Longevity problem of sterile neutrino dark matter

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Sterile neutrino dark matter of mass O(1–10) keV decays into an active neutrino and an X-ray photon, and the non-observation of the corresponding X-ray line requires the sterile neutrino to be more long-lived than estimated based on the seesaw formula: the longevity problem. We show that, if one or more of the B–L Higgs fields are charged under a flavor symmetry (or discrete R symmetry), the split mass spectrum for the right-handed neutrinos as well as the required longevity is realized. We provide several examples in which the predicted X-ray flux is just below the current bound.

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1. Introduction

Sterile neutrino is one of the plausible candidates for dark matter, and it has been extensively studied from various aspects such as the structure formation and baryogenesis [1–6]. Interestingly, sterile neutrino dark matter decays into an active neutrino and an X-ray photon through mixing with active neutrinos. So far, the corresponding X-ray line has not been observed, which places severe constraints on the mixing angle, or equivalently, its sterile Yukawa couplings.

The smallness of the neutrino Yukawa couplings can be partially understood by a simple Froggatt–Nielsen (FN) type flavor model [7] or the split seesaw mechanism [8]. One of the interesting features of these models is that the beauty of the seesaw formula [9], which relates the light neutrino masses to the ratio of the electroweak scale to the GUT (or B–L) scale, is preserved even for a split mass spectrum of the right-handed neutrinos. However, the X-ray observation requires the sterile neutrino dark matter to be more long-lived than naively expected based on the seesaw formula, and the gap becomes acute for a heavier mass. As we shall see shortly, for the sterile neutrino mass of 10 keV, the corresponding neutrino Yukawa couplings must be more than two orders of magnitude smaller than estimated based on the seesaw formula. We call this fine-tuning as “the longevity problem” of the sterile neutrino dark matter.

Taken at a face value, the longevity problem suggests an extended structure of the theory, such as an additional symmetry forbidding the neutrino Yukawa couplings. In particular, it requires a slight deviation from the seesaw formula for the sterile neutrino dark matter. In fact, it is well known that, if the sterile neutrino comprises all the dark matter, its contribution to the light neutrino mass must be negligible in order to satisfy the X-ray bounds [10,11]. The point of this Letter is to take the observational constraint seriously and construct theoretical models that could realize both the required split mass hierarchy and the longevity simultaneously by a single flavor symmetry.1

In this Letter we show that the longevity problem can be solved naturally if one or more of the B–L Higgs fields is charged under a flavor symmetry which also realizes the split mass spectrum, $M_1 \ll M_2$. The main difference from the simple FN model is that the scalar charged under the flavor symmetry has a non-zero B–L charge, and we call such mechanism achieving the split mass spectrum for the right-handed neutrinos with a sufficiently long lifetime as split flavor mechanism. The split flavor mechanism works well for both continuous and discrete flavor symmetries, and we provide several examples which solve the longevity problem and predict the X-ray flux just below the current bound.

2. Longevity problem

We consider an extension of the SM with three right-handed neutrinos, and assume the seesaw mechanism [9] throughout this Letter. The relevant interactions for the seesaw mechanism are given by

1 In Ref. [12] it was shown that the mass spectrum and the mixing angles in the so-called νSM [10] can be realized by introducing multiple flavor symmetries such as $Q_6$, $Z_2$, and $Z_3$ as well as four additional SM singlet scalars.
\[ \mathcal{L} = i\bar{N}_1 \gamma^\mu \partial_\mu N_1 - \left( \lambda_{1\alpha} \bar{N}_2 L_\alpha H + \frac{1}{2} M_1 \bar{N}_1^\dagger N_1 + \text{h.c.} \right), \]

where \( N_1, L_\alpha \) and \( H \) are the right-handed neutrino, lepton doublet and Higgs scalar, respectively, \( \lambda \) denotes the generation of the right-handed neutrinos, and \( \alpha \) runs over the lepton flavor, \( e, \mu \), and \( \tau \). The sum over repeated indices is understood. Here we adopt a basis in which the right-handed neutrinos are mass eigenstates, and \( M_1 \) is set to be real and positive. If there is a \( U(1)_{B-L} \) gauge symmetry, the breaking scale \( M \) is tied to the right-handed neutrino mass, as long as the coupling of the \( B-L \) Higgs to the right-handed neutrinos is not suppressed.

