On the interaction energy of 2D electron FQHE systems within the Chern-Simons approach

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Abstract

The interaction energy of the two-dimensional electron system in the region of fractional quantum Hall effect is considered within the Chern-Simons composite fermion approach. In the limit when Coulomb interaction is very small comparing to the cyclotron energy the RPA results are obtained for the fillings $\nu = 1/3, 1/5, 2/3, 2/5, 3/7$ and compared with the exact diagonalization results for small systems (extrapolated for infinite systems). They show very poor agreement suggesting the need for looking for alternative approaches.

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1. Introduction

One of the aims of theoretical studies of fractional-quantum-Hall-effect (FQHE) systems [1, 2] is to determine the value of the interaction energy of the system at various fractional fillings of the lowest Landau level. The exact numerical results can be found for few particle systems [3, 4, 5] and they agree very well with the predictions of the trial wave function approach of Laughlin and Jain [1, 6, 7] (Laughlin and Jain wave functions are proposed for the case when Coulomb interaction is very small comparing to the cyclotron energy and higher Landau levels can be omitted, it is also the case in related numerical work). The many-body theory for such systems is formulated within the Chern-Simons gauge theory [8, 9] which introduces the gauge field mapping fermions into fermions (so-called composite fermions obtained by attaching an even number of flux quanta to each electron). There is no small parameter in the Chern-Simons composite fermion theory [8, 10, 11, 12] (in contrast to the case of anyons in the fermion limit [13]). Nevertheless, the Chern-Simons theory gives good predictions of the transport properties of the system [8, 11]. However, in contrast to the trial wave function approach, the attempts to get the ground state energy of the system within the Chern-Simons theory are not very successful [12, 14]. In calculations of energy gaps modified approach have been used in which the relation between finite size calculations and the Chern-Simons results is assumed [8, 11, 15]. In this paper we calculate the interaction energy of the system in the Chern-Simons approach within the RPA for several values of the filling fraction \( \frac{1}{3}, \frac{1}{5}, \frac{2}{3}, \frac{2}{5}, \frac{3}{7} \) and compare them with the exact diagonalization results for few electron systems (extrapolated for infinite systems).

The composite fermion (CF) transformation consists in attaching an even number \((2p)\) of flux quanta to each electron. Such point fluxes do not change the statistics of composite particles, however, they allow to treat the 2D system of electrons in a strong magnetic field in a close analogy to the treatment in a weak magnetic field. It is motivated by the mean field approach when the sum of point fluxes is replaced by an
average flux. The corresponding average field is \( B^{Ch-S} = -2p \frac{hc}{e} \rho \) (fluxes opposite to the external flux, \( p \) is an integer, \( \rho \) – density) and the effective field acting on electrons is reduced to \( B^{eff} = B^{ex} + B^{Ch-S} \). In a close analogy to the quantum Hall effect one predicts similar effect when \( n \) Landau levels are completely filled in the effective field, i.e. \( B^{eff} = \frac{1}{n} \frac{hc}{e} \rho \). Hence, \( B^{ex} = \frac{2p_n+1}{n} \frac{hc}{e} \rho \), which means that the ”real” lowest Landau level is filled in the fraction \( \nu = \frac{n}{2p_n+1} \) (\( \nu = \frac{n}{2p_n-1} \) when the effective field is opposite to the external one). It is interesting to notice that the Laughlin fractions (of the form \( 1/m \), \( m \) – odd) can be represented in two different ways. We can add that the Chern-Simons theory results should be independent of the actual Chern-Simons parameter \( 2p \) (if even) [16]. Hence, we can use the value of \( p \) which is most suitable in a given problem. In practice, we use the value of \( p \) which gives fully filled Landau levels in the effective field (treatment of such systems is well known). Nevertheless, whatever value of \( p \) is taken the results should be the same.

