Practical application of digital control methods in ecological systems

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Abstract. The paper explains the application of regression analysis for prediction of processes in ecological systems and control of them. Theoretical analysis of the regression method structure and the main typical dependences between dependent and independent variables used for mathematical prediction modelling for statistically random processes are given. Specific aspects in significance of regression equation parameters are defined in statistical processing of the experimental data. Equation demonstrating the soil transpiration as a function of changes in fluid pressure and relative water vapor saturation is given. Comparative analysis of the absolute and relative errors of the actual and predicted values of the soil transpiration is presented. Expressions allowing one to estimate the arithmetic mean and the mean square deviation of the soil transpiration depending on the computational points are demonstrated. A method for recognising miscalculations when processing experimental data using the STATISTICA program is presented. Using the $3\sigma$ method and the tabular method, a numerical example of whether any result is miscalculated is demonstrated. Efficiency of the proposed mathematical model is shown using a specific example with account of statistical analysis of experimental results for metrological validation of them.

1. Introduction
In the era of digital transformation [1] natural and man-made objects are increasingly examined and analyzed using digital methods. The use of digital methods allows effective solutions of various complex problems both in the field of technology and in the field of science. Thanks to digital methods [2], it is possible to provide high-quality express control of processes and initiate appropriate control actions so that the parameters of these processes remain within the set limits in the presence of external disturbances.

Taking into account the multifactorial nature, crosscutting, and complexity of ecological systems, the process control in them differs significantly [3] from that in technical and other systems.

2. Problem statement
Present issues relating to the process control in ecological systems are based on mathematical models less promising in terms of accuracy and not always feasible and are of the nature of value judgments not supported by specific values that limit the implementation of digital control solutions.

The paper attempts to analyze the role of digital methods and their place in ecological systems research; to decide on the method allowing the implementation of a mathematical model more
simplified for the analysis of processes in ecological systems on the basis of comparison of the known
digital methods; to identify some features in the statistical analysis of processes taking place in the
environment and give recommendations for further practical application of the results.

3. Theoretical study

An ecological system (an ecosystem) by definition [4] is a set of populations ~of different species of
plants, animals, and microbes that interact~ with each other and their environment in such a way ~that
this set is maintained for indeterminate amount of time due to its harmonious balance. However, in
some cases the harmonious balance (homeostasis) of ecological systems is broken for some reasons
and the system can become uncontrollable and unstable (disturbance of the negative system feedback
of the). Another reason is that the processes in ecological systems tend to be random; i. e., there is
variability between the characteristic parameters both inside the ecosystem and outside it. The
uncertainty and variability of the processes in ecological systems and outside them give rise to
advanced application of statistical methods for analysis of the above processes. Therefore, there is a
necessity to have comprehensive immediate information on the current state of any ecosystem and
make an appropriate prediction regarding its equilibrium and future viability.

At present, digital diagnostics and prediction methods for various technical, social, and other
processes are the most widely used for ecosystems.

The main issues of solving problems for automatic forecasting and diagnostics of different
processes using digital statistical methods are given in [5]. This paper covers in detail such promising
digital methods as the method of univariate and multivariate regression, Bayesian method, the neural
network method, and others. When comparing the statistical methods currently known, the regression
analysis can be considered the most promising method in terms of convenience and simplicity of
implementation of the mathematical model for the process analysis as well as applicability for
prediction of parameter-variable processes.

Regression analysis. This method belonging to statistical methods for diagnostics and recognition
of processes involves the study of the dependence of a dependent variable on one or several
independent variables. Because of this, the regression analysis can be univariate and multivariate. The
equation of univariate [6] regression is as follows

\[ y = b_1 + b_2 x + \varepsilon, \]

where \( y \) is a response variable (a response function); \( x \) is a factor (a predictor); \( \varepsilon \) is a response
determination error; \( b_1 \) and \( b_2 \) are coefficients (or parameters of the regression). The average value of
the response \( E_y \) is equal to \( b_1 + b_2 x \). The deterministic component of the model is described by the
linear equation (regression dependence between variables is generally both linear and nonlinear)

\[ E_y = b_1 + b_2 x. \]

The random component is described by the distribution of \( \varepsilon \), that is, random deviations of the
observed response from the average. The purpose of the regression analysis can now be formulated
more strictly as the definition of the deterministic dependence based on the measurement data with
account of the error as an integral part of the model. In case of the linear dependence, the identification
reduces to the evaluation of \( b_1 \) and \( b_2 \).

