Entanglement of nanomechanical oscillators and two-mode fields induced by atomic coherence

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We propose a scheme via three-level cascade atoms to entangle two optomechanical oscillators as well as two-mode fields. We show that two movable mirrors and two-mode fields can be entangled even for bad cavity limit. We also study entanglement of the output two-mode fields in frequency domain. The results show that the frequency of the mirror oscillation and the injected atomic coherence affect the output entanglement of the two-mode fields.

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I. INTRODUCTION

Entanglement among mesoscopic even macroscopic systems have always been an attractive topic since the birth of quantum mechanics. Schrödinger’s cat paradox is a well known example of it (entanglement between macroscopic cat state and microscopic state). Cavity optomechanical system is an important candidate for the study of quantum mechanical features at mesoscopic even macroscopic scales. Recently, there has been considerable interest in studying entanglement in mesoscopic systems [1-9]. Refs. [3, 4] investigated the optomechanical entanglement between the field of an optical cavity and a vibrating cavity end-mirror. Many proposals were put forward to entangling two mirrors in a ring cavity [2], entangling two mirrors of two independent optical cavities driven by a pair of entangled light beams [5], entangling two mirrors of a linear cavity driven by a classical laser field [6], entangling two separated nanomechanical oscillators by injecting broad band squeezed vacuum light and laser light into the ring cavity [7]. Most recently, Ref. [8] observed strong coupling between a micromechanical resonator and an optical cavity field, which could push forward the possibility of entangling two mirrors via optical cavity field.

On the other hand, atomic medium play important role in the interaction of cavity electrodynamics. When the atomic medium is trapped in a cavity with micromechanical oscillator, some interesting phenomena have been revealed [10-13]. Genes [10] suggested cooling ground-state of micromechanical oscillators by resonant coupling of the mirror vibrations to a two-level atomic bath. Ian [11] found that two-level atoms effectively enhance the radiation pressure of the cavity field upon the oscillating mirror. Most recently, Hammerer et al [12] and Wallquist [13] have shown the possibility to achieve strong cavity-mediated coupling between a single microscopic atom and a macroscopic mechanical oscillator. In addition, Genes [14] studied tripartite entanglement among atom, mirror and cavity field. As we all know, atomic coherence results in many interesting phenomena [13], such as electromagnetically induced transparency (EIT), correlation emission laser (CEL), laser without inversion (LWI). A lot of works have been done on the entanglement directly induced by atomic coherence [14, 20]. When one of the mirrors of the cavity is movable, the atomic coherence effects are not studied before. In this paper, we propose a method to entangle two macroscopic mirrors via microscopic atomic coherence. When atomic beam with cascade configuration is injected into the two-mode cavity, the two-mode fields as well as the optomechanical oscillator are entangled. In Refs. [12, 13], authors treat the cavity field as a quantum bus and give an effective coupling between oscillator and the single atomic motion. Instead, we directly treat hybrid system. Our study show that the initial atomic coherence and the frequency of the mirror’s motion affect the entanglement of the output fields.

II. MODEL AND THE HAMILTONIAN OF THE SYSTEM

The system under study is a two mode cavity with one fixed partially transmitting mirror and two movable perfectly reflecting mirrors, sketched in Fig. 1. The atomic medium with cascade configuration is injected into the cavity and interacts with two-mode cavity fields with detuning $\Delta_i$, respectively $(i = 1, 2)$. The Hamiltonian of the hybrid system reads

$$H = \sum_{j=1,2} \hbar \omega_j a_j^\dagger a_j + \sum_{j=1,2} \frac{\hbar \omega_{m_j}}{2}(P_j^2 + Q_j^2) + \sum_{j=1,2} \hbar \chi_j Q_j a_j^\dagger a_j \sigma_{ij}^+ + \hbar g_1 \sigma_{ai} a_1^\dagger a_2^\dagger + \hbar g_2 \sigma_{bi} a_2^\dagger a_2 + \hbar c.$$

The first term describes the energy of the two cavity modes, with lowering operator $a_j$, cavity frequency $\omega_j$, and the decay rate $\kappa_j$. The second term represents the energy of the two mechanical oscillators at
frequency $\omega_{m_j}$, and $P_j$ and $Q_j$ are their position and momentum operators. The third term describes the two driving lasers with frequency $\omega_{L_i}$ and $\omega_{L_2}$, respectively. The forth is the radiation-pressure coupling with rate $\chi_j = \frac{\omega_j}{2} \sqrt{\frac{\hbar}{m_{m_j}}}$, and last summation describes the energy of the atoms, and the last term is interaction between the atom and the cavity fields, where $\sigma_{ij} = |i\rangle \langle j|$ is the spin operator of the atom.

