Diquark model for $J/\psi$ baryonic decays

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The baryonic decays of $J/\psi$ provide a new way to study the internal structure of baryons. We apply a simple diquark model to the calculation of the decay cross-sections for the reactions $J/\psi \to \bar{p}p$, $pN^*(1440)$, $N^*N^*$, $\bar{\Lambda}\Lambda$ and $\Sigma^0\Sigma^0$. The results are different from those given by the ordinary constituent quark model. Hence these reactions may provide a new check of two different pictures for the baryons.

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1 Introduction

In spite of a long history of baryon spectroscopy there are still many questions without clear answers. We know that the baryons are composed of three valence quarks, sea quarks and gluons but we are not certain if the valence quarks have constituent or current nature, if they cluster into diquarks or are well separated from each other, whether the sea quarks are in the form of $\bar{q}q$ soup or mesons etc. For a long time we have also missed direct sources of information on the properties of nucleon excitation states and could build our knowledge about them almost entirely on results from partial wave analysis of $\pi N$ scattering data. The situation is changing dramatically with new experimental data coming from facilities such as CEBAF at JLAB, ELSA in Bonn, GRAAL in Grenoble or from BEPC in Beijing.

A long-standing problem in $N^*$ physics is about the nature of the Roper resonance $N^*(1440)$ which is considered to be the first radial excitation state of the nucleon. However, various quark models have difficulty to explain its mass and electromagnetic coupling, so it was suggested that it may be a gluonic excitation of the nucleon, a hybrid baryon. An ideal tool for studying the properties of $N^*$ resonances are the decays of $J/\psi \to \bar{p}p\pi^0$ and $J/\psi \to \bar{p}p\pi^+\pi^-$ since in these processes the $\pi^0\pi^0$ and $p\pi^+\pi^-$ systems are limited to isospin 1/2 states. The $J/\psi$ decays into baryon-antibaryon states discussed in the present paper provide another way to probe the internal structure of baryons.

The $J/\psi$ decay cross sections for $\bar{p}p$, $\bar{p}N^*$, $\bar{N}^*N^*$, $\bar{\Lambda}\Lambda$ and $\Sigma^0\Sigma^0$ final states were calculated in Ref.[1, 2] by using a simple quark model. In the present paper we use the diquark model and look into the possibility of forming the final state baryons.
from the \( \bar{q}q \) (generated in \( J/\psi \) decay) and diquark-antidiquark (\( \bar{D}D \)) pairs (created as vacuum excitation). The model resembles the standard quark-pair-creation model \[3\] used extensively to describe the mechanism of hadronic decays. Our diquark model calculation gives a different prediction for these processes which can be compared with the previous calculation by the quark model\[1, 2\]. Future experimental data on these reactions may provide a check on these two different pictures for baryons.

2 Diquark model formalism

We assume that the decay of \( J/\psi \) meson leads to a creation of \( q\bar{q} \) pair and the final state baryons are formed by their coupling to the diquark-antidiquark pair produced with the vacuum quantum numbers \( J^{PC} = 0^{++} \). Restricting ourselves to the SU(3) flavour sector we have the following diquark nomenclature \[4\]

\[
(q\bar{q}') \sim (ud - du) \quad \text{scalar-isoscalar diquark} \\
(qs) \sim (us - su) \quad \text{scalar-isodoublet diquark} \\
[q\bar{q}] \sim uu \quad \text{vector-isotriplet diquark} \\
[qs] \sim (us + su) \quad \text{vector-isodoublet diquark} \\
[ss] \sim ss \quad \text{vector-isosinglet diquark},
\]

in which \( q \) (or \( q' \)) stands for either the up or down quark and we use the round and box brackets for the axial scalar and the axial vector diquarks, respectively. The effective diquark-antidiquark vacuum creation amplitude can be written as

\[
\langle \bar{D}D | V_{\text{eff}} | 0 \rangle = f_{\bar{D}D}(-1)^{T_D-M_D}(T_D M_D T_D M_D|00) \eta 
\]

A similar structure was adopted in Ref. \[5\] for the \( \bar{D}D \rightarrow \bar{q}q \) process. The coupling parameter \( f_{\bar{D}D} \) may be different for the various \( \bar{D}D \) species listed above. However, for the sake of simplicity we shall assume that this parameter does not depend on the diquark type. The phase \( \eta = 1 \) stands for the creation of scalar diquarks and \( \eta = (-1)^{1-\epsilon_D} \delta_{\epsilon_D-\epsilon_D} \) corresponds to the scalar coupling of vector diquark (with polarization \( \epsilon_D \)) and vector antidiquark (with polarization \( \epsilon_{\bar{D}} \)).