The Hamiltonian of the two-dimensional system of electrons in an external magnetic field

\[
H = \int d^2r \Psi^+(r) \left( \frac{1}{2m}(p + \frac{e}{c}A^{ex}(r))^2 \Psi(r) \right)
+ \frac{1}{2} \int d^2r d^2r' \Psi^+(r) \Psi^+(r') \frac{e^2}{\epsilon |r-r'|} \Psi(r) \Psi(r')
\]

(\( \epsilon \) – dielectric constant) can be rewritten in the following way:

\[
H = \int d^2r \Psi^+(r) \left( \frac{1}{2m}(p + \frac{e}{c}A^{ex}(r) + \frac{e}{c}A^{Ch-S}(r))^2 \Psi(r) \right)
+ \frac{1}{2} \int d^2r d^2r' \Psi^+(r) \Psi^+(r') \frac{e^2}{\epsilon |r-r'|} \Psi(r) \Psi(r')
\]

(1)

where

\[
A^{Ch-S}_{\alpha}(r) = -2p \frac{hc}{e} \int d^2r' \epsilon_{\alpha\beta} (r' - r)_{\beta} \rho(r') ,
\]

(3)

\( \rho(r) = \Psi^+(r) \Psi(r) \). The Hamiltonian \( H \) can be separated into two parts: \( H = H_0 + H_{int} \) where

\[
H_0 = \int d^2r \Psi^+(r) \frac{1}{2m}(p + \frac{e}{c}A^{ex}(r))^2 \Psi(r)
\]

(4)
is treated as the unperturbed term \( B_{\text{eff}} = \nabla \times A_{\text{eff}} = \nabla \times (A_{\text{ex}} + \bar{A}_{\text{Ch-S}}) = B_{\text{ex}} + B_{\text{Ch-S}} \), \( B_{\text{Ch-S}} \) is found by averaging point fluxes – putting the average density \( \rho \) in (3)). \( H_{\text{int}} \) is the interaction Hamiltonian [13, 17]:

\[
H_{\text{int}} = \frac{1}{2} \int \int d^2 r d^2 r' \frac{e^2}{|r - r'|} \Psi(r) \Psi(r') + H_1 + H_2 ,
\]

where

\[
H_1 = -2 \rho \frac{\hbar}{m} \int \int d^2 r d^2 r' \Psi^+(r)(p_{\alpha} + \frac{e}{c} A_{\alpha}^\text{ef})(r)\epsilon_{\alpha \beta} \frac{(r - r')_\beta}{|r - r'|^2} (\rho(r') - \rho),
\]

\[
H_2 = (2p)^2 \frac{\hbar^2}{2m} \int \int \int d^2 r d^2 r' d^2 r'' \rho(r) \frac{(r - r') (r - r'')}{|r - r'|^2 |r - r''|^2} (\rho(r') - \rho)(\rho(r'') - \rho).
\]

In this paper we consider the case when \( B_{\text{eff}} = \frac{1}{n} \frac{\hbar c}{e} \rho \), i.e. in the unperturbed state one has \( n \) completely filled Landau levels (the effective filling \( \nu^* = n \)). The first step in calculating the ground state interaction energy is the Hartree-Fock approximation. Considering the Coulomb interaction one finds the Hartree-Fock (H-F) contribution to be (we assume the presence of the positive background):

\[
E_{\text{H-F}} = -\frac{N}{2n} \frac{e^2}{\epsilon_0^{\text{eff}}} \int_0^\infty \left( \sum_{k=0}^{n-1} L_k^m \left( \frac{1}{2} r^2 \right) \right)^2 \exp \left( -\frac{1}{2} r^2 \right) dr
\]

where \( a_0^{\text{eff}} = \sqrt{\frac{\hbar c}{e B_{\text{eff}}}} \) is the effective magnetic length \( (a_0^{\text{eff}} = \sqrt{\frac{B_{\text{ex}}}{B_{\text{eff}}}} a_0^{\text{ex}} = \sqrt{2pn \pm 1} a_0^{\text{ex}}) \), \( a_0^{\text{ex}} = \sqrt{\frac{\hbar c}{e B_{\text{ex}}}} \), \( L_i^m \) – Laguerre polynomials, \( N \) – number of particles. The H-F results are presented in Table I for several filling fractions and compared with "exact" results (exact diagonalization results extrapolated for infinite systems). The difference between "exact" and Hartree-Fock results (correlation energy) increases with the decrease of the fraction, for the 1/3 state is of order of 10% (of the exact value). It seems that a higher order approximation, eg. the RPA, will give a better agreement. In the following we consider the correlation energy within the RPA, assuming that the separation between Landau levels is much larger than Coulomb interaction between particles (as it is the case in exact diagonalization methods we refer to).
2. Correlation energy

