Big Bang/ Big Crunch Cosmologies in Matrix Theory and AdS/CFT

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Abstract.
Recently several versions of gauge-gravity duality in string theory have been used to understand null and space-like singularities. In many cases, the gauge theory description allows a smooth evolution across what appears as a singularity in the gravity description.

1. Introduction
In almost all proposals for a quantum theory of gravity, usual dynamical space-time is only an approximation to some deeper structure. In String Theory, we have very concrete ideas about what these structures are in some situations. These are situations where gravitational physics has a tractable holographic description [1] in terms of a non-gravitational theory in lower number of space-time dimensions. In view of the spectacular success of the holographic principle in black hole physics, it is natural to explore whether this can be used to understand conceptual issues posed by singularities.

In String Theory, holography is a special case of a more general duality between open and closed strings. This duality implies that the dynamics of open strings contains the dynamics of closed strings. Since closed strings contain gravity, space-time questions can be posed in the open string theory which does not contain gravity and therefore conceptually easier. Under special circumstances, the open string theory can be truncated to its low energy limit - which is a gauge theory on a fixed background. In these situations, open-closed duality becomes particularly useful. The simplest example is non-critical closed string theory in two space-time dimensions. Here the holographic theory is gauged Matrix Quantum Mechanics [2]. The second class of examples involve string theory or M theory defined on spacetimes with a compact null direction. Then a sector of the theory with some specified momentum in this null direction is dual to a \( d + 1 \) dimensional gauge theory, where \( d \) depends on the number of additional (spacelike) compact directions. Using standard terminology we will call them Matrix theories [3]-[6]. Finally, the celebrated AdS/CFT correspondence [7] relates closed string theory in asymptotically anti-de-Sitter spacetimes to gauge theories living on their boundaries. In all these cases, the dynamical “bulk” spacetime (on which the closed string theory lives) is an approximation which holds in a specific regime of the gauge theory. In this regime, the closed string theory reduces to supergravity. Generically, there is no space-time interpretation, though the gauge theory may make perfect sense. This fact opens up the possibility that in regions where the bulk gravity description is singular, one may have a well defined gauge theory description and one has an answer to the question: What replaces space-time?
Treating time dependent backgrounds in string theory, particularly those with singularities, has been notoriously difficult. However, some modest progress has been made recently in both worldsheet formulations as well as holographic formulations of all the three types mentioned above. The key idea in these various types of holography are similar. One looks for toy models where the space-time background on which the closed string theory is defined is singular, but the holographic gauge theory description does not appear to be problematic. Thus, the gauge theory provides the correct description of the region which would appear singular if the gravity interpretation is extrapolated beyond its regime of validity.

In the following, we will discuss recent attempts to understand cosmological singularities using Matrix theories as well as AdS/CFT correspondence. The work on Matrix Theory is based on [9, 10]. The work on AdS/CFT is based on [11, 12, 13].

2. Matrix Big Bangs

In [14], Craps et. al. considered Type IIA string theory with string coupling \( g_s \) and string length \( l_s \), living on a flat string frame metric with a compact null direction \( x^- \) with radius \( R \)

\[
ds^2 = 2dx^+dx^- + d\vec{x} \cdot d\vec{x},
\]

and a dilaton linearly proportional to the other null direction \( x^+ \)

\[
\Phi = -Qx^+,
\]

As a supergravity solution, this background preserves half of the supersymmetries which satisfy \( \Gamma^+ \epsilon = 0 \). For \( Q > 0 \), the effective string coupling \( \bar{g}_s = g_s e^{-Qx^+} \) is small for \( x^+ \to \infty \) and one should have a perturbative spectrum, while for \( x^+ \to -\infty \) the string theory becomes strongly coupled and the corresponding Einstein metric has a null big bang like singularity.

For \( Q = 0 \), DLCQ string theory in this background with a momentum

\[
p_- = \frac{J}{\bar{R}}
\]

is dual to a 1 + 1 dimensional \( U(J) \) gauge theory - usually called Matrix String Theory [5]-[6] - living on a circle of radius \( \bar{R} \) given by

\[
\bar{R} = \frac{l_s^2}{\bar{R}};
\]

and a Yang-Mills coupling given by

\[
gYM = \frac{R}{g_s l_s^2}.
\]

The bosonic part of the gauge theory action is

\[
S = \int d\tau \int_0^{2\pi R} d\sigma \text{Tr} \left\{ \frac{1}{2gYM} F_{\tau \sigma}^2 + \frac{1}{2} (D_\tau X^i)^2 - \frac{1}{2} (D_\sigma X^i)^2 + \frac{gYM}{4} [X^i, X^j]^2 \right\}
\]

where \( X^i, i = 1 \cdots 8 \) are adjoint scalars. The above relations show that when the original string coupling is small, \( g_s \ll 1 \), the Yang-Mills coupling is large and the theory flows to the IR. The

\[
1 \text{ For discussions of cosmological singularities in the Matrix Model description of two dimensional string theory, see [8]}
\]

\[
2 \text{ For } Q < 0 \text{ we have a time-reversed situation where the big bang is replaced by the big crunch. In this paper we will exclusively deal with } Q > 0.
\]
potential term then restricts the $X^i$ to belong to a cartan subalgebra and may be therefore chosen to be diagonal

$$X^i = \text{diag}(X^i_1, X^i_2, \cdots X^i_J)$$

in a suitable gauge. The gauge field decouples, and one is left with $8J$ scalar fields $X^i_n$ in $1 + 1$ dimensions. The boundary conditions of these fields are labelled by the conjugacy classes of the group. For example, the maximally twisted sector has

$$X^i_n(\sigma + 2\pi) = X^i_{n+1}(\sigma)$$

where $X^i_{J+1} \equiv X^i_1$. In this sector we therefore have $8$ scalars on a circle of size $2\pi l_B^2$ and the action then reduces to the worldsheet action of a single string in a light cone gauge. As is appropriate in the light cone gauge, the spatial extent of the worldsheet is proportional to the longitudinal momentum $p_-$. In a similar way one has boundary conditions with cycles of smaller length - these sectors represent multiple strings. Effects of finite $g_{YM}R$ are now manifested as string interactions.

