Cosmological implications of the KATRIN experiment

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Abstract. The upcoming Karlsruhe Tritium Neutrino (KATRIN) experiment will put unprecedented constraints on the absolute mass of the electron neutrino, $m_{\nu_e}$. In this paper we investigate how this information on $m_{\nu_e}$ will affect our constraints on cosmological parameters. We consider two scenarios: one where $m_{\nu_e} = 0$ (i.e., no detection by KATRIN), and one where $m_{\nu_e} = 0.3$ eV. We find that the constraints on $m_{\nu_e}$ from KATRIN will affect estimates of some important cosmological parameters significantly. For example, the significance of $n_s < 1$ and the inferred value of $\Omega_\Lambda$ depend on the results from the KATRIN experiment.

Keywords: CMBR experiments, neutrino experiments, cosmological neutrinos

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1. Introduction

The large amount of new cosmological data in the last decade has led to what one may call the cosmological standard model. In this model the universe is close to flat, homogeneous and isotropic on sufficiently large scales, and today the energy density of the universe is dominated by dark energy (\(\sim 74\%\)), dark matter (\(\sim 22\%\)) and baryonic matter (\(\sim 4\%\)). This model is consistent with data ranging from the WMAP measurements of the anisotropies of the cosmic microwave background (CMB) radiation [1] to observations of supernovae of type 1a, galaxy distributions and several other observables (with a few exceptions, see [2]). It is often claimed that most of the data can be fitted with only six free parameters. This claim rests on the assumption of massless neutrinos, an assumption justified by the fact that adding the sum of the neutrino masses as a free parameter does not improve the fit substantially.

However, from the observation of neutrino oscillations, there is a compelling body of evidence for non-zero neutrino masses (see [3] for a review). Oscillation experiments do not give us any information on the absolute mass scale of neutrinos, only on the mass differences between the different mass eigenstates and mixing angles. The current best upper bounds on the neutrino mass from particle experiments come from the Troitsk [4] and Mainz [5] tritium beta decay experiments that found upper bounds on \(m_{\nu_e}\) of \(m_{\nu_e} < 2.2\) eV (95\% C.L.). The KATRIN experiment [6] that will start taking data in 2010, is expected to lower this limit on \(m_{\nu_e}\) by an order of magnitude to \(m_{\nu_e} < 0.2\) eV (in the case of no detection) after three years of running.

Effects of neutrino masses can also be seen in cosmological observables, and the best upper limits on the absolute scale of the neutrino mass today come from cosmology. Both CMB and the large scale structures (LSS) of the galaxy distribution are probes that
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are sensitive to the neutrino mass, the observable quantity being the sum of the three
neutrino mass eigenstates, \( M_\nu = \sum_i m_\nu_i \). The upper bounds on \( M_\nu \) from cosmology
range from \( M_\nu < 0.2 \text{ eV} \) [7] to \( M_\nu < 2.0 \text{ eV} \) [1] (95\% C.L.), depending on the data [8]
and cosmological model [9] used.

On the experimental side there is a claim of detection of the absolute scale
of the neutrino mass from the Heidelberg–Moscow neutrinoless double beta decay
experiment, with an effective electron neutrino mass of \( \langle m_\nu_e \rangle = (0.2–0.6) \text{ eV} \) (99.73\% C.L.) [10]. However, these results are regarded somewhat controversial. The cosmological
implications of this result are discussed in [11].

We know that neutrinos are massive, and since we have no current priors on the
neutrino mass in the allowed cosmological range, one should always marginalize over \( M_\nu \)
when constraining other cosmological parameters. \( M_\nu \) turns out to be partially degenerate
with several of the standard cosmological parameters, such that this marginalization over
\( M_\nu \) weakens the bounds on the other parameters in our model. Thus, any prior knowledge
of \( M_\nu \) from non-cosmological experiments will serve to break degeneracies and improve
the constraints on other cosmological parameters. The KATRIN experiment will provide
us with such a prior on \( M_\nu \) in a range that is relevant for cosmology. In this paper we
investigate how the results from KATRIN will affect our estimates of other cosmological
parameters.

