Abstract—As demand for electric vehicles (EVs) is expanding, meeting the need for charging infrastructure, especially in urban areas, has become a critical issue. One method of adding charging stations is to install them at parking spots. This increases the value of these spots to EV drivers needing to charge their vehicles. However, there is a cost to constructing these spots and such spots may preclude drivers not needing to charge from using them, reducing the parking options for such drivers. We look at two models for how decisions surrounding investment in charging stations on existing parking spots may be undertaken. First, we analyze two firms who compete over installing stations under government set mandates or subsidies. Given the cost of constructing spots and the competitiveness of the markets, we find it is ambiguous whether setting higher mandates or higher subsidies for spot construction leads to better aggregate outcomes. Second, we look at a system operator who faces uncertainty on the size of the EV market. If they are risk neutral, we find a relatively small change in the uncertainty of the EV market can lead to large changes in the optimal charging capacity.

Index Terms—Parking, electric vehicles, congestion, game theory.

I. INTRODUCTION

THOUGH sales of electric vehicles (EVs) are increasing rapidly, there are still barriers to their wide-spread adoption. A survey by AAA found that “not enough places to charge” was cited by 63% of consumers unwilling or unsure about purchasing an electric car, making it the most common reason cited [1]. While 80% of charging currently occurs at home, in many areas (like densely populated urban areas) the availability of at home charging may not be feasible. It is thus important to study ways of making more charging infrastructure available to the general public.

Federal and state governments have been pursuing multiple strategies to increase charging infrastructure. One method of increasing availability of charging is by inducing private firms to install charging stations at parking spots. For example, the city of Seattle is working with two private firms to install EV charging stations throughout the city in order to induce more EV drivers [2].

Given that these decisions affect not only the EV market but also the market for internal combustion engine (ICE) cars (in most cases are excluded from using designated charging spots), the potential effects of such policies warrants investigation. As far as we know, this is one of the first attempts to explicitly do so.

A variety of work has focused on optimizing EV charging, both in a competitive market and when a system operator is looking at global welfare [3]–[7]. In [3] the authors analyze the competition between charging stations (not necessary in a parking context) with renewable generation. The effect of EVs on the power grid and road system is studied in [6], which offers ways system planners can optimize for this. Our work differs from [3] by analyzing how firm’s investment in parking affects the market for EVs and ICEs and from [6] by focusing on parking rather than travel decisions of EV drivers along road networks.

A key feature of parking is that it is a congestible resource, i.e., the more users that park in a given location, the less desirable it is (on average) for the next user who parks there. Commonly this effect is modeled with a congestion cost term that is increasing in the number of users parking in a given location [8]. We follow this approach here and adopt a model similar to those used in the literature on competition with congestible resources, e.g., [9]–[18]. In particular, [9] most closely resembles our model, as it looks at wireless service providers pricing unlicensed and licensed spectrum where consumers suffer congestion when other consumers use the same band. We will similarly look at a case where two classes of a market (parking for drivers utilizing charging and those who do not) are priced and where consumers suffer congestion due to other consumers of the same class, but differ as each class of service can only serve one type of consumer. Work like [16]–[18] have focused on capacity competition in wireless markets, and similar to our first model [17] examines firms competing by first choosing a level of investment and then competing over prices. Unlike these wireless markets, our work examines a market where firm’s investment in capacity for one class of consumer precludes another set from utilizing it.

There is a rich literature on examining the effect of prices and competition on parking decisions by consumers and the resulting welfare effects [8], [19]–[26]. The interaction of parking and road-usage congestion is examined in [19], which shows a road-usage fee in addition to a parking fee is needed to maximize welfare. The work in [20] builds upon Vickrey’s classic bottleneck road congestion model to examine how different pricing options can help maximize welfare by reducing or eliminating congestion. The effect of setting a minimum amount of parking for urban land use, and how this reduces welfare compared to allowing the market to decide the provision of spots is studied in [23]. However, none of this work looks at situations where parking spots equipped with charging and those without interact.

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This paper focuses on the effect of government policy in two cases: in the first, two incumbent private owners of parking spaces are installing parking spots with charging stations and in the second a monopolist owns all spots but has uncertainty over the size of the market interested in charging while they park. In the first case we consider how two policies affect these firms: namely introducing subsidies for constructing EV charging spots and setting a minimum proportion of spots that should have chargers installed. This portion is built upon our earlier work [27]. We compare the effectiveness of these policies under different assumptions on how firms compete and the relative market sizes of drivers interested in charging and those who are not.

In the second case, we study the impact of stochasticity in the more valuable market (the EV market) on construction decisions by a monopolist. We assume a monopolist holds a mass of parking spots in an area with many ICE drivers. Due to the potential difficulties in forecasting the size of the future EV market, they have some uncertainty on how large the market of EVs interested in charging may be. We will show both the monopolist’s expectation and government policy can have a large impact on outcomes.

The rest of the paper is organized as follows. Section II defines the model for the competitive market; Section III analyzes the game defined for the competitive case and Section IV summarizes the results of this analysis; Section V analyzes a specific case study; Section VI defines the model for the monopolist; Section VII examines the pricing structure in this case; Section VIII examines the impact capacity decisions have on the optimal operating point; Section IX studies a numerical case study for the monopolist case; Section X concludes and looks at potential future work.

II. COMPETITIVE MARKET: MODEL

In this section we present our model for firms competing over serving EV and ICE drivers.

A. Consumers

We assume there is a mass of two classes of consumers: charging drivers who use EVs and need to charge when the park and non-charging drivers who either drive ICEs or drive an EV but do not wish to charge.\(^1\) Consumers of both classes are assumed to be non-atomic. There will be two types of parking spots: regular parking spots without charging infrastructure and spots equipped for charging EVs.

