Strangeness Equilibration at GSI Energies

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(November 21, 2018)

Abstract

We develop the notion of “broad-band equilibration” in heavy-ion processes involving dense medium. Given density-dependent \(K^-\)-masses we show that the equilibration at GSI energies claimed to hold in previous treatments down to \(\sim \rho_0/4\), can be replaced by a broad-band equilibration in which the \(K^-\)-meson and hyperons are produced in an essentially constant ratio independent of density. There are experimental indications that this also holds for AGS energies. We then proceed to argue that both \(K^+\) and \(K^-\) must get lighter in dense medium at some density \(\rho > \rho_0\) due to the decoupling of the vector mesons. As a consequence, kaon condensation in compact stars could take place before chiral restoration since the sum of bare quark masses in the kaon should lie below \(\mu_e\). Another consequence of the decoupling vector interactions is that the quasi-particle picture involving (quasi)quarks, presumably ineffective at low densities, becomes more appropriate at higher densities as chiral restoration is approached.
1 Introduction

Following work by Hagedorn [1] on production of anti-\(^{3}\)He, Cleymans et al [2] have shown that for low temperatures, such as found in systems produced at GSI, strangeness production is strongly suppressed. The abundance of \(K^+\) mesons, in systems assumed to be equilibrated, is given by [3],

\[
n_{K^+} \sim e^{-E_{K^+}/T} V \left\{ g_K \int \frac{d^3p}{(2\pi)^3} e^{-E_K/T} + g_\Lambda \int \frac{d^3p}{(2\pi)^3} e^{-(E_\Lambda - \mu_B)/T} \right\}.
\] (1)

Here the \(g\)'s are the degeneracies. Because strangeness must be conserved in the interaction volume \(V\), assumed to be that of the equilibrated system for each \(K^+\) which is produced, a particle of “negative strangeness” \(^{1}\) containing \(\bar{s}\), say, \(\bar{K}\) or \(\Lambda\), must also be produced, bringing in the \(\bar{K}\) or \(\Lambda\) phase space and Boltzmann factors. The \(K^+\) production is very small at GSI energies because of the low temperatures which give small Boltzmann factors for the \(\bar{K}\) and \(\Lambda\).

Note the linear dependence on interaction volume which follows from the necessity to include \(\bar{K}\) or \(\Lambda\) phase space.

In an extensive and careful analysis, Cleymans, Oeschler and Redlich [3] show that measured particle multiplicity ratios \(\pi^+ / p\), \(\pi^- / \pi^+\), \(d / p\), \(K^+ / \pi^+\), and \(K^+ / K^-\) – but not \(\eta / \pi^0\) – in central Au-Au and Ni-Ni collisions at (0.8-2.0)A GeV are explained in terms of a thermal model with a common freeze-out temperature and chemical potential, if collective flow is included in the description. In other words, a scenario in which the kaons and anti-kaons are equilibrated appears to work well. This result is puzzling in view of a recent study by Bratkovskaya et al [4] that shows that the \(K^+\) mesons in the energy range considered would take a time of \(\sim 40\) fm/c to equilibrate. We remark that this is roughly consistent with the estimate for higher energies in the classic paper by Koch, Müller and Rafelski [5] that strangeness equilibration should take \(\sim 80\) fm/c. Such estimates have been applied at CERN energies and the fact that emergent particle abundances are described by Boltzmann factors with a common temperature \(\sim 170 - 180\) MeV has been used as part of an argument that the quark/gluon plasma has been observed.

We interpret the result of [3] as follows. Since free-space masses are used for the hadrons involved, Cleymans et al [3] are forced to employ a \(\mu_B\) substantially less than the nucleon mass \(m_N\) in order to cut down \(\Lambda\) production as compared with \(K^-\) production, the sum of the two being equal to \(K^+\) production. This brings them to a diffuse system with density of only \(\sim \rho_0 / 4\) at chemical freeze-out. But this is much too low a density for equilibration.

We shall first show how this situation can be improved by replacing the \(K^-\) mass by the \(K^-\) energy at rest \(\omega_{K^-}^\ast \equiv \omega_{-}(k = 0) < m_K\). (The explicit formula for \(\omega_{\pm}\) is given later, see eq. (21).) In doing this, we first have to reproduce the \(K^+\) to \(K^-\) ratio found in the Ni + Ni experiments [6]:

\[
n_{K^+}/n_{K^-} \simeq 30.
\] (2)

Cleymans et al reproduce the earlier smaller ratio of \(21 \pm 9\) with \(\mu_B = 750\) MeV and \(T = 70\) MeV. How this or rather [3] comes out is easy to see. The ratio of the second term on the RHS of eq.(21) to the first term is roughly the ratio of the exponential factors multiplied by the phase

\(^{1}\)By “negative strangeness” we are referring to the negatively charged strange quark flavor. The positively charged anti-strange quark will be referred to as “positive strangeness.”
where we have used \( g_A \approx g_K \approx 2 \), \((\bar{p}_A)^2/2m \approx \frac{3}{2} T \), \( E_A = 1115 \text{ MeV} \) and \( E_K = 495 \text{ MeV} \). We are able to convert \( E_A \) to \( m_A \) and \( E_{K^-} \) to \( m_{K^-} \) because \( E_A \approx m_A + \frac{3}{2} T \) and \( E_{K^-} \approx m_{K^-} + \frac{3}{2} T \) and the thermal energies cancel out in the ratio. This works as long as the masses are more than \(~ 3T\), where the nonrelativistic approximation is valid. Inclusion of the \( \Sigma \) and \( \Xi \) hyperons would roughly increase this number by 50% with the result that the ratio of \( K^- \) to \( \Lambda \), \( \Sigma \), \( \Xi \) production is

\[
\frac{n_{K^-}}{n_{\Lambda+\Sigma+\Xi}} \approx 1/32. \tag{4}
\]

