Associated Production of $H^0\gamma$ or $H^0Z^0$ pairs at $\mu^+\mu^-$ Collisions.

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Abstract

We calculate the cross-sections for the production of Standard Model Higgs Boson in association with the neutral gauge bosons (photon and $Z^0$-boson). For the case of reaction $\mu^+\mu^- \rightarrow \gamma H^0$, complete and compact analytical expressions for the differential and total cross-sections, applicable also for the case of any (pseudo)scalar particle with mass-coupling proportionality, in particular, for an axion are given. Reaction $\mu^+\mu^- \rightarrow Z^0 H^0$ is some "generalization" of the well-known "Bjorken process" for the case of $e^+e^-$ collisions. Various distributions for both the processes above are illustrated for the energies, which will be reached at future $\mu^+\mu^-$ colliders. Study of those processes will be evidently complementary to the precision measurements at the Higgs resonance region.

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\section{Introduction}

The search for Higgs particles from various models and the study of the
dundry scenario for electroweak symmetry breaking mechanism is one of the
most important goals of future high energy colliders \[1\]. Another important
task is to make a detailed study of the basic properties of possible Higgs
particles in various models. One of the most interesting and crucial param-
eters are the values of the couplings of a Higgs bosons to other fundamental
particles \[2\]. Measurements of those couplings would allow one to make
choice between different Higgs schemes, but in this short note we restrict our
discussions to the Standard Model (SM) with a single neutral Higgs boson.

Presumably the fundamental scalar will be revealed and investigated to
some extent at the forthcoming LHC collider, but the precision will be ev-
dently insufficient for the aims above. So, it is expedient to search for the
other means to make precision measurements in a wide region of possible
Higgs boson mass values. In this respect a crucial role will be played by
colliding lepton beams. Among them, in turn, the future $\mu^+\mu^-$ colliders will
be more preferable in this respect than electron-positron one as it will be
seen below.

For the first time an idea of colliding $\mu -$ meson beams together with its
physics potential was discussed in paper \[3\]. Later this question was raised in
refs. \[4\], \[5\], \[6\]. At present concrete proposals and the technical designs for
the $\mu^+\mu^-$ colliders are being intensively discussed together with the physics
goals (see, e.g. ref.\[7\]). Up to now there are two designs for the future
$\mu^+\mu^-$-colliders:

1) $\sqrt{s} \approx 500$ GeV, $\mathcal{L} \approx 10^{33}$ cm$^{-2}$ s$^{-1}$, $L_{tot} = 50$ fb$^{-1}$/year;

2) $\sqrt{s} \approx 4$ TeV, $\mathcal{L} \approx 10^{35}$ cm$^{-2}$ s$^{-1}$, $L_{tot} = 200 - 1000$ fb$^{-1}$/year.

A muon collider has some natural advantages as compared to an $e^+e^-$
collider, including some that are important for Higgs bosons production \[8\],
\[8\]:

1) there is essentially no beamstrahlung;

2) there is substantially no bremstrahlung;

3) $\mu^+\mu^-$ collider has a higher mass of initial particles in comparison with
$e^+e^-$ collider;
4) there is no final focus problem (storage rings are used to build up the effective instantaneous luminosity);

5) rather high beam energy resolution of $R \approx 0.1\%$ is possible, if the necessary technology is built into the machine;

6) region $\sqrt{s_{\text{max}}} \geq 500$ GeV can probably be reached more easily;

7) much less storage ring diameters are required because of drastic reduction of synchrotron radiation, correspondingly, cost of this part of design is reduced substantialey.

The negatives regarding a $\mu^+\mu^-$ collider include [7], [8]:

1) the design is immature, and approximately five years of research and development projects are needed before a full–fledged proposal would be possible – in particular, cooling tests are required to see if multistage cooling will be sufficiently efficient;

2) the exact nature of the detector backgrounds, and how to manage them, is still under investigation – certainly the detector will be more expensive due to higher shielding requirement;

3) significant polarization probably implies significant loss in $L$;

4) it is not possible to have $\gamma\gamma/\gamma\mu$ facilities;

5) due to the $\mu^+\mu^-$ initial state, we have mainly $J = 1$ in comparison with almost all angular momenta for $\gamma\gamma/\gamma\mu$ facilities;

