Comments on holographic current algebras and asymptotically flat four dimensional spacetimes at null infinity

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ABSTRACT. We follow the spirit of a recent proposal to show that previous computations for asymptotically flat spacetimes in four dimensions at null infinity can be re-interpreted in terms of a well-defined holographic current algebra for the time component of the currents. The analysis is completed by the current algebra for the spatial components.
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1 Introduction

In recent work, Strominger [1, 2] has proposed to replace the computation of integrated surface charges and their algebra associated to asymptotic symmetries in gravitational or gauge theories by a local computation in terms of Ward identities and current algebras.

A formal, path integral based, derivation of local Ward identities involving Noether currents and their divergences (see e.g. [3], sections 2.4 and 2.5) is essentially equivalent to classical properties of these currents. In order to take into account quantum effects, one has to use operator product expansion techniques or use perturbative quantum field theory with due care devoted to renormalization effects (see e.g. [4]).

In this note, we start with some brief comments on specific aspects of the classical part of this construction, first for global and then for gauge currents. It is the latter that are relevant in holographic applications when performing computations on the bulk side of the correspondence. As an illustration, the cases of three dimensional asymptotically AdS spacetimes at spatial infinity [5] and asymptotically flat spacetimes at null infinity [6,7,8] are reconsidered with a special emphasis on the additional spatial current components.

We then turn to asymptotically flat spacetimes at null infinity in four dimensions. Re-interpreting previous results derived in [9] and translated to the Newman-Penrose formalism in [10] gives the algebra of the time component of the currents. This is completed by working out the algebra of the spatial components. The holographic current algebra is derived in a unified way both for the standard, globally well defined asymptotic symmetry algebra $bms_4^{\text{glob}}$ involving Lorentz transformations and supertranslation generators that can be expanded into spherical harmonics as well as for its local version $bms_4^{\text{loc}}$ [11].

The main benefit of the local formulation in terms of currents in this context is that, for the local version of the asymptotic algebra, there is no longer any problem with divergences related to poles as there is no need to explicitly integrate the time component of the currents over the sphere. Instead, one can now use contour integrals on the unit or Riemann sphere if one so wishes.

In other words, in the standard approach the existence of well-defined charges is taken as a criterion to reduce the asymptotic symmetry algebra to $bms_4^{\text{glob}}$, assuming of course that the fields that enter the time components of the currents are integrable on the sphere. Changing this criterion to the existence of a well-defined local current algebra allows one to consistently deal with $bms_4^{\text{loc}}$. 
2 Classical current algebras

2.1 Global symmetries

For an action principle that is invariant under global symmetry transformations, there is a short-cut that allows one to determine the Poisson or Dirac bracket algebra of the generators of these transformations, without the need to go through the steps of the Hamiltonian analysis.

While the latter consists in first determining the brackets of the fundamental canonical variables, then expressing the Noether charges in these variables and finally evaluating their brackets, the short-cut is well known (see e.g. [12, 13]) and goes as follows. Let $L = Ld^n x$ be the Lagrangian $n$ form of the theory. Invariance means that $\delta_X L = d_H n_X$. Here, $\delta_X \phi^i$ denotes the infinitesimal transformations of the fields $\phi^i$ and $n_X$ an $n-1$ form. The differential $d_H = dx^\mu \partial_\mu$ involves the total derivative that takes into account the space-time dependence of the fields, $\partial_\mu = \partial_{\partial x^\mu} \phi^i \partial_\phi + \ldots$.

The Noether current $j_X = j_X^\mu (d^{n-1} x)_\mu$ associated to the transformation $\delta_X$ satisfies

$$X^i i \frac{\delta L}{\delta \phi^i} = d_H j_X,$$

and can be chosen as $j_X = n_X - I_X^\mu (L)$, where $I_X^\mu (L) = (X^i \frac{\partial L}{\partial \partial_\mu \phi^i} + \ldots) (d^{n-1} x_\mu)$. The Noether current is ambiguous, $j_X \sim j_X + d_H \eta_X + t_X$ where $t_X$ is a Noether current that vanishes on-shell, while $\eta_X$ is an $n-2$ form. By applying the symmetry transformation $-\delta_X$ to (2.1) for a symmetry characterized by $X_1$, it is then straightforward to show that

$$-\delta_X j_X \sim j_{[X_1, X_2]} + K_{X_1, X_2},$$

where the Lie bracket is determined through $[X_1, X_2]^i = \delta_{X_1} X_2^i - (1 \leftrightarrow 2)$, while the classical extension $K_{X_1, X_2}$ belongs to $H^{n-1}(d_H)$ and may in general be field dependent.

For instance, working out the classical current algebra of a chiral Wess-Zumino-Witten model in this way is straightforward, while the complete Hamiltonian analysis involves first and second class constraints and is much more involved.

2.2 Gauge and asymptotic symmetries

For asymptotic symmetries, which are a subset of the bulk gauge symmetries, similar results can be shown under suitable assumptions. Some elements of the general theory and more details can be found for instance in [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].
For a gauge transformation, $\delta f^i = R^i_\alpha (f^\alpha) = R^i_\alpha f^\alpha + R^i_\alpha \partial_\mu f^\alpha + \ldots$ with possibly field dependent gauge parameters $f^\alpha$, there is a weakly vanishing Noether current $S_f = (R^i_\alpha f^\alpha \delta L / \delta f^\alpha + \ldots)(d^{n-1}x)_\mu$. It is used to construct a canonical representative for an $n-2$-form associated to gauge symmetries through

$$ k_f[\delta \phi] = I^{n-1}_{\delta \phi} S_f = \frac{1}{2} \delta \phi^i \frac{\partial}{\partial \partial_i \phi^j} \frac{\partial}{\partial dx^j} S_f + \ldots $$

(2.3)

Under standard assumptions, one can then show that, $d_H k_f [\delta \phi_s] \approx 0$, when $f^\alpha_s$ satisfy $R^i_\alpha (f^\alpha_s) \approx 0$, and when $\delta \phi_s$ satisfies the linearized field equations. Here $\approx 0$ means for all solutions of the Euler-Lagrange equations for $\phi$. The algebra of the forms $k_f[\delta \phi]$ is obtained by applying a gauge transformation, and one can show that

$$ - \delta f_2 k_{f_1} = k_{[f_1,f_2]} + \text{more}, $$

(2.4)

where $[f_1,f_2]^\alpha$ involves the structure functions of the gauge algebra. The expression for the additional terms can be worked out quite generally, but what remains depends on the particular situation that one considers. As in the global case, there will be trivial terms proportional to the equations of motion, both for $\phi$ and $\delta \phi$, and $d_H$ exact terms. The non-trivial terms include possibly field-dependent central extensions.

