There is a puzzling contradiction: direct observations favor a low-mass-density universe (0.2 \( \leq \Omega_m \leq 0.6 \)), but the only model which fits universe structure over more than three orders of magnitude in distance scale has a mix of hot (neutrino) and cold dark matter providing a critical density universe. Models of an open universe (low \( \Omega_m \)) or one adding a cosmological constant (\( \Lambda \)) to provide a critical energy density (\( \Omega_m + \Omega_\Lambda = 1 \)) have probabilities of \(< 10^{-3} \). Two-neutrino dark matter works better than having the needed \( \sim 5 \text{ eV} \) of neutrino mass in one species of neutrino, and this is consistent with the only model which fits all present indications for neutrino mass: \( \nu_\mu \rightarrow \nu_\tau \) accounting for the atmospheric anomaly (with \( \nu_\mu \) and \( \nu_\tau \) being the hot dark matter), \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) being observed by LSND, and \( \nu_e \rightarrow \nu_s \) explaining the solar \( \nu_e \) deficit. The LSND/KARMEN results are consistent with the needed mass of hot dark matter. Further support for this mass pattern is provided by the need for the sterile neutrino, \( \nu_s \), to make possible heavy-element nucleosynthesis in supernovae. It is a fascinating question as to whether the hot dark matter paradox will be resolved by better measurements or by the introduction of new physics.

1 One-, Two, or Three-Neutrino Dark Matter?

Since there are about 100/cm\(^3\) of neutrinos of each type left over from the Big Bang, if they have mass they surely are part of the dark matter of the universe. While they cannot be the major component of this missing mass, the neutrinos can have profound effects on universe structure if they have sufficient mass. So far there is only evidence for differences in mass between neutrino types, and with one exception, those differences are so small that if they are representative of mass values, the neutrinos would have little effect. We shall see, however, that there are several types of evidence that neutrino hot dark matter is quite significant.

The common view, especially among astronomers, has been that if there is hot dark matter it is due mainly to one neutrino, presumably the \( \nu_\tau \). This would be ruled out if, as fits the Super-Kamiokande data\(^{[1]} \) best, the atmospheric anomalous \( \nu_\mu/\nu_e \) ratio is due to \( \nu_\mu \rightarrow \nu_\tau \), since the mass-squared difference required is \( \Delta m^2_{\mu\tau} \sim 10^{-3} \text{eV}^2 \), whereas the needed neutrino mass is 94 \( \Omega_\nu h^2 \sim \text{eV} \) (with \( h \) the Hubble constant in units of 100 km\( \cdot \)s\(^{-1}\)\( \cdot \)Mpc\(^{-1}\)).
Other processes to explain the atmospheric results are very unlikely: $\nu_\mu \rightarrow \nu_e$ does not fit the Super-Kamiokande angular distributions, and the CHOOZ $\nu_e$ disappearance experiment rules out almost all the parameter space; $\nu_\mu \rightarrow \nu_s$ (a sterile neutrino) likely has a problem with the nucleosynthesis limit, since near maximal mixing is required, and the Super-Kamiokande results rule it out at the 95% C.L.

If hot dark matter being one active neutrino is essentially ruled out, what about using all three active neutrinos? In this case $\nu_\mu \rightarrow \nu_\tau$ explains the atmospheric anomaly, $\nu_e \rightarrow \nu_\mu$ (with $\Delta m^2_{e\mu} \lesssim 10^{-5}$ eV$^2$) provides the solar $\nu_e$ deficit, and the three nearly mass degenerate neutrinos could provide the dark matter. When this scheme was first suggested, there was a possible problem with neutrinoless double beta decay. While limits on that process have improved, theoretical ways have been found to ameliorate the problem. If results from the LSND experiment are correct, however, three-neutrino dark matter is also ruled out, since this requires $\Delta m^2_{e\mu} > 0.3$ eV$^2$, making three quite distinct mass differences, necessitating more than three neutrinos.

