Some heavy vector and tensor meson decay constants
in light-front quark model

Chao-Qiang Geng\textsuperscript{1,2,a}, Chong-Chung Lih\textsuperscript{4,b} and Chuanhui Xia\textsuperscript{1c}

\textsuperscript{1}College of Materials Science and Engineering, Chongqing Jiaotong University, Chongqing, 400074, China
\textsuperscript{2}Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 300
\textsuperscript{3}Physics Division, National Center for Theoretical Sciences, Hsinchu, Taiwan 300
\textsuperscript{4}Department of Optometry, Shu-Zen College of Medicine and Management, Kaohsiung Hsien, Taiwan 452

(Dated: June 1, 2016)

Abstract

We study the decay constants ($f_M$) of the heavy vector ($D_\ast^+, D_\ast^{s}, B_\ast^+, B_\ast^{s}$) and tensor ($D_2^\ast, D_s^\ast, B_2^\ast, B_s^\ast$) mesons in the light front quark model. With the known pseudoscalar meson decay constants of $f_{D}, f_{D_s}, f_{B}, f_{B_s}$, and $f_{B_c}$ as the input parameters to determine the light-front meson wave functions, we obtain that $f_{D_\ast^+, D_\ast^{s}, B_\ast^+, B_\ast^{s}} = (252.0^{+13.8}_{-11.6}, 318.3^{+15.3}_{-12.6}, 201.9^{+43.2}_{-41.4}, 244.2 \pm 7.0, 473.4 \pm 18.2)$ and $(264.9^{+10.2}_{-9.5}, 330.9^{+9.9}_{-9.0}, 220.2^{+49.1}_{-46.2}, 265.7 \pm 8.0, 487.6 \pm 19.2)$ MeV with Gaussian and power-law wave functions, respectively, while $f_{D_2^\ast, D_s^\ast, B_2^\ast, B_s^\ast} = (143.6^{+24.9}_{-21.8}, 209.5^{+29.1}_{-24.2}, 80.9^{+33.8}_{-27.7}, 109.7^{+15.7}_{-15.0})$ MeV with only Gaussian wave functions.

\textsuperscript{a} geng@phys.nthu.edu.tw
\textsuperscript{b} cclih@phys.nthu.edu.tw
\textsuperscript{c} chxia@cqjtu.edu.cn
I. INTRODUCTION

Meson decay constants contain useful information on the nonperturbative behavior of QCD between quarks and antiquarks inside mesons. In addition, the determinations of these helpful parameters can also be used to constrain the CKM mixing matrix elements in weak mesonic decays. In recent years, many heavy vector and tensor mesons have been experimentally discovered, such as the excited states of the charmed mesons \([1]\), observed by Babar, Belle, CLEO, FOCUS and LHCb Collaborations. Moreover, D0 \([2]\) and CDF \([3]\) Collaborations have confirmed the bottom states of \(B_1(5721)\), \(B_2(5747)\), \(B_{s1}(5830)\) and \(B_{s2}^*(5840)\). In some of these hadron states, the quantum numbers are \(I(J^P) = \frac{1}{2}(2^+)\). The investigations of these particles are clearly important in hadron physics both theoretically and experimentally. The recent experimental results on the parameters of these mesons would help us to understand the meson properties and the non-perturbative dynamics as well as the vacuum structure of QCD.

In the literature, the decay constants of heavy vector and tensor mesons are somewhat less discussed. In particular, compared to the scalar and pseudoscalar mesons, there are few theoretical works devoted to the analysis of the properties for the tensor mesons. The main purpose of this work is to examine the vector and tensor mesons decay constants simultaneously within the framework of the light-front quark model (LFQM), which has been widely used in the phenomenological study of meson physics. The LFQM is a good way for solving the nonperturbative problems of hadron physics and provides inside information about the internal structure of the bound state. The meson decay constant can be described by a two-point function and regarded as one of the simplest physical observable in the LFQM. This framework has been applied successfully to explain various properties of pseudoscalar and vector mesons \([4]\).

