On Proton Energization in Accretion Flows
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(submitted to MNRAS)

ABSTRACT: Two-temperature advection dominated accretion flow (ADAF) or hot ion tori (HIT) models help explain low luminosity stellar and galactic accreting sources and may complement observational support for black holes in nature. But low radiative efficiencies demand that ions receive a fraction $\eta \gtrsim 99\%$ of energy dissipated in the turbulent accretion. The $\eta$ depends on the ratio of particle to magnetic pressure. If compressive modes of dissipation, like magnetic mirroring, dominate incompressible modes, then even when the pressure ratio is $O(1)$, the required large $\eta$ can be attained. However the relative importance of compressive vs. incompressible modes is hard to estimate. The larger up in the turbulent cascade, the more compressible the turbulence. The relevant length scale for particle energization can be determined by equating the dominant eddy turnover time to the time for which an energy equal to that in the turbulence can be drained. Based on the large scales resulting from this estimate, it is suggested that compressive mirroring may be important. Also, regardless of the precise $\eta$ or dissipation mechanism, non-thermal protons seem natural in two-temperature discs because all dissipation mechanisms, and the use of an isotopic pressure, require wave-particle resonances that operate only on a subset of the particles. Finally, it is briefly mentioned how mirroring may help to generate an ADAF or HIT in the first place.

Key words: accretion, accretion discs; acceleration of particles; turbulence; galaxies: general; binaries: general; Galaxy: centre
1. Introduction

Magnetized accretion discs have become the most convincing physical paradigm to explain emission from the central engines of active galactic nuclei (AGN) and X-ray binary sources (Frank et al. 1992). The observed radiation comes from the energy dissipation required to maintain steady accretion of material onto the central object. As molecular viscosity is incapable of providing the required accretion rates, turbulent viscosity is necessary. For thin discs, this can be generated by shear and magnetic fields (Balbus & Hawley 1991). For thick discs, something similar may occur, though in this case angular momentum transport may ultimately require a global approach.

Nevertheless, in complement to thin disc solutions for sources requiring high radiation efficiency accretion (e.g. Frank et al. 1991), two-temperature thick advection dominated flows (ADAFs) or hot ion tori (HIT) (e.g. Ichimaru 1977; Paczynski & Bintnovatyi-Kogan 1981; Rees et al. 1982; Narayan & Yi 1995) have received much attention in an effort to explain sources requiring a low radiation efficiency. Here the ions are assumed to receive the energy dissipated by the steady accretion without having enough time to transfer their energy to the cooler electrons before falling onto the central object. Some or most of the dissipated energy is advected, not radiated, as it would have been if electrons received all of the dissipated energy. Such models have been at least partially successful in explaining quiescent galactic centres (Rees 1982; Narayan et al. 1995; Mahadevan 1998; Fabian et al. 1995, but see DiMatteo et al. 1998) and stellar X-ray binary systems (Narayan et al. 1996) with radiative efficiencies ≤ 1/100 that of thin disc solutions. When the central object is
a black hole, the advected energy is lost forever rather than re-radiated as it would be for
a neutron star. Precisely such observed differences between corresponding X-ray binary
systems have been purported to provide evidence for black hole horizons (Narayan et. al
1997).

There have been only a handful of other papers addressing how the viscous dissip-
ation might energize particles in accretion flows (Gruzinov 1997; Quataert 1997; Quataert
& Gruzinov 1998) and none addressing thermal vs. non-thermal particle distributions.
Both of these issues are extremely important for ADAF type models because: 1) a two-
temperature solution is not sufficient to explain a low radiation efficiency and 2) inter-
pretation of observations of the Galactic centre suggest that the protons are non-thermal
when an ADAF model is employed (Mahadevan 1998). The potential catastrophe for
ADAF/HIT models, if electrons are preferentially energized over protons, was partially
explored in (Bitsnovatyi-Kogan & Lovelace 1997). Because electrons cool much faster than
ions, even if 1/2 of the dissipated energy went into electrons, a two-temperature solution
would still result. But in this case, 50% of the dissipated energy would be radiated—far
too much to explain low luminosity sources. More explicitly, I define $q_t$ as the magnitude
of the energy density input rate into particles. Then $q_t = q_p^+ + q_e^+ = \eta q_t + (1 - \eta)q_t$
where $q_p^+$ and $q_e^+$ are the magnitudes of energy input rate into protons and electrons,
and $\eta$ is the fraction of $q_t$ that goes into protons. In the steady state, energy loss rates
are equal to energy gain rates so that when advection is included, we have for the pro-
tons $q_p^+ = \eta q_t = q_a^- - q_{pe} = f \eta q_t + (1 - f)\eta q_t$, where $q_a = f \eta q_t$ is the rate associated
with advection, $q_{pe}$ is the rate of transfer from ions to electrons, and $f$ is the fraction of
the proton energy loss rate associated with advection. For the electrons, we thus have
\[ q_e^+ = (1 - \eta)q_t + q_{pe} = q_t - q_a = q_t(1 - f\eta). \]
Since \( q_e^- = q_e^- \), where \( q_e^- \) is the luminosity density, the quantity \((1 - f\eta)\) must be \( << 1 \) to explain quiescent sources. Standard treatments (e.g. Rees et al. 1982; Narayan & Yi 1995) assume \( \eta = 1 \) so that 1% radiative efficiency would correspond to \( f = 0.99 \). The important questions are: (A) When can \( \eta \gtrsim 0.99 \) be justified? (B) Are protons non-thermal? and (C) Is there a faster than Coulomb coupling between electrons and protons (e.g. Begelman & Chiueh 1988) that destroys the ADAF solution? I will address (A) and (B) here.

