The Horizon Order Parameter

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Abstract
We construct a diffeomorphism invariant operator that is sensitive to how far we are from the black hole horizon. Its expectation value blows up on the horizon and it is small away from the horizon. Using this operator, we propose a non-standard effective action that, we argue, captures some of the relevant physics of quantum black holes, including the absence of the horizon at the full quantum level. With the help of a toy version of this effective action, we speculate on a possible connection between UV/IR mixing and the cosmological constant problem.
1 Introduction

Recent progress in string theory suggests that information is preserved in the process of black hole formation and evaporation. The reason is that in some cases there is a dual description for this process in terms of gauge theories that are believed to be unitary. This progress, however, does not clarify the mechanism that extracts the information back to infinity. In particular, we still do not known what is wrong with Hawking’s original argument [1] that the information is lost. Does the horizon or the singularity (or perhaps both) play the crucial role in the resolution of the black hole information puzzle? It cannot be the singularity since by the time we approach the singularity the information is already lost. It cannot be the horizon since locally there is nothing special about the horizon as all curvature invariants on the horizon are small. This is the reasoning that led Hawking to conclude that the information is lost.

The possibility that, despite the small curvature, something dramatic does happen on the horizon was studied by several authors (see e.g [3]-[11]). The argument is as follows. For a large black hole the energy of a typical Hawking particle is small at infinity - it is of the order of $1/M$. In normal situations this means that its backreaction is negligible. However, in the black hole case the energy is blue shifted by an infinite amount when traced back to the horizon. Thus no matter how large the black hole is this energy is blowing up close enough to the horizon. This is known as the transplanckian problem. The simplest and possibly strongest argument against this is that if the horizon is indeed a special hypersurface, then we should be able to write down an effective action that differs substantially from GR on the horizon and is well approximated by GR away from the horizon. But again all curvature invariants are small on the horizon so such an effective action cannot exist.

The standard answer to this criticism is that the effective action we are after must be non-local, and hence hard to find. From the gravity point of view the non-locality follows from the definition of the horizon. Roughly speaking, the horizon is defined as the boundary between the region from which massless particles can reach infinity and the region from which they cannot. This is not a local definition. From a field theory perspective this effective action, in principle, can be derived by integrating out the radiation. Since most of the radiation is of massless particles the effective action is non-local, and must involve more than just local curvature invariants.

\footnote{A possible way to overcome this was proposed recently in [2].}
Without a concrete non-local action, it is hard to test this point of view or to see whether it can be useful for other issues in quantum gravity (e.g., the cosmological constant problem). The goal of this paper is to try to improve on that answer by offering an effective action that captures at least some of the relevant physics. The key ingredient that we introduce in section 2 is the horizon order parameter. The horizon order parameter is a diffeomorphism invariant operator whose expectation value is sensitive to how far we are from the horizon. In particular, it blows up on the horizon. We show that with spherical symmetry such an operator can be defined locally. In the general case one cannot find a local horizon order parameter. Instead what we do in this case is to trade the non-locality with a new local field that we “integrate in”. In terms of this field and the metric, the horizon order parameter we propose is local.

Equipped with the horizon order parameter, we consider in section 3 a couple of possible actions that have the desirable properties described above. In section 4 we compare these scenarios with experiments. In section 5 we discuss a possible relation with the cosmological constant problem.

2 The horizon order parameter

2.1 The spherical symmetric case

As stressed in the introduction the horizon order parameter cannot be a local operator of the metric. With spherical symmetry, however, a local operator contains more information than it normally does. By measuring the expectation value of some gauge invariant operator at \( r, \theta, \phi \) we know its value at the same \( r \) but for any \( \theta \) and \( \phi \). This trivial statement combined with Gauss’s law suggests that for spherically symmetric configurations the horizon order parameter can be a local operator.

