In this paper we investigate the one-loop radiative corrections to the neutrino indices of refraction from supersymmetric models. We consider the Next-to-Minimal Supersymmetric extension of the Standard Model (NMSSM) which happens to be a better supersymmetric candidate than the MSSM for both theoretical and experimental reasons. We scan the relevant SUSY parameters and identify regions in the parameter space which yield interesting values for $V_{\mu\tau}$. If R-parity is broken there are significant differences between MSSM and NMSSM contributions contrary to the R-parity conserved case. Finally, for a non-zero CP-violating phase, we show analytically that the presence of $V_{\mu\tau}$ will explicitly imply CP-violation effects on the supernova electron (anti-) neutrino fluxes.

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I. INTRODUCTION

The charged current interaction between neutrinos and their associated leptons in medium give an effective matter potential which can lead to a resonant flavour conversion called the Mikheyev-Smirnov-Wolfenstein (MSW) effect [1,2]. Such behaviour solves elegantly the solar neutrino deficit problem first pointed out by pioneering experiment of R. Davis in the 60’s [3]. Such phenomenon is of crucial importance for the propagation of neutrinos in the supernova environment whose flux detection could yield precious information concerning the dynamics of the density profile or fundamental neutrinos properties like the hierarchy or the value of the third mixing angle. In supernovae, depending on the mass hierarchy, the electron neutrino may encounter one or two resonances via charged current, while the muon or tau neutrinos will only interact via neutral current, indistinguishably at the tree level in the Standard Model (SM). Indeed, due to the absence of muon or tau particle in such environment, coherent forward scattering may only intervene via neutral current to which all flavours are sensitive. Considering that matter interaction can also be seen as neutrino index of refraction [4,5], Botella et al. [6], after showing that correction at $O(\alpha)$ were negligible in the case of a neutral medium [7], proved that differences at one-loop in the neutrino index of refraction could arise for muon and tau neutrinos at order $O\left(\frac{\alpha \sin^2 2\theta W}{m^2_{\mu}/M^2_W}\right)$. Later, supersymmetric radiative corrections, in the case of the minimal supersymmetric model (MSSM) has been partially calculated [8] showing that it could give potentially much larger radiative effects than in the SM. It is interesting to note that such radiative corrections have been for a longtime considered as negligible or without observable consequences for the supernova environment. Such consideration was probably true when not taking into account the neutrino-neutrino interaction which has dramatically changed the vision we have of neutrino propagating in an exploding star and which has gone through an intense investigation [3,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33]. Actually few papers have shown the importance of the one-loop correction in addition with the neutrino-neutrino interaction. First it has been shown that for early time after post-bounce when the matter density profile is very high, neutrinos can encounter the $\mu-\tau$ resonance and possibly modify the $\nu_e$ flux [22,27]. Unfortunately, it has also been shown that in such case the high density in addition with a multi-angle neutrino-neutrino interaction would make those effects vanish [31]. Besides those works, only one paper [1] has shown the importance of $V_{\mu\tau}$ via the influence of the CP-violating
phase $\delta$ contained in the MNSP matrix and whose value is still unknown. Such term induces effects of a non-zero CP-violating phase on the electron neutrino fluxes inside and outside the supernova.

The goal of this paper is to calculate such corrections in the SUSY framework with and without taking into account R-parity breaking interactions. It is organized as follows: Sec.2 introduces the theoretical framework where we briefly remember how to calculate radiative corrections in this context and more importantly where we introduce the supersymmetric framework. The corrections with R-parity conservation are calculated in Sec.3 and Sec.4 is dedicated to the case where the R-parity is broken. Before concluding, we explicitly demonstrate in Sec.5 the influence of $V_{\mu\tau}$ on the $\nu_e$ flux when the CP-violating phase is non-zero.

II. THEORETICAL FRAMEWORK

A. The calculations of the radiative corrections

Neutrinos interaction through matter can be described using indices of refraction and in this case, the evolution equation of neutrinos with matter, omitting the neutrino-neutrino interaction is:

$$\frac{id}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \frac{1}{2p_\nu} U \begin{pmatrix} \Delta m^2_{12} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m^2_{32} \end{pmatrix} U^\dagger - p_\nu \begin{pmatrix} \Delta n_{\nu\mu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta n_{\tau\mu} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad (1)$$

where $U$ is the MNSP matrix, $p_\nu$ the neutrino momentum, $\Delta m^2_{ij} \equiv m^2_{\nu_i} - m^2_{\nu_j}$ and $\Delta n_{\alpha\beta} \equiv n_{\nu_\alpha} - n_{\nu_\beta}$.

To size the effect of such radiative correction on the neutrino propagation, we study the one-loop effect on the scattering amplitude matrix which describes the interaction of neutrinos with matter:

$$M(\nu f \rightarrow \nu f) = -\frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma^\rho (1 - \gamma_5) \nu_f \gamma^\rho (C^V_{\nu f} + C^A_{\nu f} \gamma_5) f. \quad (2)$$

In the calculations we will use the same approximations as in [6] and [8] i.e., neutrinos are propagating though an unpolarized medium at rest. Consequently, the neutrino index of refraction can be written as:

$$p_\nu (n_{\nu_e} - 1) = -\sqrt{2} G_F \sum_{f=u,d,e} C^V_{\nu_f} N_f, \quad (3)$$

where $N_f$ is the number density of fermion $f$ in the medium. Therefore, the interesting parameter to study in order to size the radiative correction to matter interaction is $C^V_{\nu f}$ which is defined at the tree-level by

$$C^V_{\nu f} = T_3(f_L) - 2Q_f \sin^2 \theta_W + \delta_{fJ}, \quad (4)$$

with $\sin^2 \theta_W \equiv 1 - m^2_W/m^2_Z \approx 0.23$, while $Q_f$ and $T_3(f_L)$ are respectively the electric charge and the third component of the weak isospin of the fermion $f_L$. To parametrize the loop corrections to $C^V_{\nu f}$ Botella et al. [6] have defined in some kind of an arbitrary but convenient way a new $C'^V_{\nu f}$ by

$$C'^V_{\nu f} = \rho^{\nu f} T_3(f_L) - 2Q_f \lambda^{\nu f} s^2_W. \quad (5)$$

for $f \neq \ell$ where the $\rho^{\nu f}$ includes the $f$-dependent (box) diagrams contributions. Since the $\lambda^{\nu f}$ are chosen to be independent of the $f$, in a electrically neutral medium, they will not contribute to $\Delta n_{\nu f}$. Consequently, $\Delta n_{\tau \mu}$ will only be sensitive to $\Delta \rho^f \equiv \rho^{\nu f} - \rho^{\nu \ell}$. Note that we are only interested about the difference between $n_{\nu_\mu}$ and $n_{\nu_\tau}$ indices of refraction because the loop correction to $\Delta n_{\nu\mu} = -\sqrt{2} G_F N_\nu/p_\nu$ will be negligible since electron neutrinos already encounter charged current interactions with matter. In the SM, the correction have been calculated and are found to be small:

$$\Delta n_{\tau \mu} = V_{\mu\tau} = \varepsilon V_e \approx 5.4 \times 10^{-5} V_e \quad (6)$$

where $V_{\mu\tau}$ is the effective matter potential to tau neutrinos due to one-loop corrections and $\varepsilon$ the ratio between the $V_{\mu\tau}$ and $V_e$ which yields the size of the loop correction in comparison with the charged-current matter potential for electron neutrinos $V_e$. 
**B. The supersymmetric framework**

The most significant theoretical issues of the Standard Model (SM) are the hierarchy problem for the Higgs mass and the non-unification of the gauge couplings. Supersymmetry allows us to address these problems and is an attractive candidate for new physics beyond the Standard Model (SM). In this theory a supersymmetric particle called LSP (Lightest Supersymmetric Particle) is a natural candidate for dark matter. Among various supersymmetric models the most extensively studied is the minimal supersymmetric model (MSSM).

The MSSM contains the minimum number of fields to describe the known SM particles and their superpartners. These fields can be gathered into chiral superfields and vector superfields. A chiral superfield $\hat{\Phi}$ contains a scalar field $(z)$, a fermionic field $(\psi)$ and an auxiliary field $(F)$: $\Phi = (z, \psi, F)$. A vector superfield $V$ is a multiplet which contains a bosonic field $(v^\mu)$, a fermionic field $(\lambda)$ and an auxiliary field $(D)$: $V = (v^\mu, \lambda, D)$. $F$ and $D$ are auxiliary fields, they do not have kinetic terms. They are eliminated by the minimization equations of the Lagrangian. To SM fermionic fields (leptons, quarks), we have supersymmetric scalar fields (sleptons, squarks) associated. Concerning SM vector bosons ($U(1)_Y$, $SU(2)_L$, $SU(3)_C$ gauge bosons), fermionic fields called gauginos (bino, winos, gluinos) are associated to. Finally, we have fermionic fields called higgsinos associated to Higgs bosons. Higgsinos and gauginos will mix to generate neutralinos and charginos.

