Avoided level crossing spectroscopy with dressed matter waves

André Eckardt and Martin Holthaus

ICFO-Institut de Ciencies Fotoniques, E-08860 Castelldefels (Barcelona), Spain and
Institut für Physik, Carl von Ossietzky Universität, D-26111 Oldenburg, Germany

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We devise a method for probing resonances of macroscopic matter waves in shaken optical lattices by monitoring their response to slow parameter changes, and show that such resonances can be disabled by particular choices of the driving amplitude. The theoretical analysis of this scheme reveals far-reaching analogies between dressed atoms and time-periodically forced matter waves.

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Recently it has been demonstrated experimentally that a macroscopic matter wave of ultracold bosonic atoms confined in an optical lattice can be controlled in a systematic manner by strong, off-resonant time-periodic forcing: Under suitably selected conditions, “shaking” the lattice with kilohertz frequencies mainly effectuates a modification of the tunneling matrix element connecting adjacent lattice sites. In the regime of weak interaction, this phenomenon has been inferred from the expansion of a Bose-Einstein condensate in a one-dimensional lattice geometry [1]. A subsequent experiment [2] utilizes the reduction of the tunneling matrix element to augment the relative importance of interparticle repulsion, such that the quantum phase transition from a superfluid to a filled Mott-insulator [3, 4] is induced by adiabatically varying the amplitude of the driving force [3].

These landmark experiments [1, 2] clearly confirm that there are efficient control mechanisms for ultracold atomic gases resulting from time-periodic modulation. The situation encountered here is akin to the dressed-atom approach: An atom in a laser field becomes “dressed” by that field and changes its behavior [6]. Similarly, a many-body matter wave becomes dressed in response to time-periodic forcing and acquires properties which the unforced, bare matter wave did not have.

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A system of ultracold bosonic atoms in a shaken, sufficiently deep one-dimensional optical lattice is described, in the frame of reference co-moving with the lattice, by the driven Bose-Hubbard model defined by the Hamiltonian \( H(t) = H_{\text{tun}} + H_{\text{int}} + H_{\text{drive}}(t) \) [3, 2]. With \( b_\ell \) and \( \hat{n}_\ell = b_\ell^\dagger b_\ell \) denoting the bosonic annihilation and the number operator for the Wannier state located at the site labeled by \( \ell = 1, 2, \ldots, M \), one has \( H_{\text{tun}} = -J \sum_{\ell=1}^{M} (b_\ell^\dagger b_{\ell+1} + b_{\ell+1}^\dagger b_\ell) \), where the positive hopping parameter \( J \) implements the kinetics, assumed to be exhausted by tunneling between adjacent sites. Moreover, \( H_{\text{int}} = \frac{U}{M} \sum_{\ell=1}^{M} \hat{n}_\ell (\hat{n}_\ell - 1) \) with positive interaction parameter \( U \) describes the repulsion of particles occupying the same site. Finally, \( H_{\text{drive}}(t) = K \omega \cos(\omega t) \sum_{\ell=1}^{M} \ell \hat{n}_\ell \) models time-periodic forcing with amplitude \( K \omega \) and frequency \( \omega \). With the particle number fixed to \( N \), the filling \( n \) is given by the ratio \( n = N/M \).

as witnessed by the experiments [1, 2], in a time-averaged sense the driven system governed by \( \dot{H}(t) \) behaves similar to a system described by the effective, time-independent Hamiltonian \( \hat{H}_{\text{eff}} \equiv J_0(K \omega/\hbar \omega) H_{\text{tun}} + H_{\text{int}} \), which means that the effect of the time-periodic force is captured by replacing the tunneling matrix element \( J \) by \( J_{\text{eff}} \equiv J_0(K \omega/\hbar \omega) J \), with \( J_0 \) denoting the ordinary Bessel function of order zero. this modification of the hopping matrix element is a hallmark of driven quantum tunneling [3]; it has been clearly observed for single-particle tunneling in strongly driven double-well potentials [9]. While it becomes exact for a single particle on a one-dimensional lattice endowed with nearest-neighbor coupling [10], the dynamics are considerably more involved in the many-body case described by the driven Bose-Hubbard model. due to the manifold ways to create excitations in the many-body system, the \( H_{\text{eff}} \)-description is endangered by a multitude of resonances, and holds approximately only when \( \hbar \omega \) is large compared to both energy scales which characterize the undriven system, \( U \) and \( nJ \) [3, 11, 12]. To further explore the newly emerging notion of adiabatic control of driven macroscopic matter waves [2], it is now of great importance to study such resonances in detail: When do they occur, how strong are they, are they detrimental to coherent control or can they, perhaps, even be exploited? these questions mark the scope of the present Letter. By means of numerical simulations for small systems, we first outline an experimentally feasible detection scheme which allows one to locate major excitation channels in parameter space, and to probe their strengths. we also demonstrate that the strength of such excitation channels again is subject to coherent control: A resonance can be completely quenched by an appropriate choice of the driving amplitude. in a second step, we make closer contact between the dressed-atom picture and the driven matter waves considered here by studying their quasienergy spectrum. in the final third step we explain our findings quantitatively by means of perturbation theory for Floquet states.

