**Tensile Instability and Artificial Stresses in Impact Problems in SPH**

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**Abstract.** The smooth particle hydrodynamics (SPH) is a meshless computational technique that is popular in the modeling of impact and penetration problems. However, SPH is liable to a tensile instability that manifests itself as a bunching of nodes and formation of artificial voids and no generally accepted formulation exists to counter this instability. We examine the performance of two methods that have been proposed to deal with the tensile instability— the Monaghan artificial stresses and the Godunov-type SPH. The impact and penetration of 0.5 cm radii steel spheres on 2 mm thick aluminium plate at 3.1 km/s is chosen for comparison. We show that the artificial void formation in St-Al impact is suppressed but not eliminated by using Monaghan stresses while the void formation is entirely eliminated by using Godunov-type formulation of SPH that was proposed by Parshikov and Medin.

1. Introduction

The meshless Smooth Particle Hydrodynamics (SPH) technique is widely used to model high-velocity impact and penetration problems since it handles material deformation and erosion without needing a special logic, unlike mesh-based Eulerian or Lagrangian codes [1]. However, the SPH suffers from a numerical instability in tension and consequently the regions subject to large tension exhibit 'numerical fracture' and artificial voids [2]. The underlying cause is the lack of formal consistency in SPH: it can not reproduce exactly any class of functions on a defined set of points. As a consequence, a numerical clumping instability manifests itself when nodes are mutually attracted. Essentially, the SPH kernel function $W$ is unable to keep the nodes apart once they are sufficiently close to one another. This can be a significant concern in impact and penetration problems since the elastic dynamics involves appreciable attractive forces. The instability is much less pronounced in purely hydrodynamic flows. There is no universally accepted technique for countering this instability. In this paper, we compare and contrast two different stabilizing methods that have been proposed and we apply them to a high velocity impact problem.

2. Tensile Instability

The SPH solves the continuum equations of motion by representing the continuum as a set of co-moving and disconnected nodes [3]. The nodes have mass and carry the relevant fields such as velocities, density, pressure, temperature, stresses and strains. The nodes have overlapping regions of influence that leads to node-node interaction. The equation of motion for the nodes
are obtained by discretization and interpolation. A kernel function \( W \) with required properties handles the interpolation. As the nodes are disconnected, the SPH does not require a special logic to handle topological changes that might occur in a penetration event. The discretization procedure yields the following set of equations [1]:

\[
\frac{d\rho_i}{dt} = \rho_i \sum_j \frac{m_j}{\rho_j} (v_i - v_j) \cdot \nabla W_{ij}
\]
\[
\frac{dv_i}{dt} = -\sum_j m_j \left( \frac{\sigma_i}{\rho_i^2} + \frac{\sigma_j}{\rho_j^2} + \Pi_{ij} \right) \cdot \nabla W_{ij}
\]
\[
\frac{dE_i}{dt} = -\sum_j m_j (v_i - v_j) \cdot \left( \frac{\sigma_i}{\rho_i^2} + \frac{1}{2} \Pi_{ij} \right) \cdot \nabla W_{ij}
\]

The summations run over neighboring particles, the neighborhood being defined by the support of \( W \). Here, \( v_i \) is the velocity, \( \rho_i \) is the density, \( E_i \) is the internal energy per unit mass, and \( \sigma_i \) is the stress tensor associated with the \( i \)th node. \( \Pi_{ij} \) is an artificial viscous pressure that is required to control unphysical oscillations around shocks. The kernel function \( W_{ij} = W(|x_i - x_j|, h) \) depends upon the smoothing length \( h \) that measures the zone of influence of a SPH node. For the cubic spline kernel used in this work, the kernel vanishes for \(|x_i - x_j| > 2h| [1,3] \) where \( x_i, x_j \) are the positions of \( i \)th and \( j \)th nodes resp. The smoothing length \( h \) is related to the average inter-node spacing \( d \) by \( h_{\text{fac}} = h/d \). The parameter \( h_{\text{fac}} \) is important for the stability of SPH and should be between 1 and 2 [4]. The smoothing length \( h \) may vary spatially and in that case we speak of \( h_{ij} \)–the smoothing length associated with the node \( i \) and \( h \) in above equations is replaced by \( h_{ij} = (h_i + h_j)/2 \).

