Gamma-Ray Lines from Radiative Dark Matter Decay

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Abstract

The decay of dark matter particles which are coupled predominantly to charged leptons has been proposed as a possible origin of excess high-energy positrons and electrons observed by cosmic-ray telescopes PAMELA and Fermi LAT. Even though the dark matter itself is electrically neutral, the tree-level decay of dark matter into charged lepton pairs will generically induce radiative two-body decays of dark matter at the quantum level. Using an effective theory of leptophilic dark matter decay, we calculate the rates of radiative two-body decays for scalar and fermionic dark matter particles. Due to the absence of astrophysical sources of monochromatic gamma rays, the observation of a line in the diffuse gamma-ray spectrum would constitute a strong indication of a particle physics origin of these photons. We estimate the intensity of the gamma-ray line that may be present in the energy range of a few TeV if the dark matter decay interpretation of the leptonic cosmic-ray anomalies is correct and comment on observational prospects of present and future Imaging Cherenkov Telescopes, in particular the CTA.
1 Introduction

The existence of dark matter is now established beyond reasonable doubt by a variety of independent observations [1]. These require the presence of substantial amounts of non-baryonic dark matter at vastly different scales ranging from individual galaxies to superclusters and filaments. Despite the overwhelming amount of gravitational evidence, however, no unambiguous evidence for non-gravitational dark matter interactions has been discovered to this day. Since a determination of the particle nature of the dark matter from its gravitational interactions alone is impossible, searches for non-gravitational signatures of dark matter are of paramount importance.

One of the principal approaches to its identification is the indirect detection of dark matter via searches for exotic components in the cosmic radiation produced by dark matter interactions with Standard Model particles. For weakly interacting massive particles (WIMPs), it was pointed out some decades ago that the dark matter self-annihilation processes that can yield the correct thermal relic abundance might still occur today at a rate high enough to give rise to a flux of cosmic rays and photons that may be detectable by suitable telescopes. However, the self-annihilation of dark matter is not the only possible scenario for indirect dark matter detection. Namely, the dark matter might be unstable and decay into Standard Model particles, even though it must have a very long lifetime in order to survive from its production in the early Universe to the present day. If the dark matter is not perfectly stabilized by some unbroken symmetry, however, the possibility exists that its decay products may leave visible traces in cosmic-ray fluxes. Indeed, in some well-motivated models the dark matter particles are not perfectly stable, but decay with cosmological lifetimes (see, e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]).

Among the possible indirect detection channels, measurements of cosmic-ray antimatter are particularly sensitive to exotic contributions from dark matter. Interestingly, the PAMELA telescope has recently confirmed the existence of a dramatic rise in the positron fraction extending up to energies of at least 100 GeV [14], in stark contrast with expectations from conventional models of cosmic-ray production and propagation [15]. Furthermore, the Fermi Gamma-ray Space Telescope has observed a total flux of electrons and positrons that is harder than expected [16, 17], also indicating the possible presence of an additional component of charged cosmic-ray leptons [18].

Various astrophysical explanations for these unexpected behaviors have been proposed [19, 20, 21]. Arguably more exciting, however, is the possibility that the excess of positrons and electrons is due to the annihilation or decay of dark matter particles. However, measurements of cosmic-ray antiprotons, in particular measurements of the antiproton-to-proton ratio by PAMELA [22, 23], yield stringent constraints on the frac-
tion of dark matter decays or annihilations into hadronic final states. This has lead some authors to consider ‘leptophilic’ models of dark matter [6, 24, 25, 26, 27, 28, 29, 30, 31, 32], where the dark matter is coupled predominantly or exclusively to charged leptons. In the following, we consider the possibility that the dark matter particles are indeed unstable, but decay leptonically with extremely long lifetimes. More precisely, the interpretation of the leptonic cosmic-ray anomalies observed by PAMELA and Fermi in terms of dark matter decay suggests a lifetime of the dark matter on the order of $10^{26}$ seconds (see, e.g., [33]). This lifetime exceeds the age of the Universe by nine orders of magnitude, thus leaving the dark matter sufficiently stable on cosmological timescales. Nevertheless, due to the large amounts of dark matter in the Universe, even for such enormous lifetimes the resulting fluxes can be in the observable range. Indeed, strong constraints on decaying dark matter have been derived recently from gamma-ray observations of galaxy clusters and nearby galaxies [34].

In this paper, we examine some of the effects of leptophilic models of decaying dark matter which arise at next-to-leading order in perturbation theory and show that they can have relevance to indirect dark matter searches. We will not speculate on the precise nature of the particle physics that could give rise to leptophilic dark matter decay. Instead, our approach will be to examine simple models where we assume effective interactions that describe the desired leptophilic coupling of dark matter particles to charged leptons. The salient point for us here is that even if one assumes an exclusive coupling of the dark matter to charged leptons at tree level, this behavior is only valid at leading order, while at next-to-leading order other particles, including photons and weak gauge bosons, will be produced. Indeed, these higher-order corrections have been analyzed in the past for the case of annihilating dark matter [35, 36, 37, 38, 39, 40]. It is well known that the higher-order corrections in the form of internal bremsstrahlung or from final-state radiation of weak gauge bosons can even dominate under certain conditions [36, 39].

The decay modes induced by higher-order corrections are usually suppressed by powers of the couplings and possibly loop factors, as opposed to the leading-order decay modes. This means that the resulting decay products will be difficult to detect unless they possess some distinct features. Weak gauge bosons can be produced, for instance, via final-state radiation off the charged leptons [41]. By their subsequent hadronization, the massive gauge bosons will then generate hadronic particles, including antiprotons [40]. Therefore, every leptophilic dark matter model that aims to explain the leptonic cosmic-ray anomalies also serves as a source of antiprotons.

In this work, however, we will focus on complementary constraints arising from a different decay channel induced by higher-order effects. Namely, we will study radiative two-body decays involving photons. These are particularly interesting, since they give rise
Figure 1: Tree-level diagrams contributing to the three-body decay \( \psi_{\text{DM}} \rightarrow \ell^+ \ell^- N \) of fermionic dark matter, mediated by a heavy charged scalar \( \Sigma \). Instead of the intermediate scalar \( \Sigma \), the decay can also be mediated by a vector \( V \).

to monochromatic lines in the diffuse gamma-ray spectrum or in extragalactic sources. Such lines are of utmost importance because astrophysical processes generally generate continuous gamma-ray spectra. Thus, the observation of a gamma-ray line would be a compelling signature of an underlying particle physics process. In some cases, a gamma-ray line can be produced already in tree level decays \([42]\). In the present work, we demonstrate that for leptophilic models of dark matter, the ratio between leading-order and next-to-leading-order decay modes can be large enough to produce a potentially observable gamma-ray line signal.

This paper is organized as follows. In section 2, we discuss the production of monochromatic photons from radiative two-body decays induced at the one-loop level for fermionic dark matter particles in a simple leptophilic toy model. In section 3, we examine the corresponding case for a scalar dark matter particle. Next, in section 4, we discuss observational constraints on gamma-ray lines in the GeV to TeV region and compare existing bounds with the expected signal from dark matter decay. We also comment on future observational prospects, in particular for the proposed Cherenkov Telescope Array. Finally, we present our conclusions in section 5.

2 Radiative decay of fermionic dark matter

We first regard the case that the particles comprising the dark matter are fermions which we denote by \( \psi_{\text{DM}} \). We require that the dark matter decays with a large branching fraction into pairs of charged leptons in order to explain the excess of such leptons in high-energy cosmic rays, and we assume that this is the only channel in which the dark
Figure 2: Diagrams contributing at one loop to the radiative two-body decay $\psi_{DM} \rightarrow \gamma N$, induced by a charged scalar $\Sigma$ (top row) and a vector particle $V$ (bottom row), respectively. There are two additional diagrams in each case which differ only by the direction of the charge flow.
matter decays at leading order. If the dark matter carries spin 1/2\textsuperscript{1}, Lorentz invariance requires the decay to be (at least) a three-body decay involving a third, electrically neutral fermion \( N \) for angular momentum conservation. Thus, the decay \( \psi_{\text{DM}} \to \ell^+ \ell^- N \) is the simplest one allowed by gauge and Lorentz invariance. Here, \( N \) could be a neutrino, a neutralino or a gravitino, for instance. The decays may be mediated by a virtual charged scalar particle \( \Sigma \) or by a charged vector boson \( V \), with masses \( m_\Sigma \) and \( m_V \), respectively, which are assumed to be larger than the mass of the dark matter particle. We regard the two cases separately.

In the case of an intermediate scalar, the effective Lagrangian that we use is given as the sum of a term coupling the dark matter, which we take to be a metastable Majorana fermion, to a charged lepton and a \( \Sigma \) particle, as well as a term coupling the neutral fermion to the \( \Sigma \) and a lepton field. We decompose the couplings into left- and right-handed components to allow for chiral couplings. Then the Lagrangian has the form

\[
\mathcal{L}^\Sigma_{\text{eff}} = - \bar{\psi}_{\text{DM}} \left[ \lambda_{\psi}^L P_L + \lambda_{\psi}^R P_R \right] \ell \Sigma^\dagger - \bar{N} \left[ \lambda_{\ell N}^L P_L + \lambda_{\ell N}^R P_R \right] \ell V^\dagger + \text{h.c.},
\]

(2.1)

where \( P_L = (1 - \gamma^5)/2 \) and \( P_R = (1 + \gamma^5)/2 \) are the left- and right-handed chirality projectors, respectively. The \( \lambda \)-couplings can in general be complex. To obtain the required cosmological lifetime for the dark matter, the couplings have to be super-weak or the mass \( m_\Sigma \) of the mediator has to be super-heavy. The operators of the effective Lagrangian induce three-body decays of the dark matter into a pair of charged leptons and a neutral fermion at tree level, \( \psi_{\text{DM}} \to \ell^+ \ell^- N \). The corresponding diagrams are shown in fig. 1.

In the case of a vector interaction, on the other hand, we assume an effective Lagrangian of the form

\[
\mathcal{L}^V_{\text{eff}} = - \bar{\psi}_{\text{DM}} \gamma^\mu \left[ \lambda_{\psi}^L P_L + \lambda_{\psi}^R P_R \right] \ell V^\dagger_\mu - \bar{N} \gamma^\mu \left[ \lambda_{\ell N}^L P_L + \lambda_{\ell N}^R P_R \right] \ell V^\dagger_\mu + \text{h.c.}.
\]

(2.2)

The case of mediation by a vector boson is more involved than the previous case of mediation by a scalar. In choosing a Lagrangian of this form, we assume that the essence of the gauge interaction giving rise to this Lagrangian is captured by the effective charged-vector interaction. In general, one expects neutral currents in association with the charged currents, which introduces a high degree of model dependence. For simplicity, we assume here that the decay is dominated by the charged-current interaction.

\textsuperscript{1}For the case of spin 3/2, see refs. [43, 44] on radiative gravitino decay.
2.1 Decay widths

In the following we examine decay modes of the dark matter at the tree- and one-loop level and summarize the relevant decay widths.

2.1.1 Tree-level decay: $\psi_{DM} \to \ell^+ \ell^- N$

The leading-order decay induced by the effective Lagrangian is the three-body decay $\psi_{DM} \to \ell^+ \ell^- N$. If the dark matter decays in this way, it constitutes a possible explanation for the observed cosmic-ray anomalies under certain conditions \[33\]. We present the relevant expressions for the decay widths in the following.

