Quantum Symmetries and Stringy Instantons

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The quantum symmetry of many Landau-Ginzburg orbifolds appears to be broken by Yang-Mills instantons. However, isolated Yang-Mills instantons are not solutions of string theory: They must be accompanied by gauge anti-instantons, gravitational instantons, or topologically non-trivial configurations of the $H$ field. We check that the configurations permitted in string theory do in fact preserve the quantum symmetry, as a result of non-trivial cancellations between symmetry breaking effects due to the various types of instantons. These cancellations indicate new constraints on Landau-Ginzburg orbifold spectra and require that the dilaton modulus mix with the twisted moduli in some Landau-Ginzburg compactifications. We point out that one can find similar constraints at all fixed points of the modular group of the moduli space of vacua.

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1. Introduction

Landau-Ginzburg orbifolds describe special submanifolds in the moduli spaces of Calabi-Yau models, at “very small radius.” New, stringy features of the physics are therefore often apparent in the Landau-Ginzburg theories. For example, these theories sometimes manifest enhanced gauge symmetries which do not occur in the field theory limit, where the large radius manifold description is valid. And all Landau-Ginzburg orbifolds have one stringy symmetry, the quantum symmetry which essentially counts the twisted sectors of origin of the various elementary particles. In spacetime, the quantum symmetry manifests itself as a discrete R-symmetry.

Because the quantum symmetry is intimately related to the process of constructing the orbifold, one expects that it will correspond to an exact symmetry in spacetime. However, the spacetime theories arising from Heterotic string compactification on Landau-Ginzburg orbifolds typically correspond to $E_6 \times E_8$ gauge theories, and one also expects that instanton effects associated with the observable $E_6$ and the hidden $E_8$ might lead to a non-perturbative violation of various discrete symmetries. In fact, we shall see in §2 that the quantum symmetry of many $(2,2)$ (and more generally $(0,2)$ Landau-Ginzburg theories seems naively to be broken by instantons of the observable gauge group.

This is somewhat puzzling: After all, the quantum symmetry is a direct consequence of the orbifold construction. If the string description of our theory is correct, we really expect the symmetry to be exact. The resolution of this puzzle is provided in §3, where we recall that instanton solutions of string theory must satisfy the Bianchi identity for the $H$ field. In particular, this implies that in string theory, instantons of the observable gauge group must be accompanied by anti-instantons of the hidden gauge group, gravitational instantons, or topologically non-trivial configurations of the $H$ field. This fact is quite important when considering symmetry breaking effects of Yang-Mills instantons in string compactifications.

In §4, we show that the permitted configurations with an instanton of the observable gauge group combined with an anti-instanton of the hidden $E_8$ or a gravitational instanton in fact preserve the quantum symmetry. This involves a non-trivial conspiracy between the various gauge and gravitational anomalies, and provides us with constraints on the spectra of Landau-Ginzburg orbifolds. The solutions with non-trivial $H$ field also preserve the quantum symmetry if one assigns an appropriate transformation law to the model-independent axion. We discuss some implications of these results in §5.
Cancellation of “duality anomalies” has been discussed in the context of toroidal orbifolds by several groups \([5]\). The quantum symmetry of a Landau-Ginzburg theory is somewhat analogous to the extra \(\mathbb{Z}_2\) symmetry which arises at the fixed point of the \(R \rightarrow \frac{1}{R}\) symmetry of the Teichmüller space of circle compactifications \([3]\). In this sense and others, our work represents a generalization of \([5]\) to the setting of Landau-Ginzburg orbifolds.

In particular, Ibáñez and Lüst (last reference of \([5]\)) used the constraint of the cancellation of duality anomalies to restrict the possible massless spectra which could arise in orbifold models. Using the constraints we derive below, one could make similarly powerful statements about the possible massless spectra of Landau-Ginzburg orbifolds.

Indeed, certain “bad models” (whose spectra of spacetime fermions already looked peculiar) discussed in \([3]\) violate the anomaly cancellation conditions, and so can be ruled out on this basis.

2. Quantum Symmetries and Anomalies

The Landau-Ginzburg orbifolds are distinguished by the fact that they all possess at least one discrete R-symmetry, namely the quantum symmetry which counts the twisted sector \(k\) of origin of the various physical states.

