Clustering Properties and Model Wavefunctions for Non-Abelian Fractional Quantum Hall Quasielectrons

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We present model wavefunctions for quasielectron (as opposed to quasihole) excitations of the unitary \( Z_k \) parafermion sequence (Laughlin/Moore-Read/Read-Rezayi) of Fractional Quantum Hall states. We uniquely define these states through two generalized clustering conditions: they vanish when either a cluster of \( k+2 \) electrons is put together, or when two clusters of \( k+1 \) electrons are formed at different positions. For Abelian Fractional Quantum Hall states (\( k = 1 \)), our construction reproduces the Jain quasielectron wavefunction, and elucidates the difference between the Jain and Laughlin quasielectrons. For two (or more) quasielectrons, our states differ from those constructed using Jain’s method. By adding our quasielectrons to the Laughlin state, we obtain a hierarchy scheme which gives rise to the non-Abelian non-unitary \( \nu = \frac{2}{3} \) FQH Gaffnian state.

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The connection between conformal field theory (CFT) and Fractional Quantum Hall (FQH) states\(^1\)\(^2\) provides model wavefunctions for non-Abelian ground states and their quasihole excitations. A central result of the CFT-FQH connection has been the prediction that the addition of several units of flux creates multiple degenerate pinned quasihole states which exhibit non-Abelian statistics. In particular, some of the Read-Rezayi (RR)\(^3\) series of non-Abelian states are thought to be experimentally relevant to the \( \nu = 5/2 \) and \( \nu = 12/5 \) FQH plateaus.

Despite these successes, the FQH/CFT connection has failed to produce unique model wavefunctions for the Laughlin quasielectron states. This is due to the fact that, until recently, previous attempts at introducing quasielectrons invariably necessitated using anti-holomorphic coordinates \( z^* \) (in some form) and then projecting to the lowest Landau level (LLL); this procedure can be done in several ways, leading to different polynomial wavefunctions. For the non-Abelian states, quasielectron wavefunctions are not known. Recently, several authors\(^4\) have succeeded in expressing the Jain model quasielectron wavefunctions for the Laughlin hierarchy sequence as CFT correlators. However, several Abelian quasielectron models exist (due to Laughlin, Jain, Girvin, and others), and the physical differences between them are not fully understood.

In this paper we provide an explicit construction of LLL quasielectron model wavefunctions for the \( Z_k \) RR sequence. The RR \( Z_k \) FQH ground-states are uniquely defined as the smallest degree polynomials that vanish when \( k+1 \) particles cluster together. Our purpose is to find similar physical clustering conditions (Hamiltonians) that uniquely define the 1-quasielectron state. Since quasielectrons involve the removal of flux, and hence the lowering of the total degree of the polynomial wavefunction, a 1-quasielectron wavefunction of the RR states can no longer vanish when \( k+1 \) particles come together. We find two kinds of quasielectrons: an Abelian 1-quasielectron wavefunction of the RR \( Z_k \) sequence vanishes when \( 2k+1 \) particles come together and when two clusters, each of \( k+1 \) particles are formed at different positions. A non-Abelian 1-quasielectron wavefunction vanishes when \( k+2 \) particles come together and when two clusters, each of \( k+1 \) particles are formed at different positions. For \( k = 1 \) (Laughlin states), the two clustering conditions are equivalent; our 1-quasielectron states turn out to be identical to Jain’s. The clustering conditions they satisfy explain the numerically observed energetic superiority of Jain’s quasielectron over Laughlin’s. Our many-quasielectron states differ from Jain’s. A hierarchy scheme based on the present quasielectrons gives rise to a non-Abelian (CFT non-unitary) state for the \( \nu = \frac{2}{3} \) bosonic (\( \nu = \frac{5}{2} \) fermionic) Gaffnian state\(^5\) (identical to the \( \langle k,r \rangle = (2,3) \) Jack polynomial\(^6\)). While Read has recently given general arguments\(^7\) against the idea that FQH states described by a non-unitary CFT could be gapped, we do not have a microscopic understanding of the failure of these states, such as an identification of their conjectured\(^8\) gapless bulk excitation or failure to properly screen in the non-Abelian sector. Even if they do describe critical points and not phases of matter, we consider it fruitful to investigate their properties further.

