Parametric identification of thermophysical characteristics of heat-protective decaying materials

N S Aldebenev, S Yu Ganigin, D A Demoretsky, A N Diligenskaya and M Yu Livshits*
Samara State Technical University, Molodogvardeyskaya street, 244, Samara, 443100, Russian Federation

* mikhailivshits@gmail.com

Abstract. The paper presents a method of identifying the thermophysical properties of heat-protective materials of complex multi-component composition used in structures to meet the specified requirements for their resistance to external temperature influence. The research is dedicated to the study of a thermal barrier coating material containing chemically active components that enter into a chemical decomposition process, accompanied by the heat absorption. The equivalent specific heat capacity of the material reflects its thermophysical properties, taking into account the endothermic reaction. The equivalent specific heat capacity is identified by solving the inverse heat conduction problem using the results of temperature measurement in a multi-layer structure element during the thermophysical experiment. Regularization of this incorrect inverse problem is performed when it is transformed into a mini-max parametric problem of semi-infinite optimization. The results of differential thermal analysis and known qualitative laws of endothermic reactions make it possible to narrow the set of feasible solutions to the level of a compact set of piecewise continuous functions of a special structure, which provides a reduction to the parametric optimization problem. The minimax problem is solved by an alternance optimization method. The results obtained confirm the effectiveness of the proposed method for solving applied problems.

1. Introduction
The problem of creating effective thermal protection is relevant for a wide range of applications – from the design of reusable spacecraft carriers and emergency recorders to the creation of specialized containers and safes protected from the high-temperature effects of fire [1–5]. Thermal protection of structures based on degraded heat-shielding coatings (HSC) containing components that enter into a chemical endothermic reaction when the temperature rises above the critical level has become widespread. The heat absorption during the reaction provides the effective protection of the structure inner surface from the effect of high ambient temperatures on its outer surface. The design of such heat-shielding structures is based on mathematical modeling, which in turn requires the determination of the thermophysical characteristics (TPC) of the construction materials, including the TPC of the heat-shielding coating. The paper is dedicated to the problem solution of TPC identification of heat-shielding degraded coatings. Let us briefly consider the theoretical foundations for the subject under consideration.
2. The theoretical foundation of the methodology for the identification problem solution

The temperature field \( T(x, t) \) of a wide class of objects of technological thermal physics within the open part \( D \) of the spatial area \( \overline{D}, x \in \overline{D}, D \in R^1 \) or \( \overline{D} \in R^2 \) with boundary \( S \) is described by the inhomogeneous heat conduction equation:

\[
L[T(x, t)] = f_1\left(x, t, h_{D_1}(x), h_{D_2}(t), h_0(x, t, T)\right), x \in D, t \in [0, t^{end}] \tag{1}
\]

with boundary

\[
\Gamma[T(x, t)] = f_2(h_S(x, t)), x \in S, t > 0 \tag{2}
\]

and initial conditions

\[
N[T(x, t)] = h_{\overline{D}_0}(x), x \in \overline{D}, t = 0. \tag{3}
\]

Here \( t \) is the time, \( h_{D_1}(x), x \in D \) and \( h_{D_2}(t) \) are the internal heat sources (sinks), \( h_S(x, t), x \in S \) is the space-time boundary impact on an area \( D \) from the side of its environment, \( h_{\overline{D}_0}(x), t = 0, x \in \overline{D} \) is the initial condition, \( h_0(x, t, T) \) is the characteristics or parameters expressing thermophysical properties and generally depending on the spatial coordinate \( x \), time \( t \) and/or temperature \( T(x, t) \); \( L, \Gamma, N \) are given linear or nonlinear (in general) differential operators; \( f_1, f_2 \) are known functions of their arguments that meet the ordinary smoothness requirements. The solution to the inverse heat conduction problem (IHCP) involves the determination of the vector characteristic \( h = \{h_{D_1}(x), h_{D_2}(t), h_S(x, t), h_{\overline{D}_0}(x)\} \) in the boundary value problem (1)-(3) based on a priori or experimental information on the temperature distribution \( T^*(\psi) \) by a temporal \( t \in [0, t^{end}] \) argument in some measurable area. As an area of temperature measurement in the basic formulations of typical IHCP, a finite number (usually small, including equal to one) of special coordinates \( x^* \in \overline{D} \in \Omega_1 \), corresponding to the installation sites of temperature sensors is considered [6, 7].