Since a \( K^+ \) must be produced to accompany each particle of one unit of strangeness (to conserve strangeness flavor), we then have

\[
n_{K^+}/n_{K^-} \sim 33. \tag{5}
\]

This is consistent with the empirical ratio (3). It should be noted that had we set \( \mu_B \) equal to \( m_N \), we would have had the \( K^+ \) to \( K^- \) ratio to be \(~ 280\) because it costs so much less energy to make a \( \Lambda \) (or \( \Sigma \)) rather than \( K^- \) in this case. In other words the chemical potential \( \mu_B \) is forced to lie well below \( m_N \) in order to penalize the hyperon production relative to that of the \( K^- \)s.

One can see from fig.5 in Li and Brown that without medium effects in the \( K^- \) mass, the \( K^+/K^- \) ratio is \(~ 100\), whereas the medium effect decreases the ratio to about 23. This suggests how to correctly redo the Cleymans et al’s analysis, namely, by introducing the dropping \( K^- \) mass into it.

In Appendix A we show that positive strangeness production takes place chiefly at densities greater than \( 2\rho_0 \). As the fireball expands to lower densities the amount of positive strangeness remains roughly constant. The number of \( K^+ \)’s is such as to be in equilibrium ratio \( K^+ / \pi^+ \) with the equilibrated number density of pions at \( T = 70 \text{ MeV} \), \( n_\pi \approx 0.37 T_{157}^3 \text{ fm}^{-3} \). Only in this sense do the \( K^+ \)’s equilibrate.

It is amusing to note that the “equilibrated ratio” of \(~ 30\) for the \( n_{K^+}/n_{K^-} \) holds over a large range of densities for \( T = 70 \text{ MeV} \), once density-dependent \( K^- \) masses are introduced, in that the ratio \( R \) of (3) is insensitive to density. (Remember that because of the small number of \( K^- \)’s, the number of \( K^+ \)’s must be nearly equal to the number of hyperons, \( \Lambda \), \( \Sigma^- \) and \( \Xi \), in order to conserve strangeness.) This insensitivity results because \( \omega_{K^-}^* \) decreases with density at roughly the same rate as \( \mu_B \) increases. We can write \( R \) of (3), neglecting possible changes in \( T \) and \( m_K \) in our lowest approximation, as

\[
R = \left( \frac{m_A}{m_K} \right)^{3/2} e^{(\mu_B + \omega_{K^-}^*)/T} e^{-m_A/T}. \tag{6}
\]

\(^2\)In order to reproduce this result with \( \mu_B = 750 \text{ MeV} \) and \( T = 70 \text{ MeV} \) within our approximation, we have assumed only the \( \Sigma^- \) and \( \Sigma^0 \) hyperons to equilibrate with the \( \Lambda \). This may be correct because the \( \Sigma^+ \) and \( \Xi \) couple more weakly. Inclusion of the latter could change our result slightly. Probably they should be included in analysis of the AGS experiments at higher energies where they would be more copiously produced.
As will be further stressed later, the most important point in our arguments is that the $\mu_B + \omega^\ast_{K^-}$ is nearly constant with density. This is because whereas $\mu_B$ increases from 860 MeV to 905 MeV as $\rho$ goes from $1.2\rho_0$ to $2.1\rho_0$, $\omega^\ast_{K^-}$ decreases from 370 MeV to 322 MeV, the sum $\mu_B + m^\ast_{K^-}$ decreasing very slightly from 1230 MeV to 1227 MeV. Indeed, even at $\rho = \rho_0/4$, $\mu_B + m^\ast_{K^-} \sim 1215$ MeV, not much smaller.

We believe that the temperature will change only little in the region of dropping masses because in a consistent evolution (which we do not carry out here) the scalar field energy $m_N^2\sigma^2/2$ in a mean field theory plays the role of an effective bag constant. In ref.\cite{8} this is phrased in terms of a modified Walecka theory,

$$B_{eff} = \frac{1}{2} m_N^2\sigma^2 \Rightarrow \frac{1}{2} m_N^2(M_N/g_{\sigma NN})^2,$$

(7)

the $\sigma$ going to $M_N/g_{\sigma NN}$ as the nucleon effective mass goes to zero with chiral restoration. Most of the energy with compression to higher densities goes with this effective bag constant, estimated \cite{8} to be $\sim 280$ MeV/fm$^3$, rather than heat, mocking up the behavior of a mixed phase with constant temperature. Moreover at $\rho = 2\rho_0$ where $m_N^\ast$ may be $\sim 0.5m_N$, only about 25% of the bag constant $B$ may have been achieved, so there may be some increase in temperature. We shall, at the same level of accuracy, have to replace $m_K$ in the prefactor of eq.(6) by $\omega^\ast_{K^-}$. We adjust the increase in temperature so that it exactly compensates for the decrease in prefactor so that the $K^+/K^-$ ratio is kept the same, as required by experiment \cite{6}. We then find that the temperature at $\rho = 2\rho_0$ must be increased from 70 to 95 MeV. This is roughly the change given by that in inverse slopes of $K^-$ and $K^+$ transverse momentum distributions found in going from low multiplicities to high multiplicities \cite{6}.

