On the fate of Lorentz symmetry in loop quantum gravity and noncommutative spacetimes

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ABSTRACT

Motivated by the remarkable sensitivity levels of the Lorentz-symmetry tests at some presently-running and (further improved) forthcoming experiments, I attempt a general analysis of the fate of Lorentz symmetry in quantum spacetime. In particular, I analyze the deformation of Lorentz symmetry that holds in certain noncommutative spacetimes and the way in which Lorentz symmetry is broken in other noncommutative spacetimes. I also observe that discretization of areas (and/or lengths/volumes/times) does not necessarily require departures from Lorentz symmetry, just like the discretization of angular momentum in ordinary quantum mechanics does not require departures from space-rotation symmetry. This is due to the fact that Lorentz symmetry has no implications for exclusive measurement of the area of a surface, but it governs the combined measurements of the area and the velocity of a surface. In a quantum-gravity theory Lorentz symmetry can be consistent with area discretization, but only when the observables “area of the surface” and “velocity of the surface” enjoy certain special properties. I argue that the status of Lorentz symmetry in the loop-quantum-gravity approach requires careful scrutiny, since areas are discretized within a formalism that, at least presently, does not include an observable “velocity of the surface”. In general it may prove to be very difficult to reconcile Lorentz symmetry with area discretization in theories of canonical quantization of gravity, because a proper description of Lorentz symmetry appears to require that the fundamental/primary role be played by the surface’s world-sheet, whose “projection” along the space directions of a given observer describes the observable area (just like the observable “L_\perp” is the projection of the angular-momentum, a legitimate “space-vector observable” of nonrelativistic quantum mechanics, along the “x” axis of an observer), whereas the canonical formalism only allows the introduction as primary entities of observables defined at a fixed (common) time, and the observers that can be considered must share that time variable. I also comment on the measurability of lengths/areas/volumes in theories that quantize the fields, such as the metric, that describe spacetime geometry: for example, I show that the same conceptual ingredients that lead to the description of the area of a surface as a quantum operator should also motivate a reanalysis of the operative definition of area, and, even when formally allowed, area discretization might be unobservable, in some sense “hidden” behind a fundamental limit on the measurability of areas.
1 Introduction and summary

In quantum-gravity research it is not uncommon to find hints of some departure from ordinary classical Lorentz symmetry, but the associated new effects, if at all present, would be strongly suppressed by the smallness of the Planck length ($L_p \equiv \sqrt{\hbar G/c^3} \sim 1.6 \cdot 10^{-35} m$). For quite some time the assumption that they would be unobservably small has led to scarce interest in the possibility of departures from Lorentz symmetry. This state of affairs is changing as a result of the recent realization (see, e.g., Refs. [1, 2, 3, 4, 5]) that the sensitivities of experimental tests of Lorentz symmetry are reaching the point at which even Planck-length-suppressed Lorentz-violation effects could be studied. As the debate on Lorentz symmetry in quantum spacetime is gaining depth, it is emerging that the different quantum-gravity approaches may lead to a large variety of scenarios for what concerns the fate of Lorentz symmetry.

The study of this interesting issue is slowed down by technical problems (in certain quantum pictures of spacetime it is even difficult to introduce the technical tools needed for the analysis of the rules of transformation from one inertial observer to another) and by a sort of language problem. With “language problem” here I mean that there is not even a consensus on the questions to be investigated in order to establish what happens to Lorentz symmetry; moreover, while, as mentioned, many different things happen to Lorentz symmetry in different quantum-gravity approaches, usually authors refer to all these scenarios using the single (and sometimes misleading) characterization as violations of Lorentz symmetry.

While of course the issues of interest for physics are the ones concerning the physical predictions associated with Lorentz symmetry, often Lorentz transformations are introduced only as a formal property of the technical tools that are used at the level of formalism. The formal structure of the theory and the nature of the physical predictions (for what concerns Lorentz symmetry) are very simply connected in classical spacetime, but, as I shall clarify in the following, this connection can be more subtle in certain quantum pictures of spacetime.

Hoping to provide useful tools for the mentioned “language” issue, I introduce here (in Section 5) a terminology for the description of various scenarios for the fate of Lorentz symmetry in quantum spacetime. I also propose (in Section 2 and 3) a careful physical/operative definition of a classical symmetry, which applies also to theories that are not themselves classical.

On the technical side I contribute here an analysis of the fate of Lorentz symmetry in certain noncommutative spacetimes and in loop quantum gravity. The part that concerns noncommutative spacetimes is mostly a review of recent developments, which I discuss at a rather intuitive level (while the original analyses involve relatively heavy mathematics). The part on loop quantum gravity is original. Since the aspect of loop quantum gravity which I investigate is the interplay between area/volume discretization and Lorentz symmetry, most of my observations actually apply equally well to loop quantum gravity and to any other possible canonical-quantization theory of gravity which predicts area/volume discretization.

I observe (Sections 3 and 4) that discretization of length/area/volume observables does not in general require departures from ordinary classical Lorentz symmetry, just like the discretization of angular momentum in ordinary quantum mechanics (Section 2) does not necessarily require departures from space-rotation symmetry. This is due to the fact that Lorentz symmetry has no implications for exclusive measurements...
of the area of a surface, but it governs the combined measurements of the area and the velocity of a surface. In a quantum-gravity theory Lorentz symmetry can be consistent with area discretization, but only when the observables “area of the surface” and “velocity of the surface” enjoy certain special properties. I argue (Section 7) that the status of Lorentz symmetry in the loop-quantum-gravity approach requires careful scrutiny, since areas are discretized within a formalism that, at least presently, does not include an observable “velocity of the surface”. I also observe that, in order to allow the compatibility of Lorentz symmetry with area discretization, this still unknown velocity operator should turn out to have quite a few ad hoc properties. Moreover, some of my results suggest that these hypothetical properties of the surface-velocity operator might then interfere with the role that the velocity of a surface often (always?) plays in surface-area measurements. It appears likely that in the end it will turn out to be impossible to have a pair of surface-area and surface-velocity operators that on the one hand allow the compatibility of area discretization with Lorentz symmetry and on the other hand allow the identification of meaningful measurement procedures that can endow with operative meaning the area-discretization prediction but are not affected by an in-principle measurability limit (which would render discreteness unobservable, spoiling it of its tentative status as a physical prediction). I analyze a couple of measurement procedures that had been previously considered as possible ways to render operatively meaningful the concept of loop-quantum-gravity area discretization and I find that indeed (if the surface-velocity observable has the properties required to render area discretization compatible with Lorentz symmetry) they are affected by an in-principle measurability limit which renders area discreteness unobservable.

I also formulate the hypothesis that these difficulties, if at all present (their full assessment still requires additional, more refined, analyses), might not be a genuine consequence of the physical and formal intuition that motivated the loop-quantum-gravity approach, but rather an artifact of the attempt to cast that intuition in the framework of canonical quantization, which, in light of the lack of “democracy” between time and space, might be ill suited for (an extension to the quantum realm of) general relativity. In general it may prove to be very difficult to reconcile Lorentz symmetry with area discretization in theories of canonical quantization of gravity, because a proper description of Lorentz symmetry appears to require that the fundamental/primary role be played by the surface’s world-sheet, whose “projection” along the space directions of a given observer describes the observable area, whereas the canonical formalism only allows the introduction as primary entities of observables defined at a fixed (common) time, and the observers that can be considered must share that time variable.

In my analysis (Section 6) of some examples of flat noncommutative spacetimes I focus on the aspects that are significant for establishing what happens to Lorentz symmetry. In particular, I discuss the deformation of Lorentz symmetry that holds in certain Lie-algebra noncommutative spacetimes, and the violation of Lorentz symmetry that holds in canonical noncommutative spacetimes.

2 Symmetries in classical and nonclassical physics

2.1 Physical characterization of a symmetry

Symmetries in physics are of course a characteristic of our observations. Certain limitations on the variety of phenomena we observe and certain types of relations between our
observations are what we call symmetry transformations. At the level of the mathematical formalisms we use to describe our observations one represents physical symmetries through certain properties of the mathematical tools introduced in our formalisms. But, of course, the role of formalism is secondary. Symmetries are a characterization of the phenomena that we observe and a theory will enjoy those symmetries if the processes it predicts are governed by those symmetries.

The symmetries of interest in this paper are space or spacetime symmetries, namely space-rotation symmetry and Lorentz symmetry. At the most fundamental level these symmetries are characterized by the associated symmetry transformations which describe how the same physical process appears to different observers. It is at that level that one is really forced to consider space-rotation transformations and Lorentz transformations: the laws of physics describe the processes that can occur, and an important aspect of these processes is that they are “real”/“objective”, i.e. they are observed (in principle) by all observers, so there must necessarily be some rules (which are also to be seen as laws of physics) that describe how the same physical process appears to different observers.

At least for what concerns the present study the concept of “symmetry transformations” allows a rather intuitive description: it pertains the objectivity of certain entities, in spite of the fact that different observers may obtain different sets of measurement results in their characterization of these entities. For example, in theories that are space-rotation invariant one can introduce the objective/physical concept of angular-momentum vector. Different observers (observers with different orientation of their axes) will describe this objective vector in terms of different triples of measured quantities (the components of the angular momentum). If the triples of two different observers concern the same objective angular-momentum vector they must be connected by a space-rotation transformation.

Symmetry transformations govern the way in which the same process appears to different observers, but in turn the presence of some symmetries imposes constraints on the laws of physics that apply to each of these different observers. For example, in a physical world with space-rotation symmetry the fact that two different observers must make sense, in the way prescribed by space-rotation symmetry, of their common observations also imposes, by logical consistency, some corresponding constraints on the laws of physics that apply to each observer (i.e. on the processes that each observer can observe). These constraints are “space-rotation symmetry of the laws of physics written by each observer”. In sloppy but intuitive language one could say that space-rotation symmetry has two roles: (1) it governs how the same physical process appears to different observers and (2) it imposes constraints on the processes observed by each observer.

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1Here I am attempting to be careful in describing a “symmetric theory” as a theory that predicts certain types of physical processes, rather than as a theory whose mathematical tools enjoy certain types of properties. Of course, in a given class of theories one can easily identify the properties that the mathematical tools must enjoy in order to predict the types of processes required by a given symmetry. However, it is dangerous to then identify the concept of symmetry with a given set of mathematical properties: as one moves from one class of theories to another (e.g. from theories in commutative spacetime to theories in noncommutative spacetime) it is not implausible that the same mathematical properties of some of the technical tools that compose the theory would lead, for example, to different relations between the processes predicted by the theories. As I intend to emphasize in this paper, this is a delicate issue especially for theories involving a nonclassical picture of spacetime.
A physical characterization of these different roles of space-rotation symmetry will be provided in the next Subsection. Here let me be concerned with the fact that it can at times create confusion to use the same name for these different roles of symmetries. The two roles of the concept of symmetry are directly interconnected (one follows logically form the other and vice versa) but it is nevertheless useful to keep clearly separate the different logical role of the two concepts.

The first concept of symmetry pertains the description of how the same physical process appears to different observers. I shall refer to this as the case of a “passive symmetry transformation”, to emphasize that it involves a single physical process and the symmetry transformations describe how that physical process appears to different observers.

The second concept of symmetry pertains the structure of the laws of physics that govern the physics as seen by each of the observers. It restricts the class of processes that any given observer can witness and it also governs certain connections between the different processes that a given observer can witness. I shall refer to this as the action of “active symmetry transformations”, to emphasize that it pertains different processes observed by a single observer.

As mentioned, these two concepts are intimately related. However, I will give in this paper priority to passive symmetry transformations (reflecting an intuition that they might be in some sense more fundamental at the conceptual level: it is absolutely necessary to have some rules that govern how the same physical process appears to different observers).

2.2 Space-rotation symmetry in classical mechanics

Let me characterize space-rotation symmetry through an explicit example. When one observer measures one or more components of the angular momentum of a classical system, using space-rotation symmetry some facts can immediately be deduced about how that same angular momentum appears to a second observer, whose reference axes are rotated with respect to the ones of the first observer. This is basically not much more than a statement that physical processes are real/objective (in the sense intuitively introduced above), that space does not have preferred directions and that there is no preferred observer.

Let us call \((x, y, z)\) the axes of the first observer \(O\) and \((x', y', z')\) the axes of the second observer \(O'\). Focusing, for simplicity, on the example of angular momentum, I found useful to note here some characteristic properties of space-rotation symmetry:

- A physical entity whose objectivity is codified in space-rotation symmetry transformations is the angular-momentum vector. This physical entity has the primary role both in measurement and in theory. However, each observer characterizes this vector by three (real, dimensionful) measured numbers. Each of these numbers is to be seen as the projection of the objective vector along one of the axes of the observer, and, of course, in turn these axes must be physically identified by the observer. For example, an observer may choose as “\(x\) axis” the direction of a certain magnetic field, another vector, and in that case a crucial role is played by the fact that both in measurement and in theory one can meaningfully consider the projection \(\vec{L} \cdot \vec{B}\). The observable simply denoted by “\(L_x\)” in the formalism inevitably corresponds physically to an observable obtained from two objective vectors, the angular-momentum vector \(\vec{L}\) and a
second vector such as \( \vec{B} \). When the value of \( \vec{B} \) is known one can set up a measurement procedure for \( L_x \equiv \vec{L} \cdot \vec{B} \).

- When \( O \) measures the \( x \) component of the angular momentum it is still not possible to predict the components of that angular momentum along the \((x', y', z')\) axes of \( O' \), but of course the \( x \) direction is also meaningful for \( O' \) and that information is acquired also by \( O' \). [For example, if the \( z \) and \( z' \) axes coincide and an angle \( \alpha \) characterizes the rotation of \((x', y')\) with respect \((x, y)\), then \( O' \) describes the measurement done by \( O \) as a measurement of the component of angular momentum along the direction \( \cos(\alpha)x' + \sin(\alpha)y' \).]

- When \( O \) measures all three components, along the \((x, y, z)\) axes, of the angular momentum everything can be said about all of the components of that angular momentum along the \((x', y', z')\) axes of \( O' \). The triads \((L_x, L_y, L_z)\) and \((L_{x'}, L_{y'}, L_{z'})\) are of course different, but they are related by a simple rule of transformation (a space-rotation transformation).

- When \( O \) measures the modulus of the angular momentum everything can be said about how that modulus appears to a second observer: the value of the modulus is the same for both observers.

These remarks have to do with the passive space-rotation symmetry. The active symmetry (again in the angular-momentum sector) is encoded in other related properties. For example, in a world with space-rotation symmetry a collection of systems prepared in a way that does not break that symmetry will have to enjoy, as an ensemble, the same properties along any given direction (e.g. the ensemble of measurements of \( L_x \) will have to give the same result as the ensemble of measurements of \( L_y \)). Another example of manifestation of space-rotation symmetry within the class of processes observed by a single observer is the fact that the total angular momentum of an isolated system does not change in time (space-rotation symmetry imposes a constraint on the physical processes observed by a single observer by disallowing processes in which the total angular momentum of an isolated system is not a constant of time evolution).

With time physicists have learned that these physical properties of active space-rotation-symmetry transformations and passive space-rotation-symmetry transformations are predicted by mathematical theories that involve the angular momentum vector in a certain technical way (e.g. relying on Hamiltonians that enjoy the technical/mathematical property which is known as “invariance under space-rotation transformations”).

### 2.3 Space-rotation symmetry in nonrelativistic quantum mechanics

Space-rotation symmetry is a classical continuous symmetry. Being a classical symmetry it may appear not obvious that it can be preserved upon quantization. However, ordinary non-relativistic quantum mechanics “lives” in the same spacetime as classical non-relativistic quantum mechanics. One quantizes the entities that “live” in spacetime, but spacetime is still classical. It is therefore not unplausible (and not even surprising) that one might introduce new, non-classical, rules of mechanics without modifying the classical space-rotation symmetry. One might, at first sight, be skeptical that some rules of mechanics that discretize angular momentum could enjoy a
continuous symmetry, but more careful reasoning will quickly lead to the conclusion that there is no \textit{a priori} contradiction between discretization and a continuous symmetry. In fact, as I emphasize below, the type of discretization of angular momentum which emerges in ordinary non-relativistic quantum mechanics is fully consistent with classical space-rotation symmetry.

This concept of a non-classical (\textit{e.g.} quantum) theory that enjoys classical symmetries will be more carefully introduced in Subsection 2.5. The point will be that I propose to accept that a quantum theory enjoys the classical symmetries when all the measurements that the quantum theory still allows are still subject to the rules imposed by the classical symmetry. Certain measurements that are allowed in classical mechanics are no longer allowed in quantum mechanics, and on those measurements it will of course be impossible to verify the validity of a symmetry in a quantum-mechanical theory; however, this does not amount to a violation of the classical symmetry, but merely to a reduction in the number of testable predictions of the symmetry. For the scopes of my analysis, pertaining to classical symmetries, it is convenient to reserve the term “measurement” to the situation in which the information extracted from the system is essentially classical, so in quantum mechanics I will be focusing on eigenstates of the observable under consideration. Besides the action of the classical symmetry on eigenstates, in some cases (when not dealing with eigenstates or when concerned with more than one observable, not mutually commuting) I will also consider the action of the classical symmetry on the expectation values of relevant observables. Again I will insist that on these expectation values the classical symmetry acts just as it acts on expectation values in classical contexts in which (in spite of the lack of an in-principle obstruction) one ends up not acquiring full information on the observables of interest.

This definition of classical symmetry applies in particular to certain familiar systems of ordinary quantum mechanics which enjoy classical space-rotation symmetry (these systems are already described in the literature as space-rotation symmetric systems). My definition of classical symmetry will allow me, in the later sections, to differentiate between quantum pictures of spacetime that do enjoy certain classical spacetime symmetries and pictures that do not (not even in the sense that certain systems of ordinary quantum mechanics enjoy space-rotation symmetry).

It is useful to note here certain properties that characterize the presence of classical space-rotation symmetry in ordinary nonrelativistic quantum mechanics:

- As in classical mechanics, space-rotation symmetry transformations endow the angular-momentum vector \( \vec{L} \) with physical reality/objectivity. The formalism refers most primitively to \( \vec{L} \) and it makes connection with the components of \( \vec{L} \) which observers can measure through projections completely analogous to the ones relevant in classical mechanics, such as \( L_x \equiv \vec{L} \cdot \vec{B} \).
- Just as in classical mechanics, when \( O \) measures the square-modulus \( L^2 \) of the angular momentum, \textit{everything} can be said about how that square-modulus appears to a second observer: the value of the modulus is the same for both observers. It happens to be the case that the values of \( L^2 \) are constrained by quantum mechanics on a discrete spectrum, but this of course does not represent an obstruction for the action of the continuous symmetry on invariants, such as \( \tilde{L}^2 \).
- When \( O \) measures the \( x \) component, \( L_x \), of the angular momentum it is still not possible to predict the value of any of the components of that angular momentum along the \((x', y', z')\) axes of \( O' \). This is true at the quantum level just as much as it is true at the classical level. This is another example of situation in which the fact that
quantum mechanics constrains the values of an observable, $L_x$, on a discrete spectrum is irrelevant for our symmetry considerations, since the relevant symmetry does not prescribe how that same observable is seen by another observer. (Note that if another mechanical theory, clearly different from quantum mechanics, allowed simultaneous eigenstates of $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$ and predicted discrete spectra for all of them, then the classical continuous space-rotation symmetry would inevitably fail to apply.)

- Let me make one more remark on the case in which $O$ measures the $x$ component, $\hat{L}_x$, of the angular momentum. Although nothing can be said about any of the components of that angular momentum along the $(x', y', z')$ axes of $O'$, of course the $x$ direction is also meaningful for $O'$ and that information is acquired also by $O'$. For example, the fact that a system is in an eigenstate of $\hat{L}_x$ (the component of $\vec{L}$ along the $x$ axis of a certain observer) is an objective fact that affects the observations on that system by all observers, although for some observers it will require a complicated description in terms of their natural axes. For example, if the $z$ and $z'$ axes coincide and an angle $\alpha$ characterizes the rotation of $(x', y')$ with respect $(x, y)$, then the observer $O'$ would describe an eigenstate of $\hat{L}_x$ as an eigenstate of the component of angular momentum along the direction $\cos(\alpha)\vec{x'} + \sin(\alpha)\vec{y'}$. The fact that the statement “the system is in an eigenstate of $\hat{L}_x$” must be true for all observers (or true for none) is obvious logically. Think for example of a beam of particles prepared in certain ways and then sent through a Stern-Gerlach device: if one observer sees that the way in which the beam was prepared selects eigenstates along a certain specific direction identified by the Stern-gerlach device, then all other observers will have to agree on that statement (they will also see the corresponding special behaviour of that beam going through the Stern-Gerlach device, although they might describe it in a slightly more complicated way when making reference to their own preferred axes of reference).

- In classical physics space-rotation symmetry also governs the relation between the triple measurement $(\hat{L}_x, \hat{L}_y, \hat{L}_z)$ made by $O$ and the corresponding measurement of $(\hat{L}_{x'}, \hat{L}_{y'}, \hat{L}_{z'})$ made by $O'$. [More precisely it imposes that when $O$ attributes to a system angular momentum with components $(\hat{L}_x, \hat{L}_y, \hat{L}_z)$ then $O'$ assigns to that same system angular-momentum components $(\hat{L}_{x'}, \hat{L}_{y'}, \hat{L}_{z'})$, where $(\hat{L}_{x'}, \hat{L}_{y'}, \hat{L}_{z'})$ is the appropriate $(O \rightarrow O')$ rotation of $(\hat{L}_x, \hat{L}_y, \hat{L}_z)$.] This statement is neither true nor false in quantum mechanics. In fact, quantum mechanics excludes the possibility of simultaneous classical/sharp measurement of all components of angular momentum.

