Structural properties of rotating hybrid compact stars with color-flavor-locked quark matter core and their tidal deformability

Suman Thakur, Virender Thakur\textsuperscript{a}, Raj Kumar\textsuperscript{b}, Shashi K. Dhiman\textsuperscript{c}

Department of Physics, Himachal Pradesh University, Summerhill, Shimla 171005, India

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Abstract We investigate the hybrid compact stars consisting of nucleons, hyperons and three flavor color-flavor-locked quark phase under global neutrality and chemical equilibrium conditions. The hadronic equations of state are computed within the framework of energy density functionals based on the relativistic mean field theory by employing two different model. The quark matter phase of equation of state is computed by using Quark Quasiparticle model derived from a non-relativistic energy density-functional approach. A set of hybrid equations of state for superdense hadron-quark matter is obtained and, employed to investigate the structural properties of non-rotating and rotating compact stars. The internal structure of rotating star with observed spin down frequencies, exhibiting shrinkage of soft quark core of compact stars are discussed for constant baryonic mass. We present the theoretically computed limits of radii for the spin down configurations of hybrid stars corresponding to the recently observed millisecond pulsars. The various EOSs considered in the present work are well within the recent astrophysical constraints on mass and radius measurements (Riley et al. in Astrophys J Lett 918(2):L27, 2021; Millet et al. in \texttt{http://arxiv.org/abs/2105.06979}, 2021) and dimensionless tidal deformability ($\Lambda_{1,4}$) (Abbott et al. in Phys Rev Lett 121(16):161101, 2018; Li et al. in Eur Phys J A 57(1):1–10, 2021).

1 Introduction

The theory of strong interactions, quantum chromodynamics (QCD) and ultrarelativitics heavy ion collisions predicts that at high energy densities the hadronic matter may under go deconfinement phase consisting of quarks and gluons as fundamental degree of freedom. Therefore, recently, it is an open question whether the inner core of compact stars (CS) consists of quark matter [5–9]. However, this has been suggested currently that the dense nuclear matter in the interior of stable compact stars with maximum gravitational masses $M \approx 2.0 M_\odot$ may exhibits the evidence for the presence of sizable quark matter cores [10]. Therefore, the hybrid stars phenomenology offers a unique tool to address the challenge of understanding the phase transition in the dense quantum chromodynamics. The nuclear theory studies [11–13] are mainly focusing for the understanding dense matter of compact stars (CS). The recent observations with LIGO and Virgo of GW170817 event [3,14] of Binary Neutron Stars merger and the discovery of CS with masses around $2M_\odot$ [15–20] have intensified the interest in these intriguing objects. The analysis of GW170817 has demonstrated the potential of gravitational wave (GW) observations to yield new information relating to the limits on CS tidal deformability. In addition to these astrophysical observations [21–25], the measurements of rotational frequencies of the pulsar can be employed to constraints the particle composition and behavior of Equations of State (EOS) of the dense nuclear matter. However, the direct measurement of radius and quark matter interior core of CS are still a great challenge from astrophysical interests. The upcoming high-precision x-ray space missions, such as the ongoing NICER (Neutron Star Interior Composition Explorer) [17–20,24] and the future eXTP (Enhanced X-ray Timing and Polarimetry Mission) [26] have aimed to improve the situation by simultaneous measurements of CS masses and radii with higher accuracy [18–20,27,28]. It is also expected that the limits on CS radii are to be improved by new detection of gravitational wave signals from neutron star mergers.

In fact, the precise gravitational mass and radius measurements of the neutron stars are the effective ways to constraints the EOS of high dense matter in its interiors. The quite reliable mass measurement of MSP J0740+6620 [29]...
is $2.14^{+0.10}_{-0.09} M_\odot$, highly likely to be the most massive neutron star yet observed by the Shapiro delay measurement. Recently, the simultaneous measurements of gravitational mass $M$ and equatorial circumferential radius $R_{\text{eq}}$ of PSR J0030+0451 from NICER data by Miller et al. [18] and Riley et al. [19] by using independent methods to actual map of the hot region of pulsar, have inferred $[M = 1.44^{+0.15}_{-0.14} M_\odot, R_{\text{eq}} = 13.02^{+1.24}_{-1.06} \text{km}]$ and $[M = 1.34^{+0.15}_{-0.14} M_\odot, R_{\text{eq}} = 12.71^{+1.14}_{-1.19} \text{km}]$, respectively.

Theoretically, the investigations of the observed masses and extraction of the radii of CS would allow us to reveal the particle composition and phase transition of dense nuclear matter at high densities. The lack of accurate first-principle predictions at densities beyond the nuclear matter saturation density $\rho_0 \approx 0.16 \text{ fm}^{-3}$ has so far prevented the determination of the phase of matter inside CS cores. However, attempts [10, 25, 28, 30] have been made by using microscopic many body nuclear theories to model the nuclear matter EOS capable of constructing CS, containing nucleons, hyperons and quarks, under the constraint of global $\beta$-equilibrium. The EOS including hyperons and/or quarks are typically much softer than those containing just nucleons, leading to the reduction of maximum gravitational masses of CS which may or may not be compatible with recent observations of $2 M_\odot$. However, recently there are many EOS models with hyperons and especially quark matter [28, 31] which have computed maximum gravitational mass compatible with the $2 M_\odot$.

In the low energy regime the QCD coupling constant will be significantly large and experimental data have large uncertainties [32], therefore, it is not yet possible to obtain a reliable EOS of quark matter phase from the first principle of QCD. The quark matter phase of EOSs have been treated by employing phenomenological models with some basic features of QCD, such as, the MIT bag models [33–35] with a bag constant and appropriate perturbative QCD corrections and Nambu-Jona-Lasinio with chiral symmetry and its breaking [36]. Non local chiral quark model [37] and constant speed of sound model [38]. The utmost symmetric pairing state is the color flavor locked (CFL) state, which can be realized if the mass split between light quarks and strange quarks is small [39]. Since CFL state is supposed to be more stable than ordinary quark matter. Therefore, a hybrid star is considered to be constructed with CFL quark matter in the core of CS.

The motivations of the present work are to compute a set of hybrid EOSs, where the hadron phase has been calculated within the framework of energy density functionals based on the relativistic mean field theory [30] and, the quark matter phase of EOS is computed by using Quark Quasiparticle model QQPM [40] with CFL phase [34]. The medium effects are included in the cold quark matter in terms of variation in effective mass of quarks and effective bag parameters as function of chemical potential. Further a plausible set of hybrid equations of state for hadron-quark matter is employed to determine the maximum gravitational mass, equatorial radius and rotation frequency of stable stellar configuration of hybrid stars, which satisfies the constraints provided by Keplerian limits, secular axisymmetric instability (SAI), the observational data of frequencies and maximum gravitational mass $\geq 1.97 M_\odot$ of static sequences with their radii and GW170817 event of neutron stars merger. However, the EOS model NHy2 and NmQ1 which compute gravitational mass $\leq 1.97 M_\odot$ are also presented for the comparison. We further discuss the constraint of the radii of compact star measured from GW170817 BNS event as, $R_1 = 11.9^{+1.4}_{-1.3} \text{km}$ and $R_2 = 11.9^{+1.4}_{-1.4} \text{km}$ for the EOS which support CS masses more than $1.97 M_\odot$. We present the evolutionary sequences for hybrid CS with constant baryonic mass spinning down by electromagnetic and gravitational radiations and, discuss the issues relating to phase transition from hadron to quark matter phase and size of quark matter core in CS. The internal structure of rotating star with observed spin down frequencies, exhibiting shrinkage or expansion of soft core of star are discussed for constant baryonic mass. We present the theoretically computed limits for the results of spin down configurations of hybrid stars corresponding to the recently observed millisecond pulsars, for the Love number, $k_2$ and the dimensionless tidal deformability parameter, $\tilde{\Lambda}$. We approximate the hybrid star as an axisymmetric and rigid rotating body, and resort to Einstein’s theory of general relativity for a rapidly rotating star. For more than two decades, numerical methods for (axisymmetric) rotating stellar structure have been advanced by several groups [41–48]. In this work we employed the Rotating Neutron Star RNS method and code [41, 46] to calculate the properties of rapidly rotating Hybrid CSs.

The paper has been organized as, in Sect. 2, we described the theoretical framework which is used to construct the EOSs for hybrid star. The Extended field-theoretical relativistic mean field (EFTRMF) models have been employed to describe the nucleon and hyperon phases and the quark matter phase has been obtained from the QQPM with constraints from bulk nuclear matter properties and finite nuclei. The mixed phase of hybrid EOSs is obtained by using Glendenning construction based on Gibbs conditions of equilibrium. The framework of relativistic rotation of CSs are described in Sect. 3. The Sect. 4, present results and discussions for EOSs employed to construct the non-rotating and rapidly rotating hybrid stars. The rotational evolution for a constant baryonic mass is also discussed. We have also presented and discussed the theoretical results of the dimensionless tidal deformability parameters and chirp radius of hybrid star in the context of GW170817 event of Neutron Stars merger. The conclusions of the present research work are presented in Sect. 5.
2 Theoretical framework

In this section, we discuss the theoretical approaches employed to calculate sets of EOSs of dense nuclear matter in different phases. The EFTRMF model parameter BSR3 has been successfully applied in describing the structure properties of finite nuclei, properties of bulk nuclear matter at saturation densities, asymmetric nuclear matter, hadronic and hyperonic matter at high densities [30]. These model parameters have been adopted to compute EOSs and construct neutron stars and hybrid CSs. The hybrid EOSs are comprised of two separate EOSs for each phase of matter, which are combined by utilizing a Glendenning phase transition construction [49,50].

