Tunneling and the Emergent Universe scheme in a JBD Theory

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Abstract.
In this work it is proposed an alternative scheme for an Emergent Universe scenario in the context of a Jordan-Brans-Dicke theory, where the universe is initially in a static state supported by a scalar field located in a false vacuum. The universe begins to evolve when, by quantum tunneling, the scalar field decays into a state of true vacuum.
In particular, in this work we study the stability of the static universe, under the aforementioned characteristics, and the probability of creating a true vacuum bubble. It is shown that in this model, unlike what happens in previous works, the static universe is stable under small classical perturbations.

1. Introduction
The standard cosmological model (SCM) is shown as a satisfactory description of our universe [1, 2, 3]. However, despite its great success, the SCM presents several problems related to its initial conditions at the time of the Big Bang which can be solved through the scenario of Inflationary Universe [4, 5, 6, 7]. Independent of the success of SCM and the inflationary paradigm to explain the cosmological data there are still important open questions to be answered. One of these questions is whether the universe had a definite origin, characterized by an initial singularity or if, on the contrary, it did not have a beginning, that is, it extends infinitely to the past. Theorems about space-time singularities have been developed in the context of inflationary universes, proving that the universe necessarily has a beginning. In other words, according to these theorems, the existence of an initial singularity can not be avoided even if the inflationary period occurs. We can refer to the BGV Theorem. This theorem was developed in 2003 by Arvind Borde, Alan Guth and Alex Vilenkin [8]. Vilenkin’s arguments demonstrate that null and time-like geodesics are generally incomplete in inflationary models, regardless of whether energy conditions are maintained, provided that the average expansion condition ($H > 0$) is maintained throughout of these geodesics directed towards the past. The search for cosmological models without initial singularities has led to the development of emergent universes models (UE) [9, 10, 11, 12, 13, 14, 15, 16]. However, most simple models developed in a context of General Relativity suffer from instabilities, associated with the instability of Einstein’s static universe [17].
On the other hand, Tensor-Scalar theories can give us the possibility to explore the viability of a stable UE model in which there is no initial singularity [19, 18]. The theory normally associated
with Scalar-Tensor models is the Jordan-Brans-Dicke (JBD) theory of gravitation [20, 21]. In this work we propose a scheme for an EU scenario, where the universe is initially in a stable static state. This state is supported by a scalar field located in a false vacuum. The universe begins to evolve when, by quantum tunneling, the scalar field decays to a state of true vacuum. Then, a bubble of a new phase is formed and expands as it converts the energy of vacuum and feeds the energy released to the kinetic energy of the bubble wall.

This is a preliminary report of our results in this respect, where we focus on the study of the existence of a static universe solution and its stability conditions. We also calculate the probability of creation of the bubble in this context, leaving for later the report of the study of the classic evolution of the bubble once materialized [22].

2. Emergent Universes on Jordan-Brans-Dicke Theories

In this work we consider a scheme for EU scenario where the universe is initially in a stable static state. This state is supported by a scalar field located in a false vacuum. The universe begins to evolve when, by quantum tunneling, the scalar field decays to a state of true vacuum. For this reason we will start by studying the possibility of obtaining a static and stable solution in this theory when the scalar field is in the false vacuum. Following the EU scheme we consider a closed Friedmann-Robertson-Walker metric:

$$ds^2 = dt^2 - a(t)^2 \left[ \frac{dt^2}{1-r^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right],$$

(1)

where $a(t)$ is the scale factor and $t$ represents the cosmological time. The matter content is modeled by a perfect fluid with an effective state equation given by $P_f = (\gamma - 1) \rho_f$, with $\gamma$ constant, and a scalar field (inflaton) for which

$$P_{\psi} = \frac{1}{2} \dot{\psi}^2 - U(\psi), \quad \rho_{\psi} = \frac{1}{2} \dot{\psi}^2 + U(\psi).$$

(2)

By assuming that the momentum energy tensor of ordinary matter takes the form corresponding to a perfect fluid, we obtain the field equations of our model.

$$H^2 + \frac{1}{a^2} + H \frac{\dot{\phi}}{\phi} = \frac{\rho}{3\phi} + \frac{\omega}{6} \left( \frac{\dot{\phi}}{\phi} \right)^2 + \frac{V}{3\phi}.$$

(3)

Using the Ricci’s scalar $R = 6 \left( \dot{H} + 2H^2 + 1/a^2 \right)$ we have

$$2\frac{\ddot{a}}{a} + H^2 + \frac{1}{a^2} + \frac{\ddot{\phi}}{\phi} + 2H \frac{\dot{\phi}}{\phi} + \frac{\omega}{2} \left( \frac{\dot{\phi}}{\phi} \right)^2 - \frac{V}{\phi} = -\frac{P}{\phi}.$$

