Application of Linear Equation Systems in Banking Auditing

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Abstract. In this research, we use the Euler's equation formula that constitutes a linear system and it's application in bank audits. The solution of the systems can be obtained by Novel transformation.

1-Introduction
Banking are considered one of the most important elements of the economy and monetary policy, as bank account auditors faced a distinct comparison between benefits and cost in testing bank accounts to include employee time and effort, planning auditing, and testing controls to provide sufficient evidence to provide the required level of assurance in terms of benefits, increasing the economic complexity of investors 'request. High-quality audits to preserve their money, as well as achieve greater economic return and not waste money [4].
This requires the experience and skills of a specialist to audit bank accounts, and it needs mathematical processes and special programs, one of these processes is system of differential equations which solving by integral transforms.
Integral transforms are one of the most important methods used to solve differential equations with initial conditions [7], which they havewide applications in astronomy, geometry, and physics [10]. There are many Integral transforms in literature, likelaplace, Elzaki, Shehu, Al-Temimi transform, ...etc [2,3,5,8]. In this paper, we use Novel transform which derived from the Laplace transform, and the formula identifier
\[ \Omega(\rho) = N_I(\sigma(t)) = \frac{1}{\mu} \int_0^\infty e^{-\rho t} \sigma(t) dt, \]
where \( \Omega(t) \) is a real function, \( e^{-\rho t} \) is the kernel function, and \( N_I \) is the operator of NIT
Novel transform used to solve many ordinary and partial differential equations such as arising heat problems[1,13,12].
In the second section we reviewed some definitions, properties and through utilizing the relationship between Novel and Laplace transforms. In the thirdsection applications of Euler system of equation in cost accounting are shown. Finally, in the last section, some general formula of Euler-cauchy system of equations are solved by using the Novel transform without subjected to any initial conditions.

2-Basic definitions and properties of Novel transform.
In this section, we introduce some important definitions and properties that we need it in our work.
Definition 1[9]:
If \( \sigma(t) \) is integrable function, \( t > 0 \) then Laplace transform of \( \sigma(t) \) defined by:
\[ \mathcal{L}\{\sigma(t)\} = \int_0^\infty e^{-\rho t} \sigma(t) dt, \]
Let \( L_t(\sigma(t)) = \int_0^\infty e^{-\rho t} \sigma(t) dt \), \( t > 0 \) (2.1)
where \( L_t \) is the operator of LT.
We study the relationship between Laplace transform and Novel transform exhibit a duality relationship that expressed as:
\[ \Omega(\rho) = \frac{1}{\rho} L_t(\sigma(t)) \ldots (2.2) \]

**Definition 2**[11]:
Inherent Risks - IR represents a measure of the auditor's assessment of the potential for material errors before the effectiveness of internal control is taken into account.

**Definition 3**[11]:
Control Risks - CR is a measure of the auditor's assessment of the possibility of material errors that will not be prevented or discovered by the client's internal control.

**Property 13**: Linearity property
If \( \alpha_1, \alpha_2, \ldots, \alpha_n \) are constants and, \( \sigma_1(t), \ldots, \sigma_n(t) \), have Novel transform, then:
- i) \( N_t(\alpha_1 \sigma_1(t) + \alpha_2 \sigma_2(t) + \ldots + \alpha_n \sigma_n(t)) = \alpha_1 N_t(\sigma_1(t)) + \alpha_2 N_t(\sigma_2(t)) + \ldots + \alpha_n N_t(\sigma_n(t)) \ldots (2.3) \)
- ii) \( N_t^{-1}(\alpha_1 \sigma_1(t)) + \alpha_2 \sigma_2(t) + \ldots + \alpha_n \sigma_n(t)) = \alpha_1 N_t^{-1}(\sigma_1(t)) + \alpha_2 \sigma_2(t) + \ldots + \alpha_n \sigma_n(t) \ldots (2.4) \)

**Theorem 2**: If \( N_t(\sigma(t)), t > 0, \rho > 0, \) then we have:
\[ N_t(\sigma^n(t)) = \rho^n N_t(\sigma(t)) - \rho^{n-2} \sigma(0) - \rho^{n-3} \sigma(0) - \ldots - \sigma^{(n-2)}(0) - \frac{1}{\rho} \sigma^{(n-1)} \ldots (2.5) \]

**Theorem 2**: If the Novel transform of the function \( \sigma(t) \) is given by \( N_t(\sigma(t)) \), then
\[ N_t(t^n \sigma^n) = (-1)^n \left( \frac{\rho^n d^n}{d\rho^n} N_t(\sigma) + \alpha_1 \rho^{n-1} \frac{d^{n-1}}{d\rho^{n-1}} N_t(\sigma) + \ldots + \alpha_n N_t(\sigma) \right) \ldots (2.7) \]

where \( \alpha_1, \alpha_2, \ldots, \alpha_n \) are constants that depend on the proof on the value of \( n \).