The correlation energy can be defined as follows:

$$E_c = \int_0^1 \frac{d\lambda}{\lambda} (\langle \lambda H_{int} \rangle_\lambda - \langle \lambda H_{int} \rangle_0) .$$

(9)

The expression for the correlation energy in the RPA (three-body contributions are omitted) has the form [18, 12]

$$E_{cRPA} = -\frac{1}{2}\hbar L^2 \int \frac{dq}{(2\pi)^2} \int_0^\infty \frac{d\omega}{\pi} \int_0^1 \frac{d\lambda}{\lambda} \text{Im} \text{tr}(\lambda V(q))[D_{RPA}^{RPA}(q, \omega) - D_0(q, \omega)]$$

(10)

where $D_{RPA}^{RPA}$ is the correlation function of effective field currents ($L^2$ is the area of the system):

$$D_{\mu\nu}^{RPA}(r_t, r'_t) = -\frac{i}{\hbar} < T[j^\mu(r_t), j'^\nu(r'_t)] >$$

(11)

given within the random-phase approximation (with the coupling constant $\lambda$):

$$D_{\lambda}^{RPA}(q, \omega) = [I - \lambda D_0(q, \omega)V(q)]^{-1}D_0(q, \omega).$$

(12)

The current densities are defined as:

$$\mathbf{j}(\mathbf{r}) = \frac{1}{2m} \sum_j \left\{ \mathbf{P}_j + \frac{e}{c} \mathbf{A}_j^{\text{ef}} \right\} \delta(\mathbf{r} - \mathbf{r}_j)$$

(13)

where braces denote an anticommutator, $\mathbf{j}$ is the vector part of $j^\mu$ with $\mu = 0, x, y$. We define $j^0$ as density fluctuations: $j^0 = \sum_j \delta(\mathbf{r} - \mathbf{r}_j) - \rho$. The interaction matrix $V$ is obtained from the Hamiltonian $H_{int}$ (dropping out three-body terms). We choose $\mathbf{q} = q\mathbf{x}$ and the Coulomb gauge which reduces the problem to $2 \times 2$ $D_{\mu\nu}^{RPA}$ matrix ($\mu = 0, y$).

Taking $\omega_{c\mu}^{\text{eff}} = \frac{eB_{\mu}^{\text{eff}}}{cm}$ and $a_0^{\text{eff}}$ to be frequency and length units, respectively one finds ($\hbar = 1$):

$$V(q) = \begin{pmatrix} v(q) & 0 \\ 0 & 0 \end{pmatrix} + \frac{4p\pi}{q^2} \begin{pmatrix} 2pn & -iq \\ iq & 0 \end{pmatrix}$$

(14)

where $v(q)$ is the Fourier transform of the Coulomb potential ($v(q) = \frac{2\pi e^2}{eq}$ in standard units).
Let us assume the correspondence between the two energy scales, one related to the separation between Landau levels \( \frac{e^2}{\epsilon_0} \cdot \frac{1}{\hbar \omega_{ex}} \), one to the strength of the Coulomb interaction \( \frac{e^2}{\epsilon_0} \cdot \frac{1}{\hbar \omega_{ex}} \). We introduce the dimensionless parameter:

\[
rs = \frac{e^2}{\epsilon_0} \cdot \frac{1}{\hbar \omega_{eff}}
\]

which shows the strength of the Coulomb interaction with respect to the separation between effective Landau levels \( r_{ex} = \frac{e^2}{\epsilon_0} \cdot \frac{1}{\hbar \omega_{ex}} = \frac{1}{\sqrt{2 \pi n} \pm 1} \cdot rs \). If one considers the system of electrons, the limit \( \frac{e^2}{\epsilon_0} \cdot \frac{1}{\hbar \omega_{ex}} \rightarrow 0 \) corresponds to the case when particle-hole excitations (electron excited into a higher Landau level) are negligible (hence, in exact diagonalization studies higher Landau levels can be omitted). When applying the Chern-Simons picture, however, gauge interactions are always of order of \( \hbar \omega_{ex} \), and particle-hole excitations (CF excited into an empty Landau level and a CF hole in a filled level) have to be considered. We have

\[
V(q) = \frac{4\pi}{q^2} \begin{pmatrix}
2pn(1 + \frac{q}{(2pn)^2}nr_s) & -iq \\
0 & 0
\end{pmatrix}
\]

