Quintessence and the cosmological constant
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Quintessence – the energy density of a slowly evolving scalar field – may constitute a dynamical form of the homogeneous dark energy in the universe. We review the basic idea in the light of the cosmological constant problem. Cosmological observations or a time variation of fundamental ‘constants’ can distinguish quintessence from a cosmological constant.

The idea of quintessence originates from an attempt to understand the smallness of the “cosmological constant” or dark energy in terms of the large age of the universe \[1\]. As a characteristic consequence, the amount of dark energy may be of the same order of magnitude as radiation or dark matter during a long period of the cosmological history, including the present epoch. Today, the inhomogeneous energy density in the universe – dark and baryonic matter – is about \[\rho\text{_{inhom}} \approx (10^{-3} \text{eV})^4\]. This number is tiny in units of the natural scale given by the Planck mass \[M_p = 1.22 \cdot 10^{19} \text{GeV}\]. Nevertheless, it can be understood easily as a direct consequence of the long duration of the cosmological expansion: a dominant radiation or matter energy density decreases \[\rho \sim M_p^2 t^{-2}\] and the present age of the universe is huge, \[t_0 \approx 1.5 \cdot 10^{10} \text{yr}\]. It is a natural idea that the homogeneous part of the energy density in the universe – the dark energy – also decays with time and therefore turns out to be small today\[1\].

A simple realization of this idea, motivated by the anomaly of the dilatation symmetry, considers a scalar field \[\phi\] with an exponential potential \[1\]

\[\mathcal{L} = \sqrt{g} \left\{ \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + V(\phi) \right\}\]

where

\[V(\phi) = M^4 \exp(-\alpha \phi/M),\]

with \[M^2 = M_p^2/16\pi\]. In the simplest version \[\phi\] couples only to gravity, not to baryons or leptons. Cosmology is then determined by the coupled field equations for gravity and the scalar “cosmon” field in presence of the energy density \[\rho\] of radiation or matter. For a homogeneous and flat universe they read (\[n = 4\] for radiation and \[n = 3\] for nonrelativistic matter)

\[H^2 = \frac{1}{6M^2} \left( \rho + \frac{1}{2} \dot{\phi}^2 + V \right),\]

\[\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0,\]

\[\dot{\rho} + nH \rho = 0.\]

One finds that independently of the precise initial conditions the behavior for large \[t\] approaches an exact “cosmological attractor solution” (or “tracker solution”) where the scalar kinetic and potential energy density scale proportional to matter or radiation\[1]\]

\[\phi = \frac{2M}{\alpha} \ln(t/t_0), \quad \frac{1}{2} \dot{\phi}^2 = \frac{2M^2}{\alpha^2} t^{-2},\]

\[V = \frac{2M^2}{\alpha^2} \frac{(6-n)}{n} t^{-2},\]

with the usual decrease of the Hubble parameter \[H\]

\[H = \frac{2}{n} t^{-1}, \quad \rho \sim t^{-2}.\]

This simple model predicts a fraction of dark energy or homogenous quintessence (as compared to the critical energy density \[\rho_c = 6M^2 H^2\]) which is constant in time

\[\Omega_h = \left( V + \frac{1}{2} \dot{\phi}^2 \right) / \rho_c = \rho_\phi / \rho_c = \frac{n}{2\alpha^2}.\]

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both for the radiation-dominated \((n = 4)\) and matter-dominated \((n = 3)\) universe \((\Omega_h + \rho/\rho_c) = 1\). This would lead to a natural explanation why today’s dark energy is of the same order of magnitude as dark matter.

The qualitative ingredients for the existence of the stable attractor solution \(\frac{2}{M} \equiv \alpha/\rho_c \approx 1\) are easily understood: for a large value of \(V(\phi)\) the force term in eq. \(3\), \(\partial V/\partial \phi = -(\alpha/M) V\), is large, and the dark energy decreases faster than matter or radiation. In the opposite, when the matter or radiation energy density is much larger than \(V\), the force is small as compared to the damping term \(3H \dot{\phi}\) and the scalar “sits and waits” until the radiation or matter density is small enough such that the over-damped regime ends. Stability between the two extreme situations is reached for \(V \sim \rho\).

