THE ENVIRONMENT AND DISTRIBUTION OF EMITTING ELECTRONS AS A FUNCTION OF SOURCE ACTIVITY IN MARKARIAN 421

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Abstract

For the high-frequency-peeked BL Lac object Mrk 421, we study the variation of the spectral energy distribution (SED) as a function of source activity, from quiescent to active. We use a fully automatized $\chi^2$-minimization procedure, instead of the “eyeball” procedure more commonly used in the literature, to model nine SED data sets with a one-zone synchrotron self-Compton (SSC) model and examine how the model parameters vary with source activity. The latter issue can finally be addressed now, because simultaneous broadband SEDs (spanning from optical to very high energy photon) have finally become available. Our results suggest that in Mrk 421 the magnetic field ($B$) decreases with source activity, whereas the electron spectrum’s break energy ($\gamma_{br}$) and the Doppler factor ($\delta$) increase—the other SSC parameters turn out to be uncorrelated with source activity. In the SSC framework, these results are interpreted in a picture where the synchrotron power and peak frequency remain constant with varying source activity, through a combination of decreasing magnetic field and increasing number density of $\gamma \leq \gamma_{br}$ electrons: since this leads to an increased electron–photon scattering efficiency, the resulting Compton power increases, and so does the total (= synchrotron plus Compton) emission.

Key words: BL Lacertae objects: general – BL Lacertae objects: individual (Mrk 421) – diffuse radiation – gamma rays: galaxies

1. INTRODUCTION

It is commonly thought that the fueling of supermassive black holes hosted in the cores of most galaxies by surrounding matter produces the spectacular activity observed in active galactic nuclei (AGNs). In some cases ($\lesssim 10\%$), powerful collimated jets shoot out in opposite directions at relativistic speeds. The origin of such jets is one of the fundamental open problems in astrophysics.

If a relativistic jet is viewed at a small angle to its axis, the observed emission is amplified by relativistic beaming (Doppler boosting and aberration) allowing deep insight into the physical conditions and emission processes of relativistic jets. Sources whose boosted jet emission dominates the observed emission (blazars$^3$) represent a minority among AGNs, but are the dominant extragalactic source class in $\gamma$-rays. Since jet emission overwhelms all other emission from the source, blazars are key sources for studying the physics of relativistic extragalactic jets.

The origin and nature of jets are still unclear. However, it is widely believed that jets are low-entropy (kinetic/electromagnetic) flows that dissipate some of their energy in (moving) regions associated with internal or external shocks. This highly complex physics is approximated, for the purpose of modeling the observed emission, with one or more relativistically moving homogeneous plasma regions (blobs), where radiation is emitted by a non-thermal population of particles (e.g., Maraschi et al. 1992). The high-energy emission, with its extremely fast and correlated multi-frequency variability, indicates that often a single region dominates the emission.

The jet’s broadband (from radio to $\gamma$-ray frequencies) spectral energy distribution (SED) is a non-thermal continuum featuring two broad humps that peak at IR/X-ray and GeV/TeV frequencies and show correlated luminosity and spectral changes. This emission is commonly interpreted within a synchrotron-self-Compton (SSC) model where the synchrotron and Compton peaks are produced by the same time-varying population of particles moving in a magnetic field (e.g., Tavecchio et al. 1998, henceforth T98).

One important issue that should be addressed, but has not yet been so far because of the lack of simultaneous broadband SEDs, is how the emission changes as a function of the source’s global level of activity. In particular, given an emission model that fits the data, which model parameters are correlated with source activity should be examined. In order to investigate SEDs at different levels of activity, we choose a high-frequency-peeked BL Lac (HBL) object, i.e., a blazar (1) whose relativistic jet points directly toward the observer so, owing to relativistic boosting, its SSC emission dominates the source; (2) whose Compton peak ($\gtrsim 100$ GeV) can be detected by Cherenkov telescopes; and (3) whose GeV spectrum can be described as a simple power law (unlikely other types of BL Lac objects; see Abdo et al. 2009). In addition, such an HBL source must have several simultaneous SED data sets available. Mrk 421 meets these requirements. In this paper, we study the variation of its SED with source activity, from quiescent to active.

