The Power of Duality –
Exact Results in 4D SUSY Field Theory

N. Seiberg

Department of Physics and Astronomy
Rutgers University
Piscataway, NJ 08855-0849, USA

and

Institute for Advanced Study
Princeton, NJ 08540, USA

Recently the vacuum structure of a large class of four dimensional (supersymmetric) quantum field theories was determined exactly. These theories exhibit a wide range of interesting new physical phenomena. One of the main new insights is the role of “electric-magnetic duality.” In its simplest form it describes the long distance behavior of some strongly coupled, and hence complicated, “electric theories” in terms of weakly coupled “magnetic theories.” This understanding sheds new light on confinement and the Higgs mechanism and uncovers new phases of four dimensional gauge theories. We review these developments and speculate on the outlook.

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1. Introduction

Exact solutions play a crucial role in physics. It is often the case that a simple model exhibits the same phenomena which are also present in more complicated examples. The exact solution of the simple model then teaches us about more generic situations. For example, the exact solutions of the harmonic oscillator and of the hydrogen atom demonstrated many of the crucial aspects of quantum mechanics. Similarly, the Ising model was a useful laboratory in the study of statistical mechanics and quantum field theory. Many other exactly solvable two dimensional field theories like the Schwinger model and others have led to the understanding of many mechanisms in quantum field theory, which are also present in four dimensions.

The main point of this talk is to show how four dimensional supersymmetric quantum field theories can play a similar role as laboratories and testing grounds for ideas in more generic quantum field theories. This follows from the fact that these theories are more tractable than ordinary, non-supersymmetric theories, and many of their observables can be computed exactly. Nevertheless, it turns out that these theories exhibit explicit examples of various phenomena in quantum field theory. Some of them had been suggested before without an explicit realization and others are completely new. (For a brief review summarizing the understanding as of a year ago, see [1].)

Before continuing we would like to mention that supersymmetric four dimensional field theories also have two other applications:

1. Many physicists expect supersymmetry to be present in Nature, the reason being that it appears to be the leading candidate for solving the gauge hierarchy problem. If this is indeed the case, it is likely to be discovered experimentally in the next round of accelerators. Independent of the hierarchy problem, supersymmetry plays an important role in string theory as an interesting extension of our ideas about space and time. If Nature is indeed supersymmetric, understanding the dynamics of supersymmetric theories will have direct experimental applications. In particular, we will have to understand how supersymmetry is broken, i.e. why Nature is not exactly supersymmetric.

2. Witten discovered an interesting relation between supersymmetric field theories and four dimensional topology [2]. Using this relation, the exact solutions lead to a simplification of certain topological field theories and with that to advances in topology [3].
Even though these applications are important, here we will focus on the dynamical issues and will not discuss them.

The main insight that the study of these theories has taught us so far is the role of electric-magnetic duality in strongly coupled non-Abelian gauge theories. Therefore, we will start our discussion in the next section by reviewing the duality in Abelian gauge theories, i.e. in electrodynamics. In section 3 we will present supersymmetric field theories and will outline how they are solved. This discussion will be rather heuristic. In section 4 we will summarize the results and will present the duality in non-Abelian theories. Finally in section 5 we will present our conclusions and a speculative outlook.

2. Duality in electrodynamics

2.1. The Coulomb phase

The simplest phase of electrodynamics is the Coulomb phase. It is characterized by massless photons which mediate a long range $\frac{1}{r}$ potential between external sources. In the absence of sources the relevant equations are Maxwell’s equations in the vacuum

$$\nabla \cdot E = 0$$
$$\nabla \times B - \frac{\partial}{\partial t} E = 0$$
$$\nabla \cdot B = 0$$
$$\nabla \times E + \frac{\partial}{\partial t} B = 0$$

(we have set the speed of light $c = 1$). Clearly, they are invariant under the duality transformation

$$E \rightarrow B$$
$$B \rightarrow -E$$

which exchanges electric and magnetic fields.

If charged particles are added to the equations, the duality symmetry will be preserved only if both electric charges and magnetic monopoles are present. However, in Nature we

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1 The role of electric-magnetic duality in four dimensional quantum field theory was first suggested by Montonen and Olive [4]. Then, it became clear that the simplest version of their proposal is true only in $N = 4$ supersymmetric field theories [5] and in certain $N = 2$ supersymmetric theories [6]. Here we will discuss the extension of these ideas to $N = 1$ theories [7].
see electric charges but no magnetic monopole has been observed yet. This fact ruins the
duality symmetry. We usually also ruin the symmetry by solving two of the equations
by introducing the vector potential. Then, these equations are referred to as the Bianchi
identities while the other two equations are the equations of motion. Had there also been
magnetic monopoles, this would have been impossible.

