Radiative neutrino masses in the singlet-doublet fermion dark matter model with scalar singlets

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Abstract

When the singlet-doublet fermion dark matter model is extended with additional $Z_2$–odd real singlet scalars, neutrino masses and mixings can be generated at one-loop level. In this work, we discuss the salient features arising from the combination of the two resulting simplified dark matter models. When the $Z_2$-lightest odd particle is a scalar singlet, Br($\mu \rightarrow e\gamma$) could be measurable provided that the singlet-doublet fermion mixing is small enough. In this scenario, also the new decay channels of vector-like fermions into scalars can generate interesting leptonic plus missing transverse energy signals at the LHC. On the other hand, in the case of doublet-like fermion dark matter, scalar coannihilations lead to an increase in the relic density which allow to lower the bound of doublet-like fermion dark matter.

1 Introduction

In view of the lack of signals of new physics in strong production at the LHC, there is a growing interest in simplified models where the production of new particles is only through electroweak processes, with lesser constraints from LHC limits. In particular, there are simple standard model (SM) extensions with dark matter (DM) candidates, such as the singlet scalar dark matter (SSDM) model [1, 2, 3], or the singlet-doublet fermion dark matter (SDFDM) model [4, 5, 6, 7, 8, 9]. In this kind of models, the prospects for signals at LHC are in general limited because of the softness of final SM particles coming from the small charged to neutral mass gaps of the new particles, which is usually required to obtain the proper relic density. In this sense, the addition of new particles, motivated for example by neutrino physics, could open new detection possibilities, either trough new decay channels or additional mixings which increase the mass gaps.
On those lines, scotogenic models [10], featuring neutrino masses suppressed by the same mechanism that stabilizes dark matter, are being thoroughly studied with specific predictions in almost all the current terrestrial and satellite detector experiments (For a review see for example [11]). The simplest models correspond to extensions of the inert doublet model [12, 13] with extra singlet or triplet fermions. Recently, the full list of 35 scotogenic models with neutrino masses at one-loop [14, 15], and at most triplet representations of $SU(2)_L$, was presented in [17] (and partially in [18]). The next simplest scotogenic model is possibly the one where the role of the singlet fermions is played by singlet scalars, and the role of the scalar inert doublet is played by a vector-like doublet fermion. One additional singlet fermion is required to generate neutrino masses at one-loop level. This kind of extension of the singlet dark matter model is labeled as the model T13A with $\alpha = 0$ in [17]. The extra fermion, required in order to have radiative neutrino masses, can be the singlet in the SDFDM model.

In the simplest scotogenic model [10], singlet fermion dark matter is possible but quite restricted by lepton flavor violation (LFV) [19, 20]. In contrast, we will show that in the present model the region of the parameter space, corresponding to fermion dark matter, is well below the present and near future constraints on $\text{Br}(\mu \rightarrow e\gamma)$.

On the other hand, when the lightest $Z_2$-odd particle (LOP) is one of the scalar singlets, in the regions of the parameter space compatible with constraints from LFV, we could have promising signals at colliders, thanks to the electroweak production of fermion doublets and possible large branchings into charged leptons.

The dark matter phenomenology of both the SSDM and SDFDM models has been extensively studied in the literature and recently revisited in [21]. Here we consider the possible effect of coannihilations with the scalar singlets for fermion dark matter. We will see that these coannihilations tend to increase the relic density of dark matter and may modify the viable parameter space of the model. Specifically, they allow to reduce the lower bound on the mass of the doublet-like dark matter particle from around 1.100 GeV down to about 900 GeV.

The rest of the paper is organized as follows. In the next section, we present the model. Our main results are presented in Sections 3 to 6 where we describe the correlation between the generation of neutrino masses and lepton flavor violation, new signals at colliders in the case of scalar dark matter, and new coannihilation possibilities in the case of singlet-doublet fermion dark matter. Finally, in section 7 we present our conclusions. In the Appendix we present the analytic diagonalization formulae for the mass matrix of neutral fermions.

2 The model

The particle content of the model consists of two $SU(2)_L$-doublets of Weyl fermions $\tilde{R}_u$, $R_d$ with opposite hypercharges; one singlet Weyl fermion $N$ of zero hypercharge, and a set of real scalar singlets $S_\alpha$ also of zero hypercharge. All of them are odd under one imposed $Z_2$ symmetry, under which the SM particles are even. The new particle content is summarized in Table 1. The most