In the asymptotic context, on which we will concentrate below, a typical situation is to focus on a fixed hypersurface, say $r = \text{cte} \rightarrow \infty$, with prescribed asymptotic conditions on the fields. What goes under the name of asymptotic symmetries is a suitable sub-Lie algebroid of the Lie algebroid associated to gauge symmetries. On the surface $r = \text{cte} \rightarrow \infty$, the $n-2$ form $k_f = k_f^{[uv]} (d^{n-2}) x_{uv}$ becomes to an $(n-1)-1$ form. In a coordinate system $x^\mu = (u,r,y^A)$ for instance, the components of the associated current for the lower dimensional theory are given by $(k_f^{[ur]}, k_f^{[Ar]})$. In the case where these currents are integrable in solution space, $k_f^{[ur]} \approx \delta J_f^u, k_f^{[Ar]} \approx \delta J_f^A$, their integrands $J_f^u, x^a = (u, y^A)$, provide the global current algebra of the dual boundary theory. The multiplicative normalization of $k_f$, and thus also of $J_f$, is fixed through the action of the theory, whereas the integration in solution space implies that the definition of the integrands $J_f^u$ involves the choice of a background solution.

Even though the original computations for asymptotically anti-de Sitter space-times [28, 5] have been performed in the Hamiltonian formalism, the brackets of the surface charges have been evaluated indirectly through (2.4). Indeed, a direct computation in terms of fundamental canonical variables is much more involved and is achieved, for instance, through the explicit construction of the dual boundary theory and its degrees of freedom, viz., Liouville theory [29] in the asymptotically AdS$_3$ case.
3 Standard examples in three dimensions

3.1 Solution space and transformation laws

On-shell, three dimensional spacetimes that are asymptotically AdS at spatial infinity or flat at null infinity are described by metrics of the form

$$ds^2 = \left( -\frac{r^2}{l^2} + \mathcal{M} \right) du^2 - 2dudr + 2N dud\phi + r^2 d\phi^2,$$

(3.1)

where

$$\mathcal{M} = 2(\Xi_{++} + \Xi_{--}), \quad \mathcal{N} = l(\Xi_{++} - \Xi_{--}),$$

(3.2)

with $$\Xi_{\pm\pm} = \Xi_{\pm\pm}(x^\pm), x^\pm = \frac{\Phi}{l} \pm \phi,$$ in the AdS case while $$l \to \infty$$ and

$$\mathcal{M} = \Theta, \quad \mathcal{N} = \Xi + \frac{u}{2} \partial_\phi \Theta,$$

(3.3)

with $$\Theta = \Theta(\phi)$$ and $$\Xi = \Xi(\phi)$$ in the flat case.

In the AdS case, the easiest solutions where these functions are constants,

$$\Xi_{\pm\pm} = 2G(M \pm \frac{J}{l}),$$

(3.4)

include both the BTZ black holes for which $$M \geq 0, |J| \leq Ml$$ and the AdS$_3$ spacetime which corresponds to $$M = -\frac{1}{8G}, J = 0.$$ In the flat case, the zero mode solutions are given by $$\Theta = 8GM, \Xi = 4GJ$$ and correspond for $$M \geq 0$$ to cosmological solutions.

In the former case, the asymptotic symmetry algebra forms a three-dimensional representation of the two dimensional conformal algebra, described in terms of vector fields $$\xi = Y^+ \partial_+ + Y^- \partial_-, \quad Y^\pm = Y^\pm(x^\pm),$$ equipped with the standard Lie bracket, while in the latter case, it is a representation of the bms$_3$ algebra described by vector fields $$\xi = Y \partial_\phi + (T + uY') \partial_u, \quad Y = Y(\phi), T = T(\phi).$$

By using that asymptotic symmetries preserve solutions, the gravitational computation yields the transformation laws

$$-\delta_\xi \Xi_{\pm\pm} = Y^\pm \Xi'_{\pm\pm} + 2Y^{\pm'} \Xi_{\pm\pm} - \frac{1}{2} Y^{\pm''},$$

(3.5)

respectively

$$-\delta_\xi \Theta = Y\Theta' + 2Y' \Theta - 2Y''',$$

$$-\delta_\xi \Xi = Y\Xi' + 2Y' \Xi + \frac{1}{2} T\Theta' + T' \Theta - T'''.$$

(3.6)
3.2 Current algebra

When the main purpose is to find the integrated surface charges \( Q_\xi = \int_0^{2\pi} d\phi J_\xi^u \), what is usually explicitly worked out from (2.3) is the \( k^{ur}_\xi \) and also \( J^u_\xi \) component only. In the cases at hand, when the background solution is fixed to be the BTZ black hole with \( M = 0 = J \) or the null orbifold respectively, these components are given by

\[
J^u_\xi = \frac{1}{8\pi G} [Y^+ \Xi_{++} + Y^- \Xi_{--}],
\]

\[
J^u_\xi = \frac{1}{16\pi G} [T\Theta + 2Y\Xi].
\]

Instead of a direct computation of the spatial components, they can easily be worked out from current conservation \( \partial_a J^a_\xi \approx 0 \). This gives

\[
J^\pm_\xi = \frac{1}{4\pi G} Y^\mp \Xi_{\mp\mp},
\]

in the ADS case, while \( J^\phi_\xi = 0 \) in the flat case.