We represent a partition \( \lambda \) with length \( \ell_\lambda \leq N \) as a (bosonic) occupation-number configuration \( n(\lambda) = \{ n_m(\lambda), m = 0,1,2,\ldots \} \) of each of the LLL orbitals \( \phi_m(z) = (2\pi m!)^{1/2} z^m \exp(-|z|^2/4) \) with angular momentum \( L_z = mh \) (see Fig\(^1\)), where, for \( m > 0 \), \( n_m(\lambda) \) is the multiplicity of \( m \) in \( \lambda \). It is useful to identify the “dominance rule”\(^8\) (a partial ordering of partitions \( \lambda > \mu \)) with the “squeezing rule”\(^9\) that connects configurations \( n(\lambda) \rightarrow n(\mu) \): “squeezing” is a two-particle operation that moves a particle from orbital \( m_1 \) to \( m_1^\prime \).
and another from $m_2$ to $m_2'$, where $m_1 < m'_1 < m_2 < m_2'$, and $m_1 + m_2 = m'_1 + m'_2$. $\lambda > \mu$ if $n(\mu)$ can be derived from $n(\lambda)$ by a sequence of “squeezings” (see Fig. 1). An interacting LLL polynomial $P_\lambda$ indexed by a root partition $\lambda$ is defined as exhibiting a dominance property if it can be expanded in occupation-number non-interacting states (monomials) of orbital occupations $n(\mu)$ obtained by squeezing on the root occupation $n(\lambda)$:

$$P_\lambda = m_\lambda + \sum_{\mu < \lambda} v_{\lambda \mu} m_\mu. \quad (1)$$

The $v_{\lambda \mu}$ are rational number coefficients. Partitions $\lambda$ can be classified by $\lambda_1$, their largest part. When any $P_\lambda$ is expanded in monomials $m_\mu$, no orbital with $m > \lambda_1$ is occupied. $P_\lambda$ can be interpreted as states on a sphere surrounding a monopole with charge $N_\Phi = \lambda$. Uniform (ground) states on the sphere satisfy the conditions $L^+ \psi = 0$ (highest weight, HW) and $L^- \psi = 0$ (lowest weight, LW) where $L^+ = E_0$, and $L^- = N_\Phi Z - E_2$, where $Z \equiv \sum z_i$, and $E_n = \sum z_i^n \partial / \partial z_i$. In a previous paper, we proved, by using the HW and LW conditions that the Jack polynomials (Jacks) of root occupation $n(\lambda^0(k, 2)) = [k0k0k\ldots k0k]$ and Jack parameter $\alpha_{k, r} = -(k + 1)/(r - 1)$ are the groundstate wavefunctions of the RR $Z_\lambda$ sequence. The RR quasi-hole wavefunctions are also Jacks of root occupation numbers satisfying a $(k, 2)$ of a more general $(k, r)$ Pauli principle which allows no more than $k$ particles in $r$ consecutive angular momentum orbitals. For the Jacks, the coefficients $v_{\lambda \mu}$ are explicitly known by recursion. In our construction, we require the squeezing rule be satisfied also for the quasi-electron states. Our root occupation number is reminiscent of the thin-torus description; however, we generate the full interacting polynomial wavefunction and not just the non-interacting Tao-Thouless state.

