Differentially Private Learning with Small Public Data*

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Abstract
Differentially private learning tackles tasks where the data are private and the learning process is subject to differential privacy requirements. In real applications, however, some public data are generally available in addition to private data, and it is interesting to consider how to exploit them. In this paper, we study a common situation where a small amount of public data can be used when solving the Empirical Risk Minimization problem over a private database. Specifically, we propose Private-Public Stochastic Gradient Descent, which utilizes such public information to adjust parameters in differentially private stochastic gradient descent and fine-tunes the final result with model reuse. Our method keeps differential privacy for the private database, and empirical study validates its superiority compared with existing approaches.

Introduction
With the gradual popularization of artificial intelligence and machine learning, privacy is becoming a control concern when mining sensitive data. To gain statistical knowledge while avoiding leakage of personal information, Dwork et al. (2006) presented differential privacy, which gives a mathematically rigorous definition of privacy protection and becomes a standard guarantee nowadays. Many classic learning algorithms have been modified to satisfy its requirements, such as least squares (Wang 2018), SVM (Rubinstein et al. 2009), LASSO (Talwar, Thakurta, and Zhang 2015) and neural networks (Abadi et al. 2016).

Among these algorithms, Empirical Risk Minimization (ERM) is a general framework that minimizes a given loss function over a training set. Combining with different types of regularizers, it covers a lot of specific learning approaches. Many algorithms for the ERM task with a differential privacy guarantee have been developed (Chaudhuri, Monteleoni, and Sarwate 2011; Kifer, Smith, and Thakurta 2012; Bassily, Smith, and Thakurta 2014; Wang, Ye, and Xu 2017; Zhang et al. 2017; Lee and Kifer 2018). However, unlike the usual result about $(1/\sqrt{m})$ excess risk bounds for non-private ERM where $m$ is the size of the training set (Zinkevich 2003), differentially private ERM approaches usually have additional polynomial dependences on dimension $p$ and privacy parameter $\epsilon$ (Bassily, Smith, and Thakurta 2014). These extra costs of privacy sometimes make the private ERM become a formidable job for high dimensional datasets or strict privacy requirements.

In many real-world applications, when we perform learning jobs on a private database, we can also get access to a public database about the task. For example, if a headmaster wants to evaluate teaching quality according to private questionnaires from students, some reports from his teaching supervision group which are publicity could be used as well. Besides, a doctor can study typical cases published by the centers for disease control when he analyses the epidemic trend from private medical records in his hospital, a lawyer can search non-private judgments from case information disclosure online when he looks for the patterns of judgments from his customers’ information. Thus, a natural problem is, how can we utilize these public data to help differentially private learning.

Some straightforward solutions may immediately come to mind, such as:

- Train a model on the public database at first, and replace it with the model of private data if it is incompetent.
- Train two models on the public and private database respectively, and use their ensemble as the final result.
- Add those public data into the private database and train a model together.

However, in practice, utilizing public data is non-trivial. Because on one hand, the public data usually is not sufficient to train a well-performing model solely; actually, if some domain has a sufficient amount of public data, then private data in such domain would not be highly-valued. This makes the first two solutions impractical. On the other hand, merging public data into the private database may be illegal in cases, and will cause nonnegligible communication costs; needless to say, such a process cannot handle cases where the public data is in accumulating. Thus, the third straightforward way...
Figure 1: The flowchart of PPSGD. PPSGD is a two-stage algorithm to utilize a small amount of public data when solving differentially private ERM. In its first stage, the private database is input to a differentially private SGD, while the public database is used to adjust parameters iteratively to control the noise scale. In its second stage, the final result of the first stage is taken as a pre-trained model, and the public database is used again to fine-tune it.

is not ideal either.

In this paper, we propose Private-Public Stochastic Gradient Descent (PPSGD), which is a general approach to solve differentially private ERM with additional public data. As shown in figure 1, PPSGD consists of two stages: differentially private stochastic gradient descent and model reuse. To take full advantage of the public database while avoiding overfitting, we use it in the first stage to help allocate the privacy budget which controls the rigidity of privacy enforcement, and adjust the gradient clipping to enable adaptive gradient rescaling, whereas in the second stage to fine-tune the final result. Since public data does not participate in gradient estimations directly and model reuse ensures its output will not deviate far away from the pre-trained model, we control their influence while improving the overall performance, as well as preserve differential privacy over the private database.

Our contributions can be summarized as follows:

1. We formalize the problem of learning by Empirical Risk Minimization over a private database together with a small public database, which is a common situation and promising field of differentially private approaches.

