A New Mechanism for $B^{\pm} \to \eta' K^{\pm}$ in Perturbative QCD

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Abstract

We present a new mechanism for $B^{\pm} \to \eta' K^{\pm}$, and analyze it strictly within perturbative QCD. We find that such a new mechanism may dominate non-leptonic $B$-decays to light mesons. Within a reasonable parameter space, our prediction is in good agreement with the recent CLEO data on $B^{\pm} \to \eta' K^{\pm}$. We conclude that there is no room for an abnormally large $b \to sg$ vertex from physics beyond the Standard Model.
Recently, the CLEO collaboration has reported a large \( \eta' \) yield in charmless \( B \) decays as follows \([1]\),

\[
\mathcal{BR}(B^\pm \to \eta' X_s) = (6.2 \pm 1.6 \pm 1.3) \times 10^{-4} \quad (2.0 < P_{\eta'} < 2.7\text{GeV}),
\]

and a corresponding large exclusive branching fraction \([2]\)

\[
\mathcal{BR}(B^\pm \to \eta' K^\pm) = (7.1^{+2.5}_{-2.1} \pm 0.9) \times 10^{-5}.
\]

It is another great experimental achievement in rare \( B \) decays since the measurements of \( B \to K^*\gamma \) and \( B \to X_s\gamma \) which involve the so-called QCD- and electroweak-penguins. Since then, theoretical investigations on these decays have appeared, offering several interesting interpretations of the data, both within and beyond the Standard Model (SM) \([3]\). In particular, the decay mode (2) makes itself conspicuous due to its surprisingly large branching ratio.

Now let us examine the theoretical status of the estimate of the exclusive branching fraction \( \mathcal{B}(B^\pm \to \eta' K^\pm) \). The standard theoretical framework to study non-leptonic \( B \) decays is based on the effective Hamiltonian which describes the decays at quark level,

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum V_{CKM} C_i O_i,
\]

and the BSW model \([4]\) to estimate the hadronic matrix element \( < M_1 M_2 \mid O_i \mid B > \).

An important feature of the BSW model in nonleptonic decays is the use of the factorization and spectator ansatz. It works remarkably well in the so called heavy-to-heavy transitions because the ansatz is consistent with the HQET, and therefore is almost certainly true. However, we think that it has never been tested, and so is much less justified, in heavy-to-light decays.

Even given the validity of the BSW ansatz, yet another problem exits in estimating the hadronic matrix elements like

\[
< \eta' \mid \bar{u}(1 - \gamma_5)b \mid B^- >, \quad < K^- \mid \bar{s}(1 + \gamma_5)u \mid 0 >, \\
< \eta' \mid \bar{s}\gamma_5s \mid 0 >, \quad < \eta' \mid \bar{u}\gamma_5u \mid 0 >.
\]
The Dirac equations of motion

\[
\bar{q}' = \frac{-i\partial^\mu (\bar{q}' \gamma_\mu q')}{{m_q}' - {m_{q'}}}, \quad \bar{q} \gamma^\mu q' = \frac{-i\partial^\mu (\bar{q} \gamma_\mu q')}{{m_q} + {m_{q'}}} \tag{5}
\]

are used to get the elements for \( B^\pm \to \eta' K^\pm \),

\[
< \eta' | \bar{s} \gamma_5 s | 0 > = \frac{i {m_{\eta'}}^2}{2{m_s}} f^{(ss)} \tag{6}
\]

\[
< \eta' | \bar{u} \gamma_5 u | 0 > = \frac{i {m_{\eta'}}^2}{2{m_u}} f^{(uu)} \tag{7}
\]

\[
< K^- | \bar{s} (1 + \gamma_5) b | B^- > < \eta' | \bar{u} \gamma_5 (1 - \gamma_5) u | 0 > . \tag{8}
\]

