AN INTEGRATED TOOL FOR OPTIMIZING REHABILITATION PROGRAMS OF HIGHWAYS PAVEMENT

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Abstract. Modeling pavement performance and optimizing resources represent two challenges for decision makers responsible for maintenance and rehabilitation of road networks pavement. This paper presents the developments made in a stochastic performance prediction model and optimization model as two major parts of an integrated pavement management system. Markov modeling is used to create a transition process model that is implemented to predict pavement condition throughout the life time of road networks. With the use of the Pavement Condition Index (PCI), the steps of performing the prediction of deterioration are presented, showing the process of creating the elements of Markov matrix. The obtained results are used to set the priorities for maintenance planning and budgeted cost allocations on the network level. The proposed model advises decision makers on the status of network level with the guidelines to keep road conditions in acceptable level of performance according to the predefined strategies. Genetic algorithms technique is adopted to build optimization model. Three objective functions are constructed for budgeted cost of maintenance and rehabilitation program, quality of work performed, and selected area for program implementation. A brief description of the developed pavement management systems, including the prediction and the optimization models, are presented. A numerical example is worked out to illustrate the practical use of both models.

Keywords: pavement management system, Markov modeling, multi-objective optimization, Pareto front, genetic algorithms.

1. Introduction

The total length of the road network in Egypt exceeds 65 000 km. According to the statistics of General Authority for Roads, Bridges and Land Transport, the total length of arterial road networks increased from 12 500 km in year 1972 to 47 500 km in year 2008. In Egypt, road networks are characterized by two aspects: i) they are aging and in some parts they are not functioning well, and ii) they are expanding rapidly to meet the economic growth. The share of freight transport in the road network is 95%. Keeping road networks in good condition is not possible without optimizing available limited resources. Therefore, the need to optimize maintenance planning is inevitable. The deterioration of road network assets is noticeable due to commercial and industrial activities. Keeping the workability of road networks in good condition is considered a cumbersome task which needs to be addressed on strategic level (Vanier 2001). The maintenance of such road network infrastructures requires the use of comprehensive Pavement Management Systems (PMS). The environment for running such system is characterized by the uncertainty of road condition in the future and the inaccuracy of the gathered data. A lot of efforts have been made to develop Pavement Management Systems and bridges (Abu Dabous, Alkass 2011; Amador-Jiménez, Mrawira 2011; Sobanjo 2011; Tsai, Lai 2002; Yang et al. 2009). In this paper, a framework for pavement performance prediction is developed utilizing Markov chain. The input for this model is Pavement Condition Index (PCI) which is adopted by General Authority for Roads, Bridges and Land Transport for measuring the performance of the Egyptian roads. Output from performance model is input for optimization process to provide decision makers with several alternatives for min budgeted cost with considering max quality of work performed and max percentage of area coverage. Further, the framework helps to achieve three goals: 1) minimizing budgeted cost to meet the need of strategic planners, 2) maximizing the quality of performing maintenance and rehabilitation programs, and 3) maximizing the total percentage of the network area that will be under maintenance and rehabilitation. A case study is presented to illustrate the main features of the model.
2. Pavement management systems

PMS is defined by AASHTO as "A systematic process that collects and analyzes pavement information with rational procedures that provide optimum pavement strategies based on predicted pavement attributes incorporating feedback regarding various attributes, criteria and constraints involved". In general, typical structure of a PMS consists of six main components as follows (Fig. 1):

i) **Data Input Module**: it collects and standardizes data to meet the validations requirements of the database.

ii) **Database Module**: it acts as a repository for all historical field information. This organized information is considered the base for performing any analysis or decision, pertaining to the current or future road maintenance plans. In the proposed model, the essential database attribute is the PCI for each segment which is retrieved at each inspection. This technique classifies and rates different segments of the network based on visual inspection.

