We study gaugino-mediated supersymmetry breaking in a six-dimensional $SO(10)$ orbifold GUT model where quarks and leptons are mixtures of brane and bulk fields. The couplings of bulk matter fields to the supersymmetry breaking brane field have to be suppressed in order to avoid large FCNCs. We derive bounds on the soft supersymmetry breaking parameters and calculate the superparticle mass spectrum. If the gravitino is the LSP, the $\tilde{\tau}_1$ or the $\tilde{\nu}_{\tau L}$ turns out to be the NLSP, with characteristic signatures at future colliders and in cosmology.

1 Introduction

Supersymmetric orbifold GUTs are attractive candidates for unified theories explaining the masses and mixings of fermions, see for example 2, 3, 4. Features like doublet-triplet splitting and absence of dimension-five proton decay operators, which are difficult to realise in four-dimensional grand unified theories, are easily obtained. In order to add predictions for the superparticle mass spectrum, an orbifold model has to be supplemented by a scenario for SUSY breaking. Given the higher-dimensional setup with various branes, this scenario involves in general both bulk and brane fields.

According to this reasoning, we combine an $SO(10)$ theory in six dimensions, proposed in 5, with gaugino-mediated SUSY breaking 6, 7. The orbifold compactification of the two extra dimensions has four fixed points or “branes”. On three of them, three quark-lepton generations are localised. The Standard Model leptons and down-type quarks are linear combinations of these localised fermions and a partial fourth generation living in the bulk. This leads to the observed large neutrino mixings. On the fourth brane, a field $S$ develops an $F$-term vacuum...
Table 1: Charge assignments for the symmetries $U(1)_R$ and $U(1)_X$

|   | $H_1$ | $H_2$ | $\Phi^c$ | $H_3$ | $\Phi$ | $H_4$ | $\psi_1$ | $\phi^c$ | $\phi$ | $H_5$ | $H_6$ | $S$ |
|---|-------|-------|----------|-------|-------|-------|----------|----------|-------|-------|-------|-----|
| $R$ | 0     | 0     | 0        | 2     | 0     | 2     | 1        | 1        | 1     | 1     | 1     | 0   |
| $X$ | -2a   | -2a   | -a       | 2a    | a     | -2a   | a        | -a       | a     | 2a    | -2a   | 0   |

expectation value (vev) breaking SUSY. As the gauge and Higgs fields propagate in the bulk, they feel the effects of SUSY breaking. Thus, gauginos and Higgs scalars obtain soft masses. The soft masses and trilinear couplings of the scalar quarks and leptons approximately vanish at the compactification scale. Non-zero values are generated by the running to low energies, which leads to a realistic superparticle mass spectrum. If the gravitino is the lightest superparticle (LSP), it can form the dark matter. The next-to-lightest superparticle (NLSP) is then a scalar tau or a scalar neutrino, which is consistent with constraints from big bang nucleosynthesis.

2 The Orbifold GUT Model

We consider an $N = 1$ supersymmetric $SO(10)$ gauge theory in 6 dimensions compactified on the orbifold $T^2/(\mathbb{Z}_2 \times \mathbb{Z}_2' \times \mathbb{Z}_2'')$. The theory has 4 fixed points, $O_i$, $O_{ps}$, $O_{GG}$ and $O_{fl}$, located at the corners of a “pillow” corresponding to the two compact dimensions. At $O_i$ the full $SO(10)$ survives, whereas at the other fixed points $SO(10)$ is broken to its subgroups $G_{ps} = SU(4) \times SU(2) \times SU(2)$, $G_{GG} = SU(5) \times U(1)_X$ and flipped $SU(5)$, $G_{fl} = SU(5)' \times U(1)'$, respectively. The intersection of these GUT groups yields the Standard Model group with an additional $U(1)$ factor, $G_{SM'} = SU(3) \times SU(2) \times U(1)_Y \times U(1)_X$, as an unbroken gauge symmetry below the compactification scale, which we identify with the GUT scale.

