Cavity-enhanced detection of magnetic orders in lattice spin models

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We develop a general scheme for detecting spin correlations inside a two-component lattice gas of bosonic atoms, stimulated by the recent theoretical and experimental advances on analogous systems for a single component quantum gas. Within a linearized theory for the transmission spectra of the cavity mode field, different magnetic phases of a two-component (spin 1/2) lattice bosons become clearly distinguishable. In the Mott-insulating (MI) state with unit filling for the two-component lattice bosons, three different phases: antiferromagnetic, ferromagnetic, and the XY phases are found to be associated with drastically different cavity photon numbers. Our suggested study can be straightforwardly implemented with current cold atom experiments.

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I. INTRODUCTION

Atomic quantum gases trapped in optical standing waves have become ideal systems for implementing lattice spin models after the pioneering theoretical proposal [1] and the experimental observation [2] of the superfluid (SF) to Mott insulator (MI) transition in the Bose-Hubbard model. When atoms of two-species or two-components are loaded into an optical lattice, a variety of more general effective spin models can be constructed [3, 4, 5], including the well-known anisotropic Heisenberg XXZ model. The development of noise spectroscopy [6, 7, 8, 9, 10, 11] has provided an astounding breakthrough that overcomes several significant hurdles in detecting quantum correlations, or in measuring the second order spin moments for various magnetic phases of lattice models.

Cold atoms are usually probed with time of flight methods, which measures the atomic density or matter-wave interference patterns upon being released from traps and often after significant expansions. The near resonant imaging light generally destroys the atomic state. Several quantum limited detection schemes have since been suggested, capable of quantum non-demolition detections of strongly correlated states in atomic lattice models [12, 13]. A very interesting approach relies on the enhanced detection sensitivity provided by an optical cavity, as was first proposed by Mekhov et. al. [14, 15]. The transmission spectra, calculated to the first order, or within the linear response theory of the amplitude for the probe field, assumes the initial state of atoms to remain unchanged when expectation values are taken and carries unambiguous signatures of magnetic orders in an atomic Bose-Hubbard model.

Several experimental groups have recently succeeded in the difficult first step of coupling atomic condensates into high Q optical cavities [16, 17], highlighting the prospects for creating and detecting exotic quantum phases of lattice spin models [18]. A promising new direction worthy of theoretical investigation concerns the study of atomic lattice spin models coupled with optical cavities, generalizing the single component study [14, 15]. Nonlocal quantum spin correlations of the various magnetic orders could analogously be reflected through the photon numbers and statistics.

This paper describes a scheme for detecting spin correlations in a two-species or two-component bosonic atom lattice [3, 4, 19, 20]. Our study shows that atomic spin correlations are faithfully mapped onto the transmission spectra of the cavity probe field, making them easily diagnosed through cavity QED based techniques.
II. MODEL

Our model is based on the scattering of two Raman matched incident laser beams from a lattice of effective spin 1/2 bosonic atoms \([21, 22]\). Similar to the original model \([13]\) for single component bosons, we consider \(N\) atoms with two internal states identically trapped in an optical lattice with \(M\) sites formed by far-off-resonant standing-wave laser beams. As schematically illustrated in Fig. 1, \(K < M\) lattice sites are located within the overlapped region of the two fundamental modes of the cavities. We consider two non-degenerate hyperfine states \([1]\) and \([2]\), the two stable ground states that are coupled to a common excited state \([3]\) with a blue common detuning \(\Delta\) and no differential detuning, forming a Raman coupled \(\Lambda\)-type atom model. The resonant cavity modes are denoted by matching labels with frequencies \(\omega_i\) \((l = 1, 2)\). For large detuning \(\Delta\) we adiabatically eliminate the excited state \([3]\) \([24, 25]\) and end up with two-state atoms effectively coupled in the overlapped region of two optical cavities. For a single atom, the effective coupling is described by \(\Omega a_i^\dagger b_i^\dagger b_\sigma + h.c.\) and the ac Stark shift becomes \(\delta a_i a_i^\dagger b_i^\dagger b_\sigma = 1\) with \(\delta = \eta^2/\Delta\) and \(\Omega = g_i g_2/\Delta\). The peak value for the dipole coupling with their respective cavity mode is denoted by \(g_\sigma\) for transition between \(|\sigma\rangle \leftrightarrow |\bar{\sigma}\rangle\). \(b_{1,2}(a_{1,2})\) denotes the corresponding annihilation operator for the atom (cavity photon).