Proton energization by incompressible (Quataert 1997; Gruzinov 1997; Quataert & Gruzinov 1998) and/or compressive modes of dissipation (discussed herein) both depend on the ratio of particle to magnetic pressure. Section 2 addresses the relation between magnetic, turbulent, and particle energy densities in ADAFs, relating them to the viscosity parameter \( \alpha \). Section 3 discusses the threat of electron runaway. Section 4 employs a very physical approach to acceleration by magnetic mirroring and derives the time scale for particle energy doubling for two distinct limits of the average particle speed. Section 5 discusses energization in ADAFs, first addressing why protons are likely to be non-thermal, regardless of the acceleration mode. The mirroring results are then specifically applied to ADAFs and the scale in the turbulent cascade where mirroring is favored is estimated. Because larger scales are significantly compressive, and the resulting derived scale can be large, mirroring may be important. Magnetic mirroring type processes can favor protons to the extent required for a wider range of average particle to magnetic pressure ratios than found by Quataert & Gruzinov (1998) from dissipation of incompressible Alfvén waves,
but the relative fractions of incompressible vs. compressible modes of dissipation are hard to determine. The possibility that mirroring may help provide a thermal instability which initially forms an ADAF is briefly addressed.

2. Relation Between Viscosity Parameter and Pressure Ratio

The standard parameterization of accretion disc turbulent viscosity for thin discs is (Shakura & Sunayev 1973)

\[ \nu_T = \alpha C_s H \sim V_T l_T / 3, \]  

(1)

where \( \alpha \) is the viscosity parameter and \( C_s \) and \( H \) are the sound speed and disc height, and \( V_T \) and \( l_T \) are the outer (i.e. dominant energy containing) random (turbulent) flow speed and scale. For thin discs, the scale of the turbulence is always much less than the radius of the disc. Turbulent kinetic and magnetic energy densities rapidly approach equipartition from field line stretching. A magneto-rotational shearing instability (Balbus & Hawley 1991) likely drives 3-D MHD turbulence. In a steady state, dissipation of the turbulence into particles combats the symbiotic growth of magnetic and kinetic turbulent energy. Both magnetic and kinetic turbulent energies incur a decaying power-law energy spectrum (like Kolmogorov (1941) or Kraichnan (1965)) with the largest scales of the turbulence containing the most energy. Since the sound speed is constant on all scales, the largest scales are thus the most compressive. For thin discs, the outer turbulent scale is significantly smaller than the disc radius, but for standard ADAFs, there is not such a strong scale separation (Blackman 1998).

For thin discs, we can derive a 1-to-1 link between \( \alpha \) and \( \beta_p^{-1} \equiv V_A^2/C_s^2 \equiv 6(1 - \beta_a) \),
where $V_A$ is the Alfvén speed, and $\beta_a$ is used in ADAF modeling. In the steady-state, the largest eddy turnover time $t_T = l_T/V_T$ must equal the shearing instability growth time driving the turbulence, that is: $t_T \approx R/V_\phi$, where the rotation speed $V_\phi \sim V_K$, the Keplerian speed. Since $V_A \sim V_T$ in the saturated state from field line stretching (e.g. Parker 1979; Blackman 1998) and $C_s/V_K = H/R$ from hydrostatic equilibrium, we have $\nu = \alpha C_s H \approx V_T l_T/3 \sim V_A^2 (R/3V_\phi) = V_A^2 (H/3C_s)$ which implies that

$$\alpha \approx 2(1 - \beta_a)(V_K/V_\phi) \approx 2(1 - \beta_a)$$

for thin discs. This result basically agrees with numerical simulation results (e.g. Stone et. al 1996).

For ADAFs, the ratio of $V_K/V_\phi$ can be so low that (2) is inappropriate: in this case the resulting eddy scale implied by the relation would be larger than $H \sim R$. We can instead obtain an upper limit on $\alpha$ for ADAFs that comes from the constraint

$$l_T < H \sim R. \quad (3)$$

Then from (1), the definition of $\beta_a$, and $V_T = V_A$ from field line stretching, we find

$$\alpha \leq (2/3^{1/2})(1 - \beta_a)^{1/2}. \quad (4)$$

showing that $\beta_a$ and $\alpha$ are not independent.

3. On Electron Runaway

Bistnyati-Kogan & Lovelace (1997) pose an interesting question: why can’t direct acceleration from electric fields drain energy into electrons, destroying ADAFs? I address
this here. First, note the generalized Ohm’s Law (e.g. Scudder 1986) \( \mathbf{E} = -\mathbf{V}_e/c \times \mathbf{B} + \sigma^{-1} \mathbf{J} - m_e (\mathbf{V}_e \cdot \nabla \mathbf{V}_e)/\epsilon - \nabla P_e/(\epsilon n_e) \) where \( \mathbf{B} \) is the magnetic field, \( \mathbf{J} \) is the current density, \( P_e \) is the electron pressure, \( \mathbf{V}_e \) is the bulk electron velocity, \( \sigma^{-1} \) is the resistivity, \( m_e \) is the electron mass, \( n_e \) is the density, and \( \epsilon \) is the electric charge. For the plasmas of interest, a characteristic magnitude of \( \mathbf{E} \) parallel to \( \mathbf{B} \) is given by the second last term, and using \( V_e \lesssim V_T \sim V_A \lesssim C_s \), we have \( |E|| \sim k_B T_e/ (\delta l e) \sim 2 \times 10^{-14} (T_e/10^9 K)(\delta l/10^{13} \text{cm})^{-1} \), where \( \delta l \) is the gradient length, \( T_e \) is the electron temperature and \( k_B \) is the Boltzmann constant.