As is standard in parking models we model the congestion as a function of the proportion of occupied spots [8]. This reflects that with higher occupancy each driver has to spend more time on average to find a vacant spot. We assume firms cluster spots of the same type together (i.e., those with charging and those without) and so consumers at each firm’s parking location suffer congestion due to drivers of the same class but not due to other class of drivers, nor with any at the other firm. In general we want a congestion function for each marginal consumer that is increasing in the quantity of consumers already parked, decreasing in the number of spots, and convex in the volume-capacity ratio [8]. We will use a linear congestion function, which will be the same for both classes of consumers, i.e.,

\[
S(q, N) = \epsilon q \frac{q}{N},
\]

where \(q\) is the quantity of that class of consumers already parked, \(N\) is the number of spots allocated for that class of driver, and \(\epsilon > 0\) is a constant term that represents the congestion cost if all spots are filled.\(^2\)

We do not model differences in the length of time a consumer may want to park; they simply want access to the ability to park (and will all pay the same price at a given firm to do so as discussed in the firm section below). One can think of this as a mass of drivers going to work, where they all work the same amount of time. Each consumer therefore purchases the same good, which is the ability to park during the work day.

We will assume for both classes that there is a linear downward inverse demand curve for parking determined by the valuation and size of each market, which we define for charging drivers \((P_e(q_e))\) where \(q_e\) is the quantity of EV drivers) and non-charging drivers \((P_d(q_d))\) where \(q_d\) is the quantity of drivers as

\[
P_e(q_e) = W_e(1 - \alpha q_e),
\]

\[
P_d(q_d) = W_d(1 - \beta q_d).
\]

The \(y\)-intercept of the non-charging drivers inverse demand curve is \(W_d > 0\), and the slope is \(\beta > 0\); while the \(y\)-intercept for the mass of charging drivers is \(W_e > 0\), and the slope is \(\alpha > 0\). Instead of modeling explicitly the amount of charge drivers want, for simplicity we assume that access to charging can be modeled by having charging customers gain a higher utility for parking than those that don’t charge. We capture this by assuming \(W_e > W_d\), where \(W_e\) captures both demand for the parking and the ability to charge at the parking spot.

B. Firms

We assume there are two firms (i.e., parking garage owners) that already own a mass of parking spots suitable for ICE drivers. We normalize the number of total spots to be 1 and the number of spots firm \(i\) has as \(N_i\). Specifically, we define the proportion of spots that firm 1 owns as \(N_1 = \delta\), where \(0 \leq \delta \leq 1\). This means firm 2 owns \(N_2 = 1 - \delta\) spots. Firms are unable to construct new spots, but either due to government policy or their own choice they are able to convert a portion of their spots to

\(^1\)Here we assume that the type of a driver is exogenously fixed and do not consider the case where an EV driver can strategically decide whether to charge.

\(^2\)This linear congestion function is chosen to make subsequent analysis tractable. This function has the desired properties of being convex and increasing in the occupancy ratio [8]. It can be motivated by a scenario in which all parking spots are in a straight line and where \(\epsilon\) represents the amount of disutility experienced by driving past a filled spot. On average we can imagine each driver has to drive past the occupancy ration, \(\frac{q}{N}\), of spots, and thus experiences \(\epsilon \frac{q}{N}\) disutility. Note we have not put any constraints on \(q\) and thus allow \(q > N\). This corresponds to a case where on average not only are all spots taken but there are consumers cruising around for spots (i.e. they have to drive down the line of spots multiple times).
include an EV charger and sufficient renewable generation to cover EV charging demand (for example solar panels) at cost $p$. We will call the number of spots designated for EVs at firm $i$ as $N_{ei}$ and the remaining spots as $N_{di}$.

We assume the firms are close together so on average consumers have no preference for one firm over the other.

We define the value of parking for a marginal driver of each class at each firm $i$ when the firm sets a price $m_i$ ($c_i$) for non-charging drivers (charging drivers) and $q_{di}$ ($q_{ei}$) non-charging drivers (charging drivers) are parked at firm $i$ as follows:

$$U_{ei}(q_{e1}, q_{e2}, N_{ei}, c_i) = W_e(1 - \alpha(q_{e1} + q_{e2})) - \epsilon \frac{q_{ei}}{N_{ei}} - c_i,$$

(4)

$$U_{di}(q_{d1}, q_{d2}, N_{di}, m_i) = W_d(1 - \beta(q_{d1} + q_{d2})) - \epsilon \frac{q_{di}}{N_{di}} - m_i.$$

(5)

Here, the first terms reflect the inverse demands in (2) and (3). The marginal utility of each class of driver at firm $i$ is a function of the number of spots available for that driver and the price set for them both by firm $i$ and the opposing firm $j$. The number of consumers at each firm in equilibrium is defined using the notion of a Wardrop equilibrium [28]. A Wardrop equilibrium here means that when drivers are parking at both firms, the following two conditions hold: the marginal utilities of the consumers at each garage have to be equal ($U_{ei} = U_{e_j}$, $U_{di} = U_{d_j}$) and the marginal utilities have to be 0 ($U_{ei} = 0$, $U_{e_j} = 0$, $U_{di} = 0$, $U_{d_j} = 0$). This means that consumers park until the marginal user would gain negative utility from doing so, and sort themselves at the garage at which they get the highest utility. We define $q_{di}(N_{d}, c)$ as the quantity of EV drivers served at firm $i$ and $q_{ei}(N_{e}, c)$ as the quantity of ICE drivers served at firm $i$, with both these quantities defined by the Wardrop equilibrium conditions as described above. Using these expressions, we can therefore specify the general problem faced by each firm $i$:

$$\max_{N_{ei}, N_{di}, m_i, c_i} \Pi_i = m_i q_{di}(N_{d}, m) + c_i q_{ei}(N_{e}, c) - p N_{ei}$$

s.t. $N_{ei} + N_{di} = N_i$, $N_{di} \geq 0$, $N_{ei} \geq 0.$

(6)

Note that each firm seeks to solve such a problem, and that their decisions are coupled through the resulting Wardrop equilibrium quantities.

We can get a closed solution for the quantity expressions by assuming that there exist $q_{ei} \geq 0$ and $q_{dj} \geq 0$ such that the marginal utilities at each firm are 0. Let $N_{ei} = \frac{N_i}{e}$, $N_{di} = \frac{N_i}{e}$.

These expressions were derived assuming that the marginal utility at each firm was zero for each class of user. This may not be the case if one firm charges a large enough price for a given class so that the marginal utility is negative even when no users park.