The field content of the theory is strongly constrained by requiring the cancellation of bulk and brane anomalies. The brane fields are the $16$-plets $\psi_1, \psi_2, \psi_3$. The bulk contains six $10$-plets, $H_1, \ldots, H_6$, and four $16$-plets, $\Phi, \Phi^c, \phi, \phi^c$, as hypermultiplets. Vevs of $\Phi$ and $\Phi^c$ break the surviving $U(1)_{B-L}$. The electroweak gauge group is broken by expectation values of the doublets contained in $H_1$ and $H_2$. The zero modes of $\phi, \phi^c$ and $H_5, H_6$, act as a fourth generation of down quarks and leptons and mix with the three generations of brane fields. We allocate these $16$-plets to the branes where $SO(10)$ is broken, placing $\psi_1$ at $O_{PS}$, $\psi_2$ at $O_{fl}$ and $\psi_3$ at $O_{ps}$. The three “families” are then separated by distances large compared to the cutoff scale $\Lambda$. Hence, they can only have diagonal Yukawa couplings with the bulk Higgs fields. The brane fields, however, can mix with the bulk zero modes without suppression. As these mixings take place only among left-handed leptons and right-handed down-quarks, we obtain a characteristic pattern of mass matrices. The allowed terms in the superpotential are restricted by $R$-invariance and an additional $U(1)_X$ symmetry with the charge assignments given in Tab. 1. The most general superpotential satisfying these constraints is given in [6]. It determines the SUSY-conserving mass terms and Yukawa couplings.

Soft SUSY-breaking terms are generated by gaugino mediation. A gauge-singlet chiral superfield $S$, which is localised at the fixed point $O_i$, acquires a non-vanishing vev for its $F$-term component. Supersymmetry is then fully broken and the breaking can be communicated to bulk fields by direct interactions. In the case of the gauginos, these are of the form

$$\mathcal{L}_S \supset \frac{g_4^2 h}{4 \Lambda} \int d^2 \theta S W^a W_a + \text{h.c.},$$

where $g_4$ is the four-dimensional gauge coupling and $h$ is a dimensionless coupling. Further interactions that are relevant for SUSY breaking and respect all symmetries are obtained by multiplying terms in the superpotential by $S/\Lambda$. 
Soft masses for the Higgses and for all bulk matter fields, as well as a $\mu$- and a $B\mu$-term arise from the Kähler potential. In order to obtain a $\mu$-term, we assume the global $U(1)_X$ symmetry to be only approximate and allow for explicit breaking here. Although the $\mu$-term itself is not a soft term, it is thus generated only after SUSY breaking via the Giudice-Masiero mechanism. The MSSM squarks and sleptons live on different branes than $S$. Therefore, they obtain soft masses only via loop contributions through the bulk, which are negligible here, and via renormalisation group running.

3 The Scalar Mass Matrices and FCNCs

From the superpotential one obtains $4 \times 4$ matrices of the form

$$
\begin{pmatrix}
\mu_1 & 0 & 0 & \tilde{\mu}_1 \\
0 & \mu_2 & 0 & \tilde{\mu}_2 \\
0 & 0 & \mu_3 & \tilde{\mu}_3 \\
\tilde{M}_1 & \tilde{M}_2 & \tilde{M}_3 & \tilde{M}_4
\end{pmatrix}
$$

for the down-type quark, charged lepton and Dirac neutrino masses. Here $\mu_i, \tilde{\mu}_i \sim v$ and $\tilde{M}_i \sim M_{GUT}$. While $\mu_i$ and $\tilde{\mu}_i$ have to be hierarchical, we assume no hierarchy between the $\tilde{M}_i$. The up-type quark and Majorana neutrino mass matrices are diagonal $3 \times 3$ matrices, since the corresponding fields do not have partners in the bulk. At the compactification scale, we integrate out the heavy degrees of freedom to obtain an effective theory with three generations. This requires block-diagonalising the mass matrices $m$ by transformations involving unitary matrices $U_4$ and $V_4$. In the case of the leptons, $V_4$ contributes to the observed large mixing between the left-handed fields. On the other hand, $U_4$ is close to the unit matrix, so that there is only small mixing among the right-handed fields. The situation is reversed in the down-quark sector, where the right-handed fields are strongly mixed while the left-handed ones are not.

