Free vibration of isotropic plates with various cutout configurations using finite elements and design of experiments

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Abstract. This work represents a finite element analysis of free vibration of isotropic plates with different cutout shapes, areas, locations and aspect ratios. Modal analysis was carried out using the ANSYS APDL software to evaluate the fundamental frequencies. ANSYS model was validated in the first stage and showed good agreement with the selected literature works. Furthermore, the selected cutout parameters were investigated to assess which parameter is more effective on the frequency. It is found that these parameters was overlapping, therefore Design of Experiments was conducted. Results revealed that the aspect ratio of the cutouts is the most significant factor on the fundamental frequency.

Keywords: Free vibration, Fundamental frequency, DOE, Isotropic plate, Cutout.

1. Introduction
An important parameter in the dynamic analysis of plates with cutout is the determination of their natural frequencies. This is essential as plates normally undergoes a range of dynamic excitations and is often operated under complicated conditions. Cutout is a fundamental part of nearly every structural element, including isotropic plates. The cutouts are required for component assembly, ventilation, ports for fuel and electrical lines and for weight reduction [1]. During their service life, these structures are subjected to undesirable vibrations, and again, these cutout plate structures can change the responses significantly. In the field of marine, civil and aeronautical engineering, understanding of the natural frequencies of a plate with cutouts is crucial [2]. Several parameters influence the frequencies and mode shapes of plates with cutouts, such as the cutout shape and area, aspect ratio, cutout location, boundary condition...etc. It is therefore essential to know the free vibration analysis of the plate with cutouts at various situations. In recent decades, great progress has been made in better understanding the free vibration isotropic and orthotropic plates with and without cutouts [1-15].

The natural frequencies of plates with rectangular cutouts have been investigated by Aksu and Ali [16] using experimental and two dimensional finite difference method. There was a discrepancy of theoretical natural frequencies from the experimental results and with increasing the mode shape complexity, this discrepancy increased. Srinivasa et al. [17] presented an experimental and finite element study on the free vibration of skew plates. They studied many parameters that influence the natural frequencies, such as the aspect ratio and the skew angle for isotropic plates and aspect ratio, skew angle, fiber orientation and stacking sequence of laminated composite plates. They founded that increasing the skew angle lead to increasing the natural frequencies, while the aspect ratio effect for both skew plates was low or negligible.
Merneedi et al. [2] investigated the size, position and number of rectangular and circular cutouts in a rectangular isotropic plate to determine the natural frequencies. It was evident that the value of natural frequencies reduces as the cutout moves towards the middle of the plate. Increasing the cutout size also causes the natural frequencies to decrease for the same aspect ratio. In general, the frequency values for cutout plates were lower than those for plates without cutout [2]. In another study, the same authors investigated a simply supported elliptical plate with a rectangular cutout. Their study also included the effect of cutout size and its position on the natural frequencies of the elliptical plate. They also reached the same conclusion for the cutout position and found that when the cutout size increased, the natural frequencies increased [18].

A finite element analysis was conducted by Thakare and Damale [19] to determine the modal characteristics of circular annular plates with multiple circumferential holes. Many parameters examined in this study, which are the aspect ratio (outer diameter to the inner diameter of the annular plate), plate thickness, number and diameters of circumferential holes. Results revealed that by increasing the aspect ratio and the thickness of the plate, the natural frequency was increased. Increasing the number of circumferential holes also causes the natural frequency to decrease. A free vibration analysis of isotropic dense rectangular plates has been analyzed by Serdoun and Hamza-Cherif. There were many parameters involved in the research that are aspect ratio (length to width and thickness to length) and the boundary condition [20].

Using Finite Element Analysis, Ghonasgi et al. investigated the natural frequencies of perforated plates [21]. The study included various parameters such as perforation shape and pattern, plate dimensions, and aspect ratio. Results indicated that the most affecting parameter was the aspect ratio. Wang et al studied numerically the free vibration of plates with slender and square holes [22]. By setting Young's module and mass density to zero, they treated the hole as a virtual plate. The pattern of the holes was parallel to the edges and diagonal. Results indicated that the diagonal holes had more effect on the natural frequencies and mode shapes than in the parallel holes. Sayyad and Ghalgal numerically studied the free vibration analysis of simply supported thick rectangular and square plates using the theory of exponential shear deformation. Compared to the exact theory of elasticity, they achieved good finding [23]. The free vibration of functionally graded material plates was analyzed by Do and Lee. The plates involved a complex cutout of various shapes [24]. The effect of the plate length-to-thickness ratio was investigated. Results showed that there was a certain value of length to thickness ratio that, although the ratio becomes larger, there is no increase in the natural frequency of the plate. Patil, performed a free vibration analysis for thin rectangular plate with different boundary conditions and aspect ratios [25]. Results indicated that by increasing the number of free edges and the aspect ratio, the natural frequencies decreased. Rezaiefar and Galal, investigated the fundamental frequencies of rectangular plates with different aspect ratios and boundary conditions to obtain the first three significant mode shapes [26]. Lam, presented a numerical method to study the free vibration of rectangular plates using a modified Rayleigh-Ritz method and achieved a good agreement with literature [27].

