Charmed Baryon Weak Decays with SU(3) Flavor Symmetry

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(Dated: November 15, 2018)

Abstract

We study the semileptonic and non-leptonic charmed baryon decays with $SU(3)$ flavor symmetry, where the charmed baryons can be $B_c = (\Xi_c^0, \Xi_c^+, \Lambda_c^+)$, $B'_c = (\Sigma_c^{(++,+,0)}, \Xi_c^{(+,0)}, \Omega_c^0)$, $B_{cc} = (\Xi_{cc}^{++}, \Xi_{cc}^+, \Lambda_{cc}^+)$, or $B_{ccc} = \Omega_{ccc}^{++}$. With $B_n$ denoted as the baryon octet (decuplet), we find that the $B_c \rightarrow B_n' \ell^+\nu_\ell$ decays are forbidden, while the $\Omega_c^0 \rightarrow \Omega^{-\ell^+}\nu_\ell$, $\Omega_{cc}^+ \rightarrow \Omega_{ccc}^{0\ell^+}\nu_\ell$, and $\Omega_{ccc}^{++} \rightarrow \Omega_{ccc}^{+\ell^+}\nu_\ell$ decays are the only existing Cabibbo-allowed modes for $B'_c \rightarrow B'_n' \ell^+\nu_\ell$, $B_{cc} \rightarrow B'_{cc} \ell^+\nu_\ell$, and $B_{ccc} \rightarrow B'_{ccc} \ell^+\nu_\ell$, respectively. We predict the rarely studied $B_c \rightarrow B_{n}^{(l)} M$ decays, such as $\mathcal{B}(\Xi_c^0 \rightarrow \Lambda^0 \bar{K}^0, \Xi_c^+ \rightarrow \Xi^0 \pi^+) = (8.3 \pm 0.9, 8.0 \pm 4.1) \times 10^{-3}$ and $\mathcal{B}(\Lambda_c^+ \rightarrow \Delta^{++}\pi^-, \Xi_c^0 \rightarrow \Omega^- K^+) = (5.5 \pm 1.3, 4.8 \pm 0.5) \times 10^{-3}$. For the observation, the doubly and triply charmed baryon decays of $\Omega_{cc}^+ \rightarrow \Xi_{cc}^{+} \bar{K}^0$, $\Xi_{cc}^{++} \rightarrow (\Xi_c^+ \pi^+, \Sigma_c^{++} \bar{K}^0)$, and $\Omega_{ccc}^{++} \rightarrow (\Xi_{ccc}^{++} \bar{K}^0, \Omega_{ccc}^{+\pi^+}, \Xi_c^+ D^+)$ are the favored Cabibbo-allowed decays, which are accessible to the BESIII and LHCb experiments.
I. INTRODUCTION

Since 2016, the BESIII Collaboration has richly reanalyzed the singly charmed baryon decays, such as \( \Lambda_c^+(2286) \rightarrow pK^0, \Lambda\pi^+, \Sigma^+\pi^0 \) and \( \Sigma^0\pi^+ \) \([1, 2]\), with higher precision. In addition, the Cabibbo-suppressed decays are measured for the first time, where \( B(\Lambda_c^+(2286) \rightarrow p\eta) = (1.24 \pm 0.28 \pm 0.10) \times 10^{-3} \) and \( B(\Lambda_c^+(2286) \rightarrow p\pi^0) < 3 \times 10^{-4} \) (90% C.L.) \([3]\). On the other hand, the LHCb Collaboration has recently observed the decay of \( \Xi^{++}_{cc} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+ \) \([4]\), which is used to identify one of the doubly charmed baryon triplet, \( (\Xi^{++}_{cc}, \Xi^{+}_{cc}, \Omega^{+}_{cc}) \), consisting of \( ccq \) with \( q = (u, d, s) \), respectively. These recent developments suggest the possible measurements for the spectroscopy of the singly, doubly and triply charmed baryons in the near future, despite the not-yet-observed triply charmed baryon ones. Moreover, the charmed baryon formations and their decays would reveal the underlying QCD effects, which helps us to understand the recent discoveries of the pentaquark and XYZ states that contain the charm quarks also \([5-9]\).

The spectroscopy of the charmed baryons is built by measuring their decay modes. For example, the existence of the \( \Xi^{+}_{cc} \) state was once reported by the SELEX collaboration \([10, 11]\), but not confirmed by the other experiments \([12-15]\). Until very recently, LHCb has eventually found the doubly charmed \( \Xi^{++}_{cc} \) state at a mass of \( (3621.40 \pm 0.72 \pm 0.27 \pm 0.14) \) MeV \([4]\), which is reconstructed as the two-body \( \Xi^{++}_{cc} \rightarrow \Sigma^{++}_{cc}(2455) \bar{K}^0 \) decay with the resonant strong decays of \( \Sigma^{++}_{cc} \rightarrow \Lambda^+_c \pi^+ \) and \( \bar{K}^0 \rightarrow K^- \pi^+ \), as shown by the theoretical calculation \([16]\). Note that the corresponding decay lifetime has not been determined yet. It should be interesting to perform a full exploration of all possible charmed baryon decays, and single out the suitable decay channels for the measurements.

To study the charmed baryon decays, since the most often used factorization approach in the b-hadron decays \([17-19]\) has been demonstrated not to work for the two-body \( \mathbf{B}_c \rightarrow \mathbf{B}_n M \) decays \([20, 21]\), where \( \mathbf{B}_n(c) \) and \( M \) are denoted as the (charmed) baryon and meson, respectively, one has to compute the sub-leading-order contributions or the final state interactions to take into account the non-factorizable effects \([22-26]\), whereas the QCD-based models in the \( \mathbf{B}_c \) decays are not available yet. On the other hand, with the advantage of avoiding the detailed dynamics of QCD, the approach with \( SU(3) \) flavor \((SU(3)_f)\) symmetry can relate decay modes in the \( b \) and \( c \)-hadron decays \([21, 27-36]\), where the \( SU(3) \) amplitudes receive non-perturbative and non-factorizable effects, despite the unknown sources. In this paper,
in terms of $SU(3)_f$ symmetry, we will examine the semileptonic and non-leptonic two-body $B_c$ decays, search for decay modes accessible to experiment, and establish the spectroscopy of the charmed baryon states. The analysis will explore the consequences of neglecting a decay amplitude expected to be small.

Our paper is organized as follows. In Sec. II, we develop the formalism, where the Hamiltonians, (charmed) baryon and meson states are presented in the irreducible forms under $SU(3)_f$ symmetry. The amplitudes of the semileptonic and non-leptonic decay modes are given in Secs. III and IV, respectively. In Sec. V, we discuss all possible decays and show the relationships among them as well as some numerical results, which are relevant to the experiments. We conclude in Sec. VI.

II. FORMALISM

A. The effective Hamiltonian

For the semileptonic $c \to q\ell^+\nu_\ell$ transition with $q = (d, s)$, the effective Hamiltonian at the quark-level is presented as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cq}(\bar{q}c)_{V-A} (\bar{u}_\nu v_\ell)_{V-A},$$

where $G_F$ is the Fermi constant and $V_{ij}$ are the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix elements, while $(\bar{q}_1 q_2)_{V-A}$ and $(\bar{u}_\nu v_\ell)_{V-A}$ stand for $\bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$ and $\bar{u}_\nu \gamma_\mu (1-\gamma_5) v_\ell$, respectively. For the non-leptonic $c \to su\bar{d}$, $c \to uq\bar{q}$ and $c \to du\bar{s}$ transitions, one has the effective Hamiltonian to be

$$\tilde{\mathcal{H}}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \{ V_{cs} V_{ud} (c_+ O_+ + c_- O_-) + V_{cd} V_{ud} (c_+ \hat{O}_+ + c_- \hat{O}_-) + V_{cd} V_{us} (c_+ O'_+ + c_- O'_-) \},$$

with the four-quark operators $O^{(q)}_\pm$ and $\hat{O}_\pm \equiv O^q_\pm - O^q_\pm$ written as

$$O_\pm = \frac{1}{2} [(\bar{u}d)_{V-A}(\bar{c}s)_{V-A} \pm (\bar{s}d)_{V-A}(\bar{c}u)_{V-A}],$$

$$O^q_\pm = \frac{1}{2} [(\bar{u}q)_{V-A}(\bar{q}c)_{V-A} \pm (\bar{q}q)_{V-A}(\bar{u}c)_{V-A}],$$

$$O'_\pm = \frac{1}{2} [ (\bar{u}s)_{V-A}(\bar{d}c)_{V-A} \pm (\bar{d}s)_{V-A}(\bar{u}c)_{V-A}],$$

where $V_{cd} V_{ud} = -V_{cs} V_{us}$ has been used. According to $|V_{cd} V_{ud}|/|V_{cs} V_{ud}| = \sin \theta_c$ and $|V_{cd} V_{us}|/|V_{cs} V_{ud}| = \sin^2 \theta_c$ with $\theta_c$ known as the Cabibbo angle, the operators for the
\(c \rightarrow su\bar{d}, \ c \rightarrow uq\bar{q} \) and \(c \rightarrow du\bar{s}\) transitions represent the Cabibbo-allowed, Cabibbo-suppressed and doubly Cabibbo-suppressed processes, respectively. As the scale-dependent Wilson coefficients, \(c_\pm\) are calculated to be \((c_+, c_-) = (0.76, 1.78)\) at the scale \(\mu = 1\) GeV in the NDR scheme [37, 38].

Based on SU\((3)_f\) symmetry, the Lorentz-Dirac structures for the four-quark operators in Eq. (3) are not explicitly expressed with the quark index \(q_i = (u, d, s)\) as an SU\((3)_f\) triplet, such that in Eq. (1) the quark-current side of \((\bar{q}_c)\) forms an anti-triplet \((\bar{3})\), which leads to

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} H(\bar{3})(\bar{u}_c v_\ell)_{V-A},
\]

with the tensor notation of \(H(\bar{3}) = (0, V_{cd}, V_{cs})\), where \(V_{cs} = 1\) and \(V_{cd} = -\sin \theta_c\). For the \(c \rightarrow su\bar{d}\) and \(c \rightarrow uq\bar{q}\) transitions in Eq. (2), the four-quark operators can be presented as \((\bar{q}_i q^k)(\bar{q}_j c)\), with \(\bar{q}_i q^k\bar{q}_j\) being decomposed as \(3 \times 3 = 3 + 3' + 6 + 15\). Consequently, the operators \(O_{-,-}^{(\ell)}\) (\(\hat{O}_{-,-}^{(\ell)}\)) fall into the irreducible representations of \(O_{6,15}^{(\ell)}\) (\(\hat{O}_{6,15}^{(\ell)}\)), given by

\[
\begin{align*}
O_6 &= \frac{1}{2}(\bar{u}d\bar{s} - \bar{s}d\bar{u})c, \\
O_{15} &= \frac{1}{2}(\bar{u}d\bar{s} + \bar{s}d\bar{u})c, \\
\hat{O}_6 &= \frac{1}{2}(\bar{u}d\bar{d} - \bar{d}d\bar{u} + \bar{s}s\bar{u} - \bar{u}s\bar{s})c, \\
\hat{O}_{15} &= \frac{1}{2}(\bar{u}d\bar{d} + \bar{d}d\bar{u} - \bar{s}s\bar{u} - \bar{u}s\bar{s})c, \\
O'_6 &= \frac{1}{2}(\bar{u}s\bar{d} - \bar{d}s\bar{u})c, \\
O'_{15} &= \frac{1}{2}(\bar{u}s\bar{d} + \bar{d}s\bar{u})c,
\end{align*}
\]

which are in accordance with the tensor notations of \(H(6)_{ij}\) and \(H(15)_{jk}^{15}\), with the non-zero entries:

\[
\begin{align*}
H_{22}(6) &= 2, \quad H_{23}(6) = H_{32}(6) = -2s_c, \quad H_{33}(6) = 2s_c^2, \\
H_2^{13}(15) &= H_2^{31}(15) = 1, \\
H_2^{12}(15) &= H_2^{21}(15) = -H_3^{13}(15) = -H_3^{31}(15) = s_c, \\
H_3^{12}(15) &= H_3^{21}(15) = -s_c^2,
\end{align*}
\]

respectively, with \(s_c \equiv \sin \theta_c\) to include the CKM matrix elements into the tensor notations. Accordingly, the effective Hamiltonian in Eq. (2) is transformed as

\[
\hat{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} [c_- H(6) + c_+ H(15)],
\]
where the contribution of $H(6)$ to the decay branching ratio can be 5.5 times larger than that of $H(\overline{15})$ due to $(c_-/c_+)^2 \simeq 5.5$. The simplifications resulting from the neglect of the 15-plet will be investigated below.

