Self-induced emergence of consensus in social networks: Reddit and the GameStop short squeeze

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ABSTRACT

The short squeeze of GameStop (GME) shares in mid-January 2021, primarily orchestrated by retail investors of the Reddit r/wallstreetbets community, represents a paramount example of collective coordination action on social media, resulting in large-scale consensus formation and significant market impact. In this work we characterise the structure and time evolution of Reddit conversation data, showing that the occurrence and sentiment of GME-related comments (representing how much users are engaged with GME) increased significantly much before the short squeeze actually took place. We then introduce a model of opinion dynamics where user engagement can trigger a self-reinforcing mechanism leading to the emergence of consensus on the short squeeze operation. We observe a clear phase transition from heterogeneous to homogeneous opinions as engagement grows, with the presence of hubs easing the formation of diffuse consensus. Our results shed light on the increasingly important phenomenon of self-organized collective actions taking place on social networks.

Introduction

Online social media and networks have revolutionized the way we interact with peers, share information and form opinions1, 2, giving rise to new large-scale social phenomena such as the spreading of fake news3, 4, the formation of echo chambers and polarized opinions5, 6, the organization of collective actions — from the Arab Spring7 to climate change protests8. Recently an online mass coordination on r/wallstreetbets (WSB), a community of the social media platform Reddit, was able to trigger a short squeeze of GameStop shares with a large impact on financial markets9.

Reddit is a public discussion website whose users interact with each other by submitting new posts and adding comments to existing posts or comments, thus creating tree-structured conversation threads. Reddit is composed by several independent subreddits, each dedicated to a specific topic. The subreddit WSB is a community where users (retail investors but also non-skilled traders who use no-commission mobile apps such as robinhood.com) discuss high-risk trading strategies and share their gains and losses. The hallmarks of WSB are the irreverent jargon and edgy humor used in conversations10, as well as the gambling attitude of its users who yet seem to give good investment advice11. The popularity of this forum has steadily increased in recent years and has exploded after the events of the GameStop saga.

GameStop (NYSE:GME) is a U.S. video game retailer which was struggling in recent years due to competition from digital distribution services as well as economic effects of the COVID-19 pandemic. As a result GME stock price declined (reaching an all-time low of $2.57 on April 3, 2020), leading many hedge funds to short sell the stock — meaning they would profit from its constant decrease in price. On the contrary WSB users, likely driven by the opportunity to make profit and possibly anger towards institutional investors12, coordinated with the intent to trigger a short squeeze, i.e., a rapid increase in the stock price due to the excess of demand and lack of availability. The resulting large-scale mass coordination (buying and holding GME shares) succeeded in driving up the price of GME, attracting even more users and forcing short sellers to cover their positions at large losses, thus further promoting the price rally. On January 28, 2021 GME shares reached an astounding high price of $483.00; more than 1 million of its shares were deemed failed-to-deliver, which sealed the success of the short squeeze.

Such a highly coordinated financial operation received a huge attention not only from the media and financial stakeholders but from the academic community as well. Following a popular stream of literature aimed at predicting stock market trends using social network mood13, 14 (with a recent focus on WSB and price movements of cryptocurrencies15–17), most of the scholars’ attention is being devoted to understand whether WSB activity, conversation sentiment and user interaction could be
used to predict retail trading activity and GME returns, using linear regression models or machine-learning approaches\textsuperscript{18–24}. Only a few empirical works address the more fundamental question of how coordination or consensus could spontaneously emerge in this context. Boylston et al.\textsuperscript{10} show that the jargon and humor of WSB members is a way to express and reinforce the community’s sense of identity. Semenova and Winkler\textsuperscript{24} find empirical evidence of psychological contagion among WSB users, where an initial set of investors attracts a larger and larger group of excited followers — net of any fundamental price movements. Lucchini et al.\textsuperscript{25} measure the growing user commitment to the GME operation and social identity of WSB participants, describing the key role of a committed minority of users in triggering the collective action. Anand and Patak\textsuperscript{19} go along the same line showing that it was a tiny minority of 462 most influential subredditors whose posts most impacted the GME stock price. Although a proper theoretical framework is still lacking, this empirical evidence suggests that a fundamental understanding of the GME case study requires an endogenous self-reinforcing mechanism able to trigger consensus formation. Such a mechanism can be included in classical models of opinion dynamics\textsuperscript{26–30} through a self-induced global field that drives users towards collective unity\textsuperscript{31}. However existing models that couple peer interaction to community-wide effects typically consider the presence of external fields acting in the same way on all (or groups of) users, describing the effect of conventional media or other exogenous factors\textsuperscript{32–34}. Other approaches incorporate global inertial or noisy effects that make users more stubborn or fickle, respectively\textsuperscript{35,36}. A recent model employs an endogenously-generated field that tends to align the user opinion with the one held more frequently in the past\textsuperscript{37}, in order to mimic the effect of personalised recommendations on opinion formation. Despite the presence of self-induced feedback, even this latter approach is not suitable for modeling opinion formation within the GameStop saga.