6) in many important cases the cross–sections behaviour is $\sigma \approx 1/s$ for $\mu^+\mu^-$ collisions and $\sigma \approx \text{const}$ for $\gamma\gamma/\gamma\mu$ facilities;

Almost all theoretical efforts lie in the threshold $s$–channel SM Higgs boson production as depicted on Fig. 1. But in parallel with some advantages (such as threshold behaviour), there are also a number of problems:

a) $\mu^+\mu^- \rightarrow b\bar{b}$ process requires extreme beam energy resolution, e.g. of the order of $R \approx 0.01\%$ or higher. Inspite of great possibilities of the $\mu^+\mu^-$ colliders in this domain, that may be unattainable;
b) the mass of the SM Higgs boson must be a priori reported from another sources, e.g. LHC measurements. This also may have a problem. The precision of the LHC measures supposed is of the order $\delta m_H \approx 1\% \cdot m_H$ in $\gamma\gamma$ decay mode [7]. For $m_H \approx 200 - 300$ GeV, the error on the Higgs boson mass is about $\delta m_H \approx 2 - 3$ GeV, and we have rather broad range for scanning;

c) for precision measurements too high luminosity $\mathcal{L}$ must be achieved at all the scanned energies;

d) in any case, the several final storage rings designed to maintain near-optimal $\mathcal{L}$ over a span of $\sqrt{s}$ values are to be constructed.

In view of this it would be very interesting, if there exist other processes with SM Higgs boson production, which haven’t all or part of the above-listed disadvantages.

One of such processes is a reaction $\mu^+\mu^- \rightarrow H^0\gamma$ (see Fig.2).

In the $e^+e^-$ collisions the contribution of those diagrams to the overall cross-section is extremely small in comparison with higher order diagrams with heavy particles in loops [9]. In the case of $\mu^+\mu^-$ collisions the lowest order diagrams are competetive with loop diagrams due to a greater mass of $\mu$ in comparison with the electron mass. That process may be one of the goals for the future $\mu^+\mu^-$ colliders.

Yet, a related process exists, which may be be even more interesting, namely $\mu^+\mu^- \rightarrow ZH^0$. The corresponding Feynman graphs are depicted on Fig.6. This process differs from the above in that its cross-section is not negligible at tree level due to the s-channel diagram, Fig.6-c. The contribution of the remaining two graphs, Fig.6a-b, to the cross-section is negligible for the case of $e^+e^-$ collisions, however, in the case of $\mu^+\mu^-$ collisions their contribution is finite. Moreover, only due to accounting for them it is possible to obtain the correct asymptotic behaviour of cross section, when initial particles masses are involved into calculation. This phenomenon reflects one of the fundamental property of the theory of electroweak interaction [10]. This question will be thoroughly discussed in the section 3.

The rest of the paper is organized as follows. In Section 2 we analyse the associated Higgs boson – photon production in the Standard Model. In Section 3 we investigated the prospects for the associated Higgs–Z-boson production. Section 4 contains Conclusions.
2 Associated $H^0\gamma$ production in SM

In the Standard Model the process $\mu^+\mu^- \rightarrow H^0\gamma$ is described to lowest order by the Feynman diagrams, depicted in Fig. 2.

Summing over the polarizations of the photon and averaging over the polarization of both the initials $\mu^+\mu^-$ beams, the differential cross–section of process (1) can be written as:

\[
\frac{d\sigma}{d\cos\theta} (\mu^+\mu^- \rightarrow H^0\gamma) = \frac{\pi \alpha^2}{8 \sin^2 \theta_W} \cdot \frac{m_{\mu}^2}{M_W^2} \cdot \frac{1}{s^2} \cdot \frac{1}{\beta} \cdot (s - m_H^2) \times \\
\left\{ \frac{1}{(k \cdot p_1)^2} \left[ (k \cdot p_1)(k \cdot p_2) + m_{\mu}^2 \cdot [-(k \cdot p_1) + (k \cdot p_2) - \frac{1}{2} s \beta^2] \right] \\
+ \frac{1}{(k \cdot p_2)^2} \left[ (k \cdot p_1)(k \cdot p_2) + m_{\mu}^2 \cdot [(k \cdot p_1) - (k \cdot p_2) - \frac{1}{2} s \beta^2] \right] \\
+ \frac{2}{(k \cdot p_1)(k \cdot p_2)} \left[ (k \cdot p_1)(k \cdot p_2) - \frac{1}{2} s \beta^2 \cdot (-\frac{1}{2} m_H^2 + m_{\mu}^2) \right] \right\} (1)
\]

