Prescribed performance tracking control for high-order uncertain nonlinear systems

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Abstract This paper considers the problem of prescribed performance tracking control for a class of high-order uncertain nonlinear systems with time-varying external disturbances. Using a little of priori system knowledge, a state feedback controller is designed capable of guaranteeing the prescribed performance requirements of the output tracking error, regarding the steady-state error, the convergence rate and the maximum overshoot. Compared with traditional backstepping-like approaches, no approximation techniques and hard numerical calculations are employed in the controller design. Thus, the proposed control scheme exhibits low complexity and strong robustness against system uncertainties. The effectiveness of the proposed scheme is verified by some simulation results.

Keywords Prescribed performance · High-order nonlinear systems · Tracking control · Low-complexity control

1 Introduction

In recent decades, the control problem of systems with uncertain nonlinear dynamics is a common topic among researchers. Although PID controllers are widely used as practical and simple control methods in industry, designing three parameters, proportional, integral and derivative gains of a PID controller in control systems is a challenge. To overcome these disadvantages, artificial intelligence techniques, such as fuzzy logic and neural networks, have already been found in many applications. With advantages such as independence from the system model and the ability to handle uncertain and imprecise information, adaptive control based on the backstepping technique is widely used in many control applications [1,2]. Classical and comprehensive references, such as [3], are helpful for understanding the framework of existing adaptive control approaches.

In the control domain, output tracking of uncertain nonlinear systems is one of the fundamental issues. The objective of tracking control is to drive the system output to track a prescribed reference signal asymptotically or with some prescribed accuracy. Additionally, much effort has been devoted to different types of systems via adaptive backstepping control, such as the strict-feedback nonlinear systems in [4], nonlinear pure-feedback systems in [5] and stochastic nonlinear systems in [6]. Furthermore, nonlinear systems have different unknown issues and/or performance require-
ments, including unknown control directions, actuator faults [7] and full-state constraints [8,9].

It is well known that high-order nonlinear systems have uncontrollable linearization, so adaptive control has been recognized as a challenge. Fortunately, by adding the power integrator initially proposed in [10,11], great progress in tracking control has been achieved for high-order nonlinear systems with the general form as following

\[
\begin{align*}
\dot{x}_i &= \psi_1(x_i) x_{i+1}^q_i + f_i(x_i), \quad i = 1, \ldots, n-1 \\
\dot{x}_n &= \psi_n(x_n) u^n + f_n(x_n)
\end{align*}
\]  

(1)

where \( \bar{x}_i = [x_1, \ldots, x_i]^T \in \mathbb{R}^i, i = 1, \ldots, n \) are state variables. \( u \in \mathbb{R} \) and \( y \in \mathbb{R} \) are input and output, respectively. \( f_i(x_i) \) and \( \psi_i(x_i) \) are nonlinear functions. \( q_i \geq 1 \) is the power of system (1). Using the technique of changing the supply rate, an adaptive tracking controller was designed for cascade systems with nonlinear parameterization [12]. By using a high gain function to compensate serious system uncertainties, an adaptive controller with a dynamic high gain was proposed [13]. Without imposing feasibility conditions, an adaptive practical tracking control scheme was designed for full-state constrained systems [14]. However, most results have been achieved under the assumption that the powers of system (1) are ratios of odd integers [15–18].

Man and Liu relaxed the restrictions so that the powers would not only be odd positive constants but also positive continuous functions or positive constants [19].

Despite the recent progress of tracking control for system (1), there are still shortcomings due to the following aspects: (1) A common characteristic is that only the convergence of the tracking error can be guaranteed to a residual set, and its size depends on explicit design parameters and some unknown bounded terms, but transient performance and the convergence rate cannot be prescribed. (2) The aforementioned works resorted to adaptive techniques and approximation to address the uncertain terms of the systems, which may increase extra calculations and lead to complexity of controller designs. (3) Strict conditions need to be imposed on the nonlinearities and/or the powers of high-order systems. To the authors’ knowledge, only a few works have achieved the prescribed transient performance of high-order nonlinear systems. The tracking performance is ensured within preassigned bounds regardless of high-power virtual control variables in [20,21]. A scheme of practical tracking control was developed within the funnel control framework, which caused the tracking error to forever evolve within a prescribed performance funnel in [22]. However, the powers of the high-order nonlinear systems in [20–22] are limited to positive odd integers. Therefore, relaxing restrictions on powers is still an open issue.

In order to achieve predefined transient and steady-state bounds on the output tracking errors, prescribed performance control (PPC) technology was proposed by Bechlioulis and Rovithakis in [23] and has been developed for different types of nonlinear systems [24–31]. For further development, Zhang et al. proposed a fault-tolerant control scheme that guarantees the prescribed tracking performance of systems with unknown control directions [32]. Yang et al. solved the problem of PPC for pure-feedback nonlinear systems with uncertainties [33]. Fotiadis and Rovithakis provided the PPC for multi-input multi-output nonlinear systems with discontinuous reference signals [34]. However, none of these works considered and dealt with high-order control variables of nonlinear systems.

Motivated by the above discussions, we consider the prescribed performance tracking control for a class of high-order uncertain nonlinear systems in this paper. Given any initial system condition and any output performance requirements, a tracking controller composed of only transformed error surfaces and designed performance requirements is proposed. Compared with most related works, the main contributions of this paper are threefold:

- In comparison with most existing results only guaranteeing that the tracking error converges to a bounded residual, the proposed state feedback controller can achieve the steady-state values of the tracking error, the convergence rate and the bound of the overshoot.
- Compared with typical backstepping-like approaches, the proposed method does not require adaptive techniques or approximation structures (i.e., neural networks or fuzzy logic systems) to address nonlinear uncertainties, and the explosion of complexity is avoided due to the absence of the derivatives of virtual control signals, so the proposed algorithm exhibits a low-complexity level.
- The robustness against system uncertainties is greatly extended. In fact, any system with high-order feedback form obeying certain controllability assumptions can be controlled by the proposed scheme without altering the controller structure.
The remainder of this paper is organized as follows. Section 2 provides the preliminaries of the prescribed performance control problem. Section 3 is to establish the tracking control scheme and summarize the main results of this paper. Section 4 provides simulation results. Finally, some conclusions are given in Sect. 5.