Only the scalars of the fourth generation, which are very heavy, obtain soft masses, since they are bulk fields. However, the transformations diagonalising the fermion mass matrices transmit SUSY-breaking effects from the fourth to the light generations. As some of them cause large mixing, there are soft mass matrices whose off-diagonal elements are generically of similar size as the diagonal elements in a basis where quark and lepton mass matrices are diagonal. This leads to unacceptably large flavour-changing neutral currents. We expect this problem to be generic in higher-dimensional theories with mixing between bulk and brane matter fields as long as the bulk fields can couple to the hidden sector. In the following, we shall assume that soft masses for bulk matter fields, contrary to the bulk Higgs fields, are strongly suppressed.

4 The Low-Energy Sparticle Spectrum

Imposing vanishing soft masses for the bulk matter fields, the boundary conditions at the compactification scale are those of the usual gaugino mediation scenario with bulk Higgs fields:

$$g_1 = g_2 = g_3 = g \simeq \frac{1}{\sqrt{2}},$$

$$M_1 = M_2 = M_3 = m_{1/2},$$

$$m_{\phi_L}^2 = m_{\phi_R}^2 = 0 \text{ for all squarks and sleptons } \tilde{\phi} ,$$

$$A_{\tilde{\phi}} = 0 \text{ for all squarks and sleptons } \tilde{\phi} ,$$

$$\mu, B\mu, m_{h_i}^2 \neq 0 \ (i = 1, 2).$$
As a benchmark point for our discussion, we choose $m_{1/2} = 500$ GeV, $\tan \beta = 10$ and $\text{sign}(\mu) = +1$. The LSP can be the gravitino, with a mass between 50 and 100 GeV. The values of $\mu$ and $B\mu$ are determined by the conditions for electroweak symmetry breaking. In order to find the spectrum at low energy, we have to take into account the running of the parameters. We employ SOFTSUSY for this purpose.

The 1-loop running of the gaugino masses does not depend on the scalar masses, so that their low-energy values remain virtually the same in all cases as long as we do not change $m_{1/2}$. Numerically, we find $M_1(M_Z) \simeq 200$ GeV, $M_2(M_Z) \simeq 380$ GeV, and $M_3(M_Z) \simeq 1200$ GeV. To good approximation, the lightest neutralino is the bino and the second-lightest one is the wino, unless the soft Higgs mass $m_{h_2}^2$ is sizable. In the latter case, the electroweak symmetry breaking conditions lead to a rather small $\mu$, so that there is significant mixing between the neutralinos.

One constraint on the model parameters is that the running down to the weak scale must not produce tachyons. This yields an upper bound on $m_{h_1}^2$. The bound on $m_{h_2}^2$ is due to the experimental limits on the superparticle masses. If the initial value of $m_{h_2}^2$ is too large, this mass squared crosses zero at a rather low energy, so that its absolute value at the electroweak scale is small. Consequently, the $\mu$ parameter is also small, leading to a Higgsino-like chargino with a mass below the current limit of 94 GeV. This limit on $m_{h_2}^2$ is the relevant one for almost all values of $m_{h_1}^2$. Only for very small $m_{h_1}^2$, the experimental requirement that the $\tilde{\tau}_1$ be heavier than 86 GeV becomes more restrictive. The resulting allowed region in parameter space is the gray-shaded area in Fig. 1.

In Fig. 2 we show the superparticle spectra we obtain at the 4 points in parameter space marked by dots in Fig. 1. Due to the large effects of the strong interaction, the squark masses experience the fastest running and end up around a TeV. If all scalar soft masses vanish at the GUT scale (point 1), the left-handed slepton masses change significantly in the beginning, but afterwards the evolution flattens. They reach values between 300 and 400 GeV at low energies. The flattening of the evolution is even more pronounced for the right-handed slepton masses. As a consequence, these scalars remain lighter than the lightest neutralino. This is also the case for $m_{h_2}^2 > m_{h_1}^2$ (point 3), since then the evolution of the right-handed slepton masses is slowed down further, while that of the left-handed masses is enhanced.

