Orbital Kondo effect in a parallel double quantum dot

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Abstract

We construct a theoretical model to study the orbital Kondo effect in a parallel double quantum dot (DQD). Recently, pseudospin-resolved transport spectroscopy of the orbital Kondo effect in a DQD has been experimentally reported. The experiment revealed that when interdot tunneling is ignored, two and one Kondo peaks exist in the conductance-bias curve for pseudospin-non-resolved and pseudospin-resolved cases, respectively. Our theoretical studies reproduce this experimental result. We also investigate the case of all lead voltages being non-equal (the complete pseudospin-resolved case) and found that there are at most four Kondo peaks in the curve of the conductance versus the pseudospin splitting energy. When interdot tunneling is introduced, some new Kondo peaks and dips can emerge. Furthermore, the pseudospin transport and the pseudospin flipping current are also studied in the DQD system. Since the pseudospin transport is much easier to control and measure than the real spin transport, it can be used to study the physical phenomenon related to the spin transport.

Keywords: orbital Kondo effect, parallel double quantum dot, pseudospin transport

(Some figures may appear in colour only in the online journal)

1. Introduction

The Kondo effect is an important issue in condensed-matter physics [1] and has attracted extensive attention since its discovery since it could provide a deeper understanding of the physical properties of many strongly correlated systems [2]. On the other hand, a quasi-zero-dimensional system called quantum dot (QD), of which the parameters can be modulated experimentally in a continuous and reproducible manner, offers an appropriate platform to study the Kondo problem [3–7]. Under appropriate conditions, the Kondo effect can arise from the coherent superposition of cotunneling processes [2, 6], where the spin degree of freedom plays a significant role and the electron in the QD can flip its spin. At low temperatures, the coherent superposition of many cotunneling processes could lead to the Kondo resonant state in which the spin flip occurs frequently within the QD and a very sharp Kondo peak emerges in the density of state of the QD.

After it initial discovery, the Kondo effect was proposed based on the orbital degree of freedom [8–14]. It was reported that double QD (DQD) was potentially a good candidate for realizing the orbital Kondo effect [8, 14–21]. In this situation, the energy of the orbital state in the left QD can be the same as or very close to that in the right QD. In this case, the corresponding left and right orbital states are degenerate or near degenerate and can be regarded as pseudospin degenerate states [8, 22, 23]. In real spin systems, it is difficult to manipulate the spin-up state and the spin-down state individually. In contrast, since the left and right QDs of the DQD system are separated in space, it is much easier to control both QDs, with each considered a pseudospin component [14, 18, 24–27]. As a result, the physical phenomenon related to the spin degree of freedom may also be realized in a DQD system including the pseudospin (orbital) degree of freedom.

Very recently, the pseudospin-resolved transport spectroscopy of the Kondo effect has been observed in a DQD device on the basis of an orbital degeneracy [14]. The schematic diagram of this device is shown in figure 1(a). In the experiment, the authors fabricated the parallel DQD system from an epitaxially grown AlGaAs/GaAs heterostructure. As
In this paper, we theoretically investigate the orbital Kondo effect in a parallel DQD. It should be noted that the properties of DQDs have been studied in many theoretical works [15–19]. In this work, by using the non-equilibrium Green’s function method and the equation of motion technique, the formula of the conductance for each pseudospin component and conductance formulas. In section 2, we numerically investigate the conductances and the pseudospin flipping current of the DQD in different cases. Finally, we provide conclusions in section 3.

1.1. Model and analytical results

The Hamiltonian of the DQD system as shown in figure 1(a) can be written as

\[ H = H_{\text{DQD}} + H_T + \sum_{\alpha\beta} H_{\alpha\beta}, \]

where

\[ H_{\text{DQD}} = \sum_{\alpha} \epsilon_{\alpha} d_{\alpha}^{\dagger} d_{\alpha} + U_{\alpha} d_{\alpha}^{\dagger} i_{\alpha} d_{\alpha} + (t_{c} d_{L}^{\dagger} d_{R} + \text{h.c.}), \]

\[ H_T = \sum_{\alpha\beta k} (t_{\alpha\beta} d_{\alpha k}^{\dagger} a_{\beta k} + \text{h.c.}), \]

\[ H_{\alpha\beta} = \sum_{k} \epsilon_{\alpha\beta k} a_{\alpha k}^{\dagger} a_{\alpha k}. \]

Here, \( H_{\text{DQD}} \) is the Hamiltonian of the DQD and \( d_{\alpha}^{\dagger} (d_{\alpha}) \) is the creation (annihilation) operator of the electron in the QDs with \( \alpha = L/R \) representing left and right. \( \epsilon_{\alpha} \) is the energy level of the QDs, \( U \) is the interdot electron-electron interaction, and \( t_{c} \) is the tunneling coupling. \( H_T \) denotes the tunneling between the DQD and the leads. \( a_{\alpha k}^{\dagger} (a_{\alpha k}) \) is the creation (annihilation) operator of the electron in the leads, and \( \beta = S/D \) is the source and drain. \( H_{\alpha\beta} \) describes the non-interacting leads. It should be noted that when a high magnetic field is applied to the QD, the spin splitting energy can be comparable with or even larger than the QD energy level spacing [8, 28, 29]. Compared with the low energy spin state, the opposite spin state does not affect the transport property of the system at low bias. Thus, we can neglect the spin degree of freedom. Meanwhile, we consider the low bias case with a value less than the intradot
electron–electron interaction energy $U_0$. In this case, there is only one eigenstate in each QD in the bias window. Then, we can absorb the intradot interaction $U_0$ into the energy levels $\epsilon_L$ and $\epsilon_R$ [8, 30]. As a result, both the spin degree of freedom and the intradot interaction can be ignored, leaving only the interdot interaction $U$. This approximation was adopted in [8, 30].