**B. The (charmed) baryon states and mesons**

For the singly charmed baryon states, which consist of $q_1 q_2 c$ with $q_1 q_2$ being decomposed as the irreducible representation of $3 \times 3 = \overline{3} + 6$, there exist the charmed baryon anti-triplet and sextet, given by

$$B_c = (\Xi^0_c, \Xi^+_c, \Lambda^+_c),$$

$$B'_c = \begin{pmatrix} \Sigma^{++}_c & \frac{1}{\sqrt{2}} \Sigma^+_c & \frac{1}{\sqrt{2}} \Xi^{++}_c \\ \frac{1}{\sqrt{2}} \Sigma^+_c & \Sigma^0_c & \frac{1}{\sqrt{2}} \Xi^0_c \\ \frac{1}{\sqrt{2}} \Xi^{++}_c & \frac{1}{\sqrt{2}} \Xi^0_c & \Omega^0_c \end{pmatrix},$$

respectively. Similarly, $B_{cc}$ and $B_{ccc}$ to consist of $q_{cc}$ and $ccc$ represent the doubly charmed baryon triplet and triply charmed baryon singlet, given by

$$B_{cc} = (\Xi^{++}_{cc}, \Xi^+_{cc}, \Omega^+_{cc}),$$

$$B_{ccc} = \Omega^{++}_{ccc},$$

respectively. The final states, $B_n$, $M$ and $M_c$, being the lowest-lying baryon octet, meson octet, and the charmed meson anti-triplet, are written as

$$B_n = \begin{pmatrix} \frac{1}{\sqrt{6}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}} \Lambda \end{pmatrix},$$

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^- & K^- \\ \pi^+ & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^+ & K^0 & -\sqrt{\frac{2}{3}} \eta \end{pmatrix},$$

$$M_c = (D^0, D^+, D^+_s),$$

respectively. We note that in our calculations, $\eta$ is only considered as a member of an octet, without treating it as an octet-singlet mixture to simplify the analysis. In addition, we have
the baryon decuplet, given by

$$B'_n = \frac{1}{\sqrt{3}} \left( \begin{array}{ccc} \sqrt{3} \Delta^{++} & \Delta^+ & \Sigma^+ \\ \Delta^+ & \Delta^0 & \Sigma^0 \\ \Sigma^+ & \Sigma^0 & \Xi^0 \end{array} \right),$$

$$\begin{array}{ccc} \Delta^+ & \Delta^0 & \Sigma^0 \\ \Delta^0 & \sqrt{3} \Delta^- & \Sigma^- \\ \Sigma^- & \Xi^0 & \Xi^- \end{array} \right),$$

$$\begin{array}{ccc} \Sigma^+ & \Sigma^0 & \Xi^0 \\ \Sigma^0 & \Sigma^- & \Xi^- \\ \Xi^0 & \Xi^- & \sqrt{3} \Omega^- \end{array} \right).$$

III. SEMILEPTONIC CHARMED BARYON DECAYS

In this section, we present the amplitudes for the semileptonic \( B_c^{(i)} \rightarrow B_n^{(i)} \ell^+ \nu_\ell \), \( B_{cc} \rightarrow B_{cc} \ell^+ \nu_\ell \), and \( B_{ccc} \rightarrow B_{ccc} \ell^+ \nu_\ell \) decays under \( SU(3)_f \) symmetry. In terms of \( H_{eff} \) in Eq. (4), the amplitudes of \( A(B_c^{(i)} \rightarrow B_n^{(i)} \ell^+ \nu_\ell) = \langle B_n^{(i)} \ell^+ \nu_\ell | H_{eff} | B_c^{(i)} \rangle \) are derived as \( A(B_c^{(i)} \rightarrow B_n^{(i)} \ell^+ \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cq} T(B_c^{(i)} \rightarrow B_n^{(i)})(\bar{u}_\nu \ell_\nu)_{V-A} \), where \( T(B_c^{(i)} \rightarrow B_n^{(i)}) \) are given by

$$T(B_c \rightarrow B_n) = \alpha_1 (B_n^j H^j(\bar{3})(B_c)_i),$$

$$T(B'_c \rightarrow B_n) = \alpha_2 (B_n^j H^j(\bar{3})(B'_c)_{jk} \epsilon_{dk},$$

$$T(B'_c \rightarrow B'_n) = \alpha_3 (B'_n^{ijk}) H^i(\bar{3})(B'_c)_{jk},$$

with \( SU(3) \) parameters \( \alpha_i \) \( (i = 1, 2, 3) \) associated with the \( B_c^{(i)} \rightarrow B_n^{(i)} \ell^+ \nu_\ell \) decays. Note that \( T(B_c \rightarrow B'_n) \) disappears in Eq. (12). This is due to the fact that the symmetric baryon decuplet \( (B'_n)_{ijk} \) and the anti-symmetric \( \epsilon_{ijk} \) coexist in the forms of \( (B'_n)_{ijk} H^i(\bar{3})(B_c)_{\ell^{ijk}} \) and \( (B'_n)_{ijk} H^i(\bar{3})(B_c)_{\ell^{ijk}} \), which identically vanish [32]. We also obtain the \( T \) amplitudes of the \( B_{cc} \rightarrow B_{cc}^{(i)} \ell^+ \nu_\ell \) and \( B_{ccc} \rightarrow B_{ccc} \ell^+ \nu_\ell \) decays, given by

$$T(B_{cc} \rightarrow B_{cc}) = \beta_1 H^j_q(\bar{3})(B_c)_i ^k \epsilon_{ijk}(B_{cc})^i,$$

$$T(B_{cc} \rightarrow B_{cc}') = \beta_2 H^j_q(\bar{3})(B'_c)_{ij}(B_{cc})^i,$$

$$T(B_{ccc} \rightarrow B_{cc}) = \delta_1 (B_{cc})_i H^j_q(\bar{3}),$$

with \( SU(3) \) parameters \( \beta_{1,2} \) and \( \delta_1 \), where the subscript \( q \) refers to the \( d \) or \( s \) quark in \( B_{cc} \). It is interesting to note that, for \( T(B_{ccc} \rightarrow B_{cc}) \), \( B_{ccc} = \Omega^{++}_{ccc} \) as the charmed baryon singlet has no \( SU(3) \) flavor index to connect to the final states and Hamiltonian. The full expanded \( T \) amplitudes in Eqs. (12) and (13), corresponding to the semileptonic charmed baryon decays, can be found in Table [1].
IV. NON-LEPTONIC CHARMED BARYON DECAYS

To proceed, we start with the non-leptonic charmed baryon decays, in which the charmed baryons are the singly, doubly, and triply charmed baryon states, $B_c = (B_c^{(l)}, B_{cc}, B_{ccc})$, respectively.

A. The two-body $B_c^{(l)} \rightarrow B_n^{(l)} M$ decays

In terms of $SU(3)_f$ symmetry, the amplitudes of the singly charmed $B_c^{(l)} \rightarrow B_n^{(l)} M$ decays in the irreducible forms are derived as

$$A(B_c^{(l)} \rightarrow B_n^{(l)} M) = (B_n^{(l)} M | \bar{H}_{eff} | B_c^{(l)}) = \frac{G_F}{\sqrt{2}} T(B_c^{(l)} \rightarrow B_n^{(l)} M), \quad (14)$$

where

$$T(B_c \rightarrow B_n M) =$$

$$a_1 H_{ij}(6) T^{ik}(B_i)_k^j(M)_i^j + a_2 H_{ij}(6) T^{ik}(M)_k^j(B_i)_l^j + a_3 H_{ij}(6) (B_i)_k^j(M) T^{kl}$$

$$+ a_4 (B_i)_k^j(M)_l^j H(\bar{T}^5)_j^l(B_c)_j + a_5 (B_i)_l^j(M)_j^i H(\bar{T}^5)_i^j(B_c)_k + a_6 (B_i)_l^j(M)_j^i H(\bar{T}^5)_i^j(B_c)_k,$$  \quad (15)

$$T(B_c' \rightarrow B_n' M) =$$

$$a_8 (B_i')_l^j H_{nm}(6)(M)_o^{jm} \epsilon^{kmn} + a_9 (B_i')_l^j M_j^i H(\bar{T}^5)_m^i (B_c)_n \epsilon^{kln}$$

$$+ a_{10} (B_i')_l^j H_{ijkl} (M)_m^i H(\bar{T}^5)_m^i (B_c)_n \epsilon^{kln}, \quad (16)$$

$$T(B_c' \rightarrow B_n M) =$$

$$a_{12} H_{ij}(6)(B_i')_l^j + a_{13} H_{ij}(6)(B_i')_k^l (B_n)_l^j + a_{14} H_{ij}(6)(B_i')_k^l (B_n)_l^j$$

$$+ a_{15} H_{ij}(6)(B_i')_k^l (B_n)_l^j + a_{16} (B_i')_l^j (M)_m^i H(\bar{T}^5)_m^i (B_c')_m^i \epsilon^{kmn}$$

$$+ a_{17} (B_n)_l^j (M)_m^i H(\bar{T}^5)_m^i (B_c')_m^i \epsilon^{kmn} + a_{18} (B_n)_l^j (M)_m^i H(\bar{T}^5)_m^i (B_c')_m^i \epsilon^{kimn}$$

$$+ a_{19} (B_n)_l^j (M)_m^i H(\bar{T}^5)_m^i (B_c')_m^i \epsilon^{kimn} + a_{20} (B_n)_l^j (M)_m^i H(\bar{T}^5)_m^i (B_c')_m^i \epsilon^{kimn} \epsilon_{ijk}, \quad (17)$$
and

\[ T(B'_c \rightarrow B'_n M) = \]
\[ a_{21}(B'_n)_{ikm}(M)^i_n H_{ij}(6)(B'_c)^{jk}T + a_{22}(B'_n)_{klm}(M)^l_n H_{ij}(6)(B'_c)^{jm}T \]
\[ + a_{23}(B'_n)_{ijk}(M)^ij_H(T_{ij})_m^k(B'_c)^{im} + a_{24}(B'_n)_{ijk}(M)^k_H(T_{ij})_l^m(B'_c)^{lm} \]
\[ + a_{25}(B'_n)_{ijk}(M)^ij_H(T_{ij})_m^k(B'_c)^{km} + a_{26}(B'_n)_{ijk}(M)^j_H(T_{ij})_l^m(B'_c)^{lm}, \]  

with \( T^{ij} \equiv \epsilon^{ijk}(B'_c)_{k} \). Note that the Wilson coefficients \( c_\pm \) have been absorbed in \( SU(3) \) parameters \( a_i \), which can relate all possible decay modes. The full expansions of the \( T \) amplitudes in Eqs. (15)-[18] are given in Tables [III-VII].