Hereafter we restrict ourselves to the condition that the desired characteristics belong to the corresponding sets

\[
h_{D_1}(x) \in H_{D_1}(x), h_{D_2}(t) \in H_{D_2}(t), h_S(x, t) \in H_S(x, t), h_{\overline{D}_0}(x) \in H_{\overline{D}_0}(x) \tag{4}
\]

of piecewise smooth functions. Then an optimal (variational) problem is formulated with an optimality criterion with respect to the temperature distribution \( T(x, t, h_{D_1}(x), h_{D_2}(t), h_S(x, t), h_{\overline{D}_0}(x)) \), [8, 9]:

\[
\psi = t: I_h = \max_t |T(x^*, t, h) - T^*(x^*, t)| \to \min. \tag{5}
\]

Problem (5) is included in a wide range of typical statements of IHCP: boundary, internal, initial and coefficient inverse heat conduction problems, when unknown boundary conditions, the space-time distribution function of internal heat sources, the initial state or thermophysical characteristics of the object are subject to identification.

Taking into account the specific features of a particular applied problem and a priori information about an object, as a rule, allows one to reduce the identified characteristics to a parametric form in a selected set \( H_{\overline{D}_0}(x), H_{D_2}(t) \) of a given class functions. The most typical classes are constant functions, functions that are continuous with their first derivative, polynomial functions, or functions of special structure. More universal parameterization methods are possible [8-12].

Parametric representation of the identified characteristic \( h = \{h_{D_1}(x), h_{D_2}(t), h_S(x, t), h_{\overline{D}_0}(x)\} = h(\Delta) \) allows to obtain a parametric form of the temperature distribution \( T(x, t, h_{D_1}(x), h_{D_2}(t), h_S(x, t)) = T(\Delta) \). Thus, the desired characteristic and the corresponding
temperature distribution are uniquely determined by the corresponding vector of parameters \( \Delta = (\Delta_1, \Delta_2, \ldots, \Delta_n) \).

The found parametric representation of the temperature field \( T(\Delta) \) leads to the original inverse problem of a special nonsmooth mathematical programming problem with respect to the desired vector of parameters.

Temperature measurement results are used \( T^*(x^*, t) \) on the identification time interval \( t \in [0, t_{\text{end}}] \) at a certain fixed point \( x^* \), corresponding to the spatial coordinate of the temperature sensor.

Mathematical model (1) – (3) of the temperature field, taking into account the parametrized shape of the identified influences, makes it possible to obtain an expression for the temperature on the identification interval \( T(x^*, t, \Delta) \), characterized by the vector \( \Delta = (\Delta_n)_n = 1, N \) on a set of parameters \( G_N \). Specific type of parametric description \( T(x^*, t, \Delta) \) and vector dimension value \( N \) of parameters are determined by the representation form of the general solution to the boundary value problem and the selected class of solutions in the form of a given compact set.

Thus, the preliminary parameterization of the desired characteristic \( h \in P \) on a compact set \( P \) of piecewise smooth functions of time, spatial coordinate, or temperature in the class \( P \) and the parametric representation of the temperature field \( T(\Delta) \) provide a reduction of the original problem (5) to a special nonsmooth mathematical programming problem with respect to the desired vector of parameters \( \Delta \)

\[
\psi = t: I_2(\Delta) = \max_{t}[T(x^*, t, \Delta) - T^*(x^*, t)] \rightarrow \min_{\Delta}
\]

(6)

3. The alternance method in inverse heat conduction problem

The problem of mathematical programming (6) can be solved with the use of the general scheme of the alternance method based on the special properties of optimal solutions to the problem, similar to the properties of nonlinear Chebyshev approximations. On the basis of these properties a closed system of ratios with respect to the parameters \( \Delta = \Delta_0 \) of the optimal problem solution is composed, which is further transformed into a closed system of calculation equations of desired values and unknown functional values \( I_4(\Delta_0) \)

\[
T(x^*, t_{0q}^0, \Delta_0) - T^*(x^*, t_{0q}^0) = \pm (-1)^{q+1} I_4(\Delta_0), q = 1, N
\]

(7)

According to the system of calculated ratios (14), threshold criteria \( |T(x^*, t_{0q}^0, \Delta_0) - T^*(x^*, t_{0q}^0)| \), equal to \( I_4(\Delta_0) \), are reached only at individual points of the alternance \( t_{0q}^0 \).

