THE ROSSITER–McLAUGHLIN EFFECT FOR EXOMOONS OR BINARY PLANETS

QUNTAO ZHUANG, XUN GAO, AND QINGJUAN YU
Kavli Institute for Astronomy and Astrophysics, and School of Physics, Peking University, Beijing 100871, China; yuqj@pku.edu.cn
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ABSTRACT

In this paper, we study possible signatures of binary planets or exomoons on the Rossiter–McLaughlin (R-M) effect. Our analyses show that the R-M effect for a binary planet or an exomoon during its complete transit phase can be divided into two parts. The first is the conventional one similar to the R-M effect from the transit of a single planet, of which the mass and the projected area are the combinations of the binary components; the second is caused by the orbital rotation of the binary components, which may add a sine- or linear-mode deviation to the stellar radial velocity curve. We find that the latter effect can be up to several ten m s\(^{-1}\). Our numerical simulations as well as analyses illustrate that the distribution and dispersion of the latter effects obtained from multiple transit events can be used to constrain the dynamical configuration of the binary planet, such as how the inner orbit of the binary planet is inclined to its orbit rotating around the central star. We find that the signatures caused by the orbital rotation of the binary components are more likely to be revealed if the two components of a binary planet have different masses and mass densities, especially if the heavy one has a high mass density and the light one has a low density. Similar signatures on the R-M effect may also be revealed in a hierarchical triple star system containing a dark compact binary and a tertiary star.

Key words: planets and satellites: detection – planets and satellites: dynamical evolution and stability – planets and satellites: fundamental parameters – stars: formation – white dwarfs

1. INTRODUCTION

Planets are not alone. Most of the planets in the solar system have surrounding satellites or moons; some asteroids and many of the Kuiper Belt objects (e.g., Pluto and Charon) are in binaries; and recent observations have also revealed miscellaneous exoplanetary worlds, including multi-planetary systems (e.g., Kepler-11, HD 10180, Giese 581; Lissauer et al. 2011; Lovis et al. 2011; Vogt et al. 2010; Forveille et al. 2011). Although the existence of exomoons or binary exoplanets have not been detected, various detection methods have been proposed, such as through transit light curves, transit timing and duration variations, direct imaging, microlensing, or Doppler spectroscopy of the host planet (e.g., Kipping et al. 2009; Simon et al. 2010, 2012; Cabrera & Schneider 2007; Sartoretti & Schneider 1999 and references therein). In this paper, we investigate possible observational signatures that a binary exoplanet system (if any) or an exoplanet plus one moon system would have on the Rossiter–McLaughlin (R-M) effect (McLaughlin 1924; Rossiter 1924) and how the orbital configurations of the system could be revealed through the signatures. For simplicity, below we also call the one exoplanet plus one moon system a binary planet, although the two components have very different masses. The existence of binary planets and statistics on their dynamical configurations should shed new light on the formation and evolution of planetary systems and the search for a habitable world.

When a planet transits in front of a rotating star, it blocks part of the light emitted from the stellar surface. The blocked region shifts with the transiting of the planet. As different parts of the stellar surface may have different line-of-sight velocities to the observer, the shifting of the blocked region results in either blueshift or redshift of observed stellar spectral lines and further the deviation of the inferred radial velocity of the stellar motion (i.e., the R-M effect). The deviation of the stellar radial velocity provides a way to measure the misalignment between the stellar spin and the planetary orbital angular momentum. Recent measurements of the R-M effects have found that some exoplanets are on highly inclined orbits relative to the spin of the star (e.g., retrograde or polar orbits; Winn et al. 2009; Collier Cameron et al. 2010). For similar reasons, if the transiting planet is a binary, the binary with different physical properties and orbital configurations (e.g., radii, inclination) may block the stellar surface in different ways, and hence the shift of stellar spectral lines and the deviation of its radial velocity curve may have different signatures from those expected from the transit of one single planet. The gravitation from a binary planet is also different than that from one single planet with the same total mass, but the resulting deviation in the dynamical motion of the star is generally small within each transit duration (see justification in Section 3.3 below). In this paper, we isolate and illustrate the effects on the observed stellar radial velocity due to the different light blocking geometries.

Most of the modeling of the R-M effect for planetary systems were developed for a single planet rotating around a star. The R-M effect for exomoon systems was numerically modeled by Simon et al. (2010), where the exomoon has a much smaller mass and radius than its host planet and the moon effects are modeled as a small perturbation added to the effect caused by a single planet. Their modeling includes fitting to the numerous dynamical parameters of the systems, including instantaneously and fast evolving ones. In this paper, we present a comprehensive analysis and investigation on the R-M effect for a binary planet system, where the satellite mass is not limited to be small but can be comparable to the host planet mass. We include the effects from the dynamical evolution of the binary planet system during multiple transits and their different dynamical configurations. Our detailed treatments average out the effects from some instantaneously changing dynamical angles and include the evolutionary patterns of those relatively fast changing ones, so that we can focus on the effects from different inner orbital inclinations of the binary planet relative to its orbit rotating around the central star.

1 Note that the extraction of the velocity deviation from stellar spectrum line profiles involves some detailed techniques in the modeling of the effect (e.g., Hirano et al. 2011; Albrecht et al. 2007).
The paper is organized as follows. In Section 2, we describe the geometric configuration of the system to be studied in this paper (i.e., a binary planet transiting in front of a star) and related dynamical approximations. In Section 3, we investigate how the R-M effect is affected by a binary planet and how the different orbital configurations could be inferred from the deviation of the stellar radial velocity curves, together with the transit of the stellar light curves. We explore the parameter space of binary planets that are likely to be revealed in observations. We also extend the results to hierarchical triple star systems in which a binary star (e.g., a compact stellar remnant plus a brown dwarf or planet) is transiting in front of a tertiary star gravitationally bound to the system. Section 4 contains a summary and a discussion.

