The Impact of Outdated CSI on the Finite Blocklength Performance of Relaying

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Abstract

Under the assumption of outdated channel state information (CSI) at the source, we study the blocklength-limited throughput (BL-throughput) of a two-hop relaying network. We introduce SNR weights to let the source choose a relatively lower coding rate based on the weighted outdated SNR. Both a fixed weight scheme and a dynamic weight scheme are studied. We derive the expected BL-throughput of a transmission period based on the corresponding outdated CSI and compare it with the ergodic BL-throughput over channel fading. In particular, we prove that under the dynamic SNR weight scheme the BL-throughput of an upcoming transmission is concave in the coding rate and quasi-concave in the SNR weights. In addition, under the fixed SNR weight scheme we show that the average BL-throughput over time is quasi-concave in these weights. Through numerical investigations, we show the appropriateness of our theoretical model. In addition, we study the performance difference between the dynamic and the fixed weight schemes. We show numerically that relaying outperforms direct transmission under the finite blocklength regime for cases where both systems have similar performance with respect to Shannon capacity. These results indicate that as the blocklength decreases, relaying has a increasing performance advantage over direct transmission despite its inherent reduction in the transmit times.

Index Terms

Finite blocklength regime, decode-and-forward, relaying, blocklength-limited throughput, outdated CSI.

I. INTRODUCTION

Relaying [1]-[3] is well known as an efficient way in wireless communications to mitigate fading by exploiting spatial diversity and providing better channel quality. Specifically, two-hop decode-and-forward (DF) relaying protocols significantly improve the throughput and quality of
service [4]–[7]. However, all the above studies of the advantages of relaying are under the ideal assumption of communicating arbitrarily reliably at Shannon’s channel capacity, i.e., coding is assumed to be performed using a block with an infinite length.

In the finite blocklength regime, the transmission (at a rate lower than the Shannon limit) becomes no longer arbitrarily reliably. Especially when the blocklength is short, the error probability (due to noise) is significant even if the rate is selected below the Shannon limit. Taking this into account, an accurate approximation of the achievable coding rate under the finite blocklength assumption for an AWGN channel was derived in [8] for a single-hop transmission system. The result in [8] shows that the performance loss due to finite blocklength effects is considerable and increases as the blocklength decreases. Subsequently, these initial work with AWGN was extended to Gilbert-Elliott channels [9], quasi-static fading channels [10], [11], quasi-static fading channels with retransmissions [12], [13], spectrum sharing networks [14] as well as transmissions with packet scheduling [15]. However, all these works only focus on single-hop systems.

Considering a two-hop decode-and-forward network, relaying provides a power gain as it halves the distance but at the same time halves the blocklength of the transmission (if equal time division is considered). Thus, the result in [8], namely that the loss due to finite blocklength effects increases as the blocklength decreases, actually points to a trade-off introduced by relaying between increasing the received signal strength by halving the distance vs. finite blocklength loss due to shorter time spans. This motivates us to study the relaying performance in the finite blocklength regime. In our previous work [16] and [17], we addressed in general analytical performance models for relaying with finite blocklengths under the scenario of static channels and average CSI\textsuperscript{1}. We investigated the related blocklength-limited throughput (BL-throughput) and showed that the BL-throughput is quasi-concave in the overall error probability of relaying. The work in [16] and [17] is further generalized in [18] into a scenario of quasi-static Rayleigh channels while still only average CSI is available at the source.

In this work, we extend our previous studies [16]–[18] to a fading scenario with instantaneous CSI at the source. More importantly, the feedback of instantaneous CSI is assumed to be subject

\textsuperscript{1}It should be mentioned that for a static channel the assumption of the average CSI and the assumption of the perfect CSI are the same.
to delays, i.e., resulting for example from the feedback delay. Under the above assumption, we investigate the blocklength-limited performance of relaying. Different from the average CSI scenario considered in [16]–[18], with the instantaneous CSI the source is able to adjust the coding rate accordingly for each transmission period. In addition, as the instantaneous CSI is outdated, it is possible that the outdated CSI is higher than the real/exact CSI. Hence, determining the coding rate simply based on the outdated CSI likely results in an error. At the same time, the effect of finite blocklengths also contributes to errors. Thus, the analysis of the blocklength-limited performance of such a relaying system (with finite blocklengths and outdated CSI) becomes interesting and also challenging. To the best of our knowledge, this has not been studied so far.

To address the above issue, in this work we introduce two SNR weights for these two links/hops of relaying. We propose the source to choose the coding rate based on weighted outdated SNRs (weighted by these SNR weights). This operation reduces the coding rate (determined by the source) and therefore reduces the expected error probability. Under the proposed operation, we derive the expected BL-throughput of the upcoming transmission period (based on the current outdated CSI) as well as the ergodic BL-throughput of relaying while a fixed SNR weight scheme and a dynamic SNR weight scheme are considered. In particular, we prove that under a dynamic SNR weight scheme the BL-throughput of an upcoming transmission is concave in the coding rate and quasi-concave in SNR weights. In addition, under the fixed SNR weight scheme we show that the BL-throughput over time is quasi-concave in SNR weights. Through numerical investigations, we show the appropriateness of our theoretical model. In addition, we show that there is no big difference between the fixed weight scheme and the dynamic weight scheme if there is a strong correlation between the outdated CSI and instantaneous channel state. Otherwise, dynamic weight scheme significantly outperforms the fixed scheme, especially for high SNR scenarios. Moreover, under the FBL regime, in particular for rather short blocklengths, relaying is more beneficial in comparison to direct transmission. In other words, increasing the received signal strength by relaying is more important than the time loss incurred through halving the blocklength. More importantly, this performance advantage of relaying is more significant for outdated CSI scenario than the perfect CSI scenario.

The rest of the paper is organized as follows. Section II describes the system model and briefly introduces the background theory regarding the finite blocklength regime. In Section III,
we derive the blocklength-limited performance of relaying with outdated CSI. Section IV presents our numerical results. Finally, we conclude our work in Section V.