Another requirement for this kind of study is the modeling procedure. We use a full-fledged $\chi^2$-minimization procedure instead of the “eyeball” fits more commonly used in the literature. While the latter at most prove the existence of a good solution, by finely exploring the parameter space, our procedure finds the best solution and also proves this solution to be unique.
In this paper, we investigate the SED of Mrk 421 in nine different source states. To this aim, we fit a one-zone SSC emission model (described in Section 2), using a fully automatized \( \chi^2 \)-minimization procedure (Section 3), to the data sets described in Section 4. The results are presented and discussed in Section 5.

2. BL Lac SSC EMISSION

To describe the HBL broadband emission, we use the one-zone SSC model of T98. This has been shown to adequately describe broadband SEDs of most HBL objects (e.g., Tavecchio et al. 2010) and, for a given source, both its ground and excited states (Tavecchio et al. 2001; Tagliaferri et al. 2008). The main support for the one-zone model is that in most such sources the temporal variability is clearly dominated by one characteristic timescale, which implies one dominant characteristic size of the emitting region (e.g., Anderhub et al. 2009). Moreover, one of the most convincing points of evidence favoring the SSC model is the strict correlation that usually holds between the X-ray and very high energy (VHE) \( \gamma \)-ray variability (e.g., Fossati et al. 2008): since in the SSC model the emission in the two bands is produced by the same electrons (via synchrotron and SSC mechanisms, respectively), a strict correlation is expected.

In our work, for simplicity we use a one-zone SSC model, assuming that the entire SED is produced within a single homogeneous region of the jet. As already noted, this class of models is generally adequate to reproduce HBL SEDs. However, one-zone models also face some problems in explaining some specific features of TeV blazar emission. In particular, while very large Doppler factors are often required in one-zone models, radio very long baseline interferometry observations hardly detect superluminal motion at the parsec scale (e.g., Piner et al. 2010; Giroletti et al. 2006). This led Georganopoulos \\& Kazanas (2003) and Ghisellini et al. (2005) to propose the existence of a structured, inhomogeneous, and decelerating emitting jet. Inhomogeneous (two-zone) models (e.g., Ghisellini \\& Tavecchio 2008) have also been invoked to explain the ultra-rapid variability occasionally observed in TeV blazars (e.g., Aharonian et al. 2007; Albert et al. 2007).

The emission zone is supposed to be spherical with radius \( R \) in relativistic motion with a bulk Lorentz factor \( \Gamma \) at an angle \( \theta \) with respect to the line of sight to the observer, so that special relativistic effects are cumulatively described by the relativistic Doppler factor, \( \delta = [\Gamma(1 - \beta \cos \theta)]^{-1} \). Relativistic electrons with density \( n_e \) and a tangled magnetic field with intensity \( B \) homogeneously fill the region.

The relativistic electrons’ spectrum is described by a smooth broken power-law function of the electron Lorentz factor \( \gamma \), with limits \( \gamma_1 \) and \( \gamma_2 \), break at \( \gamma_b \), and low- and high-energy slopes \( n_1 \) and \( n_2 \). This purely phenomenological choice is motivated by the observed shape of the humps in the SEDs, well represented by two smoothly joined power laws for the electron distribution. Two ingredients are important in shaping the VHE \( \gamma \)-ray part of the spectrum: (1) using the Thomson and Klein–Nishina cross sections whenever, respectively, \( \gamma \nu c < m_e c^2 \) and \( \gamma \nu c > m_e c^2 \) (with \( \nu \) being the frequency of the “seed” photon), in building the model (T98) and (2) the correction of \( \gtrsim 50 \text{ GeV} \) data for absorption by the extragalactic background light (EBL), as a function of photon energy and source distance (e.g., Mankuzhiyal et al. 2010, and references therein); for this purpose, we use the popular Franceschini et al. (2008) EBL model.