2. GEOMETRIC CONFIGURATION AND DYNAMICAL DESCRIPTION OF THE SYSTEM

Consider that a binary planet is rotating around a star (see Figure 1). For convenience, we shall call the binary planet as the “inner” binary, and call the star and the center of the mass of the binary planet as the “outer” binary. We denote the mass and the radius of the star as $m_*$ and $R_*$. The projected area of the star onto the sky is $A_* = \pi R^2_*$, and it has a surface brightness of $I_*$. The angular velocity of the stellar spin is $\Omega_*$. We denote the component masses of the inner binary as $m_1$ and $m_2$ ($m_1 \geq m_2$), and the component radii as $R_1$ and $R_2$, respectively. Here we have $m_1 + m_2 \ll m_*$. The projected areas of the two components onto the sky are $A_1 = \pi R^2_1$ and $A_2 = \pi R^2_2$, respectively (see the parameter list in Table 1).

We describe the dynamical motion of the system through the two components: (1) the orbital motion of the outer binary and (2) the orbital motion of the inner binary. The orbital motion of the inner binary includes the precession of its orbital angular momentum around the orbital angular momentum of the outer binary, as described below.

The orbital configuration of the system is indicated in a reference frame as shown in Figure 1. In Figure 1, the center of the star is located at the origin $O$. The $y$-axis is directed toward the observer, and the $z$-axis is chosen so that the stellar spin axis lies on the $y$–$z$ plane. The inclination angle of the stellar spin relative to the $y$-axis is denoted by $i_\ast$ ($0 \leq i_\ast \leq \pi$). The unit vector of the orbital angular momentum of the outer binary is denoted by $n$, and we define $\lambda$ by the angle between the $z$-axis and the projected vector of $n$ onto the $x$–$z$ plane ($0 \leq \lambda < 2\pi$). The orbital inclination angle to the observer $i$ is defined by the angle between $n$ and the $y$-axis. Thus, we have $n = (\sin \lambda \sin i, \cos i, \cos \lambda \sin i)$. The outer binary has a semimajor axis $a$ and an angular velocity $\omega = [G(m_1 + m_2)/a^3]^{1/2}$. For simplicity, the eccentricity of the outer binary $e$ is assumed to be zero, unless otherwise specified.

Similarly, we denote the unit vector of the orbital angular momentum of the inner binary by $n'$, and we define $\lambda'$ by the angle between the $z$-axis and the projected vector of $n'$ onto the $x$–$z$ plane, and define the orbital inclination angle $i'$ to the observer by the angle between $n'$ and the $y$-axis. We have $n' = (\sin \lambda' \sin i', \cos i', \cos \lambda' \sin i')$. The angle between $n$ and $n'$ is denoted by

$$\theta \equiv \arccos(n \cdot n'). \quad 0 \leq \theta \leq \pi. \quad (1)$$

The semimajor axis and angular velocity of the inner binary are denoted by $d$ and $\omega' = [G(m_1 + m_2)/d^3]^{1/2}$, respectively.
The eccentricity of the inner binary $e'$ is assumed to be zero, unless specifically discussed in some cases below. Note that the semimajor axis of the inner binary is limited by the Hill radius $d_H$ and the Roche limit $d_R$, i.e.,

$$d \lesssim d_H \equiv a \left( \frac{m_1 + m_2}{3m_\star} \right)^{1/3} \simeq 0.07 \text{AU} \left( \frac{a}{1 \text{AU}} \right)$$

$$\times \left( \frac{m_1 + m_2}{M_\star} \right)^{1/3} \left( \frac{M_\star}{m_\star} \right)^{1/3}$$

$$d \gtrsim d_R \equiv R_1 \left( \frac{\rho_1}{\rho_2} \right)^{1/3},$$

where $M_\odot$ is the solar mass, $M_\star$ is 1 Jupiter mass, and $\rho_i (= 3m_i/4\pi R_i^3, i = 1, 2)$ is the mass density of each component of the inner binary. An inner binary with a larger semimajor axis will be tidally broken up by the gravitation from the star.

In general, the orbital parameters of the outer binary and the inner binary may evolve with time under the three-body interactions of the star and the binary planet. But, under some conditions, their dynamical motion can be much simplified as follows.

1. The orbital parameters of the outer binary ($a, e, n, i, \lambda$) can be approximately constant if the semimajor axis of the inner binary is much smaller than the Hill radius $d_H$.

2. The semimajor axis of the inner binary $a$ can also be approximately constant if it is much smaller than the Hill radius. Both the inclination $i'$ and the projected angle $\lambda'$ of the inner binary may change with time due to the evolution of $n'$. The evolution of $n'$ depends on the angle $\theta$ between the orbital angular momenta of the inner and the outer binaries (i.e., $n'$ and $n$; e.g., Ford et al. 2000).

(a) If the angle $\theta$ or $180^\circ - \theta$ is small (e.g., $\lesssim 40^\circ$, the Kozai angle; Kozai 1962), the $n'$ precesses around $n$ approximately at a constant angular velocity $\Omega \sim \omega^2 \cos \theta/\omega'$, and the angle $\theta$ can also be approximately constant. The angles $i'$ and $\lambda'$ change periodically with the precession of $n'$. As one transit duration $\delta t (\sim \omega^{-1} R_*/a)$ is generally much shorter than the precession timescale of $n'$, the $n'$ ($i'$, $\lambda'$) can be approximated as constant during each transit duration.

