum stash size used (which in other words, captures the failure probability of the ORAM), the dependence of the stash size (outsourcing ratio) on bandwidth and security ($\epsilon$), the growth of the maximum stash size with the number of accesses and finally the latency of Root ORAM protocols for access over remote Amazon EC2 servers.

**Outsourcing ratio vs Bandwidth, $\epsilon$:** Fig. 7a shows the outsourcing ratio as a function of the bandwidth and security ($\epsilon$) for different values of $Z$ ($Z$ increases from 2 (lowest plane) to 5 (top plane)). When low bandwidth protocols are used (small $k$), the outsourcing ratio is relatively small. Specifically, in such regimes, for small values of $Z$, we have an outsourcing ratio of about 10X, whereas for larger $Z$ values, the ratio is almost 1000X. This enables a smartphone client to outsource about 1 TB of data to the untrusted cloud server with less than a 1 GB of local storage.

20 Given the exponentially decreasing small nature of $\delta$ we do not improve on this weak bound.

21 Locations hidden for author anonymity.
Fig. 7: Fig. 7a illustrates the outsourcing ratio as a function of \( k \) and \( p \). Different surfaces correspond to \( Z = 2 \) to \( Z = 5 \) from bottom to top respectively. The y-axis is related to the security parameter \( p \) by the equation \( p = 1 - 2^{-i} \). Fig. 7b shows for a fixed value of \( \epsilon \), the trade-off between outsourcing ratio and bandwidth (taking recursion into consideration).

required and extremely low bandwidth requirements. With higher bandwidths, much better outsourcing ratios can be achieved and this can be seen in Fig. 7.

Another interesting feature of Root ORAM is the extremely high outsourcing ratios achieved by models with almost complete tree structures i.e., \( k \approx 13 \) and 20. Though Root ORAM does not have theoretical bounds on the stash usage like other protocols such as Path ORAM or Ring ORAM, it can be seen clearly how these aspects tie together as Path ORAM is a instantiation of Root ORAM in high bandwidth, \( \epsilon = 0 \) regime, consequently having high outsourcing ratio. Similarly, using appropriate values of \( p \), we can achieve \( \epsilon = 0.001, 0.01 \) etc. for a given bandwidth with outsourcing ratios as seen in Fig. 7a.

**Real-world implementation:** Next, we discuss the results of our implementation of Root ORAM over Amazon EC2 servers. Our aim was to compute the latency overhead of a memory accesses as a function of Root ORAM parameters. Fig. 8a depicts latency as a function of the bandwidth for \( Z = 5 \) while Fig. 8b depicts the latency as a function of the constrained client bandwidth across different values of \( k \). We used the *trickle* application to constrain the bandwidth at client machines to desired values. We notice an order of magnitude difference between latencies at low and high values the bandwidth i.e., low and high values of \( k \).

### 7.3 Recursive stash reduction

Recursively storing the stash in an ORAM improves the performance exponentially. Suppose that we use Root ORAM with a specific set of parameters \( k, p, Z \) and \( \lambda \) and achieves a worst case outsourcing ratio of \( R \). The \( \epsilon, \delta \) values achieved are given by Theorem 2, bandwidth by Theorem 3 and the outsourcing ratio by the above simulations.

We can see that using \( t \) rounds of recursion, the outsourcing ratio grows exponentially to \( R^t \), whereas the security and the bandwidth increase only linearly viz., \( (\epsilon, \delta) \to (t\epsilon, t\delta) \) and bandwidth \( \to t \times \) bandwidth. This introduces further interesting design points, some of which are shown in Fig. 7b.

### 7.4 Perfect Security Design Points

Root ORAM offers a number of interesting design points even with perfect security. For \( \epsilon = 0 \), Root ORAM enables a bandwidth outsourcing ratio trade-off as can be seen in Fig. 7b. For small values of \( k \), Fig. 7b shows that we can design protocols with low bandwidth at the cost of low outsourcing ratio.

Similarly, for large values of \( k \), we can achieve extremely high outsourcing ratios at lower bandwidths by using smaller \( k \) and utilizing fake accesses. For ex: \( k \approx 15, Z = 2, \lambda = 4 \) \( \Rightarrow 75X \) overhead, a factor of 3X improvement over Path ORAM and comparable to overhead of Ring ORAM. Similarly, Root ORAM can outperform both Path ORAM and Ring ORAM at the cost of a smaller outsourcing ratio as can be seen in Fig. 7b.

### 7.5 How to choose parameters?

Given the various parameters, we describe how Root ORAM can be enabled for any particular application. As in any other differential privacy application, we set a privacy budget (an upper bound: \( \epsilon_{\text{budget}} \)) for the system. Then we allow \( \epsilon \)-DP queries till the budget is exhausted i.e., cumulative \( \epsilon \) of the queries by the composition theorem reaches \( \epsilon_{\text{budget}} \). Then we can choose two of the following three parameters independently: desired security value \( \epsilon \), the available bandwidth and the outsourcing ratio \( R \). The third parameter is determined by the choice of the other two and the optimal choice would be determined by the application requirements.

