Real Higgs singlet and the electroweak phase transition in the standard model

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Abstract

The effective potential at finite temperature is constructed within the minimal standard model when a real Higgs singlet is added on. We find that a region of parameter space exists for which one can find a first order transition strong enough to prevent the erasure due to sphalerons of baryon asymmetry, while keeping the mass of the smallest Higgs boson above the experimental lower bound of about 60 GeV.

1 Introduction

The possibility of baryogenesis at the electroweak phase transition continues to provide an active area of theoretical work [1]. Some topics in this area include: (i) constructing a more reliable effective potential by taking the infrared divergences into account, (ii) finding viable scenarios of bubble propagation at the electroweak phase transition, and (iii) examining the various extensions of the standard model (SM) that could possibly accommodate baryogenesis at the electroweak scale. This paper falls into the last category, in that we seek to find out whether the minimal extension of the SM Higgs sector - adding only a real singlet Higgs boson - can provide a first order phase transition as necessary for creating a net baryon number. And if so, whether the strength of this phase transition is
sufficient so that a subsequent washout of the baryon number by sphalerons is avoided. Quantitatively this constraint translates to
\[ v(T_c)/T_c \gtrsim 1, \tag{1} \]
where \( v(T_c) \) is the Higgs vacuum expectation value (VEV) at the first order phase transition temperature \( T_c \). Because \( v(T_c) \) is given in terms of the parameters of the Higgs potential, another important constraint for any model that aims to satisfy Eq. (1) is the experimental lower bound on the Higgs mass:
\[ m_H \gtrsim 60 \text{GeV}. \tag{2} \]

So far, we have seen the following picture emerge from this topic of effective potential construction in various models (the list below is by no means exhaustive in this area):

1) **The minimal SM**: A first order phase transition exists due to temperature induced cubic terms in the effective potential, but the strength does not appear to be enough and any baryon asymmetry created is likely to be washed out after the phase transition - that is, constraints (1) and (2) cannot be satisfied simultaneously \([4, 16]\).

2) **The SM + an extra Higgs doublet**: There seems to be a sufficient number of parameters present to satisfy both constraints \([5]\).

3) **The minimal supersymmetric model (MSSM)**: In a manner similar to that in the SM, it appeared that constraints (1) and (2) could not be satisfied simultaneously \([6]\), but a recent paper that included the higher loop corrections through the self-energies claims to have found a small parameter space where this is possible \([7]\).

4) **The MSSM + a Higgs singlet**: It appears that due to the presence of tree-level cubic terms, constraints (1) and (2) can be satisfied even without help from the temperature induced cubic terms \([8]\).

5) **The left-right symmetric model**: The minimal Higgs sector which consists of two Higgs doublets gives a similar result to the SM case \([3]\), while the alternative Higgs sector with two triplets and a bidoublet seems to require the presence of a further singlet Higgs for sufficient baryon asymmetry generation \([10]\).

6) **The SM + a complex singlet Higgs**: For the case in which the VEV of the singlet
remains zero for all temperatures it is possible to satisfy both constraints \( (1) \) and \( (2) \). In the singlet Majoron case, with the VEV of the singlet non-zero, it appears that either (i) both constraints can be satisfied simultaneously for somewhat large quartic couplings \( [12] \); or (ii) that constraint (2) may be modified through the presence of Majoron decay channels for the physical Higgs boson hence keeping the viability of this model alive \( [13] \).

In this paper, we consider a real Higgs singlet that couples only to the standard Higgs doublet. This is the smallest possible extension of the SM Higgs sector. The motivation for this work is twofold. First, we want to contribute to the above picture which basically aims to find those models compatible with electroweak baryogenesis and to try to find perhaps some consistent pattern emerging amongst the possible scenarios. Second, unlike the complex singlet case, we will have a tree-level cubic term, rather like the case of the singlet Higgs added to the MSSM. There, the tree-level cubic term appeared to play a significant role in ensuring a strong first order phase transition. Is this a promising trend to be found elsewhere too? We keep the singlet VEV non-zero thus ensuring the generality of our analysis. We also do not impose any specific restriction on the parameters, working numerically with a random selection of parameter sets to obtain results.

