DNS of starting turbulent jets with variable density

V A Ivashchenko1,2,* and R I Mullyadzhanov1,2

1Institute of Thermophysics SB RAS, Lavrentyeva str. 1, 630090 Novosibirsk, Russia
2Novosibirsk State University, Pirogova str. 2, 630090 Novosibirsk, Russia

*E-mail: v.ivashchenko@g.nsu.ru

Abstract. We study starting turbulent round jets with variable density using Direct numerical simulations (DNS). A fully developed turbulent air flow at constant Reynolds number $Re = 5300$ is injected to space, filled with air, carbon dioxide or helium with a small co-flow, with variable density being due to the mixing of different gases. We compare the celerity factor, centreline velocity and dynamics of the vortex dipole in front of the jet between all the considered cases.

1. Introduction

Turbulent jets with variable density are widely spread both in technical devices and natural phenomena, for example in steam and gas turbines, combustion processes, movement of atmospheric layers, mixing of an aircraft exhaust with surrounding air, etc. In all these cases high density gradients play an important role in mixing and entertainment processes, so researches pay a lot of attention to the study of the phenomenon of transfer, entrainment and mixing in such jets.

One of the first experiments on starting jets were performed by Abramovich et al. [1] who measured velocity in a developing submerged laminar round jet. Witze [2] noted that the spreading coordinate is well described by the time square-root. One of the first numerical works was performed by Kuo et al. [3], where processes of fuel mixing in engines were studied. During last 15 years there are also some experimental [4,5,6,7] and numerical [8] works about impulse jets, but the processes in the front vortex dipole are usually remain overboard. Our goal is to consider the evolution of the development of the vortex dipole in the starting jets.

2. Governing equations

We use non-dimensional continuity and Navier-Stokes equations in case of variable density and viscosity in a low Mach number approximation:

$$\frac{\partial u_i}{\partial x_i} = -\frac{1}{\rho} \frac{D \rho}{Dt}$$  \hspace{1cm} (1)

$$\rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j}$$ \hspace{1cm} (2)

where $u_i$ are the components of the velocity field, $p$ is the density, $p$ is the pressure, $\tau_{ij}$ is the viscous stress tensor, $\mu$ is the dynamic viscosity, $Re = UD/v$ is the Reynolds number, based on the bulk velocity, length scale and kinematic viscosity of the air. The variable density and viscosity

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appear due to the mixing of two gases with different molecular weights. The transport equation for the concentration field is as follows:

\[ \rho \left( \frac{\partial c}{\partial t} + u_j \frac{\partial c}{\partial x_j} \right) = \frac{1}{ReSc} \rho \frac{\partial}{\partial x_j} \left( \frac{\rho^2}{\partial x_j} \right) \]  

where \( c \) is the concentration field, \( \rho_{air} \) and \( \rho_{gas} \) are the densities of air and gas, respectively, \( Sc = \mu_{air}/(\rho_{air}D_{eff}) \) is the Schmidt number, based on the viscosity and density of air and their mutual diffusion coefficient. The viscosity of gas mixture is defined by the expression [9]:

\[ \mu = \sum_{j=1}^{n} \frac{X_j \mu_j}{X_j + \sum_{k=1}^{n} X_k \Phi_{jk} - X_j \Phi_{jj}} \]  

where \( X_j \) and \( \mu_j \) are the mole fraction and dynamic viscosity of \( j^{th} \) component, respectively,

\[ \Phi_{jk} = \frac{1}{4} \left( 1 + \left( \frac{\mu_j}{\mu_k} \right)^{1/2} \left( \frac{M_j}{M_k} \right)^{1/4} \right)^2 \left( \frac{2M_k}{M_j + M_k} \right)^{1/2} \]  

are coefficients characterizing the interaction of two components of the mixture. Parameters of all used gases are presented in table 1.

