Anti-Jamming Games for Multi-User Multi-Band Networks

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Abstract—For multi-user multi-band networks, a zero-sum game between the users and the jammer is considered. In the formulation of the game, the rewards of the users are modeled with various parameters including communication rate, hopping cost, and jamming loss. It is analytically shown that for any symmetric collision avoidance protocol, a staying-threshold frequency hopping and a sweeping attack establish an equilibrium. We also propose two kinds of collision avoidance protocols to ensure that at most one user communicates in a band, and provide various numerical results that show the effect of the reward parameters and collision avoidance protocols on the optimal threshold of the staying-threshold frequency hopping and the expected rewards of the users.

I. INTRODUCTION

In the era of Internet of Things (IoT), more and more autonomous devices are connected through wireless networks. However, their communications are vulnerable to various jamming attacks [1], [2], due to the broadcast and superposition nature of wireless environment. For multi-band networks, the effect of jamming can be mitigated through frequency hopping [3]–[5]. Recently, as the IoT devices and jammers become more intelligent so that they can analyze the environment and update their strategies adaptively, the competitive development of frequency-hopping policies and jamming strategies was formulated into a zero-sum game between the users and the jammer. The arms race of the game was analyzed for cognitive networks with a single secondary user [6]. In [6], the sweep attack was considered as a strategy of the jammer at some stage of the arms race, and an optimal frequency hopping policy against the sweep jamming was analytically characterized through Markov decision process (MDP). This frequency hopping policy, which we call staying-threshold policy in this paper, is to stay at the same channel if it is not jammed and has stayed there less than a certain threshold time, and otherwise hop. Furthermore, it was numerically shown that the sweep jamming and the staying-threshold policy are near an equilibrium, although the equilibrium was not directly shown. The study of characterizing optimal frequency hopping policies against the sweep jammer has been extended to single-user scenarios with some options of communication modes [7], [8]. The work [7] considered a scenario where a user can choose the communication rate among a finite set of candidates. In this setup, an optimal frequency hopping policy was shown to be the staying-threshold policy with non-increasing transmission rate until the user hops to the other channels. For multi-user networks, the interference among the users need to be considered. This makes the anti-jamming problem more complicated and most of the previous works on anti-jamming game in multi-user scenarios rely on Q-learning [9], which approximates the environment in an empirical way. In particular, the works [10], [11] considered the setting where the communication of each user consists of sensing part and transmission part. The sensing part learns the strategies of the jammer and the other users based on Q-learning, and the transmission part avoids jamming and interference based on the learning result of the sensing part. However, to the best of our knowledge, there has been no prior work providing theoretical analysis for the anti-jamming game in multi-user scenarios based on MDP formulation.

In this paper, we analyze the anti-jamming game for multi-user multi-band networks. In the formulation of the game, the rewards of the users are modeled with various parameters including communication rate, hopping cost, and jamming loss. It is analytically shown that for any symmetric collision avoidance protocol, the staying-threshold frequency hopping policy and the sweeping attack establish an equilibrium. Furthermore, we show that the optimal threshold of the staying-threshold policy can be obtained in an iterative way and the number of iterations is finite. We also propose two kinds of collision avoidance protocols to ensure that at most one user communicates in a band, and provide various numerical results that show the effect of the reward parameters and collision avoidance protocols on the optimal threshold of the staying-threshold frequency hopping policy and the expected rewards of the users.

II. PROBLEM STATEMENT

A. User model

We consider the time-slotted multi-band communication system with \( n \) sender-receiver pairs (S-R pairs). The frequency band is divided into \( M \) non-overlapping channels \( \{c_1, c_2, ..., c_M\} \). In each time slot, each sender tries to transmit its data packets to the intended receiver through one of the \( M \) channels. We assume that the communication delays are negligible. The network is illustrated in Fig. 1.

The overall transmission model in each time slot consists of the following phases.
Fig. 1: In a frequency band, each sender communicates to its receiver and a jammer interrupts the communication. \((M = 9, n = 3)\)

- Sensing phase: The S-R pairs are assumed to use an error-correcting code designed for point-to-point communications, hence an appropriate collision avoidance protocol needs to be incorporated. We give priority to the S-R pair that already occupied the channel. To do so, at the beginning of each time slot, called sensing phase, the sender who already occupied the channel broadcasts a pilot signal. Then the S-R pairs who just hopped to the channel give up the communication and randomly hop to other channels in the next time slot.

- Collision avoidance phase: If there is an S-R pair who already occupied the channel, this phase is skipped. If not, i.e., it is silent in the sensing phase, since there can be more than one S-R pairs newly hop to the channel, we apply some collision avoidance protocols to ensure that at most one S-R pair communicates in the channel. Some examples of collision avoidance protocols are provided in Section IV.

