Supernova Simulations from a 3D Progenitor Model – Impact of Perturbations and Evolution of Explosion Properties

Bernhard Müller1,2,6*, Tobias Melson3, Alexander Heger2,4,5,6, Hans-Thomas Janka3

1Astrophysics Research Centre, School of Mathematics and Physics, Queen’s University Belfast, Belfast, BT7 1NN, United Kingdom; b.mueller@qub.ac.uk
2Monash Centre for Astrophysics, School of Physics and Astronomy, Monash University, Victoria 3800, Australia
3Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85748 Garching, Germany
4School of Physics & Astronomy, University of Minnesota, Minneapolis, MN 55455, U.S.A.
5Center for Nuclear Astrophysics, Department of Physics and Astronomy, Shanghai Jiao-Tong University, Shanghai 200240, P. R. China.
6Joint Institute for Nuclear Astrophysics, 1 Cyclotron Laboratory, National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824-1321, U.S.A.

31 July 2017

ABSTRACT

We study the impact of large-scale perturbations from convective shell burning on the core-collapse supernova explosion mechanism using three-dimensional (3D) multi-group neutrino hydrodynamics simulations of an 18\(M_\odot\) progenitor. Seed asphericities in the O shell, obtained from a recent 3D model of O shell burning, help trigger a neutrino-driven explosion 330 ms after bounce whereas the shock is not revived in a model based on a spherically symmetric progenitor for at least another 300 ms. We tentatively infer a reduction of the critical luminosity for shock revival by \(\sim 20\%\) due to pre-collapse perturbations. This indicates that convective seed perturbations play an important role in the explosion mechanism in some progenitors. We follow the evolution of the 18\(M_\odot\) model into the explosion phase for more than 2 s and find that the cycle of accretion and mass ejection is still ongoing at this stage. With a preliminary value of \(7.7 \times 10^{50}\) erg for the diagnostic explosion energy, a baryonic neutron star mass of \(1.85M_\odot\), a neutron star kick of \(\sim 600\) km s\(^{-1}\) and a neutron star spin period of \(\sim 20\) ms at the end of the simulation, the explosion and remnant properties are slightly atypical, but still lie comfortably within the observed distribution. Although more refined simulations and a larger survey of progenitors are still called for, this suggests that a solution to the problem of shock revival and explosion energies in the ballpark of observations are within reach for neutrino-driven explosions in 3D.

Key words: supernovae: general – stars:massive – convection – hydrodynamics – turbulence

1 INTRODUCTION

After several decades of research, the mechanism powering supernova explosions of massive stars still remains under investigation. From the tortuous history of the field, the neutrino-driven mechanism has emerged as the most popular – and likely most prevalent – explosion scenario for core-collapse supernovae. In all but the lightest supernova progenitors (Kitaura et al. 2006; Janka et al. 2008; Fischer et al. 2010; Melson et al. 2015a), this mechanism appears to hinge on the breaking of spherical symmetry by hydrodynamic instabilities like convection (Herant et al. 1994; Burrows et al. 1995; Janka & Müller 1996) and the standing accretion shock instability (SASI; Blondin et al. 2003; Foglizzo et al. 2007; Laming 2007; Guilet & Foglizzo 2012). These instabilities improve the conditions for neutrino-driven runaway shock expansion by various means, including the “turbulent pressure” provided by non-spherical fluid motions (Burrows et al. 1995; Murphy et al. 2013; Couch & Ott 2015; Müller & Janka 2015) and the feedback of mixing on the post-shock stratification and neutrino heating and cooling (Herant et al. 1994; Janka & Müller 1996).

The computational tools for capturing these two indispensable ingredients – neutrino heating and cooling and three-dimensional (3D) fluid flow – with a high degree of confidence have become available during the last few years (see Janka et al. 2016 and Müller 2016 for an overview of recent results and simulation methodologies). Several groups have presented 3D core-collapse supernova simulations employing various algorithms for multi-group neutrino transport ranging from the very rigorous Vertex (Hanke et al. 2013; Melson et al. 2015a,b; Janka et al. 2016) and Chimera (Lentz et al. 2015) models to more approximate methods (Takiwaki et al. 2012, 2014, 2016; Müller 2015; Roberts et al. 2016).

The findings from the first generation of 3D multi-group neutrino hydrodynamics simulations present a challenge: In general, the 3D models prove more reluctant to achieve shock revival than
their axisymmetric (2D) counterparts and either marginally fail to explode (Hanke et al. 2013) or explode later than in 2D (Takiwaki et al. 2014; Melson et al. 2015b; Lentz et al. 2015). This behaviour is compatible with earlier models using a simple light-bulb approximation (Hanke et al. 2012; Couch 2013) and has been ascribed to the different turbulent energy cascade in 3D and 2D, which leads to smaller structures and stronger dissipation in the neutrino heating region in 3D (though there may be compensating effect, see Müller 2016).

The serious implication is that with the best available physics treatment, we can currently not yet obtain successful 3D models of neutrino-driven explosions across a large range of progenitor masses. Considering the paucity of first-principle 3D simulations and the fact that some of them apparently come very close to shock revival, this may reflect an unlucky choice of progenitors or minor numerical problems (such as resolution issues) rather than a fundamental problem. Nonetheless, the first generation of 3D multi-group neutrino hydrodynamics simulations certainly provides motivation for exploring missing ingredients in the neutrino-driven mechanism that could lead to more robust and earlier explosions, not least because they may also help solve the “energy problem” in many successful 2D simulations (Janka et al. 2012; Nakamura et al. 2015; O’Connor & Couch 2015), in which the explosion energy grows rather slowly and may fall short of the typical observed values. Various possible ingredients for more robust neutrino-driven explosions have recently been explored, including rapid rotation (Janka et al. 2016; Takiwaki et al. 2016) and a reduction of neutral-current neutrino-nucleon scattering opacities (Melson et al. 2015b; Horowitz et al. 2017; Burrows et al. 2016).

Another idea posits that initial asymmetries in the supernova progenitors can precipitate shock revival by boosting convection and/or the SASI (Couch & Ott 2013; Müller & Janka 2015). Such initial perturbations (mostly solenoidal velocity perturbations) at the pre-collapse stage could arise naturally in active convective burning shells, and in that sense the “perturbation-aided” neutrino-driven mechanism does not need to invoke new physics. The efficacy of this mechanism still remains unclear; however, Simulations using parameterised initial conditions for the initial perturbations using a leakage scheme (Couch & Ott 2013, 2015) or multi-group neutrino transport (Müller & Janka 2015; Burrows et al. 2016) found a range of effect sizes from nil to a significant impact on shock revival. This is mirrored by analytic estimates (Müller et al. 2016; Abdikamalov et al. 2016) for the reduction of the critical neutrino luminosity (Burrows & Goshy 1993) for shock revival due to the injection of extra turbulent kinetic energy into the post-shock region by infalling perturbations. There is consensus that the initial convective Mach number during shell burning and the spatial scale of the initial perturbations are the decisive parameters. Although these parameters can be estimated from spherically symmetric (1D) stellar evolution models (Müller et al. 2016), firm conclusions can only be drawn from supernova simulations based on 3D progenitor models. This has been attempted by Couch et al. (2015), but the conclusive force of their results is limited. On the methodological side, the approximations employed for their 3D model of silicon shell burning alter the dynamics of the convective flow (Müller et al. 2016), and their use of a leakage scheme also significantly affects the dynamics in the supernova core after collapse. Furthermore, they did not find a qualitative difference between the supernova models employing 1D and 3D initial conditions: Positive diagnostic explosion energies develop in both cases at the same time; however, the model with 3D initial conditions shows faster shock propagation from 100 ms after bounce onward. Two key questions thus remain open: Can pre-collapse perturbations make a qualitative difference for shock revival, and can they help bring the predicted explosion properties close to observed values?

In this paper, we take the next step towards answering these questions extending a cursory preview of our recent work on the perturbation-aided mechanism in Müller (2016). We present results from the first successful multi-group neutrino hydrodynamics simulation of a core-collapse supernova explosion using such a 3D progenitor model for an $18M_\odot$ star (Müller et al. 2016) and also provide a comparison to a 1D initial model and a 3D case with artificially reduced nuclear burning rates and convective velocities, respectively. The goals of our study are twofold: First, we aim to better quantify the effect of infalling initial perturbations on the post-shock flow dynamics, the neutrino heating conditions, and the critical luminosity for shock revival. With a base of comparison of only three models, we can obviously only make limited progress on this front, but our analysis is also meant as preparatory work for similar simulations that will undoubtedly emerge in the near future. Second, we seek to assess whether the inclusion of 3D initial conditions is helpful or even sufficient to bring simulations in line with observational constraints (energy, nickel mass, neutron star mass, spin, and kick). To this end, the $18M_\odot$ explosion model has been extended to more than 2.35 s after core bounce.

Our paper is structured as follows: After reviewing the initial models and the numerical methods used in our supernova simulations in Section 2, we provide a brief overview of the three different simulations in Section 3, followed by a more detailed analysis of the impact of the level of initial perturbations on the model dynamics. The explosion dynamics of the $18M_\odot$ progenitor model of Müller et al. (2016) is discussed in Section 4. In Section 5 we outline the implications and limitations of our findings and point out open questions left for future research.

1 For example, the diagnostic explosion energy already starts to grow around 20 ms after bounce even for the case of a 1D initial model, which is not even remotely similar to recent multi-group neutrino hydrodynamics simulations.
Figure 2. Slices showing the mass fraction $X_{\text{Si}}$ of silicon at the onset of collapse in models s18-3Dr (left) and s18-3D (right). Both models are characterised by 2–3 silicon rich plumes (darker shades of blue. Due to the higher convective velocities, the boundary between the oxygen shell and the carbon shell is more strongly distorted by interfacial wave breaking in model s18-3D.)

2 SETUP AND NUMERICAL METHODS

2.1 Initial Models

We compute the collapse and post-bounce evolution of a 1D initial model (s18-1D) and two 3D initial models (s18-3D and s18-3Dr) of an 18$M_\odot$ solar metallicity progenitor with a helium core mass of 5.3$M_\odot$.

s18-3D is a 3D pre-collapse model that has been obtained by simulating the final 5 minutes of oxygen shell burning in 3D (Müller et al. 2016). As Müller et al. (2016) simulated only the region between mass coordinates $m$ of 1.68$M_\odot$ and 4.07$M_\odot$ in 3D, data from the corresponding 1D stellar evolution model is used outside this domain.

In order to better analyse the response of the accretion shock to the amplitude of the pre-collapse perturbations, we have also constructed another 3D progenitor model s18-3Dr with reduced convective velocities. In this model, we assume a fixed temperature profile for computing the nuclear reaction rates during the last 5 minutes of the pre-collapse evolution (while taking composition changes into account). Otherwise the setup is identical to s18-3D.

The resulting artificial reduction of the burning rate in s18-3Dr results in smaller convective Mach numbers at collapse. This is illustrated by radial profiles of the root-mean-square average Mach number $M_{a_r}$ of fluctuations of the radial velocity $v_r$ (Figure 1),

$$M_{a_r} = \left\langle \left( v_r - \langle v_r \rangle \right)^2 \right\rangle^{1/2} \langle \xi \rangle,$$

where angled brackets denote spherical Favre (i.e. density-weighted) averages. The maximum value of $M_{a_r}$ is less than 0.04 in model s18-3Dr compared to 0.1 in s18-3D. On the other hand, the scales of the convective flow at the pre-collapse stage are similar for both models with 2–3 prominent silicon-rich plumes (Figure 2) and a convective velocity field dominated by an $\ell = 2$ mode.

In the case of the spherically symmetric model s18-1D, spherical symmetry needs to be broken by hand in the supernova simulation. To this end, we impose random seed perturbations of $\delta v_r/v_r = 5 \times 10^{-3}$ onto the radial velocity field around 110 ms after the onset of collapse.