Following the notations of Ref. \([13]\), the Hamiltonian for effective spin 1/2 bosons in a lattice coupled to two optical cavities takes the form \(H_B + H_I\), with

\[
H_I = \sum_{i=1,2} h\omega_i a_i^\dagger a_i - i\hbar \eta (a_1 e^{i\omega_1 t} - h.c.)
+ \hbar \delta \sum_{i=1}^K |u_1|^2 n_{i1} a_i^\dagger a_i + \hbar \delta_2 \sum_{i=1}^K |u_2|^2 n_{i2} a_i^\dagger a_2
+ \hbar \Omega \sum_{i=1}^K \left( A_i a_i^\dagger b_1^\dagger b_2 + h.c. \right),
\]

where \(n_{i\sigma} = b_i^\dagger b_i\) gives the number of atoms in state \(|\sigma\rangle\) at site \(i\) and \(u_{1,2}(r)\) is the mode function of the cavity with wave-vector \(k_{1,2}\). The coefficients \(A_i(\theta_1, \theta_2) = u_1^\dagger(r_i)u_2(r_i)\) due to emission/absorption or absorption/emission cycle are responsible for the geometric dependence of the effective coupling \([13]\).

With atoms assumed to occupy only the lowest Bloch band, our model generalizes the familiar Bose-Hubbard for two-components: \(H_B\) as in Eq. (1) of Ref. \([13]\) for two species. Following the work of \([13]\), we perform a linear calculation to the first order in cavity probe field, thus we leave out the dynamics of how various quantum phases of the atomic lattice are realized or dynamically created through the tuning of lattice parameters. This further justifies the neglect of atomic tunneling as well as the on-site intra- and inter-component interactions. In addition to the coupling of each atomic component with its corresponding cavity mode, Raman matched two-photon processes can transfer atoms between the two effective spin states, unless the atoms are prepared in the so-called dark state \(|\text{dark}\rangle \sim \langle a_2\rangle g_2 |1\rangle - \langle a_1\rangle g_1 |2\rangle\) corresponding to Coherent Population Trapping (CPT) \([23]\). We also assumed large detuning between cavity and atoms, to keep the actual excitations low, or negligible; thus any Raman type population transfers only affect the initial state to higher orders than the linear response theory calculation we provide. The second term in Eq. (1) describes the coherent pumping of cavity 1 at frequency \(\omega_{1p}\) with amplitude \(\eta\).

III. SEMICLASSICAL THEORY

We first consider the simplest case with no external pumping on cavity 1, i.e., \(\eta = 0\), and assume cavity mode \(a_2\) to be a classical field, or a \(c\)-number amplitude as in Ref. \([13]\). In the frame rotating with frequency \(\omega_2\), \(a_1\) evolves in time according to the Heisenberg equation

\[
\dot{a}_1 = -i(\Delta_{12} + \delta_1) \sum_{i=1}^K |u_1|^2 n_{i1} a_i - i\Omega \dot{D} a_2 - \kappa a_1, \quad (2)
\]

where \(\Delta_{12} = \omega_1 - \omega_2\) and \(\kappa\) denotes the cavity decay rate and is put in by hand. Its corresponding Langevin noise is neglected. We have defined the analogous operator \(\dot{D} = \sum_{i=1}^K S_i^\dagger\), in terms of the effective lattice spin operators \(S_i^\dagger = b_i a_i^\dagger b_i^\dagger b_\sigma + h.c.\), which obey the standard commutation relation at the same site and commute with each other on different sites. Neglecting the presumably much smaller cavity field induced ac Stark shift in comparison to \(\Delta_{12}\) or \(\kappa\) \([13]\), \(a_1\) and the photon number is easily obtained as

