Generation patterns, modified $\gamma - Z$ mixing, and hidden sector with dark matter candidates as framed standard model results

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Abstract

A descriptive summary is given of the results to-date from the framed standard model (FSM) which:

- assigns geometric meaning to the Higgs field and to fermion generations, hence offering an explanation for the observed mass and mixing patterns of quarks and leptons, reproducing near-quantitatively 17 of SM parameters with only 7.

- predicts a new vector boson $G$ which mixes with $\gamma$ and $Z$, leading to deviations from the SM mixing scheme. For $m_G > 1$ TeV, these deviations are within present experimental errors but should soon be detectable at LHC when experimental accuracy is further improved.

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• suggests the existence of a hidden sector of particles as yet unknown to experiment which interact but little with the known particles. The lowest members of the hidden sector of mass around 17 MeV, being electrically neutral and stable, may figure as dark matter constituents.

The idea is to retrace the steps leading to the above results unencumbered by details already worked out and reported elsewhere. This has helped to clarify the logic, tighten some arguments and dispense with one major assumption previously thought necessary, thus strengthening earlier results in opening up possibly a new and exciting vista for further exploration.
The FSM was initially conceived to address the generation puzzle but, for its own consistency, has led to consequences way beyond its original remits. To trace how this comes about is the aim of the present article.

1 Generation puzzle

By the generation puzzle we mean the following empirical facts [1]:

- Quarks and leptons occur in 3 generations.
- Generations have hierarchical masses, e.g. \( m_t \gg m_c \gg m_u \).
- Up and down flavoured states are not aligned giving e.g. for quarks:

\[
V_{\text{CKM}} = \begin{pmatrix}
  u \cdot d & u \cdot s & u \cdot b \\
  c \cdot d & c \cdot s & c \cdot b \\
  t \cdot d & t \cdot s & t \cdot b
\end{pmatrix} \neq I. \tag{1}
\]

For these one would wish to have an explanation [2]. The standard model (SM) takes these facts for granted, which account for some two-thirds of its many empirical parameters.

2 Rotating rank-one mass matrix

Towards understanding the generation puzzle, one first step made some years ago was R2M2 (Rotating Rank-one Mass Matrix) [3, 4, 5] which shows that if:

- the quark and lepton mass matrices are of the common form:

\[
m = m_T \alpha \alpha^\dagger \tag{2}
\]

where \( \alpha \), a vector in 3D generation space, is the same for all quarks and leptons and only the numerical coefficients \( m_T \) differ, and

- \( \alpha \) rotates as the scale changes,
then the above features can be qualitatively reproduced.

We know that coupling constants and masses can change with scale as a result of renormalization. Similarly, renormalization can cause the mass matrix which has an orientation (in generation space) to rotate with changing scales. Indeed, even in the usual formulation of the SM, the mass matrix rotates as a result of mixing [6]. The new point in R2M2 then is that it is the rotation which gives rise to the mixing rather than the other way round.

That R2M2 will give rise to mixing is easy to see. A mass matrix rotating with scale means that not only its eigenvalues but also its eigenvectors will be scale-dependent. Now the masses and state vectors of particles are supposed each to be measured at their respective mass scales. Since, for example, the up-type quarks $t, c, u$ have different masses from the down-type quarks $b, s, d$, it follows that the state vectors of the two types, say respectively $t, c, u$ and $b, s, d$, which are eigenvectors of the matrix at their own different mass scales will also be different. Hence, the CKM matrix [7] (1) will not be the identity matrix which is what is meant by mixing.

That R2M2 will lead to a hierarchical mass spectrum is also quite easy to see. The matrix (2) has only one nonzero eigenvalue, namely $m_T$ with eigenvector $\alpha$, which, for example, at scale $\mu = m_t$ gives $t = \alpha(m_t)$. The two other eigenvectors have zero eigenvalues at this $\mu$, but these are not the masses of the two lower generations $c$ and $u$ for these mass values have to be measured at their own mass scales. What happens at these lower scales is that because of the rotation of $\alpha$, there is some “leakage” of $m_t$ to the lower generations $c$ and $u$ to give each a mass, hence mass hierarchy.

Closer examination of R2M2, in fact, reveals many other detailed features which are also seen in experiment [3, 8, 9, 10, 11].

3 Framing SM for FSM

The FSM is an extension of the SM constructed to give R2M2 as a result. We recall that the standard model is a gauge theory based on the gauge symmetry $G = U(1) \times SU(2) \times SU(3)$, with the gauge (vector boson) fields, and the matter (fermion) fields as dynamical variables, to which is added a scalar Higgs field to break the $SU(2)$ symmetry, as demanded by experiment. The FSM is an extension of this set-up. However, in contrast to usual extensions met in the literature, such as GUT or SUSY, it does not seek an enlargement of the (local) gauge symmetry. It keeps the same $G$ as the SM, but instead
adds to the SM as new dynamical variables the frame vectors in the internal
symmetry space, as follows.

We recall that a gauge theory is by definition invariant under local gauge
transformations. These transformations can be and are usually represented
as matrices relating the local (spacetime point $x$-dependent) frame to a global
(spacetime point $x$-independent) reference frame. The columns of such ma-
trices are often referred to as frame vectors. The suggestion of FSM is to
add the elements of these transformation matrices or of the frame vectors as
dynamical variables to the usual SM set.

Promoting frame vectors to dynamical variables is not a new idea. Gen-
eral relativity is usually formulated in terms of the metric tensor $g_{\mu\nu}$ as
dynamical variables, but can alternatively be formulated (Einstein–Cartan
theory) \cite{12} in terms of vierbeins, $e_\mu^a$, which are frame vectors in the lan-
guage of the preceding paragraph, with $\mu$ referring to the local (co-ordinate)
frame and $a$ referring to the global reference frame, and the metric tensor
$g_{\mu\nu}$ appears then as:

$$g_{\mu\nu} = \sum_a e_\mu^a e_{\nu}^a. \quad (3)$$

So the suggestion of making frame vectors into dynamical variables in FSM
makes, in a sense, the particle theory closer in spirit to the theory of gravity
\cite{13} and might make it easier in future for the eventual unification of the two
theories.