Let us first recall the usual argument why such an operator should not exist. Consider a four dimensional Schwarzschild black hole with the familiar metric

\[
ds^2 = -(1 - \frac{2M}{r})dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_2^2,
\]

and define a new radial coordinate that measures the invariant distance from the horizon

\[
\rho = \int_{2M}^r dr \sqrt{g_{rr}} \approx \sqrt{8M(r - 2M)}.
\]
For $\rho \ll M$ we find that (2.1) can be approximated by Rindler space

$$ds^2 = -\frac{\rho^2}{16M^2}dt^2 + d\rho^2 + dY^2 + dZ^2.$$  \hspace{1cm} (2.3)

This means that there can be no diffeormorphism invariant operator that depends on $\rho$ because Rindler space is Minkowski space written in an accelerating coordinates. As we increase $M$ the region Rindler space approximates well, is getting larger. Thus it seems that at least for a large black hole there can be no horizon order parameter.

Rindler space, however, is only an approximation to the near horizon geometry. For example, the expectation value of the following operator

$$N = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma},$$  \hspace{1cm} (2.4)

vanishes in Minkowski, while in the Schwarzschild background it is

$$\langle N \rangle = 48 \frac{M^2}{r^6}.$$  \hspace{1cm} (2.5)

For a large black hole $N$ is small on the horizon. Moreover, $N$ is a smooth function of $r$ with no special features at $r = 2M$. Therefore by measuring $N$ we cannot tell whether we are approaching the horizon of some black hole. The same goes for any polynomial of the curvature.

Derivatives of the curvature are more interesting. Consider for example the simplest operator that does not vanish in a Ricci flat space

$$\tilde{N} = \nabla_{\beta}R_{\mu\nu\rho\sigma} \nabla^{\beta}R^{\mu\nu\rho\sigma}.$$  \hspace{1cm} (2.6)

The expectation value of that operator in the Schwarzschild geometry is

$$\langle \tilde{N} \rangle = 720 \frac{M^2(r - 2M)}{r^9}.$$  \hspace{1cm} (2.7)

Much like (2.5), it is a smooth function of $r$. But, unlike (2.5), here something special happens on the horizon: outside the horizon (2.7) is positive, while inside the horizon it is negative. On the horizon itself $\tilde{N}$ vanishes. This is not an accident. In static spherical symmetric situations the curvature depends only on $r$, and $g^{rr}$ switches sign on the horizon.

This suggests that $\tilde{N}$ can be used to construct the horizon order parameter. Consider for instance

$$O_1 = \frac{N^2}{\tilde{N}}.$$  \hspace{1cm} (2.8)
The expectation value of $O_1$ in the black hole background is

$$\langle O_1 \rangle = \frac{16M^2}{5(r - 2M)r^3} \approx \frac{1}{\rho^2}. \quad (2.9)$$

Thus at any classical distance away from the horizon of a large black hole, or outside a massive star, it is much smaller than one. But at Planckian distances from the horizon ($\rho \sim 1$) it is of order one. On the horizon itself it blows up.

In Ricci curved spaces $O_1$ has to be modified. For example in the case of near extremal black holes $O_1$ is of order 1 at a distance from the horizon that depends on the energy above extremality. It also blows up everywhere in AdS (and dS) spaces. To fix this we define a slightly more complicated operator

$$O_2 = \frac{NC^2}{\nabla_\beta C_{\mu\nu\rho\sigma} \nabla_\beta R_{\mu\nu\rho\sigma}}, \quad C^2 = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}, \quad (2.10)$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. $O_2$ also works as it should in Ricci curved spaces and is equivalent to $O_1$ in a Ricci flat space. It should be emphasized that both $O_1$ and $O_2$ can be measured locally.

It is easy to verify that without spherical symmetry $O_2$ does not work as it should. Namely, it blows up when it is not supposed to and it does not blow up when it is supposed to. A simple example is Kerr black hole. In that background $O_2$ blows up on the ergosphere rather than on the horizon. A different example that does not involve black holes is of two massive objects; along the line that connects their centers there is a point where $O_2$ blows up.

2.2 The general case

Without spherical symmetry the horizon order parameter cannot be a local operator. It seems hopeless to guess a non-local operator with the right properties. A more realistic approach is to add new massless degrees of freedom. The horizon order parameter can then be local with respect to these new degrees of freedom and the metric. In principle, one can integrate out the new fields and express the order parameter as a non-local operator of the metric.