In the following, Table for Novel transform of some functions inserted:

| ID | Function, \( \sigma(t) \) | \( \Psi(\mu) = \frac{1}{\rho} L(\sigma(t)) \) |
|----|--------------------------|-----------------------------------------------|
| 1  | 1                        | \( \frac{1}{\rho^2} \)                          |
| 2  | \( e^{\theta t} \)       | \( \frac{1}{\rho(\rho - \theta)} \)                |
| 3  | \( \sin \theta t \)      | \( \frac{1}{\rho^2 + \theta^2} \)                 |
| 4  | \( \cos \theta t \)      | \( \frac{1}{\rho^2 - \theta^2} \)                 |
| 5  | \( \sinh \theta t \)     | \( \frac{1}{\rho^2 - \theta^2} \)                 |
| 6  | \( \cosh \theta t \)     | \( \frac{1}{\rho^2 + \theta^2} \)                 |
| 7  | \( t^n \)                | \( \frac{n!}{\rho(\rho^{n+1})} \)                |

**Remark**: if system equations have a number of constants more than the degree of \( D \) in \( \Delta \), therefore we derive the solutions and substitute in the original system of equation, and equalize the coefficient, by doing this we get the values of the extra constants. Also using the assumption \( t = e^{\theta t} \) to obtain the solution of system in independent variable \( t \).

3- **Applications of Euler equation systems**
In this study, we will focus on control risks, the inherent risks towards the accounting system, awareness and actions of employees and managers, as it was mentioned [11,14] that these two areas are closely related to control risks and the inherent risks. Although the auditor relies to a large extent
on professional judgment in assessing risks, this does not mean that he does not rely on scientific and quantitative methods to contribute to the disclosure of the amount of each risk. The reader should not lose sight of the fact that quantitative methods are acceptable to all social circles. From what has been mentioned, mathematical systems of the Euler equation are considered one of the most appropriate means to express audit risks, as solving these systems gives accurate results for auditing and calculating the amount of risks for a particular model. For example, to express two types of risks, which are control risks and risks inherent under the influence of the time factor, where the risks of control are symbolized by \((\tau_1)\) and the inherent or inherent risks and symbolized by \((\tau_2)\), closely related to the extent of risks in the accounting system, and the awareness and actions of employees and managers symbolized by \((\omega_1)\) and \((\omega_2)\), respectively. If we assume that the total risks related to the accounting system towards both the risks of control and the inherent harm, the risks are distributed at a rate of 100% between them according to the professional judgment of the auditor, and the same is the case with the risks related to the position, awareness and actions of the employees and managers, we will have the following matrix

\[
\begin{pmatrix}
\%r \\ \%y \\ \%a
\end{pmatrix}
\begin{pmatrix}
\tau_1 \\ \tau_2
\end{pmatrix} +
\begin{pmatrix}
\%r \\ \%y \\ \%a
\end{pmatrix}
\begin{pmatrix}
\omega_1 \\ \omega_2
\end{pmatrix}
\]

From the above matrix, we have
\((\tau_1) = 0.7 \omega_1 + 0.2 \omega_2\)
\((\tau_2) = 0.3 \omega_1 + 0.8 \omega_2\)

In general, if we have \(n\) risks, the following system represents the magnitude of these risks according to the time factor:

\[
t_\omega' = A_\sigma
\]

where \(A = \frac{d\sigma}{dt}\), \(\alpha\) is a constant square matrix.

This system solution represents the amount of risk in the banking audit.

4-General Formula of Euler-Cauchy system of equations

In this section, formula of general solution for Euler-Cauchy system equations are derived consider the first order Euler-Cauchy system of equations

\[
t_\omega' = A_\sigma \cdots (4.1)
\]

where \(A = \frac{d\sigma}{dt}\), \(\alpha\) is a constant square matrix.

Firstly, from using assumption \(t=e^x, dt = e^x dx\) to reduce the Euler system to system of constant coefficients.[6]. Therefore, system(4.1) convert to the system of first order has the formula \(\sigma = \beta_\sigma\) where

\[
\sigma' = \begin{pmatrix}
d_{\omega_1} \\
\vdots \\
d_{\omega_n}
\end{pmatrix}, \beta = \begin{pmatrix}
\beta_1 \\
\vdots \\
\beta_m
\end{pmatrix}, \sigma = \begin{pmatrix}
\sigma_1 \\
\vdots \\
\sigma_n
\end{pmatrix}
\]

so, \(\cdots (4.2)\)

4-1 The formula of general solution of an in-homogeneous system of order one.