This prediction of the classical symmetry is not verifiable in ordinary quantum mechanics, but it would be improper to say that it fails. My definition of a classical symmetry in nonclassical theories will allow for these situations: the theory can still enjoy a classical symmetry even though some of the predictions of the symmetry cannot be tested because of in-principle obstructions present in the nonclassical theory. However, for the predictions that can be tested there cannot be “anomalies”: the classical symmetry will hold in the nonclassical theory only if all of its predictions that are still testable turn out to be still fully successful (and successful in the same

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\(^2\)Of course, only the properties of generic eigenstates are of interest here. The fact that one could have an eigenstate with $L_x = L_y = L_z = 0$, in the special case $L^2 = 0$, has no implications for my argument. Also note that the condition $L_x = L_y = L_z = 0$ does not involve the discretization scale $\hbar$ and is space-rotation invariant both at the classical and the quantum level $(L_x = L_y = L_z = 0 \rightarrow L_{x'} = L_{y'} = L_{z'} = 0)$. 

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sense as they are in the corresponding classical limit). Moreover, a necessary condition for space-rotation invariance is that on expectation values the symmetry must behave as expected: in a generic state of the system (or ensemble of systems) the expectation values \(\langle \hat{L}_x' \rangle, \langle \hat{L}_y' \rangle, \langle \hat{L}_z' \rangle\) should be related to the expectation values \(\langle \hat{L}_x \rangle, \langle \hat{L}_y \rangle, \langle \hat{L}_z \rangle\) by the relevant space rotation. This is indeed what happens in those ordinary quantum-mechanical frameworks which are described in the literature as enjoying space-rotation symmetry.

- One final remark that I find useful to make on the space-rotation classical symmetry of certain ordinary nonrelativistic quantum-mechanical systems concerns the role that the active space-rotations have in those theories. All previous remarks focused on “passive symmetry transformations”, i.e. how the same process/property appears to different observers. Now let me stress that a single observer can of course test the presence of (active) space-rotation symmetry. In particular, just as in happens at the classical level, the total angular momentum of an isolated system is a constant of motion within ordinary quantum mechanics (the commutator between the Hamiltonian and the total angular momentum is zero). As another example of an “active” role for space-rotation symmetry, let us consider the Stern-Gerlach device and imagine that an observer performs the first ever Stern-Gerlach experiment on a beam prepared in such a way that it has no preferred axis. That observer finds evidence of a discrete spectrum of a specific form (in that case a very simple form). A necessary condition for (active) space-rotation symmetry is that upon repeating the experiment after rotating arbitrarily the Stern-Gerlach device the same type and form of discretization is found again. The fact that a single observer cannot identify a preferred direction is a necessary condition for (active) space-rotation symmetry. This condition is clearly satisfied in ordinary quantum-mechanical systems. The careful reader will easily deduce the simple relation between these properties under active symmetry transformations and their counterparts for “passive” symmetry transformations.

### 2.4 More on the \((L_x, L_y, L_z)\) measurement

The characterization of space-rotation symmetry within ordinary quantum mechanics provided in the previous Subsection is sufficient for the purposes of the present analysis. However, my characterization of a classical symmetry (inside and outside classical physics) intends to be useful also for future studies, particularly future studies considering the fate of classical Lorentz symmetry in quantum spacetime. In this respect it may prove useful to pause here for a more careful analysis of the triple measurement \((L_x, L_y, L_z)\) within classical space-rotation symmetry.

In this Subsection I focus, for simplicity, on two observers \(O\) and \(O'\) with common orientation of the \(z\) axis and with a relative angle \(\alpha\) rotation of the axes on the \(x, y (x', y')\) plane.

As mentioned, within classical physics space-rotation symmetry transformations govern the map between a measurement \((L_x, L_y, L_z)\) made by \(O\) and the corresponding measurement of \((L_x', L_y', L_z')\) made by \(O'\). A classical beam of classical particles which is prepared in such a way that they all have the same values of \(L_x, L_y, L_z\) when studied with respect to axes \(x', y', z'\) (the natural axes of observer \(O')\) will be found to be a beam in which all particles have the same values of \(L_x', L_y', L_z'\), with \((L_x', L_y', L_z')\) being the appropriate rotation of the original triple \((L_x, L_y, L_z)\), which for the chosen pair of
would have not been possible for $L$ far from being a gaussian: it assigns nonvanishing probabilities only at work-ings of space-rotation symmetry in ordinary quantum mechanics, an important role is clearly played by the size of $\sigma$ and $L$ governed by the symmetry, like (continuous classical symmetries it is not only necessary that combinations of observables take arbitrary value within a relevant range.

As emphasized above, ordinary quantum mechanics is an example in which this aspect of space-rotation symmetry cannot be tested. The simultaneous measurement of $L_x$, $L_y$, $L_z$ is not allowed by the laws of quantum mechanics. It is not through this triple measurement $(L_x, L_y, L_z)$ that one can find space-rotation symmetry to fail (or succeed). However, the logical structure of the role of space-rotation symmetry (especially since it is a continuous symmetry) in quantum mechanics is manifest in the specific way in which quantum mechanics imposes limitations on the simultaneous measurability of $L_x$, $L_y$, $L_z$. Consider a beam of particles which have been prepared in an eigenstate of $\hat{L}^2$ and $\hat{L}_x$ and are found to all have $L^2 = 3\hbar/4$ and $L_x = \hbar/2$, where $x$ identifies the $x$ axis of observer $O$. Quantum mechanics also predicts that when $L_x$ is fully known, the values of $L_y$ and $L_z$ must be affected by a large uncertainty. The observer $O'$ could measure $L_{x'}$, finding that the same beam does not correspond to an eigenstate of $\hat{L}_{x'}$, but rather $L_{x'}$ takes values $\hbar/2$ and $-\hbar/2$ with a certain probability within the beam. This probability distribution will be characteristic of the fact that the beam is described by an eigenstate with $L_x = \hbar/2$. For example, for small angles $\alpha$ positive values $(\hbar/2)$ of $L_{x'}$ will dominate on the negative values $(-\hbar/2)$, and overall $< \hat{L}_{x'} > = \cos(\alpha)\hbar/2$.

In the narrow context here considered (and only in a handful of similar contexts) one could roughly describe in classical-physics language the prediction of ordinary quantum mechanics. Ordinary quantum mechanics roughly states that a beam of particles can be characterized by a common value of, say, $\hat{L}^2$ and $L_x$ but then inevitably $L_y$ will vary within the beam in totally random manner. Eigenstates of $\hat{L}^2, \hat{L}_x$ with $L^2 = 3\hbar/4$ and $L_x = \hbar/2$ are states in which $L_y$ is undetermined but $< L_y > = 0$ and $\sigma^2_{L_y} \equiv < L_y^2 > - < L_y >^2 = \hbar^2/4$. This point clearly plays a key role in the consistency between the discretization of angular momentum predicted by quantum mechanics and the space-rotation symmetry of quantum mechanics. For example even for small $\alpha$ in a rich beam (a beam with an infinite number of particles) there will be some small percentage of particles whose $L_{x'}$ is found to be negative, $L_{x'} = -\hbar/2$. Since $L_{x'} = \cos(\alpha)\hat{L}_x + \sin(\alpha)\hat{L}_y$, $< L_y > = 0$, and $L_x = \hbar/2$ the small percentage of particles found to have $L_{x'} = -\hbar/2$ are manifestation of the quantum-mechanical probability distribution which is strongly characterized by $\sigma_{L_y} = \hbar/2$ (although it is far from being a gaussian: it assigns nonvanishing probabilities only at $L_{x'} = \hbar/2$ and $L_{x'} = -\hbar/2$). In this entire probabilistic description, which is at the core of the workings of space-rotation symmetry in ordinary quantum mechanics, an important role is clearly played by the size of $\sigma_{L_y}$. If $\sigma_{L_y}$ had taken value, say, $\sigma_{L_y} = \hbar/1000$ it would have not been possible for $< \hat{L}_{x'} >$ to take the value $< \hat{L}_{x'} > = \cos(\alpha)\hbar/2$.

This allows us to deduce that in order for discretization to be compatible with continuous classical symmetries it is not only necessary that combinations of observables governed by the symmetry, like $(L_x, L_y, L_z)$, should not be measurable simultaneously:

\[3\text{Here I am taking abundant liberty in adopting a classical language, but the probabilistic considerations, and the role of the } \sigma_{L_y} \text{ uncertainty in those consideration, are appropriate.}\]
it is also necessary that the eigenstates of one of the relevant observables, say $\hat{L}_z$, be characterized by an appropriately large uncertainty in the other relevant observables ($\hat{L}_y$ and $\hat{L}_z$).

### 2.5 Classical symmetries in any (classical or non-classical) theory

The observations reported in the preceding Subsections are the basis for my definition of the presence of a classical symmetry in a non-classical, \textit{e.g.} quantum, theory. The role that classical space-rotation symmetry plays in certain contexts of ordinary nonrelativistic quantum mechanics will be my prototype for the role that a classical symmetry should play in a non-classical theory in order for us to state that the symmetry holds. The fact that my definition of classical symmetry applies (by construction) to the case of space-rotation symmetry in ordinary quantum mechanics assures me of the fact that the definition is not purposeless. In contexts in which a classical symmetry characterizes observations governed by a nonclassical theory “less strongly” than in the case of space-rotations in ordinary quantum mechanics it is appropriate to state that the classical symmetry is (perhaps partly or softly) violated: we should reserve the name “classical symmetry” to contexts in which the symmetry characterizes observations as strongly as in the remarks made in the Subsection 2.3.

By stating that the role that classical space-rotation symmetry plays in certain contexts of ordinary nonrelativistic quantum mechanics is my prototype for the role that a classical symmetry should play in a non-classical theory I have provided a definition which should be clear to the careful reader. It is nevertheless useful to stress here some of the points that emerged in the preceding subsections.

The basic point is that the operation of measuring one, two or more observables will always end up giving some main estimate of the observables and some uncertainties. For example, in the case of two observables, $R$ and $S$, the measurement result would be of the type $R = R_0 \pm \delta R$, $S = S_0 \pm \delta S$. The action of the classical symmetry should not be affected by the nature of the uncertainties $\delta R$, $\delta S$: the classical symmetry acts in the same way independently of whether the uncertainties are “fundamental” (due to a quantum-mechanical uncertainty principle) or due to technological/practical limitations. The most significant features of the classical symmetry emerge by considering the case $\delta R = \delta S = 0$, which is at least available (in principle) in the classical-theory limit. If the $(R,S)$ measurement is meaningful for the symmetry, when a given $(R,S)$ measurement procedure on a given system gives result $\delta R = \delta S = 0$, $R = R_0$, $S = S_0$ (a “sharp” measurement) for observer $O$, that same measurement procedure on that same system should give result $\delta R' = \delta S' = 0$, $R = R_0'$, $S = S_0'$ for observer $O'$, where $(R_0',S_0')$ is related to $(R_0,S_0)$ by the relevant $O \to O'$ symmetry transformation. If according to $O$ the measurement procedure is affected by non-zero uncertainties $R = R_0 \pm \delta R$, $S = S_0 \pm \delta S$, then according to $O'$ the measurement procedure still gives result with $R = R_0'$, $S = S_0'$ but of course also $O'$ finds non-zero uncertainties $\delta R'$, $\delta S'$. $\delta R',\delta S'$ is related to $\delta R,\delta S$ in a way affected by the structure of the symmetry transformations but the relation is not independent of the theory one is considering (in fact, the reader will be in a position to appreciate fully the strong sense in which I intend the statement “that same measurement procedure on that same system” after reading Subsections 3.3, 3.4 and 3.5.)
as clarified in Subsection 2.4, the relation also depends on the structure of the probability distributions attributed to uncertainties in the theory), whereas of course the relation between \((R_0, S_0')\) and \((R_0, S_0)\) is fully specified by the symmetry, independently of whether the theory is classical or non-classical.

It will be easy for the careful reader to verify that the properties described in the previous long paragraph are satisfied by classical space-rotation symmetry both in classical and in quantum mechanics. The properties I stated apply if, as mentioned, the \((R, S)\) measurement is “meaningful for the symmetry”. Of course, I describe as meaningful for the symmetry a measurement for which the symmetry makes definite predictions. The measurement of \(L^2\) and the measurement of \((L_x, L_y, L_z)\) are examples of measurements that are meaningful for space-rotation symmetry, while the measurements in which one only measures \(L_x\) are not meaningful for space-rotation symmetry. As clarified in the preceding Subsections, it is not an accident that the \(L^2\) measurement can be sharp (both in classical and) in quantum mechanics, since \(L^2\) is an invariant of space rotations and its discretization will therefore not interfere with continuous space-rotation symmetry transformations. As also clarified in the preceding Subsections, it is not an accident that the \((L_x, L_y, L_z)\) measurement cannot be sharp in quantum mechanics, since \((L_x, L_y, L_z)\) is not an invariant of space rotations and its discretization would have interfered with continuous space-rotation symmetry transformations. The fact that the \(L_x\) measurement can be sharp in quantum mechanics of course bears no relevance for the fate of classical space-rotation symmetry, since the measurement of \(L_x\) is not meaningful for space-rotation symmetry: the knowledge of \(L_x\) does not allow to establish anything about \(L_x', L_y'\) and \(L_z'\), independently of whether or not space-rotation symmetry is present.

Although the careful reader may find it redundant, for the benefit of leasurly readers let me stress a point about eigenstates (which is however already implicit in the remarks provided above and is therefore indeed redundant). If the \((R, S)\) measurement is meaningful for the symmetry and the theory allows the sharp measurement of \((R, S)\) (i.e. the theory allows \(\delta R = \delta S = 0\)), then the symmetry predicts without room for arbitrariness that a system measured to have \(\delta R = \delta S = 0, R = R_0, S = S_0\) for observer \(O\) must have \(\delta R' = \delta S' = 0, R = R_0', S = S_0'\) for observer \(O'\). Otherwise the non-classical theory would be allowed to violate a prediction of the classical symmetry: two observers would be analyzing the same measurement procedure and find results for \((R, S)\) that are not directly connected by the symmetry. This should not be allowed if the classical symmetry does hold in the non-classical theory, and in fact it does not happen in ordinary quantum mechanics, where classical space-rotation symmetry does hold. In the language of quantum mechanics this can be described with the statement that “eigenstates of a combination of observables \((R, S)\) which is meaningful for the symmetry must be mapped by the symmetry into states which are \((R', S')\) eigenstates”, as indeed it happens to eigenstates of \(L^2\) in ordinary quantum mechanics.

On all measurements that can be performed in a “classical sense” (e.g. by preparing/observing a suitable eigenstate, or a suitable ensemble of eigenstates) the symmetry acts just as in the classical limit: the relation between the values assigned to observables of a given system by two different observers is governed by the classical symmetry. The nonclassical theory can limit the types of “classical measurements” that can be performed (e.g. in quantum mechanics the simultaneous classical/sharp measurement of \(L_x, L_y, \) and \(L_z\) is excluded). This will not be described as a failure of the classical symmetry; however, in these cases the presence of the classical symmetry should be
reflected at the level of expectation values. This is here stated in the same sense that in ordinary quantum mechanics, as emphasized in Subsection 2.3, classical space-rotation symmetry does connect the expectation values \(< L_x' >, < L_y' >, < L_z' >\) and the expectation values \(< L_x >, < L_y >, < L_z >\) in a generic state of the system.

The analysis reported in the preceding subsections allows us to describe in general terms what are the conditions for compatibility between the presence of a classical continuous symmetry (as here defined) and the emergence of “discrete spectra” (the case in which the nonclassical theory predicts that the outcome of certain measurement procedures, the ones providing the operative definition of one of the mathematical “observables” in the formalism, can only take certain discretized values). **Discretization is consistent with the classical continuous symmetry** when it concerns observables which are invariants of the classical-symmetry transformations, and, of course, also when it concerns observables on which the continuous classical symmetry makes no prediction. A conflict emerges only when the discretized observable is directly governed by the symmetry and the symmetry predicts a continuous change of that observable in going from one observer to another. As discussed in Subsection 2.3, \(L_x\) has a discrete spectrum in quantum mechanics, but the knowledge of \(L_x\) is not governed by space-rotation symmetry (e.g., the knowledge of \(L_x\) does not allow to predict the value of \(L_{x'}\) in a classical theory with space-rotation symmetry and still does not allow to predict \(L_{x'}\) in quantum mechanics). Also \(L^2\) has a discrete spectrum in quantum mechanics and the space-rotation symmetry does govern the knowledge of \(L^2\); however, space-rotation symmetry prescribes \(L^2 = L'^2\) which is consistent with discretization (on \(L^2\) the continuous symmetry transformations we call space rotations act trivially, \(L^2\) is an invariant).

A key point for some of the considerations that are reported in the following is the fact that spacetime symmetries basically introduce some objective entities, which however lead to measurement results which are not the same for all observers: in a space-rotation-invariant world all observers, independently of the orientation of their respective \(x, y, z\) axes, agree on the angular-momentum vector of a given system, but the triple of measured numbers that each observer attributes to that angular momentum depends on the observer. The theory and the measurement procedures must make most fundamentally reference to the angular momentum vector, which is the objective entity, and its components will be identified through some other physical vectors parallel to the axes of the observer (for example, a magnetic field). These remarks apply equally well to classical mechanics and quantum mechanics; the only difference is that quantum mechanics imposes some limitations on the accuracy by which one can “measure the vector” (measure its three components for a given observer) and predicts a discretization of certain measurement results.

I close here my characterization of the presence of a classical symmetry in a non-classical theory. My characterization focuses on “passive” symmetry transformations, but the implications for active symmetry transformations can be easily deduced.

### 3 Lorentz symmetry

#### 3.1 Passive Lorentz-symmetry transformations

The discussion of space rotations, on which most of the previous Section focused, is extremely simple (so intuitive that some statements here reported for completeness
should have appeared obvious to most readers) and therefore ideally suited for the introduction of the concept of classical symmetry in a nonclassical theory that I am advocating. The main objective of this paper is however an analysis of the fate of classical Lorentz symmetry in quantum spacetime.

Such an analysis of Lorentz symmetry, could have been seen as merely academic until only a few years ago, but it should be now perceived as a high-priority objective, in light of the remarkably improved sensitivity of ongoing and forthcoming experiments, which could be sufficient [1, 2, 3, 4, 5] to detect even tiny, Planck-length suppressed, deviations from ordinary Lorentz symmetry. We even already have some tantalizing experimental hints [5], especially in the context of certain puzzling observations [6] of ultra-high-energy cosmic rays, which could be interpreted as manifestations of a Planck-length induced deviation from ordinary Lorentz symmetry.

At the conceptual level, while the analysis of space-rotations in ordinary quantum mechanics is completely elementary, the outcome of analyses of Lorentz symmetry in quantum spacetime is not at all a priori obvious. The point is that both space-rotations symmetry and Lorentz symmetry are most fundamentally properties of classical space/spacetime. Ordinary quantum mechanics, just like classical mechanics, lives in the arena provided by classical spacetime, and therefore, as long as the new rules of mechanics do not explicitly break the spacetime symmetry (and the rules of ordinary quantum mechanics do not), it is not surprising that the classical symmetries of classical flat spacetime survive that type of quantization. But quantum-gravity research is encouraging many scientists to consider one form or another of quantization of spacetime itself, so spacetime itself changes and one can expect that in general also its symmetry properties will change, as I shall show to be the case in some examples considered in this paper.

The focus on (global) Lorentz symmetry is justified by our capability to test it. Quantum-gravity research is mostly occupied with spacetimes which are far from being flat, but these theories must have a zero-curvature limit and it is that limit which we can test most accurately in ongoing and planned experiments. Think for example of approaches to the quantum-gravity problem that rely on noncommutative geometry: the most interesting formal work done on these approaches concerns non-flat spacetimes; however, if at the fundamental level spacetime geometry is proven to be noncommutative this should in particular apply to the physical contexts in which we basically deal with flat spacetime. There will be a noncommutative version of Minkowski spacetime. The symmetry properties of this noncommutative Minkowski spacetime are very significant, since they can be tested very accurately.

But let us proceed step-by-step. First we need to list a few characteristic properties of classical Lorentz symmetry.

- The entities which Lorentz symmetry endows with objective/physical/observer-independent existence are four-vectors (tensors,...) and world-volumes (world-lines, world-sheets,...). The energy-momentum four-vector is “the same” for all observers, although each observer will describe its components in a different way. Lorentz-symmetry transformations will govern the relation between the components of the energy-momentum four-vector for different observers, basically stating indeed that those different results of measurements actually describe the same objective energy-momentum. Similarly the world-sheet spanned, for example, by a physical surface is an objective entity. A given observer (with a given space/time foliation) can describe such a world-sheet as a collection of equal-time surfaces, and at each time instant...
can attribute to the surface a velocity $V$ and an area $A$. The collection of $(V,A)$ as functions of time are different for different observers, but when they refer to the same world-sheet (the same physical surface) Lorentz-symmetry transformations connect the values $(V,A)$ measured by one observer with the values $(V',A')$ measured by another observer.

