Sub-galactic scaling relations between X-ray luminosity, star-formation rate, and stellar mass

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ABSTRACT

X-ray luminosity ($L_X$) originating from high-mass X-ray binaries (HMXBs) is tightly correlated with the host galaxy’s star-formation rate (SFR). We explore this connection at sub-galactic scales spanning \(-7\) dex in SFR and \(-8\) dex in specific SFR (sSFR). There is good agreement with established relations down to $\text{SFR} \approx 10^{-3}\, M_\odot\,\text{yr}^{-1}$, below which an excess of X-ray luminosity emerges. This excess likely arises from low mass X-ray binaries. The intrinsic scatter of the $L_X$–SFR relation is constant, not correlated with SFR. Different star formation indicators scale with $L_X$ in different ways, and we attribute the differences to the effect of star formation history. The SFR derived from H$\alpha$ shows the tightest correlation with X-ray luminosity because H$\alpha$ emission probes stellar populations with ages similar to HMXB formation timescales, but the H$\alpha$-based SFR is reliable only for $\text{sSFR} > 10^{-12}\, M_\odot\,\text{yr}^{-1}/M_\odot$.

Key words: galaxies:star formation – X-rays: galaxies – X-rays:binaries

1 INTRODUCTION

Star formation throughout cosmic time has transformed the Universe. Among other things, it has illuminated it and has created the foundations for more complex forms to exist. When considered on kpc scales, star formation has shaped the phenomenology of galaxies. Two of the most fundamental characteristics of galaxies are the stellar mass ($M_\star$; past star formation) and the current/recent star formation, measured by the star-formation rate (SFR). There is a strong correlation between galaxies’ stellar masses and SFRs, i.e., the galactic main sequence (e.g., Noeske et al. 2007; Elbaz et al. 2007; Daddi et al. 2007).

Studies on sub-galactic scales can show to what extent local conditions are responsible for global scaling relations (e.g., Maragkoudakis et al. 2017; Enia et al. 2020). Comparisons on sub-galactic scales among galaxies of different types, star-formation histories (SFH), and metallicities show great differences (e.g., Boquien et al. 2014) because star formation is not homogeneously dispersed in the galactic volume (e.g., Larson et al. 2020).

X-rays probe recent and past star-formation activity and are particularly useful for characterizing star formation in obscured environments. X-ray binaries (XRBs) in particular provide a means to quantify the numbers of stellar remnants (neutron stars and black holes) otherwise hidden from view. XRBs are formed when a donor star provides mass to a compact object to which it is gravitationally bound. The mass transfer can be via Roche lobe overflow or stellar wind, and either way, the accreting mass radiates at X-ray wavelengths. Donor stars can be high-mass OB stars or low-mass stars. Based on their donor stars, systems are described as either high-mass X-ray binaries (HMXBs) or low-mass X-ray binaries (LMXBs). Collectively, the X-ray emission from all the XRBs hosted in a galaxy shows strong correlations with galaxy-wide characteristics such as SFR and stellar mass.

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Specifically, LMXB X-ray emission correlates strongly with stellar mass (e.g., Gilfanov 2004; Lehmer et al. 2010; Boroson et al. 2011; Zhang et al. 2012), and HMXB X-ray emission correlates with SFR (e.g., Grimm et al. 2003; Ranalli et al. 2003; Mineo et al. 2012a,b, 2014).

Recently there have been efforts to examine the $L_X$–SFR–$M_*$ correlations down to sub-galactic scales in the nearby Universe. The ratio of XRBs’ X-ray output to visible luminosity varies significantly when examined on small physical scales. This is witnessed by explorations of the X-ray luminosity of individual regions of a few nearby galaxies (e.g., Anastasopoulou et al. 2019) and by investigations of the X-ray luminosity functions of XRBs associated with stellar populations of different ages or metallicities (e.g., Lehmer et al. 2019).

A complication in understanding the correlation between XRBs and SFR is that there are multiple SFR indicators based on different physical mechanisms. Indicators include 1.4 GHz emission from synchrotron radiation of relativistic electrons accelerated in supernovae remnants, absorbed ultraviolet (UV) radiation heating galactic dust and being re-emitted at 24 μm, and in the far infrared, UV from high mass stars’ photospheres, emission lines from atomic gases ionized by OB stars, polycyclic aromatic hydrocarbons (PAHs) emitting from the surrounding photo-dissociation regions, etc. This results in differences between the different SFR indicators that multiple galaxy-wide studies have tried to calibrate (e.g., SFRS; Mahajan et al. 2019). The different SFR indicators probe stellar populations of different ages (e.g., Kennicutt & Evans 2012) with the ones from ionized atomic gases probing the most recent (e.g., Boquien et al. 2014; Cerviño et al. 2016).

X-ray emission is considered an emerging SFR indicator, but the correlations still suffer from stochastic and calibration effects. These effects, which are detected in galaxy-wide correlations, are increased when examined on sub-galactic scales because star formation is a local event and hence is diluted on the surface of a galaxy. Theoretical models predict X-ray luminosity variations from different stellar populations (e.g., Fabbiano et al. 2001; Mapelli et al. 2009, 2010). XRB population synthesis models show that the bulk of the X-ray output originating from XRBs is short lived ($\lesssim$20 Myr) because that the emission from HMXBs is orders of magnitude higher than that of LMXBs (e.g., Fragos et al. 2013). Therefore, in order to understand how biases arise in the X-ray luminosity, SFR, and stellar-mass correlations, it is important to examine the correlations on sub-galactic scales with different SFR indicators.

Sample selection can bias our interpretation and measurement of the aforementioned correlations. For example, Mineo et al. (2014, hereafter M14) studied the $L_X$–SFR scaling relation for a small sample of star-forming galaxies. Gilfanov (2004) and Boroson et al. (2011) studied the $L_X$–$M_*$ relation for samples of early type galaxies. Lehmer et al. (2010) introduced an $L_X$–SFR–$M_*$ scaling relation that accounts for the contribution of HMXBs (scaling with SFR) and LMXBs (scaling with stellar mass) based on samples of local as well as higher-redshift galaxies. This analysis used a sample of nearby galaxies with a large range and mix of stellar masses and SFRs.

This paper’s goal is to estimate the effect different star-forming conditions and SFHs (along with the fact that different SFR indicators probe different time-scales) may induce in the correlation and to measure the scatter in each case. The paper is organized as follows: Section 2 describes the sample of galaxies, the data/observations, and the data reduction. Section 3 describes how sub-galactic analysis was performed. The maximum likelihood fits and the results of the analysis are described in Section 4. The results of the analysis are discussed in Section 5, and the summary is in Section 6.

2 SAMPLE SELECTION AND OBSERVATIONS

2.1 Sample

Our galaxy sample is based on the Star Formation Reference Survey (SFRS; Ashby et al. 2011). The SFRS is comprised of 369 galaxies that represent all modes of star formation in the local Universe. They fully cover the 3D space of three fundamental galaxy properties: the SFR, indicated by the 60 μm luminosity; the specific SFR (sSFR), indicated by the $K_5 – F_{60}$ colour; and the dust temperature, indicated by the FIR ($F_{100}/F_{60}$) flux density ratio. The SFRS benefits from panchromatic coverage of the electromagnetic spectrum from radio to X-rays, including optical spectra of the galaxy nuclei (Maragkoudakis et al. 2017) and Hα imaging (Kouroumpatzakis et al. in prep.). The objective SFRS selection criteria let us put the sample galaxies in context of the local star-forming galaxy population.

The sample used for this work consists of 13 star-forming (non-AGN) SFRS galaxies (Table 1) for which there are Chandra data of adequate quality to study the X-ray emission down to 1 kpc$^2$ scales (Table 2) available in the archive. The sample galaxies span $\sim$4 dex in the total SFR and $\sim$3 dex in sSFR. On sub-galactic scales these ranges become $\sim$7 dex and $\sim$8 dex in SFR and sSFR respectively (Fig. 1).

2.2 Hα data

The primary SFR indicator used in this work is Hα emission, which traces gas ionized by stellar populations of ages $\lesssim$20 Myr (e.g., Murphy et al. 2011). Because the formation timescale of HMXBs is typically 10–30 Myr (e.g., Fragos et al. 2013), it is in principle well-matched to SFR probed by Hα emission.

We have obtained Hα observations with the 1.3 m telescope of the Skinakas observatory. To account for the redshift range of the SFRS sample galaxies, we used a custom-built set of filters centered at $\lambda = 6563, 6595, 6628, 6661, 6694, 6727, 6760$ Å with average FWHM = 45 Å. The exposure time for Hα observations was 1 hour. We also obtained $\sim$10 minute continuum-band exposures with a filter equivalent to SDSS r’. The Hα observations were taken between 2016 and 2019 under photometric conditions and typical seeing $\sim$1′′. Details of the observations and data will be presented by Kouroumpatzakis et al. (in prep).