In the present work, modal analysis of isotropic plates with different cutout configurations under free vibration was presented. The fundamental frequencies are presented under a wide range of cutouts locations, shapes, sizes and aspect ratios. A finite element analysis developed using ANSYS APDL.

2. Verification of the finite element model
In order to verify the present ANSYS APDL model, three literature works were selected. The first work involved experimental and numerical investigation of free vibration of glass fiber epoxy composite plates with different cut-out shape [1].

Two plates (one with circular cut-out and the other one without cut-out) were modeled. Each of these plates has been analyzed for fixed boundary condition on two opposite sides (CFCF). The dimensions of
The plates were 150 mm by 100 mm. The stacking sequence for each composite plate was \([0^\circ/90^\circ]\) with 30 layers. The thickness of a single layer was 0.15 mm and the average thickness of the composite plate was 4.5 mm. While the area of cut-out was 900 mm\(^2\).

Comparison of the natural frequencies for the present work with Jadhav and Deshmukh [1] results shown in Figure 1 a and 1 b

![Figure 1. Comparison of natural frequencies of composite plates with CFCF boundary condition.](image)

The variation of the natural frequency values between the present model with the results of Jadhav and Deshmukh was between 0.02\% and 5.03\%.

The second verification was carried out with and without rectangular cutouts to predict the dynamic characteristics of rectangular isotropic (mild steel) plates [16]. In Aksu and Ali work [6], natural frequencies have been predicted using a finite difference technique and verified experimentally. The plates in C-SS-C-SS boundary conditions were supported.
Comparison of the natural frequencies for the present work with Aksu and Ali [6] results shown in Figure 2 a and 2 b.

![Comparison of natural frequencies for CSCS boundary condition isotropic plate without cutout](image1)

![Comparison of natural frequencies for CSCS boundary condition isotropic plate with central rectangular cutout](image2)

**Figure 2.** Comparison of natural frequencies of isotropic plates with CSCS boundary condition.

Furthermore, the variation between the present model with the results of Aksu and Ali [6] ranged from 0.62% to 5.20%. The third verification was done with a rectangular isotropic plate [28]. The effect of changing the width and thickness on the plate's natural frequency was evaluated by JHA. To verify the present model, three cases of S-S-S-S aluminum plate were selected for different values of width and thickness. Case 1 \((b_1=b\text{ and } t_1=t)\), 2 \((b_2=b/2\text{ and } t_2=2\, t)\) and 3 \((b_3=b/3\text{ and } t_3=3\, t)\) respectively. The original plate size was 495 mm x 361 mm x 2.75 mm. Comparison of the natural frequencies for the present work with JHA results presented in table 1.

3. **Present study**

Isotropic plate was modeled using ANSYS APDL with dimensions of 200 mm X 200 mm (area=40000 mm²). SHELL281 element was applied to model the plate considering shell section with one layer of 20...
mm. Different geometric parameters for the cutouts are considered for the free vibration analysis of isotropic plate, which are:

1- Cutout shape; circular, rectangular and triangular.
2- Cutout area; 500 mm², 1000 mm² and 2000 mm².
3- Aspect ratio (K) which is the ratio of the horizontal cutout dimension to the vertical dimension; 1, 1.5 and 2.5.
4- Cutout center distance from the center of the main plate (X); -30 mm, 0 and 30 mm.

| Mode No. | 1       | 2       | 3       | 4       | 5       |
|----------|---------|---------|---------|---------|---------|
| Case I b₁=b and t₁=t | JHA 2007 | 79.02   | 161.37  | 233.81  | 298.52  | 316.08  |
|          | Present Work | 79.04   | 161.36  | 233.70  | 298.48  | 316.04  |
|          | % variation | -0.03   | 0.01    | 0.05    | 0.01    | 0.01    |
| Case II b₂=b/2 and t₂=2t | JHA 2007 | 467.08  | 631.19  | 904.33  | 1286.00 | 1696.90 |
|          | Present Work | 466.87  | 630.82  | 903.56  | 1284.38 | 1694.33 |
|          | % variation | 0.04    | 0.06    | 0.09    | 0.13    | 0.15    |
| Case III b₃=b/3 and t₃=3t | JHA 2007 | 1465.90 | 1709.10 | 2113.20 | 2676.30 | 3153.50 |
|          | Present Work | 1462.27 | 1704.44 | 2106.45 | 2674.91 | 3155.95 |
|          | % variation | 0.25    | 0.27    | 0.32    | 0.05    | -0.08   |

The present model’s variation with JHA results ranged from 0.01 to 0.27%.