Mediation by a scalar. In the plausible limit $m_\ell \ll m_{\psi_{DM}} \ll m_\Sigma$, the partial decay width for the decay $\psi_{DM} \to \ell^+ \ell^- N$ is given by (see app. A and ref. \[47\])

$$
\Gamma(\psi_{DM} \to \ell^+ \ell^- N) = \frac{1}{64(2\pi)^3} \frac{m_{\psi_{DM}}^5}{6m_\Sigma^2} \left\{ C_{1}^\Sigma F_1(m_N^2/m_{\psi_{DM}}^2) + C_{2}^\Sigma F_2(m_N^2/m_{\psi_{DM}}^2) \right\}.
$$

The constants $C_{1}^\Sigma, C_{2}^\Sigma$ are determined by the couplings as

$$
C_{1}^\Sigma \equiv (|\lambda_{L\ell\psi}^L|^2 + |\lambda_{R\ell\psi}^R|^2) (|\lambda_{L\ell N}^L|^2 + |\lambda_{R\ell N}^R|^2) - \eta \text{ Re} \left( \lambda_{L\ell\psi}^L \lambda_{L\ell N}^L \lambda_{R\ell\psi}^R \lambda_{R\ell N}^R \right),
$$

$$
C_{2}^\Sigma \equiv 2\eta \text{ Re} \left[ \left( \lambda_{L\ell\psi}^L \lambda_{L\ell N}^L \right)^2 + \left( \lambda_{R\ell\psi}^R \lambda_{R\ell N}^R \right)^2 \right].
$$

Here, $\eta \equiv \eta_{\psi_{DM}/\ell N} = \pm 1$ depending on the CP eigenvalues of $\psi_{DM}$ and $N$. The kinematical functions, on the other hand, are given by

$$
F_1(x) \equiv (1 - x^2)(1 + x^2 - 8x) - 12x^2 \ln(x),
$$

$$
F_2(x) \equiv \sqrt{x}(1 - x)(1 + 10x + x^2) + 6x(1 + x) \ln(x).
$$

In the hierarchical limit $m_N/m_{\psi_{DM}} \to 0$, the kinematical functions satisfy

\begin{equation}
F_1(x) \simeq 1, \quad F_2(x) \simeq \sqrt{x} \quad \text{for} \quad x \to 0,
\end{equation}

whereas in the degenerate limit $m_N/m_{\psi_{DM}} \to 1$, one gets

\begin{equation}
F_1(x) \simeq \frac{2}{5}(1 - x)^5, \quad F_2(x) \simeq \frac{1}{10}(1 - x)^5 \quad \text{for} \quad x \to 1.
\end{equation}

In the limit $m_N \ll m_{\psi_{DM}}$ the decay rate \(2.3\) corresponds to a lifetime

\begin{equation}
\tau_{\psi_{DM} \to \ell^+ \ell^- N} \simeq 6 \times 10^{26} \text{s} \left( \frac{0.1}{C_1^\Sigma} \right) \left( \frac{1 \text{ TeV}}{m_{\psi_{DM}}} \right)^5 \left( \frac{m_\Sigma}{10^{15} \text{ GeV}} \right)^4.
\end{equation}

In the case where $m_{\psi_{DM}}$ and $m_N$ are quasi-degenerate, the decay rate scales approximately like $(m_{\psi_{DM}} - m_N)^5$.

\[2\] We have cross-checked the matrix elements for the three-body decays and the decay rates in the following sections by comparing them to the results from FeynArts \[45\] and FormCalc \[46\].
Mediation by a vector. For the vector-mediated decay we find for the three-body decay rate in the limit $m_\ell \ll m_{\psi_{DM}} \ll m_V$

$$\Gamma(\psi_{DM} \to \ell^+ \ell^- N) = \frac{1}{64(2\pi)^3} \frac{4m_{\psi_{DM}}^5}{6m_V^4} \left\{ C_1^V F_1(m_N^2/m_{\psi_{DM}}^2) + C_2^V F_2(m_N^2/m_{\psi_{DM}}^2) \right\},$$

(2.11)

where the functions $F_1$ and $F_2$ are the same as in eqs. (2.6), (2.7) and

$$C_1^V \equiv (|\lambda_{\psi\ell}^L|^2 + |\lambda_{\psi\ell}^R|^2) (|\lambda_{\ell N}^L|^2 + |\lambda_{\ell N}^R|^2) + 2\eta \text{Re} \left( \lambda_{\psi\ell}^L \lambda_{\ell N}^L \lambda_{\psi\ell}^R \lambda_{\ell N}^R \right),$$

(2.12)

$$C_2^V \equiv 2\eta \text{Re} \left[ (\lambda_{\psi\ell}^L \lambda_{\ell N}^L)^2 + (\lambda_{\psi\ell}^R \lambda_{\ell N}^R)^2 \right] = C_2^\Sigma.$$  

(2.13)

For $m_N \ll m_{\psi_{DM}}$ and an analogous choice of parameters, the lifetime is smaller by a factor of four compared to eq. (2.10),

$$\tau_{\psi_{DM} \to \ell^+ \ell^- N} \simeq 1.5 \times 10^{26} s \left( \frac{0.1}{C_1^V} \left( \frac{1 \text{ TeV}}{m_{\psi_{DM}}} \right)^5 \left( \frac{m_V}{10^{15} \text{ GeV}} \right)^4. \right.$$

(2.14)

### 2.1.2 One-loop decay: $\psi_{DM} \to \gamma N$

By combining the external charged lepton lines from the tree-level diagrams into a loop, we obtain diagrams contributing to the two-body decay $\psi_{DM} \to \gamma N$ (see fig. 2). This decay mode will be suppressed with respect to the tree-level three-body decay by a loop factor and an additional power of the electromagnetic coupling. This is partially compensated by phase-space factors, however. More importantly, the two-body decay gives rise to monochromatic photons at an energy

$$E_\gamma = \frac{m_{\psi_{DM}}}{2} \left( 1 - \frac{m_N^2}{m_{\psi_{DM}}^2} \right),$$

(2.15)

which can result in a distinct observational signature at gamma-ray telescopes, as will be discussed in some detail in section 4. Interestingly, the experimental constraints on the parameters of decaying dark matter stemming from the non-observation of energetic gamma-ray lines could, despite the loop-suppression, be more stringent than the ones stemming from measurements of cosmic-ray electrons and positrons.

Based on gauge invariance, and irrespective of whether the decay is mediated by a scalar or a vector particle, the matrix element for the sum of all diagrams contributing to the radiative two-body decay can be written in the following form, introducing an effective coupling $g_{N\gamma\psi}$ [28],

$$\mathcal{M} = \frac{i g_{N\gamma\psi}}{m_{\psi_{DM}}} \bar{u}(k_1)(P_R - \eta_N \eta_{\psi_{DM}} P_L) \sigma^{\mu\nu} k_2 \epsilon_\nu u(p) = -\frac{g_{N\gamma\psi}}{m_{\psi_{DM}}} \bar{u}(k_1)(P_R - \eta_N \eta_{\psi_{DM}} P_L) \epsilon_2 k_2 \epsilon_\nu u(p),$$

(2.16)
where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$ and $\eta_\psi, \eta_N$ are the CP eigenvalues of $\psi_{DM}$ and $N$, respectively. The partial decay width for $\psi_{DM} \to \gamma N$ can then be easily calculated to be

$$
\Gamma(\psi_{DM} \to \gamma N) = \frac{g^2_{N\gamma\psi}}{8\pi} m_{\psi_{DM}} \left( 1 - \frac{m_N^2}{m_{\psi_{DM}}^2} \right)^2 .
$$

(2.17)

The effective coupling $g_{N\gamma\psi}$ encodes all the information about the interaction between dark matter and the decay products. We give explicit expressions for this coupling in the following.

**Mediation by a scalar.** We first examine the case of mediation by a charged scalar particle $\Sigma$ (top row of fig. 2). Assuming that CP is conserved in the interactions of $\psi_{DM}$ and $N$, i.e., when the $\lambda$-couplings are assumed to be real, the explicit form of the effective coupling $g_{\Sigma N\gamma\psi}$ can be expressed as follows,

$$
g_{\Sigma N\gamma\psi} = - \frac{e \eta_N m_{\psi_{DM}}}{16\pi^2} \sum_{\ell, \Sigma} Q_\ell C_\ell \left\{ m_f (\eta_{\psi_{DM}} \lambda^{L\ell N}_L \lambda^{R\ell N}_R - \eta_N \lambda^{R\ell N}_L \lambda^{L\ell N}_R) I + (\lambda^{L\ell N}_L \lambda^{L\ell_{\psi}}_L - \eta \lambda^{R\ell N}_L \lambda^{R\ell_{\psi}}_R) [\eta_{\psi_{DM}} m_{\psi_{DM}} (I^2 - K) - \eta_N m_N K] \right\} ,
$$

(2.18)

where the loop integrals $I$, $I^2$ and $K$ are defined in app. A. The sum runs over all lepton flavors $\ell \in \{e, \mu, \tau\}$, for which $Q_\ell = C_\ell = 1$. If multiple mediator particles (like left- and right-handed sleptons, $\Sigma = \tilde{\ell}_L, \tilde{\ell}_R$) are present, one also has to sum over their contributions. In principle, there can also be contributions from quarks in the loop. Then, the sum runs over quarks and leptons with electric charge $Q_q$ and color charge $C_q = 3$. However, tree-level decays into quarks can potentially lead to an overproduction of antiprotons if the relative size of the effective coupling to quarks compared to the coupling to leptons is too large. The requirement of avoiding antiproton overproduction then leads to the assumption of a leptophilic structure. For this reason, we assume throughout this work that the dark matter decays only into leptons at tree level.

If the mass of the intermediate particle $\Sigma$ is much larger than the other masses, the loop integrals take on a very simple form. The effective coupling is then given approximately by

$$
g_{\Sigma N\gamma\psi} \approx \frac{e \eta}{64\pi^2} \frac{m^2_{\psi_{DM}}}{m_{\psi_{DM}}} \left( 1 - \frac{\eta m_N}{m_{\psi_{DM}}} \right) \sum_{\ell, \Sigma} \frac{Q_\ell C_\ell}{m^2_\Sigma} \left\{ (\lambda^{L\ell N}_L \lambda^{L\ell_{\psi}}_L - \eta \lambda^{R\ell N}_L \lambda^{R\ell_{\psi}}_R) \right\} .
$$

(2.19)

For the concrete case where there is only one mediator $\Sigma$, which couples exclusively

\[3\text{Note that the superscript '2' in the integral $I^2$ is an index, not a square.}\]
to leptons, the decay rate reads
\[ \Gamma(\psi_{DM} \to \gamma N) = \frac{e^2}{8\pi} \frac{m_{\psi_{DM}}^5}{m_\Sigma^4} \left( 1 - \frac{m_N^2}{m_{\psi_{DM}}^2} \right)^3 \left( 1 - \frac{\eta m_N}{m_{\psi_{DM}}} \right)^2 \times \left[ \sum_\ell \left( \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \eta \lambda_{\ell N}^R \lambda_{\ell \psi}^R \right) \right]^2. \]  \tag{2.20}

For \( m_N \ll m_{\psi_{DM}} \), this decay width corresponds to a partial lifetime
\[ \tau_{\psi_{DM} \to \gamma N} \simeq 7 \times 10^{29} \text{ s} \frac{0.1}{\left[ \left( \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \eta \lambda_{\ell N}^R \lambda_{\ell \psi}^R \right) \right]^2 \left( \frac{1 \text{ TeV}}{m_{\psi_{DM}}} \right)^5 \left( \frac{m_\Sigma}{10^{15} \text{ GeV}} \right)^4}. \]  \tag{2.21}

**Mediation by a vector.** In the case of mediation by a charged vector boson (bottom row of fig. 2), we obtain the following expression for the effective coupling [48],
\[ g_{\psi_{DM} \to \gamma N} = \frac{e \eta m_{\psi_{DM}}}{8\pi^2} \sum_\ell \left\{ (\eta_{\psi_{DM}} \eta_N \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \lambda_{\ell N}^R \lambda_{\ell \psi}^R) \left[ \eta_{\psi_{DM}} m_{\psi_{DM}} (I^2 - J - K) + \eta_N m_N (J - K) \right] + 2 m_\ell (\eta_{\psi_{DM}} \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \eta_N \lambda_{\ell N}^R \lambda_{\ell \psi}^R) J \right\}, \]  \tag{2.22}

where we encounter an additional loop integral \( J \), which is defined in app. [A]. Again, it is possible to include quarks by the replacement \( \sum_\ell \to \sum_f Q_f C_f \). However, as discussed above, this would not correspond to a leptophilic model. In the limit \( m_\ell \to 0, m_{\psi_{DM}} \ll m_V \), the above expression simplifies to
\[ g_{\psi_{DM} \to \gamma N} \simeq \frac{3e \eta}{32\pi^2} \frac{m_{\psi_{DM}}^2}{m_V^4} \left( 1 - \frac{\eta m_N}{m_{\psi_{DM}}} \right) \sum_\ell \left( \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \eta \lambda_{\ell N}^R \lambda_{\ell \psi}^R \right). \]  \tag{2.23}