The intermediate \( \bar{q}q \) pair produced in \( J/\psi \) decay can be either in the \( 3S_1 \) or in the \( 3D_1 \) state. In the \( J/\psi \) rest system and for the initial \( J_z = J = 1 \) polarization the relative \( \bar{q}q \) wave function can be written as

\[
(p \mid \bar{q}q, J = 1 J_z = 1) = \sum_{LM} c_{LM} \phi_L(p) Y_{LM}(\hat{p}) \mid \bar{q}q, (1 - M)) 
\]

where \( \mid \bar{q}q, \mu \rangle \) stands for the spin-flavour part of the \( \bar{q}q \) wave functions and \( p \) denotes the relative \( \bar{q}q \) momentum. The radial wave functions \( \phi_L(p) \) and the coefficients \( c_{LM} \) are normalized as

\[
\sum_L \int_0^\infty dp \phi_L^2(p) = 1, \quad \sum_M c_{LM}^2 = 1, \quad (c_{00} = 1, c_{20} = \sqrt{3/10}, c_{21} = -\sqrt{3/10}, c_{22} = \sqrt{3/5}).
\]
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Table 1. The spin-flavour overlap amplitudes [see Eq. (3)] derived for various \( \bar{B}B \) final states. All coefficients \( V_q(B) \) and the summed amplitudes \( \sum f_q V_q \) are presented in units of \( f_{\bar{D}D}/2 \).

| \( \bar{B}B \) state | \( V_u(B) \) | \( V_d(B) \) | \( V_s(B) \) | \( \sum f_q V_q \) |
|----------------------|-----------------|-----------------|-----------------|-----------------|
| \( p\bar{p}N^*,N^*N^* \) | 1 - \( 1/(9\sqrt{3}) \) | -2/(9\sqrt{3}) | 0 | \( 1 - 1/(3\sqrt{3})f_u \) |
| \( \Lambda \Lambda \) | 0 | 0 | 2/3 | \( 2f_s/3 \) |
| \( \Sigma^0\Sigma^0 \) | 4/(9\sqrt{2}) | 4/(9\sqrt{2}) | -2/(9\sqrt{3}) | \( 8f_u/(9\sqrt{2}) - 2f_s/(9\sqrt{3}) \) |

The \( \bar{q}q \) pair couples with the \( \bar{D}D \) pair forming the final state baryon \( B \) and antibaryon \( \bar{B} \). The spin-flavour overlap amplitudes follow from the SU(3) quark-diquark decomposition of the baryonic states (see Appendix). It is easy to show that

\[
\langle \bar{B}B, \mu' | V_{\text{eff}} | \bar{q}q, \mu \rangle = V_q(B) \delta_{\mu'\mu},
\]

where we introduced the \( \bar{B}B \) spin triplet states in the same fashion as the \( \bar{q}q \) spin-flavour states. The amplitudes depend on both the baryon specification as well as on the flavour of the intermediate quark. For a reference we give the amplitudes relevant for the considered processes in Table 2. It is worth noting that the amplitudes vanish if they are derived for the spin-singlet \( \bar{B}B \) states in accordance with the required parity conservation.

The complete transition matrix elements for charmonium decay into \( \bar{B}B \) states (with spin projection \( \mu \)) can be written as

\[
M_\mu = \sum_{q,L,M} f_q V_q(B) c_{LM} I_{\text{space}}^{LM} \delta_{\mu'(1-M)}
\]

(4)

where the parameter \( f_q \) represents the amplitude of creating the specific \( \bar{q}q \) pair. The space integral reads

\[
I_{\text{space}}^{LM} = \frac{1}{8} \int d^3k \phi_L(k) Y_{LM}(\hat{k}) \phi_B^*(B - \frac{3}{2}k) \phi_B^*(\frac{3}{2}k - B)
\]

(5)

with \( B \) standing for the final state baryon momentum and \( \phi_B \) denoting the intrinsic spatial wave function of the baryon. We have assumed a simple gaussian spatial distribution for both the intermediate \( \bar{q}q \) state and for the baryon intrinsic wave function. Only the \( L = 0 \) and \( L = 2 \) partial waves contribute to the \( \bar{q}q \) state and we have used

\[
\phi_0(x) = n_0 \sqrt{4\pi} \left( \frac{1}{\alpha \pi^1/2} \right)^{3/2} \exp(-\frac{x^2}{8\alpha^2})
\]

and

\[
\phi_2(x) = n_2 \sqrt{4\pi} \left( \frac{1}{\alpha \pi^1/2} \right)^{3/2} \sqrt{\frac{1}{60} \frac{1}{\alpha^2}} \exp(-\frac{x^2}{8\alpha^2})
\]