A superfield is a function of spacetime coordinates and so-called superspace coordinates which appear as anticommuting Grassman variables \[^{34}\]. It can be expanded in terms of its component fields and these Grassman variables. Products of superfields can be developed resulting in products of individual particle and sparticle fields. We call superpotential all the renormalizable products of chiral superfields in a given supersymmetric model. The MSSM’s superpotential is:

$$W_{\text{MSSM}} = h_u \hat{Q} \hat{H}_u \hat{T}_R^c - h_d \hat{Q} \hat{H}_d \hat{B}_R^c - h_\tau \hat{L} \hat{H}_d \hat{L}_R^c + \mu \hat{H}_u \hat{H}_d$$  \hspace{1cm} (7)

where $\hat{H}_u, \hat{Q}, \hat{H}_d$ are chiral superfields. The fermionic part of the supersymmetric lagrangian is obtained through a procedure where the superfields of the superpotential are expanded in terms of component fields and then the superspace coordinates of the result are integrated out:

$$\mathcal{L}_{\text{MSSM}} = h_u \psi_Q \hat{H}_u \psi_{T_R}^c - h_d \psi_Q \hat{H}_d \psi_{B_R}^c - h_\tau \psi_L \hat{H}_d \psi_{L_R}^c + \mu \psi_{H_u} \psi_{H_d} + \ldots (h.c)$$ \hspace{1cm} (8)

In Supersymmetry, we need two Higgs bosons to give masses to the other particles. Despite its simplicity, there are two unexplained hierarchies, within the MSSM:

- the so-called $\mu$-problem \[^{35}\]. It arises from the presence of a mass $\mu$-term for the Higgs fields in the superpotential. The only two theoretical natural values for this parameter are either zero or the Planck energy scale. However, we need $\mu \gtrsim 100\text{GeV}$ to satisfy LEP constraints on the chargino masses and $\mu \lesssim M_{\text{Planck}}$ for a destabilization of the Higgs potential in order to have non-vanishing v.e.v. for the scalar Higgs fields ($< H_{u,d} > \neq 0$)

- The other hierarchy with an unknown origin is the one existing between the small neutrino masses (smaller than the eV scale) and the electroweak symmetry breaking scale ($\sim 100\text{GeV}$).

In this paper, our framework is the Next-to Minimal Supersymmetric extension of the Standard Model (NMSSM) \[^{36}\]. This model provides an elegant solution to the $\mu$-problem via the introduction of a new gauge-singlet superfield $S$ that acquires naturally a v.e.v. $\langle \bar{S} \rangle$ of order of the supersymmetry breaking scale, generating an effective $\mu$ parameter ($\lambda x = \mu_{\text{eff}}$) of order of the electroweak scale. Furthermore, this model explains the second hierarchy by generating two neutrino masses at tree level through R-parity breaking \[^{37}\] as we will see below.

The NMSSM’s superpotential is:

$$W_{\text{NMSSM}} = h_u \hat{Q} \hat{H}_u \hat{T}_R^c - h_d \hat{Q} \hat{H}_d \hat{B}_R^c - h_\tau \hat{L} \hat{H}_d \hat{L}_R^c + \lambda \bar{S} \hat{H}_u \hat{H}_d + \frac{\lambda}{3} \bar{S}^3$$  \hspace{1cm} (9)

Then, the fermionic part of the NMSSM lagrangian is:

$$\mathcal{L}_{\text{NMSSM}} = h_u \psi_Q \hat{H}_u \psi_{T_R}^c - h_d \psi_Q \hat{H}_d \psi_{B_R}^c - h_\tau \psi_L \hat{H}_d \psi_{L_R}^c + \lambda \bar{S} \psi_{H_u} \psi_{H_d} + \kappa \bar{S} \psi_S \psi_S + \ldots (h.c)$$ \hspace{1cm} (10)

The NMSSM contains the following particles:
values of the effective $\mu O$ which case the splitting can be $m$ than the SM ones. In the following, we shall consider in the following a large $\tilde{\tau}$ mass-squared terms. The soft SUSY breaking terms and neutral sleptons or among $\tilde{\tau}$ electroweak symmetry breaking. In this way, although squarks become significantly split from sleptons, the splittings among the masses of different slepton generations are only due to the small $\tilde{\tau}_R$ mixing. This occurs for large values of the effective $\mu$ parameter and $\tan\beta$, or for large values of the parameter $A$ of the trilinear soft terms, in which case the splitting can be $O(m_\tau + \mu_{\text{eff}}\tan\beta)$ as we can see below.

We give here the stau and smuon mass-squared matrices where we neglect splittings due to $D$-terms among charged and neutral sleptons or among $\tilde{\ell}_L$ and $\tilde{\ell}_R$. The diagonal terms are respectively the $\tilde{\ell}_R$ and $\tilde{\ell}_L$ mass-squared terms. For tau sleptons:

$$M_{\tilde{\tau}}^2 = \begin{pmatrix} m_{\tilde{\tau}}^2 & m_{\tilde{\tau}}(A_\tau + \mu_{\text{eff}}\tan\beta) \\ m_{\tilde{\tau}}(A_\tau + \mu_{\text{eff}}\tan\beta) & m_{\tilde{\tau}}^2 + m_{\tilde{\tau}_3}^2 \end{pmatrix}$$

$$M_{\tilde{\tau}}^2 = \begin{pmatrix} m_{\tilde{\tau}}^2 & m_{\tilde{\tau}}(A_\tau + \mu_{\text{eff}}\tan\beta) \\ m_{\tilde{\tau}}(A_\tau + \mu_{\text{eff}}\tan\beta) & m_{\tilde{\tau}}^2 + m_{\tilde{\tau}_3}^2 \end{pmatrix}$$

where all fields are scalar fields.
- \(m_{\tilde{\tau}_R}^2, m_{\tilde{\mu}_R}^2, m_{\tilde{\tau}_L}^2\) and \(m_{\tilde{\nu}_L}^2\) are of the order of \(M_{\text{susy}}^2 \sim 0.1 - 1\,\text{TeV}^2\),
- \((A_\tau + \mu_{\text{eff}}\tan\beta), (A_\mu + \mu_{\text{eff}}\tan\beta)\) are of the order of \(M_{\text{susy}}\),
- \(m_{\tau} = 1.8\,\text{GeV}, m_{\mu} = 105\,\text{MeV}\).

In the 2nd generation case, we can neglect the mixing between \(\tilde{\mu}_R\) and \(\tilde{\mu}_L\) because the off-diagonal terms are negligible w.r.t. the diagonal terms. But in the third generation, we have to consider \(\tilde{\tau}_1, \tilde{\tau}_2\) instead of \(\tilde{\tau}_R, \tilde{\tau}_L\).

The SUSY contribution to \(\Delta m_{\tau}^2\) can then be larger than the SM one.

In the following, to calculate the loop diagrams contributions we will use the dimensional regularization method and the vanishing external legs approximation \[38,38\] which turns out to be legitimate because of the low masses and energy of the fermions with respect to the electro-weak \((M_W)\) and SUSY \((M_{\text{susy}})\) breaking scales. When one does not use this approximation, the loop integrals to perform will have several different mass scales. Such calculations are much more complicated and need advanced mathematical methods as in \[38\]. In this framework, some specific functions will appear:

- For the self-energies and the penguin:

\[
H_0(x, y) = \sqrt{xy} \left[ \frac{x \ln x}{(x - y)(x - 1)} + \frac{x}{y} \right],
\]

\[
G_0(x, y) = \left[ \frac{x^2 \ln x}{(x - y)(x - 1)} + \frac{y^2 \ln y}{y - x} \right].
\]

- For the box diagrams:

\[
H'(x, y, z) = \sqrt{xyz} \left[ \frac{x \ln x}{(x - y)(x - z)(x - 1)} + \frac{y \ln y}{(y - x)(y - z)(y - 1)} + \frac{z \ln z}{(z - x)(z - y)(z - 1)} \right],
\]

\[
G'(x, y, z) = \left[ \frac{x^2 \ln x}{(x - y)(x - z)(x - 1)} + \frac{y^2 \ln y}{(y - x)(y - z)(y - 1)} + \frac{z^2 \ln z}{(z - x)(z - y)(z - 1)} \right].
\]

### A. Vertices

Following the notation of \[39,40\], we denote by \(N_{ij}\) by \(\chi_i^0 \equiv N_{ij} \tilde{\Psi}_j\) where \(\tilde{\Psi}^T = (\tilde{B}, \tilde{W}_3, \tilde{h}_u, \tilde{h}_d, \tilde{s})\), \(U\) and \(V\) are the \(2 \times 2\) matrices required for the diagonalization of the chargino mass matrix.

\[
\begin{align*}
\chi_i^+ & \to \nu_\tau & \simeq -igU_{i1}^* \tau_3^{\dagger} k_{\tau}^{\dagger} (1 + \gamma_5) \\
\chi_i^+ & \to \tilde{\tau}_k & \simeq -igU_{i1}^* \tau_3^{\dagger} k_{\tau}^{\dagger} (1 - \gamma_5) \\
\chi_i^+ & \to \nu_{\mu, u} & \simeq -igU_{i1}^* \tau_3^{\dagger} k_{\mu}^{\dagger} (1 + \gamma_5) \\
\chi_i^+ & \to \tilde{\mu}_L, \tilde{d}_L & \simeq -igU_{i1}^* \tau_3^{\dagger} k_{\mu}^{\dagger} (1 - \gamma_5) \\
\chi_i^+ & \to \nu_{\mu, u} & \simeq igm_{\tilde{\mu}_R, \tilde{d}_R} \sqrt{2M_W \cos \beta} (1 + \gamma_5) \\
\chi_i^+ & \to \tilde{\mu}_R, \tilde{d}_R & \simeq igm_{\tilde{\mu}_R, \tilde{d}_R} \sqrt{2M_W \cos \beta} (1 - \gamma_5)
\end{align*}
\]

**FIG. 1**: R-parity conserved vertices involving charginos and up fermions.
We see in Fig.1 and 2 that vertices between chargino $\chi_i^+$, a fermion $f$ and the right scalar partner $\tilde{f}_R$ are negligible w.r.t. vertices between chargino $\chi_i^+$, a fermion $f$ and the left scalar partner $\tilde{f}_L$ because $m_f/M_W \ll 1$ for $f \equiv \mu, u, d$. We will neglect the loops including the primary vertices below.