Thus, in the limit \( m_\ell \ll m_N \) and \( m_{\psi_{DM}} \ll m_V \) we obtain for the decay width
\[ \Gamma(\psi_{DM} \to \gamma N) = \frac{9 e^2}{8\pi} \frac{m_{\psi_{DM}}^5}{m_V^4} \left( 1 - \frac{m_N^2}{m_{\psi_{DM}}^2} \right)^3 \left( 1 - \frac{\eta m_N}{m_{\psi_{DM}}} \right)^2 \times \left[ \sum_\ell \left( \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \eta \lambda_{\ell N}^R \lambda_{\ell \psi}^R \right) \right]^2. \]  \tag{2.24}

For \( m_N \ll m_{\psi_{DM}} \), this yields a partial lifetime
\[ \tau_{\psi_{DM} \to \gamma N} \simeq 2 \times 10^{28} \text{ s} \frac{0.1}{\left[ \sum_\ell \left( \eta \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \lambda_{\ell N}^R \lambda_{\ell \psi}^R \right) \right]^2 \left( \frac{1 \text{ TeV}}{m_{\psi_{DM}}} \right)^5 \left( \frac{m_V}{10^{15} \text{ GeV}} \right)^4}. \]  \tag{2.25}

### 2.2 Intermediate scalar: intensity of the gamma-ray line

The detectability of a loop-induced gamma-ray line will depend crucially on the ratio between the three-body decays at tree level and the two-body decays at the loop level. We examine the general expressions first and then evaluate them for some specific examples.
2.2.1 General expressions

In the intermediate scalar case, the ratio between two- and three-body decay widths reads, neglecting the charged lepton masses,

$$\frac{\Gamma(\psi_{\text{DM}} \rightarrow \gamma N)}{\sum_{\ell} \Gamma(\psi_{\text{DM}} \rightarrow \ell^+ \ell^- N)} \simeq \frac{3\alpha_{\text{em}}}{8\pi} \frac{\left[\sum_{\ell} \left(\lambda_{\ell N}^L \lambda_{\ell N}^L - \eta \lambda_{\ell N}^R \lambda_{\ell N}^R\right)\right]^2}{\sum_{\ell} C_{\ell 1}^2 F_1(\lambda_{\ell N}) + C_{\ell 2}^2 F_2(\lambda_{\ell N})} \, (2.26)$$

where $x \equiv m_N^2/m_{\psi_{\text{DM}}}^2$ and the kinematical functions $F_1$ and $F_2$ were defined in eqs. (2.6), (2.7). This general expression can be used to study the intensity of the one-loop induced gamma-ray line in different scenarios. The numerical value of the prefactor is $3\alpha_{\text{em}}/(8\pi) \simeq 1/1148$.

In general, the fraction depends on the chiral and flavor structure of the couplings, the mass ratio $m_N/m_{\psi_{\text{DM}}}$ of the decay product and the dark matter particle, and the relative $CP$ parities $\eta = \pm 1$ of $N$ and $\psi_{\text{DM}}$. For many practical purposes, it turns out that the dependence on the couplings $\lambda_{\ell N/\psi}$ and on kinematics, i.e. on $x = m_N^2/m_{\psi_{\text{DM}}}^2$, can be factored according to

$$\frac{\Gamma(\psi_{\text{DM}} \rightarrow \gamma N)}{\sum_{\ell} \Gamma(\psi_{\text{DM}} \rightarrow \ell^+ \ell^- N)} \simeq \frac{3\alpha_{\text{em}}}{8\pi} \times R_{\eta}^\Sigma(\lambda_{\ell N}^L, \lambda_{\ell N}^L, \lambda_{\ell N}^R, \lambda_{\ell N}^R) \times S_{\eta}(m_N/m_{\psi_{\text{DM}}}) \, (2.27)$$

In this parametrization $R_{\eta}^\Sigma$ captures the model-dependence, whereas $S_{\eta}$ is determined entirely by kinematics.

It is interesting to consider the two limiting cases of hierachical masses, $m_N/m_{\psi_{\text{DM}}} \to 0$, and degenerate masses, $m_N/m_{\psi_{\text{DM}}} \to 1$. In the hierarchical limit, and assuming for simplicity real couplings, one explicitly obtains $S_{\eta}^{\text{hier}} = 1$ and

$$R_{\eta}^\Sigma(\lambda_{\ell N}^L, \lambda_{\ell N}^L, \lambda_{\ell N}^R, \lambda_{\ell N}^R) = \frac{\left[\sum_{\ell} \left(\lambda_{\ell N}^L \lambda_{\ell N}^L - \eta \lambda_{\ell N}^R \lambda_{\ell N}^R\right)\right]^2}{\sum_{\ell} \left[(\lambda_{\ell N}^L + \lambda_{\ell N}^R)(\lambda_{\ell N}^L + \lambda_{\ell N}^R) - \eta \lambda_{\ell N}^L \lambda_{\ell N}^R \lambda_{\ell N}^L \lambda_{\ell N}^R\right]} \, (2.28)$$

For generic couplings, $R_{\eta}^\Sigma,\text{hier}$ is roughly of order one, unless for some special cases where cancellations or chirality suppressions occur. It follows that in the hierarchical limit the two-body decays into $\gamma N$ are typically suppressed roughly by a factor $10^{-3}$ compared to the tree-level decays into $\ell^+ \ell^- N$. In the next subsection we will examine the model-dependent factor $R_{\eta}^\Sigma$ for some specific cases.

On the other hand, in the degenerate limit $m_N/m_{\psi_{\text{DM}}} \to 1$, and again assuming real couplings, one finds

$$R_{\eta}^\Sigma(\lambda_{\ell N}^L, \lambda_{\ell N}^L, \lambda_{\ell N}^R, \lambda_{\ell N}^R) = \frac{\left[\sum_{\ell} \left(\lambda_{\ell N}^L \lambda_{\ell N}^L - \eta \lambda_{\ell N}^R \lambda_{\ell N}^R\right)\right]^2}{\sum_{\ell} \left[\frac{1}{2+\eta} \left(\lambda_{\ell N}^L \lambda_{\ell N}^L + \lambda_{\ell N}^R \lambda_{\ell N}^R\right) + \frac{\eta}{2+\eta} \left(\lambda_{\ell N}^L \lambda_{\ell N}^L - \lambda_{\ell N}^R \lambda_{\ell N}^R\right)\right]} \, (2.29)$$
which is also roughly of order one for generic couplings, $R_{\eta}^{\Sigma,\text{deg}} \sim \mathcal{O}(1)$, and

$$S_{\eta}^{\text{deg}}(m_N/m_{\psi\text{DM}}) \simeq \begin{cases} 5/12 & \text{for } \eta = +1 \\ 20/(1 - m_N^2/m_{\psi\text{DM}}^2)^2 & \text{for } \eta = -1. \end{cases} \quad (2.30)$$

Thus, for the case $\eta = +1$, i.e. when $\psi_{\text{DM}}$ and $N$ have the same CP parities, we again find a typical suppression factor of the order of $10^{-3}$ for the two-body relative to the tree-level decay rate, as in the hierarchical case. Interestingly, however, when $\psi_{\text{DM}}$ and $N$ have opposite CP parities, $\eta = -1$, the two-body rate can be enhanced significantly even for a relatively mild degeneracy, as is shown in fig. 3. This enhancement is due to the fact that the decay rate $\Gamma(\psi_{\text{DM}} \to \gamma N)$ is proportional to $(m_N - m_{\psi\text{DM}})^3$, whereas the decay into leptons is suppressed like $(m_N - m_{\psi\text{DM}})^5$ [48]. Most interestingly, due to this enhancement the decay rate into $\gamma N$ can be rather large in some cases, yielding potentially very intense gamma-ray lines.

As a side remark, we note that in addition to the decay channel $\psi_{\text{DM}} \to \gamma N$ into photons, there can exist a decay mode $\psi_{\text{DM}} \to Z^0 N$ into $Z$-bosons, which can naively be expected to be of similar size. Thus, for situations where the gamma-ray line signal is strongly enhanced, an equally enhanced decay into $Z$-bosons can yield additional constraints from the antiproton flux produced by the subsequent fragmentation of the $Z$-bosons. We leave a more detailed discussion for the future [49].

For concreteness, we consider the case of purely chiral, say left-handed, couplings, and that only one mediator species is present. Then the fraction of decay rates is given by

$$\frac{\Gamma(\psi_{\text{DM}} \to \gamma N)}{\sum_{\ell} \Gamma(\psi_{\text{DM}} \to \ell^+\ell^- N)} \simeq \frac{3\alpha_{\text{em}}}{8\pi} R_{\text{chir}} \begin{cases} 1 & \text{for } m_N \to 0, \ \eta = \pm 1 \\ \frac{5}{12} & \text{for } m_N \to m_{\psi\text{DM}}, \ \eta = +1 \\ \frac{20}{(1 - m_N^2/m_{\psi\text{DM}}^2)^2} & \text{for } m_N \to m_{\psi\text{DM}}, \ \eta = -1. \end{cases} \quad (2.31)$$

In this case the model-dependent factor for the hierarchical and degenerate regimes coincides, $R_{\Sigma,\text{hier}} = R_{\Sigma,\text{deg}} = R_{\text{chir}}$, and is furthermore independent of $\eta$. Explicitly, one has

$$R_{\text{chir}} = \left[ \sum_{\ell} \lambda_{\ell N}^L \lambda_{\ell N}^L \right]^2 \sum_{\ell} \left( \lambda_{\ell N}^L \lambda_{\ell N}^L \right)^2. \quad (2.32)$$

The result for purely right-handed couplings is analogous. For a generic choice of couplings, one expects $R_{\text{chir}} \sim \mathcal{O}(1)$. Note that $R_{\text{chir}} \leq N_\ell$, where $N_\ell$ is the number of flavors participating in the decay.

For example, consider two particular cases for the flavor composition of the lepton pairs produced in the decay:
Figure 3: Ratio of the decay rates $\Gamma(\psi_{\text{DM}} \rightarrow \gamma N)/\sum_\ell \Gamma(\psi_{\text{DM}} \rightarrow \ell^+\ell^- N)$ when the decay is mediated by a scalar. The four cases correspond to single-flavor decay (red) and democratic decay into all flavors (blue), as well as $\psi_{\text{DM}}/N$ having the same CP parity (dashed, $\eta = +1$) or opposite CP parity (solid, $\eta = -1$). See eq. (2.31).

(A) Decay into a single lepton flavor: $\mu^+ \mu^-$,

(B) Flavor-democratic decay into $e^+ e^-, \mu^+ \mu^-, \tau^+ \tau^-$. 

Then one has $R_{\text{chir}}^\eta = 1$ in case (A) and $R_{\text{chir}}^\eta = 3$ in case (B).

The dependence of the ratio of decay rates on the mass ratio $m_N/m_{\psi_{\text{DM}}}$ is shown in fig. 3 for the two cases (A) and (B), and for $\eta = \pm 1$. This dependence is in fact a rather generic feature, which is independent of the details of the couplings. We emphasize again that, in the case when $\psi_{\text{DM}}$ and $N$ have opposite CP parities, $\eta = -1$, even a rather mild degeneracy between $m_N$ and $m_{\psi_{\text{DM}}}$ can lead to a considerable enhancement of the gamma-ray line signal relative to the electron/positron flux.

### 2.2.2 Examples

Now, we will discuss the branching ratio into gamma-ray lines for several specific scenarios. Namely, we consider the case where $N$ corresponds to left-handed neutrinos $\nu_L$, as well as the scenario of kinetically mixed hidden $U(1)$ gauginos, where $N$ corresponds to a neutralino.
Decay into left-handed neutrinos.

As a basic example for the scalar-mediated decay described in the previous subsection, we consider the case where the neutral fermion is a left-handed neutrino, \( N \equiv \nu_L \). Then one can set \( \lambda^L_{\nu} = 0 \) and \( m_\nu = 0 \).

