ROTATIONAL BANDS AT THE LIMIT OF ANGULAR MOMENTUM

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Abstract. The properties of rotational bands at the limit of angular momentum are discussed on the example of smooth terminating bands observed in the $A \sim 110$ mass region. The effective alignment approach is used for the study of their relative properties which provides additional insight into the properties of such bands.

With the new arrays of $\gamma$-detectors it has become possible to investigate rapidly rotating nuclei at the limit of angular momentum. The limit of angular momentum can be defined in terms of either the maximum spin at which a nucleus exists as a bound system or the limited angular momentum within specific configurations (rotational bands). In the latter case we are dealing with the concept of terminating bands. These bands, which are collective at low spin, gradually lose their collectivity and exhaust their angular momentum content approaching a pure single-particle (terminating) state of maximum spin. The existence of a maximum spin for a specific configuration is a manifestation of the finiteness of the nuclear many-fermion system where the angular momentum is built mainly from the contributions of valence particles and valence holes, while the contribution from closed shells is rather small or even negligible [1, 2]. A typical situation is that the state of maximum spin has oblate ($\gamma = 60^\circ$) or prolate ($\gamma = -120^\circ$) non-collective shape. Keeping in mind that the termination of a rotational band takes place at high spin where the pairing correlations are of small importance, its configuration can be defined by the number of particles (holes) in different $j$-shells. Since nucleons are fermions which obey the Pauli principle, one valence particle in a $j$-shell contributes with $j\hbar$, the next with $(j - 1)\hbar$ etc.

However, in most cases this gradual interplay between collective and single-particle degrees of freedom is difficult to study in experiment. One typical situation, existing for example in the $A \sim 158$ mass region (see sect. 3 of [2] for an overview of experimental situation), is that although the terminating states are seen in experiment, the rotational bands from which they originate go away from the yrast line with decreasing spin and are thus difficult to observe over a large spin range. Another even more common situation is that the rotational bands, which are yrast in some spin range, end up in terminating states residing well above the yrast line. Only recent findings of smooth terminating bands in the $A \sim 110$ mass region [3, 4] observed over a wide spin range opened a real possibility for a detailed investigation of the terminating band phenomenon. Considering the spin range over which these bands are observed and the fact that in many cases their terminating states have been definitely or tentatively seen in experiment, this mass region indeed represents a unique laboratory for theoretical study of gradual interplay between collective and single-particle degrees of freedom within the nuclear system.

A detailed theoretical investigation of smooth terminating bands has been performed within the configuration-dependent shell-correction approach with the cranked Nilsson potential [2, 4]. The main ingredients of the approach used are:

- Within the $N$-shells, virtual crossings between the single-particle orbitals are removed in an approximate way and, as a result, smooth diabatic orbitals are obtained. An additional feature is that the high-$j$ orbitals in each $N$-shell are identified after the diagonalization. As a result, it is possible to trace fixed configurations as a function of spin.

- The calculations are carried out in a mesh in the deformation space, $(\varepsilon_2, \varepsilon_4, \gamma)$. Then for each fixed configuration and each spin separately, the total energy of a nucleus is determined by a minimization in the shape degrees of freedom.

- Pairing correlations are neglected so the calculations can be considered fully realistic for spins above $I \sim 20\hbar$ in the $A \sim 110$ mass region. An additional source of possible dis-
crepancies between experiment and calculations at low spin could be the one-dimensional cranking approximation used in the present approach. Since most of configurations of interest in the $A \sim 110$ mass region are obtained by occupation of the $h_{11/2}$, $g_{7/2}$ and $d_{5/2}$ orbitals and by emptying of the proton $g_{9/2}$ orbitals, the shorthand notation $[p_1 p_2, n]^{\alpha_{tot}}$ is used for configuration labelling. In this notation $p_1$ is the number of $g_{9/2}$ proton holes, $p_2$ the number of $h_{11/2}$ protons, $n$ the number of $h_{11/2}$ neutrons and $\alpha_{tot}$ is the total signature of the configuration.

\[ \varepsilon_2 \cos(\gamma + 30^o) \]

\[ \varepsilon_2 \sin(\gamma + 30^o) \]

\[ J^{(1)} \quad J^{(2)} \]

\[ Q_t \quad [e.b] \]

Fig. 1. Top row: Evolution of potential energy surfaces (PES) for fixed configuration $[21, 3]^-$ assigned to band 1 observed in $^{110}\text{Sb}$ [3]. PES are given in steps of $8\hbar$ from near-prolate state at $I = 21^+$ up to terminating state at $I = 45^+$. The minima are shaded. The energy difference between two neighbouring equipotential lines is equal to 0.25 MeV and the last equipotential line corresponds to $3.0 \text{ MeV}$ excitation with respect of minimum. Bottom row: Comparison between theoretical results (curves without symbols) and experimental data (unlinked symbols) for band 1 in $^{110}\text{Sb}$. (a) Excitation energies given with respect to rigid rotor reference. Open circle indicates the predicted terminating state. (b) Kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia. (c) The transition quadrupole moments $Q_t$ calculated according to [4]. For more detailed discussion of description of $Q_t$ in smooth terminating bands see sect. 5.3.1 in [2].