2.2 Core-Collapse Supernova Simulations

We compute the collapse and post-bounce evolution of the three initial models with the CoCoNuT-FMT code (Müller & Janka 2015). The hydro module CoCoNuT solves the equations of general relativistic hydrodynamics in spherical polar coordinates using piecewise parabolic reconstruction (Colella & Woodward 1984), a hybrid HLLC/HLLE Riemann solver (Mignone & Bodo 2005), and second-order time integration. It employs the xCFC approximation for the space-time metric (Cordero-Carrion et al. 2009) and currently assumes a spherically symmetric metric for 3D simulations.

Neutrino transport is treated using the energy-dependent FMT scheme of Müller & Janka (2015), which makes various simplifying assumptions, but still achieves reasonable quantitative agreement with more sophisticated methods for many of the quantities relevant for the supernova explosion problem (neutrino luminosities and mean energies, heating conditions, etc.) as shown in Appendix A of Müller & Janka (2015). Technically, the FMT scheme involves the solution of the zeroth moment equation in fully decoupled energy groups under the assumption of stationarity. At high
and intermediate optical depths, the flux factor is obtained from a two-stream solution of the Boltzmann equation in such a way as to produce the correct diffusion limit. Where the flux factor exceeds 1/2, we use a two-moment closure based on the assumption maximum packing. Velocity-dependent terms in the transport equation are currently neglected (although gravitational redshift is included), and the ray-by-ray method (Buras et al. 2006a) is used to simplify the problem of multi-dimensional transport. Simplifications in the treatment of the neutrino microphysics include the omission of neutrino electron scattering, an approximate treatment of recoil energy transfer in neutrino-nucleon scattering by an effective absorption opacity, and the omission of absorption and emission processes other than nucleon bremsstrahlung (treated by an effective single-particle rate) for heavy flavour neutrinos. To better reproduce the collapse dynamics of the inner iron core, the deleptonisation scheme of Liebendörfer (2005) is used during the collapse phase. Modifications of neutrino nucleon reactions due to weak magnetism and nucleon correlations are not taken into account in the present study. For details we refer the reader to Appendix A of Müller & Janka (2015).

Our calculations are performed on a grid of $550 \times 128 \times 256$ zones in the $r$, $\theta$- and $\varphi$-direction (corresponding to an angular resolution of $1.4^\circ$) with a non-equidistant radial grid extending out to $10^5 \text{km}$.

We use the equation of state of Lattimer & Swesty (1991) with a bulk incompressibility modulus of $K = 220 \text{MeV}$ in the high-density regime. In the low-density regime, we use an EoS accounting for photons, electrons, and positrons of arbitrary degeneracy and an ideal gas contributions from 17 nuclear species. Nuclear statistical equilibrium is assumed above $5 \times 10^9 \text{K}$, and nuclear reactions below this temperature are treated using the approximate “flashing” method of Rampp & Janka (2002).

3 IMPACT OF INITIAL PERTURBATIONS ON SHOCK REVIVAL

3.1 Qualitative Impact on Shock Evolution and Hydrodynamics Instabilities

To illustrate the role of initial perturbations in models s18-3D, s18-3Dr, and s18-1D, we compare the evolution of the shock, gain, and proto-neutron star radii and the mass accretion rate $\dot{M}$ in Figure 3. We also show neutrino luminosities $L_\nu$ and mean energies $E_\nu$ for all three models in Figure 4. Meridional slices of the entropy for all three models at selected times are presented in Figure 5.

As expected, differences between the 3D models are minute at early times. A minor peculiarity of model s18-1D is the development of more violent prompt convection and shock ringing prior to 50 ms after bounce. This behaviour is connected to the imposition of random seed perturbations in s18-1D on the entire grid, i.e. also in the Fe and Si core, which is not explicitly perturbed in models s18-3D and s18-3Dr. Moreover, patching the 3D O shell burning simulation and the core of the 1D stellar evolution model together results in slight hydrostatic adjustment in model s18-3D, which slightly reduces the mass accretion rate and the electron flavour luminosity (top panel of Figure 4) compared to s18-1D and s18-3Dr. Despite these differences, the shock trajectories in the three models nonetheless converge again after this transient phase of prompt convection. 80 ms after bounce (top row Figure 5), they all show very similar shock radii and incipient neutrino-driven convection with small-scale plumes of similar size.

The evolution of the models starts to diverge around 150 ms after bounce with slightly larger shock radii in models s18-3D and s18-3Dr. This is well before the arrival of the Si/O shell interface at 200-250 ms, but convection in the O shell can already make itself felt by generating density perturbations in the stable Si shell (via g-mode excitation), which will then undergo amplification during collapse (Lai & Goldreich 2000). Shortly before the arrival of the Si/O shell interface, the density perturbations ahead of the shock can already become sizeable. To quantify the level of pre-shock density perturbations, we evaluate the root-mean-square (RMS) deviation of the density $\rho$ from its spherical average $\langle \rho \rangle$

$$\delta \rho(r) = \left( \frac{\int (\rho - \langle \rho \rangle)^2 d\Omega}{4\pi} \right)^{1/2},$$

at a radius $r$ of 250 km (Figure 6, top panel).

Prior to the arrival of the Si/O interface, the scale of the infalling density perturbations remains small, however. Only once the O shell reaches the shock do we observe large-scale density perturbations in s18-3D and s18-3Dr with angular wavenumbers $\ell \approx 2$ corresponding to the dominant convective eddies in the pre-collapse models. This is illustrated by Figure 6 (bottom panel), which shows the normalised power $\rho \nu / \rho_0$ of the density perturba-
The interaction of the infalling perturbations with the shock turns out to be very subtle upon close examination and evolves through several distinct regimes as the strength of the perturbations and the character of the post-shock flow in the unperturbed model change. Initially (second row of Figure 5 200 ms), the unperturbed model shows low-amplitude SASI oscillations with buoyancy-driven plumes emerging in some directions, which is suggestive of a transition regime between strongly SASI-dominated and convection-dominated flow. The infalling perturbations that interact with the oscillating shock are still small during this phase. Their quantitative impact on the SASI oscillation can be gauged by decomposing the angle-dependent shock position into spherical harmonics $c_{lm}$ (Figure 7),

$$
c_{lm} = \int r_\alpha(\theta, \varphi) Y_{lm}^*(\theta, \varphi) \, d\Omega.
$$

Figure 7 shows the coefficients $c_{lm}$ for the dipole mode. In the first regime of “weak perturbations” (up to about 200 ms after bounce), the quasi-periodicity of the SASI oscillations is not destroyed, and their amplitudes are unaffected or even slightly enhanced by the pre-shock perturbations.

The situation changes as strong, large-scale pre-shock density perturbations reach the shock in models s18-3D and s18-3Dr. In this regime, the models are characterised by large and long-lived high-entropy bubbles. Despite the fact that small quasi-periodic dipolar shock oscillations still persist, the fact that the bubbles clearly have higher characteristic angular wavenumbers than the typical $\ell = 1$ and $\ell = 2$ modes of the SASI in model s18-3D and in particular (bottom row of Figure 5, middle panel) suggest that the strong perturbations push the models from the SASI-dominated regime into a buoyancy-dominated regime.

To illustrate that the evolution of these bubbles is dictated by the infalling density perturbations, we show Aitoff projections of the relative perturbations in ram pressure ($P_{\text{ram}} - \langle P_{\text{ram}} \rangle$) ahead of the shock and of the angle-dependent shock radius for model s18-3Dr in Figure 8. The variations in $P_{\text{ram}}$ mostly reflect the pre-shock density perturbations; the relative variations in pre-shock velocity are smaller by more than an order of magnitude. Bubbles generally emerge below strong underdensities in the pre-shock region. The geometry of the pre-shock density perturbations is not strictly mirrored by the shock, however. The shape of the shock only adjusts to changes in the infalling perturbations with some time lag, and rather reflects the long-term average of the forcing by density and ram pressure perturbations.

In s18-3D, the formation of two high-entropy bubbles very quickly results in shock revival (around 300 ms after bounce). Due to the lower amplitude of the initial perturbations, model s18-3Dr exhibits stable shock degradation for about 200 ms and maintains somewhat higher shock radii than s18-1D before the shock eventually starts to re-expand around 500 ms after bounce. The shock is not revived in s18-1D, and continues to undergo SASI oscillations until the end of the run (625 ms after bounce).

A key point emerging from this analysis is that the beneficial effect of infalling perturbations (at least in the regime considered here) is not primarily to add “turbulent pressure” homogeneously throughout the gain region (although this does not preclude the existence of an effective phenomenological description of the reduction in critical luminosity in a quasi-spherical picture along the lines of Müller & Janka 2015 and Summa et al. 2016). The mech-
Figure 5. Entropy in the z-y-plane at 80 ms, 200 ms, 300 ms, and 400 ms after bounce (top to bottom) in models s18-3D (left column), s18-3Dr (middle column), and s18-1D (right column). At 80 ms (top row), all three models show the development of neutrino-driven convection without any noticeable influence of initial perturbations. At 200 ms (second row), infalling perturbations start to interact with large-scale SASI oscillations in s18-3D and s18-3Dr. At 300 ms (third row), s18-3D is on the way to shock-revival aided by forced-shock deformation, and infalling perturbations largely destroy SASI oscillations in s18-3Dr. At 400 ms (bottom row), model s18-3Dr also exhibit strong forced shock deformation, while model s18-1D continues to be dominated by the SASI spiral mode.
Figure 6. RMS deviation $\delta \rho / \langle \rho \rangle$ of the density from its spherical average and normalised power $\rho_{1}/\rho_{0}$ (with $\rho_{0} = \langle \rho \rangle$) in the first multipoles of the pre-shock density perturbations computed according to Equation (3) for models s18-1D (black), s18-3Dr (blue) and s18-3D (red). Since the density perturbations are evaluated at a radius of 250 km, no data are shown once the maximum shock radius exceeds this value in models s18-3Dr and s18-3D.

The mechanism of forced shock deformation crucially depends on the modification of the large-scale flow structure; and one of the key effects seems to be that variations in ram pressure facilitate the formation of stable, high-entropy bubbles that eventually reach a sufficient scale and density contrast to expand continuously due to buoyancy, which is critical for runaway shock expansion in multi-D (Dolence et al. 2013; Fernández et al. 2014; Fernández 2015). It is important to note, however, that (as shown by model s18-3Dr) there may be a considerable delay from the point when local variations in ram pressure allow high-entropy bubbles to hover stably in the accretion flow in an almost stationary manner to the point when they become sufficiently buoyant to expand continuously, which requires overcoming the ram pressure not only in the direction of the shock protrusions (Figure 5, bottom centre) at a local ram pressure minimum, but over a large solid angle with somewhat higher average ram pressure than at the local minimum.

The three simulations thus demonstrate a significant impact of the seed asphericities from O shell burning on the post-bounce evolution of the 18$M_{\odot}$ progenitor. Depending on the level of perturbations, shock revival occurs at 300 ms, 500 ms (if the O burning rate is artificially reduced), or does not occur at all for the 1D version of the progenitor. Overall, our models with 3D initial conditions behave very similar to 2D models with parameterised perturbations of comparable amplitude and scale: They confirm that the mechanism of forced shock deformation is viable in 3D and 2D alike, and initial Mach numbers of $\sim$0.1 or less are already sufficient for a large effect on shock revival provided that shell convection is dominated by large-scale $\ell \approx 2$ modes. To what extent perturbation-aided explosions are still possible in 3D under less favourable circumstances (i.e. lower Ma or higher $\ell$) remains to be determined by more systematic studies.

Figure 7. Normalised coefficients $c_{1,m}$ for the dipolar shock deformation modes with different $m$ computed according to Equation (4) for models s18-1D (black), s18-3Dr (blue), and s18-3D (red). The dipole coefficients illustrate the interaction of infalling large-scale perturbations with the SASI, which dominates the unperturbed model s18-1D. Initially (from about 80 ms to 220 ms), the infalling perturbations are not harmful to the SASI and can even enhance quasi-periodic shock oscillations. As the infalling large-scale perturbations become stronger, regular SASI oscillations are first reduced in amplitude and then give way completely to forced shock deformation with small stochastic oscillations on top.

