ONE-LOOP EFFECTIVE ACTION AROUND DE-SITTER SPACE

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abstract

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The non-local one-loop contribution to the gravitational effective action around de Sitter space is computed using the background field method with pure trace external gravitational fields and it is shown to vanish. The calculation is performed in a generic covariant gauge and the result is verified to be gauge invariant.
1. Introduction

It has been shown that quantum gravity in de Sitter space-time suffers from several pathological features [1 - 6]. In particular, the graviton propagator grows at large distances [4, 7, 8]. Although this behavior points towards an instability of de Sitter space, the precise significance of this result is still unclear. An immediate consequence is that even tree-level scattering amplitudes diverge [8]. However, the physical meaning of a quantity such as a scattering amplitude is obscure in a space with a horizon. Therefore we have looked for another quantity in which the large distance behavior of the propagator could manifest itself and which is independent of the definition of asymptotic states. As such we have chosen the one-loop effective action suitably defined to be gauge invariant.

In order to illustrate our purpose, let us consider first the simpler problem of a massless scalar field coupled to an external de Sitter gravitational background:

\[ \mathcal{L} = -\sqrt{-g}\left\{ \frac{1}{2}(D\Phi)^2 + \frac{\zeta}{12} \mathcal{R}\Phi^2 \right\} \]  

where \( \zeta = 0 \) corresponds to the massless minimal coupling and \( \zeta = 1 \) to the conformal one. The first case is expected to have similar large-distance properties with fully quantized gravity [2]. Strictly speaking, the propagator of a massless minimally coupled scalar field around a de Sitter background is singular because of the zero mode of the Laplacian operator on the four-sphere. However, dropping the zero mode, one obtains a two-point function which grows logarithmically at large distances [9] in the same way as the transverse and traceless part of the graviton propagator. Therefore one could think that this case represents a simple model for quantum gravity. In the conformally invariant coupling on the other hand, the propagator vanishes at large distances [9] and no pathological behavior is expected. We have computed in both cases the effective action for the classical gravitational field. We write

\[ g_{\mu\nu} = g_{\mu\nu}^{dS} + H_{\mu\nu} \]

where \( g_{\mu\nu}^{dS} \) is the de Sitter metric.
The effective action $\Gamma$ is a functional of $g_{\mu\nu}$. The first non-trivial term in its expansion in powers of $H_{\mu\nu}$ is the second-order one which has in an obvious notation the form:

$$\Gamma^{(2)}[H] = \int H^{\mu\nu}(x) \gamma_{\mu\nu,\mu'\nu'}^{(2)}(x, x') H^{\mu'\nu'}(x') \, dV(x) \, dV(x') \quad (1.2)$$

where $dV(x)$ is the de Sitter volume element at $x$ and indices are contracted with the de Sitter metric. We wish to compute $\gamma^{(2)}$ for $x \neq x'$ and in particular to determine its behavior for large separations of $x$ and $x'$. The computation is straightforward but, for simplicity we give here the result for $H_{\mu\nu}(x)$ restricted to the special form

$$H_{\mu\nu}(x) = \frac{1}{4} g_{\mu\nu}^{ds}(x) H(x) \quad (1.3)$$

The result is

$$\gamma^{(2)} = 2(\zeta - 1)^2[(G_{\mu\nu},\mu)\,^2 + 4\zeta(G_{\mu\mu})\,^2 + 4\zeta^2 G^2] + \text{local terms} \quad (1.4)$$

where $;\,$ denotes covariant differentiation with respect to the de Sitter background and $G(x, x')$ is the scalar propagator $\Delta_{m^2}$ defined in eq.(A.23) with $m^2 = -2\zeta$ in units such that the radius of the Euclidean four-sphere equals one.

As expected, for the conformally invariant coupling $\zeta = 1$, $\gamma^{(2)}$ is local and is given by the conformal anomaly. For the minimal coupling $\zeta = 0$, $G$, defined by the sum over all non-zero modes on $S^4$, is given by

$$G = \frac{1}{(4\pi)^2} \left[ \frac{1}{(1 - z)} - 2 \ln(1 - z) \right] \quad (1.5)$$

The invariant distance $z(x, x')$ is obtained by analytic continuation from the geodesic distance $\mu(x, x')$ on the four-sphere by $z = \cos^2(\mu/2)$. Note that $z = 1$ corresponds
to $x = x'$, while the large distance limit is given by $z \to \infty$. $\gamma^{(2)}$ is not local any more and in fact using (1.5) and (1.4) one obtains up to local terms:

$$\gamma^{(2)} = \frac{6}{(4\pi)^4} \left[ \frac{1}{(1-z)^4} + \frac{1}{(1-z)^3} + \frac{1}{(1-z)^2} \right] \quad (1.6)$$

At large $z$ the leading term is $z^{-2}$ which can be interpreted as a logarithmically divergent correction to the effective cosmological constant in the one-loop effective action. The $z^{-3}$ and $z^{-4}$ terms represent similarly the logarithmically divergent corrections to the $R$ and $R^2$ terms. We want to examine whether a similar behavior is present in the case of quantum gravity.

The plan of the paper is as follows: In the next section we define the effective action in the context of the background field method we will use throughout. Section 3 contains the actual computation of the quadratic term in the one-loop approximation for the special case of pure trace background gravitational field. The gauge invariance of the result has been explicitly verified. An important technical result is the expression of the graviton propagator in an arbitrary covariant gauge and is presented in the Appendix. We chose to work with the space-time signature $(-+++)$.