\[
a_1 = C \dot{D}, \quad a_1^\dagger a_1 = |C|^2 \dot{D}^\dagger \dot{D}, \quad (3)
\]

where \(C = -i\Omega a_2/(i\Delta_{12} + \kappa)\). The photon number \(\langle a_1^\dagger a_1\rangle\), clearly provides information about the spin correlation in the two-component boson lattice through the moments associated with the same site \(S_i^\dagger S_j\) and between the different sites \((S_i^\dagger S_j^\dagger)\). The angular dependence can become totally different due to the geometric coefficients \(A_i(\theta_1, \theta_2)\). Within the linear response, the above averages are expectation values with respect to whatever initially prescribed atomic ground state.

| Cavity phase | \(\langle a_1^\dagger a_1\rangle_{\theta_1=0}\) | \(\langle a_1^\dagger a_1\rangle_{\theta_1=\pi/2}\) |
|-------------|-----------------|-----------------|
| AF          | \(K|C|^2/2\)    | \(K|C|^2/2\)    |
| FM          | 0               | 0               |
| XY          | \((K+3K^2)|C|^2/16\) | \(K|C|^2/16\) |
| SF          | \(n_2(n_1K+1)|C|^2/16\) | \(n_2K|C|^2\) |

TABLE I: Cavity 1 photon number for the four quantum phases of the two-component Bose-Hubbard model at the diffraction maxima (minima) with \(\theta_1 = 0\) \((\theta_1 = \pi/2)\) and \(\theta_2 = 0\). For the XY phase \(\theta_A = \theta_B = \pi/3\).
The quantum phases for a two-component lattice bosons at commensurate fillings have attracted significant attention [19, 20]. The phase diagram consists of (1), 2MI where both boson components are in the MI phase; (2), SF+MI where one is SF and the other is MI; and (3), 2SF where both components are SF. Deep inside the MI phase the ground state of the system may be characterized by filling the lattice site with even or odd numbers of atoms [20]. In addition to the usual even filling phase with \( n_1 = n_2 \), a particularly interesting phase arises when the total filling factor is odd, especially at unit filling, i.e., for \( n_1 + n_2 = 1 \). This exotic phase has been extensively studied [3, 4, 19, 20] by adopting a trial wave function \( |\psi_{MI}⟩ = \prod_{i \in A,j \in B} (|\psi_A⟩)|\psi_B⟩ \), which is of a form composed of two sublattices A and B with \( |\psi_{A,B}⟩ = \cos(θ_{A,B}/2)|1,0⟩ + e^{iθ_{A,B}}\sin(θ_{A,B}/2)|0,1⟩ \). \( |n_1, n_2⟩ \) denotes the state with \( n_1 \) (\( n_2 \)) number of component-1 (2) atoms at site \( i \) and \( θ_a \) and \( ϕ_a \) are variational parameters.

Three types of spin exchange interactions are identified: (I), anti-ferromagnetic phase (AF) with \( θ_A = 0(π) \) and \( θ_B = π(0) \); (II), ferromagnetic phase (FM) with \( θ_A = θ_B = 0 \); and (III), XY phase with \( θ_A = θ_B ≠ 0 \). The 2SF phase, whose quantum state is \( Ψ_{AF} = (\sum_i b_{A,i}^\dagger)^{N_2}(\sum_i b_{A,i}^\dagger)(|0⟩) \) with \( N_{1,2} \) the total number of component-1 (2) atoms [20], will serve as a reference for presenting our results.