The paper is organized as follows. In Sec. II, we present the basic formalism of the LFQM. In Sec. III, we show our numerical results on the decay constants in the LFQM. Our conclusions are given in Sec. IV.
II. FORMALISM

In the LFQM, a neutral meson wave function is constructed in terms of its constituent quark $q$ and anti-quark $\bar{Q}$ with the total momentum $p$ and spin $S$ as \[5\],

$$ |M(p, S, S_z)\rangle = \int [dk_1][dk_2]2(2\pi)^3\delta^3(p - k_1 - k_2) \times \sum_{\lambda_1\lambda_2} \Phi_M(k_1, k_2, \lambda_1, \lambda_2)b^+_q(k_1, \lambda_1)d^+_{\bar{Q}}(k_2, \lambda_2)|0\rangle, \quad (1) $$

where

$$ [dk] = \frac{dk^+d^2k_\perp}{2(2\pi)^3}, \quad (2) $$

$M$ represents for a $P$ (pseudoscalar) or $V$ (vector) or $T$ (tensor) meson, $\Phi_M$ is the wave function of the corresponding $q\bar{Q}$ and $k_{1(2)}$ ($\lambda_{1(2)}$) is the on-mass shell LF momentum (helicity) of the internal quark. In the momentum space, $\Phi_M$ can be expressed as a covariant form \[6, 7\]

$$ \Phi_M(x, k_\perp) = \left(\frac{k_1^+k_2^+}{2[M_0^2 - (m_q - m_{\bar{Q}})^2]}\right)^{1/2} \mathfrak{F}(k_1, \lambda_1) \Gamma v(k_2, \lambda_2) \phi_M(x, k_\perp), \quad (3) $$

where $\phi_M(x, k_\perp)$ describes the momentum distribution amplitude of the bound state for the $S$ or $P$-wave meson, $(x, k_\perp)$ are LF relative momentum variables, defined by

$$ k_1^+ = xp^+, \quad k_2^+ = (1 - x)p^+, $$

$$ k_{1\perp} = xp_\perp + k_\perp, \quad k_{2\perp} = (1 - x)p_\perp - k_\perp, \quad (4) $$

and $\Gamma$ stands for

$$ \Gamma_P = \gamma_5 \quad \text{(pseudoscalar, } S = 0), $$

$$ \Gamma_V = i\{\vec{\mathcal{F}}(S_z) - \hat{\mathcal{E}} \cdot (k_1 - k_2)\} \quad \text{(vector, } S = 1), $$

$$ \Gamma_T = i\hat{\mathcal{E}}^{\mu\nu} \left\{\gamma_\mu - \frac{(k_1 - k_2)_\mu}{M_0 + m_q + m_{\bar{Q}}}\right\}(k_1 - k_2)_\nu, \quad (5) $$

with

$$ \hat{\mathcal{E}}^\mu(\pm 1) = \left[\frac{2}{p^+}\vec{\mathcal{E}}_\perp(\pm 1) \cdot \vec{p}_\perp, 0, \vec{\mathcal{E}}_\perp(\pm 1)\right], \quad \vec{\mathcal{E}}_\perp(\pm 1) = \mp(1, \pm i)/\sqrt{2}, $$

$$ \hat{\mathcal{E}}^\mu(0) = \frac{1}{M_0} \left(\frac{-M_0^2 + p_\perp^2}{p^+}, p^+, p_\perp\right). \quad (6) $$
There are several phenomenological light-front wave functions to describe the possible hadronic structures in the literature. In our work, we shall use the Gaussian-type and power-law wave functions, given by \[8, 9\]

\[\phi_P(x, k_\perp) = \phi_V(x, k_\perp) = N \sqrt{\frac{1}{N_c}} \frac{dk_z}{dx} \exp \left( -\frac{\vec{k}^2}{2\omega^2} \right), \]

\[\phi_T(x, k_\perp) = \sqrt{\frac{2}{\omega^2}} \phi_P(x, k_\perp), \]

