SIMULATION OF INTERSECTING BLACK BRANE SOLUTIONS
BY MULTI-COMPONENT ANISOTROPIC FLUID

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A family of spherically symmetric solutions with horizon in the model with multi-component anisotropic fluid (MCAF) is obtained. The metric of any solution contains \((n - 1)\) Ricci-flat “internal space” metrics and for certain equations of state \((p_i = \pm \rho)\) coincides with the metric of intersecting black brane solution in the model with antisymmetric forms. Examples of simulation of intersecting \(M2\) and \(M5\) black branes are considered. The post-Newtonian parameters \(\beta\) and \(\gamma\) corresponding to the 4-dimensional section of the metric are calculated.

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1 Introduction

Recently, spherically-symmetric \(p\)-brane solutions with horizon (see, e.g., [1] and references therein) defined on product manifolds \(\mathbb{R} \times M_0 \times \ldots \times M_n\) cause a wide interest. These solutions appear in models with antisymmetric forms and scalar fields. These and more general \(p\)-brane cosmological and spherically symmetric solutions are usually obtained by reduction of the field equations to the Lagrange equations corresponding to Toda-like systems [4]. An analogous reduction for models with multi-component anisotropic fluids was performed earlier in [6].

For cosmological-type models with antisymmetric forms without scalar fields any \(p\)-brane is equivalent to an anisotropic fluid with the equations of state:

\[
\hat{p}_i = -\hat{\rho} \quad \text{or} \quad \hat{p}_i = \hat{\rho},
\]

when the manifold \(M_i\) belongs or does not belong to the brane world volume, respectively (here \(\hat{p}_i\) is the effective pressure in \(M_i\) and \(\hat{\rho}\) is the effective density).

In this paper we find the analogues of intersecting black brane solutions in a model with multi-component anisotropic fluid (MCAF), when certain ”orthogonality” relations on fluid parameters are imposed. The one-component case was considered earlier in [12].

The paper is organized as follows. In Section 2 the model is formulated. In Section 3 general MCAF solutions with horizon corresponding to black-brane-type solutions are presented. Section 4 deals with certain MCAF analogues of intersecting black brane solutions, i.e. \(M2\) and \(M5\) black brane solutions. In Section 5 the post-Newtonian parameters for the 4-dimensional section of the MCAF-black-brane metric are calculated. In Appendix based on [1, 7] the general spherically symmetric solutions with multicomponent anisotropic fluid are considered and configurations with horizon are singled out.

2 The model

In this paper we consider a family of spherically symmetric solutions to Einstein equations with an anisotropic matter source

\[
R_N^M - \frac{1}{2} g_N^M R = k T_N^M,
\]

defined on the manifold

\[
M = \mathbb{R}_r \times (M_0 = S^{d_0}) \times (M_1 = \mathbb{R}) \times \ldots \times M_n,
\]

radial spherical time variables

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with the block-diagonal metrics

\[ ds^2 = e^{2\gamma(u)} du^2 + \sum_{i=0}^{n} e^{2X_i(u)} h_{m,n}^{(i)} dy^m dy^n. \]  

(2.3)

Here \( R_n \subseteq R \) is an open interval. The manifold \( M_i \) with the metric \( h^{[i]} \), \( i = 1, 2, \ldots, n \), is a Ricci-flat space of dimension \( d_i \):

\[ R_{m,n}^{(i)}[h^{[i]}] = 0, \]

(2.4)

and \( h^{[0]} \) is the standard metric on the unit sphere \( S^{d_0} \), so that

\[ R_{m,n}^{(0)}[h^{[0]}] = (d_0 - 1) h^{[0]}. \]

(2.5)

\( u \) is a radial variable, \( \kappa \) is the gravitational constant, \( d_1 = 1 \) and \( h^{[1]} = -dt \otimes dt \).

The energy-momentum tensor is adopted in the following form for each component of the fluid:

\[ \mathcal{T}^{(s)M}_N = \text{diag}(-\rho^{(s)}, \rho^{(s)} \delta_{k_0}, \rho^{(s)} \delta_{k_1}, \ldots, \rho^{(s)} \delta_{k_n}), \]

(2.6)

where \( \rho^{(s)} \) and \( \rho^{(s)} \) are the effective density and pressures respectively, depending on the radial variable \( u \).

