Viscous Generalized Chaplygin Gas as a Unified Dark Fluid: Including Perturbation of Bulk Viscosity

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In this paper, we continue our previous work of studying viscous generalized Chaplygin gas (VGCG) as a unified dark fluid but including the bulk viscosity perturbation. By using the currently available cosmic observational data from SNLS3, BAO, HST and recently released Planck, we gain the constraint on bulk viscosity coefficient: \( \frac{\sigma}{\eta} = 0.0000138 \pm 0.00000614 \), which is the 1.2σ region. This result shows that when considering perturbation of bulk viscosity, the currently cosmic observations favor a smaller bulk viscosity coefficient.

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I. INTRODUCTION

Several astronomical observations such as SN Ia [1], CMB [2], WL [3], etc. powerfully indicates that in the present, the overwhelming majority of cosmological total energy content is a dark sector which takes charge of the acceleration of our universe. This dark sector is generally assumed owning two different components: dark energy and dark matter. To investigate this dark sector, many cosmological models are built basing on the cosmological principle validity and the assumption of an idealized perfect fluid, which means that all components of the matter-energy in our universe are considered as perfect fluid without any viscosity. The most competitive model of dark energy is a cosmological constant model. But researches have shown that constant dark energy models are not well confirmed by both observations and theoretical considerations [20, 21]. One of the alternatives to the cosmological constant is to describe dark matter and dark energy within a unified dark fluid model. For all we know, the Chaplygin gas [15, 16, 17] was firstly presented along this line. However, the unified Chaplygin gas type models forecasted instabilities or mighty oscillations of small scale in the matter power spectrum, which disagrees with the observational data [7]. This problem may be alleviated or even avoided by the non-adiabatic perturbations. A reasonable possibility is to allow the gCG to have non-adiabatic perturbations, which is a natural assumption since it is not a pressureless fluid actually. An attempt in this direction has already been performed in [18] and [19]. Furthermore, in the recently years, more and more cosmological observations suggest that our universe is permeated by imperfect fluid, in which the negative pressure, as was argued in [14, 17], an effective pressure including bulk viscosity can play the role of an agent that drives the present acceleration of universe.

The viscous generalized Chaplygin gas (hereafter referred to as VGCG) is a widely studied model among those proposed to describe the observed accelerated expansion of the universe. In contrast to many models describing dark energy alone, the VGCG gives a unified description of dark matter and dark energy, enrolling itself in the class of so-called unified dark fluid (UDF) cosmological models see e.g. [8, 11]. A common characteristic of these papers is that only the impact of bulk viscosity on the background expansion of the universe is studied without considering perturbation of bulk viscosity. However, the perturbation analysis of the viscous cosmological models is crucially important to the evolution of cosmology. The different mentioned approaches imply a generally different dynamics at the perturbative level. Therefore, it is interesting to study the behaviour of the VGCG under perturbations.

In the present paper, we study only scalar perturbations following the notation of [14]. We will modify the pressure through Eckarts expression [22] \( p_v = -\xi u^\mu_\mu \), where bulk viscosity coefficient \( \xi \) is a non-negative quantity, and the fluid-expansion scalar \( u^\mu_\mu \) is reduces to \( 3H \) in the isotropic and homogeneous universe, where \( H = \frac{\dot{a}}{a} \) is the Hubble parameter. As a continuation of our previous work [15], here we investigate VGCG model by including bulk viscosity perturbation.

The structure of this paper is organized as follows. In the next section, we briefly introduce some basic equations of viscous generalized Chaplygin gas model. The derivation of evolution equations for density perturbation and velocity perturbations are presented in the third section. Then in the forth section, by using the MCMC method, we perform a global fitting to the currently observational data and analyze the constraint results. The discussion and conclusion are given in the final section.

