Inclusive Prompt Photons from the Color Glass Condensate at NLO

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Abstract. The cross-section for photons radiated by quarks in proton-nucleus collisions at collider energies was obtained using the Color Glass Condensate framework, in the dense-dilute kinematics regime. We observe that the inclusive photon cross-section is proportional to all-twist Wilson line correlators in the nucleus. These correlators also appear in quark-pair production; unlike the latter, photon production is insensitive to hadronization uncertainties and therefore more sensitive to multi-parton correlations in the gluon saturation regime of QCD.

1. Introduction
In hadron-hadron collision experiments, photons free-stream to the detectors without interacting strongly with the produced debris. This renders them as excellent probes for the study of gluon saturation at the very high energies of the RHIC and LHC colliders. In these regimes, prompt photon observables can be calculated pertubatively using the Color Glass Condensate (CGC) effective theory [1–5]. In the CGC, there is an emergent scale, $Q_S(x_0)$, which separates partons with a momentum fraction $x > x_0$ (fast partons) from those with $x < x_0$ (slow partons), where $x_0$ stands for an arbitrary energy measurement scale which can be evolved using the JIMWLK equation. The fast partons, highly dilated, become static sources, while the slower ones can be treated as classical Yang-Mills fields. The interplay between these two can be found by solving the classical Yang-Mills equations. Unfortunately, in this framework, a fully analytical solution for a collision of two nuclear projectiles is out of reach. However, for the case of a $p+A$ collision in the kinematic region available to LHC, one can approximate the system to the so-called dilute-dense limit in which the static sources of the projectile are much smaller than its characteristic transverse momentum, $\rho_p/k_{1\perp}^2 \ll 1$, while for the target $\rho_A/k_{2\perp}^2 \sim 1$. In this limit, the analytical solution is known, and was derived in ref. [6].

From the CGC framework, the parametric leading order (LO) process [7] for prompt photon production is given by the emission of a photon from a valence quark from the projectile after multiple-scattering off the nucleus (see LO in fig.1). Since the target is in the high occupancy regime ($gA_\mu \sim O(1)$), the $q\bar{q}g$ vertex is of $O(1)$, which makes the inclusive cross-section at LO
of $\mathcal{O}(\alpha_s)$. The next parametric order, $\mathcal{O}(\alpha_s^2\alpha_S)$, can be found by the inclusion of an incoming gluon from the proton, or strong bremsstrahlung from a quark line. The NLO processes can then be categorized in three classes, listed in fig. 1. They consist of (I) the bremsstrahlung of a photon and a gluon from valence quark line. The remaining two NLO processes consist of $q\bar{q}$ creation from a gluon of the proton, and (II) subsequent annihilation into a photon (see ref. [8]) or (III) photon emission.

It is important to note that, as stated in refs. [8,9], while at large $x$ the LO should be a good description of the cross-section, at smaller $x$, the valence quark distribution functions, $f_V$, decrease, while the gluon distribution, $f_g$, rises. In the kinematic window of $10^{-2} > x > 10^{-3}$, which is available to RHIC and LHC, the quark and gluon distributions will satisfy $f_g < \alpha_s f_V$, making the NLO dominant. Because of this, one has to include the NLO effects in the cross-section for a complete description of prompt photon emission in p+A collision. In this short report, we present the analytical results previously derived in [9] for diagram NLO-III in fig. 1.

2. Setting up the gluon field

To calculate the amplitude of this process in the dilute-dense limit, we used the Lorentz gauge, $\partial_\mu A^\mu = 0$. Within the CGC effective theory, the gluon fields for a projectile moving independently in the positive $z$ direction and a nucleus moving in the opposing direction are

$$A^\mu_p(x) = -g\delta^\mu x^- \frac{1}{\nabla^\perp_{\perp}} \rho_p(x_{\perp}) \quad \text{and} \quad A^\mu_A(x) = -g\delta^\mu x^+ \frac{1}{\nabla^\perp_{\perp}} \rho_A(x_{\perp}),$$

where $\rho_p$ and $\rho_A$ are the static (and stochastic) color charge densities of partons at large $x$. Using these results as construction blocks, one can calculate the result for both sources in the dilute-dense limit together using the Yang-Mills equations. For the complete derivation see ref. [6]. The complete result is given then by\(^1\).