2.1 Hadron phase

In the EFTRMF model the effective Lagrangian density consists of self and mixed interaction terms for \( \sigma, \omega, \rho, \sigma^+ \) and \( \phi \) mesons. The \( \sigma, \omega, \rho, \sigma^+ \) and \( \rho \) mesons are responsible for the ground state properties of the finite and heavy nuclei. The hidden strangeness scalar isoscalar \( \sigma^+ \) meson and hidden strangeness vector isoscalar \( \phi \) meson are responsible for weak hyperon-hyperon interaction to soften the EOSs. The mixed interactions terms containing the \( \rho \)-meson field enable us to vary the density dependence of the symmetry energy coefficient and neutron skin thickness in heavy nuclei over the wide range without affecting the other properties of the finite nuclei [51,52]. Neutron skin thickness is also very important in determining the structural properties of the finite nuclei [53–55]. In particular, the contribution from the self-interaction of \( \omega \)-meson performs an important role in determining the high density behavior of EOS and structure properties of compact stars [30,56]. Whereas the inclusion of self-interaction of \( \rho \)-meson affects the ground state properties of heavy nuclei and compact stars only very marginally [56]. In the present work, we use only BSR3 [30] and IOPB-I [57] parameterization to construct the hadronic phase of the Hybrid EOS. The Lagrangian density for the EFTRMF model can be written as

\[
\mathcal{L} = \mathcal{L}_{BM} + \mathcal{L}_{\sigma} + \mathcal{L}_{\omega} + \mathcal{L}_{\rho} + \mathcal{L}_{\sigma\omega\rho} + \mathcal{L}_{em} + \mathcal{L}_{e\mu} + \mathcal{L}_{YY}. \tag{1}
\]

where the baryonic and mesonic Lagrangian \( \mathcal{L}_{BM} \) can be written as,

\[
\mathcal{L}_{BM} = \sum_B [i \gamma^\mu \partial_\mu - (M_B - g_{\sigma B} \sigma)
- (g_{\sigma B} \gamma^\mu \omega_\mu + \frac{1}{2} g_{\rho B} \gamma^\mu \tau_{B,\rho} \mu)] \Psi_B. \tag{2}
\]

Here, the sum is taken over the baryon octet (B), which consists of n, p and \( A, \Sigma^- , \Sigma^0, \Sigma^+, \Xi^- \) and \( \Xi^0 \) hyperons. For the calculation of finite nuclei properties only the neutron and proton have been considered. \( t_{B\rho} \) are the isospin matrices. The Lagrangian describing self-interactions of \( \sigma, \omega, \) and \( \rho \) mesons can be written as

\[
\mathcal{L}_\sigma = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{K}{3!} g_{\sigma N}^3 \sigma^3 - \frac{K}{4!} g_{\sigma N}^4 \sigma^4. \tag{3}
\]

\[
\mathcal{L}_\omega = -\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4!} \xi \sqrt{4!} g_{\omega N} (\omega_\mu \omega^\mu)^2. \tag{4}
\]

\[
\mathcal{L}_\rho = -\frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu + \frac{1}{4!} \xi \sqrt{4!} g_{\rho N} (\rho_\mu \rho^\mu)^2. \tag{5}
\]

The field tensors \( \omega^{\mu\nu}, \rho^{\mu\nu} \) correspond to the \( \omega \) and \( \rho \) mesons and can be defined as \( \omega^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \) and \( \rho^{\mu\nu} = \partial^\mu \rho^\nu - \partial^\nu \rho^\mu \). The mixed interactions of \( \sigma, \omega, \) and \( \rho \) mesons, \( \mathcal{L}_{\sigma\omega\rho} \), can be written as

\[
\mathcal{L}_{\sigma\omega\rho} = g_{\sigma N}^2 g_{\omega N}^2 \omega_\mu \omega^\mu \left( \frac{1}{2} \alpha_1 + \frac{1}{2} \alpha_2 \right) \tag{6}
\]

where, \( \mathcal{L}_{em} \) is the Lagrangian for electromagnetic interactions and can be expressed as

\[
\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu A_\rho A_\sigma. \tag{7}
\]

The hyperon-hyperon interaction has been included by introducing two additional isoscalar meson fields (\( \sigma^+ \) and \( \phi \)) and the corresponding Lagrangian \( \mathcal{L}_{YY} \) (\( Y = \Lambda, \Sigma, \) and \( \Xi \)) can be written as

\[
\mathcal{L}_{YY} = \frac{1}{2} \left( \partial_\mu \sigma^+ \partial^\mu \sigma^+ + m_\sigma^+ \sigma^+ \cdot \sigma^+ \right) - \frac{1}{4} S_{\mu\nu} S^{\mu\nu} + \frac{1}{2} m_\phi \phi_\mu \phi^\mu
+ \sum_Y \varepsilon_{YY} \left( g_{\sigma^+ Y} \sigma^+ - g_{\phi Y} \phi^\mu \phi_\mu \right) \Psi_Y. \tag{8}
\]

The charge-neutral neutron star matter also includes leptons such as \( e^- \) and \( \mu^- \) in addition to neutrinos, protons, and hyperons at high densities. The leptonic contributions to the total Lagrangian density can be written as,

\[
\mathcal{L}_{e\mu} = \sum_{e, \mu} \varepsilon_B (i \gamma^\mu \partial_\mu - M_\ell) \Psi_\ell. \tag{9}
\]

The equation of motion for baryons, mesons, and photons can be derived from the Lagrangian density defined in Eq. (1). The energy density of the uniform matter within the framework of EFTRMF model is given by;

\[
\mathcal{E} = \sum_{j=B,\ell} \frac{1}{\rho_j} \int_{k_i}^{k_f} k^2 \sqrt{k^2 + M_j^2} dk.
\]
\[
\begin{align*}
+ \sum_B g_{\omega B} \omega \rho_B + \sum_B g_{\beta B} \tau_3 \rho_B + \frac{1}{2} m_\sigma^2 \sigma^2 \\
+ \frac{\pi}{6} g_{\sigma N}^3 \sigma^3 - \frac{\lambda}{24} g_{\sigma N}^4 \sigma^4 + \frac{\zeta}{24} g_{\omega N}^4 \omega^4 \\
- \frac{\xi}{24} g_{\rho N}^4 \rho^4 - \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{2} m_\rho^2 \rho^2 \\
- \frac{\alpha_1}{2} g_{\omega N}^2 \omega^2 \sigma^2 - \frac{1}{2} \alpha_1^2 g_{\omega N}^2 \sigma^2 \omega^2 \\
- \frac{\alpha_2}{2} g_{\omega N}^2 \rho^2 \sigma^2 - \frac{1}{2} \alpha_2^2 g_{\rho N}^2 \sigma^2 \rho^2 \\
- \frac{1}{2} \alpha_3 g_{\omega N}^2 \rho^2 \omega^2 + \sum_B g_{\psi B} \rho_B \\
+ \frac{1}{2} m_\phi^2 \phi^2.
\end{align*}
\]

The pressure of the uniform matter is given by

\[
P = \sum_{j=b, \ell} \frac{1}{3 \pi^2} \int_{k_0}^{k_j} \frac{k^4 dk}{\sqrt{k^2 + M_j^2}} = \frac{1}{2} m_\sigma^2 \sigma^2 \\
- \frac{\kappa}{6} g_{\sigma N}^3 \sigma^3 - \frac{\lambda}{24} g_{\sigma N}^4 \sigma^4 + \frac{\zeta}{24} g_{\omega N}^4 \omega^4 \\
+ \frac{\xi}{24} g_{\rho N}^4 \rho^4 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 \\
+ \frac{1}{2} \alpha_1^2 g_{\omega N}^2 \sigma^2 \omega^2 + \frac{1}{2} \alpha_1 g_{\omega N}^2 \omega^2 \sigma^2 \\
+ \frac{1}{2} \alpha_2^2 g_{\rho N}^2 \sigma^2 \rho^2 + \frac{1}{2} \alpha_2 g_{\rho N}^2 \rho^2 \sigma^2 \\
+ \frac{1}{2} \alpha_3 g_{\omega N}^2 \rho^2 \omega^2 - \frac{1}{2} m_\sigma^2 \sigma^2 \\
+ \frac{1}{2} m_\phi^2 \phi^2.
\]

Here, the sum is taken over the B, complete baryon octet which consists of nucleons, \( \Lambda, \Sigma, \Xi \) hyperons and \( l \), leptons which consists of \( e^-, \mu^- \).

The composition of nuclear matter species \( i = n, p, \Lambda, \Sigma^-, \Sigma^0, \Xi^+, \Xi^0, e^-, \mu^- \) at fixed baryon number density \( \rho_B = \sum_i B_i \rho_i \) is determined in such a way that the charge neutrality condition,

\[
\sum_i q_i \rho_i = 0, \quad (12)
\]

and the chemical equilibrium conditions

\[
\mu_i = B_i \mu_n - q_i \mu_e, \quad (13)
\]

are satisfied, where \( B_i \) and \( q_i \) denote baryon number and electric charge of the species \( i \).

The symmetry energy of symmetric nuclear matter can be expanded as a Taylor series around the nuclear saturation density \( \rho_0 \) as,

\[
E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L \left( \frac{\rho - \rho_0}{3 \rho_0} \right) + \frac{K_{\text{sym}}}{2} \left( \frac{\rho - \rho_0}{3 \rho_0} \right)^2 + \ldots, \quad (14)
\]

The nuclear matter symmetry energy coefficient \( J \), slope \( L \) and its curvature \( K_{\text{sym}} \) at nuclear saturation density \( \rho_0 \) can be defined as,

\[
J = E_{\text{sym}}(\rho_0), \quad (15)
\]

\[
L = 3\rho_0 \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \bigg|_{\rho=\rho_0}, \quad (16)
\]

\[
K_{\text{sym}} = 9 \rho_0^2 \frac{\partial^2 E_{\text{sym}}(\rho)}{\partial \rho^2} \bigg|_{\rho=\rho_0}. \quad (17)
\]

The nuclear matter incompressibility coefficient at \( \rho_0 \) can be defined as,

\[
K_0 = 9 \rho_0^2 \frac{\partial^2 E_0(\rho)}{\partial \rho^2} \bigg|_{\rho=\rho_0}, \quad (18)
\]

where \( E_0 \) is the energy per nucleon of symmetric nuclear matter, which is denoted by \(-B/A\).

The several models of relativistic mean field (RMF) effective lagrangian density consisting of nonlinear \( \sigma, \omega \) and \( \rho \) terms and cross terms have been analyzed for nucleonic matter [58] and nucleonic along with hyperonic matter [59,60] and accosted with the constraints of nuclear matter properties and astrophysical observations of CS masses [16,19,20].

Only RMF models BSR [30] with \( \zeta = 0 \) (please see Eq. (4) and NL3oW [61] can sustain the condition of maximum mass \( M \geq 2.0M_\odot \) when hyperons are included in the EOSs with appropriate meson-hyperon couplings, otherwise, the inclusion of hyperons may lead for the famous hyperon puzzle. The EOSs of dense nuclear matter is very sensitive to \( \zeta \) coupling term, as the magnitude of \( \zeta \) parameter increases the EOSs become softer. However, many RMF models [62] without the inclusion of hyperons are satisfying the constraints of astrophysical observations obtained from binary neutron star merger event GW170817. In the present work, we have used BSR3 model from BSR family [30] which can provide stiif EOS to construct CS mass around \( 2.0M_\odot \) with inclusion of hyperon. For comparison the IOPB-I model [57] has also been used in the calculation, do not satisfy constraint of CS mass \( 2.0M_\odot \) with inclusion of hyperons using same meson-hyperon couplings. The nuclear matter properties at saturation density and models parameter of BSR3 and IOPB-I along with neutron skin thickness \( \Delta r \) of \( ^{208}\text{Pb} \) nucleus are presented in Table 1.