In interaction picture, we have the Hamiltonian

$$H_1 = \hbar \delta_{\sigma a a} - \hbar 2\sigma_{cc}$$
$$+ \hbar (g_1 \sigma_{ba} a_1^\dagger + g_2 \sigma_{cb} a_2^\dagger + h.c.)$$
$$+ \hbar (\delta_1 - \Delta_1) a_1^\dagger a_1 + \hbar (\delta_2 - \Delta_2) a_2^\dagger a_2$$
$$+ i\varepsilon_1 \hbar (a_1^\dagger - a_1) + i\varepsilon_2 \hbar (a_2^\dagger - a_2)$$
$$+ \frac{\hbar^2 m_a}{2} (P_1^2 + Q_1^2) + \frac{\hbar^2 m_a}{2} (P_2^2 + Q_2^2)$$
$$+ \hbar \chi_1 Q_1 a_1^\dagger a_1 + \hbar \chi_2 Q_2 a_2 a_2$$

where $\Delta_1 = E_a - E_b - \omega_1$, $\Delta_2 = E_b - E_c - \omega_2$, $\delta_1 = E_b - E_c - \omega_1$, $\delta_2 = E_b - E_c - \omega_2$. Thus $\delta_j - \Delta_j$ in the cavity means the detuning between the classical driving fields and the cavity field. The dynamics of the system is determined by the following quantum Langevin equations

$$\dot{Q}_1 = \omega_{m_1} P_1,$$
$$\dot{Q}_2 = \omega_{m_2} P_2,$$
$$\dot{P}_1 = -\chi_1 a_1^\dagger a_1 - \omega_{m_1} Q_1 - \gamma_{m_1} P_1 + \xi_1,$$
$$\dot{P}_2 = -\chi_2 a_2^\dagger a_2 - \omega_{m_2} Q_2 - \gamma_{m_2} P_2 + \xi_2,$$
$$a_1 = -[\kappa_1 + i\Delta_1] a_1 - i g_1 \sigma_{ba} + \varepsilon_1 + \sqrt{2\kappa_1} \sigma_{a_{in}} a_1$$
$$a_2 = -[\kappa_2 + i\Delta_2] a_2 - i g_2 \sigma_{cb} + \varepsilon_2 + \sqrt{2\kappa_2} \sigma_{a_{in}} a_2,$$
$$\dot{\sigma}_{ba} = -(\gamma + i\delta_1) \sigma_{ba} - i g_1 a_1 (\sigma_{bb} - \sigma_{aa}) + i g_2 a_2^\dagger \sigma_{ca},$$
$$\dot{\sigma}_{cb} = -(\gamma + i\delta_2) \sigma_{cb} - i g_1 a_1^\dagger \sigma_{ca} - i g_2 a_2 (\sigma_{cc} - \sigma_{bb}),$$

where $\Delta_1 = \delta_j - \Delta_j + \gamma_j \omega_j (j = 1, 2)$. The quantum Brownian noise $\xi_1$ and $\xi_2$ are from the coupling of the movable mirrors to their own environment. They are mutually independent with zero mean values and have the following correlation function at temperature $T$

$$\langle \xi_j (t) \xi_k (t') \rangle = \frac{\delta_{jk} \gamma_m}{\omega_m} \int \frac{d\omega}{2\pi} e^{-i\omega (t-t')} \omega [1 + \coth(\frac{\hbar \omega}{2\kappa_B T})],$$

$$j, k = 1, 2.$$ (4)

The two cavity modes decay at the rate $\kappa_1$ and $\kappa_2$, and $a_{1in}$ ($a_{2in}$) is the vacuum radiation input noise, whose correlation functions are given by

$$\langle a_j (t) \langle a_{jin} (t') \rangle\rangle = N \delta (t - t'),$$
$$\langle a_{jin} (t) a_{jin} (t') \rangle = (N + 1) \delta (t - t'),$$

where $N = [\exp(\hbar \omega_c / k_B T) - 1]^{-1}$.

