Structure of Fluctuation Terms
in the Trace Dynamics Ward Identity

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ABSTRACT

We give a detailed analysis of the anti-self-adjoint operator contribution to the fluctuation terms in the trace dynamics Ward identity. This clarifies the origin of the apparent inconsistency between two forms of this identity discussed in Chapter 6 of our recent book on emergent quantum theory.
1. Introduction

In our recent book *Quantum Theory as an Emergent Phenomenon* [1], we developed a classical dynamics of non-commuting matrix (or operator) variables, with cyclic permutation inside a trace used as the basic calculational tool. We argued that quantum theory is the statistical thermodynamics of this underlying theory, with canonical commutation/anticommutation relations, and unitary quantum dynamics, both consequences of a generalized equipartition theorem. We also argued that fluctuation or Brownian motion corrections to this thermodynamics lead to state vector reduction and the probabilistic interpretation of quantum theory. In our analysis of fluctuation corrections, we noted that an anti-self-adjoint driving term, coming from a self-adjoint contribution to the conserved charge $\tilde{C}$ for global unitary invariance, is needed to give a stochastic Schrödinger equation that actually reduces the state vector. However, we also encountered an apparent inconsistency when such an anti-self-adjoint driving term was present, in that this term did not flip sign appropriately in going from the equation for a fermion operator $\psi$ to that for its adjoint $\psi^\dagger$. (See the discussion following Eq. (6.7a) in Chapter 6 of [1].)

Our aim in this paper is to give a detailed analysis of the origin of this apparent inconsistency. We shall show that when details that were glossed over in the treatment of Chapter 6 are taken into account, the different forms of the Ward identity are always consistent, but in certain cases the anti-self-adjoint driving terms tend to cancel. Specifically, we shall show that: (1) A self-adjoint term in $\tilde{C}$ appears when a fixed operator is used in the construction of the fermion kinetic terms, but cancels when this operator is elevated to a dynamical variable. (2) In the generic case when a self-adjoint term is present in $\tilde{C}$, the conjugate canonical momentum $p_\psi$ is no longer equal to $\psi^\dagger$. The two equations that are
analogs of the equations for $\psi$ and $\psi^\dagger$ in Eq. (6.7a) of [1] are then equations for $\psi$ and $p_\psi$, and the fact that the anti-self-adjoint driving term has the same sign in both equations is no longer an inconsistency. (3) In special cases where there are degrees of freedom with conventional fermion kinetic structure, that couple only indirectly through bosonic variables to fermion degrees of freedom that give rise to the self-adjoint term in $\tilde{C}$, the problem noted in Chapter 6 of [1] reappears. However, it is not an inconsistency in the Ward identities, but rather an indication that the $\tau$ terms, that were neglected in the approximations leading to emergent quantum theory, must play a role. In other words, in this case, the anti-self-adjoint driving term in the stochastic equation cancels to the level of the terms neglected in our approximation scheme.

This article is organized as follows. In Sec. 2 we analyze two models for bilinear fermionic Lagrangians, focusing on the structure associated with the appearance of a self-adjoint component in $\tilde{C}$. In Sec. 3, we derive the corresponding Ward identities analogous to Eq. (6.7a) of [1]. In Sec. 4, we discuss the implications of these results for the apparent inconsistency discussed in Chapter 6 of [1], leading to the conclusions briefly stated above.

2. Analysis of Models for Bilinear Fermionic Lagrangians

In this section we analyze two models for bilinear fermionic Lagrangians. The first, which generalizes the model developed in Eqs. (2.17) through (2.21) of [1], involves a fixed matrix $A_{rs}$ in the fermion kinetic term, and develops a self-adjoint contribution to $\tilde{C}$. In the second, the matrix $A_{rs}$ is elevated to a bosonic dynamical variable, in which case its contribution to $\tilde{C}$ exactly cancels the self-adjoint fermionic contribution to $\tilde{C}$.
The first model that we consider is based on the bilinear fermionic trace Lagrangian

\[
L = \text{Tr} \sum_{ra, sa, sb \in F} q^\dagger_{ra} A_{rs} (\dot{q}_{sa} + q_{sb} B_{ab}) + \text{bosonic},
\]

where the notation \( \in F \) (which will be suppressed henceforth) indicates a sum over fermionic degrees of freedom \( q_{ra} \), labeled by the composite index \( ra \), and where the purely bosonic terms are not explicitly shown. Here \( A_{rs} \) is a fixed bosonic matrix, and \( B_{ab} \) is a bosonic operator (a generalized gauge potential). Recalling our adjoint convention that for fermionic \( \chi_1, \chi_2 \), we have \((\chi_1 \chi_2)\dagger = -\chi_2^\dagger \chi_1^\dagger\), we see that the Lagrangian of Eq. (1a) is real up to a total time derivative which vanishes in the expression for the trace action, provided that

\[
A^\dagger_{rs} = A_{sr}, \quad B^\dagger_{ab} = -B_{ba}.
\]

