SecSkyline: Fast Privacy-Preserving Skyline Queries Over Encrypted Cloud Databases

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Abstract—The well-known benefits of cloud computing have spurred the popularity of database service outsourcing, where one can resort to the cloud to conveniently store and query databases. Coming with such popular trend is the threat to data privacy, as the cloud gains access to the databases and queries which may contain sensitive information, like medical or financial data. A large body of work has been presented for querying encrypted databases, which has been mostly focused on secure keyword search. In this paper, we instead focus on the support for secure skyline query processing over encrypted outsourced databases, where little work has been done. Skyline query is an advanced kind of database query which is important for multi-criteria decision-making systems and applications. We propose SecSkyline, a new system framework building on lightweight cryptography for fast privacy-preserving skyline queries. SecSkyline ambitiously provides strong protection for not only the content confidentiality of the outsourced database, the query, and the result, but also for data patterns that may incur indirect data leakages, such as dominance relationships among data points and search access patterns. Extensive experiments demonstrate that SecSkyline is substantially superior to the state-of-the-art in query latency, with up to 813× improvement.

Index Terms—Secure skyline queries, encrypted databases, secure outsourcing, cloud computing

1 INTRODUCTION

Due to the well-known benefits of cloud computing [1], [2], there has been growing popularity of enterprises or organizations leveraging commercial clouds to store and query their databases (e.g., [3], [4], [5], [6], to list a few). However, as databases may contain rich sensitive and proprietary information (like databases of medical records or financial records), deploying such database services in the cloud may raise critical privacy concerns. Therefore, there is an urgent demand that security must be embedded in such database outsourcing services, providing protection for the information-rich databases, private queries, as well as query results. In the literature, a large body of work has been presented for querying encrypted databases, which has been mostly focused on secure keyword search [7], [8], [9], [10].

In this paper, we instead focus on secure skyline queries over outsourced databases, where little work has been done. Given a query point, skyline query aims to retrieve a set of data points (called skyline points) which are not dominated by any other data point from a multi-dimensional database [11]. In particular, given a query point \( q \) and two data points \( a \) and \( b \) in the target database, \( a \) is said to dominate \( b \) if \( a \) is nearer to \( q \) than \( b \) at least in one dimension and not farther in other dimensions.

Skyline query is highly useful for multi-criteria decision-making systems in different domains, such as web information systems [12], wireless mobile ad-hoc networks [13], and geographical information systems [14], especially when it is hard to define a single distance metric with all dimensions [15].

To make the problem we focus on more concrete, we brief an example application to demonstrate how skyline query works. Consider a medical institution who outsources its database of medical records to the cloud to share its diagnosis experiences. Table 1 shows the original database \( P \), where each record (i.e., a tuple) contains values for health-related attributes about a patient, including the respiratory rate \( R \) and heart rate \( H \). A doctor from another medical organization has a patient record with \( (R = 16, H = 100) \), and wants to retrieve records from \( P \) whose conditions are similar to that of the patient based on skyline query processing. Therefore, the doctor sends a query...
The challenge that we aim to tackle in this paper is how to enable fast and privacy-preserving skyline queries over encrypted cloud databases. With respect to the above application scenario as an example, we aim to allow the cloud hosting the database \( \mathbb{P} \) in encrypted form to produce the skyline query result \( \{ p_1, p_4 \} \) in encrypted form as well. Meanwhile, besides ensuring the data content confidentiality for direct protection, it is also demanded that the cloud should be prevented from knowing data patterns which may cause indirect data leakage [15], [16], [17]. Such data patterns include the dominance relationships among database tuples, the number of database tuples that each skyline tuple dominates, and the search access patterns. Here the search pattern implies whether a new skyline query has been issued before and the access pattern reveals which database tuples are the skyline tuples.

In the literature, privacy-preserving skyline queries has recently received increasing attentions and several research endeavors have been proposed [15], [16], [17], [18], [19], [20], [21]. The state-of-the-art prior works [16], [17] that are most related to ours rely on heavy cryptosystems to craft secure protocols, leading to substantial performance overheads which heavily hinders the practical usability. In particular, even over very small-scale databases (e.g., with 1000 2-dimensional tuples), the state-of-the-art works [16], [17] still require processing latency of more than 1000 seconds and 100 seconds, respectively. Therefore, how to enable privacy-preserving skyline queries with practical performance is still challenging and remains to be fully explored.

In light of the above, in this paper, we propose SecSkyline, a new system framework that allows fast privacy-preserving skyline queries over encrypted databases outsourced to the cloud. Different from prior arts [16], [17], SecSkyline fully utilizes a lightweight cryptographic technique—additive secret sharing [22] and achieves substantially superior performance in query latency. We first conduct an in-depth examination on the procedure of skyline query processing and identify that it can be decomposed into several essential components for which we provide customized secure realizations.

Specifically, we first consider how to support secure database mapping given an encrypted query, allowing the cloud to securely map the encrypted outsourced database to the new space so as to facilitate the subsequent secure skyline tuples fetching. SecSkyline introduces an effective technique to tackle the challenging operation of computing absolute value in the secret sharing domain, securely realizing the operation of database mapping. Then, SecSkyline introduces techniques to support secure skyline fetching, allowing the cloud to obliviously fetch skyline tuples without knowing which tuples they are in the database. After that, SecSkyline provides techniques for secure skyline and dominated tuples filtering to tackle the remaining challenge, i.e., how to allow the cloud to obliviously filter out a currently found skyline tuple and the tuples dominated by it from the mapped database without knowing which tuples they are and the number of dominated tuples. The synergy of these secure components lead to the full protocol for fast privacy-preserving skyline queries developed in SecSkyline.

We implement our protocol and conduct extensive experiments on several datasets. The experiment results show that SecSkyline achieves substantial performance boost compared to the state-of-the-art works FSSP [16] and SMSQ [17]. Specifically, SecSkyline improves upon FSSP by up to 8130× and improves upon SMSQ by up to 813× in query latency. We highlight our main contributions below:

- We present SecSkyline, a new system framework for secure skyline queries over encrypted databases outsourced to the cloud, which provides strong protection for the content confidentiality of the outsourced database, the query, and the result, as well as data patterns that may incur indirect data leakages.
- We devise a suite of secure and lightweight components to support oblivious skyline query processing at the cloud, including secure database mapping, secure skyline tuples fetching, and secure skyline and dominated tuples filtering.
- We formally analyze the security of SecSkyline and conduct extensive evaluations over several datasets. The results demonstrate that under the same system model and security guarantees, SecSkyline can achieve up to 813× better query latency over the state-of-the-art [17], with promising scalability.

The rest of this paper is organized as follows. Section 2 discusses the related work. In Section 3, we introduce preliminaries. Then, we introduce our system architecture and threat model in Section 4. After that, we present the design of SecSkyline in Section 5, followed by security analysis and experiments in Sections 6 and 7, respectively. Finally, we conclude this paper in Section 8.
2 RELATED WORK

2.1 Skyline Query in Plaintext Domain

The skyline operator in the database field is first proposed by Börzsönyi et al. [23]. Since this seminal work, great efforts have been devoted to advancing the design of skyline query schemes. Kossmann et al. [24] study the online skyline using the nearest neighbor method. Papadias et al. [25] propose the branch and bound skyline algorithm, achieving performance boost in terms of efficiency and storage over prior works. The problem of skyline queries in different scenarios has also been widely studied, such as skyline on data streams [26], uncertain skyline [27], [28], and group-based skyline [29], [30]. However, all of them consider the execution of skyline queries in the plaintext domain without considering privacy protection.

