Tests of Transfer Reaction Determinations of Astrophysical S-Factors

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The $^{16}O(^{4}He,d)^{17}F$ reaction has been used to determine asymptotic normalization coefficients for transitions to the ground and first excited states of $^{17}F$. The coefficients provide the normalization for the tails of the overlap functions for $^{17}F \rightarrow ^{16}O + p$ and allow us to calculate the S-factors for $^{16}O(p,\gamma)^{17}F$ at astrophysical energies. The calculated S-factors are compared to measurements and found to be in very good agreement. This provides the first test of this indirect method to determine astrophysical direct capture rates using transfer reactions. In addition, our results yield $S(0)$ for capture to the ground and first excited states in $^{17}F$, without the uncertainty associated with extrapolation from higher energies.

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I. INTRODUCTION

Nuclear capture reactions, such as $(p,\gamma)$ and $(\alpha,\gamma)$, play a major role in defining our universe. A primary goal in nuclear astrophysics is to determine rates for capture reactions that are important in the evolution of stellar systems. However, the reactions of interest often involve radioactive targets which makes measurements quite difficult or even impossible using conventional methods. Hence techniques have been developed to determine rates by indirect methods. For example, precise information on excitation energies and particle decay widths can be used to make accurate predictions of rates which proceed by resonance capture. The only reliable method to determine a reaction rate that is dominated by direct capture has been to measure it at laboratory energies with a low energy particle beam and then extrapolate the result to energies of astrophysical interest.

Attempts have been made to use both Coulomb dissociation (C) and the determination of asymptotic normalization coefficients (ANC) from conventional nuclear transfer reactions (F) to determine S-factors for direct capture reactions, but neither technique has been tested to verify its reliability. Such tests are crucial, as stressed in the most recent evaluation of solar fusion cross section rates (V). We report here the first test of one of these two techniques to determine astrophysical S-factors; we demonstrate that the ANC inferred from a measurement of a proton transfer reaction can directly determine a $(p,\gamma)$ direct capture rate at astrophysical energies.

Direct capture reactions of astrophysical interest usually involve systems where the binding energy of the captured proton is low. Hence at stellar energies, the capture proceeds through the tail of the nuclear overlap wave function. The shape of this tail is completely determined by the Coulomb interaction, so the rate of the capture reaction can be calculated accurately if one knows its amplitude. The asymptotic normalization coefficient $C$ for the system $B \leftrightarrow A + p$ specifies the amplitude of the single-proton tail of the wave function for nucleus $B$ when the core $A$ and the proton are separated by a distance large compared to the nuclear radius. Thus, this normalization coefficient determines the corresponding direct capture rate.

The advantage of the ANC approach is that it provides a method to determine direct capture S-factors accurately from the results of nuclear reactions such as peripheral nucleon transfer which can be studied with radioactive beams and have cross sections that are orders of magnitude larger than the direct capture reactions themselves. Furthermore, direct capture S-factors derived with this technique are most reliable at the lowest incident energies, precisely where capture cross sections are smallest and most difficult to measure directly. In fact, the ANC approach even permits one to determine S-factors at zero energy, which is not possible with direct measurements except by extrapolation.

While there is little controversy that knowledge of the asymptotic normalization coefficient for a loosely bound nuclear system allows one to compute the corresponding direct capture rate, the nuclear astrophysics community has clearly indicated (P) that a test of the relationship between the transfer reaction cross section and the astrophysical S-factor is important to validate this approach. The community’s skepticism originates in the well-known model dependence found in distorted-wave Born approximation (DWBA) analyses of transfer reaction data in terms of spectroscopic factors, which is due to the uncertainty in the DWBA calculations associated with the choice of optical model potentials and single particle wave functions. By parametrizing the DWBA
cross section of a peripheral transfer reaction in terms of ANC’s, rather than spectroscopic factors, we can reduce the uncertainty associated with the choice of single particle wave functions so that it becomes small compared to that associated with the optical potential. By choosing appropriate reactions, beam energies and scattering angles, we can also minimize the uncertainty associated with the choice of optical model potentials.