We show in Appendix 10.1 that in all Nash equilibria with respect to prices given any valid choice of capacity in the overall game this never occurs and thus these closed form expressions exactly represent the Wardrop Equilibria quantities on the equilibrium path. Therefore, when we analyze the game we will use these closed form in the firm’s problem (meaning we substitute $q_{di}(N_{d}, m)$ for $q_{di}(N_{d}, m)$ and $q_{ei}(N_{e}, c)$ for $q_{ei}(N_{e}, c)$) as described in (6) in order to make deriving the equilibrium more tractable.

C. Government

We assume the government wants to induce more charging infrastructure to be constructed by firms. We will examine two methods of doing so: issuing a mandate and subsidizing construction. These are policies being currently explored and implemented.

For the mandate, we define the proportion of spots mandated for EV charging as $r$ and incorporate this into (6) by adding an additional constraint

$$N_{ei} \geq r N_i,$$

(9)

For the subsidy, we will allow the government to set a subsidy $s$ such that the cost of constructing EV spots for firms as a function of the intrinsic cost $t$ is

$$p = t - s.$$

(10)

D. Consumer Surplus and Total Welfare

Given that consumers are non-atomic, then the consumer surplus for each class of consumer is given by integrating (4) and (5) over the quantity serviced at both firms, i.e.,

$$CS_{EV} = \int_0^{q_{e1} + q_{e2}} W_e(1 - \alpha q) dq - \int_0^{q_{e1}} (\epsilon \frac{q}{N_{e1}} + c_1) dq - \int_0^{q_{e2}} (\epsilon \frac{q}{N_{e2}} + c_2) dq.$$

(11)
Note here we are assuming that drivers park sequentially so that early drivers incur a lower congestion cost than later drivers. Each firm’s welfare will simply be the total profit they receive given by the objective function in (6).

The government has to pay for any subsidies they offer firms. We define the cost of subsidies as

\[
G(N_{e1}, N_{e2}, s) = s(N_{e1} + N_{e2}.
\]

We therefore define the total welfare as the sum of consumer surpluses, the firm profits, and the government’s cost to provide subsidies, i.e.,

\[
W_{\text{total}} = CS_{\text{ICE}} + CS_{\text{EV}} + \Pi_1 + \Pi_2 - G(N_{e1}, N_{e2}, s). \tag{13}
\]

### III. Competitive Market: Sequential Spot Allocation and Pricing Game

We model the overall market interactions as a one shot, two-stage game. In the first stage, the firms simultaneously set the number of parking spots they will convert to have chargers. In the second stage they set prices having committed to the number of installations they made in the first stage. We will solve this game using backward induction, so we first examine the second stage (where parking spot quantities have already been set).

#### A. Second Stage: Price Competition

We will consider three different ways in which firms may set prices given fixed quantities of parking spaces. These will differ in the degree to which firms price discriminate and anticipate the impact of EV parking. We begin with a “two pricing case” in which firms discriminate in the prices that they charge charging and non-charging drivers accounting fully for the different preferences of these two classes. However, around 50% of public chargers are free and thus it seems many private owners of charging stations do not specifically set a price to use that service [30]. We analyze two different scenarios where this might occur. The first is where they set a single price for both classes of drivers, but optimize that price understanding charging drivers’ valuations. We call this the “optimal single price case”. In the second case, we assume that the garages had in the past (before constructing EV spots) set prices to compete over only non-charging drivers and then maintain this price even after charging stations are added to parking spots. We call this the “Naive” single price case. Each of these pricing schemes could also arise in part due to regulation that limits the prices that can be charged. We look at each case separately. A summary of the cases is shown in Table I below.

1) Two Price Case: In this case we assume that the firms each set two separate prices: one for charging spots (\(c\)) and one for the non-charging spots (\(m\)). The price for EVs who choose to charge can be thought of as the plug-in price on top of the price of parking (so \(c = m + e\), where \(e\) is this plug-in price). In principle \(e\) could vary with the amount of electricity an EV consumes while parked (and could also vary with the underlying cost of electricity in a real-time market). Here, we do not model such considerations and simply view each EV as incurring the same additional cost \(e\) (or equivalently we could view this as the average cost per EV). We also do not explicitly model the length of stay for each driver, so all drivers of the same class pay the same cost at each firm’s location. Each firm in this stage seeks to choose prices to maximize revenue. Note that in this stage the cost of building parking spaces is sunk and so we can ignore this when optimizing revenue. Also, once the spaces are allocated, the pricing decision for the two classes de-couple and thus a firm’s optimal prices in response to the price of its opponent are given by:

\[
c_i^{BR} = \arg\max_{c_i} c_iq_{e_i}(N_{e1}, N_{e2}, c_1, c_2),
\]

\[
m_i^{BR} = \arg\max_{m_i} m_iq_{d}(N_{d1}, N_{d2}, m_1, m_2).
\]

It can be shown that the objective in these optimization problems is concave in the prices and so the solution can be found by evaluating the first order optimality conditions which yields:

\[
c_i^{BR}(N_{e_j}, c_j) = \frac{W_{c}(1 + \alpha N_{e_j}c_j)}{2(\alpha W_{c}N_{e_j}^2 + 1)},
\]

\[
m_i^{BR}(N_{d_j}, m_j) = \frac{W_{d}(1 + \beta N_{d_j}m_j)}{2(\beta W_{d}N_{d_j}^2 + 1)}.\]

Hence, by solving for the fixed point of the best responses we get the following expressions for the unique equilibrium prices in the second stage:

\[
c_i^*(N_e) = \frac{2\alpha W_{d}N_{e}^2}{3\alpha^2 W_{e}^2 N_{e} N_{e_{j}}^2 + 4\alpha W_{e} N_{e_{j}}^2 + N_{e_{j}} + 4} + 2W_e, \tag{14}
\]

\[
m_i^*(N_d) = \frac{2\beta W_{d}^2 N_{e_{j}}^2}{3\beta^2 W_{d}^2 N_{e_{j}}^2 + 4\beta W_{d} N_{e_{j}}^2 + 4} + 2W_d. \tag{15}
\]

These are increasing and convex functions in \(N_{e_{j}}\), meaning as a firm invests in more spots they increase their price at a decreasing rate. If \(N_{e_{j}} = 0\), these prices become the monopolist prices.
TABLE II
FIRM CAPACITY DECISIONS

| Case                        | Capacity constraint |
|-----------------------------|---------------------|
| "Naive" mandate fulfillment | \( N_{ct} = r N_i \) |
| Optimal Capacity             | (9)                 |

(as the other firm exits the market for that class of consumer), which are \( \frac{W}{C_3} \) and \( \frac{W_d}{C_7} \) respectively.