For $m_{h_1}^2 > m_{h_2}^2$, important changes can occur in particular in the slepton spectrum. For the largest possible difference of the soft Higgs masses, the left-handed sleptons remain relatively light, with a low-energy mass below 200 GeV. Contrary to that, the right-handed slepton masses run unusually fast near the GUT scale and reach values close to 400 GeV at low energy. Thus, the NLSP is a sneutrino in this case, with a slightly heavier stau $\tilde{\tau}_1$ (cf. point 2). If $m_{h_2}^2$ is neither close to zero nor to its upper bound (point 4), the running of the right-handed slepton masses is sufficiently enhanced to lift them above the lightest neutralino mass. At the same time, the running of the left-handed slepton masses is damped weakly enough, so that they are heavier than the lightest neutralino, too. A neutralino NLSP together with a gravitino LSP heavier than a GeV is excluded by cosmology, see e.g. for the most recent analysis. Therefore, this case is only viable if the neutralino is the LSP while the gravitino is heavier. This is possible, because we only have a lower bound on the gravitino mass in gaugino mediation. The corresponding region in parameter space is the dark-gray area in Fig. 1.

Varying the high-energy gaugino mass simply leads to a rescaling of the scalar spectrum to a first approximation. If $m_{1/2}$ is increased while keeping the other soft masses fixed, the spectrum comes closer to the one obtained in the minimal case of vanishing scalar masses. The LEP bound on the lightest Higgs mass leads to a lower bound on $m_{1/2}$. If $m_{h_1}^2$ takes its maximal value, a unified gaugino mass of slightly less than 400 GeV is compatible with the LEP bound (for $m_t = 172.7$ GeV).
Figure 1: Allowed region for the soft Higgs masses. In the dark-gray area, a neutralino is lighter than all sleptons. For the points marked by the dots, the resulting superparticle mass spectrum is shown in Fig. 2.

Figure 2: Spectra of superparticle pole masses. The numbers at the bottom correspond to the points in parameter space marked by the coloured dots in Fig. 1. The high-energy boundary conditions for the soft Higgs masses were $m_{h_1}^2 = m_{h_2}^2 = 0$ (point 1), $m_{h_1}^2 = 2.7$ TeV$^2$, $m_{h_2}^2 = 0$ (point 2), $m_{h_1}^2 = 0$, $m_{h_2}^2 = 0.5$ TeV$^2$ (point 3), and $m_{h_1}^2 = 2.7$ TeV$^2$, $m_{h_2}^2 = 0.5$ TeV$^2$ (point 4), respectively. In all cases, we used $m_{1/2} = 500$ GeV, $\tan \beta = 10$ and $\text{sign} (\mu) = +1$. As the first and second generation scalars are degenerate, only the first generation is listed in the figure. Particles with a mass difference of less than about 3 GeV are represented by a single line. The heavier neutralinos and the charginos have been omitted for better readability.
A change of $\tan \beta$ leads to a change of the mass splitting between the third generation and the first two. If $\tan \beta$ is significantly smaller than 10, the value used in our benchmark scenario, the Higgs mass bound leads to severer restrictions. If $\tan \beta < 6$, this bound is violated even for maximal $m_t$ and $m_{h_1}^2$, i.e. a gaugino mass larger than 500 GeV is required. For larger values of $\tan \beta$, the lighter stau mass decreases a lot faster at lower energies. Hence, the parameter space region shrinks where the lightest neutralino is lighter than the $\tilde{\tau}_1$. For $\tan \beta = 25$, this region almost vanishes. On the other hand, the soft Higgs masses have to satisfy severer upper bounds in order to avoid tachyons and a too light stau. For $\tan \beta = 35$, the model is only viable if all soft scalar masses vanish at the GUT scale, and for $\tan \beta > 35$ the lighter stau mass always lies below its experimental limit unless the gauginos are heavier than 500 GeV. We conclude that the model favours $10 \lesssim \tan \beta \lesssim 25$.