The one-zone SSC model can be fully constrained by using simultaneous multi-frequency observations (e.g., T98). Of the nine free parameters of the model, six specify the electron energy distribution \( (n_e, \gamma_1, \gamma_b, \gamma_2, n_1, n_2) \) and three describe the global properties of the emitting region \( (B, R, \delta) \). Ideally, from observations one can derive six observational quantities that are uniquely linked to model parameters: the slopes, \( \alpha_{1,2} \), of the synchrotron bump at photon energies below and above the UV/X-ray peak are uniquely connected to \( n_1, n_2 \); the synchrotron and SSC peak frequencies, \( \nu_{b,c} \), and luminosities, \( L_{b,c} \), are linked with \( B, n_e, \delta \), and \( \gamma_b \); finally, the minimum variability timescale \( \tau_{\text{var}} \) provides an estimate of the source size through \( R \lesssim c \tau_{\text{var}}/(1+z) \).

To illustrate how important it is to sample the SED around both peaks, let us consider a standard situation before Cherenkov telescopes came online when we would have had only knowledge of the UV/X-ray (synchrotron) peak. Clearly, this would have given us information on the shape of the electron distribution but would have left all other parameters unconstrained: in particular, the degeneracy between \( B, n_e, \delta \), and \( \gamma_b \)—inherent in the synchrotron emissivity—could not be lifted without the additional knowledge of the HE/VHE \( \gamma \)-ray (Compton) peak.

Therefore, only knowledge of observational quantities related to both SED humps enables the determination of all SSC model parameters.

3. \( \chi^2 \) MINIMIZATION

In this section, we discuss the code that we have used to obtain an estimation of the characteristic parameters of the SSC model. As we recalled in the previous section, the SSC model that we assume is characterized by nine free parameters, \( n_e, \gamma_1, \gamma_b, \gamma_2, n_1, n_2, B, R, \) and \( \delta \). However, in this study we set \( \gamma_1 = 1 \), which is a widespread assumption in the literature, so this reduces the number of free parameters to eight.

The determination of the eight free parameters has been performed by finding their best values and uncertainties from a \( \chi^2 \) minimization in which multi-frequency experimental points have been fitted to the SSC model described in T98. Minimization has been performed using the Levenberg–Marquardt method (see Press et al. 1992), which is an efficient standard for non-linear least-squares minimization that smoothly interpolates between two different minimization approaches, namely the inverse Hessian method and the steepest-descent method.

The algorithm starts by making an educated guess for a starting point \( P_0 \) in parameter space with which the minimization loop is entered; at the same time a (small) constant is defined, \( \Delta \chi^2_{\text{N1}} \) (where NI stands for “Negligible Improvement”), which represents an increment in \( \chi^2 \) small enough that the minimization step can be considered to have achieved no significant improvement in moving toward the minimum \( \chi^2 \); this will be used as a first criterion for the exit condition from the minimization loop. Indeed, to be more confident that the minimization loop has reached a \( \chi^2 \) value close enough to the absolute minimum, the above condition has to be satisfied four times in a row: the number of consecutive times the condition has been satisfied is conveniently stored into the integer variable \( \text{cN1} \)—which is, thus, set to zero at startup. From the chosen value of \( P_0 \), we can compute the associated \( \chi^2_0 = \chi^2(P_0) \) and enter the minimization loop. The Levenberg–Marquardt method will determine the next point in parameter space, \( P \), where \( \chi^2 \) will be evaluated. If \( \chi^2(P) > \chi^2_0 \), the weight of the steepest-descent...

8 The rarely occurring “orphan” TeV flares, which are not accompanied by variations in the X-ray band, may arise from small, low- \( B \), high-density plasma blobs (Krawczynski et al. 2004).
method in the minimization procedure is increased, the variable $c_{\text{NI}}$ is set to zero, and we can proceed to the next minimization step. If, instead, $\chi^2(P) \leq \chi^2_0$ we further check if the decrease in $\chi^2$ is smaller than or equal to $\Delta \chi^2_{\text{NI}}$. If this is the case, the negligible-improvement counter $c_{\text{NI}}$ is increased by one: if the resulting value is $c_{\text{NI}} \geq 4$, we think we have a good enough approximation of the absolute minimum—and the algorithm ends. If, instead, in the latter test $c_{\text{NI}} < 4$, or if in the preceding test $\Delta \chi^2 > \Delta \chi^2_{\text{NI}}$, then we increase the weight of the inverse Hessian method in the minimization procedure, we set $\chi^2_0$ equal to the lower value we have just found and continue the minimization loop. For completeness and illustration, we briefly present the flow chart of the algorithm in Figure 1.