1. http://skinakas.physics.uoc.gr/
After the initial reductions (bias subtraction, flat fielding, flux calibration, etc.) the standard continuum subtraction technique was performed, based on the relative flux density of the foreground stars in the continuum and Hα images (e.g., Kennicutt et al. 2008). This comparison results in a distribution of $Hα$/SDSS $r'$ band flux density ratios for the various stars included in each frame. We used the mode of this distribution as the continuum scaling factor and its standard deviation as a measure of the uncertainty of this procedure. The rescaled continuum image was subtracted from the Hα image to generate the continuum-subtracted Hα image. In order to minimize the effect of poorly subtracted stars, their residuals were masked. These residuals were usually a result of PSF differences between the narrow band and continuum observations or colour variations arising from the variety of the foreground stars in the observed frames.

A curve of growth (CoG) technique was used to measure the net Hα flux of each galaxy while simultaneously estimating and subtracting the sky background (Fig. 2). The background was estimated by performing a linear fit to the last 5% of the CoG. This procedure was repeated iteratively while regulating the background until this part of the CoG was flat. The galaxy aperture size was defined from the point of the CoG that reaches the asymptotic line. The aperture shapes used in our analysis were based on elliptical aperture fits to the WISE 4.6 μm data of the SFRS galaxies (following a procedure similar to Jarrett et al. 2019), keeping the position angle and ellipticity constant. The photometric calibrations were based on observations of spectrophotometric standard stars (Massey et al. 1988). We included a calibration uncertainty in our analysis, estimated from the standard deviation of the standard star’s instrumental magnitudes during the observations. The Hα luminosity was converted to SFR with the Murphy et al. (2011) conversion:

$$\frac{\text{SFR}_{\text{Hα}}}{(\text{M}_\odot \text{yr}^{-1})} = 10^{-41.27} \frac{L_{\text{Hα}}}{(\text{erg s}^{-1})}$$

(MNRAS 000, 1–20 (2020))
In addition to the H\textalpha~SFR measure, we used \textit{Spitzer} IRAC non-stellar 8\,µm and MIPS 24\,µm observations (Ashby et al. 2011). 8\,µm probes PAH emission, including dust-enshrouded star formation. 24\,µm observations probe warm dust heated by UV emission from young stars. These two indicators trace star formation at longer timescales than H\textalpha~emission (e.g., Peeters et al. 2004; Rieke et al. 2009; Kennicutt & Evans 2012). The annuli used for the 8\,µm and MIPS 24\,µm analysis were the same as the H\textalpha~ones. The background was subtracted as measured by an annulus outside the galaxy aperture, accounting for any contribution from foreground stars or background AGN. In the case of the IRAC 8\,µm, the stellar continuum was subtracted by rescaling the 3.6\,µm images, using the formula from Helou et al. (2004):

\[
 f_{8\text{\,µm,PAH}} = f_{8\text{\,µm}} - 0.26 f_{3.6\text{\,µm}} .
\]  

Then the non-stellar 8\,µm luminosity was converted to SFR using the calibration of Pancoast et al. (2010):

\[
 \frac{\text{SFR}_{8\text{\,µm,PAH}}}{(M_{\odot} \text{ yr}^{-1})} = 6.3 \times 10^{-10} \frac{L_{8\text{\,µm}}}{L_{\odot}} .
\]  

The MIPS 24\,µm luminosity was converted to SFR using the calibration of Rieke et al. (2009):

\[
 \frac{\text{SFR}_{24\text{\,µm}}}{(M_{\odot} \text{ yr}^{-1})} = 10^{-42.69} \frac{L_{24\text{\,µm}}}{(\text{erg s}^{-1})} .
\]  

The IRAC 3.6\,µm observations were used to estimate total stellar masses. The observed flux density was converted to stellar mass using the Zhu et al. (2010) mass-to-light ratio calibration.

\[
 \frac{M_{*}}{M_{\odot}} = 10^{0.23+1.14(g-r)} \frac{\nu L_{\nu,3.6\text{\,µm}}}{L_{\odot}} .
\]  

where $g$ and $r$ are total galaxy Petrosian AB magnitudes from SDSS DR12 (Alam et al. 2015). We used each galaxy’s integrated emission $g - r$ colour for all of its sub-galactic regions.

### 2.4 X-ray Data

The \textit{Chandra} data were reduced with CIAO v.4.9 and CALDB v.4.7.3. The raw data were reprocessed in order to apply the latest calibrations and screened for background flares. Then from the clean event files, we extracted images in the full (F: 0.5–8\,keV), soft (S: 0.5–2\,keV), and hard (H: 2–8\,keV) bands and calculated the corresponding monochromatic exposure maps (at energies of 3.8, 1.5, and 3.8\,keV respectively).

For each galaxy we also extracted its integrated spectrum using the CIAO \texttt{dmextract} command. The extraction aperture was the same as the H\textalpha~apertures. Corresponding response and ancillary response files were also calculated with the CIAO \texttt{specextract} tool. Background spectra were extracted from source-free regions within each field. The X-ray spectra were fitted with spectral models including power-law, thermal plasma (APEC; Smith et al. 2001), and when needed, Gaussian emission-line components. The spectral analysis was performed using Sherpa v.4.9. The spectra were binned to have at least 20 counts per bin in order to use the $\chi^2$ statistic. The best-fit model parameters for the integrated spectra of each galaxy are presented in Table 2. The details of the spectral analysis will be presented by Sell et al. (in prep.).

The integrated flux of each galaxy was measured by...
integrating the best-fit spectral models. In order to account for uncertainties in the spectral parameters, the `sample_flux` Sherpa task was used. This task samples model parameters from the covariance matrix of the best-fit model, and for each sample it calculates the corresponding model integrated flux. This yielded the probability density distribution of the model flux and the corresponding uncertainties on the spectral parameters. In addition, for each sample of spectral parameters, the expected number of counts was calculated by folding the model through the ancillary response function (Davis 2001) of the corresponding spectrum. The ratio of the model integrated flux to the estimated source counts yielded the count-rate to flux conversion factor, while the distribution of this ratio gave the uncertainty of the conversion factor as a result of the uncertainty in the model parameters.

### Table 3. Number of sub-galactic regions per galaxy at each physical scale.

| Galaxy/Surface (kpc²) | 1x1 | 2x2 | 3x3 | 4x4 |
|-----------------------|-----|-----|-----|-----|
| NGC03245              | 71  | 25  | 15  | 9   |
| UGC05720              | 54  | 20  | 9   | 9   |
| NGC03353              | 26  | 10  | 8   | 7   |
| NGC03656              | 256 | 74  | 40  | 24  |
| NGC04194              | 169 | 51  | 29  | 25  |
| NGC05204              | 24  | 9   | 8   | 5   |
| NGC05474              | 73  | 22  | 11  | 9   |
| NGC05585              | 91  | 23  | 17  | 9   |
| NGC05584              | 593 | 159 | 80  | 50  |
| MCG-6-32-070           | 1292| 337 | 164 | 100 |
| NGC05691              | 54  | 19  | 10  | 8   |
| NGC05879              | 202 | 62  | 30  | 15  |
| NGC06090              | 199 | 57  | 31  | 19  |
| Total                 | 3104| 868 | 452 | 289 |

In order to explore the correlations between SFR, stellar mass, and X-ray luminosity on sub-galactic scales, we defined grids of different physical scales following the same approach as Maragkoudakis et al. (2017). Physical scales of 1x1, 2x2, 3x3, and 4x4kpc² were considered. The minimum physical scale was dictated by the MIPS 24µm PSF (FWHM of centered point spread function =2′′6), which corresponds to a scale of ~1kpc for the most distant galaxy (NGC 6090) in our sample (3′′14 for 1 kpc regions). One additional reason for not considering smaller scales is that the SFR indicators suffer from severe stochasticity at scales ≲1 kpc (e.g., Kennicutt & Evans 2012). Another reason is to ensure that the natal kicks neutron stars (and possibly black holes) receive will not add significant scatter to the relations we find. These kicks can result in a considerable velocity for the surviving binary systems (e.g., Podsialowski et al. 2004), displacing XRBs from their formation sites. This could increase the scatter in the sub-galactic correlations between SFR and X-ray luminosity. Typical center-of-mass velocities measured for HMXBs are in the 15–30km s⁻¹ range (e.g., van den Heuvel et al. 2000; Coe 2005; Antoniou & Zezas 2016). However, for a travel time of ~20 Myr (i.e., the time between formation of the compact object and the onset of the X-ray emitting phase, e.g., Politakis et al. 2020), even the upper end of the velocity range gives a distance no more than ~600 pc from the formation site of an HMXB. In the case of LMXBs, their long formation timescales (≳1 Gyr) mean that they trace the old stellar populations of a galaxy, which are more evenly distributed. Therefore, the natal kicks will not affect the statistical association of LMXBs with the older stellar populations.