**Notations** The following notations are adopted throughout this paper. $\mathbb{R}^i$ denotes the set of all real $i$-dimensional vectors; $\mathbb{R}_+$ denotes the set of all nonnegative real numbers; $\lceil x \rceil^q = \text{sign}(x)|x|^q$, where $\text{sign}(x)$ denotes its sign function.

## 2 Preliminaries

### 2.1 Prescribed performance

The prescribed performance control means that the output tracking error converges to a predefined arbitrary small residual set with convergence rate no less than a certain prespecified value and displays maximum overshoot less than a predefined value [25]. In this respect, consider a measurable tracking error $\varepsilon(t) : \mathbb{R}_+ \to \mathbb{R}$ and a prescribed function $\gamma(t) = (\ell_0 - \ell_\infty)e^{-\mu t} + \ell_\infty$ with $\ell_0$, $\ell_\infty$ and $\mu$ being positive constants, and $\ell_0 > \ell_\infty$. Given any initial condition $\varepsilon(0)$, choosing $\ell_0$ such that $\ell_0 > |\varepsilon(0)|$, if the following inequality

$$-\gamma(t) < \varepsilon(t) < \gamma(t), \quad \forall t \geq 0$$

holds, the prescribed performance of the tracking error $\varepsilon(t)$ is achieved. Clearly, $-\gamma(0)$ and $\gamma(0)$ represent the lower bound of an undershoot and the upper bound of an overshoot of $\varepsilon(t)$, respectively. The scalar $\mu$ represents the convergence rate, and $\ell_\infty$ represents the maximum allowable size of $\varepsilon(t)$ at the steady state, which can be set arbitrarily small. Thus, the appropriate choice of prescribed function $\gamma(t)$ reflects performance characteristics of the tracking error $\varepsilon(t)$.

### 2.2 Motivation example

In this section, we investigate a practical example with high order, which is rather representative although relatively simple.

Consider an underactuated unstable two degrees of freedom mechanical system previously considered in [35]. The mechanical system contains a mass $m_1$ on a horizontal smooth surface and an inverted pendulum $m_2$ supported by a massless rod as shown in Fig. 1. The mass $m_1$ is interconnected to the wall by a linear spring and to the inverted pendulum by a nonlinear spring. Let $x$ be the displacement of mass $m_1$ and $\theta$ be the angle of the pendulum from the vertical at $x = 0$ and $\theta = 0$. The springs are unstretched. A control force $u$ acts on $m_1$. The equations of motion for the system are described by

$$\begin{align*}
\ddot{\theta} &= \frac{g}{l} \sin \theta + \frac{k_s}{m_2 l} [x - l \sin \theta]^q \cos \theta \\
\ddot{x} &= -\frac{k}{m_1} x - \frac{k_s}{m_1} [x - l \sin \theta]^q + \frac{u}{m_1},
\end{align*}$$

where $g = 9.8m/s^2$ is the acceleration of gravity, $l$ is the length of rod, $k$ and $k_s$ are spring coefficients. Using the changes of coordinates

$$x_1 = \theta, \quad x_2 = \dot{\theta}, \quad x_3 = (x - l \sin \theta)(\cos \theta)^{1/m}, \quad x_4 = \dot{x}_3,$$
where \((\theta, \dot{\theta}, x, \dot{x}) \in (-\pi/2, \pi/2) \times \mathbb{R}^3\), system (2) can be transformed into
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{k_x}{m_2} x_3^{q_1} + \frac{g}{l} \sin x_1 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{u}{m_1} + f(x_1, x_2, x_3, x_4),
\end{align*}
\] (3)
with \(f(0, 0, 0, 0) = 0\).

2.3 Problem statement

Consider the following high-order nonlinear systems
\[
\begin{align*}
\dot{x}_i &= \psi_i(x_1)[x_{i+1}]^{q_i} + f_i(x_i) + q_i(t), & i &= 1, \ldots, n - 1 \\
\dot{x}_n &= \psi_n(x)[n]^{q_n} + f_n(x) + q_n(t) \\
y &= x_1
\end{align*}
\] (4)
where \(\overline{x}_i = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i\) and \(x = \overline{x}_n\) are measurable system states; \(u \in \mathbb{R}\) and \(y \in \mathbb{R}\) are control input and system output, respectively. For all \(i = 1, \ldots, n\), the nonlinear functions \(f_i, \psi_i : \mathbb{R}^i \to \mathbb{R}\) are unknown but locally Lipschitz; \(q_i(t) \in \mathbb{R}\) are bounded unknown piecewise continuous functions; \(q_i \geq 1\) are the powers of system (4). In this work, there exists at least one power \(q_i > 1\).

Control Objective Given desired tracking trajectory \(y_r(t)\) and any initial condition \(x(0) \in \mathbb{R}^n\), design a low-complexity state feedback controller for system (4) such that: (1) the system output \(y\) tracks \(y_r(t)\) with prescribed performance; (2) all the signals of the closed-loop systems remain bounded.

In order to design the required control scheme, the following assumptions and lemmas are needed.

Assumption 1 For control coefficients \(\psi_i(\overline{x}_i)\), \(i = 1, \ldots, n\), there exists unknown constant \(\overline{\psi}_i > 0\) such that \(\overline{\psi}_i = \psi_i(\overline{x}_i)\).

Assumption 2 The desired trajectory signal \(y_r(t)\) and its first-order derivative \(\dot{y}_r(t)\) are bounded.

