Spurious microwave crosstalk in floating superconducting circuits

Peng Zhao,† Yingshan Zhang,† Xuegang Li,† Jiaxiu Han,† Huikai Xu,† Guangming Xue,†, ‡ Yirong Jin,† and Haifeng Yu†

1Beijing Academy of Quantum Information Sciences, Beijing 100193, China

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Crosstalk is a major concern in the implementation of large-scale quantum computation since it can degrade the performance of qubit addressing and cause gate errors. Finding the origin of crosstalk and separating contributions from different channels is essential for figuring out crosstalk mitigation schemes. Here, by performing circuit analysis of two coupled floating transmon qubits, we demonstrate that, even if the stray coupling, e.g., between a qubit and the drive line of its nearby qubit, is absent, microwave crosstalk between qubits can still exist due to the presence of a spurious crosstalk channel. This channel arises from free modes, which are supported by the floating structure of transmon qubits, i.e., the two superconducting islands of each qubit with no galvanic connection to the ground. For various geometric layouts of floating transmon qubits, we give the contributions of microwave crosstalk from the spurious channel and show that this channel can become a performance-limiting factor in qubit addressing. This research could provide guidance for suppressing microwave crosstalk between floating superconducting qubits through the design of qubit circuits.

I. INTRODUCTION

Integrating a growing number of qubits without sacrificing quantum gate performance is a key task in the implementation of large-scale quantum computers with superconducting qubits [1]. One of the main obstacles that needs to be overcome in this task, is crosstalk, including classical crosstalk due to unintended classical electromagnetic couplings [2–8] and quantum crosstalk arising from residual quantum coupling [5, 9–12], which can make qubit addressing a challenge [9] and degrade gate performance in multiqubit quantum processors [13]. In this context, the progress in understanding and mitigating crosstalk has made indispensable contributions to the impressive achievements toward developing large-scale superconducting quantum computing over the past decade.

One of the most ubiquitous crosstalk for superconducting quantum processors is the microwave crosstalk, which describes that microwave drives applied to one qubit can cause unintended drives felt by the others. To address this issue, various active cancelation methods, i.e., one first characterizes it and then cancels it actively with a compensation drive, have been demonstrated [14, 15]. Nevertheless, mitigating the crosstalk at device level may complement existing active approaches and further reduce the needed physical resource, especially for large-scale quantum processors. Indeed, previous works show that the crosstalk can be suppressed at the device level, but only if the origin of the crosstalk and contributions from different crosstalk channels are well understood [2, 6].

Generally, to implement universal control over qubits in quantum processors, at least a single drive line (drive channel) per qubit is needed. Hence, as shown in Fig. 1, there are three possible microwave crosstalk channels in superconducting quantum processors, including (Type-1) stray coupling between the dedicated drive lines of qubits, the direct (Type-2) and the indirect (Type-3) coupling between one qubit and the drive line of others. For large-scale quantum processors, a high density of control wiring is required, thus the crosstalk through the first two channels could become more serious [2, 4, 6]. Generally, the two channels can be mitigated by improving the physical isolation between drive lines and qubits [4]. For the Type-3 channel, however, its physical origin and mitigation seems to be more nontrivial. In principle, its origin can be modeled by the indirect coupling between one qubit and the drive line of the others through an effective circuit network [16]. Physically, the circuit network could arise from free modes, e.g., between a qubit and the drive line of its nearby qubit, as such a direct coupling capacitor or a bus coupler. Microwave crosstalk can be arisen from: (Type-1) the stray coupling between the two drive lines; (Type-2) the direct or (Type-3) indirect coupling between one qubit and the drive line of its neighbor.

FIG. 1: Three possible channels of microwave crosstalk between two coupled superconducting qubits. Each qubit (cross-shaped) has a dedicated drive line (T-shaped) for single-qubit addressing. The two qubits are coupled via a coupler circuit (strip-shaped), such as a direct coupling capacitor or a bus coupler. Microwave crosstalk can be arisen from: (Type-1) the stray coupling between the two drive lines; (Type-2) the direct or (Type-3) indirect coupling between one qubit and the drive line of its neighbor.

In this work, we present a spurious microwave crosstalk...
channel (Type-3) enabled by the presence of free modes in floating transmon circuits [19–22]. By performing circuit analysis of coupled floating transmon qubits with different geometric layouts [23–25], we show that the microwave crosstalk contributed from this spurious channel can become non-negligible, thus potentially limiting the performance of qubit addressing. More importantly, this crosstalk channel only depends on the qubit circuit itself, thus acting as an intrinsic channel, which can exist even when the stray coupling, e.g., between drive lines and qubits, is absent. This feature also suggests that this spurious channel can be mitigated through qubit circuit design.