Quasielectron states satisfy only the HW condition $L^+ \psi = 0$ and should represent a small local perturbation of the otherwise featureless ground state density. We now present the root occupation $n(\lambda)$ and a set of clustering conditions which uniquely define the quasielectron wavefunctions. We start with the Abelian 1-quasielectron added to the $\nu = k / 2$ Jacks $J_{k, s, 2}^{\lambda^0(1)}(z_1, \ldots, z_N)$ (RR groundstates) of root occupation $n(\lambda^0_{k, s, 2}) = [k0k0k\ldots k0k]$. By analogy with the Abelian quasiholes, this should be a state of total angular momentum $L = N/2$. We add 3 fluxes to the groundstate and obtain the occupation number: $n = [000k0k0k\ldots k0k]$. The Abelian 1-quasielectron state is obtained by adding $2 \cdot k$ particles in the zero'th orbital (North Pole): $n(\lambda^0_{k, s, qp}) = [2 \cdot k0k0k0k\ldots k0k]$. Simple counting gives us $N_\Phi = k(N - k) - 1$, the correct flux for an $N$ particle $\nu = k$ Read-Rezayi state with an Abelian 1-quasielectron. Away from the north pole, the quasielectron root occupation relaxes to the bulk sequence $[k0k\ldots k0k]$. The root occupation and the HW condition do not define the 1-quasielectron polynomial wavefunction uniquely. We now search for a way to uniquely define the polynomial. In a previous paper we showed that the HW condition on the Jacks gives an infinite set of Jacks at $\alpha = -(k + 1)$ of occupation numbers $n(\lambda^0_{k, s, 2}) = [n_00^s+1k0k0k0k\ldots]$ with $n_0 = (k + 1)(s + 1) - 1$, and $s \geq 0$ a positive integer. For $s = 0$ these are the RR FQH groundstates. For $s \geq 1$, we have $n_0 > k$ and hence these configurations contain an excess of charge at the north pole, and heal in the bulk to the RR ground-state configurations. However, the Abelian $s$-quasielectron state in the $Z_\lambda$ sequence should have $N_\Phi = k(N - k) - s$, the orbital occupation $n(\lambda^0_{k, s, 2})$ contains too much charge at the north pole. To obtain the correct $N_\Phi$, we must “subtract” $s$ particles from the zero orbital of the occupation sequence $n(\lambda^0_{k, s, 2}) = [n_00^s+1k0k0k0k\ldots]$ of the Jacks given in 14, to obtain the root occupation configuration $n(\lambda^0_{k, s, qp}) = [k \cdot (s + 1)^{s+1}k0k0k0k\ldots]$. At the explicit, first quantized wavefunction level, this “subtraction” can be done by symmetrization and padding of the Jack polynomial $J_{k, s, 2}^{\lambda^0_{k, s, qp}}$ 15, but a simpler expression will be presented shortly. Defined in this way, the $s$-quasielectron state shares a clustering property with the orbital occupation $n(\lambda^0_{k, s, qp})$ that we obtained in 14: it vanishes when $s+1$ clusters of $k+1$ same-position particles are formed. Being HW states dominated by $n(\lambda^0_{k, s, qp})$, they also vanish when $k \cdot (s + 1) + 1$ particles come together at the same point as the $s + 2$’s power of the difference between coordinates 14. The angular momentum of the Abelian $s$-quasielectron configurations above is $l(\lambda^0_{k, s, qp}) = L_z(\lambda^0_{k, s, qp}) = s/2$. The above root configurations define the maximum angular momentum Abelian $s$-quasielectron states (bunched up at the North Pole) of the Laughlin, Read-Moore and Read-Rezayi sequence. Hence, our HW Abelian ($s = 1$) 1-quasielectron state is uniquely defined as the smallest
degree polynomial satisfying the clustering conditions:

\[ P(z_1, z_2, ..., z_N) = 0 \]

\[ P(z_1, z_2, ..., z_N) \sim \prod_{i=2}^{N} (z_1 - z_i)^3 (2) \]

