Analysis of Complex Electromagnetic Structures by Hybrid FDTD/WCIP Method

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ABSTRACT

This paper proposes a hybrid full-wave analysis using Finite-Difference Time-Domain (FDTD) and Wave Concept Iterative Process (WCIP) methods, developed to analyze locally arbitrarily shaped microwave structures and Multilayer Planar structure. Using the equivalence principle, the original problem can be decomposed into two sub regions and solve each sub region separately. An interpolation scheme is proposed for communicating between the FDTD fields and WCIP wave, which will not require the effort of fitting the WCIP mesh to the FDTD cells in the interface region. This method is applied to calculate the scattering parameters of arbitrary (3-D) microwave structures. Applying FDTD to 3D discontinuity and WCIP to the remaining region preserves the advantages of both WCIP flexibility and FDTD efficiency. A comparison of the results with the FDTD staircasing data verifies the accuracy of the proposed method.

Keywords: FDTD; Hybrid Finite-Difference Time-Domain (FDTD); Hybrid Techniques; WCIP Method

1. Introduction

The WCIP method is a widely used numerical technique for characterizing various electromagnetic problems.

It has long been recognized that the accuracy and efficiency of the method can be dramatically improved through the use inhomogeneous layers and for planar structures with a curved boundary due to the staircasing approximation.

Several researches have been developed to overcome these difficulties in the WCIP method. The effort spent by the researchers to develop the WCIP method is restricted to study the homogeneous layers [1].

One attempt to study the inhomogeneous layers is presented in [2,3]. This consists in hybridizing the WCIP method with a differential formulation using the differential operator in conjunction with the Maxwell equation in their local form to link the waves on the two sides of the inhomogeneous layer [4].

Hybrid methods, which combine the desirable features of two or more different techniques, are developed to analyze complex electromagnetic problems that cannot be resolved conveniently, and/or accurately, by using them individually [5].

This paper proposes a new hybrid approach by introducing an interpolation scheme for communication between the FDTD field and the WCIP wave in the interface region. This approach employs a combination of Fourier transformation and iteration process, and exchanges information on the field values, back and forth, between the FDTD sub problem and the WCIP region. The iterative technique is rapidly convergent only when the mutual interaction between the two sub domains is relatively weak. The comparison between the scattering parameter results obtained with the proposed hybrid method, and FDTD staircasing data verifies the accuracy of this analysis.

2. Formulation

The idea of the hybrid approach consists to combines the above two methods in a manner that retains the advantages of both. Then, the hybrid method is implemented as follows:

First, the partition of the global domain in two sub domains:
- Interior model or FDTD domain (3D discontinuity domain);
- Exterior model or spectral domain (homogeneous domain).

Second, for the physical behavior, it is necessary to develop accurate procedures to support the Interaction between these two models, is fulfilled by enforcing boundary conditions, i.e., the continuity of the tangential fields on the equivalent surfaces S.

The processes started, by dividing D the computational
domain, into two sub domains DWCIP and DFDTD corresponding to the WCIP and FDTD regions, respectively, such that \( D = \text{DWCIP} \cup \text{DFDTD} \).

Next, mesh these two regions using structured meshes, respectively, with common nodes shared at the interface but with no overlapping (\( \text{DWCIP} \cap \text{DFDTD} = \emptyset \)).

The structured grid of the WCIP is well suited to conforms the FDTD mesh at the interface, and this allows us to limit the number of unknowns in the FDTD region.

The FDTD and WCIP models describing the corresponding region are coupled together through the boundary conditions on the surfaces \( S \):

\[
\frac{\hat{n} \times \mathbf{E}_1}{Z_1} = \frac{\hat{n} \times \mathbf{E}_2}{Z_2} \quad (1)
\]

\[
\hat{n} \times \mathbf{H}_1 = \hat{n} \times \mathbf{H}_2 \quad (2)
\]

Now, it is important to turn to the marching scheme that has been implemented in our algorithm. The FDTD technique a well-known and has been used widely, and hence, we do not need to quote his formulation in this work [6,7].

### 2.1. Formulation of the Wave Concept Iterative Method

The Wave Concept Iterative Process (WCIP) counts among the most recent and the most efficient iterative methods.

It was developed as an instrument for the study of guide wave and planar circuits, its applicability extends to all range of geometrical dimensions of the scattering obstacle. The WCIP approach consists in separating the structure under study into interfaces with upper and lower homogeneous media.