Integrating out the massive right-handed neutrinos yields the seesaw formula for the light neutrino mass:

\[
(m_\nu)_{\alpha\beta} = \lambda_{1\alpha} \lambda_{1\beta} v^2 / M_1,
\]

where \( v \equiv \langle H^0 \rangle \simeq 174 \text{ GeV} \) is the vacuum expectation value (VEV) of the Higgs field. The solar and atmospheric neutrino oscillation experiments showed that there are two neutrinos having small non-zero masses, and the mass splittings are given by \( \Delta m^2_{\text{sol}} \approx 8 \times 10^{-5} \text{ eV}^2 \) and \( \Delta m^2_{\text{atm}} \approx 2.3 \times 10^{-3} \text{ eV}^2 \). The seesaw mechanism then suggests that a typical mass scale of the right-handed neutrinos or the \( B-L \) breaking scale is around \( 10^{15} \text{ GeV} \), close to the GUT scale, for \( \lambda_{1\alpha} \sim 1 \). Furthermore, the baryon asymmetry of the Universe can be generated via leptogenesis by out-of-equilibrium decays of such heavy right-handed neutrinos [13].

The above argument does not necessarily mean that all the right-handed neutrinos should have a mass of \( 10^{15} \text{ GeV} \), and one (or more) of them could be much lighter than the others. In particular, if the lightest one is stable in a cosmological time scale, it will contribute to the dark matter density. Thus an interesting scenario is that sterile neutrinos have a split mass spectrum \( M_1 \ll M_{2,3} \) so that the lightest one becomes dark matter while the other two implement leptogenesis. Intriguingly, this may explain why there are three generations, as emphasized in Ref. [8].

In the simple FN model or the split seesaw mechanism, \( N_i \) transforms differently from \( N_1 (i = 2, 3) \) under some symmetry or has an exponentially different localization property due to slightly different bulk masses, respectively. The mass and Yukawa couplings of the lightest right-handed neutrino \( N_1 \) are then suppressed as

\[
M_1 = x^2 M,
\]

\[
|\lambda_{1\alpha}| = x_\alpha,
\]

where \( x \ll x_\alpha \ll 1 \) represents the suppression factor, and \( M \) is the \( U(1)_{B-L} \) breaking scale. The relation \( x \sim x_\alpha \) arises from the assumption that the suppression mechanism is independent of the \( U(1)_{B-L} \) symmetry and its breaking. The light neutrino masses are still related to the ratio of the electroweak scale to the GUT (or \( B-L \)) scale, since the dependence on \( x \) and \( x_\alpha \) is canceled in the seesaw formula (2) as long as \( x \sim x_\alpha \).

On the other hand, the mixing angle between \( N_1 \) and active neutrinos is given by

\[
\theta^2 \equiv \frac{\sum |\lambda_{1\alpha}|^2 v^2}{M_1^2} = 10^{-5} x^2 \left( \frac{m_{\text{seesaw}}}{0.1 \text{ eV}} \right) \left( \frac{M_1}{10 \text{ keV}} \right)^{-1},
\]

where we have defined \( \epsilon^2 \equiv \sum x_\alpha^2 / x^2 \), and \( m_{\text{seesaw}} \) denotes the typical neutrino mass induced by the seesaw mechanism,

\[
m_{\text{seesaw}} = \frac{v^2}{M} \approx 0.03 \text{ eV} \left( \frac{M}{10^{15} \text{ GeV}} \right)^{-1}.
\]

