Electromagnetic instabilities in rotating magnetized viscous objects

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Accepted 2009 August 18. Received 2009 July 27; in original form 2009 May 6

ABSTRACT

In this paper, we study electromagnetic streaming instabilities in the thermal viscous regions of rotating astrophysical objects, such as magnetized accretion discs, molecular clouds, their cores and elephant trunks. The results obtained can also be applied to any regions of interstellar medium, where different equilibrium velocities between charged species can arise. We consider a weakly ionized multicomponent plasma consisting of neutrals and magnetized electrons, ions and dust grains. We take into account the effect of perturbation of collisional frequencies as a result of the density perturbations of species. We obtain general expressions for the perturbed velocities of species involving the thermal pressure and viscosity when perturbations propagate perpendicular to the background magnetic field. The dispersion relation is derived and investigated for axisymmetric perturbations. New compressible instabilities generated as a result of different equilibrium velocities of different charged species are found in the cold and thermal limits either when the viscosity of neutrals can be neglected or when it is important. The viscosity of magnetized charged species is negligible for the perturbations considered that have wavelengths much larger than the Larmor radius of species. At the same time, the neutrals are shown to be immobile in electromagnetic perturbations when their viscosity is sufficiently large.

Key words: accretion, accretion discs – instabilities – magnetic fields – plasmas – waves.

1 INTRODUCTION

In a series of papers (Nekrasov 2007, 2008a,b, 2009a,b), a general theory has been developed for electromagnetic compressible streaming instabilities in multicomponent rotating magnetized objects, such as accretion discs and molecular clouds. For accretion discs, the different equilibrium velocities of the different species (electrons, ions, dust grains and neutrals) have been found using momentum equations, taking into account the anisotropic thermal pressure and collisions of charged species with neutrals (Nekrasov 2008a,b, 2009b). For molecular clouds, their cores and elephant trunks, the different equilibrium velocities of charged species have been presumed to be a result of the action of the background magnetic field (Nekrasov 2009a). It has been shown that the differences between equilibrium velocities generate compressible streaming instabilities that have growth rates much larger than the rotation frequencies. In the papers mentioned above, it has been suggested that these new fast instabilities are a source of electromagnetic turbulence in accretion discs and molecular clouds.

Streaming instabilities can also be generated in neutral media. Recently, it has been suggested that the streaming instability, which originates as a result of the difference between the equilibrium velocities of small neutral solids and gas, is a possible source that could contribute to planetesimal formation (Youdin & Goodman 2005). These authors have numerically treated this hydrodynamic instability in the Keplerian disc for interpenetrating streams coupled via drag forces. The growth rates of instability have been found to be much smaller than the dynamical time-scales. The particle density perturbations generated by this instability could seed planetesimal formation without self-gravity.

In the papers by Nekrasov devoted to electromagnetic streaming instabilities, the viscosity has not been considered. However, numerical simulations of the magnetorotational instability show that this effect can influence the magnitude of the saturated amplitudes of perturbations and, correspondingly, the turbulent transport of the angular momentum (e.g. Masada & Sano 2008; Pessah & Chan 2008). The interstellar medium, molecular clouds and accretion discs around young stars are weakly ionized objects, where collisional effects play a dominant role. The ratio of the viscosity to the resistivity (the magnetic Prandtl number) for astrophysical objects takes a wide range of values. In particular, in accretion discs around compact X-ray sources and active galactic nuclei, the magnetic Prandtl number varies by several orders of

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2 BASIC EQUATIONS

In this paper, we investigate weakly ionized astrophysical objects or their definite regions, where all the charged species are magnetized, that is, when, in particular, their cyclotron frequencies are larger than their collisional frequencies with neutrals. Then, the momentum equations for charged species and neutrals, including viscous terms, have the form (Braginskii 1965):

\[
\frac{\partial \mathbf{v}_j}{\partial t} + \mathbf{v}_j \cdot \nabla \mathbf{v}_j = -\nabla U - \frac{\nabla P_j}{m_j} + \frac{q_j}{m_j} \mathbf{E} + \frac{q_j}{m_j c} (\mathbf{v}_j \times \mathbf{B})_\perp - \nu_{j\perp} (\mathbf{v}_j - \mathbf{v}_n) \\
+ \mu_{j\parallel} \nabla \cdot \mathbf{v}_j + \mu_{j\perp} \nabla^2 \mathbf{v}_j + \mathbf{v}_n (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n,
\]

(1)

\[
\frac{\partial \mathbf{v}_n}{\partial t} + \mathbf{v}_n \cdot \nabla \mathbf{v}_n = -\nabla U - \frac{\nabla P_n}{m_n} - \sum_j \nu_{n\parallel} (\mathbf{v}_n - \mathbf{v}_j) + \mu_n \left( \nabla^2 \mathbf{v}_n + \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}_n) \right).\]

(2)