2. The model

We are interested in studying quantum gravity around de Sitter space. So, we consider the lagrangian density:

$$\mathcal{L} = -\sqrt{-g} \left( R - 2\Lambda \right) \quad (2.1)$$

with $\Lambda$ the cosmological constant. It is known [10 - 12] that in the background field method the effective-action is automatically invariant under the so-called background gauge transformations and that it becomes invariant also with respect to quantum gauge transformations if one chooses the background field to be a solution of the classical equations of motion [12].
We thus expand the metric as

\[ g_{\mu\nu} = g^{dS}_{\mu\nu} + H_{\mu\nu} + h_{\mu\nu} \]  

(2.2)

where \( g^{dS}_{\mu\nu} + H_{\mu\nu} \equiv \bar{g}_{\mu\nu} \) is the full gravitational background and \( h_{\mu\nu} \) the quantum fluctuating field. \( H_{\mu\nu}(x) \) is restricted to be a solution of the classical equations of motion

\[ \bar{\mathcal{R}}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{\mathcal{R}} + \Lambda \bar{g}_{\mu\nu} = 0 \]  

(2.3)

These equations show that in quantum gravity the requirement of gauge invariance relates the possible terms in the effective action as a function of the background field. In other words, the computation of \( \Gamma^{(2)} \) will not allow us to identify the behavior of each individual term. This problem becomes more severe if one considers, for simplicity, pure trace background fields as in eq. (1.3). At the linearized level the equations of motion (2.3) yield:

\[ H_{;\mu;\nu} = - g^{dS}_{\mu\nu} H \]  

(2.4)

These equations are too restrictive. The only solution is a superposition of the five conformal Killing fields of de Sitter space. With this form of \( H \) the effective action becomes just a function of five real variables and is found to be local. We conclude that the pure trace part of \( \Gamma^{(2)} \) for the theory (2.1) cannot teach us anything about the large distance properties of the theory.

A possible way to overcome this difficulty is to consider the case of quantum gravity coupled to matter fields which add in the right hand side of eq. (2.3) the matter energy momentum tensor. In this note we shall consider the simple problem of gravity coupled to a massive scalar field \( \Phi \) which does not change the
large distance properties of the theory. We thus consider the model

\[ L = -\sqrt{-g} \left[ (1 + a\Phi)\mathcal{R} - 2\Lambda(1 + 2a\Phi) - \frac{1}{2}(D\Phi)^2 - \mu^2\Phi^2 \right] \] (2.5)

Notice that the term linear in \( \Phi \) is multiplied by the trace of the field equation (2.3). This guarantees that the equations of motion derived from \( L \) above still admit the solution \([g_{\mu\nu}(x) = g^{dS}_{\mu\nu}(x), \Phi = 0]\). We are going to study perturbatively the fluctuations around it. As for the graviton field, we also decompose the scalar field \( \Phi \) into a background \( \phi \) and a fluctuating part \( \varphi \).

The effective action is now a functional of \( H_{\mu\nu} \) and \( \phi \). The computation of the full \( \Gamma^{(2)} \) is rather lengthy, so, as a first step, we shall again consider the simpler case for which \( H_{\mu\nu}(x) \) is a pure trace \( H \). \( H \) and \( \phi \) satisfy the linearized equations of motion

\[ H(x) = -4a \phi(x) \] (2.6)

\[ (\Box + \frac{\mu^2 - 12a^2}{1 - 3a^2}) \phi(x) = 0 \] (2.7)

which admit in de Sitter space a complete set of wave solutions.

3. The effective action

3.1. The Feynman rules

We now proceed with the computation of the effective action of the model (2.5). The quadratic part of (2.5) in the quantum fields, diagonalized by the field
redefinition

\[ h_{\mu\nu} \rightarrow h_{\mu\nu} - a\varphi \bar{g}_{\mu\nu} \quad (3.1) \]

takes the form (in our units \( \Lambda = 3 \)):

\[
\mathcal{L}^Q = -\sqrt{-g} \left[ \frac{1}{4} (\bar{D}_\mu h)^2 + \frac{1}{2} (\bar{D}_\mu h^{\mu\nu})^2 - \frac{1}{4} (\bar{D}_\lambda h_{\mu\nu})^2 - \frac{1}{2} (\bar{D}_\mu h)(\bar{D}_\nu h^{\mu\nu}) + \frac{3}{2} h_{\mu\nu}^2 - \frac{3}{4} h^2 \right. \\
+ \frac{1}{8} \mathcal{R}(h^2 - 2 h_{\mu\nu}^2) + \frac{1}{2} \mathcal{R}_{\mu\nu\lambda\rho} h^{\mu\rho} h^{\lambda\nu} - \frac{1}{2} \mathcal{R}_{\mu\nu\lambda\rho} h^{\mu\rho} h^{\lambda\nu} \\
- \frac{1}{2} (1 + 3 a^2)(\bar{D}_\mu \varphi)^2 - \frac{1}{2} (\mu^2 - 12 a^2) \varphi^2 \left. \right] \\
(3.2)
\]

In (3.2) the covariant derivatives \( \bar{D} \), the Riemann tensor and the contractions of indices are computed with the full background gravitational field \( \bar{g}_{\mu\nu} \). \( h \) is the trace of \( h_{\mu\nu} \).

In the Appendix we compute the graviton and the ghost propagators \( G_{\mu\nu\mu'\nu'}(z) \) and \( Q_{\mu\nu}(z) \), respectively, in a general class of gauges defined in terms of the redefined graviton field (3.1) by

\[ \mathcal{L}_{GF} = -\frac{1}{2\alpha} \sqrt{-\bar{g}} (\bar{D}_\nu h_{\mu\nu} - \xi \bar{D}_\mu h)^2 \quad (3.3) \]

To leading order in the quantum fields, the redefined graviton transforms under an infinitesimal general coordinate transformation with parameters \( \epsilon_\mu(x) \) according to

\[ \delta h_{\mu\nu} = \bar{D}_\nu \epsilon_\mu + \bar{D}_\mu \epsilon_\nu + a \bar{g}_{\mu\nu} \epsilon^\lambda \bar{D}_\lambda \phi \]

and this in turn leads to the corresponding quadratic ghost Lagrangian:

\[ \mathcal{L}_{\text{ghost}} = -\sqrt{-\bar{g}} (\bar{D}^\mu \bar{\eta}^\nu) [\bar{D}_\mu \bar{\eta}_\nu + \bar{D}_\nu \bar{\eta}_\mu - 2\xi \bar{g}_{\mu\nu} \bar{D}_\lambda \bar{\eta}^\lambda + a(1 - 4\xi) \bar{g}_{\mu\nu} \bar{\eta}^\lambda \bar{D}_\lambda \phi] \quad (3.4) \]

The result for the graviton propagator can be cast in the form:

\[ G = G^{(0)} + \alpha G^{(1)} \quad (3.5) \]

with \( G^{(0)} \) given in eqs. (A.10), (A.15), (A.16) and \( G^{(1)} \) in (A.19). The calculations
simplify if one uses the value $\xi = 1/4$. As shown in the Appendix, this is one of the special values for which the propagator on $S^4$ has a zero-mode. However, a similar zero-mode appears in the ghost propagator $Q$ in eq. (A.24). We will explicitly show below that they cancel in the one-loop calculation [13]. Notice, that this problem of zero modes is unavoidable for any value of $\xi$ for, even when the graviton equation has no zero modes, that of the ghost always has. In the intermediate steps we shall use, as propagators, the inverse of the corresponding wave operators in the non-zero-mode sub-spaces.