The fields in the Yang-Mills theory are the low energy degrees of freedom of open string field theory on D1 branes. Holography is realized as the metamorphosis of the fields $X^i$ of the YM theory into transverse coordinates in ten dimensional space-time. Note that this space-time interpretation is valid only when $g_s \ll 1$. For finite $g_s$ the Yang-Mills theory of course makes perfect sense - but there is no natural space-time interpretation of the nonabelian degrees of freedom.

In [14] it was argued that a similar Matrix String Theory may be written down for $Q \neq 0$. The line of reasoning which leads to this is similar to the Sen-Seiberg argument [15], but more subtle - as explained in [14]. The action is a simple modification of (6)

$$S = \int d\tau \int_{0}^{2\pi R} d\sigma \text{Tr}\{e^{-Q\tau} F_{\tau\sigma}^2 + \frac{1}{2} (D_{\tau} X^i)^2 - \frac{1}{2} (D_{\sigma} X^i)^2 + \frac{g_{YM}^2}{4} e^{Q\tau} [X^i, X^j]^2\}$$

Since this is essentially the action of $J$ D1 branes in the light cone gauge, $\tau$ is the same as the coordinate $x^+$ in the background. Thus, in the far future in light cone time, the gauge theory is strongly coupled, while near the singularity at $x^+ \to -\infty$ the gauge theory is weakly coupled. This means that while the theory has a nice interpretation as a space-time theory with dynamical gravity in the future, such an interpretation breaks down at $\tau \to -\infty$ - precisely the place where there is a null singularity. Here all the $J^2$ degrees of freedom are relevant and might "resolve" the singularity.

2.1. IIB Big Bangs

The Type IIB version of this background shows a richer structure [10]. The background is once again given by (1) and (2) where both $x^-$ and $x^8$ are compact,

$$x^- \sim x^- + 2\pi R, \quad x^8 \sim x^8 + 2\pi R_B$$

The usual DLCQ Matrix Theory logic then implies that string theory in the sector with $p_- = J/R$ and $p_8 = 0$ is described by a $SU(J)$ $2+1$ dimensional Yang-Mills theory of $J$ D2 branes [4], [5]. This gauge theory lives on a $T^2$ with sides

$$R_\rho = g_B l_B^2 R, \quad R_\sigma = l_B^2 R$$

where $g_B, l_B$ are the string coupling and the string length of the original IIB theory. The dimensional coupling constant of the Yang-Mills is

$$G_{YM}^2 = \frac{R}{R_\sigma R_\rho} = \frac{RR_B^2}{g_B l_B^4}$$
We will call this theory "Matrix Membrane Theory".

The action of this matrix membrane theory is given by

$$S = \int d\tau \int_0^{2\pi R_\tau} d\sigma \int_0^{2\pi R_\sigma} d\rho \mathcal{L}$$

(13)

where

$$\mathcal{L} = \text{Tr} \left\{ \frac{1}{2} [(D_\tau X^a)^2 - (D_\sigma X^a)^2 - e^{2Q\tau} (D_\rho X^a)^2] + \frac{1}{2(GYM e^{Q\tau})^2} [F_{\tau\tau}^2 + e^{2Q\tau} (F_{\rho\rho}^2 - F_{\rho\sigma}^2)] 
+ \frac{(GYMe^{Q\tau})^2}{4} [X^a, X^b]^2 \right\},$$

(14)

where $X^a, a = 1 \cdots 7$ are now seven scalar fields and $F_{\mu\nu}$ denotes the gauge field strength. Note that there is a factor of $e^{Q\tau}$ with each $\partial_{\rho}$ or a covariant vector component $V_{\mu}$, in addition to a factor of $e^{Q\tau}$ for each $GYM$.

For $Q = 0$ and $g_B \ll 1$ this action reproduces the worldsheet action for Type IIB strings in the light cone gauge. In this limit the commutator terms force the fields to be diagonal. The gauge field strengths can be dualized to a scalar which we will call $X^8$, so that we have a 2+1 dimensional action of eight scalar fields. Finally, since for small $g_B$ we have $R_\rho \ll R_\sigma$, the action reduces to a 1+1 dimensional action which may be then identified with the Green-Schwarz light cone worldsheet theory. Once again sectors of boundary conditions describe up to $J$ strings with the spatial extent of the worldsheets proportional to their longitudinal momenta.