2.2 Secure Skyline Query Processing

Bothe et al. [18] initiate the first study on secure skyline query processing. They introduce a preliminary approach that relies on a mechanism which multiplies vectors via secret matrices for protection. Their approach does not provide formal and rigorous security guarantees. Recently, Liu et al. propose the FSSP scheme [16] (which first appeared in [15]), and Ding et al. propose the SMSQ scheme [17]. Both these recent schemes provide strong cryptographic guarantees for the databases, the skyline queries, as well as the query results. However, as mentioned above, FSSP [16] and SMSQ [17] rely on the use of heavy cryptosystems and incur substantial performance overheads, which heavily affect their practical usability. In contrast, SecSkyline is a new system design for fast privacy-preserving skyline queries over encrypted databases hosted in the cloud, which fully builds on lightweight cryptography and achieves performance substantially better than the state-of-the-art prior schemes FSSP [16] and SMSQ [17].

There are some works [19], [21] focusing on privacy-preserving skyline query under application scenarios different from ours. Specifically, the work [21] studies privacy-preserving user-defined skyline queries, focusing on a different and simplified case of constrained subspace skyline queries, where the client specifies a constrained region to search. The underlying skyline query targeted in our security design as well as the prior works [16], [17] is generic and much more challenging. In addition, it is noted that the scheme in [21] does not offer protection for the access pattern. Wang et al. [19] focus on the support for verifiability with respect to location-based skyline queries where the client is only allowed to customize its skyline queries with two spatial attributes. In addition, to achieve affordable online query latency, the scheme in [19] requires the data owner to pre-compute the dominance relationships with respect to non-spatial attributes among database tuples before outsourcing them to the cloud. An additional work by Wang et al. [20] proposes a trusted hardware-based approach for privacy-preserving skyline query. Such approach requires to put additional trust on trusted hardware vendors. Moreover, in recent years, various attacks against trusted hardware have been proposed [31], [32], [33], [34], which pose severe threats to trusted hardware-based secure systems, but the solution in [20] does not consider these attacks. Hence, the state-of-the-art prior works that are most related to ours are [16], [17].

3 PRELIMINARIES

3.1 Skyline Query

Definition 1. Given a database \( P = \{p_1, \ldots, p_n\} \), where each database tuple \( p_i \) is an \( m \)-dimensional vector, i.e., a tuple where each dimension corresponds to an attribute. Let \( p_a \) and \( p_b \) be two different tuples in \( P \). We say \( p_a \) dominates \( p_b \), if and only if \( \forall j \in [1, m], p_a[j] \leq p_b[j] \) and \( \exists j \in [1, m], p_a[j] < p_b[j] \). Then the skyline tuples are tuples that are not dominated by any other tuple.

Given a query tuple, the skyline query targeted in this paper aims to retrieve from a database tuples that are not dominated by any other tuple [15], [35]. The formal definition of skyline query considered in this paper is given below [16]:

Definition 2. Given a query tuple \( q \) and a database \( P = \{p_1, \ldots, p_n\} \), where \( q \) has the same dimension as each tuple in \( P \). Let \( p_a \) and \( p_b \) be two different tuples in \( P \). We say \( p_a \) dynamically dominates \( p_b \) with respect to \( q \) if and only if \( \forall j \in [1, m], |p_a[j] - q[j]| \leq |p_b[j] - q[j]| \) and \( \exists j \in [1, m], |p_a[j] - q[j]| < |p_b[j] - q[j]| \). A skyline tuple with respect to \( q \) is a tuple that is not dominated by any other tuple. The set of skyline tuples under \( q \) is denoted by \( SK_q \).

Algorithm 1 shows the plaintext-domain processing of the skyline query [15], [16]. Given a database \( P \) and a query tuple \( q \), the first step is to map the database \( P \) to a new database (referred to as mapped database) with respect to \( q \) (i.e., lines 1-6). Then, for each tuple in the initial mapped database (i.e., \( T^{(0)} \)), the sum over its all attributes (i.e., lines 7-9) is computed. Skyline tuples are selected from the mapped
and, additive secret sharing in a two-party respectively, some operations can be performed by the parties. For the skyline query, the power of the cloud is divided into two cloud servers (denoted by $C_1$ and $C_2$) who can be hosted by independent cloud service providers, e.g., Google, AWS, and Microsoft in practice. Such a two-server model has also been adopted in state-of-the-art prior works on privacy-preserving skyline queries [15], [16], [17], as well as in other application domains [37], [38], [39], [40], [41], [42], [43], [44], [45]. In addition to the adoption in academia, the two-server model has also gained increasing traction in industry. For example, Mozilla initiates a secure telemetry data collection service on Firefox under the two-server model [46]; Apple and Google collaboratively provide users with automated alerts about potential COVID-19 exposure, while providing strong privacy guarantees [47]. SecSkyline follows such trend and contributes a new design for enabling fast privacy-preserving skyline queries over encrypted cloud databases.

### 3.2 Additive Secret Sharing

Additive secret sharing [22] is a lightweight encryption technique that allows some secure computation. Given a secret value $x \in \mathbb{Z}_2$, additive secret sharing in a two-party setting works by splitting it into two secret shares $(x)_1 \in \mathbb{Z}_2$ and $(x)_2 \in \mathbb{Z}_2$. For $l > 1$, $x = (x)_1^l + (x)_2^l$ in $\mathbb{Z}_2$ and such sharing is referred to as arithmetic sharing. For $l = 1$, $x = (x)_1 + (x)_2$ in $\mathbb{Z}_2$, and such sharing is referred to as binary sharing. Each share alone reveals no information about $x$. The shares are to be held by two parties $P_1$ and $P_2$ respectively for subsequent secure computation. We write $[x]^A$ and $[y]^A$ respectively to clearly distinguish between arithmetic sharing and binary sharing in the above form.

With the shares of two secret values $x$ and $y$ held by two parties $P_1$ and $P_2$ respectively, some operations can be performed securely among them. We use arithmetic sharing to illustrate the secure computation. Note that in binary sharing, the only differences are that addition/subtraction operations are replaced by XOR ($\oplus$) and multiplication operations are replaced by AND ($\otimes$).

In particular, the addition/subtraction between two secret-shared values $[x]^A$ and $[y]^A$ only requires local computation at each party, i.e., $z = (x)_1 + (y)_1$, $i \in \{1, 2\}$. Also, the scalar multiplication between a public value $\eta$ and a secret-shared value $[x]^A$ also only requires local computation, i.e., $z = \eta \cdot (x)_1$. The multiplication between two secret-shared values $[x]^A$ and $[y]^A$, however, requires one round of online communication. Specifically, to compute $[z]_w$ where $z = xy$, $P_1$ and $P_2$ need to additionally have as input a secret-shared Beaver triple $([u]^A, [v]^A, [w]^A)$ which can be prepared offline [36], where $w = uw$. Then, each party first locally computes $(z)_1 = (x)_1 - (w)_1$, $(z)_2 = (y)_1 - (w)_1$, and then reveal $e$ and $f$ to each other. Finally, $P_1$ and $P_2$ locally compute the secret shares of $z$ by $(z)_1 = e \cdot f + f \cdot (w)_1 + e \cdot (w)_1 + (w)_1$ and $(z)_2 = f \cdot (w)_2 + e \cdot (w)_2 + (w)_2$, respectively. For simplicity, we write $[z]^A = [x]^A \cdot [y]^A$ to denote such secure multiplication. In addition, the NOT operation (denoted by $\neg$) in binary secret sharing domain can be realized by letting one of $P_1$ and $P_2$ locally flip the share it holds, e.g., $\neg(x)_1 = (\neg x)_1^A$, $\neg(x)_2 = (\neg x)_2^A$.

