Fuzzy Optimization Model for Waste Load Allocation in a River with Total Maximum Daily Load (TMDL) Planning

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Abstract: In traditional waste load allocation (WLA) decision making, water quality-related constraints must be satisfied. Fuzzy models, however, can be useful for policy makers to make the most reasonable decisions in an ambiguous environment, considering various surrounding environments. We developed a fuzzy WLA model that optimizes the satisfaction level by using fuzzy membership functions and minimizes the water quality management cost for policy decision makers considering given environmental and socioeconomic conditions. The fuzzy optimization problem was formulated using a max–min operator. The fuzzy WLA model was applied to the Yeongsan River basin, which is located in the southwestern part of the Korean Peninsula and Korean TMDLs were applied. The results of the fuzzy model show that the pollutant load reduction should be increased in the Gwangju 1 and Gwangju 2 wastewater treatment plants (WWTPs) and in subcatchments with high pollutant load. In particular, it is necessary to perform advanced wastewater treatment to decrease the load of 932 kg ultimate biochemical oxygen demand (BODu)/day in the large-capacity Gwangju 1 WWTP and reduce the BODu emission concentration from 4.3 to 2.7 mg/L during the low-flow season. The satisfaction level of the fuzzy model is a relatively high at 0.81.

Keywords: fuzzy WLA model; ambiguity; fuzzy membership function; satisfaction level; pollution load reduction; water quality management

1. Introduction

The general waste load allocation (WLA) model allocates the pollution load for each sewage treatment plant or each subcatchment by obtaining a solution to the optimization problem consisting of the constraints on river water quality and the objective function for the watershed water quality management cost. The existing WLA decision-making problem optimizes the objective function while necessarily satisfying the constraints on water quality and the wastewater treatment efficiency. Various types of traditional WLA models have been developed using optimization techniques such as linear programming, nonlinear programming, and genetic algorithm, and these models mainly focus on economic factors such as water quality management costs [1–3]. In addition to the economic goal of cost minimization, two types of inequalities among waste dischargers in the basin were considered for WLA. The first inequality was calculated with the ER-Gini coefficient, based on the environmental resources and discharge load in each subbasin. The second type of inequality involved the fairness in the distribution of treatment efforts among waste dischargers [4].

Moreover, since a fuzzy WLA model involves the application of fuzzy logic [5] to WLA and promotes sensible decision making in an ambiguous environment from a subjective perspective,
these models can be used by policy makers to make rational decisions considering the political, social and economic characteristics of a region.

For a long time, the one and only tool used to establish logic descriptions was binary logic. Everything in logic was written in terms of TRUE and FALSE. The problem with this simplistic description is that it does not consider the uncertainty and imprecision of human knowledge [5]. The automation scientist, L.A. Zadeh, constructed a new logic concept based on fuzzy sets. This approach allows us to consider imprecision and uncertainty in human knowledge, as well as the progressive transitions between states. The main difference between classical logic and fuzzy logic is the existence of a progressive transition between TRUE and FALSE [5,6].

Fuzzy models have been actively applied to allocation and investment problems in the water resource sector for some time. Using the fuzzy max–min decision model, water resource allocation problems were analyzed. The developed model optimized water resources and balanced competing water conflicts among different consumers [7]. A multipreference-based interval fuzzy-credibility constrained programming (MIFCP) was developed to plan a regional-scale water resource management system (RWMS). Solutions for multiple water resources, multiple water-receiving cities and multiple water-using departments for decision makers with various attitudes and credibility levels were examined in their study [8]. An integration of stochastic dynamic programming (SDP) and fuzzy integer goal programming (FIGP) model framework was proposed to address problems involving multiobjective-multicriteria sequential decision making under the budgetary and sociotechnical uncertainties inherent in water resource investment planning [9].

A fuzzy waste load allocation model (FWLAM) was developed for water quality management in a river system using fuzzy multiple-objective optimization. An important feature of this model is its ability to incorporate the requirements and conflicting objectives of various pollution control agencies and dischargers. The vagueness associated with specifying water quality criteria and fraction removal levels was modeled in a fuzzy framework [10]. A suitable adaptation of the model proposed by Fujiwara et al. was later used for FWLAM modeling [11]. The water quality calculation in the FWLAM model was performed using a recursive rule and the Streeter–Phelps equation. Elleuch et al. developed a model based on fuzzy multicriteria decision-making (FMCDM) methods and mathematical optimization programming (MOP) to solve a water allocation problem [12]. The political process of establishing effluent charges and minimum acceptable qualities, in the form of either effluent or stream quality standards, involves the participation of each group of interested individuals within a river basin. To consider the effect of political influence in a water quality model, relative weights can be defined and used in the objective function. By varying the relative weights, an analyst can establish potential alternatives from the infinite set of possible alternatives [13]. In this way, the ambiguous characteristics associated with environmental policy decisions can be applied to river water quality management via a mathematical model with fuzzy logic.