Introducing the canonical momentum defined by

\[
p_{sa} = \frac{\delta L}{\delta \dot{q}_{sa}} = \sum_r q^\dagger_{ra} A_{rs},
\]

the trace Hamiltonian defined by

\[
H = \text{Tr} \sum_{sa} p_{sa} \dot{q}_{sa} - L,
\]

has fermionic terms given explicitly by

\[
H = -\text{Tr} \sum_{sab} p_{sa} q_{sb} B_{ab}.
\]

From this we find the equations of motion

\[
\begin{align*}
\dot{q}_{sa} &= -\frac{\delta H}{\delta p_{sa}} = -\sum_b q_{sb} B_{ab}, \\
\dot{p}_{sa} &= -\frac{\delta H}{\delta q_{sa}} = \sum_b B_{ba} p_{sb}.
\end{align*}
\]
where in the first line we have used the cyclic permutation rule for fermionic variables,
\[ \text{Tr}\chi_1\chi_2 = -\text{Tr}\chi_2\chi_1. \]

Although the trace Lagrangian in Eq. (1a) involves the fixed non-commutative matrix 
\( A_{rs} \), this does not appear explicitly in the trace Hamiltonian, and so the conditions for global
unitary invariance of the theory are fulfilled. Consequently, there is a conserved Noether 
charge \( \tilde{C} \) given by
\[ \tilde{C} = \tilde{C}_F + \tilde{C}_B. \tag{3a} \]

The bosonic part \( \tilde{C}_B \) is given by
\[ \tilde{C}_B = \sum_{r \in B} [q_r, p_r], \tag{3b} \]
and is anti-self-adjoint in the generic case with the bosonic canonical variables \( q_r, p_r \) either
both self-adjoint or both anti-self-adjoint. The fermionic part \( \tilde{C}_F \) is given by
\[ \tilde{C}_F = -\sum_{ra} (q_{ra}p_{ra} + p_{ra}q_{ra}) \tag{4a} \]
and by using Eq. (2a) we find that \( \tilde{C}_F \) has a self-adjoint part \( \tilde{C}_{sa}^F \) given explicitly by
\[ \tilde{C}_{sa}^F = \frac{1}{2}(\tilde{C}_F + \tilde{C}_F^\dagger) = \frac{1}{2} \sum_{rsa} [A_{rs}, q_{sa}q_{ra}^\dagger]. \tag{4b} \]
Using the equations of motion of Eq. (2d), we find that \( \tilde{C}_F \) has the time derivative
\[ \dot{\tilde{C}}_F = -\sum_{rab} [B_{ab}, p_{ra}q_{rb}] = -\sum_{rsab} [B_{ab}, q_{sa}^\dagger A_{sr}q_{rb}], \tag{4c} \]
from which we see that \( \dot{\tilde{C}}_F \) is anti-self-adjoint, as required by the fact that it must cancel
against the anti-self-adjoint contribution coming from \( \dot{\tilde{C}}_B \). Thus the self-adjoint part of \( \tilde{C}_F \)
given in Eq. (4b) is separately conserved. This can also be verified directly by using Eq. (2d)
and its adjoint, together with Eq. (1b), as follows:

\[
\dot{\bar{C}}_{sa} F = \frac{1}{2} \sum_{rsa} \left[ A_{rs} q_{sa} q_{ra} + q_{sa} \dot{q}_{ra} \right] = -\frac{1}{2} \sum_{rsa} \left[ A_{rs} q_{sb} B_{ab} q_{ra} + q_{sa} B_{ab}^\dagger \dot{q}_{rb} \right] = -\frac{1}{2} \sum_{rsab} \left[ A_{rs} q_{sb} B_{ab} q_{ra}^\dagger - q_{sa} B_{ba} q_{rb}^\dagger \right] = 0 .
\]  

(4d)

In writing the Ward identities to be discussed in the next section, several auxiliary quantities related to the above discussion will be needed. First of all, we will need a self-adjoint operator Hamiltonian \( H \), the trace of which gives the trace Hamiltonian \( \text{Tr} H \).