The hyperonic models of EOSs consist of hyperons \( Y = \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^0, e^-, \mu^- \) hidden strangeness isoscalar scalar \( \sigma^* \) meson and hidden strangeness vector isoscalar \( \phi \) meson. The vector meson hyperon coupling parameters \( g_{\gamma Y} \) are calculated using SU(6) symmetry model as [30,63];

\[
\frac{1}{2} g_{\omega N} = \frac{1}{2} g_{\omega \omega} = \frac{1}{2} g_{\omega \Sigma} = g_{\omega \Xi}, \quad 2 g_{\phi \Lambda} = 2 g_{\phi \Sigma} = g_{\phi \Xi} = \frac{3}{2} g_{\phi N} = g_{\phi N} \quad \text{and the following coupling parameters as} \quad g_{\rho N} = \frac{1}{2} g_{\rho \Sigma} = g_{\rho \Xi}, \quad g_{\rho \Lambda} = 0.
\]

The \( \sigma^* \) meson does not cou-
The mesons are taken to be $m_{\rho} = 782.5$ MeV, $m_{\omega} = 763$ MeV, $m_{\sigma}^* = 975$ MeV, and $m_{\phi} = 1020$ MeV for BSR3 parameterization. The masses of the mesons are taken to be $m_{\rho} = 782.187$ MeV, $m_{\omega} = 762.468$ MeV, $m_{\sigma}^* = 975$ MeV, and $m_{\phi} = 1020$ MeV for IOPB-I parameterization. The mass of nucleon, $M_N = 939$ MeV, and the masses of hyperons, $M_{\Lambda} = 1116$ MeV, $M_{\Sigma} = 1193$ MeV, and $M_{\Xi} = 1313$ MeV. The values in parentheses, $\Sigma, \Sigma^+, \Sigma^0, \Xi, \Xi^+$, $\Xi^0$, are multiplied by $10^2$.

### Table 1

| EOS     | $\Delta r$ | $g_0N$ | $g_{\omega N}$ | $g_{\sigma N}$ | $\sigma$ | $\lambda$ | $\xi_{\Sigma}$ | $\xi_{\Sigma^0}$ | $\xi_{\Sigma^+}$ | $\xi_{\Xi}$ | $\xi_{\Xi^0}$ | $\xi_{\Xi^+}$ | $\mu$   |
|---------|-------------|--------|----------------|----------------|---------|----------|---------------|----------------|----------------|-------------|-------------|-------------|---------|
| BSR3    | 0.20        | 10.442 | 13.5223        | 11.1257        | 2.4304  | -0.0427  | 0.1812        | 1.5977         | 2.9666        | 1.2530      | 0.0972      | 497.8348    | 4.80    |
| IOPB-I  | 0.221       | 10.3966| 13.3509        | 11.1257        | 1.8584  | -0.7621  | 0.0           | 0.0            | 0.0            | 0.0          | 500.4870    |         |

The value of $g_{\omega N}$ is taken from SU(6) model. The values of $U^N_f$ potentials are chosen from the experimental results of hypernuclei: $U^N_A = -28$ MeV, $U^N_{\Sigma} = +30$ MeV, and $U^N_{\Xi} = -18$ MeV [64]. In the present work we define the constant ratio $X_{\omega N} = \omega g_{\omega N}/g_{\sigma N}$, here $m$ represent $\omega$ and $\sigma$ and the values $a_\omega = a_{\sigma} = 1$ and $a_{\omega N}$ = 2. In order to satisfy the constraints of astrophysical observation of CS mass 2.0M⊙ with hyperon in nuclear matter of EOSs, we choose $X_{\omega N} = 0.8$ and the values of various hyperon-meson coupling parameters are obtained as, $g_{\omega N} = 0.4714$, $g_{m N} = 0.9428$, $g_{\sigma N} = -1.6666$, $g_{\sigma N} = -1.3333$, $g_{\omega N} = 0.8333$, $g_{\omega N} = 0.4166$, $g_{\rho N} = g_{\rho N} = 0.30 = g_{\rho N} = g_{\rho N} = 1 = g_{\rho N} = g_{\rho N} = 0.5$ where $Y_1$ represents the hyperons as, $\Lambda, \Sigma^+$, $\Sigma^0$, $\Sigma^+$, $\Xi^0$; $Y_2$ representing the hyperons $\Sigma^-$, $\Sigma^0$.

The theoretically computed values of binding energy per nucleon B/A, nuclear matter incompressibility coefficient $K_{sym}$, effective nucleon mass defined as, $M_N^{sym} = M_N - g_{\sigma N} \sigma - g_{\tau N} \tau^*$, symmetry energy coefficient (J) and its slope (L) at nuclear saturation density $\rho_0$ for BSR3 parameter set is $B/A = 16.086$ MeV, $K_0 = 230.556$ MeV, $M_N^{sym} = 0.604$, $J = 32.65$ MeV and $L = 70.58$ MeV at $\rho_0 = 0.1496$fm$^{-3}$ and for IOPB-I parameter set is $B/A = 16.10$ MeV, $K_0 = 226.65$ MeV, $M_N^{sym} = 0.593$, $J = 33.30$ MeV and $L = 63.85$ MeV at $\rho_0 = 0.149$fm$^{-3}$.

The nuclear matter density $\rho = \rho_p + \rho_n$ and density dependence of the symmetry energy (Eq. (14)) is strongly influence the EOS of neutron star matter and therefore, the mass-radius relationship of neutron star. The experimental constraints have been extracted for the EOS [65] of symmetric nuclear matter (SNM) and pure neutron matter (PNM). The SNM and PNM EOSs obtained by employing BSR3 parameters [30] are well within the limits of EOS [65]. Theoretically, the constraints for J and slope L parameters have been obtained from nuclear masses and binding energies [66], the comprehensive analysis [67,68] by considering excitation energies to isobaric analog states and charge invariance with measurements of skin thickness, elastic and Quasiparticle energy spectrum. The total thermodynamical

2.2 Quark matter phase

In the last few years the study of color superconducting phase in quark-gluon plasma has been drawing a great interest in discussing the possible states of quark matter. At QCD perturbative regime the attractive quark interaction introduces instabilities in the Fermi surface, producing a gap in the quasiparticle energy spectrum.
potential density of CFL quark matter may written as \[ \Omega(\mu, \mu_e) = \Omega_{QPM}(\mu) + \Omega(\mu_e) + \Omega_{CFL}(\mu), \] (20)
where \( \mu \) is the average chemical potential of quarks, the first term in Eq. (20) is obtained from the quarks using QQPM model, \( \Omega(\mu_e) \) is the thermodynamical potential of electrons and, the \( \Omega_{CFL}(\mu) \) is the contribution from CFL pairing gap of quark phase. The contribution to first term in Eq. (20) is given by [40],
\[
\Omega_{QPM}(\mu) = -\frac{1}{\pi^2} \sum_i \frac{1}{8} \left[ v_i \sqrt{v_i^2 + m_i^4} + m_i^4 \ln \left( \frac{v_i^2 + m_i^4 + m_i^2}{m_i^4} \right) \right], \tag{21}
\]
where \( v_i \) is the fermi momentum of particle type \( i = u, d \) and \( s \). The contribution to second term in Eq. (20) is given by,
\[
\Omega(\mu_e) = \frac{1}{\pi^2} \int \sqrt{v^2 - m_e^2} v^2 \ln \left( \frac{v^2 + m_e^2 - \mu_e}{v^2} \right) dv. \tag{22}
\]
The CFL gap pairing term contribution \[33,75\] is given by,
\[
\Omega_{CFL}(\mu) = -\frac{3\Delta^2 \mu^2}{\pi^2}, \tag{23}
\]
where \( \Delta \) is CFL color superconducting gap parameter of CFL phase of quark matter. Medium effects play an important role in describing the properties of quark matter via the concept of effective masses. Effective mass of quarks is taken to be \[76–78\],
\[
m_i^4 = m_{i0}^4 + \sqrt{m_{i0}^2 \frac{m_i^2}{2} + \frac{g^2 \mu_i^2}{6\pi^2}}, \tag{24}
\]
where \( m_{i0}, \mu_i \) and \( g \) is the current quark mass, quark chemical potential, strong interaction coupling constant respectively. In actual practice the strong coupling constant is running \[79–81\] which is having a phenomenological expression such as
\[
g^2(T = 0, \mu_i) = \frac{48}{29} \frac{1}{\Lambda^2} \left[ \ln \left( \frac{0.8 \mu_i^2}{A^2} \right) \right]^{-1}, \tag{25}
\]
where \( \Lambda \) is the QCD scale-fixing parameter. In the present calculation, the value is taken to be 200 MeV. In Fig. 1, the variation of coupling strength parameter with chemical potential is shown. It is observed that range of coupling strength parameter is from 3 to 5 for the chemical potential varying in the range of 510–320 MeV approximately. Also, it is evident from the figure that interaction strength parameter decreases with increase in chemical potential \[82\]. At zero temperature, number densities for all three flavor of quark considered are the same and can be obtained as,
\[
n_i = \frac{d_i v_i^3}{6\pi^2} + \frac{2\Delta^2 \mu}{\pi^2}, \tag{26}
\]
where \( d_i \) is the degeneracy factor with \( 2(\text{spin}) \times 3(\text{color}) = 6 \) for quarks and \( v_F = v_u = v_d = v_s \).
\[
v_F = \sqrt{\left( 2\mu - \sqrt{\mu^2 + \frac{m_u^2 - m_d^2}{3}} \right) - m_u^2}^{1/2}, \tag{27}
\]
where \( \mu = \frac{\mu_u + \mu_d + \mu_s}{3} \) is common fermi momentum of the quark system which depends on the mass of the three quark flavors. For \( m_u = m_d = 0 \), common fermi momentum becomes,
\[
v_F = 2\mu - \sqrt{\mu^2 + \frac{m_u^2}{3}}. \tag{28}
\]
For the quark matter the energy density becomes,
\[
\mathcal{E}_{QM} = \frac{1}{\pi^2} \sum_i \frac{1}{8} \left[ v_i \sqrt{v_i^2 + m_i^4} + m_i^4 \ln \left( \frac{v_i^2 + m_i^4 + m_i^2}{m_i^4} \right) \right] + B^*, \tag{29}
\]
where \( B^* \) is the effective bag function and, can be written as,
\[
B^* = B_0 + \sum_i B_i(\mu_i). \tag{31}
\]
The introduction of \( B^* \) is done to show the automatic confinement characteristic in the model, where \( B_0 \) denotes the usual MIT bag constant which corresponds to the energy difference between the perturbative vacuum inside the bag and the true
vacuum at the outside. The \( B_i(\mu_i) \), chemical potential dependent bag constant term need to be determined. The effective bag constant \( B^\ast \) is chemical potential dependent, Eq. (29), all thermodynamical relations must be reformulated [83].