We applied the same sub-region grids to all the observables: IRAC 3.6 µm (used to measure the stellar mass), Hα, IRAC 8 µm, MIPS 24 µm, (used to measure the SFR), and the Chandra data in the soft, hard, and full bands. At this stage, regions with signal-to-noise (S/N)≤3 in the IRAC 3.6 µm data were discarded. This is why the number of sub-galactic regions does not increase geometrically for smaller physical scales. The resulting maps of stellar mass, SFR, sSFR, and X-ray luminosity were used to correlate these parameters in sub-galactic regions. Figure 3 shows an example. Table 3 lists the number of regions in each of the galaxies.

In order to calculate the X-ray emission in each sub-galactic region, the observed number of counts was measured using the CIAO `dmextract` tool on the Chandra images in each of the three bands. Because most regions had ≤5 counts above the background, the background could not simply be subtracted as estimated from a source-free region outside the galaxy. Instead, the BEHR² code (Park et al. 2006) was used, which gives the posterior probability distribution of the source intensity based on the formulation of van Dyk et al. (2001), accounting for the Poissonian nature of the source and background counts. BEHR also takes into account differences in the effective area between the source and background regions. A non-informative Jeffreys’ prior on the source intensities was adopted. This approach allowed a reliable estimate of the intensity of the X-ray emission even in regions with weaker signals than formal detections. It also accounted for effective area variations across the galaxy’s surface based on the exposure maps of each galaxy.

In order to calculate the X-ray luminosity for each sub-galactic region, the posterior distribution of the source counts (calculated as described in Section 2.4) was folded with the distribution of count-rate to flux conversion factors. This conversion depends on the X-ray spectrum (e.g., Zezas et al. 2006). Because each sub-galactic region has typically ≤20 total counts, no independent spectral analysis could be performed. Instead the spectrum of each sub-galactic region was assumed to be the same as the galaxy integrated spectrum. This is a reasonable assumption for the 10 galaxies fitted with an absorbed power-law spectrum and not requiring any additional thermal component. The X-ray emission of these galaxies typically has Γ≥1.6 (Section 2.4, Table 2). Therefore their spectra are dominated by XRBs, which on average have X-ray spectra with photon indices 1.5Γ≤3. The three galaxies that require a thermal component may have spatial variations in the relative intensity of the thermal and the power-law components. Assuming that the spectral parameters of each of the two components are on average the same in the different sub-galactic regions, the X-ray colour

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2 Bayesian Estimation of Hardness Ratios; [http://hea-www.harvard.edu/astrostat/BEHR/index.html](http://hea-www.harvard.edu/astrostat/BEHR/index.html)
Figure 3. Sub-galactic maps at 1 kpc² scale for NGC 5879, illustrating the character of the data. Top left: the IRAC 3.6 μm image used to measure the stellar mass. Top center: the stellar mass map derived from the IRAC 3.6 μm observations. Top right: The SFR map based on the Hα observations. Bottom left: The SFR map derived from the IRAC 8 μm observations. Bottom center: The sSFR map that results from combining the Hα and the IRAC 3.6 μm observations. Bottom right: The full (0.5–8 keV) X-ray luminosity map based on the Chandra imaging. Bars to the right of each image show the mapping from grey scale to physical quantity.

Table 4. Mean (of the distribution of all the sub-galactic regions) thermal contribution of the thermal-plasma component in the full band $L_{0.5-8\text{ keV}}$ luminosity for each galaxy.

| Galaxy/Surface ($\text{kpc}^2$) | 1×1 | 2×2 | 3×3 | 4×4 |
|-------------------------------|-----|-----|-----|-----|
| NGC 4194                     | 0%  | 0%  | 0%  | 0%  |
| NGC 3245                     | 18.5% | 19.6% | 23.1% | 33.5% |
| UGC 5720                     | 8.5%  | 6.9%  | 16.1% | 5.8%  |

$C \equiv \log(S) - \log(H)$ of each region can be used to infer their relative contribution in the full band. $C$ was calculated with the BEHR method. Figure 4 shows the relation between $C$ and the relative contribution of the power-law to total (power-law + thermal) components. Based on $C$, the corresponding total flux for each region and the flux arising only from the power-law component (which is relevant for the XRBs) were calculated. The mean thermal contribution for these galaxies is shown in Table 4.

The calculation of the X-ray luminosity for each region was performed by sampling the posterior distribution of the net counts and the corresponding distribution of count-rate to flux conversions. The resulting X-ray luminosity distributions are non-Gaussian, usually positively skewed for low-emission regions.

In order to compare our results with the scaling relations of M14 and with results from the Chandra deep surveys, we also calculated the luminosities in each sub-galactic region in the soft and the hard bands. Because the thermal emission included in the soft band can also be correlated with recent star formation, we opted not to subtract the thermal component. Therefore the count-rate to flux conversion factors in the soft band were calculated as described above, i.e. without correcting for the thermal component. In the case of the X-ray emission above 2 keV, which is dominated by the power-law component, we simply used the best-fit photon index for each galaxy and its corresponding uncertainty to calculate the distribution of the count-rate to flux conversion factors.

4 RESULTS

4.1 Maximum likelihood fits

In order to measure the correlation between X-ray luminosity, SFR, and stellar mass, we performed maximum likeli-
where $\sigma$ indicates a Gaussian random variable with $\mu = 0$ and standard deviation $\sigma$. The results are reported in Table 6 and described in Section 4.2.

In order to disentangle the contribution of HMXBs and LMXBs in the X-ray luminosity of the sub-galactic regions, we performed a joint X-ray luminosity, SFR, and stellar mass maximum likelihood fit. The model was parameterized as

$$\log L_X = \log(10^\alpha+\log \text{SFR} + 10^\beta-\log M_\star) + \sigma, \quad (8)$$

where $\alpha$ and $\beta$ are the scaling factors of the X-ray luminosity resulting from the young and the old stellar populations (associated with HMXBs and LMXBs respectively), and $\sigma$ is again a Gaussian random variable accounting for intrinsic scatter in the data. The fit results are given in Table 7 and described in Section 4.3. The implementation of the maximum likelihood method is described in more detail in Appendix A.

### 4.2 Correlations between X-ray luminosity and SFR

Figure 5 presents the correlations between X-ray luminosity and SFR using the H$\alpha$, 8 $\mu$m, and 24 $\mu$m SFR indicators. Our analysis used the observed H$\alpha$ and X-ray luminosities, i.e., not corrected for absorption. This is because we are interested in deriving empirical relations between observable quantities. The sample galaxies show small inclinations (minimum minor-to-major axis ratio $= 0.52$, median $= 0.87 \pm 0.14$—Table 1), suggesting low intrinsic absorption. The median extinction for these thirteen galaxies (Maragkoudakis et al. 2018) is 0.36 mag based on their integrated or nuclear spectra. Translating the typical $A_V$-to-hydrogen column density conversion $N_H/A_V = 1.9 \times 10^{21}$ atoms cm$^{-2}$ mag$^{-1}$ (Gorenstein 1975 with cross sections from Morrison & McCammon 1983) to H$\alpha$ with a Cardelli et al. (1989) extinction curve (with $R_V = 3.1$) gives $N_H/A_H = 1.55 \times 10^{21}$ atoms cm$^{-2}$ mag$^{-1}$. This makes the absorption in H$\alpha$ and at 1 keV similar within $\sim 30\%$.

The best-fit $L_X$–SFR results are presented in Table 6. Overall correlations between these two quantities are flatter than the reference correlation of M14. There are also differences in the slopes depending on the star-formation indicator considered: H$\alpha$-based SFR shows systematically steeper slopes, while correlations on the 8 $\mu$m-based and the 24 $\mu$m-based emission show shallower slopes. There are also systematic trends depending on the spatial scales considered. While the correlations are shallower than linear in all cases, larger spatial scales tend towards linearity. The shallower slopes are mainly driven by regions in the extremely low SFR regime, which show an X-ray luminosity excess in comparison to the linear relation of M14 and the best maximum likelihood fits from this work.