From eq. (2.26) it directly follows, that in the limit \( m_\ell \ll m_{\psi_{DM}} \ll m_\Sigma \) the ratio reads
\[
\frac{\Gamma(\psi_{DM} \to \gamma\nu)}{\sum_\ell \Gamma(\psi_{DM} \to \ell^+\ell^-\nu)} \simeq \frac{3\alpha_{\text{em}}}{8\pi} \frac{\left[ \sum_\ell \lambda^R_{\ell\psi} \lambda^R_{\ell\psi} \right]^2}{\sum_\ell \left( |\lambda^L_{\ell\psi}|^2 + |\lambda^R_{\ell\psi}|^2 \right) |\lambda^R_{\ell\psi}|^2}.
\]

As long as only one virtual scalar particle is relevant for the three-body decay, the last factor in this expression is bounded from above by the number of lepton flavors \( N_\ell \) that contribute to the decay. For \( N_\ell = 3 \), this corresponds to a branching ratio into monochromatic photons smaller than \( 3 \times 10^{-3} \).

Hidden-gaugino dark matter.

In supersymmetric scenarios with an extra \( U(1)_X \) gauge group in the hidden sector, which kinetically mixes with the Standard Model \( U(1)_Y \), the particles \( \psi_{DM} \) and \( N \) could be associated with the hidden gaugino and the lightest MSSM neutralino, respectively. If the kinetic mixing parameter \( \theta \) is extremely small, the hidden gaugino could constitute decaying dark matter [9]. For a bino-like lightest neutralino and \( \Sigma \equiv \tilde{\ell}_L \) being a left-handed slepton, the couplings are approximately given by
\[
\lambda^L_{\ell\psi} \simeq \frac{g'}{\sqrt{2}} Y^L_{\ell} \theta, \quad \lambda^L_{\ell N} \simeq \frac{g'}{\sqrt{2}} Y^L_{\ell},
\]
where \( \theta \sim 10^{-24} \) is the mixing angle of hidden gaugino and bino, fixed by the requirement of a lifetime of the order of \( 10^{26} \) s, and \( Y^L_{\ell} = +1 \).

For a hidden gaugino that decays into a bino-like neutralino, the decay rates are given in app. A of ref. [9]. One can also obtain these rates using the expressions derived above. In particular, we have to sum over two ‘mediators’ \( \Sigma \) for each flavor:
- \( \Sigma = \tilde{\ell}_L: \lambda^L_{\ell\psi} = \frac{g'}{\sqrt{2}} Y^L_{\ell} \theta, \quad \lambda^L_{\ell N} = \frac{g'}{\sqrt{2}} Y^L_{\ell}, \quad \lambda^R_{\ell\psi} \simeq 0, \quad \lambda^R_{\ell N} \simeq 0 \)
- \( \Sigma = \tilde{\ell}_R: \lambda^R_{\ell\psi} = \frac{g'}{\sqrt{2}} Y^R_{\ell} \theta, \quad \lambda^R_{\ell N} = \frac{g'}{\sqrt{2}} Y^R_{\ell}, \quad \lambda^L_{\ell\psi} \simeq 0, \quad \lambda^L_{\ell N} \simeq 0 \).

In addition, there are corresponding contributions from (s)quarks. We assume \( m_f \gg m_{\psi_{DM}} \), and neglect the mixing of \( f_{L,R} \) for simplicity. Note that there are additional contributions from chargino loops [48] that are suppressed by the fourth power of the
inverse chargino mass. We assume that the squarks are much heavier than the sleptons, and we furthermore assume that all slepton masses are degenerate. Finally, if we take the limit $m_\ell \to 0$, we get

$$\Gamma(\psi_{\text{DM}} \to \gamma N) \simeq \frac{e^2 g^4 g'^2}{8\pi (32\pi^2)^2} \frac{m_{\psi_{\text{DM}}}^5}{16m_\ell^4} \left(1 - \frac{m_N^2}{m_{\psi_{\text{DM}}}^2}\right)^3 \left(1 - \eta \frac{m_N}{m_{\psi_{\text{DM}}}}\right)^2 \times [3 (1 - 4\eta)]^2.$$ (2.36)

If the bino and the hidden gaugino have the same $CP$ eigenvalue, one has $\eta = +1$, otherwise $\eta = -1$.

The three-body decay rate can be obtained from eq. (2.3), which can be easily generalized to also account for neutrinos and quarks in the final state. Note that in general one has to add the matrix elements for the decays mediated by $\Sigma = \tilde{\ell}_L$ and $\Sigma = \tilde{\ell}_R$, and compute the decay rate from the square of the summed matrix elements. However, it turns out that all ‘interference’-terms are suppressed by the bino–higgsino mixing, which we neglect here. Thus, it is possible to add the decay rates directly. Note that there is an additional contribution from a $Z^0$ on the intermediate line [47], which is subdominant for the parameter range considered in ref. [9]. Therefore, we also neglect it here for simplicity. We assume, as above, degenerate sleptons. The decay rate summed over three generations of charged leptons and neutrinos is thus (assuming $m_\nu \simeq m_\ell$)

$$\sum_{\ell, \nu} \Gamma(\psi_{\text{DM}} \to \ell \bar{\ell} N) \simeq \frac{g^4 g'^2}{64(2\pi)^3} \frac{m_{\psi_{\text{DM}}}^5}{24m_\ell^4} \times 3 \times 18 \times (F_1 + 2\eta F_2).$$ (2.37)

In the hierarchical limit $m_N \ll m_{\psi_{\text{DM}}}$, the kinematical factor approaches unity, $F_1 + 2\eta F_2 \to 1$. In the degenerate limit $m_N \to m_{\psi_{\text{DM}}}$, one finds $F_1 + 2\eta F_2 \to (2 + \eta)(1 - m_N^2/m_{\psi_{\text{DM}}}^2)^5/5$.

The ratio of decays into $\gamma N$ to the decays into charged leptons is thus given by

$$\frac{\Gamma(\psi_{\text{DM}} \to \gamma N)}{\sum_{\ell} \Gamma(\psi_{\text{DM}} \to \ell^+ \ell^- N)} \simeq \frac{3\alpha_{\text{em}}}{8\pi} \left[\frac{3(1 - 4\eta)]^2}{51} \left(1 - x\right)^3 \left(1 - \eta \sqrt{x}\right)^2 \right] \equiv R_\eta \equiv S_\eta,$$ (2.38)

with $x = m_N^2/m_{\psi_{\text{DM}}}^2$. For the hidden gaugino, the decays $\psi_{\text{DM}} \to Z^0 N$ and $\psi_{\text{DM}} \to h^0 N$ can also be important since they occur at tree level. Their rates are given in eqs. (A.1) and (A.3) of ref. [9].

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4 The reason is the following: If we consider a pure bino-slepton-lepton interaction, the slepton $\tilde{\ell}_L$ couples only to left-handed leptons and the slepton $\tilde{\ell}_R$ only to right-handed ones. Thus both channels are ‘orthogonal’ in the limit where neutralino and slepton mixing are neglected.
According to eq. (2.38), the model-dependent factor \( R_\eta \) is here given by \( R_+ = 1.6 \) for \( \eta = +1 \) and \( R_- = 4.4 \) for \( \eta = -1 \), respectively, and hence of order one. The kinematical factor \( S_\eta \) is precisely of the form that was discussed in section 2.2.1 where we found that the two-body decay rate may gain significantly in importance relative to the three-body decay rate if the masses of the hidden gaugino \( \psi_{\text{DM}} \) and the neutralino \( N \) are near-degenerate and the two particles have opposite CP parities.

### 2.3 Intermediate vector: intensity of the gamma-ray line

In the case of mediation by a vector, the ratio between two- and three-body decay rates is

\[
\frac{\Gamma(\psi_{\text{DM}} \rightarrow N\gamma)}{\sum \Gamma(\psi_{\text{DM}} \rightarrow \ell^+\ell^- N)} \approx \frac{27 \alpha_{\text{em}}}{8\pi} \frac{\left[ \sum \ell \left( \frac{L_{\ell N}L_{\ell\psi} - \eta R_{\ell N}R_{\ell\psi}}{m_{\psi_{\text{DM}}}^2} \right) \right]^2 (1-x)^3 (1-\eta \sqrt{1-x})^2}{\sum \ell \left[ C_1^2 F_1(x) + C_2^2 F_2(x) \right]},
\]

(2.39)

where \( x \equiv m_N^2/m_{\psi_{\text{DM}}}^2 \). As before, it is useful to consider the hierarchical limit \( m_N/m_{\psi_{\text{DM}}} \rightarrow 0 \), and the degenerate limit \( m_N/m_{\psi_{\text{DM}}} \rightarrow 1 \), for which it is possible to capture the dependence on the couplings in a factor \( R_\eta^V \) and on kinematics in a model-independent factor \( S_\eta \),

\[
\frac{\Gamma(\psi_{\text{DM}} \rightarrow N\gamma)}{\sum \Gamma(\psi_{\text{DM}} \rightarrow \ell^+\ell^- N)} \approx \frac{27 \alpha_{\text{em}}}{8\pi} \times R_\eta^V \left( \lambda_{\ell N}^L, \lambda_{\ell\psi}^L, \lambda_{\ell N}^R, \lambda_{\ell\psi}^R \right) \times S_\eta \left( m_N/m_{\psi_{\text{DM}}} \right).
\]

(2.40)

The kinematical factors \( S_\eta \) are identical to the case of scalar mediation, see eq. (2.30).

For the model-dependent factors, one finds

\[
R_{\eta}^{V,\text{hier}}(\lambda_{\ell N}^L, \lambda_{\ell\psi}^L, \lambda_{\ell N}^R, \lambda_{\ell\psi}^R) = \frac{\left[ \sum \ell \left( \lambda_{\ell N}^L \lambda_{\ell\psi}^L - \eta \lambda_{\ell N}^R \lambda_{\ell\psi}^R \right) \right]^2}{\sum \ell \left[ (\lambda_{\ell N}^L \lambda_{\ell\psi}^L + \lambda_{\ell\psi}^R \lambda_{\ell N}^R) + 2\eta \lambda_{\ell N}^L \lambda_{\ell\psi}^L \lambda_{\ell N}^R \lambda_{\ell\psi}^R \right]},
\]

(2.41)

in the hierarchical case, and

\[
R_{\eta}^{V,\text{deg}}(\lambda_{\ell N}^L, \lambda_{\ell\psi}^L, \lambda_{\ell N}^R, \lambda_{\ell\psi}^R) = \frac{\left[ \sum \ell \left( \lambda_{\ell N}^L \lambda_{\ell\psi}^L - \eta \lambda_{\ell N}^R \lambda_{\ell\psi}^R \right) \right]^2}{\sum \ell \left[ (\lambda_{\ell N}^L \lambda_{\ell\psi}^L + \lambda_{\ell\psi}^R \lambda_{\ell N}^R) + \frac{2}{2+\eta} \lambda_{\ell N}^L \lambda_{\ell\psi}^L \lambda_{\ell N}^R \lambda_{\ell\psi}^R \right]},
\]

(2.42)

in the degenerate case. For purely chiral, e.g. left-handed, couplings one finds that \( R_{\eta}^{V,\text{hier}} = R_{\eta}^{V,\text{deg}} = R_{\eta}^{\text{chir}} \) coincides with the expression (2.32) for the scalar case, as does the kinematical factor. For a generic set of couplings, \( R_\eta^V \) is roughly of order one.

Note that the prefactor of the ratio of decay rates, eq. (2.40), for mediation by a vector is larger by a factor of nine compared to mediation by a scalar, eq. (2.27). Thus, in the hierarchical case \( m_N/m_{\psi_{\text{DM}}} \rightarrow 0 \) as well as in the degenerate case with \( \eta = +1 \).
one finds a ratio between two-body and tree-level decay of the order of $10^{-2}$, one order of magnitude larger than for the scalar case. In addition, when $\eta = -1$ the gamma-ray line is further enhanced for $m_N/m_{\psi_{\text{DM}}} \to 1$ by the kinematic effect discussed in section 2.2.1.

Therefore, there are scenarios with dark matter decay mediated by heavy vectors where a gamma-ray line can be fairly intense, despite being loop-suppressed, while at the same time being in agreement with the electron/positron measurements.