In this article, we describe a measurement of the $^{16}O(^3He, d) ^{17}F$ reaction, from which we determine the ANC’s for the $\frac{1}{2}^+$ ground state and the $\frac{3}{2}^+$ first excited state in $^{17}F$. We then use our measured ANC’s to calculate, with no additional normalization factors, the S-factors for the $^{16}O(p, \gamma)^{17}F$ reaction at astrophysical energies. Such a determination of the S-factors for $^{16}O(p, \gamma)^{17}F$ from ANC’s measured in proton transfer reactions is an ideal test case for this indirect method because the results can be compared to existing direct measurements of the capture cross sections. Furthermore, the $^{16}O(p, \gamma)^{17}F$ reaction has substantial similarities to the $^7Be(p, p)^8B$ reaction, which is the source of all high energy neutrinos produced in the sun. We will report determinations of the S-factor for $^7Be(p, p)^8B$ using this technique in future publications. It will also be straightforward to utilize this procedure to determine S-factors at astrophysical energies for other cases that include significant direct capture components.

II. $^{17}F \leftrightarrow ^{16}O + p$ ASYMPTOTIC NORMALIZATION COEFFICIENTS

For a peripheral transfer reaction, ANC’s are extracted from the measured angular distribution by comparison to a DWBA calculation. Consider the proton transfer reaction $a + A \rightarrow c + B$, where $a = c + p$ and $B = A + p$. The experimental cross section is related to the DWBA calculation by

$$\frac{d\sigma}{d\Omega} = \sum l_{B|B} (C_{Ap|B})^2 (C_{cpl|a})^2 R_{l_{B|B}l_{a|a}},$$

(1)

where

$$R_{l_{B|B}l_{a|a}} = \frac{\tilde{\sigma}_{l_{B|B}l_{a|a}}}{b_{Ap|B}^2 b_{cpl|a}^2}.$$  

(2)

$\tilde{\sigma}$ is the calculated DWBA cross section and the $b$’s are the asymptotic normalization constants for the single particle orbitals used in the DWBA. The sum in Eq. (2) is taken over the allowed angular momentum couplings, and the $C$’s are the ANC’s for $B \rightarrow A + p$ and $a \rightarrow c + p$. The normalization of the DWBA cross section by the ANC’s for the single particle orbitals makes the extraction of the ANC for $B \rightarrow A + p$ insensitive to the parameters used in the single particle potential wells, in contrast to traditional spectroscopic factors. See [4] for additional details.

DWBA calculations of the $^{16}O(^3He, d)^{17}F$ reaction populating the $^{17}F$ first excited state indicate that the sensitivity of the extracted ANC to the choice of optical model potentials is minimized near 0°. There exists a previous study of the $^{16}O(^3He, d)^{17}F$ reaction at $E_{^3He} = 25$ MeV [1] that reported cross sections at 9 angles over the range $\theta_{cm} \approx 6 - 36^\circ$. The limited small-angle coverage makes an attempt to infer the $^{17}F$ first excited state ANC from that experiment very imprecise. We have now measured the $^{16}O(^3He, d)^{17}F$ reaction at $E_{^3He} \approx 29.7$ MeV primarily to determine the angular distribution carefully at small angles, thus minimizing the systematic uncertainty in the extracted ANC. However, by obtaining data at a second beam energy, we can also do a combined analysis to reduce our sensitivity to the choice of optical potentials even further.

Two separate measurements were performed, one optimized to determine the absolute cross section with a minimum of uncertainty and the other to obtain a detailed angular distribution at small angles. The reaction was measured at laboratory angles between $6.5^\circ$ and $25^\circ$ using a momentum-analyzed 29.75 MeV $^3He$ beam from the U-120M isochronous cyclotron of the Nuclear Physics Institute (NPI) of the Czech Academy of Sciences incident on a Mylar target. The target thickness was measured to be $134 \mu g/cm^2$ by scanning with well-collimated alpha particles from $^{241}Am$, $^{238}Pu$ and $^{244}Cm$. Reaction products were observed by a pair of detector telescopes, consisting of 150 $\mu$m thick $\Delta E$ and 2000 $\mu$m thick $E$ Si surface barrier detectors, with solid angles of 0.23 msr. One of the telescopes was rotated about the target during the measurements while the other was fixed at $\theta_L = 18.2^\circ$. Elastic scattering and several reaction channels were measured simultaneously in both telescopes to provide a continuous calibration of the beam energy, reaction angle and target thickness. The beam current was integrated by a Faraday cup biased to 1 kV. Absolute cross sections were determined to $\pm 4.5\%$, using procedures developed at NPI to minimize overall normalization uncertainties.