### 4 Problem Statement

#### 4.1 System Architecture

Fig. 2 illustrates the system architecture of SecSkyline. There are three kinds of entities: the data owner, the client, and the cloud. The data owner can be an organization (e.g., a medical institution), who has a database $P$ and wants to offer skyline query services to clients (e.g., doctors in a hospital). To leverage the well-known benefits of cloud computing [1], [2], the data owner intends to store the database $P$ in the cloud, who then helps provide skyline query services for the client. Due to privacy concerns, it is demanded that security must be embedded in such cloud-empowered service, safeguarding the database $P$, skyline query $q$, as well as the corresponding query result $SK_q$.

For high efficiency, in SecSkyline we resort to a lightweight cryptographic technique—additive secret sharing—for fast encryption of the database and skyline query and for supporting subsequent secure processing in the cloud, through a customized design. To be compatible with the working paradigm of additive secret sharing, the power of the cloud in SecSkyline is divided into two cloud servers (denoted by $C_1$ and $C_2$) who can be hosted by independent cloud service providers, e.g., Google, AWS, and Microsoft in practice. Such a two-server model has also been adopted in state-of-the-art prior works on privacy-preserving skyline queries [15], [16], [17], as well as in other application domains [37], [38], [39], [40], [41], [42], [43], [44], [45]. In addition to the adoption in academia, the two-server model has also gained increasing traction in industry. For example, Mozilla initiates a secure telemetry data collection service on Firefox under the two-server model [46]; Apple and Google collaboratively provide users with automated alerts about potential COVID-19 exposure, while providing strong privacy guarantees [47]. SecSkyline follows such trend and contributes a new design for enabling fast privacy-preserving skyline queries over encrypted cloud databases.

#### 4.2 Threat Model

Similar to the state-of-the-art prior works on privacy-preserving skyline queries [16], [17] as well as other works in the two-server setting [39], [41], [48], [49], we assume a semi-honest and non-colluding adversary model where each cloud server honestly follows our protocol, yet may individually attempt to learn the private information from the execution of (dynamic) skyline queries. Following the prior works [16], [17], we consider the data owner and the client as trustworthy parties, who will honestly follow the protocol specification.

Under the above threat model and following the state-of-the-art prior works [16], [17], SecSkyline aims to protect against the cloud servers (i) the content of the database $P$, skyline query $q$, and query result $SK_q$, (ii) the dominance relationships among database tuples, (iii) the number of database tuples that each skyline tuple dominates, and (iv) search access patterns. Following the standard definitions in searchable encryption [50], we describe the search access patterns in secure skyline queries as follows.
Definition 3. Search Pattern. For two skyline queries \( q \) and \( q' \), define \( \Sigma(q, q') \in \{0, 1\} \), where \( \Sigma(q, q') = 1 \) if and only if the two queries are identical, and otherwise \( \Sigma(q, q') = 0 \). Here, “identical” means that all corresponding attribute values of \( q \) and \( q' \) are identical. Let \( Q = \{q_1, \ldots, q_t\} \) be a non-empty sequence of skyline queries. The search pattern reveals an \( r \times r \) (symmetric) matrix with element \((i, j)\) equal to \( \Sigma(q_i, q_j) \).

In short, the search pattern implies whether a new skyline query has been issued before.

Definition 4. Access Pattern. Given a skyline query \( q \) on the database \( P \), the access pattern reveals the indexes of skyline tuples with respect to \( q \) in \( P \).

In practice, the access pattern reveals which database tuples are the skyline tuples with respect to a given query.

5 The Design of SecSkyline

5.1 Overview

At a high level, the protocol in SecSkyline proceeds through the following phases. First, in an initialization phase, the data owner adequately encrypts each tuple in its database \( P \) under arithmetic additive secret sharing and produces \([P]^A\). The data owner then sends the secret shares \([P]^A\) and \([P]^B\) to \( C_1 \) and \( C_2 \), respectively. Subsequently, it comes to the online query phase, where the client first encrypts its skyline query tuple \( q \) through arithmetic sharing and sends the secret shares \([q]^A\) and \([q]^B\) to the cloud servers \( C_1 \) and \( C_2 \), respectively. Hereafter, for simplicity of presentation, we will write \( C_{1,2} \) to represent the two cloud servers \( C_1 \) and \( C_2 \). Upon receiving the encrypted query \([q]^A\), \( C_{1,2} \) securely process the encrypted skyline query over \([P]^A\) as per the customized design of SecSkyline.

To allow \( C_{1,2} \) to perform the skyline query processing (i.e., Algorithm 1) in an oblivious manner, we first conduct an in-depth examination on the whole procedure and decompose it into several essential components, for which we provide customized secure realizations. Specifically, we identify and devise the following secure components for supporting secure skyline queries.

- **Secure database mapping secMap.** Given the encrypted database \([P]^A\) and query \([q]^A\), SecSkyline provides secMap to have \( C_{1,2} \) securely map the encrypted database \([P]^A\) to the encrypted mapped database \([T]^A\) with respect to the query \([q]^A\). From the process in Algorithm 1, lines 1-6, we observe that the challenge here is to securely calculate the absolute value \( t[j] = |p[j] - q[j]| \) in the secret sharing domain. Therefore, we design a tailored protocol to allow \( C_{1,2} \) to securely evaluate the encrypted absolute value \(|a - b|^A\) when they hold the secret shareings \([a]^A\) and \([b]^A\).

Our solution to this challenge is based on the following observation

\[
|a - b| = (a < b) \cdot (b - a) + (a < b) \cdot (a - b),
\]

where \((\cdot)\) represents the NOT operation and \((a < b) = 1 \in Z_2 \) if \( a < b \) and \((a < b) = 0 \in Z_2 \) if \( a \geq b \). Given this observation, what needs to be considered is how to securely realize the computation of \((a < b)\) as well as the NOT operation in the secret sharing domain. As mentioned in Section 3.2, the NOT operation on a secret-shared bit can be simply achieved by letting one of \( C_{1,2} \) (\( C_1 \) undertakes this in SecSkyline) locally flip the share it holds. So it remains to be considered how to allow \( C_{1,2} \) to securely evaluate \( a < b \) with the secret shareings \([a]^A\) and \([b]^A\).

Next, we introduce the detailed design of secMap in Section 5.2, secFetch in Section 5.3, and secFilt in Section 5.4. Afterwards, in Section 5.5, we give the complete protocol in SecSkyline for secure skyline query processing at the cloud, which relies on the synergy of the three secure components devised in SecSkyline.

5.2 Secure Database Mapping

Secure database mapping secMap aims at allowing \( C_{1,2} \) to securely map the encrypted database \([P]^A\) to the encrypted mapped database \([T]^A\) with respect to the query \([q]^A\). From the process in Algorithm 1, lines 1-6, we observe that the challenge here is to securely calculate the absolute value \( t[j] = |p[j] - q[j]| \) in the secret sharing domain. Therefore, we design a tailored protocol to allow \( C_{1,2} \) to securely evaluate the encrypted absolute value \(|a - b|^A\) when they hold the secret shareings \([a]^A\) and \([b]^A\).