The coupling of one neutralino $\chi_i^0$ to one fermion and the associated left (right) scalar partner $G_{fL(R)}$ is:

$$G_{fL} = Q_f \sin \theta_W N_{j1}^* + \frac{e_f^L}{\cos \theta_W} N_{j2}^*$$

where $f \equiv \nu, e, u, d$.

$$G_{fR} = \text{sign}(m_{\chi_i^0}) \left[ Q_f \sin \theta_W N_{j1} + \frac{e_f^R}{\cos \theta_W} N_{j2} \right]$$

where $f \equiv e, u, d$.

The coupling of left (right) scalar partners to the $Z^0$-boson is:

$$e_{fL(R)}^L = T_3(f_{L(R)}) - Q_f \sin^2 \theta_W.$$

The coupling of scalar partners in their mass basis to the $Z^0$-boson \([11]\) is:

$$e_{fM}^{kL} = T_3(f) \sum_{i=1}^3 \Gamma_{fL_i}^{kL} \Gamma_{fL}^{iL} - Q_f \sin^2 \theta_W \delta^{kh}$$

where $\hat{\tau}_L = \Gamma_{fL}^{kL} \hat{\tau}_k$. The coupling of charginos to the $Z^0$-boson \([11, 32]\), is:

$$\mathcal{O}_{ij}^{L} = -V_{i1} V_{j1}^* - \frac{1}{2} V_{i2} V_{j2}^* + \delta_{ij} \sin^2 \theta_W$$

and

$$\mathcal{O}_{ij}^{R} = -U_{i1} U_{j1}^* - \frac{1}{2} U_{i2} U_{j2}^* + \delta_{ij} \sin^2 \theta_W.$$
FIG. 4: R-parity conserved vertices with $Z^0$-boson.

### B. Self-energy diagrams

The computation of the supersymmetric contribution to $\Delta \rho$ requires the evaluation of the Feynman diagrams. The self-energy, penguin and box contributions to $\Delta \rho^f$ can be written as

$$\Delta \rho^f = \Delta \rho_p + \frac{\Delta \rho_{box}}{T_3(f_L)}.$$  

We consider here the contributions to neutrino and antineutrino scattering involving self-energy diagrams.

The first contribution implies a slepton-chargino loop (Fig. 5). We diagonalize the stau mass matrix and use the vertices of Fig.(1). We neglect the $\tilde{\mu}_R$ contribution.

$$\Delta \rho_{\tilde{\nu}}(\Sigma) = -\frac{\alpha}{4\pi} \sum_{j=1}^2 U_{\nu j}^2 \left[ \sum_{k=1}^2 \frac{\Gamma^3_{\tau L}}{\gamma^0} \left\{ G_0(X_{\chi^+_j \tau_L}, 1) + \ln \frac{m^2_{\tau_L}}{\mu^2} \right\} - \left\{ G_0(X_{\chi^+_j \bar{\mu}_L}, 1) + \ln \frac{m^2_{\bar{\mu}_L}}{\mu^2} \right\} \right].$$  

(13)

where $X_{ab} = \frac{m^2_{ab}}{m^2_{\chi^0}}$. Then, we have a neutralino-sneutrino loop (Fig. 6).

$$\Delta \rho_{\tilde{\nu}}(\Sigma) = -\frac{\alpha \Gamma}{4\pi} \sum_{j=1}^2 C^2_{\nu j} \left\{ G_0(X_{\chi^0_{\nu j} \bar{\nu}_L}, 1) + \ln \frac{m^2_{\bar{\nu}_L}}{\mu^2} - (\bar{\nu}_\tau L \rightarrow \bar{\nu}_{\mu L}) \right\}. $$  

(14)

### C. Penguin diagrams

We consider here the contributions to (anti)neutrino scattering involving penguins. We use the vertices of Fig.(4).

$\Delta \rho_p(\tilde{\ell})$ implies a chargino-slepton loop in which the slepton couples to the $Z^0$-boson (Fig. 5). In $\Delta \rho_p(\tilde{\nu})$, the chargino couples to the $Z^0$-boson (Fig. 6). $\Delta \rho_p^L(\tilde{\ell})$ implies a neutralino-sneutrino loop in which the sneutrino couples to the $Z^0$-boson (Fig. 6) and in $\Delta \rho_p^L(\tilde{\chi}^0)$ the neutralino couples to $Z^0$-boson (Fig. 6).
\[ \Delta \rho_p(\tilde{\ell}) = \frac{\alpha W}{4\pi} \sum_{i=1}^{2} U_{i1}^2 \left[ \sum_{j,k=1}^{2} \Gamma_{\tau L \tau L}^{j3} C_{j3}^{k} \left\{ G_0(X_{\tau_j \chi_i^+}^+, X_{\tilde{\tau}_k \chi_i^+}^+) + \ln \frac{m_{\tilde{\tau}_k}^2}{\mu^2} \right\} \right. \]

\[ \left. - c_{\ell L}^j \left\{ G_0(X_{\tau_j \tilde{\mu}_L}, 1) + \ln \frac{m_{\tilde{\mu}_L}^2}{\mu^2} \right\} \right], \quad (15) \]

\[ \Delta \rho_p^L(\nu) = \frac{\alpha W}{4\pi} \sum_{j=1}^{5} G_{\nu L}^j 2 \left\{ G_0(X_{\nu_L^j \tilde{\nu}_L^j}, 1) + \ln \frac{m_{\tilde{\tau}_k}^2}{\mu^2} \right\} - \left( \nu_{\tau L} \rightarrow \tilde{\nu}_{\mu L} \right) \right), \quad (16) \]

\[ \Delta \rho_p(\chi^+) = \frac{\alpha W}{4\pi} \sum_{j,j=1}^{2} U_{i1} U_{j1} \times \left\{ \sum_{k=1}^{2} \Gamma_{\tau L \tau L}^{k3} 2 \left\{ 2 \mathcal{O}_{ij}^{L} H_0(X_{\tau_j \tilde{\tau}_k}, X_{\tilde{\mu}_L \tilde{\mu}_L}) - \mathcal{O}_{ij}^{R} \left( G_0(X_{\tau_j \tilde{\tau}_k}, X_{\tilde{\mu}_L \tilde{\mu}_L}) + \ln \frac{m_{\tilde{\tau}_k}^2}{\mu^2} \right) \right\} \right. \]

\[ \left. - \left( 2 \mathcal{O}_{ij}^{L} H_0(X_{\tau_j \tilde{\tau}_k}, X_{\tilde{\nu}_L \tilde{\nu}_L}) - \mathcal{O}_{ij}^{R} \left( G_0(X_{\tau_j \tilde{\tau}_k}, X_{\tilde{\nu}_L \tilde{\nu}_L}) + \ln \frac{m_{\tilde{\tau}_k}^2}{\mu^2} \right) \right) \right], \quad (17) \]
\[ \Delta \rho_p^L(\chi^0) = \frac{2\alpha_W}{\pi} \sum_{i,j=1}^2 \frac{G^i_{\nu L} G^j_{\nu L}}{m_W^2} \left( N_{i4} N_{j4} - N_{i3} N_{j3} \right) \times \left[ 2H_0(X_{\chi^0 j}^{\nu L}, X_{\chi^0 j}^{\nu L}) + \left\{ G_0(X_{\chi^0 j}^{\nu L}, X_{\chi^0 j}^{\nu L}) + \ln \frac{m^2_{\nu L}}{m^2_{\nu L}} \right\} \right] - \left( \tilde{\nu}_{\tau L} \rightarrow \tilde{\nu}_{\mu L} \right). \]  

We have neglected the loops involving \( \tilde{\mu}_R \).

When we sum all the contributions (Fig.5, 6, 7) involving the charged Higgs boson \( \tilde{\tau}_R \), we find:

\[ \Delta \rho_p^{H^+} \simeq -\frac{\alpha_W}{4\pi} \frac{m_W^2}{M_W^2} g^2 \beta y \left[ \frac{1}{1 - y} + \frac{\ln y}{(1 - y)^2} \right]. \]

D. Box diagrams

The charge conjugation operators in the vertices of Fig.2 imply that all box diagrams involving charginos induce radiative corrections to neutrino scattering only, despite the fact that the neutrino fermionic lines of Fig.7 and Fig.17 are oriented on the left. As we did before, we diagonalize the tau mass matrix and we neglect the \( \tilde{\mu}_R \) contribution. The box diagrams involving charginos (resp. Fig.7, 17, 17) are:

\[ \Delta \rho_{\text{box}}^{\nu}(\chi^+) = \Delta \rho_{\text{box}}^{\mu}(\chi^+) \left( \tilde{\nu}_e \rightarrow \tilde{\nu}_L \right), \]

\[ \Delta \rho_{\text{box}}^{\mu}(\chi^+) = \Delta \rho_{\text{box}}^{\nu}(\chi^+) \left( \tilde{\nu}_{\tau} \rightarrow \tilde{\nu}_{\mu} \right), \]

The box diagrams involving neutralinos with crossed fermionic lines will only contribute to neutrino scattering because the neutrino fermionic lines are oriented on the right. Here, \( \tilde{f}_R \) contributions are non-negligible.