This model can describe various properties of the parallel DQQ, including the properties illustrated in the recent experiment of [14] and could also be used to study the pseudospin transport. Here, we briefly introduce the concept of the pseudospin transport in the parallel DQD. As we know, the electron has two spin states: spin up and spin down. The pseudospin, however, the situation is completely different. When the electron is in the left (right) QD and the electron in the right (left) QD, we call it the pseudospin up (down) state. The transport related to the pseudospin is called pseudospin transport. Here, we briefly introduce the concept of pseudospin splitting energy $\Delta_0$ into the energy levels $\epsilon_L$ and $\epsilon_R$.

In this case, there is $\epsilon_L - \epsilon_R = \Delta_0$ and the pseudospin splitting energy $\Delta_0$ can be manipulated by the gates $P_L$ and $P_R$, the pseudospin splitting energy $\Delta_0 = \epsilon_L - \epsilon_R$ can be adjusted in a wide range. The pseudospin flipping strength $t_0$ in the DQQ can also be tuned by the gates $C_D$ and $C_S$. It can be open or closed by simply tuning the gate voltages. It is difficult to maintain the real spin in the lead in order to keep its direction, and the spin flipping exists inevitably. Conversely, the pseudospin can keep its 'direction' steadily outside the DQD because the electrons in the left lead cannot tunnel into the right lead and vice versa. Therefore, the pseudospin flipping current in the DQQ can be accurately measured in the experiment.

Next, we use the standard equation of motion technique to solve the retarded Green’s function [8, 33–35]. The equation of motion is:

$$\varepsilon \langle [A|B] \rangle' = \langle [\hat{A}, \hat{B}] \rangle + \langle [\hat{A}, H]|\hat{B} \rangle \rangle', \tag{2}$$

where $\hat{A}$ and $\hat{B}$ are arbitrary operators, and $\langle [A|B] \rangle'$ is the standard notation of the retarded Green’s function. Since higher order Green’s functions appear in the calculations of equations of motion, a decoupling scheme is needed. The decoupling scheme in this work has the following rules: (1) If we use $X$ to represent the leads operator $d_{\alpha \uparrow}^{\dagger}$ and $a_{\alpha \uparrow}^{\dagger}$, and use $Y$ to represent the DQQs operator $d_{\alpha \uparrow}$ and $d_{\alpha \downarrow}^{\dagger}$, then we take $\langle XY \rangle = 0$. (2) If the two-particle Green’s function involves two leads operators, then we take $\langle XX_1X_2 Y(d_{\alpha \uparrow}^{\dagger}) \rangle' = \langle XX_1X_2 \rangle \langle Y|d_{\alpha \uparrow}^{\dagger} \rangle \rangle'$. (3) If the two-particle Green’s function involves only one lead operator, which is $\langle XX_1X_2 |d_{\alpha \uparrow}^{\dagger} \rangle \rangle'$, we continue to apply the equation of motion until all the two-particle Green’s functions contain two leads operators. This decoupling scheme has been used in previous papers [8, 33].

Moreover, because the methods of derivation we used are similar to [8], we omit the detailed derivation and only show the results in this paper. It should be pointed out that, although we use the same calculation method as [8], the research subject and conclusions are totally different. [8] described the series DQQs, while the present work refers to the parallel DQQs. Unlike the series DQQs in [8], the conductance can hold a pseudospin-resolved character in the parallel DQQs system when the interdot tunnelling is $t_0 = 0$. The pseudospin transport and the pseudospin flipping current can also be studied in the present model, while $t_0 \neq 0$ and the calculation present a new method to measure and control the pseudospin transport in the parallel DQQs system.

In addition, it is worth mentioning that although the equation of motion method based on the non-equilibrium Green’s function cannot quantitatively give the intensity of the Kondo effect, it can give the qualitative physics and the positions of the Kondo peaks. Using the equation of motion in equation (2), we can obtain the matrix equation

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \cdot \begin{pmatrix} \langle [d_{\alpha \uparrow}|d_{\alpha \uparrow}^{\dagger}] \rangle' \\ \langle [d_{\alpha \uparrow}|d_{\alpha \uparrow}^{\dagger}] \rangle' \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}, \tag{3}$$

where

$$C_{11} = \varepsilon - \varepsilon_L - \Sigma_{LS}^0 - \Sigma_{LD}^d + U_A|B (\hat{i}, \hat{A} R \Sigma^d + \Sigma_{RS}^2 + \Sigma_{RD}^2 + \Sigma_{LS}^d + \Sigma_{LD}^d),$$

$$C_{12} = -t_0 + U_A|B (\hat{i}, \hat{A} R (\Sigma_{LS}^d + \Sigma_{LD}^d + \Sigma_{RS}^d) + \Sigma_{RD}^2 + \Sigma_{LS}^d + \Sigma_{LD}^d + \Sigma_{RD}^d),$$

$$C_{21} = -t_0 + U_A|B (\hat{i}, \hat{A} L (\Sigma_{RS}^d + \Sigma_{RD}^2 + \Sigma_{LS}^d) + \Sigma_{RD}^2 + \Sigma_{LS}^d + \Sigma_{RS}^d),$$

$$C_{22} = \varepsilon - \varepsilon_R - \Sigma_{RS}^0 - \Sigma_{RD}^2 + U_A|B (\hat{i}, \hat{A} L (\Sigma_{RS}^d + \Sigma_{RD}^2 + \Sigma_{LS}^d + \Sigma_{RS}^d + \Sigma_{RD}^d).$$