As a result of solving the equation system (7), the desired vector of parameters is found \( \Delta_0 \), which allows to implement the parametric identification of unknown quantities \( h \) in a parametrically specified class of functions. Found number values \( N \) and vector of parameters \( \Delta \) implement the approximation of the desired characteristic with the required accuracy at the identification interval.

As the number increases \( N = 1, 2, \ldots \) a sequence of functional values is made \( I_2^N \) (6), converging, under ideal conditions, to the minimum possible value \( I_2^{\text{inf}} \). In this case, for any value \( N \) it is ensured that the solutions belong to the selected compact set and, thus, the correct statement of problem (6) is preserved.

In practical applications, the sequence of parametric optimization problems (6) is solved for increasing values \( N = 1, 2, \ldots N^* \) up to a certain value \( N^* \), that ensures the accuracy requirements, according to a given value of the functional \( I_2^0(\Delta_0) \).

4. The problem of heat-shielding materials TPC identification

The stated general method for solving IHCP is used to identify the thermophysical properties of a heat-shielding material having a complex multicomponent composition.
The design and optimization of the weight and size characteristics of a heat-shielding structure is based on the thermophysical characteristics of materials [1–5].

Modern approaches to the creation of effective thermal protection coatings provide for the use of chemically unstable components in the HSC, the heating of which causes their interaction accompanied by an endothermic reaction.

In this case, the amount of heat to be absorbed by the HSC during the endothermic reaction and providing the specified requirements for the fire resistance of the structure is determined by the decision of the corresponding IHCP [6,7]. To formalize this problem, let us introduce into consideration the equivalent time-varying heat capacity of the HSC \( C_e(T) \), providing the equivalent real heat content of the HSC, when it is heated, accompanied by an endothermic reaction and a change in the phase composition of the components.

The article reflects the study of the thermophysical characteristics of HSC, which has a complex multicomponent composition, which includes, in addition to the basic components, two chemically active fillers that degrade with the effect of heat absorption at temperatures 120-150 °C and 300-320 °C respectively.

HSC, due to its efficiency, is not a thermally thin body; therefore, it is difficult to identify its TPC, because the process of decomposition of unstable components begins in the HSC layer not simultaneously, but gradually as the critical temperature for each filler is reached during the heating of the layer. Due to the low thermal conductivity of the HSC composition, large temperature gradients appear in this layer, which leads to the fact that, during heating, the HSC material has a different phase composition and, as a consequence, different thermophysical characteristics.

The inverse heat conduction problem for determining the TPC of the investigated HSC was solved on the basis of experimental information on the temperature distribution in the prototypes obtained as a result of a series of experiments.

5. Differential thermal analysis

A materials laboratory testing [13,14] of the HSC fillers for each of the two stages of the endothermic reaction was carried out separately.

As endothermic fillers by differential thermal analysis (DTA) on a DTA GSM instrument, which determines the temperature difference between two test components that have different decomposition temperatures. For the analysis, weighed portions of 50 mg of the test substances were formed. The determination of the endothermic effect was carried out using thermocouples of the DTA instrument placed inside the test substances. Temperature deviation is monitored during tube heating \( T_{\text{sub}} \) inside the substance from the substance \( T_{\text{tb}} \) of the outer tubes. The experimental setup is shown in figure 1. The experiment stops when the temperature inside the substance is equal to the temperature of the surface of the tube, which means the end of the endothermic reaction. The results of the experiment were recorded using the software of DTA GSM and are presented in figure 2 for each of the reaction stages. The axis of ordinates shows the deviation of the temperature \( \Delta T = T_{\text{sub}} - T_{\text{tb}} \) inside the sample \( T_{\text{sub}} \) from the temperature of the tube wall \( T_{\text{tb}} \). The axis of abscissa shows the values of the tube wall temperature \( T_{\text{tb}} \), which increases at a rate 5 °C/min (0.083 K/sec).

