Bat algorithm for multilevel image thresholding based on Otsu and Kapur's entropy

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Abstract. Multilevel image thresholding has attracted plenty of attention in the past decades. Otsu and Kapur's entropy-based methods are often applied to search the optimal bi-thresholding. These techniques are also suitable for multilevel thresholds. However, it takes a lot of computation to solve the multilevel threshold problem. To address this problem, in this paper, a recently proposed bat algorithm is used to find the appropriate multilevel thresholds, in which Otsu and Kapur's entropy is regarded as its fitness functions. Evaluation of image segmentation effect is performed using the peak-to-signal ratio (PSNR) and structural similarity (SSIM) index. The experiment results show that Otsu based method is more suitable for multi-level threshold image segmentation.

Keywords: Bat algorithm; multilevel thresholds; Otsu; Kapur's entropy.

1. Introduction
Threshold image segmentation methods are often used in gray image segmentation. Usually, there are two means involved in image segmentation: bi-level threshold and multi-level threshold segmentation [1]. Bi-level threshold image segmentation is suitable for dividing images into target and background categories. When the target image contains multiple valuable targets, it is difficult to separate multiple targets and backgrounds with a single threshold. Furthermore, it takes a lot of time to calculate multiple thresholds. The traditional exhaustive search method is difficult to solve the multi-level threshold problems [2]. Nowadays, more and more swarm intelligence algorithms have been proposed to solve optimization problems such as fuzzy gravity search algorithm (FGSA) [3], black window optimization (BWO) [4], differential evolution (DE) [5], whale optimization algorithm (WOA) [6] and bat algorithm (BA) [7].

Bat algorithm due to its simple structure and fewer control parameters and good robustness, and has been widely used in various fields. So we use the bat algorithm to solve the multi-level threshold optimization problem, two kinds of metrics are used as the fitness function of the segmented image such as: Otsu and Kapur's entropy. In the experiments, PSNR and SSIM will be used to measure the effect of comparing the two objective functions for image segmentation.
2. Multilevel thresholding

Multilevel thresholding method for gray image segmentation is to find a suitable combination of thresholds to segment an image. The desirable thresholds are solved by optimizing a fitness function. Maximum between-class variance (Otsu) and Kapur's entropy-based techniques are widely used to solve the optimal multilevel thresholding. Let \( p_i = p_0, p_1, ..., p_{L-1} \) be the probability distribution of the L gray levels for a gray image, where \( L \) is restricted in the \([0, L-1]\). The probability distribution \( p_i \) can be calculated by:

\[
N = \sum_{i=0}^{L-1} n(i) \quad p_i = n(i)/ N
\]

where \( n(i) \) denotes the count times of gray level \( i \), and \( N \) represents all the pixels of a grayscale image.

2.1. Otsu method

Otsu has been proposed for gray image segmentation for a long time. Let us consider that there are “\( m \)” thresholds. These threshold subdivide the image into “\( m+1 \)” parts: A threshold divides the image into two parts, two thresholds to get three target regions, it is easy to extend the threshold to more than one. The objective function based on maximizing Otsu can be expressed as [8]:

\[
F_o(t_1, t_2, ..., t_m) = \delta_1^2 + \delta_2^2 + ... + \delta_m^2
\]

where

\[
\delta_i^2 = \omega_i (\mu_i - \mu_\tau)^2, \quad \omega_i = \sum_{i=0}^{L-1} p_i/ \omega_s;
\]

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\( \sigma_1^2, \sigma_2^2, ..., \sigma_m^2 \) are the variances. \( \omega_0, \omega_1, ..., \omega_m \) are the each part probabilities. \( \mu_0, \mu_1, ..., \mu_m \) are the mean levels of each segmented class. Besides, \( \mu_\tau \) is the average intensity for the total image and satisfies the following conditions:

\[
\omega_0 \mu_0 + \omega_1 \mu_1 + ... + \omega_m \mu_m = \mu_\tau,
\]

\[
\omega_0 + \omega_1 + ... + \omega_m = 1
\]

The optimal multilevel thresholds \([t_1, t_2, ..., t_m] \) for a specific image can be calculated by maximizing the fitness function:

\[
[t_1, t_2, ..., t_m] = \text{arg max} (F_o)
\]