The scattered photons are explicitly tabulated in Table I. For a 1D optical lattice of a spatial period \( d = λ/2 \) and with atoms trapped at sites centered at \( x_j = jd \), the mode functions are \( u_{1,2}(r) = \exp(\imath|k_{1,2}|d \sin θ_{1,2}) \) for a traveling wave and/or \( u_{1,2}(r) = \cos(|k_{1,2}|d \sin θ_{1,2}) \) for a standing wave form. Atoms in the FM phase do not scatter because the two coupling paths to the excited state [3] destructively cancels as in the dark state. For the notation we use, the FM state corresponds to all atoms staying in state [1], then a semi-classical light amplitude \( ⟨a_2⟩ g_2 \) clearly will not be able to cause any scattering. While the initial atomic states of the AF and XY phases under the single excitation of a semi-classical light are not any more dark states, they will scatter. These features thus completely characterize the many-body spin correlations of the quantum phases for the two-component Bose-Hubbard model.

To map quantum fluctuations of lattice spins faithfully onto the probe cavity photon statistics, we define a noise function \( R(θ_1, θ_2) = (D^\dagger D) - (D^\dagger D) \), whose angular distribution is compared in Fig. 2 for all four quantum phases. The structure in the angular distribution comes from the summation of the geometric coefficients from different sites, reflecting both the on-site and off-site lattice spin correlations. In the SF phase with \( n_1 = n_2 = 1/2 \), the respective noise functions are completely different for the two choices of cavity modes. For the traveling wave, the noise function is zero for the FM phase, but takes nonzero values and is isotropic for the XY and AF phases. The angular dependence for the standing wave mode case is richer than that for the traveling waves. The structures in the angle dependence can be attributed to dependence on the summation of the geometric coefficients, and physically due to both on-site and off-site lattice spin correlations.

FIG. 2: (Color online) The angular distribution of \( R(θ_1, θ_2) \) for the four quantum phases evaluated for different choices of cavity mode functions: the left (right) panels are for two traveling (standing) waves and for \( θ_2 = 0 \) (\( θ_2 = 0.1π \)). We have assumed \( N = M = 2K = 40 \) and in the SF phase \( n_1 = n_2 = 1/2 \). For the XY phase \( θ_A = θ_B = π/3 \).

IV. QUANTIZED MODEL

We next consider the more general case with coherent pumping for cavity 1 at frequency \( ω_{p1} \) [13]. The dissipations for both cavities are assumed the same with the associated Langevin noise terms neglected in the Heisenberg operator equations. Within a linearized calculation, we decorrelate the atomic and field operators and replace in the Heisenberg equations for \( a_{1,2} \) the atomic operators by their respective expectation values, which leads to \( ⟨a_1^\dagger⟩ ⟨a_1⟩ = ⟨a_1^\dagger⟩ ⟨a_1⟩^2 \). To simplify our result, we further assume \( |u_{1,2}(r)|^2 = 1 \), which occurs for the diffraction maxima with \( A_1 = 1 \) at \( θ_1 = 0 \) or the minima with \( A_1 = (−1)^i \) at \( θ_1 = π/2 \) when the 1D lattice is lined up at \( θ_2 = 0 \). The cavity photons are found to be

\[
⟨a_1^\dagger⟩ ⟨a_1⟩ = η^2(κ^2 + ζ_1^2)/B, \quad ⟨a_2^\dagger⟩ ⟨a_2⟩ = η^2 ω^2 B/2, \quad (4)
\]