The fits discussed above are based on the full band X-ray data, which provide the maximum S/N ratio for each sub-region. However, full band fluxes can be subject to differential absorption and residual thermal emission. In order to address the importance of these we also calculated the $L_X$–SFR scaling relations in the soft and hard bands, the latter being a cleaner probe of the X-ray emission produced by XRBs. The results are presented in Table 6 and illustrated in Figure 6. The soft band shows weaker correlation with SFR in all cases. The hard-band fits have similar slopes to
Figure 5. $L_{X,0.5-8\text{keV}}$ as a function of SFR for three different SFR indicators ($\text{H}\alpha$, $8\mu\text{m}$, and $24\mu\text{m}$ from left to right) and for four different sub-galactic scales ($1\times1$, $2\times2$, $3\times3$, and $4\times4\text{kpc}^2$ from top to bottom). All regions within all the sample galaxies are included in the fits and are represented by black error bars (including uncertainties only in the X-ray luminosity for clarity). The red dashed-dotted line represents the maximum likelihood best fit for $\log L_X = a \log \text{SFR} + b + \sigma$ (Eq. 7) for all sub-galactic region in the sample. Parameters $a$, $b$, and $\sigma$ are given in Table 6. The shaded area represents the estimates for the intrinsic scatter $\sigma$. The blue error bars represent mode values of the distributions of points included in bins of 1 dex of SFR. The M14 correlation is drawn with a dashed black line. Underneath each panel, the black error bars represent for each sub-galactic region the ratio of the measured $L_X$ to the value expected based on the best-fit model (red dashed-dotted line).
the full band, a fact that reinforces the usefulness of the full band $L_X$–SFR correlation on sub-galactic scales despite the potential complication of differential absorption. The hard band shows significantly lower scatter than the full and the soft band in all cases. The hard band–Hα correlation shows the tightest correlation and slopes closest to one. Especially in the case of the Hα-based relation, we find remarkably similar results between the hard and the full bands. In the case of 2x2 and 3x3 kpc$^2$ 24 $\mu$m fits, the hard band shows a shallower fit than the full and soft band. This is due to the rejection of the low-S/N regions in the 24 $\mu$m MIPS data. These regions have very low SFR, reducing the range of SFR and causing the low-SFR locus to be less populated, thereby driving the flatter fits.

In order to explore galaxy-to-galaxy variations of the scaling relations, the model described by Eq. 7 was fitted to each individual galaxy of our sample. The best-fit slopes and intercepts for the fits for each sub-galactic scale and SFR indicator are plotted in Figure 7. We see a broad range of intercepts and slopes, with some galaxies showing no correlation (slope=0) and others having slope steeper than 1. As expected, there is significant correlation between the best-fit slopes and intercepts. The best-fit parameters for most cases show large uncertainties ($\sim 1$ dex) as result of the small number of regions (Table 3) used to derive each correlation. This is particularly evident as we consider increasing spatial scales. However, we do see significant differences between the best-fit slopes and intercepts for the different galaxies, particularly in the case of the smaller physical scales, where differences are not masked by large uncertainties. These variations illustrate the stochasticity in the $L_X$–SFR correlation, arising from the differences in the SFHs and stellar populations of the galaxies.

### 4.3 Joint correlations between X-ray luminosity, SFR and stellar mass

The sSFR is a metric of the relative contribution of the young and old stellar populations in the mass assembly of the galaxy. Because HMXBs are associated with young, and LMXBs with old stellar populations, the sSFR is a proxy for the relative contribution of these two XRB populations in the overall X-ray emission of a galaxy. Figure 8 illustrates these correlations projected on the $L_X$–SFR–$M_*$ plane. For almost all cases, we find excellent agreement with the $z<0.5$ (Lehmer et al. 2016; hereafter L16) relation for the integrated properties of galaxies, even though the results presented here consider sub-galactic scales and extend these relations to $\sim 2$ dex lower sSFR. The agreement is better for larger scales, with smaller scales tending to give larger $\sigma$ (Eq. 8). As in the case for the $L_X$–SFR correlations, the scatter is smallest for the Hα SFR indicator.

### 5 DISCUSSION

#### 5.1 Comparisons between different SFR indicators

There is growing evidence that the X-ray emission of XRB populations evolves as a function of time (e.g., Fragos et al. 2013; Antoniou et al. 2019b; Lehmer et al. 2019). HMXBs in particular are a short-lived population, and therefore their abundance depends on SFH. Several recent studies have started to explore the sensitivity of SFR inferred from different SFHs. For example, Hα traces $\sim 10$ Myr stellar populations whereas 8 $\mu$m and 24 $\mu$m traces $\gtrsim 200$ Myr stellar populations. However, what is not clear yet is how the X-ray scaling relations depend on the SFH of the population responsible for the X-ray emission, because previous works have used indiscriminately different SFR indicators even for different galaxies in the same scaling relations. Such variation may contribute to the observed scatter.

Our observations show a systematic difference in the $L_X$–SFR correlations between the different SFR indicators. The Hα SFR indicator gives a steeper, more linear slope and the lowest scatter, indicating that it is better correlated with the XRBs’ X-ray emission than the 8 $\mu$m and the 24 $\mu$m indicators. The Hα emission traces the ionizing radiation from stellar populations with ages (e.g., Kennicutt & Evans 2012; Boquien et al. 2014; Cerviño et al. 2016) similar to the formation timescale of the HMXBs (e.g., Bhattacharya & van den Heuvel 1991; Tauris & van den Heuvel 2006; Fragos et al. 2013). In contrast, the 8 and 24 $\mu$m bands’ connection with HMXBs is diluted (Fig. 9) by the much larger age range those SFR indicators reflect.

The X-ray emission from LMXBs begins to dominate over that from HMXBs for stellar populations older than $\gtrsim 80$ Myr (Fig. 9), even though the bulk of their population forms at much later times. In regions dominated by a young stellar population, the IR indicators will be dominated by the same young stellar populations traced by the Hα emission, which also host the HMXB populations. On the other hand, for regions with star-forming activity extending beyond 100 Myr, the IR indicators will include contributions from older stellar populations than those traced by the Hα emission. These older stellar populations do not include HMXBs (e.g., Fragos et al. 2013), resulting in increasing scatter.

In order to obtain at least a qualitative picture of the X-ray luminosity scaling relations’ dependence on SFH, we performed a simple simulation study where we calculated the X-ray luminosity, SFR, and stellar mass under different assumptions for the SFH. The top panel of Fig. 9 presents the X-ray output of a stellar population from the model of Fragos et al. (2013) as a function of age along with the age sensitivities (response functions) of the three SFR indicators considered here.3

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3 The response functions were calculated by modeling the evolution of the Hα, 8 $\mu$m, and 24 $\mu$m emission for an instantaneous burst of star formation. In order to subtract the stellar continuum from the 8 $\mu$m emission, we also calculated the ratio of the flux in the 3.6 $\mu$m and 8 $\mu$m Spitzer-IRAC bands for the same decaying population without including any dust contribution. These calculations were performed with CIGALE v.2018.0.1 (Boquien et al. 2019). The stellar populations were modeled after the BC03 (Bruzual & Charlot 2003) models assuming solar metallicity. We considered models with Salpeter (Salpeter 1955) or Chabrier (Chabrier 2003) IMFs, values for the absorption $E(B-V) = 0.3, 1.0, 2.0$, nebular component ionization parameter $U = -1.0, -2.0, -3.0, -4.0$, and two dust emission models: those of Dale et al. 2014 and Draine & Li 2007. We explored different values of the $\alpha$ parameter in the (Dale et al. 2014) dust model and of the PAH mass fraction (gph) and limiting ionization field ($U_{\text{max}}$) for the Draine et al. (2014) models. The response func-
Figure 6. Best maximum-likelihood fits to the $L_X$–SFR relations (see Table 6). The different lines correspond to fits for the soft (S; 0.5–2 keV; green dotted), hard (H; 2–8 keV; blue dashed-dotted), and full (F; 0.5–8 keV; red) bands. Fits for the different SFR indicators ($H\alpha$, 8 $\mu$m, and 24 $\mu$m) are shown in the columns from left to right at four sub-galactic scales (1 $\times$ 1, 2 $\times$ 2, 3 $\times$ 3, and 4 $\times$ 4 kpc$^2$) from top to bottom. The shaded areas of similar colours represent the intrinsic scatter $\sigma$ for each band. For comparison the M14 correlation is drawn with a black dashed line in all panels.