The correlation function \( D_0 \) is [19]:

\[
D_0(q, \omega) = \frac{n}{2\pi} \begin{pmatrix}
q^2\Sigma_0 & -iq\Sigma_1 \\
q\Sigma_1 & \Sigma_2
\end{pmatrix}
\]

where

\[
\Sigma_j = \frac{e^{-x}}{n} \sum_{m=n}^{\infty} \sum_{l=0}^{n-1} \frac{m-l}{(\omega)^2 - (m-l-i\eta)^2} \frac{l!}{m!} x^{m-l-1}[L_i^{m-l}(x)]^{2-j} \\
\times [(m-l-x)L_i^{m-l}(x) + 2x \frac{dL_i^{m-l}(x)}{dx}]^j
\]

and \( x = \frac{q^2}{2} \). Then one obtains:

\[
D^{RPA}(q, \omega) = \frac{n}{2\pi det} \begin{pmatrix}
q^2\Sigma_0 & -iq\Sigma_s \\
q\Sigma_s & \Sigma_p
\end{pmatrix}
\]

where \( det = det(I - D^0V) = (1 - 2pn\Sigma_1)^2 - (2pn)^2\Sigma_0(1 + \Sigma_2) - nr_sq\Sigma_0, \Sigma_s = \Sigma_1 - 2pn\Sigma_1^2 + 2pn\Sigma_0\Sigma_2, \Sigma_p = (2pn)^2\Sigma_1^2 + \Sigma_2 - (2pn)^2\Sigma_0\Sigma_2 + qnr_s(\Sigma_1^2 - \Sigma_0\Sigma_2) \).
Collective modes are determined by the poles of the correlation function $D^{RPA}$. In Figures 1-2 we plot the results for $\nu = 1$ (the direct result and the $p = 1$ CF approach result) and similar results for $\nu = 1/3$ are presented in Figures 3-4, in Figure 5 the 3/7 case is presented.

The RPA correlation energy will be found using the dispersion relation of collective modes. In units of $\frac{e^2}{\epsilon^2 p a^0}$ the correlation energy can be expressed as follows \cite{18} ($\frac{e^2}{\epsilon^2 p a^0} = r_s \hbar \omega_c^\text{eff}$):

$$E_{c}^{RPA} = \frac{N}{2nr_s} \int_0^\infty dq \int_0^\infty \frac{d\omega}{\pi} \text{Im}(\ln det + \text{tr}(V(q)D_0))$$

which equals

$$E_{c}^{RPA} = \frac{N}{2nr_s} \int_0^\infty dq \int_0^\infty \frac{d\omega}{\pi} \text{Im}(\ln det + 2pn(2p\Sigma_0 + 2\Sigma_1) + nr_sq\Sigma_0).$$

We have:

$$\int_0^\infty dx \int_0^\infty \frac{d\omega}{\pi} \text{Im} \Sigma_0(x, \omega) = -\frac{1}{2} \sum_{m=1}^\infty \frac{1}{m} + \frac{1}{2}(S_n - 1)$$

($S_n = \sum_{j=1}^{n} \frac{1}{j}$). This term is divergent but combined with other terms in the integral (21) has to give a finite value. Additionally

$$\int_0^\infty dx \int_0^\infty \frac{d\omega}{\pi} \text{Im} \Sigma_1(x, \omega) = 0.$$ 

The last integral in (21)

$$\frac{N}{2} \int_0^\infty q^2 dq \int_0^\infty \frac{d\omega}{\pi} \text{Im} \Sigma_0.$$ 

will be calculated separately for different $\nu^* = n$.