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II. BASIC EQUATIONS OF VISCOUS GENERALIZED CHAPLYGIN GAS MODEL

In an isotropic and homogeneous universe, we consider the standard Friedmann-Robertson-Walker metric,
\[ ds^2 = -dt^2 + a^2(t) \left[ \frac{1}{1 - k r^2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]. \]  
(1)

For the sake of simplicity, we choose the flat geometry \( k = 0 \), which is also favored by the update result of the cosmic background radiation measurement. The general stress-energy-momentum tensor is
\[ T_{\mu\nu} = (\rho + p)U_{\mu}U_{\nu} + pg_{\mu\nu}. \]  
(2)

To consider the effect of bulk viscosity, we modify the pressure only by redefining the effective pressure \( p_{eff} \), according to \( p_{eff} = p + p_\nu = p - 3H\zeta \), we re-write the viscous energy-momentum tensor as:
\[ T_{\mu\nu}^{\text{viscous}} = \rho U^\mu U^\nu + \left( p - 3\zeta \frac{\dot{a}}{a} \right) (g^{\mu\nu} + U^\mu U^\nu) \equiv p_{eff} g^{\mu\nu} + (p_{eff} + \rho)U^\mu U^\nu. \]  
(3)

From the equation above, we see that the effect of bulk viscosity is to change the pressure \( p \) to an effective pressure \( p_{eff} = p - 3\zeta \dot{a}/a \). The physical interpretation is clear that a viscous pressure can play the role of an agent that drives the present acceleration of the universe. Note that the possibility of a viscosity dominated late epoch of the universe with accelerated expansion was already mentioned by Padmanabhan and Chitre in [24].

Using the GCG equation of state \( p_{\nu} = -A/\rho_{\nu}^{\alpha} \), which yields an analytically solvable cosmological dynamics if the universe is GCG dominated, we obtain the equation of state (EoS) of viscous GCG (VGCG) model is given in the form of
\[ p_{\nu \text{VGCG}} = -A/\rho_{\nu \text{VGCG}}^{\alpha} - 3H\zeta, \]  
(4)

this EoS includes the GCG model as its special case when \( \zeta = 0 \); when \( \zeta \neq 0 \), for the normal form \( \zeta = \zeta_0 \rho_{\nu \text{VGCG}}^{\frac{1}{\alpha}} \), we have the equation of state EoS
\[ p_{\nu \text{VGCG}} = -A/\rho_{\nu \text{VGCG}}^{\alpha} - \sqrt{3}\zeta_0 \rho_{\nu \text{VGCG}}, \]  
(5)

where \( A, \zeta_0 \) and \( \alpha \) are model parameters. Applying the energy conservation of VGCG, one can deduce its energy density as
\[ \rho_{\nu \text{VGCG}} = \rho_{\nu \text{VGCG}0} \left[ \frac{B_s}{1 - \sqrt{3}\zeta_0} + (1 - \frac{B_s}{1 - \sqrt{3}\zeta_0}) a^{-3(1+\alpha)(1-\sqrt{3}\zeta_0)} \right]. \]  
(6)

where \( B_s = A/\rho_{\nu \text{VGCG}0}^{1+\alpha} \), \( \alpha \) and \( \zeta_0 \) are model parameters. Form Eq. (6), one can find that \( 0 \leq B_s \leq 1 \) and \( \zeta_0 < \frac{1}{\sqrt{3}} \) are demanded to keep the positivity of energy density. If \( \alpha = 0 \) and \( \zeta_0 = 0 \) in Eq. (6), the standard ACDM model is recovered. Taking VGCG as a unified component, one has the Friedmann equation
\[ H^2 = H_0^2 \left\{ (1 - \Omega_b - \Omega_r - \Omega_k) \left[ \frac{B_s}{1 - \sqrt{3}\zeta_0} + (1 - \frac{B_s}{1 - \sqrt{3}\zeta_0}) a^{-3(1+\alpha)(1-\sqrt{3}\zeta_0)} \right] \right\} + \Omega_b a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2}, \]  
(7)

where \( H \) is the Hubble parameter with its current value \( H_0 = 100 \text{km s}^{-1}\text{Mpc}^{-1} \), and \( \Omega_i \) \((i = b, r, k)\) are dimensionless energy parameters of baryon, radiation and effective curvature density respectively. In this paper, we only consider the spatially flat FRW universe.