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1The general realization of the Weinberg operator at two-loops have been undertaken in [16]
1/2 Spin

other hand, the

The specific signs on the right hand side of eq. (1) were chosen so that the terms in the mass matrix

where

and

$H = (0 \ (h + v) / \sqrt{2})^T$ as the SM Higgs doublet with $\tilde{H} = i \sigma_2 H^*$ and $v = 246 \text{ GeV}$. In the scalar potential, we assume that the $M_2^S$ matrix has only positive entries and $(M_2^S)_{\alpha \beta} + \lambda^{S_H} v^2 = 0$ for $\alpha \neq \beta$, which means $S_\alpha$ are mass eigenstates with masses $m_{S_{\alpha}}^2 = (M_2^S)_{\alpha \alpha} + \lambda^{S_H} v^2$ and $m_{S_{\alpha}} < m_{S_{\alpha+1}}$. On the other hand, the $Z_2$-odd fermion spectrum is composed by a charged Dirac fermion $\chi^- = (\psi_L^- , \psi_R^-)^T$ with a tree level mass $m_{\chi} = M_D$, and three Majorana fermions arisen from the mixture between the neutral parts of the $SU(2)_L$ doublets and the singlet fermion. By defining the fermion basis through the vector $\Xi = (N , \psi_L^0 , (\psi_R^0)^\dagger)^T$, the neutral fermion mass matrix reads

$$M^X = \begin{pmatrix} M_N & -m_\lambda \cos \beta & m_\lambda \sin \beta \\ -m_\lambda \cos \beta & 0 & -M_D \\ m_\lambda \sin \beta & -M_D & 0 \end{pmatrix},$$

where

$$m_\lambda = \frac{\lambda v}{\sqrt{2}}, \quad \lambda = \sqrt{\lambda_u^2 + \lambda_d^2}, \quad \tan \beta = \frac{\lambda_u}{\lambda_d}. \quad (3)$$

The specific signs on the right hand side of eq. (1) were chosen so that the terms in the mass matrix $M^X$ follow the same conventions as in the neutralino mass matrix [22]. From this follows that the supersymmetric case corresponds to the pure bino-higgsino limit with $m_{\lambda} = m_Z \sin \theta_W$ leading to $\lambda = g^\prime / \sqrt{2}$. The Majorana fermion mass eigenstates $X = (\chi_1 , \chi_2 , \chi_3)^T$ are obtained through the rotation matrix $N$ as $\Xi = N X$ such that

$$N^T M^X N = M^X_{\text{diag}}, \quad (4)$$

with $M^X_{\text{diag}} = \text{Diag}(m_1^\lambda , m_2^\lambda , m_3^\lambda)$ and $m_n^\lambda$ being the corresponding masses (no mass ordering is implied). In which follows we assume CP invariance and therefore $N$ can be chosen real. The analytical

| Symbol | $(SU(2)_L , U(1)_Y)$ | $Z_2$ | Spin |
|--------|----------------------|-------|------|
| $S_\alpha$ | (1, 0) | − | 0 |
| $N$ | (1, 0) | − | 1/2 |
| $\tilde{R}_u$ | (2, +1/2) | − | 1/2 |
| $R_d$ | (2, −1/2) | − | 1/2 |

Table 1: $\alpha$-set of scalars and Weyl fermions of the model.
diagonalization of the neutral fermion mass matrix is carried out in Appendix A. For the subsequent analysis, it will be convenient to have some approximate expressions in the limit of small doublet-fermion mixing ($m_\lambda \ll M_D, M_N$). Expanding the analytical expressions for the eigensystem of eq. (4) given in Appendix A, up to order $m_\lambda^2$, the fermion masses are

$$m_1^\chi = M_N + \frac{M_D \sin(2\beta) + M_N}{M_N^2 - M_D^2} m_\lambda^2 + \mathcal{O}(m_\lambda^4)$$

$$m_2^\chi = M_D + \frac{\sin(2\beta) + 1}{2(M_D - M_N)} m_\lambda^2 + \mathcal{O}(m_\lambda^4)$$

$$m_3^\chi = -M_D + \frac{\sin(2\beta) - 1}{2(M_D + M_N)} m_\lambda^2 + \mathcal{O}(m_\lambda^4).$$

(5)

Approximate expressions for the mixing matrix are also given in that Appendix.

3 One-loop neutrino masses

By assigning a null lepton number to the new fields in the model\(^2\), the only lepton-number violating term in the Lagrangian eq. (1) is the one with coupling $h_{i\alpha}$. Hence, the introduction of real singlet scalars allows to generate non-zero neutrino masses at one-loop level through the diagram shown in figure 1. The resulting one-loop neutrino mass matrix was presented in the interaction basis in [15] and [23], and more recently in the limit $\lambda_d = 0$ and $M_N \to 0$ in [24]. Instead, we work out the calculations in the more convenient mass-eigenstate basis, in which the neutrino mass matrix takes the form

$$M_{ij}^\nu = -\sum_{\alpha} h_{i\alpha} h_{j\alpha} \frac{1}{16\pi^2} \sum_{n=1}^{3} (N_{3n})^2 m_{\chi_n} B_0 \left(0; m_{\chi_n}^2, m_{S_{\alpha}}^2\right),$$