The challenge is to find these new massless degrees of freedom. Recall that there is a simple way to tell whether we are inside or outside an eternal Schwarzschild black hole. There are four Killing vectors: one in the Schwarzschild time direction and three that
are associated with rotations of the $S^2$. Inside the black hole all the Killing vectors are space-like while outside one of them is time-like. That Killing vector is null on the Horizon. Therefore, the norms of the Killing vectors contain the information on which side of the horizon we are. In the case of a dynamical formation of black holes, in general, there are no exact Killing vectors. But there are approximated Killing vectors. The no-hair theorem ensures that with the evolution of the collapsing matter they become increasingly exact. Most importantly, the approximated Killing vector that becomes null on the horizon asymptotes to an exact Killing vector exponentially fast. This plays a crucial role in Hawking’s derivation that the black hole radiation depends only on the total mass, angular momentum and charge, and not on the details of the matter that forms the black hole.

This suggests that the new degrees of freedom we need to add is a vector field $V^\mu$. Our task is to find an action for $V^\mu$ whose equations of motions are solved by the vector field described above. Namely, a vector field that asymptotes to a Killing vector that becomes null on the horizon. We denote that vector field by $\bar{V}^\mu$. The norm of $\bar{V}^\mu$ will be used to construct the horizon order parameter. This means that $V^\mu$ is not a gauge field. In order to find that action we use the following properties of the Killing vectors in the black hole geometry. First, $V^\mu$ is a Killing vector if the following symmetric tensor vanishes

$$G^\mu_\nu = \nabla^\mu V_\nu + \nabla^\nu V_\mu. \quad (2.11)$$

We do not want to impose $G^\mu_\nu = 0$ as a constraint since, as we argue above, in the generic case of black hole formation there are no non-trivial solutions to that constraint. To find approximated Killing vectors we need to find vectors that minimize $G^\mu_\nu$. The simplest action that does that is

$$S_1 = -\int d^4x \sqrt{g}(G^2 + \lambda G^\mu_\mu). \quad (2.12)$$

The constraint $G^\mu_\mu = 0$ is needed to ensure that the Hamiltonian in bounded from below.

We still have to fix $V^\mu$. A naive attempt to do this is to impose that at infinity $V^2 = -1$. That works for the eternal Schwarzschild black hole since it fixes

$$V^\mu = (1, 0, 0, 0), \quad (2.13)$$

in the coordinate system used in eq.(2.1). In general, however, there are problems with that proposal. The most obvious one is that there is no reason why the time-like
Killing vector at infinity should agree with the null Killing vector on the horizon. A simple example is Kerr black hole: At infinity the Killing vector that satisfies $V^2 = -1$ is (2.13). But the Killing vector that becomes null on the horizon of a Kerr black hole is

$$V^\mu = (1, 0, 0, \Omega_H),$$

(2.14)

where $\Omega_H$ is the horizon angular velocity that depends on the mass and angular momentum of the black hole. That Killing vector is actually space-like at infinity. Moreover, there are situations in which imposing $V^2 = -1$ as a boundary condition simply does not make sense.

In short, the direction and magnitude of $V^\mu$ should be determined by the dynamics at the neighborhood of the horizon and not by the asymptotic region. One way to do this is to take advantage of the fact that $\bar{V}^\mu$ is the only Killing vector for which $\nabla^\mu(\bar{V}^2)$ is pointing, on the horizon, in the same direction as $\bar{V}^\mu$ (see fig. 1). Then one can define on the horizon a scalar, $\kappa$, via the relation

$$\nabla^\mu(\bar{V}^2) = -2\kappa \bar{V}^\mu,$$

(2.15)

and show that $\kappa$ is a constant on the horizon (that is known as the horizon surface gravity). In the normalization, that we do not use, in which the $t$ component of $\bar{V}^\mu$ at infinity is one, $\kappa$ is the temperature associated with the black hole. The analog of the fact that $\kappa$ is constant on the horizon is that the temperature is constant in thermal equilibrium. This holds for any black hole, including black holes with angular momentum.