Limits on the neutrino mass when combining results from KATRIN and WMAP have
been studied in a recent paper by Høst et al [12]. Our emphasis in this paper is on how
other cosmological parameters are affected when the results from KATRIN are used as an
external prior.

Section 2 contains a short review on the effect of massive neutrinos in cosmology. In
section 3 we will present the data and methods that we will use in our analysis, including
the assumed priors from the KATRIN experiment. Then we will present our results in
section 4. A comparison of \( \chi^2 \) values found when introducing the KATRIN priors is
presented in section 5. Finally we summarize and conclude in section 6.

2. Cosmology with massive neutrinos

All our results are derived within the standard cosmological paradigm of a flat ΛCDM
model, using the following free parameters: \( \{ \Omega_i h^2, \Omega_m, \log(10^{10} A_s), n_s, \tau, M_\nu \} \). Here
\( \Omega_i \) denotes the energy density of energy component \( i \) (\( m = \text{matter} \), \( b = \text{baryons} \), \( \Lambda = \text{cosmological constant} \), CDM = \text{cold dark matter}) relative to the total energy density
of a spatially flat universe. The matter density, \( \Omega_m \), is the sum of all non-relativistic
components, such that \( \Omega_m = \Omega_\text{CDM} + \Omega_b + \Omega_\nu \). The parameter \( h \) is the dimensionless
Hubble parameter, defined by \( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( A_s \) denotes the amplitude of the
primordial fluctuations, while \( n_s \) gives the tilt of the primordial power spectrum. Finally, \( \tau \)
is the optical depth at reionization. For more details on the parameter definitions, see the
description of the CosmoMC code [13]. The effect of massive neutrinos on cosmological
observables is parameterized by \( M_\nu \), the sum of the neutrino masses, and is related to
the neutrino energy density by the simple relation [14] \( \Omega_\nu h^2 = M_\nu / 93.14 \text{ eV} \). We will
also extend the parameter space by including \( w \), the equation of state parameter of dark
energy, as a free parameter. We will assume \( w \) to be constant. This parameter may be
interesting to study, as it is fundamental in the understanding of the nature of dark energy,
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Figure 1. CMB (left panel) and LSS (right panel) power spectra with different values of $M_\nu$. Here $\Omega_m$ is held constant, and increasing $M_\nu$ has been compensated by decreasing $\Omega_{\text{CDM}}$ correspondingly.

and since it is known to be slightly correlated with $M_\nu$ [15]. It should be stressed that this analysis rests on the assumption of a standard thermal background of three weakly interacting neutrino species. Alternatives to this picture are studied in e.g. [16] and [17].

Recent reviews of the role of massive neutrinos in cosmology can be found in [14, 18]. In this section we will only give a brief description of the most important effects of $M_\nu$ on relevant cosmological observables. We will throughout this work assume that the neutrino mass eigenstates are degenerate, such that $M_\nu = 3m_\nu$. In the mass range that we will operating in, it has been shown that this is a valid simplification when it comes to cosmological observables [19].

Effects on the CMB from massive neutrinos manifest themselves mainly on the level of background evolution. In the neutrino mass ranges relevant to us, the neutrinos will still be relativistic at the time of matter–radiation equality, and must be regarded as a radiation component when it comes to the background evolution of the universe. Increasing $M_\nu$ (and thus also $\Omega_\nu$), keeping $\Omega_m$ constant, will thus postpone the time of matter–radiation equality. This will enhance the acoustic peaks in the CMB power spectrum and give a small horizontal shift of the peaks to larger scales. This effect is shown in the left panel of figure 1. To compensate for this effect, one can increase $\Omega_m$ and decrease $H_0$. It is already obvious that $M_\nu$ will be correlated with both $H_0$ and $\Omega_m$ (and thus also $\Omega_{\Lambda}$ when we stick to the assumption of spatial flatness). Another effect comes from neutrino free streaming, which will smooth out gravitational wells on scales below an $M_\nu$-dependent neutrino free streaming scale [20, 21]. On scales smaller than this, the acoustic oscillations will be enhanced, increasing the height of the peaks in the CMB power spectrum.

