Acoustic identification of the elastic properties of porous and nonporous superconducting materials \( \text{DyBa}_{2-x}\text{Sr}_x\text{Cu}_3\text{O}_{7-\delta} \)

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Abstract. This work involves the development of a method using the principle of acoustic microscopy to determine the elastic properties of high-temperature superconducting materials, the method is applied to analyze the variation of the elastic properties of the superconducting alloy \( \text{DyBa}_{2-x}\text{Sr}_x\text{Cu}_3\text{O}_{7-\delta} \) for different variations of the concentration parameters \((x = 0, x = 0.3\) and \(x = 0.6\)), porosity and temperature. The method is based on the reconstruction of the reflection coefficient calculated from the acoustic signature of the signal received by the microscope during the exploration of the superconducting material for different concentrations. This permitted the determination of the velocities of the surface and volume waves from the modeled reflection coefficient. On the other hand, the elastic parameters of the material such as Young's, shear and bulk moduli were also deduced.

1. Introduction
The goal of this study is to develop a method that can improve knowledge on the underlying mechanisms that control high-temperature superconductivity and how to evaluate the enhancement of this property when some parameter of the material is modified. The nature of high-temperature superconductivity mechanisms still remains an open question. In non-destructive testing, the elastic constants of solids have been investigated by measuring the velocity and attenuation of transmitted or/and reflected ultrasound waves. Since there is essentially no difference between thermal motion (random vibrations in solids especially metals) and elastic waves it is possible to get information on the order of the mean free path of the electrons in solids.

Therefore, by recovering the elastic constants we can gain a better understanding about the interactions between electrons and the atomic lattice, lattice instabilities and phase transitions (the contribution of the free electrons to the sound attenuation vanishes completely due to recombination of the free electrons into Cooper pairs). Among the multiple techniques used in Non-Destructive Testing (NDT), we have chosen to do the study using acoustic microscopy technique as proposed by Quate et al [1,2,3] and further developed by Briggs et al [3].

The acoustic microscopy measurement technique is based on the emission and reception of ultrasonic waves [4], it allows the determination of the different elastic parameters of the
superconducting material, to understand the mechanism of superconductivity [5], using the reflection coefficient and acoustic signature for this type of material at ultrasonic frequencies.

2. Theory
This work is done at high frequency (600 MHz) and therefore to allow the transmission and consequently the propagation of the ultrasonic waves in the solid, a coupling fluid was used as propagation medium. This is because the high frequency ultrasonic acoustic waves are very quickly damped by air. Water was employed as the coupling fluid. The acoustic wave propagating in the coupling fluid in the direction of the solid is considered as incident wave. It arrives at the surface of the solid \((DyBa_{2}Sr_{x}Cu_{1-y}O_{7-x})\) at an angle of incidence \(\theta\). Part of the incident wave is transmitted into the solid while the other is reflected towards the liquid. The boundary conditions at the solid-fluid interface allow us to determine the reflection coefficient which is given by the following expression [6, 7 and 8]:

\[
R(\theta) = \frac{Z_t \cos^2 2\theta + Z_s \sin^2 2\theta - Z_o}{Z_t \cos^2 2\theta + Z_s \sin^2 2\theta + Z_o},
\]

where \(Z_o\) is the acoustic impedance of the fluid, \(Z_t\) and \(Z_s\) are respectively the longitudinal and transverse acoustic impedances of the solid. The impedances are given by,

\[
Z_0 = \frac{\rho f c_f}{\cos(\theta)}, \quad Z_L = \frac{\rho c_L}{\cos(\theta_L)}, \quad Z_S = \frac{\rho c_s}{\cos(\theta_s)}.
\]

where \(\rho_f\) is the fluid density and \(c_f\) is the wave velocity in the fluid, \(c_L\) is the longitudinal wave velocity, \(c_s\) is the transverse wave velocity, \(\theta_L\) and \(\theta_S\) are the longitudinal and transverse wave angles in the solid. The relationship between the angles are given by,

\[
k_0 \sin(\theta) = k_L \sin(\theta_L) = k_S \sin(\theta_S),
\]

where \(k_0\) is the longitudinal wave number, \(k_L\) and \(k_S\) are the longitudinal and shear wave numbers respectively in the solid.

The expression for the reflection coefficient \(R(\theta)\) is used to determine the acoustic signature \(V(z)\) according to the following equation [8]:

\[
V(z) = \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} p^*(\theta) R(\theta) \exp(2 j k_0 z \cos \theta) \sin \theta \cos \theta d\theta
\]

where \(\theta_{\text{max}}\) is the maximum aperture of the lens, \(p^*(\theta)\) is the pupil function.