$$A^\mu(q) = A^\mu_p(q) + \frac{ig}{q^2 + i\eta^\perp} \int_{k_{\perp}} \int_{x_{\perp}} e^{i(q_{\perp} - k_{\perp})x_{\perp}} \left[ C^\mu_U(q,k_{\perp})[U(x_{\perp}) - 1] + C^\mu_V(q)V(x_{\perp}) - 1 \right] \frac{\rho_p(k_{\perp})}{k_{\perp}^2},$$

where $A^\mu(q) = A^\mu_p(q)T^a$ and $T^a$ are the generators of $SU(N_c)$ in the adjoint representation.

We can dissect eq. (2) into its components. $A^\mu_p(q)$ corresponds to the field of a projectile travelling alone. The vectors $C^\mu_U$ and $C^\mu_V$ appear as factors for the multiple scattering of the gluon from the proton off the nucleus. $k_{\perp}$ here is the momentum exchanged from the proton,

\(^1\) Here we use a shorthand notation $f_{k_{\perp}} \equiv \int \frac{d^2k_{\perp}}{(2\pi)^2}$ and $f_{x_{\perp}} \equiv \int d^2x_{\perp}$.
while $k_{2\perp} \equiv q_{\perp} - k_{1\perp}$ is exchanged from the nucleus. These vectors are explicitly given by

\begin{align}
C^+_U(q, k_{1\perp}) &= -\frac{k_{1\perp}^2}{q^- + i\epsilon}, \quad C^-_U(q, k_{1\perp}) = \frac{(q_{\perp} - k_{1\perp})^2 - q_{\perp}^2}{q^2}, \quad C^+_V(q, k_{1\perp}) = -2k_{1\perp}, \\
C^+_V(q) &= 2q^+, \quad C^-_V(q) = 2\frac{q_{\perp}^2}{q^2} - 2q^-, \quad C^+_V(q) = 2q_{\perp}.
\end{align}

(3)

Using $C^\mu_V = C^+_V + C^-_V/2$, we can relate the two vector functions above to the Lipatov effective vertex $[10–12]$.

The remaining ingredients in eq. (2) are the Wilson lines $U$ and $V$, where

\begin{align}
U(x_{\perp}) &= \mathcal{P}_+ \exp \left[ ig \int dz^+ A^- (z^+, x_{\perp}) \cdot T \right], \\
V(x_{\perp}) &= \mathcal{P}_+ \exp \left[ iq \int dz^+ A^- (z^+, x_{\perp}) \cdot T \right],
\end{align}

(4) (5)

These objects are color rotation matrices in the adjoint representation, which arise from multiple scatterings of nuclear gluons over the proton gluon field. In the above, the operator $\mathcal{P}_+$ denotes light cone time-ordering in the $x^+$ direction. The uncommon $1/2$ factor in the $V$ Wilson line is due to $V$ being a gauge artefact. Therefore, terms including $V$ will not be present in the final result for the cross-section. It is interesting to remark that to cancel these contributions, one needs to include a singular gluon field, which accounts for the creation of the pair inside the nucleus. A complete discussion of these cancellations is given for $q\bar{q}$ production in ref. [6] and for this process in ref. [9].

Finally, for the computation of process III in fig.1, one needs to understand how quarks propagate in the presence of the field $A_A$. The fermion propagator can be expressed as in ref. [13],

\begin{equation}
G_F(p, k) = (2\pi)^4 \delta^4(k) G^0_F(p) + G^0_F(q + k) T(k, p) G^0_F(p)
\end{equation}

(6)

where $T(k, p)$ is an effective vertex which contains the multiple scatterings of the quark off the nucleus. It is given here explicitly by

\begin{equation}
T(k, p) = \begin{cases} 
2\pi \delta(k^+) \gamma^+ \int_{x_{\perp}} e^{ik_{\perp} \cdot x_{\perp}} \left[ \hat{U}(x_{\perp}) - 1 \right] & (p^+ > 0), \\
-2\pi \delta(k^+) \gamma^+ \int_{x_{\perp}} e^{ik_{\perp} \cdot x_{\perp}} \left[ \hat{U}^\dagger(x_{\perp}) - 1 \right] & (p^+ < 0),
\end{cases}
\end{equation}

(7)

for a momentum exchange $k$. $\hat{U}(x_{\perp})$ is the Wilson line in the fundamental representation.

