Studying $K\pi$ S-wave scattering in K-matrix formalism

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(Dated: November 3, 2002)

Abstract

We generalize our previous work on $\pi\pi$ scattering to $K\pi$ scattering, and re-analyze the experiment data of $K\pi$ scattering below 1.6 GeV. Without any free parameter, we explain $K\pi I = 3/2$ S-wave phase shift very well by using $t$-channel $\rho$ and $u$-channel $K^*$ meson exchange. With the $t$-channel and $u$-channel meson exchange fixed as the background term, we fit the $K\pi I = 1/2$ S-wave data of the LASS experiment quite well by introducing one or two $s$-channel resonances. It is found that there is only one $s$-channel resonance between $K\pi$ threshold and 1.6 GeV, i.e., $K^*_0(1430)$ with a mass around 1438 \(\sim\) 1486 MeV and a width about 346 MeV, while the $t$-channel $\rho$ exchange gives a pole at $(450 - 480i)$ MeV for the amplitude.

PACS numbers: 14.40.Aq, 11.80.Gw, 13.75.Lb
I. INTRODUCTION

The assignment of the scalar mesons has been a long standing problem. Recently the existence of the low-lying $\pi\pi$ scalar state $\sigma$ has been well established, i.e., $f_0(400-1200)$ as listed by the Particle Data Group (PDG) [1]. Now the PDG lists five well-established isoscalar $0^{++}$ mesons: $f_0(400-1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, which are obviously too many for a standard $q\bar{q}$ nonet. Given the existence of two isovector scalars, $a_0(980)$ and $a_0(1450)$, two scalar nonets have been suggested [2, 3]: an unconventional one composed of $\sigma$, $\kappa$, $a_0(980)$ and $f_0(980)$ and the conventional $q\bar{q}$ nonet composed of $f_0(1370)$, $K^*_0(1430)$, $a_0(1450)$ and $f_0(1500)$ or $f_0(1710)$.

However, the existence of the $\kappa$ is still in controversy. Evidence for this resonance has been claimed within certain models [4, 5, 6, 7], whilst other studies dispute this [8, 9]. Recently, a less model-dependent analysis of the LASS $K\pi$ scattering data between 825 MeV and 2 GeV by Cherry and Pennington [10] concludes that there is no $\kappa(900)$, but a very low mass $\kappa$ well below 825 MeV cannot be ruled out.

In fact the phase shifts of $K\pi$ S-wave scattering at low energies [11, 12] look very similar to those of $\pi\pi$ S-wave scattering. In our previous study on $\pi\pi$ scattering in the K-matrix formalism [13], the negative phase shifts for the isotensor $\pi\pi$ S-wave were naturally explained by the $t$-channel $\rho$ meson exchange while the broad $f_0(400-1200)$ structure in the isoscalar $\pi\pi$ S-wave was decomposed into a $t$-channel $\rho$ meson exchange part dominating at the low energy end plus an additional $s$-channel wide resonance $f_0(1670)$. Considering the similarity between the $K\pi$ scattering and the $\pi\pi$ scattering, it is nature to extend our previous work on the $\pi\pi$ scattering to the $K\pi$ scattering. We find that the negative phase shifts of the $K\pi I = 3/2$ S-wave can be very well reproduced by the $t$-channel $\rho$ and $u$-channel $K^*$ meson exchange without any free parameter. With the $t$-channel and $u$-channel meson exchange fixed as the background term, the positive smoothly rising phase shifts for the $K\pi I = 1/2$ S-wave can be well fitted by introducing one or two additional $s$-channel resonances. It is found that there is only one $s$-channel resonance between $K\pi$ threshold and 1.6 GeV, i.e., $K^*_0(1430)$ with a mass around $1438 \sim 1486$ MeV and a width about 346 MeV, while the $t$-channel $\rho$ exchange gives a pole at $(450 - 480i)$ MeV for the amplitude.
II. FORMALISM

For the pseudoscalar-pseudoscalar-vector coupling, we use the SU(3)-symmetric lagrangian [14]

\[ \mathcal{L}_{PPV} = -\frac{1}{2} i G_V \text{Tr}([P, \partial_\mu P]V^\mu), \]  

(1)

where \( G_V \) is the coupling constant, \( P \) is the 3 \( \times \) 3 matrix representation of the pseudoscalar meson octet, \( P = \lambda^a P^a, a = 1, \ldots, 8 \) and \( \lambda^a \) are the 3 \( \times \) 3 generators of SU(3). A similar definition of \( V^\mu \) is used for the vector meson octet.

