Formation, diffusion, and accreting pollution of DB white dwarfs

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ABSTRACT

Context. Over 1500 DBZ or DZ white dwarfs (WDs) have been observed so far, and polluted atmospheres with metal elements have been found among these WDs. The surface heavy element abundances of known DBZ or DZ WDs show an evolutionary sequence. Cooling, diffusion, and accretion are important physical processes in WD evolution which can alter the element abundances of the WD surface.

Aims. Using the stellar evolutionary code, we investigated the DB WD formation and the effects of input parameters – e.g., mixing length parameter ($\alpha$), thermal mixing efficiency ($\eta$), and the metallicity (Z) – on the structures of these DB WDs. The impacts of the convective zone mass ($M_{cvz}$), cooling timescales, diffusive timescales ($\tau_{diff}$), and the mass-accretion rate ($\dot{M}$) on the element abundances of the WDs’ surfaces are discussed. By comparing the theoretical model results with observations, we try to understand the evolutionary sequence of the heavy element abundance on DBZ WD surfaces.

Methods. By using Modules for Experiments in Stellar Evolution, we created DB WDs, and simulated the element diffusion due to high gravitational fields and the metal-rich material accretion coming from the planet disrupted by the WD. Then, we calculated the element abundances of these DB WDs for a further comparison with observations.

Results. In our models, the input parameters ($\alpha_{MLT}$, $\eta$, and $Z$) have a very weak effect on DB WD structures, including interior temperatures, chemical profiles, and convective zones. They hardly affect the evolution of the heavy elements on the surface of DB WDs. The mass-accretion rate and the effective temperature of DB WDs determine the abundances of heavy elements. The evolutionary sequence of the 47 Ca element for about 1500 observed DB or DBZ WDs cannot be explained by the model with a constant mass-accretion rate, but it is very consistent with the model in which the mass-accretion rate decreases by one power law when $T_{\text{eff}} > 10$ kK and it slightly increases by another power law when $T_{\text{eff}} < 10$ kK.

Conclusions. The observed DB WD evolutionary sequence of heavy element abundances originates from WD cooling and the change in the mass-accretion rate.

Key words. white dwarfs – stars: evolution – accretion, accretion disks

1. Introduction

It is well known that single stars with an initial mass between $\sim 1$ and $8\ M_{\odot}$ finally evolve into white dwarfs (WDs). Due to high gravitational fields ($\log g \sim 8\ \text{cm\ s}^{-2}$), the heavy elements on WDs’ surfaces would diffuse downward during WD cooling. Usually, the timescale of diffusion ($\tau_{\text{diff}}$) at the photosphere is approximately several to $10^3$ years (Koester 2009), which is much shorter than the cooling timescale ($\tau_{\text{cool}} \sim 10^7$ yr, e.g., Shapiro & Teukolsky 1983; Zhu et al. 2019; Li et al. 2017, 2020). Therefore, cool WDs should have pure hydrogen (H) or helium (He) atmospheres. The former is referred to as a DA WD, while the latter is called a DB WD. However, Zuckerman et al. (2003) pointed out that more than 25% of WDs are polluted by metal elements such as Mg, Fe, and Na. WDs are referred to as DAZ or DBZ WDs if their spectra show H or He lines with heavy element lines. When only heavy elements lines are displayed in the spectra, WDs are categorized as DZ type. Three different ways are proposed as three possible sources for the surface heavy elements of the WDs, such as primordial or fallback stellar material, interstellar medium, or debris disk produced by WD tidally disrupting rocky objects (i.e., planets, e.g., Farihi 2016).

The pollution of WDs has been explained by ongoing accretion of planetary debris. A number of pieces of observational evidence show the infrared emission from debris disk around polluted WDs (Jura 2003; Farihi et al. 2009; Girven et al. 2012; Vanderburg et al. 2015), thus these polluted WDs become unique laboratories for studying the interior composition of exoplanets (Zuckerman et al. 2007; Koester et al. 2014; Jura & Young 2014). The chemical abundances detected on the surface of polluted WDs reflect the equilibrium between accretion for metal-rich material and diffusive sedimentation (Koester 2009; Bauer & Bildsten 2018).