Frame vectors as dynamical variables, or framons as we shall call them,
have a special property that the gauge bosons and matter fermions do not
have. Being transformation matrices between the local frame and a global ref-
ence frame, they depend naturally on both the local and global frames, and
transform when either the local or the global reference frame is changed. This
is like the vierbeins in gravity, which is seen above to carry two indices, one
local and one global, but unlike the gauge bosons and matter fermions which
transform only under local gauge transformations. Since physics should be
invariant under changes in both the local and global reference frame, it means
that for a theory with the SM gauge symmetry $G = U(1) \times SU(2) \times SU(3)$,
the action when framons are included as dynamical variables should be in-
vARIANT under not only (local) $G$ but also a global $\tilde{G} = \tilde{U}(1) \times \tilde{SU}(2) \times \tilde{SU}(3)$. This additional requirement does not affect those terms originally present in
the SM involving only the gauge bosons and matter fermions fields, since
these fields are themselves invariant under $\tilde{G}$, but will put restrictive con-
strains on the new terms involving the new framon fields, and help to specify the theory.

The SM gauge symmetry $G = U(1) \times SU(2) \times SU(3)$ being a product of three simple symmetries (modulo some discrete identifications which do not concern us here), there are several different ways to represent the gauge transformations as matrices, depending on whether for the product of each pair we take the sum or the product representation. To fit the SM framework, the FSM [13, 14, 15] opted for the choice:

$$1 \times (2 + 3)$$

for the local $G$, where $1$ is for $U(1)$, $2$ means the doublet for $SU(2)$ and $3$ the triplet for $SU(3)$, but

$$\tilde{1} \times \tilde{2} \times \tilde{3}$$

for the global $\tilde{G}$. This choice is the closest, in fact the only one close, to the SM in structure and happens also to require the smallest number of framon fields to be introduced [14].

Specifically, this means that the framon matrix breaks up into two parts:

- [FF] the “flavour framon”: $\alpha \Phi$,
- [CF] the “colour framon”: $\beta \Phi$.

The factors $\alpha$ and $\beta$ are global (space-time $x$-independent) quantities, where $\alpha$ transforms as a triplet under $\tilde{SU}(3)$, and $\beta$ transforms as a doublet under $\tilde{SU}(2)$. Next, $\Phi$ is a scalar field (space-time scalar $x$-dependent quantity), as well as a $2 \times 2$ matrix whose rows transform as (local flavour) $SU(2)$ doublets but whose columns as (global flavour) $\tilde{SU}(2)$ anti-doublets. Similarly, $\Phi$ is a scalar field as well as a $3 \times 3$ matrix whose rows transform as (local colour) $SU(3)$ triplets but whose columns as (global colour) $\tilde{SU}(3)$ anti-triplets.

The two columns of $\Phi$ are flavour doublet scalar fields either of which can play the role of the standard Higgs field in the electroweak theory. The SM, however, requires only one such. To conform with this requirement, the FSM imposes the following orthonormal condition:

$$\phi_r^2 = -\epsilon_{rs}(\phi_s^1)^*$$

on $\Phi$ so that one column can be eliminated leaving only the other to be identified with the standard Higgs field. This reduces further the number
of scalar fields introduced by framing, in the same spirit as the “minimal”
choice (4) the framom representation. Now the possibility to impose such a
condition on the framom field is unique to the group $SU(2)$, not adaptable to
colour $SU(3)$ nor to $SU(N)$ for any larger $N$, having to do with the possibility
of embedding $SU(2)$ in $\mathbb{R}^4$ [14], and (6) will for this reason be referred to as
the minimal embedding condition.

4 First FSM results

Once this form for the framom is written down, several advantages of the
FSM scheme become immediately apparent.

- The standard Higgs field in electroweak theory being now identified as
  a column of the framom $\Phi$ is thus given a geometric significance (namely
  as a frame vector) which is missing in the usual formulation of the SM.

- There has appeared a global 3-fold symmetry $\tilde{SU}(3)$ which can be taken
  as fermion generations, and if so gives to the latter also a geometric
  significance (as the “dual” to colour) which is missing also in the usual
  formulation.

- The mass matrices of quarks and leptons appear automatically in the
  form (2), where $\alpha$ can be taken here without any loss of generality as
  a real unit vector [14]. Coming from the flavour framom $[\text{FF}]$ playing
  the role of the Higgs scalar, $\alpha$ is naturally independent of the type of
  quark or lepton to which the mass matrix refers, as is wanted in R2M2.

- The colour framom $[\text{CF}]$ carries both local colour and global colour
  (generation) indices and, when appearing in loop diagrams, automati-
  cally generates rotation of $\alpha$ with changing scales, as is wanted in
  R2M2 to give hierachical mass and mixing patterns.

The first two items are bonuses while the last two are the stated aims of the
FSM. But will the scheme really work as hoped?

The requirement of the doubled invariance under $G \times \tilde{G}$ already men-
tioned restricts the framom action sufficiently for simple loop diagrams to be
calculated, and so the question posed can immediately be put to the test.
This is done in [15] where a renormalization group equation (RGE) for the
scale dependence of $\alpha$ to one-framon-loop is derived. The equation itself depends on some parameters and when applied, depends on some integration constants, making it 7 real adjustable parameters in all. This is then required to fit experiment and Table 1 is obtained as the result. One sees there that the masses and mixing elements measured in experiment have all been fitted quite well, with most fitted within 1.5 $\sigma$ and none too wild, despite their intricate variations in size over a wide range².

As already noted, R2M2 being already incorporated, FSM can be expected to give the qualitative features correctly. But this does not by any means guarantee that the values of the mass and mixing parameters can be correctly reproduced. For these, the details of the rotation trajectory of $\alpha$ matter, namely the shape of the curve it traces on the unit sphere as well as the variable speed with respect to change of scale at which it moves along this curve, and it looks nontrivial that the FSM seems to have got it right.

