B decays to excited charm mesons

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We review several aspects of the phenomenology of P-wave $qQ$ mesons: mass splittings, effective strong couplings and leptonic constants. We also describe a QCD sum rule determination to order $\alpha_s$ of the form factor $\tau_{1/2}(y)$ governing the semileptonic $B$ decays to the charm doublet with $J^P = (0_{1/2}^+, 1_{1/2}^+)$. 

1. Spectroscopy and strong decays of mesons containing one heavy quark

The spectroscopy of hadrons containing one heavy quark $Q$, in the infinite $m_Q$ limit, is simplified by the decoupling of the heavy quark spin $s_Q$ from the angular momentum of the light degrees of freedom (quarks and gluons) $\vec{s} = \vec{J} - \vec{s}_Q$. In this heavy quark theory (HQET) the states are classified by both $\vec{J}$ and $\vec{s}_Q$, and the hadrons corresponding to the same $s_Q$ belong to degenerate doublets. In the case of mesons, the low-lying states $s_Q^P = \frac{1}{2}^-$ correspond to the pseudoscalar $0^-$ and vector $1^-$ mesons ($B, B^*$; $D, D^*$), the $s$-wave states of the constituent quark model. The four states corresponding to orbital angular momentum $L = 1$ can be classified in two doublets: $J^P = (0_{1/2}^+, 1_{1/2}^+)$ and $J^P = (1_{3/2}^+, 2_{3/2}^+)$, which differ by the values $s_Q^P = \frac{1}{2}^+$ and $s_Q^P = \frac{3}{2}^+$, respectively. The charmed $2_{1/2}^+$ state, denoted as $D_2^+(2460)$, has been experimentially observed: $m_{D_2^+} = 2458.9 \pm 2.0$ MeV, $\Gamma_{D_2^+} = 23 \pm 5$ MeV and $m_{D_2^{*+}} = 2459 \pm 4$ MeV, $\Gamma_{D_2^{*+}} = 25.4^{+8}_{-7}$ MeV for the neutral and charged states, respectively. The state $1_{3/2}^+$ can be identified with $D_1(2420)$, with $m_{D_1} = 2422.2 \pm 1.8$ MeV and $\Gamma_{D_1} = 18.9^{+4.6}_{-3.5}$ MeV, neglecting a possible $1_{1/2}^+$ component allowed by the finite value of the charm quark mass. Experimental evidence of beauty $s_Q^P = \frac{3}{2}^+$ states has also been reported. The narrow width of both the states $2_{3/2}^+$ and $1_{3/2}^+$ is due to their $d$-wave suppressed strong transitions, governed by strong coupling constants that can be determined using experimental information.

The not yet observed charm doublet $J^P = (0_{1/2}^+, 1_{1/2}^+)$ ($D_0, D_1^*$) has $s$-wave strong transitions. Analyses of the couplings governing the two-body hadronic decays to pions can be done by QCD sum rules, in a theoretical framework where both the QCD heavy flavour-spin symmetry and chiral symmetry are implemented. Three effective couplings, in the infinite $m_Q$ limit, are relevant: $g$ (for the vertex $D^* D \pi$), $g'$ (for $D_1^* D_0 \pi$) and $h$ (for $D_1^* D \pi$). The numerical results $g = 0.2 - 0.4$, $g' = 0.07 - 0.13$ and $|h| = 0.4 - 0.8$ allow to predict $\Gamma(D_0^0 \to D^+ \pi^-) \simeq 180$ MeV and $\Gamma(D_1^{*0} \to D^{*+} \pi^-) \simeq 165$ MeV, and the mixing angle $\alpha \simeq 16^\circ$ between $D_1^*$ and $D_1$. The predictions for the beauty sector are $\Gamma(B_0 \to B^{0} \pi^-) \simeq \Gamma(B_{10}^{*0} \to B^{*+} \pi^-) \simeq 360$ MeV.

2. Semileptonic B decays to excited states and universal form factors

The decay matrix elements governing $B \to (D_0, D_1^*) \ell \bar{\nu}$ can be parameterized by six form factors which allow to compute the physical observables such as, e.g., the spectrum of the momentum transfer to the lepton pair and of the charged lepton energy:

$$<D_0(v')|\bar{c}\gamma_\mu \gamma_5 b|B(v)> \propto \frac{g_+(y)(v + v')_\mu + g_-(y)(v - v')_\mu}{\sqrt{m_B m_{D_0}}},$$

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\[
\left< D_1^*(v', \epsilon) | \bar{c} \gamma_\mu (1 - \gamma_5) b | B(v) \right> = \sqrt{m_D m_{D_1^*}}
\]
\[
g_{V_1}(y) \epsilon^*_\mu + \epsilon^* \cdot v \left[ g_{V_2}(y) \epsilon_\mu + g_{V_3}(y) \epsilon^*_\mu \right] - i g_A(y) \epsilon_{\mu \alpha \beta \gamma} \gamma^\alpha v^\beta v^\gamma ;
\]
\(v\) and \(v'\) are the meson four-velocities \((y = v \cdot v')\).