Motivated by this we define the following vector field

$$B^\mu = V^\mu + \nabla^\mu(V^2),$$

(2.16)

and add to $S_1$

$$S_2 = \int d^4x \sqrt{g} C^2 \frac{B^4}{V^4}.$$

(2.17)

There are several reasons for the appearance of $C^2$ (that was defined in (2.10)) in $S_2$. First, we want to avoid destabilization of conformally flat spaces. Second, in the black hole case $C^2 \sim M^2/\rho^6$ and hence it suppresses the contribution from the large $r$ region. Thus $V^\mu$ is fixed by the near horizon region. Even in the neighborhood of the horizon $C^2 \ll 1$. This means that $S_2$ is a perturbation with respect to $S_1$. This perturbation breaks the degeneracy among the Killing vectors and fixes the direction and magnitude of $V^\mu$. 
Figure 1: On the horizon (the dashed line) $V^\mu$ and $\nabla^\mu(V^2)$ point in the same direction. Throughout the near horizon region their sum is null.

To see this, note that if $V^\mu$ is space-like then $B^4 > V^4$. The reason is that for all Killing vectors outside the black hole $\nabla^\mu(V^2)$ is a space-like vector that increases the norm of $B^\mu$ relative to $V^\mu$. For the same reason, when $V^\mu$ is time-like $B^4 < V^4$. This fixes the direction of $V^\mu$. The magnitude of $V^\mu$ is also fixed because the first term in (2.16) is linear in $V^\mu$ while the second is quadratic in $V^\mu$. Indeed expanding (2.17) around $V^\mu = 0$ in the black hole background, we find a tachyonic mode that is stabilized when $B^2 = 0$. Near the horizon of a Kerr black hole $B^2$ vanishes for

$$V^\mu = \frac{1}{2\kappa} (1, 0, 0, \Omega_H), \quad \kappa = \frac{\sqrt{M^2 - a^2}}{2M(M + \sqrt{M^2 - a^2})}.$$  \hspace{1cm} (2.18)

It is important to emphasis that while eq.(2.15) holds only on the horizon itself, $B^2$ vanishes throughout the near horizon region. Namely, $B^2 \ll V^2$ as long as $\rho \ll M$ (see fig.1).

The direction of (2.18) is as expected. Since $\kappa$ is small one might worry that the backreaction of $V^\mu$ is too large to be considered as a perturbation of the black hole geometry. The $C^2$ in $S_2$ ensures that this does not happen and that the energy in the $V^\mu$ field is much smaller than $M$. 
The horizon order parameter is defined to be

\[ \mathcal{O} = \frac{F^2}{V^2}, \quad \text{where} \quad F_{\mu \nu} = \nabla_{\mu}V_{\nu} - \nabla_{\nu}V_{\mu}. \] (2.19)

On the horizon \( V^2 \) vanishes while \( F^2 \) is a constant (that is related to the surface gravity) and therefore \( \mathcal{O} \) behaves as required. That is, \( \mathcal{O} \) blows up as we approach the horizon. For example in the Schwarzschild case, \( V^\mu \) is given by (2.18) with \( a = \Omega_H = 0 \) and the expectation value of \( \mathcal{O} \) is

\[ \langle \mathcal{O} \rangle = \frac{2M^2}{(r - 2M)r^3} \approx \frac{1}{\rho^2}. \] (2.20)

Up to a constant of order one this expression is the same as \( \langle \mathcal{O}_1 \rangle \) (see eq. (2.9)). In particular, it is much smaller than one at any classical distance from the horizon, and at Planckian distances from the horizon it is of order one. The advantage of \( \mathcal{O} \) over \( \mathcal{O}_1 \) is that for \( \mathcal{O} \) this also holds in the absence of spherical symmetry.

