Implications of Combined Solar Neutrino Observations and Their Theoretical Uncertainties

S. A. Bludman, N. Hata, D. C. Kennedy†, and P. G. Langacker

University of Pennsylvania, Philadelphia, PA 19104
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Abstract

Constraints on the core temperature ($T_c$) of the Sun and on neutrino-oscillation parameters are obtained from the existing solar neutrino data, including the recent GALLEX and Kamiokande III results. (1) A purely astrophysical solution to the solar neutrino problem is strongly disfavored by the data: the best fit in a cooler Sun model requires an 8% reduction in $T_c$, but the $\chi^2$ test rejects this hypothesis at 99.99% C.L. This suggests neutrino physics (mass, mixings) that goes beyond the standard electroweak theory. (2) Assuming the Standard Solar Model (SSM) and matter-enhanced neutrino oscillations, the MSW parameters are constrained to two small regions: non-adiabatic oscillations with $\Delta m^2 = (0.3 - 1.2) \times 10^{-5}$ eV$^2$, $\sin^2 2\theta = (0.4 - 1.5) \times 10^{-2}$, or large mixing-angle oscillations with $\Delta m^2 = (0.3 - 4) \times 10^{-5}$ eV$^2$, $\sin^2 2\theta = 0.5 - 0.9$. The non-adiabatic solution gives a considerably better fit. For $\nu_e$ oscillations into sterile neutrinos, the allowed region (90%) is constrained to non-adiabatic oscillations. As long as the SSM is assumed, the neutrino mixing angles are at least four times larger, or considerably smaller, than the corresponding quark mixing angles. (3) Allowing both MSW oscillations and a non-standard core temperature, a) the experiments determine the core temperature at the 5% level: $T_c = 1.03^{+0.03}_{-0.05}$ (90% C.L.) relative to the SSM, which is consistent with the SSM prediction. b) When $T_c$ is used as a free parameter, the allowed MSW region is broadened: a cooler Sun ($T_c = 0.95$) allows $\Delta m^2, \sin^2 2\theta$ implied by the supersymmetric SO(10) grand unified theory (GUT), while a warmer Sun ($T_c = 1.05$) extends the allowed parameter space into values suggested by intermediate-scale SO(10) GUTs, for which the $\nu_\tau$ may be cosmologically relevant. Superstring-inspired models are consistent with all solutions. (4) From the narrowed parameter space, we predict the neutrino spectral shape which should be observed in the Sudbury Neutrino Observatory (SNO). Expected rates for SNO, Super-Kamiokande, and BOREXINO are also discussed. Throughout the calculation we use the Bahcall-Pinsonneault SSM (1992) with helium diffusion, and include nuclear and astrophysical uncertainties in a simplified, but physically transparent way.

† Present address: Fermi National Accelerator Laboratory, P.O. Box 500 MS106, Batavia, IL 60510
I. INTRODUCTION: EXPERIMENTAL STATUS AND SSM IMPROVEMENTS

Since solar neutrinos were first detected two decades ago, the observed neutrino flux has always been a factor of 1.5 to 4 times less than that predicted by the Standard Solar Model (SSM). When both experimental and SSM uncertainties are included, the Homestake chlorine (Cl) experiment, Kamiokande, and GALLEX rates respectively are approximately 6σ, 3σ, and 2σ below the SSM predictions. Although the most recent GALLEX deficit is only 35%, significant differences from the SSM predictions persist. In this paper, we consider both astrophysical solutions (a cooler Sun model) and particle-physics solutions (MSW) to the solar neutrino problem. We show that no reasonable change in the SSM can reconcile the quoted Homestake, Kamiokande, and GALLEX results. On the other hand the MSW effect, which assumes neutrino mass differences $\Delta m^2$ and mixing $\sin^2 2\theta$, accommodates all data: it does not require discarding any of the experiments, nor stretching the SSM beyond its uncertainties. Once MSW is admitted, the data determine $T_c$ at the 5% level, yielding a value consistent with the SSM. We also discuss possible MSW parameters when a non-standard core temperature is assumed.

There are four measurements of the solar neutrinos so far. The chlorine experiment at Homestake [1] is mainly sensitive to $^8$B and $^7$Be neutrinos, and reports \[^\dagger\] an observed rate $2.1\pm 0.3$ SNU ($0.28\pm 0.04$ of the SSM), while the SSM prediction is $8.0\pm 1.0$ SNU [2]. (Quoted errors are all 1 σ in this paper.) A direct counting measurement by the Kamiokande Collaboration observes Čerenkov light from recoil electrons scattered by $^8$B neutrinos. The combined result of Kamiokande II [3] and 220 days of Kamiokande III [4] is $0.49\pm 0.08$ of the central value of the SSM; there is an additional 14% uncertainty in the SSM prediction. A unique opportunity to observe low energy pp neutrinos, which come from the main reaction responsible for the energy generation in the Sun, is provided by the SAGE and GALLEX gallium experiments. The GALLEX result is $83\pm 21$ SNU ($0.63\pm 0.16$ of the SSM) [5], again well below the SSM value $132^{+7}_{-6}$ SNU. SAGE has published a lower value $20\pm 38$ SNU based on their first 5 extractions [6]. The later SAGE runs clearly suggest a higher rate, but no cumulative rate has been published. We will therefore exclude the SAGE results from our analysis.