In any case, we see that $R$ will depend but little on density. This near cancellation of changes in the factors is fortunate because the $K^- + p \leftrightarrow \Lambda$ reaction, operating in the negative strangeness sector, is much stronger than the positive strangeness reactions, so the former should equilibrate to densities well below $2\rho_0$ and we can see that the “apparent equilibration” might extend all the way down to $\rho \sim \rho_0/4$.

The near constancy of $R$ with density also explains the fact that the $K^+/K^-$ ratio does not vary with centrality \cite{5}. Although $R$ is the ratio of $\Lambda$’s to $K^-$’s, both of which are in the negative strangeness sector, nonetheless, the number of $K^+$’s must be equal to the sum of the two and since the $\Lambda$’s are much more abundant than the $K^-$’s, $R$ essentially represents the $K^+/K^-$ ratio.

Detailed transport results of Bratkovskaya and Cassing (see Fig.4) show the last scattering of the detected $K^-$ to be spread over all densities from $\rho_0/2$ to $3\rho_0$, somewhat more of the last scatterings to come from the higher density. This seems difficult to reconcile with a scenario of the $K^-$ numbers being decided at one definite density and temperature, but given our picture of dropping masses, one can see that the $K^+/K^-$ ratio depends little on density ($\mu_B$) at which the $K^-$ last interacts. In any case we understand from our above argument that the apparent density of equilibration can be chosen to be very low in a thermal description and still get more or less correct $K^+/K^-$ ratio.

\footnote{We are grateful to Helmut Oeschler for pointing this out to us.}
Figure 1: Calculations by Bratkovskaya and Cassing (private communication) which show the density of origin and that of the last interaction of the $K^-$ mesons.

2 The Top-Down Scenario of $K^\pm$ Production

Brown and Rho [9] discussed fluctuations in the kaon sector in terms of a simple Lagrangian

$$\delta \mathcal{L}_{KN} = \frac{-6i}{8f_{\omega}^2} (\overline{N}\gamma_0 N) \overline{K} \partial_t K + \frac{\Sigma_{KN}}{f_{\omega}^2} (\overline{N}N) \overline{K} K + \cdots \equiv \mathcal{L}_\omega + \mathcal{L}_\sigma + \cdots$$

(8)

It was suggested there that at high densities, the constituent quark or quasi-quark description can be used with the $\omega$-meson coupling to the kaon viewed as a matter field (rather than as a Goldstone boson). Such a description suggests that the $\omega$ coupling to the kaon which has one non-strange quark is $1/3$ of the $\omega$ coupling to the nucleon which has three non-strange quarks. The $\mathcal{L}_\omega$ in the Lagrangian was obtained by integrating out the $\omega$-meson as in the baryon sector. We may therefore replace it by the interaction

$$V_{K^\pm} \approx \pm \frac{1}{3} V_N.$$

(9)

In isospin asymmetric matter, we shall have to include also the $\rho$-meson exchange with the vector-meson coupling treated in the top-down approach.

For the top-down scenario, we should replace the chiral Lagrangian [8] by one in which the “heavy” degrees of freedom figure explicitly. This means that $\frac{1}{2f_{\omega}^2}$ in the first term of (8) should be replaced by $g^*_{\omega}^2/m_{\omega}^2$ and $\frac{\Sigma_{KN}}{f_{\omega}^2}$ in the second term by $\frac{2}{3}m_K g_\sigma^2/m_{\sigma}^2$ assuming that both $\omega$ and $\sigma$ are still massive. We will argue in the next section that while the $\omega$ mass drops, the ratio $g^*_{\omega}^2/m_{\omega}^2$ should stay constant or more likely decrease with density and that beyond certain density above nuclear matter, the vector fields should decouple. On the other hand, $g_\sigma$ is not scaled in the mean field that we are working with; the motivation for this is given in Brown, Buballa and Rho [8] who construct the chiral restoration transition in the mean field in the Nambu-Jona-Lasinio model. Thus

$$\frac{\Sigma_{KN}}{f^2} \approx \frac{2}{3} m_K \frac{g_\sigma^2}{m_{\sigma}^2}.$$
In this framework, $m_\sigma^*$ is the order parameter for chiral restoration which drops à la BR scaling \cite{10}:

$$\frac{m_\sigma^*}{m_\sigma} \equiv \Phi(\rho) \simeq \frac{1}{1 + y\rho/\rho_0}$$

with $y \simeq 0.28$, at least for $\rho \lesssim \rho_0$ \cite{4}.

Once the vector is decoupled, a simple way to calculate the in-medium kaon effective mass, equivalent to using the $L_\sigma$, is to consider the kaon as fluctuation about the "$\sigma$"-axis in the V-spin formalism \cite{11} as depicted pictorially in Fig.2. The Hamiltonian for explicit chiral symmetry breaking is

$$H_{\chi SB} = \Sigma_{KN} \langle NN \rangle \cos(\theta) + \frac{1}{2} m_K^2 f^2 \sin^2(\theta)$$

$$\simeq \Sigma_{KN} \langle NN \rangle (1 - \frac{\theta^2}{2}) + \frac{1}{2} m_K^2 f^2 \theta^2$$

where the last expression is obtained for small fluctuation $\theta$. Dropping the term independent of $\theta$, we find

$$m_{K^\pm}^2 = m_K^2 \left(1 - \frac{\Sigma_{KN} \langle NN \rangle}{f^2 m_K^2}\right).$$

Using eq. (10) we obtain

$$m_{K^\pm}^2 = m_K^2 \left(1 - \frac{2 g_\sigma^2 \langle NN \rangle}{3 m_\sigma^2 m_K}\right).$$

