Femtosecond time synchronization of optical clocks off of a flying quadcopter

Hugo Bergeron1,2,3, Laura C. Sinclair1,2,3, William C. Swann1,2, Isaac Khader1,2, Kevin C. Cossel1,2, Michael Cermak1,2, Jean-Daniel Deschênes1,2 & Nathan R. Newbury1,2

Future optical clock networks will require free-space optical time-frequency transfer between flying clocks. However, simple one-way or standard two-way time transfer between flying clocks will completely break down because of the time-of-flight variations and Doppler shifts associated with the strongly time-varying link distances. Here, we demonstrate an advanced, frequency comb-based optical two-way time-frequency transfer (O-TWTFT) that can successfully synchronize the optical timescales at two sites connected via a time-varying turbulent air path. The link between the two sites is established using either a quadcopter-mounted retroreflector or a swept delay line at speeds up to 24 ms−1. Despite 50-ps breakdown in time-of-flight reciprocity, the sites’ timescales are synchronized to <1 fs in time deviation. The corresponding sites’ frequencies agree to ~10−18 despite 10−7 Doppler shifts. This work demonstrates comb-based O-TWTFT can enable free-space optical networks between airborne or satellite-borne optical clocks for precision navigation, timing and probes of fundamental science.
Optical clock networks promise advances in precision navigation, time distribution, coherent sensing, relativity experiments, dark matter searches, and other areas. Such networks will need to compare and synchronize clocks over free-space optical links between moving airborne or satellite-borne clocks. However, current comb-based optical two-way time–frequency transfer (O-TWTFT) cannot support femtosecond clock synchronization in the presence of motion. Even modest closing velocities between clocks lead to many picoseconds of non-reciprocity in the two-way optical time-of-flight, and correspondingly large time synchronization errors. Here, we demonstrate an advanced comb-based O-TWTFT to synchronize clocks without penalty, despite strong effective closing velocities.

There are multiple challenges in implementing sub-femtosecond two-way frequency distribution between moving clocks via free-space optical links. These challenges reflect and extend those faced by rf time–frequency transfer over free space and optical time–frequency transfer via fiber optics. First, because of turbulence and diffraction, the received free-space signals will be weak, vary strongly, and suffer frequent fades. These turbulence-induced effects are far less for rf links, because of the longer wavelength, or for fiber-optic links, because of the stable medium. Previous comb-based O-TWTFT has nevertheless overcome these turbulence effects to achieve femtosecond synchronization. Here, we focus on the second critical challenge. Namely, the clock sites can move rapidly, leading to strong Doppler shifts and a complete breakdown in the reciprocity of the two-way time-of-flight. Consider even a terrestrial velocity of 30 m s⁻¹. The fractional Doppler shift of 10⁻⁷ must be suppressed by 10¹¹ to synchronize clocks to 10⁻¹⁸ in frequency. At this same velocity, the non-reciprocal time-of-flight of 3 ps (due to the finite speed-of-flight) must be suppressed by 10⁴ to synchronize clocks to below 1 fs in time. This level of suppression is orders of magnitude beyond that achieved in rf time–frequency transfer. Moreover, it must be achieved, despite recurrent turbulence-induced signal fades.

Here, we demonstrate an advanced comb-based O-TWTFT that synchronizes clocks to within femtoseconds, despite motion. We synchronize two optical timescales connected via a quadrupole-mounted retroreflector or swept delay line over turbulent air paths at speeds up to 24 m s⁻¹. The synchronized clocks agree to ~10⁻¹⁸ in frequency, despite 10⁻⁷ Doppler shifts, and to < 1 fs in time deviation, despite 50-ps breakdown in time-of-flight reciprocity.

Results Advanced O-TWTFT System. We synchronize two sites A and B each with a clock, or optical timescale, defined by the labeled pulses from a 200-MHz fiber frequency comb phase-locked to a ~195-THz local optical oscillator. (For a full atomic clock, this optical oscillator would be locked to an atomic transition.) Site A acts as the master site. Site B is synchronized to it by adjusting the phase of the site B frequency comb. Because a fully flyable optical clock/oscillator is currently unavailable, both sites are fixed and we instead change the distance between sites by bouncing the optical signals off a quadrupole-mounted retroreflector or a rapidly swept delay line. In either case, the link also includes the 2- or 4-km free-space turbulent air path. As shown in Fig. 1, the link is folded to enable verification by a single short fiber link that directly connects the sites to provide out-of-loop verification of the time synchronization. All O-TWTFT information traverses the 2-4-km open-path link as if the two clock sites were, in fact, separated by this distance.