- Communication phase: The communication of the S-R pair, who already occupied the channel or who was allowed to communicate through the collision avoidance protocol, takes place.

- Jamming detection phase: The S-R pair, who just finished the communication, judges whether the communication has been jammed. We assume that the jamming hypothesis test is correct with high probability.

- Action phase: At the end of the time slot, each S-R pair determines whether to hop or not based on its current and past states (not allowed to transmit, if allowed to transmit, successfully communicated or jammed).

If an S-R pair determines to hop, the next channel is selected uniformly at random among all \(M\) channels. Also, each S-R pair is assumed to share a sufficiently long pseudo-random sequence, so that they can hop to the same channel. The overall transmission in each time slot is described in Fig. 2.

In Appendix 1, we consider the case where the the next channel is selected uniformly at random among the other \(M - 1\) channels except the current channel.

Fig. 2: Transmission protocol: \((a)\) and \((b)\) are the S-R pairs’ protocol corresponding to silent and non-silent sensing phases, respectively, and \((c)\) is the jammer’s protocol.

Fig. 3: Sweep jamming example for \(M = 9\) and \(m = 3\). The shaded channels represent the channels scanned by the jammer.

B. Jammer model

A jammer scans \(m < M\) channels in each time slot to test whether the S-R pairs are communicating or not as shown in Fig. 1. We assume that \(m\) divides \(M\), and let \(T := M/m\). If the jammer detects some communications in the scanned channels, then it attacks all the channels where the communications take place by transmitting sufficiently large Gaussian noise so that the receivers cannot decode the data packets with high probability. We assume that the channel scanning and detection at the jammer takes more time than the sensing and collision avoidance, i.e., the sensing and collision avoidance phases are not affected by the jamming attack. The jamming model is described in Fig. 2.

C. Arms race

The S-R pairs and the jammer play a zero-sum game, i.e., the S-R pairs and the jammer try to maximize and minimize the “reward”, respectively. This reward, precisely defined in Section III, is set to take into account the communication throughput, jamming loss, hopping cost, and the priority of the current compared to the future. The zero-sum game can be described as an arms race [6]. In the arms race, when the jammer changes the jamming strategy to minimize the reward, the S-R pairs change the frequency hopping policy to maximize it, and vice versa. In some cases, as the arms race continues, the game reaches an equilibrium, where the jamming strategy and the frequency hopping policy no longer change. In this paper, we analyze how the arms race proceeds and examine when the game reaches an equilibrium.

The arms race starts with a naive jamming strategy, the random jamming. The random jamming changes the set of
targeted $m$ channels in each time slot uniformly at random. The combating hopping policy corresponding to the random jamming would be the minimal hopping policy where the S-R pairs hop only when they are not allowed to communicate (to avoid collision among the S-R pairs). Note that an S-R pair stays in the same channel even if it is jammed, since the random jamming newly and randomly chooses the set of $m$ channels for each time slot and hence there is no reason to hop by paying the hopping cost.

In response to the minimal hopping policy of the S-R pairs, the best strategy for the jammer is the sweep jamming. In the sweep jamming, the jammer scans all the $M$ channels in any window of $T$ consecutive time slots. To do so, the pattern of choosing $m$ channels across the time is fixed to shown in Fig. 3. We analyze the arms race that proceeds after the sweep jamming and show that the game reaches an equilibrium in Section IV.