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Here we resort to the strategy of secure bit decomposition in the secret sharing domain [51], [52]. Specifically, given \( a, b \in Z_2 \) under two’s complement representation, the most significant bit (MSB) of \( a - b \) (denoted as \( msb(a - b) \)) can indicate whether \( a < b \) or not. Namely, if \( a - b < 0 \), \( msb(a - b) = 1 \) and otherwise \( msb(a - b) = 0 \). Secure extraction of the MSB in the secret sharing domain can be achieved by securely realizing a parallel prefix adder (PPA), which only requires basic \( \oplus \) and \( \otimes \) operations in the secret sharing domain. Fig. 3 illustrates an 8-bit PPA for
MSB extraction. In [51], a concrete construction for secure MSB extraction based on PPA was provided, which allows two parties holding the secret sharings of two values \(a\) and \(b\) as input to obtain the secret sharing of the MSB of \(a - b\). Let \(\text{SecExt}\) denote the secure MSB extraction protocol, for which we have \([\text{msb}(a - b)]^B = \text{SecExt}(a^A, b^A)\). For more details on the construction \(\text{SecExt}\), we refer the readers to [51]. It is noted that the output \([\text{msb}(a - b)]^B\) from \(\text{SecExt}\) is in binary secret sharing domain. However, we need to obtain \([a - b]_b\) as the result according to the computation in Eq. 1. So we need to consider how to perform multiplication between \([\text{msb}(a - b)]^B\) i.e., \([a < b]_B\) and \([b - a]_b\) as well as \([-\text{msb}(a - b)]^B\) i.e., \([- (a < b)]]_B\) and \([a - b]_b\). That is, given the secret sharings \([x]^B\) and \([y]^A\), we want to obtain \([x \cdot y]^A\). Inspired by [52], \(\text{SecSkyline}\) deals with the multiplication of secret-shared values in different domains as follows.

1. \(C_1\) draws a random value \(r_1 \in \mathbb{Z}_{2^l}\) and constructs two messages: \(m_0 := (\mu + (x)^A_1 \cdot (y)^A_1) - r_1, \mu \in \{0, 1\}\), and then sends \(m_0, m_1, c_1\) to \(C_2\).
2. \(C_2\) chooses \(m_1\) according to the secret share \((x)^B_1\) it holds. That is, \(C_2\) chooses \(m_0\) if \((x)^B_1 = 0\) and \(C_2\) chooses \(m_1\) if \((x)^B_1 = 1\). Then, \(C_2\) holds the intermediate value \(x \cdot (y)^A_1 - r_1\) and \(C_1\) holds \(r_1\).
3. For the secret share \((y)^A_2\), \(C_2\) acts as the sender and \(C_1\) acts as the receiver to repeat step 1) and 2). Then \(C_1\) holds the intermediate value \(x \cdot (y)^A_2 - r_2\) and \(C_2\) holds the random value \(r_2\) it draws.
4. \(C_1\) and \(C_2\) respectively compute the shares of \([x \cdot y]^A_2\) by: \(\langle x \cdot y \rangle^A_1 = r_1 + x \cdot (y)^A_2 - r_2\), \(\langle x \cdot y \rangle^A_2 = r_2 + x \cdot (y)^A_1 - r_1\). It is easy to see that \(\langle x \cdot y \rangle^A_1 + \langle x \cdot y \rangle^A_2 = x \cdot (y)^A_1 + (y)^A_2 = x \cdot y\).

Finally, \(C_1\) and \(C_2\) can obtain the secret sharing \([x \cdot y]^A\). Let \(\text{MultiBA}\) denote such secret-shared multiplication, for which we have \([x \cdot y]^A = \text{MultiBA}(x^B, y^A)\). Analogously, \(\text{MultiBA}\) can also be applied on the secret-shared component-wise multiplication between a binary secret-shared value \([x]^B\) and an arithmetic secret-shared vector \([v]^A\), for which we have \([x \cdot v]^A = \text{MultiBA}(x^B_1, [v]^A)\), where \(x \cdot v\) is a vector from component-wise multiplication. With the above secure operations, we present the details of secure database mapping in Algorithm 2.

Algorithm 3. Secure Skyline Fetching \(\text{secFetch}\)

**Input:** The encrypted sum vector \([s]^A\), encrypted mapped database \([T]^A\), and encrypted original database \([P]^A\).

**Output:** The encrypted minimum \([s\text{Min}]^A \in [s]^A\), and the encrypted skyline tuples \([l]^A\) and \([p]^A\).

1: Initialization: \([s\text{Min}]^A = [s[1]^A], [l]^A = [l[1]^A \in [T]^A, [p]^A = \[p]^A \in [P]^A\).

2: for \(i = 2 \to n\) do
3: \([v]^B = \text{SecExt}(s[i]^A, [s\text{Min}]^A)\).
4: \([w]^B = [-v]^B\).
5: \([s\text{Min}]^A = \text{MultiBA}([v]^B, s[i]^A) + \text{MultiBA}([w]^B, [s\text{Min}]^A)\).
6: \([l[i]^A = \text{MultiBA}([v]^B, [l[1]^A] + \text{MultiBA}([w]^B, [l[1]^A]\).
7: \([p]^A = \text{MultiBA}([v]^B, [p]^A) + \text{MultiBA}([w]^B, [p]^A)\).
8: end for
9: return \([s\text{Min}]^A, [l]^A\), and \([p]^A\).

5.3 Secure Skyline Fetching

After mapping the encrypted database \([P]^A\) to \([T]^A\) with respect to \([q]^A\), \(C_{(1,2)}\) need to obliviously fetch the skyline tuples \([l]^A\) from \([T]^A\) and the skyline tuples \([p]^A\) corresponding to \([l]^A\) from \([P]^A\). For simplicity of presentation, we next introduce how to allow \(C_{(1,2)}\) to obliviously fetch one \([l][1]^A\) from \([T]^A\) and its corresponding \([p]^A\) from \([P]^A\).

According to the plaintext-domain shown in Algorithm 1, secure skyline fetching first needs to compute the attribute sum for each tuple \([l][1]^A \in [T]^A\). The summation operation is naturally supported in the secret sharing domain, namely

\[
[s[i]]^A = \sum_{j=1}^{m} [t[j]^A],
\]

where \([s[i]]^A\) represents the attribute sum for tuple \([l][1]^A\). With this, \(\text{SecSkyline}\) devises a component \(\text{secFetch}\) to allow \(C_{(1,2)}\) to obliviously fetch the skyline tuple \([l][1]^A\) from \([T]^A\) and its corresponding \([p]^A\) from \([P]^A\). Note that \(t\) refers to the tuple which has the minimum attribute sum in \(T\). Therefore, the first challenge of secure skyline fetching in the secret sharing domain is how to allow \(C_{(1,2)}\) to obliviously fetch the minimum value from a set of secret-shared values without knowing which and what value it is.