3. Results

To calculate the correlation energy (21) one needs to know the zeros of the determinant det (collective modes). For $n = 1$ (the effective filling $\nu^* = 1$ – Laughlin fractions) we have an infinite set of modes with shortwavelength behaviour like $\omega_m(q \to \infty) = m –$ Figs.1-4 \cite{10}. It can be shown that \cite{15}:

$$\int_0^\infty \frac{d\omega}{\pi} \text{Im} \ln det = \sum_{m=1}^{\infty} (\omega_m - m) = \sum_{m=1}^{\infty} \Delta \omega_m$$

(25)
The integral (24) equals:

\[ \int_0^\infty q^2 dq \int_0^\infty \frac{d \omega}{\pi} \text{Im} \Sigma_0 = \int_0^\infty \sqrt{2x} \sum_{m=1}^\infty \frac{e^{-x} x^{m-1}}{m!} = \sqrt{2} \int_0^\infty \sqrt{x}(1 - e^{-x}) . \]  

(26)

Again, this integral is divergent, but combined with (25) and (22) will give a finite result. We write:

\[ \int_0^\infty q^2 dq \int_0^\infty \frac{d \omega}{\pi} \text{Im} \Sigma_0 = -\frac{1}{2} \sqrt{2\pi} \sum_{m=1}^\infty \frac{1}{2m m!} \prod_{i=1}^m (2i - 1) \]  

(27)

and then (in units of \( \frac{\varepsilon^2}{\epsilon a_0^3} \))

\[ \frac{E^{\text{RPA}}}{N} = \frac{1}{2r_s} \sum_{m=1}^\infty \left( \int \Delta \omega_m(x) dx - (2p)^2 \frac{1}{2m} - \frac{1}{2} r_s \sqrt{2\pi} \prod_{i=1}^m \frac{2i-1}{2i} \right) . \]  

(28)

In the case when \( n \geq 2 \) every root (of collective modes) higher than the first is splitted into two (for \( m > 1, \omega^{-}_m(q \to \infty) = m = \omega^{+}_m(q \to \infty) \) – Figure 5). One has

\[ \int_0^\infty \frac{d \omega}{\pi} \text{Im} \ln \det = \Delta \omega_1 + \sum_{m=2}^\infty (\Delta \omega^{-}_m + \Delta \omega^{+}_m) \]  

(29)

and the correlation energy for \( n = 2 \) is given by (in units of \( \frac{\varepsilon^2}{\epsilon a_0^3} \))

\[ \frac{E^{\text{RPA}}}{N} = \frac{1}{4r_s} \left( \int \Delta \omega_1(x) dx - 8p^2 \right) + \frac{1}{4r_s} \sum_{m=2}^\infty \left[ \int (\Delta \omega^{-}_m(x) + \Delta \omega^{+}_m(x)) dx - (4p)^2 \frac{1}{2m} \right] + \frac{p^2}{r_s} \]

\[ - \frac{1}{8} \sqrt{2\pi} \sum_{m=1}^\infty [(1 - \delta_{m1}) - m + \frac{(2m+3)(2m+1)}{4(m+1)}] \prod_{i=1}^m \frac{2i-1}{2i} . \]  

(30)

We have also found the expression for \( E^{\text{RPA}}_c \) for \( n = 3 \) (applied for \( \nu = 3/7 \)).

The value of interaction energy related to Coulomb interaction (in the limit of \( r_s \to 0 \)) is:

\[ \frac{\Delta E^{\text{RPA}}_c}{N} = \lim_{r_s \to 0} \left( \frac{E^{\text{RPA}}_c(r_s)}{N} - \frac{E^{\text{RPA}}_c(NC)}{N} \right) , \]  

(31)

NC stands for the case with no Coulomb interaction. For \( n = 1 \) one has

\[ \frac{\Delta E^{\text{RPA}}_c}{N} = \lim_{r_s \to 0} \frac{1}{2r_s} \sum_{m=1}^\infty \left( \int (\omega^{r_s}_m(x) - \omega^{NC}_m(x)) dx - \frac{1}{2} r_s \sqrt{2\pi} \prod_{i=1}^m \frac{2i-1}{2i} \right) . \]  

(32)

The main problem in calculating (32) is the calculation of the integrals and the convergence in summation over \( m \). The integrals in (32) (and similar for \( n = 2 \) and \( n = 3 \))
have been calculated numerically using $k$-point Gauss-Laquerre integration. It was verified that the summation over $m$ converges well and the sums have been truncated at $2k$ terms. In order to find the limit $r_s \to 0$ we considered small values of $r_s$. It appears that for $r_s$ of order of $10^{-4} - 10^{-5}$ the RPA energies become practically independent of $r_s$ and for that range the results are given in Tables II-IV.