The linear univariate regression in the form of \( y = b_1 + b_2 x \) can be found in many physical laws,
engineering (uniformly accelerated motion of a material point, Ohm's law, Darcy's law for water
filtration in the soil, Hooke's law for elastic deformations, and etc.), biology (dissolution rate of a
substance in the blood). However, due to the fact that in natural objects the property (characteristic) of
the controlled environment often depends on a large number of factors, the multiple regression
analysis is used in such a situation.

For example, in case of two independent variables, the linear equation describes a plane
\[
y = b_1 + b_2 x_1 + b_3 x_2 + \varepsilon,
\]
where \( y \) is a response function; \( x_1 \) and \( x_2 \) are factors (predictors); \( b_1, b_2, \) and \( b_3 \) are regression parameters or coefficients; \( \varepsilon \) is a response determination error.

The average response \( E_y \) is equal to \( b_1 \) if \( x_1 = 0 \) and \( x_2 = 0 \). The rate of change of the average response along the axes \( x_1 \) and \( x_2 \) is equal to \( b_2 \) and \( b_3 \), respectively. Thus, \( b_2 \) is a change in the \( E_y \) depending on \( x_1 \) at a fixed value of \( x_2 \) and \( b_3 \) is a change in \( E_y \) depending on \( x_2 \) at a fixed value of \( x_1 \).

It is common practice, that the parameters in the regression analysis are estimated by the method of least squares \([7]\), i.e., by minimizing the sum of squared differences between the values of the observed and expected variables. In the geometric interpretation, this means the following: the regression plane is selected so that the sum of squares of vertical distances between the observed values and the plane is minimal. The method of least squares analytically means that one should find the minimum sum of squared deviations of a certain function of the desired variables.

The essence of the method of least squares (the ordinary one) is to find such \( b \) parameters at which the sum of squared deviations (errors for regression models, which are often called regression excesses) \( \varepsilon \) will be minimal; i.e., the regression analysis uses the probabilistic models of dependencies between variables

\[
y_i = f(x_i + b) + \varepsilon_i,
\]
where \( \varepsilon_i \) are so-called random errors of the model.

The methods for finding the minimum values of random errors for sampling predictor-response values may be different, but they are based on the method of least squares. It is necessary to find such a value of the response-predictor relation, at which the errors (excesses or errors of the regression model) will be minimal.

As is known, nonlinear dependences between the parameters of various processes in nature are more common than linear ones, and, therefore, in such cases, the processing of experimental data on natural objects requires nonlinear regression \([8]\), which allows the procedure of experimental data approximation \([9]\). The problem of approximation reduces to the sequential execution of the following two operations. The first one is to determine a dependence type for \( y = \varphi(x) \) (a procedure that is generally carried out by experiments and according to certain rules). And the second one is to define numerical values of unknown parameters of the function selected \((b_1, b_2, b_3... b_n)\), at which the problem of smoothing the experimental data is solved in the best way (analysis of approximation errors for points \( y_1, y_2, y_3... y_n \)). Nonlinear regressions are divided into two classes \([10]\): regressions that are nonlinear in the explanatory variables included in the analysis but linear in the estimated parameters and regressions that are nonlinear in the estimated parameters.

Regressions on the explanatory variables can be represented as:

- polynomials of different powers such as \( y = a + b_1 x + b_2 x^2 + b_3 x^3 + \varepsilon \);  
- equilateral hyperbola such as \( y = a + \frac{b}{x} + \varepsilon \).