In order to obtain the steady state solution of Eq. (4), we calculate the last two equations to the first order in $g_i$ ($i = 1, 2$), i.e., using linear approximation theory [15], which means that in the last equation of (3), for the terms that the $\sigma_{ij}$ multiply a $|a\rangle \langle a|$, we use zero order $\langle \sigma_{ij}\rangle$ substitute of $\sigma_{ij}$. We assume that the atoms are injected into the cavity with the state $\rho_a = \rho_{a_{in}|a\rangle \langle a| + \rho_{a_{in}}|0\rangle \langle 0| + \rho_{a_{in}}|0\rangle \langle 0|$, at injection rate $r_a$. The last two equation of (4) can be rewritten as

$$\dot{\sigma}_{ba} = -(\gamma + i\delta_1) \sigma_{ba} + i g_1 r_a \rho_{a_{in}} a_1 + i g_2 r_a \rho_{a_{in}} a_2^\dagger,$$
$$\dot{\sigma}_{cb} = -(\gamma + i\delta_2) \sigma_{cb} - i g_1 r_a \rho_{a_{in}} a_1^\dagger - i g_2 r_a \rho_{a_{in}} a_2.$$ (6)

By combining it with Eq. (3), we finally have the steady-state mean values of the system as

$$P_1^s = 0, P_2^s = 0,$$
$$Q_1^s = \frac{-\chi_1 |a_1|^2}{\omega_{m_1}}, Q_2^s = \frac{-\chi_2 |a_2|^2}{\omega_{m_2}},$$
$$a_1^s = \frac{s_2^* e_1 + e_1^* u_1}{u_1 u_2 + s_1 a s_2},$$
$$a_2^s = \frac{s_1^* e_2 - e_2^* u_2}{u_1^* u_2 + s_2 a s_1}$$

with

$$u_l = \frac{g_1 g_2 r_a \rho_{a_{in}}}{\gamma + i\delta_l}, l = 1, 2,$$
$$s_{1a} = \kappa_1 + i \Delta_1 - \frac{g_1^2 r_a \rho_{a_{in}}}{\gamma + i\delta_1},$$
$$s_{2c} = \kappa_2 + i \Delta_2 + \frac{g_2^2 r_a \rho_{a_{in}}}{\gamma + i\delta_2}. $$ (8)

From the expressions of (7) and (8), we see that if $g_1 = g_2 = 0$ (no atoms within the cavity), the steady value of $a_j^s$ ($j = 1, 2$) will have the same form with (1), i.e.,

$$a_j^s = \frac{\varepsilon_j}{\kappa_j + i \Delta_j}.$$ If the injected atoms have no coherence between their levels, that is to say, $\rho_{ca} (0) = 0 (u_l = 0)$. By writing the last terms of $s_{1a}$ and $s_{2c}$ into real and imaginary parts, we know that the existence of atomic medium only affect the effective decay rate of the photon.
and the radiation pressure of the cavity field upon the two mirrors. However, if the injected atoms have coherence between the level $|a\rangle$ and $|c\rangle$, i.e., $\rho_{ac}^{(0)} \neq 0$, the two mode fields will be correlated so that the two mirrors will be dependent each other, see (7) and (8).