Lemma 1 [36] For any constant \(q \geq 1\), and any real-valued functions \(\omega_1\) and \(\omega_2\), the following inequality
\[
|\omega_1|^q - |\omega_2|^q \leq q (2^{q-2} + 2) (|\omega_1 - \omega_2|^q + |\omega_1 - \omega_2| |\omega_2|^{q-1})
\]
holds.

Lemma 2 [10] For any constants \(q_1 > 0\), \(q_2 > 0\) and \(c > 0\), and any real-valued functions \(\omega_1\) and \(\omega_2\), the following inequality
\[
|\omega_1|^{q_1} |\omega_2|^{q_2} \leq c \frac{q_1}{q_1 + q_2} |\omega_1|^{q_1 + q_2} + c \frac{q_2}{q_1 + q_2} |\omega_2|^{q_1 + q_2}
\]
holds.

Remark 1 Compared with related control results for high-order nonlinear systems, the restrictions on high powers and system nonlinearities are relaxed in this work, which are described as follows: 1) The powers were assumed only being odd integers in [11,12,15,16,39] and only being ratios of odd integers in [13,14,17,18,37,38]. In this work, there is no restriction on powers except that \(q_i \geq 1\). 2) In [13–15], the nonlinear functions \(f_i(\overline{x}_i)\), \(i = 1, \ldots, n\) of system (4) were assumed to satisfy \(f_i(\overline{x}_i) \leq \overline{f}_i(\overline{x}_i)(|x_1|^{q_1} + \cdots + |x_i|^{q_i})\), where \(\overline{f}_i(\overline{x}_i)\) are known positive functions. In this work, the nonlinearities \(f_i(\overline{x}_i)\) are completely unknown, and Assumption 1 is a relaxed version for the control coefficients \(\psi_i(\overline{x}_i)\).

Remark 2 When \(q_i = 1\), system (4) is regarded as a general form of strict-feedback nonlinear systems [4, 8]. If there exists at least one power satisfying \(q_i > 1\), system (4) is called the high-order nonlinear systems. Compared with some related works of tracking control, this work only requests the knowledge that the desired trajectory signal \(y_r(t)\) and its first-order derivative \(\dot{y}_r(t)\) are bounded, and none of the high-order derivatives of \(y_r(t)\) are required.

Next, we shall design a computationally efficient controller and prove that the controller can fulfill the control objective.

2.4 Control design

Define the function \(F : (-1, 1) \to \mathbb{R}\) with the form \(F(\bullet) = \ln\left(\frac{1+\bullet}{1-\bullet}\right)\). In order to ensure a satisfactory tracking behavior, a group of performance functions are introduced
\[
y_i(t) = (\ell_i - \ell_{i,\infty}) e^{-\mu_i t} + \ell_{i,\infty}, \quad i = 1, \ldots, n.
\]
(5)
where \(\ell_{i,0} > \ell_{i,\infty} > 0\) and \(\mu_i > 0\). The parameters \(\ell_{i,\infty}\) and \(\mu_i\) denote the maximum steady-state error.
and the required minimum exponential convergence rate, respectively, and the parameter $\ell_{i,0}$ needs to meet some initial conditions described in the following control design process. From (5), we have

$$\begin{align*}
\ell_{i,0} & \leq \gamma_i(t) \leq \ell_{i,\infty} \\
\frac{1}{\ell_{i,0}} & \leq \frac{1}{\gamma_i(t)} \leq \frac{1}{\ell_{i,\infty}} \\
\mu_i(\ell_{i,0} - \ell_{i,\infty}) & \leq \dot{\gamma}_i(t) \leq 0, \quad i = 1, \ldots, n.
\end{align*}$$

(6)

Therefore, $\gamma_i(t)$, $1/\gamma_i(t)$ and $\dot{\gamma}_i(t)$ are bounded.

Given a desired trajectory signal $x_f(t)$ and any initial system condition $x(0) \in \mathbb{R}^n$, the detailed control design process is given as follows.

**Step I** Select a performance function $\gamma_1(t)$ satisfying $\gamma_1(0) > |x_1(0) - y_r(0)|$, and design the first virtual control signal as

$$\alpha_1(x_1, t) = -\beta_1 F \left( \frac{x_1 - y_r(t)}{\gamma_1(t)} \right)$$

(7)

with $\beta_1$ being a positive control gain.

**Step II** Select a performance function $\gamma_2(t)$ satisfying $\gamma_2(0) > |x_2(0) - \alpha_1(x_1(0), 0)|$, and design the second virtual control signal as

$$\alpha_2(x_1, x_2, t) = -\beta_2 F \left( \frac{x_2 - \alpha_1(x_1, t)}{\gamma_2(t)} \right)$$

(8)

with $\beta_2$ being a positive control gain.

**Step III** Select performance functions $\gamma_i(t)$ satisfying $\gamma_i(0) > |x_i(0)|$

$$\alpha_{i-1}(x_1(0), \ldots, x_{i-1}(0), 0), \quad i = 3, \ldots, n - 1,$$

repeat step II for all remaining virtual control signals

$$\alpha_i(x_1, \ldots, x_i, t) = -\beta_i F \left( \frac{x_i - \alpha_{i-1}(x_1, \ldots, x_{i-1}, t)}{\gamma_i(t)} \right)$$

(9)

with $\beta_i$ being a positive control gain.

**Step IV** Finally, design the control input as

$$u(x_1, \ldots, x_n, t) = -\beta_n F \left( \frac{x_n - \alpha_{n-1}(x_1, \ldots, x_{n-1}, t)}{\gamma_n(t)} \right)$$

(10)

by selecting a performance function $\gamma_n(t)$ satisfying $\gamma_n(0) > |x_n(0) - \alpha_{n-1}(x_1(0), \ldots, x_{n-1}(0), 0)|$, where $\beta_n$ is a positive constant.