2) Optimal Single Pricing Case: In this case we assume that each firm only sets one price that both charging drivers and non-charging drivers pay to park (i.e., the plug-in price is always 0). Each firm in this stage seeks to choose a single price \( m \) to maximize revenue. Note again that in this stage the cost of building parking spaces is sunk and so we can ignore this when optimizing revenue. Thus a firm’s optimal price in response to the price of its opponent are given by:

\[
m_i^{BR} = \arg\max_{m_i} m_i[q_{ei}(N_e, m) + q_{di}(N_d, m)].
\]

Again this objective can be shown to be concave in price, and therefore we can find the solution by examining the first order optimality conditions. Define

\[
g_i^m(N_e, N_d, m_j) = W_e \left[ \beta W_d \left( 1 + \frac{N_d}{N_{di}} \right) + \frac{1}{N_{dj}} \right] (1)
+ \alpha N_{ej}(m_j) + W_d \left[ \alpha W_e \left( 1 + \frac{N_e}{N_{ei}} \right) + \frac{1}{N_{ej}} \right] (1 + \beta N_{dj}(m_j)),
\]

\[
h_i^m(N_e, N_d, m_j) = (\alpha W_e N_{ij} + 1) \left( \beta W_d \left( 1 + \frac{N_d}{N_{di}} \right) \right)
+ \frac{1}{N_{dj}} + \left( \frac{\beta W_d N_{ej}}{N_{di}} + 1 \right) \left( \alpha W_e \left( 1 + \frac{N_e}{N_{ei}} \right) + \frac{1}{N_{ej}} \right).
\]

Using these, the best response price for each firm is:

\[
m_i^{BR}(N_e, N_d, m_j) = \frac{g_i^m(N_e, N_d, m_j)}{2h_i^m(N_e, N_d, m_j)}.
\] (16)

This expression can be viewed as a weighted sum of the price they would set for the non-charging demand and the charging demand independently by how valuable (given the capacity they and the opposing firm have set) each market is to them. Note this expression is equal to the two price case for non charging or charging customers if \( N_{ei} = 0 \) or \( N_{di} = 0 \), respectively. As before we can solve for the fixed point of these best responses to get an expression for the equilibrium prices as a function of the number of parking spots for each class of driver \((m_i^*(N_e, N_d))\). The expression contains many terms and so is omitted here.

3) “Naive” Single Pricing Case: In this case we also assume firms charge one price for parking. However firms are “naive” about the entry of EVs into the market and so maintain the parking prices they had set before they had constructed EV chargers. Thus the best response function for the single price they charge is

\[
m_i^{BR} = \arg\max_{m_i} m_i[q_{di}(N_{di} = \delta, N_{d2} = 1 - \delta, m_1, m_2)].
\]

Note in this case the price is independent of any decision made in the first stage of the game and any parameters of the charging drivers market as the objective function assumes \( N_{di} = \delta, N_{d2} = 1 - \delta \). Again, we can solve this using the first-order optimality conditions to derive the best responses and then solve for the fixed point. Defining \( N_i^* = \frac{N_i}{c} \), this yields

\[
m_i^* = \frac{2\beta W_i^2 N_i^* + 4W_i^2 N_i^* + 2W_d^2}{3\beta^2 W_i^2 N_i^* + 4W_i^2 N_i^* + 4W_d^2 N_i^* + 4}.
\] (17)

The price for both firms is monotonic decreasing in the size of the non-charging drivers \( \beta \).

B. First Stage: Capacity Competition

Next, we turn to the first stage in which firms decide on how many EV spaces to construct accounting for the resulting price equilibrium in the second stage. We will look at two scenarios of how firms may respond to the government mandate in this stage. In the pricing stage, we assumed in some cases firms did not fully internalize their ability to price charging drivers differently than non-charging drivers. We will do something similar in this stage. In one case we assume firms “naively” follow the mandate while in the other they will more intelligently choose the number of spaces to optimize their objective function in (6).

In the first scenario, the firms simply meet the mandate (i.e., adding \( N_{ei} = r N_i \) as a constraint in (6)). In the second scenario, both firms optimize over the number of EV spots taking the mandate as a lower bound (adding (9) as a constraint in (6)). Table II summarizes these two scenarios.

IV. COMPETITIVE MARKET: RESULTS

We now summarize the main results from analyzing the overall equilibrium under the different scenarios.

Proposition 1: For the “naive” mandate fulfillment case, a sub-game perfect Nash equilibrium exists and is unique for all pricing schemes. In the optimal capacity case, a sub-game perfect Nash equilibrium exists if \( \alpha W_i \leq 1 \), \( \beta W_d \leq 1 \) and \( \epsilon = 1 \) for the two price case and for any values in the “naive” single price case.

This proposition shows that under certain scenarios a unique sub-game perfect Nash equilibrium exists. In general a closed form expression for this equilibrium appears to be difficult to obtain. In Section V we will instead numerically analyze properties of these equilibria. The uniqueness of the equilibria given in this theorem provides a guarantee that our results will not depend on the algorithm used to numerically find equilibria. The proof for this is provided in the appendix.

Proposition 2: If the total number of spots across both firms for a class of driver is fixed, then the quantity of that class of

\footnote{The constraints on \( \alpha W_i, \beta W_d \), and \( \epsilon \) were chosen as they more cleanly allowed us to show the revenue functions for both firms were strictly concave. This proposition does not preclude equilibria existing when these exogenous variables do not satisfy these constraints. These bounds can be interpreted as guaranteeing that the disutility caused by congestion at one firm if all drivers parked at a it is larger than the the utility the highest valued consumer gets from parking or parking and charging.}
driver serviced is maximized if when these spots are divided equally across the firms.

This makes intuitive sense. If there is a fixed number of spots for a given class of driver, the price will be lowest when firms have equal market power and thus compete the most. As a direct corollary to this we have:

**Corollary 1:** Consumer welfare is maximized when each firm has the same initial endowment \((\delta = 0.5)\) and there is a unique equilibria.