5 Conclusions

We have discussed gaugino-mediated SUSY breaking in a six-dimensional SO(10) orbifold GUT where quarks and leptons are mixtures of brane and bulk fields. The couplings of bulk matter fields to the SUSY breaking brane field have to be suppressed in order to avoid flavour-changing neutral currents. The compatibility of the SUSY breaking mechanism and orbifold GUTs with brane and bulk matter fields is a generic problem which requires further studies.

The parameters relevant for the superparticle mass spectrum are the universal gaugino mass, the soft Higgs masses, $\tan \beta$ and the sign of $\mu$. We have analysed their impact on the spectrum and determined the region in parameter space that results in a viable phenomenology. The model favours moderate values of $\tan \beta$ between about 10 and 25. The gaugino mass at the GUT scale should not be far below 500 GeV in order to satisfy the LEP bound on the Higgs mass. Typically, the lightest neutralino is bino-like with a mass of 200 GeV, and the gluino mass is about 1.2 TeV. Either the right-handed or the left-handed sleptons can be lighter than the neutralinos. The corresponding region in parameter space grows with $\tan \beta$. In this region, the gravitino is the LSP with a mass around 50 GeV. The $\tilde{\tau}_1$ or the $\tilde{\nu}_L$ is the NLSP. A sneutrino NLSP has the advantage that constraints from big bang nucleosynthesis and the cosmic microwave background are less stringent. For a stau NLSP, on the other hand, there exists the exciting possibility that its decays may lead to the discovery of the gravitino in future collider experiments.

References

1. W. Buchmüller, J. Kersten, K. Schmidt-Hoberg, JHEP 02, 069 (2006) hep-ph/0512152.
2. Y. Kawamura, Prog. Theor. Phys. 103, 613 (2000) hep-ph/9902423.
3. L. J. Hall, Y. Nomura, Phys. Rev. D 64, 055003 (2001) hep-ph/00103125.
4. A. Hebecker, J. March-Russell, Nucl. Phys. B 613, 3 (2001) hep-ph/0106166.
5. T. Asaka, W. Buchmüller, L. Covi, Phys. Lett. B 563, 209 (2003) hep-ph/0304142.
6. D.E. Kaplan, G.D. Kribs, M. Schmaltz, Phys. Rev. D 62, 035010 (2000) hep-ph/9911293.
7. Z. Chacko, M.A. Luty, A.E. Nelson, E. Ponton, JHEP 01, 003 (2000) hep-ph/9911323.
8. G.F. Giudice, A. Masiero, Phys. Lett. B 206, 480 (1988).
9. W. Buchmüller, K. Hamaguchi, J. Kersten, Phys. Lett. B 632, 366 (2006) hep-ph/0506105.
10. B.C. Allanach, Comput. Phys. Commun. 143, 305 (2002) hep-ph/0104145.
11. Particle Data Group, S. Eidelman et al, Phys. Lett. B 592, 1 (2004).
12. D.E. Kaplan, T.M.P. Tait, JHEP 06, 020 (2000) hep-ph/0004200.
13. M. Schmaltz, W. Skiba, Phys. Rev. D 62, 095004 (2000) hep-ph/0004210.
14. D.G. Cerdeño, K.-Y. Choi, K. Jedamzik, L. Roszkowski, R. Ruiz de Austri, hep-ph/0509275 (2005).
15. J.L. Feng, S. Su, F. Takayama, *Phys. Rev.* D **70**, 075019 (2004) [hep-ph/0404231](https://arxiv.org/abs/hep-ph/0404231).
16. W. Buchmüller, K. Hamaguchi, M. Ratz, T. Yanagida, *Phys. Lett.* B **588**, 90 (2004) [hep-ph/0402179](https://arxiv.org/abs/hep-ph/0402179).