Dirac was the first to study the possible existence of magnetic monopoles in the
quantum theory. He derived the famous Dirac quantization condition which relates the
electric charge $e$ and the magnetic charge $g$:

$$eg = 2\pi$$

(we have set Planck’s constant $\hbar = 1$). This relation has many important consequences.
One of them is that since duality exchanges electric and magnetic fields, it also exchanges

$$e \leftrightarrow g.$$  

Since the product $eg = 2\pi$ is fixed, it relates weak coupling ($e \ll 1$) to strong coupling
($g \gg 1$). Therefore, even if we could perform such duality transformations in quantum
electrodynamics, they would not be useful. Electrodynamics is weakly coupled because $e$
is small. Expressing it in terms of “magnetic variables” will make it strongly coupled and
therefore complicated.

However, one might hope that in other theories like QCD, such a duality transforma-
tion exists. If it does, it will map the underlying “electric” degrees of freedom of QCD,
which are strongly coupled, to weakly coupled “magnetic” degrees of freedom. We would
then have a weakly coupled, and therefore easily understandable, effective description of
QCD.

### 2.2. The Higgs phase

When charged matter particles are present, electrodynamics can be in another phase
— the superconducting or the Higgs phase. It is characterized by the condensation of a
charged field $\phi$

$$\langle \phi \rangle \neq 0.$$  

This condensation creates a gap in the spectrum by making the photon massive. This
phenomenon was first described in the context of superconductivity, where $\phi$ is the Cooper
pair. It has since appeared in different systems including the weak interactions of particle physics where $\phi$ is the Higgs field.

The condensation of $\phi$ makes electric currents superconducting. Its effect on magnetic fields is known as the Meissner effect. Magnetic fields cannot penetrate the superconductor except in thin flux tubes. Therefore, when two magnetic monopoles (e.g. the ends of a long magnet) are inserted in a superconductor, the flux lines are not spread. Instead, a thin flux tube is formed between them. The energy stored in the flux tube is linear in its length and therefore the potential between two external magnetic monopoles is linear (as opposed to the $\frac{1}{R}$ potential outside the superconductor). Such a linear potential is known as a confining potential.

Mandelstam and 'tHooft considered the dual of this phenomenon: if instead of electric charges, magnetic monopoles condense, then magnetic currents are superconducting while electric charges are confined. Therefore, confinement is the dual of the Higgs mechanism. They suggested that confinement in QCD can be understood in a similar way by the condensation of color magnetic monopoles.

To summarize, we see that duality exchanges weak coupling and strong coupling. Therefore, it exchanges a description of the theory with small quantum fluctuations with a description with large quantum fluctuations. Similarly, it exchanges the weakly coupled phenomenon of the Higgs mechanism with the strong coupling phenomenon of confinement.

Such a transformation between variables which fluctuate rapidly and variables which are almost fixed is similar to a Fourier transform. When a coordinate is localized, its conjugate momentum fluctuates rapidly and vice versa. A duality transformation is like a Fourier transform between electric and magnetic variables.

It should be stressed, however, that except in simple cases (like electrodynamics without charges and some examples in two dimensional field theory) an explicit duality transformation is not known. It is not even known whether such a transformation exists at all. As we will show below, at least in supersymmetric theories such a transformation does exist (even though we do not have an explicit description of it).

3. The Dynamics of Supersymmetric Field Theories

In supersymmetric theories the elementary particles are in representations of supersymmetry. Every gauge boson, a gluon, is accompanied by a fermion, a gluino, and every matter fermion, a quark, is accompanied by a scalar, a squark. The theory is specified
by a choice of a gauge group, which determines the coupling between the gluons, and a matter representation, which determines the coupling between the quarks and the gluons. For example, in QCD the gauge group is $SU(N_c)$ ($N_c$ is the number of colors) and the matter representation is $N_f \times (N_c + N_c)$ ($N_f$ is the number of flavors).

An important object in these theories is the superpotential, $W(q)$. It is a holomorphic (independent of $\bar{q}$, the complex conjugate of $q$) gauge invariant function of the squarks, $q$. It determines many of the coupling constants and interactions in the theory including the Yukawa couplings of the quarks and the squarks and the scalar potential, $V(q, \bar{q})$. For example, a mass term for the quarks appears as a quadratic term in the superpotential.