This requires a deviation from the seesaw formula (2) for the sterile neutrino dark matter \( N_1 \), and the gap becomes acute for a heavier \( M_1 \). Note that the muon alpha bounds on \( M_1 \) reads \( M_1 \geq 8 \text{ keV} \) (99.7% C.L.), assuming the non-resonant production for the sterile neutrino dark matter. Therefore \( \epsilon \) must be much smaller than unity, which implies the neutrino Yukawa couplings \( \lambda_{1\alpha} \) should be suppressed by about \( \epsilon \) with respect to that estimated from the seesaw formula. For instance, for \( M_1 = 10 \text{ keV} \), we need \( \epsilon \) smaller than \( 2 \times 10^{-3} \). If \( x_\alpha / x \) takes a value of order unity randomly as in the neutrino mass anarchy [22,23], it would require a fine-tuning of order \( \epsilon^3 \sim 10^{-8} \). We call this fine-tuning problem as the longevity problem. Importantly, the problem cannot be resolved in the split seesaw mechanism or the simple FN model. As we shall see in the next section, the split mass spectrum as well as the required longevity can be naturally explained if one or more of the \( B-L \) Higgs is charged under a flavor symmetry; the key is to combine the flavor symmetry with the \( B-L \) symmetry.

Fig. 1. X-ray bounds on the mixing angle \( \sin^2 2\theta_1 \) as a function of the sterile neutrino mass \( M_1 \). In the left panel, the dashed green lines show the value of \( \sin^2 2\theta_1 \) estimated by Eq. (5) for \( \epsilon = 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1} \) and 1, from bottom to top, respectively. The upper-right (pink) shaded region is excluded by the X-ray observations [4], while the upper-left (yellow) shaded region is excluded by the dark matter overproduction via the Dodelson-Widrow mechanism [14,15]. Note that the yellow region becomes viable if there is a late-time entropy production. The dotted blue line shows the analytic fit to the X-ray constraint given by Eq. (7). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this letter.)

Through the mixing \( \theta_1 \), the sterile neutrino decays into three active neutrinos, and also radiatively into active neutrino plus photon. The latter process is strongly constrained by the non-observation of the corresponding X-ray line [4] (see also Refs. [16–18]), leading to a tight upper bound on the mixing angle as shown Fig. 1. The bound can be conveniently parameterized by

\[
\sin^2 2\theta_1 \lesssim 1.0 \times 10^{-10} \left( \frac{M_1}{10 \text{ keV}} \right)^{-5}.
\]

Therefore, \( \epsilon \) should be much smaller than unity to satisfy the X-ray bound for \( M_1 \gtrsim 1 \text{ keV} \):

\[
\epsilon \lesssim 2 \times 10^{-3} \left( \frac{m_{\text{seesaw}}}{0.1 \text{ eV}} \right)^{-1} \left( \frac{M_1}{10 \text{ keV}} \right)^2.
\]

The bound is relaxed for the production from the singlet Higgs decay [19,20] or the resonant production which works in the presence of large lepton asymmetry [21].
problem. We consider an extension of the SM with three right-handed neutrinos. As reference values we take two B–L Higgs fields, where we have taken $n = 3$ and $\Lambda = M_\psi$ under the assumption that $\Phi$ and $\Phi'$ have VEVs of a similar size.

3. Split flavor mechanism

In this section, we present a modified seesaw model which realizes the split mass spectrum for $N_1$ while solving the longevity problem. We consider an extension of the SM with three right-handed neutrinos $N_i = (N_{i1}, N_{i2})$ for $i = 2, 3$, the $U(1)_{B–L}$ gauge symmetry, and two B–L Higgs fields $\Phi$ and $\Phi'$ whose VEVs provide masses to the sterile neutrinos. The reason why two B–L Higgs fields are needed will be clarified soon. Here we adopt a flavor basis for $N_1$, but the mixing between $N_1$ and $N_i$ is suppressed in the models considered below. We will introduce a flavor symmetry, under which only the fields in the seesaw sector are charged, and the SM fields are assumed to be neutral. The role of the flavor symmetry is to suppress both the mass and mixings of $N_1$ to satisfy the X-ray bound (7), and the key is to assign a flavor charge on one (or more) of the B–L Higgs fields. As reference values we take $M_1 \approx 10$ keV and $M_2 \approx 10^{14}$–$10^{15}$ GeV, but it is straightforward to further impose a usual FN flavor symmetry, e.g., in order to make $N_2$ much lighter than $N_3$.

