Generalized Reduced Gradient Approach for Solving Periodic Heterogeneous Vehicle Routing Problem with Side Constraints

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Abstract. In order to transport customers' request economically, such as, goods, meals etc it is frequently the delivery company depend on a model which is called vehicle routing problem (VRP). This model consists of a customer population with deterministic demands, and a central station which acts as the base of a set of vehicles. The purpose is to design the vehicle routes starting and terminating at the assigned central station, such that the customer demands is met, within a time windows This paper develops an optimization model for the management of periodic deliveries of a given commodity. Due to the geographic location of customers the company requires to have diverse type of vehicles with diverse capacity to serve the commodity for customers. This kind of problem is called Periodic Heterogeneous Vehicle Routing Problem (PHVRP). The purpose is to schedule periodically the deliveries according to feasible combinations of delivery days and to determine the scheduling of fleet and driver and routing policies of the vehicles. The aim is to minimize the total costs of all routes over the planning horizon. We propose a combined approach of reduced gradient and heuristic algorithm to solve the problem.

1. Introduction
In logistic system there is a familiar combinatorial model known as Vehicle Routing Problem (VRP). The challenge is to choose the best selection of routes used by a variety of means of transport to meet the needs of a number of customers. The Vehicle Routing Problem (VRP) can be described as a controlled graph $G = (V, A)$, where $V = \{V_1, V_2, ..., V_n\}$ is a number of customers and $A \subseteq \{(v_i, v_j) : i \neq j, v_i, v_j \in V\}$ is a category of routes to be travelled. Optimum collection of paths, consisting of a cyclic connection of arcs starting and finishing at Depot ($v_1$), is chosen to support the specified number of clients. The objective of the problem is to reduce overall transport costs (in relation to transport times or distances) and maintenance costs (in relation to the number of vehicles used).

In [1] submitted to the VRP in 1959. They used the model in the truck dispatching problem, which was structured to uncover an optimum routing strategy for a number of vehicles to satisfy a wide range of customers. The basic concept of the whole issue is based on the Hamilton model, which is essentially designed to meet the requirements where every transport starts and ends at the point of departure, and it is specifically prohibited for any consumer to be visited more than once by a single vehicle. Until now, several scholars have been working in this field to
find different variations and new methodologies. Several important surveys for VRP have been published, such as [2]–[5] and in books [6], [7].

Considering the timing of the routing of vehicles, the VRP is known as the periodic VRP (PVRP). In PVRP, we can provide a visit schedule for each client within a certain period of time. Each vehicle leaves the facility, handles a group of customers on a daily basis, while its working time or magnitude is reached, returns to the warehouse. The challenge is to reduce the overall length of the distances that cars periodically navigate on a time scale. In reality, such a dilemma is very significant, such as, for example, the delivery to pastry companies [8], the distribution of blood stocks [9], or the purchase of raw particles for the manufacture of vehicle parts [10].

The review of PVRP and its variants may be described in [11]. The difficulty of the issue is high, and therefore most of the technique used is based on heuristic methods, although [12] an accurate technique has been suggested. In [13] a hybrid heuristic and exact technique was adopted to tackle PVRP.

Another vital addition to the VRP, where it is crucial to determine the composition of the vehicle. This form is known as the Heterogeneous Vehicle Routing Problem (HVRP). The problem lies in the seminar paper [14] released in 1984, which has been extended to a wide range of scientific fields. In terms of how to address the problem [15], it offered a specific review of the articles on the basis of lower bound strategies and heuristics. The researchers evaluated the efficacy of the heuristics shown on the metrics prior to 2008. In [16] the study of exact algorithms and differences in numerical results on VRPs and HVRPs was conducted, while in [17] the study explored the issue from a wide range of industrial points of view with mixed vessel structure and routing in naval and road transport.

2. Problem Description

In the context of the decision, the issue with the transport scheme is to select a decent means of moving vehicles in such a way as to reduce the overall operating costs. Address the challenge, it is better to get the graphic concept to visualize the issue. Let $G = (V,A)$ be the directed graph, where $V = \{0,1,...,n\}$ is the vertex set, and let $A = \{(i,j) : i, j \in V, i \neq j\}$ be the set of directed edges, indicating the routes between the vertex. $V = \{0\}$ is the vertex of the store. $V_c = V \{0\}$ is a set of client locations. It is assumed that each pinnacle $i \in V_c$ has a demand $q_i \geq 0$ on a regular basis, a service timeframe $s_i \geq 0$, a timeframe $[e_i,l_i]$ where $e_i$ is the first period service, and $l_i$ is the final time, and a certain frequency of $f_i$ interventions is expected to be achieved to fit one of the authorized visit-day cycles. The timeframe for the transport period to depart and return to the facility is specified by $[e_0,l_0]$. Let the number of accessible vehicles of all kinds be $K = \{1,...,k\}$. The fleet of vehicles of each category with capacity $Q_k$ shall be located at the collection point.