This follows from Proposition 2. When \(\delta = 0.5\), it can be shown in all cases where equilibria are unique that each firm chooses the same number of spots for charging and non-charging drivers. This cannot happen if \(\delta \neq 0.5\) as in this case the same number of spots cannot be constructed for both classes of drivers: at best this can only occur for one class. Again this result is intuitive, as consumer welfare is maximized when no firm has market power over another.

**Proposition 3:** The best response prices in either market for the two price case is decreasing as the slope of demand in that market increases (i.e. as \(\alpha\) or \(\beta\) increases). The best response price in the “naive” single price case increases as \(\beta\) increases.

**Proof:** We prove this by showing that in the two price case the derivative of best response price for EV drivers is negative with respect to \(\alpha\), \(\forall \alpha \geq 0\) and the derivative of best response price for non-charging drivers is negative with respect to \(\beta\), \(\forall \beta \geq 0\).

\[
\frac{\delta c^*_i}{\delta \alpha} (Ne) = -\left( \frac{3\alpha W_e^2 N_e(N_e W_e + N_e) + 4\alpha W_e(N_e + N_e^r) + 4)^2}{e^2} \right) \leq 0,
\]

\[
\frac{\delta m^*_d}{\delta \beta} (Nd) = -\left( \frac{3\beta W_d^2 N_d(N_d W_d + N_d) + 4\beta W_d(N_d + N_d^r) + 4)^2}{e^2} \right) \leq 0.
\]

The “Naive” pricing case’s best response price function is equivalent to the non-charging drivers best price function of the two price case but with quantities of firm \(i\) fixed as \(N_i\) and firm \(j\) fixed as \(N_j\).

This means that the smaller the market, the more each firm has an incentive to compete by lowering price. In the most extreme case where \(\alpha = 0\) or \(\beta = 0\), the firms do not compete at all (as there are an infinite number of consumers with the same valuation) and set the same price as a monopoly servicing that market.

V. COMPETITIVE MARKET: NUMERICAL CASE STUDY

Next we present a numerical case study to see how differing levels of government mandate impact the market. We first define some values of interest.

The first is the average price consumers of each class pay in equilibrium. For given prices for charging spots \(c_i\), the average price is defined as:

\[
c_{ave} = \sum_{i=1}^{2} \frac{q_i c_i}{q_1 + q_2}.
\]

An analogous expression defines the average price of non-charging with spots.

We are also interested in how competitive each market is. One way to measure this is the Herfindahl Index, a widely used measure of market concentration [31]. Given the quantity serviced \(q_i\) at each firm \(i\), this is defined as

\[
H = \sum_{i=1}^{2} \left( \frac{q_i}{q_1 + q_2} \right)^2,
\]

where larger values of \(H\) indicate a less competitive market.

A. “Naive” Mandate Fulfillment

In our first set of numerical results, we look at all three pricing approaches in the “Naive” mandate fulfillment case. For all plots we set \(W_e = 1\), \(\beta = 1\), \(W_d = 1.25\), \(\alpha = 5\), \(e = 1\), \(p = 0\), \(t = 0\), and \(\delta = 0.6\). In this case we have a large non-charging driver market and a relatively small charging driver market, although the charging users have a larger utility for parking. We assume one of the garages is larger than the other (controlling 60% of the parking spots). We then use the same values except set \(\alpha = 2\). This can be viewed as the charging market expanding (the slope of demand decreases) while all other market conditions stay the same.

We plot the average EV charging plus parking price, total firm profits, total welfare, total congestion (sum of congestion at all firms) and the Herfindahl index of both markets as a function of the government mandate \((\tau)\). The plots for \(\alpha = 5\) are shown in Fig. 1 and the plots for \(\alpha = 2\) are in Fig. 2. The values are plotted for all three pricing cases that were described above. We note some interesting features. First, the average EV charging plus parking price can be lower for the two price case than in any of the single price cases if the EV mandate is high enough when \(\alpha = 5\) (the smaller charging market). However, when the charging market is larger (\(\alpha = 2\)), the average EV charging plus parking price in the two price case is never lower than in the naive single price case but can still be slightly lower than the optimal single price case when \(r\) is large. Also, for \(\alpha = 5\) the optimal single price can be lower than the naive price when \(r\) is large but not for \(\alpha = 2\). Intuitively, when the charging market is small and a large number of spots are allocated to EV’s the parking owners need to lower prices. We can also see that total firm profits can be larger in the naive pricing case than in any other case when \(\alpha = 5\) and the EV mandate is large enough, though this is not the case for \(\alpha = 2\). The intuition here is that although individually each firm would rather set two prices, fixing the price to that of the market before EVs enter reduces the ability of firms to compete with each other, which can increase aggregate profits.

Note in both cases the optimal total welfare occurs approximately when the mandate is equal to the size of the charging market (\(\tau\)). At this value, naive single pricing maximizes welfare even though it generates lower firm profits. Apparently, the
increased consumer surplus from lower prices makes up for the loss of profits, yielding improved welfare. Lastly, we note that the competitiveness of the non-charging market is lower in the two prices case than with the naive pricing when the EV mandate is set to the size of the EV market \((\alpha)\) for both \(\alpha = 2\) and \(\alpha = 5\). This is capturing the trade-off of inducing EV parking spots: if firms start competing on charging drivers, they may compete less over non-charging drivers and thus make the market less competitive for this class of consumers.

**B. Optimal Capacity**

We now examine the case where firms choose both \(N_e\) and prices. We restrict ourselves to the “two price” case where we have shown an equilibrium exists. We will assume the intrinsic cost of constructing spots is \(t\), and the cost firms pay to construct them \(p\) is a function of \(t\) and government subsidies. This will impact aggregate welfare.

We set \(\mu = 1\), \(\alpha = 1\), \(W_d = 0.9\), \(\beta = 0.33\), \(\epsilon = 1\), \(t = 0.1\), while varying \(\delta\) between 0.5 to 0.9. We examine the effect of a government having access to two policy levers. With the first lever the government can move from having no construction mandate to a mandate with \(r = 0.33\) (i.e. where each firm has to add charging to at least 33% of their spots). The second lever moves the government from no subsidy to fully subsidizing charging infrastructure construction (i.e. setting \(p = 0\)). We plot the welfare in Fig. 3 as a function of \(\delta\) with the four
possible combinations of either pulling or not pulling these government policy levers as defined in Table III.