Consider the system

\[
\sigma' = A_\sigma + U(x) \cdots (4.3)
\]

The previous system can be expressed in the following matrices

\[
\sigma' = \frac{d\sigma}{dx} = \begin{pmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{pmatrix}, \sigma = \begin{pmatrix}
\sigma_1 \\
\vdots \\
\sigma_n
\end{pmatrix}
\]

\[
\rho N_1(\sigma_1) = \frac{\sigma_1(0)}{\rho} = a_{11}N_1(\sigma_1) + a_{12}N_1(\sigma_2) + \cdots + a_{1n}N_1(\sigma_n) + N_1(u_1(x))
\]
\[ \rho N_1(\omega_2) - \frac{\omega_2(0)}{\rho} = a_{21} N_1(\omega_1) + a_{22} N_1(\omega_2) + \cdots + a_{2n} N_1(\omega_n) + N_1(u_2(x)) \]

\[ \vdots \]

\[ \rho N_1(\omega_n) - \frac{\omega_n(0)}{\rho} = a_{m1} N_1(\omega_1) + a_{m2} N_1(\omega_2) + \cdots + a_{mn} N_1(\omega_n) + N_1(u_n(x)) \]

without subjected to any initial conditions, and \( a_{11}, a_{12}, a_{21} \) and \( a_{nn} \) are constant. \( u_1(x), u_2(x), \ldots, u_n(x) \) are functions of \( x \).

Also, we can use Grammar Rule to find \( \sigma(x) \):

\[ N_1 \sigma(x) = \frac{X(\rho)}{Y(\rho)} \ldots (4.4) \]

Where \( Y(\rho) \) is matrix of \( nxn \) of \( \rho \) represents denominator of N.T of the function \( \sigma(x) \), and \( X(\rho) \) is also matrix of \( nxn \) of \( \rho \) and its degree smaller than the degree of \( Y(\rho) \).

The above system are not subjected to any initial conditions then by taking inverse Novel Transform of (4.4), we get

\[ \sigma(x) = N_1^{-1} \left\{ \frac{X(\rho)}{Y(\rho)} \right\} \ldots (4.5) \]

By the same way, we can find the formula of solutions \( \sigma(x), \ldots, \sigma_n(x) \) represented the set of solution of system (4.3), but have a number of constants more than the degree of \( D \) in \( \Delta \), therefore we derive the solutions and substitute in the original system of equation, and equalize the coefficient, by doing this we get the values of the extra constants. Also using the assumption \( t=e^x \) to obtain the solution of system (4.1) in independent variable \( t \).

**Remark:** If \( U(x) = 0 \) then the system (4.3) is homogenous system.

**4.2 The formula of general solution of non-homogeneous system of order two.**

A non-homogeneous system has the formula \( \sigma'' = A \sigma' + C \sigma + U \ldots (4.6) \)

Where \( \sigma'' = \begin{bmatrix} \frac{d^2 \sigma_1}{dx^2} \\ \frac{d^2 \sigma_2}{dx^2} \\ \vdots \\ \frac{d^2 \sigma_n}{dx^2} \end{bmatrix}, A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, H' = \begin{bmatrix} \frac{dH_1}{dx} \\ \frac{dH_2}{dx} \\ \vdots \\ \frac{dH_n}{dx} \end{bmatrix}, C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix}, \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix}, U(t) = \begin{bmatrix} u_1(x) \\ u_2(x) \\ \vdots \\ u_n(x) \end{bmatrix} \]

The solve the system (4.2.1) by take N.T then

\[ \rho^2 N_1(\omega_2) - \omega_2(0) - \frac{\omega_2'(0)}{\rho} = a_{21} \rho N_1(\omega_1) - a_{11} \frac{\omega_1(0)}{\rho} + a_{12} \rho N_1(\omega_2) - a_{12} \frac{\omega_2(0)}{\rho} + \cdots + a_{1n} \rho N_1(\omega_n) - a_{1n} \frac{\omega_n(0)}{\rho} + a_{1n} \rho N_1(\omega_1) + a_{1n} \frac{\omega_1(0)}{\rho} + \cdots + c_{11} N_1(\omega_1) + c_{11} N_1(\omega_2) + \cdots + c_{1n} N_1(\omega_n) + N_1(u_1) \]