The expressions of the above notations are listed as follows:

$$\Sigma_{0}^{a \beta} = \sum_{k} \frac{|a_{\alpha \beta}|^2}{\varepsilon - \varepsilon_{a \beta k}} = -\frac{i}{2} \Gamma_{a \beta},$$

$$\Sigma_{a \beta}^{2 \beta} = (\varepsilon + \varepsilon_L - \varepsilon_R + \varepsilon_{a \beta k})(\varepsilon - \varepsilon_L + \varepsilon_R - \varepsilon_{a \beta k}) - 4t_0^2,$$

$$\Sigma_{1 \alpha}^{a \beta} = \sum_{k} \frac{|a_{\alpha \beta}|^2}{(\varepsilon - \varepsilon_{a \beta k})U_{a \beta k}} - \frac{F_{1 \alpha}^{a \beta} (\varepsilon_{a \beta k}),}{\varepsilon_{a \beta k}}$$

$$\Sigma_{2 \alpha}^{a \beta} = \sum_{k} \frac{|a_{\alpha \beta}|^2}{(\varepsilon - \varepsilon_{a \beta k})U_{a \beta k}} - \frac{2t_0^2}{(\varepsilon - \varepsilon_{a \beta k})\bar{\varepsilon}_{a \beta k}},$$

$$\Sigma_{3 \alpha}^{a \beta} = \sum_{k} \frac{2t_0^2}{\varepsilon_{a \beta k}} - \frac{|a_{\alpha \beta}|^2 F_{1 \alpha}^{a \beta} (\varepsilon_{a \beta k}),}{\varepsilon_{a \beta k}}.$$
In the above equations, $F_{\alpha\beta}(\epsilon_{\alpha\beta}) = 1$ and $F_{\alpha\beta}^{\dagger}(\epsilon_{\alpha\beta}) = f_{\alpha\beta}(\epsilon_{\alpha\beta})$, where $f_{\alpha\beta}(\epsilon_{\alpha\beta}) \equiv 1/\{\exp[(\epsilon_{\alpha\beta} - \mu_{\alpha\beta})/k_BT] + 1\}$ is the Fermi distribution function and $\mu_{\alpha\beta} = eV_{\alpha\beta}$ is the chemical potential of the lead $\alpha\beta$. $\alpha$ and $\beta$ denote different right-left positions. That is, if $\alpha$ is left, $\beta$ is right; if $\alpha$ is right, $\alpha$ is left. $\Sigma^{1\alpha}_{\alpha\beta}$, $\Sigma^{2\alpha}_{\alpha\beta}$, $\Sigma^{\beta\alpha}_{\alpha\beta}$, $\Sigma^{\beta\beta}_{\alpha\beta}$, $\Sigma^{\bar{\alpha}\alpha}_{\alpha\beta}$, and $\Sigma^{\bar{\alpha}\beta}_{\alpha\beta}$ are the higher-order self-energies.

Taking the limit of $U \to \infty$, equation (3) can be simplified and the elements of the matrices are replaced by:

\[ C_{1\alpha} = \varepsilon - \varepsilon_L - \Sigma^{0\alpha}_{\alpha\beta} - \Sigma^{\alpha\alpha}_{\alpha\beta} - \Sigma^{\alpha\beta}_{\alpha\beta} - \Sigma^{\alpha S}_{\alpha\beta}, \]
\[ C_{2\alpha} = -t_{\alpha}, \]
\[ C_{2\beta} = \varepsilon - \varepsilon_K - \Sigma^{0\beta}_{\alpha\beta} - \Sigma^{\beta\alpha}_{\alpha\beta} - \Sigma^{\beta\beta}_{\alpha\beta} - \Sigma^{\beta S}_{\alpha\beta}, \]
\[ \Gamma_D = 1 - n_R, \quad \Gamma_S = \langle d_{\alpha}^{\dagger}d_{\alpha} \rangle, \quad \Gamma_L = \langle d_{\beta}^{\dagger}d_{\beta} \rangle, \quad \Gamma_R = \langle d_{\beta}^{\dagger}d_{\alpha} \rangle. \]

Using the non-equilibrium Green’s function, the current from the lead $\alpha\beta$ flowing into the system can be obtained as [8]:

\[ J_{\alpha\beta} = \frac{-e}{\pi} \int d\epsilon f_{\alpha\beta}(\epsilon) ImG^{>\alpha\beta}_{\alpha\beta} - e \Gamma_{\alpha\beta}(\langle d_{\alpha}^{\dagger}d_{\alpha} \rangle). \tag{4} \]