The temperature curve changes of the studied composition of the first reaction stage are shown in figure 2a. It shows that the endothermic reaction in the substance begins 22 minutes after the heating start and proceeds in the temperature range \( \Delta T = 112-133 \) °C during 4.2 minutes.

The maximum endothermic effect (deviation of the temperature in the substance from the heating temperature) is observed at \( T_{\text{tb}} = 120 \) °C (\( \Delta T = 6.8 \) °C).

The specific endothermic effect of the first stage chemical reaction is: \( \Delta H_{\text{reaction}}^0 = 97.89 \) kJ (for the decomposition of 1 mole of a substance, 97.89 kJ of heat is spent).

The temperature curve changes in the medium of the studied composition of the second reaction stage are shown in figure 2b. It shows that the endothermic reaction in the test substance begins 60 minutes after the heating start and proceeds in the temperature range \( T_{\text{tb}} = 300…382 \) °C during 16.4 minutes.
The maximum endothermic effect (deviation of the temperature in the substance from the heating temperature) is observed at $T_{th} = 365 \, ^\circ C$ ($\Delta T = 19,8 \, ^\circ C$).

The specific endothermic effect of the second stage chemical reaction is: $\Delta H^0_{reaction} = 175,9 \, kJ$ (for the decomposition of 1 mole of a substance 175,9 kJ of heat is spent).

Figure 2. DTA results for: (a) the first, and, (b) the second chemically active components of the HSC

6. Thermophysical experiment

For a preliminary assessment of the effectiveness of the use of HSC, a series of thermophysical experiments was carried out in the thermal protection structure using three variants of the model design. In all cases, the test models were multi-layer structures of steel plates and a thin air gap and differed in the filling of the HSC layer or its absence.

As a model, an experimental element of a real multi-layer structure with dimensions is studied: $2H \times L, L = 0.263 \, m, H = 0.075 \, m, x_1 = 0.185 \, m, x_2 = 0.23 \, m, x_3 = 0.239 \, m, x_4 = 0.242 \, m, x_5 = 0.251 \, m, r = 0.038 \, m, s = 0.02 \, m$, (figure 3 a, b). The outer part of the structure consists of two steel plates separated by a thin air gap and connected by a longitudinal stiffener. The prototype from the lateral surfaces is insulated with a layer of asbestos, and from the outer surface it is exposed to the action of an open flame. A multicomponent heat-shielding coating is applied to the inner shell and must provide the specified requirements for resistance to external temperature effects. The maximum permissible temperature $T(x, y, t)$ on the outer surface of the containment should not exceed 180 $^\circ C$ for reasons of structural reliability, fire and explosion safety of the contents protected by the structure. To control the temperature conditions, the temperature of the outer surface is measured (point A in figure 3), as well as the temperature at point B of the inner wall of the shell using thermocouples (accuracy class 0.2).

The first design option does not contain a heat-shielding coating (figure 3a), the second and third design options contain HSC of different composition (figure 3b). In the second variant, an inert material is used as HSC, which does not contain endothermic components, and in the third, a
multicomponent composition of HSC is used, containing chemically active fillers that enter into an endothermic decomposition reaction.

Figure 3. Test layouts in a preliminary thermophysical experiment:
- a – without HSC
- b – with HSC

In the course of the experiments, the outer surface of the model was under the conditions of an open flame, the temperature of which at the initial stage was maintained at about 900 °C, after which the experimental conditions were tightened, and the flame temperature exceeded 1200 °C (figure 4).

Heating continued until reaching the maximum permissible temperature of 180 °C at point B. The experimental results are shown in figure 4. The results of the experiment make it possible to visually assess the effectiveness of the HSC. The introduction of decomposing components (curve 5) into the composition, which is accompanied by heat absorption, makes it possible to increase the time to reach the critical temperature of 180 °C at the control point. Thus, the results of a series of thermophysical experiments have confirmed the effectiveness of using chemically active materials as fillers for HSC and substantiate the use of a chemically active HSC with two decomposing components in a heat-shielding structure.