For concreteness, let us focus attention on the quintic hypersurface in \(\mathbb{C}P^4\). The Landau-Ginzburg theory is a point of enhanced symmetry in the Kähler moduli space. One normally says that the Landau-Ginzburg orbifold has a \(\mathbb{Z}_5\) quantum symmetry, but since one needs to include both NS and R sectors for the left-movers, there are actually 10 sectors in the Landau-Ginzburg orbifold \([6]\). So one might better think of the quantum symmetry as \(\mathbb{Z}_{10} = \mathbb{Z}_2 \times \mathbb{Z}_5\). Actually, this definition of the quantum symmetry is a little awkward because the different components, under the decomposition \(E_6 \supset SO(10) \times U(1)\), of a given \(E_6\) representation transform with different weight under this \(\mathbb{Z}_{10}\) symmetry. To fix this, we can compose this symmetry with an element of the center of \(E_6\), to obtain a \(\mathbb{Z}_{30} = \mathbb{Z}_3 \times \mathbb{Z}_{10}\) symmetry which acts homogeneously on \(E_6\) multiplets. In the language of \([6]\), this \(\mathbb{Z}_{30}\) is generated by

\[
S_Q = e^{2\pi i(3k-2q)/30}
\]  

\(\text{(2.1)}\)

\(^1\) Of course, this \(\mathbb{Z}_2\) is just a subgroup of the extra \(SU(2)\) Kac-Moody symmetry which arises at the self-dual radius.
where \( k = 0, \ldots, 9 \) labels the sector number of the Landau-Ginzburg orbifold, and \( q \) is the left-moving \( U(1) \) charge. The charges of the various massless multiplets under \( S_Q \) are listed in Table 1 as integers \( \in \mathbb{Z}/30\mathbb{Z} \). In the table, \( C \) and \( R \) refer to the 101 complex structure moduli and the Kähler modulus of the quintic. \( S \) and \( S' \) represent, respectively, groups of 200 and 24 other massless \( E_6 \) singlets, corresponding to elements of \( H^1(\text{End}(T)) \). The former arise in the untwisted sector, the latter in a twisted sector of the Landau-Ginzburg orbifold \([6]\).

| \( S_Q \) | 27 | 27 | C, S | R | S' | 78 | \( W \) |
|---|---|---|---|---|---|---|---|
|   | −2 | 8 | 0 | 6 | 6 | 0 | −6 |

**Table 1**: Charges \( \in \mathbb{Z}/30\mathbb{Z} \) of the spacetime matter multiplets, gluons, and the spacetime superpotential \( W \) under \( S_Q \), the “quantum” R-symmetry present at the Landau-Ginzburg point.

On the world sheet, \( S_Q \) simply enforces the fact that sector number is conserved mod 10 in correlation functions. It is easy to see that, in spacetime, \( S_Q \) generates a discrete R-symmetry, under which the spacetime superpotential has charge \(-6 \mod 30\). That is, one should add 3 to the entries in Table 1 to obtain the \( S_Q \)-charge of the corresponding right-handed fermions in these chiral multiplets, and subtract 3 to obtain the \( S_Q \) charge of the right-handed \( E_6 \) gluinos. Clearly \( 27^3 \) and \( 27^3 \) are couplings in the superpotential allowed by the discrete \( R \) symmetry, whereas, say, \( 27^2 27^2 \) is not.

This \( R \)-symmetry and its analogues in other \( (2,2) \) Landau-Ginzburg theories are very useful in proving the existence of a large class of \( (0,2) \) deformations. The analogous quantum symmetries of the non-deformation \( (0,2) \) models considered in \([4]\) provide a stringent self-consistency check on the assumption that those models are conformally invariant. These matters will be discussed elsewhere \([7]\).

Our present interest is simply to check that this discrete \( R \) symmetry is nonanomalous. It is easy to find the charge of the \( E_6 \) instanton-induced ’t Hooft effective Lagrangian \( \mathcal{L}_{\text{eff}} \) under \( S_Q \):

\[ 2 \]

The states found in \([8]\) were the massless spacetime fermions; for a fermion which comes from the \((k + 1)^{st}\) twisted sector, its scalar superpartner comes from the \( k^{th} \) twisted sector.

\[ 3 \]

Though we have chosen to discuss the anomaly in terms of the (nonperturbative) ’t Hooft effective Lagrangian, one could equally well phrase the discussion of the anomaly in terms of the noninvariance of the (nonlocal) one loop effective Lagrangian. That is, if the anomaly doesn’t vanish, the symmetry is broken already in string perturbation theory.
Charge of \(L_{\text{eff}} = 2 \cdot \left[(-3) \cdot c(78) + 101 \cdot (1) \cdot c(27) + (11) \cdot c(27)\right] \mod 30 \) \(\text{(2.2)}\)

\[= 600 \mod 30 = 0 \mod 30\]

where \(c(78) = 12, c(27) = c(27) = 3\) for \(E_6\).