The second term is important even in MIT bag model when employing density dependent bag constant \( B(n_i) \), in this case, the extra term was \( n_i d B/\mathbf{d} n_i \) [34,84], and considered in the calculation of EOS of hadron-quark phase transition in nuclear dense matter. The \( \mu_i \)-dependent part of effective bag function is calculated at zero temperature as [83],

\[
B_i(\mu_i) = - \int^{\mu_i}_{m_i^e} \frac{\partial \Omega_i}{\partial m_i} \mathbf{d} m_i. 
\]

With the above quark mass formulae and thermodynamic treatment, one can get the properties of bulk CFL quark matter. In the CFL phase, the three flavor of quarks satisfy the following conditions:

(i) they have equal fermi momenta, which minimize the free energy of the system.

(ii) they have equal number densities \( n_i \), as a consequence of the first condition, which means that \( n_i = n_B \), with \( n_B \) as baryon number density and \( \mu_i = \mu \). Therefore, CFL phase is neutralized without any electrons.

For quark matter to exist in stable beta-equilibrium at zero external pressure, the Witten-Bodmer conjecture [5,85] has to be satisfied. It has been conjectured that the strange quark matter may be the true state of hadronic matter [85], because its energy per baryon could be less than the most stable \( ^{56}\text{Fe} \) nucleus. In the present calculation the parameters \( \Delta \) and \( B_0 \) are constrained by requiring the minimum energy per baryon of quark matter smaller than 930 MeV. Therefore, the quark matter phase of EOSs have been computed by employing Quark Quasiparticle model by considering the color superconducting phase with CFL, \( \Delta = 48 \) MeV and bag constant \( (B_0)^{\frac{3}{2}} \leq 146 \) MeV in the quark matter phase of hybrid EOS to support maximum gravitational masses of hybrid CS \( M_{\odot} \geq 1.97M_{\odot} \). Whereas in the present calculation we consider all three quark masses as, \( m_u = m_d = 4 \) MeV/c\(^2\) and \( m_s = 95 \) MeV/c\(^2\).

2.3 Mixed phase

We construct the EOS of mixed phase (MP) made up of the hadron phase (HP) and CFL quark matter phase by employing the Glendenning construction [49,50] for hybrid compact star. The evolution of mixed phase is favored when the surface tension and coulomb interaction between hadronic and quark matter is smaller [75] and negligible. As the calculation of surface tension is very model dependent [75,86], so for higher values of surface tension, the phase transition is sharp to be constructed with Maxwell construction and for the low values phase transition is continuous to be constructed with Glendenning construction. Since the value of surface tension is not established yet, both of the methods of mixed phase construction are equally valid. But, we adopted the Gibbs construction here. The equilibrium chemical potential of the mixed phase corresponding to the intersection of the two surfaces representing hadron and quark matter phase can be calculated from the Gibbs condition for mechanical and chemical equilibrium at zero temperature which reads as,

\[
P_{HP}(\mu_e, \mu_n) = P_{CFL}(\mu, \mu) = P_{MP}, 
\]

where \( P_{HP} \), \( P_{CFL} \), and \( P_{MP} \) are the pressures of hadron phase, CFL phase and mixed phase, respectively. In mixed phase, we consider chemical equilibrium at the hadron-quark interface as well as inside each phase [87], so that Eq. (13) implies

\[
\begin{align*}
\mu_u + \mu_e &= \mu_d = \mu_s, \\
\mu_p + \mu_e &= \mu_n = \mu_A = \mu = \mu_s + 2\mu_d, \\
\mu_S^- + \mu_p &= 2\mu_n, \quad \mu_S^- + \mu_p &= 2\mu_n.
\end{align*}
\]

The \( \Sigma^+ \) and \( \Sigma^0 \) hyperons do not appear at the relevant densities in the present research work, because the values of \( g_{\Sigma N} \) and \( g_{\Sigma Y} \) are obtained by fitting the expressions for the hyperon-nucleon potential, Eq. (19) and optimally smaller values of nuclear matter incompressibility coefficient \( K = 230.556 \) MeV and \( 222.65 \) MeV for BSR3 and IOPB-I parameterizations, respectively. However, all species of hyperons appear well below \( 7\rho_0 \) [88] for the TM1 parameterization of the FTRMF model, may be due to the large value of the nuclear matter incompressibility coefficient \( K = 281 \) MeV. Alternatively, the appropriately calibrated parameters of FTRMF models, not only do the variations in the properties of the neutron stars but the chemical compositions of nuclear matter EOS can also become different. In the mixed phase the local charge neutrality condition is replaced by the global charge neutrality which means that both hadron and quark matter are allowed to be charged separately. The condition of the global charge neutrality determine the volume fraction \( \chi \) of CFL phase and, can be obtained by using,

\[
\chi \rho^{CFL}_c + (1-\chi)\rho^{HP}_c = 0,
\]

where, the \( \rho^{CFL}_c \) and \( \rho^{HP}_c \) are the charge densities of CFL phase and hadron phase of dense matter, respectively. The value of the \( \chi \) increases from zero in the pure hadron phase to \( \chi = 1 \) in the pure quark phase. The energy density \( \varepsilon_{MP} \) and the baryon density \( \rho_{MP} \) of the mixed phase can be calculated as,

\[
\varepsilon_{MP} = \chi \varepsilon_{CFL} + (1-\chi)\varepsilon_{HP},
\]

\[
\rho_{MP} = \chi \rho_{CFL} + (1-\chi)\rho_{HP}.
\]
The particle fractions profile of hadrons and leptons in the β-equilibrium hadron matter phase and neutral CFL matter as a function of pressure. The vertical dashed magenta lines A and B represent the particle structure of sharp phase transition at which the pressures of neutral CFL matter and hadron matter are equal.

The mixed phase of EOSs have been computed by employing the procedure explained above and Eqs. (33–38). In Fig. 2, we present the particle fractions of the hadrons and leptons in the homogeneous electrically neutral hadron phase and neutral CFL matter as a function of pressure for NHymQ4 EOS. The vertical dashed magenta lines A and B represent the particle structure of sharp phase transition at which the pressures of neutral CFL matter and hadron matter are equal. In the mix phase region, the CFL quark matter slowly appear along the dashed vertical line A, as the hadrons and leptons particle fraction disappear along dashed vertical line B. The particle fraction of leptons start decreasing as the negatively charged hyperon Σ⁻ appear in hadronic matter at its threshold baryon density 0.36 fm⁻³.

3 Rotating relativistic stars

The structure of a rapidly rotating CS is different from the static one, since the rotation can strongly deform the star. We assume CS are stationarily rotating, and have circular, axisymmetric structure. Therefore the space-time metric used to model a rotating star can be expressed as [48],

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \]
\[ = -e^{\gamma} a^2 dt^2 + e^{\beta} (dr^2 + r^2 d\theta^2) \]
\[ + e^{\gamma} a^2 r^2 \sin^2 \theta (d\phi - \omega dt)^2, \]
\[ = g_{tt} dt^2 + 2g_{t\phi} dtd\phi + g_{\phi\phi} d\phi^2 + g_{rr} (dr^2 + r^2 d\theta^2), \]

where the potentials \( \gamma, \rho, \beta, \omega \) are functions of \( r \) and \( \theta \) only. The matter inside the star is approximated by a perfect fluid so that the energy-momentum tensor is given by,

\[ T^{\mu\nu} = (E + P) u^\mu u^\nu + P g^{\mu\nu}, \]

where \( E, P, u^\mu \) and \( g^{\mu\nu} \) are the energy density, pressure, four-velocity and metric tensor (defined in Eq. (40)) respectively.

In order to solve Einstein’s field equation for the potentials \( \gamma, \rho, \beta \) and \( \omega \), we have used the RNS code [44,46,48] for calculating the properties of a rotating star.

4 Results and discussions

4.1 Equations of state and non-rotating hybrid stars

In this section, we present the results for a set of EOSs of hybrid stars in the hadron phase, quark phase and mixed phase. In epitome, we employed Baym-Pethick-Sutherland EOS [89] for low density regime from outer crust baryon number density, \( \rho_b = 6.3 \times 10^{-12} \) fm⁻³ up to the pasta phase \( \rho_b = 9.4 \times 10^{-12} \) fm⁻³ that includes the data and description of the neutron drip. The crust region and core region of EOSs have been matched by employing the cubic interpolation method that offers true continuity between crust and core. As such in the present matching of EOS, we have generated 10 new data points between the regions of BPS EOS and our EOS, where 5 data endpoints from both side of the segments are considered for matching of EOS. In order to describe the EOS in liquid core of hybrid stars from its inner crust \( \rho_{\text{crust}} \) up to outer core \( \rho_b \approx 0.35 \) fm⁻³, we used an improved and consistent nuclear matter EOS in β-equilibrium based on Extended Field Theoretical Relativistic Mean Field model parameterizations BSR3 [30] and IOPB-I [57]. The description for the construction of mixed phase is discussed in Sect. 2C.