A crucial point in our implementation is that from T98 we can only obtain a numerical approximation to the SSC spectrum, in the form of a sampled SED. On the other hand, at each step of the loop (see Figure 1) the calculation of $\chi^2$ requires the evaluation of the SED for all the observed frequencies. Usually, the model function is known analytically, so these evaluations are a straightforward algebraic process. In our case, instead, we know the model function only through a numerical sample and it is unlikely that an observed point will be one of the sampled points coming from the implementation of the T98 model. Nevertheless, it will in general fall between two sampled points, which allows us to use interpolation to approximate the value of the SED.

At the same time, the Levenberg–Marquardt method requires the calculation of the partial derivatives of $\chi^2$ with respect to the eight fitted SSC parameters. Contrary to the usual case, in which all derivatives can be obtained analytically from knowledge of the model function, in our case they have also been obtained numerically by evaluating the incremental ratio of $\chi^2$ with respect to a sufficiently small, dynamically adjusted increment of each parameter. This method could have introduced a potential inefficiency in the computation, due to the recurrent need to evaluate the SED at many, slightly different points in parameter space, this being the most demanding operation in terms of CPU time. For this reason, we set up an algorithm to minimize the number of calls to T98 across different iterations.

4. DATA SETS

From the literature, we then select nine SED data sets corresponding to different emission states (low to high) of the HBL source Mrk 421.

State 1 and state 2 (Acciari et al. 2009) multi-wavelength campaigns were triggered by a major outburst in 2006 April that was detected by the Whipple 10 m telescope. A prompt campaign was not possible because of visibility constraints on XMM-Newton, so simultaneous multi-wavelength observations took place during the decaying phase of the burst. The optical/UV and X-ray observations were carried out using XMM-Newton’s optical monitor and EPIC-pn detector, respectively. The MAGIC and Whipple telescopes were used for the VHE $\gamma$-ray observations. Strictly simultaneous observations lasted ~4 hr for state 1, and more than 3 hr for state 2.

State 3 (Rebillot et al. 2006) reports multi-wavelength observations during 2002 December and 2003 January. The campaign was initiated by X-ray and VHE flares detected by the all-sky monitor of the Rossi X-Ray Timing Explorer (RXTE) and the 10 m Whipple telescope. Whipple and HEGRA-CT1 were used for the VHE observations during the campaign. Even though Whipple observed the source from 2002 December 4 to 2003 January 15 and HEGRA-CT1 on 2002 November 3 to December 12 only the data taken during nights with simultaneous X-ray observations are used in this paper to construct the SED. Optical flux is the average flux of the data obtained from the Boltwood Observatory’s 1.2 m telescope, KVA telescope, and WIYN telescope during the campaign period.

State 4 and state 9 (Blazejowski et al. 2005) observations were taken from a comparatively longer time campaign in 2003 and 2004. The X-ray flux obtained from RXTE was grouped into low-, medium-, and high-flux groups. For each X-ray observation in a given group, Whipple VHE $\gamma$-ray data that had been observed within an hour of the X-ray data were selected. State 4 (i.e., medium flux) was observed between 2003 March 8 and May 3 whereas state 9 (i.e., high flux) was observed on 2004 April 16–20. Optical data sets obtained with Whipple Observatory’s 1.2 m telescope and Boltwood Observatory’s 0.4 m telescope were also selected based on the same grouping method. The optical data measured during the whole campaign were not simultaneous with the other multi-wavelength data; however, the optical flux was not found to vary significantly during the campaign, so its highest and lowest values are taken to be reliable proxies of the actual values.

State 5 and state 7 (Fossati et al. 2008) data were taken on 2001 March 18–25 during a multi-wavelength campaign. State 7 denotes the peak of the March 19 flare, whereas state 5 denotes a post-flare state on March 22 and 23. In both cases, the X-ray and VHE $\gamma$-ray data were obtained with, respectively, RXTE and the Whipple telescope. The lowest and highest optical fluxes obtained during the whole campaign with the 1.2 m
Figure 2. Best-fit one-zone SSC models for nine data sets referring to different emission levels of the HBL source Mrk 421. Source states are ordered by increasing model luminosity and data have been obtained as follows: state 1 (Acciari et al. 2009), state 2 (Acciari et al. 2009), state 3 (Rebillot et al. 2006), state 4 (Blazejowski et al. 2005), state 5 (Fossati et al. 2008), state 6 (Acciari et al. 2009), state 7 (Fossati et al. 2008), state 8 (Donnarumma et al. 2009), and state 9 (Blazejowski et al. 2005).