\[ \rho^2 N_1(\omega_2) - \omega_2'(0) = a_{21} \rho N_1(\omega_1) - a_{21} \frac{\omega_1(0)}{\rho} + a_{22} \rho N_1(\omega_2) - a_{22} \frac{\omega_2(0)}{\rho} + \cdots + a_{2n} \rho N_1(\omega_n) - a_{2n} \frac{\omega_n(0)}{\rho} + a_{2n} \rho N_1(\omega_1) + a_{2n} \frac{\omega_1(0)}{\rho} + \cdots + c_{21} N_1(\omega_1) + c_{21} N_1(\omega_2) + \cdots + c_{2n} N_1(\omega_n) + N_1(u_2) \]
From eq. (5.2) we get:
\[
\rho^2 N_1(\omega_1) - \omega_1 \frac{\omega_1'(0)}{\rho} = a_{11} \rho N_1(\omega_1) - a_{11} \frac{\omega_1(0)}{\rho} + a_{12} \rho N_1(\omega_2) - a_{12} \frac{\omega_2(0)}{\rho} + \cdots + a_{mn} \rho N_1(\omega_n) - a_{mn} \frac{\omega_n(0)}{\rho} + N_1(u_1)
\]
where \( \omega_1(0), \omega_2(0), \cdots, \omega_n(0) \) and \( \omega_1'(0), \omega_2'(0), \cdots, \omega_n'(0) \) are initial conditions not defined, the after simplification the terms:
\[
\left( \rho^2 - a_{11} \rho - c_{11} \right) N_1(\omega_1) - (a_{12} \rho + c_{12}) N_1(\omega_2) - \cdots - (a_{1n} \rho + c_{1n}) N_1(\omega_n) = \omega_1(0) + \frac{\omega_1(0)}{\rho} - a_{11} \frac{\omega_1(0)}{\rho} - a_{12} \frac{\omega_2(0)}{\rho} - \cdots - a_{1n} \frac{\omega_n(0)}{\rho} + N_1(u_1)
\]
\[
\left( \rho^2 - a_{22} \rho - c_{22} \right) N_1(\omega_2) - (a_{21} \rho + c_{21}) N_1(\omega_1) - \cdots - (a_{2n} \rho + c_{2n}) N_1(\omega_n) = \omega_2(0) + \frac{\omega_2(0)}{\rho} - a_{21} \frac{\omega_1(0)}{\rho} - a_{22} \frac{\omega_2(0)}{\rho} - \cdots - a_{2n} \frac{\omega_n(0)}{\rho} + N_1(u_2)
\]
\[
\left( \rho^2 - a_{mn} \rho - c_{mn} \right) N_1(\omega_n) - (a_{m1} \rho + c_{m1}) N_1(\omega_1) - \cdots - (a_{m2} \rho + c_{m2}) N_1(\omega_2) = \omega_n(0) + \frac{\omega_n(0)}{\rho} - a_{m1} \frac{\omega_1(0)}{\rho} - a_{m2} \frac{\omega_2(0)}{\rho} - \cdots - a_{mn} \frac{\omega_n(0)}{\rho} + N_1(u_n)
\]
Now, we suppose:
\[
N_1(\omega_1) = \frac{X_1(\rho)}{Y_1(\rho)} = Y_1(\rho) 
eq 0
\]
By the similar method, we find:
\[
N_1(\omega_2) = \frac{X_2(\rho)}{Y_2(\rho)}, \quad Y_2(\rho) 
eq 0
\]
\[
N_1(\omega_n) = \frac{X_n(\rho)}{Y_n(\rho)}, \quad Y_n(\rho) 
eq 0
\]
Where \( X_1(\rho), X_2(\rho), \cdots, X_n(\rho), Y_1(\rho), Y_2(\rho), \cdots, Y_n(\rho) \) are polynomials of \( \rho \).
Since the degree of \( X_n(\rho) \) in \( \rho \) is less than the degree of \( Y_n(\rho) \).

by taking inverse to both sides of eq. we get:
\[
\omega_1 = N^{-1} \left( \frac{X_1(\rho)}{Y_1(\rho)} \right), \quad \omega_n = N^{-1} \left( \frac{X_n(\rho)}{Y_n(\rho)} \right)
\]
Then \( \omega_1, \omega_2, \cdots, \omega_n \) represents the general solution of system (4.6).

5. Applications
In this section, we show the usefulness and the effectiveness of Novel transform to find exact solutions of the Euler system of equations.