### 3 Radiative decay of scalar dark matter

We now consider the case that the dark matter particle is a (pseudo-)scalar which we denote by $\phi_{\text{DM}}$. In this case, the symmetries allow for the decay into a pair of charged leptons at tree level, $\phi_{\text{DM}} \to \ell^+ \ell^-$. We describe this by an effective Lagrangian that describes a direct interaction between dark matter and charged leptons,

$$\mathcal{L}_{\text{eff}} = -\bar{\ell} \left[ \lambda_{\ell \phi}^L P_L + \lambda_{\ell \phi}^R P_R \right] \ell \phi_{\text{DM}} + \text{h.c.}.$$  

(3.1)

If the dark matter particle is a parity eigenstate, one has $\lambda_{\ell \phi}^L = \lambda_{\ell \phi}^R$ for a scalar and $\lambda_{\ell \phi}^L = -\lambda_{\ell \phi}^R$ for a pseudo-scalar.
\section*{Tree-level decay of scalar dark matter}

Figure 5: Tree-level decay of scalar dark matter.

\section*{Two-body decay at the one-loop level}

Figure 6: Diagrams contributing to the two-body decay of scalar dark matter into two photons at the one-loop level.
3.1 The decay $\phi_{DM} \to \ell^+ \ell^-$

The effective Lagrangian (3.1) will give rise to the tree-level decay shown in fig. 5. The corresponding decay width is

$$\Gamma(\phi_{DM} \to \ell^+ \ell^-) = \frac{1}{16\pi m_{\phi_{DM}}} |M|^2 \sqrt{1 - \frac{4m_{\ell}^2}{m_{\phi_{DM}}^2}}, \quad (3.2)$$

where $m_{\phi_{DM}}$ and $m_{\ell}$ are the mass of the dark matter and the charged leptons, respectively, and the amplitude is given by

$$|M|^2 = m_{\phi_{DM}}^2 \left( |\lambda_{\ell\phi}^L|^2 + |\lambda_{\ell\phi}^R|^2 \right) - 2m_{\ell}^2 |\lambda_{\ell\phi}^L + \lambda_{\ell\phi}^R|^2. \quad (3.3)$$

Thus, in the case of equal left- and right-handed couplings, $\lambda_{\ell\phi}^L = \lambda_{\ell\phi}^R \equiv \lambda_{\ell\phi}$, one gets

$$\Gamma(\phi_{DM} \to \ell^+ \ell^-) = \frac{|\lambda_{\ell\phi}|^2}{8\pi} m_{\phi_{DM}} \left( 1 - \frac{4m_{\ell}^2}{m_{\phi_{DM}}^2} \right)^{3/2}. \quad (3.4)$$

For $m_{\ell} \ll m_{\phi_{DM}}$ this corresponds to a lifetime

$$\tau_{\phi_{DM} \to \ell^+ \ell^-} \simeq 2 \times 10^{26} \text{s} \left( \frac{10^{-26}}{|\lambda_{\ell\phi}|} \right)^2 \left( \frac{1 \text{ TeV}}{m_{\phi_{DM}}} \right). \quad (3.5)$$

3.2 The decay $\phi_{DM} \to \gamma \gamma$

By combining the external lepton lines into a loop, decays into two monochromatic photons radiated off the charged lepton loop are induced at the quantum level (see fig. 6).

For equal left- and right-handed couplings, $\lambda_{\ell\phi}^L = \lambda_{\ell\phi}^R \equiv \lambda_{\ell\phi}$, and in the limit $m_{\ell} \ll m_{\phi_{DM}}$ (see app. B and \[50, 51\]),

$$\Gamma(\phi_{DM} \to \gamma \gamma) = \frac{m_{\phi_{DM}}^3}{16\pi} \left( \frac{e^2}{16\pi^2} \right)^2 \left| \sum_{\ell} \frac{\lambda_{\ell\phi}}{m_{\ell}} A_f(\tau_\ell) \right|^2, \quad (3.6)$$

where for the relevant limit $\tau \gg 1$ one has

$$A_f(\tau) \simeq \frac{1}{\tau} \left\{ 2 - \frac{1}{2} \left( \ln(4\tau) - i\pi \right) \right\}. \quad (3.7)$$

Thus, when taking only one lepton species into account, we obtain for the ratio between the decay into photons and charged leptons

$$\frac{\Gamma(\phi_{DM} \to \gamma \gamma)}{\Gamma(\phi_{DM} \to \ell^+ \ell^-)} \simeq \frac{\alpha_{\text{em}}^2}{2\pi^2} \frac{m_{\ell}^2}{m_{\phi_{DM}}^2} \left| 2 - \frac{1}{2} \left( \ln(4\tau_\ell) - i\pi \right)^2 \right|^2 \left( \frac{1 \text{ TeV}}{106 \text{ MeV}} \right)^2. \quad (3.8)$$
We see that for scalar dark matter, the decay into two photons is highly suppressed by the factor $m_\ell^2/m_{\phi_{\text{DM}}}^2$ compared to the decay into a pair of charged leptons. In addition to this helicity-suppression factor there appears a factor $\alpha_\text{em}^2/\pi^2$ as opposed to $\alpha_\text{em}/\pi$ for fermionic dark matter, since the loop contains two photon vertices, and both the tree-level and one-loop decays are two-body decays. The same suppression factors occur for pseudo-scalar dark matter and for the decay into massive gauge bosons. Thus, there appears to be no hope of detecting a gamma-ray line in this case. For more general expressions for the decay rates, see app. B.

4 Observational constraints

The observation of a cosmic gamma-ray line at TeV energies would be a strong hint for the dark matter interpretation of the PAMELA/Fermi LAT $e^\pm$ anomalies. On the other hand, the non-observation of gamma-ray lines can be used to constrain the above leptophilic models, which induce these lines at one loop, as discussed above. The gamma-ray lines that originate from dark matter decay inside the Milky Way halo could be observed in the isotropic diffuse gamma-ray flux. Furthermore, lines may be observable in the flux from nearby galaxies and galaxy clusters.

At intermediate energies, satellite instruments such as Fermi LAT are a very sensitive probe for gamma-ray lines in the Galactic flux. At higher energies, Imaging Atmospheric Cherenkov Telescopes (IACTs) provide important information. For the future, the proposed Cherenkov Telescope Array (CTA) is expected to improve the flux sensitivity of current IACTs (MAGIC, H.E.S.S., VERITAS) by an order of magnitude. We put some emphasis on IACTs, since these instruments are capable of probing the high energy ranges relevant to the dark matter interpretation of PAMELA/Fermi LAT.

4.1 Fermi LAT

The flux of monochromatic gamma rays from the decay of dark matter in the Milky Way halo is given by a line-of-sight integral over the dark matter distribution [52]. This component of the gamma-ray flux is explicitly given by

$$\frac{dJ_{\text{halo}}^{\psi_{\text{DM}} \to \gamma N}}{dE} = \frac{\Gamma(\psi_{\text{DM}} \to \gamma N)}{4\pi m_{\psi_{\text{DM}}}} \frac{\delta (E_\gamma - E)}{\text{I.o.s.}} \int_{l.o.s.} dl \rho_{\text{DM}}^{\text{MW}} (l),$$

where $\Gamma(\psi_{\text{DM}} \to \gamma N)$ denotes the partial decay width of dark matter particles for two-body decays involving a photon and a neutral particle $N$. When the neutral particle is massless, we will write $\nu$ instead of $N$ in the following. Furthermore, $m_{\psi_{\text{DM}}}$ is the mass...
of the dark matter particle, $E_\gamma$ is the energy of the produced gamma-ray line as given by eq. (2.15), while $\rho_{\text{DM}}^{\text{MW}}$ is the Milky Way’s dark matter halo density profile. We adopt the NFW profile here, which has the form

$$\rho_{\text{DM}}^{\text{MW}}(r) = \frac{\rho_c}{r/r_c (1 + r/r_c)^2},$$

(4.2)

with the parameters $\rho_c = 0.35 \text{ GeV cm}^{-3}$ and $r_c = 20 \text{ kpc}$ [53, 54], leading to a local dark matter density of $0.4 \text{ GeV/cm}^3$ [55]. The gamma-ray flux from dark matter decay inside the Galactic halo has only a mild angular dependence and can be considered as isotropic for our purposes (for details on anisotropies in the Galactic gamma-ray flux from dark matter decay, see refs. [52, 56]). The extragalactic contribution stemming from the decay of dark matter at cosmological distances is generally fainter than the Galactic flux, and we will neglect this component here.

The Fermi LAT collaboration has conducted a negative search for Galactic gamma-ray lines in the diffuse flux in the energy range from 30 to 200 GeV [54]. For the halo profile (4.2), we plot the resulting $2\sigma$ limits on the partial decay width corresponding to $\psi_{\text{DM}} \to \gamma\nu$ in fig. [7]. Most interestingly, the Fermi LAT observations can constrain the dark matter decay into photons at the one-loop level if the total dark matter lifetime is of the order $10^{26}$ seconds. Thus, the Fermi LAT bounds on gamma-ray lines can be relevant for dark matter scenarios with $m_{\psi_{\text{DM}}} \simeq 300 – 400 \text{ GeV}$, which can provide a possible explanation for the rise in the positron fraction observed by PAMELA (see, e.g. [57]).

### 4.2 Imaging Atmospheric Cherenkov Telescopes

IACTs are important tools to constrain scenarios with dark matter masses in the multi-TeV range. One property of these instruments is that the atmospheric showers induced by cosmic-ray electrons or gamma rays cannot be distinguished easily, since both particle species initiate similar electromagnetic cascades in the atmosphere. The large cosmic-ray electron flux hence comprises an irreducible background for high energy gamma-ray observations. Since the electron background is expected to be very isotropic — in contrast to the gamma rays — it can be removed by calculating differences between fluxes that are observed in different neighboring regions of the sky. As a result, IACTs are best suited to observe localized sources, whereas diffuse signals such as those resulting from dark matter decay are more difficult to discern from the background unless they exhibit sharp spectral features. Constraints on the gamma-rays from decaying dark matter can be derived in two different ways. First, one can observe point-like sources like M31. Second, by using the observed electron+gamma-ray flux (potentially also contaminated
by unrejected protons), one can derive upper limits on the Galactic halo signal from dark matter decay. If the statistics are good enough, one could even hope to see spectral features in the electron+gamma-ray flux, or translate their non-observation into bounds on the corresponding dark matter decay width. This will be described in the context of the CTA below.

The HEGRA collaboration has published constraints on the gamma-ray line flux from M31 [58]. These bounds can be converted into 99% C.L. limits on the decay width of dark matter into gamma-ray lines. HEGRA observed a region with an opening-angle of \( \theta_{\text{obs}} = 0.105^\circ \), corresponding to the inner 1.4 kpc region of M31. The expected flux of gamma rays from dark matter decay from M31 can be derived as follows. We define \( \theta \) to be the angle between the line-of-sight and the ray that passes through our position and the center of M31. Each angle \( \theta \) then corresponds to an ‘impact parameter’ \( R \). If \( D \) is the distance to the target (\( D = 770 \) kpc in case of M31), we have \( R \approx D \theta \). The gamma-ray flux from dark matter decay in M31 within the opening angle \( \theta_{\text{obs}} \) is then

\[
\frac{dJ_{\gamma}^{\text{M31}}}{dE} = \frac{\Gamma(\psi \rightarrow \gamma N)}{4\pi m_{\chi}} \delta(E_{\gamma} - E) \frac{\theta_{\text{obs}}}{2\pi} \int_0^{\infty} d\theta \sin \theta \int ds \rho_{\text{DM}}^\text{M31}(\sqrt{s^2 + R^2}),
\]

where the first integral is over the solid-angle, whereas the second integral is over the line-of-sight. For the dark matter density profile of M31 we adopt the NFW profile with values given in ref. [59], \( \rho_c = 2.0 \) GeV/cm\(^3\) and \( r_c = 8.31 \) kpc. The other profiles from ref. [59] lead to similar constraints. The signal from decaying dark matter has a relatively large angular extent due to the linear dependence on the halo profile, and can leak into the off-region which is used to estimate the background fluxes of the IACT. The details of this effect depend on the details of the adopted off-region and are different for each observation. Here and below, we incorporate this effect simply by subtracting from eq. (4.3) a flux corresponding to the dark matter-induced flux emitted at \( \theta = 2 \theta_{\text{obs}} \). This should lead to correct bounds within a factor of two. Our results are shown in fig. 7.

