Line spectrum extraction method based on hidden Markov model

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To solve the difficult problem of line spectrum detection with a low signal-to-noise ratio, a line spectrum extraction algorithm based on a hidden Markov model (HMM) is proposed. The forward–backward algorithm was used to track single and multiple spectral lines in a lofargram, and a new algorithm for the state transition probability matrix was proposed to handle the large amount of HMM calculation and the uncertainty difficulty in processing real signals. Finally, a median fitting algorithm was used to correct the few outlier points in the line spectrum estimation process. Simulation and sea trial data showed that the algorithm had good line spectrum tracking ability for a low signal-to-noise ratio.

Introduction: The radiated noise signals [1] of underwater targets often contain rich components of line spectra. These line spectra are usually caused by mechanical and propeller noise [2], so they are rich in target information.

The commonly used line spectrum extraction methods [3–5] often make use of the fact that the spectral level of a line spectrum is higher than that of a continuous spectrum by 10 dB or more [6]. However, for the case of weak targets, this method performs poorly. Because the line spectrum can be submerged in the noise background, its spectral level may be lower than that of the continuous spectrum. Li et al.’s [7] proposal of a fusion method to extract frequency features proved to be an effective algorithm. Paris et al. [8] proposed a series of methods for applying a hidden Markov model (HMM) to model line spectrum extraction, but the amount of calculation was large and the probability of state transition was difficult to determine when processing actual signals. To solve the problem of the difficulty of extracting the line spectrum under a low signal-to-noise ratio, the large amount of calculation of the HMM algorithm and the difficulty of determining the probability of state transition, a line spectrum extraction method based on an HMM is proposed.

Basic theory of HMM:

HMM expression: The HMM [9, 10] algorithm is a relatively mature algorithm that has been studied for many years. The state sequence of an HMM can be described as \( X = (x_1, x_2, x_3, \ldots, x_n) \), and the observation sequence is \( Y = (y_1, y_2, y_3, \ldots, y_n) \), where \( x_k \) is the frequency number of the lofargram and \( y_k \) can usually be expressed by the power of the lofargram; i.e., \( y_k = P_x \).

The HMM model is usually represented by a triple \( \lambda = (\pi, A, B) \), where \( A \) is the state transition matrix, \( A = (a_{ij}(g))_{M \times M} \) is the number of frequency points, and \( a_{ij}(g) \) is

\[
a_{ij}(g) = \Pr (x_k = g | x_{k-1} = i) = e^{-\frac{1}{2} \| x_k - Hx_{k-1} \|^2_2},
\]

where \( g \) is the frequency number at the current moment, \( e \) is the frequency number at the previous moment, \( c \) is a normalization factor, \( R \) is the covariance matrix that determines the frequency distribution of the line spectrum, \( H \) is the state matrix, \( \det \) indicates a determinant, and \( \| x_k - x_{k-1} \|^2_2 \) represents the Mahalanobis distance.

\( B \) is the observation probability matrix, \( B = (b_{ij})_{k \times M} \), where

\[
b_{ij}(y_k) = \frac{L \left( \frac{y_k - \mu_i}{\sigma_i} \right)}{\sum_{m=0}^{M-1} L \left( \frac{y_k - \mu_m}{\sigma_m} \right)},
\]

\( \xi \) is the signal-to-noise ratio at time \( k \), \( \sigma \) is the standard deviation of noise, and \( L \) represents the Bessel function. However, under normal circumstances, the signal-to-noise ratio is unknown, so \( b_{ij} \) can be expressed as

\[
b_{ij}(y_k) = \frac{P_{yk}}{\sum_{m=0}^{M-1} P_{ym}},
\]

where \( P_{yk} \) represents the amplitude of the \( j \)th frequency at time \( k \).

Forward–backward algorithm: The forward–backward (FB) algorithm is a local optimization algorithm that is the maximum posterior probability estimation of the state. The FB algorithm can therefore use the observation data \( Y \) to estimate the possible state \( \hat{x}_k \) of the line spectrum, expressed as

\[
\hat{x}_k = \arg \max_{i \in I_s} \left\{ \Pr (x_k = i | Y_k) \right\}.
\]

The FB algorithm can implement the recursive calculation in the forward and backward directions using \( a \) and \( b \), respectively. The forward recursion formula can be expressed as

\[
\left\{ \begin{array}{l}
\tilde{a}_k (j) = b_{j}(y_k) \sum_{i=0}^{M-1} a_{ij} \tilde{a}_{i-1}(i), \\
a_{ik}^* = \Pr (x_k = j | Y_k) = \frac{\tilde{a}_k (j)}{\sum_{i=0}^{M-1} \tilde{a}_k (i)}.
\end{array} \right.
\]

Similarly, when there are multiple spectral lines, the forward recursion formula is

\[
\tilde{b}_k (j) = \sum_{i=0}^{M-1} b_{ji} \beta_{i-1}(i).
\]

Then, according to (5) and (6), the FB algorithm can be expressed as

\[
\eta_k (j) = \frac{a_{ik}^* \beta_{i-1}(j)}{\sum_{i=0}^{M-1} a_{ik}^* \beta_{i-1}(j)}.
\]

The line spectrum estimated according to the FB algorithm is

\[
\hat{x}_k = \arg \max_{j \in I_s} \left\{ \eta_k (j) \right\}.
\]

Similarly, when there are multiple spectral lines, the forward recursion formula of the HMM algorithm evolves into

\[
a_{ik}^* (j) = b_{ij}(y_k) \sum_{j=0}^{M-1} a_{ij} \beta_{i-1}^* (j),
\]

where \( k = 2, 3, \ldots, K \). The backward recursion formula is

\[
\beta_{i-1}^* (j) = \sum_{j=0}^{M-1} \beta_{ii}^* (j) \beta_{i+1}^* (j),
\]

where \( k = K-1, K-2, \ldots, 1 \).