We consider here the box diagrams involving neutralinos for neutrino scattering (Fig.7):

\[ \Delta \rho_{\text{box}}^{\tilde{f}_L}(\chi^0) = -\frac{\alpha_W}{2\pi} \sum_{j,k=1}^5 \frac{G^j_{\nu L} G^k_{\nu L}}{m_{\tilde{f}_L}^2} G_{f_L}^j G^k_{f_L} \left[ G^j(\tilde{\nu}_{\tau}, \tilde{\nu}_{\mu}, \tilde{\nu}_{\mu}, \tilde{\nu}_{\mu}) \right] \]

FIG. 7: R-parity conserved box diagrams.
with right sfermions ($\tilde{e}_R, \tilde{u}_R, \tilde{d}_R$):

$$\Delta \rho_{\text{box}}^{\tilde{f}_R}(\chi^0) = \frac{\alpha_W}{\pi} \sum_{j,k=1}^{5} G^k_{\nu L} G^j_{\nu L} G^k_{f_R} \frac{M^2_{\tilde{f}_R}}{m^2_{f_R}} \left[ H'(X_{\chi^0_j \tilde{f}_R}, X_{\chi^0_k \tilde{f}_R}, X_{\nu_\tau \tilde{f}_R}) - (\tilde{\nu}_\tau \to \tilde{\nu}_\mu) \right]$$

Note that the contributions from all box diagrams involving antineutrino scattering are identical to the previous contributions, only the forms of the boxes will be different. For instance if a ladder box was contributing to antineutrinos then the corresponding box for neutrinos will have crossed fermionic lines and vice-versa.

### E. Numerical results

We make here a low-energy study where we take first generation sleptons degenerate with the second generation ones, and only allow the third generation sleptons to have a different mass. It is a possibility that sfermion masses may dynamically align along the directions, in flavour space, of the fermion masses, suppressing FCNC but allowing large mass splittings [43].

![FIG. 8: $\varepsilon$ as a function of $\mu$. We fix $\lambda = 0.4$ and $\kappa = 0.5$. $M_1 = 66\text{GeV}$, $M_2 = 133\text{GeV}$ and $M_3 = 500\text{GeV}$. The figure on the left represents the normal supersymmetric hierarchy ($m_{\tilde{\tau}} = 300\text{GeV}$ and $m_{\tilde{\mu}} = 200\text{GeV}$), the figure on the right represents the inverted supersymmetric hierarchy ($m_{\tilde{\tau}} = 200\text{GeV}$ and $m_{\tilde{\mu}} = 300\text{GeV}$).](image)

We consider two experimentally allowed cases for the sleptons with a splitting between the third and the second generation:

- $m_{\tilde{\tau}} = 300\text{GeV}$ and $m_{\tilde{\mu}} = 200\text{GeV}$ (normal supersymmetric hierarchy)
- $m_{\tilde{\tau}} = 200\text{GeV}$ and $m_{\tilde{\mu}} = 300\text{GeV}$ (inverted supersymmetric hierarchy)

We also assume that squarks are much heavier than sleptons: $M_{\tilde{Q}} = 1\text{TeV}$. These are the effects of gluino masses in the renormalization group evolution of scalar masses.

The splitting among the sleptons of the second and third generations is mainly responsible for the size of $\Delta \rho$.

The purpose of this Section is the investigation of the supersymmetric parameter space in some specific cases. To this end we made a subroutine to the Fortran code NMHDECAY, which is available on the NMSSMTools web page [40], [44], [45]. This subroutine computes the different R-parity conserved supersymmetric contributions to $\varepsilon$. Note that $\varepsilon$ is the same for antineutrinos, as in the SM.

By making scans over the supersymmetric parameter space, we can obtain, in some regions of the parameter space, divergences for either $m_{\chi^+_{j}} = m_{\tilde{\tau}_2}$, $m_{\chi^0_{j}} = m_{\tilde{\mu}_L}$ or $m_{\chi^0_{i}} = m_{\tilde{\nu}_\tau}$. This is because we assume vanishing external legs.

In supersymmetry, there are many parameters. $\varepsilon$ doesn’t depend very much on $\lambda$ and $\kappa$ so we fix them as we usually do in supersymmetry: $\lambda = 0.4$ and $\kappa = 0.5$. We allow some other parameters to vary: $\tan \beta$, $M_A$ and $\mu_{\text{eff}}$.

$\tan \beta$ is the ratio $v_u/v_d$ where $v_u$ and $v_d$ are the v.e.v. of the scalar Higgs fields $H_u$ and $H_d$. $M_A$ is an effective supersymmetric parameter somewhat equivalent to the second pseudoscalar Higgs mass in our regions of the parameter space.
We show two different illustrative situations motivated by high-energy models for the gaugino masses:

- \( M_1 = 66 \text{GeV}, \ M_2 = 133 \text{GeV} \text{ and } M_3 = 500 \text{GeV} \) (cf Fig.(8))
- \( M_1 = 150 \text{GeV}, \ M_2 = 300 \text{GeV} \text{ and } M_3 = 1 \text{TeV} \) (cf Fig.(9))

In Fig.(8), \( \tan \beta \) varies between 2 and 15, \( M_A \) varies between 579 and 2000 GeV, \( \mu \) varies between 300 and 800 GeV. We see in Fig.(8) that contrary to the SM case, \( V_{\mu \tau} \) can be either positive or negative.

In Fig.(9), \( \tan \beta \) varies between 2 and 18 (for the figure on the left) and between 2 and 9.6 (for the figure on the right), \( M_A \) varies between 500 and 1000 GeV, \( \mu \) varies between 200 and 564 GeV.

On the other hand, as we can see in the Fig.(9), \( \varepsilon \) can go up to \( 2 \times 10^{-2} \) in this region of the parameter space. The sign of \( \mu \) does not have an important impact on the maximal value that \( \varepsilon \) can reach.

We finally mention that other extensions of the SM may also lead to sizable effects upon \( \Delta n_{\tau \mu} \). In the next section, we will consider supersymmetric \( R \)-parity violating interactions which can have important effects on the neutrino indices of refraction already at the tree-level [46].

**IV. R-PARITY BREAKING SUSY CORRECTIONS**

**A. Introduction**

The superpotential \( W_{\text{NMSSM}} \) is not the most general superpotential we want to write because there exist some other gauge invariant couplings that we didn’t take into account. These new couplings break a discrete symmetry called R-parity. This symmetry requires that an interaction must have an even number of SUSY particles. If R-parity is broken, the LSP will no longer be stable [47]. R-parity violating interactions also violate lepton number \( (L) \) or baryon number \( (B) \). To be as general as possible, the superpotential has to contain the following terms:

\[
W_{R_P} = \sum_{i,j,k} \left( \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k^c + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k^c + \frac{1}{2} \lambda'_{ij k} \epsilon_{\alpha \beta \gamma} \hat{U}_i^{c \alpha} \hat{D}_j^{c \beta} \hat{D}_k^{c \gamma} + \mu_i \hat{H}_u \hat{L}_i + \lambda_i \hat{S} \hat{H}_u \hat{L}_i \right)
\]

(24)

The lagrangian \( L_{R_P} \) can be derived using the common procedure [37, 48].

If we consider this part of the superpotential:

\[
W_{\text{masses}} = W_{\text{NMSSM}} + \mu_i \hat{H}_u \hat{L}_i + \lambda_i \hat{S} \hat{H}_u \hat{L}_i
\]

(25)

we can generate two neutrino masses at tree-level [37] by mixing neutrinos and neutralinos. By considering a See-Saw-like mechanism, the mass matrix is:
\[ M_{\tilde{\chi}^0} = \begin{pmatrix} M_{NMSSM} & \xi_{R_P}^T \\ \xi_{R_P} & 0_{3\times3} \end{pmatrix} \]

where \( M_{NMSSM} \) is the R-parity conserved neutralino mass matrix and \( \xi_{R_P} \) is the part of the mass matrix induced by R-parity violation which mix neutralinos and neutrinos. We will assume here: \( v_i/v_{u,d} \ll 1, |\mu_i/\mu| \ll 1 \) and \( |\lambda_{i}/\lambda| \ll 1 \) in order to reproduce the neutrino phenomenology. \( v_i \) are the v.e.v. of the sneutrinos.

This model is self-consistent because we give here a way to generate the neutrino masses contrary to many other scenarios of radiative corrections on neutrino indices of refraction.

In the following, \( \nu_i = N_{ij} \tilde{\psi}_j \) where \( \tilde{\psi} = (\tilde{B}, \tilde{W}_3, \tilde{h}_u, \tilde{h}_d, \tilde{s}, \nu_e, \nu_\mu, \nu_\tau) \), \( i = 1,3 \) and \( j = 1,8 \). \( \nu_e, \nu_\mu \) and \( \nu_\tau \) represent the neutrinos in their flavour basis, \( \nu_i \) represent the neutrinos in their mass basis. \( \chi^0_i = N_{(i+3)j} \tilde{\psi}_j \) where \( i = 1..5 \) and \( j = 1..8 \). \( \chi^0_i \) are the five NMSSM’s neutralinos.

### B. Tree-level corrections

Because of R-parity breaking, new interactions are possible between SUSY particles. This yields new one-loop corrections but also tree level corrections contrary to the R-parity conserved case. Such tree level corrections in the Supernova context have been partially studied in \cite{46, 49}. Since this work focuses on \( V_{\mu\tau} \), we only concentrate on graphs involving \( \mu \) and \( \tau \) neutrinos. Moreover, we only consider neutrinos which do not change of flavour after the R-parity breaking interaction, therefore we are not concerned by off-diagonal term in the matter Hamiltonian.