- Composition of velocities. Consider a particle which, according to observer $O$, has velocity $\vec{V}$. Lorentz symmetry governs the relation between $\vec{V}$ and the velocity $\vec{V}'$ that another observer $O'$ will measure for that same particle, if the relative $O-O'$ velocity is known.

- Time dilatation. Consider a muon which moves at speed $V$ with respect to observer $O$, and $O$ measures the decay time $\tau$ of the muon. Lorentz symmetry governs the relation between $\tau$ and the decay time $\tau'$ that another observer $O'$ will measure for that same muon, if the relative $O-O'$ velocity is known.

- Length contraction. Consider a thin straight bar (a collection of particles in rigid motion) which moves at speed $V$ with respect to observer $O$. $O$ measures the length $L$ of the bar. Lorentz-symmetry transformations govern the relation between $L$ and the length $L'$ that another observer $O'$ will measure for that same bar, if the relative $O-O'$ velocity is known. This remark also applies in particular to wavelengths. It also affects in an obvious way the contraction of areas and volumes.

- Kinematical thresholds. Consider the situation in which observer $O$ has two ideal photon lasers such that the energy of the emitted photons can be tuned with arbitrarily high accuracy over an extremely wide range of energies. $O$ points the two lasers one toward the other, in order to study head-on collisions, keeps one of the lasers tuned at a fixed small energy $\epsilon$ and increases the energy of emission of the second laser gradually from zero up to the value $E$ (the threshold energy) for which some production of electron-positron pairs starts to occur. Lorentz symmetry also governs the way in which this threshold arrangement of the experimental setup appears to a second observer $O'$ moving with respect to $O$ at some speed $V_0$ along the axis of the collision. Specifically, for known $V_0$, Lorentz symmetry governs the relation between $\epsilon,E$ and the corresponding energies $\epsilon',E'$ that $O'$ measures as emission energies of the lasers at the given threshold condition realized by $O$ (I am, for simplicity, assuming that only $O$ is allowed to tune the lasers). In addition, Lorentz symmetry predicts that, independently of $V_0$, the product $\epsilon.E$ will have the same value as the product $\epsilon'.E'$ and these products will give the square of the electron mass: $\epsilon'.E' = \epsilon.E = m_e^2$.

3.2 Active Lorentz-symmetry transformations

The list of characteristic properties of classical Lorentz symmetry in the preceding Subsection mainly focused on “passive” Lorentz-symmetry transformations (however, the careful reader will notice that the description of the kinematical thresholds involved both active and passive Lorentz transformations.) Let me mention here a couple of examples of active roles for Lorentz symmetry. The wavelength independence of the speed of light (and the associated form of the photon dispersion relation, $E^2 = c^2p^2 + c^4m^2$) is a prediction associated with active Lorentz-symmetry transformations since

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5The way in which we establish experimentally kinematical thresholds does not follow the schematization adopted here for simplicity. In particular, the ideal lasers I consider are not available to us. However, the basic point is correctly portrayed by my simplified scheme.
it is a relation between the results of different measurements done by a single observer (speed-of-light measurements at different energies: all photons have the same speed independently of energy).

Another example in which active Lorentz-symmetry transformations play a role is the muon decay time, already mentioned above for what concerns passive Lorentz-symmetry transformations. The fact that the same muon has different decay times for different observers (and the transformation rules that connect those time measurements) is a prediction associated with passive Lorentz-symmetry transformations. The fact that the two muons with different energy (for a given observer \( O \)) will have decay times that typically (on average, see below) differ by amounts dictated by Lorentz-symmetry transformations is a prediction associated with active Lorentz-symmetry transformations.

It is particularly clear in the case of the muon decay time that the two manifestations (active and passive) of Lorentz symmetry are deeply and simply connected. But the muon decay time also allows us to point out a certain difference between active and passive symmetry transformations. In fact, the muon lifetime has a “statistical” component, e.g., a muon at rest “lives” on average \( 2.2 \cdot 10^{-6} \) s, but in an ensemble of muons at rest some live shorter than \( 2.2 \cdot 10^{-6} \) s, some live longer. In the situation I described, an observer measuring the decay times of two muons with different energies (and therefore different speeds), Lorentz symmetry cannot predict exactly what is the relation between the two decay times. However, if a single observer has a large number of muons at energy \( E \) and another large number of muons at energy \( E' \) she will be able to see that Lorentz symmetry transformations predict accurately the relation between the average lifetimes of the two groups of muons. When a single muon is available and it is observed by two observers the situation is slightly different: for one observer (which could be the rest-frame observer) the given muon will “live” a certain time \( t^* \) which might or might not coincide with the lifetime \( \tau \), according to the other observer, if indeed Lorentz-symmetry transformations apply, that same muon will “live” a corresponding time \( t'^* \). There is of course no statistical consideration that applies to the context of a single muon observed by two observers, while statistical considerations do play a role when comparing the lifetimes of two muons being observed by a single observer.

The list of examples of instances in which active or passive Lorentz-symmetry transformations play a role is endless. The examples I discussed probably illustrate a wide enough ensemble of situations. But, before turning to the fate of Lorentz symmetry in quantum spacetime, it is perhaps useful to describe in greater detail certain features of passive Lorentz-symmetry transformations. This is done in the next three Subsections.

### 3.3 Time dilatation

In order to illustrate the way in which passive Lorentz-symmetry transformations govern the rules of time dilatation it is sufficient to analyze a simple clock (for special-relativity experts this analysis is by now a textbook exercise, but I repeat it here since it is useful for one of the points I intend to raise about Lorentz symmetry). Let us consider two observers, \( O \) and \( O' \), each with its own spaceship, in a situation such that the relative position and the relative velocity of the spaceships are both pointing in the same direction (a configuration which is effectively one-dimensional), which the observers choose to identify with their respective \( z \) axes. Let us then mark “\( A' \)” and
“B” two points on O’s spaceships (the rest frame): A is a point on the z axis, while B is off of the z axis and such that the segment that joins A and B is orthogonal to the z axis (and therefore orthogonal to the direction of relative motion of the two observers). The first step is for O and O’ to measure the distance \( AB \), to which they will end up attributing the same value (lengths orthogonal to the boost direction are unaffected by boosts). Assume then that ideal mirrors are placed at A and B, so that light can bounce back and forth between A and B. This constitutes a “light clock”. Assuming nothing else but the constancy of the speed of light (postulated) time dilatation follows straightforwardly. For observer O the time interval corresponding to each tick of the light-clock is \( \tau = 2 \frac{AB}{c} \). For the second observer, O’, the light clock is moving. The speed of the light used by the light clock is the same for the two observers but the distance travelled between ticks has different values for the two observers: that distance is \( 2 \frac{AB}{c} \) for the first observer, while for the second observer it has value \( \sqrt{1 - \frac{V^2}{c^2}} \). We conclude that, whereas the first (rest) observer attributes to each tick of the light clock a time interval \( \tau = 2 \frac{AC}{c} \), the second observer attributes to each tick of the light clock a time interval \( \tau' = 2 \frac{AC}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{\tau}{\sqrt{1 - \frac{V^2}{c^2}}} \), as predicted by special-relativistic time dilatation.

A key point for the understanding of some predictions of passive Lorentz-symmetry transformations is that the instruments used by the first observer are also admissible instruments for the second observer. Here I have discussed time dilatation using a single clock. The two observers will of course agree on the readout of the instrument (the objective/observer-independent number of “ticks” done by the light clock, which, with suitable electronics, could correspond to a number shown by the light clock); however, while the number of ticks of the light clock is the same for the two observers, the time interval that the two observers assign to each tick is different.

### 3.4 Length contraction

In order to illustrate the way in which passive Lorentz-symmetry transformations govern the rules of length contraction it is sufficient to analyze a simple gedanken length-measurement procedure (again a textbook exercise which is useful for one of the points I want to raise about Lorentz symmetry). Let us consider again our two observers, O and O’, with their spaceships. The setup is identical to the one adopted in the previous subsection: the relative position and the relative velocity of the spaceships are both pointing in the same direction, which the observers choose to identify with their respective z axes, and the previously-introduced points A and B coincide with the mirrors of a light clock. In addition now let us mark a third point, “C”, which, like A, is on the z axis. O and O’ want to measure the distance \( AC \) (the length of a segment placed in direction parallel to the relative \( OO' \) motion). The procedure of measurement of the

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6Note that for observer O’ the light clock (which is at rest with respect to O) is moving with velocity V, which of course coincide with the \( OO' \) relative velocity. While O sees the trajectory of the light beam as going back and forth along a straight light, O’ describes the trajectory of the light beam as a “zig-zag”: for example, when bounced back from B toward A the light beam, according to observer O, goes in an oblique direction, and while the light beam progresses toward A, the point A keeps moving (it is at rest with respect to O but moves with velocity V with respect to O’).
distance $\overline{AC}$ is structured as a time-of-flight measurement: an ideal mirror is placed at $C$ and the distance is measured as the half of the time needed by a photon wave packet, “the probe”, sent from $A$ toward $C$ to be back at $A$ (after reflection by the mirror). Timing is provided by the digital light-clock (involving the points $A$ and $B$) which I have already analyzed in the previous subsection. The rest-frame observer, $O$, will measure $\overline{AC}$ as the length $L = c \cdot N \cdot \tau / 2$, where $N$ is the number of ticks done by the digital light-clock during the $A \rightarrow C \rightarrow A$ journey of the probe. ($\tau$ is again the time interval corresponding to each tick of the light-clock, which, as discussed in the previous subsection, has the value $\tau = 2 \overline{AC} / c$.)

Again, also in this more complex measurement procedure, measuring a distance, it is worth emphasizing (since it is a key point for the understanding of passive Lorentz-symmetry transformations) that the instruments used by observer $O$ (the one on the rest-frame spaceship) are also admissible instruments for the second observer: the light gun is actually described in the same way by the two observers since, according to Lorentz symmetry, the speed of the emitted photons is independent of the speed of the emitting gun; moreover, as shown in the previous subsection, an accurate light clock at rest for observer $O$ is also an accurate moving clock for observer $O'$. So the second observer, $O'$ can “look at” the measurement procedure adopted by the first observer and adopt it as its own measurement procedure. As already emphasized in the previous subsection, in looking at this same measurement procedure the two observers will of course agree on the number of “ticks” done by the light clock during the probe’s two-way journey. That number of ticks is an objective fact (possibly a number shown by the light clock when triggered to stop upon the return of the probe at $A$). Of course, while the number of ticks of the light clock is the same for the two observers, the time interval that the two observers assign to each tick is different, and in fact we found in the previous subsection that according to the second observer, $O'$, each tick of the light clock corresponds to the time interval $\tau' = 2 \overline{AC} / \sqrt{c^2 - V^2} = \tau / \sqrt{1 - V^2 / c^2}$.

The other aspect of the measurement procedure that takes different form for the two observers is the relation between the time needed by the probe for its two-way journey and the length of the bar. The first observer sees the bar at rest, so she uses the relation $T = N\tau = 2L/c$. The second observer sees the bar moving with velocity $V$ and the two parts of the two-way journey of the probe are, for the second observer, of different length: one part has length $cL'/(c - V)$ and the other part has length $cL'/(c + V)$, where I denoted with $L'$ the distance $\overline{AC}$ according to observer $O'$. So for the second observer the relation between the time needed by the probe for its two-way journey and the length of the bar takes the form

$$T' = N\tau' = \frac{L'}{c-V} + \frac{L'}{c+V} = \frac{2cL'}{c^2 - V^2}.$$  \hspace{1cm} (1)

Using the relation between $\tau$ and $L$ and the relation between $\tau'$ and $\tau$ derived above this leads to

$$L' = \frac{c^2 - V^2}{c} N\frac{\tau'}{2} = \sqrt{1 - \frac{V^2}{c^2}} L .$$  \hspace{1cm} (2)

\footnote{For part of the journey of the probe the fact that the bar is moving shortens the probe’s trip toward the next extremity of the bar, for the other part of the journey the opposite occurs.}
This is length contraction. It is often said that the same distance has different value for different observers, it is contracted in the boosted frame with respect to its value in the rest frame. Some authors appear to assume that this exclusively means that two different measurement procedures, one done by observer $O$ and another one done by observer $O'$, would give the different results for the distance/length. As shown in this subsection, FitzGerald-Lorentz contraction has a stronger implication: the same measurement procedure is witnessed by the two observers, giving the same experimental readouts (such as the readout of the light clock in my examples), but the analysis of the measurement is different for the two observers and leads to different conclusions about the value of the distance/length being measured.

3.5 Kinematic thresholds

In the previous Subsection length contraction was analyzed applying the concept of passive Lorentz-symmetry transformation in the strongest sense: I did not just consider the same length as seen by two observers, I also considered the case in which the two observers use the same devices. This is the sense in which passive symmetry transformations describe how the same measurement procedure appears to two different observers. This is, in this author’s opinion, a very important aspect of the classical symmetries under consideration in this paper: in the analysis of situations in which two observers share the same measurement procedure several objective statements arise which can be of guidance for the analysis.

It is worth making another example, in addition to length contraction. Let me look again at the kinematic-threshold procedure already considered above and analyze it more carefully as a measurement procedure shared by two observers. As mentioned, I imagine that observer $O$ has two ideal photon lasers such that the energy of emitted photons can be tuned with arbitrarily high accuracy over an extremely wide range of energies. The two lasers are pointed along an axis in such a way to produce head-on collisions. The second observer $O'$ moves with velocity $V_0$ (with respect to $O$) along the direction of the axis of the head-on collisions.

Before starting the measurement procedure $O$ will need to calibrate her lasers. It is not sufficient that they can be accurately tuned, it must also be possible to establish which energy they are emitting when tuned in a certain way. Let us imagine that they are constructed in such a way that the energy of the photons emitted can be tuned at any of an infinity of energy levels, all equally spaced in energy, so that the calibration procedure will basically amount to establishing the $\Delta E$ gained each time that the laser is tuned up to the next discrete level. This calibration will be easily done by $O$ before the measurement procedure: she will place a device that measures the energy of photons in front of the laser, change once from one level of tuning to the next, and this will be sufficient for the calibration. I assume for simplicity that $O$ finds that both devices have the same calibration $\Delta E$ (this will happen if the two devices are built in exactly the same way and they are both at rest with respect to $O$). If $O$ and $O'$ want to share the lasers (if they want to be able to make use simultaneously of the same measurement procedure) it is necessary for $O$, who has the lasers on her spaceship, to be kind enough to send some photons from her lasers toward some energy-measurement devices that belong to $O'$. It is convenient for $O'$ to prepare these calibration devices at rest (with respect to $O'$). Moreover, $O'$ should take into account that the two lasers point in opposite directions on a spaceship ($O$’s spaceship) which is moving with respect to
and therefore $O'$ should place the calibration devices accordingly (in particular, at least one of the calibration devices might have to be placed outside his spaceship). $O'$ will find that the two lasers have different calibrations, $\Delta E_a'$ and $\Delta E_b'$. The relations between $\Delta E$ and $\Delta E_a', \Delta E_b'$ are an experimental result for $O'$ but of course we can predict them to be governed by Lorentz transformations of energy, and depend only on $V_0$ (and the fact that one laser emits its photons with velocity parallel to the relative $O-O'$ velocity while the other laser emits in the opposite direction).

At this point both observers have a calibration of the lasers and everyone is ready for the measurement procedure. $O$ will have the task of tuning the lasers, since they are on her spaceship (but actually $O'$ could use a remote-control device), but everything that happens will be witnessed by both observers. $O$ tunes one of the lasers to a fixed level “$n$”, which according to her calibration corresponds to the small energy $\epsilon = n \Delta E$, and increases the energy of emission of the second laser gradually from zero up to the value $E = N \cdot \Delta E$ for which some production of electron-positron pairs starts to occur (the threshold). The numbers $n$ and $N$ and the fact that some electrons start to be produced when the second laser reaches the $N$ level of tuning are objective facts, on which of course both observers agree. The only difference in the way in which the measurement procedure is perceived by the two observers is the calibration. From the point of view of $O'$ the threshold is reached in a situation that corresponds to having one laser tuned at energy $\epsilon' = n' \Delta E_a'$ and the other laser at energy $E' = N' \Delta E_b'$.

Lorentz symmetry governs various aspects of this experimental setup. Specifically, for known $V_0$, Lorentz symmetry governs the relation between $\epsilon, E$ and $\epsilon', E'$ (but this part is not specific to threshold experiments); moreover, Lorentz symmetry predicts that, independently of $V_0$, the product $\epsilon \cdot E$ will have the same value as the product $\epsilon' \cdot E'$ and these products will give the square of the electron mass: $\epsilon \cdot E = \epsilon' \cdot E' = m_e^2$.

The threshold condition $\epsilon \cdot E = m_e^2$ is of course the comparison of two relativistic invariants: the invariant $m_e^2$ of the emerging electron-positron pair and the invariant $\epsilon \cdot E$ of the system of two photon colliding head on. The fact that the production of an electron-positron pair is an objective fact that can be witnessed by two (or more) observers imposes that the threshold condition be a fully invariant statement when satisfied for one inertial observer it must satisfied also for all other inertial observers. This is of course true in Special Relativity, as a result of the properties here reviewed of Lorentz symmetry. However, this point has wider validity: the threshold condition must be an invariant statement in all physical theories of particle collisions, since it is unacceptable that some observers would see two photons disappearing in an electron-positron pair while others would see the two photons crossing each other without particle production.

\footnote{In principle an invariant statement does not need to be based on one of the relativistic invariants of the relativistic theory. For example a logically consistent threshold condition for the process $\gamma + \gamma \rightarrow e^+ + e^-$ can be formulated by making reference to center-of-mass frame. The condition could state that in the center-of-mass frame the kinematics of the process must enjoy a certain property. An observer for whom the center of mass of the process is not at rest would then have to first boost the observed energies and momenta to center of mass frame and then apply the condition. In ordinary special relativity one could say that the condition for energy-momentum conservation is to be imposed in the center-of-mass frame, but, since the ordinary Lorentz transformations preserve the energy-momentum-conservation conditions, this automatically implies that energy-momentum conservation is satisfied in every frame. In some alternative relativistic theory it would be logically consistent to introduce a threshold condition in the center-of-mass frame which is not preserved by the transformation rules.}
4 Comparison of space-rotation symmetry and Lorentz symmetry

In the previous two sections some aspects of space-rotation symmetry and Lorentz symmetry have been revisited. My emphasis has been on the way in which these symmetries characterize the relations between certain experimental results. Some analogies have emerged, which I want to summarize here using as examples the angular momentum vector \( \vec{L} \) and its three components \((L_x, L_y, L_z)\), for what concerns space-rotation symmetry, and the world-sheet \( \mathcal{W} \) and its associated properties, equal-time area \( A \) and surface velocity \( V \), for what concerns Lorentz symmetry. I have observed that in order to predict the value of \( L_x' \), the component of the objective entity \( \vec{L} \) along a certain direction (possibly identified with a magnetic field that specifies the “\( x \) axis” of a second observer \( O' \)) the observer \( O \) must do a triple measurement, a measurement of \( L_x, L_y \) and \( L_z \). The value of \( L_x' \) cannot be predicted if \( O \) only measures \( L_x \). This was a key conceptual ingredient for my analysis of the compatibility between angular-momentum discretization and continuous space-rotation symmetry.

Analogously Lorentz symmetry makes definite predictions for \( A' \) (the equal-time surface area according to observer \( O' \)) when \( V \) and \( A \) (the surface velocity and the equal-time surface area according to observer \( O \)) are known. If instead only \( A \) is known nothing can be said about \( A' \). Discretization of areas can therefore be compatible with Lorentz symmetry if \( V \) is appropriately underdetermined on \( A \) eigenstates (just like \( L_x \) discretization in ordinary quantum mechanics is accompanied by \( L_x, L_y \) noncommutativity such that \( L_y \) is undetermined on \( L_x \) eigenstates in a way appropriate for the preservation of space-rotation symmetry).

However, the analogy between the analysis of the interplay between Lorentz symmetry and area discretization and the analysis of the interplay between space-rotation symmetry and angular-momentum discretization must not be pushed too far. On the Lorentz-symmetry side from the world-sheet \( \mathcal{W} \) an observer \( O \) with a specific choice of time axis (a clock) obtains both the observable “velocity of the surface” \( V \) and “area of the surface” \( A \). On the space-rotation-symmetry side from the angular-momentum vector \( \vec{L} \) an observer \( O \) with a specific choice of \( x \) axis (a magnetic field) obtains only the observable \( L_x \). Noncommutativity of \( L_x \) with \( L_{x'} \) looks more like the noncommutativity of \( A \) and \( A' \), rather than the noncommutativity of \( V \) and \( A \). In fact, \( L_x \) and \( L_{x'} \) are projections of the same objective entity, angular momentum vector, \( \vec{L} \) along two (space) axes, just like \( A \) and \( A' \) are “projections” of the objective entity, world-sheet, \( \mathcal{W} \) that are specified by two choices of (time) axis. However, given \( \mathcal{W} \), a single choice of time axis allows to specify both \( V \) and \( A \); moreover, the knowledge of \( V \) and \( A \) for one choice of time axis allows to predict the values of \( A' \) (and \( V' \)) for other choices of time axis. Instead if only one (space) axis is introduced the projection of \( \vec{L} \) along that axis only gives us one observable, \( L_x \), and the knowledge is not sufficient to predict the value of \( L_{x'} \). Of course, I am using the same term “projection” to describe what are very different operations on the angular-momentum vector and on the world-sheet. This however might have important implications about how a change of “projection axis” should affect the observables that are significant from the perspective of the symmetries.