The set of EOSs employed in present work is shown in Table 2, where Nucl1 and Nucl2 are the nucleonic matter EOSs computed with parameters BSR3 and IOPB-I, respectively. The EOSs NHy1 and NHy2 are represented by the compositions of nucleons and hyperons, where hyperons appeared at a threshold baryon number density \( \rho_b \approx 0.35 \) fm⁻³. The EOSs namely NmQ1, NmQ2 and NmQ3 are composed of nucleons and quarks in β-equilibrium with mixed phase where quark matter phases appear at \( \approx 3 \rho_0 \). Finally, the EOS NHymQ4 has particle composition of nucleons, hyperons and quarks in β-equilibrium where the \( \Lambda \) hyperon appeared at the threshold density of \( \rho_b \approx 0.35 \) fm⁻³ and the CFL quark matter phase transition occur at \( \rho_b = 0.64 \) fm⁻³. In Table 2, we list our EOSs and their particles composition, the brief description of theoretical models of dense matter and its parameters, the maximum gravitational mass, \( M_G(M_\odot) \) of the compact stars, radius of the compact stars, \( R_{\text{max}} \), central energy density \( \varepsilon_c \) corresponding to maximum gravitational mass, the values of \( R_{1.4} \) and \( A_{1.4} \) tidal deforma-
Table 2  The equations of state and their particles composition, the brief description of theoretical models of dense matter and their parameters along with the values of CFL gap parameter $\Delta$, bag constant $(B_0)^{1/4}$ are displayed. The maximum gravitational mass $M_{G}(M_\odot)$, radius corresponding to maximum mass $R_{\text{max}}$ and central energy density, $\varepsilon_c$ corresponding to maximum gravitational mass of the non-rotating compact stars, the values for $R_{1.4}$ radius and $\Lambda_{1.4}$ tidal deformability parameter at $1.4M_\odot$ mass of compact stars are also presented. The data of EOSs are available with authors through email.

| No. | EOS          | Model Parameters                                  | $M$ ($M_\odot$) | $R_{\text{max}}$ (km) | $\varepsilon_c$ ($\times 10^{15}$ g/cm$^{-3}$) | $R_{1.4}$ (km) | $\Lambda_{1.4}$ |
|-----|--------------|---------------------------------------------------|-----------------|------------------------|-----------------------------------------------|----------------|----------------|
| 1   | Nucl1        | n,p; BSR3                                          | 2.38            | 12.02                  | 1.950                                         | 13.57          | 632.94         |
| 2   | Nucl2        | n,p;IOPB-I                                         | 2.16            | 12.07                  | 1.920                                         | 13.52          | 638.38         |
| 3   | NHy1         | n, p, $\Lambda, \Sigma, \Xi$; BSR3                | 2.00            | 11.58                  | 2.203                                         | 13.57          | 632.94         |
| 4   | NHy2         | n, p, $\Lambda, \Sigma, \Xi$, IOPB-I              | 1.84            | 11.76                  | 2.080                                         | 13.52          | 612.69         |
| 5   | NmQ1         | n, p, u, d, s, $\Delta = 48$ MeV, BSR3             | 1.97            | 11.88                  | 2.045                                         | 13.70          | 609.94         |
| 6   | NmQ2         | n, p, u, d, s, $\Delta = 48$ MeV, BSR3             | 2.04            | 12.32                  | 1.862                                         | 14.32          | 573.38         |
| 7   | NmQ3         | n, p, u, d, s, $\Delta = 48$ MeV, IOPB-I           | 1.99            | 11.91                  | 2.031                                         | 13.37          | 500.43         |
| 8   | NHymQ4       | n, p, $\Lambda, \Sigma, \Xi$, +u,d,s quarks        | 2.01            | 12.05                  | 1.923                                         | 13.57          | 542.13         |

The maximum gravitational mass of non-rotating hybrid stars varies in the range of 2.38–1.84$M_\odot$ and radius varies as 11.58–12.32 km, whereas the EOS models Nucl1, Nucl2, NHy1, NmQ2, NmQ3 and NHymQ4 satisfies the constraints of the recently extracted astrophysical observations of gravitational mass and radius of compact stars [29,90–93]. The computed values of $R_{1.4}$ varies as 13.37–14.32 km and their tidal deformability $\Lambda_{1.4}$ lies in range from 500–638. These results reasonably satisfy the constraints anticipated from the 90 % confidence limit on $\Lambda_{1.4} \leq 800$ extracted from the GW170817 signal [94] translates into a corresponding upper limit on the radius of a 1.40$M_\odot$ neutron star of $R_{1.4} \leq 13.76$ km [91]. Adopting these constraints, EOS NmQ2 is ruled out by these radius constraint and EOS and NHy2 is ruled out by the maximum mass limit of 2.0$M_\odot$. EOSs namely Nucl2, NHy1, NmQ3 and NHymQ4 satisfy the recent mass-radius measurement constraints of the PSR J0740 + 6620 by the NICER detectors [1,2]. The EOS NmQ1 is in close agreement with mass constraint of [1,2] but ruled out by the radius constraints.

In Fig. 3, we present the variation of theoretically computed pressure (MeVfm$^{-3}$) as a function of energy density $\varepsilon$ (MeVfm$^{-3}$) for the EOSs. In the upper panel, we present the EOSs Nucl1, Nucl2, NmQ1, NmQ2 and NmQ3 represented by black solid curve, brown dotted curve, orange double-dash-dotted curve, red small-dashed curve and indigo long-dashed dotted curve respectively. The open colored circles represent the boundaries of mixed phase consisting of nucle-
ons and quarks in $\beta$-equilibrium. Further, the pressure at densities $2\rho_0$ and $6\rho_0$ of Nuc1 EOS are $5.25 \times 10^{34}$ dyne/cm$^2$ and $9.648 \times 10^{35}$ dyne/cm$^2$, respectively, are comparable with the recently measured pressure at same densities [3]. However, in the present work the magnitude of pressure is decreasing at $6\rho_0$, as the EOSs become softer with the appearance of hyperons and quark matter phases and, the pressure at $6\rho_0$ for EOS of NHymQ4 model becomes $4.3 \times 10^{35}$ dyne/cm$^2$.

In Fig. 4, we present the results for relationship between gravitational mass and radius of non-rotating compact star for various EOSs. The region excluded by causality, light green solid line and rotation constraints of neutron star XTE J1739-285 solid maroon line are given. The mass limits of pulsars PSR J1614-2230 and PSR J0348+0432 are plotted for comparison. The limits on compact star mass and radius from Özel’s analysis of EXO 0748-676 with $1\sigma$ (dark solid black line) and $2\sigma$ (extended black line) error bars are also shown. The gravitational mass limit $1.74 \pm 0.04M_\odot$ of pulsar PSR J1903+0327 [21] obtained with relativistic analysis of timing observations of the pulsar are also presented. An accurately determined mass, $M = 1.54 \pm 0.03M_\odot$ ($1\sigma$ error) of the millisecond pulsar PSR J1909-3744 [140] is also presented. The pink and grey regions separated by black dotted lines represent M-R relationship corresponding to the constraints of $1\sigma$ and $2\sigma$, respectively and, yellow solid line represents the M-R relationship obtained from EOS QMC+Model A [95].

The gravitational mass limit $1.74 \pm 0.04M_\odot$ of pulsar PSR J1614-2230 and PSR J0348+0432 are plotted for comparison. The limits on compact star mass and radius from Özel’s analysis of EXO 0748-676 with $1\sigma$ (dark solid black line) and $2\sigma$ (extended black line) error bars are also shown. The gravitational mass limit $1.74 \pm 0.04M_\odot$ of pulsar PSR J1903+0327 [21] obtained with relativistic analysis of timing observations of the pulsar are also presented. An accurately determined mass, $M = 1.54 \pm 0.03M_\odot$ ($1\sigma$ error) of the millisecond pulsar PSR J1909-3744 [140] is also presented. The pink and grey regions separated by black dotted lines represent M-R relationship corresponding to the constraints of $1\sigma$ and $2\sigma$, respectively and, yellow solid line represents the M-R relationship obtained from EOS QMC+Model A [95].

The gravitational mass limit $1.74 \pm 0.04M_\odot$ of pulsar PSR J1614-2230 and PSR J0348+0432 are plotted for comparison. The limits on compact star mass and radius from Özel’s analysis of EXO 0748-676 with $1\sigma$ (dark solid black line) and $2\sigma$ (extended black line) error bars are also shown. The gravitational mass limit $1.74 \pm 0.04M_\odot$ of pulsar PSR J1903+0327 [21] obtained with relativistic analysis of timing observations of the pulsar are also presented. An accurately determined mass, $M = 1.54 \pm 0.03M_\odot$ ($1\sigma$ error) of the millisecond pulsar PSR J1909-3744 [140] is also presented. The pink and grey regions separated by black dotted lines represent M-R relationship corresponding to the constraints of $1\sigma$ and $2\sigma$, respectively and, yellow solid line represents the M-R relationship obtained from EOS QMC+Model A [95].

The rotational frequency is a directly measurable physical quantity of the pulsars, and the Keplerian (mass-shedding) frequency $f_K$ is one of the most studied physical quantities for rotating stars [98–101]. In Table 3, we present the structural properties of rotating compact stars at Keplerian frequency. We present the values of maximum gravitational mass $M_{\text{max}}(M_\odot)$ of rigidly rotating star, equatorial radius $R_{\text{max}}$(km), central energy density $E_c(\times 10^{15}$ g/cm$^{-3}$), the maximum Keplerian frequency $f_K$ (Hz) and the empirical approximation of maximum frequency $f_{\text{max}}$. The empirical formula [102] of $f_{\text{max}}$ is the correspondence between two extremely configurations of the static configuration with a maximum allowable mass, $M_{\text{max}}$, $R_{\text{max}}$, and stably rotating configuration with a maximum allowed frequency, can be written as,

$$f_{\text{max}} = f_0 \left( \frac{M_{\text{max}}}{M_\odot} \right)^{\frac{1}{2}} \left( \frac{R_{\text{max}}}{10 \text{ km}} \right)^{-\frac{1}{2}},$$

where $M_{\text{max}}$ and $R_{\text{max}}$ are the maximum gravitational mass and radius of the static configuration, respectively. The $f_0$ is a constant frequency equals to $1.22$ kHz, which does not depend on the theoretical model of EOS. This formula for $f_{\text{max}}$ is valid for all CSs composition of hadrons in $\beta$-equilibrium, self-bound quark matter stars and the hybrid compact stars. The Fig. 5, represents the theoretical results for the variation of gravitational mass as a function of equatorial radius for various model of EOSs. In Fig. 6, we presented...
Table 3 The structural properties of rotating compact stars, the maximum gravitational mass $M_{\text{max}}(M_\odot)$ and its corresponding circumferential radius $R_{\text{max}}$(km), central energy density $\mathcal{E}_c(\times 10^{15}\text{g/cm}^{-3})$, the maximum Keplerian frequency $f_K$(Hz), the approximate value of maximum frequency $f_{\text{max}}$ as defined in Eq. (42) are shown.

| Nucl1 | Nucl2 | NHy1 | NHy2 | NmQ1 | NmQ2 | NmQ3 | NHymQ4 |
|-------|-------|------|------|------|------|------|--------|
| $M_{\text{max}}/M_\odot$ | 2.87 | 2.60 | 2.39 | 2.20 | 2.32 | 2.43 | 2.37 | 2.39 |
| $R_{\text{max}}$(km) | 16.06 | 16.26 | 16.22 | 16.57 | 16.42 | 16.96 | 16.31 | 16.62 |
| $\mathcal{E}_c(\times 10^{15}\text{g/cm}^{-3})$ | 1.67 | 1.70 | 1.75 | 1.67 | 1.72 | 1.60 | 1.72 | 1.63 |
| $f_K$(Hz) | 1488 | 1396 | 1352 | 1261 | 1313 | 1275 | 1313 | 1304 |
| $f_{\text{max}}$(Hz) | 1428 | 1352 | 1381 | 1294 | 1306 | 1262 | 1321 | 1290 |

Fig. 5 The variation of gravitational mass as a function of equatorial radius with Keplerian configurations for various EOSs.