The SSM itself has undergone several refinements over the years. There are presently

\[^\dagger\] We ignore the possibility of time dependence in the Homestake data.
at least four SSMs [2, 3, 8, 4], which, for the same physics input, agree with each other within 1% [2] and agree with the sound speed calculated in p-mode helioseismology within 2% [10]. The latest published solar model by Bahcall and Pinsonneault improves on earlier calculations by using the most recent OPAL calculated opacities, meteoritic iron abundances, and updated nuclear-reaction cross sections [2]. The new model includes the effects of helium diffusion so that the observed $^3$He surface abundance is obtained. Their prediction for the convective zone boundary is in striking agreement with helioseismology data [11]. The surface abundances of the $^7$Li and $^7$Be are still over-estimated, but the surface abundances have negligible effect on the neutrino production in the core.

II. COOLER SUN WILL NOT EXPLAIN THE OBSERVED NEUTRINO FLUXES

The production of high energy $^8$B neutrinos and intermediate energy $^7$Be neutrinos is directly proportional to the $^3$He($\alpha, \gamma$)$^7$Be and $^7$Be(p,$\gamma$)$^8$B nuclear cross sections and depends very sensitively on the solar temperature in the innermost 5% of the Sun’s radius. This inner core temperature is not probed by existing p-wave helioseismological observations, but is determined by the radiative opacities throughout the Sun, which are believed to be calculated to within a few percent [2].

Many nuclear and astrophysical explanations have been proposed to explain the solar neutrino deficit. One possibility is to change input parameters, such as lowering the $^7$Be(p,$\gamma$)$^8$B cross section or reducing the opacity. Another is to invoke mechanisms that are not included in the SSM, such as a large core-magnetic field, a rapidly rotating core, or hypothetical WIMPs which carry away energy from the core. The net effect is generally a lowering of the core temperature and thus a reduction of the nuclear burning taking place there.

It is therefore reasonable to examine the core temperature as a diagnostic of a whole class of astrophysical explanations of the neutrino deficit, although the temperature profile is the result, not an input, of a solar model [12]. We choose the central temperature ($T_c$) as a phenomenological parameter representing different conceivable solar models. The approximate correlation of the neutrino fluxes with $T_c$ was obtained by Bahcall and Ulrich [7] by examining 1000 self-consistent SSMs with randomly distributed input
parameters, and is given as simple power laws [7]:

\[ \phi(\text{pp}) \sim T_c^{-1.2}, \quad \phi(^7\text{Be}) \sim T_c^8, \quad \phi(^8\text{B}) \sim T_c^{18}. \] (1)

For each experiment we therefore parameterize rates relative to the SSM as functions of \( T_c \):

\[
R_{\text{Cl}} = (1 \pm 0.033)[0.775 \times (1 \pm 0.100) \times T_c^{18} + 0.150 \times (1 \pm 0.036) \times T_c^8 \\
+ \text{small terms}]
\]

\[
R_{\text{Kam}} = (1 \pm 0.100) \times T_c^{18}
\]

\[
R_{\text{Ga}} = (1 \pm 0.04)[0.538 \times (1 \pm 0.002) \times T_c^{-1.2} + 0.271 \times (1 \pm 0.036) \times T_c^8 \\
+ 0.105 \times (1 \pm 0.100) \times T_c^{18} + \text{small terms}],
\]

where \( T_c \) is the central temperature relative to the SSM \( (T_c = 1 \equiv 15.67 \times 10^6 \text{ K}) \). The Bahcall-Pinsonneault solar model including diffusion is used throughout the paper, unless otherwise mentioned. The uncertainty in the overall factors for Cl and Ga is due to the detector reaction cross sections. The uncertainties in each neutrino flux include only nuclear physics uncertainties in production cross sections; astrophysical uncertainties are absorbed into a variable \( T_c \). These flux uncertainties are properly correlated for the three experiments.

The \( T_c \) dependence of the Kamiokande, Cl, and Ga detectors are each shown in Fig. 1, and the best fits for various combinations of the data are summarized in Table I. If each experiment were fit alone, Kamiokande and Homestake require a reduction of \( T_c \) by 4\( \pm \)1% and 10\( \pm \)1% respectively. For GALLEX alone, \( T_c \) must be reduced by 14\( \pm \)4%, but still does not completely fit the data, because the negative exponent of \( T_c \) in the pp flux works against the reduction of the total rate. In order to match the observed luminosity, the pp chain has to compensate the energy loss due to reductions of the other reactions. As a result the \( T_c \) reduction fails to fit the central value of the GALLEX rate, although it is compatible with the upper end of the range. The three separate \( T_c \) fits are respectively 3\( \sigma \), 6\( \sigma \), and 3\( \sigma \) below the SSM prediction \( T_c = 1 \pm \Delta T_c \), where the SSM uncertainty \( \Delta T_c = 0.0057 \) is estimated from the 1000 SSM calculations in Ref. [13].