While aggregate welfare is decreased only slightly for all values of $\delta$ when any government action is taken, we see a large decrease in non-charging consumer surplus (almost a 50% drop-off) in all cases of government actions, but with a large increase in charging drivers’ welfare (up to an almost 400% increase) for all values of $\delta$. Effectively the government is only somewhat inefficiently trading off non-charging welfare for charging drivers’ welfare by either forcing or inducing (through subsidies) construction of EV chargers. Having both a large subsidy and relatively large mandate does not have much more impact beyond using just one policy given: they both produce similar effects independently and together. However, subsidies lead to government welfare loss and increases both firm profits at all levels of $\delta$, while mandates lead to lower firm profits and no cost to the government. While the magnitude of change in welfare of those who charge versus those that don’t hold for other exogenous variables, the same sort of qualitative behavior holds.

VI. MONOPOLIST WITH STOCHASTIC DEMAND: MODEL

We now turn to our second model in which there is uncertainty in the demand for EV drivers interested in charging. We consider a similar model as in Section II, with two changes. First, we consider a monopolist instead of a competitive market. Second, the demand function of both types of drivers will change.\footnote{Most importantly the size of the EV market will be stochastic. In future work we plan to analyze the effects a stochastic EV market has when firms with spots are competing with each other.}

A. Consumers

We will assume there is a mass of non-charging drivers all with the same valuation for parking, $W_d$. This mass is sufficiently large (essentially unbounded) such that at any positive price, the monopolist can fill up its parking spots until the marginal value is zero. For the charging drivers market, we similarly assume all drivers have the same valuation for parking, $W_c > W_d$, but there are only $Q_{\text{max}}$ EV consumers interested in charging, where $Q_{\text{max}}$ depends on the state of the world. Specifically, we define a probability distribution $\pi = \{\pi_1, \pi_2, \ldots, \pi_n\}$ over a set $q = \{q_1, q_2, \ldots, q_n\}$, so that $Q_{\text{max}} = q_i$ with probability $\pi_i$. The monopolist sets both the capacity ($N_e$) and prices for non-charging and charging drivers ($m$ and $c$, respectively) before this distribution is realized. We can therefore write the marginal utility of non-charging drivers and charging drivers (as a function of $Q_{\text{max}}$) as

$$U_d(q_d, N_e, m) = W_d - \epsilon \frac{q_d}{1 - N_e} - m,$$

$$U_c(q_c, N_e, c, Q_{\text{max}}) = W_c - \epsilon \frac{q_c}{N_e} - c.$$

s.t. $q_c \leq Q_{\text{max}}$

B. Monopolist

We assume there exists a monopolist who already owns a mass of parking spots without EV charging. Like in the previous model, we normalize this number of spots to be 1. This monopolist cannot construct new parking spots, but is able to convert a portion of their spots to include an EV charger at cost $p$. This means choosing a number of EV spots 0 $\leq N_e \leq 1$, leaving $1 - N_e$ spots for non-charging drivers. The monopolist then chooses a price $c$ for charging drivers and $m$ for non-charging drivers (i.e., we are only examining the “Two Price” case as described in the competitive model). They do this before realizing what state of the world they are in (i.e., before the size of the EV market is realized).

Assuming $m \leq W_d$ and $c \leq W_c$, the demand for non-charging drivers and charging drivers (as a function of the realization $Q_{\text{max}}$), respectively, is

$$D_d(N_e, m) = (1 - N_e) \frac{W_d - m}{\epsilon},$$

$$D_c(N_e, c, Q_{\text{max}}) = \min\left\{N_e \frac{W_c - c}{\epsilon}, Q_{\text{max}}\right\}.$$ 

The monopolist is risk neutral and wants to maximize their expected profit from the non-charging market and the charging drivers market, which will we define, respectively, as

$$\Pi^d(m, N_e) = mD_d(N_e, m),$$

and

$$\Pi^c(c, N_e, Q_{\text{max}}) = cD_c(N_e, c, Q_{\text{max}}).$$

The optimal capacity is

$$m^* = \arg\max_{m} \Pi^d(m, N_e),$$

$$c^* = \arg\max_{c} \Pi^c(c, N_e, Q_{\text{max}}).$$

We now turn to our second model in which there is uncertainty in the demand for EV drivers interested in charging. We consider a similar model as in Section II, with two changes. First, we consider a monopolist instead of a competitive market. Second, the demand function of both types of drivers will change.\footnote{Most importantly the size of the EV market will be stochastic. In future work we plan to analyze the effects a stochastic EV market has when firms with spots are competing with each other.}

Table III: Optimal Capacity Results

|          | No Subsidy | Subsidy |
|----------|------------|---------|
| No Mandate | (a)        | (b)     |
| Mandate   | (c)        | (d)     |

Figure 3. Plots for optimal capacity.
\begin{equation}
E[\Pi^e(c, N_e)] = \sum_{i=1}^{n} c \pi_i D_e (N_e, c, q_i). \tag{20}
\end{equation}

Combining (19) and (20) as well as the cost of constructing spots, the overall profit maximization problem of the monopolist is:

\begin{align}
\max_{N_e, m, c} &\quad \Pi_d(m, N_e) + E[\Pi^e(c, N_e)] - p N_e \\
\text{subject to} &\quad 0 \leq N_e \leq 1.
\end{align} \tag{21}

Similar to the competitive model, we analyze this problem by viewing the monopolist’s decision in two stages: first choosing \(N_e\) and then choosing prices \(m\) and \(c\).

\section{Monopolist With Stochastic Demand: Pricing}

In this section, we analyze the prices the monopolist chooses as a function of \(N_e\). We note that once the proportion of EV spots is chosen, the decisions to price non-charging and charging drivers are independent as they cause no congestion to each other and the cost of the spots is sunk, and so we examine each separately.

\subsection{Pricing for Non-Charging Drivers}

We can find the optimal non-charging driver price by solving

\begin{equation}
\max_m \quad \Pi^d(m) = m(1 - N_e) \frac{W_d - m}{\epsilon}.
\end{equation}

This problem is concave in \(m\) and so we can find the solution by examining the first order optimality conditions which gives

\begin{equation}
m^*(N_e) = \frac{W_d}{2}. \tag{22}
\end{equation}

Substituting (22) back into the optimization problem gives the following expression for non-charging revenue as a function of \(N_e\):

\begin{equation}
\Pi^d(N_e) = \frac{(1 - N_e)W_d^2}{4\epsilon}. \tag{23}
\end{equation}

We can see this is simply a linear function in \(N_e\), which increases in the non-charging drivers’ utility \((W_d)\) and decreases in \(\epsilon\).