That leaves two-neutrino dark matter. This scheme requires four neutrinos, with the solar deficit explained by $\nu_e \rightarrow \nu_\mu$, both neutrinos being quite light, the atmospheric effect due to $\nu_\mu \rightarrow \nu_\tau$, which share the dark matter role, and the LSND $\nu_\mu \rightarrow \nu_e$ demonstrating the mass difference between these two nearly mass-degenerate doublets. Note that the solar $\nu_e \rightarrow \nu_\mu$ is for the small mixing angle (or “just-so” vacuum oscillation) solution, so $\nu_s$ does not affect nucleosynthesis. The original motivation for this mass pattern preceded LSND and was simply to provide some hot dark matter, given the solar and atmospheric phenomena. If LSND is correct, it becomes the unique pattern.

This neutrino scheme was the basis for simulations which showed that two-neutrino dark matter fits observations better than the one-neutrino variety. The latter produces several problems at a distance scale of the order of $10 h^{-1}$ Mpc, particularly overproducing clusters of galaxies. Whether the $\sim 5$ eV of neutrino mass is in the form of one neutrino species or two makes no difference at very large or very small scales, but at $\sim 10 h^{-1}$ Mpc the larger free streaming length of $\sim 5/2$ eV neutrinos washes out density fluctuations and hence lowers the abundance of galactic clusters. In every aspect of simulations done subsequently, the two-neutrino dark matter has given the best results. For example, a single neutrino species, as well as low universe density models, overproduce void regions between galaxies, whereas the two-neutrino model agrees well with observations.
2 Evidence from Universe Structure

Cosmic microwave background radiation observations of the first Doppler peak favor the total energy density of the universe having the critical value ($\Omega = 1$) of a flat universe. Such a flat universe has the only time-stable value of density and is expected in all but rather contrived models of an early era of exponential expansion, or “inflation”. Until recently it has usually been assumed that $\Omega = \Omega_m = 1$; i.e., the energy density is the matter density, and the universe will expand forever at an ever decreasing rate. Now evidence points to $0.3 \leq \Omega_m \leq 0.6$, however, based on a number of observations: high-redshift Type Ia supernovae, evolution of galactic clusters, high baryon content of clusters, lensing arcs in clusters, and dynamical estimates from infrared galaxy surveys. On this basis it has become popular to assume $\Omega_m \approx 0.3$, but $\Omega = 1$ through the addition of a vacuum energy density, often designated as a cosmological constant, $\Lambda$. The model with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ is in trouble with some determinations of the age of the universe, and lensing measurements require $\Omega_\Lambda < 0.74$ at the two standard deviation level.

The evidence for low $\Omega_m$ does not include a global look at universe structure. Gawiser and Silk\textsuperscript{9}, however, used all the published data from the cosmic microwave background and galaxy surveys which covered three orders of magnitude in distance scale and came to a different conclusion. They compared the data with ten models of universe structure, but of concern here are only three of these, the other seven giving extremely poor fits. In two low-density models the parameters were varied to get best fits, resulting in $\Omega_m = 0.5$, of which the baryons contribute $\Omega_b = 0.05$, and the rest is cold dark matter. One of these is an open universe model (OCDM) having $\Omega = \Omega_m = 0.5$, and the other (LCDM) has $\Omega = 1$ with $\Omega_\Lambda = 0.5$. The third model (CHDM) has $\Omega_m = 1$, of which $\Omega_\nu = 0.2$ is in neutrinos, and $\Omega_b = 0.1$ in baryons, with the main component being cold dark matter, that which was nonrelativistic at the time it dropped out of equilibrium in the early universe. The probabilities of the fits were CHDM = 0.09, OCDM = 2.9 $\times$ 10$^{-5}$, and $\Lambda$CDM = 1.1 $\times$ 10$^{-5}$. If one dubious set of data is removed, the APM cluster survey (which disagrees with galaxy power spectra), these probabilities become CHDM = 0.34, OCDM = 6.7 $\times$ 10$^{-4}$, and $\Lambda$CDM = 4.3 $\times$ 10$^{-4}$.

Had it been possible to extend the fit to even smaller scales, the discrepancy between CHDM and the others would have been even greater, but this is the non-linear regime requiring simulations. The CHDM model with two neutrinos gives an excellent fit\textsuperscript{10} to the data at this extended scale, whereas the others deviate even more strongly than in the linear region.