Obviously finding the minimum value from several values essentially needs comparison followed by swapping based on the comparison. This can be securely realized as follows. First, given the secret sharings \([a]^A\) and \([b]^A\) held by \(C_1\) and \(C_2\), we can first leverage \(\text{SecExt}\) to obtain the secret-shared result \([v]^B\) of comparison between \(a\) and \(b\), i.e., \([v]^B = \text{SecExt}([a]^A, [b]^A)\). Then, the smaller value among \(a\) and \(b\) can be obliviously selected via

\[
\min(a, b)^A = \text{MultiBA}([v]^B, [a]^A) + \text{MultiBA}([v]^B, [b]^A),
\]

where \([v]^B = [-v]^B\). With this as a basis, we are able to compute the minimum attribute sum in the secret sharing domain, as well as obliviously fetch the corresponding skyline tuple \(l\) from \([T]^A\) and the corresponding tuple \(p\) from \([P]^A\). In particular, when securely switching two attribute sums based on the secret-shared comparison result, we can perform secure switching of the two associated secret-shared tuples from \([T]^A\) and \([P]^A\) as well. The details of
secure skyline fetching is presented in Algorithm 3. Note that there may be more than one tuple whose attribute sum is equal to the smallest value in \( s \), but only one of them needs to be fetched in the current round, because the remainders are the skyline tuples to be processed in the subsequent rounds. Therefore, SecSkyline lets \( C_{12} \) obliviously choose the first one that has the minimum attribute sum by performing

\[
\varphi^B = \text{SecExt}([s[i]]^A, [sMin]^A),
\]

where \( \varphi = 1 \) if and only if \( s[i] < sMin \), which keeps \( t \) and \( p \), unchanged when \( s[i] = sMin \).

Note that when implementing Algorithm 3, we can use the trick of divide-and-conquer [53] to boost the performance during secure minimum computation. For example, the minimum in a vector \( [v]^A \) of four elements can be calculated by:

\[
\min(\min([v[1]]^A, [v[2]]^A), \min([v[3]]^A, [v[4]]^A)),
\]

where \( \min([v[1]]^A, [v[2]]^A) \) and \( \min([v[3]]^A, [v[4]]^A) \) can be calculated in parallel, saving communication rounds.

### 5.4 Secure Skyline and Dominated Tuples Filtering

So far we have introduced how \( C_{12} \) obliviously fetch the encrypted skyline tuple \([t]_A^4\) from the encrypted mapped database \([T]_A^4\). Then we should consider how to allow \( C_{12} \) to obliviously filter out \([t]_A^4\) and the tuples dominated by \([t]_A^4\) from \([T]_A^4\) without knowing which tuples they are, i.e., hiding the access pattern and the dominance relationships. Therefore, we devise a component \text{secFilt} (as given in Algorithm 4) for secure skyline and dominated tuples filtering.

**Challenges.** There are two challenges to be tackled in secFilt: 1) how to allow \( C_{12} \) to obliviously locate the skyline tuple and dominated tuples in \([T]_A^4\)? 2) how to allow \( C_{12} \) to obliviously filter out these tuples?

**Addressing the First Challenge.** SecSkyline first defines two encrypted (binary) labels \([\text{isFirstSky}])^B\) and \([\text{isDom}])^B\) for each tuple \([t]_A^4 \in [T]_A^4\), where \( \text{isFirstSky} = 1 \) indicates that \( t \) is the skyline tuple in the current round and \( \text{isDom} = 1 \) indicates that \( t \) is a tuple dominated by the skyline tuple. Then \( C_{12} \) can obliviously mark whether tuple \( t \) needs to be filtered out by calculating

\[
[\Phi_i]^B = [\text{isFirstSky}])^B + [\text{isDom}])^B.
\]

It is noted that since \( \text{isFirstSky} \), and \( \text{isDom} \), cannot both be equal to 1, \( \Phi_i = 0 \) indicates that both \( \text{isFirstSky} \), and \( \text{isDom} \), are equal to 0 and \( t \) does not need to be filtered out, and \( \Phi_i = 1 \) indicates that \( \text{isFirstSky} \) or \( \text{isDom} \), is equal to 1 and \( t \) needs to be filtered out. Next, we introduce how \( C_{12} \) obliviously evaluate \([\text{isFirstSky}])^B\) and \([\text{isDom}])^B\) for each tuple \([t]_A^4 \in [T]_A^4\).

We first introduce how \( C_{12} \) obliviously evaluate \([\text{isFirstSky}])^B\) for each tuple \([t]_A^4 \in [T]_A^4\), i.e., whether \( t \) is the skyline tuple. Recall that in Algorithm 3, the skyline tuple has the minimum attribute sum \( sMin \). So SecSkyline lets \( C_{12} \) obliviously evaluate whether \( t \)'s attribute sum (i.e., \( [s^{(k)}]_i^4 \) in the current round \( k \)) is equal to \( sMin \). Specifically, SecSkyline first lets \( C_{12} \) securely compare \([sMin]^A\) and \([s^{(k)}]_i^4\) by

\[
[\sigma_i]^B = \text{SecExt}([sMin]^A, [s^{(k)}]_i^4),
\]

where \( \sigma_i = 1 \) indicates \( sMin \geq [s^{(k)}]_i^4 \). Note that \( sMin \) is the minimum value in \( s^{(k)} \), and thus \( \sigma_i = 1 \) means \( [s^{(k)}]_i^4 = sMin \). However, since there may be more than one value in \( s^{(k)} \) that is equal to \( sMin \), \( [s^{(k)}]_i^4 = sMin \) indicates that \( t \) may be the skyline tuple \( t \). Recall that in Algorithm 3, SecSkyline lets \( C_{12} \) obliviously fetch the first tuple \([t]_A^4\) whose \([s^{(k)}]_i^4\) is minimum in \([s^{(k)}]^A\) (i.e., \([sMin]_A^4\)) as the skyline tuple \([t]_A^4\). Therefore, SecSkyline provides a delicate security design to allow \( C_{12} \) to only set \([\text{isFirstSky}]_j^B = [1]^B\) for the first tuple \([t]_A^4\) that satisfies \([s^{(k)}]_i^4 = sMin^A\) as follows:

\[
[\text{isFirstSky}]_j^B = [\sigma_j]^B \otimes [\neg \text{flag}]^B,
\]

\[
\text{flag} = [\text{flag}]^B \oplus [\text{isFirstSky}]_j^B,
\]

where \([\text{flag}]^B\) is an auxiliary variable and set as \([0]^B\) at the beginning of the current round. \( \text{isFirstSky} = 1 \) indicates that tuple \( t \) is the required skyline tuple \( t \).

The correctness is analyzed as follows. \([\text{flag}]^B = [0]^B\) at the beginning. When the first \([s^{(k)}]_i^4 = sMin^A\) appears, \( C_{12} \) obliviously set \([\sigma_j]^B = [1]^B\). Since \([\neg \text{flag}]^B = [1]^B\), \( C_{12} \) obliviously set \([\text{isFirstSky}]_j^B = [1]^B\) (i.e., Eq. 5), which marks the required skyline tuple \( t \). After that, \( C_{12} \) obliviously set \([\text{flag}]^B = [1]^B\) (i.e., Eq. 6), and \([\text{flag}]^B\) remains equal to \([1]^B\) in the following loops. \([\text{flag}]^B = [1]^B\) prevents \( C_{12} \) from setting \([\text{isFirstSky}]_j^B = [1]^B\) for other tuples because \([\neg \text{flag}]^B = [0]^B\) in the following loops.

