Spin-polarized electron transport in ferromagnet/semiconductor heterostructures:
Unification of ballistic and diffusive transport

R. Lipperheide and U. Wille
Abteilung Theoretische Physik, Hahn-Meitner-Institut Berlin,
Glienicker Straße 100, D-14109 Berlin, Germany
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A theory of spin-polarized electron transport in ferromagnet/semiconductor heterostructures, based on a unified semiclassical description of ballistic and diffusive transport in semiconductor structures, is developed. The aim is to provide a comprehensive framework for systematically studying the interplay of spin relaxation and transport mechanism in spintronic devices. A key element of the unified description of transport inside a (nondegenerate) semiconductor is the thermballistic current consisting of electrons which move ballistically in the electric field arising from internal and external electrostatic potentials, and which are thermalized at randomly distributed equilibration points. The ballistic component in the unified description gives rise to discontinuities in the chemical potential at the boundaries of the semiconductor, which are related to the Sharvin interface conductance. By allowing spin relaxation to occur during the ballistic motion between the equilibration points, a thermoballistic spin-polarized current and density are constructed in terms of a spin transport function. An integral equation for this function is derived for arbitrary profile of the electrostatic potential and for arbitrary values of the momentum and spin relaxation lengths. Detailed consideration is given to field-driven spin-polarized transport in a homogeneous semiconductor. An approximation is introduced which allows one to convert the integral equation into a second-order differential equation that generalizes the standard spin drift-diffusion equation. The spin polarization in ferromagnet/semiconductor heterostructures is obtained by invoking continuity of the current spin polarization and matching the spin-resolved chemical potentials on the ferromagnet sides of the interfaces. Allowance is made for spin-selective interface resistances. Examples are considered which illustrate the effects of transport mechanism and electric field.

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I. INTRODUCTION

Considerable attention has been devoted in the past few years to the study of spin-polarized electron transport in hybrid nanostructures composed of different types of material, such as nonmagnetic or magnetic semiconductors, normal metals, ferromagnets, and superconductors. The motivation behind these efforts derives from the desire to understand the principles of operation, to assess the performance, and to explore the field of possible applications, of solid-state devices relying on the control and manipulation of electron spin (“spintronic devices”). Particular emphasis in spintronics research is currently placed on the study of spin-polarized transport in heterostructures composed of nonmagnetic semiconductors and two (metallic or semiconducting) ferromagnetic contacts. Structures of this kind are considered promising candidates for future technological applications. For the actual design of spintronic devices, a detailed theoretical understanding of spin-polarized transport in ferromagnet/semiconductor heterostructures is indispensable. Up to now, a number of pertinent studies have been undertaken, which mostly rely on the drift-diffusion model.

Schmidt et al. describe the spin polarization by the same diffusion equation for the chemical potential as used to treat spin-polarized transport in ferromagnet/normal-metal heterojunctions. For a metallic-ferromagnet/semiconductor heterojunction with a perfect interface (no interface resistance or spin scattering), they find that, as a consequence of the large conductance mismatch, the injected current spin polarization is very low. Filip et al. and Rashba suggest that efficient spin injection can be obtained by introducing spin-selective interface resistances, for example, in the form of tunnel barriers. This idea is pursued in a number of detailed theoretical investigations in which the interface resistances are taken into account either phenomenologically by introducing discontinuities in the chemical potentials at the interface, or explicitly by treating the Schottky barrier arising from band bending in the interface depletion region. Yu and Flatté have derived a drift-diffusion equation for the density spin polarization, which allows, in particular, the effect of applied electric fields to be studied. A formalism taking into account the effect of the electron-electron interaction on spin-polarized transport in metals and doped (degenerate) semiconductors in the diffusive regime has been introduced by D’Amico and Vignale and has subsequently been generalized to include the effect of applied electric fields. Spin injection under conditions where, in the semiconductor, ballistic transport prevails over drift-diffusion has been studied by Kravchenko and Rashba within a Boltzmann equation approach; they find that in the absence of spin-selective interface resistances, the Sharvin interface conductance controls the injection efficiency. Spin injection across a Schottky barrier, arising from thermionic emission as well as tunneling injection,
has been treated by Shen et al.\[26\] within a Monte Carlo model. Phase-coherent transport in the ballistic limit has been studied, e.g., in Refs. 27, 28, 29, 30, 31, 32, 33.

It emerges that the theory of spin-polarized electron transport in ferromagnet/semiconductor heterostructures has reached a level of considerable sophistication. Nevertheless, we believe that certain aspects of the semiconductor part of the transport problem require a more detailed, unified treatment, such as the interplay of spin relaxation and transport mechanism all the way from the diffusive to the ballistic regime, and the effects of the spatial variation of the electrostatic potential profile. In the present work, we provide a comprehensive framework for systematically dealing with these aspects.

The starting point is our unified semiclassical description of (spinless) ballistic and diffusive electron transport in parallel-plane semiconductor structures\[34\] in which the idea of a thermoballistic transport mechanism was introduced. The latter relies on the concept of a “thermoballistic current” inside the semiconducting sample. This current consists of electrons which move ballistically in the electric field arising from internal (built-in) and external electrostatic potentials, and which are thermalized at randomly distributed equilibration points (with mean distance equal to the mean free path, or momentum relaxation length) due to coupling to the background of impurity atoms and phonons. The current-voltage characteristic for nondegenerate systems as well as the zero-bias conductance for degenerate systems are expressed in terms of a reduced resistance; for arbitrary momentum relaxation length and arbitrary potential profile, the latter quantity is determined from a resistance function which is obtained as the solution of an integral equation. The thermoballistic chemical potential and current are derived from this solution as well. The chemical potential exhibits discontinuities at the boundaries of the semiconductor, which are related to the Sharvin interface conductance.

In order to develop, within the unified description, a theory of spin-polarized electron transport in (nondegenerate) semiconductors, we introduce the “thermoballistic spin-polarized current” which generalizes the thermoballistic current of Ref. 34 so as to allow spin relaxation to take place during the ballistic motion between the equilibration points. The thermoballistic spin-polarized current is constructed in terms of a “spin transport function” that determines the spin polarization inside the semiconductor for arbitrary potential profile and arbitrary values of the momentum and spin relaxation lengths. This function satisfies an integral equation which follows from the balance equation connecting the thermoballistic spin-polarized current and density. The appearance of an integral equation in the unified description of electron transport (with or without account of the spin degree of freedom) reflects the nonlocal character of the transport across the ballistic intervals between the equilibration points. For electron transport in a homogeneous semiconductor without space charge, driven by an external electric field, the integral equation for the spin transport function can be converted, in an approximation that is adequate for demonstrating the principal effects of the transport mechanism, into a second-order differential equation that generalizes the standard spin drift-diffusion equation. The spin polarization along a ferromagnet/semiconductor heterostructure is obtained by invoking continuity of the current spin polarization at the interfaces and matching the spin-resolved chemical potentials on the ferromagnet sides of the latter, with allowance for spin-selective interface resistances.

As a prerequisite to developing a theory of spin-polarized electron transport in semiconductors within the unified transport model of Ref. 34, we have to modify and complete the formulation given in that reference. This will be done in the next section. In Sec. III, the spin degree of freedom is introduced into the unified description. The integral equation for the spin transport function inside a semiconducting sample is derived, and the generalized spin drift-diffusion equation is obtained. Spin-polarized transport in heterostructures formed of a nonmagnetic, homogeneous semiconductor and two ferromagnetic contacts is treated in Sec. IV. We demonstrate the procedure for the calculation of the current and density spin polarizations across a heterostructure in the zero-bias limit and of the injected spin polarizations for field-driven transport. Various examples are considered which illustrate the effects of transport mechanism and electric field and exhibit the relation of the unified description to previous descriptions by other authors. In Sec. V, the contents of the paper are summarized and our conclusions as well as an outlook towards applications and extensions of the present work are presented. In the Appendix, details of the extended formulation of the unified transport model outlined in Sec. II are worked out.

II. UNIFICATION OF BALLISTIC AND DIFFUSIVE TRANSPORT IN SEMICONDUCTORS

The unified description of (spinless) ballistic and diffusive electron transport developed in Ref. 34 has yielded, for the nondegenerate case, the current-voltage characteristic for a semiconducting sample enclosed between two plane-parallel contacts. There, the discontinuity of the chemical potential has been placed at the interface at one or the other end of the sample; this gave rise to an ambiguity in the behavior of the chemical potential inside the sample. In order, nevertheless, to obtain a unique current-voltage characteristic, the reduced resistance determining the latter was subjected to a heuristic symmetrization procedure (see Sec. IV.C of Ref. 34).

In the following, we extend in a systematic way the unified description by treating simultaneously the effects of the two interfaces on an equal footing. In this way, we arrive at unique chemical potentials, cur-
Fermi energy (symbol $E_F$) was used for the chemical potential as defined, e.g., in Ref. 35. Furthermore, $v_e = (2\pi m^*\beta)^{-1/2}$ is the emission velocity, $N_c = 2(2\pi m^*/\beta h^2)^{3/2}$ is the effective density of states at the conduction band edge, $m^*$ is the effective mass of the electrons, and $\beta = (k_B T)^{-1}$. The currents (2.2) and (2.3) contain only the electrons transmitted across the sample, i.e., the electrons with sufficient energy to surmount the potential barrier determined by

$$E_c^m(x', x'') = E_c^m(x', x'') - E_c^0;$$

(2.4)

Here, $E_c^m(x', x'')$ is the maximum value of the potential profile $E_c(x)$ in the interval $[x', x'']$, and $E_c^0$ is its overall minimum across the sample. The profile $E_c(x)$ is, in general, a self-consistent solution of a nonlinear Poisson equation involving the conduction band edge potential and the external electrostatic potential.

Expressions (2.2) and (2.3) for the currents injected at the left and right ends of a ballistic interval follow from Eqs. (20) and (21) of Ref. 34 if classical transmission probabilities are used. Tunneling effects can be included by replacing the classical probabilities with their quantal (WKB) analogues, as done, e.g., in Ref. 36. In the present paper, we do not consider this possibility.

It is convenient to rewrite Eq. (2.1) in the form

$$J(x', x'') = e^{-\beta E_c^m(x', x'')} \left[ J^{(x')} - J^{(x'')} \right];$$

(2.5)

the quantity

$$J^{(x')} = v_e N_c e^{-\beta E_c^m(x', x'')},$$

(2.6)

is the current injected at the left end $x'$ of the ballistic interval into the right direction in a flat profile $E_c(x) = E_c^0$, and similarly for the current $J^{(x'')}$. The ballistic density, at position $x$ in the interval $[x'', x']$, of transmitted electrons injected at the left end at $x'$ is given by

$$n^l(x', x'': x) = \frac{N_c}{2} \left( \frac{2\beta}{\pi m^*} \right)^{1/2}$$

$$\times \int_0^\infty dp e^{-\beta [p^2/2m^* + E_c(x) - \mu(x')]$$

$$\times \Theta(p^2/2m^* + E_c(x) - E_c^m(x', x''))$$

$$= \frac{N_c}{2} C(x', x''; x) e^{-\beta [E_c^m(x', x'') - \mu(x')]},$$

(2.7)

where

$$C(x', x''; x) = e^{\beta [E_c^m(x', x'') - E_c(x)]}$$

$$\times \text{erfc} \left( \beta (E_c^m(x', x'') - E_c(x))^{1/2} \right);$$

(2.8)

the function $\text{erfc}(x)$ is the complementary error function and $0 < C(x', x''; x) \leq 1$ (the conserved current...
and \[2.2\] and \[2.3\] are, of course, independent of \(x\). Analogously, the ballistic electron density in the transmitted current injected at the right end at \(x'\) is

\[
n^r(x', x''; x) = \frac{N_e}{2} C(x', x''; x) e^{-\beta[E^r_{nm}(x', x'') - \mu(x')]}.
\]

(2.9)

Dividing the current \[2.3\] by the density \[2.1\] or \[2.2\], we obtain for the average velocity \(v(x', x''; x)\) at position \(x\) of the electrons injected at either end of, and transmitted across, the interval \([x', x'']\)

\[
v(x', x''; x) = \frac{J_{l,r}(x', x'')}{n(x', x''; x)} = \frac{2v_c}{C(x', x''; x)},
\]

(2.10)

which depends only on the potential profile. For constant profile, one has \(C(x', x''; x) = 1\), and the electrons move with the average velocity \(2v_c\) from one or the other end to its opposite. This is the average velocity of the injected electrons also in the case of a position-dependent profile. However, some of these electrons are reflected back, so that the average velocity at position \(x\) of those electrons which have sufficient energy to pass over the top of the potential profile must be higher than \(2v_c\), namely, equal to the velocity given by Eq. \[2.10\].

In analogy to Eq. \[2.1\], the sum of the densities \(n^r(x', x''; x)\) and \(n^r(x', x''; x)\) is the ballistic density \(n(x', x''; x)\) of transmitted electrons in the interval \([x', x'']\),

\[
n(x', x''; x) = \frac{N_e}{2} C(x', x''; x) e^{-\beta E^r_{nm}(x', x'')} \\
\times \left[ e^{\beta \mu(x')} + e^{\beta \mu(x'')} \right].
\]

(2.11)

The density \(n(x', x''; x)\), like the current \(J(x', x'')\), comprises only electrons that participate in the transport.