For \( E|| \) to produce ER, it would have to exceed the Dreicer electric field (Dreicer 1962; Holman 1985; Bitsnovatyi-Kogan & Lovelace 1997) \( E_D = e^2 \ln \Gamma/\lambda_D^2 = 1.8 \times 10^{-7} (\ln \Gamma/20)(n_e/10^{12} \text{cm}^{-3})^{1/2} (T_e/10^9 \text{K})^{-1} \text{St-Volt/cm} \), using \( \lambda_D \sim 6.65 (T_e^{1/2}/n_e) \text{cm} \), for the Debye length. Whether \( E|| > E_D \) depends on the size of \( \delta l \). For AGN, \( E_D \) is only exceeded on scales \( 10^6 \) times smaller than the turbulent outer scale. For stellar size X-ray binary ADAF systems, the outer scale is \( \sim 10^7 \text{cm} \), so in principle ER is possible throughout the flow. But the accelerated electrons can never produce a current which induces a magnetic field in excess of the inferred ambient field. This gives an upper limit (Holman 1985) to the size of field gradients that generate ER, namely \( \delta l \leq 8 (B/10^4 \text{G})(n_e/10^{12} \text{cm}^{-3})^{-1} (T_e/10^9 \text{K})^{-1/2} (E_D/E||) \text{cm} \). For all relevant accretion discs, this scale in the cascade is always way below that at which magnetic mirroring, employed in the next section, could have already drained most of the energy in the cascade. Nevertheless, if mirroring is not important, or equivalently, if a significant component of the turbulence cascades to incompressible scales before draining, then the cascade may
proceed down to this scale where ER, or other (e.g. Quataert & Gruzinov 1998) electron energization processes may be important.

However, the more extreme ER of Bistnovati-Kogan & Lovelace (1997) is not likely. They employ the mean electric field, obtained by coarsely averaging \( \mathbf{E} = \langle \mathbf{E} \rangle + \mathbf{E}_T \) over the turbulent scales \( l_T \). The dominant terms in this mean Ohm’s law are then \( \langle \mathbf{E} \rangle \simeq -\langle \mathbf{V} \times \mathbf{B}_T \rangle - \langle \mathbf{V} \rangle \times \langle \mathbf{B} \rangle \) where the turbulent EMF (Parker 1979) is, in kinematic theory, \( \langle \mathbf{V} \times \mathbf{B}_T \rangle = (\alpha_d/c)\langle \mathbf{B} \rangle - \beta_d \nabla \times \langle \mathbf{B} \rangle \) where \( \alpha_d \) is a pseudo-scalar helicity, and \( \beta_d \) is the turbulent diffusivity. A representative magnitude of \( \langle \mathbf{E} \rangle \) using \( |\alpha_d| \sim V_T/3 \), is \( \langle \mathbf{E} \rangle \sim V_T\langle B \rangle/3c \). Assuming that \( \langle B \rangle \sim B \), then \( \langle \mathbf{E} \rangle \sim 3 \times 10^3(V_T/10^{10}\text{cm/sec})(B/10^4\text{G}) \text{St-Volt/cm} \). Thus \( \langle \mathbf{E} \rangle \gg E_D \) and one might be tempted to conclude that runaway electron acceleration is extreme. But particles do not actually see \( \langle \mathbf{E} \rangle \), since the average is only defined on scales larger than \( l_T \). Thus extreme ER should not occur.

4. Energization by Magnetic Mirroring

4.1 Basic Physical Picture

Fermi energization, or magnetic mirroring of particles off magnetic compressions (Fermi 1949; Spitzer 1962) provides a means of dissipation of compressive turbulent energy into particles. Consider a field, \( B_T \), which represents the field on the largest turbulent scale, superimposed on which is a smaller scale compression, so the total field in the compression is \( B = B_T + \delta B \). These compressions travel along field lines at speeds \( \sim V_A \), and can transfer energy to particles: consider what happens as the particle traveling along \( B_T \) interacts with the compression \( B_T + \delta B \). Since the magnetic force is perpendicular
to the particle velocity, as long as the magnetic gradient scale $>>$ particle gyro-radius (adiabatic approximation), the particle’s angular momentum and energy are conserved in the frame of a magnetic compression at rest. Denoting quantities in this frame by a prime and working in the non-relativistic limit, the energy and angular momentum magnitudes are given by $u' = mv'^2/2$ and $j' = mv'_\perp r_g = m^2 cv'_\perp r^2/(eB)$, where $m$ is the particle mass, $v', v'_\perp$ are the total speed and speed perpendicular to the field and $r_g$ is the gyro-radius. The constancy of both $u'$ and $j'$ implies that $v'^2_{\perp}/B = v'^2_{\perp} \sin^2 \phi'/B$ is also constant. Thus, because $v'$ is constant, $\sin^2 \phi' \propto B = B_T + \delta B$, or

$$\sin^2 \phi' = \sin^2 \phi'_T(B_T + \delta B)/B_T.$$ (5)

When $\sin \phi' = 1$, the particle reflects. Thus there exists a minimum pitch angle the particle must have with respect to the ambient $B_T$ such that it can reflect upon entering the compression. This is given by

$$\sin^2 \phi'_{T,\text{min}} \equiv \sin^2 \phi'_{\text{min}} = B_T/(B_T + \delta B)$$ (6)

(see also figure 1). In the lab frame, the moving compression then boosts the velocity component of a given particle parallel to $B_T$. For energy to be gained from repeated reflections, the boost must be rapidly isotropized by particle generated waves (Eilek & Hughes 1991; Larosa et al. 1996) as discussed further in section 5. Assuming isotropy in the lab frame, in the frame of a moving magnetic fluctuation the velocity distribution is centered around $\sim V_A$. The minimum angle for mirroring by a magnetic compression then gives a minimum speed that particles need to reflect:

$$v_{\text{min}} = V_A\sin \phi'_{\text{min}} = V_A B_T^{1/2}/(B_T + \delta B)^{1/2} \sim V_A \text{ for } \delta B/B_T < 1,$$ (7)
as seen in figure 1.