On the basis of DTA (see Figure 2) and the results of thermophysical experiments (see figure 4), the possible stages of the HSC state are identified:

- I - initial state characterized by constant specific density \( \rho_5 = \rho_5^{(I)} = \text{const} \), heat-conduction coefficient \( \lambda_5 = \lambda_5^{(I)} = \text{const} \) and specific heat capacity \( C_5 = C_5^{(I)} = \text{const} \) of thermal protective composition, which determine the regular temperature condition of the sample;
- II - the first stage of the endothermic reaction, at which there is a sharp increase in the specific heat capacity \( C_5 = C_5^{(II)}(t) \), followed by a sharp decline;
- III - an intermediate steady state of HSC after the end of the decomposition stage of the first endothermic filler and the beginning of the decomposition of the second active component.
Here, due to a change in its phase composition, the component under study is characterized by other, different from the previous, constant values of the specific heat capacity $C_5 = C_5^{(III)} = \text{const}$, density $\rho_5 = \rho_5^{(III)} = \text{const}$ and heat-conduction coefficient $\lambda_5 = \lambda_5^{(III)} = \text{const}$;

- IV – the process of decomposition of the second chemically active component, accompanied first by an increase and then a decrease in the value of the equivalent specific heat $C_5 = C_5^{(IV)}(t)$;

- V – final state described by constant values of thermophysical characteristics $C_5 = C_5^{(V)} = \text{const}$, $\rho_5 = \rho_5^{(V)} = \text{const}$, $\lambda_5 = \lambda_5^{(V)} = \text{const}$ and established upon completion of the endothermic reaction.

7. **Mathematical model of the HSC thermal conductivity process**

To identify the TPC of HSC, it is necessary to concretize the general mathematical model of the boundary value problem (1) - (3). For the temperature distribution in the test sample, the boundary value problem (1) – (3) is considered, which takes the form of a two-dimensional boundary value problem of unsteady heat conduction, containing a system of two-dimensional heat conduction equations for temperature $T_i(x, y, t), i = 1, 5$ in every $i$ structural element (figure 3b), where the third layer (steel shell) contains three elements $(x, y) \in \Omega_3 \div \{3a, 3b, 3c\}$, and the fourth layer consists of two sections of the air gap $(x, y) \in \Omega_4 \div \{4a, 4b\}$. Every $i$ element $(i = 1, 5)$ is characterized by specific heat capacity $C_i$, material density $\rho_i$, heat-conduction coefficient $\lambda_i$ and the corresponding boundary conditions. Thus, the mathematical model for temperature $T_i(x, y, t)$ is described by the system of Fourier equations:

![Figure 4](image-url)
\[
C_i \rho_i \frac{\partial T_i}{\partial t} = \lambda_i \left( \frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} \right), i = 1, 5: 0 < t \leq t_{end}
\]

\(i = 1: (x, y) \in [0, x_1] \times [0, H] \quad i = 2: (x, y) \in [x_1, x_2] \times [0, H] \quad i = 5: (x, y) \in [x_2, x_3] \times [0, H]\)

\(i = 3: (x, y) \in [x_3, x_4] \times [0, H] \quad (x, y) \in [x_4, x_5] \times [r, r + s] \quad (x, y) \in [x_5, L] \times [0, H]\)

\(i = 4: (x, y) \in [x_4, x_5] \times [0, r] \quad (x, y) \in [x_4, x_5] \times [r + s, H]\)

Initial conditions (3) for all structural elements are taken uniform and constant:

\[
T(x, y, 0) = T_0 = \text{const} ; \quad 0 \leq x \leq L, 0 \leq y \leq H.
\]

Boundary conditions are formulated. Axial symmetry condition is specified on the boundary \(y = 0\),

\[
\frac{\partial T_i(x, 0, t)}{\partial y} = 0
\]