The scenario with constant \(\Omega_h\) is, however, too simple to be consistent with observational data. In Fig. 1, we have collected our present knowledge about the time evolution of \(\Omega_h\). From present observations one concludes that today’s fraction of dark energy is rather large

\[
\Omega_h^0 = 0.6 - 0.7.
\]

On the other hand, structure formation would be hindered by a too large amount of dark energy \(\Omega_h\). One infers an approximate upper bound for the average fraction of dark energy during structure formation \(\Omega_h^\text{eff}\)

\[
\Omega_h^\text{eff} \lesssim 0.1 - 0.2.
\]

As a consequence, the fraction of dark energy must have increased in the recent epoch since the formation of structure. This implies a negative equation of state for quintessence \(\Omega_h \approx 7\). \(\Omega_h^\text{eff} \approx 0.5\) and can lead to a universe whose expansion is presently accelerating, as suggested by the redshifts of distant supernovae \(\Omega_h \approx 7\). For a phenomenological analysis of the observations that lead to the constraints depicted in Fig. 1, we refer to \(\Omega_h \approx 7\) and the other talks at this meeting.

The pure exponential potential in eq. \(4\) cannot account for the recent increase in \(\Omega_h\). Possible modifications of the basic idea of quintessence include the use of other potentials \(\Omega_h \approx 7\), the coupling of quintessence to dark matter \(\Omega_h \approx 7\), nonstandard scalar kinetic terms \(\Omega_h \approx 7\) or the role of nonlinear fluctuations \(\Omega_h \approx 7\). We note that these ideas may not be unrelated, since the presence of large fluctuations can modify the effective field equations \(\Omega_h \approx 7\) e.g. change the effective cosmion potential and kinetic term) and lead to a coupling between quintessence and dark matter \(\Omega_h \approx 7\).

In view of the still very incomplete theoretical understanding of the origin of quintessence the choice of an appropriate effective action for the cosmon is mainly restricted by observation. For comparison with observation and a discussion of naturalness of various approaches \(\Omega_h \approx 7\) we find it convenient to work with a rescaled cosmon field such that the scalar field Lagrangian reads

\[
\mathcal{L}(\phi) = \frac{1}{2} (\partial \phi)^2 k^2 (\varphi) + \exp[-\varphi].
\]

Here and in what follows all quantities are measured in units of the reduced Planck mass \(\tilde{M}_p\), i.e., we set \(\tilde{M}_p^2 \equiv M_p^2/(8\pi) \equiv (8\pi G_N)^{-1} = 2M^2 = 1\). The Lagrangian of Eq. \(10\) contains a simple exponential potential \(V = \exp[-\varphi]\) and a non-standard kinetic term with \(k(\varphi) > 0\). The exponential potential in eq. \(1\) corresponds to a constant \(k = 1/\sqrt{2\alpha}\).

This parameterization has the advantage of a one to one correspondence between the value of \(\varphi\) and the contribution to dark energy from the cosmon potential \(V\). At present, we have \(\varphi \approx 276\) and the recent history of quintessence is directly connected to the behavior of \(k(\varphi)\) for \(\varphi\) smaller but in the vicinity of this value. For many (‘tracker’) scenarios of quintessence, there is a direct relation between the functions \(k(\varphi)\) and \(\Omega_h(\varphi)\), the latter being the one which is most easily accessible to observation and therefore the most appropriate one for phenomenological discussions and comparison between different observations.