Having failed to achieve any deeper understanding of the possible role of this (possibly even insignificant) point, I am however tempted to conjecture that it might be
related to some of the observations which, on the measurement-analysis side I reported in the previous section. In Subsections 3.3, 3.4 and 3.5 I observed that some measurement procedures that are relevant for Lorentz symmetry can be shared (simultaneously witnessed) by different observers. For example, the measurement procedure in Subsection 3.4 allows (at once) that both observer $O$ and observer $O'$ measure the distance $AC$, finding results $L$ and $L'$ respectively. As the careful reader can easily verify, the same argument applies to area measurement (although the analysis of the measurement procedure is somewhat more complex). Instead for the measurement of components of angular momentum I was unable to find a similar situation: I couldn’t find a context in which a single measurement procedure intended for the measurement of $L_x$ ended up also giving a measurement of $L_{x'}$. This might related with the fact that the measurement procedures for lengths, areas (and similar) usually (necessarily?) assume the knowledge of the velocity of the segment (surface) whose length (area) is being measured. These observations, however tentative, appear to be potentially relevant for developing an intuition for what to expect of a quantum-spacetime theory on the subject of lengths/areas measurement and Lorentz symmetry.

5 Various scenarios for the fate of Lorentz symmetry in quantum spacetime

In order to proceed in the spirit of my analysis of space rotations in ordinary quantum mechanics we need to identify from the previous Section some Lorentz-symmetry characteristic measurements, measurements for which Lorentz symmetry governs the relations between the numerical values obtained by different observers. For the case of space-rotation symmetry my discussion focused on the measurement $(L_x, L_y, L_z)$, simultaneous measurement of the three components of the angular momentum of a system, the measurement of $L_x$ only, and the measurement of $\sqrt{L^2}$. The measurement of $(L_x, L_y, L_z)$ is relevant for classical space-rotation symmetry, through the associated prediction of $(L_{x'}, L_{y'}, L_{z'})$, but it is not an allowed measurement in quantum mechanics (not in the classical sense, which would require simultaneous eigenstates of all three operators), so at the quantum level the measurement of $(L_x, L_y, L_z)$ cannot be used to test space-rotation symmetry. However, one can test the validity of the space-rotation transformation rules on expectation values of $(L_x, L_y, L_z)$, and in ordinary quantum mechanics this test is successful (the symmetry does hold). The measurement of $L_x$ is allowed both at the classical and at the quantum level, but space-rotation symmetry makes no prediction concerning this measurement: the knowledge of $L_x$ does not allow to reconstruct $L_{x'}$ ($x'$ being the $x$ axis of another, rotated, observer). The measurement of $L_x$ is not meaningful for Lorentz-symmetry transformations: Lorentz symmetry does not govern the relation between the numerical values of $L_x$ and $L_{x'}$. The measurement of $\sqrt{L^2}$ is allowed both at the classical and at the quantum level, and space-rotation symmetry describes it as an invariant (no effect of the classical continuous symmetry on the discretization).

Let us consider a few measurements that are relevant on the Lorentz-symmetry side. I start with the observables length, $L$, area, $A$, volume, $\Omega$ and time, $\tau$. If the observer $O$ measures only the length of a bar (but not its velocity) Lorentz symmetry makes no prediction on the value that another observer $O'$ would attribute to that
length. Similarly for the area of a surface and the volume of an object. And, again similarly, if the observer $O$ measures only the value $\tau$ of the ticks of a light clock (but not the velocity of the light clock) Lorentz symmetry makes no prediction for the value $\tau'$ of the corresponding measurement done by $O'$. If the observer $O$ measures both the length of the bar and its velocity, a $(V, L)$ measurement, then Lorentz symmetry makes a definite prediction. Lorentz-symmetry (active) transformations dictate that actually
\[ (V, L) = (V, \sqrt{1 - V^2/c^2}L_0), \]
where $L_0$ is the rest length of the bar. For given relative $OO'$ velocity, $V_0$, Lorentz symmetry predicts the velocity $V'$ of the bar with respect to $O'$, and predicts that $L' = \sqrt{1 - V'^2/c^2}L_0 = \sqrt{(c^2 - V'^2)/(c^2 - V^2)}L$. So Lorentz symmetry is fully operative on the $(V, L)$ measurement: it predicts the transformation $(V, L) \rightarrow (V', L')$. Completely analogous remarks apply to the measurements $(V, A)$, $(V, \Omega)$ and $(V, \tau)$.

The measurements $(V, L)$, $(V, A)$, $(V, \Omega)$, $(V, \tau)$ are relevant for Lorentz symmetry just like the measurement $(L_x, L_y, L_z)$ is relevant for space-rotation symmetry: the classical symmetry makes definite predictions for the laws of transformation of these measurements and they are not invariants (the numerical values of the observables do change between inequivalent classes of observers). An example of Lorentz-symmetry invariant is of course $E^2 - c^2p^2$ ($E,p$ energy-momentum of a particle). The measurement of $E^2 - c^2p^2$ is relevant for Lorentz symmetry just like the measurement of $L^2 \equiv L_x^2 + L_y^2 + L_z^2$ is relevant for space-rotation symmetry: the classical symmetry makes definite predictions that these measurements correspond to invariants (the numerical values of the observables is the same for all inertial observers).

The understanding of the fate of Lorentz symmetry in quantum spacetime requires us to establish if, and in which way, the spacetime quantization affects these physical predictions for non-invariant Lorentz-symmetry-meaningful measurements such as $(V, L)$, $(V, A)$, $(V, \Omega)$, $(V, \tau)$ and for invariant Lorentz-symmetry-meaningful measurements such as $E^2 - c^2p^2$. Even before considering specific quantum-gravity proposals it is possible to discuss in general terms a few scenarios for the fate of Lorentz symmetry in quantum gravity.

### 5.1 Classical Lorentz symmetry preserved

Of course, it is plausible that classical Lorentz symmetry applies to the (still to be established) correct theory of quantum gravity in the same sense that space-rotation symmetry applies in ordinary quantum mechanics. This is actually the most natural expectation for quantum-gravity theories in which spacetime is not really quantized, in the sense that these theories still rely on a classical background spacetime and admit classical Minkowski spacetime as a possible background. An example of quantum-gravity theory which reflects this expectation is string theory. In fact, in string theory among the admitted spacetime backgrounds it is still possible to choose spacetimes that are completely classical. In that case physical processes still occur in a classical (background) spacetime arena, and spacetime is only “quantized” in the sense that some

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9For simplicity, but without any true loss of generality, I am basically discussing a one-dimensional configuration: the bar’s end points both are on the $x$ axis of $O$ and the relative $OO'$ velocity is also along that $x$ axis.
new particles (notably, the graviton) are allowed to propagate in this fundamentally classical spacetime and mediate gravitational interactions. Of course, departures from Lorentz symmetry are not necessary as long as a theory (as in the case of string theory) still admits the possibility of a background spacetime that is exactly Minkowski.

In other quantum-gravity approaches classical Minkowski spacetime will only emerge as an approximate description of a fundamentally nonclassical spacetime; for example, certain noncommutative versions of Minkowski spacetime are perceived just like classical Minkowski spacetime by long-wavelength particles, while they are fully nonclassical for short-wavelength particles. In those cases Lorentz symmetry will only be an approximate symmetry, emerging in the low-energy limit. At present this is not the case in string theory, but it is noteworthy that the issue of the “spacetime background” is probably the one on which most progress must be sought within the string-theory research programme. The idea of a background-dependent approach to the quantum-gravity problem is not satisfactory on many grounds. Future developments in the string-theory programme might eliminate the need to make reference to a background spacetime, and at that stage it might be interesting to reassess the status of exactly classical Minkowski spacetime within string theory.

5.2 Classical Lorentz symmetry not preserved

The situation is quite different in theories that really change the fundamental description of spacetime, such as theories that invoke some form of spacetime discretization or spacetime noncommutativity. In these instances it is actually more natural to expect that the fate of Lorentz symmetry be nontrivial, that classical Lorentz symmetry would not apply to such quantum-gravity theories, at least not in the strong sense in which classical space-rotation symmetry applies to ordinary quantum-mechanics theories. The point is that classical space-rotation symmetry “survived” (in the sense clarified in Section 2) the advent of ordinary quantum mechanics because space-rotation symmetry pertains to spacetime and ordinary quantum mechanics still relies on a fully classical spacetime. The idea of spacetime quantization would instead truly modify the structure of spacetime and it is therefore natural to expect (as verified in certain specific toy-model examples) that the spacetime symmetry we call Lorentz symmetry would be affected by the spacetime quantization. In the next Section I discuss some examples of noncommutative versions of Minkowski spacetime and clarify that departures from ordinary Lorentz symmetry are rather natural. Also the idea of spacetime discretization naturally leads to the expectation of a nontrivial fate for the continuous classical Lorentz symmetry, but our intuition is still not reliable in these contexts. Clearly a simple-minded rigid discretization of Minkowski spacetime would not be consistent with the continuous classical Lorentz symmetry [7], but it is difficult to develop some intuition for more sophisticated ways to introduce discreteness in spacetime structure. An interesting attempt to introduce a non-trivial discretization of spacetime structure has emerged from the “loop quantum gravity” [3, 8, 10, 11] research programme. Section 7 is devoted to this loop-quantum-gravity discretization scenario and there I shall argue that the present understanding/interpretation of certain loop-quantum-gravity results appears to be in conflict with classical Lorentz symmetry, but I shall also argue that at present it is not clear whether this is a genuine feature of the theory or perhaps just an indication that the relevant results should be interpreted and analyzed more carefully.
5.3 Deformation of the classical Lorentz symmetry

In the preceding Subsection I made the point that in approaches that rely on a genuinely nonclassical spacetime it is natural (though perhaps not necessary) to find that the fate of classical Lorentz symmetry is nontrivial, i.e. that classical Lorentz symmetry would not be an exact classical symmetry at the quantum-spacetime level. The present Subsection comments on a scenario for the fate of Lorentz symmetry in quantum spacetime which this author finds rather appealing: the scenario in which one basically still has the same conceptual structure of Lorentz symmetry (for example with six symmetry generators) but Lorentz transformations are deformed in such a way as to have two observer-independent scales, a length scale $\lambda$ (possibly given by the Planck length) and (again, as in ordinary Lorentz symmetry) the speed-of-light constant.

This deformed-symmetry scenario, which is being called “Doubly Special Relativity”, was proposed by this author in Ref. [12], where a first example of such deformed symmetries was also constructed. A second example was more recently constructed by Maguejio and Smolin [13], and even more recently Kowalski-Glikman and Nowak [14] and Lukierski and Nowicki [15] have reported progress in the construction of a larger class of such deformed symmetries.

It appears that the idea of deformed Lorentz symmetry is indeed realized in certain nonclassical pictures of spacetime, at least in certain noncommutative versions of Minkowski spacetime. I shall comment on this in greater detail in the next Section. Here I want to give a physical characterization of a deformation of Lorentz symmetry, in the spirit I have adopted throughout this paper. A deformation of Lorentz symmetry would still be characterized by transformation rules and invariants of the transformation rules, but their structure would be different from the one of ordinary Lorentz symmetry. For example, just like Lorentz symmetry predicts that $E^2 - c^2 \vec{p}^2$ is an energy-momentum invariant, the deformation of Lorentz symmetry described in Ref. [12] predicts that

$$10\lambda^{-2}c^{-2}(e^{\lambda E/c} + e^{-\lambda E/c} - 2) - c^2 p^2 e^{\lambda E/c}$$

is an invariant. In ordinary Lorentz symmetry one has the observer-independent scale $c$ that plays the role of speed of massless particles of any energy and maximum attainable velocity, whereas in the deformation of Lorentz symmetry described in Ref. [12] one has the observer-independent scale $\lambda$ that plays the role of speed of low-energy massless particles, and the observer-independent scale $\lambda$ that plays the role of inverse of the maximum attainable momentum. These are physical characteristics of the deformed symmetry that could be tested experimentally and one can also attempt to construct theories whose predictions are consistent with these characteristics of the symmetry.

Notice that a deformation does not involve any loss of symmetry and does not involve any changes in the type of rules that we associate to the concept of symmetry, i.e. a deformation of Lorentz symmetry can still be a classical symmetry according

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10Focusing on this specific form of the dispersion relation, while not necessary, can be motivated by previous arguments in the study of quantum algebras [16, 17] and of an approach to the study of noncritical string theory [18]. The quantum-algebra results are found [13] (at least for the one-particle sector [12]) to provide an acceptable description of DSR transformations (doubly-special-relativity transformations), in the same sense that the preexisting Lorentz transformations provided the correct mathematical language for Einstein’s Special Relativity. DSR transformations can accordingly be called quantum algebra or “$\kappa$-deformed” [16, 17] transformations, just like we refer to the special-relativistic transformation rules of Einstein’s physical theory as Lorentz transformations.
to the definitions here given. This idea might find application in quantum pictures of spacetime that enjoy classical symmetries: in this scenario the transition from classical to quantum spacetime would require that the symmetries (still being classical in nature) reflect/preserve the new (quantum) structure introduced in spacetime by quantization. An example of this scenario will be discussed in Subsection 6.1.

5.4 Lost Lorentz-symmetry

The idea of a deformation of Lorentz symmetry, considered in the previous Subsection, does not involve any loss of symmetry, but it is of course legitimate (although rather painful conceptually) to consider the possibility that the transition from classical to quantum spacetime would actually cause a loss of symmetry.

Let me provide a physically-characterized example of loss of symmetry by considering again the dispersion relation. In the previous Subsection I implicitly considered two dispersion relations, the standard \( E^2 - c^2\vec{p}^2 = c^4m^2 \) and the dispersion relation \( \lambda c^{-2}(e^{\lambda E/c} + e^{-\lambda E/c} - 2) - c^2\vec{p}^2 e^{\lambda E/c} = c^4m^2 \) characteristic of a deformed symmetry scenario (if the laws of transformations between inertial observers are accordingly deformed \( \text{(12)} \)). An example of dispersion relation that would signal symmetry loss is \( E^2 - c^2\vec{p}^2 + Eu_0 - \vec{p} \cdot \vec{u} = c^4m^2 \), with \( u \) a given four vector that transforms from observer to Lorentz transformations, so that the values of \( u_0, u_x, u_y \) and \( u_z \) are not identical to the values of \( u'_0, u'_x, u'_y \) and \( u'_z \). Of course, the presence of \( u \) allows one to identify a preferred class of inertial observers (specified by a chosen value of the components of \( u \) along the \( t, x, y, z \) axis of those observers), signaling a loss of Lorentz symmetry.

A key point here is that, according to my definitions, a genuine loss of Lorentz symmetry will be present only if an object such as the \( u \) of my example is an intrinsic property of spacetime. We already know (even experimentally) that in a perfectly Lorentz invariant theory living in a perfectly Lorentz invariant spacetime one can have cases in which the dispersion relation involves some external four-vector or tensor. For example, we know that a Lorentz-invariant theory of the propagation of light in certain media does predict a modification of the dispersion relation, often even involving preferred directions (the preferred directions of the dispersion relation reflect the preferred directions of the medium). For the case of light propagating in a medium we can still (and should) speak of a Lorentz-symmetric theory living in a Lorentz-symmetric spacetime in which the specific system under study (in particular the medium) is not invariant under Lorentz transformations. It is clearly a different situation when spacetime itself has preferred directions or anyway allows the identification of a preferred class of inertial observers. In such cases it is appropriate to speak of loss of Lorentz symmetry.

5.5 Classical Lorentz symmetry spontaneously broken

An important class of scenarios in which there is loss of Lorentz symmetry is the one in which Lorentz symmetry is spontaneously broken. Here of course I have in mind the field-theory/particle-physics mechanism of spontaneous symmetry breaking. It is not easy to imagine a similar mechanism applied to spacetime structure, especially because we lack a true understanding of the concept of spacetime vacuum (we can
perhaps attempt to describe the concept of empty spacetime, but it is much harder
to imagine some sort of minimum-energy spacetime, since we do not even have an a priori concept of energy for contexts in which a background spacetime is not provided ab initio). However, it is not unplausible that the correct theory of spacetime physics would enjoy Lorentz symmetry in the sense that different spacetime solutions that are connected by a Lorentz transformation are equally likely, but then the most likely solutions ("the vacuum") would not themselves enjoy Lorentz symmetry. (This of course requires that the appropriate concept of "most likely spacetime solution" does not identify a single spacetime solution, but rather identifies a 6-parameter family of degenerate solutions, all mapped one into the other by Lorentz transformations.) The example of a four-vector $u$ discussed in the previous Subsection could emerge in such a spontaneous-symmetry-breaking scenario: the "vacuum solutions" would be characterized by $u$, and all forms of $u$ that are connected by a Lorentz transformation to a certain $u^*$ would all be equally likely, but then Nature would have chosen a specific vacuum, breaking the degeneracy.

5.6 Fuzzy Lorentz symmetry?

My a priori discussion of possible scenarios for the fate of Lorentz symmetry in quantum spacetime cannot aim for completeness. Nature may well host a scenario which this author has not managed to even imagine. A characteristic of all scenarios I have considered up to this point is that they still rely on the concept of a classical symmetry (in the sense introduced in this paper). The classical symmetry is deformed or even violated (symmetry loss) but the questions one would ask (the properties that characterize the symmetry concept) are formulated classically in the sense of Section 2. To this author it is not even clear whether one should/could contemplate anything different from this. One is confronted with similar conceptual challenges when trying to analyze the conceptual framework of ordinary quantum mechanics without relying on a classical apparatus (what is a nonclassical apparatus? what would be a nonclassical interpretation of the readout of a measurement device?). At least for the context of passive symmetry transformations, which is more constrained by demands of the objectivity of physical processes, it is difficult to think about alternatives to the conceptual framework of classical symmetries. Think for example of the description of the length-contraction experiment on which I focused in Subsection 3.3. There the two observers not only measure the same length but they also rely on a single measurement procedure (which however they interpret in a different way). The two observers even agree on the readout that gives the result of that length measurement experiment (which is the number $N$ of ticks of the light clock), and they obtain a different result for the length measurement simply because their relative motion affects the calibration attributed to the devices and the description of the measurement procedure. The (passive) symmetry transformation connects the interpretation of $N$ for one observer with the interpretation of $N$ for another observer, and it is difficult to imagine that the relation between these interpretations would not be classical in the sense here advocated. It might only be possible in theories predicting some new limitations (of an appropriate type) on the mechanism of calibration of devices (or perhaps an absolute limitation on the measurability of the relative velocity of two observers).

Still, it is tempting to conjecture here that it might be possible for a quantum theory of spacetime to accommodate some sort of nonclassical, "fuzzy", symmetry concept.
Perhaps a symmetry concept which only applies to ensembles of observations and not
to a single observation. However, this possibility is indeed challenged\textsuperscript{11} by the analysis
of contexts, such as the length measurement in Subsection 3.3, in which a single measure-
ment is meaningful for two observers and therefore the symmetry transformation
should predict how that single measurement procedure appears to the two observers.

5.7 Challenge to quantum-gravity theories

In this Section devoted to a brief description of various scenarios for the fate of Lorentz
symmetry in quantum spacetime it appears to be appropriate to for-
mulate a challenge
to quantum-gravity theories. A large research effort is focusing on the quantum-gravity
problem, but only a relatively small percentage of these studies concerns the fate of
Lorentz symmetry. The recent progress (and the expected progress) of sensitivity of
tests of Lorentz symmetry (see, e.g., Refs. \cite{1, 2, 3, 4, 5, 19}) renders this state of affairs
unjustifiable. If any deviation from ordinary (classical-spacetime) Lorentz symmetry
is hosted by a quantum-gravity theory, it will most likely turn out that the associated
predictions can be tested with very high accuracy.