The subsequently developed modified fuzzy waste load allocation model (MFWLAM) is a stochastic model that considers the moments (mean, variance and skewness) of water quality indicators and incorporates uncertainty due to the randomness of input variables along with uncertainty due to imprecision [14]. Additionally, a two-stage fuzzy chance-constrained programming approach was developed for water resource management under dual uncertainties [15]. In the model, the concept of distribution with fuzzy probability is presented to express uncertainties. An interval-parameter chance-constrained fuzzy multi-objective programming (CFMOP) model was developed by Liu et al. to assist with water pollution control within a sustainable wetland management system under uncertainty [16]. Additionally, an inexact stochastic-fuzzy programming model was proposed for irrigation water resource allocation and land resource utilization management considering multiple uncertainties [17]. In the model, uncertainties can be directly integrated into the optimization process by treating parameters and coefficients as interval values, fuzzy sets, random variables, and their combinations.
The existing deterministic WLA techniques have mostly been used to derive optimal solutions that fully satisfy the objective functions and constraints regarding the cost of reducing pollutants and river water quality in making decisions to achieve water quality objectives. In regions where seasonal fluctuations in river flows and water temperatures are severe, such as in Korea, the self-purification capacity and physical dilution associated with the flow rates of streams vary greatly, and the seasonal fluctuations in river water quality are also severe. In addition, the treatment efficiency of each wastewater treatment plant in a basin is not constant due to seasonal changes in rainfall and temperature. A WLA model was developed that can reflect these ambiguous characteristics of river water quality and the wastewater treatment efficiency and contribute to water quality management policy decisions at the basin scale. The fuzzy model presented in this study considers the satisfaction of the cost of reducing the pollution load in a basin to a certain extent. In addition, the satisfaction level of river water quality is considered in comparison with water quality goals and acceptable water quality criteria at certain points. In this study, a fuzzy WLA model was developed that optimizes the satisfaction level of pollution load reduction and river water quality, and an optimization problem is explored using the max-min operator. This fuzzy model was applied to the Yeongsan River, one of Korea’s four major rivers.

2. Materials and Methods

2.1. Study Area

The fuzzy model presented in this study was applied to the Yeongsan River, which is located in the southwestern part of the Korean Peninsula (Figure 1) and is highly polluted by domestic wastewater discharged from the metropolitan city of Gwangju. Korean total maximum daily loads (TMDLs) are applied to this river. The area covered by the fuzzy model is a drainage zone that includes the target points of TMDLs, Yeongbon A, Yeongbon B, and Whangyong A. The drainage area of the study area is 533 km², and many people live in the middle and lower streams of the Yeongsan River. The Yeongbon B point is located at the border between Gwangju city and Jeollanam-do. The Yeongbon A point was 1.44 mg/L (6.44 mg/L ultimate biochemical oxygen demand (BOD); BODu). Additionally, the average BOD concentration at the Yeongbon A point was 1.44 mg/L. Currently, the water quality at Yeongbon B has deteriorated considerably [18].

![Figure 1. The Yeongsan River basin study area.](image-url)
As such, the pollution level at Yeongbon B is increasing due to the inflow of wastewater treatment effluent from the Gwangju 1 and Gwangju 2 wastewater treatment plants (WWTPs) into the Yeongsan River between Yeongbon A and Yeongbon B. The proportion of sewage-treated effluent in the flow of the Yeongsan River in this section is large. The average flow rates at Yeongbon A and Yeongbon B in the low-flow season are 5.40 and 20.17 m$^3$/s, respectively. Notably, 6.405 and 1.248 m$^3$/s of treated wastewater are introduced from the Gwangju 1 and Gwangju 2 WWTPs, respectively, in the section between Yeongbon A and Yeongbon B.

