Saturation of $E_T/N_{ch}$ and Freeze-Out Criteria in Heavy-Ion Collisions

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Abstract. The pseudorapidity densities of transverse energy, the charged particle multiplicity and their ratios, $E_T/N_{ch}$, are estimated at mid-rapidity, in a statistical-thermal model based on chemical freeze-out criteria, for a wide range of energies from GSI-AGS-SPS to RHIC. It has been observed that in nucleus-nucleus collisions, $E_T/N_{ch}$ increases rapidly with beam energy and remains approximately constant at about a value of 800 MeV for beam energies from SPS to RHIC. $E_T/N_{ch}$ has been observed to be almost independent of centrality at all measured energies. The statistical-thermal model describes the energy dependence as well as the centrality independence, qualitatively well. The values of $E_T/N_{ch}$ are related to the chemical freeze-out criterium, $E/N \approx 1$ GeV valid for primordial hadrons. We have studied the variation of the average mass ($\langle \text{MASS} \rangle$), $N_{\text{decays}}/N_{\text{primordial}}$, $N_{ch}/N_{\text{decays}}$ and $E_T/N_{ch}$ with $\sqrt{s_{NN}}$ for all freeze-out criteria discussed in literature. These observables show saturation around SPS and higher $\sqrt{s_{NN}}$, like the chemical freeze-out temperature ($T_{ch}$).

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1. Introduction

The final state particles in relativistic heavy-ion collisions hardly remember about their primordial origins, as they are subjected to many rescatterings in the hadronic stage. This has given rise to the interpretation of hadron production in terms of thermal and statistical models which assume chemical and kinetic freeze-out of the particles. All relativistic heavy-ion experiments have so far confirmed the validity of $E/N \approx 1$ GeV as a freeze-out criterion, with $E$ and $N$ being, respectively the total energy and particle number of the primordial hadronic resonances before they decay into stable hadrons. These quantities cannot be determined directly from experiment unless the final state multiplicity is low and hadronic resonances can be identified, which is not the case in relativistic heavy-ion collisions. It is thus not straightforward to link $E/N$ to directly measurable quantities. In this paper, we establish an approximate connection between $E/N$ and the ratio of the pseudo-rapidity density of transverse energy and that of the charged particle yield, $\left( \frac{dE_T/d\eta}{dN_{ch}/d\eta} \right) \equiv E_T/N_{ch}$, at mid-rapidity, for beam energies ranging from about 1 AGeV up to 200 AGeV. In this energy range, $E_T/N_{ch}$ at first increases rapidly from SIS to AGS, then saturates to a value of about 800 MeV at SPS energies and remains constant up to the highest available RHIC energies [1]. The present analysis of $E_T/N_{ch}$ uses the hadron resonance gas model (thermal model). Our analysis starts by relating the number of charged particles seen in the detector to the number of primordial hadronic resonances and the transverse energy to the energy $E$ of primordial hadrons. The present status of $E/N$ could be found in Ref. [4].

In this paper, all thermal model calculations were performed using the THERMUS package [2]. At high energies the chemical freeze-out temperature saturates at a value of about 160 - 170 MeV as shown in Figure 1(a) and at the same time the baryon chemical potential becomes very small [3]. As a consequence, several other quantities also become independent of beam energy. The average mass of hadronic resonances saturates at approximately the $\rho$ mass at high energies as shown in Figure 1(b). The ratio of all hadrons after resonance decays to the number of directly emitted hadrons at chemical freeze-out saturates at a value of about 1.7 as shown in Figure 2. All of these are direct consequences of the saturation of the freeze-out temperature observed in Figure 1(a) for increasing beam energies and the associated convergence of the baryon chemical potential to zero.

2. Freeze-out

A theoretical description of the whole duration of evolution of the fireball produced in heavy ion collisions is difficult as different degrees of freedom are important at various stages of the evolution. The thermal model uses the hadronic degrees of freedom at the latest stages of the evolution of the fireball when the chemical composition of different particle species stops changing (chemical freeze-out) and then the particle mean free path becomes larger than the system size and thus the system is frozen...
kinetically (thermal freeze-out). Freeze-out could be a complicated process involving duration in time and a hierarchy where different kinds of particles and reactions switch-off at different times, giving rise to the concept of "differential freeze-out". By kinetic arguments, it is expected that the reactions with lower cross-sections switch-off at higher densities/temperatures compared to reactions with larger cross-sections. Hence, the chemical freeze-out (corresponds to inelastic reactions) occurs earlier in time compared to the kinetic freeze-out (corresponds to elastic reactions). In line to this argument, one can think of strange or charmed particles decoupling from the system earlier than the lighter hadrons. A series of freeze-outs could be imagined corresponding to particular reaction channels [9]. However, we will focus on chemical and kinetic freeze-outs in our discussions.