The boundary conditions on the interface are represented by the diffraction operator, \( S \), and in the homogeneous media by \( \Gamma \) the reflection operator. They are defined in spatial and modal domains, respectively.

The two conditions for existence of the Wave Iterative formulation are: first, the partition of the global domain in two subdomains:

- Spatial domain (interfaces or lumped elements);
- Spectral domain (medium or propagation).

The Wave Concept method described here is based on full wave transverse formulation, where the dual quantities current density and electric field are considered.

The incident \( \mathbf{A} \) and reflected \( \mathbf{B} \) waves are calculated from the tangential electric \( \mathbf{E} \) and magnetic \( \mathbf{H} \) fields, on the interface:

\[
\left\{ \begin{array}{l}
A_i = \frac{1}{2\sqrt{Z_{0i}}} \left( E_i + Z_{0i} J_i \right) \\
B_i = \frac{1}{2\sqrt{Z_{0i}}} \left( E_i - Z_{0i} J_i \right)
\end{array} \right. \quad (3)
\]

where indicates the medium 1 or 2 corresponding to a given interface \( \Omega \). \( Z_{0i} \) is the characteristic impedance of the same medium \( (i) \) and \( J_i \) is the surface current density vector given as

\[
J_i = H_i \wedge n_i \quad (4)
\]

where \( n_i \) is the outward vector normal to the interface.

In general case the planar structures are modeled in the WCIP method by a thin metallic plate at the interface \( \Omega \) between two medium, enclosed by a rectangular waveguide with transversal cut. The \( \Gamma \) operator is described in the modal domain and assigns the propagation boundary conditions at upper and lower interface.

The S operator expressed in the spatial domain assigns boundary conditions at the interface plane and represents the different possible sub domains (dielectric, metal and source). Then, the air-dielectric interface plane \( \Omega \) is divided into cells (named pixels), forming a uniform grid, used to discretize each sub domain.

The spatial and modal waves are directly deduced from each other with the help of Fast Modal Transformation (FMT) and its inverse transform (IFMT).

The decomposition of the electromagnetic wave in guided modes propagating in waveguide with electric or magnetic wall (TE and TM modes) takes place by the use of this Fast Modal Transformation.

The procedure is repeated until convergence of the input admittance \( Y_{in} \) and the frequency parameter of the structure is obtained (The convergence is obtained once in \( Y_{in} \) does not vary as the number of iterations increases) [8].

### 2.2. Hybrid WCIP-FDTD Algorithm

The hybrid method combines the method of WCIP in the frequency domain to solve the homogeneous problem and the FDTD method to handle the inhomogeneous dielectric object [9,10]. This approach employs a combination of Fourier transformation and iteration, and exchanges information on the field values, back and forth, between the homogeneous dielectric sub problem and the dielectric inhomogeneous region. The iterative technique is rapidly convergent only when the mutual interaction between the two sub regions relatively weak.

The hybridization technique is based upon the use of the concept of surface impedance boundary conditions it begins by dividing the original problem into two separate ones. The first one of these, which contains the inhomogeneous or 3D discontinuity region, is solved by using the FDTD scheme, while the second, which deals with the homogeneous sub region, is handled via the WCIP scheme.

A time-stepping solution procedure is implemented as follows.

The initial condition of the hybrid approach:
Initial incident wave in the WCIP region can be expressed as:

$$A_i^{(n)} = 0$$  \hspace{1cm} (5)

The situation is equivalent to an ABC with free space impedance.

Initial excitation in the FDTD region can be expressed as:

$$B_i^{(n)} = 0$$  \hspace{1cm} (6)

Begin the iteration process of the hybrid method by using the excitation source in the FDTD region and running the conventional sub region. The FDTD region is surrounded by an impedance bounding surface (an equivalence-principle surface) which is used to relaunch the inward and outward traveling fields between the two domains.

Next, the FDTD algorithm is applied in the relatively subregion, the equivalence principle is implemented in the 3D-FDTD computer code and all fields in this subregion is obtained.

Then updating the fields at the interface surface between the two subregions, and will be used in the reconstructing the incident wave in the WCIP region.

$$A_i = \frac{1}{2\sqrt{Z_{ii}}}(E_i + Z_0J_i)$$  \hspace{1cm} (7)

At the interface S of the FDTD domain the field relation defined as follows:

$$E = Z_0J$$  \hspace{1cm} (8)

The surface current density vector given as:

$$J = \hat{n} \times h$$  \hspace{1cm} (9)

$\hat{n}$ is the outward vector normal to the interface. $Z_0$ a characteristic impedance.