2. We propose PPSGD, which starts with stochastic gradient descent for truncated concentrated differential privacy, and we dynamically adjust privacy budget and gradient clipping to control noise intensity and individual contribution actively.

3. Experiments over both synthesis datasets and real-world datasets validate the superiority of our method and also manifest a new application of model reuse.

The rest of this paper is organized as follows. We discuss related works and introduce backgrounds in sections 2 and 3 respectively. In section 4, we present our main method. Finally, Section 5 reports the results of empirical study and Section 6 concludes.

Related Work
Differentially private ERM has been studied a lot. Chaudhuri, Monteleoni, and Sarwate (2011) gave two approaches to solve this issue: output perturbation and objective perturbation. Kifer, Smith, and Thakurta (2012) generalized objective perturbation to approximately differential privacy. Zhang et al. (2017) improved output perturbation according to the new stability result of gradient descent. Song, Chaudhuri, and Sarwate (2013) introduced stochastic gradient descent (SGD) to differentially private ERM. Bassily, Smith, and Thakurta (2014) gave an SGD based private algorithm and proved its optimality. Then, many gradient based methods have been proposed, such as Wang, Ye, and Xu (2017) proposed differentially private stochastic variance reduced descent, Lee and Kifer (2018) used adaptive privacy budgets in gradient descent for concentrated differential privacy.

Model reuse focuses on the problem that how can we reuse pre-trained models to help the learning process which typically lacks enough data to train a model directly. It is a key component in Learnware (Zhou 2016). Many methods of model reuse have been proposed, such as Li, Tsang, and Zhou (2012) tried to optimize a performance measure by reusing models trained for other performance measures. Tommasi, Orabona, and Caputo (2013) designed a biased regularizer based on the pre-trained models. Segev et al. (2016), Yang et al. (2017), and Wu, Liu, and Zhou (2019) reused forest models, deep models, and multiclass models respectively. Moreover, some theoretical analyses about model reuse have been established in recent years (Kuzborskij and Orabona 2017; Du et al. 2017), which show that model reuse enjoys a fast rate of generalization bound when the pre-trained models are good enough.

To the best of our knowledge, there are a few works that take into account public data in differentially private learning. Papernot et al. (2016) used the private database to train teacher models at first, then learned a student model with generative adversarial networks by noisy voting among all of the teachers over a public but unlabeled database. Avent et al. (2017) proposed a hybrid model of local differential privacy \(^1\) and classic differential privacy. Their approach operates with the hybrid differential privacy model for computing heavy hitters. Feldman et al. (2018) introduced amplification by iteration, where the privacy guarantees of elements used in early iterations of an iterative private process are stronger than elements used in subsequent iterations if intermediate results are hidden. Thus, the noise needed to add to an SGD algorithm could be shrunken to protect privacy if the public elements are arranged in the last few iterations. Moreover, their method requires the loss function to be smoothness and indeed ensures local differential privacy.

Preliminaries
Differentially private, which formulates as definition 1, is based on neighboring databases. Let \( D = \{ d_1, d_2, ..., d_m \} \) be a private database of \( m \) elements drawn from universe \( D \).

\(^1\)Local differential privacy (Kasiviswanathan et al. 2008) allows database participants to perturb their data by themselves. It is more strict than differential privacy.
A database $D'$ is a neighbor of $D$ (denote as $D' \sim D$) if $|D'| = m$ and $D'$ has exactly one different element from $D$.

**Definition 1.** (Dwork, Roth, and others 2014) For $\epsilon > 0$ and $\delta \geq 0$, a randomized algorithm $A$ that maps $D^m$ into some range $S$ is said to be $(\epsilon, \delta)$-differential privacy, if for all neighboring databases $D, D' \in D^m$ and for any subset $S \subseteq S$, we have

$$\Pr(A(D) \in S) \leq e^\epsilon \Pr(A(D') \in S) + \delta.$$  

To give a meaningful privacy guarantee, one usually sets $\epsilon < 1$ and $\delta \ll 1/m$. When $\delta = 0$, $A$ is said to be pure $\epsilon$-differential privacy. In this paper, we only consider approximately differential privacy case, i.e., we suppose $\delta > 0$.

Truncated concentrated differential privacy (tCDP) (Bun et al. 2018) is a variation of differential privacy. We use it in our algorithm to perform composition easily. The definition of tCDP is given in the following, where privacy guarantee is stronger when $\rho$ is smaller or $\omega$ is larger.