where \( B = κ^4 + κ^2(ζ_1^2 + ζ_2^2 + 2ω^2/κ^2) + (ζ_1 ζ_2 − α^2)^2 \), \( α = Σ_k A_1 \langle S_k^+ \rangle \), and \( ζ_l = ∆_p + δ_l Σ_i \langle n_i \rangle \). The detuning \( ∆_p = ω_1 − ω_2 \) are assumed the same for \( l = 1, 2 \) because \( ∆_12 ≪ Ω, \omega_1, ∆_p \). If the cavity coupling is assumed identical, we end up with \( δ_1 = Ω = δ_2 \). Equation (4) shows that probe photon numbers depend on the average values of on-site atom numbers \( ⟨n_σ⟩ \) and the lattice spin operators \( ⟨S_σ^±⟩ \). A crucial term for spin correlation \( α^2 \) appears in the expression for \( ⟨a_2^\dagger⟩ ⟨a_2⟩ \). At the diffraction minima or maxima \( α = 0 \) so that no photon will be detected from cavity 2 except for the XY
phase. This then allows for simplified expressions of the scattered photon numbers $\langle a_1^\dagger a_1 \rangle$ from the AF and FM phases into $\langle a_1^\dagger a_1 \rangle = \eta^2/(\kappa^2 + \zeta^2)$, which only depends on the detuning $\Delta_{1p}$ and atom numbers for component-1 in the overlapped $K$-sites $N^K_1 = \sum^K_n \langle n a_1 \rangle$. When $\alpha \neq 0$, however, $\langle a_1^\dagger a_1 \rangle$ for the XY phase at the diffraction maxima depends on two parameters $\zeta_1$ and $\zeta_2$ including the detunings $\Delta_{1p}$, $\Delta_{2p}$, and the number of atoms for both components in the overlapped region of $K$-sites. Measuring photon numbers $\langle a_1^\dagger a_1 \rangle$ thus gives sufficient information to distinguish magnetic orders or quantum phases of the two-component Bose-Hubbard model.

An especially interesting property concerns the dependence of the probe photon numbers on the detuning $\Delta_{1p}$, as is illustrated in Fig. 3 for the four quantum phases. For the FM and AF phases, we find Lorentzians with width $\kappa$ and shifted by $\delta N^K$ as in the classical result of a single component Bose-Hubbard model [15]. In contrast, for the SF phase the photon number distribution is an envelope of a comb for a good cavity ($\kappa = 0.1\delta$) while a smooth broadened contour for a bad cavity ($\kappa = \delta$). In the SF case, individual atoms are completely delocalized over all sites causing significant number fluctuations over each site within the $K$-site region. The corresponding quantum state is a superposition of Fock states containing all possible distributions of $N^K_1$ atoms for component-1 at $K$ sites, which gives rise to scattering terms from all possible atomic distributions. For the XY phase, the double peaked feature provides evidence for different population of atoms in the two internal states, with the relative heights of the two peaks being controlled by the variational parameters $\theta_{A,B}$. This structure in the cavity X phase is essentially identified with the so-called superfluid counterflow (SCF) phase, which can be qualitatively understood as a paired superfluid vacuum (PSF) phase, a strongly correlated superfluid ground state already predicted from numerical simulations [13]. These distinct features of the transmission spectrum we discuss for the various quantum phases form the basis for easily detecting and differentiating the corresponding magnetic orders in the two-component Bose-Hubbard model.

Like the original cavity scheme of Mekhov et. al. [14, 15], the scheme we propose, is constructed to detect high order moments. The different phases (in the sense of quantum states of matter) of a two-component lattice boson gas are resolved from the statistics of scattered photons or pseudo-spins. In this sense, it is analogous to the so-called noise spectroscopy of quantum gases [6, 28], albeit somewhat superior due to the enhanced collection efficiency aided by cavities. The Ramsey spectroscopy [27], as proposed by Kuklov, measures the first order moments of atomic pseudo-spins. The SCF state or the paired condensation phase is a special case, where the order parameters are simply field operators themselves. Thus their presence can be probed by the Ramsey spectroscopy measurement of the relative phase (in the sense of amplitude and phase).

![FIG. 3: (Color online) Cavity 1 photon numbers as a function of cavity-probe detuning for the four quantum phases: AF (red dashed dot), FM (blue dashed), XY (pink dotted), and SF (green solid). In our simulation we use $K = 20$ for all phases and in the SF phase $n_1 = n_2 = 1/2$. For the XY phase $\theta_A = \theta_B = 0.6\pi$.](image)

V. CONCLUSIONS

In summary we have generalized the model of a single component atomic lattice gas described by the Bose-Hubbard model coupled to near resonant optical cavities to the case of a two-component Bose-Hubbard model. We have shown conclusively through the probe cavity photon numbers and its spectra dependence on various system parameters that different quantum phases of the two-component Bose-Hubbard model can be easily distinguished and confirmed. Our results shine new light on atomic lattice gases coupled to cavity QED systems.