In relation to Table 1 two points are particularly noteworthy:

- [a] The QCD action is well known to admit CP-violation via a so-called theta-angle term [16] of topological origin which, if admitted with $\theta$ naturally of order unity, would lead to CP-violations in strong interactions many orders above what is seen in, for example, the neutron dipole moment. It is also well known that this so-called strong CP problem can be solved if the quark mass matrix has zero eigenvalues, but this seems to contradict the empirical observation that all known quarks have nonzero mass. Now it happens that in R2M2, and hence also the FSM, the quark mass matrix (2) does have zero eigenvalues but yet all quarks have finite masses by virtue of the “leakage mechanism” already mentioned, so that the strong CP-problem can be solved as above by transforming away the theta-angle term. However, the effect of eliminating the theta-angle term is transmitted by the rotation of $\alpha$ to the CKM matrix to give it a Kobayashi-Moskawa CP-violating phase. Moreover, a $\theta$ of order unity is shown to give a KM phase or Jarlskog invariant [17] of the right size [9, 10]. In other words, the FSM offers a simultaneous solution to both the strong CP-problem and to the question why a KM phase of a certain size should appear in the

²This means effectively that the FSM has, to this accuracy, replaced by 7 adjustable parameters 17 of the SM’s empirical parameters, although not all of these latter have been measured.
|                  | Expt (June 2014) | FSM Calc | Agree to | Control Calc |
|------------------|------------------|----------|----------|--------------|
| **INPUT**        |                  |          |          |              |
| $m_c$            | 1.275 ± 0.025 GeV| 1.275 GeV| < 1σ     | 1.2755 GeV   |
| $m_{\mu}$        | 0.10566 GeV      | 0.1054 GeV| 0.2%    | 0.1056 GeV   |
| $m_e$            | 0.511 MeV        | 0.513 MeV| 0.4%    | 0.518 MeV    |
| $|V_{us}|$         | 0.22534 ± 0.00065| 0.22493  | < 1σ     | 0.22468      |
| $|V_{ub}|$         | 0.00351$^{+0.00015}_{-0.00014}$| 0.00346  | < 1σ     | 0.00346      |
| $\sin^2 2\theta_{13}$ | 0.095 ± 0.010  | 0.101    | < 1σ     | 0.102        |
| **OUTPUT**       |                  |          |          |              |
| $m_s$            | 0.095 ± 0.005 GeV| 0.169 GeV| QCD      | 0.170 GeV    |
| (at 2 GeV)       | (at $m_s$)       |          | running  |              |
| $m_u/m_d$        | 0.38—0.58        | 0.56     | < 1σ     | 0.56         |
| $|V_{ud}|$         | 0.97427 ± 0.00015| 0.97437  | < 1σ     | 0.97443      |
| $|V_{cs}|$         | 0.97344 ± 0.00016| 0.97350  | < 1σ     | 0.97356      |
| $|V_{tb}|$         | 0.999146$^{+0.000021}_{-0.000046}$| 0.99907  | 1.65σ    | 0.999075     |
| $|V_{cd}|$         | 0.22520 ± 0.00065| 0.22462  | < 1σ     | 0.22437      |
| $|V_{cb}|$         | 0.0412$^{+0.0011}_{-0.0005}$| 0.0429  | 1.55σ    | 0.0429       |
| $|V_{ts}|$         | 0.0404$^{+0.0004}_{-0.0004}$| 0.0413  | < 1σ     | 0.0412       |
| $|V_{td}|$         | 0.00867$^{+0.00029}_{-0.00031}$| 0.01223  | 41 %     | 0.01221      |
| $|J|$             | $(2.96^{+0.20}_{-0.16}) \times 10^{-5}$| $2.35 \times 10^{-5}$ | 20 %     | $2.34 \times 10^{-5}$ |
| $\sin^2 2\theta_{12}$ | 0.857 ± 0.024  | 0.841    | < 1σ     | 0.840        |
| $\sin^2 2\theta_{23}$ | > 0.95         | 0.89     | > 6%     | 0.89         |

Table 1: Calculated fermion masses and mixing parameters compared with experiment, reproduced from [15]
CKM matrix for quarks.\footnote{It is known that in the weak lagrangian a similar topological term can be rotated away without any physical consequences. Hence if CP violation in the leptonic sector were also due to a topological term as in QCD, then it would seem to indicate that there is no CP violating Dirac phase in the PMNS matrix for leptons $[15]$. However, there may be other sources of CP violation, and in any case we know that there is the possibility of CP violation due to Majorana phases.}

- [b] It is a crucial empirical fact that $m_u < m_d$, which is what makes the proton lighter than the neutron and therefore stable, or otherwise we ourselves would not be here. But this looks anomalous, given that the up-type quarks of the heavier generations are heavier than their down-type counterparts, namely $m_t \gg m_b$ and $m_c > m_s$. The FSM fit in Table 1 however, gives the right answer $m_u < m_d$, indeed even to the ratio $m_u/m_d$. This comes about as follows. In Figure 1 is shown the trajectory of $\alpha$ on the unit sphere as obtained in the FSM fit of $[15]$. There is a change in the (normal) curvature between $c, s$ and $u, d$, which is what gives in $[15]$ this “anomaly”. It is a special intrinsic property of the RGE for $\alpha$ derived from the FSM which cannot be envisaged from [R2M2] alone, and is to play another significant role later.

## 5 Questions of consistency

Taken in all then, the FSM seems to have done the job it was intended to do in reproducing the empirical mass and mixing patterns of quarks and leptons. Even if taken just as a parametrization of the data, it is competitive with any seen in the literature. But this is not enough. In constructing the FSM, new assumptions have been made, and new assumptions imply new physics. One is obliged therefore to ascertain whether the new physics predicted by the FSM is consistent with present experiments, and if so, whether it can be checked further by experiments in future.

Now the new ingredients introduced by FSM are the framons, of which the flavour framon [FF] has already been identified with the standard Higgs scalar. So what is newly introduced by the FSM over the SM are just the colour framons [CF]. But these represent 9 new complex degrees of freedom. Then why have we not been made aware of them? This is not immediately
Figure 1: The rotation trajectory of $\alpha$ on the unit sphere as determined by the fit in [15]. Note the change in sign of normal curvature around the pole.
answerable because the colour framons are coloured and colour is confined, so that colour framons cannot propagate freely as particles in space. They can however combine with one another and with other coloured objects via colour confinement into colour neutral bound states, and these can appear then as particles. A framon can combine with an antiframon in s-wave to form a colourless scalar bound states which we shall call generically $H$, or in $p$-wave to form vector bound states which we call generically $G$, which via the covariant derivative will bring in the colour gluon. Or a framon can combine with a coloured fermion to form fermionic bound states which we call generically $F$. In this case, an immediate question is:

- **[Q1]** Why have we not seen these particles $H, G, F$?

This is a question that FSM has to answer to remain viable, but it can only do so when enough has been learnt about the properties of $H, G$ and $F$.

To this end, let us ask ourselves another question which needs to be asked for internal consistency in any case, namely

- **[Q2]** In the flavour theory, the introduction of the Higgs scalar (the flavour framon here) with non-zero vacuum expectation value breaks the $SU(2)$ gauge symmetry and gives masses to the quarks, the leptons, the vector bosons $W, Z$ and the scalar Higgs boson $h$. By analogy then, why does the introduction of the colour framon, also with nonzero vacuum expectation value to break the generation symmetry, not break the colour gauge symmetry and give massive fermions, vector bosons and Higgs scalar bosons in analogy to the above?