The analysis of the classical theory starts by studying the minima of the scalar potential $V(q, \bar{q})$. It is often the case that the potential has many different degenerate minima. Then, the classical theory has many inequivalent ground states. Such a degeneracy between states which are not related by a symmetry is known as an “accidental degeneracy.” Typically in field theory such an accidental degeneracy is lifted by quantum corrections. However, in supersymmetric theories, the degeneracy often persists in the quantum theory. Therefore, the quantum theory has a space of inequivalent vacua which can be labeled by the expectation values of the squarks $\langle q \rangle$.

This situation of many different ground states in the quantum theory is reminiscent of the situation when a symmetry is spontaneously broken. However, it should be stressed, that unlike that case, where the different ground states are related by a symmetry, here they are inequivalent. Physical observables vary from one vacuum to the other – they are functions of $\langle q \rangle$.

Our problem is to solve these theories not only as a function of all the parameters (like quark masses) but, for every value of the parameters, also as a function of the ground state $\langle q \rangle$.

As in Landau-Ginzburg theory, the best way to organize the information is in a low energy effective Lagrangian, $L_{\text{eff}}$. It includes only the low lying modes and describes their interactions. This effective theory is also specified by a gauge group and a matter representation. Since the theory is supersymmetric, the interactions of the light particles are characterized by a superpotential, $W_{\text{eff}}$.

The key observation is that this effective superpotential can often be determined exactly by imposing the following constraints  

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2 We limit ourselves to theories where supersymmetry is not spontaneously broken.
1. In various limits of the parameter space or the space of ground states the theory is weakly coupled. In these limits $W_{\text{eff}}$ can be determined approximately by weak coupling techniques (examples are the instanton calculations of [9-11]).

2. $W_{\text{eff}}$ must respect all the symmetries in the problem. It is important to use also the constraints following from symmetries which are explicitly broken by the various coupling constants. Such symmetries lead to selection rules.

3. $W_{\text{eff}}$ is holomorphic in the light fields and the parameters. This constraint is the crucial one which makes supersymmetric theories different from non-supersymmetric ones. The use of holomorphy in [8] generalized previous related ideas [12-16]. The superpotential determined in this way teaches us about the light particles and their interactions. This information determines the phase structure of the theory and the mechanisms for phase transitions.

By repeating this process for different theories, i.e. different gauge groups and matter representations, a rich spectrum of phenomena has been found. In the next section we will describe some of them.

4. Results – duality in non-Abelian theories

4.1. $SU(N_c)$ with $N_f \times (N_c + \overline{N}_c)$

Here the physical phenomena depend crucially on the number of massless quarks [9,17,13,18,15]:

I. $N_f \geq 3N_c$

In this range the theory is not asymptotically free. This means that, because of screening, the coupling constant becomes smaller at large distances. Therefore, the spectrum of the theory at large distance can be read off from the Lagrangian – it consists of the elementary quarks and gluons. The long distance behavior of the potential between external electric sources is of the form

$$V \sim \frac{1}{R \log R}.$$ 

The logarithm in the denominator shows that the interactions between the particles are weaker than in Coulomb theory. Therefore, we refer to this phase of the theory as a free electric phase.
We should add here that, strictly speaking, such a theory is not well defined as an interacting quantum field theory. However, it can be a consistent description of the low energy limit of another theory.

II. $\frac{3}{2}N_c < N_f < 3N_c$

In this range the theory is asymptotically free. This means that at short distance the coupling constant is small and it becomes larger at longer distances. However, in this regime rather than growing to infinity, it reaches a finite value – a fixed point of the renormalization group.

Therefore, this is a non-trivial four dimensional conformal field theory. The elementary quarks and gluons are not confined there but appear as interacting massless particles. The potential between external electric sources behaves as

$$V \sim \frac{1}{R}$$

and therefore we refer to this phase of the theory as the non-Abelian Coulomb phase.

It turns out that there is an equivalent, “magnetic,” description of the physics at this point. It is based on the gauge group $SU(N_f - N_c)$, with $N_f$ flavors of quarks and some gauge invariant fields. We will refer to this gauge group as the magnetic gauge group and to its quarks as magnetic quarks. This theory is also in a non-Abelian Coulomb phase because $\frac{3}{2}(N_f - N_c) < N_f < 3(N_f - N_c)$. The surprising fact is that its large distance behavior is identical to the large distance behavior of the original, “electric,” $SU(N_c)$ theory. Note that the two theories have different gauge groups and different numbers of interacting particles. Nevertheless, they describe the same fixed point. In other words, there is no experimental way to determine whether the $\frac{1}{R}$ potential between external sources is mediated by the interacting electric or the interacting magnetic variables. Such a phenomenon of two different Lagrangians describing the same long distance physics is common in two dimensions and is known there as quantum equivalence. These four dimensional examples generalize the duality in finite $N = 4$ supersymmetric theories and in finite $N = 2$ theories to asymptotically free $N = 1$ theories.