Neutrino masses affect the LSS power spectrum in an even more distinct way. Again, massive neutrinos will suppress structure growth on scales below a free streaming scale given by [14]

$$k_{nr} = 0.010 \sqrt{\frac{M_\nu \Omega_m}{1 \text{ eV}}} h \text{ Mpc}^{-1}.$$  (1)
The smaller $M_\nu$, the larger scales will be affected, and the larger $M_\nu$, the more suppression of power on the scales affected. The effect of neutrino mass on the matter power spectrum can be seen in figure 1. Again, $\Omega_m$ is kept constant, and increasing $M_\nu$ has been compensated for by decreasing $\Omega_{CDM}$ correspondingly.

3. Data and methods

3.1. Cosmological data

Our analysis include both observations of CMB, LSS, SN1a, information about baryonic acoustic oscillations (BAO) in the matter power spectrum and constraints from the cluster mass function from weak gravitational lensing. We have also applied priors on $H_0$ and $\Omega_b$.

The CMB data used in our analysis, comes from the temperature [22] and polarization [23] data from the 3 year data release from the WMAP team. The WMAP experiment is a satellite based full-sky survey of the CMB temperature anisotropies and polarization. In our analysis of the WMAP data we have used the Fortran 90 likelihood code\(^1\) provided with the data release.

We have used LSS data from the Sloan Digital Sky Survey (SDSS) luminous red galaxy (LRG) sample [24]. As SN1a data we have used the sample from the Supernova Legacy Survey (SNLS) [25]. Other probes of the matter distribution that we have applied come from the measurement of the baryonic acoustic peak (BAO) in the matter power spectrum and the cluster mass function (CMF) from weak gravitational lensing. The BAO constraint comes from the SDSS-LRG sample [26], and we have adopted the fit function from [27],

$$A_{BAO} = 0.469 \left( \frac{n_s}{0.98} \right)^{-0.35} (1 + 0.94 f_\nu) \pm 0.017,$$

where

$$A_{BAO} \equiv \left[ D_M(z)^2 \frac{z}{H(z)} \right]^{1/3} \sqrt{\Omega_m H_0^2} \frac{z}{z},$$

and $D_M(z)$ is the comoving angular diameter distance at redshift $z$.

Handles on parameters governing the clustering of matter are also provided by the cluster mass function. The cluster mass function from weak gravitational lensing, as measured in [28], gives constraints on a combination of $\Omega_m$ and $\sigma_8$ (the root mean square mass fluctuations in spheres of radius $8h^{-1}$ Mpc). We have adopted the fit function for $\chi^2_{CMF}$ from [8],

$$\chi^2_{CMF} = 10.000u^4 + 6726u^3 + 1230u^2 - 4.09u + 0.004,$$

where $u = \sigma_8(\Omega_m/0.3)^{0.37} - 0.67$.

A prior on the Hubble parameter, $h = 0.72 \pm 0.08$ [29] comes from the Hubble Space Telescope (HST) key project. From big bang nucleosynthesis (BBN) we adopt a prior on the baryon density today, $\Omega_b h^2 = 0.022 \pm 0.002$ [30]–[32].

Throughout the entire work we also apply a top-hat prior on the age $t_0$ of the universe: $10 \text{ Gyr} < t_0 < 20 \text{ Gyr}$.

\(^1\) http://lambda.gsfc.nasa.gov; version v2p2
3.2. Constraints from KATRIN

The KATRIN [6] experiment measures the energy distribution of electrons from tritium beta decay. The exact shape of the end of this spectrum will depend on how much of the energy that is bound in the outgoing electron neutrinos, and thus also be a probe of the electron neutrino mass. If KATRIN does not detect $m_{\nu_e}$, they are expected to place an upper limit on $m_{\nu_e} < 0.2$ eV (90% C.L.). They expect to reach an uncertainty of $\sigma_{m_{\nu_e}^2} \approx 0.025$ eV$^2$.

