Nucleon structure functions with dynamical (2+1)-flavor domain wall fermions

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RBC and UKQCD-DWF collaborations produced some dynamical DWF ensembles using QCDOC’s:

- $24^3 \times 64 \times 16$ (2.7fm across), $16^3 \times 32 \times 16$ (1.8 fm), ...
- Iwasaki gauge action, $\beta = 2.13$, and Domain-Wall Fermions (DWF) quarks, $M_5 = 1.8$,
- $m_{\text{strange}}a = 0.04$, $m_{\text{ud}}a = 0.03, 0.02, 0.01$ and 0.005, with $a^{-1} \sim 1.7$ GeV.

Best ever hadron structure calculations: flavor and chiral symmetries and lattice volume.

- Very accurate determination of kaon bag parameter, $B_{K}^{\text{MS}}(2\text{GeV}) = 0.524(10)(28)$,
- beginning to see SU(3) chiral perturbation failure, e.g. NLO corrections $\sim 0.5\times$LO.

Here we report some low moments of isovector nucleon structure functions calculated by Takeshi Yamazaki, Huey-Wen Lin, Shoichi Sasaki, Tom Blum, James Zanotti, Robert Tweedie, ... and are now nearly final.

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RBC/UKQCD $N_f = 2 + 1$ dynamical DWF ensemble: from $\Omega^-$ and $K$ masses we estimate

- Lattice cutoff $a^{-1} = 1.73(2)$ GeV, or physical volume is $(2.74(3)\text{fm})^3$;
- physical strange mass is $0.035(1)+0.003$ in lattice units.

The best ever kaon and nucleon calculations in regard of good flavor and chiral symmetries.

- $m_\pi = 0.67, 0.56, 0.42$ and $0.33$ GeV, $m_N = 1.55, 1.39, 1.22$ and $1.15$ GeV.
- Very accurate constraints on CKM matrix: $B_K^{\text{MS}}(2\text{GeV}) = 0.524(10)(28)$, $K_{l3} f_+(0) = 0.964(5)$, ...
- Beginning to tell where SU(3) chiral perturbation fails, e.g. NLO corrections $\sim 0.5\times \text{LO}$. 
The isovector form factors are defined in the following:

\[ \langle p|V_\mu(0)|p \rangle = \bar{u}_p \left[ \gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu F_2(q^2)/2m_N \right] u_p, \]
\[ \langle p|A_\mu(0)|p \rangle = \bar{u}_p \left[ \gamma_\mu \gamma_5 G_A(q^2) + i q_\mu \gamma_5 G_P(q^2) \right] u_p, \]

where \( V_\mu = \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d \) and \( A_\mu = \bar{u}\gamma_\mu \gamma_5 u - \bar{d}\gamma_\mu \gamma_5 d \) are isovector vector and axial vector currents.

Nucleon structure functions are measured in deep inelastic scatterings (and RHIC/Spin):

- **DIS**

\[ \left| \frac{A}{4\pi} \right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}, \quad W^{\mu\nu} = W^{\{\mu\nu\}} + W^{[\mu\nu]} \]

- unpolarized: \( W^{\{\mu\nu\}}(x, Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x, Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu\right) \left(P^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x, Q^2)}{\nu} , \]

- polarized: \( W^{[\mu\nu]}(x, Q^2) = i \varepsilon^{\mu\nu\rho\sigma} q_\rho \left( \frac{S^\sigma}{\nu^2} (g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P^\sigma}{\nu^2} g_2(x, Q^2) \right), \)

with \( \nu = q \cdot P, \quad S^2 = -M^2, \quad x = Q^2/2\nu. \)

