Suppressing the lower Multiplpes in the CMB Anisotropies

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I. INTRODUCTION

The most appealing cosmological scenario emerged in the mid 1980s as the flat cold dark matter model with a nearly scale free initial power spectrum, characterized by a gravitational potential $k^3\Phi(k) = A_s k^{n_s-1}$, with $n_s \approx 1$. Besides the usual ingredients like baryon content $\Omega_B$ and a present expansion rate $H_0$, the model has only a single free parameter, namely the amplitude of initial scalar perturbations $A_s$. However, the wealth of observational results over the past decade has forced the inclusion of an extra parameter in the form of an effective cosmological constant $\Omega_\Lambda$.

Inflation plays a crucial role in explaining the flatness and homogeneity required by the observations, as well as providing a theory of the primordial spectrum of cosmological perturbations \cite{1,2,3,4,5,6,7,8,9,10,11}. The simplest single field models of inflation \cite{12,13,14} predict a flat universe with primordial perturbations which slightly deviate from exact scale invariance, $n_s \approx 0.95 \pm 0.03$ or so \cite{15}, while more elaborate models with more parameters can go beyond this interval. For instance in the simplest models of hybrid inflation with two fields one can have $n_s > 1$ \cite{16}. A long period of inflation (with more than 65 e-folds of expansion) generates a huge patch which is much larger than the presently observable universe.

CMB experiments manifest a striking agreement with this standard cosmological model \cite{17}. However, an interesting result of the WMAP data, which confirms earlier COBE DMR observations, is a lower amount of power on the largest scales when compared to that predicted by the standard $\Lambda$CDM models \cite{1,2,3}. Accurate Monte Carlo simulations of the WMAP observations indicate that only 0.7% of the realizations of the models studied \cite{18} have less power than the observed quadrupole. This number, which approximates the probability of observing a smaller quadrupole than the one observed by WMAP in a realization of standard $\Lambda$CDM model, should not be compared to the posterior probability for the theoretical quadrupole to be equal or higher to the one predicted by standard $\Lambda$CDM model, given the small value measured by WMAP. The latter is about 5% for $\Lambda$CDM best fit quadrupoles (as can be easily estimated using a $\chi^2$ distribution for the quadrupole), and the difference between the two has been the source of some confusion in the literature \cite{19}.

The small posterior probability for $\Lambda$CDM model may still be taken as a hint that some other model may provide a better fit to WMAP data. One could dismiss this result by attributing it to cosmic variance. We may simply live in a patch of the universe where the low $\ell$ multi...
tipleles happens to be small, for no special reason. One may even argue that with many independent measurements (many bins) it would be very surprising if the results of at least one of these measurements would not deviate strongly from theoretical expectations. Indeed, WMAP shows several such peculiarities at various values of $\ell$. Therefore it would be premature to interpret the smallness of the low multipoles as a serious problem of the standard model, particularly in light of its great success in explaining and predicting major features of our universe in great detail. Indeed this is the attitude adopted by Ref. [3] who mention the “intriguing” smallness of the low multipoles, but overall interpret their results as providing a strong confirmation of the standard inflationary paradigm. However, the deficit in power observed first by COBE DMR and now by WMAP is an interesting result, paradigm. However, the deficit in power observed first by COBE DMR and now by WMAP is an interesting result, and it is tempting to look for possible explanations for such an effect. In this paper we discuss some candidates for the mechanism by which the low $\ell$ multipoles may be suppressed. In other words, we shall investigate possible modifications of the standard model which can give much better probability to have small power in low $\ell$ multipoles.

We distinguish between two possibilities in suppressing the low $\ell$ amplitudes. One is related to the physics of the inflationary phase in the early universe. Theoretically we have significant freedom in the design of inflationary potentials and, consequently, in the shape of the primordial power spectra. As we will show, one can also use the freedom in choosing initial conditions at the onset of inflation. This also provides a mechanism to affect the primordial power spectra. Both effects can be related through the existence of a stage when the velocity of the scalar field is not negligible, either during or prior to the observed $65$-e-fold stage of inflation, and hence the perturbations cannot be described using the usual slow-roll parameter approximation. The drawback of this approach is clear; it is simply a tuning of parameters to obtain a feature close to the present horizon scale. A similar problem is encountered in other attempts.

As a second option, the suppression could be due to late universe physics under the influence of an effective cosmological constant. This is probably the most interesting possibility, because it relates the suppression of CMB anisotropy power at horizon scales $\sim H^{-1}$ to the smallness of the cosmological constant, which becomes dominant precisely at times $\sim H^{-1}$. The realization of such a mechanism would then link two seemingly unrelated coincidence problems.

This paper is organized as follows. In Sections II and III we comment on late and early universe physics effects as a cause for the suppression. In Section IV we present a simple example of an inflationary model used to obtain a spectrum with a cutoff. In Section V we fit two separate cutoff spectra to the WMAP observations in an attempt to constrain the scale of the cutoff. We discuss our results in Section VI.