The acoustic signature \(V(z)\) is composed of periodic signals with a periodicity \(\Delta z\) of interferences that give information on the acoustic surface wave. This period depends on the velocity of the surface waves according to:

\[
V_z = V_{\text{ini}} \left[1 - \left(1 - \frac{V_{\text{ini}}}{2f \Delta z}\right)^2\right]^{\frac{1}{2}}
\]
The acoustic signature is associated with the values of the elastic parameters characterizing the material such as Young's modulus ($E$), shear modulus ($G$), bulk modulus ($B$) according to Eqns. (4, 5 and 6). It is also related to the structure and nature of the material [7,9].

\[
E = \rho V_S^2 \frac{3V_L^2 - 4V_T^2}{V_L^2 - V_T^2}
\]

\[
G = \rho V_S^2
\]

\[
B = \frac{E}{3(1-2\nu)}
\]

\[
\nu = \frac{2V_T^2 - V_L^2}{2(V_L^2 - V_T^2)}
\]

where $\rho$ is the density of material, $V_T$ is the transverse wave velocity, $V_L$ is the longitudinal velocity and $\nu$ is the Poisson ratio.

For the porous material with a porosity rate $p$, wave velocities are modified. The change is computed by replacing in the reflection coefficient function the modified propagation wave velocities of the volume waves and the density of the porous material by [10]:

\[
V_L = V_{L0}(1-p)^m
\]

\[
V_T = V_{T0}(1-p)^m
\]

\[
\rho = \rho_{0}(1-p)
\]

where $m$ is an empirical constant and the subscript 0 denotes a material with zero porosity.

3. Results and Discussion

3.1. Study of the superconducting material $\text{DyBa}_{2-x}\text{Sr}_x\text{Cu}_3\text{O}_{7-\delta}$ ($x = 0, 0.3, 0.6$):

Our study is based on the modeling of the reflection coefficient as a function of the angle of incidence according to Eqn. (1) and the acoustic signature $V(z)$ according to Eqn. (2). A computer program was developed to compute the acoustic signature $V(z)$ using the reflection coefficient as a function of the angle of incidence ($R(\theta)$) computed using the measured longitudinal and shear wave velocities (Eqn. 1). The velocities were measured for different concentration parameter values ($x = 0, x = 0.3$ et $x = 0.6$).

3.1.1. The reflection coefficient $R(\theta)$:

The values of the longitudinal and transverse wave velocities of the superconducting material $\text{DyBa}_{2-x}\text{Sr}_x\text{Cu}_3\text{O}_{7-\delta}$ as a function of the temperature were determined from the curves of the changes in the reference velocities found in [11]. The reference velocities were first recalculated then the reflection coefficients $R(\theta)$ for different values of temperature (T) from $80 \degree$ K to $230 \degree$ K, were determined. An example of some reflection coefficient curves (variation of the amplitude and phase of $R(\theta)$) for this material at $80 \degree$ K for different concentrations are shown in Figure (1).
It can be seen in Figure (1), that the amplitude (modulus) of the reflection coefficient varies from 0 to 1 and has two critical angles, corresponding to the angles $\theta_L$ (longitudinal critical angle) and $\theta_S$ (transverse critical angle). The phase curve contains two peaks, one very small corresponds to the longitudinal critical angle $\theta_L$ and the other, the largest, corresponds to the critical Rayleigh angle $\theta_R$.

For the different values of the parameter x, there is a significant shift of the critical angles to the lower angles for all modes with respect to the critical angles relative to the material which corresponds to the zero concentration value (x = 0).

3.1.2. The acoustic signature $V(z)$:

The determination of the reflection coefficient enables the determination of the acoustic signature $V(z)$. The acoustic signature curves for different concentration values at 80 °K are depicted in Figure (2).
It will be noted that, depending on the parameter x, different curves of the acoustic signature are obtained with a decrease in the period of the pseudo-oscillations, which indicate a change in the velocities of the modes which propagate in the superconducting material studied.