\section{Effective actions}

Now that we have defined the horizon order parameter, we can consider different effective actions that are sensitive to the location of the black hole horizon. There are various interesting scenarios to study. Below we focus on the scenario that we believe is the most plausible scenario.

\subsection{The no-horizon scenario}

Motivated by 't Hooft’s S-matrix for black holes (for a review see [12]) a scenario was proposed in [9] (see also [7]) according to which at the quantum level there is no horizon to the black hole. The line of argument that led to this proposal is as follows. Using the conformal anomaly and energy-momentum conservation one can calculate, subject to some reasonable assumptions, the one-loop contribution to the expectation value of the energy-momentum tensor associated with the Hawking radiation [13] (for a review see e.g. [14]). The result of this calculation is in accord with Hawking’s assumption that for a large black hole nothing special happens on the horizon. Namely, away from the horizon there is an out-going flux of positive energy that is associated with the on-shell Hawking radiation. Near the horizon, however, there is an in-going flux of negative energy that corresponds to the partners of the Hawking particles. As these
particles are simply falling through the horizon their backreaction is regular. In fact, for massless particles it is possible to find the exact solution to Einstein’s equations near the horizon including the backreaction [15].

A classical system is fully described by the one-point functions. In a quantum theory, however, the one-point functions contain only part of the information. Moreover, there are cases in which this information is actually misleading. This was illustrated beautifully in [16] where in a consistent quantum field theory, states were constructed such that the one-point function of the energy momentum tensor violates causality at arbitrarily large distances.

It is certainly possible that the one-point functions of [13] are misleading as well. This is supported by the following argument. If the process of black hole creation and evaporation is described by some complicated unitary S-matrix amplitude then the out-going particles should be treated on equal footing as the in-going particles. In particular, since the ingoing particles are on-shell, the out-going particles must be on-shell as well. This implies that the out-going radiation should be associated with a positive out-going energy momentum flux everywhere, not just at infinity (much like the flux associated with the in-going particles is on-shell everywhere). The backreaction in this case is such that a test particle falling towards the black hole will always be causally connected with $I^+$ [9]. As strange as this might seem, this is a consequence of CPT; by time reversal this statement is equivalent to the claim, that is obviously correct, that the Hawking particles are causally connected with $I^-$. The result of [13] is expected to be obtained by first averaging over the final states and then calculating the expectation value. Thus in this picture the black hole horizon is an artifact of the semi-classical approximation that does not exists at the full quantum level.

Concrete support for the no-horizon scenario comes from the D5-D1 system. This system is understood so well that a precise quantum geometry can be associated with each quantum state of the black hole. Quite remarkably, Lunin and Mathur showed that these quantum geometries do not admit horizons [17, 18, 19]. The horizon appears only as a classical concept after averaging over the quantum geometries. The techniques used in [17, 18] are special to this system and are valid when the black hole in question is basically a BTZ black hole. Nevertheless, these results clearly illustrate that the no-horizon scenario, as radical as it might appear from a semi-classical point of view, is a possibility that should be considered seriously.
As a first step towards realizing the no-horizon scenario for realistic black holes, it would be useful to have an effective action that captures some of the relevant physics. With the help of the horizon order parameter we can make some proposals. The most straightforward one is

\[ S = S_1 + S_2 + \int d^4 x \sqrt{g} \left( R + \exp(O) R^2 + \exp(2O) R^3 + \ldots \right). \]  

(3.1)

This proposal is inspired by string theory. In string theory the Planck scale is not a universal constant since it depends on the local value of the dilaton. As a result the stringy loops corrections to GR depend on the dilaton. Here we have replaced the dilaton with \( O \) because, unlike the dilaton, the expectation value of \( O \) blows up on the horizon. This implies that the effective Planck scale grows as we approach the horizon, and therefore the corrections to GR become increasingly important. From eq.(2.20) we see that at a finite distance from the horizon the effective Planck scale is of the order of the size of the horizon itself. At that point all terms are equally important. This means that the horizon becomes a non-local sphere where the non-locally is of the order of the size of the sphere itself. All the information is supposed to be encoded in this non-local sphere.