Here, the index \( j = e, i, d \) denotes the electrons, ions and dust grains, respectively, and the index \( n \) denotes the neutrals. In equations (1)–(3), \( q_j \) and \( m_{j,n} \) are the charge and mass of species \( j \) and neutrals, \( \mathbf{v}_{j,n} \) is the hydrodynamic velocity, \( n_{j,n} \) is the number density, \( P_{j,n} = n_{j,n} T_{j,n} \) is the thermal pressure, \( T_{j,n} \) is the temperature, \( \nu_{j\perp} = \gamma_j m_{j,n} \nu_{j\perp} \) is the collisional frequency of species \( j \) (neutrals) with neutrals (species \( j \)), where \( \gamma_j = \langle \sigma v \rangle_j / (m_j + m_n) \) \( (\langle \sigma v \rangle_j \) is the rate coefficient for momentum transfer). The coefficients of the kinematic viscosity are \( \mu_{j\parallel} = c_\parallel \mu_{j\parallel} / 3, \mu_{j\perp} = c_\perp \mu_{j\parallel} / \alpha_{j\perp} T_{j,n} \) and \( \mu_n = c_\parallel \mu_n / \alpha_{n\parallel} T_{j,n}^2 \). Here, \( \mu_{j\parallel} = \nu_{j\parallel} / \nu_{j\parallel} \) \( \nu_{j\parallel} \) is the neutral–neutral collisional frequency), \( \alpha_{j\perp} = q_j B_0 / m_{j,n} c_\parallel \) is the cyclotron frequency, \( \tau_{j\parallel} = \nu_{j\parallel}^{-1} \) and \( \tau_{j\perp} = \left(T_{j,n}/m_{j,n} \right)^{1/2} \) is the thermal velocity. Numerical coefficients for the electrons and ions are \( c_\parallel = 0.73, c_\perp = 0.96, c_\parallel = 0.5, c_\perp = 0.51 \) and \( c_\perp = 0.3 \) (Braginskii 1965). Furthermore, \( U = -GM/R \) is the gravitational potential of the central object having mass \( M \) (when it presents), \( R = (r^2 + z^2)^{1/2} \), \( G \) is the gravitational constant, \( E \) and \( B \) are the electric and magnetic fields, and \( c \) is the speed of light in vacuum. The magnetic field \( \mathbf{B} \) includes the external magnetic field \( B_{0\text{ext}} \) of the central object and/or interstellar medium, the magnetic field \( \mathbf{B}_{0\text{int}} \) of the stationary current in a steady state, and the perturbed magnetic field. We use the cylindrical coordinate system \((r, \theta, z)\), where \( r \) is the distance from the symmetry axis \( z \) and \( \theta \) is the angle in the azimuthal direction. The background magnetic field is assumed to be directed along the \( z \)-axis, \( \mathbf{B}_0 = B_{0\text{ext}} \hat{z} + B_{0\text{int}} \mathbf{e}_z \), and the sign \( \perp \) denotes the direction across the \( z \)-axis. In equations (1) and (2), the condition \( \alpha_{j\perp} \gg \nu_{j\perp} \) is satisfied in the viscous terms. For unmagnetized charged particles of species \( j \), \( \alpha_{j\perp} \ll \nu_{j\perp} \), the viscosity coefficient has the same form as that for neutrals. We adopt the adiabatic model for the temperature evolution when \( P_{j,n} \ll n_{j,n}^2 T_{j,n} \), where \( T_{j,n} \) is the adiabatic constant. In general, we consider the two-dimensional case in which \( \nabla = (\partial / \partial r, \partial / \partial r \partial \theta, 0) \).

The other basic equations are the continuity equation

\[
\frac{\partial n_{j,n}}{\partial t} + \nabla \cdot n_{j,n} \mathbf{v}_{j,n} = 0,
\]

(4)

the Faraday equation

\[
\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}.
\]

(5)
and Ampere’s law
\[ \nabla \times B = \frac{4\pi}{c} j. \]
where \( j = \sum q_i n_i v_i \). We consider the wave processes with typical time-scales much larger than the time the light spends to cover the wavelength of perturbations. In this case, we can neglect the displacement current in equation (6), which results in quasi-neutrality for both the electromagnetic and purely electrostatic perturbations.

3 EQUILIBRIUM

We suppose that in equilibrium the electrons, ions, dust grains and neutrals rotate in the azimuthal direction of the astrophysical object (accretion disc, molecular cloud, its cores, elephant trunk, etc.) with different velocities \( n_{j0} \), in general. These velocities can depend on the radial coordinate. The stationary dynamics of the charged species is determined by the action of the background magnetic field and collisions with neutrals. In their turn, the neutrals also experience a collisional coupling with charged species influencing their equilibrium velocity. Some specific cases of equilibrium have been investigated by Nekrasov (2007, 2008a,b, 2009b). The stationary velocities of charged species in the background electric field and gravitational field of the central mass in the absence of collisions have been used in Nekrasov (2007). Nekrasov (2008a) considered the weak collisional coupling of neutrals with light charged species (electrons and ions) and weak and strong collisional coupling between neutrals and heavy dust grains. Nekrasov (2008b, 2009b) investigated equilibrium where the neutrals have a strong coupling with light charged species as well as with heavy dust grains.

In the papers cited above, it has been shown that the different charged species have different stationary velocities. Because of this effect, electric currents exist in the equilibrium state, which generate their magnetic fields.

4 LINEAR APPROXIMATION: GENERAL EXPRESSIONS

In this paper, we do not consider electromagnetic perturbations connected with background pressure gradients. Thus, we exclude drift waves from our consideration. We take into account the induced reaction of neutrals on the perturbed motion of charged species. The neutrals can be involved in electromagnetic perturbations, if the ionization degree of medium is sufficiently high. We also include the effect of perturbation on the collisional frequencies as a result of the density perturbations of charged species and neutrals. This effect emerges when there are different background velocities of species. Then the momentum equations (1)–(3) in the linear approximation take the forms:

\[
\frac{\partial v_{j1\perp}}{\partial t} + \nu_{j0} \cdot \nabla v_{j1\perp} = -\frac{\gamma_j}{m_j} n_{j0} \nabla n_{j1} - \frac{q_j}{m_j c} (v_{j0} \times B_{1\perp}) + \frac{q_j}{m_j c} (v_{j1} \times B_{0\perp}) - \nu_{j0} (v_{j1\perp} - v_{j0\perp}) - \nu_{j0} n_{j1} (v_{j0} - v_{j0\perp}) + \mu_j \nabla \cdot v_{j1} + \mu_j \nabla^2 v_{j1\perp},
\]

\[
\frac{\partial v_{j1z}}{\partial t} + \nu_{j0} \cdot \nabla v_{j1z} = \frac{q_j}{m_j} E_{1z} + \frac{q_j}{m_j c} (v_{j0} \times B_{1z}) - \nu_{j0} (v_{j1z} - v_{j0z}) + 4 \mu_j \nabla^2 v_{j1z},
\]

\[
\frac{\partial v_{j1n}}{\partial t} + \nu_{j0} \cdot \nabla v_{j1n} = -\frac{\gamma_j}{m_j} n_{j0} n_{j1} - \sum_j \nu_{j0} (v_{j1n} - v_{j0n}) - \sum_j \nu_{j0} n_{j1} (v_{j0n} - v_{j0}) + \mu_j \nabla \cdot v_{j1n} + \frac{1}{3} \nabla \cdot (v_{j1} - \nabla \cdot v_{j0}).
\]

Here, \( c_{kj} = (\nu_j T_{j,0}/m_{j,0})^{1/2} \) is the sound velocity \( [\gamma_j = 2(3) \text{ for the two(1)-dimensional perturbations}] \), \( v_{j0} = \gamma_j m_j n_{j0} \) and \( v_{j0} = \gamma_j m_j n_{j0} \).