**Remark 3** The design process of control signals in (7)–(10) reveals that: (1) The control design does not incorporate any prior knowledge regarding either the actual system nonlinearities or some corresponding bounding functions. (2) Adaptive techniques and approximation structures (i.e., neural networks or fuzzy logic systems) are not employed to deal with unknown system nonlinearities. (3) None of the high-order derivatives of intermediate control signals are required in this control design. Accordingly, the explosion of complexity is avoided without using command filters. Comparing with the tracking problem [11–18], no hard calculations exist in the control signals (7)–(10). Thus, a low-complexity control law is designed in this work.

### 3 Performance analysis

We first define $e_i = x_i - y_r$, $e_i = x_i - \alpha_{i-1}$, $i = 2, \ldots, n$ and the normalized state errors as follows:

$$_{i} = \frac{e_i}{\gamma_i(t)}, \quad i = 1, \ldots, n.$$  

(11)

From (7)–(10), the virtual control signals and the control law are written as

$$\alpha_i(x_1, \ldots, x_i, t) = \alpha_i(e_i) = -\beta_i \ln \frac{1 + e_i(t)}{1 - e_i(t)}$$

$$i = 1, \ldots, n - 1,$$

$$u(x_1, \ldots, x_n, t) = u(e_n) = -\beta_n \ln \frac{1 + e_n(t)}{1 - e_n(t)}.$$

(12)  

(13)

It follows from (11) that

$$\begin{align*}
x_1 &= \epsilon_1 \gamma_1(t) + y_r(t) \\
x_i &= \epsilon_i \gamma_i(t) + \alpha_i(e_{i-1}), \quad i = 2, \ldots, n.
\end{align*}$$

(14)

Taking derivative of $\epsilon_i$ with respect to time and invoking (4), yields

$$\dot{\epsilon}_1 = \Phi_1(t, \epsilon_1, \epsilon_2)$$

$$= \frac{1}{\gamma_1(t)} \left\{ \psi_1(e_1 \gamma_1(t) + y_r(t)) \right\}$$

$$+ f_1(e_1 \gamma_1(t) + y_r(t)) + g_1(t) - \dot{y}_r(t) - \dot{\gamma}_1(t) \epsilon_1 \}.$$  

(15)

$$\dot{\epsilon}_i = \Phi_i(t, \epsilon_1, \ldots, \epsilon_{i-1})$$

$$= \frac{1}{\gamma_i(t)} \left\{ \psi_1(e_i \gamma_i(t) + y_r(t), \ldots, \epsilon_{i-1}) \right\}$$

$$+ f_i(e_i \gamma_i(t) + y_r(t), \ldots, \epsilon_{i-1}) + g_i(t)$$

$$- d_{\alpha_{i-1}} \frac{d}{d \epsilon_{i-1}} \Phi_{i-1}(t, \epsilon_1, \ldots, \epsilon_{i-1}) - \dot{\gamma}_i(t) \epsilon_i \}.$$  

(16)
\[ \dot{e}_n = \Phi_n(t, e_1, \ldots, e_n) \]
\[ = \frac{1}{\gamma_n(t)} \left[ \psi_n(e_1 y_1(t) + y_r(t), \ldots, e_n y_1(t) + \alpha_{n-1}(e_{n-1})) u(e_n) q_n + f_n(e_1 y_1(t) + y_r(t), \ldots, e_n y_1(t) + \alpha_{n-1}(e_{n-1})) q_n + \frac{d\alpha_{n-1}}{d\varepsilon_{n-1}} \Phi_{n-1}(t, e_1, \ldots, e_n) - \dot{\gamma}_n(e_n) \right] \]  \tag{17}

The normalized error vector \( \dot{e} = [e_1, e_2, \ldots, e_n]^T \) can be written as

\[ \dot{e} = \Phi(t, e) = \begin{bmatrix} \Phi_1(t, e_1, e_2) \\ \vdots \\ \Phi_n(t, e_1, \ldots, e_n) \end{bmatrix} \]  \tag{18}

Define the open set \( \Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_n \) with \( \Omega_i = (-1, 1), i = 1, \ldots, n \), and the signal \( t \rightarrow \zeta_i(t) \) with

\[ \zeta_i(t) = \ln \frac{1 + e_i(t)}{1 - e_i(t)}, \quad i = 1, \ldots, n. \]  \tag{19}

**Performance Analysis Philosophy** If \( e_i \in \Omega_i, \zeta_i(t) \) is well defined. On the other hand, if \( \zeta_i \in \mathcal{L}^\infty, i = 1, \ldots, n \), one can deduce that \( \dot{e} \) evolves strictly within a compact subset of \( \Omega \). Furthermore, the boundedness of all the signals in the closed-loop systems can be guaranteed. Therefore, the existence and uniqueness of a maximal solution \( \dot{e} : [0, T_m) \rightarrow \Omega \) of (18) will be ensured firstly. Then, we prove that \( \zeta_i \in \mathcal{L}^\infty, i = 1, \ldots, n \). Consequently, the control objective can be achieved by the proposed control scheme.

Next, we summarize the main results of this paper as follows.

**Theorem 1** Consider the high-order nonlinear system (4) under Assumption 1, and the desired trajectory \( y_r(t) \) obeys Assumption 2. Given any initial conditions satisfying \( |e_i(0)| < 1, i = 1, \ldots, n \), the proposed control scheme (7)–(10) guarantees that

1. The system output \( y \) tracks the desired trajectory \( y_r(t) \) with prescribed performance, i.e.,

\[ |e_1(t)| < y_1(t), \quad t \geq 0. \]  

2. All the closed-loop signals remain bounded for \( t \geq 0 \).

**Proof** The proof of Theorem 1 is divided into two parts.

In Part I, the existence and uniqueness of a maximal solution \( \dot{e}(t) : [0, T_m) \rightarrow \Omega \) of (18) is guaranteed (i.e., \( e(t) \in \Omega, \forall t \in [0, T_m) \)). In Part II, we prove that the boundedness of all the closed-loop signals of (12)–(17) can be guaranteed and \( e(t) \) evolves in a compact subset of \( \Omega \) for all \( t \in [0, T_m) \). Further, we obtain \( T_m = +\infty \).