The results for $G$ and $Q$ are:

$$G^{(0)} = G_{TT}^{(0)} + G_{PT}^{(0)},$$

with the transverse traceless part $G_{TT}^{(0)}$ given in eq. (A.15) and the pure trace part $G_{PT}^{(0)}$ given by:

$$G_{PT}^{(0)} = g_{\mu\nu} g_{\mu'\nu'} \sum_{n \neq 1} \frac{\varphi_n \varphi'_n}{6(\lambda_n^{(0)} + 4)} + \beta_1 \sum_{i=1}^{5} \chi_{1i\mu\nu} \chi'_{1i\mu'\nu'}$$

The part containing the longitudinal pieces $G^{(1)}$ and $Q$ take the form:

$$G^{(1)} = \sum_{n \geq 1} \frac{2}{\lambda_n^{(1)}} V_{n\mu\nu} V'_{n\mu'\nu'} + \frac{16}{9} \sum_{n \geq 2} \frac{\varphi_{n;\mu;\nu'} \varphi'_{n;\mu';\nu'}}{\lambda_n^{(0)}(\lambda_n^{(0)} + 4)^2}$$

$$Q = \sum_{n \geq 1} \frac{\xi_{\mu\nu} \xi_{\mu'\nu'}}{\lambda_n^{(1)}} + \frac{2}{3} \sum_{n \geq 2} \frac{\varphi_{n;\mu;\nu'} \varphi'_{n;\mu';\nu'}}{\lambda_n^{(0)}(\lambda_n^{(0)} + 4)} + \frac{15}{\beta_2} \sum_{j=1}^{15} K_{j\mu} K'_{j\mu'}$$

where $\beta_{1,2}$ are arbitrary parameters, which should not appear in any physical quantity. This is a further check of gauge invariance. The $\chi_{1i}^{\mu\nu}$ are the five zero-modes of the graviton kinetic operator $\mathcal{D}_{inv} + (1/\alpha)\mathcal{D}_{GF}$ in the gauge $\xi = 1/4$, and they are given in the Appendix. $\chi_{1i}^{\mu\nu} = (1/2) g^{\mu\nu} \phi_{1i}$ with $\partial_{\mu} \phi_{1i}/2$ the five proper conformal Killing fields of de Sitter space. $K_{j\mu}$ denote collectively its fifteen
conformal Killing fields: the ten Killing vectors $\xi^i_\mu$ (i=1,2,...10), together with the above five proper conformal Killing vectors. They are all zero-modes of the ghost kinetic operator in the $\xi = 1/4$ gauge. Note that because of the zero-modes equations (A.5) and (A.22) get modified to:

$$D_{inv}G^{(0)} + D_{GF}G^{(1)} = 1 - \sum_{i=1}^{5} \chi_{1i}\chi_{1i}'$$

$$D_{GF}G^{(0)} = D_{inv}G^{(1)} = 0$$

$$D_{gh}Q = 1 - \sum_{j=1}^{15} K_jK_j'$$  \hspace{1cm} (3.10)

We wish to study the large distance behavior of the non-local part of the effective action $\gamma^{(2)}$. So, it will not be necessary to consider diagrams with $\phi$ internal lines, since these are suppressed, and furthermore, for the evaluation of the graviton contribution we shall only need the three-point vertices $H-h-h$ and $\phi-h-h$.

The $H-h-h$ vertex comes from $\mathcal{L}^E = -\sqrt{-g}(\mathcal{R} - 2\Lambda)$ as well as from the gauge-fixing term. We find:

$$V_{Hhh}^E = -\frac{1}{4} H \left[ \frac{1}{4} h^{\mu\beta;\alpha} h_{\mu\beta;\alpha} - \frac{1}{2} h^{\mu\alpha;\beta} h_{\alpha\beta;\mu} + \frac{1}{2} h_{;\alpha} h^{\beta;\beta} - \frac{1}{4} h_{;\alpha} h^{\alpha} - (h^{\alpha\mu} h_{\mu\beta})_{;\beta}\right]$$

$$+ \frac{1}{2} (h^{\alpha\beta} h_{;\beta;\beta}) + \frac{1}{2} (h^{\alpha\beta} h_{;\beta;\alpha})$$  \hspace{1cm} (3.11)

$$V_{Hhh}^{GF} = \frac{1}{2\alpha} H \left[ \frac{3}{4} (h^{T;\beta})^2 + \frac{1}{2} h^{T;\beta} h^{T;\mu}_{;\mu;\beta} \right]$$

$$= \frac{1}{2\alpha} H \left[ \frac{3}{4} (h^{T;\beta})^2 + \frac{1}{2} h^{T;\beta} (D_{GF} h)_{;\alpha\beta} \right] \equiv \frac{1}{\alpha} \mathcal{V}^{(1)}$$  \hspace{1cm} (3.12)

where $h^{T;\mu} = h^{T;\mu} - (1/4) g^{dS}_{\mu\mu} h$ denotes the traceless part of $h^{T;\mu}$ and the operator $D_{GF}$ is given in eq. (A.4). The expressions for the various pieces of the vertex are by definition up to the factor $\sqrt{-g_{dS}}$ the corresponding terms in the expansion of the lagrangian density.
Similarly we obtain the $\phi - h - h$ vertex:

$$\mathcal{V}_{\phi hh} = -a \phi \left[ \frac{1}{4} (h_{;\mu})^2 + \frac{1}{2} (h^{\alpha\beta}_{;\alpha})^2 - \frac{1}{4} (h^{\alpha\beta\mu})^2 - \frac{1}{2} h^{\beta}_{;\beta} h^{\alpha}_{;\alpha} - \frac{1}{2} (h_{;\mu} h^{\beta\mu})_{;\alpha} \\ - \frac{3}{2} (h^{\alpha}_{;\alpha} h^{\beta}_{;\mu})_{;\mu} + (h^{\alpha\beta}_{;\alpha})_{;\beta} - \frac{1}{2} (h_{;\alpha})^{;\alpha} + (h^{\alpha\beta}_{;\alpha\beta})^{;\mu} \\ + \frac{1}{2} (h h^{\alpha\beta}_{;\alpha})_{;\alpha} + h^2 - h^2 \right]$$

