THE LOW ENERGY EFFECTIVE THEORY AND NUCLEON STABILITY

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We show that the Standard Model Lagrangian, including small neutrino masses, has an anomaly-free discrete $Z_6$ symmetry. Anomaly cancellation requires the number of family to be $3 \ mod \ 6$. This symmetry can ensure the stability of the nucleon even when the threshold of new physics $\Lambda$ is low as $10^2$ GeV. All $\Delta B = 1$ and $\Delta B = 2$ ($B$ is the baryon number) effective operators are forbidden by the $Z_6$ symmetry. $\Delta B = 3$ operators are allowed, but they arise only at dimension 15. We suggest a simple mechanism for realizing reasonable neutrino masses and mixings even with such a low scale for $\Lambda$.

The Standard Model (SM) has been highly successful in explaining all experimental observations in the energy regime up to a few hundred GeV. However, it is believed to be an effective field theory valid only up to a cutoff scale $\Lambda$. Non-renormalizable operators which are gauge invariant but suppressed by appropriate inverse powers of $\Lambda$ should then be considered in the low energy effective theory. The dimension 5 operator $\ell \ell HH/\Lambda_L$ ($\ell$ is the lepton doublet) which violates lepton number ($L$) by two units is the lowest dimensional of such operators. Experimental evidence for neutrino masses suggests the effective scale of $L$-violation is around $\Lambda_L \sim 10^{14} - 10^{15}$ GeV. The $d = 6$ operator $QQQ\ell/\Lambda_B^2$ violates both baryon number ($B$) and lepton number and leads to the decay of the nucleon. The current limits on proton lifetime are $\tau_p > 5 \times 10^{33}$ yrs for $p \rightarrow e^+ \pi^0$ 1. These limits imply that $\Lambda_B > 10^{15}$ GeV. Grand Unified Theories with or without supersymmetry generate such $B$-violating operator with $\Lambda_B \sim 10^{15} - 10^{16}$ GeV. These theories are currently being tested through nucleon decay. Any new physics with a threshold $\Lambda$ less than the GUT scale will thus be constrained by both proton lifetime and neutrino masses.

As we know the SM effective lagrangian does not have a continuous anomaly-free symmetry that can suppress baryon number and lepton number violating processes. This is our reason for focusing on discrete sym-
metries. It is preferable that such symmetries have a gauge origin\(^2\) since all global symmetries are expected to be violated by the quantum gravitational effects. Discrete gauge symmetries have been utilized in suppressing nucleon decay\(^3\) as well as in addressing other aspects of physics such as solving the \(\mu\) problem\(^4\) of supersymmetry, fermion mass hierarchy problem\(^5\) and the stability of the axion\(^4,6\). A \(Z_3\) baryon parity was found in Ref. \([3]\) that suppresses nucleon decay. In order for it to have a gauge origin, complicated particle content were introduced.

We pointed out the SM lagrangian has a discrete \(Z_6\) gauge symmetry which forbids all \(\Delta B = 1\) and \(\Delta B = 2\) baryon violating effective operators. This can be seen as follows. The SM Yukawa couplings incorporating the seesaw mechanism to generate small neutrino masses is

\[
L_Y = Qu^c H + Qd^c H^* + \ell e^c H^* + \ell \nu^c H + M_R \nu^c \nu^c .
\]

Here we have used the standard (lefthanded) notation for the fermion fields and have not displayed the Yukawa couplings or the generation indices. This lagrangian respects a \(Z_6\) discrete symmetry with the charge assignment as shown in Table 1. From Table 1 it is easy to calculate the \(Z_6\) crossed anomaly coefficients with the SM gauge groups. We find the \(SU(3)_C\) and \(SU(2)_L\) anomalies to be: 
\[A_{[SU(3)_C]_2 \times Z_6} = 3N_g\] and 
\[A_{[SU(2)_L]_2 \times Z_6} = N_g\]
where \(N_g\) is the number of generations. The condition for a \(Z_N\) discrete group to be anomaly-free is: 
\[A_i = \frac{N}{2} \mod N\]
where \(i\) stands for \(SU(3)_C\) and \(SU(2)_L\). For \(Z_6\), this condition reduces to \(A_i = 3 \mod 6\), so when \(N_g = 3\), \(Z_6\) is anomaly-free. The significance of this result is that unknown quantum gravitational effects will respect this \(Z_6\). It is this feature that we utilize to stabilize the nucleon. Absence of anomalies also suggests that the \(Z_6\) may have a simple gauge origin.