### 7.6 Summary

We have shown the practicality of Root ORAM through theoretical analysis, simulations and real-world experiments. Theoretically, we have analyzed the bandwidth and security for different parameter values. Experimentally, we have shown the dependence of the outsourcing ratio and access latency on Root ORAM parameters; demonstrating the fea-
Figure 8: Real-world implementations over Amazon EC2. Fig. 8a show the latency as a function the bandwidth for $N = 2^{20}$ and three block sizes viz., 1 KB, 4 KB and 16 KB. Fig. 8b shows the latency as a function of the constrained/limited application bandwidth for 4 KB block sizes. There is a significant difference between the latencies for across $k$ values for constrained bandwidth applications.

sibility of multi-dimensional design space and order of magnitude performance improvement.

8. RELATED WORK

Since the formalization of the concept of an Oblivious RAM, in a seminal paper by Goldreich and Ostrovsky [11], the research community has made substantial progress in making ORAM practical by improving their performance [4, 9, 16, 17, 20–22]. Recent work has also shown the promise of using ORAMs as a critical component in developing protocols for Secure Multi-Party Computation [9].

A recent benchmark for ORAMs has been the Path ORAM protocol [22]. It builds upon previous hierarchical constructions such as [21] and gives theoretical bounds on stash usage. Root ORAM generalizes the construction of Path ORAM to provide a tunable framework offering DP-ORAM guarantees. Root ORAM also reintroduces the eviction scheme of dummy/fake accesses. This was first looked into by Shi et al. [19] and formalized in Ren et al. [18]. The latter also highlights the potential pitfalls in proposing eviction schemes that are not provably secure while demonstrating the security consequences of one such scheme. Root ORAM avoids that by using dummy accesses indistinguishable from real accesses while also giving rigorous bounds on the security.

Another novel concept that was recently introduced in the ORAM domain was that of the XOR technique to reduce online bandwidth. Online bandwidth, first formalized by Boneh et al. in [3] was reduced to $O(1)$ using the XOR technique by Dautrich et al. [4]. The XOR technique can be extended to Root ORAM as well, which will further influence the protocol design space.

Two optimizations for [19] were provided by Gentry et al. [9]. Concretely, they show the benefits of using a tree structure with multiple leaves instead of 2 as in the case of a binary tree. This idea is in similar spirit as that of Root ORAM though these differ considerably in terms of their working. The higher-degree tree in [9] is a complete higher-degree tree (i.e degree of each node is the same) where as in the Root ORAM paper, the tree is binary till the last level and only the last level nodes have a higher degree. This leads to very different dynamics of the two schemes. Similarly, Root ORAM uses fake accesses as its eviction process, different from [9].

ORAM has been implemented and shown to be feasible at a chip level in prototypes such as the Ascend architecture [8] and the Phantom architecture [14]. But unlike the case of chip-level implementations where trusted local cache is expensive, most other applications have more client space. In fact, in today’s settings, it is feasible to have client storage of the order of 1 GB for outsourced data of about 1 TB [2].

In short, Root ORAM is the only protocol with tunable security-bandwidth-outsourcing ratio construction. Similarly, none of the previous works deal with statistical privacy in the ORAM context, hence the formalization of differentially private ORAMs is an important contribution of this work.

9. LIMITATIONS AND FUTURE WORK

To enable the design of practical ORAM schemes for applications with stringent bandwidth constrain, it is desirable to have statistical privacy and Root ORAM demonstrates the first step in this direction by introducing a tunable framework that provides rigorous differential privacy guarantees. This opens up a number of research ideas which remain unexplored in the current work.

First, we would like to explore the integration of techniques that leverage server-side computation in the Root ORAM architecture, such as the XOR technique [9]. Such an approach can trade-off bandwidth at the cost of server-side computation and can further influence the design space of differentially private ORAMs. Second, we would like to explore the effect of varying the ratio of number of blocks outsourced to the size of the server-side storage. Gentry et al. [9] have explored similar techniques in the case of Path ORAM, and it would be interesting to combine these techniques with Root ORAM.

Our experimental results have demonstrated the required stash size for various parameters of Root ORAM. However, we acknowledge that Root ORAM lacks rigorous theoretical guarantees on stash usage. In future work, it would be interesting to rigorously bound the stash usage of Root ORAM.
10. CONCLUSIONS

To summarize, we present Root ORAM, a tunable family of ORAM protocols which provide a multi-dimensional trade-off between bandwidth (performance), security and local storage requirements. We introduce and formalize the notion of a differentially private ORAM, which to our knowledge is the first of its kind.

We evaluate the protocol using theoretical analysis, simulations, and real world implementation on Amazon EC2. We theoretically prove that Root ORAM provides the rigorous privacy guarantees of differential privacy. We experimentally demonstrate that the stash size used by Root ORAM is bounded for realistic values of parameters. Overall, Root ORAM can serve as an enabler for real-world deployment of oblivious RAM by providing novel design points that provide an order of magnitude performance improvement over current state-of-the-art.

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