At first sight, this seems a totally unrelated question but it will soon be seen to be just another way of posing [Q1], and that the recognition of this equivalence will lead us a long way towards understanding the properties of the $H, G, F$ that we seek.

That [Q2] is indeed equivalent to [Q1] can be seen by a deep and subtle fact which has taken the perspicacity of ’t Hooft [19] first to point out. In this illuminating paper, ’t Hooft, among other things, made the observation that the standard (Salam-Weinberg) electroweak theory, which is usually said to have its flavour $SU(2)$ symmetry spontaneously broken, has a “mathematically equivalent” interpretation as a theory in which the $SU(2)$ theory is confining and exact, what is broken being only a global (spacetime $x$-independent) symmetry, say $\tilde{SU}(2)$, associated with it which the electroweak
In this alternative interpretation, which we shall call the confinement picture, the Higgs boson \( h \) appears as a flavour neutral bound state of the original flavoured Higgs scalar field \( \phi \) with its conjugate \( \phi^\dagger \) in the \( s \)-wave, the massive vector bosons \( W, Z \) appear as the same but now in the \( p \)-wave, while the quarks and leptons appear as flavour neutral bound states of \( \phi \) with the fundamental flavour doublet fermion fields. This means, first, that the presence of scalar fields with nonzero vacuum expectation value does not by itself preclude the theory being confining, answering thus the first half of \([Q2]\). Secondly, we see that as above interpreted in the confinement picture, the \( h, W, Z \) and \( q, \ell \) in the flavour theory would be the exact analogues of respectively the \( H, G, \) and \( F \) in the colour theory, only with flavour and colour interchanged. Or, in other words, \([Q2]\) is just \([Q1]\) rephrased, as claimed.

6  Flavour-colour parallel and the dichotomy of matter

Now, for the FSM, ’t Hooft’s confinement picture for the electroweak theory is a veritable eye-opener in that it reveals a very close parallel between the two nonabelian component theories of the FSM, a parallel that was at first entirely unsuspected. This is schematised in Figure 2, where we see that both flavour and colour are now confined and both of them are framed by scalar framion fields. Besides, both the flavour and colour framons have nonzero vacuum expectation values, which lead to the breaking of both the global symmetries, first of the \( \tilde{SU}(2) \) symmetry giving the two up-down flavours, and second, of the \( \tilde{SU}(3) \) symmetry giving the three generations.

The cited vacuum expectation value of the flavour framon (that is the Higgs scalar) is well known, while the cited vacuum expectation value of the colour framon will appear later on. Then the flavour framon combines with its own conjugate and with flavoured fermions via flavour confinement to form the particles \( h, W, Z, q, \ell \) on the left, while the colour framons combine parallelly via colour confinement to form the particles \( H, G, \) and \( F \) on the right.

At first sight, this close parallel between the two sides may seem worrisome, since we are used to the conception of the flavour theory as describing the weak interactions and the colour theory as describing the strong. But this conception is a little loose and needs to be scrutinized anew in the present
|                      | **Flavour Theory** | **Colour Theory** |
|----------------------|--------------------|------------------|
| Gauge Symmetry (local) | $SU(2)$            | $SU(3)$          |
| Confinement?         | Confined, exact    | Confined, exact  |
|                      | (`t Hooft’s picture) | (general consensus) |
| Framon scalar        | $\Phi$ (flavour framons $\Rightarrow$ standard Higgs) | $\Phi$ (colour framons new for FSM) |
| Symmetry doubled     | $SU(2) \times \tilde{SU}(2)$ | $SU(3) \times \tilde{SU}(3)$ |
|                      | local              | local            |
|                      | global             | global           |
| Framon vev $\neq 0$  | $\zeta_W$ (246 GeV) | $\zeta_S$ ($\sim$ TeV) |
| Global symmetry broken | $\tilde{SU}(2)$ broken | $\tilde{SU}(3)$ broken |
|                      | $\Rightarrow$ up-down flavour | $\Rightarrow$ 3 generations |
| Framon bound states  | by $SU(2)$ confinment (`t Hooft) | by colour $SU(3)$ confinement |
|                      | $\rightarrow h, W, Z, (q, \ell)$ | $\rightarrow H, G, F$ |
|                      |                     | $Q$ co-quarks $\Rightarrow L$ co-leptons |
| Higher level bound states | $QQ, QQ$ co-hadrons by flavour confinement | $q\bar{q}, qqq$ hadrons by colour confinement |

Figure 2: Comparing the flavour and colour theories in FSM
context. When we said above that flavour interactions are weak while colour interactions are strong, what we really meant was that the particles bound by flavour confinement that we know, namely what we shall call for simplicity the “weak particles” $h, W, Z$, and $q, \ell$ interact weakly while those particles bound by colour confinement that we know, namely the hadrons, interact strongly. But this is not comparing like with like in terms of the parallel set out in Figure 2. The analogues of $h, W, Z$ and $q, \ell$ on the left, we agreed, are the $H, G, F$ on the right, not the hadrons. Hadrons are constructs of a very different sort. They are bound states, indeed also by colour confinement as are the $H, G, F$, but not of a colour framon with its conjugate or with something else, but of quarks and antiquarks which, though coloured, are themselves already flavour neutral composites of a flavour framon bound to other flavoured constituent via flavour $SU(2)$ confinement. Hadrons are thus, in a sense, higher-level constructs, not the parallel of the weak particles $h, W, Z$, and $q, \ell$. The correct parallel of hadrons instead would be flavour neutral bound states by flavour confinement of some $F$s carrying flavour, if such exist, which we may call co-quarks $Q$ and anti-co-quarks $\bar{Q}$, resulting in say $Q\bar{Q}$ or $QQ$ which we may call co-hadrons (co-mesons or co-baryons). The parallel between the left- and right-hand sides of Figure 2 is still maintained so long as the $H, G, F$ are point-like and weakly interacting like the weak particles, as will be shown later to be the case, while co-hadrons can be bulky and strongly interacting, like hadrons. From the perspective of Figure 2 what gives us the wrong impression that flavour interactions are weak and colour strong is just that of the bound states confined by flavour, we know in experiment only the weak particles $h, W, Z$, and $q, \ell$, and of the bound states confined by colour, we know in experiment only the hadrons. In other words, what remains a mystery is still just [Q1], and so long as that is understood, as we hope to do later, the parallel exhibited in Figure 2 holds good.