Dupuis et al. (1992) initially explored the metal traces in WDs, and they also investigated the diffusion of metals accreted onto WDs (Dupuis et al. 1993a,b). Koester & Wilken (2006) calculated the diffusion timescales for some metals in DAZ WDs’ atmosphere, and they estimated the accretion rates for 38 DAZ WDs. Koester (2009) extended the above works to DAZ, DBZ, and DZ WDs. Considering diffusion and thermal mixing, Wachlin et al. (2017) and Bauer & Bildsten (2018) simulated the trace of metals for DAZ WDs. Bauer & Bildsten (2019) discuss the effects of the mixing processes, including convection, gravitational sedimentation, overshoot, and thermal instability on the diffusion. Using new WD envelope models and diffusion, Koester et al. (2020) investigated the atmospheres of carbon-rich WDs. In the theoretical models, the metal abundances of several polluted WDs can be explained well if suitable accretion rates...
are assumed (e.g., Koester 2009; Bauer & Bildsten 2018). They also predict that the metals would slowly settle downward as soon as accretion stops. However, as is shown in Koester (2009) and Bauer & Bildsten (2018), the diffusion timescales of WDs increase when their effective temperature \(T_{\text{eff}}\) decreases. For a WD with \(T_{\text{eff}} < 10\,\text{kK}\), \(T_{\text{diff}}\) is longer than about \(10^8\) yr. Therefore, it is not sufficient to check long-timescale diffusion theory by only comparing theoretical results with several known cool WDs. A comparison involving a large observational sample of DB WDs with different \(T_{\text{eff}}\)'s becomes necessary.

Thanks to many large sky surveys, the number of observed WDs are dramatically increasing (e.g., Gaia Collaboration 2016, 2018; Chambers et al. 2016; Blouin et al. 2019). Up to now, there are more than 60000 WDs in The Montreal White Dwarf Database (Dufour et al. 2017), of which 1023 of them are DBZ or DZ WDs (Coutu et al. 2019). Observationally, Dufour et al. (2007) showed the spectroscopic and photometric data of 147 DZ WDs with \(T_{\text{eff}}\) between about 6 kK and 12 kK. Based on SDSS DR10 and 12, Koester & Kepler (2015) analyzed the data of 1107 DBZ WDs whose effective temperatures are between about 50 kK and 11 kK. Hollands et al. (2017) identified 231 cool DZ WDs with \(T_{\text{eff}}\) lower than 9 kK in SDSS DR12. They discuss the distribution of \(\log[\text{Ca/He}]\) versus \(T_{\text{eff}}\) for the three samples (see Fig. 11 of Hollands et al. 2017). At about \(T_{\text{eff}} \approx 10\,\text{kK}\), Ca abundances rapidly decrease with \(T_{\text{eff}}\) declining. Koester & Kepler (2015) suggest that this trend should be relative to the mass-accretion rates. However, Ca abundances of DZ WDs with \(T_{\text{eff}}\) between 10 kK and 8 kK increase by about 100 times. Hollands et al. (2017) consider that this sharp increase might result from the decrease in the convective zone mass \(M_{\text{cvz}}\) or the increase in \(r_{\text{diff}}\). The second downwards trend of a Ca abundance with \(T_{\text{eff}}\) appears between 9 kK and 4 kK. Hollands et al. (2018) suggest that the trend is relative to \(M_{\text{cvz}}\) or \(r_{\text{diff}}\).

| Cases   | \(\alpha_{\text{MLT}}\) | \(\alpha_{\text{th}}\) | \(Z\) |
|--------|-----------------|-----------------|------|
| Case 1 | 1.8             | 1               | 0.02 |
| Case 2 | 0.8             | 1               | 0.02 |
| Case 3 | 1.8             | 0               | 0.02 |
| Case 4 | 1.8             | 1000            | 0.02 |
| Case 5 | 0.8             | 1               | 0.001|

Notes. The first column gives the case number. Columns 2, 3 and 4 show the values of input parameters \(\alpha_{\text{MLT}},\ \alpha_{\text{th}}\) and \(Z\), respectively.

