Methods for the detection of gravitational waves from sub-solar mass ultracompact binaries

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We describe detection methods for extensions of gravitational wave searches to sub-solar mass compact binaries. Sub-solar mass searches were previously carried out using Initial LIGO, and Advanced LIGO boasts a detection volume approximately 1000 times larger than Initial LIGO at design sensitivity. Low mass compact binary searches present computational difficulties, and we suggest a way to rein in the increased computational cost while retaining a sensitivity much greater than previous searches. Sub-solar mass compact objects are of particular interest because they are not expected to form astrophysically. If detected they could be evidence of primordial black holes (PBH). We consider a particular model of PBH binary formation that would allow LIGO/Virgo to place constraints on this population within the context of dark matter, and we demonstrate how to obtain conservative bounds for the upper limit on the dark matter fraction.

I. INTRODUCTION

Advanced LIGO [1] and Advanced Virgo’s [2] detections of gravitational waves from compact binary coalescences (CBC) have ushered in the dawn of gravitational wave astronomy. To date, there have been 5 detections of binary black hole mergers [3–6] and 1 detection of a binary neutron star system [7], each of which has expanded our knowledge of the properties and populations of compact objects in our universe. Advanced LIGO and Advanced Virgo’s success in detecting traditional sources of gravitational waves suggest that ground based interferometers could be powerful new tools in observing the dark universe. We describe considerations for extensions of traditional compact binary searches to the sub-solar mass regime, and provide motivation for these searches in the context of dark matter. In particular, we consider a model where a uniform distribution of monochromatic primordial black holes (PBH) make up a fraction of the dark matter. We examine the model’s robustness and demonstrate how it can place constraints on the abundance of PBHs for different sub-solar mass populations.

II. ANALYSIS TECHNIQUES

LIGO compact binary searches rely on matched filtering to extract candidate signals from the noise by correlating known gravitational waveforms with the data. Compact binary searches currently require $O(10^5) - O(10^6)$ templates to adequately recover arbitrary signals placed in the parameter spaces considered thus far (binary systems with a total mass of $2M_\odot - 600M_\odot$ [8,9]). The addition of fully precessing waveforms in future observing runs could increase this by yet another factor of 10, though for now this remains computationally infeasible.

The difficulty of CBC searches scales with both the number and length of the waveforms used as matched filter templates, which could present a problem for sub-solar mass searches. Here we focus on the effect of the number of templates in the template bank which is expected to scale (roughly) as:

$$N \propto m_{\text{min}}^{-8/3} f_{\text{min}}^{-8/3}$$

where $m_{\text{min}}$ is the minimum mass included in the search and $f_{\text{min}}$ denotes the starting frequency of the template waveforms [10]. Previous Advanced LIGO searches have searched for binaries with components as light as $1M_\odot$ [8,11]; extending these searches to lower masses could easily lead to a $10 - 100$ time increase in difficulty compared to offline analyses in Advanced LIGO’s first observing run. Below we propose increasing $f_{\text{min}}$ to mitigate the increased computational costs associated with low mass extensions of compact binary searches, and we calculate the expected loss in sensitivity that this brings.

A. Estimates of sensitivity

Second-generation ground-based gravitational wave detectors such as Advanced LIGO and Advanced Virgo are sensitive over a broad range of frequencies ($\sim 10 - 10000$ Hz) but they are most sensitive near $100$ Hz [12]. Compact binary pipelines exploit this sensitivity and typically analyze a subset of the total bandwidth. In Advanced LIGO’s first observing run, frequencies spanning $10 - 2048$ Hz were analyzed [13]. This is an excellent approximation for standard CBC searches; the majority of the signal-to-noise ratio (SNR) is accumulated at lower
frequencies and very little sensitivity is lost by cutting the analysis at 2048 Hz. This is an even better approximation for sub-solar mass compact binaries since the frequency evolution of a binary goes as [14]:

$$\dot{f} \propto M^{5/3} f^{11/3}$$

where

$$M = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

is the chirp mass of the system. Sub-solar mass systems therefore are not only long lived, but also spend a long time in LIGO’s most sensitive band compared to heavier binaries. This suggests that it may be possible to analyze an even more reduced frequency band than previous searches while retaining a significant amount of SNR.