Example (1): To find the general solution of the Euler systems of equation
\[
\tau \sigma' = A \sigma \text{ where } \tau \sigma' = \begin{pmatrix} \frac{d\sigma_1}{dx} \\ \frac{d\sigma_2}{dx} \end{pmatrix}, \quad A = \begin{pmatrix} 4 & 5 \\ -4 & -4 \end{pmatrix} \quad (5.1)
\]

Solution: Using the formula (4.3) in previous section.
we reduce the Euler system to system of differential equation with constant coefficients independent \( x \).
\[
\bar{\sigma}'(x) = A \bar{\sigma}(x)
\]
Taking Novel transform to both sides of equation,
\[
\rho N_1(\bar{\sigma}_1) - \frac{\bar{\sigma}_1(0)}{\rho} = 4N_1(\bar{\sigma}_1) + 5N_1(\bar{\sigma}_2) \quad (5.2)
\]
\[
\rho N_1(\bar{\sigma}_2) - \frac{\bar{\sigma}_2(0)}{\rho} = -4N_1(\bar{\sigma}_1) - 4N_1(\bar{\sigma}_2) \quad (5.3)
\]
From eq. (5.2) we get
From eq. (1) we get we reduce the Euler system to system of differential equation with constant coefficients
\[ N_f(\sigma_1) = \frac{\alpha_1(0)}{\rho} + 5N_f(\sigma_2) \]
\[ N_f(\sigma_2) = \frac{\alpha_1(0)}{\rho(\rho - 4)} + \frac{5\alpha_2(0)}{(\rho - 4)} \]
we substitute the equation (5.4) in (5.3) we get
\[ N_f(\sigma_2) = -\frac{4\alpha_1(0)}{\rho(\rho^2 + 4)} + \frac{(\rho - 4)\alpha_2(0)}{\rho(\rho^2 + 4)} \]
take inverse to both side we get
\[ \sigma_2(x) = -2r_1 \sin(2x) + r_2 \cos(2x) - 2c_2 \sin(2x) \]
\[ \sigma_2(\ln(t)) = -2r_1 \sin(2 \ln(t)) + r_2 \cos(2 \ln(t)) - 2c_2 \sin(2 \ln(t)) \]
Since \( e^x = t \) then \( x = \ln(t) \).

Then \( \sigma_1, \sigma_2 \) represents the general solution of system (5.1) and \( r_1, r_2 \) are not defined. which equation the degree of 2 in \( \Delta \) since
\[ \Delta = \begin{vmatrix} (D - 4) & -5/4 \\ 4 & (D + 4) \end{vmatrix} = (D - 4)(D + 4) + 20 \]
\[ = D^2 + 4 \] this mean the polynomial of degree 2 .

**Example (2):** To find the general solution of the Euler systems of equation
\[ t\dot{\sigma} = A\sigma + U(t) \cdots (5.5) \]

where \( \dot{\sigma} = \begin{pmatrix} \frac{\alpha_1}{dx} \\ \frac{\alpha_2}{dx} \end{pmatrix} \), \( A = \begin{pmatrix} 3 & 3 \\ -1 & -1 \end{pmatrix} \), \( \sigma = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \), \( U(x) = \begin{pmatrix} e^x \\ 0 \end{pmatrix} \)

Solution: Using the formula (4.3) in pervious section.
we reduce the Euler system to system of differential equation with constant coefficients \( \dot{\sigma} = A\sigma + U(x) \) such that \( t\dot{\sigma} = \sigma(x) \), \( e^x = t \)
taking Novel transform to both sides of equation,
\[ \rho N_f(\sigma_1) - \frac{\alpha_1(0)}{\rho} = 3N_f(\sigma_1) + 3N_f(\sigma_2) + N_f(e^x) \cdots (5.6) \]
\[ \rho N_f(\sigma_2) - \frac{\alpha_2(0)}{\rho} = -N_f(\sigma_1) - N_f(\sigma_2) + N_f(1) \cdots (5.7) \]
From eq. (1) we get
\[ (\rho - 3)N_f(\sigma_1) = 3N_f(\sigma_2) + \frac{1}{\rho(\rho - 1)} + \frac{\alpha_1(0)}{\rho} \]
dived to both side on \( (\rho - 3) \)
\[ N_f(\sigma_1) = \frac{3N_f(\sigma_2)}{(\rho - 3)} + \frac{1}{\rho(\rho - 1)(\rho - 3)} + \frac{\alpha_1(0)}{\rho(\rho - 3)} \cdots (5.8) \]
we substitute the equation (5.8) in (5.7) we get
\[ N_f(\sigma_2) = \frac{\rho^2 - 5\rho + 3}{\rho^3(\rho - 1)(\rho - 2)} + \frac{\rho\alpha_2(0)}{\rho^2(\rho - 2)} - \frac{3\alpha_2(0)}{\rho^2(\rho - 2)} = \frac{\alpha_1(0)}{\rho^2(\rho - 2)} \cdots (5.9) \]
by using partition fractions for item
\[
N_1(\sigma_2) = \left( \frac{1}{\rho(\rho - 1)} - \frac{3}{4} \frac{\sigma_1(0)}{\rho^2} - \frac{3}{2} \frac{\sigma_2(0)}{\rho^3} \right) + \frac{\sigma_2(0)}{\rho(\rho - 2)} - \left( -\frac{3}{2} \frac{\sigma_2(0)}{\rho^2} + \frac{3}{2} \frac{\sigma_2(0)}{\rho(\rho - 2)} \right)
\]