For \( N_q = \frac{2}{\nu}(N - k) - 1 \), the counting developed in [14] gives exactly \( N + 1 \) linearly independent polynomials satisfying Eq. (2). They correspond to the different \( l_z \)'s of the \( l = \frac{N}{\nu} \) multiplet of states. The HW state \( (l, l_z) = (\frac{N}{\nu}, \frac{N}{\nu}) \) satisfies a more stringent clustering condition than Eq. (2): \( P(z_1, z_2, ..., z_N) = \prod_{i=2}^{N} (z_1 - z_i)^3 (2) \)

\[ \prod_{i=2}^{N} (z_1 - z_i)^3 J_{s}(k+1) (z_2, ..., z_N) \] where \( n(\lambda^0(k, 2)) = [k0k0...k0] \) and \( J_{s}(k+1) (z_2, ..., z_N) \) is the RR \( Z_k \) ground-state for \( N - k \) particles. An alternate definition which also uniquely fixes the HW 1-quasielectron state is requiring that it satisfies HW, dominance, and angular momentum \( l \). For the Laughlin, \( (k, r) = (1, 2) \), \( \nu = 1/2 \), state we find that the 1-quasielectron HW wavefunction \( P_{\lambda^0(1, 1)} \) involves one symmetrization over a Jack found in [14]:

\[ P_{\lambda^0(1, 1)} (z_1, ..., z_N) = \text{Sym}J_{s}(2) (z_1, z_2, ..., z_N) \] (3)

Model HW wavefunctions for the \( s \)-qp state of maximum angular momentum \( l = s \frac{N}{\nu} \) are obtained by further symmetrization over the Jacks of [14]: \( P_{\lambda^0(s, s)} (z_1, ..., z_N) = \text{Sym}J_{s}(2) (z_1, z_2, ..., z_N) \). For \( k > 1 \), similar expressions can be obtained [13]. However, we found that our wavefunctions can be written in compact form using an operator first introduced by Jain [10]:

\[ O(\partial_1, ..., \partial_N, z_1, ..., z_N) = \text{Det} \begin{pmatrix} \partial_1 & \partial_2 & ... & \partial_N \\ 1 & 1 & ... & 1 \\ z_1 & z_2 & ... & z_N \\ ... & ... & ... & ... \\ z_1^{N-2} & z_2^{N-2} & ... & z_N^{N-2} \end{pmatrix} \]

where \( \text{Det} \) denotes the determinant. We find our HW Abelian 1-quasielectron states of the RR \( Z_k \) sequence, as defined by symmetrization over Jacks, are identical to:

\[ P_{\lambda^0(1, 1)} (z_1, ..., z_N) = \frac{1}{\Delta} OJ_{s}(k+1) (z_1, ..., z_N) \] (4)

where \( \Delta = \prod_{i<j} (z_i - z_j) \) is the VanderMonde determinant and \( J_{s}(k+1) (z_1, ..., z_N) \) is the Jack polynomial FQH ground-state of the RR \( Z_k \) sequence [6]. The right hand side of Eq. (3) is a symmetric polynomial as the determinant operator \( O \) is antisymmetric in the \( z_i \)'s. We have checked that \( P_{\lambda^0(s, 1)}^{\pm} \) in Eq. (3) exhibits a dominance property Eq. (1) with the root occupation \( n(\lambda^0(s, 1)) = [2, k00k00k0...k0] \), and satisfies the clustering conditions in Eq. (2). The \( l_z = -N/2...N/2 \) multiplet can be obtained by successively applying the \( L^- \) operator on \( P_{\lambda^0(s, 1)}^{\pm} \). These states also satisfy the clustering conditions in Eq. (2). The density profiles for the Read-Moore \( \nu = 1 \) and the Read-Rezayi \( \nu = \frac{2}{3} \) quasielectron are plotted in Fig. [2]. For \( k = 1 \) Laughlin states, by Eq. (2) our quasielectron wavefunctions can be seen to be identical to Jain’s [16]. Our definition of the quasielectron through the clustering conditions Eq. (2) provides a physical explanation for the numerical finding [17] that Jain’s quasielectron has a lower energy than Laughlin’s [18]. We found that Laughlin’s original quasielectron wavefunction satisfies the second of the clustering conditions in Eq. (2) but not the first one. We have checked that Jain’s quasielectron has a lower energy than Laughlin’s due to the fact that it satisfies one extra clustering condition.