B. The doubly charmed \( B_{cc} \rightarrow B_{n}^{(i)} M_c \) and \( B_{cc} \rightarrow B_{c}^{(i)} M \) decays

In the doubly charmed baryon decays, the \( T \) amplitudes of \( B_{cc} \rightarrow B_{n} M_c \) and \( B_{cc} \rightarrow B_{c} M \) are written as

\[ T(B_{cc} \rightarrow B_{n} M_c) = \]
\[ b_{1}(B_{cc})^{ij}(M_c)^j_i H_{ij}(6) + b_{2}(B_{cc})^{ij}(M_c)^j_i H_{jk}(6) \]
\[ + b_{3}(B_{cc})^{ij}(B_{n})^{j}_i H(T_{ij})^m_i H_{jk}(6) + b_{4}(B_{cc})^{ij}(M_c)^j_i H(T_{ij})^m_i \epsilon_{ijk}, \]

and

\[ T(B_{cc} \rightarrow B_{c} M) = \]
\[ b_{5}(B_{cc})^{ij}(B_{c})^{j}_i H_{ij}(6) + b_{6}(B_{cc})^{ij}(M_c)^j_i H_{jk}(6) \]
\[ + b_{7}(B_{cc})^{ij} H(T_{ij})^{jk}(B_{c})^{m}_j H(T_{ij})^l_i + b_{8}(B_{cc})^{ij} H(T_{ij})^{jk}(B_{c})^{m}_j H(T_{ij})^l_i \epsilon_{ijk}, \]

where \( B_{n} \) and \( B_{c}^{i} \) represent the octet and decuplet of the the baryon states in Eqs. (10) and (11), respectively. It is interesting to note that measuring the processes in Eq. (20) can be a test of the smallness of the 15-plet. For the \( B_{cc} \rightarrow B_{c}^{(i)} M \) decays, the \( T \) amplitudes are expanded as

\[ T(B_{cc} \rightarrow B_{c} M) = \]
\[ b_{9}(B_{cc})^{ij} H(T_{ij})^{jk}(B_{c})^{m}_j H(T_{ij})^l_i \epsilon_{ijk} + b_{10}(B_{cc})^{ij} H(T_{ij})^{jk}(B_{c})^{m}_j H(T_{ij})^l_i \epsilon_{ijk}, \]

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and

\[ T(B_{cc} \to B'_c M) = b_{11}(B_{cc})^i(B'_c)_{jk}(M)^k_i H(15)^j_k + b_{12}(B_{cc})^i(B'_c)_{jl}(M)^j_l H(15)^i_k 
+ b_{13}(B_{cc})^i(B'_c)_{jk}(M)^k_i H(15)^j_l + b_{14}(B_{cc})^i(B'_c)_{ij}(M)^j_i H_{km}(6) \epsilon^{mjl} 
+ b_{15}(B_{cc})^i(B'_c)_{jk}(M)^k_i H_{im}(6) \epsilon^{mjl}. \]  \( \text{(22)} \)

The full expansions of the \( T \) amplitudes in Eqs. (19)-(22) are given in Tables VIII and IX.

C. The triply charmed \( B_{ccc} \to B_{cc} M \) and \( B_{ccc} \to B'_c M_c \) decays

For the triply charmed baryon decays, there are three types of decay modes, that is, \( B_{ccc} \to B_{cc} M, B_{ccc} \to B'_c M_c, \) and \( B_{ccc} \to B_c M_c. \) The corresponding \( T \) amplitudes are given by

\[ T(B_{ccc} \to B_{cc} M) = d_1(B_{cc})^i(M)^j_k H(15)^j_i, \]
\[ T(B_{ccc} \to B'_c M_c) = d_2(B'_c)^i(M_c)^j_k H(15)^i_j, \]
\[ T(B_{ccc} \to B_c M_c) = d_3(B_c)^i(M_c)^j_k H(6)^i_j, \] \( \text{(23)} \)

where \( B_{ccc} = \Omega_{ccc}^{++} \) as the charmed baryon singlet has no \( SU(3) \) flavor index to connect to the final states and \( H(6, 15). \) The full expansions of the \( T \) amplitudes in Eq. (23) are given in Table X.

V. DISCUSSIONS

A. Semileptonic charmed baryon decays

By taking \( \mathcal{B}(\Lambda_c^+ \to \Lambda^0 e^+ \nu_e) = (3.6 \pm 0.4) \times 10^{-2} \) as the experimental input, and relating the possible \( B_c \to B_n \ell^+ \nu_\ell \) decays with the \( SU(3) \) parameter \( \alpha_1 \) in Table I, the branching ratios of the Cabibbo-allowed decays are predicted to be

\[ \mathcal{B}(\Xi^0_c \to \Xi^- e^+ \nu_e) = (11.9 \pm 1.6) \times 10^{-2}, \]
\[ \mathcal{B}(\Xi^+_c \to \Xi^0 e^+ \nu_e) = (3.0 \pm 0.5) \times 10^{-2}, \] \( \text{(24)} \)
while the Cabibbo-suppressed ones are evaluated as

\[ B(\Xi^0_c \to \Sigma^- e^+ \nu_e) = (6.0 \pm 0.8) \times 10^{-3}, \]
\[ B(\Lambda^+_c \to n e^+ \nu_e) = (2.7 \pm 0.3) \times 10^{-3}, \]
\[ B(\Xi^+_c \to \Sigma^0 e^+ \nu_e) = (0.8 \pm 0.1) \times 10^{-3}, \]
\[ B(\Xi^+_c \to \Lambda^0 e^+ \nu_e) = (2.5 \pm 0.4) \times 10^{-4}, \]

(25)

where we have taken \((\tau_{\Xi^0_c}, \tau_{\Xi^+_c}, \tau_{\Lambda^+_c}) = (1.12^{+0.13}_{-0.10}, 4.42 \pm 0.26, 2.00 \pm 0.06) \times 10^{-13}\) s and 
\(s_c = 0.2248\). Our result of \(B(\Lambda^+_c \to n e^+ \nu_e)\) in Eq. (25) agrees with that in Ref. [21] by \(SU(3)_f\) symmetry also. The \(B_c \to B'_n \ell^+ \nu_\ell\) decays are forbidden modes, reflecting the fact that the \(B_c\) and \(B'_n\) states are the uncorrelated anti-symmetric triplet and symmetric decuplet, respectively, which can be viewed as the interesting measurements to test the broken symmetry.

In Table [I] we illustrate the possible \(B'_c \to B'_n(\ell^+ \nu_\ell)\) decays, where \(B'_c\) stands for the singly charmed baryon sextet in Eq. (5). We remark that currently it is hard to observe the weak decays with \(B_c = (\Sigma^+_c, \Xi^+_c, \Omega^+_c)\) and \(B'_c = (\Xi^+_c, \Omega^+_c)\), as the \(\Sigma_c\) and \(\Xi'_c\) decays are dominantly through the strong and electromagnetic interactions, with \(B(\Sigma_c \to \Lambda_c \pi) \approx 100\%\) and \(\Xi'_c \to \Xi_c \gamma\), respectively. In contrast, the \(\Omega^+_c\) state that decays weakly can be measurable. In particular, the \(\Omega^+_c \to \Omega^- \ell^+ \nu_\ell\) decay with \(\Omega^- = sss\) becomes the only possible Cabibbo-allowed \(\Omega^+_c\) case \([32]\), whereas the \(\Omega^0_c \to B_n \ell^+ \nu_\ell\) decays with the baryon octet are forbidden. This is due to the fact that, via the Cabibbo-allowed \(c \to s \ell^+ \nu_\ell\) transition, the \(\Omega^0_c\) baryon consists of \(ssc\) transforms as the \(sss\) state, and has no association with the the baryon octet. In the Cabibbo-suppressed \(css \to dss\) transition, one has the \(\Omega^0_c \to \Xi^0_c(\ell^- \ell^+ \nu_\ell)\) decays with \(\Xi^+\) and \(\Xi^-\) from both baryon octet and decuplet.

For \(B_{cc} \to B'_c(\ell^+ \nu_\ell)\), it is found from Table [II] that

\[ \Gamma(\Xi^+_{cc} \to \Xi^0_c(\ell^+ \nu_\ell)) = \Gamma(\Xi^{++}_{cc} \to \Xi^{0+}_{cc}(\ell^+ \nu_\ell)), \]

(26)

which respect the isospin symmetry. Like the singly charmed \(\Omega^0_c\) cases, the Cabibbo-allowed \(\Omega^+(ccs) \to css\) transition forbids the \(\Omega^+_c \to B_c \ell^+ \nu_\ell\) decays, but allows \(\Omega^+_c \to \Omega^0_c \ell^+ \nu_\ell\) with \(\Omega^0_c = css\). The Cabibbo-suppressed \(\Omega^0_{cc}(ccs) \to cds\) transition permits \(\Omega^+_c \to (\Xi^-_c, \Xi^0_c)(\ell^+ \nu_\ell)\).

In the \(B_{ccc} \to B_{cc}(\ell^+ \nu_\ell)\) decays, \(SU(3)_f\) symmetry leads to two possible decay modes, of which the branching ratios are related as

\[ s_c^2 B(\Omega^{++}_{ccc} \to \Omega^+_c \ell^+ \nu_\ell) = B(\Omega^{++}_{ccc} \to \Xi^{0+}_{cc}(\ell^+ \nu_\ell)), \]

(27)
suggesting that the Cabibbo-allowed $\Omega_{c\bar{c}c}^{++} \rightarrow \Omega_{c\bar{c}}^{+}f^+\nu_{\ell}$ decay is more accessible to experiment.

B. Non-leptonic charmed baryon decays

- The $B_c \rightarrow B_n^{(0)} M$ decays

In the $\Lambda_c^+ \rightarrow B_n M$ decays, the PDG [1] lists six *Cabibbo-favored* channels, in addition to two *Cabibbo-suppressed* ones, whereas no absolute branching fractions for the $\Xi_c^{0,+}$ decays have been seen [1]. Being demonstrated to well fit the measured values of $B(\Lambda_c^+ \rightarrow B_n M)$ [36], $SU(3)_f$ symmetry can be used to study the $\Xi_c^{0,+} \rightarrow B_n M$ decays. For example, according to the data in the PDG [1], it is given that

$$B(\Xi_c^+ \rightarrow \Xi_c^{0+}) = (7.2 \pm 3.5) \times 10^{-3},$$

$$B(\Xi_c^0 \rightarrow \Lambda^0 K^0) = (8.3 \pm 3.7) \times 10^{-3},$$

which result in

$$B_I(\Xi_c^+ \rightarrow \Xi_c^{0+}) = (7.2 \pm 3.5) \times 10^{-3},$$

$$B_I(\Xi_c^0 \rightarrow \Lambda^0 K^0) = (8.3 \pm 3.7) \times 10^{-3},$$

by bringing the predictions of Eq. (24) into the relations. On the other hand, the $SU(3)$ parameters for $B_c \rightarrow B_n M$ have been extracted from the observed $B(\Lambda_c^+ \rightarrow B_n M)$ data, given by [36]

$$(a_1, a_2, a_3) = (0.257 \pm 0.006, 0.121 \pm 0.015, 0.092 \pm 0.021) \text{ GeV}^3,$$

$$(\delta_{a_2}, \delta_{a_3}) = (79.0 \pm 6.8, 35.2 \pm 8.8)^\circ,$$

where $\delta_{a_2,a_3}$ are the relative phases from the complex $a_2$ and $a_3$ parameters, and $a_1$ is fixed to be real. Besides, we follow Ref. [21] to ignore $a_{4,5,...,7}$ from $H(T_5)$, which are based on $(c_-/c_+)^2 = 5.5$ from $H_{eff}$ in Eq. (7), leading to the estimation of $B(\Lambda_c \rightarrow \Sigma^+ K^0)$ with the $(10-15)^\%$ deviation from the data [36]. By using $SU(3)$ parameters in Eq. (30), we obtain

$$B_{II}(\Xi_c^+ \rightarrow \Xi_c^{0+}) = (8.0 \pm 4.1) \times 10^{-3},$$

$$B_{II}(\Xi_c^0 \rightarrow \Lambda^0 K^0) = (8.3 \pm 0.9) \times 10^{-3}.$$
agreements for $\Xi_c^+ \rightarrow \Xi^0\pi^+$ and $\Xi_c^0 \rightarrow \Lambda^0K^0$ clearly support the approach with the $SU(3)_f$ symmetry.