This paper is organized as follows. In Sec. II, we analyze the quantum circuit of two direct-coupled floating transmon qubits (in the Appendix we further extended our analysis to the case, where floating transmon qubits are coupled via a grounded or floating bus coupler) and show that the presence of the free modes can induce a spurious microwave crosstalk channel. In Sec. III, for transmon qubits with different qubit geometric layouts, we give the contributions of microwave crosstalk from the spurious channel. In Sec. IV, we give discussions on the relation of the free-mode-mediated spurious crosstalk illustrated in two recent works [26, 27] and show that the present work can be viewed as complements to the two earlier works, extending the free-mode mediated interactions from “quantum regime” (for inter-qubit coupling) to “semi-classical regime” (for classical microwave crosstalk). Finally, in Sec. V, we provide a summary of our work.

II. SPURIOUS MICROWAVE CROSSTALK CHANNEL

To understand the origin of the spurious microwave crosstalk channel, we consider a superconducting circuit comprising two direct-coupled floating transmon qubits ($Q_1$ and $Q_2$), and one of the two qubits ($Q_1$) is coupled capacitively to a voltage source, as shown in Fig. 2(a). When expressed in terms of the node flux variables $\Phi_j$ and $\phi_j = 2\pi \Phi_j / \Phi_0$ with $\Phi_0$ the magnetic flux quantum, the circuit Lagrangian is given by (details on its derivation can be found in Appendix A) [28, 29]

$$\mathcal{L} = \frac{1}{2} \Phi^T C \Phi + E_{J1} \cos(\phi_{1m}) + E_{J2} \cos(\phi_{2m}),$$

(1)

where $\Phi = (\Phi_d \Phi_{1p} \Phi_{1m} \Phi_{2p} \Phi_{2m})^T$ with $\Phi_{1p(m)} = \Phi_1 \pm \Phi_2$ and $\Phi_{2p(m)} = \Phi_3 \pm \Phi_4$, $E_{J1}$ and $E_{J2}$ are the Josephson energies, and $C$ denotes the capacitance matrix of the circuit. Accordingly, the circuit Hamiltonian can be constructed as

$$H = \sum_j Q_j \dot{\Phi}_j - E_{J1} \cos(\phi_{1m}) - E_{J2} \cos(\phi_{2m})$$

with the charge variables $Q_j = \partial \mathcal{L} / \partial \dot{\Phi}_j$. Here, the modes associated with the variables $Q_{1m}$ and $Q_{2m}$ are the qubit modes, which have both the charge energy and the potential energy [22], while the modes associated with $Q_{1p}$ and $Q_{2p}$, and which don’t have potential terms in the Hamiltonian, are the free modes [19–21]. From the derivation and also mentioned in previous works [19–21], the presence of the free modes are supported by the floating structure of transmon qubits, i.e., the two superconducting islands of the qubit have no galvanic connection to the ground, as shown in Fig. 2(a), contributing to an additional quantum degree of freedom.

Since free modes actually don’t participate in the circuit dynamics [19–21], one can drop the charge terms corresponding to the two free modes and rewrite the circuit Hamiltonian as

$$H_T = \frac{1}{2} Q_T^T C^{-1} Q_T - E_{J1} \cos(\phi_{1m}) - E_{J2} \cos(\phi_{2m}),$$

(2)

with $Q_T = (Q_d Q_{1m} Q_{2m})^T$. Here, $C_T$ denotes the reduced capacitance matrix, which can also be used to describe a circuit system consisting of two direct-coupled grounded transmon qubits, as shown in Fig. 2(b) (see Appendix A for details). Here, for illustration purpose, considering a typical case where all the island capacitors take a same capacitance, i.e., $C_{g1} = C_{g2} = C_{g3} = C_{g4} = C_g$ and both the island capacitance $C_g$ and the shunt capacitance $C_q$ largely exceed the coupling capacitances, i.e., $\{C_g, C_q\} \gg \{C_d, C_{c1}, C_{c2}\}$, the matrix $C_T$ can be approximated by

$$C_T \approx \begin{pmatrix}
C_d & -C_q & -C_d(C_g - C_{c2}) \\
-C_d & C_q + C_{c2} & -C_d(C_g - C_{c2}) \\
-C_d(C_g - C_{c2}) & -C_d(C_g - C_{c2}) & C_g + C_{c2}
\end{pmatrix}.$$  

(3)

Inspecting $C_T$, one can find that both the matrix elements $[C_T]_{d,1m}$ and $[C_T]_{d,2m}$, which describe the coupling between the two qubit modes and the external voltage source, take nonzero values. This means that although in the original circuit, only one of the two qubits is coupled to the voltage source, here, both of the two qubits are coupled to the voltage source simultaneously. As a consequence, after dropping

![Figure 2](image-url)
the free modes, the full circuit shown in Fig. 2(a) can be transformed into an equivalent circuit shown in Fig. 2(b). The most striking result from the equivalence is that due to the presence of the free modes, a spurious microwave crosstalk channel can exist, even if the stray coupling between qubits and drive lines is absent.