In accord with Brown and Rho \cite{9} we are proposing that eq.(13) should be used for low densities, in the Goldstone description of the $K^\pm$, and that we should switch over to eq.(14) for higher densities. It is possible that the $m_K$ appearing in (14) should be replaced by $m_\sigma^*$ for self-consistency but the dropping of $m_\sigma^2$ makes the $m_{K^\pm}^*$ of (14) decrease more rapidly than that of (13) so that eq.(13) with $\langle NN \rangle$ set equal to the vector density $\rho$, a much used formula valid to linear order in density,

$$m_{K^\pm}^2 \approx m_K^2 \left(1 - \frac{\rho \Sigma_{KN}^4}{f^2 m_K^2}\right)$$

\footnote{\[y\] may well be different from this value for $\rho > \rho_0$. In fact the denominator of $\Phi(\rho)$ could even be significantly modified from this linear form. At present there is no way to calculate this quantity from first principles.}
obviously gives too slow a decrease of \(m_K\) with density.

Although the above are our chief points, there are two further points to remark. One, even without scaling, our vector interaction on the kaon is still too large. Two, more importantly, there is reason to believe in the large \(\Sigma_{KN}\) term,

\[
\Sigma_{KN} \sim 400 \text{ MeV.}
\] (16)

This comes from scaling of the pion sigma term

\[
\Sigma_{KN} = \left(\frac{m_u + m_s}{m_u + m_d}\right) \langle N|\bar{u}u + \bar{s}s|N\rangle \Sigma_{\pi N},
\] (17)

Taking \(m_s \sim 150\) MeV, \(m_u + m_d \sim 12\) MeV, \(\Sigma_{\pi N} = 46\) MeV and \(\langle N|\bar{s}s|N\rangle \sim \frac{1}{3}\langle N|\bar{d}d|N\rangle\) from lattice calculations \([12]\), one arrives at (16).

Other authors, in adjusting the \(\Sigma\) term to fit the kaon-nucleon scattering amplitudes, have obtained a somewhat smaller \(\Sigma_{KN}\). This can be understood in the chiral perturbation calculation of C.-H. Lee \([13]\) where the only significant effect of higher chiral order terms can be summarized in the “range term” \(^5\), namely \(\Sigma_{KN}\) is to be replaced by an effective \(\Sigma\),

\[
(\Sigma_{KN})_{\text{eff}} = (1 - 0.37\omega_K^2/m_{K}^2)\Sigma_{KN}.
\] (18)

It should be pointed out that although the \(\Sigma_{KN}\) is important at low densities, \(\omega_K\) decreases with \(m_K^*\), this “range-term” correction becomes less important at higher densities. This effect – which is easy to implement – is included in the realistic calculations.

### 3 Partial Decoupling of the Vector Interaction

#### 3.1 Evidence

There are both theoretical and empirical reasons why we believe that the vector interaction should decouple at high density.

1. We first give the theoretical arguments. We know of three theoretical reasons why the vector coupling \(g_\omega^*\) should drop with density.

   - The first is the observation by Song et al \([14]\) that describing nuclear matter in terms of chiral Lagrangian in the mean field requires the ratio \(g_\omega^*/m_\omega^*\) to at least be roughly constant or even decreasing as a function of density. In fact to quantitatively account for non-linear terms in a mean-field effective Lagrangian, a dropping ratio is definitely favored \(^6\). For instance, as discussed in \([14]\), the in-medium behavior of the \(\omega\)-meson field is encoded in the “FTS1” version of the non-linear theories of ref. \([15]\). In fact, because of the attractive quartic \(\omega\) term in the FTS1 theory, the authors of \([15]\) have (for the parameter \(\eta = -1/2\) favored by experiments) \(g_\omega^2/m_{\omega}^2 \simeq 0.8g_{\omega}^2/m_{\omega}^2\) as modification of the quadratic term when rewritten in our notation. In other words, their vector mean field contains a partial decoupling already at \(\rho \approx \rho_0\) although they do not explicitly scale \(g_\omega\) as we do.

\(^5\)In the language of heavy-baryon chiral perturbation theory, this corresponds to the “1/m” correction term.

\(^6\)Since the non-linear terms – though treated in the mean field – are fluctuation effects in the effective field theory approach, this represents a quantum correction to the BR scaling.
Historically, Walecka-type mean field theories with only quadratic interactions (i.e., linear Walecka model) gave compression moduli $K \sim 500$ MeV, about double the empirical value. This is cured in nonlinear effective field theories like FTS1 by higher-dimension non-renormalizable terms which effectively decrease the growth in repulsion in density. As suggested in [14], an effective chiral Lagrangian with BR scaling can do the same (by the increase in magnitude of the effective scalar field with density) but more economically and efficiently.

- The second reason is perhaps more theoretical. Kim and Lee [16] have shown recently that in an effective QCD Lagrangian with baryons, pions and vector mesons put together in hidden gauge symmetric theory, the ratio $g^V/m^V$ (where $V$ stands for hidden gauge bosons) falls very rapidly with baryon chemical potential. The main agent for this behavior is found in [16] to be the pionic one-loop contribution linked to chiral symmetry which is lacking in the mean-field treatment for BR scaling [10]. One may argue on a more profound ground that the vector decoupling in approaching chiral phase transition is a flow to a fixed point. It has been argued recently by Harada and Yamawaki [17] that chiral symmetry restoration may correspond to a “vector manifestation” of chiral symmetry where the octet of Goldstone pions and the octet of longitudinal vector mesons belong to the representation $(8, 1) \oplus (1, 8)$. At this point, the vector coupling goes zero à la Georgi’s vector limit as does the vector meson mass [13]. The above arguments were made for the $\rho$ which has a simple interpretation in terms of hidden gauge symmetry but it will apply to the $\omega$ if the nonet symmetry continues to hold in nuclear medium. It is difficult to be quantitative as to how fast the ratio falls but it is clear that the drop is substantial already near normal nuclear matter density.

