The Nuclear g_{9/2} Shell - Comparison of our work with an old B.H. Flowers Paper

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Abstract

In an old paper [1] B.H. Flowers discussed calculations for odd A nuclei in the g_{9/2} shell. He finds when he varies the parameters of a certain interaction that the lowest energy state is always a seniority v=1 state with angular momentum J=9/2^+. However experiments show about half the states have J=9/2^+ lower and the other ones have J=7/2^+ lower. More recently we considered selected nuclei between Z=40 and Z=50. For lower odd N Zr isotopes we find J=7/2^+ comes below J=9/2^+ while for odd Z isotones near Z=50 the opposite is true i.e. J=9/2^+ comes below J=7/2^+. We justify using different interactions in the 2 regions by considering nuclear deformation near the Z=40 region. A plea to find missing energy levels is made.

1. Discussion

A slight paraphrase of the abstract of the paper of B.H. Flowers [1] I here shown:

“Wave functions and energy level diagrams for 3 identical particles in the g_{9/2} are calculated. The J=7/2^+ level of seniority three is shown never to be the ground state; this is in conflict with experimental findings and suggests a breakdown of the jj coupling approximation. “

Another slight paraphrase on the experimental situation:
“Now Goldhaber and Sunyar [2] report in about half the cases in which the J=9/2 (s=1) and J=7/2 (s=3) (of the g_{9/2}^n configuration for n=3,5,7) can be identified about half the J=7/2 level lies lower than the J=9/2 level.”

Flowers used an interaction V(r)=const. *exp(-(r/a)^2 . He notes the g_{9/2} radial wave function is of the form constant r^4 exp(-1/2 (r/a_0)^2) . He plots results as a function of the dimensionless parameter (a/a_0)^2 . He found that for g_{9/2}^n no matter what value this parameter assumes the J=9/2+ level came out lower than J=7/2+.

In more recent work by Escuderos and Zamick [3] much attention was given to their finding that for identical particles of the g_{9/2}^4 configuration there was a unique J=4 state with seniority 4; likewise J=6. This was a surprise because seniority was supposed not to be a good quantum number in the g_{9/2} shell. Only in f_{7/2} and below was seniority a good quantum number.

There is a second part of the paper that deals with g_{9/2}^n for odd n. This has not received as much attention but there are some good points to be made.

2. The g_{9/2} Model Space

In the work of Escuderos and Zamick [3] they consider nuclei consisting of proton holes relative to 100Sn and neutrons relative to 80Zr. We call this the g_{9/2} model space rather than the g_{9/2} shell because it is known the 2 extremes above are quite different. Most researchers including the present author are quite comfortable with employing the shell model for proton holes relative to 100Sn. But as was shown in Hartree-Fock calculations [4], 80 Zr is highly deformed-beta
is around 0.4. This was reinforced by Zheng and Zamick [5] who also considered excited n- particle-n-hole bands. This will play a key in explaining the observation by Flowers [1] that not all odd A in this region have J= 9/2± states. Loosely speaking we can say that 100Sn is a good closed shell whilst 80Zr is a bad one.

In the work of Escuderos and Zamick [3] single j (g9/2 ) calculations are performed for the following nuclei:

97Ag - 3 proton holes
95Rh – 5 proton holes
93Tc - 7 proton holes or 3 protons.
83Zr - 3 neutrons
85Zr – 5 neutrons
87Zr – 7 neutrons or 3 neutron holes

We first discuss experiment. We refer to Fig1 of ref [3.]

For 97 Ag, 95Rh and 93Tc the ground state has J=9/2 +. For the latter 2 nuclei the J=7/2+ states are over half an MeV above. Surprisingly the J= 7/2+ state has not been identified in 97 Ag, even to this day. But we are sure it is not the ground state.

In the Zr isotopes we must mention that there are 2 negative parity states below the positive parity ones- J=1/2− and 5/2−. This underscores the fact that 80Zr is not a good closed shell.

In 83Zr and 85 Zr the J=7/2+ state lies below J=9/2+ whilst in. 87 Zr J=9/2+ is lower.
In a shell model calculation with a fixed charge independent interaction and limited to a single j shell the spectra for n proton holes relative to $^{100}$Sn would be the same as that of n neutrons relative to $^{80}$Zr. In order to get different spectra one must use different interactions for the 2 cases.

Near $^{100}$Sn one can get the 2 body interaction matrix elements from experiment i.e. the spectrum of 98 Cd -a 2 proton hole system. We call this $V(^{98}$Cd). In Fig 4,5 and 6 get the robust result that with this interaction $J=9/2^+$ is calculated to be lower than $J=7/2^+$ for $^{97}$Cd, $^{95}$Rh and $^{93}$Tc, in agreement with experiment.

As just mentioned if one used $V(^{98}$Cd) for Zr isotopes we would get the spectra as for the above nuclei. To explain the Zr region we must use a drastically different interaction.

In ref [3] we noted that the quadrupole-quadrupole interaction Q.Q yields the result that $J=7/2^+$ comes below $J=9/2^+$. One can use a linear combination of Q.Q and a delta interaction to fit all nuclei ,those close to $^{100}$Sn and those close to $^{80}$Zr. In the $^{100}$Sn region one has much more delta while in the $^{80}$Zr region one needs much more Q.Q. The latter seems reasonable -in a deformed region Q.Q dominates over delta.

Another point of interest from Ref [3] is the energy splitting $E(21/2^+) – E(3/2^+)$ for n=3 and 5. Here $J=21/2$ is the highest angular momentum for $(g_{9/2})^3$ and $J= 3/2$ is the lowest. With a seniority conserving interaction this splitting is the same for n=3 and n=5. With a Q.Q interaction the splitting is equal but opposite for n=3 and n=5. Our calculations show that this splitting should be positive in $^{97}$Cd and $^{95}$Rh but positive for $^{83}$Zr and negative for $^{85}$Zr.
Unfortunately no J=3/2$^+$ states have been identified experimentally for any of the nuclei in mentioned here. We hope this work will stimulate a search for such states.

Although the Q.Q interaction correctly gives the J=7/2 J=9/2 splitting for $^{83}$Zr it is too much to ask a $(g_{9/2})^3$ to give a detailed spectrum. Other approaches such as that of Huttmeier et al. [6] are required.

Besides references to the second part of [3] we add those to the first part following ref[6].

Closing remarks:

Flowers [1] put his finger on an important problem-getting the right level ordering in the g$_{9/2}$ shell. He in particular focused on the J=9/2$^+$ and J=7/2$^+$ levels in odd A nuclei. We find that to solve this requires fairly drastic measures. This is mainly due to the fact that $^{100}$Sn is spherical but $^{80}$Zr is highly deformed. In g$_{9/2}$ model space one needs quite different interactions in the A=80 region than what one uses in the A=100 region.

We also hope that a surprising number of missing levels be found experimentally in the near future. This will help to sharpen our analysis of this region of the periodic table.
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