Upper limits on the gamma-ray flux from the Perseus galaxy cluster were presented by the MAGIC collaboration in ref. [60]. For the density profile of the Perseus cluster we take the NFW profile with \( r_c = 384 \) kpc and \( \rho_c = 0.04 \) GeV cm\(^{-3}\), the observational angle is \( \theta_{\text{obs}} = 0.15^\circ \), and the distance to the Perseus cluster is 78 Mpc. The resulting 95% C.L. bounds are shown in fig. 7. Since the energy threshold of the MAGIC telescope is very low, we can constrain gamma-ray lines with energies down to 100 GeV.

The H.E.S.S. collaboration has published measurements of the electron flux at TeV energies [61, 62]. The measured electron flux may be contaminated with diffuse gamma

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\(^5\)We take the limits corresponding to \( \Gamma = -2.5 \) from tab. 4 of ref. [60].
Figure 7: Lower bounds on the inverse decay width of dark matter decaying into gamma-ray lines via $\psi_{DM} \rightarrow \gamma \nu$ are shown as black lines. The bounds on this decay channel come from line searches in M31 by HEGRA, from line searches in the diffuse flux by Fermi LAT and from observations of the Perseus cluster by MAGIC. Further bounds can be derived from the $(\gamma + e^-)$ observations of H.E.S.S. Our estimates of the reach of the future CTA in measurements of the flux from M31 or spectral variations in the diffuse $\gamma + e^-$ flux are shown as red lines.

Table 1: Benchmark scenarios. In the first three cases, the three-body decay produces only electrons. In the last four cases, the three-body decay produces muons. The gamma-ray line intensity of these scenarios is illustrated in fig. 12.
Figure 8: Same as fig. 7. In addition, the orange and gray shaded regions show the parts of the parameter space that are relevant for the dark matter explanation of the PAMELA/Fermi $e^\pm$ anomalies with the flavor-democratic decay channel $\psi_{\text{DM}} \rightarrow \ell^+ \ell^- \nu$.

The intermediate particle is assumed to be a scalar, in which case the branching ratio into monochromatic photons can be as large as $\text{BR}(\psi_{\text{DM}} \rightarrow \gamma \nu) \simeq 3 \times 3\alpha_{\text{em}}/(8\pi)$, which we assume here.
Figure 9: Same as fig. 8, but assuming that the intermediate particle is a vector, in which case the branching ratio into monochromatic photons can be as large as \( \text{BR}(\psi_{\text{DM}} \rightarrow \gamma \nu) \simeq 3 \times 27\alpha_{\text{em}}/(8\pi) \), which we assume here.

Figure 10: Same as fig. 8 but for decay into \( \psi_{\text{DM}} \rightarrow \mu^+ \mu^- \nu \). The intermediate particle is assumed to be a scalar, leading to a maximal branching ratio of \( \text{BR}(\psi_{\text{DM}} \rightarrow \gamma \nu) \simeq 3\alpha_{\text{em}}/(8\pi) \), which we assume here.
Figure 11: Same as fig. 8, but for decay into $\psi_{\text{DM}} \rightarrow \mu^+ \mu^- \nu$. The intermediate particle is assumed to be a vector, leading to a maximal branching ratio of $\text{BR}(\psi_{\text{DM}} \rightarrow \gamma \nu) \simeq 27\alpha_{\text{em}}/(8\pi)$, which we assume here.

rays by no more than $\approx 50\%$ [61]. This fact allows to translate the electron flux into upper bounds on gamma-ray lines from dark matter decay in the Galactic halo. For energies above 1 TeV, we derived 2$\sigma$-bounds from the fluxes shown in fig. 3 of ref. [61]. For energies below 1 TeV, where the H.E.S.S. results overlap with the Fermi LAT measurements of the electron flux, 1$\sigma$-upper limits on the amount of diffuse gamma rays were derived by comparing the H.E.S.S. and the Fermi LAT electron fluxes in ref. [63]. These upper limits can also be used as bounds on gamma-ray lines. Our results are shown in fig. 7.

**Prospects for the CTA.** We will now briefly discuss observational prospects for the future Cherenkov Telescope Array (CTA, see ref. [64] for a recent discussion). The expected 2$\sigma$-limit from M31 that the CTA could produce can be roughly estimated by

$$\langle J_{\text{dim}}^{\text{M31}} \rangle_{\text{on}} \lesssim \frac{\max(2\sqrt{N_{\text{on}}}, 3.1)}{T A_{\text{eff}}},$$

where $N_{\text{on}}$ denotes the number of measured events in the on-region, $T$ is the measurement time, $A_{\text{eff}}$ denotes the effective area of the instrument (we take $A_{\text{eff}} \approx 2 \text{km}^2$ at 5 TeV, and let it scale with the energy as in ref. [65], fig. 17a), and $\langle J_{\text{dim}}^{\text{M31}} \rangle_{\text{on}}$ is the gamma-ray flux from M31 averaged over the on-region. As on-region, we take a circle with $1.0^\circ$ radius around the center of M31, and for the off-region we assume that the solid-angle of the
Figure 12: Same as fig. 7 but with different scaling of the axes to allow for non-vanishing $m_N$. The black squares and red dots show the predictions for the different benchmark scenarios summarized in tab. 1. Black squares correspond to scenarios with $\eta = +1$, while red dots correspond to $\eta = -1$. The last benchmark point in tab. 1 lies outside of the shown parameter region.
off-region is much larger than the on-region, \( \Omega_{\text{off}} \gg \Omega_{\text{on}} \). If we assume that only the background is observed, and no signal is coming from M31, \( N_{\text{on}} \) can be estimated by

\[
\bar{N}_{\text{on}} = \Omega_{\text{on}} T A_{\text{eff}} (J_{e^-} + \epsilon_r J_p) .
\]  

(4.5)

Here, \( J_{e^-} \) and \( J_p \) denote the cosmic-ray electron and proton fluxes, respectively, and \( \epsilon_r \) is the rejection factor of protons. Fluxes have to be integrated over an energy range that corresponds to the energy resolution of the detector (around 10\%, taken from ref. [64], fig. 23, scenario E). For the cosmic-ray electron and proton fluxes at high energies we take

\[
\frac{dJ_{e^-}}{dE} = 1.17 \times 10^{-11} \left( \frac{E}{\text{TeV}} \right)^{-3.9} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}^{-1},
\]

(4.6)

from ref. [61], and

\[
\frac{dJ_p}{dE} = 8.73 \times 10^{-9} \left( \frac{E}{\text{TeV}} \right)^{-2.7} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}^{-1},
\]

(4.7)

from ref. [65], respectively, in agreement with the cosmic-ray measurements. Furthermore, at energies below 1 TeV, the electron flux becomes somewhat harder with a spectral index of \( \simeq -3.1 \), and we replace eq. (4.6) with a flux fitting the results of ref. [16] in this energy regime. Taking into account also other cosmic-ray species beside the protons would only have minor impact on our results. The proton rejection factor is set to \( \epsilon_r \approx 10^{-2} \) [61, 65], and we use as observational time of M31 \( T = 20 \) h. Our estimates for the limits that the CTA could produce from M31 observations in the future are shown in fig. 7 by the lower dashed line. They are almost two orders of magnitude better than the limits derived from the HEGRA observation. This is mainly due to the increased effective area of CTA, but also due to the larger on-region that we adopted in our estimates. For decaying dark matter it is not optimal to search for point-source signals, as was done in the HEGRA analysis, for example. Using a larger on-region typically leads to better results.

Instead of using the spatial variations of the observed cosmic-ray flux to derive constraints on dark matter decay in extragalactic sources, one can also derive constraints from the non-observation of spectral line features in the diffuse flux, which could come from dark matter decaying into gamma-ray lines in the Galactic halo. In this case it is best to consider data from large fractions of the sky, to maximize the statistics. The expected “halo”-bound that the CTA will presumably reach then follows from

\[
\langle J_{\text{dm}}^\text{halo} \rangle_{\text{sky}} \leq \frac{2\sqrt{N}}{T A_{\text{eff}} \Omega},
\]

(4.8)

where \( \langle J_{\text{dm}}^\text{halo} \rangle_{\text{sky}} \) denotes the gamma-ray flux coming from dark matter decaying in our Galactic halo, averaged over all angles. We assume that the data will be good enough to
estimate the background by fitting a power-law to the observed flux at energies close to the line, similar to the analysis in ref. \cite{54}, and we neglect the statistical uncertainties in the background estimate.\footnote{Note that this is different from our treatment of the H.E.S.S. electron flux, where we only required that the predicted line signal is below the observed fluxes, without any attempt to subtract a power-law background.} In eq. (4.8), \(N\) is the total number of observed events, including electrons, gamma-rays and protons that pass the cuts. The region \(\Omega\) is taken to be as large as possible to maximize the statistics (we assume \(\Omega = \pi (3^\circ)^2\)), and as observational time we take \(T = 1000\) h. As above, we integrate over energy bands which correspond to the anticipated energy resolution of CTA. Our resulting estimates for the bounds that CTA could obtain observing the diffuse flux resolution of CTA. Our resulting estimates for the bounds that CTA could obtain observing the diffuse flux are shown in fig. 7 by the upper dashed line. As can be seen from this figure, the bounds on gamma-ray lines from dark matter decay that can be put by looking at spectral variations in the observed diffuse fluxes can be even stronger than the ones that can be derived from flux limits on point-like sources like M31.

### 4.3 Discussion

**The case \(m_N \to 0\).** We first discuss the case where dark matter decays into a photon and a massless particle. In fig. 7 we present a collection of the lower bounds on the inverse decay width for two-body decays into a monochromatic photon and a massless particle as determined by the methods described in the previous subsection. For dark matter masses between 100 and 400 GeV, the line searches in the diffuse Galactic flux by the Fermi LAT constitute the strongest constraints. At higher energies, Cherenkov telescopes provide important information. As far as constraints from particular sources are concerned, we show the constraints from HEGRA observations of M31 and MAGIC observations of the Perseus cluster. We also plot the constraints from the diffuse electron flux observed by H.E.S.S. Lastly, we show our estimates for the reach of the future CTA which could improve current limits by almost two orders of magnitude at energies above a few hundred GeV.

In fig. 8 we show the same constraints together with shaded regions indicating the part of the parameter space relevant to PAMELA and Fermi for the gamma-ray lines induced by the decay \(\psi_{\text{DM}} \to \ell^+\ell^-\nu\). The orange regions correspond to the fit to the positron fraction as measured by PAMELA, whereas the dark gray regions correspond to the fit to the total \(e^\pm\) flux as measured by Fermi LAT. In both cases, the lighter shades indicate the 5\(\sigma\) confidence level around the best-fit point, while the darker shades indicate the 3\(\sigma\) confidence level. We only regard the data points above 10 GeV, which are not significantly affected by solar modulation. For the background fluxes of secondary
electrons and positrons, we assume the ‘model 0’ backgrounds [18] as parametrized in [33]. In the energy range of interest, we assume that the primary electron flux is given by a simple power law. At each point in the \( (m_{\text{DM}}, \tau_{\text{DM}}) \)-plane, we then allow the power-law index of the primary electron flux to vary between \(-3.0\) and \(-3.3\), whereas the normalization is fitted to the data. We find that the relevant parameter space is not constrained by current instruments, but could be constrained by CTA in the future.

The same plot is shown in fig. 9, but assuming that the decay is mediated by an intermediate vector particle, in which case the branching ratio can be as large as \(3 \times 10^{-27} \alpha_{\text{em}}/(8\pi)\), which we assume in the figure. This is about an order of magnitude larger than in the case of mediation by a scalar, and one can see that in this case the CTA can indeed constrain a significant part of the parameter space relevant to the dark matter interpretations of PAMELA and Fermi. Analogously, in figs. 10, 11 we show the corresponding plots for the lines induced by the decay \(\psi_{\text{DM}} \rightarrow \mu^+\mu^-\nu\), in the cases of an intermediate scalar and an intermediate vector particle, respectively. In these scenarios the expected line signal is somewhat weaker.