In Table II we present the RPA results in respective units of $\frac{e^2}{\epsilon_0 a_r}$. It can be seen that the results for the same value of $p$ and $|\nu^*|$ are very close one to the other. They are not the same for the same filling (as they should be). We observe rather strong dependence on $p$ (at a given effective filling $\nu^*$).

In Table III and Table IV the RPA results are compared with the exact diagonalization results. The best agreement with the ”exact” values is found for the series $3/7, 2/5, 1/3$ (fractions going down from $1/2$). The $1/3$ state gives the best result, the difference between the RPA and the exact values is of order of 20% (of the exact result).

An interesting example is the system of electrons at the real filling $\nu = 1$. In the limit $r_s^{\text{ex}} \to 0$ the RPA correlation energy is zero. The qualitative difference is found within the RPA for the CF approach. Performing the CF transformation one finds the system at the effective filling $\nu^* = -1$ (the effective field is opposite to the external one). We plot collective modes for the two descriptions in Figures 1-2. Calculating the RPA result (as it is described above) we find a finite CF result for the correlation energy ($r_s \to 0$).

Similar situation can be found at the filling $\nu = 1/3$. Then the effective filling is $\nu^* = 1$ ($p = 1$) or $\nu^* = -1$ ($p = 2$) and the Hartree-Fock contributions are the same in the two descriptions. The RPA collective modes spectra look very similar – Figures 3-4 (the agreement is exact at $q \to 0$) but the values of interaction energy differ a lot (Table III and Table IV).

In summary the agreement between the RPA interaction energies and the exact results are far from the expected one and the analysis needs an extension by including three-body contributions (three-body density-density correlation function [18]) which
seems to be very complicated. An alternative approach may be obtained within the new formalism developed by Shankar and Murthy [20] (which has a direct relation with the Laughlin trial wave function). In Ref. [21] they showed how to calculate the energy gaps and their results agree reasonably with numerical results [22].

4. Conclusions

The values of Coulomb interaction energies for the 2D electron system in the region of FQHE are calculated within the Chern-Simons theory in the RPA for several fractional fillings. The results are obtained in the limit \( \frac{e^2}{4\epsilon a_0} \left( \frac{1}{\hbar \omega_{\text{exc}}} \right) \rightarrow 0 \) (i.e., when Coulomb interaction is very small comparing to the separation between Landau levels) and compared with the exact diagonalization results (the results for few particle systems extrapolated for infinite systems). The best agreement is found for fractions going down from \( \frac{1}{2} \) (3/7, 2/5, 1/3), for the best 1/3 result the difference between the RPA and the exact results is of order of 20% (of the exact value). A qualitative difference is obtained for \( \nu = 1 \) \( p = 1 \) CF description when a finite RPA correlation energy is found. Also the result for \( \nu = 1/3 \) \( p = 2 \) is very different from \( \nu = 1/3 \) result obtained for \( p = 1 \). Our analysis needs an extension to a higher order approximation including three-body contributions. An alternative approach may be obtained within the new formalism developed by Shankar and Murthy [20], their results for energy gaps [21] are in reasonable agreement with numerical results [22].
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Figure 1:
Collective modes for $\nu = 1, r_s = 1$.

Figure 2:
Collective modes for the filling $\nu = 1$ given within the $p = 1$ CF description ($\nu^* = -1$),
$r_s = 1$.

Figure 3:
Collective modes for $\nu = 1/3, r_s = 1$.

Figure 4:
Collective modes for the filling $\nu = 1/3$ given within the $p = 2$ CF description ($\nu^* = -1$),
$r_s = 1$.