The terms proportional to \( n_{j1} \) in equation (7) and \( n_{j1} \) in equation (9) describe the effect of perturbation of the collisional frequencies \( v_{j1} = \nu_{j0} (n_{j1}/n_{j0}) \) and \( v_{j1} = \nu_{j0} (n_{j1}/n_{j0}) \) due to number density perturbations. The index 1 denotes quantities of the first order of magnitude. The neutrals participate in the electromagnetic dynamics only as a result of collisional coupling with the charged species. On the left-hand sides of equations (7)–(9), we do not take into account the terms of the form \( v_{j1\perp} \cdot \nabla v_{j0\perp} \). These terms without neutral dynamics have been used by Nekrasov (2007, 2008a). The neutral dynamics has been included by Nekrasov (2008b, 2009b). In all specific cases that have been considered, the terms mentioned above were negligible. From the general expressions for perturbed velocities given in Nekrasov (2008b, 2009b), it can be seen that these Coriolis terms can be neglected under a sufficient condition \( \omega_{j,sc} \gg 2 \Omega_{j,sc} \), where \( \omega_{j,sc} \) in the present case are given below and \( \Omega_{j,sc} \) are the rotation frequencies of species in equilibrium.

The continuity equation (4) in the linear regime is

\[
\frac{\partial n_{j1\perp}}{\partial t} + \nu_{j0} \cdot \nabla n_{j1\perp} + n_{j0\perp} \cdot \nabla v_{j1\perp} = 0.
\]

We further apply the Fourier transform to equations (7)–(10), presuming perturbations of the form \( \exp(ikr + im\theta - i\omega t) \). Below, we find expressions for the perturbed velocities of neutrals and charged species.
4.1 Perturbed velocity and number density of neutrals

Using the $z$ component of equation (9), we find the Fourier amplitude of the induced longitudinal velocity of neutrals $v_{n1z}$, 

$$-i\omega_k v_{n1z} = \sum_j v_{nj} v_{j1z},$$

(11)

where

$$\omega_k = \omega_0 + i \nu + i \eta k^2,$$

$$\omega_0 = \omega - k \cdot v_0,$$

$$v_0 = \sum_j v_{nj},$$

$$k = \{k, \omega\} = \{k_z, k_\perp = m/r, \omega\} \quad \text{and} \quad k_\perp^2 = k_z^2 + k_\|^2.$$  

Here and below, we omit, for simplicity, the index 0 for $v_0^0$ and $v_0^i$.

From equations (9) and (10), we obtain the induced transverse velocity and induced perturbed number density of neutrals as

$$-i\omega_k v_{n1\perp} = \sum_j v_{nj} \eta j n  \left[ k \frac{\omega_0}{\omega_{n\perp}} - (v_0 - v_j) \right] + \sum_j v_{nj} v_{j1\perp},$$

(12)

$$-i\omega_k n_{n1\perp} = \sum_j v_{nj} \eta j n  \left[ k \frac{\omega_0}{\omega_{n\perp}} - (v_0 - v_j) \right],$$

(13)

where

$$\omega_{n\perp} = \omega_k - k_\perp^2 \eta_0, \quad \eta_0 = \frac{\epsilon_m}{\omega_0} - i \frac{1}{3} \mu_0.$$  

We see from equation (12) that the perturbed velocity of neutrals is induced by the perturbed number density and velocity of charged species. Equation (13) shows that the density perturbation of neutrals is defined by the density perturbations of charged species. When $\omega_{n\perp} \sim v_0$, we obtain $n_{n1\perp}/n_0 \sim \eta j n/n_0$. Note that to obtain the correct expressions (12) and (13) it is necessary to take into account perturbation on the collisional frequency of neutrals with charged species.

4.2 Longitudinal perturbed velocity of charged species

Let us now find the longitudinal perturbed velocity of charged species $v_{j1z}$. From equations (5) and (8) in the linear approximation and by using equation (11), we obtain

$$\omega_k \alpha_j v_{j1z} = \sum_j v_{nj} v_{j1z} = i \omega_k F_{j1z},$$

(14)

where

$$\omega_j = \omega_j + i \nu_j + i 4 \mu_j k^2,$$  

$$\alpha_j = \omega - k \cdot v_0,$$

$$F_{j1z} = \frac{q_j}{m_j} \frac{\omega_0}{\omega} E_{1z}.$$  

Equation (14) shows that because of the collisions of neutrals with charged species and charged species with neutrals, the charged species experience feedback on their perturbations. If neutrals collide with two charged species only, for example, with electrons and ions, the solutions of equation (14) for $e$ and $i$ have the form

$$D_e v_{e1z} = i \alpha_{e}\alpha_{j} F_{j1z} - i v_{en} v_{ne} F_{j1z},$$

$$D_i v_{i1z} = i \alpha_{e}\alpha_{j} F_{j1z} - i v_{in} v_{ne} F_{j1z},$$

where

$$\alpha_{e} = \omega_k \alpha_{j} + v_0 v_{nj},$$

$$D_e = \omega_k \omega_0 \alpha_{j} + \omega_k v_{en} v_{ne} + \omega_0 v_{ne} v_{ne}. $$

Substituting $F_{j1z}$ in equations (15), we obtain

$$D_j v_{j1z} = i (\alpha_{e} - n_0 \alpha_{j} E_{1z}),$$

(16)

where $n_0 = k_\| c/\omega$ and

$$a_{e} = \frac{q_j}{m_j} - v_{en} v_{ne} \frac{q_j}{m_1},$$

$$b_{e} = \frac{q_j}{m_j} \frac{v_{en}}{c} - v_{en} v_{ne} \frac{q_j}{m_1} \frac{v_0}{c},$$

$$a_{i} = \frac{q_j}{m_j} \frac{v_0}{c} - v_{en} v_{ne} \frac{q_j}{m_1},$$

$$b_{i} = \frac{q_j}{m_j} \frac{v_0}{c} - v_{en} v_{ne} \frac{q_j}{m_1} \frac{v_0}{c}. $$

If neutrals collide with ions and dust grains, the index $e$ in equations (15) and (16) must be substituted by the index $d$. 
4.3 Transverse perturbed velocity and perturbed number density of charged species