3.2 Spectrum of Pre-Shock Perturbations

Müller & Janka (2015) outlined the mechanism of perturbation-aided neutrino-driven explosions qualitatively, invoking a causal chain from near-solenoidal initial perturbations, which are converted into density perturbations via during the infall (Lai & Goldreich 2000; Takahashi & Yamada 2014), and increase shock deformation and the violence of the turbulent post-shock flow (Müller & Janka 2015; Couch & Ott 2013). Using analytic estimates (Müller et al. 2016) and Abdikamalov et al. (2016) also made attempts to
Figure 8. Aitoff projections of the relative variations in pre-shock ram pressure \( (P_{\text{ram}} - \langle P_{\text{ram}} \rangle) / \langle P_{\text{ram}} \rangle \) at a radius of 170 km (encoded on a red-white-blue colour scale) and isocontours of the angle-dependent shock position \( r_{\text{sh}} \) (in units of km) in model s18-3Dr at post-bounce times of 326 ms, 408 ms, 428 ms, and 482 ms. The poles lie on the \( z \)-axis, and the central meridian is aligned with the \( x-z \)-plane. As model s18-3Dr remains in the regime of forced shock deformation for about 200 ms, it nicely illustrates the adjustment of the shock geometry to changes in the infalling perturbations. The shock initially develops three protrusions in the equatorial plane and one at the South pole due to the low density and ram pressure ahead of the shock (top). The three protrusions in the equatorial plane remain stable despite the emergence of stronger underdensities ahead of the shock in other spots (second panel), although one of these original protrusions gradually disappears in favour of a strong bulge in the Western hemisphere (third and fourth panel).

Figure 9. Spectra of pre-shock density perturbations \( \hat{\rho}/\rho_0 \) at a radius of 350 km (red) for model s18-3D (top, RMS average between 260 ms and 360 ms after bounce) and s18-3Dr (bottom, RMS average between 300 ms and 400 ms) compared to the spectra of pre-collapse density perturbations (blue) and the radial Mach number in the O shell. The spectrum \( \hat{M}_a \) of the radial Mach number is computed according to Equation (6). The solid black curves define a corridor of \( \pm 0.25 \hat{M}_a \) around 0.5 \( \hat{M}_a \) (dashed curve); the spectrum of pre-shock density perturbations mostly falls within this corridor. For comparison, the spectra of the pre-shock perturbations in model s18-1D are also included (magenta) for the respective time frame. Note that the pre-collapse spectra of density and velocity perturbations are RMS averages over radii from 3352 km to 4126 km.

Previous simulations of perturbation-aided shock revival have only provided limited insights on how initial perturbations in the progenitor translate quantitatively into pre-shock perturbations and then into more violent turbulent motions. The new models allow us to better test the validity of the proposed analytic approaches to the perturbation aided mechanism.

Regarding the conversion of initial velocity perturbations into
density perturbations via advective-acoustic coupling, our simulations are in line with the simple estimate $\delta \rho / \rho \sim M_{\mathrm{conv}}$ of Müller & Janka (2015) for the pre-shock density perturbations: Once the O shell reaches the shock, we find – roughly as predicted – $\delta \rho / \rho \approx 0.1$ for s18-3D and $\delta \rho / \rho \approx 0.04$ for s18-3D (top panel of Figure 6). $\delta \rho / \rho$ is only a very gross measure for the pre-shock density perturbations, however. It is also susceptible to short-wavelength perturbations that grow from random noise in model s18-1D (where we encounter values of $\delta \rho / \rho$ of up 0.03) and could also arise for purely numerical reasons (e.g. odd-even modes). The relatively large values of $\delta \rho / \rho$ in models s18-3D and s18-3D near the arrival of the Si/O interface at the shock are also due to such high-wavenumber perturbations.

Since small-scale perturbations do not have a significant impact on the shock (Müller & Janka 2015), it is more appropriate to consider the spectrum $\rho_1$ of pre-shock perturbations instead of $\delta \rho / \rho$. Our simulations suggest that the spectra of the pre-shock density perturbations and the initial velocity perturbations in the O shell are remarkably similar, at least in a time-average sense. Figure 9 shows normalised time-averaged spectra $\dot{\rho}_1 / \rho_0$ of the pre-shock density perturbations with

$$\dot{\rho}_1(r) = \left( \frac{\int_{V_\ell} \rho_1^2(r) \, dr}{t_2 - t_1} \right)^{1/2},$$

for relevant phases before shock revival in models s18-3D and s18-3Dr to illustrate this. We compare this to the spectrum $M_{\lambda,t}$ of the Mach number of radial motions in the middle of the O shell, which we define as

$$M_{\lambda,t} = \left( \sum_{n=\ell} \int Y_{n,\ell}(\theta,\varphi) \rho_\ell(r) / c_s \, d\Omega \right)^{1/2},$$

where we have normalised the velocity by the local sound speed. To obtain a smoother spectrum, we compute RMS averages of $M_{\lambda,t}$ for the region between 3352 km and 4126 km in the progenitor (although we find $M_{\lambda,t}$ to vary little in the interior of the O shell).

To illustrate the generation of density perturbations from vorticity perturbations due to advective-acoustic coupling during collapse, we also show the RMS averaged spectrum of initial density in this region in Figure 9. Spectra of the pre-shock density perturbations in model s18-1D are also included for comparison.

With only a few exceptions, the pre-shock density perturbations fall nicely within the range,

$$\dot{\rho}_1 / \rho_0 = (0.25-0.75)M_{\lambda,t},$$

for both s18-3D and s18-3Dr over a wide range of angular wavenumbers $\ell$. On average, the spectrum of pre-shock density perturbations is well described by $\dot{\rho}_1 / \rho_0 \sim M_{\lambda,t}/2$.

The similarity of the initial velocity spectrum and the spectrum of pre-shock density perturbations is in line with the simple explanation of density perturbations by “differential infall” in Müller & Janka (2015), which suggests $\dot{\rho}_1 / \rho_0 \sim M_{\lambda,t}/2$ for all eddy scales. Interestingly, the simple relation between $\dot{\rho}_1$ and $M_{\lambda,t}/2$ has not been established so far by studies of linear perturbations in Bondi accretion in a general context (Kovalenko & Ermin 1998; Foglizzo 2001, 2002) or specifically in core-collapse supernovae (Lai & Goldreich 2000; Takahashi & Yamada 2014). Instead, these works suggest that stronger density perturbations are created for higher wavenumbers, e.g. $\rho_1 \propto (\ell + 1)$ (Lai & Goldreich 2000) and $\rho_1 \propto \ell$ (Takahashi & Yamada 2014). The crucial problem here is that some assumptions of these linear studies need to be adjusted in order to model the growth of perturbations during core collapse accurately: Lai & Goldreich (2000) and Takahashi & Yamada (2014) do not account for the subdominant role of compressive velocity perturbations at the pre-collapse stage (Müller & Janka 2015). They also assume stationary Bondi flow as a background solution and neither account for the small initial infall velocity (much slower than free-fall) of perturbed convective shells nor for the density gradient in the progenitor. The problem of linear perturbation growth in collapsing supernova cores therefore needs to be revisited to provide a theoretical basis for Equation (7).

Figure 10 also clearly shows the excess power in pre-shock density perturbations at small $\ell$ in s18-3D and s18-3Dr compared to s18-1D. While s18-1D develops non-negligible pre-shock density perturbations at late times, these lie predominantly in the range $\ell = 10-20$ and are dynamically less significant than the strong large-scale perturbations in the other two simulations. For further analysis, it is convenient to introduce a dimensionless number to characterise the power of perturbations in the relevant region of small $\ell$ by a single dimensionless number to replace the problematic metric $\delta \rho / \rho$. To this end, we define the scale-weighted power $^3$ in perturbations $\mathcal{R}$ as

$$\mathcal{R}(r) = \left( \frac{\sum_{\ell=1}^{m} (\dot{\rho}_1 / \rho_0^2) \ell^2}{\ell^2} \right)^{1/2},$$

Because of the similarity of $\rho_1$ and $M_{\lambda,t}$, $\mathcal{R}$ is also tightly related to the typical Mach number and angular wavenumber of the convective shell in question, i.e. we have

$$\mathcal{R} \approx \frac{M_{\lambda,t}}{2\ell_{\ell_{\ell_{\ell}}}}$$

as shown in Figure 10.

---

3 Note that this expression naturally generalises the identity $\delta \rho / \rho = (\sum_{\ell=1}^{m} (\rho_1 / \rho_0)^2)^{1/2}$ between the RMS value of the density perturbations in real space and the power in spectral space (which follows from Parseval’s identity for spherical harmonics and also motivates the use of RMS values for the perturbations in real and spectral space elsewhere).
3.3 Turbulent Kinetic Energy in the Gain Region

Previous studies of perturbation-aided shock revival (Couch & Ott 2013; Müller & Janka 2015; Couch et al. 2015) have traced the beneficial impact of the pre-collapse perturbations to the increased violence of non-spherical flows in the post-shock region, which increases the mass \( M_{\text{gain}} \) in the gain region and the volume-integrated neutrino heating rate \( \dot{Q}_\nu \) (solid). Our simulations conform to this picture. Figure 11 compares \( M_{\text{gain}} \) and \( \dot{Q}_\nu \) for the three different models as well as the kinetic energy contained in turbulent motions. The turbulent kinetic energies in radial \( (E_{\text{kin,r}}) \) and non-radial \( (E_{\text{kin,\theta,\phi}}) \) motions are computed separately as integrals over the gain region \( V_{\text{gain}} \).

\[
E_{\text{kin,r}} = \frac{1}{2} \int_{V_{\text{gain}}} \rho (v_r - \langle v_r \rangle)^2 \, dV \tag{10}
\]

\[
E_{\text{kin,\theta,\phi}} = \frac{1}{2} \int_{V_{\text{gain}}} \rho (v_\theta^2 + v_\phi^2) \, dV, \tag{11}
\]

where \( v_r, v_\theta, \) and \( v_\phi \) denote the velocity components and \( \langle v_r \rangle \) is the spherical Favre (i.e. density-weighted) average of the radial velocity.

We also calculate effective Mach numbers \( M_a \) and \( M_{a,\theta,\phi} \) for the radial and transverse components of the turbulent velocity fluctuations as the relevant parameters characterising the importance of non-radial fluid flow in the gain region. These are defined as

\[
M_a^2 \equiv \frac{1}{c_s^2(\langle r_{\text{sh}} \rangle)} \frac{2E_{\text{kin,r}}}{M_{\text{gain}}} \approx \frac{3(\langle r_{\text{sh}} \rangle)}{GM} \frac{2E_{\text{kin,r}}}{M_{\text{gain}}}, \tag{12}
\]

and analogously for \( M_{a,\theta,\phi} \). For convenience we use the post-shock value of the sound speed \( c_s \) at the average shock radius \( \langle r_{\text{sh}} \rangle \) for computing effective Mach numbers and approximate \( c_s^2(\langle r_{\text{sh}} \rangle) \approx GM/(3\langle r_{\text{sh}} \rangle) \) as in Müller & Janka (2015). Generally find the expected hierarchy between the three models, i.e. \( E_{\text{kin,r}}, E_{\text{kin,\theta,\phi}}, \dot{Q}_\nu \).

Because of the small velocities, the Newtonian expression for the kinetic energy can be used. Special and general relativistic corrections to the volume element are implicitly included in Equations (10) and (11) as well as in all other volume integrals used in this study.

There is, in fact, some motivation for using this approximation; it is more closely connected to the idea that the ratio of turbulent stresses and the pre-
and $M_{\text{gain}}$ are higher for stronger initial perturbations during the accretion phase with the exception of brief intervals where s18-1D shows larger shock radii than the two other models due to modulations in SASI activity.