**Part I** The performance functions have been chosen satisfying \( y_1(0) > |x_1(0) - y_r(0)| \) and \( y_r(0) > |x_2(0) - \alpha_{i-1}(x_{i-1}(0), 0)|, i = 2, \ldots, n, \) so \( |e_i(0)| < 1 \) can be guaranteed, that is, \( \dot{e}(0) \in \Omega \). Therefore, the set \( \Omega \) is nonempty. Additionally, \( y_1(t) \) and \( y_r(t) \) are bounded and continuous differentiable, the functions \( f_i \) and \( \psi_i \) are locally Lipschitz and piecewise continuous in their arguments, and the signals \( \alpha_i \) and \( u \) are smooth over \( \Omega \). Thus, \( \Phi(t, e) \) is bounded and continuous in \( t \), and locally Lipschitz in \( e \) over the set \( \Omega \).

By Theorem 54 in [40], there exists a unique maximal solution \( e : [0, T_m) \rightarrow \Omega \) of (18) such that \( e(t) \in \Omega \) for all \( t \in [0, T_m) \).

**Part II** In Part I, \( \dot{e}(t) \in \Omega \) is ensured, which leads to \( \zeta_i(t) \) in (15) being well defined for all \( t \in [0, T_m) \). Then, (12) and (13) can be rewritten as follows:

\[ \alpha_i(e_i) = -\beta_i \zeta_i(t), \quad i = 1, \ldots, n - 1, \]  \tag{20}
\[ u(e_n) = -\beta_n \zeta_n(t). \]  \tag{21}

Notice that the following discussions are all based on the time interval \( [0, T_m) \), and \( |e_i(t)| < 1 \) for \( t \in [0, T_m) \).

**Step 1** Define the Lyapunov function \( V_1 = \frac{1}{2} \zeta_1^2 \). Taking derivative of \( V_1 \) with respect to time, we get

\[ \dot{V}_1 = \frac{2\zeta_1}{(1 - e_1^2)^2} \left( \psi_1([x_2]^q_1 - [\alpha_1]^q_1) + \psi_1[\alpha_1]^q_1 \right. \\ + \left. f_1 + q_1(t) - y_r(t) - \dot{y}_r(t)e_1 \right). \]  \tag{22}

Using Lemma 1, it follows that

\[ |[x_2]^q_1 - [\alpha_1]^q_1| \leq q_1(2q_1 - 2) ([x_2 - \alpha_1]^q_1 \\ + |x_2 - \alpha_1||[\alpha_1]^q_1| + \gamma_2(t)|e_2|^q_1 + \gamma_1(t)|e_1|^q_1), \]  \tag{23}

which together with Assumption 1 leads to

\[ \dot{V}_1 \leq \frac{2\psi_1}{(1 - e_1^2)^2} \left( f_1 + q_1(t) - y_r(t) - \dot{y}_r(t)e_1 \right) \\ + \frac{2\zeta_1}{(1 - e_1^2)^2} \left( \dot{\gamma}_1(t) + \dot{\alpha}_1(t) \right) \]  \tag{24}

with

\[ \Delta_{1,1} = q_1(2q_1 - 2) \gamma_2^q_1(t)|e_2|^q_1 + \]
which together with (24)–(25) leads to
\[
\Delta_{1,2} = q_1(2^{q_1-2} + 2)\gamma_2(t)|\varepsilon_2|.
\]

We establish the boundedness of $\Delta_{1,1}$ and $\Delta_{1,2}$ as follows:

1. Clearly, $\gamma_1(t) \in L^\infty$, $\dot{\gamma}_1(t) \in L^\infty$, $\varphi_1(t) \in L^\infty$ and $\varepsilon_1 < 1$.
2. By Assumption 2, $\gamma_1(t) \in L^\infty$, $\dot{\gamma}_1(t) \in L^\infty$.
3. Notice that $x_1 = \varepsilon_1\gamma_1(t) + y_1(t)$, one has $x_1 \in L^\infty$.
4. In view that $f_1(x_1)$ and $\psi_1(x_1)$ are locally Lipschitz in $x_1$ with $x_1 \in L^\infty$, one can get $f_1(x_1) \in L^\infty$ and $\psi_1(x_1) \in L^\infty$.

Based on the above analysis, the variables $\Delta_{1,1}$ and $\Delta_{1,2}$ are bounded for all $t \in [0, T_m)$. Therefore, there exist positive constants $\eta_{i,1}$ and $\eta_{i,2}$ such that $\Delta_{1,1} \leq \eta_{i,1}$ and $\Delta_{1,2} \leq \eta_{i,2}$. According to (20), we obtain
\[
|\xi_t|\alpha_1^{q_1-1} = \beta_1^{q_1-1}|\xi_t|^{q_1},
\]
\[
\xi_t|\alpha_1^{q_1} = -\beta_1^{q_1}|\xi_t|^{q_1} + 1.
\]

By Lemma 2, it follows that
\[
\Delta_{1,1}|\xi_1| \leq \eta_{i,1}|\xi_1| \leq \frac{\beta_1^{q_1}}{4}|\xi_1|^{q_1} + \frac{4}{\eta_{i,1}} + \frac{1}{\beta_1}
\]
\[
\Delta_{1,2}|\xi_1|\alpha_1^{q_1-1} \leq \eta_{i,2}\beta_1^{q_1-1}|\xi_1|^{q_1} \leq \frac{\beta_1^{q_1}}{2}|\xi_1|^{q_1} + 1
\]
\[
\frac{2}{\beta_1^{q_1}}|\xi_1|^{q_1} + 1
\]
which together with (24)–(25) leads to
\[
\dot{V}_1 \leq \frac{2\psi_1}{(1-\varepsilon_1^2)}(\eta_1 - \frac{\beta_1^{q_1}}{4}|\xi_1|^{q_1} + 1)
\]
with
\[
\eta_1 = \frac{\frac{1}{4} + \frac{1}{\beta_1}}{\eta_{i,1}} + \frac{\frac{2}{\beta_1^{q_1}} + \frac{1}{\eta_{i,1}}}.
\]

