Novel Equalities for Stability Analysis of Asynchronous Sampled-Data Systems

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This work was supported in part by the National Research Foundation of Korea (NRF) Grant funded by the Korean Government (MSIT) under Grant NRF-2020R1G1A1010423, and in part by the Soonchunhyang University Research Fund under Grant 20191072.

ABSTRACT

This paper is concerned with the stability analysis problems of asynchronous sampled-data systems with use of an input-delay approach, where a sampled-data system can be reformulated into a time-delay system having incremental delays. For these systems, this paper introduces a new looped-functional to utilize both integral states and their interval-normalized ones. Utilization of these two types of integral state variables in a construction of stability conditions has been effective in reducing the conservatism. Further, generalized equalities are proposed for the case utilizing these two types of integral states. The sampled-data system formulation consists of a sampled state, a system state and its derivative. Here, the sampled state between two consecutive sampling instants is a constant value and thus can be eliminated by an integration with Legendre polynomials of positive degrees. Consequently, the system formulation turns into the equalities consisting of state variables, sampled-state variables, integral state variables and their interval-normalized ones without additional slack matrices. Based on the proposed equalities, a large computational burden can be reduced while reducing the conservatism. The effectiveness of the proposed approaches is demonstrated via three numerical examples for the stability analysis of asynchronous sampled-data systems.

INDEX TERMS

Asynchronous sampled-data system, stability analysis, integral inequality, equalities, looped-functional, linear matrix inequalities.

I. INTRODUCTION

In the past decades, networked control systems (NCSs) have attracted considerable attention [1]–[5]. The NCSs consist of several distributed plants connected through digital communication networks and thus have several advantages such as reduced wiring, simple maintenance, and long distance control. However, because of limited network resources, a heavy temporary computational burden in a processor can corrupt the constancy of sampling periods. Additionally, packet losses in a wireless communication network and transmission delays cannot guarantee a stability of the systems. Such phenomena make the NCSs time-varying and asynchronous. It is thus an important issue to develop robust stability criteria for sampled-data systems with time delays and asynchronous samplings [6].

Since sampled-data control only requires the information of a system at the sampling instants, increasing sampling interval reduces a computational burden of overall systems. It is thus an important topic to increase a sampling period while maintaining a system stability. Therefore, sampled-data systems with periodic/asynchronous samplings have been extensively researched in the literature [7]–[15]. In the case of periodic samplings, tools for stability analysis and controller synthesis problems have been well established [11]. However, in the case of asynchronous samplings, there still exist open problems. To deal with the stability analysis of asynchronous sampled-data systems, there have been three approaches according to modelings of a sampled-data system: impulsive models [7], [9], discrete-time systems [8], [10], input-delay systems [12], [13], [15], [16]. Among them, this paper focuses on the input-delay system approaches, where the original sampled-data system is reformulated into a time-delay system with an incremental delay. Therefore, the reformulated system can be analyzed by the Lyapunov-Krasovskii (L-K) theorem which has been widely used for the analysis of time-delay systems [17]–[20]. Recently, a framework based on a looped-functional was proposed for the
analysis of sampled-data systems [21], [22]. Differently from the L-K functional satisfying positivity conditions, the looped-functional consists of a quadratic Lyapunov function and the differentiable functional satisfying the looping conditions. This additional functional is zero at the sampling instants, and thus the looped-functional approach can relax the positivity conditions on the functional.

Commonly, a construction of the functional has played key roles in reducing the conservatism of stability criteria, which results in development of many functionals with use of various quadratic functions and integral quadratic functions [13], [15], [21]. Therefore, the time derivative of the looped-functional also contains integral quadratic functions which cannot be directly utilized as stability conditions expressed in terms of linear matrix inequalities (LMIs). To derive and relax LMIs guaranteeing the negativity of a derivative of the functional, integral inequalities [17], [20] and zero equalities [12], [13], [15], [18] have been proposed. The zero equalities can be obtained from the relation among system state variables such as a system state \(x(t)\), two consecutive sampled states \(x(t_k), x(t_{k+1})\), integral states \(\int_{r_k}^{r_{k+1}} x(r) dr\), \(k \in [0, +\infty)\), and interval-normalized integral states \(\int_{t_k}^{t_{k+1}} x(r) dr\), \(k \in [0, +\infty)\), which come from the L-K functional satisfying positivity conditions, or fromLegendre polynomials and a sampled-data system formulation. Between two consecutive sampling instants, a sampled state variable \(x(t)\) in the system formulation is a constant value, and thus it can be eliminated by an integration with Legendre polynomials of positive degrees. Consequently, the system formulation turns into the equalities consisting of state variables, sampled-state variables, integral state variables and their interval-normalized versions without additional slack matrices. In the case utilizing the two types of integral state variables, large computational burden can be reduced while reducing the conservatism of stability criteria.

- Less conservative stability criteria for asynchronous sampled-data systems are developed in terms of LMIs by employing the proposed looped-functional and the equalities. Compared to the recent work [15], the proposed stability criteria have less number of variables while reducing the conservatism.

The proposed stability criteria can deal with the cases of both periodic and asynchronous samplings. Three numerical examples of the stability analysis for sampled-data systems are given to show the effectiveness of the proposed methods in terms of maximum sampling intervals for given minimum sampling intervals.

**Notations:** \(X > 0\) \((X \geq 0)\) means that \(X\) is a real symmetric positive definite matrix (positive semidefinite). \(\text{col}(x_1, x_2, \ldots, x_n)\) means \([x_1^T, x_2^T, \ldots, x_n^T]^T\). \(X \oplus Y\) means \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}. \(\text{He}(M)\) denotes \(M + M^T\). \(\mathbb{N}\) and \(\mathbb{R}^n\) denote the set of positive integer and the \(n\)-dimensional Euclidean space, respectively. \(\mathbb{Z}_{\geq 0}\) denotes the set of non-negative integer. \(\mathbb{R}^{n \times m}\) is the set of all \(n \times m\) real matrices. \(S_n\) and \(S_n^+\) represent the set of symmetric matrices and the set of positive definite matrices of \(\mathbb{R}^{n \times n}\), respectively. The matrix \(I_n\) represents the identity matrix in \(\mathbb{R}^{n \times n}\). The notation \(0_{m,n}\) stands for the matrix in \(\mathbb{R}^{n \times m}\) whose entries are zero and, when no confusion is possible, the subscript will be omitted. Define \(K\), as the set of differentiable functions from an interval of the form \([0, T]\) to \(\mathbb{R}^n\).