As in the case of \(B \to (D, D^*) \ell \bar{\nu}\) transitions \(\downarrow\), in the limit \(m_{b,c} \to \infty\) the form factors \(g_i(y)\) can be related to a single universal function \(\tau_{1/2}(y)\) through short-distance coefficients, perturbatively calculable, which depend on the heavy quark masses \(m_{b,c}\) on \(y\) and on a renormalization scale \(\mu \uparrow\). Such relations, at the next-to-leading logarithmic accuracy in \(\alpha_s\), are reported in \(\uparrow\). A \(\mu\) independent quantity \(\tau_{1/2}^{pert}(y)\) can also be defined.

Similarly, the eight form factors relevant to the decays \(B \to D_1 \ell \bar{\nu}\) and \(B \to D_2 \ell \bar{\nu}\) can be related to the universal function \(\tau_{3/2}(y)\) \(\uparrow\). The main difference with respect to the universal Isgur-Wise \(B \to D, D^*\) form factor \(\xi(y)\), constrained by the heavy quark symmetry at the zero recoil point \(\xi(1) = 1\), is that one cannot invoke symmetry arguments to predict the normalization of neither \(\tau_{1/2}(y)\) nor \(\tau_{3/2}(y)\).

The QCD sum rule determination of \(\tau_{1/2}\) including \(\mathcal{O}(\alpha_s)\) corrections has been recently considered \(\uparrow\) (analogous corrections for the Isgur-Wise form factor \(\xi\) were previously determined in \(\downarrow\)). Notice that this a genuinely field theoretical approach in the based on QCD; The relevant three-point correlator is

\[
\Pi_\mu(\omega, \omega', y) = i^2 \int dx \, dz \, e^{i(k' \cdot x - k \cdot z)}
\]

\[
< 0 | T [ J^\nu_\mu(x), \hat{A}_\mu(0), J^0_\nu(z) ] | 0 > ,
\]

with \(\hat{A}_\mu = \bar{h}_Q \gamma_\mu \gamma_5 h^\nu_Q\), the \(b \to c\) weak axial current in HQET, \(J^\nu_\mu = \bar{q} h_Q^\nu \gamma_\mu\) and \(J^0_\nu = \bar{q} i \gamma_5 h_Q^\nu\) local interpolating currents for the \(D_0\) and \(B\). The residual momenta \(k, k'\), obtained by the expansion of the heavy meson momenta in terms of the four-velocities: \(P = m_Q v + k, P' = m_Q v' + k'\), are \(\mathcal{O}(\Lambda_{QCD})\), and remain finite in the heavy quark limit.

The analyticity of \(\Pi(\omega, \omega', y)\) in the variables \(\omega = 2v \cdot k\) and \(\omega' = 2v' \cdot k'\) at fixed \(y\), allows to express the correlator in terms of hadronic contributions, from poles at positive real values of \(\omega\) and \(\omega'\) and from a continuum of states. The lowest-lying contribution corresponding to the \(B\) and \(D_0\) poles, introduces the form factor \(\tau_{1/2}\) through the relation:

\[
\Pi_{pole}(\omega, \omega', y) = \frac{-2 \tau_{1/2}(y, \mu) F(\mu) F^+(\mu)}{(2 \Lambda - \omega - i\epsilon)(2 \Lambda + \omega' - i\epsilon)} ,
\]

where \(\mu\) is a renormalization scale and \(F(\mu), F^+(\mu)\) are the current-particle matrix elements

\[
< 0 | J^\nu_\mu B(v) > = F(\mu) \quad (2)
\]

\[
< 0 | J^0_\nu B_0(v) > = F^+(\mu) . \quad (3)
\]

Notice that \(F(\mu)\) is related to the \(B\)-meson leptonic decay constant \(f_B\). The mass parameters \(\Delta = M_B - m_b\) and \(\Delta^+ = M_{D_0} - m_c\) identify the position of the poles in \(\omega\) and \(\omega'\), and can be interpreted as the binding energies of the \(0^-\) and \(0^+\) states.

\[
\Pi_\mu(\omega, \omega', y)\]

can alternatively be computed in QCD in the Euclidean region in terms of perturbative and nonperturbative contributions

\[
\Pi = \Pi^{pert} + \Pi^{np} ,
\]

where \(\Pi^{np}\) represents the series of power corrections, determined by quark and gluon vacuum condensates. A sum rule for \(\tau_{1/2}\) is obtained by matching the hadronic and the QCD representation of the correlator in a suitable range of Euclidean values of \(\omega\) and \(\omega'\). By the same method, considering two-point correlators, the constants \(F(\mu)\) and \(F^+(\mu)\) can be determined.