This set of SPH equations is liable to the tensile instability. An example may be seen in the high-velocity impact of mild steel spheres of 0.5 cm radii onto 0.2 cm thick aluminium plate at 3.1 km/s (St-Al impact) [1,5] (fig. 1 shows the initial setup of the nodes). The fig. 2 shows target and projectile 8 \( \mu \)s post-impact. Voids are seen between the target and the front part of the projectile and also inside the projectile. These voids are not seen in the free-Lagrange simulations of the impact that were done by Howell and Ball [5] and thus may be a numerical artifact.

![Figure 1.](image1.png)  
**Figure 1.** Geometry and the initial node placement for St-Al impact. The circular projectile (red nodes), on the left, has a radius of 0.5 cm. On the right is the target plate (green nodes) having a thickness of 0.2 cm.

![Figure 2.](image2.png)  
**Figure 2.** The St-Al impact by conventional SPH showing void formation and tensile instability. The projectile is shown in red and target in green. The distances are in cm. The configuration is shown 8 \( \mu \)s post-impact at 3.1 km/s.
3. Monaghan Stresses

Monaghan [2] proposed to deal with the tensile instability by adding certain artificial stresses $R_f i j$ to the momentum equation:

$$\frac{d v_i}{d t} = - \sum_j m_j (\frac{\sigma_i}{\rho_i^2} + \frac{\sigma_j}{\rho_j^2} + \Pi_{ij} + R_f i j) \cdot \nabla W_{ij}$$

(2)

where

$$R = -0.15 \frac{\sigma_i + \sigma_j}{\rho_i^2 + \rho_j^2}$$

and

$$f_{ij} = \left[ W(|x_i - x_j|, h_{ij}) \right]^4.$$  

Since $h_{ij}/h_{fac}$ is just the average nodal spacing at the neighbourhood of particles $i, j$, the term $f_{ij}$ expresses a force that increases as the particle separation decreases. As it seeks to correct a numerical instability, the $f_{ij}$ is expressed in terms of the kernel function.

The Monaghan stress have been shown to work for a variety of low-velocity problems [2] but have not been tried on high-velocity impact and penetration problem. The fig. 3 shows the St-Al impact configuration produced using the Monaghan stresses. The voids have reduced in size but they are still very prominent. Thus the Monaghan stresses do not eliminate tensile instability in the high-velocity impacts.

4. Godunov-type SPH

Implementation of Godunov-type scheme into SPH [6] (CON) offers an alternative way to tackle the tensile instability. CON casts the problem of computing the derivatives $\nabla v, \nabla \sigma$, that occur in the momentum and energy balance equations of SPH, into the solution of a Riemann problem [7]. The Riemann problem directly solves the conservation equations at a discontinuity that is supposed to lie between two given nodes. The solution of the Riemann problem yields the values of hydrodynamic fields at the contact surface. In conventional SPH, the field values at the contact surface are just the mean values of the fields at interacting nodes. The CON procedure does not require artificial viscosity. Godunov-type methods are commonplace for gas dynamic applications. However, the applications to materials with strength has been rare, due to increased complexity of formulation. Shown below is the St-Al impact with CON. In figure 4, there are no visible voids, either in projectile front or inside. Thus the tensile instability in SPH is dealt better with CON than with the Monaghan artificial stresses at high impact velocities. CON also offers freedom from setting arbitrary artificial viscosity parameters, a subject of some controversy in conventional SPH [8].
The absence of visible tensile instability in CON simulation may merely indicate a slower growth of instability that has not had sufficient time to get itself manifest in this particular case. Further studies with CON on different impact scenarios with different dimensions of projectile and target and impact velocities are needed to definitively rule out tensile instability in CON.

5. Conclusion
Impact and penetration of 0.5 cm radii steel spheres on 2 mm thick Aluminium plate at 3.1 km/s (St-Al impact) has been studied through meshless smooth particle hydrodynamics (SPH) technique. Conventional SPH is liable to a tensile instability that manifests itself as a bunching of nodes and formation of artificial voids. The St-Al impact as modeled through conventional SPH shows evidences of tensile instability. Monaghan proposed adding certain artificial stresses to SPH equations to counter tensile instability. We show that the artificial void formation in St-Al impact is suppressed but not eliminated by using Monaghan stresses while the void formation is entirely eliminated by using an alternative Godunov-type formulation of SPH that has been proposed by Parshikov and Medin. More studies with different impact scenarios are required to definitively rule out the presence or absence of tensile instability in the Godunov-type SPH formulation.

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