Inverse parameter identification of an anisotropic plasticity model for sheet metal

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Abstract. The increasing economic and ecological demands on the mobility sector require efforts to reduce resource consumption in both the production and utilization phases. The use of lightweight construction technologies can save material and increase energy efficiency during operation. Multi-material systems consisting of different materials and geometries are used to achieve weight reduction. Since conventional joining processes reach their limits in the connection of these components, new methods and technologies are necessary in order to be able to react versatilely to varying process and disturbance variables. For fundamental investigations of new possibilities in joining technology, numerical investigations are helpful to identify process parameters. To generate valid results, robust and efficient material models are developed which are adapted to the requirements of versatile joining technologies, for instance to the high plastic strains associated with self-piercing riveting. To describe the inherent strain-induced plastic orthotropy of sheet metal an anisotropic Hill-plasticity model is formulated. Tensile tests for different sheet orientations are conducted both experimentally and numerically to adjust the anisotropic material parameters by inverse parameter identification for aluminium EN AW-6014 and steel HCT590X. Then, the layer compression test is used to validate the model and the previously identified parameters.

1. Introduction
In order to achieve resource efficiency objectives also in the mobility sector, the use of lightweight constructions is a feasible option. One aspect of this is the use of multi-material systems consisting of materials with different mechanical properties and varying geometries as for example the sheet thickness. A central challenge of such components is the joining of the individual components, as conventional joining methods reach their limits because of unequal stiffness, thermal expansion and chemical incompatibilities. Therefore, new joining techniques and processes are required to face upcoming challenges in joining technology [1]. To cover the variety of joining capabilities, the methods must be versatile and able to adapt to process and disturbance variables. To set up a process route and for a better understanding of the process especially for future modifications, numerical investigations are essential. This requires increasingly more accurate and robust material models that especially for joining of sheet metal need to capture large elasto-plastic deformations together with the strain-induced plastic orthotropy of the sheets.
This anisotropy notably affects the following processing, such as bending, joining or the observable earing in deep drawing. Various anisotropy models for plasticity have been proposed in the past, see for instance [2] for a descriptive overview of Hill and Barlat models, where herein the classical Hill48 model is utilised to assess whether it is capable of describing the given materials. The necessary anisotropy parameters are determined from tensile tests with different sheet orientations by evaluating the Lankford or r-values.

The layer compression test (or stack compression test) can be used to characterise the material under biaxial loading and achieves higher plastic strains. This is beneficial to identify proper plastic hardening relations and especially suitable for usually thin sheet metal [3]. The stack of circular blanks is loaded in normal direction and enables the observation of the orthotropic plastic deformation in the longitudinal and transversal sheet directions. Several studies have already been conducted to analyse the layer compression test, such as the description of the experimental setup and evaluation in [3] and investigations regarding the factors of influence [4]. Moreover, current work concerns the simplification of the evaluation procedure [5] as well as the optimisation of the test setup [6]. Also numerical simulations of the experimental setup in finite element software have been performed such as [7] in LS-DYNA, or [4] with their in-house FEM-code I-form. Important challenges arise therein in the correct modelling of friction, the influence of manufacturing tolerances and the comparison to measured experimental results. Recent work on single layer sheet metal compression tests [6] highlights the influence of the friction state and also, among other things, compares the usage of liquid lubricant and solid PTFE foil. Parameter identification and inverse modelling were performed, for instance for bulk titanium alloy Ti-6Al-4V [8], for an aluminium alloy AA5182 and the deep drawing steel DC06 [7], and for 6060-aluminium and AZ31-magnesium [9]. In [10] the methodology has been extended to hot forming conditions, which has similarly been simulated in LS-DYNA [11].