So far we have focused on the bosonic (\( m = 0 \)) \( Z_k \) FQH states. For integer \( m \geq 1 \), the Read-Rezayi sequence at filling \( \nu = k/(km + 2) \) has the wavefunction \( \Psi_{\text{RR}}^m = \prod_{i<j} (z_i - z_j)^m J_{\lambda^0(k, 2)}^{\nu} \). The HW quasielectron wavefunction is \( \psi_{\nu}^{\nu} (z_1, ..., z_N) = \prod_{i<j=1}^{N} (z_i - z_j)^m P_{\lambda^0(1, 1)} (z_1, ..., z_N) \). The above construction of the quasielectron trivially generalizes to the entire \( (k, r) \) Jack sequence of FQH states introduced in [6].

We now construct the non-Abelian quasielectron states for the RR \( Z_k \) sequence. A non-Abelian fractionalized quasielectron will always be accompanied by a non-
Abelian fractionalized quasihole, and will be composed of an electron bound to a fractionalized quasihole. As the Abelian quasihole has angular momentum \( l = \frac{N}{k} \), each fractionalized non-Abelian quasihole (and fractionalized quasiparticle) has \( l = \frac{N}{k} \). The basic neutral excitation of the system is a fractionalized 1–quasielectron 1–quasihole state at the same flux as the FQH RR ground state \( N_k = \frac{2}{\nu}(N - k) \). As a fractionalized quasielectron and quasihole are distinguishable particles, angular momentum addition gives multiplets of states \( l_1 \equiv \frac{N}{k} \bigoplus \frac{N}{k} = \frac{N}{k} - 1, \frac{N}{k} - 2, \ldots, 2, 1, 0 \) (the \( l = 1 \) state will be missing). The HW \( l = \frac{N}{k} \) state corresponds to completely separating the fractionalized quasielectron at the North Pole from the fractionalized quasihole at the South Pole. It is uniquely defined by the dominated polynomial of root occupation number \( n(k_0, 1_{qp} - 1_{qh}) = [k + 10k - 11k - 11k - 12k] \), satisfying the clustering conditions:

\[
P(z_{k+1}, \ldots, z_0; z_2, \ldots, z_{2k+3}, 2k+4, \ldots, N) = 0; \quad P(z_{k+1}, \ldots, z_0, z_{2k+3}, 2k+4, \ldots, z_N) = 0 \quad (5)
\]

The HW \((l = l)\) states of the \( l = \frac{N}{k} - 1, \ldots, 2, 0\) multiplets can be uniquely defined by imposing HW, along with first clustering condition in Eq.\((6)\) on dominated polynomials with root occupation numbers:

\[

l = \frac{N}{k} - 1; \quad [k + 10k - 11k - 11k - 1];
\]

\[
l = \frac{N}{k} - 2; \quad [k + 10k - 11k - 11k - 10k];
\]

\[
l = \frac{N}{k} - 3; \quad [k + 10k - 11k - 10k];
\]

\[
l = \frac{N}{k} - 4; \quad [k + 10k - 10k];
\]

\[
l = \frac{N}{k}; \quad [k + 10k];
\]

\[

The second clustering condition in Eq.\((6)\) is then automatically satisfied. Alternatively, Eq.\((7)\) and the second clustering in Eq.\((6)\) also uniquely define the Hilbert space of 1–quasielectron 1–quasihole states, although in this case further angular momentum projection is needed to obtain \( \vec{L} \) eigenstates. For \( k = 1, z_2 \) is not different from \( z_3, \ldots, z_N \), and the non-Abelian clustering conditions become identical to the Abelian ones (the Laughlin states support only Abelian excitations). We can “energetically” justify our quasielectron-quasihole wavefunctions. As they cannot vanish when \( k + 1 \) particles come together (this condition defines the RR \( Z_k \) ground-state and pure quasiholes), the lowest “energy” configuration that one can create is to require the wavefunction vanish in a \( k + 2 \) particle cluster.

