Mathematical model of oscillations of a railway tank car with partial filling under shunting collision

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Abstract. This article is devoted to simulating longitudinal vibrations of the tank car in case of partial filling with liquid during shunting collision. A brief overview of studies of the movement of solid bodies with liquid was performed. The design scheme of cars, represented by a 4-mass system with elastically-dissipative connections, has been developed. A system of differential equations of car motion according to the main equation of dynamics has been compiled. The mathematical model of liquid cargo oscillations based on shallow water theory is derived. In integration, the characteristic method and Euler method are applied. It is envisaged to transfer parameters between the mathematical model of cars (tank acceleration) and the mathematical model of liquid load (pressure forces on the bottoms). The methodology is implemented in the C++ Builder programming environment. According to the developed method, a study of the shunting collision with the tank at incomplete filling was carried out. The dependence of the reaction in the automatic coupling on time has two bursts. The maximum reaction is also dependent on the impact rate and the degree of filling of the tank.

1. Introduction

A serious cause of damage to railway tanks should be sought in the dynamic loading of the tank under longitudinal vibrations, especially at shunting. During operation, there is an obvious tendency for collision velocities and car weights. In addition, in conditions of incomplete filling, the process of fluctuations of the liquid consignment and the occurrence of water hammer becomes an important factor. All this can lead to the appearance of unacceptably high forces in the car elements, including those acting on the barrel, and to emergencies.

In this paper, we consider the process of longitudinal oscillations of a tank [22, 25, 26] in the case of partial filling of the barrel with fluid. Various options of shunting collision are the characteristic modes of oscillations of this kind. A feature of modeling vibrations at incomplete filling is the need to take into account fluid vibrations inside the barrel, for which hydrodynamic equations are used. Equations are transformed using the method of characteristics.

A mathematical model of the car’s oscillations during shunting collisions is also being formed. The relationship between the fluid model and the mechanical part model is based on the mutual transfer of parameters.

In the field of studies of the motion of rigid bodies with a cavity filled with a fluid, as one of the fundamental researches, it is necessary to mention the work of N.E. Zhukovsky, who solved in 1885 a problem for the case of complete filling with an ideal fluid.
Since the second half of the 20th century, this problem has received significant development in the field of rocketry and various types of transport.

The main principles of the linear theory were presented in [1, 2]. Some nonlinear problems were considered in [3–8].

In the field of car building, we should mention the studies conducted in [9] on transverse fluid vibrations in tank barrels. The longitudinal dynamics of tanks taking into account incomplete loading was studied in [10].

In these and similar studies, the movement of fluid in a tank barrel is presented as oscillations of a pendulum with empirically determined parameters.

A detailed experimental study of the dynamics of tanks taking into account fluid oscillations was carried out in [11].

The objective of the study is to develop a methodology for determining the dynamic loading of railway tanks during shunting collisions by modeling their longitudinal vibrations taking into account incomplete filling.

The tasks of work are:

- the development of a mathematical model of longitudinal oscillations of the car during shunting collision;
- the development of a mathematical model of liquid consignment oscillations;
- the development of a computer program based on the developed models;
- the simulation of the process of shunting the collision of a tank.

2. Methods

Colliding cars during tank shunting collisions is represented by a system of material points with elastic-dissipative connections. The design scheme of a tank is shown in Fig. 1. An example corresponds to the most common case of a shunting collision - a collision of a tank into a braked wagon coupling. The tank is approximated by a system of four material points, and the braked coupler is presented as a single-mass model of a gondola car in back pressure.

![Figure 1. The design scheme for shunting collision.](image)

- $m_1$ is the mass of a gondola car standing in the back pressure; $m_2$ is the mass of the running tank frame; $m_3$ is the own weight of the barrel; $m_4$, $m_5$ are the masses of tank bogies; $R_1$ is inter-car connection of a standing car and its non-linear response; $R_2$ is the relationship between the standing car and the oncoming tank and its non-linear response; $c_3$ is longitudinal stiffness of the tank throat brace; $c_4$, $c_5$ are rigidity of the longitudinal bonds between the frame and the bogies; $P_{\text{left}}$, $P_{\text{right}}$ are resultant forces of fluid pressure on the left and right bottoms of the barrel, respectively; $V_0$ is the initial velocity of the tank.

The tank moves at an initial velocity, hits a standing car, and automatically couples with it.
In calculation, the following assumptions are made.
1. When determining the parameters of the oscillating fluid, the barrel is assumed to be absolutely rigid.
2. The forces of the vertical interaction of the elements of the design scheme are neglected.
3. When considering fluid oscillations, the barrel is assumed to be stationary, and the acceleration is applied to the fluid, equal to the acceleration of the barrel and directed in the opposite direction.
4. The fluid is considered incompressible.
5. There is no internal friction in the fluid.
6. Fluid oscillations are described using the theory of shallow water.
7. The fluid density is assumed to be constant.
8. The gas pressure above the fluid surface inside the barrel is taken equal to the atmospheric one.