Essential physics related to smooth terminating bands can be illustrated on the example of band 1 recently observed in $^{110}\text{Sb}$ [3]. This band was predicted as an especially favoured intruder band in $^{110}\text{Sb}$, yrast over the spin range $25\hbar - 45\hbar$, see Fig. 13 of [3], one year before experimental data on this nucleus became available. For spin values $I \geq 25\hbar$, where the role of pairing correlations is negligible, the evolution of excitation energies as a function of spin within the band is in excellent agreement with theoretical predictions, see Fig. 1a. At spin $I \geq 38\hbar$, the $(E - E_{RLD})$ curve shows an upbend. This feature indicates that the last spin units before termination are built at high energy cost; unfavoured termination. This high energy cost is determined mainly by [3]: (i) the difficulty to align the proton $g_{9/2}$ holes when they are surrounded by aligned particles and (ii) the fact that the neutron $g_{7/2}d_{5/2}$ subshells are
essentially half-filled, i.e. the spin contribution from the last particles in these subshells is close to zero. Experimentally, it is seen from the increase of the $\gamma$-ray energy spacings with increasing spin which implies that the dynamic moment of inertia $J^{(2)}$ decreases to values which are only a fraction of the rigid-body value. At rotational frequencies $\hbar\omega > 0.7$ MeV, where the role of pairing correlations is negligible, the calculations reproduce this drop of $J^{(2)}$ rather well, see Fig. 1b. Furthermore, at these frequencies the calculated kinematic moment of inertia $J^{(1)}$ is in good agreement with experiment. The smooth terminating bands show a continuous transition from high collectivity to a pure particle-hole (terminating) state which is illustrated in top row of Fig. 1. This transition is associated with a gradual loss of collectivity as shown in Fig. 1c. Unfortunately, precise measurements of transition quadrupole moments $Q_t$ in smooth terminating bands are not available at present.

![Fig. 2. Smooth terminating bands observed in $^{109,110}$Sb and in $^{108}$Sn [3, 4, 5]. The assigned configurations are indicated after band labels. The difference in the configurations of observed bands related to specific orbitals is indicated by arrows of different types. The correspondence between the type of arrow and orbital is given in right bottom panel. The orbitals are labeled by the main quantum number, $N_{rot}$, by sign of signature $\alpha$ given as superscript and by the position of the orbitals within the specific signature group of the $N_{rot}$ shell given as subscript, counted from the $Z = 50$ and $N = 50$ spherical shell gaps. In this mass region, $N_{rot} = 4$ corresponds to the $g_{7/2}$ $d_{5/2}$ subshell and $N_{rot} = 5$ corresponds to the $h_{11/2}$ subshell.]

One should note that in many cases, as for example in $^{109}$Sb, the observed bands are not linked to the low spin level scheme and the interpretation that they are observed up to termination is partly based on comparison with the $(E - E_{RLD})$ curves for yrast and near-yrast configurations obtained in model calculations. In other cases when such bands have been linked to the level scheme, as for example $^{108}$Sn(2) and $^{110}$Sb(1) bands, the transitions depopulating the terminating states have been established only tentatively [5, 8]. As will be shown below, in such a situation the relative properties of unlinked and linked bands can be very useful in order to establish if the present interpretation is consistent or not. We use an effective alignment approach [9] successfully applied for an interpretation of superdeformed bands observed in the $A \sim 150$ mass region employing present model [10] and cranked relativistic mean field theory [11]. Effective alignment of two bands (A and B) is simply the difference between their spins at constant rotational frequency $\hbar\omega$ (or $\gamma$-transition energy $E_\gamma$):

$$i_{eff}^{B,A}(\hbar\omega) = I^B(\hbar\omega) - I^A(\hbar\omega).$$  (1)

Experimentally, $i_{eff}$ includes both the alignment of the single-particle orbital and the effects associated with changes in deformation, pairing etc. between two bands. This approach exploits the fact that spin is quantized, integer for even nuclei and half-integer for odd nuclei and furthermore constrained by signature. One should note that with the configurations and specifically
the signatures fixed, the relative spins of observed bands can only change in steps of $\pm 2\hbar \cdot n$ ($n$ is integer number).