This can be constructed from the self-adjoint part of any cyclic permutation of the factors in Eq. (2c), and so is not unique. We will adopt the simplest choice, with fermionic terms given by the expression

\[
H = H^\dagger = -\frac{1}{2} \sum_{sab} \left[ p_{sa} q_{sb} B_{ab} + B_{ab} p_{sa} q_{sb} \right] .
\]  

(5a)

Because this is a function only of the dynamical variables but not of the fixed bosonic matrix \( A_{rs} \), under a unitary transformation of the dynamical variables \( p_{sa} \to U^\dagger p_{sa} U, q_{sb} \to U^\dagger q_{sb} U, B_{ab} \to U^\dagger B_{ab} U \), the Hamiltonian \( H \) of Eq. (5a) has the attractive feature of being unitary covariant, \( H \to U^\dagger H U \). An alternative expression for the operator Hamiltonian \( H \), that yields the same trace Hamiltonian \( \text{Tr} H \), is given by

\[
\frac{1}{2} \sum_{sab} \left[ q_{sb} B_{ab} p_{sa} + (q_{sb} B_{ab} p_{sa})^\dagger \right] = \frac{1}{2} \sum_{sab} \left[ q_{sb} B_{ab} p_{sa} + \sum_{ru} A_{sr} q_{ra} B_{ba} p_{ub} A_{us}^{-1} \right] ,
\]  

(5b)

but since this explicitly involves both \( A_{sr} \) and its inverse \( A_{us}^{-1} \), it is a less natural choice than Eq. (5a) (it is not a unitary covariant, as well as being less tractable), and we will not use it in the discussion that follows.
We will also need to evaluate the anticommutator expression

\[ i_{\text{eff}} \tilde{C}_{\text{eff}} \equiv \frac{1}{2} \{ \tilde{C}, i_{\text{eff}} \} \equiv -\hbar(1 + \mathcal{K} + \mathcal{N}) \quad , \tag{5c} \]

where \( i_{\text{eff}} \) and \( \hbar \) are the effective imaginary unit and Planck constant given by the ensemble expectation \( \langle \tilde{C} \rangle_{AV} = i_{\text{eff}} \hbar \) (see Eq. (4.11b) of [1]), and where \( -\hbar \mathcal{K} \) and \( -\hbar \mathcal{N} \) are respectively the \( c \)-number and operator parts of the fluctuating part of \( i_{\text{eff}} \tilde{C}_{\text{eff}} \). At this point we introduce the specialization that the fixed matrix \( A_{rs} \) commutes with \( i_{\text{eff}} \),

\[ [i_{\text{eff}}, A_{rs}] = 0 \quad , \tag{6a} \]

as a consequence of which, by the cyclic identities, we have

\[ \text{Tr}_{i_{\text{eff}}} \tilde{C}_{F}^{sa} = \frac{1}{2} \sum_{rsa} \text{Tr} [i_{\text{eff}}, A_{rs}] q_{sa} q_{ra}^{\dagger} = 0 \quad . \tag{6b} \]

This implies that it is consistent to ignore the self-adjoint part of \( \tilde{C} \) in forming the canonical ensemble. Since ensemble expectations are then functions only of \( i_{\text{eff}} \), a second consequence of Eq. (6a) is that

\[ \langle \tilde{C}_{F}^{sa} \rangle_{AV} = \frac{1}{2} \sum_{rsa} [A_{rs}, \langle q_{sa} q_{ra}^{\dagger} \rangle_{AV}] = 0 \quad , \tag{6c} \]

which implies that even in the presence of \( \tilde{C}_{F}^{sa} \), we can still define an effective imaginary unit by the phase of the ensemble expectation of \( \tilde{C} \).