\(i = 1; 2; 3: (x, y) \in [x_3, x_4] \times [0, H] \quad [x_5, L] \times [0, H]\)

\(i = 4: (x, y) \in [x_4, x_5] \times [0, r] \quad i = 5\)

The high-temperature effect of an open flame on the outer surface is given in the form of a complex heat exchange containing radiation and convective components

\[
\lambda_3 \frac{\partial T_3(L, y, t)}{\partial x} = \varepsilon \sigma \left( T_3^4(L, y, t) - T_{am}^4(t) \right) + \alpha (T_3(L, y, t) - T_{am}(t))
\]

\(0 < y < H\)

where \(\varepsilon\) – emissivity factor, \(\sigma\) – Stefan-Boltzmann constant, \(\alpha\) – heat-exchange coefficient, \(T_{am}(t)\) – environment temperature.

The boundary conditions on the remaining external surfaces correspond to the thermal insulation conditions:

\[
\frac{\partial T_i(0, y, t)}{\partial x} = 0
\]

\(0 < y < H, \quad \frac{\partial T_i(x, H, t)}{\partial y} = 0\)

\(i = 1; 2; 3: (x, y) \in [x_3, x_4] \times [0, H] \quad [x_5, L] \times [0, H]\)

\(i = 4: (x, y) \in [x_4, x_5] \times [r + s, H] \quad i = 5\)

On the inner contact surfaces of the layers, the conjugation conditions formulated for spatial variable \(x\) are satisfied:

\[
T_i(x, k, y, t) = T_j(x, k, y, t)
\]

\[-\lambda_i \frac{\partial T_i(x, k, y, t)}{\partial x} = -\lambda_j \frac{\partial T_j(x, k, y, t)}{\partial x}\]

\(i = 1, j = 2, k = 1, 0 \leq y \leq H\)

\(i = 5, j = 3, k = 3, 0 \leq y \leq H\)

\(i = 3, j = 4, k = 4, 0 \leq y \leq r\)

\(i = 3, j = 4, k = 5, 0 \leq y \leq r\)

\(i = 3, j = 4, k = 5, r + s \leq y \leq H\)

\(i = 3, j = 4, k = 5, r + s \leq y \leq H\)

and, similarly, for variable \(y\):
\[ T_3(x, r, t) = T_4(x, r, t) \]
\[ -\lambda_3 \frac{\partial T_3(x, r, t)}{\partial y} = -\lambda_4 \frac{\partial T_4(x, r, t)}{\partial y} \]  
(14)

The nonlinear direct boundary value problem of heat conduction (8) – (14) can be solved by known numerical methods if all the thermophysical characteristics included in it are known. The solution of this boundary value problem allows, using the appropriate optimization procedures, to determine the design parameters that provide effective thermal protection.

8. Thermophysical characteristics identification based on the IHCP solution

On the basis of the mathematical model (8) – (14), the task of designing a structure with given heat-shielding properties is posed, the solution of which provides for the stage of identifying the thermophysical parameters of the HSC. To solve the identification problem, IHCP is considered on the basis of model (8) - (14), where the equivalent specific heat capacity is to be determined \( C^e_5(T) \) (i=5) HSC (element 5 in figure 3b), reflecting its heat content, taking into account endothermic reactions, based on the experimentally obtained temperature \( T(x^*, y^*, t) \) at point B \( x = x^*, y = y^* \). TPC of other structural elements is considered to be known with the following data:

- Heating end time \( T_{\text{end}} \approx \) 2820 sec; operating temperature range of HSC \( [T_{\text{beg}}, T_{\text{end}}] \approx [20^0C, 800^0C] \), where \( T^*(x^*, y^*, t) \) — experimental temperature values at the sample point \( (x^*, y^*) \); \( T(x^*, y^*, C^e_5(T), t) \) — the values obtained by numerical solution to the boundary value problem (8) – (14) at the same point.