Consider the following scenario: A system conforming to the undriven Bose-Hubbard model \( \hat{H}_{\text{tun}} + \hat{H}_{\text{int}} \) is prepared in its ground state for \( U/J = 0.1 \). then a drive \( \hat{H}_{\text{drive}}(t) \) is switched on, with an amplitude increasing linearly in time, and a high frequency \( h \omega/J = 20 \). Since this
drive is sufficiently off-resonant, one expects the system to adiabatically follow the ground state of $\hat H_{\text{eff}}$. After the working amplitude $K_\omega$ has been reached, it is held constant. Then the interaction parameter $U$ is ramped up at constant rate $\eta \equiv \dot{U}/J$ (with $T = 2\pi/\omega$) into the regime where resonances should make themselves felt. In a laboratory experiment this can be done, e.g., by increasing the transversal confinement used to create the effective one-dimensional geometry. We have simulated this protocol for a small system with $N = 5$ particles on $M = 7$ lattice sites. Starting in the ground state at interaction strength $U/J = 0.1$, a drive of frequency $\hbar \omega/J = 20$ has been linearly ramped up within 50 cycles $T = 2\pi/\omega$ to the working amplitude $K_\omega$, before $U$ is increased at various rates $\eta \equiv \dot{U}/J = 0.3$ (black), 0.1 (blue), 0.03 (magenta), 0.01 (green), 0.003 (red), 0.001 (brown). We plot the squared overlap $P_{\text{eff}}(t)$ of the instantaneous ground state of $\hat H_{\text{eff}}$ with the actual time-evolved state versus $U(t)/\hbar \omega$ at integer $t/T$. For large $U(t)/\hbar \omega$, $P_{\text{eff}}$ decreases with decreasing $\eta$. For $K_\omega/\hbar \omega = 2.5$ there is strong resonant excitation at $U/\hbar \omega = 2/3$ (a). For $K_\omega/\hbar \omega = 3.4$ this resonance is quenched, and another one around $U/\hbar \omega = 1$ becomes active (b).

![FIG. 1: Exact time-evolution of $N = 7$ particles on $M = 7$ lattice sites. Starting in the ground state at interaction strength $U/J = 0.1$, a drive of frequency $\hbar \omega/J = 20$ has been linearly ramped up within 50 cycles $T = 2\pi/\omega$ to the working amplitude $K_\omega$, before $U$ is increased at various rates $\eta \equiv \dot{U}/J = 0.3$ (black), 0.1 (blue), 0.03 (magenta), 0.01 (green), 0.003 (red), 0.001 (brown). We plot the squared overlap $P_{\text{eff}}(t)$ of the instantaneous ground state of $\hat H_{\text{eff}}$ with the actual time-evolved state versus $U(t)/\hbar \omega$ at integer $t/T$. For large $U(t)/\hbar \omega$, $P_{\text{eff}}$ decreases with decreasing $\eta$. For $K_\omega/\hbar \omega = 2.5$ there is strong resonant excitation at $U/\hbar \omega = 2/3$ (a). For $K_\omega/\hbar \omega = 3.4$ this resonance is quenched, and another one around $U/\hbar \omega = 1$ becomes active (b).](image-url)
by an integer multiple of $\hbar$ strength of resonant coupling and determines the degree of the quasienergy copies corresponding to the ground

$U/J$ cited states with increasing $\omega/J$ emerge if eigenstates of $\hat{U}/J$ patterns which appear when $r$ able to $\hat{U}/J$ pronounced avoided crossings when $\hat{U}/J$ undergoes a wide avoided crossing with such a band

of particle-hole excitations of $\hat{U}/J$ of quasienergy levels belonging to different copies of the

FIG. 2: (a) Quasienergy spectrum of a driven Bose-Hubbard system with $N = M = 5$, $h \omega/J = 20$, and $K_{\omega}/h \omega = 2$ versus $U/J$. Bands with different slopes belong to different types of particle-hole excitations of $\hat{H}_{\text{eff}}$. Resonant coupling of such bands results in avoided crossings. The isolated quasienergy level, highlighted in (b), emerges from the ground state of the undriven system. Clearly visible are the avoided crossings at $U/h \omega \approx 2/3$ ($U/J \approx 13$) and $U/h \omega \approx 1$ ($U/J \approx 20$) which have been detected dynamically in Fig. 1, whereas there are no avoided crossings at $1/3$ and $1/2$.