iii) **Performance Prediction Module**: role of this module is to predict future network condition based on the available information. The prediction module is either based on deterministic or probabilistic approach. In the proposed model, the focus of research is the probabilistic approach which represents the real life situation and provides a better accuracy with respect to the future condition of the network.

iv) **Optimization and Analysis Module**: the output obtained from the performance prediction module is fed to this module to calculate the different options for future maintenance programs. The cost is estimated based on different scenarios. Different outputs are based on the selection of different resource options.

v) **Planning and Implementation Module**: it tracks and reports the programs for maintenance plans including the budgets and resources. All cost components are detailed and reported via this module.

vi) **Reporting and Feedback Module**: this module plays a major role in developing and upgrading the system. In addition, it communicates with all network stakeholders regarding any reporting requests.

3. Markov chain

Markov chain is a stochastic process that handles the uncertainty condition of road system performance through time. Applying Markov chain models for asset management systems has proved to be reliable in different applications (Adedimila et. al. 2009; Black et. al. 2005; Orcesi, Cremona 2010; Puz, Radic 2008; Yang et. al. 2005). Several efforts have been made utilizing Markov chain to optimize maintenance and replacement decisions of bridges’ components (Golabi, Shepard 1997; Jiang et al. 2000; Madanat 1993). Markov chain has been adopted for developing performance deterioration model for bridge deck, taking into consideration the history of deterioration and maintenance (Madanat, Robelin 2007). This stochastic process is an indexed collection of random variables \( \{ X_t \} \) for \( t \) runs through a given set of non-negative integers \( (T) \) (Hillier 2000). The Markov process is considered to have Markovian property if conditional probability of any future event is independent of the past and depends only upon the present state; for any matrix to be considered a Markov Matrix or Transition Matrix the following two properties should be valid (Janssen, Manca 2006).

\[
P_{ij} \geq 0 \text{ for all } i, j \in T, \tag{1}
\]

\[
\sum P_{ij} = 1 \text{ for all } i, j \in T, \tag{2}
\]

where \( p \) – the probability of transition from one state \( i \) to another state \( j \), Fig. 2 shows the graphical representation of the transition between two states.
Using Markov model, the transition matrix represents the probability of change from state $i$ to state $j$ over time period; the one step transition matrix $P$ is a two dimensional matrix of size $4 \times 4$ to represent states of the road condition which are Excellent, Good, Fair, and Bad as per Eq (3). The Transition Matrix after some time $n$ is calculated using Eq (4) as per Janssen and Manca (2006).

$$
P = \begin{bmatrix}
P_{11} & P_{12} & P_{13} & P_{14} \\
P_{21} & P_{22} & P_{23} & P_{24} \\
P_{31} & P_{32} & P_{33} & P_{34} \\
P_{41} & P_{42} & P_{43} & P_{44}
\end{bmatrix},$$

(3)

$$
P_n = P^n$$

(4)

The transition states, i.e., the values of the matrix, are classified into three types as follows:

- **Transient**: if upon entering this state, the process may never return to this state again.
- **Recurrent**: if upon entering this state the process will definitely return to this state again.
- **Absorbing**: if upon entering this state, the process will never leave the state again.

As such, it is possible to predict the deterioration of the pavement through a fixed period of time which is the period between two consecutive inspections (Black et al. 2005). The adaptation of this method requires large amount of data to decrease errors and to gain reliable results.

4. Model development

The methodology that has been followed in the proposed research includes three major stages (Awad 2010). These are: 1) data collection, 2) developing performance prediction module, and 3) optimization module. The details of these stages are described hereinafter.

4.1. Data collection

Road network consists of several roads. The road comprises smaller units that are called road segments. In this research, the length of road segment is considered two kilometres long with adopting the actual data that are collected by GARBLT to capture the characteristics of road segments. The data represents the PCI for each segment within a given period, the total number of segments in state $i$ before the transition; $p_{ij}$ – the probability of transition from state $i$ to state $j$ between two successive inspections without any maintenance.

$$
p = \frac{n_{ij}}{n_i},$$

(6)

$$\sum p_{ij} = 1 \text{ for } 1 \leq i \leq 4,$$

(7)

where $n_{ij}$ – the number of transitions from state $i$ to state $j$ within a given period, $n_i$ – the total number of segments in state $i$ before the transition; $p_{ij}$ – the probability of transition from state $i$ to state $j$ between two successive inspections without any maintenance.