### III. MDP FORMULATION

For each S-R pair, the process of finding an optimal frequency hopping policy can be modeled as a Markov decision process (MDP) \[12\], under the environment determined by the hopping policies of the other S-R pairs and the jamming strategy. The MDP model of an S-R pair is described by a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{U}, \gamma)$. First, the set $\mathcal{S}$ of states is given as $\{J, I, 1, 2, \ldots, K_m\}$. The state $S_t \in \mathcal{S}$ at time $t$ is determined at the end of the jamming detection phase. It becomes $K \in [1 : K_m]$ if the S-R pair has successfully communicated for $K$ consecutive times up to time $t$, where $K_m$ denotes the largest possible number of consecutive times the communication of the S-R pair is successful. This $K_m$ is affected by the jamming strategy and it can be infinite for some jamming strategies. The state $S_t$ is $I$ if the S-R pair did not get the chance to communicate at time $t$, and it is $J$ if the S-R pair tried to communicate but it was jammed. The set $\mathcal{A}$ of actions that the S-R pair can take is $\{s, h\}$, i.e., stay in the same channel or hop uniformly at random. In the action phase of time $t$, the S-R pair chooses an action $A_t \in \mathcal{A}$ based on its current state $S_t$. Next, the probabilistic transition function $\mathcal{P} : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0, 1]$ represents the probability of the next state $S_{t+1}$ given the current state $S_t$ and action $A_t$. The transition function depends on the jamming strategy and the collision avoidance protocol. We note that the set of possible next states is not reduced depending on the current state and action. If $A_t = h$, then $S_{t+1} \in \{J, I, 1\}$ regardless of $S_t$ as shown in Fig. 4. The transitions for the case of $A_t = s$ are represented in Fig. 5. If $A_t = s$ and $S_t \in [1 : K_m - 1]$, then $S_{t+1} \in \{S_t + 1, J\}$ because the S-R pair will try to communicate at time $t + 1$ since it has occupied the channel, and $S_{t+1}$ will be $S_t + 1$ if the communication is successful and $S_{t+1}$ will be $J$ otherwise. If $A_t = s$ and $S_t = K_m$, then $S_{t+1} = J$ from the definition of $K_m$. Now, the S-R pair gets a reward at the end of time $t$, based on $S_t$ and $A_t$, according to the reward function $\mathcal{U} : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$. If the communication was successful in time $t$, the S-R pair gets a reward $R > 0$ proportional to the communication throughput. If the S-R pair tried to communicate but it was jammed, the S-R pair just wasted some communication resource such as transmit power and gets a reward $-L < 0$. If the S-R pair decides to hop at the end of time $t$, it needs to pay a hopping cost $C > 0$, because the sender cannot start communication immediately right after it hops to new channel due to the settling time, e.g., the settling time is about 7.6 ms in Atheros chipset card \[3\]. Another reason behind the hopping cost is due to the addition of the collision avoidance phase. Note that the collision avoidance phase is skipped if the S-R pair stays in the same channel.

By taking into account all the aforementioned factors, the reward $U_{t+1}$ at time $t + 1$ (right after time $t$) for state $S_t$ and action $A_t$ is given as follows:

\[
U_{t+1}(S_t, A_t) = \begin{cases} 
R & \text{for } S_t \in [1 : K_m], A_t = s, \\
R - C & \text{for } S_t \in [1 : K_m], A_t = h, \\
-L - C & \text{for } S_t = J, A_t = h, \\
-C & \text{for } S_t = I, A_t = h.
\end{cases}
\]

(1)

The policy $\pi : \mathcal{S} \to \mathcal{A}$ of each S-R pair specifies the action $\pi(S)$ that the S-R pair will choose in state $S$. We define the set of all possible policies as $\Pi$. If an S-R pair starts communication at $t = 0$, the value function corresponding to policy $\pi$ with initial state $S$ is defined by

\[
V_\pi(S) = E_\pi \left[ \sum_{t=0}^{\infty} \gamma^t U_{t+1}(S_t, A_t) \middle| S_0 = S \right],
\]

(2)

where $0 < \gamma < 1$ is the discount factor, the parameter that captures the importance of the current reward compared to the future rewards. Hence, $V_\pi(S)$ corresponds to the expected discounted sum of rewards if the S-R pair determines its action based on policy $\pi$ when the initial state $S_0$ is $S$. Similarly, the
action-value function corresponding to policy \( \pi \), starting with state \( S \) and action \( A \), is defined by

\[
Q_\pi(S, A) = E_\pi \left[ \sum_{t=0}^{\infty} \gamma^t U_{t+1}(S_t, A_t) \bigg| S_0 = S, A_0 = A \right].
\]

The value function and the action-value function can be derived using Bellman expectation equation (3).

The maximum action-value function \( V_\pi(S) \) and the maximum action-value function \( Q_\pi(S, A) \) over all possible policies \( \pi \) are denoted by \( V^*(S) \) and \( Q^*(S, A) \), respectively. They have the following relationship:

\[
V^*(S) = \max_{A \in A} Q^*(S, A).
\]

If policy \( \pi^* \) satisfies \( V^*_{\pi^*}(S) \geq V_\pi(S) \) for any policy \( \pi \) and state \( S \in S \), then \( \pi^* \) is said to be optimal. An optimal policy \( \pi^* \) also satisfies \( Q^*_{\pi^*}(S, A) \geq Q_{\pi}(S, A) \) for any \( \pi, S \in S \), and \( A \in A \). It can be easily checked that if a policy \( \pi^* \) satisfies the following, it is optimal.

\[
\pi^*(S) \in \arg\max_{A \in A} Q^*(S, A)
\]

Hence, we can find the optimal frequency hopping policy from the maximum action-value function \( Q^*(S, A) \). A standard approach to derive \( Q^*(S, A) \) is to use the following Bellman optimality equation (4):

\[
Q^*(S_t, A_t) \overset{(a)}{=} U_{t+1}(S_t, A_t) + \gamma \sum_{S_{t+1} \in S} p(S_{t+1}|S_t, A_t) V^*(S_{t+1})
\]

\[
\overset{(b)}{=} U_{t+1}(S_t, A_t) + \gamma \sum_{S_{t+1} \in S} \max_{A_{t+1} \in A} p(S_{t+1}|S_t, A_t) Q^*(S_{t+1}, A_{t+1}),
\]

where \( p(S_{t+1}|S_t, A_t) \) is from the probabilistic transition function \( \mathcal{P} \). Here, (a) is since the maximum action value function can be expressed by using the next reward and the probabilistic sum of the next maximum value functions and (b) is due to (4).