We now focus on the \( s \)-quasielectron states, \( s > 1 \) and first treat the \( k = 1, \nu = \frac{1}{\nu} \) Laughlin state. We have previously described the root occupation for the HW \( s \)-quasielectron states at maximum angular momentum \( l = s \frac{N}{k} \), as well as their explicit wavefunction in terms of symmetrization over a series of Jack polynomials defined in \([14]\). We now want to find the root occupation and clustering conditions for the minimum angular momentum \( s \)-quasielectron states. Consistency arguments favor maintaining that the state vanishes when 3 particles come together (second clustering condition for 1-quasiparticle in Eq.\((5)\)). The generalization of the first clustering condition in Eq.\((5)\) is that the state vanishes when we form \( s + 1 \) distinct clusters of 2–particles:

\[
P(z_1, z_2, z_3, \ldots, z_N) \sim (z_1 - z_2)^{2k+1}
\]

\[
	\times \prod_{i=2k+2}^{N}(z_2 - z_i)^2(z_1 - z_i)
\]

\[
\quad (7)
\]

The root occupation numbers for the Moore-Read ground-state and it’s quasiparticle excitations are shown in Fig.\([3]\).

Just as in the Abelian case, there are several ways to define the non-Abelian 1–quasielectron 1–quasihole states, which lead to the same result. Requiring HW, dominance with respect to the root occupation numbers Eq.\((6)\) and the first of the clustering conditions in Eq.\((5)\) uniquely defines the states. The second clustering condition in Eq.\((6)\) is then automatically satisfied. Alternatively, Eq.\((7)\) and the second clustering in Eq.\((6)\) also uniquely define the Hilbert space of 1–quasielectron 1–quasihole states, although in this case further angular momentum projection is needed to obtain \( \vec{L} \) eigenstates. For \( k = 1, z_2 \) is not different from \( z_3, \ldots, z_N \), and the non-Abelian clustering conditions become identical to the Abelian ones (the Laughlin states support only Abelian excitations). We can “energetically” justify our quasielectron-quasihole wavefunctions. As they cannot vanish when \( k + 1 \) particles come together (this condition defines the RR \( Z_k \) ground-state and pure quasiholes), the lowest “energy” configuration that one can create is to require the wavefunction vanish in a \( k + 2 \) particle cluster.

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\[
P(z_1, z_2, z_3, \ldots, z_N) \sim (z_1 - z_2)^{2k+1}
\]

\[
	\times \prod_{i=2k+2}^{N}(z_2 - z_i)^2(z_1 - z_i)
\]

\[
\quad (7)
\]

This is the most consistent set of clustering conditions generalizing the Eq.\((5)\). For \( s > 1 \), these states differ

\[\text{Non-Abelian Quasiparticle-Quasihole}\]

\[\nu = 1 \text{ Moore-Read} \]

\[\nu = \frac{3}{5} \text{ Read-Rezayi} \]

\[\begin{array}{ll}
N = 6 & \quad N = 8 \\
N = 9 & \quad N = 10
\end{array}\]

\[\text{FIG. 3: Exact HW non-Abelian quasielectron-quasihole density profiles, in units of } kN/4\pi(N - k)^2 \text{, for the Moore-Read} (k = 2) \text{ and Read-Rezayi} (k = 3) \text{ N-particle states on the sphere. The fractionalized quasielectron is at the north pole while the fractionalized quasihole is at the south pole. In the thermodynamic limit, the region in the middle of the sphere at density 1 will dominate the density function.}\]
from Jain’s. The HW state is the minimum degree polynomial in $N$ variables satisfying Eq.(8). We find it exhibits a dominance property with the root configuration:

$$n(\lambda_0^0_{s,s}) = [2002002...2002001010101...0101]$$ (9)