\subsection{Pricing for Charging Drivers}

Fixing \(N_e\), we can break down the charging driver quantity serviced in each realization into three broad cases based on the \(c\) the monopolist sets. We will define each case by what quantity of charging drivers \(q_e\) they service for a given \(Q_{\text{max}} \in q\).

The first two cases are uniquely defined by a target realization \(q_t \in q\). Case 1 occurs when the monopolist sets a price such that \(q_e = Q_{\text{max}}\) in realizations where \(Q_{\text{max}} \leq q_t\), while \(q_e = q_t\) when \(Q_{\text{max}} > q_t\). In other words the monopolist sets a charging price such that they exactly service the target quantity when there is a large enough demand for it, and fully service all realizations smaller than the target. Case 2 occurs when the monopolist sets a price such that \(q_e = Q_{\text{max}}\) when \(Q_{\text{max}} \leq q_t\) and \(q_e = q_t\) for some \(q_t < q_{t+1}\). This means the monopolist serves exactly \(Q_{\text{max}}\) for any realization where \(Q_{\text{max}} \leq q_t\), but serves some amount between the target realization and the next highest realization in all states of the world such that \(Q_{\text{max}} > q_t\).

Finally the monopolist can set a price such that \(q_e < q_t\) in all realizations of \(Q_{\text{max}}\), which we can call Case 3. These cases are summarized in Table IV.

For a given number of possible realizations \(n\), the possible cases can be ordered as follows (this is increasing in the size of the maximum number of charging EVs served over all realizations):

\begin{itemize}
  \item [Case 3, Case 1 \((t = 1)\), Case 2 \((t = 1)\), Case 1 \((t = 2)\), ... , Case 2 \((t = n - 1)\), Case 1 \((t = n)\)].
\end{itemize}

This allows us to describe any feasible set of quantities the monopolist serves with \(n\) target quantities for Case 1 (where \(t = 1, \ldots, n\) denotes that the monopolist’s target quantity is \(q_t\)) and \(n - 1\) target quantities for Case 2 (here \(t\) denotes the largest realization that the monopolist serves). Given the threshold structure of EV demand in (18), note that for any realization where \(q_t < Q_{\text{max}}\), the monopolist will serve the \(q_t\) in any realization with a larger quantity of charging drivers.

We now analyze the pricing in each of these cases (with a given target realization \(q_t\) where applicable) for a fixed \(N_e\).

\textbf{Case 1:} \(q_e = Q_{\text{max}}\) \textit{iff} \(Q_{\text{max}} \leq q_t\), \textit{else} \(q_e = q_t\). In this case, for a given target quantity \(q_t\), the monopolist sets the price such that they get exactly \(q_t\) in any realization where the number of arrivals is greater than or equal to \(q_t\). By setting (18) equal to \(q_t\) and solving for \(c\), we find the price is

\begin{equation}
c_t^*(N_e, t) = W_e - \epsilon \frac{q_t}{N_e}. \tag{24}
\end{equation}

\textbf{Case 2:} \(q_e = Q_{\text{max}}\) \textit{iff} \(Q_{\text{max}} \leq q_t\), \textit{else} \(q_e = q_t\) \((q_t < q_{t+1})\). In this case the monopolist wants to set the optimal price which is low enough to get a quantity larger than \(q_t\) while large enough to get some amount less than \(q_{t+1}\). This can be expressed as the following optimization problem

\begin{equation}
\max_c \quad \Pi^c_h(c) = c \left( \sum_{i=1}^{L} \pi_i q_i + \sum_{j=1}^{N} \pi_j N_j \frac{W_e - c}{\epsilon} \right) \\
\text{subject to} \quad W_e - \epsilon \frac{q_t}{N_e} < c < W_e - \epsilon \frac{q_{t+1}}{N_e}. \tag{25}
\end{equation}
order optimality conditions, which gives us

$$c^*_1(N_e, t) = \frac{W_e}{2} + \frac{\epsilon}{2} \sum_{j=1}^{t} \pi_j q_j.$$  

(26)

This price can be viewed as the price if there was no constraint on the size of the EV market \(\frac{W_e}{N_e}\) (see Case 3) plus a term which is decreasing in the target \(t\).

Case 3: \(q_e < q_1 \forall q_{\text{max}}\). In this case, the monopolist sets the optimal price as a function of \(N_e\) such that they get \(q_e < q_1\) in any realization. For this case to be feasible, the monopolist has to set a high enough price such that \(D_e(N_e, c, q_1) < q_1\). Hence the pricing scheme in this case is equivalent to the following optimization problem

$$\begin{align*}
\text{maximize} & \quad \Pi^*_1(c) = cN_e \frac{W_e - c}{\epsilon} \\
\text{subject to} & \quad c > W_e - \epsilon \frac{q_1}{N_e}.
\end{align*}$$  

(27)

Again we can relax the constraint to be \(c \geq W_e - \epsilon \frac{q_1}{N_e}\), and note that if this constraint is ever tight we are not in this case. If this occurs we will say that this case is no longer feasible, as case 2 is strictly superior. When the constraint is not tight, we find the optimal price is

$$c^*_1(N_e, t) = \frac{W_e}{2}. \quad (28)$$

This is the same pricing scheme as for the non-charging drivers (a constant price independent of \(N_e\)). This makes sense as assuming ex ante the monopolist sets a price such that they serves \(q_e < q_1\) EV drivers is the same as being unconstrained by the size of the charging market.