- Finally, close to chiral restoration in temperature, there is clear evidence from QCD for an equally rapid drop, specifically, from the quark number susceptibility that can be measured on the lattice [19]. The lattice calculation of the quark number susceptibility dealt with quarks and the large drop in the (isoscalar) vector mean field was found to be due chiefly to the change-over from hadrons to quarks as the chiral restoration temperature is approached from below. The factor of 9 in the ratio $g_{\omega NN}/g_{\omega QQ}$ (where $Q$ is the constituent quark) should disappear in the change-over. Now since the electroweak properties of a constituent quark (quasiquark) are expected to be the same as those of a bare Dirac particle with $g_A = 1$ and no anomalous magnetic moment (i.e., the QCD quark) [21] with possible corrections that are suppressed as $1/N_c$, [21], there will be continuity between before and after the chiral transition. This is very much in accordance with the “Cheshire-Cat picture” developed elsewhere [22]. In fact, it is possible to give a dynamical (hadronic) interpretation of the above scenario. For instance in the picture of [23], this may be understood as the “elementary” $\omega$ strength moving downwards into the “nuclear” $\omega$, the $[N^*(1520)N^{-1}]J=1,I=0$ isobar-hole state involving a single-quark spin flip [24]. The mechanism being intrinsic in the change-over of the degrees of freedom, we expect the same phenomenon to hold in density as well as in temperature. The upshot of this line of argument is that the suppression of the vector coupling is inevitable as density approaches the critical density for chiral transition.
We believe that the different behavior of vector and scalar mean fields, the latter to be discussed below, follows from their different roles in QCD. With the vector this is made clearer in the lattice calculations of the quark number susceptibility which involves the vector interactions. In Brown and Rho [19], it is shown that as the description changes from hadronic to quark/gluonic at \( T \sim T_c \), the critical temperature for chiral restoration, the vector interaction drops by an order of magnitude, much faster than the logarithmic decrease due to asymptotic freedom. We expect a similar feature in density, somewhat like in the renormalization-group analysis for the isovector vector meson \( \rho \) of Kim and Lee [16]. The scalar interaction, on the other hand, brings about chiral restoration and must become more and more important with increasing density as the phase transition point is approached.

2. From the empirical side, the most direct indication of the decoupling of the vector interaction is from the baryon flow [25] which is particularly sensitive to the vector interaction. The authors of [25] find a form factor of the form

\[ f_V(p) = \frac{\Lambda_V^2 - \frac{1}{2}p^2}{\Lambda_V^2 + p^2} \]  

with \( \Lambda_V = 0.9 \text{ GeV} \) is required to understand the baryon flow. Connecting momenta with distances, one finds that this represents a cutoff at

\[ R_{\text{cutoff}} \sim \sqrt{6} \frac{\Lambda_V}{6} \sim 0.5 \text{ fm}. \]  

Furthermore it is well known that the vector mean field of the Walecka model must be modified, its increase as \( E/m_N \) removed, at a scale of \( \sim 1 \text{ GeV} \). The reason for this is presumably that inside of \( R \sim 0.5 \text{ fm} \), the finite size of the solitonic nucleon must be taken into account. A repulsion still results, but it is scalar in nature as found in [26] and for which there are direct physical indications [27].

3.2 Kaons at GSI

The \( K^+ \) and \( K^- \) energies in the top-down scenario are given by

\[ \omega_\pm = \pm \frac{1}{3} \frac{\omega_+}{m_K} V_N + \sqrt{k^2 + m_K^*}. \]  

Although at high densities it will decouple, the term linear in \( V_N \) that figures in the range correction in \( (\Sigma_{KN})_{\text{eff}} \) will give a slightly different effective mass for \( K^+ \)and \( K^- \)before decoupling. Although the large distance vector mean field must arise from vector meson exchange, this must be cut off at a reasonably large distance, say, \( \sim 0.5 \text{ fm} \) as indicated by the baryon-flow mentioned above.

For the GSI experiments with temperature \( \sim 75 \text{ MeV} \), the momentum is \( |p| \sim m_N/2 \), and

\[ f_V^2(p) \sim 0.6. \]  

\[ \text{(22)} \]
We therefore propose to use
\[
\frac{V_K}{m_K} = \frac{3}{8} \left( \frac{f_V^2}{m_K^2} \right)^2 \rho \approx 0.07.
\] (23)

This is small. Furthermore we assume it to be constant above \( \rho_0 \).

The above arguments could be quantified by a specific model. For example, as alluded to above, the low-lying \( \rho \)- and \( \omega \)-excitations in the bottom-up model can be built up as \( N^* \)-hole excitations [23]. At higher densities, these provide the low-mass strength. One might attempt to calculate the coupling constants to these excitations in the constituent-quark (or quasi-quark) model, which as we have suggested would be expected to be more applicable at densities near chiral restoration. Riska and Brown [24] find the quark model couplings to be a factor \( \sim 2 \) lower than the hadronic ones [28].

4 Schematic Model

4.1 First try

On the basis of our above considerations, a first try in transport calculation might use the vector potential with the Song scaling [14] as \( g_\omega/m_\omega^* = \text{constant} \) and the effective mass
\[
m_K^* = m_K^2 \left( 1 - \frac{\rho\langle \Sigma_{KN} \rangle_{\text{eff}}}{f^2 m_K^2} \right)
\] (24)

with \( \Sigma_{KN} = 400 \text{ MeV} \) and \( \langle \Sigma_{KN} \rangle_{\text{eff}} \) given by eq. (18). While as argued above the vector coupling will decouple at very high densities, as \( \omega_K \) drops, the vector potential will become less important even at moderate densities since the factor \( \omega_K/m_K \) comes into the coupling of the vector potential to the kaon.

