Adhesional rolling behavior of micro-cylinder: Experimental observation of adhesional contacts and measurement of critical rolling resistance

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Abstract
There has been much research offering theoretical explanations of rolling behavior considering the effect of adhesion, which is essential when the object is small. However, there has been a shortage of experimental verifications because objects are too small to manipulate and observe directly. This study measured the moment and observed the contact area before, during, and after rolling with an experimental setup on a relatively large scale. The experiment setup consisted of two glass cylinders which were placed in between two PDMS blocks. This paper reports three main findings: the results suggest that viscosity influences the transition of states of the contact. The critical rolling resistance is dependent on the radius of the cylinders and the applied weight to the cylinders. The results support the theoretical expectation of the value of a critical rolling resistance, which is the product of the radius of the cylinders and the work of adhesion. This study offers experimental foundations to develop theories of rolling of micro-cylinders.

1. Introduction
Micromanipulation techniques are a way to align and assemble microelements in the production of electronic parts such as micro-LEDs [1]. It is not feasible to conduct such manipulation by end-effectors, e.g. grippers, because adhesional forces are dominant at such a small scale [2]. This is because the adhesional force is proportional to the first power of the size of the object, while the gravitational force is proportional to the third power.

Therefore, researchers have proposed methods of micromanipulation, which include mechanical force [3–6], capillary force [7, 8] and electrostatic force [9]. One of the methods is to control a needle-shaped tool in a specific orbit. The method uses the adhesional force of the needle’s tip to selectively destroy the contact between the device and a target object [3].

It is also of importance to understand the mechanism of adhesional contact between an elastic particle and a substrate. For instance, the Johnson-Kendall-Roberts (JKR) model of elastic contact under a compressive load is a fundamental theory for micromanipulation [10, 11]. The contact has been also investigated in terms of distributions of the contact stress [12–14].

It has been of interest to investigate the critical condition of rolling. There has been theoretical and experimental research on adhesive contact between a sphere and a substrate [15–17], and between a cylinder and a substrate [18–20]. Dominik et al theoretically proposed the existence of a rolling resistance beyond which an object irreversibly rotates [21]. Peri and Cetinkaya and Ding et al experimentally verified the presence of a critical rolling resistance of a microsphere [20, 22]. Saito et al proposed a critical condition for rolling where a cylinder starts rolling once a given an applied torque exceeds a critical value based on finite element method (FEM) simulations [18]. Figure 1 shows the estimated rolling behavior under an applied torque. As the applied torque
gradually increases, the cylinder does not rotate but inclines, and contact width decreases. Once the applied moment exceeds the critical rolling resistance, the cylinder begins to roll as a result of diminishing contact width, while total energy is balanced. The FEM simulation calculates that the critical rolling moment is the product of the work of adhesion and the cylindrical radius.

Although there has been theoretical research on rolling behavior, the condition for rolling to begin has not been experimentally investigated. Therefore, this study aims at experimentally measuring the moment at which an object begins to roll, and at comparing the measured moment with values obtained by the FEM simulations.

2. Experimental setup and method

2.1. Experiment setup

Figure 2 shows an experimental setup to measure moments and to observe the contact surface. Two cylinders (radius: 5 mm, length: 30 mm) are sandwiched between two Poly-dimethylsiloxane (PDMS) blocks. The upper block is in contact with a load cell which is connected to a motorized stage. The displacement of the motorized stage applies force to the upper block. As the load cell measures the applied force, $F[N]$, tractional force, $F/2[N]$, acts as a couple of forces. The moment applies to the contact of the cylinder and the PDMS blocks per unit length is shown by the following formula.

$$
M_{\text{measured}} = \frac{FR}{2L}
$$

The contact surface is observed by a microscope (Hirox, KH-1300) with optical lens (Hirox, MX-400z) from the bottom of the experimental setup. The experiment setup is covered by a precise air-condition system which can control temperature and humidity independently. During the experiment, the temperature is kept at $25 \pm 0.5$ degrees Celsius and humidity is kept at $30 \pm 0.5$ percent. The difference of contact states between upper and lower cylinders due to gravitational force can be negligible because the weight of the cylinders is significantly small compared to the applied weight.

PDMS is suitable for experiments on elastic bodies because it is transparent and has a small effect of viscoelasticity. PDMS blocks are made by mixing Sylgard 184 (Dow Corning Corp.) at 10:1, and then degassing at 500 hPa for 1 h, and then curing at 25 degrees Celsius for 48 h. The material of the cylinder is glass because glass has a smooth surface which enables compliant contact, and glass can be processed accurately. Glass is often used for experiments on contact with PDMS [23–28].
2.2. Definition of the point of roll start

The point of roll start is generally defined as the occurrence of a new contact at the leading edge of contact and destruction of contact at the rear edge [29]. In this experiment, the critical point cannot directly be identified based on the definition because there are results where only the leading edge of the contact is progressing, and results where rolling occurs once an equilibrium is reached (see experimental results in section 3.1.). In addition, the torque measurement does not show clear inflection points. Therefore, the point of roll start is estimated based on the expected behaviors of the cylinders as in figure 3 [18]. The previous study assumes that the cylinder rotates once an applied moment reaches a critical rolling resistance under the condition that the moment is gradually applied. Under the pushing process, the cylinders are expected to incline to a certain point and then start rolling. In the returning process, the cylinders decline to a certain point and then detach from the load cell.