Harvard-Smithsonian telescope on Mt. Hopkins were used in the SEDs for states 5 and 7, respectively.

State 6 (Acciari et al. 2009) observations were taken, using the same optical and X-ray instruments as states 1 and 2, during a decaying phase of an outburst in 2008 May. The VHE \( \gamma \)-ray data were taken with VERITAS. There are \( \sim 2.5 \) hr of strictly simultaneous data.

State 8 (Donnarumma et al. 2009) data were taken during a multi-wavelength campaign on 2008 June 6: VERITAS, RXTE and Swift/BAT, and WEBT provided the VHE \( \gamma \)-ray, X-ray, and optical data, respectively.

5. RESULTS AND DISCUSSION

To each of the data sets, we apply our \( \chi^2 \)-minimization procedure (see Figure 1). The best-fit SSC models are plotted alongside the SED data in Figure 2.

Obtaining truly best-fit SSC models of simultaneous blazar SEDs is crucial to measure the SSC parameters describing the emitting region. The obtained optimal model is proven to be unique because the SSC manifold is thoroughly searched for the absolute \( \chi^2 \) minimum. Furthermore, the very nature of our procedure ensures that there is no obvious bias affecting the resulting best-fit SSC parameters.

Nevertheless, as it has also been noted (Andrae et al. 2010), there may be caveats related with the \( \chi^2 \) fitting, especially when applied to non-linear models such as the present one. For this reason, it is important to try to understand the goodness of the fit with methods other than the value of \( \chi^2 \) (reported in Table 1). Following Andrae et al. (2010), we have applied the Kolmogorov–Smirnov (K-S) test for normality of the residuals of all our SED fits. A standard application of the test shows that in all cases the residuals are not normally distributed: the K-S test thus fails at the 5% significance level. It is, of course, crucial to understand the reason for this behavior of our SSC fits with respect to the K-S test. Let us start with some general remarks about the modeling of blazar emission and their observations.

First, the one-zone SSC model contains two distinct physical processes in the same region, i.e., synchrotron emission and its Compton up-scattered counterpart, that manifest themselves as essentially separate components at very different energies; on the other hand, additional subtle effects may enter the modeling of blazar emission, so that the SSC model may be only an approximation of the real thing. (A more refined K-S analysis suggests exactly this—see below.) Second, our blazar data sets do cover the (far apart) energy ranges spanned by, respectively, synchrotron emission and its Compton up-scattered counterpart, that manifest themselves as essentially separate components at very different energies; on the other hand, additional subtle effects may enter the modeling of blazar emission, so that the SSC model may be only an approximation of the real thing. (A more refined K-S analysis suggests exactly this—see below.) Second, our blazar data sets do cover the (far apart) energy ranges spanned by, respectively, synchrotron emission and its Compton up-scattered counterpart, that manifest themselves as essentially separate components at very different energies; on the other hand, additional subtle effects may enter the modeling of blazar emission, so that the SSC model may be only an approximation of the real thing. (A more refined K-S analysis suggests exactly this—see below.)
In Tables 1 and 2, we report the best-fit SSC parameters. Source activity (measured as the total luminosity of the best-fit SSC model) appears to be correlated with $B$, $\gamma_{br}$, and $\delta$ (see Figure 3, top)—and to be uncorrelated with the remaining SSC parameters. The bolometric luminosity used in these plots has been obtained directly from the fitted SED. In more detail, after determining the numerical approximation to the SED $\log[vF(v)]$, the parameters being fixed at their best values obtained with the previously described minimization procedure, we have performed $L = \int_{v_{\text{min}}}^{v_{\text{max}}} v F(v) dv$, with $v_{\text{min}}$, $v_{\text{max}}$ set at 2.5 decades, respectively, below the synchrotron peak and above the Compton peak. In this way, we make sure to perform the integral over all the relevant frequencies in a way that is independent from any change in the location of these peaks.