II. LATE UNIVERSE CASE

Observations of late-time acceleration and low power on largest scales challenge our simplest theoretical expectations. In both cases the effects become manifest on scales comparable to the present size of the horizon, and at a time equal to the present age of the universe. This is seen as a coincidence problem: “Why now?”.

Indeed, CMB observations suggest that the spectrum of density perturbations is nearly flat on scales even a few times smaller than the present size of the observable universe. Therefore, if we were born $10^9$ instead of $1.4 \times 10^{10}$ years after the big bang, we would see CMB anisotropies that do not display any suppression of low multipoles (unless we were living in an unusual part of the universe due to cosmic variance). Similarly, if we were living $10^9$ years after the big bang, the value of $\Omega_\Lambda$ would be less than $1\%$.

It is thus tempting to search for a physical mechanism relating these two problems. This may require a non-trivial modification of gravity at the horizon scale. For example, it would be appealing to have a mechanism that would screen not only the cosmological constant, but also inhomogeneities on the scale $O(H^{-1}) \sim \ell$, on which the effects of the cosmological constant become manifest. Another idea is related to possible nonperturbative effects due to eternal inflation. For example, under certain assumptions concerning the choice of measure in the theory of eternal inflation, one can come to a paradoxical conclusion that we should live in a center of a spherically symmetric distribution of matter. This could lead to suppression of the large-angle anisotropy without affecting the small-angle effects.

However, these possibilities are very speculative. As a simpler alternative one may investigate whether the late time evolution of the metric fluctuations $\Phi$ could be responsible for the suppression via the Integral Sachs–Wolfe (ISW) effect.

The observed large-angle CMB temperature anisotropies are generated by scalar perturbations to the FRW geometry, which in the longitudinal gauge can be written as

$$ds^2 = a^2(\eta) (1 + 2\Phi) d\eta^2 - (1 - 2\Phi) dz^2. \quad (1)$$

The anisotropy has a contribution of $\frac{1}{4} \Phi'$ sources at the last scattering time hypersurface $\eta_l$. In addition, an integral contribution arises, in the case where time derivative $\Phi'$ is non-vanishing. Here $a(\eta)$ denotes the scale factor, $\eta$ the conformal time, and $z$ a prime denotes differentiation wrt $\eta$. Consider a scale free primordial spectrum $|\Phi_k|^2 = \frac{A^2}{k^3}$. Then, for the CMB angular power spectrum amplitudes we can adopt the result of $\Phi$

$$\left\langle \frac{\Delta T}{T} \right\rangle^2 = \frac{2\ell + 1}{\ell (\ell + 1)} \frac{A^2}{36\pi^2} K^2_{\ell}, \quad (2)$$
where the coefficient $K^2_t$ is given by

$$K^2_t = 2\ell(\ell + 1) \int_0^\infty \frac{dk}{k} \left[ j_\ell(k) - j_\ell(k) \right]^2 + 6\int_{n_c}^{n_0} \frac{d\eta f_k}{\eta},$$

where $f_k$ is $\Phi_k$ normalized to $\Phi_k/k$. The flat CDM model gives $\Phi = 0$ and $K^2_t = 1$ at large scales. The flat $\Lambda$CDM model with pure cosmological constant $\Omega_\Lambda \sim 0.7$ gives instead $K^2_t > 1$, amplifying the power above the Sachs–Wolfe plateau $[34]$. However, the coefficients $K^2_t$ may be smaller than unity with particular solutions for the late time evolution of $\Phi$. This can be realized if the interference between the two terms in the integral $[34]$ is destructive, which in turn constrains the spectrum and evolution of the metric perturbation $\Phi_k(\eta)$. This may require special models of the effective cosmological constant, or the addition of isocurvature fluctuations in the quintessence field $[35]$. We will address these possibilities in a future investigation.

### III. EARLY UNIVERSE CASE

Here we will describe modifications of the simplest inflationary models that can account for the suppression of the low $\ell$ multipoles. As we will see, these modifications have a common cause, which is the presence of a primordial or intermediate regime where some of the slow roll approximations are not valid (without necessarily meaning the interruption of inflation). This results in a departure from the slow roll approximation when one evaluates the power spectrum originated during inflation.

#### A. Changing the Potentials

There have been many suggestions in the past on how to alter the amplitude of density perturbations produced during inflation, see e.g. $[27, 28, 29, 30, 31, 32]$. The simplest idea is to fine-tune the shape of the potential. In general, this procedure is far from being trivial. The amplitude of metric perturbations at the horizon crossing in the slow-roll approximation is given by

$$k^{3/2} \Phi_k = \frac{V^{3/2}(\phi)}{\sqrt{6V'(\phi)}},$$

where in the r.h.s. $\dot{\phi}(t)$ is a function of $k$, $k \sim a(t)H(t)$, we use the units $M_p = (8\pi G)^{-1/2} = 1$, and $V'$ denotes the derivative wrt $\phi$. This formula is convenient to gain a heuristic insight of the problem. Once we start bending the potential, change occurs both in $V(\phi)$ and in $V'(\phi)$, and the relation between $\dot{\phi}$ and the momentum $k$ of perturbations changes as well.