In this paper we address the challenge of understanding and modeling how consensus on the GME operation emerged in the WSB community. We analyse discussions on WSB from September 01, 2019 to February 01, 2021 (see Methods), characterising how the forest of tree-like conversation threads grew as the GME saga unfolded. We measure user engagement towards GME through the occurrence and mean sentiment of GME-related conversations. These variables increased significantly far before January (in particular, the frequency of GME in conversations peaks in correspondence of the major events in the GameStop saga) and thus provide early signs of the collective action. We assume that a high and widespread engagement with the GME collective operation increases the likelihood that users themselves become committed and will actively participate to the short squeeze — since its success strongly depends on the number of participants. We model opinion dynamics in this scenario with users forming their opinions either by interacting with peers or by following a global field, which is self-induced by the current status of the community and whose strength is determined by the level of user engagement. Analytical mean-field solution of the model display a phase transition from a disordered state (where no opinion prevails) to full consensus as user engagement grows. Model simulations on statistically validated social networks of WSB users, extracted from their ‘reply-to’ interaction patterns, show that the most connected users play a key role in triggering the emergence of consensus when engagement is low. Notably the transition becomes abrupt when, as data suggests, the community grows together with the level of consensus reached.

Results and Discussion

Reddit conversation patterns. Figure 1A shows the typical structure of a Reddit post with the comment section underneath. Figure 1B highlights how this structure can be translated into a forest of trees: each post corresponds to the root of a tree, while comments to this post or to other comments in the same thread represent the tree branches. Figure 1C shows how these trees can be used to extract a network of user-user ‘reply to’ interactions (which we shall discuss later on). Visual inspection of daily forests, each containing all trees rooted in a post published on the given date, gives a first idea of how the structure of WSB conversation looks like and how it has evolved over time. Figure 1F shows the forest of Jan 21, 2021 — the day before the short squeeze was initiated, while Figure 1G shows the forest of March 19, 2020, when WSB was not as popular and its activity much less intense. We see how the daily forest has grown substantially in terms of overall number of comments as well as number and size of trees. Note in particular how each daily forest is characterized by a giant tree: the Daily Discussion Thread, created with the purpose of summarizing the events of the day and planning future actions\textsuperscript{16}, where users are encouraged to comment by WSB rules. Other very large trees are often present, such as What Are Your Moves Tomorrow, whereas the GME Mega-thread appears in the daily discussions of January. Figure 1D shows the monthly histograms of the number of trees by size, whose power law trends end at large sizes due to deviations produced by such mega-threads. Moreover, the overall number of conversation trees grows in time. The reason is not only an increasing number of WSB users who join the discussion but also their increasing activity in terms of number of contributed posts or comments. This is shown by the monthly histograms of the number of users by number of contributions shown in Figure 1E. Further analyses on conversation trees are reported in the Supplementary Materials S2.

Conversation content and sentiment. We now turn to the analysis of the content of WSB conversations and how it evolves in time. Since WSB is a community of traders, the occurrence of stock tickers in the text of posts and comments represents a
first indicator of what is a popular conversation topic. The total number of occurrences for the various stock tickers follows a power law (Figure S3.1), with some very large outliers — GME in particular is the most frequent one. In order to detect statistically significant occurrences in time we compute their daily Z-scores (see Methods). Figure 2A shows the Z-scores for GME, compared to the average Z-score of all tickers\textsuperscript{38}. We clearly see how the peaks given by significant Z-scores correspond to major events of the GameStop saga: (2020-06-09) GME Q1 earning reports; (2020-09-21) RC Ventures increases its stake in GME to 9.98%; (2020-10-08) GameStop announces a multiyear strategic partnership with Microsoft; (2020-12-08) GME Q3 earning reports, with 257% increase in e-commerce revenues; (2021-01-11) GME announced a new Board of Directors;
Figure 2. Content and sentiment of WSB conversations across the GME saga. In the following plots we do not show daily data for weekends, when activity on WSB is lower as the stock exchange is closed. A) Z-score for the occurrences of ‘GME’ in WSB conversations, compared to the mean Z-score for the occurrence of all stock tickers (shaded area). GME peaks correspond to major events in the GameStop saga. We contextualize these events by also reporting the Z-score of GME trading volumes and the OHLC (average of open-high-low-close) share price of GME. B) For other three representative stocks (AMC, MSFT, PLTR): Z-score of the ticker occurrences in conversations, Z-score of trading volumes and OHLC price. C) Mean sentiment (and standard deviation of the mean) of comments containing ‘GME’, with respect to the same quantity computed on all comments. GME OHLC price is also reported for illustrative purposes.

(2021-01-19) Citron Research predicted that GME’s price would fall and belittled GME buyers on Twitter. Notably these peaks become higher in time, signaling that the community’s interest towards GameStop has grown substantially until January, when GME monopolizes the conversation on WSB. Additionally these peaks mostly coincide with those for the Z-score of GME trading volume (i.e., the number of shares traded daily), pointing to a strong relation between the two variables. A similar but weaker signal can be found regarding conversations about other stocks, as shown in the examples reported in Figure 2B. AMC (AMC Entertainment Holdings Inc) is a penny stock that similarly to GME was suffering due to the COVID-19 pandemic and was then subject to a short squeeze in mid February 2021. This event is not covered by our data, but we can already see a significant signal of AMC occurrences at the end of January. MSFT (Microsoft Corporation) is instead a more solid stock with a constant and regular price growth; in this case we do not observe significant occurrences. At last PLTR (Palantir Technologies Inc) had its public debut at the end of September 2020, yet it is remarkable that a significant interest from WSB users was
present in the previous months. The peak in the second half of November 2020 was due to a new contract of the company with the U.S. Army, a price jump of +170% with Citron Research labeling the stock as a gambling deal.