Differential equations of the car’s movement based on the fundamental dynamics equation (Newton’s second law) [12–14] is:

\[
\begin{align*}
    m_1 \ddot{x}_1 &= R_1 - R_1; \\
    m_2 \ddot{x}_2 &= R_2 + R_3 + R_4 - R_2; \\
    m_3 \ddot{x}_3 &= P_{right} - R_{left} - R_3; \\
    m_4 \ddot{x}_4 &= -R_4; \\
    m_5 \ddot{x}_5 &= -R_5,
\end{align*}
\]

(1)

where \(R_1=F_{ad}(x_1, \dot{x}_1); R_2=F_{ad}(x_2-x_1, \dot{x}_2-\dot{x}_1); R_3=c_3(x_2-x_3); R_4=c_4(x_4-x_2); R_5=c_5(x_5-x_2)\) are the inter-mass responses.

After substituting the responses into the system of equations (1), we obtain

\[
\begin{align*}
    \ddot{x}_1 &= \frac{F_{ad}(x_2-x_1, \dot{x}_2-\dot{x}_1) - F_{ad}(x_1, \dot{x}_1)}{m_1}; \\
    \ddot{x}_2 &= \frac{c_1(x_1 - x_2) + c_4(x_4 - x_2) + c_5(x_5 - x_2)}{m_2}; \\
    \ddot{x}_3 &= \frac{-P_{right} - R_{left} - c_1(x_1 - x_2)}{m_3}; \\
    \ddot{x}_4 &= \frac{-c_4(x_4 - x_2)}{m_4}; \\
    \ddot{x}_5 &= \frac{-R_5}{m_5}(x_4 - x_3).
\end{align*}
\]

(2)

Under initial conditions:

\[
\begin{align*}
    t = 0; \\
    \dot{x}_1 = \dot{x}_2 = \dot{x}_3 = \dot{x}_4 = \dot{x}_5 = 0; \\
    \ddot{x}_1 = 0; \ddot{x}_2 = \ddot{x}_3 = \ddot{x}_4 = \ddot{x}_5 = V_0.
\end{align*}
\]

(3)

In cases of other shunting collision patterns, a similar calculation scheme (Figure 1) and a mathematical model (2), (3) can be applied. In particular, if a collision into a free-standing car is considered, then in all equations \(R_1=0\).
If the collision of the tank is considered at point-blank, then the first equation is excluded from the system (1). Besides, it is assumed that: \( x_i = 0; \dot{x}_i = 0 \).

Mathematical models of inter-car couplings in the form of dependences \( R = F_{ac}(x, \dot{x}) \) are given in [15–18].

The mathematical model of car vibrations described above is supplemented by the mathematical model of liquid consignment oscillations. The design scheme for considering fluid vibrations is shown in Figure 2.

![Figure 2. The design scheme for oscillating fluid](image)

In the Cartesian coordinate system \( Oxyz \), the Ox axis is parallel to the longitudinal axis of the barrel, Oy is the vertical axis, Oz is the transverse horizontal axis. The figure also indicates: \( h \) is the surface level of the fluid; \( u \) is the longitudinal velocity of the fluid at an arbitrary point; \( a \) is the acceleration acting on the fluid; \( dx \) is an infinitely small section of the barrel with fluid.

We apply the shallow water theory [19], according to which the fluid velocity at all points of the cross-section is the same.

The continuity equation for a small portion of the fluid, separated by cross-sections corresponding to \( x \) and \( (x + dx) \) is written as

\[
\left. \left( \frac{\partial}{\partial \tau} \int_{e}^{h} \Delta z \cdot dy \right) + \left( \frac{\partial}{\partial \tau} \int_{e}^{h} \Delta z \cdot dy \right) \right|_{x+dx}^{x} - \left. \left( \frac{\partial h}{\partial \tau} \right) \right|_{x+dx}^{x} dx = 0, \tag{4}
\]

Where \( \tau \) is time; \( \frac{\partial h}{\partial \tau} \) is the fluid velocity in the vertical direction; \( \Delta z \) is the width of the barrel; \( e \) is the bottom level.