Jain’s $s-$quasielectron state is a polynomial that also exhibits the dominance property in Eq. (10) with the root occupation $s = 2$ $20101101010101010101...0101$, as will be shown in a future publication [19]. It satisfies the clustering conditions [19]:

$$P(z_1, z_2, z_3, z_4, ..., z_N) = 0;$$
$$P(z_1, z_1, z_3, z_4, ..., z_N) \sim \prod_{i=2}^N (z_1 - z_i)^2$$ (10)

but, unlike our $s-$ quasielectron state, is not uniquely defined by them. The exponent of the $s > 1$ quasielectron second clustering condition in the Jain state Eq.(10) is different from the exponent in the $s = 1$ quasielectron Jain state Eq. (2). The exponent in our clustering condition Eq. (8) remains the same for both the $s = 1$ and $s > 1$ quasielectron states. For small number of electrons with Coulomb interaction, Jain’s and our Jack quasielectron states have the same energy for $s = 1$ (by virtue of being identical), and similar energies for $s > 1$ (Monte Carlo energy for our Jack 2 quasiparticle bosonic state is $4.71 \pm 0.03 (z^2_3)$ whereas Jain’s is $4.68 \pm 0.01 (z^2_3)$ for $N = 6$ electrons).

A hierarchical scheme of adding our Jack quasielectrons on top of the $\nu = 1/2$ Laughlin state takes one to a non-Abelian, non-unitary $\nu = 2/3$ bosonic state. By adding $s = N/2$ Jack quasielectrons in the Laughlin state one obtains the HW state of root occupation $[002002...002]$. We find this is the $(k, r) = (2, 3)$ Jack polynomial state [3], initially discovered in [20] and called the Gaffnian. Adding $s = N/2$ of Jain’s quasielectrons to the Laughlin state, we obtain the Abelian Jain $\nu = 2/3$ state [4]. We now focus on the differences between the two states. As the usual expression for the Jain $\nu = 2/3$ state on the plane [4] does not lead to an $\vec{L} = 0$ state, we first construct it on the sphere, and then stereographically project to the plane. We write below the root occupations of the Gaffnian and Jain’s $\nu = 2/3$ states (the Jain state root configuration will be explained in a future manuscript [19]):

Jack $\frac{2}{3}$: [2002002002002...002002002002] (11)
Jain $\frac{2}{3}$: [2010110101010101010101...0101]

For a small number of particles, the Jain and the Jack $\nu = \frac{2}{3}$ are very similar (they are identical for $N = 4$ particles). This explains their close energy and large common overlap observed in [21]. From the root configuration above, we can see that Jain state is the $2-$quasielectron excitation of the Gaffnian Hamiltonian.

Finally, we conjecture that the non-Abelian $s$-quasielectron $s$-quasihole RR Hilbert space is spanned by polynomials with $N_\phi = \frac{1}{2} (N - k)$ satisfying the clustering conditions: 1. $P(z_1, z_2, z_3, z_4, ..., z_N) = 0$; 2. $P(z_1, z_2, z_3, z_4, ..., z_N) \sim \prod_{i=3}^N (z_1 - z_i)2k+1$; 3. $P(z_1, z_2, z_3, z_4, ..., z_N) = 0$.

In this paper we have generalized the clustering conditions that define the RR FQH ground-states and quasiholes to include the Abelian and non-Abelian quasielectron excitations. For the Laughlin state, the Jack 1-quasielectron excitations are identical to Jain’s but they differ for $s > 1$ quasielectrons. In particular, a hierarchy scheme based on adding Jack quasielectrons in the Laughlin state leads one to a non-Abelian non-unitary $\nu = \frac{2}{3}$ (or $\nu = \frac{2}{3}$ fermionic) state, the Gaffnian [20] or the Jack (2, 3) state [3].

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Note Recently, during the editing process of this manuscript, an operator describing the non-Abelian quasiparticle state in the Moore-Read $(Z_2)$ state was proposed in [21] (the full polynomial wavefunction is not given in [21] but is advertised in a longer, upcoming version of that manuscript). It would be interesting to investigate whether the two methods give identical polynomials for the Moore-Read quasiparticle state, as they do for the quasiparticle of the Laughlin state.
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