(6)
where the factors $n_L$ must satisfy the normalization condition $\sum_L n_L^2 = 1$. As there is no evidence that the strength of $\bar{q}q$ formation process depends on $L$ we simply assume $n_0 = n_2 = \sqrt{1/2}$ in our calculation. The harmonic oscillator eigenfunctions are used for the baryons in their center-of-mass system, e.g.

$$
\phi_p(x) = \sqrt{4\pi} \left( \frac{1}{\beta \pi^{1/2}} \right)^{3/2} \exp\left(-\frac{x^2}{12\beta^2}\right)
$$

$$
\phi_{N^*}(x) = \sqrt{4\pi} \sqrt{\frac{12}{5}} \left( \frac{1}{\beta \pi^{1/2}} \right)^{3/2} \left(1 - \frac{x^2}{18\beta^2}\right) \exp\left(-\frac{x^2}{12\beta^2}\right)
$$

for the proton and the $N^*(1440)$ resonance. The momentum $x$ stands for the relative $\bar{q}q$ momenta and for the quark-diquark momenta in Eqs. (6) and (7), respectively. The harmonic oscillator parameters $\alpha$ and $\beta$ characterize the size of the relevant interaction and we varied them to fit the experimental data.

It was already noted in Ref. [1] that relativistic description seems to be more appropriate for the spatial distribution of the final state baryon clusters. Then the integral (5) includes the jacobian due to Lorentz transformation from the baryon CMS to the laboratory ($J/\psi$ at rest) system and the internal quark-diquark momenta are transformed appropriately as well (see [1, 2] for more details). We have performed the calculation for both the nonrelativistic as well as the relativistic baryon wave functions.

The space integrals were computed numerically by using the program RIWIAD from CERN Program Library. The decay cross-section for $J/\psi \rightarrow \bar{B}B$ was then constructed from the amplitudes $M_{\mu}$ as

$$
\frac{d\Gamma(J/\psi \rightarrow \bar{B}B)}{d\Omega} = \frac{1}{32\pi^2} \frac{|B|^2}{M_\psi} \sum_{\mu} |M_{\mu}|^2
$$

with $\Omega$ denoting the solid angle of $B$ and $M_\psi$ standing for the charmonium mass.

### 3 Results and discussion

There are three parameters in our calculation. The harmonic-oscillator parameters $\alpha$ and $\beta$ determine the shape of the angular distribution

$$
\frac{d\Gamma(J/\psi \rightarrow \bar{B}B)}{d\Omega} = N_{\bar{B}B} (1 + a_B \cos^2 \theta)
$$

i.e. the computed parameter $a_B$. The constant $N_{\bar{B}B}$ is directly related to the experimental branching ratio of $J/\psi \rightarrow \bar{p}p$ and can be used to fix the overall normalization of the computed rates. Finally, we define our third parameter as the rate $g = f_s/f_u$ and naturally assume $f_d = f_u$. Clearly, the rate $g$ affects the relative numbers of strange and nonstrange baryons produced in $J/\psi$ decay. As the creation of $\bar{q}q$ pair in charmonium decay is flavor blind one may simply assume $g = 1$. However, the variation of $g$ also accounts effectively for some differences between creation of
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Table 2. The computed characteristics $\Gamma_{\bar{B}B}/\Gamma_{\bar{p}p}$ and $a_B$ of the $J/\psi \rightarrow \bar{B}B$ decay rates.

| $\bar{B}B$ state | nonrelativistic case $\Gamma_{\bar{B}B}/\Gamma_{\bar{p}p}$ $a_B$ | relativistic case $\Gamma_{\bar{B}B}/\Gamma_{\bar{p}p}$ $a_B$ | experiment $\Gamma_{\bar{B}B}/\Gamma_{\bar{p}p}$ $a_B$ |
|------------------|-----------------------------|-----------------------------|-----------------------------|
| $\bar{p}p$       | 1.00                        | 1.00                        | 1.00                        | 0.61(11)                  |
| $\bar{p}N^*$     | 0.90                        | 0.53                        | 0.76                        | 0.68                      | --                        |
| $N^*N^*$         | 1.05                        | 0.18                        | 0.99                        | 0.32                      | --                        |
| $\bar{\Lambda}\Lambda$ | 0.56                        | 0.50                        | 0.54                        | 0.54                      | 0.61(9)                  |
| $\bar{\Sigma}^0\Sigma^0$ | 0.41                        | 0.50                        | 0.39                        | 0.56                      | 0.60(11)                  |

various $\bar{D}D$ species, i.e. between different amplitudes $f_{\bar{D}D}$. In the calculation we adopted the value $g = 0.9$ suppressing partly the formation of strange baryons in the final state. Our assumptions and the structure of the amplitudes $V_q(B)$ allow to sum over $q$ in Eq. (4). The summed amplitudes $\sum_q f_q V_q(B)$ are given in the last column of the Table 2.