So we see that, for the quintic, the quantum R-symmetry is not broken by \(E_6\) instantons \[\text{[6]}\]. Unfortunately, this will be far from true in many other examples.

In the case of more general (2,2) or (0,2) theories, the quantum R-symmetry is generated by

\[S_Q = e^{2\pi i (kr - 2q)/2mr} \text{ \(\text{(2.3)}\)}\]

where \(k = 0, 1, \ldots, 2m - 1\) is the sector number, and \(r\) is the “rank of the vacuum gauge bundle” \(r = 3, 4, 5\) for spacetime gauge group \(G = E_6, SO(10),\) or \(SU(5)\). In particular, \(r = 3\) for all (2,2) theories. We are interested in checking the transformation properties of the ’t Hooft effective Lagrangian induced by \(G\) instantons under \(\text{(2.3)}\), just as we did for the quintic. If \(L_{\text{eff}}\) is not invariant, then instantons break the quantum symmetry – that is, it is anomalous.

Rather generally, the computation of the transformation properties of the ’t Hooft effective Lagrangian can be rephrased as the computation of a new index for Landau-Ginzburg orbifolds. As usual, the index can be phrased as a trace over massless \((L_0 = \bar{L}_0 = 0)\) states in the right-moving Ramond sector

\[A_1 = \sum_k Tr_R \left( q^2 \left(\frac{1}{2r} (rk - 2q) (-1)^{F_R} \right) \right) \mod 2mr \text{ \(\text{(2.4)}\)}\]

\[= \sum_k Tr_R \left( \frac{q^2}{2} k (-1)^{F_R} \right) \mod 2mr \text{ \(\text{(2.5)}\)}\]

where, again, \(k\) is the sector number, \(q\) is the left-moving \(U(1)\) charge, and \(F_R = \bar{q} + 1/2\) is the right-moving fermion number. The factor of \(1/2r\) in \(\text{(2.4)}\) is necessary because the worldsheet \(U(1)\) current \(J\) has been normalized so that \(J(z) \cdot J(w) \sim \frac{r}{(z-w)^2}\), while the current which goes into the vertex operator for the gauge boson is canonically normalized to have a central term of \(1/2r\). One easily finds \(\text{(2.3)}\) from \(\text{(2.4)}\) by realizing that \(\sum_k Tr_R (q^3 (-1)^{F_R})\) measures the gauge anomaly which vanishes. Note that one should include the \((16 - 2r)\) free left-moving Majorana-Weyl “gauge” fermions when taking the
sum (2.5). For example in an $E_6$ theory, the various $E_6$ representations can be decomposed as representations of the maximal $SO(10) \times U(1)$ subgroup, and each state of the Landau-Ginzburg theory should be multiplied by the dimension of the $SO(10)$ representation of the corresponding spacetime particle. If the index (2.5) doesn’t vanish, then the quantum R-symmetry is broken by instantons.

The index that we have introduced is but the first term in a modular form

$$F(q) = \sum_k Tr_R \left( \frac{q^2}{2} k (-1)^{F_R} q^{L_0-c/24} \right)$$

with coefficients in $\mathbb{Z}/2mr\mathbb{Z}$. This is a refinement for Landau-Ginzburg orbifolds of the elliptic genus [9,10,11,12].

We find that in many models, including well known (2,2) compactifications, the quantum symmetry is broken by instantons of the observable gauge group. Consider, for example, the (2,2) Landau-Ginzburg theory with superpotential

$$W = \sum_i (S^3_i + S_i T^3_i) + S_3^3.$$  

This is the Landau-Ginzburg description of the (1,16,16,16) Gepner model [13] with exceptional modular invariants for the 16s. The Calabi-Yau phase of this theory was first discussed by Schimmrigk [14]. This model has been of some interest in the literature because, after suitable orbifolding, it yields a quasi-realistic three generation model. The quantum symmetry of the string theory compactified on (2.7) is, naively, broken by $E_6$ instantons.