2.2. Kapur's entropy

The same as the Otsu method, if they are \( m \) number of thresholds \((t_1, t_2, ..., t_m) \) to be selected an image. The fitness function based on maximizing Kapur's entropy can be expressed by the following formula [9]:

\[
F_k(t_1, t_2, ..., t_m) = K_0 + K_1 + ... + K_m
\]

where
\[ K_i = -\sum_{i=0}^{K_i} (p_i / \omega_k) \ln(p_i / \omega_k), \omega_k = \sum_{i=0}^{K_i} p_i; \]
\[ K_j = -\sum_{j=0}^{K_j} (p_j / \omega_l) \ln(p_j / \omega_l), \omega_l = \sum_{j=0}^{K_j} p_j; \]
\[ \vdots \]
\[ K_m = -\sum_{i=0}^{K_m} (p_m / \omega_m) \ln(p_m / \omega_m), \omega_m = \sum_{i=0}^{K_m} p_i. \]

K0, K1, ..., Km are the Kapur’s entropies. \(\omega_0, \omega_1, ..., \omega_m\) are the each probabilities of every segmented part. The optimal multilevel thresholds \([t_1, t_2, ..., t_m]\) can be measured by maximizing the following fitness function:
\[ \{[t_1, t_2, ..., t_m]\} = agr \max(F_x) \]

3. Bat algorithm

Bat algorithm is a widely used meta-heuristic algorithm which imitates the predation behavior of bats [10]. In nature, bat obstacles and discovery targets are achieved through echolocation. By emitting sound waves of a certain frequency to the surroundings, when the sound waves encounter obstacles, the sound waves will be reflected. The longer the distance from the obstacles, the stronger the reflected sound waves, and vice versa. The bats can use this kind of echolocation. Then determining the location and distance of obstacles.

During predation, bats going somewhere with velocity \(v_i\) at position \(x_i\) with initial frequency \(f_{min}\), wavelength \(\lambda\) and loudness \(A_0\). Then continuously adjusting the emission frequency \(r_i\) of the sound waves by judging the distance from the target. Bats constantly change their position and flying speed while searching for food. Suppose at a certain time \(t\), a bat is flying at position \(x_i^*\) with speed \(v_i^*\) cities, then the next position is updated according to the following equations:
\[ f_i = f_{min} + (f_{max} - f_{min}) \beta \]
\[ v_i^* = v_i^* + (x_i^* - x_i^*) f_i \]
\[ x_i^* = x_i^* + v_i^* \]

Where \(\beta \in [0, 1]\) is a random number that obeys a uniform distribution function. Besides, \(x_i^*\) is the present global best location among the whole population. \(f_{min}\), \(f_{max}\) is the minimum frequency and maximum frequency, respectively. Usually, \(f_{min}=0\) and \(f_{max}=1\). In the process of flight, individual bats will fly randomly and explore the surrounding environment, and the process can be expressed by the formula:
\[ x_i^{new} = x_i^* + \varepsilon A_i \]

Where \(\varepsilon \in [-1, 1]\) is a random number, while \(A_i\) is the mean loudness of all the optimal individual at step \(t\). The principles for updating loudness \(A_i\) and pulse rate emission \(r_i\) can be defined as follows:
\[ A_i^{new} = \alpha A_i, r_i^{new} = r_i^{new} [1 - \exp(-\gamma r)] \]

Where \(\alpha\) and \(\gamma\) are predefined numbers.

4. Experimental study

4.1. Parameter Setting

The experiments are conducted on 4 standard pictures. They are often used in image segmentation which are named airfield, crowd, dollar, and plane, as shown in Fig. 1. In the experiments, the population size of BA is set to 25, the number of loop iterations is set to 200, all the images are segmented into 2, 3, 4, and 5 thresholds.
4.2. Quality measures

The popular performance measure index, PSNR is used to estimate the image effect after segmentation. The PSNR is calculated by [6]:

\[
PNSR = 20\log_{10}\left(\frac{255}{RMSE}\right) \quad (dB)
\]  \hspace{1cm} (14)

Where RMSE is the root-mean-squared error, expressed as:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}(I(i, j) - S(i, j))^2}{M \times N}}
\]  \hspace{1cm} (15)

Where \( I \) and \( S \) are original and segmented images of size, \( M \times N \), respectively.

