Towards physics-based operational modeling of the unsteady wind turbine response to atmospheric and wake-induced turbulence

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Abstract. The objective of the present work is to develop a tool able to predict, in a computationally affordable way, the unsteady wind turbine power production and loads as well as its wake dynamics, as a function of the turbine dynamics and incoming wind conditions. Based on the lessons learned from a previous study about the characterization of the unsteady wake dynamics, the framework for an operational wake model is presented. The approach relies on an underlying vorticity-based skeleton consisting of different components, such as a regularized Vortex Sheet Tube (VST) and Vortex Dipole Line (VDL). Physically based evolution equations, accounting for the various flow phenomena occurring in the wake (such as advection, turbulent diffusion/core spreading, source/sink terms, etc.), are then derived. Once calibrated, the wake model is shown to be in good agreement with results of high-fidelity Large Eddy Simulations (LES) obtained using an Immersed Lifting Line-enabled Vortex Particle-Mesh method.

1. Introduction
The challenge for wind energy is to lower the levelized cost of energy (LCOE) in order to make it competitive compared to other types of energies. From a mechanics/aerodynamics perspective, this can be achieved by reducing as much as possible the material fatigue generating loads and/or by increasing the total power production. To that end, there is growing interest in global wind farm control, which would provide a superior global efficiency with respect to that resulting from current control strategies mostly focusing on individual turbines. At this level, one of the key aspects is the improvement of the response of wind turbine operations in wind farms to highly unsteady phenomena (wind gusts, meandering wakes impinging downstream rotors, emergency shutdown, etc.). In order to anticipate the incoming unsteady flow in the context of model-predictive control, very accurate and computationally efficient wake modeling is required. This goes beyond common steady state approaches mainly designed for the a priori optimization of wind farm layouts [1]. Many research efforts have been made in order to address this shortcoming, e.g. the Dynamic Wake Meandering (DWM) model [2], FLORIDyn [3], QBlade [4], etc. Yet, to the authors’ opinion, these models still lack strong physical grounds, i.e. by assuming passive advection of the wake for the former or by lacking the transition from distinct tip vortices to far wake turbulence for free vortex wake models such as QBlade.
The present work aims to provide a wake modeling framework more in line with the underlying wake physics and vortex dynamics. Hence a vorticity-based skeleton is likely the best suited candidate for such a model. Based on a previous study about the wake characterization using such a vortex skeleton [5], the present paper focuses on the development of dynamical equations for the time evolution of the parameters describing the wake model components. The target is to develop an operational model capable of accurately predicting in real- or fast-time the behavior of a wind turbine and its wake with respect to upstream conditions; including atmospheric turbulence as well as turbulent wakes from upstream turbines.

First a description of the model and its components is provided in Section 2. Section 3 depicts the Immersed Lifting Line-enabled Vortex Particle-Mesh method that is used to calibrate and validate the wake model. The results are then presented in Section 4. Finally, conclusions are drawn in Section 5 along with perspectives of future developments.

2. Wake model formulation

We here consider the flow past a wind turbine of diameter \( D \) (radius \( R = D/2 \)) with the hub centered at \( x = 0 \). The upstream flow \( U \) is aligned with the rotor axis \( \hat{e}_z \), and it is assumed statistically uniform. The turbulence intensity is defined by \( I = \frac{\omega_{\text{RMS}}}{U} \), where \( \omega_{\text{RMS}} \) is the root mean square of the axial velocity. The usual way to characterize the operational point of the wind turbine is given by the tip speed ratio \( \lambda = \frac{\Omega R}{U} \), with \( \Omega \) being the rotor rotational speed.

2.1. General description

Based on the wake characterization results obtained in [5], we use a vortex skeleton consisting of the following components (see Fig. 1) in order to model the wake of the wind turbine:

- The rotor operation is modeled using Blade Element Momentum (BEM) theory [6, 7].
- The near wake is represented by a regularized Vortex Sheet Tube (VST) of radius \( R_{\text{VST}} \), azimuthal circulation per unit length \( \gamma_{\text{VST}} \) and mollification parameter \( \sigma_{\text{VST}} \) (here using a high order algebraic regularization, see [8, 9, 10]). This element can be related to the “tangential vorticity cylinder” from Branlard and Gaunaa [11].
- For the far wake, we use a regularized Vortex Dipole Line (VDL) of impulse per unit length \( i_{\text{VDL}} \) and mollification parameter \( \sigma_{\text{VDL}} \).