Let $Q_k$ signify weight power for each fleet $k \in K$. We assume that the number of modes of transport is equivalent to the number of drivers. Express the number of consumers (nodes) to $N = \{1,2,...,n\}$. Symbolize the $\{0,n+1\}$ facility. Every vehicle starts at the depot and ends at $\{n+1\}$. Each client $i \in N$ shall specify the number of days to be accessed. Customer $i \in N$ requests service on a daily basis with $q_i$ demand in weight, over time window $[a_i,b_i]$. Note that we have defined $q_0^i = 0$ for the facility $i \in \{0,n+1\}$. Indicate the collection of chosen vehicles for the user i by $K_i (K_i \in K)$ and the additional service period per pallet by $e$ if the consumer does not have access to the desired vehicle.

3. Mathematical Modelling

Sets:

$DI$ internal drivers,
DE  external drivers,
D  $D = DL + DE$ all drivers,
K  vehicles to be used,
N  customers,
$N_0$ customers and depot $N_0 = \{0, n+1\} \cup N$,
$K_i$ suitable transportation for customer $i \in N$,

Parameters:
$M_k$ Amount of transportation of type $k \in K$ available
$Q_k$ Capacity of fleet $k \in K$ in terms of weight
$[a_i, b_i]$ The beginning and the end of visit time at customer $i \in N_0$,
$q_i$ The weight demand of customer $i \in N_0$,
$[g_1, h_1]$ The time to start and the time to finish for driver $l \in D$
$\alpha_i$ The amount to pick up from customer $i$
$\beta_i$ The amount to deliver regarding to customer $i$
$CV^k$ Payment (fixed) for any type of fleet $k \in K$ from center
$CVA^k_i$ Payment of vehicle $k \in K$ to travel along the route $(i,j) \in K$
$CVN^k_i$ Payment regarding to the visit of a customer by different fleet $k \in K$
$CDI^k_l$ Payment for internal driver $l \in DI$ using fleet $k \in K$
$CDE^k_l$ Payment referring to external driver $l \in DE$ for fleet $k \in K$
$CPQ^k_j$ Payment for picking up quantities for customer $j \in N$ applying fleet $k \in K$
$CDD^k_j$ Payment for delivering quantities for $j \in N$ customer applying fleet $k \in K$

Variables:
$x^k_{ij}$ Equivalents to 1 if fleet $k \in K$ travels from node $i \in N_0$ to $j \in N_0$ , otherwise 0
$w^k_i$ Equivalents to 1 if customer $i \in N_0$ is visited by an unpreferable vehicle, otherwise 0
$y^k_i$ The time for vehicle $k \in K$ to begin service at node $i \in N_0$
$z^k_i$ Equivalents to 1 if vehicle $k \in K$ is assigned to internal driver $l \in D I$ , otherwise 0
$\theta^k_j$ The amount of picking up of customer $j$ demand done by fleet $k \in K$
$\sigma^k_j$ The amount of deliveries to meet customer $j$ demands done by fleet $k \in K$

The model for this problem is written mathematically with the following expressions:
The aim is to minimize the operational costs

$$\begin{align}
\text{Min} & \sum_{k \in K} \sum_{j \in N_0} CV^k x^k_{0j} + \sum_{k \in K} \sum_{(ij) \in A} CV A^k_{ij} x^k_{ij} + \sum_{i \in N_0} \sum_{k \in K} CVN^k_i w^k_i + \sum_{l \in DI} \sum_{k \in K} CDI^k_l y^k_l + \\
& \sum_{l \in DE} \sum_{k \in K} CDE^k_l z^k_l + \sum_{j \in N} \sum_{k \in K} CPQ^k_j \theta^k_j + \sum_{j \in N} \sum_{k \in K} CDD^k_j \sigma^k_j \\
\end{align}$$