The combined observations cannot be fit by any single \( T_c \). The larger Kamiokande rate relative to the Cl rate especially contradicts the \( T_c \) dependence shown in Fig. 1.
The simultaneous fit of Kamiokande and Cl yields $T_c = 0.92 \pm 0.01$, but the $\chi^2$ value is so large ($\chi^2 = 13.78$) as to exclude the fit at the >99.99% C.L. The combined Kamiokande and GALLEX results yield a marginally consistent $T_c$: the best fit is $T_c = 0.96 \pm 0.01$ with $\chi^2 = 2.64$ (89.3% C.L.). When all three experiments are fit simultaneously, $T_c = 0.92 \pm 0.01$ but the $\chi^2$ test rejects the cooler Sun hypothesis at 99.99% C.L. This strong rejection of the cooler Sun is driven mainly by the contradiction between Kamiokande and Homestake and, secondarily, the low GALLEX result.

In our fit, $T_c$ is allowed to vary in a range wider than that for which the power laws (Eqn. (1)) were derived. We assume that, even for large changes of $T_c$, the power laws yield a reasonable approximation to the core-temperature dependence of the neutrino fluxes. It should be noted that our conclusions do not depend on a specific choice of the exponents given in Eqn. (1). Provided that the $^7$Be flux is less temperature dependent than the $^8$B flux, the Cl rate is expected to be larger than the Kamiokande rate, contradicting the data [14]. Even if both flux components had the same temperature exponent (=18), we find that $T_c = 0.93 \pm 0.01$ and $\chi^2 = 10.21$ for 1 degree of freedom: the $\chi^2$ test excludes the fit at 99.91% C.L., with similar conclusions for other exponents. Based on these observations one can conclude that the cooler Sun model, which is a nearly universal feature of astrophysical solutions to the solar neutrino problem, is strongly disfavored by the data: the central value of the GALLEX result and the combined result of Homestake and Kamiokande are each incompatible with the cooler Sun hypothesis.

III. MSW FIT TO THE COMBINED OBSERVATIONS

While modifications of the solar model cannot accommodate the data, an attractive solution is proposed from particle physics. Matter-enhanced neutrino oscillation (MSW effect), first proposed by Wolfenstein, then applied to solar neutrinos by Mikheyev and Smirnov, offers a natural explanation of the observed solar neutrino deficit without requiring any ad-hoc mechanisms [15]. This MSW mechanism assumes new properties of neutrinos, mass and mixings, to convert electron-neutrinos to other species when the neutrinos propagate through the Sun, making possible a large reduction of the $\nu_e$-counting rate. The MSW assumption of neutrino mass and mixing is a natural extension of the Standard Model, and requires no ad-hoc features such as a large magnetic
moment. Unlike vacuum oscillations, it does not require fine-tuning. If the MSW oscillation takes place in the Sun, the determination of the neutrino mass and mixing will provide a clue to grand unified theories, which naturally lead to parameters in the relevant region \([14, 17]\).

The MSW effect depends on two parameters: the mass-squared difference \(\Delta m^2\) and the vacuum mixing angle \(\theta\) between \(\nu_e\) and another neutrino species into which it converts. The conversion occurs in a wide parameter space that covers four orders of magnitude both in \(\Delta m^2\) and \(\sin^2 2\theta\): a triangle-shaped region in the \(\sin^2 2\theta\) vs. \(\Delta m^2/E\) plane, surrounded by \(\Delta m^2/E \leq 2 \times 10^{-5} \text{ (eV}^2/\text{MeV})\) and \(\sin^2 2\theta \cdot \Delta m^2/E \geq 10^{-9} \text{ (eV}^2/\text{MeV})\), where \(E\) is neutrino energy. Typical survival-probability contours are shown in the \(\Delta m^2\) vs. \(\sin^2 2\theta\) plane (MSW diagram) after integrations over the neutrino production site and the neutrino energy, including the detector cross sections \([16]\).

The MSW diagrams show three physically-distinct regions: the adiabatic region (the horizontal upper arm of the triangle), the non-adiabatic region (the diagonal arm), and the large-mixing region (the right, vertical arm). For the adiabatic solution, the MSW resonance takes place in the core of the Sun where the neutrinos are produced; the density is high enough for the higher-energy neutrinos to resonate and be depleted while the lower-energy ones survive. For the non-adiabatic solution, the higher-energy neutrinos survive more because of non-adiabatic (Landau-Zener) jumping. In the large-mixing region, which connects smoothly to vacuum oscillation, the neutrino spectrum is equally reduced over the whole spectrum. In the middle of the isosnu triangle, almost 100% conversion of \(\nu_e\) occurs.