II. SYSTEM MODEL

A. System Description

We consider a simple relaying scenario with a source $S$, a destination $D$ and a decode-and-forward (DF) relay $R$ as schematically shown in Fig. 1. The entire system operates in a slotted fashion where time is divided into transmission periods of length $2m$ (symbols). Each transmission period contains two frames (each frame with length $m$), which are referred to as backhaul frame and relaying frame. In the backhaul frame, the source sends a data block to the relay. Afterwards, if the relay decodes the block successfully, it forwards the block to the destination in the subsequent relaying frame.

Channels are assumed to experience a time-varying random fading due to moving objects in the multi-path environment. We assume a correlated, slow-fading Rayleigh-distributed process with Jakes power spectrum density [19]. Although the fading process is assumed to be correlated, consecutive transmission periods are assumed to have independent fading gains due to the large time duration in between. In other words, the channel fading is constant during the duration of each transmission period, i.e., satisfying a block-fading model.

During a transmission period, first a backhaul frame is employed, followed by a relaying frame. Considering a transmission period $i$, channels of the backhaul link and the relaying link are denoted by $h_{1,i}$ and $h_{2,i}$. In addition, $h_{1,i}$ and $h_{2,i}$ are assumed to be independent and identically distributed (i.i.d.). The received SNR at the relay of the backhaul frame and the received SNR at the destination of the relaying frame are denoted by $\gamma_{1,i}$ and $\gamma_{2,i}$. Hence, we have $\gamma_{k,i} = \tilde{\gamma}_k h_{k,i}^2$, $k = 1, 2$, where $\tilde{\gamma}_k$ is the average SNR of link $k$ (either the backhaul link or the relaying link). Moreover, the source is assumed to have outdated instantaneous CSI $\hat{h}_{1,i}$.

Fig. 1. Example of the considered DF relaying scenario.
and \( \hat{h}_{2,i} \) of the two links, which is obtained by sampling the channel \( n \) symbols prior to each upcoming transmission period. According to the sequence of the backhaul frame and relaying frame in each transmission period, the time gaps (between sampling the channel and transmitting a packet by the channel) of a backhaul frame and a relaying frame are \( n \) and \( m + n \). Based on the results of [19], \( h_{k,i}, k = 1, 2 \) conditioned on \( \hat{h}_{k,i} \) follows a complex Gaussian distribution: 
\[
h_{k,i} | \hat{h}_{k,i} \sim \mathcal{CN} \left( \rho_k \hat{h}_{k,i}, (1 - \rho_k^2) \right),
\]
where \( \rho_k^2 \), \( k = 1, 2 \) are correlation coefficients for the backhaul link and the relaying link between the outdated CSI and the exact CSI. Under the assumption of a Jakes’ model, \( \rho_1 = J_0(2\pi f_{S-R} n) \) and \( \rho_2 = J_0(2\pi f_{R-D} (n + m)) \), where \( f_{S-R} \) and \( f_{R-D} \) stands for the Doppler frequency experienced on the backhaul link and the relaying link. In addition, \( J_0(\cdot) \) denotes the zero-order Bessel function of the first kind. Based on the outdated CSI \( \hat{h}_{k,i} \), the outdated SNRs are given by 
\[
\hat{\gamma}_{k,i} = \bar{\gamma}_{k,i} \hat{h}_{k,i}^2, k = 1, 2.
\]

B. Blocklength-Limited Performance of a Single-Hop Transmission Scenario with Perfect CSI

For the real additive white Gaussian noise (AWGN) channel, [8, Theorem 54] derives an accurate approximation of the coding rate of a single-hop transmission system. With blocklength \( m \), block error probability \( \varepsilon \) and SNR \( \gamma \), the coding rate (in bits per channel use) is given by:
\[
r \approx \frac{1}{2} \log_2 (1 + \gamma) - \sqrt{\frac{V_{\text{real}}}{m}} Q^{-1}(\varepsilon),
\]
where \( Q^{-1}(\cdot) \) is the inverse Q-function, and as usual, the Q-function is given by 
\[
Q(w) = \int_w^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt.
\]
In addition, \( V_{\text{real}} \) is the channel dispersion of a real Gaussian channel which is given by 
\[
V_{\text{real}} = \frac{\gamma}{2} \frac{\gamma+2}{(1+\gamma)^2} (\log_2 e)^2.
\]

Under a quasi-static fading channel model, each channel state is assumed to be static during a transmission period. For example, in each transmission period a quasi-static fading channel with fading coefficient \( h \) can be viewed as an AWGN channel with channel gain \( |h|^2 \). Therefore, the above result with a real AWGN channel can be reasonably extended to a complex quasi-static fading channel model in [10]–[14]: With a received SNR \( \gamma \), the coding rate of a transmission period (in bits per channel use) is given by:
\[
r = \mathcal{R}(\gamma, \varepsilon, m) \approx \mathcal{C}(\gamma) - \sqrt{\frac{V_{\text{comp}}}{m}} Q^{-1}(\varepsilon),
\]
where \( \mathcal{C}(\gamma) \) is the Shannon capacity function of a complex channel with received SNR \( \gamma \): 
\[
\mathcal{C}(\gamma) = \log_2 (1 + \gamma).
\]
In addition, the channel dispersion of a complex Gaussian channel is twice the one of a real Gaussian channel: 
\[
V_{\text{comp}} = 2V_{\text{real}} = \gamma \frac{\gamma+2}{(1+\gamma)^2} (\log_2 e)^2 = \left(1 - \frac{1}{(1+\gamma)^2} \right) (\log_2 e)^2 \text{s}.
\]
\[ \varepsilon_{k,i} = \int_{0}^{+\infty} P[\gamma_{k,i}, r_i] P(\gamma_{k,i}, r_i, m) d\gamma_{k,i} = \int_{0}^{+\infty} \frac{I_0(2\rho_k \sqrt{\gamma_{k,i} / \bar{\gamma}_{k,i}(1 - \rho_k^2)}) \exp(-\frac{\gamma_{k,i} + \rho_k^2 \gamma_{k,i}}{\bar{\gamma}_{k,i}(1 - \rho_k^2)})}{2\bar{\gamma}_{k,i}(1 - \rho_k^2)} \frac{C(\gamma_{k,i}) - r_i}{\sqrt{V_{\text{comp}} / m}} \log_2 e d\gamma_{k,i} \]