**II. PRELIMINARIES**

This section revisits several lemmas, a definition, and a system formulation to derive main results. Consider the following sampled-data system.

\[
\dot{x}(t) = Ax(t) + A_dx(t_k), \quad \forall t \in [t_k, t_{k+1}),
\]

(1)

where \(x(t) \in \mathbb{R}^n\) is the system state, \(A, A_d \in \mathbb{R}^{n \times n}\) are system matrices, and \(t_k\) is the sampling instant such that \(\cup_{k \in \mathbb{N}}[t_k, t_{k+1}) = [0, +\infty)\). The sampling interval is defined as

\[
t_{k+1} - t_k = h_k \in [h_m, h_M], \quad \forall k \in \mathbb{N}.
\]

(2)

In [21], it was shown that integrating the differential equation (1) yields

\[
x(t) = \Upsilon(t - t_k)x(t_k), \quad \forall t \in [t_k, t_{k+1}),
\]

(3)

\[
\Upsilon(t) = e^{At} + \int_0^t e^{A(t-s)}drA_d.
\]

(4)
Under the periodic sampling $h$, the dynamics in (3) and (4) become
\[ x(t_{k+1}) = Y(h)x(t_k). \] (5)
Thus, under the periodic sampling, the system (1) is asymptotically stable if and only if $\Gamma(h)$ has all eigenvalues inside the unit circle. In the case of time-varying sampling interval $h_k$, however, this method does not hold. Thus, an input-delay approach with the following looped-functional has been widely utilized for the stability analysis of asynchronous sampled-data systems.

**Lemma 1 (Looped-Functional [21]):** Let $0 < T_1 < T_2$ be two positive scalars and $V : \mathbb{R}^n \rightarrow \mathbb{R}^+$ be a differentiable function for which there exist positive scalars $\mu_1 < \mu_2$ and $p$ such that
\[ \forall x \in \mathbb{R}^n, \quad \mu_1|x|^p \leq V(x) \leq \mu_2|x|^p. \] (6)
Then, the following statements are equivalent.
1) The increment of the Lyapunov function is strictly negative for all $k \in \mathbb{N}$ and $T_k \in [T_1, T_2]$, i.e.,
\[ \Delta_0 V(k) = V(\chi_k(T_k)) - V(\chi_k(0)) < 0. \] (7)
2) There exists a continuous and differentiable functional $V_0 : [0, T_2] \times \mathbb{K} \rightarrow \mathbb{R}$ which satisfies, for all $z \in \mathbb{K}$,
\[ \forall T \in [T_1, T_2], \quad V_0(T, z(\cdot)) = V_0(0, z(\cdot)) \] (8)
and such that, for all $k \in \mathbb{N}$, $T_k \in [T_1, T_2]$, $\tau \in [0, T_k]$,
\[ \dot{V}_0(T_k, \chi_k) = \frac{d}{d\tau}[V(\chi_k(\tau)) + V_0(\tau, \chi_k)] < 0. \] (9)
Moreover, if one of these two statements is satisfied, then the solutions of the system are asymptotically stable.

**Definition 1 (Legendre Polynomials [17]):** For scalars $a, b \in \mathbb{R}$, Legendre polynomials over the integral interval $[a, b]$ can be defined as follows.
\[ L_i(r) = \sum_{j=0}^{i} \frac{\binom{r-a}{j}}{\binom{b-a}{j}} \quad \forall r \in [a, b], \] (10)

This polynomial function satisfies the following properties.
1) $L_0(b) = 1$, $L_k(a) = (-1)^k$,
2) $\int_a^b L_k(r)L_i(r)dr = \begin{cases} 0 & \text{if } k \neq l, \\ \frac{b-a}{2k+1} & \text{if } k = l. \end{cases}$ (11)

**Lemma 2 [23]:** For scalars $m \in \mathbb{Z}_{\geq 0}$, $a, b \in \mathbb{R}$, let $x(r) \in \mathbb{R}^{n \times m}$ be an integrable function: $[x(r)]r \in [a, b]$.
Then we have
\[ \int_a^b (r-a)^m x(r)dr = m! \int_a^b \int_{r_1}^{r} \cdots \int_{r_m}^{r} x(r_{m+1}) \cdots dr_m \cdots dr_2 dr_1, \] (12)
where $r_0 = a$.

Integral inequalities have played essential roles to develop stability criteria for time-delay systems in terms of LMIs. The following lemma contains major two general integral inequalities.

**Lemma 3 (Integral Inequalities [17], [20]):** For scalars $a, b \in \mathbb{R}$, let $x(t) \in \mathbb{R}^n$ be a continuous function: $x(t) : t \in [a, b]$. Then, for integers $m, k \in \mathbb{N}$, an arbitrary vector $\zeta(t) \in \mathbb{R}^{2kn}$, a positive definite matrix $R \in \mathbb{S}^+_n$, and a matrix $S \in \mathbb{R}^{kn \times (m+1)n}$, the following inequalities hold:
1) Orthogonal polynomials based integral inequality [20]
\[ -\int_a^b x^T(r)R\dot{x}(r)dr \leq (b-a)\zeta^T(t)S\bar{R}^{-1}S^T\zeta(t) + \text{He}\left\{\zeta^T(t)S^T\Gamma_m(a, b)\right\}, \] (13)
2) Bessel-Legendre (B-L) inequality [17]
\[ -\int_a^b x^T(r)R(x(r))dr \leq -\frac{1}{b-a}T_{m-1}(a, b)\bar{R}_{m-1} \bar{Y}_{m-1}(a, b), \] (14)
where, for a non-negative integer $i \in \mathbb{Z}_{\geq 0}$,
\[ \Gamma_i(a, b) = \text{col}\{\Gamma_0(a, b), \Gamma_1(a, b), \ldots, \Gamma_i(a, b)\}, \] (15)
\[ T_i(a, b) = \text{col}\{T_0(a, b), T_1(a, b), \ldots, T_i(a, b)\}, \] (16)
\[ \bar{R}_i = R \oplus 3R \oplus \ldots \oplus (2i + 1)R, \] (17)
\[ \bar{Y}_i(a, b) = \begin{cases} x(b) - x(a) & \text{if } i = 0, \\ -\sum_{j=0}^{i-1} \frac{(b-a)}{j+1}J_j(a, b) & \text{for } i \in \mathbb{N}^+ \end{cases} \] (18)
\[ T_i(a, b) = \sum_{j=0}^{i} \frac{1}{j+1}U_j(a, b), \] (19)
\[ U_i(a, b) = \frac{(i+1)!}{(b-a)^{i+1}} \] (20)
\[ \times \int_{r_0}^{r_1} \cdots \int_{r_m}^{r} x(r_{m+1})dr_{m+1} \cdots dr_1, \] (21)
\[ (b-a)\bar{R}_i(a, b). \] (22)