In the language we have been using, the quantum symmetry of (2.7) is generated by

$$S_Q = e^{2\pi i (3k-2q)/54}.$$  

To see that this symmetry is anomalous, recall that the compactification on (2.7) yields 35 left-handed generations ($27$s of $E_6$) and 8 left-handed anti-generations ($\overline{27}$s of $E_6$). In the orbifold, the 35 left-handed generations have $3k-2q = -1$. Six of the left-handed anti-generations have $3k-2q = 19$, while one has $3k-2q = 13$ and one has $3k-2q = 25$. Remembering also that the left-handed gaugino in the $78$ of $E_6$ has $3k-2q = 3$, and using $c(27) = c(\overline{27}) = 3$, $c(78) = 12$, we see that the total anomaly is

$$3 \cdot 12 + 35 \cdot (-1) \cdot 3 + (6 \cdot (19) + (13) + (25)) \cdot 3 = 387 = 9 \mod 27.$$  

Actually, by doing the sum over only left-handed fermions, we are computing half of $A_1$ and requiring that it vanish mod $mr$. 

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Therefore, the quantum R-symmetry is broken by instanton effects in the spacetime theory.

There are many simpler examples which exhibit this anomaly. For example it is a simple matter to compute (2.4) for the $r = 3$ theories corresponding to the (2,2) compactifications on the Calabi-Yau hypersurfaces in $W_{1,1,1,3}$ and $W_{1,1,1,4}$. In the first case, with $m = 7$, one finds that $A_1$ is equal to $324 \mod 42 \neq 0$ and that the quantum $\mathbb{Z}_7$ symmetry is completely broken by instantons. A similar computation shows that in the second case the quantum $\mathbb{Z}_8$ is broken to a $\mathbb{Z}_2$.

The fact that non-perturbative effects seem to be breaking the quantum symmetry of these Landau-Ginzburg orbifolds is very disturbing – one would expect, because of its role in constructing the orbifold, that the quantum symmetry should be an exact symmetry of the compactified string theory. We shall see in §4 that in fact it is – but to understand why string theory evades the effects of $E_6$ instantons, we must first remind ourselves of some pertinent facts about anomaly cancellation.

3. The Green-Schwarz Mechanism and Stringy Instantons

The cancellation of spacetime and worldsheet anomalies is one of the most delicate features of string theory. The stringy modification of the minimal $N = 1, D = 10$ supergravity theory coupled to super Yang-Mills theory which allows for the cancellation of all gauge and gravitational anomalies [15] involves assigning non-trivial gauge and local Lorentz transformation properties to the antisymmetric tensor field $B$. Under an infinitesimal gauge transformation with parameter $\Lambda$ and an infinitesimal local Lorentz transformation with parameter $\theta$, the $B$ field transforms as

$$
\delta B = tr(Ad\Lambda) - tr(\omega d\theta)
$$

where $A$ is the gauge connection and $\omega$ is the spin connection. The transformation law (3.1) which cancels the space-time anomalies is also responsible for the cancellation of $\sigma$-model anomalies in the heterotic string, which may occur when one considers nontrivial maps of the worldsheet to spacetime [10].

(3.1) implies that the gauge invariant field strength $H$ of $B$ obeys the Bianchi identity

$$
dH = tr F_1^2 + tr F_2^2 - tr R^2 .
$$

Here $H$ is a 3-form and $F_{1,2}$ and $R$ are (Lie-algebra valued) two-forms in spacetime. $tr F_i^2$ and $tr R^2$ represent the first Pontryagin classes $p_1(V_i, \mathbb{R})$ and $p_1(T, \mathbb{R})$ of the $E_8 \times E_8$ gauge
bundle $V_1 \oplus V_2$ and the tangent bundle $T$ of our 4-manifold. This equation leads to an important topological restriction for string propagation on compact spaces, which is closely related to cancellation of local and global anomalies in string theory \[16,17\].

The case of interest to us is strings propagating in four-dimensional Minkowski space. In order to satisfy (3.2), we can consider at least three types of configurations.