## 2 The effective potential

We add a real singlet Higgs field \( S \) to the minimal SM with its doublet Higgs field \( \phi \). As usual, \( \phi \sim 2(1) \) under \( SU(2)_L \otimes U(1)_Y \) while \( S \sim 1(0) \). Hence the most general potential at zero temperature is:

\[
V_o(\phi, S) = \lambda_\phi (\phi^\dagger \phi)^2 - \mu_\phi^2 \phi^\dagger \phi + \frac{\lambda_S}{2} S^4 - \frac{\mu_S^2}{2} S^2 - \frac{\alpha}{3} S^3 + 2\lambda (\phi^\dagger \phi) S^2 - \frac{\sigma}{2} (\phi^\dagger \phi) S.
\]

(3)

Expanding each Higgs field about constant background fields, we obtain the tree-level potential,

\[
V_{\text{tree}}(u, v) = \frac{\lambda_\phi}{4} u^4 - \frac{\mu_\phi^2}{2} u^2 + \frac{\lambda_S}{4} v^4 - \frac{\mu_S^2}{2} v^2 - \frac{\alpha}{3} v^3 + \frac{\lambda}{2} u^2 v^2 - \frac{\sigma}{4} u^2 v,
\]

(4)
where \( \phi(x) = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} 0 \\ u \end{array} \right] + \Phi(x) \) and \( S(x) = v + \eta(x) \). Note that \( V_{\text{tree}}(u, v) \) is symmetric in \( u \rightarrow -u \) but not in \( v \rightarrow -v \). Hence we will restrict our attention to the \( u \geq 0 \) section of the \((u, v)\) plane. Setting \( \nabla V_{\text{tree}}(u, v) = 0 \) gives the following equations for the points of zero gradient:

\[
\begin{align*}
(-\mu^2_{\phi} + \lambda_{\phi} u^2 + \lambda v^2 - \sigma v/2)u &= 0 \\
-\mu^2_{S}v + \lambda_{S} v^3 + \lambda v^2 - \sigma u^2/4 - \alpha v^2 &= 0.
\end{align*}
\]

(5)

There are up to six different solutions to Eq. (5) in \( u \geq 0 \) part of the \((u, v)\) plane. Some of these are saddle points, while others are local minima or maxima. Up to two local minima can exist, with the only constraint on these being that the global minimum \( \equiv (\kappa_1, \kappa_2) \) obeys \( \kappa_1 = 246 \text{ GeV} \).

The field dependent Higgs masses are given by,

\[
m_{\phi,S}^2(u, v) = \frac{1}{2} \left[ -\mu^2_{\phi} - \mu^2_{S} + (3\lambda_{\phi} + \lambda)u^2 + (3\lambda_{S} + \lambda)v^2 - \left( \frac{\sigma}{2} + 2\alpha \right)v \right]
\pm \left\{ \frac{1}{4} \left[ -\mu^2_{\phi} + \mu^2_{S} + (3\lambda_{\phi} - \lambda)u^2 - (3\lambda_{S} - \lambda)v^2 - \left( \frac{\sigma}{2} - 2\alpha \right)v \right]^2 + (2\lambda v - \sigma v/2)^2u^2 \right\}^{1/2},
\]

\[
m_g^2(u, v) = \lambda_{\phi} u^2 + \lambda v^2 - \sigma v/2 - \mu^2_{\phi},
\]

(6)

where \( m_g^2(u, v) \) is the contribution from goldstone bosons. Because \( S \) does not couple to the gauge bosons or fermions, their field dependent masses are the same as in the minimal SM:

\[
m_W^2(u) = g^2 u^2/4, \quad m_Z^2(u) = (g^2 + g'^2) u^2/4, \quad m_A^2(u) = 0, \quad m_t^2(u) = Y^2 u^2/2,
\]

(7)

where \( Y \) is the top Yukawa coupling constant.

Using standard methods, we obtain the 1-loop effective potential \[14\]

\[
V(u, v) = V_{\text{tree}}(u, v) + V_1^{(0)}(u, v) + V_1^{(T)}(u, v)
\]

(8)

where

\[
V_1^{(0)}(u, v) = \sum_i \frac{n_i}{64\pi^2} m_i^4(u, v) \left[ \ln \frac{m_i^2(u, v)}{m_0^2} - \frac{3}{2} \right]
\]

(9)
and
\[ V_1^{(T)}(u, v) = \sum_i n_i \frac{n_i}{2\pi^2} T^4 I_\pm \left( \frac{m_i^2(u, v)}{T^2} \right). \] (10)