Therefore, $\dot{V}_1 < 0$ when $|\xi_1|^{q_1} > 4\eta_1/\beta_1^{q_1}$. By the definition of $V_1$, there holds
\[
|\xi_t| \leq \bar{\xi}_1 = \max\left\{|\xi_t(0), \frac{4\eta_1}{\beta_1^{q_1}} + \frac{1}{\beta_1}\}, t \in [0, T_m)\right\}.
\]

It follows from (20) that $|\alpha_1| \leq \beta_1\bar{\xi}_1$. From (14), by the boundedness of $\alpha_1, \gamma_1(t)$ and $\varepsilon_2, x_2 \in L^\infty$ holds.

Taking the inverse logarithmic function of (19), one has
\[
-1 < e^{-\varepsilon_1} - \varepsilon_1 = \varepsilon_1 \leq \varepsilon_1(t) \leq \bar{\varepsilon}_1 = \frac{e^{\varepsilon_t} - 1}{e^{\varepsilon_1} + 1} < 1, t \in [0, T_m),
\]
that is, $\varepsilon_1 \in [\varepsilon_1, \bar{\varepsilon}_1] \subset (-1, 1)$. Finally, taking derivative of $\alpha_1$ with respect to time, we have
\[
\dot{\alpha}_1 = \frac{d\alpha_1}{d\varepsilon_1}\Phi_1(t, \varepsilon_1, \varepsilon_2) = -\frac{2\beta_1}{(1-\varepsilon_1^2)}\Phi_1(t, \varepsilon_1, \varepsilon_2)
\]
\[
= -\frac{2\beta_1}{(1-\varepsilon_1^2)}(\psi_1|x_2|^q + f_1 + \varphi_1(t) + \dot{\gamma}_1(t))
\]
\[
-\dot{\gamma}_1(t)|\varepsilon_1|.
\]

Owing to the fact that $\psi_1, f_1, \varphi_1, x_2, \gamma_1(t), \dot{\gamma}_1(t)$ and $\dot{\gamma}_t$ are bounded, $\dot{\alpha}_1 \in L^\infty$ holds.

**Step i (i = 2, \ldots, n - 1)** Define the Lyapunov function $V_i = \frac{1}{2}\bar{\xi}_i^2$. Taking derivative of $V_i$ with respect to time gives
\[
\dot{V}_i = \frac{2\psi_i}{(1-\varepsilon_i^2)}(\psi_i|x_{i+1}|^q + f_i + \varphi_i(t) - \dot{\alpha}_i-1
\]
\[
-\dot{\gamma}_i(t)).
\]

Applying recursively in line with the aforementioned proof of Step 1, it yields
\[
\dot{V}_i \leq \frac{2\psi_i}{(1-\varepsilon_i^2)}(\Delta_{i,1}|\xi_1| + \Delta_{i,2}|\xi_1| |\alpha_1|^{q_1-1})
\]
\[
+ \frac{2\psi_i}{(1-\varepsilon_i^2)}|\alpha_1|^{q_1},
\]
\[
(32)
\]
where
\[
\Delta_{i,1} = q_i(2^{q_i-2} + 2)\gamma_{i+1}(t)|\varepsilon_{i+1}|^{q_i}
\]
\[
+ \frac{1}{4} + \frac{1}{\beta_1}
\]
\[
\Delta_{i,2} = q_i(2^{q_i-2} + 2)\gamma_{i+1}(t)|\varepsilon_{i+1}|^{q_i}.
\]

Similar to Step 1, $\Delta_{i,1} \in L^\infty$ and $\Delta_{i,2} \in L^\infty$ are ensured recursively, i.e., there exist positive constants $\eta_{i,1}$ and $\eta_{i,2}$ such that $\Delta_{i,1} \leq \eta_{i,1}$ and $\Delta_{i,2} \leq \eta_{i,2}$. According to (20), one has
\[
|\xi_i| |\alpha_i|^{q_1-1} = \beta_1^{q_1-1}|\xi_i|^{q_1},
\]
\[
\xi_i|\alpha_i|^{q_1} = -\beta_1^{q_1}|\xi_i|^{q_1} + 1.
\]

Similar to (28), there holds
\[
\dot{V}_i \leq \frac{2\psi_i}{(1-\varepsilon_i^2)}(\eta_i - \frac{\beta_1^{q_1}}{4}|\xi_i|^{q_1} + 1)
\]
\[
(34)
\]
with

\[
\eta_i = \frac{\epsilon_1^{1+\frac{1}{\eta_n}}}{\beta_1} + \frac{\epsilon_2^{1+\frac{1}{\eta_n}}}{\beta_1}.
\]

Thus, when \(|\xi_i|^{1+\eta_i} > 4\eta_i/\beta_i^\eta_i\), \(\dot{V}_i < 0\). By the definition of \(V_i\), we have

\[
|\xi_i| \leq \bar{\xi}_i = \max \left\{ |\xi_i(0)|, \left(\frac{4\epsilon_i}{\beta_i^\eta_i}\right)^{\frac{1}{\eta_i}} \right\}, \quad t \in [0, T_m).
\]

(35)

Correspondingly, it follows from (19) that

\[
-1 < \frac{e^{-\bar{\xi}_i} - 1}{e^{-\bar{\xi}_i} + 1} = \xi_i \leq \epsilon_i(t) \leq \bar{\epsilon}_i = \frac{e^{-\bar{\xi}_i} - 1}{e^{-\bar{\xi}_i} + 1} < 1,
\]

for \(i = 2, \ldots, n - 1\). (36)