The relation between these two integral inequalities was shown in [20]. For the same integrand such as $\dot{x}^T(r)R\dot{x}(r)$ or $x^T(r)R(x(r))$, the B-L inequality is a special case, which provides the tightest upper bound, of the orthogonal polynomials based integral inequality adopting $\zeta(t) = \Gamma_m(a, b)$ in (16). However, the B-L inequality inevitably provides reciprocally convex upper bounds with respect to the length $(b-a)$ of the integral interval $[a, b]$, which leads to non-convex stability conditions. On the other hand, the orthogonal polynomials-based integral inequality provides the upper bound in terms of convex conditions with respect to $(b-a)$. Thus, it might be less conservative in the case of time-varying $(b-a)$ at a price of more computational burden for calculating the slack matrix $S$ in (14). To reduce the conservatism and computational burden, this paper simultaneously utilize the two integral inequalities (14) and (15). Also, the two types of integral state variables (21) and (22) occurred in the upper bounds of the integral inequalities are also employed.
III. STABILITY ANALYSIS OF ASYNCHRONOUS SAMPLED-DATA SYSTEMS

This section derives stability criteria for asynchronous sampled-data systems by utilizing a new looped-functional and new generalized equalities. First, we introduce a new looped-functional \( V(t) \) such that

\[
V(t) = V_0(t) + V_k(t),
\]

where

\[
V_0(t) = x^T(t)Px(t),
\]

\[
V_k(t) = 2 \left[ \begin{array}{c}
(t_{k+1} - t)(x(t) - x(t_k)) \\
(t_{k+1} - t) x(t_k) \\
(t_{k+1} - t) x(t_{k+1}) \\
(t_{k+1} - t) \int_t^{t_{k+1}} x(r)dr
\end{array} \right]^T Q
\]

\[
+ X \left[ \begin{array}{c}
(t_{k+1} - t)(x(t) - x(t_k)) \\
(t_{k+1} - t) x(t_k) \\
(t_{k+1} - t) x(t_{k+1}) \\
(t_{k+1} - t) \int_t^{t_{k+1}} x(r)dr
\end{array} \right] + 2 \left[ \begin{array}{c}
(t_{k+1} - t)(x(t) - x(t_k)) \\
(t_{k+1} - t) x(t_k) \\
(t_{k+1} - t) x(t_{k+1}) \\
(t_{k+1} - t) \int_t^{t_{k+1}} x(r)dr
\end{array} \right]^T T
\]

\[
 R_2, R_3, R_4 = 0,
\]

\[I_m(a, b) \text{ is defined in (21) and } l^i_1 \text{ is defined in (10). The proof of Lemma 4 is provided in the appendix.}
\]

**Remark 1:** To reduce the conservatism of stability criteria, this paper introduces a new looped-functional containing two types of integral quadratic functions, where integrands are \( \hat{x}(r) R_2 \hat{x}(r) \) and \( x^T(r) R_3 x(r) \), respectively. The proposed functional is more general than that of [13], which can be shown as follows. For matrices \( Q_1, Q_2 \in \mathbb{R}^{2n \times 2n}, X \in \mathbb{R}^{nxn} \), and \( Z \in \mathbb{S}_{2n} \), if the matrices in the functional (23) are defined by

\[
Q = \begin{bmatrix}
I_1 & -I_1 & 0 \\
I_1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
Q_2 = \begin{bmatrix}
0 & I_1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
X = X \oplus (Z/2) \oplus 0_{n,n},
\]

\[
R_3 = X^T R_3 X = 0,
\]

the Lyapunov functional (23) reduces to the functional of [13]. Due to such generality, the stability criteria based on the proposed functional has the same or less conservatism than those of [13], which will be shown in numerical examples.

**Remark 2:** By simultaneously utilizing the two types of integral state variables (21) and (22), the B-L inequality in Lemma 3 can be modified as follow.

\[
-\int_a^b x^T(r) R x(r) dr \leq - \frac{1}{2} \text{He}[(\Gamma_{m-1}^T (a, b) \bar{R}_{m-1} Y_{m-1} (a, b))].
\]

This modification provides the upper bound of integral quadratic functions containing the integrand \( x^T(r) R x(r) \) without additional slack matrices, the reciprocally convexity and the convexity.

Second, we propose the following lemma.

**Lemma 4:** Let \( x(r) \in \mathbb{R}^n \) be a state of the sampled-data system (1): \( \{ x(r) : r \in [t_k, t_{k+1}) \} \). For scalars \( m \in \mathbb{Z}_{\geq 0} \), \( a, b \in [t_k, t_{k+1}) \), the following equality holds:

\[
\mathbb{H}_m(a, b) = D_m^{-1} \left( E_m(a, b) \right) = \begin{bmatrix}
A \otimes A & \ldots & A \otimes G_m \otimes \mathbb{H}_m(a, b)
\end{bmatrix},
\]

where

\[
\mathbb{H}_m(a, b) = \text{col} \{ I_0(a, b), \ldots, I_{m-1}(a, b) \},
\]

\[
U_m(a, b) = \text{col} \{ U_0(a, b), \ldots, U_{m-1}(a, b) \},
\]

\[
E_m(a, b) = \begin{bmatrix}
x(b) + x(a) \\
\vdots \\
x(b) - (-1)^m x(a)
\end{bmatrix},
\]

\[
D_m = \begin{bmatrix}
l^1_0 & 0 & \ldots & 0 \\
l^1_1 & l^2_0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
l^m_0 & l^m_1 & \ldots & l^m_m
\end{bmatrix},
\]

\[
G_m = \begin{bmatrix}
l^0_0 & l^0_1 & \ldots & 0 \\
l^1_0 & l^1_1 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
l^m_0 & l^m_1 & \ldots & l^m_m
\end{bmatrix},
\]

\[
I_m(a, b) \text{ is defined in (21) and } l^i_1 \text{ is defined in (10). The proof of Lemma 4 is provided in the appendix.}
\]

**Remark 3:** In the literature [12], [13], [15], [18], the following two types of zero equalities have been employed to fully utilize the relations among the state variables in a construction of the stability criteria.

1) In [15], [18], the relation between integral state variables \( U_0(a, b), U_1(a, b) \) and their interval-normalized versions \( U_0(a, b), I_1(a, b) \) yields

\[
0 = U_1(a, b) - (b - a) I_1(a, b), \quad (i = 0, 1).
\]

2) In [12], [13], [15], single and double integration of the system formulation (1) on the integral interval
\([a, b] \in ([t_k, t], [t, t_{k+1}])\) respectively yields
\[
0 = x(b) - x(a) = -(b - a) A_0 x(t_k) + A_d x(t_k),
\]

(32)
\[
0 = x(b) - I_0 x(a) = -(b - a) \left( A_1 x(t_k) + \frac{1}{2} A_d x(t_k) \right).
\]

(33)
In the case utilizing the two types of integral state variables \(U_i(a, b)\) and \(I_i(a, b)\) \((i = 1, 2)\), these zero equalities can be represented as follows.
\[
0 = x(b) - x(a) - A U_0 x(a, b)
\]

(34)
\[
0 = x(b) - I_0 x(a, b) - (b - a) A_d x(t_k),
\]

(35)
The common difficulties when utilizing these zero equalities come from computational burden. Since the zero equalities (31)-(35) are convex with respect to the length \((b - a)\) of the integral interval \([a, b]\), additional slack matrices are needed to make LMIs. Such observations motivate our work to eliminate convex terms in the zero equalities (34) and (35). Between two consecutive sampling instants, the sampled state variable \(x(t_k)\) in the system formulation (1) is a constant value, and thus it can be eliminated by an integration with Legendre polynomials of positive degrees. By fully utilizing Legendre polynomials (9) and the system formulation (1), generalized equalities are derived without the convexity. Consequently, an augmented integral state term \(I_m x(a, b)\) can be represented as combination of \(x(t_k), x(t_{k+1})\) and \(U_{m}(a, b)\) without any slack matrices and the convexity.