Small angle data at laboratory angles between $1^\circ$ and $11^\circ$ were obtained using a molecular $^3He - d$ beam from the Texas A&M University K500 superconducting cyclotron incident on a 540 $\mu g/cm^2$ Mylar target. The angular spread of the beam on target was $\approx 0.1^\circ$ after passing through the Texas A&M Beam Analysis System. Reaction products were detected at the focal plane of the Multipole Dipole Multipole magnetic spectrometer using the modified Oxford detector. The detector consists of a 50 cm long gas ionization chamber to measure the specific energy loss of particles in the gas and their focal plane positions at four resistive wires, separated by 16 cm steps along the particles’ trajectories, followed by an NE102A plastic scintillator to measure the residual energy. The $^3He$ energy in the molecular beam was determined from the crossover between the $^{12}C(^3He, n)^{12}N$ and $^{16}O(^3He, \alpha)^{15}O$ reactions,
observed simultaneously off the Mylar target. It was 29.71 MeV, tuned to match the measurements carried out at the NPI. The beam angle was determined to ±0.1° from the crossover between the $^1H(^3He,^3He)^2H$ and $^{12}C(^3He,^9He)^{12}C$(4.44 MeV) reactions, also observed simultaneously off the Mylar target. The charge in the beam was collected in a Faraday cup and provided the normalization between angles. Additional details regarding the experimental procedures may be found in [6].

The absolute normalization of the Texas A&M cross section measurements was determined by matching the ground and first excited state yields to those determined at NPI in the angular region where the two data sets overlap. The matching procedure introduced an additional ±1.1% uncertainty in the absolute normalization of the small angle cross section measurements. The combined angular distributions for the ground and first excited states are shown in Fig. 1. DWBA calculations were carried out with the finite range code PTOLEMY [1], using the full transition operator. Seven different optical potentials were studied for the $^3He$ - $^{16}O$ entrance channel. Six came from an extensive study of $^3He$ elastic scattering on s - d shell nuclei at 25 MeV [15], with small (<0.5%) adjustments in the depths to account for the energy-dependence of the real and imaginary volume integrals [15]. One came from a global fit [19]. The potentials include three different families of discrete ambiguities, characterized by the real volume integral, and contain both volume and surface imaginary forms. In general, the calculations with potentials including volume imaginary terms reproduced our measured angular distributions slightly better. Eventually, the potentials with the intermediate real volume integrals, which were identified as the “physical” family in [18], were adopted. The deep potentials predicted a forward maximum for the $^{17}F$ excited state that varied too slowly with angle compared to our measured angular distribution. Some of the shallow potentials gave reasonable fits to our measured angular distributions at 29.7 MeV but did a poor job reproducing the 25 MeV data [13]. Five d - $^{17}F$ exit channel potentials were studied. Three came from various global fits [20], and two came from fits to d - $^{17}O$ elastic scattering [21]. One global potential predicted a forward maximum that varied too slowly with angle, while the two d - $^{17}O$ potentials gave very poor fits. The remaining global potentials reproduced the measured angular distributions well and were adopted. The single particle orbitals were calculated in Woods-Saxon potentials with $r_0$ in the range 1.15 - 1.35 fm and $a_0$ in the range 0.55 - 0.75 fm. Over this full range, the extracted $^{17}F$ ANC's varied by only ±1.5% and ±4% for the ground and first excited states, respectively, demonstrating the insensitivity of the extracted ANC's to assumptions about the $^{17}F$ wave functions in the nuclear interior. In contrast, the more traditional spectroscopic factors varied by ±45% and ±19%.