We now find the transverse perturbed velocity of charged species $v_{jlk}$. From equation (7), we obtain two equations for components $v_{jlk}$ and $v_{j0k}$:

\[-i\omega_{j}v_{jlk} = -i k_{r} c_{j}^{2} \frac{n_{j0k}}{n_{j0}} + \omega_{j}^{2} v_{j0k} + G_{jlk},\]

\[-i\omega_{j}v_{j0k} = -i k_{r} c_{j}^{2} \frac{n_{j0k}}{n_{j0}} - \omega_{j}^{2} v_{j0k} + G_{j0k}.\]  \hspace{1cm} (17)

Here, the following notations are introduced:

\[\omega_{j} = \omega_{j0} + i\nu_{j} + i\mu_{j} k_{+}^{2},\]

\[\omega_{j}^{2} = \omega_{j0}^{2} - \mu_{j}^{2} k_{+}^{2},\]

\[c_{j}^{2} = c_{j0}^{2} - i\alpha_{j}\mu_{j},\]

\[G_{jtr,tk} = F_{jtr,tk} + Q_{jtr,tk},\]

\[F_{jtr} = \frac{q_{i}}{m_{j}} \left[ E_{tr} + \frac{v_{j0}}{c} (n_{i} E_{10k} - n_{j} E_{10k}) \right],\]

\[Q_{jtr} = v_{j0} v_{n10k},\]

\[Q_{j0k} = v_{j0} v_{n10k} - v_{j0} n_{n10} (v_{j0} - v_{j0}).\]

In the expression for $F_{jtr}$, we have used equation (5). Solutions of equations (17) have the following form

\[D_{j+} v_{jlk} = i \omega_{j+} (1 - \delta_{j0}) G_{j10k} + i (i \omega_{j}^{2} + \omega_{j0}^{2} \delta_{j0}) G_{j01k},\]

\[D_{j+} v_{j0k} = i \omega_{j+} (1 - \delta_{j0}) G_{j10k} + i (-i \omega_{j}^{2} + \omega_{j0}^{2} \delta_{j0}) G_{j01k},\]  \hspace{1cm} (18)

where

\[D_{j+} = \omega_{j+}^{2} (1 - \delta_{j0}) - \omega_{j0}^{2},\]

and $\delta_{j0} = k_{r} c_{j}^{2} / \omega_{j0} \omega_{j+}, I, m = r, \theta, \delta_{j0} = \delta_{j\theta} + \delta_{j0 \theta}$. The number density perturbation, $n_{jlk} = n_{j0} k \cdot v_{jlk} / \omega_{j}$, is

\[\omega_{j} D_{j+} n_{j0k} \frac{n_{j1k}}{n_{j0}} = (i k_{r} \omega_{j+} + k_{0} \omega_{j0}) G_{j10k} + (i k_{0} \omega_{j+} - k_{0} \omega_{j0}) G_{j01k}.\]  \hspace{1cm} (19)

In the absence of viscosity, expressions (18) and (19) coincide with the corresponding expressions of Nekrasov (2009b).

5 AXISYMMETRIC PERTURBATIONS

Below, we consider axisymmetric perturbations with $k_{r} \neq 0$ and $k_{\theta} = 0$. In this case, equations (18) and (19) take the forms:

\[D_{j+} v_{jlk} = i \omega_{j+} \frac{q_{i}}{m_{j}} E_{jlk} + \frac{q_{i}}{m_{j}} \frac{k_{r} v_{j0}}{\omega} (\omega_{j0}^{2} - \omega_{j}^{2}) E_{j0k} + i \omega_{j+} v_{j0} v_{n10k} - v_{j0} \omega_{j}^{2} \frac{n_{n10}}{n_{j0}} (v_{j0} - v_{j0}),\]

\[D_{j+} v_{j0k} = \left[ i \omega_{j+} (1 - \delta_{j0}) + \omega_{j0}^{2} \frac{k_{r} v_{j0}}{\omega} \right] \frac{q_{i}}{m_{j}} E_{j0k} + \omega_{j0}^{2} \frac{q_{i}}{m_{j}} E_{j10k} + \omega_{j0}^{2} v_{j0} v_{n10k} + i \omega_{j+} v_{j0} (1 - \delta_{j0}) \frac{n_{n10}}{n_{j0}} (v_{j0} - v_{j0}).\]  \hspace{1cm} (20)

\[\frac{n_{j0k}}{n_{j0}} = \frac{k_{r} v_{j30k}}{\omega}.\]  \hspace{1cm} (21)

From equation (12), we can find components of the perturbed velocity of neutrals in the present case:

\[v_{n10k} = i \frac{1}{\omega_{j+}} \sum_{j} v_{j0} v_{j10k},\]

\[v_{n10k} = -i \frac{1}{\omega_{j+}} \sum_{j} v_{j0} \frac{n_{j10}}{n_{j0}} (v_{j0} - v_{j0}) + i \frac{1}{\omega_{j+}} \sum_{j} v_{j0} v_{j10k} \frac{n_{n10}}{n_{j0}}.\]  \hspace{1cm} (22)
Substituting these expressions into equations (20) and using equations (13) and (21), we derive equations that contain only the perturbed velocities of charged species:

\[
D_{j \perp} v_{j \perp \delta k} = i \omega_{j \perp} q_j \frac{m_j}{\omega_{j \perp}} E_{1 \perp k} + \frac{q_j}{m_j} \left( i \omega_{j \perp} \frac{k_e v_{j \perp}}{\omega_{j \perp}} - \omega_{j \perp}^* \right) E_{1 \parallel k} - \frac{\omega_{j \perp} v_{j \parallel}^*}{\omega_{j \perp}} \sum_i \nu_{i \perp} v_{i \perp \delta k}
\]

\[
- \frac{i \omega_{j \perp} \omega_{j \perp}^*}{\omega_{j \perp}} \sum_i \nu_{i \perp} v_{i \perp \delta k} + \frac{i \omega_{j \perp} \omega_{j \perp}^*}{\omega_{j \perp}} \sum_i \nu_{i \perp} v_{i \perp \delta k} \left[ \frac{k_e (v_{j 0} - v_{j \perp})}{\omega_{j \perp}} + \frac{k_e (v_{j 0} - v_{j \perp})}{\omega_{j \perp^*}} \right]
\]

\[
D_{j \perp} v_{j \parallel \delta k} = \left[ i \omega_{j \perp} (1 - \delta_{j \perp}^*) + \omega_{j \perp}^* \frac{k_e v_{j \perp}}{\omega_{j \perp}} \right] q_j \frac{m_j}{\omega_{j \perp}} E_{1 \parallel k} + \frac{\omega_{j \perp} v_{j \perp}^*}{\omega_{j \perp}} \sum_i \nu_{i \perp} v_{i \perp \delta k}
\]

\[
+ \frac{\omega_{j \perp} v_{j \perp}^*}{\omega_{j \perp}} (1 - \delta_{j \perp}) \sum_i \nu_{i \perp} v_{i \perp \delta k} \left[ \frac{k_e (v_{j 0} - v_{j 0})}{\omega_{j \perp}} + \frac{k_e (v_{j 0} - v_{j \perp})}{\omega_{j \perp^*}} \right]
\]