(b) If the angle $\theta$ is approximately in the range of $40^\circ$ to $140^\circ$, the Kozai mechanism affects the evolution of the orbital parameters of the inner binary, in which the angle $\theta$ and the eccentricity $e'$ exchange at a period $\sim 2\pi \omega' / \omega^2$ due to angular momentum transfer and the conservation of the quantity $C_K \equiv \sqrt{1 - e'^2 \cos \theta}$ (see an example shown in Figure 4 below). During the oscillation, the eccentricity of the inner binary can be induced to high values close to 1, and thus the inner binary is likely to be destroyed by collision of its two components. If other moons exist in the system, one component of the binary is also likely to be ejected from the system or collide with one of the moons. The Jovian system is such an example influenced by the Kozai mechanism, and almost all the inclinations of their moons are out of the angle range. Here we ignore other effects from the planet (e.g., tides, the general relativistic effect), which would limit the influence of the Kozai mechanism. The limitation would become relatively significant for a system with small $d$ but large $a$. For example, Uranus is far from the sun and its inner moons (with $R_i/a \simeq 2.4 \times 10^{-4}$ and $a/d \sim 5 \times 10^3$) are on polar orbits relative to its orbital plane surrounding the sun (e.g., Murray & Dermott 1999).

Below we do not focus on case (b).

3. TRANSIT OF A BINARY PLANET AND ITS SIGNATURES ON THE R-M EFFECT

3.1. Transit Light Curves

When a binary planet transits in front of a star (see a schematic diagram in Figure 2), the stellar light curves may be imprinted...
with signatures of the binary planet. We illustrate one example of the normalized transit light curve in Figure 3 (see panel (a)). The observational transit curve is obtained by \( L \equiv \int \int I(x, z) dxdz \), where \( \int I(x, z) \) is the observational stellar surface brightness at position \((x, z)\). During the transit, part of the stellar surface is blocked by the planets, and we set

\[
I(x, z) = \begin{cases} 
I_{\ast}, & \text{unblocked region with } x^2 + z^2 \leq R_*^2, \\
0, & \text{otherwise}. 
\end{cases}
\]

For simplicity, the limb-darkening effect of the stellar surface brightness is ignored in this paper. We use a full three-body simulation to obtain the dynamical motion of the system. The dynamical motion of the binary planet relative to the star determines the shifting of the blocked region during the transit. The following phases during the transit are illustrated in Figures 2 and 3.

1. **Ingress phase.** At the beginning of the transit, at least one component of the binary planet starts to block the stellar light, but the projected areas of the two components have not been fully enclosed by the projected stellar surface (e.g., the “AC” part in Figure 3).

2. **Complete transit phase of the binary planet.** Both the projected areas of the two components have been fully enclosed by the projected stellar surface (e.g., the CE” part in Figure 3). Generally, the two planets are more likely to spend most of the transit time in that phase when \( i \) is close to \( \pi / 2 \). The \( d < 2 R_* \) can be roughly taken as a condition for the occurrence of this phase during the transit. If the binary planet has a sufficiently large \( \omega' \), it is likely that during the transit their orbital evolution leads to the evolution of their projected areas from non-overlap to overlap, and to non-overlap again, for which a “bulge” (the “DD” part in Figure 3) is shown in the light curve.

3. **Egress phase.** At least one component of the binary planet has transited to the other end of the projected stellar surface and its projected area is not fully enclosed by the projected stellar surface again (e.g., the “EF” part in Figure 3).

“EF” flat part of the light curve in Figure 3(a) represents the period in which one component has fully moved out of the projected stellar surface, but the other one is still completely inside.

The transit of a binary is different from that of a single body. As illustrated above, the binary may enter or exit the transit one by one, or the projected areas of the two components may overlap, so that some special features can be shown in the transit light curve (e.g., some step changes or “bulges”).

Some properties of the binary planet (e.g., radii of its two components, and its angular velocity and semimajor axis if its \( \omega' \) is sufficiently fast; Sato & Asada 2009) can be extracted from the features. In addition, the transit duration variation and the transit timing variation measured from the light curves have also been proposed to obtain the exomoon mass and the semimajor axis of the moon’s orbit (Kipping 2009a, 2009b). Below we illustrate that the orbital configuration of the binary planet can be further constrained by the evolution curve of the observational stellar radial velocity.

Note that the velocity in Figure 3 (similarly in Figure 5 below) is expressed in a dimensionless quantity, where the stellar parameters involved in the normalization could be non-trivial to estimate in realistic systems and would be done through other independent methods and abundant knowledge in stellar astrophysics.

### 3.2. Stellar Radial Velocity Anomaly and Orbital Configuration of a Binary Planet

As mentioned in the Introduction, a binary planet may leave its signature on the observational radial velocity of the star as well as on its light curve. The apparent stellar radial velocity anomaly due to the blocking of the stellar light is given by

\[
\Delta v_s = -K \frac{\int \int xI(x, z)dxdz}{\int \int I(x, z)dxdz}
\]

(e.g., Ohta et al. 2005; Winn et al. 2005), where \( K \equiv \Omega_* \sin i_* \) is the line-of-sight component of the spin angular velocity and...
may be observationally constrained from the stellar spectrum. Applying Equation (5) to the example shown in Figure 3(a), we obtain the evolution curve of its corresponding radial velocity anomaly in Figure 3(b) (see also Figure 1 in Simon et al. 2009).

Below we demonstrate how the dynamical configuration of the binary planet is incorporated into the evolution curve of the stellar radial velocity anomaly. For simplicity, we consider the complete transit phase of the binary planet. We analyze the case in which the projected areas of the two planets do not overlap, and the non-overlap is likely to be true during most of the transit time especially if $R_1 + R_2 \ll d$ or $|\pi/2 - i'| \gtrsim (R_1 + R_2)/d$. Thus, the stellar radial velocity anomaly can be simplified as follows:

$$\Delta v_o = K \frac{x_1 A_1 + x_2 A_2}{A_1 - A_2},$$

where $x_1 = x_o + x_1'$ and $x_2 = x_o + x_2'$ are the $x$-coordinates of the center of each planet, respectively (see Figure 2(h)),

$$x_o' = a(1 - \sin^2 \lambda \sin^2 i')^{1/2}\sin[\omega(t - t_0)] \quad \text{if } e = 0 \tag{7}$$