In this paper material parameters are identified by inverse parameter identification of tensile tests also determining the Lankford values and validating the resulting model by the layer compression test. The experiments are carried out with two typical materials from the automotive industry with different mechanical properties. First, a galvanized dual-phase steel HCT590X is considered. This type of material is characterised by a high tensile strength and strong strain hardening. As second material an aluminium alloy EN AW-6014 is investigated, which is a precipitation hardening aluminium, magnesium and silicon alloy with high strength and energy absorption capability. Thus it is widely used for structural components in the automotive industry.

Section 2 briefly summarises the anisotropic plasticity model and its implementation in the commercial FEM software LS-DYNA MPP D R10.2.0. In section 3, the experimental and numerical setup of the tensile test is described that is used in section 4 to identify the plastic hardening and anisotropy parameters for both materials. For the validation of the determined parameters in section 6 the layer compression test is used as outlined in section 5.

2. Material modelling
The material model is implemented as a user-defined material (UMAT) into the commercial software LS-DYNA. Comparisons of the present implementation with the existing LS-DYNA MAT_122_3D for Hill48 [12], proved our UMAT to be accurate as well as robust and more flexible with respect to the hardening. An Updated Lagrangian formulation is used together with a hypoelastic based approach. With this approach, similar to [13] for ABAQUS, we can capture large deformations together with the related geometric nonlinearity. In this incremental framework the small strain increment $\Delta \varepsilon$ is computed from the velocity gradient [14] and provided as input for the user routine, leading to the following formulation, similar to [15] with FORGE2005. More details on the corresponding rate formulation can be found in the LS-DYNA theory manual [16] and in great detail in [17].
2.1. Elasto-plasticity

In a continuum mechanics framework elasto-plasticity can be described by the total strain energy \( \Psi \), where the contribution \( \Psi^{\text{hard}} \) describes the plastic hardening and is formulated in section 2.3.

\[
\Psi = \frac{1}{2} \kappa \text{tr}(\varepsilon^p)^2 + \mu |\varepsilon^{\text{dev}} - \varepsilon^p|^2 + \Psi^{\text{hard}}(\alpha)
\]  

(1)

In contrast to pure elasticity governed by the volumetric and deviatoric (dev) parts of the elastic strains \( \varepsilon^e \), the bulk modulus \( \kappa \) and the shear modulus \( \mu \), constitutive models for plasticity have to take the history dependency into account. The latter is described by the plastic hardening variable \( \alpha \) and the second order plastic strain tensor \( \varepsilon^p \).

2.2. Anisotropic Hill-yield function

The anisotropy for plasticity is introduced by the standard Hill48 anisotropic yield function [18] as outlined in [19] in tensor notation

\[
\Phi = ||T||_H - \sqrt{\frac{2}{3}} [\sigma_{y,0} - R] \leq 0
\]

with \( ||(\bullet)||_H \equiv \sqrt{\langle \bullet, H : \bullet \rangle} \)  

(2)

(3)

Eq. (2) is similar to classical von Mises or J2-plasticity besides the generalisation of the stress norm, where the deviatoric four order Hill tensor \( H \) is used instead of the deviatoric unit tensor. With \( H \) the material anisotropy, in this case a plastic orthotropy, is introduced, which can be represented as a fully symmetric tensor \( (H_{ijkl} = H_{klji} = H_{jikl} = H_{ijlk}) \). When the axes of orthotropy coincide with the Cartesian coordinate system the non-zero entries of \( H \) can be stated as follows, where 1, 2 and 3 describe the rolling, transverse and normal (thickness) direction, respectively.