Then

$$A_{ii}^{(i)} = \frac{a}{\sqrt{Z_0}}E_i(x,y)$$  \hspace{1cm} (10)

e_i(x,y) = \text{the electric fields on interface surface S of the FDTD model.}

Apply the FFT on the expression of the incident wave to pass the time domain to frequency domain, and can be used in the initial condition in the WCIP processes.

Starting the WCIP process, it has already been seen that $B$ waves can be determined if $A$ waves are known and vice versa. The reflected wave $B$ can be computed by applying the basically boundary conditions of the WCIP technique in the homogeneous media represented by the reflection operator.

$$B = \hat{r}A$$  \hspace{1cm} (11)

After convergence of the iterative process of the WCIP approach. The reflected waves $B$ is modeled by equivalent source of the internal impedance $Z_0$ and are defined as follows:

$$B = \frac{1}{2\sqrt{Z_0}}(E - Z_0J) = \frac{E_0}{2\sqrt{Z_0}}$$  \hspace{1cm} (12)

$$E_0 = 2\sqrt{Z_0} \cdot B$$  \hspace{1cm} (13)

The continuity boundary conditions at the equivalent surface according to the two sub regions are written as [12,13]:

**Metallic domain**

$$E_x = E_y = 0$$  \hspace{1cm} (14)

$$J = -\frac{E_0}{Z_0}$$  \hspace{1cm} (15)

**Dielectric domain**

$$J = \frac{E_0}{Z_{ii} + Z_{si}}$$  \hspace{1cm} (16)

$$E = \frac{E_0}{Z_{ii} + Z_{si}}$$  \hspace{1cm} (17)

$H$, $E$ magnetic and electric field.

$Z_{ii}$ a characteristic impedance of the medium ($i$).

Applying the IFFT of the electric fields at the interface $S$ according to the WCIP scheme, the magnetic fields can then be evaluated from the electric field along the interface between WCIP region and FDTD region and are used as the excitation source for the sub region solved by the FDTD procedure. The procedure can now be repeated to continue the iteration process shown in Figure 1. The solution is checked at each iteration step until a steady state solution is obtained and illustrated by Figure 2.

Accurate solution to the hybrid problem can be achieved by a minimal cost of iterations verified in Figure 3.

3. Numerical Results

Wave concept iterative process and Finite Difference-Time Domain (FDTD) techniques have been used to characterize essentially commonly found discontinuities.

Specifically, a microstrip short circuit, a Short-Circuited Stubs microstrip Filter, and a rectangular dielectric resonant antenna were analyzed and their electrical performance was studied.

This hybrid method was applied in the first steps to characterize the cylindrical via-hole grounds in microstrip. The computational domain is split into two regions, the homogeneous dielectric and no metallic discontinuity region is replaced by WCIP region and the discontinuity
region is replaced by the FDTD region, as shown in Figure 1. Since the microstrip and ground plane coincide with the top and the bottom boundaries of the FDTD region and, the Dirichlet boundary conditions are applied to the top and the bottom as well as the via-hole cylinder wall [11]. The hybrid technique reduces the computational cost in the FDTD analysis and increases the computational efficiency.

The parameters of the first analyzed via-hole grounded microstrip structure presented in Figure 4 are as follows: via-hole diameter is 0.6 mm, microstrip width is 2.3 mm, and substrate thickness is 0.794 mm. lastly, the substrate has a low dielectric constant ($\varepsilon_r = 2.3$).

The derived results from the present method agreed very well. It has been found that a via at the center of a microstrip line provide a good dc connection at higher frequencies resulting in substantial coupling between the
This hybrid method can be applied toward analyzing 3-D locally arbitrarily shaped structures accurately and efficiently. The next example is a three short-circuited stubs distributed highpass filter presented in Figure 6, designed using the conventional technique [12-14]. For which the design parameters are: Cut-off frequency, $f_c = 1.3$ GHz, the dielectric Constant is $\varepsilon_r = 2.2$ and the height of substrate is $h = 1.57$ mm, Characteristic impedance of terminating microstrip line, $Z_0 = 50$ Ω, Guided wavelength $\lambda_{gc} = 167.24$ mm, corresponding width of the microstrip = 4.83 mm, Number of stub elements, $n = 3$ [14].

The electrical characteristics of the ground connection vs. frequency as evaluated by FDTD and WCIP are shown in Figure 7 and demonstrate very good agreement between the two methods. The slight discrepancy between the values can be attributed to numerical errors associated with both techniques.

