Superradiance of Degenerate Fermi Gases in a Cavity

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Cavity QED

* Discrete spectrum -> one or a few relevant cavity modes
* Possible strong coupling at SINGLE photon level

Electronic dipole coupling between single atoms and light field

\[ h_{dip} = E \cdot d \]

With canonical quantization

\[ h_{dip} = \sum_{k, \epsilon} g_{k, \epsilon} (i a_{k, \epsilon} e^{ik \cdot r} + h.c.) \]

\[ g_{k, \epsilon} = \sqrt{\frac{\omega_k}{2 \Omega}} \epsilon \cdot d \sim \sqrt{\omega_k} \frac{e^2}{a_0} \sqrt{\frac{a_0^3}{\Omega}} \sim 10 \text{ MHz} \]

\[ e a_0 \sim 10^{14} \text{ Hz} \]

\[ 1 \mu m^3 \]
Strong coupling

Decay rate

$\kappa \sim 1 \text{ MHz}$

For a resonant cavity mode whose frequency is equal to the electronic excitation energy of the atom,

\[
\begin{align*}
\frac{i}{\partial t} a_{k, \epsilon} &= -i \kappa a_{k, \epsilon} - i g_{k, \epsilon} \sigma_- e^{-i k \cdot r} \\
\frac{i}{\partial t} \sigma_- &= i g_{k, \epsilon} a_{k, \epsilon} e^{i k \cdot r},
\end{align*}
\]

$\sigma_- = |g\rangle \langle e|$
What for many atoms?

Correlations between atoms can be built by the mediated interactions via the same cavity field.
Realizing long range interactions ???

- Atoms in optical cavities
- Atoms with magnetic dipole moments (Dy)—small magnitude
- Rydberg atoms—limited lifetime
- Molecules with electric dipole moments
Superradiance in BEC with a single cavity mode

Red far-detuned pumping lasers

$$\Delta_a = \omega_p - \omega_a$$

Cavity detuning $$\Delta_c = \omega_p - \omega_c < 0$$

K. Baumann & el at, Nature 464, 1301 (2010); PRL 107, 140402 (2011);
R. Mottl & el at, Science 336, 1570 (2012)
Superradiance in BEC

Explained by linear stability analysis of the Gross-Pitaevskii equation, K. Baumann & el at, Nature 464, 1301 (2010)

Transition related to the density correlations of the atomic gases
Superradiance in Degenerate Fermi Gases

Consider spinless fermions, no direct interatomic interactions

\[ H = \int d\mathbf{r} \left[ \psi^+ (\mathbf{r}) h_0 \psi (\mathbf{r}) \right] - \Delta_c a^+ a \]

\[ h_0 = h_{at} + \eta (\mathbf{r}) (a^+ + a) + U (\mathbf{r}) a^+ a, \]

\[ h_{at} = \frac{P^2}{2m} + \frac{\Omega_p^2}{\Delta_a} \cos^2 (k_0 y) \]

\[ \eta (\mathbf{r}) = \eta_0 \cos (k_0 x) \cos (k_0 y), \]

\[ U (\mathbf{r}) = \frac{g^2}{\Delta_a} \cos^2 (k_0 x) \]

\[ \eta_0 = \frac{g \Omega_p}{\Delta_a} \]

Pumping laser

Rabi frequency

Cavity mode coupling

Pumping laser mode

Cavity mode

Pumping laser mode

P > P_{cr}
Nature of Superradiance

\[ H = \int d\mathbf{r} \left[ \psi^+(\mathbf{r}) h_0 \psi(\mathbf{r}) \right] - \Delta_c a^+ a \]

\[ h_0 = h_{at} + \eta(\mathbf{r})(a^+ + a) + U(\mathbf{r}) a^+ a, \quad \eta(\mathbf{r}) = \eta_0 \cos(k_0 x) \cos(k_0 y) \]

Equation of motion:

\[ i \frac{\partial a}{\partial t} = -(\tilde{\Delta}_c + i \kappa) a + \eta_0 \Theta \]

Density order:

\[ \Theta = \int d\mathbf{r} n(\mathbf{r}) \eta(\mathbf{r}) / \eta_0 \]

Effective cavity detuning

\[ \tilde{\Delta}_c = \Delta_c - \int d\mathbf{r} n(\mathbf{r}) U(\mathbf{r}) < 0 \]
Mean Field Theory