Normalizing the DWBA calculations to the data and accounting for the ANC's for the single particle orbitals and the known ANC for $^3He \rightarrow d + p$ [22,23] provides $C^2$ for $^{17}F \rightarrow ^{16}O + p$. Fits over several angular ranges, from $\theta_{cm} = 2 - 6^\circ$ to $\theta_{cm} = 2 - 30^\circ$, gave ANC's consistent to within 2%. The final ANC's were determined from fits to the forward angle peaks ($\theta_{cm} = 2 - 9^\circ$) to minimize the sensitivity to the choice of optical model parameters. Table I shows the adopted optical model parameter combinations that gave the smallest and largest ANC's. It is worth noting that most optical potentials that gave poor fits nonetheless gave ANC's that also fell within this range. The corresponding fits to the ground and first excited state angular distributions are shown in Fig. 1. The fits to the $^{17}F$ first excited state near the minimum and the weak population that we observe for the $^{17}F \frac{1}{2}^-$ second excited state and $\frac{3}{2}^-$ third excited state set upper limits on the contributions due to compound nuclear effects and multi-step reactions at <1%. Our final adopted ANC is $C^2_{d_{1/2}} = 1.08 \pm 0.10$ fm$^{-1}$ for the ground state. The uncertainty includes ±4.8% from the absolute normalization and angle accuracies, plus the statistics of the fits, and ±7.6% associated with the choice of optical model parameters and single particle orbital, as well as ambiguities in the reaction mechanism. Our final adopted ANC is $C^2_{s_{1/2}} = 6490 \pm 680$ fm$^{-1}$ for the first excited state. The corresponding contributions to

![FIG. 1. Angular distributions for the ground and first excited states of $^{17}F$ from the $^{16}O(^3He,d)^{17}F$ reaction. The dashed curves are DWBA fits using optical potential set I in Table I, and the solid curves use set II.](image)
TABLE I. Adopted optical potentials. Sets I and II gave the smallest and largest ANC’s for the two transitions, with other optical potential combinations giving ANC’s in between. The $d$ potentials are specified for the $^{17}\text{F}$ first excited state. The energy-dependent terms were slightly different for the ground state. All energies are in MeV, distances are in fm, and ANC’s are in fm$^{-1}$.

| Set | $V$ | $r$ | $a$ | $W_S$ | $W_D$ | $r_I$ | $a_I$ | $V_{LS}$ | $r_{LS}$ | $a_{LS}$ | $r_C$ | $C^2_{d_{5/2}}$ | $C^2_{s_{1/2}}$ |
|-----|-----|-----|-----|-------|-------|-------|-------|--------|--------|--------|------|---------------|---------------|
| I: $^3\text{He}$ | 185.03 | 1.15 | 0.672 | 11.75 | 1.511 | 0.748 | | 6.69 | 1.07 | 0.66 | 1.3 | 1.00 | 5980 |
| I: $d$ | 85.87 | 1.17 | 0.746 | 0.60 | 12.17 | 0.67 | | 6.69 | 1.07 | 0.66 | 1.3 | 1.00 | 5980 |
| II: $^3\text{He}$ | 183.33 | 1.15 | 0.659 | 7.93 | 2.142 | 0.695 | | 5.2 | 0.85 | 0.475 | 1.3 | 1.16 | 7000 |
| II: $d$ | 83.92 | 1.13 | 0.80 | 12.0 | 1.442 | 0.714 | | 5.2 | 0.85 | 0.475 | 1.3 | 1.16 | 7000 |

its uncertainty are ±5.4% and ±9.0%.

