Reconstruction of single muon tracks in Cherenkov water detector NEVOD

E A Kovlyaeva, S S Khokhlov, V A Khomyakov, R P Kokoulin, A A Petrukhin, V V Shutenko, I I Yashin
National Research Nuclear University MEPhI, Moscow, Russia
E-mail: E.A.Kovyliaeva@mail.ru

Abstract. The technique of single muon track reconstruction in Cherenkov water detector NEVOD is presented. The technique is based on the maximum likelihood method, in which probabilities of PMT response are used. The reconstruction technique has been tested on real experimental events selected by the coordinate detector DECOR. The angular distribution of detected muons has been estimated.

1. Introduction
Cherenkov water detector (CWD) NEVOD is a multifunctional experimental facility with volume about 2000 m$^3$ intended for detection of all basic components of cosmic rays on the Earth’s surface [1] (figure 1).

![Figure 1. Experimental complex NEVOD-DECOR.](image1)

![Figure 2. Detecting system: 1 - quasispherical modules, 2 - scintillation counters.](image2)

The NEVOD detecting system is formed by a spatial lattice, in the knots of which quasispherical optical modules (QSM) are located. Modules are combined into vertical strings of 3 or 4 QSM in each (figure 2). The distances between the modules are 2.5 m along the detector, and 2.0 m across it and over the depth. Every QSM includes six photomultiplier tubes (PMTs) with flat cathodes oriented along the axes of the orthogonal coordinate system and allows to detect Cherenkov radiation from any direction with practically the same efficiency.

A nearly cubic shape and structure of the detector provide similar conditions for the detection of Cherenkov light produced by particles arriving from any direction, and creates a detecting system with properties of a 4π-detector. For calibration of PMTs, the calibration telescope system (CTS) is used.
(figure 2). To expand setup research possibilities, the coordinate detector DECOR [2] was constructed in frame of the Russian–Italian cooperation around the Cherenkov water detector (figure 1). DECOR represents a modular multi-layer system of plastic streamer tube chambers with resistive cathode coating. The side part of DECOR includes eight vertically suspended eight-layer assemblies (supermodules, SMs) of chambers with the total sensitive area 70 m². Chamber planes are equipped with two-coordinate external strip readout system that allows to localize charged particle tracks with about 1 cm accuracy in both coordinates (X, Y). Angular accuracy of reconstruction of muon tracks crossing the SM is better than 0.7° and 0.8° for projected zenith and azimuth angles, respectively.

The technique of single muon track reconstruction in CWD NEVOD is based on amplitude analysis of PMT responses.

2. Technique of single muon track reconstruction

The technique of single muon track reconstruction is based on the maximum likelihood method. The reconstruction can be reduced to the determination of unknown muon track parameters x, y, z, θ, φ by maximization of the likelihood function:

\[
L = \sum_{i=1}^{N_{hit}} \ln P_{hit,i} + \sum_{i=N_{hit}+1}^{N_{PMT}} \ln P_{no-hit,i}
\]

where \(N_{hit}\) is the number of hit PMTs in the spatial lattice of CWD; \(N_{PMT} = 546\) is the total number of PMTs in the detector; \(P_{hit}\) is a hit probability of PMT, which gives response to the event; \(P_{no-hit}\) is no-hit probability of PMT, which doesn’t give response to the event. If direct Cherenkov radiation from a hypothetical track illuminates PMTs, i.e. \(\cos \alpha > 0\) (α is the angle of Cherenkov radiation incidence on PMT cathode) probabilities were calculated according to Poisson distribution: 

\[
\begin{align*}
    P_{hit} &= 1 - e^{-N_{ph.e.}} \\
    P_{no-hit} &= N_{PMT} e^{-N_{ph.e.}},
\end{align*}
\]

where \(N_{ph.e.}\) is the average number of photoelectrons knocked out by Cherenkov photons from the cathode surface. If direct Cherenkov radiation from a hypothetical track does not illuminate PMT (\(\cos \alpha < 0\)), probabilities are set to be constants: \(P_{hit} = 0.01\) and \(P_{no-hit} = 0.99\).

The dependence of \(N_{ph.e.}\) on the distance \(d\) between the track and the centre of PMT has been studied. The signals are analyzed from PMT-1 (the PMT normals are parallel to the Y-axis of CWD coordinate system, see figure 1) at azimuth angle close to 90° and from PMT-3 at angles close to 270°. In this geometry, the average incidence angle of Cherenkov light is close to the angle of Cherenkov radiation in water \(\theta_c\). The scheme of direct Cherenkov photon detection by PMT-1 is shown in figure 3. PMTs of QSM are indicated by numbers.