Order parameter: \( \Theta = \int d\mathbf{r} \langle n(\mathbf{r}) \rangle \cos(k_0 x) \cos(k_0 y) \)

Steady solution: \( 0 = i \frac{\partial \langle a \rangle}{\partial t} = - (\tilde{\Delta}_c + i \kappa) \langle a \rangle + \eta_0 \Theta \)

\[ \langle a \rangle = \frac{\eta_0 \Theta}{\tilde{\Delta}_c + i \kappa} \]

Free energy:
\[
F = - \frac{1}{\beta} \ln \text{Tr} e^{-\beta H} = - \left[ \frac{\tilde{\Delta}_c}{\tilde{\Delta}_c^2 + \kappa^2} + \eta_0^2 \chi \frac{4 \tilde{\Delta}_c^2}{(\tilde{\Delta}_c^2 + \kappa^2)^2} \right] (\eta_0 \Theta)^2
\]

\[ \chi = - \frac{1}{2 \beta \eta_0^2} \text{Tr} \left[ G_0 \eta(\mathbf{r}) G_0 \eta(\mathbf{r}') \right] > 0 \]

Single particle Green's function

Density susceptibility to modulation \( \eta(\mathbf{r})/\eta_0 \)
Transition Condition

\[ \eta_{cr}^0 = \frac{1}{2} \sqrt{\frac{\tilde{\Delta}_c^2 + \kappa^2}{(-\tilde{\Delta}_c)} \chi} \]

In terms of the single particle states \( \phi_k \)

\[ \chi = \frac{1}{2 \eta_0^2} \sum_{k, k'} \left| \int dr \phi_k^*(r) \phi_{k'}(r) \eta(r) \right|^2 \frac{n(\epsilon_k) - n(\epsilon_{k'}^\prime)}{\epsilon_k - \epsilon_{k'}^\prime} \]

Also applies to BEC
1d @ T=0

\[ \eta(r) \sim \cos(k_0 x) \]

Normalized and dimensionless susceptibility

\[ f = \chi \frac{E_r}{N_{at}} \quad E_r = k_0^2 / 2m \]

\[ f = \frac{k_0}{8k_F} \ln \left| \frac{k_0 + 2k_F}{k_0 - 2k_F} \right| \]

Fermi surface nesting
Pauli blocking
Boson Gas

Filling = n/2k_0
\[ 2d @ T=0 \]

\[ \eta(r) \sim \cos(k_0 x) \cos(k_0 y) \]

\[ h_{at} = \frac{P^2}{2m} + \frac{\Omega_p^2}{\Delta_a} \cos^2(k_0 y) \]

\[ V_0 = \frac{\Omega_p^2}{\Delta_a} \]

Filling = \( \frac{n}{4k_0^2} \)

Pauli blocking

Fermi surface nesting
3d @ T=0

$f (b) 3d$

$V_0/E_r$

- 1
- 2
- 4
- 8

Boson Gas
Superradiance in free space

Bosons: Ketterle's group
Science 285, 571 (1999)

Fermions: Zhang Jing's group
PRL 106, 210401 (2011)
FIG. 4: (a) and (c): The phase diagram for two-dimension case, in terms of effective detuning $\tilde{\Delta}_c/E_r$ and pumping lattice depth $V_0/E_r$. Different lines in (a) represent phase boundary with different fillings. (b) Critical $V_0/E_r$ as a function of filling $\nu$ for $\tilde{\Delta}/E_r$ fixed at $2 \times 10^3$. $\kappa/E_r = 250$ for (a) and (b); $\kappa/E_r = 4085$ for (c) and $U_0 N_{at}/E_r = 1 \times 10^3$ for (a-c).
Ongoing Project

Interplay between BEC-BCS crossover and superradiance
\[ F = -\frac{1}{\beta} \ln \text{Tr} e^{-\beta H} = -\left[ \frac{\tilde{\Delta}_c}{\tilde{\Delta}_c^2 + \kappa^2} + \eta_0^2 \chi \frac{4 \tilde{\Delta}_c^2}{(\tilde{\Delta}_c^2 + \kappa^2)^2} \right] (\eta_0 \Theta)^2 \]
Preliminary Result

\[(k_F/k_0)^3 = \nu = 0.1, \nu = 0.5, \nu = 1.0\]

Origin of the maximum

\[\frac{1}{k_0 a_s}\]
Thank you