Table 1. All cases in the present paper are simulated.

MESA, which is a helium dominated atmosphere table for DB WDs, is used to calculate the DB WD atmosphere boundary. The size of the convective zone depends on the mixing length parameter \(\alpha_{\text{MLT}}\). In order to discuss its effect, we consider \(\alpha_{\text{MLT}}\) to be 0.8 and 1.8 in different simulations.

Schatzman (1945) suggested that the high gravitational fields in cool WDs should result in the downward diffusion of heavy elements. By resolving the Burgers equations which give multi-component fluid’s evolutions (Burgers 1969), Thoul et al. (1994) investigated the element diffusion in the interior of the Sun. Using the approach in Thoul et al. (1994), MESA can calculate the chemical diffusion in the stellar interior (Paxton et al. 2015, 2018). The diffusion coefficients that originated from Paquette et al. (1986) and were updated by Stanton & Murillo (2016) are used in our models.

Similarly, metallicity \((Z)\) can also affect stellar evolutions and WD properties. Here, Table 1 gives all cases in which different input parameters are considered.

3. Formation and structures of DB WDs

Many observations have shown that there are some H elements in the atmospheres of DBZ or DZ WDs (Voss et al. 2007; Koester & Kepler 2015; Coutu et al. 2019). However, the ratios of the H to He abundance estimated by these observations are lower than about \(10^{-2}\). These H elements may be continuously accreted by DBZ or DZ WDs from the interstellar medium (Voss et al. 2007; Koester & Kepler 2015; Coutu et al. 2019). Therefore, there might be no H elements left in the atmospheres of DB WDs when they form.

Usually, the range of DBZ or DZ WDs’ masses is between about 0.4 and 1.0\(M_{\odot}\), and their mass distribution has a peak around 0.6\(M_{\odot}\) (e.g., Han 1998; Han et al. 2000; Coutu et al. 2019). Taking 0.6\(M_{\odot}\) DB WD created by the main sequence (MS) star under input parameters in case 1 as an example, we provide all steps involved to create DB WDs in the following.

Firstly, as is illustrated by the black line in the top left panel of Fig. 1, the 3.5\(M_{\odot}\) MS star begins to evolve normally, that is, H starts to burn in the stellar core. The mass-loss rate \((\dot{M})\) is calculated by the “Dutch” scheme (Paxton et al. 2011), in which \(\dot{M}\) of hot and cool stars is provided by Nieuwenhuijzen & de Jager (1990), Nugis & Lamers (2000), Vink et al. (2001), Glebbeek et al. (2009), and Reimers (1975). The element mixing is mainly determined by convection and
thermohaline instability. At this phase, in order to save CPU time, we did not consider gravitational diffusion.

Secondly, as is shown by the red line in the top left panel of Fig. 1, we artificially enhanced the mass-loss rate up to $10^{-4} \, M_\odot \, \text{yr}^{-1}$ when the He-core mass was larger than 0.6 $M_\odot$. The H-rich envelope is rapidly stripped, and the star evolves into a He star. As the red lines in the top right panel of Fig. 1 show, the H abundance ($X(H)$) on the stellar surface decreases from about 0.7 to roughly lower than $10^{-15}$, while $X(He)$ increases up to about 0.98.

Thirdly, the WD cooling is involved, which is given by the green line. At this time, the H element almost is lost, and the He element is the lightest one, and it floats upward toward the stellar surface by gravitational settling. A DB WD is created.

The bottom left panel of Fig. 1 shows the evolution in the HR diagram for the star with different $\alpha_{\text{MLT}}$, $\alpha_{\text{fn}}$, and $Z$. Obviously, the effects of input parameters on evolutionary tracks are negligible. The bottom right panel gives the change in $X(Ca)$ on the stellar surface. The $X(Ca)$ on the stellar surface starts to reduce because of gravitational sedimentation at the WD cooling phase.