3.1. Non-supersymmetric case

We adopt a $Z_4$ flavor symmetry under which only $\Phi'$ and $N_1$ are charged while the others are singlet:

\[
\begin{array}{ccccccc}
\text{U(1)$_{B–L}$} & \Phi & \Phi' & N_1 & N_i & L_a & H \\
Z_4 & 2 & -2n & 1 & -1 & -1 & 0 \\
\end{array}
\]

with $n$ being a positive integer, and $i = 2, 3$. Then the seesaw sector is described by

\[
-\Delta L = \frac{1}{2} \kappa_{ij} \Phi^a \bar{N}_{ij}^a N_i \lambda_{ij} \bar{N}_{i1} L H + \frac{1}{2} \kappa_1 (\Phi^{2n-1} \Phi'^{2})^a N_1^a \\
+ \frac{\lambda}{2} (\Phi \Phi')^a \bar{N}_{i1} L H + \text{h.c.},
\]

(9)

for a cut-off scale $\Lambda$. Here $\kappa_{ij}, \kappa_1, \lambda_i$ and $\lambda$ are numerical coefficients of order unity, and we have dropped the lepton flavor indices. Note that the term $\Phi^{n+1} \Phi' N_i N_i$ has been omitted as it can be removed by redefining $N_1$ without any significant effects on the above interactions.

The $U(1)_{B–L}$ gauge symmetry is spontaneously broken when $\Phi$ and $\Phi'$ develop a non-zero VEV. Here we assume $\langle \Phi \rangle \gtrsim \langle \Phi' \rangle$. As a result, the mass of the two heavy right-handed neutrinos is set by $M = \langle \Phi \rangle$, and the light neutrino masses are nicely explained by the seesaw mechanism. The above neutrino interactions lead to the mass and mixing of the $N_i$ as

\[
M_1 \approx \frac{M}{\Lambda} \left(\frac{\langle \Phi' \rangle}{\Lambda}\right)^2 M_1,
\]

(10)

\[
\lambda_{1\alpha} \approx \frac{M}{\Lambda} \left(\frac{\langle \Phi' \rangle}{\Lambda}\right),
\]

(11)

implying $\epsilon \approx M/\Lambda$. Therefore the suppression of $\epsilon$ is achieved for $M \ll \Lambda$, and consequently the active-sterile neutrino mixing is estimated to be

\[
\theta_1^2 \approx 2 \times 10^{-12} \left(\frac{m_{\text{seesaw}}}{0.1 \text{ eV}}\right) \left(\frac{M_1}{10 \text{ keV}}\right)^{-1} \left(\frac{M}{10^{15} \text{ GeV}}\right)^2,
\]

(12)

where we have set $\Lambda$ to be the Planck scale, $M_p = 2.4 \times 10^{18}$ GeV, in the second equality. Note that the mixing angle depends on $n$ only through $M_1$. For instance, in the case of $n = 3$, $M_1$ is around 10 keV when both $\Phi$ and $\Phi'$ have a VEV around $10^{15}$ GeV. Fig. 2 shows the property of $N_1$ for the case with $n = 3$, assuming that $\Phi$ and $\Phi'$ have VEVs of a similar size. Also, $M_1 \sim 10$ keV can be realized for $n = 1$ or 2 if $\langle \Phi' \rangle$ is at an intermediate scale, which is possible because there is no dynamical reason to relate $\langle \Phi \rangle$ to $\langle \Phi' \rangle$ in contrast to supersymmetric cases.

It is possible to consider a general discrete symmetry $Z_k$ under which only $\Phi'$ and $N_1$ are charged. A proper $Z_k$ charge assignment makes $N_1$ have a small Yukawa coupling induced from the term $(\Phi^a \Phi'^b)^* N^a_1 N^b_1 L H$ after B–L breaking. Here $\Phi'$ carries a B–L charge equal to $-2a/b$ for coprime positive integers $a$ and $b$. Then it is obvious that $M_1$ always receives contribution from $\Phi(\Phi^a \Phi'^b)^* N^a_1 N^b_1$. If it is the dominant contribution, one obtains $\epsilon \sim 1$ as in the simple FN model, and thus the longevity problem is not solved. This holds also when one uses a global $U(1)$ instead of $Z_k$. We note that a suppression of $\epsilon$ can be achieved by taking a $Z_k$ charge assignment such that $N_1$ gets a mass dominantly either from $(\Phi^a \Phi'^b)^* N^a_1 N_1$ or from $\Phi(\Phi^a \Phi'^b)^* N^a_1 N_1$.