A set \( V \) of physically realizable functions is considered under constraints

\[ C^e_5(T) \in V, T \in [t_{\text{beg}}, t_{\text{end}}] \]  
(15)

and the problem is posed of finding an equivalent specific heat capacity \( C_5 = C^e_5(T) \), that ensures the fulfillment \( (0, t_{\text{end}}) \) of the minimax relation on the identification interval (6), which in the considered case will take the form:

\[ I(C^e_5) = \max_{t \in [0, t_{\text{end}}]} |T^*(x^*, y^*, t) - T(x^*, y^*, C^e_5(T), t)| \rightarrow \min_{C^e_5(T) \in V}. \]  
(16)

The search for solutions is carried out on the set \( V \) of piecewise continuous measurable functions with the following data: heating end time \( t_{\text{end}} = 2820 \) sec; operating temperature range of HSC \( [T_{\text{beg}}, T_{\text{end}}] \approx [20^0C, 800^0C] \), where \( T^*(x^*, y^*, t) \) — experimental temperature values at the sample point \( (x^*, y^*) \); \( T(x^*, y^*, C^e_5(T), t) \) — the values obtained by numerical solution to the boundary value problem (8) – (14) at the same point.

At the stages of decomposition of the components, it is advisable to approximate the desired characteristic \( C^e_5(T) \) (see figure 2) by a shifted Gaussian function \( c + k \exp(-(T - T^*)^2 \alpha^{-1}) \), on the basis of which a set of piecewise continuous functions of the following form is considered:
Such an approximation makes it possible, during formalization, to combine the stage of decomposition of the filler, accompanied by a significant change $C_{5}^{eq}(T)$, with subsequent stages of permanence $C_{5}^{eq}(T) = c_{5j} = \text{const}, j = 0, 2$.

Thus, for an effective solution of the IHCP, a parametrization is performed, which, in accordance with the general methodology, narrows the area $V$ of the existence of the sought functions. The values of temperature $T^{(1)}$ and $T^{(2)}$ are determined on the basis of a general description of the desired function as the moments of the onset of a sharp increase $C_{5}^{eq}(T)$ in the active stages of the endothermic reaction. The value $T^{end}$ is specified according to the calculated temperature field for a point $(x_{3}, 0)$ on the outer boundary of the HSC. Thus, the desired characteristic $C_{5}^{eq}(T)$ is uniquely determined by the vector of unknown parameters $\Delta = \Delta^{(N)} = [c_{50}, c_{51}, c_{52}, k_{1}, k_{2}, T_{1}, T_{2}, \alpha_{1}, \alpha_{2}]$ with the dimension $N = 9$.

Dependency approximation (17) $C_{5}^{eq}(T)$ in the selected class $V(T)$ of functions accurate to the vector of parameters $\Delta$ provides the parameterization of the temperature field $T_{i}(x, y, \Delta, t), i = 1, 5$ as a function of the argument $C_{5}^{eq}(\Delta)$ (17) in the numerical solution of the problem (8) – (14), by determining the temperature as a function of the vector of parameters $\Delta$ at any fixed point $(x^{*}, y^{*})$.

Parametric representation $T_{i}(x, y, \Delta, t)$ allows you to go from the original IHCP setting (16) to an equivalent mathematical programming problem of the form (6) relative to the vector $\Delta$, which in this case will take the form

$$I^{*}(\Delta) = \max_{t \in [0, T^{end}]} |T^{*}(x^{*}, y^{*}, t) - T(x^{*}, y^{*}, \Delta, t)| \to \min_{\Delta}$$  

(18)

The solution of the mathematical programming problem (15), (17), (18) is performed by the alternance optimization method.

The main basic relations of the alternance method (7) [10] put in correspondence the number $R$ of alternance points, in which the limiting relations are fulfilled (18), which have alternating character, with the number of unknown parameters, which include parameters $\Delta$ of the desired characteristic and the value of the maximum deviation $I(\Delta)$. On this basis, a closed system of algebraic equations relative to the unknown components of the vector is compiled $\Delta = \Delta^{(N)} = [c_{50}, c_{51}, c_{52}, k_{1}, k_{2}, T_{1}, T_{2}, \alpha_{1}, \alpha_{2}]$ as a result of the solution of which the required parameters are found that ensure the minimization of the functional $I^{*}(\Delta)$ (18). Often, when solving applied problems, the requirement to achieve a global minimum $I^{*}(\Delta)$ turns out to be excessive and can be replaced by the condition:

$$\max_{t \in [0, T]^{end}} |T^{*}(x^{*}, y^{*}, t) - T(x^{*}, y^{*}, \Delta, t)| \leq \varepsilon_{dop},$$  

(19)

where $\varepsilon_{dop}$ – predetermined permissible temperature discrepancy at the control point $(x^{*}, y^{*})$.