![FIG. 10: R-parity broken Tree-level corrections.](image)

Here, we give a specific contribution to antineutrino scattering (Fig.10a) with left sleptons \( \tilde{\ell}_{L,i} \):

\[ \Delta \rho_a(\tilde{\ell}_{L,i}) = -\frac{3}{g^2} \sum_{i=1}^{3} \left( \frac{\lambda_{31}^2 - \lambda_{21}^2}{M_{W}^2} \right) \frac{M_{W}^2}{m_{\tilde{\ell}_{L,i}}^2} \]  

(27)

With left down squarks \( \tilde{d}_{L,i} \) (Fig.10b):

\[ \Delta \rho_b(\tilde{d}_{L,i}) = \Delta \rho_a(\tilde{\ell}_{L,i}) \left( \lambda_{k11} \rightarrow \lambda_{k11} \right) \]

(28)

The right sleptons \( \tilde{\ell}_{R,i} \) and the right down squarks \( \tilde{d}_{R,i} \) do not contribute.

The following contributions induce corrections to both neutrino and antineutrino scattering:
for diagram (Fig.10c):

$$\Delta \rho_c(\chi^0) = - \sum_{j,k=1}^5 \left( N_{(j+3)8} N_{(i+3)8} - N_{(j+3)7} N_{(i+3)7} \right) \left( N_{(j+3)4} N_{(i+3)4} - N_{(j+3)3} N_{(i+3)3} \right)$$

(29)

for diagram (Fig.10d):

$$\Delta \rho_d(\tilde{\ell}_{L,i}) = \Delta \rho_a(\tilde{\ell}_{L,i})$$

(30)

for diagram (Fig.10e):

$$\Delta \rho_e(\tilde{d}_{L,i}) = \Delta \rho_b(\tilde{d}_{L,i})$$

(31)

For (Fig.10d) and (Fig.10e), the right sleptons $\tilde{\ell}_{R,i}$ and the right down squarks $\tilde{d}_{R,i}$ do not contribute.

C. Self-energy corrections

We willingly leave certain factors not simplified to be able to compare with other corrections more rapidly. All the contributions will be equal for neutrino and antineutrino scattering.

For Fig.11:

$$\Delta \rho(\Sigma) = - \frac{\alpha_W}{8\pi} \sum_{i,j=1}^3 \left( \lambda_{2,ij}^2 - \lambda_{3,ij}^2 \right) \left\{ G_0(X_{\ell_i,\ell_{R,j}}, 1) + \ln \frac{m_{\tilde{\ell}_{R,j}}^2}{\mu^2} \right\}$$

(32)

For Fig.11:

$$\Delta \rho(\Sigma) = \Delta \rho(\Sigma) \left( \lambda_{kij} \rightarrow \lambda'_{kji}, \tilde{\ell}_{R,j} \rightarrow \tilde{d}_{R,j} \right)$$

(33)
In the R-parity breaking scenario, there are NMSSM specific self-energies due to the term $\lambda_i \hat{S} \hat{H}_u \hat{L}_i$ in the superpotential. This term induce one new coupling between a neutrino, one Higgs (scalar or pseudoscalar) and a neutralino. We present here the contributions for scalar Higgses and for pseudoscalar Higgses.

For both Fig.(11g) and (11i), the contribution from scalar higgses:

$$\Delta \rho^h(\Sigma) = -\frac{\alpha_W}{8\pi} \frac{\lambda^2}{g^2} \sum_{i=1}^{3} \sum_{j=1}^{3} \left( N_{(i+3)3}^2 S^2_{j3} + N_{(i+3)3}^2 S^2_{j3} \right) \left[ G_0(X_{\chi^0 h_j}, 1) + \ln \frac{m^2_{h_j}}{\mu^2} \right]$$

from pseudoscalar higgses:

$$\Delta \rho^p(\Sigma) = -\frac{\alpha_W}{8\pi} \frac{\lambda^2}{g^2} \sum_{i=1}^{3} \sum_{j=1}^{2} \left( N_{(i+3)3}^2 P^2_{j1} + N_{(i+3)3}^2 P^2_{j3} \right) \left[ G_0(X_{\chi^0 a_j}, 1) + \ln \frac{m^2_{a_j}}{\mu^2} \right]$$

D. penguin type diagrams

![Diagram of R-parity broken penguin type diagrams](image)

FIG. 12: R-parity broken penguin type diagrams.

We consider here the penguin diagrams. All the contributions will be equal for neutrino and antineutrino scattering.

For the diagram of Fig.(13) and Fig.(13a):

$$\Delta \rho_{ij}^\nu (\ell) = -\frac{\alpha_W}{16\pi} \sum_{i,j=1}^{3} \frac{\lambda^2_{2ij} - \lambda^2_{3ij}}{g^2} \left[ G_0(\tilde{x}_{L_{ij}}, \ell, 1) + \ln \left( \frac{m^2_{\ell}}{\mu^2} \right) \right] ,$$

$$\Delta \rho_{ij}^d (d) = \Delta \rho_{ij}^\nu (\ell) (\lambda_{kji} \rightarrow \lambda_{kji}^\nu, \tilde{x}_{L_{ij}} \rightarrow \tilde{x}_{L_{ij}}, \ell_i \rightarrow d_i)$$

For the diagram of Fig.(13b) and Fig.(13c):

$$\Delta \rho_{ij}^\nu (\tilde{x}_L) = -\frac{\alpha_W}{4\pi} \sum_{i,j=1}^{3} \frac{\lambda^2_{2ij} - \lambda^2_{3ij}}{g^2} \left[ G_0(\tilde{x}_{L_{ij}}, \tilde{x}_L, 1) + \ln \left( \frac{m^2_{\ell_{ij}}}{\mu^2} \right) \right]$$

$$\Delta \rho_{ij}^d (\tilde{x}_L) = \Delta \rho_{ij}^\nu (\tilde{x}_L) (\lambda_{kji} \rightarrow \lambda_{kji}, \tilde{x}_{L_{ij}} \rightarrow \tilde{x}_{L_{ij}}, \ell_i \rightarrow d_i)$$

For all the previous penguin diagrams, the right sleptons $\tilde{\ell}_{R,j}$ and the right squarks $\tilde{d}_{R,j}$ do not contribute.

We have below NMSSM specific penguins also due to the term $\lambda_i \hat{S} \hat{H}_u \hat{L}_i$ in the superpotential.

For the diagram of Fig.(13d):

$$\Delta \rho^h(\chi^0) = -\frac{\alpha_W}{8\pi} \frac{\lambda^2_{2} - \lambda^2_{3}}{g^2} \sum_{j=1}^{3} \sum_{m=1}^{3} N_{(i+3)3}^2 N_{(j+3)3}^2 S^2_{m1} \left( N_{(i+3)4} N_{(j+3)4} - N_{(j+3)3} N_{(i+3)3} \right) \times \left[ 2H_0(X_{\chi^0 h_m}, X_{\chi^0 h_m}) + \left\{ G_0(X_{\chi^0 h_m}, X_{\chi^0 h_m}) + \ln \frac{m^2_{m}}{\mu^2} \right\} \right]$$

(38)
For the diagram of Fig. (13):
\[
\Delta \rho_p^\nu (\chi^0) = - \frac{\alpha_W}{8\pi} \left( \frac{\lambda_2^2 - \lambda_3^2}{g^2} \right) \sum_{i,j=1}^{5} \sum_{m,n=1}^{2} N_{(i+3)5} N_{(j+3)5} P_{m1}^2 \left( N_{(j+3)4} N_{i+3)4} - N_{(j+3)3} N_{(i+3)3} \right) \times \left[ 2H_0(X_{\chi^0_i a_n}, X_{\chi^0_j a_n}) + \left\{ G_0(X_{\chi^0_i a_n}, X_{\chi^0_j a_n}) + \ln \frac{m^2}{\mu^2} \right\} \right],
\]
(39)

We have two identical contributions from the diagram of Fig. (13) and Fig. (13) and we obtain:
\[
\Delta \rho_p^\nu = - \frac{\alpha_W}{4\pi} \left( \frac{\lambda_2^2 - \lambda_3^2}{g^2} \right) \sum_{i=1}^{5} \sum_{m=1}^{3} \sum_{n=1}^{2} N_{(i+3)5} S_{m1} P_{m1} A_{M}^{m1} \left\{ G_0(X_{h_m a_i}, X_{a_n a_i}) + \ln \frac{m^2}{\mu^2} \right\}
\]
(40)
where \( A_{M}^{m1} = S_{m1} P_{m1} - S_{m2} P_{m2} \) following the notation of [41].

E. Box diagrams

1. R-parity broken box diagrams

We present in this section the box diagrams corrections. Each box diagram correction involves Fierz transformations to obtain a similar form as the SM tree level interaction. The box diagrams with crossed fermions lines (Fig.14).
Fig. 14.2, Fig. 14.2, Fig. 14.2, Fig. 14.2, Fig. 14.2) will only contribute to neutrinos, meanwhile the uncrossed fermion lines (Fig. 14.1, Fig. 14.1, Fig. 14.1, Fig. 14.1, Fig. 14.1) will contribute to antineutrinos. However, the contributions to antineutrino scattering will be exactly the same as the contributions to neutrino scattering. We show below the contributions to neutrino scattering.