(6)

where $B_0 \left(0; m_{\chi_n}^2, m_{S_{\alpha}}^2\right)$ is the $B_0$ Passarino-Veltman function [26] and $(N_{mn})$ are matrix elements of the rotation matrix $N$. By using the identity

$$\sum_{n=1}^{3} (N_{3n})^2 m_{\chi_n}^2 = (M^\chi)_{33} = 0,$$

(7)

\(\text{\textsuperscript{2}}\text{If complex singlets instead real singlets are considered, an accidentally conserved lepton number would have been obtained in the Lagrangian, and such a case vanishing neutrino masses are expected.}\)
we obtain the expected cancellation of divergent terms coming from the mass independent term in $B_0$, leading to the finite neutrino mass matrix

$$M_{ij}^\nu = \sum_\alpha \frac{h_{i\alpha} h_{j\alpha}}{16\pi^2} \sum_{n=1}^3 (N_{3n})^2 m_{\chi n} f(m_{S\alpha}, m_{\chi n}) ,$$

$$= \sum_\alpha h_{i\alpha} \Lambda_\alpha h_{j\alpha}$$

$$= (h A h^T)_{ij} ,$$

with $f(m_1, m_2) = (m_1^2 \ln m_1^2 - m_2^2 \ln m_2^2)/(m_1^2 - m_2^2)$, $\Lambda = \text{Diag}(\Lambda_1, \Lambda_2, \Lambda_3)$ and

$$\Lambda_\alpha = \frac{1}{16\pi^2} \sum_{n=1}^3 (N_{3n})^2 m_{\chi n} f(m_{S\alpha}, m_{\chi n}) .$$

The flavor structure of the neutrino mass matrix $M_{ij}^\nu$, given by eq. (10), allows us to express the Yukawa couplings in terms of the neutrino oscillation observables (ensuring the proper compatibility with them) through the Casas-Ibarra parametrization introduced in [27, 28]. Thus, by using an arbitrary complex orthogonal rotation matrix $R$, the Yukawa couplings $h_{i\alpha}$ are given by

$$h^T = D_{\sqrt{\Lambda^{-1}}} R D_{\sqrt{m_\nu}} U^\dagger ,$$

where $D_{\sqrt{m_\nu}} = \text{Diag}(\sqrt{m_{\nu1}}, \sqrt{m_{\nu2}}, \sqrt{m_{\nu3}})$, $D_{\sqrt{\Lambda^{-1}}} = \text{Diag}\left(\sqrt{\Lambda_1^{-1}}, \sqrt{\Lambda_2^{-1}}, \cdots\right)$ and $U$ is the PMNS [29] neutrino mixing matrix. Henceforth we will consider the case of three scalar singlets, $\alpha = 1, 2, 3$, where the Yukawa couplings take the form

$$h_{i\alpha} = \sqrt{m_{\nu1}} R_{\alpha 1} U_{i1}^* + \sqrt{m_{\nu2}} R_{\alpha 2} U_{i2}^* + \sqrt{m_{\nu3}} R_{\alpha 3} U_{i3}^* .$$

In the above equation, the $3 \times 3$ matrix $\mathcal{R}$ can be casted in terms of three rotation angles $\theta_{23}, \theta_{13}, \theta_{12}$, which are assumed to be real. It is worth mentioning that for the case two scalar singlets $\alpha = 1, 2$ a viable scenario is also possible with the remarks that one massless neutrino is obtained. To fully exploit the generality of $h_{i\alpha}$ couplings obtained from (12), we stick to the case with three scalar singlets.

In summary, the set of input parameters of the model are the scalar masses $m_{S\alpha}$, $M_N$, $M_D$, $\lambda$, $\tan\beta$, the lightest neutrino mass $m_{\nu1}$, the three rotation angles present in $\mathcal{R}$ and $\lambda^{SH}_{\alpha\beta\gamma\delta}$. With no lose of generality we assume for the latter to be small $\lambda^{SH}_{\alpha\beta\gamma\delta} \lesssim 0.01$, except for the case of scalar dark matter where $\lambda^{SH}_{11}$ is set to give the proper relic density.

In order to have an approximate expression for $\Lambda_\alpha$ in terms of this set of input parameters, we can use the identity (7) to obtain

$$\Lambda_\alpha = \frac{1}{16\pi^2} \left\{ N_{31}^2 m_1^\chi [f(m_{S\alpha}, m_1^\chi) - f(m_{S\alpha}, m_3^\chi)] + N_{32}^2 m_2^\chi [f(m_{S\alpha}, m_2^\chi) - f(m_{S\alpha}, m_3^\chi)] \right\} .$$

\(^3\text{The couplings } \lambda^{SH}_{\alpha\beta\gamma\delta} \text{ are irrelevant for phenomenological purposes.}\)
The expression for the matrix elements $N_{31}^2$ at $O(m_{\chi}^2)$ are given in the Appendix A. Since $N_{31}^2$ and $f(m_{S_\alpha}, m_{S_2}) - f(m_{S_\alpha}, m_{S_3})$ are already $O(m_{\chi}^2)$, we can use the leading order values for the other masses and mixings parameters to obtain

$$\Lambda_\alpha \approx \frac{1}{16\pi^2} \left\{ N_{31}^2 M_N [f(m_{S_\alpha}, M_N) - f(m_{S_\alpha}, M_D)] + \frac{1}{2} M_D [f(m_{S_\alpha}, m_{S_2}) - f(m_{S_\alpha}, m_{S_3})] \right\} + O(m_{\chi}^4).$$