The sum is over all the particles in the model, with \( m_i^2(u, v) \) as given in Eqs. (3) and (7). The quantities \( n_i \) account for the degrees of freedom of each particle, and \( m_{0i} \) is the mass of the particle at zero temperature. The functions \( I_\pm \) are given by
\[ I_\pm(y) = \int_0^\infty dx x^2 \ln(1 \mp e^{\sqrt{x^2+y}}), \] (11)
where \( I_+ \) is used for bosons and \( I_- \) for fermions. This one loop result can be improved by summing up the dominant higher order loop corrections, a procedure that introduces the self-energies \( \Pi_i(0) \) for the appropriate particle degrees of freedom \([14, 15]\). By including only the leading high temperature correction and expanding \( I_\pm \) in powers of \( m_i/T \), we obtain
\[ V_1^{(T)}(u, v) = \sum_i n_i \left[ \frac{m_i^2(u, v)T^2}{24} - \frac{3m_i^4(u, v)}{2 \cdot 64\pi^2} \right] - \sum_j n_j \frac{M_j^3(u, v)T}{12\pi}, \] (12)
where \( M_j^2(u, v) = m_j^2(u, v) + \Pi_j(0) \) for \( j = g, W_L, W_T, Z_T, \gamma_T \) [where subscripts \( L \) and \( T \) refer to longitudinal and transverse modes respectively], and for \( j = \phi, S \), one can use the \( M_j^2(u, v) \) as the diagonal elements of the initial mass matrix modified by the addition of the self-energies of the Higgs bosons.

This finite temperature effective potential (FTEP) is quite similar to that of Enqvist et al. \([13]\) except that we have the additional terms \(-\frac{\alpha}{3}v^3 - \frac{\sigma}{4}u^2v\). They found a weak first order phase transition in the \( u = 0 \) direction followed by another first order phase transition in the \( v \approx \) constant direction induced by finite temperature cubic terms. Clearly the same scenario should be viable in this model also, as we have extra parameters \( \alpha \) and \( \sigma \). But the presence of these parameters should give us an extra effect. Because no Majoron is present in this model, we cannot modify constraint \([2]\) as was done by Enqvist et al. So we must rely on these extra parameters to allow us to satisfy both of the constraints.
3 Analysis of the finite temperature effective potential

As a first step, we omit the temperature induced cubic terms altogether. This will give us a clear indication of the magnitude of the role played by the $\alpha$ and $\sigma$ terms, since no first order phase transition can exist without the temperature induced cubic terms within the minimal SM or the singlet Majoron model. The temperature independent 1 loop potential terms will be neglected for simplicity (they only contribute a few percent to the overall potential in the region $m/T < 1$). Now the FTEP can be written down explicitly:

$$V(u, v) = V_{\text{tree}}(u, v) + T^2 \left( \frac{6\lambda_\phi + \lambda}{24} u^2 + \frac{3\lambda_S + 4\lambda}{24} v^2 - \frac{\sigma + \alpha}{12} v \right), \quad (13)$$

where we have used $g = 0.652$, $g' = 0.352$ and $Y = 0.73$ ($\Rightarrow m_t = 127 \text{ GeV}$; this choice for $Y$ is illustrative only, and corresponds to the most likely value for the top mass within the context of the minimal SM).

There are seven free parameters ($\lambda_\phi, \lambda_S, \lambda, \mu^2_\phi, \mu^2_S, \alpha, \sigma$), with one constraint that the global minimum at zero temperature is set at $u = 246 \text{ GeV}$. To take this into account more easily, we express ($\mu^2_\phi, \mu^2_S, \alpha, \sigma$) in terms of ($u_1, v_1, u_2, v_2$), the values of VEV’s at which the gradient of $V_{\text{tree}}(u, v)$ is zero. We then choose $u_1 = 246 \text{ GeV}$ at zero temperature. We also choose to concentrate on those cases where $v_1 = -v_2$ which makes the pattern of VEV formation described in Figs. 1 to 4 more realisable. This leaves us with 5 parameters: ($\lambda_\phi, \lambda_S, \lambda, u_2, v_2$). We then choose random sets of these in the following ranges:

$$10^{-3} < \lambda_\phi, \lambda_S < 10^{-1}, \quad -0.5 < \lambda < 0.5, \quad 100 \text{ GeV} < u_2, v_2 < 300 \text{ GeV}. \quad (14)$$

The range in $\lambda_\phi, \lambda_S$ is chosen so that $V(u, v)$ which comes partly from expanding in these parameters makes sense. The range in $\lambda$ is chosen to be general, with one constraint that $\frac{\lambda_\phi}{4} + \frac{\lambda_S}{4} + \frac{\lambda}{2} > 0$ to ensure that $V_{\text{tree}}$ is bounded below along $u = v$ line. The VEV’s $u_2, v_2$ are essentially unconstrained and we have simply chosen them to be at the same scale as $u_1$.