This flexibility makes the MSW effect phenomenologically robust. It can preferentially suppress the high energy (\(8\)B) neutrinos, or the low energy (\(7\)Be and pp) neutrinos more. It can deplete the lower energy part of the \(8\)B and \(7\)Be spectrum while keeping the pp flux, as suggested by the experiments (Fig. 14).

In the MSW calculations, we use the Parke formula \([18]\) instead of solving the Schrödinger equation for the oscillations numerically. The jump probability given by the exponential Landau-Zener formula \(P_j = e^{-\chi}\) is used \([19]\), where \(\chi = \pi h \sin^2 \theta \Delta m^2/E\), \(E\) is the neutrino energy, and \(h = (-d \log n_e/dr)^{-1}\) is the electron-density scale height evaluated at the resonance point. This formula agrees with the exact solution for large mixing angles and with the linear Landau-Zener approximation \([20]\) for the small mixing region. The spatial distribution of neutrino production in the Sun is taken from Bahcall
and Pinsonneault. The detector cross sections are taken from Bahcall and Ulrich. For Kamiokande, the $\nu_\mu$ (or $\nu_\tau$) contribution for flavor oscillations, the energy threshold, the energy resolution, and the trigger efficiency are all properly included.

In fitting the data, theoretical uncertainties of the SSM are treated with care, using a simple and transparent parameterization. For each flux component the nuclear cross section uncertainties of every reaction are added quadratically to the detector cross section uncertainties and to the astrophysical (non-nuclear) uncertainties. The latter is represented by the uncertainty in the central temperature $\Delta T_c$ times the exponent defined in Eq (1). The theoretical uncertainty $\Delta T_c = 0.0057$ is chosen to yield flux uncertainties consistent with those given in Ref. 2, and to be consistent with estimates from the 1000 SSM Monte-Carlo calculations of Bahcall and Ulrich. The correlations of the uncertainties among the experiments and flux components are properly taken into account. Our calculations were compared with other studies which utilize 1000 Monte-Carlo SSMs; the agreement is excellent.

Effects of the astrophysical uncertainties on the MSW effect were also examined. Both the uncertainties from the neutrino production profile, which affects the matter mixing angle at the neutrino production, and from the electron-density scale height, which enters in the jump probability, were found to be small.

Survival-probability contours and 90%-C.L. allowed regions are shown for each experiment in Fig. 2, 3(a), 4 and 5. The Homestake allowed region (Fig. 2) does not precisely trace the iso-probability contour because of the difference in $^7$Be and $^8$B theoretical uncertainties. Fig. 3(a) and 3(b) show the Kamiokande result for flavor oscillations and oscillations to sterile neutrinos, respectively.

The combined result (90% C.L.) of Homestake, Kamiokande II+III, and GALLEX is displayed in Fig. 6(a). The confidence-level region is defined from $\chi^2$ values that satisfy $\chi^2(\sin^2 2\theta, \Delta m^2) = \chi^2_{\text{min}} + 4.6$, which is valid in the approximation that the allowed regions are ‘ellipses’ on the log $\sin^2 2\theta$–log $\Delta m^2$ plane. Including the GALLEX observations, the allowed MSW parameters are either $\Delta m^2 = (0.3 - 1.2) \times 10^{-5}$ eV$^2$, $\sin^2 2\theta = (0.4 - 1.5) \times 10^{-2}$ (non-adiabatic solution), or $\Delta m^2 = (0.3 - 4) \times 10^{-5}$ eV$^2$, $\sin^2 2\theta = 0.5 - 0.9$ (large-mixing solution). The best fits of $\Delta m^2$ and $\sin^2 2\theta$ along with $\chi^2$ value for each region are listed in the second and third columns of Table II. The experiments prefer the non-adiabatic solution to the large-angle solution. The non-adiabatic MSW solution yields a good fit ($\chi^2=0.56$); but in the large-mixing region, the $\chi^2$ value is
large ($\chi^2 = 3.52$): it is allowed at the 90% C.L. by the definition above, but for 1 degree of freedom (= 3 experiments − 2 parameters) this region is excluded at 95% C.L. The allowed regions at the 68, 90, and 95% C.L. are shown in Fig. 6 (b): there is no parameter allowed in the large-angle region at 68% C.L. The combined fit without theoretical uncertainties from the SSM and detector cross sections is displayed in Fig. 6 (c). Comparison with Fig. 6 (a) shows the noticeable effect of SSM uncertainties. Our Fig. 6 (a) practically agrees with that obtained by the GALLEX group [5], who included the day-night effect and $\nu_e$ regeneration in the Earth, which we have neglected.

Allowed regions for various combinations of any two experiments are shown in Fig. 7 (Kamiokande and Homestake), Fig. 8 (Kamiokande and GALLEX), and Fig. 9 (Homestake and GALLEX).