We assume that the following "conservation laws"

\[ \nabla M \mathcal{T}^{(s)M}_N = 0 \]

(2.7)

are valid for all components.

We also impose the following equations of state

\[ \tilde{\rho}^{(s)} = \left(1 - \frac{2U_i^{(s)}}{d_i}\right) \rho^{(s)}, \]

(2.8)

where \( U_i^{(s)} \) are constants, \( i = 0, 1, \ldots, n \).

The physical density and pressures are related to the effective ones (with "hats") by the formulae

\[ \rho^{(s)} = -\rho^{(s)}_1, \quad \rho^{(s)}_u = -\rho^{(s)}_1, \quad \rho^{(s)}_i = \tilde{\rho}^{(s)}_i \quad (i \neq 1). \]

(2.9)

In what follows we put \( \kappa = 1 \) for simplicity.

### 3 Spherically symmetric solutions with horizon

We will make the following assumptions:

1°. \( U_0^{(s)} = 0 \iff \tilde{\rho}^{(s)}_0 = \tilde{\rho}^{(s)}_0 \),

2°. \( U_1^{(s)} = 1 \iff \tilde{\rho}^{(s)}_1 = -\rho^{(s)}_1 \),

3°. \( (U^{(s)}, U^{(s)}) = U_i^{(s)} G_{ij} U_j^{(s)} > 0, \quad (U^{(s)}, U^{(l)}) = 0, \quad s \neq l \),

where

\[ G_{ij} = \delta_{ij} + \frac{1}{2 - D}, \]

(3.2)

are components of the matrix inverse to the matrix of the minisuperspace metric \( \tilde{g} \)

\[ (G_{ij}) = (d_i \delta_{1j} - d_i d_j), \]

(3.3)

and \( D = 1 + \sum_{i=0}^{n} d_i \) is the total dimension.

The orthogonality condition 3° is an integrability condition (see Appendix). The conditions 1° and 2° in p-brane terms mean that brane "lives" in a time manifold \( M_1 \) and does not "live" in \( R_n \times M_0 \). The assumptions 1° and 2° are natural ones from the point of view of state equations (2.8), so we can rewrite the energy-momentum tensor (2.6) as following:

\[ \mathcal{T}^{(s)M}_N = \text{diag}(-\rho^{(s)}_0, \rho^{(s)}_1 \delta_{k_0}, -\rho^{(s)}_1 \delta_{k_1}, \rho^{(s)}_2 \delta_{k_2}, \ldots, \rho^{(s)}_n \delta_{k_n}). \]

(3.4)
Under the conditions (2.8) and (3.1) we have obtained the following black-hole solutions to the Einstein equations (2.1):

\[
\begin{align*}
    ds^2 &= J_0 \left( \frac{dr^2}{1 - 2\mu/r^d} + r^2 d\Omega^2_{d-1} \right) - J_1 \left( 1 - \frac{2\mu}{r^d} \right) dt^2 + \sum_{i=2}^{n} J_i h_{m,n_i}^{[i]} dy^{m_i} dy^{n_i}, \\
    \rho^{(s)} &= -\frac{A_s}{H_s^2 J_0 r^{2d_0}}, \quad A_s = -\frac{1}{2} \nu_s^2 d^2 P_s (P_s + 2\mu),
\end{align*}
\]

which may be verified from [3] and by analogy with the \( p \)-brane solution [4]. For direct derivation of the solution see Appendix. Here \( d = d_0 - 1 \),

\[
d\Omega^2_{d_0} = h_0^{[0]} dy^{m_0} dy^{n_0}
\]
is the spherical element,

\[
J_s = \prod_{s=1}^{m} H_s^{-2\nu_s^2 U(s)}, \quad H_s = 1 + P_s/r^d;
\]

\( P_s > 0, \mu > 0 \) are integration constants and

\[
U^{(s)} = G^{ij} U^{(s)}_{ij} = \sum_{d_0}^{d} \frac{U^{(s)}_{ij}}{d_0} + \frac{1}{2 - D} \sum_{j=0}^{n} U^{(s)}_{ij},
\]

\[
\nu_s = (U^{(s)}, U^{(s)})^{-1/2}.
\]