Here, we treat VGCG as a unified dark fluid which interacts with the remaining matter purely through gravity. With assumption of pure adiabatic contribution to the perturbations, the adiabatic sound speed for VGCG is
\[ c_{s,ad}^2 = \frac{\rho_{\nu \text{VGCG}} \dot{\rho}_{\nu \text{VGCG}}}{\rho_{\nu \text{VGCG}}^2} = -\alpha w_{eff} - \sqrt{3}\zeta_0, \]  
(8)

where \( w_{eff} \) is the EoS of VGCG in the form of
\[ w_{eff} = -\frac{B_s}{B_s + (1 - B_s) a^{-3(1+\alpha)}} - \sqrt{3}\zeta_0. \]  
(9)

From the above equation, one can find that in order to protect the sound of speed from negativity , \( \alpha \geq 0 \) is required because of the non-positive values of \( w_{eff} \).

We studied the perturbation evolution equations of VGCG in order to research the effects on CMB anisotropic power spectrum. In the synchronous gauge, using the conservation of energy-momentum tensor \( T_{\mu\nu}^{(\text{viscous})} \) in Eq. (5) as zero, one has the perturbation equations of density contrast and velocity divergence for VGCG
\[ \dot{\delta}_{\nu \text{VGCG}} = -(1 + w_{eff}) (\theta_{\nu \text{VGCG}} + \frac{\dot{a}}{a}) - 3H(c_s^2 - w_{eff}) \delta_{\nu \text{VGCG}}, \]  
(10)

\[ \dot{\theta}_{\nu \text{VGCG}} = -H(1 - 3c_s^2) \theta_{\nu \text{VGCG}} + \frac{c_s^2}{1 + w_{eff}} k^2 \delta_{\nu \text{VGCG}} - k^2 \sigma_{\nu \text{VGCG}}, \]  
(11)

following the notation of Ma and Bertschinger [34]. For the perturbation theory in gauge ready formalism, please see [35]. The shear perturbation \( \sigma_{\nu \text{VGCG}} = 0 \) is assumed and the adiabatic initial conditions are adopted in our calculation. When the EoS of a pure barotropic fluid is negative, it has an imaginary adiabatic sound speed which causes instability of the perturbations, for example the \( w = constant \) quintessence dark energy model. The way to overcome this problem is to allow an entropy perturbation and to assume a positive or null effective speed of sound, which we will give a detailed study in the following.
III. PERTURBATION EQUATIONS

A. perturbed Metric and Energy-momentum Tensor

Scalar perturbations of the flat FRW metric are given in the following form

\[ ds^2 = a^2 \{ -(1 + 2\phi)dt^2 + 2\partial_iB drdx^i \}
+ [\{(1-2\phi)\delta_{ij} + 2\partial_i\partial_j\phi)]dxdy, \]

where \( a \) is the scale factor, \( \tau \) is the conformal time, \( x_i \) are the spatial coordinates and \( \phi \) and \( \psi \) are the metric perturbations. The background four-velocity is \( u^\mu = a^{-1}(1,0,0,0) \), which can be derived as follows,

\[ u^\mu = \frac{d\tau}{ds} = \frac{1}{a} \frac{dt}{ds} = \frac{1}{a} \delta^\mu_0, \]

The spatial part is the perturbation, we can set it as \( \partial^i\nu \) for scalar perturbation only. Then using the equality \( g_{\mu\nu}u^\mu u^\nu = -1 \), one has

\[ u^0 = \pm \frac{1}{\sqrt{g_{00}}} = \pm \frac{1}{a}(1 - \phi), \]

so one has the four-velocity of the fluid

\[ u^\mu = a^{-1}(1 - \phi, \partial^i\nu), \]

\[ u_\mu = g_{\mu\nu}u^\nu = a(1 - \phi, \partial^i[v + B]), \]

where \( v \) is the peculiar velocity potential. The local volume expansion rate is \( \theta = k^2(v + B) \). Then one has the expansion rate \( \theta = k^2(v + B) \). Let \( u^\mu \) as the energy-frame four-velocity (zero momentum flux relative to \( u^\nu \)). The energy density is its eigenvalue of this four-velocity, i.e., \( T^\mu_\nu u^\mu = -\rho u^\nu \). The energy-momentum tensor can be written as

\[ T^\mu_\nu = (\rho + p_{\text{eff}})u^\mu u^\nu + p_{\text{eff}}\delta^\mu_\nu, \]

where \( \rho = \bar{\rho} + \delta\rho \) and \( p = \bar{p} + \delta p \). The effective pressure \( p_{\text{eff}} \) is given as

\[ p_{\text{eff}} = \rho - \zeta(\nabla^\mu u_\mu) = \rho - \frac{3}{a} \zeta. \]