With these relations, the amplitude for \( B^\pm \to \eta' K^\pm \) reads

\[
\mathcal{M} = \frac{G_F}{\sqrt{2}} (V_{tb} V_{ts}^*) \left[ a_2 + a_1 \frac{M_B^2 - {m_{\eta'}}^2}{M_B^2 - {m_K}^2} \frac{F_0^{B \to \eta'} (m_K^2)}{F_0^{B \to K^-} (m_{\eta'}^2)} f_{K^-} \right]
\]

\[
- V_{tb} V_{ts}^* \left[ 2a_3 - 2a_5 + (a_3 - a_5 + a_4 + \frac{a_6 {m_{\eta'}}^2}{m_s (m_b - m_s)}) f^{(ss)}_{\eta' b} \right]
\]

\[
+ (a_4 + \frac{2a_6 {m_K}^2}{(m_s + m_u) (m_b - m_u)} \frac{M_B^2 - {m_{\eta'}}^2}{M_B^2 - {m_K}^2} \frac{F_0^{B \to \eta'} (m_K^2)}{F_0^{B \to K^-} (m_{\eta'}^2)} f_{\eta' K^-} ) \right] \}
\]

\[
< K^- | \bar{s} \gamma_\mu (1 - \gamma_5) b | B^- > < \eta' | \bar{u} \gamma_\mu (1 - \gamma_5) u | 0 > . \tag{9}
\]

Assuming the Dirac equation is valid for bounded fermions, the factor \( \frac{{m_{\eta'}}}{m_s} \) in the amplitude enhances the contribution of operator \( O_6 \), which is very difficult to understand.

In an alternative way, we turn to estimate the amplitude using the perturbative QCD hard scattering formalism [5], which has also been extensively applied to \( B \)-decays [6]. The diagrams to be calculated are depicted in Fig. 1. However, we find that these amplitudes are rather small due to cancellations between the diagrams and the small \( b \to gg \) vertex [7]. Nevertheless, before one goes beyond the SM, the contributions in the SM should be examined carefully and exhausted. In what follows, we discuss a new type of mechanism for \( B \)-decays to light mesons in detail.

The new mechanism is depicted in Fig. 2. This mechanism is motivated by the fact that both the recoil between \( \eta' \) and \( K^- \) and the energy released in the process are large. The gluon from \( b \to sg \) vertex carries energy about \( M_B/2 \) and then materializes to \( \eta' \) and emits another hard gluon to balance color and momentum. The momentum squares of the gluons...
scale as $\propto M_B^2$

$$
k_1^2 = ((1 - x)P_B - yP_K)^2 \cong M_B^2 - (1 - x)y(M_B^2 - M_{\eta'}^2),$$

and $k_2^2 = (xP_B - (1 - y)P_K)^2 \cong -x(1 - y)M_B^2,$

(10)

where $x$ and $y$ are the momentum fractions carried by the collinear quarks shown in Fig. 2. For self-consistency of the co-linear picture used here, the terms $\sim x^2 M_B^2$ are neglected since they are at the same level of the transverse momentum square of the quarks in the bound state $B$ meson. Using mean values $<y> \sim \frac{1}{2}, <x> \sim \epsilon_b$ with $\epsilon_b \sim 0.05 - 0.1$, we get $<k_1^2> \sim 12\text{ GeV}^2$ and $<k_2^2> \sim 1\text{ GeV}^2$, which are large enough to justify a perturbative calculation.

The soft contributions are parameterized in terms of wave functions of the bound states. In the spirit of Ref. [6], we neglect the transverse components of quarks and take the wave functions for $B^-$ and $K^-$ as

$$
\Psi_B = \frac{1}{\sqrt{2}} \sqrt{3} I_C \phi_B(x)(P_B + M_B)\gamma_5 \ , \\
\Psi_K = \frac{1}{\sqrt{2}} \phi_K(y)\gamma_5 q_K \ ,
$$

(11)

where $I_C$ is an identity in color space. In QCD, the integration of the distribution amplitude is related to the meson decay constant

$$
\int \phi_K(y)dy = \frac{1}{2\sqrt{3}} f_K \ , \quad \int \phi_B(x)dx = \frac{1}{2\sqrt{3}} f_B .
$$

(12)

We write down the amplitude of Fig. 2 as

$$
\mathcal{M} = \int dx dy \phi_B(x)Tr \left[ \gamma_5 q' \Gamma_\mu (P/ + M_B)\gamma_5 \gamma_\mu \right] \frac{4\epsilon^{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} C_{eff} C_F g_s^3 \phi_K(y)}{\sqrt{2} \sqrt{2} (k_1 \cdot k_2) k_1^2 k_2^2} ,
$$