As $N_f$ is reduced (e.g. by giving masses to some quarks and decoupling them) the electric theory becomes stronger – the fixed point of the renormalization group occurs at larger values of the coupling. Correspondingly, the magnetic theory becomes weaker. This can be understood by noting that as $N_f$ is reduced, the magnetic gauge group becomes smaller. This happens by the Higgs mechanism in the magnetic theory.
III. $N_c + 2 \leq N_f \leq \frac{3}{2}N_c$

In this range the electric theory is very strongly coupled. However, since $3(N_f - N_c) \leq N_f$, the equivalent magnetic description based on the gauge group $SU(N_f - N_c)$ is not asymptotically free and it is weakly coupled at large distances. Therefore, the low energy spectrum of the theory consists of the particles in the dual magnetic Lagrangian [7]. These magnetic massless states are composites of the elementary electric degrees of freedom. The massless composite gauge bosons exhibit gauge invariance which is not visible in the underlying electric description. The theory generates new gauge invariance! We will return to this phenomenon in the conclusions.

This understanding allows us to determine the long distance behavior of the potential between magnetic sources:

$$V \sim \frac{1}{R \log R}.$$ 

Since the magnetic variables are free at long distance, we refer to this phase as a free magnetic phase.

IV. $N_f = N_c + 1, N_c$

As we continue to decouple quarks by giving them masses and thus reducing $N_f$, the magnetic gauge group is Higgsed more and more. Eventually it is completely broken and there are no massless gauge bosons. This complete Higgsing of the magnetic theory can be interpreted as complete confinement of the electric variables. We thus see an explicit realization of the ideas of Mandelstam and 'tHooft about confinement.

For $N_f = N_c + 1$ this confinement is not accompanied by chiral symmetry breaking, while for $N_f = N_c$ chiral symmetry is also broken [18]. In the electric language we describe the spectrum in terms of gauge invariant fields. These include massless mesons and baryons. In the magnetic language these are elementary fields. The idea that some of the composites in QCD, in particular the baryons, can be thought of as solitons was suggested in [19]. Here we see an explicit realization of a related idea – the baryons are magnetic monopoles composed of the elementary quarks and gluons.

V. $N_f < N_c$

In this range the theory of massless quarks has no ground state [7,15].
4.2. $SO(N_c)$ with $N_f \times N_c$

In the $SU(N_c)$ theories there is no invariant distinction between Higgs and confinement \cite{20}. This is not the case in theories based on $SO(N_c)$ with $N_f \times N_c$ and therefore, they lead to a clearer picture of the dynamics. In particular, here the transition from the Higgs phase to the Confining phase occurs with a well defined phase transition.

Many of the results in these $SO(N_c)$ theories \cite{9,21-23,7,24} are similar to the results in $SU(N_c)$ showing that these phenomena are generic. Here the duality map is

$$SO(N_c) \times N_f \times N_c \longleftrightarrow SO(N_f - N_c + 4) \times N_f \times (N_f - N_c + 4)$$

Let us consider three special cases of the duality:

1. For $N_c = 2$, $N_f = 0$ the map is

$$SO(2) \cong U(1) \longleftrightarrow SO(2) \cong U(1)$$

which is the ordinary duality of electrodynamics. Therefore, our duality is compatible with and generalizes this duality.

2. For $N_c = 3$, $N_f = 1$ the map is:

$$SO(3) \leftrightarrow SO(2) \cong U(1)$$

This theory was first analyzed in \cite{22}. Here it is possible to understand the duality in more detail than in the more general case and in particular to identify the matter fields in the magnetic theory as magnetic monopoles. This theory has $N = 2$ supersymmetry but this did not play a role in our discussion. However, using the extra supersymmetry one can derive more exact results about the massive spectrum of the theory \cite{22}.

3. For $N_f = N_c - 2$ the map is

$$SO(N_c) \times (N_c - 2) \times N_c \longleftrightarrow SO(2) \cong U(1) \times (N_c - 2) \times 2$$

which generalizes the previous example to situations without $N = 2$ supersymmetry. These three examples are simple because the magnetic theory is Abelian. However, in general the magnetic $SO(N_f - N_c + 4)$ gauge group is non-Abelian.