### 4.2 Time Scale for Energy Doubling

Different regions will have $B_T$ aligned in different directions but consider $B_T$ in one region of size $l_T$ in which $B_T$ is assumed constant. Following (Larosa et al. 1996), define $\tau_r \equiv U_r(dU_r/dt)^{-1} = N\delta t$ as the time scale for the average reflected particle energy $U_r$ to double, where $N$ is the number of required reflections and $\delta t$ is the time between reflections. Since only a fraction $F$ are reflected, the energization time averaged for all particles is then

$$\tau = U/dU/dt = (\delta t/F)(NU/U_r), \quad (8)$$

where $U$ is the average particle energy averaged over all particles. We need to estimate $N, \delta t, F, U/U_r$.

To understand the role of $F$, two limits *not* usually distinguished *must* be considered separately: For particles with speeds $v_{\text{min}} < v < V_A$, all reflections are “head-on” since the particles can never catch up to the fluctuations which move at speeds along the field lines $\sim V_A$ (see figure 1). For $v >> V_A$, there are both “catch-up” and head-on reflections. Let $L1$ and $L2$ label separate regimes where the average particle speed, $v_{\text{ave}}$, satisfies for $L1: v_{\text{ave}} >> V_A \sim v_{\text{min}}$ and $L2: v_{\text{ave}} \sim v_{\text{min}} \sim V_A$. That $v_{\text{min}} \sim V_A$ follows from the assumption that $\delta B < B_T$ in (7). $L1$ is appropriate for electrons in a thermal equilibrium system whose magnetic pressure is not dominant. $L2$ is appropriate for protons in a thermal equilibrium system, or for both protons and electrons in plasma of $\beta_p \sim 1$ with the ratio of proton to electron temperature $T_p/T_e \gtrsim 1000$—like ADAFs or HIT. Define the corresponding energization times $\tau_{L1}$ and $\tau_{L2}$ for the two limits. Limit $L1$ is the
standard “stochastic Fermi acceleration” limit, and the energization time for L1 derived below simply, agrees with other treatments (e.g. Miller 1985, Melrose 1986). The limit L2 produces a different formula.

To proceed further, I compute velocity moments of reflected particles in the two cases L1 and L2 for thermal and non-thermal distributions. This is necessary for computing \( F \). For a power-law distribution, \( d g_{nt} = (\lambda - 1)(v/v_0)^{-\lambda}d(v/v_0) \), where \( d g_{nt}/d(v/v_0) \) is the distribution function, \( v_0 \) is the lower cutoff on the power-law, and \( \lambda > 3 \) will be assumed (to avoid the appearance of logarithms). Integrating over the reflected particles gives

\[
\int_{v_{min}}^{\infty} d g_{nt} = (v_{min}/v_0)^{1-\lambda}, \tag{9}
\]

if \( v_0 < v_{min} \). If \( v_0 \geq v_{min} \), then \( v_0 \) replaces \( v_{min} \) as the lower integral bound. The average velocity of reflected particles is then \( v_{r,ave}|_{nt} = (\lambda - 1)v_0(v_{min}/v_0)^{2-\lambda}/(\lambda - 2) \), when \( v_0 < v_{min} \). In the thermal case, \( d g_{th} = (4/\pi^{1/2})(v^2/v_{ave}^2)Exp[-v^2/v_{ave}^2]d(v/v_{ave}) \), so that

\[
\int_{v_{min}}^{\infty} d g_{th} = 1 - Erf[v_{min}/v_{ave}] + (v_{min}/v_{ave})Exp[-v_{min}^2/v_{ave}^2]. \tag{10}
\]

I now use (9) and (10) to determine \( F \), the fraction of particles that reflect. Generally,

\[
F = \int_{v_{min}}^{\infty} f dg, \tag{11}
\]

where \( f \) is the fraction of particles that reflect at a given speed. The \( f \) is the “area” of the sphere (see Figure 1) corresponding to particles which can be reflected, divided by the total area, \( 4\pi v^2 \):

\[
f = 2\pi \int_{v_{||}=-}^{v_{||}+} v_\bot [1 + (dv_\bot/dv_{||})^2]^{1/2}dv_{||}/4\pi v^2 = 2\pi v|v_{||}|v_{||}^{v_{||}+}/4\pi v^2, \tag{12}
\]
where ||(⊥) indicates parallel (perpendicular) to $B_T$, and the second equality comes from using the equation for the circle centered at $V_A$ for the particle speed, $v_\perp^2 = v^2 - (v_\parallel - V_A)^2$.

The bounds $v_{\parallel\pm}$ are determined by finding the abscissa values at which the line defining $\phi_{\min}$ intersects (Figure 1) the circle defined by $v_{ave}$. Setting the equation for the lines, $v_\perp = v_\parallel \tan^2 \phi_{\min}$, equal to that of the circle gives

$$v_{\parallel\pm} = V_A \cos \phi'_{\min} \pm \cos \phi'_{\min} (v_{ave}^2 - V_A \sin^2 \phi'_{\min})^{1/2}.$$  

(13)

For $L_1$ (i.e. $v_{ave} \gg V_A$), using (13) in (12) gives $f \sim \cos \phi'_{\min} \sim (\delta B / B_T)^{1/2} \sim 2(2\pi v^2 \cos \phi'_{\min})/4\pi v^2 = \cos \phi'$. This can be pulled out of the integral in (11). Then because $v_{min}/v_{ave} << 1$ in this limit,

$$F_{L_1} = f = \cos \phi'_{\min},$$  

(14)

for both the non-thermal and thermal cases.