The system uses a layered approach: TWTFT with a modulated communication channel for picosecond-level time transfer followed by TWTFT with coherent frequency comb pulses for femtosecond-level time transfer. Frequency-comb TWTFT uses linear-optical sampling (LOS) to achieve femtosecond uncertainty (with a 5-nanosecond ambiguity given by the ~200-MHz comb repetition rate). LOS requires the repetition rate of the two pulse trains transmitted across the link to differ by Δνt ~2 kHz, which leads to inclusion of a third transfer comb X at Site A (see Fig. 1b.) The timing data from the communication channel and frequency-comb transfer are input into synchronization algorithms that, unlike ref. 13, resolve the 5-ns ambiguity on the comb pulse-by-pulse to generate four calculated timestamps from the motion of the retroreflector and correspondingly large time synchronization errors. Here, we focus on the second critical challenge. Namely, the clock sites can move rapidly, leading to strong Doppler shifts and a complete breakdown in the reciprocity of the two-way time-of-flight. Consider even a terrestrial velocity of 30 m s⁻¹. The fractional Doppler shift of 10⁻⁷ must be suppressed by 10¹¹ to synchronize clocks to 10⁻¹⁸ in frequency. At this same velocity, the non-reciprocal time-of-flight of 3 ps (due to the finite speed-of-flight) must be suppressed by 10⁴ to synchronize clocks to below 1 fs in time. This level of suppression is orders of magnitude beyond that achieved in rf time–frequency transfer. Moreover, it must be achieved, despite recurrent turbulence-induced signal fades.

Here, we demonstrate an advanced comb-based O-TWTFT that synchronizes clocks to within femtoseconds, despite motion. We synchronize two optical timescales connected via a quadrupole-mounted retroreflector or swept delay line over turbulent air paths at speeds up to 24 m s⁻¹. The synchronized clocks agree to ~10⁻¹⁸ in frequency, despite 10⁻⁷ Doppler shifts, and to < 1 fs in time deviation, despite 50-ps breakdown in time-of-flight reciprocity.

Velocity-dependent reciprocity breakdown. For the case realized here experimentally, the two clocks are connected via a retroreflector moving at closing velocity V/2 away from the clocks. The retroreflector is at a distance Lc(t) from site A and Lb(t) from site B. This scenario mimics time transfer via a moving, intermediate clock site—the solution presented here could be generalized to the alternate scenario of a stationary clock A and moving clock B with inclusion of the time dilation effect and choice of reference frame. Because of motion, the time-of-flight is non-reciprocal and given by

$$ T_{A-B} - T_{B-A} = \frac{V}{c}(T_{AB} - T_{BA} + \Delta T_{AB}) + \frac{V}{c^2}(L_A - L_B) $$

(2)

to first order in V/c, where c is the speed of light. The first term arises because the link distance is sampled at different times (asynchronously) by the pulses traveling each direction since they do not necessarily depart their transceiver at the same time. The second term exists even for synchronous sampling and arises from the motion of the retroreflector. It can be derived from geometric considerations or more formally via Lorentz transformations. At a modest V = 30 m s⁻¹, 4-km link, and an asynchronous sampling of $T_{AB} - T_{BA} + \Delta T_{AB} \approx 0.5$ ms, the first term in (2) yields a non-reciprocal time-of-flight of 50 ps. At the same velocity and for a reflector located adjacent to one clock site ($L_A - L_B \approx 4$ km), as is the case with the swept delay line,
the second term in (2) yields a non-reciprocal time-of-flight of 1.3 ps. Note that for a symmetrically located retroreflector, as is approximately true for the quadcopter, this term vanishes. We include these two non-reciprocity corrections in (1) to <100 as uncertainty by using the available O-TWTFT data to calculate the speed to 20 μm s⁻¹ uncertainty at 1-s averaging time. The speed is found from the rate-of-change of the measured time-of-flight (calculated via a different combination of timestamps) over three consecutive measurements. Errors due to acceleration are suppressed to <0.1 fs at even our maximum experimental acceleration of 7 g, both by the fast update rate (~2 kHz) of the O-TWTFT system and the use of centered derivatives to determine the speed at the correct time.