respectively, where \(\omega\) is the scale parameter, \(N_c\) is the number of colors, \(N = \frac{4(\pi/\omega)^2}{3}\), \(\vec{k} = (k_\perp, k_z)\), \(k_z\) is defined through

\[x = \frac{E_q + k_z}{E_q + E_Q}, \quad 1 - x = \frac{E_Q - k_z}{E_q + E_Q}, \quad E_i = \sqrt{m_i^2 + \vec{k}^2} \]

by

\[k_z = \left(x - \frac{1}{2}\right) M_0 + \frac{m_q^2 - m_Q^2}{2M_0}, \quad M_0 = E_q + E_Q, \]

\[dk_z/dx = E_qE_Q/(1-x)M_0, \text{ and } A = m_qx + m_Q(1-x). \]

The pseudoscalar and vector mesonic decay constants are defined by

\[
\langle 0 | A^\mu | P \rangle = if_{P} P^\mu, \\
\langle 0 | V^\mu | V \rangle = f_{V} M_{V} e^{\mu},
\]

where \(A^\mu = \bar{q} \gamma^\mu \gamma^5 Q\) and \(V^\mu = \bar{q} \gamma^\mu Q\), respectively. For an \(3P_2\) tensor meson with \(J^{PC} = 2^{++}\), the decay constant cannot be produced through the local \(V - A\) and tensor currents. But, it can be created from the local currents involving covariant derivatives \[10, 11\]:

\[
\langle 0 | J_{\mu\nu} | T \rangle = f_{T} M_{T}^2 e^{\ast}_{\mu\nu},
\]

where

\[
J_{\mu\nu} = \frac{i}{2} [\bar{q}_1 \gamma_\mu \hat{D}_\nu q_2 + \bar{q}_1 \gamma_\nu \hat{D}_\mu q_2].
\]

and

\[
\hat{D}_\mu = [\hat{D}_\mu - \hat{D}_\mu], \\
\hat{D}_\mu = \partial_\mu - ig \frac{\lambda^a A^a_\mu}{2}, \\
\hat{D}_\mu = \partial_\mu + ig \frac{\lambda^a A^a_\mu}{2}.
\]
The polarization tensor $\epsilon_{\mu\nu}$ for a massive spin-two particle can be constructed out of the polarization vector of a massive vector state $^{11,12}$, given by

$$
\epsilon_{\mu\nu}(\pm 2) = \epsilon_{\mu}(\pm 1)\epsilon_{\nu}(\pm 1),
$$

$$
\epsilon_{\mu\nu}(\pm 1) = \sqrt{\frac{1}{2}}[\epsilon_{\mu}(\pm 1)\epsilon_{\nu}(0) + \epsilon_{\mu}(0)\epsilon_{\nu}(\pm 1)],
$$

$$
\epsilon_{\mu\nu}(0) = \sqrt{\frac{1}{6}}[\epsilon_{\mu}(+1)\epsilon_{\nu}(-1) + \epsilon_{\mu}(-1)\epsilon_{\nu}(+1)] + \sqrt{\frac{2}{3}}\epsilon_{\mu}(0)\epsilon_{\nu}(0). 
$$

From the definitions of the meson decay constants, one has

$$
\langle 0|A^\mu|P(p)\rangle = -\sqrt{N_c} \int \frac{d^4k_1}{(2\pi)^4} A_P \text{Tr} \left[ \Gamma_P \frac{i(-k_1 + m_q)}{k_1^2 - m_q^2 + i\epsilon} A^\mu \frac{i(p - k_1 + m_Q)}{(p - k_1)^2 - m_Q^2 + i\epsilon} \right],
$$

$$
\langle 0|V^\mu|V(p)\rangle = -\sqrt{N_c} \int \frac{d^4k_1}{(2\pi)^4} A_V \text{Tr} \left[ \Gamma_V \frac{i(-k_1 + m_q)}{k_1^2 - m_q^2 + i\epsilon} V^{\mu} \frac{i(p - k_1 + m_Q)}{(p - k_1)^2 - m_Q^2 + i\epsilon} \right],
$$