To do so, one does not have to use complicated potentials. One can use, for instance, the simple generic renormalizable potential

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{3}\phi^3 + \frac{1}{4}\lambda\phi^4.$$  

By an appropriate choice of parameters we can generate significant features – dips or peaks – in the spectrum, as demonstrated in $[37]$ with exact solutions of the gauge invariant mode-by-mode equations for fluctuations. Obviously, we must tune the parameters to place the dip of the power spectrum $\Phi_k$ around the present day cosmological horizon.

The situation is even simpler in the hybrid inflation scenario, which has more parameters by construction. The value of $V(\phi)$ is mainly determined by the cosmological constant $V_0$ that does not change significantly during inflation. In this case, in order to make the amplitude of the potential smaller (greater) one should simply increase (decrease) the slope of the potential.

Suppose, for example, that we have $\Phi \sim 10^{-5}$ (as we should) in the theory where the potential looks like $V_0 + \alpha \phi$ at low $\phi$, corresponding to the number of e-folds $N_e \simeq 55$. Suppose also that at larger $\phi$ the slope gradually increases and becomes 10 times greater on the scale corresponding to $60 < N_e < 65$. Then on this scale the amplitude of perturbations will become 10 times smaller. This should lead to a primordial spectral index $\Phi_k$ with an amplitude which is strongly suppressed at low $k$, which will give us a desirable effect in the CMB anisotropy. Again, this requires some fine-tuning of the position of the place where the slope of the potential changes. An analogous spectrum was found in $[38]$ through a numerical mode-by-mode computation in the model $V(\phi) \approx V_0 + \lambda\phi^4$.

Moreover, potentials of this type naturally appear in the simplest versions of hybrid inflation in supergravity (F-term inflation $[39]$). If one considers N=1 supergravity with a minimal Kähler potential and a very simple superpotential $W = S(\kappa\phi^2 - \mu^2)$, one finds a potential of the following general form $[40]$:

$$V(\phi) \approx V_0(1 + \alpha \ln \phi) + \lambda\phi^4 + ...$$

Note that the potential is steep at small and at large $\phi$, and relatively flat for some intermediate values of $\phi$. This leads to the cutoff of the spectrum at very short distances, as well as on the very large scale. A similar behavior was advocated in $[2, 3, 4]$ based on the WMAP data and LSS+$L_\alpha$ data. For certain natural values of parameters the infrared cutoff of the spectrum occurs on the scale of the horizon. This is exactly what one needs to explain the suppression of the low $\ell$ multipoles of CMB.

#### B. Kinetic regime

It is usually assumed that the inflaton scalar field is moving very slowly at the beginning of inflation. This picture goes back to the early days of inflationary
paradigm, when the field was supposed to be trapped in the minimum of the effective potential \[13, 14\]. Therefore the initial speed of the field was supposed to be zero. In the language of density perturbations, \(\frac{\delta \rho}{\rho} \sim a^{3/2} \sim \frac{H^2}{\dot{\phi}}\), this implied that the density perturbations produced at the beginning of inflation were very large, in complete agreement with the usual statement that the spectrum of density perturbations in simplest one-field inflationary models is red.

However, in chaotic inflation \[14\] the initial speed of the field \(\phi\) can be quite large. The field eventually slows down when it approaches the inflationary regime, which is an attractor in the phase space of all possible trajectories \((\phi(t), \dot{\phi}(t))\). For a small subset of initial conditions the field may approach the inflationary trajectory relatively late, so that at the onset of the last 65 e-folds of inflation the field will have higher speed than is usually expected. In this case the amplitude of metric perturbations at the scale of the horizon will be smaller than expected, which in turn will result in a lower variance of CMB anisotropies at small \(\ell\). This marginal inflation requires a tuning of the initial conditions, and not of the shape of the potential.

A similar situation may occur in hybrid inflation. The simplest potential for two-field hybrid inflation is \[16\]

\[
V(\phi, \sigma) = \frac{\lambda}{4} (\sigma^2 - v^2)^2 + \frac{g^2}{2} \sigma^2 \phi^2 + \frac{1}{2} m^2 \sigma^2. \quad (7)
\]

The point where \(\phi = \phi_c = \frac{\sqrt{3} \lambda}{g} v\) and \(\sigma = 0\) is a bifurcation point. For \(\phi > \phi_c\) the squares of the effective masses of both fields \(m^2_{\phi} = g^2 \sigma^2 - \lambda \nu^2 + 3 \lambda \sigma^2\) and \(m^2_\sigma = m^2 + g^2 \sigma^2\) are positive and the potential has a minimum at \(\sigma = 0\). For \(\phi < \phi_c\) the potential has a maximum at \(\sigma = 0\). The global minimum is located at \(\phi = 0\) and \(|\sigma| = v\). Inflation in this model occurs while the \(\phi\) field rolls slowly from large values toward the bifurcation point.