Besides assessing the content of posts/comments by WSB users we also look at their sentiment. As discussed in the introduction, this variable has been often pointed out as a predictor of market movements. We thus perform text sentiment analysis using VADER (Valence Aware Dictionary and Sentiment Reasoner)\textsuperscript{39}, a python tool that assigns to each piece of text a score between -1 (very negative) and +1 (very positive). In line with other studies\textsuperscript{19,27}, we adapt the VADER dictionary to the peculiar jargon and sarcasm used by WSB members (see Methods and Supplementary Materials S4). Figure 2C shows an intensive sentiment indicator, i.e., the mean sentiment of all daily posts/comments that mention GME. We see that the signal is initially quite noisy due to the low number of GME-related comments until mid-October; Then as early as the beginning of December it starts to grow significantly (both with respect to its previous trend and to the mean sentiment of all comments), far before the short squeeze of January. Overall we can associate these empirical evidences to a growing engagement of users with GME, which in turn represents an early sign of consensus formation in the community concerning the short squeeze operation. In light of these results, we now work out a model in which user engagement with a collective cause can influence opinion dynamics and foster the emergence of consensus or cooperation thanks to a self-induced feedback mechanism.

**Voter model with self-induced global feedback.** We build on one of the most popular theoretical frameworks of opinion dynamics: the voter model\textsuperscript{40,41}. In the standard voter dynamics, \( N \) users are placed on the nodes of a network and are endowed with a binary opinion \( s \in \{-1, +1\} \). Starting at \( t = 0 \) from an initially disordered configuration where each user \( i \in N \) has opinion \( s_i(0) = \pm 1 \) with equal probability, at each time step \( \delta t = \frac{1}{N} \) a user is chosen at random and copies the opinion of one of its neighbors. The magnetization or order parameter \( \text{m}(t) = \frac{1}{N} \sum s_i(t) \) represents the average opinion at time \( t \), or equivalently the level of consensus reached, with \( \text{m}(t) \approx 0 \) and \( \text{m}(t) = \pm 1 \) indicating no consensus and full consensus, respectively. The standard model has been studied extensively on different population structures and has been adapted to a variety of different situations\textsuperscript{27,42}. We are interested in a model formulation where, depending on how much users are engaged with a collective cause, they are more keen on assuming a given opinion if that opinion is popular within the community. Mathematically speaking, the model should include a tunable self-induced field acting on all users simultaneously. Following an approach formally similar to\textsuperscript{37}, we define the update rule as:

\[
\begin{cases} \left. s_i(t + \delta t) \right|_{s_i(0) = \pm 1} = \begin{align*} s_j(t) & \quad \text{with probability } \frac{1 - \lambda}{k_i} \\ e(t) & \quad \text{with probability } \lambda \end{align*} \end{cases}
\]

This expression, visually represented in Figure 3A, has the following meaning. When user \( i \) is selected for the update, with probability \( 1 - \lambda \) she copies the state of a random neighbor \( j \) (i.e., each neighbor is selected with probability \( \frac{1}{k_i} \), where \( k_i \) is the degree or number of neighbors of \( i \)). Instead, with probability \( \lambda \) she follows a global field given by the random variable \( e(t) \pm 1 \). In order to have a self-induced field depending on the current level of consensus, we pose that the probability of \( e(t) = +1 \) is

\[
P_1[e(t)] = \frac{e^m(t)}{1 + e^m(t)}
\]

When there is no consensus at all (i.e., \( m(t) = 0 \)) we have \( P_1[e(t)] = \frac{1}{2} \): the global field acts randomly on each user and is equivalent to a white noise term. Instead \( m(t) \rightarrow +1 \) leads to \( P_1[e(t)] \rightarrow 1 \) and analogously \( m(t) \rightarrow -1 \) to \( P_1[e(t)] \rightarrow 0 \): the global field is increasingly able to align users with the majority opinion. \( c \geq 1 \) is a control parameter that we associate with the level of user engagement: the higher the value of \( c \), the less consensus is required for users to align with \( m(t) \).

We can understand the behavior of the model for different values of \( c \) through its analytical mean-field solution (see Methods). Figure 3B shows sample realizations of the stochastic temporal dynamics of the magnetization (for a fixed value of \( \lambda = 0.1 \)), while Figure 3C shows the drift term \( \nu(m) \) rescaled by \( \lambda > 0 \) as a function of \( m \). For \( c = 1 \) we have \( P_1[e(t)] = \frac{1}{2} \): the global field is always white noise that keeps the system in the initial disordered configuration, as in the noisy voter model\textsuperscript{43,44}. This is due to the drift and magnetization having always opposite sign: the process is mean-reverting and the only equilibrium point is \( m^* = 0 \). Such equilibrium remains stable also when \( c > 1 \), though the drift towards it becomes less intense. At the singular point \( c = e^2 \) we have \( P_1[e(t)] = \frac{1}{2} [1 + \tanh m(t)] \) and the drift vanishes in the region around \( m = 0 \): the initial stochastic dynamics becomes purely diffusive, as in the standard voter model\textsuperscript{27}. Finally for \( c \) above this threshold the drift pushes the system away from \( m = 0 \) with a speed that grows with \( \lambda \): the dynamics quickly reaches a new stable equilibrium point that becomes closer to full consensus as \( c \) grows. Looking at the stable states \( m^* \) of the dynamics as a function of \( c \) (inset of Figure 3C) we see that the system exhibits an explicit second order phase transition from disorder to order. These results are confirmed by numerical simulations of the model on Erdős-Rényi random graphs (Figure 3D). Only for very small values of \( \lambda < 0.1 \), for which the interaction between peers is largely dominant, network effects make the transition less sharp. Furthermore, in this region the dynamics is very slow so the system keeps memory of its initial configuration for a long time. This produces
Figure 3. Voter model with self-induced global feedback. A) Schematic representation of the update rule of the model. At each time step, a user takes on the opinion of either a randomly chosen neighbor (with probability $1 - \lambda$) or is influenced (according to a control parameter $c$) by the current level of consensus in the community, namely the magnetization $m$. B) Sample stochastic realizations of the model dynamics: temporal evolution of the magnetization $m$ for different values of the parameter $c$ setting the strength of the global feedback, for $\lambda = 0.1$. C) Re-scaled drift term $v(m)$ of the model dynamics (according to the mean-field approximation, for $\lambda \neq 0$) as a function of the magnetization of the system. Inset: stable equilibrium points $|m^*|$ as a function of $c$. D) Phase diagram of the model simulated on Erdős-Rényi random graphs of $N = 10000$ nodes and average degree $\langle k \rangle = 20$.