where the following notations were introduced

\[
(k \cdot p_{1,2}) = \frac{1}{4} (s - m_H^2)(1 \mp \beta \cos \theta), \\
\beta = 1 - \frac{4m_{\mu}^2}{s} \quad (2)
\]

with $\sqrt{s}$ as the c.m. energy and $\theta$ – the scattering angle of the photon. After introducing in addition to the usual $\beta$ the notation

\[
\beta_H = 1 - \frac{4m_{\mu}^2}{m_H^2} \quad (3)
\]

and integration over $\cos \theta$ in the $[-1, 1]$ limits, the cross–section acquires the following final form:

\[
\sigma (\mu^+\mu^- \rightarrow H^0\gamma) = \frac{\pi \alpha^2}{2 \sin^2 \theta_W} \cdot \frac{m_{\mu}^2}{M_W^2} \cdot \frac{1}{s^2} \cdot \frac{1}{\beta} \cdot \frac{1}{s - m_H^2} \times \\
\left\{ -2m_H^2 s \beta_H^2 + (s^2 \beta^4 + m_H^4 \beta_H^2) \cdot \frac{1}{\beta} \cdot \ln \frac{1 + \beta}{1 - \beta} \right\} (4)
\]
All the calculations have been performed with nonzero muon mass.

The cross-sections for the process $\mu^+\mu^- \rightarrow H^0\gamma$ are shown in Fig. 3 as a function of the Higgs boson mass for the three center of mass energies: $\sqrt{s} = 500\ \text{GeV}$, $\sqrt{s} = 1\ \text{TeV}$, $\sqrt{s} = 4\ \text{TeV}$. At 500 GeV the cross-section is of the order of $\sigma = 2 \div 3 \times 10^{-2}\ \text{fb}$ for the light Higgs boson masses. At $\sqrt{s} = 1.5\ \text{TeV}$ the cross-section for light Higgs boson drops by a factor of $\approx 4$ compared to the previous case.

Fig. 4 exhibits the dependence of the cross-section on the center of mass energy for several values of the Higgs boson mass. The cross section decreases smoothly with the increasing energy; it scales approximately as $\ln s/s$ at high energies.

The asymptotic behaviour of the cross-section under condition $\sqrt{s} \rightarrow \infty$ and $s \gg m^2_H$ can be written as:

$$
\sigma_{as}(\mu^+\mu^- \rightarrow H^0\gamma) = \frac{\pi\alpha^2}{2\sin^2\theta_W} \cdot \frac{m^2_{\mu}}{M^2_W} \cdot \frac{\ln(s/m^2_{\mu})}{s} \quad (5)
$$

Finally, Fig. 5 shows the angular distribution $d\sigma/d\cos\theta$ for several Higgs boson mass values at c.m. energy of 500 GeV. The distribution is forward-backward symmetric and does not depend very strongly on the Higgs boson mass.

With the yearly integrated luminosity of $L \approx 10^3\ \text{fb}^{-1}$ expected at future $\mu^+\mu^-$ colliders, one could collect 20 to 30 $H^0\gamma$ events (detector efficiency is supposed equal 1, and acceptance $-4\pi$). The signal, which mainly consists of a photon and $b\bar{b}$ pairs in the low Higgs mass range or $WW/ZZ$ pairs for Higgs masses larger than $\approx 200\ \text{GeV}$, is extremely a clean. The backgrounds should be very small since the photon must be very energetic and the $b\bar{b}$ or $WW/ZZ$ pairs should peak at an invariant mass $M_H$. Therefore, despite of the low rates, a clean signal gives a good possibility to detect these events.