**Velocity-dependent systematic timing shifts.** The Doppler shifts of the received comb light are large (10⁻⁷, or 20 MHz, at V = 30 m s⁻¹) and changing as V is not constant. These Doppler shifts can couple with the system dispersion to cause distortions in the measured heterodyne signal between the incoming and local comb light. These distortions can lead to picosecond-level timing errors in the calculated timestamps. To avoid this, we calculate the cross-ambiguity function²⁷ between the measured heterodyne signal and a frequency-shifted template of the expected zero velocity waveform. We find its peak in real time (< 300 μs) to <100 as uncertainty by use of a Fourier transform algorithm and the Nelder–Mead search algorithm²⁸.

**Synchronization feedback via an adaptive Kalman filter.** The final synchronization algorithms are implemented in a digital signal processing platform to generate an estimate of Δt_AB in real time at a 2-kHz measurement rate. Under strong turbulence, signal fades block the exchange of comb pulses and the communication link, but these fades are usually of short duration, e.g., a few milliseconds. With motion, these signal fades can extend over longer durations because of the challenges of tracking the moving platform. Indeed, for the quadcopter data shown later, we suffer signal fades of many seconds. To synchronize despite these signal fades, we implement a Kalman-filter-based loop filter as in
The Kalman filter provides optimal hold-over behavior during fades. It uses a two-element state vector, modeling, respectively, time error and frequency error. The state transition matrix is simply two integrators in order to accurately account for the random frequency walk of the cavity-stabilized oscillators ($1/f^4$ phase noise). The measurement noise is modeled as white noise with a 5-femtosecond standard deviation at the 2-kHz update rate. The Kalman filter’s output is sent to a standard.

**Fig. 2** Synchronization over 4 km with the in-line swept delay line. **a** The time-of-flight and closing velocity are retrieved from the O-TWTFT data. The closing velocity varied from 0 m s$^{-1}$ to 24 m s$^{-1}$. The clock time offset is the out-of-loop verification. During active synchronization (i.e., no long fades) the standard deviation is 1.1 fs. All data are at the 2.2 kHz update rate. **b** Expanded view. The clocks’ time offset is shown for all time (cyan) and only during active synchronization, i.e., no turbulence-induced fades (black line).

**Fig. 3** Synchronization results for a link to the flying quadcopter. The results shown are the time-of-flight (optical pathlength), closing velocity, and clocks’ time offset as measured by the out-of-loop verification channel. The clock’s time offset is given for periods of active synchronization (gray dots) at the full -2 kHz update rate and for all times (cyan dots) at a 10 Hz sampling rate. The latter clearly shows the walkoff of the clocks during longer duration fades. The walkoff of the clock offset can exceed ~100 femtoseconds for fades longer than 1 s. The standard deviation is 3.7 fs for active synchronization at the -2 kHz update rate (also see Supplementary Movie 1).
Results of time synchronization with quadcopter and swept delay line. Figure 2 shows time synchronization between the two sites A and B over a link that includes both 4 km of turbulent air and the swept delay line operated at closing velocities from 0 m s\(^{-1}\) to ± 24 m s\(^{-1}\). The time-of-flight, closing velocity, and calculated time offset are all returned from the O-TWTFT signals. In parallel, the clocks’ time offset (i.e., arrival time of labeled optical comb pulses) is measured by the out-of-loop verification. When actively synchronized (outside of signal fades >20 ms), the clock times agree with a standard deviation of 1.1 fs at the full 2.2 kHz update rate. During brief signal fades due to atmospheric turbulence, the clocks’ times walk off randomly as previously mentioned (cyan trace of Fig. 2b), but are resynchronized when the signal is reacquired.

Synchronization to a quadcopter-mounted retroreflector is shown in Supplementary Movie 1 and Fig. 3. The quadcopter provided a maximum 500-m optical pathlength change and a 20 m s\(^{-1}\) (quadcopter-limited) maximum speed. Again, we see femtosecond-level synchronization with no evidence of speed-dependent bias. These data do show much longer fades due to the additional challenge of tracking the moving quadcopter, accomplished as in ref. 30.