The correlations presented in Fig. 9 are the average of the results from the different models. A more detailed discussion of the response functions and the parameters they depend on will be presented in Kouroumpatzakis et al. (in prep). Similar investigations for various SFR indicators have been presented in previous works (e.g., Cerviño et al. 2016; Boquien et al. 2014) but for different SFR indicators than those used here or for more complex SFHs, which complicate the disentanglement of the contribution of different stellar populations to the measured SFR.
Figure 7. Best-fit slopes and intercepts of the sub-regions in each individual galaxy. SFR indicators ($\text{H} \alpha$, 8 µm, and 24 µm) are in columns from left to right, and four sub-galactic scales (1x1, 2x2, 3x3, and 4x4 kpc$^2$) are from top to bottom. The points are colour-coded based on each galaxy’s integrated emission $K_{S} - F_{60}$ colour, a proxy for their sSFR.
Figure 8. $L_{\text{X},0.1-3.8\text{keV}}/\text{SFR}$ as a function of sSFR with the use of three different SFR indicators ($H\alpha$, 8 $\mu$m, and 24 $\mu$m from left to right) and for four different sub-galactic scales (1x1, 2x2, 3x3, and 4x4 kpc$^2$ from top to bottom). All regions of all the sample galaxies are represented by grey points. The red dashed curve represents the best fit for a log $L_{\text{X}} = \log(10^{\alpha + \log \text{SFR}} + 10^{\beta + \log M_\star}) + \sigma$ model (Eq. 8). The shaded area represents the calculated intrinsic scatter $\sigma$. The blue error bars represent the modes and 1$\sigma$ uncertainties of the distributions of points in 1 dex bins of sSFR. The L16 relation for zero redshift is plotted with a black dashed-dotted curve.

Based on the XRB luminosity evolution and the SFR indicator response functions, we can quantify the dependence of the $L_{\text{X}}$–SFR relations on the SFH and the SFR indicator used. To demonstrate this effect we considered five different SFHs (see Fig. 9). The total stellar mass is:

$$M_\star = \int_0^t \text{SFH}(t')dt',$$

(9)

and the “effective” SFR for each indicator, which accounts for their sensitivity to older or younger stellar populations.
is:

$$\text{SFR}_X = \frac{\int t' R_X(t') \text{SFR}(t') dt'}{\int R_X(t') dt'} \quad .$$

Figure 9 shows five example SFHs, and results for each one are presented in Table 8. We expect variations in the SFR for the different SFH scenarios only if the SFR changes within the time window of each indicator (e.g., largest difference the time window of each indicator (e.g., largest difference

$$\text{SFR}_X = \frac{\int t' R_X(t') \text{SFR}(t') dt'}{\int R_X(t') dt'} \quad .$$

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$$\text{SFR}_X = \frac{\int t' R_X(t') \text{SFR}(t') dt'}{\int R_X(t') dt'} \quad .$$

Table 5. Maximum likelihood fits of the $L_X$–SFR relation with SFR-dependent scatter.

| Scale      | $a$     | $b$     | $\sigma_1$ | $\sigma_2$ | $\alpha$ | $\sigma$ | $\alpha_1$ | $\sigma_1$ | $\alpha_2$ | $\sigma_2$ |
|------------|---------|---------|-------------|-------------|-----------|----------|-------------|-----------|-------------|-----------|
| 1×1 kpc$^2$ | 0.50±0.02 | 38.87±0.05 | 0.14±0.01 | 1.13±0.04 | 0.44±0.01 | 38.99±0.05 | 0.18±0.02 | 1.43±0.05 | 0.46±0.03 | 38.87±0.09 | 0.14±0.02 |
| 2×2 kpc$^2$ | 0.60±0.03 | 39.15±0.15 | 0.04±0.04 | 0.87±0.10 | 0.51±0.03 | 39.11±0.07 | 0.01±0.04 | 0.91±0.07 | 0.46±0.07 | 38.93±0.18 | 0.11±0.04 |
| 3×3 kpc$^2$ | 0.66±0.03 | 39.42±0.15 | 0.03±0.08 | 0.87±0.15 | 0.63±0.05 | 39.26±0.10 | 0.21±0.04 | 1.19±0.12 | 0.58±0.07 | 39.22±0.10 | 0.09±0.03 |
| 4×4 kpc$^2$ | 0.76±0.06 | 39.57±0.16 | 0.05±0.06 | 0.80±0.13 | 0.69±0.06 | 39.35±0.14 | 0.15±0.06 | 1.13±0.10 | 0.75±0.06 | 39.45±0.11 | 0.09±0.04 |

Table 6. Maximum likelihood fits of the $L_X$–SFR relation with constant scatter.

| Scale      | $a$     | $b$     | $\sigma$ | $\alpha$ | $\sigma_1$ | $\sigma_2$ |
|------------|---------|---------|----------|-----------|-------------|-------------|
| Full $L_X$ | 0.60±0.01 | 39.07±0.03 | 0.84±0.01 | 0.45±0.02 | 39.04±0.05 | 0.96±0.01 | 0.54±0.02 | 39.10±0.04 | 0.92±0.02 |
| Soft $L_X$ | 0.34±0.03 | 37.87±0.07 | 1.05±0.01 | 0.45±0.01 | 38.24±0.03 | 0.94±0.01 | 0.40±0.02 | 38.13±0.05 | 1.06±0.02 |
| Hard $L_X$ | 0.73±0.02 | 39.65±0.04 | 0.39±0.02 | 0.38±0.01 | 39.24±0.05 | 0.48±0.03 | 0.33±0.02 | 38.83±0.08 | 0.69±0.03 |

NOTE: Model $\log L_X = \alpha \log \text{SFR} + b + \epsilon(\text{SFR})$, where $\epsilon$ is a Gaussian random variable with mean $\mu = 0$ and standard deviation $\sigma = \sigma_1 \log \text{SFR} + \sigma_2$ for the full (0.5-8 keV) X-ray band.

Although the 8 and 24 $\mu$m SFR-indicator response functions track fairly well the HMXB X-ray luminosity as a function of time (Fig. 9), they can be affected by emission from stars older than those that can form HMXBs. Thus these indicators can overestimate the SFR when a stellar population is dominated by older stars and has larger $t_{\text{eff}}$ (Table 8). In addition, because $\geq 60$ Myr populations do not contribute to the formation of HMXBs (e.g., Fragos et al. 2013; Garofali et al. 2018; Antoniou et al. 2019a), the $L_X$–SFR scaling relations based on the 8 and 24 $\mu$m indicators will result in lower scaling factors for galaxies with SFHs not dominated by a recent burst. Therefore, $Hr$ is the most appropriate proxy to trace the young HMXB populations as demonstrated by the tighter $Hr$-based scaling relations (Table 6).

All of the SFR indicators can break down in regions with extremely low SFR. In such regions, UV photons originating from A-type or post-AGB stars may give significant contributions. The UV luminosity emitted by a stellar population is the sum of the emission from young and old stars. The $Hr$ SFR indicator is based on the number of Lyman continuum photons, assuming that all the Lyman photons are absorbed by the gas (case-B recombination; Osterbrock
The results are shown in Figure 10. Older stellar populations make no significant contribution to the ionizing-photon budget in regions with sSFR $\gtrsim 10^{-12} M_\odot$ yr$^{-1}$/$M_\odot$.

Even this upper limit assumes that all UV photons from the older stellar populations contribute to the ionization of the interstellar medium, but in real spiral galaxies, many such photons escape. Therefore, the derived limiting sSFR is a conservative limit for trustworthy SFRs from young stellar populations, but lower sSFR than this value cannot be reliably measured by H$\alpha$. This limiting sSFR is insensitive to the SFRH. The corresponding limiting SFR of course depends on stellar mass. At the sSFRs of the most actively star-forming regions in our sample the ionizing photon production rate exceeds that of the old by $\Delta$ex. For the present study, as shown in Fig. 1, at most 3.5% of the regions (and fewer for the regions smaller than 4x4 kpc$^2$) have sSFR$<10^{-12} M_\odot$ yr$^{-1}$/$M_\odot$, indicating that UV photons from older stellar populations do not affect our present conclusions.