Clearly there are many other actions with similar properties. To find the one that describes best the near horizon physics we have to make contact with the backreaction of the Hawking radiation and with the black hole entropy. In our discussion we used familiar ingredients, like surface gravity and null Killing vectors, but we certainly have not made this connection precise. It is likely that a better understanding of these issues will modify, possibly substantially, the proposed horizon order parameter and effective action.

An unusual feature of (3.1) is that it involves ratios of polynomials of the fields. This is certainly beyond the Wilsonian effective action framework, and it raises many questions about naturalness and quantum corrections. In particular, in our case, unlike in the Wilsonian framework, it is not even clear which degrees of freedom are integrated out. This is closely related to the fact that at this stage, the precise relation with the backreaction of the Hawking radiation and the black hole entropy is unclear. At the moment we do not have good answers to questions of this kind. The only comment we would like to make is that the fact that we are forced to work outside the framework of the Wilsonian effective action should not come as a surprise. In the Wilsonian approach the number of degrees of freedom is proportional to the volume of the system whereas
according to the holographic principle [21, 22] the number of degrees of freedom should be proportional to the area that surrounds the system.\textsuperscript{2} Put differently, a Wilsonian effective action includes only polynomials of the curvature and its derivatives. These are clearly not sensitive to the location of the horizon. A closely related comment is about UV/IR mixing. It is hard to imagine a realization of the holographic principle without some kind of UV/IR mixing. Indeed this is what is happening here. Naive power counting suggests that the terms added to GR in (3.1) are irrelevant. Nevertheless, as we approach the horizon they become as important as the Einstein-Hilbert term. Of course this is not to say that the scenario proposed here is the correct way to realize the holographic principle, but it does seem to involve some of the expected features.

An issue that is often raised in relation to the no-horizon scenario is that information can be lost long before the horizon is formed. Therefore the fact that an actual horizon is not formed is not going to help with retrieving the information. Let us illustrate this puzzle and its resolution in a Gedanken experiment. Consider a spherically symmetric null shell that carries some information and is not energetic enough to form a black hole by itself. In Fig.1 that shell is denoted by $A$. At a later time there is an additional spherically symmetric null shell, $B$, that has enough energy to form a black hole. By the time the horizon is formed the information contained in $A$ is already lost. Namely, point $c$ and $\mathcal{I}^+$ are not causally connected. In the no-horizon scenario shell $B$ follows the same trajectory as before until it reaches $r = 2M$. At this point the shell becomes completely non-local, and it floats while emitting radiation which slowly decreases its mass. Shell $A$ reaches $r = 0$ and as before it reexpands. However, instead of hitting a space-like singularity it will hit shell $B$. At all times shell $A$ is causally connected with $\mathcal{I}^+$. A more general way to say this, that is valid away from the thin shell limit, is the following. Before reaching the singularity shell $A$ has to cross the apparent horizon. In the standard picture nothing special happens there so the shell continues towards the singularity. In the no-horizon scenario, however, the apparent horizon and its interior (including the would be singularity) are effectively replaced by a giant non-local ball.

\textsuperscript{2}The AdS/CFT correspondence avoids this issue since the area of AdS is proportional to its volume. In that regard the most concrete example we have so far to the holographic principle is misleading.
Figure 2: (a) The standard picture in which the information in the shell A is lost long before the horizon was formed. (b) In the no-horizon scenario shell A is always causally connected with $\mathcal{I}^+$. 

3.2 Black hole as a superconductor

Equipped with the horizon order parameter there are other types of effective actions that one might want to explore. An especially entertaining scenario is one in which the region behind the horizon is in a different phase. In this scenario the black hole is the gravitational analog of a superconductor: outside the black hole we are in the Coulomb phase while the region behind the horizon is in the Higgs phase. One way to realize the gravitational analog of the Higgs phase is through ghost condensation [20]. We want the kinetic term to have the wrong sign only inside the horizon so the action should have a term of the form

$$F(\mathcal{O})(\partial \phi)^2,$$

where $F$ is some odd function of $\mathcal{O}$. In this scenario the information can fall behind the horizon, but since the dispersion relation in the “Higgs” phase is not Lorentz invariant [20] the information can escape back to infinity.
4 Comparison to experiment

With the growing evidence for black holes one might suspect that the no-horizon scenario is ruled out by experiment. In this section we briefly review these experiments and discuss their relevance to our scenario.