With the last two approximate formulas for masses in (5), and the $N_{31}^2$ mixing in (26), we have

$$16\pi^2 \frac{\Lambda_\alpha}{m_{\chi}^2} \approx \left( \frac{M_D \cos \beta + M_N \sin \beta}{M_D^2 - M_N^2} \right)^2 M_N [f(m_{S_\alpha}, M_N) - f(m_{S_\alpha}, M_D)]$$

$$+ \frac{M_D^2 [M_D \sin (2\beta) + M_N]}{(M_D^2 - M_N^2) (M_D^2 - m_{S_\alpha}^2)^2} \left\{ M_D^2 - m_{S_\alpha}^2 \left[ \log \left( \frac{M_D^2}{m_{S_\alpha}^2} \right) + 1 \right] \right\} + O(m_{\chi}^2).$$

To illustrate the dependence in $\tan \beta$ of $\Lambda_\alpha$, we consider the following set of input masses (SIM) compatible with singlet scalar dark matter:

$$m_{S_1} = 60 \text{ GeV} \quad m_{S_2} = 800 \text{ GeV} \quad m_{S_3} = 1500 \text{ GeV}$$

$$m_N = 100 \text{ GeV} \quad m_D = 550 \text{ GeV}.$$  \hspace{1cm} (14)

The results for $\lambda = 5 \times 10^{-3}$ are shown in figure 2(a). For large values of $\tan \beta$, the $\Lambda_\alpha$ are positive. However, there are specific values of $\tan \beta$ for which each $\Lambda_\alpha$ goes to zero and turn to negative values as illustrated by the red lines in the plot. The specific point with $\beta = \pi/6$ is illustrated by the yellow stars in the figure.

4 Lepton flavor violation

The size of the lepton flavor violation (LFV) is controlled by the lepton number violating couplings $h_{i\alpha}$. From the approximate expression for $\Lambda_\alpha$ in (13) and the analysis of the previous section, we
will show that these couplings are inversely related to the Yukawa coupling strength $\lambda$. Since in SDFDM the observed dark matter abundance is typically obtained for $\lambda \gtrsim 0.1$ [9], the lepton flavor observables are not expected to give better constraints than the obtained from direct detection experiments. Therefore we will focus our discussion of LFV in regions of the parameter space where $S_1$ is the dark matter candidate.

It is well known LFV processes put severe constraints on the LFV couplings and in general on the model’s parameter space. One of the most restrictive LFV processes is the radiative muon decay $\mu \to e\gamma$, which in the present model is mediated by same particles present in the internal lines of the one-loop neutrino mass diagram. The corresponding expression for the branching ratio reads

$$\text{Br}(\mu \to e\gamma) = \frac{3}{4} \frac{\alpha_{\text{em}}}{16\pi G_F^2} \left| \sum_\alpha h_{1\alpha} F \left( \frac{M_D^2 / m_{S_\alpha}^2}{m_{S_\alpha}} \right) h_{2\alpha}^* \right|^2,$$

where

$$F(x) = \frac{x^3 - 6x^2 + 3x + 2 + 6x\ln x}{6(x - 1)^4}.$$  

(15)

With the implementation of the model in the BSM-Toolbox [30] of SARAH [31, 32], we have cross-checked the one-loop results for both neutrino masses and $\text{Br}(\mu \to e\gamma)$. Moreover, with the SARAH FlavorKit [33], we have also checked that the most restrictive lepton flavor violating process in the scan to be described below, is just $\text{Br}(\mu \to e\gamma)$. From eq. (9), we obtain

$$M_{12}^\nu = \sum_\alpha h_{1\alpha} \Lambda_\alpha h_{2\alpha} \sim \text{constant.}$$  

(17)

Comparing this result with the corresponding combination of couplings in the expression for $\text{Br}(\mu \to e\gamma)$ in eq. (15), we expect that for a set of fixed input masses $\text{Br}(\mu \to e\gamma)$ turns to be inversely proportional to $\Lambda_\alpha^2$. This is illustrated in figure 2(b) for $\lambda = 5 \times 10^{-3}$, where the scatter plot of $\text{Br}(\mu \to e\gamma)$ is shown for the same range of $\tan\beta$ values than in figure 2(a). In such a case, once $h_{1\alpha}$ are obtained from the Casas-Ibarra parametrization, the specific hierarchy of $\Lambda_\alpha$ fix the several contributions to $\text{Br}(\mu \to e\gamma)$. The dispersion of the points is due to the 3-σ variation of neutrino oscillation data [34] used in the numerical implementation of the Casas-Ibarra parametrization, along with the random variation of the parameters of $\mathcal{R}$. The minimum value of $\text{Br}(\mu \to e\gamma)$ around $\tan\beta = 1$ corresponds to the maximum value of $\Lambda_\alpha$, while the maximum values happen at the cancellation points of each $\Lambda_\alpha$. In the subsequent analysis, and for a fixed SIM and $\lambda$, we allow for cancellations only by two orders of magnitude from the maximum value of each $\Lambda_\alpha$.