**Definition 2.** (Bun et al. 2018) For $\rho > 0$ and $\omega > 1$, a randomized algorithm $A$ that maps $D^m$ into some range $S$ satisfies $(\rho, \omega)$-tCDP, if for all neighboring databases $D, D' \in D^m$ we have

$$\forall \alpha \in (1, \rho) \quad D_\alpha(A(D)) || A(D') \leq \rho \alpha,$$

where $D_\alpha(\cdot)$ is the Rényi divergence of order $\alpha$ (Rényi and others 1961).

In this paper, we mainly use the following properties.

**Theorem 1.** (Bun et al. 2018) Given an arbitrary algorithm $A$, the Gaussian mechanism which adds i.i.d. $N(0, \sigma^2)$ noise to each coordinate of $A$‘s output ensures $(\rho, \infty)$-tCDP, where $\sigma = \sqrt{\Delta_2(A)}$, $\Delta_2(A) = \sup_{D, D'} ||A(D) - A(D')||_2$. Among them, $\Delta_2(A)$ is known as L2 sensitivity, it measures the maximal difference of $A$‘s outputs while changing a single element.  

**Theorem 2.** (Bun et al. 2018) If an algorithm $A$ satisfies $(\rho, \omega)$-tCDP, for $\delta \geq \frac{1}{\exp((\omega - 1)^2 \rho)}$, $A$ satisfies $(\epsilon, \delta)$-differential privacy with $\epsilon = \rho + 2\sqrt{\rho \log(1/\delta)}$.

**Theorem 3.** (Bun et al. 2018) Let $A_1 : D^m \rightarrow S_1$ satisfies $(p_1, \omega)$-tCDP and $A_2 : D^m \times S_1 \rightarrow S_2$ be such that $A_2(1, s_1)$ satisfies $(p_2, \omega_2)$-tCDP for all $s_1 \in S_1$. Define $A_3 : D^m \rightarrow S_2$ by $A_3(D) = A_2(D, A_1(D))$. Then $A_3$ satisfies $(\rho_1 + p_2, \min(\omega_1, \omega_2))$-tCDP.

**Theorem 4.** (Bun et al. 2018) Let $A_1 : D^m \rightarrow S$ satisfy $(\rho, \omega)$-tCDP and $A_2 : D^M \rightarrow S$ be such that $A_2(D) = A_1(D_S)$ where $D_S \in D^m$ is sampled uniformly randomly from $D$. Let $s = m/M$, if $\rho, s \in (0, 0.1], \log(1/s) \geq 3\rho(2 + \log_2(1/\rho))$ and $\omega \geq \log(1/s)/(2\rho)$, then $A_2$ satisfies $(13\rho^2, \log(1/s)/(4\rho))$-tCDP.

**PPSGD**

In this section, we first provide our settings formally, then propose Private-Public Stochastic Gradient Descent (PPSGD).

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2Bun et al. proposed a more ingenious Sinh-normal mechanism for tCDP, but here we use the Gaussian mechanism to maintain a fair competition with comparative methods in experiments though our work can be easily generalized to the Sinh-normal mechanism.

**Settings and Notations**

Let the data universe $D = \mathcal{X} \times \mathcal{Y}$ where $\mathcal{X} \subseteq \mathbb{R}^p$ is the instance space and $\mathcal{Y} \subseteq \mathbb{R}$ is the label set. Given a loss function $l$, to minimize the expected loss of an unknown distribution $\mathcal{D}_X \times \mathcal{D}_Y$ over $\mathcal{X} \times \mathcal{Y}$, Empirical Risk Minimization solves $w$ such that

$$\min_{w \in W} f(w, D) = \frac{1}{m} \sum_{i=1}^{m} l(w, x_i, y_i),$$

where $D = \{(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)\}$ is a database drawn from $\mathcal{D}_X \times \mathcal{D}_Y$. We assume $W$ is a closed convex set, $l(w, x_i, y_i)$ is convex and $L$-Lipschitz for all $x \in \mathcal{X}, y \in \mathcal{Y}$.

As a private algorithm, we require $w$ differentially private for $D$. Besides, we assume a public database $D = \{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$ is available, which is drawn from the same distribution of $D$. The public database can be used without the concern of privacy, but may be insufficient to solve the objective $w$ directly.