The standard argument for the existence of black holes is that there are dark objects that are simply too massive to be supported by the nuclear force. Their mass is known by studying the trajectories of nearby stars. Since these trajectories are at large distances from the horizon they do not involve large gravitational field, and, in particular, they do not test the no-horizon scenario. There are, however, more recent observations that do involve large red-shift and are more relevant to the scenarios discussed here.

Some observations are based on the fact that the red-shift at the location of the accretion disk for a Kerr black hole is

\[
1 + z = \sqrt{\frac{1 + a^2/r^2 + 2Ma^2/r^3}{1 + a^2/r^2 - 2M/r}}.
\] (4.1)

The Schwarzschild limit (\(a = 0\)) is not very interesting in that regard. The accretion disk is located at \(r = 6M\) and we find that the red-shift is pretty small, \(z \sim 0.2\). More importantly, this probes the geometry at \(r \geq 6M\), and not the near horizon geometry. In the extreme case (\(a = M\)) the accretion disk is touching the horizon at \(r = a = M\) and the maximal red-shift is infinite. Experimentally large red-shift in the iron X-ray lines have been observed in some Seyfert galaxies [23]. For a large enough \(a\) the accretion disk is located between the ergosphere and the horizon. Therefore, these observations could rule out scenarios in which the order parameter is \(O_2\) (or \(O_1\)) and not \(O\). This is in agreement with our theoretical expectations.

More evidence for black holes comes from the fact that type I X-ray bursts are quite common in accreting neutron starts, but have never been detected in accreting black hole candidates (for a recent review see [24]). This fits neatly with GR because type I bursts are due to thermonuclear explosions taking place when enough gas accretes on the surface of the neutron star. For black holes this does not happen since there is no surface for the gas to accrete on. In the scenario presented here the horizon can be viewed as such a surface. Naively this suggests that careful measurement of the X-ray burst can test the no-horizon scenario. Unfortunately measurements of this kind
cannot test this scenario. The reason is simple. Indeed in this scenario the gas accretes on the horizon. However, the local temperature on that "surface" is of the order of the Planck scale that makes any thermonuclear activity irrelevant.

5 The cosmological constant problem

It is likely that a better understanding of black holes at the quantum level will revolutionize our understanding of quantum gravity in general and it might even shed new and unexpected light on various issues in cosmology (for suggestions see [25, 26, 27]). In particular, an intriguing speculation is that UV/IR mixing plays an important role in the resolution of the cosmological constant problem. The rough idea is that due to large scale non-local effects, the macroscopic cosmological constant might be much smaller than the microscopic cosmological constant. Where the macroscopic cosmological constant is the one we observe, and the microscopic cosmological constant is determined by the Planck or SUSY scale. Phrased slightly differently that point of view has been advocated by Banks [28, 29].

Since the effective action we proposed involves UV/IR mixing it is interesting to see if it provides new insights to the cosmological constant problem. Unfortunately, with our current understanding of the horizon order parameter, it is premature to have a rigorous discussion on this issue. Instead, with the help of a toy action, that is related to the actions considered above, we illustrate, quite heuristically, how UV/IR mixing could have interesting applications to the cosmological constant problem.

The toy effective action we wish to consider is

\[ \int d^4x \sqrt{g} \left( R + R \frac{N^2}{\square N} + \ldots \right), \quad N = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}. \]  

(5.1)

Since the second term is irrelevant with respect to the Einstein-Hilbert term, it is expected not to matter much when the curvature is small in Planck units. Take for example a spatially flat FRW universe

\[ ds^2 = -dt^2 + a^2(t) \sum_{i=1}^{3} dx_i^2, \]  

(5.2)

with \( a(t) \sim t^b \). In that case \( R \sim 1/t^2 \) whereas the second term scales like \( 1/t^4 \), and as expected, it is irrelevant at late times. In that respect, the second term in (5.1) behaves like an ordinary higher loop correction to GR. Furthermore, one can verify
that the graviton propagator in flat space is intact and that this effective action does not contradict the solar system experiment.