We then went through these randomly chosen sets of parameters and tested each set for the following conditions, rejecting the set if any of these conditions was not met:
1) The global minimum at zero temperature is at \((u_1, v_1)\), where \(u_1 = 246\) GeV.
2) The mass of the lightest Higgs boson \(\geq 60\) GeV [constraint (2)].
3) A first order phase transition exists for some temperature \(T_c\).
4) \(m_i/T_c < 1\) at the points in the \((u, v)\) plane where a first order phase transition occurs.

Figs. 1 to 4 show the most promising pattern of VEV formation (as temperature changes) that we found in terms of satisfying both constraints (1) and (2). (The second set of parameters in Table 1 is used to generate these figures.) Above the transition temperature the global minimum lies at \((U_1^T, V_1^T)\) where \(U_1^T > 0\) and \(V_1^T < 0\). At the temperature \(T_c\) (which is of order 100 GeV), two degenerate minima form at points \((U_1^{Tc}, V_1^{Tc})\) and \((0, V_2^{Tc})\), where \(V_2^{Tc} > 0\), and there is a barrier between the two minima. A first order phase transition occurs as the global minimum “tunnels” from \((U_1^{Tc}, V_1^{Tc})\) point to \((0, V_2^{Tc})\) point. Table 1 shows some of the allowed sets of parameters and the values of some relevant quantities. It reveals how constraints (1) and (2) can both be met. In general we find that as \(T \to \infty\), the global minimum \(\to (0, V_\infty)\), where \(V_\infty \neq 0\). If we set \(\alpha = -\sigma\), \(V_\infty = 0\) is achieved. However, we find that within the allowed sets of parameters, \(V_\infty\) is small [\(\sim O(10)\)]. At any rate because the electroweak-breaking VEV \(U_1^T\) has jumped to zero at \(T_c\), electroweak symmetry restoration does take place.

Qualitatively, the reason for this FTEP to allow such a strong first order phase transition lies in the asymmetry in \(v \to -v\). Two valleys, roughly parallel to the \(u-\)axis, form as \(T\) nears \(T_c\), in such a way as to allow two degenerate minima. In this regard the tree level cubic terms \(\alpha\) and \(\sigma\) are essential. Note that for most of the sets of parameters in Table 1, we find that in addition to the usual electroweak phase transitions, another first order phase transition purely in \(v\) direction can occur, at temperatures higher than the electroweak phase transition temperature (see the caption for Fig. 3). However this additional first order phase transition is not a requirement for successful baryogenesis and the last set of parameters in Table 1 is an example where only one first order phase transition occurs (for this set, \(v_2 = 200\) GeV).

What would happen if we were to include the temperature dependent cubic terms?
Table 1: Representative parameter values for which the conditions (15) are met. $V_c$ is the distance between the two degenerate minima in the $(u, v)$ plane at $T_c$. The parameters $\lambda_{\phi, S}$ and $\lambda$ are dimensionless while all the other quantities have dimension GeV.

| $\lambda_{\phi}$ | $\lambda_S$ | $\lambda$ | $v_1$ | $\sigma$ | $\alpha$ | $m_{\phi, S}$ | $V_c/T_c$ |
|-------|-------|-------|------|--------|--------|---------------|-----------|
| 0.071 | 0.082 | -0.0234 | -125 | -12.81 | 12.20 | 61, 102 | 170/107 |
| 0.063 | 0.073 | 0.0250 | -154 | -13.26 | 3.58 | 73, 88 | 390/93 |
| 0.087 | 0.090 | 0.0114 | -245 | -9.37 | 1.24 | 103, 108 | 510/117 |
| 0.096 | 0.082 | 0.0499 | -146 | -17.27 | 5.10 | 75, 109 | 430/97 |
| 0.050 | 0.077 | 0.0168 | -117 | -12.43 | 8.34 | 67, 79 | 360/70 |
| 0.057 | 0.0920 | 0.0026 | -171 | -16.39 | 4.64 | 73, 95 | 360/85 |
| 0.057 | 0.0920 | 0.0026 | -107 | -16.54 | 16.76 | 68, 92 | 360/63 |