We have also examined possibilities of oscillations to a sterile neutrino [24, 25]. If $\nu_e$ oscillates into a sterile neutrino instead of $\nu_\mu$ or $\nu_\tau$, the term $n_e - n_n/2$ enters the MSW equation in place of $n_e$. (Here $n_e$ and $n_n$ are the local electron and neutron densities in the Sun [24].) Also, for the Kamiokande detector there is no neutral current contribution from the converted neutrinos. The result for $\nu_e$ oscillations into sterile neutrinos is displayed in Fig. 3 (b) for Kamiokande, and in Fig. 10 and Table III for the combined fit. (There is no significant change for the Homestake and GALLEX experiments.) Because of the constraints by the Kamiokande observations, there is no solution in the large-angle region at 90% C.L., and the allowed parameters are limited in the non-adiabatic region. Even in the non-adiabatic region, the best fit yields $\chi^2 = 3.17$ and is excluded at 92% C.L. for 1 degree of freedom. This is because of the smaller Homestake rate relative to Kamiokande: the absence of the neutral current events in Kamiokande requires a larger electron survival probability than for the flavor-oscillation case, and therefore widens the discrepancy between the two experiments.

The precise determination of MSW parameters will allow us to draw some theoretical conclusions in Sec. V and to make predictions for next-generation neutrino experiments in Sec. VI.

IV. SIMULTANEOUS FIT OF MSW AND CORE TEMPERATURE

We have also studied the possibilities of having both MSW oscillations and a non-standard solar model by allowing $T_c$ to be a completely free parameter. (We use the
same nuclear and detector uncertainties as in the SSM case.) The data are fit simultaneously to three parameters $\Delta m^2$, $\sin^2 2\theta$, and $T_c$. The $\chi^2$ plot is displayed as a function of $T_c$ (Fig. 11), where $\chi^2$ is minimized for each $T_c$ with respect to $\Delta m^2$ and $\sin^2 2\theta$ in the allowed region in the MSW diagram. By the $\chi^2$ fit, the data determine the core temperature at the 5% level. The best fits are $T_c = 1.03^{+0.03}_{-0.05}$ (90% C.L.) in the non-adiabatic region and $T_c = 1.05^{+0.01}_{-0.07}$ in the large-mixing region, in good agreement with the SSM prediction $T_c = 1 \pm 0.0057$. The consistency between the data and the SSM is encouraging. Moreover, even allowing the MSW conversion and the other uncertainties, the observations determine the core temperature to within 5% [26].

The allowed region for the 3-parameter fit is shown in Fig. 12. For each $\Delta m^2$ and $\sin^2 2\theta$, the $\chi^2$ is minimized with respect to $T_c$, and $\chi^2(\sin^2 2\theta, \Delta m^2) = \chi^2_{\text{min}} + 4.6$, determines the 90% C.L. allowed region, where $\chi^2_{\text{min}}$ is a minimum with respect to all 3 parameters. By allowing $T_c$ to be a free parameter, the allowed region is widened, now stretching from $\sin^2 2\theta = 4 \times 10^{-4} \sim 1$ and $\Delta m^2 = (0.2 - 4) \times 10^{-5}$ eV$^2$. The best fit parameters are shown in Tables II and III. The $T_c$ dependence of the region is seen in Fig. 13 (a) and Fig. 13 (b), which shows the 90% C.L. contours when $T_c$ is fixed at 1.05 and 0.95 respectively. The higher temperature Sun allows a region between the two islands allowed in the SSM case (Fig. 6 (a)), while the cooler Sun pushes the parameter-space outward.

V. THEORETICAL IMPLICATION OF FITTED MSW PARAMETERS

The best fit MSW parameters from Fig. 6 (a) and Fig. 11 are summarized in Table II for the SSM ($T_c = 1$) and for non-SSM in which $T_c$ is an adjustable parameter. The results for sterile-neutrino oscillations are listed in Table III. The $\Delta m^2$ range is consistent with the general expectations of grand unified theories [14] or string-inspired models [17], but the mixing angles are not in agreement with the expectation $\theta_{\text{lepton}} \sim \theta_{\text{CKM}}$ of the simplest GUTs [14]. If we accept the SSM, the observed neutrino mixing are at least four times larger (or considerably smaller) than the quark mixings, $\sin^2 2\theta = 0.18$ and $< 2 \times 10^{-3}$ for $u-c$ and $u-t$ quarks, respectively. If we allow a warm Sun, then Cabibbo mixing with $\Delta m^2 \simeq (0.8 - 2) \times 10^{-5}$ eV$^2$ is possible; this suggests $m_{\nu_{\mu}} \simeq (3 - 4) \times 10^{-3}$ eV and an SO(10) GUT with intermediate-scale symmetry breaking. If we allow a cool Sun, then $\Delta m^2 \simeq (0.5 - 1.5) \times 10^{-5}$ eV$^2$ is possible, suggesting $m_{\nu_{\tau}} \simeq (2 - 4) \times 10^{-3}$ eV
and a SUSY SO(10) GUT (with large Higgs representations so that the seesaw scale is close to the unification scale).