For the diagram of Fig. 14.2 with \( \tilde{\ell}_{L,i} \rightarrow d_{R,m} \):

\[
\Delta \rho_{\text{box}}^{\nu^2}(\tilde{\ell}_L - \tilde{\ell}_R) = \frac{\alpha_W}{8\pi} \sum_{i,j,k,m=1}^{3} \frac{M_W^2}{m_{\tilde{d}_{R,m}}} \frac{(\lambda_{3ij}\lambda_{3ki} - \lambda_{2ij}\lambda_{2ki})\lambda_{j1m}\lambda_{k1m}^*}{g^4} G'(X_{\ell,j,d_{R,m}}, X_{\ell,k,d_{R,m}}, X_{\ell_{L,i},d_{R,m}}).
\]

(41)

For the diagram of Fig. 14.2 with \( \tilde{d}_{L,i} \rightarrow \tilde{\ell}_L,m \) and the diagram of Fig. 14.2 with \( \tilde{d}_{L,i} \rightarrow \tilde{u}_L,m \):

\[
\Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_L - \tilde{\ell}_L) = \Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_L - \tilde{d}_R) (\lambda \rightarrow \lambda', \lambda_{j1m}' \lambda_{k1m}' \rightarrow \lambda_{j1m}' \lambda_{k1m}', l_{j,k} \rightarrow d_{j,k}, \tilde{d}_{R,m} \rightarrow \tilde{\ell}_L,m),
\]

(42)

\[
\Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_L - \tilde{u}_L) = \Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_L - \tilde{\ell}_L) (\lambda_{m1j}' \lambda_{m1k}' \rightarrow \lambda_{m1j}' \lambda_{m1k}' \tilde{\ell}_L,m \rightarrow \tilde{u}_L,m).
\]

For the diagram of Fig. 14.2 with \( \tilde{d}_{R,i} \rightarrow \tilde{\ell}_L,m \) and the diagram of Fig. 14.2 with \( \tilde{d}_{R,i} \rightarrow \tilde{u}_L,m \):

\[
\Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_R - \tilde{\ell}_L) = \frac{\alpha_W}{4\pi} \sum_{i,j,k,m=1}^{3} \frac{M_W^2}{m_{\tilde{d}_{L,m}}} \frac{(\lambda_{3ij}^* \lambda_{3ki}^* - \lambda_{2ij}^* \lambda_{2ki}^*)\lambda_{j1m}' \lambda_{k1m}''}{g^4} H'(X_{\ell_j,d_{L,m}}, X_{\ell_k,d_{L,m}}, X_{\tilde{d}_{R,i},\tilde{\ell}_L,m}),
\]

(43)

\[
\Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_R - \tilde{u}_L) = \Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_R - \tilde{\ell}_L) (\lambda_{m1j}' \lambda_{m1k}' \rightarrow \lambda_{m1j}' \lambda_{m1k}' \tilde{\ell}_L,m \rightarrow \tilde{u}_L,m).
\]

For the diagram of Fig. 14.2 with \( \tilde{\ell}_{R,i} \rightarrow \tilde{d}_{L,m} \) and for the diagram of Fig. 14.2 with \( \tilde{\ell}_{R,i} \rightarrow \tilde{u}_{L,m} \):

\[
\Delta \rho_{\text{box}}^{\nu^2}(\tilde{\ell}_R - \tilde{\ell}_L) = \frac{\alpha_W}{4\pi} \sum_{i,j,k,m=1}^{3} \frac{M_W^2}{m_{\tilde{d}_{L,m}}} \frac{(\lambda_{ij} \lambda_{3ki} - \lambda_{ij} \lambda_{2ki})\lambda_{j1m}' \lambda_{k1m}''}{g^4} H'(X_{\ell_j,d_{L,m}}, X_{\ell_k,d_{L,m}}, X_{\tilde{d}_{R,i},\tilde{\ell}_L,m}),
\]

(44)

\[
\Delta \rho_{\text{box}}^{\nu^2}(\tilde{\ell}_R - \tilde{u}_L) = \Delta \rho_{\text{box}}^{\nu^2}(\tilde{\ell}_R - \tilde{\ell}_L) (\lambda_{m1j}' \lambda_{m1k}' \rightarrow \lambda_{m1j}' \lambda_{m1k}' \tilde{\ell}_L,m \rightarrow \tilde{u}_L,m).
\]

For the diagram of Fig. 14.2 with \( \tilde{d}_{R,i} \rightarrow \tilde{\ell}_{L,m} \) and for the diagram of Fig. 14.2 with \( \tilde{d}_{R,i} \rightarrow \tilde{u}_{L,m} \):

\[
\Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_R - \tilde{\ell}_L) = \frac{\alpha_W}{4\pi} \sum_{i,j,k,m=1}^{3} \frac{M_W^2}{m_{\tilde{d}_{R,m}}} \frac{(\lambda_{3ij} \lambda_{3ki} - \lambda_{2ij} \lambda_{2ki})\lambda_{j1m}'' \lambda_{k1m}'}{g^4} H'(X_{\ell_j,d_{R,m}}, X_{\ell_k,d_{R,m}}, X_{\tilde{d}_{R,i},\tilde{\ell}_L,m}),
\]

(45)

\[
\Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_R - \tilde{u}_L) = \Delta \rho_{\text{box}}^{\nu^2}(\tilde{d}_R - \tilde{\ell}_L) (\lambda_{j1m}' \lambda_{k1m}'' \rightarrow \lambda_{j1m}' \lambda_{k1m}'' \tilde{\ell}_L,m \rightarrow \tilde{u}_L,m).
\]

For the diagram of Fig. 14.2 with \( \tilde{\ell}_{L,i} \rightarrow \tilde{d}_{R,m} \):

\[
\Delta \rho_{\text{box,corr}}^{\nu^2}(\tilde{d}_L - \tilde{d}_R) = \frac{\alpha_W}{8\pi} \sum_{i,j,k,m=1}^{3} \frac{M_W^2}{m_{\tilde{d}_{R,m}}} \frac{(\lambda_{3ij} \lambda_{3ki} - \lambda_{2ij} \lambda_{2ki})\lambda_{j1m}' \lambda_{k1m}''}{g^4} \epsilon_{\sigma\gamma\delta} \delta_{\alpha\gamma\beta} H'(X_{d_{L}^\gamma,d_{R}^\alpha}, X_{d_{L}^\gamma,d_{R}^\alpha}, X_{d_{L}^\gamma,d_{R}^\alpha}).
\]

(46)

For the diagram of Fig. 14.2 with \( \tilde{d}_{R,i} \rightarrow \tilde{\ell}_{L,m} \):

\[
\Delta \rho_{\text{box,corr}}^{\nu^2}(\tilde{d}_R - \tilde{\ell}_L) = \frac{\alpha_W}{8\pi} \sum_{i,j,k,m=1}^{3} \frac{M_W^2}{m_{\tilde{d}_{R,m}}} \frac{(\lambda_{3ij} \lambda_{3ki} - \lambda_{2ij} \lambda_{2ki})\lambda_{j1m}' \lambda_{k1m}''}{g^4} \epsilon_{\delta\gamma\alpha} \epsilon_{\gamma\beta\alpha} H'(X_{d_{R}^\gamma,d_{L}^\alpha}, X_{d_{R}^\gamma,d_{L}^\alpha}, X_{d_{R}^\gamma,d_{L}^\alpha}).
\]

(47)
The indices of $\epsilon$ denote SU(3) color indices where $\epsilon$ is antisymmetric, $\epsilon_{\alpha\beta\gamma} = \epsilon_{\beta\gamma\alpha} = \epsilon_{\gamma\alpha\beta}$. For example, for the diagram of Fig. (14q2) with $d_{L,i} - \tilde{u}_{R,m}$, the down quark on the left carries the index $\beta$ and the down quark on the right carries the index $\sigma$:

$$\Delta \rho_{\text{box,}\beta\sigma}^{q^2}(d_L - \tilde{u}_R) = \frac{\alpha W}{16\pi} \sum_{i,j,k,m=1}^3 \frac{M_W^2}{m^2_{\tilde{u}_{R,m}}} \frac{(\lambda'_{3ij} \lambda_{3ik} - \lambda'_{2ij} \lambda_{2ik})\lambda_{m1j}\lambda_{m1k}}{g^4} \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha\beta\gamma}$$

(48)

$$\left[ H' (X_{d_j^\dagger,\tilde{u}_{R,m}}, X_{d_{L,i}}, X_{\tilde{u}_{R,m}}) - \frac{G'}{2} (X_{d_j^\dagger,\tilde{u}_{R,m}}, X_{d_{L,i}}, X_{\tilde{u}_{R,m}}) \right].$$

The diagram of Fig. (14q2) with $\tilde{d}_{L,i} - \tilde{\nu}_{L,m}$ can be deduced from the previous one by removing the $\epsilon$ factors and by taking a factor 4 and $\lambda' \rightarrow \lambda$, $\tilde{u}_R \rightarrow \tilde{\nu}_L$:

$$\Delta \rho_{\text{box}}^{q^2}(\tilde{d}_L - \tilde{\nu}_L) = \frac{\alpha W}{4\pi} \sum_{i,j,k,m=1}^3 \frac{M_W^2}{m^2_{\tilde{u}_{L,m}}} \frac{(\lambda'_{3ij} \lambda_{3ik} - \lambda'_{2ij} \lambda_{2ik})\lambda_{m1j}\lambda_{m1k}}{g^4}$$

(49)

$$\left[ H' (X_{d_j^\dagger,\tilde{u}_{L,m}}, X_{d_{L,i}}, X_{\tilde{u}_{L,m}}) - \frac{G'}{2} (X_{d_j^\dagger,\tilde{u}_{L,m}}, X_{d_{L,i}}, X_{\tilde{u}_{L,m}}) \right].$$

Finally, for the diagram of Fig. (14q2) with $\tilde{\ell}_{L,i} - \tilde{\nu}_{L,m}$:

$$\Delta \rho_{\text{box}}^{q^2}(\tilde{\ell}_L - \tilde{\nu}_L) = \Delta \rho_{\text{box}}^{q^2}(\tilde{d}_L - \tilde{\nu}_L) (\lambda' \rightarrow \lambda, d_{j,k} \rightarrow \ell_{j,k}, \tilde{d}_{L,i} \rightarrow \tilde{\ell}_{L,i}).$$

(50)

The rest of diagrams do not contribute because of a different form as the correct tree level SM form given in Eq. (2).