To quantify the crosstalk through the spurious channel, we consider the crosstalk strength defined as $M_{ij} = 20 \log_{10}(\Omega_i/\Omega_j)$, expressed in units of dB. Here, $\Omega_j$ denotes the magnitude of the microwave drive applied to a target qubit (e.g., $Q_1$), while $\Omega_i$ represents the magnitude of the crosstalk felt by the other nearby qubit (e.g., $Q_2$). For a grounded transmon qubit coupled to an external voltage source $V_d$ via a coupling capacitor $C_d$, the magnitude of the drive can be approximated by $\Omega = C_d Q_{xp} V_d/C_q$ with $Q_{xp} = \sqrt{\hbar/2Z_q}$ the zero-point charge fluctuations and $Z_q = \sqrt{L/C}$ the qubit impedance [30]. $L$ and $C$ correspond to the qubit inductance and capacitance, respectively. For illustration purposes only, we further assume that both qubits have the same qubit inductances and capacitances, thus in the system shown in Fig. 2(a), the strength of the spurious microwave crosstalk from $Q_1$ to $Q_2$ can be expressed by

$$M = 20 \log_{10} \left( \frac{\left| C_r \right|_{d,2m}}{\left| C_r \right|_{d,1m}} \right) = 20 \log_{10} (R),$$

with the ratio $R$ given by

$$R = \left| \frac{C_{g1} C_{c1} - C_{g2} C_{c2}}{(C_{g3} + C_{g4})(C_{c2} + C_{g2}) + C_{g2}(C_{c1} + C_{c2})} \right|. \quad (4)$$

As shown in Eq. (5), the spurious crosstalk only depends on the qubit circuit parameters, thus acting as an intrinsic crosstalk channel. Moreover, since in coupled qubit circuits, the coupling capacitors, e.g., $C_{c1}$ and $C_{c2}$, generally have a rather small capacitance, which is typical of the order of a few fF’s or less, the spurious crosstalk can be suppressed through increasing the island capacitance. However, depending on the qubit geometric layout, the island capacitance can take a wide range of values, typically, ranging from a few fF to 100 fF. Hence, as we will show below, the spurious crosstalk can become non-negligible, thus limiting the performance of qubit addressing.

While here we focus on the direct-coupled qubit system, in Appendices B and C, we also extend the above analysis to the indirect-coupled qubit systems, including floating qubits coupled through a grounded bus or a floating bus. We find that the spurious crosstalk channel disappears for floating qubits coupled via the grounded bus, while it still exists for qubits coupled via the floating bus. This opposite conclusions further demonstrate that the presence of the spurious crosstalk is mediated by the free modes in the qubit circuit. In addition, for floating qubits coupled by the grounded bus, the analysis also shows that the grounded bus can also feel the drive applied to the floating qubit, i.e., a spurious crosstalk channel between the floating qubit and the grounded bus is existent. Thus, we conclude that for the crosstalk from $Q_1$ to $Q_2$ in the system shown in Fig. 2(a), this spurious crosstalk is in fact mediated by the free mode supported by the driven qubit $Q_1$, and it will still exist even if $Q_2$ is a grounded one.

![FIG. 3: Different qubit geometric layouts of floating transmon qubits.](image)

The drive line (for single-qubit addressing) and coupler (for coupling two qubits) can be coupled capacitively to the same superconducting island or two opposite islands, thus giving rise to two different geometric layouts, i.e., the same-island layout and the opposite-island layout. (a) The typical floating transmon qubit with symmetric islands. (b) The coaxial transmon (coaxmons) with asymmetric islands. (c) The floating merged-element transmon qubit (MET). In (a) and (b), the capacitance between the island and the ground (island capacitance), in general, largely exceeds the coupling capacitance between qubits (coupling capacitance), while in (c) the island capacitance can be comparable with or even be smaller than the coupling capacitance.