We find solutions of equations (23) for the case in which charged species are magnetized, that is, when the Lorentz force dominates inertia, thermal and drag forces. More specifically, we suppose that

\[
\frac{\omega_{j \perp} (1 - \delta_{j \perp})}{\omega_{j \perp}} \ll \frac{k_e v_{j \perp}}{\omega_{j \perp}} \ll \frac{\omega_{j \perp}}{\omega_{j \perp^*}}.
\]

It follows from these inequalities that the condition \( \omega_{j \perp}^* \gg \omega_{j \perp} (1 - \delta_{j \perp}) \) must be satisfied. From the latter, we obtain that \( \omega_{j \perp}^* \gg \omega_{j \perp} \) and \( \omega/\omega_{j \perp} \gg k_e^2 \rho_j^2 \), where \( \rho_j = \epsilon_{j \perp}/\omega_{j \perp} \) is the Larmor radius. Thus, the viscosity of charged species will be negligible for our consideration given below. Besides, \( \epsilon_{j \perp}^2 \sim 2 \epsilon_{j \parallel}^2 \) because we consider that \( \omega \ll v_{j \perp} \). Under conditions (24) the main perturbed velocities of charged species (denoted by the upper index 0) are the electric and magnetic (due to the Lorentz force) drifts:

\[
v_{j \perp \delta k}^0 = \frac{q_j}{m_j} \omega_{j \perp} E_{1 \perp k},
\]

\[
v_{j \parallel \delta k}^0 = - \frac{q_j}{m_j \omega_{j \perp}} \left( \omega_{j \perp} E_{1 \perp k} + k_e v_{j \perp} \frac{E_{1 \parallel k}}{\omega_{j \perp}} \right).
\]

Substituting expressions (25) into the collisional terms in equations (23), we obtain the following expressions for the transverse velocities of charged species:

\[
D_{j \perp} v_{j \perp \delta k} = \frac{q_j}{m_j} \sigma_{j \perp \delta k} E_{1 \perp k} + \frac{q_j}{m_j} \frac{1}{\omega_{j \perp}} \left[ -\sigma_{j \perp 0} + i \sigma_{j \perp \delta k} \frac{k_e v_{j \perp}}{\omega_{j \perp}} + i \sigma_{j \perp \delta k} \frac{k_e v_{j 0}}{\omega_{j \perp}} \right] E_{1 \parallel k},
\]

\[
D_{j \parallel} v_{j \parallel \delta k} = \frac{q_j}{m_j} \sigma_{j \parallel \delta k} E_{1 \parallel k} + \frac{q_j}{m_j} \frac{1}{\omega_{j \perp}} \left[ i \sigma_{j \parallel \delta k} - \omega_{j \perp}^* \delta_{j \perp} + \sigma_{j \parallel \delta k} \frac{k_e v_{j 0}}{\omega_{j \perp}} + \sigma_{j \parallel \delta k} \frac{k_e v_{j 0}}{\omega_{j \perp^*}} \right] E_{1 \parallel k},
\]

where the following notations are introduced:

\[
\sigma_{j \perp \delta k} = \sigma_{j \perp 0} + \frac{\nu_{j \perp}^* v_{j \perp}}{\omega_{j \perp}}, \quad \sigma_{j \parallel \delta k} = \sigma_{j \parallel 0} + \frac{\nu_{j \perp}^* v_{j \perp}}{\omega_{j \perp}}, \quad \sigma_{j \perp 0} = \sigma_{j \perp 0} + \frac{\nu_{j \perp}^* v_{j \perp}}{\omega_{j \perp}}.
\]

Below, we derive the dispersion relation and find its solutions.

### 6 DISPERSION RELATION

From the Faraday and Ampere laws (5) and (6), we find the following:

\[
j_{j \perp k} = 0, \quad n_j^2 E_{1 \perp k} = \frac{4 \pi i}{\omega} j_{j \perp k}, \quad n_j^2 E_{1 \perp k} = \frac{4 \pi i}{\omega} j_{j \perp k}.
\]

Here, \( j_{j \perp k} = \sum q_j n_j v_{j \perp k} \), \( j_{j \perp k} = \sum q_j n_j v_{j \perp k} \), and \( n_j^2 = k_e^2 c^2/\omega^2 \). We see that perturbations with polarizations of the electric field along and across the background magnetic field are split. We do not consider perturbations with longitudinal polarization because the current \( j_{j \perp} \) does not contain equilibrium velocities in the axisymmetric case (see equation 16). Using expressions (26), we find the transverse electric currents:

\[
\frac{4 \pi i}{\omega} j_{j \perp k} = -\varepsilon_{j \perp} E_{1 \perp k} - \varepsilon_{j \parallel} E_{1 \parallel k}, \quad \frac{4 \pi i}{\omega} j_{j \perp k} = \varepsilon_{j \parallel} E_{1 \perp k} + \varepsilon_{j \parallel} E_{1 \parallel k},
\]

\[
\quad \varepsilon_{j \perp} E_{1 \perp k} = -\varepsilon_{j \parallel} E_{1 \parallel k} + \varepsilon_{j \parallel} E_{1 \parallel k}.
\]
Here
\[ \varepsilon_{tr} = \sum_j \frac{\omega_j^2}{\omega D_{ij}} \sigma_{yj}, \quad \varepsilon_{th} = \sum_j \frac{\omega_j^2}{\omega D_{ij}} \left( i\sigma_{\mu j} + \sigma_{\nu j} \frac{k_\perp v_j}{\omega} + \sigma_{\iota j} \frac{k_r v_j}{\omega} \right), \]
\[ \varepsilon_{ho} = \sum_j \frac{\omega_j^2}{\omega D_{ij}} \left( i\sigma_{yj} - \sigma_{\mu j} \frac{k_r v_j}{\omega} \right), \]
\[ \varepsilon_{\iota 0} = \sum_j \frac{\omega_j^2}{\omega D_{ij}} \left[ -\sigma_{\mu j} \frac{k_r v_j}{\omega} + \omega_{\iota j} \delta_{yj} + \left( i\sigma_{\mu j} - \sigma_{\nu j} \frac{k_r v_j}{\omega} \right) \frac{k_r v_0}{\omega} \right]. \]
Here, \( \omega_0 = (4\pi n_0 k_B^2 / m) / m \) is the plasma frequency.