We have found\(^7\) a simple and economic embedding of \(Z_6\) into a \(U(1)\) gauge symmetry associated with \(I^3_R + L_i + L_j - 2L_k\). Here \(L_i\) is the \(i\)th family lepton number and \(i \neq j \neq k\). No new particles are needed to cancel gauge anomalies. With the inclusion of righthanded neutrinos \(I^3_R = Y - (B - L)/2\)
is an anomaly-free symmetry. \( L_i + L_j - 2L_k \), which corresponds to the \( \lambda_8 \) generator acting in the leptonic \( SU(3) \) family space, is also anomaly-free. The charges of the SM particles under this \( U(1) \) are: \( Q_i = (0,0,0) \), \( u_i^c = (-1, -1, -1) \), \( d_i^c = (1, 1, 1) \), \( l_i = (-4,2,2) \), \( e_i^c = (5,-1,-1) \), \( \nu_i^c = (3,-3,-3) \), \( H = 1 \). This charge assignment allows all quark masses and mixings as well as charged lepton masses. When the \( U(1) \) symmetry breaks spontaneously down to \( Z_6 \) by the vacuum expectation value of a SM singlet scalar field \( \phi \) with a charge of 6, realistic neutrino masses and mixings are also induced\(^7\).

From Table 1 it is easy to see that the \( Z_6 \) discrete symmetry allowed only \( \Delta B = 3 \) effective operators with lowest-dimension \( d = 15 \) and forbids all \( \Delta B = 1 \) and \( \Delta B = 2 \) operators. \( \Delta B = 3 \) and \( d = 15 \) operator will lead to “triple nucleon decay” processes where three nucleons in a heavy nucleus undergo collective decays. We choose a specific operator \( Q^5 \bar{d}^4 \bar{\ell}/\Lambda^11 \) as an example to study the process \( pnn \rightarrow e^+ + \pi^0 \) triple nucleon decay process. In this case the triple nucleon decay lifetime can then be estimated to be

\[
\tau \sim \frac{16\pi f_\pi^2 \Lambda^{22} R^6}{P^2 \beta^6 M^3 H},
\]

where \( \beta \simeq 0.014 \text{ GeV}^3 \) is the matrix element to convert three quarks into a nucleon \(^8\), \( f_\pi = 139 \text{ MeV} \) is the pion decay constant, \( P \) is the probability for three nucleons in Oxygen nucleus to overlap in a range the size of Tritium nucleus, \( R \) is the ratio between the radii of Tritium nucleus and Oxygen nucleus. By putting the current limit on proton lifetime of \( 3 \times 10^{33} \) yrs, we obtain: \( \Lambda \sim 10^2 \text{ GeV} \). Thus we see the \( Z_6 \) symmetry ensures the stability of the nucleon.

If the threshold of new physics is low as a few TeV, neutrino mass induced through the effective operator \( \ell\ell HH/\Lambda \) will be too large. We found a mechanism by which such operators can be suppressed by making use of a discrete \( Z_N \) symmetry (with \( N \) odd) surviving to low scale.

Consider the following effective operators in the low energy lagrangian:

\[
L \supset \ell\ell HH S^6/\Lambda^7 + \frac{S^{2N}}{\Lambda^{2N-4}}.
\]

Here \( S \) is a singlet field which has charge \((1,3)\) under \( Z_N \times Z_6 \) while \( \ell \) has charge \((-3,2)\). (The \( Z_6 \) charges of SM particles are as listed in Table 1.) In this case, if \( \Lambda = 10 \text{ TeV} \) and \( S = 10^2 \text{ GeV} \), the neutrino mass is of order \( O(0.1) \text{ eV} \), which is consistent with the mass scale suggested by the atmospheric neutrino oscillation data.
Two explicit examples of the $Z_N$ symmetry with $N = 5$ and 7 are shown in Table 2. These $Z_N$ symmetries are free from gauge anomalies. In the $Z_5$ example, the crossed anomaly coefficients for $SU(3)_C$ and $SU(2)_L$ are $5N_g$ and $5N_g/2$ respectively showing that $Z_5$ is indeed anomaly-free. For $Z_7$, these coefficients are $7N_g$ and $7N_g/2$, so it is also anomaly-free.

| Field | $Q$ | $u^c$ | $d^c$ | $\ell$ | $e^c$ | $H$ | $S$ |
|-------|-----|-------|------|-------|------|-----|-----|
| $Z_5$ | 1   | 4     | 4    | 2     | 3    | 0   | 1   |
| $Z_7$ | 1   | 6     | 6    | 4     | 3    | 0   | 1   |

Table 2. $Z_N$ charge assignment for $N = 5$ and 7.

It is interesting to ask if the $Z_N$ can be embedded into a gauged $U(1)$ symmetry. A simple possibility we have found is to embed this $Z_N$ into the anomalous $U(1)_A$ symmetry of string origin with the anomalies cancelled by the Green-Schwarz mechanism \(^9\). Consider $U(1)_{B-L}$ without the right handed neutrinos but with the inclusion of vector-like fermions which have the quantum numbers of $5(3)$ and $\bar{5}(2)$ under $SU(5) \times U(1)_A$. This $U(1)_A$ is anomaly-free by virtue of the Green-Schwarz mechanism. When this $U(1)_A$ breaks down to $Z_5$, the extra particles get heavy mass and are removed from the low energy theory which is the $Z_6 \times Z_5$ model.

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