From the ballistic current \[2.6\], the thermoballistic current \(J(x)\) at position \(x\) inside the semiconductor is constructed by summing up the weighted contributions of the ballistic intervals \([x', x'']\) for which \(x_1 \leq x' < x < x'' \leq x_2\) [see Eq. \(23\) of Ref. \[24\]]

\[
J(x) = w_1(x_1, x_2, l) \left[ J_1 - J_2 \right] + \int_{x_1}^{x} \frac{dx'}{l} \left[ J_3(x') - J_2 \right] + \int_{x}^{x_2} \frac{dx'}{l} \left[ w_2(x', x, l) \right] \left[ J_1 - J(x') \right] + \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} \left[ w_3(x', x'', l) \left[ J(x') - J(x'') \right] \right] ,
\]

(2.12)

with

\[
w_n(x', x''; l) = p_n((x'' - x')/l) e^{-\beta E^r_{nm}(x', x'')}
\]

(2.13)

\((n = 0, 1, 2, 3)\), where \(l\) is the momentum relaxation length of the electrons, which comprises the effect of relaxation due to electron scattering by impurity atoms and phonons. The probabilities \(p_n(x/l)\) of occurrence of the ballistic intervals depend on the dimensionality of the transport [note that \(w_n(x', x''; l)\) is symmetric with respect to an interchange of \(x'\) and \(x''\)]. In Eq. \[2.12\], the quantities \(J_{l,r} = J(x_{l,r})\) are fixed by the chemical potentials \(\mu_{l,r} = \mu(x_{l,r})\) on the contact sides of the interfaces at \(x_{l,r}\), i.e., immediately outside of the sample,

\[
J_{l,r} = v_c N_e e^{-\beta_E^r_{nm}(x_{l,r})}.
\]

(2.14)

For later convenience, we introduce a symbolic operator \(\mathcal{W}(x', x''; l)\) to write expression \[2.12\] in the condensed form

\[
J(x) = \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} \mathcal{W}(x', x''; l) \left[ J(x') - J(x'') \right],
\]

(2.15)

which, in view of Eqs. \[2.6\] and \[2.14\], may also be written as

\[
J(x) = v_c N_e e^{-\beta_E^0} \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} \mathcal{W}(x', x''; l) \left[ e^{\beta \mu(x')} - e^{\beta \mu(x'')} \right].
\]

(2.16)

Analogously, we introduce the thermoballistic density inside the semiconductor, \(n(x)\), as

\[
n(x) = \frac{N_e}{2} e^{-\beta_E^0} \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} \mathcal{W}(x', x''; l) x \left[ e^{\beta \mu(x')} + e^{\beta \mu(x'')} \right],
\]

(2.17)

where

\[
\mathcal{W}(x', x''; l) = \mathcal{W}(x', x''; l) C(x', x''; x).
\]

(2.18)

Again, the current \(J(x)\) and the density \(n(x)\) comprise only electrons that participate in the transport.

The thermoballistic current \[2.12\] by itself is not, in general, conserved, but together with the background current \[2.12\] \(J_{\text{back}}(x)\) it adds up to the conserved physical current \(J\):

\[
J(x) + J_{\text{back}}(x) = J = \text{const}.
\]

(2.19)

The background current is confined within the sample and, therefore, must vanish when integrated over the latter, which implies that the thermoballistic current \(J(x)\) averaged over the sample yields the physical current \(J\),

\[
\frac{1}{x_2 - x_1} \int_{x_1}^{x_2} dx J(x) = J.
\]

(2.20)

The non-conservation of the thermoballistic current can be viewed as arising from a source term \(Q(x)\) associated with the gain of thermoballistic electron density due to the coupling to the background, as expressed by the equation

\[
\frac{dJ(x)}{dx} = Q(x).
\]

(2.21)
In the background, the source term appears as a sink term describing the loss of electron density,

$$Q^{\text{back}}(x) = -Q(x) .$$  \hfill (2.22)

Again, since the background electrons are confined to the sample, the integral of $Q^{\text{back}}(x)$ over the sample must vanish and, therefore, also that of the thermoballistic source term $Q(x)$,

$$\int_{x_1}^{x_2} dx \, Q(x) = 0 .$$ \hfill (2.23)

Owing to Eq. (2.21), this implies

$$J(x_1^+) = J(x_2^-) \equiv \kappa J ,$$ \hfill (2.24)

that is, the thermoballistic current entering at one end of the sample, $x = x_1^+$, must be the same as the one leaving at the other end, $x = x_2^-$. The quantity $\kappa$ has been introduced for later convenience; it normalizes the thermoballistic current on the sample sides of the ferromagnet/semiconductor interfaces to the physical current $J$. We remark that, in analogy to the thermoballistic current, the thermoballistic density as well as other thermoballistic quantities introduced later in the development all have their background complement.

Condition (2.20) has been used in Ref. 34 to obtain the fundamental integral equation for the determination of the thermoballistic current. Condition (2.24) is new, and provides us with an extension of the formalism of Ref. 34 which allows us to establish a unique thermoballistic chemical potential inside the sample, as will be shown in the following.

B. Thermoballistic chemical potential, current, and density

Substituting expression (2.12) in condition (2.20), we obtain

$$\frac{x_2 - x_1}{l} J = \left[ u_1(x_1, x_2; l) + \int_{x_1}^{x_2} \frac{dx'}{l} u_2(x_1, x'; l) \right] J_1 - \left[ u_1(x_1, x_2; l) + \int_{x_1}^{x_2} \frac{dx'}{l} u_2(x_2, x'; l) \right] J_2 + \int_{x_1}^{x_2} \frac{dx'}{l} \left[ u_2(x', x_2; l) - u_2(x_1, x'; l) \right] J(x') + \int_{x_1}^{x_2} \frac{dx''}{l} u_3(x', x''; l) J(x') ,$$ \hfill (2.25)

where

$$u_n(x', x''; l) = \frac{x'' - x'}{l} w_n(x', x''; l)$$ \hfill (2.26)

[note that $u_n(x', x''; l)$ is antisymmetric with respect to an interchange of $x'$ and $x''$]. Equation (2.25) is a basic condition on the function $J(x)$, and hence, via Eq. (2.20), on the chemical potential $\mu(x)$, whose determination allows all relevant transport quantities to be obtained.

For given current $J$, only the value of the chemical potential at one of the interfaces with the contacts can be prescribed. If, at the interface at $x_1$, we prescribe the value of $\mu_1$, i.e., that of $J_1$ [case (i)], we can find the value of $J(x)$ at the other interface at $x_2$ by re-expressing in Eq. (2.20) $J_2$ as $J(x_2)$ and replacing $x_2$ with the variable $x$, thereby obtaining an integral equation for the function $J(x)$ in the range $x_1 < x < x_2$. If, now, $J(x_2)$ is required to assume a preassigned value for which we re-introduce the symbol $J_2$, then the current $J$ on the left-hand side of Eq. (2.25) is fixed at some value $J_1$. We denote the associated solution of the integral equation by $J_1(x)$.

On the other hand, prescribing the value $\mu_2$ for the chemical potential at the interface at $x_2$ [case (ii)], we re-express in Eq. (2.20) $J_1$ as $J(x_1)$. Then, replacing $x_1$ with the variable $x$, we obtain an integral equation for the function $J(x)$ in the range $x_1 < x < x_2$. With $J(x_1)$ required to assume a preassigned value $J_1$, the current $J$ is now fixed at the value $J_2$. The associated solution of the integral equation is denoted by $J_2(x)$.

To determine the functions $J_1, J_2(x)$ explicitly, we proceed in the following way. In case (i), we define the “resistance function” $\chi_1(x)$

$$\chi_1(x) = \frac{J_1 - J_1(x)}{J_1} ; \quad \chi_1(x_1) = 0 ,$$ \hfill (2.27)

and obtain from Eq. (2.25), following the procedure outlined above,

$$\frac{x - x_1}{l} = - \left[ u_1(x_1, x; l) + \int_{x_1}^{x} \frac{dx'}{l} u_2(x', x; l) \right] \chi_1(x) + \int_{x_1}^{x} \frac{dx'}{l} \left[ u_2(x', x; l) - u_2(x_1, x'; l) \right] \chi_1(x') + \int_{x_1}^{x} \frac{dx''}{l} u_3(x', x''; l) \chi_1(x') = 0 ,$$ \hfill (2.28)

which is a linear, Volterra-type integral equation for $\chi_1(x)$. Letting $x \to x_1^+$ in Eq. (2.28), we find, using the properties of $u_n(x', x''; l)$,

$$\chi_1(x_1^+) = \frac{J_1 - J_1(x_1^+)}{J_1} = e^{\beta E_i(x_1)-E_i^0} \neq \chi_1(x_1) .$$ \hfill (2.29)

With this discontinuity incorporated in it, the solution $\chi_1(x)$ is unique and continuous for $x_1 < x < x_2$. The solution of Eq. (2.28) can be obtained in closed form under special conditions; in the general case, one solves this equation efficiently by discretization and numerical propagation, using the initial value $\chi_1(x_1^+)$ given by Eq. (2.29).

In case (ii), we define the resistance function

$$\chi_2(x) = \frac{J_2(x) - J_2}{J_2} ; \quad \chi_2(x_2) = 0 ,$$ \hfill (2.30)
which satisfies the integral equation

\[
\frac{x_2 - x}{l} = \left[ u_1(x, x_2; l) + \int_x^{x_2} \frac{dx'}{l} u_2(x, x'; l) \right] \chi_2(x) \\
- \int_x^{x_2} \frac{dx'}{l} \left[ u_2(x', x_2; l) - u_2(x, x'; l) \right] + \int_x^{x_2} \frac{dx''}{l} u_3(x', x''; l) \right] \chi_2(x') = 0.
\]

(2.31)

The solution \( \chi_2(x) \) is discontinuous at \( x = x_2 \):

\[
\chi_2(x_2^-) = \frac{J_2(x_2^-) - J_2}{J_2} = e^{\beta[E_c(x_2) - E_a]} \neq \chi_2(x_2) .
\]

(2.32)

It follows from Eqs. (2.28) and (2.31), using the properties of \( u_n(x', x''; l) \), that the functions \( \chi_1(x) \) and \( \chi_2(x) \) are related by

\[
\chi_2(x) = \chi_1^*(x_0 - x) ,
\]

(2.33)

where \( x_0 = x_1 + x_2 \); the asterisk attached to \( \chi_1 \) indicates that this function is to be calculated using the reverse of the profile \( E_c(x) \), given by \( E_c^*(x) = E_c(x_0 - x) \). If the profile is symmetric, \( E_c^*(x) = E_c(x) \), the functions \( \chi_1(x) \) and \( \chi_2(x) \) are the reverse of one another, \( \chi_2(x) = \chi_1(x_0 - x) \).

The two functions \( J_{1,2}(x) \) are not, in general, equal and yield different chemical potentials \( \mu_{1,2}(x) \) via Eq. (2.28). Then, in view of Eq. (2.34), Eq. (2.28) implies \( \mu_1(x^+) \neq \mu_1 \), and the chemical potential \( \mu_1(x) \) is discontinuous at the interface at \( x = x_1 \), i.e., its value on the semiconductor side of the interface is not equal to its value at the interface itself. Analogously, \( \mu_2(x^+) \neq \mu_2 \).

The ambiguity thus found is a generalization of the ambiguity of the chemical potential in the ballistic limit \( l/S \rightarrow \infty \) where it may either be associated with the current injected at \( x_1 \), in which case it is discontinuous at \( x = x_1 \) (Sharvin resistance at the interface at \( x = x_1 \)), or with the current injected at \( x_2 \), in which case it is discontinuous at \( x = x_2 \) (Sharvin resistance at the interface at \( x = x_2 \)). In the Appendix, details of the construction of a unique thermoballistic chemical potential \( \mu(x) \), current \( J(x) \), and density \( n(x) \) in terms of the two solutions \( \chi_1, \chi_2 \) are presented. Here, we only summarize the results.

A quantity \( \chi \) is introduced as

\[
\chi = \tilde{a}_1 \chi_1(x_2) + \tilde{a}_2 \chi_2(x_1) ,
\]

(2.34)

where \( \tilde{a}_1 + \tilde{a}_2 = 1 \). The coefficients \( \tilde{a}_1 \) and \( \tilde{a}_2 \) are given by

\[
\tilde{a}_{1,2} = \frac{a_{1,2}}{a} ,
\]

(2.35)

where

\[
a_1 = \int_{x_1}^{x_2} \frac{dx'}{l} \left\{ w_2(x_1, x'; l) \left[ \chi_2(x') - \chi_2(x_1) \right] \right\}.
\]

(2.36)

Thus, \( \tilde{a}_1 = \tilde{a}_2 = 1/2 \).

For a symmetric potential profile \( E_c(x) \), we have \( \tilde{a}_1 = \tilde{a}_2 = 1/2 \).

The current \( J(x) \) is expressed as

\[
J(x) = \frac{1}{2} \left( J_1 + J_2 \right) - J \chi_-(x) ,
\]

(2.39)

where

\[
\chi_-(x) = \tilde{a}_1 \left[ \chi_1(x) - \frac{1}{2} \chi_1(x_2) \right] - \tilde{a}_2 \left[ \chi_2(x) - \frac{1}{2} \chi_2(x_1) \right] ,
\]

(2.40)

\((x_1 \leq x \leq x_2)\). The currents \( J_1 \) and \( J_2 \) satisfy the relation

\[
J_1 - J_2 = J \chi .
\]

(2.41)

With the use of Eq. (2.41), the current-voltage characteristic is then obtained in the form

\[
J = \nu e N_c e^{-\beta E_p} \frac{1}{\chi} \left( 1 - e^{-\beta V} \right) ,
\]

(2.42)

where

\[
V = \frac{\mu_1 - \mu_2}{e}
\]

(2.43)

is the voltage bias between the contacts, and \( E_p = E_c^m(x_1, x_2) - \mu_1 \); the quantity \( \tilde{\chi} \), given by

\[
\tilde{\chi} = e^{-\beta E_c^m(x_1, x_2)} \chi ,
\]

(2.44)

is the “reduced resistance”\(^{34}\). It replaces, in the present extended unified description, expression (58) of Ref. 34, which was obtained, in a heuristic way, by taking the mean value of the reduced resistances corresponding to case (i) and (ii), respectively. To determine \( \tilde{\chi} \) in the diffusive and ballistic regimes, we evaluate the functions \( \chi_1(x) \) and \( \chi_2(x) \) by following the development given in the Appendix of Ref. 34. In the diffusive regime \( l/S \ll 1 \), we find \( \chi_1(x_2) = \chi_2(x_1) = \chi \), which leads to

\[
\tilde{\chi} = \left[ e^{\beta \mu(x)} \right]^{-1} \frac{1}{l} \int_{x_1}^{x_2} \frac{dx}{l} e^{-\beta E_c^m(x_1, x_2) - E_c(x)} ,
\]

(2.45)

where the values of \( p_0(0) \) for one-, two-, and three-dimensional transport are given in Sec. II of Ref. 34. In the ballistic limit \( l/S \rightarrow \infty \), we have \( \tilde{\chi} = 1 \).

According to Eqs. (2.28), (2.31), (2.39), and (2.41), the thermoballistic chemical potential \( \mu(x) \) is given by

\[
e^{-\beta \mu(x)} = \left[ \frac{1}{2} \frac{\chi-\chi^p}{\chi} \right] e^{\beta \mu_1} + \left[ \frac{1}{2} + \frac{\chi-\chi^p}{\chi} \right] e^{\beta \mu_2} = \eta + 2 \frac{\chi-\chi^p}{\chi} \eta_- .
\]

(2.46)
are the respective Sharvin interface conductances, with \(\hat{n}_{1,2} = \bar{n}(x_{1,2})\). The discontinuities of the functions \(\exp(\beta \mu(x))\) and \(\bar{n}(x)\) are proportional to \(J\). In the diffusive regime \(l/S \ll 1\), where, according to Eqs. \ref{2.24} and \ref{2.25}, \(J \propto l/S\), the discontinuities approach zero together with \(l/S\).