The results of our calculation are presented in the Table 3 for both the nonrelativistic and the relativistic approaches. We show the relative decay rates $\Gamma_{\bar{B}B}/\Gamma_{\bar{p}p}$, where $\Gamma_{\bar{B}B} = 4\pi N_{\bar{B}B}(1 + a_B/3)$, and the angular distribution coefficients $a_B$ in comparison with the available experimental data. As the reliable data are limited only to the $\bar{p}p$ channel we further assumed $\alpha = \beta$ and made an one parameter fit to the measured value of $a_\rho$. The results shown in the Table 3 were obtained for $\alpha = \beta = 0.4$ GeV and for $\alpha = \beta = 0.22$ GeV in the nonrelativistic and the relativistic cases, respectively. Although the assumption of using the same size parameters for both the intermediate $\bar{q}q$ state and for the space distribution of baryon clusters may not be sound the fitted values compare well with those used in other quark models [1, 8].

The results are compatible with available experimental data on $\bar{p}p$, $\bar{\Lambda}\Lambda$ and $\bar{\Sigma}^0\Sigma^0$ channels within two standard deviations. However the predicted results for the $\bar{p}N^*(1440)$ and $N^*N^*$ are quite different from those given by the simple quark model [1] which predicts rates for these two channels that are at least two times larger than the values presented here. Future experimental results on these two channels will be helpful for examining various model predictions and to improve our understanding of internal quark structure of these baryons. If the experimental $N^*$ production rates turn out to be much lower than the quark (and diquark) model predictions a large component of $\pi N$ could contribute to the $N^*$ internal structure. On the other hand, too high production rates would indicate some other mechanisms playing a role.

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Appendix

In this section we show the quark-diquark wave functions of baryons used in
the present work. Suppressing color anti-symmetrization factors the spin-up and
spin-down proton states read

\[ |p_{\pm}\rangle = \frac{1}{3\sqrt{2}} \left\{ \pm[ud]_{0}u_{\pm} \mp \sqrt{2}[uu]_{0}d_{\pm} \mp \sqrt{2}[ud]_{\pm1}u_{\mp} \pm 2[uu]_{\pm1}d_{\mp} \right\} + \frac{1}{\sqrt{2}}(ud)u_{\pm} \].

The \( N^* \) resonance has exactly the same spin-flavour structure and we can write
for the \( \Lambda \) and \( \Sigma^0 \):

\[ |\Lambda_{\pm}\rangle = \frac{1}{2\sqrt{3}} \left\{ \pm[ds]_{0}u_{\pm} \mp [us]_{0}d_{\pm} \mp \sqrt{2}[ds]_{\pm1}u_{\mp} \pm \sqrt{2}[us]_{\pm1}d_{\mp} + (ds)u_{\pm} \right\} + (us)d_{\pm} - 2(ud)s_{\pm} \].

\[ |\Sigma^0_{\pm}\rangle = \frac{1}{6} \left\{ \pm2[ud]_{0}s_{\pm} \mp [us]_{0}d_{\pm} \mp [ds]_{0}u_{\pm} \pm 2\sqrt{2}[ud]_{\pm1}s_{\mp} \pm \sqrt{2}[us]_{\pm1}d_{\mp} \right\} \pm \sqrt{2}[ds]_{\pm1}u_{\mp} \mp \frac{1}{2} \left\{ (us)d_{\pm} + (ds)u_{\pm} \right\} \].

References

[1] B.S. Zou et al. - Euro. Phys. J. A 11 (2001) 341.
[2] R.G.Ping, H.C.Chiang, B.S.Zou, Phys. Rev. D66 (2002) 054020.
[3] A. Le Yaouanc, L. Oliver, O. Pene, J.C. Raynal - Phys. Rev. D 8 (1973) 2223.
[4] U. Vogl, W. Weise - Prog. Nucl. Part. Phys. 27 (1991) 195.
[5] A.Cieplý, M.P.Locher, B.S.Zou - Zeitsch. f. Phys. A345 (1993) 41.
[6] DM2 Collaboration, Nucl. Phys. B192 (1987) 653.
[7] Particle Data Group, Phys. Rev. D66 (2002) 010001.
[8] E.S. Ackleh, T. Barnes, E.S. Swanson - Phys. Rev. D54 (1996) 6811.