**General Framework**

We start by discussing how to utilize public data in differentially private ERM. Specifically, we focus on gradient based iterative algorithms (Bassily, Smith, and Thakurta 2014) which usually performs

$$w_{t+1} = w_t - \eta \nabla f(w_t, \cdot) + n_t$$

for $t = 1, 2, \ldots, T$, where $\nabla f(w_t, \cdot)$ stands for the gradient and $n_t$ is a noise vector added to protect privacy. Generally, three aspects of (2) can be improved with public data: getting a more accurate estimator of $\nabla f(w_t, \cdot)$, minimizing the scale of $n_t$, and increasing $T$.

Note that public elements are drawn from the same distribution as private elements, adding them into mini-batches could improve the accuracy of gradient estimators definitely. However, since the public database is relatively small, we have to either utilize public information in only a few iterations or use each public element repeatedly. In the former case, public data has little influence on the overall performance. In the latter case, there will be serious overfitting.

Differential privacy is immune to post-processing. That is, we cannot make a differential privacy algorithm’s output less differentially private without additional knowledge about the private database. Thus, the public database seems unable to contribute to the overall privacy guarantee. However, since noise vectors $n_t$ are spread in every iteration, as shown in (Lee and Kifer 2018), the performance of iterative private algorithms can be improved significantly by replacing the noise by reweighting methods, similar to what happened before, serious overfitting still destroys the overall performance when emphasizing too much on the public database.

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Algorithm 1 PPSGD

**Input:** Private database \(\{x_i, y_i\}_{i=1}^m\), public database \(\{\hat{x}_i, \hat{y}_i\}_{i=1}^n\), loss function \(l\), privacy parameters \((\epsilon, \delta)\), learning rate \(\eta\), batch size \(s\), parameters \(\phi, \alpha, \varphi, \beta, \lambda\)

**Output:** Model \(w\)

1. Calculate \((\rho_{\text{total}}, \omega_{\text{total}})\) according to (3)
2. Initialize \(w_0, \rho_t\) and \(C_0\)
3. \(t = 0\)
4. **while** \((\rho_{\text{total}} \geq \rho_t)\) **do**
   5. Uniformly randomize \(S_t \subset [m]\) with \(|S_t| = s\)
   6. \(g_t = \frac{1}{s} \sum_{i \in S_t} \nabla l(w_t, x_i, y_i)\)
   7. Calculate \(\rho_t^\wedge, \omega_t^\wedge\) according to (4)
   8. \(g_t = g_t + N\left(0, \frac{\rho_t^\wedge}{\epsilon t}\right)\) \(\epsilon\)
   9. \(w_{t+1} = w_t - \eta g_t\)
   10. \(\rho_{\text{total}} = \rho_{\text{total}} - \rho_t\)
   11. \(g_t^{\text{public}} = \frac{1}{s} \sum_{i \in [n]} \nabla l(w_{t+1}, \hat{x}_i, \hat{y}_i)\)
   12. \(\text{mse}_t = \frac{2\rho_t^\wedge}{\epsilon t}\)
   13. **if** \((\phi \cdot \text{norm}(g_t^{\text{public}}) < \sqrt{\text{mse}_t})\) **then**
      14. \(\rho_{t+1} = (1 + \alpha)\rho_t\)
   15. **else**
   16. \(\rho_{t+1} = \rho_t\)
   17. **end if**
   18. **if** \((\phi \cdot \text{norm}(g_t^{\text{public}}) < C_t)\) **then**
      19. \(C_{t+1} = (1 - \beta)C_t\)
   20. **else**
   21. \(C_{t+1} = C_t\)
   22. **end if**
   23. \(t = t + 1\)
   24. **end while**
25. \(w^* = \arg\min_w \frac{1}{n} \sum_{i \in [n]} l(w, \hat{x}_i, \hat{y}_i) + \lambda \|w - w_t\|^2\)
26. **return** \(w^*\)

Gradient Estimation and Privacy Guarantee

The **while** loop in line 4 is the first stage of figure 1, i.e., the differentially private SGD. As mentioned before, we use tCDP in our algorithm since its composition property is more straightforward for our dynamic privacy budget allocation. Theorem 2 points out that tCDP is a stronger model than approximately differential privacy. Thus, for given parameters \((\epsilon, \delta)\), we can solve \((\rho_{\text{total}}, \omega_{\text{total}})\) with

\[
\begin{align*}
\rho_{\text{total}} &= \epsilon + 2 \log(1/\delta) - 2 \sqrt{\log(1/\delta)(\epsilon + \log(1/\delta))} \\
\omega_{\text{total}} &= \sqrt{\log(1/\delta)/\rho_{\text{total}} + 1}
\end{align*}
\]

in line 1 and ensure that each \((\rho_{\text{total}}, \omega_{\text{total}})\)-tCDP algorithm satisfies \((\epsilon, \delta)\)-differential privacy.