Figure 3 shows a schematic estimation of measured torque and displacement of the measuring unit bases on the theoretical model [18]. In figure 4, the lines are drawn only for illustrative purposes. It is expected that the moment increases to the point of roll start as the distance increases. Then, the moment may be kept or increased after the point of roll start. If the rolling becomes steady, the torque should become stable. When there are other effects such as viscosity, the torque may increase during the rolling phase. The trend continues until the load cell reaches the maximum displacement. During return of the load cell, it is expected that the moment decreases until the load cell detaches from the PDMS block. After that, the measured moment should stay at zero as the load cell is not in contact with anything. The displacement of the PDMS block can be measured as the length at which the load cell detaches from the block in returning. The displacement of the PDMS block is considered to...
be equal to the distance that the cylinders rotate. Therefore, the displacement of the point of roll start is defined by subtracting the rolling distance of the cylinders from the maximum displacement of the PDMS blocks. This definition is justified based on the observation of the contact surface, which is described in section 3.1.

2.3. Observing adhesion surface and measuring the torque
The video microscope observes the contact surface. The torque is calculated with Formula (1) based on the force measured by the load cell.

The motorized stage is shifted 100 μm while pushing the PDMS and returned the distance of 100 μm. The displacement of the load cell is given in two conditions. One of the conditions is shifting the motorized stage 5 μm and waiting for 20 s. Under the other condition, the motorized stage is shifted at 5 μm per second without a waiting time. Based on the result, which is described in section 3.1.1., the condition of ’shifting 5 μm and waiting for 20 s’ is chosen in the following experiments because the previous studies assume a fully elastic body.

In order to investigate the influence of a given displacement on the measured moments, the maximum displacement of the motorized stage is given within the range between 10 μm and 150 μm, namely 10, 20, 30, 50, 75, 100, 125, and 150 μm.

2.4. Measuring critical rolling resistance
The measurement of rolling resistance is conducted to explore the dependency on applied weight and radius of the cylinders. Aiming at identifying the dependency of applies weight, the experiment is conducted ten times without applied weight and five times each with applied weights of 10 g, 20 g, 30 g, 40 g, and 50 g.

The experiment is also conducted to explore the influence of the cylinders’ radius with three conditions; 0.002, 0.005, and 0.01 m. The size of the PDMS blocks is also altered. When the cylinders’ radius is 0.002 m, the length of the cylinders (L) was 0.012 m, and the size of the PDMS blocks is 0.012 × 0.012 × 0.002. When the cylinders’ radius is 0.01 m, the length of the cylinders (L) was 0.06 m, and the size of the PDMS blocks was 0.06 × 0.06 × 0.01m.

3. Result and discussion

3.1. Investigating the transition in the state of contact surface
3.1.1. Observation of adhesion surface
Figure 5 shows an observation of the contact surface in the two conditions, which are either the 100 μm displacement of the motorized stage is given without a waiting time or with a waiting time of 20 s. In the condition without waiting time (figure 5(b)), the rear edge of the contact surface is slightly advanced in the returning phase. The length of the advancement is longer in the condition with waiting time (figure 5(a)) than in the condition without waiting time (figure 5(b)). The difference might have been caused by viscosity which delayed the system in reaching an energy equilibrium state. The authors choose to use the condition shifting 5 μm and waiting for 20 s in the following experiments because the previous studies assume a fully elastic body.

The result shows that the width of the contact surface increased when a moment is applied, which is not consistent with the simulation result [18]. The experimental observation can be explained by several reasons such as that there is a delay to reach the energy equilibrium state due to the effect of viscosity, energy is dissipated, and effective adhesion work is increased by tangential force [30]. The result also implies that it is impossible to
pinpoint a point of roll start based on the general definition, which is an occurrence of new contact at the leading edge of contact and destruction of contact at the rear edge [29]. Therefore, the authors choose to use the definition of point of rolling as described in section 2.2.