We write each operators of the system as the sum of their steady-state mean value and a small fluctuation with zero mean value. The fluctuations can be calculated analytically by using the linearization approach of quantum optics [9, 13, 21], provided that the nonlinear effect between the cavity field and the movable mirrors is weak. We can linearize the fluctuations around the steady state using Eqs.(4) and obtain a set of linear quantum Langevin equations for the fluctuation operators. We define $f = (\delta Q_1, \delta P_1, \delta Q_2, \delta P_2, \delta X_1, \delta Y_1, \delta X_2, \delta Y_2, \delta U_1, \delta U_2, \delta V_1, \delta V_2)^T$. A set of linear quantum Langevin can be written as

$$\dot{f} = Af + B$$

where

$$A = \begin{bmatrix}
0 & \omega_{m_1} & 0 & 0 & 0 & -\chi_1 X_1^t & 0 & 0 & 0 & 0 & 0 & 0 \\
-\omega_{m_1} & -\gamma_{m_1} & 0 & 0 & 0 & -\chi_1 Y_1^s & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \omega_{m_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\omega_{m_2} & -\gamma_{m_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\chi_1 X_1^s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\chi_1 X_1^t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\chi_1 Y_1^s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\chi_1 Y_1^t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \chi_2 Y_2^s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\chi_2 Y_2^t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

$$B = (0, 0, 0, \sigma_{\xi_1}, \sqrt{2\kappa_1} \delta X_{1m}, \sqrt{2\kappa_1} \delta Y_{1m}, \sqrt{2\kappa_2} \delta X_{2m}, \sqrt{2\kappa_2} \delta Y_{2m}, 0, 0, 0, 0)^T.$$ We have defined

$$X_j = \frac{1}{\sqrt{2}}(a_j + a_j^\dagger), 
Y_j = \frac{1}{\sqrt{2}i}(a_j - a_j^\dagger), 
j = 1, 2,(10)$$

$$U_1 = \frac{1}{\sqrt{2}}(\sigma_{ba} + \sigma_{ab}), 
U_2 = \frac{1}{\sqrt{2}i}(\sigma_{ba} - \sigma_{ab}),$$

$$V_1 = \frac{1}{\sqrt{2}}(\sigma_{ba} + \sigma_{ab}), 
V_2 = \frac{1}{\sqrt{2}i}(\sigma_{cb} - \sigma_{bc}).$$

A is the drift matrix. The system is stable only if the real part of all the eigenvalues of the matrix A are negative, which is also the requirement of the validity of the linearization method. Because of the atomic cascade form, the system can be considered as an amplifying system, that is to say, for some parameters, the eigenvalues of drift matrix A can have a positive real part. So, we must carefully eliminate the parameters region in order to retain it within the stability conditions. Due to the twelve dimensions of drift matrix A, it is not easy to obtain the analytic expression of the requirement as proposed by Duan [22] and Simon [23]. According to [22], a state is entangled if the summation of the quantum fluctuations in the two EPR-like operators $X$ and $Y$ satisfy the following inequality

$$\langle \Delta X \rangle^2 + \langle \Delta Y \rangle^2 < 2.$$ (11)

A. Entanglement of the nanomechanical oscillators and the two-mode fields within the cavity

We investigate the nature of linear quantum correlations among fields and mirrors by considering the steady state of the correlation matrix of quantum fluctuations in this multipartite system. The quantum noises $\xi$ and $\eta$ are zero-mean quantum Gaussian noises and the dynamics has been linearized, as a consequence, the steady state of the system is a zero-mean multi-partite Gaussian state. We defined $V \gamma(\infty) = \frac{1}{2} [\langle \xi_1(\infty) \xi_j(\infty) \rangle + \langle \eta_1(\infty) \eta_j(\infty) \rangle]$, the element of covariance matrix. One can obtain various correlation information from it, and we will show the entanglement of the two mirrors as well as the two-mode fields after obtaining the covariance matrix.

We now recall entanglement criteria of continuous variable proposed by Duan [22] and Simon [23]. According to [22], a state is entangled if the summation of the quantum fluctuations in the two EPR-like operators $X$ and $Y$ satisfy the following inequality

$$\langle \Delta X \rangle^2 + \langle \Delta Y \rangle^2 < 2.$$ (11)