### TABLE I: Summary of crossstalk ($R$) in floating transmon qubits with different qubit geometric layouts. Here $r \equiv C_c/C_g$ denotes the ratio of the coupling capacitance to the island capacitance, and hereafter we refer to it as the capacitance ratio.

| Geometric layout | Asymmetric | Symmetric |
|------------------|------------|-----------|
|                  | $(\lambda \neq 1)$ | $(\lambda = 1)$ |
| Same-island      | \(C_{c1} \) | \(C_{c1} \) |
| \(C_{c1} = C_{c2}, C_{c2} = 0\) | \(\frac{(\lambda + 1)C_g + C_{c1}}{\lambda + 1 + r} = \frac{r}{2 + r}\) | \(\frac{2C_g + C_{c1}}{2C_g + 3C_{c1}}\) |
| Opposite-island  | \(C_{c2} \) | \(C_{c2} \) |
| \(C_{c1} = 0, C_{c2} = C_c\) | \(\frac{r}{\lambda(\lambda + 1) + (2\lambda + 1)r} = \frac{r}{2 + 3r}\) | 

### III. SPURIOUS MICROWAVE CROSSTALK IN TYPICAL FLOATING TRANSMON CIRCUITS

The above discussion indicates that the spurious crosstalk channel is intrinsic, and its strength only depends on the qubit circuit parameters, i.e., the concrete qubit geometric layout. Here, to study this geometric dependence, we consider that for the coupled qubit system shown in Fig. 2(a), $C_{g1} = C_{g3} = C_g$ and $C_{g2} = C_{g4} = \lambda C_g$, thus giving rise to

$$R = \left| \frac{\lambda C_{c1} - C_{c2}}{(\lambda + 1)(C_{c2} + \lambda C_g) + \lambda(C_{c1} + C_{c2})} \right|. \quad (6)$$

Thus, according to the ratio $\lambda$ of the two island capacitances (hereafter we refer to it as the island asymmetric ratio), there are two main island geometric layouts for floating transmon qubits, i.e., symmetry layout and asymmetry layout. For ex-
ample, the traditional floating transmon qubits have two same islands [23], as shown in Fig. 3(a), thus acting as a symmetric one, while for the coaxial transmon [24], as shown in Fig. 3(b), its two islands have different geometric designs, thus it belongs to the asymmetric one.

Additionally, note that in principle, the drive line and the coupler (e.g., the capacitors \( C_{c1} \) and \( C_{c2} \)) could be coupled to one of the two islands or both of the two islands simultaneously, as shown in Fig. 2(a). However, for practically implemented floating qubit circuits, as shown in Fig. 3, generally, the drive line and the coupler are dominantly coupled to one of the two islands. Hence, as shown in Fig. 3, here we consider two coupling geometric layouts, i.e., the drive line could be coupled capacitively to the same island or the opposite island with respect to the coupling island, which is coupled capacitively to the other nearby qubits. For example, in Fig. 2(a), when \( C_{c1} \neq 0 \) and \( C_{c2} = 0 \), the coupling geometric layout is a same-island one, while for \( C_{c1} = 0 \) and \( C_{c2} \neq 0 \), it is an opposite-island case.

According to the above mentioned island geometric layout (symmetric v.s. asymmetric) and coupling geometric layout (same-island v.s. opposite-island), Table I lists the expressions of the crosstalk strengths for four different qubit geometric layouts. Accordingly, Figure 4 shows the crosstalk strengths as a function of the capacitance ratio \( r \), i.e., the ratio of the coupling capacitance to the island capacitance. As shown in Fig. 4(a), for the traditional floating transmon qubits, where the island capacitance is generally far larger than the coupling capacitance, thus the spurious crosstalk can be entirely suppressed below, e.g., \(-30 \) dB, regardless of the coupling geometric layout. However, for the floating merged-element transmon qubit (MET) with a symmetric layout [25], where the island capacitance can be comparable to or even smaller than the coupling capacitance, the spurious channel can become the dominated microwave crosstalk source. In this situation, the magnitude of the spurious crosstalk can even be comparable to that of the target drive. In addition, compared to the case of the same-island layout, qubits with the opposite-island layout, especially for the floating MET qubits, can show a less pronounced spurious crosstalk, typically below \(-10 \) dB.

Figure 4(b) shows the spurious crosstalk strength for qubits with the asymmetric island layout. One can find that generally larger island asymmetric ratio \( \lambda \) can mitigate the spurious crosstalk, and can become even more noticeable when taking the opposite-island layout. Thus, for the coaxial transmon shown in Fig. 3(b), when the drive line is coupled to the center island and inter-qubit coupling is realized via the outer island, the spurious crosstalk could be readily suppressed below \(-50 \) dB. Moreover, while the spurious channel acts as a significant source of the crosstalk for the MET qubits with the symmetric island layout, by taking the opposite-island layout and increasing the island asymmetric ratio, the spurious crosstalk is promising to be pushed below \(-25 \) dB.

