On the validity of Lorentz invariance relations between parton distributions

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Abstract

Lorentz invariance relations connecting twist-3 parton distributions with transverse momentum dependent twist-2 distributions have been proposed previously. These relations can be extracted from a covariant decomposition of the quark-quark correlator. It is argued, however, that the derivation of the Lorentz invariance relations fails if the path-ordered exponential is taken into account in the correlator. The model independent analysis is supplemented by an explicit calculation of the corresponding parton distributions in perturbative QCD with a quark target, and in a simple spectator model. We also clarify the status of a specific calculation of time-reversal even parton distributions in light-cone gauge.

1 Introduction

Transverse-momentum dependent ($k_{\perp}$-dependent) parton distributions and twist-3 parton distributions play an important role in describing various hard processes like semi-inclusive deep inelastic scattering or the Drell-Yan process. In particular, these functions enter certain spin and/or azimuthal asymmetries [1, 2], and recently results for such asymmetries have been reported by the HERMES and CLAS collaborations [3, 4].

In Refs. [5, 6, 7] so-called Lorentz invariance relations (LI-relations) were introduced, which connect twist-3 distributions with moments of $k_{\perp}$-dependent twist-2 distributions. The LI-relations impose important constraints on the distribution functions, which allow one to eliminate unknown structure functions in favor of known ones. Moreover, these relations could be very useful for deriving the evolution of moments of $k_{\perp}$-dependent distribution functions [8].

Here, we argue that the LI-relations are violated because in their derivation a dependence on a light-cone vector was neglected [9]. Our model independent analysis is supplemented by explicit model results. Two specific LI-relations for time-reversal even (T-even) distributions have already been questioned in [10], where the involved functions were computed in light-front Hamiltonian QCD using a dressed quark target. However, the treatment in Ref. [10] is not fully gauge invariant, which motivated us to revisit the calculation.

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2 Model independent analysis

The derivation of the LI-relations in Refs. [5, 6, 7] is based on the consideration of the quark-quark correlator for the nucleon (characterized by its momentum $P$ and a spin vector $S$)

$$\Phi_{ij}(P,k,S|\text{path}) \equiv \frac{1}{(2\pi)^4} \int d^4\xi \ e^{ik\cdot\xi} \langle P,S|\bar{\psi}_j(0)W(0;\xi|\text{path})\psi_i(\xi)|P,S\rangle,$$

(1)

where the Wilson line ensuring gauge invariance is given by

$$W(0;\xi|\text{path}) = \exp\{-ig\int_0^\xi ds\ A_\mu(s)\}_{\text{path}}.$$

(2)

Note that the correlator in (1) doesn’t enter a factorization theorem for a physical process. Taking into consideration the constraints due to hermiticity and parity the correlator in Eq. (1) can be decomposed in the most general Lorentz invariant way according to

$$\Phi_{ij}(P,k,S|\text{path}) = M A_1 + \not{P} A_2 + \not{k} A_3 + \frac{i}{2M}[\not{P},\not{k}] A_4 + \ldots,$$

(3)

where the $A_i = A_i(k^2, k \cdot P)$ are unknown coefficient functions. We have limited the list of structures to those relevant for an unpolarized target. The factors of the nucleon mass $M$ in (3) were introduced in order that all $A_i$ have the same dimension.

On the other hand, the ($k_\perp$-dependent) parton distributions entering the hadronic part of physical processes are defined through a correlation function $\Phi_{ij}(x,k_\perp,S|\text{path})$ by taking projections with appropriate Dirac matrices. This correlator is connected to the one in (1) by means of

$$\Phi_{ij}(x,k_\perp,S|\text{path}) = \int dk^-\Phi_{ij}(P,k,S|\text{path})|_{k^+ = xP^+}.$$  

(4)

The path on the lhs in (4) is fixed when doing a proper factorization, and will be specified below. The path on the rhs has to be chosen such that after the $k^-$-integration it matches with the one on the lhs. Eq. (4) immediately relates the parton distributions with the amplitudes $A_i$. One finds, e.g.,

$$f_1(x,k_\perp^2) = 2P^+ \int dk^- \left( A_2 + x A_3 \right),$$

(5)

$$h_1^+(x,k_\perp^2) = 2P^+ \int dk^- \left( - A_4 \right),$$

(6)

$$h(x,k_\perp^2) = 2P^+ \int dk^- \left( \frac{k \cdot P - xM^2}{M^2} A_4 \right),$$

