High Saturation Power Josephson Parametric Amplifier with GHz Bandwidth

O. Naaman, D. G. Ferguson, and R. J. Epstein
Northrop Grumman Systems Corp., Baltimore, Maryland 21240, USA
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We present and simulate of a Josephson parametric amplifier with bandwidth exceeding 1.6 GHz, and with high saturation power approaching -90 dBm at a gain of 23.8 dB. An improvement by a factor of roughly 50 in bandwidth over the state of the art is achieved by using well-established impedance matching techniques. An improvement by a factor of roughly 100 in saturation power over the state of the art is achieved by implementing the Josephson nonlinear element as an array of rf-SQUIDs with a total of 40 junctions. WRSpice simulations of the circuit are in excellent agreement with the calculated gain and saturation characteristics.

Josephson parametric amplifiers have been in extensive use over the past few years, providing quantum limited noise performance at gains exceeding 20 dB, and enabling high fidelity qubit readout\(^1\)\(^2\)\(^3\)\(^4\) of squeezed microwave field generation\(^5\)\(^6\) weak measurement\(^7\)\(^8\) and feedback control\(^9\)\(^10\). However, state-of-the-art devices of the eponymous JPA\(^1\)\(^1\) generation, high fidelity qubit readout, use over the past few years, providing quantum limited power.

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An improvement by a factor of roughly 100 in saturation power over the state of the art in bandwidth and -90 dBm, is achieved by using well-established impedance matching techniques. A 7.5 GHz amplifier with 25 dB of gain, a linear inductance of 90 pH, and an effective negative resistance load \( |R_{sq}| \) of 10 \( \Omega \), could have a maximum bandwidth of about 3.5 GHz. With a three-pole physically realizable matching network, one can theoretically achieve up to 60% of that bandwidth.

In what follows, we will use the so-called pumppistor model\(^12\) to obtain an expression for \( Y_{sq}(\omega) \), and design a three-pole bandpass network to match the effective load to 50 \( \Omega \). Figure 2(a) shows the overall topology of the resulting circuit, whose gain characteristics are shown in Fig. 2. We will outline a design procedure that is quite general to Josephson amplifiers, however, we will concentrate on a particular nonlinear element shown in Figure 2(b)—a parallel arrangement of two rf-SQUID arrays, which we have previously demonstrated to be capable of carrying up to -53 dBm of power\(^13\) and should therefore allow amplification of up to -90 dBm input signals with 20 dB of gain without saturation. The relatively high junction \( I_c \) in the array can support higher mode currents than a typical JPA, and the low-inductance shunt of each of the junctions eliminates phase slips that are likely to occur otherwise when the amplifier is biased and pumped.

The design of the matching network follows Ref. 14. We choose to implement a three-pole coupled-resonator bandpass network; larger number of poles gives only

\(^{15}\)ofer.naaman@ngc.com
FIG. 1. (a) Schematic of the amplifier circuit including a band-pass matching network embedding the pumped Josephson nonlinearity, represented by $Y_{sp}$. The signal port is shown on the right and labeled $Z_0$. $Y_{ext}$ is the admittance seen by the nonlinearity out through the matching network. (b) Schematic of the particular implementation of the active element in our amplifier, built with two rf-SQUID arrays in parallel. Each array has $N$ sections that contain a junction with critical current $I_c$ shunted by inductors $L_1$ and $L_2$. The pump is coupled inductively to the loop and the self inductance of the coupling transformer, $L_m$ is treated as a parasitic in our calculation.

marginal improvement in bandwidth per pole. The set of filter prototype coefficients \{$g_i$\} that describe the network, which were calculated specifically to accommodate negative-resistance loads, are tabulated in Refs. 12 and 21 for specified gain and ripple characteristics. Once the center frequency of the network (which we take to coincide with half the pump frequency $\omega_p = \omega_{fp}/2$) is specified, as well as its desired bandwidth, gain, and ripple, the only remaining free parameter in the design is the capacitance shunting the Josephson element. All other parameters, particularly the dc flux bias to the Josephson element and the amplitude of the pump tone, will be constrained and can be calculated from the design equations.