Given the state-of-the-art values of the microwave crosstalk in multiqubit quantum processors, typically ranging from \(-25 \) dB to \(-30 \) dB for neighboring control lines [31, 32], we conclude that: (i) for the traditional floating transmon qubits with symmetric island layout, as shown in Fig. 3(a), the spurious crosstalk can be ignored safely if the island capacitance largely exceeds the coupling capacitance. (ii) for the floating transmon qubits with asymmetric island layout, such as the coaxial transmon shown in Fig. 3(b), taking a larger asymmetric ratio could help to suppress the spurious crosstalk. This suppression can become more prominent when employing the opposite-island layout. (iii) for the floating MET qubits with symmetric island layout, as shown in Fig. 3(c), the spurious crosstalk is less readily suppressed. Nevertheless, similar to the coaxial transmon, the combination of the asymmetric island layout and the opposite-island layout will hopefully lead to adequate suppression of the spurious crosstalk.

IV. DISCUSSION

Recently, two independent works show that the free mode can mediate an indirect inter-qubit coupling [26, 27]. In Ref. [26], Sete et al. have demonstrated that for qubits coupled via a floating tunable coupler (i.e., an additional frequency-tunable floating transmon qubit), the free mode due to the floating structure of the coupler can induce an indirect inter-qubit coupling, for which its strength only depends on the circuit capacitance, rather than the coupler frequency. Thus, combined with the coupler mediated dispersive coupling, tunable coupling between qubits can be realized by adjusting the coupler frequency [26, 33]. Similarly, in Ref. [27], Yanay et al. have shown that in an array of floating transmon qubits,
the free mode can mediate interactions between next-nearest neighbor qubits and even beyond next-nearest neighbors. The strength of the mediated interaction can be engineered via circuit design. Similarly, our results show that the free mode can also mediate cross-driving, enabling the presence of an additional intrinsic crosstalk channel. Hence, our analysis can be viewed as complements to the two earlier works, extending the free-mode mediated interactions from “quantum regime” (for inter-qubit coupling) to “semi-classical regime” (for spurious microwave crosstalk). Moreover, by extending to this semi-classical regime, it is also reasonable to expect that the floating transmon qubit can also relax through the control lines of its coupled neighbors.

V. CONCLUSION

In summary, by performing circuit analysis of two coupled floating transmon qubits, we have demonstrated that the free mode, which is supported by the floating structure of qubits, can induce a spurious microwave crosstalk channel. Depending on the actual qubit geometric layout, the spurious crosstalk can become non-negligible, and can even limit the performance of qubit addressing. Thus, to ensure higher-fidelity qubit addressing, the spurious crosstalk needs to be carefully considered. To address it, we have also shown that for various typical floating transmon circuits, the spurious crosstalk can be largely suppressed through circuit design.

Although our present analysis of the spurious crosstalk focuses on the floating transmon qubits, we expect that the analysis and many of the results may also be applied to other types of superconducting circuits with floating islands, such as flux qubits [34] and fluxonium qubits [35].

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Appendix A: Direct Capacitor

Here, we consider two floating qubits coupled via direct capacitors, as shown in Fig. 2(a). The Lagrangian of the circuit can be expressed in terms of the node flux variables \( \Phi_n \) with \( n = \{d, 1, 2, 3, 4\} \), as denoted in Fig. 2(a), and is given as [29]

\[
L' = \frac{1}{2} \dot{\Phi}'^T C' \dot{\Phi}' + E_{J1} \cos(\phi_1 - \phi_2) + E_{J2} \cos(\phi_3 - \phi_4)
\]

(A1)

where \( E_{J1} \) and \( E_{J2} \) are the Josephson energies, \( \phi_n = 2\pi \Phi_n / \Phi_0 \) with \( \Phi_0 \) the magnetic flux quantum, \( \Phi' = (\Phi_d \Phi_1 \Phi_2 \Phi_3 \Phi_4)^T \), and \( C' \) the capacitance matrix, given as

\[
C' = \begin{pmatrix}
C_d & -C_d & 0 & 0 & 0 \\
-C_d & C_{\Sigma_1} & -C_q & -C_{\Sigma_1} & 0 \\
0 & -C_q & C_{\Sigma_2} & 0 & -C_{\Sigma_2} \\
-C_{\Sigma_1} & 0 & C_{\Sigma_3} & -C_q \\
0 & 0 & -C_{\Sigma_2} & -C_q & C_{\Sigma_4}
\end{pmatrix}
\]

(A2)

with

\[
C_{\Sigma_1} = C_q + C_{g1} + C_{c1} + C_d, \\
C_{\Sigma_2} = C_q + C_{g2} + C_{c2}, \\
C_{\Sigma_3} = C_q + C_{g3} + C_{c1}, \\
C_{\Sigma_4} = C_q + C_{g4} + C_{c2}.
\]

(A3)