VIII. MONOPOLIST WITH STOCHASTIC DEMAND: CAPACITY

Next, we turn to the Monopolist’s optimal choice of \(N_e\). Using the results of the previous section, we can formulate this as

$$\begin{align*}
\text{maximize} & \quad m^*(N_e) D_e(N_e, m^*) \\
& \quad + c^*(N_e, t) \sum_{r=1}^{\epsilon} \pi_r D_e(N_e, c^*(N_e, t), q_r) - pN_e \\
\text{subject to} & \quad 0 \leq N_e \leq 1.
\end{align*}$$  

(29)

At each value of \(N_e\), the optimal charging plus parking price \(c^*(N_e, k)\) is defined by whichever pricing case and target quantity pair leads to the highest profit (at a given \(N_e\), any given case could lead to this). The optimal non-charging parking price \(m^*(N_e)\) is defined by (22) regardless of case (and in fact is also independent of \(N_e\)). The overall optimal profit of the monopolist is defined by the best choice of EV spots \(N^*_e\) that solves this problem. This choice also solves (21) (where the optimal \(m\) and \(c\) are determined by this choice \(N^*_e\)). We now examine an important property of \(N^*_e\).

**Theorem 1.** Assume \(\frac{W_e}{N_e} > c_1\) and \(\epsilon(W_e^2 - W_e^2) > p\). Any optimal pricing strategy for the monopolist is to set a price \(c\) such that for some \(t\), \(1 \leq t \leq n\), the monopolist services exactly \(q_t\) for every realization of the distribution such that \(q_t \geq q_1\), and exactly \(q_t\) for any realization where the quantity of charging drivers is \(q_t < q_1\).

In other words, this shows that under the assumptions in this theorem the optimal price is always in Case 1. Hence, to determine the optimal \(N_e\), we only need to consider the \(n\) prices corresponding to Case 1. This means the monopolist can solve for the optimal capacity (and get the corresponding price) with an algorithm that is \(O(n)\) with respect to the cardinality of the support of the arrival distribution. We can prove this by examining the structure of (29) given how \(c^*(N_e)\) is defined in each case. The proof for this is found in the appendix.

IX. MONOPOLIST WITH STOCHASTIC DEMAND: CASE STUDY

We now examine how changes in the monopolist’s belief about the market of EVs interested in charging can affect the capacity they add. In Fig. 4 we plot the optimal revenue function for the monopolist as a function of \(N_e\) with \(W_e = 1.25\), \(W_d = 1\), \(\epsilon = 1\), \(n = 3\), \(t = 0.01\) and \(p = 0.01\). Note these values are slightly different from ones used in Section V, but the same general trends hold when these exogenous variables have different magnitudes. In all cases We divide the pricing into the cases defined by different \(k\) values as described before. The overall optimal \(N^*_e\) in each plot is marked by a vertical blue line. In each plot, we vary the distribution of EV arrivals as noted below. As indicated in Theorem 1, in each plot the optimal number of spots is in a Case 1 pricing regime. but the target quantity \(k\) is different and noted.

In plot (a), we set \(q = [0.1, 0.15, 0.3]\) and \(\pi = [0.4, 0.33, 0.27]\) and find \(N^*_e = 0.196\) with \(k = 1\). In plot (b), we set \(q = [0.1, 0.15, 0.3]\) and \(\pi = [0.31, 0.33, 0.36]\) and find that the optimal exists at \(N^*_e = 0.279\) with \(k = 2\). This is the same as plot (a) but with a small change in the arrival distribution (a 0.06 decrease in \(\pi_1\) and a corresponding increase in \(\pi_3\)). This small change leads to an almost 50% increase in the number of EV spots constructed. In plot (c), we set \(q = [0.1, 0.15, 0.5]\) and \(\pi = [0.31, 0.33, 0.36]\) and find \(N^*_e = 0.278\) for \(k = 2\) again. This is the same as plot (b) but with \(q_3\) increasing by 0.2. We can see that even though there is now a possible realization where many more charging drivers enter the market than in plot (b), the monopolist optimally does not plan any differently as \(N^*_e\) independent of \(q_3\).

In plot (d), we set \(q = [0.1, 0.15, 0.5]\) and \(\pi = [0.2, 0.1, 0.7]\) and find \(N^*_e = 0.860\) with \(k = 3\). A change in the distribution of EVs interested in charging (not the quantities in each state) leads to a large change in the number of spots constructed. In

\[\text{The first assumption } \frac{W_e}{N_e} > c_1 \text{ guarantees that even if the the monopolist equips all spots with charging stations, the optimal price they set for charging consumers (} \frac{W_e}{N_e} \text{) will lead to at least } q_t \text{ customers served. The assumption } \epsilon(W_e^2 - W_d^2) > p \text{ guarantees that the cost of adding charging to parking spots (} p \text{) is less than the additional revenue the firm can get from the higher prices they get set for customers who want to charge versus those that don’t.}\]
We set $q = [0.1, 0.15, 0.5]$, $\pi = [0.4, 0.33, 0.27]$, $p = 0.01$. And find $N^*_e = 0.284$ with $k = 2$. This is a small change from plot (d) (the same quantity distribution but with $\tau_1$ 0.05 higher and $\pi_3$ correspondingly smaller). A small change in the distribution of arrivals in this case leads to more than a 3 times larger number of EV spots constructed. Lastly, in plot (f) we have the government subsidize the construction of EV spots ($t = p$) with no change in the distribution of arrivals and find $N^*_e = 0.857$ with $k = 3$. We see that government subsidies can have a large impact on equilibrium behavior as it can shift which case is optimal for the monopolist. It also affects the number of spots within cases as we can see from (34). This makes it a tool that always increases the number of EV spots created, unlike the distributions of arrivals (which as we saw may not impact the number of spots constructed).

**X. Conclusions**

We proposed a model where EVs interested in charging, ICE drivers and non-charging EV drivers utilize the same fixed quantity of a resource (parking) and EV drivers have a higher valuation due to the need to charge. However, to serve EV drivers interested in charging, operators require additional investment leading to these spots only being available to these consumers. We have used this model to examine a case where there is competition or another where there is uncertainty in future EV demand. The right set of policies (subsidies and mandates) and the correct levels are dependent on how competitive the market is when firms are more cognizant of the ability to add capacity and price these two concurrent markets. With a single monopolist, we showed that errors in forecasting the EV demand for charging while parking can have significant impact on the charging capacity installed, suggesting techniques for improving such forecasts are worth considering in future work. This also suggests that risk neutrality may not be the best way to optimize over this uncertainty, as a different objective function could lead to a capacity that targets multiple sizes of the EVs interested in charging instead of the one target as in our work.

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