$$
\langle 0|J_{\mu\nu}|T(p)\rangle = -\sqrt{N_c} \int \frac{d^4k_1}{(2\pi)^4} A_T \text{Tr} \left[ \Gamma_T \frac{i(-k_1 + m_q)}{k_1^2 - m_q^2 + i\epsilon} J^{\mu\nu} \frac{i(p - k_1 + m_Q)}{(p - k_1)^2 - m_Q^2 + i\epsilon} \right], 
$$

where $\Lambda_M$ is a vertex function, related to the momentum distribution amplitude of the $q\bar{Q}$ Fock state. From Eqs. $^{3}$ and $^{15}$, we find the vertex function as

$$
\Lambda_M = \left( \frac{k_1^+ k_2^+}{2[M_0^2 - (m_q - m_Q)^2]} \right)^{\frac{1}{2}} \phi_M(x, k_\perp),
$$

where we have used the light-front variables in Eq. $^{14}$. Then, the explicit expressions of the meson decay constants are given by $^{13,14}$

$$
f_P = 4\sqrt{3N_c} \int dx \frac{d^2k_\perp}{2(2\pi)^3} \phi_P(x, k_\perp) \frac{\mathcal{A}}{\sqrt{\mathcal{A}^2 + k_\perp^2}},
$$

$$
f_V = 4\sqrt{3N_c} \int dx \frac{d^2k_\perp}{2(2\pi)^3} \phi_V(x, k_\perp) \frac{1}{\sqrt{\mathcal{A}^2 + k_\perp^2}} \times \left\{ x(1-x)M_0^2 + m_q m_Q + k_\perp^2 \\
+ \frac{2W}{k_\perp^2} [m_q^2 + k_\perp^2 - m_Q^2 + k_\perp^2 - (1-2x)M_0^2] \right\},
$$

$$
f_T = 4\sqrt{N_c} \int dx \frac{d^2k_\perp}{2(2\pi)^3} \phi_T(x, k_\perp) \frac{1}{x(1-x)\sqrt{\mathcal{A}^2 + k_\perp^2}} \times \left\{ 2k_\perp^2 [k_\perp^2 (2x-1)^2 + \mathcal{A}^2] \\
+ (2x-1)(k_\perp^2 + m_q m_Q) [(x-1)m_q^2 + x m_Q^2 + (2x-1) k_\perp^2] \\
+ \frac{1}{2W} [16 k_\perp^4 x (1-x) (m_q + m_Q) \\
+ (1-2x)^2 (x m_q + m_Q)(k_\perp^2 + m_q m_Q) - m_Q (k_\perp^2 + m_q^2)] \\
+ \sqrt{\mathcal{A}^2} + k_\perp^2 \right\},
$$

where $\mathcal{A} = m_q x + m_Q (1-x)$, $\mathcal{B} = m_q x - m_Q (1-x)$ and $W = M_0 + m_q + m_Q$. 


III. NUMERICAL RESULTS

A. Vector meson decay constants

In the numerical calculation, we take the known decay constants of the pseudoscalar mesons ($P$) and quark masses to evaluate the scalar parameters of $\omega_P$. For the meson wave functions, we first use the Gaussian-type wave function in Eq. (7a) and then the power-law one in Eq. (7b). For the latter, we only briefly summarize our results. We start from the decay constants of $f_D$ and $f_{Ds}$ from the PDG [15], given by

$$f_D = 204 \pm 5 \text{ MeV}, \quad f_{Ds} = 257.5 \pm 4.6 \text{ MeV}.$$  \hfill (18)