is the $x$-coordinate of the center of mass of the binary planet, $t_0$ is set so that $x_o' = 0 \text{ at } t = t_0$,

$$x_1' = d_1(1 - \sin^2 \lambda' \sin^2 i')^{1/2}\sin[\omega'(t - t_0) + \phi] \tag{8}$$

and

$$x_2' = -d_2(1 - \sin^2 \lambda' \sin^2 i')^{1/2}\sin[\omega'(t - t_0) + \phi] \tag{9}$$

are the $x$-coordinates of the two planets relative to their center of mass, $d_1$ and $d_2$ are the distances of the two planets to their center of mass, and $\phi$ represents the orbital phase of the inner binary. The $y$-coordinates of the planets along the line of sight do not appear explicitly in Equation (6), but they are involved in the expression through the angles $\omega'(t - t_0)$ and $\phi$. Applying Equations (7)–(9) to Equation (6), we have

$$\Delta v_o = \Delta v_o' + \Delta v_b, \tag{10}$$

where

$$\Delta v_o' = K \frac{a(A_1 + A_2)}{A_1 - A_2} (1 - \sin^2 \lambda \sin^2 i')^{1/2}\sin[\omega(t - t_0)], \tag{11}$$

$$\Delta v_b = K \frac{d_1 A_1 - d_2 A_2}{A_1 - A_2} (1 - \sin^2 \lambda' \sin^2 i')^{1/2}\sin[\omega'(t - t_0) + \phi]. \tag{12}$$

The $\Delta v_o$ is composed of the terms, $\Delta v_o'$ and $\Delta v_b$.

1. The $\Delta v_o'$ gives the radial velocity anomaly as if the transiting body is a single body with the values of its mass and projecting area being the total ones of the binary and contains the orbital configuration of the outer binary, i.e., the angles $\lambda$ and $i$. This R-M effect due to the transit of a single planet has been used to extract those angles of some realistic exoplanetary systems. Together with some assumption or observational evidence on the possible distribution of the inclination of the stellar spin $i_o$, the orbital inclination of the planet relative to the stellar spin (i.e., the angle between $n$ and $n_o$) can be further statistically constrained (e.g., Winn et al. 2009). This inclination has also been measured in a number of realistic systems through the photometric anomalies exhibited in transit light curves, which are interpreted as passages of the planet over dark starspots (e.g., Sanchis-Ojeda & Winn 2011).

2. The $\Delta v_b$ comes from the relative motion of the inner binary, for which the sine mode of Equation (12) represents its periodical orbital motion. The information on the orbital configuration of the inner binary ($\lambda', i'$) comes from $\Delta v_b$, and we focus on the effects of this term in this paper.

Within one transit duration, as $\omega \delta t \sim R_e/a \ll 1$, the $\Delta v_o'$ in Equation (11) can be simplified to be linear with time as follows:

$$\Delta v_o' = K \frac{a(A_1 + A_2)}{A_1 - A_2} (1 - \sin^2 \lambda \sin^2 i')^{1/2}\omega(t - t_0). \tag{13}$$

If the sine mode in $\Delta v_b$ can be identified in the observations, its period and amplitude can be used to constrain the value of $\omega'$ and the geometric configuration ($\lambda', i'$). If $\omega' \delta t \sim (\omega'/\omega)(R_e/a) \ll 1$, the $\Delta v_b$ in Equation (12) can be reduced to be also linear with time as follows:

$$\Delta v_b = K \frac{d_1 A_1 - d_2 A_2}{A_1 - A_2} (1 - \sin^2 \lambda' \sin^2 i')^{1/2} \times [\sin \phi + \omega'(t - t_0) \cos \phi], \tag{14}$$

and the slope of the $\Delta v_o$–$t$ curve during the complete transit phase is given by

$$k \equiv \frac{d\Delta v_o}{dt} = k_o' + k_b \cos \phi, \tag{15}$$

where

$$k_o' = K \frac{a(A_1 + A_2)}{A_1 - A_2} (1 - \sin^2 \lambda \sin^2 i')^{1/2} \tag{16}$$

and

$$k_b = K \frac{d_1 A_1 - d_2 A_2}{A_1 - A_2} (1 - \sin^2 \lambda' \sin^2 i')^{1/2} \omega'. \tag{17}$$

The $k_o'$ is constant with time as $\lambda$ and $i$. The $k_b$ depends on the orbital configuration of the inner binary ($\lambda', i'$), which is usually constant within one transit duration, as mentioned in Section 2.

Even if the eccentricity of the outer binary $e$ is non-zero, but has a low or moderate value (e.g., $e = 0.3$, so that the linear approximation of $\omega \delta t$ in Equation (13) is still valid), the expressions for $\Delta v_o'$ and $k_o'$ can be modified simply by replacing $a$ with $g a$ in Equations (13) and (16), where the factor

$$g = (1 - e^2)^{-1/2}[1 + e \cos(\alpha + \angle P O Q)], \tag{18}$$

the angle $\alpha$ is defined by $\tan \alpha = \sin \lambda \cos i / \cos \lambda$ and $\cos \alpha = -\cos \lambda'/(1 - \sin^2 \lambda \sin^2 i')^{1/2}$, and the meaning of the angle $\angle P O Q$ is indicated in Figure 1.

In a longer time period $\Delta t(\gg \delta t)$, the evolution of $\lambda'$ and $i'$ is determined by the precession of $n$ around $n'$ and the value of the angle $\theta$. In this case, we discuss the linear mode in the following two regimes.

