The $\Delta B = -\Delta Q$ transitions and $B_d \leftrightarrow \bar{B}_d$ oscillations

G.V. Dass$^a$ and K.V.L. Sarma$^b,*$

$^a$Department of Physics, Indian Institute of Technology, Powai, Mumbai, 400 076, India

$^b$Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai, 400 005, India

Abstract

We estimate the product of the relative strength of the $\Delta B = -\Delta Q$ amplitude in the decay $B^0_d \rightarrow D^* \ell \bar{\nu}_\ell$ and the width-difference parameter $y$. For this we have used the data on time-dependence of $B^0_d \bar{B}^0_d$ oscillations in $Z$ decays and the fraction of like-sign dilepton events at the $\Upsilon(4S)$.

PACS: 13.20.He, 11.30.Er.

Keywords: Semileptonic decay rule; Neutral bottom mesons; Mixing; Oscillations.

* E-mail: kvls@theory.tifr.res.in; fax: 091 22 215 2110
There is now a considerable body of experimental evidence in favour of $B_d^0\bar{B}_d^0$ oscillations. This comes from the studies of charge correlations in $Z \to b\bar{b}$ decays at the $Z$ resonance [1] and also in hadron collisions wherein the bottom flavour is produced [2]. The general strategy adopted by the various experimental groups (see e.g., Ref [3]) can be briefly stated as follows: (a) tag the bottom flavour at production (when $Z$ decays) by the charge of a high $p_T$ lepton, or by the “jet charge”, (b) in the opposite side hemisphere, look for $B^0$ meson decay events that contain, say, $D^{*\pm}$ and $\ell^\pm$, (c) measure the displacement of the decay vertex from the production vertex, (d) estimate the parent $B^0$ momentum, and (e) determine the duration of propagation of the neutral beon. From such measurements it has been shown that the fraction of events arising from $B_d\bar{B}_d$ mixing has a sinusoidal time-dependence characteristic of oscillations.

In these analyses, it has been implicitly assumed that the quark model rule $\Delta B = \Delta Q$ is valid in the semileptonic decays of neutral $B$ mesons. Here we wish to examine whether any information on this rule can be gleaned from the oscillation data obtained at the $Z$ resonance. To this end we make use of the mass difference $\Delta m$ extracted from oscillation data and the like-sign dilepton fraction $\chi_d$ obtained in the decay of $\Upsilon(4S)$ state. We are able to extract the product of the relative strength $\rho$ of the $\Delta B = -\Delta Q$ amplitude in the $D^{*\ell\nu}$ mode of semileptonic $B_d^0$ decay and the width difference parameter

$$y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}; \quad \Gamma = \frac{\Gamma_2 + \Gamma_1}{2}. \quad (1)$$

In regard to the determination of the initial flavour (also known as production tag) we consider, for illustration, the jet-charge technique (see, e.g., [4]). Consider the jet produced by a $b$ quark coming from $Z$ decay. This jet can show up as having the normal jet charge ($Q_J = -1/3$) with a probability $b_n$, or an abnormal jet charge ($Q_J = +1/3$) with a probability $b_a$. The abnormal jet charge arises in the Standard Model due to the $b$ fragmenting into a $\bar{B}_d$ or $\bar{B}_s$ which undergoes oscillation to a neutral meson with positive bottom flavour. Therefore the probabilities associated with jet production are

$$b_n = \text{Prob}(b \to J^{-1/3}), \quad b_a = \text{Prob}(b \to J^{+1/3}), \quad \quad (2)$$

$$\bar{b}_n = \text{Prob}(\bar{b} \to J^{+1/3}), \quad \bar{b}_a = \text{Prob}(\bar{b} \to J^{-1/3}). \quad \quad (3)$$
wherein the superscripts denote the jet charges. Requiring $CP$ invariance would imply the relations $b_n = \bar{b}_n$ and $b_a = \bar{b}_a$.