By using the first equation in Eq. (17) with the Gaussian-type wave function in Eq. (7a), taking the decay constants in Eq. (18) and inputing the quark masses of $m_u = m_d = 0.25$ and $m_s = 0.38$ in GeV, we obtain the parameters $\omega_D$ and $\omega_{Ds}$ as functions of the charm quark mass $m_c$, shown in Fig. 1. In Fig. 2, by assuming the parameters of $\omega_D^*$ and $\omega_{Ds}^*$ are same as $\omega_D$ and $\omega_{Ds}$ with $m_{u,s} = (0.25, 0.38)$ GeV, we plot the decay constants of $f_D^*$ and $f_{Ds}^*$ as functions of $m_c$ in the LFQM, respectively. From the figure, we see that the decay constants decrease with $m_c$ but the changes are mild. Consequently, from Fig. 2 with $m_c = 1.5 \sim 1.8$ GeV, we find

$$f_{D^*} = 252.0^{+13.8}_{-11.6} \text{ MeV}, \quad f_{Ds} = 318.3^{+15.3}_{-12.6} \text{ MeV},$$  \hfill (19)

which lead to the ratios of the vector and pseudoscalar meson decay constants as

$$\frac{f_{D^*}}{f_D} = 1.232^{+0.074}_{-0.064}, \quad \frac{f_{Ds}}{f_{Ds}} = 1.236^{+0.063}_{-0.054},$$  \hfill (20)

respectively. Note that the uncertainties in Eqs. (19) come from those of Eq. (18) and $m_c$, while the errors in Eqs. (20) result from the combinations of those in Eqs. (18) and (19).

From the Belle experimental results [16] and the lattice QCD calculations [17] of $f_B$, $f_{Bs}$ and $f_{Bc}$ [20], given by

$$f_B = 185 \pm 35 \text{ MeV}, \quad f_{Bs} = 224 \pm 5 \text{ MeV}, \quad f_{Bc} = 434 \pm 15 \text{ MeV},$$  \hfill (21)

we can fix $\omega_B$, $\omega_{Bs}$, and $\omega_{Bc}$, respectively. Our results are shown in Figs. 3 and 4 with $m_{u,s,c} = (0.25, 0.38, 1.5)$ GeV. In Figs. 5 and 6 we present the decay constants of $f_{B^*}$, $f_{B^*_s}$ and $f_{B^*_c}$ as functions of $m_b$ in the LFQM. Obviously, these decay constants are insensitive
FIG. 1. Scalar parameters $\omega_P$ ($P = D$ and $D_s$) as functions of $m_c$ in the LFQM with $m_q = 0.25$ and $m_s = 0.38$ in GeV.

FIG. 2. $f_{D^*}$ and $f_{D^*_s}$ as functions of $m_c$ in the LFQM.

to the change of $m_b$ as seen from the figures. Similarly, we can derive the ranges of the decay constants $f_{B^*}$ and $f_{B^*_s}$ to be

$$f_{B^*} = 201.9^{+43.2}_{-41.4} \text{ MeV}, \quad f_{B^*_s} = 244.2 \pm 7.0 \text{ MeV}, \quad f_{B^*_c} = 473.4 \pm 18.2 \text{ MeV}. \quad (22)$$

Note that the large error in Eq. (22) for $f_{B^*}$ is originated from the one in Eq. (21) for $f_B$. 

7
Subsequently, we get the ratios of the vector and pseudoscalar meson decay constants as

\[ \frac{f_{B^*}}{f_B} = 1.09^{+0.31}_{-0.30}, \quad \frac{f_{B_s^*}}{f_{B_s}} = 1.09 \pm 0.04, \quad \frac{f_{B_c^*}}{f_{B_c}} = 1.09 \pm 0.06. \]  

In Table I, we summarize our results with both Gaussian and power-law meson wave functions for the vector meson decay constants. In the table, we also show the other related theoretical values in the literature [18-25]. From the table, we find that our numerical values
FIG. 5. $f_{B^*}$ and $f_{B_s^*}$ as functions of $m_b$ in the LFQM.