GUT predictions for neutrino masses are much less robust than for mixing angles and mass ratios. We therefore regard the seesaw model \[27\] only as a crude guide to neutrino masses. In the SUSY GUT case, all neutrino masses are cosmologically and astrophysically insignificant. In the non-SUSY SO(10) GUT, the seesaw model suggests cosmologically interesting $\nu_\tau$ masses. Since $m_t/m_c \sim 100$ and $m_\tau/m_\mu = 17$ we have $m_{\nu_\tau} \simeq 0.4 - 0.8 \ (40 - 80) \text{ eV}$ for a linear (quadratic) seesaw model with up-quark masses and $m_{\nu_\tau} \simeq 0.07 - 0.14 \ (1 - 2) \text{ eV}$ for a linear (quadratic) seesaw model with charged lepton masses. In carrying out these extrapolations, we have taken $m_{\nu_\mu} \simeq (2 - 4) \times 10^{-3} \text{ eV}$ and, for the up-quark case, have included a factor two to renormalize mass ratio from from GUT to low energy scales \[14\]. (This factor increases beyond two non-linearly for large value of the top quark mass because of the Higgs corrections to the top mass \[28\].) We see that only a intermediate scale quadratic seesaw mechanism can give cosmologically significant $\nu_\tau$ mass \[14\]. String-inspired models can generate an intermediate seesaw scale via effective non-renormalizable operators \[17\]. There are no clear mixing angle predictions in such models, but for $\nu_e \rightarrow \nu_\mu$ oscillations consistent with either MSW solution, the $\nu_\tau$ may again be cosmologically significant.

VI. PROSPECTS FOR FUTURE EXPERIMENTS

One can predict the results of future solar neutrino observations from the parameters of the combined fit (Fig. 6(a)). The $\nu_e$ survival probability is shown as a function of energy in Fig. 14 for each of the allowed regions. The predicted observed rates for the high-energy $^8\text{B}$ $\nu_e - e$ scattering (SNO \[29\] and Super-Kamiokande \[30\]), the $\nu_e - d$ reaction (SNO), and the $\nu_e - e$ scattering from $^7\text{Be}$ neutrinos (BOREXINO \[31\]) are listed in Table IV \[32\]. The detector cross sections as well as the proposed energy resolution of the detectors are included in the calculation.

The measurement of the charged current reaction $\nu_e + d \rightarrow e + p + p$, which is planned for the first-year operation of SNO, will be a clear diagnostic of the MSW; the distortion of the energy spectrum is characteristic of most particle physics solutions of the solar neutrino problem and cannot be caused by any astrophysical effects operative in the Sun \[33\]. Figure 15 shows the predicted energy spectra for both the non-adiabatic
and the large-mixing solution, along with estimated statistical errors equivalent to a
2-year operation (6000 total events). The non-adiabatic spectrum is very similar to
the predicted spectrum for sterile neutrinos. If there is a non-adiabatic MSW effect, as
suggested by the best fit (Table II), the spectral distortion will (a) confirm the MSW
effect, and (b) discriminate between the two presently allowed solutions. Of course, the
neutral to charged current ratio would also establish MSW oscillations into $\nu_\mu$ or $\nu_\tau$
(but not into a sterile $\nu$).

The measurements of the neutral current events by SNO and BOREXINO, and
their ratio relative to the charged current events would provide definite evidence for
MSW flavor oscillations. The neutral current mode can determine the core temperature
precisely even in the presence of the MSW effect: assuming 10% systematic errors, the
high sensitivity of the $^8$B neutrino flux ($\sim T^{18}_c$) allows a determination of $T_c$ at the 0.5%
level.

VII. SUMMARY

Existing Homestake, Kamiokande, and GALLEX experiments strongly disfavor astrophysical solutions invoking a cooler Sun. For the matter-enhanced neutrino oscil-
lations, the data constrain the parameter space to two small regions, one in the non-
adiabatic region (which is preferred by the data) and one in the large-mixing region.
The fit for oscillations into a sterile neutrino allows only non-adiabatic oscillations at
90% C.L. Allowing a non-standard core temperature along with MSW oscillations we
find that the data constrain the core temperature at the 5% level, yielding values consist-
tent with the SSM. The warmer Sun ($T_c=1.05$) allows the parameter space predicted by
the SO(10) GUT with an intermediate-breaking scale (for which the $\nu_\tau$ may be cosmo-
logically relevant), while the cooler Sun stretches the allowed parameters into a region
predicted by simple supersymmetric SO(10) GUTs. Superstring-inspired models are
consistent with all solutions. Predictions are made for future solar neutrino detectors.

