Geometric models of multi-parametric technological processes for estimation some inverse control problems

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Abstract. This paper is devoted to the geometric and computer simulation of multi-parameter systems. We adduce description of original method and algorithm which may be used for inverse control problem solution. We describe the multidimensional geometric model based on experimental data. We base our method on the set of one-dimensional spline approximations which form a non-linear frame of hyper-surface. The method, algorithm and software of visualization have been used in researching heat-insulating packets of clothing industry. Control of technological processes connected with the manufacturing of the packets is considered as an inverse problem. As the result we received new scientific-based parameters of the heat-insulated packets and these parameters may be used in clothes design and for the control of manufacturing technology.

1. Introduction

Design of geometric models for investigation of multi-parametric technological systems is one of important problems of applied geometry [1]. Geometric models have a certain scientific value if they represent not only those properties of specific technological processes which are interested for researchers but if they may be adapted to some class of similar processes [2, 3]. The processes may belong to different branches of industry. All geometric models may be considered as consisting of two parts. First part is usually a model of class of processes and it is usually considered in multidimensional space of input and output parameters. The second part is a model of multidimensional space on the plane.

Standard methods and software generate a set of special functions which simulate “input – output” correspondences with given precision. Initial data of the model consist of experimental data and some additional information which diminish the set of models. It is widely used in Computer Aided Simulation at present [4]. The second part is realized by various projective methods and operations of section in multidimensional spaces.

Geometric problems of technological process simulation may be classified as problems of structure and parametric identification. If we have a solution of some identification problem we may solve the direct problem that is to find the values of output parameters if the input parameters are given. But control of technological process requires the solution of inverse problem that is to find the values of input parameters if the output parameters are given. At this point we encounter various cases from infinity set of admissible solutions to a finite number of ones. After all, the set of solutions may be empty.

In this paper we present a new method aimed at the solution of multidimensional inverse problem. Our method diminishes the difficulties of structure and parametric identification and it allows us to solve the inverse problem of technological process control more effective.

2. General considerations

The inverse control problem in the Euclidean multidimensional space may be written as \( AX = Y \), where \( A: X \rightarrow Y \times X \) is a set of operators, \( A = \{A_1, ..., A_k\} \); \( X \) is a space of input parameters, \( \dim X = k \); \( Y \) is a space of output parameters, \( \dim Y = m \). If we know the set \( A \), we consider the direct problem as a structure and parametric identified problem. It is necessary to note...
that all values of \( y_i \) mean the values of output parameters at some points of the domain where the values of input parameters exist. But on investigating the real processes operator \( A_i \) is usually determined approximately and we must use some computing algorithms [5, 6].

Our approach to the inverse control problem is as follows:
1) We avoid the structure identification of operator \( A_i \) at all. It allows us to diminish the influence of input data errors and to avoid the unfounded complication of the model;
2) We avoid a piecewise representation of input and output parameter domains and we do not find parameter identifications in each sub-domain;
3) The operator \( A_i \) is considered over the rectangular box area of regular point set and it is determined by experimental data only;
4) All approximations have limited dimensions, namely \( \dim A_i = 1 \). This fact allows us to simplify all our computing algorithms, to reduce computing errors and it permits us to carry out the visualization of the process by means of standard software [7].