Analysis of time and frequency instability. Figure 4 shows the time and modified Allan deviations. For these data, the swept delay line was operated for ~20 min at ± 24 m s\(^{-1}\) to 0 m s\(^{-1}\) and the swept delay line operated at closing velocities from 0 m s\(^{-1}\) to ± 24 m s\(^{-1}\). The time-of-flight, closing velocity, and calculated time offset are all returned from the O-TWTFT signals. In parallel, the clocks’ time offset (i.e., arrival time of labeled optical comb pulses) is measured by the out-of-loop verification. When actively synchronized (outside of signal fades >20 ms), the clock times agree with a standard deviation of 1.1 fs at the full 2.2 kHz update rate. During brief signal fades due to atmospheric turbulence, the clocks’ times walk off randomly as previously mentioned (cyan trace of Fig. 2b), but are resynchronized when the signal is reacquired.

Synchronization to a quadcopter-mounted retroreflector is shown in Supplementary Movie 1 and Fig. 3. The quadcopter provided a maximum 500-m optical pathlength change and a 20 m s\(^{-1}\) (quadcopter-limited) maximum speed. Again, we see femtosecond-level synchronization with no evidence of speed-dependent bias. These data do show much longer fades due to the additional challenge of tracking the moving quadcopter, accomplished as in ref. 30.

Analysis of time and frequency instability. Figure 4 shows the time and modified Allan deviations. For these data, the swept delay line was operated for ~20 min at ± 24 m s\(^{-1}\) to 0 m s\(^{-1}\) and the swept delay line operated at closing velocities from 0 m s\(^{-1}\) to ± 24 m s\(^{-1}\). The time-of-flight, closing velocity, and calculated time offset are all returned from the O-TWTFT signals. In parallel, the clocks’ time offset (i.e., arrival time of labeled optical comb pulses) is measured by the out-of-loop verification. When actively synchronized (outside of signal fades >20 ms), the clock times agree with a standard deviation of 1.1 fs at the full 2.2 kHz update rate. During brief signal fades due to atmospheric turbulence, the clocks’ times walk off randomly as previously mentioned (cyan trace of Fig. 2b), but are resynchronized when the signal is reacquired.

Synchronization to a quadcopter-mounted retroreflector is shown in Supplementary Movie 1 and Fig. 3. The quadcopter provided a maximum 500-m optical pathlength change and a 20 m s\(^{-1}\) (quadcopter-limited) maximum speed. Again, we see femtosecond-level synchronization with no evidence of speed-dependent bias. These data do show much longer fades due to the additional challenge of tracking the moving quadcopter, accomplished as in ref. 30.

Analysis of time and frequency instability. Figure 4 shows the time and modified Allan deviations. For these data, the swept delay line was operated for ~20 min at ± 24 m s\(^{-1}\) to 0 m s\(^{-1}\) and the swept delay line operated at closing velocities from 0 m s\(^{-1}\) to ± 24 m s\(^{-1}\). The time-of-flight, closing velocity, and calculated time offset are all returned from the O-TWTFT signals. In parallel, the clocks’ time offset (i.e., arrival time of labeled optical comb pulses) is measured by the out-of-loop verification. When actively synchronized (outside of signal fades >20 ms), the clock times agree with a standard deviation of 1.1 fs at the full 2.2 kHz update rate. During brief signal fades due to atmospheric turbulence, the clocks’ times walk off randomly as previously mentioned (cyan trace of Fig. 2b), but are resynchronized when the signal is reacquired.

Synchronization to a quadcopter-mounted retroreflector is shown in Supplementary Movie 1 and Fig. 3. The quadcopter provided a maximum 500-m optical pathlength change and a 20 m s\(^{-1}\) (quadcopter-limited) maximum speed. Again, we see femtosecond-level synchronization with no evidence of speed-dependent bias. These data do show much longer fades due to the additional challenge of tracking the moving quadcopter, accomplished as in ref. 30.

Analysis of time and frequency instability. Figure 4 shows the time and modified Allan deviations. For these data, the swept delay line was operated for ~20 min at ± 24 m s\(^{-1}\) to 0 m s\(^{-1}\) and the swept delay line operated at closing velocities from 0 m s\(^{-1}\) to ± 24 m s\(^{-1}\). The time-of-flight, closing velocity, and calculated time offset are all returned from the O-TWTFT signals. In parallel, the clocks’ time offset (i.e., arrival time of labeled optical comb pulses) is measured by the out-of-loop verification. When actively synchronized (outside of signal fades >20 ms), the clock times agree with a standard deviation of 1.1 fs at the full 2.2 kHz update rate. During brief signal fades due to atmospheric turbulence, the clocks’ times walk off randomly as previously mentioned (cyan trace of Fig. 2b), but are resynchronized when the signal is reacquired.