Since orbital decay is slow for sub-solar mass ultracompact binaries, inspiral only waveforms are a very good approximation of the signal received on earth. The amplitude of the waveform can be written as [15]:

$$\tilde{h}(f) = \frac{1}{D} \left( \frac{5\pi}{24c^3} \right)^{1/2} (GM)^{5/6} (\pi f)^{-7/6}$$

and the average recovered signal to noise ratio is given by:

$$\langle \rho \rangle = \int_{f_{\min}}^{f_{\max}} \frac{\tilde{h}(f)^2}{S_n(f)} \, df$$

where $S_n(f)$ denotes the single sided power spectral density, informally referred to as the “noise curve”. $f_{\min}$ is determined by either the low frequency noise floor or the starting frequency of the template waveform (whichever is greater) and $f_{\max}$ is determined by the frequency of the innermost stable circular orbit ($f_{ISCO}$) or the ending frequency of the template waveform (whichever is less) where $f_{ISCO}$ is defined as:

$$f_{ISCO} = \frac{c^3}{6\sqrt{6\pi GM_{\text{total}}}}$$

For a $1 M_\odot - 1 M_\odot$ binary, $f_{ISCO} \approx 2200$ Hz. The frequency monotonically increases for lighter total mass systems; for a sub-solar mass search, $f_{\max}$ is determined by the bandwidth of the template waveforms.

We can substitute the waveform amplitude into the equation for SNR and rearrange to find the horizon distance for a given $\langle \rho \rangle$ (or equivalently, the SNR recovered at some fiducial distance):

$$D_{\max} \propto \frac{1}{\langle \rho \rangle} M^{5/6} \sqrt{\int_{f_{\min}}^{f_{\max}} \frac{f^{-7/3}}{S_n(f)} \, df}$$

which is dependent on the noise curves, the chirp mass of the binary, and the frequency band of the analysis. This allows us to compare LIGO’s sensitivity for frequency bands that do not encompass the full sensitive range. We choose the $f \in (10Hz, 2048Hz)$ band as a point of comparison. The fraction of SNR retained is then:

$$f_{\text{SNR}} = \frac{D(f_{\min}, f_{\max})}{D(10Hz, 2048Hz)}$$

Note that this fractional reduction is independent of the mass of the binary. This presents an important trade off in sub-solar mass searches: increasing $f_{\min}$ drives the difficulty of a search down, but it also causes the search to lose sensitivity. This drop in SNR is equivalent to a fractional decrease in LIGO’s average range, which means that the observed volume (and therefore the expected number of detections at a given chirp mass) is smaller by a factor of $f_{SNR}^3$. Thus even a 3% loss in SNR would represent a detection volume nearly 10% smaller. The sensitive volume retained as a function of $f_{\min}$ and $f_{\max}$ is shown in Fig. 1.

![Fig. 1. The fractional volume retained for various values of $f_{\max}$ and as a function of $f_{\min}$. The green, red, and blue lines correspond to upper cut-off frequencies of 2048, 1024, and 512 Hz respectively. Note that there is very little difference between the various $f_{\max}$ values; this is because there is more than an order of magnitude more noise at these frequencies than the ~ 100 Hz region and very little SNR is accumulated there. All values are measured relative to the band $f \in (10 Hz, 2048 Hz)$.

B. Sensitive distance

Initial LIGO previously carried out searches for compact binaries with components as light as 0.2 $M_\odot$ [16].
Using the relations outlined above and the fact that current Advanced LIGO searches extend to 1M⊙ and \( f_{\text{min}} = 10 \text{ Hz} \), we can estimate the reduction in frequency band and sensitivity required to keep the cost of a sub-solar mass search comparable to current Advanced LIGO searches. Equation 1 shows that we expect similar scalings in both \( m_{\text{min}} \) and \( f_{\text{min}} \). Thus if we decrease the lower mass bound of previous Advanced LIGO searches by a factor of 5, we need to increase \( f_{\text{min}} \) by a factor of 5 as well to keep the number of templates approximately constant. We estimate that in order to modify current searches to extend down to this mass we would need to increase \( f_{\text{min}} \) to \( \sim 50 \text{ Hz} \). This amounts to a loss of 10\% in SNR and range, and therefore a loss of \( \sim 30\% \) in volume and detection rate. Even with this loss in sensitivity, LIGO would remain incredibly sensitive to sub-solar mass ultracompact binaries.