\[
H_{1111} = \frac{2}{3} [G_{xx} + H_{xy}] = \frac{2}{3} h_{11}^2 ; \quad H_{1222} = -\frac{2}{3} H_{xy} = \frac{1}{3} [-h_{11}^2 - h_{22}^2 + h_{33}^2] \\
H_{2222} = \frac{2}{3} [H_{xy} + F_{yz}] = \frac{2}{3} h_{22}^2 ; \quad H_{2233} = -\frac{2}{3} F_{y,z} = \frac{1}{3} [h_{11}^2 - h_{22}^2 - h_{33}^2] \\
H_{3333} = \frac{2}{3} [F_{yz} + G_{zz}] = \frac{2}{3} h_{33}^2 ; \quad H_{3311} = -\frac{2}{3} G_{xx} = \frac{1}{3} [-h_{11}^2 + h_{22}^2 - h_{33}^2] \\
H_{1212} = \frac{N_{xy}}{3} = \frac{1}{6} h_{12}^2 ; \quad H_{2323} = \frac{L_{yz}}{3} = \frac{1}{6} h_{23}^2 ; \quad H_{3131} = \frac{M_{xz}}{3} = \frac{1}{6} h_{31}^2
\]

These entries can be computed from the Hill coefficients \( h_{\alpha\beta} \equiv \sigma_{y,0\alpha}/\sigma_{y,0} \) as described in section 4.1, which relate the yield stresses in direction \( \alpha \beta \) to a reference yield stress \( \sigma_{y,0} \) or alternatively from factors \( F_{yz}, G_{zz}, H_{xy}, M_{xz}, N_{xy}, L_{yz} \) [18], where indices have been added for clarity. Regardless, both can, for instance, be determined from the Lankford-/r-values for different rolling directions, see section 4.1, or directly from the yield stresses for different sheet orientations \( \theta \), see e. g. the comparison in [13].

2.3. Nonlinear isotropic hardening

The isotropic plastic hardening of both steel and aluminium is described by the following nonlinear hardening law in Eq. (4), which is a combination of linear hardening with the hardening modulus \( K \) and Voce-/saturation-type hardening. \( \Delta \sigma_{y,\infty} \) is the increment in the yield stress that will be approached asymptotically adjustable by the exponent \( \omega \).

\[
R(\alpha) = -\frac{\partial \Psi^{\text{hard}}}{\partial \alpha} = -K \alpha - \Delta \sigma_{y,\infty} [1 - \exp(-\omega \alpha)]
\]

(4)

2.4. Evolution equations and algorithmic implementation

Starting from the anisotropic yield function in Eq. (2) the evolution equations for the internal variables, namely the plastic hardening variable \( \alpha \) and the plastic strain tensor \( \varepsilon^p \) are stated with the Lagrange multiplier \( \lambda^p \) as

\[
\dot{\alpha} = \sqrt{\frac{2}{3}} \lambda^p, \quad \dot{\varepsilon}^p = \lambda^p n \quad \text{with} \quad n(T) = \frac{H : T}{||T||_H}
\]

(5)
The evolution equations are integrated by an implicit Euler-Backward scheme. However, we need to pay attention to the fact that the evolution direction \( n \) is stress-dependent and changes during the local iterations due to the anisotropy. This complicates the computation of the stress tensor \( T \) and also the derivation of the local and global consistent tangent operators (not shown here for brevity) with the unit tensors \( I \) (fourth order) and \( I \) (second order).

\[
T = \left[ I + \frac{2\mu \Delta \lambda_p}{\sqrt{2/3} (\sigma_y - R)} \right]^{-1} : T^t \quad \text{with} \quad T^t = \kappa \text{trace}(\varepsilon) I + 2\mu \left[ \varepsilon^{\text{dev}} - \varepsilon_p^n \right]
\]

To sum up, the UMAT receives the current strain increment \( \Delta \varepsilon \) and from the last load or pseudo time step the history \( \{ \alpha_n, \varepsilon^n_\alpha \} \) and stress \( T_n \). These are rotated (LS-DYNA option \( AOPT=2 \)) into the local material coordinate system and used to update the stress, history as well as the stress-strain tangent, where the latter is internally denoted as \( es \). Due to the nonlinearity in the hardening relation and the anisotropy, the unknown plastic Lagrange multiplier increment \( \Delta \lambda_p \) is determined iteratively by a full Newton-Raphson scheme to satisfy the yield condition in Eq. (2). To directly implement the above tensor-based algorithm in the LS-DYNA-Fortran90 routines, a tensor toolbox for Fortran [20] is used.