As seen from Table[11] for the $B_c \rightarrow B'_nM$ decays, one has that

$$B(\Lambda_c^+ \rightarrow \Delta^{++}K^-) = \frac{1}{s_c^2}B(\Lambda_c^+ \rightarrow \Delta^{++}\pi^-)$$

$$= \frac{1}{s_c^2R_+}B(\Xi_c^+ \rightarrow \Delta^{++}K^-)$$

$$= \frac{3}{s_c^4R_+}B(\Xi_c^+ \rightarrow \Sigma^{'+}K^0) = \frac{1}{s_c^4R_+}B(\Xi_c^+ \rightarrow \Delta^{++}\pi^-), \quad (32)$$

and

$$B(\Xi_c^0 \rightarrow \Omega^-K^+) = 3B(\Xi_c^0 \rightarrow \Xi^{'-}\pi^+)$$

$$= \frac{3}{4s_c^2}B(\Xi_c^0 \rightarrow \Sigma^{'+}\pi^+, \Xi^{'-}K^+)$$

$$= \frac{1}{s_c^4}B(\Xi_c^0 \rightarrow \Delta^{-}\pi^+) = \frac{1}{3s_c^4}B(\Xi_c^0 \rightarrow \Sigma^{'-}K^+), \quad (33)$$

with $R_{+(0)} = \tau_{\Xi_c^{+(0)}}/\tau_{\Lambda_c^+}$, whose amplitudes are commonly proportional to $2a_8 + a_9$ and $2a_8 - a_9$, respectively. Besides, we obtain

$$B(\Xi_c^0 \rightarrow \Sigma^{'+}K^-) = \frac{1}{s_c^2}B(\Xi_c^0 \rightarrow \Delta^+K^-, \Sigma^{'+}\pi^+)$$

$$= \frac{1}{s_c^4}B(\Xi_c^0 \rightarrow \Delta^+\pi^-) = \frac{1}{2s_c^4}B(\Xi_c^0 \rightarrow \Delta^0\pi^0),$$

$$B(\Xi_c^+ \rightarrow \Sigma^{'+}\bar{K}^0, \Xi^0\pi^+) = \frac{R_0}{s_c^4}B(\Lambda_c^+ \rightarrow \Delta^+K^0), \quad (34)$$

corresponding to $T \propto 2a_8 - a_9 - 2a_{11}$ and $a_{11}$, respectively. Currently, apart from $B(\Lambda_c^+ \rightarrow \Delta^{++}K^-)$, it is measured that $B(\Xi_c^0 \rightarrow \Omega^-K^+) = (0.297 \pm 0.024) \times B(\Xi_c^0 \rightarrow \Xi^{'-}\pi^+) [1]$, such that we can estimate $B(\Xi_c^0 \rightarrow \Omega^-K^+)$ with the input of $B(\Xi_c^0 \rightarrow \Xi^{'-}\pi^+) = (1.6 \pm 0.1) \times 10^{-2}$.

Subsequently, with the the two branching ratios, given by

$$B(\Lambda_c^+ \rightarrow \Delta^{++}K^-) = (1.09 \pm 0.25) \times 10^{-2} [1],$$

$$B(\Xi_c^0 \rightarrow \Omega^-K^+) = (4.8 \pm 0.5) \times 10^{-3}, \quad (35)$$

and the relations in Eqs. (32) and (33), we predict that

$$B(\Lambda_c^+ \rightarrow \Delta^{++}\pi^-) = (5.5 \pm 1.3) \times 10^{-3},$$

$$B(\Xi_c^0 \rightarrow \Delta^{++}K^-) = (1.2 \pm 0.3) \times 10^{-3},$$

$$B(\Xi_c^+ \rightarrow \Sigma^{'+}K^0, \Delta^{++}\pi^-) = (2.1 \pm 0.5, 6.2 \pm 1.5) \times 10^{-5}, \quad (36)$$
and

\[
\mathcal{B}(\Xi^0_c \to \Xi^- \pi^+) = (1.6 \pm 0.2) \times 10^{-3},
\mathcal{B}(\Xi^0_c \to \Sigma'^- \pi^+ (\Xi'^- K^+)) = (3.2 \pm 0.3) \times 10^{-4},
\mathcal{B}(\Xi^0_c \to \Delta^- \pi^+, \Sigma'^- K^+) = (1.2 \pm 0.1, 3.7 \pm 0.4) \times 10^{-5}.
\] (37)

We remark that, if \( H(15) \) is negligible, one has \( \mathcal{B}(\Xi^0_c \to \Omega^- K^+) \approx R_0 \mathcal{B}(\Lambda'^+ \to \Delta'^+ K^-) \) with \( R_0 = 0.56 \pm 0.07 \), which agrees with the value of \( 0.44 \pm 0.11 \) from Eq. (35).

• The \( \mathcal{B}^t_c \to \mathcal{B}^{(t)}_n M \) decays

From Table \[IV \] to Table \[VII \], we show the \( \mathcal{B}^t_c \to \mathcal{B}^{(t)}_n M \) decays with \( \mathcal{B}^t_c = (\Sigma_c, \Xi_c, \Omega_c) \). Experimentally, we have that \[
\mathcal{B}(\Omega^0_c \to \Omega^- \pi^+) = 0.41 \pm 0.19 \pm 0.04, \] (38)

where \( \Omega^0_c \to \Omega^- \pi^+ \) and \( \Omega^0_c \to \Omega^- e^+ \nu_e \) are identified from Tables \[I \] and \[VII \] as Cabibbo-allowed processes, with \( \Omega^- \) belonging to the baryon decuplet \( \mathcal{B}^t_c \). On the other hand, as the only Cabibbo-allowed \( \Omega^0_c \to \mathcal{B}^{(t)}_n M \) mode, \( \Omega^0_c \to \Xi^0 K^0 \) has not been measured yet, which calls for the other accessible decay modes. Although it seems that there is no relation for \( \Omega^0_c \to \mathcal{B}^{(t)}_n M \) in Table \[VI \], if \( H(15) \) is ignorable, we have

\[
\mathcal{B}(\Omega^0_c \to \Sigma^+ K^-) = 2 \mathcal{B}(\Omega^0_c \to \Sigma^0 K^-),
\mathcal{B}(\Omega^0_c \to \Xi^- \pi^+) = 2 \mathcal{B}(\Omega^0_c \to \Xi^0 \pi^+),
\] (39)

for the Cabibbo-suppressed processes, and

\[
\mathcal{B}(\Omega^0_c \to \Sigma^\pm \pi^\mp) = \mathcal{B}(\Omega^0_c \to \Sigma^0 \pi^0),
\mathcal{B}(\Omega^0_c \to \Xi^0 K^0) = \mathcal{B}(\Omega^0_c \to \Xi^- K^+),
\mathcal{B}(\Omega^0_c \to p K^-) = \mathcal{B}(\Omega^0_c \to n K^0),
\] (40)

for the doubly Cabibbo-suppressed ones, which can be regarded to recover the isospin symmetry.

For \( \Omega^0_c \to \mathcal{B}^t_n M \), as seen in Table \[VII \], it is found that

\[
\mathcal{B}(\Omega^0_c \to \Delta^+ K^-) = \mathcal{B}(\Omega^0_c \to \Delta^0 K^-),
\mathcal{B}(\Omega^0_c \to \Omega^- K^+) = \frac{1}{s_c^2} \mathcal{B}(\Omega^0_c \to \Xi^- K^+). \] (41)
In addition, ignoring $H(15)$, we derive the relations with the recovered isospin symmetry, given by

\[
\mathcal{B}(\Omega_c^0 \rightarrow \Sigma^+ K^-) = \mathcal{B}(\Omega_c^0 \rightarrow \Sigma^0 K^0),
\]
\[
\mathcal{B}(\Omega_c^0 \rightarrow \Xi^- \pi^+) = \mathcal{B}(\Omega_c^0 \rightarrow \Xi^0 \pi^0), \tag{42}
\]

and

\[
\mathcal{B}(\Omega_c^0 \rightarrow \Sigma'^+ \pi^-) = \mathcal{B}(\Omega_c^0 \rightarrow \Sigma^0 \pi^0),
\]
\[
\mathcal{B}(\Omega_c^0 \rightarrow \Xi'^- K^+) = \mathcal{B}(\Omega_c^0 \rightarrow \Xi^0 K^0), \tag{43}
\]

for the Cabibbo- and doubly Cabibbo-suppressed decays, respectively.

- The $\mathcal{B}_{cc} \rightarrow \mathcal{B}_{n}^{(t)} M_c$ decays

For the possible $\mathcal{B}_{cc} \rightarrow \mathcal{B}_{n} M_c$ decays in Table VIII, the Cabibbo-allowd decay modes can be related to the (doubly) Cabibbo-suppressed ones, given by

\[
\Gamma(\Xi_{cc}^+ \rightarrow \Sigma^+ D^+) = \frac{1}{s_c^2} \Gamma(\Xi_{cc}^+ \rightarrow pD^+) = \frac{1}{s_c^2} \Gamma(\Xi_{cc}^+ \rightarrow pD_s^+),
\]
\[
\Gamma(\Xi_{cc}^+ \rightarrow \Sigma^+ D^0) = \frac{1}{s_c^2} \Gamma(\Xi_{cc}^+ \rightarrow pD^0) = \frac{1}{s_c^4} \Gamma(\Omega_{cc}^+ \rightarrow pD^0),
\]
\[
\Gamma(\Xi_{cc}^+ \rightarrow \Xi^0 D_s^+) = \frac{1}{s_c^4} \Gamma(\Xi_{cc}^+ \rightarrow nD_s^+),
\]
\[
\Gamma(\Omega_{cc}^+ \rightarrow \Xi^0 D^+) = \frac{1}{s_c^4} \Gamma(\Xi_{cc}^+ \rightarrow nD_s^+),
\]
\[
\Gamma(\Xi_{cc}^+ \rightarrow \Sigma^0 D^+) = \frac{1}{2s_c^2} \Gamma(\Xi_{cc}^+ \rightarrow nD^+). \tag{44}
\]

By keeping $b_{1,2}$ from $H(6)$ and disregarding $b_{3,4}$ from $H(15)$, similar to the demonstrations for $\mathcal{B}_c \rightarrow \mathcal{B}_{n}^{(t)} M$, we obtain additional relations such as

\[
\Gamma(\Xi_{cc}^+ \rightarrow \Sigma^0 D^+) = 3\Gamma(\Xi_{cc}^+ \rightarrow \Lambda^0 D^+),
\]
\[
\Gamma(\Xi_{cc}^+ \rightarrow \Lambda^0 D^+) = 4\Gamma(\Xi_{cc}^+ \rightarrow \Xi^0 D_s^+) = \frac{3}{2s_c^4} \Gamma(\Omega_{cc}^+ \rightarrow \Lambda^0 D_s^+),
\]
\[
\Gamma(\Omega_{cc}^+ \rightarrow pD^0) = \Gamma(\Omega_{cc}^+ \rightarrow nD_s^+). \tag{45}
\]