We will now present inhomogeneous dielectric structure to validate the hybrid technique to modeling a multilayer structure.

The present method was applied to characterize the rectangular dielectric resonant antenna structure [15-20] shown in Figure 8.

The parameters of the final example: the high permittivity ($\varepsilon_{rd} = 48$) rectangular Dielectric Resonator. The microstrip line is fabricated on a substrate of dielectric constant $\varepsilon_{rs} = 4.28$ and thickness $h_s = 1.6$ mm. The length of the microstrip line is chosen to be twice the resonator length ($l_r = 2l_d = 6.8$ cm), the width of the microstrip line $w_f = 0.3$ cm, and the Ground plane dimensions: $l_g = 9$ cm, $w_g = 4$ cm.

A firstly we simulate the FDTD cell size is $\Delta x = 0.2$ mm, $\Delta y = 0.625$ mm, $\Delta z = 0.703$ mm and grid size is $60 \times 64 \times 128$.

In the WCIP the cell size is $\Delta y = 0.625$ mm, $\Delta z = 0.703$ mm and the grid size is $64 \times 128$.

In order to evaluate the precision factor of the hybrid method, we use the cell with very fine sizes, to explain the influence of the discretization of the divergence of the method at high frequencies. So we must assign a simulation with very small cell size and compare it with
Table 1. Comparison between the FDTD solution and the Hybrid FDTD-WCIP technique regarding the computational time for the different structure illustrated in this work.

| Structure       | FDTD Method | Hybrid FDTD-WCIP | Time of the simulation |
|-----------------|-------------|------------------|------------------------|
|                 | Number of cells | Time of the simulation | Number of cells | Time of the simulation | T_FDTD | T_HYBRID |
| HP Filter       | 491,520     | 58 min 52 s      | 114688                | 22 min 20 s          | 2.6358 |
| DRA antenna     | 1,966,080   | 243 min 03 s     | 90112                 | 89 min 20 s          | 2.714  |

Figure 9. Comparison of the results obtained by the proposed hybrid method with the FDTD simulation results.

much more facility in modelling of the structures and especially to make simulation with a minimum of time. We succeeded in formulating a three-dimensional and fast numerical method.

A hybrid FDTD-WCIP method is implemented in this letter using an iterative solution approach. Numerical results show that the hybrid method is accurate and efficient especially in terms of memory usage.

4. Conclusions

It is estimated whereas we will solve many problems by using the hybrid method which will allow us to have

REFERENCES

[1] S. Wane, D. Bajon and H. Baudrand, “A New Full-Wave Hybrid Differential-Integral Approach for Investigation of Multilayer Structures Including Nonuniformly Doped Diffusions,” IEEE Transactions on Microwave Theory and Techniques, Vol. 3, No. 53, 2005, pp. 200-214.

[2] N. Sboui, L. Latrach, A. Gharallah, H. Baudrand and A. Gharbi, “A 2D Design and Modeling of Micro Strip Structures on Inhomogeneous Substrate,” Wiley Interscience, New York, 2008.

[3] M. Glaoui, H. Zairi and H. Trabelsi, “A New Computationally Efficient Hybrid fdtLM-WCIP Method,” International Journal of Electronic, Vol. 96, No. 5 2009, pp. 537-548.

[4] E.-X. Liu, E.-P. Li and L.-W. Li, “Analysis of Multilayer Planar Circuits by a Hybrid Method,” IEEE Microwave Theory and Wireless Component Letters, Vol. 16, No. 2, 2006, pp. 66-68.

[5] H. Trabelsi, A. Gharallah and H. Baudrand, “Analysis of Microwave Circuits Including Lumped Elements Based
Analysis of Complex Electromagnetic Structures by Hybrid FDTD/WCIP Method

[6] A. Taflove and M. E. Brodwin, “Numerical Solution of Steady-State Electromagnetic Scattering Problems Using the Time-Dependent Maxwell Equations,” *IEEE Transactions on Microwave Theory and Techniques*, Vol. 23, No. 8, 1975, pp. 623-630. doi:10.1109/TMTT.1975.1128640

[7] K. S. Kunz and R. J. Luebbers, “The Finite Difference Time Domain Method for Electromagnetics,” CRC Press, Boca Raton, 1993.