However, Bellman optimality equation does not have a closed form solution due to the non-linearity. Instead, we can derive \( Q^*(S, A) \) in an iterative way by using value iteration [13]. If the environment (the transition function) of the MDP is not fully known to the S-R pairs, one possible approach to obtain \( Q^*(S, A) \) is to use the Q-learning [2], which approximates the unknown transition function in an empirical way.

Note that the starting state \( S_0 \) is a random variable for our model, where \( S_0 \in \{1, J, I\} \). Hence, as a criterion for evaluating the jamming strategy or the S-R pairs’ hopping policy, we use the expected discounted sum of rewards (EDSR) corresponding to the policy \( \pi \) defined by

\[
\bar{U}_\pi = E_{S_0} \left[ E_\pi \left[ \sum_{t=0}^{\infty} \gamma^t U_{t+1}(S_t, A_t) \bigg| S_0 \right] \right].
\]

The maximum EDSR \( \bar{U}^* \) is achieved by an optimal policy \( \pi^* \).

IV. ARMS RACE IN MULTI-USER ENVIRONMENT

In this section, we analyze the arms race for the multi S-R pair case \((n \geq 1)\) where one or more S-R pairs communicate in the presence of a jammer. We continue the arms race by analyzing an optimal frequency hopping policy against the sweep jamming described in Section [13] and show that the frequency hopping policy and the sweep jammer establish an equilibrium.

In multi S-R pair environment, each S-R pair has an inactive probability \(0 \leq \theta \leq 1\), defined as the probability that an S-R pair who just hops to a channel is not allowed to communicate in the communication phase. Trivially, \( \theta = 0 \) if \( n = 1 \). The inactive probability of an S-R pair depends on its environment determined by frequency hopping policies of the S-R pairs, collision avoidance protocols, and jamming strategies. Because we consider symmetric collision avoidance protocols with respect to the S-R pairs, we assume that the frequency hopping policies of the S-R pairs are also symmetric, i.e., the S-R pairs use the same policy. To find an optimal policy \( \pi^* \) and the corresponding inactive probability \( \theta^* \) against the sweep jamming for the given collision avoidance protocol, we start with an initial pair \((\pi_1, \theta_1)\), where \( \pi_1 \) is an arbitrarily chosen frequency hopping policy and \( \theta_1 \) is the corresponding inactive probability. Next, we can find a frequency hopping policy \( \pi_2 \) from Bellman optimality equation based on the transition function assuming \( \theta_1 \), and let \( \theta_2 \) denote the inactive probability corresponding to \( \pi_2 \). This iterative update proceeds and let \( f \) and \( g \) denote the update functions, i.e., \( \pi_{i+1} = f(\pi_i) \) and \( \theta_{i+1} = g(\theta_i) \). Note that \( \pi^* \) is an optimal policy if \( \pi^* = f \circ g(\pi^*) \).

The following theorem characterizes \( \pi = f(\theta) \) against the sweep jamming, for any inactive probability \( \theta \). It can be proved similarly as [6] Proposition 1 with some modifications taking into account the inactive probability \( \theta \).

**Theorem 1.** For the sweep jamming, an optimal frequency hopping policy \( \pi = f(\theta) \) under the environment described by an inactive probability \( \theta \) is as follows:

\[
\pi(S) = \begin{cases} 
S & \text{for } S \in [1 : K(\theta)], \\
\text{null} & \text{otherwise}
\end{cases}
\]

for some staying-threshold \( K(\theta) \in [0 : T - 1] \). Note that if \( K(\theta) = 0 \), the set \([1 : K(\theta)]\) is an empty set and hence the policy is to always hop.