III. S-FACTORS FOR $^{16}\text{O}(p,\gamma)^{17}\text{F}$

The relation of the ANC’s to the direct capture rate at low energies is straightforward [2]. The cross section for the direct capture reaction $A + p \rightarrow B + \gamma$ can be written as

$$\sigma = \lambda |< I_{Ap}^B(r) | \hat{O}(r) | \psi_i^{(+)}(r) > |^2,$$

where $\lambda$ contains kinematic factors, $I_{Ap}^B$ is the overlap function for $B \rightarrow A + p$, $\hat{O}$ is the electromagnetic transition operator, and $\psi_i^{(+)}$ is the incident scattering wave. If the dominant contribution to the matrix element comes from outside the nuclear radius, the overlap function may be replaced by

$$I_{Ap}^B(r) \approx C W_{-\eta,t+1/2}(2\kappa r),$$

where $C$ defines the amplitude of the tail of the radial overlap function $I_{Ap}^B$, $W$ is the Whittaker function, $\eta$ is the Coulomb parameter for the bound state $B = A + p$, and $\kappa$ is the bound state wave number. For $^{16}\text{O}(p,\gamma)^{17}\text{F}$, the necessary $C$’s are just the ANC’s determined from the $^{16}\text{O}(^3\text{He}, d)^{17}\text{F}$ transfer reaction studies in Sect. II. Thus, the direct capture cross sections are directly proportional to the squares of these ANC’s. In fact, the $^{16}\text{O}(p,\gamma)^{17}\text{F}$ reaction populating the very weakly bound $^{17}\text{F}$ first excited state provides an extreme test of the connection between the ANC measured in a transfer reaction and the S-factor measured in direct capture. The approximation of Eq. (4) is excellent at large radii, but the proximity of the node in the $2s_{1/2}$ wave function makes it rather poor near the nuclear surface. In contrast, Eq. (4) provides a good description of the $^{17}\text{F}$ ground state $1d_{5/2}$ wave function even in the vicinity of the nuclear surface.

Following the prescription outlined above, the S-factors for $^{16}\text{O}(p,\gamma)^{17}\text{F}$ were calculated with no free parameters. The results are shown in Fig. 2. Both $E1$ and $E2$ contributions have been included in the calculations, but the $E1$ components dominate. The capture of protons by $^{16}\text{O}$ at low energies occurs at very large distances $r$ due to the extremely small proton separation energy of

![FIG. 2. A comparison of the experimental S-factors to those determined from the ANC’s found in $^{16}\text{O}(^3\text{He}, d)^{17}\text{F}$. The solid data points are from [9], and the open boxes are from [10]. The solid lines indicate our calculated S-factors, and the dashed lines indicate the ±1σ error bands. Note that the experimental ground state S-factor may be contaminated by background at energies below 500 keV [25].](image-url)
Thus, we find that the calculated capture cross sections are sensitive neither to the behavior of the overlap functions at small \( r \), nor to the nuclear interaction between \( ^{16}O \) and \( p \) in the initial state. We find that \( S(0) = 0.40 \pm 0.04 \text{ keV·b} \) for populating the \( ^{17}F \) ground state and \( S(0) = 9.8 \pm 1.0 \text{ keV·b} \) for populating the first excited state. The uncertainties in these calculated zero-energy S-factors come almost entirely from those in the corresponding ANC’s determined above. There is no uncertainty associated with ambiguities in an extrapolation from higher incident energies to zero energy, and there is very little theoretical uncertainty, since the capture reaction is almost purely peripheral at very low incident energies. In the astrophysical domain, the energy dependence of the capture cross sections is determined entirely by the initial Coulomb scattering wave functions and the kinematic factors, while their magnitudes are fixed by the ANC’s. The theoretical uncertainty in the S-factors is less than 2\% at an energy of 1 MeV. This was estimated by repeating the calculation while completely neglecting the nuclear interaction in the initial state. Hence, the uncertainty in S at small energies is due just to the uncertainties in the ANC’s measured above. However, as the energy increases above 1 MeV, the calculated S-factors become more sensitive to the behavior of the overlap functions at smaller \( r \) and to the details of the nuclear interaction in the initial state. In that case, the simple direct radiative capture model used here breaks down, and a microscopic approach including antisymmetrization is needed. This effect has been studied for \( ^7Be(p,\gamma)^8B \) in [10].