In the Gell-Mann representation, the lagrangian can be expressed as

\[ \mathcal{L}_{PPV} = 2 G_V \epsilon_{ijk} \pi^i \partial_\mu \pi^j \rho^k, \]  

(2)

where \( \epsilon_{ijk} \) are the antisymmetric structure constants of SU(3). For example,

\[ \mathcal{L}_{\pi\pi\rho} = 2 G_V \epsilon_{ijk} \pi^i \partial_\mu \pi^j \rho^k, \]  

(3)

\[ \mathcal{L}_{\pi K K^*} = i G_V \left\{ \left( \partial_\mu K \right) \bar{\tau} K^{*\mu} - K^{*\mu} \bar{\tau} \partial_\mu K \right\} \cdot (\bar{\tau} - \bar{KK}^{*\mu} \bar{\tau} K) \]  

(4)

where \( \bar{\tau} \equiv (\pi_1, \pi_2, \pi_3) \), \( K^{*\mu} \equiv \left( \begin{array}{c} K^{*+} \\ K^{*0} \end{array} \right) \), \( K \equiv \left( \begin{array}{c} K^+ \\ K^0 \end{array} \right) \), \( K^{*\mu} \equiv \left( K^{*+\mu}, K^{*0\mu} \right) \), and \( \bar{\tau} = (\tau_1, \tau_2, \tau_3) \) are usual Pauli matrices acting on the kaon iso-spinors.

For \( K\pi \) scattering, amplitude \( T \) can be written in terms of two invariant amplitudes \( T^+ \) and \( T^- \) by [13]

\[ T_{\beta\alpha} = \delta_{\beta\alpha} T^+ + \frac{1}{2} [\tau_\beta, \tau_\alpha] T^-, \]  

(5)

where \( \alpha, \beta \) are the isospin indices of the pions. Using isospin projection operators gives

\[ 3T^+ = T^{1/2}(s, t, u) + 2T^{3/2}(s, t, u), \]  

\[ 3T^- = T^{1/2}(s, t, u) - T^{3/2}(s, t, u). \]  

(6)

where \( s, t, u \) are the usual Mandelstam variables.

The partial-wave amplitudes are obtained from the full amplitude by the standard projection formula [13, 15]

\[ T_l(s) = \frac{1}{16\pi^2} \frac{1}{2} \int_{-1}^{+1} d(\cos \theta) P_l(\cos \theta) T(s, t, u) \]  

\[ = \frac{1}{16\pi^2} \frac{1}{4p^2} \int_{-4p^2}^0 dt P_l \left[ 1 + \frac{t}{2p^2} \right] T(s, t, u), \]  

(7)
where $P_l(x)$ is the Legendre function and $p = \sqrt{[s - (m_\pi + m_K)^2][s - (m_\pi - m_K)^2]/(2\sqrt{s})}$.

Our normalization is such that the unitarity relation for partial-wave amplitude reads

$$\text{Im}T_l(s) = \rho_1(s)|T_l(s)|^2,$$

with $\rho_1(s) = 2\rho/\sqrt{s}$.

We start with the Born term of the $K\pi$ scattering amplitude by $\rho$ meson and $K^*$ meson exchange and follow the $K$-matrix formalism as in Refs.\[13, 16, 17\]. Fig. 1 is the Feynman diagram of the $K\pi$ scattering Born term.

**FIG. 1:** The Born term of $K\pi$ scattering.

### A. $s$-channel and $u$-channel $K^*$ meson exchange amplitude

The Born term for the $K^*$ meson exchange ((a) and (c) of Fig. 1) is

$$T^{1/2}(s, t, u) = g_{\pi KK^*}^2 \left[ \frac{3(t-u)}{m_{K^*}^2-s} + \frac{3(m_\pi^2-m_{K^*}^2)^2}{s(m_{K^*}^2-s)} + \frac{s-t}{m_{K^*}^2-u} - \frac{(m_\pi^2-m_{K^*}^2)^2}{m_{K^*}^2(m_{K^*}^2-u)} \right], \quad (8)$$

$$T^{3/2}(s, t, u) = -2g_{\pi KK^*}^2 \left[ \frac{s-t}{m_{K^*}^2-u} - \frac{(m_\pi^2-m_{K^*}^2)^2}{m_{K^*}^2(m_{K^*}^2-u)} \right], \quad (9)$$

where $g_{\pi KK^*} = G_\rho$ is the coupling constant. Their $S$-wave projections are