Here we have adopted this uncertainty for two cases, one assuming $m_{\nu_e} = 0$ eV (i.e., no detection by KATRIN), and one assuming $m_{\nu_e} = 0.3$ eV (giving $M_\nu = 0.9$ eV). Further we have assumed the Gaussian distribution of $m_{\nu_e}^2$ around these values [6, 12], and used this as a prior in our cosmological parameter analysis.

3.3. Method

Employing the publicly available Markov chain Monte Carlo code CosmoMC [13] we have studied our seven-parameter model for two combinations of datasets; first using only WMAP data, and then adding LSS, SN1a data and priors from HST, BBN, BAO and CMF. In both cases we have compared the results from using only cosmological data, and from adding priors from KATRIN in the case of $m_{\nu_e} = 0$ and 0.3 eV. First we will assume $w = -1$ (cosmological constant).

Yet more freedom in the cosmological model might be added by including $w$ as a free parameter, yielding a more general form of the dark energy component. We will also include $w$ in our analysis, assuming it to be constant.

4. Results

4.1. A seven-parameter model

Starting out, we considered the simplest case using only the standard seven-parameter universe model, as explained in section 2, and WMAP data only. Then we added the assumed KATRIN priors for $m_{\nu_e} = 0$ eV and $m_{\nu_e} = 0.3$ eV as explained in section 3.2. The results are summarized in figure 2 and table 1. One easily sees that the different priors from KATRIN indeed affect some of the parameter constraints.

Evidently the inferred value of $h$ depend heavily on which prior we assume on $m_{\nu_e}$. A larger $M_{\nu}$ requires a smaller $h$. This degeneracy is not very surprising, as both $M_{\nu}$ and $H_0$ tend to shift the positions of the acoustic peaks in the CMB power spectrum, as mentioned in section 2. Note that we have assumed no prior on $H_0$ in this case. Also $\sigma_8$ is highly dependent on the priors on $m_{\nu_e}$. We saw in section 2 that $M_{\nu}$ alters the height of the peaks in the CMB power spectrum, and it will therefore be strongly correlated with $\sigma_8$ which is an amplitude parameter.

Effects on $n_s$ may be even more interesting. The significance of $n_s < 1$ is important, as $n_s \lesssim 1$ is a generic prediction of most inflation models. In the case of $m_{\nu_e} = 0$ eV, we find a significance of 2.7$\sigma$ for $n_s < 1$, while this increases to 3.8$\sigma$ when applying a KATRIN prior in the case of $m_{\nu_e} = 0.3$ eV. This degeneracy between $M_{\nu}$ and $n_s$ stems from the fact that increasing $M_{\nu}$ tends to increase the amplitude of the acoustic peaks.
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Figure 2. Marginalized parameter distributions when using WMAP data (black solid lines), compared to the resulting distributions when adding KATRIN data with \( m_{\nu_e} = 0 \) eV (red dashed lines) and \( m_{\nu_e} = 0.3 \) eV (blue dotted lines).

Table 1. Limits on cosmological parameters with different priors on the neutrino mass when using WMAP data only. In the left column are the results from having no priors on \( M_{\nu} \) (black solid lines in figure 2), the middle column shows the results when using the assumed KATRIN prior in the case of \( m_{\nu_e} = 0 \) eV (red dashed lines in figure 2), and the rightmost column gives the results with an assumed KATRIN prior for \( m_{\nu_e} = 0.3 \) eV.

| \( m_{\nu_e} = \text{free} \) | \( m_{\nu_e} = 0 \) eV | \( m_{\nu_e} = 0.3 \) eV |
|-----------------------------|------------------|------------------|
| \( \Omega_b h^2 \)         | 0.0216 ± 0.0008  | 0.0220 ± 0.0007  | 0.0215 ± 0.0007  |
| \( h \)                    | 0.65 ± 0.06      | 0.70 ± 0.04      | 0.63 ± 0.03      |
| \( \tau \)                 | 0.085 ± 0.029    | 0.089 ± 0.029    | 0.085 ± 0.028    |
| \( n_s \)                  | 0.940 ± 0.022    | 0.956 ± 0.016    | 0.939 ± 0.016    |
| \( \sigma_8 \)             | 0.61 ± 0.09      | 0.71 ± 0.06      | 0.60 ± 0.05      |
| \( \Omega_\Lambda \)       | 0.66 ± 0.07      | 0.73 ± 0.04      | 0.65 ± 0.05      |
| \( \Omega_m \)             | 0.34 ± 0.07      | 0.27 ± 0.04      | 0.35 ± 0.05      |
| \( z_{re} \)               | 11.2 ± 2.6       | 11.3 ± 2.6       | 11.2 ± 2.7       |
| \( M_{\nu} \) (eV)         | <1.75 (95% C.L.) | <0.58 (95% C.L.) | 0.87 ± 0.10      |