Keplerian frequency as a function of gravitational mass for EOSs used in the present work. It can be observed from Fig. 6 that the Keplerian frequency increases monotonically both for hadronic star as well as hybrid star as a function of gravitational mass ($M_G$). The maximum gravitational mass and Keplerian frequencies of the hybrid compact stars decreases as the hyperons and quarks appears in the inner core of star.

The stability criterion of static CS can be performed by the analysis of the extremum i.e. Mathematical Maxima (MaM) of a basic parameter $M_G$ of static stars as a function of any variable which parameterize the static stellar sequences such as central energy density, pressure or circumferential radius $R_{eq}$. The onset of instability for static configuration is determined by the condition, $\left(\frac{\partial M_G}{\partial \mathcal{E}_c}\right)_J = 0$. The curves of gravitational mass and radius relationship shown in Fig. 4 should stop at its extremum which gives us maximum gravitational mass of the static configurations. For rotating neutron stars, the onset of the secular axisymmetric instability (SAI) is determined by the following relation [48, 103],

$$\left(\frac{\partial M_G}{\partial \mathcal{E}_c}\right)_J = 0. \tag{43}$$

In order to understand SAI conditions defined in Eq. (43), we have considered a sequences of stellar models based on NHymQ4 model of EOS, in Fig. 7 left panel the gravitational mass is plotted as a function of central energy density for the fixed values of the angular momentum $J$. Along the different curves of fixed $J$ the gravitational mass increases with increasing central energy density to gain its maximum value. The blue dashed line is representing the SAI, where $\left(\frac{\partial M_G}{\partial \mathcal{E}_c}\right)_J = 0$ and, separate the regions of stable sequences and unstable sequences of the hybrid CS. The green solid line represents the Kepler configuration of stellar model and, above this curve there is no equilibrium for rigidly rotating hybrid stars.

In the right panel Fig. 7, the gravitational mass is plotted as a function of circumferential equatorial radius at various fixed rotation frequencies. The stable configurations are constrained by the Kepler and secular axisymmetric instability conditions at large and small radius, respectively. At a low frequency 465.1 Hz, the lower boundary of mass $M$ is fixed by the Kepler condition and upper boundary by SAI condition. As the frequency of the known pulsars increases start-
The baryonic mass of evolutionary sequences are kept fixed with a step size of 0.2M⊙. Along the evolutionary sequences, the radius of a compact star shrinks to its limiting value corresponding to the static configuration.

4.3 Phase transition caused by rotational evolution

The probability of a phase transition to quark phase which is caused by the rotational evolution has been extensively discussed in the literature [102, 109–112]. For a constant baryonic mass, a rotating star loses its rotational energy by magnetic dipole radiation, which makes the compact star spin down and the central density increases. When the central density of a compact star reaches a critical value, the phase transition from the hadronic matter to quark phase will take place, and star converts to a hybrid star. As the compact star continues spinning down and the central density continues increasing, more and more quark matter phase appears in the core of the hybrid star. The Fig. 8 depicts the profile of the pressure of a compact star as a function of circumferential radius R eq of star using EOS NHymQ4 with baryonic mass M B = 2.26M⊙, for four spin frequencies in the range as, 0 ≤ f ≤ 716.4 Hz. The circumferential radius of a star at the equator may be defined as

\[ R_{eq} = r_e(\exp(\gamma - \rho)/2) \]  

(44)

here \( \gamma \) and \( \rho \) are the metric potentials used in Eq. (40) of the space time metric and \( r_e \) is the coordinate radius at the equator. For the frequency of fastest rotating stars 716 Hz [106], it is found that the hybrid star does not contain quark matter inside the core and it is composed of neutrons, protons and hyperons only. The stellar radius shrinks along with the regions of particle compositions of hyperons and mixed phase with the spin down frequencies and, however, the region of neutrons and protons expanded and CFL quark matter core of small size is evident at the frequency 465.1 Hz of pulsar [104]. At asymptotically slow rotation rates, the...
compact star represents a dense CFL quark phase that extends up to the radius 2.1 km and, is surrounded by a mixed phase followed by hyperonic and nucleonic phase in the region between 2.1 km ≤ Req ≤ 12.03 km. Figure 8 gives a true reflection of the internal structure of the compact stars as it spins-up or down, if the dynamical timescales are much larger than the timescales required for the nucleation of the CFL quark matter phase. Therefore, the hybrid star constructed by using NHymQ4 EOS have mass 2.0M⊙ with radius 12 km, may predict the evidence of quark matter core of size 2.1 km.

4.4 The ms pulsars and limits of radii

Recently, many attempts have been made to constrain the EOSs model with the extracted radii [18, 19, 25, 113–115] of compact stars from astrophysical observations by making use of various spectroscopic and timing methods. In this work we have attempted to predict the limits of radii for catalogue of compact stars from astrophysical observations by making use of various spectroscopic and timing methods. In this work we have attempted to predict the limits of radii for catalogue of compact stars as it spins-up or down, if the dynamical timescales are much larger than the timescales required for the nucleation of the CFL quark matter phase. Therefore, the hybrid star constructed by using NHymQ4 EOS have mass 2.0M⊙ with radius 12 km, may predict the evidence of quark matter core of size 2.1 km.

Table 4 List of ms pulsars with measured gravitational mass and less than 10 ms spin-period

| No. | Pulsar name   | Spin-period [frequency] (ms [Hz]) | Mass (M⊙)          | References                  |
|-----|---------------|-----------------------------------|--------------------|-----------------------------|
| 1   | J1903+0327    | 2.15 [465.1]                      | 1.667±0.021        | [21,104]                    |
| 2   | J2043+1711    | 2.40 [416.7]                      | 1.410±0.21         | [22–24]                     |
| 3   | J0337+1715    | 2.73 [366.0]                      | 1.4378±0.0013      | [116]                       |
| 4   | J1909-3744    | 2.95 [339.0]                      | 1.540±0.027        | [117,118]                   |
| 5   | J1614-2230    | 3.15 [317.5]                      | 1.928±0.017        | [15,24]                     |
| 6   | J1946+3417    | 3.17 [315.5]                      | 1.833±0.028        | [25,119]                    |
| 7   | J0751+1807    | 3.48 [287.4]                      | 1.640±0.15         | [118,120]                   |
| 8   | J2234+0611    | 3.58 [279.3]                      | 1.393±0.013        | [25,121]                    |

4.5 Tidal deformability

The tidal influences of its companion in BNS system will deform CS in binary system and, the resulting change in the
Table 5: The theoretically computed limits for the results of spin down sequences of hybrid star for gravitational mass $M_c(M_g)$, baryonic mass $M_B$, equatorial radii $R_{eq}$(km), redshift $Z$, moment of inertia $I$, ratio of rotational to gravitational energies $T/W$, dimensionless angular momentum $c/J/GM$ and ratio of the polar radii to equatorial radii. These limits for results have been obtained corresponding to the observed rotating frequencies $f$ at 465.1 Hz, 416.7 Hz, 366.0 Hz, 339.0 Hz, 317.5 Hz, 315.5 Hz, 287.4 Hz and 279.3 Hz corresponding to pulsars PSR J1903+0327 [21,104], PSR J2043+1711 [15,24], PSR J0337+1715 [116], PSR J1909-3744 [117,118], PSR J1614-2230 [15,24], PSR J1946+3417 [25,119], PSR J0751+1807 [117,120] and PSR J2234+0611 [25,121]. The corresponding values of central energy density $\epsilon_c = (x \times 10^{15} \text{gcm}^{-3})$ and baryon number density $\rho_c(x \times 10^{15} \text{gcm}^{-3})$ are also shown in the table.

| Frequency [Hz] | $\epsilon_c$ | $M_B$($M_g$) | $R_{eq}$(km) | $Z$ | $I(\times 10^{45} \text{gcm}^2)$ | $T/W$ | $c/J/GM$ | $R_p/R_{eq}$ |
|---------------|--------------|--------------|-------------|-----|------------------|-------|-----------|-------------|
| 465.1         | 7.11         | 0.75±0.03    | 1.84±0.03   | 13.95±0.04 | 0.26±0.00        | 3.23±0.04 | 0.02       | 0.77±0.01   | 0.93        |
| 416.7         | 5.69         | 0.59±0.12    | 1.53±0.25   | 13.94±0.14 | 0.20±0.03        | 1.85±0.37 | 0.02       | 0.58±0.11   | 0.93        |
| 366.0         | 5.94         | 1.61±0.00    | 1.56±0.00   | 13.87±0.00 | 0.21±0.03        | 1.87±0.01 | 0.01       | 0.49±0.00   | 0.95        |
| 339.0         | 6.25         | 0.65±0.01    | 1.68±0.03   | 13.77±0.00 | 0.23±0.01        | 2.05±0.04 | 0.01       | 0.50±0.01   | 0.96        |
| 317.5         | 7.11         | 1.22±0.06    | 2.18±0.02   | 12.98±0.09 | 0.34±0.01        | 2.51±0.00 | 0.01       | 0.57±0.00   | 0.98        |
| 315.5         | 9.14         | 1.00±0.05    | 2.06±0.04   | 13.33±0.08 | 0.30±0.01        | 2.46±0.02 | 0.01       | 0.55±0.00   | 0.97        |
| 317.5         | 7.23         | 0.77±0.13    | 1.81±0.19   | 13.59±0.25 | 0.24±0.03        | 2.26±0.19 | 0.00       | 0.45±0.04   | 0.97        |
| 287.4         | 5.82         | 0.60±0.01    | 1.51±0.02   | 13.70±0.00 | 0.20±0.00        | 1.77±0.02 | 0.00       | 0.35±0.00   | 0.97        |

Fig. 9: The theoretical limits of the radii of various well known millisecond pulsars is presented as a function of their rotational frequencies. The theoretical limits of the radii and the gravitational mass of these pulsars have been computed by using NHymQ4 EOS.