Let us first try to satisfy (3.2) using only gauge instantons. Actually, when searching for such instanton solutions, we should consider a Euclidean continuation and compactification of Minkowski space to a four-dimensional torus. The relevant property of this space for us is that $tr R^2 = 0$. So the condition (3.2) reduces to

$$tr F_1 \wedge F_1 + tr F_2 \wedge F_2 = 0$$ (3.3)

Of course a single instanton of either the first or second $E_8$ cannot possibly satisfy (3.3)! So we see that the instanton configurations of the observable $E_6$, whose effects we previously considered, will never appear by themselves in string theory. They must be accompanied by anti-instantons of the hidden $E_8$, in such a way that (3.3) is satisfied.

We can also find solutions of (3.2) which combine gauge instantons of the observable gauge group with gravitational instantons in such a manner that

$$tr R \wedge R = tr F_1 \wedge F_1$$ (3.4)

is satisfied. Similarly, solutions of this sort with the gauge instantons of the observable gauge group replaced by those of the hidden $E_8$ must be considered.

Finally, we can build solutions of (3.2) in which we take a single instanton of the observable gauge group, and choose a non-trivial configuration of the $H$ field in such a way that (3.2) is still satisfied. This is only possible in noncompact (Euclidean) spacetimes. Such configurations have been discussed in \[18\] in $\mathbb{R}^4$ and more recently have been reincarnated as “gauge fivebranes” \[19\], which is what we will call them. In fact, the configurations which satisfy (3.3) can presumably be constructed by taking two widely separated gauge fivebranes, one built around an instanton of the observable gauge group and the other built around an anti-instanton of the hidden gauge group.

One could also study configurations with non-trivial $H$ fields, gauge, and gravitational instantons \[19\]; we shall only consider the more basic types of solutions discussed above. If those basic configurations do not break a symmetry, the more elaborate solutions will also preserve it.

Therefore, the questions of interest become: What are the symmetry breaking effects of instanton-anti-instanton pairs, with an instanton from the observable $E_6$ and an anti-instanton from the hidden $E_8$, what are the effects of gauge instanton/gravitational instanton pairs, and what are the effects of gauge fivebranes?
4. Quantum Symmetries Revisited

The computation of quantum symmetry breaking due to instantons of the observable gauge group has already been discussed in §2. The effect of hidden $E_8$ instantons on the quantum symmetry is even easier to analyze.

The only fermi fields charged under the hidden $E_8$ which arise in twisted sectors of the Landau-Ginzburg orbifold are the gluinos in the $248$ of $E_8$; the left-handed gluinos come from the $k = 1$ twisted sector [3]. Therefore, the ’t Hooft effective Lagrangian induced by instantons of the second $E_8$ has $S_Q$ charge

$$A_2 = -2 \cdot c(248) \cdot r \mod 2mr = -60r \mod 2mr . \quad (4.1)$$

Considering only configurations with $E_8$ anti-instantons for every instanton of the observable gauge group, we see that the resulting effective Lagrangian will then have $S_Q$ charge

$$A_1 - A_2 = 60r + \sum_k Tr_R \left( \frac{q^2}{2} k(-1)^{F_R} \right) \mod 2mr . \quad (4.2)$$

It is easy to check that (4.2) in fact vanishes for the theories whose quantum symmetries appeared to be broken by instantons of the observable gauge group. For example, for the 27 generation $(1, 16, 16, 16)$ Gepner model discussed in §2, one sees that (4.2) is equal to $-774 + 180 \mod 54 = -594 \mod 54 = 0$. Similarly, for the hypersurface in $W \mathbb{P}^4_{1,1,1,1,3}$, (4.2) is equal to $324 + 180 \mod 42 = 504 \mod 42 = 0$. In fact, the anomaly cancellation works in all of the theories we have checked.

Similarly, one finds that a single gravitational instanton contributes (up to an overall normalization)

$$A_3 = -452r + \sum_k Tr_R \left( (rk - 2q)(-1)^{F_R} \right) \mod 2mr \quad (4.3)$$

to the anomaly. The $-452r$ comes from the contribution of the $E_8$ gauginos $(-496r)$, the dilatino $(2r)$, and the gravitino $(42r)$ (to fix the contributions it is enough to know that the left-handed gauginos and gravitino and the right-handed dilatino come from the $k = 1$ twisted sector). The fact that the gravitino has the same chiral anomaly as $-21$ Weyl fermions is familiar from the study of gravitational index theorems [20]. As in (2.4), one must include the $16 - 2r$ free “gauge” fermions when taking the sum (4.3); their sole effect is to multiply the contribution of a fermion transforming in the $R$ representation of the observable gauge group by $\dim(R)$. 