3. Calculation of the cross-section

In the Lorentz gauge, there are a total of 14 non-vanishing diagrams that contribute to the amplitude of the process. For the complete set diagrams, and a detailed derivation, see ref. [9]).

Summing over all of them, one obtains the amplitude,

\begin{equation}
\mathcal{M}^\mu(p, q, k_{\perp}) = -q_f g^2 \int_{k_{\perp} k_{\perp}} \int_{x_{\perp} y_{\perp}} \frac{\rho^\mu_V(k_{1\perp})}{k_{1\perp}^2} e^{ik_{\perp} \cdot x_{\perp} + i(P_{\perp} - k_{\perp} - k_{1\perp}) \cdot y_{\perp}} \\
\times \bar{u}(q) \left\{ T^\mu_F(k_{1\perp}) U(x_{\perp}) \right\}_{ba} l^b + T^\mu_{gq}(k_{\perp}, k_{1\perp}) \hat{U}(x_{\perp}) t^a \hat{U}^\dagger(y_{\perp}) \right\} v(p),
\end{equation}

(8)

where we use the notation $P_{\perp} = p_{\perp} + q_{\perp} + k_{\perp}$. Here $T^\mu_F$ stands for the sum of the fermionic structures proportional to $U(x_{\perp})_{ba}$, scattering over a gluon line, while $T^\mu_{gq}$ for the ones.
proportional to $U(x_u)U(x_d)\dagger$, scattering over both quark and antiquark lines. Therefore, one can easily identify that these two pieces in eq. (8) are the amplitudes of two independent processes: (i) a gluon from the proton goes through the nucleus and multiple scatters before splitting into a $q\bar{q}$ pair, and (ii) the gluon splits into a $q\bar{q}$ dipole, which pierces the nucleus and multiple scatters from it.

It is interesting to remark that the amplitude matches exactly to the present one when calculated in the light-cone gauge [9].

One can then find the cross-section by squaring the amplitude, averaging over the proton and nuclear sources and finally integrating over the impact parameter. The differential cross-section for this process is then explicitly given by

$$
\frac{d\sigma^\gamma}{d^2k_{\perp\gamma}d\eta_{\gamma}} = \frac{\alpha_e^2 q^2_\perp}{16\pi^2 C_F} \int_{k_{1\perp}}^{\infty} \frac{q^+ dp^+}{p^+} \int_{k_{1\perp},k_{2\perp},q_{\perp},p_{\perp}} (2\pi)^2 \delta(2)(P_{\perp} - k_{1\perp} - k_{2\perp}) \frac{\varphi_p(k_{1\perp})}{k_{1\perp}^2 k_{2\perp}^2}
$$

$$
\times \left\{ \int_{k_{1\perp},k_{1\perp}'} \text{tr} \left[ (g + m)T_{q\bar{q}}(k_{\perp},k_{1\perp})(m - p)^0 T^\dagger_{q\bar{q}}(k_{\perp},k_{1\perp})\gamma^0 \right] \phi^{q\bar{q}}_A(k_{\perp} - k_{1\perp} - k_{2\perp}; k_{1\perp}' - k_{2\perp} - k_{1\perp}') \right. 
$$

$$
+ \int_{k_{1\perp}} \text{tr} \left[ (g + m)T_{q\bar{q}}(k_{\perp},k_{1\perp})(m - p)^0 T^\dagger_{q\bar{q}}(k_{\perp},k_{1\perp})\gamma^0 \right] \phi^{q\bar{q}}_A(k_{\perp} - k_{1\perp} - k_{2\perp}; k_{2\perp}) + h.c.
$$

$$
+ \text{tr} \left[ (g + m)T_g(k_{1\perp})(m - p)^0 T^\dagger_g(k_{1\perp})\gamma^0 \right] \phi^{g,g}_A(k_{2\perp}) \right\}.
$$