3.1.2. Influence of the condition of moving the load cell on the measured moment

Figure 6 shows that the measured moment and displacement of the measuring unit in the two conditions. The two conditions show different trends. The line of measured (a) represents the condition of shifting the motorized stage 5 μm and waiting for 20 s, while the line of measured (b) represents the other condition, the motorized stage is shifted at 5 μm per second without a waiting time. With the definition of roll start, as mentioned earlier, there are two possible points of roll start. In the measured condition (b), the moment at the point of roll start is higher than in condition (a). This implies that the cylinder was pushed farther before reaching an energy equilibrium state. Consequently, the attitude angles of the cylinders may have exceeded the theoretical critical rolling angle. The result suggests viscosity impacts the rolling behavior, which results in a higher degree of measured critical rolling resistance in condition (b).

3.1.3. Influence of given maximum displacement on measured torque

Figures 7 and 8 show displacement-measured torque curves under different given maximum displacements of the load cell. The displacements are given in the way of shifting 5 μm and waiting for 20 s. Figure 7 is the result of relatively small displacements, namely 10, 20, 30, and 40 μm. The value of the measured force during pushing is close to the value of that during the return. The width of the contact surface do not change significantly during the experiment. The results suggest that the cylinders do not rotate but incline, which supports the theoretical assumption [18]. The value of returning is slightly smaller than the value of pushing, which might be because of the contact between the load cell and the upper PDMS block. As adhesions have hysteresis [31], the hysteresis increases the value of the adhesional force between the load cells and the PDMS block.

Figure 8 shows the result of relatively large displacements, which are 50, 75, 100, 125, 150 μm. A shift of the contact surface is observed, which suggests that the cylinders rotate during the experiment. The distance of rolling increases along with the given displacement of the motorized stage. This means that, under the conditions, a larger given displacement of the motorized stage corresponds to a larger coordinate at which the upper PDMS block detaches from the load cell, namely displacement of the PDMS block due to the cylinders’ rolling.

Figure 8 also shows that the displacement of the upper PDMS block is smaller than the given displacement of the motorized stage. The results support the previously mentioned influence of viscosity, which is that the attitude angles of the cylinders may exceed a critical rolling angle because the cylinder is pushed farther before reaching an energy equilibrium state.

In summary, these results show that the cylinders firstly incline, and then rotate, which is consistent with the assumption of the theory [18]. It supports the definition of the point of roll start as described in section 2.2.
Figure 9 shows the displacement-torque curves for both expected and experimental. The horizontal line is the displacement of the load cell, and the force is measured by the load cell. The load cell is displaced by shifting 5 μm and waiting for 20 s. Based on the assumed behavior of the cylinders, the force is expected to increase linearly during the inclination phase (figure 9-(2)), and then the force becomes stable during the rotation phase (figure 9-(3)). The theoretical line is drawn to illustrate how the theory expects the torque to change rather than to show quantitative results. Therefore, the authors do not have an intention to suggest that the theoretical critical rolling torque is calculated as shown in figure 9.

The experimental line has a saw-like shape because the viscosity delays reaching an energy equilibrium state during the waiting time. Figure 9 suggests that there are two phases during the experiment. In the initial stage, while the cylinders incline, the value of the measured force linearly increases. In the second stage, the rolling phase, the measured force continuously increases. The gradient of the second stage is less than that of the first stage. The authors consider that the reasons for the torque’s increase in the second stage are the impact of...
viscosity (i.e. figure 6). The waiting time, 20 s, might not have been long enough for the system to reach an energy equilibrium state due to the viscosity.

3.1.4. Estimation of transitions of contact states

Figure 10 shows an estimation of the transition of contact state between the cylinder and plate based on the observation of the contact surface and the measurement of the moment. In figure 10, \( \theta_{\text{roll}} \) is the attitude angle of the cylinder when starting to roll, \( \theta_d \) is the angular displacement due to rolling, and \( \theta_{\text{ret}} \) is the angle of the cylinder which exceeds \( \theta_{\text{roll}} \) due to viscosity.

The theory bases on the assumption of a perfectly elastic body shows that the cylinder inclines without rolling while the contact width becomes smaller under the condition that the applied moment is smaller than the critical rolling resistance \( M_{\text{roll}} \) [18]. When the applied moment is removed, the system returns to the state before the moment was applied. Once the moment exceeds \( M_{\text{roll}} \), the attitude angle becomes \( \theta_{\text{roll}} \) and rolling

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**Figure 9.** The measured moment and a theoretically expected line.

**Figure 10.** The transition of contact states between a cylinder and a plate.
starts. As the state transition to rolling happens while maintaining an energy equilibrium state, the moment requires to keep the cylinder rolling is also considered \( M_{roll} \). If the moment is removed after rolling, the inclination of the cylinder is reduced by \( \theta_{roll} \), and becomes the angular displacement \( \theta_d \) due to continued rolling.