4 Simulation of intersecting black branes

The solution from the previous section for MCAF allows to simulate the intersecting black brane solutions [1] in the model with antisymmetric forms without scalar fields. In this case the parameters \( U^{(s)}_{ij} \) have the following form:

\[
U^{(s)}_{ij} = \begin{cases} 
    d_i, & p^{(s)}_{ij} = -\rho^{(s)}, \quad i \in I(s); \\
    0, & \rho^{(s)}, \quad i \notin I(s).
\end{cases}
\]

Here \( I(s) = \{i_1, \ldots, i_k \} \subset \{1, \ldots, n \} \) is the index set [3] corresponding to brane submanifold \( M_{i_1} \times \cdots \times M_{i_k} \).

The orthogonality constraints 3\(^{\circ}\) (3.1) lead us to the following dimension of intersection of brane submanifolds [3]:

\[
d_{I(s) \cap I(t)} = \frac{d_{I(s)} d_{I(t)}}{D - 2},
\]

where \( d_{I(s)} \) and \( d_{I(t)} \) are dimensions of \( p \)-brane world-volumes, \( s, l = 1, \ldots, m, \) \( s \neq l \).

Due to relations (4.1) and 1\(^{\circ}\), (3.1) we can rewrite (3.6) as follows:

\[
\rho^{(s)} = -\frac{A_s}{H_s^2 \prod_{l=1}^{m} H_l^{2/(D - d_{I(s)} - 2)} \ r^{2d_0}},
\]

and investigate the behavior of the density as a radial function. For the single fluid the density is regular and positive at zero when the parameter \( d \) (see the previous section) is equal to \( d^* = D - d_{I(s)} - 2 \). In this case the brane submanifold fills the total manifold (2.2) except \( R_s \times S^{d_0} \). When \( d < d^* \) the density is infinite at zero.

For multi-component fluid all densities are finite at \( r = 0 \), if (and only if)

\[
\sum_{s=1}^{m} \frac{1}{D - d_{I(s)} - 2} \geq \frac{1}{d}.
\]

Moreover, all \( \rho^{(s)}(0) > 0 \) when the equality in (4.4) takes place.

As an example we consider simulation by MCAF of intersecting \( M2 \cap M5, M2 \cap M2, M5 \cap M5 \) configurations in \( D = 11 \) supergravity. The metric for all cases reads:
\[ds^2 = J_0 \left[ \frac{dr^2}{1 - 2\mu/r} + r^2 d\Omega_5^2 - (H_{(I)})^{-1} (H_{(II)})^{-1} \left\{ \left( 1 - \frac{2\mu}{r} \right) dt^2 + h_{m_2}^{[2]} dy^m dy^n \right\} + H_{(I)}^{-1} h_{m_3}^{[3]} dy^m dy^n + H_{(II)}^{-1} h_{m_4}^{[4]} dy^m dy^n + h_{m_5}^{[5]} dy^m dy^n \right], \tag{4.5}\]

where we can express the factor \( J_0 = H_{(I)}^{2/(D-d_{(I)})-2} H_{(II)}^{2/(D-d_{(II)})-2} \); the first brane world-volume is \( M_1 \times M_2 \times M_3 \), the second one is \( M_1 \times M_2 \times M_4 \).

a). For MCAF, corresponding to intersecting of \( M2 \) (with index \( I \) in (3.6)) and \( M5 \) (with index \( II \)) branes the dimensions are following \( d_1 = d_2 = d_3 = 1 \), \( d_4 = 4 \) and \( J_0 = H_{(I)}^{1/3} H_{(II)}^{2/3} \).

The densities \( \rho^{(I)} \), \( \rho^{(II)} \) are infinite at zero when \( d = 1 \) and for \( d = 2 \) they are finite: \( \rho^{(I)}(0) = (P_1 + 2\mu)/H_{(I)}^{4/3} H_{(II)}^{2/3} \), \( \rho^{(II)}(0) = (P_{II} + 2\mu)/H_{(I)}^{4/3} H_{(II)}^{2/3} \). It is interesting to note that in the extremal limit \( \mu \to 0 \), \( \rho^{(I)}(0) = \rho^{(II)}(0) \).

b). For MCAF, equivalent to two electrical \( M2 \) branes intersecting on the time manifold we get \( d_3 = d_4 = 2 \), \( d_2 = 0 \). Here \( J_0 = H_{(I)}^{1/3} H_{(II)}^{1/3} \).