1. If $\Omega \Delta t \ll 1$ (i.e., $\Delta t$ is much shorter than the precession timescale, but covers multiple transit events), $k_b$ is still roughly constant. The distribution of the phases $\phi/2\pi$ follows the distribution of $n \omega' / \omega$ (n: integer). In general, unless $\omega' / \omega$ is an integer, $\phi$ is uniformly distributed between 0 and $2\pi$, and we can get the average of the slopes over the multiple transits as follows:

$$\bar{k} = k_o'.$$
which can be used to constrain the sky-projected angle $\lambda$. The rms of the slopes is

$$\sqrt{\langle (k - \bar{k})^2 \rangle} = \frac{1}{2} k_b.$$  \hfill (20)

The value of $\bar{k}$ is a function of $\lambda'$ and $i'$ (see Equation (1)), and Equation (20) can be used as an observational constraint for statistical determination of the probability distribution of $\bar{k}$.

2. If $\Omega \Delta \tau \gtrsim 1$, $k_b$ changes due to the precession of $n$ and the variation of $\lambda'$ and $i'$. Here we only discuss the case with $0 \lesssim \theta \lesssim 40^\circ$ or $140^\circ \lesssim \theta \lesssim 180^\circ$. As seen from Figure 4, the evolution pattern of $f \equiv (1 - \sin^2 \lambda' \sin^2 i')^{1/2}$ is different with different $\theta$. Note that the evolution pattern for $\pi - \theta$ ($0 \lesssim \theta \lesssim 40^\circ$) can be obtained by reversing the time in the pattern for $\theta$, as the precession direction for $\theta > 90^\circ$ is along $-n$. The slope of $k$ should distribute within the envelope of $f$, and the magnitude of the variation of $f$ increases with increasing $\theta$ (for $\theta < 90^\circ$). Hence, the distribution of the slopes can be used to constrain the value of $\theta$ (or $90^\circ - \theta$). We illustrate such an example in Figures 5(a)–(e), by doing full three-body numerical simulations on dynamical evolution of a binary planet rotating around a star and obtaining its multiple transiting events.

Similarly, if the stellar radial velocity anomaly due to a binary planet $\Delta v_b$ is in a sine mode in Equation (12), observations of multiple transit events within a longer time $\Delta t$ can be useful to obtain the evolution of the amplitude of the sine mode, the evolution of the orbital configuration of the inner binary ($\lambda'$, $i'$), and also further statistically constrain the angle $\theta$ (see the example shown in Figures 3(a)–(b)).

In addition, in some cases, the binary planet is not in the complete transit phase, but only one component is in the transit. For example, this may occur in the ingress or egress phase; and if $i > 2 R_*$, it is also more likely that each component transits in front of the star one by one. In these cases, it is easy to generalize the analysis above and obtain the deviation in the R-M effect due to the transit of each component by setting the projected area of the other component to zero in the formula above.

Note that in the modeling of most of the other exomoon detection methods, the orbit of the exomoon surrounding the planet is assumed to be co-aligned with the orbit of the planet surrounding the star. Sato & Asada (2010) consider how the special step or overlap features shown in transit light curves (cf. Figure 3(a)) can be used to infer the orbital inclination of an exomoon, in which the relevant cases are for the condition that $\delta i$ is close to $90^\circ$. The transit duration variations derived by Kipping (2009b) involve the different inclination parameters of an exomoon but are limited to $\omega/\delta t \ll 1$ or the linear mode.
and ignore the evolution of the dynamical systems (i.e., the precession of \( n' \)).

3.3. Systems Likely to be Revealed by Observations

To have the signatures of a binary planet detectable in observations, the change of the stellar radial velocity anomaly due to the binary planet should be significantly large during each transit. As analyzed above, the deviation \( \Delta \upsilon_{b} \) is composed of two parts, \( \Delta \upsilon_{bO} \) and \( \Delta \upsilon_{b} \) (see Equation (10)), and both have the contributions from a second planet or an exomoon.

For the first part \( \Delta \upsilon_{bO} \), the contribution from each component of the binary can be estimated by

\[
\Delta \upsilon_{bO} \approx \text{kr}(\frac{v_{b}}{2\pi \Omega})^{2} (\Omega_{r} \sin i)^{2}
\]

which is the same if the two components have identical projected areas; and the special features (e.g., step change EE) shown in Figure 3 have the same orders of magnitude as that estimated and may serve as some characteristic signals of binary planet candidates in the R-M effect.

For the second part \( \Delta \upsilon_{b} \), which reveals the dynamical configuration of the inner binary, we used some individual systems with specific dynamical parameters to indicate its effect in the above section. To see a general parameter space of binary planet systems that are likely to be revealed in \( \Delta \upsilon_{b} \) by observations, we define the following velocity change:

\[
\delta \upsilon_{b} \equiv K \left| \frac{d_{1}A_{1} - d_{2}A_{2}}{A_{*} - A_{1} - A_{2}} \right| \omega \delta t, \quad \text{if } \omega \delta t < 1,
\]

\[
\delta \upsilon_{b} \approx KR_{s} \left( \frac{d}{A_{s}} \right) \left( \frac{A_{1} + A_{2}}{A_{*}} \right) f_{\delta \upsilon_{b}},
\]

where

\[
f_{\delta \upsilon_{b}} = \left[ \frac{1}{1 + m_{1}/m_{2}} - \frac{1}{1 + (m_{1}/m_{2})^{2/3}} \right], \quad \text{if } \frac{A_{1} + A_{2}}{A_{*}} \ll 1.
\]

Equation (22) represents the maximum change of the stellar radial velocity anomaly due to the inner binary in the linear mode of Equation (14), i.e., \( k_{b} \delta t \); and Equation (24) represents the amplitude of the sine mode shown in Equation (12). Note that the \( \delta \upsilon_{b} \) in both Equations (22) and (24) is a defined variable. Although the expressions are obtained through the limits at \( \omega \delta t \ll 1 \) and \( \omega \delta t \gg 1 \), their values at \( \omega \delta t \) should be in the transition of the two limits and the equations above work for the order-of-magnitude estimates and the purpose of the paper. As seen from the equations above, not only the planet areas/sizes are involved in the R-M effect as for a single planet, but their masses or mass densities are also incorporated in the effect for a binary planet due to the relative motion of the two components of the binary, as the hidden areas of the stellar surface are affected by the relative positions of the binary
components and the relative distance of each component from the center of mass of the binary (cf. Equations (8) and (9)) is determined by its two component masses. Given the mass ratios of the two components, the radius ratios can also be expressed through their mass density ratios. Thus, the amplitudes of the R-M effects indicated in Figures 6–8 are expressed through the extra dimensions in multiple panels. The involvement of the extra (mass density) dimension in the study is useful especially considering that recent Kepler observations have revealed the mass density of planets do span a large range.