2.2. Fuzzy Optimization Model

In this study, a fuzzy water quality management model was developed considering the satisfaction level regarding the cost of reducing the pollution load and improving river water quality. The cost of reducing the pollutant load in the watershed was determined based on the amount of pollutant reduction in the entire watershed. The reduction in the pollutant loads at each WWTP and in each subwatershed was calculated from the wastewater discharge and removal efficiency values. In this study, the maximum reduction efficiency of the Gwangju 1 and Gwangju 2 WWTPs was 99.4% when the Gwangju WWTP effluent BOD$_5$ was 1 mg/L, and the maximum efficiency of the Jangseong and Damyang WWTPs was 99.5%, which is the treatment efficiency of the Damyang WWTP in 2016. For other small wastewater treatment facilities, 95% was applied. For diffuse pollution treatment facilities, a treatment efficiency of 25% for BOD and TP was applied, with a dry pond used as the standard treatment facility [19,20].

In the fuzzy WLA model, the membership function and satisfaction level for water quality and watershed management costs are defined and formulated to construct the objective function and constraint equations of the WLA model. This study used a linear membership function that is easy to apply and efficient for water quality management problems. The satisfaction level is set to 0 when the objective function is below a condition that cannot be satisfied in terms of the treatment cost and water quality, and the satisfaction level is set to 1 when the condition is met; otherwise, this value is assumed to vary linearly between the upper and lower limits.

In this study, the satisfaction level for river water quality and pollution load reduction, that is, the water quality management cost, is represented by a linear membership function. The maximum possible level, $C_c^{Max}$ (maximum pollutant abatement in the river basin) is assigned a membership value of 0. $C_c^{Min}$ is the case in which the current pollution load is reduced at the WWTPs and in each subbasin, and the corresponding membership value is 1 (Figure 2). The membership level associated with pollutant abatement in the river basin after WLA is calculated as follows:

$$\mu_c = \begin{cases} 
1 & \text{if } C_c \leq C_c^{Min} \\
\frac{C_c^{Max} - C_c}{C_c^{Max} - C_c^{Min}} & \text{if } C_c^{Min} < C_c < C_c^{Max} \\
0 & \text{if } C_c \geq C_c^{Max}
\end{cases}$$

(1)

where

- $C_c^{Max}$ = maximum pollutant abatement in the river basin
- $C_c^{Min}$ = minimum pollutant abatement in the river basin
- $C_c$ = pollutant abatement in the river basin after waste load allocation

The maximum permissible level, $L^P$ is assigned a membership value of 0 at the water quality check points in the river basin. Therefore, the membership value is 0 when the water quality of the target point is $L^P$ or higher. Alternatively, if the target water quality at a water quality checkpoint is satisfied,
that is, when the water quality at the target point is $L^T$ or less, the membership value is 1 (Figure 3). In the WLA process, the membership level at a water quality checkpoint is calculated as follows:

$$\mu_w = \begin{cases} 1 & \text{if } L_j \leq L^T_j \\ \frac{L^T_j - L_j}{L^T_j - L^P_j} & \text{if } L^P_j < L_j < L^T_j \\ 0 & \text{if } L_j \geq L^P_j \end{cases}$$  

(2)

where

- $L^T_j$ = target water quality at the water quality checkpoint
- $L^P_j$ = permissible water quality at the water quality checkpoint
- $L_j$ = calculated water quality concentration after waste load allocation, mg/L

The pollution load reduction, which is the objective function in the general deterministic WLA model, and the river water quality, which is expressed by the constraint equation in the model, are expressed by the following membership functions:

$$\mu_c = \left(\sum_{i=1}^{n} q_i O_i X_{i,max} - \sum_{i=1}^{n} q_i O_i X_{i,min}\right) / \left(\sum_{i=1}^{n} q_i O_i X_{i,max} - \sum_{i=1}^{n} q_i O_i X_{i,min}\right)$$  

(3)

$$\mu_{w_j} = \left(\frac{t^j_p - L_j}{t^j_p - t^j_{L_j}}\right) j = 1, 2, \cdots, n$$  