identifiable through their slopes. While in Fig. 2 (a) quasienergy levels belonging to different copies of the $\hat{H}_{\text{eff}}$-spectrum hardly “notice” each other for interaction strengths $U/J$ much smaller than $h \omega/J = 20$, there are pronounced avoided crossings when $U/J$ becomes comparable to $h \omega/J$, prominently exemplified by the complex patterns which appear when $U/J$ is an integer multiple of $h \omega/J$. Such avoided crossings indicate resonances which emerge if eigenstates of $\hat{H}_{\text{eff}}$ are energetically separated by an integer multiple of $h \omega$; their size quantifies the strength of resonant coupling and determines the degree of deviation from the $\hat{H}_{\text{eff}}$-description.

Fig. 2 (b) shows a detail of Fig. 2 (a), focusing on one of the quasienergy copies corresponding to the ground state of $\hat{H}_{\text{eff}}$. After separating from the bands of excited states with increasing $U/J$, thus indicating the superfluid-to-Mott insulator transition [2, 3], this level crosses several bands associated with different copies of the $\hat{H}_{\text{eff}}$-spectrum without being notably affected, until it undergoes a wide avoided crossing with such a band

at $U/J \approx 2h \omega/J \approx 13$, and subsequently an even wider one around $U/J \approx h \omega/J = 20$. These avoided crossings explain the excitation observed in Fig. 1. The dynamical detection scheme illustrated by that figure relies on the adiabatic principle for Floquet states [12]. With increasing $U$, the state $|\psi(t)\rangle$ adjusts itself to the slowly changing parameter and thus follows the quasienergy level corresponding to the ground state of $\hat{H}_{\text{eff}}$, until it reaches an avoided crossing too wide to be passed diabatically. Then an incomplete Landau-Zener transition to the anticrossing state excites the system. According to Landau-Zener estimates, and in agreement with the simulations depicted in Fig. 1, the excitation probability increases exponentially with both the width of the anticrossing and decreasing parameter speed. Thus, the method of detecting resonances in dressed matter waves by monitoring their response to slow parameter changes can be regarded as a kind of avoided level crossing spectroscopy.

Note that in contrast to the regime of linear response, suitable for probing properties of the undriven system, here we consider the excitation of a system which has already been strongly modified by the driving force, in a manner described by $\hat{H}_{\text{eff}}$. Moreover, besides the wide, “active” avoided quasienergy crossings there also is a host of tiny avoided crossings, reflecting the high density of quasienergies in each Brillouin zone, so that effectively adiabatic dynamics on the level of $\hat{H}_{\text{eff}}$ actually includes fully diabatic Landau-Zener tunneling through these narrow anticrossings. In an infinitely large system with a truly dense quasienergy spectrum, the existence of a well-defined adiabatic limit cannot, thus, be expected [17]. However, realistic parameter variations take place on finite time-scales, in all likelihood making the system “blind” against such small features of the spectrum.

We now formalize our reasoning. For each admissible set $\{n_{\ell}\}$ of site-occupation numbers, we employ the usual Fock states $|\{n_{\ell}\}\rangle \equiv \prod_{\ell} |n_{\ell}\rangle$ for constructing an orthonormal basis of Floquet-Fock states $|\{n_{\ell}\}, \tilde{m}\rangle \equiv |\{n_{\ell}\}\rangle \exp[-iK_{\omega} \sin(\omega t) \sum_{\ell} n_{\ell} \exp(i\tilde{m}_{\ell}\omega t)]$ in $\mathcal{H} \otimes \mathcal{T}$, with $\tilde{m}$ serving as “photon” index for distinguishing different Brillouin zones. Invoking the scalar product $\langle \cdot | \cdot \rangle \equiv \frac{1}{T} \int_{0}^{T} dt \langle \cdot | \cdot \rangle$, the quasienergy operator $\hat{\tilde{Q}} \equiv \hat{\tilde{Q}}_{0} + \hat{\tilde{Q}}_{1}$ of the driven Bose-Hubbard model possesses the matrix elements

$$
\begin{align*}
\langle \{n_{\ell}'\}, \tilde{m}' | \hat{\tilde{Q}}_{0} | \{n_{\ell}\}, \tilde{m} \rangle &= \delta_{\tilde{m}',\tilde{m}} \langle \{n_{\ell}'\} | \{\tilde{m} \omega \hat{H}_{\text{int}} + j_{0} \hat{H}_{\text{tun}}\} | \{n_{\ell}\} \rangle, \\
\langle \{n_{\ell}'\}, \tilde{m}' | \hat{\tilde{Q}}_{1} | \{n_{\ell}\}, \tilde{m} \rangle &= (1 - \delta_{\tilde{m}',\tilde{m}}) j_{n_{\ell}' - \tilde{m}'} \langle \{n_{\ell}'\} | \hat{H}_{\text{tun}} | \{n_{\ell}\} \rangle,
\end{align*}
$$