$$K_S^{1/2}(s) = -\frac{1}{2}K_S^{3/2}(s)$$

$$= G_2 \left\{ -1 + \frac{2(s-m_\pi^2-m_{K^*}^2)+m_{K^*}^2-(m_\pi^2-m_{K^*}^2)^2/m_{K^*}^2}{4p^2} \times \ln \frac{m_{K^*}^2+s-2(m_\pi^2+m_{K^*}^2)}{m_{K^*}^2+s-2(m_\pi^2+m_{K^*}^2)-4p^2} \right\}, \quad (10)$$

where $G_2 = g_{\pi KK^*}^2/(16\pi)$. $K$-matrix unitarization is introduced by

$$T_S^{l=1/2}(s) = \frac{K_S^{l=1/2}(s)}{1 - i\rho_1(s)K_S^{l=1/2}(s)}, \quad (11)$$

$$T_S^{l=3/2}(s) = \frac{K_S^{l=3/2}(s)}{1 - i\rho_1(s)K_S^{l=3/2}(s)}. \quad (12)$$
Now we calculate the coupling constant $G_2$. Considering $I = 1/2$ $P$-wave amplitude

$$T^I_{P=1/2}(s) = \frac{K^I_{P=1/2}(s)}{1 - i\rho_1(s)K^I_{P=1/2}(s)},$$

(13)

where $K^I_{P=1/2}$ is the $I = 1/2$ $P$-wave Born amplitude

$$K^I_{P=1/2}(s) = \frac{1}{4p^2} \int_{-4p^2}^{0} dt \left\{ G_2 \left[ \frac{3(t-u)}{m_{K^*}^2 - s} + \frac{3(m_{\pi}^2 - m_{K}^2)^2}{s(m_{K^*}^2 - s)} \right] + \frac{s-t}{m_{K^*}^2 - u} \left( \frac{(m_{\pi}^2 - m_{K}^2)^2}{m_{K^*}^2(m_{K^*}^2 - u)} \right) \times \left[ 1 + \frac{t}{2p^2} \right] \right\}. \quad (14)$$

Near the $K^*$ pole at $s \approx m_{K^*}^2$, we have

$$K^I_{P=1/2}(s) \approx \frac{G_2 4p^2}{m_{K^*}^2 - s}, \quad (15)$$

thus,

$$T^I_{P=1/2}(s) = \frac{G_2 4p^2}{m_{K^*}^2 - s - i\rho_1(s)G_2 4p^2}. \quad (16)$$

Comparing with the standard Breit-Wigner formula, we obtain

$$M_{K^*} \Gamma_{K^*} = \rho_1(s)4p^2 G_2 |_{s=M_{K^*}}, \quad (17)$$

which leads to $G_2 = 0.21$ with the $K^*$ mass $M_{K^*} = 891.66$ MeV and width $\Gamma_{K^*} = 50.8$ MeV from Ref. [1].

The ratio of coupling constants is $g_{\rho\pi\pi}/g_{\piKK^*} \approx 1.9$ using the $g_{\rho\pi\pi}$ value of Refs. [13, 16]: $g_{\rho\pi\pi}^2/(32\pi) = 0.364$. It agrees well with the value from SU(3) symmetry: $g_{\rho\pi\pi}/g_{\piKK^*} = 2$.

In order to explain the $K\pi$ $I = 3/2$ $S$-wave experimental data, a form factor is needed to take into account the off-shell behavior of the exchanged mesons. For $t$ and $u$-channel exchange, we use a form factor of conventional monopole type at each vertex:

$$F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}, \quad (18)$$

where $m$ and $q$ are the mass and four-vector momentum of exchanged mesons, and the cutoff parameter $\Lambda = 1500$ MeV, the same value as the $\pi\pi$ scattering in Ref. [13].

After adding the form factor, $K^I_{S=1/2}(s)$ and $K^I_{S=3/2}(s)$ becomes

$$K^I_{S=1/2}(s) = \frac{1}{4p^2} \int_{-4p^2}^{0} dt \left\{ G_2 \left[ \frac{3(t-u)}{m_{K^*}^2 - s} + \frac{3(m_{\pi}^2 - m_{K}^2)^2}{s(m_{K^*}^2 - s)} \right] + \left( \frac{\Lambda^2 - m_{K^*}^2}{\Lambda^2 - u} \right)^2 \left[ \frac{s-t}{m_{K^*}^2 - u} - \frac{(m_{\pi}^2 - m_{K}^2)^2}{m_{K^*}^2(m_{K^*}^2 - u)} \right] \right\}.$$
\[
\frac{1}{4p^2} \int_{-4p^2}^{0} dt \left\{ -2G_2 \left( \frac{\Lambda^2 - m_{K^*}^2}{\Lambda^2 - u} \right)^2 \left[ \frac{s - t}{m_{K^*}^2} - \frac{(m_\pi^2 - m_K^2)^2}{m_{K^*}^2(m_{K^*}^2 - u)} \right] \right\}
\]