This story changes interestingly when $Q \neq 0$. The mass scale associated with the Kaluza Klein modes in the $\rho$ direction is given by $M_{KK} \sim \frac{R}{g_B^2 l_B^4}$ while the mass scale which determines the non-abelian dynamics is $GYM$ given in (12). Thus for $R_B \gg l_B$ the KK modes are much lighter than the Yang-Mills scale. In our present time-dependent context, these scales become time-dependent and it follows from the coupling and the $\partial_{\rho}$ terms in (14) that the KK modes are expected to decouple much later than the time when the non-abelian excitations decouple. Therefore, there is a regime where we can ignore the non-abelian excitations, but cannot ignore the KK modes. In this regime, the Matrix Membrane Lagrangian density is given by

$$L_{\text{diag}} = \frac{1}{2} \sum_{l=1}^{8} (\partial_{\tau} X^l)^2 - (\partial_{\sigma} X^l)^2 - e^{2Q\tau} (\partial_{\rho} X^l)^2 - 2\mu^2 \sum_{l=1}^{8} (X^l)^2$$

(15)

It is tempting to argue that as $\tau \to \infty$ the Kaluza-Klein modes in the $\rho$ direction become infinitely massive, so that the theory becomes 1+1 dimensional and exactly identical to the Green-Schwarz string action in this background. However, this is too hasty since we have a time-dependent background here and energetic arguments do not apply.

Instead, we should ask whether any state at an early time evolves into a state of the perturbative fundamental string - i.e. states which do not carry any momentum in the $\rho$ direction. The modes of the field $Y^I(\rho, \sigma, \tau)$ which are positive frequency at early times are given by

$$\varphi^{(in)}_{m,n} = \left\{ \frac{R}{8\pi^2 l_B^2 g_B} \right\}^{1/2} \frac{1}{\Gamma(1 - i\omega_m/Q)} e^{i(\omega_m n + \omega_m n R) / Qg_B l_B^2} J_{-i\omega_m} (\kappa_n e^{Q\tau})$$

(16)

where

$$\omega_m^2 = \frac{m^2 R^2}{l_B^2}, \quad \kappa_n = \frac{nR}{Qg_B l_B^2}$$

(17)

while those which are appropriate at late times are

$$\varphi^{(out)}_{m,n} = \left\{ \frac{R}{16\pi l_B^4 g_B Q} \right\}^{1/2} e^{i(\omega_m n + \omega_m n R) / Qg_B l_B^2} H_{-i\omega_m}^{(2)} (\kappa_n e^{Q\tau})$$

(18)
The problem at hand is identical to that of a bunch of two dimensional scalar field (living on $\tau, \sigma$ spacetime) with time dependent masses. It is well known that such time dependence leads to particle production or depletion [16], [17],[18]. Because of standard relations between the Hankel function $H^{(2)}_\nu(z)$ and the Bessel function $J_\nu(z)$ there is a non-trivial Bogoliubov transformation between these modes which imply that the vacua defined by the in and out modes are not equivalent. In fact, the out vacuum $|0 >_{\text{out}}$ is a squeezed state of the “in” particles. In other words, if we require that the final state at late times does not contain any of the KK modes, the initial state must be a squeezed state of these modes. The occupation number of the in modes in the out state is thermal

$$\alpha < 0 | a^{\dagger}_{m,n} & a_{m,n} | 0 >_{\text{out}} = \frac{1}{e^{\frac{2\pi m\alpha}{Q}} - 1}$$

(19)

Note that the Bogoliubov coefficients and number densities depend only on $m$ for all $n \neq 0$. This follows from the fact that $n$- dependence may be removed by shifting the time $\tau$ by $\log(\kappa_n)$. However, the modes with $n = 0$ need special treatment. Indeed, in the $n \to 0$ limit the “in” modes (16) go over to standard positive frequency modes of the form $e^{-i\omega m \tau}$ as expected. In this limit, however, the out modes (18) contain both positive and negative frequencies. This is of course a wrong choice, since for these $n = 0$ modes there is no difference between “in” and “out” states. In fact, the “out” modes (18) have been chosen by considering an appropriate large time property for nonzero $n$ and do not apply for $n = 0$. In other words, the squeezed state contains only the $n \neq 0$ modes.

The operators $a^{\dagger}_{m,n}$ in fact create states of $(p,q)$ strings in the original Type IIB theory [4]. To see this, let us recall how the light cone IIB fundamental string states arise from the $n = 0$ modes of the Matrix Membrane. In this sector, the action is exactly the Green-Schwarz action. The oscillators $a^{\dagger}_{m,0}$ defined above are in fact the world sheet oscillators and create excited states of a string. The gauge invariance of the theory allows nontrivial boundary conditions, so that $m$ defined above can be fractional. Equivalently the boundary conditions are characterized by conjugacy classes of the gauge group. The longest cycle corresponds to a single string whose $\sigma$ coordinate has an extent of $2\pi J_\nu^2$ which is the same as $2\pi J_\nu^2$ as it should be in the light cone gauge. Shorter cycles lead to multiple strings - the sum of the lengths of the strings is always $2\pi J_\nu^2$, so that there could be at most $J$ strings. Note that $m$ is the momentum in the $\sigma$ direction: a state with net momentum in the $\sigma$ direction in fact corresponds to a fundamental IIB string wound in the $x^-$ direction. This may be easily seen from the chain of dualities which led to the Matrix Membrane.

As shown in [4], following the arguments of [20, 19, 21], $SL(2, Z)$ transformations on the torus on which the Yang-Mills theory lives become the $SL(2, Z)$ transformations which relate $(p,q)$ strings in the original IIB theory. In particular, the oscillators $a^{\dagger}_{m,0}$ create states of a D-string.