It is interesting to note that, in contrast with $\mathcal{B}_{cc} \rightarrow \mathcal{B}_{n} M_c$, the $\mathcal{B}_{cc} \rightarrow \mathcal{B}_{n}^{(t)} M_c$ decays are suppressed, where the amplitudes in Eq. (19) consist of $b_{5,6}$ from $H(15)$ only, resulting in
contributions 5.5 times smaller than $H(6)$. According to Table VIII, one gets that

$$\Gamma(\Xi^{++}_{cc} \to \Sigma'^+ D^+) = \Gamma(\Omega^{+}_{cc} \to \Xi^0 D^+)$$

$$= \frac{1}{s_c^2} \Gamma(\Xi^{++}_{cc} \to \Delta^+ D^+, \Sigma'^+ D^+_s) = \frac{1}{s_c^4} \Gamma(\Xi^{++}_{cc} \to \Delta^+ D^+_s) = \frac{1}{s_c^4} \Gamma(\Xi^{++}_{cc} \to \Delta^0 D^+_s),$$

$$\Gamma(\Xi^{+}_{cc} \to \Sigma'^+ D^0) = \Gamma(\Xi^{+}_{cc} \to \Xi^0 D^+_s)$$

$$= \frac{1}{s_c^2} \Gamma(\Xi^{+}_{cc} \to \Delta^+ D^0) = \frac{1}{s_c^2} \Gamma(\Omega^{+}_{cc} \to \Sigma'^+ D^0) = \frac{1}{s_c^4} \Gamma(\Omega^{+}_{cc} \to \Delta^+ D^0, \Delta^0 D^+), \quad (46)$$

and

$$\Gamma(\Xi^{+}_{cc} \to \Sigma'^0 D^+)$$

$$= \frac{1}{2s_c^2} \Gamma(\Xi^{+}_{cc} \to \Delta^0 D^+), \quad (47)$$

• The $\mathbf{B}_{cc} \to \mathbf{B}_c M$ decays

In the $\mathbf{B}_{cc} \to \mathbf{B}_c M$ decays, the Cabibbo-allowed amplitudes are composed of $SU(3)$ parameters $b_{7,8}$ from $H(6)$, instead of $b_{9,10}$ from $H(15)$, which indicate that the decays are measurable. In fact, the decay mode of $\Xi^{++}_{cc} \to \Xi^+_c \pi^+$ has been suggested to be worth measuring by the model calculation [16]. Here, we connect these Cabibbo-allowed decays to be

$$\Gamma(\Xi^{++}_{cc} \to \Xi^+_c \pi^+) = \Gamma(\Omega^{+}_{cc} \to \Xi^+_c \bar{K}^0),$$

$$\Gamma(\Xi^{+}_{cc} \to \Xi^0_c \pi^+) = \Gamma(\Xi^{+}_{cc} \to \Lambda^+_c \bar{K}^0),$$

$$\Gamma(\Xi^{+}_{cc} \to \Xi^+_c \pi^0) = 3\Gamma(\Xi^{+}_{cc} \to \Xi^+_c \eta), \quad (48)$$

which are the most accessible decay modes to the experiments. We note that the accuracy of the prediction involving $\eta$ is limited by the assumption that $\eta$ is a pure octet. Next, the Cabibbo-suppressed decays are related as

$$\Gamma(\Xi^{++}_{cc} \to \Xi^+_c K^+) = 4\Gamma(\Xi^{++}_{cc} \to \Lambda^+_c \pi^+) = 8\Gamma(\Xi^{++}_{cc} \to \Lambda^+_c \pi^+),$$

$$\Gamma(\Xi^{+}_{cc} \to \Xi^0_c K^+) = 4\Gamma(\Omega^{+}_{cc} \to \Xi^0_c \pi^+) = 8\Gamma(\Omega^{+}_{cc} \to \Xi^+_c \pi^0),$$

$$\Gamma(\Xi^{+}_{cc} \to \Xi^+_c K^0) = 4\Gamma(\Omega^{+}_{cc} \to \Lambda^+_c \bar{K}^0). \quad (49)$$

For the doubly Cabibbo-suppressed ones, only when $a_{9,10}$ from $H(15)$ are negligible, we can find that

$$\Gamma(\Xi^{++}_{cc} \to \Lambda^+_c K^+) = \Gamma(\Xi^{++}_{cc} \to \Lambda^+_c K^0),$$

$$\Gamma(\Omega^{+}_{cc} \to \Xi^0_c K^+) = \Gamma(\Omega^{+}_{cc} \to \Xi^+_c K^0). \quad (50)$$
There are three kinds of relations in the $B_{cc} \rightarrow B_c^* M$ decays, given by

$$
\Gamma(\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^{++} K^0) = \frac{2}{s_c^2} \Gamma(\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^{++} \pi^0) = \frac{1}{s_c^2} \Gamma(\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^{++} K^0) = \frac{1}{s_c^2} \Gamma(\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^{++} K^0),
$$

$$
\Gamma(\Omega_{cc}^+ \rightarrow \Omega_{cc}^0 \pi^+) = 2 \Gamma(\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++} \pi^+) = \frac{2}{s_c^2} \Gamma(\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^+ K^0) = \frac{1}{s_c^2} \Gamma(\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^+ K^0),
$$

$$
\Gamma(\Omega_{cc}^+ \rightarrow \Xi_{cc}^{++} \bar{K}^0) = 2 \Gamma(\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{++} K^0). \tag{51}
$$

Note that, $\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^{++} \bar{K}^0$, with the strong decays of $\Sigma_{cc}^{++} \rightarrow \Lambda_4^+ \pi^+$ and $K^* \rightarrow K^+ \pi^+$, corresponds to the observation of $\Xi_{cc}^{++} \rightarrow \Lambda_4^+ K^+ \pi^+$ \cite{4, 16}. Since the vector meson octet ($V$) is nearly the same as the pseudo-scalar meson ones ($M$) in Eq. \(10\), the non-leptonic charmed baryon decays with $V$ and $M$ have similar $SU(3)$ amplitudes. Therefore, as the counterpart of $\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^{++} \bar{K}^0$ observed by LHCb, $\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^{++} \bar{K}^0$ is promising to be observed. Moreover, with the amplitudes that contain $2a_{14} + 2a_{15}$ from $H(6)$ to give larger contributions, provided that the two terms have a constructive interference, it is possible that the decays of $\Xi_{cc}^{++} \rightarrow (\Sigma_{cc}^{++} \bar{K}^0, \Xi_{cc}^{0\pi})$ can be more significant than that of $\Xi_{cc}^{++} \rightarrow \Sigma_{cc}^{++} \bar{K}^0$.

- $B_{cc} \rightarrow B_{cc}M$ and $B_{ccc} \rightarrow B_{cc}^{(0)} M_c$ decays

In Table \[\Xi\] the $B_{ccc}$ state is indeed the singlet of $\Omega_{ccc}^{++}$, and the $B_{ccc} \rightarrow B_{cc}M$ decays have two types, given by

$$
\Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} \bar{K}^0) = \frac{2}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} \pi^0) = \frac{2}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} \eta) = \frac{1}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} K^0),
$$

$$
\Gamma(\Omega_{ccc}^{++} \rightarrow \Omega_{ccc}^{0\pi^+}) = \frac{1}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} \pi^+) = \frac{1}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} \eta) = \frac{1}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} K^0),
$$

$$
\Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} \bar{K}^0) = 2 \Gamma(\Xi_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} K^0). \tag{52}
$$

where $T$'s are proportional to $d_1 - 2d_2$ and $d_1 + 2d_2$, respectively, with $d_1(2)$ from $H(15(6))$. The $\Omega_{ccc}^{++} \rightarrow B_{cc}^{(0)} M_c$ decays can be simply related, given by

$$
\Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ D^+) = \frac{1}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ D^+, \Lambda_4^+ D^+) = \frac{1}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Lambda_4^+ D^+),
$$

$$
\Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} D^+, \Sigma_c^+ D^+) = \frac{1}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{ccc}^{++} D^+, \Sigma_c^+ D^+) = \frac{1}{s_c^2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^+). \tag{53}
$$

Note that the decay modes with $B_{cc}$ and $B_{cc}'$ are in accordance with $d_{1,3}$ from $H(6)$ and $H(15)$, respectively, such that it is possible that the Cabibbo-allowed $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ D^+$ decay can be more accessible to the experiments.
VI. CONCLUSIONS

We have studied the semileptonic and non-leptonic charmed baryon decays with \( SU(3)_f \) symmetry. By separating the Cabibbo-allowed decays from the (doubly) Cabibbo-suppressed ones, we have provided the accessible decay modes to the experiments at BESIII and LHCb. We have predicted the rarely studied \( B_c \to B^{(t)}_n \ell^+ \nu_\ell \) and \( B_c \to B^{(t)}_n M \) decays, such as 
\[
B(\Xi^0_e \to \Xi^- e^+ \nu_e, \Xi^+_e \to \Xi^0 e^+ \nu_e) = (11.9 \pm 1.6, 3.0 \pm 0.5) \times 10^{-2}, \quad B(\Xi^+_e \to \Lambda^0 K^0, \Xi^+_e \to \Xi^0 \pi^+) = (8.3 \pm 0.9, 8.0 \pm 4.1) \times 10^{-3}, \quad B(\Lambda^+_c \to \Delta^{++} \pi^-, \Xi^0_c \to \Omega^- K^+) = (5.5 \pm 1.3, 4.8 \pm 0.5) \times 10^{-3}
\]
We have found that the \( B_c \to B^{(t)}n \ell^+ \nu_\ell \) decays are forbidden due to the \( SU(3)_f \) symmetry. On the other hand, the \( \Omega^0_c \to \Omega^- \ell^+ \nu_\ell, \Omega^{+}_{cc} \to \Omega^{0}_{cc} \ell^+ \nu_\ell, \) and \( \Omega^{++}_{ccc} \to \Omega^{+}_{ccc} \ell^+ \nu_\ell \) decays have been presented as the only existing Cabibbo-allowed cases in \( B^{(t)}_n \ell^+ \nu_\ell, B_{cc} \to B^{(t)}_c \ell^+ \nu_\ell, \) and \( B_{ccc} \to B^{(t)}_{cc} \ell^+ \nu_\ell, \) respectively, where only \( \Omega^0_c \) from \( B^{(t)}_c \) decays weakly. Moreover, being compatible to \( \Omega^+_{cc} \to \Xi^+_{cc} K^0, \) the doubly charmed \( \Xi^{++}_{cc} \to \Xi^+_{cc} \pi^+ \) decay is favored to be measured, which agrees with the model calculation. As the counterpart of \( \Xi^{++}_{cc} \to \Sigma^{++}_{cc} K^{*0}, \) which is observed as the resonant \( \Xi^{++}_{cc} \to (\Sigma^{++}_{cc} \to \Lambda^+_c \pi^+ (K^{*0} \to) K^- \pi^+) \) four-body decays, \( \Xi^{++}_{cc} \to \Sigma^{++}_{cc} K^0 \) is promising to be seen. Finally, the triply \( \Omega^{++}_{ccc} \to (\Xi^{++}_{ccc} K^0, \Omega^{++}_{cc} \pi^+, \Xi^+_c D^+) \) decays are the favored Cabibbo-allowed decays.

ACKNOWLEDGMENTS

This work was supported in part by National Center for Theoretical Sciences, MoST (MoST-104-2112-M-007-003-MY3), and National Science Foundation of China (11675030).