Müller & Janka (2015) pointed out, however, that this hierarchy alone only furnishes indirect evidence that the excitation of non-radial fluid motions in the presence of infalling perturbations is more efficient than in the presence of neutrino heating as a driver of convection (or of an advective-acoustic amplification cycle for SASI modes) alone. For this it is useful to consider efficiency factors $\eta_{\text{conv},r}$ for the conversion of neutrino heating into turbulent kinetic energy,

$$\eta_{\text{conv},r} = \frac{E_{\text{kin},r}/M_{\text{gain}}}{\langle (\rho_{\text{sh}} - \rho_{\text{gain}})(Q_{\nu}/M_{\text{gain}}) \rangle^{2/3}},$$

$$\eta_{\text{conv},r,\theta,\phi} = \frac{E_{\text{kin},r,\theta,\phi}/M_{\text{gain}}}{\langle (\rho_{\text{sh}} - \rho_{\text{gain}})(Q_{\nu}/M_{\text{gain}}) \rangle^{2/3}}.$$  

(13)

(14)

In the non-perturbed case in 2D, Müller & Janka (2015) found $\eta_{\text{conv},r,\theta,\phi} \approx 0.5$ once the non-spherical instabilities had reached saturation, which can be understood as the result of a balance between buoyant driving and turbulent dissipation that obtains in the gain region (Murphy et al. 2013). As already observed in Müller (2016), the efficiency factor in non-perturbed 3D models is considerably smaller ($\eta_{\text{conv}} \sim 0.2 \ldots 0.35$; Figure 12) than in the 2D case. This is likely due to a combination of several factors: The forward turbulent cascade in 3D results in smaller eddy structures (Hanke et al. 2012; Murphy et al. 2013; Couch 2013) and implies a smaller eddy dissipation length. It is also conceivable that turbulent 3D flow more efficiently transports heat for a given average convective velocity, so that marginal instability against convective overturn can be maintained by less violent convective motions than in 2D. The results of Murphy et al. (2013), who found a higher ratio of the convective heat flux (or convective luminosity in their terminology) to the volume-integrated neutrino heating rate points in this direction.

Interestingly, Müller & Janka (2015) detected no clear impact of the perturbation amplitudes on $\eta_{\text{conv},r,\theta,\phi}$: all their perturbed 2D models showed small fluctuations around $\eta_{\text{conv},r,\theta,\phi} = 0.5$ similar to the unperturbed baseline model.

This counterintuitive finding can be resolved by considering the contributions to the turbulent kinetic energy from radial and transverse velocity fluctuations separately. Figure 11 shows that the infalling perturbations appear to selectively boost radial velocity fluctuations, whereas model s18-1D shows $E_{\text{kin},r} \approx E_{\text{kin},\theta,\phi}$ for fully developed SASI. This conclusion is reinforced by plots of the efficiency factors $\eta_{\text{conv},r}$ and $\eta_{\text{conv},r,\theta,\phi}$ in Figure 12. Prior to and around shock revival, the perturbations increase $\eta_{\text{conv},r}$ considerably above the typical value of $\eta_{\text{conv},r} \approx 0.27$ in s18-1D after the accretion of the Si/O shell interface. This is consistent with the change in geometry from an $\ell = 1$ spiral SASI in s18-1D with rapid mass motions around the proto-neutron star to 2-4 stable high-entropy bubbles that are supported by (radial) buoyancy forces. Since it is the radial velocity fluctuations that regulate turbulent heat transport within the gain region and determine the momentum carried by convective bubbles colliding with the shock, it is natural to consider the increase in $\eta_{\text{conv},r}$ as the relevant factor for achieving better conditions in models with strong initial perturbations.

Having identified $\eta_{\text{conv},r}$ as a suitable metric that reflects the stronger activity of turbulent motions in the gain region due to the infalling perturbations (while factoring out feedback effects due to the larger shock radius and increased neutrino heating), it is tempting to ask whether our models can be used to validate analytic predictions (Müller et al. 2016; Abdikamalov et al. 2016) for this phenomenon. At the current stage, this seems premature, however. A comparison with the theory of Abdikamalov et al. (2016) is not yet meaningful! Although their study is based on an elaborate framework for the interaction of perturbations with the shock in the linear approximation (Ribner 1953), it considers only the interaction of vorticity and entropy perturbations with the shock and disregards the action of buoyancy on the shocked perturbations, which is likely the dominant effect for the injection of extra turbulent kinetic energy (Müller et al. 2016). The approach of Müller et al. (2016) simplifies the interaction of the perturbations with the shock, but in principle admits a comparison with our numerical results. Their prediction can be encapsulated in a correction term in the equation for the turbulent kinetic energy due to the work of buoyancy on the shocked perturbations. Phrased in our variables, they predict

$$\frac{E_{\text{kin},r}}{M_{\text{gain}}} = \eta_{\text{conv},r} \left[ \frac{\langle (\rho_{\text{sh}} - \rho_{\text{gain}}) Q_{\nu} \rangle}{M_{\text{gain}}} \right]^{2/3} (1 + \psi)^{2/3}.$$  

(15)

Here the correction term $\psi$ is determined by the ratio of buoyant energy generation $\dot{Q}_{\text{pot}}$ from the shocked perturbations and the neutrino heating rate $\dot{Q}_{\nu}$, and by the postulated ratio of turbulent dissipation lengths in the non-perturbed and strongly perturbed regime,

$$\psi = \frac{\pi (\rho_{\text{sh}}) \dot{Q}_{\text{pot}} / (M_{\text{gain}})}{(\rho_{\text{sh}} - \rho_{\text{gain}}) \dot{Q}_{\nu} / M_{\text{gain}}}.$$  

(16)

$\dot{Q}_{\text{pot}}$ depends on the density fluctuations, the accretion rate $\dot{M}$ and the difference in gravitational potential between the shock and gain radius,

$$\dot{Q}_{\text{pot}} = \frac{M \dot{\rho}}{\rho} \left( \frac{GM}{\rho_{\text{sh}}} - \frac{GM}{\rho_{\text{gain}}} \right).$$  

(17)

Since $\dot{\rho}/\rho$ is weighted with $t^{-1}$ in Equation (16), it is natural to relate the term $\dot{Q}_{\text{pot}}/t$ to $R$ for a spectrum of incident perturbations.

$$\frac{\dot{Q}_{\text{pot}}}{t} = \frac{M R}{\ell} \left( \frac{GM}{\rho_{\text{sh}}} - \frac{GM}{\rho_{\text{gain}}} \right).$$  

(18)

To test the prediction of Müller et al. (2016), we re-compute $\eta_{\text{conv},r}$ based on Equations (15), (16), and (18), to check whether the correction factor $(1 + \psi)^{2/3}$ allows us to recover a "universal" value of $\eta_{\text{conv},r}$. The corrected values of $\eta_{\text{conv},r}$ are included in Figure 12. We also show the evolution of $\psi$ (Figure 10) as a measure for the relative importance of buoyant energy generation by infalling perturbations.

The result is ambiguous. During the quasi-stationary evolution after the infall of the Si/O interface, the correction factor brings $\eta_{\text{conv},r}$ in s18-3D nicely in line with s18-1D, and it also reduces the excursion to 0.4 in s18-3D around 300 ms to about the long-term average of $\eta_{\text{conv},r} \approx 0.27$. Before and during the infall of the Si/O shell interface, the non-stationarity of the models precludes a reasonable comparison. Moreover, it is not clear whether the transition from the SASI-dominated regime (s18-1D) to a buoyancy-dominated regime (s18-3D and s18-3Dr) offsets the comparison. At present, we can only conclude that the factor for the turbulent kinetic energy proposed by Müller et al. (2016) is not implausible. Firm conclusions can only be based on a larger set of models, and more idealised simulations controlled setup (in the vein of...
Figure 13. Ratio $\tau_{\text{adv}}/\tau_{\text{heat}}$ of the advection and heating time-scales $\tau_{\text{adv}}$ and $\tau_{\text{heat}}$ for models s18-1D (black), s18-3Dr (blue), and s18-3D (red). The critical value for runaway shock expansion $\tau_{\text{adv}}/\tau_{\text{heat}} = 1$ is denoted by a dashed line. Thanks to forced shocked deformation by infalling large-scale perturbations, models s18-3D and s18-3Dr undergo shock revival while the critical ratio is still well below unity for s18-1D.

Figure 14. Evolution of models s18-1D (black), s18-3Dr (blue), and s18-3D towards the critical curve in the $M\dot{M}/L_{\nu}E_{\nu}^2$-plane. The dependency of the critical heating functional $L_{\nu}E_{\nu}^2$ on the gain radius (Equation (19)) is taken into account by means of a correction factor $\xi = (r_{\text{gain}}/100 \text{ km})^{-2/5}$. A lower limit for the theoretical critical curve (dashed) is obtained by anchoring the power law $(L_{\nu}E_{\nu}^2)_{\text{crit}}(\xi) \propto (MM)^{2/5}$ at the endpoint of the trajectory of s18-1D, which has not undergone shock revival yet at this junction, but is close to the threshold. Red and blue dots denote points on the trajectory of model s18-1D corresponding to the onset of the explosion in s18-3D and s18-3Dr.

Fernández et al. (2014; Fernández 2015) may be more informative for this purpose.

3.4 Impact on Explosion Conditions

From their different estimates for the additional generation for turbulent kinetic energy due to infalling perturbations, Müller et al. (2016) and Abdikamalov et al. (2016) proceeded further to predict the reduction of the critical neutrino luminosity for explosion in terms of the parameters of the initial perturbations. Specifically, Müller et al. (2016) postulated a reduction of the critical luminosity by $-0.15\psi_f$, which according to their estimates would amount to 12-24% for model s18-3D. In view of our discussion in Section 3.3, it is obvious that we cannot validate the quantitative relations and the underlying physical arguments proposed in the studies. Instead we can only provide a plausibility check by examining the impact of the perturbations on the explosion conditions in our models. Following Müller & Janka (2015), we quantify the effect of the perturbations by considering both the critical ratio of the advection and heating time-scales $\tau_{\text{adv}}$ and $\tau_{\text{heat}}$ (Janka 1998, 2001; Thompson 2000; Buras et al. 2006b; Murphy & Burrows 2008; Fernández 2012) and the trajectories of the models in the generalised $M - L_{\nu}$ plane.

Figure 13 shows the time-scale ratio $\tau_{\text{adv}}/\tau_{\text{heat}}$ for the three different models. It is evident that the perturbations result in a major enhancement in heating conditions: Disregarding some fluctuations in models s18-1D, s18-3D and s18-3Dr show significantly higher values than s18-1D while the convective O shell is accreted and reach the critical threshold $\tau_{\text{adv}}/\tau_{\text{heat}} = 1$ at times of 340 ms and 500 ms after bounce respectively. The typical values for model s18-1D are only $\sim 0.5$ at these times. Incidentally, Figure 13 also demonstrates that the slightly lower mass accretion rate in s18-3D compared to s18-1D and s18-3Dr (see discussion in Section 3.1) is not responsible for earlier shock revival in this model; the time-scale ratio $\tau_{\text{adv}}/\tau_{\text{heat}}$ is practically identical to s18-3Dr until the Si/O interface reaches the shock.

The time-scale criterion is closely related to the concept of the critical neutrino luminosity (Burrows & Goshy 1993) for shock revival (Janka 2012). Disregarding multi-D effects (and assuming a roughly constant binding energy at the gain radius), Müller & Janka (2015) showed that it is tantamount to a critical condition

$$\left( L_{\nu}E_{\nu}^2 \right)_{\text{crit}}(\xi) \propto (MM)^{2/5},$$

where $L_{\nu}$ is the total electron flavour luminosity, $E_{\nu}$ is an appropriately defined average mean energy for $\nu_e$ and $\bar{\nu}_e$, and $M$ is the proto-neutron star mass. Since s18-1D is close to reaching the critical explosion condition $\tau_{\text{adv}}/\tau_{\text{heat}} = 1$, we can anchor the critical curve defined by Equation (19) at (or slightly above) the end point of trajectory of this model in the $M\dot{M}/L_{\nu}E_{\nu}^2$-plane (Figure 14). The explosions in models s18-3D and s18-3Dr occur roughly 22% and 16% below the (theoretically inferred) critical curve. This reduction of the critical luminosity is of a similar order as the estimate of 0.15$\psi_f$ of Müller et al. (2016), though more on the high side. These effects are sizeable and comparable to the differences between spherically symmetric and multi-dimensional models (Murphy & Burrows 2008; Hanke et al. 2012; Couch 2013; Dolence et al. 2013). For s18-3D we must, however, bear in mind that the explosion is triggered right when the Si/O shell interface arrives, i.e. as soon as the shock experiences strong forcing by infalling perturbations. It cannot be excluded that shock revival could be achieved even earlier if similarly strong perturbations were already present when $\tau_{\text{adv}}/\tau_{\text{heat}}$ is still down at values of $\sim 0.25$. On the other hand, the effect of the infalling perturbations may be magnified during a highly non-stationary phase of rapidly declining $\dot{M}$ and transient shock expansion. For s18-3D, diagnosing the reduction of the critical luminosity is therefore particularly problematic. Despite these uncertainties, it is clear, however, that the various diagnostics point to a reduction of the critical luminosity in the tens-of-percent range due convective seed perturbations for our particular 18$M_\odot$ progenitor.