The current algebra can then be deduced directly from the expression for the currents, the asymptotic symmetry algebra and the transformation laws of the fields,

\[
-\delta_{\xi_2} J^a_\xi_1 \approx J^a_{[\xi_1,\xi_2]} + K^a_{\xi_1,\xi_2} + \partial_b L^a_{[\xi_1,\xi_2]},
\]

where for AdS3,

\[
K^\pm_{\xi_1,\xi_2} = \frac{1}{16\pi G} [Y^+ Y^\mp \Theta - (1 \leftrightarrow 2)],
\]

\[
L^{[\mp\mp]}_{\xi_1,\xi_2} = \frac{1}{4\pi G} [Y^+ Y^\mp \Xi_{\mp\mp} - \frac{1}{2} Y^+ Y^\mp \Xi_{\mp\mp} + \frac{1}{4} Y^+ Y^\mp \Xi_{\mp\mp}],
\]

while

\[
K^u_{\xi_1,\xi_2} = \frac{1}{16\pi G} [Y^u T^u_2 + T^u_1 Y^u_2 - (1 \leftrightarrow 2)], \quad K^\phi_{\xi_1,\xi_2} = 0,
\]

\[
L^{[a\phi]}_{\xi_1,\xi_2} = \frac{1}{16\pi G} [(Y_1 T_2 + T_1 Y_2)\Theta + 2(Y_2 \Xi_1 - T_1 Y_2) + 2T^u_2 Y_1 Y_2 + Y_1 T^u_2],
\]

in the flat case.

4 Four dimensional asymptotically flat spacetimes at \( \mathcal{I}^+ \)

4.1 Solution space and transformation laws

Our conventions are as in [10]. More details can be found for instance in the reviews [30, 31, 32, 33].
On the space-like cut of $\mathcal{I}^+$, we use coordinates $\zeta, \bar{\zeta}$, the metric $\hat{ds}^2 = \hat{\gamma}_{AB}dx^Adx^B = 2P^{-2}d\zeta d\bar{\zeta}$ and the volume element $\hat{d}^2\Omega^\varphi = (id\zeta \wedge d\bar{\zeta})P^{-2}$. For the unit sphere, we have $\zeta = \cot\frac{\varphi}{2} e^{i\varphi}$ in terms of standard spherical coordinates and $P_S = \frac{1}{\sqrt{2}}(1 + \zeta \bar{\zeta})$ so that $d^2\Omega_S = \sin \theta d\theta \wedge d\varphi = \frac{2d\zeta \wedge d\bar{\zeta}}{i(1 + \zeta \bar{\zeta})^2}$.

The covariant derivative on the 2 surface is then encoded in the operator

$$\bar{\partial}^s \eta^s = P^{1-s} \bar{\partial} (P^s \eta^s), \quad \bar{\partial} \eta^s = P^{1+s} \partial (P^{-s} \eta^s), \quad (4.1)$$

where $\bar{\partial}, \partial$ raise respectively lower the spin weight by one unit and satisfy

$$[\bar{\partial}, \partial] \eta^s = \frac{s}{2} R \eta^s, \quad (4.2)$$

with $R = 4P^2 \partial \ln P$, $R_S = 2$. The spin weights of the various quantities are summarized in table[1]. Note that a field $\eta$ of spin weight $s$ and conformal weight $w$ transforms as

$$\delta \bar{\gamma} \gamma \eta = [\gamma \partial + \bar{\gamma} \bar{\partial}h \partial \gamma + \bar{\eta} \partial \gamma] \eta, \quad (4.3)$$

where the conformal dimensions are given by $(h, \bar{h}) = (\frac{s-w}{2}, \frac{-s-w}{2})$.

Let $\gamma = P^{-1} \gamma$ and $\bar{\gamma} = P^{-1} \bar{\gamma}$. The conformal Killing equations and the conformal factor then become

$$\partial \bar{\gamma} \gamma = 0 = \bar{\partial} \gamma, \quad \psi = (\partial \gamma + \bar{\partial} \bar{\gamma}). \quad (4.4)$$

It follows for instance that

$$\bar{\partial} \gamma \partial = -\frac{R}{2} \gamma, \quad \partial^2 \psi = \partial^3 \partial - \frac{1}{2} \bar{\gamma} \partial R, \quad \bar{\partial} \partial \psi = -\frac{1}{2} [\partial (R \gamma) + \bar{\partial} (R \bar{\gamma})]. \quad (4.5)$$

Let $f = T + \frac{1}{2} \mu \psi$ and $\xi = f \partial u + \gamma \partial + \bar{\gamma} \bar{\partial}$ with $\gamma, \bar{\gamma}$ conformal Killing vectors. The asymptotic symmetry algebra is described by the vector fields $\xi$ on $\mathcal{I}^+$, parametrized by $T, \gamma, \bar{\gamma}$, and equipped with the standard Lie bracket. More explicitly, $\tilde{\xi} = [\xi_1, \xi_2]$ is parametrized by $\tilde{T}, \tilde{\gamma}, \tilde{\bar{\gamma}}$ where

$$\tilde{\gamma} = \gamma_1 \partial \gamma_2 - (1 \leftrightarrow 2), \quad \tilde{\bar{\gamma}} = \bar{\gamma}_1 \bar{\partial} \bar{\gamma}_2 - (1 \leftrightarrow 2),$$

$$\tilde{T} = (\gamma_1 \partial + \bar{\gamma}_1 \bar{\partial}) T_2 - \frac{1}{2} \psi_1 T_2 - (1 \leftrightarrow 2). \quad (4.6)$$

The part of solution space that is relevant for the asymptotic current algebra on $\mathcal{I}^+$ is given by fields $\sigma^0, \Psi_2, \Psi_1$ and their complex conjugates and are denoted collectively by $\chi$. In this framework, $\sigma^0$ is the news function. For convenience, one also introduces

$$\Psi_3^0 = -\bar{\partial} \sigma^0 - \frac{1}{4} \bar{\partial} R, \quad \Psi_4^0 = -\bar{\partial} \sigma^0. \quad (4.7)$$

Above there is a mistake of a factor 2 for $d^2\Omega^\varphi$ in [10] after equation (6.5). For simplicity of notations, we have also removed the bar over the real scalar curvature $R$ of the metric $d\hat{s}^2$. 