The general energy-momentum tensor is

\[ T^0_0 = -\bar{\rho} - \delta\rho, \]

\[ T^i_i = (\bar{\rho} + \bar{p}_{\text{eff}})\partial_i(v + B) = (\bar{\rho} + \bar{p}_{\text{eff}})(v_i + B_i), \]

\[ T^0_i = -\bar{\rho} + \bar{p}_{\text{eff}} v_i, \]

\[ T^i_j = \frac{\bar{p}_{\text{eff}}}{\partial_j v_i}. \]

Then one has the background energy-momentum tensor

\[ T^0_0 = -\bar{\rho}, \quad T^i_i = 0, \quad T^0_i = 0, \quad T^i_j = \frac{\bar{p}_{\text{eff}}}{\partial_j v_i}. \]

Thus perturbed energy-momentum tensor can be written as

\[ \delta T^0_0 = -\delta\rho, \quad \delta T^0_i = (\bar{\rho} + \bar{p}_{\text{eff}})(v_i + B_i), \]

\[ \delta T^i_i = -\bar{\rho} + \bar{p}_{\text{eff}} v_i, \quad \delta T^i_j = \frac{\bar{p}_{\text{eff}}}{\partial_j v_i}. \]

B. Calculation of Christoffel symbols

The formula of the Christoffel symbols is given

\[ \Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} (g_{\alpha\nu,\beta} + g_{\beta\nu,\alpha} - g_{\alpha\beta,\nu}), \]

where \( , \) stand for derivative, Greek letters \( \mu, \nu, \alpha, \beta \) take the values 0,1,2,3. In the following, the prime "\( \prime \)" stand for derivative with respect to the conformal time \( \tau \). So one has the following equations

\[ \Gamma^0_{00} = \mathcal{H} + \phi', \]

\[ \Gamma^0_{ii} = \phi_\tau + \mathcal{H} B_i, \]

\[ \Gamma^i_{ij} = \mathcal{H} \delta_{ij} - [\psi' + 2\mathcal{H}(\psi + \phi)] \delta_{ij} + \partial_i \partial_j (E' - B + 2\mathcal{H}E), \]

\[ \Gamma^i_{0i} = \partial^i (\phi + B' + \mathcal{H}B), \]

\[ \Gamma^i_{j0} = \mathcal{H} \delta^i_j - \psi' \delta^i_j + \partial_i \partial^j E', \]

\[ \Gamma^j_{jk} = -\mathcal{H} \partial^i B \delta_{ij} + \delta_{jk} \partial^i \psi - \delta_{ik} \partial_j \psi - \delta_{ij} \partial_k \psi + \partial_j \partial_k E'. \]

So, the nonzero Christoffel symbols are shown in the following, the background items are

\[ \Gamma^0_{00} = \mathcal{H}, \quad \Gamma^0_{ij} = \mathcal{H} \delta_{ij}, \quad \Gamma^1_{j0} = \mathcal{H} \delta^1_j, \]

the perturbed items are

\[ \delta \Gamma^0_{00} = \phi', \quad \delta \Gamma^0_{0i} = \partial^i (\phi + B' + \mathcal{H}B), \]

\[ \delta \Gamma^0_{ii} = \partial_i \phi + \mathcal{H} \partial_i B, \quad \delta \Gamma^i_{j0} = -\psi' \delta^i_j + \partial_i \partial^j E', \]

\[ \delta \Gamma^i_{jk} = -\mathcal{H} \partial^i B \delta_{jk} + \delta_{jk} \partial^i \psi - \delta_{ik} \partial_j \psi - \delta_{ij} \partial_k \psi + \partial_j \partial_k E'. \]