The first pole of the network, resonator $Z_1$ in Fig. \(\Pi(a)\), is comprised of the Josephson element’s linear inductance $L_a(\Phi_{dc})$ in parallel with a shunt capacitance $C_1$. The choices for $C_1$ and $\omega_0$ determine both the required flux bias $\Phi_{dc}$ operating point such that $L_a(\Phi_{dc}) = 1/\omega_0^2C_1$, as well as the impedance of the resonator, $Z_1 = \sqrt{L_a(\Phi_{dc})/C_1}$. The other two poles of the network are built with passive LC resonators whose frequencies are $\omega_0$, and impedances are $Z_2 = \sqrt{L_{p2}/C_2}$ and $Z_3 = \sqrt{L_{p3}/C_3}$ (the values of the resonator shunt capacitances will be modified below from $C_{1\ldots3}$ to $C_{p1\ldots3}$). The resonators are coupled via admittance inverters $J_{ij}$ whose values are calculated from $g_i$ according to $J_{ij} = w/L_{ij}Z_3g_3g_4$, where $w$ is the fractional bandwidth, $Z_0 = 50\,\Omega$, and with the additional constraint,\footnote{\textsuperscript{15}}

\begin{equation}
    w \times \frac{|R_{sq}|}{Z_1} \leq g_i. \tag{1}
\end{equation}

We choose the impedance of the passive resonator $Z_3$ to satisfy $Z_3 = wZ_0/g_3g_4$; this allows us to eliminate the last inverter $J_{34}$. The impedance $Z_2$ can be chosen arbitrarily, and we set $Z_2 = \sqrt{Z_1Z_3}$. We can now implement all admittance inverters as capacitive pi-sections\footnote{\textsuperscript{22}} with $C_{ij} = J_{ij}/\omega_0$ to obtain the circuit shown in Fig. \(\Pi(a)\), where $C_{p1} = C_1 - C_{12}$, $C_{p2} = C_2 - C_{12} - C_{23}$ and $C_{p3} = C_3 - C_{23}$, and all component values are determined by the above. The admittance seen by the nonlinear element looking through the matching network out to the 50 Ω environment, $Y_{ext}(\omega)$ in Fig. \(\Pi(a)\), can be evaluated at the center of the band:

\begin{equation}
    Y_{ext}(\omega) = j\omega_0 C_1 + \left(\frac{J_{12}}{J_{23}}\right)^2 \frac{1}{Z_0}. \tag{2}
\end{equation}

The matching network design is now complete, and without assuming anything about the particular form of the Josephson nonlinearity, it is quite general and can be used to broadband amplifiers based on dc-SQUIDs, Josephson dipole elements\footnote{\textsuperscript{21}} and rf-SQUID arrays (Fig. \(\Pi(b)\)) alike. However, we still need to find the optimal pump amplitude $\Phi_{ac}$ and calculate the gain profile, which require knowledge of the admittance $Y_{sq}$. We find this admittance by use of the pumpistor model of Ref.\footnote{\textsuperscript{18}} for a flux-pumped nonlinearity in 3-wave mixing operation, which we can write as $Y_{sq}(\omega_s) = 1/j\omega_s L_a + 1/j\omega_s (L_b + L_c)$, where $\omega_s$ is the signal frequency, $L_a = L_T(\Phi_{dc})$ is the linear inductance of the Josephson element at the operating point, and

\begin{equation}
    L_b = \frac{4L_T^3(\Phi_{dc})}{L_T^2(\Phi_{dc})^2 \Phi_{ac}^2} \tag{3}
\end{equation}

\begin{equation}
    L_c = \frac{4i\omega L_T^3(\Phi_{dc})Y_{ext}(\omega)}{L_T^2(\Phi_{dc})^2 \Phi_{ac}^2} \tag{4}
\end{equation}

where $L_T'(\Phi_{dc})$ is the flux derivative of the inductance evaluated at the operating point, and $\omega_i = \omega_p - \omega_s$ is the idler frequency. If the amplifier is built using a simple dc-SQUID with a total critical current $I_c$, then $L_T(\Phi) = h/2eI_c \cos(\pi/\Phi_{0})$. Our nonlinear element, shown in Fig. \(\Pi(b)\), is constructed from two arrays of rf-SQUIDs, each of the array’s $N$ stages composed of a junction with critical current $I_c$ shunted by linear inductors $L_1$ and $L_2$. In this case we have for the two arrays in parallel:\footnote{\textsuperscript{11}}

\begin{equation}
    L_T(\delta(\Phi_{dc})) = \frac{N(L_1 + L_2)(L_J + L_1L_2\cos\delta)}{2} \times \frac{1}{L_J + (4L_1 + L_2)\cos\delta}, \tag{5}
\end{equation}

The Josephson nonlinearity, represented by $Y_{sp}$, is thus shunted by $I_c$-shunted $L_1$ and $L_2$ in parallel.
where \( L_j = \hbar/2eI_c \), \( \Phi_0 \) is the flux quantum, and \( \delta_0(\Phi_{dc}) \) is given implicitly by

\[
\left( \frac{1}{L_1} + \frac{1}{L_2} \right) \delta_0 + \frac{1}{L_j} \sin \delta_0 = \frac{\pi \Phi_{dc}}{N \Phi_0} \left( \frac{1}{L_1} + \frac{2}{L_2} \right).
\] (7)

Using Eqs. (2)-(7), we can find the pump amplitude \( \Phi_{ac} \) for which \( R_{sq} = 1/\Re \{ Y_{sq}(\omega_0) \} \) satisfies the constraint in Eq. (1) at the center of the band.