3. Theory
To solve the inverse control problem we must have the space \( X \) of input parameters and \( m \) spaces \( Y \) of output ones. The total space is \((k + m)\)-dimensional Euclidean space. A regular point set \( S = \{ (x_{1,i}), \ldots, (x_{k,i}) \} \), where \( i = 1, \ldots, k \), \( j = 1, \ldots, k_i \), and \( k_i \) may be of different values. The set of correspondences \( A : X \to Y \times X \) is given discretely and all correspondences are determined at the knots of the set \( S \) only. Out of these points the correspondences exist but they are not determined. Hence, in the space \( Y \times X \) we have regular functional point sets \( T_i = \{ y_{1,i} \times S, \ldots, y_{m,i} \times S \} \) with the operators \( A_i \), which are not identified. All correspondences \( S \to T_i \subset T \) are determined by numeration of the knots. Two knots correspond to each other if they have the same numeration. Approximation of partial operator \( A_i \) is realized by one-dimensional plane spline-function only and this function exists in the space \( X \times y_i \). These approximations take place in the planes \( x_i \times y_i \) and they are determined by means of orthogonal mapping in \( k + 1 \)-dimensional space \( X \times y_i \). The order of each plane spline-function is not more than three and that is enough for practical simulation. As a result, each knot of the set \( T = \{ y_1 \times S, \ldots, y_m \times S \} \) belongs to \( k \) one-dimensional space spline-functions and these space spline-functions generate \( m \) one-dimensional non-linear frames of \( k \)-dimensional surfaces in \( m \) spaces \( X \times y_i \). Let us consider only one value \( y_{i,0} \) of output parameter \( y_i \). Then hyper-plane \( y_i = y_{i,0} \) separates the point set \( \{ y_i > y_{i,0} \} \subset T_i \) from the set \( \{ y_i < y_{i,0} \} \subset T_i \). Correspondently, the set of knots \( S = \{ (x_{1,i}), \ldots, (x_{k,i}) \} \) is separated into two sub-sets \( S_1 = S(y_i > y_{i,0}), S_2 = S(y_i < y_{i,0}) \). The border of the sets \( S_1 \) and \( S_2 \) is a hyper-surface in the space \( X \). The correspondence which is given by numeration of the knots allows us to select some knots from each side of the hyper-surface and also to do the same along each parameter axis of the space \( X \). That kind of approximation is characterized by compact one-dimensional patterns that take into account the information at the knots only. In simplest case of linear approximation it is enough to have two knots located on either side of the hyper-surface. In the case of quadratic approximation it is necessary to have three knots and in the case of cubic approximation it is enough to have four knots. Special locations of the knots may be various in regard to hyper-surface. Also, the patterns may be various in regard to the parameter axis. Visual appreciations of the knot location graphs are the criterion of the pattern choosing. Approximation by the patterns forms the stage of partial operator identification.

Let us consider the solution of the inverse control problem now. Since all spline approximations are determined by their mappings the solution of inverse problem is fulfilled by means of standard algorithms which give us the solution of system consisting of two equations. If we shall fulfill that repeatedly we shall get a domain \( U_i \) of admissible values of input parameters and in this domain each value of input parameter will be equal to \( y_{i,0} \). The domain \( U_i \) is determined discretely at the knots another then the knots of domain \( S_i \). We may approximate the domain \( U_i \) as required.
The set of function lattices $T = \{y_1 \times S, \ldots, y_m \times S\}$ and partial mappings $A_i: \{S \rightarrow y_i \times S\}$ lead to existence $m$ domains $U_i$ of admissible values in the space $X$. If $m < k$ we analyze the mutual location of $U_i$ with the purpose of finding some general domain $U_1 \cup U_2 \cup \ldots \cup U_m$. If $m = k$ the general domain $U_1 \cup U_2 \cup \ldots \cup U_m$ degenerates into finite number of points. If $m > k$ the general domain $U_1 \cup U_2 \cup \ldots \cup U_m$ may be empty.

In order to use our theory at practice we developed the special software in which the basic geometric operations are realized. The software contains a set of plane splain approximation algorithms for given sets of points. The principal method of data representation is a tabular one. More detailed information about software one can find in [8].

4. Heat-insulating packets of clothing industry

Let us consider one applied problem of light industry. The problem is connected with determination of optimal parameters for the packets of clothing products. Such products are applied for protection from low temperature.

We considered two types of packet structures: three-layered and five-layered types (figure 1). To construct the geometric model of the packets we investigated 56 samples. Input parameters of the model were as follows:
1) The sizes of one packet module as non-varied parameters;
2) Various down-held materials and their combinations as varied parameters;
3) The mass of down filler into the module, temperature of the air and the speed of the wind as continues parameters.

![Figure 1. Tree-layered down packet: 1, 2 are down-held clothes, 3 is a partition, 4 is a down mixture](image)

The sizes of the module were equal to $120 \times 300$ mm. The down-held materials were various jacket fabrics and lining cloth. The upper part of the packet was manufactured from the jacket fabric, but the bottom part was made from lining cloth. Mass of down filler was varied from 2.5 up to 20.0 g. Temperature of the air corresponded to autumn and winter seasons and it was varied from $-25^\circ C$ up to $-5^\circ C$. Speed of the wind was varied from 5 up to 10 m/s.