In addition, the R-M effect for the transit of a single planet is related with the orbital semimajor axis, but the semimajor axis \(a\) of the outer binary is involved in the effect for a binary planet as shown in Equation (23) because the transit time \(\delta t\) is affected by \(a\) and a longer transit time leads to a larger change of the relative position of the binary components and further a larger deviation in the stellar radial velocity.

As seen from Equation (26), the value of \(\delta v_b\) is significant only if the two components of the binary planets are different, especially if the heavy one has a high mass density and the
Figure 7. Same as in Figure 6, except that the parameter $R^* / a = 0.05$ is set in this figure. The cyan dashed line shown in some panels represents the Roche limit, calculated from Equation (3); and the parameter space of a binary planet should lie to the left of the line. The blue dotted line is a reference line for $d = R_1 + R_2$, and a binary planet should also lie to the left of the line. As in Figure 6, the gray shaded areas indicate the parameter space that a binary planet cannot survive dynamically. Both the cyan dashed line and the blue dotted line are not shown in Figure 6, as they lie beyond the upper bound of the $x$-axis of each panel.

light one has a low-mass density. If the two components have the same mass and mass density, they have the same projecting area and their motion is symmetric, and thus we have $\delta v_b = 0$ with $f_{\delta v_b} = 0$, although in this case the contribution from $\Delta v_{O'}$ (Equation (11)) can be large due to the combination of the projected areas of the two single components. In addition, the value of $\delta v_b$ is significant only if the area of the binary planet $A_1 + A_2$ is significantly large, as $\delta v_b$ is proportional to $(A_1 + A_2) / A_*$. By using Equations (23) and (25), we show how $\delta v_b$ depends on other various properties of the system in Figures 6 and 7. We set the parameters $KR_* = 5 \text{ km s}^{-1}$ and $(A_1 + A_2) / A_* = 0.01$ in both the figures. We set $R_e / a = 0.005$ and 0.05 in Figures 6 and 7, respectively. If $R_e = R_2$, $R_e / a = 0.005$ corresponds to the distance of the Earth from the Sun ($a = 1$ AU), and $R_e / a = 0.05$ corresponds to some typical semimajor axis of hot Jupiters discovered in the vicinity of a star ($a = 0.1$ AU). The contours of $\delta v_b$, as a function of $(m_1 + m_2) / m_*$ and $a / d$, are shown by black solid curves. In each figure, we display how the contours change for binary planets with different mass ratios $(m_1 / m_2 = 1, 10, 100, 1000)$ and different mass density ratios $(\rho_1 / \rho_2 = 5, 1, 0.2)$. As seen from the figures, in the region below the black dotted curve (i.e., $\omega \delta t < 1$), $\delta v_b$ increases with increasing $(m_1 + m_2) / m_*$ and $a / d$, as $\omega \delta t$ does in the same tendency (see Equation (23)); and in the region above the curve, $\delta v_b$ increases with decreasing $a / d$ (given $R_e / a$), as the
difference of the line-of-sight rotation velocity of a star covered by each component is likely to be relatively large for a wide binary planet (with large $d$). By comparing the contours obtained for different mass density ratios, the figures also illustrate that the values of $\delta v_0$ can be significant (e.g., up to 1 m s$^{-1}$ or even several ten m s$^{-1}$) only if the two components of the binary planets are different, especially for high $\rho_1/\rho_2$ ratios (see panels (1a)–(1c)), as analyzed above. Given $\rho_1/\rho_2$, the $\delta v_0$ at bottom panels (e.g., (1d), (2d), (3d)) have relatively low values, which indicate the difficulty to detect the effect of a too small exomoon. Figure 7 has a higher $R_*/a$ than Figure 6. On the one hand, the curve of $\delta v_0/\delta t = 1$ in Figure 7 (black dotted curve) shifts downward; and thus, although the contours of $\delta v_0$ below the black dotted curve are the same in both of the figures, the region above the curve has relatively low $\delta v_0$ in Figure 7. On the other hand, a higher $R_*/a$ would imply a shorter period of the binary rotating around the star and thus a higher probability to do multiple transit observations to get a better statistics for the system. In addition, for different values of $KR_*$ and $(A_1 + A_2)/A_*$, the contour values of $\delta v_0$ in the figures should be adjusted simply by multiplying them by a factor of $(KR_*/5 \text{ km s}^{-1}) \cdot (A_1 + A_2)/0.01 A_*$.

Based on the results above, we discuss the magnitude of the $\delta v_0$ in the following examples of binary planet or exomoon systems. We assume that the central star is a solar-like star (with solar mass and radius).

1. An Earth–moon system (with $m_1/m_2 \simeq 81$, $\rho_1/\rho_2 \simeq 1.6$, $R_*/a \simeq 0.005$, $a/d \simeq 400$) may cause a $\delta v_0$ up to 0.1 cm s$^{-1}$, e.g., due to the small sizes of the Earth and the moon (with $(A_1 + A_2)/A_* \simeq 10^{-4}$), which is too small to be detected. Note that according to the estimate by Equation (21), the contribution of the moon to the deviation $\Delta v_0'$ can be up to 1 cm s$^{-1}$ (consistent with the value shown in Simon et al. 2010, where the effect of $\Delta v_0$ is not discussed).