The state $|0 >_{\text{out}}$ therefore contain excited states of these $(p,q)$ strings. The number of such strings depends on the choice of the conjugacy classes characterizing boundary conditions. Since each $(m,n)$ quantum number is accompanied by a partner with $(-m, -n)$ this state does not carry any F-string or D-string winding number. Finally this squeezed state contains only $n \neq 0$ modes, i.e. they do not contain the states of a pure F-string. We therefore conclude that in this toy model, the initial state has to be chosen as a special squeezed state of unwound $(p,q)$ strings near the big bang to ensure that the late time spectrum contains only perturbative strings.

2.2. pp-wave Big Bangs

The nonabelian degrees of freedom of Matrix String Theory or Matrix Membrane theory become important near the “singularity”. In the background considered above, this theory has one length scale - given by the Yang-Mills coupling $G^{-1}_YM$. It would be worthwhile to find similar situations
with an additional length scale with the hope that tuning the dimensionless ratio would allow us to go to a regime where some class of nonabelian configurations become important. One such example is provided by pp-waves \([10], [9]\). The string frame metric \((1)\) is now modified to \(^3\)

\[
ds^2 = 2dx^+dx^- - 4\mu^2[(x^1)^2 + \cdots (x^6)^2](dx^+)^2 - 8\mu x^7 dx^8 dx^+ + [(dx^1)^2 + \cdots (dx^8)^2]
\]

(20)

The dilaton remains the same as \((2)\), and there is an additional 5-form field strength

\[
F_{+1234} = F_{+5678} = \mu e^{Qx^+}
\]

(21)

For \(Q = 0\), the matrix membrane theory has been considered in \([24]\). The detailed action in this background has been derived in \([23] \) and \([26]\). The matrix membrane action for \(Q \neq 0\) now has additional terms \([10]\)

\[
\mathcal{L} = \text{Tr} \left\{ \frac{1}{2}((D_\tau X^a)^2 - (D_\sigma X^a)^2 - e^{2Q\tau}(D_\rho X^a)^2) + \frac{1}{2(G_{YM} e^{Q\tau})^2}[F_{\rho \tau}^2 + e^{2Q\tau}(F_{\rho \tau}^2 - F_{p \rho}^2)] - 2\mu^2[(X^1)^2 + \cdots (X^6)^2 + 4(X^7)^2] + \frac{(G_{YM} e^{Q\tau})^2}{4}[X^a, X^b]^2 - \frac{4\mu}{(G_{YM} e^{Q\tau})}e^{Q\tau}X^7 F_{\rho \sigma} - 8\mu i(G_{YM} e^{Q\tau})X^7 [X^5, X^6] \right\},
\]

(22)

The new length scale is now \(\mu\).

Let us briefly recall the physics of this model for \(Q = 0\). When the original IIB theory is weakly coupled, \(g_B \ll 1\) with \(\frac{\mu^2}{\pi R_B^2} \sim O(1)\), the effective coupling constant of this YM theory is strong. Then, along the lines of the discussion in the previous subsection, the action becomes identical to the worldsheet action for Green-Schwarz string in the pp-wave background \(^4\). In fact, as shown in \([24]\), integrating out the Kaluza Klein modes in the \(\rho\) direction generates string couplings with exactly the correct strength.

It is straightforward to see that one could rescale the fields and the coordinates to write the lagrangian \(\mathcal{L}\) in the form

\[
\mathcal{L} = \frac{\mu}{G_{YM}^2} \mathcal{L} (\mu = 1, G_{YM} = 1)
\]

(23)

Therefore, in the limit \(\lambda \gg 1\) the Yang-Mills theory becomes weakly coupled and nonabelian classical solutions play a significant role. These classical solutions are fuzzy ellipsoids discussed in \([23, 25]\) similar to fuzzy spheres in M theory and Type IIA pp-waves \([27]\),

\[
X^5 = 2\sqrt{2}\frac{\mu^2}{\pi} J^1, \\
X^6 = 2\sqrt{2}\frac{\mu^2}{\pi} J^2, \\
X^7 = 2\frac{\mu^2}{\pi} J^3
\]

(24)

where \(J^a\) obey the SU(2) algebra, and the remaining matrices \(X^i\) vanish. These solutions have vanishing light cone energy and can be shown \([23, 25]\) to preserve all 24 supercharges of the M-theory background. A detailed study of all the 1/2 BPS states of this model appear in \([23]\).

In the original Type IIB description they are fuzzy D3 branes with a topology \(S^2 \times S^1\) where the \(S^1\) factor is the compact space direction.

For \(Q \neq 0\) the coupling is always weak near \(\tau \to -\infty\) so that these fuzzy ellipsoids proliferate. As \(\tau\) increases the coupling gets stronger and one would expect that they should not be present,

\(^3\) The coordinates used here make a space-like isometry explicit \([22]\)

\(^4\) The dualization required to convert the gauge field to a scalar involves a time dependent rotation \([23]\).
leaving behind only perturbative abelian degrees of freedom representing the fundamental string. This indeed happens. The size of the ellipsoids is now time dependent: with some initial size the equations of motion may be used to examine the size at later times. Numerical results [10] show that with generic initial conditions, the size oscillates with an amplitude decaying fast with time. In other words, at late times we are left with only the abelian configurations which can be now interpreted as fundamental strings. The phenomenon of production/depletion of \((p, q)\) strings is identical to the \(\mu = 0\) case described in the previous subsection.

2.3. Outlook
The key feature of holographic models of this type is that conventional space-time is an emergent phenomenon in a very special regime. In matrix theories, this is the regime where the gauge theory coupling is strong so that the fields of the theory can be interpreted as space-time coordinates of a point on a fundamental string. In the toy models of cosmology described above, such an interpretation appears to be valid at late times. If we forcibly extrapolate this interpretation to early times we encounter a singularity. At this singularity, however, the holographic gauge theory is weakly coupled: as such a space-time interpretation is not valid in any case. Since the coupling is weak there is a good chance that we have a well defined time evolution.