Thus, \(|\alpha_i| \leq \beta_i \bar{\xi}_i\), \(i = 2, \ldots, n - 1\) and \(x_{i+1} \in \mathcal{L}^\infty\). Finally, taking derivative of \(\alpha_i\) with respect to time yields

\[
\dot{\alpha}_i = \frac{d\alpha_i}{d\xi_i} \Phi_i(t, \xi_i, \ldots, \xi_i)
= -\frac{2\beta_i}{(1 - e^{-\bar{\xi}_i})\gamma_i(\epsilon_i)} \left(\psi_i[x_{i+1}]^{\eta_i} + f_i + g_i(t)\right)
- \dot{\alpha}_{i-1} - \dot{\gamma}_i(t) \epsilon_i.
\]

(37)

By (36), \(\epsilon_i \in [\bar{\xi}_i, \bar{\bar{\xi}}_i] \subset (-1, 1)\). By the fact that the terms on the right-hand side of (37) are bounded, the boundedness of \(\dot{\alpha}_i\) is implied.

**Step 3** Define the Lyapunov function \(V_n = \frac{1}{2} \xi_n^2\). Taking derivative of \(V_n\) with respect to time gives

\[
\dot{V}_n = \frac{2\epsilon_n}{(1 - e^{-\bar{\xi}_n})\gamma_n(t)} \left(\psi_n[u]^{\eta_n} + f_n + g_n(t)\right)
- \dot{\alpha}_{n-1} - \dot{\gamma}_n(t) \epsilon_n
\leq \frac{2\psi_n}{(1 - e^{-\bar{\xi}_n})\gamma_n(t)} \left(|\xi_n|\Delta_n - \beta_n^{\eta_n}|\xi_n|^{\eta_n+1}\right)
\]

(38)

with \(\Delta_n = |f_n + g_n(t) - \dot{\alpha}_{n-1} - \dot{\gamma}_n(t) \epsilon_n| / \psi_n\). Similar to the proof in Step 1, there exists positive constant \(\eta_n\) such that \(\Delta_n \leq \eta_n\). By Lemma 2, we have

\[
|\xi_n|\Delta_n \leq |\xi_n|\eta_n \leq \frac{\beta_n^{\eta_n}}{2} |\xi_n|^{\eta_n+1} + \frac{2\epsilon_n}{(1 - e^{-\bar{\xi}_n})\gamma_n(t)} \left|\xi_n\right|^{\eta_n+1},
\]

(39)

which together with (38) leads to

\[
\dot{V}_n \leq \frac{2\psi_n}{(1 - e^{-\bar{\xi}_n})\gamma_n(t)} \left(\bar{\eta}_n - \frac{\beta_n^{\eta_n}}{2} |\xi_n|^{\eta_n+1}\right)
\]

(40)

with \(\bar{\eta}_n = 2\epsilon_n \eta_n^{1+\frac{1}{\eta_n}}/\beta_n\). From (40), \(\dot{V}_n < 0\) when \(|\xi_n|^{1+\eta_n} > 2\epsilon_n \eta_n^{1+\eta_n}/\beta_n\). Considering \(V_n = \frac{1}{2} \xi_n^2\), we have

\[
|\xi_n| \leq \bar{\xi}_n = \max \left\{ |\xi_n(0)|, \left(\frac{2\epsilon_n}{\beta_n} \right)^{\frac{1}{\eta_n}} \right\}, \quad t \in [0, T_m).
\]

(41)

Thus, the control input \(u\) defined in (21) is bounded. Correspondingly, (19) also leads to

\[
-1 < \frac{e^{-\bar{\xi}_n} - 1}{e^{-\bar{\xi}_n} + 1} = \xi_n \leq \epsilon_n(t) \leq \bar{\epsilon}_n = \frac{e^{-\bar{\xi}_n} - 1}{e^{-\bar{\xi}_n} + 1} < 1,
\]

\[
t \in [0, T_m).
\]

(42)

From (30), (36) and (42), \(e(t) \in \Omega_e = [\bar{\xi}_1, \bar{\bar{\xi}}_1] \times \cdots \times [\bar{\xi}_e, \bar{\bar{\xi}}_e] \subset \Omega\) for all \(t \in [0, T_m]\), which means that the solutions of (15)–(17) remain bounded within a compact subset of \(\Omega\). According to Theorem 3.3 in [41], \(T_m = +\infty\) can be obtained.

Owing to the analysis given in Step 1–Step 3, all the closed-loop signals remain bounded and \(e(t) \in \Omega_e\) for all \(t \geq 0\). Moreover, invoking (30), we have

\[
-\gamma_1(t) < \frac{e^{-\bar{\xi}_1} - 1}{e^{-\bar{\xi}_1} + 1} < \gamma_1(t) < \gamma_1(t),
\]

\[
\forall t \geq 0.
\]

(43)

Thus, the output tracking with prescribed performance is achieved.