Synchronization to a quadcopter-mounted retroreflector is shown in Supplementary Movie 1 and Fig. 3. The quadcopter provided a maximum 500-m optical pathlength change and a 20 m s\(^{-1}\) (quadcopter-limited) maximum speed. Again, we see femtosecond-level synchronization with no evidence of speed-dependent bias. These data do show much longer fades due to the additional challenge of tracking the moving quadcopter, accomplished as in ref. 30.

Analysis of time and frequency instability. Figure 4 shows the time and modified Allan deviations. For these data, the swept delay line was operated for ~20 min at ± 24 m s\(^{-1}\) to 0 m s\(^{-1}\) and the swept delay line operated at closing velocities from 0 m s\(^{-1}\) to ± 24 m s\(^{-1}\). The time-of-flight, closing velocity, and calculated time offset are all returned from the O-TWTFT signals. In parallel, the clocks’ time offset (i.e., arrival time of labeled optical comb pulses) is measured by the out-of-loop verification. When actively synchronized (outside of signal fades >20 ms), the clock times agree with a standard deviation of 1.1 fs at the full 2.2 kHz update rate. During brief signal fades due to atmospheric turbulence, the clocks’ times walk off randomly as previously mentioned (cyan trace of Fig. 2b), but are resynchronized when the signal is reacquired.

Synchronization to a quadcopter-mounted retroreflector is shown in Supplementary Movie 1 and Fig. 3. The quadcopter provided a maximum 500-m optical pathlength change and a 20 m s\(^{-1}\) (quadcopter-limited) maximum speed. Again, we see femtosecond-level synchronization with no evidence of speed-dependent bias. These data do show much longer fades due to the additional challenge of tracking the moving quadcopter, accomplished as in ref. 30.
overall dispersion in the transceivers was reduced 30-fold by use of dispersion compensation, the transceiver calibration was performed by a custom, integrated optical time domain reflectometer (OTDR). The rf system was redesigned to reduce multipath reflections due to impedance mismatches, the rf group delay were measured and digitally canceled prior to interferogram detection, and the digital signal processing hardware was redesigned to support the more extensive, real-time synchronization algorithms. A more detailed description of the redesigned transceiver is given in ref. 34.

**Synchronization algorithm.** The presence of motion in the O-TWTFT link required entirely new synchronization algorithms and implementation. The derivation of the four effective timestamps is lengthy and provided in ref. 34. We briefly outline a different derivation here that leads to a single master synchronization equation, but does not provide the same physical insight as the use of the virtual calculated timestamps of Eq. (1).

At site A, the comb pulses arrive at the local defined reference plane at times \( t_A \), where \( n_A \) labels the pulses, \( f_i \) is the nominal defined repetition frequency and \( \tau_A \) is the overall time offset that includes any integrated out-of-loop measurement directly yields

\[
\Delta t_{\text{cal}}(t_A(t)) = \frac{2\Delta \tau_{\text{ref}}}{\Delta f_{\text{ref}}} + \frac{\Delta f_{\text{ref}}}{\Delta f_{\text{ref}}} \Delta t_{\text{LOS}}(t_A(t))
\]

where \( \Delta t_{\text{cal}} \) is the correction term, \( \Delta f_{\text{ref}} \) is the frequency going into the light as it couples to the relative chirp between the comb pulses thereby distorting the interferogram shape and causing systematic timing shifts. As discussed in the article, to avoid this, we use the ambiguity function, which suppresses this systematic to below 100 as.