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Table 1: Spin and conformal weights

| s | 2   | 2   | -2  | -1  | 0   | 1   | -1  | 0   |
|---|-----|-----|-----|-----|-----|-----|-----|-----|
| w | -1  | -2  | -3  | -3  | -3  | -3  | 1   | 1   |

In these terms, the evolution equations are

\[ \dot{\Psi}_3^0 = \partial \Psi_4^0, \quad \dot{\Psi}_2^0 = \partial \Psi_3^0 + \sigma^0 \Psi_4^0, \quad \dot{\Psi}_1^0 = \partial \Psi_2^0 + 2 \sigma^0 \Psi_3^0, \tag{4.8} \]

which have to be supplemented by the additional on-shell relation,

\[ \Psi_2^0 - \bar{\Psi}_2^0 = \partial^2 \sigma^0 - \bar{\partial}^2 \bar{\sigma}^0 + \sigma^0 \bar{\sigma}^0 - \sigma^0 \bar{\sigma}^0, \tag{4.9} \]

and the transformation laws are

\[
\begin{align*}
-\delta_\xi \sigma^0 &= [f \partial_u + \gamma \bar{\partial} + \bar{\gamma} \partial + \frac{3}{2} \partial \gamma - \frac{1}{2} \bar{\partial} \bar{\gamma}] \sigma^0 - \partial^2 f, \\
-\delta_\xi \bar{\sigma}^0 &= [f \partial_u + \gamma \bar{\partial} + \bar{\gamma} \partial + 2 \partial \gamma] \bar{\sigma}^0 - \bar{\partial}^2 \bar{\sigma}, \\
-\delta_\xi \Psi_4^0 &= [f \partial_u + \gamma \bar{\partial} + \bar{\gamma} \partial + \frac{1}{2} \partial \gamma + \frac{5}{2} \bar{\partial} \bar{\gamma}] \Psi_4^0, \\
-\delta_\xi \Psi_3^0 &= [f \partial_u + \gamma \bar{\partial} + \bar{\gamma} \partial + \partial \gamma + 2 \bar{\partial} \bar{\gamma}] \Psi_3^0 + \bar{\partial} f \Psi_4^0, \\
-\delta_\xi \Psi_2^0 &= [f \partial_u + \gamma \bar{\partial} + \bar{\gamma} \partial + \frac{3}{2} \partial \gamma + \frac{3}{2} \bar{\partial} \bar{\gamma}] \Psi_2^0 + 2 \bar{\partial} f \Psi_2^0, \\
-\delta_\xi \Psi_1^0 &= [f \partial_u + \gamma \bar{\partial} + \bar{\gamma} \partial + 2 \partial \gamma + \bar{\partial} \bar{\gamma}] \Psi_1^0 + 3 \bar{\partial} f \Psi_2^0. 
\end{align*} \tag{4.10} \]

### 4.2 Current algebra in the absence of news

Let us concentrate on the sphere \( P = P_S \), or the Riemann sphere with \( P = P_R = 1 \), so that \( \partial R = 0 = \bar{\partial} R \). Let us also assume that there is no news, \( \frac{\partial}{\partial u^n} \sigma = 0 \) for all \( n \geq 1 \), which implies in particular also that \( \Psi_4^0 = 0 = \Psi_3^0 \) and that their time derivatives vanish. Furthermore, the complex conjugates of all these expressions also vanish. The transformations that leave these conditions invariant have to satisfy \( \bar{\partial}^2 \psi = 0 = \bar{\partial}^2 \bar{\psi} \). Both on the Riemann and the unit sphere, this is equivalent to \( \bar{\partial}^3 \gamma = 0 = \bar{\partial}^3 \bar{\gamma} \) and then to \( \gamma = P^{-1} \bar{\gamma}, \bar{\gamma} = P^{-1} \gamma \) with \( \bar{\partial}^3 \bar{\gamma} = 0 = \bar{\partial}^3 \gamma \). It follows that among the superrotations, only the standard Lorentz transformations corresponding to \( l_{1} = P^{-1} \bar{\partial}, l_0 = P^{-1} \bar{\gamma}, l_1 = P^{-1} \bar{\bar{\gamma}} \partial \) and their complex conjugates remain. The remaining dynamical equations simplify and read \( \dot{\Psi}_2^0 = 0, \dot{\Psi}_1^0 = \partial \Psi_2^0 \), together with their complex conjugates, and also...
\(\Psi^0_2 = \Psi^0_2 = \frac{\partial^2}{\partial x^2} \sigma^0 - \partial_x \sigma^0\). In the following, absence of news means that all the above conditions are satisfied.

Note that one could also impose \(\partial^2 T = 0 = \bar{\partial}^2 T\), in which case the supertranslations reduce to ordinary translations and the asymptotic symmetry algebra becomes the Poincaré algebra. We will not make these additional assumptions here.

In the absence of news, the time component of the current, which is real, is given by (4.18) where the second term proportional to \(\dot{\sigma}^0\) is absent. Current conservation in the form
\[
\partial_u J^\mu_\xi + \bar{\partial} J^\mu_\xi \approx 0,
\]
then leads to \(J_\xi\) given in (4.21), where the second, the third and the last two terms vanish according to the assumptions made in the present section. This is equivalent to the standard conservation law
\[
\partial_u f^0_\xi \approx 0,
\]
provided that \(f^0_\xi = P^{-2} J^u_\xi, f^\xi_\xi = P^{-1} J_\xi, f^\xi_\xi = P^{-1} \bar{J}_\xi\).