The performance prediction module encompasses two main elements: Transition Matrix ($P$) and Initial Vector Matrix ($V_0$). These two elements are deemed essential to provide the status of the network condition after any number of inspections. The Initial Vector Matrix is essentially the initial condition vector that represents the current road condition records.

| PCI       | Road Condition | Maintenance Type |
|-----------|----------------|------------------|
| 100 ≥ PCI ≥ 85 | Excellent | Routine          |
| 84 ≥ PCI ≥ 70   | Good       | Preventive       |
| 69 ≥ PCI ≥ 40   | Fair       | Rehabilitation   |
| 39 ≥ PCI ≥ 0    | Bad        | Complete         |

Table 1. Road network conditions vs. maintenance types
status of the road (Morcous 2005). It is built by calculating the percentage of each state to the total number of road segments for the data of the initial year as per Eqs (8) and (9):

\[ V_0 = [V_1 \ V_2 \ V_3 \ V_4], \] (8)

\[ V_i = \frac{\sum v_i}{N} \text{ for } 1 \leq i \leq 4, \] (9)

where \( N \) – the number of all segments in the network; \( V_1, V_2, V_3, V_4 \) – the percentage of road segments that are in Excellent, Good, Fair and Bad conditions, respectively.

As such, the vector matrix or the future condition of the network over time is obtained. To plot the performance of the road, a scale from 1 to 4 is used to quantify the vector matrix into one single value that is used as an index to draw the curve. This performance index \( I \) is calculated by applying Eq (10):

\[ I = 4V_1 + 3V_2 + 2V_3 + V_4. \] (10)

where the value \( V_i \) – used to represent the percentage of each state to the total area of the network. The performance index \( I \) varies from 1 to 4. For example, Excellent road network has a performance index close to 4 (Fig. 3), whereas, Bad road network has a performance index close to 1. The result should provide an input to predict the future requirements of road network including the allocation of resources by using these values as the upper limit of each state that requires maintenance.

5. Optimization module

Optimization module works to achieve three goals: 1) minimizing budgeted cost to meet the need of strategic planners, 2) maximizing the quality of performing maintenance and rehabilitation programs, and 3) maximizing the total percentage of the network area that will be under maintenance and rehabilitation. Minimizing budgeted cost is major demand by strategic planners and different proposals should be made available to empower the decision making process. The main goal is to optimize budgeted cost to receive endorsement of network stakeholders (government, legislatures, etc.) on one of these proposals. But this item is not the only objective and it is not evaluated without setting two other major factors, the quality of performance and the percentage of area covered to the total area of the network. Maximizing quality ensures that final output from M&R programs should be made according to acceptable standard. Maximizing area percentage that is targeted by M&R programs ensures high road serviceability and safety and decreases the deferred backlog. In order to achieve these goals, the framework of the M&R includes six types of programs as per Table 2. It is assumed that these programs are selected concurrently and that none of them is to be eliminated. Achieving these objectives requires the formulation of an optimization problem that is modeled via genetic algorithms chromosome. The chromosome handles the presence of the six programs together and is capable to provide different scenarios for planning of M&R programs. The chromosomes consist of eighteen genes. The first six genes (1–6) represent the resources required to apply these programs, whereas, the second six genes (7–12) are the quality of each option. The quality is defined as the quality of performing these programs upon each resource selection. percentage of each type of project to the total pavement area of the network; The last six genes (13–18) represent the percentage of each sub area that is to be selected for M&R to the total proposed area for M&R (Fig. 4).