For models without strong initial perturbations, Summa et al. (2016) and Janka et al. (2016) showed that the effects of a varying
4 EXPLOSION PHASE

We have continued model s18-3D to 2.35 s after bounce to allow for a tentative evaluation of the explosion and remnant properties in a 3D model of a perturbation-aided neutrino-driven explosion.

The evolution of the shock radius and mass shell trajectories for the entire simulation are shown in the bottom panel of Figure 15. By the end of the simulation, the minimum shock radius exceeds 10,000 km, i.e. the shock has already traversed the entire O/Si shell and reached the C/O shell. The shock and the ejecta retain a pronounced global asymmetry at this stage with considerably stronger shock expansion in the y-direction of the computational grid (Figure 16) with a maximum shock radius of 27,000 km. The shock geometry imprinted by forced shock deformation at the time of shock revival is largely preserved until these late times aside from some minor adjustments; the direction of fastest shock propagation corresponds to one of the Si-rich convective updrafts in the progenitor shown in Figure 2.

4.1 Explosion Energy

Even at the end of the simulation, the cycle of mass accretion and mass ejection due to neutrino heating is still ongoing, and we can therefore only obtain tentative estimates for the final explosion properties as in previous 2D and 3D explosion models with multi-group neutrino transport. In lieu of the final explosion energy, one typically considers the “diagnostic” energy $E_{\text{diag}}$ (Buras et al. 2006b; Suwa et al. 2010; Müller et al. 2012; Bruenn et al. 2016) as an estimator, which is calculated by integrating the net total (kinetic+internal+gravitational) energy $\epsilon_{\text{tot}}$ per unit mass over the region where it is positive (i.e. where matter is formally unbound,

$$E_{\text{diag}} = \int_{\Omega_{\text{out}}} \rho \epsilon_{\text{tot}} \, dV,$$

(20) where the volume element is implicitly taken to include general relativistic corrections.

The proper definition of $\epsilon_{\text{tot}}$ is not straightforward, especially in general relativity. Müller et al. (2012) suggested computing $\epsilon_{\text{tot}}$ as

$$\epsilon_{\text{tot}} = \epsilon + v^2/2 + \Phi,$$

(22) in terms of the pressure $P$, density $\rho$, internal energy density $\epsilon$ (excluding rest-mass differences between different nuclear species), Lorentz factor $W$ and lapse function $\alpha$ in the relativistic case. In the Newtonian limit, this reduces to the familiar form

$$\epsilon_{\text{tot}} = \epsilon + v^2/2 + \Phi,$$

(23) Calculating $E_{\text{diag}}$ based on Equations (21,22) effectively amounts to double-counting the potential energy of the ejecta that is due to their own self-gravity (and not to the gravitational field of the proto-neutron star) and therefore overestimates the energy needed to expel the ejecta to infinity. In the Newtonian case, Buras et al. (2006a,b) circumvented this problem by computing $\Phi$ for a given mass shell from the enclosed mass only, but this is not a suitable solution in our case because it would imply discarding GR corrections to the gravitational field of the proto-neutron star. To avoid the problem of double-counting, we can, however, subtract the Newtonian potential $\Phi_{\text{grav.out}}$ generated by the shells outside a given radius $r$.

$$\epsilon_{\text{tot}} = \alpha[(c^2 + \epsilon + P/\rho)W^2 - P/\rho] - Wc^2 - \Phi_{\text{grav.out}}(r),$$

(24)
Figure 16. Volume rendering of the entropy in model s18-3D 2 s after bounce. The expanding neutrino-heated bubbles (red/yellow) retain a pronounced global asymmetry at this stage and drive faster shock (translucent cyan surface) expansion in the direction of positive y and x. The global geometry is still very similar to the stage of shock revival shown in Figure 5.

where

$$\Phi_{\text{grav, out}} = \int_{r}^{\infty} 4\pi r^2 G \rho dr'.$$  \hspace{0.5cm} (25)

This is justified because relativistic corrections to the outside potential are negligible in the region in question ($r > 100 \text{ km}$).

The diagnostic explosion energy computed from Equations (20), (24), and (25) is shown in the top panel of Figure 15. Model s18-3D exhibits a steady increase in diagnostic energy, even though the rate of increase slows down at about 700 ms. By the end of the simulation, the model has reached a value of $E_{\text{diag}} = 7.7 \times 10^{50} \text{ erg}$, which is still increasing at a rate of $1-2 \times 10^{50} \text{ erg s}^{-1}$. $E_{\text{diag}}$ does not include any correction for the binding energy of the material outside the shock (for which Bruenn et al. 2013, 2016 introduced the convenient term “overburden”). Taking all the material outside a radius of 10,000 km with negative $e_{\text{tot}}$ into account, we obtain an overburden of $2.6 \times 10^{50} \text{ erg}$. The true correction is likely somewhat smaller because part of this bound material will still be channelled onto the proto-neutron through the downflows, so that its negative total energy will not contribute to the energy budget of the ejecta. We can therefore put a relatively confident lower limit of $5 \times 10^{50} \text{ erg}$ on the explosion energy of s18-3D. The true value is likely higher, the continuing slow increase of the diagnostic energy will probably equalise the correction from the overburden within a second, so that a final explosion energy of $7-8 \times 10^{50} \text{ erg}$ appears reasonable.

In agreement with the argument of Marek & Janka (2009) and earlier long-time simulations of supernova explosions in 2D and 3D
(Müller 2015; Bruenn et al. 2016), the growth of the diagnostic energy at late times is essentially determined by the mass outflow rate $M_{\text{out}}$ of neutrino-heated ejecta, which are first lifted to $\eta_{\text{out}} \approx 0$ and then obtain most of their net positive energy from nucleon recombination. Other contributions to the energy budget of the ejecta (e.g. nuclear burning and the accumulation of weakly bound material by the shock) are subdominant at late times. Figure 17 demonstrates that

$$\frac{dE_{\text{out}}}{dt} \approx \epsilon_{\text{rec}} M_{\text{out}}$$

(26)

describes the evolution of the diagnostic energy well at late times with a value of $\epsilon_{\text{rec}} = 6$ MeV that accounts for incomplete recombination and some turbulent mixing between the outflows and downflows.

What is quite remarkable about the late-time evolution of the diagnostic energy in model s18-3D is that neutrino heating proves very efficient at driving outflows. Based on the assumption that the volume-integrated neutrino heating rate $Q_\nu$ and the specific binding energy at $|\epsilon_{\text{gain}}|$ at the gain radius are relevant parameters regulating the outflow rate $M_{\text{out}}$, Müller (2015) suggested the ratio

$$\eta_{\text{out}} = \frac{M_{\text{out}}|\epsilon_{\text{gain}}|}{Q_\nu}$$

(27)

as an appropriate efficiency parameter for mass ejection. Müller (2015), however, already observed efficiency parameters $\eta_{\text{out}} > 1$.
during some phases of their 11.2M⊙ explosion model. Model s18-3D can maintain an appreciable growth of the explosion energy despite modest neutrino heating rates of only a few 10⁶ erg s⁻¹. Since the typical binding energy at the turnaround point is not much larger than the recombinination energy \( E_{\text{rec}} \), the effective “return of investment” for using up the neutrino heating to unbind the ejected material is large, and the time derivative of the explosion energy is formally a large fraction of the neutrino heating rate. At late times, we find (top panel of Figure 17),

\[
E_{\text{diss}} \approx 0.5Q, \tag{30}
\]

which is even more extreme than in the 3D long-time simulation of Müller (2015) and considerably higher than in 2D simulations, where \( E_{\text{diss}} \approx (0.15-0.2)Q \) (Müller et al. 2012; Bruenn et al. 2016) is more typical.

Though this diagnosis reveals a plausible cause for the steady growth of \( E_{\text{diss}} \) in model s18-3D, the crucial role of turbulent mixing between the downflows and outflows raises further questions that need to be addressed with more sophisticated analysis methods and high-resolution simulations: Why do the efficiency parameters \( \eta_{\text{out}} \) and \( \eta_{\text{out,100}} \) grow after the first few hundreds of milliseconds of the explosion phase? Could this come about because turbulent mixing between the downflows and outflows becomes more effective as the expanding structures in the post-shock region are resolved by a larger number of grid points and the effective numerical Reynolds number increases? Is there a simple physical explanation for the distribution of turnaround points? It is clear that we cannot provide better answers based on a single, moderately resolved 3D simulation, but the late-time behaviour of model s18-3D furnishes further evidence that – contrary to the problem of shock revival – the effects of 3D turbulent flow could be beneficial in the explosion phase (Handy et al. 2014; Melson et al. 2015a; Müller 2015) and help to achieve explosion energies in line with observational constraints.

### 4.2 Ejecta Composition

With a small set of nuclear species and a very simplified treatment of nuclear burning, we can only draw limited conclusions on the composition of the ejecta in model s18-3D; obtaining the detailed nucleosynthesis would require post-processing the model and is beyond the scope of the current paper. Moreover, our use of the FMT method for neutrino transport introduces uncertainties in the electron fraction of the neutrino-processed ejecta. Nonetheless, a closer look at the ejecta composition in s18-3D is useful both as a tentative plausibility check for our simulation and as an indicator of potential problems and crucial sensitivities of 3D explosion models.

The contribution \( \Delta M_C \) of important α- and iron-group nuclei to the mass of the ejecta is shown in Figure 19. During the early phase of the explosion, the intermediate post-shock temperatures are sufficiently high for burning the shocked material into NSE, which produces about 0.03M⊙ of ⁵⁶Ni. A considerable fraction of the synthesised iron group material does not remain unbound, however, and is eventually channelled around the expanding neutrino-heated bubbles and accreted onto the proto-neutron star. About half of the ⁵⁶Ni synthesised by explosive burning is thus lost again by a post-bounce time of 1 s. For the same reason, only about half of the intermediate-mass elements synthesised in the O shell (S, Si, etc.) in the progenitor are eventually ejected (see Figures 1 and 14 in Müller et al. 2016 for the initial composition of the O shell, which contains about 0.2M⊙ of ²⁸Si). By contrast, shocked material from the almost completely processed C shell mostly retains positive ve-

---

6 As a sanity check, we verified, for example, that the rate of ejection \( dn_{\text{out}}/dt \) of tracer particles through the outer boundary of the analysis region reproduces the mass outflow rate \( \dot{M}_{\text{out}} = n_{\text{out}} \dot{m}_{\text{gain}} \) if we assign each particle a mass \( M_{\text{gain}}/N \), where \( N \) is the number of tracers.
Most of the eventually ejected iron-group material in model s18-3D is produced by freeze-out from NSE in the neutrino-heated bubbles. Together with the $^{56}$Ni produced by explosive burning, roughly 0.06$M_\odot$ of iron-group material are thus ejected by the end of the explosion. The relatively high content of iron-group nuclei in the neutrino-driven ejecta (compared to only 0.03$M_\odot$ of $^4$He) is the result of turbulent mixing between the bubbles and the colder downflows, which implies that the freeze-out from NSE occurs at relatively low entropies.