The full scan of the input masses up to 2 TeV, with $m_{S_1} > 53\text{ GeV}$ [21] as the dark matter candidate, $M_D > 100\text{ GeV}$ to satisfy LEP constraints, and $10^{-2} \leq \tan\beta \leq 10^2$, give to arise the dark-gray plus light-gray regions in figure 3. In particular, the $\lambda$ variation for the SIM with $\beta = \pi/6$, denoted by yellow stars in figure 2(a), is illustrated with the white dots in figure 3. The corresponding dashed line is obtained for the best-fit values of the neutrino oscillation data and $\mathcal{R}$ fixed to the identity. The horizontal dotted line in the plot corresponds to the current experimental bound for $\text{Br}(\mu \to e\gamma) < 5.7 \times 10^{-13}$ at 90% CL [35]. The upper part of the light-gray region is restricted by our imposition to avoid too strong cancellation in $\Lambda_\alpha$. We check that for all the sets of input masses in the random scan, this cancellation region always happens when $\tan\beta < 1$. In this way, points
with $\tan \beta > 1$ are absent from the light-gray region, as labeled in figure 3. For the same reason, in the dark-gray region there are not points with $\Lambda_\alpha \ll \Lambda_\beta \sim \Lambda_\gamma$ ($\alpha \neq \beta \neq \gamma$). We can check for example that points with $\Lambda_1 \ll \Lambda_2 < \Lambda_3$ are absent inside the dark-gray region of figure 3.

The lower part of the dark-gray region is saturated by the values of $M_N,m_\lambda \ll M_D$ could be probed up to $M_D \lesssim 600 - 700$ GeV for the 14-TeV run of the LHC with 3000 fb$^{-1}$.

On the other hand, in the case of the singlet scalar dark matter, the main production processes associated with the new fermions remain the same, but there are new signals from the mediation, or presence in the final decay chains, of the new scalars. The most promising possibility is the dilepton plus missing transverse energy signal coming from the production of charged fermions decaying into leptons and the lightest scalar. This signal can be important when $\lambda$ is not too large, $\lambda \lesssim 0.1$, and $M_N \gtrsim M_D$. For a fixed set of input parameters, the random phases in the Casas-Ibarra can be chosen to have all the possibilities in the lepton flavor space associated with the coupling $h_{i1}$, with $i = e, \mu, \tau$. In view of that, we will focus in the best scenario where $\text{Br}(\chi^\pm \to e^\pm S_1) \approx 1$. The Feynman diagram for the processes is displayed in figure 4.

The mass of the charged Dirac fermion $\chi^\pm$, can be constrained from dilepton plus missing transverse energy searches at the LHC. In [36], this kind of signals was used by the ATLAS collaboration to establish bounds on the slepton masses from the search for $pp \to \tilde{l}^+\tilde{l}^- \to l^+l^-\chi^0\chi^0$, where $\tilde{l}^\pm$ are the sleptons, $\chi^0$ are the neutralinos and $l^-$ is $e^-$ or $\mu^-$. Purely left-handed sleptons produced and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{brmueg.png}
\caption{\text{Br}(\mu \to e\gamma) in terms of the Yukawa coupling strength $\lambda$ for the SIM in eq. (14) with $\beta = \pi/6$, and the general scan described in the text.}
\end{figure}
decaying this way, have been excluded up to masses of about 300 GeV at 95% CL, from the data with integrated luminosity of 20.3 fb$^{-1}$ and the $pp$ collision energy of 8 TeV. This corresponds to an excluded cross section of 1.4 fb at NLO calculated with PROSPINO [37].

In the present model, the charged fermion field may decay in the mode $\chi^\pm \rightarrow l_i^\pm S_0^\alpha$ which are proportional to the Yukawa couplings $h_{i1}$. Therefore, a similar final state as in the slepton pair production is obtained through the process $pp \rightarrow \chi^+\chi^- \rightarrow l^+l^-S_1^0S_1^0$, as can be seen in figure 4.

![Feynman diagram](image)

**Figure 4:** Feynman diagram for $pp \rightarrow \chi^+\chi^- \rightarrow l^+l^-S_1^0S_1^0$.

In this case, the excluded cross section of this process can be estimated from:

$$\sigma(pp \rightarrow l^+l^-S_1^0S_1^0) = \sigma(pp \rightarrow \chi^+\chi^-) \times \text{Br}(\chi^\pm \rightarrow l^\pm S_0^\alpha)^2,$$

where $\sigma(pp \rightarrow \chi^+\chi^-)$ is the pair production cross section of charged Dirac fermion, and $\text{Br}(\chi^\pm \rightarrow l^\pm S_0^\alpha)$ is the branching fraction for $\chi^\pm \rightarrow l^\pm S_0^\alpha$ mode.