There are roughly two classes of quintessence models - one where the dark energy plays a role in the early cosmology, i.e. at last scattering and before (early quintessence) and the other where it is negligible except for a relatively recent epoch.
Figure 1. Fractional dark energy contribution as a function of redshift $z$. The shaded regions indicate bounds coming from Big Bang Nucleosynthesis (BBN), the cosmic microwave background (CMB), structure formation and the Supernovae Type Ia (SNe Ia) observations. The dashed curve shows a cosmological constant with $\Omega_\Lambda^0 = 0.7$, the straight line a Leaping Kinetic Term quintessence model with the same amount of dark energy today.

(recent quintessence). Recent quintessence corresponds to a function $k(\varphi)$ that becomes tiny for $\varphi \ll 276$. An example for early quintessence is given by leaping kinetic term (LKT) quintessence [21], where the effective value of $k$ jumps from a small to a larger value - possibly as a back-reaction to structure formation or some other ‘recent’ cosmological event.

$$k(\varphi) = k_{\min} + \tanh(\varphi - \varphi_1) + 1$$
(with $k_{\min} = 0.1$, $\varphi_1 = 276.6$). \hspace{1cm} (11)

The cosmological evolution of this model is depicted in Fig. 3. The LKT model completely avoids the explicit use of very large or very small parameters and realizes all the desired features of quintessence [21]. The homogeneous dark energy density tracks below the radiation or matter component in the early universe ($k = 0.1$, corresponding to $\Omega_h = 0.04$) and then suddenly comes to dominate the evolution when $k$ rises to a value $k \gtrsim 1$. With a tuning on the percent level (the value of $\varphi_1$ has to be appropriately adjusted) realistic present-day values of $\Omega_h^0$ and $\omega_h^0$ can be realized. In the above example, one finds $\Omega_h^0 = 0.70$.
and $w^0 = -0.80$. Note that, due to the extended tracking period, the late cosmology is completely insensitive to the initial conditions. For models of the type discussed in [16], $k(\varphi)$ diverges near $\varphi_1$, leading to a similar cosmology.

In this talk, we turn back to the original question, if quintessence can explain why the dark energy vanishes asymptotically for large time, or, equivalently, if quintessence is related to a solution of the cosmological constant problem. In terms of the cosmon potential this concerns the question if the property $V(\chi \to \infty) \to 0$ is natural.

We address this issue in the context of some (unknown) unified theory with a dynamical unification scale $M_{\text{GUT}}$. Similar to grand unified theories, higher dimensional theories and superstring theories, the unification scale can be associated to the expectation value of a scalar field $\chi$. This field will play the role of the cosmon. We restrict the discussion to the (four dimensional) system of gravity and the cosmon with (euclidian action)

$$S = \int d^4x \sqrt{g} \left[ -\frac{1}{12} f^2(\chi) \chi^2 \mathcal{R} + \frac{1}{2} Z_\chi(\chi) \partial_\mu \chi \partial_\nu \chi g^{\mu\nu} + V(\chi) \right]. \quad (12)$$

The function $f(x)$ is slowly varying such that the dynamical Planck mass is essentially $\propto \chi$. The fate of the cosmological constant depends on the behavior of the potential $V(\chi)$ for $\chi \to \infty$. Here, we only make the assumption that the combination $f^4 \chi^4 / V$ diverges for $\chi \to \infty$ and is a monotonic function of $\chi$. Solving the field equations for this system [22], one finds a stable solution where $\chi$ grows to infinity for large time.

The interpretation of cosmology may be somewhat simpler, if we rescale the metric such that the Planck mass becomes a constant, $\bar{g}_{\mu\nu} = (6 \bar{M}_P^2 / f^2) g_{\mu\nu}$. The action (12) becomes then precisely the one underlying our previous discussion ($\bar{M}_P = 1$)

$$S = \int d^4x \sqrt{\bar{g}} \left[ -\frac{1}{2} \mathcal{R} + \frac{1}{2} k^2(\varphi) \partial_\mu \varphi \partial^\mu \varphi + \exp(-\varphi) \right]. \quad (13)$$