We then searched the data plotted in the top row of Figure 2 for possible correlations. The linear-correlation coefficients turn out to be 0.67, 0.64, and 0.54, which confirm linear correlations with confidence levels of 4.8%, 6.3%, and 13.3%, respectively. As an additional test (given the relatively low statistics of our data sets), we checked that the K-S test confirms normality of the fit residuals.\footnote{The same approach involving a Monte Carlo generated empirical distribution for the null-hypothesis described just above has also been used in all these cases.}

All the parameters derived through our automatic fitting procedure are within the range of SSC parameters found in the literature for HBLs in general (e.g., Tavecchio et al. 2001, 2010; Tagliaferri et al. 2008; Celotti & Ghisellini 2008) and for Mrk 421 in particular (e.g., Bednarek & Protheroe 1997; Tavecchio et al. 1998; Maraschi et al. 1999; Ghisellini et al. 2002; Konopelko et al. 2003; Fossati et al. 2008). In particular, large Doppler factors such as those derived in our extreme cases, $\delta > 50$, have been occasionally derived (e.g., Konopelko et al. 2003; Fossati et al. 2008; see also the discussion in Ghisellini et al. 2005).

As is seen from Figure 3, top, $\gamma_{br}$ and $B$ are correlated, respectively, directly (left) and inversely (right) with $L$. This may be explained as follows. An increase of $\gamma_{br}$ implies an effective increase of the energy of most electrons (or, equivalently, of the density of $\nu < \nu_{br}$ electrons). To keep the synchrotron power and peak roughly constant (within a factor of three; see Figure 2), $B$ must decrease. This improves the photon–electron scattering efficiency, and the Compton power increases. The total (i.e., synchrotron plus Compton) luminosity will be higher. So a higher $\gamma_{br}$ implies a lower $B$ and a higher emission state. The $\delta–L$ correlation (Figure 3, top middle) results from combining
Figure 3. Top: variations of the SSC parameters magnetic field $B$, Lorentz factor $\delta$, and $\gamma_{\text{br}}$ as a function of the model’s bolometric luminosity. The other SSC parameters that are left to vary ($R$, $n_e$, $\gamma_2$, $n_2$) only show a scatter plot with the luminosity. Bottom: correlations between $B$, $\delta$, and $\gamma_{\text{br}}$.

A deeper insight on emission physics can be obtained by plotting the three $L$-dependent parameters (Figure 3, bottom). The $B-\gamma_{\text{br}}$ anticorrelation, with $\Delta \log B \simeq -2 \Delta \log \gamma_{\text{br}}$ (Figure 3, bottom left), derives from the synchrotron peak, $\nu_s \propto B \gamma_s^2$, staying roughly constant (see Figure 2). For fixed synchrotron and Compton peak frequencies in a relativistically beamed emission, the $B-\delta$ relation is predicted to be inverse in the Thomson limit and direct in the Klein–Nishina limit (e.g., Tavecchio et al. 1998). Because $\nu_s$, $\nu_c$ do not greatly vary from state to state in our data (see Figure 2), the correlation in Figure 3 (bottom right) suggests that the Compton emission of Mrk 421 is always in the Thomson limit. The $\delta-\gamma_{\text{br}}$ correlation (Figure 3, bottom middle) results from a corollary of the condition of constant $\nu_s$, $\nu_c$ emitted by a plasma in bulk relativistic motion toward the observer.

Our fits show clear trends among some of the basic physical quantities of the emitting region, the magnetic field, the electron Lorentz factor at the spectral break, and the Doppler factor (see Figure 3). In particular, $B$ and $\gamma_{\text{br}}$ follow a relation $B \propto \gamma_{\text{br}}^{-2}$, while $B$ and $\delta$ are approximately related to $B \propto \delta^{-2}$.