Returning to Eq. (5c), we now specify conditions to make the separation into terms \( \mathcal{K} \) and \( \mathcal{N} \) unique. In [1] a normal ordering prescription in the emergent field theory was invoked, but here we stay within the underlying trace dynamics, and impose the natural conditions that \( \mathcal{K} \) and \( \mathcal{N} \) are respectively the \( c \)-number part, and the traceless part, of Eq. (5c). Then as a consequence Eq. (6b), the self-adjoint part \( \tilde{C}_{F}^{sa} \) makes a vanishing contribution to \( \mathcal{K} \), which
therefore is a real number, while the operator $\mathcal{N}$ receives an anti-self-adjoint contribution $\mathcal{N}^{asa}$ given by

$$-\hbar\mathcal{N}^{asa} = i\text{eff}\, \tilde{C}^{asa}_{\text{eff}}. \quad (6d)$$

Let us turn now to a second model for the bilinear fermionic trace Lagrangian, which has a similar structure to that of Eq. (1a), but with the matrix $A_{rs}$ now itself a dynamical variable. Since $\dot{A}_{rs}$ is no longer zero, to get a trace Lagrangian that is real up to time derivative terms, we must redefine the fermion kinetic part of Eq. (1a) according to

$$\mathbf{L} = \text{Tr} \sum_{rsab} \left[ q^\dagger_{ra} A_{rs} (q_{sa} + q_{sb} B_{ab}) + \frac{1}{2} q^\dagger_{ra} \dot{A}_{rs} q_{sa} \right] + \text{bosonic} \quad . \quad (7a)$$

The canonical momentum $p_{ra}$ is unchanged in form, but now there is a bosonic canonical momentum $P_{rs}$ conjugate to $A_{rs}$ given by

$$P_{rs} = \frac{\delta \mathbf{L}}{\delta \dot{A}_{rs}} = -\frac{1}{2} \sum_a q_{sa} q^\dagger_{ra} + \frac{\delta \mathbf{L}_{\text{bosonic}}}{\delta \dot{A}_{rs}} \quad . \quad (7b)$$

Since $(q_{sa} q^\dagger_{ra})^\dagger = -q_{ra} q^\dagger_{sa}$, the canonical momentum $P_{rs}$ now has the adjoint behavior $P^\dagger_{rs} \neq P_{sr}$, and as a consequence, the contribution of the canonical pair $A_{rs}, P_{rs}$ to $\tilde{C}$ is no longer anti-self-adjoint, but instead has a self-adjoint part

$$\left( \sum_{rs} [A_{rs}, P_{rs}] \right)^{sa} = \frac{1}{2} \sum_{rs} [A_{rs}, P_{rs} - P^\dagger_{sr}] = -\frac{1}{2} \sum_{rs} [A_{rs}, q_{sa} q^\dagger_{ra}] \quad , \quad (7c)$$

which exactly cancels the self-adjoint fermionic contribution of Eq. (4b). Thus, when the matrix $A_{rs}$ is elevated to a dynamical variable, the Noether charge $\tilde{C}$ is purely anti-self-adjoint.

3. Fluctuation Terms in the Trace Dynamics Ward Identities

We proceed now to work out the implications of the Lagrangian of Eq. (1a) for the trace dynamics Ward identities. To make contact with Eqs. (6.7a) of [1], we shall not
need the most general form of these identities, but only the statement that the quantities $D_{q_{ra eff}}$ and $D_{p_{ra eff}}$ vanish when sandwiched between general polynomial functions of the “eff” projections of the dynamical variables, and averaged over the zero source canonical ensemble. The ensemble equilibrium distribution is given by $\rho = Z^{-1} \exp(-\lambda \text{Tr}_{\text{eff}} \tilde{C} - \tau H)$, with $\lambda$ and $\tau$ parameters characterizing the ensemble, and with $Z$ (the “partition function”) the ensemble normalizing factor. For a fermionic $x_u$, the expression $D_{x_{u eff}}$ is given by

$$D_{x_{u eff}} = -\tau \dot{x}_{u eff} \text{Tr} \tilde{C}_{i eff} W_{\text{eff}}$$

$$+ [i \text{eff} W_{\text{eff}}, x_{u eff}] + \sum_{s, \ell} \omega_{us} \epsilon_{\ell} \left( W_{s}^{Re} \frac{1}{2} \{ \tilde{C}, i \text{eff} \} W_{s}^{Lt} \right)_{\text{eff}}.$$  

(8)

Here $\omega_{us}$ is a matrix with element -1 when $s$ is the label of the variable $x_s$ conjugate to $x_u$, and 0 otherwise, and $W$ is a general self-adjoint bosonic polynomial in the dynamical variables. The quantities in the final term are defined by writing the variation of $W$ when the variable $x_s$ is varied (which we denote by $\delta x_s W$) in the form

$$\delta x_s W = \sum_{\ell} W_{s}^{Lt} \delta x_s W_{s}^{Re} ,$$  

(9a)

where $\ell$ is a composite index that labels each monomial in the polynomial $W$, as well as each occurrence of $x_s$ in the respective monomial term. In this notation we have

$$\frac{\delta W}{\delta x_s} = \sum_{\ell} \epsilon_{\ell} W_{s}^{Re} W_{s}^{Lt} ,$$  

(9b)

with $\epsilon_{\ell}$ the grading factor appropriate to $W_{s}^{Re}$ and to $W_{s}^{Lt} x_s$ (which must both be of the same grade since we have defined $W$ to be bosonic).