The problem of our investigation was to determine the total thermal resistance of the packet by the various temperature of the air. The total thermal resistance is denoted by $R_{sum} (m^2 \cdot K/Wt)$, the mass of down filler is denoted by $m$ (g). In order to solve the problem we prepared the special device which allowed us to carry out all experiments by the real climatic conditions.

5. Determination of packet parameters by means of geometric model

Our geometric model of the packet was created by means of special developed software “Hyperdescent”. The result of modeling was one-dimensional lattice frame of the hyper-surface which simulated interconnection of total thermal resistance and varied parameters of the packet.
Let us to determine the optimal mass of down filler which secures $R_{\text{sum}} = 0.6$ by $T = -10^\circ\text{C}$; $T = -20^\circ\text{C}$. Visual geometric model one can see in figure 2. The upper part of the figure (figure 2A) shows the graphs of splain-functions which describe the total thermal resistance depending on the mass of down filler. The temperature of the air is denoted by $T_b$. If the mass of down filler increases up to 12.5 … 15.0 the total thermal resistance increases too. Further increase in the mass of down filler leads to fall

**Figure 2.** Geometric model of the heat-insulating packet structure and the solution of inverse control problem
in total thermal resistance. We can explain that property by increase of down filler closeness and by fall in its porosity. As a result we have increase in thermal conductivity of down mixture and the packet as a whole.

The bottom part of figure (figure 2B) shows the solution of the inverse control problem. We have to determine the packet parameters in order to secure the values of $R_{\text{sum}}$ and $T_b$ which are given. The results of solution are shown in table 1. The structure of the packet is denoted as follows: the symmetric three-layered packet is denoted by $P_1$; the jacket clothes are denoted by $T_2$, $T_3$ and $T_4$; the lining cloth is denoted by $T$.

| $R_{\text{sum}}$, m$^2$K/Wt | $T_b$, °C | $m$, g | Packet structure |
|-----------------------------|------------|--------|------------------|
| 0.4                         | -5         | 14.7   | $P_1$            |
|                             | -10        | 16.4   |                  |
| 0.5                         | -10        | 30.6   | $P_1$            |
|                             |            | 13.1   | $T_2$-$P_1$-$T$  |
|                             |            | 9.7    | $T_3$-$P_1$-$T$  |
|                             |            | 13.1   | $T_4$-$P_1$-$T$  |
|                             | -15        | 33.9   | $P_1$            |
|                             |            | 20.0   | $T_2$-$P_1$-$T$  |
|                             |            | 12.5   | $T_3$-$P_1$-$T$  |
|                             |            | 20.0   | $T_4$-$P_1$-$T$  |
| 0.6                         | -15        | 40.0   | $P_1$            |
|                             |            | 30.0   | $T_2$-$P_1$-$T$  |
|                             |            | 20.8   | $T_3$-$P_1$-$T$  |
|                             |            | 30.0   | $T_4$-$P_1$-$T$  |
|                             | -20        | 45.8   | $P_1$            |
|                             |            | 38.1   | $T_2$-$P_1$-$T$  |
|                             |            | 32.5   | $T_3$-$P_1$-$T$  |
|                             |            | 36.9   | $T_4$-$P_1$-$T$  |
|                             | -25        | 49.44  | $P_1$            |
|                             |            | 43.9   | $T_2$-$P_1$-$T$  |
|                             |            | 38.9   | $T_3$-$P_1$-$T$  |
|                             |            | 43.9   | $T_4$-$P_1$-$T$  |

6. Results and conclusions
Taking as a basis our theoretical developments and practical experiments we have the following statements:
1) There is rather effective algorithm of inverse control problem solution based on original geometric model and applied software;
2) All theoretical and practical results may be applied for designing the light industry products having a lot of parameters;
3) The heat-insulated packets received new scientific-based parameters which may be used in clothes design and for the control of manufacturing technology.
7. References

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