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Table Captions

Table I. The $T_c$ fits for various combinations of the Homestake (Cl), Kamiokande II+III (Kam), and GALLEX results. Listed are the best fit value of $T_c$ with 90% confidence level (C.L.) error, $\chi^2$ values, and C.L. of excluding the fits.

Table II. The best fit of the Homestake, Kamiokande II+III, and GALLEX results. In the $T_c \neq 1$ (SSM) column, shown are the best fit of $\Delta m^2$ and $\sin^2 2\theta$ with the $\chi^2$ value for each of the allowed MSW regions. The $T_c = \text{free}$ column is for the three parameter fit ($\Delta m^2$, $\sin^2 2\theta$, and $T_c$). The $T_c$ errors are at 90% C.L.

Table III. The best fit of the Homestake, Kamiokande II+III, and GALLEX results for sterile-neutrino oscillations. In the $T_c \neq 1$ (SSM) column, shown are the best fit of $\Delta m^2$ and $\sin^2 2\theta$ with the $\chi^2$ value for each of the allowed MSW regions. The $T_c = \text{free}$ column is for the three parameter fit ($\Delta m^2$, $\sin^2 2\theta$, and $T_c$). The $T_c$ errors are at 90% C.L.

Table IV. Predicted rates for future solar neutrino detectors, relative to the SSM expectations. The rates are listed for each of the allowed regions obtained from the best fit (Fig. 6(a)).
Figure Captions

Figure 1. The approximate $T_c$ dependence of the neutrino counting rates (relative to the SSM) for the Cl, Ga, and Kamiokande experiments, according to the power laws (Eqn. (1)). $T_c$ is relative to the SSM value ($T_c = 1 = 15.67 \times 10^6$ K = 1.35 keV).

Figure 2. The $\nu_e$ survival probability contours (solid lines) for Cl experiments and the allowed region obtained from the Homestake result (90% C.L., shaded region). The contours are for survival probabilities of 0.1, 0.2, $\cdots$, 0.9, starting with the innermost solid line. The calculation of the allowed region includes the experimental errors, the detector cross section uncertainties, and the SSM flux errors. The allowed region slightly deviates from the iso-probability contours because of the difference in the $^7$Be and $^8$B flux uncertainties.

Figure 3 (a). The $\nu_e$ survival probability contours (solid lines) for Kamiokande experiments and the allowed region for the Kamiokande II and III (220 days) result (90% C.L., shaded region). This is for flavor oscillations into $\nu_\mu$ or $\nu_\tau$. The contours are for effective survival probabilities of 0.2, 0.3, $\cdots$, 0.9, which include the effects of neutral current scattering. (There is no 0.1 contour.) The calculation includes the energy threshold, the energy resolution, and the trigger efficiency. The allowed region includes the SSM uncertainties of the $^8$B flux.

Figure 3 (b). Same as Fig. 3 (a), except that it is for oscillations into a sterile neutrino. The contours are for survival probabilities of 0.1, 0.2, $\cdots$, 0.9. Compared to flavor oscillations, lack of a neutral current contribution increases the $\nu_e$ survival probability required by the data and therefore pushes the allowed region outward of the triangle.

Figure 4. The $\nu_e$ survival probability contours (solid lines) for Ga experiments and the 90%-C.L. allowed region for the GALLEX result. The calculation of the allowed region includes the experimental errors, the detector cross section uncertainties, and the SSM flux errors. The contours are for survival probabilities of 0.1, 0.2, $\cdots$, 0.9.

Figure 5. Same as Fig. 4, except the allowed region is for SAGE.

Figure 6 (a). The allowed region for Homestake (dotted region), Kamiokande II+III (solid line), and GALLEX (dashed line) results. The shaded region is the combined fit of the
three experiments (90% C.L.).

Figure 6 (b). The allowed region of combined Homestake, Kamiokande II+III, and GALLEX results at 68% (shaded region), 90% (dashed lines), and 95% (dotted lines) C.L.

Figure 6 (c). The allowed region of the combined results with (dotted line) and without (shaded regions) theoretical uncertainties from the SSM and the detector cross sections.

Figure 7. The combined fit of the Kamiokande II+III and Homestake results (90% C.L.).

Figure 8. The combined fit of the Kamiokande II+III and GALLEX results (90% C.L.).

Figure 9. The combined fit of the Homestake and GALLEX results (90% C.L.).

Figure 10. The fit to oscillations to sterile neutrinos at 68% (shaded), 90% (dashed line) and 95% (dotted line) C.L. The data are the combined result of the Homestake, Kamiokande II+III, and GALLEX.

Figure 11. The $\chi^2$ plot as a function of $T_c$ in the three parameter fit ($T_c, \Delta m^2, \sin^2 2\theta$) of the Homestake, Kamiokande, and GALLEX. For each $T_c$, the $\chi^2$ is minimized with respect to $\Delta m^2$ and $\sin^2 2\theta$. The data determine $T_c$ within $0.98 \leq T_c \leq 1.06$ (90% C.L.), which is consistent with the SSM value.