C. evolution equations for density perturbation and velocity perturbations

In this section, we will given the derivation process of perturbed energy-momentum equations. From the formula

\[ \delta \nabla^\mu T^\mu_\sigma = \delta g^{\mu\sigma} \nabla^\mu T^\sigma_\sigma + \bar{g}^{\mu\sigma} \delta \nabla^\mu T^\sigma_\sigma, \]

\[ \delta \nabla^\mu T^\mu_\sigma = \delta g^{\mu\sigma} \nabla^\mu T^\sigma_\sigma + \bar{g}^{\mu\sigma} \delta \nabla^\mu T^\sigma_\sigma, \]

and

\[ \nabla^\sigma T^\mu_\nu = T^\mu_\sigma + \Gamma^\mu_{\sigma\nu} T^\sigma_\sigma - \bar{\Gamma}^\mu_{\sigma\nu} \bar{T}^\sigma_\sigma, \]

\[ \nabla^\sigma T^\mu_\nu = T^\mu_\sigma + \Gamma^\mu_{\sigma\nu} T^\sigma_\sigma - \bar{\Gamma}^\mu_{\sigma\nu} \bar{T}^\sigma_\sigma, \]
we obtain the following perturbed energy-momentum equation
\[
\delta\nabla_j T^0_0 = \frac{1}{a^2} \{ (\rho' + 3\bar{H}(\rho + \rho_{\text{eff}}) - 3(\bar{\rho} + \bar{p}_{\text{eff}}))\psi' \\
+ (\bar{\rho} + \bar{p}_{\text{eff}})\frac{\partial}{\partial v'}(\varphi + 2\bar{H}(\varphi + \phi))\} \delta_{ij} + (\bar{\rho} + \bar{p}_{\text{eff}})\nabla^2(E' + 2\bar{H}\varphi - B),
\]
(36)
And in the same way, make use of the following results
\[
\delta\nabla_j T^0_0 = -((\bar{\rho} + \bar{p}_{\text{eff}})\psi)' \\
- (\bar{\rho} + \bar{p}_{\text{eff}})\frac{\partial}{\partial v'}(\varphi + 2\bar{H}(\varphi + \phi)),
\]
(37)
we also have the following equation
\[
\delta\nabla_\mu T^{\mu\nu} = \frac{1}{a^2} \{ (\rho' + 3\bar{H}(\rho + \rho_{\text{eff}}) - 3(\bar{\rho} + \bar{p}_{\text{eff}}))\psi' \\
+ 4\bar{H}(\bar{\rho} + \bar{p}_{\text{eff}})(\varphi + \phi) + \delta p_{\text{eff}} \\
- \{ (\bar{\rho} + \bar{p}_{\text{eff}})\nabla^2(v + E')\} \delta_{ik},
\]
(38)
If the fluid is conservation, i.e. \(\rho' \geq 3\bar{H}(\rho + \bar{p}_{\text{eff}}) = 0\), the above perturbed equations can be rewritten as
\[
\delta\nabla_\mu T^{\mu0} = \frac{1}{a^2} \{ (\rho' + 3\bar{H}(\rho + \rho_{\text{eff}}) - 3(\bar{\rho} + \bar{p}_{\text{eff}}))\psi' \\
+ (\bar{\rho} + \bar{p}_{\text{eff}})\nabla^2(v + E'),
\]
(39)
\[
\delta\nabla_\mu T^{\mu i} = \frac{1}{a^2} \{ (\rho' + 3\bar{H}(\rho + \rho_{\text{eff}})(\varphi + \phi) + \delta p_{\text{eff}} \\
+ 4\bar{H}(\bar{\rho} + \bar{p}_{\text{eff}})(\varphi + \phi) + \delta p_{\text{eff}}\} \delta_{ik},
\]
(40)
where \(\bar{p}_{\text{eff}} = \bar{p} - \frac{2}{a}H\bar{\zeta}\).
\[
\delta p_{\text{eff}} = \delta\rho - \frac{\zeta}{a}(\nabla_u w^u) - \zeta(\nabla_u w^u)' \\
= \delta\rho - \frac{3\bar{H}}{a}\delta\phi - \frac{\zeta}{a}(\nabla^2(v + E') - (3\psi' + 3\bar{H}\phi)).
\]
(41)
To solve the above equations or make the complete, we need the relations between \(\delta\rho\) and \(\delta p\). The sound speed \(c_{s,\text{eff}}^2\) of a fluid or scalar field, is the propagation speed of pressure fluctuation in the rest frame
\[
c_{s,\text{eff}}^2 = \frac{\delta p_{\text{eff}}}{\delta\rho}|_{rf},
\]
(42)
where \(|_{rf}\) denotes the rest frame. For scalar field \(\phi\), the rest frame is defined as the hypersurfaces \(\delta\phi = 0\), i.e. \(\phi = \text{constant}\). So, one has \(\delta\phi = 0\) and \(\delta p_{\text{eff}} = \delta(\bar{\rho}'a^{-2}\varphi^2 + V) = a^{-2}\phi\delta\phi' = \delta\rho_{\phi}\). Thus the sound speed of scalar field equals to the speed of light, is independent the form of \(V(\phi)\)
\[
\delta\phi|_{rf} = 0 \Rightarrow c_{s,\phi}^2 = 1.
\]
(43)
The ”adiabatic sound speed” for any medium is defined as
\[