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TABLE I. The $T$ amplitudes ($T$-amps) related to the semileptonic charmed baryon decays.

| $B_c \to B_a$ | $T$-amp | $B'_c \to B_a$ | $T$-amp | $B'_c \to B'_a$ | $T$-amp |
|---------------|---------|----------------|---------|----------------|---------|
| $\Xi^+_c \to \Xi^-$ | $\alpha_1$ | $\Xi^+_c \to \Xi^-$ | $\sqrt{\frac{3}{2}} \alpha_2$ | $\Xi^+_c \to \Xi^-$ | $\sqrt{\frac{3}{2}} \alpha_3$ |
| $\Xi^+_c \to \Xi^0$ | $\alpha_1$ | $\Xi^+_c \to \Xi^0$ | $\sqrt{\frac{3}{2}} \alpha_2$ | $\Xi^+_c \to \Xi^0$ | $\sqrt{\frac{3}{2}} \alpha_3$ |
| $\Lambda^+_c \to \Lambda^0$ | $-\sqrt{\frac{3}{2}} \alpha_1$ | $\Sigma^+_c \to \Sigma^-$ | $-\alpha_2$ | $\Sigma^+_c \to \Sigma^-$ | $\sqrt{\frac{3}{2}} \alpha_3$ |
| $\Sigma^+_c \to \Sigma^0$ | $-\alpha_2$ | $\Sigma^+_c \to \Sigma^0$ | $\alpha_2$ | $\Sigma^+_c \to \Sigma^0$ | $\sqrt{\frac{3}{2}} \alpha_3$ |
| $\Sigma^+_c \to \Sigma^+$ | $\alpha_2$ | $\Sigma^+_c \to \Sigma^+$ | $\alpha_2$ | $\Sigma^+_c \to \Sigma^+$ | $\sqrt{\frac{3}{2}} \alpha_3$ |
| $\Sigma^0_c \to \Lambda^0$ | $-\alpha_1 s_c$ | $\Sigma^0_c \to \Lambda^0$ | $-\alpha_1 s_c$ | $\Sigma^0_c \to \Lambda^0$ | $-\alpha_1 s_c$ |
| $\Sigma^+_c \to \Lambda^0$ | $-\alpha_1 s_c$ | $\Sigma^+_c \to \Lambda^0$ | $-\alpha_1 s_c$ | $\Sigma^+_c \to \Lambda^0$ | $-\alpha_1 s_c$ |
| $\Lambda^+_c \to n$ | $-\alpha_1 s_c$ | $\Lambda^+_c \to n$ | $-\alpha_1 s_c$ | $\Lambda^+_c \to n$ | $-\alpha_1 s_c$ |
| $\Omega^0_c \to \Xi^-$ | $-\alpha_3 s_c$ | $\Omega^0_c \to \Xi^-$ | $-\alpha_3 s_c$ | $\Omega^0_c \to \Xi^-$ | $-\alpha_3 s_c$ |

| $B_{ccc} \to B_{cc}$ | $T$-amp | $B_{ccc} \to B_c$ | $T$-amp | $B_{ccc} \to B'_c$ | $T$-amp |
|---------------|---------|----------------|---------|----------------|---------|
| $\Omega^{++}_{ccc} \to \Omega^+_c$ | $\delta_1$ | $\Omega^{++}_{ccc} \to \Omega^+_c$ | $\beta_1$ | $\Omega^{++}_{ccc} \to \Omega^+_c$ | $\sqrt{\frac{3}{2}} \beta_2$ |
| $\Omega^{++}_{ccc} \to \Xi^+_c$ | $\delta_1 s_c$ | $\Omega^{++}_{ccc} \to \Xi^+_c$ | $\beta_1 s_c$ | $\Omega^{++}_{ccc} \to \Xi^+_c$ | $\sqrt{\frac{3}{2}} \beta_2$ |

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TABLE II. The $B_c \to B_s M$ decays, where the notations of CA and (D)CS T-amps stand for Cabibbo-allowed and (doubly) Cabibbo-suppressed $T$ amplitudes, which are the same as those in the following tables.

| $\Xi^0_{c}$ | CA T-amp | $\Xi^0_{c}$ | CS T-amp | $\Xi^0_{c}$ | DCS T-amp |
|--------------|-----------|--------------|-----------|--------------|-----------|
| $\Sigma^+ K^-$ | $2(a_2 + \frac{a_3+a_6}{2})$ | $\Sigma^+ \pi^-$ | $-2(a_2 + \frac{a_3+a_6}{2})s_c$ | $p\pi^-$ | $-2(a_2 + \frac{a_3+a_6}{2})s_c^2$ |
| $\Sigma^0 K^0$ | $-\sqrt{2}(a_2 + a_3$ | $\Sigma^0 \pi^0$ | $-(a_2 + a_3$ | $\Sigma^0 K^0$ | $\sqrt{2}(a_2 + \frac{a_3-a_6}{2})s_c^2$ |
| $\Xi^0\pi^0$ | $-\sqrt{2}(a_3 - a_3$ | $\Xi^0 K^0$ | $2(a_1 - a_3$ | $n\pi^0$ | $\sqrt{2}(a_2 - \frac{a_3-a_6}{2})s_c^2$ |
| $\Xi^0\eta$ | $\frac{\sqrt{2}}{3}(a_2 - 2a_2 - a_3$ | $\Sigma^0\eta$ | $\sqrt{\frac{2}{3}(a_1 + a_2 - a_3$ | $n\eta$ | $\sqrt{\frac{2}{3}(2a_1 - a_2 - 2a_3$ |
| $\Xi^0 K^+$ | $2(a_1 + \frac{a_3-a_6}{2})s_c$ | $\Xi^0 K^+$ | $-\sqrt{2}(a_1 + a_2 + a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_1 + \frac{a_3-a_6}{2})s_c^2$ |
| $\Lambda^0 K^0$ | $-\sqrt{2}(2a_1 - a_2 - a_3$ | $\Sigma^0\eta$ | $\sqrt{\frac{2}{3}(a_1 + a_2 - a_3$ | $\Sigma^0\eta$ | $\sqrt{\frac{2}{3}(a_1 + a_2 - a_3$ |
| $\Xi^0\pi^+$ | $2(a_3 + \frac{a_3-a_6}{2})s_c$ | $\Xi^0 K^-$ | $\sqrt{2}(a_3 + a_2 - a_3$ | $\Xi^0 K^-$ | $\sqrt{2}(a_1 + a_2 - 2a_3$ |
| $\Xi^0 K^+$ | $\sqrt{2}(a_1 + a_2 - a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_1 + a_2 - 2a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_1 + a_2 - 2a_3$ |
| $\Xi^0\eta$ | $\sqrt{\frac{2}{3}(a_1 + a_2 - a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_1 + a_2 - 2a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_1 + a_2 - 2a_3$ |
| $\Xi^0 K^+$ | $\sqrt{2}(a_3 + a_2 - a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_3 + a_2 - a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_3 + a_2 - a_3$ |

| $\Lambda^+_c$ | CA T-amp | $\Lambda^+_c$ | CS T-amp | $\Lambda^+_c$ | DCS T-amp |
|--------------|-----------|--------------|-----------|--------------|-----------|
| $\Sigma^+ K^0$ | $\sqrt{2}(a_1 - a_2 - a_3$ | $\Sigma^+ K^0$ | $\sqrt{2}(a_1 - a_2 - a_3$ | $\Sigma^+ K^0$ | $\sqrt{2}(a_1 - a_2 - a_3$ |
| $\Sigma^0 K^0$ | $-\sqrt{2}(a_1 - a_2 - a_3$ | $\Sigma^0 K^0$ | $\sqrt{2}(a_1 - a_2 - a_3$ | $\Sigma^0 K^0$ | $\sqrt{2}(a_1 - a_2 - a_3$ |
| $\Xi^0 K^+$ | $\sqrt{2}(a_2 - a_2 - a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_2 - a_2 - a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_2 - a_2 - a_3$ |
| $\Xi^0 K^+$ | $\sqrt{2}(a_2 - a_2 - a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_2 - a_2 - a_3$ | $\Xi^0 K^+$ | $\sqrt{2}(a_2 - a_2 - a_3$ |

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| \( \Xi_0^0 \) | \( \Xi_0^+ \) | \( \Xi_0^- \) | \( \Xi_0^+ \) | \( \Xi_0^- \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \Sigma^+ K^0 \) | \( \Delta^+ K^0 \) | \( \Sigma^0 K^0 \) | \( \Delta^+ K^0 \) | \( \Sigma^0 K^0 \) |
| \( \sqrt{\frac{4}{3}}(2a_8 - a_9 - 2a_{10}) \) | \( \sqrt{\frac{4}{3}}(2a_8 - a_9 - 2a_{10})s_c \) | \( \sqrt{\frac{4}{3}}(2a_8 - a_9 - 2a_{10})s_c \) | \( \sqrt{\frac{4}{3}}(2a_8 - a_9 - 2a_{10})s_c \) | \( \sqrt{\frac{4}{3}}(2a_8 - a_9 - 2a_{10})s_c \) |
| \( \Sigma^0 K^0 \) | \( \Delta^+ K^0 \) | \( \Sigma^0 K^0 \) | \( \Delta^+ K^0 \) | \( \Sigma^0 K^0 \) |
| \( \sqrt{\frac{4}{3}}(2a_8 - a_9) \) | \( \sqrt{\frac{4}{3}}(2a_8 - a_9) \) | \( \sqrt{\frac{4}{3}}(2a_8 - a_9) \) | \( \sqrt{\frac{4}{3}}(2a_8 - a_9) \) | \( \sqrt{\frac{4}{3}}(2a_8 - a_9) \) |
| \( \Xi^0 \) | \( \Xi^+ \) | \( \Xi^- \) | \( \Xi^+ \) | \( \Xi^- \) |
| \( \Xi^0 K^+ \) | \( \Xi^+ K^+ \) | \( \Xi^- K^+ \) | \( \Xi^+ K^+ \) | \( \Xi^- K^+ \) |
| \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11}) \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) |
| \( \Delta^+ K^0 \) | \( \Delta^+ K^0 \) | \( \Delta^+ K^0 \) | \( \Delta^+ K^0 \) | \( \Delta^+ K^0 \) |
| \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11}) \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) |
| \( \Xi^0 \) | \( \Xi^+ \) | \( \Xi^- \) | \( \Xi^+ \) | \( \Xi^- \) |
| \( \Xi^0 K^+ \) | \( \Xi^+ K^+ \) | \( \Xi^- K^+ \) | \( \Xi^+ K^+ \) | \( \Xi^- K^+ \) |
| \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11}) \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) | \( \sqrt{\frac{8}{3}}(2a_8 + a_9 + 2a_{11})s_c \) |

TABLE III. The \( B_c \rightarrow B'_m M \) decays.
### TABLE IV. The $\mathbf{B}_c' \to \mathbf{B}_n M$ decays, where $\mathbf{B}_c' = (\Sigma^{++}, \Sigma^+, \Sigma^0)$. 