Thus, slack matrices are only needed in the zero equalities obtained from the relation between the two types of integral state terms (31) and the single integration of the system formulation (32).

Before deriving stability criteria for sampled-data systems, several vectors and matrices are defined as follows.
\[
\xi(t) = \begin{bmatrix} x(t_k) \\ x(t) \\ x(t_{k+1}) \\ I_1(t_k, t) \\ I_1(t_k, t_{k+1}) \\ I_1(t, t_{k+1}) \end{bmatrix},
\]

(36)
\[
e_i = \begin{bmatrix} 0_{n \times (i-1) qr} & I_n & 0_{n \times (q-i) n} \end{bmatrix} i \in [1, 9] \cap \mathbb{Z},
\]

(37)
\[
e_0 = A e_2 + A_d e_1,
\]

(38)
\[
\xi(t) = \begin{bmatrix} \gamma_1 x(t_k) & \text{if } [a, b] = [t_k, t] \\ \gamma_2 x(t_k) & \text{if } [a, b] = [t, t_{k+1}] \end{bmatrix},
\]

(39)
Here, \(\xi(t)\) is an arbitrary vector in Lemma 1. According to the integral interval \([a, b]\), an arbitrary vector is chosen by defining arbitrary matrices \(\gamma_1\) and \(\gamma_2\) in (39).

Based on the proposed looped-functional (23) and the definitions (36)-(39), we have the following theorem.

**Theorem 1:** Given scalars \(h_m\) and \(h_M\), matrices \(\gamma_1, \gamma_2 \in \mathbb{R}^{n \times q}\), and a positive integer \(q \in \mathbb{N}\), the system (1) is stable if there exist matrices \(P, R_i\) \((i = 1, 2, 3, 4) \in \mathbb{S}_n^+\), such that

\[
Q \in \mathbb{R}^{4n \times 5n}, \ X \in \mathbb{R}^{4n \times 4n}, \ S_1, S_2 \in \mathbb{R}^{q \times 3n}, \ M \in \mathbb{R}^{9n \times 6n}
\]

such that

\[
\begin{bmatrix}
\Lambda_1(h_m) & \sqrt{h_m} \gamma_1^T S_1 \\
\sqrt{h_m} S_1^T \gamma_1 & -R_1
\end{bmatrix} < 0,
\]

(40)
\[
\begin{bmatrix}
\Lambda_2(h_m) & \sqrt{h_m} \gamma_2^T S_2 \\
\sqrt{h_m} S_2^T \gamma_2 & -R_2
\end{bmatrix} < 0,
\]

(41)
\[
\begin{bmatrix}
\Lambda_1(h_M) & \sqrt{h_M} \gamma_1^T S_1 \\
\sqrt{h_M} S_1^T \gamma_1 & -R_1
\end{bmatrix} < 0,
\]

(42)
\[
\begin{bmatrix}
\Lambda_2(h_M) & \sqrt{h_M} \gamma_2^T S_2 \\
\sqrt{h_M} S_2^T \gamma_2 & -R_2
\end{bmatrix} < 0,
\]

(43)
where

\[
\mathcal{Z}_0 = \begin{bmatrix} e_0^T P e_2 + \gamma_1^T S_1 L_1 + \gamma_2^T S_2 L_2 \\
-\frac{1}{2} \bar{L} L + \frac{1}{2} \bar{L} \bar{L} \bar{L} \bar{L} \bar{L} \end{bmatrix}.
\]

(44)
\[
\mathcal{Z}_1(h_k) = \begin{bmatrix}
-\gamma_2 e_2 + e_3 \\
e_0 - h_k e_2 + e_7
\end{bmatrix}^T Q \begin{bmatrix} e_2 \\ e_1 \\ e_3 \\ e_6 \\ e_7 \end{bmatrix}
\]

(45)

\[
\mathcal{Z}_2(h_k) = \begin{bmatrix}
h_k e_2 - e_3 \\
h_k e_6
\end{bmatrix}^T X \begin{bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \\ e_6 \\ e_7 \end{bmatrix}
\]

(46)
\[
\mathcal{Z}_3(h_k) = \begin{bmatrix}
h_k e_6 + e_7
\end{bmatrix}^T X \begin{bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \\ e_6 \\ e_7 \end{bmatrix}
\]

(47)
Defining the matrices, the proof of Theorem 1 is provided in the appendix. It will be shown in numerical examples.

Given scalars \( \gamma_1 \) and \( \gamma_2 \), matrices \( \gamma_1 = \gamma_2 = I \) provides the most general upper bound among those obtained with choices of \( \xi(t) \). In this case, stability conditions might be less conservative than the others. However, since the dimensions of slack matrices \( S_1 \) and \( S_2 \) increase, a number of variables also increases. According to the choices of the matrices \( \gamma_1 \) and \( \gamma_2 \), there exist trade-offs between a number of variables and the conservatism of stability criteria, which will be shown in numerical examples.

Remark 4: It has been noted that increasing the degree \( m \) of the integral inequalities (14) and (15) contributes to reducing the conservativeness of stability criteria at a price of more computational burden. With increasing the degree of the integral inequalities, the proposed equalities is effective in reducing a number of variables. For a non-negative integer \( i \in \mathbb{N} \), let us define

\[
\tilde{I}_i(t) = \begin{bmatrix} I_i(t, t) \\ I_i(t, t_k+1) \end{bmatrix} \in \mathbb{R}^{2n},
\]

\[
\tilde{U}_i(t) = \begin{bmatrix} U_i(t, t) \\ U_i(t, t_k+1) \end{bmatrix} \in \mathbb{R}^{2n},
\]

\[
\tilde{I}_i(t) = col\{I_0(t), \tilde{I}_1(t), \ldots, \tilde{I}_i(t)\} \in \mathbb{R}^{2(i+1)n},
\]

\[
\tilde{U}_i(t) = col\{\tilde{U}_0(t), \tilde{U}_1(t), \ldots, \tilde{U}_i(t)\} \in \mathbb{R}^{2(i+1)n}.
\]

Then, according to availability of the proposed equalities, the augmented vector \( \xi(t) \) and the dimension of the slack matrix \( M \) in Theorem 1 can be defined as follows.