Expressions (2) – (4) obtained for the cross-section of the process $\mu^+\mu^- \rightarrow H^0\gamma$ are applicable, on the equal foot, to the case of any other (pseudo)scalar particles production. Moreover it might happen that namely muon colliders will be most suitable and crucial means for their searches. Foremost it refers to the axion. This particle was postulated in papers [11] and [12] as a consequence of the strong $CP$–violation problem solution [13], [14]. The numerous fruitless searches of that pseudoscalar (for review see, e.g. ref. [15]) produced a widely accepted opinion, that this hypothetical particle is extremely light and weakly interacting one (“invisible axion”, [16]).
However in a recent paper \cite{17} the solution of strong $CP$–violation problem in $QCD$ has been proposed, which may lead to a heavy axion, $M_a \leq 1$ TeV. Its interaction with usual matter is induced by mixing with axial Higgs boson. For example, in the case of fermions it has the form $\mathcal{L}_{\text{int}} \sim \text{const} \cdot m_f \cdot (\alpha \vec{a} \gamma_5 f)$. A mixing parameters are model dependent but might not be negligible small, therefore this interaction can lead to observable effects. In that case the muon colliders might be irreplaceable tool for the axion search aim.

### 3 Associated $HZ$ production in SM

Another interesting process for the Higgs investigations is $\mu^+\mu^- \rightarrow ZH^0$. At first sight it is analogous to the process considered in the preceding section. However, it possesses the additional very interesting features, which display the deepest properties of the nonabelian gauge theories with spontaneous symmetry breaking. First of all, one finds the difference in the numbers of Feynman graphs, corresponding to both of aforementioned processes. For the second of them they are drawn on Fig.6. To the third diagram of this set, Fig.6-c, corresponds the so called Bjorken process, considered early for the case of $e^+ e^-$ collisions \cite{18} (see also \cite{19}). All those calculations had been done in the limit where the masses of initial particles were neglected. Now, with the accounting for those masses the cross-section reveals a very interesting feature: despite of its $s$–channel character it does not fall at very high energy but approaches a constant limit. At the same time its angle distribution is flat, indicating that it comes entirely from the $J = 0$ plane wave. It is obvious, that this behaviour contradicts unitarity condition, which requires $\sigma_{J=0} \leq s^{-1}$ at high energy. The contradiction is removed if in calculation procedure all the three diagrams of Fig.6 are accounted for. Because the whole contribution of $t$–channel diagrams of Fig.6 is proportional to the initial particles mass it might be considered as an additional argument in favor of the $\mu$–meson collider.

In the course of cross-section calculation for the process $\mu^+\mu^- \rightarrow ZH^0$ without neglecting the masses of initial $\mu^\pm$ much more complicated expressions arise so we confine ourselves by numerical computation with the aid of the Monte Carlo method for integration on phase space of final particles to obtain the total cross-section and various distributions.
The main formula for the Monte Carlo calculations is

\[ \sigma = \int f(\Phi) \, d\Phi \]  

where \( f(\Phi) \) denotes the matrix element squared (any cut can be easily implemented by putting \( f(\Phi) = 0 \) in the unwanted region of the phase space), and \( d\Phi \) is the 2–body phase space integration element.

In view of vital importance of remarks, made in the beginning of this section it is expedient to discuss separately contributions to the cross-section of the first two diagrams of Fig.6 from the one hand side and the third one from the another hand side. Fig.7 (lower curve) shows the c.m. energy dependence of contribution to the cross-section of the sum of the first two diagrams, Fig.6a-b, along with the contribution of whole set of diagrams (upper curve). Already at the relatively not too high energy the contribution of t–channel diagram plus u–channel one approaches the limiting value equal to \( \approx 1.2 \times 10^{-2} \, fb \).

Now, let us calculate the cross-section corresponding to the diagram Fig.6-c alone, accounting for masses of initial muons. Asymptotics of this process at \( \sqrt{s} \to \infty \) is as follows:

\[ \sigma^{(c)}(\mu^+\mu^- \to ZH^0)|_{m_\mu \neq 0} = \frac{2\pi \alpha^2}{\sin^4(2\theta_W)} \cdot g_A^2 \cdot \frac{m_\mu^2}{m_Z^2} \]  

(7)

It is seen, that despite of the fact that this diagram is the pure s–channel one, the corresponding cross-section is not falling at high energy, but approaches a constant limit, whose value is also equal to \( \approx 1.2 \times 10^{-2} \, fb \). There is sense to draw attention to the absence of vector coupling in the expression obtained. At last, the interference term between sum of t– and u–channel diagrams of Fig.6 and those of s–channel gives the contribution to the full cross-section, which is equal to \( \approx 2.4 \times 10^{-2} \, fb \). Therefore, we see that only accounting for all of three diagrams on Fig.6 with finite muon mass gives the correct asymptotic behaviour of cross section.