**Proof.** Let us first state the transition function for given inactive probability \( \theta \), \( p_\theta(S_{t+1}|S_t, A_t) \) in the presence of the sweep jammer. The set \( S \) of states is given as \( \{I, J, 1, 2, \cdots, T - 1\} \), i.e., \( K_m = T - 1 \) since the sweep jammer scans all the \( M \) channels in any window of \( T \) consecutive time slots. The transition function for \( A_t = h \) is given as follows:

\[
p_{\theta}(I|S, h) = \theta, \quad p_{\theta}(J|S, h) = \frac{(1 - \theta) m}{M}, \quad p_{\theta}(1|S, h) = \frac{(1 - \theta) (M - m)}{M},
\]
for any $S \in S$ since $m$ channels out of all $M$ channels are jammed in each time $t$ and an S-R pair who just hops to a channel get the chance to communicate in the communication phase with probability $1 - \theta$. The transition function for $A_t = s$ is given as

$$p_\theta(J|K, s) = \frac{m}{M - Km},$$

$$p_\theta(K + 1|K, s) = 1 - \frac{m}{M - Km},$$

for $K \in [1 : T - 1]$, since the jammer already scanned other $mK$ channels during $K$ consecutive successful communications of the S-R pair, and it attacks $m$ channels out of remaining $M - mK$ channels.

To derive a policy obtained from Bellman optimality equation and using the described transition function, we need to derive the maximum action-value function for the given $\theta$, $Q_\theta^*(S, A)$. Since it is in general difficult to obtain the closed form solution of $Q_\theta^*(S, A)$, we prove the theorem from some monotonicity properties of $Q_\theta^*(S, A)$.

First we show that $Q_\theta^*(K, h)$ is a constant regardless of $K \in [1 : T - 1]$. Note that $Q_\theta^*(K, h)$ for $K \in [1 : T - 1]$ is given as

$$Q_\theta^*(K, h) \overset{(a)}{=} R - C + \gamma \sum_{S_{t+1} \in \{1, J\}} p_\theta(S_{t+1}|S_{t+1}, h)V_\theta^*(S_{t+1}),$$

$$\overset{(b)}{=} R - C + \gamma \sum_{S_{t+1} \in \{1, J\}} p_\theta(S_{t+1}|1, h)V_\theta^*(S_{t+1}),$$

where (a) is from the Bellman optimality equation and (b) is since the transition function for $A_t = h$ does not depend on the current state. Hence, $Q_\theta^*(K, h)$ has the same value regardless of $K \in [1 : T - 1]$.

Similarly, $Q_\theta^*(J, h)$ can be written as

$$Q_\theta^*(J, h) = -L - C + \gamma \sum_{S_{t+1} \in \{1, J\}} p_\theta(S_{t+1}|1, h)V_\theta^*(S_{t+1}).$$

By subtracting (16) from (17), the following equation holds for $K \in [1 : T - 1]$

$$Q_\theta^*(J, h) = Q_\theta^*(K, h) - R - L.$$

Now, let us show that $Q_\theta^*(K, s)$ is strictly decreasing in $K \in [1 : T - 1]$ by induction. To that end, we first show $Q_\theta^*(T - 2, s) > Q_\theta^*(T - 1, s)$ as follows:

$$\overset{(a)}{=} \gamma \sum_{S_{t+1} \in \mathcal{S}} (p_\theta(S_{t+1}|T - 1, s) - p_\theta(S_{t+1}|T - 2, s))V_\theta^*(S_{t+1})$$

$$= \gamma (V_\theta^*(J) - p_\theta(T - 1|T - 2, s)V_\theta^*(T - 1)$$

$$- p_\theta(T - 1|T - 2, s)V_\theta^*(J))$$

$$= \gamma \cdot p_\theta(T - 1|T - 2, s)(V_\theta^*(J) - V_\theta^*(T - 1))$$

where (a) is from the Bellman optimality equation, (b) is due to (4), and (c) is by (13) and $Q_\theta^*(J, h) = V_\theta^*(J)$.

Next, under the assumption $Q_\theta^*(K - 1, s) > Q_\theta^*(K, s)$ for $K \in [3 : T - 1]$, $Q_\theta^*(K - 2, s) > Q_\theta^*(K - 1, s)$ can be proved as follows:

$$Q_\theta^*(K - 1, s) - Q_\theta^*(K - 2, s)$$

$$\overset{(a)}{=} \gamma \sum_{S_{t+1} \in \mathcal{S}} (p_\theta(S_{t+1}|K - 1, s) - p_\theta(S_{t+1}|K - 2, s))V_\theta^*(S_{t+1})$$

$$= \gamma (\gamma (p_\theta(K|K - 1, s)V_\theta^*(K) + p_\theta(J|K - 1, s)V_\theta^*(J)$$

$$- p_\theta(K - 1|K - 2, s)V_\theta^*(K - 1) - p_\theta(J|K - 2, s)V_\theta^*(J))$$

$$= \gamma \cdot (((p_\theta(K - 1|K - 2, s) - p_\theta(K|K - 1, s)))V_\theta^*(J)$$