The thermoballistic current \(J(x)\) is obtained in terms of \(\chi_{\pm}(x)\) and \(\chi\) by substituting expression \ref{2.22} in combination with Eq. \ref{2.24} in Eq. \ref{2.25} [or, more explicitly, in Eq. \ref{2.22}],

\[
J(x) = J \left\{ \begin{array}{l}
w_1(x_1, x_2; l) \chi \\
\quad + \int_{x_1}^{x} \frac{dx'}{l} w_2(x', x_2; l) \left[ \frac{\chi}{2} - \chi_{-}(x') \right] \\
\quad + \int_{x}^{x_2} \frac{dx'}{l} w_2(x_1, x'; l) \left[ \frac{\chi}{2} + \chi_{-}(x') \right] \\
\quad + \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} w_3(x', x''; l) \left[ \chi_{-}(x'') - \chi_{-}(x') \right] \end{array} \right.
\]

The thermoballistic density \(n(x)\) is found in a similar fashion from Eq. \ref{2.21},

\[
n(x) = \frac{J}{2 e \nu} \left\{ \chi \coth(\beta e V/2) \mathfrak{M}(x_1, x_2; l; x) \\
\quad - \int_{x_1}^{x} \frac{dx'}{l} w_2(x', x_2; l; x) \left[ \frac{\chi}{2} + \chi_{-}(x') \right] \\
\quad + \int_{x}^{x_2} \frac{dx'}{l} w_2(x_1, x'; l; x) \left[ \frac{\chi}{2} - \chi_{-}(x') \right] \\
\quad - \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} w_3(x', x''; l; x) \left[ \chi_{-}(x'') + \chi_{-}(x') \right] \right\},
\]

where

\[
\mathfrak{M}(x_1, x_2; l; x) = w_1(x_1, x_2; l; x) \\
\quad + \int_{x_1}^{x} \frac{dx'}{l} w_2(x', x_2; l; x) \\
\quad + \int_{x}^{x_2} \frac{dx'}{l} w_2(x_1, x'; l; x) \\
\quad + \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} w_3(x', x''; l; x)
\]

and

\[
w_n(x', x''; l; x) = w_n(x', x''; l) C(x', x''; x) \ .
\]

In the zero-bias limit \(V \to 0\), expression \ref{2.26} reduces to the form \(n(x) = \bar{n}_1 \mathfrak{M}(x_1, x_2; l; x)\), from which the physical meaning of the function \(\mathfrak{M}(x_1, x_2; l; x)\) becomes apparent.

In the diffusive regime \(l/S \ll 1\), we have

\[
e^{\beta \mu(x)} = \eta_+ - \frac{I_1(x_1, x) - I_2(x, x_2)}{l e(x_1, x_2)} \eta_-, \]

where

\[
I_1(x', x'') = \int_{x'}^{x''} dz e^{E_{c}(z)} \ ,
\]

and

\[
J(x) = J = -\frac{\nu}{e} \bar{n}(x) \frac{dp(x)}{dx} + \frac{1}{\beta} \frac{d\bar{n}(x)}{dx} \ ,
\]

The discontinuities of the functions \(\exp(\beta \mu(x))\) and \(\bar{n}(x)\) are proportional to \(J\). In the diffusive regime \(l/S \ll 1\), where, according to Eqs. \ref{2.24} and \ref{2.25}, \(J \propto l/S\), the discontinuities approach zero together with \(l/S\).

The thermoballistic current \(J(x)\) is obtained in terms of \(\chi_{\pm}(x)\) and \(\chi\) by substituting expression \ref{2.22} in combination with Eq. \ref{2.24} in Eq. \ref{2.25} [or, more explicitly, in Eq. \ref{2.22}],

\[
J(x) = J \left\{ \begin{array}{l}
w_1(x_1, x_2; l) \chi \\
\quad + \int_{x_1}^{x} \frac{dx'}{l} w_2(x', x_2; l) \left[ \frac{\chi}{2} - \chi_{-}(x') \right] \\
\quad + \int_{x}^{x_2} \frac{dx'}{l} w_2(x_1, x'; l) \left[ \frac{\chi}{2} + \chi_{-}(x') \right] \\
\quad + \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} w_3(x', x''; l) \left[ \chi_{-}(x'') - \chi_{-}(x') \right] \end{array} \right.
\]

The thermoballistic density \(n(x)\) is found in a similar fashion from Eq. \ref{2.21},

\[
n(x) = \frac{J}{2 e \nu} \left\{ \chi \coth(\beta e V/2) \mathfrak{M}(x_1, x_2; l; x) \\
\quad - \int_{x_1}^{x} \frac{dx'}{l} w_2(x', x_2; l; x) \left[ \frac{\chi}{2} + \chi_{-}(x') \right] \\
\quad + \int_{x}^{x_2} \frac{dx'}{l} w_2(x_1, x'; l; x) \left[ \frac{\chi}{2} - \chi_{-}(x') \right] \\
\quad - \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} w_3(x', x''; l; x) \left[ \chi_{-}(x'') + \chi_{-}(x') \right] \right\},
\]

where

\[
\mathfrak{M}(x_1, x_2; l; x) = w_1(x_1, x_2; l; x) \\
\quad + \int_{x_1}^{x} \frac{dx'}{l} w_2(x', x_2; l; x) \\
\quad + \int_{x}^{x_2} \frac{dx'}{l} w_2(x_1, x'; l; x) \\
\quad + \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} w_3(x', x''; l; x)
\]

and

\[
w_n(x', x''; l; x) = w_n(x', x''; l) C(x', x''; x) \ .
\]
Further,
\[ J(x) = J = 2v_e n_c e^{-\beta E^e_c(x_1, x_2)} \eta_- \] (2.62)
and
\[ n(x) = n(x) = N_c C(x_1, x_2; x) e^{-\beta E^m_c(x_1, x_2)} \eta_+ . \] (2.63)
As our model demands, expressions \[ Eq. (2.53) \] and \[ Eq. (2.11) \] with \( x' = x_1 \) and \( x'' = x_2 \).

C. Field-driven transport in a homogeneous semiconductor

To illustrate the formalism developed so far, we now consider electron transport in a homogeneous semiconductor without space charge. The electrons are assumed to be driven by a (constant) electric field \( \mathcal{E} \) directed antiparallel to the \( x \)-axis, i.e., they are moving in a linearly falling potential of the form
\[ E_c(x) = E_c(x_1) - e|\mathcal{E}|(x - x_1) , \] (2.64)
in which case
\[ C(x', x''; x) \equiv C(\epsilon(x - x')) = e^{\epsilon(x-x')} \text{erfc}(\epsilon(x-x')^{1/2}) , \] (2.65)
where \( \epsilon = \beta e|\mathcal{E}| \). The latter quantity is related to the voltage bias \( V \) via \( eS = \beta eV \), so that
\[ \bar{n}_1 = \bar{n}_2 = \bar{n} , \] (2.66)
where use has been made of Eqs. \ref{eq:2.13} and \ref{eq:2.14}.

In order to calculate the thermoballistic chemical potential \( \mu(x) \) from Eq. \ref{eq:2.11} [or, equivalently, the thermoballistic equilibrium electron density \( \bar{n}(x) \) from Eq. \ref{eq:2.15}], the thermoballistic current \( J(x) \) from Eq. \ref{eq:2.12}, and the thermoballistic density \( n(x) \) from Eq. \ref{eq:2.14}, we first have to solve the integral equations \ref{eq:2.28} and \ref{eq:2.29} numerically for the functions \( \chi_1(x) \) and \( \chi_2(x) \), respectively. For convenience, we use in the integral equations the probabilities \( p_n(x/l) \) in their one-dimensional form, \( p_n(x/l) = e^{-x/l} \) [see Eq. (10) of Ref. \ref{34}], so that from Eq. \ref{eq:2.13} and \ref{eq:2.14}
\[ u_n(x', x''; l) = \frac{x'' - x'}{l} e^{-|x'' - x'|/l} e^{-\epsilon|x_2 - \min(x', x'')} \] (2.67)
(this simplification has only minor effect, see Ref. \ref{34}).
Owing to the scaling properties of the function \( C(x', x''; x) \) given by Eq. \ref{eq:2.16} and of the function \( u_n(x', x''; l) \) given by Eq. \ref{eq:2.29}, the results of the calculations can be expressed essentially in terms of three dimensionless quantities, for example, \( x/S, eS, \) and \( l/S \). In Figs. \ref{fig:2} and \ref{fig:3} we show the dependence of \( \mu(x), \bar{n}(x), J(x), \) and \( n(x) \) on \( x/S \) (assuming \( x_1 = 0 \)) for \( eS = 1 \) and various values of \( l/S \).

From Fig. \ref{fig:2} (lower panel), it is seen that in the diffusive limit \( l/S \rightarrow 0 \) the chemical potential \( \mu(x) \) decreases linearly with \( x/S \) and is continuous at the interfaces; this behavior persists in the case of arbitrary \( eS \), where \( \beta[\mu(x) - \mu_1] = -e\epsilon \) for \( 0 \leq x/S \leq 1 \). In combination with the identical decrease of the potential profile \( E_c(x) \), this implies a position-independent equilibrium density \( \bar{n}(x) \) (see upper panel of Fig. \ref{fig:2}). When \( l/S \) rises towards the ballistic limit \( l/S \rightarrow \infty \), discontinuities of \( \mu(x) \) develop at the interfaces, which increase in magnitude, and the slope of \( \mu(x) \) becomes smaller. This results in a rise of the equilibrium density \( \bar{n}(x) \) across the sample. As a function of \( eS \), the discontinuities of \( \mu(x) \) become smaller in magnitude if \( eS \rightarrow \infty \), and larger if \( eS \rightarrow 0 \), such that in the latter case \( \mu(x) \) becomes independent of \( x \) for \( 0 < x/S < 1 \).

The ratio \( J(x)/J \) shown in Fig. \ref{fig:4} is close to unity across the whole sample. As the ballistic limit \( l/S \rightarrow \infty \) is approached, \( J(x)/J \) becomes more and more symmetric about \( x/S = 1/2 \), and \( J(x)/J \rightarrow 1 \) in the full range \( 0 \leq x/S \leq 1 \). A somewhat peculiar behavior of \( J(x) \) is observed in the diffusive regime \( l/S \ll 1 \). Here, \( J(x)/J \) is very close to unity inside the sample, except for the immediate vicinity of the ends at \( x/S = 0 \) and \( x/S = 1 \), where some structure develops, and \( J(x)/J \) converges towards a value smaller than unity when \( x/S \rightarrow 0 \) or \( x/S \rightarrow 1 \). It is important to keep in mind the latter feature when, within the unified treatment of spin-
polarized transport, the injected spin polarization at ferromagnet/semiconductor interfaces is defined (see Sec. IV.C). When $eS$ is varied, the qualitative behavior of $J(x)/J$ persists for all values of $l/S$ considered.

In Fig. 4, we show the ratio $n(x)/n_0$, where $n_0$ is the constant value that $n(x)$ [as well as $\bar{n}(x)$] assume in the diffusive limit $l/S \to 0$. For increasing $l/S$, i.e., as the ballistic contribution to the transport mechanism increases, the ratio $n(x)/n_0$ decreases as a whole. This effect can be interpreted as reflecting the fact that ballistically, the density decreases rapidly as the velocity rises along the sample [see Eq. (2.10)]; this is impeded at the equilibration points, which lie very dense when $l/S \ll 1$ [slow decrease of $n(x)$] and are widely spread when $l/S \gg 1$ [rapid decrease of $n(x)$]. For $l/S > 1$, the behavior of $n(x)$ is largely determined by the function $C(\epsilon x)$ in the ballistic limit $l/S \to \infty$, we have $n(x) = (\bar{n}/2) (1 + e^{-\epsilon S}) C(\epsilon x)$. Again, the qualitative behavior of $n(x)$ does not change when $eS$ is varied.

The zero-bias limit $\epsilon \to 0$ can be treated analytically. The solution of Eq. (2.28) is found to be

$$\chi_1(x) = 1 + \frac{x - x_1}{2l}. \quad (2.68)$$

Since $\chi_2(x) = \chi_1(x_0 - x)$ and $\tilde{a}_1 = \tilde{a}_2 = 1/2$ to zeroth order in $eS = \beta eV$, we obtain

$$\chi = 1 + \frac{S}{2l}, \quad \chi_-(x) = \frac{x - x_0/2}{2l}. \quad (2.69)$$

From Eq. (2.42), the thermoballistic current $J(x)$ is found to be constant, $J(x) = J$. For the current $J$, we have from Eqs. (2.42) and (2.48), to first order in $\beta eV$,

$$J = \frac{2l}{2l + S} v_e \bar{n} \beta eV, \quad (2.70)$$

where $\bar{n}$ is the common value of the thermoballistic equilibrium density at either end of the sample [see Eq. (2.66)]. Then, the conductance $G = eJ/V$ becomes

$$G = \frac{2l}{2l + S} \bar{G}, \quad (2.71)$$

where $\bar{G}$ is the Sharvin interface conductance [see Eq. (2.31)]. Relation (2.71) generalizes the Ohm conductance $G = (2l/S)\bar{G} = \sigma/S$ (valid in the diffusive regime, $l/S \ll 1$), where $\sigma = 2\beta e^2 v_e \bar{n}$ is the conductivity. The thermoballistic equilibrium density $\bar{n}(x)$ and the thermoballistic density $n(x)$ are obtained from expressions (2.48) and (2.53), respectively, as

$$\bar{n}(x) = n(x) = \bar{n}, \quad (2.72)$$

i.e., they are, in the present case of zero bias, both independent of position and equal to the equilibrium density at the ends of the sample.

III. SPIN-POLARIZED TRANSPORT WITHIN THE UNIFIED DESCRIPTION

Having established, in the preceding section, a unified description of spinless electron transport in semiconductors in terms of a unique thermoballistic chemical potential, we will now extend this scheme by including the spin degree of freedom. We allow spin relaxation to take place during the motion of the electrons across the ballistic intervals. Spin relaxation is generally governed by the equation of balance connecting the spin-polarized current with the spin-polarized density. In the unified
description, it is the thermoballistic current and density which enter into this equation. The solution of the balance equation is found in terms of a spin transport function that is related to the spin-resolved thermoballistic chemical potentials.