In each iteration, according to (2), our primary tasks are estimating \(\nabla f(w_t, \cdot)\) and sampling \(n_t\). For the former task, we randomly select a subset \(S_t\) of size \(s\) and calculate its average gradient in line 6, where

\[
\Pi_{i:|S_t|} x = \min(1, C_t/\|x\|) \cdot x
\]

is a project operator, it ensures the norm of each gradient not greater than the gradient clipping threshold \(C_t\).

Theorem 1, i.e., the Gaussian mechanism of tCDP, states that noise vector \(n_t\) can be sampled from a Gaussian distribution based on its L2 sensitivity and the privacy budget. After the gradient clipping, it’s clear that the average gradient has an L2 sensitivity of \(2C_t/s\). For another, we allocate \(\rho_t\) privacy budget for the \(t\)-th iteration, i.e., the \(t\)-th update of (2) should satisfies \((\rho_t, \omega_{\text{total}})\)-tCDP. Note the subtle difference that the privacy guarantee of PPSGD acts on the whole private database while we estimate \(\nabla f(w_t, \cdot)\) on a small subset \(S_t\) only. Fortunately, Theorem 4 proposes amplification of sampling. That is, in order to ensure \((\rho_{\text{total}}, \omega_{\text{total}})\)-tCDP before random sampling, we only need to satisfy \((\rho_t^\wedge, \omega_t^\wedge)\)-tCDP for \(S_t\) where

\[
\rho_t^s = \frac{\rho_t}{13(s/\rho_t)^2}, \quad \omega_t^s = \frac{\log(m/s)}{2\rho_t^s}.
\]

Thus, we can sample \(n_t\) in line 8 according to Theorem 1.

For the composition between iterations, the following theorem states that PPSGD satisfies the privacy requirement.

**Theorem 5.** Given any public database and parameters required in algorithm 1, PPSGD is \((\epsilon, \delta)\)-differential privacy for the private database.

**Proof.** As mentioned before, after adding noise in line 8, the \(t\)-th execution of lines 5 to 8 satisfies \((\rho_t, \omega_{\text{total}})\)-tCDP for the private database. Since line 10 and the condition of the **while** loop guarantee that \(\sum_t \rho_t \leq \rho_{\text{total}}\), according to theorem 3, the composition of all executions of lines 5 to 8 satisfies \((\rho_{\text{total}}, \omega_{\text{total}})\)-tCDP. At the meanwhile, only lines 5 to 8 access the private database in algorithm 1. So, according to the post-processing property of differential privacy (Theorem 3), we have PPSGD also satisfies \((\rho_{\text{total}}, \omega_{\text{total}})\)-tCDP.

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3Several conditions are required for amplification of sampling, details are specified in Theorem 4.
Finally, review (3) and theorem 2, PPSGD is $(\epsilon, \delta)$-differential privacy for the private database since it satisfies the stronger $(\rho_{total}, \omega_{total})$-tCDP.

**Privacy Budget and Gradient Clipping**

As mentioned before, we adjust the privacy budget $\rho_i$ and gradient clipping threshold $C_i$ to control the noise scale. While performing this job, the central role of the public database is to estimate the norm of $\nabla f(w_r, \cdot)$, which is in line 11. Then, we calculate the mean square error caused by the noise, i.e., the dimension $p$ times the variance of the noise in each coordinate. We adjust the privacy budget based on the ratio of gradient’s norm and the square root of the noise in each coordinate. We adjust the privacy budget based on the ratio of gradient’s norm and the square root of the noise in each coordinate. We need no additional knowledge about the private database, partially private SGD and the public database are used. Since there is no outlier in the public database, using the maximum threshold is calculated. Note that the gradient clipping is a pre-trained model and reused as a biased regularizer to ensure $\|w\|$ does not deviate far away from it.

During model reuse, only the final result of the differentially private SGD and the public database are used. Since we need no additional knowledge about the private database, we can use an arbitrary optimization method without any noise at this stage. What’s more, the empirical error in the public database is also a good estimator of the generalization error, due to several fast rate convergence bounds (Kuzborskij and Orabona 2017).