$$- V_\theta^*(K - 1)) + p_\theta(K|K - 1, s)(V_\theta^*(K) - V_\theta^*(K - 1)))$$

$$\overset{(b)}{=} \gamma \cdot p_\theta(K|K - 1, s)(V_\theta^*(K) - V_\theta^*(K - 1))$$

$$\leq 0,$$

where (a) is by Bellman optimality equation, (b) is by (4) and (18), and (c) is because $Q_\theta^*(K - 1, s) > Q_\theta^*(K, s)$ from the assumption and $Q_\theta^*(K - 1, h) = Q_\theta^*(K, h)$. By induction, we conclude that $Q_\theta^*(K, s)$ is strictly decreasing in $K$.

Since $Q_\theta^*(K, h)$ and $Q_\theta^*(K, s)$ are constant and strictly decreasing in $K$, respectively, there exists a threshold $K(\theta) \in [1 : T - 1]$ such that $Q_\theta^*(K, s) \geq Q_\theta^*(K, h)$ for $K \in [1 : K(\theta)]$ and $Q_\theta^*(K, s) \leq Q_\theta^*(K, h)$ for $K \in [K(\theta) + 1 : T - 1]$, unless $Q_\theta^*(1, s) < Q_\theta^*(1, h)$. If $Q_\theta^*(1, s) < Q_\theta^*(1, h)$, let $K(\theta) = 0$. From (5), we conclude that a policy obtained by Bellman optimality equation in inactive probability $\theta$ is staying for $K \in [1 : K(\theta)]$ and hopping for $K \in [K(\theta) + 1 : T - 1]$, with the interpretation that if $K(\theta) = 0$, the set $[1 : K(\theta)]$ is an empty set and hence the policy is to hop for all $K \in [1 : T - 1]$.

Let us call the frequency hopping policy in Theorem 1 the staying-threshold policy. The staying-threshold $K(\theta)$ can be obtained by deriving the maximum action-value function through value iteration and then applying (5). Here, $K(\theta)$ depends on the various model parameters. It increases in $M$ and decreases in $m$ since the transition function in $\theta$, $p_\theta(J|K, s)$ decreases in $M$ and increases in $m$ for any $K \in [1 : T - 1]$. It decreases in both $R$ and $L$ since large $R$ and $L$ means that the difference in the rewards for the jammed and successful communication is large. Also, $K(\theta)$ increases in $C$ since large hopping cost means that the received reward when the S-R pair hops to the next channel is small. Finally, it increases in $\theta$ as proved in Theorem 2. In Fig. 5, the tendency of $K^\ast$ in $L$ and $m$ is shown.

Now, the following theorem shows that an optimal policy $\pi^\ast$ and the corresponding inactive probability $\theta^\ast$, i.e., $\pi^\ast = f(\theta^\ast)$ and $\theta^\ast = g(\pi^\ast)$, can be obtained through a finite number of
which implies that we can find an optimal policy \( \pi \) through the iterative update procedure.

**Proof.** Let us first derive that \( g \circ f \) is an increasing step function under the assumption that \( g \circ f \) is an increasing function. Since a policy induced by an update function \( f \) is the staying-threshold policy \( K \in [0 : T - 1] \) as shown in Theorem 1, the cardinality of set \( \{ g \circ f(\theta) | 0 \leq \theta \leq T \} \) is equal or less than \( T \). Hence, \( g \circ f \) is an increasing step function where the number of the steps is upper-bounded by \( T \). We assume that the step function has the following \( i \leq T \) steps: \( [1, \theta'_1], [\theta'_1, \theta'_2], ..., [\theta'_i, 1] \). Step \( [\theta'_{k-1}, \theta'_k] \) \((k \leq i)\) corresponds to policy \( \pi^*_k \) and the different step induces the different policy.

Next, we derive that an optimal frequency hopping policy is obtained through less than \( T \) updates of a pair \((\pi, \theta)\) by using \( f(\theta) \) and \( g(\pi) \). Let the update start in \((\pi_1, \theta_1)\) and \((\pi_j, \theta_j)\) is updated to \((\pi_{j+1}, \theta_{j+1})\), i.e., \( \pi_{i+1} = f(\theta_i) \) and \( \theta_{i+1} = g(\pi_{i+1}) \). For convenience, we denote the update as \((\pi_1, \theta_1) \rightarrow (\pi_{j+1}, \theta_{j+1})\). If \( \theta_{j} \geq \theta_{j+1} \) \((or \leq)\), then \( \theta_{j+1} \geq \theta_{j+2} \) \((or \leq)\) since \( g \circ f(\theta_j) \geq g \circ f(\theta_{j+1}) \) \((or \leq)\). Hence, \( \theta_j \) is increasing or decreasing on \( j \). By monotonic property of \( \theta_j, k(j) \) corresponding to \( \pi^*_{k(j)} \) also monotonic on \( j \). Since an optimal policy holds \( (f \circ g)(\pi^*) = \pi^* \) and \( i \leq T \), there exist \( s \in [1 : T - 1] \) such that \( (f \circ g)^s(\pi_1) = (f \circ g)^{s+1}(\pi_1) \) for any initial staying-threshold policy \( \pi_1 \).