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So to summarize, the full anomaly in the quantum symmetry is proportional to

\[ (-\frac{1}{24} A_3 p_1(T) + A_1 p_1(V_1) + A_2 p_1(V_2)) \mod 2mr. \] (4.4)

The normalization of the gravitational contribution to the anomaly relative to the gauge contribution is the same \(-\frac{1}{24}\) which appears in the Atiyah-Singer index theorem for the twisted spin complex \[\text{(21)}\].

We are interested in the vanishing of (4.4) for instanton configurations with gravitational instanton number \(n_1, n_2\) observable gauge group instantons, and \(n_3\) hidden \(E_8\) instantons. The configurations which satisfy (3.2) with trivial \(H\) field necessarily have

\[ p_1(T) = p_1(V_1) + p_1(V_2) . \] (4.5)

However, compact spin four-manifolds without boundary have

\[ p_1(T) \in 24\mathbb{Z} . \] (4.6)

That is, the index of the Dirac operator on such a manifold is an integer.

Therefore, to insure that the solutions with non-trivial gauge and gravitational configurations do not violate the quantum symmetry, one needs to check

\[ A_1 - A_2 = 0 \mod 2mr \] (4.7a)
\[ A_3 - 24A_2 = 0 \mod 2mr \] (4.7b)
\[ A_3 - 24A_1 = 0 \mod 2mr . \] (4.7c)

These are simply the conditions for the cancellation of the anomaly in the case where one of the three first Pontryagin classes in (1.3) vanishes. If (4.7a, b, c) are satisfied, then the quantum symmetry is non-anomalous for any choice of instanton satisfying (4.5).\footnote{In fact, the conditions (4.7a, b, c) are not all independent; one can see that if (4.7a) and either of (4.7b, c) are satisfied, then all three conditions are satisfied.}

We have checked that (4.7a, b, c) are satisfied by all the models that we have studied. For example, in the Landau-Ginzburg phase of the hypersurface in \(\mathbb{P}^4_{1,1,1,1,3}\), one finds \(A_3 \simeq 6 \mod 42\) while \(A_1 \simeq 30 \mod 42\) and \(A_2 \simeq 30 \mod 42\), so (4.7a, b, c) are indeed satisfied.
The effects of gauge fivebranes, or solutions with non-trivial $H$ field in general, are slightly more subtle. Recall that the 't Hooft effective Lagrangian generated by an instanton in string theory has the general form

$$\mathcal{L}_{\text{eff}} \sim e^{-8\pi^2/g^2} e^{ia/f_a} \psi_1 \ldots \psi_N$$

(4.8)

where $a$ is the model-independent axion, $g$ is the string-coupling, and the $\psi_i$ denote some space-time fermion fields. So notice that even if $\mathcal{L}_{\text{eff}}$ seems naively (based on the transformation properties of the $\psi_i$) to violate the quantum symmetry, by requiring that the axion $a$ transform by an appropriate shift one can always restore invariance of $\mathcal{L}_{\text{eff}}$.\footnote{This method has been employed by other authors in similar contexts \cite{5,22}.} Note that in order for this trick to work, it is crucial that the full anomaly be proportional to $dH = trF_1^2 + trF_2^2 - trR^2$. That is, it is crucial that the anomaly (4.4) vanish whenever (4.5) is satisfied, or equivalently that (4.7a, b, c) all vanish. Only then can one assign a transformation law to $a$, which cancels the anomaly.

But assuming that the anomalies vanish, as they do in the sensible (2,2) and (0,2) theories we have checked, one can also cancel the symmetry breaking effects of gauge fivebranes by requiring that $a$ transform by a shift under the quantum symmetry. This has an interesting consequence for the dilaton-axion moduli space, which we mention in \S 5.

5. Discussion

We have seen that that one encounters certain puzzling facts when one naively extrapolates field theory non-perturbative effects to a stringy context. These puzzles are naturally resolved when proper account is taken of the condition (3.2), which solutions of string theory must satisfy.