finite-time hysteresis loops that slow down both the emergence of consensus from a disordered configuration and its dissolution from an ordered one (see Supplementary Materials S6).

Consensus on the WSB user network. We now study how the model behaves on user-user interaction networks extracted from WSB conversation data. We build a network for each month by placing a directed link between two users $i$ and $j$ weighted by the number of times $i$ commented on $j$’s posts/comments during that period (Figure 1C). We then extract the most significant connections using the disparity filter $^{45}$ with significance level $\alpha = 0.1$ (see Methods). We focus on the four months preceding the GME short squeeze, from October 2020 to January 2021. Due to the explosion of activity in WSB, the network in January has many more nodes than those in the previous months; however, the application of the disparity filter makes their density of connections comparable (see Table 1). In particular, all four networks display a power law distribution of the connectivity (Figure 4A), with some deviations caused by many super-hubs appearing in January. Model simulations on these networks reported in Figure 4B (see the Supplementary Materials S7 for further details) show that the degree heterogeneity of real user
Figure 4. WSB user-user network and emergence of consensus. A) Degree distributions of the monthly user interaction (‘reply-to’) networks, statistically validated using the disparity filter. B) Phase transition of the magnetization, obtained by simulating the model (for $\lambda = 0.1$) on the monthly networks, as compared to the transition observed on Erdős-Rényi graphs (black line). C) Number of daily active users (who contributed at least one post/comment) and daily OHLC price of GME shares. D) Phase transition of the extensive order parameter of the model (total magnetization for a community that grows as an exponential of $m$) according to the mean-field solution and to numerical simulations on the January user network.

interactions leads to the emergence a non-negligible level of consensus also for very small values of user engagement $c$. This vanishing of the transition point is reminiscent of what occurs for other processes on scale-free networks, such as epidemic spreading\textsuperscript{46} and coordination games\textsuperscript{47}. Notably, the curves for the networks of October, November and December collapse onto each other, as opposed to that of January which is smoother due to the presence of more numerous and more connected super-hubs that ease the formation of an initial consensus. However magnetization alone does not allow for an appropriate comparison between networks of different sizes, since it represents the average opinion and is therefore an intensive variable. Equally important is the extent of consensus in terms of number of users. Indeed the success of the short squeeze required a large number of investors who bought and held GME shares. Another factor to take into account is the steep growth of WSB users number in correspondence with the short squeeze, as shown in Figure 4C. All together these observations suggest to consider an extensive order parameter, namely the total sum of opinions within a population that grows with the level of consensus reached: $M^* = m^* N_0 e^{\theta|m^*|}$ (see Methods). As shown in Figure 4D this extensive magnetization features an abrupt transition, properly describing a sudden and large-scale formation of consensus. For a user engagement level $c$ that grows linearly in time (see Figure 2C), this transition is qualitatively similar to the sharp surge of GME price (see Figure 4C), which ultimately represents the best proxy for the success of the short squeeze.

Conclusion

The empirical and theoretical results presented in this work can be useful to better understand the dynamics of consensus formation and collective actions on social networks. These phenomena have become increasingly relevant in recent years and have entered the financial domain with the GME case. This event is unlikely to remain isolated, particularly in the current financial context which sees the growing influence of retail and non-professional investors due to the emergence of
commission-free trading and leverage platforms. While the ethical aspects of the “democratization of trading and investing” and the “David vs Goliath” contrast between small investors versus hedge funds can be widely debated, their effects on market quality are certainly tangible.

An inherent limitation of our empirical analyses is that we have focused on a single unprecedented financial mass action. Although we have briefly shown some similar case studies as well as counterexamples, a more in-depth analysis of several (possibly future) events of the same type can help corroborate or falsify our findings. From the theoretical viewpoint, it would be interesting to study the effect of a self-induced global field added to other popular models of opinion dynamics, such as the majority-vote model or the threshold model. At last there is the very practical question of understanding how the dynamics of mass coordination reflect quantitatively on financial markets movements. All these issues certainly represent interesting directions for future research.

Methods

**Dataset.** We retrieved Reddit conversation data from Pushshift, an API that regularly copies activity data of Reddit and other social networks. We queried the service to retrieve information about WSB posts and comments (summarized in Table S1.1) from September 01, 2019 to February 01, 2021. Note that our dataset covers the days following the short squeeze (25, 26, 27 January 2021) which were released by the service only on August 31, 2021. Overall our data contains 22 099 235 comments and 865 597 posts. The dataset was cleaned by removing posts/comments by Reddit bots (Table S1.2) as well as by "[deleted]" users (i.e., users who deleted their account before Pushshift could acquire their contributions). This latter operation was performed only for the analyses that required a unique userID (i.e., user activity statistics and user-user interaction networks), but not for those analyses that considered each post or comment on its own (i.e., tree statistics, ticker occurrences and sentiment). Data on stock price and traded volumes (Table S1.3) for GME and other tickers were retrieved from the API service of polygon.io.