Compose the equation of fluid motion for a small element \( dx \):

\[
\rho \left. \frac{\partial}{\partial \tau} \int_{e}^{h} \Delta z \cdot dy \right|_{x+dx}^{x} = \rho \left. \left( u \cdot u \right) \int_{e}^{h} \Delta z \cdot dy \right|_{x}^{x+dx} - \rho \left. \int_{e}^{h} P \Delta z \cdot dy \right|_{x}^{x+dx} + P_{0} \left. \frac{\partial h}{\partial x} dx \Delta z \right|_{x}^{x+dx} - P_{z} \left. \frac{\partial \varepsilon}{\partial x} dx \Delta z \right|_{x}^{x+dx} + a \rho \left. \int_{e}^{h} \Delta z \cdot dy \right|_{x}^{x+dx} dx = 0, \tag{5}
\]

Where \( P_{z} = P_{0} + \rho g (h - y) \) is the fluid pressure; \( P_{0} \) is the gas pressure above the fluid surface; \( P_{z} \) is the fluid pressure at the bottom level.

Dividing equations (4) and (5) by \( dx \), for an arbitrary point we obtain...
The system of partial differential equations (6) describes fluid oscillations. The initial conditions are: \( t=0; \quad u=0; \quad h=h_0 \).

The system of equations (6) is presented in matrix form
\[
\begin{bmatrix}
u x
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{u}}{\partial t} = \bar{f}.
\end{bmatrix}
\]

Where \( \bar{u} = \{ h; u \} \) is the vector of the unknowns; \([A],[B]\) are matrices of coefficients (second order), the form of which depends on the sectional shape of the barrel; \( \bar{f} = \{ f_1; f_2 \} \) is the vector of the right side, depending on the shape of the cross-section and the geometry of the tank bottom.

For further transformations of the system of equations (7), we apply the method of characteristics [20, 23, 24].

The total differential of the vector of unknowns is written as a function of two variables:
\[
d\bar{u} = \frac{\partial \bar{u}}{\partial x} dx + \frac{\partial \bar{u}}{\partial t} dt
\]

substituting (8) into equation (7), we get:
\[
\frac{\partial \bar{u}}{\partial x} \left( [A]dt - [B]dx \right) = \bar{f}dt - [B]d\bar{u}.
\]

For the resulting system of equations (9), we compose the equations of characteristics and relations on the characteristics. To obtain the characteristic equation, we equate the determinant of the system to zero:
\[
\begin{vmatrix}
[A]dt - [B]dx
\end{vmatrix} = 0.
\]

Expanding the determinant, we obtain the equations of characteristics in the following form:
\[
\frac{dx}{dt} = -\frac{C}{2|B|} \pm \sqrt{C^2 - 4|A||B|},
\]

where \( C = a_{11}b_{11}b_{22} + a_{12}b_{21} + b_{12}a_{21} \).

Relationships on the characteristics are determined from the condition:
\[
\begin{vmatrix}
([A]dt - [B]dx), \left( \bar{f}dt - [B]d\bar{u} \right)
\end{vmatrix} = 0,
\]

in which in equation (10) one column is replaced by a vector of the right-hand side of the system of equations (7). Expanding the determinant (12), considering equation (11), we obtain the relations on the characteristics in the following form:
\[
(D_1 \pm D_2)du + (E_1 \pm E_2)dh = (F_1 \pm F_2)dt,
\]

where \( D_1 = -a_{11}b_{11} + a_{12}b_{21} - \frac{C}{2}; \quad D_2 = |B|\sqrt{C^2 - 4|A||B|}; \)

\( E_1 = -a_{11}b_{21} + a_{12}b_{11} - \frac{C}{|B|}b_{11}b_{21}; \quad E_2 = 2b_{11}b_{21}\sqrt{C^2 - 4|A||B|}; \)
\[ F_1 = -a_1 f_2 + a_2 f_1 + \frac{C}{2|B|} \left( -b_1 f_2 + b_2 f_1 \right); \]
\[ F_2 = \left( -b_1 f_2 + b_2 f_1 \right) \sqrt{C^2 - 4|A||B|} \]

The integration of equations (11) and (13) is based on the difference scheme in the form of the Euler method. The general order of integration is as follows. Along the length of the barrel, on the \( \Omega x \) axis, a set of points (sections) are selected; each section has its values of velocity \( u \) and fluid level \( h \). This set of points at the initial time makes up the initial time layer.

Writing the equations of characteristics according to (11) for every two adjacent points, we determine the points of the next time layer from the conditions of the intersection of the characteristics between themselves or with the boundary.

Finally, the values of velocity \( u \) and level \( h \) at each point of the new layer are determined through the relations on the characteristics (13). As a result, the new time layer becomes the starting one, and the cycle of calculations is repeated.

The boundaries of the fluid are the cross-sections of the beginning and end of the free surface along the longitudinal axis of the barrel. Various options for the position of the boundaries (boundary conditions) are possible depending on the configuration of the fluid free surface. In the adopted barrel scheme, the end sections of the bottom are represented as vertical flat walls, and the approximation to the real geometry of the elliptical bottoms is achieved by the variable in length bottom and ceiling level.