## A. Balance equation and transport mechanism

In a stationary situation, the total electron current $J = J_T(x) + J_L(x)$ composed of its spin-resolved parts $J_T(x)$ and $J_L(x)$ is conserved, whereas the spin-polarized current $J_{\uparrow\downarrow}(x) = J_\uparrow(x) - J_\downarrow(x)$, or rather its off-equilibrium part $\hat{J}_{\uparrow\downarrow}(x) = J_{\uparrow}(x) - J_{\downarrow}(x)$, where $\hat{J}_{\uparrow\downarrow}(x)$ is the relaxed part of the spin-polarized current, is connected with the off-equilibrium spin-polarized density $\hat{n}_{\downarrow}(x)$ through the balance equation

$$\frac{d\hat{J}_{\uparrow\downarrow}(x)}{dx} + \frac{\hat{n}_{\downarrow}(x)}{\tau_s} = 0. \quad (3.1)$$

Here, $\hat{n}_{\downarrow}(x)$ is defined in analogy to $\hat{J}_{\uparrow\downarrow}(x)$, and $\tau_s$ is the spin relaxation time. For a complete description of spin-polarized transport, the balance equation (3.1) is to be supplemented with a relation between the current and the density, which reflects the specific transport mechanism.

In the ballistic limit, the electron currents $J_{\uparrow\downarrow}(x)$ are proportional to the densities $n_{\uparrow\downarrow}(x)$ of the electrons participating in the transport,

$$J_{\uparrow\downarrow}(x) = v(x) n_{\uparrow\downarrow}(x), \quad (3.2)$$

where $v(x)$ is the average velocity of the electrons at position $x$ [we disregard spin splitting of the conduction band edge potential $E_c(x)$, so that $v(x)$ is independent of spin]. This relation holds also for the off-equilibrium spin-polarized current and density,

$$\hat{J}_{\uparrow\downarrow}(x) = v(x) \hat{n}_{\downarrow}(x). \quad (3.3)$$

Use of this equation in Eq. (3.1) yields

$$\frac{d\hat{J}_{\uparrow\downarrow}(x)}{dx} + C(x) \frac{\hat{J}_{\uparrow\downarrow}(x)}{l_s} = 0, \quad (3.4)$$

where $l_s = 2v_c\tau_s$ is the (ballistic) spin relaxation length, which comprises the overall effect of the various underlying microscopic spin relaxation mechanisms and where Eq. (2.11) has been used (omitting the positions $x', x''$ of the end points) to express $v(x)$ in terms of $C(x)$, i.e., of the potential $E_c(x)$ in which the electrons move.

In the diffusive regime, the off-equilibrium spin-polarized current and density are connected by the relation

$$\hat{J}_{\downarrow}(x) = -\nu \frac{e}{\beta} \left[ \hat{n}_{\downarrow}(x) \frac{dE_c(x)}{dx} + \frac{1}{\beta} \frac{d\hat{n}_{\downarrow}(x)}{dx} \right] \quad (3.5)$$

[see Eq. (2.53)]. We then find from Eq. (3.1)

$$\frac{d^2\hat{n}_{\downarrow}(x)}{dx^2} + \beta \frac{dE_c(x)}{dx} \frac{d\hat{n}_{\downarrow}(x)}{dx} + \beta \frac{d^2E_c(x)}{dx^2} \hat{n}_{\downarrow}(x) - \frac{1}{l_s^2} \hat{n}_{\downarrow}(x) = 0, \quad (3.6)$$

where

$$L_s = \sqrt{(l_s)l_s} \quad (3.7)$$

is the spin diffusion length.

In the unified description, the total current and density inside the semiconducting sample are taken to be the thermoballistic current $J(x)$ and density $n(x)$, between which no direct relation generally exists. Instead, Eqs. (2.10) and (2.11) express these two quantities separately in terms of the chemical potential $\mu(x)$. The connection between the off-equilibrium thermoballistic spin-polarized current $\hat{J}_{\uparrow\downarrow}(x)$ and density $\hat{n}_{\uparrow\downarrow}(x)$ can be established along similar lines, as will be described in the following.

## B. Thermoballistic spin-polarized current and density

In order to include spin relaxation in the unified description, we begin by introducing the thermoballistic equilibrium densities $\bar{n}_{\uparrow\downarrow}(x')$ for spin-up and spin-down electrons at an equilibrium point $x'$. It is convenient to express $\bar{n}_{\uparrow\downarrow}(x')$ in terms of the spin-independent thermoballistic equilibrium density $\bar{n}(x')$ and a “spin fraction” $\alpha_{\uparrow\downarrow}(x')$ via

$$\bar{n}_{\uparrow\downarrow}(x') = \bar{n}(x') \alpha_{\uparrow\downarrow}(x'), \quad (3.8)$$

with $\alpha_\uparrow(x') + \alpha_\downarrow(x') = 1$. In analogy to Eq. (2.45), we define spin-resolved thermoballistic chemical potentials $\mu_{\uparrow\downarrow}(x')$ via

$$\bar{n}_{\uparrow\downarrow}(x') = N_e e^{-\beta\left[E_c(x') - \mu_{\uparrow\downarrow}(x')\right]}, \quad (3.9)$$

which implies

$$e^{\beta\mu_{\uparrow\downarrow}(x')} = e^{\beta\mu(x')} \alpha_{\uparrow\downarrow}(x'). \quad (3.10)$$

The spin fraction $\alpha_{\uparrow\downarrow}(x')$ also enters into the definition of the spin-resolved ballistic current $J_{\uparrow\downarrow}(x', x''')$ injected at the left end at $x'$ of the interval $[x', x'']$,

$$J_{\uparrow\downarrow}(x', x''') = v_c N_e e^{-\beta\left[E_c(x'') - \mu_{\uparrow\downarrow}(x')\right]} = J'(x', x''') \alpha_{\uparrow\downarrow}(x'). \quad (3.11)$$

We emphasize that expression (3.11) for $J_{\uparrow\downarrow}(x', x''')$ holds only at the left end, since this current is not conserved owing to spin relaxation, and becomes position-dependent inside the interval. There, we write it in the form

$$J_{\uparrow\downarrow}(x', x'''') = J'(x', x'''') \alpha_{\uparrow\downarrow}(x', x'''') \quad (3.12)$$
The function $\alpha_{\uparrow\downarrow}(x', x''; x)$ is the spin fraction at position $x$ of spin-up (spin-down) electrons injected into the ballistic interval $[x', x'']$ at its left end at $x'$, with $\alpha_{\uparrow}(x', x''; x) + \alpha_{\downarrow}(x', x''; x) = 1$. Here, the dependence on the position $x''$ (at the end of the ballistic interval $[x', x'']$ opposite to that at position $x'$ where the electrons are injected) is due to the effect of the potential barrier embodied in the function $E^n_{m}(x', x'')$ in expression (2.2).

We now introduce the “spin fraction excess” $\tilde{\alpha}_\uparrow(x') = \alpha_{\uparrow}(x') - \alpha_{\uparrow}(x')$ and the off-equilibrium spin fraction excess $\tilde{\alpha}_\uparrow(x') = \alpha_{\uparrow}(x') - \tilde{\alpha}_\uparrow(x')$ and $\tilde{\alpha}_\downarrow(x')$, where $\tilde{\alpha}_\uparrow = \beta_\uparrow - \beta_\downarrow$, and $\tilde{\alpha}_\downarrow$ are the relaxed parts of the spin fractions ($\tilde{\alpha}_\downarrow = 0$ for nonmagnetic semiconductors). With the off-equilibrium spin fraction excess $\tilde{\alpha}_\uparrow(x', x''; x)$ defined in an analogous way, we write the off-equilibrium ballistic spin-polarized current $\hat{J}_\uparrow(x', x''; x)$ as

$$\hat{J}_\uparrow(x', x''; x) = J_\uparrow(x', x'') \tilde{\alpha}_\uparrow(x', x''; x).$$

(3.14)

The spin relaxation of the electrons injected at the left equilibration point $x''$ into the ballistic interval $[x', x'']$ is governed by Eq. (3.12), which, owing to Eq. (3.12), becomes a differential equation for $\tilde{\alpha}_\uparrow(x', x''; x)$,

$$\frac{d\tilde{\alpha}_\uparrow(x', x''; x)}{dx} + C(x', x''; x) \frac{\tilde{\alpha}_\uparrow(x', x''; x)}{l_{s_x}} = 0. \tag{3.15}$$

The solution of Eq. (3.15) is

$$\tilde{\alpha}_\uparrow(x', x''; x) = \tilde{\alpha}_\uparrow(x') e^{-\mathcal{E}(x', x'': x'; x)/l_{s_x}}, \tag{3.16}$$

where

$$\mathcal{E}(x', x''; z_1, z_2) = \int_{z_1}^{z_2} dz' C(x', x''; z'), \tag{3.17}$$

with $z_1 = \min(z_1, z_2)$ and $z_2 = \max(z_1, z_2)$. We then have for the off-equilibrium ballistic spin-polarized current at position $x$ of electrons injected at $x'$

$$\hat{J}_\uparrow(x', x''; x) = J_\uparrow(x', x'') \tilde{\alpha}_\uparrow(x') e^{-\mathcal{E}(x', x'': x'; x)/l_{s_x}}; \tag{3.18}$$

analogously, we find

$$\hat{J}_\downarrow(x', x''; x) = J_\downarrow(x', x'') \tilde{\alpha}_\downarrow(x') e^{-\mathcal{E}(x', x'': x'; x)/l_{s_x}}, \tag{3.19}$$

for the off-equilibrium ballistic spin-polarized current injected at $x''$.

Separating out the relaxed part, we now write the (net) ballistic spin-polarized current, in analogy to Eq. (2.11), in the form

$$J_\uparrow(x', x''; x) = \hat{J}_\uparrow(x', x''; x) + J_\uparrow(x', x'') \tilde{\alpha}_\uparrow, \tag{3.20}$$

with the off-equilibrium ballistic spin-polarized current

$$\hat{J}_\uparrow(x', x''; x) = v_e N_e e^{-\beta E^n_{m}(x', x'')} \times \left[ A(x') e^{-\mathcal{E}(x', x'': x'; x)/l_{s_x}} - A(x'') e^{-\mathcal{E}(x', x'': x''; x)/l_{s_x}} \right]; \tag{3.21}$$

and

$$\hat{J}_\downarrow(x', x''; x) = \hat{J}_\downarrow(x', x''; x) + J_\downarrow(x', x'') \tilde{\alpha}_\downarrow, \tag{3.22}$$

at the equilibration point $x$ $(x_1 \leq x \leq x_2)$. Using the relation

$$\mu_-(x') = \frac{1}{2} \ln \left( \frac{1 + \alpha_-(x')}{{\alpha}_-(x')} \right), \tag{3.23}$$

between the splitting $\mu_-(x') = \mu_-(x') - \mu_-(x')$ of the spin-up and spin-down chemical potentials and the spin fraction excess $\alpha_-(x')$, which follows from Eq. (3.11), we have

$$A(x') = e^{-\beta [E^n_{m}(x')]} \times \left[ \tanh \left( \frac{\beta \mu_-(x')}{2} \right) - \tanh \left( \frac{\beta \mu_-(x')}{2} \right) \right]. \tag{3.24}$$

For the ballistic spin-polarized density, we have, in analogy to Eq. (3.21),

$$n_-(x', x''; x) = n_-(x', x''; x) + n(x', x''; x) \tilde{\alpha}_-, \tag{3.25}$$

where

$$n_-(x', x''; x) = \frac{N_e}{2} C(x', x''; x) e^{-\beta E^n_{m}(x', x'')} \times \left[ A(x') e^{-\mathcal{E}(x', x'': x'; x)/l_{s_x}} + A(x'') e^{-\mathcal{E}(x', x'': x''; x)/l_{s_x}} \right] \tag{3.26}$$

is the off-equilibrium ballistic spin-polarized density [see Eqs. (3.11) and (3.21)].

For the thermoballistic spin-polarized current $J_\uparrow(x')$ passing through the point $x$, we find

$$J_\uparrow(x) = \hat{J}_\uparrow(x) + J_\uparrow(x) \tilde{\alpha}_\uparrow, \tag{3.27}$$

where the off-equilibrium thermoballistic spin-polarized current $\hat{J}_\uparrow(x)$ is obtained from the off-equilibrium ballistic spin-polarized current $\hat{J}_\uparrow(x)$ by summing up the weighted contributions of the ballistic intervals,

$$\hat{J}_\uparrow(x) = v_e N_e \int_{x_1}^{x_2} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} \mathcal{W}(x', x''; l) \times \left[ A(x') e^{-\mathcal{E}(x', x'': x'; x)/l_{s_x}} - A(x'') e^{-\mathcal{E}(x', x'': x''; x)/l_{s_x}} \right] \tag{3.28}$$

with

$$\mathcal{W}(x', x''; l) \times \left[ A(x') e^{-\mathcal{E}(x', x'': x'; x)/l_{s_x}} - A(x'') e^{-\mathcal{E}(x', x'': x''; x)/l_{s_x}} \right] \tag{3.29}$$

($x_1 < x < x_2$). Similarly, the thermoballistic spin-polarized density $n_-(x)$ at the point $x$ is

$$n_-(x) = \hat{n}_-(x) + n(x) \tilde{\alpha}_\uparrow, \tag{3.30}$$
where the off-equilibrium thermoballistic spin-polarized density \( \hat{n}_-(x) \) is obtained from Eq. 3.26 as

\[
\hat{n}_-(x) = \frac{N_e}{2} \int_{x_1}^{x_2} dx' \left[ A(x') e^{-\mathcal{C}(x',x';x',x)/l_s} + A(x'') e^{-\mathcal{C}(x',x'';x,x'')/l_s} \right]
\]

\[
\times \left[ A(x') e^{-\mathcal{C}(x',x';x',x)/l_s} + A(x'') e^{-\mathcal{C}(x',x'';x,x'')/l_s} \right] (3.30)
\]

\((x_1 < x < x_2)\).

In the diffusive regime, \( l/l_s \ll 1, l/S \ll 1 \), the integrals over \( x' \) and \( x'' \) in Eqs. (3.28) and (3.30) can be evaluated explicitly, yielding

\[
\hat{J}_-(x) = -2v_e N_e l e^{-\beta \left| E_c(x) - E_c^0 \right|} \frac{dA(x)}{dx}, (3.31)
\]

and

\[
\hat{n}_-(x) = N_e e^{-\beta \left| E_c(x) - E_c^0 \right|} A(x). (3.32)
\]

Eliminating the function \( A(x) \) from these two equations, we obtain the standard drift-diffusion relation (3.5).