Take inverse Novel transform to side
\[
\sigma_2(x) = e^x - \frac{3}{4} e^{2x} - \frac{1}{4} + \frac{3}{2} x + r_2 e^{2x} + \frac{3}{2} r_2 - \frac{3}{2} r_2 e^{2x} + \frac{1}{2} r_1 - \frac{1}{2} r_2 e^{2x}
\]

\[
\sigma_2(ln(t)) = e^{ln(t)} - \frac{3}{4} e^{2ln(t)} - \frac{1}{4} + \frac{3}{2} ln(t) + r_2 e^{2ln(t)} + \frac{3}{2} r_2 - \frac{3}{2} r_2 e^{2ln(t)} + \frac{1}{2} r_1 - \frac{1}{2} r_2 e^{2ln(t)}
\]

we substitute the equation (5.9) in (5.8) we get
\[
N_1(\sigma_1) = \frac{3}{\rho(\rho-1)(\rho-3)} \left( \frac{1}{\rho(\rho - 1)} - \frac{\sigma_1(0)}{\rho^2} - \frac{3}{2} \frac{\sigma_2(0)}{\rho^3} \right) + \frac{\sigma_2(0)}{\rho(\rho - 2)} - \left( -\frac{3}{2} \frac{\sigma_2(0)}{\rho^2} + \frac{3}{2} \frac{\sigma_2(0)}{\rho(\rho - 2)} \right) + \frac{r_1}{\rho(\rho - 2)}
\]

\[
N_1(\sigma_1) = \frac{4}{\rho(\rho-3)(\rho-1)} - \frac{9}{4} \frac{\sigma_2(0)}{\rho(\rho-3)(\rho-2)} - \frac{3}{4} \frac{\rho^2(\rho-3)}{\rho(\rho-3)(\rho-2)} + \frac{9}{2} \frac{\sigma_2(0)}{\rho(\rho-3)(\rho-2)} - \frac{3}{2} \frac{\sigma_2(0)}{\rho(\rho-3)(\rho-2)} + \frac{1}{\rho(\rho-3)(\rho-2)} \frac{\sigma_1(0)}{\rho(\rho-3)(\rho-2)} + \frac{1}{\rho(\rho-3)(\rho-2)} \frac{\sigma_1(0)}{\rho(\rho-3)(\rho-2)}
\]

By using partition fractions for item
\[
N_1(\sigma_1) = \left( \frac{-2}{\rho(\rho - 1)} + \frac{2}{\rho(\rho - 3)} \right) - \left( -\frac{9}{4} \frac{\sigma_2(0)}{\rho(\rho-3)(\rho-2)} + \frac{9}{4} \frac{\sigma_2(0)}{\rho(\rho-3)(\rho-2)} \right) - \left( -\frac{1}{2} \frac{\sigma_2(0)}{\rho(\rho-3)(\rho-2)} + \frac{3}{2} \frac{\sigma_2(0)}{\rho(\rho-3)(\rho-2)} \right) + \left( -\frac{3}{2} \frac{\sigma_2(0)}{\rho(\rho-3)(\rho-2)} + \frac{3}{2} \frac{\sigma_2(0)}{\rho(\rho-3)(\rho-2)} \right) + \frac{1}{\rho(\rho-3)(\rho-2)} \frac{\sigma_1(0)}{\rho(\rho-3)(\rho-2)} + \frac{1}{\rho(\rho-3)(\rho-2)} \frac{\sigma_1(0)}{\rho(\rho-3)(\rho-2)}
\]

Taking inverse Novel Transform of both side we get
\[
\sigma_1(x) = -2 e^x + \frac{9}{4} e^{2x} - \frac{1}{4} + \frac{3}{2} x + \frac{3}{2} r_2 e^{2x} - \frac{3}{2} r_2 - \frac{1}{2} r_1
\]