There are several caveats in this general story. The success of Matrix Theory generally depends on supersymmetry. Even though the backgrounds considered have half of the supersymmetries, the matrix theory does not. One of the consequences of this is that a potential for the fields \(X^a\) could be generated which spoils the interpretation in terms of space-time coordinates. This issue has been investigated in [28] and [29]. Indeed there is a potential at one loop. However it turns out that at late times the potential vanishes fast, indicating that \(X^a\) become moduli.

An important question relates to backreaction. Sometimes null singularities of the type described here are unstable under perturbations. In the past, orbifold singularities of this type have been investigated as possibly consistent backgrounds for perturbative string theory. However, it was soon found that these null singularities turn spacelike under small perturbations - large curvatures develop invalidating the use of perturbative string theory [34]. In our case, the significance of such an instability, if present, is rather different. Here the string theory is in any case strongly coupled near the singularity and there is no question of a perturbative description. Rather the correct description is provided by a weakly coupled Yang-Mills theory. The question now is to find out the meaning of a bulk instability in the gauge theory. It remains to be seen if this causes any problem even though the coupling is weak. This issue is particularly significant for variations of this model based on null branes [31]. For other discussions of Matrix Theory in such backgrounds, see [32].

Perhaps the most important question is about continuation through the singularity. Even though the holographic theory is weakly coupled near the null singularity, the hamiltonian expressed in terms of the conjugate momenta have a singular behavior as one approaches this region - and it is not clear whether there is an unambiguous prescription to continue back in time beyond this point. Recently [33] has put forward an interesting proposal to address this issue.

3. Singularities and AdS/CFT
In its simplest setting, the correspondence implies IIB string theory on \(AdS_5 \times S^5\) with a constant 5-form flux is dual to 3+1 dimensional \(N = 4\) supersymmetric \(SU(N)\) Yang-Mills theory which

\[\text{In [29] it is claimed that the potential in fact vanishes. However it turns out that the quantity which is computed in this paper is a time averaged potential rather than the time dependent potential [30].}\]
lives on the boundary of $AdS_5$. If $R_{AdS}$ denotes the radius of the $S^5$ as well as the curvature length scale of $AdS_5$ and $g_s$ denotes the string coupling, the coupling constant $g_{YM}$ and the rank of the gauge group $N$ of the Yang Mills theory are related by

$$\frac{R_{AdS}^4}{l_s^4} = 4\pi g_{YM}^2 N \quad g_s = g_{YM}^2$$

(25)

This immediately implies that the gauge theory describes classical string theory in the 't Hooft limit

$$N \to \infty \quad g_{YM} \to 0 \quad g_{YM}^2 N = \text{finite}$$

(26)

The low energy limit of the closed string theory - supergravity - is a good approximation only in the strong coupling regime $g_{YM}^2 N \gg 1$. For small $g_{YM}^2 N$ supergravity and hence conventional space-time is not a good description of the gauge theory dynamics. Finite $N$ corrections correspond to string loop effects.

One could, therefore, have a situation where a spacetime with cosmological singularities involves a time (or null) dependent dilaton which becomes large and negative near the singularity. This means that the effective string coupling becomes small. The above relations suggest that the dual gauge theory becomes weakly coupled. In this situation it may be possible to study the dual gauge theory in a controlled fashion, even though the bulk gravity description breaks down.

I will describe one attempt in this direction developed in collaboration with Adel Awad, Jeremy Michelslon, K. Narayan and Sandip Trivedi [11, 12, 13]. Our ultimate aim is to determine which class of cosmological singularities can be "resolved" by formulating them in terms of the gauge theory dual. We will argue that null singularities are indeed resolved in this sense. Our conclusions for spacelike singularities are not definite at the moment.

Some of these results on null singularities contained in these papers have been independently obtained by C.S. Chu and P.M. Ho [35].

There have been several other attempts to understand cosmological singularities using the AdS/CFT correspondence. One interesting approach is that of [36] which shows how to find signatures of space-like singularities inside AdS black holes in the CFT. Another significant line of work involves spacelike singularities which result from the instability of certain deformed AdS space-times [37]. It has been recently argued that in some cases the gauge theory remains well defined and provides a passage through the singularity [38].

### 3.1. The set-up and the strategy

In the following we will consider evolution in real time or along a null direction $x^+$. In this section, however, we will refer to both as "time".

Consider $N = 4$ gauge theory on $R^{1,3}$ at strong 't Hooft coupling in its ground state. The dual of this is supergravity in $AdS_5 \times S^5$ in Poincare coordinates. Now let us turn on a source in the gauge theory, thereby deforming the Hamiltonian. At early times, the source is weak and its variation is small. The AdS/CFT correspondence then tells us that the gravity dual of this is a non-normalizable deformation of $AdS_5 \times S^5$. As time evolves, the gauge theory state evolves according to this deformed hamiltonian. The gravity background evolves according to the supergravity equation of motion.