The vanishing of the anomalies (4.7a, b, c) for Landau-Ginzburg orbifolds is non-trivial. It is giving us some interesting constraints on the twisted sectors the matter fields and moduli must come from in Landau-Ginzburg compactifications. For instance, in the quintic, it is crucial in the anomaly cancellation that 24 of the $E_6$ singlets related to neither complex structure nor Kähler structure deformations arise from the $k = 3$ twisted sector instead of the $k = 1$ sector, where one might have naively expected to find them \cite{3}. Furthermore, in models like the 27-generation model and the $\mathbb{W}P_4^{1,1,1,1,3}$ model, where “extra” singlets arise
at the Landau-Ginzburg radius, both the number of such extra singlets and the twisted sectors in which they appear are constrained by the cancellation of the gravitational anomalies \((4.7b, c)\). Indeed, were these extra singlets \textit{not} to arise, then the anomalies would not cancel in these models!

Clearly, this situation is much more general than Landau-Ginzburg theories, or the orbifold theories studied in \[3\]. Rather generally, one has a moduli space \(\mathcal{M}\) of string vacua, which is obtained as the quotient of the “naive” moduli space by the action of a modular group \(\mathcal{G}\). In general, \(\mathcal{M}\) contains orbifold “points,” which are left fixed by various subgroups \(\mathcal{G}_0 \subset \mathcal{G}\). \(\mathcal{G}_0\) then acts as an automorphism of the conformal field theory corresponding to the fixed point set. In favourable circumstances, one even generates all of \(\mathcal{G}\) in this fashion. Demanding that these symmetries be nonanomalous, \textit{i.e.} that the modular group \(\mathcal{G}\) be a good symmetry of string theory, is a powerful restriction on the spectrum of the string theory. It constrains not just the states which are massless generically in \(\mathcal{M}\), but also those states which become massless only at the fixed points.

The fact that, in those models where \(A_{1,2,3}\) do not separately vanish mod \(2mr\), one must transform the model-independent axion under the quantum symmetry is also interesting. Roughly speaking, it means that the T-duality which acts on the Kähler moduli space also acts non-trivially on the dilaton. So the true moduli space is a quotient

\[
\mathcal{M} = \frac{\mathcal{D} \times \mathcal{R}}{\mathbb{Z}_m}
\]

of the product of the dilaton and Kähler moduli spaces, where the quantum symmetry acts nontrivially on both factors. For believers in strong-weak coupling duality (see \[23\] and \[24\] and references therein), this means that beyond the \(SL(2, \mathbb{Z})\) symmetry which acts on the dilaton-axion modulus alone, one must further quotient the dilaton-axion moduli space by a \(\mathbb{Z}_m\) action. Since, in general, these do not commute, the full modular group is a semidirect product.

We also noted before that, if the anomalies \((4.7a, b, c)\) do not vanish for a given compactification, then the quantum symmetry will be broken (one cannot cancel \((4.4)\) by shifting the model-independent axion). Although these anomalies do indeed vanish for all the \((2,2)\) theories that we have checked, they do not vanish automatically for \((0,2)\) theories.

\footnote{The \(SL(2, \mathbb{Z})\) symmetry is generated by \(\tau \to \tau + 1\) and \(\tau \to -1/\tau\), where \(\tau = \frac{a}{2\pi f_a} + \frac{4\pi i}{g^2}\). The \(\mathbb{Z}_m\) symmetry acts on \(\tau\) by \(\tau \to \tau + n/m\), for some \(n\).}
The anomaly cancellation conditions are satisfied, in the cases we have checked, for the (0,2) theories with “generically sensible” spectra, like the $Y_{W5;4,4}$ model discussed in [4].

However, for models whose spectra exhibit unexpected pathologies like the $Y_{W4;10}$ model in [4], (4.2) does not always vanish. In particular, the $Y_{W4;10}$ model suffers from an anomalous quantum symmetry. We are tempted to conclude that this is yet another way of seeing that this “bad model” does not give rise to a consistent string vacuum. This is a stronger conclusion than was ventured in [4]. There, we noted that for very special choices of the defining data, we could obtain a sensible-looking spectrum for a Landau-Ginzburg theory with unbroken $E_6$ gauge symmetry. But now we see that even that theory suffers from an anomaly in the quantum symmetry. Hence, since the quantum symmetry should be a good symmetry of string theory, we must reject the $Y_{W4;10}$ model, even at its $E_6$-symmetric point.

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