It is important to note that the precise composition of the ejecta from the iron-group would certainly differ from the one plotted in Figure 19, where $^{56}$Fe is the dominant iron-group nucleus. This is not only an artefact of the small number of iron-group nuclei in our NSE table, but in all likelihood related to the FMT transport solver. The FMT scheme tends to produce somewhat higher luminosities for $v_\nu$ than for $v_e$ and a slowly widening gap between the mean energies of these two neutrino species at late times (Figure 4). As a result, the neutrino-heated ejecta tend to become slightly neutron-rich, different from simulations relying on more sophisticated transport solvers (Müller et al. 2012; Bruenn et al. 2016; Wanajo et al. 2017), where the neutrino-heated ejecta tend to be proton-rich for massive progenitors. With an electron fraction $Y_e$ slightly lower than $Y_e = 0.5$ in the neutrino-heated ejecta, $^{54}$Fe (which is not included in our NSE table) and then $^{56}$Fe replaces $^{56}$Ni as the dominant nucleus in NSE (Hartmann et al. 1985). Since the overall explosion dynamics and the entropy of the neutrino-heated bubbles is less sensitive to subtle differences in electron neutrino and antineutrino emission, we expect, to first order, that the effect of a more rigorous transport treatment would be to shift the $^{56}$Fe to $^{56}$Ni, which dominates the nucleosynthesis in freeze-out from NSE/QSE for $Y_e > 0.5$. The true mass of ejected $^{56}$Ni in s18-3D could therefore well be of order $\gtrsim 0.05M_\odot$.

Considering the modelling uncertainties (small set of 17 nuclei in NSE, fixed freeze-out temperature of $5.8 \times 10^9$ K, flashing prescription below NSE temperatures, uncertainties in $Y_e$), the production of a few $0.01M_\odot$ of iron-group elements does not present any significant conflict with observational constraints: Based on observed correlations between explosion energy and nickel mass ($\propto M_{\text{exp}}$), and then $^{56}$Fe replaces $^{56}$Ni as the dominant nucleus in NSE (Hartmann et al. 1985). Since the overall explosion dynamics and the entropy of the neutrino-heated bubbles is less sensitive to subtle differences in electron neutrino and antineutrino emission, we expect, to first order, that the effect of a more rigorous transport treatment would be to shift the $^{56}$Fe to $^{56}$Ni, which dominates the nucleosynthesis in freeze-out from NSE/QSE for $Y_e > 0.5$. The true mass of ejected $^{56}$Ni in s18-3D could therefore well be of order $\gtrsim 0.05 M_\odot$.

Figure 20. Baryonic mass $M_{\text{bary}}$ of the proto-neutron star (top panel), $v_x$, $v_y$, and $v_z$-components (thin black, red, and blue curves) and absolute value (thick curve) of the inferred kick velocity $v_{\text{kick}}$ computed according to Equation (32) (middle panel) and the proto-neutron star angular momentum $\mathbf{J}$ (bottom panel).

Figure 21. Time-integrated contributions of the different flux and force terms to the kick velocity of the proto-neutron star (pressure anisotropy $F_{\text{p},\text{ff}}$: black, advective momentum flux $F_{\text{adv}}$: red, asymmetrical gravitational tug $F_{\text{grav}}$: blue, anisotropic neutrino emission: violet). The sum of these contributions is shown in green and compared to the estimate based on momentum conversion (cyan, Equation 32). $x$, $y$, and $z$-components are shown in the top, middle, and bottom panel. Note that the sustained accretion of matter results in large contributions from the gravitational tug and the advection of momentum onto the neutron star that almost cancel. The cancellation is not perfect because of the gravitational tug exerted by the actual ejecta, which determines the net kick.


Figure 20. Baryonic mass $M_{\text{bary}}$ of the proto-neutron star (top panel), $v_x$, $v_y$, and $v_z$-components (thin black, red, and blue curves) and absolute value (thick curve) of the inferred kick velocity $v_{\text{kick}}$ computed according to Equation (32) (middle panel) and the proto-neutron star angular momentum $\mathbf{J}$ (bottom panel).

Figure 21. Time-integrated contributions of the different flux and force terms to the kick velocity of the proto-neutron star (pressure anisotropy $F_{\text{p},\text{ff}}$: black, advective momentum flux $F_{\text{adv}}$: red, asymmetrical gravitational tug $F_{\text{grav}}$: blue, anisotropic neutrino emission: violet). The sum of these contributions is shown in green and compared to the estimate based on momentum conversion (cyan, Equation 32). $x$, $y$, and $z$-components are shown in the top, middle, and bottom panel. Note that the sustained accretion of matter results in large contributions from the gravitational tug and the advection of momentum onto the neutron star that almost cancel. The cancellation is not perfect because of the gravitational tug exerted by the actual ejecta, which determines the net kick.
4.3 Neutron Star Properties

The evolution of the baryonic mass, kick, and spin of the proto-neutron star up to that point is shown in Figure 20; a detailed description of how we evaluate the neutron star kick and spin is given further below in this section. As for the explosion energy and the nickel mass, we can only give tentative limits for the final neutron star properties since accretion is still ongoing at the end of the simulation.

4.3.1 Neutron Star Mass

The baryonic proto-neutron star mass $M_{\text{bary}}$, however, appears to be close to its asymptotic limit already. Later than 1.2 s after bounce the accretion rate onto the proto-neutron star typically fluctuates between $0.01 M_\odot \text{s}^{-1}$ and $0.02 M_\odot \text{s}^{-1}$. With $M_{\text{bary}} = 1.865 M_\odot$; at the end of the simulation, this suggests a final neutron star of $M_{\text{NS}} \leq 1.9 M_\odot$ even if we allow for another 2 s of accretion. This is also in line with the mass shell trajectories in Figure 15, which show an emerging mass cut at around 2000 km; the mass coordinate for which the spherically averaged velocity vanishes is $M_{\text{bary}} = 1.887 M_\odot$. Using the fit formula of Lattimer & Yahil (1989) and Lattimer & Prakash (2001) for the neutron star binding energy $E_{\text{bind}}$,

$$E_{\text{bind}} \approx 0.084 M_\odot c^2 \frac{M_{\text{grav}}}{M_\odot} c^2,$$

in terms of the gravitational mass $M_{\text{grav}}$, we thus estimate a final value of $M_{\text{grav}} \leq 1.67 M_\odot$ (assuming $M_{\text{bary}} = 1.9 M_\odot$). Although higher than the typical masses in double neutron star binaries (Schwab et al. 2010; Özel et al. 2012; Özel & Freire 2016), such a neutron star birth mass is not implausible. It is well within the mass distribution of slow pulsars that likely have undergone little accretion ( Özel & Freire 2016), and more detailed analyses of the evolutionary pathways of some binary systems also suggest some neutron stars are born at least with $M_{\text{grav}} = 1.7 M_\odot$ (Tauris et al. 2011).

4.4 Neutron Star Kick

Since we model the interior of the proto-neutron star at densities larger than $10^{11} \text{ g cm}^{-3}$ in spherical symmetry, we need to infer the kick and spin indirectly via balance equations. For the kick, one can invoke momentum conservation (Scheck et al. 2006) and obtain the kick velocity $v_{\text{kick}}$ by assuming that the neutron star neutron momentum, and the momenta $p_\alpha$ and $p_s$ of the ejecta and the emitted neutrinos add up to zero, i.e.

$$v_{\text{kick}} = - \frac{p_\alpha + p_s}{M},$$

where $M$ is the inertial mass of the neutron star. In the relativistic case, this is non-trivial because one needs the inertial mass of the neutron star to obtain the kick velocity from the momentum. By dint of the equivalence principle, we can identify the inertial mass with the gravitational mass $M_{\text{grav}}$, but, strictly speaking, $M_{\text{grav}}$ can only be defined for isolated systems. For practical purposes, the gravitational field of the ejecta can still be considered as sufficiently weak to regard the proto-neutron star as isolated, however. We can thus define the approximate gravitational and inertial mass $M_{\text{grav}}$ enclosed within a radius $r_0$ (assumed to lie outside the neutron star surface) in terms of the derivative of the lapse function in the xCFC metric,

$$M_{\text{grav}} = \frac{c^2 r_0^2}{G} \frac{\partial \alpha}{\partial r}(r_0),$$

which is analogous to the Newtonian case where the enclosed mass is related to the surface integral of the gravitational acceleration $\mathbf{g}$,

$$\int \mathbf{g} \cdot d\mathbf{A} = -4\pi G M.$$

For our analysis, we use $r_0 = 50$ km, which adds little mass outside the proto-neutron star, but is sufficiently large to justify the computation of an effective gravitational mass from the local gravitational acceleration.

The kick velocity computed from Equation (32) (middle panel of Figure 20) reaches more than 600 km s$^{-1}$ and is oriented opposite to the biggest neutrino-heated plume (Figure 16 and bottom left panel of Figure 5) along the negative y-direction of the grid. Different from long-time simulations with parameterised gray transport in 2D (Scheck et al. 2006) and 3D (Wongwathanarat et al. 2013), where accretion onto the neutron star had ended after at most 1 s, the kick velocity shows little sign of asymptoting to its final value as late as 2 s after bounce. At this stage, we can only conclude that the final kick will lie on the high side of the observed kick distribution (Hobbs et al. 2005; Faucher-Giguère & Kaspi 2006; Ng & Romani 2007), but not to the degree that there is a conflict with observations. Even if the kick were to increase at the current rate for another 2 s, model s18-3D would still be within the bounds of the observed distribution that extends to $\geq 1000$ km s$^{-1}$ (Chatterjee et al. 2005). Neither does the behaviour of model s18-3D appear too unusual in terms of kick velocity compared to extant parametric and self-consistent simulations in 2D (Scheck et al. 2004, 2006; Nordhaus et al. 2010, 2012; Bruenn et al. 2016) and 3D (Wongwathanarat et al. 2010, 2013) if we leave aside the slower saturation of the kick.

This slow saturation is nonetheless the key to an interesting peculiarity of model s18-3D. It can be explained naturally by the relatively slow-paced propagation of the shock in our simulation and the persistence of stronger global accretion asymmetries than in the 3D models of Wongwathanarat et al. (2010, 2013): Whereas the models shown in Wongwathanarat et al. (2013) exhibit average
shock radii of $\sim 15,000 \, \text{km}$ about 1.3 s after bounce and already show at least the onset of an isotropic neutrino-driven wind (see their Figure 3), the average shock radius at this time is smaller than $\sim 10,000 \, \text{km}$ in our models, and downflows onto the proto-neutron star still persist. This implies that the acceleration by the “gravitational tug” of the ejecta as well as impulsive momentum transfer by the accretion downflows can be maintained longer in our simulations.

Considering the pronounced global asymmetries and the relatively slow shock propagation in model s18-3D, it is in fact somewhat surprising that the kick is not too different from the 3D models of Wongwathanarat et al. (2010, 2013) in our case. The reason for this emerges from a closer analysis of the different effects contributing to the kick: Following Scheck et al. (2006), Nordhaus et al. (2010), and Wongwathanarat et al. (2013), one can write the time derivative of the neutron star momentum $\vec{p}_{\text{PNS}}$ in terms of the gravitational force $F_{\text{grav}}$ exerted onto the proto-neutron star by material outside $r_0$, the net pressure force $F_P$ on the volume enclosed by $r_v$, the advective momentum flux $F_{\text{adv}}$ into this region, and the backreaction term $d\vec{p}_/dt$ due to anisotropic neutrino emission:

$$d\vec{p}_{\text{PNS}}/dt = F_{\text{grav}} + F_P + F_{\text{adv}} - d\vec{p}_/dt.$$  

(35)

Here, $F_P$ and $F_{\text{adv}}$ are evaluated as

$$F_P = - \int r v^2 \rho \chi d\Omega,$$  

(36)

$$F_{\text{adv}} = - \int r^2 \rho \chi v \times \vec{r} d\Omega,$$  

(37)

and $F_{\text{grav}}$ is computed using the monopole approximation for the gravitational field of the neutron star (but not the ejecta),

$$F_{\text{grav}} = \int \frac{GM_{\text{PNS}} \vec{r}}{r^3} \rho dV,$$  

(38)

where $M_{\text{PNS}}$ is calculated according to Equation (33). We compute the backreaction term from the flux of neutrinos through the outer boundary of the grid,

$$d\vec{p}_/dt = \int \vec{F}_n \cdot d\Omega,$$  

(39)

where $\vec{F}_n$ is the sum of the frequency-integrated neutrino energy flux for all species.\footnote{Since the FMT scheme involves an approximate solution to the stationary transport equation, it is more appropriate not to include the momentum nominally carried by the radiation field on the grid in the total momentum budget.}

Figure 21 shows the time-dependent contributions of $F_{\text{grav}}$, $F_P$, and momentum flux $F_{\text{adv}}$ to the components of the kick velocity (and incidentally shows that the two different ways evaluating the kick via Equation (35) or via momentum balance (Equation 32) are consistent with each other). It is noticeable that the contribution of the gravitational tug and the advective momentum flux almost cancel for our choice of $r_v = 50 \, \text{km}$ with $F_{\text{grav}}$ slightly outweighing $F_{\text{adv}}$. Since the evaluation of the flux and force terms is sensitive to $r_v$ (Nordhaus et al. 2010), this can, to some degree, be viewed as accidental, but it nonetheless enunciates a physical peculiarity of models with sustained accretion onto the proto-neutron star. The crucial point for understanding the near-cancellation of these two terms is that a sizeable contribution of the asymmetric gravitational tug in s18-3D comes from material that is either not accelerated to positive velocities by the shock at all or moves so slowly that it eventually falls back onto the proto-neutron star. Accretion of this material will then almost exactly cancel the proto-neutron star momentum generated by gravitational acceleration by virtue of Newton’s third law (discounting the small effect of momentum transfer from the downflows to the outflows by hydrodynamic forces).