The variants of behavior of the densities are presented on Figure 1. When \( d = 3 \) both functions are regular and positive at zero (the middle branch).

c). For two \( M5 \) branes the dimension of intersection is \( 4 \) and \( d_0 = d_3 = d_4 = 2 \), \( d_2 = 3 \), \( d_5 = 0 \) and \( J_0 = H_{(I)}^{4/3} H_{(II)}^{2/3} \). The only possibility here is \( d = 1 \) and the fluid densities are infinite at zero.

\section{Physical parameters}

\subsection{Gravitational mass and post-Newtonian parameters}

Here for simplicity we put \( d_0 = 2 \) (\( d = 1 \)). Consider the 4-dimensional space-time section of the metric (3.5).

Introducing a new radial variable by the relation

\[ r = R \left( 1 + \frac{\mu}{2R} \right)^2, \tag{5.1} \]

we rewrite the 4-section in the following form:

\[ ds^2(4) = g_{\mu\nu} dx^\mu dx^\nu = \left( \prod_{s=1}^{m} H_s^{-2\nu_s^2 V} \right) \left[ - \left( 1 - \frac{\mu}{2R} \right)^2 \prod_{s=1}^{m} H_s^{-2\nu_s^2} \right] dt^2 + \left( 1 + \frac{\mu}{2R} \right)^4 \delta_{ij} dx^i dx^j, \tag{5.2} \]

\( i, j = 1, 2, 3 \). Here \( R^2 = \delta_{ij} x^i x^j \).

The post-Newtonian (Eddington) parameters are defined by the well-known relations

\[ g_{00}^{(4)} = -(1 - 2V + 2\beta V^2) + O(V^3), \tag{5.3} \]

\[ g_{ij}^{(4)} = \delta_{ij}(1 + 2\gamma V) + O(V^2), \tag{5.4} \]
\( i, j = 1, 2, 3 \). Here \( V = GM/R \) is the Newtonian potential, \( M \) is the gravitational mass and \( G \) is the gravitational constant. From (5.2)-(5.4) we obtain:

\[
GM = \mu + \sum_{s=1}^{m} \nu_{s}^{2} P_{s}(1 + U^{(s)0}), \tag{5.5}
\]

and

\[
\beta - 1 = \frac{1}{2(GM)^{2}} \sum_{s=1}^{m} \nu_{s}^{2} P_{s}(P_{s} + 2\mu)(1 + U^{(s)0}), \tag{5.6}
\]

\[
\gamma - 1 = -\frac{1}{GM} \sum_{s=1}^{m} \nu_{s}^{2} P_{s}(1 + 2U^{(s)0}). \tag{5.7}
\]

For fixed vector \( U^{(s)} \) the parameter \( \beta - 1 \) is proportional to the ratio of two physical parameters: the anisotropic fluid density parameter \( A_{s} \) (see (3.15)), and the gravitational radius squared \( (GM)^{2} \).

### 5.2 The Hawking temperature

The Hawking temperature of a black hole may be calculated using the relation from (3) and has the following form:

\[
T_{H} = \frac{d}{4\pi(2\mu)^{1/d}} \prod_{s=1}^{m} \left( \frac{2\mu}{2\mu + P_{s}} \right) \nu_{s}^{2}. \tag{5.8}
\]

## 6 Conclusions

Here we have obtained a family of spherically symmetric solutions with horizon in the model with multi-component anisotropic fluid with the equations of state (2.8) and the conditions (3.1) imposed. The metric of any solution contains \((n-1)\) Ricci-flat "internal" space metrics. For certain equations of state (with \( p_{i} = \pm \rho \)) the metric of the solution may coincide with the metric of intersecting black branes (in a model with antisymmetric forms without dilatons). Here the examples of simulating of intersecting \( M2 \) and \( M5 \) black branes in \( D = 11 \) supergravity are considered.