(3.13)

The vertices (3.11) and (3.13) are combined using the equation of motion of the background fields (2.6). The result is:

$$\mathcal{V}^{(0)} = -\frac{1}{4} H \left[ -\frac{1}{2} h^{\alpha\beta}(\Box - 2) h_{\alpha\beta} + \frac{1}{2} h(\Box + 1) h + h^{\beta\nu} h^{\alpha}_{\beta;\alpha\nu} - \frac{1}{2} h^{\alpha\beta} h_{;\alpha;\beta} - \frac{1}{2} h h^{\alpha\beta}_{;\alpha;\beta} \right]$$

where $D_{\text{inv}}$ is the wave operator given in eq. (A.4) and corresponds by definition to the invariant part of the action (A.1).

Finally, the $H - ghost - antighost$ vertex is obtained from the expansion of eq. (3.4) and is given by:

$$\mathcal{V}^{(G)} = H \left[ -\frac{1}{2} \eta^{\alpha\beta} (\eta_{\alpha;\beta} + \eta_{\beta;\alpha}) + \frac{1}{4} \eta^{\alpha}_{;\alpha} \eta^{\beta}_{;\beta} \right]$$

(3.15)

A comment about notation is in order: Having expanded to linear order with respect to the background fluctuations $H$ and $\phi$ and to second order in the quantum fields, the expressions for the vertices contain only contractions and covariant differentiations with respect to the de Sitter background. Thus, in the rest of the present paper we will simplify our notation and use $g_{\mu\nu}$ for the de Sitter metric and both $; \mu$ or $D_{\mu}$, depending on notational convenience, for the covariant derivative with respect to it.
3.2. The $\alpha$–independence

Using the propagators and the vertices above we calculate the one-loop effective action. It is easier to organize the calculation as an expansion in powers of the gauge parameter $\alpha$. In this way the gauge invariance of the result will be manifest.

The first term is proportional to $\alpha^{-2}$. It is generated by the diagram having two $V^{(1)}$ vertices given in eq. (3.12), and two $G^{(0)}$ propagators. This term vanishes because $V^{(1)}$ is proportional to $h^{T\mu\nu,\nu}$ and the traceless part of $G^{(0)}$ has no longitudinal piece.

The second term is proportional to $\alpha^{-1}$. It is the sum of two diagrams, one with two $V^{(1)}$ vertices, one $G^{(0)}$ and one $G^{(1)}$ propagators and another one with one $V^{(1)}$ and one $V^{(0)}$ vertices and two $G^{(0)}$ propagators. The second can be shown to vanish with an argument similar to the one given above. After a straightforward calculation, the first one is found to be proportional to

\[ G^{(0)}_{\mu\nu,\rho,\nu,\rho}G^{(1)}_{\rho,\rho} \]

where $\mathcal{T}G^{(1)}_{\rho,\rho}$ denotes the traceless part of $G^{(1)}$ with respect to both pairs of indices. Using the expression (3.8) one can show that $\mathcal{T}G^{(1)}_{\rho,\rho}$ satisfies the following identity:

\[ \mathcal{T}G^{(1)}_{\rho,\rho} = -1_{\rho,\rho} + \sum_{j=1}^{15} K_{j_{\rho,\rho}} K'_{j_{\rho,\rho}} \]

(3.16)

where $1_{\rho,\rho}$ is the $\delta$ function for vectors in de Sitter space. Taking two more derivatives with respect to $\nu, \nu'$, symmetrizing in the two pairs of indices, subtracting the traces and using the defining properties of the conformal Killing fields, one can easily show that this diagram also vanishes, up to possible local terms.

In order to finish with the explicit demonstration of the $\alpha$–invariance of our result, we consider the terms with positive powers of $\alpha$ in the effective action. First, the one with $\alpha^{2}$ arises entirely from the graph with two $V^{(0)}$ vertices (3.14) and
two $G^{(1)}$ propagators. Its contribution is proportional to $\mathcal{D}_{\text{inv}} G^{(1)}$ and it is zero due to eq. (3.10).

Finally, two graphs contribute to the $\mathcal{O}(\alpha)$ terms in the effective action. Firstly, the one with one vertex $\mathcal{V}^{(0)}$, one vertex $\mathcal{V}^{(1)}$ and two propagators $G^{(1)}$. Like in the previous graph the vertex $\mathcal{V}^{(0)}$ with two longitudinal propagators attached to it vanishes identically. Secondly, the graph with two vertices $\mathcal{V}^{(0)}$, one propagator $G^{(1)}$ and one $G^{(0)}$. Ignoring the contractions which vanish by the previous argument we find that its contribution is proportional to

$$G^{(1)} \mathcal{D}'_{\text{inv}} \mathcal{D}_{\text{inv}} G^{(0)}$$

Using eq. (3.10) and the relation $\mathcal{D}_{\text{inv}} \chi_{1i} = 0$, it is straightforward to show that the result vanishes up to local terms.

This concludes our explicit demonstration of the $\alpha$—independence of the $\mathcal{O}(H^2)$ part of the one-loop effective action we are computing. The use of the equations of motion (2.6) was crucial for the above result [12]. Had we allowed for generic $H, \phi$ backgrounds we would have obtained an $\alpha$—dependent answer.

3.3. The $\mathcal{O}(\alpha^0)$ Term

Finally, as we will show below the remaining $\mathcal{O}(\alpha^0)$ term of the non-local part of the one-loop effective action density $\gamma^{(2)}$ is zero. Since we are interested here only in the non-local piece of $\gamma^{(2)}$, we shall systematically ignore the local contributions in all intermediate steps.

Four graphs contribute to the above quantity:

1) The graph with both vertices $\mathcal{V}^{(0)}$ and two graviton propagators $G^{(0)}$.