1) First, if Lemma 4 is utilized, the interval-normalized integral state term \( I_{i-1}(a, b), (a, b) \in \{(t_k, t), (t, t_k+1)\} \) can be represented as combination of \( x(t) \), \( x(t_k) \), \( x(t_{k+1}) \) and \( \tilde{U}_i(a, b) \), and thus there is no need to include \( \bar{I}_{i-1}(a, b) \) into \( \xi(t) \). In this case,

\[
\xi(t) = col\{x(t_k), x(t), x(t_{k+1}), \tilde{I}_i(t), \tilde{U}_i(t)\} \in \mathbb{R}^p,
\]

\[
p = (2i + 7)n,
\]

\[
M \in \mathbb{R}^{p \times 2(i+1)n}.
\]

2) Second, if the equalities (32)-(35) and those obtained from their multiple integration are utilized,

\[
\xi(t) = col\{x(t_k), x(t), x(t_{k+1}), \tilde{I}_i(t), \tilde{U}_i(t)\} \in \mathbb{R}^p,
\]

\[
p = (4i + 7)n,
\]

\[
M \in \mathbb{R}^{p \times 4(i+1)n}.
\]

In the construction of less conservative stability criteria, the dimension of the augmented vector \( \xi(t) \) also affects the sizes of the slack matrices \( S_1, S_2 \in \mathbb{R}^{p \times (i+2)n} \). Thus, in the first and the second cases, the number of variables from the slack matrices \( S_1, S_2 \), and \( M \) is \((6i^2 + 33i + 42)n^2\) and \((20i^2 + 59i + 42)n^2\), respectively. The difference between number of variable is \((14i^2 + 26i)n^2\), where \( i \) is the highest degree of the integral state terms. If the integral inequalities of the degree \( m \) in Lemma 3 is employed, the highest degree of the integral state term is \( m - 1 \). Consequently, compared to the previous approaches, Lemma 4 reduces the number of variables \((14(m - 1)^2 + 26(m - 1))n^2\). Since this gap is an increasing function of \( m \in \mathbb{N} \), the proposed approach is more effective when the degree \( m \) increases.

Utilizing Lemma 3 of the degree \( m = 3 \), the following theorem is obtained.

Theorem 2: Given scalars \( h_m \) and \( h_M \), matrices \( \gamma_1, \gamma_2 \in \mathbb{R}^{11n \times 5q} \), and a positive integer \( q \in \mathbb{N} \), the system (1) is stable if there exist matrices \( P, R_i, i = 1, 2, 3, 4, 5 \) \( S_1, S_2 \in \mathbb{R}^{n \times n}, \mathcal{Q} \in \mathbb{R}^{4n \times 5n}, \mathcal{X} \in \mathbb{R}^{4n \times 4n} \), \( \mathcal{S}_1, \mathcal{S}_2, \mathcal{X} \in \mathbb{R}^{4n \times 4n} \), \( M \in \mathbb{R}^{11n \times 5n} \).
such that
\[
\begin{bmatrix}
\Lambda_1(h_m) & \sqrt{h_m}S_1 \\
\sqrt{h_m}S_1^T & -R_1
\end{bmatrix} < 0,
\]
\[
\begin{bmatrix}
\Lambda_2(h_m) & \sqrt{h_m}S_2 \\
\sqrt{h_m}S_2^T & -R_2
\end{bmatrix} < 0,
\]
\[
\begin{bmatrix}
\Lambda_1(h_M) & \sqrt{h_M}S_1 \\
\sqrt{h_M}S_1^T & -R_1
\end{bmatrix} < 0,
\]
\[
\begin{bmatrix}
\Lambda_2(h_M) & \sqrt{h_M}S_2 \\
\sqrt{h_M}S_2^T & -R_2
\end{bmatrix} < 0,
\]
where
\[
L_1 = \begin{bmatrix}
e_2 - e_1 \\
e_2 + e_1 - 2\bar{e}_6 \\
e_2 + e_1 - 6\bar{e}_6 - 6\bar{e}_8 \\
e_2 + e_1 - 12\bar{e}_6 + 30\bar{e}_8 - 20e_4
\end{bmatrix},
\]
\[
L_2 = \begin{bmatrix}
e_3 - e_2 \\
e_3 + e_2 - 2\bar{e}_7 \\
e_3 + e_2 - 6\bar{e}_7 - 6\bar{e}_9 \\
e_3 + e_2 - 12\bar{e}_7 + 30\bar{e}_9 - 20e_5
\end{bmatrix},
\]
\[
L_3 = \begin{bmatrix}
e_6 \\
e_6 - e_8 + e_{10} \\
e_6 - 3e_8 + 2e_{10}
\end{bmatrix},
\]
\[
L_4 = \begin{bmatrix}
e_7 + e_9 \\
e_7 - 3e_9 + 2e_{11}
\end{bmatrix},
\]
\[
\bar{L}_3 = \begin{bmatrix}
\bar{e}_6 + e_8 \\
\bar{e}_6 + e_8 + e_{10}
\end{bmatrix},
\]
\[
\bar{L}_4 = \begin{bmatrix}
\bar{e}_7 + e_9 \\
\bar{e}_7 - 3e_9 + 2e_5
\end{bmatrix},
\]
\[
O_1(h_k) = \begin{bmatrix}
e_2 - e_1 - A\bar{e}_6 \\
e_6 \\
e_8 \\
e_{10}
\end{bmatrix},
\]
\[
A_d e_1 \\
e_6 \\
e_8 \\
e_{10}
\end{bmatrix},
\]
\[
O_2(h_k) = \begin{bmatrix}
e_2 - e_1 - A\bar{e}_6 \\
e_6 \\
e_8 \\
e_{10}
\end{bmatrix},
\]
\[
A_d e_1 \\
e_6 \\
e_8 \\
e_{10}
\end{bmatrix},
\]
\[
\bar{R}_1 = R_1 \oplus 3R_1 \oplus 5R_1 \oplus 7R_1,
\]
\[
\bar{R}_2 = R_2 \oplus 3R_2 \oplus 5R_2 \oplus 7R_2,
\]
\[
\bar{R}_3 = R_3 \oplus 3R_3 \oplus 5R_3,
\]
\[
\bar{R}_4 = R_4 \oplus 3R_4 \oplus 5R_4,
\]
\[
\bar{e}_6 = (e_2 + e_1 + A(e_6 - e_8))/2,
\]
\[
\bar{e}_7 = (e_3 + e_2 + A(e_7 - e_9))/2,
\]
\[
\bar{e}_8 = (2e_2 + e_1 + A(e_8 - e_{10}))/2,
\]
\[
\bar{e}_9 = (2e_3 + e_2 + A(e_7 - e_{11}))/2,
\]
and \(\gamma_1\) and \(\gamma_2\) are defined as follows.
1) If \(\xi(t) = \cos\{x(a), x(b), \mathbb{I}_{m}(a, b)\}\),
\[
\gamma_1 = \cos\{e_1, e_2, \bar{e}_6, \bar{e}_8, e_4\},
\]
\[
\gamma_2 = \cos\{e_2, e_3, \bar{e}_7, \bar{e}_9, e_5\}.
\]
2) If \(\xi(t) = \xi(t)\),
\[
\gamma_1 = \gamma_2 = I_{11n}.
\]

The proof of Theorem 2 is provided in the appendix.