\[
= -2G_2 \left\{ \frac{m_{K^*}^2 - \Lambda^2}{A - 4p^2} \times \left[ 1 + \frac{s}{A} - \frac{(m_\pi^2 - m_K^2)^2}{m_{K^*}^2 A} \right] \right. \\
+ \frac{s + B - (m_\pi^2 - m_K^2)^2/m_{K^*}^2}{4p^2} \ln \frac{B(A - 4p^2)}{A(B - 4p^2)} \right\},
\]

where \( A = \Lambda^2 + s - 2(m_\pi^2 + m_K^2), B = m_{K^*}^2 + s - 2(m_\pi^2 + m_K^2) \).

**B. \( t \)-channel \( \rho \) meson exchange amplitude**

The Born term for the \( \rho \) meson exchange (see Fig 1(b)) is

\[
T^{\text{Born}}(I = 1/2) = 2g_{\pi\pi\rho}g_{\rho KK} \frac{s - u}{m_\rho^2 - t},
\]

\[
T^{\text{Born}}(I = 3/2) = -g_{\pi\pi\rho}g_{\rho KK} \frac{s - u}{m_\rho^2 - t}.
\]

Their \( S \)-wave projections are

\[
K_{S}^{1/2}(s) = -2K_{S}^{3/2}(s)
\]

\[
= 2G_1 \left\{ -1 + \frac{2(s - m_\pi^2 - m_K^2) + m_\rho^2}{4p^2} \ln \frac{m_\rho^2(\Lambda^2 + 4p^2)}{\Lambda^2(m_\rho^2 + 4p^2)} \right\},
\]

where \( G_1 = g_{\pi\pi\rho}^2/(32\pi) = 0.364 \) \cite{13, 10}. Because we cannot obtain \( g_{\rho KK} \) from experiment, \( SU(3) \) symmetry \( g_{\pi\pi\rho} = 2g_{\rho KK} \) is used.

After introducing form factor,

\[
K_{S}^{1/2}(s) = -2K_{S}^{3/2}(s)
\]

\[
= 2G_1 \left\{ \frac{2(s - m_\pi^2 - m_K^2)}{\Lambda^2} + \frac{1}{\Lambda^2 + 4p^2} - \frac{2(s - m_\pi^2 - m_K^2) + m_\rho^2}{4p^2} \ln \frac{m_\rho^2(\Lambda^2 + 4p^2)}{\Lambda^2(m_\rho^2 + 4p^2)} \right\}.
\]
C. Amplitude of s-channel S-wave resonances

The phase shift is known to be elastic below 1300 MeV. The threshold for the $K\eta'$ channel is at 1453 MeV and the $K\eta$ channel is only weakly coupled to the $K\pi$ channel [8, 11]. Considering the $K\pi$ and $K\eta'$ channels, the explicit form is

$$T = \frac{M \Gamma_{K\pi} / \rho_1(M^2)}{M^2 - s - i [M \Gamma_{K\pi} / \rho_1(M^2) + M \Gamma_{K\eta'} / \rho_2(M^2)]}$$

(25)

where $\rho_2(s) = \sqrt{s - (m_{\eta'} + m_K)^2][s - (m_{\eta'} - m_K)^2]/s$ is the phase space factor of $K\eta'$.

When fitting the experimental data, we first try introducing one s-channel resonance and then try introducing two such resonances.

III. NUMERICAL RESULTS AND DISCUSSION

As in Refs. [13, 16], we use Dalitz-Tuan method to combine various components given in the last section to get the full partial wave amplitudes and corresponding phase shifts.