We use the vertex $\mathcal{V}^{(0)}$ (3.14) and perform the contractions to obtain:

$$\gamma^{(2)}_{\mathcal{V}_0 \mathcal{V}_0} = \frac{1}{16} \left[ G^{(0)} \mathcal{D}'_{\text{inv}} \mathcal{D}_{\text{inv}} G^{(0)} + (\mathcal{D}_{\text{inv}} G^{(0)}) (\mathcal{D}'_{\text{inv}} G^{(0)}) \right]$$

Using (3.10) and $\mathcal{D}_{\text{inv}} \chi_{1i} = 0$ one can show that the first term vanishes and $\gamma^{(2)}_{\mathcal{V}_0 \mathcal{V}_0}$
simplifies to

$$\gamma^{(2)}_{V_0V_0} = \frac{1}{16} (D_{inv} G^{(0)})(D'_{inv} G^{(0)})$$

(3.17)

With the $G^{(0)}$ and $D_{inv}$ given in (3.6), (A.15), (3.7) and (A.4) we find

$$D_{inv} G^{(0)} = \sum_{n \geq 0} h_{n\mu\nu}^T h_{n\mu'\nu'}^T - \frac{1}{3} (D_\mu D_\nu - g_{\mu\nu}(\Box + 3)) g_{\mu'\nu'} \sum_{n \neq 1} \frac{\varphi_n \varphi'_n}{\lambda_n^{(0)} + 4}$$

(3.18)

and consequently:

$$\gamma^{(2)}_{V_0V_0} = \frac{1}{16} (\sum_{r=1}^5 \varphi_1 \varphi'_1)^2 + \frac{1}{16} (\sum_{n \geq 0} h_{nTT} h_{n(PT)})^2$$

(3.19)

2) The graph with one vertex $V^{(0)}$, the other $V^{(1)}$ and $G^{(0)}$ and $G^{(1)}$ for the two graviton propagators.

The vertex $V^{(1)}$ is given in (3.12). It contains two terms but upon contraction with $V^{(0)}$ given in (3.14) the first term vanishes because it contains divergences of the transverse and traceless $G^{(0)}_{TT}$. The second simplified by the use of $D_{inv} G^{(1)} = 0$ leads to:

$$\gamma^{(2)}_{V_0V_1} = -\frac{1}{8} (D'_{inv} G^{(0)})(D_{GF} G^{(1)})$$

(3.20)

which combined with (3.10) and (3.18) gives:

$$\gamma^{(2)}_{V_0V_1} = \frac{1}{8} \left[ (D'_{inv} G^{(0)})(D_{inv} G^{(0)}) - \left( \sum_{r=1}^5 \varphi_1 \varphi'_1 \right)^2 \right]$$

$$= \frac{1}{8} (\sum_{n \geq 0} h_{nTT} h_{n(PT)})^2$$

(3.21)

3) The graph with both vertices $V^{(1)}$ and two graviton propagators $G^{(1)}$. 
We carry out the contractions and we obtain
\[
\gamma^{(2)}_{V_1 V_1} = \frac{9}{32} (T G^{(1)}_{T \mu \nu \rho} : \rho \rho')^2 + \frac{1}{16} G^{(1)} \mathcal{D}_{G \mu} \mathcal{D}_{G \nu} G^{(1)} + \frac{1}{16} (\mathcal{D}_{G \mu} G^{(1)}) (\mathcal{D}_{G \nu} G^{(1)}) \\
+ \frac{3}{8} G^{(1)}_{T \mu \nu \rho} : \rho' (\mathcal{D}_{G \nu} G^{(1)})_{\mu \rho' \alpha} : \alpha'
\]

(3.22)

where \( G^{(1)}_{T \mu \nu \rho} \equiv G^{(1)}_{T \mu \nu \rho} - (1/4) g^{\mu \rho'} g_{\alpha' \beta'} G^{(1)}_{\mu \rho \beta} \) denotes the traceless part of \( G^{(1)} \) with respect to the primed indices. We then use (3.10) and (A.4) to show that the second and fourth term of \( \gamma^{(2)}_{V_1 V_1} \) vanish, while by (3.10) and (3.16) the remaining terms lead to

\[
\gamma^{(2)}_{V_1 V_1} = \frac{1}{16} \left( (\mathcal{D}_{\mu} G^{(0)})(\mathcal{D}_{\nu} G^{(0)}) - \left( \sum_{r=1}^{5} \varphi_{1r} \varphi'_{1r} \right)^2 \right) + \frac{9}{32} \left( \sum_{j=1}^{15} K_{j\mu} K'_{j\mu'} \right)^2 \\
= \frac{1}{16} \left( \sum_{n \geq 0} h^{TT}_n h'_{TT} \right)^2 + \frac{9}{32} \left( \sum_{j=1}^{15} K_{j\mu} K'_{j\mu'} \right)^2
\]

(3.23)

Adding the results (3.19), (3.21) and (3.23) of the three diagrams above, we obtain the total contribution of the graviton loop:

\[
\gamma^{(2)}_{\text{graviton}} = \frac{1}{4} \left( \sum_{n \geq 0} h^{TT}_n h'_{TT} \right)^2 + \frac{1}{16} \left( \sum_{r=1}^{5} \varphi_{1r} \varphi'_{1r} \right)^2 + \frac{9}{32} \left( \sum_{j=1}^{15} K_{j\mu} K'_{j\mu'} \right)^2
\]

(3.24)

4) Finally, the ghost-loop i.e. the graph with both vertices \( \psi^{(G)} \) and two ghost propagators \( Q \) given in eqs. (3.15) and (3.9) respectively. Its contribution is

\[
\gamma^{(2)}_{\text{ghost}} = -\left( D_{(\mu} D^{(\mu'} Q_{\nu')} \right)^2 - \frac{1}{16} \left( D_{(\mu} D_{\mu'} Q^{\mu'} \right)^2 + \frac{1}{2} \left( D_{(\mu} D^{\mu'} Q_{\nu})_{\mu'} \right)^2 \\
= - \left[ D_{(\mu} D^{(\mu'} Q_{\nu')} - \frac{1}{4} g_{\mu \nu} D^{\alpha} D^{\nu} Q_{\alpha'} - \frac{1}{4} g^{\mu \nu} D_{\alpha} D^{\nu} Q_{\alpha'} \right] + \frac{1}{16} g_{\mu \nu} g^{\mu'} g^{\nu'} D^{\alpha} D^{\alpha} Q_{\alpha'} - \frac{5}{8} g_{\mu \nu} g^{\mu'} g^{\nu'} \sum_{r=1}^{5} \varphi_{1r} \varphi'_{1r}
\]