In the expressions of the current and the coefficients $D_{ij}$, $(\langle d_{\alpha}^{\dagger}d_{\alpha} \rangle)$ is determined self-consistently. From the relation $(\langle d_{\alpha}^{\dagger}d_{\alpha} \rangle) = -i \int (d\epsilon/2\pi) G^{<\alpha\alpha}_{\alpha\alpha}(\epsilon)$ with the lesser Green’s function $G^{<\alpha\alpha}_{\alpha\alpha}(\epsilon)$, the self-consistent equations can be exactly derived [8]:

\[ -t_{\alpha}(d_{\alpha}^{\dagger}d_{\alpha}) + t_{\beta}(d_{\beta}^{\dagger}d_{\beta}) - i\Gamma_{\alpha\beta}(d_{\alpha}^{\dagger}d_{\beta}) - i\Gamma_{\alpha\beta}(d_{\beta}^{\dagger}d_{\alpha}) = \int \frac{d\epsilon}{2\pi} (\Gamma_{\alpha\beta}(\epsilon) + \Gamma_{\alpha\beta}(\epsilon)^{\dagger}) G^{<\alpha\alpha}_{\alpha\alpha}(\epsilon), \tag{5} \]
\[ -\varepsilon_{\alpha} + \varepsilon_{\beta} - \frac{i}{2}\Gamma_{\alpha\beta} - \frac{i}{2}\Gamma_{\alpha\beta}^{\dagger} - \frac{i}{2}\Gamma_{\alpha\beta} - \frac{i}{2}\Gamma_{\alpha\beta}^{\dagger} \langle d_{\alpha}^{\dagger}d_{\beta} \rangle = \int \frac{d\epsilon}{2\pi} (\Gamma_{\alpha\beta}(\epsilon) + \Gamma_{\beta\alpha}(\epsilon)^{\dagger}) G^{<\alpha\alpha}_{\alpha\alpha}(\epsilon), \tag{6} \]
\[ -t_{\beta}(d_{\beta}^{\dagger}d_{\beta}) + t_{\alpha}(d_{\alpha}^{\dagger}d_{\alpha}) - i\Gamma_{\alpha\beta}(d_{\beta}^{\dagger}d_{\alpha}) - i\Gamma_{\alpha\beta}(d_{\alpha}^{\dagger}d_{\beta}) = \int \frac{d\epsilon}{2\pi} (\Gamma_{\alpha\beta}(\epsilon) + \Gamma_{\beta\alpha}(\epsilon)^{\dagger}) G^{<\beta\beta}_{\beta\beta}(\epsilon), \tag{7} \]
\[ -\varepsilon_{\beta} - \varepsilon_{\alpha} - \frac{i}{2}\Gamma_{\alpha\beta} - \frac{i}{2}\Gamma_{\alpha\beta}^{\dagger} - \frac{i}{2}\Gamma_{\alpha\beta} - \frac{i}{2}\Gamma_{\alpha\beta}^{\dagger} \langle d_{\beta}^{\dagger}d_{\alpha} \rangle = \int \frac{d\epsilon}{2\pi} (\Gamma_{\alpha\beta}(\epsilon) + \Gamma_{\beta\alpha}(\epsilon)^{\dagger}) G^{<\beta\beta}_{\beta\beta}(\epsilon), \tag{8} \]

In this section, we first discuss the case of negligible interdot tunneling, then generalize our study to the case of finite interdot tunneling. Finally, we study the pseudospin flipping current in the DQD. In our calculations, we have taken $\Gamma_{\alpha\beta} = \Gamma_{\beta\alpha} = \Gamma_{\beta\beta} = \Gamma_{\alpha\alpha} = 1$ in all cases.

2. Numerical results and analysis

In this section, we first discuss the case of negligible interdot tunneling, then generalize our study to the case of finite interdot tunneling. Finally, we study the pseudospin flipping current in the DQD. In our calculations, we have taken $\Gamma_{\alpha\beta} = \Gamma_{\beta\alpha} = \Gamma_{\beta\beta} = \Gamma_{\alpha\alpha} = 1$ in all cases.

2.1. The numerical results without interdot tunneling

In this subsection, we focus on the case without any interdot tunneling. Before we discuss the conductance of the DQD, there is one thing that should be emphasized. When the interdot tunneling is ignored, i.e. $t_{\alpha} = 0$, there is no pseudospin flipping and we can obtain the results of $G_{\alpha\beta} = G_{\beta\alpha} = G_{\alpha\beta}$ and $G_{\beta\alpha} = G_{\beta\beta}$. When finite interdot tunneling exists, i.e. $t_{\alpha} \neq 0$, all of the four conductances may not be the same, which is determined by the structure of the DQD’s energy levels and voltages. In addition,
since the characters of the four conductances are similar, we only analyse $G_{LS}$. Figure 1(b) shows the conductance $G_{LS}$ as functions of $\bar{\varepsilon}$ and $\Delta \varepsilon$, where $\bar{\varepsilon} = \frac{\varepsilon + \varepsilon_R}{2}$ and $\Delta \varepsilon = \varepsilon_L - \varepsilon_R$. The different colors represent different values of the conductance. We can see a bright peak emerging at $\Delta \varepsilon = 0$, which is the zero-bias Kondo resonant peak.