3. Tensile test: experimental and numerical setup
The tensile test setup shown in figure 1 combines the experimental and numerical design and is carried out according to DIN 6892-1 [21]. It is the most commonly used method for material characterisation in sheet metal forming having the distinction of a simple test setup and sample preparation. To investigate the anisotropy of the sheet material, the samples are extracted under 0°, 45° and 90° to the rolling direction by laser cutting. In addition, the test setup is frictionless and an uniaxial stress state is induced into the specimen. Depending on the material characteristics, high plastic strains can be achieved, however they can only be evaluated reliably as long as the strain state remains homogeneous. For recording the experimental data, a GOM Aramis is used, which is an optical measuring system. Since it calculates the strain on the basis of images, an area of a stochastic pattern on the specimens is required.

![Figure 1. Tensile test. Experimental and numerical setup.](image)

The geometry is taken from [21] as the A50 specimen discretised with 4 elements in the thickness direction. Only the shown free part with a length of 84 mm is modelled, where the left end is fully constrained. On the right the load is prescribed as a Dirichlet boundary condition \( u_x \) with constrained contraction, where \( u \) describes the displacement vector. In the parallel part of the specimen a structured mesh is chosen with 16 elements in \( y \)-direction and a biased longitudinal refinement, focussing the elements in the centre part (element size (0.26...1.30) mm). With the use of the LS-DYNA element formulation \( ELMFORM=1 \) in the implicit solver and a structured mesh, the necking takes place in the middle of the specimen.

4. Inverse parameter identification
For steel a Young’s modulus \( E = 210000 \) MPa and Poisson’s ratio \( \nu = 0.3 \) are chosen, whereas the elastic parameters for aluminium are taken as \( E = 63000 \) MPa and \( \nu = 0.34 \). The remaining ten parameters for plasticity and its anisotropy are listed in table 1, where the hardening
parameters are determined by inverse parameter identification in the subsequent sections and the anisotropy coefficients are computed from the Lankford values.

4.1. Determination of anisotropy coefficients

For each material the following three Lankford values are evaluated from the tensile tests, at an elongation of 10 mm (20 % strain)

\[
\begin{align*}
\text{steel: } & r_0 = (0.8232 \pm 0.0077) \quad r_{45} = (0.8958 \pm 0.0172) \quad r_{90} = (0.9888 \pm 0.0030) \\
\text{aluminium: } & r_0 = (0.8060 \pm 0.0118) \quad r_{45} = (0.4637 \pm 0.0063) \quad r_{90} = (0.6105 \pm 0.0248)
\end{align*}
\]  

(7)  (8)

With these, the Hill coefficients are computed from [22] and are listed in table 1.

\[
\begin{align*}
h_{11} & = 1 \\
h_{12} & = T_{xy} = h_{33} \sqrt{\frac{1}{2r_{45}+1}} \\
h_{22} & = \sqrt{\frac{r_{90}[r_0+1]}{r_{90}[r_0+1]}} \\
h_{33} & = \sqrt{\frac{r_{90}[r_0+1]}{r_0+r_{90}}}
\end{align*}
\]

Herein, the convention \((h_{11} = 1)\)\(\approx(H_{xy} + G_{xx} = 1)\) is used, leading to the known relations between the Hill coefficients and the Lankford values. Other definitions such as \((h_{33} = 1)\)\(\approx(F_{yz}+G_{xz} = 1)\) (as in the LS-DYNA internal Hill48 model) lead to different equations, but can be derived from instance from [23].

The out-of-plane anisotropy coefficients \(h_{23}\) and \(h_{31}\) are assumed to be identical to the isotropic case (definition for isotropy: normal components \(h_{ii} = 1\), shear components \(h_{ij} = 1/\sqrt{3}\)) as in [13], also because they would be challenging to determine experimentally.