Rather interestingly, the relation connecting $B$ and $\gamma$ is naturally expected within the context of the simplest electron acceleration scenarios (e.g., Henri et al. 1999). In this framework, the typical acceleration timescale, $t_{\text{acc}}$, is proportional to the gyroradius: $t_{\text{acc}}(\gamma) \propto r_L/c$, where $r_L = \gamma m_e c/(eB)$ is the Larmor radius. On the other hand, acceleration competes with radiative (synchrotron and inverse Compton (IC)) cooling. In Mrk 421, characterized by comparable power in the synchrotron and IC components, we can assume that $t_{\text{cool}}(\gamma) \approx t_{\text{syn}} \propto 1/\gamma B$. The maximum energy reached by the electrons is determined by setting these two timescales equal, i.e.,

$$t_{\text{acc}}(\gamma_{\text{max}}) = t_{\text{cool}}(\gamma_{\text{max}}) \rightarrow \gamma_{\text{max}} \propto B^{-1/2},$$

in agreement with the relation derived by our fit. It is therefore tempting to associate $\gamma_{\text{br}}$ with $\gamma_{\text{max}}$ and explain the $B \propto \gamma_{\text{br}}^{-2}$ relation as resulting from the acceleration/cooling competition.

If the above inference is correct, one can explain the variations of $\gamma_{\text{br}}$ as simply reflecting the variations of $B$ in the acceleration region. In turn, the variations of $B$ could be associated either with changes in the global quantities related to the jet flow or with some local process in the jet (i.e., dissipation of magnetic energy through reconnection). The second relation mentioned above, i.e., $\delta$ versus $B$, seems to point to the former possibility. Let us assume $\delta \sim \Gamma$ (since we are probably observing the Mrk 421 jet at a small angle with respect to the line of sight). A general result of jet acceleration models is that, during the acceleration phase, the jet has a parabolic shape, $R \propto d^{1/2}$ (with $d$ the distance from the central black hole), and the bulk Lorentz factor $\Gamma$ increases with $d$ as $\Gamma \propto d^{1/2}$ (e.g., Vlahakis & Königl 2004; Komissarov et al. 2007). On the other hand, if the magnetic flux is conserved, $B \propto R^{-2} \propto d^{-1}$ and thus we find $B \propto \Gamma^{-2}$. Albeit somewhat speculative, these arguments suggest that the trends displayed in Figure 3 are naturally expected in the general framework of jet acceleration.

The correlations in the bottom panels of Figure 3 seem to be tighter than those in the top panels of Figure 3. The larger scatter affecting the latter owes to the fact that the electron density, $n_e$, that also enters the definition of SSC luminosity, shows no correlation with $B$, $\gamma_{\text{br}}$, and $\delta$—hence it slightly blurs the latter’s plots with luminosity.

One further note concerns error bars. Our code returns $1\sigma$ error bars. To our best knowledge, this is the first time that
formal errors of SED fits are obtained in a rigorous way. As an example of the soundness of the method, note that the obtained values of \( \delta \) are affected by the largest errors when the distribution of the VHE data points is most irregular (e.g., states 4 and 7).

We note that the variability of Mrk 421 markedly differs from that of (e.g.) the other nearby HBL source, Mrk 501. The extremely bursting state of 1997 showed a shift of \( \nu_e \) and \( \nu_c \) by two and one orders of magnitude, respectively, suggesting a Klein–Nishina regime for the Compton peak (Pian et al. 1998). Based on the data analyzed in this paper, Mrk 421 displays (within our observational memory) a completely different variability pattern. However, one important similarity may hold between the two sources: based on eyeball-fit analysis of Mrk 501’s SED in different emission states using an SSC model similar to the one used here, Acciari et al. (2011) suggest that \( \gamma_a \) does vary with \( L \). If this correlation is generally true in blazars, the implication is that particle acceleration, providing fresh high-energy electrons within the blob, must be one defining characteristic of excited source states.

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five slightly randomized measurements around a maximum, but now the maximum of the function \(-2x^4 + 16x^2 - 14\) and with much larger uncertainties than the first five.

By minimizing \( \chi^2 \), we can fit the above set of 10 randomly generated data points versus the function \( p(x; a, b, c) \). After obtaining the desired fitted parameters, we calculate the residuals and, using the K-S test, check if they are normally distributed. The K-S test rejects normal distribution of the entire set of residuals at the 5% significance level. On the other hand, if we separately apply the K-S test to the first, i.e., \( i = 1, \ldots, 5 \), and the second, \( i = 6, \ldots, 10 \), subset of residuals, the test confirms that the residuals are normally distributed. This exactly replicates the behavior we obtained in the SED fits.

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