2. Ganymede is the biggest moon in the solar system. A Jupiter–Ganymede system (with $m_1/m_2 \simeq 1.3 \times 10^4$, $\rho_1/\rho_2 \simeq 0.7$) may lead to a $\delta v_0$ only up to 5 cm s$^{-1}$ (for $R_*/a \simeq 0.001$, $a/d \simeq 800$).

3. For a Jupiter–rocky moon system (e.g., with $\rho_1/\rho_2 = 0.2$): a moon or a satellite with mass $m_2 < 0.1 m_1$ has a low $\delta v_0$ less than $\sim 1$ m s$^{-1}$ (cf. panels (3b)–(3d) in Figures 6 and 7). A Jupiter–Earth system (with $m_2 \simeq 3 \times 10^{-3} m_1$) has a low $\delta v_0$ only in the range of 1–10 cm s$^{-1}$ (see panels (3c) and (3d)). However, if the rocky moon/satellite has a mass close to that of Jupiter (see panel (3a)), the $\delta v_0$ can be up to 1–10 m s$^{-1}$, which may be detectable by the current techniques. Note that this is an exotic case, as most of the planets revealed by the Kepler whose mass densities are higher than Earth’s have a mass not exceeding several ten Earth mass so far.

4. A binary Jupiter-like planet system (e.g., with component masses $m_1 = 10 M_J$ and $m_2 = 1$ or $0.1 M_J$, and with Jupiter-like mass densities $\rho_1 = \rho_2$) may have a $\delta v_0$ ranging from 1 m s$^{-1}$ to 30 m s$^{-1}$, although hot binary Jupiter systems (with relatively high $R_*/a$) have relatively low $\delta v_0$. The $\delta v_0$ can be even larger if $\rho_1 > \rho_2$. For such a system, if any, its signatures on the R-M effect may be detected by future observations.

5. The CoRoT-9 system has a solar-like central star and its orbiting exoplanet CoRoT-9b has the mass and radius close to those of Jupiter. The CoRoT-9b is one of the longest period transiting Jupiter ($\sim 95$ days) that has so far been confirmed and has a semimajor axis $a \simeq 0.4$ AU (Deeg et al. 2010). Its $R_*/a \simeq 0.01$ is between the cases shown in Figures 6 and 7. As inferred from the figures, if the CoRoT-9b has a satellite $m_2 \gtrsim 10^{-3} m_1$, its $\delta v_0$ can range from 5 cm s$^{-1}$ to 10 m s$^{-1}$, depending on the detailed satellite properties (i.e., mass, density, and its distance to the CoRoT-9b).

By using Equations (13), (16), (22), and (24), the ratio of the two parts in the deviation $\Delta v_*$ (see Equation (10)) is about

$$\frac{\delta v_0}{k_\alpha \delta t} \simeq 0.1 \left( \frac{\delta v_0}{5 \text{ km s}^{-1}} \right) \left( \frac{5 \text{ km s}^{-1}}{KR_*} \right) \left( \frac{0.01 A_*}{A_1 + A_2} \right). \quad (27)$$

As mentioned above, both of the two parts have the contribution from a second planet or an exomoon, and the ratio of the two contributions can be estimated by $(\delta v_0/5 \text{ km s}^{-1})(5 \text{ km s}^{-1}/KR_*)(10^{-3} A_*/A_1)$.

Note that a binary planet has a different gravitational effect on the stellar motion from a single planet. The quadrupole moment
of the gravitational force from the binary planet is
\[
\Delta F \sim \frac{Gm_*}{a^2} \cdot \frac{m_1 d_1^2 + m_2 d_2^2}{a^2},
\]
and its effect on the dynamical motion over each transit duration can be estimated by
\[
\frac{\Delta F \delta t}{m_*} \sim 1 \text{ cm s}^{-1} \left( \frac{v_*}{100 \text{ m s}^{-1}} \right) \left( \frac{10d}{a} \right)^2 \left( \frac{10m_2}{m_1} \right) \left( \frac{10R_*}{a} \right),
\]
which is generally negligible. The \( v_* \) is the velocity of the star relative to the center of mass of the system. It would be interesting to investigate the long-term dynamical effect of the binary planet on the stellar radial velocity, but this is beyond the scope of this paper.

### 3.4. Application to Hierarchical Triple Star Systems

Figures 6 and 7 can be applied to a hierarchical triple star system (e.g., for the parameter space \((m_1 + m_2)/m_* \geq 10^{-3}\), in which a dark binary star is transiting in front of a tertiary star (e.g., Carter et al. 2011). Here by “dark” we mean that the light emission from the binary is ignored as planets for simplicity. It is easy to generalize the analysis above to include the light emission from the binary. If the transiting binary is a compact object (e.g., white dwarf or neutron star) plus a planet/brown dwarf, the factor \( f_{\delta \nu_b} \) is up to 1, and the \( \delta \nu_b \) can be much larger than that of binary planet system. Figure 8 illustrates such a case. As seen from the figure, the \( \delta \nu_b \) is high, up to \( 10^2 \text{ m s}^{-1} \). Thus, such hierarchical triple star systems may be revealed through the R-M effects by future observations.

Figure 9 illustrates two examples of the transit light curves and the radial velocity anomaly curves for the triple star systems with dynamical parameters located in the parameter space shown in Figure 8. As seen from the figure, the radial velocity anomaly caused by the rotational motion of the planet/brown dwarf around the compact object is much more significantly displayed either through the bulge/hill/trough features during the ingress/egress phase (for panel (a)) or in the sine-like curve during the complete transit phase. The values of \( \Delta \nu_b \) can be comparable to \( \Delta \nu_O \). It is plausible to expect that the features of these curves are very useful to infer the dynamical configurations of the systems, as illustrated for binary planets above. A more detailed
discussion on extracting the configurations is beyond the scope of the paper.