Figures 3 to 5 illustrate the geometry of the isotropic plate with various configuration of the cutouts. Therefore, a total number of tests were 81 tests.

The isotropic plate material is aluminum with the following properties; Young's modulus is 70 GPa, Poisson's ratio is 0.33 and density is 2700 kg/m³.

![Figure 3. Isotropic plate with circular cutout.](image_url)
One of the most critical aspects of ANSYS model analysis is the mesh generation. Few elements may lead to inaccurate results, in other hand; large number of elements may require long time to solve. A mesh convergence study was therefore conducted. Figure 6 illustrates a case study of the mesh convergence of isotropic plate with rectangular cutout; the cutout area is $500 \text{mm}^2$, the aspect ratio $K=1$ and the cutout...
distance $X=0$. Therefore, for all cutout configurations, the isotropic plates were meshed by 2 mm size elements.

![Mesh convergence study of isotropic plate with rectangular cutout, cutout area=500mm$^2$, K=1 and X=0.](image)

**Figure 6.** Mesh convergence study of isotropic plate with rectangular cutout, cutout area=500mm$^2$, K=1 and X=0.

Modal analysis has been applied for all plates and the mode shapes were extracted using Block Lanczos method. Plates were analyzed for clamped-simply supported condition (CSCS). Two opposite edges clamped (at $y=0$ and $y=b$) and other two simply supported (at $x=0$ and $x=a$), as shown in Figure 7.

![Geometry the isotropic plate](image)

**Figure 7.** Geometry the isotropic plate

For this study, the selected element is SHELL281, which is appropriate for analyzing thin to moderately thick shell structures. This element has eight nodes with six degrees of freedom at each node namely three translations and three rotation in the nodal x, y and z directions respectively.

![SHELL281 Geometry](image)

**Figure 8.** SHELL281 Geometry
4. Results and Discussion

4.1 Finite Element Analysis

The fundamental frequencies of the isotropic plates with different shape of the cutouts, areas, aspect ratios and distances from the center is presented in table 2.

Table 2. Fundamental frequencies of the isotropic plates with various cutouts.

| Area (mm²) | K | X (mm) | Frequency (Hz) |
|------------|---|--------|----------------|
| 500        | 1 | -30    | 3231.07, 3218.92, 3199.31 |
| 500        | 1 | 0      | 3218.77, 3200.46, 3171.13 |
| 500        | 1 | 30     | 3231.07, 3218.93, 3199.36 |
| 500        | 1.5| -30    | 3225.69, 3216.26, 3191.24 |
| 500        | 1.5| 0      | 3209.95, 3195.7, 3160.51 |
| 500        | 1.5| 30     | 3225.69, 3216.23, 3191.24 |
| 500        | 2.5| -30    | 3176.49, 3177.32, 3109.49 |
| 500        | 2.5| 0      | 3138.26, 3139.24, 3061.89 |
| 500        | 2.5| 30     | 3176.51, 3177.34, 3109.45 |
| 1000       | 1  | -30    | 3232.63, 3211.33, 3183.25 |
| 1000       | 1  | 0      | 3228.66, 3199.82, 3159.7 |
| 1000       | 1  | 30     | 3232.63, 3111.43, 3183.25 |
| 1000       | 1.5| -30    | 3227.67, 3210.41, 3160.59 |
| 1000       | 1.5| 0      | 3216.25, 3193.7, 3136.96 |
| 1000       | 1.5| 30     | 3227.67, 3210.33, 3160.54 |
| 1000       | 2.5| -30    | 3153.33, 3149.25, 2997.22 |
| 1000       | 2.5| 0      | 3118.98, 3114.39, 2985.46 |
| 1000       | 2.5| 30     | 3153.33, 3149.23, 2997.04 |
| 2000       | 1  | -30    | 3245.55, 3206.52, 3167.44 |
| 2000       | 1  | 0      | 3299.44, 3257.04, 3207.29 |
| 2000       | 1  | 30     | 3245.55, 3206.45, 3167.47 |
| 2000       | 1.5| -30    | 3246.69, 3212.01, 3105.62 |
| 2000       | 1.5| 0      | 3280.46, 3246.48, 3157.56 |
| 2000       | 1.5| 30     | 3246.69, 3211.99, 3105.69 |
| 2000       | 2.5| -30    | 3125.89, 3103.64, 2773.8 |
| 2000       | 2.5| 0      | 3154.51, 3130.82, 2934.34 |
| 2000       | 2.5| 30     | 3125.88, 3103.63, 2773.73 |

Figure 9 demonstrates that for plates with various shapes of central cutouts with aspect ratio of 1.5 with an area of 500 and 1000 mm² possess the lower fundamental frequencies and these frequencies increases as the cutout relocate from the center. While for plates with central cutouts of 2000 mm² the minimum frequency was achieved.
Figure 9. Effect of cutout location on the fundamental frequencies.