The pair production of charged Dirac fermions can be calculated in the pure-higgsino limit of the minimal supersymmetric standard model. The NLO cross section calculated with PROSPINO is displayed in figure 5 as a function of the charged Dirac fermion.

For points in the parameter space where the Casas-Ibarra solution is chosen such that $\text{Br}(\chi^\pm \rightarrow e^\pm S_1) \approx 1$, and assuming the same efficiency as for the dilepton plus missing transverse energy signal coming from left-sleptons in eq. (18), the charged Dirac fermions of the present model can be excluded up to 510 GeV, as illustrated in figure 5.

Note that many points in the scan of figure 3 with $\lambda \lesssim 0.1$ and featuring $m_{S_1} \ll M_D$, could be excluded by this LHC constraint. However, a detailed analysis of the restriction from the Run I of the LHC, in the full parameter space of the model, is beyond the scope of this work.

### 6 Singlet-doublet fermion dark matter

In this model, the role of the dark matter particle can be played by either the lightest of the fermions $\chi_{LOP}$ or the lightest of the scalars $S_1$. In the latter case, the present model resembles the singlet scalar DM model [1, 2, 3] as long as the other $Z_2$-odd particles do not contribute to the total annihilation cross section of $S_1$, namely through to the addition of new (co)annihilation channels. Therefore, by
choosing a non degenerate mass spectrum and small Yukawa couplings (which is in agreement with neutrino masses) the effects of these particles on dark matter can be neglected. Hence we expect that the dark matter phenomenology to be similar to that of the SSDM [38].

On the other hand, regarding the case of fermion DM, the present model includes the singlet doublet fermion DM model [4, 5, 6, 7, 8, 9]. In such scenario, when the dark matter candidate is mainly singlet (doublet) the relic density is in general rather large (small). In particular, a pure doublet has the proper relic density for $M_D \sim 1$ TeV [5, 9, 39] with decreasing values as $M_D$ decreases. Nonetheless, in the present model we have the additional possibility of coannihilations between the $Z_2$-odd scalars and fermions. In this work, we explore at what extent coannihilation with scalars may allow to recover pure-doublet DM regions with $M_D \lesssim 1$ TeV and $\lambda \lesssim 0.3$, while keeping the proper relic density. Hereafter, we focus in that specific region.

In the simple radiative seesaw model with inert doublet scalar dark matter, the coannihilations with singlet fermions can enhance rather than reduce the relic density, as shown in [40]. That work also presented a review of the several models [41, 42, 43, 44, 45] where such an enhancement also occurs. In particular, supersymmetric models where the neutralino is higgsino-like were considered in [45] and it was shown that slepton coannihilations not only lead to an increase in the relic density but also to an enhancement in the predicted indirect detection signals. Below, we show that the singlet scalars can play the role of the sleptons in our generalization of the higgsino-like dark matter with radiative neutrino masses.

The interactions of the scalars $S_\alpha$ are described by the $h_{i\alpha}$, $\lambda_{\alpha\beta}^{SH}$ terms in eq. (1). It turns out that Yukawa interactions are suppressed by neutrino masses ($h_{i\alpha} \lesssim 10^{-4}$) and the same occurs for the interaction with the Higgs boson if we impose $\lambda_{\alpha\beta}^{SH} \lesssim 10^{-2}$. In this way the coannihilating scalars $S_\alpha$ act as as parasite degrees of freedom at freeze-out leading to an increase of the singlet-doublet fermion relic density.
Figure 6: Regions consistent with the observed relic density for $\lambda = 0.3$ and $\tan \beta = 2$. The solid cyan line corresponds to the observed relic density without coannihilations which was shown to be compatible with the current direct detection bounds from LUX [46] in [9]. The effect of the coannihilations with the new scalars is shown for a mass degeneracy of 0.1 to 10% between the scalars and the DM candidate. The dark-gray region corresponds to coannihilations with one scalar singlet, while the dark plus light-gray regions correspond to coannihilations with two scalar singlets.