**Experiments**

In this section, we empirically evaluate the performance of PPSGD and compare it to the following baselines:

1. Differentially private algorithms which merge the public database into private database and train a model together.

   Specifically, we consider following algorithms:

   (a) SgdAdv (Bassily, Smith, and Thakurta 2014) adds equivalent noise in each iteration of an SGD algorithm and applies advanced composition theorem.

   (b) Agd (Lee and Kifer 2018) is a gradient descent algorithm for concentrated differential privacy with dynamic adaption of the privacy budget.

   (c) OutPert (Zhang et al. 2017) adds Gaussian noise after a non-private gradient descent.

   (d) ObjPert (Kifer, Smith, and Thakurta 2012) adds a noise term to its optimization objective, then solves the new objective non-privately.

2. PNSGD (Feldman et al. 2018, Corollary 30) arranges public data at the end of the training set and uses amplification by iteration to shrunk noise.

3. OnlyPub runs non-private SGD on the public database.

4. AgdAvg is the ensemble method takes average over Agd and OnlyPub.

Also, we denote NonPriv to represent the non-private SGD, which does not consider privacy preserving.

We use two loss functions in our experiments: hinge loss and square loss. Hinge loss is formulated as

$$l_{\text{hinge}}(w, x, y) = \max(0, 1 - yw^T x),$$

and square loss is formulated as

$$l_{\text{square}}(w, x, y) = (y - w^T x)^2.$$  

As in most cases, we add an L2 regularizer $\|w\|_2^2$ to both of the loss functions. Since the regularizer is independent of private data, adding its gradient after added noise is no harm for the privacy guarantee.

For data preprocessing, we normalize each feature into a unit norm. Since our experiments are not for parameter selection, we use a non-private SGD to select the coefficient of the regularizer for each dataset. SgdAdv and OutPert are quite sensitive to their parameters, we run these algorithms in different parameter settings and report the best result. When using the square loss, we estimate the norm of $w$ non-privately and thus give an upper bound of the gradient for ObjPert and OutPert. For Agd and SgdAdv, we set their gradient clipping threshold as a priori. ObjPert, OutPert, and PNSGD have additional requirements on the loss function such as continuous Hessian or smoothness, which are violated by the hinge loss. For simplicity, we ignore these conditions directly. It might overestimate the performance of these algorithms, but without influencing on other approaches including PPSGD.

For both synthesis datasets and real-world datasets, our experiments are repeated for 20 times, and average accuracy or mean square error is presented.4

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4Detailed experimental setups, results, and Matlab codes for PPSGD can be found at http://www.lamda.nju.edu.cn/code_PPSSGD.ashx.
Figure 2: Accuracy comparisons of various parameter settings. Synthesis datasets with 9950 private samples, 50 public samples and 50 dimensions is used.

**Synthesis Datasets and Effects of Parameters**

There are 5 prime internal parameters in PPSGD. Among them, $\phi$ and $\alpha$ control the allocation of privacy budget, $\varphi$ and $\beta$ control the decreasing of gradient clipping threshold, $\lambda$ is the weight of the biased regularizer in model reuse. In this subsection, we evaluate the effects of parameters on a toy problem where samples are generated with 100 dimensional Gaussian distribution, each training set has 9950 private samples and only 50 public samples.

Part (a) of figure 2 shows how accuracy changes with various values of $\phi$. It's can be found that an extremely small $\phi$ will increase the privacy budget too fast and degrade the performance. For a relatively large $\epsilon$, the performance is stable with the increasing of $\phi$. But a extremely large $\phi$ hurts performance since the noise conceals the gradient.

The impact of parameter $\varphi$ is shown in part (b) of figure 2. For hinge loss, we find that $\varphi$ hardly affect the performance since its gradient is

$$\nabla_w l_{\text{hinge}}(w, x, y) = \begin{cases} -y & 1 > yw^T x \\ 0 & \text{otherwise} \end{cases},$$

which means that the gradients of misclassified samples remain unchanged during the learning process. For square loss, a moderate $\varphi \approx 5$ could enhance the performance because its gradient is

$$\nabla_w l_{\text{square}}(w, x, y) = 2(w^T x - y)x,$$

which usually decreases for majority samples.

As shown in part (c) of figure 2, $\lambda$ has more delicate effects on the performance. An excessively small $\lambda$ could lose the performance due to overfitting, while an overlarge $\lambda$ invalidate model reuse. Different $\lambda$ is preferred according to the privacy parameter $\epsilon$.