3. Results and Discussions

The transverse energy density in pseudorapidity, $dE_T/d\eta \equiv E_T$, has two components, the hadronic one, $E_T^{\text{had}}$, and the electromagnetic one, $E_T^{\text{em}}$, coming from the electromagnetic particles (photons, electrons and positrons). Electromagnetic calorimeters are used to measure $E_T^{\text{em}}$, whereas hadronic calorimeters or the Time Projection Chamber with experimental correction for long-lived neutral hadrons are used to measure $E_T^{\text{had}}$ [10] [11]. The energy of a particle is defined as being the kinetic energy for nucleons, for anti-nucleons as the total energy plus the rest mass and for all other particles as the total energy [10] [12].

Experiments have reported a constant value of the ratio $E_T/N_{\text{ch}} \sim 0.8$ GeV from SPS to RHIC [10] [13], with the ratio being almost independent of centrality of the
collision for all measurements at different energies. In all cases, the value of \( E_T/N_{\text{ch}} \) has been taken for the most central collisions at mid-rapidity. At the end of this paper we consider the centrality dependence of \( E_T/N_{\text{ch}} \). When this ratio is observed for the full range of center of mass energies, it shows two regions \[13\]. In the first region from lowest \( \sqrt{s_{NN}} \) to SPS energy, there is a steep increase of the \( E_T/N_{\text{ch}} \) ratio with \( \sqrt{s_{NN}} \). In this regime, the increase of \( \sqrt{s_{NN}} \) causes an increase in the \( \langle m_T \rangle \) of the produced particles. In the second region, SPS to higher energies, the \( E_T/N_{\text{ch}} \) ratio is very weakly dependent on \( \sqrt{s_{NN}} \).

To estimate \( E_T/N_{\text{ch}} \) in the thermal model, we relate the number of charged particles, \( N_{\text{ch}} \), to the number, \( N \), of primordial hadrons. To estimate the charged particle multiplicity at different center of mass energies from the thermal model, we proceed as follows. First we study the variation of the ratio of the total particle multiplicity in the final state, \( N_{\text{decays}} \), and that in the primordial i.e. \( N_{\text{decays}}/N \) with \( \sqrt{s_{NN}} \). This ratio starts from one, since there are only a few resonances produced at low beam energy and becomes almost independent of energy after SPS energy with a saturated value of around 1.7. The excitation function of \( N_{\text{decays}}/N \) is shown in Figure 2(a). Secondly, we have studied the variation of the ratio of charge particle multiplicity in the final state \( (N_{\text{ch}}/N_{\text{decays}}) \) with \( \sqrt{s_{NN}} \). This is shown in Figure 2(b). The \( N_{\text{ch}}/N_{\text{decays}} \) ratio starts around 0.4 at lower \( \sqrt{s_{NN}} \) and shows an energy independence at SPS and higher energies. At lower SIS energy, the baryon dominance at mid-rapidity makes \( N_{\text{ch}}/N_{\text{decays}} \sim N_{\text{proton}}/N_{(\text{proton+neutron})} \) which has a value of 0.45 for Au-Au collisions.

As the next step, we connect the transverse energy \( E_T \) to the the energy of the primordial hadrons \( E \). In the hadronic resonance gas model there is a sum over all hadrons; furthermore, taking into account the experimental configuration which leads to adding the mass of the nucleon for anti-nucleons and subtracting the same for nucleons one has

\[
\langle E_T \rangle \equiv V \sum_{i=\text{nucleons}} \frac{d^3p_i}{(2\pi)^3} (E_i - m_N) \sin \theta_i \ f(E_i) \\
+ V \sum_{i=\text{anti-nucleons}} \frac{d^3p_i}{(2\pi)^3} (E_i + m_N) \sin \theta_i \ f(E_i) \\
+ V \sum_{i=\text{all others}} \frac{d^3p_i}{(2\pi)^3} E_i \sin \theta_i \ f(E_i),
\]

\[
= \frac{\pi}{4} \left[ \langle E \rangle - m_N \langle N_B - N_{\bar{B}} \rangle \right].
\]  

(1)

Here \( \theta \) is the polar angle of a particle with the beam direction and \( f(E) \) is the statistical distribution factor. The above equation relates the transverse energy measured from the data and that estimated from the thermal model. In the limit of large beam energies one has

\[
\lim_{\sqrt{s_{NN}} \to \infty} \frac{\langle E_T \rangle}{N_{\text{ch}}} = \frac{\langle E_T \rangle}{0.6 N_{\text{decays}}},
\]
Saturation of $E_T/N_{ch}$ and Freeze-Out Criteria in Heavy-Ion Collisions