3.2. Supersymmetric case

The seesaw mechanism can be embedded into a supersymmetric framework. For the anomaly cancellation, $\Phi$ and $\Phi'$ must be vector-like under $U(1)_{B–L}$. Interestingly enough, it is then possible to suppress $M_1$ as well as the active-sterile neutrino mixing by both supersymmetry (SUSY) breaking effects and a flavor symmetry. We will also show that a discrete R-symmetry can do the job.

3.2.1. Discrete flavor symmetry

Let us first consider a $Z_k$ flavor symmetry with $k \geq 3$, under which only $\Phi'$ and $N_1$ transform non-trivially and the others are neutral:

\[
\begin{array}{ccccccc}
\text{U(1)$_{B–L}$} & \Phi & \Phi' & N_1 & N_i & L_a & H \\
Z_k & -2 & 2 & 1 & 1 & -1 & 0 \\
\end{array}
\]

with $H_2$ being the up-type Higgs doublet superfield. Such discrete symmetry acting on one of the B–L Higgs fields was considered in the B–L Higgs inflation models [24]. Note that $N_1, \Phi$ and $\Phi'$ are left-chiral superfields, and in particular, the fermionic component of $N_i$ is the left-handed anti-neutrino. That is why the
B–L charge assignment on these fields is different from the non-supersymmetric case.

With the above charge assignment, the relevant terms in the Kähler and super-potentials of the seesaw sector are given by

\[ \Delta K = \frac{\Phi^*}{A} N_1 N_1 + 2 A^{-3} N_1 N_1 + \text{h.c.,} \]
\[ \Delta W = 2 \frac{\Phi N_1 N_1 + N_1 L H_u + (\Phi \Phi')^{k-1} - 2 A^{-2k-4} N_1 L H_u}{A^{2k-4} N_1 N_1}, \]

where we have omitted coupling constants of order unity.\(^3\) Though we have not considered here, one may impose a U(1)\(_R\) symmetry under the assumption that it is broken by a small constant term under the assumption that it is broken by a small constant term in the superpotential, i.e. by the gravitino mass \(m_{3/2}\), as we shall see shortly, in such case, both of the terms in \(\Delta K\) can be further suppressed by \(m_{3/2}\) if the superpotential is to possess the term \(\Phi N_1 N_1\). Note here that the gravitino mass represents the explicit \(U(1)_R\) breaking by two units.

To examine the property of sterile neutrino dark matter, it is convenient to integrate out the \(U(1)_{B-L}\) sector. The \(U(1)_{B-L}\) is broken along the D-flat direction \(|\Phi|^2 = |\Phi'|^2 = M^2\), which is stabilized by higher dimension operators, or by a radiative potential induced by the \(\lambda_i\) interaction. For \(M\) much larger than the gravitino mass \(m_{3/2}\), the effective theory of neutrinos is written as

\[ \Delta W_{\text{eff}} = \frac{1}{2} k M N_1 N_1 + \lambda_1 N_1 L H_u + \frac{1}{2} M_1 N_1 N_1 \]
\[ + \lambda_{1a} N_1 L H_u, \]

at energy scales around and below \(M\), where the sterile neutrino \(N_1\) obtains

\[ M_1 = \frac{m_{3/2} A^3}{\Lambda^3} + M^{2k-3} A^{2k-4}, \]
\[ \lambda_{1a} = \frac{m_{3/2}}{A} + M^{2k-2} A^{2k-4}. \]

omitting numerical coefficients of order unity. Here the terms proportional to \(m_{3/2}\) arise from \(\Delta K\) after redefining \(N_1\) to remove mixing terms \(N_1 N_1\) in the effective superpotential. In contrast to the non-supersymmetric case, there are two important effects here. One is the holomorphic nature of the superpotential, and the other is the SUSY breaking effects represented by the gravitino mass.