We will apply the above expressions when $W$ is taken as the Hamiltonian $H$ with fermionic terms given by Eq. (5a). For the fermionic variations of $H$ we find

$$\delta H = -\frac{1}{2} \sum_{sab} (\delta p_{sa} q_{sb} B_{ab} + p_{sa} \delta q_{sb} B_{ab} + B_{ab} \delta p_{sa} q_{sb} + B_{ab} p_{sa} \delta q_{sb}) ,$$  

(10a)
from which we can read off the factors $W_s^{R1}, W_s^{R2b}$, and $\epsilon_1$ needed in Eq. (8). For example, when $x_u$ is the variable $q_{sa}$, the index $s$ in Eq. (8) labels the canonical conjugate variable $p_{sa}$. Referring to Eq. (9a), we see that the composite index $\ell$ takes the respective values 1 and 2, $b$ for the two factor orderings in Eq. (10a), with

$$ W_s^{R1} = -\frac{1}{2} \sum_b q_{sb} B_{ab} , \quad W_s^{L1} = 1 , \quad \epsilon_1 = -1 \quad ,$$

$$ W_s^{R2b} = -\frac{1}{2} q_{sb} , \quad W_s^{L2b} = B_{ab} , \quad \epsilon_{2b} = -1 \quad .$$

The corresponding expressions when $x_u$ is the variable $p_{sa}$ have a similar structure that can be easily read off from the terms in Eq. (10b) in which $\delta q_{sb}$ appears. Assembling the various pieces of Eq. (8), and using Eqs. (5c) and (9b), we get the following two formulas,

$$\mathcal{D}q_{ra eff} = -\tau \dot{q}_{ra eff} \text{Tr}(\tilde{C}^{asa} + \tilde{C}^{asa})i_{eff}H_{eff} + i_{eff}[H_{eff}, q_{ra eff}]$$

$$-\hbar(1 + K)\dot{q}_{ra eff} + \frac{1}{2} \hbar \sum_b (q_{rb} \{ B_{ab}, N^{asa} + N^{asa} \})_{eff} \quad ,$$

$$\mathcal{D}p_{ra eff} = -\tau \dot{p}_{ra eff} \text{Tr}(\tilde{C}^{asa} + \tilde{C}^{asa})i_{eff}H_{eff} + i_{eff}[H_{eff}, p_{ra eff}]$$

$$-\hbar(1 + K)\dot{p}_{ra eff} - \frac{1}{2} \hbar \sum_b (\{ B_{ba}, N^{asa} + N^{asa} \} p_{rb})_{eff} \quad ,$$

where we have explicitly separated $\tilde{C}$ and $N$ into self-adjoint (superscript sa) and anti-self-adjoint (superscript asa) parts. Taking the adjoint of the first of these equations, and remembering that $B_{ab}^\dagger = -B_{ba}$, we also get for comparison the formula

$$(\mathcal{D}q_{ra eff})^\dagger = -\tau \dot{q}_{ra eff}^\dagger \text{Tr}(\tilde{C}^{asa} - \tilde{C}^{asa})i_{eff}H_{eff} + i_{eff}[H_{eff}, q_{ra eff}]$$

$$-\hbar(1 + K)\dot{q}_{ra eff}^\dagger - \frac{1}{2} \hbar \sum_b (\{ B_{ba}, N^{asa} - N^{asa} \} q_{rb})_{eff}^\dagger \quad .$$