It must be emphasised that despite this seeming peculiarity, model s18-3D is fully in line the established theory of hydrodynamical neutron star kicks in 3D. The net kick is still due to the slight preponderance of the gravitational tug over the advective momentum flux, which is eventually bound to become stronger as accretion ceases. Model s18-3D merely illustrates the difficulty of peeling out the net gravitational tug of the eventual ejecta by a fixed-volume analysis of the forces and fluxes onto the proto-neutron star.

In accordance with Nordhaus et al. (2010) and Wongwathanarat et al. (2013) we find the contribution from anisotropic neutrino emission to the kick to be of minor importance. It contributes only $30 \, \text{km} \, \text{s}^{-1}$ to the dominant component of the kick in the $y$-direction. Although one might expect sustained strong global asymmetry of the accretion flow during the explosion phase to be favourable for a large kick by anisotropic neutrino emission, this expectation is thwarted by the lateral redistribution of matter at the base of the gain region and the decline of the accretion rate.

Finally, a word is in order about the potential implications of two approximations made in our model, namely the use of a non-moving spherical inner core and a spherical metric. The effects of both approximations are very similar, since they imply that the proto-neutron star becomes a “momentum sink” whose momentum budget in the simulation is not captured accurately. The effects on the momentum budget of the ejecta are less severe since the gravitational potential of the proto-neutron star does not deviate strongly from spherical symmetry, and since any higher multipoles decrease rapidly with higher powers of $r$. While long-time simulations without a spherically symmetric core and a full 3D metric remain desirable, previous studies suggest that these approximation do not constitute severe limitations: In their 2D simulations with the VULCAN code, Nordhaus et al. (2012) found that evaluating the momentum of the proto-neutron star using Equation (35) (assuming a spherically symmetric potential of the neutron star) yields results in very good agreement with the direct evaluation of the neutron star in their hydrodynamical simulation, which allowed for free movement of the proto-neutron star across the grid, included a 2D potential, and maintained momentum conservation up to truncation error. Scheck et al. (2006) also investigated potential problems with a “momentum sink” in the centre by performing neutrino hydrodynamics simulations in an accelerated reference frame that tracks the motion of the proto-neutron star, and found no major differences compared to the naive implementation of the non-moving neutron star core as a momentum sink.

4.5 Neutron Star Spin

The evolution of the angular momentum of the proto-neutron star is more remarkable against the background of extant 3D explosion models with parameterised neutrino heating and cooling or gray transport (Fryer & Young 2007; Wongwathanarat et al. 2010, 2013; Rantsiou et al. 2011). Following Wongwathanarat et al. (2010, 2013), we evaluate the proto-neutron star angular momentum $\vec{J}_{\text{PNS}}$ by integrating the flux of angular momentum across the sphere with radius $r_v$,

$$d\vec{J}_{\text{PNS}}/dt = \int \alpha r^4 \chi v \times r d\Omega.$$  

(40)
Again we do not expect that neglecting higher-order multipoles of the gravitational field has a major impact on the angular momentum of the accreted material because the deviations of the gravitational field of the proto-neutron star from spherical symmetry are very small (which has prompted the use of the multipole approximation also in earlier studies such as Rantsiou et al. 2011). Exchange of momentum and angular momentum by gravitational torques between the ejecta and the material accreted after shock revival (less than 0.1\(M_\odot\)) is likely also of minor importance, though simulations with a full 3D metric are warranted in the future.

\(J_{\text{PNS}}\) grows quickly immediately after the onset of the explosion to reach values of order \(-6\times10^{37}\) g cm² s⁻¹. Assuming a final neutron star mass and radius of \(M = 1.67M_\odot\) and \(R = 12\) km respectively, we obtain a moment of inertia of about \(I \approx 2\times10^{45}\) g cm² from the fit formula of (Lattimer & Schutz 2005).

\[
I \approx 0.237M_{\text{grav}}R^2 \left[ 1 + 4.2 \left( \frac{M_{\text{grav}} \text{ km}}{M \text{ R}} \right) + 90 \left( \frac{M_{\text{grav}} \text{ km}}{M \text{ R}} \right)^4 \right].
\]  

(41)

This implies a spin period at birth of around 20 ms, i.e. close two the lower end of the period distribution of young pulsars (Muslimov & Page 1996; Marshall et al. 1998). The inferred distribution of birth periods is somewhat model-dependent, but likely centred at longer periods of several hundred milliseconds (Faucher-Giguère & Kaspi 2006; Perna et al. 2008; Noutsos et al. 2013), although on the low side Popov & Turolla (2012) still obtain typical periods of \(-100\) ms. Similar to the kick, model s18-3D thus appears to represent an outlier rather than the norm in terms of the neutron star spin, but is not in conflict with observational constraints.

Whether or not we classify the neutron star spin in s18-3D as unusually fast or not, there is a clear contrast to extant 3D simulations (Wongwathanarat et al. 2010, 2013; Rantsiou et al. 2011) as the spin period is about 10 times shorter than for the fastest rotator in Wongwathanarat et al. (2010). This is again a consequence of the long duration of accretion after the onset of shock revival. Under these conditions, the spin-up mechanism of Spruit & Phinney (1998), i.e. the stochastic transfer of angular momentum to the proto-neutron star by downflows that strike the proto-neutron star slightly off-centre, becomes more effective for several reasons: More impulsive spin-up events occur and the individual events transfer more angular momentum if late-time accretion is stronger. Furthermore, the plumes striking the proto-neutron star at late times originate from large radii, and can have relatively large angular momentum even if their initial non-radial velocity is small.

It is important to note that an appreciable amount angular momentum is actually transferred onto the proto-neutron star even at late times. While the absolute value of \(J_{\text{PNS}}\) remains roughly constant from about 0.7 s after bounce, the direction of the spin axis of the proto-neutron star still undergoes considerable re-orientation. The \(y\)-component of the angular momentum vector changes most appreciably at late times. This results in a relatively stable decrease of the angle \(\alpha\) between the proto-neutron star kick and spin from 80° at 0.7 s to about 40° at the end of the simulation, i.e. a trend towards increasing spin-kick alignment (Figure 22).

Model s18-3D would need to be extended longer to determine whether this trend continues and clear alignment is reached, and even if this is the case, a tendency towards spin-kick alignment in a single simulation may still prove a statistical fluke. Moreover, the proto-neutron star initially acquires angular momentum in the direction perpendicular to the kick after shock revival, and if there is a systematic alignment mechanism acting afterwards, it is not clear whether it would generally be sufficiently effective to “right” the proto-neutron star later. A further caveat concerns late-time fallback, which may alter the direction of the neutron star spin on much longer time-scales. If fallback occurs predominantly from directions where the shock is weaker (i.e. in the direction of the kick), this would destroy spin-kick alignment even if the the fallback mass is quite small.

Nonetheless, it is worth speculating whether there could be a physical reason behind this trend: The preferential ejection of mass into the \(y\)-direction (Figure 16) suggests that there is a weak preference for accretion to occur in the \(x-z\)-plane, i.e. the plane perpendicular to the kick direction. Since the angular momentum imparted onto the proto-neutron star by downflows is perpendicular to their direction, this could imply that the downflows change \(J_y\) more strongly than \(J_x\) and \(J_z\) on average. Even after averaging over many impulsive events, one expects that such an asymmetry between the angular momentum components parallel and perpendicular to the kick direction subsists. It should be noted that this suggested mechanism for spin-kick alignment is distinct from the one initially posited by Spruit & Phinney (1998), who claimed that stochastic angular momentum transfer by downflows is sufficient for spin-kick alignment. Their suggestion may not work generically (Wang et al. 2007) and is problematic because it hinges on the isotropy of the impulsive events in the rotating frame of the neutron star. By contrast, the mechanism we just outlined is based on the realisation that angular momentum transfer by downflows onto the proto-neutron star is non-isotropic and not completely stochastic, but exhibits an asymmetry related to the explosion geometry (and hence to the direction of the kick).

This hypothesis will need to be investigated further based on a larger sample of 3D simulations in the future. At present, the evolution of theproto-neutron star spin only opens up an interesting perspective. It suggests that sustained accretion may offer an alternative road towards short neutron star birth periods of a few tens of milliseconds other than spin-up by the standing accretion shock instability (Blondin & Mezzacappa 2006; Guilet & Fernández 2014; Kazarini et al. 2016) or the assumption of pre-collapse rotation rates of \(-100\) s (Heger et al. 2005; Ott et al. 2006). This is especially interesting considering that asteroseismic measurements of core rotation in evolved low-mass stars (Cantiello et al. 2014) suggest that current stellar evolution models still underestimate the efficiency of angular momentum transport in stellar interiors, which implies that pre-collapse rotation rates of supernova progenitors may be even lower than currently (Heger et al. 2005) predicted. Even spin-up mechanisms for strongly braked cores during late evolutionary stages, such a stochastic spin-up by internal gravity waves could likely not explain neutron star spin periods shorter than a few hundred milliseconds (Fuller et al. 2015). The possibility that late-time accretion may explain the spin-kick alignment posited by several observational papers (Lai et al. 2001; Ng & Romani 2007; Noutsos et al. 2015; Rankin 2015) is still more speculative, but remains noteworthy since many alternative mechanisms (see Janka 2017 for an overview) are still somewhat problematic (such as the invocation of jets, which cannot achieve sufficiently large kicks) or remain more schematic, such as the putative spin-kick alignment for explosions triggered by a spiral SASI mode (Janka 2017).

5 SUMMARY AND CONCLUSIONS

In this paper, we presented the first 3D multi-group neutrino hydrodynamics simulations based on initial conditions from 3D models of O shell burning. We studied the impact of the perturbations on the dynamics in the supernova core for an 18\(M_\odot\) star (s18-3D,
Müller et al. 2016) and a model with artificially reduced nuclear burning rates and smaller convective velocities in the O shell (s18-3Dr). For comparison, we conducted a run based on the corresponding 1D progenitor with small random seed perturbations.

In the simulations starting from 3D initial conditions a neutrino-driven explosion develops around 0.3 s (s18-3D) and 0.5 s (s18-3Dr) after bounce thanks to the mechanism of forced shock deformation whereas the shock is not revived in the control model s18-1D before the end of the simulation 0.645 s after bounce. Different from the 3D leakage models of Couch et al. (2015), the initial perturbations from convective shell burning thus have a significant and qualitative impact on the fate of the progenitor. When s18-3D and s18-3Dr explode, the heating conditions in model s18-1D are far below the runaway threshold with values of the critical timescale ratio $\tau_{\text{adv}}/\tau_{\text{heat}}$ are only around 0.5. This implies that at least for some progenitors, the perturbation-aided neutrino-driven mechanism could be an important part of a solution to the problem of shock revival.