Current algebra in the absence of news is then explicitly given by
\[
- \delta_{\xi_2} J^u_\xi \approx J^u_{[\xi_1, \xi_2]} + \partial \mathcal{L}_{\xi_1, \xi_2} + \bar{\partial} \mathcal{L}_{\xi_1, \xi_2},
\]
\[
- \delta_{\xi_2} J_\xi \approx J_{[\xi_1, \xi_2]} - \partial_u \mathcal{L}_{\xi_1, \xi_2} + \bar{\partial} \mathcal{M}_{\xi_1, \xi_2},
\]
where \(\mathcal{L}_{\xi_1, \xi_2}\) is obtained from (4.20) and \(\mathcal{M}_{\xi_1, \xi_2} = -\mathcal{L}_{\xi_1, \xi_2}\) from (4.26) by dropping all terms that vanish in the absence of news. It is understood that the algebra for \(\bar{J}_\xi\) is obtained by the one for \(J_\xi\) through complex conjugation. This is equivalent to the standard form
\[
- \delta_{\xi_2} f^a_\xi \approx f^a_{[\xi_1, \xi_2]} + \partial_b L^{[ab]}_{\xi_1, \xi_2},
\]
provided that \(L^{[\mu \nu]}_{\xi_1, \xi_2} = P^{-1} \mathcal{L}_{\xi_1, \xi_2}, L^{[\mu \nu]}_{\xi_1, \xi_2} = P^{-1} \mathcal{L}_{\xi_1, \xi_2}, L^{[\bar{\nu} \bar{\mu}]}_{\xi_1, \xi_2} = \mathcal{M}_{\xi_1, \xi_2}\). In particular, this requires \(\mathcal{M}_{\xi_1, \xi_2}\) to be purely imaginary.

Consistency conditions between conservation and current algebra can be deduced by using that \(\partial_u J^u_\xi = \frac{\partial}{\partial u} J^u_\xi - \partial_{\bar{u}} J^u_\xi\), where the partial derivative denotes the explicit \(u\) dependence contained in \(\xi\), and also \(J^u_{[\xi, \partial_u]} = J^u_{-\bar{u} \xi}\). It then follows from (4.11) and (4.13) that
\[
\bar{\partial}(\bar{J} + L_{\bar{\partial}_\xi, u}) + \partial(\bar{J} + L_{\bar{\partial}_\xi, \bar{u}}) \approx 0.
\]
This condition is satisfied because (4.20) implies that \(\bar{J} = -\mathcal{L}_{\bar{\partial}_\xi, u}\).

4.3 Current algebra in the presence of news

As in standard applications (see e.g. [34]), current algebra is more involved when currents are not conserved. This is the case in the presence of news. We will also allow for an
arbitrary, \( u \)-independent, \( P(\zeta, \overline{\zeta}) \) in the metric on the space-like cut of \( \mathcal{I}^+ \). The presence of news implies in addition that \( \mathcal{K}_{\zeta}^{[ur]} \), \( \mathcal{K}_{\zeta}^{[Ar]} \), are no longer integrable.

Let
\[
\Theta_{\xi}^{\mu} (\delta \chi) = \frac{1}{8 \pi G} \left[ f \bar{\sigma}^0 \delta \sigma^0 + \text{c.c.} \right],
\] (4.16)
and
\[
\mathcal{K}_{\xi_1, \xi_2}^{\mu} = \frac{1}{8 \pi G} \left[ \left( \frac{1}{2} \bar{\sigma}^0 f_1 \delta^2 \psi_2 + \frac{1}{4} f_1 \bar{\sigma} f_2 \bar{\sigma} R - (1 \leftrightarrow 2) \right) + \text{c.c.} \right],
\] (4.17)
be expressions that vanish in the absence of news. The computations of [9], translated to the Newman-Penrose formalism in [10], state that the time component of the current
\[
\mathcal{J}_{\xi}^{\mu} = - \frac{1}{8 \pi G} \left[ (f (\Psi_0^0 + \sigma^0 \bar{\sigma}^0) + \mathcal{Y}(\Psi_1^0 + \sigma^0 \bar{\sigma}^0) + \frac{1}{2} \bar{\sigma} (\sigma^0 \bar{\sigma}^0)) + \text{c.c.} \right].
\] (4.18)
satisfies
\[
- \delta_{\xi_2} \mathcal{J}_{\xi_1}^{\mu} + \Theta_{\xi_2}^{\mu} (- \delta_{\xi_1} \chi) \approx \mathcal{J}_{\xi_1, \xi_2}^{\mu} + \mathcal{K}_{\xi_1, \xi_2}^{\mu} + \partial \mathcal{L}_{\xi_1, \xi_2} + \overline{\partial \mathcal{L}_{\xi_1, \xi_2}},
\] (4.19)
for some \( \mathcal{L}_{\xi_1, \xi_2} \). Since it is relevant for current algebra, the explicit expression is now worked out. It is given by
\[
\mathcal{L}_{\xi_1, \xi_2} = \mathcal{Y}_2 \mathcal{J}_{\xi_1}^{\mu} - f_2 \mathcal{J}_{\xi_1}^{\mu} - \frac{1}{8 \pi G} \left[ \left( \frac{1}{2} [\partial \mathcal{Y}_1 + \overline{\partial \mathcal{Y}_1}] \partial f_2 - \frac{1}{2} \mathcal{Y}_1 \bar{\sigma}^2 f_2 - \overline{\mathcal{Y}_1} \bar{\sigma} f_2 \right) \bar{\sigma}^0 \right.
\[
- \frac{1}{2} \mathcal{Y}_1 \bar{\sigma} f_2 \sigma^0 + (- \mathcal{Y}_1 \bar{\sigma} f_2 + \overline{\mathcal{Y}_1} \bar{\sigma} f_2) \partial \bar{\sigma}^0 - f_1 \bar{\sigma} f_2 \bar{\sigma}^0 \right].
\] (4.20)
where
\[
\mathcal{J}_{\xi}^{\mu} = \frac{1}{8 \pi G} \left[ \mathcal{Y}(\Psi_0^0 + \frac{1}{2} [\sigma^0 \bar{\sigma}^0 - \sigma^0 \bar{\sigma}^0]) - \frac{1}{2} \partial (\partial \mathcal{Y} - \overline{\partial \mathcal{Y}}) \bar{\sigma}^0 \\
+ \frac{1}{2} (\partial \mathcal{Y} - \overline{\partial \mathcal{Y}}) \bar{\sigma} \sigma^0 + f \Psi_3^0 + \bar{\sigma} f \bar{\sigma}^0 \right].
\] (4.21)
In order to identify the spatial component of the current \( \mathcal{J}_{\xi}^{\mu} \), we have used that the non-conservation of \( J_{\xi}^{\mu} \) can be obtained from (4.19) for \( \xi_1 = \zeta \) and \( \xi_2 = \overline{\partial u} \). Indeed, this gives
\[
\partial_u \mathcal{J}_{\xi}^{\mu} + \overline{\partial \mathcal{J}_{\xi}^{\mu}} + \overline{\partial \mathcal{J}_{\xi}^{\mu}} \approx \Theta_{\partial u}^{\mu} (\delta \chi),
\] (4.22)
when \( \mathcal{J}_{\xi}^{\mu} = - \mathcal{L}_{\xi, \overline{\partial u}} \).