FIG. 6. $f_{B_s^*}$ as functions of $m_b$ in the LFQM.

with the power-law wave functions are slightly higher than those with the Gaussian ones. In addition, we can see that our results for $f_{D^{(*)}, B^{(*)}}$ are consistent with those from the Lattice QCD [10] and QCD sum rules (QCDSR) in Refs. [21, 23], but larger than the ones in Ref. [24]. We note that $f_{B^{(*)}}/f_B < 1$ in Ref. [24]. For $f_{B_s^*}$, our predicted values are all larger than those in Refs. [20, 22]. By comparing with Ref. [18], we see that our predictions for $f_{D^*}$, $f_{D_s^*}$ and $f_{B_s^*}$ are consistent each other within errors, but those for $f_{B_s^*}$ and $f_{B_c^*}$ are
not. The main reasons for the differences are that the author in Ref. [18] used a different set of input parameters such as quark masses and decay constants of the pseudoscalar mesons. Finally, we remark that if we take the sharp parameters \( \omega_V \) of the vector mesons to be different from \( \omega_P \) of the pseudoscalar ones, e.g. \( \omega_V \sim (1 + 5\%) \omega_P \), the corresponding vector meson decay constants will increase about 5% for the same set of input parameters. It is clear that our assumption of \( \omega_V \sim \omega_P \) is a consequence of the heavy quark limit, in which \( f_P = f_V \) is expected [26–28], so that it may only be applied to the heavy mesons as it is obvious breaking down for the light mesons, such as the case of \( \pi \) and \( \rho \).

### B. Tensor meson decay constants

Similar to the vector meson cases, if we take the parameters of \( \omega_T \) are the same as the corresponding ones for the pseudoscalar mesons, we may calculate the decay constants of the tensor mesons \( D_2^* \), \( D_{s2}^* \), \( B_2^* \) and \( B_{s2}^* \). In this part of the study, we shall concentrate on the Gaussian-type of the meson wave functions in Eq. (7a). Note that the relation in Eq. (7c) has been demonstrated only with the Gaussian wave functions [29]. Explicitly, we obtain

\[
\begin{align*}
    f_{D_2^*} &= 143.6^{+24.9}_{-21.8} \text{ MeV}, \quad f_{D_{s2}^*} = 209.5^{+29.1}_{-24.2} \text{ MeV}, \\
    f_{B_2^*} &= 80.9^{+33.8}_{-27.7} \text{ MeV}, \quad f_{B_{s2}^*} = 109.7^{+15.7}_{-15.0} \text{ MeV},
\end{align*}
\]  

(24)
where $m_{u,s,c,b} = 0.25, 0.38, 1.6$ and $4.8$ in GeV have been used to evaluate the center values. Consequently, we find the ratios of the two related tensor meson decay constants to be

$$\frac{f_{D_s^2}}{f_{D_s^2}} = 1.5 \pm 0.3, \quad \frac{f_{B_s^2}}{f_{B_s^2}} = 1.4^{+0.6}_{-0.5}. \quad (25)$$

In Figs 7 and 8, we show the tensor decay constants of $D_{2,s_2}$ ($B_{2,s_2}$) as functions of $m_{c(b)}$. One can see that the decay constants are enhanced if $m_{c(b)}$ increases.

![Figure 7](image.png)

**FIG. 7.** $f_{D_s^2}$ and $f_{D_s^2}$ as functions of $m_c$ in the LFQM.

In Table II, we list our results for the tensor meson decay constants in the LFQM along with those in QCDSR [30]. From the table, we observe that our predicted value for $D_s^*$ is close to that in QCDSR, whereas the other ones are about 20% smaller. It is interesting to note that our results in the LFQM can match with those in QCDSR if larger quark masses of $m_{c,b}$ are used.