- The same structure functions appear in RHIC/Spin and Drell-Yang, which may also provide \( \langle 1|\delta q \rangle \) or \( \langle P, S|\bar{\psi} i \gamma_5 \sigma_{\mu\nu} \psi|P, S \rangle \).
Moments of the structure functions are accessible on the lattice:

\[ 2 \int_0^1 dx x^{n-1} F_1(x, Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\frac{\mu^2}{Q^2}, g(\mu)) \langle x^n \rangle_q(\mu) + O(1/Q^2), \]

\[ \int_0^1 dx x^{n-2} F_2(x, Q^2) = \sum_{f=u,d} c_{2,n}^{(q)}(\frac{\mu^2}{Q^2}, g(\mu)) \langle x^n \rangle_q(\mu) + O(1/Q^2), \]

\[ 2 \int_0^1 dx x^n g_1(x, Q^2) = \sum_{q=u,d} e_{1,n}^{(q)}(\frac{\mu^2}{Q^2}, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu) + O(1/Q^2), \]

\[ 2 \int_0^1 dx x^n g_2(x, Q^2) = \frac{1}{2n+1} \sum_{q=u,d} [e_{2,n}^{(q)}(\frac{\mu^2}{Q^2}, g(\mu)) d_n^{(q)}(\mu) - 2e_{1,n}^{(q)}(\frac{\mu^2}{Q^2}, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu)] + O(1/Q^2) \]

- \( c_1, c_2, e_1, \) and \( e_2 \) are the Wilson coefficients (perturbative),
- \( \langle x^n \rangle_q(\mu), \langle x^n \rangle_{\Delta q}(\mu) \) and \( d_n(\mu) \) are forward nucleon matrix elements of certain local operators,
- so is \( \langle x^n \rangle_{\delta q}(\mu) \) which may be provided by polarized Drell-Yang and RHIC Spin.
Unpolarized \( (F_1/F_2) \): on the lattice we can measure: \( \langle x \rangle_q, \langle x^2 \rangle_q \) and \( \langle x^3 \rangle_q \).

\[
\frac{1}{2} \sum_s \langle P, S | \mathcal{O}^q_{\{\mu_1 \mu_2 \cdots \mu_n\}} | P, S \rangle = 2 \langle x^{n-1} \rangle_q(\mu)[P_{\mu_1} P_{\mu_2} \cdots P_{\mu_n} + \cdots - \text{(trace)}]
\]

\[
\mathcal{O}^q_{\mu_1 \mu_2 \cdots \mu_n} = \bar{q} \left[ \frac{i}{2} \right]^{n-1} \gamma_{\mu_1} \not{D}_{\mu_2} \cdots \not{D}_{\mu_n} - \text{(trace)} \right] q
\]

Polarized \( (g_1/g_2) \): on the lattice we can measure: \( \langle 1 \rangle_{\Delta q} (g_A), \langle x \rangle_{\Delta q}, \langle x^2 \rangle_{\Delta q}, d_1, d_2, \langle 1 \rangle_{\delta q} \) and \( \langle x \rangle_{\delta q} \).

\[
-\langle P, S | \mathcal{O}^{5q}_{\{\sigma_{\mu_1 \mu_2 \cdots \mu_n}\}} | P, S \rangle = \frac{2}{n+1} \langle x^n \rangle_{\Delta q}(\mu)[S_{\sigma} P_{\mu_1} P_{\mu_2} \cdots P_{\mu_n} + \cdots - \text{(traces)}]
\]

\[
\mathcal{O}^{5q}_{\sigma_{\mu_1 \mu_2 \cdots \mu_n}} = \bar{q} \left[ \frac{i}{2} \right]^n \gamma_{5} \gamma_{\sigma} \not{D}_{\mu_1} \cdots \not{D}_{\mu_n} - \text{(traces)} \right] q
\]

\[
\langle P, S | \mathcal{O}^{[5]q}_{\{\sigma_{\mu_1 \mu_2 \cdots \mu_n}\}} | P, S \rangle = \frac{1}{n+1} q^n(\mu)[(S_{\sigma} P_{\mu_1} - S_{\mu_1} S_{\sigma}) P_{\mu_2} \cdots P_{\mu_n} + \cdots - \text{(traces)}]
\]

\[
\mathcal{O}^{[5]q}_{\sigma_{\mu_1 \mu_2 \cdots \mu_n}} = \bar{q} \left[ \frac{i}{2} \right]^n \gamma_{5} \gamma_{[\sigma} \not{D}_{\mu_1]} \cdots \not{D}_{\mu_n} - \text{(traces)} \right] q
\]