Note that only the velocity deficit is modeled here, i.e. swirl effects related to axial vorticity, such as the root vortex emanating from the hub, are ignored. Therefore, in the current model formulation no vorticity is present at the rotor disc. For more details about this vortex skeleton, refer to [5]. In this study, it was shown through fitting of the model parameters that this skeleton is particularly well suited to represent the velocity deficit profiles measured around the centerline of a wind turbine wake (considering long and short time averages). Several evolution trends have also been identified for these parameters as a function of \( \lambda \) and \( I \), e.g. low variability of the radius \( R_{\text{VST}} \), increase of the core size \( \sigma_{\text{VST}} \), etc. Based on these observations, we here focus on the development of a physical temporal evolution model for these parameters, which can be expressed in the form of a set of ODE’s. The objective is to capture unsteady phenomena arising from dynamic inflow conditions (atmospheric and wake-induced turbulence from upstream turbines). This is the subject of the following section.

2.2. Rotor model: Blade Element Momentum theory (BEM)

The BEM inputs are the blade geometry and polar characterization, as well as the wind speed \( U \), the collective blade pitch \( \beta \) and the rotor rotation speed \( \Omega \). The latter two are obtained from the static response curve of the wind turbine as a function of the wind speed (a realistic wind
turbine controller could be used instead). Note that the wind speed required by the BEM is a “freestream” velocity which precludes the contribution of the wake. Since the vortex skeleton framework makes a clear distinction between the wakes coming from different wind turbines, one can easily evaluate the wind speed “as seen” by the designated turbine (i.e. rotor-averaged velocity) by simply excluding the own wake vorticity contribution.

One can then also obtain the circulation distribution resulting from the BEM

$$\Gamma(r) = \frac{1}{2} U_{rel}(r) c(r) C_l(r) ,$$

with $U_{rel}$ the relative velocity norm observed at the blade (in the plane normal to the blade), $c$ the local chord and $C_l$ the lift coefficient.

2.3. Near wake model: Vortex Sheet Tube (VST)

The evolution equations that are derived here rely on the conservation of a quantity $\phi(x, t)$ integrated over a material line

$$\frac{D(\phi dl)}{Dt} = \frac{\partial (\phi dl)}{\partial t} + (\mathbf{u} \cdot \nabla)(\phi dl) = 0 ,$$

where $\mathbf{u}$ is the velocity field and $dl(x, t) = dl \hat{e}_l$ is an infinitesimal material line element ($\hat{e}_l$ is the unit orientation vector of the line element and $dl$ its length). Applying this to a particular material line $L(t)$ described by $X_n(\lambda, t)$ and neglecting the line curvature, the previous equation becomes

$$\frac{\partial f}{\partial t} + \left( \frac{\partial l}{\partial \lambda} \right)^{-1} \frac{\partial}{\partial \lambda} (u_l f) = 0 ,$$

with $f(\lambda, t) \triangleq \phi(X_n(\lambda, t), t)$, $l = l(\lambda, t)$ an arc-length coordinate and $u_l$ the projection of $\mathbf{u}$ on $\hat{e}_l$. $\lambda$ is a “normal” Lagrangian coordinate in the sense that it characterizes the path $X_n(\lambda, t)$ resulting from the advection using the velocity normal to the line $\mathbf{u}_n \triangleq \mathbf{u} - u_l \hat{e}_l$. From a numerical perspective, the advection is hence decomposed into a normal and a tangential contribution, where the first part is handled in a Lagrangian way, and the second part of the advection is treated in an Eulerian fashion by solving Eq. (1) over the control points $X_n(\lambda_j, t) (j = 1 \ldots N)$. Figure 1: Sketch of the wake model components along with the component parameters: Vortex Sheet Tube (VST) and Vortex Dipole Line (VDL).
The application of this framework to the wake model requires us to assume an axisymmetric wake with a straight centerline along \( \hat{e}_{z} \). The VST is considered to be a material surface. In the polar frame of reference \((r, z)\), Eq. (1) can be applied over the tube surface described by \( X_{n} = R^{VST}_{T} \hat{e}_{r} + c^{VST}_{z} \hat{e}_{z} \).