Since the ultimate goal is comparison to experimental results, the fate of Lorentz
symmetry must be analyzed in quantum-gravity theories giving priority to physical is-
sues (the nature and magnitude of the predicted effects) rather than formalism issues.
It is in this respect that there is a clear set of challenges to quantum-gravity theories.
For example, we are reaching extremely high sensitivity \textsuperscript{12} to the study of the prop-
agation over cosmological distances of short-duration bursts of photons. In classical
physics the distance travelled, $L$, would be classical, the photons would be point-like
and the photons would follow the classical trajectory along $L$, so, according to classical
physics, a group of such photons which were emitted simultaneously at time $t = 0$
would complete the journey simultaneously at time $t = L/c \equiv T$. Ordinary (known)
quantum properties of matter (in classical spacetime) already modify this picture: the
structure of quantum mechanics imposes that the time of emission of a particle with
energy uncertainty $\delta E$ can only be specified with accuracy $1/\delta E$, and there is of course
a corresponding limitation on how accurately the simultaneity of the times of arrival
can be established, but the relation $T = L/c$ will emerge if appropriate averaging over
a large number of observations is performed. The quantum properties of the particles
(still assuming classicality of the spacetime) introduce a nonsystematic effect, an uncer-
tainty: $T = L/c \pm \delta T_{QM}$. I have here a clear opportunity for a well-defined challenge
to quantum-gravity theories: how does a given quantum-gravity theory af-
fect this prediction? In order to be covered on all possible fronts we should be open
to the possibility of both systematic and nonsystematic quantum-gravity effects. This
can be captured in the formula

$$T = (L/c + \Delta T_{QG}) \pm \delta T_{QM} \pm \delta T_{QG}, \tag{3}$$

\textsuperscript{11}Since the same length-measurement procedure can be shared by two observers it would be para-
doxical if, for example, one observer was to conclude, after repeated measurements, that she is dealing
with a length eigenstate, while the other observer would conclude that he is dealing with a superposi-
tion of different eigenstates. Both observers see the same readouts $N$ of the light clock and they must
therefore agree on whether or not they are dealing with a length eigenstate.
with self-explanatory notation. A nonvanishing prediction for $\Delta T_{QG}$ would require a departure from classical Lorentz symmetry. $\Delta T_{QG}$, if nonzero, would likely be energy-dependent (at low energies we have good data that strongly support $\Delta T_{QG} = 0$) and this would affect the propagation over cosmological distances of short-duration bursts of photons in an obvious way: one would expect a systematic energy-dependent time-of-arrival difference in the analysis of the short duration bursts in different energy channels of our detectors. The possibility that $\Delta T_{QG} = 0$ but the given quantum-gravity theory predicts a nonvanishing value for $\delta T_{QG}$ does not necessarily require a deviation from classical ordinary Lorentz symmetry. The careful reader will easily realize that, with the definition here adopted of a classical symmetry in a nonclassical theory, it is necessary to analyze the properties of a given $\delta T_{QG}$ picture in order to establish whether or not it implies a deviation from classical ordinary Lorentz symmetry. It might well be that $\delta T_{QG} \neq 0$ in a way that still satisfies the conditions for classical ordinary Lorentz symmetry in the given quantum-gravity theory. A $\delta T_{QG} \neq 0$ is to be expected with rather natural assumptions about quantum gravity: just like the quantum properties of matter (the relevant photons) in classical spacetime introduce a nonvanishing $\delta T_{QM}$, the quantum properties of spacetime should introduce a nonvanishing $\delta T_{QG}$. The magnitude and structure (e.g. the dependence on the energy of the particles involved) of $\delta T_{QG}$ will vary strongly from one quantum-gravity theory to another. A nonvanishing $\delta T_{QG}$ would of course affect the propagation over cosmological distances of short-duration bursts of photons. A nonvanishing energy-dependent value of $\delta T_{QG}$ could be established by searching for differences in the time spread of the burst in different energy channels of our detectors. A nonvanishing distance-travelled-dependent value of $\delta T_{QG}$ could be established by searching for differences in the time spread of bursts with otherwise similar characteristics but reaching us from different distances. If $\delta T_{QG}$ is distance-independent and energy-independent it might be hard to find experimental evidence for it (in that case a natural estimate for $\delta T_{QG}$ would be $\delta T_{QG} \sim t_p$, and the Planck time $t_p$ is so small that we would never find evidence for such a $\delta T_{QG}$).

The study of the propagation over cosmological distances of short-duration bursts of photons is clearly a key challenge to quantum-gravity theories. Another example of important challenge for quantum-gravity theories comes from the analysis of the energy thresholds for certain particle-production processes, such as the electron-positron pair production in photon-photon collisions, which I considered in the preceding Section. Classically two photons can have sharply-defined energies, say $E$ and $\epsilon$, and the process $\gamma + \gamma \rightarrow e^+ + e^-$ is allowed when $E\epsilon \geq m_e^2$ and it is absolutely forbidden if $E\epsilon < m_e^2$. Within ordinary quantum mechanics (here the relevant formalism is field theory in Minkowski spacetime) one would most naturally consider photons prepared with energies $E$ and $\epsilon$, with $E_0 - \Delta < E < E_0 + \Delta$ and $\epsilon_0 - \delta < \epsilon < \epsilon_0 + \delta$, but one still has a definite prediction: the process is allowed if $E_0\epsilon_0 \geq m_e^2 - \Delta\epsilon_0 - \delta E_0$ while it is forbidden if $E_0\epsilon_0 < m_e^2 - \Delta\epsilon_0 - \delta E_0$. Here there is another clear opportunity for a well-defined challenge to quantum-gravity theories: how does a given quantum-gravity theory affect this prediction? Assume we prepare the photons just as we do now, with $E_0 - \Delta < E < E_0 + \Delta$ and $\epsilon_0 - \delta < \epsilon < \epsilon_0 + \delta$. Will the condition $E_0\epsilon_0 \geq m_e^2 - \Delta\epsilon_0 - \delta E_0$ still hold? or will it take the form $E_0\epsilon_0 \geq m_e^2 + \Delta_{threshold,QG} - \Delta\epsilon_0 - \delta E_0$? A nonvanishing value of $\Delta_{threshold,QG}$ would be predicted [2] in most quantum pictures of spacetime whose symmetries are not described by ordinary classical Lorentz symmetry (in these spacetimes even pair production by classical photons, with ideally
sharp definition of their energies, would be governed by $E_0 \epsilon_0 \geq m_e^2 + \Delta_{\text{threshold, QG}}$ rather than $E_0 \epsilon_0 \geq m_e^2$). Even quantum-gravity theories in which ordinary classical Lorentz symmetry holds (in the sense here advocated) might predict a threshold of the form $E_0 \epsilon_0 \geq m_e^2 + \Delta_{\text{threshold, QG}} - \Delta \epsilon_0 - \delta E_0$, but there the threshold deformation should be attributed to some sort of new uncertainty principle \[20\]. In both cases it might not be hard to obtain stringent limits on the predicted deviations from the classical-spacetime threshold; in fact, the analysis \[3\] of ultra-high-energy cosmic rays and other observations that are primarily of interest in astrophysics can be used \[4, 5, 21\] to test the idea of threshold deformation with extremely high sensitivity.

6 On the fate of Lorentz symmetry in noncommutative spacetime

Noncommutative geometry is being used more and more extensively in attempts to unify general relativity and quantum mechanics. Some quantum-gravity approaches explore the possibility that noncommutative geometry might provide the correct fundamental description of spacetime, while in other approaches noncommutative geometry turns out to play a role at the level of the effective theories that describe certain aspects of quantum gravity.

For the issues here under consideration it is the case of noncommutative versions of flat (Minkowski) spacetime that is of interest. Of course, a flat noncommutative spacetime could not possibly provide a full solution to the quantum-gravity problem, but if it turns out to be true that noncommutative geometry is the correct language/formalism for the description of the fundamental structure of spacetime then in particular the spacetimes that we perceive as (approximately) flat and classical should also be described by noncommutative geometry and noncommutative versions of Minkowski spacetime might therefore be relevant.

Two simple examples \[22\] are “canonical noncommutative spacetimes” ($\mu, \nu, \beta = 0, 1, 2, 3$)

$$[x_\mu, x_\nu] = i \theta_{\mu\nu}$$

and “Lie-algebra noncommutative spacetimes”

$$[x_\mu, x_\nu] = i C_{\mu\nu}^\beta x_\beta .$$

These two simple examples of noncommutative spacetimes\[22\] are also useful for illustrative purposes, since they provide well-defined models in which some of the scenarios for

\[12\] In this study I will only be concerned with the simple case in which noncommutative geometry takes the form of noncommuting coordinates, whose commutators are either constant or depend on the coordinate themselves. This is mostly in the spirit of an approach to noncommutative geometry that originates from the quantum-group research programme \[23\]. Another interesting case of noncommutative version of Minkowski spacetime, which is not considered here (but will be analyzed in a forthcoming publication \[24\]), is the Snyder spacetime \[25, 26\], in which however the commutators of the coordinates are expressed in terms of elements of the Lorentz algebra. Noncommutative geometry is also being developed following another approach that originates primarily from original work by Connes \[27\], but in that approach nothing significant as emerged concerning noncommutative versions of Minkowski spacetime.
the fate of Lorentz symmetry in quantum spacetime considered in the previous Section are realized.

On the Lie-algebra side I will focus for simplicity on the $\kappa$-Minkowski \cite{16, 17} spacetime ($l, m = 1, 2, 3$)

$$[x_m, t] = \frac{i}{\kappa} x_m, \quad [x_m, x_l] = 0,$$

which is one of the most studied\footnote{\$\kappa$-Minkowski spacetime is a Lie-algebra spacetime that clearly enjoys classical space-rotation symmetry (while boosts are deformed). I think there could be justifiable interest in the possibility of studies of other flat space-rotation-invariant spacetimes based on the Lie-algebra-type algebraic relations, such as $[x_i, x_j] = i L^i_j k^i_j x_k$ which would also naturally lead to a theory with classical space rotations and deformed boosts. However, at least within the presently-adopted mathematical framework for these noncommutative geometries, from the algebraic relations $[x_i, x_j] = i L^i_j k^i_j x_k$ one is naturally led \cite{28} to the description of spheres rather than flat spacetimes.} noncommutative to classical Minkowski spacetime.

### 6.1 $\kappa$-Minkowski noncommutative spacetime

Detailed analyses of $\kappa$-Minkowski spacetime can be found in Refs. \cite{16, 17, 29}. Here I just want to provide an intuitive characterization of the fate of Lorentz symmetry in this noncommutative version of Minkowski spacetime. A first point is that the law of composition of momenta is deformed and nonlinear in $\kappa$-Minkowski. This is encoded in the so-called coproduct. An intuitive way to see this is through the introduction of the Fourier transform. It turns out \cite{29, 30, 31} that in the $\kappa$-Minkowski case the correct formulation of the Fourier theory requires a suitable ordering prescription\footnote{There is of course an equally valid alternative ordering prescription in which the time-dependent exponential is placed to the left (while we are here choosing the convention with the time-dependent exponential to the right).} for wave exponentials:

$$e^{ik\mu x_\mu} \equiv e^{ikm x_m} e^{ik0 x_0}.$$ \hspace{1cm} (7)

These wave exponentials are actual solutions of a $\kappa$-Minkowski wave equation \cite{29}. While wave exponentials of the type $e^{ip\mu x_\mu}$ would not combine in a simple way (as a result of the $\kappa$-Minkowski noncommutativity relation), for the ordered exponential one finds

$$(: e^{ip\mu x_\mu} :) (e^{ik\nu x_\nu} :) = e^{i(p+k)\mu x_\mu}.$$ \hspace{1cm} (8)

The notation “$+:+$” here introduced reflects the behaviour of the mentioned “coproduct”, composition of momenta\footnote{Here I use the vague expression “composition of momenta”. In physics we need to compose momenta in various situations, e.g. when we combine two plane waves into one and when we impose energy-momentum conservation in multiparticle processes. Using the bare coproduct in the law of composition of plane waves appears to be appropriate in light of the property \cite{6}, but using the bare coproduct in the law of conservation of energy-momentum would lead to a statement of energy-momentum conservation which is not objective for all inertial observers \cite{22}: for a particle-producing collision process $a + b \rightarrow c + d$ laws of the type $(p_a + p_b)\mu = (p_c + p_d)\mu$ are inconsistent with the} in $\kappa$-Minkowski spacetime:

$$p_\mu + k_\mu \equiv \delta_{\mu,0}(p_0 + k_0) + (1 - \delta_{\mu,0})(p_\mu + e^{i\delta_{\mu,0} k_\mu}).$$ \hspace{1cm} (9)
As argued in Refs. [12] the nonlinearity of the law of composition of momenta should require an absolute (observer-independent) momentum scale, just like upon introducing a nonlinear law of composition of velocities (in going from Galilei/Newton relativity to Einstein relativity) one must introduce the absolute observer-independent scale of velocity $c$. The inverse of the noncommutativity scale $\lambda$ plays the role of this absolute momentum scale. This of course requires [12] transformation laws for energy-momentum between different observers which have two invariants, $c$ and $\lambda$, while ordinary Lorentz transformations have only one invariant. An example of laws of transformation that enjoy this property was used as illustrative example in Refs. [12], in which the idea of deformed Lorentz symmetry was introduced. A key point is that the deformed Lorentz transformations form group. They actually are a nonlinear representation of the Lorentz group itself. While ordinary Lorentz transformations leave invariant the combination $E^2 - c^2 p^2$, the deformed transformation rules leave invariant the combination

$$C_\lambda(E, \vec{p}^a) = \frac{c^2}{\lambda^2} (e^{\lambda E/c} + e^{-\lambda E/c} - 2) - c^2 \vec{p}^a e^{\lambda E/c}.$$  \hspace{1cm} (10)

The dispersion relation $E^2 = c^2 p^2 + c^4 m^2$ is accordingly replaced by the new (deformed) dispersion relation implicitly defined by the requirement $C_\lambda(E, \vec{p}^a) = C_\lambda(m, 0)$.

In work that preceeded Refs. [12], some examples of Hopf algebras that could represent deformed infinitesimal symmetry transformations had been worked out, but it was believed [33] that these algebra structures would not be compatible with a genuine symmetry group of finite transformations. In Refs. [12] it was proposed that one should look for deformed transformation laws that form a genuine group and it was shown that one example of the Hopf algebras that mathematical physicists had been developing did allow the emergence of a group of finite transformations (while the same is not true for other examples of these Hopf algebras). Interest in the proposal [12] of deformed Lorentz symmetry is growing, and very recently other examples of the same type of deformed transformation rules have been constructed in Refs. [13, 14, 15].

### 6.2 Canonical noncommutative spacetime

Also in the case of canonical noncommutative spacetime an intuitive characterization of the fate of Lorentz symmetry can be obtained by looking at wave exponentials. The Fourier theory in canonical noncommutative spacetime is based on simple wave exponentials $e^{ip^\mu x_\mu}$ and from the relevant noncommutativity relations one finds that

$$e^{ip^\mu x_\mu} e^{ik^\nu x_\nu} = e^{-\frac{1}{2} p^\mu \theta_{\mu\nu} k^\nu} e^{i(p+k)^\mu x_\mu},$$  \hspace{1cm} (11)

i.e. the Fourier parameters $p_\mu$ and $k_\mu$ combine just as usual, with the only new ingredient of the overall phase factor that depends on $\theta_{\mu\nu}$. The fact that momenta combine relevant, $\kappa$-deformed, laws of transformation for the $p^\mu$’s of the four particles (the condition $(p_a+p_b)^\mu = (p_c+p_d)^\mu$ can be imposed in a given inertial frame but it will then be violated in other inertial frames!). This point has strangely been missed in the whole of the $\kappa$-deformation literature (see, e.g., Ref. [1], [13]), but now, in light of the recent doubly-special-relativity proposal [1], it must be seen as a top-priority problem for the $\kappa$-deformation programme.
in the usual way reflects the fact that the transformation rules for energy-momentum from one (inertial) observer to another are still the usual, undeformed, Lorentz transformation rules. However, the product of wave exponentials depends on $p^\mu \theta_{\mu\nu} k^\nu$: it depends on the “orientation” of the energy-momentum vectors $p^\mu$ and $k^\nu$ with respect to the $\theta_{\mu\nu}$ tensor. The $\theta_{\mu\nu}$ tensor plays the role of a background that identifies a preferred class of inertial observers. Different particles are affected by the presence of this background in different ways, as shown by the results [34, 35, 36] of the study of field theories in canonical noncommutative spacetimes.

The situation, for what concerns Lorentz transformations, is actually very familiar. We know well many contexts in which the presence of a background selects a preferred class of inertial observers. This is reflected, for example, in the fact that the dispersion relation for light travelling in water, in certain crystals, and in other media is modified. The study of field theories in canonical noncommutative spacetimes shows [34, 35, 36] that the $\theta_{\mu\nu}$ background induces effects that are somewhat similar to the ones induced by the presence of a crystal, including the effect of birefringence of light.

The $\theta_{\mu\nu}$ tensor “breaks” Lorentz symmetry in the same sense that any medium breaks Lorentz symmetry: the theory is still fundamentally Lorentz invariant but the Lorentz invariance is manifest only when different observers take into account the different form that the background, in this case the $\theta_{\mu\nu}$ tensor, takes in their respective reference systems. While the single (dimensionful) deformation parameter $\lambda$ of the $\kappa$-Minkowski spacetime is observer-independent, i.e. takes the same value for all observers, the $\theta_{\mu\nu}$ matrix behaves like a Lorentz tensor: the elements of the $\theta_{\mu\nu}$ matrix take different values for different observers. If the observers only take into account the transformation rules for the energy-momentum of the particles involved in a process the results are not the ones predicted by Lorentz symmetry; in particular, the dispersion relation depends on the background. In fact, the dispersion relations found in the study of field theories in canonical noncommutative spacetimes acquire [34, 35, 36] a dependence on $p^\mu \theta_{\mu\nu} p^\nu$, $E^2 = c^2 p^2 + c^4 m^2 + f(p^\mu \theta_{\mu\nu} p^\nu)$, with the function $f$ that depends on the spin and charges of the particle.

Concerning the construction of field theories in canonical noncommutative spacetimes there are some interesting issues. It has emerged that field theories constructed in strict analogy with the way we construct them in commutative spacetimes do not host the familiar mechanism of Wilson decoupling between ultraviolet and infrared degrees of freedom [34, 35, 36]. This connection between ultraviolet and infrared is not necessarily troublesome [37, 38], moreover, to this author it is not at all obvious that in these noncommutative geometries one should necessarily construct field theories in strict analogy with what usually done in commutative spacetime. Since the $\theta_{\mu\nu}$ tensor can be used to single out a preferred class of inertial observers one could for example introduce a maximum momentum for that class of inertial observers, and one could even specify the laws of physics only according to that class of preferred observers. The other observers would of course witness the same physical phenomena but would describe them as Lorentz transformations of the phenomena seen by the preferred class of inertial observers.

### 6.3 Doplicher-Fredenhagen-Roberts noncommutative spacetimes

Canonical noncommutative spacetimes are characterized by a single tensor $\theta_{\mu\nu}$, which of course takes different form in different reference systems, breaking Lorentz symmetry.
It is possible to see the emergence of this $\theta_{\mu\nu}$ background as a result of a phenomenon of spontaneous symmetry breaking. In fact, canonical noncommutative spacetimes could emerge from more general spacetime theories in which the $\theta_{\mu\nu}$ tensor is not fixed a priori. A research line that originates in a 1994 paper by Doplicher, Fredenhagen and Roberts [39] is aiming for a theory in which $\theta_{\mu\nu}$ is itself associated with a dynamical element of the theory. Since no preferred $\theta_{\mu\nu}$ is introduced a priori, the theory is fundamentally Lorentz invariant (in the ordinary, undeformed, sense). As usual, the fact that the dynamical equations of the theory enjoy a certain symmetry, in this case Lorentz symmetry, does not imply that the solutions of the theory be Lorentz invariant. It is plausible that the “vacuum” of the theory would be characterized by a specific tensor $\theta_{\mu\nu}$ (but the Lorentz invariance of the theory would then impose a large degeneracy of this vacuum, since acting with a symmetry on a vacuum one must find another vacuum), in which case one would talk of spontaneous symmetry breaking. It is even conceivable that all physical states (not only the vacua) would be characterized by a nonvanishing value of $\theta_{\mu\nu}$, so that in each physically viable realization of flat spacetime there would be a preferred class of observers, again a mechanism of spontaneous symmetry breaking (although of novel type).

In these cases the theory, i.e. the equations of dynamics, would be Lorentz invariant but the spacetimes predicted by the theory would break Lorentz invariance. The theory would be fundamentally Lorentz invariant, but the energy-momentum dispersion relations (possibly different for different particles) would never be of the type $E^2 = c^2 p^2 + c^4 m^2$, signaling that the symmetry is spontaneously broken. At the fundamental level the theory could only predict a general formula for the dispersion relation, involving the dynamical variable $\theta_{\mu\nu}$, then in a specific spacetime (possibly an eigenstate of $\theta_{\mu\nu}$) the dispersion relation would take a specific form (which would “break” Lorentz invariance in the sense discussed above).

Of course, a theory that hosts $\theta_{\mu\nu}$ as a dynamical variable might also not have spontaneous Lorentz-symmetry breaking, if, for example, “the vacuum” of the theory was characterized by the condition $\theta_{\mu\nu} = 0$ or “the vacuum” was obtained as a large “democratic” superposition of states characterized by all possible values of the $\theta_{\mu\nu}$ matrix.

7 On the fate of Lorentz symmetry in loop quantum gravity

The two ideas for a nonclassical description of spacetime that are being extensively considered in the quantum-gravity literature are the ideas of noncommutativity and of

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16 As mentioned, it is still not clear whether the quantum-gravity problem necessarily requires a spacetime picture that is fundamentally nonclassical in the sense here advocated. In particular, within the popular string-theory approach to the quantum-gravity problem the underlying spacetime picture is still not fundamentally nonclassical; in fact, in string theory among the admitted spacetime backgrounds it is still possible to choose spacetimes that are completely classical. In that case physical processes still occur in a classical (background) spacetime arena, and spacetime is only “quantized” in the sense that some new particles (notably, the graviton) are allowed to propagate (and mediate gravitational interactions) in this fundamentally classical spacetime.
discretization. The observations reported in the previous Section confirm that the fate of Lorentz symmetry is naturally nontrivial (in one or another way) in noncommutative versions of Minkowski spacetime. Also the idea of spacetime discretization naturally leads to the expectation of a nontrivial fate for the continuous classical Lorentz symmetry, but discretization can be introduced in spacetime physics in many ways and it is difficult to make very general considerations. As mentioned, a simple-minded rigid discretization of Minkowski spacetime would clearly not be consistent with the continuous classical Lorentz symmetry \[7\], but it is not \textit{a priori} obvious that the same would happen in more sophisticated ways to introduce discreteness in spacetime structure. In this Section I consider the best developed approach to spacetime discretization: the one that emerged from research work on “loop quantum gravity” \[8, 9, 10, 11\].