We now have all circuit element values, the dc flux operating point and the optimal amplitude of the pump. To calculate the gain profile of the amplifier, we have to evaluate, at each signal frequency \( \omega_s \), the admittance at the idler frequency \( Y_{st}(\omega_p - \omega_s) \), and from it calculate the admittance of the pumped nonlinearity \( Y_{sq}(\omega_s) \). We then calculate the impedance of the whole amplifier as seen from the 50 \( \Omega \) signal port to the right of Fig. 1(a), \( Z_{amp}(\omega_s) \), and the gain in dB is given by

\[
G(\omega_s) = 20 \times \log_{10} \left| \frac{Z_{amp}(\omega_s) - Z_0}{Z_{amp}(\omega_s) + Z_0} \right|.
\] (8)

As a concrete example, we design an amplifier with a center frequency of 7.5 GHz. We target a design with a gain of 25 dB and with 0.5 dB gain ripple. From tables in Refs. 15,20, we find the coefficients \( g_1 = 0.6068 \), \( g_2 = 0.6742 \), \( g_3 = 0.3836 \), and \( g_4 = 0.8992 \). We choose \( w = 0.25 \) for the fractional bandwidth parameter, and an initial shunt capacitance of \( C_1 = 5.2 \) pF. Each rf-SQUID array is implemented with 20 Josephson junctions with \( I_0 = 35 \) \( \mu \)A, and the array inductors are \( L_1 = 1.45 \) pH and \( L_2 = 3.52 \) pH. The loop enclosing the two arrays is coupled to the pump via a 10 pH transformer with a self inductance of \( L_m = 20 \) pH. Following the above procedure we find the dc flux operating point \( \Phi_{dc} = 0.26 \) \( \Phi_0 \) per junction (total of 10.38 \( \Phi_0 \) over the entire loop of 20 \( N = 40 \) junctions), and the pump amplitude that gives \( R_{sq} = -9.15 \) \( \Omega \) at \( \omega_0 \) and satisfies Eq. (1) is found to be \( \Phi_{ac} = 0.073 \) \( \Phi_0 \) per junction for a total of 2.93 \( \Phi_0 \) over the entire 40-junction loop. With reference to Fig. 1(a), the coupling capacitors evaluate to \( C_{12} = 1.19 \) pF and \( C_{23} = 0.5 \) pF, and the final shunt capacitors are \( C_{p1} = 4.01 \) pF, \( C_{p2} = C_{p3} = 85 \) fF. The passive resonator inductances are \( L_{p2} = 254 \) pH, and \( L_{p3} = 769 \) pH.

The solid curve in Figure 2 shows the resulting gain characteristics of the amplifier as calculated using Eq. 8 and the pumpistor model with Eq. (3)-(7). We see that the amplifier has a maximum gain of 22.8 dB, a 3 dB bandwidth of 1.63 GHz, and gain ripple of 0.74 dB. We have additionally simulated the full nonlinear circuit in WRSpice in the time domain using transient analysis. We extracted the reflection coefficient of the circuit from the voltage waveform at the port of the amplifier by numerical I-Q demodulation, allowing us to separate the reflected waves from the incident signal. The results of these simulations are shown as circles in Fig. 2 and are in excellent agreement with the calculated response. In the simulation, we found that the optimal pump amplitude and dc flux were both higher than estimated within the pumpistor model by roughly 20% and 30% respectively, which is not surprising as the pumpistor is a linearized model while the Spice simulation captures the full nonlinearity of the circuit.

In Figure 3, we use the Spice simulation to characterize the large-signal response of the circuit and estimate its 1 dB compression power, independent of any theory for the saturation mechanism. The simulated data are shown in the figure as circles for a signal frequency of 7.3 GHz, and we obtain a saturation power of \( P_{sat} = -92.2 \) dBM. We compare the simulated data to a calculation within the pumpistor model framework, in which we assume a pump depletion saturation mechanism, and which is shown in Fig. 3 as solid line. Our calculation is based on a power balance equation for the pump, recognizing that the conversion of pump photon into signal and idler photons present the pump mode with an effective loss channel. The loaded pump power \( P_{load} \) available to drive the amplification process is \( P_{load} = P_{av} - P_{loss} \), where \( P_{av} \) is the available power from the pump port, and \( P_{loss} = \frac{1}{2} G_{load} (2P_{in} + wh \omega_0^2) \). Here the last term is the input quantum noise power over the amplifier band, \( P_{in} \) is the input signal power, and \( G_{load} \) is the loaded gain evaluated with \( P_{load} \) drive. The available and loaded pump powers relate to the respective pump amplitudes.
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will require separate designs for the spatially and spec-
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