The function $\varphi_p(k_{1\perp},Y_p)$ is the unintegrated gluon distribution function of the proton and is here given by $2\pi N_C C_F g^2(\rho_p'(k_{1\perp})\rho_p'(k_{1\perp})) = \delta^{ab} k_{1\perp}^2 \varphi_p(k_{1\perp},Y_p)$. The functions $\phi^{q\bar{q}}_A, \phi^{q\bar{q}}_A$ and $\phi^{g,g}_A$ (see refs. [9, 14]) are unintegrated gluon distribution functions of the nucleus. These are the Fourier transforms of four, three and two point Wilson line correlators and correspond to all twist corrections to the gluon distributions. In these unintegrated distributions are encoded the saturation effects which should be observable in the inclusive photon cross-section.

The expression in eq. (9) can be used to calculate the correlation of a quarkonium state, like the $J/\psi$ with a single photon. The information of such a correlation function would give new and valuable information on non trivial aspects of saturation, like angular dependence of final states. However, such a measurement is unfortunately still out of reach, even at LHC energies. Nonetheless, the inclusive prompt photon cross-section has been measured in the RHIC kinematic window, and new measurements are expected at RHIC and LHC. For this, we need to integrate over the $q\bar{q}$ pair phase space to find

$$
\frac{d\sigma^\gamma}{d^2k_{\gamma\perp}d\eta_{\gamma}} = \frac{\alpha_e^2 q^2_\perp}{16\pi^2 C_F} \int_{0}^{\infty} \frac{dq^+ dp^+}{p^+} \int_{k_{1\perp},k_{2\perp},q_{\perp},p_{\perp}} (2\pi)^2 \delta(2)(P_{\perp} - k_{1\perp} - k_{2\perp}) \frac{\varphi_p(k_{1\perp})}{k_{1\perp}^2 k_{2\perp}^2}
$$

$$
\times \left\{ \int_{k_{1\perp},k_{1\perp}'} \text{tr} \left[ (g + m)T_{q\bar{q}}(k_{\perp},k_{1\perp})(m - p)^0 T^\dagger_{q\bar{q}}(k_{\perp},k_{1\perp})\gamma^0 \right] \phi^{q\bar{q}}_A(k_{\perp} - k_{1\perp} - k_{2\perp}; k_{1\perp}' - k_{2\perp} - k_{1\perp}') \right. 
$$

$$
+ \int_{k_{1\perp}} \text{tr} \left[ (g + m)T_{q\bar{q}}(k_{\perp},k_{1\perp})(m - p)^0 T^\dagger_{q\bar{q}}(k_{\perp},k_{1\perp})\gamma^0 \right] \phi^{q\bar{q}}_A(k_{\perp} - k_{1\perp} - k_{2\perp}; k_{2\perp}) + h.c.
$$

$$
+ \text{tr} \left[ (g + m)T_g(k_{1\perp})(m - p)^0 T^\dagger_g(k_{1\perp})\gamma^0 \right] \phi^{g,g}_A(k_{2\perp}) \right\},
$$

which is the final result for this process.

2 The average of a source operator $\mathcal{O}$ is given by $\langle \mathcal{O}[\rho_p,\rho_A] \rangle = \int D\rho_p D\rho_A W_p[\rho_p] W_A[\rho_A] \mathcal{O}[\rho_p,\rho_A]$. One can see that for this cross-section, the averages factor out in the proton and nuclear ones.
4. Conclusions

We have presented here our derivation for the cross-section of the process $g \rightarrow q \bar{q} \gamma$ in the CGC formalism. With the result in eq. (9) we conclude the calculation of single prompt photon production at NLO in the CGC formalism. The NLO processes studied here are sensitive to saturation, and therefore are an important complement to quarkonium production studies [15,16], especially because of appearance of the same multi-parton correlators in the cross-sections for both processes. Moreover, this study may contribute to the understanding of the excess signal of soft photons in hadron-hadron collision experiments [17,18]. Future work will be oriented to the numerical resolution of the inclusive photon cross-section.

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