To emphasize LIGO’s improvement in sensitivity even with this reduction in recovered SNR, consider the most recent search for sub-solar mass compact objects carried out in Initial LIGO’s third and fourth science runs. The lowest mass binary considered in this search remained visible at a range of \( \sim 4 \text{ Mpc} \) [17]. The estimate outlined here suggests that at the same mass and over a reduced frequency band, Advanced LIGO has a range of \( \sim 45 \text{ Mpc} \) which corresponds to a sensitive volume more than 1000 times greater. The massive increase in physical volume accessible by Advanced LIGO vastly outweighs any loss in sensitivity due to a moderately reduced frequency band (provided analyzable time remains approximately the same).

C. Approximation of the merger rate for null-results

In Equation 7 we defined the horizon distance of the detector. This represents the maximum distance for which an optimally located and oriented source would be recovered with some \( \langle \rho \rangle \). In general, however, detectors will measure a weaker response to a gravitational wave depending on the location and orientation of the binary. This reduction is described by the antenna patterns, \( F_+ \) and \( F_\times \), which always take values less than or equal to 1 and are related to the signal observed on earth through:

\[
h = F_+ h_+ + F_\times h_\times
\]  

(9)

Averaging the detector response over both location and orientation of the binary reduces the strain recovered (and therefore the distance to a binary with some fiducial \( \langle \rho \rangle \)) by a factor of 2.26 [24–26]. This can be used to define the average range of the detector as:

\[
D_{\text{avg}} = \frac{D_{\text{max}}}{2.26}
\]  

(10)

The average sensitive distance allows us to approximate limits on the coalescence rate from null results for a general gravitational wave search. The loudest event statistic formalism [27] states that we can constrain the binary merger rate for a specific mass bin, \( i \), to 90\% confidence with:

\[
R_{90,i} = \frac{2.3}{\langle VT \rangle_i}
\]  

(11)

We can estimate the sensitive volume-time for a particular observing run using the earlier range approximation:

\[
\langle VT \rangle_i = \frac{4}{3} \pi D_{\text{avg}}^3 T
\]  

(12)

where \( T \) is the analyzable live-time of the two detectors. This method provides an excellent approximation of the sensitive 4-volume. The remaining plots in this paper use this procedure to estimate LIGO rates and LIGO sensitivity in the sub-solar mass region.

D. Non-spinning waveforms

While reducing the frequency band is one way to mitigate the increased computational cost of sub-solar mass
searches, non-spinning waveforms also offer an easy way to reduce the difficulty by potentially 1–2 orders of magnitude. There are some theoretical justifications for non-spinning searches: some models predict sub-solar mass black holes to be predominately slowly spinning \[28\], and LIGO’s previous detections have been consistent with low \( \chi_{\text{eff}} \) binaries. Regardless, a completely non-spinning binary is clearly a non-physical assumption. The efficacy of using non-spinning waveforms to recover spinning waveforms has been examined before \[29\]–\[31\]. In particular, \[29\] examined neutron star systems and found that non-spinning templates recovered aligned spin binary neutron stars to the desired level only for \(-0.2 \lesssim \chi_{\text{eff}} \lesssim 0\).

We performed a similar test on a population of \(0.5M_\odot - 0.5M_\odot\) binary black holes. We created a non-spinning template bank covering component masses \(m_i \in (0.3M_\odot, 0.7M_\odot)\) using TaylorF2 waveforms \[30\]–\[32\]. We then injected 10,000 spinning signals that were purely aligned or anti-aligned with the orbital angular momentum and had dimensionless spin values of \(|\chi_i| < 0.5\) into fake data. We then calculated the overlap between our non-spinning template waveforms and the spinning signals. We find results similar to those of \[29\]: at low spin, there is a large overlap between the template waveforms and the injected, spinning signals. At higher spins, however, the maximum overlap rapidly falls off, implying that LIGO would miss a significant fraction of the signals with appreciable spin. In fact, we find that the non-spinning bank used here recovers signals well provided \(\chi_{\text{eff}} > -0.08\) or \(\chi_{\text{eff}} < 0.2\). As \(\chi_{\text{eff}}\) deviates from these values, the fraction of signals missed grows rapidly. A spinning template bank is therefore necessary if sub-solar mass ultracompact binaries are either born with appreciable spin components or accrete enough matter to develop substantial spin. We are currently examining the effects of spin on the computational cost of sub-solar mass CBC searches, as well as other possible ways to mitigate the increased difficulty.