From equations (27) and (28), we obtain the dispersion relation:
\[ \varepsilon_{tr} n_t^2 = \varepsilon_{tr} \varepsilon_{\iota 0} - \varepsilon_{\iota 0} \varepsilon_{ho}. \]
Below, we find solutions of equation (29) in limiting cases.

### 6.1 Cold species

First, we consider the case in which neutrals and charged species are sufficiently cold, \( \omega^2 \gg k_\perp^2 c_s^2 \) and \( \omega v_m \gg k_r^2 c_s^2 \) (it is obvious that the former is the main condition). We also suppose that \( v_m \gg v_\perp \gg \omega \). Under these conditions, the values \( \sigma \) have a simple form:
\[ \sigma_{yj} = \sigma_{\mu j} = \sigma_{\nu j} = \sigma_{\iota j} = 0, \]
\[ \omega_{\iota j} = \omega_{\iota j} = 0, \]
and the main terms. Then the dispersion relation (29) takes the form:
\[ \omega^2 \sum_j \frac{\omega_j^2 v_m^2 v_\perp^2}{\omega_{\iota j}^2} = v_h \sum_j \frac{\omega_j^2 v_m^2}{\omega_{\iota j}^2} k_r^2 c_s^2 - \frac{1}{2} \sum_j \frac{\omega_j^2 v_m}{\omega_{\iota j}} \frac{\omega_j^2 v_\perp}{\omega_{\iota j}} \frac{k_r^2 (v_\perp - v_\parallel)^2}{\omega_{\iota j}^2}. \]

We see that the streaming instability is possible when the difference of the equilibrium velocities of charged and neutral species is sufficient to exceed the threshold defined by the first term on the right-hand side of equation (30).

Let us consider, for example, equation (30) for the electron–ion plasma (without dust grains). Then this equation will be
\[ \omega^2 = \frac{k_r^2 c_s^2}{m_n v_m} \left( m_e v_m^2 / m_n v_m^2 \right)^2, \]
where \( c_s = (B^2 / 4\pi n_e) / m_e \) is the Alfvén velocity. When obtaining this equation, we have used \( m_n v_m \gg m_e v_m \) (see, for example, Section 7). The first term on the right-hand side of equation (31) describes the magnetosonic waves in a weakly ionized plasma. The second term can generate the streaming instability of the hydrodynamic kind. This term does not change the order of the dispersion relation for perturbations under consideration that results in the appearance of the threshold for the absolute instability to develop (see also Nekrasov 2007).

### 6.2 Thermal species

In the case of thermal species, we suppose conditions \( \mu_k k_\perp^2 \gg \omega \) and \( k_\perp^2 c_s^2 \gg \omega v_m \) to be satisfied. Then we have that \( k_\perp^2 c_s^2 \gg \omega^2 \) (we do not consider the intermediate case for neutrals, \( \mu_k k_\perp^2 \ll \omega \ll k_s c_m \)). We also adopt, as above, that \( v_m \gg v_\perp \). However, the condition \( v_m \gg \omega \) is now not necessary. Then the values \( \sigma \) take the forms
\[ \sigma_{yj} = i k v_m \mu_k k_j^2 / \omega_{\iota j}, \quad \sigma_{\mu j} = \omega_{\iota j}, \quad \sigma_{\nu j} = i k v_m \omega_{\iota j}, \quad \sigma_{\iota j} = -i k v_m \omega_{\iota j}, \]
where \( k = v_n / (v_n + k_\perp^2 / \omega_{\iota j}) \) and \( d_j = k_\perp^2 c_s^2 / \omega_{\iota j} \). Substituting these values in \( \varepsilon_{tr, \iota} \) and \( \varepsilon_{tr, \iota} \), we find the expression \( \varepsilon_{tr, \iota} \varepsilon_{\iota 0} - \varepsilon_{\iota 0} \varepsilon_{ho} \):
\[ \varepsilon_{tr, \iota} = \varepsilon_{\iota 0} = \varepsilon_{ho} \varepsilon_{\iota 0} = \left( i k v_m \right) \sum_j \frac{\mu_k k_j^2}{\omega_{\iota j}} \frac{\lambda_j d_j + k_\perp^2 c_s^2 / \omega_{\iota j} \frac{\lambda_j d_j}{2k_\perp^2}}{2\omega^2}. \]

Here, \( \lambda_j = -\omega_j v_m^2 / \omega_{\iota j} \omega_{\iota j} \). When deriving equation (32), we have used \( \sum_j \omega_{\iota j}^2 \omega_{\iota j} / \omega D_{ij} = -i \sum_j \lambda_j d_j \) because of the condition of quasi-neutrality. The first term on the right-hand side of this equation is the main term, according to the current conditions, among other terms that are independent of background velocities. We see from equation (32) that the viscosity of neutrals plays an important role in the case under consideration.