(1)

Subject to several constraints

$$\sum_{j \in N_0} x^k_{0j} = 1, \forall k \in K$$

(2)

$$\sum_{k \in K} \sum_{j \in N} x^k_{ij} = 1, \forall i \in N_0$$

(3)

Equations (2 - 3) are to guarantee that only a vehicle with any type returns and move away from each customer.
\[
\sum_{i \in N} x_{ij}^k - \sum_{i \in N} x_{ji}^k = 1; \forall j \in N, \forall k \in K
\] (4)

This expression is an equation which is required to preserve the continuation of the all vehicle route for every time period.

\[
\sum_{j \in N_0} x_{0j}^k \leq M_k, \forall k \in K
\] (5)

Expression (5) symbolizes that every customer is tackled only by the vehicle of type k, which is available at the depot.

\[
\sum_{j \in N_0} x_{1j}^k \leq 1; \forall k \in K
\] (6)

\[
\sum_{i \in N, i \geq 1} x_{il}^k \leq 1; \forall k \in K
\] (7)

Equations (6-7) express the readiness of vehicles by restricting the distance of route, corresponding to fleet k of each type, which leaves from and comes back to the central, at most one.

\[
\sum_{i \in N} q_i \sum_{j \in N_0} x_{ij}^k \leq Q_k; \forall k \in K
\] (8)

Equations (8) guarantee that for every delivery does not surpass the space requirement of each vehicle.

\[
\sum_{k \in K} \sum_{j \in N_0} x^i_{ijk} = 1; \forall i \in N, t \in T
\] (9)

\[
\sum_{k \in K} \sum_{j \in N_0} x^t_{ijk} = w^t_i; \forall i \in N, t \in T
\] (10)

\[
\sum_{i \in N} q_i x^t_{ijk} \leq Q_k; \forall k \in K, t \in T
\] (11)

\[
b_i \geq v^t_{ik} \geq a_i; \forall i \in N, k \in K, t \in T
\] (12)

\[
v^t_{0k} \geq \sum_{l \in D} (g^t_l, y^t_{lk}); \forall k \in K, t \in T
\] (13)

\[
v^t_{n+1,k} \leq \sum_{l \in D} (h^t_l, y^t_{lk}); \forall k \in K, t \in T
\] (14)

\[
\sum_{k \in K} \theta^t_{jk} = \alpha^t_j; \forall j \in N, t \in T
\] (15)

\[
\sum_{k \in K} \sigma^t_{jk} = \beta^t_j; \forall j \in N, t \in T
\] (16)

\[
x^t_{ijk}, w^t_i, z^t_{ik}, y^t_{lk} \in \{0, 1\}; \forall i, j \in N_0, l \in D, k \in K, t \in T
\] (17)

\[
v^t_{ik}, u^t_{ik}, r^t_i, s^t_l \geq 0; \forall i, j \in N_0, l \in D, k \in K, t \in T
\] (18)
\[ \theta_{jk}, \sigma_{jk} \in \{0, 1, 2, \ldots\}; \forall j \in N, k \in K, t \in T \] (19)

Equation (9) expresses that only one particular vehicle can deliver a customer on each time scheduled. Expressions (10) guarantee that a customer is visited by a special vehicle. Equations (11) ensure that the capacities of each vehicle satisfy in weight and volume.

Equations (12-13) give the elapsed driving time. Equations (14) are to guarantee that the elapsed driving time never surpasses an upper limit. Equations (15 – 16) are to express the pickup and delivery for each customer. Expressions (17-19) characterize the binary and positive variables applied in the model.

4. Outline of The Algorithm
Step 1. (Test for convergence in the current subspace). If \( \|h\| > \text{TOLRG} \) go to step 3.
Step 2. Estimate Lagrange multipliers, add one super basic.
Step 3. Calculate direction of search, \( p = Zp_a \).
Step 4. Do ratio test.
Step 5. Do Line search.
Step 6. Calculate reduced gradient, \( \bar{h} = Z^T g \).
Step 7. Change basis if necessary; delete one super basic.

5. Conclusions
The aim of this paper is to develop a model of Periodic Vehicle Routing with Time Windows, Fleet and Driver Scheduling, Pick-up and Distribution Problems. This dilemma has an additional restriction, i.e. the limitation of the number of vehicles. We propose an algorithm that uses the nearest heuristic approach to solve the problem.

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