The source we will consider corresponds to a time (or null-time) dependent coupling of the gauge theory. The background metric on which the gauge theory is defined is always flat. When the time variation is small, the gravity dual is the non-normalizable mode of the dilaton field whose value on the Poincare boundary is the logarithm of the gauge coupling. As time evolves, however, the dilaton starts back reacting to the other fields and the background should be given by a solution of the full nonlinear equations of motion.
We will consider a situation in which the gauge theory ’t Hooft coupling decreases from its large value at early times to zero at some time. If we extrapolate the supergravity solution from early times to late times, the value of the dilaton would go to $-\infty$, so that the derivative of the dilaton field would diverge. Generically this would also mean that the curvature components would become large as well - this would be a singularity. However such an extrapolation is not really justified. Since the effective coupling of the gauge theory is now weak, a supergravity approximation to the dual theory is invalid and there is no notion of conventional space-time. In fact this is typical of relativistic singularities : smooth initial conditions lead to regions where the Einstein equations break down due to high curvature and large tidal forces.

If we had a good knowledge and control over nonperturbative closed string theory in such backgrounds we could have tried to ask whether the theory makes sense at the time when supergravity breaks down. Unfortunately we do not have this knowledge. However in this case there is a good chance that we can make sense of the dual gauge theory since the theory lives on flat space-time and has a bounded coupling. In fact near the singularity the coupling becomes arbitrarily weak and one might hope that weak coupling perturbation expansion is reliable. In the following we will provide examples where the gauge theory can be in fact studied in a controlled fashion, and will find that when the bulk has null singularities there is no obvious obstruction to continue light front time evolution beyond the place where the coupling vanishes.

3.2. The Null Solutions

The “null solutions” are given by the Einstein frame metric

$$ds^2 = \frac{1}{w^2} \left[ dw^2 - 2dy^+dy^- + dy^2 + \frac{1}{4} w^2 (\Phi')^2 (dy^+)^2 \right] + d\Omega_5^2$$

and a dilaton $\Phi(y^\pm)$ and 5 form field strength $F_5 = \omega_5 + \ast \omega_5$ (where $\omega_5$ is the volume form on the $S^5$ and $\ast$ denotes ten dimensional hodge dual). In the above we have expressed all quantities in units of the AdS scale.

These are solutions for any $\Phi(y^\pm)$ which we are free to choose. As mentioned in the previous section we will choose a solution which has a constant $e^{\Phi}$ for $y^\pm \to \pm \infty$ and becomes small at some point $y^+ = 0$. A convenient choice is

$$e^{\Phi(y^+)} = g_s [1 - \alpha \ e^{-(y^+)/a^2}]$$

Note this $e^{\Phi}$ and its derivatives are smooth everywhere and attains a minimum value of $(1 - \alpha)$ at $y^+ = 0$.

For $\alpha = 1$, $e^{\Phi}$ vanishes here and therefore $(\partial_+ \Phi)$ diverges. This is a null singularity. Even though all scalar invariants constructed out of the curvature tensor are bounded, tidal forces between neighboring geodesics diverge at $y^+ = 0$ [13].

The boundary of the space-time (27) is at $w = 0$. The boundary metric is flat. In fact the surface $w = w_0$ is flat for any $w_0$. This may be seen by a trivial change of coordinates.

3.3. Penrose-Brown-Henneaux Transformations

It is well known that Weyl transformations on the boundary of asymptotically AdS space-times are equivalent to a class of bulk diffeomorphisms called Penrose-Brown-Henneaux transformations [41, 42, 43, 44, 46]. This allows us to rewrite our null solutions in a form
in which the boundary is *conformally flat* rather than *flat*. The PBH transformations are given by

\[ z = w \, e^{f(y^+)/2}, \quad x^- = y^- - \frac{1}{4} w^2 (\partial_+ f), \quad x^+ = y^+, \quad \vec{x} = \vec{y} \]  

(29)

where

\[ \frac{1}{2} (f')^2 - f'' = \frac{1}{2} (\partial_+ \Phi)^2. \]

(30)

and prime denotes derivative with respect to \( x^+ \). The new form of the metric is

\[ ds^2 = \frac{1}{z^2} \left[ dz^2 + e^{f(x^+)} (-2dx^+dx^- + dx_1^2 + dx_2^2) \right] + d\Omega_b^2 \]

(31)

The new coordinates provide a new foliation of the space-time. The boundary \( w = 0 \) is naively the same as the original boundary \( z = 0 \). However, it is well known that AdS/CFT requires an infra-red cutoff in the bulk which corresponds to a ultraviolet cutoff in the dual gauge theory. For any such finite cutoff \( \epsilon \), the boundary \( w = \epsilon \) is not the same as \( z = \epsilon \).

### 3.4. More General Solutions

The form of the solution (31) may be readily generalized. In fact a Einstein frame metric

\[ ds^2 = \frac{1}{z^2} \left[ dz^2 + \tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu \right] + d\Omega_b^2 \]

(32)

and a dilaton \( \Phi(x) \) (i.e. a function of the \( x^\mu \) only), together with the standard 5 form field strength is a solution of ten dimensional supergravity provided

\[ \tilde{R}_{\mu\nu} = \frac{1}{2} \partial_\mu \Phi \partial_\nu \Phi, \quad \tilde{\nabla}^2 \Phi = 0 \]

(33)

where \( \tilde{R}_{\mu\nu} \) is the Ricci tensor constructed from the metric \( \tilde{g}_{\mu\nu}(x) \). In other words a solution of 3 + 1 dimensional dilaton gravity can be lifted to a solution of 10 dimensional supergravity.

### 3.5. Solutions with Spacelike Singularities: Kasner-like spacetimes

The observation of the previous subsection may be used to write down supergravity solutions with spacelike singularities.