\[ x = \frac{\gamma_{k,i}}{\rho_k^2 \gamma_{k,i} + 1} = \frac{1}{\sqrt{2\pi}} \int_{0}^{+\infty} x I_0(\frac{x \rho_k \sqrt{\gamma_{k,i}}}{\sqrt{\bar{\gamma}_{k,i}(1 - \rho_k^2)}}) e^{-\frac{x^2}{2} - \frac{\rho_k^2 \gamma_{k,i}}{\gamma_{k,i}(1 - \rho_k^2)} + \frac{1}{4} x^2} dx 
\]

\[ = \frac{1}{\sqrt{2\pi}} \int_{0}^{+\infty} x I_0(\frac{x \rho_k \sqrt{\gamma_{k,i}}}{\sqrt{\bar{\gamma}_{k,i}(1 - \rho_k^2)}}) e^{-\frac{x^2}{2} + \frac{\rho_k^2 \gamma_{k,i}}{\gamma_{k,i}(1 - \rho_k^2)} + \frac{1}{4} x^2} dx. \]

Then, for a single hop transmission under a quasi-static fading channel, with blocklength \( m \) and coding rate \( r \) (during a transmission period), the decoding (block) error probability at the receiver is given by:

\[ \varepsilon = \mathcal{P}(\gamma, r, m) \approx Q \left( \frac{C(\gamma) - r}{\sqrt{V_{\text{comp}} / m}} \right). \]  

(3)

Considering the channel fading, the expected/average error probability over channel fading is given by \([10]:\)

\[ \mathbb{E}_\gamma [\varepsilon] = \mathbb{E}_\gamma [\mathcal{P}(\gamma, r, m)] \approx \mathbb{E}_\gamma \left[ Q \left( \frac{C(\gamma) - r}{\sqrt{V_{\text{comp}} / m}} \right) \right], \]  

(4)

where \( \mathbb{E}_\gamma [\cdot] \) is the expectation over the distribution of channel SNR \( \gamma \).

In the remainder of the paper, we investigate the considered relaying system in the finite blocklength regime by applying the above approximations. As these approximations have been shown to be accurate for a sufficiently large value of \( m \) \([8]\), for simplicity we will assume them to be equal in our analysis and numerical evaluation where we consider a sufficiently large value of \( m \) at each hop of relaying.

### III. Blocklength-Limited Performance of Relaying with Outdated CSI

As the CSI is outdated, if the source determines the coding rate directly based on it, it is likely that the exact channel SNR is lower than the outdated one. This introduces a significant additional error probability. Therefore, we introduce SNR weights, i.e., SNR back-offs, to let the source choose a relatively lower coding rate which is obtained by the weighted outdated SNR. In the following, we consider two different schemes of determining the SNR weights.
A. Dynamic SNR Weight Scheme

Under the dynamic scheme, the source determines two appropriate SNR weights for the backhaul link and the relaying link of each transmission period based on the instantaneous outdated CSI. Denote the determined weights at transmission period $i$ of either the backhaul link or the relaying link by $\eta_{k,i}$ ($k = 1, 2$), where $0 < \eta_{k,i} \leq 1$. As the performance of relaying is subject to the bottleneck link of the system (which is either the backhaul link or the relaying link), the coding rate $r_i$ of transmission period $i$ could therefore be obtained by:

$$r_i = R(\min\{\eta_{1,i}, \hat{\gamma}_{1,i}, \eta_{2,i}, \hat{\gamma}_{2,i}\}, \Delta, m),$$

where $\Delta$ is a constant parameter (a given error probability target of the bottleneck link). Hence, with the exact SNRs $\gamma_{1,i}$ and $\gamma_{2,i}$ the overall error probability of relaying during transmission period $i$ is given by:

$$\varepsilon_{R,i}(r_i) = 1 - (1 - \varepsilon_{1,i})(1 - \varepsilon_{2,i}),$$

(5)

where $\varepsilon_{k,i} = \mathcal{P}(\gamma_{k,i}, r_i, m), i = 1, 2$. Based on (5), we immediately have the expected overall error probability conditioned on the outdated CSI $\hat{\gamma}$. It is the expected value of (5) over the channel fading:

$$\bar{\varepsilon}_{R,i}(r_i) = \varepsilon_{1,i} + \varepsilon_{2,i} - \varepsilon_{1,i}\varepsilon_{2,i}.$$

(6)

In (6), $\bar{\varepsilon}_{k,i}, k = 1, 2$ are the expected error probability of either the backhaul link and the relaying link, which are with expected values based on the outdated channel SNRs $\hat{\gamma}_{k,i}, k = 1, 2$.

According to [19], with correlation coefficients $\rho_k^2$, the conditional probability density function (PDF) of the SNR of link $k$ during transmission period $i$ is given by:

$$\mathbb{P}[\gamma_{k,i}|\hat{\gamma}_{k,i}] = \exp\left(-\frac{\gamma_{k,i} + \rho_k^2 \hat{\gamma}_{k,i}}{\hat{\gamma}_{k,i}(1 - \rho_k^2)}\right) \cdot I_0 \left(\frac{2\rho_k \sqrt{\gamma_{k,i} \hat{\gamma}_{k,i}}}{\hat{\gamma}_{k,i}(1 - \rho_k^2)}\right),$$

(7)

where $I_0$ is the modified Bessel function of the first kind.

Then, $\bar{\varepsilon}_{k,i}, k = 1, 2$ can be obtained by averaging $\varepsilon_{k,i}$ over the above PDF. We provide the derivation of $\bar{\varepsilon}_{k,i}$ in (2), where $\alpha(x, r_i) = \frac{C(x^2 \hat{\gamma}_{k,i}(1 - \rho_k^2)/2 - r_i)}{\sqrt{1 - 2c(x^2 \hat{\gamma}_{k,i}(1 - \rho_k^2)/2)\log_2 e}}$ and $x_{\hat{\gamma}_{k,i}} = \sqrt{2/(1 - \rho_k^2)}$.