**IV. NUMERICAL EXAMPLES**

This section illustrates the effectiveness of the proposed approaches in terms of allowable maximum sampling intervals and numbers of variables.

**Example 1:** Consider the system (1) with
\[
A = \begin{bmatrix}
-2 & 0 \\
0 & -0.9
\end{bmatrix},
\]
\[
A_d = \begin{bmatrix}
-1 & 0 \\
-1 & -1
\end{bmatrix}.
\]

The conservatism of stability criteria for sampled-data systems has been checked in terms of an allowable maximum sampling interval \(h_M\) for a given minimum sampling interval \(h_m\). A detailed comparison to other results of the literature are shown in Table 1 and Table 2, where the numbers of variables and the allowable maximum sampling intervals are listed, respectively. In the case of periodic sampling, an analytic bound obtained by the eigenvalue analysis (5) for this example is \(0, 3.2715\). In the case of aperiodic sampling, it can be seen that the stability criteria in Theorem 1 and Theorem 2 have the less conservatism than those of [12]–[15]. In [15], the two types of zero equalities (32)-(31) were simultaneously utilized, which leads to less conservative stability criteria than those of [12]–[15] at a price of a more number of variables. Compared to the result of [15], Theorem 1 and Theorem 2 with (85) and (86) have the less number of variables while reducing the conservatism. Theorem 2 utilizes

| Methods | Number of variables |
|---------|---------------------|
| [12]    | 12n^2 + 3n         |
| [16]    | 10.5n^2 + 2.5n     |
| [13]    | 64.5n^2 + 2.5n     |
| [15]    | 161.5n^2 + 4.5n    |
| Theorem 1 with (60) and (61) | 110.5n^2 + 2.5n |
| Theorem 1 with (62) | 146.5n^2 + 2.5n |
| Theorem 2 with (85) and (86) | 158.5n^2 + 2.5n |
| Theorem 2 with (87) | 214.5n^2 + 2.5n |

**TABLE 1.** Number of variables \((n\) is the dimension of the state vector \(x(t) \in \mathbb{R}^n\))

**TABLE 2.** The allowable maximum sampling interval \(h_M\) under a given minimum sampling interval \(h_m\) for Example 1.
higher inequalities than others, and thus number of variables increase with reduction of conservatism. However, utilizing the proposed methods, Theorem 2 has still less number of variables than that of [15].

**Example 2:** Consider the system (1) with

\[
A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}.
\]

Table 3 and Table 1 list the allowable maximum sampling intervals and numbers of variables, respectively. In the literature [14], [15], [24], it has been noted that this system is stable for any constant sampling time. In the case of aperiodic sampling, it can be seen that the stability criteria in Theorem 1 and Theorem 2 have the less conservatism than those of [12]–[15].

**Example 3:** Consider the system (1) with

\[
A = \begin{bmatrix} 0 & 1 \\ -2 & 0.1 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}.
\]

Table 4 and Table 1 list the allowable maximum sampling intervals and numbers of variables, respectively. In the case of periodic sampling, analytic bound obtained by the eigenvalue analysis (5) for this example is [0.2007, 2.020]. In the case of aperiodic sampling, it can be seen that the stability criteria in Theorem 1 and Theorem 2 have the same or less conservatism than those of [12]–[15]. In this example, Theorem 1 has the same conservatism as that of [15] whereas the number of variables is reduced.

### V. Conclusion

In this paper, we have proposed novel equalities and a new looped-functional to develop the less conservative stability criteria for asynchronous sampled-data systems with reduction of the number of variables. In the construction of stability conditions, two types of integral state variables are fully utilized based on the proposed equalities and the functional. Compared to the existing results from the literature, notable improvements have been obtained. Since the proposed methods mainly have dealt with the sampled-data systems, they cannot be directly utilized for the general time-delay systems. However, in the future work, the proposed methods can be applied to sampled-data controller synthesis problems of various systems such as fuzzy systems with sampled data and synchronization of chaotic Lur’e systems with time delays.

### APPENDIX

This section provides proofs of Lemma 4, Theorem 1, and Theorem 2. Also, a method calculating number of variables listed in Table 1 is provided.

**A. PROOF OF LEMMA 4**

From the sampled-data system formulation (1), the following equality holds for \(a, b \in [t_k, t_{k+1})\):

\[
\int_a^b L_i(r)\dot{x}(r)dr = A \int_a^b L_i(r)x(r)dr + A_d \int_a^b L_i(r)dx(t_k),
\]

(91)

Based on Definition 1, Lemma 2, and Lemma 3, the following equalities hold:

\[
\int_a^b L_i(r)\dot{x}(r)dr = L_i(a, b),
\]

(92)

\[
\int_a^b L_k(r)x(r)dr = \sum_{j=0}^{i} (l_j^i/(j+1))U_j(a, b).
\]

(93)

Due to the property (12), for integers \(i \geq 1\), combining (91)-(93) yields the following equality:

\[
\sum_{j=0}^{i-1} l_{j+1}^i f_j(a, b) = x(b) - (-1)^i x(a) - A \sum_{j=0}^{i} (l_j^i/(j+1))U_j(a, b), \forall i \geq 1,
\]

(94)

Augmentation of (94) for \(i \in [1, m] \cap \mathbb{Z}\) yields

\[
D_m \cup_m (a, b) = E_m(a, b)
\]

\[
-\underbrace{A \otimes A \cdots A} \otimes A G_m \cup_{m+1}(a, b),
\]

(95)

where \(D_m\) is a triangular matrix and its diagonal components are nonzeros. Thus, the matrix \(D_m\) is nonsingular. It ends the proof.