Obviously, such a close parallel between the flavour and colour sectors in the FSM will have far-reaching consequences. First, it leads to a conception which we might call:

- [DoM] The flavour-colour dichotomy of matter, where the material world is partitioned into two sectors related to each other by having the roles of flavour and colour interchanged. One (the standard sector) already known to us is composed of the weak particles $h, W, Z$ and $q, \ell$ as building blocks, while the other (the “hidden” sector) is composed of their colour analogues $H, G, F$ as building blocks. From these
building blocks on either side, higher-level constructs can be built. For example, the quarks $q$ are coloured and so can form hadrons by colour confinement. Then these hadrons have (soft) nuclear forces between them, and can combine via such forces to give complex nuclei. Further, some of these nuclei being charged, they can combine by electromagnetic interaction with charged leptons to form atoms and molecules, and eventually us. Similar constructs are in principle possible also with the co-quarks $Q$ and co-leptons $L$, forming a co-sector which is possibly as complex and as vibrant in interactions within itself as our own standard sector.

This dichotomy is illustrated in Table 2. Notice that the hidden sector there is labelled “hidden” only because we have not seen in experiment any of the particles listed in that sector. But we have yet to understand theoretically in FSM why it is that we have not seen them, namely again the answer to [Q1]. This is a task that we shall have to come back to later.

| Building blocks | Standard Sector | “Hidden Sector” |
|----------------|----------------|-----------------|
|                | $(h), (W, Z), (q, \ell)$ | $H, G, F$ |
|                | point-like, perturbative interactions | point-like, perturbative interactions |
| Bound states of above by colour confinement | $q\bar{q}$: mesons (bosons) | $QQ$ co-mesons (bosons) |
|                | $qqq$ baryons (fermions) | $QQ$ co-baryons (bosons) |
|                | bulky, non-perturbative soft interactions | bulky, non-perturbative soft interactions? |
| Bound states of above by flavour confinement | nuclei | co-nuclei? |
| Bound states by soft interactions | | |
| Bound states by e.m. | atoms, molecules, . . . , us | co-atoms, co-molecules? |

Table 2: The Dichotomy of Matter (according to the FSM)
7 Transfer of technology between sectors

Now, it is unexpected, indeed surprising, that the FSM, which was constructed originally to understand the generation puzzle for quarks and leptons in our sector should have led us to such a dichotomy with a “hidden sector”. But once there, the “hidded sector” would be far from unwelcome when we recall that more than half our world is made up of dark matter the nature of which is still hidden from us [20]. However, merely to suggest the existence of such a sector is not much use unless one can also suggest the means for studying it, to explain why it should be hidden in the first place, and then, having done so, to find ways to probe into its hidden secrets. And most gratifyingly, the parallel depicted in Figure 2 also allows us to do so via what we might call:

• [ToT] Transfer of technology between the standard and “hidden” sectors, meaning that the same machinery used to investigate $h, W, Z$ and $q, \ell$ on the left of Figure 2 can be applied also to their counterparts $H, G, F$ on the right, allowing one to explore the “hidden sector” in depth.\(^4\)

In particular:

• The perturbative method used so succesfully to study the properties and interactions of the weak particles $h, W, Z$, and $q, \ell$ in the standard sector should be applicable also to their analogues $H, G, F$ in the hidden sector.

For the weak particles in the flavour theory, perturbation theory is usually carried out in the symmetry-breaking picture, but the calculation is basically the same in the confinement picture we favour, only interpreted differently, as has been shown by ’t Hooft [19], and Banks and Rabinovici [21]. Further, according to ’t Hooft, the perturbative method in the electroweak theory is permissible if the vacuum expectation value of the scalar Higgs field $\zeta_W (\sim 246$ GeV) is large. So it should be permissible also in the colour theory in FSM

\(^4\)Although the “hidden sector” being still unknown, the transfer of technology will mostly go one way at present, namely from the known standard sector to the other, the transfer can go in principle the other way also. Indeed, in [15] one had already an example where it was from the study of radiative corrections to the $F$ self-energy in the “hidden sector” that the RGE for the rotation of $\alpha$ was derived, which was what gave the result for quarks and leptons in Table 1.
where the vacuum expectation value of the scalar framon is even larger $\zeta_S \sim \text{TeV}$.

If this is true, it leads immediately to the following results:

- It post-justifies the one-loop calculation carried out in [15] which gave the result in Table 1. This calculation was initially carried out merely as the simplest one could do, without it being then considered whether a perturbative approach was in fact justified.

- It justifies the entry in Table 2 that the particles $H, G, F$ are point-like. This was our conclusion for the weak particles $h, W, Z, q, \ell$ in the flavour theory because perturbation theory applied, and so by the same token, the conclusion should hold also for the particles $H, G$ and $F$ in the colour theory. Now, previously, in [22] an intuitive physical argument was suggested to support what was then a conjecture that the $H, G, F$ are point-like for having little soft interactions based on the fact that their framon constituents have short life-times. This is now seen to be unnecessary since the point-likeness of the $H, G, F$ is supposedly already guaranteed by their perturbative nature. By dispensing then with that argument as an assumption, one has put the conclusion on a firmer basis and confirms the parallel drawn up in Figure 2 above. However, this does not by itself invalidate that qualitative argument, which may still be retained as a useful intuitive picture of why quark bound states via colour confinement (hadrons) have soft interactions while framonic bound states via the same colour confinement have none.

- It opens up a huge vista of coming explorations of the hidden sector using perturbative methods which can in principle rival in detail and complexity that for our standard sector and turn into a major industry.

8 $G$-modified mixing

The last item listed is of course exciting, but before we get carried away, let us first perform the initial steps to test whether such a programme is at all sensible and likely to bear fruit. As a first step, let us investigate the mass spectra and interaction vertices of the $H$s, $G$s, and $F$s. As in the familiar flavour theory, these can be obtained by expanding the framon action in fluctuations of the framon about its vacuum expectation value. This one can
do when one recalls that the framon action is strongly constrained by the requirement of the double invariance under $G \times \tilde{G}$ essentially fixing its form, only dependent on some parameters. Though lengthy and cumbersome, the calculation is fairly straightforward and is reported in [22]. As an example, only that for the mass spectrum of the $G$s is outlined here, which is in some sense the simplest but has particular physical significance.