Depending on the values of \(M\), \(\lambda_{1a}\), \(m_{3/2}\) and \(k\), there are various possibilities. To simplify our analysis, let us focus on the case of the reference values, \(M \sim 10^{15}\) GeV and \(\lambda_{1a} \approx M_p\). Then \(M_1 \sim 10^{10}\) keV is realized for \(m_{3/2} \lesssim O(100)\) TeV and \(k \gtrsim 5\), for which the neutrino Yukawa coupling \(\lambda_{1a}\) receives the dominant contribution from the SUSY breaking effect, i.e., from the first term in Eq. (16). Note also that \(M_1\) is determined entirely by the SUSY breaking effect for \(k \gtrsim 6\). In the following we consider \(m_{3/2} \sim 100\) TeV and \(k \gtrsim 6\). The \(\epsilon\) parameter and active-sterile neutrino mixing angle then read

\[ \epsilon \approx 4 \times 10^{-4} \left( \frac{m_{3/2}}{100 \text{ TeV}} \right)^{3/2} \left( \frac{M_1}{10 \text{ keV}} \right)^{-1/2}, \]
\[ \theta_{1i}^2 = \frac{\epsilon^2 m_{\text{seesaw}}}{M_1} \approx 10^{-12} \left( \frac{m_{\text{seesaw}}}{0.1 \text{ eV}} \right) \left( \frac{m_{3/2}}{100 \text{ TeV}} \right)^{3/2} \left( \frac{M_1}{10 \text{ keV}} \right)^{-1/2}. \]

Thus, the observational constraint (7) is naturally satisfied if the gravitino mass is smaller than or comparable to 100 TeV. In particular, the predicted X-ray flux is just below the observational bound for \(m_{3/2} \sim 100\) TeV. See Fig. 3, where the contours of \(M_1\) and \(\theta_{1i}^2\) are shown in the \((M, m_{3/2})\) plane. On the other hand, the squarks and sleptons acquire soft SUSY breaking masses in the range between about \(m_{3/2}/8\pi^2\) and \(m_{3/2}\), depending on mediation mechanism. It is interesting to note that the gravitino mass around 100 TeV leads to TeV to sub-PeV scale SUSY, which can accommodate a SM-like Higgs boson at 126 GeV within the minimal supersymmetric SM (MSSM).

Lastly we comment on the case with an approximate global \(U(1)_R\) broken by a constant superpotential term. The neutrino interactions are then further constrained. For instance, let us consider the case where \(N_1\) and \(L_{\alpha}\) have the same \(R\) charge equal to one while \(\Phi, \Phi', H_u\) are neutral. Then both terms in \(\Delta K\) are further suppressed by the gravitino mass. As a result, the sterile neutrino mass as well as the neutrino Yukawa couplings are determined by the ratio of the \(B-L\) breaking scale to the cut-off scale, and the effect of SUSY breaking is negligibly small. That is to say, \(M_1\) and \(\lambda_{1a}\) receive the dominant contributions from the second terms in (15) and (16), respectively. For the reference values \(M \sim 10^{15}\) GeV and \(\lambda_{1a} = M_p\), \(k\) must be equal to 5 to realize \(M_1 \sim 10^7\) keV unless \(m_{3/2}\) is extremely heavy (say, \(10^{11}\) GeV or heavier). Then the neutrino Yukawa couplings will become extremely small so that sterile neutrino dark matter becomes practically stable and the predicted X-ray flux is negligibly small. Although not pursued here, it may be interesting to consider the case of \(k < 5\) where a sterile neutrino dark matter is much heavier than 10 keV.

3.2.2. Discrete R symmetry

Next let us consider a case of discrete R symmetry. The discrete R symmetry has been extensively studied from various cosmological and phenomenological aspects. See, e.g., Refs. [28–33]. Now we

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\(^3\) Instead of the discrete symmetry, one can take a global \(U(1)\) symmetry under which \(\Phi\) and \(N_1\) have the same charge and the other fields are neutral. Then the terms in \(\Delta K\) are still allowed while the last two terms in \(\Delta W\) are forbidden. The Nambu-Goldstone boson associated with \(U(1)\) may contribute to dark radiation [25–27].