(4)

$$L_j = L^O_j + \sum_{i=1}^{n} T_{ij} (86.4 q_i O_i) j = 1, 2, \cdots, n$$  

(5)

where

- $\mu_{w_j}$ = membership function of river water quality (BOD)
- $\mu_c$ = membership function of pollution load abatement
- $L^T_j$ = target water quality at checkpoint $j$, mg/L
- $L^P_j$ = permissible water quality at checkpoint $j$, mg/L
- $X_{i,max}$ = maximum pollutant removal rate for point source $i$
- $X_{i,min}$ = minimum pollutant removal rate for point source $i$
- $L^O_j$ = present water quality at checkpoint $j$, mg/L
- $L_j$ = calculated water quality at checkpoint $j$ after waste load allocation, mg/L
- $T_{ij}$ = transfer coefficient (water quality variation at point $j$ associated with pollution source $i$), (mg/L/kg/d)
- $q_i$ = flow from point source $i$, m$^3$/s
- $O_i$ = untreated water quality from point source $i$, mg/L

![Figure 2. Membership function of pollution load abatement.](image-url)
As shown above, the river water quality and pollution load reduction cost are expressed as membership functions, and finally, the lowest satisfaction level among the satisfaction levels of all water quality and reduction costs is selected to optimize the satisfaction level $\lambda$ [21,22]. Based on Zimmermann’s approach [23,24], using a max–min as the operator, the following crisp model is formulated to solve the fuzzy WLA problem:

$$\begin{align*}
\text{Maximize} & \quad \lambda \\
\text{Subject to} & \quad \lambda \leq \mu_c \\
& \quad \lambda \leq \mu_{w,j} \quad j = 1, 2, \ldots, n \\
& \quad X_{Min}^i \leq X_j \leq X_{Max}^i \quad i = 1, 2, \ldots, m \\
& \quad L_j^T \leq L_j \leq L_j^P \quad j = 1, 2, \ldots, n \\
& \quad 0 \leq \lambda \leq 1
\end{align*}$$

(6)

In this study, the BOD of the river was calculated by formulating the BOD reduction process as the self-purification ability of the river. Additionally, the transfer coefficient, which represents the change in water quality at downstream points due to the reduction in the pollution load in the upstream, was calculated using the QUAL2K model [4]. The optimization problem for the fuzzy model was solved with a genetic algorithm (GA) [25]. In the GA, the population size was set to 90, the number of generations was set to 800, and the crossover probability was set to 0.5. The water quality and flow data for sewage treatment plants and streams used in this study were obtained during the low-water period from October 2016 to February 2017.

3. Results and Discussion

3.1. Calibration of the Water Quality Model

The results of water quality calibration for the Yeongsan River and Whangyonggang River under low-flow conditions in 2016 are shown in Figures 4 and 5. The water quality calculation results reflect the water quality measurements for the Yeongsan and Whangyonggang rivers in 2016. In particular, these results match the TMDL measurement results at water quality target points such as Yeongbon A, Yeongbon B, and Whangyong A. The BOD calibration results for the main stream of the Yeongsan River are shown in Figure 4. The Yeongsan River BODu concentration suddenly increases immediately
after the Yongcheon (a point 5.9 km from the upstream boundary) joins and suddenly increases again immediately following the Orecheon (at 10.3 km) and Jeongamgang (at 13.3 km) confluence points. Because the river flow is small in this area, the river water quality is sensitive to external pollutants. The BOD concentration increases as the effluents from the Gwangju 1 WWTP (at 30.8 km) and Gwangjucheon Stream (at 31 km) join the Yeongsan River, and the BOD concentration gradually decreases as the Whangyonggang River (at 38.1 km) joins.

**Figure 4.** Calibration results for BOD in the main stem of the Yeongsan River: BODu is ultimate biochemical oxygen demand, and “observed” represents the water quality measurement results for the Yeongsan River in the low-flow season in 2016. “2016 Calibrated” represents calculated BODu after calibration in the low-flow season in 2016.

**Figure 5.** Calibration results for BOD in the Whangyonggang River: “Observed” represents the water quality measurement results for the Whangyonggang River in the low-flow season in 2016.