**B. PROOF OF THEOREM 1**

The time-derivative of the functional (23) is obtained as follows

\[
\dot{V}_0(t) = 2\dot{x}^T(t)Px(t),
\]

(96)
\[ \dot{V}_k(t) = 2 \begin{bmatrix} \left( t_{k+1} - t \right) \dot{x}(t) \\ \left( t - t_k \right) \dot{x}(t) \\ \left( t - t_k \right) x(t) \end{bmatrix}^T Q \begin{bmatrix} x(t) \\ x(t_k) \\ x(t_{k+1}) \end{bmatrix} + \int_{t_k}^{t_{k+1}} x(t) \dot{x}(t) \, dt + \int_{t_k}^{t_{k+1}} x(t) \dot{r}(t) \, dt \]

where

\[ \dot{V}_1(t_k, t_{k+1}) = - \int_{t_k}^{t_{k+1}} x^T(t) R_1 \dot{x}(t) \, dt \]

\[ \dot{V}_2(t_k, t_{k+1}) = - \int_{t_k}^{t_{k+1}} x^T(t) R_3 x(t) \, dt \]

Utilizing the modified B-L inequality (24) of Lemma 3 at the degree \( m = 2 \), the upper bounds of integral quadratic functions (99) can be obtained as follows.

\[ - \int_{t_k}^{t_{k+1}} x^T(t) R_3 x(t) \, dt \leq - \frac{1}{2} \xi^T(t) \left[ L_3^T \tilde{R}_3 L_3 \right] \xi(t), \quad (102) \]

\[ - \int_{t_k}^{t_{k+1}} x^T(t) R_4 x(t) \, dt \leq - \frac{1}{2} \xi^T(t) \left[ L_4^T \tilde{R}_4 L_4 \right] \xi(t). \quad (103) \]

The interval-normalized integral state variables \( I_0(t_k, t) \) and \( I_0(t, t_{k+1}) \) can be represented as follows.

\[ I_0(t_k, t) = \bar{e}_6 \xi(t), \]

\[ I_0(t, t_{k+1}) = \bar{e}_7 \xi(t). \]

Here, Utilizing Lemma 4, \( \bar{e}_i, i = 6, 7 \) can be obtained as follows.

\[ D_1 = 2I, \quad D_1^{-1} = (1/2)I, \]

\[ \bar{e}_6 = D_1^{-1}(e_2 + e_1 + A(e_6 + e_8)), \]

\[ \bar{e}_7 = D_1^{-1}(e_3 + e_2 + A(e_7 + e_9)), \]

which results in (47) and (48). Due to the equalities (22) and (32), the following zero equalities can be obtained at \( [a, b] = [t_k, t] \) and \( [a, b] = [t, t_{k+1}] \), respectively.

\[ 0 = \left\{ \begin{array}{l} e_2 - e_1 - A e_6 \\ e_6 \\ e_8 \\ e_4 \end{array} \right\} - (t - t_k) \left[ A_d e_1 \right] \xi(t). \quad (104) \]

\[ 0 = \left\{ \begin{array}{l} e_3 - e_2 - A e_7 \\ e_7 \\ e_9 \\ e_8 \end{array} \right\} - (t_{k+1} - t) \left[ A_d e_1 \right] \xi(t). \quad (105) \]

Combining (104) and (105) with the slack matrix \( M \) yields

\[ \dot{V}(t) \leq \xi^T(t) \Xi(t, h_k) \xi(t), \quad (107) \]

where

\[ \Xi(t, h_k) = \left( \frac{t - t_k}{h_k} \right) \left( \Lambda_1(h_k) + \Omega_1(h_k) \right) + \left( \frac{t_{k+1} - t}{h_k} \right) \left( \Lambda_2(h_k) + \Omega_2(h_k) \right). \]

\[ \Omega_1(h_k) = h_k M_{11} + h_k M_{12} \]

\[ \Omega_2(h_k) = h_k M_{21} + h_k M_{22}. \]

Since \( \xi(t, h_k) \) is convex in \( t \in [t_k, t_{k+1}] \), \( \Xi(t, h_k) < 0 \) if and only if \( \Xi(t_k, h_k) < 0 \) and \( \Xi(t_{k+1}, h_k) < 0 \). Based on schur complement, \( \Xi(t_k, h_k) < 0 \) and \( \Xi(t_{k+1}, h_k) < 0 \) are guaranteed if the conditions (40)-(43) hold. It ends the proof.
C. PROOF OF THEOREM 2
Let us define \( \xi(t) \) and \( e_i \) instead of (36) and (37), respectively.

\[
\xi(t) = \text{col} \left\{ \begin{bmatrix} x(t_k) \\ x(t) \\ x(t_{k+1}) \\ I_2(t_k, t) \\ I_2(t, t_{k+1}) \end{bmatrix} \right\}, \quad (111)
\]

\[
e_i = \begin{bmatrix} I_{n \times (i-1)m} \\ I_{n \times (1-i)m} \end{bmatrix} \quad i \in [1, 11] \cap \mathbb{Z}. \quad (112)
\]

Interval-normalized integral state variables \( I_1(t_k, t) \) and \( I_1(t, t_{k+1}) \) are represented as follows.

\[
I_1(t_k, t) = \begin{bmatrix} I_0(t_k, t) \\ I_1(t_k, t) \end{bmatrix} = \begin{bmatrix} \bar{e}_6 \\ \bar{e}_8 \end{bmatrix} \xi(t),
\]

\[
I_1(t, t_{k+1}) = \begin{bmatrix} I_0(t, t_{k+1}) \\ I_1(t, t_{k+1}) \end{bmatrix} = \begin{bmatrix} \bar{e}_7 \\ \bar{e}_9 \end{bmatrix} \xi(t),
\]

Utilizing Lemma 4, \( \bar{e}_i \), \( i = 6, 7, 8, 9 \) can be obtained as follows.

\[
D_2 = \begin{bmatrix} 2I & 0 \\ -6I & 6I \end{bmatrix}, \quad D_2^{-1} = \begin{bmatrix} \frac{1}{3}I & 0 \\ \frac{2}{3}I & \frac{1}{3}I \end{bmatrix},
\]

\[
\bar{e}_6 = D_2^{-1} \begin{bmatrix} e_2 + e_1 + A(e_6 - e_8) \\ e_2 - e_1 - A(e_6 - 3e_8 + 2e_{10}) \end{bmatrix},
\]

\[
\bar{e}_8 = D_2^{-1} \begin{bmatrix} e_2 + e_1 + A(e_6 - e_8) \\ e_2 + e_1 + A(e_6 - e_{10}) \end{bmatrix},
\]

\[
\bar{e}_7 = D_2^{-1} \begin{bmatrix} e_3 + e_2 + A(e_7 - e_9) \\ e_3 - e_2 - A(e_7 - 3e_9 + 2e_{11}) \end{bmatrix},
\]

\[
\bar{e}_9 = D_2^{-1} \begin{bmatrix} e_3 + e_2 + A(e_7 - e_9) \\ e_3 + e_2 + A(e_7 - e_{11}) \end{bmatrix},
\]

which results in (81)-(84). The rest of the proof process is the same as those of Theorem 1 and thus is omitted for brevity.

