Domains of Disoriented Chiral Condensate

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Abstract

The probability distribution of neutral pion fraction from independent domains of disoriented chiral condensate is characterized. The signal for the condensate is clear for a small number of domains but is greatly reduced for more than three.
There has been much interest recently in the possible production of a disoriented chiral condensate in high energy hadronic or heavy ion collisions [1–3]. It has been proposed that this condensate can be modeled as a region of coherent classical pion field described either in the linear or non-linear sigma model. These studies predict a striking probability distribution for the neutral pion fraction, \( f \), coming from the condensate,

\[
f = \frac{n_0}{n_{\text{total}}},
\]

where \( n_0 \) is the number of observed neutral pions. If the field is constant in the formation region, one finds a probability distribution of

\[
P(f) = \frac{1}{2\sqrt{f}}.
\]

This is markedly different from the standard statistical distribution which, for large \( n_{\text{total}} \), we expect to be

\[
P(f) = \delta(f - \frac{1}{3}).
\]

Recently Anselm and Ryskin have shown [2] that a region of classical pion field varying rapidly in space and time will also lead to an easily recognized and characteristic probability distribution for the neutral fraction. Although in this case that distribution is not easily given in closed form and is not quite so dramatic as for the uniform field case.

Previous studies have concentrated on a single classical field region or single domain of coherent condensate. It is likely that, particularly in a heavy ion collision, more that one domain or region of chiral condensate will form with an independent “direction” for the chiral condensate in each domain. In that case the observed neutral pion fraction, \( f \), will be an average over regions. In this note we discuss the probability of neutral pion fraction that emerges from averaging over independent production domains both for the case of regions of constant field and for regions of rapidly varying field as given by Anselm and Ryskin. As we average over many regions the probability distribution will tend to the statistical one of Eq. (3), but the physically interesting case is more likely one of a only a few domains.
We wish to average \( f \) over \( N \) independent regions. In each single region we take the probability of \( f \) to be given by \( P_i(f) \) and for simplicity we take the \( N \) regions to have equal weight. The probability of finding neutral fraction \( f \) averaged over the \( N \) regions is given by

\[
P_N(f) = \int df_1 \ldots df_N \delta \left( f - \frac{f_1 + f_2 + \ldots f_N}{N} \right) P_1(f_1)P_1(f_2)\ldots P_1(f_N).
\]

This can be transformed into a recursion relation,

\[
P_N(f) = \frac{N}{N-1} \int P_{N-1} \left( \frac{Nf - f_N}{N-1} \right) P_1(f_N)df_N.
\]

This relation is particularly helpful in computing \( P_N \) stepwise in \( N \). Note that the recursion relation guarantees that the average value of \( f \) is the same for any \( N \). The average is \( 1/3 \) both for the constant field case and for the rapidly varying field case studied by Anselm and Ryskin.

First let us consider the case of uniform pion field in each domain. In this case \( P_1(f) \) is given by Eq. (2). For two domains the probability can be found analytically and we find

\[
P_2(f) = \frac{\pi}{2}
\]

for \( f < 1/2 \) and

\[
P_2(f) = \frac{\pi}{2} - 2 \arccos \left( \frac{1}{\sqrt{2f}} \right)
\]

for \( f > 1/2 \). This result was previously calculated in [3]. Beyond two regions the integrals are most easily done numerically. The results of \( N = 1 \ldots 8 \) are shown in Fig. 1. The approach to a gaussian distribution centered at \( 1/3 \) is clear. This is a consequence of the central limit theorem. The standard deviation of the distribution is proportional to \( 1/\sqrt{N} \) and decreases as \( N \) gets larger, finally reducing the distribution at very large \( N \) to a delta function at \( 1/3 \).

What is also clear is that it would require a high statistics experiment to distinguish four or more domains from the gaussian of incoherent pion production.

The case of rapid variation of the pion field in each domain is shown in Fig. 2. For \( N = 1 \) we have the result of Anselm and Ryskin calculated numerically. This result assumes the
variation of the field can be averaged over. We then use the recursion relation to obtain
the higher $P_N$ up to $N = 5$. The approach to a gaussian distribution is again clear. The
experimental challenge with many domains is more daunting in this case, since even two or
three domains will be very difficult to distinguish from the random case.

One can mix domains from the Anselm Ryskin case with constant domains or complicate
things more by giving unequal weight to domains. It is clear that so long as there are fewer
than three domains it should be possible to extract a signal of the chiral condensate, but if
there are more than three it will be difficult.

We have shown how the neutral fraction of pions from independent domains of chiral
condensate can be characterized. It should be possible to see a signal of the condensate
so long as the number of domains is not large, even if that number varies from collision to
collision, but more than one domain will make the signal less dramatic.

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FIGURES

FIG. 1. Probability density, $P(f)$, for neutral pion fraction $f$, with multiple random domains of static disoriented chiral condensates. Number of domains ranges from 1 to 8.

FIG. 2. Probability density, $P(f)$, for neutral pion fraction $f$, with multiple random domains of rapidly time-dependent disoriented chiral condensates. Number of domains ranges from 1 to 5.