Note, that usually the calculations were \( e^+ e^- \) oriented with electron mass neglected, so the Yang-Mills character of theory was enough to secure the situation. In our case the Higgs mechanism is urgently needed. Concerning the process considered the tuning compensation would allow for studying
a new physics or to feel the existence of more complicated Higgs sector. Evidently muon colliders will deliver a unique possibility to study interactions of Higgs scalar within the lepton sector.

Fig. 8 shows the angular distribution \( d\sigma/d\cos\theta \) for c.m. energy of 500 GeV, 1 TeV, 4 TeV and for a Higgs boson mass \( M_H = 100 \) GeV in all the three cases. The distribution is forward–backward symmetric and does not depend very strongly on the Higgs boson mass. It reveals a typical behaviour of the scalar particle emitted when fermion-antifermion pair collide and fuse into vectorial one (Z – boson in the case at hand), i.e. it prefers to fly at 90°. Explicitly, the corresponding piece of differential cross-section behaves as \( \approx a - b \cdot \cos\theta \) with \( a \) and \( b \) being positive.

Fig. 9 exhibits the dependence of the cross–section on the center of mass energy for several values of the Higgs boson mass. The cross–section increases rapidly with the opening of the phase space and then drops smoothly with the increasing energy; it scales approximately as \( 1/s \) at high energies. Explicitly, the asymptotic behaviour for \( \sqrt{s} \to \infty \) of the cross–section is as follows:

\[
\sigma^{as}(\mu^+\mu^- \to H^0 Z) = \frac{1}{3} \cdot \frac{\pi \alpha^2}{\sin^4(2\theta_W)} \cdot (g_V^2 + g_A^2) \cdot \frac{1}{s}
\]  (8)

The cross–sections for the process \( \mu^+\mu^- \to H^0 Z \) are shown in Fig.10 as a function of the Higgs boson mass value for three representative center of mass energy, \( \sqrt{s} = 500 \) GeV, \( \sqrt{s} = 1 \) TeV, and \( \sqrt{s} = 4 \) TeV. At 500 GeV the cross–section is of the order of \( \sigma = 10^{-1} pb \) for the light Higgs masses; it drops out slightly with increasing \( M_H \) due to the lack of the phase space. At \( \sqrt{s} = 1 \) TeV the cross–section for light Higgs boson drops by a factor of \( \approx 9 \) compared to the previous case, but the decrease with increasing \( M_H \) is slower.

With the yearly integrated luminosity of \( \mathcal{L} \approx 10^3 \) fb\(^{-1} \) expected at future \( \mu^+\mu^- \) colliders, one could collect a sufficient number of \( H^0 Z \) events for thorough investigations of this process (detector efficiency is supposed equal to 1, and acceptance – 4\( \pi \)). The signal, which mainly consists of a Z–boson products and \( b\bar{b} \) pairs in the low Higgs mass range or \( WW/ZZ \) pairs for Higgs masses larger than \( \approx 200 \) GeV, is rather clean. The backgrounds should be rather small since the Z–boson must be very energetic and the \( b\bar{b} \) or \( WW/ZZ \) pairs should peak at an invariant mass \( M_H \). Therefore, the clean signal gives a good possibility for extensive study of these events.
4 Conclusions

We have calculated the cross–sections for the production of the Standard Model Higgs boson in association with a photon and Z–boson in $\mu^+\mu^-$ collisions in the lowest order of the perturbation theory. We have given the complete and compact analytical expression for $\mu^+\mu^- \rightarrow H^0\gamma$ process with detailed Monte Carlo simulations. For the case of $\mu^+\mu^- \rightarrow H^0Z$ process, we presented analytically only asymptotyc expressions for the cross–section of the process; all the histograms were produced by means of Monte Carlo simulation.

We have then illustrated the size of the cross–sections for energies, which will be reached at future $\mu^+\mu^-$ colliders. The cross–section for $\mu^+\mu^- \rightarrow H^0\gamma$ process is, in general, small, but much more intensive compared with the corresponding signal for the case of $e^+e^-$ collisions (at tree level), and rather clean. With an integrated yearly luminosity of $\mathcal{L} \approx 1000$ fb$^{-1}$ expected at future $\mu^+\mu^-$ colliders, we can detect those signals despite the low rates. Process $\mu^+\mu^- \rightarrow H^0Z$, in turn, is easily detectable and gives some opportunity to study the Higgs boson interaction in the lepton sector. From the theoretical point of view it demonstrates the efficiency of modern electroweak interaction scheme.