The “loop quantum gravity” approach is perhaps the most ambitious of all quantum-gravity approaches. While this approach, not unlike all other quantum-gravity approaches, is not immune from the presence of “conceptual shortcuts” \[17\] it is the only approach that does not rely on an \textit{a priori} spacetime background. Spacetime-background independence is a very natural (but technically challenging) requirement for theories, quantum-gravity theories, attempting to address the “conceptual tension” between quantum mechanics and classical general relativity.

In fact, general relativity is a background-independent description of spacetime dynamics.

Of course, for a background-independent approach an important task is the one of describing those physical contexts in which a background spacetime does emerge\[18\]. This task has not yet been accomplished in loop quantum gravity, but progress in this direction might be forthcoming. In general it is not surprising that the analysis of this ambitious theory turns out to be extremely difficult. Only very few characteristic predictions have been obtained. Among these predictions the most celebrated are the ones concerning some natural area and volume “observables” (here intended in the technical/mathematical sense) and the spectrum of these observables, which turns out to be discrete. The discretization of the spectrum of geometry/spacetime observables is the most fundamental characterization of a discretized picture of spacetime.

I shall not review here the loop quantum gravity approach, not even for what concerns the derivation and analysis of the area and volume operators. These topics are very effectively and pedagogically reviewed in some recent publications (see, \textit{e.g.}, Refs. \[8, 9, 10, 11\]). For the purposes of my analysis it is sufficient to comment here on some qualitative aspects of those results. I am in fact exclusively interested in the way in which discretization of areas and volumes might affect Lorentz symmetry.

In spite of the fact that the classical-Minkowski limit has not yet been found in loop quantum gravity (actually at present the programme is still attempting to identify a suitable limiting procedure for the emergence of classical spacetimes; see, \textit{e.g.},

\[17\] Readers familiar with the “loop quantum gravity” approach will realize that, among other things, I am here concerned with the fact that the diffeomorphism invariance originally sought by this approach has never been truly realized, since the approach still basically requires an \textit{a priori} space/time split. Only invariance under 3-dimensional space diffeomorphisms is genuinely maintained. Also mysterious (and suspicious) is the role that the “Immirzi parameter” \[8, 9, 10, 11\] plays in the formalism. Moreover, the lack of a natural scheme for the introduction of nongeometric degrees of freedom (\textit{e.g.}, Standard-Model particles) is of course cause of serious concern.

\[18\] The majority and the simplest of our observations are naturally described as processes occurring in a classical (and often nearly flat) spacetime arena, but the loop quantum gravity approach is still unable to describe that simplest type of phenomena.
Ref. [40] this concern about Lorentz symmetry is not premature since the area/volume discretization results are understood as completely general: those discretizations are a general prediction of the loop quantum gravity approach, which should in particular apply to the zero-curvature (Minkowski/quasi-Minkowski) limit. While a dedicated study of this issue is still lacking, at conferences and from the introductory remarks of review papers it appears that two intuitions are emerging: according to one of these intuitions one expects that, in spite of the worrisome appearance, these discretizations should turn out to be compatible with classical ordinary Lorentz symmetry, because at the level of the tools introduced in the formalism it appears that nothing could have spoiled the symmetry; while the other intuition assumes that it would be impossible to reconcile the discretization of these spectra with continuous (classical) transformations of Lorentz type.

I argue here that both intuitions rely on false premises. The intuition which is favourable to the survival of classical ordinary Lorentz symmetry focuses exclusively on properties of the formal tools introduced in the formalism (while, as emphasized in the previous Sections, it is at the level of the physical predictions that Lorentz symmetry should be analyzed) and neglects some key differences (see below) between space rotations and Lorentz boosts that are relevant for a canonical quantum theory. The intuition which is hostile to the survival of classical ordinary Lorentz symmetry assumes that discrete spectra are inevitably inconsistent with the presence of continuous classical symmetries, while the case of classical space-rotation symmetry in ordinary quantum mechanics (here reviewed in Section 2) shows that there are counter-examples for this, however intuitive, argument.

Some insight on this delicate (and crucial, especially considering the mentioned developing experimental situation) issue can be gained using as guidance the comparison between the description of angular-momentum discretization in ordinary quantum mechanics and the description of area/volume discretization in loop quantum gravity. As clarified in Section 2, angular-momentum discretization in ordinary quantum mechanics is consistent with classical space-rotation symmetry because the $L^2$ discretization involves an observable on which the classical symmetry acts trivially (an invariant), and $L_z$ discretization (just like the discretization of $L_y$ and $L_z$) involves an observable on which the classical symmetry does not act at all. As emphasized in Sections 3 and 4, Lorentz-symmetry transformations do not act on the area observable $A$. If the observer $O$ measures the area $A$ of the surface of a table, but does not measure the velocity $V$ of the table, Lorentz symmetry is unable to predict the size of that area according to another observer $O'$ moving at velocity $V_0$ with respect to $O$. If instead the observer $O$ measures both the area of the surface of the table and its velocity, a $(V, A)$ measurement, then Lorentz symmetry makes a definite prediction: it establishes that (assuming velocities are all aligned) $(V, A) = (V, \sqrt{1 - V^2/c^2} A_0)$, where $A_0$ is the rest area of the surface of the table and Lorentz symmetry also establishes that, denoting with $V_0$ the relative $OO'$ velocity, $A' = \sqrt{1 - V'^2/c^2} A_0 = \sqrt{(c^2 - V^{'2})/(c^2 - V^2)} A$.

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19As emphasized in Section 2, when the observer $O$ measures $L_z$ in a space-rotation invariant theory it is still not possible to make a definite prediction for the components $L_x'$, $L_y'$, $L_z'$ that are relevant for another observer $O'$. In order to predict $L_x'$ (and/or $L_y'$ and/or $L_z'$) one must do three measurements: $L_x$, $L_y$ and $L_z$.

20Since my remarks apply equally both to the area and the volume operators, for short I will often refer only to the area operator.
where \( V'(V_0, V) \) is the velocity of the table with respect to \( O' \). So Lorentz symmetry is fully operative on the \((V, A)\) measurement, while it bears no relevance when only \( A \) is measured.

Lorentz-symmetry transformations act on the double measurement \((V, A)\) in the same sense that space-rotation-symmetry transformations act on the triple measurement \((L_x, L_y, L_z)\). Just like classical space-rotation symmetry is consistent with \( L_x \) discretization if and only if (see Section 2) the theory also predicts that a sharp measurement of \( L_x \) imposes a large uncertainty on \( L_y \) and \( L_z \), ordinary classical Lorentz symmetry is consistent with \( A \) discretization if and only if the theory also predicts that a sharp measurement of \( A \) imposes a large uncertainty on \( V \).

This point I am raising concerning the \((V, A)\) measurement and the fact that area discretization is not necessarily in conflict with Lorentz symmetry should be distinguished from a previous argument, due to Snyder [25], also relevant for the possible compatibility of Lorentz symmetry with spacetime discreteness. Snyder proposed and analyzed a specific type (with coordinate commutators not expressible as functions of the coordinates themselves) of flat noncommutative spacetime, finding that Lorentz symmetry was maintained and that the commutation relations would induce coordinate discreteness (which, I observe, should be discussed cautiously, in terms of distances and other diffeomorphism-invariant entities). In the Snyder spacetime the sharp measurement of one of the coordinates would in general impose an uncertainty on the time coordinate (and the other coordinates). One clear difference is that my argument concerns area (and associated surface velocity) measurements, while Snyder’s concerns space coordinate (and associated time coordinate) measurements. A more subtle, but perhaps even more important, difference is that Snyder only verified Lorentz symmetry as a property of the formalism; Snyder introduced Lorentz-symmetry transformations as a set of transformations for the “coordinate observables”, and only emphasized that the spectra of the coordinate operators are the same for all observers. Instead here I discussed Lorentz symmetry at the level of the predictions of the theory, and in particular I emphasized that some of the measurement procedures that are governed by Lorentz symmetry can be shared by different observers (see Subsections 3.3, 3.4 and 3.5) and that this might render insufficient any general argument on the spectra of symmetry-connected observables (one might have to perform a dedicated analysis of the action of Lorentz symmetry on eigenstates of some of the observables).

From these observations it follows that area discretization does not in itself represent a violation of ordinary classical Lorentz-symmetry, and therefore in each quantum-gravity approach predicting area discretization a dedicated analysis of the fate of Lorentz symmetry is needed. I shall argue that the present level of development of loop quantum gravity is insufficient for obtaining a definite answer to this important question, especially in light of the fact that an observable “area of the surface” was introduced without the introduction of an observable “velocity of the surface”, which, from a Lorentz-symmetry perspective, should naturally accompany the surface area observable. From my analysis it emerges that a natural path for the realization of ordinary Lorentz symmetry in loop quantum gravity can be found through the analogy of the status of space-rotation symmetry in quantum mechanics. In order to achieve this goal a suitable observable “velocity of the surface” must be found, and some of its relevant properties must be analyzed. In particular, one should find that in surface-area eigenstates the velocity of the surface is affected by a large uncertainty (large enough to comply with the requirements sketched in Subsection 2.4). But even if this
task was accomplished \( i.e. \) if indeed the analogy between space-rotations in quantum mechanics and Lorentz transformations in loop quantum gravity was successful in the sense of finding satisfactory commutation relations between surface velocity and surface area) the status of Lorentz symmetry in loop quantum gravity would still require further investigation. In fact, from the perspective of a canonical quantum theory, such as the loop quantum gravity here considered, the analogy between space-rotation symmetry and Lorentz symmetry has several limitations. The special role that time plays in a canonical theory imposes that one should consider observers connected by a Lorentz boost in a way that is significantly different from the one appropriate for observers connected by a space rotation: one can roughly say that a space rotation acts within a given Hilbert space (a given choice of the time variable) whereas Lorentz boosts connect a given canonical theory (a given Hilbert space and time variable) with another different canonical theory. In light of this difference it is not even conceivable that the status of Lorentz symmetry in a canonical theory could be truly analogous to the status of space-rotation symmetry in a canonical theory. On the contrary it appears plausible that at some level of analysis the fact that the canonical theory is forced to make direct/primitive reference to equal-time surfaces, rather than to the surface’s world sheet, might have significant implications. As emphasized in the previous sections, in the preservation of space-rotation symmetry in quantum mechanics a key role is played by the fact that the theory and our description of measurement procedures are most primitively formulated in terms of the physical entities, like the angular-momentum vector, whose objectivity is codified by space-rotation symmetry transformations. The observable simply denoted by \( L_x \) in the formalism inevitably corresponds physically to an observable obtained from two “objective/physical vectors”, the angular-momentum vector \( \vec{L} \) and a second “projecting” vector \( \vec{B} \). Meaningful measurement procedures can be devised when the value of \( \vec{B} \) is known. A faithful description of Lorentz symmetry might require that the theory be formulated in terms of the world-sheet (world-line, world-volume,...) and that the observable equal-time area be introduced as an appropriate “projection” of the world sheet (a “projection” that identifies the time variable with respect to which one is defining equal-time surfaces). A conventional canonical quantum theory cannot be formulated in this way, since it requires an \textit{a priori} time variable/parameter. This problem of the “absence of the world-sheet” might already be hidden in the nature of the “area observable” presently adopted in loop quantum gravity: in fact that observable refers directly to an equal-time coordinate area (although it is clear to loop-quantum-gravity experts that the surface should be eventually meaningfully identified through some fields/particles) and it is not seen as some projection of a world-sheet: it is indeed like introducing in ordinary quantum mechanics as most primitive observable \( L_x \), without any room for identifying this \( L_x \) as a projection of the type \( \vec{L} \cdot \vec{B} \).

I shall also argue that the problem of the “absence of the world-sheet” is also significant from the perspective of transforming the formal result of a discrete spectrum for the area observable into a physical prediction. A necessary (actually not sufficient) condition for theorists to make a physical prediction of discretization is to indicate at least one measurement procedure in which this discretization is (at least in principle) observable, and analyze the chosen measurement procedure within the adopted theory in order to verify that the full theory is consistent with the observability of the discreteness. Since loop quantum gravity is not formulated in terms of world sheets one runs
into a situation which is alarmingly different from the one in which one endows angular-momentum discretization with the status of a genuine physical prediction: for example, the Stern-Gerlach device realizes physically the projection (also allowed/admitted by the formalism) of the vector $\vec{L}$ along the direction of another vector (the direction of the magnetic field $\vec{B}$ which characterizes the measurement procedure). So it appears that no specific measurement procedure can be suggested by the formal result, and, in addition, I shall show that the analysis of some standard/familiar area-measurement procedures suggests a paradoxical situation in which area discretization would not be observable, even if rigorously introduced at the level of formalism.

My observation that, from the perspective of a canonical quantum theory, such as the loop quantum gravity here considered, the analogy between space-rotation symmetry and Lorentz symmetry can only be adopted in a partial/limited way is also based on some aspects of the measurement procedures we presently adopt to give operative meaning to the relevant observables. I must stress here again that some of the measurement procedures that are governed by Lorentz symmetry can be shared by different observers (see Subsections 3.3, 3.4 and 3.5) and that this might render insufficient any general argument on the spectra of symmetry-connected observables. In the length measurement of Subsection 3.4 both observer $O$ and observer $O'$ measure time (which is there used to measure length) using the same clock. In the analysis of that length-defining procedure (and of analogous procedures for the measurement of other spacetime entities, like areas) it appears impossible to contemplate the possibility that observer $O$ would characterize the situation with a length eigenstate while the observer $O'$ characterizes the situation with a superposition of length eigenstates: both observers establish whether or not they are dealing with a length eigenstate using the same clock readouts! This appears to be different from the case of measurement procedures for angular-momentum components: a Stern-Gerlach device measures one and only one component of angular momentum. (Instead the length-measuring setup of Subsection 3.4 measures both the “equal-time area projection to observer $O$ of the surface’s world-sheet” and the “equal-time area projection to observer $O'$ of the surface’s world-sheet”.) I shall also emphasize another difference: while the value of the, say, $L_y$ component of angular momentum is not relevant for procedures that measure $L_x$, and therefore the noncommutativity of $L_x$ and $L_y$ does not affect the analysis of the measurement procedure (it only affects the logical structure of the results of different measurement procedure), the knowledge of the velocity of the surface appears to be needed in order to introduce meaningful area-measurement procedures, and therefore a noncommutativity of surface-velocity and surface-area would have severe implications for the analysis of area-measurement procedures.

These remarks set the agenda for my analysis of the status of Lorentz symmetry in loop quantum gravity, an admittedly rich agenda. The delicate nature of some of the points I intend to raise forces me to organize the remaining subsections of this section in a way that does not have a nice flow. Rather than recognizing a logical order in the sequence of the subsections the reader should attempt to recognize that most subsections provide material in support for the remarks anticipated in these opening paragraphs of this section devoted to loop quantum gravity. In addition, some of the subsections report observations that are not directly relevant for the line of reasoning I proposed in these opening remarks, but instead they intend to provide to the reader some material useful for comparison with other recent studies. I start with some general remarks on the way in which a meaningful investigation of Lorentz symmetry can be performed in a dynamical (and quantum) theory of spacetime.
7.1 Investigating Lorentz symmetry in dynamical quantum theories of spacetime

The focus of this paper is global Lorentz symmetry, which can be rigorously investigated in the nondynamical flat noncommutative spacetimes considered in the preceding section, but is not a natural element of a dynamical quantum theory of spacetime. Global Lorentz symmetry does not even have a truly fundamental role in classical general relativity, where it only emerges on what should be seen as very special solutions of the dynamical equations (or as an approximate symmetry of spacetimes that are well approximated by Minkowski in a region of small size). The role of Lorentz symmetry is likely to be even less on the forefront of the structure of a quantum dynamical theory of spacetime. However, a quantum-gravity theory should be able to describe the contexts with which we are observationally familiar, in which spacetime is to a good approximation flat and classical. Most of my considerations concern these quasi-Minkowski spacetimes, which must be admitted by quantum-gravity theories. Although they might play a relatively marginal role in the conceptual/technical structure of quantum-gravity theories, these spacetimes are likely to provide our best experimental-testing ground for quantum-gravity theories [41].

Even in a “quasi-Minkowski” spacetime one might wonder whether Lorentz-symmetry transformations (or some predictable departure from their structure) should play a role in quantum theories of spacetime. However, I am adopting here the working assumption that a meaningful test of Lorentz symmetry should be possible in quantum gravity. A quantum-gravity theory must have a classical-spacetime $L_p \rightarrow 0$ limit and a further zero-curvature $R \rightarrow 0$ limit should introduce a corresponding role for Minkowski spacetime in quantum gravity. Of course, in that limit, which I am here schematically describing as a $L_p \rightarrow 0$, $R \rightarrow 0$ limit, ordinary Lorentz symmetry should hold. If then the theory allows us to consider (both formally and operatively/experimentally) situations in which the $R \rightarrow 0$ condition is maintained but the $L_p \rightarrow 0$ condition is softened ($L_p$ small but nonzero) there are basically two possibilities: either Lorentz symmetry still holds exactly or there are small ($L_p$-suppressed) departures from Lorentz symmetry. The analysis I am reporting in this paper hopes to provide useful elements for the investigation of quantum theories of spacetime in this respect, for establishing whether $L_p$-suppressed departures from Lorentz symmetry are to be expected.

7.2 Procedures for area measurement

Since I am here considering the status of Lorentz symmetry in loop-quantum-gravity, the first point I must be concerned with is whether the formal result of area discretization can be adopted as a truly physical prediction. In loop quantum gravity there is indeed an operator which in the classical limit represents the area of a surface and at the quantum level turns out to have discrete eigenvalues. Does this quantum operator represent physical areas also at the quantum level?

It is of course not sufficient to call an operator “area” for it to qualify as the description of physical areas. We should at least specify one class of area-measurement procedures for which we predict that the outcome of the measurements would reflect the discretization. We should also analyze the proposed measurement procedure and verify that there are no logical obstructions for that discretization to be revealed. (I stress that this is not a point about our technological ability to reveal the discretization:
in order to formulate a physical prediction it is sufficient that the adopted measurement procedure does not have any in principle obstructions for revealing the discretization.)

The prediction of $L_x$ discretization in ordinary quantum mechanics is more than a formal result: it refers to specific angular-momentum measurement procedures, of which the Stern-Gerlach procedure is a prototype. The interpretation of angular-momentum discretization within ordinary quantum mechanics is very simple: the same observable quantity (i.e. the quantity measurement through the same operative procedure) that is called angular momentum in classical mechanics also exists in quantum mechanics and it can be measured exactly in the same way, but according to quantum mechanics the results of these measurement procedures, which in classical mechanics could take any arbitrary value, can only take certain discrete values. Also in classical physics we could use a Stern-Gerlach-type device to measure the angular momentum of a charged spinning “classical particle” (small ball). The loop-quantum-gravity results on area/volume discretization are presently being discussed as if they were to be interpreted in complete analogy with the known angular-momentum discretization, as if we should be able to adopt the same operative definition of area we presently adopt, the same area-measurement procedures we presently adopt, and for those measurement procedures the theory would predict discrete outcomes.

Angular-momentum discretization is a general prediction of ordinary quantum mechanics. It applies also to macroscopic systems, but there we lack the needed experimental accuracy to reveal it. If the loop-quantum-gravity area discretization is to be interpreted just as the angular-momentum discretization in ordinary quantum mechanics it should mean that area-measurement procedures performed on macroscopic surfaces (macroscopic with respect to the Planck distance scales) should also give sharp discretized outcomes, which we would have not noticed because of lack of the needed experimental accuracy.

But I conjecture that the area operator of loop quantum gravity cannot be interpreted as a new description of the same area observable we are all familiar with in classical physics, and that the loop-quantum-gravity discretization of area is unobservable. I repeat: here I do not mean that, because of the small discretization scale, we might never be able to reveal the effect. I mean that the theory itself should predict that this discretization cannot be observed, that there be an in-principle obstruction to its observation. If this conjecture is correct, the associated reanalysis of the area and volume operators might also lead to a reassessment of the status of Lorentz symmetry.

My conjecture is based in part on the fact that the same physical intuition which motivated the description of areas at the formalism level in terms of a quantum operator should also motivate a reanalysis of area-measurement procedures, and this reanalysis suggests that there should be an absolute limitation on the measurability of areas. This sort of new uncertainty principle would also render unobservable, and therefore unphysical, area discretizations of the type discussed in loop quantum gravity, with Planckian area quantum.

In building up to this intuition let me start with a given procedure for the measurement of areas. Unfortunately, in spite of the relevance of the area results at the formal level, and therefore the need to endow with physical meaningfulness those results through a defining measurement procedure, not much has been done on this point in the loop-quantum-gravity literature. One noticeable exception is the deservedly popular study reported in Ref. [42]. The measurement procedure there adopted is relevant for the case in which the matter fields that specify the surface whose area is being measured are taken to form a metal plate, and the area $A$ of this metal plate is measured
using an electromagnetic device that keeps a second metal plate at a small distance \( d \) and measures the capacity \( C \) of the capacitor formed by the two plates. The primary measurement would be the capacity, and the sought area would be evaluated through the relation\(^{21}\)

\[
A = \frac{d}{\varepsilon_0} C .
\]  

(12)

This is as good as any other candidate area-measurement procedure on which to explore the physical interpretation of the loop-quantum-gravity area-discretization result. It could be for area discretization in loop quantum gravity the analog of what the Stern-Gerlach setup is for angular-momentum discretization in ordinary quantum mechanics. In the Stern-Gerlach setup one measures the angular momentum through the measurement of the position of arrival of the particles on a screen/detector, and the prediction is that those positions take discretized values, reflecting angular-momentum discretization. In the area-measurement procedure considered in Ref. [42] one measures the area through the measurement of the capacity \( C \). Is then the prediction of the loop-quantum-gravity approach that those capacity readouts can only take certain discrete values? This is one of the hypotheses raised in Ref. [42], but in this respect several considerations are in order.