Defining \(\mu\)-independent constants \(F^+\) and \(F\), we obtain: \(\Lambda^+ = 1.0 \pm 0.1\) GeV and \(F^+ = 0.7 \pm 0.2\) GeV \(\uparrow\); \(\Lambda = 0.5 \pm 0.1\) GeV and \(F = 0.45 \pm 0.05\) GeV \(\uparrow\). The difference \(\Delta = \Lambda^+ - \Lambda\) corresponds to the difference \(m_{D_0} - m_D\), with \(D\) and \(D_0\) spin averaged states of the \(\frac{1}{2}^-\) and \(\frac{1}{2}^+\) doublets. The central value \(\Delta = 0.5\) GeV predicts \(m_{D_0} \simeq 2.45\) GeV with an uncertainty of about 0.15 GeV.

Neglecting \(\mathcal{O}(\alpha_s)\) corrections QCD sum rules would give: \(F^+ = 0.46 \pm 0.06\) GeV \(\uparrow\) and \(F^+ = 0.40 \pm 0.04\) GeV \(\uparrow\) and \(\Lambda^+ = 1.05 \pm 0.5\) GeV or \(\Lambda^+ = 0.90 \pm 0.10\) GeV \(\uparrow\), by various choices of the interpolating currents. \(\alpha_s\) corrections are thus sizeable for the couplings \(\downarrow, \uparrow\). The same is true for \(\Pi^{pert}\) in \(\uparrow\).
Relativistic quark models give $F^+ \simeq 0.6 - 0.7$ GeV$^{3/2}$, whereas lower values are obtained: $F^+ \simeq 0.235$ GeV$^{3/2}$ if non relativistic models are employed [10].

The numerical result of the sum rule for $\tau_{1/2}$ including next-to-leading $\alpha_s$ corrections is depicted in fig. 1; the shaded region essentially represents the theoretical uncertainty of the calculation. Expanding the form factor near $y = 1$ as

\[ \tau_{1/2}(y) = \tau_{1/2}(1) \left( 1 - \rho_{1/2}^2 (y-1) + c_{1/2} (y-1)^2 \right) \]

we get: $\tau_{1/2}(1) = 0.35 \pm 0.08$, $\rho_{1/2}^2 = 2.5 \pm 1.0$ and $c_{1/2} = 3 \pm 3$.

QCD sum rules at $\mathcal{O}(\alpha_s = 0)$ gave the results $\tau_{1/2}(1) \simeq 0.25$ and $\rho_{1/2}^2 \simeq 0.4$, which shows that $\alpha_s$ corrections, although not negligible, do not dramatically affect the final result.

As for $\tau_{3/2}(y)$, a QCD sum rule analysis at the leading order in $\alpha_s$ gives $\tau_{3/2}(1) \simeq 0.28$ and $\rho_{3/2}^2 \simeq 0.9$. Determinations of the universal form factors by constituent quark models give results in a quite wide range: $\tau_{3/2}(1) = 0.06 - 0.40$, $\tau_{3/2}(1) = 0.3 - 0.7$ [11].

Using $V_{cb} = 3.9 \times 10^{-2}$ and $\tau(B) = 1.56$ ps, from (5) we obtain $\mathcal{B}(B \to D_0 \ell \bar{\nu}) = (5 \pm 3) \times 10^{-4}$ and $\mathcal{B}(B \to D^*_1 \ell \bar{\nu}) = (7 \pm 5) \times 10^{-4}$.

According to this result only a small fraction of the inclusive semileptonic $B \to X_c$ decays is represented by $B$ transitions into the $s_1^P = 1^+$ charmed doublet. One cannot exclude that such processes might be in the reach of future $B$-facilities [11]. At present, data on semileptonic $B \to D^{**}$ decays exist only for the $s_1^P = 1^+$ doublet, since the $s_1^P = 3^+$ doublet is not distinguished from the non-resonant charmed background due to the large width.

We conclude by observing that predictions derived within HQET must always be supported by the calculation of $1/m_Q$ as well as radiative corrections. The role of both depend on the specific situation one is faced with. For the form factor $\tau_{1/2}(y)$, using QCD sum rules in the framework of HQET, we have obtained that, similar to the case of the Isgur-Wise function, radiative corrections are quite under control while they affect considerably the value of the leptonic constants.

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