### 4. Discussion

The formulas of Eqs. (11a) and (11b), which so far involve no approximations, are the analogs within the model of Eq. (1a) of the similar formulas given in Eqs. (6.7a) of ref. [1]. They differ from Eqs. (6.7a) in a number of respects.
1. First of all, the structure of the term involving $\mathcal{N}$ is different from what appears in [1] because the simplest choice for the self-adjoint operator Hamiltonian $H$, when the matrices $A_{rs}$ and $B_{ab}$ are non-trivial operators, has the structure of Eq. (5a), in both terms of which $p_{sa}$ stands to the left of $q_{sb}$. When $B_{ab} = im\delta_{ab}$, corresponding to a mass term, this reduces to $H = -im \sum_{sa} p_{sa} q_{sa}$, which when $A_{rs} = \delta_{rs}$ further reduces to $H = -im \sum_{sa} q_{sa}^\dagger q_{sa}$, which does not have the commutator structure assumed on an ad hoc basis in Eq. (6.6) of [1]. As a result, in Eq. (11b) the creation operator $q_{rb}^\dagger$ automatically stands on the right, and the assumption made in [1], that $\mathcal{N}$ is normal ordered, is not necessary.

2. In the treatment here, we have based the separation of the fluctuation term into $\mathcal{K}$ and $\mathcal{N}$ terms on a decomposition into $c$-number and traceless parts, rather than an invocation of normal ordering. As a result, we saw that $\mathcal{K}$ receives no anti-self-adjoint contribution, and so is a real (rather than a complex) number. In terms of the discussion of Chapter 6 of [1], this means that the model of Eq. (1a) does not lead to energy-driven reduction, which requires a nonzero imaginary part of $\mathcal{K}$. Localization-driven reduction, which arises from the anti-self-adjoint part of $\mathcal{N}$, is still allowed.

3. In the generic case when $A_{rs}$ is not equal to $\delta_{rs}$ in any sector, the canonical momentum $p_{sa}$ is not the same as the adjoint $q_{sa}^\dagger$. So even when the $\tau$ terms in Eqs. (11a) and (11b) are dropped, there is no contradiction arising from the fact that $\mathcal{N}_{asa}$ appears in the second equation of Eq. (11a) and in Eq. (11b) with opposite signs. Thus, in the generic case, the inconsistency discussed following Eqs. (6.7a) of [1] is not present.
4. However, there is a specialization of Eqs. (11a) and (11b) in which an analog of the problem noted in ref. [1] persists. Suppose we divide the fermionic degrees of freedom $q_{ra}$ into two classes I and II, based on the value of the index $r$, and take $A_{rs}$ to be block diagonal within the two classes. For the class I degrees of freedom, we take $A_{rs}$ to be nontrivial, so that $p_{ra} \neq q_{ra}^\dagger$. For the class II degrees of freedom, we take $A_{rs} = \delta_{rs}$, so that $p_{ra} = q_{ra}^\dagger$. Then if we restrict Eqs. (11a) and (11b) to $r$ values for class II degrees of freedom, we see that the second equation in Eq. (11a) has the same structure as Eq. (11b), except that the terms involving $\tilde{C}^{sa}$ and $N^{asa}$ both have opposite signs in the two equations. Hence taking the difference between the second equation in Eq. (11a) and Eq. (11b), we get for $r$ in class II,

$$D q_{ra}^{\dagger} - (D q_{ra}^{\dagger})^\dagger = -2\tau q_{ra}^{\dagger} \text{Tr} \tilde{C}^{sa} i_{eff} H_{eff}$$

$$-\hbar \sum_b \{ B_{ba}, N^{asa} \} q_{rb}^{\dagger} \text{eff}. \quad (12)$$

This expression must vanish when inserted (sandwiched between polynomials in the variables) in canonical ensemble averages. Hence in this case, which is a somewhat more general version of the model formulated in Eq. (6.6) of [1], the terms involving $N^{asa}$ must effectively average to be of the same order of magnitude as the $\tau$ terms, which were neglected in the approximation scheme of Chapter 5 of [1]. There is no inconsistency in the Ward identities of Eqs. (11a) and (11b), or in Eq. (12) that is derived from them, but in this case one cannot consistently drop the $\tau$ terms and reinterpret these equations as operator equations at the level of the emergent quantum theory.

To conclude, we have reexamined the apparent inconsistency arising from Eqs. (6.7a) of [1], taking into account details that were not sufficiently carefully dealt with there. We
see that in the generic case one can still use operator analogs of Eqs. (11a) and (11b) as the basis for a state vector reduction model. But we have also seen that there is a tendency for the needed anti-self-adjoint driving term to cancel, suggesting some caution, and also more speculatively, suggesting a reason why one might expect the state vector reduction terms to be small corrections to the basic emergent Schrödinger equation.

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References

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