The experimental results show that the contact width is either kept or widened when the moment is applied to the cylinders (cf figure 5). It is caused by an increase in the effective adhesion force due to tangential force and a delay of deformation at the rear end of contact due to viscosity. When the moment is removed, the angle of the cylinder basically decreases to the original states. However, the angles and the contact width may become larger than the original ones because there are several angles at which energy is locally stable. Once the applied moment exceeds the critical rolling resistance, \( M_{roll} \), the cylinders rotate. It takes time to reach an equilibrium state due to viscosity. If the applied moment increases before the system reaches an equilibrium state, the attitude angle exceeds \( \theta_{roll} \) and becomes the sum of \( \theta_{roll} \) and \( \theta_{ret} \). Once the moment is removed after rolling, the angle decreases from the sum of \( \theta_{roll} \) and \( \theta_{ret} \) and becomes the angular displacement due to rolling, \( \theta_d \). The distance of rolling and \( \theta_d \) is less than the theoretical calculation.

This estimation is consistent with the expectation by research on a rigid cylinder which is constantly rolling on a viscoelastic plate [32]. The study expects significant energy dissipation near the rear edge of the contact area. The theory is supported by the observation results on the contact area, which is the leading edge of the contact advances first, and the fracture of the contact interface at the trailing edge follows.

The theoretical research assumes a perfectly elastic body on analysis of the behavior of an object which is about to roll with consideration of adhesion. However, there is no perfect elastic body in the real world. As the experimental results strongly show that the behavior at the beginning of rolling is affected by viscosity, it is of importance to develop a theory taking viscosity into account.

3.2. Measurement of critical rolling resistance
3.2.1. The dependency of critical rolling resistance on applied weight
Figure 11 shows a scatter plot of the critical rolling resistance and applied weight. The plot shows that critical rolling resistance is proportional to applied weight. Differences in surface conditions may have caused variability in the values. The line and the equation in figure 11 are a linear approximation of the measured points by the least-squares method.

\[
M_{measured} = 1.046 \times 10^{-6} w + 1.864 \times 10^{-4}
\]  

Based on the approximation and the weights of the PDMS block and the cylinders, critical rolling resistance under no applied weight is calculated to be \( 1.78 \times 10^{-4} \text{ Nm/m} \). Based on the value of the work of adhesion (0.03 N/m) in the related study of adhesion work of contact between glass and PDMS [23], critical rolling resistance, which is the product of the radius of a cylinder and the work of adhesion, can be simulated to be \( 1.50 \times 10^{-4} \text{ Nm/m} \). The experimental result is consistent with the simulated result. The experimental value is larger than the simulated value because the effective work of adhesion due to tangential force may have increased.
3.2.2. The dependency of critical rolling resistance on the radius of the cylinders

Figure 12 shows the result of the experiments with the three cylinders radius conditions; 0.002, 0.005, 0.010 m. The result shows that the measured rolling resistance increased along with the increase in the radius of cylinders, which is consistent with the related study \[18\].

In changing the radius of the cylinders, the weight of the cylinders and the PDMS blocks are changed. It is crucial to confirm that the difference is caused by the radius rather than the weights. The critical rolling resistance is calculated based on the aforementioned Formula (2). The formula enables calculation of the critical rolling resistance for radius 0.005 m with the applied weights of the cylinders of radius 0.002 m and the corresponding PDMS, and radius of 0.01 m and the corresponding PDMS. When the radius is 0.002 m, the weight of the cylinders is 0.355 g and 0.346 g, and the weight of the PDMS is 0.377 g. Therefore, the simulated applied weight to the cylinders of radius 0.005 m is 2.70 g. With the aforementioned Formula (2), the critical rolling resistance is calculated to be $1.73 \times 10^{-4}$ Nm/m. The value is more than the average value of the measured critical rolling resistance of radius 0.002 m: $1.28 \times 10^{-4}$ Nm/m. When the radius is 0.01 m, the weight of the cylinders is 40.7 g and 41.3 g, and the weight of the PDMS is 37.5 g. Therefore, the simulated applied weight to the cylinders of radius 0.005 m is 59.75 g. With the aforementioned Formula (2), the critical rolling resistance is calculated to be $2.33 \times 10^{-4}$ Nm/m. The value is more than the average value of the measured critical rolling resistance of radius 0.002 m: $3.10 \times 10^{-4}$ Nm/m. The calculation confirms that critical rolling resistance is dependent on the radius of the cylinders.

4. Conclusions

This study reports the observation of the contact surface and measurement of the moment on glass cylinders which are sandwiched by PDMS blocks before and during the rolling of the cylinders. The observation suggests that viscosity influences the transition of states of the contact. The measured value of the moment closely corresponds to the critical rolling resistance simulated by FEM analysis, which suggests critical rolling resistance is the product of the radius of the cylinders and the work of adhesion. The result also suggests that the critical rolling resistance is dependent on the radius of the cylinders and the applied weight to the cylinders. This paper experimentally suggests that it is of importance to develop a theory including the influence of viscosity.

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