2. Cancelling diagrams

---

**FIG. 15:** R-parity broken radiative corrections which cancel out. The Feynman graphs with uncrossed scalar lines on the left cancel with the Feynman graphs with crossed scalar lines on the right.

It is interesting to notice that amongst the possible radiative corrections induced by the R-parity breaking interactions, some of them cancel. Contrary to the previous box diagrams, all box diagrams in Fig. 15 should contribute equally to neutrinos and anti-neutrinos because the fermionic lines do not link the neutrinos to the fermion present in the external leg. The particles exchanged between the two fermionic lines here are all scalar.
To understand the consequences, we write the tensorial part of the scattering amplitude of a ladder scalar line box diagram (Fig.15.1) and the corresponding crossed scalar line box diagram (Fig.15.2):

For the left diagram we have:

\[
M_1 = \bar{u}(k_1) \left[ \int \frac{dq}{(2\pi)^4 i} \lambda_{3k_1} \frac{1 + \gamma_5 i(q + m_{\ell_1})}{2} i\lambda_{3j_1} \frac{1 - \gamma_5}{2} u(p_1) \right] \frac{1}{(p_1 - q)^2 - m^2_{\ell_{L,j}}} i \frac{1}{(k_1 - q)^2 - m^2_{\ell_{L,k}}} \]

while for the right diagram we have:

\[
M_2 = \bar{u}(k_1) \left[ \int \frac{dq}{(2\pi)^4 i} \lambda_{3k_1} \frac{1 + \gamma_5 i(q + m_{\ell_1})}{2} i\lambda_{3j_1} \frac{1 - \gamma_5}{2} u(p_1) \right] \frac{1}{(p_1 - q)^2 - m^2_{\ell_{L,j}}} i \frac{1}{(k_1 - q)^2 - m^2_{\ell_{L,k}}} \]

Therefore , in the approximation of zero external legs \((p_1, p_2, k_1, k_2 = 0)\), we have \(M_1 = -M_2\). From a physical point of view, it is actually quite natural that such corrections cancel out when considering vanishing external legs, the Feynman box diagrams are anti-symmetric in this case since scalar particles are exchanged and the sign of the \(\nu_m\) neutrino impulse is opposite.

V. IMPLICATIONS ON THE SUPERNOVA NEUTRINO FLUXES

In the supernova environment the density is sufficiently high that neutrinos encounter not only the so called high resonance associated with \(\theta_{13}\) and the low resonance associated with \(\theta_{12}\) but also the \(\mu - \tau\) resonance whose precise conditions depend on the hierarchy \([50]\). The importance of such correction term has been investigated in \([23, 27]\) showing that such resonance could influence the electron neutrino flux. This radiative correction term may also influence the electron (anti-) neutrino in an indirect way. Indeed, \(V_{\mu\tau}\) breaks the symmetry between muon neutrinos and tau neutrinos and creates a CP-violation dependence for the electron (anti-) neutrino survival probability and consequently on the electron (anti-) neutrino flux. Such a phenomenon was numerically observed in \([51]\). The addition of the non-linear neutrino-neutrino interaction induces larger effects on the electron neutrino fluxes up to a level of 10\% inside the supernova \([29]\). In this section, we analytically demonstrate that the inclusion of \(V_{\mu\tau}\) (or more generally the inclusion of an interaction term in the total Hamiltonian breaking the symmetry between mu and tau neutrinos) will make the electron survival probability dependent upon the CP-violation phase \(\delta\). The fact that in the SUSY framework, \(V_{\mu\tau}\) can be up to a factor 2 × 10^{-2} \(V_e\) implies sizeable effects on the electron neutrino fluxes seen in a detector on Earth.

1. The factorization

In a dense environment the neutrino evolution equations with one-loop correction to matter interactions are given by:

\[
i \frac{\partial}{\partial t} \begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix} = \left[ U \begin{pmatrix} E_3 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_1 \end{pmatrix} U^\dagger + \begin{pmatrix} V_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V_{\mu\tau} \end{pmatrix} \right] \begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix}
\]

where \(\Psi_\alpha\) denotes a neutrino in a flavour state \(\alpha = e, \mu, \tau\), \(E_{i=1,2,3}\) being the energies of the neutrino mass eigenstates, and \(U\) the unitary Maki-Nakagawa-Sakata-Pontecorvo matrix

\[
U = T_{23}T_{23}T_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{12} & s_{12} \cos \theta_{12} \\ 0 & -s_{12} & c_{12} \cos \theta_{12} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \cos \theta_{13} \\ 0 & 1 & 0 \\ -s_{13} \sin \theta_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

where \(c_{ij} = \cos \theta_{ij}\) and \(s_{ij} = \sin \theta_{ij}\) with \(\theta_{12}, \theta_{23}\) and \(\theta_{13}\) the three neutrino mixing angles. The presence of a Dirac \(\delta\) phase in Eq.\([53]\) renders \(U\) complex and introduces a difference between matter and anti-matter. The neutrino
interaction with matter is taken into account through an effective Hamiltonian which corresponds, at tree level, to the diagonal matrix \( H_m = \text{diag}(V_e, V_\mu, V_\tau) \), where the \( V_e(x) = \sqrt{2G_F}N_e(x) \) potential, due to the charged-current interaction, depends upon the electron density \( N_e(x) \) (note that the neutral current interaction introduces an overall phase only). Following the derivation of [29, 51], to obtain explicit relations between probabilities and the CP-violating phase, it is convenient to work within a new basis where a rotation by \( T_{23} \) is performed. In this basis, one can factorize the \( S \) matrix, defined by \( \text{diag}(1, 1, e^{i\delta}) \), out of the Hamiltonian, so that:

\[
\frac{i}{\hbar} \frac{\partial}{\partial t} \begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix} = \hat{H}_T(\delta) \begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix} = S \hat{H}'_T(\delta) S^\dagger \begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix}
\]

Contrary to the case where only tree level matter interaction is considered, the \( S \) matrix does not commute with the matter Hamiltonian in the \( T_{23} \) basis. This fact implies that:

\[ \hat{H}_T(\delta) \neq S \hat{H}_T(\delta = 0) S^\dagger, \]

and, therefore, that

\[ \tilde{U}_m(\delta) \neq S \tilde{U}_m(\delta = 0) S^\dagger. \]

2. **Consequence on the electron neutrino survival probability**

Nevertheless, the factorization of the \( S \) matrices is always possible but the Hamiltonian \( \hat{H}'_T(\delta) \) will depend on \( \delta \). We can rewrite Eq. (55) in the evolution operator formalism to be:

\[
\frac{i}{\hbar} \frac{\partial}{\partial t} \begin{pmatrix} A_{ee} & A_{e\mu} & A_{e\tau} e^{i\delta} \\ A_{e\mu}^\dagger & A_{\mu\mu} & A_{\mu\tau} e^{i\delta} \\ A_{e\tau} e^{-i\delta} & A_{\mu\tau} e^{-i\delta} & A_{\tau\tau} \end{pmatrix} = \hat{H}'_T(\delta) \begin{pmatrix} A_{ee} & A_{e\mu} & A_{e\tau} e^{i\delta} \\ A_{e\mu}^\dagger & A_{\mu\mu} & A_{\mu\tau} e^{i\delta} \\ A_{e\tau} e^{-i\delta} & A_{\mu\tau} e^{-i\delta} & A_{\tau\tau} \end{pmatrix}
\]

Defining the Hamiltonian \( \hat{H}'_T(\delta) \) by\(^2\):

\[ \hat{H}'_T(\delta) = \begin{pmatrix} a & b & c \\ b & d & (e - g e^{i\delta}) \\ c & (e - g e^{i\delta}) & f \end{pmatrix} \]

we can rewrite the evolution equations for the amplitudes of the first column of the evolution operator, which corresponds to the creation of an electron neutrino \( \nu_e \) initially:

\[
\frac{id}{dt} A_{ee} = a A_{ee} + b A_{e\mu} + c A_{e\tau} e^{-i\delta}
\]

\[
\frac{id}{dt} A_{e\mu} = b A_{ee} + d A_{e\mu} + (e - g e^{i\delta}) A_{e\tau} e^{-i\delta}
\]

\[
\frac{id}{dt} A_{e\tau} e^{-i\delta} = c A_{ee} + (e - g e^{i\delta}) A_{e\mu} + f A_{e\tau} e^{-i\delta}
\]

Similarly, we can write the same equation for the amplitudes \( B_{\alpha\beta} \) when \( \delta \) is taken to be zero and look at the difference between the amplitudes that depend on \( \delta \) and those which do not. The basic idea here is to prove that, because of the

\(^2\) Note that the terms \( a, b, c, d, e, f \) and \( g \) are real.
one-loop correction term $V_{\mu\tau}$, the function $(A_{ee} - B_{ee})$ can not remain zero function by showing that its derivative is non zero.

\[
i \frac{d}{dt}(A_{ee} - B_{ee}) = a (A_{ee} - B_{ee}) + b (A_{\tilde{\mu}} - B_{\tilde{\mu}}) + c (A_{\tilde{\tau}} - B_{\tilde{\tau}})
\]