\(^4\) This may provide a motivation to consider a SUSY at around 100 TeV, which is consistent with the recent discovery of the SM-like Higgs boson of mass \(\sim 126\) GeV. If the SUSY breaking was much higher, the sterile neutrino could not be dark matter because of its too short lifetime. Note that the decay rate is proportional to \(M_1^2\).
show that the split flavor mechanism can be implemented by the discrete $R$ symmetry with the following charge assignment,

\[
\begin{array}{cccccc}
\text{charge} & \phi & \phi' & N_1 & N_i & L_u & H_u \\
U(1)_{3L} & -2 & 2 & 1 & 1 & -1 & 0 \\
Z_R & 0 & p & q & 1 & 1 & 0 \\
\end{array}
\]

where $p$ and $q$ are integers mod $k$. To simplify our analysis, we assume that the cut-off scale for higher dimensional operators is given by the Planck scale, $M_p$, and the $B-L$ breaking scale $M$ is about $10^{15}$ GeV. The gravitino mass is assumed to be below PeV scale.

Note that the discrete $Z_R$ symmetry ($k > 3$) is explicitly broken by the constant term in the superpotential, $(W) \simeq M_3/2M_p^2$. Therefore, the mass $M_1$ and neutrino Yukawa couplings $\lambda_{14}$ generically receive two contributions; one is invariant under $Z_R$, and the other is not invariant and is proportional to the gravitino mass.

The sterile neutrino mass $M_1 \sim 10$ keV is numerically close to $M^2/M^4_p$ or $m_{32}/M^3_p$, and the mass of this order can be generated if one or more of the following operators are allowed:

\[
\Delta K = \frac{\Phi \Phi'^*}{M^3_p} - N_1 N_1 + \text{h.c.},
\]

\[
\Delta W = \frac{\Phi \Phi \Phi'^*}{M^3_p} N_1 N_1 \quad \text{or} \quad m_{32}/M^3_p N_1 N_1.
\]

Similarly, the neutrino Yukawa coupling of the desired magnitude can be induced from the following operators,

\[
\Delta K = \frac{\Phi'^*}{M_p} - N_1 N_1 + \text{h.c.},
\]

\[
\Delta W = \frac{(\Phi \Phi')^2}{M^4_p} N_1 L H_u \quad \text{or} \quad \frac{m_{32}}{M_p} N_1 L H_u.
\]

In order for one or more of the above operators to give the dominant contribution to $M_1$ and $\lambda_{14}$, the following operators must be forbidden by the discrete $R$-symmetry:

\[
\Delta K_{\text{forbidden}} = \frac{\Phi'^*}{M_p} N_1 N_1 + \text{h.c.},
\]

\[
\Delta W_{\text{forbidden}} = \Phi N_1 N_1 + \frac{\Phi \Phi'^*}{M^3_p} N_1 N_1 + \frac{(\Phi \Phi')^2}{M^4_p} N_1 N_1 + N_1 L H_u + \frac{\Phi \Phi'^*}{M^3_p} N_1 L H_u,
\]

which puts constraints on $p$ and $q$.

To summarize, we need to find a set of $(k, p, q)$ satisfying

\[
2p - 2q \equiv 0 \quad \text{or} \quad 3p + 2q \equiv 2 \quad \text{or} \quad p + 2q \equiv 0,
\]

\[
p - q - 1 \equiv 0 \quad \text{or} \quad 2p + q + 1 \equiv 2 \quad \text{or} \quad q + 1 \equiv 0,
\]

\[
p - 2q \not\equiv 0, \quad 2q \not\equiv 2, \quad q + 1 \not\equiv 2, \quad p + 2q \not\equiv 2,
\]

\[
p + q + 1 \not\equiv 2, \quad 2p + 2q \not\equiv 2,
\]

where all the equations are mod $k$. Some of the solutions of the above conditions are$^5$

\[
(k, p, q) = (5, 2, 2), (5, 4, 3), (7, 3, 2), (7, 5, 4), (7, 5, 5), \quad (7, 6, 6), \ldots
\]

$^5$ If we forbid a SUSY mass $\Phi \Phi'$ in the superpotential, the solutions with $p = 2$ should be excluded.

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Fig. 4. Contours of the sterile neutrino mass $M_1$ (solid (blue in the web version)) and the mixing angle $\theta^2_1$ (dashed (green in the web version)) in the $M$--$m_{32}/M_p$ plane for the case of the discrete $R$ symmetry.