![Figure 3. The scheme of direct Cherenkov photon detection by PMT-1.](image)

The obtained dependence \(N_{ph.e.}(d)\) is shown in figure 4. The dependence was fitted by the following function (for detailed description refer to [3]):

\[
N_{ph.e.}(d) = 10.7 \cdot (e^{-d/0.7} + 0.5 \cdot e^{-d/3.8}) / (d + 0.075).
\]
In general, for any PMT in the detector the number of photoelectrons is calculated by the following approximate formula:

$$N_{ph.e.}(d, \alpha) = \frac{N_{ph.e.}(d)}{\cos \theta_e} \cdot \cos \alpha.$$  

(3)

An example of the dependence of the likelihood function (1) on the azimuth angle for “OneTrack” event is shown in figure 5. The vertical line in this figure corresponds to azimuth angle $\varphi$ value reconstructed by DECOR. As one can see from this figure, the likelihood function has a pronounced global maximum distorted by local extremums. To find the maximum of likelihood function, direct search method of Hooke-Jeeves was used [4]. Unlike the gradient method, in this method information about the function itself and not about the derivatives of the function is used. The advantage of this method is that it requires very low machine capacity. Basic track is an initial parameter for the search by means of Hook-Jeeves method. To find the basic track, $N$ random tracks were considered in the search space (CWD volume). Likelihood function was calculated for each of these tracks. The track, for which the likelihood function reaches its maximum value, is taken as the basic one.

![Figure 4](image1.png) ![Figure 5](image2.png)

**Figure 4.** Experimental dependence of the average number of photoelectrons on the distance and its fit (2).  

**Figure 5.** The dependence of the likelihood function on the azimuth angle.

3. The results of experimental event reconstruction

To check the accuracy of the reconstruction, the developed technique has been tested on experimental events selected by the coordinate detector DECOR. An example of reconstructed “OneTrack” event is shown in figure 6. Muon tracks reconstructed by each SM of DECOR separately are indicated as “1”; the average muon track “OneTrack” according to the data of DECOR is marked as “2”; the track reconstructed by the maximum likelihood method is indicated as “3”. The dependence of average angle values between the reconstructed track and “OneTrack” direction on the number of random tracks $N$ used for the selection of initial basic track in Hook-Jeeves method is shown in figure 7.

![Figure 6](image3.png) ![Figure 7](image4.png)

**Figure 6.** Example of reconstructed “OneTrack” event.  

**Figure 7.** The dependence of the average angle between reconstructed track and “OneTrack” direction on the number of random tracks $N$. 


As it is seen from the figure, the accuracy of angle reconstruction is improving with increasing number of tracks $N$. However, when comparing values of the angles for $N=500$ and $N=1000$, it turns out that CPU time of calculation increases 2 times, but angular accuracy improves only by 8%. On the basis of these considerations, $N=500$ was chosen as an optimal parameter for Hook-Jeeves method.

4. Measurement of the angular distribution of muons at the Earth surface
Events of experimental run (14.11.2011) were reconstructed by means of the developed technique. Event selection condition during the run was the threshold on the number of hit QSM > 35. Zenith angular distribution of reconstructed muons is shown in figure 8. This distribution was fitted by a power function $\cos^\beta \theta$, where $\beta = 1.61 \pm 0.04$. Azimuth angular distribution of reconstructed muons for three intervals of zenith angle is shown in figure 9.

Excess in muon rate for $\varphi = 90^\circ$ и $\varphi = 270^\circ$ is determined by the geometry of CWD NEVOD (figure 1). Along-going muons pass a longer way in water to the detecting lattice, thus Cherenkov light from them illuminates on average more PMTs.

5. Conclusion
Developed technique of single muon track reconstruction can provide hodoscopic operating mode of the CWD NEVOD. The accuracy of experimental event reconstruction better than $5^\circ$ has been reached.

Acknowledgments
The work was performed in Scientific and Educational Centre NEVOD in frame of the leading scientific school NSh-6817.2012.2 with the support of the Russian Ministry of Education and Science (contract no. 16.518.11.7053).

References
[1] Aynutdinov V M et al 1995 Proc. 24th ICRC (Roma) vol 1 p 1076
[2] Amelchakov M B et al 2001 Proc. 27th ICRC (Hamburg) vol 3 p 1267
[3] Khokhlov S S et al 2012 Proc. 23rd ECRS (Moscow) Submitted to J. Phys.: Conf. Series
[4] Hooke R and Jeeves T A 1961 Direct search solution of numerical and statistical problems
  *Journal of the Association for Computing Machinery (ACM)* 8 212