As for the decay tag, we take the time $t = 0$ when the primary decay $Z \to b\bar{b}$ (we ignore $Z$ decays involving multiple $b\bar{b}$ pairs) occurs. Let the $b$ quark produce a $B_d^0$ meson which undergoes flavour oscillations during its propagation and decays semileptonically at time $t$. We shall, for definiteness, focus on the decay mode $B \to D^{*}(2010)\ell\nu$. The corresponding normal and abnormal decay rates are denoted by

\begin{align}
\bar{B}_n(t) &\equiv \Gamma(\bar{B}_d^0(t) \to D^{*+} \ell^- \bar{\nu}_\ell), \\
\bar{B}_a(t) &\equiv \Gamma(\bar{B}_d^0(t) \to D^{*-} \ell^+ \nu_\ell),
\end{align}

where $\bar{B}_d^0(t)$ denotes the state at time $t$ that evolved from a state that was pure $\bar{B}_d^0$ at $t = 0$. In the Standard Model the abnormal rate arises from $B_d\bar{B}_d$ oscillations. Also, for an initial $\bar{b}$ producing a $B_d^0$, the corresponding normal and abnormal decay rates are

\begin{align}
B_n(t) &\equiv \Gamma(B_d^0(t) \to D^{*-} \ell^+ \nu_\ell), \\
B_a(t) &\equiv \Gamma(B_d^0(t) \to D^{*-} \ell^- \bar{\nu}_\ell).
\end{align}

Again $CP$ invariance implies the conditions $B_n(t) = \bar{B}_n(t)$ and $B_a(t) = \bar{B}_a(t)$.

In what follows, we shall ignore second order $CP$ violations by neglecting terms that are bilinear in the differences $[B_n(t) - \bar{B}_n(t)]$, $[B_a(t) - \bar{B}_a(t)]$, $(b_n - \bar{b}_n)$ and $(b_a - \bar{b}_a)$.

The number of unmixed events in which a bottom jet is observed on one side and the particle pair $(D^*\ell)$ is present on the other side, following the primary decay $Z \to b\bar{b}$, can be written (including the $\bar{b}b$ configuration) as

\begin{align}
N_{\text{unmixed}} &= (b_nB_n + b_aB_a) + (\bar{b}_n\bar{B}_n + \bar{b}_a\bar{B}_a) \\
&\approx \frac{1}{2}(b_n + \bar{b}_n)(B_n + \bar{B}_n) + \frac{1}{2}(b_a + \bar{b}_a)(B_a + \bar{B}_a).
\end{align}

The last step neglects $CP$ violations of second-order. The number of mixed events which ought to have abnormal charge either at production or at decay (but not at both), is given by

\begin{align}
N_{\text{mixed}} &= (b_nB_a + b_aB_n) + (\bar{b}_n\bar{B}_a + \bar{b}_a\bar{B}_n)
\end{align}
\[
\simeq \frac{1}{2}(b_n + \bar{b}_n)(B_a + \bar{B}_a) + \frac{1}{2}(b_a + \bar{b}_a)(B_n + \bar{B}_n);
\]

(11)

again, the last step neglects \(CP\) violations of second order.

The observable of interest is the charge-correlation function defined by

\[
C_Q(t) = \frac{N_{\text{unmixed}} - N_{\text{mixed}}}{N_{\text{unmixed}} + N_{\text{mixed}}}. \tag{12}
\]

This takes a factorized form when we substitute Eqs. (9) and (11)

\[
C_Q(t) \simeq K \frac{(B_n + \bar{B}_n) - (B_a + \bar{B}_a)}{(B_n + \bar{B}_n) + (B_a + \bar{B}_a)}. \tag{13}
\]

The only assumption behind this simple relation is the neglect of second and higher order \(CP\)-violation effects. The time-independent constant \(K\) refers to the jet production tag, while the remaining factor refers to the decay tag (\(K\) is obtained from the decay tag factor by simply replacing \(B_i\) by \(b_i\) and \(\bar{B}_i\) by \(\bar{b}_i\)). Hence for convenience in studying the decay time distribution, we define

\[
C'_Q(t) \equiv \frac{1}{K}C_Q(t). \tag{14}
\]

In the framework of the Standard Model, it is reasonable (and customary) to assume \(y \simeq 0\) and that the mass-eigenstates and \(CP\)-eigenstates of the neutral beons are nearly the same (which means \(|q/p| \simeq 1\) in the usual notation, \(B_{1,2} = pB \pm q\bar{B}\)). We thus get the standard probabilities per unit time \(P_{u,m}(t)\) for finding, respectively, a \(B_d\) or a \(\bar{B}_d\) at time \(t\) having started with an initial \(B_d\) \[3\].