The VST can be seen as a collection of vortex rings. For an inviscid flow, their circulation \( \gamma^{VST} \) must be conserved in time. Therefore, \( \partial / \partial t \) of \( \gamma^{VST} \) can be written as:

\[
\frac{\partial \gamma^{VST}}{\partial t} = \frac{\partial}{\partial t} \left( \int \gamma^{VST} dl \right)
\]

This equation is extended with a turbulent stress term that is assumed to be proportional to \( \gamma^{VST} \). Turbulent mixing is taken into account by adding source and sink terms based on an eddy viscosity model. The circulation equation is thus extended with a turbulent stress term that is assumed to be proportional to \( \gamma^{VST} / (\sigma^{VST})^{2} \), and the mollification parameter equation is complemented using a core-spreading technique [12], similarly to what is done for free vortex wake models [13]. In summary, the following ODE's are obtained:

\[
\begin{align*}
\frac{\partial R^{VST}}{\partial t} &= u_{n} \cdot \hat{e}_{r} \\
\frac{\partial R^{VST}}{\partial t} &= u_{n} \cdot \hat{e}_{z} \\
\frac{\partial \gamma^{VST}}{\partial t} + \left( \frac{\partial}{\partial \lambda} \right)^{-1} \frac{\partial}{\partial \lambda} \left( \mu_{t} \gamma^{VST} \right) &= -\nu^{VST}_{\alpha, \gamma} \frac{\gamma^{VST}}{(\sigma^{VST})^{2}} \\
\frac{\partial (\sigma^{VST} R^{VST})}{\partial t} + \left( \frac{\partial}{\partial \lambda} \right)^{-1} \frac{\partial}{\partial \lambda} \left( \mu_{t} (\sigma^{VST} R^{VST}) \right) &= 2 \nu^{VST}_{\alpha, \sigma} \left( \frac{R^{VST}}{\sigma^{VST}} \right).
\end{align*}
\]

The eddy viscosity is modeled in the spirit of Ainslie [14] by separating the contributions from the wake turbulence and from the ambient turbulence (i.e. atmosphere and impinging wakes from upstream turbines):

\[
\begin{align*}
\nu^{VST}_{\alpha, \gamma} &= \nu^{VST}_{\gamma, a} \nu^{VST}_{a} \gamma^{VST} + \nu^{VST}_{\alpha, \gamma} \nu^{VST}_{a} \\
\nu^{VST}_{\alpha, \sigma} &= \nu^{VST}_{\sigma, a} \nu^{VST}_{a} + \nu^{VST}_{\alpha, \sigma} \nu^{VST}_{a},
\end{align*}
\]

where \( \nu^{VST} \) are calibrated model coefficients. The wake contribution is modeled as:

\[
\nu^{VST}_{a} = F \left( \Delta U_{e} / \eta_{BD} \right) \sigma^{VST} \Delta U_{e},
\]

with \( \Delta U_{e} \) the local wake deficit and \( F(\cdot) \) a filter function accounting for the lack of equilibrium between the mean velocity field and the wake generated turbulent field [14]. The filter function is here adapted compared to its original definition [14]: a \( S \)-shaped function, going from 0.1 to 1 in the far wake, is used, and the original filter length scale \( D \) is replaced by the tip vortex breakdown length \( \eta_{BD} \) obtained by Sorensen et al [15] through stability analysis of the tip vortex system. The ambient contribution is modeled as \( \nu^{VST}_{a} = 1 \nu^{VST}_{a} \), which also slightly differs from the original definition [14].

The boundary conditions for Eqs. (2) are obtained by extracting information from the BEM rotor model. The tip vortex core size emanating from the rotor is modeled through:

\[
\sigma^{VST}(0, t) = \sigma^{VST}_{T} \Delta \alpha^{VST}_{T} \frac{\Gamma_{b}}{\frac{d \Gamma}{d \sigma}_{T} \bigg|_{\sigma = \Gamma_{b}}},
\]

where \( \Gamma_{b} = \max_{\Gamma} (\Gamma) \) and \( \alpha^{VST}_{T} \) is a calibrated coefficient. The VST radius and circulation per unit length are obtained through a momentum balance around the rotor actuator disc:

\[
R^{VST}(0, t) = R^{VST}_{T} \Delta \sqrt{\frac{2T}{\rho N_{b} \Gamma_{b} \Omega}} \quad \text{and} \quad \gamma^{VST}(0, t) = \gamma^{VST}_{T} \Delta = \frac{N_{b} \Gamma_{b} \Omega}{2\pi U_{TV}},
\]
with $T$ the rotor thrust obtained by the BEM, $\rho$ the air density, $N_b$ the number of blades and $U_{TV}$ the tip vortex advection velocity induced by the vortex skeleton at the rotor ($r = R_{VST}(0, t)$). This result can be compared to [16], where the link between the circulation per unit length and the thrust is obtained using the pressure jump across an actuator disc. Note that the equation for $\gamma_{VST}(0, t)$ is implicit, but no iteration is performed here thanks to the unsteady framework (the $\gamma_{VST}$ value from previous time step can be used to evaluate $U_{TV}$).