We then introduce how \( C_{12} \) obliviously evaluate \([\text{isDom}]_j^B\) for each tuple \([t]_A^4 \in [T]_A^4\), i.e., whether \( t \) is a dominated tuple. According to the definition of dominance (i.e., Definition 1), given two tuples \( a \) and \( b \), we say \( a \) dominates \( b \) if \( \forall j, a[j] \leq b[j] \) and \( \exists j, a[j] < b[j] \). Therefore, if \( \forall j, a[j] \leq b[j] \), we have either \( a \) dominates \( b \) or \( a \) is identical to \( b \). Therefore, SecSkyline defines an encrypted (binary) label \([\delta_j]^B\) to mark the above dominance relationship. \( C_{12} \) obliviously evaluate \([\delta_j]^B\) for each tuple \([t]_A^4 \in [T]_A^4\) by first comparing each attribute of the skyline tuple \([t]_A^4\) and \([t]_A^4\)
\[ \delta_{i,j}^B = \text{SecExt}([t, \{j\}], B, [n]) \]

and then aggregating the comparison results to \( \delta^B \) by

\[ \delta^B = [\delta_1^B] \oplus \cdots \oplus [\delta_m^B], \] (7)

where \( \delta_i = 1 \) if and only if \( \forall j \neq i \), \( t \), \( j \leq t \), \( i \). Therefore, \( \delta_i = 1 \) if \( t \) is a dominated tuple or otherwise \( \delta_i = 0 \). The above process is described at lines 6-9 of Algorithm 4. Subsequently, \( C_{(1,2)} \) securely evaluate \( \text{isDom}_i^B \) for \( [t, \{i\}]^A \) by

\[ \text{isDom}_i^B = [\delta_i^B] \oplus [\sigma_i^B], \]

where \( \text{isDom}_i = 1 \) indicates that tuple \( t \) is a dominated tuple, and \( [\delta_i^B] \) and \( [\sigma_i^B] \) are calculated by Eq. 7 and Eq. 4, respectively. We give the correctness analysis as follows. Firstly, if and only if both \( \delta_i = 1 \) and \( \sigma_i = 1 \), we have \( \text{isDom}_i = 1 \). \( \delta_i = 1 \) indicates that \( t \) is a dominated tuple or \( t \), \( i \), \( t \), \( i \), and \( \sigma_i = 1 \) rules out the possibility of \( t \). Therefore, if and only if the tuple \( t \) is a dominated tuple, we have \( \text{isDom}_i = 1 \).

So far, \( C_{(1,2)} \) have obliviously marked the skyline tuple by \( \text{isFirstSky} = 1 \) and the dominated tuples by \( \text{isDom}_i = 1 \). Therefore, \( C_{(1,2)} \) can securely evaluate \( [\Phi]^B \) for each tuple \( [t, \{i\}]^A \) by Eq. 3 to obliviously mark whether tuple \( t \) needs to be filtered out.

Addressing the Second Challenge. We should then consider how to tackle the second challenge, namely, how to allow \( C_{(1,2)} \) to obliviously filter out tuples which have \( \Phi_i = 1 \) underlying the \( [\Phi]^B \) (recall Eq. 3). A naive method is to let \( C_{(1,2)} \) directly open each tuple's \( \Phi_i \). However, such simple method will leak which tuple is the skyline tuple and the dominance relationships, which easily violates the security requirement for access pattern protection.

Instead, SecSkyline achieves oblivious tuple filtering via a different strategy. Specifically, SecSkyline lets \( C_{(1,2)} \) obliviously set \( s^{(k+1)}[i] \) to a pre-set system-wide maximum value \( \nu_{\text{MAX}} \) to mark for filtering if \( \Phi_i = 1 \), and obliviously keep \( s^{(k+1)}[i] \) unchanged if \( \Phi_i = 0 \). Formally, \( C_{(1,2)} \) perform the following

\[ s^{(k+1)}[i] = \text{MultiBA}([\Phi]^B, [\nu_{\text{MAX}}]^A) + \text{MultiBA}([\Phi_i]^B, [s^{(k)}[i]]^A). \]

The secret sharing of \( \nu_{\text{MAX}} \) can be prepared by \( C_{(1,2)} \) offline. Note that \( s^{(k+1)}[i] = \nu_{\text{MAX}} \) will prevent \( C_{(1,2)} \) from selecting \( t \) (or \( p_r \)) as the skyline tuple in the following rounds. Therefore, the filtering strategy will not degrade the skyline query accuracy.

5.5 Putting Things Together

In this section, we introduce how to synthesize the above three secure components to enable secure skyline query processing over encrypted cloud databases in SecSkyline. We first encapsulate them as follows:

- Secure database mapping \( [T]^A = \text{secMap}([P]^A, [q]^A) \), which inputs the encrypted original database \([P]^A\) and skyline query request \([q]^A \), and then outputs the initial encrypted mapped database \([T]^A \).
- Secure skyline fetching \( ([s_{\text{Min}}]^A, [T]^A, [p_r]^A) = \text{secFetch}([s^{(0)}]^A, [T]^A, [P]^A) \), which inputs the encrypted attribute sum vector \([s^{(0)}]^A \), mapped database \([T]^A \), and the original database \([P]^A \) output from the previous round, and then outputs the encrypted minimum attribute sum \([s_{\text{Min}}]^A \), \( [T]^A \), and \([p_r]^A \).
- Secure skyline and dominated tuples filtering \( ([s^{(k+1)}]^A = \text{secFilt}([T]^A, [t]^A, [s_{\text{Min}}]^A, [s^{(k)}]^A) \), which inputs the encrypted mapped database \([T]^A \), minimum sum \([s_{\text{Min}}]^A \), and attribute sum vector \([s^{(k)}]^A \), and then outputs the updated attribute sum vector \([s^{(k+1)}]^A \).

Algorithm 5 gives the complete construction for secure skyline query processing in SecSkyline, which is the secure instantiation of Algorithm 1 and relies on the coordination of the above three secure components. The only challenge in the design of Algorithm 5 is how to allow \( C_{(1,2)} \) to decide whether to terminate the secure search process without leaking other information. Our solution is to let \( C_{(1,2)} \) first securely compare \([s_{\text{Min}}]^A\) and \( \nu_{\text{MAX}} \)

\[ \text{isStop}^B = \neg \text{SecExt}([s_{\text{Min}}]^A, [\nu_{\text{MAX}}]^A), \]

and then open the flag \( \text{isStop} \), where \( \text{isStop} = 1 \) indicates that the smallest element in \([s^{(k)}]^A \) is \( \nu_{\text{MAX}} \). Therefore, \( C_{(1,2)} \) can know that all elements in \([s^{(k)}]^A \) have been filtered out, and then terminate the search process.

5.6 Complexity Analysis

It is noted that the performance of SecSkyline is dominated by three main components: 1) secure database mapping (secMap), 2) secure skyline fetching (secFetch), and 3) secure skyline and dominated tuples filtering (secFilt). In practice, the cost of these components is dominated by the secure MSB extraction operation SecExt(\( \cdot \)). Therefore, we first separately analyze their complexities by counting SecExt(\( \cdot \)) during their execution. For Algorithm 2, the cost of secMap is dominated by \( n \cdot m \) secure MSB extraction operations SecExt(\( \cdot \)). For Algorithm 3, secFetch requires \( n - 1 \) secure MSB extraction operations SecExt(\( \cdot \)). For Algorithm 4, secFilt requires \( n \cdot m + n \) secure MSB extraction operations SecExt(\( \cdot \)).