Then \( \sigma_1, \sigma_2 \) represents the general solution of system (5.5) and \( r_1, r_2 \) are not defined. Which equation the degree of 2 in \( \Delta \) since
\[
\Delta = \begin{vmatrix} (D - 3) & -3 \\ 1 & (D + 1) \end{vmatrix}
\]
\[= (\rho - 3)(\rho + 1) + 3\]

**Example (3):** To find the general solution of the Euler systems of equation \[t^2\omega'' = +A\omega \cdots (5.10)\]

Where \[t^2\omega'' = \left(\begin{array}{c}
\frac{d\omega_1}{dx^2} \\
\frac{d\omega_2}{dx^2}
\end{array}\right), \quad A= \left(\begin{array}{cc}
-3 & -2 \\
4 & 3
\end{array}\right), \quad \omega = \left(\begin{array}{c}
\omega_1 \\
\omega_2
\end{array}\right)\]

Solution: Using the formula (4.6) in pervious section.

we reduce the Euler system to system of differential equation with constant coefficients \[\omega''(x) = A\omega(x)\]

Taking Novel transform to both sides of equation, it is will be:

\[\rho^2N_1(\omega_1) - \omega_1(0) - \frac{\omega_1'(0)}{\rho} = -3N_1(\omega_1) - 2N_1(\omega_2) \cdots (5.11)\]

\[\rho^2N_1(\omega_2) - \omega_2(0) = 4N_1(\omega_1) + 3N_1(\omega_2) \cdots (5.12)\]

Where

\[(\rho^2+3)N_1(\omega_1) = -2N_1(\omega_2) + \omega_1(0) + \frac{\omega_1'(0)}{\rho}, \quad \omega_1(0) = \frac{\omega_1'(0)}{\rho}\]

\[(\rho^2-3)N_1(\omega_2) = 4N_1(\omega_1) + \omega_2(0) + \frac{\omega_2'(0)}{\rho}\]

From eq.(5.11) we get \(\omega_2(0)\)

\[N_1(\omega_1) = \frac{\omega_1(0)}{\rho^2 + 3}N_1(\omega_2) + \frac{\omega_2(0)}{\rho^2 + 3} + \frac{\omega_1'(0)}{\rho} \cdots (5.13)\]

we substitute in eq.(5.13) in (5.12) we get

\[N_1(\omega_2) = -\frac{2}{\rho^2 + 3}N_1(\omega_2) + \frac{\omega_1(0)}{\rho^2 + 3} + \frac{\omega_1'(0)}{\rho} \cdots (5.13)\]

By using partion fractions for item and take inverse Novel transform to side we get

\[\omega_2(x) = 2r_1 \cosh(x) - 2r_1 \cos(x) + 2r_1 \sinh(x) - 2r_1 \sin(x) + 2r_2 \cosh(x) - r_2 \cos(x) + 2r_2 \sinh(x) - r_2 \sin(x)\]

we substitute \(N_1(\omega_2)\) in eq. (5.13) we get

\[N_1(\omega_1) = \frac{-2\rho^2 - 6}{\rho^2 + 1}(\rho^2 + 3)\omega_2(0) + \frac{-2\rho^2 - 6}{\rho^2 + 1}(\rho^2 + 3)\omega_2'(0)\]

\[-\frac{\omega_1(0)}{\rho^2 + 1}(\rho^2 + 3)\omega_1(0) - \frac{\omega_1'(0)}{\rho^2 + 1}(\rho^2 + 3)\omega_1'(0)\]

By using partion fractions for item and take inverse Novel transform to side we get

\[\omega_1(x) = -r_2 \cosh(x) + r_2 \cos(x) + r_1 \sinh(x) + r_1 \sin(x) - r_1 \cosh(x) + 2r_1 \cos(x) - r_1 \sin(x) + 2r_1 \sin(x)\]

Then \(\omega_1, \omega_2\) represents the general solution of system(5.10) and \(r_1, r_2\) are not defined

**Example (4):** To find the general solution of the Euler systems of equation \[t^2\omega'' = +At\omega + B\omega + U(t) \cdots (5.14)\]

Where \[t^2\omega'' = \left(\begin{array}{c}
\frac{d\omega_1}{dx^2} \\
\frac{d\omega_2}{dx^2}
\end{array}\right), \quad A= \left(\begin{array}{cc}
0 & 2 \\
-2 & -1
\end{array}\right), \quad t\omega = \left(\begin{array}{c}
\frac{d\omega_1}{dx} \\
\frac{d\omega_2}{dx}
\end{array}\right), \quad B = \left(\begin{array}{cc}
-1 & 0 \\
1 & 2
\end{array}\right), \quad \omega = \left(\omega_1, \omega_2\right), \quad U(x) = \left(\frac{2e^x}{7}\right)\]