A particularly interesting class of solutions is of the Kasner form

\[ ds^2 = \frac{1}{z^2} \left[ dz^2 + e^{\Phi(t)} \left( -2dt^2 + dx^1_1^2 + \cdots + dx^n_2^2 \right) \right] \sum_i p_i = 1 \]

\[ e^{\Phi(t)} = |t| \sqrt{2(1-\sum_i p_i^2)} \]

(34)

The string coupling - and therefore the Yang-Mills coupling - still goes to zero at the spacelike singularity at \( t = 0 \), but diverges at early or late times. This solution is therefore not of the kind we are really looking for. However as we will soon see, we can indeed construct solutions with bounded dilaton and in these solutions, the behaviour near the singularity is Kasner type.

We will concentrate on solutions with \( p_1 = p_2 = p_3 = \frac{1}{3} \). Redefining the time coordinate, this solution may be written in the form

\[ ds^2 = \frac{1}{z^2} \left[ dz^2 + \frac{2t}{3} \left( -dt^2 + (dx^1)^2 + \cdots + (dx^3)^2 \right) \right] \]

\[ e^{\Phi(t)} = |t| \sqrt{3} \]

(35)
This solution has a spacelike singularity at $t = 0$.

Since the boundary metric on $z = 0$ is conformally flat, there should be a PBH transformation which leads to a foliation with a flat boundary. This is indeed true. The solution for $t > 0$ becomes

$$ds^2 = \frac{1}{\rho^2} \left[ d\rho^2 - \frac{(16T^2 - 5\rho^2)^2}{256T^4} dT^2 + \frac{(16T^2 - \rho^2)^2}{256T^4} (16T^2 + 5\rho^2)^2 \left( (dx^1)^2 + \cdots (dx^3)^2 \right) \right]$$  (36)

The PBH transformations are detailed in [13].

It is clear that in this new foliation defined by slices of constant $\rho$, the boundary $\rho = 0$ has a flat metric. However these coordinate system has a coordinate singularity at $\rho = 4T$, but may be extended beyond this point.

Furthermore, these solutions have a curvature singularity at $z = \infty$ at any finite time, though the singularity goes away at early and late times. The bulk Ricci scalar is given by In the global geometry, the Poincare horizon is a product of a null plane times a $S^2$. This singularity appears at one point on this null plane. The rest of the Poincare horizon is non-singular.

3.6. Time dependent solutions with bounded couplings

We now discuss solutions with bounded dilatons which behave like the Kasner solutions near the singularity. The Einstein frame metric is given by

$$ds^2 = \frac{1}{z^2} \left[ dz^2 + (1 - \frac{1}{\tau^4})(-d\tau^2 + \tau^2 d\xi^2 + \tau^2 \sinh^2 \xi d\Omega^2_{10}) \right]$$  (37)

The dilaton is given by

$$e^{\Phi(\tau)} = \left[ \frac{\tau^2 - 1}{\tau^2 + 1} \right]^{\sqrt{3}}$$  (38)

The four dimensional space-time inside the curly bracket is the Milne portion of Minkowski space. The singularity is at $\tau = 1$ with diverging curvature components and curvature invariants. In the asymptotic past (or future) the background is $AdS_5 \times S^5$.

Like the Kasner solutions, these solutions generically have curvature singularities at $z = \infty$. The global nature of this singularity is similar to the Kasner type solution. In particular, at early times $\tau \to -\infty$ there is no such singularity.

Since constant $z$ slices are conformally flat, we can perform a PBH transformation to go over to a foliation which leads to a flat boundary metric. In this case, the PBH transformations are not known, but may be worked out systematically in an expansion around the boundary. The details of this procedure is explained in [13].

3.7. Energy-Momentum Tensors

As explained in the previous sections, the backgrounds described above have natural gauge theory duals. In choices of foliations which leads to a flat boundary, the gauge theory is the usual $N = 4$ theory in $R^{3,1}$ with a coupling which is time or null dependent.

Our overall setup required that this gauge theory should be in the vacuum state in the far past (either in terms of time or in terms of light front evolution). We wanted to ensure this by requiring that the supergravity backgrounds are asymptotically $AdS_5 \times S^5$ and at finite times non-normalizable deformations. In the gauge theory this is expected to lead to addition of source terms to the hamiltonian, but not to excitation to an excited state.

Since addition of any amount of a normalizable deformation to a non-normalizable deformation retains the non-normalizable nature, it is sometimes tricky to ensure that we really have the gauge theory in its vacuum state in the far past. To verify that this is indeed the case,
we can calculate the expectation value of the energy momentum tensor and see if this differs from zero. In the far past, the ’t Hooft coupling is large, so this calculation can be performed using standard techniques of Holographic Renormalization Group [44, 45, 46]. The details of this calculation are given in [13]. The final result for the stress tensor are as follows

(i) For the null solutions \( < T^\mu_\nu > = 0 \).

(ii) For the Kasner solutions

\[
<T^\mu_\nu> = \frac{N^2}{512 \pi^2 T^4} \text{diag} (9, 13, 13, 13) \tag{39}
\]

(iii) For the solutions of the form of (37) we have

\[
<T^\mu_\nu> = \frac{N^2}{2 \pi^2 (\tau^4 - 1)^2} \text{diag} \left( 12 - 3 \tau^4, 4 + 9 \tau^4, 4 + 9 \tau^4, 4 + 9 \tau^4 \right) \tag{40}
\]

Thus at early times (40) vanishes as \( 1/\tau^{12} \), while near the singularity \( \tau = 1 \) the answer becomes identical to the Kasner solution.