and transversity \( (h_1) \):

\[
\langle P, S | \mathcal{O}^{\sigma q}_{\rho \nu \{\mu_1 \mu_2 \cdots \mu_n\}} | P, S \rangle = \frac{2}{m_N} \langle x^n \rangle_{\delta q}[(S_{\rho} P_{\nu} - S_{\nu} P_{\rho}) P_{\mu_1} P_{\mu_2} \cdots P_{\mu_n} + \cdots - \text{(traces)}]
\]

\[
\mathcal{O}^{\sigma q}_{\rho \nu \mu_1 \mu_2 \cdots \mu_n} = \bar{q} \left[ \frac{i}{2} \right]^n \gamma_{5} \sigma_{\rho \nu} \not{D}_{\mu_1} \cdots \not{D}_{\mu_n} - \text{(traces)} \right] q
\]

Higher moment operators mix with lower dimensional ones: Only \( \langle x \rangle_q, \langle 1 \rangle_{\Delta q}, \langle x \rangle_{\Delta q}, d_1, \) and \( \langle 1 \rangle_{\delta q} \) can be measured with \( \vec{P} = 0 \).
Our formulation follows the standard one:

- Two-point function: \( G_N(t) = \text{Tr}[(1 + \gamma_t)\sum \langle \vec{x} B_1(x) B_1(0) \rangle] \), using \( B_1 = \epsilon_{abc}(u^T_a C\gamma_5 d_b)u_c \) for proton.

- Three-point functions: appropriate operator inserted between source and sink.

- Source and sink are separated by 12 lattice spacings. 4 source positions.

- Gaussian smearing is employed to enhance ground-state signals.

- All the matrix elements except \( d_1 \) are iso-vector.

Number of configurations and pion mass

| \( m_f a \) | # of config.'s | meas. interval | \( N_{\text{sources}} \) | \( m_\pi \) (GeV) | \( m_N \) (GeV) |
|---|---|---|---|---|---|
| 0.005 | 932 | 10 | 4 | 0.33 | 1.15 |
| 0.01 | 356 | 10 | 4 | 0.42 | 1.22 |
| 0.02 | 98 | 20 | 4 | 0.56 | 1.39 |
| 0.03 | 106 | 20 | 4 | 0.67 | 1.55 |

Good chiral and flavor symmetries of DWF help:

- negligible unwanted mixings,

- straight-forward non-perturbative renormalization: RBC-standard RI-MOM (Rome-Southampton).

In particular, ratios such as \( g_A/g_V \) or \( \langle x \rangle_{u-d}/\langle x \rangle_{\Delta u-\Delta d} \) are naturally renormalized.
Two possibly important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

Source/sink time separation:

- If too short, too much contamination from excited states, but if too long, the signal is lost.

In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.

In this study we choose separation 12 or 1.37 fm: lighter quarks should help maintaining the signal.
Spatial volume: in Lattice 2007 Takeshi Yamazaki reported significant finite-size effect in axial charge:

- measured in neutron $\beta$ decay, $g_A/g_V = 1.2695(29)$, decides neutron life.

Our DWF on quenched and LHPC DWF on MILC calculations are presented for comparison.

- Heavier quarks: consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
  - elastic form factors demand big volumes.
- Does not necessarily mean inelastic structure functions do: need further investigation.
RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\text{res}} = 0.00315(2)$, $m_{\text{strange}} = 0.04$.

Mass signal: $m_f = 0.005$

Bare three-point functions: $\langle x \rangle_{u-d}$ (left) and $\langle x \rangle_{\Delta u-\Delta d}$ (right), for $m_f = 0.005$ (red +) and 0.01 (blue ×):

Similar in quality with 2-flavor, time separation 12.
RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\text{res}} = 0.00315(2)$, $m_{\text{strange}} = 0.04$,

- $m_\pi = 0.67, 0.56, 0.42$ and $0.33$ GeV; $m_N = 1.55, 1.39, 1.22$ and $1.15$ GeV,

Ratio, $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u-\Delta d}$, of momentum and helicity fractions (naturally renormalized on the lattice), consistent with experiment, no discernible quark-mass dependence.