where $j_{\nu} \equiv J_{\nu}(K_{\omega}/h \omega)$ indicates the Bessel function of order $\nu$ evaluated at $K_{\omega}/h \omega$, and $s \equiv \sum_{\ell} \ell(n_{\ell} - n_{\ell}')$, giving $s = +1$ ($s = -1$) if $\hat{H}_{\text{tun}}$ transfers one particle by one site to the left (right) [11, 12]. This splitting of
the quasienergy operator is performed such that $\hat{Q}_0$ acts within each subspace with fixed “photon” number $\hat{m}$ in a manner conforming to $H_{\text{eff}}$, whereas $Q_1$ describes the coupling between these subspaces.

Let us assume that $U$ is comparable to $\hbar\omega$ while $\hbar\omega \gg n|J|$ and treat $Q_1$ by perturbation theory. For $U \gg n|J|$ the ground state of $H_{\text{eff}}$ is approximately given by the extreme Mott-insulator state $|\text{MI}\rangle = |\{n_\ell = n\}\rangle$ with $n$ particles localized at each site. Excited states differ from $|\text{MI}\rangle$ by particle-hole excitations of energy $U$; these excitations form bands with widths on the order of $\sim n|J|$ due to tunneling of the particles and holes “on top” of $|\text{MI}\rangle$. Thus, near $U = \alpha\hbar\omega$ with integer $\alpha = 1, 2, \ldots$ the drive is resonant with respect to the creation of a single particle-hole pair; eigenstates of $\hat{Q}_0$ differing from $|\text{MI}\rangle$ by one particle-hole pair and $\alpha$ “photons” are degenerate with $|\text{MI}\rangle$ and couple directly (i.e., in first order) via $Q_1$ by matrix elements of size $\sim \sqrt{n(n+1)}J_{\lambda\Delta\hat{m}}$. This coupling leads to the large avoided level/level crossings visible in Fig. 2 at $U/J$ close to 20 and 40.

In second order, the simultaneous creation of two particle-hole pairs via (quasi-)energetically distant intermediate states is taken into account. Intriguingly, second-order coupling between states differing from $|\text{MI}\rangle$ by $\beta$ “photons” and two separate particle-hole excitations of total energy $2U$, expected near $U = \beta\hbar\omega/2$ with $\beta = 1, 3, 5$ (omitting first-order resonances), vanishes completely due to destructive interference between paths involving different intermediate states. This explains why there is no avoided crossing at $U/J \approx 10$ in Fig. 2. However, there are non-vanishing second-order processes creating two overlapping particle-hole pairs, having two particles or holes sitting at the same site. Assuming unit filling $n = 1$, the only possibility is to place both particles at the same site, costing the excitation energy $3U$. For such excitations near $U = \gamma\hbar\omega/3$ with $\gamma = 1, 2, 4, 5, \ldots$, we find coupling constants $c_\gamma \propto \sqrt{(n+1)(n+2)}/\hbar\omega$ with strengths $c_\gamma \equiv \sum_{\gamma_{\hat{m}J}} \langle \gamma_{\hat{m}J}|(\gamma_{\hat{m}J}\hat{m}^\dagger(J_{\lambda\Delta\hat{m}}\hat{m}^\dagger + J_{\lambda\Delta\hat{m}}\hat{m}^\dagger) - (2U/\hbar\omega - \gamma - \hat{m}^\dagger)^{-1} - (U/\hbar\omega + \hat{m}^\dagger)^{-1}\rangle$ which vanish for odd $\gamma$. The plot of $c_\gamma$ depicted in Fig. 3 testifies that these strengths depend in an oscillating manner on the driving amplitude. In particular, it is possible to adjust that amplitude such that the resonant coupling strength vanishes. For instance, the zero of $c_2$ at $K_\omega/\hbar\omega \approx 3.4$ is the reason for the resonance quenching illustrated in Fig. 1. In $\nu$th order, coupling matrix elements generally are $\sim nJ(nJ/\hbar\omega)^{\nu-1}$; however, we have hardly noticed third-order effects in our numerical simulations. Thus, degenerate-state perturbation theory in $H \otimes T$ systematically uncovers the hierarchy of resonances which, in a system with slowly changing parameters, become observable order by order with decreasing parameter speed.

To conclude, we have outlined a scheme for probing resonances which endanger the adiabatic control of macroscopic matter waves achievable through time-periodic forcing 2. The theoretical analysis of this scheme reveals far-reaching conceptual similarities between dressed atoms and dressed matter waves in shaken optical lattices, thus opening up wide new grounds between quantum optics and matter-wave physics.

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* email: andre.eckardt@icfo.es

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