Preliminary tests have shown that a few additional adjustments of the model are required. First, since we do not explicitly model the hub jet and its breakdown, as was done in [5], $\gamma_{VST} = \gamma_{VST}^T$ is here fixed for $z \leq z_0 = C_{BD} l_{BD}$ with $C_{BD}$ a calibrated coefficient. Secondly, the material surface hypothesis for the VST is well suited to describe the initial wake expansion. As soon as $\sigma_{VST} / R_{VST}$ becomes significant, this hypothesis becomes inappropriate. Hence, the slope of $R_{VST}$ is prescribed for $z > z^*$ ($z^*$ is such that $\frac{\partial R_{VST}}{\partial z}(z^*) = 0)$:

$$\frac{\partial R_{VST}}{\partial z} = k_{R_{VST}} \max(0, I - I_{th}) ,$$

with $k_{R_{VST}}$ and $I_{th}$ calibrated coefficients (note that the discontinuity in the $R_{VST}$ derivative at $z^*$ is not problematic since the radius slope is small in practice).

2.4. Far wake model: Vortex Dipole Line (VDL)

The ODE's describing the time of evolution of $i_{VDL}$ and $\sigma_{VDL}$ are similar to those of the VST. This is due to the fact that the VST and VDL models become equivalent when the ratio $\sigma_{VST} / R_{VST}$ becomes large. One then obtains that $i_{VDL} = \pi (R_{VST})^2 \gamma_{VST}$. From an operational perspective, the VDL is far less computationally expensive to evaluate compared to the VST. Yet, it provides a satisfactory approximation of the velocity profile in the far wake and can therefore replace the VST. As the scope of the present paper solely consists in the calibration of the wake model, the VDL component is left out here.

3. Wake flow simulation using a Vortex Particle-Mesh method (VPM)

The wake flow is obtained through simulations of a wind turbine evolving in a turbulent wind field using an Immersed Lifting Line-enabled Vortex Particle-Mesh (VPM) method, as presented in Chatelain et al. [17]. The VPM flow solver relies on the vorticity formulation of the Navier-Stokes equations. Advection is handled in a Lagrangian fashion using vorticity-carrying particles, and all the remaining spatial differential operations, such as the solution of the Poisson equation to obtain the velocity, the vortex stretching term, as well as the subgrid-scale modeling term are efficiently computed on an underlying grid [18], thus in a Eulerian manner (information being interpolated back and forth between the particles and the grid using high order interpolation schemes). The presence of the blades is modeled using an Immersed Lifting Line approach.

The approach is very well suited to efficiently capture detailed flow physics of wind turbines, from the near wake to the far wake. This is enabled by the relaxation of usual linear stability constraints (CFL criterion for advection problems) and by low numerical dispersion and numerical diffusion errors. Moreover, the computational domain is very compact, thanks to the Hockney-Eastwood algorithm [19] that allows unbounded outer conditions on the lateral domain faces.

Adopting the same procedure as described in [5], the inflow turbulence is accounted for using vortex particles carrying the vorticity deduced from Mann’s synthetic atmospheric turbulence model [20].

4. Results

The wake model described in Section 2 is calibrated using simulation results of the flow past the NREL-5MW offshore wind turbine [21].
4.1. Wake flow characterization
The flow characterization procedure is fully described in [5]. Basically, in order to quantify the behavior of the wake deficit, the characterization starts by identifying the unsteady wake centerline. Indeed, as the ambient turbulence intensity increases, the wake meandering phenomenon plays a more and more significant role in the wake dynamics, see Fig. 2, and the centerline is subject to a transversal fluctuation (lateral and vertical). The centerline computation can be applied to short and long time-averaged velocity fields. Here, the target is to obtain an unsteady description of the wake deficit with an averaging period ∆T that filters out the fine grain turbulence but preserves the fluctuations related to the wake meandering and large atmospheric gusts. Next, the azimuthally averaged axial velocity profiles are computed around the centerline at several z/D stations, similarly to [22]. The envelope of these profiles, as well as their time average are shown in Fig. 3 at z/D = 4 and z/D = 7 at I = 6.6% and I = 15%. The calibration of the wake model is performed based on this information.