Now, we derive the increasing property of \( g \circ f \). It can be checked as follows. First, Theorem 1 shows that \( f(\theta) \) is a staying-threshold policy with staying-threshold \( K(\theta) \). This threshold \( K(\theta) \) increases in \( \theta \) since the risk of not getting the chance to communicate in the communication phase right after the hopping action increases in \( \theta \). It is also shown in Fig. 7. Next, for staying-threshold policy \( \pi \) with threshold \( K, g(\pi) \) increases in \( K \). To see this, first note that the number of the S-R pairs selecting hopping action decreases in \( K \) and hence the probability that multiple S-R pairs are in the same channel also decreases. Hence, as \( K \) increases, the total number of channels occupied by the S-R pairs increases. Therefore, if an S-R pair takes the hopping action, the probability that it hops to a channel that has been already occupied by other S-R pair(s), i.e., the probability of collision \( g(\pi) \), also increases in \( K \). Also, we can check that a function \( g \circ f : [0, 1] \rightarrow [0, 1] \) is an increasing function in Fig. 8.

**Remark 1.** In single S-R pair environment \((n = 1)\), we can obtain the optimal frequency hopping policy \( \pi^* \) by only one update procedure since the inactive probability \( \theta \) is fixed to 0, i.e., \( s = 1 \).

Theorem 2 shows that an optimal policy against the sweep jamming is a staying-threshold policy, and the optimal threshold \( K^* \) can be obtained by at most \( T - 1 \) iterative updates. The optimality of the staying-threshold policy of an S-R pair can be explained as follows. The probability that the S-R pair is jammed increases as it stays the same channel longer. Hence, it is better to hop to the next channel if the risk of being jammed by staying the same channel outweighs the hopping cost and the risk of not getting the chance to communicate in the communication phase in the newly entering channel.

![Fig. 6: Staying-threshold versus L for m = 2, 4, 6 in the sweep jamming environment with parameters by R = 5, C = 5, n = 1, M = 60, and γ = 0.9.](image)

![Fig. 7: Staying-threshold K(θ) versus θ for m = 2, 4, 6 in the sweep jamming environment with parameters R = 5, C = 5, L = 20, M = 60, and γ = 0.9.](image)

![Fig. 8: Inactive probability θ versus g ◦ f(θ) with the random protocol, the sweep jamming, parameters R = 5, C = 5, L = 20, n = 10, M = 60, m = 3, and γ = 0.9.](image)
The staying-threshold $K^*$ is a critical state in which staying is more beneficial to the S-R pair than hopping.

Now we further show that the staying-threshold policy and the sweep jamming policy establish an equilibrium in multi S-R pair environment, by continuing the arms race and proving that the next round of the arms race does not change the EDSR. Against the staying-threshold policy of the S-R pair, the jammer can try the smarter attack [6], selecting $m$ target channels uniformly at random out of all $M$ channels except the channels scanned in the recent $K^*$ time slots. It can be analytically shown that the smarter attack is an optimal jamming strategy against the staying-threshold policy (although our result in Theorem 3 implies that it is not "uniquely" optimal). It was numerically shown in [6] that for $n = 1$, the smarter jamming slightly decreases the EDSR compared to the sweep jamming, for the staying-threshold policy. Here we analytically show that the EDSR corresponding to the staying-threshold policy is the same for both the sweep jamming and the smarter jamming for general multi-user scenario, i.e., $n \geq 1$.

**Theorem 3.** In multi S-R pair case with the staying-threshold policy, the EDSR of an S-R pair is the same for both the sweep jamming and the smarter jamming environments.

**Proof.** The EDSR for a given pair of frequency hopping policy and jamming strategy can be derived from the Bellman expectation equation [13], which only requires the transition functions associated with visitable state-action pairs. For the staying-threshold policy with threshold $K^*$, the Bellman expectation equations corresponding to both the jamming strategies require only the transition functions $p(I,h)$, $p(J,h)$, $p(K,s)$ for $K \in [1 : K^*]$, and $p(K^*+1,h)$. Note that these transition functions are the same for both the sweep jamming and the smarter jamming, since they attack non-overlapping channels in any window of $K^*$ consecutive time slots and hence the inactive probability is the same in both the jamming strategies. Consequently, we conclude that the EDSR of a S-R pair is the same in both the jamming environments.