Using a similar method, we also created DB WDs with 0.4 and 0.8 $M_\odot$, which are shown in Fig. 2. The changes in $X(H)$, $X(He)$, and $X(Ca)$ on these WD surfaces are given as well.

As the bottom left panel of Fig. 1 shows, the cooling tracks of DB WDs are hardly affected by the input parameters. Similarly, the effects of these input parameters on DB WD internal structures can be negligible.

In Fig. 3, we found that the profiles of the temperature, convective velocity ($v_{\text{conv}}$), He abundance ($X(He)$), and the abundance ratio of Ca to He ($X(Ca)/X(He)$) for the 0.6 $M_\odot$ DB WD at the same effective temperature are similar. Due to the strong gravitational diffusion of the WD, heavy elements sink down and light He element floats up. For example, $X(Ca)$ on the DB WD’s surface has decreased to $10^{-15}$ from an initial $10^{-5}$, while a heavy He envelope with a mass of about 0.02 $M_\odot$ forms around the WD surface. Figure 4 gives the profiles of 0.4 and 0.8 $M_\odot$ DB WDs for case 1. Obviously, in our model, the He layer mass is affected by the WD’s mass. It changes from about 0.1 to 0.01 $M_\odot$ when $M_{WD}$ increases from 0.4 to 0.8 $M_\odot$.

In Fig. 5, we provided the change in the convective-zone mass ($M_{\text{cvz}}$) around the WD surface with $T_{\text{eff}}$. For the 0.6 $M_\odot$ DB WD shown in the left panel of Fig. 5, the effects of input parameters on $M_{\text{cvz}}$ can be negligible. The main reasons are as follows. Firstly, the mixing length parameter $\alpha_{\text{MLT}}$ has a weak effect on $M_{\text{cvz}}$ because a high density of WDs results in a small pressure scale height. For example, it is about 10 cm for a WD with $T_{\text{eff}} = 6$ kK. Secondly, the thermohaline mixing hardly affects the convective zone of DB WDs, while it can significantly affect the $M_{\text{cvz}}$ of DA WDs (Wachlin et al. 2017; Bauer & Bildsten 2018). Compared with the latter ($10^{-15}$–$10^{-11}$ $M_\odot$ when $T_{\text{eff}} > 10$ kK, Koester 2009; Wachlin et al. 2017), the $M_{\text{cvz}}$ of DB is extremely massive, and between about $10^{-9}$–$10^{-7}$ $M_\odot$. A thick convective zone of DB WDs dilutes the effects of thermohaline mixing, which has been discussed by Bauer & Bildsten (2019). Bauer & Bildsten (2019) also mentioned that the mean molecular weight of a DB WD is more than two times that of a DA WD, which dilutes thermohaline mixing effects. Thirdly, metallicity has no effect on $M_{\text{cvz}}$ because the heavy elements rapidly diffuse downward due to the strong gravitational field of WDs.

Compared with $M_{\text{cvz}}$ of 0.6 $M_\odot$ DB WD calculated by Benvenuto & Althaus (1997) and Koester (2009), the $M_{\text{cvz}}$ in this work is similar to both of their results when $T_{\text{eff}} > 14$ kK. We note that $M_{\text{cvz}}$ in this work is between that in Benvenuto & Althaus (1997) and in Koester (2009) when $T_{\text{eff}} < 14$ kK. The right panel of Fig. 5 shows $M_{\text{cvz}}$ in the models of 0.4 and 0.8 $M_\odot$ DB WDs. Compared with the results of Benvenuto & Althaus (1997), $M_{\text{cvz}}$ is more massive in this work. The differences mainly result from the following possible aspects.

Firstly, in Benvenuto & Althaus (1997), the He layer mass of DB WDs is between about $10^{-2}$ and $10^{-6}$ $M_\odot$. However, in
Structures of 0.6 \( M_\odot \) DB WDs in different effective temperatures \((T_{\text{eff}})\) for cases 1, 2, 4, and 5, respectively. The different \( T_{\text{eff}} \)s are given by different lines.