Figure 5:
Collective modes for $\nu = 3/7, r_s = 1$. 
| $\nu$ | H-F  | Exact   |
|-------|------|---------|
| 1     | −0.627 | −0.627  |
| 2/3   | −0.497 | −0.519  |
| 5/11  | −0.406 | −0.451* |
| 4/9   | −0.402 | −0.447* |
| 3/7   | −0.396 | −0.443  |
| 2/5   | −0.385 | −0.433  |
| 1/3   | −0.362 | −0.412  |
| 1/5   | −0.280 | −0.328  |

Table I: The Hartree-Fock and exact interaction energies (per particle) in respective units of $\frac{e^2}{\epsilon a_0^2}$. The ”exact” results are taken from Refs. [3, 4] where the results (in spherical systems) for few particles ($N \leq 12$) were extrapolated for infinite systems ($N \to \infty$). Two numbers with stars are obtained within the Jain CF approach [7]. The ”exact” 2/3 result is found via particle-hole symmetry [8].
\[
\begin{array}{cccc|cccc}
\nu & \nu^* & p & \Delta E^{RPA} \\
 & & & N \\
 & & & r_s = 10^{-4} & r_s = 10^{-5} \\
k = 10 & k = 15 & k = 20 & k = 10 & k = 15 & k = 20 \\
3/7 & 3 & 1 & -0.531 & -0.525 & -0.520 & -0.531 & -0.525 & -0.520 \\
2/5 & 2 & 1 & -0.360 & -0.357 & -0.355 & -0.360 & -0.357 & -0.355 \\
1/3 & 1 & 1 & -0.247 & -0.247 & -0.246 & -0.247 & -0.247 & -0.246 \\
1/5 & 1 & 2 & -0.548 & -0.544 & -0.542 & -0.548 & -0.544 & -0.542 \\
1 & -1 & 1 & -0.230 & -0.230 & -0.230 & -0.230 & -0.230 & -0.230 \\
1/3 & -1 & 2 & -0.585 & -0.581 & -0.580 & -0.585 & -0.581 & -0.580 \\
2/3 & -2 & 1 & -0.358 & -0.354 & -0.352 & -0.358 & -0.354 & -0.352 \\
\end{array}
\]

Table II. The RPA correlation energies in units of \( e^2 / a_F \). For the \( k = 10 \) case of \( \nu = 3/7 \) the summation goes over 15 modes. For negative effective fillings we used instead \( \nu^* > 0 \) \( p < 0 \) (\( \nu < 0 \)).

\[
\begin{array}{cccc|cc}
\nu & \nu^* & p & E^{HF} / N & \Delta E^{RPA} / N & E^{HF} / N + \Delta E^{RPA} / N \\
& & & \text{exact diagonalization} \\
1 & 0 & -0.627 & 0 & -0.627 & -0.627 \\
3/7 & 3 & 1 & -0.396 & -0.197 & -0.593 & -0.443 \\
2/5 & 2 & 1 & -0.385 & -0.159 & -0.544 & -0.433 \\
1/3 & 1 & 1 & -0.362 & -0.142 & -0.504 & -0.412 \\
1/5 & 1 & 2 & -0.280 & -0.242 & -0.522 & -0.328 \\
\end{array}
\]

Table III. The interaction energies in respective units of \( e^2 / a_F \) (note the change of units with respect to Table II). The RPA values in the fifth column are taken from \( k = 20 \) results of Table II.
| $\nu$ | $\nu^*$ | $p$ | $\frac{E_{H-F}}{N}$ | $\frac{\Delta E_{RPA}}{N}$ | $\frac{E_{H-F}}{N} + \frac{\Delta E_{RPA}}{N}$ | exact diagonalization |
|-----|--------|-----|------------------|------------------|------------------|------------------|
| 1   | -1     | 1   | -0.627           | -0.230           | -0.857           | -0.627           |
| 1/3 | -1     | 2   | -0.362           | -0.335           | -0.697           | -0.412           |
| 2/3 | -2     | 1   | -0.497           | -0.203           | -0.700           | -0.519           |

Table IV. The interaction energies in units of $\frac{e^2}{\epsilon a_0^2}$ obtained for negative effective fillings.
Figure 2
Figure 3
Figure 4
Figure 5