No finite-size effect seen, in contrast to $g_A/g_V$ which is also naturally renormalized on the lattice.
RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\text{res}} = 0.00315(2)$, $m_{\text{strange}} = 0.04$,

- $m_{\pi} = 0.67, 0.56, 0.42$ and $0.33$ GeV; $m_N = 1.55, 1.39, 1.22$ and $1.15$ GeV,

Momentum fraction, $\langle x \rangle_{u-d}$, with NPR, $Z_{\text{MS}}^{\text{MS}}(2\text{GeV}) = 1.15(4)$.

Absolute values have improved, trending to the experimental values, with NPR, $Z_{\text{MS}}^{\text{MS}}(2\text{GeV}) = 1.15(4)$.

Light quarks or finite volume? Need further investigation.

Why differ from LHPC/MILC? NPR? Unitarity? Source/sink?
RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2) \text{ GeV}$, $m_{\text{res}} = 0.00315(2)$, $m_{\text{strange}} = 0.04$,

- $m_{\pi} = 0.67, 0.56, 0.42$ and $0.33 \text{ GeV}$; $m_N = 1.55, 1.39, 1.22$ and $1.15 \text{ GeV}$,

Helicity fraction, $\langle x \rangle_{\Delta u - \Delta d}$, with NPR, $Z_{\text{MS}(2\text{GeV})} = 1.15(3)$,

Absolute values have improved, trending to the experimental values, with NPR, $Z_{\text{MS}(2\text{GeV})} = 1.15(3)$.

Light quarks or finite volume? Need further investigation.

Why differ from LHPC/MILC? NPR? Unitarity? Source/sink?
RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\text{res}} = 0.00315(2)$, $m_{\text{strange}} = 0.04$,

- $m_{\pi} = 0.67, 0.56, 0.42$ and $0.33$ GeV; $m_N = 1.55, 1.39, 1.22$ and $1.15$ GeV,

Transversity fraction, $\langle 1 \rangle_{\delta u - \delta d}$, with preliminary NPR, $Z^\text{MS}(2\text{GeV}) = 0.783(3)$.
RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\text{res}} = 0.00315(2)$, $m_{\text{strange}} = 0.04$,

- $m_{\pi} = 0.67, 0.56, 0.42$ and $0.33$ GeV; $m_N = 1.55, 1.39, 1.22$ and $1.15$ GeV,

Twist-3, $d_1$, yet to be renormalized.

Chirally well-behaved, small, and in consistency with Wandzura-Wilczek relation.
Conclusions

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical calculations with, $a^{-1} = 1.73(2) \text{ GeV}$, $(2.74(3)\text{fm})^3$ box, $m_{\text{res}} = 0.00315(2)$, $m_{\text{strange}} = 0.04$, $m_{\pi} = 0.67, 0.56, 0.42$ and $0.33 \text{ GeV}$; $m_{N} = 1.55, 1.39, 1.22$ and $1.15 \text{ GeV}$:

While elastic form factors such as the axial charge, $g_A$, suffer large finite-volume effect,

• a similarly naturally renormalized ratio, $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta_u-\Delta_d}$, of momentum and helicity fractions does not show such effect.
  
  – It is consistent with experiment, and
  
  – does not show any discernible quark-mass dependence.

• Lightest points show an encouraging trend toward experiments in both momentum and helicity fractions.
  
  – Light quark or finite volume? Plan to check the smaller 1.8-fm box.
  
  – But they are different from corresponding LHPC/MILC results. NPR? Unitarity? Source/sink?

• Transversity moment is obtained.

• Twist-3 moment, $d_1$, is chirally well-behaved, small, and consistent with Wandzura-Wilczek relation.

Structure function renormalizations are now complete: typically 15-20 % effect.
Exploring auxiliary determinant for much larger volume.