By following the discussion in [40], the maximum enhancement of the relic density is achieved when $\Delta S_\alpha = (m_{S_\alpha} - m'^{\chi}_{\text{LOP}})/m'^{\chi}_{\text{LOP}}$ becomes negligible. Accordingly one can write

$$\frac{\Omega^{S_\alpha}}{\Omega^0} \approx \left( \frac{g_0 + g_{S_\alpha}}{g_0} \right)^2,$$

where $\Omega^{S_\alpha}$ ($\Omega^0$) denotes the relic density with (without) including $S_\alpha$ coannihilations, $g_{S_\alpha}$ represents the total number of internal degrees of freedom related to the scalars participating in the coannihilation process and $g_0$ is the total number of internal degrees of freedom when $\Delta S_\alpha \gg 1$. When the DM particle is pure doublet ($M_D \sim 1$ TeV and $M_N \gg M_D$) the fermion masses are $m_1^\chi = M_N$, $m_{2,3}^\chi \approx m_\chi^\pm = M_D$ and therefore $g_0 = g_{\chi_2} + g_{\chi_3} + g_{\chi^\pm} = 8$. Since each real scalar have one degree of freedom we have $g_{S_\alpha} = 1, 2, 3$ depending on the number of scalars coannihilating from which it follows that the maximum enhancement is $\Omega^{S_\alpha}/\Omega^0 = 1.27, 1.56, 1.89$, respectively. This enhancement results in that for the present model with doublet-like DM and $\lambda \lesssim 0.3$ the $M_D$ required to explain the correct relic density lies in the range $[0.9, 1.1]$ TeV instead of taking a single value as in the SDFDM model. The values inside this range, arise due to a no mass degeneracy between the fermions and scalars. In figure 6 we show the effect of coannihilations on the relic density of $m'^{\chi}_{\text{LOP}}$ for a mass degeneracy of 0.1 to 10% between scalar singlets and the DM candidate and for $\lambda = 0.3$ and $\tan \beta = 2$. In particular, in the light-gray region we plot the coannihilations with two scalars to facilitate the comparison with the results in [45] for higgsino-like dark matter coannihilating with a right-handed stau ($g \approx 2$ in their plots). As expected, the upper limit in the LOP mass is about 20% smaller with respect to the case without coannihilation, and we could expect

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4The relic density is calculated with the BSM-Toolbox chain: SPheNo 3.3.6 [47]-MicrOMEGAs 4.1.7 [48, 49].
similar enhancements for indirect DM searches as in [45] for \( g \approx 2 \). Note that the impact of the \( S_\alpha \) coannihilations when \( M_D, M_N < 1 \) TeV, is reduced because in such case the dark matter particle is a mixture of singlet and doublet (well-tempered DM [50]), and the non-negligible splitting among the fermion particles \( \chi \) leads to a non-zero Boltzmann suppression. We have checked that the same results are obtained when \( \lambda \lesssim 0.3 \).

With regard to DM direct detection in the pure-doublet DM scenario discussed above, it is not restricted by the current LUX [46] bounds as long as \( \tan \beta > 0 \). This is due to the existence of zones, known as blind spots, where the spin independent cross section vanishes identically and they occur only for positive values of \( \tan \beta \) [9]. In consequence, the recovered pure-doublet DM regions are still viable in light of the present results of direct searches of dark matter.

7 Conclusions

We have combined the singlet-doublet fermion dark matter (SDFDM) and the singlet scalar dark matter (SSDM) models into a framework that generates radiative neutrino masses. The required lepton number violation only happens if the scalars are real. We have then explored the novel features of the final model in flavor physics, collider searches, and dark matter related experiments. In the case of SSDM, for example, the singlet-doublet fermion mixing cannot be too small in order to be compatible with lepton flavor violating (LFV) observables like \( \text{Br}(\mu \to e\gamma) \), while in the case of fermion dark matter the LFV constraints are automatically satisfied. The presence of new decay channels for the next to lightest odd particle opens the possibility of new signals at the LHC. In particular, when the singlet scalar is the lightest odd-particle and the singlet-like Majorana fermion is heavier than the charged Dirac fermion, the production of the later yields dilepton plus missing transverse energy signals. For large enough \( e^\pm \) or \( \mu^\pm \) branchings, these signals could exclude charged Dirac fermion masses of order 500 GeV in the Run I of the LHC. Finally, the effect of coannihilations with the scalar singlets was studied in the case of doublet-like fermion dark matter. In that case, it is possible to obtain the observed dark matter relic density with lower values of the LOP mass.

8 Acknowledgments

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\footnote{Note that \( \tan \beta > 0 \) corresponds to \( \tan \theta < 0 \) in notation of [9].}
A Analytic formulas for masses and mixing matrix of neutral fermions

The characteristic equation of the mass matrix (2) is [9]:
\[
\begin{pmatrix}
(M^\chi_{\text{diag}})^2 - M_D^2
\end{pmatrix}
\begin{pmatrix}
M_N - (M^\chi_{\text{diag}})
\end{pmatrix}
+ \frac{1}{2} m^2 \chi \left[M^\chi_{\text{diag}} + M_D \sin 2\beta\right] = 0.
\]