According to the definitions \( \gamma_1 \) and \( \gamma_2 \), the integer \( q \) changes.

\[
q = \begin{cases} 3n & \text{for (60) and (61)} \\ 9n & \text{for (62)} \end{cases}
\]

In Theorem 2, the positive integer \( q \) is defined by

\[
q = \begin{cases} 4n & \text{for (85) and (86)} \\ 11n & \text{for (87)} \end{cases}
\]

D. NUMBER OF VARIABLES
Stability conditions in Theorem 1 and Theorem 2 consist of slack matrices which will be chosen by LMI solvers. As listed in Table 1, number of variables in slack matrices have been calculated to compare computational burden. For a symmetric matrix \( P \in \mathbb{S}_n^+ \) and a positive integer \( n \in \mathbb{N} \), a number of variables in \( P \) is \( \frac{n(n+1)}{2} \). For a matrix \( M \in \mathbb{R}^{n \times m} \) and positive integers \( n, m \in \mathbb{N} \), a number of variables in \( M \) is \( nm \). Thus, the numbers of variables in Theorem 1 and Theorem 2 can be summarized in Table 5. In Theorem 1, the positive integer \( q \) is defined by

\[
q = \begin{cases} 3n & \text{for (60) and (61)} \\ 9n & \text{for (62)} \end{cases}
\]

In Theorem 2, the positive integer \( q \) is defined by

\[
q = \begin{cases} 4n & \text{for (85) and (86)} \\ 11n & \text{for (87)} \end{cases}
\]

According to the definitions \( \gamma_1 \) and \( \gamma_2 \), the integer \( q \) changes.

**REFERENCES**

[1] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, “A survey of recent results in networked control systems,” Proc. IEEE, vol. 95, no. 1, pp. 138–162, Jan. 2007.

[2] S. Zampieri, “Trends in networked control systems,” IFAC Proc. Volumes, vol. 41, no. 2, pp. 2886–2894, 2008.

[3] A. Seuret, H. Özbay, C. Bonnet, and H. Mounier, Low-Complexity Controllers for Time-Delay Systems: Cham, Switzerland: Springer, 2014.

[4] H.-B. Zeng, Z.-L. Zhai, H.-Q. Xiao, and W. Wang, “Stability analysis of sampled-data control systems with constant communication delays,” IEEE Access, vol. 7, pp. 111–116, 2019.

[5] H.-H. Lian, P. Deng, S.-P. Xiao, and H.-Q. Xiao, “New results on stability analysis for sampled-data control systems with nonuniform sampling and communication delays,” IEEE Access, vol. 8, pp. 86696–86705, 2020.

[6] D. Zhang, P. Shi, Q.-G. Wang, and L. Yu, “Analysis and synthesis of networked control systems: A survey of recent advances and challenges,” ISA Trans., vol. 66, pp. 376–392, Jan. 2017.

[7] L.-S. Hu, J. Lam, Y.-Y. Cao, and H.-H. Shao, “A linear matrix inequality (LMI) approach to robust H2 sampled-data control for linear uncertain systems,” IEEE Trans. Syst., Man Cybern., B, Cybern., vol. 33, no. 1, pp. 149–155, Feb. 2003.

[8] L. Hetel, J. Daafouz, and C. Jung, “Stabilization of arbitrary switched linear systems with unknown time-varying delays,” IEEE Trans. Autom. Control, vol. 51, no. 10, pp. 1668–1674, Oct. 2006.

[9] P. Naghshtabrizi, J. P. Hespanha, and A. R. Teel, “Exponential stability of impulsive systems with application to uncertain sampled-data systems,” Syst. Control Lett., vol. 57, no. 5, pp. 378–385, May 2008.

[10] Y. S. Suh, “Stability and stabilization of nonuniform sampling systems,” Automatica, vol. 44, no. 12, pp. 3222–3226, Dec. 2008.

[11] T. Chen and B. A. Francis, Optimal Sampled-Data Control Systems. London, U.K.: Springer, 2012.

[12] A. Seuret and F. Gouaisbaut, “Wirtinger-based integral inequality: Application to time-delay systems,” Automatica, vol. 49, no. 9, pp. 2860–2866, Sep. 2013.

[13] H.-B. Zeng, K. L. Teo, and Y. He, “A new looped-functional for stability analysis of sampled-data systems,” Automatica, vol. 82, pp. 328–331, Aug. 2017.

[14] T. H. Lee and J. H. Park, “Stability analysis of sampled-data systems via Free-Matrix-Based time-dependent discontinuous Lyapunov approach,” IEEE Trans. Autom. Control, vol. 62, no. 7, pp. 3653–3657, Jul. 2017.

[15] S. H. Lee, P. Selvaraj, M. J. Park, and O. M. Kwon, “Improved results on H∞ stability analysis of sampled-data systems via looped-functionals and zero equalities,” Appl. Math. Comput., vol. 373, May 2020, Art. no. 125003.

[16] A. Seuret and C. Briat, “Stability analysis of uncertain sampled-data systems with incremental delay using looped-functionals,” Automatica, vol. 55, pp. 274–278, May 2015.
[17] A. Seuret and F. Gouaisbaut, “Complete quadratic Lyapunov functionals using Bessel-Legendre inequality,” in Proc. Eur. Control Conf. (ECC), Jun. 2014, pp. 448–453.

[18] S. Y. Lee, W. I. Lee, and P. Park, “Improved stability criteria for linear systems with interval time-varying delays: Generalized zero equalities approach,” Appl. Math. Comput., vol. 292, pp. 336–348, Jan. 2017.

[19] H. Yang, Y. Jiang, and S. Yin, “Fault-tolerant control of time-delay Markov jump systems with \( \dot{I} \) stochastic process and output disturbance based on sliding mode observer,” IEEE Trans. Ind. Informat., vol. 14, no. 12, pp. 5299–5307, Dec. 2018.

[20] S. Y. Lee, W. I. Lee, and P. Park, “Orthogonal-polynomials-based integral inequality and its applications to systems with additive time-varying delays,” J. Franklin Inst., vol. 355, no. 1, pp. 421–435, Jan. 2018.

[21] A. Seuret, “A novel stability analysis of linear systems under asynchronous samplings,” Automatica, vol. 48, no. 1, pp. 177–182, Jan. 2012.

[22] C. Briat and A. Seuret, “A looped-functional approach for robust stability analysis of linear impulsive systems,” Syst. Control Lett., vol. 61, no. 10, pp. 980–988, Oct. 2012.

[23] S. Y. Lee, W. I. Lee, and P. Park, “Polynomials-based integral inequality for stability analysis of linear systems with time-varying delays,” J. Franklin Inst., vol. 354, no. 4, pp. 2053–2067, Mar. 2017.

[24] C.-Y. Kao, “An IQC approach to robust stability of aperiodic sampled-data systems,” IEEE Trans. Autom. Control, vol. 61, no. 8, pp. 2219–2225, Aug. 2016.

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