Assuming successful suppression of this velocity-induced bias and following similar analysis as in ref. 13, the three times as measured against the timescale \( t_{\text{LOS}} \) in the system, however, the timestamps are instead measured against the local timescale at site A or B. Therefore, we use the relationships

\[
\begin{align*}
\Delta f_{\text{ref}} &= f_i - f_i(t_A(t)) - f_i(t_B(t)) \\
\Delta f_{\text{ref}} &= f_i(t_A(t)) - f_i(t_B(t)) \\
\Delta f_{\text{ref}} &= f_i(t_B(t)) - f_i(t_A(t))
\end{align*}
\]

as measured with respect to the timescale \( t_{\text{LOS}} \). In the system, however, the timestamps are instead measured against the local timescale at site A or B. Therefore, we use the relationships

\[
\begin{align*}
\Delta t_{\text{cal}}(t_A(t)) &= \frac{2\Delta \tau_{\text{ref}}}{\Delta f_{\text{ref}}} + \frac{\Delta f_{\text{ref}}}{\Delta f_{\text{ref}}} \Delta t_{\text{LOS}}(t_A(t)) \\
\Delta t_{\text{cal}}(t_B(t)) &= \frac{2\Delta \tau_{\text{ref}}}{\Delta f_{\text{ref}}} + \frac{\Delta f_{\text{ref}}}{\Delta f_{\text{ref}}} \Delta t_{\text{LOS}}(t_B(t)) \\
\Delta t_{\text{cal}}(t_B(t)) &= \frac{2\Delta \tau_{\text{ref}}}{\Delta f_{\text{ref}}} + \frac{\Delta f_{\text{ref}}}{\Delta f_{\text{ref}}} \Delta t_{\text{LOS}}(t_B(t))
\end{align*}
\]

From (3) and the substitution mentioned afterward, it is clear we are interested in the asymmetric

\[
T_{A \rightarrow B}(t) = T_{A \rightarrow B}(t) - T_{A \rightarrow B}(t) - T_{A \rightarrow B}(t).
\]

We must extrapolate the time-of-flight to a common measurement time. To this end and first in \( V/c \), we write

\[
T_{A \rightarrow B}(t - t_A) = T_{A \rightarrow B}(t - t_A) + O(V/c)^2.
\]

Combined with (4), this yields the breakdown in reciprocity,

\[
T_{A \rightarrow B}(t - t_A) - T_{A \rightarrow B}(t - t_A) = O((V/c)^2) + O(V/c)^2.
\]

With this information, we can solve to find the time offset,

\[
\Delta t_{\text{cal}} = \frac{2\Delta \tau_{\text{ref}}}{\Delta f_{\text{ref}}} + \frac{\Delta f_{\text{ref}}}{\Delta f_{\text{ref}}} \Delta t_{\text{LOS}}(t_A(t))
\]

where we introduce two calibration terms, \( \Delta t_{\text{cal}} \) and \( \Delta t_{\text{cal}} \), discussed below, and drop terms of order \( (V/c)^2 \) and \( (V/c)^3 \). Therefore, we introduce a third, transfer comb \( B \) at the master site with repetition rate \( f_i + \Delta f \) and time offset \( \Delta t_{\text{cal}} \), also phase-locked to the master optical oscillator.

We measure three heterodyne signals between the master, transfer, and remote combs, each consisting of a series of consecutive interferograms, i.e., short arbitrary ad hoc timescale; in the end, we are only concerned with their differences. For verification purposes, we locate the reference plane for both sites at the end of the optical fiber that is used for the out-of-loop time synchronization measurement, so that this out-of-loop measurement directly yields

\[
\Delta t_{\text{cal}} = \frac{2\Delta \tau_{\text{ref}}}{\Delta f_{\text{ref}}} + \frac{\Delta f_{\text{ref}}}{\Delta f_{\text{ref}}} \Delta t_{\text{LOS}}(t_A(t))
\]

the successive interferograms since we will combine these timing data with the times \( t_{\text{ref}} \) and \( \Delta t_{\text{cal}} \), and of the interferograms. The time \( t \) is some common ad hoc arbitrary timescale that is mathematically convenient, but drops out of the final synchronization equation. The first interferogram, \( I_{\text{AS}}(t) \), is generated from the local heterodyne mixing of the master and transfer comb at the master site. The middle interferogram, \( I_{\text{BS}}(t) \), is the heterodyne signal between the transmitted remote comb pulses and the transfer comb at the remote site. The third interferogram, \( I_{\text{CS}}(t) \), is the heterodyne signal between the transmitted transfer comb and the remote comb at the remote site. We are ultimately interested in the times \( t_{\text{AS}}(t) \), \( t_{\text{BS}}(t) \), and \( t_{\text{CS}}(t) \) that define the center of the successive interferograms since we will combine these timing data with the communication-based two-way time–frequency transfer to evaluate \( \Delta t_{\text{cal}} \). The interferograms’ waveform depends on the Doppler shift of the incoming light as it couples to the relative chirp between the comb pulses thereby distorting the interferogram shape and causing systematic timing shifts. As discussed in the article, to avoid this, we use the ambiguity function, which suppresses this systematic to below 100 as.