**IV. CONCLUSIONS**

We have studied the decay constants of the heavy vector ($D^*, D_s^* B^*, B_s^*, B_c^*$) and tensor ($D_s^*, D_s^* B_s^*, B_s^*$) mesons in the LFQM. In our study, we have used the known pseudoscalar meson decay constants of $f_D, f_{D_s}, f_B, f_{B_s}$ and $f_{B_c}$ and quark mass $m_{u,d,s}$ and $m_{c(b)}$ as the input parameters to determine the values of the scale parameters of $\omega_P$. 
in the light-front wave functions. By taking $\omega_{D_s}$ and $\omega_{B_{s,c}}$ in both Gaussian and power-law wave functions being the same as the corresponding $\omega_{D_s}$ and $\omega_{B_{s,c}}$, we have calculated the decay constants of the vector $D_s^*$ and $B_{s,c}^*$ mesons, respectively. Explicitly, we have found that $f_{D_s^*D_s^*,B_{s,c}^*,B_{s,c}^*} = (252.0^{+13.8}_{-11.6}, 318.3^{+15.3}_{-12.6}, 201.9^{+43.2}_{-41.4}, 244.2 \pm 7.0, 473.4 \pm 18.2)$ and $(264.9^{+10.2}_{-9.5}, 330.9^{+9.9}_{-9.9}, 220.2^{+40.1}_{-46.2}, 265.7 \pm 8.0, 487.6 \pm 19.2)$ MeV with Gaussian and power-law wave functions, respectively. Similarly, we have obtained $f_{D_s^*D_s^*,B_{s,c}^*,B_{s,c}^*} = (143.6^{+24.9}_{-21.8}, 209.5^{+29.1}_{-24.2}, 80.9^{+33.8}_{-27.7}, 109.7^{+15.7}_{-15.0})$ MeV with only Gaussian wave functions.

FIG. 8. $f_{B_s^*}$ and $f_{B_{s,c}^*}$ as functions of $m_b$ in the LFQM.

TABLE II. Tensor meson decay constants of $f_{D_s^*}, f_{D_s^*}, f_{B_s^*}$ and $f_{B_{s,c}^*}$ (MeV) in the LFQM and QCDSR [30].

|        | LFQM       | QCDSR [30] |
|--------|------------|------------|
| $f_{D_s}$ | $143.6^{+24.9}_{-21.8}$ | $183 \pm 20$ |
| $f_{D_s^*}$ | $209.5^{+29.1}_{-24.2}$ | $222 \pm 22$ |
| $f_{B_s}$ | $80.9^{+33.8}_{-27.7}$ | $111 \pm 10$ |
| $f_{B_{s,c}}$ | $109.7^{+15.7}_{-15.0}$ | $134 \pm 11$ |
V. ACKNOWLEDGMENTS

The work was supported in part by National Center for Theoretical Sciences, National Science Council (NSC-101-2112-M-007-006-MY3 and NSC-102-2112-M-471-001-MY3) and MoST (MoST-104-2112-M-007-003-MY3).

[1] E. S. Swanson, Phys. Rept. 429 (2006) 243; B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 90 (2003) 242001; P. Krokovny et al. [Belle Collaboration], Phys. Rev. Lett. 91 (2003) 262002; D. Besson et al. [CLEO Collaboration], Phys. Rev. D68 (2003) 032002; B. Aubert et al. [BABAR Collaboration], Phys. Rev. D69 (2004) 031101; J. Link et al. (FOCUS Collaboration), Phys. Lett. B586 (2004) 11; R. Aaij et al. (LHCb Collaboration), Phys. Lett. B698 (2011) 14.

[2] V. M. Abazov et al, Phys. Rev. Lett. 99 (2007) 172001; V. Abazov et al, Phys. Rev. Lett. 100 (2008) 082002.

[3] T. Aaltonen et al, Phys. Rev. Lett. 102 (2009) 102003; T. Aaltonen et al, Phys. Rev. Lett. 100 (2008) 082001.

[4] W. Jaus, Phys. Rev. D41 (1990) 3394; W. Jaus, Phys. Rev. D44 (1991) 2851; W. Jaus, Phys. Rev. D60 (1999) 054026.

[5] C. C. Lih and C. Q. Geng, Phys. Rev. C85 (2012) 018201.

[6] K. G. Wilson, T. S. Walhout, A. Harindranath, W. M. Zhang, R. J. Perry and S. D. Glazek, Phys. Rev. D49, 6720 (1994); C. Q. Geng, C. C. Lih and W. M. Zhang, Phys. Rev. D57 (1998) 5697; Phys. Rev. D62 (2000) 074017.