Under the two-hop relaying scenario, if the coding rate at each hop during transmission period $i$ is $r_i$, the (source-to-destination) equivalent coding rate during the period is actually $r_i/2$. Therefore, the expected BL-throughput of relaying during transmission period $i$, i.e., the expected effectively transmitted information (the number of correctly received bits at the destination) per
channel use, is given by:

\[ C_{BL,i}(r_i) = r_i(1 - \bar{\varepsilon}_{R,i}(r_i))/2. \]  

(8)

So far, we derived the expected BL-throughput of relaying for an upcoming transmission period \( i \) based on the current outdated CSI (the feedbacked CSI for period \( i \)). We then have the following proposition:

**Proposition 1.** Consider a relaying scenario with correlated and slow-fading Rayleigh channels where only outdated CSI are available at the source. If the coding rate is determined by the weighted SNR while the SNR weights for transmission period \( i \) satisfy \( \eta_{k,i} \leq \rho_k^2, k = 1, 2 \), \( C_{BL,i} \) the expected BL-throughput of the upcoming transmission period \( i \) is quasi-concave in the coding rate \( r_i \).

**Proof.** See Appendix A.

Recall that the coding rate is chosen by the source based on \( \min \{ \eta_{1,i}, \hat{\gamma}_{1,i}, \eta_{2,i}, \hat{\gamma}_{2,i} \} \). According to (1), the coding rate is strictly increasing in \( \min \{ \eta_{1,i}, \hat{\gamma}_{1,i}, \eta_{2,i}, \hat{\gamma}_{2,i} \} \) and therefore increasing in \( \eta_{1,i} \) or \( \eta_{2,i} \). Combining this with Proposition 1, we have an important corollary:

**Corollary 1.** Consider a relaying scenario with correlated and slow-fading Rayleigh channels where only outdated CSI are available at the source. If the coding rate is determined by the weighted SNR while these SNR weights for transmission period \( i \) satisfy \( \eta_{k,i} \leq \rho_k^2, k = 1, 2 \), \( C_{BL,i} \) the expected BL-throughput of the upcoming transmission period \( i \) is quasi-concave in either \( \eta_{1,i} \) or \( \eta_{2,i} \).

**Proof.** See Appendix B.

Finally, based on the expected BL-throughput for each transmission period, the ergodic BL-throughput of relaying, which is actually the expectation value of \( C_{BL,i} \) over the distribution of \( r_i \), can be obtained by: 

\[ C_{BL} = \mathbb{E}_{r_i}[C_{BL,i}(r_i)]. \]

Due to the channels' Rayleigh fading behavior, \( C_{BL} \)
is:
\[ C_{BL} = \mathbb{E}_{\eta_i} [C_{BL,i}(r_i)] = \int_0^\infty \int_0^\infty C_{BL,i}(\mathcal{R}(\min\{\eta_1,\hat{\gamma}_1,\eta_2,\hat{\gamma}_2\}, \varepsilon^*, m)) e^{-\frac{\gamma_1}{\eta_1} - \frac{\gamma_2}{\eta_2}} d\hat{\gamma}_1 d\hat{\gamma}_2 \]
\[ = \frac{1}{\gamma_1 \gamma_2} \int_0^\infty \int_{\frac{\eta_1 \gamma_1}{\eta_2 \gamma_2}}^\infty C_{BL,i}(\mathcal{R}(\eta_1,\hat{\gamma}_1, \varepsilon^*, m)) e^{-\frac{\gamma_1}{\eta_1} - \frac{\gamma_2}{\eta_2}} d\hat{\gamma}_2 d\hat{\gamma}_1 \]
\[ + \frac{1}{\gamma_1 \gamma_2} \int_0^\infty \int_{\frac{\eta_2 \gamma_2}{\eta_1 \gamma_1}}^\infty C_{BL,i}(\mathcal{R}(\eta_2,\hat{\gamma}_2, \varepsilon^*, m)) e^{-\frac{\gamma_1}{\eta_1} - \frac{\gamma_2}{\eta_2}} d\hat{\gamma}_1 d\hat{\gamma}_2 \]  
\[ (9) \]

**B. Fixed SNR Weight Scheme**

In this subsection, we consider the scheme with fixed SNR weights. We denote the weights by \( \eta_1 \) and \( \eta_2 \). Hence, once \( \eta_1 \) and \( \eta_2 \) are determined by the system initialization, they will be constant during all transmission periods. It should be mentioned that under the fixed weight scheme the coding rate is not fixed as the coding rate is influenced by both the weight and the instantaneous outdated CSI. In particular, the coding rate \( r_i \) of transmission period \( i \) is obtained by: \( r_i = \mathcal{R}(\min\{\eta_1 \hat{\gamma}_1,i,\eta_2 \hat{\gamma}_2,i\}, \varepsilon^*, m) \). According to (1), the coding rate is strictly increasing in \( \min\{\eta_1 \hat{\gamma}_1,i,\eta_2 \hat{\gamma}_2,i\} \) and therefore monotonically increasing in \( \eta_1 \) and \( \eta_2 \). In addition, (8) also holds for the fixed SNR weight scheme (while the expressions of \( r_i \) under dynamic and fixed schemes are different). Finally, under the fixed SNR weight scheme \( C_{BL} \) the BL-throughput over time is given by:

\[ C_{BL} = \mathbb{E}_{\eta_i} [C_{BL,i}(r_i)] = \frac{1}{\gamma_1 \gamma_2} \int_0^\infty \int_{\frac{\eta_1 \gamma_1}{\eta_2 \gamma_2}}^\infty C_{BL,i}(\mathcal{R}(\eta_1,\hat{\gamma}_1, \varepsilon^*, m)) e^{-\frac{\gamma_1}{\eta_1} - \frac{\gamma_2}{\eta_2}} d\hat{\gamma}_2 d\hat{\gamma}_1 \]
\[ + \frac{1}{\gamma_1 \gamma_2} \int_0^\infty \int_{\frac{\eta_2 \gamma_2}{\eta_1 \gamma_1}}^\infty C_{BL,i}(\mathcal{R}(\eta_2,\hat{\gamma}_2, \varepsilon^*, m)) e^{-\frac{\gamma_1}{\eta_1} - \frac{\gamma_2}{\eta_2}} d\hat{\gamma}_1 d\hat{\gamma}_2. \]  
\[ (10) \]

For the fixed SNR weight scheme, we have the following Proposition:

**Proposition 2.** Consider a relaying scenario with correlated and slow-fading Rayleigh channels where only outdated SNRs are available at the source. If the coding rate is determined by the weighted SNR while these SNR weights are constant over time and satisfy \( \eta_k \leq \rho_k^2, k = 1, 2 \), then \( C_{BL} \) the BL-throughput is quasi-concave in either \( \eta_1 \) or \( \eta_2 \).