First of all let me emphasize that of course this area-measurement setup cannot follow instantaneously the time evolution of the surface (and of its area). Typically the time of measurement required by area-measurement is at least of order \( T \sim \sqrt{A/c} \). For this area-measurement procedure based on capacity measurement this estimate of the time of measurement is found by considering the time needed by the capacity to respond to a sudden change in the area of the plate being measured. After such a sudden transition from one size of the area to another the distribution of electrons on the surface of the metal plate will have to readjust over the whole surface until a new equilibrium is reached, and this transition from the previous equilibrium configuration to the new one must take at least a time of order \( \sqrt{A/c} \) (since \( c \) is the limiting speed for information). In a theory where area eigenstates are states in which the metric is sharply defined, if the theory also predicts that correspondingly the time derivative of the metric is largely uncertain, this area measurement setup would not be able to give the value of the area of the surface at a certain specific time \( t^* \) but would rather give the average size of that area over a time interval of order \( \sqrt{A/c} \). The net result would be that there would be no trace of the discretization.

Also notice that the formula (12) which is at the core of this measurement procedure (just like the relation between external magnetic field, relevant component of the angular momentum and point of arrival on the screen is at the core of the Stern-Gerlach procedure) implicitly assumes that the surface be absolutely flat. In a theory in which area eigenstates were not allowed to be exactly flat the discretization of the area would be masked (hidden) by the fact that this measurement procedure makes some averaging over the fluctuations with respect to exact flatness that the quantum theory predicts.

Also notice that this measurement procedure basically assume that one can measure accurately the velocity of the (metallic) surface whose area is being measured. In fact it

\(^{21}\)In Eq. (12) the presence of \( \varepsilon_0 \) reflects the simplifying assumption that the measurement be performed in absolute vacuum. This simplification does not affect the validity of my remarks.
appears to be necessary to assure that the two surfaces that compose the capacitor are parallel (constant distance $d$) and that they be centered with respect to one another.22

Finally let me make a point which I already stressed in Refs.\cite{43}. In this area-measurement procedure based on capacity measurement it is necessary to measure the distance between the plates. If $d$ is not known sharply then the relation between $C$ and $A$ becomes fuzzy and the discretization of $A$ may become unobservable. This observation appears to be relevant for theories, such as loop quantum gravity, in which there appears to be a well-defined area operator but some difficulties are encountered in describing distances. It would be paradoxical\cite{43} for a theory predicting that distances cannot be measured with absolute accuracy to predict that the results of this capacity-based area-measurement procedure should be sharp (and discretized).

7.3 Toward a loop-quantum-gravity description of fuzzyness

As I hope it emerged from the line of analysis advocated in this paper, if it was true that the measurement of a flat area described in the preceding subsection would give discretized results and one was able to measure rather accurately the velocity of the surface (as it appears to be required by that measurement procedure) then Lorentz symmetry should be necessarily violated as explained in the preceding sections. My conjecture is that more refined analyses of the loop-quantum-gravity formalism should find that the discretization of areas is not a physical prediction of the formalism, that it would not show up in any procedure for the measurement of areas. If such developments did come about one should then reassess the status of Lorentz symmetry in loop quantum gravity. I am raising the possibility that the present loop-quantum-gravity results on area/volume discretization should not be intended as a genuine discretization (at least not in the sense we presently understand angular-momentum discretization). They should rather be an awkward way in which the theory renders us aware of a new absolute limitation on the accuracy with which areas can be measured.

The other possibility (which I find to be less likely, but is certainly plausible) that emerges from this study is that instead area discretization is indeed a physical prediction of loop quantum gravity and Lorentz symmetry is preserved, but, in light of the arguments here presented, in this case it is inevitable that the formalism should also predict that the surface velocity is largely undetermined on surface-area eigenstates and in addition one should face the challenges of: (i) identifying an area-measurement procedure which is not affected by the surface-velocity uncertainty, and (ii) understand the role that measurement procedures such as the ones described in Subsections 3.3, 3.4, 3.5, in which the readouts of the same measurement procedure are meaningful for different inertial observers, should have in the theory (they should somehow become disallowed, otherwise they would impose that area eigenstates are area eigenstates for all observers, with the consequence, in which case the discretization scale of the area spectrum could not be observer-independent.

\footnote{22If the second surface (the one that belongs to the measuring device) is much larger than the surface whose area is being measured one should be concerned about “boundary effects” since the formula \cite{13} actually assume a highly symmetric configuration (it strictly applies to infinite parallel metallic plates). If the two surfaces are roughly of the same size any relative velocity would of course affect the capacity. Moreover, for a charged metallic plate which is not at rest one should worry about associated magnetic fields.}
I will, for short, refer to these two possibilities as the “fuzzyness” scenario and the “area/velocity noncommutativity” scenario.

In the previous subsection I have provided some physical arguments, through the analysis of a measurement procedure, that support my conjecture in favour of the “fuzzyness” scenario. Here I want to provide some remarks on the formalism that go in the same direction.

A first point that needs careful consideration is the fact that the area “measured” by the loop-quantum-gravity area operator is an area defined by corresponding conditions on coordinates, rather than the area of a surface physically/meaningfully identified by some field/matter distributions. Some work in the direction of such a meaningful identification has been done, but the problem appears to be dangerously entangled with the general open problem of loop quantum gravity for what concerns the introduction of nongravitational degrees of freedom (such as realistic descriptions of the standard model of particle physics). Lacking this technical ingredient one might be tempted to adopt the viewpoint in which a surface is meaningfully identified by some conditions on the boundary coordinates, if these coordinates are intended as physical distances from the axes of the “laboratory” of an observer. However, this interpretation is, in my opinion, rather troublesome when the tetrads are promoted, as done in loop quantum gravity, to the status of quantum variables. This should intuitively mean that, in a given “state of spacetime” the laboratory axes (the laboratory frame) cannot be identified with the same sharpness as in classical physics. One might end up evaluating the sharp spectrum of a “coordinate area” in a framework where the physical meaning of those coordinates is not itself sharp. Even the “coordinate area”, with coordinates intended as physical distances from the natural reference axes of a laboratory of course should involve some suitable field/matter distributions that identify those axes. As observed in Refs. [44], reference axes can be physically identified with absolute precision in classical physics, where one could for example use particles of negligibly small mass rendering gravitational evolution negligible. Reference axes can also be physically identified with absolute precision in ordinary (no-gravitation) quantum mechanics, where one would instead choose to identify the axes using very massive particles for which the uncertainty principle is negligible. But in quantum gravity axes identified by very massive particles would have “problems” due to gravity and axes identified by nearly-massless particles would have “problems” due to the uncertainty principle. This line of reasoning provides support for the “fuzzyness” scenario even when not focusing on the nature of material reference systems: in fact the argument also applies to the analysis of a ring of particles used to identify a surface. In order to measure the area of that surface we need that the particle be in rigid motion for the time needed to complete the measurement procedure, which should be at least a time of order $\sqrt{A/c}$. In order to suppress the gravitational interactions among the particles (which could lead to area deformation on time scales shorter that the time of measurement) one would like particles of very small mass, but then Heisenberg’s uncertainty principle would introduce a large uncertainty in the motion of the particles and the area would vary on very short time scales anyway. If we take particles of large mass the uncertainty principle would not cause problems, but rigid motion would be in conflict with the strong gravitational fields being generated by the massive particles. So in a quantum theory of gravity a surface which is physically/meaningfully identified by a ring of particles will be deformed (at a level significant for Planck-scale accuracy of area measurement) on time scales that are shorter than the time needed by the measurement.
Another issue that should be addressed in the formalism is the one concerning the role that the time derivatives of the metric play in the procedures used to give operative meaning to the concept of area. One could study how the analysis of Lorentz symmetry is affected by the fact that the concept of area of a surface involves the metric, while the concept of velocity of an area involves (in an appropriately weak sense, see below) the time derivatives of the metric. In a theory predicting nontrivial commutation relations between the metric and its derivatives this might generate the type of surface-velocity/surface-area noncommutativity which I have shown here to be needed in order to have a Lorentz-symmetric description of area discretization. This possibility certainly deserves detailed investigation. It is probably reasonable to conjecture that nontrivial commutation relations between the metric tensor and its time derivative will introduce some level of noncommutativity between the velocity of the area and the area itself, but probably the induced velocity uncertainty in area eigenstates would not be large enough to rescue classical continuous Lorentz symmetry (not large enough in light of the argument here discussed in Subsection 2.4). The point is that the time derivatives of the metric tensor do not properly contribute to the velocity of the area in the sense relevant for Lorentz-symmetry transformations. The relevant physical property described by Lorentz symmetry concerns measurements of rigidly-moving surfaces (surfaces whose area does not change significantly during the time of measurement) in which one simultaneously measures the area and the velocity of the surface. In Minkowski spacetime it doesn’t matter which point on the surface we choose in defining the velocity of the surface. At the instant \( t^* \) at which the measurement is performed one is free to choose this point for the identification of the velocity as the origin of the reference system adopted by the inertial observer, and in this case the time derivatives of the metric tensor should not contribute significantly to the velocity of the area. They rather contribute to the deformation of the surface as a function of time (a sort of velocity of deformation of the surface, velocity that describes how quickly the area changes in time). For what concerns Lorentz symmetry, the most important consequence of nontrivial commutation relations between the metric tensor and its time derivative should be a possible absolute limit on rigid motion of a surface. This also fits the intuition [44], described above, that in a theory in which both the uncertainty principle and gravitational interactions are included, as expected for quantum-gravity theories, rigid motion would not be allowed. The study of the time derivatives of the metric tensor might simply allow us to establish that, as conjectured above, the area spectrum derived in loop quantum gravity would not describe a new phenomenon of discretization of the familiar concept of area, but would rather reflect an absolute limitation on our present idealization of the operative definition of the area of a surface of a moving body. Since it appears inevitable that the measurement of an area of (roughly determined) size \( A \) will require at least a time of measurement \( T_{\text{meas}} = \sqrt{A}/c \), if areas cannot move rigidly (i.e., if the theory predicts that the area of a surface must vary on very short, perhaps Planckian, time scales) how well could we measure them? If the spectrum is quantized with \( L_p^2 \) discretization scale would we be able (even at the gedanken-experiment level) to measure the area accurately enough to find evidence of this discretization? If not, in which sense would the discretization be “real”?

Again on the point of the noncommutativity between metric and its derivatives I want to observe that this should most intuitively contribute to some sort of uncertainty between the shape of the boundary of the surface (which specifies the area) and the
time variation of that shape (which specifies a limit on rigid motion). Instead the compatibility of Lorentz symmetry with area discretization requires, as here shown, a certain type of noncommutativity between surface velocity and surface area, which raises a few puzzles. One of these puzzles is that we already know that the velocity of the surface will become totally uncertain if we measure the position of the table (this follows straightforwardly from ordinary quantum mechanics), and now it might be surprising (and perhaps hard to implement technically) if the uncertainty of the velocity of the surface is also subject to limitations linked to the uncertainty in the area of the surface.

Concerning procedures for area measurement, as mentioned, besides the issue of the needed time of measurement it might also be significant the role that length measurements have in the measurement of areas, and this in turns provides me another link between properties of the loop-quantum-gravity formalism and the “fuzzyness” scenario. While the discrete spectrum of the area and volume operators is (in the sense here discussed) a well-established prediction of the loop-quantum-gravity formalism, the status of the length observable in loop quantum gravity is still unsettled. It is difficult (perhaps impossible) to devise an area-measurement procedure that truly avoids the use of length/distance measurements. If in loop quantum gravity the concept of length turned out to emerge as “inherently fuzzy” this would then affect any area measurement that involved a length measurement: this type of area measurement would be subject to the fuzzyness of lengths, and if the fuzzyness scale was larger than the area discretization scale the discretization would become unphysical/unobservable.

Looking at the type of spacetime geometries that correspond to area-operator eigenstates one can find additional encouragement to interpret area/volume discretization in loop quantum gravity as a manifestation of a new limitation on the measurability of area, rather than as a genuine discretization of the results of the area-measurement procedures we presently use to define areas operatively. As discussed, e.g., in Ref., these spacetime geometries do not look anything like a quasi-classical spacetime. At present the concept of area is well understood in classical spacetime. Quantum-gravity theories should first of all tell us how the concept of area can be introduced (and the properties it acquires) in spacetimes which are nearly, but not exactly, classical. Area-operator eigenstates do not admit this interpretation. A compelling quasi-classical (and particularly quasi-Minkowski) limit has not yet emerged in the quantum-gravity literature, but it appears likely that these quasi-classical spacetimes would be described in terms of a large superposition of area eigenstates. Measuring area in such quasi-classical geometries, the only ones that are likely to be accessible to us, we would never find evidence of area discretization, but only of some sort of fuzzyness of the area. Again this argument suggests that the discretization that emerged in the loop-quantum-gravity literature might not have the same physical meaning as other, more familiar, examples of discretization, such as angular-momentum discretization. When we talk of angular-momentum discretization we are still describing angular momentum in the same way as

\[ \text{Here I am puzzled by the fact that usually we see the position and velocity of the surface as properties of the COM system, while area and positions of the boundary points are seen as properties of the motion relative to the COM. The two aspects of the problem usually decouple and instead the consistency of Lorentz symmetry with area discretization appears to require that there be a link between the uncertainty of the area (a property of the motion of the boundary points with respect to the COM) and the uncertainty of the velocity of the surface (a property of the COM).} \]
done in classical physics, and we are introducing a new property (discretization) without changing the classical concept of angular momentum. The loop-quantum-gravity area discretization might instead not admit interpretation as a prediction of discrete results for measurements of the familiar area observable, but rather a manifestation of the fact that at the quantum-gravity level that simple-minded concept of area (as presently defined at the operative level) is no longer applicable.

7.4 On the role of the surface velocity in area-measurement procedures

Some of the points I raised on the loop-quantum-gravity area operator came from the observation that Lorentz symmetry acts on the \((V, A)\) measurement (simultaneous measurement of the area and the velocity of a rigidly-moving surface), just like space-rotation symmetry acts on the \((L_x, L_y, L_z)\) measurement (simultaneous measurement of the three components of angular momentum). In this Subsection I want to raise the possibility that connection between \(V\) and \(A\) measurements in a \((V, A)\) measurement might be stronger than the connection between \(L_x, L_y,\) and \(L_z\) measurements in a \((L_x, L_y, L_z)\) measurement.

For the point raised in this Subsection a key ingredient of the Stern-Gerlach procedure is that the corresponding \(L_x\) measurement does not depend on the values of \(L_y\) and \(L_z\): in the Stern-Gerlach procedure by measuring the point of arrival of an electron on a screen one can reliably deduce the value of \(L_x\) independently of the values of \(L_y\) and \(L_z\). The fact that \(L_y\) and \(L_z\) are not known does not affect the precision of the \(L_x\) measurement which in fact can be absolutely sharp (in principle). The point is that in the Stern-Gerlach setup the equation that translates the measured “point of arrival of the particle on the screen” into an \(L_x\) measurement does not depend in any way on the values of \(L_y\) and \(L_z\), so the measurement of \(L_x\) can be sharp even when little or nothing is known about \(L_y\) and \(L_z\). This statement of course holds true both in classical and in quantum mechanics. In this sense the \(L_x, L_y,\) and \(L_z\) measurements are truly independent measurements.

Between \(V\) and \(A\) measurement there appears to be a stronger connection: it might be impossible to measure \(A\) without knowing \(V\). This at least is the indication that emerges from a couple of examples of area-measurement procedures. Let us imagine that we measure the area of the surface of a table using the time-of-flight of two light bursts (I am assuming for simplicity that I have previously established that the table is rectangular, so that by measuring two sides I can obtain the area). These would be two length measurements of the type described in Subsection 3.3. The area should be obtained from two time-of-flight measurements \(T_1\) and \(T_2\). But it is not sufficient to measure \(T_1\) and \(T_2\) in order to obtain an area measurement: it is necessary to know the velocity of the table! If the table is at rest the area will be deduced from the \((T_1, T_2)\) measurement as \(A = T_1 \cdot T_2 \cdot c^2 / 4\). But if the area is moving with speed \(V\) along the direction of the \(T_1\) measurement procedure one would instead deduce from the \((T_1, T_2)\) measurement that\(^{24}\) \(A = T_1 \cdot T_2 \cdot (c^2 - V^2) / 4\).

\(^{24}\)This assumes that the \(T_1\) measurement is itself independent of the knowledge of the speed of the table. In practice it is most natural to set the clocks in rigid motion with the table, and then the \(T_1\) readout would have to be corrected in a \(V\)-dependent way by the observer, since the relevant clock is
So, within this time-of-flight measurement procedure, it is somewhat paradoxical to assume that one could get a sharp area measurement without relying on a sharp surface-velocity measurement. How could genuine area eigenstates not be simultaneously surface-velocity eigenstates? But if surface-area eigenstates are also surface-velocity eigenstates then the discretization of areas would naturally be incompatible with Lorentz symmetry.

In the case of $L_x$ measurement (a la Stern-Gerlach) there is instead no paradox in assuming that $L_x$ would be found experimentally to have discretized sharp values even when $L_y$ and $L_z$ do not. There is no paradox in the ordinary-quantum-mechanics result of $\hat{L}_x$ eigenstates which are not $\hat{L}_y$ and $\hat{L}_z$ eigenstates.

The $V$ dependence may well be an artifact of the measurement procedure here considered, and in fact I will not be able to argue for this dependence in general. However, as emphasized in Subsection 7.2, the need to know the velocity of the surface is also present in the area-measurement procedure based on a capacity measurement which had already been considered in Ref. [42]. The same holds in every area-measurement procedure I considered: all of the ones I could think of involved (more or less implicitly, but in an inevitably substantial way) a $V$ dependence of the map between the quantities actually measured (times, length, capacities,...) and the area one would attempt to evaluate through those measurements. Interesting insight could be gained if the community took the challenge of devising an area measurement procedure truly free from dependence on the velocity of the surface whose area is being measured.

In closing this subsection, since I have here introduced a second area-measurement procedure, let me stress again some of the points I already emphasized in Subsection 7.2, in the discussion of the capacity-based measurement procedure. Both area-measurement procedures satisfy my expectation that the time of measurement required by area-measurement procedures is at least of order $T \sim \sqrt{A/c}$. This I already discussed for the capacity-based measurement procedure in Subsection 7.2, and it is completely obvious in the area-measurement procedure based on time-of-flight measurements.

Clearly also the area measurement procedure based on time-of-flight measurements, just like the area measurement procedure based on capacity measurement, relies on a distance measurement. (And, as emphasized already, the status of distances in loop quantum gravity is still being debated: it may well be impossible to find an area operator with good eigenstates.)

Is worth emphasizing that also the area measurement procedure based on time-of-flight measurements, just like the area measurement procedure based on capacity measurement, relies on the assumption that the surface be absolutely flat. If the surface was only approximately flat the map between the $(T_1, T_2)$ readout and the area of interest would only be an approximate map, just like for the other measurement procedure the relation between the capacity readouts and the area of the surface is only approximate if the surface is not exactly flat.

Notice that some of the apparent obstructions to the observability of the claimed area discretization that are encountered in these measurement procedures have their not at rest with respect to the observer. The additional $\sqrt{1-V^2/c^2}$ dependence does not change the nature of the argument I am making, and I can therefore adopt the simplifying assumption that $T_1$ is measured in a $V$-independent way.
root in the same physical intuition which provided motivation for quantization of geometry. For example, at the level of formalism loop quantum gravity implements the intuition that the spacetimes we perceive (with low-energy probes) as classical are actually only approximately classical, but then the fact that the metric is not classical affects the maps \((T_1, T_2) \to A\) and \(C \to A\) that are relevant for the two measurement procedures here considered. And those maps are affected in a “fuzzy” (uncontrolled) way: the connection between the readout (respectively \((T_1, T_2)\) and \(C\)) and the quantity we want to measure \((A)\) is no longer sharp, its validity is only approximate, so that an absolute limit on the accuracy of the area measurement emerges.

### 7.5 Fuzzyness and quantum-gravity measurement theory

If indeed, as here conjectured, loop quantum gravity ends up providing us the first theoretical framework for spacetime fuzzyness, this result should probably be interpreted as a result of the (partial) diffeomorphism invariance of the approach. For decades there has been a portion of the quantum-gravity community contemplating the possibility that quantum gravity, as a result of the associated diffeomorphism invariance, might require a new measurement theory. These ideas have not captured the attention of the quantum-gravity community as a whole, probably because the arguments used to support them are often presented in a sloppy way.