Clearly there is a serious conflict between the observations indicating a
low value of $\Omega_m$ and the degree of structure in the universe as a function of
distance scale. Two recent developments raise some doubts about the present
popular interpretation of the data. The most compelling evidence for $\Lambda$ comes
from the observation of distant supernovae Ia, but recent measurements on
nearby SNIa show they take over two days longer to reach peak brightness
than do distant SNIa. This indicates that these may not be the “standard can-
dles” required for the conclusions reached, but rather that evolutionary effects
mimic the need for $\Lambda$. The second straw in the wind is a geometric measure-
ment of the distance to galaxy NGC4258 which disagrees with the standard
Cepheid ladder of distances by $\sim 15\%$. This lowering of the distance scale
would reduce the age of the universe, more in line with larger values of $\Omega_m$.
It is, however, a single determination. Better measurements will be available
soon, and these may resolve the conundrum, but if the difference sharpens,
this could lead to important new physics.

3 Evidence from the LSND Experiment

While new galaxy surveys and cosmic microwave background experiments will
be so precise that they can provide a good measure of neutrino mass and the
number of neutrino types contributing to dark matter, the ultimate answer
must come from laboratory experiments. Neutrino oscillation experiments
can provide only mass differences, but if the difference is sufficient, the case
for cosmologically significant neutrino dark matter will be settled. Only one
experiment has given evidence for such dark matter, but so far the range of
possible mass difference between $\nu_e$ and $\nu_\mu$ is quite large: $0.3 \text{ eV}^2 \le \Delta m^2_{e\mu} \le 10\text{eV}^2$.

In its 1996 publication LSND claimed a signal in $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ on the basis
that 22 events of the type $\bar{\nu}_\mu p \rightarrow e^+ n$ were seen, using a stringent criterion
to reduce accidental coincidences between $e^-$ or $e^+$ and $\gamma$ rays mimicking
the 2.2-MeV $\gamma$ from $np \rightarrow d\gamma$, whereas only 4.6 $\pm$ 0.06 events were expected.
The probability of this being a fluctuation is $4 \times 10^{-8}$. Note especially that
these data were restricted to the energy range 36 to 60 MeV to stay below
the $\bar{\nu}_\mu$ endpoint and to stay above the region where backgrounds are high
due to the $\nu_e^{12}\text{C} \rightarrow e^- X$ reaction. In plotting $\Delta m^2$ vs. $\sin^2 2\theta$, however,
events down to 20 MeV were used to increase the range of $E/L$, the ratio of
the neutrino’s energy to its distance from the target to detection. This was
done because the plot employed was intended to show the favored regions of
$\Delta m^2$, and all information about each event was used. The likelihood analysis
applied did not have a Gaussian likelihood distribution, since its integral is
infinite, but the likelihood contour labeled “90%” was obtained by going down
a factor of 10 from the maximum, as in the Gaussian case. The contours in
the LSND plot have been widely misinterpreted as confidence levels—which
they certainly are not—because they were plotted along with confidence-level
limits from other experiments.

Recently the difficult, computer-intensive analysis in terms of real con-
fidence levels has been done. The likelihood for a grid in \((\sin^2 2\theta, \Delta m^2)\)
space, including backgrounds, has been computed and compared with numer-
ous Monte Carlo experiments to obtain a 90\% confidence region. While the
equivalency varies from point to point in the \(\Delta m^2 - \sin^2 2\theta\) plane, a typical
value for the 90\% confidence level is down a factor of 20 from the likelihood
maximum. Thus the LSND allowed regions are considerably broader in \(\sin^2 2\theta\)
than in the plots published so far, and other experiments are less constraining
of allowed \(\Delta m^2\) regions.

The confusion of comparing likelihood levels for LSND with confidence
levels from other experiments may be exacerbated by using the 20–36 MeV
region for the LSND data. While this higher background energy range makes
some difference for the 1993–5 data, it could have had an appreciable effect for
the parasitic 1996–7 runs, which were at a low event rate. This decreased the
ratio of signal/background events, since the main background is from cosmic
rays. This could raise the low end of the supposed signal energy spectrum,
making the higher \(\Delta m^2\) values desirable for dark matter appear less likely.