**Proof.** See Appendix C. \( \square \)

So far, both the dynamic weight scheme and the fixed weight scheme have been discussed. It should be mentioned that Proposition 1 and Proposition 2 indicate: The expected BL-throughput
of a transmission period can be optimized based on the current outdated CSI. In other words, the expected BL-throughput can be optimized per transmission period by choosing appropriate SNR weights. Therefore, the dynamic weight scheme definitely outperforms the scheme with fixed weight. Nevertheless, the cost of this advantage is complexity, i.e., the source under the dynamic scheme needs to search the optimal weights for each transmission period while the one under the fixed scheme only needs to substitute the fixed weights and the outdated CSI to \( r_i = \mathcal{R}(\min\{\eta_1\hat{\gamma}_{1,i}, \eta_2\hat{\gamma}_{2,i}\}, \varepsilon^*, m) \). Moreover, Proposition 3 indicates that the BL-throughput under the fixed weight scheme can also be optimized by determining SNR weights. Then, straightforward questions arise: What is the performance difference between these two schemes? If the gain of searching and adapting the weights per transmission period is significant? We will provide a numerical analysis to compare them in the next section.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we first present numerical results to illustrate our analytical model. Subsequently, we compare the dynamic weight scheme with the fixed weight scheme. Later on, we evaluate the studied relaying system while the direct transmission scheme is treated as a comparison scheme. During the numerical analysis, we consider the following parameterization of the system model: Firstly, in the simulation we consider an outdoor urban scenario and the distances of the backhaul, relaying and direct links are set to 100 m, 100 m and 200 m. Secondly, we utilize the well-known COST [20] model (which is a commonly-used model in LTE) for calculating the path-loss while the center frequency is set to 2 GHz. Thirdly, we set the transmit power \( p_{tx} \) to 35 dBm (we vary the transmit power in Fig. 4 and Fig. 7) and noise power to -90 dBm, respectively. Lastly, the blocklength at each hop of relaying is set to 300 (symbols) while the choice of this \( m = 300 > 100 \) is motivated by [8, Fig. 2] where the relative difference of the approximate and the exact achievable rates is less than 2% for the cases with \( m \geq 100 \).

A. Appropriateness of Our Analytical Model

In this subsection, we show the appropriateness of our analytical model based on numerical results. We first study under the dynamic SNR weight scheme the relationship between the expected BL-throughput of an upcoming transmission period (based on the corresponding outdated CSI) and the SNR weights. The result is shown in Fig. 2. First of all, the figure illustrates our
analytical model that the BL-throughput of a transmission period $i$ (conditioned on the outdated CSI) is quasi-concave in SNR weights $\eta_{1,i}$ and $\eta_{2,i}$. Hence, by choosing appropriate values for $\eta_{1,i}$ and $\eta_{2,i}$ the BL-throughput of transmission period $i$ can be optimized. Secondly, the figure also shows that the expected BL-throughput of the upcoming period is actually subject to both $\eta_{1,i}$ and $\eta_{2,i}$. For example, if $\eta_{1,i}$ is chosen correctly, then the choice of $\eta_{2,i}$ can be arbitrary big (not arbitrary small). The explanation is as follows. The case (BL-throughput being only influenced by $\eta_{1,i}$) corresponds to the situation where the bottleneck link (for determining the coding rate) is the backhaul link. At the same time, $\eta_{2,i}$ (the SNR weight of the relaying link) does not influence the SNR of the backhaul link and therefore does not influence the coding rate. In other words, there is no impact of one link’s weight on the BL-throughput of the upcoming transmission period if this link is not the bottleneck link. It should be mentioned that reducing the weight of a link likely makes this link become the bottleneck link. Then, the BL-throughput of the upcoming transmission period is influenced by this weight.

Secondly, we consider the fixed weight scheme in Fig. 3 where we show the relationship between the BL-throughput $C_{BL}$ and the SNR weights $\eta_1$ and $\eta_2$. The figure matches well with our analytical model (Proposition 3) that $C_{BL}$ is quasi-concave in $\eta_1$ or $\eta_2$.
Fig. 3. Fixed SNR weight scheme ($\rho_1^2 = 0.7$ and $\rho_2^2 = 0.5$): The BL-throughput over transmission periods vs. SNR weights.

B. Relaying Performance Investigation

1) Dynamic SNR weight scheme vs. fixed SNR weight scheme: The comparison between these two weight schemes is shown in Fig. 4 where we consider three scenarios with different channel

Fig. 4. The ergodic BL-throughput of relaying with outdated CSI: Dynamic SNR weight scheme vs. fixed SNR weight scheme.
correlation coefficients. We observe that there is no big difference between the dynamic scheme and the fixed scheme if the correlation between the channel estimate and the real channel state is large, e.g., channels vary extremely slow or/and the transmission period are extremely short. Otherwise, the dynamic scheme outperforms the fixed one. Moreover, especially for the high SNR range does the dynamic scheme outperform the static scheme.

2) **Convergence: Relaying with finite blocklengths vs. Relaying with infinite blocklengths:** We show the convergence comparison in Fig. 5 where we vary channel correlation coefficients. In the figure, the maximal BL-throughput and the maximal outage capacity (based on the Shannon capacity) under the outdated CSI scenario are obtained by choosing optimal \( \eta_{1,i} \) and \( \eta_{2,i} \) for each transmission period \( i \). We find that the performance loss (due to a finite blocklength) in a relaying network is considerable under the perfect CSI scenario but is negligible under the outdated CSI scenario. In other words, relaying is more efficient under the outdated CSI scenario than the perfect CSI scenario.