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[1] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. 81, 3108 (1998).
[2] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415, 39 (2002).
[3] L.-M. Duan, E. Demler, and M. D. Lukin, Phys. Rev. Lett. 91, 090402 (2003).
[4] A.B. Kuklov and B.V. Svistunov, Phys. Rev. Lett. 90, 100401 (2003); A. Kuklov, N. Prokof'ev, and B. Svistunov, Phys. Rev. Lett. 92, 030403 (2004); ibid 92, 050402 (2004).

[5] C. Lee, Phys. Rev. Lett. 93, 120406 (2004).

[6] E. Altman, E. Demler, and M. D. Lukin, Phys. Rev. A 70, 013603 (2004).

[7] S. Fölling, F. Gerbier, A. Widera, O. Mandel, T. Gericke, and I. Bloch, Nature 434, 481 (2005).

[8] M. Greiner, C. A. Regal, J. T. Stewart, and D. S. Jin, Phys. Rev. Lett. 94, 110401 (2005).

[9] I. Carusotto and E. J. Mueller, J. of Phys. B 37, S115 (2004).

[10] I. Carusotto, J. of Phys. B: At. Mol. Opt. Phys. 39, S211 (2006).

[11] Q. Niu, I. Carusotto, and A. B. Kuklov, Phys. Rev. A 73, 053604 (2006).

[12] K. Eckert, O. Romero-Isart, M. Rodriguez, M. Lewenstein, and E. S. Polzik, A. Sanpera, Nature Physics 4, 50 (2008).

[13] K. Eckert, L. Zawitkowski, A. Sanpera, M. Lewenstein, and E. S. Polzik, Phys. Rev. Lett. 98, 100404 (2007).

[14] I. B. Mekhov, C. Maschler, and H. Ritsch, Nature Physics 3, 319 (2007).

[15] I. B. Mekhov, C. Maschler, and H. Ritsch, Phys. Rev. Lett. 98, 100402 (2007).

[16] F. Brennecke, T. Donner, S. Ritter, T. Bourdel, M. Köhl, and T. Esslinger, Nature 450, 268 (2007); Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, and J. Reichel, Nature 450, 272 (2007).

[17] D. Jaksch, S. A. Gardiner, K. Schulze, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 86, 4733 (2001).

[18] J. Larson, B. Damski, G. Morigi, and M. Lewenstein, Phys. Rev. Lett. 100, 050401 (2008).

[19] E. Altman, W. Hofstetter, E. Demler, and M. D. Lukin, New J. Phys. 5, 113 (2003).

[20] A. Isacsson, M.-C. Cha, K. Sengupta, and S. M. Girvin, Phys. Rev. B 72, 184507 (2005).

[21] C. J. Myatt, E. A. Burt, R. W. Ghrist, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. 78, 586 (1997); D. S. Hall, M. R. Matthews, J. R. Ensher, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. 81, 1539 (1998); ibid., 1543 (1998)

[22] P. Maddaloni, M. Modugno, C. Fort, F. Minardi, and M. Inguscio, Phys. Rev. Lett. 85, 2413 (2000)

[23] E. Arimondo, Progress in Optics 35, 257 (1996).

[24] M. Alexanian and S. K. Bose, Phys. Rev. A 52, 2218 (1995).

[25] C. C. Gerry and J. H. Eberly, Phys. Rev. A 42, 6805 (1990); D. A. Cardimona, V. Kovanis, M. P. Sharma and A. Gavrielides, Phys. Rev. A 43, 3710 (1991).

[26] M. Rodriguez, S.R. Clark, and D. Jaksch, Phys. Rev. A 75, 011601(R)(2007).

[27] A. Kuklov, N. Prokof’ev and B. Svistunov, Phys. Rev. A 69, 025601 (2004).

[28] I. B. Mekhov, C. Maschler, and H. Ritsch, Phys. Rev. A 76, 053618 (2007).