Nevertheless, when a joint analysis is made of the LSND and KARMEN
experiments even using the 20–36 MeV range for LSND, the region around
6 eV\(^2\) is as probable as the banana-shaped region at lower \(\Delta m^2\), as shown
in Fig. 1. Frequently theorists consider only the latter, whereas the \(\bar{\nu}_\mu \rightarrow \nu_e\)
LSND data favors the higher mass region. Of course the \(\nu_\mu \rightarrow \nu_e\) data, which
uses \(\nu_\mu\) from \(\pi^+\) decay-in-flight and detects \(\nu_e\) by \(\nu_e^{12}\mathrm{C} \rightarrow e^-\mathrm{X}\) has higher
backgrounds and hence much poorer statistics than the \(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\) with \(\bar{\nu}_\mu\) from
\(\mu^+\) at rest. In addition to the \(\Delta m^2\) issue, the important point of Fig. 1 is
that although the KARMEN data are consistent with background, the joint
analysis of the \(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\) data from the two experiments shows an appreciable
region for a signal. KARMEN is continuing to take data, and LSND should
have an improved analysis available soon.

4 Evidence from Supernova Nucleosynthesis

Support for the double doublet of neutrinos with sufficient mass in two of the
neutrinos to provide significant hot dark matter comes from an unusual source:
the creation of heavy nuclei by supernovae. Initially the reverse appeared to
be the case, since this \(r\) process of rapid neutron capture, which occurs in the
Figure 1. Filled in areas are 90% and 95% confidence regions based on the product of the KARMEN and LSND Feldman-Cousins likelihood ratios. Also shown is the Feldman-Cousins 90% confidence region for LSND alone (“L”). Left of the “K”, “N”, and “B” curves are exclusion regions of KARMEN, NOMAD, and Bugey.
outer neutrino-heated ejecta of Type II supernovae, seemed to place a limit on the mixing of $\nu_\mu$ and $\nu_e$. Energetic $\nu_\mu$ ($\langle E \rangle \approx 25$ MeV) coming from deep in the supernova core could convert via an MSW transition to $\nu_e$ inside the region of the $r$-process, producing $\nu_e$ of much higher energy than the thermal $\nu_e$ ($\langle E \rangle \approx 11$ MeV). The latter, because of their charge-current interactions, emerge from farther out in the supernova where it is cooler. Since the cross section for $\nu_e n \to e^- p$ rises as the square of the energy, these converted energetic $\nu_e$ would deplete neutrons, stopping the $r$-process. Calculations of this effect limit $\sin^2 2\theta$ for $\nu_\mu \to \nu_e$ to $\lesssim 10^{-4}$ for $\Delta m^2_{\text{e}\mu} \gtrsim 2$ eV$^2$, in conflict with compatibility between the LSND result and a neutrino component of dark matter.

Since the work of reference 15, serious problems have been found with the $r$ process itself. First, recent simulations have revealed the $r$-process region to be insufficiently neutron-rich, since about $10^2$ neutrons is required for each seed nucleus, such as iron. This was bad enough, but the recent realization of the full effect of $\alpha$-particle formation has created a disaster for the $r$ process. At a radial region inside where the $r$ process should occur, all available protons swallow up neutrons to form the very stable $\alpha$ particles, following which $\nu_e n \to e^- p$ reactions reduce the neutrons further and create more protons which make more $\alpha$ particles, and so on. The depletion of neutrons by making $\alpha$ particles and by $\nu_e n \to e^- p$ rapidly shuts off the $r$ process, and essentially no nuclei above $A = 95$ are produced.

What is required to solve this problem is to remove the $\nu_e$ flux at the $r$ process site, but there still has to be a very large $\nu_e$ flux at a smaller radius for material heating and ejection. This apparent miracle can be accomplished if there is (1) a sterile neutrino, (2) approximately maximal $\nu_\mu \to \nu_\tau$ mixing, (3) small $\nu_\mu \to \nu_e$ mixing, and (4) an appreciable ($\gtrsim 2$ eV$^2$) mass-squared difference between $\nu_s$ and the $\nu_\mu-\nu_\tau$. This is precisely the neutrino mass pattern required to explain the solar and atmospheric anomalies and the LSND result, plus providing some hot dark matter!