For $L_2$, ($v_{ave} \sim v_{min}$) (12) and (13) give $f \lesssim \cos \phi'_{\min}$. This can be pulled out of the integral in (11). For the non-thermal case, using (9) for the remaining integrand, I obtain

$$F_{L_2} \lesssim \cos \phi'_{\min} [(v_{min}/v_{ave})(\lambda - 1)/(\lambda - 2)]^{1-\lambda}.$$  

(15)

For the $L_2$ thermal case, I use (10) instead of (9), noting that the first two terms in (10) approximately cancel, giving

$$F_{L_2} \sim \cos \phi'_{\min} (v_{min}/v_{ave}) \exp[-v_{min}^2/v_{ave}^2].$$  

(16)

Now consider $\delta t = \delta l / \langle |v_\parallel| \rangle$, the time between reflections, where $\delta l$ is the length scale of the fluctuation $\delta B$ and $\langle |v_\parallel| \rangle$ is the average magnitude of the reflected particles’ velocity.

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parallel to $B_T$. For L1, $\langle |v|| \rangle = v_{||+}/2 = (v_{ave}/2)\cos \phi_{min}' \sim (v_{ave}/2)(\delta B/B_T)^{1/2}$ where the latter similarity follows for $\delta B/B_T < 1$ in (7). Thus

$$\delta t_{L1} = (2\delta l/v_{ave})(B_T/\delta B)^{1/2}. \quad (17)$$

For L2 $\langle |v|| \rangle \sim v_{||+}/2 \sim (V_A/2)\cos^2 \phi_{min} \sim (V_w/2)(\delta B/B_T)$, so that

$$\delta t_{L2} = (2\delta l/V_w)(B_T/\delta B). \quad (18)$$

Consider now $N(U/U_r)$ appearing in (8). For L1, $U/U_r \sim 1$, as the energy of reflected particles is of order the average energy of all particles, but we must determine $N$. For L1, the energy gain is stochastic (Fermi 1949; Spitzer 1962; Eilek & Hughes 1991; Larosa et al 1996) as the particles incur random walks through momentum space and

$$N_{L1} = U^2/\langle \delta U_+ \rangle^2, \quad (19)$$

where $\langle \delta U_+ \rangle$, is the average energy gain by a particle from a head-on reflection. For L2, there are mainly head-on reflections, so that $U_r/\langle \delta U_+ \rangle \lesssim N_{L2} \lesssim U_r^2/\langle \delta U_+ \rangle^2$. Since a lower bound on $\tau_r$ suffices, I employ $N_{L2} \gtrsim U_r/\langle \delta U_+ \rangle$. This means that For L2,

$$[NU/U_r]_{L2} \gtrsim U/\langle \delta U_+ \rangle. \quad (20)$$

I now need $\langle \delta U_+ \rangle$ for both L1 and L2. The $\langle \delta U_+ \rangle$ is determined by energy and momentum conservation before and after a mirroring. This gives $\langle \delta U_+ \rangle \sim 2mV_A\langle |v|| \rangle$. For L1, using the value of $\langle |v|| \rangle$ calculated above then gives $\langle \delta U_+ \rangle \sim mV_A v_{ave}(\delta B/B)^{1/2}$, and thus

$$[NU/U_r]_{L1} = N_{L1} = (v_{ave}^2/4V_A^2)(B_T/\delta B), \quad (21)$$

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while for L2, using the appropriate \(|v|\) calculated above (18) gives

\[
[NU/U_r]_{L2} \gtrsim U/\langle \delta U_+ \rangle = (v_{ave}^2/2V_A^2)(B_T/\delta B).
\] (22)

Collecting the calculations of \(N(U/U_r), \delta t,\) and \(F\) for L1 in (8) gives

\[
\tau_{L1} = (\delta t_{L1}/F_{L1})[NU/U_r]_{L1} = (\delta l/4V_A)(v_{ave}/V_A)(B_T/\delta B)^2 = (l_T/4V_A)(v_{ave}/V_A)(\delta l/l_T)^{1/2}
\] (23)

where the last equality follows from assuming a Kraichnan (1965) spectrum \((\delta B/B) = (\delta l/l_T)^{1/4}\) relating the magnetic to scale fluctuations. Eq. (23) describes “stochastic Fermi” energization (Miller 1985; Melrose 1986). Similarly for L2, using the appropriate above results for \(N(U/U_r), \delta t,\) and \(F\) in (8), I obtain

\[
\tau_{L2} = (\delta t_{L2}/F_{L2})[NU/U_r]_{L2} \gtrsim (\delta l/V_A)(v_{ave}/V_A)^3(B_T/\delta B)^{3/2} \exp[V_A^2/v_{ave}^2]
\]

\[
= (l_T/V_A)(v_{ave}/V_A)^3(\delta l/l_T)^{5/8} \exp[V_A^2/v_{ave}^2],
\] (24)

for the thermal case, while for the non-thermal case with \(v_{min} > v_0,\) and \(\lambda > 3\)

\[
\tau_{L2} \gtrsim (l_T/V_A)(v_{ave}/V_A)^{3-\lambda}(\delta l/l_T)^{5/8},
\] (25)

where the Kraichnan (1965) relation has again been used.

We see that each of the energy doubling times (23)-(25) depend only the particle average speeds \(v_{ave}\) and not on their particle mass. But if electrons and protons have the same \(v_{ave},\) the protons have \((m_p/m_e)\) more energy. Thus each of (23)-(25) shows that electrons take \((m_e/m_p)\) longer to drain the same amount of energy. When electrons and
protons do not have the same $v_{ave}$ one population could be in L2 and the other in L1 and the comparison of energy doubling times becomes more subtle. This is because although in L1 there are many more reflections possible than in L2 L1 has both energy gaining and energy losing reflections (i.e. note the smaller shaded area and absence of symmetry in figure 1a compared to figure 1b). Thus the energization is second order as expected for stochastic acceleration. For L2 however, while there are less reflections, they are mainly head-on (i.e. energy gaining). These two effects (less reflections but mainly energy gaining=L1 vs. more reflections but both energy gaining and energy losing =L2) compete and the $\beta_p$ regime for which protons vs. electrons dominate the drain then also depends on the particle distribution of that population in the L2 limit. Another complication comes if the populations have the same $v_{ave}$ but different distribution functions. Then one must compare (24) and (25). We will study some of these cases more specifically in the next section.