From the above theorem, we conclude that the staying-threshold policy and the sweep jamming strategy establish an equilibrium in multi S-R pair environment: a staying-threshold frequency hopping maximizes the EDSR against the sweep jamming, and the sweep jamming minimizes the EDSR against the staying-threshold frequency hopping. Figs. 9 and 10 illustrate the arms race between the S-R pairs and the jammer.

Now, let us introduce some examples of the collision protocols. The simplest protocol would be all-hopping protocol [3]. In this protocol, every sender who newly hops to the channel broadcasts a pilot signal at a random moment. If there are more than one pilot signals, all the S-R pairs give up the communication and randomly hop to the other channels in the next time slot. Although this protocol is simple, it has the disadvantage that no one uses the channel if there are more than two S-R pairs hopping to the channel. To resolve this issue, we consider random protocol [3]. In the random protocol, each sender broadcasts a pilot signal at a random instant during the collision avoidance phase, and the S-R pair who first broadcasts the signal occupies the channel, i.e., it is allowed to communicate.

In multi S-R pair environment, the threshold $K^*$ in the staying-threshold policy also depends on the number $n$ of S-R pairs and the collision avoidance protocols. It increases in $n$ since the inactive probability increases in $n$. To explain the relation between $K^*$ and the introduced collision avoidance protocols, we present some bounds on the inactive probability $\theta_{\text{min}} \leq \theta \leq \theta_{\text{max}}$ of the S-R pairs for each collision avoidance protocol in the following propositions.

**Proposition 1.** For the all-hopping collision avoidance protocol and any arbitrary frequency hopping policy, the inactive probability of an S-R pair is bounded as follows:

$$1 - (1 - 1/M)^{n-1} \leq \theta \leq (n - 1)/M.$$
communication phase is the probability that the S-R pair is allowed to communicate in the
when

where

p

θ

the communication collision. Here, now we consider that the random protocol is used to avoid
S-R pairs that already occupied their channels, i.e., the S-R
Proposition 1 since we give priority to communicate to the
Proof.

The proof of θ

is similar with Proposition 1 but now we consider that the random protocol is used to avoid
the communication collision. Here, θ

is derived as:

θ

= 1 − (1 − 1/M)

i=1

i + 1

(31)
since the probability that the S-R pair and another S-R pair are allocated to the different channels is 1 − 1/M and the S-R pairs hop with i.i.d. process. The largest θ, θ

max

is induced when the other S-R pairs select the staying action, i.e, at most one S-R pair occupies a channel. Here, θ

max

is given as:

θ

max

= n − 1

M ,

(32)
since n − 1 channel are already allocated to the other S-R pairs.

Proposition 2. For the random collision avoidance protocol and any arbitrary frequency hopping policy, the inactive probability of an S-R pair is bounded as follows:

\[
\sum_{i=1}^{n-1} i \binom{n-1}{i} \left(\frac{1}{M}\right)^i \left(1 - \frac{1}{M}\right)^{n-1-i} \leq \frac{n-1}{M}.
\]

Proof. The proof of θ

min

is similar with Proposition 1 since we give priority to communicate to the S-R pairs that already occupied their channels, i.e., the S-R pairs choosing the staying action.

We note that the collision avoidance protocols have different lower-bounds on the inactive probability. The random protocol has smaller θ

min

than the all-hopping protocol, since for the random protocol, if there are multiple S-R pairs hopping to the same channel, an S-R pair uses the channel. Since Fig. 8 shows that K(θ) increases in θ, K∗ is bounded as K(θ

min

) ≤ K∗ ≤ K(θ

max

). Fig 11 illustrates these bounds on K∗ for the all-hopping and the random protocols, from which we can see that the lower and upper bounds are quite tight. Finally, Fig.12 shows the EDSRs at the equilibrium for the all-hopping protocol and the random protocol. We can see that EDSR using the random protocol is higher, and the gap from the EDSR

Fig. 11: The bounds on K∗ for the all-hopping and the random collision avoidance protocols with parameters R = 5, C = 5, L = 20, M = 60, m = 3, and γ = 0.9.

Fig. 12: The EDSRs at the equilibrium for the all-hopping and the random collision avoidance protocols with the parameters R = 5, C = 5, L = 20, M = 60, m = 5, and γ = 0.9.

using the all-hopping protocol increases as the number of S-R pairs increases. This is because, as the number of S-R pairs increases, the probability that there are multiple S-R pairs in the same channel increases and hence the use of an efficient collision avoidance protocol becomes more critical.

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