Interaction of cosmic background neutrinos with matter of periodic structure

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We study coherent interaction of cosmic background neutrinos (CBNs) with matter of periodic structure. The mixing and small masses of neutrinos discovered in neutrino oscillation experiments indicate that CBNs which have very low energy today should be in mass states and can transform from one mass state to another in interaction with electrons in matter. We show that in a coherent scattering process a periodic matter structure designed to match the scale of the mass square difference of neutrinos can enhance the conversion of CBNs from one mass state to another. Energy of CBNs can be released in this scattering process and momentum transfer from CBNs to electrons in target matter can be obtained.

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CBN is one of the major predictions of the big-bang cosmology. The big-bang cosmology predicts that the temperature of CBNs, if they are relativistic today, is around 1.96 K \cite{1} (\(\sim 10^{-3}\) eV) in the present universe. The average number density of CBNs per species in the present time is predicted to be \(\bar{n}_\nu = 56 \text{ cm}^{-3}\). \(\tag{1}\)

The detection of CBNs is extremely difficult due to the extremely low energy and the small density of CBNs in the present universe.

In this article we study the scattering of massive CBNs with matter of periodic structure. We point out that a periodic structure of matter, when matching with the scale of the mass square difference of massive CBNs, can help to convert CBN of one mass state to another and enhance the probability of CBNs scattering with matter. Net momentum transfer from CBNs to target matter can be achieved in this coherent scattering process.

In the following we first have a brief review of present knowledge of neutrino masses and mixing. Then we study the scattering and conversion of CBNs in a target matter of periodic profile. We discuss momentum transfer from the wind of CBNs to target matter. Finally we conclude.

Neutrino oscillation experiments have shown that neutrinos have very small masses and flavor mixing. Two mass square differences of neutrinos have been measured in oscillation experiments \cite{2}:

\[
\Delta m^2_{21} = 7.50_{-0.20}^{+0.19} \times 10^{-5} \text{ eV}^2, \quad (2)
\]

\[
|\Delta m^2_{32}| = 2.32_{-0.08}^{+0.12} \times 10^{-3} \text{ eV}^2. \quad (3)
\]

Flavor mixing of neutrinos is expressed as

\[
\nu_1 = \sum_i U_{1i} \nu_i, \quad (4)
\]

where \(\nu_1\) is neutrino state in flavor base and \(\nu_i\) neutrino state in mass base. Neutrino mixing matrix \(U\) can be parameterized using three mixing angles \(\theta_{12,23,13}\) and a CP violating phase \(\delta\). In solar, atmospheric and long baseline neutrino oscillation experiments two mixing angles have been measured: \(\sin^2 2\theta_{12} \approx 0.86\), \(\sin^2 \theta_{23} \approx 0.50\) \cite{2}. Recent observation of oscillation of reactor antineutrinos in Daya Bay experiment \cite{3}, confirmed by RENO experiment \cite{4}, shows that \(\sin^2 2\theta_{13} \approx 0.092\). According to all these measurements of neutrino oscillation we can conclude that no element of \(U\) is zero.

Neutrino masses are also measured in \(\beta\) decay experiment and in cosmological observations. \(\beta\) decay experiment gives an upper bound on the mass of electron anti-neutrino \cite{2}

\[
m_{\bar{\nu}_e} \lesssim 2.0 \text{ eV} \quad (5)
\]

The observations of the anisotropy of Cosmic Microwave Background Radiation and the Sloan Digital Sky Survey give a constraint on the total mass of neutrinos \cite{2}

\[
\sum_i m_i \lesssim 0.8 \text{ eV} \quad (6)
\]

Taken all these measurements of neutrino masses into account, possible patterns of neutrino masses are: Normal Hierarchy (NH) with \(m_1 < m_2 < m_3\), \(m_2 \approx \sqrt{|\Delta m^2_{32}|} \approx 0.9 \times 10^{-2} \text{ eV}\) and \(m_3 \approx \sqrt{|\Delta m^2_{21}|} \approx 0.05 \text{ eV}\); Inverted Hierarchy with \(m_3 < m_1 < m_2\), \(m_{1,2} \approx \sqrt{|\Delta m^2_{32}|} \approx 0.05 \text{ eV}\); Quasi-Degeneracy (QD) with \(m_1 \approx m_2 \approx m_3 \lesssim 0.3 \text{ eV}\), \(m_i \gg \sqrt{|\Delta m^2_{32}|}\).

Massive CBNs with \(m \gg 10^{-4} \text{ eV}\) should be non-relativistic in the present time and should be in mass states. Massive CBNs with low velocity should also be clustered in galactic halos or in...
cluster halos. For different neutrino mass patterns massive neutrinos are different and the contents of non-relativistic CBNs are different too. According to the above discussion of neutrino mass patterns we can figure out that the CBNs which are non-relativistic today include $\nu_{2,3}$ for NH, $\nu_{1,2}$ for IH and all $\nu_{1,2,3}$ for QD. In the following we will consider interaction of massive CBNs with matter.

The interaction of neutrinos relevant to our analysis is the interaction which is non-universal in neutrino flavors. Neglecting radiative corrections neutral current interaction of neutrinos with matter is universal in flavors and is irrelevant to later analysis. The relevant interaction is given by the charged current interaction of neutrino with electron:

$$\Delta \mathcal{L} = -\frac{4G_F}{\sqrt{2}} \bar{\nu}_e \gamma^{\mu} \nu_\mu \bar{e}_L \gamma_{\mu} e_L,$$

(7)

where $e_L$ and $\nu_e$ are the fields of electron and electron neutrino with left-chirality. $G_F$ is the Fermi constant. Using (7) one can find that in an unpolarized target of matter at rest the coherent interaction of neutrino with matter gives a potential term to electron neutrino:

$$\Delta \mathcal{L} = -\sqrt{2} G_F N_e \bar{\nu}_e \gamma^0 \nu_e,$$

(8)

where $N_e$ is the number density of electron in matter. (8) describes coherent scattering of neutrino with matter in which neutrino coherently scatters with many electrons in matter. Momentum transfer from neutrino, if not zero, is distributed to very large numbers of electrons participating actively in the scattering process and momentum transfer to a single electron can be taken zero. This case is exactly what we study in later discussion.