Next, we study the conductance in detail. Figures 2(a) and (b) show $G_{LS}$ as a function of the bias voltage $V_{LS}$, while figures 2(c) and (d) illustrate $G_{LS}$ as a function of the pseudospin splitting energy $\Delta \varepsilon$. In figure 2(a), we keep $V_{LS} = V_{RS}$ and $V_{LD} = V_{RD} = 0$, implying that the chemical potentials of both pseudospin up and down electrons are changed simultaneously. When $\Delta \varepsilon = 0$, the Kondo peak emerges at $V_{LS} = 0$; when $\Delta \varepsilon \neq 0$, the Kondo peak splits into two peaks at $V_{LS} = \pm \Delta \varepsilon$. This phenomenon is similar to the splitting of the spin Kondo peak of a single QD in the magnetic field, and $\Delta \varepsilon$ is equivalent to the Zeeman energy due to the magnetic field. This is the pseudospin-non-resolved Kondo effect. In figure 2(b), we keep $V_{RS} = V_{LD} = V_{RD} = 0$ and change $V_{LS}$ only. Since the chemical potential of the pseudospin up electron in the source wire only is changed, a single peak emerges at $V_{LS} = \Delta \varepsilon$. This is the pseudospin-resolved effect. The results in figures 2(a) and (b) are in good agreement with the recent experiment [14].

Next, we discuss the relationship between the conductance $G_{LS}$ and the pseudospin splitting energy $\Delta \varepsilon$, which is shown in figures 2(c) and (d). In figure 2(c), we keep $V_{LD} = V_{RD} = 0$ and $V_{LS} = V_{RS}$. If $V_{LS} = V_{RS} = 0$, there exists only one Kondo peak, which is located at $\Delta \varepsilon = 0$. This is well-known in the spin Kondo system. While $V_{LS} = V_{RS} \neq 0$, the Kondo peak is divided into three peaks with their positions located at $\Delta \varepsilon = 0, \pm V_{LS}$. In figure 2(d), we keep $V_{RS} = V_{LD} = V_{RD} = 0$ and change $V_{LS}$ alone. In contrast to figure 2(c), only two peaks are found at $\Delta \varepsilon = 0$, $V_{LS}$ in figure 2(d) when $V_{LS} \neq 0$, and the original peak at $\Delta \varepsilon = -V_{LS}$ disappears because the chemical potential of the pseudospin down electron in the source wire is exactly zero. It should be noted that since the spin-up and spin-down chemical potentials in the real spin system are difficult to manipulate separately, it is difficult to observe these phenomena, as shown in figures 2(c) and (d). However, these phenomena are easy to observe in the parallel DQD system as it is easy to manipulate the chemical potentials and the splitting of the pseudospin degree of freedom.

The Kondo peaks in figure 2 can be explained in terms of the cotunneling processes shown in figure 3. It should be pointed out that the Kondo effect can be captured by the fourth or higher-order perturbation processes with respect to the tunneling between dot and leads. As can be seen from figure 3, when the electric state in the parallel DQDs returns to its original state, it has experienced four tunneling processes (shown by two red lines and two blue lines).
These four tunneling processes make up two cotunneling processes, and each cotunneling process is a second order perturbation process. Notice that only the combination of two cotunneling processes can lead to the Kondo effect. A similar explanation, which interprets the Kondo effect by cotunneling processes, has been used in many previous papers [8, 15, 36–38]. Figure 3(a) plots a cotunneling process that leads to the main Kondo resonance when $V_{LS} = V_{LD} = V_{RD} = 0$ and $\Delta \varepsilon = 0$ (blue lines in figures 2(a) and (b)). The blue and red arrows illustrate the correlative tunneling events, respectively. To be specific, we first consider an electron in the right QD. This electron can tunnel from the right QD into the lead RD. Then, another electron in the lead LS with energy $V_{RD}$ can tunnel into the left QD. These two tunneling events are shown by the blue arrows. After this, the left QD is occupied and the right QD is empty, where the system energy is the same as that in the beginning state. The red arrows show another two similar tunneling events, where an electron in the left QD tunnels into the lead LD and then another electron in the lead RS tunnels into the right QD. With the above four tunneling events, although the system recovers to the beginning state, the electrons travel from the left (right) source lead through the left (right) QD to the left (right) drain lead. When many of these cotunneling processes take coherent superposition at low temperature, a Kondo resonance will appear. This leads to the main Kondo peak at $\Delta \varepsilon = 0$ in figure 2(c), when $V_{LS} = V_{RS} = 0$ and $V_{RD} = V_{RD} = 0$ and $\Delta \varepsilon = 0$ (blue lines in figures 2(c) and (d)). Figure 3(b) explains the emergence of three peaks when $V_{LS} = V_{RS} = 0$ in figure 2(c). Whether $\Delta \varepsilon = 0$, $\Delta \varepsilon > 0$, or $\Delta \varepsilon < 0$, the electrons can travel through both QDs because of the cotunneling processes shown in figure 3(b), and thus three peaks appear in figure 2(c). On the other hand, it should be pointed out that the energy is conserved in the cotunneling processes. Therefore, for $\Delta \varepsilon = 0$ in figure 3(b), when an electron in the right QD tunnels into the lead RD, another electron in the lead LS with the energy $V_{RD}$ can tunnel into the left QD. For $\Delta \varepsilon > 0$ ($\Delta \varepsilon < 0$), the condition of $V_{LS} - \varepsilon_L = V_{RD} - \varepsilon_R$ ($V_{LD} - \varepsilon_L = V_{RS} - \varepsilon_R$) should be preserved due to the energy conservation in the cotunneling processes. Since we keep $V_{LD} = V_{RD} = 0$ and $V_{LS} = V_{RS}$, the Kondo peaks can emerge at $\Delta \varepsilon \equiv \varepsilon_L - \varepsilon_R = \pm V_{LS} = \pm V_{RS}$ (see figure 2(c)). Figure 3(c) explains the emergence of two peaks when $V_{LS} \neq 0$ in figure 2(d). For $\Delta \varepsilon = 0$ and $\Delta \varepsilon > 0$, the electrons can pass through both QDs. However, for $\Delta \varepsilon < 0$, the energy obtained from the electron jumping from the right source lead RS to the right QD cannot support the tunneling event from the left QD to the drain lead LD (shown by the red dash-dotted lines). Therefore, the electrons
cannot travel through the DQD for $\Delta \varepsilon < 0$. As a result, no Kondo peak appears at $\Delta \varepsilon = -V_L$, and there are only two Kondo peaks at $\Delta \varepsilon = 0$ and $\Delta \varepsilon = V_L$ in figure 2(d).