\[
B_d \rightarrow B_d : \quad P_u(t) = \frac{\Gamma}{2} e^{-\Gamma t}(1 + \cos \Delta mt), \tag{15}
\]

\[
B_d \rightarrow \bar{B}_d : \quad P_m(t) = \frac{\Gamma}{2} e^{-\Gamma t}(1 - \cos \Delta mt). \tag{16}
\]

Starting with a \(\bar{B}_d\) at time \(t = 0\) also leads to the same unmixed and mixed probabilities per unit time.

For describing the subsequent decays of the beons, we shall appeal to \(CPT\) invariance to relate the semileptonic partial widths of \(B_d\) and \(\bar{B}_d\) mesons. If there is a single hadron in the final semileptonic channel, there is no strong phase due to final state interactions and the corresponding decay rates of \(B_d\) and \(\bar{B}_d\) into individual conjugate channels are equal.
Thus the time-dependence describing the decays into exclusive single-hadron semileptonic channels will be

\[ B_n = \bar{B}_n = R \, P_n(t), \]  
\[ B_a = \bar{B}_a = R \, P_a(t), \]  

(17)

(18)

with the same constant of proportionality \( R \) in the normal and abnormal cases. In this way we are led to the function

\[ C'_Q(t) = \cos \Delta m t \]  

(19)

which may be fitted to the observed (proper) time dependence for extracting the mixing parameter \( \Delta m \).

Now we examine how the above analysis gets modified when \( \Delta B = -\Delta Q \) amplitudes are present. Focusing on the specific channel \( D^*(2010)\ell\nu \), we define the ratios

\[ \rho = \frac{<D^{*-}\ell+\nu|T|\bar{B}>} {<D^{*-}\ell+\nu|T|B>}, \quad \bar{\rho} = \frac{<D^{*+}\ell^+\nu|T|B>} {<D^{*+}\ell^+\nu|T|\bar{B}>}. \]  

(20)

\( CPT \) invariance (the validity of which we assume throughout) implies the relation \( \bar{\rho} = \rho^* \). On the other hand, \( CP \) invariance implies \( \rho = \bar{\rho} \). However when we allow for the presence of \( \Delta B = -\Delta Q \) amplitudes, it is reasonable to allow also for possible \( CP \) violations. We do this by retaining the terms which are linear in the \( CP \) violating quantities \( \rho - \bar{\rho} \) and \( (|p|^2 - |q|^2) \) (where \( p \) and \( q \) are the coefficients which define the propagation states as mixtures of flavour states).

We reevaluate the charge correlation function by neglecting terms that are second (and higher) order small in \( \rho \) or/and \( CP \) violations, to obtain

\[ C'_Q(t) \simeq \frac{\cos \Delta m t}{\cosh y \Gamma t + 2 \Re \rho \sinh y \Gamma t}. \]  

(21)

Instead of fitting the data to the above function, we consider the corresponding time-integrated version

\[ C' = \frac{a}{1 + 2 y \Re \rho}, \quad a \equiv \frac{1 - y^2}{1 + x^2}, \]  

(22)

and obtain information on \( y \Re \rho \). For the parameter \( a \) we use the relation connecting it to the like-sign dilepton fraction \( \chi_d \):

\[ \chi_d = \frac{1 - a}{2}. \]  

(23)
From the experiments at Υ(4S) by ARGUS [7] and CLEO [8] collaborations, we have the average value [9]
\[ \chi_d = 0.156 \pm 0.024 . \] (24)

By definition \( \chi_d \) is CP even and hence unaffected by first order CP violations; it is also unaffected by terms which are linear in \( \rho \) [5]. Therefore, substituting for \( a \), we obtain

\[ C' = \frac{1 - 2\chi_d}{1 + 2y\text{Re}\rho} . \] (25)

A determination of the combination \( y\text{Re}\rho \) is thus possible provided we know \( C' \). We observe that the experimental value for \( x_d \) deduced from \( B\bar{B} \) oscillations at \( Z \) does give us \( C' \) through the relation

\[ C' = \frac{1}{(1 + x_d^2)} \] (26)

because \( y = 0 \) and \( \rho = 0 \) are assumed in the experimental determinations of \( x_d \).