on scales smaller than the neutrino free streaming scale. This can be compensated by decreasing \( n_s \) which will give a larger tilt on the primordial power spectrum. Modifications in the distributions of \( \Omega_m \) and \( \Omega_\Lambda \) are also evident. This happens since increasing \( M_{\nu} \) will shift the time of matter–radiation equality and thus amplify the acoustic peaks, and this effect can be compensated by increasing \( \Omega_m \) (and thus also reducing \( \Omega_\Lambda \)).
Figure 3. Marginalized parameter distributions when using data from WMAP + LSS + SN1 + HST + BBN + BAO + CMF (black solid lines), compared to the resulting distributions when adding KATRIN data with $m_{\nu_e} = 0$ eV (red dashed lines) and $m_{\nu_e} = 0.3$ eV (blue dotted lines).

Next, we added data from LSS, SN1, HST, BBN, BAO and CMF to the WMAP data. Doing this, the limit on $M_\nu$ from cosmology alone is in the same range as we get from KATRIN in the case of $m_{\nu_e} = 0$, thus we would not expect a large effect from adding the KATRIN prior in this case. If, however, we apply the $m_{\nu_e} = 0.3$ eV scenario, we would expect to see some effects. This is indeed the case, as we can see from figure 3 and table 2.

The effect of adding a KATRIN prior with $m_{\nu_e} = 0.3$ eV is most pronounced for the $\Omega_m$, $\Omega_\Lambda$, $\sigma_8$ and $h$. The shifts in the distributions can be explained by much of the same arguments as in the case where we used WMAP data only. A difference can be seen in the effect on $n_s$. When using only WMAP data, a larger $M_\nu$ pulled $n_s$ to lower values, while here $n_s$ is shifted to slightly larger values. This can be understood by the effect of $M_\nu$ on the matter power spectrum, where a larger $M_\nu$ suppresses small scale fluctuations. This can be compensated by increasing $n_s$.

4.2. Constraining $w$

Next we redo the analysis, including $w$ as a free parameter. The resulting constraints on $w$ can be seen in figure 4 and table 3.

When using WMAP data only, the constraints on $w$ are relatively weak. One usually needs to include e.g. SN1a or LSS data to get tight constraints on this parameter, as the
Figure 4. Marginalized parameter distributions of $w$ when using data from WMAP only (left panel) and WMAP + LSS + SN1a + HST + BBN + BAO + CMF (right panel). Black solid lines show the distributions for cosmological data only. This is compared to the resulting distributions when adding KATRIN data with $m_{\nu_e} = 0$ eV (red dashed lines) and $m_{\nu_e} = 0.3$ eV (blue dotted lines).

Table 2. Limits on cosmological parameters with different priors on the neutrino mass using the full range of data sets WMAP+LSS+SN1a+HST+BBN+BAO+CMF. In the left column are the results from having no priors on $M_\nu$ (black solid lines in figure 2), the middle column shows the results when using the assumed KATRIN prior in the case of $m_{\nu_e} = 0$ eV (red dashed lines in figure 2), and the rightmost column gives the results with an assumed KATRIN prior for $m_{\nu_e} = 0.3$ eV.