In fact there is no solution for which both $M_1$ and $\lambda_{14}$ are generated by the $Z_R$ invariant operators. That is to say, either or both of them should be generated by the SUSY breaking effect proportional to the gravitino mass.

Let us focus on the case of $(k, p, q) = (5, 4, 3)$. Then the relevant terms in the superpotential are given by

\[
\Delta W = \frac{1}{2} \Phi N_1 N_1 + N_1 L H_u + \frac{1}{2} \frac{m_{32}}{M_p} N_1 N_1 + \frac{(\Phi \Phi')^2}{M^4_p} N_1 L H_u,
\]

where we have dropped numerical coefficients of order unity. The other interactions in the Kähler and super-potentials are either forbidden or irrelevant for the following discussion. The mass and neutrino Yukawa couplings for $N_1$ are given by

\[
M_1 \approx 10 \text{ keV} \left( \frac{m_{32}}{100 \text{ TeV}} \right) \left( \frac{M}{10^{15} \text{ GeV}} \right)^3,
\]

\[
\lambda_{14} \approx 10^{-14} \left( \frac{M}{10^{15} \text{ GeV}} \right)^4,
\]

from which one finds

\[
\epsilon \simeq 3 \times 10^{-4} \left( \frac{m_{32}}{100 \text{ TeV}} \right)^{-2} \left( \frac{M}{10^{15} \text{ GeV}} \right)^3,
\]

using the D-flat condition, $(\Phi) = (\Phi') = M$. Therefore the mass $M_1$ is close to 10 keV and $\epsilon \sim 10^{-3}$ for the reference values $M = 10^{15}$ GeV and $\Lambda = M_p$. Finally, the mixing angle reads

\[
\theta^2_1 \approx 2 \times 10^{-12} \left( \frac{m_{\text{seesaw}}}{0.1 \text{ eV}} \right) \left( \frac{M_1}{10 \text{ keV}} \right)^{-3} \left( \frac{m_{32}}{100 \text{ TeV}} \right)^{-3}.
\]

We show the contours of $M_1$ and the mixing angle $\theta^2_1$ in the $M$--$m_{32}/M_p$ plane in Fig. 4. It is interesting to note that $m_{32}/M_p \sim 100$ TeV and $M \sim 10^{15}$ GeV lead to the sterile neutrino mass $M_1 \sim 10$ keV with the predicted X-ray line flux just below the current bound.
4. Discussion and conclusions

We have so far focused on the mass and mixing angles of the sterile neutrinos. In order for the lightest sterile neutrino $N_1$ to account for the observed dark matter, a right amount of $N_1$ must be produced in the early Universe. One possibility is that the $N_1$ is produced via the s-channel exchange of the B–L gauge boson $[8,34]$. The number to entropy ratio of the sterile neutrino produced by this mechanism is roughly estimated as

$$\frac{n_{N_1}}{s} \sim 10^{-4} \left( \frac{g_\ast}{100} \right)^{1/2} \left( \frac{M}{10^{15} \text{ GeV}} \right)^{-4} \left( \frac{T_R}{5 \times 10^{13} \text{ GeV}} \right)^3,$$

where $g_\ast$ counts the relativistic degrees of freedom at the reheating, and $T_R$ denotes the reheating temperature. Also, a right amount of the baryon asymmetry can be created via thermal leptogenesis due to the two heavy right-handed neutrinos $N_2$ and $N_3$ for such high reheating temperature $[35,36]$.$^6$

In this Letter we have quantified the longevity problem and proposed the split flavor mechanism as a possible solution. In this mechanism, we have introduced a single flavor symmetry (or discrete $R$ symmetry) under which one or more of the B–Liggs is charged. As a result, the split mass spectrum for the sterile neutrinos as well as the longevity required for the lightest sterile neutrino dark matter are realized. The key is to combine the B–L symmetry with the flavor symmetry. We have provided several examples in which the lightest sterile neutrino has a mass of $O(1–10) \text{ keV}$ and the predicted X-ray flux is just below the current bound. Therefore it may be possible to test our models in the future X-ray observations.

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$^6$ The thermal leptogenesis in the neutrino mass anarchy hypothesis was studied in Ref. [37].