5. Application to Accretion Flows

5.1 Why Protons Are Likely Non-Thermal in ADAFs

The discussion of section 4 is one approach to the mirroring or Fermi energization process. Others include stochastic magnetic pumping (e.g. Hall & Sturrock 1967) and transit time pumping (e.g. Stix 1962, described as the magnetic analogue of Landau damping). Achterberg (1981) showed that all small amplitude ($\delta B << B_T$) approaches in L1 to mirroring in a turbulent plasma can also be described by quasi-linear diffusion of particles in momentum space, from magnetosonic wave particle resonances at the Cherenkov resonance
\( (\omega_w - k_\parallel v\cos\phi) = 0 \). The relevant waves have frequencies \( \omega_w \ll \) particle gyrofrequencies (i.e. very long wavelengths compared to the gyro-radii) which is equivalent to the adiabatic approximation discussed in section 4.

This resonance requires a minimum particle speed \( v_{\text{min}} \sim V_A \) and also a minimum \( \sin\phi' \), as derived in section 4. The required minimum in \( \sin\phi' \) means that in order for particles to undergo repeated reflections and gain energy, their momentum must be rapidly isotropized on a time scale shorter than the time between reflections, which itself must be shorter than the largest eddy turnover time. The largest eddy turnover time is in turn shorter than the ADAF infall time, given by \( t_{\text{in}} \sim 1.8 \times 10^{-5} M r^{3/2}/\alpha \), where \( M \) is the mass in units of \( M_\odot \), and \( r \) is the radius in Schwarzschild units. But for ADAFs, Coulomb isotropization is not fast enough: the time scale for momentum isotropization from Coulomb collisions is of order the time scale for thermalization and is given by (e.g. Spitzer 1960; Mahadevan & Quataert 1997)

\[
t_{pp} = (2\pi)^{1/2}(n_p\sigma_T\ln\Lambda)^{-1}(m_p/m_e)^2(kT_p/m_pc^2) \sim 10^{-2} \alpha(\beta_a/0.5)^{3/2}M\dot{M},
\]

where \( n_p \) is the proton number density, \( \sigma_T \) is the Thomson cross section, \( \ln\Lambda \) is the Coulomb logarithm, and \( \dot{M} \) is the accretion rate is units of the Eddington value, \( 1.4 \times 10^{18} M\text{g/sec.} \)

Setting (26) equal to \( t_{\text{in}} \) shows that protons can only be Coulomb thermalized/isotropized well outside of the dominant energy emission location. i.e. for \( r \gtrsim 100 \) (Mahadevan & Quataert 1997). (In fact, this feature is fundamental to enabling an ADAF solution.) Thus the isotropization requires an additional kind of wave-particle resonance.

Unlike the mirroring waves, the required isotropizing waves have wavelengths of order
the particle gyro-radius. Some, or all these small wavelength (Whistler, Alfvén or magneto-sonic) waves can be generated by the particles themselves and then they do not transfer energy to the particles. Some fraction may also be generated directly from the turbulence in which case they can transfer energy to the particles. This latter possibility is explored in (e.g. Quataert & Gruzinov 1998) as the primary means by which the turbulence dissipates into particles. The resonances occur when the wave frequency in the particle frame is an integer multiple of the particle gyrofrequency, that is \( \omega - k_{||} v \cos \phi - N \Omega^* = 0 \), where \( \Omega^* \equiv e B c / E_p \) and \( E_p \) is the total proton rest+kinetic energy. Quataert & Gruzinov (1998) show that Whistlers are not damped by protons. The short wavelength Alfvén waves \( (\omega = k_{||} V_A \lesssim \Omega_g \equiv e B / (m_i c) < \Omega^*) \) are the most relevant for isotropization and have the approximate resonance condition \( -e B c / E_p - k_{||} v \cos \phi = 0 \). The condition \( |\cos \phi| < 1 \) then leads to the injection condition \( E_p > (V_A / v) m_i c^2 (\Omega_g / \omega) \). For \( \omega \sim \Omega_g \), this leads to \( v_{\text{min}} \sim V_A \) for protons– similar to the requirement of the mirroring waves derived earlier.

So both types of resonant waves–long wavelength mirroring waves and short wavelength Alfvén waves (whether they accelerate or just isotropize)– have a proton minimum speed requirement of order \( V_A \). The inefficiency of Coulomb thermalization, and the need for wave particle resonances to dissipate the turbulence means that a significant non-thermal particle population should be produced. Since ADAFs are most commonly modeled with \( \beta_a \sim 0.5 \), non-thermal protons will likely be a fraction \( \sim O(1) \) of the population. The inefficiency of Coulomb collisions in ensuring a non-thermal population is fundamental. Even when stochastic Fermi energization (the L1 limit), can be shown to rigorously lead to a power-law distribution in the energized particles (e.g. Eilek & Hughes 1991)
efficient Coulomb scattering would thermalize the distribution. The fact that Coulomb collisions are inefficient, as shown above, precludes redistribution of energy over the full population of protons.