4.2. Wake model calibration and validation
Given the importance of accounting for the unsteady meandering of the wake centerline, the present model has been designed such that the centerline can handle a transversal displacement. This will be performed by introducing atmospheric turbulence into the computational domain, following an approach similar to that of the DWM model. However, as a first validation step, the model performance is here assessed with respect to time-averaged wake velocity profiles and diagnostics (yet, computed around the time dependent centerline as measured in the VPM simulations).

We here investigate the influence of the turbulence intensity I at optimal tip speed ratio λ = 7.55. Two cases are considered: I = 6.6% (typical offshore value) with ∆TU/D = 2.1 and I = 15% (typical onshore value) with ∆TU/D = 2.0. The model coefficients have been calibrated such that the velocity deficit profiles from the wake model match those of the VPM simulations. In the present study, we consider the time average of the time-filtered axial velocity profiles computed around the unsteady centerline from the VPM simulation results. The inflow velocity for the wake model is constant and the turbulence is simply characterized by I.
The advection terms in Eqs. (2) are discretized using first order upwind finite differences and the ODE's are integrated using the Euler time integration scheme. The time integration stops when a steady state is reached, based on a residual threshold criterion. The resulting axial velocity profiles are shown in Fig. 3 at two different axial positions downstream of the rotor. Figure 4 provides an overview of the spatial evolution of some wake characteristics, i.e. the wake radius (corresponding to the radial location associated to a deficit recovery of 25%, 50%, 75% and 90%) and the axial velocity at specific radial stations ($r/D = 0, 0.5, 0.75$ and 1).

There is a good agreement between the model and the VPM results, especially in the far wake. Despite the significantly higher variability of the case with I = 15%, the calibrated model provides a very accurate representation of the wake deficit profiles, even in the far wake. As to the axial velocity in the near wake, the accuracy of the model is lower. This is due to the fact that the hub jet is not intended to be explicitly captured by the model. Yet, the model is able to predict the wake flow in a wide range of atmospheric turbulence conditions.

![Figure 3: Axial velocity profiles: I = 6.6% (left) and I = 15% (right); VPM profile time average (dark green), VPM profile envelope (light green) and wake model (blue).](image-url)

The computational time related to the evaluation of the model is not addressed in detail here since the current implementation of the model stands as a proof of concept Matlab tool and is thus non-optimized as such. The Biot-Savart velocity evaluation is quite efficient, though, since it is performed in a Fortran subroutine. Nevertheless, the CPU time and complexity related to the method lies between a simple BEM algorithm and a typical free vortex wake method. As an indication, a typical run takes a couple of minutes on a modern desktop PC (single process) to
reach a steady state (i.e. 15 to 25\(D/U\) of physical time), depending on the considered operating point, flow condition and initial condition.

5. Conclusion and perspectives
An operational wake model has been presented for the prediction of the unsteady wake dynamics induced by the flow past a wind turbine subjected to different turbulent inflow conditions. The model is able to accurately reproduce the time-averaged wake behavior for different turbulence intensities, as obtained by high-fidelity Large Eddy Simulations using an Immersed Lifting Line-enabled Vortex Particle-Mesh method. The next calibration step will consist in comparing the model predictions with VPM simulations by also varying the tip speed ratio. Thanks to the unsteady nature of the model based on a set of ODE’s, it is expected that the model will reproduce reasonably well low frequency wake deficit variations induced by an unsteady freestream velocity signal. This shall be confirmed by future validation studies, also using field measurement data.

As to perspectives of future developments, in order to further improve the unsteady response, the equilibrium state hypothesis in the momentum balance underlying the BEM theory could be relaxed by accounting for unsteady effects and turbulent fluctuations. Moreover, improved
BEM methods taking into account non uniform inflow conditions, as well as skewed/yawed inflow [6, 16] might be used. Another important feature will be the relaxation of the axisymmetric wake hypothesis by allowing a lateral displacement of the centerline in order to reproduce wake meandering and wake misalignment. One could also consider additional wake model components accounting for swirl, e.g. using elements similar to those of [11].

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