(7)

where $f_1$ is the usual unpolarized quark distribution, while $h_1^+$ and $h$ are T-odd distributions [6, 7]. Comparing now these expressions for the parton distributions, one can find LI-relations between those distributions which contain the same $A_i$’s. We list here the most important ones [5, 6, 7],

$$g_T(x) = g_1(x) + \frac{d}{dx}g_1^{(1)}(x),$$

(8)

$$h_L(x) = h_1(x) - \frac{d}{dx}h_1^{(1)}(x),$$

(9)

$$f_T(x) = -\frac{d}{dx}f_1^{(1)}(x),$$

(10)

$$h(x) = -\frac{d}{dx}h_1^{(1)}(x),$$

(11)
with
\[ g^{(1)}_{1T}(x) = \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{2M^2} g_{1T}(x, \vec{k}_\perp^2), \text{ etc.} \] (12)

All distributions on the \( \text{lhs} \) in Eqs. (8)–(11) are of twist-3, whereas the functions on the \( \text{rhs} \) appear unsuppressed in observables. The two LI-relations in (8,9) connect T-even parton distributions, the ones in (10,11) contain T-odd distributions.

In Ref. [9] we have argued, that the LI-relations are invalid. The crucial observation is that the decomposition in Eq. (3) is incomplete because the presence of the gauge link leads to a dependence on an additional light-like vector. This point becomes obvious when keeping in mind the appropriate gauge link structure of the correlator
\[ \Phi_{ij}(x, k_\perp, S|\text{path}) = \frac{1}{(2\pi)^3} \int d\xi d^2\vec{\xi}_\perp e^{i(k^+\xi^- - \vec{k}_\perp \cdot \vec{\xi}_\perp)} \langle P, S|\bar{\psi}_j(0)W(0; \xi|\text{path})\psi_i(\vec{k})|P, S\rangle, \] (13)
with \( \vec{\xi} = (0, \xi^-, \vec{\xi}_T) \) and the Wilson line [11, 12, 13, 14]
\[ W(0; \vec{\xi}|\text{path}) = W(0, 0, 0, 0, \xi_{\perp}, 0, \infty, \vec{\xi}_T) \times W(0, 0, 0, \infty, \vec{\xi}_\perp, 0, 0, 0, 0) \] (14)

To parameterize this gauge link a light-like vector \( n \) (in addition to the light-cone direction given by the target momentum) is needed, which has to show up also in the unintegrated correlator in Eq. (1) due to the connection in (4). Therefore, more covariant structures in (3) with new coefficient functions \( B_i \) show up [9, 15]
\[ \Phi_{ij}(P, k, S|\text{path}) = M A_1 + P A_2 + k A_3 + \frac{i}{2M} [P, k] A_4 + \ldots \] (15)
\[ + \frac{M^2}{P \cdot n} B_1 + \frac{iM}{2P \cdot n} [P, \gamma_5] B_2 + \frac{iM}{2P \cdot n} [k, \gamma_5] B_3 + \gamma_5 \gamma_\mu P\gamma_\nu n_\mu k_\sigma B_4 + \ldots, \]
where again we only listed the structures that appear in the case of an unpolarized target. The new terms typically modify the expressions of the parton distributions in terms of the coefficient functions. As an example we consider the two functions entering the LI-relation in (11) for which we now find
\[ h_1^+(x, \vec{k}_\perp^2) = 2P^+ \int dk^- \left( - A_4 \right), \] (16)
\[ h(x, \vec{k}_\perp^2) = 2P^+ \int dk^- \left( \frac{k \cdot P - xM^2}{M^2} A_4 + (B_2 + xB_3) \right). \] (17)

Obviously, the presence of the amplitudes \( B_2 \) and \( B_3 \) in the expression for \( h \) spoils this particular LI-relation. By considering structures which depend on the target polarization the violation of the relations in (8)-(10) can be shown as well.

### 3 Model calculations

We now want to supplement our model independent analysis by an explicit model calculation of the relevant parton distributions.

For the two relations in (8,9) all involved distributions have already been computed for a dressed quark target in light-front Hamiltonian QCD in Ref. [10], and a violation of the LI-relations has been observed. However, the transverse Wilson line at the light-cone infinity in Eq. (14), whose importance

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1The last term in the second line in (15) has been overlooked in Ref. [9] as pointed out recently in [15]. This term, however, does not influence the discussion of the LI-relations.
for light-cone gauge calculations has been realized only afterwards [13, 14], was not taken into account in [10]. As a consequence, the results for the moments of the $k_\perp$-dependent distributions $g_{1T}$ and $h_{1L}^T$ in Eqs. (8,9) might change in a fully gauge invariant treatment. We have repeated this pQCD calculation for a quark target in Feynman gauge, where the Wilson line at the light-cone infinity doesn’t contribute. Only the two parts of the link in Eq. (14) which run along the $\xi^-$-direction are relevant. By introducing intermediate states in the correlator in (13) one can make an expansion up to first order in $\alpha_s$,