| Table 1. Parameters of considered gases at 20°C |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \rho \), kg/m³ | \( \mu \cdot 10^8 \), Pa·sec | \( D \), cm²/sec | \( M \), g/mole |
| Air             | 1.293           | 1812            | 0.17            | 28.98           |
| Helium          | 0.179           | 1946            | 0.62            | 4.002           |
| Carbon dioxide  | 1.977           | 1463            | 0.21            | 44.01           |

3. Results

We consider an air jet at a fixed Reynolds number \( Re = 5300 \), injected to the ambient space filled with air, helium or carbon dioxide. The geometry of the main computational domain represents a cylinder with a size of \( 15D \times 12D \), where \( D \) is the diameter of a the supplying pipe, see Fig. 1. The inflow conditions are generated in a separate periodic pipe computation. The computational mesh contains more than 120 million points and is schematically shown in Fig. 2. The computational code was previously verified in our group [10].

The coordinate of spreading is time dependent as square root and is shown in figure 3. Lines for “air – air” and “air – CO₂” pairs are very close to the line from literature [8], where “air – air” jet at much higher Reynolds number was investigated. It means that the ratio of densities plays more important role in the spreading of the jet than Reynolds number effect.

We are interested in the development of the frontal vortex dipole and the effect of the variable density. Figure 4 shows the streamwise velocity in the cross-sections through the front vortex dipole at different moments of time. There are visible qualitative differences between the first three pairs (“air – air”, “air – carbon dioxide”, “air – helium”) and additionally carried out simulation, which is called “inverse” in Fig. 4. In the “inverse” case a fully developed turbulent helium flow at the Reynolds number \( Re = 5300 \) is used as the inflow while the ambient space is filled with air. The density ratio for the “inverse” case is \( \rho_{hel}/\rho_{air} = 0.14 \) that leads to the development of the Rayleigh–Taylor-type instability on the interface between two gases of different densities, when helium is pushing the air. The same instability is observed for the “air – carbon dioxide” case, but since the density ratio is only \( \rho_{air}/\rho_{gas} = 0.65 \), it quickly disappears and merges with the core of the vortex dipole.
Figure 1. Geometry of the main computational domain.

Figure 2. Example of the mesh in two sections.

Figure 3. The dependence of the coordinate of jet’s spreading from the time for all 3 cases. Reference data are from [8].

Figure 4. The comparison of streamwise velocity in spanwise section inside the front of the jet in different time for all pair of gases.
Conclusion
The direct numerical simulations of starting turbulent jets with variable density and viscosity were performed. The Reynolds number was fixed to Re = 5300. The inflow conditions were provided by a fully developed pipe flow. Different density ratios lead to the difference in the jet spreading velocity as well as the level of turbulent fluctuations, including the processes in the front vortex dipole. The Rayleigh–Taylor-type instability was observed and the additional case was considered to show the effects of variable density. Further statistics and quantitative analyses will help to understand the physical mechanisms of mixing and entertainment in starting turbulent round jets.

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References
[1] Abramovich S & Solan A 1973 Journal of Fluid Mechanics 59 (4) 791–801
[2] Witze P O 1980 Sandia Labs., Livermore, CA (USA) SAND-80-8617
[3] Kuo T W & Bracco F V 1982 Turbulent and Spray Jets (SAE Paper) 820038
[4] Joshi A & Schreiber W 2006 Experiments in fluids 40 (1) 156–60
[5] Bajpai S & Tirumkudulu M S 2008 The European Physical Journal B 61 (3) 293–7
[6] Abani N & Ghandhi J B 2012 Journal of Fluids Engineering 134 (6) 061202
[7] Vuorinen V, Wehrfritz A, Duwig C & Boersma B J 2014 Fuel 130 241–50
[8] Ghasemi A, Pereira A & Li X 2017 Flow, Turbulence and Combustion 98 (1) 83–108
[9] Gordon S, & McBride B J 1994 NASA Report NASA-RP-1311
[10] Ryzhenkov V, Ivashchenko V, Vinuesa R, Mullyadzhanov R 2016 Journal of Physics: Conference Series 754 062009