(13)

where $\Gamma_\mu$ is the effective $b \rightarrow s g$ vertex known for years [8]

$$
\Gamma^a_\mu = \frac{G_F g_s}{\sqrt{2} 4\pi^2} V_{ts}^* V_{tb} \epsilon^{a\mu\nu\rho\sigma} \left\{ F_1^i (k_2^2 \gamma_\mu - k_{1\mu} k_1^2) L - F_2^i \sigma_{\mu\nu} k_{1\nu} m_b R \right\} .
$$

(14)

We have used a Lorentz and gauge invariant amplitude $<g^a (k_1, \epsilon_1) g^b (k_2, \epsilon_2) | \eta'>$ in Eq. (13), with the shorthand notation $G_{\mu\nu} \sim k_\mu \epsilon_\nu - \epsilon_\mu k_\nu$, and it reads [9, 10]

$$
<g^a g^b | \eta'> = \delta^{ab} A_{\eta'} F(k_1^2, k_2^2) \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^1 G_{\rho\sigma}^2
$$

(15)
with $A_{q'} = \frac{2C_{eff}}{m_{q'}^2}$ and the form factor $F(k_1^2, k_2^2) = \frac{m_{q'}^2}{2k_1^2k_2^2}$ which will be extracted form the $J/\Psi$ decays. $C_F$ is a color factor $C_F = \frac{1}{\sqrt{3}} Tr[T_aT_b] \delta_{ab} = \frac{4}{3}$.

Finally we obtain

$$M = \frac{G_F}{\sqrt{2}} \alpha_s^2 C_{eff} C_F 32 (V_{ts}V_{tb}^*)$$

$$\times \int dx dy \phi_B(x)\phi_K(y) \frac{F_{i_1}^2 F_i}{k_1 \cdot k_2 k_1^2 k_2^2 k_1^2} [p \cdot k_1 q \cdot k_2 - p \cdot k_2 q \cdot k_1 + F_{i_2}^2 M_B m_b [q \cdot k_2^2 - q \cdot k_1^2 k_1 \cdot k_2]], \quad (16)$$

where the momenta $k_1, k_2, p, q$ can be read off from Fig. 2. We note that from Eqs. (10,16) the singularities of the gluons-propagators are located at the point $y = 1$, which is just the end point for the wave function $\phi_K(y)$. However, it is well known that the value of bound state wave function at its end point is exactly zero in QCD.

We can extract $C_{eff}$ form the data $J/\Psi \rightarrow \eta' \gamma$ to remove ambiguities:

$$\frac{\Gamma(J/\Psi \rightarrow \eta' \gamma)}{\Gamma(J/\Psi \rightarrow e^+ e^-)} = \frac{4 \alpha_s^4 (m_\Psi) \alpha_e}{\pi M_\Psi^2} \left(8 \sqrt{3} C_{eff}\right)^2 \frac{x \cdot |\hat{H}^{PS}(x)|^2}{54}, \quad (17)$$

where the $x \cdot |\hat{H}^{PS}(x)|^2$ can be found in [9]. For $x = 1 - \frac{m_{\eta'}^2}{M_\Psi^2}$, $x \cdot |\hat{H}^{PS}(x)|^2 = 54.8$. We extract $C_{eff} = 0.075$ GeV. In order to get quantitative estimates, we take the wave function as [3]

$$\phi_B(x) = \frac{f_B}{2 \sqrt{3}} \delta(x - \epsilon_B), \quad \text{and} \quad \phi_K(y) = \sqrt{3} f_K y (1 - y), \quad (18)$$

and $\tau_B = 1.62$ ps, $f_B = 140$ MeV, $f_K = 113$ MeV (corresponding $f_\pi = 92$ MeV) and $| V_{ts}V_{tb}^*| = 0.044$. For the strong coupling at the energy scale $< k_2^2 >$, we respect the choice $\alpha_s = 0.38$ in Ref. [3]. The main uncertainty of our estimate exists in the distribution function $\phi_B(x)$. At present, it can not be derived from the first principle of QCD. However, it is easy to understand that the distribution function is peaked sharply at one point due to the heavy $b$ quark carrying most of the momentum of the $B$ meson. The peaking position is expected at $x = \frac{M_B - M_b}{M_B} \sim \frac{\Lambda_{QCD}}{M_B}$.