| Parameter | $m_{\nu_e} =$ free | $m_{\nu_e} = 0$ eV | $m_{\nu_e} = 0.3$ eV |
|-----------|---------------------|---------------------|---------------------|
| $\Omega_b h^2$ | $0.0220 \pm 0.0006$ | $0.0220 \pm 0.0006$ | $0.0221 \pm 0.0007$ |
| $h$ | $0.70 \pm 0.02$ | $0.71 \pm 0.02$ | $0.68 \pm 0.02$ |
| $\tau$ | $0.086 \pm 0.029$ | $0.084 \pm 0.028$ | $0.100 \pm 0.029$ |
| $n_s$ | $0.956 \pm 0.014$ | $0.955 \pm 0.014$ | $0.958 \pm 0.015$ |
| $\sigma_8$ | $0.70 \pm 0.03$ | $0.71 \pm 0.03$ | $0.66 \pm 0.04$ |
| $\Omega_A$ | $0.74 \pm 0.02$ | $0.74 \pm 0.02$ | $0.71 \pm 0.03$ |
| $\Omega_m$ | $0.26 \pm 0.02$ | $0.26 \pm 0.02$ | $0.29 \pm 0.03$ |
| $z_{re}$ | $10.9 \pm 2.6$ | $10.8 \pm 2.6$ | $12.3 \pm 2.5$ |
| $M_\nu$ (eV) | $<0.55$ (95% C.L.) | $<0.47$ (95% C.L.) | $0.58^{+0.15}_{-0.17}$ |

The main effect of $w$ is to change the expansion history at late times. We see, however, that the lower limit on $w$ improves from $w > -2.5$ to $w > -1.8$ (95% C.L.) when including the KATRIN prior for $m_{\nu_e} = 0$ eV. This happens since increasing $w$ will decrease the late-time expansion rate of the universe and thus shift the peaks in the CMB power spectrum to the left. This can be compensated by reducing $M_\nu$, which will shift the peaks back to the right.
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Table 3. Limits on the equation of state of dark energy, $w$, with different priors on the neutrino mass. The first row are the results from using WMAP data only. The second row comes from using the full range of data sets WMAP + LSS + SN1a + HST + BBN + BAO + CMF.

| Cosmological data | $m_{\nu_e}$ = free | $m_{\nu_e}$ = 0 eV | $m_{\nu_e}$ = 0.3 eV |
|-------------------|---------------------|-------------------|---------------------|
| WMAP              | $-1.3^{+0.5}_{-0.6}$ | $-1.1 \pm 0.4$     | $-1.3 \pm 0.5$       |
| WMAP++            | $-1.06^{+0.10}_{-0.11}$ | $-1.01 \pm 0.07$   | $-1.18 \pm 0.09$     |

Table 4. $\Delta \chi^2$ of the models with a KATRIN prior on $M_\nu$ relative to the models with no prior on $M_\nu$. WMAP++ refers to the analysis with using all data sets WMAP + LSS + SN1a + HST + BBN + BAO + CMF as described in the text. $N_{\text{par}}$ refers to the number of free parameters in the models.

| Cosmological data | Model          | $\Delta \chi^2$ ($m_{\nu_e}$ = 0 eV) | $\Delta \chi^2$ ($m_{\nu_e}$ = 0.3 eV) | $N_{\text{par}}$ |
|-------------------|----------------|--------------------------------------|----------------------------------------|-----------------|
| WMAP              | $\Lambda$CDM + $M_\nu$ | $-0.01$                             | $-0.18$                               | 7               |
| WMAP++            | $\Lambda$CDM + $M_\nu$ | $-0.02$                             | $-4.08$                               | 7               |
| WMAP              | $\Lambda$CDM + $M_\nu + w$ | $-0.13$                             | $-0.01$                               | 8               |
| WMAP++            | $\Lambda$CDM + $M_\nu + w$ | $-0.07$                             | $-1.59$                               | 8               |

When using the full range of cosmological data sets, the results become more interesting, as the constraints on $w$ are tighter in this case. Here, the most interesting effect occurs in the case of adding a KATRIN prior for $m_{\nu_e} = 0.3$ eV, which makes $w = -1$ excluded by approximately $2\sigma$. That means that a positive KATRIN detection of $m_{\nu_e}$ in this range, would be an indication of $w < -1$, which would be very interesting from a cosmological point of view.