To identify and remove the free modes in the circuit, we consider the following transformation of the node flux variables with respect to the transformation matrix [28]

\[
S = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & -1
\end{pmatrix}
\]

(A4)

After performing the transformation, the circuit Lagrangian now reads

\[
L = \frac{1}{2} \dot{\Phi}'^T C' \dot{\Phi}' + E_{J1} \cos(\phi_{1m}) + E_{J2} \cos(\phi_{2m}).
\]

(A5)

Here, the transformed node flux variables is \( \Phi = S \Phi' = (\Phi_d \Phi_{1p} \Phi_{1m} \Phi_{2p} \Phi_{2m})^T \), where \( \Phi_{1p(m)} = \Phi_1 \pm \Phi_2 \) and \( \Phi_{2p(m)} = \Phi_3 \pm \Phi_4 \), and the corresponding capacitance matrix is now given as \( C = S^{-1} C' S^{-1} \). Then, the circuit Hamiltonian can be expressed as [29]

\[
H = \sum_j Q_j \dot{\phi}_j - E_{J1} \cos(\phi_{1m}) - E_{J2} \cos(\phi_{2m}),
\]

(A6)

where \( Q_j = \partial L / \partial \dot{\phi}_j \) denotes the charge variable, which is conjugated to the node flux variable \( \phi_j \) with \( j = \{d, 1p, 1m, 2p, 2m\} \). Here, the modes associated with \( Q_{1p} \) and \( Q_{2m} \), and which don’t have any potential energies in the circuit Hamiltonian, are the free modes [19–21].

As discussed in previous works, the free modes actually don’t participate in the circuit dynamics [19–21]. Hence, one can drop the terms associated with the free modes in the circuit Hamiltonian in Eq. (A6), giving rise to

\[
H_r = \frac{1}{2} Q_r^T C_r^{-1} Q_r - E_{J1} \cos(\phi_{1m}) - E_{J2} \cos(\phi_{2m})
\]

(A7)

where \( Q_r = (Q_d Q_{1m} Q_{2m})^T \), \( C_r \) denotes the reduced capacitance matrix, which can also be used to describe a system consisting of two direct-coupled grounded transmon qubits, as shown in Fig. 2(b). Here, for illustration purpose and to avoid the extremely cumbersome expression of \( C_r \), we consider
that all the island capacitor take the same capacitance, i.e., $C_{g1} = C_{g2} = C_{g3} = C_{g4} = C_g$. In this case, the expression of $C_r$ is given by (since $C_r$ is a symmetric matrix, hereafter, the elements in the lower triangular parts of the capacitance matrix, denoted by ■, are not given explicitly)

$$
C_r = \begin{pmatrix}
\tilde{C}_{\Sigma d} & -\frac{C_d C_g (C_{g1} + 2 C_{g2} + 3 C_{g2})}{K} & \cdots & \cdots & -\frac{C_d C_g (C_{g1} - C_{g2})}{K} \\
\vdots & \tilde{C}_{\Sigma q1} & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \vdots & \ddots & \ddots \\
-\frac{C_d C_g (C_{g1} - C_{g2})}{K} & \cdots & \cdots & \cdots & \tilde{C}_{\Sigma q2}
\end{pmatrix},
$$

where

$$
K = C_{g1} (C_d + 4 C_g) + 4 C_g (C_g + C_{g2}) + C_d (2 C_g + C_{g2}),
$$

$$
\tilde{C}_{\Sigma d} = \frac{4 C_d C_g (C_{g1} + C_g + C_{g2})}{K} \tag{A9}
$$

$$
\tilde{C}_{\Sigma q1} = \left[ C_d \left( 2 C_g^2 + (2 C_q + 3 C_{g2}) C_g + C_q C_{g2} \right)
+ C_g \left( 2 C_g^2 + (4 C_q + 3 C_{g2}) C_g + 4 C_q C_{g2} \right)
+ C_{g1} C_d (C_g + C_{g2})
+ C_{g1} C_g (3 C_g + 4 (C_q + C_{g2})) \right] / K
$$

$$
\tilde{C}_{\Sigma q2} = \left[ C_d \left( (2 C_g^2 + (2 C_q + C_{g2}) C_g + C_q C_{g2} \right)
+ C_g \left( 2 C_g^2 + (4 C_q + 3 C_{g2}) C_g + 4 C_q C_{g2} \right)
+ C_{g1} C_d (C_g + C_{g2})
+ C_{g1} C_g (3 C_g + 4 (C_q + C_{g2})) \right] / K
$$