We note that \( T, v_{\text{conv}}, X(\text{He}), \) and \([\text{Ca}/\text{He}]\) represent the temperature, convective velocity, He abundance, and the abundance ratio of Ca to He, respectively.

Secondly, in Benvenuto & Althaus (1997) and Koester (2009), \( M_{\text{cvz}} \) is defined by the thermal time scale. However, \( M_{\text{cvz}} \) is defined by the Ledoux criterion in our results. As discussed in Koester (2009), \( M_{\text{cvz}} \) can differ by orders of magnitude because of different definitions.

4. Accreting pollution of DB WDs

The bottom right panels of Figs. 3 and 4 show that \([\text{Ca}/\text{He}]\) on the surface of a DB WD decreases to about \(10^{-2} M_\odot\) due to gravitational settling when \(T_{\text{eff}} > 20 \text{ kK}\). Chayer et al. (1995a) suggested that some element diffusion can be prevented by radiative levitation when the WD temperature is higher than 20 kK (Chayer et al. 1995b; Chayer 2014). In Fig. 6, we performed a test for the 0.6 \( M_\odot \) DB WD as follows: The gravitational settling is not included when the \( T_{\text{eff}} \) of a cooling WD is higher than 20 kK, but it is included when \( T_{\text{eff}} < 20 \text{ kK}\). We find that \([\text{Ca}/\text{He}]\) rapidly decreases, and we can not explain the observations. Therefore, the heavy elements observed on the DB WD’s surfaces must originate from other sources. The rocky objects tidally disrupted by a DB WD are a possible source (Farihi 2016).

4.1. Metal-rich material accretion

In general, the element abundances on the surface of an accreting DB WD not only depend on the WD properties, but also on mass-accretion rates \((\dot{M})\) and the chemical abundances of accreted material. In order to match the observed properties of G29–38 in Xu et al. (2014), Bauer & Bildsten (2018) assumed that the mass fractions of Fe, O, Mg, Si, and Ca were in accreted materials 0.307, 0.295, 0.199, 0.153, and 0.046, respectively. We adopted the above mass fractions.
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By resolving the Burgers equations, MESA can calculate the chemical diffusion of an accreting WD. Figure 7 shows the evolution of [Ca/He] on the surface of 0.6 $M_\odot$ DB WD with a mass-accretion rate of $10^8$ g s$^{-1}$ when $T_{\text{eff}} = 20$ kK. It takes about 10$^7$ yr to reach an accretion-diffusion equilibrium for the accreting DB WD. When the accretion stops, the Ca element diffuses downward within a diffusive timescale of about 10$^6$ yr, which is similar to what was found by Koester (2009). Obviously, input parameters ($\alpha_{\text{MLT}}, \alpha_{\text{th}},$ and $Z$) have weak effects on the surface [Ca/He]. The main reasons are similar to those for $M_{\text{cvz}}$.

Figure 8 gives the evolution of [Ca/He] on the 0.6$M_\odot$ DB WD with different $\dot{M}_a$ at different $T_{\text{eff}}$. The timescale of reaching accretion-diffusion equilibrium is about 10$^4$ for all models. The mass-accretion rate and the effective temperature greatly affect the element abundances of an accreting DB WD. When $\dot{M}_a$ decreases from $10^{10}$ to $10^4$ g s$^{-1}$, [Ca/He] reduces from about $10^{-5}$ to $10^{-10}$. This means that the metal abundance of an accreting WD is approximately in proportion to the mass-accretion rate. In fact, Dupuis et al. (1992) and Koester (2009) assumed that the element abundances observed in polluted WDs should

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Fig. 4. Similar to Fig. 3, but for 0.4 and 0.8 $M_\odot$ DB WDs for case 1.