In the current \( \hat{J}_-(x) \) and density \( \hat{n}_-(x) \), the spin relaxation in each ballistic interval is described in terms of the values of the spin transport function \( A(x) \) at the end points \( x = x' \) and \( x = x'' \). Since \( \hat{J}_-(x) \) and \( \hat{n}_-(x) \) are linearly connected with \( A(x) \), they are linearly connected with each other. Thus, it appears that the function \( A(x) \) [and not the chemical-potential splitting \( \mu_-(x) \)] is the key quantity for treating spin transport within the unified description. It remains to find an equation for the determination of this function.

C. Integral equation for the spin transport function

The required equation is provided by the basic balance equation, Eq. (3.1), which we now read in terms of the off-equilibrium thermoballistic spin-polarized current (3.28) and density (3.30) of the unified transport description. Since the derivative with respect to \( x \) of the terms in the brackets of expression (3.28) for \( \hat{J}_-(x) \) is compensated by the term \( \hat{n}_-(x)/\tau_s \) (this reflects the fact that spin relaxation in the ballistic intervals has already been taken into account), only the derivative on the limits of integration in expression (3.28) remains, and we have

\[
\int_x^{x_2} \frac{dx'}{l} \mathcal{W}(x, x'; l) \left[ A(x) - A(x') e^{-\mathcal{C}(x,x')/l_s} \right] - \int_x^{x_1} \frac{dx'}{l} \mathcal{W}(x', x; l) \left[ A(x') e^{-\mathcal{C}(x',x)/l_s} - A(x) \right] = 0,
\]

\[
(3.33)
\]

where

\[
\mathcal{C}(x', x'') = \mathcal{C}(x', x''; x', x''). (3.34)
\]

With the action of the symbolic operator \( \mathcal{W}(x', x''; l) \) explained by comparison of Eqs. (2.12) and (2.15), Eq. (3.33) reads explicitly

\[
\mathcal{W}_2(x_1, x; l, l_s) A_1 + \mathcal{W}_2(x, x_2; l, l_s) A_2
\]

\[- W(x_1, x_2; x; l) A(x)
\]

\[+ \int_{x_1}^{x_2} \frac{dx'}{l} \mathcal{W}_3(x', x; l, l_s) A(x') = 0,
\]

\[
(3.35)
\]

where

\[
\mathcal{W}_n(x', x''; l, l_s) = w_n(x', x''; l) e^{-\mathcal{C}(x', x'')/l_s}, (3.36)
\]

\[
W(x_1, x_2; x; l) = w_2(x_1, x; l) + w_2(x, x_2; l)
\]

\[+ \int_{x_1}^{x_2} \frac{dx'}{l} w_3(x', x; l), (3.37)
\]

and \( A_{1,2} = A(x_{1,2}) \). Equation (3.35) is a linear, Fredholm-type integral equation for the spin transport function \( A(x) \). Its solution for \( x_1 < x < x_2 \) determines the spin-polarized electron transport inside the semiconducting sample, and is obtained in terms of the values \( A_1 \) and \( A_2 \) at the interfaces at the ends of the sample. The latter are determined by the off-equilibrium spin fraction excesses \( \hat{\alpha}_-(x_{1,2}) \) and the chemical potentials \( \mu_{1,2} = \mu(x_{1,2}) \) in the contacts [see Eq. (3.22)]. The function \( A(x) \) is not, in general, continuous at the interfaces, \( A(x_1^+) \neq A_1, A(x_2^-) \neq A_2 \) ("Sharvin effect"), as will be demonstrated in Sec. III.D by way of a particular example. The discontinuities of \( A(x) \) arise from the joint effect of the discontinuities of the spin-independent thermoballistic chemical potential \( \mu(x) \) and those of the spin fraction excess \( \alpha_-(x) \) [or, equivalently, of the spin-resolved thermoballistic chemical potentials \( \mu_{1,2}(x) \)].

Substituting \( A(x) \) in Eqs. (3.28) and (3.30), we obtain the off-equilibrium thermoballistic spin-polarized current \( \hat{J}_-(x) \) and density \( \hat{n}_-(x) \), respectively; the thermoballistic spin-polarized current \( J_-(x) \) and density \( n_-(x) \) then follow from Eqs. (3.27) and (3.29), respectively. Dividing by the corresponding total thermoballistic current and density, Eqs. (2.14) and (2.11), respectively, we get the current spin polarization

\[
P_\uparrow(x) = \frac{J_-(x)}{J(x)} \frac{\hat{J}_-(x)}{\hat{J}(x)} + \hat{\alpha}_- (3.38)
\]

and the density spin polarization

\[
P_\uparrow(x) = \frac{n_-(x)}{n(x)} = \frac{\hat{n}_-(x)}{n(x)} + \hat{\alpha}_- (3.39)
\]

inside the sample. These polarizations are written in terms of the thermoballistic current and density; however, we take their magnitudes to be also those of the physical polarizations, for the following reason. The underlying assumption of our approach is that the equilibration process, i.e., the coupling between the thermoballistic and background currents, is independent of spin (this
is clearly true for the D’yakonov-Perel’ spin relaxation mechanism but remains to be examined for the other mechanisms. Therefore, the relative spin content is the same in these two currents, and thus equal to that of their sum, viz., the physical current. Hence, we may take the polarizations $P_f(x)$ and $P_n(x)$ of Eqs. (3.38) and (3.39), respectively, for the physical polarizations.

The integral equation (3.38) constitutes the central result of the present work. It allows the calculation of the spin polarization in semiconductors for any value of the momentum and spin relaxation lengths as well as for arbitrary band edge potential profile. The fact that we are led, in the unified description of spin-polarized transport, to an integral equation is connected with the introduction of the momentum relaxation length $l$ as an independent parameter of arbitrary magnitude, which gives rise to nonlocal ballistic effects. The basic parameters controlling the transport in the unified description are the equilibrium densities $\bar{n}_e$, $\bar{n}_h$, the momentum relaxation length $l$, and the spin relaxation length $l_s$, whereas in the standard drift-diffusion model one uses the conductivity $\sigma$ and the spin diffusion length $L_s$.

D. Differential equation for the spin transport function

In order to interpret our unified description of spin-polarized transport and relate it to previous, less general descriptions, we consider in the following the case of field-driven transport in a homogeneous semiconductor without space charge. As in Sec. II.C, we take the probabilities $p_n(x/l)$ in their one-dimensional form. Then, Eq. (3.38), can be converted into an integrodifferential equation for the spin transport function $A(x)$. In an approximation which is adequate for the present purposes, the latter equation reduces to a second-order differential equation.

1. General form and diffusive regime

For a potential of the form (2.64), the integral equation (3.38) reduces, with the help of Eqs. (2.65), (2.67), (3.17), and (3.36), to

$$f(x - x_1) A_1 + f_2(x_2 - x) A_2 - f(x - x_1) A(x) + \int_{x_1}^{x_2} \frac{dx'}{l} f_1(x - x') A(x') + \int_x^{x_2} \frac{dx'}{l} f_2(x' - x) A(x') = 0 ,$$  

(3.40)

where

$$f_1(x) = e^{-[\epsilon + 1/l + \epsilon(x)/l_s]x} ,$$  

(3.41)

$$f_2(x) = e^{-[1/l + \epsilon(x)/l_s]x} ,$$  

(3.42)

and

$$f(x) = \frac{1}{1 + \epsilon l} \left\{ 2 + \epsilon l \left[ 1 + e^{-(\epsilon + 1/l)x} \right] \right\} ,$$  

(3.43)

with $0 < \epsilon(\zeta) \leq 1$, $\epsilon(\zeta) \to 1$ for $\zeta \to 0$, and $\epsilon(\zeta) \sim 2(\pi\zeta)^{-1/2}$ for $\zeta \to \infty$.

By supplementing the inhomogeneous integral equation (3.40) with the equations obtained by forming its first and second derivative with respect to $x$, and eliminating from this set of equations the quantities $A_1$ and $A_2$, we can convert Eq. (3.40) into a homogeneous integrodifferential equation for the spin transport function $A(x)$. [This procedure could also be applied to Eq. (3.39), but does not seem to be helpful in the general case]. Now, the latter equation can be simplified by replacing the function $\epsilon(\zeta)$ with a position-independent average value $\overline{\epsilon}$, so that the coefficient functions $f_1(x)$ and $f_2(x)$ in Eq. (3.40) reduce to pure exponentials. With this approximation, the integrodifferential equation for $A(x)$ becomes a second-order differential equation of the form

$$b_0(x) \frac{d^2 A(x)}{dx^2} + b_1(x) \frac{dA(x)}{dx} + b_2(x) A(x) = 0 ,$$  

(3.45)

where

$$b_0(x) = 2 + \epsilon l \left[ 1 + b(x) \right] ,$$  

(3.46)

$$b_1(x) = \epsilon (2 + \epsilon l) \left[ 1 - b(x) \right] ,$$  

(3.47)

$$b_2(x) = \frac{1}{l \overline{\epsilon}^2} \left\{ 2 \left[ l^2 - \tilde{l}^2 + \epsilon l \left( \tilde{l} - \tilde{l} \right) \right] + \epsilon l \left( \tilde{l} - l \right) \left( l + \overline{l} + \epsilon l \overline{l} \right) \left[ 1 + b(x) \right] \right\} ,$$  

(3.48)

with

$$b(x) = e^{-\epsilon(x + 1/l)(x - x_1)} ,$$  

(3.49)

and

$$\frac{1}{l} = \frac{1}{l} + \frac{1}{l_s} , \quad \frac{1}{\tilde{l}} = \frac{1}{l} + \frac{1}{l_s} , \quad \tilde{l}_s = \frac{L_s}{\overline{\epsilon}} .$$  

(3.50)

Since, owing to the presence of the factor $e^{-x/l}$ in the functions $f_1(x)$ and $f_2(x)$, only the values of $\epsilon(\epsilon x)$ within the range $0 \leq x \leq l$ contribute appreciably, we choose $\overline{\epsilon}$ as the average of $\epsilon(\epsilon x)$ over an $x$-interval of length equal to the momentum relaxation length $l$,

$$\overline{\epsilon} = \frac{1}{l} \int_0^l dx \epsilon(\epsilon x) = \frac{1}{\epsilon l} \int_0^\epsilon d\zeta \ln(\epsilon l/\zeta) \epsilon(\zeta) .$$  

(3.51)

In the right-hand integral of this equation, the range of small $\epsilon x$, where $\epsilon(\zeta) \approx 1$, is emphasized because of the weight factor $\ln(\epsilon l/\zeta)$. For large $\epsilon l$ (in the ballistic regime...
and/or for strong fields), the variation of \( \epsilon(x) \) with \( x \) becomes essential, and a more detailed study of the validity of the approximation leading to Eq. (3.41) will be necessary. For the present purpose of solely demonstrating the principal effects of the transport mechanism, we consider this approximation, in conjunction with the choice (3.51) for \( \tilde{c} \), to be sufficiently accurate.

In the diffusive regime characterized by the conditions \( l/l_s \ll 1 \), \( l/S \ll 1 \), and \( \ell \ll 1 \), we have \( \tilde{c} = 1 \) and \( \ell = \bar{L} \). Thus the solution of Eq. (3.34) reduces to

\[
\frac{d^2 A(x)}{dx^2} + \epsilon A(x) = 0. \tag{3.52}
\]

In view of Eq. (3.32), Eq. (3.52) can be rewritten in terms of \( \hat{n}_- (x) \) and then agrees with Eq. (3.6), and with Eq. (2.8) of Yu and Flatté, if the intrinsic spin diffusion length \( L \) of that reference is identified with the spin diffusion length \( L_s = \sqrt{1/l_s} \). It thus turns out that Eq. (3.34) generalizes the usual spin drift-diffusion equation to the case of arbitrary values of the ratio \( l/l_s \).

2. Zero-bias limit

In the zero-bias limit \( \epsilon \to 0 \), the integral equation (3.40) reduces to

\[
e^{-(x-x_1)/l} A_1 + e^{-(x_2-x)/l} A_2 - 2A(x) + \int_{x_1}^{x_2} dx' e^{-(x-x')/l} A(x') = 0, \tag{3.53}
\]

from which one derives the differential equation

\[
\frac{d^2 A(x)}{dx^2} - \frac{1}{L_s^2} A(x) = 0; \tag{3.54}
\]

here,

\[
L = \sqrt{l/l_s} = \frac{L_s}{\sqrt{1 + l/l_s}}. \tag{3.55}
\]

is the generalization of the spin diffusion length (3.50), which includes ballistic effects via the renormalization factor \( 1/\sqrt{1 + l/l_s} \). The length \( L \) becomes equal to the spin diffusion length proper, \( L = L_s \), in the diffusive regime where \( l = \bar{L} \), and to the spin relaxation length, \( L = l_s \), in the ballistic limit \( l/l_s \to \infty \) where \( l = l_s \).

Equation (3.54) has the general solution, for \( x_1 < x < x_2 \),

\[
A(x) = C_1 e^{-(x-x_1)/L} + C_2 e^{-(x_2-x)/L}. \tag{3.56}
\]

With this expression substituted for \( A(x) \) in Eq. (3.53), the set of two equations resulting from writing down this equation for \( x = x_1 \) and \( x = x_2 \), respectively, can be solved for the coefficients \( C_{1,2} \),

\[
C_2 = -\frac{1}{D} \left[ (1 - \gamma) A_1 - (1 + \gamma) e^{S/L} A_2 \right], \tag{3.58}
\]

where

\[
D = (1 + \gamma)^2 e^{S/L} - (1 - \gamma)^2 e^{-S/L}, \tag{3.59}
\]

with

\[
\gamma = \frac{L}{l_s} = \frac{\bar{L}}{l} = \sqrt{\frac{l}{l + l_s}} \leq 1. \tag{3.60}
\]

It follows from Eqs. (3.55)–(3.60) that the function \( A(x) \) is discontinuous at \( x = x_{1,2} \),

\[
\Delta A_1 \equiv A(x_1^+) - A_1 = -\frac{1}{2} \left( g A_1 - h A_2 \right), \tag{3.61}
\]

\[
\Delta A_2 \equiv A_2 - A(x_2^-) = -\frac{1}{2} \left( h A_1 - g A_2 \right), \tag{3.62}
\]

where

\[
g = \frac{2\gamma}{D} \left[ (1 + \gamma) e^{S/L} + (1 - \gamma) e^{-S/L} \right] \leq 1 \tag{3.63}
\]

and

\[
h = \frac{4\gamma}{D} \leq \frac{2}{1 + \gamma} e^{-S/L}. \tag{3.64}
\]

In the diffusive regime, one has \( L = L_s = l_s \sqrt{l/l_s} \), and hence \( \gamma = 0 \), and therefore

\[
A(x) = A_1 e^{-(x-x_1)/L_s} + A_2 e^{-(x_2-x)/L_s}. \tag{3.65}
\]

In the ballistic limit, one has \( L = l_s \) and \( \gamma = 1 \), so that

\[
A(x) = \frac{1}{2} \left[ A_1 e^{-(x-x_1)/l_s} + A_2 e^{-(x_2-x)/l_s} \right]. \tag{3.66}
\]

The discontinuity of \( A(x) \), e.g., at \( x = x_1 \), is \( \Delta A_1 = A_1 e^{-S/L} \) in the diffusive regime, and \( \Delta A_1 = \frac{1}{2} \left[ -A_1 + A_2 e^{S/l_s} \right] \) in the ballistic limit.