For the $K\pi I = 3/2$ S-wave scattering, the phase shift is negative with magnitude slowly increasing as the center-of-mass energy increases as shown in Fig.2. There is no s-channel quark-antiquark resonance contribution allowed for isospin $I = 3/2$. So the only contribution here is the $t$-channel $\rho$ and $u$-channel $K^*$ meson exchanges. With the cutoff parameter $\Lambda = 1.5$ GeV fixed as the same as in $\pi\pi$ scattering [13], we get the prediction for the $K\pi I = 3/2$ S-wave phase shift as shown by the solid line in Fig.2(b) without introducing any free parameters, which reproduces data nicely. To show the effect of off-shell form factor, the results without form factor are shown in Fig.2(a). The $t$-channel $\rho$ exchange and $u$-channel $K^*$ exchange give very similar contribution to the $K\pi I = 3/2$ S-wave phase shift as shown by the long-dashed line and dotted line, respectively. Here the $t$-channel $\rho$ exchange gives a much larger contribution than the $u$-channel $K^*$ meson exchange. The sum of these two contributions is shown by the dashed line and is obviously not enough to reproduce the experimental data. Some contribution from s-channel resonance(s) is definitely needed. By
FIG. 2: $I = 3/2$ $K\pi$ $S$-wave phase shift. Data are from Refs. [12] (dots) and [18] (circles). Theoretical curves are for $t$-channel $\rho$ exchange (dotted line), $u$-channel $K^*$ exchange (dashed line), and the sum (solid line). (a) without form factor and (b) with form factor and $\Lambda = 1.5$ GeV.

fixing the $t$-channel $\rho$ exchange and the $u$-channel $K^*$ exchange as background contribution, we fit the LASS data [11] first by introducing one $s$-channel resonance (dot-dashed line) and then by introducing two $s$-channel resonances (solid line). The fitted parameters for the $s$-channel resonance(s) and the corresponding $\chi^2$ for two cases are listed in Table I.
FIG. 3: The $I = 1/2$ $K\pi$ $S$-wave phase shift and amplitude. The experimental data for $\delta_{1/2}$ and $T_{1/2}$ are from Ref. [11] (dotted), Ref. [12] (boxed) Ref. [19] (circled) Ref. [20] (diamond). The long-dashed line is for $K^*$ meson exchange, the dotted line is for $\rho$ meson exchange, the dashed line is the sum of $K^*$ and $\rho$ meson exchange, the dot-dashed line includes one $s$-channel resonance and the solid line includes two resonances.
TABLE I: Fitted parameters for the s-channel resonances and the corresponding $\chi^2$ for two cases: with one resonance (first line) and with two resonances (second line). Values for mass and width are in unit of GeV.

| $M_1$ | $\Gamma_{K\pi}^{(1)}$ | $\Gamma_{K\eta'}^{(1)}$ | $M_2$ | $\Gamma_{K\pi}^{(2)}$ | $\Gamma_{K\eta'}^{(2)}$ | $\chi^2$ |
|-------|----------------|----------------|-------|----------------|----------------|-------|
| 1.438 | 0.345          | 0.001          | —     | —              | —              | 86/45 |
| 1.486 | 0.346          | 0.000          | 1.668 | 0.150          | 0.491          | 57/45 |

It is natural that the fit with two s-channel resonances gives a smaller $\chi^2$ value. But from Fig.3 we see that both cases with one or two s-channel resonances give quite good fit to the data. For the case of two resonances, the second resonance is very broad and has a mass above the upper energy limit (1.6 GeV) of the data, and could be an effective tail of resonances above 1.6 GeV. In both cases, there is only one s-channel resonance between the $K\pi$ threshold and 1.6 GeV, corresponding to the PDG well established $K^*(1430)$ resonance. The fitted mass and width for the $K^*(1430)$ depend on whether we introduce one or two s-channel resonances, with mass around $1438 \sim 1486$ MeV and width about 346 MeV, which is very close to the value (1450, 350) MeV by Tornqvist and Roos [21] with a different formalism.

For the $t$-channel $\rho$ meson exchange amplitude, we find a pole at (0.45-0.48i) GeV. This is consistent with the conclusion by Cherry and Pennington that there is no $\kappa(900)$, but a very low mass $\kappa$ well below 825 MeV cannot be ruled out.

In summary, the $K\pi I = 3/2$ S-wave phase shift can be well reproduced by the $t$-channel $\rho$ and $u$-channel $K^*$ meson exchange while the $K\pi I = 1/2$ S-wave phase shift are dominated by the s-channel $K^*_0(1430)$ resonance and the $t$-channel $\rho$ exchange with a pole at (450-480i) MeV. The $\kappa(450)$ has a similar nature as $\sigma(400)$[13, 16]: both are produced by the t-channel $\rho$ exchange and are very broad with a width around 1 GeV.

Acknowledgments

This work was supported in part by the Major State Basic Research Development Program (G20000774), CAS Knowledge Innovation Project (KJCX2-SW-N02) and by National
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