4.2. Identification of plastic hardening parameters

LS-OPT is an optimisation software from LS-DYNA and used to identify the first sets St-I and Al-I of plasticity parameters by matching the experimental and numerical results for the tensile test along the rolling direction \(\theta = 0^\circ\). Because the necking shall also be considered, we use the already computed anisotropic plasticity parameters from section 4.1, which should improve the simulation of the triaxial stress state during necking. In LS-OPT a metamodel-based optimisation strategy is used with a domain reduction (SRSM with default settings), thus initially larger ranges for the parameters can be given, which are gradually shrunk around the currently best parameter set [24]. Initial parameters and ranges are chosen as follows for steel: \(\sigma_{y,0} = 400 \text{ MPa} \in (100...500) \text{ MPa}, \; K = 250 \text{ MPa} \in (50...800) \text{ MPa}, \; \Delta\sigma_{y,\infty} = 250 \text{ MPa} \in (0...600) \text{ MPa}, \; \omega = 20 \in (0...50); \) and for aluminium: \(\sigma_{y,0} = 150 \text{ MPa} \in (100...200) \text{ MPa}, \; K = 100 \text{ MPa} \in (50...300) \text{ MPa}, \; \Delta\sigma_{y,\infty} = 150 \text{ MPa} \in (0...300) \text{ MPa}, \; \omega = 10 \in (0...30). \) To avoid an "overall best fit" character of overestimating the experimental results in some areas while underestimating them in the remaining parts, the optimisation only considers displacements up to 8 mm for steel and up to 11 mm for aluminium for the identification of the plastic parameters.

The comparison of the load-displacement responses in figure 2 indicates that the used saturation-type hardening law in section 2.3 is sufficient to approximate the observed material response up to necking. It can even partly capture the force decrease due to geometric softening and the saturation of the plastic hardening.

Table 1 provides now all necessary parameters to describe the plastic hardening as well as the related anisotropy. Usually a fine tuning of the entire parameter set within the experimental admissible range would be advantageous at this point to consider the data from the remaining sheet orientations. However, as it is well known for plasticity [2] and sheet metal [25], the Hill48 anisotropy formulation shows weaknesses in describing the stress and strain anisotropy simultaneously. Because we herein identified the anisotropy coefficients from the Lankford values, which are determined from the anisotropic strains, we expect a better description of the anisotropic deformation, as later shown in figure 4 for the layer compression test. Consequently, we put up with the false depiction of the stress anisotropy with an error of up to 5 % for the different sheet orientations in the tensile test.
Table 1. Identified hardening and anisotropy parameters.

| Behaviour   | Symbol | Description            | steel HCT590X | EN AW-6014 | Unit |
|-------------|--------|------------------------|---------------|------------|------|
|             |        |                        | set St-I      | set Al-I   |      |
| plasticity  | $\sigma_y,0$ | Yield stress          | 391.183       | 141.394    | MPa  |
|             | $K$    | Hardening modulus      | 677.777       | 220.602    | MPa  |
|             | $\Delta\sigma_y,\infty$ | Yield stress increment | 217.928       | 111.986    | MPa  |
|             | $\omega$ | Plastic saturation     | 47.0388       | 26.17      | –    |
| anisotropy  | $h_{11}$ | Hill coeff. longitudinal | 1            | –          | –    |
|             | $h_{22}$ | Hill coeff. transversal | 1.0494       | 0.9216     | –    |
|             | $h_{33}$ | Hill coeff. in thickness | 0.9975       | 0.8823     | –    |
|             | $h_{12}$ | Hill coeff. shear 12   | 0.5970        | 0.6355     | –    |
|             | $h_{23} = h_{31}$ | Hill coeff. shear 23, 31 | $\frac{1}{\sqrt{3}} \approx 0.5774$ | –         | –    |

Figure 2. Load-displacement diagram for $\theta = 0^\circ$. Comparison of experimental (—) and numerical (---) results for parameter set St-I and Al-I, respectively. The experimental results also contain the experimental scatter as blue shaded area around the mean value.