IV. SPIN-POLARIZED TRANSPORT IN FERROMAGNET/SEMICONDUCTOR HETEROSTRUCTURES

We now turn to the unified description of spin-polarized electron transport in heterostructures formed of a semiconductor and two ferromagnetic contacts (cf. Fig. 11). We treat the ferromagnets as fully degenerate Fermi systems. The semiconductor is taken to be nonmagnetic [i.e., \( \bar{\alpha} = 0 \), and hence \( \bar{\alpha}_-(x') = \bar{\alpha}_-(x) \)] and homogeneous without space charge. We disregard spin-flip scattering at the interfaces, but spin-selective interface resistances are included in our description by introducing discontinuities into the spin-resolved chemical potentials, in the same way as in previous descriptions of magnetic tunneling.\[15,16,17,18\] Within
the drift-diffusion model. Of course, for realistic applications, it is necessary to treat the effect of interface barriers explicitly using potential profiles $E_s(x)$ which, generally, must be calculated self-consistently from a nonlinear Poisson equation. In some cases, however, it may be sufficient to perform non-self-consistent calculations using appropriately modeled band edge profiles.

In any case, this would require the spin transport function $A(x)$ to be determined by numerically solving the integral equation (3.35). This task will be deferred to future work.

In order to obtain the position dependence of the spin polarization across the heterostructure, the current spin polarization and the chemical potential in the semiconductor and the ferromagnets can be equated directly there. On the other hand, in the presence of ballistic contributions, the thermoballistic chemical potential $μ(x)$ and the spin transport function $A(x)$ are not continuous at the interfaces. The discontinuities at the interfaces are taken into account when the functions $μ(x)$ and $A(x)$ inside the semiconductor are calculated in terms of their values $μ_{1,2}$ and $A_{1,2}$, respectively. The latter values are to be equated with the values of the corresponding quantities in the ferromagnet.

We begin with a brief summary of the standard description (see, e.g., Ref. [19]) of the spin polarization in the ferromagnets.

### A. Current spin polarization in the ferromagnets

In the (semi-infinite) left ferromagnet located in the range $x < x_1$, the spin-up and spin-down chemical potentials $μ_{↑,↓}(x)$ are given by

$$μ_{↑,↓}(x) = \frac{e^2J}{σ_1} (x_1 - x) \pm \frac{C_1}{σ_{↑,↓}^{(1)}} e^{−(x_1−x)/L_σ^{(1)}},$$

where $L_σ^{(1)}$ is the spin diffusion length. The quantities $σ_{↑,↓}^{(1)}$ are the conductivities for spin up and spin down, which are independent of position, and $σ_1 = σ_{↑}^{(1)} + σ_{↓}^{(1)}$.

We then have $C_1 = σ_{↑}^{(1)} μ_{↑}(x_1) = −σ_{↓}^{(1)} μ_{↓}(x_1)$, and therefore $C_1 = (σ_{↑}^{(1)} σ_{↓}^{(1)}/σ_1) μ_−(x_1)$, where $μ_−(x) = μ_{↑}(x) − μ_{↓}(x)$.

$$J_{↑,↓}(x) = −\frac{σ_{↑,↓}^{(1)}}{e^2} \frac{dμ_{↑,↓}(x)}{dx},$$

we now find for the current spin polarization

$$P_J(x) = P_1 − \frac{G_1}{2e^2J} μ_−(x_1) e^{−(x_1−x)/L_σ^{(1)}},$$

where $P_1 = (σ_1^{(1)} σ_{↑}^{(1)})/σ_1$ is the relaxed (current or density) spin polarization in the left ferromagnet, and

$$G_1 = \frac{4σ_{↑}^{(1)} σ_{↓}^{(1)}}{σ_1 L_σ^{(1)}} = \frac{σ_1}{L_σ^{(1)}} (1 − P_1^2)$$

is a transport parameter of the ferromagnet, which has the dimension of interface conductance. Analogously, we obtain

$$P_J(x) = P_2 + \frac{G_2}{2e^2J} μ_−(x_2) e^{−(x−x_2)/L_σ^{(2)}},$$

for the current spin polarization in the right ferromagnet located in the range $x > x_2$.

In the absence of spin-selective interface resistances, the chemical-potential splitting $μ_−(x)$ is continuous at the interface, $μ_−(x_1) = μ_−(x_2)$ and $μ_−(x_2) = μ_−(x_2^±)$, where $μ_−(x_1,2)$ are its values at the interface itself. The latter are to be set equal to the corresponding values in the semiconductor, which yields

$$[μ_−(x_1,2)]_{\text{ferromagnet}} = [μ_−(x_1,2)]_{\text{semiconductor}} = \frac{1}{β} \ln \left(\frac{1 + α_{1,2}}{1 − α_{1,2}}\right),$$

where the right-hand part of this equation follows from Eq. (3.20) for $x' = x_1,2$, and $α_{1,2} = α_−(x_1,2)$. For the current spin polarizations at the interfaces, $P_J(x_1,2)$, we have from Eqs. (4.3) and (4.5)

$$P_J(x_1) = P_1 − \frac{G_1}{2βe^2J} \ln \left(\frac{1 + α_1}{1 − α_1}\right),$$

$$P_J(x_2) = P_2 + \frac{G_2}{2βe^2J} \ln \left(\frac{1 + α_2}{1 − α_2}\right),$$

which are to be set equal to the corresponding polarizations of the semiconductor.

Spin-selective interface resistances $ρ_{↑,↓}^{(1)}$ are introduced via discontinuities of the spin-resolved chemical potentials on the contact sides of the interfaces. At $x = x_1$, for example, the discontinuity has the form

$$μ_{↑,↓}(x_1) − μ_{↑,↓}(x_1) = e^2J_{↑}(x_1) ρ_{↑}^{(1)}.$$  

The corresponding interface resistance is located between $x = x_1$ and $x_1$ (in the ferromagnetic contact), and thus is adjacent to the Sharvin interface resistance between $x = x_1$ and $x_1^+$ (in the semiconductor). The quantity $μ_−(x_1)$ to be substituted in Eq. (4.3) is obtained, using Eqs. (1.3), (4.2), and (4.3), as

$$μ_−(x_1) = \frac{1}{1 + G_1 ρ_1^{(1)}/4} \left\{ μ_−(x_1) + \frac{e^2J}{2} \left[P_1 ρ_1^{(1)} + ρ_−^{(1)}\right] \right\},$$

(4.10)
where $\rho_{\pm}^{(1)} = \rho_{\pm}^{(1)} + \rho_{\parallel}^{(1)}$. The connection of $\mu_-(x_1)$ with the spin fraction excess $\alpha_1$ is, as before, given by Eq. (4.10). The same procedure applies mutatis mutandis to the interface at $x = x_2$.

**B. Spin polarization across a heterostructure in the zero-bias limit**

In the zero-bias limit $J \to 0$, we now demonstrate the procedure for calculating the current and density spin polarizations across a ferromagnet/semiconductor heterostructure.

Evaluating expressions (4.28) and (4.30), respectively, with $A(x)$ given by Eq. (3.50), we find for the thermoballistic spin-polarized current in the semiconductor

$$J_-(x) = -2v_n N_c l \frac{dA(x)}{dx}, \quad (4.11)$$

and for the thermoballistic spin-polarized density

$$n_-(x) = N_c A(x) \quad (4.12)$$

($x_1 < x < x_2$). For zero bias, one has $J(x) = J = \text{const}$, and $n(x) = \bar{n} = \text{const}$. [see Eqs. (2.71) and (2.72)], so that, by combining expressions (4.11) and (4.12), we obtain the relation

$$P_J(x) = -\frac{2v_n \bar{n}}{J} \frac{dP_n(x)}{dx}. \quad (4.13)$$

between the current and density spin polarizations. Furthermore, $J_-(x)$ and $n_-(x)$ both satisfy Eq. (3.54), and so do the polarizations $P_J(x)$ and $P_n(x)$ given by Eqs. (4.38) and (4.39), respectively. Differentiation of Eq. (4.13) then yields, together with Eq. (3.51) for $P_n(x)$,

$$P_n(x) = -\frac{l_s J}{2v_n \bar{n}} \frac{dP_J(x)}{dx}. \quad (4.14)$$

From Eqs. (3.38) with $J(x) = J$, we find, using Eqs. (3.50) and (4.11), the explicit form of the current spin polarization as

$$P_J(x) = \frac{2v_n N_c l}{EJ} \left[ C_1 e^{-(x-x_1)/L} - C_2 e^{-(x_2-x)/L} \right]. \quad (4.15)$$

The density spin polarization is obtained from Eqs. (3.50) and (4.12) [or, equivalently, from Eqs. (4.14) and (4.15)]

$$P_n(x) = \frac{N_c}{n} \left[ C_1 e^{-(x-x_1)/L} + C_2 e^{-(x_2-x)/L} \right]. \quad (4.16)$$

The coefficients $C_{1,2}$ in Eqs. (4.15) and (4.16) can be expressed via Eqs. (3.67) and (4.60), using Eq. (4.22), in terms of the spin fraction excesses $\alpha_{1,2}$ on the contact sides of the interfaces.

In order to determine the quantities $\alpha_{1,2}$, we consider the current spin polarization (4.15) on the semiconductor sides of the interfaces,

$$P_J(x^-) = \frac{G}{\beta e^2 J} (\alpha_1 - \alpha_2), \quad (4.17)$$

$$P_J(x^-) = \frac{G}{\beta e^2 J} (\alpha_1 - \alpha_2); \quad (4.18)$$

here, $G$ is the Sharvin interface conductance given by Eq. (4.21), and the coefficients $g$ and $h$ are given by Eqs. (3.63) and (3.64), respectively. As mentioned before, $P_J(x_1) = P_J(x^-_1)$ and $P_J(x_2) = P_J(x^+_2)$, and the connection with the polarization in the contacts is made by equating expressions (4.17) and (4.18), and expressions (4.18) and (4.18) [note that, if spin-selective interfacial resistances are included, expression (4.17) for $P_J(x_1)$ is to be replaced with the general expression obtained by using expression (4.10) for $\mu_-(\vec{x})$ in Eq. (3.3), and analogously for $P_J(x^+_2)$]. In the zero-bias limit, when $J(x_1,2) = J$ [or $\kappa = 1$ in Eq. (2.28)], we have $|\alpha_{1,2}| \ll 1$, and this procedure then results in the system of coupled linear equations

$$\left( g + \tilde{G}_1 \right) \alpha_1 - \alpha_2 = \frac{\beta e^2 J}{G} P_1, \quad (4.19)$$

$$g_1 - \left( g + \tilde{G}_2 \right) \alpha_2 = \frac{\beta e^2 J}{G} P_2, \quad (4.20)$$

where

$$\tilde{G}_{1,2} = \frac{G_{1,2}}{G}. \quad (4.21)$$

The solutions of Eqs. (4.19) and (4.20) are found to be

$$\alpha_1 = \frac{\beta e^2 J}{g \Delta} \left[ g + \tilde{G}_2 \right] - h P_1, \quad (4.22)$$

$$\alpha_2 = \frac{\beta e^2 J}{g \Delta} \left[ h P_1 - \left( g + \tilde{G}_1 \right) \right] P_2, \quad (4.23)$$

where

$$\Delta = g + \tilde{G}_1 - h^2. \quad (4.24)$$

Expressions (4.22) and (4.23) determine the spin fraction excesses $\alpha_1$ and $\alpha_2$ in terms of the current $J$, of the polarizations $P_1$ and $P_2$ in the left and right ferromagnets, respectively, and of material parameters, such as the conductivities $\sigma_{1,2}$ and the spin diffusion lengths $l_s^{(1,2)}$ of the ferromagnets (via $\tilde{G}_{1,2}$), and the momentum relaxation length $\ell$ and the spin relaxation length $l_s$ of the semiconductor as well as its length $L$ (via $g$ and $h$) and the equilibrium density $\bar{n}$ (via $G$). Since the quantities $\alpha_{1,2}$ are proportional to the current $J$, the current spin
polarization \( P_J(x) \) is independent of \( J \), while the density spin polarization \( P_n(x) \) is proportional to \( J \).

The current spin polarization along the entire heterostructure, \( P_J(x) \), is now obtained as follows. In the semiconductor, it is given by expression (1.15), with \( C_{1,2} \) calculated from \( \alpha_{1,2} \) as explained there. In the ferromagnets, the expressions for the current spin polarization are provided by Eqs. (4.16) and (4.18), respectively, where the quantities \( \mu_- (x_1^+) \) and \( \mu_- (x_2^-) \) are calculated from Eq. (4.14) and from its analogue for \( \mu_- (x_2^+) \), respectively. Analogously, the density spin polarization \( P_n(x) \) in the semiconductor is given by expression (1.13). We do not write down the density spin polarizations in the ferromagnets, but only mention that they do not, in general, match the polarizations \( P_n(x_1^+) \) and \( P_n(x_2^-) \) on the semiconductor sides of the interfaces.