Table 7. The maximum likelihood fit results for $L_X$--sSFR--$M_*$ relation.

| Scale | $\alpha$ | $\beta$ | $\sigma$ | $\alpha$ | $\beta$ | $\sigma$ | $\alpha$ | $\beta$ | $\sigma$ |
|-------|----------|--------|--------|----------|--------|--------|----------|--------|--------|
|       | $L_X$    |        |        | $L_X$    |        |        | $L_X$    |        |        |
| 1x1 kpc$^2$ | 39.03$^{+0.06}_{-0.06}$ | 30.18$^{+0.04}_{-0.03}$ | 0.83$^{+0.02}_{-0.02}$ | 39.53$^{+0.07}_{-0.07}$ | 30.51$^{+0.02}_{-0.02}$ | 0.89$^{+0.01}_{-0.01}$ | 39.43$^{+0.06}_{-0.10}$ | 30.31$^{+0.04}_{-0.04}$ | 0.96$^{+0.02}_{-0.02}$ |
| 2x2 kpc$^2$ | 39.61$^{+0.05}_{-0.06}$ | 29.54$^{+0.06}_{-0.07}$ | 0.66$^{+0.03}_{-0.03}$ | 39.69$^{+0.06}_{-0.13}$ | 30.06$^{+0.10}_{-0.05}$ | 0.97$^{+0.02}_{-0.03}$ | 39.09$^{+0.04}_{-0.11}$ | 30.19$^{+0.06}_{-0.16}$ | 0.97$^{+0.03}_{-0.03}$ |
| 3x3 kpc$^2$ | 39.38$^{+0.09}_{-0.22}$ | 29.78$^{+0.07}_{-0.07}$ | 0.73$^{+0.05}_{-0.05}$ | 39.91$^{+0.06}_{-0.07}$ | 29.29$^{+0.27}_{-0.26}$ | 0.93$^{+0.02}_{-0.02}$ | 39.72$^{+0.05}_{-0.15}$ | 29.82$^{+0.10}_{-0.25}$ | 0.83$^{+0.01}_{-0.01}$ |
| 4x4 kpc$^2$ | 39.47$^{+0.21}_{-0.20}$ | 29.63$^{+0.13}_{-0.13}$ | 0.89$^{+0.07}_{-0.08}$ | 39.87$^{+0.16}_{-0.02}$ | 28.94$^{+0.35}_{-0.41}$ | 1.02$^{+0.04}_{-0.06}$ | 39.64$^{+0.13}_{-0.02}$ | 29.47$^{+0.17}_{-0.08}$ | 1.03$^{+0.02}_{-0.02}$ |

Table 8. Results of the calculations based on the model of Fragos et al. 2013, the $H\alpha$, 8 $\mu$m, 24 $\mu$m response functions, and the SFHs shown in Figure 9.

|        | MW | M51 | C(1) | LMC | RB |
|--------|----|-----|------|-----|----|
| log $M_*(M_\odot)$ | Eq. 9 | 10.74 | 11.02 | 11.00 | 0.03 | 10.44 |
| SFR$_{H\alpha}$ (M$_\odot$ yr$^{-1}$) | Eq. 10 | 3.17 | 4.30 | 10.00 | 0.40 | 6.00 |
| SFR$_{8\mu m}$ (M$_\odot$ yr$^{-1}$) | 3.34 | 4.72 | 10.00 | 0.39 | 13.24 |
| SFR$_{24\mu m}$ (M$_\odot$ yr$^{-1}$) | 3.35 | 4.74 | 10.00 | 0.39 | 13.53 |
| $t_{eff}$ (H$\alpha$; Myr) | Eq. 11 | 10 | 9 | 9 | 9 |
| $t_{eff}$ (8 $\mu$m; Myr) | 709 | 569 | 594 | 272 | 194 |
| $t_{eff}$ (24 $\mu$m; Myr) | 716 | 574 | 601 | 276 | 204 |
| log $L_X$ (erg s$^{-1}$) | Eq. 12 | 42.99 | 43.15 | 43.45 | 42.03 | 43.73 |
| log $L_X$ (erg s$^{-1}$) | 42.95 | 43.09 | 43.42 | 42.01 | 43.72 |
| log $L_X$ (erg s$^{-1}$) | 41.97 | 42.27 | 42.35 | 40.85 | 42.43 |
| log $\alpha'/\beta'$ (H$\alpha$; M$_\odot$ yr$^{-1}$/M$_\odot$) | Eq. 13 | -10.25 | -10.40 | -10.01 | -9.44 | -9.66 |
| log $\alpha'/\beta'$ (8 $\mu$m; M$_\odot$ yr$^{-1}$/M$_\odot$) | -10.23 | -10.36 | -10.01 | -9.45 | -9.33 |
| log $\alpha'/\beta'$ (24 $\mu$m; M$_\odot$ yr$^{-1}$/M$_\odot$) | -10.23 | -10.36 | -10.01 | -9.45 | -9.32 |

$\&$ Ferland (2006). The 8 $\mu$m indicator is based on the number of photons at somewhat longer UV wavelengths, while the 24 $\mu$m indicator is based on the UV luminosity, assuming all the energy is absorbed by dust and reradiated.

In order to quantify the contribution of older stellar populations when measuring extremely low SFRs from H$\alpha$, we calculated separately the SFRs that would be measured for the old and young populations in the aforementioned CIGALE simulations for the five SFH scenarios. We considered as young stars with ages $<100$ Myr and the rest as old. Lyman-continuum photons produced by each population were converted to the equivalent SFR via the Kennicutt (1998) factor. Dividing by stellar mass gave the equivalent sSFR. The results are shown in Figure 10. Older stellar populations make no significant contribution to the ionizing-photon budget in regions with sSFR $\gtrsim 10^{-12} M_\odot$ yr$^{-1}$/M$_\odot$. Even this upper limit assumes that all UV photons from the older stellar populations contribute to the ionization of the interstellar medium, but in real spiral galaxies, many such photons escape. Therefore, the derived limiting sSFR is a conservative limit for trustworthy SFRs from young stellar populations, but lower sSFR than this value cannot be reliably measured by H$\alpha$. This limiting sSFR is insensitive to the SFRH. The corresponding limiting SFR of course depends on stellar mass. At the sSFRs of the most actively star-forming regions in our sample the ionizing photon production rate exceeds that of the old by $\Delta$ex.
X-ray luminosity sub-galactic scaling relations

Figure 9. Upper panel: Bolometric X-ray luminosity per $M_\star$ (in units of $10^9 M_\odot$, green line) of a stellar population as a function of the population’s age from Fragos et al. (2013). Contributions of HMXBs are shown by the blue dashed-dotted line and of LMXBs by the red dashed line. Response functions for $H\alpha$ and 8 $\mu$m are shown with grey dashed and black dashed-dotted lines respectively, and their scales are shown on the right ordinate. The 24 $\mu$m response function is indistinguishable from the 8 $\mu$m one. Bottom panel: Measured SFR as a function of lookback time for five indicative SFHs. The SFHs comprise one representing an early-type spiral galaxy, for which we used the Milky Way’s (MW) SFH (Xiang et al. 2018), the SFH of the Large Magellanic Cloud (LMC) as a proxy for a dwarf galaxy dominated by a recent star-formation episode (Harris & Zaritsky 2009), the SFH of M51 (Eufrasio et al. 2017) as a galaxy with a peak of star formation around 200 Myr ago, the SFH of a galaxy with a recent star-formation burst (RB), formulated as a double exponential model (Boquien et al. 2019) with $t_0 = 4000$ Myr, $t_1 = 3000$ Myr, $\tau_0 = 1000$ Myr, $\tau_1 = 1000$ Myr, and $\kappa = 10$, and a galaxy with constant SFR throughout its history with SFR = $10^3 M_\odot$ yr$^{-1}$ (labeled “C(1)”) for reference. These SFHs are presented with red dashed, blue dashed-dotted, green dotted, yellow, and purple lines respectively. Gray dashed and black dash-dotted lines show the response functions from the upper panel.

Figure 10. Upper panel: Inferred Hα-based SFR separating the contribution of young and old stellar populations as a function of sSFR for three stellar population masses. The lines are based on CIGALE simulations using the five SFHs presented in Fig. 9 with young and old stellar populations separated at 100 Myr. Solid lines represent total SFR, and dashed lines represent the contribution of the old stellar populations. Results are nearly identical for all SFHs. Bottom panel: ratio $\text{SFR}_{\text{total}}/\text{SFR}_{\text{old}}$ as a function of sSFR.

The X-ray luminosity for each SFH scenario (Fig. 9) is:

$$L'_X(v) = \int \left( \frac{L_X(v')}{M_\star} \right)_\nu \text{SFH}(v') \, dv' ,$$

(12)

where $v$ indicates the particular XRB population (HMXBs, LMXBs, XRBs), and $M_\star$ is the total stellar mass of the parent stellar population of the XRBs. The results of these calculations show $\geq 0.85$ dex differences in the X-ray luminosity produced by the HMXBs and LMXBs regardless of the SFH assumed. This difference is larger for SFHs with more intense and more recent star-formation episodes.