| $\Sigma_c^{++}$ | CA T-amp | $\Sigma_c^{++}$ | CS T-amp | $\Sigma_c^{++}$ | DCS T-amp |
|-----------------|----------|-----------------|----------|-----------------|-----------|
| $\Sigma_c^+ + \pi^0$ | $2a_{13} + a_{16} + a_{17}$ | $\Sigma_c^+ + \pi^0$ | $(2a_{13} + a_{16} + a_{17})s_c$ | $\Sigma_c^+ + \pi^0$ | $(2a_{13} + a_{16} + a_{17})s_c^2$ |
| $\Sigma_c^+ + \pi^0$ | $2a_{13} + a_{16} + a_{17}$ | $\Sigma_c^+ + \pi^0$ | $(2a_{13} + a_{16} + a_{17})s_c$ | $\Sigma_c^+ + \pi^0$ | $(2a_{13} + a_{16} + a_{17})s_c^2$ |
| $\Sigma_c^+ + \eta$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Sigma_c^+ + \eta$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Sigma_c^+ + \eta$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ |
| $\Sigma_c^0 + \pi^0$ | $-a_{13} - a_{14} - a_{15}$ | $\pi^0$ | $-a_{13} - a_{14} - a_{15}$ | $\pi^0$ | $-a_{13} - a_{14} - a_{15}$ |
| $\Sigma_c^0 + \pi^0$ | $-a_{13} - a_{14} - a_{15}$ | $\pi^0$ | $-a_{13} - a_{14} - a_{15}$ | $\pi^0$ | $-a_{13} - a_{14} - a_{15}$ |
| $\Xi_c^0 + K^0$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Xi_c^0 + K^0$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Xi_c^0 + K^0$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ |

| $\Sigma_c^+$ | CA T-amp | $\Sigma_c^+$ | CS T-amp | $\Sigma_c^+$ | DCS T-amp |
|-----------------|----------|-----------------|----------|-----------------|-----------|
| $\Sigma_c^+ + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ | $\Sigma_c^+ + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ | $\Sigma_c^+ + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ |
| $\Sigma_c^0 + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ | $\Sigma_c^0 + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ | $\Sigma_c^0 + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ |
| $\Sigma_c^0 + \eta$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Sigma_c^0 + \eta$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Sigma_c^0 + \eta$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ |
| $\Sigma_c^0 + \pi^0$ | $-a_{13} - a_{14} - a_{15}$ | $\pi^0$ | $-a_{13} - a_{14} - a_{15}$ | $\pi^0$ | $-a_{13} - a_{14} - a_{15}$ |
| $\Sigma_c^0 + \pi^0$ | $-a_{13} - a_{14} - a_{15}$ | $\pi^0$ | $-a_{13} - a_{14} - a_{15}$ | $\pi^0$ | $-a_{13} - a_{14} - a_{15}$ |
| $\Xi_c^0 + K^0$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Xi_c^0 + K^0$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Xi_c^0 + K^0$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ |
| $\Sigma^0 + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ | $\Sigma^0 + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ | $\Sigma^0 + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ |
| $\Sigma^0 + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ | $\Sigma^0 + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ | $\Sigma^0 + \pi^0$ | $2a_{13} + 2a_{14} + a_{15}$ |
| $\Xi^0 + K^0$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Xi^0 + K^0$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ | $\Xi^0 + K^0$ | $\sqrt{2}(2a_{13} + 2a_{14} + 2a_{15})$ |
TABLE V. The $B_c^+ \to B_hM$ decays, where $B_c^+ = (\Xi_c^{+}, \Xi_c^{0}, \Omega_c^0)$.

| $\Xi_0^{0}$ | CA T-amp | $\Xi_0^{0}$ | CS T-amp | $\Xi_0^{0}$ | DCS T-amp |
|-------------|----------|-------------|----------|-------------|------------|
| $\Sigma^+K^-$ | $\sqrt{2}(2a_{15} - a_{16} - a_{18} - a_{19})$ | $\Sigma^+\pi^0$ | $-a_{14} + a_{15} + a_{16}$ | $\Sigma^+K^0$ | $\sqrt{2}(2a_{14} - a_{20})s_c^2$ |
| $\Sigma^0K^0$ | $-a_{13} + a_{14} + 2a_{16} + a_{18} + a_{19}$ | $\Sigma^0\pi^0$ | $-a_{14} + a_{15} + a_{16}$ | $\Sigma^0K^0$ | $\sqrt{2}(a_{14} + a_{19} + a_{20})s_c^2$ |
| $\Xi_0^{0}$ | $a_{13} - a_{14} + 2a_{16} + a_{18} + a_{19}$ | $\Xi_0^{0}$ | $s_{12}(a_{14} + a_{15} + a_{18} + 3a_{19} + 3a_{20})s_e$ | $\Xi_0^{0}$ | $s_{12}(a_{14} + a_{19} + a_{20} + 4a_{19})s_e^2$ |
| $\Xi^-\pi^+$ | $a_{13} - a_{14} + 2a_{16} + a_{18} + a_{19}$ | $\Xi^-K^0$ | $a_{13} - a_{14} + 2a_{14} + 2a_{15}$ | $\Xi^-K^0$ | $a_{14} - a_{20} + a_{21} - a_{19}s_e^2$ |

| $\Lambda^0K^0$ | $a_{13} - a_{14} + 2a_{14} + 2a_{15}$ | $\Lambda^0K^0$ | $a_{13} - a_{14} + 2a_{14} + 2a_{15}$ | $\Lambda^0K^0$ | $a_{13} - a_{14} + 2a_{15} + a_{16}$ |

| $\Omega_0^{0}$ | CA T-amp | $\Omega_0^{0}$ | CS T-amp | $\Omega_0^{0}$ | DCS T-amp |
|-------------|----------|-------------|----------|-------------|------------|
| $\Xi_0^{0}$ | $2a_{13} - a_{16} - a_{17}$ | $\Xi_0^{0}$ | $a_{14} - a_{15} - a_{16} - a_{18} - a_{19}s_e$ | $\Xi_0^{0}$ | $a_{14} + a_{19} + a_{20}s_e^2$ |
| $\Sigma^+K^-$ | $-a_{14} - a_{15} + a_{16}$ | $\Sigma^0\pi^0$ | $a_{13} - a_{14} + 2a_{16} + a_{18} + a_{19}$ | $\Sigma^0\pi^0$ | $a_{12}a_{14}^2s_e^2$ |
| $\Sigma^0K^0$ | $a_{13} - a_{14} + 2a_{16} + a_{18} + a_{19}$ | $\Xi_0^{0}$ | $a_{13} - a_{14} + 2a_{14} + 2a_{15}$ | $\Xi_0^{0}$ | $a_{14} + a_{19} - a_{20}s_e^2$ |
| $\Xi_0^{0}$ | $a_{13} - a_{14} + 2a_{16} + a_{18} + a_{19}$ | $\Xi_0^{0}$ | $a_{13} - a_{14} + 2a_{14} + 2a_{15}$ | $\Xi_0^{0}$ | $a_{14} + a_{19} + a_{20}s_e^2$ |

| $\Xi^-\pi^+$ | $a_{13} - a_{14} + 2a_{14} + 2a_{15}$ | $\Xi^-K^0$ | $a_{13} - a_{14} + 2a_{14} + 2a_{15}$ | $\Xi^-K^0$ | $a_{13} - a_{14} + 2a_{15} + a_{16}$ |

| $\Lambda^0K^0$ | $a_{13} - a_{14} + 2a_{14} + 2a_{15}$ | $\Lambda^0K^0$ | $a_{13} - a_{14} + 2a_{14} + 2a_{15}$ | $\Lambda^0K^0$ | $a_{13} - a_{14} + 2a_{15} + a_{16}$ |
TABLE VI. The $B'_c \rightarrow B'_0 M$ decays, where $B'_c = \langle \Sigma^{++}, \Sigma^+, \Sigma^0 \rangle$.

| $\Sigma^+_c$ | CA T-amp | $\Sigma^+_c$ | CS T-amp | $\Sigma^+_c$ | CDS T-amp |
|-------------|-----------|-------------|-----------|-------------|-----------|
| $\Delta^{++}K^0$ | $-(2a_{21} - a_{23})$ | $\Delta^{++}K^0$ | $-\sqrt{\frac{3}{2}}(2a_{21} - a_{23})s_c$ | $\Delta^{++}K^0$ | $(2a_{21} - a_{23})s_c^2$ |
| $\Sigma'^{\pm}+\sqrt{\delta}(2a_{21} + a_{23})$ | $\pm(2a_{21} + a_{23})s_c$ | $\Sigma'^{\pm}+\sqrt{\delta}(2a_{21} + a_{23} + 2a_{25})s_c$ | $\Sigma'^{\pm}+\sqrt{\delta}(2a_{21} + a_{23})s_c^2$ |
| $\Sigma^0_c$ | CA T-amp | $\Sigma^0_c$ | CS T-amp | $\Sigma^0_c$ | CDS T-amp |
| $\Delta^{+}K^0$ | $-\sqrt{\frac{3}{2}}(2a_{22} - a_{26})$ | $\Delta^{+}K^0$ | $\sqrt{\frac{3}{2}}(2a_{22} - a_{26})s_c$ | $\Delta^{+}K^0$ | $\sqrt{\frac{3}{2}}(2a_{22} - a_{26})s_c^2$ |
| $\Delta^{+}K^0$ | $-\sqrt{\frac{3}{2}}(4a_{21} + 2a_{23} - 2a_{26})$ | $\Delta^{+}K^0$ | $\sqrt{\frac{3}{2}}(4a_{21} + 2a_{23} - 2a_{26})s_c$ | $\Delta^{+}K^0$ | $\sqrt{\frac{3}{2}}(4a_{21} - a_{23} - a_{26})s_c^2$ |
| $\Sigma'^{\pm}\eta$ | $\sqrt{\frac{3}{2}}(a_{22} + a_{24})$ | $\Sigma'^{\pm}\eta$ | $\sqrt{\frac{3}{2}}(a_{22} + a_{24} + a_{25})s_c$ | $\Sigma'^{\pm}\eta$ | $\sqrt{\frac{3}{2}}(a_{22} + a_{24} + a_{25})s_c^2$ |
| $\Sigma'^{\pm}\eta$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})$ | $\Sigma'^{\pm}\eta$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c$ | $\Sigma'^{\pm}\eta$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c^2$ |
| $\Sigma'^{0}\pi^+$ | $\sqrt{\frac{3}{2}}(2a_{21} + a_{22} + a_{23} + a_{24} + a_{25})$ | $\Sigma'^{0}\pi^+$ | $\sqrt{\frac{3}{2}}(a_{22} + 2a_{24} + 2a_{25})s_c$ | $\Sigma'^{0}\pi^+$ | $\sqrt{\frac{3}{2}}(a_{22} + 2a_{24} + 2a_{25})s_c^2$ |
| $\Xi'^{0}\pi^+$ | $\sqrt{\frac{3}{2}}(a_{22} + a_{24})$ | $\Xi'^{0}\pi^+$ | $\sqrt{\frac{3}{2}}(a_{22} + a_{24})s_c$ | $\Xi'^{0}\pi^+$ | $\sqrt{\frac{3}{2}}(a_{22} + a_{24})s_c^2$ |
| $\Xi'^{0}\pi^+$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})$ | $\Xi'^{0}\pi^+$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c$ | $\Xi'^{0}\pi^+$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c^2$ |
| $\Xi^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24})$ | $\Xi^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24})s_c$ | $\Xi^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24})s_c^2$ |
| $\Xi^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})$ | $\Xi^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c$ | $\Xi^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c^2$ |

| $\Sigma^0_c$ | CA T-amp | $\Sigma^0_c$ | CS T-amp | $\Sigma^0_c$ | CDS T-amp |
|-------------|-----------|-------------|-----------|-------------|-----------|
| $\Delta^{+}K^0$ | $-\sqrt{\frac{3}{2}}(2a_{21} - a_{23})$ | $\Delta^{+}K^0$ | $\sqrt{\frac{3}{2}}(2a_{21} - a_{23})s_c$ | $\Delta^{+}K^0$ | $\sqrt{\frac{3}{2}}(2a_{21} - a_{23})s_c^2$ |
| $\Delta^{+}K^0$ | $-\sqrt{\frac{3}{2}}(3a_{21} + 2a_{23} - 2a_{26})$ | $\Delta^{+}K^0$ | $\sqrt{\frac{3}{2}}(3a_{21} + 2a_{23} - 2a_{26})s_c$ | $\Delta^{+}K^0$ | $\sqrt{\frac{3}{2}}(3a_{21} + 2a_{23} - 2a_{26})s_c^2$ |
| $\Sigma'^{0}\pi^+$ | $\frac{3}{2}(a_{22} + a_{24})$ | $\Sigma'^{0}\pi^+$ | $\frac{3}{2}(a_{22} + a_{24})s_c$ | $\Sigma'^{0}\pi^+$ | $\frac{3}{2}(a_{22} + a_{24})s_c^2$ |
| $\Sigma'^{0}\pi^+$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})$ | $\Sigma'^{0}\pi^+$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c$ | $\Sigma'^{0}\pi^+$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c^2$ |
| $\Xi'^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24})$ | $\Xi'^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24})s_c$ | $\Xi'^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24})s_c^2$ |
| $\Xi'^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})$ | $\Xi'^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c$ | $\Xi'^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24} + a_{25})s_c^2$ |