For the mechanical oscillator we define $X_m = Q_1 + Q_2, Y_m = P_1 - P_2$ while for the two mode fields $X_f = X_1 - X_2, Y_f = Y_1 + Y_2$. The criterion in Eq.(11) can be directly detected in experiment via homodyne measurements [24]. Simon’s criterion has more direct relation
observe that the larger value of the coupling, the deeper existence, the two sideband move towards its middle. In when \( \Delta = 2 \), for a physical state, the covariance matrix must obey the Robertson-Schrödinger uncertainty principle,

\[
V + \frac{i}{2} \beta \geq 0,
\]

where \( \beta = \begin{bmatrix} J & 0 \\ 0 & J \end{bmatrix} \) with \( J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \) when we define vector \( f = (\delta q_1, \delta p_1, \delta q_2, \delta p_2)^T \) for two-mode fields \[24\]. If a state is separable, partial transpose matrix \( \tilde{V} \) (obtained from \( V \) just by taking \( p_j \) in \( -p_j \)) still obey the inequality in \[12\]. The inequality requires that all the symplectic eigenvalues of the transposed matrix are larger than 1/2 in terms of the definition of Eq.(10). The symplectic eigenvalues can be calculated from the square roots of the ordinary eigenvalues of \(- \left( \beta \tilde{V} \right)^2 \) \[20, 27\]. So, if the smallest eigenvalue is smaller than 1/2, the transposed mode is then inseparable. For the two-mode Gaussian states, the violation of the inequality is a sufficient and necessary condition for the existence of entanglement between the transposed mode and the remaining modes.

Neglecting the frequency dependence as it was pointed in \[4\], under the condition of Markovian approximation, the frequency domain treatment is equivalent to the time domain derivation considered in \[10\]. When the stability conditions (all real parts of eigenvalues of matrix \( \Lambda \) are negative) are satisfied, the steady state correlation matrix of the quantum fluctuation meet the following Lyapunov equation \[8\]

\[
AV + V A^T = -D,
\]

where \( D = Diag[0, \gamma_m, (2N+1), 0, \gamma_m, (2N+1), \kappa_1(2N+1), \kappa_1(2N+1), \kappa_2(2N+1), \kappa_2(2N+1), 0, 0, 0, 0] \).

For simplicity, we choose all parameters of the two mirrors and the two mode cavities are the same, for example \( g_1 = g_2, \omega_{m_1} = \omega_{m_2}, \omega_{L_1} = \omega_{L_2}, \) etc.. Then the effective detuning \( \Delta_1 = \Delta_2 = \Delta \). In Fig. 2a, we choose the atoms injected into the cavity with initial state \( |\Psi_0(0)\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |\bar{c}\rangle) \) which is the best coherence of the two levels. We numerically show the entanglement of the movable mirrors as well as the two mode fields induced by atomic coherence. Fig. 2a show that with the increasing of the coupling between cavity and the atoms, the deep squeezing in fluctuation move towards the side of small value of \( \Delta \). As already shown in \[12, 28\], many features of the radiation-pressure interaction in the cavity can be understood by considering that the driving laser light is scattered by the vibrating cavity boundary mostly at the first Stokes \( \omega_0 - \omega_m \) and anti-Stokes \( \omega_0 + \omega_m \). Therefore, the optomechanical interaction will be enhanced when the cavity is resonant with one of the two sidebands, i.e., when \( \Delta = \omega_m \), where entanglement between cavity field and mirror is enhanced. Here, due to the atomic medium existence, the two sideband move towards its middle. In addition, within the region of parameter in Fig. 2a we can observe that the larger value of the coupling, the deeper squeezing in the fluctuation. Undoubtedly, the entanglement is resulted from the interaction between atoms and the fields as well as the atomic level coherence, therefore, the larger coupling \( g_i \) the deeper squeezing within our parameters. The enhancement of \( g_i \) can be substituted by increasing the injection rate of the atoms. Fig. 2b show that the two-mode fields are entangled for nearly the whole region of detuning \( \Delta \) (from 0 to 1). Using the same group of parameters, from the solution of Eq.(13) we reconstruct two-partite covariance matrixes for mirrors and two-mode fields and show their smallest eigenvalues under partial transpose, shown in Fig.3. One can see that Fig.3 are almost the same in shape with Fig.2, that is to say, the entanglement judged with two criteria are equal. By showing the entanglement with two criteria, we prove our correction of calculation and convince the entanglement existence.