SSIM index is used to compare the structural changes of the segmented image with the original image which can be denoted as the following formula:

\[
SSIM_{(I,S)} = \frac{(2\mu_I\mu_S + C_a)(2\delta IS + C_s)}{(\mu_I^2 + \mu_S^2 - C_a)(\delta_I^2 + \sigma_S^2 + C_s)}
\]  \hspace{1cm} (16)

Where \( Ca=(KaL)^2, Cb=(KbL)^2 \), and other parameters are introduced in reference [4].

4.3. The experimental results-based bat algorithm

The experimental results of the bat algorithm, and compared to different fitness functions are listed in Table 1 and Table 2. Table 1 indicates the optimal threshold values solved by the bat algorithm based on Otsu and Kapur's entropy. The results in table 1 indicate that the bat algorithm can effectively solve the suitable thresholds when using different objective functions. Besides, the optimal thresholds solved by the two different objective functions are not the same, and all the thresholds are relatively uniform. Table 2 gives the PSNR, and SSIM values, we can conclude that with the number of the threshold increases, the SSIM and the PSNR values are increased for all test images on different objective functions. From the analysis of the results, it can be seen that the Otsu method has advantages in the indicator SSIM and PSNR. Therefore, the image segmentation method based on Otsu has obvious advantages compared with Kapur's entropy.
Table 1. The optimal threshold values acquired by the bat algorithm using Otsu and Kapur's entropy

| Test images | \( t \) | Otsu Threshold values | Kapur's entropy Threshold values |
|-------------|-------|----------------------|----------------------------------|
| Airfield    | 2     | 92 165              | 93 165                           |
| 3           | 88 155 213 |                  | 67 118 175                       |
| 4           | 74 124 167 216 |              | 61 108 154 200                    |
| 5           | 56 101 141 177 218 |            | 56 97 135 171 205                 |
| 2           | 84 155 |                  | 100 172                          |
| crowd       | 3     | 71 120 185          | 82 135 195                        |
| 4           | 65 101 147 203 |              | 73 111 154 194                    |
| 5           | 61 86 119 158 205 |            | 67 100 136 172 205                |
| 2           | 104 174 |               | 95 175                           |
| Dollar      | 3     | 86 137 187          | 82 139 195                        |
| 4           | 73 114 152 193 |              | 38 90 143 196                     |
| 5           | 46 83 122 158 194 |         | 31 72 113 155 198                 |
| 2           | 99 158 |                  | 54 154                           |
| plane       | 3     | 77 129 175          | 51 109 165                        |
| 4           | 66 110 154 185 |              | 47 87 127 168                     |
| 5           | 33 77 115 156 185 |            | 43 73 107 139 170                 |

Table 2. The PSNR and SSIM measures of Otsu and Kapur's entropy

| Test images | \( t \) | Otsu | Kapur's entropy |
|-------------|-------|-----|----------------|
| PSNR | SSIM | PSNR | SSIM |
| Airfield | 2     | 20.8416 | 0.4767 | 20.8426 | 0.4321 |
| 3     | 23.2175 | 0.5251 | 21.9792 | 0.5266 |
| 4     | 25.2072 | 0.6163 | 24.6556 | 0.5994 |
| 5     | 26.5676 | 0.6865 | 25.3221 | 0.6012 |
| 2     | 21.5588 | 0.5759 | 22.0584 | 0.3484 |
| crowd | 3     | 22.8132 | 0.6288 | 22.9589 | 0.4379 |
| 4     | 23.6349 | 0.6680 | 23.6015 | 0.4955 |
| 5     | 24.4793 | 0.6989 | 24.5127 | 0.5362 |
| 2     | 23.1852 | 0.7284 | 23.3413 | 0.6007 |
| Dollar | 3     | 25.0371 | 0.7674 | 24.3618 | 0.6481 |
| 4     | 25.7097 | 0.7868 | 25.0399 | 0.6588 |
| 5     | 26.4281 | 0.7951 | 25.7165 | 0.7902 |
| 2     | 21.1647 | 0.5953 | 21.4054 | 0.2474 |
| plane | 3     | 21.2086 | 0.6110 | 21.7246 | 0.3140 |
| 4     | 22.0481 | 0.6251 | 21.9331 | 0.3504 |
| 5     | 22.4532 | 0.6315 | 21.8442 | 0.6539 |

5. Conclusion
In this paper, we used the bat algorithm to search the multi-level threshold for 4 test images, and taken Otsu and Kapur's entropy as its fitness functions. The experimental results show that Otsu based method is more suitable for multi-level threshold image segmentation according to the PSNR and SSIM results.

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