Subsequently, the first objective function is formulated to minimize the cost of maintenance and rehabilitation programs as follows:

\[ \text{Cost} = T(\%A \cdot C^{R_A} + \%B \cdot C^{R_B} + \%C \cdot C^{R_C} + \%D \cdot C^{R_D} + \%E \cdot C^{R_E} + \%F \cdot C^{R_F}), \] (11)

where \( T \) – total area of the network, \( \%A, \%B, \%C, \%D, \%E \& \%F \): percentage of the area that requires program type A, B, C, D, E & F, respectively; \( C^{R_A}, C^{R_B}, C^{R_C}, C^{R_D}, C^{R_E}, C^{R_F} \)

Fig. 3. Performance prediction curve

Fig. 4. Representation of chromosomes’ genes
The second objective function is formulated to maximize quality of performed maintenance and rehabilitation works as follows:

\[ \text{Average Quality} = \frac{\sum_{j} q^i_j}{6}, \]  

(12)

where \(q^i_j\) – quality of program \(i\) with budgeted cost \(c\) for option \(j\) which is one of three budgeted cost options for each program.

The third objective function is formulated to maximize the total percentage of area selected for maintenance.

\[ \text{Percentage Area} = \sum_{i=1}^{6} a^i_j, \]  

(13)

where \(a^i_j\) – percentage of area \(a\) of program \(i\) with budgeted cost \(c\) for option \(j\) which is one of three budgeted cost options for each program.

These three functions are optimized using Non-dominated Sorting Genetic Algorithm (Deb 2001). The main objective of the optimization module is to optimize the total budgeted cost of the program considering the quality of each maintenance program and the area of each program. The advantage of this module is its ability to expand, to include further parameters of the road and to customize the list of programs to meet agencies requirements. Finally, this module ensures that maintenance of road network is adjusted to the limits of budgeted cost with maintaining standard quality of performance. Detail description of optimization module is found elsewhere (Awad 2010). In addition to its simplicity, the model is capable of being dynamically modified to include further parameters like user cost.

6. Model implementation

The proposed model was implemented using Microsoft Access 2000 and VB.net for facilitating data entry, processing of the gathered information, and generating reports. First, the user inputs the gathered data from field which contains the identification of the road segment (RoadSegment), PCI for that segment (PCI), and inspection year (Year). The database of the model is designed to perform several queries that calculate the elements of the Transition Matrix at initial state \((P_0)\) and Initial Vector Matrix \((V_0)\). These results are automatically populated in a user interface, depicted in Fig. 5. Running the model provides the Transition Matrix after any number of inspections and the corresponding Vector Matrix. Finally, the information of the deterioration model is generated and is represented in a graphical format. Output from performance model is used as input for optimization model. Fig. 6 shows the dataflow in the proposed pavement management system.

### Table 2. Description of maintenance and rehabilitation programs

| Program   | Program Description | Before Program Status | After Program Status |
|-----------|---------------------|-----------------------|----------------------|
| A         | Complete reconstruction | Bad                   | Excellent            |
| B         | Major rehabilitation | Fair                  | Excellent            |
| C         | Minor rehabilitation | Fair                  | Good                 |
| D         | Major maintenance   | Good                  | Excellent            |
| E         | Minor maintenance   | Good                  | Good                 |
| F         | Routine             | Excellent             | Excellent            |

Fig. 5. Performance prediction module user interface of the proposed model

Fig. 6. Dataflow of the proposed system
7. Case study

7.1. Case description

This section describes a hypothetical case to clarify the use of optimization module. The final results from this example are near-optimum solutions for the optimized budgeted cost, percentage of quality of work performed, and selected area percentage of total area covered. Table 3 lists the data that are obtained from Helwan/El-Saf rural road from years 1999 to 2005. The input data of the case are listed in Tables 4 to 6 for budgeted cost, percentage quality of work performed, and percentage area for each maintenance program. As shown, there are six maintenance and rehabilitation programs with three options for each one. Each option provides different scenario. The purpose of this optimization is to minimize budgeted cost and to maximize percentage quality and percentage covered area for road segments that have a total area of 120,000 m².