Two previous measurements of \( ^{16}O(p,\gamma)^{17}F \) have determined the capture cross sections to the ground and first excited states separately [11,12]. The experimental results for the S-factors populating the \( ^{17}F \) ground and excited states are also shown in Fig. 3. It is clear from Fig. 3 that the agreement between the experimental results and the predictions based on our measured ANC’s is indeed very good for proton energies below 1 MeV. At these energies, the \( ^{16}O(p,\gamma)^{17}F \) S-factors derived from the analysis of our \( ^{16}O(\ ^3He,d)^{17}F \) measurements agree with the corresponding direct experimental results to better than 9\%.

Our calculated S-factors for \( ^{16}O(p,\gamma)^{17}F \) in Fig. 3 are very similar to the S-factors calculated for the same reaction in [11]. The energy dependencies are virtually identical. For both states, we calculate the S-factor to be slightly larger than those in [11], which provides us with a somewhat better representation of the ground state S-factor and a slightly poorer representation for the first excited state. It is important to recognize that the procedures used in the two calculations are very different, even though their final results are quite similar. In [11], the \( ^{17}F \) ground and first excited states were assumed to be good single particle states outside a closed \( ^{16}O \) core. Electron scattering data were used to specify the density distribution of \( ^{16}O \), which provided the input for a folding model calculation of the low energy \( p-^{16}O \) potential with DDM3Y. The central and spin-orbit terms in the potential were renormalized separately, for both even and odd partial waves, by fitting the \( ^{17}F \) bound state energies and comparing to detailed data on low energy \( p+^{16}O \) elastic scattering. Finally, the direct capture rates were calculated with no additional free parameters. This level of detail was necessary to reproduce the \( ^{16}O(p,\gamma)^{17}F \) S-factors at proton energies higher than we consider here.

The ANC technique is quite different, much simpler, and based on much less experimental input. Our measured \( ^{16}O(^3He,d)^{17}F \) angular distributions determined the ANC’s for \( ^{17}F \rightarrow ^{16}O+p \) experimentally. These specify the amplitudes of the tails of the \( ^{17}F \rightarrow ^{16}O+p \) overlap functions. We then normalized single particle orbitals to the measured ANC’s, and used them to calculate the corresponding direct radiative capture S-factors. So long as one restricts oneself to the low energies typically of greatest importance to nuclear astrophysics, the only input required by this technique is the experimentally measured value of the ANC. In practice, the close agreement between our calculated S-factors and those in [11] indicate that the body of experimental data used to specify the \( p+^{16}O \) potential in [11] ultimately was sufficient to determine the \( ^{17}F \) ANC’s indirectly. However, the ANC approach may also be used to determine S-factors for direct radiative capture from peripheral transfer reaction data in cases, such as radioactive targets, for which much less experimental data are available than for \( ^{16}O \).

**IV. CONCLUSION**

In conclusion, the \( ^{16}O(p,\gamma)^{17}F \) S-factors derived from the analysis of our \( ^{16}O(^3He,d)^{17}F \) measurements agree with the corresponding direct experimental results to better than 9\%. This demonstrates the practicality of determining accurate S-factors for very low energy direct capture reactions from measurements of the corresponding asymptotic normalization coefficients in peripheral proton transfer reactions. This technique can be extended to other systems, including measurements with radioactive beams. The production of \(^7B\) in the sun via the \(^7Be(p,\gamma)^8B\) reaction is an ideal example. While this reaction is relatively unimportant in the production of energy, it provides the only source of high energy neutrinos. Hence its rate is crucial to interpreting measurements from solar neutrino detectors [13]. At stellar energies this reaction is completely dominated by direct capture which occurs at large radii. Indeed, even before this demonstration of the accuracy of this indirect technique, there has been an attempt [14] to determine the \(^7Be(p,\gamma)^8B\) S-factor from a measurement of the \(^8B\rightarrow^7Be+p\) ANC in the reaction \(^2H(^7Be,^8B)n\). But interpretation of that result suffered from significant uncertainties in the choice of optical potentials [14], at least in part due to the very low energies involved. The \(^8B\rightarrow^7Be+p\) ANC can also be measured in \(^7Be,^8B\)
transfer reactions at higher energies with heavier targets, where the uncertainties due to the choice of optical potentials are much reduced. We will report cross sections for this reaction using $^{10}\text{B}$ and $^{14}\text{N}$ targets in future publications.

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