Adding the gauge boson cubic terms $\sim g^3 T(u^2)^{3/2}$ may change the shape of the potential along the valleys but not the degenerate nature of the two valleys themselves as these terms are independent of $v$. The Higgs boson cubic terms $\sim \frac{T}{12\pi^2}[(3\lambda_S + \lambda)v^2 + \ldots + c_i^2 T^2]^{3/2}$, where the $c_i^2 T^2$ piece comes from self-energy effects, are much more complicated and they will depend on $v$. Within the minimal SM, including the self-energy pieces in the Higgs boson cubic terms introduces an upper bound on the Higgs boson mass, above which the phase transition becomes second order [16]. The existence of this upper bound [in addition to the lower bound on the Higgs mass that comes from satisfying constraint (1)] further restricts the available parameter space. Therefore one may wonder if the sets of allowed parameters we found here may actually become excluded as soon as the Higgs self-energy terms are included. But we are again saved by the tree-level cubic terms, as the upper bound restriction no longer applies when terms of the form $\alpha v^3$ exist (for any non-zero value of $\alpha$). Hence adding the Higgs self-energy terms will not make the first order phase transition go away. It will of course change the actual value of the allowed parameters. To get some idea on the quantitative effect on the parameters when these Higgs boson cubic terms are included, we fitted cubic polynomials in $v$ to these terms.
(around the regions of degenerate minima at $T_c$). The result is consistent with what one may expect with a naive estimate of $\sim \frac{T}{12\pi} (3\lambda S + \lambda)^{3/2} v^3$. For the values of the parameters shown in Table 1, there is at most an additional term of $\sim -0.3v^3$ coming from the Higgs boson cubic terms. This means that these terms make less contribution for larger values of $\alpha$ (for example, when $\alpha = 12.20$, $-0.3v^3$ term is about 7% of the tree-level cubic term $-\alpha v^3/3$). We thus conclude that our qualitative conclusions obtained without inclusion of the temperature-dependent cubic terms are robust, and that our approximate effective potential is quantitatively accurate to about the 10% level near the degenerate minima at temperatures around $T_c$.

4 Conclusion

We have investigated the minimal extension of the SM Higgs sector at finite temperature by writing down the effective potential when a real singlet Higgs boson is included. We found that one can find sets of parameters such that a strong first order phase transition that will prevent a washout of possible baryon asymmetry can exist, while satisfying the lower bound on the experimentally observed Higgs mass. Our main point is that by having tree-level cubic terms in the Higgs potential these conditions are more easily satisfied. When one considers the difficulties faced by models that rely solely on the temperature dependent cubic terms in satisfying these constraints, tree-level cubic terms provide perhaps a good rough measure of the potential viability of the models for electroweak baryogenesis.

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We note that a first-order electroweak phase transition is a necessary but not sufficient condition to generate adequate baryon asymmetry. Other issues such as adequate CP-violation and appropriate bubble-wall propagation are also important, but these are beyond the scope of our present analysis.
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**Figure captions**

Fig.1: A contour plot of $V(u,v)$ at $T = 0$. The position of global minimum is denoted by the dot at $(u,v) = (246, -154)$ (all units are in GeV). There are saddle points at $(0, -162)$ and $(0, 211)$. There is a local minimum at $(168, 154)$ which will move towards $(0, 200)$ in a second order phase transition as $T$ increases. (The second set of parameters in Table 1 is used to generate these figures.)
Fig. 2: A contour plot of $V(u, v)$ at the transition temperature, 93 GeV. The positions of degenerate global minima are denoted by the dots at $(0, 200)$ and $(160, -152)$.

Fig. 3: A contour plot of $V(u, v)$ at $T = 200$ GeV. The local minimum at $(160, -152)$ at $T = 93$ has moved to $(0, -130)$ in a second order phase transition. After this, another first order phase transition has occurred from $(0, 156)$ to $(0, -130)$, making the latter the global minimum now. This extra first order phase transition is however not necessary for adequate baryogenesis.

Fig. 4: A contour plot of $V(u, v)$ at $T = 400$ GeV. The global minimum is now at $(0, -58)$. As $T$ increases further, this global minimum will settle towards $(0, -30)$. 