We have also calculated the post-Newtonian parameters \( \beta \) and \( \gamma \) corresponding to the 4-dimensional section of the metric. The parameter \( \beta - 1 \) is written in terms of ratios of the physical parameters: the anisotropic fluid parameter \( |A_{s}| \) and the gravitational radius squared \( (GM)^{2} \). An open problem is to generalize the formalism to the case when dilaton scalar fields are added into consideration.

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## Appendix

### A Lagrange representation

The "conservation law" equation (2.7) may be written, due to relations (2.3) and (2.6) in the following form:

\[
\dot{\rho}^{(s)} + \sum_{i=0}^{n} d_{i} \dot{X}^{i} (\rho^{(s)} + \dot{\rho}^{(s)} + P_{s}^{(s)}) = 0. \tag{A.1}
\]

Using the equation of state (2.8) we get

\[
\dot{\rho}^{(s)} = -A_{s} e^{2U^{(s)X^{1}} - 2\gamma_{0}}, \tag{A.2}
\]

where \( \gamma_{0}(X) = \sum_{i=0}^{n} d_{i}X^{i} \), and \( A_{s} \) are constants.
The Einstein equations (2.1) with the relations (2.8) and (A.2) imposed are equivalent to the Lagrange equations for the Lagrangian

\[ L = \frac{1}{2}e^{-\gamma+\gamma_0(X)}G_{ij}\dot{X}^i\dot{X}^j - e^{\gamma-\gamma_0(X)}V, \]

where

\[ V = \frac{1}{2}d_0(d_0 - 1)e^{2U_i^{(0)}X^i} + \sum_{s=1}^{m} A_s e^{2U_i^{(s)}X^i} = \sum_{s=0}^{m} A_s e^{2U_i^{(s)}X^i}, \]

is the potential and the components of the minisupermetric \( G_{ij} \) are defined in (3.3).

\[ U_i^{(0)}X^i = -X^0 + \gamma_0(X), \quad U_i^{(0)} = -\delta_i^0 + d_i, \quad A_0 = \frac{1}{2}d_0(d_0 - 1), \]

\( i = 0, \ldots, n. \)

For \( \gamma = \gamma_0(X) \), i.e. when the harmonic time gauge is considered, we get the set of Lagrange equations for the Lagrangian

\[ L = \frac{1}{2}G_{ij}\dot{X}^i\dot{X}^j - V, \]

with the zero-energy constraint imposed

\[ E = \frac{1}{2}G_{ij}\dot{X}^i\dot{X}^j + V = 0. \]

It follows from the restriction \( U_0^{(s)} = 0 \) that

\[ (U^{(0)}, U^{(s)}) = U_i^{(0)}G^{ij}U_j^{(s)} = 0. \]

Indeed, the contravariant components \( U_i^{(0)} = G^{ij}U_j^{(0)} \) are the following ones

\[ U_i^{(0)} = -\frac{\delta_i^0}{d_0}. \]

Then we get \( (U^{(0)}, U^{(s)}) = U_i^{(0)}U_i^{(s)} = -U_0^{(s)}/d_0 = 0 \). In what follows we also use the formula

\[ \frac{1}{U_0^{(0)}} = (U^{(0)}, U^{(0)}) = \frac{1}{d_0} - 1 < 0, \]

for \( d_0 > 1. \)

In what follows we will make the following assumption on indices: \( s = 1, \ldots, m \) and \( \alpha = 0, \ldots, m. \)

**B General spherically symmetric and cosmological-type solutions**

When the orthogonality relations (A.8) and 3. of (3.1) are satisfied the Euler-Lagrange equations for the Lagrangian (A.4) with the potential (A.4) have the following solutions (see relations from (3) adopted for our case):

\[ X^i(u) = -\sum_{\alpha=0}^{m} \frac{U_i^{(\alpha)}}{(U^{(\alpha)}, U^{(\alpha)})}\ln |f_\alpha(u - u_\alpha)| + c^i u + \bar{c}^i, \]

where \( u_\alpha (\alpha = 0, \ldots, m) \) are integration constants; and vectors \( c = (c^i) \) and \( \bar{c} = (\bar{c}^i) \) are orthogonal to the \( U_i^{(\alpha)} = (U^{(\alpha)})^i \), i.e. they satisfy the linear constraint relations