Fig. 5. Mass of convective zone ($M_{\text{cvz}}$) versus the WD’s effective temperature ($T_{\text{eff}}$). The left panel is for the 0.6 $M_\odot$ DB WD in different cases, while the right panel is for 0.4, 0.6, and 0.8 $M_\odot$ DB WDs in case 1. Theoretical results from Benvenuto & Althaus (1997) and Koester (2009) are shown by different symbols. B97 and K09 refer to Benvenuto & Althaus (1997) and Koester (2009), respectively.

Fig. 6. Evolutions of log [Ca/He] during 0.6 $M_\odot$ DB WD cooling, in which the gravitational diffusion is not involved when $T > 20$ kK. Black, red, and green dots represent observations from Koester & Kepler (2015), Dufour et al. (2007), and Hollands et al. (2017), respectively.

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be accretion-diffusion equilibrium, and they suggested that the mass fraction of the $i$th element ($X_{cvz,i}$) in the convective zone is given by

$$M_{cvz} \frac{dX_{cvz,i}}{dt} = M_i - \frac{X_{cvz,i} M_{cvz}}{\tau_{diff,i}}. \quad (1)$$

where $X_{cvz,i}$ and $M_i$ are the $i$th element abundance in the convective zone and the mass-accretion rate of $i$th element, respectively. Here, $\tau_{diff,i}$ is the $i$th element diffusive timescale, which can be estimated by

$$\tau_{diff,i} = \frac{M_{cvz}}{4 \pi R_{cvz}^2 \rho_{cvz} v_{diff,i}}, \quad (2)$$

where $R_{cvz}$ and $\rho_{cvz}$ are the radius and the mass density at the bottom of the convective zone, respectively. Here, $v_{diff,i}$ is the $i$th element velocity of downward sedimentation at bottom of the convective envelope. If $\tau_{diff}$ is much shorter than the WD lifetime, Koester (2009) gave the relation between the mass-accretion rate and the element abundance by

$$X_{cvz,i} = \frac{M_i}{M_{cvz}} \tau_{diff,i}. \quad (3)$$

Obviously, our result is consistent with Eq. (3).

However, the change in [Ca/He] with $T_{\text{eff}}$ is complex. When $T_{\text{eff}}$ decreases from 20 to 10 kK, [Ca/He] reduces by about three orders of magnitude. When it decreases from 10 to 8 kK, [Ca/He] slightly enhances. When it decreases from 8 to 6 kK, [Ca/He] reduces by about one order of magnitude again. This change can be explained by the relation of $T_{\text{eff}}$ and $M_{\text{cvz}}$(see Fig. 5).

Figure 9 gives the diffusion downward of the Ca element on the surface of 0.6 $M_\odot$ DB WD after a lasting 10$^6$ yr accretion at different $T_{\text{eff}}$. The evolution of [Ca/He] with $T_{\text{eff}}$ is similar to that in Fig. 8. In fact, Fig. 9 indicates the timescale of Ca element diffusion, that is $\tau_{\text{diff,ca}}$. Obviously, it deeply depends on $T_{\text{eff}}$. In our model, $\tau_{\text{diff,ca}}$ increases from about 10$^5$ to 10$^9$ yr when $T_{\text{eff}}$ decreases from 20 to 6 kK. However, $\tau_{\text{diff,ca}}$ in Koester (2009) increases from about 10$^4$ to 10$^9$ yr.

Based on Eq. (2), $\tau_{\text{diff,ca}}$ depends on $M_{\text{cvz}}$, $R_{\text{cvz}}$, $\rho_{\text{cvz}}$, and $v_{\text{diff,ca}}$. Figure 10 shows the profiles of $X$(Ca), $v_{\text{diff,ca}}$, opacity($\kappa$), and $l_{\text{cool}}$ around the surface of 0.6 $M_\odot$ DB WDs with different $M_i$ and $T_{\text{eff}}$. Obviously, when comparing model of 10$^8$ with that of 10$^6$ g s$^{-1}$, the mass-accretion rate can affect $X$(Ca), but it does not change the internal structure of the accreting DB WD, including the opacity, the mass density and the radius. The reason is that the matter accreted by a DB WD quickly di

Fig. 7. Evolution of [Ca/He] on the surface of 0.6 $M_\odot$ DB WD with a mass-accretion rate of 10$^6$ g s$^{-1}$ when $T_{\text{eff}} = 20$ kK. Accretion ceases after 10$^6$ yr. The different lines represent different cases which are shown in the bottom left zone.