3) **Relaying vs. direct transmission:** Further, we show in Fig. 6 the ergodic BL-throughput under the fixed SNR weight scheme while the performance of direct transmission is also plotted as a comparison scheme. To make a fair comparison, we have the following assumptions in the numerical analysis: i. During each transmission period, relaying and direct transmission have the
same (equivalent) coding rate and have the same resource consumption, i.e., the blocklength of direct transmission is twice as large as the blocklength at each hop of relaying. ii. The channel correlation coefficient of direct transmission equals the one of the backhaul link of relaying, i.e., $\rho_{DT}^2 = \rho_1^2 > \rho_2^2$. Therefore, the delay by which the CSI feedback of the second hop of relaying subjected is longer than direct transmission. iv. By adjusting the transmit power of direct transmission we make relaying and direct transmission have the same performance in the infinite blocklength regime, i.e., $C(\bar{\gamma}_{DT}) = C(\min\{\bar{\gamma}_1, \bar{\gamma}_2\})/2$, where $\bar{\gamma}_{DT}$ is the expected SNR of direct transmission. Under the above setup, we compare relaying and direct transmission. We show the result in Fig. 6. Although having similar performance in the infinite blocklength regime, shorter blocklength at each hop and more outdated CSI (of the second hop), relaying outperforms direct transmission in the finite blocklength regime for both the perfect CSI case and the outdated CSI case. This is actually the performance advantage of relaying (which we also observed in our previous work [16]–[18] where the CSI is not outdated). More importantly, in this work we find that under the outdated CSI scenario this performance advantage of relaying

Fig. 6. The ergodic BL-throughput of relaying ($\rho_1^2 = 0.7$ and $\rho_2^2 = 0.5$) in comparison to direct transmission ($\rho_{DT}^2 = 0.7$).
is more significant than under the perfect CSI scenario.

4) An explanation of the performance advantage of relaying: An explanation of relaying outperforming direct transmission in the finite blocklength regime in term of the BL-throughput is provided as follows. As we know, finite blocklength introduces a performance loss (comparing the BL-throughput with the Shannon capacity). More importantly, this performance gap is influenced by the SNR. If we call the ratio of the performance loss over the Shannon capacity "performance loss ratio", i.e., \( \frac{C - C_{\text{BL}}}{C} \), this performance loss ratio is also influenced by the SNR. In the following figure, we show the relationship between the performance loss ratio and the SNR for a single-hop link under an AWGN channel. Based on the figure, we find that increasing the SNR significantly reduces the performance loss ratio. As relaying has a significantly higher SNR at each hop in comparison to the direct link, relaying significantly mitigates the performance loss due to a finite blocklength. As a result, although relaying halves the blocklength, it shows a performance advantage over direct transmission in the finite blocklength regime. This explanation can also be extended to the considered quasi-static channels: Relaying has a low performance loss ratio for each channel state, therefore the average performance loss ratio over channel fading of relaying is lower than the direct transmission. As a result, although having similar Shannon capacity, the BL-throughput of relaying is higher than direct transmission.
V. CONCLUSION

In this work, we studied the finite blocklength performance of relaying under outdated CSI at the source. We showed that under the dynamic SNR weight scheme the instantaneous BL-throughput of a transmission period is concave in the coding rate and quasi-concave in SNR weights. In addition, under the fixed SNR weight scheme we showed that the average BL-throughput over time is quasi-concave in the SNR weights. By numerical analysis, we showed the appropriateness of our analytical model. Moreover, we concluded a set of guidelines for the design of efficient relaying systems (in the finite blocklength regime) from numerical analysis. Firstly, for a given scenario the performance can be optimized by adjusting the SNR weights. Secondly, the distinction in fixed and dynamic weight scheduling matters a lot whenever the correlation is weak and the SNR is high. More importantly, we showed that the lower the blocklength the more relaying pays off in comparison to direct transmission, and that this is especially relevant for the design of low latency systems. This performance advantage is due to the fact that relaying has a higher SNR at each hop (than the direct link) which is more relevant for the system performance than the halving of the transmission periods per single hop.

In our previous work [18] of relaying with the average CSI, we observed the performance advantage of a relaying system where a spatial diversity is exploited, i.e., the destination receives signals from both the source and the relay. Therefore, the performance advantage of relaying shown in this paper (where the signal from the source directly to the destination is not considered) further indicates that even without exploiting spatial diversity relaying still pays off in the finite blocklength regime due to having a higher SNR at each hop. Combining the results of this paper with our previous work [16]–[18], we finally conclude that applying relaying is a promising way to improve the performance of low latency systems, especially for latency critical applications e.g., haptic communication, automation and control applications and cyber physical systems, where blocklengths are significantly short.
APPENDIX A

PROOF OF THE THEOREM 1

Proof. Based on (8) and (6), we immediately have \( C_{\text{BL},i}(r_i) = (1 - \bar{\varepsilon}_{1,i}(r_i))(1 - \bar{\varepsilon}_{2,i}(r_i))r_i/2 \). Therefore, the first and second derivatives of \( C_{\text{BL},i} \) with respect to \( r_i \) are given by:

\[
\frac{\partial C_{\text{BL},i}}{\partial r_i} = \frac{(1 - \bar{\varepsilon}_{1,i}(r_i))(1 - \bar{\varepsilon}_{2,i}(r_i))}{2} - \frac{\partial \bar{\varepsilon}_{1,i}(1 - \bar{\varepsilon}_{2,i}(r_i))r_i}{2}
\]

\[
= - \frac{\partial \bar{\varepsilon}_{2,i}(1 - \bar{\varepsilon}_{1,i}(r_i))r_i}{2},
\]

\[
\frac{\partial^2 C_{\text{BL},i}}{\partial^2 r_i} = - \frac{\partial \bar{\varepsilon}_{1,i}(1 - \bar{\varepsilon}_{2,i}(r_i))}{\partial r_i} - \frac{\partial \bar{\varepsilon}_{2,i}(1 - \bar{\varepsilon}_{1,i}(r_i))}{\partial r_i} - \frac{\partial^2 \bar{\varepsilon}_{1,i}(1 - \bar{\varepsilon}_{2,i}(r_i))r_i}{2} + \frac{\partial \bar{\varepsilon}_{1,i}}{\partial r_i} \frac{\partial \bar{\varepsilon}_{2,i}}{\partial r_i}.\]

In the following, we prove Theorem 1 by showing \( \frac{\partial^2 C_{\text{BL},i}}{\partial^2 r_i} < 0 \).