In general, if the four lead voltages $V_{LS}$, $V_{LD}$, $V_{RS}$, and $V_{RD}$ are not equal to each other, there are four Kondo peaks with their positions at $\Delta \varepsilon = V_{LS} - V_{RS}$, $\Delta \varepsilon = V_{LS} - V_{RD}$, $\Delta \varepsilon = V_{LD} - V_{RS}$, and $\Delta \varepsilon = V_{LD} - V_{RD}$, respectively. It should be noted that although there can be four Kondo peaks in the curve of the conductance as a function of the pseudospin splitting energy $\Delta \varepsilon$, there are at most two Kondo peaks in the curve of the conductance versus the voltage, e.g. $G_{LS}$ versus $V_{LS}$. When some of the four lead voltages have identical value, some Kondo peaks will overlap and then the number of the peaks can be reduced, as shown in figure 2. Figure 4(a) displays $G_{LS}$ versus $\Delta \varepsilon$, with $V_{LS} = 0.3$, $V_{RS} = 0.2$, and $V_{LD} = V_{RD} = 0$, in which four Kondo peaks clearly exhibit. Figure 4(b) shows $G_{LS}$ versus the voltage $V_{LS}$ by fixing $V_{RS} = 0.2$, $V_{LD} = 0$, and $V_{RD} = -0.1$ with different pseudospin splitting energy $\Delta \varepsilon$. Here, two Kondo peaks emerge. It is worth mentioning that the conductance $G_{LS}$ at $\Delta \varepsilon = -0.2$ is obviously larger than the other cases. This is due to the fact that when $\Delta \varepsilon = -0.2$, $\Delta \varepsilon = V_{LD} - V_{RD}$ is maintained, regardless of the voltage $V_{LS}$. This means that the Kondo resonance always occurs, so a very large conductance $G_{LS}$ can be observed at low temperatures.

2.2. The effect of the interdot tunneling

When the interdot tunneling coupling $t_c$ is considered, we can generalize the experimental results of [14]. Before the discussion of the conductance, we first analyze the cotunneling processes at $t_c \neq 0$. Figure 5(d) shows the change of the energy level of the DQD in the presence of $t_c$. When $t_c \neq 0$, the energy levels in the left and right QDs hybridize into molecular states. That is, $\epsilon_L$ and $\epsilon_R$ can be recombined into $\epsilon^\pm = \frac{\epsilon_L + \epsilon_R \pm \Delta \varepsilon}{2}$, which expands to the entire device at $\Delta \varepsilon = 0$ [8, 39, 40], where $\Delta E = \sqrt{\Delta \varepsilon^2 + 4t_c^2}$. Then, there are four kinds of cotunneling processes in the DQD (see figure 5(d)). (1) The electron originally occupying $\varepsilon^-$ tunnels to the lead RD (LD), and another electron at $V_{RD} + \Delta E$ ($V_{LD} + \Delta E$) in the lead LS (RS) tunnels to $\varepsilon^+$. (2) The electron at state $\varepsilon^+$ tunnels to the lead LD (RD), and another electron at $V_{LD} - \Delta E$ ($V_{RD} - \Delta E$) in the lead RS (LS) tunnels to $\varepsilon^-$. (3) The electron at the state $\varepsilon^+$ tunnels to the drain lead LD (RD), and another electron at $V_{LD}$ ($V_{RD}$) in the source lead LS (RS) tunnels to $\varepsilon^+$. (4) The electron at $\varepsilon^-$ tunnels to the lead LD (RD), and another electron at $V_{LD}$ ($V_{RD}$) in the lead LS (RS) tunnels to $\varepsilon^-$. Here, although the cotunneling processes may be similar to that discussed in [8], the conductance is totally different. In [8], the system is a serial DQD. When $t_c = 0$, since there is no transport coupling between the two QDs, $I$ and $dI/dV$ are exactly zero. In the present system, because each QD is connected to its own source and drain leads, $I$ and $dI/dV$ are nonzero, no matter $t_c = 0$ or $t_c \neq 0$.