When the island capacitance $C_g$ and the shunt capacitance $C_q$ is far larger than the coupling capacitance, i.e., $\{C_g, C_q\} \gg \{C_{g1}, C_{g2}, C_{g2}\}$, the reduced capacitance matrix $C_r$ can be approximated by

$$
C_r \approx \begin{pmatrix}
C_d & -\frac{C_d}{2} & \cdots & \cdots & -\frac{C_d}{2}

-\frac{C_d}{2} & C_g + \frac{C_q}{2} & \cdots & \cdots & -\frac{C_d}{2} \\
-\frac{C_d}{2} & \frac{C_g}{4} + \frac{C_q}{4} & \ddots & \ddots & \ddots \\
\cdots & \cdots & \ddots & \ddots & \ddots \\
\cdots & \cdots & \cdots & \ddots & \ddots \\
-\frac{C_d}{2} & \frac{C_q}{4} + \frac{C_q}{4} & \cdots & \cdots & C_g + \frac{C_q}{2}
\end{pmatrix}
$$

$$
\tag{A10}
$$

$$
\tag{A11}
$$

$$
\tag{A12}
$$

$$
\tag{A13}
$$

\section*{Appendix B: Grounded Bus}

Here, as shown in Fig. 5(a), we consider that two floating transmon qubits coupled via a grounded bus. Following the same procedure given in Appendix A, the circuit in Fig. 5(a) can be reduced to the circuit in Fig. 5(b), where two grounded transmon qubits are coupled via a grounded bus. After removing free modes, the reduced capacitance matrix of the current circuit is given by (here, the charge variables are $Q_r = (Q_{d1} Q_{1m} Q_1 Q_{2m})^T$, and the corresponding flux variables are $\Phi_r = (\Phi_d \Phi_{1m} \Phi_1 \Phi_{2m})^T$ with $\Phi_{1m} = \Phi_1 - \Phi_2$ and $\Phi_{2m} = \Phi_3 - \Phi_4$)

$$
C_r = \begin{pmatrix}
\tilde{C}_{\Sigma d} & -\frac{C_d C_g}{C_{d} + C_{g} + 2 C_{g}} & \cdots & \cdots & -\frac{C_d C_g}{C_{d} + C_{g} + 2 C_{g}}

\vdots & \tilde{C}_{\Sigma q1} & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \vdots & \ddots & \ddots \\
\vdots & \vdots & \vdots & \ddots & \ddots \\
-\frac{C_d C_g}{C_{d} + C_{g} + 2 C_{g}} & \cdots & \cdots & \cdots & \tilde{C}_{\Sigma q2}
\end{pmatrix},
$$

$$
\tag{B1}
$$

FIG. 5: (a) Schematic circuit diagram for two floating transmon qubits $(Q_1$ and $Q_2$) coupled via a grounded bus $(Q_d)$, where a dedicated drive line (left) is coupled capacitively to one of the two qubits, i.e., $Q_1$. After removing free modes, the circuit can be transformed into an equivalent circuit shown in (b), where two grounded transmon qubits are coupled via a grounded bus. Note here that unlike the results shown in Fig. 2, the spurious crosstalk between the two qubits disappears, giving rise to $C_{d2} = 0$. This can be explained by the fact that (1) there is no direct coupling between the two qubits; (2) the grounded bus does not support the presence of free mode; Thus, there are no free modes to mediate the spurious crosstalk between the two qubits.
where

\[ \tilde{C}_{\Sigma_t} = \frac{C_d(C_c + 2C_g)}{C_d + C_c + 2C_g}, \]

\[ \tilde{C}_{\Sigma_{q1}} = \frac{C_d(C_q + C_g) + C_c(C_q + C_g) + C_g(C_g + 2C_q)}{C_d + C_c + 2C_g}, \]

\[ \tilde{C}_{\Sigma_{q2}} = \frac{C_c(C_q + C_g) + C_g(C_g + 2C_q)}{C_c + 2C_g}, \]

and

\[ \tilde{C}_{\Sigma_d} = \frac{(C_c + 2C_g)(2C_qC_t + C_f(4C_q + C_t))}{(C_c + 2C_g)(C_d + C_c + 2C_g)} + \frac{C_d}{(C_c + 2C_g)(C_d + C_c + 2C_g)} \]

When considering that \( \{C_g, C_q, C_t\} \gg \{C_d, C_c\} \), the above matrix can be approximated by

\[ C_r \approx \begin{pmatrix} C_d & -\frac{C_d}{2} & -\frac{C_dC_g}{2} & 0 \\ C_q + \frac{C_g}{2} & C_q & -\frac{C_g}{2} & 0 \\ C_t & -\frac{C_g}{2} & C_t & -\frac{C_g}{2} \\ 0 & 0 & 0 & C_q + \frac{C_g}{2} \end{pmatrix} \]  

As shown in Eq. (B1), for qubits coupled via the grounded bus, the spurious crosstalk disappears, i.e., the matrix element, which represents the coupling between the drive source \( V_d \) and the \( Q_2 \), takes the value of 0, i.e., \( [C_r]_{d,2m} = -C_{d2} = 0 \). This is to be expected since the grounded bus does not support the presence of free mode, thus there are no free modes to mediate the spurious crosstalk between the two qubits.