The effect on GW phasing can be parameterized by the dimensionless tidal deformability parameter, $\Lambda_i = \lambda_i/M_f^2$, $i = 1, 2$. For each CS, its quadrupole moment $Q_{j,k}$ must be related to the tidal field $\epsilon_{j,k}$ caused by its companion as, $Q_{j,k} = -\lambda \epsilon_{j,k}$, where, $j$ and $k$ are spatial tensor indices. The dimensionless tidal deformability parameter $\Lambda$ of a static, spherically symmetric compact star depends on the neutron star compactness parameter $C$ and a dimensionless quadrupole Love number $k_2$ as, $\Lambda = (2k_2/3)C^{-5}$. The $\Lambda$ critically parameterize the deformation of CS under the given tidal field, therefore it should depend on EOS of nuclear dense matter. When the orbital separation are very small at the frequencies in the BNS systems, the tidal corrections are added to the tidal energy and luminosity linearly to the point-particle energy and luminosity. The leading-order tidal corrections are Newtonian effects and, are known as 5PN (Post Newtonian) and next-to-leading-order 6PN corrections to the energy and luminosity [123, 124]. These leading-order tidal corrections required to be included in the waveform model employed for analysis of GW signals from advanced LIGO and Virgo GW detectors at the high frequencies, as discussed for the various wave forms by Abbott et al. [14].

To measure the Love number $k_2$ along with the evaluation of the TOV equations we have to compute $y_2 = y(R)$ with initial boundary condition $y(0) = 2$ from the first-order differential equation [125–128] simultaneously.
Fig. 10 (Color online) (left panel) The tidal deformability (Λ) and (right panel) the dimensionlessLove number (k2) with respect to gravitational mass for different EOSs.

\[ y' = \frac{1}{r} \left[ -r^2 \left\{ 4\pi G \left( 1 - \frac{2Gm}{r} \right)^{-1} \left\{ 5\varepsilon + 9P + \frac{\varepsilon + P}{c_s^2} \right\} - 6 \left( 1 - \frac{2Gm}{r^2} \right)^{-1} - 2G \left( 1 - \frac{2Gm}{r} \right)^{-1} \left( m + 4\pi Pr^3 \right)^2 \right\} ight] - \frac{y}{1 - \frac{2Gm}{r}^{-1} \left( 1 + 4\pi Gr^2(P - \varepsilon) \right)} \]  

Here, \( G \) and \( c_s \) are the gravitational constant and speed of sound respectively. First, we get the solutions of Eq. (45) with boundary condition, \( y_2 = \gamma(R) \), then the electric tidal Love numbers \( k_2 \) is calculated from the expression as,

\[ k_2 = \frac{8}{5} C^5 (1 - 2C)^2 [2C(\gamma_2 - 1) - \gamma_2 + 2] \left\{ 2C(4(\gamma_2 + 1)C^4 + (6\gamma_2 - 4)C^3 + (26 - 22\gamma_2)C^2 + 3(5\gamma_2 - 8)C - 3\gamma_2 + 6) \right\} - 3(1 - 2C)^2 (2C(\gamma_2 - 1) - \gamma_2 + 2) \log \left( \frac{1}{1 - 2C} \right)^{-1} \]  

Here, \( C = m/R \) is the compactness of the star.

Figure 10 right panel presents the dimensionless tidal Love number \( k_2 \) as a function of gravitational mass of the compact star. The value of \( k_2 \) suddenly decreases with increasing gravitational mass after getting maximum value. The value of \( k_2 \) is low at higher and lower gravitational masses of the compact star and, indicating that the quadrupole deformation is maximum for intermediate ranges of masses. Figure 10 left panel presents the dimensionless tidal deformability as a function of gravitational mass for the CS with selected eight EOSs. It is found that dimensionless tidal deformability decreases with increase in gravitational mass of compact stars. The obtained value of \( \Lambda_{14} \) for 1.4 \( M_\odot \) for various EOSs considered in the present work is from 500 to 638, which is consistent with recent constraints proposed for tidal deformability \( \Lambda_{14} \) from the GW170817 event [3, 4, 94]. The EOSs NmQ2, NmQ3 and NHymQ4 satisfy the constraints on \( \Lambda_{14} = 190^{+260}_{-190} \) [3]. The values of \( \Lambda_{14} \) for EOSs Nucl1, Nucl2, NHy1, NHy2 and NmQ1 lie in the range 609.94 - 638.38. The EOS Nucl1 and Nucl2 are consistent with the constraints on dimensionless tidal deformability obtained using Bayesian analysis \( \Lambda_{14} = 500^{+186}_{-267} \) [4]. The EOSs NHy1, NHy2 and NmQ1 are slightly off from the results of tidal deformability reported in [3]. Finally, in Fig. 11 we plot the tidal deformability parameters \( A_1 \) and \( A_2 \). These tidal deformability parameters have relationship with the neutron star binary companion with a high mass \( M_1 \) and a low mass \( M_2 \), respectively associated with GW170817 event [14]. The dark green band represents the tidal parameters computed by using the representative set of EOSs shown in Table 2. While the other tidal parameters curves are calculated with RMF models IUFSU [129] solid blue curve, TM1 [130] solid violet curve and G2 [131] solid red curve are calculated by using our computer code. The solid brown curve and orange curve corresponding to the APR4 [132] and SLy [133] models are taken from Fig. (1) of Ref. [3] for comparison. The curves of tidal parameters are ending at \( A_1 = A_2 \) boundary represented by light green sliding line. The black, magenta and turquoise lines represents
50 % (dashed) and 90 % (solid) credible levels for posteriors obtained using EOS insensitive relations taken from figure 1 of Ref. [3]. The brown shading corresponds to unphysical region ($A_2 \leq A_1$). It can be seen from the figure that the results for tidal deformability parameter for our set of eight EOSs are reasonably consistent with the 90 % credible region suggested in Ref. [3]. This might be attributed to the fact that soft EOSs gives smaller value of tidal deformabilities and are favoured over stiff EOSs. Our set of EOSs comprise of soft, stiff and moderate nature. Also, EOSs like G2 and IUFSU lies within the 90 % credible region due to the soft nature of these EOSs. TM1 EOS lies outside the credible region reflecting its stiff behaviour. This means that Posterior is suggesting more support for the softer EOS than prior. The tidal parameters results are obtained by varying the high mass $M_1$ independently in the range as, $1.36 \leq M/M_\odot \leq 1.61$ and, obtained the low mass partner $M_2$ of the neutron star merger by keeping the chirp mass $M_{\text{chirp}} = (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}$ fixed at the observed value, $1.186 M_\odot$ [14]. The low mass $M_2$ of the neutron binary is obtained in the range $1.16 \leq M/M_\odot \leq 1.36$, as presented in the Table 6 for NHymQ4 EOS model.

The EOS describe the structural properties of cold matter CS in global charge $\beta$ equilibrium relating to its variables such as gravitational mass and circumferential radius as shown in Fig. 4. For a fixed CS mass, CSs with large radii are supporting stiff EOS and, for same mass with small radii have a soft EOS. Therefore, in the present work Table (2), the EOS models NHy1, NmQ2 and NHymQ4 support maximum mass $\approx 2.0 M_\odot$ with their respective radius $11.58$ km, $12.32$ km, $12.05$ km, representing stiff, soft and moderate EOSs, respectively. These models of EOSs are satisfying the measurements of radii from LIGO and Virgo data [3] that requires the EOS which supports NS masses larger than $1.97 M_\odot$ as required from the astrophysical observations at 90% credible level. While many accurate CS mass measurements have been made, but corresponding radius measurements are still awaited [134]. The simultaneous CS mass-radius measurements or equivalently mass-Λ measurements can highly constrain the EOS of nuclear dense matter in the high density regimes [124,135–138].

The $A_1 \approx (R_1/M_1)^5$ and $A_2 \approx (R_2/M_2)^5$ are not being measured properly by advanced GW detectors due their correlation, whereas the tidal deformability parameter $\tilde{\Lambda}$ which is linear combination of ($A_1$, $A_2$) is precisely measured [14]. Recently, LIGO and VIRGO detectors precisely measured the value of $\tilde{\Lambda}$ of the BNS GW170817 [14]. The accumulated phase contribution caused due to the deformation from two stars of a binary is included in the inspiral signal as the combined dimensionless tidal deformability. The prominent tidal contribution $\tilde{\Lambda}$ to the GW phase evolution is a mass–two tidal parameters. It first appears at 5PN order. The weighted dimensionless tidal deformability $\tilde{\Lambda}$ of the BNS of masses $M_1$ and $M_2$ have relationship with $\tilde{\Lambda} = A_1 = A_2$ for $M_1 = M_2$ as defined [14,123],

$$\tilde{\Lambda} = \frac{16}{13} \left( \frac{(M_1 + 12 M_2) M_1^4}{(M_1 + M_2)^5} A_1 + \frac{(M_2 + 12 M_1) M_2^4}{(M_1 + M_2)^5} A_2 \right),$$

with weighted tidal correction $\Delta \tilde{\Lambda}$ appearing at 6PN order only [124],

$$\Delta \tilde{\Lambda} = \frac{1}{2} \left[ \sqrt{1 - 4\eta \left( 1 - \frac{13272}{1319} \eta + \frac{8944}{1319} \eta^2 \right) (A_1 + A_2)} + \left( 1 - \frac{15910}{1319} \eta + \frac{32850}{1319} \eta^2 + \frac{3380}{1319} \eta^3 \right) (A_1 - A_2) \right],$$

where $\eta = M_1 M_2/M^2$ is the symmetric mass ratio and $M = M_1 + M_2$ is the total mass. The tidal parameters $A_1$ and $A_2$ satisfy the condition $M_1 \geq M_2$.