This completes the proof. \(\square\)

### 4 Simulation results

Consider an underactuated unstable two degrees of freedom mechanical system previously considered in [35]. This model has been transformed into system (2) in Sect. 2.2. Introducing uncertain time-varying disturbance \(\varphi(t)\) to system (2), we have

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{k_x}{m_2} |x_3|^q + \frac{g}{l} \sin x_1 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{u}{m_1} - \frac{k_x}{m_1} (x_3 + l \sin x_1) - \frac{k_x}{m_1} |x_3|^q + l x_2^2 \sin x_1 \\
&\quad + g \sin x_1 \cos x_1 - \frac{k_x}{m_2} |x_3|^q \cos^2 x_1 + \varphi(t)
\end{aligned}
\]

(44)

Obviously, system (44) is in the form of (4), and Assumption 1 holds. The control objective is to design a
Table 1 The control gains and the parameters of performance functions

| $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\ell_1$ | $\ell_{1,\infty}$ | $\ell_2$ | $\ell_{2,\infty}$ | $\ell_3$ | $\ell_{3,\infty}$ | $\ell_4$ | $\ell_{4,\infty}$ |
|---------|---------|---------|---------|--------|---------------|--------|---------------|--------|---------------|--------|---------------|
| 5       | 2       | 4       | 5       | 9.9    | 0.1          | 10     | 0.3           | 10     | 0.2           | 9.8    | 0.2           |

Fig. 2 Output $y(t)$ and desired reference $y_r(t)$

![Output y(t) and desired reference y_r(t)](image)

Fig. 3 Output error $e_1$ and prescribed performance function $\gamma_1(t)$

![Output error e_1 and prescribed performance function gamma_1(t)](image)

State-feedback controller $u$ to force the output $y$ tracking the trajectory $y_r = \sin(1.5t) - 0.3\cos t$. For the simulation, we assume that $m_1 = 1$, $m_2 = 0.5$, $l = 10$, $k = 5$, $k_s = 10$, $q_1 = 3$, $\varrho(t) = \sin(0.2t)$. The initial conditions of the state variables are chosen as $x_1(0) = 2$, $x_2(0) = 0$, $x_3(0) = 0$, $x_4(0) = 0$.

We require that the steady state of the output tracking error $y - y_r(t)$ is no more than 0.1 and the minimum speed of convergence is obtained by the exponential $e^{-3t}$, and select $\gamma_1(0) = 9.9 > 2|x_1(0) - y_r(0)|$. Therefore, the performance function is $\gamma_1(t) = (9.9 - 0.1)e^{-3t} + 0.1$. Following the design procedure presented in Sect. 2.1, the intermediate performance functions are selected as $\gamma_2(t) = (10 - 0.3)e^{-3t} + 0.3$, $\gamma_3(t) = (10 - 0.2)e^{-t} + 0.2$ and $\gamma_4(t) = (9.8 -$
Fig. 4 State error $e_2$ and prescribed performance function $\gamma_2(t)$

Fig. 5 State error $e_3$ and prescribed performance function $\gamma_3(t)$

The control gains and the parameters of performance functions are designed in Table 1.

To show the superiority of the proposed method, the method reported in [11] is also employed to control this model. All the results are obtained under the same initial conditions. The simulation results are described in Figs. 2, 3, 4, 5, 6 and 7. The tracking trajectories are plotted in Fig. 2. The output tracking error $y - y_r(t)$ and the prescribed performance function $\gamma_1(t)$ are illustrated in Fig. 3, while the control input $u$ is plotted in Fig. 7. The trajectories of state error $e_i$ and the prescribed performance functions $\gamma_i(t), i = 2, 3, 4,$ are illustrated in Figs. 4, 5 and 6, respectively. Obviously, the output tracking has achieved the prescribed transient and steady-state error bounds.

The simulation results reveal that the proposed control scheme can guarantee the tracking performance and the closed-loop stability despite unknown nonlinearities and unknown time-varying external disturbances. Figures 2 and 3 reveal that the control scheme in this work achieves better tracking performance than the method reported in [11].

Remark 4 The control schemes in [11, 12, 15] only guaranteed the ultimate convergence of the tracking error, but not concerned with its maximum overshoot or arrival time. Fortunately, choosing appropriate per-
Fig. 6 State error $e_4$ and prescribed performance function $\gamma_4(t)$

![Graph showing State error $e_4$ and prescribed performance function $\gamma_4(t)$](image)

**Fig. 7** The control input $u$

![Graph showing The control input $u$](image)

**Table 2** The control gains and the parameters of performance functions

| $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\ell_1$ | $\ell_{1,\infty}$ | $\ell_2$ | $\ell_{2,\infty}$ | $\ell_3$ | $\ell_{3,\infty}$ | $\ell_4$ | $\ell_{4,\infty}$ |
|-----------|-----------|-----------|-----------|---------|----------------|---------|----------------|---------|----------------|---------|----------------|
| 3         | 1.5       | 3         | 2         | 9.9     | 0.1           | 10      | 0.3           | 10      | 0.2           | 9.8     | 0.2           |

Performance functions, the control design in this work can achieve practical tracking with prescribed maximum overshoot and transient performance.

**Remark 5** Inspecting of (29), (35) and (41) reveals that $\xi_i$ would become smaller by enlarging control gains. Then, the size of the sets $[\xi_i, \bar{\xi}_i], i = 1, \ldots, n$ in (30), (36) and (42) would be shrunk. We alter the control gains to verify these performances, while the performance functions are not changed. The altered control gains and the parameters of performance functions are shown in Table 2. The tracking error is presented in Fig. 8. Although the tracking error becomes large, the prescribed transient and bounds of steady-state error are still preserved.
5 Conclusions

In this paper, a general framework of tracking control is established for high-order uncertain nonlinear systems with unknown powers and unknown external disturbances. A low-complexity state feedback controller is designed to achieve prescribed performance with respect to trajectory oriented metrics. Contrary to some existing related results, the restrictions on system powers and nonlinearities are relaxed; thus, the considered problem has stronger theoretical and practical values. The proposed control method is of low complexity for that no approximation structures and iterative calculations of derivatives are employed to the control design. In addition, the tracking performance is isolated from the selection of control selection, so further simplification is exhibited in the proposed control law. In the future, the proposed methodology would be extended to fault-tolerant control for linear stochastic systems [42], event-triggered fault detection for nonlinear discrete-time switched systems [43] and the switched large-scale nonlinear delay systems [44].

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Data Availability The simulation data that support the findings of this study are available within the article.

Declarations

Conflict of interest The authors declare that there is no conflict of interest regarding the publication of this paper.

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