**Ticker occurrences and Z-scores.** To measure the popularity of a given stock in WSB conversations we computed \( x_s(t) \), the count of how many times the ticker symbol of the stock \( s \) (e.g., ‘GME’ for GameStop) appears as a regular expression in the raw text of posts/comments of day \( t \). The mean \( \mu_s(t) = \frac{1}{t} \sum_{t'=1}^{t} x_s(t') \) and variance \( \sigma_s^2(t) = \frac{1}{t} \sum_{t'=1}^{t} [x_s(t') - \mu_s(t)]^2 \) of the time series \( x_s(t) \) (starting from March 01, 2020) are used to obtain the Z-score \( Z_s(t) = (x_s(t) - \mu_s(t))/\sigma_s(t) \). We applied a 5-day moving average to these values in order to obtain a less noisy signal. The baseline \( \bar{Z}(t) \) is the average Z-score of all stocks on day \( t \) (computed over tickers with a symbol of at least three characters and appearing more than 10 times over the whole time interval).

**Sentiment analysis and VADER lexicon.** VADER (Valence Aware Dictionary and sEntiment Reasoner) is an algorithm that assigns a piece of text with a compound score between −1 (very negative) and +1 (very positive). VADER is sensitive to both the polarity and intensity of the text, taking into account punctuation and word shape (ALL CAPS) used to add emphasis, degree modifiers that alter intensity (boosters such as "very" and dampeners such as "kind of"), slang and acronyms. VADER is based on a lexicon of words and emojis, each with an associated score ranging from -4 to +4 according to its meaning (from negative to positive). We adapted VADER to the typical jargon and sarcasm of WSB users by adding to its lexicon the words reported in Table S4.1.

**Voter model with self-induced global field.** Analytic mean-field solution of the model (full calculations in the Supporting Materials S5) leads to the following stochastic differential equation for the evolution of the magnetization of the system:

\[
\frac{dm}{dt} = v(m) + \sqrt{D(m)}dW
\]

where \( W \) is the standard Wiener process while the drift and diffusion coefficients are

\[
v(m) = \lambda \left[ f_e(m) - m \right]
\]

\[
D(m) = \frac{1}{N} \left\{ (1 - \lambda)(1 - m^2) + \lambda [1 - mf_e(m)] \right\}
\]

with \( f_e(m) = 2P_1(e) - 1 = \frac{e^{m-1}}{1 + e^{m-1}} \). The formal solution for \( m \simeq 0 \) is

\[
m_t = \frac{e^{-\lambda(1-\ln \sqrt{\tau})t}}{\sqrt{2N\lambda(1-\ln \sqrt{c})}} W_{2^\lambda\ln(1-\ln \sqrt{c})} - 1
\]

For \( c < e^2 \) the sign of the drift coefficient is always opposite to the sign of \( m \) and the only equilibrium point is \( m = 0 \). For \( c \geq e^2 \) zero is no longer a stable point, for as soon as \( m \neq 0 \) the drift pushes the system towards a new equilibrium. \( \lambda \) sets
the speed at which the new stationary state is reached: the larger $\lambda$ the stronger the drift so the quicker the system will reach equilibrium. The critical value $c = e^2$ corresponds to a purely diffusive process driven by $D(m)$, which is of order $O(1/N)$ and thus can always be neglected except at the critical point. $D(m)$ is what drives the system out of the initial equilibrium state $m = 0$, but as soon as $m \neq 0$ the drift kicks in, governing the evolution of $m$.

Model simulations always start from an initial disordered configuration where each user is randomly assigned opinion $s = \pm 1$ with equal probability, such that $m(0) \approx 0$. The dynamics is run until the magnetization reaches the stationary value $m^*$. Values shown in plots are averaged over 1000 independent runs. The extensive magnetization $M^* = m^* N$ is defined using a population size $N$ that grows exponentially with $|m^* d|$, from a baseline level $N_0$. Parameters used for $N = N_0 e^{q|m^*|}$ are $N_0 = 10000$ (roughly the number of active users before January, see Figure 3C) and $q = 6$, in order to have $N \approx 200000$ when $|m^*| \approx 0.5$.

**User network construction.** For each month we reconstruct the network of social interactions by considering only posts and comments contributed during that month. Each user who contributed at least one of these posts/comments is represented as a node; each weighted directed link $w_{ij}$ represents the number of times user $i$ commented on posts/comments by user $j$. In order to filter out the less informative links and keep only those that are more likely to represent a significant interaction, we firstly removed all users who commented just once and then extracted the network backbone through the disparity filter$^{45}$. This algorithm assesses the statistical significance of links with respect to a null model where the weights of the links originating from a node are produced by a random assignment from a uniform distribution. Specifically, a link is deemed statistically significant if it satisfies $\alpha_{ij} = 1 - (k_i - 1) \int_0^{w_{ij}/s_i} (1 - x)^{k_i - 2} dx < \alpha$, where $\alpha$ is the significance level and $k_i$ and $s_i = \sum_j w_{ij}$ respectively the degree and strength of node $i$. The statistically validated network is then a binary undirected network made up of those links which at least in one direction satisfy the former condition (in the case where a node $i$ with $k_i = 1$ is connected to a node $j$ with $k_j > 1$, the link is kept only if node $j$ satisfies the criterion). Note that in the case of a directed network the incoming and outgoing links associated with a node must be considered separately. At last, in our simulations we considered only the largest connected component of the network (see Table 1).