The situation is quite different in a universe that is dominated by a cosmological constant. The solution to Einstein’s equations in such a universe is a de-Sitter spacetime. The usual higher loops corrections are not going to change this. They could change the exact relation between the cosmological constant and the radius of curvature of de-Sitter. If the cosmological constant is not Planckian but is fixed by the SUSY breaking scale, then even the relation between the cosmological constant and the radius of curvature is, to a good approximation, fixed by the Einstein’s equations. With the deformations considered here this is not the case. Not only do we not get de-Sitter as a solution, but the solution we get is not highly curved. That is, the curvature of spacetime at late times is much smaller than Λ. The reason is that although the second term in (5.1) is supposed to be irrelevant, it actually blows up when the curvature is a non-zero constant. This excludes de-Sitter and AdS as solutions regardless of how large the cosmological constant is. Instead, we find a solution that asymptotes at late times to

\[ a(t) = t^{b(t)}, \quad b(t) = \frac{5}{3} - \frac{1}{\Lambda^2 t^4}. \]  

(5.3)

The curvature associated with that solution scales like \(1/t^2\). Thus at late times we get a smooth solution although the cosmological constant is large. Put differently, (5.3) describes an accelerating universe, but the acceleration does not depend on \(\Lambda\) at all:

\[ \frac{\ddot{a}}{a} = \frac{10}{9 t^2}. \]  

(5.4)

Notice that we did not have to introduce a new scale to the action in order to get this.

Eq.(5.4) describes a universe with \(p/\rho = w = -0.6\). This value is ruled out by the Supernova observation [30, 31]. But, unlike the problems that will be raised momentarily, this should not be considered as a serious problem because other, more complicated, actions yield an expansion that is consistent with [30, 31].

The cosmological constant in the equation of motions is balanced not by a large curvature, as it is in GR, but rather by the fact that the derivatives of the curvature invariants are even smaller than the curvature itself. The problem with that is that this balancing is extremely fragile. There are other terms that can be added to (5.1),

\[ ^{3}\text{It is easy to generalize this in a way that it vanishes for AdS}_5 \times S^5 \text{ when non-trivial fluxes are involved.} \]
that are naively irrelevant, like $R\left(\frac{N^2}{\Omega N}\right)^2$, but change the solution (5.3) drastically. Thus we have not made a real progress with the naturalness of the cosmological constant. The only positive aspect of this is that the corrected solutions have at late times low curvature as well.

There are other, more practical, problems with this toy model. Eq. (5.3) is valid all the way to times as early as $\sim 1/\sqrt{\Lambda}$. This contradicts early universe observations. For example, the Hubble constant at nucleosynthesis is so different than in the standard big-bang scenario that it is hard to see how abundances of light element could agree with observation. This scenario is problematic even at late times. In the presence of a large cosmological constant the equations of motions are balanced by having a small $\Box N$. This has to be done locally everywhere in the universe. But this is inconsistent with the solar system experiment, where $\Box N$ is determined by the sun rather than by the cosmological evolution. Simply put, we indeed get a low curvature universe (5.3) but there is still a large energy density due to the cosmological constant. This energy screens the gravitational effects of all other forms of matter or radiations.

To conclude, this toy model has many problems and it is far from solving the cosmological constant problem. It does illustrate, however, the role UV/IR mixing might play in the resolution to the cosmological constant problem.

6 Summary

The black hole information puzzle has generated over the years fascinating ideas about quantum black holes, large scale non-locality and holography. Without effective actions that complement these ideas it is hard to study or even state them concretely. In this paper we attempted to make a small step in that direction by constructing an effective action that is sensitive to the location of the black hole horizon. We hope that concrete progress will be made using actions of this kind.

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