[7] W. Jaus, Phys. Rev. D41 (1990) 3394; D44 (1991) 2851.

[8] C. Q. Geng, C. C. Lih and W. M. Zhang; Mod. Phys. Lett. A15 (2000) 2087; C. C. Lih, C. Q. Geng and W. M. Zhang, Phys. Rev. D59 (1999) 114002; C. Q. Geng, C. C. Lih and C. C. Liu, Phys. Rev. D62 (2000) 034019; C. H. Chen, C. Q. Geng, C. C. Lih and C. C. Liu, Phys. Rev. D75 (2007) 074010.

[9] Chien-Wen Hwang, Phys. Lett. B530 (2002) 93.

[10] K. Azizi, H. Sundu, J. Y. Sungu, N. Yinelek, Phys. Rev. D88 (2013) 036005; Phys. Rev. D88 (2013) 099901(E); K. Azizi, H. Sundu, A. Trkan and E. V. Veliev, J. Phys. G 41 (2014)
035003.

[11] Hai-Yang Cheng, Kwei-Chou Yang, Phys. Rev. D83 (2011) 034001.

[12] E. R. Berger, A. Donnachie, H. G. Dosch, O. Nachtmann, Eur. Phys. J. C14 (2000) 673;

[13] H.Y. Cheng, C.Y. Cheung and C.W. Hwang, Phys. Rev. D55 (1997) 1559. W. Jaus, Phys. Rev. D60 (1999) 054026.

[14] Ho-Meoyng Choi, Phys. Rev. D75 (2007) 073016; Ho-Meoyng Choi, Chueng-Ryong Ji, Phys. Rev. D75 (2007) 034019; Ho-Meoyng Choi, Chueng-Ryong Ji, Phys.Rev. D89 (2014) 033011.

[15] K. A. Olive et al. (Particle Data Group), Chin. Phys. C38 (2014) 090001.

[16] I. Adachi et al. [Belle Collab], Phys. Rev. Lett. 110 131801 (2013); M. J. Baker, J. Bordes, C. A. Dominguez, J. Penarrocha and K. Schilcher, JHEP 1407 (2014) 032.

[17] R.J. Dowdall [HPQCD Collaboration] et al, Phys. Rev. Lett. 110 22, (2013) 222003.

[18] Chien-Wen Hwang, Phys. Rev. D81 (2010) 114024.

[19] Damir Becirevic, Francesco Sanfilippo, Silvano Simula and Cecilia Tarantino, JHEP 1202 (2012) 042.

[20] B. Colquhoun, C. T. H. Davies, J. Kettle, J. Koponen, and A. T. Lytle, Phys. Rev. D91 (2015) 114509.

[21] Z. G. Wang, Eur. Phys. J. C 75 (2015) 427.

[22] Z. G. Wang, Eur. Phys. J. A 49 (2013) 131.

[23] Wolfgang Lucha, Dmitri Melikhov and Silvano Simula, Phys. Lett. B735 (2014) 12-18.

[24] Wolfgang Lucha, Dmitri Melikhov and Silvano Simula, Phys. Rev. D91 (2015) 116009.

[25] Patrick Gelhausen, Alexander Khodjamirian, Alexei A. Pivovarov, Denis Rosenthal, Phys. Rev. D88 (2013) 014015; Phys. Rev. D 89 (2014) 099901 (E); Phys. Rev. D 91 (2015) 099901 (E).

[26] M. Neubert, Phys. Rev. D 46, 1076 (1992).

[27] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D 69, 074025 (2004).

[28] C. W. Hwang, Phys. Rev. D 86, 094031 (2012).

[29] Martin A. DeWitt, Ho-Meoyng Choi, Chueng-Ryong Ji, Phys. Rev. D68 (2003) 054026.

[30] Zhi-Gang Wang and Zun-Yan Di, Eur. Phys. J. A50 (2014) 143.