The linear-optical sampling (LOS) detection used in comb-based O-TWTFT requires the pulse trains of the two combs transmitted across the link have repetition rates differing by \( \Delta f \lesssim 2 \text{ kHz} \). Therefore, we introduce a third, transfer comb \( B \) at the master site with repetition rate \( f_i + \Delta f \) and time offset \( \Delta t_{\text{cal}} \), also phase-locked to the master optical oscillator.

The calibration term, \( \Delta t_{\text{cal}} \), nominally reflects a time delay in the transceiver between the reference plane and the incoming pulse detection. However, in reality, each transceiver consists of multiple optical and rf paths between, for example, the optical oscillator, the frequency combs, the optical detection of the arriving frequency comb pulses, the various analog-to-digital converters, and throughout the communication-based O-TWTFT. Without motion, all these paths can be lumped into a single overall time delay. With motion and the resulting Doppler shifts, some combination of delays must be corrected for velocity. As a result, the overall transceiver must be calibrated via a built-in rf-domain optical time domain reflectometer (OTDR) that measures the various required delays. In a simplified view, the net result is that the calibration becomes velocity-dependent as

\[
\Delta t_{\text{cal}} = \Delta t_{\text{cal}} + O((V/c)^2).
\]

**Data availability**

The data that support the findings of this paper are available from the corresponding author upon reasonable request.

Received: 10 December 2018 Accepted: 28 March 2019
Published online: 18 April 2019

**References**

1. Chleb, J. F. & Shillue, B. Precision timing control for radioastronomy: maintaining femtosecond synchronization in the Atacama Large Millimeter Array. *IEEE Control Syst. 26*, 19–26 (2006).

2. Gill, P. When should we change the definition of the second? *Philos. Trans. R. Soc. A 369*, 4109–4110 (2011).

3. Schiller, S. Feasibility of giant fiber-optic gyroscopes. *Phys. Rev. A* 87, 033823 (2013).

4. Derevianko, A. & Pospelov, M. Hunting for topological dark matter with atomic clocks. *Nat. Phys. 10*, 933–936 (2014).

5. Stadnik, Y. V. & Flambaum, V. V. Searching for dark matter and variation of fundamental constants with laser and maser interferometry. *Phys. Rev. Lett. 114*, 161301 (2015).

6. Kolkowitz, S. et al. Gravitational wave detection with optical lattice atomic clocks. *Phys. Rev. D* 94, 124043 (2016).

7. Takano, T. et al. Geopotential measurements with synchronously linked optical lattice clocks. *Nat. Photonics 10*, 662–666 (2016).

8. Lisdat, C. et al. A clock network for geodesy and fundamental science. *Nat. Commun. 7*, 12443 (2016).

9. Delva, P. et al. Test of special relativity using a fiber network of optical clocks. *Phys. Rev. Lett. 118*, 221102 (2017).