From Eq.10,16,18, it is easy to see that the integration in Eq.16 is infrared safe because the pole in $1/k_2^2$ is canceled by the $K$ meson wavefunction and $k_1^2$ is always positive. To confirm that $| \mathcal{M} |$ is dominated by hard gluon contributions, we define $R$ as the ratio of
with cut on $k_2^2$ to that without cut. Taking $\Lambda_{QCD} \approx 200 \text{MeV}$, we list the numerical results of $R$ in Table I to show that the integration in Eq.16 is indeed dominated by hard gluon contributions. For somewhat complete investigation on this mechanism, the next to leading order contributions (high twist) should be included, however, it is complicated. We shall discuss it elsewhere.

The numerical results of $BR(B^\pm \to \eta'K^\pm)$ are displayed in Fig. 3 as a function of the peaking position parameter $\epsilon_B$.

We can see that our predictions are in a good agreement with experimental results in the region of $\epsilon_B = 0.05 \sim 0.07$. Furthermore, if the contributions of Eq. (9) estimated from the conventional way (which may contribute up to $BR(B^\pm \to \eta'K^\pm) = (1 \sim 4) \times 10^{-5}$ [11,12]) are taken into account, the SM predictions turn out to be in agreement with the CLEO data within the $1\sigma$ level in the whole parameter range of $\epsilon_B = 0.05 \sim 0.1$. We conclude that $BR(B^\pm \to \eta'K^\pm)$ is not "surprisingly large", and the mechanism in the Standard Model presented here seems sufficient for explaining it.

Note added: After finishing this paper, we became aware that a similar idea appeared in hep-ph/9710509 [13]. But our physical picture, calculation method and conclusion are all different from theirs, which used a gluon-diffusion picture and calculated it by using effective Hamiltonian method. However, we here present a hard scattering picture and solve it on perturbative QCD. Acknowledgment We thank H.Y. Cheng and M. Drees for careful reading of the manuscript and valuable comments. D.Du and Y.D.Yang are supported in part by the National Natural Science Foundation and the Grant of State Commission of Science and Technology of China. The work of CSK was supported in part by Non-Directed-Research-Fund, KRF in 1997, in part by the CTP, Seoul National University, in part by Yonsei University Faculty Research Fund of 1997, in part by the BSRI Program, Ministry of Education, Project No. BSRI-97-2425 and in part by the KOSEF-DFG large collaboration project, Project No. 96-0702-01-01-2.
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Figure 1: *Time and space like strong penguin diagrams for $B^\pm \rightarrow \eta'K^\pm$ in usual way. Diagrams suppressed by $V_{ub}$ are neglected. Blobs depict $K$ and $\eta'$*
Figure 2: New diagram for $B^\pm \rightarrow \eta' K^\pm$. $x$, $1-x$, $y$ and $1-y$ are momentum fractions carried by the quarks.
Figure 3: \( \text{BR}(B^\pm \to \eta'K^\pm) \) as a function of \( \epsilon_B \). The horizontal thick solid lines show the CLEO measurement (with \( \pm 1\sigma \) error bar) and the thin curve is the contribution from the new mechanism presented here.
| Cut                  | $R(\epsilon_B = .05)$ | $R(\epsilon_B = .06)$ | $R(\epsilon_B = .07)$ |
|----------------------|------------------------|------------------------|------------------------|
| $k_2^2 > \Lambda_{QCD}^2$ | 96%                    | 97%                    | 98%                    |
| $k_2^2 > 4\Lambda_{QCD}^2$ | 76%                    | 87%                    | 95%                    |
| $k_2^2 > 9\Lambda_{QCD}^2$ | 50%                    | 58%                    | 64%                    |

Table 1: Numerical results for $R$ for different cuts and $\epsilon_B$. 