7.2. Case analysis

The optimization module is triggered to evaluate its performance in searching large space of possible solutions. A number of optimization parameters are defined including; number of generations (G = 200), population size (S = 20), crossover (C = 0.6), and mutation (M = 0.02) values as described hereinafter. The results shown in Figs 7 to 9 are obtained, indicating Pareto set. The final Pareto set of chromosomes and the corresponding Budgeted Cost (LE), Quality (%), and Area (%) are listed in Table 7 and depicted in Fig. 10.

### Table 3. Helwan/El-Saf rural road data

| Segment ID | From, km | To, km | Year | 1999 | 2000 | 2002 | 2005 |
|------------|----------|--------|------|------|------|------|------|
| 1          | 0        | 2      |      | 74   | 79   | 94   | 43   |
| 2          | 2        | 4      |      | 79   | 46   | 100  | 65   |
| 3          | 4        | 6      |      | 82   | 77   | 96   | 28   |
| 4          | 6        | 8      |      | 77   | 59   | 96   | 63   |
| 5          | 8        | 10     |      | 85   | 91   | 96   | 42   |
| 6          | 10       | 12     |      | 87   | 82   | 81   | 55   |
| 7          | 12       | 14     |      | 69   | 88   | 81   | 77   |
| 8          | 14       | 16     |      | 80   | 72   | 55   | 71   |
| 9          | 16       | 18     |      | 81   | 72   | 84   | 80   |
| 10         | 18       | 20     |      | 85   | 82   | 50   | 100  |
| 11         | 20       | 22     |      | 87   | 100  | 100  | 87   |

### Table 4. Budgeted cost of resources' options

| Program | Cost of option 1 (LE/m²) | Cost of option 2 (LE/m²) | Cost of option 3 (LE/m²) |
|---------|--------------------------|--------------------------|--------------------------|
| A       | 30                       | 35                       | 37                       |
| B       | 20                       | 24                       | 28                       |
| C       | 14                       | 16                       | 18                       |
| D       | 9                        | 12                       | 14                       |
| E       | 6                        | 7                        | 8                        |
| F       | 2                        | 3                        | 5                        |
8. Conclusions

This paper presented the development of pavement prediction model and optimization model within the frame of pavement management system. As a stochastic approach, Markov chain is applied to provide valuable information about the state of the pavement in the future. The process is applicable on the network level geared towards furnishing decision-makers with a tool for assessing road conditions. Therefore, reasonable maintenance budgets for analyzed network(s) are allocated. Road maintenance should be based on strategic decision that adopts PMS. The paper presented the methodology that was followed in the proposed research including: 1) data collection, and 2) developing performance prediction module 3) optimization module. Furthermore, the optimization model is dynamically modified to include further pavement related parameters to enable better selection of programs by decision makers. The road network conditions are classified into four classes; Excellent, Good, Fair and Bad. The model was implemented using Microsoft Access 2000 and VB.net.

The paper presented an optimization framework that helps to achieve three goals: 1) minimizing budgeted cost to meet the need of strategic planners, 2) maximizing the quality of performing maintenance and rehabilitation programs, and 3) maximizing the total percentage of the network area that will be under maintenance and rehabilitation (M&R). In order to achieve these goals, the framework of the M&R includes six types of programs. Achieving these goals requires the formulation of an optimization problem that is modeled via genetic algorithms chromosome. The chromosome handles the presence of the six programs together and is capable to provide different scenarios for planning of M&R programs. A case study was presented to demonstrate the capabilities of the proposed model and its ability in identifying near-optimum Pareto solutions. The case was obtained from Helwan/El-Saf rural road from years 1999 to 2005 for a total area of 120 000 m². The Pareto fronts have been plotted in 2D and 3D to demonstrate the near-optimum feasible M&R programs. This research is extendable for future integration of user cost as a major factor in the optimization model. Additional efforts are recommended for optimizing the program with considering