Our schematic model (24) gives roughly the same mass as used by Li and Brown [7] to predict kaon and antikaon subthreshold production at GSI. For \( \rho \sim 3 \rho_0 \) it gives \( m_K^* \sim 230 \text{ MeV} \), less than half the bare mass. We predict somewhat fewer \( K^\pm \)-mesons because the (attractive) vector interaction is largely reduced if not decoupled. Cassing et al [29] have employed an \( m_K^* \) somewhat lower than that given by eq.(24) to describe a lot of data.

The experimental data verify that our description is quite good up to the densities probed, i.e., \( \sim 3 \rho_0 \). In order to go higher in density, we switch to our top-down description which through eq.(10) involves \( g_\sigma^2/m_\sigma^2 \). Although the scalar interaction could have roughly the same form factor as the vector, cutting it off at \( \sim 0.5 \text{ fm} \) as mentioned above, we believe that this will be countered by the dropping scalar mass \( m_\sigma^* \) which must go to zero at chiral restoration (viewed as an order parameter). Treating the scalar interaction linearly as a fluctuation (as in (10)) cannot be expected to be valid all the way to chiral restoration but approaching the latter the \( \sigma \)-particle becomes the “dilaton” in the sense of Weinberg’s “mended symmetry” [30, 31] with mass going to zero (in the chiral limit) together with the pion.

At high densities at which the vector interaction decouples, the \( K^+ \) and \( K^- \) will experience nearly the same very strong attractive interactions. This can be minimally expressed through the effective mass \( m_K^* \). At low densities where the vector potential not only comes into play but slightly predominates over the scalar potential, the \( K^+ \) will have a small repulsive interaction with nucleons. It is this interaction, extrapolated without medium effects by Bratkovskaya et al [4] which gives the long equilibration time of 40 fm/c. However, clearly the medium effects will change this by an order of magnitude.
4.2 Implication on kaon condensation and maximum neutron-star mass

While in heavy-ion processes, we expect that taking \( m^*_\sigma \) to zero (or nearly zero in the real world) is a relevant limiting process, we do not have to take \( m^*_\sigma \) to zero for kaon condensation in neutron stars, since the \( K^- \) mass \( m^*_K \) must be brought down only to the electron chemical potential \( \mu_e \simeq E_F(e) \), the approximate equality holding because the electrons are highly degenerate. We should mention that it has been suggested that the electron chemical potential \( \mu_e \) could be kept low by replacing electrons plus neutrons by \( \Sigma^- \) hyperons (or more generally by exploiting Pauli exclusion principle with hyperon introduction) in neutron stars \[32\]. In this case, the \( \mu_e \) might never meet \( m^*_K \).

Hyperon introduction may or may not take place, but even if it does, the scenario will be more subtle than considered presently. To see what can happen, let us consider what one could expect from a naive extrapolation to the relevant density, i.e., \( \rho \sim 3 \rho_0 \), based on the best available nuclear physics. The replacement of neutron plus electron will take place if the vector mean field felt by the neutron is still high at that density. The threshold for that would be

\[
E^\rho_F + V_N + \mu_e \simeq M_{\Sigma^-} + \frac{2}{3} V_N + S_{\Sigma^-}
\]

(25)

where \( E^\rho_F \) is the Fermi energy of the neutron, \( M_{\Sigma^-} \) the bare mass of the \( \Sigma^- \) and the \( S_{\Sigma^-} \) the scalar potential energy felt by the \( \Sigma^- \). Here we are simply assuming that the two non-strange quarks of the \( \Sigma^- \) experience \( 2/3 \) of the vector mean field felt by the neutron. Extrapolating the FTS1 theory \[15\] and taking into account in \( V_N \) the effect of the \( \rho \)-meson using vector dominance , we find \( E^\rho_F + V_N \sim 1064 \text{ MeV} \) at \( \rho \approx 3 \rho_0 \). From the extended BPAL 32 equation of state with compression modulus 240 MeV \[33\], the electron chemical potential comes out to be \( \mu_e \simeq 214 \text{ MeV} \). So the left-hand side of (25) is \( E^\rho_F + V_N + \mu_e \sim 1258 \text{ MeV} \). For the right-hand side, we use the scalar potential energy for the \( \Sigma^- \) at \( \rho \approx 3 \rho_0 \) estimated by Brown, Lee and Rapp \[34\] to find that \( M_{\Sigma^-} + \frac{2}{3} V_N + S_{\Sigma^-} \sim 1240 \text{ MeV} \). The replacement of neutron plus electron by \( \Sigma^- \) looks favored but only slightly.

What is the possible scenario on the maximum neutron star mass if we continue assuming that the calculation we made here can be trusted? A plausible scenario would be as follows. \( K^- \) condensation supposedly occurs at about the same density and both the hyperonic excitation (in the form of \( \Sigma N^{-1} \) – where \( N^{-1} \) stands for the nucleon hole – component of the “kaesobar” \[24\]) and \( K^- \) condensation would occur at \( T \sim 50 \text{ MeV} \) relevant to the neutron-star matter. Now if as is likely the temperature is greater than the difference in energies between the two possible phases, although the hyperons will be more important initially than the kaons, all of the phases will enter more or less equally in constructing the free energy of the system. In going to higher density the distribution between the different phases will change in order to minimize the free energy. Then it is clear that dropping from one minimum to another, the derivative of the free energy with density – which is just the pressure – will decrease as compared with the pressure from any single phase. This would imply that the maximum neutron star mass calculated with either hyperonic excitation or kaon condensation alone must be greater than the neutron star mass calculated with inclusion of both.