### 3.2. Waste Load Allocation Results

The average BOD\textsubscript{5} levels in the inflow and effluent at the Gwangju 1 WWTP in the low-flow season were 164.3 and 2.7 mg/L, respectively, and as of 2016, the BOD treatment efficiency was 98.36\%. The average BOD\textsubscript{5} influent and effluent quality levels at the Gwangju 2 WWTP were 132.3 and 1.9 mg/L, respectively, and as of 2016, the BOD treatment efficiency was 98.58\%. At the Gwangju 1 and Gwangju 2 WWTPs, 6.405 and 1.248 \text{m}^{3}/\text{s} of effluent flow into the Yeongsan River, respectively. In this study, it was assumed that the effluent at the Gwangju 1 and Gwangju 2 WWTPs can be treated to achieve a BOD\textsubscript{5} level of 1 mg/L.
The target water quality of the Korean 2020 BODu TMDL is 3.48, 3.80, and 7.59 mg/L at Whangyong A, Yeongbon A, and Yeongbon B, respectively, but the average BODu concentrations during the low-flow season in 2016 were 2.82, 2.27, and 6.44 mg/L at Whangyong A, Yeongbon A, and Yeongbon B, respectively. Since the target water quality was achieved in the low-flow season in 2016, the target water qualities for BODu in the fuzzy model considered in this study were 2.0, 1.5, and 4.0 mg/L for Whangyong A, Yeongbon A, and Yeongbon B, respectively. For the permissible water quality in the fuzzy model, the average measured water quality in 2016 was applied.

The allocation results of the application of optimization methods are provided in Table 1. The satisfaction level of the fuzzy WLA model was calculated to be 0.81, and this satisfaction value is relatively high [10,26,27]. The reason for this high satisfaction value is that the effect of reducing pollutant loads is largely reflected in river water quality. The pollution load reduction across the entire basin was also relatively small based on WLA results, which suggests that the river water quality is close to the target water quality; thus, the satisfaction value was large. The water quality distribution in the Yeongsan River calculated by the fuzzy allocation model is shown in Figures 6 and 7. The predicted water quality results are improved from those in 2016, and the water quality improvement at Yeongbon B on the main stream of the Yeongsan River is greater than that at Yeongbon A. The BODu levels at Yeongbon A, Yeongbon B, and Whangyong A were predicted to be 1.62, 4.70, and 2.15 mg/L, respectively.

**Table 1.** Allocated biochemical oxygen demand (BOD) load calculated by the fuzzy waste load allocation model.

| Point Source | Reduction Rate (%) | Allocated Load (kg BODu/day) | Remark               |
|--------------|--------------------|------------------------------|----------------------|
| PS1          | 31.11              | 46.4                         |                      |
| PS2          | 7.77               | 3.4                          |                      |
| PS3          | 99.41              | 13.3                         | Jangseong WWTP       |
| PS4          | 36.57              | 27.7                         |                      |
| PS5          | 73.76              | 146.3                        | Wangdongcheon        |
| PS6          | 14.35              | 60.3                         |                      |
| PS7          | 51.27              | 117.7                        |                      |
| PS8          | 6.96               | 10.0                         |                      |
| PS9          | 21.59              | 207.0                        | Seobangcheon         |
| PS10         | 66.94              | 73.5                         |                      |
| PS11         | 50.47              | 245.6                        | Yongcheon            |
| PS12         | 99.55              | 1.8                          | Damyang WWTP         |
| PS13         | 34.76              | 28.8                         |                      |
| PS14         | 63.06              | 241.6                        | Orecheon             |
| PS15         | 2.91               | 29.8                         | Jeongamgang          |
| PS16         | 73.71              | 32.3                         |                      |
| PS17         | 52.34              | 56.8                         |                      |
| PS18         | 37.06              | 15.9                         |                      |
| PS19         | 80.54              | 14.6                         |                      |
| PS20         | 37.94              | 123.1                        |                      |
Table 1. Cont.

| Point Source | Reduction Rate (%) | Allocated Load (kg BODu/day) | Remark          |
|--------------|--------------------|------------------------------|-----------------|
| PS21         | 17.39              | 14.9                        |                 |
| PS22         | 14.03              | 12.2                        |                 |
| PS23         | 54.62              | 121.7                       |                 |
| PS24         | 98.96              | 932.1                       | Gwangju 1 WWTP  |
| PS25         | 2.91               | 23.0                        |                 |
| PS26         | 25.84              | 26.8                        |                 |
| PS27         | 99.25              | 142.5                       | Gwangju 2 WWTP  |
| PS28         | 3.94               | 14.9                        |                 |
| PS29         | 35.53              | 27.7                        |                 |
| Total        |                    | 2811.7                      |                 |