The space of initial conditions now is \((\phi(t), \dot{\phi}(t); \sigma(t), \dot{\sigma}(t))\). In general evolution begins with large values of \(\phi\) and \(\sigma\), at the boundary with Planck energy density. At large \(\phi\) the dominant contribution to \(V\) is given by the term \(g^2 \phi^2 \sigma^2/2\), so the Planck boundary is given by a set of four hyperbola \[12\]

\[
g|\phi||\sigma| \sim 1. \quad (8)
\]

We will assume that initially \(|\phi| > |\sigma|\) and take \(g \ll 1\). The field \(\phi\) on this branch can take any value up to \(g^{-1}\) (for greater values of \(\phi\) the field \(\sigma\) would have a super-Planckian mass \(g \dot{\phi}\)). Suppose for definiteness that \(|\phi|\) is close to its upper bound, so that the initial value of the field \(\sigma\) at the Planck boundary is of order 1 or somewhat greater. This allows for a short stage of chaotic inflation supported by the potential of the field \(\sigma\) when this field rolls down toward \(\sigma = 0\).

After the first stage of inflation, the field \(\sigma\) rapidly oscillates, with the frequency just slightly smaller than the Planck mass, and with the amplitude \(\sigma(t)\) decreasing as \(a^{-3/2}\). The oscillations induce a large contribution to the effective mass of the field \(\phi\): \(m^2_\phi = m^2 + g^2 (\sigma^2)\).

Here \((\sigma^2) \sim a^{-3}\) stays either for the average square of the amplitude of the oscillations of the field, or for the fluctuations of this field produced during its decay to particles \(\phi\) and \(\sigma\).

In the beginning, \(g^2 (\sigma^2) \gg m^2\), so the field moves with a much greater speed than what one could naively expect by looking at the effective potential of the field \(\phi\) and ignoring its interactions with the field \(\sigma\) \[42, 43\]. This leads to a strong suppression of density perturbations produced at the first stages of hybrid inflation. However, eventually the term \(g^2 (\sigma^2)\) drops down and we enter the standard hybrid inflation regime producing perturbations with nearly flat spectrum. Since the term \(g^2 (\sigma^2)\) drops down very rapidly, as \(a^{-3}(t)\), the transition between the flat spectrum produced at late stages of inflation and the strongly suppressed spectrum at the early stages of inflation occurs very abruptly, within a single e-fold of inflation. This is exactly what we want in order to explain the absence of the low \(\ell\) multipoles.

In terms of the slow roll parameters \(\epsilon = \frac{\dot{\phi}^2}{2H^2}, \delta = -\frac{\dot{\phi}}{H\phi}\), one has \(n_s - 1 = -2\delta - 4\epsilon\). \(\delta\) Suppression of the spectrum at small \(k\) implies that \(n_s - 1\), is large for \(k\) corresponding to the scale of the present horizon. This means that in our examples some of the slow-roll parameters \(\epsilon, \delta, \) etc., at some early (or intermediate) stage of inflation were large. This can be attributed to the time derivatives \(\dot{\phi}, \ddot{\phi}, \) etc. Thus, departure from the slow roll approximation for calculating power spectrum is related to the regime where kinetic terms cannot be neglected.

We would like to mention also, that there is a new possibility of modulated cosmological perturbations from inflation \[14, 15\], where one may use a different type of tuning to get desirable spectra from inflation, while leaving the inflaton potential unchanged.

### IV. ILLUSTRATION OF THE CUTOFF SPECTRUM

For illustrative purposes, we now study in a more detailed way one of the simplest possibilities we have discussed above, namely the existence of a period of fast rolling of the inflaton field \(\phi\) (with strong kinetic domination) before the last stage of inflation. The period of inflation is assumed to last for about 60 to 65 e-folds (the exact length depending on the details of reheating) such that the stage of fast roll can leave an imprint precisely at the largest scales we presently observe. As a working
assumption, we start with a homogeneous and flat universe already at the stage of fast roll, so that one may have to postulate the existence of a previous period of inflation at earlier times. Apart for this requirement, we leave the details of the earlier universe unspecified, and we rather concentrate on the evolution of the background and of the cosmological perturbations starting from the fast roll regime.