| Month   | $N_{tot}$ | $E_{tot}$ | $N_b$ | $E_b$  | $N_g$ | $E_g$ | $\langle k \rangle$ | $\langle k^2 \rangle / \langle k \rangle$ | $\gamma$ |
|---------|-----------|-----------|-------|--------|-------|-------|-----------------|---------------------------------|------|
| October | 35850     | 56426     | 4235  | 11212  | 3542  | 8524  | 4.8                  | 23.9                             | 1.82 |
| November| 47536     | 761374    | 5765  | 14888  | 4730  | 11307 | 4.7                  | 27.5                             | 2.12 |
| December| 57822     | 913388    | 6675  | 18057  | 5474  | 13689 | 4.9                  | 45.6                             | 2.07 |
| January | 357039    | 3359500   | 17740 | 38365  | 12232 | 29504 | 4.7                  | 231.1                            | 2.30 |

Table 1. Size of the monthly user-user interaction networks. The subscript _tot_ stands for the unfiltered data, _b_ for the backbone extracted with the disparity filter, _g_ for the largest connected component of the graph. Network statistics reported for these latter networks are the average degree $\langle k \rangle$, the average excess degree $\langle k^2 \rangle / \langle k \rangle$ and the slope $\gamma \approx -\ln P(k)/\ln k$.

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Supplementary Materials

S1 Dataset informations

| Column     | Description                                                                 |
|------------|-----------------------------------------------------------------------------|
| Author     | Username                                                                    |
| Author ID  | ID that uniquely identifies each Reddit user                                |
| Comment ID | ID that uniquely identifies each comment                                    |
| Submission ID | ID of the post under which the comment was made                             |
| Parent ID  | ID of the post or ID of the comment to which the given comment is a reply  |
| Text       | Text of the comment                                                         |
| UTC        | Epoch Unix timestamp of the comment                                         |

**Table S1.1.** Metadata downloaded from Pushshift for each Reddit comment.

| Bot name                                      |
|-----------------------------------------------|
| WSBVoteBot                                    |
| RemindMeBot                                   |
| GenericRedditBot                              |
| ReverseCaptioningBot                         |
| LimbRetrievalBot                             |
| NoGoogleAMPBot                                |
| RepostSleuthBot                               |
| GetVideoBot                                   |
| CouldWouldShouldBot                           |

**Table S1.2.** Reddit bots removed from the database.

| Column     | Description                                                  |
|------------|--------------------------------------------------------------|
| Time       | Day timestamp of Stock Index                                 |
| Open       | Daily Opening Value of Stock Index                           |
| Close      | Daily Closing Value of Stock Index                           |
| High       | Daily High Value of Stock Index                              |
| Low        | Daily Low Value of Stock Index                               |
| Volume     | Daily Volume of Stock Transactions                           |

**Table S1.3.** Metadata downloaded from polygon.io for each stock ticker.

S2 Tree structures: lifetime and ramification

To study how long a WSB conversation thread may last, we consider for each tree how many comments are written on each day from the creation of the root post onwards. As shown in the left panel of Figure S2.1, most trees have a lifetime of 1 day, due to Reddit giving more relevance and visibility to newly written posts rather than older ones. Yet in some cases a small trail of comments can last even many days after the creation of the post. Another interesting tree feature to look at is the percentage of leaf comments (*i.e.*, comments without a further reply), which is inversely related to the height of the tree (*i.e.*, the maximal length of a branch). As shown in the right left panel of Figure S2.1, the number of leaves increases substantially in January, pointing to a less-structured conversation in this period. This pattern can be explained by users commenting mostly to cheer on each other during the short squeeze, contributing to a collective discussion rather than establishing structured one-to-one conversations**25,55**.
Figure S2.1. Left panel: fraction of comments written on each day from the creation of the root post onwards, averaged over all trees generated in a given month. Right panel: time series for the fraction of leaf comments, averaged over all trees of a given day.

S3 Ticker occurrences

Figure S3.1. Histogram of the total number of occurrences of the various stock tickers in the text of WSB posts and comments. GME (in yellow) is the most frequent ticker. The black line has slope -1.23.
S4 Modified VADER lexicon

As pointed out by other studies before ours, “VADER tends to overpredict neutral sentiment for WSB conversations, especially for posts labeled as ‘positive’ by human annotators. We speculate that this is due to a high proportion of out-of-vocabulary words [...] which VADER generally scores as neutral, as well as in-vocabulary words with WSB-specific senses that convey different sentiment polarity or intensity than they would in a generic social media context.”21. A possible solution is to manually assign weights to several idiosyncratic slang-origin terms popular on WSB.19. We followed this route by adding to the VADER lexicon a new group of words reported in Table S4.1 together with their associated scores.

| Word         | Score | Emoji |
|--------------|-------|-------|
| rocket       | 4.0   | yes   |
| moon(ing)    | 4.0   | no    |
| diamond      | 4.0   | no    |
| gem stone    | 4.0   | yes   |
| hold(ing)    | 4.0   | no    |
| tendies      | 4.0   | no    |
| yolo         | 4.0   | no    |
| retard(s-ed) | 2.0   | no    |
| autist(s)    | 2.0   | no    |
| degenerate(s)| 2.0   | no    |
| ape(s)       | 2.0   | no    |
| gorilla(s)   | 2.0   | yes   |
| bear(s)      | -2.0  | no    |
| paper        | -4.0  | no    |

Table S4.1. Words (with possible suffixes) and corresponding emojis added to the VADER lexicon.