$$\Phi_{ij}(x, k_\perp, S | \text{path}) = \frac{1}{(2\pi)^2} \int d\xi^- d^2 \vec{\xi}_\perp e^{i(k^+ + \vec{k}_\perp \cdot \vec{\xi}_\perp)}$$

$$\times \left[ \langle q; P, S, a| \bar{\psi}_j(0) W(0, 0, 0, 0)| 0 \rangle \langle 0 | W(0, \infty, \vec{\xi}_\perp, 0, -\vec{\xi}_\perp) \psi_i(\bar{\xi}) | q; P, S, a \rangle + \sum_{r, b} \int \frac{d^3 l}{(2\pi)^3} E_l \langle q; P, S, a| \bar{\psi}_j(0) W(0, 0, 0, 0)| 0 \rangle \langle 0 | W(0, \infty, \vec{\xi}_\perp, 0, -\vec{\xi}_\perp) \psi_i(\bar{\xi}) | q; P, S, a \rangle + \ldots \right].$$

Up to $\mathcal{O}(\alpha_s)$ we only need to consider gluons as particles in the intermediate state. The one-loop calculation is UV-divergent, and we regularize the expressions by calculating in $4 - 2\varepsilon$ dimensions. The resulting parton distributions are extracted from $\Phi_{ij}(x, k_\perp, S | \text{path})$ by appropriate projections and integration over $\vec{k}_\perp$. For the parton distributions in Eq. (8) we obtain

$$g_1(x) = \delta(1 - x) + \frac{\alpha_s}{2\pi} C_F \frac{1 + x^2}{1 - x} \frac{1}{\varepsilon} + \ldots,$$

$$g_{1T}(x) = \delta(1 - x) + \frac{\alpha_s}{2\pi} C_F \frac{1 + 2x - x^2}{1 - x} \frac{1}{\varepsilon} + \ldots,$$

$$g_{1T}^{(1)}(x) = -\frac{\alpha_s}{2\pi} C_F x(1 - x) \frac{1}{\varepsilon} + \ldots.$$  

The dots in Eqs. (19)–(21) indicate virtual radiative corrections and/or higher orders in $\alpha_s$. The virtual contributions contain an explicit factor $\delta(1 - x)$ and remove the singularity for $g_1$ and $g_{1T}$ at $x = 1$ (see, e.g., Ref. [10]). In order to check the LI-relations it is sufficient to consider the UV-divergent part of the real radiative corrections which are proportional to $1/\varepsilon$. With the results in (19)–(21) one finds that the LI-relation (8) is fulfilled at leading order, but the $\alpha_s$-corrections spoil the relation. By means of an analogous calculation it can be shown that relation (9) is violated as well. The expressions in (19)–(21) completely agree with the ones obtained in Ref. [10], where a UV-cutoff has been used to regulate the divergent $\vec{k}_\perp$-integral. In particular, the results for $g_{1T}$ and $h_{1L}^T$ coincide. From this observation we conclude that for our specific calculation the transverse Wilson line at the light-cone infinity is irrelevant in light-cone gauge. Whether this result holds in general for T-even parton distributions remains to be seen.

The situation for the LI-relations (10,11) which involve T-odd functions is simpler. In this case we can use results which already exist in the literature in addition to arguments based on T-invariance. In the framework of a simple diquark spectator model for the nucleon it has been shown that $f_{1T}(x, \vec{k}_\perp)$ doesn’t vanish [16, 12, 13]. On the other hand, the $\vec{k}_\perp$-independent parton distribution $f_{1T}(x)$ is zero because of T-invariance [17, 18]. Therefore, the relation (10) is violated in this model. The same reasoning can be used for the LI-relation (11): an explicitly non-vanishing result for $h_{1L}^T(x, \vec{k}_\perp)$ has been obtained in the diquark spectator model [19, 20], while T-invariance rules out a non-zero $h(x)$. Finally, we mention that by considering the T-odd case we can easily give even stronger support to the picture that the presence of the light-cone vector $n$, and consequently the presence of the amplitudes $B_i$, is at the origin of the violation of the LI-relations. If one would define the $k_\perp$-dependent parton
distributions with a single straight Wilson line connecting the two quark fields, then neither the correlator in Eq. (13) nor the one in (1) would contain a $n$-dependence. Hence, the second line in (15) containing the amplitudes $B_i$ would be absent, and the LI-relations should be valid. For the relations (10,11) one observes readily that this expectation is indeed correct, because both sides of the relations would vanish because of T-invariance.

**Note added:** It is interesting, that the very last $n$-dependent term in Eq. (15) gives rise to a new twist-3 T-odd parton distribution [15] and, therefore, to an observable effect. The spectator model calculations of the beam single spin asymmetry for semi-inclusive deep inelastic scattering in Refs. [21, 22] suggested the existence of this new function.

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