For cascade atomic configuration system, the process of radiation two-mode fields are correspondence to parametric-down-conversion process. So, one can easy understand the two-mode entanglement. For fixed mirror, it has been known that two cavity modes are entangled when they interact with cascade three-level atoms \[16\]. Here, we show further that when one mirror is movable, the two-mode entangled continuous-variable state still hold. In our numerical simulation, we find that there is a threshold coupling between atoms and fields (under these group of parameters), below that we have no two entangled mirrors, which means that the appropriate coupling between atoms and cavity is favorable if we would have entangled mirrors. Thus, the radiation fields emitted from cascade atoms can be entangled, and the

![FIG. 2: (Color online) (a) shows the entanglement of the mechanical mirror where \( g_1 = 2 \times 1.5 \times 10^7 H z \) (solid), \( 2 \pi \times 1.7 \times 10^8 H z \) (dash-dot), \( 2 \pi \times 2.0 \times 10^8 H z \) (dashed). (b) plot the entanglement of the two mode fields where \( g_1 = 2 \pi \times 2.2 \times 10^8 H z \). For all the plots, the other parameters are \( L = 5 m m, m = 20 m g, \kappa_1 = \kappa_2 = 2 \pi \times 215 K H z, \omega_{m_1} = \omega_{m_2} = 2 \pi \times 10 M H z \). The wavelength of the laser \( \lambda = 810 m m \) with power \( 10 m W \), and the mechanical quality factor \( Q_l = \frac{2 m g}{k_m} = 6700, \) \( r a = 2000, r = 1.3 M H z, \delta_1 = \delta_2 = 4 M H z, T = 42 \mu K \).](image)
entanglement of the two-mode field can be transferred into the mechanical mirror because of the radiation-pressure interaction. As a consequence, we have both the entanglement of two-mode fields and two mirrors.

B. The output two-mode field entanglement with optomechanical oscillator

Because one can not direct access to the intracavity fields, if one would like to use the entanglement, he need to export the entangled fields first; thus only the output entanglement of the optical cavity has practical meaning. In addition, by means of spectral filters, one can always select many different traveling output modes originating from a single intracavity mode, which offer us the opportunity to easily produce and manipulate a multipartite system, eventually possessing multipartite entanglement. So, in this section we study the entanglement of the system, eventually possessing multipartite entanglement from a single intracavity mode, which offer us the opportunity to easily produce and manipulate a multipartite system. In Fig.4a, we notice that deep squeezing (usually means that the maximum entanglement) happen when $\omega = 0$, that is to say, when the output frequency is equal to that of cavity frequency, we have maximum entanglement. It is very interesting to observe that the deep squeezing are affected by the frequency of optomechanical oscillator. The larger frequency of the mirror, the smaller entanglement. We can deduce that if the mirrors is fixed, we should have the deepest squeezing in fluctuation of the output fields. The more higher frequency of the mirrors, the more entanglement can be transferred from the fields to the mirrors so that the smaller entanglement is leaved in the output fields. In Fig.4b, we have one valley of deep squeezing when the output frequency is equal to that of cavity frequency, and the squeezing spectrum can be calculated from the correlation matrix. The spectrum is defined in a frame of

In Fig.4, we have maximum entanglement. It is very interesting to observe that the deep squeezing are affected by the frequency of optomechanical oscillator. The larger frequency of the mirror, the smaller entanglement. We can deduce that if the mirrors is fixed, we should have the deepest squeezing in fluctuation of the output fields. The more higher frequency of the mirrors, the more entanglement can be transferred from the fields to the mirrors so that the smaller entanglement is leaved in the output fields.