The solutions to the cubic equation in \((M^\chi_{\text{diag}})^2\) are:
\[
m_1^\chi = z_2 + \frac{M_N}{3}, \quad m_2^\chi = z_1 + \frac{M_N}{3}, \quad m_3^\chi = z_3 + \frac{M_N}{3}.
\]

where
\[
z_1 = \left(-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}\right)^{1/3} + \left(-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}\right)^{1/3}
\]
\[
z_2 = -\frac{z_1}{2} + \sqrt{\frac{z_1^2}{4} + \frac{q}{z_1}}
\]
\[
z_3 = -\frac{z_1}{2} - \sqrt{\frac{z_1^2}{4} + \frac{q}{z_1}}
\]
\[
p = -\frac{1}{3} M_N^3 - (M_D^2 + m^2_{\chi})
\]
\[
q = -\frac{2}{27} M_N^3 - \frac{1}{3} M_N (M_D^2 + m^2_{\chi}) + \left[M_N M_D^2 - m^2_{\chi} \sin(2\beta) M_D\right].
\]

Notice that \(q^2/4 + p^3/27 < 0\) and therefore, we have three real masses \(m^\chi_i\) \((i = 1, 2, 3)\).

Expanding the eigensystem in eq. (4) by assuming that \(N_{ii} \neq 0\), we have
\[
M^\chi_{ii} N_{ji} + M^\chi_{ji} N_{ii} = -(M^\chi_{11} - m^\chi_i)
\]
\[
(M^\chi_{22} - m^\chi_i) N_{ji} + M^\chi_{32} N_{ji} = -M^\chi_{12}
\]
\[
M^\chi_{23} N_{ji} + (M^\chi_{33} - m^\chi_i) N_{ji} = -M^\chi_{13},
\]
where
\[
N_{1i} = \left[1 + \left(\frac{N_{2i}}{N_{ii}}\right)^2 + \left(\frac{N_{3i}}{N_{ii}}\right)^2\right]^{-1/2}.
\]

Using the matrix \(M^\chi\) given in the eq. (2), we get the ratios
\[
\frac{N_{2i}}{N_{ii}} = -\frac{m\chi \cos \beta}{m^\chi_i} + \frac{M_D [m^\chi_i (M_N - m^\chi_i) + m^2_{\chi \sin \beta + M_D \cos \beta}]}{m^\chi_i m\chi (m^\chi_i \sin \beta + M_D \cos \beta)},
\]
\[
\frac{N_{3i}}{N_{ii}} = -\frac{[m^\chi_i (M_N - m^\chi_i) + m^2_{\chi \cos \beta}]}{m\chi (m^\chi_i \sin \beta + M_D \cos \beta)}.
\]

The analytic formulas for the neutralino masses and the neutralino mixing matrix was analyzed in [51].
A.1 Approximate mixing matrix

By using the analytical expressions for the mixing ratios of eq. (23) with the approximate eigenvalues (5) in eq. (22), we obtain

\[
N_{11}^2 = 1 - \frac{[M_D^2 + M_N^2 + 2M_D M_N \sin(2\beta)] m_\lambda^2}{(M_D^2 - M_N^2)^2} + O(m_\lambda^4)
\]

\[
N_{12}^2 = [\sin(2\beta) + 1] m_\lambda^2 + O(m_\lambda^4)
\]

\[
N_{13}^2 = - \frac{[\sin(2\beta) - 1] m_\lambda^2}{2 (M_D + M_N)^2} + O(m_\lambda^4).
\]

(24)

\[
N_{21}^2 = \frac{m_\lambda^2 (\sin \beta M_D + \cos \beta M_N)^2}{(M_N^2 - M_D^2)^2} + O(m_\lambda^4)
\]

\[
N_{22}^2 = \frac{1}{2} - \frac{m_\lambda^2 (\sin \beta + \cos \beta) [\cos \beta M_N - \sin \beta (M_N - 2M_D)]}{4M_D (M_N - M_D)^2} + O(m_\lambda^4)
\]

\[
N_{23}^2 = \frac{1}{2} + \frac{m_\lambda^2 (\cos \beta - \sin \beta) [\sin \beta (2M_D + M_N) + \cos \beta M_N]}{4M_D (M_D + M_N)^2} + O(m_\lambda^4).
\]

(25)

\[
N_{31}^2 = \left(\frac{M_D \cos \beta + M_N \sin \beta}{M_N^2 - M_D^2}\right)^2 m_\lambda^2 + O(m_\lambda^4)
\]

\[
N_{32}^2 = \frac{1}{2} - \frac{[M_N \sin \beta - (M_N - 2M_D) \cos \beta] (\cos \beta + \sin \beta)}{4M_D (M_N - M_D)^2} m_\lambda^2 + O(m_\lambda^4)
\]

\[
N_{33}^2 = \frac{1}{2} - \frac{[M_N \sin \beta + (M_N + 2M_D) \cos \beta] (\cos \beta - \sin \beta)}{4M_D (M_N + M_D)^2} m_\lambda^2 + O(m_\lambda^4).
\]

(26)

In particular, with eq. (5) and the expressions for \(N_{3i}^2\), the identity (7) is satisfied up to terms of order \(O(m_\lambda^4)\).

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