Solution: Using the formula (4.6) in pervious section.
we reduce the Euler system to system of differential equation with constant coefficients
\[ \sigma'(x) = A\sigma'(x) + B\sigma(x) + U(x) \]
Taking Novel transform to both sides of equation, it is will be:
\[ \rho^2 N_1(\sigma_1) - \frac{\sigma_1(0)}{\rho} - \frac{2\sigma_2(0)}{\rho} = 2\rho N_1(\sigma_2) - \frac{2\sigma_2(0)}{\rho} - N_1(\sigma_1) + N_1(2e^x) \cdots (5.15) \]
\[ 0 = -2\rho N_1(\sigma_1) + \frac{2\sigma_2(0)}{\rho} - \rho N_1(\sigma_2) + \frac{\sigma_2(0)}{\rho} + N_1(\sigma_1) + 2(\sigma_2) + N_1(7) \cdots (5.16) \]

From eq.(5.16) we get
\[ N_1(\sigma_2) = \frac{2\rho + 1}{\rho(\rho - 2)} N_1(\sigma_1) + \frac{7}{\rho^2(\rho - 2)} + \frac{2\sigma_1(0)}{\rho(\rho - 2)} + \frac{\sigma_2(0)}{\rho(\rho - 2)} \cdots (5.17) \]
we substitute eq. (5.17) in eq. (5.15) and some simplification we get
\[ N_1(\sigma_1) = \frac{18}{(\rho + 2)(\rho^2 - 1)} + \frac{\sigma_1(0)}{\rho(\rho - 2)} + \frac{4\sigma_2(0)}{\rho^2(\rho - 2)} + \frac{(\rho + 2)\sigma_1(0)}{\rho(\rho - 2)} \]

By using partion fractions for item and take inverse Novel transform to both side we get
\[ \sigma_1(x) = -12e^{-2x} \]
\[ + 12 \cosh(x) \]
\[ - 6 \sinh(x) \]
\[ + r_1 \cosh(x) + \frac{4}{3} r_2 e^{-2x} - \frac{4}{3} r_2 \cosh(x) \]
\[ + \frac{8}{3} r_2 \sinh(x) - \frac{4}{3} r_1 e^{-2x} + \frac{4}{3} r_1 \cosh(x) \]
\[ - \frac{5}{3} r_1 \sinh(x) \]
we substitute \( \sigma_1(x) \) in eq. (5.17) we get
\[ N_1(\sigma_2) = \frac{24}{(\rho^2 - 4)} - \frac{12}{\rho(\rho - 4)} + \frac{-24\rho^2 + 24\rho - 6}{\rho(\rho - 2)(\rho^2 - 1)} + \frac{\sigma_1(0)}{\rho(\rho - 1)} - \frac{\sigma_2(0)}{\rho(\rho^2 - 4)} + \frac{\sigma_2(0)}{\rho(\rho^2 - 4)} \]
\[ + \frac{11}{3} \rho^2 - \frac{20}{3} \rho + 3 \sigma_2(0) \]
\[ + \frac{\sigma_1(0)}{\rho(\rho - 2)(\rho^2 - 1)} \]
\[ - \frac{4}{3} \sigma_2(0) \]
\[ + \frac{\sigma_1(0)}{\rho(\rho^2 - 4)} + \frac{\sigma_1(0)}{\rho(\rho^2 - 1)} \]
\[ + \frac{(\rho^2 - 4)}{\rho(\rho - 2)(\rho^2 - 1)} \]
\[ + \frac{7}{\rho(\rho^2 - 2)} \]

By using partion fractions for item and take inverse Novel transform to both side we get
\[ \sigma_2(x) = 24 \cosh(2x) - 6 \sinh(2x) - 18e^{2x} - 6 \cosh(x) + 12 \sinh(x) + r_1 \sinh(x) \]
\[ - \frac{8}{3} r_2 \cosh(2x) + \frac{2}{3} r_2 \sinh(2x) + r_2 e^{2x} + \frac{8}{3} r_2 \cosh(x) - \frac{4}{3} r_2 \sinh(x) \]
\[ + \frac{3}{7} r_2 \cosh(2x) - \frac{3}{7} r_2 \sinh(2x) - r_1 e^{2x} - \frac{5}{3} r_1 \cosh(x) + \frac{4}{3} r_1 \sinh(x) - \frac{7}{2} \]
\[ + \frac{2}{e^{2x}} \]

Then \( \sigma_1, \sigma_2 \) represents the general solution of system(5.14) and \( r_1, r_2 \) are not defined

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