Figure 12. The allowed MSW region of the combined result of Homestake, Kamiokande II+III, and GALLEX, using $T_c$ as a free parameter. As a result of allowing $T_c$ to change, the 90% C.L. region widens. Also shown are $\Delta m^2$, $\sin^2 2\theta$ predicted by SUSY SO(10) GUTs (shaded), intermediate-scale SO(10) GUTs (thick line), and string-inspired SUSY models with nonrenormalizable operators (shaded line). In each model the predictions for $\Delta m^2$ are not robust and easy to change. In the string-inspired model, $\sin^2 2\theta$ is also changeable.

Figure 13 (a). The allowed MSW region of the combined result of Homestake, Kamiokande II+III, and GALLEX when $T_c$ is fixed at 1.05 (a warmer Sun). As a result of the high $T_c$, the two regions of the SSM fit (Fig. 6 (a)) merge into one, allowing the parameters predicted by SO(10) GUTs with intermediate-breaking scales.

Figure 13 (b). The allowed MSW region of the combined result of Homestake, Kamiokande
II+III, and GALLEX when \( T_c \) is fixed at 0.95 (a cooler Sun). As a result of the low \( T_c \), the allowed region is pushed outward of the MSW region, allowing the parameters predicted by supersymmetric SO(10) GUTs.

Figure 14 The \( \nu_e \) survival probabilities as a function of energy for the two regions obtained by the best fit (Fig. 6 (a)). The solid line is for the non-adiabatic region and the dashed line is for the large-mixing region.

Figure 15. The predicted spectral shape of charged-current events at SNO. The solid line is for the allowed parameter space in the non-adiabatic region, and the dashed line is for the large-mixing region. The errors are equivalent to a 2-year operation (6000 events). The distortion of the spectrum in the non-adiabatic branch will confirm the MSW effect and differentiate the two allowed regions (Fig. 6(a)).
Table I: $T_c$ Fit for Kamiokande II+III, Homestake, and GALLEX

|          | $T_c \pm \Delta T_c$ | $\chi^2$ | C.L.  |
|----------|----------------------|----------|-------|
| Kam      | 0.961 ± 0.010        | 0        | —     |
| Cl       | 0.901 ± 0.015        | 0        | —     |
| GALLEX   | 0.860 ± 0.042        | 1.31     | —     |
| Kam+Cl   | 0.921 ± 0.013        | 13.78    | >99.99|
| Kam+GALLEX | 0.960 ± 0.010   | 2.64     | 89.26 |
| Cl+GALLEX | 0.902 ± 0.013        | 1.49     | 77.56 |
| Kam+Cl+GALLEX | 0.920 ± 0.012 | 15.46    | 99.99 |

Table II: Best Fit of Kamiokande II+III, Homestake, and GALLEX

|          | $T_c = 1$ | $T_c = \text{free}$ |
|----------|-----------|---------------------|
|          | Non-adiabatic | Large-mixing | Non-adiabatic | Large-mixing |
| $\sin^2 2\theta$ | 6.7×10^{-3} | 0.73 | 7.7×10^{-3} | 0.31 |
| $\Delta m^2 (\text{eV}^2)$ | 6.1×10^{-6} | 1.1×10^{-5} | 8.9×10^{-6} | 1.1×10^{-5} |
| $T_c$ | 1±0.0057 | 1±0.0057 | 1.03$^{+0.03}_{-0.05}$ | 1.05$^{+0.01}_{-0.07}$ |
| $\chi^2$ | 0.56 | 3.52 | 0. | 0. |

Table III: Best Fit for Oscillations into Sterile Neutrino

|          | $T_c = 1$ | $T_c = \text{free}$ |
|----------|-----------|---------------------|
|          | Non-adiabatic | Large-mixing | Non-adiabatic | Large-mixing |
| $\sin^2 2\theta$ | 9.7×10^{-3} | 0.83 | 7.5×10^{-3} | 0.15 |
| $\Delta m^2 (\text{eV}^2)$ | 3.8×10^{-6} | 4.8×10^{-6} | 4.0×10^{-6} | 3.5×10^{-6} |
| $T_c$ | 1±0.0057 | 1±0.0057 | 0.99$^{+0.16}_{-0.02}$ | 1.15$^{+0.005}_{-0.18}$ |
| $\chi^2$ | 3.17 | 8.95 | 3.13 | 4.91 |
Table IV: Predicted Rates for Future Detectors

| Rate / SSM | Non-adiabatic | Large-mixing |
|------------|---------------|--------------|
| SNO (Charged Current) | 0.15 - 0.5 | 0.15 - 0.3 |
| SNO ($\nu - e$ scattering) | 0.3 - 0.6 | 0.3 - 0.45 |
| Super-Kamiokande | 0.3 - 0.6 | 0.3 - 0.45 |
| Borexino ($^7$Be$\nu - e$) | 0.15 - 0.6 | 0.3 - 0.6 |