Our simulations confirm that the perturbation-aided mechanism works in a similar fashion in 3D as in the earlier parameterised 2D models of Müller & Janka (2015). At least in the case of perturbations from O shell burning, the beneficial role of the perturbations consists in providing slowly-varying large-scale “forcing” of the shock due to the anisotropic pre-shock density and ram pressure that arise from the convective velocity field in the progenitor by advective-acoustic coupling during the collapse. This facilitates the formation of large neutrino-heated bubbles whose geometry is dictated by the pre-shock density perturbations.

At present, some of the links in the chain from convective seed perturbations in the progenitor to the excitation of more violent non-spherical flow in the post-shock region and the concomitant reduction of the critical luminosity for shock revival can still only be understood in qualitative terms. The first element of the perturbation-aided mechanism, i.e. the generation of pre-shock density perturbations during the collapse phase, appears to be relatively straightforward; the spectrum of pre-shock density perturbations closely reflects the turbulent velocity spectrum in the infalling convective shell in our models.

It is much more difficult to pinpoint how these pre-shock perturbations affect the violence of non-spherical motions in the gain region qualitatively. Although strongly perturbed models generally show higher turbulent kinetic energies and average Mach numbers in the gain region (Couch & Ott 2013; Couch et al. 2015; Müller & Janka 2015), this is largely due to the positive feedback effects that occur for slight changes in the shock radius and the neutrino heating conditions. Our models allow us to better pin down the impact of the infalling perturbations: They primarily result in a more efficient excitation of turbulent motions in the radial direction (but not the transverse direction) than would be expected by neutrino heating alone. This is consistent with the idea that the additional turbulent driving is provided by the buoyancy of the shocked density perturbations (Müller et al. 2016). Upon closer inspection, the comparison of models s18-3D and s18-3Dr to s18-1D proves more complicated: Part of the differences between the simulations may be stochastic in nature, as the SASI-dominated models s18-1D exhibits long-term fluctuations in the heating conditions. At early times, weak perturbations in s18-3D and s18-3Dr can apparently either boost or dampen shock oscillations under different circumstances. Moreover, some of the differences between the runs may be related to the disruption of the SASI spiral mode in s18-3D and s18-3Dr so that one ought to be careful before extrapolating our results to the convection-dominated regime, where the interaction between infalling perturbations and the shock may be somewhat different. One of the key differences in the convection-dominated regime could be that the buoyancy-driven modes excited by the infalling perturbations are already unstable to begin with and could be triggered by weaker infalling perturbations, whereas advection stabilises these modes in the SASI-dominated regime (Foglizzo et al. 2006) unless the flow is strongly perturbed (Scheck et al. 2008).

For these reasons, it is not yet possible to validate quantitative models for the additional turbulent driving and the concomitant reduction of the critical luminosity for shock revival (Müller et al. 2016; Abdikamalov et al. 2016), and it must be emphasised that the current analytic approaches to the interaction of perturbations with the shock are undoubtedly too simple to capture the complex nuances found in models s18-3D and s18-3Dr. Still, the inferred reduction of the critical luminosity by about 22% and 16% for s18-3D and s18-3Dr compared to s18-1D is roughly compatible with the analytic model of Müller et al. (2016), which may retain some usefulness as a reasonable fit for extant 2D and 3D simulations with large-scale initial perturbations.

We studied the explosion dynamics of model s18-3D by extending this run to more than 2.35 s after bounce. Although this makes s18-3D the longest 3D simulation with multi-group neutrino transport, this is still no sufficient to obtain final values for the explosion energy, the nickel mass, and the mass, kick, and spin of the proto-neutron star. Different from parameterised long-term simulations based on grey transport (Wongwathanarat et al. 2010, 2013; Handy et al. 2014), the cycle of accretion and mass ejection is still ongoing in our simulation even at these late times, and we cannot capture the transition to the neutrino-driven wind phase yet (cp. Bruenn et al. 2016), which may still occur on even longer time scales. Nonetheless, model s18-3D exhibits interesting and encouraging trends that strengthen the paradigm of neutrino-driven supernova explosions.

The diagnostic explosion energy reaches $7.7 \times 10^{50}$ erg in model s18-3D at the end of simulations and is still increasing at an appreciable rate. Even if we subtract the residual binding energy of the shells ahead of the shock, the explosion is clearly sufficiently energetic to unbind the envelope, and we can place a relatively firm lower limit of $5 \times 10^{50}$ erg on the final exploding energy. 3D turbulence plays a crucial role for the stable growth of the explosion energy, as accreted material from the downflows is halted and mixed into the outflows at relatively large radii so that little neutrino heating is required to unbind it.

The prospective explosion energy is well within the observed range for Type IIP supernovae of $1-40 \times 10^{50}$ erg (Kasen & Woosley 2009; Pejcha & Prieto 2015) and already reasonably close to the “typical” value of $\sim 9 \times 10^{50}$ erg (Kasen & Woosley 2009) (although such “typical” values have yet to be determined for volume-limited samples). Because of the simplified treatment of nuclear burning and neutrino transport, the nickel mass remains rather uncertain, but is likely of the order of $\sim 0.05M_{\odot}$, i.e. there is no apparent conflict with observational constraints.

The parameters of the proto-neutron star also appear to lie within the the observed spectrum, although the kick and spin have not reached their final values yet. The proto-neutron star mass, however, has almost converged. Barring the possibility of late-time fallback, it is not likely to exceed $M_{\text{grav}} = 1.67M_{\odot}$. This would put model s18-3D above the peak of the neutron star mass distribution (Ozel et al. 2012; Ozel & Freire 2016), but is again not an implausible value. The kick velocity of $\sim 600 \text{ km s}^{-1}$ is already sizable at the end of the simulation, and may still considerably exceed that value. Interestingly, continued accretion through the downflows imparts
considerable angular momentum onto the proto-neutron star, resulting in a spin period of about 20 ms. With all due caveats about the limitations of model s18-3D this suggests that a reasonable spectrum of neutron star birth spin periods could be obtained even with slower progenitor core rotation than predicted by current stellar evolution models (Heger et al. 2005). Moreover, we observe a trend towards spin-kick alignment at late times. It is not completely clear, however, whether this is accidental or the result of prefered accretion through the plane perpendicular to the largest neutrino-heated bubble and the kick direction.

In summary, our simulations provide further evidence for the viability of the neutrino-driven mechanism in 3D beyond the extent successful explosion models with rigorous (Melson et al. 2015a,b; Lentz et al. 2015) and more simplified neutrino transport (Takiwaki et al. 2012; Müller 2015; Roberts et al. 2016): At least for some progenitors, large-scale initial perturbations can apparently negate the penalty of slightly less optimistic heating conditions in 3D. With relatively early shock revival around the infall of the Si/O shell interface, plausible explosion and remnant parameters can then be reached.

Obviously, however, model s18-3D is only a step towards a solution of the problem of shock revival and the “energy problem” that has plagued many supernova simulations in the past. For one thing, the 18M☉ progenitor considered here provides very favourable conditions for a perturbation-aided explosion, as it exhibits high nuclear generation rates in an extended O shell that admits large-scale convective modes. Such conditions are not found in all supernova progenitors, and hence only future simulations will be able to determine whether convective seed perturbations are generally of major relevance for shock revival.

Model s18-3D also provides some hints that more refined simulations and additional physics are still needed for better agreement with observational constraints: While s18-3D exhibits plausible explosion and remnant properties, it apparently does not represent a typical core-collapse supernova yet: Its explosion energy is slightly below average, the neutron star mass lies in the high-mass tail of the distribution, and so does the neutron star kick. It is possible that the 18M☉ progenitor is simply not a typical case, but there is some observational evidence that suggests otherwise. Based on the observed loose correlation between the ejecta mass and explosion energy of Type IIP supernovae (Poznanski 2013; Chugai & Utrobin 2014; Pejcha & Prieto 2015), one would expect progenitors in this mass range to explode with more than 1051 erg like SN 1987A, which had ~1.4 × 1051 erg (Utrobin 2005) for a very similar helium core mass.

It is easy to conceive of effects that could make model s18-3D somewhat more energetic and perhaps also shift the proto-neutron star mass and spin towards more typical values considering the limitations of our current simulation. Both a more rigorous treatment of the neutrino transport and higher numerical resolution could move the explosion parameters further in the right direction. In the current implementation of the FMT scheme, we neglect, for example, the effect of nucleon correlations at high densities, which lead to faster PNS cooling (Hudepohl et al. 2010) and at least slightly more favourable heating conditions (Horowitz et al. 2017) (though the effect is likely less pronounced than claimed by Burrows et al. 2016). This is especially noteworthy because correlation effects will become more important as the neutrinospheric densities increase in models that accrete as long as s18-3D. Higher resolution could also be beneficial during the explosion phase if it leads to faster mixing between the downflows and outflows and larger turnaround radii for the ejecta. It could also help to entrain more of the 56Ni synthesised by explosive burning in the shock into the neutrino-heated ejecta and thereby increase the nickel mass. In the pre-explosion phase, better resolution may be required to more accurately capture the drag on the high-entropy bubbles seeded by the inflating perturbations via forced shock deformation. Further effects are conceivable and cannot be extensively discussed here. It is clear that considerable challenges remain before we can demonstrate that the neutrino-driven mechanism can explain the bulk of core-collapse supernova explosions by means of well resolved state-of-the-art simulations employing rigorous neutrino transport across a wide range of progenitors. Our current results merely underscore that 3D supernova modelling is now on a promising track towards this ultimate goal.

ACKNOWLEDGEMENTS

We acknowledge support by the Australian Research Council through an ARC Future Fellowship FT160100035, STFC grant ST/P000312/1 (BM), an ARC Future Fellowship FT120100636 (AH), and at Garching by the Deutsche Forschungsgemeinschaft through the Excellence Cluster Universe EXC 153 and the European Research Council through grant ERC-AdG No. 341157-COCO2CASA. This research was undertaken with the assistance of resources from the National Computational Infrastructure (NCI), which is supported by the Australian Government and was supported by resources provided by the Pawsey Supercomputing Centre with funding from the Australian Government and the Government of Western Australia. This work used the DiRAC Data Centric system at Durham University, operated by the Institute for Computational Cosmology on behalf of the STFC DiRAC HPC Facility (www.dirac.ac.uk); this equipment was funded by a BIS National E-infrastructure capital grant ST/K00042X/1, STFC capital grant ST/K00087X/1, DiRAC Operations grant ST/K003267/1 and Durham University. DiRAC is part of the UK National E-Infrastructure. This material is based upon work supported by the National Science Foundation under Grant No. PHY-1430152 (JINA Center for the Evolution of the Elements).

REFERENCES

Abdikamalov E., Zhaksylykov A., Radice D., Berdibek S., 2016, MNRAS, 461, 3864
Blondin J. M., Mezzacappa A., 2006, ApJ, 642, 401
Blondin J. M., Mezzacappa A., DeMarino C., 2003, ApJ, 584, 971
Bruenn S. W., et al., 2013, ApJ, 767, L6
Bruenn S. W., et al., 2016, ApJ, 818, 123
Buras R., Rampp M., Janka H.-T., Kifonidis K., 2006a, A&A, 447, 1049
Buras R., Janka H.-T., Rampp M., Kifonidis K., 2006b, A&A, 457, 281
Burrows A., 2013, Reviews of Modern Physics, 85, 245
Burrows A., Goshy J., 1993, ApJ, 416, L75
Burrows A., Hayes J., Fryxell B. A., 1995, ApJ, 450, 830
Burrows A., Vartanyan D., Dolence J. C., Skinner M. A., Radice D., 2016, preprint, arXiv:1611.05859
Cantiello M., Mankovich C., Bildsten L., Christensen-Dalsgaard J., Paxton B., 2014, ApJ, 788, 93
Chatterjee S., et al., 2005, ApJ, 630, L61
Chugai N. N., Utrobin V. P., 2014, Astronomy Letters, 40, 291
Cullen P., Woodward P. R., 1984, J. Comp. Phys., 54, 174
Cordero-Carrillo I., Cerda-Durán P., Dimmelmeier H., Jaramillo J. L., Novak J., Gourgoulhon E., 2009, Phys. Rev. D, 79, 024017
Couch S. M., 2013, ApJ, 775, 35
Couch S. M., Ott C. D., 2013, ApJ, 778, L7