S5 Mean-field solution of the Voter model with self-induced field

We consider \( N \) users distributed over the nodes \( i \) of a network. Each user can assume two states \( s_i = \pm 1 \), that correspond to two different opinions (such as \emph{join the short squeeze} or \emph{do nothing}). The dynamics takes place as follows. Initially each opinion is set to \( s_i = \pm 1 \) with equal probability. At each time step \( t \), a given individual \( i \) is selected at random and

- with probability \( 1 - \lambda \), she follows the usual voter dynamics and copies the opinion \( s_j(t) \) of a randomly selected neighbor \( j \) (out of her \( k_i \) neighbors);
- with complementary probability \( \lambda \), she follows a global field that is self-induced by the global state of the community (see below), and takes the opinion \( e(t) = \pm 1 \) assuming positive value with a probability \( P_1[e(t)] \).

In formula:

\[
 s_i(t + \delta t) = \begin{cases} 
 e_i(t) \text{ with probability } \lambda \\
 s_j(t) \text{ with probability } \frac{1 - \lambda}{k_i}, 
\end{cases}
\]  

(S5.1)

where \( \delta t = 1/N \) and \( j \) is one of the neighbors of \( i \). We assume that

\[
P_1[e(t)] = \frac{c m(t)}{1 + c m(t)}.
\]

(S5.2)

where \( m(t) = \frac{1}{N} \sum_i s_i(t) \) is the magnetization (i.e., the average opinion) and \( c \geq 1 \) is the control parameter that sets how easily users tend to align with \( m(t) \) (Figure S5.1, upper left panel). Indeed for \( c = 1 \) we get \( P_1[e(t)] = P_{-1}[e(t)] = 1/2 \): the global field is pure noise and the model is equivalent to the noisy voter model. Instead when \( c > 1 \), \( P_1[e(t)] \) is larger than 1/2 for \( m > 0 \) and smaller for \( m < 0 \) (and quickly converges to \( \pm 1 \) for \( c \gg 1 \), respectively).

The evolution of a system following eq. (S5.1) depends on the topology of the network defining the interactions among users, since the probability of being in a particular state depends on the state of neighboring nodes. We can however study the
Figure S5.1. Upper left panel: $P_1[e(t)]$ as a function of $m$ for different values of $c$. Upper right panel: drift coefficient $v(m)$ rescaled by $\lambda$ as a function of $m$ for different values of $c$. Lower panels: diffusion coefficient $D(m)$ as a function of $m$ for different values of $c$ and $\lambda$.

system in the mean field approximation, which assumes that neighboring states are independent. In this case the conditional probability of finding a neighbor in a particular state given the state of the selected node can be approximated by the fraction of users in that state out of the entire population. In practice this is equivalent to considering a complete graph structure.

We denote by $N_\uparrow$ the number of users in state +1, while $N_\downarrow = N - N_\uparrow$ is the number of users in the opposite state:

$$N_\uparrow(t) = \sum_i \left( \frac{1 + s_i(t)}{2} \right) = \frac{N}{2} \left[ 1 + m(t) \right]$$

$$N_\downarrow(t) = \sum_i \left( \frac{1 - s_i(t)}{2} \right) = \frac{N}{2} \left[ 1 - m(t) \right]$$

(S5.3)
Then the updating rule of eq. (S5.1) becomes

\[
    s_i(t + \delta t) = \begin{cases} 
        e(t) & \text{with probability} \quad \lambda \\
        +1 & \text{with probability} \quad (1 - \lambda) \frac{N_i(t)}{N} \\
        -1 & \text{with probability} \quad (1 - \lambda) \frac{N_{\overline{i}}(t)}{N} 
    \end{cases} 
\]  

(S5.4)

\[
    \text{hence each time node } i \text{ is selected her opinion evolves according to}
\]

\[
    s_i(t) \rightarrow s_i(t + \delta t) = \begin{cases} 
        +1 & \text{with probability} \quad (1 - \lambda) \left(1 + \frac{m(t)}{2}\right) + \lambda P_1[e(t)] \\
        -1 & \text{with probability} \quad (1 - \lambda) \left(1 - \frac{m(t)}{2}\right) + \lambda \{1 - P_1[e(t)]\} 
    \end{cases} 
\]  

(S5.5)

We now drop the explicit dependence of quantities on \( t \). The probability of a spin-flip of a single user \( i \) is given by

\[
    R(s_i = -1 \rightarrow s_i = +1) = \frac{1}{N} \left(1 - \frac{s_i}{2}\right) \left(1 - \lambda\right) \left(1 + \frac{m}{2}\right) + \lambda P_1[e] \\
    L(s_i = +1 \rightarrow s_i = -1) = \frac{1}{N} \left(1 + \frac{s_i}{2}\right) \left(1 - \lambda\right) \left(1 - \frac{m}{2}\right) + \lambda \{1 - P_1[e]\} 
\]  

(S5.6)

where the prefactor stems from the fact that the \( i \)th spin is selected with probability \( 1/N \). Summing the probabilities over all users we get the transition rates for the magnetization:

\[
    R(m) = \left(1 - \lambda\right) \left(1 - m^2\right) + \lambda \left(1 + m\right) P_1(e) \\
    L(m) = \left(1 - \lambda\right) \left(1 + m^2\right) + \lambda \left(1 - m\right) \{1 - P_1(e)\} 
\]  

(S5.7)