Regressions that are nonlinear in the estimated parameters can be represented as:

- power regressions such as \( y = ax^b + \varepsilon \);  
- exponential regressions such as \( y = ab^x + \varepsilon \).

The selection of a regression function is usually performed by the particular science on which basis the problems of establishing relations between the variables arise. The practice shows that the more the numbers of observations are, the more satisfactory results are. In each specific case, one should try the possibility of applying the linear regression at least in a limited area of change of variables.

For the first-class regression often referred to as quasi-linear regression the least squares method can be used, and, therefore, all the initial assumptions of the linear regression analysis and the properties of the least squares method – regression parameter estimates – remain valid.
The second-class regression does not allow the use of the ordinary least squares method. The solution the resulting system of nonlinear equations involves iterative methods [11] or approximation of the parameters of the desired dependence. In some cases, the linear transformation of the regression function is also employed, which allows the statistical linear regression criteria to be applied to the transformed parameters.

4. Practical relevance

Based on the above regression analysis, let us consider an example showing the possibility of using this method for diagnosing and predicting processes in ecological systems.

In [12], the dependence of the relative soil transpiration $T/T_0$ on the fluid pressure $p_F$ and relative water vapor saturation $W_{rel}$ is examined on the basis of linear multivariate regression. Experimental results demonstrating the change in soil transpiration depending on changes in fluid pressure and relative water vapor saturation are presented. To compose a multidimensional regression equation, the STATISTICA program has been used in this case. The following summary table (table 1) has been obtained with the changes in fluid pressure from 2.5 to 3.2, relative water vapor saturation from 72 to 30, and relative soil transpiration from 1 to 0.76.

| Regression Summary for Dependent Variable: $T/T_0$ |
|---------------------------------------------------|
| $R = 0.89901628$ $R^2 = 0.80823026$ Adjusted $R^2 = 0.71234540$ |
| $F(2,4) = 8.4292$ $p < 0.03678$ Std.Error of estimate: 0.04660 |

| Beta   | Std.Err. | $B$   | Std.Err. | $t(4)$ | $p$-level |
|--------|----------|-------|----------|--------|-----------|
| Intercept | 0.955250 | 1.046450 | 0.912848 | 0.412965 |
| $p_F$  | -0.263579 | 0.802285 | -0.0926 | 0.295027 | -0.328535 | 0.758987 |
| $W_{rel}$ | 0.642561 | 0.802285 | 0.003423 | 0.004274 | 0.800914 | 0.468055 |

According to the table, the multiple regression equation will be as follows:

$$T / T_0 = 0.955 - 0.097 p_F + 0.003 W_{rel}.$$  

The table shows that the equation is highly reliable according to the Fisher criterion (the significance point is less than 0.05, $p = 0.03678$). The analysis of significance of the regression equation parameters using the $t$-test has revealed that the parameters $b_1$ (the free member), $b_2$ (the coefficient before $p_F$), and $b_3$ (the coefficient before $W_{rel}$) are not significant at the accepted significance point (below 0.05). This means that we have the right to use the calculated equation to determine the relative transpiration based on $p_F$ and relative water vapor saturation $W_{rel}$, indicating that the equation is valid only at the significance point of 0.5.

The quality of the model found, i.e., how reliable the model is, is estimated by comparing the actual and predicted values. In other words, the discrepancy between the response of the dependence found and the value obtained in the experiment is known as a modeling error $\Delta$. The error can usually be represented as absolute and relative errors [13]. The absolute error is the deviation of the $\sigma$ value predicted by the model from the actually observed one:

$$\Delta_{abs} = y_{exp} - y_{calc}$$  

The relative error that is the ratio of the absolute error to the experimental value is expressed as:

$$\Delta_{rel} = (\Delta_{abs} / y_{exp}) \times 100\%$$

Using equations (1) and (2) makes it possible to calculate $\Delta_{abs}$ and $\Delta_{rel}$. For example, at a minimum value of relative soil transpiration of 0.76 the absolute error is equal to 0.02 and the relative error is equal to 2%.
According to the results of the calculated data at each point under study, the values of $\bar{x}$ and $\sigma$ can be estimated using the following formulas [14]:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$  \hspace{1cm} (3)

$$\sigma = \left( \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \right)^{\frac{1}{2}}$$  \hspace{1cm} (4)

where $\bar{x}$ is the arithmetic mean; $\sigma$ is the mean square deviation; $x_i$ is the result of the $i$-th change in the relative transpiration of the soil; $n$ is the number of calculated points.