Fig. 8. Evolution of [Ca/He] on the surface of 0.6 $M_\odot$ DB WD with different mass-accretion rates at different $T_{\text{eff}}$.

Fig. 9. Similar to Fig. 8, but for the evolution of [Ca/He] on the surface of 0.6 $M_\odot$ DB WD which just experiences lasting 10$^6$ yr accretion at different $T_{\text{eff}}$. The mass-accretion rates are shown in the top middle region of every panel.
Fig. 10. Profiles of Ca abundance ($X$(Ca)), opacity ($\kappa$), Ca diffusive velocity ($v_{\text{diff, Ca}}$), convective velocity ($v_{\text{conv}}$), mass density ($\rho$), and radius ($R$) around the surface of 0.6 $M_\odot$ DB WDs with mass-accretion rates of $10^8$ g s$^{-1}$, but different effective temperatures which are represented by different lines.

Fig. 11. Similar to Fig. 10, but for $\dot{M} = 10^6$ g s$^{-1}$.

internal structure of a DB WD, which depends on the cooling duration presented by $T_{\text{eff}}$

Combining Figs. 5, 10, and 11, with DB WD cooling from $T_{\text{eff}} = 20$ kK to 6 kK, $M_{\text{cVZ}}$ increases from about $10^{-8}$ to $10^{-5} M_\odot$. $\rho_{B, \text{cVZ}}$ also increases from $\sim 1$ to $10^3$ g cm$^{-3}$, $v_{\text{diff, Ca}}$ at the base of the surface convection zone decreases from about $10^{-6}$ to $10^{-10} \text{ cm s}^{-1}$, while $R_{B, \text{cVZ}}$ keeps constant. Therefore, $\tau_{\text{diff, Ca}}$ increases from about $10^3$ to $10^6$ yr. This means that, compared to $\tau_{\text{cool}}$, $\tau_{\text{diff, Ca}}$ cannot be neglected when $T_{\text{eff}} < 10$ kK, that is, Eq. (3) is not suitable for cool polluted DB WDs. Of course, one should note, in our model, $v_{\text{diff, Ca}}$ has irregular oscillations when $T_{\text{eff}} < 8$ kK. Koester (2009) did not show $v_{\text{diff, Ca}}$. However, $v_{\text{diff, Ca}}$ may result in a great difference of $\tau_{\text{diff, Ca}}$ between the present paper and Koester (2009).

4.2. Accretion pollution with the power law

As Fig. 6 shows, the [Ca/He] of about 1500 DB WDs observed by Dufour et al. (2007), Koester & Kepler (2015), and Hollands et al. (2017) must be explained by accretion pollution.
In the panels a, b, and c of Fig. 12, we gave the evolutional tracks and green dots represent observations from Koester & Kepler (2015). Panels (d) is for the mass-accretion rate given by Eq. (5). Black, red and green dots represent observations from Köester & Kepler (2015), Dufour et al. (2007) and Hollands et al. (2017), respectively.

In order to model the metal abundances in GD 362’s atmosphere, Jura et al. (2009) considered that $M_d$ should decrease by the following power law:

$$M_d = M_{\text{disk}} e^{-t/t_{\text{disk}}},$$  \hspace{1cm} (4)

where $M_{\text{disk}}$ is the mass of planet disrupted by a WD and $t_{\text{disk}}$ is a characteristic timescale of the accretion disk. Jura et al. (2009) found that all of GD 362’s distinctive properties can be explained if $M_{\text{disk}}$ is between about $10^{25}$ and $10^{26}$ g, in which the range of $t_{\text{disk}}$ is between about $2 \times 10^8$ and $10^9$ yr.