In addition, note that the matrix element \( [C_r]_{d,1} \) gets a nonzero value, as shown in Eq. (B1). This means that there exists a spurious crosstalk channel between the qubit \( Q_1 \) and the grounded bus \( Q_1 \) (in Fig. 5(b), the corresponding virtual drive line is not presented explicitly). Thus, we can conclude that this spurious crosstalk channel is mediated by the free mode supported by the qubit \( Q_1 \).

**Appendix C: Floating Bus**

Here, as shown in Fig. 6(a), we consider that two floating transmon qubits coupled via a floating bus [26]. Following the same procedure given in Appendix A, the circuit in Fig. 6(a) can be transformed into an equivalent circuit shown in Fig. 6(b), where two grounded transmon qubits are coupled via a coupler circuit combining a grounded bus and a capacitor [26, 33]. Accordingly, the reduced capacitance matrix of the present circuit is given by (here, the charge variables are \( \Phi_r = (Q_d, Q_{1m}, Q_{1q}, Q_{2m})^T \), and the corresponding flux variables are \( \Phi_r = (\Phi_d, \Phi_{1m}, \Phi_{1q}, \Phi_{2m})^T \) with \( \Phi_{1m} = \Phi_1 - \Phi_2 \), \( \Phi_t = \Phi_3 - \Phi_4 \), and \( \Phi_{2m} = \Phi_5 - \Phi_6 \)).

\[
\begin{pmatrix}
\tilde{C}_{\Sigma_d} & -\frac{C_dC_g}{2} & -\frac{C_dC_g}{2} & 0 \\
-\frac{C_dC_g}{2} & C_q + \frac{C_g}{2} & -\frac{C_g}{2} & 0 \\
-\frac{C_dC_g}{2} & -\frac{C_g}{2} & C_t & -\frac{C_g}{2} \\
0 & 0 & 0 & C_q + \frac{C_g}{2}
\end{pmatrix}
\]  

where

\[
K = 2C_b(C_c + 2C_g)(C_c + C_d + 2C_g) + C_c(4C_g(C_d + 2C_g) + C_c(C_d + 4C_g)),
\]

\[
\tilde{C}_{\Sigma_d} = \frac{2C_d(C_c + 2C_g)(2C_qC_c + C_b(C_c + 2C_g))}{K},
\]

\[
\tilde{C}_{\Sigma_{q1}} = \frac{2C_b(C_c + 2C_g)(C_c(C_c + C_g) + C_d(C_g + C_q) + C_g(C_g + 2C_q) + C_g(C_g + 2C_q))}{K},
\]

\[
\tilde{C}_{\Sigma_{q2}} = \left[2C_b(C_c + 2C_g)(C_c(C_c + C_g) + C_d(C_g + C_q) + C_g(C_g + 2C_q) + C_g(C_g + 2C_q) + C_c(C_d(C_g + C_q) + C_g(C_g + 2C_q) + C_c(C_d(C_g + C_q) + C_g(C_g + 2C_q)) + C_c(C_d(C_g + C_q) + C_g(3C_g + 4C_q)))\right]/K.
\]
the above matrix can be approximated by

\[
C_r \approx \begin{pmatrix}
C_d & -\frac{C_d}{2} & -\frac{C_d}{2} & -\frac{C_d}{2} & -\frac{C_d}{2} \\
-\frac{C_d}{2} & C_q + \frac{C_d}{2} & -\frac{C_d}{4} & \frac{C_d}{4} & -\frac{C_d}{4} \\
-\frac{C_d}{2} & -\frac{C_d}{4} & C_t + \frac{C_d}{2} & \frac{C_d}{8} & -\frac{C_d}{8} \\
-\frac{C_d}{2} & \frac{C_d}{4} & -\frac{C_d}{8} & C_q + \frac{C_d}{8} \\
-\frac{C_d}{2} & -\frac{C_d}{4} & -\frac{C_d}{8} & -\frac{C_d}{8} & C_t + \frac{C_d}{2}
\end{pmatrix} .
\] (C7)

As shown in Eq. (C1), for qubits coupled via the floating bus, the spurious crosstalk between the two qubits exists due to the presence of the free mode supported by the floating bus. Moreover, as demonstrated in previous works [26, 27], the free mode, which is supported by the floating bus, can also mediate an indirect coupling between the two qubits.
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