Such a mass-mixing pattern creates two level crossings. The inner one, which is outside the neutrinosphere (beyond which neutrinos can readily escape) is near where the $\nu_\mu,\tau$ potential $\propto (n_{\nu_e} - n_n/2)$ goes to zero. Here $n_{\nu_e}$ and $n_n$ are the numbers of $\nu_e$ and neutrons, respectively. The $\nu_{\mu,\tau} \to \nu_s$ transition which occurs depletes the dangerous high-energy $\nu_{\mu,\tau}$ population. Outside of this level crossing, another occurs where the density is appropriate for a matter-enhanced MSW transition corresponding to whatever $\Delta m^2_{\text{e}\mu}$ LSND is observing. Because of the $\nu_{\mu,\tau}$ reduction at the first level crossing, the dominant process in the MSW region reverses from the deleterious $\nu_{\mu,\tau} \to \nu_e$, becoming $\nu_e \to \nu_{\mu,\tau}$ and dropping the $\nu_e$ flux going into the $r$-
process region. For an appropriate value of $\Delta m^2_{\text{e}\mu}$, the two level crossings are separate but sufficiently close so that the transitions are coherent. Then in the limits of adiabatic transitions and near maximal $\nu_\mu - \nu_\tau$ mixing, the neutrino flux emerging from the second level crossing is $\frac{1}{4} \nu_\mu$, $\frac{1}{4} \nu_\tau$, and $\frac{1}{2} \nu_s$, with no $\nu_e$ at all. Calculations show the transitions to be adiabatic, and the atmospheric observations require near maximal mixing, so the $\nu_e$ flux is certainly sufficiently depleted to allow a successful $r$ process, especially as the $\bar{\nu}_e$ flux is unaffected, so that $\bar{\nu}_e p \rightarrow e^+ n$ enhances the neutron number. It should be emphasized that this mechanism is quite robust, not depending on details of the supernova dynamics, especially as it occurs quite late in the explosive expansion.

It is essential that the two level crossings be in the correct order, and this provides a requirement on $\Delta m^2_{\text{e}\mu}$, since the MSW transition depends on density and hence on radial distance from the protoneutron star. Detailed calculations have been made for $\Delta m^2_{\text{e}\mu} \sim 6 \text{ eV}^2$, which works very well. Possibly $\Delta m^2_{\text{e}\mu}$ as low as $2 \text{ eV}^2$ or maybe even $1 \text{ eV}^2$ would work, but that is speculative. At any rate, the mass difference needed in this scheme, which is the only one surely consistent with all manifestations of neutrino mass and which rescues the $r$ process, implies appreciable hot dark matter.

5 Conclusions

A neutrino component of dark matter appears very probable, both from the astrophysics and particle physics standpoints. Despite abundant evidence for $\Omega_m < 1$, the one model which fits universe structure has $\Omega_m = 1$, with $20\%$ neutrinos and most of the rest as cold dark matter. Open universe and low-density models with a cosmological constant give extremely bad fits. This conflict should be the source of future progress, but since there are $10^2/\text{cm}^3$ of neutrinos of each active species left over from the early universe, the ultimate answer on neutrino dark matter will come from determinations of neutrino mass. While the solar and atmospheric evidences for neutrino mass are important, the crucial issue is the much larger mass-squared difference observed by the LSND experiment. In the mass region needed for dark matter, no other experiment excludes the LSND result, and a joint analysis of the LSND and KARMEN experiments shows this region has good probability.

The resulting mass pattern, $\nu_e \rightarrow \nu_s$ for solar, $\nu_\mu \rightarrow \nu_\tau$ for atmospheric, and $\nu_\mu \rightarrow \nu_e$ for LSND, requires a sterile neutrino and provides two-neutrino ($\nu_\mu$ and $\nu_\tau$) dark matter. This form of dark matter fits observational data better than the one-neutrino variety. Furthermore, the four-neutrino pattern, and especially the sterile neutrino, provides a robust way to make possible
the production of heavy elements by supernovae.

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