III. POTENTIAL CONSTRAINTS ON PRIMORDIAL BLACK HOLE ABUNDANCE

While there is a large population of compact objects below one solar mass, the only objects compact enough for detection by LIGO are black holes and neutron stars. Other compact objects begin to coalesce at too low of an orbital frequency to produce gravitational waves in the sensitive band of ground-based interferometers. Neither black holes nor neutron stars are expected to form below one solar mass via known astrophysical mechanisms, though there are models that propose alternative ways to form black holes at this mass \[33\]–\[34\]. It is interesting to consider the possibility that sub-solar mass black holes are formed via primordial processes and could be a component of the dark matter. In the event of either a detection or null-result LIGO can provide estimates on the merger rate, so it is therefore necessary to model the binary formation rate for primordial black holes in order to connect LIGO with primordial populations. Here we describe the sensitivity of one particular model to changes in input parameters, as well as the response of constraints on the dark matter fraction, \(f_{\text{PBH}} \equiv \Omega_{\text{PBH}}/\Omega_{\text{DM}}\), to changes in merger rate constraints that could be provided by LIGO. We motivate this model as a way to provide a conservative limit on \(f_{\text{PBH}}\).

We consider a model of (initially) uniformly distributed, monochromatic black holes formed in the early universe. A pair of nearest neighbor black holes will start to decouple from the background cosmological expansion and form a binary when the mean energy density in a volume encompassing the two exceeds the background energy density. A third, closest black hole to the binary injects angular momentum in the system by applying tidal forces, which ensures that the two black holes will orbit rather than collide head-on. The resulting expression for the merger rate of primordial black hole binaries in the local universe is given by:

\[
\text{event rate} = n_{\text{PBH}} \frac{dP}{dt} \bigg|_{t=t_0}, \tag{13}
\]
where $dP$ is given by:

$$
 dP = \begin{cases} 
 3 f_{PBH}^{3/8} \left[ f_{PBH}^{29/37} \left( \frac{t}{t_c} \right)^{3/8} - \left( \frac{t}{t_c} \right)^{3} \right] \frac{dt}{t}, & t < t_c \\
 3 f_{PBH}^{3/8} \left[ f_{PBH}^{29/37} \left( \frac{t}{t_c} \right)^{-1} - \left( \frac{t}{t_c} \right)^{3} \right] \frac{dt}{t}, & t \geq t_c
\end{cases}
$$

and $n_{PBH}$ by:

$$
 n_{PBH} = \frac{3H_0^2 \Omega_{PBH}}{8\pi G M_{PBH}}
$$

with $Q = 3/170 \left( GM_{PBH} \right)^{-3}$, $G$ the gravitational constant, $z_{eq}$ the redshift at matter-radiation equality, and $M_{PBH}$ the mass of each individual black hole in this population. $\alpha$ and $\beta$ are constants of $O(1)$ that depend on the dynamics of binary formation and are typically set to 1. This model has been extensively studied in [37, 39].

This model provides a direct connection between LIGO and PBHs via an expected merger rate which is solely a function of the age of the universe, $t_0$, given some $M_{BH}$ and $f_{PBH}$. The merger rate is not analytically invertible, but if gravitational wave observations provide a constraint on the merger rate for black holes of a particular mass, then it can be numerically solved to obtain an upper limit on $f_{PBH}$ for that mass bin. Similar procedures have been considered before in [37, 39].

It is important to consider the robustness of this model and the relative strictness of the constraints it provides. First, consider the effects of varying $\alpha$ and $\beta$. Numerical simulations suggest that realistic values are $\alpha = 0.4$, $\beta = 0.8$ [37]. Though not immediately evident from the above equation, smaller values of $\alpha$ and $\beta$ lead to larger expected rates and therefore more stringent estimates of the upper limit of $f_{PBH}$. The dependence of the expected rate on $\alpha$ and $\beta$ is shown explicitly in Figure 4. As $\alpha$ and $\beta$ dip below 1, the expected merger rate increases. It is a simple extension to approximate how the constraints on $f_{PBH}$ are affected by variations of $\alpha$ and $\beta$. We can use the procedure outlined in [11] to approximate the upper limit on the merger rate, which we then invert to find limits on $f_{PBH}$.

We present bounds under this approximation for $\alpha = \beta = 1$ and $\alpha = 0.4$, $\beta = 0.8$ in Figure 5a. This figure shows a general feature of the model: as either $\alpha$ or $\beta$ is decreased, the constraint on $f_{PBH}$ for a given upper bound on the merger rate becomes tighter. Thus $\alpha = \beta = 1$ provides a more conservative limit on $f_{PBH}$.