| $\Xi^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24})$ | $\Xi^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24})s_c$ | $\Xi^{0}K^0$ | $\frac{3}{2}(a_{22} + a_{24})s_c^2$ |
|-------------|-----------|-------------|-----------|-------------|-----------|
| $\Xi_c^{+}$ | CA T-amp | $\Xi_c^0$ | CS T-amp | $\Xi_c^0$ | CDS T-amp |
|---|---|---|---|---|---|
| $\Sigma^+ K^0$ | $\frac{\Xi_c^0}{(2a_{21} + a_{23} + a_{25})}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ |
| $\Xi^0 K^0$ | $\frac{\Xi_c^0}{(2a_{22} - 2a_{25} + a_{26})}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ |

| $\Xi_c^0$ | CA T-amp | $\Xi_c^0$ | CS T-amp | $\Xi_c^0$ | CDS T-amp |
|---|---|---|---|---|---|
| $\Sigma^+ K^0$ | $\frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ |
| $\Xi^0 K^0$ | $\frac{\Xi_c^0}{(2a_{22} - 2a_{25} + a_{26})}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24}) \frac{s_c}{6}}$ |

| $\Omega_c^0$ | CA T-amp | $\Omega_c^0$ | CS T-amp | $\Omega_c^0$ | CDS T-amp |
|---|---|---|---|---|---|
| $\Xi^0 K^0$ | $-\sqrt{\frac{1}{3}}(2a_{21} - a_{23} - 2a_{25})$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24} + a_{25}) \frac{s_c}{2}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24} + a_{25}) \frac{s_c}{2}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24} + a_{25}) \frac{s_c}{2}}$ |
| $\Omega^+ - 2a_{21} + a_{23}$ | $\Xi^0 K^0$ | $-\sqrt{\frac{1}{3}}(2a_{21} - a_{23} - 2a_{25})$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24} + a_{25}) \frac{s_c}{2}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24} + a_{25}) \frac{s_c}{2}}$ | $\Delta^+ K^0 - \frac{\Xi_c^0}{(2a_{22} - a_{24} + a_{25}) \frac{s_c}{2}}$ |
| $\text{B}_{cc} \to \text{B}_{n} M_{c}$ | CA $T$-amp | $\text{B}_{cc} \to \text{B}_{d} M_{c}$ | CS $T$-amp | $\text{B}_{cc} \to \text{B}_{n} M_{c}$ | DCS $T$-amp |
|---|---|---|---|---|---|
| $\Xi_{cc}^{++} \to \Sigma^{+} D^{+}$ | $2b_2 - b_4$ | $\Xi_{cc}^{++} \to \Sigma^{+} D^{+}$ | $-(b_2 + b_4)s_c$ | $\Xi_{cc}^{++} \to p D^{+}$ | $(b_2 - b_4)x^2_c$ |
| $\Xi_{cc}^{++} \to \Sigma^{+} D^{0}$ | $2b_1 - b_3$ | $\Xi_{cc}^{++} \to \Sigma^{+} D^{0}$ | $(2b_2 - b_4)s_c$ | $\Xi_{cc}^{++} \to n D^{+}$ | $(2b_2 + b_4)s_c^2$ |
| $\Xi_{cc}^{++} \to \Sigma^{0} D^{+}$ | $-\sqrt{2}(b_1 + b_2$ | $\Xi_{cc}^{++} \to \Sigma^{0} D^{0}$ | $\sqrt{2}(b_2 + b_3 + b_4)s_c$ | $\Omega_{cc}^{++} \to \Sigma D^{+}$ | $(b_3 + b_4)s_c^2$ |
| $\Xi_{cc}^{++} \to \Xi^{0} D^{+}$ | $2b_1 + b_3$ | $\Xi_{cc}^{++} \to \Xi^{0} D^{0}$ | $(2b_1 - b_3)s_c$ | $\Omega_{cc}^{++} \to \Xi D^{+}$ | $(2b_1 + b_3)s_c^2$ |
| $\Xi_{cc}^{++} \to \Lambda^{0} D^{+}$ | $\sqrt{2}(b_1 + b_2 + b_4)$ | $\Xi_{cc}^{++} \to \Lambda^{0} D^{0}$ | $2(b_1 + b_2 + b_4)s_c$ | $\Omega_{cc}^{++} \to \Lambda D^{+}$ | $(2b_1 + b_3)s_c^2$ |
| $\Omega_{cc}^{+} \to \Xi^{0} D^{+}$ | $2b_2 + b_4$ | $\Omega_{cc}^{+} \to \Xi^{0} D^{0}$ | $-\sqrt{2}(b_1 + b_2 + b_4)s_c$ | $\Omega_{cc}^{++} \to \Xi^{0} D^{+}$ | $-\sqrt{2}(b_1 + b_2)$ |
| $\Xi_{cc}^{++} \to \Delta^{+} D^{+}$ | $2\sqrt{3}b_5$ | $\Xi_{cc}^{++} \to \Delta^{+} D^{0}$ | $-2\sqrt{3}b_5 s_c$ | $\Xi_{cc}^{++} \to \Delta^{+} D^{+}$ | $-2\sqrt{3}b_5 s_c^2$ |
| $\Xi_{cc}^{++} \to \Delta^{+} D^{0}$ | $2\sqrt{3}b_6$ | $\Xi_{cc}^{++} \to \Delta^{+} D^{0}$ | $2\sqrt{3}b_5 s_c$ | $\Xi_{cc}^{++} \to \Delta^{+} D^{0}$ | $-2\sqrt{3}b_5 s_c^2$ |
| $\Xi_{cc}^{++} \to \Sigma^{0} D^{+}$ | $\sqrt{6}(b_5 + b_6)$ | $\Xi_{cc}^{++} \to \Sigma^{0} D^{0}$ | $-\sqrt{6}(b_5 b_6)s_c$ | $\Omega_{cc}^{++} \to \Delta^{0} D^{+}$ | $-2\sqrt{6}(b_5 + b_6)s_c^2$ |
| $\Xi_{cc}^{++} \to \Xi^{0} D^{+}$ | $2\sqrt{3}b_5$ | $\Xi_{cc}^{++} \to \Xi^{0} D^{0}$ | $\sqrt{6}(b_5 + b_6)s_c$ | $\Omega_{cc}^{++} \to \Xi^{0} D^{+}$ | $-\sqrt{6}(b_5 + b_6)s_c^2$ |
| $\Xi_{cc}^{++} \to \Xi^{0} D^{+}$ | $2\sqrt{3}b_6$ | | | | |
| B_{cc} \rightarrow B_{c}^0 M | CA T-amp | B_{cc} \rightarrow B'_{c} M | CA T-amp | B_{cc} \rightarrow B_{c}^0 M | CS T-amp | B_{cc} \rightarrow B'_{c} M | DCS T-amp |
|-----------------|---------|-----------------|---------|-----------------|--------|-----------------|--------|
| \Xi_{cc}^+ \rightarrow \Xi_{cc}^+ K^0 | 2b_7 | \Xi_{cc}^+ \rightarrow \Sigma_{cc}^+ \pi^0 | \frac{1}{2}b_{12} + b_{13} | \Xi_{cc}^+ \rightarrow \Sigma_{cc}^+ K^0 | b_{11} + b_{13} - 2b_{14} | \Xi_{cc}^+ \rightarrow \Sigma_{cc}^+ K^0 | -(b_{11} + b_{13} - 2b_{14})s_c^2 |
| \Xi_{cc}^+ \rightarrow \Lambda_{cc}^+ K^0 | 2b_8b_9 | \Xi_{cc}^+ \rightarrow \Xi_{cc}^+ K^0 | -b_{7}s_c | \Xi_{cc}^+ \rightarrow \Sigma_{cc}^0 K^0 | +b_{14}b_9s_c | \Xi_{cc}^+ \rightarrow \Xi_{cc}^0 K^0 | -(b_{11} + b_{13})s_c^2 |
| \Xi_{cc}^+ \rightarrow \Xi_{cc}^0 \pi^0 | -\sqrt{2}(b_7 + b_8) | \Xi_{cc}^+ \rightarrow \Xi_{cc}^0 \pi^0 | \sqrt{2}(b_7 - 4b_8)s_c + 2b_9s_c | \Xi_{cc}^+ \rightarrow \Xi_{cc}^0 K^0 | 2b_9s_c | \Xi_{cc}^+ \rightarrow \Xi_{cc}^0 K^0 | -(b_{11} + b_{13})s_c^2 |
| \Xi_{cc}^+ \rightarrow \Xi_{cc}^+ \eta | \sqrt{2}(b_7 + b_8) | \Xi_{cc}^+ \rightarrow \Xi_{cc}^+ \eta | -\sqrt{2}(b_7 + b_8) + 4b_8) | \Xi_{cc}^+ \rightarrow \Xi_{cc}^+ \eta | 4b_9s_c | \Xi_{cc}^+ \rightarrow \Xi_{cc}^+ \eta | 4b_9s_c |
TABLE X. The $B_{ccc} \rightarrow B_{cc} M$ and $B_{cc} \rightarrow B_{c}^{(i)} M_c$ decays.

| $B_{ccc} \rightarrow B_{cc} M$ | $B_{ccc} \rightarrow B_{cc} M$ | $B_{ccc} \rightarrow B_{cc} M$ | $B_{ccc} \rightarrow B_{cc} M$ |
|---|---|---|---|
| CA T-amp | CS T-amp | DCS T-amp |
| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^0$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \pi^0$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^0$ |
| $d_1 - 2d_2$ | $\sqrt{2}(d_1 - 2d_2) s_c$ | $-(d_1 - 2d_2) s_c$ |
| $\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{+} \pi^+$ | $\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{+} \eta$ | $\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{+} K^+$ |
| $d_1 + 2d_2$ | $-(d_1 + 2d_2) s_c$ | $(d_1 + 2d_2) s_c$ |

| $B_{ccc} \rightarrow B_{c}^{(i)} M_c$ | $B_{ccc} \rightarrow B_{c}^{(i)} M_c$ | $B_{ccc} \rightarrow B_{c}^{(i)} M_c$ |
| CA T-amp | CS T-amp | DCS T-amp |
| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} D^+$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} D^+_s$ | $\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^+_s$ |
| $2d_4$ | $2d_4 s_c$ | $2d_4 s_c$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} D^+_s$ | $\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^+_s$ | $\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^+_s$ |
| $\sqrt{2}d_3 s_c$ | $\sqrt{2}d_3 s_c$ | $-\sqrt{2}d_3 s_c$ |

| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} D^+_s$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} D^+_s$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} D^+_s$ |
| $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} D^+_s$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} D^+_s$ | $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} D^+_s$ |
| $\sqrt{2}d_3 s_c$ | $\sqrt{2}d_3 s_c$ | $-\sqrt{2}d_3 s_c$ |