Alternatively, one can obtain the spatial components directly by evaluating the \( k^{[Ar]} \)-components of the surface charge \( n - 2 \) form. This is done explicitly in the appendix. It also provides an expression for the non-integrable part,
\[
\Theta_{\xi}^{\mu} (\delta \chi) = \frac{1}{8 \pi G} \mathcal{Y} [\bar{\sigma}^0 \delta \sigma^0 + \sigma^0 \bar{\sigma}^0] = \mathcal{Y} \Theta_{\partial u}^{\mu} (\delta \chi).
\] (4.23)
One can now work out the current algebra for the spatial component,
\[ - \delta_{\xi_2} J_{\xi_1} + \Theta_{\xi_2} (\delta_{\xi_1} \chi) \approx J_{[\xi_1, \xi_2]} + K_{[\xi_1, \xi_2]} - \partial_u L_{[\xi_1, \xi_2]} + \overline{\delta M}_{\xi_1, \xi_2}, \tag{4.24} \]
where
\[ K_{[\xi_1, \xi_2]} = \frac{1}{8\pi G} \left[ \frac{1}{2} \epsilon^{0} \mathcal{Y}_{12} \partial^{2} \psi_{2} + \frac{1}{2} \epsilon^{0} \mathcal{Y}_{1} \partial^{2} \psi_{2} + \frac{1}{2} \epsilon^{2} \psi_{1} \partial f_{2} \right. \]
\[ + \frac{1}{4} \delta R \mathcal{Y}_{1} \partial f_{2} + \frac{1}{4} \delta R \mathcal{Y}_{1} \partial f_{2} - (1 \leftrightarrow 2) \right], \tag{4.25} \]
and
\[ \overline{M}_{\xi_1, \xi_2} = (\overline{\mathcal{Y}}_{2} J_{\xi_1} + \frac{1}{8\pi G} \left[ \frac{1}{2} \delta (\partial \mathcal{Y}_{1} - \partial \bar{\mathcal{Y}}_{1}) \partial f_{2} - \frac{1}{2} \partial \mathcal{Y}_{1} \partial \overline{\partial} f_{2} \right] - \text{c.c.}). \tag{4.26} \]
If one defines in addition \( \theta_{u}^{\xi} = P^{-2} \Theta_{u}^{\xi} \), \( \theta_{i}^{\xi} = P^{-1} \Theta_{i}^{\xi} \), and also \( K_{[\xi_1, \xi_2]}^u = P^{-2} K_{[\xi_1, \xi_2]}, K_{[\xi_1, \xi_2]}^i = P^{-1} K_{[\xi_1, \xi_2]} \), the total current algebra takes the form
\[ - \delta_{\xi_2} f_{\xi_1}^a + \theta_{\xi_2}^b (\delta_{\xi_1} \chi) \approx f_{[\xi_1, \xi_2]}^a + K_{[\xi_1, \xi_2]}^a + \partial_u L_{[\xi_1, \xi_2]}^{[ab]} \tag{4.27} \]
The time component of the field-dependent central term has been shown to satisfy the cocycle condition
\[ K_{[\xi_1, \xi_2]}^{u} - \delta_{\xi_3} K_{[\xi_1, \xi_2]}^{u} + \text{cyclic}(1, 2, 3) = \overline{\delta N}_{\xi_1, \xi_2, \xi_3} + \overline{\delta N}_{\xi_1, \xi_2, \xi_3}, \tag{4.28} \]
for some \( N_{\xi_1, \xi_2, \xi_3} \), which is now worked out to be
\[ N_{\xi_1, \xi_2, \xi_3} = - f_{3} K_{\xi_1, \xi_2} + \text{cyclic}(1, 2, 3). \tag{4.29} \]
This is completed by showing that the spatial component satisfies
\[ K_{[\xi_1, \xi_2]} - \delta_{\xi_3} K_{[\xi_1, \xi_2]} + \text{cyclic}(1, 2, 3) = - \partial_u N_{\xi_1, \xi_2, \xi_3} + \overline{\delta O}_{\xi_1, \xi_2, \xi_3}, \tag{4.30} \]
where
\[ \overline{O}_{\xi_1, \xi_2, \xi_3} = \left( \frac{1}{8\pi G} \mathcal{Y}_{3} \left[ \frac{1}{2} \epsilon^{0}(\mathcal{Y}_{1} \partial^{2} \mathcal{Y}_{2} - \mathcal{Y}_{2} \partial^{2} \mathcal{Y}_{1}) + \frac{1}{2} \partial^{2} \mathcal{Y}_{1} \partial f_{2} - \overline{\partial}^{2} \mathcal{Y}_{1} \partial f_{2} \right] \right. \]
\[ - \text{c.c.} \left. + \text{cyclic}(1, 2, 3). \right] \tag{4.31} \]
If one defines \( N_{\xi_1, \xi_2, \xi_3}^{[\mu \xi]} = P^{-1} N_{\xi_1, \xi_2, \xi_3}, \overline{N}_{\xi_1, \xi_2, \xi_3}^{[\mu \xi]} = P^{-1} \overline{N}_{\xi_1, \xi_2, \xi_3}, N_{\xi_1, \xi_2, \xi_3}^{[\xi]} = \overline{O}_{\xi_1, \xi_2, \xi_3}, \)
the complete field dependent extension is thus a current of the dual boundary theory that satisfies
\[ K_{[\xi_1, \xi_2]}^{\mu} - \delta_{\xi_3} K_{[\xi_1, \xi_2]}^{\mu} + \text{cyclic}(1, 2, 3) = \partial_u N_{\xi_1, \xi_2, \xi_3}^{[ab]}, \tag{4.32} \]
The current algebra (4.27) is valid both in the case of \( b m s_{4}^{\text{loc}} \), in which case one may choose for example to expand \( Y, \bar{Y}, PT \) in Laurent series, and for \( b m s_{4}^{\text{glob}} \), which is explicitly obtained by using \( \hat{P} = P_{S} \) and restricting oneself to Lorentz transformations as described in the beginning of section 4.2 while simultaneously expanding \( T \) in spherical harmonics. In this case, all components of the extension \( K_{[\xi_1, \xi_2]} \) are easily seen to vanish.
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A Direct derivation of spatial components