(3.25)

It is easy to verify that:

\[
D^{(\mu} D^{(\nu') \nu')} = -\frac{1}{2} \sum_{n \geq 1} V_{n \mu} V_{n \nu'} - \frac{1}{3} \sum_{n \geq 2} \left( D^{(\nu'} \varphi_n \right) (D^{\mu} D^{\nu') \nu'}) + \frac{1}{2} g^{\mu' \nu'} g^{\nu'} \sum_{r=1}^{5} \varphi_{1r} \varphi'_{1r}
\]

Subtracting the traces under both pairs of indices in order to form the quantity in
the second line of eq. (3.25) and using the completeness relation (A.14) we find for the non-local part of the ghost contribution

$$\gamma^{(2)}_{\text{ghost}} = -\frac{1}{4} \left( \sum_{n \geq 0} h^T_n h^{TT}_n \right)^2$$

(3.26)

Note that the unphysical parameters $\beta_{1,2}$ have disappeared from $\gamma^{(2)}_{\text{graviton}}$ and $\gamma^{(2)}_{\text{ghost}}$.

3.4. THE ZERO-MODE CONTRIBUTION

It is crucial to realise at this point that the sum of (3.24) and (3.26) is not the full answer. We are effectively computing the path-integral over the metric, the scalar field and the ghosts of the exponential of the action (3.2) + (3.3) + (3.4). For $\xi = 1/4$ and $H_{\mu\nu}$ pure trace the ghost and the graviton kinetic operators have zero-modes. To linear order in $H$, they take the form

$$\eta^\mu_{(j)} = (1 + \frac{1}{4}H)K^\mu_j \quad j = 1, 2, ..., 15$$

(3.27)

and

$$h^{\mu\nu}_{(r)} = (1 + \frac{1}{4}H)\chi^{\mu\nu}_{1r} \quad r = 1, 2, ..., 5$$

(3.28)

respectively.

To treat properly these zero-modes special attention is required [14]. Namely, one performs the path-integration over the subspace orthogonal to the zero-modes but multiplies by an appropriate power of the determinant of the matrix of inner products of the zero-modes. The inner products, fixed to be ultralocal, are defined by:

$$\mathcal{M}_{j k}^{g h} \equiv \langle \eta_{(j)} | \eta_{(k)} \rangle = \int d^4x \sqrt{-\bar{g}} \bar{g}^{\mu\nu} \eta_{(j)}^\mu \eta_{(k)}^\nu$$

$$\mathcal{M}_{r s}^{g r} \equiv \langle h_{(r)} | h_{(s)} \rangle = \int d^4x \sqrt{-\bar{g}} \bar{g}^{\mu\lambda} \bar{g}^{\nu\rho} h_{(r)}^\mu h_{(s)}^\nu$$

(3.29)

where $\bar{g}_{\mu\nu} = g_{\mu\nu}^{dS}(1 + \frac{1}{4}H)$ denotes the full background metric. The functional
integral is then multiplied by the factor

\[ \text{det}^{-1} \mathcal{M}^h \text{det}^{-\frac{1}{2}} \mathcal{M}^{gr} \]

where the minus sign in the second factor can be traced to the fact that the action of Euclidean gravity is unbounded from below [15]. We expand this factor in powers of \( H \) using eqs (3.27) and (3.28) and we find that the contribution to the non-local part of \( \gamma^{(2)} \) cancels precisely the last two terms of eq.(3.24).

Adding all contributions, we obtain our result that the non-local part of \( \gamma^{(2)} \) is zero! Thus, we conclude that the pathological large-distance behavior of the graviton propagator on the background de Sitter space does not manifest itself in the Weyl non-invariant quadratic part of the effective action in the one loop approximation.

What is the significance of this result? It may be an accident of the order in which we are working and it may disappear at higher orders. On the other hand, it may be an indication that quantum gravity around a de Sitter background behaves effectively like a conformally invariant theory. In such a case the local terms should be related to the conformal anomaly. In a pure Einstein theory, if one wants to compute the various terms in the anomalous equation for the trace of the energy-momentum tensor, one faces the problem of isolating the individual contributions which was mentioned in section 2. Our method of coupling the theory to a scalar field may provide a solution.
APPENDIX

In this Appendix we compute the graviton propagator for the class of gauges given by (3.3), as well as the propagator of the corresponding Faddeev-Popov ghosts. We shall work in the Euclidean continuation of de Sitter space which is $S^4$ and whose metric will be denoted by $g_{\mu\nu}$ throughout this section. The quadratic part of the lagrangian including the gauge-fixing is

$$L^Q = -\sqrt{-g} \left[ \frac{1}{4} (h_{\mu\nu})^2 + \frac{1}{2} (h_{\mu\nu}^{\mu\nu})^2 - \frac{1}{4} (h_{\mu\nu})_\lambda^2 - \frac{1}{2} h_{\mu\nu} h_{\mu\nu}^{\mu\nu} - \frac{1}{2} (h_{\mu\nu})^2 - \frac{1}{4} h^2 ight. \left. + \frac{1}{2\alpha} (h_{\mu\nu}^{\mu\nu} - \xi h_{\mu\nu})^2 \right]$$  \hspace{1cm} (A.1)

We expand the propagator $G_{\mu\nu\mu'\nu'}(x, x')$ in a power series of $\alpha$ and write (omitting the indices)

$$G = G^{(0)} + \alpha G^{(1)} + \alpha^2 G^{(2)} + .... \hspace{1cm} (A.2)$$

$G$ satisfies the equation:

$$(D^{-1} + \frac{1}{\alpha} D_{GF}) (G^{(0)} + \alpha G^{(1)} + \alpha^2 G^{(2)} + ....) = 1 \hspace{1cm} (A.3)$$

where $D_{inv}$ and $D_{GF}$ denote the differential operators of the gauge invariant and gauge fixing parts of the quadratic lagrangian (A.1) respectively:

$$(D_{inv} h)_{\mu\nu} = -\frac{1}{2} (\Box - 2) h_{\mu\nu} + \frac{1}{2} (\Box + 1) g_{\mu\nu} - D_\mu D^\alpha h_\nu^{\alpha} - \frac{1}{2} g_{\mu\nu} D^\alpha D^\beta h_{\alpha\beta} - \frac{1}{2} D_\mu D_\nu h$$

$$(D_{GF} h)_{\mu\nu} = D_{(\mu} D^{\alpha} h_{\nu)}^{\alpha} - \xi D_\mu D_\nu h - \xi g_{\mu\nu} (D^\alpha D^\beta h_{\alpha\beta} - \xi h) \hspace{1cm} (A.4)$$

As usual two indices inside parentheses are symmetrized, i.e. $T^{(\alpha\beta)} \equiv (T^{\alpha\beta} + T^{\beta\alpha})/2$. 