Figure 10 shows that by increasing the aspect ratio of the cutouts the fundamental frequency decreases. There is little change from K=1 to 1.5, but there is a noticeable decrease in the aspect ratio K=2.5.
In Figure 11, the effect of the cutout area is more evident for $K=1$ and 1.5 for all plates with central cutouts.
To determine the effective parameter, it was difficult to observe which parameter is the most effect on the frequency, so DOE was performed.

4.2 Design of Experiments
Design of experiments performed using Minitab software to obtain full information on all variables influencing the frequencies of plates with cutout. Taguchi method selected because it is an effective...
method to minimize the number of experiments to give a full information about all the factors that affect the response. For the current work, the design consists of four factors and three levels each, so nine runs required (Taguchi L9 design). Figure 12 illustrate the main effect plot for fundamental frequencies. It is clear that the aspect ratio of the cutouts possess the higher effect followed by the cutouts shape with small effect of the size and location of the cutouts. As shown in Figure 13, the aspect ratio of the cutouts is the mostly significant factor compared to the other parameters. Contour plots of fundamental frequencies vs. cutout area, location, shape and aspect ratio are presented in Figures 14 to 16.

![Main Effects Plot for FREQUENCY](image)

**Figure 12.** Main effects plot of the fundamental frequencies (*1=Circular, 2=Rectangular and 3=Triangular).

**Regression Analysis: FREQUENCY versus Shape, Area, K, X**

| Source | df | Adj SS | Adj MS | F-Value | P-Value |
|--------|----|--------|--------|---------|---------|
| Regression | 4  | 27952.6 | 6988.2 | 4.23 | 0.096 |
| Shape | 1  | 2477.7 | 2477.7 | 5.99 | 0.027 |
| Area | 1  | 161.9 | 161.9 | 0.10 | 0.768 |
| K | 1  | 1874.1 | 1874.1 | 11.58 | 0.027 |
| X | 1  | 256.9 | 256.9 | 0.16 | 0.710 |
| Error | 4  | 6486.3 | 1621.5 |
| Total | 8  | 43393.7 |

| R-sq | R-sq(adj) | R-sq(pred) |
|------|-----------|------------|
| 0.90999 | 0.81798 | 0.60000 |

| Term | Coef | SE Coef | T-Value | P-Value | VIF |
|------|------|---------|---------|---------|-----|
| Constant | 3376.0 | 36.4 | 93.88 | 0.000 | 1.00 |
| Shape | -37.1 | 16.4 | -2.26 | 0.097 | 1.00 |
| Area | -0.0068 | 0.0215 | -0.33 | 0.748 | 1.00 |
| K | 73.3 | 21.5 | 3.39 | 0.027 | 1.00 |
| X | 0.219 | 0.348 | 0.62 | 0.710 | 1.00 |

**Regression Equation**

\[
\text{FREQUENCY} = 3376.0 - 37.1 \text{Shape} - 0.0068 \text{Area} - 73.3K + 0.219X
\]

**Figure 13.** Regression analysis of fundamental frequencies versus shape, area, K and X.
Figure 14. Contour plot of fundamental frequencies vs. cutout area and location.

Figure 15. Contour plot of fundamental frequencies vs. cutout area and shape.

Figure 16. Contour plot of fundamental frequencies vs. cutout shape and aspect ratio.
5. Conclusion
A free vibration of isotropic plates with different cutout configurations was presented using the finite element method. Modal analysis was conducted using ANSYS APDL. In order to validate the ANSYS model, three literature case studies were selected and showed a very good agreement. Fundamental frequencies of isotropic plates were presented and examined for various cutout parameters. Three cutout shapes, circular, rectangular and triangular, were selected. Three cutout areas, three aspect ratios and three center distances from the center of the isotropic plate were modeled for each cutout shape. From the results, it was obvious that the cutout aspect ratio had a drastic effect on the fundamental frequency of all cutout shapes and areas, especially at K=2.5. For small and medium cutout size, the frequency increased when the cutouts shifted from the plate center. Whilst in the case of large cutout, the frequency decreased when it shifted from the plate center for all cutout shapes. Generally, the effect of the cutout parameters on the fundamental frequency was overlapped and it was difficult to determine which parameter is the most effective. Therefore, design of experiments (DOE) was conducted to determine which parameter is the most effect on the frequency. It was clearly observed that the aspect ratio of the cutouts is the most significant factor on the fundamental frequency.

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