The mass matrix for the vector bosons is obtained by expanding the kinetic energy term of scalar framon fields in their fluctuations about their vacuum values to leading order, meaning in this case the substitution of the framon fields by their vacuum values. We are familiar in the standard electroweak theory how the calculation there gives diagonal masses to the $SU(2)$ fields $B_i^\mu$ labelled by the Pauli matrices $\tau_i$ to give the massive vector bosons $W^i$, except for the third component $B_3^\mu$ which mixes with the $U(1)$ field $A_\mu$ to give the electromagnetic field $\gamma_\mu$ and $Z_\mu$. Not surprisingly, the same machinery applied to the FSM, extended to include both the flavour and colour sectors, gives a mass matrix for the vector bosons which is still diagonal when the $SU(2)$ fields remain labelled by the Pauli matrices while the $SU(3)$ fields are labelled by the Gell-Mann matrices $\lambda_k$, except now for an extended and modified mixing among $A_\mu, B_3^\mu$ and $C_8^\mu$.

Because of this latter mixing, two critical questions immediately arise. First, 

- Will the photon remain massless? Or else the FSM theory will be ruled out right away given that the photon mass has already been checked to hardly disputable accuracy.

Fortunately the answer is yes: one can keep the photon massless if one chooses the electric charges of the colour framons judiciously, as Salam and Weinberg did for the flavour framon (Higgs scalar) when constructing the electroweak theory. The appropriate choice here in the colour case is $-\frac{1}{3}, -\frac{1}{3}, +\frac{2}{3}$ respectively for the 3 columns of $\Phi$ in [CF], where the third column is that aligned with $\alpha$ and the first and second columns are perpendicular to it [22, 23]. This choice is similar to the Salam-Weinberg theory of the charges $-\frac{1}{2}, +\frac{1}{2}$ for the two columns of $\Phi$ in [FF] except that in this case, the orientation of the columns need not be specified, basically again because of (6). These charges were in fact needed to complete the specification of the framon representations in $G \times \tilde{G}$ but were missed out deliberately before in [FF] and [CF] because they could not then be specified. Now that they are settled, they can be inserted there for completeness.
Secondly,

- **What about the new mixing for the Z?** The extra mixing above will mean deviations from the standard Weinberg mixing scheme which has already been tested to great accuracy in experiment. Will these deviations then lead to violations of present experimental bounds?

This question cannot be answered yet in full given that the FSM has not been developed sufficiently for loop diagrams (radiative corrections) to be calculated in general, which will be needed for checking with experiment in depth. However, initial tests can be devised as follows. Assuming, as is generally accepted, that the SM agrees with experiment to within present bounds, and that the deviations of the FSM from the SM in loop corrections are of higher order, we compare the tree-level results of the FSM and the SM and if the difference is within experimental bounds, we conclude that the FSM results are also within present experimental bounds. This criterion has been applied in [23] to the following most urgent cases:

- (a) $m_Z - m_W$,
- (b) $\Gamma(Z \rightarrow \ell^+\ell^-)$,
- (c) $\Gamma(Z \rightarrow q\bar{q})$

where the deviations at tree level of the FSM from the SM are evaluated. These all depend on $m_G$, the mass of a new vector boson $G$ (or equivalently the vacuum expectation value of the colour framon $\zeta_S \sim 2 \times m_G$) as the only parameter. And it is found that so long as $m_G > 1$ TeV, then the FSM deviations from the SM at tree level will all remain inside the present already very stringent experimental bounds [23]. There are some subtle cancellations which have allowed this to happen and point perhaps to a deeper reason for the agreement not yet understood.

One can turn the argument around, of course, and treat these deviations as new physics to be tested when experimental accuracy further improves. In this direction, an observation on the $W$ mass may be noteworthy. The FSM predicts, via the extra mixing with the new boson $G$, a smaller value for the mass shift $m_Z - m_W$ than the SM. This means that starting with the better measured $m_Z$ to predict $m_W$, as it is usually done, one would obtain a larger value for $m_W$ in the FSM than in the SM. The present experimental situation as recently summarized by ATLAS [24] is shown in Figure 3 where
Figure 3: The ATLAS measurement of the W boson mass and the combined values measured at the LEP and Tevatron colliders compared to the Standard Model prediction (mauve) and the FSM predictions (green) at $\zeta_S = 2.0$ TeV (left) and $\zeta_S = 1.5$ TeV (right).

It is seen that successive measurements at LEP, the Tevatron, and the LHC all actually give central values for $m_W$ bigger than the SM prediction, as the FSM suggests, although the excess is only 1–2 $\sigma$, and therefore statistically not yet significant. But if in future experimental accuracy for $m_W$ further improves, then it would be meaningful to ask whether the excess predicted by the FSM really exists. The FSM prediction depends on $\zeta_S$, the vacuum expectation value of the colour framon which works out to be about $2m_G$.

In Figure 3, the FSM prediction is shown in green for $\zeta_S = 2$, and 1.5 TeV, (corresponding to $m_G \sim 1.0$ and 0.75 TeV respectively). The FSM predictions actually seem to give a better fit than the SM to the present available data.
The deviations found in [23] of the FSM from SM in the decay widths of $Z \rightarrow \ell^+\ell^-$ and $Z \rightarrow q\bar{q}$ may also appear as new physics when experimental accuracy improves. Obviously, however, as far as the change in mixing of the vector bosons ($G$-modified mixing) from the SM to the FSM is concerned, the prime new physics target would be the discovery of the vector boson $G$ itself. We shall postpone discussions of this till later when further facts are known.

9 Masses and interactions of the $H$, $G$ and $F$

Let us turn back now to the exploration of the “hidden” sector, continuing with the mass spectrum of the $G$s. We noted already that their mass-squared matrix is nearly diagonal except for the mixing of $G_8$ already studied. The diagonal values are as listed below:

\[
\begin{align*}
\frac{1}{6}g_3^2\zeta_S^2(1 - R) & \quad \text{for } K = 1, 2, 3; \\
\frac{1}{12}g_3^2\zeta_S^2(2 + R) & \quad \text{for } K = 4, 5, 6, 7; \\
\frac{1}{6}g_3^2\zeta_S^2(1 + R) & \quad \text{for } K = 8,
\end{align*}
\]

where $g_3$ is the colour coupling and $\zeta_S$ the vacuum expectation value of the colour framon. They are very similar to those in the familiar flavour case except for the appearance of factors depending on a parameter $R$. This parameter is a ratio which measures the relative strengths of symmetry-breaking versus symmetry-restoring terms in $\tilde{SU}(3)$ which has no analogue in $\tilde{SU}(2)$ because of the condition (6) imposed on the flavour framon for reasons already stated.

