Dual Abrikosov vortex between confined charges

Srinath Cheluvaraja a, Richard W. Haymaker a and Takayuki Matsuki b

a Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana, 70803 USA
b Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, Japan

We show that the dual Abrikosov vortex between quark and antiquark in Abelian Projected SU(2) gauge theory is insensitive to truncation of all loops except the large monopole cluster noted by Hart and Teper. As the transverse distance increases the discrepancy decreases, suggesting that the London penetration depth determined by tail is invariant under the truncation of short loops.

1. Introduction

In 1992 the LSU group demonstrated the viability of measuring the profile of the dual Abrikosov vortex between quark and antiquark as a signal of the spontaneous U(1) gauge symmetry breaking and hence confinement in Abelian projected SU(2) gauge theory. More recently Bali et.al. did a large-scale simulation confirming the picture to a higher resolution. Gubarev et.al. improved the fit to the same lattice data by employing a lattice Ginzburg-Landau-Higgs model rather than the continuum version used in prior fits.

In addition to the encouragement from the above mentioned improved lattice results we are interested in revisiting this study for a number of reasons. (i) In a recent work by DiCecio, Hart and Haymaker using a Ward Identity we were able to define a conserved U(1) current and hence a precise definition of field strength and charge density. This particular definition has not been used prior to this in determining the profile of the vortex. (ii) The Abrikosov vortex is an explicit consequence of spontaneous U(1) breaking of the vacuum and provides a connection between this and other confinement studies. (iii) Hart and Teper have shown that monopoles in Abelian projected SU(2) theory fall into two groups: there is one very large percolating cluster of connected monopole loops and the remaining monopoles form much smaller clusters or small simple loops. They showed that truncating all but the percolating cluster preserves the string tension. We verify here that this picture is preserved under the same truncation. (iv) Greensite et.al. have noted connections between Z(2) vortex dynamical variables and U(1) monopoles and this approach provides an added opportunity to further these connections.

2. Review of Ginzburg-Landau-Higgs effective theory for Modeling of lattice data

Consider the classical lattice field strength (in lattice units).

\[ \hat{F}_{\mu\nu}(m) = \sin \theta_{\mu\nu}. \]

We consider a constrained Higgs field

\[ \Phi(m) = v e^{i \chi(m)}, \quad v = 1. \]

Under these conditions the electric current is

\[ \hat{J}_\mu(m) = \sin \{ \theta_\mu(m) + \chi_\mu(m + \mu) - \chi_\mu(m) \}. \]

For small \( \theta \) we get the London relations

\[ \hat{F}_{\mu\nu}(m) \equiv \hat{F}_{\mu\nu}(m) - \Delta_\mu^+ \hat{J}_\nu^c(m) + \Delta_\nu^+ \hat{J}_\mu^c(m) = 0. \]

For small \( \theta \mod 2\pi \) at the origin we obtain a vortex with \( N \) units of flux.

\[ \hat{F}_{\mu\nu}(m) = 2\pi N \delta_{m_1,0} \delta_{m_2,0}. \]
This assumes an infinite Higgs mass $M_H$. With a finite mass there is a transition region of size $\sim 1/M_H$ in the core of the vortex but the above London relation holds outside the core. This suggests that we look for this relation far from the source. The quarks need not be far apart to check this.

3. Precise lattice Abelian flux

We can define the classical lattice field strength and conserved current through the relation

$$\Delta_\mu F_{\mu\nu} = J_\nu,$$

$$0 = \Delta_\mu \Delta_\nu F_{\mu\nu} = \Delta_\nu J_\mu.$$

Zach et al. noted that these relations can be derived from a Ward identity for lattice averages of the U(1) gauge theory

$$ea^2 F_{\mu\nu} = \frac{\langle \sin \theta_{\mu\nu} \sin \theta_W \rangle}{\langle \cos \theta_W \rangle}.$$

$$J_\mu = J^\text{ext}_\mu.$$

Evaluating the divergence of the electric field on a time-like line of a Wilson loop gives