In the thermodynamic limit \( N \rightarrow \infty \) the probability density \( P(m,t) \) of a voter model dynamics evolves according to a diffusion process described by the Fokker-Plank equation\(^9\), whose drift and diffusion coefficients are

\[
    v(m) = \frac{\delta m}{\delta t} \left[R(m) - L(m)\right] \\
    D(m) = \frac{\delta m^2}{2\delta t} \left[R(m) + L(m)\right] 
\]  

(S5.8)

Considering that a single update occurs in a time \( \delta t = 1/N \) and the variation of \( m \) in a time step is equal to \( \delta m = 2/N \), we can substitute Equations (S5.7) in Equations (S5.8) and obtain

\[
    v(m) = \lambda \left[f_c(m) - m\right] \\
    D(m) = \frac{1}{N} \left((1 - \lambda)(1 - m^2) + \lambda \left[1 - m f_c(m)\right]\right) 
\]  

(S5.9)

where

\[
    f_c(m) = 2P_1(e) - 1 = \frac{c^m - 1}{c^m + 1} 
\]  

(S5.10)

If \( P(m,t) \) follows a Fokker-Plank equation then the corresponding value of \( m \) evolves according to a stochastic differential equation of the form\(^7\)

\[
    dm = v(m)dt + \sqrt{D(m)}dW 
\]  

(S5.11)

where \( dW \) is the standard Wiener process. Concerning the drift coefficient (Figure S5.1, upper right panel) we have \( v(m) = 0 \) for \( \lambda = 0 \); otherwise it scales linearly with \( \lambda \). For \( c = 1 \) the drift has sign opposed to \( m \), hence the system is always driven towards the stable point \( m = 0 \). However for growing \( c \) the drift decreases, and after the threshold value \( c^* \) the point \( m = 0 \) becomes unstable and a stable point \( |m|^* > 0 \) appears. Instead the diffusion coefficient (Figure S5.1, bottom panels) is always \( O(1/N) \) when \( m \approx 0 \), and can be neglected otherwise unless \( c \) and \( \lambda \) are both close to 1. The drift term is however responsible for the deviation from the initial equilibrium state \( m = 0 \).
We can study the system around \( m \approx 0 \) (i.e., the initial configuration) using the first order approximation \( f_c(m) \approx m \ln \sqrt{c} \). In this case the stochastic differential equation for \( m \) becomes

\[
\begin{align*}
\frac{dm(t)}{dt} &\approx -\lambda(1 - \ln \sqrt{c})m + \sqrt{\frac{1 - m^2}{N}[1 - \lambda(1 - \ln \sqrt{c})]} dW \\
&\approx -\lambda(1 - \ln \sqrt{c})m + \left(1 - \frac{m^2}{2}[1 - \lambda(1 - \ln \sqrt{c})]\right) \frac{dW}{\sqrt{N}}
\end{align*}
\] (S5.12)

and we can see how the drift term changes sign for \( c = e^2 \). By discarding terms of order \( O(m^2) \) we are left with

\[
\frac{dm(t)}{dt} \approx -\lambda m(1 - \ln \sqrt{c}) dt + \frac{dW}{\sqrt{N}}
\] (S5.13)

which represents an Ornstein-Uhlenbeck process whose formal solution, given \( m(0) = 0 \), is

\[
m_t = \frac{e^{-\lambda(1 - \ln \sqrt{c})t}}{\sqrt{2N\lambda(1 - \ln \sqrt{c})}} W_{e^{2\lambda(1 - \ln \sqrt{c})t-1}}
\] (S5.14)

We can see now how the critical value \( c = e^2 \) corresponds to a pure Weiner process (the traditional purely diffusive voter dynamics) that characterizes the transition between the disordered and ordered phases.

**S6 Hysteresis as a finite time phenomenon**

Here we study the model dynamics at finite time steps, i.e., before the system may reach its equilibrium state. This is done by simulating the model for \( T \) time steps at a given value of \( c \); the final state of the system is then used as the starting configuration for a simulation with a different value of \( c \), and so forth. We change \( c \) in steps of 0.1, both for increasing (forward) and decreasing (backward) order. Figure S6.1 reveals the presence of a hysteresis region in the phase transition, which becomes broader for small \( \lambda \), meaning that in this region the system keeps a long memory of its previous state. This phenomenon is relevant for systems that are inherently out-of-equilibrium, like the process of opinion formation on a social network. Hysteresis yet disappears if \( T \) is large enough to allow the system to reach its equilibrium state \( |m^*| \).

**S7 Model simulations for varying \( \lambda \)**

As explained in the main text, the value of \( \lambda \) sets how important is the local peer interaction with respect to the global action of the self-induced field. In the case of Erdős-Rényi networks (Figure 3D), for high values of \( \lambda > 0.1 \) we observe that the mean-field solution properly describes the critical behavior of the system regardless of the topology of the underlying network; instead for low values \( \lambda < 0.1 \) there are consistent deviations due to the structure of the network. The same observations hold for model simulations on empirical user-user networks, we report in Figure S7.1 the cases \( \lambda = 0.05 \) (left panel) and \( \lambda = 0.3 \) (right panel), while the plot of Figure 4B in the main text refers to \( \lambda = 0.1 \).
Figure S6.1. Magnetization $|m^*(T)|$ averaged over 1000 runs of model simulations on Erdős-Rényi networks with $N = 10000$ nodes and average degree $\langle k \rangle = 20$, with a finite time horizon $T$.

Figure S7.1. Phase transition of the magnetization, for $\lambda = 0.05$ (left panel) and $\lambda = 0.3$ (right panel) obtained by simulating the model on the monthly networks, as compared to the transition observed on Erdős-Rényi graphs (black line).