The dispersion relation (29) takes the form:
\[ \omega^2 \left[ 2 \sum_j \frac{\omega_j^2 v_m^2}{\omega_{\iota j}^2} c_s^2 - \frac{1}{2} \sum_j \frac{\omega_j^2 v_m^2}{\omega_{\iota j}^2} v_\perp c_s^2 + \frac{1}{2} \sum_j \frac{\omega_j^2 v_m^2}{\omega_{\iota j}^2} v_\perp c_s^2 \right] = \frac{1}{2} \sum_j \frac{\omega_j^2 v_m^2}{\omega_{\iota j}^2} v_\perp c_s^2 (v_\perp - v_\parallel)^2. \]
Thus, the streaming instability exists for the sufficiently short wavelengths of perturbations when the viscosity of neutrals and thermal pressure of species play a role.
We now consider equation (33) for the electron–ion plasma (as above). In this case, we obtain
\[
\omega = \frac{m_e}{m_i} \sqrt{\frac{(v_{\|0} - v_{\|0})^2}{(c_{\text{Ai}}^2 + c_i^2)}}.
\]
(34)
where \(c_{\text{Ai}} = (B_0^2/4\pi n_e m_i)^{1/2}\) is the ion Alfvén velocity and \(c_i = \sqrt{\gamma_s(T_e + T_i)/m_i}\) is the ion sound velocity. We see from equation (34) that the growth rate \(\gamma = \text{Im}\omega\) can be sufficiently large in comparison to the rotation frequency in media where \(c_{\text{Ai}}^2 + c_i^2\) is not too large as compared to \((v_{\|0} - v_{\|0})^2\).

The instability described by equation (33) belongs to the class of dissipative instabilities because the dispersion relation contains the corresponding imaginary term, which is proportional to the collisional frequency. Without additional damping mechanisms, the threshold of this dissipative-streaming instability is absent.

7 DISCUSSION

Let us discuss some consequences that follow from the results obtained above. From equation (13), we see that the relative number density perturbation of neutrals is of the same order of that for charged species, if \(n_{hi} \sim n_e\). However, for \(k^2 c_{\text{in}}^2 \gg \omega n_{hi}\), a neutral fluid can be considered as incompressible. In the latter case, the perturbed radial velocity of neutrals in the axisymmetric perturbations is also negligible (see equation 22). At the same time, the perturbed azimuthal velocity of neutrals is of the order of that for charged species for small viscosity of neutrals, \(v_\phi \gg \mu_n k^2 \gamma \omega\), and is negligible for large viscosity of neutrals, \(v_\phi \ll \mu_n k^2 \gamma \omega\) (see equation 22). Thus, the neutrals are immobile in perturbations when their viscosity is large.

As a specific example, we consider the case of the dense regions of molecular cloud cores where the number density of neutrals is \(n_e \sim 10^6\) cm\(^{-3}\) and \(n_i/n_e \sim 10^{-7}\) (Caselli et al. 1998; Ruffle et al. 1998). For simplicity, we omit the index \(0\) by \(n_0\). The ratio of mass densities of dust grains, \(\rho_0 = m_n \rho_n\), and neutrals, \(\rho_0 = m_n \rho_n\), in the interstellar medium is of the order of \(10^{-2}\) (e.g. Abergel et al. 2005). From these relations, we obtain that \(n_\phi \ll n_i\) at \(n_\phi \gg 10^5 m_n\). For usually adopted \(m_0 = 2.33 m_p\), where \(m_0\) is the proton mass, and for \(\sigma = 1\) g cm\(^{-3}\), where \(\sigma_0\) is the density of grain material, we obtain that \(n_\phi \gg 10^5 n_v = 3.9 \times 10^{-19}\) g is satisfied at \(n_\phi \gg 4.53 \times 10^{-3}\) um, where \(n_\phi\) is the grain radius. The typical values of grain radius are \(r_\phi \sim 0.01–1\) um (Mendis & Rosenberg 1994; Wardle & Ng 1999). However, in spite of the fact that \(n_\phi \ll n_i\), the mass density of dust grains \(\rho_\phi \sim 7.77 \times 10^5\) \(\rho_i = m_0 n_0\).

Further, we take the magnetic field \(B_0 = 150\) \(\mu\)G and the radius and charge of dust grains \(r_\phi = 0.01\) um and \(q_\phi = q_e\) (we consider one typical type of dust grain). In this case, \(m_\phi = 4.19 \times 10^{-18}\) g and \(n_\phi = 0.93 \times 10^{-3}\) cm\(^{-3}\). Then, we obtain the following values for the plasma and cyclotron frequencies of charged species: \(\omega_{pe} = 5.64 \times 10^5\) s\(^{-1}\), \(\omega_{ce}(>0) = 2.64 \times 10^5\) s\(^{-1}\), \(\omega_{pe} = 24\) s\(^{-1}\), \(\omega_{ce} = 4.78 \times 10^2\) s\(^{-1}\). The rate coefficients for momentum transfer by elastic scattering of electrons and ions with neutrals are \(\sigma v_{\|0} = 4.5 \times 10^6\) s\(^{-1}\), \(\sigma v_{\|0} = 1.9 \times 10^8\) cm\(^{-3}\) s\(^{-1}\), and \(\sigma v_{\parallel0} = 4.5 \times 10^6\) s\(^{-1}\), \(\sigma v_{\|0} = 1.9 \times 10^8\) cm\(^{-3}\) s\(^{-1}\) (Draine, Robber & Dalgarno 1983). We take \(T_0 = 300\) K. Then we obtain \(v_{\|0} = 1.42 \times 10^3\) s\(^{-1}\), \(v_{\|0} = 1.37 \times 10^3\) s\(^{-1}\). The collisional frequency of dust grains with neutrals has the form \(v_{\|0} \approx 6.7 \mu n_\phi r_\phi^2 v_{\|0}/m_\phi\) (below, we take \(T_0 \approx T_\phi\)) (e.g. Wardle & Ng 1999). Using the parameters given above, we obtain \(v_{\|0} = 6.24 \times 10^3\) s\(^{-1}\). The collisional frequency of neutrals with charged species is \(v_{\|0} = 6.24 \times 10^1\) s\(^{-1}\).