We are mainly interested in the spectrum of primordial perturbations. We first perform an exact mode-by-mode numerical computation in the context of chaotic inflationary potential and large initial velocity for the inflaton field. We will then compute the spectrum in a simpler idealized situation, with an instantaneous transition between a regime of kinetic domination and a nearly de-Sitter stage. In this last case one can derive a simple analytical result which reproduces very well the exact spectrum obtained in the numerical evolution.

We start with a quadratic inflaton potential, \( V(\phi) = m^2 \phi^2 / 2 \), and initial conditions \( \phi_{\text{in}}(t_0) = 18.0 \, M_p \), \( \left. (d\phi/dt)_{\text{in}} \right|_{t_0} \approx -10 \, m_\phi \, \phi_{\text{in}} \), giving about 61 e-folds of inflation. Concerning the perturbations, we work in momentum space and focus on the Mukhanov variable \(\Phi_k\)

\[
v_k \equiv a \left( \delta \phi_k + \frac{\phi'}{h} \Phi_k \right)
\]

where \(\delta \phi_k\) denotes the perturbations in the inflaton field, \(\Phi\) is the metric perturbation given in Eq. (10), prime denotes derivative with respect to conformal time \(\eta\), and \(h \equiv a'/a\), \(a\) being the scale factor of the universe. The evolution of \(v_k\) is given by (see e.g. [17] for details)

\[
v'' + \left( k^2 - \frac{z''}{z} \right) v = 0 , \quad z \equiv \frac{a \phi'}{h}
\]

Due to the choice of initial conditions, the system is initially in a kinetic dominated regime, with \(h \propto \phi'\). As a consequence, we have initially

\[
a \approx \sqrt{1 + 2 \, h_0 \, \eta}, \quad \frac{z''}{z} \approx -\frac{h_0^2}{(1 + 2 \, h_0 \, \eta)^2}
\]

with \(h_0\) denoting the value of \(h\) at the time \(\eta = 0\), where we have normalized \(a = 1\).

As long as the approximation (11) is valid, Eq. (10) is solved by

\[
v_k(\eta) = \sqrt{\frac{\pi}{8 \, h_0^2}} \, \sqrt{1 + 2 \, h_0 \, \eta} \, H_0^{(2)}(k \eta + \frac{k}{2 \, h_0})
\]

where \(H_0^{(2)}\) is a specific Hankel’s function and where we have chosen (and appropriately normalized) the solution of Eq. (11) which reduces to \(\exp(-i \, k \, \eta / \sqrt{2} \, k)\) at very short wavelengths, \(k \gg 1 / |\eta| \).

In the numerical integration we have solved exactly the evolution equations for the background quantities \(\phi\) and \(a\), together with Eq. (10) for the perturbation mode \(v_k\). We have used Eq. (12) only to set the value of \(v_k\) and its derivative at the initial time \(\eta = 0\). We have plotted in the last panel of fig. 2 the final spectrum for the metric perturbation \(\Phi_k\).

Modes with very large \(k\) are well inside the horizon during the fast roll of \(\phi\), and hence they are quite insensitive to the evolution of the background at this stage. Their horizon crossing occurs during inflation, and the usual computation (giving nearly scale invariant spectrum) applies. At wavelengths comparable with the size of the horizon \(H_s^{-1}\) during the onset of inflation we find instead a strong departure from scale invariance. The amplitude of the spectrum is oscillating for \(k \gtrsim H_s\), followed by a sharp decrease at \(k \sim H_\ast\). By an appropriate choice of the number of e-folds of inflation, it is possible to relate such wavelengths to the size of the present horizon, with a consequent decrease of the observed CMB anisotropies at large scales.

This result is well reproduced in a highly simplified situation, with an instantaneous transition between the regimes of kinetic domination and nearly de-Sitter expansion. The scale factor \(a\) in the idealized model evolves as

\[
a \approx \sqrt{1 + 2 \, H \, \eta}, \quad \eta \leq 0
\]

\[
a \approx \frac{1}{1 - H \, \eta}, \quad \eta \geq 0
\]

where we have now set \(\eta = 0\), and \(a = 1\) at the transition, while \(H\) denotes the (physical) Hubble parameter during inflation.

The evolution equation for the Mukhanov variable \(v_k\) is again given by Eq. (11), with \(z''/z \approx a''/a\) in both regimes. During kinetic domination, the evolution of \(v_k\) is again given by Eq. (12) with the quantity \(h_0\) replaced by \(H\). During the de-Sitter stage one finds instead

\[
v = C \, e^{-i (k \eta - k / H)} \left( 1 - \frac{i \, k}{k \eta - k / H} \right) +
\]

\[
+ \, D \, e^{i (k \eta - k / H)} \left( 1 + \frac{i \, k}{k \eta - k / H} \right)
\]

and the coefficients of the two modes can be obtained by requiring continuity of \(v\) and \(v'\) at the transition,

\[
C = \frac{e^{-i k / H}}{\sqrt{32 \, H / \pi}} \left[ H_0^{(2)}(k \eta + \frac{k}{2 \, H}) - \left( \frac{H}{k} + i \right) H_1^{(2)}(k \eta + \frac{k}{2 \, H}) \right]
\]

during a previous inflationary stage, and assuming an adiabatic transition into the kinetic dominated regime (adiabaticity may be preserved due to a high effective mass for the field during these previous times, as for example in the hybrid inflationary case described in the previous section). An analogous initial state was considered for example in [18], where the production of gravitational waves was studied in a model with a radiation dominated stage followed by inflationary expansion.