| Table 5. Thresholds for quality options vs. budgeted cost |
|-----------------------------------------------------------|
| Program | Quality, % | |
|         | Option (1) | Option (2) | Option (3) |
| A       | 80         | 85         | 88         |
| B       | 80         | 88         | 92         |
| C       | 80         | 85         | 88         |
| D       | 80         | 85         | 88         |
| E       | 80         | 85         | 90         |
| F       | 80         | 85         | 90         |

| Table 6. Thresholds for percentage area options vs. budgeted cost |
|---------------------------------------------------------------|
| Gene Name | Area percentage |
|           | Option (1) | Option (2) | Option (3) |
| A         | 31         | 28         | 30         |
| B         | 22         | 20         | 20         |
| C         | 10         | 14         | 12         |
| D         | 10         | 8          | 10         |
| E         | 16         | 12         | 15         |
| F         | 8          | 6          | 4          |

| Table 7. Estimated objective functions of Pareto solutions |
|--------------------------------------------------------|
| Pareto front | Program option | Program area, % | Objective functions |
|              | A | B | C | G | E | F | A | B | C | G | E | F | Quality, % | Area, % | Budgeted cost (LE*) |
| QR           | 2 | 2 | 3 | 3 | 3 | 28 | 20 | 12 | 10 | 15 | 4   | 88.2 | 89       | 24 492000   |
|              | 1 | 1 | 2 | 2 | 3 | 3 | 31 | 22 | 14 | 10 | 15 | 4   | 85.5 | 96       | 21 444000   |
|              | 1 | 1 | 2 | 2 | 2 | 2 | 31 | 22 | 14 | 8  | 15 | 4   | 85.5 | 94       | 21 204000   |
|              | 1 | 1 | 2 | 2 | 2 | 2 | 31 | 22 | 14 | 8  | 15 | 6   | 84.2 | 96       | 21 036000   |
|              | 1 | 1 | 2 | 2 | 2 | 2 | 31 | 22 | 14 | 8  | 12 | 6   | 83.3 | 93       | 20 856000   |
|              | 1 | 1 | 2 | 1 | 2 | 2 | 2 | 2   | 14 | 10 | 12 | 6   | 82.5 | 85       | 20 496000   |
|              | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 3   | 22 | 10 | 10 | 10 | 4   | 81.7 | 93       | 20 196000   |
|              | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 31 | 22 | 10 | 10 | 16 | 6   | 80.8 | 95       | 20 028000   |
|              | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 3   | 22 | 10 | 10 | 16 | 8   | 80.0 | 97       | 19 944000   |
| AB           | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 31 | 22 | 14 | 10 | 16 | 8   | 80.8 | 100      | 20 232000   |
|              | 1 | 1 | 1 | 1 | 1 | 1 | 3 | 3   | 22 | 10 | 10 | 16 | 6   | 80.0 | 97       | 19 944000   |
| AQ           | 2 | 2 | 3 | 3 | 3 | 28 | 20 | 12 | 10 | 15 | 4   | 88.2 | 89       | 24 492000   |
|              | 3 | 3 | 2 | 3 | 2 | 30 | 20 | 14 | 10 | 15 | 6   | 88.0 | 95       | 25 716000   |
|              | 1 | 1 | 2 | 3 | 3 | 3 | 31 | 22 | 14 | 10 | 15 | 4   | 85.5 | 96       | 21 444000   |
|              | 1 | 1 | 2 | 2 | 3 | 1 | 31 | 22 | 14 | 8  | 15 | 8   | 83.3 | 98       | 20 952000   |
|              | 1 | 1 | 2 | 2 | 3 | 1 | 31 | 22 | 14 | 10 | 16 | 8   | 82.2 | 100      | 20 832000   |

* 8 LE = 1 EUR
other deterioration models of different infrastructure in the same location of the road network.

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