Table 1 shows that among the four large sewage treatment plants in the Yeongsan River basin, the required pollutant load reductions are smallest at the Jangseong WWTP and Damyang WWTP. As of 2016, the BOD treatment efficiencies of the Gwangju 1 and Gwangju 2 WWTPs were 98.36% and 98.58%, respectively, and the corresponding treatment efficiencies allocated in the fuzzy WLA model presented in Table 1 were 98.96% and 99.25%, respectively. The Gwangju 1 and Gwangju 2 WWTPs need to provide more treatment now than they did in the past. In addition, significant pollutant load reductions must be achieved in the Orecheon and Yongcheon watersheds, which flow directly into the main stream of the Yeongsan River. Additional reductions should be emphasized in the Wangdongcheon watershed of the Whangyonggang River basin and the Seobangcheon subbasin of the Gwangjucheon Stream basin. For the pollution sources in the basin, the Gwangju 1 WWTP has the largest required reduction of 932 kg BODu/day; additionally, the reduction at the Gwangju 2 WWTP should reach 142 kg BODu/day. For the Gwangju 1 WWTP, the mean BODu concentration of the treated effluent during the low-flow season was 4.3 mg/L (the conversion factor for BOD₅ to BODu is 1.582), but this value should be lowered to 2.7 mg/L through additional advanced treatment. In addition to the large-scale sewage treatment plants, it is necessary to significantly reduce the BOD load in the subcatchments in which the pollutant load is large. It is necessary to reduce the loads by 241, 245, 146, and 207 kg BODu/day in the Orecheon, Yongcheon, Wangdongcheon and Seobangcheon subbasins of the Whangyonggang River and Gwangjucheon Stream, respectively.

Figure 6. Water quality distribution in the Yeongsan River after waste load allocation (WLA) application according to the fuzzy model. The water quality goal is the target water quality for BODu in the fuzzy WLA model proposed in this study. The allocated BOD load is the water quality distribution in the Yeongsan River calculated from the fuzzy WLA model.
was explored to maximize this satisfaction value based on water quality and cost reduction satisfaction. Water resources, civic groups, government officials, and local residents to comprehensively consider water resource acquisition problems, flood management, aquatic ecosystem management, income levels, the population and the industrial status in the area. In the WLA approach, it is necessary to consider different natural resources and socioeconomic factors for each region in the future.

In river basin management, it is necessary to promote water quality governance (WQG) involving the central government and other regional entities. Transitional countries facing rapid institutional adjustment, the restructuring of regulations, and political and economic changes are faced with specific internal decentralization demands and external policy formulations. The application of new water policies may fail again if a top-down governance model is proposed that limits the ability of local governance entities to effectively govern water quality [28]. Kayser et al. identified common challenges in drinking water quality governance (DWQGo) in Brazil, Ecuador, and Malawi. While research on water quality has often focused on specific technical solutions, this research provides an analysis of water governance structures, relationships, and processes that are critical to the delivery of safe drinking water over time [29].

In Korea, top-down water management policies are being implemented first, leading to controversy over the effectiveness of projects such as the Four Major Rivers Project. In the management of large and medium-sized rivers, the central government should shift their policy-making method for national projects to a governance method that involves local communities. Integrated water management should involve WQG in a basin and consider the opinions of experts in water quality management and water resources, civic groups, government officials, and local residents to comprehensively consider water resource acquisition problems, flood management, aquatic ecosystem management, income levels, the population and the industrial status in the area. In the WLA approach, it is necessary to consider different natural resources and socioeconomic factors for each region in the future.

4. Conclusions

In this study, a fuzzy WLA model was developed considering the pollutant load reduction cost and satisfaction level associated with river water quality in a basin. The river water quality and pollution load reduction cost were expressed as membership functions, and a fuzzy WLA problem was explored to maximize this satisfaction value based on water quality and cost reduction satisfaction values. Because the effect of reducing the pollution load in the fuzzy WLA model is greatly reflected in the river water quality, the overall pollution load reduction in the WLA results is relatively small, and the river water quality is close to the target water quality; therefore, the satisfaction value of the fuzzy model is large. The Gwangju 1 WWTP in Gwangju city, the largest of the WWTPs in the Yeongsan River basin, needs to achieve significant reductions in pollutant loads, and the Gwangju 2 WWTP and Orecheon subbasin also need to reduce the amount of pollutant load significantly. This fuzzy

**Figure 7.** Water quality distribution in the Whangyonggang River after WLA application according to the fuzzy model. The allocated BOD load is the water quality distribution in the Whangyonggang River calculated from the fuzzy WLA model.
model can be used as a tool to make decisions considering political, economic and social factors in the target basin in an ambiguous environment where it is unclear whether the target water quality will be achieved.

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