These $SO(N_c)$ theories also exhibit many new phenomena, which are not present in the $SU(N_c)$ examples. The most dramatic of them is oblique confinement \cite{25,26}, driven by the condensation of dyons (particles with both electric and magnetic charges). This phenomenon is best described by another equivalent theory – a dyonic theory. Therefore, these theories exhibit electric-magnetic-dyonic triality \cite{24}.
5. Conclusions

To conclude, supersymmetric field theories are tractable and many of their observables can be computed exactly\(^3\). The main dynamical lesson we learn is the role of electric-magnetic duality in non-Abelian gauge theories in four dimensions. This duality generalizes the duality in Maxwell’s theory and in \( N = 4 \) \(^4\) and certain \( N = 2 \) supersymmetric theories \(^5\).

The magnetic degrees of freedom are related to the underlying electric degrees of freedom in a complicated (non-local) way. They are the effective degrees of freedom useful for describing the long distance behavior of the theory. These variables give a weak coupling description of strong coupling phenomena such as confinement.

Our analysis led us to find new phases of non-Abelian gauge theories, like the non-Abelian Coulomb phase with its quantum equivalence and the free magnetic phase with its massless composite gauge bosons.

Outlook

We would like to end by suggesting some future directions for research and speculate about them.

1. The exploration of supersymmetric models is far from complete. There are many models whose dynamics are not yet understood. It is likely that there are new phenomena in quantum field theory which can be uncovered here.

2. Of particular importance are chiral theories whose matter content is not in a real representation of the gauge group. Only a few of these have been analyzed \(^6\). These theories are also interesting as they can lead to dynamical supersymmetry breaking. Finding a nice model of dynamical supersymmetry breaking which is phenomenologically acceptable is an important challenge. For some recent work in this direction see \(^7\).

3. An important question is to what extent these results are specific to supersymmetric theories. It would be very interesting to extend at least some of these ideas to non-supersymmetric theories and to find the various phases and the mechanisms for the phase transitions without supersymmetry. One way such a study can proceed is by perturbing a supersymmetric theory whose solution is known by soft breaking terms.

\(^3\) Although we did not discuss them here, we would like to point out that many other examples were studied \(^8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27\) exhibiting many new interesting phenomena.
When these terms are small they do not affect the dynamics significantly and the non-supersymmetric perturbed theory is qualitatively similar to the unperturbed theory (for a recent attempt in this direction see [48]). This makes it clear that the phenomena we found can be present in non-supersymmetric theories as well. Alternatively, one can start with ordinary non-supersymmetric QCD with a large enough number of flavors $N_f$, where a non-trivial fixed point of the renormalization group exists [49] and the theory is in a non-Abelian Coulomb phase. It is not yet known for which values of $N_f$ this phase exists. It is possible that there is a dual magnetic description of this phase and perhaps even a non-Abelian free magnetic phase exists for some values of $N_f$. One might be tempted to speculate that perhaps the confinement of ordinary QCD can be described as a Higgs phenomenon in these variables. Then, perhaps some of the massive particles in the spectrum of QCD (like the $\rho$ meson or the $A_1$) can be identified as “Higgsed magnetic gluons.”

4. The duality points at a big gap in our current understanding of gauge theories. We do not have an explicit transformation relating the underlying electric degrees of freedom to their magnetic counterparts. Finding such an explicit transformation will be extremely interesting. If such a transformation does not exist within the standard framework of local quantum field theory, perhaps a reformulation of quantum field theory will be needed.

5. The duality reflects new gauge invariance, which is not obvious in the fundamental description of the theory. In hindsight this is not surprising because gauge symmetry is not a symmetry. It is merely a redundancy in the description. Perhaps we should conclude that gauge symmetries might not be fundamental! They might only appear as long distance artifacts of our description of the theory. If so, perhaps some of the gauge symmetries of the standard model or even general relativity are similarly long distance artifacts. Then, the corresponding gauge particles are the “magnetic” degrees of freedom of more elementary “electric” variables. In order not to violate the theorem of [50], the underlying theory cannot have these symmetries as global symmetries. In particular, this can be the case for gravity only if the underlying theory is topological.

6. Recently, there has been enormous interest in dualities and non-perturbative effects in string theory [51]. Some of the phenomena found in string theory are similar to and generalize those found in field theory. It would be nice to use some of the techniques which turned out to be useful in field theory in string theory as well.
Finally we should note that there are obvious relations between these directions. For example, progress in understanding the origin of the duality can help the study of supersymmetric chiral theories and lead to phenomenological theories of supersymmetry breaking.

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