10. Grotti, J. et al. Geodesy and metrology with a transportable optical clock. *Nat. Phys. 14*, 437–441 (2018).
11. Riehle, F. Optical clock networks. Nat. Photonics 11, 25–31 (2017).
12. Mehlstäubler, T. E., Grosche, G., Lisdat, C., Schmidt, P. O. & Denker, H. Atomic clocks for geodesy. Rep. Prog. Phys. 81, 064401 (2018).
13. Deschênes, J.-D. et al. Synchronization of distant optical clocks at the femtosecond level. Phys. Rev. X 6, 021016 (2016).
14. Sinclair, L. C. et al. Synchronization of clocks through 12 km of strongly turbulent air over a city. Appl. Phys. Lett. 109, 151104 (2016).
15. Sinclair, L. C. et al. Comparing optical oscillators across the air to milliradians in phase and 10$^{-17}$ in frequency. Phys. Rev. Lett. 120, 050801 (2018).
16. Kirchner, D. Two-way time transfer via communication satellites. Proc. IEEE 79, 983–990 (1991).
17. Fujieda, M. et al. Carrier-phase two-way satellite frequency transfer over a very long baseline. Metrologia 51, 253 (2014).
18. Warriner, J., Beckman, R., Celano, T., Miller, M. & Howe, P. Real-Time Two-Way Time Transfer to Aircraft (Air Force Research Lab Wright-Patterson AFB OH, Dayton, Ohio USA 2007).
19. Kim, J., Cox, J. A., Chen, I. & Kärtner, F. X. Drift-free femtosecond timing synchronization of remote optical and microwave sources. Nat. Photon 2, 733–736 (2008).
20. Ning, B. et al. High-precision distribution of highly stable optical pulse trains with 8.8 x 10$^{-19}$ instability. Sci. Rep. 4, 5109 (2014).
21. Krehlik, P. Sliewczyński, L., Buczek, Ł., Kołodziej, J. & Lipiński, M. Ultrastable long-distance fibre-optic time transfer: active compensation over a wide range of delays. Metrologia 52, 82 (2015).
22. Bercy, A. et al. Two-way optical frequency comparisons at 5 x 10$^{-21}$ relative stability over 100-km telecommunication network fibers. Phys. Rev. A 90, 061802 (2014).
23. Calosso, C. E. et al. Frequency transfer via a two-way optical phase comparison on a multiplexed fiber network. Opt. Lett. 39, 1177–1180 (2014).
24. Xin, M. et al. Attosecond precision multi-kilometer laser-microwave network. Light Sci. Appl. 6, e16187 (2017).
25. Lin, Z. et al. Development of sub-100 femtosecond timing and synchronization system. Rev. Sci. Instrum. 89, 014701 (2018).
26. Khader, I. et al. Time synchronization over a free-space optical communication channel. Optica 5, 1542–1548 (2018).
27. Richards, M. A. Fundamentals of Radar Signal Processing, 2nd Edition (McGraw-Hill Education, New York, NY, USA 2014).
28. Nelder, J. A. & Mead, R. A simplex method for function minimization. Comput. J. 7, 308–313 (1965).
29. Bergeron, H. et al. Tight real-time synchronization of a microwave clock to an optical clock across a turbulent air path. Optica 3, 441 (2016).
30. Cossel, K. C. et al. Open-path dual-comb spectroscopy to an airborne retroreflector. Optica 4, 724–728 (2017).
31. Robert, C., Conan, J.-M. & Wolf, P. Impact of turbulence on high-precision ground-satellite frequency transfer with two-way coherent optical links. Phys. Rev. A 93, 033860 (2016).
32. Belmonte, A., Taylor, M. T., Hollberg, L. & Kahn, J. M. Effect of atmospheric anisoplanatism on earth-to-satellite time transfer over laser communication links. Opt. Express 25, 15676–15686 (2017).
33. Swann, W. C. et al. Measurement of the impact of turbulence anisoplanatism on precision free-space optical time transfer. Phys. Rev. A 99, 023855 (2019).
34. Sinclair, L. C. et al. Femtosecond optical two-way time-frequency transfer in the presence of motion. Phys. Rev. A 99, 023844 (2019).

Acknowledgements
This work was funded by the National Institute of Standards and Technology (NIST) and the Defense Advanced Research Projects Agency (DARPA) PULSE program. We thank Prem Kumar, Martha Bodine, Jennifer Ellis, and Kyle Beloy for helpful discussions.

Author contributions
H.B., L.C.S., J.D.D., and N.R.N. determined the effects of motion and designed the algorithms. H.B. and J.D.D. implemented the digital signal processing. H.B., J.D.D., I.K., L.C.S., and W.C.S. acquired and analyzed the data. K.C.C. designed and implemented the tracking terminal necessary for quadcopter operation. W.C.S. designed and implemented the swept delay line. M.C. and K.C.C. assisted with hardware implementation and quadcopter operation. L.C.S., J.D.D., and N.R.N. prepared the paper.

Additional information
Supplementary Information accompanies this paper at https://doi.org/10.1038/s41467-019-09768-9.

Competing interests: The authors declare no competing interests.

Reprints and permission information is available online at http://npg.nature.com/reprintsandpermissions/

Journal peer review information: Nature Communications would like to thank the Jungwon Kim and the other anonymous reviewers for their contribution to the peer review of this work. Peer review reports are available.

Publisher’s note: Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.