Now this ratio $R$ figures prominently in the RGE for the rotation of $\alpha$ which was used in [15] to give the fit in Table 1. Since both these quantities are connected with how the symmetry $\tilde{SU}(3)$ is broken, it is not surprising that their scale-dependences are correlated. Thus, corresponding to the trajectory of $\alpha$ of Figure 1, one has obtained in [15] the scale-dependence of $R$ shown in Figure 4. Given that this dependence is quite as strong as that of $\alpha$ which was used to fit the quark and lepton spectra, it seems reasonable to take account also of the parallel scale-dependence of $R$ in studying the spectra of the $G$s. In that case, the matrix elements in (7) depend on scale and we have to question at what scale or scales are the physical masses of the $G$s are to be evaluated.
Figure 4: Dependence of $R$ on scale $\mu$ [15]

The general consensus is that when the mass matrix is scale-dependent in QFT the physical masses of particles are to be evaluated each at their own mass scales, meaning that they are each to be solutions of the equation:

$$m_x(\mu) = \mu,$$

(8)

where $m_x(\mu)$ is the scale-dependent eigenvalue of the mass matrix corresponding to the particle $x$ under consideration.

Apply now this criterion to the $G$s. From Figure 4 we see that $R$ is small, 0.02 or less, for $\mu > m_Z$, so that the eigenvalues in (7) are all nearly degenerate. Further, from the analysis given above for the change in mass shift $m_Z - m_W$ due to $G$-modified mixing, $\zeta_S > 2$ TeV, which means that the physical masses obtained from (8) will be in excess of 1 TeV and nearly degenerate, apart from two exceptions. First, $G_8$ will be pushed higher by a small amount via mixing with $\gamma$ and $Z$ to form $G$. Secondly, and much more dramatically, the mass matrix elements of $G_1, G_2, G_3$ in (7) all carry a factor $1 - R$ which according to Figure 4 vanishes at $\mu$ around 17 MeV. This means that the equation (8) is bound to have another solution just above 17 MeV. Now it has been suggested in [15] in a parallel case for leptons and quarks that whenever a second lower solution exists for (8), then it should
Table 3: Suggested spectrum of the $G$ states

| Particle | State | Mass       |
|----------|-------|------------|
| $G^0$   | mixture of $G_8, Z$ and $\gamma$ | $\geq 1.1$ TeV |
| $G^+$   | $\frac{1}{\sqrt{2}}[G_4 + iG_5]$ | $\geq 1.0$ TeV |
| $G^-$   | $\frac{1}{\sqrt{2}}[G_4 - iG_5]$ |           |
| $G'^+$  | $\frac{1}{\sqrt{2}}[G_6 + iG_7]$ |           |
| $G'^-$  | $\frac{1}{\sqrt{2}}[G_6 - iG_7]$ |           |
| $G^0_1$ |       | $\sim 17$ MeV |
| $G^0_2$ |       |             |
| $G^0_3$ |       |             |

be taken as the physical solution since the higher solution will be unstable against decay into the lower. Indeed, this assertion is what gives the result $m_u < m_d$ much vaunted in [b] above. If it is accepted here also, then we have the spectrum for $G$s summarized in Table 3. Notice the characteristic separation of the spectrum into two groups, one, say $G_{\text{heavy}}$ with masses of the order TeV, and the other $G_{\text{light}}$ with masses $\sim 17$ MeV, where those in $G_{\text{light}}$ are all electrically neutral, which will be of significance for discussions later.

The mass spectra of the $H$s and $F$s have been similarly investigated, for the $H$s by expanding the framon potential and for the $F$s the Yukawa terms. Apart from some parameters with precise values yet unknown, the $H$ spectrum poses little problem, dividing as did that for the $G$s into two

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However, a serious word of caution is needed for the $17$ MeV prediction for the mass of $G_{\text{light}}$, and later on for $H_{\text{light}}$. We recall that its derivation is based on strict adherence to the following 3 criteria: (i) physical mass of a particle is to be measured at its own mass scale, i.e. solution of (8), (ii) validity of the fit in [15] giving Table 1, and (iii) when there are two solutions to (8), one takes the lower. None of these three theoretical criteria is beyond reasonable doubt. Thus with these alone to deduce a mass of $17$ MeV from a system with a natural scale of order TeV is a little audacious. On the other hand, for phenomenological support, one can quote, first, the apparent success in Table 1, where using the same three criteria, one gets right the lightest generation properties: (i) $m_u, m_d, m_e$ all of order MeV, (ii) $m_u < m_d$, both of which facts will otherwise be very hard to understand. Secondly, there is also the coincidence in mass with the Atomki anomaly to be mentioned later. How much trust one can put on this prediction is a question which needs to be asked again after more consideration.
groups, one $H_{\text{heavy}}$ with masses probably in excess of TeV, and one $H_{\text{light}}$ with mass $\sim 17$ MeV. Again the members of $H_{\text{light}}$ are all electrically neutral. The spectrum for the $F$s, however, is more problematic because, with no known geometrical significance for the fermion fields, one is unsure what the fermion fields are which should enter into the Yukawa couplings. In addition, the co-neutrinos among the $F$s can be affected by a see-saw mechanism [25] similar to that for neutrinos in the standard sector, and so may end up with masses considerably lower than may appear at the tree level. The spectrum for $F$s suggested in [22] is thus model-dependent, standing only to be adjusted when more information becomes available. As it stands, however, it divides also into a $F_{\text{heavy}}$ and a $F_{\text{light}}$ group as do the spectra for $G$ and $H$.

Expanding further the framon action to higher orders, one obtains interaction vertices for the $H$s, $G$s and $F$s. This is done in [22] giving some 10 pages of vertices between the various $H$s, $G$s, and $F$s. An outstanding feature in these results is that despite the many vertices coupling the $H$s, $G$s and $F$s among themselves, there are very few which link them to the weak particles $h$, $W$, $Z$, and $q, \ell$ in the standard sector. Indeed, the only examples found for the latter type of couplings are due to the mixing already mentioned of $G_8$ with the $\gamma$ and $Z$ of the standard electroweak theory and a similar but somewhat more complex mixing of certain $H$ states with the electroweak theory Higgs state $h_W$. This lack of couplings linking the two sectors comes about mainly because of the choice (4) for the representation of the framon made at the beginning. For instance, this choice implies that the framon kinetic energy term, which is the most prolific in spawning vertices, is a sum of two terms, one for the flavour and one for the colour sector, with only the $U(1)$ gauge field $A_\mu$ linking the two. This lack of couplings between the two sectors will figure prominently later when physical consequences are considered.