c_{a,\text{eff}}^2 = \frac{p_{\text{eff}}'}{\rho'} = w_{\text{eff}} + \frac{w_{\text{eff}}'}{\rho'/\rho}.
\]
(44)
The rest frame (the zero momentum gauge or comoving orthogonal gauge) is the comoving \(|_{rf} = 0\) orthogonal (\(B|_{rf} = 0\)) frame, so that
\[
T^0_0|_{rf} = 0 = T^0_0|_{rf}.
\]
(45)
We make a gauge transformation, \(x^\mu \rightarrow x^\mu + (\delta\tau, \delta\varphi\delta\alpha)\), from the rest frame gauge to a general gauge
\[
v + B = (v + B)|_{rf} + \delta\tau, \quad \delta p = \delta p|_{rf} - \rho'\delta\tau, \quad \delta\rho = \delta\rho|_{rf} - \rho'\delta\tau.
\]
(46)
Thus, one has \(\delta\tau = v + B\) and
\[
\delta\rho = \delta p_{\text{eff}}(v + B) = \delta p_{\text{eff}}|_{rf} - \rho'\delta\tau = c_{s,\text{eff}}^2\delta\rho + \delta p_{\text{ad}},
\]
(47)
where \(\delta p_{\text{ad}} = (c_{s,\text{eff}}^2 - c_{a,\text{eff}}^2)\delta\rho\) is the intrinsic non-adiabatic perturbation in the fluid. When the fluid is conservation, i.e. \(\rho' = -3\bar{H}(\bar{\rho} + \bar{p}_{\text{eff}})\). By using the relation \(\theta = -k^2(v + B)\) in Fourier space, one has
\[
\delta p_{\text{eff}} = c_{s,\text{eff}}^2\delta\rho + (c_{s,\text{eff}}^2 - c_{a,\text{eff}}^2)(v + B) = c_{s,\text{eff}}^2\rho\theta + (c_{s,\text{eff}}^2 - c_{a,\text{eff}}^2)\bar{H}(\bar{\rho} + \bar{p}_{\text{eff}})\theta^2.
\]
(48)
We define the density contrast \(\delta = \delta\rho/\rho\), then one has the evolution equations for density perturbation and velocity perturbation for a generic conservation fluid are
\[
\delta' + 3\bar{H}(c_{s,\text{eff}}^2 - w_{\text{eff}})\delta + (1 + w_{\text{eff}})(\theta - 3\psi') = 0,
\]
(49)
\[
\theta' + \bar{H}(1 - 3c_{s,\text{eff}}^2)\theta - \frac{c_{s,\text{eff}}^2 k^2 \delta}{1 + w_{\text{eff}}} - k^2\phi = 0.
\]
(50)
In the synchronous gauges, one has
\[ \phi = \beta'' + \frac{a'}{a} \beta', \quad (51) \]
\[ \psi = -\frac{h}{6} - \frac{1}{3} \nabla^2 \beta - \frac{a'}{a} \beta'. \quad (52) \]
Therefore, \( k^2 \phi = 0 \), \( -3 \psi' = \frac{h'}{h} \), finally, we have the following evolution equations for density perturbation and velocity perturbation.
\[ \delta' = -(1 + w_{\text{eff}})(\theta + \frac{h'}{2}) - 3\mathcal{H}(c_{s,\text{eff}}^2 - w_{\text{eff}})\delta, \quad (53) \]
\[ \theta' = -\mathcal{H}(1 - 3c_{s,\text{eff}}^2)\theta + \frac{c_{s,\text{eff}}^2 k^2 \delta}{1 + w_{\text{eff}}}, \quad (54) \]
Following the formalism for a generalized dark matter [27], one can recast Eqs. (53), and (54) into
\[ \delta' = -(1 + w_{\text{eff}})(\theta + \frac{h'}{2}) + \frac{w'_{\text{eff}}}{1 + w_{\text{eff}}} \delta - 3\mathcal{H}(c_{s,\text{eff}}^2 - c_{a,\text{eff}}^2)\delta + 3\mathcal{H}(1 + w_{\text{eff}})\frac{\theta}{k^2}, \quad (55) \]
\[ \theta' = -\mathcal{H}(1 - 3c_{s,\text{eff}}^2)\theta + \frac{c_{s,\text{eff}}^2 k^2 \delta}{1 + w_{\text{eff}}}, \quad (56) \]
where
\[ w_{\text{eff}} = -\frac{B_s}{B_s + (1 - B_s)a^{-3(1 + \alpha)}} - \sqrt{3}\zeta_0, \quad (57) \]
\[ c_{a,\text{eff}}^2 = w_{\text{eff}} - \frac{w'_{\text{eff}}}{3\mathcal{H}(1 + w_{\text{eff}})}, \quad (58) \]
\[ c_{s,\text{eff}}^2 = c_s^2 - \frac{\sqrt{3}}{2} \zeta_0 - \frac{\zeta_0}{\sqrt{3}\mathcal{H}\delta}(\theta + \frac{h'}{2}), \quad (59) \]
\[ c_{a,\text{eff}}^2 = c_{s,\text{eff}}^2 - \frac{w_{\text{eff}}}{\Gamma_{\text{nad,eff}}} \delta_{\text{rest}}, \quad (60) \]
\[ \Gamma_{\text{nad,eff}} = \frac{\delta_{\text{nad}}}{p_{\text{eff}}}, \quad (61) \]
\[ \delta_{\text{rest}} = \delta + 3\mathcal{H}(1 + w)\frac{\theta}{k^2}. \quad (62) \]