Then we analyze the overall complexity of invoking SecExt(\( \cdot \)) in the complete protocol shown in Algorithm 5. Note that it is dominated by the While loop, which terminates when all tuples in \([T]^A \) are filtered out, as indicated by the Boolean flag \( \text{isStop} \). The number of loops is \( k + 1 \) where \( k \) is the number of skyline tuples returned for the skyline query, i.e., the size of \([S_{\text{Sky}}]^A \). In addition, the computation of \([\text{isStop}^B]\) in each loop requires a secure MSB extraction operation SecExt(\( \cdot \)) and the \((k+1)\)-th loop early stops at line 8. Therefore, we can conclude that the overall execution of our protocol requires \( n \cdot m + k \cdot n \cdot (2 + m) + n \) secure MSB extraction operations SecExt(\( \cdot \)).
Algorithm 5. The Complete Construction for Secure Skyline Query Processing in SecSkyline

Input: The encrypted database \([P]^A\) and skyline query \([q]^A\).
Output: The encrypted resulting set of skyline tuples \([SK_q]^A\).
1: Initialization: \([SK_q]^A = \emptyset, [s]^A = [0]^A\), isStop = 0.
2: \([T]^A = \text{secMap}([P]^A, [q]^A)\).
3: \([s(i)]^A = \sum_{j=1}^{m} [t(j)]^A\), for \(i \in [1,n]\). / / \(n\) is the number of tuples; \(m\) is the dimension of tuples.
4: \(k = 0\).
5: while \(\neg\text{isStop}\) do
6: \(([sMin]^A, [t.]^A, [p.]^A) = \text{secFetch}([s(i)]^A, [T]^A, [P]^A)\).
7: \([\text{isStop}]^B = \neg\text{SecExt}([sMin]^A, [vMAX]^A)\).
8: \(C_{(1,2)}\) open the flag isStop to decide whether to stop the process.
9: \([SK_q]^A = \text{append}([P]^A)\).
10: \([s(k+1)]^A = \text{secMap}([T]^A, [t.]^A, [sMin]^A, [s(i)]^A)\).
11: \(k = k + 1\).
12: end while
13: return \([SK_q]^A\).

6 Security Analysis

We now analyze the security of SecSkyline. Our analysis follows the standard ideal/real world paradigm [54]. We first define the ideal functionality \(F\) for the secure skyline query processing:

- **Input.** The data owner provides to \(F\) the database \(P\) and a client submits a query \(q\).
- **Computation.** After receiving \(P\) and \(q\), \(F\) retrieves the skyline tuples \(SK_q\) of \(P\) with respect to \(q\).
- **Output.** \(F\) returns \(SK_q\) to the client.

Let \(\Pi\) represent a protocol for secure skyline query processing that realizes the ideal functionality \(F\). The security of \(\Pi\) is formally defined as follows:

**Definition 5.** Let \(A\) be an adversary who has the view of a corrupted server during the execution of \(\Pi\). Let \(\Pi^{\text{Real}}\) denote \(A\)’s view in the real world. We say that \(\Pi\) is secure in the semi-honest and non-colluding setting, if for every PPT adversary, \(a\) a PPT simulator \(S\) s.t. \(\Pi^{\text{Real}} = \Pi^{\text{Ideal}}\). That is, the simulator \(S\) can simulate a view for the adversary, which is indistinguishable from its view in the real-world.

**Theorem 1.** In the semi-honest and non-colluding adversary model, SecSkyline can securely realize the ideal functionality \(F\) according to Definition 5.

**Proof.** Recall that in the framework of SecSkyline, i.e., Algorithm 5, which consists of several components: 1) secure database mapping (secMap); 2) secure skyline fetching (secFetch); 3) secure skyline and dominated tuples filtering (secFlt). Since each of them is invoked in order as per the processing pipeline and their inputs and outputs are secret shares, we can conclude that SecSkyline is secure if the simulator for each component exists [55], [56], [57]. We use \(\text{Sim}^{C_i}_{\text{secMap}}\) to represent the simulator which generates \(C_i\)’s view in the execution of component \(X\) on corresponding input and output. It is noted that the roles of \(C_1\) and \(C_2\) in these components are symmetric. So it suffices to analyze the existence of simulators for \(C_1\).

- **\(\text{Sim}^{C_1}_{\text{secMap}}\).** It is noted that secMap (i.e., Algorithm 2) consists of three meta operations, i.e., secure MSB extraction (line 4), secure bit flipping (line 5), and secure multiplication between a binary secret-shared value and an arithmetic secret-shared value (line 6). Since these operations are invoked in turn and their inputs are secret shares, we analyze the existence of their simulators in turn. Since the secure MSB extraction consists of basic binary secret sharing operations (i.e., AND \(\otimes\) and XOR \(\oplus\)), its simulator clearly exists. Note that the secure bit flipping operation only requires local computation and \(C_1\) receives nothing during its execution. Therefore, its simulator clearly exists. We then analyze the existence of the simulator for the secure multiplication between a binary secret-shared value \([x]^B\) and an arithmetic secret-shared value \([y]^A\). It is noted that we only need to analyze the case where \(C_1\) acts as the receiver, because in the case where \(C_1\) acts as the sender, \(C_1\) receives nothing. At the beginning of the operation, \(C_1\) has \([x_1]^B\) and \([y_1]^A\) and later receives two messages \(m_1 := (\mu \oplus (x_2)^B) \cdot (y_2)^A - r_2, \mu \in \{0,1\}\) from \(C_2\). Therefore, we need to prove that the messages are uniformly random in the view of \(C_1\). Note that the random value \(r_2\) generated by \(C_2\) is uniformly random in the view of \(C_1\). This implies that \(m_{(1,2)}\) are also uniformly random in \(C_1\)’s view since \(r_2\) is independent of other values used in the generation of \(m_{(1,2)}\) [58]. Therefore, the simulator \(\text{Sim}^{C_1}_{\text{secFlt}}\) exists.

- **\(\text{Sim}^{C_1}_{\text{secFetch}}\).** It is noted that secFetch (i.e., Algorithm 3) consists of secure MSB extraction, secure bit flipping, secure multiplication between a binary secret-shared value and an arithmetic secret-shared value, and basic secret sharing operations. Meanwhile, these operations are invoked in turn and their inputs are secret shares. Therefore, based on the above analysis, the simulator \(\text{Sim}^{C_1}_{\text{secFetch}}\) exists.

- **\(\text{Sim}^{C_1}_{\text{secMap}}\).** Similarly, \(\text{Sim}^{C_1}_{\text{secMap}}\) exists, since the meta operations of secMap (i.e., Algorithm 4) are same as secMap and secFetch, and they as are invoked in turn and their inputs are secret shares.

The proof of Theorem 1 is completed.

We now explicitly analyze why SecSkyline can hide search access patterns as follows.

- **Hiding the search pattern.** Given an encrypted skyline query request \([q]^A\), each cloud server \(C_i, i \in \{1,2\}\) only receives the share \([q]^A\). According to the security of additive secret sharing, it is ensured that encrypting the same query multiple times will produce different secret shares that are indistinguishable from uniformly random values. Therefore, given the security of additive secret sharing [22], \(C_{(1,2)}\) cannot determine whether a new skyline query has been issued before. Therefore, SecSkyline can hide the search pattern.

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Hiding the access pattern. The access pattern in fact indicates whether a tuple in the original database \([P]\) or the mapped database \([T]\) is a skyline tuple. That is, it refers to which tuples will appear in the query result \([SK_q]\). Since the skyline tuples are obliviously fetched in secFetch and obliviously filtered out in secFilt, \(C_{[1,2]}\) cannot know which tuples are the skyline tuples. Therefore, SecSkyline can hide the access pattern.