A (unfortunately) popular sloppy description of the reason why a new measurement theory should be expected is based on two points: (a) the present measurement theory requires an apparatus external to the system but “nothing is external to the gravitational field”, and (b) the present measurement theory requires decoupling between system and apparatus but everything interacts with the gravitational field. This is clearly an unsatisfactory line of argument. In fact Rovelli in Ref. [42] stressed that the logical structure of measurement theory does not really require an apparatus that is “spatially external” to the gravitational field, it just requires a separation between degrees of freedom which are being studied (the system) and degrees of freedom that are used to study them (the apparatus). So point (a) does not provide good motivation for a new measurement theory. Rovelli [42] also stressed that the present measurement theory does not require decoupling between system and apparatus, on the contrary the apparatus must interact with the system in order to be able to extract the sought information about the system. In fact, measurements of electromagnetic fields are done using charged probes. From this viewpoint the fact that all probes are charged gravitationally may be seen as convenient rather than armful for our present measurement theory. So also point (b) does not provide good motivation for a new measurement theory.

The fact that most researchers advocating a new quantum-gravity measurement theory resort to the weak (wrong) arguments (a)+(b) has strongly penalized the understanding of this crucial point by the quantum-gravity community as a whole. Sadly the correct and strong argument in favour of a new quantum-gravity measurement theory should be well known since some 70 years: already in the mid 1930s Bronstein realized [45, 46] that a key point of our present measurement theory is the availability of the limit in which the charge that the probes have with respect to the fields being measured has vanishingly small effects as compared to the inertial mass of the probes. In the appropriate sense (after appropriate dimensional rescaling) a sharp measurement (such as the ones required to establish the discreteness of the spectrum
of a relevant observable) is only possible in the limit in which the ratio between the charge of the probe and the inertial mass of the probe is vanishingly small. This has been studied in detail for what concerns measurement of electromagnetic fields, where it was established \[ e/m_i \to 0 \]. Brostein and Salomon (and several other authors in more recent times, see e.g. Refs. [44, 48]) realized that the Equivalence Principle renders this limit unaccessible in the case of measurement of gravitational fields, since the Equivalence Principle imposes that the ratio between gravitational charge and inertial mass cannot be varied at all: it is fixed to 1. This obstruction that the Equivalence Principle imposes to our present measurement theory leads to the expectation that geometric observables cannot be measured sharply, that there should be an absolute limit to their measurability, that there should be some fundamental fuzziness of geometric observables. The considerations I made in this Section on the loop-quantum-gravity area operator provide an explicit example of the difficulties of sharp measurement in quantum gravity.

7.6 On the Rovelli-Speziale operators

In a study that progressed in parallel with the one I am here reporting, Rovelli and Speziale have obtained some results [49] that are relevant for some of the points I raised in this Section. From my perspective, Rovelli and Speziale report progress in realizing a first level of analogy between the role of Lorentz symmetry in loop quantum gravity and the role of space rotations in ordinary quantum mechanics. For my arguments one concludes that such an analogy requires noncommutativity between surface area and surface velocity and this is indeed (although somewhat implicitly) the type of structure that Rovelli and Speziale encountered. They construct [49] an operator \( A' \) which could plausibly (although this is a delicate point) describe the boosted area as seen in the unboosted frame that defines the canonical theory: if the canonical theory adopts the time variable of observer \( O \) it will also naturally host as an observable the (equal-time) area \( A \) of a surface, then this same surface (more properly the same world-sheet) will also define an equal-time area \( A' \) for a boosted observer \( O' \). \( O \) and \( O' \) “live” in different canonical theory (characterized by different Hilbert spaces) since their spacetime foliation (and therefore their time variable) are different, but the comparison of the operators \( A \) and \( A' \) in different canonical theories is not a natural concept. Rovelli and Speziale propose to rewrite the operator \( A' \) in terms of operators of the canonical theory of observer \( O \). At least in a certain limit (which in particular involves infinitesimal boosts) they have a definite proposal for the form of \( A' \) in the canonical theory of \( O \). They then show that \( A' \) does not commute with \( A \) and they argue that this might render Lorentz symmetry compatible with area discreteness. In fact, their operator \( A' \) when written in terms of observable of the observer \( O \) is basically (of course) a function of \( A \) and of the surface velocity \( V \), therefore the fact that \( A' \) does not commute with \( A \) follows from the fact that \( V \) does not commute with \( A \).

In Lorentz-symmetric canonical theories with area discreteness the observation that \( V \) should not commute with \( A \) is perhaps more fundamental than the observation that \( A' \) should not commute with \( A \). In fact, the noncommutativity of \( V \) and \( A \) can be formulated as a property of legitimate observables in the same canonical theory, while the noncommutativity of \( A' \) and \( A \) must always rely on some alchemy that allows to compare observables that live in different canonical theories. Both \( V,A \)
noncommutativity and \( A', A \) noncommutativity appear to allow \( A \) discretization (after all, in a Lorentz-symmetric theory, \( A' \) should roughly be an operator function of \( V \) and \( A \), although I provide below some elements of caution on this point). While my result on \( V, A \) noncommutativity followed exclusively from the analysis of the way in which Lorentz symmetry governs the results of measurements\(^25\), the observation on \( A', A \) noncommutativity reported in Ref. \([49]\) is a technical/algebraic argument drawn in the framework of the formalism of quantum mechanics (applied to spacetime). Therefore the discussion of \( A', A \) noncommutativity proposed by Rovelli and Speziale follows more closely the spirit of the already-mentioned classic paper \([25]\) by Snyder, in which the compatibility of Lorentz symmetry with spacetime discretization was first argued, with the important difference that Snyder analyzed Lorentz-symmetry transformations of distances (generously interpreting Snyder’s reference to coordinates), while Rovelli and Speziale generalized that result to the case of areas.

Besides introducing a candidate description of the operator \( A' \) in the canonical theory of observer \( O \), Ref. \([49]\) also introduces some transformation operators \( M \) that attempt to reproduce Lorentz-symmetry transformations connecting the operators \( A \) and \( A' \). Under the assumption that \( M \) is unitary Rovelli and Speziale conclude that the spectrum of \( A \) and \( A' \) must be the same.

The operators \( A', M \) introduced by Rovelli and Speziale represent an important development for the loop-quantum-gravity approach, which indeed provides encouragement for a first level of analogy between the role of Lorentz symmetry in loop quantum gravity and the role of space rotations in ordinary quantum mechanics. Their analysis relies on some conjectures, especially concerning the unitarity of \( M \) and some implicit assumptions about a surface-velocity operator. The surface-velocity operator has not yet been rigorously introduced in loop quantum gravity but clearly governs the properties of the \( A' \) operator (from the perspective of observer \( O \)). If these conjectures prove to be correct the first level of analogy between the role of Lorentz symmetry in loop quantum gravity and the role of space rotations in ordinary quantum mechanics will have been established and we will be left with the more delicate issues which I also raised, together with \( V, A \) noncommutativity, in this section: those aspects of Lorentz symmetry in a canonical theory which instead appear to require that the analogy with space rotations in ordinary quantum mechanics be limited.

An example of conjectured properties whose verification deserves attention is found in the interesting but delicate point of the study reported in Ref. \([49]\) which describes the operator \( A' \) that “belongs” to observer \( O' \) in terms of operators of the \( O \) observer. This description is obtained through a limiting procedure (infinitesimal boosts...). This attempt to describe the operators of \( O' \) in terms of operators of \( O \) could be relevant for my concern about the problem of the absence of the world-sheet from the formalism. Lorentz symmetry naturally invites us to make direct reference to the world-sheet and construct different equal-time observables by suitable projections, but the canonical theory would describe each of these projections as leaving in different theories (different Hilbert spaces, different canonical theories characterized by different time

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\(^25\)In fact my result applies to any theory predicting discretized areas, whether or not the theory makes use of the formalism of quantum mechanics. What I mean by noncommutativity does not require the algebraic concept of commutation relations between quantum-theory operators. It simply corresponds to a statement about the results of measurements: in a Lorentz-symmetric theory with area discretization \( V \) cannot be sharply measured when \( A \) is sharply measured.
variable/parameter). If we really succeed in giving a faithful description of the operators of all observers in the terms of the operators of one observer, perhaps the need of the world-sheet will be relaxed. However, especially in light of its importance for the physical problem here of interest, this hypothesis must still be investigated in greater depth. I fear that the canonical formalism will not allow us to make genuine predictions concerning Lorentz symmetry: rather than showing that the measurements done by $O'$ on the same world-sheet give the results expected according to Lorentz symmetry (in comparison with the results obtained by $O$ on that same world-sheet) the canonical formalism will only allow us to enforce by hand a description of the operator $A'$ as a certain function (roughly given by a function of $V$ and $A$) of the operators that live in the canonical theory of observer $O$. In this case we would risk to misinterpret Lorentz symmetry as a prediction of the theory, while instead we are just making use of our knowledge of the rules of Lorentz-symmetry transformations to introduce in the $O$'s canonical theory some observables (functions of $V$ and $A$) which do not genuinely describe the Lorentz-transformed observables (they might not truly describe observations performed by $O'$, but rather describe some complicated functions of natural observations, such as $V$ and $A$, conducted by $O'$). In order to gain insight on these issues it may be useful to push further the approach proposed by Rovelli and Speziale. What happens if finite boosts are considered? Is the description of $A'$ in terms of $O$’s operators still satisfactory? Can one set up a satisfactory Heisenberg (or Schroedinger) picture if $O$ must make use of both “his own operators” and “$O'$ operators”? [For example, in the Schroedinger picture we want time-independent observables and time-dependent states, but it might be hard to describe $A'$ as time-independent from the $O$ perspective, since according to $O$ the surface described by $A'$ is not equal-time.]

As mentioned, I feel that, together with these additional investigations of the technical aspects of the Rovelli-Speziale approach, the analysis reported in Ref. [49] can provide a natural starting point for the investigation of some of the delicate issues I raised in connection with the limits that one must impose to the analogy between the role of Lorentz symmetry in loop quantum gravity and the role of space rotations in ordinary quantum mechanics. In particular, Rovelli and Speziale find encouragement for the idea that area eigenstates of observer $O$ might be mapped into states that are not area eigenstates for observer $O'$. This renders even more urgent the point I raised concerning the fact that Lorentz symmetry (unlike space-rotation symmetry) also governs some measurements (see Subsections 3.3, 3.4, 3.5) in which two observers share the measurement procedure: both observers use the same readout (although they adopt different calibrations) to measure the same area. How can then two observers disagree on whether the results can be characterized with an area eigenstate?

### 7.7 Field theory and covariant reformulations of loop quantum gravity

Some of the concerns I discussed here and some of the points that must still be addressed, even after the important developments in the loop-quantum-gravity formalism reported by Rovelli and Speziale [49], have to do with the interplay between space/time transformations, like Lorentz boosts, and the structure of a canonical quantum theory, with the peculiar role attributed to the time variable/coordinate/parameter.

It is difficult to formulate a conjecture on the outlook of these issues in a covariant reformulations of loop quantum gravity, such as the ones being already explored...
relying on the so-called “sum over histories” formalism \[8\]. Perhaps this covariant re-
formulation will lead to a satisfactory description of equal-time observables in terms of
world-sheets, world-lines and their relations (intersections...).

Some insight might be gained by using as guidance the success of field theory in
background Minkowski spacetime, which is indeed a quantum theory with Lorentz
symmetry. The guidance provided by field theory will however be limited, since field
theory lives in classical and nondynamical spacetime and of course it makes no quantum
predictions concerning spacetime observables (such as areas, volumes...).

7.8 Specifically on discretization of rest area

In the preceding subsections I have presented my case for area fuzzyness in loop quan-
tum gravity. For completeness let me return to the hypothesis (which however I dis-
favour) that the area discretization is observable. And let me focus on the special case
in which the spectrum discussed in the loop-quantum-gravity literature would only
apply to the case of an area at rest.

As clarified above, Lorentz symmetry should be abandoned if that same spectrum
applied to areas of all velocities (sharply determined velocities). It is instead possible to
assume that the presently-adopted spectrum applies only to areas at rest (sharply de-
termined to have zero velocity). However, it would then be necessary to assume/predict
that boosted observers (observers that assign a nonvanishing velocity to the area) would
see a boosted picture of that spectrum (including a Lorentz-Fitzgerald contraction of
the discretization scale). The area operator and its spectrum should depend on the
velocity of the area. One might be tempted to think otherwise, for example assuming
that Lorentz symmetry might be preserved if the same physical surface could be an
area eigenstate for one observer and not an area eigenstate for another observer (this
would in principle allow that the spectrum be the same for all observers, even the
ones in motion with respect to the area); however, consistency with classical Lorentz
symmetry imposes that an area eigenstate with well-specified velocity of the surface
(in particular, “at rest”, \(V = 0\)) must be an area eigenstate with well-specified velocity
for all other inertial observers.

I find that the idea the the area operator which is being adopted by the loop-
quantum-gravity research programme would refer to the rest area of a surface is peculiar
also from a technical perspective: the structure of that operator only makes reference
to some space coordinates that delimit the area. It includes no reference to the fields
that physically identify that surface and it makes no reference to the time evolution of
the surface. As required by canonical quantization, it is just an operator that refers to
quantities defined at a fixed time. The operator does not specify in any way where the
surface will be at a later time. At a later time (however small is the time difference)
the surface could well be still identified by those coordinates, but it might equally well
have moved somewhere else, and in both cases the area operator would just take the
fixed-time section of the world-sheet identified by the surface and calculate its area
according to a certain prescription.
7.9 On Lorentz-symmetry deformations and on the Gambini-Pullin mechanism for induced violations of Lorentz symmetry

In closing this Section of the fate of Lorentz symmetry in loop quantum gravity I find appropriate to emphasize the differences between the type of departure from Lorentz symmetry I considered here and certain other studies of departures from Lorentz symmetry. I have been concerned with the status of Lorentz symmetry at the fundamental theory level. In a series of papers initiated by a study by Gambini and Pullin [2, 50] deviations from Lorentz symmetry motivated by loop quantum gravity had already been considered, but these are of totally different nature. Gambini and Pullin rely on the introduction of a background of a “weave state” [51]. The type of Lorentz-symmetry violation they discuss is the very familiar one encountered whenever a background is introduced. It is a departure from Lorentz symmetry which is induced by the presence of the background, rather than being truly fundamental in the theory. The Gambini-Pullin mechanism is the one we are familiar with from the study of the behaviour of light in water, crystals, and other media and also from theory work on certain fixed-background spacetime (such as the canonical noncommutative spacetimes discussed in the preceding Section). The Gambini-Pullin mechanism in fact is not in any way related with the discretization of areas/volumes. On the contrary my considerations on the fate of Lorentz symmetry in loop quantum gravity concern the truly fundamental level of analysis of the theory. I am not considering any special choice of background. I am looking for some fundamental implications of the spacetime discretization encountered in the loop-quantum-gravity formalism.

The nature of my considerations on the fate of Lorentz symmetry in loop quantum gravity is also not directly connected with the proposal I put forward [12] of deformations of Lorentz symmetry. While, as discussed in Subsection 6.1, those deformations should provide the exact/fundamental description of the symmetries of certain noncommutative spacetimes, I expect that their applicability to loop quantum gravity, if any, should be confined to contexts in which some level of coarse-graining of the fundamental spacetime picture has been advocated. It appears to be plausible that a deformation of Lorentz symmetry might emerge in the study of the limiting procedure that the loop-quantum-gravity research programme must identify in order to make contact with classical spacetimes. In fact, while the classical limit must be described by ordinary classical symmetries (in particular classical flat spacetimes should have Lorentz invariance), if one stops the limiting procedure a bit before the classical limit, where spacetime would already look “quasi-classical”, it is plausible that a deformation of Lorentz symmetry would play a role. In this author’s opinion the classical-spacetime limit should correspond to the limit in which the particles that probe spacetime have extremely-low energy (extremely-large wavelengths). For probes of finite but low energy (“moderately-large” wavelength) spacetime would be perceived as “quasi-classical” (and possibly characterized by a deformation of Lorentz symmetry), but in the low-energy limit a truly classical spacetime would emerge (together with its ordinary classical symmetries, including Lorentz invariance of flat spacetimes).
8 Closing remarks

It is not accidental that in this study the number of issues that have been raised is much larger than the number of issues for which an answer has been provided. While it is not difficult to analyze the fate of classical spacetime symmetries in theories in which nonclassical properties are only assigned to non-spacetime degrees of freedom, the analysis of spacetime symmetries in nonclassical pictures of spacetime itself involves a large number of subtle issues. In some cases there is not even an *a priori* intuitive way to rephrase the questions we usually ask of a spacetime symmetry in classical spacetime. Perhaps the best example of this is the concept of spontaneous breaking of spacetime symmetries: we are familiar with spontaneous symmetry breaking in particle-physics theories, where the concept of “vacuum” is clear to us, but we lack any depth in the understanding of the “spacetime vacuum”.

In the type of noncommutative geometries here considered, using some recent mathematical-physics results, it is now clear that classical Lorentz symmetry does not hold. For canonical spacetimes, the simplest case from the technical perspective, we even have a rather deep understanding of the relevant type of violation of Lorentz symmetry, but, unless we are willing to accept the existence of truly preferred observers, we must devote urgent attention to the search of spontaneous-symmetry-breaking mechanisms that might support it. In $\kappa$-Minkowski Lie-algebra spacetime it is clear that the appropriate concept of transformation rules between inertial observers will require the concept of deformed Lorentz symmetry (a group of finite transformations) introduced in Refs. [12], and it is clear that (at least in the one-particle sector [12]) infinitesimal symmetry transformations should be described by a Hopf algebra of the type considered in Refs. [13, 17]. But the precise description of the symmetries of $\kappa$-Minkowski still requires further study; in particular, certain mathematical-physics studies [17, 14, 15] appear to argue that several Hopf algebras are equally good candidates as tools for the description of the symmetries of $\kappa$-Minkowski spacetime. This is clearly unacceptable physically: we are allowed to introduced meaningfully the concept of “symmetry of a spacetime” only if we are able to associate to a given spacetime one and only one mathematical structure which describes its symmetries. This is another issue that deserves urgent investigation, particularly since preliminary estimates [11] of the departures from ordinary Lorentz symmetry required by $\kappa$-Minkowski suggest that forthcoming experiments [19] should be able to test the prediction.

The case of loop quantum gravity is even more interesting. While the examples of noncommutative spacetimes I considered have relatively narrow objectives (at best providing an effective description of spacetime in an appropriate quasi-classical and quasi-flat limit) the loop-quantum-gravity research programme is constructing a full ambitious description of quantum gravity. As the issue of Lorentz symmetry gains importance in light of the developing experimental situation, the loop-quantum-gravity research programme discovers to be unprepared to provide this key prediction. As shown here the loop-quantum-gravity analysis of the concept of area in quantum spacetime is insufficient for providing guidance on this important issue, in spite of the emergence of discretization. It is on this issue of the description of areas in loop quantum gravity and the associated analysis of Lorentz transformations that new technical and conceptual developments appear to be most urgently needed.

Concerning loop quantum gravity my analysis has provided both material in support of preservation of ordinary Lorentz symmetry and material in support of loss of Lorentz symmetry. My point that area discretization is not in general inconsistent with
Lorentz symmetry might be of encouragement for the idea that Lorentz symmetry be preserved in loop quantum gravity, but it opens the challenge of a proper definition of the observable “velocity of the surface”. While at first sight the construction of this observable does not appear to be confronted with in-principle obstructions, some of the observations I reported here about the interplay between surface velocity and surface area in the way in which Lorentz symmetry is realized in our (present) observations may suggest that some obstacle for Lorentz symmetry might be eventually encountered in loop quantum gravity. I have analyzed various sources of concern for the compatibility of Lorentz symmetry with the type of area discretization predicted by loop quantum gravity, which indeed are all in some way connected with the interplay between surface velocity and surface area, and from a physical/operative perspective originate from the following two challenges: (i) the need to identify a area-measurement procedure which is not affected by the surface-velocity uncertainty (otherwise the non-commutativity between surface velocity and surface area, needed for the compatibility between area discretization and Lorentz symmetry, will end up rendering unobservable the discreteness of the area spectrum), and (ii) the need to understand the role that measurement procedures such as the ones described in Subsections 3.3, 3.4, 3.5, in which the readouts of the same measurement procedure are meaningful for different inertial observers, should have in a theory in which area eigenstates for one inertial observer are predicted to be mapped into superpositions of area eigenstates for another inertial observer. (On the one hand eigenstates for $O$ must go into superpositions of eigenstates of $O'$ if area is discretized but the expectation values of the area observable satisfy the usual FitzGerald-Lorentz contraction, on the other hand measurement procedures such as the ones described in Subsections 3.3, 3.4, 3.5 appear to require that eigenstates of length/area/volume should be eigenstates for all inertial observers.) 

The construction and careful analysis of the observable “surface velocity” in loop quantum gravity is the key to the understanding of the fate of Lorentz symmetry in that theory and is also important for establishing whether or not area discreteness is, at least in principle, observable. Through the understanding of the properties of this, still unknown, surface-velocity observable we could establish whether its commutation relations with the surface-area observable are of a type that may render area discreteness compatible with Lorentz symmetry, and, perhaps, we could also gain some insight on the issues related with my concerns for the operative definition of the area observable. Very little of what I observed in Section 7 is specific to loop quantum gravity: my observations apply to any canonical quantum theory of gravity with area discretization. My observations all originate from the fact that, in the example of the area observable, the primary/fundamental entity from the Lorentz-symmetry perspective is the surface’s world-sheet, but in a canonical quantum theory one can only meaningfully introduce observables that are defined at a fixed time. The programme of investigation of loop quantum gravity for which the present study provides motivation might also provide insight on the wider subject of the interplay between area/volume discretization and Lorentz symmetry in other canonical quantum theories of gravity.

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