The linear equation of (10) can be solved in the frequency domain by Fourier transformation with the solution $f(\omega) = (-i\omega - A)^{-1}B$. In the interaction picture $\omega$ represents the detuning from the cavity frequency. Considering the relation of (11), we have

$$f^{\text{out}}(\omega) = C(-i\omega - A)^{-1}B - E,$$

where $C = \text{diag}(0, 0, 0, 0, \sqrt{2}k_1, \sqrt{2}k_2, \sqrt{2}k_3, 0, 0, 0, 0, 0)$, $E = [0, 0, 0, 0, \delta X_{1\text{in}}(\omega), \delta Y_{1\text{in}}(\omega), \delta X_{2\text{in}}(\omega), \delta Y_{2\text{in}}(\omega), 0, 0, 0, 0]^T$. We can finally obtain the output correlation matrix $Y_{ij}^{\text{out}}(\omega) = \frac{1}{2}[(f_i^{\text{out}}(\omega))f_j^{\text{out}}(\omega') + f_j^{\text{out}}(\omega')f_i^{\text{out}}(\omega)],$ and the squeezing spectrum

$$S_{\text{OUT}}(\omega) = \frac{1}{2} \left\{ \delta X_f(\omega)\delta X_f(\omega') + \delta X_f(\omega')\delta X_f(\omega) \right \} + \delta Y_f(\omega)\delta Y_f(\omega') + \delta Y_f(\omega')\delta Y_f(\omega')$$

FIG. 3: (Color online) The entanglement of the mechanical mirrors and two-mode fields via the smallest eigenvalues of partial transposed matrix. All the lines and parameters are correspond to Fig.2.

FIG. 4: (Color online) (a) polls the output squeezing spectrum for several values of frequency of the mirrors where $\omega_{m_1} = \omega_{m_2} = 2\pi \times 10 \text{MHz}$ (solid) $2\pi \times 15 \text{MHz}$ (dashed). (b) shows the squeezing spectrum under different input-atomic state. For solid line $\rho_{aa} = \rho_{cc} = \rho_{ac} = 0.5$, for dashed line $\rho_{aa} = \frac{1}{5}, \rho_{cc} = \frac{5}{7}, \rho_{ac} = \frac{1}{7}$. In Fig.4b, $\omega_{m_1} = \omega_{m_2} = 2\pi \times 10 \text{MHz}$. For all the line in Fig.4, $g_1 = g_2 = 2\pi \times 2.0 \times 10^7 \text{Hz}$, $\Delta = 0.8\omega_{m_1}$, and the other parameters are the same with Fig.2.
in Fig. 4). But for the input atomic state $\rho = \frac{1}{2}(|a\rangle\langle a| + 2|c\rangle\langle c|)$, the maximum squeezing have two valley. As to the split of the squeezing spectrum, we can understand it from initial atomic state. The initial population in $|a\rangle$ and $|c\rangle$ are no longer equal so that the average of steady state $|X_1^s|, |Y_1^s|)$ for the two modes are not yet equal. Then, the fluctuation the $\delta X_1$ ($\delta Y_1$) and $\delta X_2$ ($\delta Y_2$) can be different so that the squeezing spectrum of the product of the fluctuation change from one degeneracy valley into two maximum squeezing. The split of the squeezing spectrum from one valley into two minima also have been shown in the case of asymmetric loss for each mode, i.e., $\kappa_1 \neq \kappa_2$. Our group had revealed the similar split resulting from asymmetric detuning and asymmetric atomic initial state.

During above numerical simulation, we choose parameters of the system based on recent experiment for optomechanical system, also considering the parameters used in theoretical papers. As to the coupling between the atoms and the cavity, we use $g = g_1 = g_2 = 2\pi \times 2.8 \times 10^3 \text{Hz}$ ( in Fig.4). Comparing with recent experiment $g/\pi = 50 \text{MHz}$ [31], the coupling is much more weak. Moreover, the ratio $g/\kappa$ is only 1.3, see Fig.4, which can be realized in recent experiment technique.

III. CONCLUSION

We proposed a scheme via three-level cascade atom to entangle two-mode fields as well as optomechanical oscillator. Our study show that intracavity fields and two movable mirrors are entangled respectively for realizable coupling between the cavity fields and the atoms. The output two-mode fields entanglement is affected by the frequency of the mirror motion, the larger frequency of the mirror, the larger entanglement of the output fields. Because the entanglement is resulted from the atomic coherence, the initial atomic state play important role on the output entanglement of the two-mode fields. For the best coherence of the initial atomic state, the output spectrum appears one deep squeezing while for other initial atomic state, the one deep squeezing split into two deep squeezing.

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