After normalization, the equation becomes

\[
\left\{ \begin{array}{l}
\tilde{a}_{ik}^* (j) = \frac{a_{ik}^* (j)}{\sum_{i=0}^{M-1} a_{ik}^* (j)}, \\
\tilde{b}_{i+1}^* (j) = \frac{\beta_{ii} (j)}{\sum_{i=0}^{M-1} \beta_{i}^* (j)}.
\end{array} \right.
\]

At the same time, to eliminate the coupling between the forward and backward probabilities, their weights must be obtained:

\[
\tilde{a}_{ik}^* (j) = \frac{1}{I_{r=1,p=1} \left[ 1 - a_{ik}^* (j) \right]},
\]

\[
\tilde{b}_{i+1}^* (j) = \frac{I_{r=1,p=1} \left[ 1 - B_{ik}^* (j) \right]}{B_{ik}^* (j)}.
\]
In Equation (12), \( \alpha_{t}^{\omega,j} \) and \( \beta_{t}^{\omega,j} \) are the weighting factors of the forward and backward probabilities, respectively. To further eliminate the mutual influence of multiple spectral lines, the state probability around the \( L \)th spectral line must be integrated to obtain

\[
\begin{align*}
\alpha_{t}^{\omega,j} (i) &= \prod_{r=1}^{L} \left[ 1 - \sum_{s=t-r+1}^{t-r+j} \alpha_{r}^{\omega,s} (j) \right] \\
\beta_{t}^{\omega,j} (i) &= \prod_{r=1}^{L} \left[ 1 - \sum_{s=t-r+j+1}^{t-r+1} \beta_{r}^{\omega,s} (j) \right],
\end{align*}
\]

where \( v_{a} \) and \( v_{b} \) are the numbers of the frequency points that must be integrated around the \( L \)th spectral line in the forward and backward recursive algorithms. Substituting Equation (13) into Equation (11), the equation becomes

\[
\begin{align*}
\alpha_{t}^{\omega,j} (i) &= \frac{\alpha_{t}^{\omega,j} (i) \alpha_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i)}{\sum_{j=0}^{M-1} \alpha_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i)}, \\
\beta_{t}^{\omega,j} (i) &= \frac{\beta_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i) \alpha_{t}^{\omega,j} (i) \alpha_{t}^{\omega,j} (i)}{\sum_{j=0}^{M-1} \beta_{t}^{\omega,j} (i) \alpha_{t}^{\omega,j} (i) \alpha_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i)},
\end{align*}
\]

so the FB algorithm can finally be expressed as

\[
\eta_{t}^{0} (i) = \frac{\alpha_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i)}{\sum_{j=0}^{M-1} \alpha_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i) \beta_{t}^{\omega,j} (i)},
\]

The estimated value of the \( L \)th spectral line is

\[
\hat{x}_{t} = \arg \max_{i \in \mathbb{N}} \left\{ \eta_{t}^{0} (i) \right\}.
\]
and the signal-to-noise ratio was 1 s, the total duration was 500 s, the noise was Gaussian white noise, including three spectral lines at frequencies of 100, 400, and 700 Hz.

Multiple spectral lines: The case of multiple spectral lines was then performed well even for a line spectrum with weak energy.

Verification of sea trial data: The performance of the algorithm was verified with data received from the sea trial test. Because the actual processing required the filtering of the algorithm, only part of the frequency band was extracted. Figure 5a shows the spectrogram of the first segment of the processed data. Only the line spectrum in the 100–400 Hz frequency band was studied, and the time duration of the data was 100 s. The figure shows many spectral lines, including some indistinct, weak spectral lines, e.g. at 332 Hz. Figure 5b shows the spectrogram after processing with the algorithm proposed in this paper. The spectral lines were extracted mostly intact, including the weak spectral line at 332 Hz that was successfully extracted after the algorithm processing.

Figures 6a and 6b show the original spectrogram of the second segment of data and the line spectrum extracted with the proposed algorithm, respectively. As can be seen from the figures, the proposed algorithm performed well in extracting the line spectrum. The algorithm performed well even for a line spectrum with weak energy.

Summary: To address the difficulty of line spectrum extraction under a low signal-to-noise ratio, an improved HMM-based line spectrum extraction algorithm is proposed. By improving the calculation method of the state transition probability and the FB recursion calculation, the proposed algorithm greatly reduced the amount of calculation and improved the practicality of the algorithm. To solve the problem of discrete outliers that tended to appear in line spectrum estimates at low signal-to-noise ratio, a median fitting method was proposed that was validated with simulation and sea trial data. The line spectrum extraction algorithm still performs well when the signal-to-noise ratio is as low as −22 dB.

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