The case \(m_N \sim m_{\psi_{\text{DM}}}\). Let us now turn to the case where the mass \(m_N\) of the neutral fermion produced in the tree-level decay \(\psi_{\text{DM}} \rightarrow \ell^+\ell^-N\) is comparable in size to the dark matter mass itself. This possibility can occur, for example, within the leptophilic model discussed in section 2.2.2, where \(\psi_{\text{DM}}\) is the hidden gaugino of an unbroken \(U(1)\)-symmetry, and \(N\) is a neutralino [9].

As was shown in section 2.2.1 the decay channel \(\psi_{\text{DM}} \rightarrow \gamma N\) is kinematically enhanced compared to the three-body decay when \(m_N \sim m_{\psi_{\text{DM}}}\), provided that \(\psi_{\text{DM}}\) and \(N\) have opposite CP parities \((\eta = -1)\). Thus, such scenarios can be tested particularly well via the loop-induced gamma-ray line signal. In order to infer the observational constraints, it is convenient to consider the ratio \(m_{\psi_{\text{DM}}} / \Gamma(\psi_{\text{DM}} \rightarrow \gamma N)\), which determines the magnitude of the observable flux. For the case of scalar-mediated decay, and purely chiral couplings, it is given by (see eq. 2.31)

\[
\Gamma(\psi_{\text{DM}} \rightarrow \gamma N)^{-1}m_{\psi_{\text{DM}}} \approx \frac{1}{R_{\text{chir}}} \left( \frac{\Gamma(\psi_{\text{DM}} \rightarrow \ell^+\ell^-)^{-1}m_{\psi_{\text{DM}}}}{10^{26} \text{s} \times 2.5 \text{ TeV}} \right) \times \begin{cases} 
3 \times 10^{20} \text{s TeV} & \text{for } m_N \rightarrow 0, \eta = \pm 1 \\
7 \times 10^{20} \text{s TeV} & \text{for } m_N \rightarrow m_{\psi_{\text{DM}}}, \eta = +1 \\
1.4 \times 10^{28} \text{s TeV} \left( \frac{2E_\gamma}{m_{\psi_{\text{DM}}}} \right)^2 & \text{for } m_N \rightarrow m_{\psi_{\text{DM}}}, \eta = -1 
\end{cases}
\]

(4.9)

where \(R_{\text{chir}} = 1\) for three-body decays into a single lepton flavor, and \(R_{\text{chir}} = 3\) for flavor-democratic three-body decays. In the case of an intermediate vector, the right-hand side is smaller by a factor nine, implying a nine times larger gamma-ray flux. From the last
line, it is apparent that in the case of opposite CP parities, the monochromatic gamma-ray flux is enhanced for large values of $m_{\psi_{\text{DM}}}$, when keeping the photon energy $E_\gamma$ fixed. Note that a similar enhancement of the decay channel $\psi_{\text{DM}} \rightarrow Z^0 N$, which may also be induced at the loop-level, could lead to complementary constraints from the antiproton flux produced by the fragmentation of the $Z$-boson, which we do not discuss here.

In order to illustrate this result, we consider a number of benchmark scenarios for which $m_{\psi_{\text{DM}}}$ and $m_N$ are of comparable size, with parameters chosen as shown in table 1. All the benchmark scenarios reproduce the PAMELA positron data, and all except scenarios 1, 2 and 3 additionally reproduce the electron spectrum measured by Fermi. Note that the maximum lepton energy in the three-body decay $\psi_{\text{DM}} \rightarrow \ell^+ \ell^- N$ coincides with the energy of the monochromatic photons, $E_{\text{max}} = E_\gamma$.

The gamma-ray line signal induced by the one-loop decay $\psi_{\text{DM}} \rightarrow \gamma N$ is shown in fig. 12 for the various benchmark scenarios. Clearly, the scenarios 1 and 4 are in conflict with the gamma-ray line searches performed by Fermi and HEGRA, respectively. Thus, despite the fact that the dark matter couples only to leptons at tree-level, the gamma-ray line signal induced by one-loop corrections has an intensity that is detectable by present gamma-ray telescopes. In other words, scenarios 1 and 4 can be ruled out as possible explanations of the high-energy positron excess, because the loop-induced radiative decay produces a gamma-ray line that should have been already detected. This shows that the higher-order corrections are indeed relevant and have to be taken into account. In contrast, the other benchmarks are in agreement with present bounds on gamma-ray lines. For example, in scenario 3 the partial lifetime for the radiative decay is larger compared to scenario 1, and lies slightly above the current Fermi bounds. Scenario 6 can be tested in the future by the CTA. Since there is no kinematic enhancement of the decay $\psi_{\text{DM}} \rightarrow \gamma N$ in the case $\eta = +1$, the intensity of the gamma-ray line is comparably weak in scenarios 2 and 5. For example, scenario 5 differs from 6 just by the sign of $\eta$, but is much more difficult to probe by the CTA. Finally, for benchmark point 7, we assume that the couplings of the leptons to $\psi_{\text{DM}}$ and to $N$ have opposite chirality, in which case the loop is strongly suppressed and there is no hope of detecting a gamma-ray line signal.

5 Conclusions

We have analyzed the radiative decay of dark matter particles in view of the leptonic cosmic-ray anomalies reported by PAMELA and Fermi LAT. Assuming an effective description of leptophilic dark matter decay, we have pointed out that the lines induced at the quantum level may be observable and can be used to constrain models of decaying dark matter. In the case of scalar dark matter, two-body decays into photons
are strongly helicity-suppressed and thus unobservable. In the case that the dark matter particles carry spin 1/2, however, the radiative decay rate is typically suppressed compared to the tree-level decays by some two to three orders of magnitude. Interestingly, the corresponding partial lifetimes for decays into monochromatic photons can then be in the observable range, in particular for dark matter masses of a few hundred GeV, where stringent constraints from Fermi LAT apply. Thus, in some cases the loop-induced gamma-ray line yields constraints that can be competitive with the constraints on charged cosmic rays. At higher energies, constraints from Cherenkov telescopes exist. At present, these constraints are only relevant for certain scenarios for which the radiative two-body decay is kinematically enhanced compared to the three-body decay channel. However, we have pointed out that the proposed CTA should be able to improve on the existing bounds significantly and probe a relevant part of the parameter space which is presently unconstrained.

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A Decay widths for fermionic dark matter

A.1 The decay $\psi_{\text{DM}} \rightarrow \ell^+ \ell^- N$

The differential decay rate for this process is given by

$$d\Gamma(\psi_{\text{DM}} \rightarrow \ell^+ \ell^- N) = \frac{1}{(2\pi)^3} \frac{1}{64m_{\psi_{\text{DM}}}^3} |M_t + M_u|^2 dt \, ds . \quad (A.1)$$

Note that there is a relative minus sign between the $t$- and $u$-channel amplitudes due to the exchange of two anticommuting fermions that is not present by a naive application of the Feynman rules for the two diagrams. Neglecting the lepton mass, one obtains for
the squared amplitude

\[
|M_t + M_u|^2 = \left( |\lambda_{L t}^\ell| - |\lambda_{R t}^\ell| \right) \left( |\lambda_{L N}^\ell| + |\lambda_{R N}^\ell| \right)
\times \left( \frac{(t - m_N^2)(t - m_{\psi DM}^\ell) - u + (u - m_N^2)(m_{\psi DM}^\ell - u)}{(t - m_N^2)(t - m_{\psi DM}^\ell) - u + (u - m_N^2)(m_{\psi DM}^\ell - u)} \right)
\times \sqrt{(s - t)} (u - m_N^2 - s)
\times \left( \frac{m_{\psi DM}^N m_N s}{(t - m_N^2)(u - m_N^2)} \right) \right) \times \left( \frac{(t - m_N^2)(m_{\psi DM}^\ell - t) + (u - m_N^2)(m_{\psi DM}^\ell - u) - s(t + u)}{(t - m_N^2)(u - m_N^2)} \right), \quad (A.2)
\]

where

\[
s = (q_1 - p_1)^2, \quad t = (q_1 - p_2)^2, \quad u = (q_1 - p_3)^2 = m_{\psi}^2 + m_N^2 + 2m_\ell^2 - s - t. \quad (A.3)
\]

Again, \( \eta = \eta_{\psi DM} \eta_N = \pm 1 \) depending on the CP eigenvalues of \( \psi_{DM} \) and \( N \). The integration limits for the Mandelstam variables are given by

\[
0 \leq s \leq (m_{\psi DM} - m_N)^2 \quad (A.4)
\]

and

\[
t_{1,2} = \frac{1}{2} \left( m_{\psi DM}^2 + m_N^2 - s - \sqrt{\lambda(m_{\psi DM}^2, m_N^2, s)} \right), \quad (A.5)
\]

where

\[
\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc. \quad (A.6)
\]

We can perform the kinematical integrations in the limit \( m_\Sigma \gg t, u \), in which case the Mandelstam variables in the denominator can be neglected. We then get

\[
\Gamma(\psi_{DM} \to \ell^+ \ell^- N) = \frac{1}{64(2\pi)^3} \frac{m_{\psi DM}^5}{m_N^4} \left\{ \left( (|\lambda_{L t}^\ell| - |\lambda_{R t}^\ell|) |\lambda_{L N}^\ell| - |\lambda_{R N}^\ell| \right)^2 \right.
\]

\[
- \eta \Re \left( \lambda_{L t}^\ell \lambda_{L N}^\ell \lambda_{R t}^\ell \lambda_{R N}^\ell \right) F_1(x)
\]

\[
+ 2\eta \Re \left( \lambda_{L t}^\ell \lambda_{L N}^\ell \lambda_{R t}^\ell \lambda_{R N}^\ell \right) F_2(x) \right\}, \quad (A.7)
\]

where \( x \equiv m_N^2/m_{\psi DM}^2 \) and \( F_1(x), F_2(x) \) are defined in eqs. (2.6), (2.7).

In the case of mediation by a vector, the matrix element for vanishing lepton mass
\[ |M_t + M_u|^2 = 4 \left( |\lambda_{\psi\ell}^L \lambda_{\psi N}^L|^2 + |\lambda_{\psi\ell}^R \lambda_{\psi N}^R|^2 \right) \times \left[ \frac{(u - m_N^2)(m_{\psi DM}^2 - u)}{(t - m_\psi^2)^2} + \frac{(t - m_N^2)(m_{\psi DM}^2 - t)}{(u - m_\psi^2)^2} \right] + 4 \left( |\lambda_{\psi\ell}^L \lambda_{\psi N}^R|^2 + |\lambda_{\psi\ell}^R \lambda_{\psi N}^L|^2 \right) s(t + u) \left[ \frac{1}{(t - m_\psi^2)^2} + \frac{1}{(u - m_\psi^2)^2} \right] + 8 \eta \left\{ \text{Re} \left[ (\lambda_{\psi\ell}^L \lambda_{\psi N}^L)^2 + (\lambda_{\psi\ell}^R \lambda_{\psi N}^R)^2 \right] \frac{m_{\psi DM} m_{NS}}{(t - m_\psi^2)(u - m_\psi^2)} \right\} + 2 \text{Re} \left[ (\lambda_{\psi\ell}^L \lambda_{\psi N}^L \lambda_{\psi\ell}^R \lambda_{\psi N}^R)^2 \right] \frac{s(t + u)}{(t - m_\psi^2)(u - m_\psi^2)} \right\} . \] (A.8)

In the limit \( m_{\psi DM} \ll m_\psi \) we get for the decay rate

\[ \Gamma(\psi_{DM} \to \ell^+ \ell^- N) = \frac{1}{64(2\pi)^3} \frac{4m_{\psi DM}^5}{6m_\psi^4} \left\{ \left[ (|\lambda_{\psi\ell}^L|^2 + |\lambda_{\psi\ell}^R|^2) (|\lambda_{\psi N}^L|^2 + |\lambda_{\psi N}^R|^2) \right] \frac{m_{\psi DM}}{m_{NS}} \frac{1}{m_\psi^2} \left[ F_1(x) \right] + 2 \eta \text{Re} \left[ (\lambda_{\psi\ell}^L \lambda_{\psi N}^L \lambda_{\psi\ell}^R \lambda_{\psi N}^R)^2 \right] F_2(x) \right\} . \] (A.9)

### A.2 The decay \( \psi_{DM} \to \gamma N \)

There are four scalar-mediated diagrams at the one-loop level contributing to the decay \( \psi_{DM} \to \gamma N \), which are shown in fig. [2] Due to gauge invariance, in the case of CP-conserving interactions, the matrix element corresponding to the sum of the four diagrams can be written in the form

\[ \mathcal{M} = \frac{ig_{N\gamma\psi}}{m_{\psi DM}} \bar{u}(k_1) \left( P_R - \eta_N \eta_\psi P_L \right) \sigma^{\mu\nu} k_2 \epsilon_\mu^* u(p) \]

\[ = - \frac{g_{N\gamma\psi}}{m_{\psi DM}} \bar{u}(k_1) \left( P_R - \eta_N \eta_\psi P_L \right) k_2 \epsilon_\mu^* u(p) , \] (A.10)

where \( \sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2 \) and \( \eta_{\psi DM}, \eta_N \) are the CP eigenvalues of \( \psi_{DM} \) and \( N \), respectively. This is manifestly gauge invariant in the sense that it satisfies the Ward identity: the matrix element vanishes when replacing \( \epsilon_\mu^* \rightarrow k_2 \) since the photon is on-shell.