IV. COSMOLOGICAL CONSTRAINTS FROM DATA SETS: SNLS3, BAO, PLANCK AND HST

In this section, we apply the Markov Chain Monte Carlo method to investigate the observational constraint on viscous generalized Ghaplyin gas model which included bulk viscous perturbation to obtain the parameters space. The MCMC method is based on the publicly available cosmoMC package [41], which has been modified to include the dark fluid perturbation in the CAMB [42] code which is used to calculate the theoretical CMB power spectrum. To get the converged results, in MCMC calculation we stop sampling by checking the worst e-values [the variance(mean)/mean(variance)] of 1/2 chains] R - 1 of the order 0.01. In the following calculations, we take the total likelihood \( \mathcal{L} \propto e^{-\chi^2/2} \) to be the product of the separate likelihoods of SNLS3, BAO, Planck and HST. Then the \( \chi^2 \) is given as
\[ \chi^2 = \chi^2_{\text{SNLS3}} + \chi^2_{\text{BAO}} + \chi^2_{\text{Planck}} + \chi^2_{\text{HST}}, \quad (63) \]
with the following 8-dimensional parameter space:
\[ P \equiv (\omega_b, 1000h_{\text{MC}}, \tau, B_s, \zeta_0, n_s, \log(10^{10}A_s)). \quad (64) \]