A metric of the relative contribution of HMXB and LMXB populations in the integrated X-ray luminosity is the ratio $(\alpha/\beta)$ used in Eq. 8. Given that $L_{X,HMXB} = \alpha \text{SFR}$ and $L_{X,LMXB} = \beta M_\star$, we can calculate the theoretically expected $\alpha'/\beta'$ ratio from the X-ray luminosity of the LMXB and HMXB populations given an SFH (Eq. 12).

$$\left( \frac{\alpha'}{\beta'} \right)_X = \frac{L_{X,HMXB}}{\text{SFR}_X} \left( \frac{L_{X,LMXB}}{M_\star} \right)$$

(13)

for each SFR indicator (Eq. 10). The results for these calculations are presented in Table 8. The continuous SFH gives $\alpha'/\beta' = 10^{-10.01} M_\odot$ yr$^{-1}/M_\odot$. LMC-like or RB-like SFHs, with a recent star-formation episode, show $\alpha'/\beta' > 10^{-10.01} M_\odot$ yr$^{-1}/M_\odot$. On the other hand, MW and M51, which comprise far older stellar populations, show $\alpha'/\beta' < 10^{-10.01} M_\odot$ yr$^{-1}/M_\odot$, indicating a larger contribution of LMXBs to the total X-ray luminosity.
5.2 Distributions of X-ray luminosity for regions with different sSFR

If the X-ray emission arises from a population of HMXBs, it would be expected to scale linearly with SFR. The scaling factor depends on the formation efficiency of HMXBs and their integrated luminosity per unit SFR, which is a function of their age (Figure 9, Section 5.1). Therefore, the galaxy-wide scaling relations are expected to extend to lower SFR even on sub-galactic scales if the average properties of the stellar populations (age and metallicity) are the same. Any deviations from this linear relation or change in slope indicates a different R XB population. As discussed in Section 4.2, we observe an excess of X-ray emission in the low SFR regime compared to the extrapolation of the linear $L_X$–SFR relation from higher SFR. The excess can be quantified as the ratio of the measured luminosity to the one expected from the linear scaling relation of M14,

$$L_{X,\text{excess}} = \frac{L_X}{L_{X,\text{M14}}(\text{sSFR})} \quad .$$

Fig. 11 shows histograms of the excess in regions of different sSFR. The modes and 68.3% confidence intervals of these distributions are presented in Table 9. Regions with lower sSFR exhibit systematically higher excess, including the highest values seen. The bin of sSFR $\leq 10^{-12}$, in particular, isolates sub-galactic regions with very low current star formation, where no massive young stars and consequently HMXBs are expected. At these sSFRs, the dominant source of X-ray emission is expected to be LMXBs (e.g., Pancoast et al. 2010).

In regions encompassing large enough stellar mass, the collective emission of cataclysmic variables (CVs) and coronally active binaries (ABs) may have non-negligible contribution, particularly at the very low integrated X-ray luminosities probed ($\leq 10^{35.5}$ erg s$^{-1}$). The relation between the X-ray luminosity from these components ($L_{X,\text{stellar}}$) and K-band luminosity (Boroson et al. 2011) is:

$$L_{X,\text{stellar}}(\text{erg s}^{-1}) = 9.5^{+2.1}_{-1.1} \times 10^{27} L_K$$

where $L_K$ is in solar luminosities (a proxy of the total stellar mass they encompass). Because in this work we used 3.6 $\mu$m as a proxy of stellar mass, we converted 3.6 $\mu$m to K-band luminosities. For most of the regions, CVs’ and ABs’ stellar contribution to the X-ray luminosity is less than observed by more than 1 dex (98%, 95%, 91%, and 90% of the 1$x$1, 2$x$2, 3$x$3, and 4$x$4 kpc$^2$ regions respectively), even for regions with extremely high stellar mass (Fig 12). However, there are a handful of regions where the calculated stellar X-ray luminosity is comparable to the observed X-ray luminosity, but they also exhibit high relative uncertainties. This minority of regions is not sufficient to explain the observed X-ray luminosity excess. Alternatives being insufficient, the bulk of the X-ray luminosity excess found in the low SFR regime must come from LMXB emission.

4 The 3.6 $\mu$m to K-band magnitudes were calibrated and converted using the complete SFRS. The linear correlation found is:

$$m_K = 1.876 \pm 0.1 + 1.10 \pm 0.01 m_{3.6}$$

5.3 Comparison with galaxy-wide scaling relations

Sub-galactic regions show a shallower slope of $L_X$–SFR (Table 6, Fig. 5) compared to the M14 relation for all cases considered in this work. This is driven by regions with high X-ray luminosity at SFR $\leq 10^{-3}$ $M_\odot$ yr$^{-1}$, particularly at the smallest physical scales. For reference, the lowest SFR used in the derivation of the galaxy-wide scaling relation was $\sim 10^{-1}$ $M_\odot$ yr$^{-1}$, whereas our analysis extends to 5 dex lower SFR. The X-ray emission of these regions arises from an unresolved population of LMXBs (Section 5.2). The inclusion of the stellar mass as a parameter (Eq. 8) accounts for the LMXB contribution, particularly in regions with low SFR or those dominated by older stellar populations (low sSFR). As a result we obtain good fits with linear scaling of the X-ray luminosity with respect to both the SFR and stellar mass.

Even though our $L_X$–SFR–$M_*$ fits follow a different approach from L16, by fitting sub-galactic regions and including an intrinsic scatter term (Eq. 8), our results are in good agreement (Fig. 8) with only small differences in the best-fit parameters. The main difference is that we find significant intrinsic scatter. We interpret the scatter as the result of stochastic effects. In all cases, the Hα SFR indicator gives...
the lowest scatter and the best agreement with the relation of L16 (despite their use of UV and far-IR instead of Hα-based SFR tracers). However, we do find differences with the Lehmer et al. 2019 scaling relations, which are based on integration of the XRB luminosity functions (XLFs) derived for different sSFR regimes. More specifically, while for the largest physical scales (4 × 4 kpc$^2$) and the scaling with SFR (parameter $\alpha$) in the $L_X - \text{SFR}$–$M_*$ fit (Eq. 8) we find good agreement for all SFR indicators used, in the case of smaller physical scales, we find increasing $L_X - \text{SFR}$ scaling factors (Table 7). This can be explained by the local variations of stellar populations between the different regions, which results in localised variations of the $L_X/\text{SFR}$ scale factor (e.g., Section 5.1). This effect in combination with stochastic sampling of the XLF results in a few regions with high X-ray luminosity (because of the presence of very young populations and/or luminous individual sources) and therefore small $L_X$ and SFR uncertainties, that can drive the fits to steeper slopes. At larger scales, local variations in the X-ray emission and stellar populations are averaged out, and the scaling relations approach the galaxy-wide relations. On the other hand, the $L_X - M_*$ scaling (parameter $\beta$ in Eq. 8) is consistent with Lehmer et al. (2019) for most SFR indicators and spatial scales we consider. The smoother spatial distribution of the older stellar populations and the weak $L_X$–age dependence of the X-ray binaries associated with them results in more uniform sampling regardless of physical scales and therefore consistent $L_X - M_*$ scaling factors through the different physical scales.

5.4 Intrinsic scatter & stochasticity

The wide range of SFRs and stellar masses probed in our study (Fig. 1, Table 1) is ideal for examining the intrinsic scatter under conditions found in nearby galaxies. This scatter could be the result of (a) Poisson sampling of sparsely populated luminosity functions or (b) time variability of the XRBs (e.g., Gilfanov 2004). Such scatter has been previously reported in galaxy-wide scaling relations, particularly at lower SFRs (e.g., Mineo et al. 2014; Lehmer et al. 2019). However, as discussed in Section 5.1, an additional source of scatter could be stellar population differences through their effect on the inferred SFR and the age-dependent X-ray output of stellar populations.

There is intrinsic scatter in the sub-galactic $L_X - \text{SFR}$ (Table 6) and $L_X - \text{SFR} - M_*$ (Table 7) correlations. However, we do not find any evidence for anti-correlation of the intrinsic scatter with the SFR (Table 5) as would be expected from stochasticity or time variability. This could be the result of the large uncertainties in the SFR and X-ray luminosity measurements for the individual regions at low SFR, which could mask any such trend. On the other hand, the overall intrinsic scatter we measure both in the $L_X$–SFR and the $L_X$–SFR–$M_*$ relations (typically 0.5–1.0 dex) is larger than the scatter observed in the galaxy-wide relations (e.g., <0.37 dex in L16). This additional scatter could be the result of bright X-ray sources in some of the individual regions. However, typically less than 3% of the regions in each galaxy of our sample encompass individually detected X-ray sources, making them an unlikely source for the increased scatter on sub-galactic regions.

One parameter that is particularly important on sub-galactic scales is local variations of the stellar populations, such as those resulting from the spiral structure, localized star-formation episodes, sequential star formation, and metallicity gradients. XRB population synthesis models show that the X-ray emission for an ensemble of XRBs is a strong function of the age and metallicity of their parent stellar populations (e.g., Fragos et al. 2013; Dray 2006; Linden et al. 2010; Lehmer et al. 2019). This is supported by observational studies of the XRB populations associated with different stellar generations (e.g., Antoniou & Zezas 2016; Antoniou et al. 2019a) or populations of different metallicity (e.g., Mapelli et al. 2010; Prestwich et al. 2013; Douma et al. 2015; Brorby et al. 2016). On galaxy-wide scales, any local variations of the stellar populations and the corresponding X-ray emission can be smeared out giving an average $L_X/\text{SFR}$ value for the entire galaxy. On the other hand, local variations of the stellar populations within a galaxy (which can vary in age from a few Myr for very young star-forming regions to several Gyr for interarm regions) can result in very different X-ray emission efficiency as discussed in Section 5.1.