7 EXPERIMENTS

7.1 Experiment Setup
We implement our protocol in C++. All experiments are conducted on a machine with 8 AMD Ryzen 7 5800H CPU cores and 16 GB RAM running 64-bit Windows 10. In our experiments, the two cloud servers are simulated by threads and executed in parallel on the same machine. The network delay is set to 1 ms. Similar to the previous works \([16], [17]\), we use three synthetic datasets and a real-world NBA dataset. Specifically, we generate independent (INDE), correlated (CORR), and anti-correlated (ANTI) datasets following \([16]\). We also build a real-world dataset about NBA players based on the data from the Kaggle dataset\(^1\), where each player has six attributes: minutes, points, rebounds, assists, blocks, and steals. The results reported in our experiments are the average over 100 skyline queries unless otherwise stated.

7.2 Evaluation on Accuracy
We first report the accuracy of SecSkyline to demonstrate the effectiveness of our design. Specifically, we first implement the plaintext dynamic skyline query algorithm (i.e., Algorithm 1). Then, over different datasets, we randomly generate 1000 skyline queries and use the plaintext algorithm and SecSkyline to search the skyline tuples with respect to these skyline queries. We use the skyline tuples output by the plaintext algorithm as the baseline to evaluate the accuracy of SecSkyline. If the skyline query results returned by SecSkyline exactly match that returned by the plaintext baseline, there is no accuracy loss so the accuracy of SecSkyline is 100%. The experiment results on different datasets are summarized in Table 3, where \(n\) represents the number of tuples and \(m\) represents the number of dimensions. It can be observed that SecSkyline outputs exactly the same skyline tuples as the plaintext algorithm, which validates the effectiveness of SecSkyline.

| Dataset (\(n=1000, m=6\)) | CORR | INDE | ANTI | NBA |
|---------------------------|------|------|------|-----|
| Accuracy                  | 100% | 100% | 100% | 100% |

7.3 Evaluation on Performance

7.3.1 Evaluation on Query Latency
We now examine the query latency of SecSkyline. That is, given an encrypted skyline query, we evaluate how long it takes \(C_{[1,2]}\) to obliviously execute dynamic skyline search on the encrypted database and output encrypted skyline tuples. We start with evaluating SecSkyline on different datasets with the number of dimensions \(m = 2\), for varying the number of tuples \(n\), and summarize the results in Fig. 4. It is noted that since the NBA dataset only has 2500 tuples, the curve about its results is shorter than other datasets. From Fig. 4, it can be observed that under the same \(m\), the query latency is small even with \(n\) ranging from 1000 to 11000. Specifically, as \(n\) increases from 1000 to 11000, the query latency on different datasets with \(m = 2\) increases from about 1 to 2.15 seconds. Moreover, it can be observed that the query latency on ANTI datasets is larger than others. The reason is that the dataset tuples in ANTI datasets show weaker correlation, and thus skyline queries on them produce more skyline tuples, which require more rounds of secure skyline tuples search. Then, over different datasets and with the number of tuples \(n = 1000\), we evaluate SecSkyline for varying the number of dimensions \(m \in \{2, 3, 4, 5, 6\}\), and summarize the results in Fig. 5. It can be observed that under the same \(n\), as \(m\) increases, the query latency grows quickly. Specifically, as \(m\) increases from 2 to 6, the query latency on different datasets with \(n = 1000\) increases from about 1 to 27 seconds.

7.3.2 Evaluation on Communication Performance
We now examine the online communication performance of SecSkyline. That is, given an encrypted skyline query, the amount of data communicated between \(C_{[1,2]}\) to conduct the secure skyline query processing as per our design and output encrypted skyline tuples. Note that we use the same experiment setting as that in Section 7.3.1, and summarize the results in Figs. 6 and 7. According to Fig. 6, as \(n\) increases from 1000 to 11000, the communication cost over different datasets with \(m = 2\) increases from about 6 to 144 MB; According to Fig. 7, as \(m\) increases from 2 to 6, the
communication cost over different datasets with \( n = 1000 \) increases from about 6 to 524 MB. So the number of dimensions \( m \) heavily affects the communication cost.

7.4 Scalability Evaluation

To demonstrate the scalability of SecSkyline on large-scale datasets, we now report the computation cost of SecSkyline on larger datasets and different numbers of threads. Specifically, we first evaluate SecSkyline on different datasets under \( m = 2 \) and single thread, for varying number of tuples \( n \in \{2 \times 10^5, 3 \times 10^5, 4 \times 10^5, 5 \times 10^5, 6 \times 10^5\} \), and summarize the results in Fig. 8. It can be observed that even on \( 6 \times 10^5 \) tuples, the query latency is still on the order of seconds, which should be tolerable for the client. We then evaluate SecSkyline on the CORR dataset under \( m = 2 \), for varying number of tuples \( n \in \{2 \times 10^5, 3 \times 10^5, 4 \times 10^5, 5 \times 10^5, 6 \times 10^5\} \) and the number of threads \( \chi \in \{1, 2, 4\} \). The results are summarized in Fig. 9. To reduce the time cost, we use the data partitioning method [16] to support parallel execution of our protocol. In the experiment, we divide the dataset into \( \chi \) sub-datasets and distribute them to \( \chi \) sub-threads. Each sub-thread runs SecSkyline protocol independently to compute the skyline tuples of the sub-dataset and sends the candidate set of skyline tuples to a main thread. After receiving the results from any two sub-threads, the main thread merges them into a new dataset and assigns it to an unoccupied sub-thread to securely search the skyline tuples. This process is repeated until the encrypted skyline tuples of the last merged dataset has been securely found, and these tuples are the ultimate encrypted skyline tuples returned to the client.

7.5 Comparison to State-of-the-art Prior Works

As reported in the state-of-the-art prior works, the protocols FSSP [16] and SMSQ [17] run secure skyline queries over small-scale datasets (e.g., CORR, INDE, and ANTI with size \( n = 1000, m = 2 \)) in at least 1000 seconds and 100 seconds\(^2\), respectively. Meanwhile, we note that they consider the query latency to be the sum of computation time and memory copying time between threads, and do not consider network latency. In contrast, even considering the network latency, SecSkyline only requires 0.79 seconds on CORR dataset, 0.65 seconds on INDE dataset, and 1.11 seconds on ANTI dataset, which is \( 90 \sim 154 \times \) and \( 901 \sim 1538 \times \) faster than SMSQ [17] and FSSP [16], respectively. For the larger datasets (e.g., CORR, INDE, and ANTI with size \( n = 10000, m = 2 \)), FSSP and SMSQ require at least 10000 seconds and 1000 seconds, respectively. In contrast, even considering the network latency, SecSkyline only requires 1.54 seconds on CORR dataset, 1.23 seconds on INDE dataset, and 2.15 seconds on ANTI dataset, which is \( 465 \sim 813 \times \) and \( 4651 \sim 8130 \times \) better than SMSQ [17] and FSSP [16], respectively.

8 CONCLUSION

In this paper, we design, implement, and evaluate SecSkyline, a new system framework enabling fast privacy-preserving skyline query over outsourced encrypted cloud databases. SecSkyline is fully based on the lightweight secret sharing technique, and is derived from a delicate synergy of three proposed secure components, including secure database mapping, secure skyline fetching, and secure skyline and dominated tuples filtering. Extensive experiments over multiple datasets show that SecSkyline greatly improves upon state-of-the-art prior works [16], [17] in query latency, with up to \( 8130 \times \) improvement over FSSP [16] and up to \( 813 \times \) improvement over SMSQ [17].
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