In Table 6, we present the BNS masses $M_1(M_\odot)$, $M_2(M_\odot)$ and their corresponding radii $R_1$, $R_2$ in km, dimensionless tidal deformability parameters ($A_1$, $A_2$), weighted dimensionless tidal deformability $\tilde{\Lambda}$, tidal correction $\Delta \tilde{\Lambda}$ and chirp radius $R_c$ in km for EOS model NHymQ4. The radius of chirp mass is $2 M_{\text{chirp}} \tilde{\Lambda}^{1/5}$. The BNS masses vary as $M_1 \in (1.36, 1.61) M_\odot$ and $M_2 \in (1.16, 1.36) M_\odot$ with their corresponding baryon number densities $(0.328 \text{ fm}^{-3}, 0.423 \text{ fm}^{-3})$ and $(0.303 \text{ fm}^{-3}, 0.328 \text{ fm}^{-3})$, respectively for hybrid star NHymQ4 EOS, while the threshold baryon number den-
deformabilities ($\Lambda$-hyperons is $0.35 \text{ fm}^{-3}$ and quark matter phase transition appears at $0.64 \text{ fm}^{-3}$). It indicates that the hybrid star EOS may not be favorable to describe GW170817 measurements, although satisfying other constraints. The magnitudes $R_1$ and $R_2$ given in the Table 6 are marginally satisfying the upper limits of LIGO and Virgo measurements of radii corresponding to EOS that supports the neutron stars with masses larger than $1.97M_\odot$. Therefore, the baryon number densities relevant for the medium-mass neutron stars involved in GW170817 may be too small to really test the quark matter phase transition of the EOS, as can be seen from Fig. 11 in which the different EOS essentially lead to similar results, or directly from Fig. 4. It is noticed that the values of $\Lambda \leq 800$ in the low-spin priors, which is very well consistent with the recent observations [3,14]. The weighted tidal deformation is found to be lie in the range $691 \leq \tilde{\Lambda} \leq 729$ and the chirp radius is in the range $8.77 \text{ km} \leq R_c \leq 8.86 \text{ km}$ for equal and unequal-mass binary neutron stars, as shown in Table 6.

| $M_1$ | $M_2$ | $R_1$ | $R_2$ | $\Lambda_1$ | $\Lambda_2$ | $\tilde{\Lambda}$ | $\Delta \tilde{\Lambda}$ | $R_c$ |
|------|------|------|------|-----------|-----------|-------------|----------------|------|
| 1.36 | 1.36 | 13.28 | 13.24 | 741.89    | 728.088   | 727          | -4.67          | 8.86 |
| 1.38 | 1.35 | 13.28 | 13.28 | 676.33    | 786.749   | 721          | 15.31          | 8.85 |
| 1.40 | 1.33 | 13.28 | 13.28 | 624.53    | 862.664   | 729          | 33.11          | 8.86 |
| 1.42 | 1.31 | 13.28 | 13.28 | 576.26    | 935.128   | 727          | 49.16          | 8.86 |
| 1.44 | 1.29 | 13.28 | 13.28 | 527.95    | 1001.14   | 721          | 63.83          | 8.85 |
| 1.46 | 1.27 | 13.28 | 13.28 | 483.09    | 1083.51   | 724          | 80.80          | 8.85 |
| 1.48 | 1.26 | 13.28 | 13.28 | 445.14    | 1173.22   | 723          | 97.01          | 8.85 |
| 1.50 | 1.24 | 13.28 | 13.28 | 410.63    | 1262.94   | 721          | 113.84         | 8.85 |
| 1.52 | 1.22 | 13.28 | 13.28 | 376.12    | 1345.76   | 712          | 125.07         | 8.82 |
| 1.54 | 1.21 | 13.28 | 13.28 | 345.07    | 1459.63   | 722          | 142.23         | 8.85 |
| 1.56 | 1.19 | 13.24 | 13.28 | 317.46    | 1566.6    | 717          | 156.53         | 8.84 |
| 1.58 | 1.18 | 13.21 | 13.28 | 289.86    | 1666.67   | 717          | 169.18         | 8.84 |
| 1.60 | 1.17 | 13.21 | 13.28 | 265.70    | 1766.74   | 701          | 180.26         | 8.80 |
| 1.61 | 1.16 | 13.14 | 13.28 | 251.90    | 1821.95   | 691          | 186.95         | 8.77 |

5 Conclusions

In the present research work, we have obtained a plausible set of hybrid EOSs for superdense hadron-quark matter which satisfies the constraints provided by finite nuclei, bulk nuclear matter, the observational data of astrophysical interest of CS including the limits of tidal deformabilities obtained from GW170817 BNS merger. We employ Baym-Pethick-Sutherland EOS [89] from outer crust low density regions to the pasta phase of the neutron drip. The inner crust region and core region of EOSs have been matched by employing the cubic interpolation method that offers true continuity between crust and core. The high density region of EOSs are constructed within the EFTRMF approach for hadronic matter and QQPM model for quark matter.

The EOS of hadronic matter have been computed by using BSR3 and IOPB-I parameterizations. The quark matter phase of EOSs have been computed by employing Quark Quasiparticle model by considering the color superconducting phase with CFL gap parameters $\Delta = 48 \text{ MeV}$ and bag constant $(B_0)_{\frac{3}{2}} \leq 146 \text{ MeV}$ in the quark matter phase of hybrid star EOS. Assuming the phase transition under the Gibbs construction. Further, we considered in region of baryons densities as, $0.1 \text{ fm}^{-3} \leq \rho \leq 2 \rho_0$, the EOS is composed of neutrons and protons in beta equilibrium, $2 \rho \leq \rho \leq 4 \rho_0$ hyperons appears and $4 \rho_0 \leq \rho \leq 1.6 \text{ fm}^{-3}$ the quark matter appears in $\beta$-equilibrium. The set of EOSs is used to determine the maximum gravitational mass, circumferential equatorial radius, rotational frequency of stable stellar configuration and tidal deformability of hybrid stars. The hybrid EOS NHymQ4 has been employed to investigate the phase transition caused by the rotational evolution, limits for the radii of ms pulsars and tidal deformability parameters of hybrid CS.

In the present calculation, we obtained a bound for maximum gravitational mass $1.84 M_\odot \leq M_G \leq 2.38 M_\odot$ and radius $11.58-12.32 \text{ km}$ as shown in Table 2. These limits of mass and radii are well within the recently extracted limits on mass and radii [18,19,90–92]. We have also discussed the structural properties of rotating compact stars at Keplerian frequency and, presented the values of maximum gravitational mass $M_{\text{max}}(M_\odot)$ of rigidly rotating star its corresponding circumferential radius $R_{\text{max}}(\text{km})$, central energy density $\varepsilon_c(\times 10^{15} \text{ g/cm}^3)$, the maximum Keplerian frequency $f_K (\text{Hz})$ and the empirical approximation of maximum frequency $f_{\text{max}}$ [102]. It is evident from Fig. 6 that the Keplerian frequency increases monotonically both for hadronic star as well as hybrid star as a function of gravit-
tional mass ($M_G$). The maximum gravitational mass and Keplerian frequencies of the hybrid compact stars decreases as the hyperons and quarks appears in core of star. Further, the particle composition of EOS NHymQ4 model has motivated us to employ this stellar model to investigate the phenomena of rotation, phase transition and its evolution, internal structure, tidal deformabilities and tidal corrections of hybrid CS. We have discussed the SAI conditions defined in Eq. (43) by calculating a sequences of stellar models with NHymQ4 along the different fixed values of angular momentum J in Fig. 7, the gravitational mass increases with increasing central energy density to gain its maximum value. Two separate regions of stable sequences and unstable sequences of the hybrid CS are obtained by using SAI condition. We obtained uniformly rotating sequences of hybrid CS at the known pulsars frequency, $f = 465.1$ Hz [104], 641.9 Hz [105], 716.4 Hz [106] and 1122 Hz. The stable configurations are constrained by the Kepler and secular axisymmetric instability conditions at large and small radius, respectively. Static sequences are connected to their Keplerian sequences via their evolutionary sequences, which corresponds to the rotational evolution at constant baryonic mass of step size $0.2M_⊙$. Along the evolutionary sequences, the radius of a compact star shrinks to its limiting value corresponding to the static configuration.

The phase transition induced by the spin-down of pulsars with a constant baryonic mass $2.26M_⊙$, on the internal radius of four spin frequencies in the range as, $0 < f < 716.4$ Hz. The stellar radius shrinks along with the regions of particle compositions of hyperons and mixed phase with the spin down frequencies and, however, the region of neutrons and protons expanded and quark matter core of small size is indicated at the frequency $465.1$ Hz of pulsar in Fig. 8. At asymptotically slow rotation rates, the compact star represents a dense CFL quark matter phase that extends up to the radius 2.1 km. This is surrounded by a mixed phase followed by hyperonic and nucleonic phase in the region between 2.1 km $< R < 12.03$ km. Therefore, the hybrid star constructed by using NHymQ4 EOS have mass $2.0M_⊙$ with radius 12 km, may predict the evidence of quark matter core of size 2.1 km in hybrid CS.

We computed the catalogue of the recently observed pulsars in terms of their structural properties. The results of spin-down sequences of hybrid star are presented in Table 5 for gravitational mass $M_G (M_⊙)$, equatorial radii $R_{eq} (km)$, baryonic mass $M_B$, redshift $Z$, moment of inertia $I$, ratio of rotational to gravitational energies $T/W$, dimensionless angular momentum $cJ/GM_G^2$ and ratio of the polar radii to equatorial radii $R_p/R_e$. For ms pulsars in Table 4 with gravitational mass more than $1.4M_⊙$, we have extracted the theoretical limits of the radii as a function of their rotating frequencies by using NHymQ4 EOS of hybrid star. The limits of radii varies as $13.70 \text{ km} \leq R_{eq} \leq 14.16$ km corresponding to rotational frequencies lying in the range from $279.3 \text{ Hz} \leq f \leq 465.1$ Hz which is very close with the recently extracted bounds on radii of the compact stars [12,139]. Whereas for the spinning hybrid star with gravitational mass around $2.0M_⊙$, we predict the limits of radii as, $12.09 \text{ km} \leq R_{eq} \leq 12.75$ km corresponding to rotational frequencies lying in the range from $279.3 \text{ Hz} \leq f \leq 465.1$ Hz.

Finally, we estimate the Love number and tidal deformability for the set of EOSs. The tidal deformability parameters do not differ much from each other for set of EOSs (Fig. 10) and lie within the range as, $245 \leq A_1 \leq 732$ and $727 \leq A_2 \leq 1821$ and slightly consistent with the associated components BNS of GW170817 event [3]. Further, by investigating the consequences of dimensionless tidal deformability $A_{1,4} \leq 800$ provided by LIGO-Virgo collaboration, we extract a limit of the stellar radius of neutron star of canonical mass i.e. $R_{1.4} \text{ as } 13.37-14.32$ km. The limits of radii measurements for binary neutron stars based on EOSs supporting NS masses more than 1.97$M_⊙$ are $R_1 = 11.9^{+1.4}_{-1.3}$ and $R_2 = 11.9^{+1.4}_{-1.3}$ from GW170817 event [3] implies that the EOS of dense nuclear matter at high densities may be soft and, the evolution from stiff to soft EOS may indicating that the hybrid star EOS may not be favorable to describe GW170817 measurements, although NHymQ4 EOS satisfying other constraints. Therefore, the baryon number densities relevant for the medium-mass neutron stars involved in GW170817 may be too small to really test the quark matter phase transition of the EOS, as can be seen from Fig. 11 in which the different EOSs essentially lead to similar results, or directly from Fig. 4 or from discussions of Table 6.

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