Figure 5(a) shows the conductance $G_{LS}$ as a function of the voltage $V_{LS}$ by changing $V_{LS}$ and $V_{RS}$ simultaneously, i.e. $V_{LS} = V_{RS}$. For $t_c = 0$, the Kondo peaks are located at $V_{LS} = \pm \Delta \varepsilon$. When $t_c$ is increased, the two Kondo peaks move to $V_{LS} = \pm \Delta E$. Thus, they can emerge in larger $|V_{LS}|$ with increasing $t_c$. These two peaks correspond to the first and second kind of cotunneling processes as discussed in the above paragraph. In addition, another small Kondo peak and dip emerges at $V_{LS} = 0$, which is attributed to the third and fourth kind of cotunneling processes. Note that in the third and fourth kind of cotunneling processes, the original and final electrons are at the same molecular state. Thus, the Kondo peak and dip is always fixed around $V_{LS} = 0$. Figure 5(b) shows $G_{LS}$ as a function of $V_{LS}$ when only $V_{LS}$ is changed and $V_{LD} = V_{RS} = V_{RD} = 0$. At $t_c = 0$, there is only one Kondo peak at $V_{LS} = \Delta \varepsilon$, which is the pseudospin-resolved Kondo peak observed in the experiment of [14]. However, when $t_c$ is increased, this peak moves to $V_{LS} = \Delta E$. Furthermore, the Kondo peak at $V_{LS} = -\Delta E$ also emerges, and its height becomes higher and higher. The reason for this is that at $t_c \neq 0$, the electron at the molecular state $\varepsilon^-$ ($\varepsilon^+$) can tunnel to both left and right drain leads, and the electron in the left and right source leads can tunnel to the molecular state $\varepsilon^-$ ($\varepsilon^+$). This is different from $t_c = 0$, in which the electron at level $\epsilon_L$ ($\epsilon_R$) can only tunnel to one drain lead LD (RD). Additionally, a small peak and a small dip emerge around $V_{LS} = 0$ because of the third and fourth kinds of cotunneling processes. Figure 5(c) shows the dependence of $G_{LS}$ on temperature $T$. It can be clearly seen that with increasing $T$, the height of the Kondo peak becomes increasingly lower. At $T = 0.5$, all of the Kondo peaks disappear.
Next, we investigate the conductance \( G_{LS} \) as a function of the pseudospin splitting energy \( \Delta \varepsilon \) at different \( t_c \). In figure 6(d), the voltages are set to \( V_{LS} = 0.2 \) and \( V_{RS} = V_{LD} = V_{RD} = 0 \). At \( t_c = 0 \), there are two Kondo peaks at \( \Delta \varepsilon = 0 \) and \( \Delta \varepsilon = V_{LS} \). With increasing \( t_c \), the original peak at \( \Delta \varepsilon = V_{LS} \) moves toward \( \Delta \varepsilon = 0 \) and the height is decreased, as this Kondo peak is now located at \( \sqrt{\Delta \varepsilon^2 + 4t_c^2} = V_{LS} \). The other Kondo peak emerges at the symmetric place of the other side of \( \Delta \varepsilon \). In addition, the peak at \( \Delta \varepsilon = 0 \) broadens and the height is declined. If \( t_c \) is gradually increased, the height of the peak at \( \Delta \varepsilon = 0 \) is decreased. At the same time, the two peaks at the opposite sides of \( \Delta \varepsilon \) move toward \( \Delta \varepsilon = 0 \) and eventually mix together at \( \Delta \varepsilon = 0 \). Thus, there is only one broadening peak around \( \Delta \varepsilon = 0 \). If \( t_c \) is further increased, the height of this broadening peak decreases until this peak vanishes. This is attributed to the fact that the two quantum dots become a whole when \( t_c \) is considerably large. Neither the degeneracy of the pseudospin nor the Kondo effect exist. Figures 6(a)–(c) are the two-dimensional plots of the conductance \( G_{LS} \) versus \( \Delta \varepsilon \) and \( \bar{t} \), with \( t_c = 0 \), 0.07, and 0.2, respectively. The change of the color in figures 6(a)–(c) clearly shows the process discussed above. As a comparison, figure 6(e) shows \( G_{LS} \) as a function of the pseudospin splitting energy \( \Delta \varepsilon \) when \( V_{LS} = V_{RS} = 0.2 \) and \( V_{LD} = V_{RD} = 0 \). It is clear that at \( t_c = 0 \), except for the peak at \( \Delta \varepsilon = 0 \), there are two Kondo peaks at both sides of \( \Delta \varepsilon \). When \( t_c \) is increased, the peak at \( \Delta \varepsilon = 0 \) becomes lower and broader, and the peaks at both sides move toward \( \Delta \varepsilon = 0 \) and eventually mix together. If \( t_c \) is gradually increased, the last peak becomes lower till it disappears.