Here, $\varphi$ is related to $\chi$ by

$$\varphi = \ln \frac{f^4(\chi) \chi^4}{36V(\chi)} \quad (14)$$

and

$$k^2(\varphi) = 3 \frac{Z_\chi}{f^2} \left[ 1 + \left( \frac{\partial \ln f}{\partial \ln \chi} \right)^2 \right] \times \left( 1 + \frac{\partial \ln f}{\partial \ln \chi} - \frac{1}{4} \frac{\partial \ln V}{\partial \ln \chi} \right)^{-2}. \quad (15)$$
is a finite non-zero positive function. In particular, no additive constant is present in the cosmon potential.

We conclude that the cosmological constant vanishes independently of the details of \( V(\chi) \) - for example we may add a constant in Equation 12 - provided the ‘cosmon condition’ holds\(^3\)

\[
\lim_{\chi \to \infty} \left( \frac{f^4 \chi^4}{V} \right) \to \infty
\] (16)

holds\(^3\). The theoretical status of the cosmological constant cannot be settled unless we know the properties of the unified theory. At least, the asymptotic vanishing of the cosmological constant is associated to some generic behavior \(^4\) and needs no tuning of parameters to many decimal places. The status of naturalness of the cosmion condition is linked to the role of quantum fluctuations. Here, we point out only two observations:

(i) The quantum effects on the potential \( V(\chi) \) are dominated by fluctuations with momenta around the unification scale \( \chi \). Fluctuations of the fields of the standard model, the cosmon or graviton with momenta much smaller than \( \chi \) play a negligible role. The fluctuations of the modes relevant around the unification scale may be described by a higher dimensional theory or string theory such that simple extrapolations from four dimensional quantum fluctuations may be quite inappropriate. Arguments that arbitrarily select parts of the fluctuation effects (like quantum fluctuations of the standard model fields with momenta below a fixed scale) and associate their size with the ‘natural size’ of the total fluctuation effect are known to be grossly misleading in other circumstances, like critical behavior in statistical physics. For example, the ‘quartic coupling’ \( \lambda \propto V/\chi^4 \) may have a ‘renormalization group flow’ to zero for \( \chi \to \infty \) and therefore become very small at large \( \chi \), whereas arbitrarily selected ‘individual contributions’ of quantum fluctuations are much larger.

(ii) The naive guess that the quantum corrections to \( V \) should be \( \propto \chi^4 \) by dimensional reasons tacitly assumes that dilatation symmetry is a quantum symmetry. This is, however, not very likely and one expects a dilatation anomaly. If this manifests itself in the form of a non-vanishing anomalous dimension \( A \) for \( \lambda \), one rather has \( V \propto \chi^{4-A} \).

As an important consequence of the dilatation anomaly, also the gauge couplings and the mass ratio between the nucleon and the Planck mass typically become dependent on \( \chi \). An example for varying gauge couplings is provided by a term (with \( F_{\mu\nu} \) the field strength of the gauge fields)

\[
\mathcal{L}_F = \frac{1}{4} Z_F(\chi) F_{\mu\nu} F^{\mu\nu},
\] (17)

with anomalous dimension

\[
\eta_F = \frac{\partial \ln Z_F}{\partial \ln \chi}.
\] (18)

Due to the cosmological variation of \( \chi \) this leads to a time dependence of couplings \(^5\) and a composition dependent gravity-like long range force \(^5\). The time variation of fundamental couplings is a generic prediction of quintessence, albeit the size of the effect is not known. The recently reported cosmological time variation of the fine structure constant \(^5\) corresponds to \( \eta_F = -4 \times 10^{-6}A \). Confirmation of this result would be a very clear signal for quintessence - in distinction to the cosmological constant for which no such variation is expected.

Acknowledgment: The authors would like to thank A. Hebecker, J. Jackel, M. Lilley, and M. Schwindt for collaboration on the content of this work. Part of it is based on refs. \(^2\), \(^{10}\).

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