Parts (d) and (e) of figure 2 show that parameters $\alpha$ and $\beta$ have almost no impact on the performance. Unlike Agd (Lee and Kifer 2018), PPSGD adjusts the privacy budget and gradient clipping without privacy cost. Thus, it can adjust parameters successively even meets a small $\alpha$ or $\beta$.

When using PPSGD in more situations (such as the real-world datasets in the next subsection), we can find the effects of parameters are relatively robustness for PPSGD. That is, their best settings are similar in various dimensions, training set sizes, and privacy requirements. To sum up, we set $\phi = 10$, $\alpha = \beta = 0.3$, $\varphi = 100$ for hinge loss, $\varphi = 5$ for square loss, and $\lambda \in \{0.01, 0.1, 1\}$. Figure 3 shows the comparisons between PPSGD and baselines. As we can see, the performance of PPSGD is superior to other compared methods with various $\epsilon$.

**Real World Datasets**

In this subsection, we evaluate the performance of PPSGD on 8 real-world datasets. The specific information of each dataset are represented in table 1. We maintain the parameter settings of last subsection for PPSGD, except set $\varphi = 10$ for square loss when $\epsilon \geq 0.3$. In each experiment, 80 percent of samples are randomly selected to the training set and other samples compose the testing set. Within each (private) training set, we randomly select 0.1 percent (0.01 percent for

| Table 1: Characteristics of real-world datasets. |
|-----------------|-----------------|-----------------|
| Classification dataset | # Sample | # Feature | % Positive |
| adult-a | 32561 | 123 | 24.1 |
| ipums-br | 38000 | 52 | 50.6 |
| ipums-us | 39928 | 57 | 51.3 |
| magic04 | 19020 | 10 | 64.8 |
| mini-boo-ne | 130064 | 50 | 28.1 |
| skin | 245057 | 3 | 20.8 |
| Regression dataset | # Sample | # Feature | Variance |
| cadata | 20640 | 8 | 0.23 |
| stability | 10000 | 12 | 0.15 |
datasets skin and mini-boo-ne) of samples to form the public dataset. That is, the size of each public dataset always is less than 50.

Tables 2 summarizes the results on real-world datasets. Generally, the performance of PPSGD is superior or highly competitive to other compared private approaches. The public data is insufficient, which makes AgdAvg and OnlyPub behave badly. Compare to simply merge public data into the private database, our framework achieves significant performance gains in most cases.

Note that the performance of OutPert and ObjPert decreases dramatically when using the square loss, this may because the gradient of square loss is much more difficult to be bounded compared to hinge loss. For Agd and SgdAdv, we use a gradient clipping threshold much smaller than the Lipschitz constant as a priori, which retains their performance benefits. This illustrates the power of our dynamic gradient clipping, which could diminish the noise intensity adaptively. PNSGD doesn’t perform well in our experiments, although it can utilize public samples directly. This might be due to it only uses each sample in one round, which is necessary to derive its theoretical bounds. Also, we emphasize that PNSGD ensures a kind of local differential privacy, which makes its performance difficult to compete directly with other methods.