(4)

and the field equation for the JBD field is

$$\ddot{\phi} + 3H \dot{\phi} = \frac{\rho - 3P}{2\omega + 3} + \frac{2}{2\omega + 3} \left[ 2V - \phi V' \right],$$

(5)

where $V' = dV(\phi)/d\phi$, $\rho = \rho_f + \rho_\psi$, and $P = P_f + P_\psi$. The points mean derivatives with respect to cosmological time. In the context of JBD theory solutions are closed universes characterized by the conditions $a = a_0 = \text{Const.}, \quad \dot{a}_0 = \ddot{a}_0 = 0 \quad \text{y} \quad \phi = \phi_0 = \text{Const.}, \quad \dot{\phi}_0 = \ddot{\phi}_0 = 0$. Then by
replacing these conditions in the field equations (3), (4) and (5) the static solution for a universe dominated by a general perfect fluid is obtained if the following conditions are satisfied:

\[ a_0^2 = \frac{3}{V_0'}, \quad \rho_0 = -U_0 - V_0'\phi_0, \quad \gamma = 2\frac{\phi_0}{a_0^2\rho_0}, \]

where \( V_0 = V(\phi_0) \) y \( V_0' = (dV(\phi)/d\phi)_{\phi=\phi_0} \), \( \rho_{f0} = \rho_0 \) y \( \rho_{\phi0} = U_0 \). These equations connect the equilibrium values of the scale factor and the JBD field with the energy density and the JBD potential at the equilibrium point. We now study the stability of this solution against small homogeneous and isotropic perturbations. In order to do this, we consider small perturbations around the static solution for the scaling factor and the JBD field. We set

\[ a(t) = a_0 [1 + \alpha(t)], \]

and

\[ \phi(t) = \phi_0 [1 + \beta(t)]. \]

Then, we have

\[ \rho(t) = \rho_0 + \delta\rho \approx \rho_0 - 3\gamma\rho_0\alpha(t), \]

where \( \alpha \ll 1 \) y \( \beta \ll 1 \) are small perturbations.

By introducing the expressions (9), (10) and (11) into equations (4) and (5), and retaining terms at a linear order in \( \alpha \) and \( \beta \), we obtain the following coupled equations

\[ \ddot{\alpha} - \left[ \frac{1}{a_0^2} + \frac{3(\gamma - 1)}{2a_0^2} - \frac{\beta}{2a_0^2} \right] \alpha - \frac{\beta}{a_0^2} = 0, \]

and

\[ (3 + 2\omega)\ddot{\beta} - \left( \frac{6}{a_0^2} - 2\phi_0V_0'' \right) \beta + (4 - 3\gamma)\frac{6}{a_0^2}\alpha = 0, \]

where \( V_0'' = (d^2V(\phi)/d\phi^2)_{\phi=\phi_0}. \)

From this system of coupled differential equations we can obtain the frequencies for small oscillations

\[ \omega_{\pm}^2 = \frac{1}{a_0^2(3 + 2\omega)} \left[ a_0^2\phi_0V_0'' - 6 + \omega(2 - 3\gamma) \right] \]

\[ \pm \sqrt{\left[ -6 + a_0^2\phi_0V_0'' + 2\omega - 3\omega\gamma \right]^2 + 2(3 + 2\omega) \left( -6 + a_0^2\phi_0V_0'' [3\gamma - 2] \right)}. \]

The static solution is stable if \( \omega_{\pm}^2 > 0 \). Then assuming that the parameter \( \omega \) satisfies the constraint, \( (3 + 2\omega) > 0 \), we find that the following inequalities must be fulfilled in order to have a stable static solution

\[ \frac{2}{3} < \gamma < \frac{4}{3}, \]

\[ -\frac{3}{2} < \omega < -18 \frac{(\gamma - 1)}{(2 - 3\gamma)^2}, \]

and

\[ 2(6 + \omega) - 3(3 + \omega)\gamma + \sqrt{3}|4 - 3\gamma| \sqrt{3 + 2\omega} < a_0^2\phi_0V_0'' < \frac{6}{3\gamma - 2}. \]

From these inequalities we can conclude that for a universe dominated by a perfect standard fluid (with \( \gamma > 2/3 \)) and a scalar field localized in a false vacuum, it is possible to find a solution where the universe is static and stable.
3. Bubble Nucleation

We calculate the probability of creation of the bubble in a context of JBD Theory. The calculation of the decay of the false vacuum was solved by Coleman and Callan [23, 24] and the incorporation of gravitational effects was studied by Coleman and De Luccia [25]. However, the latter analysis is not applicable here because of the non-standard gravitational action. In order to avoid these difficulties, we reformulate the theory by perform a Weyl rescaling in order to remove the coupling of \( \phi \) with \( R \). We consider the following action

\[
S = \frac{1}{16\pi} \int d^4x \sqrt{-\tilde{g}} \left[ \phi R - \frac{\omega}{\phi} \nabla_\mu \phi \nabla^\mu \phi + V(\phi) + \mathcal{L}_m \right].
\]