The spatial components of the currents are computed in two steps. We will start with the conventions of [7] and then we will use the dictionary given in [10] to translate the result into the Newman-Penrose formalism used in section 4. The non-integrable piece is identified as in [23] and treated in the current algebra as in [9].

Asymptotically flat metrics solving Einstein’s equation are of the form

\[ ds^2 = e^{2\beta} V r du^2 - 2e^{2\beta} dudr + g_{AB}(dx^A - U^A du)(dx^B - U^B du), \]  

where

\[ g_{AB} = r^2 \gamma_{AB} + r C_{AB} + \frac{1}{4} \gamma_{CD} C^D C^C + o(r^{-\epsilon}). \]  

The 2D metric \( \gamma_{AB} \) is fixed and corresponds to a choice of \( P \). Indices on \( C_{AB} \) are raised with the inverse of \( \gamma_{AB} \) and \( C^A = 0 \). To the order we need, the other metric components are given by

\[ \beta = -\frac{1}{32} r^{-2} C^A C^B + o(r^{-3-\epsilon}), \]  

\[ g_{uA} = \frac{1}{2} \overline{D}_B C^B_A + \frac{2}{3} r^{-1} \left[ N_A + \frac{1}{4} \gamma_{CD} C^C_B \right] + o(r^{-1-\epsilon}), \]  

\[ \frac{V}{r} = -\frac{1}{2} R + r^{-1} 2 M + o(r^{-1-\epsilon}), \]  

where \( \overline{D}_A \) is the covariant derivative associated to \( \gamma_{AB} \) and \( R \) is its scalar curvature. In these conventions, \( M(u, x^A) \), \( N_A(u, x^A) \) and \( C_{AB}(u, x^A) \) parametrize the part of the solution space relevant for the asymptotic current algebra. They will be related below to the fields \( \sigma^0 \), \( \Psi^0_1 \) and \( \Psi^0_2 \).

The BMS4 algebra parametrized by \( \xi = f \partial_u + Y^A \partial_A \), \( f = T + \frac{1}{2} u \overline{D}_A Y^A \), is represented by space-time vector fields as follows:

\[
\begin{align*}
(4) \xi^u &= f, \\
(4) \xi^A &= Y^A + I^A, \quad I^A = -\partial_B f \int_r^\infty dr' (e^{2\beta} g^{AB}), \\
(4) \xi^r &= -\frac{1}{2} r (\overline{D}_A (4) \xi^A - \partial_A f U^A). 
\end{align*}
\]
The associated $n - 2$ form $k_{\xi}^{[\mu\nu]}$ is then computed using the expression given in [24],

$$k_{\xi}^{[\mu\nu]}[h, g] = \frac{1}{16\pi G} \sqrt{-g} \left[ (4) \bar{\xi}^\mu D_\rho h^{\rho\mu} - (4) \bar{\xi}^\nu D_\rho h^{\rho\nu} + (4) \bar{\xi}_\rho D^\nu h^{\rho\mu} \right. 
+ \left. \frac{1}{2} h D^\nu(4) \bar{\xi}^\mu + \frac{1}{2} h^{\rho\nu}(D^\mu(4) \bar{\xi}_\rho - D_\rho(4) \bar{\xi}^\mu) - (\mu \leftrightarrow \nu) \right], \quad (A.7)$$

where $h_{\mu\nu}$ stands for a variation of the metric $\delta g_{\mu\nu}$ preserving the asymptotic structure. The information relevant for the asymptotic current algebra is given by the components $k_{\xi}^{[ur]}$ and $k_{\xi}^{[Ar]}$ evaluated in the limit $r \to \infty$. The former has already been computed in [9],

$$k_{\xi}^{[ur]} = \frac{\sqrt{\gamma}}{16\pi G} \lim_{r \to \infty} \left[ r Y^A D_B \delta C^B_A - \frac{1}{8} D_C Y^A C^{AB} \delta C_{AB} + 4 f \delta M \right. 
- \left. \frac{1}{2} f D_A D_B \delta C^{AB} + \frac{1}{2} f \delta C^{AB} \partial_u C_{AB} - \frac{1}{2} D_A f D_B \delta C^{AB} + 2 Y^A \delta N_A \right]. \quad (A.8)$$