The effective coupling \( g_{N\gamma\psi_{DM}}^\Sigma \) for an intermediate scalar can be given in terms of loop integrals as follows,

\[ g_{N\gamma\psi_{DM}}^\Sigma = - \frac{\epsilon \eta_N m_{\psi DM}}{16\pi^2} \sum_{f, g, \Sigma} Q_f C_f \left\{ m_f (\eta_{\psi DM} \lambda_{\psi N}^L \lambda_{\psi\ell}^L \eta_N \lambda_{\psi N}^R \lambda_{\psi\ell}^R) I + (\lambda_{\psi N}^L \lambda_{\psi\ell}^L - \eta_{\psi DM} \eta_N \lambda_{\psi N}^R \lambda_{\psi\ell}^R) \eta_{\psi DM} m_{\psi DM} (I^2 - K) - \eta_N m_{NS} K \right\} , \] (A.11)
where the sum runs over all fermions $f$ and all mediators $\Sigma$ that contribute in the loop. The loop integrals are written in terms of Feynman parameters as

$$ I = \frac{1}{\Delta} \int_0^1 \frac{dx}{1-x} \log X $$ \quad (A.12) \\
$$ I^2 = \frac{1}{\Delta} \int_0^1 dx \log X $$ \quad (A.13) \\
$$ K = -\frac{1}{\Delta} \int_0^1 dx \left( 1 + \frac{B}{\Delta x(1-x)} \log X \right) $$ \quad (A.14) \\

where

$$ \Delta \equiv m_{\psi}^2 - m_N^2 $$ \quad (A.15) \\
$$ B \equiv m_x^2 + m_\Sigma^2 (1-x) - m_{\psi}^2 x(1-x) $$ \quad (A.16) \\
$$ X \equiv m_x^2 + m_\Sigma^2 (1-x) - m_{\psi}^2 x(1-x) $$ \quad (A.17) \\

In the limit $m_{\psi}, m_N \ll m_\Sigma$, the loop integrals take on the simplified form \[48\]

$$ I = \frac{1}{m_\Sigma^2} f(m_x^2/m_\Sigma^2) $$ \quad (A.18) \\
$$ I^2 = -\frac{1}{2m_\Sigma^2} f_2(m_x^2/m_\Sigma^2) $$ \quad (A.19) \\
$$ K = \frac{1}{2} f^2 $$ \quad (A.20) \\

where the functions $f$, $f_2$ are defined as

$$ f(x) = \frac{1}{1-x} \left[ 1 + \frac{1}{1-x} \ln(x) \right] $$ \quad (A.21) \\
$$ f_2(x) = \frac{1}{(1-x)^2} \left[ 1 + x + \frac{2x}{1-x} \ln(x) \right] $$ \quad (A.22) \\

The expression for the effective coupling then assumes the form

$$ g_{N\gamma\psi}^N \simeq -\frac{e \eta N m_{\psi} \chi}{16\pi^2} \sum_{\ell,\Sigma} Q_\ell C_\ell \left\{ m_\ell \left( \eta_{\psi_{\ell,\psi}^L} \lambda_{\ell}^L \lambda_{\ell}^R - \eta_N \lambda_{\ell}^L \lambda_{\ell}^R \right) \frac{f(m_x^2/m_\Sigma^2)}{m_\Sigma^2} \right\} $$

$$ - \left\{ \lambda_{\ell}^L \lambda_{\ell}^R - \eta_{\psi_{\ell,\psi}^L} \eta_N \lambda_{\ell}^L \lambda_{\ell}^R \right\} \frac{\eta_{\psi_{\ell,\psi}^L} m_{\psi_{\ell,\psi}^L} - \eta_N m_N}{4m_\Sigma^2} f_2(m_x^2/m_\Sigma^2) $$

$$ \simeq \frac{e \eta}{64\pi^2} m_{\psi_{\ell,\psi}^L} \left( 1 - \frac{m_{\psi}}{m_{\psi_{\ell,\psi}^L}} \right) \sum_{\ell,\Sigma} Q_\ell C_\ell \left\{ \left( \lambda_{\ell}^L \lambda_{\ell}^L - \eta \lambda_{\ell}^L \lambda_{\ell}^R \right) \right\}, $$ \quad (A.23) \\

where in the last line we have taken $m_\ell \to 0$. 

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In the case of an intermediate vector, the same integrals \( I, I^2, K \) (with the replacement \( m_\Sigma \to m_V \) in the constants \( B, X \)) appear, together with the additional integral

\[
J = \frac{1}{\Delta} \int_0^1 \frac{dx}{x} \log X ,
\]

which simplifies in the limit \( m_{\psi\text{DM}}, m_N \ll m_\Sigma \) to

\[
J = \frac{1}{m_V^2} \log (1 - x) - I = -\frac{1}{m_V^2} f^V(m_\ell^2/m_V^2) ,
\]

where in this case the kinematical functions are defined as

\[
f^V(x) = \frac{1}{1 - x} \left[ 1 + \frac{x}{1 - x} \log(x) \right] ,
\]

\[
f_2^V(x) = \frac{1}{(1 - x)^2} \left[ 1 - \frac{5x}{3} + \frac{2x(1 - 2x)}{3(1 - x)} \log(x) \right] .
\]

Furthermore, one finds

\[
I^2 - J - K = K - J = \frac{3}{4m_V^2} f^V(x) .
\]

The effective coupling in terms of loop integrals is given by

\[
g_{N\gamma\psi}^V = \frac{\epsilon \eta_N m_{\psi\text{DM}}}{8\pi^2} \sum_\ell \left\{ (\eta_N \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \lambda_{\ell N}^R \lambda_{\ell \psi}^R) \right\} \eta_{\psi\text{DM}} m_{\psi\text{DM}} (I^2 - J - K) + \eta_N m_N (J - K) + 2m_\ell (\eta_{\psi\text{DM}} \lambda_{\ell N}^L \lambda_{\ell \psi}^R - \eta_N \lambda_{\ell N}^R \lambda_{\ell \psi}^L) J \right\} ,
\]

For \( m_{\psi\text{DM}} \ll m_V \) this expression then simplifies to

\[
g_{N\gamma\psi}^V \approx -\frac{\epsilon \eta_N m_{\psi\text{DM}}}{8\pi^2} \sum_\ell \left\{ 2m_\ell (\eta_N \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \lambda_{\ell N}^R \lambda_{\ell \psi}^R) \frac{f^V(m_\ell^2/m_V^2)}{m_V^2} \right. - \left. 3 (\eta_N \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \lambda_{\ell N}^R \lambda_{\ell \psi}^R) \eta_{\psi\text{DM}} m_{\psi\text{DM}} - \eta_N m_N \right\} f_2^V(m_\ell^2/m_V^2) \frac{m_{\psi\text{DM}}}{4m_V^4} \}
\]

\[
\approx \frac{3\epsilon \eta m_{\psi\text{DM}}}{32\pi^2 m_V^2} \left( 1 - \frac{\eta m_N}{m_{\psi\text{DM}}} \right) \sum_\ell \left( \lambda_{\ell N}^L \lambda_{\ell \psi}^L - \eta \lambda_{\ell N}^R \lambda_{\ell \psi}^R \right) \right) ,
\]

where in the last line we have taken \( m_\ell \to 0 \).

The decay rate in both cases is finally given by

\[
\Gamma(\psi_{\text{DM}} \to \gamma N) = \frac{\left( g_{N\gamma\psi}^V \right)^2}{8\pi} m_{\psi\text{DM}} \left( 1 - \frac{m_N^2}{m_{\psi\text{DM}}^2} \right)^3 .
\]
For the scalar in the case of one mediator coupled to leptons we get in the limit $m_\ell \ll m_N$ and $m_{\psi_{DM}} \ll m_\Sigma$,

$$\Gamma(\psi_{DM} \rightarrow \gamma N) = \frac{e^2}{8\pi (64\pi^2)^2} \frac{m_{\psi_{DM}}^5}{m_\Sigma^4} \left(1 - \frac{m_N^2}{m_{\psi_{DM}}^2}\right)^3 \left(1 - \frac{\eta m_N}{m_{\psi_{DM}}}\right)^2 \times \left[ \sum_{\ell} \left(\lambda_{\ell N}^L \lambda_{\ell N}^L - \eta \lambda_{\ell N}^R \lambda_{\ell N}^R\right) \right]^2,$$

(A.32)

whereas for the vector we have, in the limit $m_\ell \ll m_N$ and $m_{\psi_{DM}} \ll m_V$,

$$\Gamma(\psi_{DM} \rightarrow \gamma N) = \frac{1}{8\pi (8\pi^2)^2} \frac{9e^2}{16m_V^4} \frac{m_{\psi_{DM}}^5}{m_{\psi_{DM}}^2} \left(1 - \frac{m_N^2}{m_{\psi_{DM}}^2}\right)^3 \left(1 - \frac{\eta m_N}{m_{\psi_{DM}}}\right)^2 \times \left[ \sum_{\ell} \left(\lambda_{\ell N}^L \lambda_{\ell N}^L - \eta \lambda_{\ell N}^R \lambda_{\ell N}^R\right) \right]^2.$$

(A.33)

### B Decay widths for scalar dark matter

In this appendix we present the expressions for the decay width of the radiative decay of scalar dark matter into two photons, $\phi_{DM} \rightarrow \gamma\gamma$.

For $\lambda_{\ell}^L = \lambda_{\ell}^R \equiv \lambda_{\ell}$ the decay rate reads \cite{50,51}

$$\Gamma(\phi_{DM} \rightarrow \gamma\gamma) = \frac{m_{\phi_{DM}}^3}{4\pi} \left(\frac{e^2}{16\pi^2}\right)^2 \left| \sum_{\ell} \frac{\lambda_{\ell}}{m_\ell} A_f(\tau_\ell) \right|^2,$$

(B.1)

where $\tau_\ell \equiv m_{\phi_{DM}}^2/(4m_\ell^2)$ and

$$A_f(\tau) = \frac{2}{\tau + (\tau - 1)f(\tau)} / \tau^2$$

(B.2)

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau}, & \tau \leq 1 \\ -\frac{1}{4} \left[ \ln \frac{1 + \sqrt{1 - 1/\tau}}{1 - \sqrt{1 - 1/\tau}} - i\pi \right]^2, & \tau > 1 \end{cases}.$$  

(B.3)

In the case of interest here, $\tau_\ell \gg 1$. Then we can approximate

$$A_f(\tau) \simeq \frac{1}{\tau} \left\{ 2 - \frac{1}{2} \ln(4\tau) - i\pi \right\}^2.$$  

(B.4)

In this limit, and taking only one lepton species into account, the decay rate is given by

$$\Gamma(\phi_{DM} \rightarrow \gamma\gamma) \simeq \frac{1}{16\pi} \frac{|\lambda_{\ell}|^2}{m_{\phi_{DM}}} \left(\frac{e^2}{16\pi^2}\right)^2 \frac{4m_\ell^2}{m_{\phi_{DM}}^2} \times \left\{ 2 + \frac{\pi^2}{2} - \frac{1}{2} \ln^2(4\tau_\ell) \right\}^2 + \pi^2 \ln^2(4\tau_\ell).$$  

(B.5)
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