Note that at least the isotropizing waves are also *implicitly* built into ADAFs because the standard models presume isotropic pressure and this would be impossible without wave-particle resonances. In fact, the plasma must be “collisional” in the sense of wave-particle interactions, even though it is “collisionless” with respect to Coulomb collisions. In short, non-thermal protons should be a generic prediction of ADAFs. This is consistent with observations (Mahadevan 1998), which can distinguish between thermal and non-thermal proton distributions in an ADAF framework (and so far are not too sensitive to the proton power law index.) In principle, similar arguments could be applied to electrons with more stringent resonance conditions. However Mahadevan & Quataert (1996) and Ghisselini et al. (1998) argue that synchrotron self-absorption can thermalize weakly relativistic and non-relativistic electrons (at least those not produced from pion decay) under ADAF conditions during an in-fall time. Thus I assume (25) applies to ADAF/HIT protons in the steady state, and (24) to electrons.

5.3 Reflecting Waves, Scales of Dissipation, and when Mirroring Preferentially Energizes Protons vs. Electrons

Magnetosonic waves are the dominant mirrorers in the low amplitude limit ($\delta B \ll B_T$), as Alfvén waves are incompressible and compression is required for mirroring, though the relevant compression speed along the field lines is always $\sim V_A$ regardless of the
wave mode. Achterberg (1981) considered a magnetically dominated plasma at a single temperature, and focused only electrons. Here we are interested in a two-temperature plasma and are considering both electrons and ions. In general, both slow and fast waves may participate in the mirroring.

Though magnetosonic waves dominate in the low amplitude limit, this is not necessarily true in the large amplitude limit ($\delta B \sim B_T$). Because the largest scales of turbulence in discs are the most compressive, the large amplitude limit is relevant when the scale on which the mirroring can compete with the cascade of energy from larger to smaller turbulent scales is a large fraction of the outer turbulent scale $l_T$. In this case, the energy could be compressively drained into particles before it reaches smaller scales in the cascade where incompressible modes of dissipation dominate. Since large amplitude Alfvén waves are compressible (Alfvén & Falthammar 1963), even they could then contribute to the mirroring. Such Alfvén waves could even steepen to form shocks and perhaps shock-Fermi acceleration would be relevant. This must be considered in future work. as will see that in fact the relevant mirroring scales can be large.

I now proceed to estimate the scales on which the favored particles are energized and when protons vs. electrons are favored. For low luminosity sources, ADAFs require $1 - f\eta \leq 0.01$, implying an accretion efficiency $\leq 1\%$ of that for thin discs (e.g. Rees et al. 1982; Narayan & Yi 1995). The respective energization times then need to satisfy

$$\tau_p/\tau_e \leq \zeta \equiv T_p(1 - f\eta)/T_e \lesssim 10, \quad (27)$$

where the subscript $p(e)$ indicates ions (electrons). Since the turbulence cascades from
large to small scales, I compare the scales of energy drain for protons, \((\delta l)_p\), and electrons, \((\delta l)_e\), for which (27) is satisfied. The larger of the two length scales then determines the dominant drain. Using (25) for ADAF protons and setting it equal to the eddy turnover time \(l_T/V_A\) gives

\[
(\delta l)_p/l_T \sim (V_A/v_{p,ave})^{(24-8\lambda)/5},
\]

where \(v_{p,ave}\) is the average proton speed. For electrons, setting \(\zeta\) times (24) equal to \(l_T/V_A\) gives

\[
(\delta l)_e/l_T \sim \zeta^{-8/5}(V_A/v_{e,ave})^{24/5}Exp[-8V_A^2/5v_{e,ave}^2].
\]

Then we can see that

\[
(\delta l)_p/(\delta l)_e = \zeta^{8/5}k^{24/5}\beta_p^{4\lambda/5}Exp[8/(5k^2\beta_p)].
\]

This is \(> 1\) for a range of parameters applicable to ADAFs (e.g. \(\beta_p \sim k \sim O(1), \zeta \sim 10\)). The same conclusion results when the particle distributions are either both thermal or non-thermal. Protons can be favored to the required extent when compressive modes dominate the turbulent dissipation.

Let us determine the scale on which the protons are dissipated. From (28) we see that when \(V_A \sim v_{ave,p}\), it is not hard to have \((\delta l)_p/l_T \sim O(1)\), (recall \(\lambda > 3\)). This means that the compressive modes may be very important and much of the energy in the turbulence may drain before approaching the incompressible scales where Quataert & Gruzinov (1998) is applicable. In general, the small amplitude limit may not be fully appropriate in describing energy dissipation in ADAFs.
Now consider a thermal plasma with $T_p = T_e$, which corresponds to radii outside the ADAF region (Narayan & Yi 1995) or to a thin, precursor disc. In this case, no matter which particles initially receive the energy, Coulomb collisions redistribute this energy between electrons and protons. However, whether protons vs. electrons receive the dissipated energy determines the heating rate. When $\beta_p \sim O(1)$, the relevant limits of interest are (23) for electrons and (24) for protons. Using $v_{e,\text{ave}} = (m_p/m_e)^{1/2}v_{p,\text{ave}}$, the dissipation scale ratio becomes

$$(\delta l)_p/(\delta l)_e \sim (m_p/16m_e)\beta_p^{-7/5}\exp[-8/(5\beta_p)].$$

This is $< 1$ for $\beta_p \lesssim 0.25$ and $> 1$ for $\beta_p > 0.25$. For $\beta_p >> 1$, both electrons and protons are in the limit of (23), for which $(\delta l)_p/(\delta l)_e \sim m_p/m_e$. For $\beta_p < m_e/m_p$, both electrons and protons are in the limit of (24) for which $(\delta l)_p/(\delta l)_e \sim (m_p/m_e)^{4/5}$. In sum, electrons are favored only for the range $m_e/m_p \lesssim \beta_p \lesssim 0.25$, while for $\beta_p$ outside this range protons are favored. (The conditions of low $\beta_p$ for which electrons are favored may be found in solar flares (e.g. Larosa et al. 1996) and some thin accretion disc coronae models (Field & Rogers 1993).