\[ U_i^{(0)}(c) = U_i^{(0)}(c^i) = -c^0 + \sum_{j=0}^{n} d_j c^j = 0, \]

\[ U_i^{(0)}(\bar{c}) = U_i^{(0)}(\bar{c}^i) = -\bar{c}^0 + \sum_{j=0}^{n} d_j \bar{c}^j = 0, \]

\[ U_i^{(s)}(c) = U_i^{(s)}(c^i) = 0, \]

\[ U_i^{(s)}(\bar{c}) = U_i^{(s)}(\bar{c}^i) = 0. \]
Here
\[ f_\alpha(\tau) = R_\alpha \frac{\text{sh}(\sqrt{\alpha} \tau)}{\sqrt{\alpha}}, \quad C_\alpha \neq 0, \quad \eta_\alpha = +1, \]
\[ R_\alpha \frac{\text{ch}(\sqrt{\alpha} \tau)}{\sqrt{\alpha}}, \quad C_\alpha > 0, \quad \eta_\alpha = -1, \]
\[ R_\alpha \tau, \quad C_\alpha = 0, \quad \eta_\alpha = +1, \]
\[ R_\alpha = \sqrt{2|A_\alpha|/\nu_\alpha^2}, \quad \eta_\alpha = -\text{sign}(A_\alpha/\nu_\alpha^2); \] and parameters \( \nu_\alpha \) are defined in (3.10) and (A.10), \( \alpha = 0, \ldots, m. \)

The zero-energy constraint, corresponding to the solution (B.1) reads
\[ E = \frac{1}{2} \sum_{\alpha=0}^m C_\alpha \left( \frac{U^{(\alpha)}}{U^{(\alpha)}} \right) + \frac{1}{2} G_{ij} \epsilon^i \epsilon^j = 0. \]  

From (B.1) we get the following relation for the metric (see also (3.3), (A.9) and (A.10))
\[ g = e^{2\epsilon_0 u + 2\epsilon_0} \left( \prod_{\alpha=0}^m f_\alpha e^{2\nu_\alpha U^{(\alpha)} \alpha} \right) \left\{ d\tau \otimes d\tau + f_0^2 h[0] \right\} + \sum_{i \neq 0} e^{2\epsilon_0 u + 2\epsilon_0} \left( \prod_{\alpha=0}^m f_\alpha e^{2\nu_\alpha U^{(\alpha)} \alpha} \right) h[i], \]  

where \( f_\alpha = f_\alpha(u - u_\alpha) \) (here we use the relations \( d_iU^i + \frac{U_a}{a_0} = U^0 \) and (A.10)).

**Solutions with horizon.** For integration constants we put
\[ \epsilon^i = 0, \]
\[ \epsilon^i = \bar{\mu} \sum_{\alpha=0}^m \frac{U^{(\alpha)} i}{U^{(\alpha)}}, \]
\[ C_\alpha = \bar{\mu}^2, \]
where \( \bar{\mu} > 0, \alpha = 0, \ldots, m. \)

We also introduce new radial variable \( r = r(u) \) by relations
\[ \exp(-2\bar{\mu} u) = 1 - \frac{2\mu}{\bar{\mu} d}, \quad \mu = \bar{\mu}/d > 0, \quad d = d_0 - 1, \]
and put \( u_0 = 0; \ u_s < 0, \ A_s < 0, \ s = 1, \ldots, m, \)
\[ \frac{\sqrt{2|A_s|}}{\bar{\mu} \nu_s} \text{sh} \beta_s = 1, \quad \beta_s \equiv \bar{\mu}|u_s|. \]  

Now the parameter \( P_s \) may be introduced \( (P_s > 0) \) by the following relation:
\[ \frac{\mu}{\text{sh} \beta_s} = P_s e^{\beta_s} = \sqrt{P_s(P_s + 2\mu)}, \]  

and, hence,
\[ -A_s = \frac{1}{2} \nu_s^2 \bar{\mu} P_s (P_s + 2\mu), \]  

see (A.3). The relations of the Appendix imply the formulae (3.5), (3.6) for the solution from Section 3.

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List of captions for illustrations

Figure 1. The variants of behavior of $\rho(r)$ for $M_2 \cap M_2$ intersection.