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3 The choice (12) can be motivated by taking an adiabatic vacuum
result of a (nearly) scale invariant spectrum at short wavelengths (Eq. (15) and (16)). This is to be compared with the related by a first is derived from the simple model discussed above have fit the WMAP data with two template cutoffs. The dial power spectrum with a feature at large scales we fig. 2.

We consider the spectrum of Q ≡ v/a which becomes constant at late time, as indicated by the leading order contribution to equation (14) for η → 1/H

\[ P_Q \equiv \frac{k^3}{2 \pi^2} |Q|^2 \rightarrow \frac{H^2}{2 \pi^2} k |C - D|^2 \]

At short wavelengths (k \gg H) one recovers the standard result of a (nearly) scale invariant spectrum

\[ |C| \approx \frac{1}{\sqrt{2k}}, \quad |D| \ll |C| \]

\[ P_Q \approx \left( \frac{H}{2 \pi} \right)^2 \]

At super-horizon scales, the two modes Q_k and \Phi_k are related by a k independent rescaling so that the spectrum given by Eqs. (16) and (16) directly translates into the spectrum of \Phi, up to an overall normalization factor.

The spectrum obtained by this simple analytical calculation is shown in figure 1. We see that it reproduces very well the spectrum of \Phi obtained with the exact numerical evolution, which is reported in the last panel of fig. 2.

V. CMB ANISOTROPIES WITH A CUT-OFF

To determine how strongly the data favour a primordial power spectrum with a feature at large scales we have fit the WMAP data with two template cutoffs. The first is derived from the simple model discussed above and includes the detailed oscillatory effect at the transition between the two regimes in the initial spectrum. The second is based on an exponential cutoff of the form

\[ k^2 |\Phi(k)|^2 = A_s (1 - e^{-k/k_0})^\alpha k^{-n_s - 1}, \quad (18) \]

where we have chosen α to fit the rise of the model with α = 3.35. A_s is the overall amplitude of the scalar perturbations. This parametrization does not include features in the power spectrum which are dependent on the specific model considered, but which are too small to change qualitative features in the CMB power spectrum.

Due to the nature of the TE cross-correlation signal reported by WMAP, we chose to fit our models to the TT data only. We believe that it is still premature to draw too strong conclusions by making use of the E-type polarization signal, which has only been measured through its correlation with the much larger temperature signal by WMAP. Once the WMAP E-type observations have reached a sufficiently high signal–to–noise level to unambiguously measure a polarization signal free of systematics, its use in constraining models will be much safer. 4

We use a modified version of the publicly available CMBFAST 51 code to compute CMB spectra with various \kappa_c in the two cutoffs. We generate grids of models by varying three parameters which affect the shape of the spectrum at the lowest multipoles, namely the spectral index of the primordial power spectrum \n_s, the energy density of cosmological constant component in units of the critical density \Omega_\Lambda, and the present day wavenumber of the cutoff in the spectrum \kappa_c in units of Mpc^{-1}. The grid values are regular in the parameters and have a resolution of 32, 20, and 32 grid points with ranges [0.83, 1.04], [0.6, 0.85], and [2.0 \times 10^{-3}, 1.0 \times 10^{-5}] respectively.

At each point in the grid we use subroutines derived from those made available by the WMAP team to evaluate the log likelihood with respect to the WMAP data 51. Other parameters that are not expected to affect the shape of the spectrum at ℓ < 30 are held fixed at the following values: \Omega_b h^2 = 0.022, \Omega_m h^2 = 0.135 and τ_c = 0.17 8. In addition we consider only flat models with \Omega_{tot} = 1. The present value of the Hubble constant for each model is then fixed for each choice of \Omega_\Lambda in the grid. As we are not interested in the overall amplitude of the primordial perturbations we marginalize over

4 An analysis with inclusion of TE data was first performed in 22, and, successively to 22 and to the first version of the present manuscript, in 16. The results of 16 are similar to ours, but their conclusions concerning the problem of low ℓ multipoles are slightly more optimistic due to the strong emphasis put on the TE data. We caution that taking into account both cosmic variance and the correlation with the much larger TT signal would result in a significant increase of the error bars on the TE data (see fig. 2 of ref. 21, where this is explicitly shown for a particular model), possibly affecting claims which strongly depend on their use.
a range of values in the amplitude of the power spectrum. We only consider scalar perturbations in this work.