In particular, we define the new fields \( \tilde{g} \) y \( \tilde{\phi} \) as

\[
g_{\mu\nu} = \Omega^{-2}(x) \tilde{g}_{\mu\nu}, \quad \tilde{\phi} = \sqrt{\frac{2\omega + 3}{16\pi G}} \ln (\phi/\phi_0),
\]

where \( \phi_0^{-1} = G \), here \( G \) is the value of the Newton constant observed in the present. We choose the conform factor \( \Omega(x) \) so that \( \Omega = \sqrt{G\phi} \). Finally we see that the action in the rescaled theory can be expressed as:

\[
S[\tilde{g}, \tilde{\phi}, \psi] = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{1}{16\pi G} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{\phi} \tilde{\nabla}^\mu \tilde{\phi} + W(\tilde{\phi}) \right. \\
+ \exp \left(-4\sqrt{\frac{\pi G}{2\omega + 3}} \tilde{\phi}\right) \frac{1}{2} \tilde{g}^{\mu\nu} \nabla_\mu \psi \nabla^\nu \psi - \exp \left(-8\sqrt{\frac{\pi G}{2\omega + 3}} \tilde{\phi}\right) U(\psi) \right].
\]

We note that we have a model where we removed the non-standard coupling between \( \phi \) and \( R \), where the gravitational coupling constant is independent of time, also have a minimally coupled field \( \tilde{\phi} \). And an inflaton field now coupled to \( \tilde{\phi} \).

Now that we have removed the coupling of \( \phi \) to \( R \) we can calculate the nucleation rate in this rescaled theory. Due to Weyl’s rescaling the two line elements \( ds^2 = dt^2 - a^2(t)dl^2 \) and \( ds^2 = dt^2 - a^2(t)dl^2 \) are related by \( ds = \Omega ds \), we also have \( d\bar{t} = dl \), hence

\[
\frac{dt}{d\bar{t}} = \frac{a(t)}{\dot{a}(t)} = \Omega(t).
\]

In the rescaled theory the probability of nucleation per unit of physical volume per time is given by

\[
\dot{\Gamma}(\bar{t}) = \frac{dP(\bar{t})}{dV(\bar{t})d\bar{t}},
\]

where \( P(\bar{t}) \) is the bubble probability of nucleation and \( V(\bar{t}) \) is the physical volume measured in the rescaled system at time \( \bar{t} \). Therefore, the equivalent rate in the original theory is

\[
\Gamma(t) \equiv \frac{dP(t)}{dV(t)dt} = \frac{dP(t)}{(\Omega^3d\bar{V})} = \Omega^{-4}(t)\dot{\Gamma}(\bar{t}),
\]

where we have used the equation (24) and the fact that \( P(t) = P[\bar{t}(t)] \).

In JBD theories, the gravitational coupling and the time dependence of the JBD field might cause the bubble nucleation rate to deviate from its value to a flat space. In order to eliminate any time dependence coming directly from JBD field, we consider the limit \( G \to 0 \) in the theory.
of the equation (22) adopted by [26]. At this limit the term gravitational in action can be omitted. The term, which contains the kinetic energy of the JBD field, should also be neglected. To make this we replace $\tilde{\phi}$ with
\[
\tilde{\Phi} = \sqrt{\frac{16\pi G}{2\omega + 3}} \tilde{\phi}.
\]
(27)
The term containing the kinetic energy of the JBD field now has the form
\[
\frac{1}{2} \left( \frac{2\omega + 3}{16\pi G} \right) \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{\Phi} \tilde{\nabla}_\nu \tilde{\Phi}.
\]
(28)
Then this term is scaled precisely in the same way as the term containing the Ricci scalar as $G \to 0$.

Finally we are left with the last two terms of the equation (22). Only the field $\psi$ remains as a dynamic quantity. Without their derivative terms, the metric and JBD field now enter essentially as externally specified non-dynamic quantities. While it has been shown that if it is assumed that the background gravitational field remains classical, static and is not perturbed during the decay, Coleman-De Luccia’s prescription can be justified as a direct extension of the formalism of flat space, the problem general of back-reaction of the geometry against a quantum (non-causal) decay has not been answered.