The latter are now shown to be given by

$$k_{\xi}^{[Ar]} = \frac{\sqrt{\gamma}}{16\pi G} \lim_{r \to \infty} \left[ - r Y^B \partial_u \delta C^B_A - 2 Y^A \delta M - \frac{1}{2} \delta C^{AB} D_B D_C Y^C + D_B f \partial_u \delta C^{BA} \right. 
- \left. \frac{1}{2} f D_B \partial_u \delta C^{BA} - \frac{R}{2} \delta C^A_B Y^B - \frac{1}{2} Y^A D_B D_C \delta C^{BC} + \frac{1}{2} Y^B D^A D_C \delta C^C \right. 
- \left. \frac{5}{8} Y^A C^{BC} \partial_u \delta C_{BC} + Y^B C^{CA} \partial_u \delta C_{BC} - \frac{1}{8} Y^A \delta C^{BC} \partial_u C_{BC} \right. 
+ \left. Y^B \delta C^{CA} \partial_u C_{BC} + \frac{1}{4} (D^B Y^A - D^A Y^B + 2 D_D Y D^B Y^A) D_C \delta C^C \right]. \quad (A.9)$$

In order to translate these results into the Newman-Penrose formalism, we use the dictionary [10],

$$\bar{\xi}^\xi = 2 \bar{\sigma}^0, \quad C_{\xi}^\xi = 2 \sigma^0, \quad \bar{\mathcal{Y}} = P^{-1} Y\bar{\xi}, \quad \mathcal{Y} = P^{-1} \bar{Y}\xi, \quad (A.10)$$

$$2M = - \Psi^0_2 - \Psi_2^0 - \partial_u (\sigma^0 \bar{\sigma}^0), \quad (A.11)$$

$$PN_{\xi} = - \bar{\Psi}^0_1 - \bar{\sigma}^0 \bar{\sigma}^0 - \frac{3}{4} \bar{\partial} (\sigma^0 \bar{\sigma}^0), \quad \bar{N}_{\xi} = \bar{N}_{\xi}, \quad (A.12)$$

together with the definition of $\bar{\partial}$ and $\mathcal{\bar{\partial}}$ given in equation (4.1). Straightforward computation then leads to

$$k_{\xi}^{[ur]} = \frac{P^{-2}}{8\pi G} \lim_{r \to \infty} \left[ r \bar{Y} \delta \bar{\sigma}^0 - \frac{1}{2} f \delta \bar{\sigma}^0 - \frac{1}{2} \partial f \bar{\sigma}^0 - f \delta \Psi^0_1 - f \sigma^0 \bar{\sigma}^0 - \frac{1}{4} \mathcal{Y} \delta (7 \sigma^0 \bar{\sigma}^0 + 3 \bar{\sigma}^0 \sigma^0) - \frac{1}{4} \bar{\partial} \mathcal{Y} (\sigma^0 \bar{\sigma}^0 + \bar{\sigma}^0 \sigma^0) + c.c. \right], \quad (A.13)$$
\[ k^r_\xi \mid \xi = \frac{P^{-1}}{8\pi G} \lim_{r \to \infty} \left[ -r \bar{\mathcal{Y}} \delta \sigma^0 + \mathcal{Y} \delta \Psi^0 - \frac{1}{2} \bar{\mathcal{Y}} \delta \overline{\sigma^0} + \frac{1}{2} \mathcal{Y} \overline{\delta \sigma^0} + \bar{\delta} \delta \sigma^0 \\
- \frac{1}{2} f \delta \sigma^0 - \frac{1}{2} \mathcal{Y} \delta \sigma^0 - \frac{1}{4} \mathcal{Y} \sigma^0 \delta \overline{\sigma^0} + \frac{3}{4} \mathcal{Y} \overline{\sigma^0} \delta \sigma^0 \\
+ \frac{1}{4} \left( 3 \overline{\delta \mathcal{Y}} + \overline{\mathcal{Y}} \right) \delta \sigma^0 - \frac{1}{2} \mathcal{Y} \left( \delta \mathcal{Y} + \overline{\mathcal{Y}} \right) \delta \sigma^0 + \frac{3}{4} \mathcal{Y} \delta \sigma^0 \overline{\delta \sigma^0} + \frac{7}{4} \mathcal{Y} \delta \overline{\sigma^0} \delta \sigma^0 \right], \quad (A.14) \]

and the last component \( k^{\bar{r}}_\xi \) being the complex conjugate of the right hand side of (A.14).

Expressions (A.13)-(A.14) diverge as \( r \) goes to infinity. This is not a problem though as the divergent part can be absorbed in an exact form \( \partial_\rho \eta_{\xi}^{[\mu \nu \rho]} \). Defining \( \eta_{\xi}^{[ur \xi]} = P^{-1} N_{\xi}^u \), \( \eta_{\xi}^{[ur \xi]} = P^{-1} N_{\xi}^u \) and \( \eta_{\xi}^{[r \xi]} = N_{\xi} \), we get

\[ P^2 k^{[ur]}_\xi = \delta J^u_\xi + \Theta^u_\xi \mid \delta \chi \mid + \bar{\partial} N^u_\xi + \bar{\partial} N^u_\xi, \quad (A.15) \]
\[ P k^{[r \xi]} = \delta J^r_\xi + \Theta^r_\xi \mid \delta \chi \mid - \partial_\rho N^r_\xi + \bar{\partial} N_\xi, \quad (A.16) \]

where the currents \( J^u_\xi, J^r_\xi \) are given in (4.18), (4.21), while the non-integrable pieces \( \Theta^u_\xi \mid \delta \chi \mid, \Theta^r_\xi \mid \delta \chi \mid \) are given in (4.16), (4.23), and

\[ N^u_\xi = \frac{1}{8\pi G} \lim_{r \to \infty} \left[ r \overline{\mathcal{Y}} \delta \sigma^0 - \frac{1}{2} \bar{\mathcal{Y}} \delta \overline{\sigma^0} - \frac{1}{4} \mathcal{Y} \sigma^0 \overline{\delta \sigma^0} + \overline{\sigma^0} \delta \sigma^0 \right], \quad (A.17) \]
\[ \overline{N}_\xi = \frac{1}{8\pi G} \left[ \frac{1}{2} \overline{\mathcal{Y}} \delta \sigma^0 - \frac{1}{2} \mathcal{Y} \overline{\delta \sigma^0} \right]. \quad (A.18) \]

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