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Order by order in $\alpha$ equation (A.3) gives:

$\alpha^{-1} : \quad \mathcal{D}_{GF} G^{(0)} = 0$

$\alpha^0 : \quad \mathcal{D}_{\text{int}} G^{(0)} + \mathcal{D}_{GF} G^{(1)} = 1$

$\alpha^1 : \quad \mathcal{D}_{\text{int}} G^{(1)} + \mathcal{D}_{GF} G^{(2)} = 0$

$\alpha^2 : \quad \mathcal{D}_{\text{int}} G^{(2)} + \mathcal{D}_{GF} G^{(3)} = 0$  \hspace{1cm} (A.5)

By definition $G^{(0)}$ is the graviton propagator calculated for $\alpha = 0$ i.e. by imposing on the graviton quantum field the Landau-type gauge condition

$$h_{\mu\nu} = \xi h^{\mu}$$  \hspace{1cm} (A.6)

Its computation is done independently as follows: start with the graviton decomposition

$$h_{\mu\nu} = h_{TT\mu\nu} + A^{T}_{(\mu;\nu)} + B_{\mu;\nu} + \frac{1}{4} g_{\mu\nu}(-\Box B + h)$$  \hspace{1cm} (A.7)

and impose the gauge condition to obtain:

$$A^{T}_{\mu} = 0, \quad 3(\Box + 4)B + (1 - 4\xi)h = 0$$  \hspace{1cm} (A.8)

Substitute (A.7) together with (A.8) into the quadratic action to rewrite it in terms of the independent fields $h_{TT\mu\nu}$ and $B$ as:

$$\mathcal{L}^Q = -\sqrt{-g} \left[ \frac{1}{2} h_{TT\mu\nu}(-\Box + 2)h_{TT\mu\nu}^{TT\mu\nu} + \frac{3}{(4\xi - 1)^2} B((1-\xi)(\Box + 3)^2(\Box + 4)B \right]$$  \hspace{1cm} (A.9)

Accordingly, the graviton propagator is decomposed as

$$G^{(0)} = G_{TT}^{(0)} + G_{PT}^{(0)}$$  \hspace{1cm} (A.10)

It is convenient to introduce at this point [7] the scalar, transverse-vector and spin-2 (transverse traceless symmetric tensor) eigenfunctions of the $\Box$ operator on
the sphere $S^4$ of radius $\rho$. They are given by

\[ \square \varphi_n^i(x) = \lambda_n^{(0)} \varphi_n^i(x) \]

\[ \square \xi_{n}^{\mu}(x) = \lambda_n^{(1)} \xi_{n}^{\mu}(x) \]  \hspace{1cm} (A.11)

\[ \square h_{TT}^{\mu\nu}(x) = \lambda_n^{(2)} h_{TT}^{\mu\nu}(x) \]

The corresponding eigenvalues and their degeneracies are:

\[ \lambda_n^{(0)} = -\frac{1}{\rho^2} n(n + 3) \quad g_n^{(0)} = \frac{1}{6} (n + 1)(n + 2)(2n + 3) \]

\[ \lambda_n^{(1)} = -\frac{1}{\rho^2} (n^2 + 5n + 3) \quad g_n^{(1)} = \frac{1}{2} (n + 1)(n + 4)(2n + 5) \]  \hspace{1cm} (A.12)

\[ \lambda_n^{(2)} = -\frac{1}{\rho^2} (n^2 + 7n + 8) \quad g_n^{(2)} = \frac{5}{6} (n + 1)(n + 6)(2n + 7) \]

We shall also need the following tensor functions:

\[ \chi_n^{\mu\nu} = \frac{1}{2} g^{\mu\nu} \varphi_n \quad n = 0, 1, 2, ... \]

\[ W_n^{\mu\nu} = \frac{\varphi_n^{\mu;\nu} - \frac{1}{4} g^{\mu\nu} \square \varphi_n}{\sqrt{\lambda_n^{(0)} (\frac{3}{4} \lambda_n^{(0)} + 3\rho^{-2})}} \quad n = 2, 3, 4, ... \]  \hspace{1cm} (A.13)

\[ V_n^{\mu\nu} = \frac{\xi_n^{(\mu;\nu)}}{\sqrt{-\frac{1}{2} \lambda_n^{(1)} (\lambda_n^{(1)} + 3\rho^{-2})}} \quad n = 1, 2, 3, ... \]

which satisfy the completeness relation

\[ \sum_{n=0}^{\infty} h_{n}^{TT} h_{n}^{TT'} + \sum_{n=1}^{\infty} V_n V'_n + \sum_{n=2}^{\infty} W_n W'_n + \sum_{n=0}^{\infty} \chi_n \chi'_n = 1 \]  \hspace{1cm} (A.14)

where primed functions or tensors with primed indices are meant to be evaluated at the point $x'$. In terms of these, the expressions for the two parts of the graviton
propagator $G^{(0)}$ in the Landau gauge (A.6) are respectively (in our units $\Lambda = 3$ which corresponds to $\rho = 1$):

$$G^{(0)}_{TT} = \sum_{n=0}^{\infty} \frac{-2}{\lambda_n^{(2)} - 2} h_n^{TT} h_n^{TT'}$$  \hspace{1cm} (A.15)

$$G^{(0)}_{PT} = \sum_{n=0}^{\infty} \frac{[(4\xi - 1)\varphi_n^{\mu;\nu} + g^{\mu\nu}((1 - \xi)\Box + 3)\varphi_n][((4\xi - 1)\varphi_n'^{\mu';\nu'} + g^{\mu';\nu'}((1 - \xi)\Box' + 3)\varphi_n']}{6(\lambda_n^{(0)} + 4)((1 - \xi)\lambda_n^{(0)} + 3)^2}$$  \hspace{1cm} (A.16)

Notice that the $\lambda_n^{(0)} + 4$ factor in the denominator vanishes for $n = 1$. However $\varphi_1$ satisfies $\varphi_{1;\mu;\nu} = -g_{\mu\nu}\varphi_1$ and consequently the $n = 1$ term in the sum of (A.16) is actually zero.