When processing experimental data using the STATISTICA program, there may occur miscalculations of the data because of computer failure. Special methods are used for objective solution to the issue of whether any result is miscalculated. Two methods are the most widespread: the 3 $\sigma$ (three sigma) method (criterion) and the tabular method (the Smirnov-Grubbs test).

The 3 $\sigma$ method [15] is based on the assumption that the results of single calculations of $x$ can deviate from their arithmetic mean $\bar{x}$ by no more than 3$\sigma$. To find out a miscalculation, it is necessary to find the absolute value of the difference $A$ between the proposed miscalculation $x_m$ and the arithmetic mean of a number of calculations [16] according to the expression:

$$A = |x_m - \bar{x}|;$$

and compare the obtained $A$ value with 3$\sigma$. If the condition is met

$$A < 3\sigma$$  \hspace{1cm} (5)

the value of $x_m$ is not miscalculated. If condition in equation (5) is not met, then $x_m$ is miscalculated and it should be discarded.

The tabular method is used based on the assumption that the calculation results are equally accurate and subject to the normal distribution law. When using this method [17] in practice, special tables are compiled [18]. The table contains the values of some table coefficients $W_t$ calculated depending on the confidence probability $P$ and the number of calculations $n$. To find the expected miscalculation of $x_m$, first one should calculated $W$ using the following formula [19]:

$$W = \frac{|x_m - \bar{x}|}{\sigma}$$  \hspace{1cm} (6)

Then the value of $W$ obtained according to equation (6) from the calculated data is compared with a table value $W_t$. If the condition $W < W_t$ is met, the value of $x_m$ is not miscalculated. If it turns out that $W > W_t$, the value of $x_m$ should be discarded.

According to the calculated and experimental data [12] establishing the dependence of the relative soil transpiration $T/T_0$ on the fluid pressure $p_F$ and relative water vapor saturation $W_{rel}$, it is possible to identify the presence (absence) of miscalculations. To do this, first calculate the arithmetic mean using equation (3):

$$\bar{x} = \frac{5.95}{7} = 0.85.$$  

The number of calculations is taken as $n = 7$ here. After that, let us determine the mean square deviation $\sigma$ using equation (4): $\sigma = 0.1$. 


To identify a possible miscalculation by method 3, \(\sigma = 0.1\), which is compared with the difference \(x_m - \bar{x} = 0.93 - 0.85 = 0.08 < 0.3\), where \(x_m\) is the expected miscalculation and it is equal to 0.93. Since 0.08 is less than 0.3, the calculated value of 0.93 is not a miscalculation, and this value should stay in the calculation series.

To implement the tabular method, it is possible to write \(W = |0.93 - 0.85| / 0.1 = 0.8\). From the table data \(W_i = f(p,n) = 2.1\) (for the confidence probability \(P = 0.95\) and \(n = 7\) [20]). Since 0.8 < 2.1, 0.93 is not a miscalculation, this value should stay in the calculation series.

5. Conclusion
The study allows of making the following conclusions:
- The applicability of the regression analysis (of digital control methods) which provides simple and reliable mathematical modelling for prediction of random processes is shown;
- The main regularities and features of the regression method for process prediction and statistical processing of the current data on characteristic parameters of the processes are analysed;
- Using the experimental data, the paper demonstrates the practical application of a mathematical model for process prediction for an ecological object with account of metrological validation of the model.

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