As a result of solving a mathematical programming problem (15), (17), (19) based on gradient numerical methods using special software the vector of parameters was determined $\Delta^{0}$, that unambiguously identifies the equivalent specific heat capacity of the HSC, the results of which are presented in figures 5, 6.

The specific heat capacity function of the HSC reconstructed on the basis of the values of the parameters vector $\Delta^{0} = \Delta^{(N)} = [c_{50}, c_{51}, c_{52}, k_{1}, k_{2}, T_{1}, T_{2}, \alpha_{1}, \alpha_{2}]$ found as a result of solving the problem (15), (17), (19) is shown in figure 5.
The obtained parametric representation of the desired function $C_{5eq}^C(T)$ allows to carry out thermophysical studies on the studied identification interval, in which the absolute error in the deviation of the temperature distribution does not exceed 7.5 °C and is 4.16 %, which is quite satisfactory for engineering calculations (figure 6a). The permissible deviation of the calculated temperature from the experimental one obtained at the control point B is set from the conditions of technical benefit and is within 10 %.

The calculated temperature curves in the test sample are shown in Figure 6b and confirm the efficiency of the HSC.

9. Conclusion
A solution to the problem of identifying the thermophysical characteristics of heat-shielding coatings with decomposing components is obtained. The use of the alternance method for minimax optimization of the error in the deviation of the calculated temperature from that obtained...
experimentally at the control points of the heat-protective coating, carried out in a uniform estimation metric in a given class of functions, provides high identification accuracy.

The obtained results can be used to solve urgent problems of designing thermal protection containing chemically active components.

Acknowledgements
This work was supported by the Russian Foundation for Basic Research (projects No. 20-08-00240, No. 18-08-00565)

References
[1] Okhapkin A S 1985 J. Eng. Phys. 49, 6, 1469–1473
[2] Alifanov O M, Cherepanov V V 2010 J. Eng. Phys. 83, 4, 770–782
[3] Alifanov, O M, Budnik S A, Nenarokomov A V, Netelev A V, Titov D M 2010 The 61th International Astronautical Congress (Prague, Czech Republic), 10
[4] Dec J A, Braun R D, Lamb B 2012 J Thermophys Heat Tr. 26, 2, 201–212
[5] Reznik S V, Prosuntsov P V, Mikhailovskii K V 2015 J. Eng. Phys. 88, 3, 594–601
[6] Alifanov, O M 1988 Inverse Heat Transfer Problems (Moscow: Mashinostroenie)
[7] Beck J.V, Blackwell B and St. Clair C R 1985 Inverse Heat Conduction: Ill-posed Problems (New York, John Wiley and Sons)
[8] Rapoport E Ya and Pleshivtseva Yu E 2009 RF Avtometriya 45, 5, 103–112
[9] Diligenskaya A N, Rapoport E Y 2016 J. Eng. Phys. 89, 4, 1008–1013
[10] Rapoport E Ya 2000 Alternance Method in Optimization Applications (Moscow: Nauka)
[11] Livshits M Y, Nenashev A V, Borodulin B B 2020 Studies in Systems, Decision and Control, 260, 79–88
[12] Livshits M Y, Nenashev A V 2019 RF. J. Samara State Tech. Univ., Ser. Phys. Math. Sci. 23, 2, 361–377
[13] Fatima Zohra El Wardi , Abou-bakr Cherki , Soumia Mounir , Abdelhamid Khabbazi , Youssef Maalouf 2019 Energy Procedia 157, 480–491
[14] Ying Li, Lu Zhang, Rujie He, Yongbin Ma, Keqiang Zhang, Xuejian Bai, Baosheng Xu, Yanfei Chen 2019 Aerospace Science and Technology 91, 607-616