An additional source of scatter could be local variations of absorption. In order to correct for this one would need spatially resolved extinction and $N_H$ maps from X-ray spectral fits in each sub-galactic region, which are not available for these data (c.f. Section 2.4). Furthermore as discussed in Section 4.2, the absorption in Hα and soft X-rays is similar, which reduces the effect of differential extinction across the galaxies.

A general trend is that scaling relations based on the Hα emission show lower scatter than the relations based on the 8 μm and 24 μm SFR indicators. Hα emission traces the stellar populations with ages ~10 Myr (Fig. 9; Table 8) which are most relevant to the HMXBs (which have lifetimes ≤30 Myr; Section 4.2). On the other hand, although the IR-based SFR indicators still trace young stellar populations, they are sensitive to a much wider range of ages. Therefore, they are
not a clean proxy for the star-formation episodes that produced the HMXBs. This mismatch between the formation timescales of the HMXBs and the star-formation timescales probed by the different SFR indicators could be the origin of the larger scatter we measure in the sub-galactic scaling relations in comparison to the galaxy-wide relations. This is because sub-galactic regions may have significant variations in their SFHs compared to the overall galaxy averages.

6 SUMMARY

We present scaling relations between $L_X$–SFR–$M_\star$ on sub-galactic scales using a maximum likelihood method that takes into account the posterior (not necessarily Gaussian) uncertainty distributions of all the data. In this way we obtain unbiased scaling relations by including in our analysis regions that have extremely low SFRs, stellar masses and X-ray luminosities which otherwise would be omitted. This analysis extends the $L_X$–SFR and the $L_X$–SFR–$M_\star$ relations down to SFRs $\approx 10^{-6} M_\odot$ yr$^{-1}$, and sSFRs $\approx 10^{-14} M_\odot$ yr$^{-1}$/$M_\odot$. These are 5 dex and 2 dex lower in SFR and sSFR respectively than existing galaxy-wide scaling relations. In the $L_X$–SFR correlation, slopes are shallower than linear on all sub-galactic scales (1x1, 2x2, 3x3, and 4x4 kpc$^2$) and by all SFR indicators (H$\alpha$, 8 $\mu$m, and 24 $\mu$m) used in this work. This shallower slope is driven by regions with high X-ray luminosity at low SFR ($\lesssim 10^{-3} M_\odot$ yr$^{-1}$), probably due to a population of LMXBs. For larger sub-galactic regions, correlations of $L_X$–SFR converge to the integrated galactic emission relations.

The full-band X-ray luminosity fits are very similar to those of the hard band. Although the use of the full X-ray band increases the scatter in the correlations, it integrates more flux and therefore can be very useful for low-X-ray-luminosity objects. The extended relations we present can be used to model the X-ray output of extremely low-SFR luminosity objects. The extended relations we present can acknowledge support from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie RISE action, grant agreement No 691164 (ASTROSTAT). AZ also acknowledges support from Chandra grant GO2-3111X.

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APPENDIX A: MAXIMUM LIKELIHOOD FITTING METHOD

We fit a linear model with intrinsic scatter to the SFR and X-ray luminosity of the regions of all galaxies. Specifically, we consider the errors-in-variables regression model:

\[ x_i = x'_i + \eta_i \]
\[ y_i = y'_i + \zeta_i \]

(A1)

\[ y'_i = ax'_i + b + \epsilon(x'_i) \]

where \( x_i \) and \( y_i \) are the observed log SFR and log \( L_X \) of the \( i \)-th region, while \( x'_i \) and \( y'_i \) are the respective intrinsic values; \( \eta_i \) and \( \zeta_i \) denote the error distributions on \( x_i \) and \( y_i \) respectively. For the intrinsic scatter \( \epsilon \) we consider two cases: (i) constant:

\[ \epsilon(x'_i) = \sigma, \quad \text{where } \sigma \geq 0 \]

(A2)

and (ii) including a term linear in the logarithm of SFR:

\[ \epsilon(x'_i) = \max\{0, \sigma_1 x'_i + \sigma_2\} \]

(A3)

where the ‘max’ function ensures that the intrinsic scatter is non-negative.

Assuming independent measurements, the posterior probability of the model parameters, \( \tilde{\rho} = (a, b, \sigma_1, \sigma_2) \):

\[ \pi(\tilde{\rho}) \prod_i P(x_i, y_i|\tilde{\rho}) \]

(A4)

where the prior is the product of the priors of each parameter

\[ \pi(\tilde{\rho}) = \pi(a)\pi(b)\pi(\sigma) \]

or

\[ \pi(a)\pi(b)\pi(\sigma_1)\pi(\sigma_2) \]

(A5)

and the data likelihood is the marginalization of the likelihood considering all possible values for the intrinsic SFR and X-ray luminosity

\[ P(x_i, y_i|\tilde{\rho}) = \int P(x_i|\tilde{\rho}), P(y_i|x'_i, \tilde{\rho}) dx'_i dy'_i \]

(A6)

Considering that (i) the observed values depend only on the measurement errors and the intrinsic values, (ii) the intrinsic values depend only on the intrinsic model, and (iii) the errors on \( x_i \) and \( y_i \) are independent, the integrand of (A6) becomes

\[ P(x_i|x'_i, \eta'_i) P(y_i|x'_i, \zeta_i) P(y'_i|x'_i, \tilde{\rho})P(x'_i|\tilde{\rho}) \]

(A7)

where the probability of \( x_i \) and \( y_i \) was computed using the

\[ \eta_i \text{ and } \zeta_i \text{ are not normally distributed because they represent the logarithmic transformation of the truncated Gaussian errors on SFRs (zero-truncated) and the logarithm of the X-ray luminosity (Section 3) } \]
corresponding distributions of $\eta_i$ and $\zeta_i$, the prior on $x'_i$ was chosen to be uniform between two bounds $x'_{\text{min}}$ and $x'_{\text{max}}$ (ensuring that they enclose all the observed values $x_i$ and $3\sigma$ around them), and the probability of $x'_i$ was given by the normal distribution density considering the intrinsic mean and scatter:

$$P\left(x'\right) = \frac{1}{\sqrt{2\pi\sigma_{x'_i}}} \exp\left[-\frac{\left(x'_i - \text{median}_{x'_i}\right)^2}{2\sigma_{x'_i}^2}\right].$$

The model parameters $a$, $b$, and $\sigma$ (or $\sigma_1$ and $\sigma_2$) were estimated by sampling the posterior distribution using the Markov Chain Monte Carlo technique. Specifically, we used the emcee 3.0rc2 package for Python 3 (Foreman-Mackey et al. 2013) with uniform priors for the model parameters, sufficiently wide to not be very informative but narrow enough to aid the convergence of the MCMC chains, i.e., $a \in [0, 2], b \in [38, 41], \sigma \in [0, 2], \sigma_1 \in [-1, 1]$ and $\sigma_2 \in [0, 2]$. The priors were also used to sample the initial positions of the Markov chains.

In order to fit the scaling with both the SFR and the stellar mass, i.e.,

$$\log L_X = \log(10^{a \log SFR + b} + 10^{\beta \log M_\star}) + \sigma,$$

we employed the intrinsic mean model

$$y'_i = \log \left(10^{a \log s_f + b} + 10^{\beta \log m_i}\right),$$

where $m_i$ is the logarithm of the stellar mass of the $i$-th region with error distribution $\xi_i$ with respect to its intrinsic value:

$$m_i = m'_i + \xi_i.$$

Now, the datum likelihood is a triple integral,

$$P(x_i, m_i, y_i | \vec{p}) = \iiint P(x_i, m_i, y_i, x'_i, y'_i | \vec{p}) \, ,$$

but using the same assumptions as before (i.e., independent measurements), the integral is the same as in equation A7 with an additional multiplicative PDF term for the stellar mass measurement $P(m_i | m'_i, \xi_i)$.

Results are shown in Figures 5, 6, 7, and 8 and Tables 5, 6, and 7. An example of the results of the fits is shown in Fig. A1.

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Figure A1. The marginal posterior distributions of the three parameters of the model: $\log L_X = a \log SFR + b + \sigma$ in the case of H$\alpha$ SFR and $1 \times 1$ kpc$^2$ sub-galactic regions.