10 Exploring the hidden sector—a few first steps

Given the mass spectra of the $H$s, $G$s and $F$s and the interaction vertices between them, one has then the basic ingredients to develop the perturbation theory for the $H$s, $G$s and $F$s in the hidden sector. There may, of course, be unforeseen difficulties, but the perturbation theory can in principle be devel-
oped to a similar degree of sophistication as for the standard sector including loops for radiative corrections and so on. In this perspective, therefore, what has been done so far has merely scratched the surface of what is accessible, and not so evenly even at that. At tree level, little is examined beyond the mass spectra and interaction vertices of the $H$s, $G$s and $F$s. And as for loops, the only venture made so far has been that reported in [15] giving the RGE for the rotation of $\alpha$. Nevertheless, it is worthwhile to pause awhile for breath and take stock of the physical situation revealed by that little which has been found.

Our first task is to return to the question [Q1] why we have all along been unaware of the particles $H$, $G$ and $F$, or why the hidden sector has so far been hidden from us. Let us see whether we can now venture an answer. The analysis of the mass spectra and interaction vertices above has shown us that the hidden sector is as heavily populated and as vibrant in interactions within itself as our standard sector, in fact perhaps even more so. However, there are only restricted communications between the two sectors so that we who live in the standard sector may have difficulty knowing about what we call the “hidden sector”, as they also who abide in the other sector may have difficulty knowing about us. Nevertheless, some of the $H$s, $G$s and $F$s, such as $G^0$ and $G^\pm, G'^\pm$, would still manifest themselves, if they exist; thus $G^0$ by decaying into $\ell^+\ell^-$, for example, and the other charged states by exchanging a photon with our standard charged particles. However, one might argue that these particles being heavy with masses of order TeV would have decayed away by our epoch even if they were present in the early universe so that none will occur now naturally.\footnote{This is assumed to be the case even though, with insufficient knowledge of the $F$ spectrum, it has not been possible to work out the decay process of the $H$s, $G$s and $F$s in every case.} If we wish to see them, we shall have to produce them in the laboratory, and this is not easy, given their very high masses and their reluctance in coupling to normal matter with which we do our experiments. What can occur naturally are the low mass states, that is, what we call the $H_{\text{light}}$, $G_{\text{light}}$ and $F_{\text{light}}$ some of which are likely to be stable. But these being electrically neutral, with little interaction with ordinary matter, will behave like dark matter to us. One can then claim for these reasons that the whole sector has so far been hidden from us, answering thereby [Q1].

Suppose this is true, then knowing now how the particles in the hidden sector hide themselves, can we not find some gaps in their defence to get at
them? Based just on what is known, one can suggest the following:

- The most promising is probably the vector boson $G$. If the tempting indications of Figure 3 are taken seriously, the $G$ mass would not be much above 1 TeV. Its known mixing with $\gamma$ and $Z$ means that it will decay into $\ell^+\ell^-$ pairs with a known width (depending only on its mass) which is already calculated [23]. Its production cross section at LHC seems calculable and is under investigation. Its total width depends on the projected mass spectra and couplings of the $H$, $G$ and $F$ which are already available though not yet fully tested. With these bits of information, one may soon be able to make practicable suggestions for $G$ to be searched for as a $\ell^+\ell^-$ bump at the LHC. Its discovery would not only serve as a detailed check of the FSM scheme but also, according to this, a window into the “hidden” sector, for it will be into the dark matter candidates ($G_{\text{light}}$ and $H_{\text{light}}$ states of mass around 17 MeV or co-neutrinos of mass < 8 MeV) that $G$ will mostly decay.

- The next most promising is probably $G_3$, with predicted mass 17 MeV, which can decay into $e^+e^-$ via a photon attached to a framon loop. Now, it so happens that coincidentally at this predicted mass, an anomaly has been reported by the Atomki collaboration [26] in the decays: $Be_8^* \rightarrow Be_8 + e^+e^-$. Not enough work has been done yet on $G_3$ to ascertain whether it can explain the Atomki anomaly, and the anomaly itself needs to be independently confirmed, but the coincidence is intriguing. One notes that the prediction of 17 MeV is deduced from the result of [15] which predates the Atomki report of the anomaly. (Note, however, the remark in footnote 5.) In any case, independently of Atomki, it is worthwhile studying further this mass region around which there has already been a lot of activity, prompted by such phenomena as the $g - 2$ anomaly [27] or the proton radius puzzle [28].

- The remaining states $G_1, G_2$ in $G_{\text{light}}$ (and their counterparts in $H_{\text{light}}$) at 17 MeV appear stable, unless some co-neutrinos can acquire masses via a seesaw mechanism low enough for these to decay into. In any case, there will be dark matter candidates at these low masses for experiment to search for. They are probably beyond the reach of LUX [29], LZ [30], and other experiments which concentrate on the multi-GeV region but may be accessible with new techniques such as SENSEI
Although light, these dark candidates are expected to occur in abundance. First, being binary objects, they would be statistically easier to form in the early universe than baryons which would require the coincidence of three quarks for their formation. Secondly, these dark matter candidates occur frequently as decay products of higher $H, G, F$ states. And so, though light, they may make up a sizeable fraction of the missing mass. However, much more work will be needed to ascertain whether they may make up the bulk of it. One novel feature of these particles as dark matter is that they are predicted by a model not specifically designed for dark matter itself but one constructed for another purpose, and which model already prescribes a great deal about how these particles will behave, given that the action is known and perturbative calculations apply.

- The charged particles in the “hidden” sector couple to the photon as usual so that these could be pair-produced in a $e^+e^-$ collider provided the energy is high enough. They will give rise to step increases of the ratio $R$ as usual in $e^+e^-$ collisions. Now the model Yukawa coupling proposed tentatively in [22] suggests masses of these charged $F$'s of the order TeV, in which case they would be above the $e^+e^-$ colliders being planned. But this may not be the case for other choices of the fermion fields in constructing the Yukawa couplings.

In brief, it is unexpected, indeed rather amazing, that constructed originally for addressing the generational puzzle in our own standard sector, the FSM has provided us, in addition, with a lead into the mysterious dark matter world. It is made even more intriguing by the fact that, the physics of the two sectors being closely interlocked, what is known about our standard sector provides also the technology for exploring the other. And the first few steps along the indicated direction have already yielded some items of new physics testable by experiment. Thus, if there is truth in what is said, we may soon be probing into a vast new world which has so far been hidden from us.

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