In order to demonstrate the effect of the transport mechanism (characterized by the magnitude of the ratios \( l/l_s \) and \( L/S \)), we show in Fig. 5 the zero-bias current spin polarization \( P_J(x) \) for a symmetric ferromagnet/semiconductor/ferromagnet heterostructure with sample length \( S = 1 \) \( \mu \)m at \( T = 300 \) K as a function of \( x \) for various values of the momentum relaxation length \( l \). For the parameters of the ferromagnets, we adopt from Ref. 14 the values \( \sigma_1 = \sigma_2 = 10^5 \) \( \Omega^{-1} \) cm\(^{-1} \) for the bulk conductivities and \( L_s (1) = L_s (2) = 60 \) nm for the spin diffusion lengths; the bulk polarizations are chosen as \( P_1 = P_2 = 0.8 \). For the material parameters

![Figure 5](image.png)

**FIG. 5:** The zero-bias (\( \epsilon \to 0 \)) current spin polarization \( P_J(x) \) along a symmetric ferromagnet/semiconductor/ferromagnet heterostructure with \( S = 1 \) \( \mu \)m for the indicated values of the momentum relaxation length \( l \), calculated from Eqs. (4.16), (4.17), and (4.18) with \( x_1 = 0 \) and \( x_2 = S \). The solid curves correspond to zero interface resistance, the dashed curves to interface resistances of \( 10^{-7} \) \( \Omega \) cm\(^2 \) for spin-up electrons and \( 5 \times 10^{-7} \) \( \Omega \) cm\(^2 \) for spin-down electrons, respectively. For the remaining parameter values, see text.

of the semiconductor, we take the values \( m^* = 0.067 m_e \) for the effective electron mass, \( l_s = 2.5 \) \( \mu \)m for the ballistic spin relaxation length (corresponding to n-doped GaAs; see Refs. 44 and 45), and \( \bar{n} = 5.0 \times 10^{17} \) cm\(^{-3} \) for the equilibrium electron density. Clearly, in a specific semiconducting system, the value of the momentum relaxation length \( l \) is fixed. Therefore, when varying \( l \), we are considering the above parameter values to be representative for a whole class of semiconductors (regarded as nondegenerate; at room temperature, this should be an acceptable working hypothesis) that differ in the strength of impurity and phonon scattering and hence in the magnitude of \( l \).

The momentum relaxation length \( l \) affects the results shown in Fig. 4 in a twofold way. (i) It determines the conduction in the semiconductor. For small values of \( l \), the conductance of the latter is small, and thus the conductance mismatch with the ferromagnets is large, leading to a small injected current spin polarization \( P_J(0) \). (ii) It determines the generalized spin diffusivity length \( L = [l_s/(1 + l/l_s)]^{1/2} \), which acts as the polarization decay length, so that for small \( l \) the polarization dies out rapidly inside the semiconductor. The degree of polarization may be raised considerably all along the semiconductor when the value of \( l \) is increased up to a length of the order of the sample length, in which case the ballistic component becomes prevalent. Figure 5 also shows that, by introducing appropriately chosen spin-selective interface resistances, one may offset the suppression of the injected polarization due to the conductance mismatch for small \( l \); however, the rapid decay of the polarization inside the semiconductor cannot be prevented in this way.

For the case of Fig. 5 we show in Figs. 6 and 7, respectively, the zero-bias current spin polarization \( P_J(x) \) for various values of the equilibrium density \( \bar{n} \) and the spin relaxation length \( l_s \). It is seen that varying \( \bar{n} \) has about the same overall effect on \( P_J(x) \) as varying the momentum relaxation length \( l \), whereas varying \( l_s \) affects mainly the rate of decay of \( P_J(x) \).

C. Injected spin polarization for field-driven transport

We introduce the “injected spin polarization” as the spin polarization at one of the interfaces, e.g., at \( x = x_1 \), generated by the bulk polarization \( P_l \) of the left ferromagnet regardless of the influence of the right ferromagnet. More precisely, we define the injected current spin polarization as the current spin polarization \( P_J(x_1^+) \) given by Eq. (4.35) in the limit \( S/L \rightarrow \infty \). Similarly, the injected density spin polarization is defined as the polarization \( P_n(x_1^+) \) of Eq. (4.35) in the same limit. The injected spin polarization at \( x = x_1^+ \) provides the initial value of the left-generated polarization in the semiconductor, which propagates into the region \( x > x_1 \) while being degraded by the effect of spin relaxation.

We now consider the injected spin polarization for ele-
electron transport driven by an external electric field, i.e., a potential profile of the form (2.64).

1. General case

In order to obtain the spin transport function \( A(x) \), we have to solve Eq. (3.45) numerically under the condition \( A(x) \propto \exp(-x/\lambda) \) for \( x \to \infty \). The decay length \( \lambda \) is determined by solving Eq. (3.45) in the range \( x-x_1 \gg (\epsilon+1/l)^{-1} \) where the function \( b(x) \) in the coefficient functions \( b_0(x) \), \( b_1(x) \), and \( b_2(x) \) can be disregarded,

\[
\lambda = \left\{ \frac{\epsilon}{2} + \left[ \frac{\epsilon^2}{4} + \frac{1+\epsilon l}{l^2} - \frac{1+\epsilon l}{l^2} \right]^{1/2} \right\}^{-1} .
\] (4.25)

It can be shown that for any combination of parameter values, \( \lambda \) is a real number.

For calculating the injected current spin polarization from Eq. (3.35), we determine the thermoballistic spin-polarized current at the interface, \( J(x_1^+) \), from Eq. (3.28). Using Eq. (3.22) and fixing the normalization of the function \( A(x) \) in terms of \( A_1 \) with the help of Eq. (3.40), we find

\[
J_-(x_1^+) = v_e \bar{n} \Gamma J \alpha_1 ,
\] (4.26)

where

\[
\Gamma_J = \frac{A(x_1^+)-\bar{A}}{A(x_1^+)-\bar{A}/2} \quad (4.27)
\]

and

\[
\bar{A} = \int_{x_1}^{\infty} \frac{dx}{l} e^{-(x-x_1)/l} A(x) . \quad (4.28)
\]

To find the total thermoballistic current at the interface, \( J(x_1^+) \), we go back to Eq. (2.24). Expressing the current \( J \) in the form

\[
J = \frac{1}{\bar{x}} v_e \bar{n} , \quad (4.29)
\]

which follows, for \( \epsilon > 0 \) and \( S/L \to \infty \), from the current-voltage characteristic with \( \beta eV = \epsilon S \) and \( N_e \exp(-\beta E_p) = \bar{n} \), we obtain

\[
J(x_1^+) = \frac{\kappa}{\bar{x}} v_e \bar{n} . \quad (4.30)
\]

This expression is conveniently evaluated by using for \( \kappa \) and \( \bar{x} \) the closed-form representations

\[
\kappa = \frac{1+\epsilon l}{2 + \epsilon l} , \quad \bar{x} = \frac{(1+\epsilon l)^2}{\epsilon l(2 + \epsilon l)} , (4.31)
\]

which have been inferred from the results of systematic numerical calculations for fixed \( \epsilon l > 0 \) and very large values of \( S/L \). For the injected current spin polarization, we now find

\[
P_J(x_1^+) = \frac{J_-(x_1^+)}{J(x_1^+)} = \frac{\bar{x}}{\kappa} \Gamma J \alpha_1 , \quad (4.32)
\]
which, by continuity, is equal to $P_J(x_1)$. Setting the right-hand side of Eq. (4.32) equal to expression (4.17) or to the more general expression including spin-selective interface resistances; see the remark following Eqs. (4.17) and (4.18) for the injected spin polarization in terms of the contact parameters, we arrive at

$$P_1 - \frac{\tilde{G}_1}{2} \kappa \ln \left( \frac{1 + \alpha_1}{1 - \alpha_1} \right) = \tilde{\chi} \Gamma_J \alpha_1 .$$

(4.33)

This is a nonlinear equation for $\alpha_1$ which is to be solved for given values of the parameters $\epsilon$, $P_1$, $\tilde{G}_1$, $\tilde{n}$, $l$, and $l_s$.

Turning to the calculation of the injected density spin polarization, we determine the thermoballistic spin-polarized density at the interface, $n_-(x_1^+)$, from Eq. (3.30), using again Eqs. (3.22) and (3.40), and obtain

$$n_-(x_1^+) = \frac{n}{2} \Gamma_n \alpha_1 .$$

(4.34)

where

$$\Gamma_n = \frac{A(x_1^+)}{A(x_1^+) - A/2} .$$

(4.35)

For the total thermoballistic density at the interface, $n(x_1^+)$, we find from Eq. (2.53), using Eqs. (2.48), (2.52), and (4.19),

$$n(x_1^+) = \frac{n}{\chi} \left( 1 - \frac{\kappa J}{2\nu_c n} \right) = \tilde{n} \left( 1 - \frac{\kappa}{2\tilde{\chi}} \right) .$$

(4.36)

The injected density spin polarization now follows as

$$P_n(x_1^+) = \frac{n_-(x_1^+)}{n(x_1^+)} = \frac{\tilde{\chi}}{2\tilde{\chi} - \kappa} \Gamma_n \alpha_1 ,$$

(4.37)

where the spin fraction excess $\alpha_1$ is again to be determined by solving Eq. (4.32).

Figure 8 shows the injected current spin polarization $P_J(x_1)$ for $S/L \rightarrow \infty$ as a function of the electric-field parameter $\epsilon$ for various values of the momentum relaxation length $\ell$; the remaining parameter values are the same as in Fig. 6. In calculating $P_J(x_1)$ from Eq. (4.32), we have used $\kappa = 1$. This choice has been made because expression (4.32) with $\kappa$ given by Eq. (4.31) does not represent a meaningful injected polarization in the diffusive limit (see below); instead, one must set $\kappa = 1$ in this limit. For simplicity, we have used this value throughout. In conformity with the drift-diffusion results of Ref. 19, the injected polarization generally rises with increasing $\epsilon$; however, as in Fig. 5 the main effect is due to the variation of $l$.

2. Diffusive regime

In order to relate our treatment of the injected current spin polarization at ferromagnet/semiconductor interfaces to previous treatments within the drift-diffusion model, in particular to that of Yu and Flatté,\textsuperscript{18} we consider the diffusive regime, $l/l_s \ll 1$ and $\ell \ll 1$, in some detail. In that regime, the spin transport function $A(x)$ is determined by Eq. (4.32), whose solution is $A(x) \propto \exp(-x/L_s)$, with the field-dependent spin diffusion length $L_s$ given by

$$\frac{1}{L_s} = \frac{\epsilon}{2} + \left( \frac{\epsilon^2}{4} + \frac{1}{L_s^2} \right)^{1/2} .$$

(4.38)

We then obtain $\Gamma_J = 2l/L_s$ and $\Gamma_n = 2$. Furthermore, from Eqs. (4.31) for $\ell \ll 1$, we find $\kappa = 1/2$ and $\tilde{\chi} = 1/2l$.

At this point, some analysis is required regarding the definition of the injected current spin polarization in the diffusive regime. In the definition introduced above, first the functions $J_-(x)$ and $J(x)$ are evaluated for $x \rightarrow x_1^+$, and subsequently the diffusive limit is approached. This procedure results, in particular, in the value $\kappa = J(x_1^+)/J = 1/2$. A closer look at the function $J(x)/J$ (see Fig. 3), however, shows that in the diffusive regime this function is virtually equal to unity inside the semiconducting sample and tends to smaller values only within a (very short) distance of order $l$ from the interfaces. Therefore, it is indicated here to define the injected current spin polarization in terms of a position $x > l$ inside the sample, where $J(x)/J = 1$ is the relevant value for the propagation of the spin polarization into the semiconductor. Thus, in the diffusive regime, we adopt the effective value $\kappa = 1$ in the calculation of
the injected spin polarization. While in the ballistic limit (and now also in the diffusive regime) the choice \( \kappa = 1 \) is unique, in the range of intermediate \( l \)-values a meaningful definition of the injected spin polarization requires an appropriate choice of the position inside the sample at which the thermoballistic current and spin-polarized current are to be evaluated.

With the choice \( \kappa = 1 \), the injected current spin polarization in the diffusive regime is obtained from Eq. (3.54) as

\[
P_j(x_1) = \frac{1}{\epsilon L_s} \alpha_1 ,
\]

where the spin fraction excess \( \alpha_1 \) is now to be calculated from Eq. (4.33) with \( \kappa = 1 \). Since \( \kappa \ll \xi \) for \( cl \ll 1 \), the injected density spin polarization in the diffusive regime follows from Eq. (4.37) as

\[
P_s(x_1^+) = \alpha_1 ,
\]

where \( \alpha_1 \) again must be calculated from Eq. (4.33) with \( \kappa = 1 \).

Comparing our results for the injected spin polarization in the diffusive regime to the results of Yu and Flatte, based on standard drift-diffusion theory, we find that the field-dependent spin diffusion length \( L_s \) given by Eq. (4.33) agrees with the “up-stream” spin diffusion length \( L_u \) given by Eq. (2.23b) of Ref. 19, provided the intrinsic spin diffusion length \( L_s \) of that reference is identified with the spin diffusion length \( L_s = \sqrt{\Omega} \) of the present work. Then, by expressing the conductivity of the semiconductor in Eq. (3.5) of Ref. 19 (with the interface resistances set equal to zero) as \( \sigma_s = 2e^2v_0n_1l \), we recognize the equivalence of that equation with our Eq. (4.33) in the diffusive regime. This, in turn, implies that the injected current and density spin polarizations of either work are formally identical. Numerical calculations have confirmed this result.

3. Zero-bias limit

We now consider the injected current spin polarization in the zero-bias limit, in which \( |\alpha_1| \ll 1 \) and \( J(x) = J \), i.e., \( \kappa = 1 \). Here, the spin transport function \( A(x) \) is determined by Eq. (4.41), i.e., \( A(x) \propto \exp(-x/L) \), so that \( \Gamma_x = 2x/(1 + \gamma) \equiv \gamma_J \) and \( \Gamma_s = 2/(1 + \gamma) \equiv \gamma_n. \) From Eq. (4.33), we then have

\[
\alpha_1 = \frac{P_1}{\chi \gamma_J (1 + G_1/\gamma J \mathcal{G})}.
\]

Combining this with Eq. (4.32), we find for the injected current spin polarization

\[
P_j(x_1) = \frac{1}{1 + G_1/\gamma J \mathcal{G}} P_1 .
\]

It is instructive to consider expression (4.42) in the diffusive and ballistic regimes.

In the diffusive regime \( l/l_s \ll 1 \), we have \( \gamma_J = 2\sqrt{l/l_s} \) and, therefore,

\[
P_j(x_1) = \frac{1}{1 + G_1/\gamma J \mathcal{G}} P_1 .
\]

Here, \( \mathcal{G}_0 = 2G/\sqrt{l/l_s} = \sigma_0/L_s \), where Eq. (4.37) has been used, and \( \sigma_0 = 2G \) is the conductivity of the semiconductor [see the remarks following Eq. (4.4)]. The quantity \( \mathcal{G}_0 \) is seen to be the semiconductor analogue of the ferromagnet parameter \( G_1 \) defined by Eq. (4.4). Choosing \( P_1 = 0.8 \) and adopting the value \( \sigma_1 = 10^3 \Omega^{-1}\text{cm}^{-1}, L_s = 60 \text{ nm}, \sigma_0 = 10 \Omega^{-1}\text{cm}^{-1}, \) and \( L_s = 2 \mu\text{m} \), we have \( G_1/\mathcal{G}_0 = 1.2 	imes 10^3 \) and hence \( P_j(x_1) \approx 0.6 \times 10^{-3} \). The large value of the ratio \( G_1/\mathcal{G}_0 \) reflects the “conductance mismatch” which appears to be the determining parameter of the injected spin polarization in the diffusive regime.