The reason for this dependence on $M_\nu$, can partially be explained by the shift of the peaks in the CMB power spectrum. But maybe more important is the fact that a smaller $w$ will require a smaller $\Omega_\Lambda$ to accommodate the accelerated expansion observed in the SN1a data. This will in turn increase $\Omega_m$, which will allow for larger neutrino masses, as explained earlier.

5. Goodness of fit

Another interesting issue is the change in $\chi^2$ when adding the KATRIN priors. For each combination of cosmological model and data sets, we have calculated the change in $\chi^2$ by

$$\Delta \chi^2 = 2 \ln \mathcal{L}(\text{no prior on } M_\nu) - 2 \ln \mathcal{L}(\text{KATRIN prior on } M_\nu),$$

where $\mathcal{L}$ is the likelihood of the best-fit sample in each combination of data sets on cosmological model. Thus $\Delta \chi^2$ is a measure of how much worse the fit to the data becomes by introducing the KATRIN priors in the different cases. The resulting values of $\Delta \chi^2$ are summarized in table 4.

We see that in the case of $m_{\nu_e} = 0$ eV, the changes in $\chi^2$ are small compared to the models with no prior on $M_\nu$. This is not surprising, as values of $M_\nu \approx 0$ eV fit the cosmological data very well. When using the full range of data sets in our seven-parameter model, the situation becomes slightly worse, giving $\Delta \chi^2 = -4.08$. Introducing
$w$ as a free parameter in our model, the change in $\chi^2$ by introducing the $m_{\nu_e} = 0.3$ eV is not that severe anymore, giving $\Delta \chi^2 = -1.59$. This can be understood by the well-known degeneracy between $M_\nu$ and $w$.

6. Discussion and conclusions

In this paper we have investigated whether constraints on $m_{\nu_e}$ from the KATRIN experiment will affect our knowledge on cosmological parameters. This has been done for two scenarios, one where $m_{\nu_e} = 0$ eV, and one where $m_{\nu_e} = 0.3$ eV. We have carried out the analysis both with a simple seven-parameter model with a cosmological constant, and extending the parameter space to include the equation of state for dark energy, $w$, as a free parameter.

When using WMAP data only, we find that knowledge from the KATRIN experiment will contribute significantly to constrain a wide range of cosmological parameters, regardless of which of the $m_{\nu_e}$ scenarios we use. For instance will the significance of $n_s < 1$ depend on what KATRIN tells us about $m_{\nu_e}$. Other parameters that are sensitive to the value of $m_{\nu_e}$ are $\Omega_m$, $\Omega_{\Lambda}$, $\sigma_8$ and $H_0$.

Adding more cosmological data sets, both from SN1a, galaxy catalogues and other priors, the situation changes a bit. In this case $M_\nu$ is strongly constrained from above by cosmology alone, such that an additional KATRIN prior in the case of $m_{\nu_e} = 0$ eV has little effect on our cosmological parameter constraints. However, if KATRIN measures a neutrino mass of $m_{\nu_e} = 0.3$ eV, there will be significant shifts in several of the parameter distributions. One should also note that several of the extra cosmological data sets added here may be affected by uncontrolled systematics (see [8]). Therefore, having cosmological constraints from WMAP+KATRIN without any additional cosmological data sets will be interesting regardless of the possibility to add other cosmological data sets to obtain similar results.

In the case of $w$ the most interesting result occurs in the scenario of a KATRIN defection of $m_{\nu_e} = 0.3$ eV and using the full range of data sets. In this case, $w < -1$ is favored at a 2$\sigma$ level. It should also be mentioned that there are degeneracies between parameters from different cosmological inflation models and neutrino masses (see [33]). This means that a KATRIN prior on $M_\nu$ will be important also for constraining inflationary models.

To conclude, we find that the expected limits on $m_{\nu_e}$ from KATRIN will be a useful input to constrain cosmological models, regardless of the value of $m_{\nu_e}$. If KATRIN detects a non-zero value of $m_{\nu_e}$, this would be especially interesting.

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