At the undisturbed ("at rest") state of the fluid, as well as relatively small deviations from it, the free surface intersects with the vertical end walls. In this case, the coordinates of the boundaries are constant: \( x = 0 \) for the left boundary and \( x = l \) for the right one (\( l \) is the length of the barrel). The velocity in the longitudinal direction at each such boundary is zero. From these considerations, the fluid level in the boundary section at the next time layer is determined.

However, with an increase in the deviation of the fluid from the "at rest" state, the nature of the boundaries conditions can change. If the surface of the fluid in the boundary section reaches the bottom, then with the continued outflow of liquid from this boundary, the bottom of the barrel may begin to expose, that is, the boundary of the fluid will begin to move at a certain velocity from the end wall of the barrel. In this case, it is first necessary to determine the position of this boundary based on the velocity of the boundary point of the fluid. Then, using the equations of the method of characteristics, new values of the parameters \( u \) and \( h \) for the boundary point are determined.

An option of the state of the liquid boundary is possible when the free surface reaches the ceiling level on the end wall. If under this condition, the flow of fluid to the given boundary continues, then the boundary of the free surface begins to move from the end vertical wall of the barrel. In other words, the end of the barrel begins to be filled with fluid "to the ceiling", and the filled area expands. Note that the accepted theory of shallow water implies the impossibility of fluid movement in the filled end region of the barrel. The part of the fluid that is outside the boundary of the free surface becomes temporarily “turned off” from the oscillation process. Thus, in this case, as well, the boundary conditions are considered for the boundary section of the free surface. From the law of conservation of matter in the case of an incompressible fluid, it follows that the boundary moves at a velocity equal in magnitude to the velocity of the boundary point of the free surface and in the opposite direction. With this in mind, we can determine the position of the boundary for a new time layer, and then, using the equations of the method of characteristics, find the parameters of \( u \) and \( h \) for a given boundary point. Below, similar conditions separately for the left and right boundaries are obtained.

To calculate the pressure, we apply the calculation scheme shown in Figure 3. Here it is assumed that the acceleration \( a \) caused by the barrel oscillations is directed to the right.
The pressure at an arbitrary point is determined by the depth value in the direction of resulting acceleration ($\vec{g} + \vec{a}$):

$$p = P_0 + \rho h' \sqrt{\vec{g}^2 + a^2},$$

where $h' = \frac{h - y}{\sin \alpha}$; $\tan \alpha = \frac{g}{a}$.

To find the total pressure force on the right bottom, it is necessary to take the integral over the height of the bottom:

$$P_{right} = \int_0^h \left( P_0 + \rho h' \sqrt{\vec{g}^2 + a^2} \right) \Delta z \cdot dy. \quad (14)$$

If the fluid surface level corresponding to a certain value of $y$ coincides with the surface of the barrel, then $P_0 = 0$ is taken in (14).

If the acceleration $a$ is directed to the left, then the pressure $P_{right}$ is equal to zero.

A similar approach is used to calculate the pressure force $P_{left}$ on the left bottom.

3. Results and discussion

The proposed technique is implemented on a PC in the form of a program C++ in the C++ Builder programming environment.

Using this program, three options of calculations of the shunting collision process were conducted. Figures 4-6 show the results of calculations of an impact through an intermediate car at point-blank, the design scheme of which is shown in Figure 1.
From the results obtained, it is seen that the process of force changing in an automatic coupler at incomplete filling has two bursts associated with the accumulation of liquid consignment on the bottoms: first, compressive force (in the collision direction, from 0.5 to 1.5 s), and then tensile force (in the opposite direction, from 4 to 4.7 s), and with a level of filling of 80%, the maximum tensile force exceeds the compressive one.

The dependences of the maximum response values in an automatic coupler on the collision velocity and the level of tank filling were also obtained.

The graphs of the response dependence on velocity monotonously increase, and the dependence of the tensile response on the level of filling has a maximum at 80% of filling. When the barrel is filled to 66-93%, the tensile forces exceed the compressive ones. That is, under these conditions, the burst from the outflow of liquid consignment becomes more significant than the burst from the inflow to the right bottom.
4. Conclusions
1. A mathematical model is proposed that describes the process of oscillations of tank cars under shunting collisions taking into account incomplete fluid filling.
2. A method is designed to assess the dynamic loads acting on the car elements.
3. It is revealed that the process of force changing in an automatic coupler at incomplete filling has two bursts associated with the accumulation of liquid consignment on the bottoms: first a compressive force, and then a tensile one; at 80% of filling level the maximum tensile force exceeds the compressive one.
4. The application of this methodology allows reducing the cost of full-scale testing in the design of tank cars.

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