4. DISCUSSION

The properties of binary planets (e.g., mass, size, and semi-major axis) can be constrained through their signatures on the transit light curves and stellar radial velocity curves. Our analysis of the R-M effect for a binary planet during its complete transit phase shows that effect is composed of two parts. The first part is the conventional one similar to the R-M effect from the transit of a single planet with the combined masses and projected areas of the binary components (Equation (11)); and the second part is caused by the orbital rotation of the binary components, which may add a sine- or linear-mode deviation to the stellar radial velocity curve (Equation (12)).

In this paper we focus on the discussion on the second part, and we find both the evolution of the orbital rotating phases and the precession of the binary orbital plane may lead to different amplitudes of the deviations in different transit events of the same system. The resulting distribution and dispersion of the deviations in multiple transit events can be used to extract the orbital configuration of the binary planet or even the inclination of its orbital plane relative to the plane of its center of mass rotating around the star (e.g., Figure 5).

The second part of the R-M effect is more likely to be revealed if the binary components have different masses and mass densities, especially if the heavy one has a high-mass density and the light one has a low density. For example, our calculations show that the signature can be up to several or several ten $m \cdot s^{-1}$ with the mass ratio $m_1/m_2$ up to $10^3$, if the mass density ratio $\rho_1/\rho_2 = 5$. A small and rocky exomoon with $m_3 < 0.1 m_1$ would cause a low $\delta v_b$ less than $m \cdot s^{-1}$. A strong signature may be caused if at least one of the components of the binary planet is a giant planet. Note that stellar noise produced by oscillations, granulation phenomena, and activities could contribute to the change of stellar radial velocities with an amplitude up to $m \cdot s^{-1}$ (e.g., Dumusque et al. 2011), but they have their own variation periods and patterns to be distinguished from the effect of binary planets. Some statistical methods can be developed to extract smaller signals from the noise.

If a long observation time would cover multiple transits of a binary planet, better statistics on the distribution of the deviation in the R-M effect could be potentially obtained. To trace the evolution of the geometric configuration of the system, we need at least one period of the orbital angular momentum of the inner binary precessing around that of the outer binary. The precession period is roughly about $\omega'/\omega$ times the orbital period of the outer binary. For the parameter space shown in Figures 6 and 7, $\omega'/\omega$ is generally less than 200 (cf. the $\omega' t = 1$ curve, which can also be taken as a reference line for $\omega'/\omega \simeq (R_*/a)^{-1}$).

The misalignment between the plane of the binary planet and its rotating plane around the star is one of fundamental parameters of the dynamical system. A large misalignment is likely to cause a large dispersion or different distribution of the deviations in the R-M effect. Recent measurements have discovered that the orbit of a planet may be highly inclined to the stellar spin. Different mechanisms have been proposed for the formation of those misaligned orbits, e.g., Kozai capture, planet–planet scattering, resonance capture by planet migration (e.g., Murray-Clay & Schlichting 2011; Nagasawa et al. 2008; Fabrycky & Tremaine 2007; Yu & Tremaine 2001). Similarly, the different configurations/inclinations of binary planets (see $\theta$ defined in Table 1), if detected in future, should be also useful in constraining formation mechanisms of binary planets or exomoons, and shed new light on our understanding of the diversity of planetary systems. For example, different moon formation mechanisms may lead to different kinematic distributions. Moons formed from the disk material surrounding a planet are predicted to have prograde orbits, while those formed from gravitational capture/impacts/exchange interactions can have either prograde or retrograde orbits (Jewitt & Haghighipour 2007 and references therein). It is also likely that the orbit of a moon could be affected by the later evolution of the system (e.g., by the later inner/outer migration of the outer/inner planets).

Regarding a binary planet system with comparable component masses, the study of their formation theory is starting (e.g., Podsiadlowski et al. 2010), and one of the most exciting steps would be to discover a realistic system in observations in the near future.

To discover binary planets and exomoons becomes promising and practical with future developments in instruments, which also lay the foundation for finding the signatures discussed in this paper. For example, planned ground-based surveys such as the Large Synoptic Survey Telescope may detect thousands to tens of thousands of planetary transit candidates; and the space missions, PLANetary Transits and Oscillations of stars and Transiting Exoplanet Survey Satellite, aim to find transiting planets around relatively bright stars, making it easier to confirm discoveries using follow-up radial velocity measurements. Hopefully, follow-up observations could provide some binary planet or exomoon candidates. The astro-comb technique is aiming to achieve a precision as high as 1 cm s$^{-1}$ in astronomical radial velocity measurements (Li et al. 2008). Kipping et al. (2012) also proposed a systematic search for exomoons around transiting exoplanet candidates observed by the Kepler mission.

We studied the basic signatures of binary planets that are likely to be revealed in the R-M effect. To understand the roles of the crucial parameters played in the signatures, we have made some approximations in our study, which could be improved or adapted to realistic systems in future work, for example, the study could be extended to a general case in which the center of mass of the binary planet is on an eccentric orbit. The study would become complicated if an exoplanet has multiple moons. In this case, the small moons would contribute little to the deviation in the R-M effect, and the one with a relatively large radius and located at a relatively far distance from the primary planet would imprint the most significant effect. The limb-darkening effect can reduce the amplitude of the R-M effect by 20%–40% (Simon et al. 2010), and also affect the shape of the radial velocity anomaly (for both the linear and the sine cases studied in this paper). This effect could be corrected with the aid of the observational transit light curves in reality. A statistical method to map the reconstruction of relevant parameters of a binary planet system is beyond the scope of this paper, but would need to be explored in details in future.

After extending the results to a hierarchical triple star system containing a dark binary and a tertiary star, the deviation in the R-M effect would be large enough to be detected especially if the dark binary is composed of a compact object and a brown dwarf/planet, which may put further constraints on the gravitational configuration of triple star systems and provide insights on their formation and evolution.
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