5. Layer compression test: experimental and numerical setup
The layer compression test shown in figure 3 is an adaptation to the conventional compression test based on DIN 50105 [3]. The test setup is designed to analyse the material behaviour of sheet materials in a (deviatoric) biaxial stress state. In order to examine the material, plates are cut out of the sheet plane by laser cutting and stacked above each other. Here, the rolling direction of the specimens must be considered during alignment of the plates. Size and number of the specimen depend on the initial sheet thickness and must obtain a height-diameter ratio of at least one, see figure 3. During the investigation the specimen are compressed by 50%. To reduce the friction between the upsetting tool and the stack a Teflon (PTFE) foil is inserted between both. The plastic strain of the specimen is again recorded with the optical measurement system GOM Aramis. With the layer compression test plastic strains up to 0.7 can be achieved frequently depending on the material characteristics. A disadvantage of the presented test setup is the elaborate sample preparation. The elastic press frame deflection must be taken into account and compensated when conducting the tests and analysing the results.
The numerical setup is depicted on the left in figure 3 and is directly based on the LS-DYNA model by [7]. The stack with different number of layers and sheet thicknesses as listed in figure 3 is discretised as shown in figures 3 and 4. Both the upper and lower tool are modelled as rigid bodies and discretised by shell elements to introduce the frictional contact with a static friction coefficient of $FS=0.01$. The latter choice was identified in [7] as suitable for their identical setup with different specimen materials. Further details on the setup can be found in [7], where the loading has also been computed with an explicit solver, whereas the elastic springback uses an implicit computation. In contrast to the tensile test in section 3, the explicit solver is used here for loading to keep the established contact setup from [7], which has proven robust.

6. Model validation

The previously determined parameter sets in table 1 are eventually validated by means of the layer compression test. The material model is equipped with set St-I and Al-I and applied to the test setup in section 5 rendering the results shown in figure 4 for the system response and the deformation of the middle specimen simultaneously. For both the numerical (•) and experimental (□) test, the final state of the stack (height reduction after elastic springback over the maximum process force) is indicated in the force-displacement response.

For the steel HCT590X the deformations of the middle specimen in figure 4a are matched well. In contrast, the elliptical shape of the aluminium specimen in figure 4b is underestimated in transversal direction, but overestimated in the rolling direction. This could be adjusted with the biaxial anisotropy, which can be associated with the ratio between $r_0$ and $r_{90}$ [6]. Moreover, the Hill48 model is known to be not well suited for aluminium, compare [25].
7. Summary and outlook

For an anisotropic finite plasticity material model, parameters have successfully been identified and validated. The classical Hill48-model proved sufficient to capture yielding in the sheet thickness direction and adequately describe the in-plane plastic deformations. Based on the results given in section 6, we can identify further challenges that need to be considered in detail. The plasticity parameters have been determined from uniaxial tensile tests in section 4. However, in the layer compression test a quasi-biaxial loading is present, which might affect the plastic behaviour and the flow curve as indicated in [7], when comparing different experimental results as basis for the simulation of the layer compression test. Section 4.2 emphasises again the inability of the standard Hill48 model to simultaneously capture the present anisotropy in the stress and deformation. More suitable anisotropy formulations such as Barlat91 [26] could be applied to resolve this limitation [2] and also improve the representation of aluminium. Fitting the plastic hardening parameters to the results from the layer compression test would also enable a comparison between the flow curve in tension and compression. Differences in the results for these load states can be attributed to process-induced damage that will be introduced later by a coupled plasticity-damage model. The latter will require a novel user-defined material, where the presented implementation marks a first important step towards this goal.

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