Our choice of parameters is motivated by our interest in motivating the requirement for a cutoff in the spectrum and not by an attempt at a precise determination of cosmological parameters, an exercise which should be the focus of much more exhaustive investigations. Residual correlations with parameters that have been fixed such as $\tau_c$ will be subsumed by our tilt and amplitude parameters which should be viewed as mildly effective quantities. In addition our use of a prior on $\Omega_\Lambda$ (see below) is motivated by a simple mimicking of Large Scale Structure (LSS) and Type–Ia supernovae (SN1a) constraints external to the WMAP results.\footnote{The authors of \cite{45} erroneously compare our analysis to their “WMAP TT only” results. More appropriately, our simple choice of parameters and priors is closer to a “WMAPext TT + 2dF + SN1a” combination. The latter requires a very different parameter grid compared to the “WMAP TT only” data, whose best fit model lies out in the tail of the LSS+SN1a motivated priors \cite{45}.}

In Fig. 2 we show the resulting best–fit models we obtain from the grid for both cutoff models considered. The top panel shows a standard (no cutoff) spectrum model with the same parameters as those obtained in the best–fit for the two models ($n_s = 0.9587$ and $\Omega_\Lambda = 0.732$). We have displayed the noise contribution to the uncertainties as errors on the binned WMAP results. The cosmic variance contribution to the errors is shown as upper and lower contours around the model. We have displayed the $1\sigma$ confidence limits for a lognormal distribution as this approximates more clearly the significance of the low quadrupole and octupole signal. The middle two panels show the best–fit for the model cutoffs. We can see that both models fit the lower multipoles better then the standard spectrum by reducing power on the largest scales. The bottom panel shows the two template spectra used in the fits shifted to an arbitrary cutoff scale of $k_c = 1$ for a $n_s = 1$ model.

The bottom panel shows the two template spectra used in the fits shifted to an arbitrary cutoff scale of $k_c = 1$ for a $n_s = 1$ model. It is important to note that even a step–like cutoff in the initial spectrum will not produce a step–like feature in the CMB spectrum due to the convolution of the initial spectrum in Eq. 3. Thus it is difficult to reproduce the sharp drop–off that occurs in the angular power spectrum.

Although we fit for all TT multipoles in the range $2 \leq \ell \leq 900$, most of these simply provide an accurate calibration of the overall amplitude of the model as we are keeping the matter content fixed. We therefore concentrate on the lowest multipoles in considering the significance of our best–fit models. For $\ell \leq 10$ we find that the probability of exceeding the observed $\chi^2$ is $5.6\%$ in the case of the standard model, $21.7\%$ when using the template model cutoff, and $33.4\%$ with the exponential cutoff of Eq. 18.

In Fig. 3 we marginalize over the range in $\Omega_\Lambda$ with a Gaussian prior $\Omega_\Lambda = 0.70 \pm 0.03$ and plot the resulting $\chi^2$ values as a function of $n_s$ and cutoff scale $k_c$. The contours shown are for $\Delta \chi^2$ values giving one, two, and three $\sigma$ contours for two parameter Gaussian distributions. We find that the distribution for the cutoff scale has a tail as $k_c \rightarrow 0$ close to the $2\sigma$ level. This is evident from the fact that the models asymptotically approach the scale invariant case which has a low but not vanishing probability with respect to the observations.\footnote{The authors of \cite{45} erroneously compare our analysis to their “WMAP TT only” results. More appropriately, our simple choice of parameters and priors is closer to a “WMAPext TT + 2dF + SN1a” combination. The latter requires a very different parameter grid compared to the “WMAP TT only” data, whose best fit model lies out in the tail of the LSS+SN1a motivated priors \cite{45}.}
we find \( k_c = 5.3^{+0.9}_{-0.6} \times 10^{-4}\text{Mpc}^{-1} \) with a 95% upper limit of \( k_c < 7.02 \times 10^{-4}\text{Mpc}^{-1} \). Similarly for the exponential cutoff model we find \( k_c = 4.9^{+1.3}_{-1.6} \times 10^{-4}\text{Mpc}^{-1} \) with a 95% upper limit of \( k_c < 7.4 \times 10^{-4}\text{Mpc}^{-1} \). It is important to note that the confidence limits depend on the integration measure adopted. In this case we have taken a measure linear in \( k_c \); however an alternative would be to take a measure in \( \ln k_c \) which may be more suitable for the distribution of the cutoff scales given by inflation theories. This second option, which gives higher weights to lower wavenumbers, will crucially depend on the value chosen for the largest scales, \( \ln k_c^{\min} > -\infty \). In the context of inflationary models it is natural to limit to those scales which left the horizon when the energy density of the universe was close to Planckian.