![Graphs showing effective alignments](image)

Fig. 3. Effective alignments, $i_{\text{eff}}$ (in units $\hbar$), extracted from experiment (unlinked large symbols) and corresponding calculated configurations (linked small symbols). The experimental effective alignment between bands A and B is indicated as “A/B” and it measures the effect of additional particle. The $i_{\text{eff}}$ values are shown at the transition energies of the shorter band indicated by an asterisk (*). The compared configurations differ by the occupation of the orbitals indicated on the panels. The points corresponding to transitions depopulating terminating states are encircled. The experimental points which, as follows from the analysis of $J(2)$ of compared bands, appears to be affected either by pairing interactions or by unpaired band crossings are shown on shaded background.

In the present study, we exploit the fact that bands 1 and 2 in $^{108}\text{Sn}$ and band 1 in $^{110}\text{Sb}$ are linked to the level scheme. The configuration assignment for observed bands, see Fig. 2, and the spin assignment for unlinked bands of $^{109}\text{Sb}$ are the same as the one given in original articles [5, 7, 8]. Considering the reasonable agreement between experiment and calculations in the spin range where the pairing correlations are of small importance, see Fig. 3, the following conclusions can be drawn:

- The interpretation of smooth terminating bands observed in $^{109,110}\text{Sb}$ and $^{108}\text{Sn}$ [4, 6, 8] based on the features of experimental and theoretical $(E - E_{\text{RLD}})$ curves is consistent with the present study. The theoretical interpretation using the parabola-like behaviour of the $(E - E_{\text{RLD}})$ curves of the bands which terminate in unfavoured way appears reliable. In addition, the present analysis gives strong confidence in the spin assignment for unlinked bands 1-4 in $^{109}\text{Sb}$ thus indicating that they have indeed been observed up to their terminating states.

- The effective alignment approach can be used also for the analysis of smooth terminating bands giving an additional and very sensitive tool for the interpretation of the observed bands. However, compared with the case of superdeformed bands the changes in deformation between two bands play a much more important role and, as a consequence, the effective alignment does not necessary reflect the alignment of single-particle orbital. This
can be illustrated by the fact that the effective alignment in the $^{108}\text{Sn}(2)^{\pi}_{\text{bot}}/^{109}\text{Sb}(1)$ pair is $\approx 1h$ lower than pure single-particle alignment of the lowest $\pi h_{11/2}$ orbital, see Fig. 10 in [3].

- It is more difficult to reproduce the $i_{\text{eff}}$ values for the transitions depopulating terminating states. These transitions link the states having largest difference in equilibrium deformation between two neighbouring states within a band. For example, in this mass region this difference could reach $\sim 30^\circ$ for $\gamma$-deformation. As a result, these $i_{\text{eff}}$ values are more sensitive both to the accuracy of reproduction of prolate-oblate energy difference in the liquid-drop part and the parametrization of the Nilsson potential than the $i_{\text{eff}}$ values for transitions at lower spin.

Detailed theoretical investigations within the configuration-dependent shell-correction approach performed recently [12, 13] indicates that similar bands could be observed in the $A \sim 70 - 80$ mass region. One interesting feature in this region is the way in which some of these bands can evolve at $I \geq I_{\text{max}}$. At low spins in the cranked harmonic oscillator, configurations can be defined by the number of particles in the different $N$-shells and the maximum spin within each $N$-shell is easily deduced. At large deformations, $\beta \sin(\frac{\pi}{3} - \gamma) \geq 0.511$, however, the $N$-shells mix strongly and then with increasing rotational frequency, more angular momentum is continuously pumped into the configuration so that it can continue as a collective rotation beyond $I_{\text{max}}$ [14]. In a realistic nuclear potential for the nuclei in the $A \sim 70 - 80$ mass region, the $f_{7/2}$ high-$j$ subshell splits off from the other $N = 3$ subshells and, at low spin, the number of particles in $f_{7/2}$ and in other $N = 3$ subshells can be specified separately. With increasing angular momentum, however, all the $N = 3$ subshells mix very strongly and according to our calculations, such a configuration can, in a similar way as for the pure oscillator, continue as collective beyond the maximum spin $I_{\text{max}}$ as defined from the distribution of the valence nucleons over the $j$-shells at low spin. One should note that in a realistic nuclear potential, such a situation arises at considerably smaller deformation compared with the strong mixing of $N$-shells in the pure oscillator. For example, at low spin, the configurations of interest in the $A \sim 70 - 80$ mass region are triaxial close to oblate with $\gamma = -40^\circ$ and $\varepsilon_2 \approx 0.3$.

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