In parallel with threshold s–channel Higgs boson production, the processes $\mu^+\mu^- \rightarrow H^0\gamma$ and $\mu^+\mu^- \rightarrow H^0Z$ may be used for the search for mass of the SM Higgs boson.

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Figures captions

Fig.1. Diagram for $s$–channel Higgs boson production is shown.

Fig.2. Diagrams for $\mu^+\mu^- \rightarrow H^0\gamma$ are shown.

Fig.3. The cross section for $\mu^+\mu^- \rightarrow H^0\gamma$ is given as a function of Higgs boson mass for collider energies of $\sqrt{s} = 500$ GeV, $\sqrt{s} = 1$ TeV and $\sqrt{s} = 4$ TeV.

Fig.4. The cross section for $\mu^+\mu^- \rightarrow H^0\gamma$ is given for several values of Higgs boson mass. Curves correspond to $M_H = 100$, $M_H = 150$, and $M_H = 200$ GeV.

Fig.5. The angular distributions $d\sigma/d\cos\theta$ for $\mu^+\mu^- \rightarrow H^0\gamma$ are shown for collider energies of 500 GeV and several values of $M_H$.

Fig.6. Diagrams for $\mu^+\mu^- \rightarrow ZH^0$ are shown.

Fig.7. The cross-section for $\mu^+\mu^- \rightarrow ZH^0$ resulting from the sum of the diagrams drawn in Fig.6a-b is shown (lower curve) along with the cross-section resulting from full set of diagrams drawn in Fig.6 (shown at upper curve).

Fig.8. The angular distributions for $\mu^+\mu^- \rightarrow ZH^0$ are shown for collider energies of 500 GeV, 1 TeV and 4 TeV. $M_H = 100$ GeV in all the three cases.

Fig.9. The cross–section for $\mu^+\mu^- \rightarrow ZH^0$ is given for several values of the Higgs boson mass, $m_H = 100, 150 and 200$ GeV

Fig.10. The cross–section for $\mu^+\mu^- \rightarrow ZH^0$ is given as a function of Higgs boson mass for collider energies of 500 GeV, 1 TeV and 4 TeV.
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\[
\mu^+ \mu^- \rightarrow H^0 \gamma
\]
$\mu^+ \mu^- \rightarrow H^0 \gamma$

- $m_H = 100$ GeV
- $m_H = 150$ GeV
- $m_H = 200$ GeV
\( \sqrt{s} = 500 \text{ GeV} \quad \mu^+ \mu^- \rightarrow H^0 Z^0 \quad m_\mu = 100 \text{ GeV} \)

\( \sqrt{s} = 1000 \text{ GeV} \)

\( \sqrt{s} = 4000 \text{ GeV} \)
\[ \mu^+ \mu^- \rightarrow H^0 Z^0 \]

- \( m_H = 100 \text{ GeV} \)
- \( m_H = 150 \text{ GeV} \)
- \( m_H = 200 \text{ GeV} \)
$\mu^+ \mu^- \rightarrow H^0 Z^0$

- $\sqrt{s} = 500$ GeV
- $\sqrt{s} = 1000$ GeV
- $\sqrt{s} = 4000$ GeV
\( \mu^+ \mu^- \rightarrow H^0 Z^0 \)

\( m_H = 100 \text{ GeV} \)
\( \sqrt{s} = 500 \text{ GeV} \quad \mu^+ \mu^- \rightarrow H^0 Z^0 \quad m_\mu = 100 \text{ GeV} \)

\( \sqrt{s} = 1000 \text{ GeV} \)

\( \sqrt{s} = 4000 \text{ GeV} \)
$\mu^+ \mu^- \rightarrow H^0 Z^0$

- $m_\mu = 100$ GeV
- $m_\mu = 150$ GeV
- $m_\mu = 200$ GeV
$\mu^+ \mu^- \rightarrow H^0 \ Z^0$

- $\sqrt{s} = 500$ GeV
- $\sqrt{s} = 1000$ GeV
- $\sqrt{s} = 4000$ GeV

$\sigma_{tot}, \text{ pb}$ vs. $m_{h}, \text{ GeV}$