When faced with such ambiguities it is useful to refer to the difference in likelihoods between different values of \( k_c \). This allows an approximate estimate of the significance of having a cutoff without depending on any measure used in deriving confidence limits. As an example the difference in \( \chi^2 \) between the peak in the distributions and at \( k_c = 0 \) is found to be \( \Delta \chi^2 = 5.3 \) for the exponential cutoff and \( \Delta \chi^2 = 4.2 \) for the inflation model. This indicates that the case with no cutoff is just over 2σ below our best-fit cutoff model.

VI. DISCUSSION

Both COBE DMR and WMAP observations indicate a low value for the power on largest scales. Interestingly, these scales are of the same order as the scale corresponding to the time of dominance of the cosmological constant. If one ignores the possible interpretation of the suppression of low multipoles due to cosmic variance (which is perhaps the simplest and the least dramatic interpretation), one faces a new coincidence problem in addition to the usual one required to explain the late-time acceleration of the universe (i.e. \( \Omega_m \sim 2\Omega_{\Lambda} \) today). In this paper we briefly discussed the possibility that the late–time evolution of the metric perturbations \( \Phi(t) \) may reduce the power in the low multipoles through integrated Sachs–Wolfe effect. This may require a combined investigation of adiabatic and isocurvature perturbations in certain models of dark energy.

The main emphasis of our paper was related to modifications of the simplest models of inflation due to the possible existence of a kinetic stage, when the velocity of the scalar field was not negligible. The breaking of the slow roll approximation which could occur at this stage may cause a significant suppression of the large scale density perturbations. In order to affect the low \( \ell \) multipoles, this stage should occur in the beginning of the last 65 e-fold period of inflation. Clearly, this requires either fine tuning of parameters in the inflaton potential, or fine-tuning of initial conditions which are necessary to obtain a desirable modification of the spectrum close to the present horizon scale.
The final part of the paper has the aim to quantify with a simple example the level of suppression required by the data. A stage of kinetic domination before the observable stage of inflation can result in a sharp cutoff in the spectrum of primordial perturbations.

Fits to the observations reveal that the data favour a cutoff in the spectrum although the significance of the “detection” is less than 3σ. This is a reflection of the fact that the $k_c = 0$ case (i.e. standard ΛCDM model) is not a terrible fit to the data. The price of obtaining a better fit is the addition of an extra parameter in the theory which should probably be done under stronger observational evidence than we currently have.

On the other hand, we should note that when we attempt to evaluate all possible theories that could produce observational evidence than we currently have. A better fit is the addition of an extra parameter in the space of all possible cutoff parameters equally distributed in momentum space. Meanwhile if we would assume equal prior distribution in space of parameters of inflationary models, this would translate into an approximately equal distribution in terms of ln $k$ rather than $k$. This remark also applies to other attempts to explain the suppression of low multipoles [21, 22, 23, 24] and constitutes a choice of prior on $k_c$, a problem encountered when constraining many of the cosmological parameters (e.g. a choice of uniform prior on $H_0$ as opposed to in $\Omega_\Lambda$). However, as we have discussed, a possibly less ambiguous quantity is the likelihood ratio between different values of the cutoff since this does not depend on any underlying theoretical distribution. We have quantified this in the previous section.

We should also bear in mind that in any case, observations tell us the values of the parameters, and we should accept them even if they seem unusually fine-tuned. For example, many coupling constants in the standard theory of electroweak interactions are of the order $10^{-1}$. Meanwhile the coupling constant of the electron and the Higgs field, which is responsible for the electron mass, is $2 \times 10^{-6}$. The smallness of this parameter is notable, but usually we do not assume that all coupling constants are random variables and do not calculate the probability to have such a small coupling constant on the basis of its comparison to other coupling constants in the theory. We still do not know why this coupling constant is so incredibly small, but it does not make the standard theory of electroweak interactions unacceptable. Moreover, according to the standard model, the neutrino must be massless, meanwhile the recent data suggest that this is not the case. This means that we need to extend the standard model by making it even more complicated. However, this does not cast any doubts concerning the basic idea of gauge invariance and spontaneous symmetry breaking, which made the electroweak theory internally consistent.

High energy physicists never claim that they fine-tune the parameters of the theory, they simply fit the data. Until very recently cosmologists did not need to do the same and had the luxury to debate which values of the parameters seemed more natural. We are now entering the age of precision cosmology, and we need to adjust our attitude accordingly.

During the last 20 years, the inflationary scenario emerged as the simplest and possibly even unique way of constructing an internally consistent cosmological theory. The main result of our paper is that, if needed, we can fit the parameters of the inflationary theory and make it consistent with the small amplitude of the large-angle anisotropy of the CMB. It is incomparably easier to do so than to explain all features of the spectrum of the CMB, as well as the observed homogeneity, isotropy and flatness of the universe without using inflation.

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