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Figure 2. Saturation of $N_{\text{decays}}/N$ (a) and $N_{ch}/N_{\text{decays}}$ (b) with $\sqrt{s_{NN}}$. In (a) the results from various freeze-out criteria are indicated. In (b) the different freeze-out criteria give results that are indistinguishable.

\begin{align}
\frac{E}{N} &= \frac{\pi}{4} \frac{1}{0.6} \frac{E}{1.7N}, \\
&= 0.77 \frac{E}{N}, \\
&\approx 0.83 \text{ GeV}.
\end{align}

(2)

This value of $E_T/N_{ch}$ is close to the values measured at RHIC energies and is independent of collision species, centrality of the collisions and $\sqrt{s_{NN}}$. It should be noted that the measured $E_T$ will be affected by the transverse collective flow and by the difference between chemical freeze-out and kinetic freeze-out temperatures and therefore the description presented here is only a qualitative one. An analysis including flow was presented in Figure 17 of the review article by Kolb and Heinz [16] who show that this improves the agreement with the data at SPS and RHIC beam energies. A detailed comparison in the framework of a specific model with a single freeze-out temperature, has been made in Ref. [17].

At higher energies, when $\mu_B$ nearly goes to zero, the transverse energy production is mainly due to the meson content in the matter. The intersection points of lines of constant $E_T/N_{ch}$ and the freeze-out line give the values of $E_T/N_{ch}$ at the chemical freeze-out. Hence at freeze-out, given the values of $E_T/N_{ch}$ from the experimental measurements we can determine $T$ and $\mu_B$ of the system [11].

For the most central collisions, the variation of $E_T/N_{ch}$ with center of mass energy is shown in Figure 3(a). The data have been taken from Ref. [1], and are compared with the corresponding calculation from the thermal model with chemical freeze-out. We have
Saturation of $E_T/N_{ch}$ and Freeze-Out Criteria in Heavy-Ion Collisions

![Graph](image)

Figure 3. (a) Comparison between experimental data for $E_T/N_{ch}$ with $\sqrt{s_{NN}}$ and the thermal model using $E/N = 1.08$ GeV as well as other freeze-out conditions [5, 6, 7, 8], (b) The variation of $E_T/N_{ch}$ with $N_{part}$ for 130 GeV Au+Au collisions at RHIC [14] with corresponding thermal model estimates.

checked explicitly that other freeze-out criteria discussed in the literature give almost identical results for the behavior of $E_T/N_{ch}$ as a function of $\sqrt{s_{NN}}$; this is the case for the fixed baryon plus anti-baryon density condition [5] and also for fixed normalized entropy density condition, $s/T^3 = 7$ [6, 7, 8]. As is shown in Figure 3(b), centrality behavior of $E_T/N_{ch}$ is well reproduced by the thermal hadronic resonance gas model.

Taking the arguments by Heinz et al. [18], if the freeze-out is a kinetic process, it is controlled by the competition between local scattering (moving the system towards equilibrium) and global expansion (driving the system out of equilibrium). The resulting freeze-out temperature is therefore sensitive to the fireball expansion rate which depends on collision centrality. Hence the kinetic decoupling temperature ($T_{\text{kin}}$) should depend on centrality. Such an centrality dependence has been observed for $T_{\text{kin}}$, whereas $T_{ch}$ has been observed to be independent of centrality [19]. The centrality independence of $T_{ch}$ has been interpreted as due to the chemical decoupling of hadron abundances being driven by a phase transition during which the chemical reaction rates decrease abruptly, leaving the system in a chemically frozen-out state at the end of the transition. Thus we get a universal $T_{ch}$ which is insensitive to the collective dynamics but depends on the thermodynamic parameters of the phase transition. The observation of $E_T/N_{ch}$ saturating at a universal value and being independent of centrality of the collisions could also be related to the quark-hadron phase transition through chemical freeze-out. The saturation value of $E_T/N_{ch}$ can also be taken as its value for a pre-hadronic state, as $T_{ch} \sim T_C$. Irrespective of the initial conditions (controlled by system size and beam energy), at higher energies, the system evolves to the same chemical freeze-out condition.
4. Summary

In conclusion, we have discussed the connection between $E_T/N_{ch}$ and the ratio of primordial energy to primordial particle multiplicity, $E/N$, from the thermal model. This model, when combined with chemical freeze-out criteria explains the data over all available measurements for the $\sqrt{s_{NN}}$ and centrality behavior of $E_T/N_{ch}$. $E_T/N_{ch}$ being related to $T_{ch}$ is associated with the quark-hadron phase transition. It has to be noted that variables like $E_T/N_{ch}$, the chemical freeze-out temperature $T_{ch}$, $N_{\text{decays}}/N_{\text{primordial}}$ and $N_{ch}/N_{\text{decays}}$ discussed in this paper, show saturation starting at SPS and continuing to higher center of mass energies.

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