$$\Delta \cdot (ea^2 E) \big|_{n=n_0} = e^2 = \frac{1}{\beta_{\text{U(1)}}},$$

and zero otherwise. In the generalization to Abelian projected SU(2) in the maximal Abelian gauge, multiplication of a link by a group element requires an associated gauge transformation to maintain the gauge condition. Taken together we obtain

$$ea^2 F_{\mu\nu} = \frac{\langle \frac{1}{2} \text{tr}(i\sigma_3 D_{\mu\nu}) \sin \theta_W \rangle}{\langle \cos \theta_W \rangle},$$

where the link variable is separated into the diagonal and off-diagonal parts

$$U_\mu = D_\mu + O_\mu,$$

and $D_{\mu\nu}$ is a plaquette constructed from the diagonal parts. The subsequent current defined by the divergence gets contributions from the external source, the charged fields, the gauge fixing and from ghosts.

$$J_\mu = J^\text{ext}_\mu + J^\text{dyn}_\mu + J^\text{gf}_\mu + J^\text{ghosts}_\mu.$$

Separating the diagonal parts of the links gives the photon part of the action

$$S = \beta \sum \frac{1}{2} \text{tr}(D_{\mu\nu}) + \cdots \approx \beta \sum \langle \cos \phi \rangle^4 \cos \theta_{\mu\nu} + \cdots \approx \frac{1}{e^2} \sum \cos \theta_{\mu\nu} + \cdots,$$

where we used the fact that $\langle \cos \phi \rangle$ has small fluctuations in this gauge.

$$\frac{1}{e^2} \approx \beta \langle \cos \phi \rangle^4 = 2.5115 \times (0.9331(2))^4,$$

$$e^2 \approx 0.53.$$

This identifies the U(1) charge that determines the electric flux quantization which will be discussed in a subsequent paper.

The divergence of the electric current measured on a time-like Wilson line in the classical limit gives

$$\Delta \cdot (ea^2 E) \big|_{n=n_0} = \frac{1}{\beta_{\text{SU(2)}}} = 0.40 \text{ (bare)}.$$

In the quantum case, the charged field dresses the bare charge and gives in this case

$$= 0.51 \text{ measured (dressed)},$$

showing that there is significant screening even at the shortest distances.

4. Percolating monopole cluster

Hart and Teper showed that for large volumes, the monopole currents fall into two distinct classes. There is one large percolating cluster that permeates the whole lattice volume and it gives the full string tension. On this cluster, scaling is observed for the current density and magnetic screening mass. The remaining loops are localized and appear to give no contribution to the string tension.

Fig. 1 gives the profile of the curl of the monopole current as a function of the transverse distance from the source, with and without the truncation of all but the percolating cluster.
Fig. 2 shows the blowup of the tail region. We see that as the transverse distance increases, the effect of the truncation is suppressed indicating that the London penetration depth is due to the percolating cluster alone.

REFERENCES

[1] V. Singh, D. A. Browne and R. W. Haymaker, Phys. Lett. B306 (1993) 115.
[2] G. S. Bali, C. Schlicher and K. Schilling, Prog. Theor. Phys. Suppl. 131 (1998) 645.
[3] F. V. Gubarev, E.-M Ilgenfritz, M. I. Polikarpov, and T. Suzuki, Phys. Lett. B468 (1999) 134.
[4] G. DiCecio, A. Hart and R. Haymaker, Phys. Lett. B441 (1998) 319.
[5] J.M. Carmona, M. D’Elia, A. Di Giacomo, B. Lucini and G. Paffuti, Phys. Rev. D64 (2001) 114507.
[6] A. Hart and M. Teper, Phys. Rev. D58 (1998) 014504.
[7] J. Ambjorn, J. Giedt and J. Greensite, JHEP 0002 (2000) 033.
[8] M. Zach, M. Faber, W. Kainz and P. Skala Phys. Lett. B358 (1995) 325.
[9] G. Poulis, Phys. Rev. D54 (1996) 6974.

Figure 1. Solid symbols are the curl of the monopole current vs. the transverse distance from the center of a Wilson loop; $\beta = 2.5115$, lattice $20^4$, $3 \times 3$ loop with fat space links. The open symbols are calculated from truncated monopole loops.
Figure 2. Rescaled Fig. 1. Lines are not fits.