The pivot scale of the initial scalar power spectrum \( h_{\text{pivot}} = 0.05 \text{Mpc}^{-1} \) is used and the priors to model parameters is taken as follows: the physical baryon density \( \omega_b = (\Omega_b h^2) \in [0.005, 0.1] \); the ratio of the sound horizon and angular diameter distance \( 1000h_{\text{MC}} \in [0.5, 10] \); the optical depth \( \tau \in [0.01, 0.8] \); the model parameters \( \alpha \in [0, 0.1], B_s \in [0, 1] \) and \( \zeta_0 \in [0, 0.01] \); the scalar spectral index \( n_s \in [0.5, 1.5] \), and logarithm of the amplitude of the initial power spectrum \( \log(10^{10}A_s) \in [2.7, 4] \). In addition, the hard coded prior on the cosmic age 10Gyr < \( t_0 \) < 20Gyr is imposed. Also, the weak Gaussian prior on the physical baryon density \( \omega_b = 0.02 \pm 0.002 \) from big bang nucleosynthesis and new Hubble constant \( H_0 = 74.2 \pm 3.6 \text{km/s/Mpc}^{-1} \) [14] are adopted. Notice that the current dimensionless energy density of VGCG \( \Omega_{\text{VGCG}} \) is not included in the model parameter space \( P \), because it is a derived parameter in a spatially flat \( (k = 0) \) FRW universe. To study the evolutions of the perturbation, we should fix the background evolution. To realize that, we use the cosmic observations from the type Ia supernovae SNLS3, cosmic microwave background radiation from recently released Planck, baryon acoustic oscillation from Sloan Digital Sky Survey and the WiggleZ data points and High-redshift SN observations from Hubble Space Telescope. For the detailed description, please see Refs. [46, 47].

**TABLE 1:** The mean values of model parameters with 1\( \sigma \), 2\( \sigma \), and 3\( \sigma \) errors from the combination SNLS3+BAO+Planck+HST.
The best fitting values of the cosmological parameters and the mean values of model parameters with 1σ, 2σ and 3σ regions in VGCG model from the combination SNLS3+BAO+Planck+HST are listed in Table I. Correspondingly, the contour plots are shown in Figure 1. We find that the minimum \( \chi^2 \) is \( \chi^2_{\text{min}} = 5115.878 \). From Table I and Figure 1, we obtain the constraint on the bulk viscosity coefficient: \( \zeta_0 = 0.0000138 +0.00000145+0.0000212 \) in 1, 2, 3σ regions respectively, it is obvious that we obtain a tighter constraint than our previous results in [15] due to the bulk viscosity perturbation is included. From [15], we know that the value of bulk viscosity impacts the CMB power spectra by showing the first peak is depressed as the value of bulk viscosity impacts the CMB power spectra by showing the first peak is depressed as the value of bulk viscosity decreases, this can be embodied in the CMB anisotropic power spectra by showing the first peak is depressed as observed in the figure 2 in [15].

To show the effect of bulk viscosity perturbation to the efficient state parameter \( w_{eff} \) and the efficient adiabatic sound speed \( c^2_{a,eff} \), we plot the the evolution curves of \( c^2_{a,eff} \) and \( w_{eff} \) with respect to scale factor \( a \) in Figure 2 and Figure 3 respectively, which corresponding to VGCG1 model (not considering bulk viscosity perturbation) and VGCG2 model (including bulk viscosity perturbation). From Figure 2, one can conclude that VGCG2 model provides a more smaller efficient adiabatic sound speed (which approximately equal to zero) than VGCG1 model. It is well known that almost zero adiabatic sound speed which being characterized by the perturbation of density contrast is important for large scale structure formation. So, VGCG2 model make it possible to form large scale structures in our universe. From the upper panel of Figure 3, one can see that the two VGCG models behave like cold dark matter with almost zero EoS at early epoch \( (a < 0.2) \), and behave like dark energy with EoS \( w_{eff} < 0 \) at late time, which pushes the universe into an accelerated phase. Furthermore, from the under panel of Figure 3, which enlarged the upper panel (from \( a = 2 \) to the end ), we can conclude that VGCG1 model behaves like phantom \( (w_{eff} > -1) \) at present, behaves like quintessence \( (w_{eff} < -1) \) in the distant future. However, unlike VGCG1 model, VGCG2 model behaves like quintessence at present and in the distant future, which will avoid our universe to be terminated by a cosmic doomsday. Therefore, it is more necessary and reasonable to include the perturbation of bulk viscosity when we study of cosmic evolution. In conclusion, VGCG2 model (including bulk viscosity perturbation) being proposed here is a more competitive model than the one we studied previously.

V. DISCUSSION AND CONCLUSION

In this paper, we have revisited the viscous generalized Chaplygin gas (VGCG) model by including perturbation of bulk viscosity. We derived the cosmological
FIG. 3: The evolution of $w_{\text{eff}}$ with respect to scale factor $a$. The solid curve correspond to VGCG2 model (including bulk viscosity perturbation); the dashed curve correspond to VGCG1 model (not considering bulk viscosity perturbation).

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