2.3. Pseudospin transport and pseudospin flipping current

In this subsection, we discuss the pseudospin transport in the DQD system. As we know, the direction of the real spin can be changed in the electron transport process. As a result, a steady spin current cannot be held easily. On the other hand, the measurement of the spin current is also difficult. Thus, it limits the development of the research field on the spin transport. The orbital Kondo effect, which is a pseudospin Kondo effect, can be regarded as the counterpart of the spin Kondo effect. The Kondo effect, whose emergence is initially related to the spin degree of freedom, can also be realized in the system with the orbital degree of freedom. This indicates that we can use the system, including the orbital degree of freedom, to study physical properties that are difficult to observe with the spin degree of freedom. In the DQD system, the current flow in the leads LS, RS, LD, and RD is easy to measure, which means that the pseudospin current is also easy to measure. Furthermore, the pseudospin flipping only occurs in the QDs, and its flipping strength is controllable and tunable. When the current flows in the leads, it cannot tunnel from the left side (LS and LD) to the right side (RS and RD), which indicates that the pseudospin current is conserved in the leads. Thus, it is possible and convenient to use the orbital degree of freedom to study the properties related to the spin degree of freedom.
It should be pointed out that, when $t_c \neq 0$, the currents in the leads LS, LD, RS, and RD may not be equal to each other, but they still satisfy the relation $I_{LS} + I_{RS} = I_{LD} + I_{RD}$ due to the electric current conservation. Here, we note that the positive direction of the current flows into the DQD for the source leads and goes out from the DQD for the drain leads. Besides, we introduce the pseudospin flipping current $I_t$, which describes the current from the right QD to the left one. The relation between $I_t$ and the four wire currents are $I_{LS} + I_t = I_{LD}$ and $I_{RS} - I_t = I_{RD}$. Thus, $I_t$ can be expressed as:

$$I_t = \frac{(I_{LD} - I_{RD}) - (I_{LS} - I_{RS})}{2} = \frac{I^{\text{spin}}_{LS/D} - I^{\text{spin}}_{RS/D}}{2},$$

where $I^{\text{spin}}_{S/D} = I_{LS/D} - I_{RS/D}$ is the pseudospin current in the source/drain lead. Figures 7(a) and (b) illustrate the pseudospin flipping current $I_t$ as a function of the voltage $V_{LS}$. In figure 7(a), only $V_{LS}$ is changed, and in figure 7(b), both $V_{LS}$ and $V_{RS}$ are changed with $V_{LS} = V_{RS}$. It is clear that when only $V_{LS}$ is changed, the pseudospin flipping current $I_t$ is considerable compared with the current in the four leads because the pseudospin-up chemical potential $eV_{LS}$ is not equal to the pseudospin-down potential $eV_{RS}$, i.e., a pseudospin bias $V^{\text{spin}}_{S} = V_{LS} - V_{RS}$ exists. On the other hand, when both $V_{LS}$ and $V_{RS}$ are changed, the pseudospin flipping current $I_t$ is negligible, because the pseudospin bias $V^{\text{spin}}_{S/D} = V_{LS/D} - V_{RS/D}$ is zero. These calculations demonstrate that if we deal with the pseudospin-resolved transport spectroscopy in the DQD system, a steady pseudospin current can be induced. The magnitude of this pseudospin current is not small and is particularly it is easy to control and measure.

Finally, to see the characteristics of the pseudospin flipping in the DQD more clearly, we calculate the flipping conductance, which is defined as

$$G_t(V_{LS}, V_{LD}, V_{RS}, V_{RD}) = \lim_{V \to 0} \frac{I_t\left(V_{LS} + \frac{V}{2}, V_{LD} - \frac{V}{2}, V_{RS}, V_{RD}\right)}{V},$$

(11)

Notice that in the above definition, only the left source and drain voltages are changed by $\pm V/2$. Figures 7(c) and (d) show $G_t$ as a function of $V_{LS}$, with $\Delta \mu = 0$ and 0.2, respectively. The results exhibit the following features: (1) regardless of whether $V_{RS}$ is changed with $V_{LS}$, the Kondo peaks and dips of $G_t$ emerge at $V_{LS} = 0$ and $V_{LS} = \pm \Delta \varepsilon$; (2) with increasing $t_c$, $G_t$ is enhanced in general; (3) the dips are much sharper when $V_{LS}$ is changed only, which also indicates that we can focus on the pseudospin-resolved transport spectroscopy when we study the pseudospin flipping current in the parallel DQD systems.

3. Conclusions

In this paper, we investigate the orbital Kondo effect in a parallel double quantum dot. When the interdot tunneling coupling $t_c$ is zero, we explain the pseudospin-resolved results...
Figure 7. (a) and (b) show the current in the leads LS, LD, RS, and RD, and the pseudospin flipping current $I_t$ as a function of $V_{LS}$ with the pseudospin splitting energy $\Delta \varepsilon = 0.2$ and $t_c = 0.2$. In (a), only $V_{LS}$ is changed; in (b), both $V_{LS}$ and $V_{RS}$ are changed with $V_{LS} = V_{RS}$. (c) and (d) show the pseudospin flipping conductance $G_t$ as a function of $V_{LS}$ with $\Delta \varepsilon = 0$ and $\Delta \varepsilon = 0.2$, respectively. All the unchanged source and drain voltages are set to zero, and the temperature stays $T = 0.001$, and $\bar{\varepsilon} = -5.0$.

observed in a recent experiment [14]. We find that three Kondo peaks and two Kondo peaks in the curve of the conductance exist versus the pseudospin splitting energy for the pseudospin-non-resolved case and the pseudospin-resolved case, respectively. When the interdot coupling $t_c$ is nonzero, the levels in the separated quantum dots can hybridize into the molecular levels, and new Kondo peaks emerge. In addition, the pseudospin flipping current and conductance are also studied, and both show the Kondo peaks and dips. We point out that the present pseudospin system has many advantages in comparison to the real spin system. In the pseudospin system, the chemical potential of each pseudospin component, the pseudospin splitting energy, and the coupling strength can be well controlled and tuned. Furthermore, the pseudospin current is conserved in the source and drain leads, and the pseudospin-up and pseudospin-down currents can individually be measured. Therefore, we believe that these results can be observed in the present technology.

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