In order to eliminate this problem, we neglect during this calculation the gravitational back-reaction of the bubble on the geometry of space-time. Since gravitational effects may be small only if the bubble size is much smaller than the curvature of space, we take the metric as flat, and write the action of this theory as
\[
\bar{S}[\psi] = \int d^4x \left[ \frac{1}{2} b \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \psi \tilde{\nabla}_\nu \psi - b^2 U(\psi) \right],
\]
(29)
where we have defined
\[
b \equiv \exp \left( -4 \sqrt{\frac{\pi G}{2\omega + 3}} \tilde{\phi} \right).
\]
(30)
Now we would like to calculate the rate of nucleation of a bubble in this theory, or more precisely, to determine its dependence on the value $b$. Without the present gravitational interaction we can use the methods of the references [23, 24], although with some necessary modifications due to the non-standard form of the kinetic energy term.

The nucleation rate $\bar{\Gamma}$ is given by
\[
\bar{\Gamma}(t) = A \exp \left( -S_E \right),
\]
(31)
where $S_E$ is Euclidean action. It is shown in [26] that the prefactor $A$ has a net factor of $b^2$ with respect to a theory in which $b = 1$, this entails that $\bar{\Gamma}(t) = b^2 \Gamma_0$, where $\Gamma_0$ is the nucleation rate for a normal scalar field theory with potential $U(\psi)$.

Since $b$ is a function of the time-dependent JBD field, then we have a time-dependent nucleation rate in the rescaled theory. However, this time dependence disappears if we return to the original theory. The equations (20) and (21), together with the definition of $\phi_0$ and $\Omega$ imply that $b = \Omega^2$.

Using the equation (26) we find that the nucleation rate of the original theory is
\[
\Gamma(t) = \Omega^{-4}(t) \bar{\Gamma}(\tilde{t}) = (b^{-2}) (b^2 \Gamma_0) = \Gamma_0,
\]
(32)
and it is then independent of time on the approach with which we are working.

From the results obtained by [27] we have the imaginary part of the action for a universe with a closed geometry $k = 1$ is
\[
\text{Im}[S] = \frac{27\sigma^4}{4\epsilon^3} \left[ 1 - \frac{1}{2} r_0^2 \right],
\]
(33)
where $\epsilon$ is the difference of energy density between the two phases (latent heat), $\sigma$ the energy density of the wall and $r_0 = 3\sigma/(\epsilon a_0)$ is the nucleation radius of the bubble when the space is flat and static.

Then, the nucleation rate $\Gamma_0$ for our model will be

$$\Gamma_0 \approx \exp \left( -2 Im [S] \right) \approx \exp \left[ -\frac{27\sigma^4\pi}{2\epsilon^3} \left( 1 - \frac{81\gamma^2 \rho^2 \epsilon^2}{8 \phi_0^2 \epsilon^4} \right) \right]. \quad (34)$$

We can conclude that the bubble nucleation rate, within this approximation, is essentially that of the theory used by Coleman-De Luccia to which a correction of the order of $r_0^2$ has been added.

Once the nucleation rate was obtained, we have to study the evolution of the true vacuum bubble after its nucleation. To calculate this evolution we will consider the response of the background. In particular, we will follow the scheme developed by [28] regarding the conditions of joining the formalism of Darmois-Israel ([29, 30]) in JBD Theories. At this respect, we note that the bubble is formed in a region of the initial static universe, which has a perfect fluid as content of matter. As the bubble expands, this matter accumulates on the outer side of its wall. In this sense, the bubble could collapse or, on the contrary, continue to expand until it fills the entire space. On the other hand, inside the bubble it constitutes our universe, according to the scheme of open inflation, so it is important to determine whether the bubble once materialized collapses or expands in this model. We have postponed the report of the study of the classical evolution of the bubble, but preliminary results show that once the bubble materializes it grows without collapsing [22].

4. Conclusions

In this work we propose an scheme for an Emergent Universe scenario, where the universe is initially in a truly static state. This state is supported by a scalar field which is located in a false vacuum. The universe leaves the static state when, by quantum tunneling, the scalar field decays into a state of true vacuum.

In our model the matter content, in the initial static universe, is modeled by a perfect fluid and by an inflaton scalar field located in the false vacuum.

In particular, in Section 2, we search for static universe solutions for our emergent universe model. Then we study the stability of these solutions, finding the conditions that must be satisfied in order to obtain a static universe solution that is stable under small isotropic and homogeneous perturbations. These results show that our emergent universe model, based on Jordan-Brans-Dicke theory, presents static-universe solutions that do not suffer from classical instabilities, improving the results of [27].

In the section 3 we calculate the nucleation rate of the true vacuum bubble using the approximation developed by [26]. According to this approach, the calculation of the bubble nucleation rate in a context of Jordan-Brans-Dicke theories, can be reduced to the case of a coupling constant independent of time. This allows us to use the results obtained by [27] in our case. The calculation of the nucleation rate of the true vacuum bubble was performed applying these results to our model.

We have left for later the report of the study of the classical evolution of the bubble once materialized [22], but preliminary results show that once the bubble materializes it grows without collapsing. Then this model is viable and an interesting alternative for EU to be studied in detail.

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