However, based on Fig. 8 and panels a–c of Fig. 12, with DB WD cooling, a decreasing mass-accretion rate with a power law results in a continued decrease in [Ca/He]. Therefore, it cannot explain the observations. According to our model, the mass-accretion rate should decrease when $T_{\text{eff}} > 10$ kK, but it should increase when $T_{\text{eff}} < 10$ kK. Considering that the $T_{\text{eff}}$ of WDs mainly depends on $t_{\text{cool}}$ and it can be compared with the observations, we assumed that $M_d$ should change by the following power law:

$$M_d = \begin{cases} 
10^{14} \times 10^{\left( \frac{2.6 \log T_{\text{eff}}}{3.5} \right)}, & \text{g s}^{-1} \text{ when } T_{\text{eff}} > 10 \text{kK} \\
10^3 \times 10^{\left( \frac{\alpha_{\text{MLT}}}{1.8} \right)}, & \text{g s}^{-1} \text{ when } T_{\text{eff}} < 10 \text{kK}.
\end{cases}$$  \hspace{1cm} (5)

Panel d of Fig. 12 gives the evolution of [Ca/He] with $T_{\text{eff}}$ for DB WDs with different $M_{\text{WD}}$ and a power-law $M_t$ described by Eq. (5). Our results are consistent with the observations for DB WDs. The $t_{\text{disk}}$ of an accretion disk composed purely of dust is higher than $10^9$ yr (Farhi et al. 2008). Usually, the cooling timescale of a DB WD from 20 kK to 10 kK is about $10^8 - 10^9$ yr, and it is about $10^9$ yr from 10 kK to 5 kK. This means that a DB WD can accrete a disk, produced by itself, disrupting a planet during the whole cooling phase. The decrease in the mass-accretion rate when $T_{\text{eff}} > 10$ kK results from the viscous dissipation of accretion disk (Jura et al. 2009). However, we did not find any model to explain its enhancement when $T_{\text{eff}} < 10$ kK. If Eq. (5) basically represents the true trend of the mass-accretion rates, this indicates that the accretion disk produced by a WD disrupting a planet may have a complex structure.

5. Conclusions

In order to explain the evolutionary sequence of heavy element abundances observed among 1500 DB or DZ WDs, we used MESA to create DB WDs with masses of 0.4, 0.6, and 0.8 $M_\odot$ by artificially stripping envelope once. The H-rich envelope was stripped when stars evolved into red giants. We investigated the effects of input parameters ($\alpha_{\text{MLT}}, \alpha_{\text{th}},$ and $Z$) on DB WD structures. Due to the small pressure scale height, thick convective zone, or mean molecular weight of DB WDs, these input parameters have a weak effect on DB WD structures, including interior temperatures, chemical profiles, and convective zones. Therefore, they hardly affect the evolution of heavy elements on the surface of DB WDs.

Due to high gravitational fields of DB WDs, the element diffusion in the theoretical model is too fast to explain the observations. Therefore, the heavy elements on the DB WDs’ surfaces may originate from the pollution by accreting the planet disrupted by these WDs. They mainly depend on the mass-accretion rates and the effective temperatures of DB WDs. In our model, a constant mass-accretion rate cannot explain the evolutionary sequence of a Ca element for about 1500 observed DB or DZ WDs. However, it is consistent well with the model in which the mass-accretion rate decreases by one power law when $T_{\text{eff}} > 10$ kK and slightly increases by another power law when $T_{\text{eff}} < 10$ kK. The observed DB WD evolutionary sequence of heavy element abundances originates from WD cooling and the change in the mass-accretion rate.

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Fig. 12. Effective temperature vs. log [Ca/He] for DB WDs. Panels (a), (b), and (c) represent models with different mass DB WDs and a constant accretion rate ($10^{6}, 10^{7}$ and $10^{8}$ g s$^{-1}$), respectively. Panels (d) is for the mass-accretion rate given by Eq. (5). Black, red and green dots represent observations from Köester & Kepler (2015), Dufour et al. (2007) and Hollands et al. (2017), respectively.