![Figure 4](image)

FIG. 4. Merger rate dependence on $\alpha$ and $\beta$ for a fixed dark matter fraction ($f = 0.5$) and primordial black hole mass ($M_{BH} = 1.0 M_\odot$), shown in units of $Gpc^{-3}yr^{-1}$. The expected merger rate strictly increases as either $\alpha$ or $\beta$ are changed from 1.0. Similar behavior is observed independent of the black hole mass or dark matter fraction. This implies that the constraints on the dark matter fraction that are typically published assuming $\alpha = \beta = 1$ are conservative for this model.

Of course, allowing $\alpha$, $\beta$ to increase beyond 1 yields looser constraints. At the time that two PBHs become gravitationally bound to one another, $\alpha$ describes the ratio between the semi-major axis of the binary and the initial physical separation of the two PBHs at the moment they become bound. It is therefore unphysical to expect $\alpha > 1$. $\beta$ helps to determine the minimum ellipticity of the binary; for $\beta > 1$, the ellipticity becomes imaginary. $\alpha = \beta = 1$ therefore provides the most conservative rate estimate for this model.

Another important consideration is the sensitivity of this model to errors in observational measurements of the merger rate. We can propagate errors in rates measurements through to the dark matter fraction. From our upper limit on the merger rate estimate, we find that $f_{PBH} \approx .28$ at 0.2$M_\odot$ and $f_{PBH} \approx .04$ at 1.0$M_\odot$. If we allow for a 50% error in the merger rate estimate that this procedure provides we still find $f \in (0.17, 0.37)$ and $f \in (0.03, 0.06)$ for the respective mass bins, thus demonstrating that the constraints are relatively insensitive to even large errors in the upper bound on the merger rate.

There are several other assumptions made in this model that we do not attempt to quantify, but instead provide a brief qualitative argument on their effects. First, we have assumed that primordial black holes are uniformly distributed in space. In reality, we expect...
PBHs to cluster to some extent which would change the expected event rate for PBH binary mergers. Clustering would tend to increase the amount of binary coalescences, however, so the expected event rate would rise and therefore the maximum permissible fraction, \( f_{\text{PBH}} \), would decrease. Therefore a spatially uniform distribution of PBHs provides a conservative bound on \( f_{\text{PBH}} \). We also ignore the binary’s evolution between formation and coalescence, as well as the possibility of late-universe binary formation. For a discussion of these effects, which appear to be sub-dominant (though they also drive the expected rate up), see [45]. A potentially larger effect comes from the assumption of a purely monochromatic distribution of black holes. Though the framework for this formation model has been extended to the unequal mass case in [35], we have not considered those effects here. Finally, we also ignore the effects of spin on binary formation.

As Advanced LIGO approaches design sensitivity, its horizon distance should increase by a factor of 2 – 3 [46]. This, coupled with the more observation, means that LIGO could conceivably have a (cumulative) sensitive \( \langle VT \rangle O(10) \) times larger than what was observed in [44]. Figure 5(b) shows projections for how continued null results could contribute to constraints on \( f_{\text{PBH}} \) for this mass range. Ground-based interferometers have the unique ability to strengthen bounds in the sub-solar mass regime by systematics independent of previous microlensing observations [40, 41, 47]. This is especially important in light of recent criticisms [48] and studies of the model dependencies of these surveys [49].

IV. FUTURE PROSPECTS AND DISCUSSION

There are many areas in which sub-solar mass searches can improve on the suggestions outlined here. The most obvious are extensions to lower masses and spinning binaries, each of which presents its own challenges. Lower masses require denser template banks and they persist in LIGO’s sensitive band longer. One possible solution could be to alter the width of the frequency band considered for different mass bins, thus stitching together a suitable template bank. Spin is more difficult to incorpo-
rate; early tests seem to imply at least a factor of 10 more
templates would be required for fully spinning binaries.
Examining smaller component spins, such as $\chi_i < 0.3$,
could remain computationally feasible and help to mit-
igate the rapid fall off in sensitivity that non-spinning
banks currently experience for moderate to high spin sys-
tems. We are actively pursuing extensions in these areas.

More careful PBH population modeling is also a ne-
cessity. In particular, a careful consideration of extended
PBH distributions will offer more accurate and general
merger rate predictions. Not only will this allow for more
precise constraints, but it will also be useful in examin-
ing the feasibility of detecting preferred PBH distribu-
tions peaked in this mass range. While this paper has
demonstrated that the model considered typically pro-
vides a conservative estimate of the bounds on $f_{\text{PBH}}$, a
more general formalism will allow testing of different in-
flationary models.

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