We consider, mainly to illustrate our procedure, the oscillation data in \( Z \) decays using the \( D^*\ell/Q_J \) method. This method uses the jet charge \( Q_J \) for production tag and hence deals with a bigger event sample than that using lepton tag. Also, it uses for the decay tag, the semi-exclusive channel \( B_d \to D^*(2010)\ell\nu X \) for which contamination from the semileptonic decays of \( B_s \) and charged \( B \) is expected to be minimal. Recently the OPAL group [10] had reported a value for \( \Delta m \) using this method; it is based on a sample of 1200 \( D^{*\pm}\ell^\mp \) candidate events of which 778 \( \pm 84 \) are expected to be from \( B_d^0 \) decays. Multiplying this \( \Delta m \) by the average lifetime of the \( B_d \) meson [9], we obtain

\[ x_d = (0.539 \pm 0.060 \pm 0.024) \text{ps}^{-1} \cdot (1.56 \pm 0.06) \text{ps} \] (27)
\[ = 0.84 \pm 0.11. \] (28)

Assuming that the observed events are all due to the 3-body channel \( B_d \to D^*(2010)\ell\nu \), we are therefore led to conclude that

\[ y\text{Re}\rho = \frac{1}{2}[(1 - 2\chi_d)(1 + x_d^2) - 1] \] (29)
\[ = 0.09 \pm 0.07. \] (30)
A more accurate determination of this and other such quantities should be possible with the future dilepton data at the asymmetric $B$ Factories.

To conclude, we have shown that the magnitude of the product $y \text{Re } \rho$ cannot exceed 0.21 at 90% CL. This limit depends on the dilepton data at $\Upsilon(4S)$ and the $B_d \bar{B}_d$ oscillation data from $Z$ decays in which the decaying neutral boson is tagged by the pair $D^*(2010)^{\mp} \ell^\pm$. 
References

[1] ALEPH Collab., D. Busculic et al., CERN preprint CERN-PPE/96-102, To be submitted to Z. Phys. C; DELPHI Collab., P. Abreu et al., CERN preprint CERN-PPE/96-06; OPAL Collab., G. Alexander et al., CERN preprint CERN-PPE/96-074; L3 Collab., M. Acciarri et al., Phys. Lett. B 383 (1996) 487; L. Gibbons, Talk at the XXVIII International Conference on High Energy Physics, Warsaw, Poland, July 1996.

[2] CDF Collab., B. Todd Huffman, Report at Beauty 96, Rome, Italy, 1996, Fermilab preprint FERMILAB-Conf-96/312-E.

[3] S.L. Wu, Talk at the 17th International Symposium on Lepton-Photon Interactions at High Energies, Beijing, China, 1995, CERN preprint CERN-PPE/96-082 (1996).

[4] OPAL Collab., R. Akers et al., Phys. Lett. B 327 (1994) 411.

[5] G.V. Dass and K.V.L. Sarma, Phys. Rev. Lett. 72 (1994) 191; (E) 1573.

[6] L.B. Okun, V.I. Zakharov and B.M. Pontecorvo, Lett. Nuovo Cim. 13 (1975) 218; G.V. Dass and K.V.L. Sarma, Int. J. Mod. Phys. A 7 (1992) 6081; 8 (1993) (E) 1183.

[7] ARGUS Collab., H. Albrecht et al., Phys. Lett. B 324 (1994) 249; Z. Phys. C 55 (1992) 357.

[8] CLEO Collab., J. Bartelt et al., Phys. Rev. Lett. 71 (1993) 1680.

[9] Particle Data Group, R.M. Barnett et al., Phys. Rev. D 54 (1996) 1.

[10] OPAL Collab., G. Alexander et al., CERN preprint CERN-PPE/96-074 June 1996, To be submitted to Z. Phys. C.

[11] G.V. Dass and K.V.L. Sarma, Phys. Rev. D 54 (1996) 5880.