(61)

\[
i \frac{d}{dt}(A_{\tilde{\mu}} - B_{\tilde{\mu}}) = b (A_{ee} - B_{ee}) + d (A_{\tilde{\mu}} - B_{\tilde{\mu}}) + e (A_{\tilde{\tau}} e^{-i\delta} - B_{\tilde{\tau}}) + g (A_{ee} - B_{ee})
\]

\[
i \frac{d}{dt}(A_{\tilde{\tau}} e^{-i\delta} - B_{\tilde{\tau}}) = c (A_{ee} - B_{ee}) + e (A_{\tilde{\mu}} - B_{\tilde{\mu}}) + f (A_{\tilde{\tau}} e^{-i\delta} - B_{\tilde{\tau}}) - g (A_{\tilde{\mu}} e^{-i\delta} - B_{\tilde{\mu}})
\]

(62)

Let us now take a closer look at Eqs. (61). The initial condition we are interested in, namely a $\nu_e$ created initially means that initially the amplitudes $A$ and $B$ are:

\[
\begin{pmatrix}
\Psi_e \\
\Psi_\mu \\
\Psi_\tau
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}
\Rightarrow
\begin{pmatrix}
A_{ee} \\
A_{\tilde{\mu}} \\
A_{\tilde{\tau}} e^{-i\delta}
\end{pmatrix}
= 
\begin{pmatrix}
B_{ee} \\
B_{\tilde{\mu}} \\
0
\end{pmatrix}
\]

(63)

When $g = 0$ ($V_{\mu\tau} = 0$), it is easy to see that inserting the initial conditions into Eqs. (61) will imply that the functions $f_e = A_{ee} - B_{ee}$, $f_\mu = A_{\tilde{\mu}} - B_{\tilde{\mu}}$ and $f_\tau = A_{\tilde{\tau}} e^{-i\delta} - B_{\tilde{\tau}}$ will be equal to zero. By discretizing time we see, by recurrence, that if those functions are zero at beginning, they will be equal to zero at all time. But when $g \neq 0$ ($V_{\mu\tau} \neq 0$) we have to look also at the evolution of the functions $\hat{f}_\mu = A_{\tilde{\mu}} e^{-i\delta} - B_{\tilde{\mu}}$ and $\hat{f}_\tau = A_{\tilde{\tau}} - B_{\tilde{\tau}}$. Their respective evolution equation can be easily derived from Eq. (60) to yield:

\[
i \frac{d}{dt}(A_{\tilde{\mu}} e^{-i\delta} - B_{\tilde{\mu}}) = b (A_{ee} e^{-i\delta} - B_{ee}) + d (A_{\tilde{\mu}} e^{-i\delta} - B_{\tilde{\mu}}) + e (A_{\tilde{\tau}} e^{-2i\delta} - B_{\tilde{\tau}}) + g (A_{ee} e^{-i\delta} - B_{ee})
\]

\[
i \frac{d}{dt}(A_{\tilde{\tau}} - B_{\tilde{\tau}}) = c (A_{ee} e^{i\delta} - B_{ee}) + e (A_{\tilde{\mu}} e^{i\delta} - A_{\tilde{\mu}}) + f (A_{\tilde{\tau}} - B_{\tilde{\tau}}) - g (A_{\tilde{\mu}} - B_{\tilde{\mu}})
\]

(64)

Initially, the derivatives are:

\[
i \frac{d}{dt}(A_{\tilde{\mu}} e^{-i\delta} - B_{\tilde{\mu}})(t = 0) = i \frac{d}{dt}(A_{\tilde{\mu}})(t = 0) = b(e^{-i\delta} - 1)
\]

\[
i \frac{d}{dt}(A_{\tilde{\tau}} - B_{\tilde{\tau}})(t = 0) = = i \frac{d}{dt}(A_{\tilde{\tau}})(t = 0) = c(e^{i\delta} - 1)
\]

(65)

We just proved that since the functions $\hat{f}_\mu$ and $\hat{f}_\tau$ are non constant zero functions, the functions $f_\mu$ and $f_\tau$ won’t be zero as well. But does it implies that the function $f_e$ is non zero at all time? No, because the contributions from $\hat{f}_\mu$ and $\hat{f}_\tau$ could cancel in the evolution equation (61) of $f_e$. To precisely study the evolution of $f_e$, we discretize time such as $t = N * \Delta t$ with $N \in \mathbb{N}$ and

\[
\frac{d}{dt} f_e = \frac{f_e(t + \Delta t) - f_e(t)}{\Delta t}
\]

(66)

Using Eqs. (61) and the time discretization we see that: At $t = \Delta t$:

\[
\hat{f}_\mu(\Delta t) = \frac{b(e^{-i\delta} - 1)}{i} \Delta t
\]

\[
\hat{f}_\tau(\Delta t) = \frac{c(e^{i\delta} - 1)}{i} \Delta t
\]

(67)

which implies that: At $t = 2\Delta t$:

\[
f_\mu(2\Delta t) = gb(e^{-i\delta} - 1)\Delta^2 t
\]

\[
f_\tau(2\Delta t) = -gc(e^{i\delta} - 1)\Delta^2 t
\]

(68)
leading at $t = 3\Delta t$

$$f_e(3\Delta t) = \frac{1}{i} \left( gbc(e^{-i\delta} - 1) - gbc(e^{i\delta} - 1) \right) \Delta^3 t$$

$$= \frac{1}{i} gbc(e^{-i\delta} - e^{i\delta}) \Delta^3 t$$

$$= -2gbc\sin\delta\Delta^3 t$$

(69)

This last formula proves that the function $f_e$ is not the constant zero function when $\delta \neq 0^3$, therefore $A_{ee} \neq B_{ee}$ and consequently, for $V_{\mu\tau} \neq 0$:

$$P(\nu_e \rightarrow \nu_e, \delta \neq 0) \neq P(\nu_e \rightarrow \nu_e, \delta = 0)$$

(70)

Therefore, when $\delta$ is non-zero, it has an influence on the value of $P(\nu_e \rightarrow \nu_e, \delta)$. The luminosity of a neutrino emitted initially as a flavour $\alpha$ is

$$L_{\nu_e}(r, E_\nu) = \frac{L^0_{\nu_e}}{T^3_{\nu_e}(E_{\nu_e})F_2(\eta)} \frac{E^2_{\nu_e}}{1 + \exp(E_{\nu_e}/T_{\nu_e} - \eta)}$$

(71)

where $F_2(\eta)$ is the Fermi integral, $L^0_{\nu_e}$ and $T_{\nu_e}$ are the luminosity and temperature at the neutrinosphere. The $\nu_e$ and $\bar{\nu}_e$ fluxes will depend on $\delta$ even when the luminosities $L_{\nu_e}$ and $L_{\bar{\nu}_e}$ are taken equal at the neutrino-sphere:

$$\phi_{\nu_e}(\delta) = L_{\nu_e} P(\nu_e \rightarrow \nu_e, \delta) + L_{\bar{\nu}_e} \left( P(\nu_\mu \rightarrow \nu_e) + P(\nu_\tau \rightarrow \nu_e) \right)$$

$$= L_{\nu_e} P(\nu_e \rightarrow \nu_e, \delta) + L_{\bar{\nu}_e} \left( 1 - P(\nu_e \rightarrow \nu_e, \delta) \right)$$

$$= (L_{\nu_e} - L_{\bar{\nu}_e}) P(\nu_e \rightarrow \nu_e, \delta) + L_{\bar{\nu}_e}$$

(72)

This analytical derivation proves that, if the dependence on $\delta$ of the evolution operator cannot be factorized then the electron neutrino survival probability depends on $\delta$. Nevertheless, this implication had been observed numerically. We can easily generalize this derivation to other interactions that would distinguish non-standard neutrino interaction [52]. Therefore we can state that as soon as the medium effect on supersymmetric parameters. Such value could be highly important in the calculation of neutrino fluxes from core-collapse supernovae as it can induce sizeable effects upon the electron (anti-)neutrino fluxes.

VI. CONCLUSIONS

In this paper, we have investigated the radiative correction on the $\mu - \tau$ neutrino indices of refraction coming from beyond standard physics. In the NMSSM, we have shown that the sign of $V_{\mu\tau}$ depends upon the hierarchy of the sleptons masses and therefore could be negative contrary to the Standard Model case. After writing and adding a subroutine to a low-energy code taking into account all current constraints on SUSY we showed that $\varepsilon$ can increase up to the order of $2 \times 10^{-2}$ depending on the supersymmetric parameters. Such value could be highly important in the calculation of neutrino fluxes from core-collapse supernovae as it can induce sizeable effects upon the electron (anti)-neutrino fluxes. In a second part we have calculated all contributions from R-parity breaking interactions on the radiative corrections $V_{\mu\tau}$. Taking into account such interactions, we showed that NMSSM distinguishes from MSSM in this case and bring new possible contributions. The next step in this type of calculations would be to see the consequences of these values of $V_{\mu\tau}$ on the supernova neutrino fluxes and to use it in order to survey the supersymmetric parameter space [53]. Secondly, we would have to calculate corrections with gravitino loops and all contributions from R-parity breaking interactions. Another interesting possibility would be to calculate the radiative corrections for the neutrino-neutrino interaction. Such calculation has been recently done in the SM and SUSY framework could yield potentially much bigger effect. Finally, as an application for $V_{\mu\tau}$ we demonstrated that the inclusion of such a term implies that the electron (anti-) neutrino survival probability, and consequently the electron (anti-) neutrino fluxes, will depend upon $\delta$. The consequences of such dependence and the fact that $V_{\mu\tau}$ can be up to $2 \times 10^{-2} V_e$ will be studied in a future work.

Note that with such formula, $f_e$ is also equal to zero when $\delta = \pi$, but going to the forth step will show, after a tedious but straightforward calculation, that $f_e$ is non zero even when $\delta = \pi$. [54]
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