Recall that \( \varepsilon_{k,i} = P(\gamma_{k,i}, r_i, m_j, j = 1, 2 \text{. According to (11), we have:} \)

\[
\frac{\partial \varepsilon_{k,i}}{\partial r_i} = \frac{m^\frac{1}{2} \exp \left( - \frac{m(C(\gamma_{k,i}) - r_i)^2}{2(1 - 2^{-2C(\gamma_{k,i})}(\log_2 e))^2} \right)}{\sqrt{2\pi}(1 - 2^{-2C(\gamma_{k,i})})^\frac{3}{2} \log_2 e} > 0,
\]

\[
\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} = \frac{m^\frac{3}{2} (C(\gamma_{k,i}) - r_i) \exp \left( - \frac{m(C(\gamma_{k,i}) - r_i)^2}{2(1 - 2^{-2C(\gamma_{k,i})})(\log_2 e)^3} \right)}{\sqrt{2\pi}(1 - 2^{-2C(\gamma_{k,i})})^\frac{3}{2} (\log_2 e)^3}.
\]

Based on (12), we have:

\[
\frac{\partial \varepsilon_{k,i}}{\partial r_i} = \int_0^{\infty} P[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{\partial \varepsilon_{k,i}}{\partial r_i} d\gamma_{k,i} > 0,
\]

\[
\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} = \int_0^{\infty} P[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i}.
\]

According to the distribution of \( \gamma_{k,i} \) conditioned on \( \hat{\gamma}_{k,i} \), we immediately obtain the median of the distribution, which is \( \rho_k^2 \hat{\gamma}_{k,i} \). In other words, the following equation holds:

\[
\int_0^{\rho_k^2 \hat{\gamma}_{k,i}} P[\gamma_{k,i} | \hat{\gamma}_{k,i}] d\gamma_{k,i} = \int_{\rho_k^2 \hat{\gamma}_{k,i}}^{\infty} P[\gamma_{k,i} | \hat{\gamma}_{k,i}] d\gamma_{k,i} = \frac{1}{2}.
\]

As \( \eta_k < \rho_k^2, k = 1, 2 \), the following inequality holds: \( r_i < C(\min_{j=1,2}(\eta_k \hat{\gamma}_{k,i})) \leq C(\min_{j=1,2}(\rho_k^2 \hat{\gamma}_{k,i})) \). Hence, \( C(\gamma_{k,i}) - r_i > 0 \) and therefore \( \frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} > 0 \) if \( \gamma_{k,i} \in [\rho_k^2 \hat{\gamma}_{k,i} + \infty) \). In other words, \( \int_{\rho_k^2 \hat{\gamma}_{k,i}}^{\infty} P[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i} > 0 \) and \( \int_{\rho_k^2 \hat{\gamma}_{k,i}}^{\infty} P[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i} > 0 \).
Considering equality (17), we have:

\[
\int_{\rho_k^2 \hat{\gamma}_{k,i}}^{+\infty} \mathbb{P}[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i} = \int_{\rho_k^2 \hat{\gamma}_{k,i}}^{+\infty} \mathbb{P}[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{m^2}{2\pi} \left( C(\gamma_{k,i}) - r_i \right) e^{-\frac{m(C(\gamma_{k,i}) - r_i)^2}{2(1 - 2C(\gamma_{k,i}))^2} \left( \log_2 e \right)^3} d\gamma_{k,i}
\]

\[
> \int_{0}^{\rho_k^2 \hat{\gamma}_{k,i}} \mathbb{P}[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{m^2}{2\pi} \left( C(\gamma_{k,i}) - r_i \right) e^{-\frac{m(C(\gamma_{k,i}) - r_i)^2}{2(1 - 2C(\gamma_{k,i}))^2} \left( \log_2 e \right)^3} d\gamma_{k,i}
\]

\[
= \int_{0}^{\rho_k^2 \hat{\gamma}_{k,i}} \mathbb{P}[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i}
\]

(18)

So far, it has been shown that

\[
\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} = \int_{0}^{\rho_k^2 \hat{\gamma}_{k,i}} \mathbb{P}[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i}
\]

\[
> \int_{0}^{\rho_k^2 \hat{\gamma}_{k,i}} \mathbb{P}[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i} + \int_{0}^{\rho_k^2 \hat{\gamma}_{k,i}} \mathbb{P}[\gamma_{k,i} | \hat{\gamma}_{k,i}] \frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i}
\]

(19)

\[
> 0.
\]

According to (3), the error probability of a link is higher than 0.5 only if the coding rate is higher than the Shannon capacity. Recall that the coding rate chosen by the source satisfies

\[
r_i < C \left( \min_{j=1,2} \{ \eta_k \hat{\gamma}_{k,i} \} \right) \leq C \left( \min_{j=1,2} \{ \rho_k^2 \hat{\gamma}_{k,i} \} \right) .
\]

This makes the expected error probability of each single link (during transmission period \(i\)) be lower than 0.5, i.e., \(\varepsilon_{k,i} < 0.5\), \(j = 1, 2\). Based on (12), we have:

\[
\frac{\partial^2 \varepsilon_{1,1}}{\partial^2 r_i} < \frac{\partial^2 \varepsilon_{2,1}}{\partial^2 r_i} \frac{r_i}{4} - \frac{\partial^2 \varepsilon_{2,1}}{\partial^2 r_i} \frac{r_i}{4} + \frac{\partial^2 \varepsilon_{1,1}}{\partial^2 r_i} \frac{\partial^2 \varepsilon_{2,1}}{\partial r_i} \frac{r_i}{2}
\]

\[
< \left( \frac{2 \partial^2 \varepsilon_{1,1}}{\partial^2 r_i} - \frac{\partial^2 \varepsilon_{1,1}}{\partial^2 r_i} - \frac{\partial^2 \varepsilon_{2,1}}{\partial^2 r_i} \right) \frac{r_i}{2}
\]

\[
\leq \left( 2 \frac{\partial^2 \varepsilon_{1,1}}{\partial^2 r_i} - \frac{\partial^2 \varepsilon_{2,1}}{\partial^2 r_i} \right) \frac{r_i}{2}
\]

(20)
Based on Proposition 1, bottleneck link, i.e., 
\min_{\{\eta_{i}\}} \partial C_{BL,i} |_{\eta_{i}} \leq 0 if \partial^{2}C_{BL,i} |_{\eta_{i}} < 0 if \partial^{2}C_{BL,i} |_{\eta_{i}} > 0$.