| Dataset | Loss function | $\epsilon$ | PPSGD | SgdAdv | Agd | OutPert | ObjPert | PNSGD | AgdAvg | OnlyPub | NonPriv |
|---------|---------------|-----------|--------|--------|------|---------|---------|-------|--------|---------|--------|
| adult-a | hinge         | 0.1       | 0.7882 | 0.7597 | 0.7863 | 0.7645 | 0.7643 | 0.7597 | 0.7872 | 0.7441 | 0.8401 |
|         |               | 0.5       | 0.8241 | 0.7864 | 0.8225 | 0.8130 | 0.8288 | 0.7957 | 0.8029 | 0.7706 | 0.8412 |
|         | square        | 0.1       | 0.7941 | 0.6842 | 0.7689 | 0.6600 | 0.5332 | 0.7557 | 0.7729 | 0.7706 | 0.8412 |
|         |               | 0.5       | 0.8231 | 0.7816 | 0.8229 | 0.7683 | 0.6106 | 0.7598 | 0.7899 | 0.7706 | 0.8412 |
| ipums-br | hinge        | 0.1       | 0.7170 | 0.6083 | 0.7078 | 0.7056 | 0.7000 | 0.5121 | 0.6862 | 0.6411 | 0.7638 |
|         |               | 0.5       | 0.7588 | 0.7137 | 0.7472 | 0.7392 | 0.7559 | 0.6714 | 0.6966 | 0.6411 | 0.7638 |
|         | square        | 0.1       | 0.7112 | 0.5784 | 0.6914 | 0.5459 | 0.5205 | 0.5205 | 0.6865 | 0.6363 | 0.7623 |
|         |               | 0.5       | 0.7463 | 0.7032 | 0.7443 | 0.6781 | 0.6047 | 0.6367 | 0.6787 | 0.6363 | 0.7623 |
| ipums-us | hinge        | 0.1       | 0.7137 | 0.5956 | 0.7022 | 0.7119 | 0.6994 | 0.5546 | 0.7036 | 0.5958 | 0.7775 |
|         |               | 0.5       | 0.7701 | 0.7167 | 0.7556 | 0.7491 | 0.7668 | 0.6602 | 0.7180 | 0.6254 | 0.7766 |
|         | square        | 0.1       | 0.7192 | 0.5828 | 0.6831 | 0.5430 | 0.3363 | 0.5332 | 0.6750 | 0.6363 | 0.7686 |
|         |               | 0.5       | 0.7639 | 0.7016 | 0.7517 | 0.6864 | 0.6314 | 0.6330 | 0.6868 | 0.6363 | 0.7686 |
| magic04 | hinge         | 0.1       | 0.7549 | 0.6999 | 0.7622 | 0.7480 | 0.7577 | 0.6831 | 0.7497 | 0.6981 | 0.7837 |
|         |               | 0.5       | 0.7842 | 0.7655 | 0.7771 | 0.7665 | 0.7810 | 0.7331 | 0.7456 | 0.6411 | 0.7408 |
|         | square        | 0.1       | 0.7576 | 0.6366 | 0.7563 | 0.6841 | 0.6162 | 0.6423 | 0.7408 | 0.7150 | 0.7762 |
|         |               | 0.5       | 0.7712 | 0.7555 | 0.7724 | 0.7506 | 0.7388 | 0.7224 | 0.7499 | 0.7150 | 0.7762 |
| mini-boo-ne | hinge      | 0.1       | 0.7605 | 0.7731 | 0.7151 | 0.7184 | 0.7551 | 0.7151 | 0.7370 | 0.6950 | 0.8252 |
|          |              | 0.5       | 0.8045 | 0.7662 | 0.7151 | 0.7584 | 0.8049 | 0.7151 | 0.7366 | 0.6950 | 0.8252 |
|          | square        | 0.1       | 0.7829 | 0.7253 | 0.7255 | 0.5904 | 0.5854 | 0.7153 | 0.7443 | 0.7198 | 0.8248 |
|          |              | 0.5       | 0.8134 | 0.7746 | 0.7424 | 0.7424 | 0.7199 | 0.6534 | 0.7153 | 0.7589 | 0.7198 |
| skin     | hinge         | 0.1       | 0.9069 | 0.9079 | 0.9065 | 0.9078 | 0.9061 | 0.8933 | 0.9068 | 0.8977 | 0.9077 |
|          |               | 0.5       | 0.9072 | 0.9078 | 0.9069 | 0.9069 | 0.9071 | 0.9071 | 0.9071 | 0.9071 | 0.9077 |
|          | square        | 0.1       | 0.9079 | 0.9079 | 0.9072 | 0.9066 | 0.9071 | 0.8606 | 0.9074 | 0.8529 | 0.9079 |
|          |               | 0.5       | 0.9077 | 0.9079 | 0.9074 | 0.9069 | 0.9048 | 0.9048 | 0.9048 | 0.9048 | 0.9048 |

Table 2: Performance comparisons on real-world datasets, where the upper part shows results of classification tasks (evaluated by accuracy) and the lower part shows results of regression tasks (evaluated by mean square error). The best private result on each dataset, loss function and $\epsilon$ is bolded.

Conclusion and Future Works

In this paper, we consider differential privacy learning by Empirical Risk Minimization where a public database could be helpful. We propose a simple yet effective approach named Private-Public Stochastic Gradient Descent (PPSGD). PPSGD includes two main stages, where the public database is used to adjust optimization parameters dynamically in the differentially private SGD and treated as the training set in model reuse. Empirical studies validate that our method can utilize small public data and perform better than existing approaches.

There are several future directions for our setting. Firstly, it is interesting to explore other approaches to utilize public data to improve the performance of differentially private learning. Secondly, the public database may play a greater role in local differential privacy, such as recognizing and resisting outliers caused by the huge noise. Thirdly, some more ingenious model reuse approaches have been proposed recently, which may be helpful to differentially private learning with a wider range of public data.

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