On the other hand, in the ballistic limit \( l/l_s \to \infty \), we have \( \gamma_J = \gamma = 1 \), so that

\[
P_j(x_1) = \frac{1}{1 + G_1/\mathcal{G}} P_1 .
\]

Here, the Sharvin interface conductance \( \mathcal{G} \) takes the place of the quantity \( \mathcal{G}_x \) in Eq. (4.43). Assuming \( m^* = 0.067m_e \) and \( T = 300 \text{ K} \), we have \( \mathcal{G} = 3.2 \times 10^{11} \Omega^{-1}\text{m}^{-2} \) for \( \bar{n} = 5 \times 10^{17} \text{ cm}^{-3} \) and \( \mathcal{G} = 0.64 \times 10^{10} \Omega^{-1}\text{m}^{-2} \) for \( \bar{n} = 10^{16} \text{ cm}^{-3} \). This results in \( P_j(x_1) \approx 0.3 \) and \( 0.8 \times 10^{-2} \), respectively. The first example, where the large Sharvin interface conductance entails a large injected spin polarization, is fictitious since the high doping concentration needed to obtain an electron density of \( 5 \times 10^{17} \text{ cm}^{-3} \) (for example, in GaAs) would imply such small values of the momentum relaxation length \( l \) that ballistic transport is all but ruled out. Only semiconducting materials with unusually large mobilities would make this a realistic case. The second example with the lower electron density of \( 10^{16} \text{ cm}^{-3} \) would be more favorable to a ballistic transport mechanism, but leads to a very small injected spin polarization; this confirms the conclusion of Kravchenko and Rashba, stating that spin injection is suppressed even in the ballistic regime unless spin-selective interface resistances are introduced.

V. CONCLUDING REMARKS

We have developed a unified semiclassical theory of spin-polarized electron transport in heterostructures formed of a nondegenerate semiconductor and two ferromagnetic contacts. In this theory, the spin polarization inside the semiconductor is obtained for a general transport mechanism that covers the whole range between the purely diffusive and purely ballistic mechanisms and is controlled by the momentum relaxation length of the electrons.

The basis of the present work is provided by our previously developed unified model of (spinless) electron transport in semiconductors, in which diffusive and ballistic
transport are combined in the concept of the thermoballistic electron current. As a prerequisite to the extension of the spinless unified model to spin-polarized transport, we have modified and completed its formulation in such a way that an unambiguous description of electron transport in terms of a uniquely defined thermoballistic chemical potential is achieved. From the chemical potential, the unique thermoballistic current and density are obtained; numerical calculations show that, for typical parameter values, the thermoballistic current is close to the physical current.

In order to treat spin-polarized transport in semiconductors within the unified description, we have introduced a thermoballistic spin-polarized current and a thermoballistic spin-polarized density by allowing spin relaxation to take place during the ballistic electron motion. These are expressed in terms of a spin transport function which comprises in a compact form the information contained in the spin-resolved thermoballistic chemical potentials. Using the balance equation that connects the thermoballistic spin-polarized current and density, we have derived an integral equation for the spin transport function, from which the latter can be calculated in terms of its values at the interfaces of the semiconductor with the contacts. The spin transport function determines, in conjunction with the spin-independent thermoballistic chemical potential, all spin-dependent quantities in the semiconductor, in particular, the position dependence of the current and density spin polarization. The spin polarization all across a ferromagnet/semiconductor heterostructure is determined by making use of the continuity of the current spin polarization at the contact-semiconductor interfaces and connecting the spin-resolved chemical potentials there. Thereby, a unified description of spin-polarized transport emerges that provides a basis for the systematic study of the interplay of spin relaxation and transport mechanism in heterostructures relevant to spintronic applications.

To interpret the formalism developed here and to relate it to previous, less general formulations, we have considered spin-polarized electron transport in a homogeneous semiconductor without space charge, driven by an external electric field. Within a judicious approximation, the integral equation for the spin transport function can then be reduced to a second-order differential equation which generalizes the standard spin drift–diffusion equation to the case of arbitrary values of the ratio of momentum to spin relaxation length. In the zero-bias limit, the position dependence of the spin polarizations across a heterostructure is obtained in closed form.

The generalized spin drift–diffusion equation has been used in calculations of the current spin polarization across a symmetric ferromagnet/semiconductor/ferromagnet heterostructure with material parameters in the range of interest for spintronic devices. The dependence on the transport mechanism in the semiconductor has been exhibited by varying the momentum relaxation length over several orders of magnitude. It was found that the ballistic regime favors sizeable (large) spin polarizations. The same picture emerges from calculations of the injected current spin polarization as a function of an applied electric field. While the field also serves to raise the polarization in the semiconductor, the main effect still is due to the variation of the momentum relaxation length, i.e., to the influence of the ballistic component of the transport mechanism. In order to exploit the potentiality of varying the transport mechanism with the aim to improve the efficiency of spintronic devices, the identification and design of novel semiconducting materials is called for.

In the present work, emphasis has been placed on a careful elaboration of the formalism underlying the unified description of spin-polarized electron transport in ferromagnet/semiconductor heterostructures. In the illustrative calculations, we have restricted ourselves to the simplest cases. In future work, applications of the present formalism will have to be based on the solution of the fundamental integral equation for the spin transport function in its general form. These should include the treatment of magnetic semiconducting samples (characterized, in the unified description, by nonzero values of the relaxed spin fraction excess) and of interface barriers, like Schottky or tunnel barriers (represented by appropriately chosen potential profiles). As to possible extensions of the theory, setting up a formalism for the treatment of degenerate semiconductors appears to have first priority.

**APPENDIX: UNIQUE THERMOBALLISTIC FUNCTIONS**

In this Appendix, we present details of the construction of a unique thermoballistic chemical potential $\mu(x)$, current $J(x)$, and density $n(x)$ in terms of the solutions $\chi_1(x)$ and $\chi_2(x)$ of Eqs. (2.28) and (2.31), respectively.

Evaluating expression (2.12) with the function $J_1(x)$ following from the solution of Eq. (2.28) [case (i)], with $J_2$ set equal to $J_1(x_2)$, we obtain the thermoballistic current

$$J_1(x) = w_1(x_1, x_2; l) [J_1 - J_1(x_2)]$$

$$+ \int_{x_1}^{x_2} \frac{dx'}{l} w_2(x', x_2; l) [J_1(x') - J_1(x_2)]$$

$$+ \int_x^{x_2} \frac{dx'}{l} w_2(x_1, x'; l) [J_1(x') - J_1(x)]$$

$$+ \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} w_3(x', x''; l) [J_1(x') - J_1(x'')]$$

(A.1)

and, similarly, for case (ii), the current

$$J_2(x) = w_1(x_1, x_2; l) [J_2(x_1) - J_2]$$

$$+ \int_{x_1}^{x} \frac{dx'}{l} w_2(x', x_2; l) [J_2(x') - J_2]$$
which are not, in general, equal. This ambiguity is removed by introducing a unique thermoballistic current \( J^{(u)}(x) \) as a superposition of the currents \( J_1(x) \) and \( J_2(x) \),

\[
J^{(u)}(x) = \hat{a}_1 \frac{J}{J_1} J_1(x) + \hat{a}_2 \frac{J}{J_2} J_2(x) ,
\]

where

\[
J_{1,2} = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} dx \, J_{1,2}(x) .
\]

The current \( J^{(u)}(x) \) has to satisfy Eq. (2.24),

\[
\frac{1}{x_2 - x_1} \int_{x_1}^{x_2} dx \, J^{(u)}(x) = J .
\]

Similarly, Eq. (2.24) is to be replaced with

\[
J^{(u)}(x^+_1) = J^{(u)}(x^-_2) \equiv \kappa J .
\]

The conditions (A.3) and (A.4) determine the coefficients \( \hat{a}_1 \) and \( \hat{a}_2 \). We find from Eq. (A.3), using Eqs. (A.3) and (A.4),

\[
\hat{a}_1 + \hat{a}_2 = 1 .
\]

In order to apply condition (A.6), we evaluate the currents (A.1) and (A.2) at the ends of the sample, using Eqs. (2.27) and (2.30),

\[
\frac{J_1(x^+_2)}{J_1} = w_1(x_1, x_2; l) \chi_1(x_2) + \int_{x_1}^{x_2} \frac{dx'}{l} w_2(x_1, x'; l) \chi_1(x') ,
\]

\[
\frac{J_2(x^+_2)}{J_2} = w_1(x_1, x_2; l) \chi_2(x_1) + \int_{x_1}^{x_2} \frac{dx'}{l} w_2(x_1, x'; l) [\chi_2(x_1) - \chi_2(x')] ,
\]

\[
\frac{J_1(x^-_2)}{J_1} = w_1(x_1, x_2; l) \chi_1(x_2) + \int_{x_1}^{x_2} \frac{dx'}{l} w_2(x', x_2; l) [\chi_1(x_2) - \chi_1(x')] ,
\]

\[
\frac{J_2(x^-_2)}{J_2} = w_1(x_1, x_2; l) \chi_2(x_1) + \int_{x_1}^{x_2} \frac{dx'}{l} w_2(x', x_2; l) \chi_2(x') .
\]

Employing Eqs. (A.3), (A.6), and (A.8)–(A.11), we obtain the coefficients \( \hat{a}_1, \hat{a}_2 \) as given by Eqs. (2.30)–(2.33).

Now, introducing, in analogy to Eq. (A.3),

\[
J^{(u)}(x) = \hat{a}_1 \frac{J}{J_1} J_1(x) + \hat{a}_2 \frac{J}{J_2} J_2(x) \quad \text{for } x_1 < x < x_2,
\]

and, in addition,

\[
J^{(u)}(x_1) = \hat{a}_1 \frac{J}{J_1} J_1 + \hat{a}_2 \frac{J}{J_2} J_2(x_1) ,
\]

\[
J^{(u)}(x_2) = \hat{a}_1 \frac{J}{J_1} J_1(x_2) + \hat{a}_2 \frac{J}{J_2} J_2 ,
\]

we may write the unique thermoballistic current \( J^{(u)} \), using Eqs. (A.1), (A.2) and (A.12), (A.13), in a symbolic form analogous to expression (2.34),

\[
J^{(u)}(x) = \int_{x_1}^{x_2} \frac{dx'}{l} \int_{x_1}^{x_2} \frac{dx''}{l} \chi(x', x''; l) \chi(x', x''; l) J^{(u)}(x') \chi(x''; l) .
\]

here, the values \( J^{(u)}(x_{1,2}) \) are to be identified with their physical values \( J_{1,2} \) in the contacts,

\[
J^{(u)}(x_{1,2}) = J_{1,2} .
\]

In line with the definition (2.30) of the current \( J(x) \) in terms of the chemical potential \( \mu(x) \), we now define a unique chemical potential \( \mu^{(u)}(x) \) via relation (A.12) by

\[
e^{\beta \mu^{(u)}(x)} = \frac{1}{V_e N_e} e^{\beta E_e} J^{(u)}(x) .
\]

for \( x_1 \leq x \leq x_2 \), where now

\[
\mu^{(u)}(x_{1,2}) = \mu_{1,2} .
\]

The chemical potential \( \mu^{(u)}(x) \) is the key quantity in the extended unified description of electron transport inside the sample. In terms of \( \mu^{(u)}(x) \), the unique ballistic current across the interval \( [x', x''] \) appearing in the expression for the thermoballistic current \( A.15 \) is given by Eq. (2.1).

For the explicit calculation of the chemical potential \( \mu^{(u)}(x) \), we use Eq. (A.16) in Eqs. (A.13) and (A.14). We then find, with the help of Eqs. (2.27) and (2.30),

\[
\left( 1 - \hat{a}_1 \frac{J}{J_1} \right) J_1 - \hat{a}_2 \frac{J}{J_2} J_2 = \hat{a}_2 J \chi_2(x_1) ,
\]

\[
- \hat{a}_1 \frac{J}{J_1} J_1 + \left( 1 - \hat{a}_2 \frac{J}{J_2} \right) J_2 = - \hat{a}_1 J \chi_1(x_2) .
\]
By subtracting Eq. (A.20) from Eq. (A.19), we obtain Eq. (2.31). On the other hand, adding Eqs. (A.19) and (A.20) results in
\[
\hat{a}_1 \frac{J}{J_1} J_1 + \hat{a}_2 \frac{J}{J_2} J_2 = \frac{1}{2} (J_1 + J_2)
\]
\[
+ \frac{J}{2} \left[ \hat{a}_1 \chi_1(x_2) - \hat{a}_2 \chi_2(x_1) \right].
\]
(A.21)

Then, expressing the current \( J^{(u)}(x) \) in terms of the quantities \( J_{1,2} \) and the functions \( \chi_{1,2}(x) \) by combining Eq. (A.12) with Eqs. (2.27) and (2.30),
\[
J^{(u)}(x) = \hat{a}_1 \frac{J}{J_1} [J_1 - J_1 \chi_1(x)]
\]
\[
+ \hat{a}_2 \frac{J}{J_2} [J_2 + J_2 \chi_2(x)]
\]
\[
= \hat{a}_1 \frac{J}{J_1} J_1 + \hat{a}_2 \frac{J}{J_2} J_2
\]
\[
- J \left[ \hat{a}_1 \chi_1(x) - \hat{a}_2 \chi_2(x) \right],
\]
(A.22)

we obtain Eq. (2.39) with \( \chi_{-}(x) \) given by Eq. (2.40). Using Eqs. (2.39) and (2.41) to eliminate the total current \( J \) as well as Eqs. (A.17) to go over to the unique chemical potential \( \mu^{(u)}(x) \), we find
\[
e^{\beta \mu^{(u)}(x)} = \left[ \frac{1}{2} - \frac{\chi_{-}(x)}{\chi} \right] e^{\beta \mu_1} + \left[ \frac{1}{2} + \frac{\chi_{-}(x)}{\chi} \right] e^{\beta \mu_2},
\]
(A.23)

where \( \chi \) is defined by Eq. (2.24). The corresponding thermoballistic current \( J^{(u)}(x) \) and density \( n^{(u)}(x) \) are obtained by substituting expression (A.22) in Eq. (2.16) and (2.17), respectively.

In the main body of the paper, we always deal with the unique chemical potential, current, and density, and omit the superscript \( u \); we have already adhered to this convention when referring from the Appendix to the equations of Sec. II.B.

* Electronic address: wille@hmi.de

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