For large enough $m$, we have:

\[4 \left( \frac{\partial \varepsilon_{k,i}}{\partial r_{i}} \right)^{2} \sim O \left( \frac{8 \left( 1 - 2^{-2C(\gamma_{k,i})} \right) \left( \log_{2} e \right)^{2}}{\pi (\gamma_{k,i} - r_{i})^{2}} \right)\]

\[= O \left( \frac{1}{m} \right) \cdot \frac{8 \left( 1 - 2^{-2C(\gamma_{k,i})} \right) \left( \log_{2} e \right)^{2}}{\pi (\gamma_{k,i} - r_{i})^{2}} \]

\[< O \left( \frac{1}{m} \right) \cdot \frac{8 \left( \log_{2} e \right)^{2}}{\pi (\gamma_{k,i} - r_{i})^{2}}, \]

\[\frac{\partial^{2} \varepsilon_{k,i}}{\partial^{2} r_{i}} \sim O \left( \frac{2}{\sqrt{2\pi} \left( 1 - 2^{-2C(\gamma_{k,i})} \right)^{\frac{1}{2}} \left( \gamma_{k,i} - r_{i} \right) \log_{2} e} \right)\]

\[= O \left( m^{\frac{1}{2}} \right) \cdot \frac{2}{\sqrt{2\pi} \left( 1 - 2^{-2C(\gamma_{k,i})} \right)^{\frac{1}{2}} \left( \gamma_{k,i} - r_{i} \right) \log_{2} e} \]

\[> O \left( m^{\frac{1}{2}} \right) \cdot \frac{2}{\sqrt{2\pi} \left( \gamma_{k,i} - r_{i} \right) \log_{2} e}.\]

Obviously, $\frac{\partial^{2} C_{BL,i}}{\partial^{2} r_{i}} < 0$ holds if $m \gg \frac{4 \left( \log_{2} e \right)^{2}}{\sqrt{2\pi} \left( \gamma_{k,i} - r_{i} \right)^{2}} \approx \frac{4.5}{\left( \gamma_{k,i} - r_{i} \right)^{2}}$. Note that $C(\gamma_{k,i}) - r_{i} \gg 0$ and recall that we consider $m$ with a practical value i.e., is significantly longer than 100. Hence, $C_{BL,i}$ is concave in $r_{i}$. 

\[\square\]

**APPENDIX B**

**PROOF OF THE PROPOSITION 2**

**Proof.** The coding rate for transmission period $i$ is determined based by the outdated CSI of the bottleneck link, i.e., $\min \{\eta_{l}, \gamma_{1,i}, \eta_{2}, \gamma_{2,i}\}$. According to (1), we learn that $r_{i}$ is strictly increasing in $\min \{\eta_{l}, \gamma_{1,i}, \eta_{2}, \gamma_{2,i}\}$. Hence, when the source determines the coding rate $r_{i}$, high values of $\eta_{l}$ and $\gamma_{2,i}$ lead to a big $r_{i}$. In other words, $r_{i}$ is monotonically increasing in $\eta_{k,i}, k = 1, 2$.

$\Rightarrow \forall x_{k} < y_{k}, x_{k}, y_{k} \in (0, \rho_{k}]$ and $\lambda_{k} \in [0, 1]$, we have $r_{i} |_{\eta_{k,i} = x_{k}} < r_{i} |_{\eta_{k,i} = \lambda_{k} x_{k} + (1 - \lambda_{k}) y_{k}} < r_{i} |_{\eta_{k,i} = y_{k}}$, where $k = 1, 2$.

Based on Proposition 1, $C_{BL,i}$ is concave in $r_{i}$,

$\Rightarrow \min \left\{ C_{BL,i} \left( r_{i} |_{\eta_{k,i} = x_{k}} \right), C_{BL,i} \left( r_{i} |_{\eta_{k,i} = y_{k}} \right) \right\} \leq C_{BL,i} \left( r_{i} |_{\eta = \lambda_{k} x_{k} + (1 - \lambda_{k}) y_{k}} \right).

\Rightarrow C_{BL,i}$ is quasi-concave in $\eta_{k,i}$, where $k = 1, 2$ and $0 < \eta_{k,i} \leq \rho_{k}^{2}$. 

\[\square\]

**APPENDIX C**

**PROOF OF THE PROPOSITION 3**

**Proof.** According to the proof of Proposition 2, $r_{i}, i = 1, 2, ..., +\infty$ is monotonically increasing in $\eta_{k}, k = 1, 2$. 
⇒ ∀ \( x_{k,i} < y_{k,i}, i = 1, 2, \ldots, +\infty, x_{k,i}, y_{k,i} \in (0, \rho_k] \) and \( \lambda_{k,i} \in [0, 1] \), we have \( r_i|_{\eta_k=x_{k,i}} < r_i|_{\eta_k=\lambda_{k,i}x_{k,i}+(1-\lambda_{k,i})y_{k,i}} < r_i|_{\eta_k=y_{k,i}} \), where \( k = 1, 2 \).

As shown in Proposition 1, \( C_{BL,i} \) is concave in \( r_i \). Hence, \( C_{BL} = \sum_i C_{BL,i} \) is concave in \( r = (r_1, r_2, \ldots, r_i, \ldots) \).

⇒ \( \min \left\{ \sum_i C_{BL,i}(r_i|_{\eta_k=x_{k,i}}) ; \sum_i C_{BL,i}(r_i|_{\eta_k=y_{k,i}}) \right\} \leq \sum_i C_{BL,i}(r_i|_{\eta_k=\lambda_{k,i}x_{k,i}+(1-\lambda_{k,i})y_{k,i}}) \).

⇒ \( C_{BL} \) is quasi-concave in \( \eta_k \), where \( 0 < \eta_k \leq \rho_k^2 \) and \( k = 1, 2 \).

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