[8] M. Titaouine, A. G. Neto, H. Baudrand and F. Djahli, “Analysis of Frequency Selective Surface on Isotropic/Anisotropic Layers Using WCIP Method,” *ETRI Journal*, Vol. 29, No. 1, 2007, pp. 36-44. doi:10.4218/etrij.07.0106.0123

[9] M. A. Mangoud, R. A. Abd-Alhameed and P. S. Excell, “Simulation of Human Interaction with Mobile Telephones Using Hybrid Techniques over Coupled Domains,” *IEEE Transactions on Microwave Theory and Techniques*, Vol. 48, No. 11, 2000, pp. 2014-2021. doi:10.1109/22.884190

[10] M. Al Sharkawy, V. Demir and A. Z. Elsherbeni, “The Iterative Multi-Region Algorithm Using a Hybrid Finite Difference Frequency Domain and Method of Moment Techniques,” *Progress in Electromagnetics Research*, Vol. 57, 2006, pp. 19-32. doi:10.2528/PIER05071001

[11] D. Koh, H.-B. Lee and T. Itoh, “A Hybrid Full-Wave Analysis of Via-Hole Grounds Using Finite-Difference and Finite-Element Time-Domain Methods,” *IEEE Transactions on Microwave Theory and Techniques*, Vol. 45, No. 12, 1997, pp. 89-92.

[12] G. Ramzi, Z. Hassan, H. Tabelsi and H. Baudran, “Tunable Lowpass Filters Using Folded Slot Etched in the Ground Plan,” *Progress in Electromagnetics Research C*, Vol. 7, 2009, pp. 65-78. doi:10.2528/PIERC09012706

[13] F. Lacroux and B. Jecko, “Contribution à la Modélisation D’éléments Localisés Pour les Simulations Electromagnétiques en Transitoire. Application en Millimétrique et au Transport D’énergie Sans Fil,” Thesis, Limoges University, Limoges, 2005.

[14] J. Garcia-Garcia, J. Bonache and M. Ferran “Application of Electromagnetic Bandgaps to the Design of Ultra-Wide Bandpass Filters with Good Out-of-Band Performance,” Progress in Electromagnetics Research Symposium, Singapore, 2003.

[15] E.-X. Liu, E.-P. Li and L.-W. Li, “Analysis of Multilayer Planar Circuits by a Hybrid Method,” *IEEE Microwave Theory and Wireless Component Letters*, Vol. 16, No. 2, 2006.

[16] R. A. Abd-Alhameed, P. S. Excell, J. A. Vaul and M. A. Mangoud, “Computation of Radiated and Scattered Fields Using Separate Frequency Domain Moment-Method Regions and Frequency-Domain MOM-FDTD Hybrid Methods,” *Proceedings of IEEE Conference Antennas and Propagation*, York, 31 March-1 April 1999, pp. 53-56.

[17] W. Thiel, K. Sabet and L. P. Katehi, “A Hybrid Mom/FDTD Approach for an Efficient Modelling of Complex Antennas on Mobile Platforms,” *Proceedings of the European Microwave Conference*, 7-9 October 2003, pp. 719-722.

[18] Z. Huang, K. R. Demarest and R. G. Plumb, “An FDTD/MOM Hybrid Technique for Modelling Complex Antennas in the Presence of Heterogeneous Grounds,” *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 37, No. 6, 1999, pp. 2692-2698. doi:10.1109/36.803416

[19] E. X. Liu, E.-P. Li and L.-W. Li, “Hybrid FDTD-MPIE Method for the Simulation of Locally Inhomogeneous Multilayer LTCC Structure,” *IEEE Microwave and Wireless Components Letters*, Vol. 15, No. 1, 2005, pp. 42-44. doi:10.1109/LMWC.2004.840981

[20] S. Mochizuk, S. Watanabe, M. Taki and Y. Yamanaka, “A New Hybrid MOM/FDTD Method for Antennas Located off the Yee’s Lattice,” 2004 URSI EMTS, International Symposium on Electromagnetic Theory, Pisa, 23-27 May 2004, pp. 436-438.

[21] R. K. Mongia and A. Ittipiboon, “Theoretical and Experimental Investigations on Rectangular Dielectric Resonator Antennas,” *IEEE Transactions on Antennas and Propagation*, Vol. 45, No. 9, 1997, pp. 1348-1356. doi:10.1109/8.623123

[22] H. Rogier, F. Olyslager and D. De Zutter, “A New Hybrid FDTD-BIE Approach to Model Electromagnetic Scattering Problems,” *IEEE Microwave and Wireless Components Letters*, Vol. 8, No. 3, 1998, pp. 138-140. doi:10.1109/75.661141