In fact, these energy momentum tensors are results of anomalous transformation properties under Weyl transformations. This is because we found that for conformally flat (rather than flat) boundary choices, the energy momentum tensor vanishes for all the solutions we considered [13]. The fact that the flat space energy momentum tensor vanishes for all times for null backgrounds is a reflection of the absence of particle production in space-times with null isometries, while the nonzero values above reflect particle production.

### 3.8. Properties of the Gauge Theory

Since all the above backgrounds admit a choice of boundary which is flat, the dual gauge theory is also defined on a 3+1 dimensional flat space. The coupling of the theory is however space-time dependent,

\[
g_{YM}(x) = \bar{g}_{YM} e^{\Phi(x)} \tag{41}
\]

We have chosen examples where this space-time dependent coupling is always bounded and go to zero near the “singularity”. Since the theory is weakly coupled near the singularity one might be tempted to conclude that in all these cases one could use the gauge theory to continue time evolution (or light front evolution) to the far future where the spacetime becomes \( AdS_5 \times S^5 \) again.

However such a conclusion is rather hasty. This is because the way the coupling enters in the lagrangian is as an overall factor. For example the gauge field term is now

\[
\mathcal{L} \sim \frac{1}{\bar{g}_{YM}^2} e^{-\Phi(x)} \text{Tr} F_{\mu\nu} F^{\mu\nu} \tag{42}
\]

To get a canonical kinetic term we need to make a field redefinition

\[
A_\mu \to A_\mu e^{\Phi(x)/2} \tag{43}
\]

This would get rid of factors of \( e^{-\Phi} \) from the kinetic term, and introduce positive powers of \( e^\Phi \) in the interaction terms. However, it is clear that this would also generically introduce factors of \( (\partial_\mu \Phi) \) in terms which are quadratic in the fields. There are no such factors in the nonlinear terms. In our backgrounds \( \partial_\mu \Phi \) diverge at the place of bulk singularity. Thus even though the interaction terms are small here, the kinetic term may have infinite contributions. This would make an analysis of the theory rather subtle.
It turns out that these troublesome factors are present only for the gauge field kinetic term. Similar factors in the fermion part of the action vanish due to the Majorana condition and the scalar quadratic term does not have a factor of $e^{-\Phi}$ to begin with. After the field redefinition the kinetic term for the gauge field becomes

$$\text{Tr} \left[ -F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \left\{ (\nabla \Phi)^2 A_\nu A^\nu - (\partial_\mu \Phi)(\partial_\nu \Phi) A_\mu A^\nu \right\} + (\nabla^2 \Phi) A_\nu A^\nu + 2(\partial_\nu \Phi) A^\mu \partial_\mu A_\nu \right]$$

(44)

For the null backgrounds, $\Phi(y^+)$ only. In this case one can choose the light cone gauge $A_\perp = 0$. In this gauge the extra terms in (44) vanish and the kinetic term is standard. It may be also checked that the constraints are identical to those in a constant coupling theory. Thus a perturbative light front quantization of the theory is standard. It may be also checked that the constraints are identical to those in a constant coupling theory. Thus a perturbative light front quantization of the theory is standard. The nonlinear terms always involve positive powers of $e^\Phi$ or derivatives of $e^\Phi$, all of which may be chosen to vanish near the singularity by a suitable choice of $\Phi$. We therefore expect that the gauge theory is well defined and one could use this theory to evolve the system in a light front evolution beyond the “singularity” at $y^+ = 0$.

For backgrounds where the dilaton depends on time, e.g. the ones with spacelike singularities in the bulk, the extra terms in (44) do not vanish. Now a suitable choice of gauge is $A_0 = 0$. In this gauge the transverse components of the gauge fields $A_i$ acquire a time dependent mass given by

$$m^2(t) = \frac{1}{4}((\nabla \Phi)^2 - 2\nabla^2 \Phi)$$

(45)

As we have seen before, near the singularity the dilaton approaches its value for the symmetric Kasner solution. In a choice of time where the singularity appears at $t = 0$ this mass term becomes

$$m^2(t) \sim -\frac{3 + 2\sqrt{3}}{4t^2}$$

(46)

The mass is therefore tachyonic and becomes infinitely strong at $t = 0$.

This has an interesting effect on an initial vacuum state. The wave functional of the vacuum state at early times is a gaussian in the field variables with some width. In the presence of this time dependent tachyonic mass, the width of this wave functional increases as a power to time as we approach $t = 0$, and at the same time the overall magnitude decreases. Thus large values of the fields become highly probable. If we started out with a slightly excited state at $t = -\infty$, one would find that the average value of the field will also diverge.

Such a spread of the wave function may be considered as a signature of the bulk singularity and it appears that it is not meaningful to continue time evolution beyond this point. However, this happened because the coupling $e^\Phi$ actually vanished at $t = 0$, leading to an infinite $(\partial_\mu \Phi)$. An infinite spread would be avoided if we choose a source such that $e^\Phi$ does not quite vanish at $t = 0$, but becomes very small. As yet, we do not have the corresponding supergravity solution. In fact if we modify the dilaton profile only near the singularity it is pointless to look for modifications of supergravity solutions since the couplings are small and the curvature are Planck scale anyway. Such a modified solution would not have a diverging curvature anywhere, but the curvatures will become Planck scale - and this would be a singularity for all physical purposes. The question is whether such a modified dilaton profile leads to an eventual decrease of the spread of the wave function, thereby leading to a smooth passage across $t = 0$. This issue is under current investigation and will be reported soon [47].

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