The critical angles of the propagating modes at oblique angles of incidence take into account: the longitudinal critical angle, the transverse critical angle and the velocity in the liquid $V_{\text{Liquide}}$. This makes it possible to obtain the velocities of these modes for the material studied using the Snell-Descartes law [8]:

$$V_L = \frac{V_{\text{Liquide}}}{\sin(\theta_L)}, \quad V_S = \frac{V_{\text{Liquide}}}{\sin(\theta_S)}.$$ 

The following figures show the variation of the longitudinal and transverse velocities as a function of the temperature for different values of concentration x:

Figure 3: Variation of the longitudinal and transverse wave velocities ($V_L$ and $V_T$ respectively) as a function of temperature for the material DyBa$_2$SrCu$_{1-x}$O$_{y}$ – (a) and (b) (x=0), (c) and (d) (x=0.6).

It is found that the velocities of the modes which propagate in the superconducting material study decrease with the increase of the temperature. The increase in the concentration x causes the increment of these velocities. These results compare very well with those in references (Yahya, 2008).

By introducing the values of the longitudinal and transverse velocities into the equations (4, 5 and 6), the elastic parameters ($E$, $G$, $B$) of the superconducting material considered as a function of the temperature can be determined for the three concentration values ($x = 0$, $x = 0.3$ et $x = 0.6$). The results obtained are presented in the following figures:
Figure 4: Variation of the elastic moduli (E, G, B) of $DyBa_{2-x}Sr_xCuO_8-d$ as a function of the temperature for the concentration $x = 0$. 
Figure 5: Variation of the elastic moduli (E, G, B) of $DyBa_{2-x}Sr_{x}Cu_{3}O_{7-\delta}$ as a function of the temperature for the concentration $x = 0.3$

Figure 6: Variation of the elastic moduli (E, G, B) of $DyBa_{2-x}Sr_{x}Cu_{3}O_{7-\delta}$ as a function of the temperature for the concentration $x = 0.6$

3.1.3. The reflection coefficient for porous $R(\theta)$:
For the porous material the variations of the reflection coefficient were determined in the same way as previously with the replacement of the propagation velocities of the volume waves and the density of the porous material from the relations (7, 8 and 9). The variation of the reflection coefficient as a function of the porosity is shown in Figure (7).
There is a significant shift of the critical angles for all modes to the upper angles with respect to the critical angles of the non-porous material (p = 0%), which implies a decrease in the velocities of these modes. The reflection function gives the acoustic signature of the superconducting material for the parameter value (x = 0) and for different porosities (Figure 8).

Figure 7: Variation of the reflection coefficient $R(\theta)$ of porous $\text{DyBa}_{2-x}\text{Sr}_x\text{Cu}_3\text{O}_{7-\delta}$ for $x=0$ - (a) Amplitude, (b) Phase.

Figure 8: Acoustic signature of porous $\text{DyBa}_{2-x}\text{Sr}_x\text{Cu}_3\text{O}_{7-\delta}$ for $p=0\%$ and $p=11.7\%$ (x = 0).

From the acoustic signature $V(z)$, the velocities of the modes propagating in the porous superconducting material studied and the elasticity moduli as a function of the porosity for the three parameters ($x = 0$, $x = 0.3$ and $x = 0.6$) are determined (Figure 9).
Figure 9: Variation of the elastic constants (E and B) of the porous $DyBa_{2-x}Sr_xCu_3O_{7-δ}$ as function of porosity and the three concentration parameters ($x = 0, 0.3$ and $0.6$) (a) E, (b) B.

Figure 9 shows that the elastic constants decrease with porosity for example, $x = 0$ and $p = 0\%$ Young’s modulus $E = 142,05\text{GPa}$, while for $x = 0$ and $p = 11.7\%$, $E = 110,91\text{GPa}$. The decrease in these parameters are due to the increase of porosity (Equations 8, 9) resulting in the diminution of the longitudinal and transversal wave speeds and the reduction of the amplitude level of the received signal [10].

4. Conclusion
The change of concentration (parameter $x$) of the superconducting material $DyBa_{2-x}Sr_xCu_3O_{7-δ}$ leads to a variation in the velocities of the modes propagating in this material, i.e. the increase of the concentration $x$ leads to the increment of the longitudinal and transverse mode velocities. As a result, the moduli of elasticity change when the concentration number $x$ of the superconducting material varies from 0 to 0.3 and then to 0.6.

For the porous superconducting material, the velocities are influenced by the increase of the porosity rate, which results in a decrease in the values of the elastic moduli of this material (Young’s modulus, shear modulus and the bulk modulus. From the knowledge of these moduli, the longitudinal and transverse wave velocities, the acoustic Debye temperature can be calculated [11]. The acoustic Debye temperature of a superconducting material can provide information about the role of phonons in its superconducting mechanism.

5. References
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