Having computed $G^{(0)}$ we may use (A.5) to solve for the remaining pieces of the graviton propagator. First of all notice that by definition $G^{(0)}$ satisfies $\mathcal{D}_{\Gamma}G^{(0)} = 0$.

To determine $G^{(1)}$ we start with its decomposition in terms of the functions defined in (A.13)

$$G^{(1)} = \sum_n (b_n V_n V_n' + c_n W_n W_n' + d_n \chi_n \chi_n' + e_n (W_n \chi_n' + W_n' \chi_n))$$  \hspace{1cm} (A.17)

and determine the unknown coefficients so as to satisfy equations (A.5). It is straightforward to verify that the solution of the latter leads to:

$$b_n = \frac{2}{\lambda_n^{(1)} + 3} \hspace{1cm} n \geq 1$$

$$d_n = \frac{\frac{1}{4}\lambda_n^{(0)}}{((1 - \xi)\lambda_n^{(0)} + 3)^2}$$

$$c_n = \frac{\frac{3}{4}\lambda_n^{(0)} + 4}{(1 - \xi)\lambda_n^{(0)} + 3}$$

$$e_n = \frac{[\frac{3}{4}\lambda_n^{(0)}(\lambda_n^{(0)} + 4)]^{\frac{1}{2}}}{2((1 - \xi)\lambda_n^{(0)} + 3)^2}$$  \hspace{1cm} (A.18)

$$G^{(k)} = 0 \text{ for all } k > 1$$

Inserting the above coefficients into the expression (A.17) we obtain the useful
form of $G^{(1)}$

$$G^{(1)} = \sum_{n \geq 1} \left( \frac{2}{\lambda_n^{(1)} + 3} V^\mu_n V_n^{\mu'} + \frac{\varphi_n^{\mu;\nu} \varphi_n^{\mu';\nu'}}{\lambda_n^{(0)} ((1 - \xi) \lambda_n^{(0)} + 3)^2} \right) \quad (A.19)$$

The propagator $G = G^{(0)} + \alpha G^{(1)}$ is well-defined except for the set of special values of $\xi = (k^2 + 3k - 3)/k(k + 3)$, $k = 1, 2, ...$ [13].

In this class of gauges, the quadratic part of the Fadeev-Popov lagrangian (3.4) is

$$L^Q_{\text{ghost}} = -\sqrt{-g} \ \bar{\eta}^{\mu;\nu} (\eta_{\mu;\nu} + \eta_{\nu;\mu} - 2\xi g_{\mu\nu} \eta^\lambda;\lambda) \quad (A.20)$$

We decompose the ghost field into transverse and longitudinal parts according to

$$\eta^\mu = \eta^\mu_T + \eta^\mu_L$$

and write the quadratic part of the ghost lagrangian

$$L^Q_{\text{ghost}} = \sqrt{-g} \left[ \bar{\eta}^\mu_T (\Box + 3) \eta_T^\mu - 2\bar{\eta} ((1 - \xi) \Box + 3) \Box \eta \right] \quad (A.21)$$

We notice that the differential operators of $L^Q_{\text{ghost}}$ have zero modes. In the transverse part these zero modes are given by the ten Killing vectors of the four-sphere, $\xi_0^\mu$ of eq.(A.11), and they are present for all values of $\xi$. The longitudinal part has zero modes only for the set of special values of $\xi$ mentioned above for which also the graviton propagator has zero modes.

As a result, the ghost propagator $Q_{\mu\nu'}$ is defined only in the non-zero-mode subspace and satisfies:

$$\mathcal{D}_{gh}^{\mu} Q_{\nu\nu'} = \{(\Box + 3)\delta_{\nu}^{\nu'} - 2[(1 - \xi)\Box + 3]D_{\mu}D^{\nu'}\}Q_{\nu\nu'} = 1_{\mu\nu'} - \sum_{i=1}^{10} \xi_i^\mu \xi_i^{\mu'} \quad (A.22)$$

Its explicit form is:

$$Q_{\mu\nu'} = Q^T_{\mu\nu'} - \frac{1}{6} \left( \Delta_0 - \Delta_{\frac{3}{1-\xi}} \right)_{\mu;\nu'}$$

where $\Delta_{m^2}$ denotes the propagator of a minimally coupled scalar field of mass-
squared $-m^2$

$$\Delta m^2 = \sum_{n=0}^{\infty} \frac{\varphi_n \varphi'_n}{\lambda_{\mu}^{(0)} + m^2}$$  \hspace{1cm} (A.23)

The corresponding summation in $\Delta_0$ starts from $n = 1$. $Q_{\mu \mu'}^T$ is written in terms of the transverse parts of the vector eigenfunctions (A.11) as

$$Q_{\mu \mu'}^T \equiv \sum_{n \geq 1} \frac{\xi_{n \mu} \xi_{n \mu'}}{\lambda_{n}^{(1)} + 3} + \beta \sum_{i=1}^{10} \xi_{0 \mu} \xi_{0 \mu'}$$

where $\beta$ is an arbitrary parameter which should not appear in any physical quantity.

Using the above expression the ghost propagator takes its final form:

$$Q_{\mu \mu'} = \sum_{n \geq 1} \frac{\xi_{n \mu} \xi_{n \mu'}}{\lambda_{n}^{(1)} + 3} - \frac{1}{2} \sum_{n \geq 1} \frac{\varphi_{n \mu} \varphi'_{n \mu'}}{(1 - \xi)\lambda_{n}^{(0)} + 3} + \beta \sum_{i=1}^{10} \xi_{0 \mu} \xi_{0 \mu'}$$ \hspace{1cm} (A.24)

It is straightforward to carry out the summations over the modes in the various propagators and express them in terms of the invariant distance $z(x, x')$ and its derivatives [7]. For our purposes though the use of these explicit formulas is not particularly illuminating.

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