The story will be quite different if the vector field decouples. We showed in Section \[3.1\] that the isoscalar vector mean field must drop by a factor \( \lesssim 9 \) in the change-over from

\[7\]There is nothing that would suggest that the effective Lagrangian valid up to \( \rho \sim \rho_0 \) will continue to be valid at \( \rho \sim 3 \rho_0 \) without addition of higher mass-dimension operators, particularly if the chiral critical point is nearby. So this exercise can be taken only as indicative.
nucleons to quasiquarks as variables as one approaches the chiral restoration density. Hyperons will disappear during this drop. It is then inevitable that the kaon will condense before chiral restoration and that the kaon condensed phase will persist through the relevant range of densities which determine the maximum neutron star mass.

5 Concluding Remarks

By now there is a general consensus that the light-quark hadrons must behave differently in medium than in free space. This is understood in terms of a vacuum change induced by medium à la QCD. In this paper, we are re-confirming this property by arguing that not only the kaon mass but also its coupling to vector mesons should drop in matter with density. In particular, with the introduction of medium effects the apparent equilibration found in strangeness production at GSI can be increased from the baryon number density of \( \sim \frac{1}{4} \rho_0 \) up to the much more reasonable \( \sim 2 \rho_0 \).

From the baryon flow analysis we have direct indications that the vector interaction decouples from the nucleon at a three-momentum of \( |p| \sim 0.9 \text{ GeV/c} \) or at roughly \( 0.5 m_{NC} \). In colliding heavy ions this is reached at a kinetic energy per nucleon of \( \sim \frac{1}{2} m_{NC}^2 \) which means a temperature of 78 MeV when equated to \( \frac{3}{2} T \). This is just the temperature for chemical freeze-out at GSI energies. We have given several theoretical arguments why the vector coupling should drop rapidly with density.

Once the vector mean field, which acts with opposite signs on the \( K^+ \) and \( K^- \) mesons is decoupled, these mesons will feel the same highly attractive scalar meson field. Their masses will fall down sharply; e.g., from eq.(24) with proposed parameters,

\[
\frac{m^*_{K^-}}{m_K} \sim 0.5
\]

at \( \rho \sim 3 \rho_0 \) and possibly further because of the dropping \( m^*_\sigma \). The differing slopes of \( K^+ \) and \( K^- \) with kinetic energy will then develop after chemical freeze-out, as suggested by Li and Brown [7].

In this paper we have focused on the phenomenon at GSI energies. Here the chief role that the dropping \( K^- \)-mass played was to keep the combination \( \mu_B + \omega^*_{K^-} \) nearly constant so that low freeze-out density in the thermal equilibration scenario became irrelevant for the \( K^+/K^- \) ratio. We suggest that the same scenario applies to AGS physics, where the freeze-out density in the thermal equilibration picture comes out to be \( \sim 0.35 \rho_0 \) [35]. In increasing this density to \( \sim 0.57 \rho_0 \), which we consider more reasonable, \( \mu_B + \omega^*_{K^-} \) changes from 992 MeV only to 1026 MeV, an \( \sim 3 \% \) change, when only the density dependence is put into \( \omega^*_{K^-} \). It is likely to change less when the temperature dependence, that we are now working on, is added. In fact, there is no discernible dependence on centrality in the \( K^-/K^+ \) ratios measured at 4A GeV, 6A GeV, 8A GeV and 10.8A GeV [36]. From this it follows either that the ratio of produced \( K^- \) to hyperons is nearly independent of density or that the negative strangeness equilibrates down to a lower freeze-out density and then disperses. Given the relative weakness of the strange interactions, we believe the former to be the case. In fact, we suggest that near constancy with multiplicity of the \( K^-/K^+ \) ratio found experimentally be used to determine temperature dependence of \( \omega^*_{K^-} \) in the region of temperatures reached in AGS physics. As was done for GSI energies, the temperature can be obtained from the inverse slopes of the kaon and antikaon distributions of \( p_{\perp} \) and then the temperature dependence of \( \omega^*_{K^-} \) can be added to the density.
dependence as in [37], in such a way that \( \mu_B + \omega_{K^-} \) stays roughly constant as function of density. At least this can be done in the low-density regime considered in [35] when the approximation of a Boltzmann gas is accurate enough to calculate \( \mu_B \). Our “broad-band equilibration”; i.e., the production of the same, apparently equilibrated ratio of \( K^- \)-mesons to hyperons over a broad band of densities, avoids complications in the way in which the \( K^- \) degree of freedom is mixed into other degrees of freedom at low density [38]. Most of the \( K^- \)-production will take place at the higher densities, as shown in Fig. 1, where the degrees of freedom other than \( K^- \) have been sent up to higher energies by the Pauli principle.

Unless the electron chemical potential in dense neutron star matter is prevented from increasing with density (as might happen if the repulsive \( \Sigma \)-nuclear interaction turns to attraction at \( \rho > \rho_0 \)), kaon condensation will take place before chiral restoration. This has several implications at and beyond chiral restoration. For instance, its presence would have influence on the conjectured color superconductivity at high density, in particular regarding its possible coexistence with Overhauser effects, skyrmion crystal and other phases with interesting effects on neutron star cooling.