Under the current conditions, the values \(\omega_{pe}^2 \gamma v_{\|0}/\omega v_{\|0}^2\), which are contained in equations (30) and (33), are \(\omega_{ce}^2 \gamma v_{\|0}/\omega v_{\|0}^2 = 6.4 \times 10^{-3}\) s\(^{-1}\), \(\omega_{pe}^2 \gamma v_{\|0}/\omega v_{\|0}^2 = 3.44\) s\(^{-1}\), and \(\omega_{ce}^2 \gamma v_{\|0}/\omega v_{\|0}^2 = 120.8\) s\(^{-1}\). Thus, we can write equation (30) in the form
\[
\frac{\omega_{pe}^2 \gamma v_{\|0}/\omega v_{\|0}^2}{\omega_{ce}^2 \gamma v_{\|0}/\omega v_{\|0}^2} = 2.24 \times 10^{-3}\text{s}^{-1}.
\]
where \(c_{\text{Ad}} = c_{\text{Ad}}/\omega v_{\|0}\) is the dust Alfvén velocity. Supposing that \(|v_{\|0} - v_{\|0}| \gg (v_{\|0}/v_{\|0})^{1/2} c_{\text{Ai}}\) (the sign || denotes an absolute value), the growth rate of instability \(\gamma\) will be equal to
\[
\gamma = \left(\frac{v_{\|0}^2}{v_{\|0}^2}ight)^{1/2} c_{\text{Ad}}^2 k_\| |v_{\|0} - v_{\|0}| |.
\]
(35)
This solution exists if
\[
\frac{v_{\|0}^2}{c_{\text{Ad}}^2 (v_{\|0}^2)} \left|\frac{v_{\|0} - v_{\|0}}{c_{\text{Ad}}^2 (v_{\|0}^2)}\right| > \frac{c_{\text{Ai}}^2 (v_{\|0}^2)}{c_{\text{Ad}}^2 (v_{\|0}^2)}.
\]
From these inequalities, it follows that \(\gamma = 2\pi c_{\text{Ad}}/v_{\|0}\), where \(\lambda_r = 2\pi/k_{\perp}\) is the wavelength of perturbations. For the parameters given above, we obtain \(c_{\text{Ad}} = 1.73\) km s\(^{-1}\), \(c_{\text{Ai}} = 600\) km s\(^{-1}\), \(c_{\text{Ad}} = 6.8\) km s\(^{-1}\), \(|v_{\|0} - v_{\|0}| > 10\) km s\(^{-1}\), \(|\lambda_r| > 1.73 \times 10^{10}\) km. The estimation of the growth rate (35) at \(|v_{\|0} - v_{\|0}| = 15\) km s\(^{-1}\) and \(\lambda_r = 5 \times 10^{10}\) km is \(\gamma = 3 \times 10^{-10}\) s\(^{-1}\). The condition of magnetization (24) is satisfied for ions as well as for dust grains. The case of unmagnetized dust grains has been considered in Nekrasov (2009a).

For thermal species and/or short wavelength perturbations, \(\mu_n k_{\perp}^2 \gg \omega\) and \(k_{\perp}^2 c_{\text{in}}^2 \gg \omega v_{\|0}\), we consider the electron–ion plasma because conditions \(k_{\perp}^2 c_{\text{in}}^2 \gg \omega v_{\|0}\) and \(k_{\perp} v_{\|0}/\omega \ll c_{\text{Ad}}/v_{\|0}\) (see inequalities 24) are incompatible for solution (33) and the parameters given above. In the absence of dust grains, equation (33) has the form (34). The condition \(k_{\perp}^2 c_{\text{in}}^2 \gg \gamma v_{\|0}\) and the right inequality (24) for ions \(k_{\perp} v_{\|0}/\gamma \ll c_{\text{Ad}}/v_{\|0}\) (for electrons these conditions are weaker) are compatible if
\[
\frac{m_e v_{\|0}}{m_i v_{\|0}} < \frac{\beta}{2(1+\beta)} \frac{\sigma v_{\|0}^2}{v_{\|0}^2}
\]
where $\beta = 12\pi n_i (T_e + T_i) / B_0^2$. For the number densities $n_i$ and $n_n$ used above, this inequality is also not satisfied. At $\beta \ll 1$ it can be written in the form,

$$\frac{\omega_{ci}}{\nu_{en}} c \gg 1.41 \left( \frac{m_i v_n}{m_e v_m} \right)^{1/2}.$$

For $n_n \sim 10^4 \text{ cm}^{-3}$ and $n_i / n_n \sim 10^{-6}$, this condition is satisfied.

We have excluded drift waves in our study. This can be done if the frequency spectra of these waves and perturbations under consideration are different. As is known, the frequency of drift waves, for example, in the electron–ion plasma, is equal to $\omega = (T_e/m_i \omega_{ci} n_i) |k \times \nabla n_i|$. For axisymmetric perturbations, this frequency is equal to zero, if the number density is uniform in the azimuthal direction. If $\partial n_i / n_i \partial \theta \sim L_\theta^{-1} \neq 0$ in any region, then $\omega \sim \omega_{ci} k L_\theta^{-1}$. Comparing this expression, for example, with the growth rate (34), we see that under the following condition

$$\left( \frac{\nu_{en}}{\nu_{ci}} \right)^{1/2} \frac{|\nu_{ci} - \nu_{en}|}{c_{Ai}} \gg (k \cdot L_\theta)^{1/2} \frac{\rho_i}{L_\theta}$$

($\beta \ll 1$), the growth rate is larger than the drift frequency. The last condition can easily be satisfied because $\rho_i$ is much smaller than $L_\theta$.

8 CONCLUSION

In this paper, we have studied electromagnetic streaming instabilities in the thermal viscous regions of rotating astrophysical objects, such as magnetized accretion discs, molecular clouds, their cores and elephant trunks. However, the results obtained can be applied to any regions of interstellar medium, where different background velocities between charged species can arise.

We have considered a weakly ionized multicomponent plasma consisting of electrons, ions, dust grains and neutrals. The cyclotron frequencies of charged species are presumed to be larger than their collisional frequencies with neutrals. The axisymmetric perturbations across the background magnetic field have been investigated. We have taken into account the effect of perturbation of collisional frequencies due to density perturbations of species. New compressible instabilities generated by the different equilibrium velocities of species have been found in the cold and thermal limits either when the viscosity of neutrals can be neglected or when it is important. For the perturbations considered, the viscosity of magnetized charged species is negligible.

In dense accretion discs, the ions are unmagnetized while the electrons remain magnetized (e.g. Wardle & Ng 1999). In this case, the viscosity of ions can play the same role as that for neutrals. However, our present model does not describe such objects.

The electromagnetic streaming instabilities studied in this paper can be a source of turbulence in weakly ionized magnetized astrophysical objects in regions where thermal and viscous effects can play a role.

ACKNOWLEDGMENT

The insightful and constructive comments and suggestions of the anonymous referee are gratefully acknowledged.

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