5.4 Can Mirroring Help Form an ADAF?

It is sometimes believed that purely a low enough accretion rate is enough to form an ADAF/HIT. However, unless the disc is already thick, the critical accretion rate below which electrons and protons do not couple by Coulomb collisions on an infall time as computed for a standard thin disc is far too low to be physically relevant. For a thin disc system to evolve into an ADAF, a mechanism is needed to form a thick disc first. This
may occur by thermal instability and mirroring may help. The condition (e.g. Pringle 1981) for thermal instability is

\[ \frac{d \ln [q_t]}{dT} > \frac{d \ln [q_e^-]}{dT}. \]  

(32)

If the instability proceeds from within an optically thick disc, then we must compare blackbody emission to the heating. In the regime \( \beta_p \lesssim 0.25 \) for the thermal disc, electrons are favored as shown in above, and (23) is applicable. Taking the inverse of (23) for electrons, multiplying by \( v_{ave}^2 \propto T \) and differentiating gives \( \frac{d \ln [q_t]}{dT} = \frac{1}{2T} \). If the emission is blackbody, then

\[ \frac{d \ln [q_e^-]}{dT} = \frac{4}{T}, \]  

(33)

and the instability is not favored. For \( \beta_p \gtrsim 0.25 \) protons are favored and using (24) gives

\[ \frac{d \ln [q_t]}{dT} = T^{-1}(1/\beta_p - 1/2T) \]  

and still, even for \( 0.2 < \beta_p \lesssim 0.5 \) the thermal instability cannot ensue.

However, it is more likely that the formation of an ADAF would proceed by thermal instability within the very surface layer of the thin disc, and successive layers would eventually evaporate from the surface to form the thick ADAF disc. The particle distribution in the very surface layer could be non-thermal. To see how mirroring might help, in the limit where the protons dominate the energy drain (\( \beta_p \gtrsim 0.2 \)), I invoke (25) for protons, take its reciprocal, and multiply by \( v_{ave}^2 \propto T_p = T \) to obtain the quantity proportional to \( q_t \). Then \( \frac{d \ln [q_t]}{dT} = (\lambda - 2)/2T \). For \( \lambda > 8 \) this can satisfy (32) when the emission is blackbody (33). For Bremsstrahlung \( \frac{d \ln [q_e^-]}{dT} = 3/2T \), and (32) can be satisfied when \( \lambda > 5 \).
6. Conclusions

Dissipation of turbulence in presumed ADAF sources must preferentially energize protons by a factor $\eta > 99\%$ over electrons if ADAFs are to account for the observed low luminosities. Compressive magnetic mirroring can in principle favor protons to the extent required for a ratio of particle to magnetic pressure $\beta_p > 0.25$. This is less stringent than the requirements of incompressible modes which demand $\beta_p >> 1$. However, it is not easy to estimate what fraction of energy is dissipated in compressive vs. incompressible modes. An estimate of the scale at which mirroring can compete with the transfer of energy down the turbulent cascade seems to indicate that for $\beta_p \sim 1$, the scale can be quite a large fraction of the outer turbulent scale—which for ADAFs/HIT can be a large fraction of the disc size (Blackman 1998). This has two implications: 1) a significant fraction of the energy may be dissipated in compressive modes and 2) the small amplitude approach to dissipation may not be valid. In the small amplitude limit, the relevant waves involved in the energy transfer to particles are magnetosonic waves, as Alfvén waves are not compressive and will not be damped by mirroring. However on the larger scales in the turbulent cascade, the large amplitude limit is relevant. Since large amplitude Alfvén waves are compressive, they too may be involved in compressive modes of dissipation. This presents additional complications for future work. Magnetic reconnection may also be a complication, however reconnection itself generates turbulence, and possibly shock or direct acceleration processes which may also favor protons, but it is important to know on what scale the reconnection is occurring.
Regardless of the fraction of energy dissipated in compressive or incompressible modes, the fact ADAFs are “Coulomb collisionless” on the radial infall time scale seems to make a non-thermal proton population inevitable. The required dissipation of turbulence must proceed through wave particle interactions, all of which act on only a subset of the particles. Since ADAF models presume an isotropic pressure tensor, wave-particle resonances are implicitly assumed to play a role in ADAFs because Coulomb isotropization is necessarily too slow. The presence of a non-thermal proton population seems to indeed be indicated by observations of the Galactic centre (Mahadevan 1998) when modeled with an ADAF. A remaining fundamental problem which still needs more attention is the question of a faster than Coulomb coupling between particles (Begelman & Chiueh 1988) even if the protons could receive the dissipated energy.

Acknowledgments: Thanks to M. Rees for stimulating discussions and insights.

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FIGURE 1 CAPTION

Particle speed diagram for magnetic mirroring. The magnetic compression is assumed to move at velocity $\sim -V_A$ along the field line and so an isotropically distributed population of velocities in the lab frame has a spherical distribution centered around $v'_\parallel = V_A$. The angle $\phi'_\text{min}$ and the speed $v_{\text{min}}$ bound the respective minima needed for a particle to reflect at the magnetic compression. The weaker the compression the larger these minima. The area inside the shaded region between the two circles represents the particle speed region which can be reflected. Approximate schematics of the regimes a) L2 ($v_{\text{ave}} \sim V_A$) and b) L1 ($v_{\text{ave}} \gg V_A$) of the text are shown. L2 is relevant for $\beta_a \sim 1$ ADAFs. Note that only the region L1 strictly corresponds to stochastic energization since in this case the number of “catch-up” and head on reflections are about equal whereas region L2 has primarily “head on” (albeit less total) reflections.