The phenomenon of vector decoupling, if confirmed to be correct, will have several important spin-off consequences. The first is that it will provide a refutation of the recent claim [39] that in the mean-field theory with a kaon-nuclear potential given by the vector-exchange (Weinberg-Tomozawa) term – both argued to be valid at high density – kaon condensation would be pushed up to a much higher density than that relevant in neutron-star matter. Our chief point against that argument is that the vector decoupling and the different role of scalar fields in QCD (e.g., BR scaling) described in this article cannot be accessed by the mean field reasoning used in [39] or by any other standard nuclear physics potential models. The second consequence that could be of a potential importance to the interpretation of heavy-ion experiments is that if the vector coupling [4] rapidly diminishes with density, the strong-coupling perturbation calculation of the vector response functions used in terms of “melting” vector mesons to explain [40], e.g., the CERES dilepton data must break down as one approaches the decoupling point that is out of reach of strong-coupling perturbative methods. It in turn provides yet one more justification (in addition to what has been already offered [41]) to the quasi-particle description in dense matter encoded in BR scaling and exploited in this paper.

Finally we should stress that the scenario presented in this paper – which is anchored on Brown-Rho scaling – should not be considered as an alternative to a possible quark-gluon scenario currently favored by the heavy-ion community. It is more likely a sort of “dual” description of the same physics along the line that the Cheshire-Cat Principle [22] embodies that would continue to apply at higher energies.

Acknowledgments

The initial part of this work was done while we were visiting Korea Institute for Advanced Study whose hospitality is acknowledged. We are grateful for comments from Bengt Friman and Chang-Hwan Lee. One of the authors (GEB) is indebted to Elena Bratkovskaya, Peter Braun-

\[8\] While the decoupling of the isoscalar vector interaction near the chiral restoration is highly plausible by the disappearance of a factor of 9 in the change-over from hadrons to quarks, the decoupling of the isovector vector interaction seems to be a lot more complex as indicated by the RG analysis of Kim and Lee [16]. What is not difficult to see is that their decoupling should occur at the same point as indicated in the quark number susceptibility [19].
Appendix A

In this appendix we show how our argument that gives a correct $K^+/K^-$ ratio can reproduce the $K^+/{\pi^+}$ ratio. Let us leave $T = 70$ MeV and choose $\rho \sim 2\rho_0$ as educated guesses. We are thereby increasing the equilibration density by a factor $\sim 8$. We then calculate the baryon density for this $\mu_B$ and $T = 70$ MeV and find $\rho = 2\rho_0$ which checks the consistency.

According to Brown et al. [37] the $K^+$ production under these conditions will come chiefly from $BB \to N\Lambda K$, with excited baryon states giving most of the production. From the solid curve for $\rho/\rho_0 = 2$ in fig.9 of [37] we find $\langle \sigma v \rangle \sim 2 \times 10^{-3} \text{ mb} = 2 \times 10^{-4} \text{ fm}^2$. The rate equation reads

$$\frac{d\Psi_K}{dt} = \frac{1}{2} (\sigma_{BB}^{BY K} v_{BB}) \rho_B^2 = \frac{d\rho}{dt} = \frac{1}{9} \times 10^{-4} \text{ fm}^{-4} \quad (A.1)$$

where $B$, $Y$ and $K$ stand, respectively, for baryon, hyperon and kaon. For this, we have taken $\langle \sigma_{BB}^{BY K} v_{BB} \rangle$ from fig.9 of [37] and $\rho = 2\rho_0 = \frac{4}{3} \text{ fm}^{-3}$. Choosing a time $t = 10 \text{ fm}/c$ we obtain

$$n_{K^+} \sim \delta \Psi_K t = \frac{2}{9} \times 10^{-3} \text{ fm}^{-3}. \quad (A.2)$$

Now equilibrated pions have a density

$$n_\pi = 0.37(T/197\text{MeV})^3 \text{ fm}^{-3} = 0.016 \text{ fm}^{-3} \quad (A.3)$$

for $T = 70$ MeV. From (A.2) and (A.3) we get

$$n_{K^+}/n_\pi \simeq 0.0069 \quad (A.4)$$

which is slightly below the “equilibrated value” of 0.0084 of Table 1 of Cleymans et al. [3]. Production of $K^+$ by pions may increase our number by $\sim 25\%$ [12].

Our discussion of $K^+$ production in this Appendix is in general agreement with earlier works by Ko and collaborators [42, 43]. In fact, if applied at the quark level, the vector mean field is conserved through the production process in heavy-ion collisions, so it affects only the strangeness condensation in which there is time for strangeness non-conservation. These earlier works establish that at the GSI energies the $K^+$ content remains roughly constant once the fireball has expanded to $\sim 2\rho_0$, so that in this sense one can consider this as a chemical freeze-out density.

It should be noted that in the papers [29, 37, 12, 8] and others, the net potential – scalar plus vector – on the $K^+$-meson is slightly attractive at $\rho \sim 2\rho_0$ even though the repulsive vector interaction is not decoupled. Since in our top-down description the vector interaction can be thought of as applied to the quark (matter) field in the $K^+$, the total vector field on the initial components of a collision is then the same as on the final ones, so the vector mean fields have effectively no effect on the threshold energy of that process. Our proposal in this paper is that the vector mean field on the $K^+$ should be below the values used by the workers in [29, 37, 12, 43] due to the decoupling. However, in comparison with [29], our total potential on the $K^+$ at $\rho = 2\rho_0$ is $\sim -85 \text{ MeV}$, as compared with their $\sim -30 \text{ MeV}$. We have not redone the calculation of [37] to take into account this difference.
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