In the mass base (8) can be rewritten as

$$\Delta \mathcal{L} = -\bar{\nu}_e U^{*}_{ei} U_{ej} \bar{v}_j \gamma^0 v_i,$$

(9)

where $i, j = 1, 2, 3$ and $V_e = \sqrt{2} G_F N_e$. According to (9) neutrino in one mass state $\nu_i$ can transform to another mass state $\nu_j$ in interaction with electrons in matter. Consider such a transition $\nu_i \rightarrow \nu_j$ in a target of matter. For a uniform incident flux of neutrino $\nu_i$ the cross section for the $\nu_i \rightarrow \nu_j (\bar{v}_i \rightarrow \bar{v}_j)$ conversion in matter is

$$\sigma = \frac{1}{2E_i \nu_i} \int \frac{d^3k_j}{(2\pi)^3} \frac{1}{2E_j} 2\pi \delta(E_i - E_j) \left| M \right|^2$$

$$\times \left| \int \frac{d^3x}{\Omega} \bar{V}_e(x) U^{*}_{ei} U_{ej} e^{-i (\bar{k}_i - k_j) \cdot x} \right|^2,$$

(10)

where $E_i$ and $E_j$ are the energies of the initial $\nu_i$ and final $\nu_j$ respectively, $\nu_i$ the velocity of the initial $\nu_i$ relative to the target, $\bar{k}_i$ and $k_j$ the initial and final momenta. $\Omega$ is the volume of the target. $\left| M \right|^2$ is the matrix element squared:

$$\left| M \right|^2 = k_i^0 k_j^0 + \bar{k}_i \cdot \bar{k}_j$$

for unpolarized Dirac type neutrinos. In later discussion we will concentrate on massive $\nu_i$ CBNs which are non-relativistic and are clustered in present time. Since the direction of motion is changed in clustering process but the spin of neutrino is not, the clustered CBNs can be considered mixed with left and right helicities. So we can take $|\bar{k}_i| \ll k_i^0$ and use $\left| M \right|^2 = k_i^0 k_j^0$ in this case. For Majorana neutrino the neutrino and anti-neutrino are identical and $\left| M \right|^2$ is replaced by $\left| M \right|^2 = 2(k_i^0 k_j^0 + \bar{k}_i \cdot \bar{k}_j - m_i m_j)$. It is velocity suppressed if $\nu_i$ and $\nu_j$ are both non-relativistic. For relativistic $\nu_j$ and non-relativistic $\nu_i$ one can use the approximation $\left| M \right|^2 = 2E_i E_j$. For simplicity we will concentrate on Dirac type neutrino and use $\left| M \right|^2 = E_i E_j$ in later discussion.

We consider a target of matter which is constant in $x$ and $y$ directions and periodic in $z$ direction, as shown in Fig. 1. The potential term in such kind of matter profile satisfies

$$V_e(z + d) = V_e(z), \quad \frac{dV_e}{dx} = \frac{dV_e}{dy} = 0,$$

(11)

where $d$ is the period of the matter profile. A general potential term of such kind periodic structure can be expressed using Fourier transformation as

$$V_e(x) = \sum_n V_n e^{i \mathbf{q}_n \cdot \mathbf{x}},$$

(12)

where $n$ is an integer, $\mathbf{q}_n = q_n \hat{z}$ and $q_n = 2n\pi/d$. $V_n$ satisfies: $V_n^* = V_{-n}$.

If the path length of neutrino in target is constant the cross-section can be expressed as $\sigma = Sp$ where $p$ is the probability of $\nu_i \rightarrow \nu_j$ conversion and $S$ is the geometric cross section of the target of matter. Implementing (12) into (10) and integrating over space coordinates we find that the conversion probability is

$$p_n = \frac{|k^2_i| \left| M \right|^2}{4 E_i^2 \nu_i E_j} |V_n| \bar{V}_e(z) \left| U_{ei} U_{ej} \right|^2 \frac{4 \sin^2(\Delta_n \mathbf{L}_z)}{(\Delta_n \mathbf{L}_z)^2},$$

(13)

FIG. 1: Scattering of neutrino with matter of periodic structure.
When the resonant conversion happens the cross-section and it means that non-relativistic velocity of CBNs is a bit larger and can reach 1000 km/s. Since CBNs are in virial equilibrium in the local cluster halo, the velocity of CBNs is proportional to $\nu$.

We can see that the probability is proportional to $|V_n|^2$ if $|\Delta_n L_z| > 1$. If $|\Delta_n L_z| < 1$ the conversion is resonantly enhanced and the probability is proportional to $|V_n|^2$.

At the resonant point of $\nu_i - \nu_j$ conversion we can find

$$k^2_j = k^2_i - k^2_j - q_n = 0$$  \hspace{1cm} (15)

This condition of resonant conversion is expressed in short as

$$\bar{k}_i - \bar{k}_j - \bar{q}_n = 0. \hspace{1cm} (16)$$

When the resonant conversion happens the cross-section and the probability $p_n$ are proportional to $|V_n|^2$ of one particular $n$ and other $V_{n' \neq n}$ effectively contribute zero.

Using energy conservation we can find that

$$|\bar{k}_j| = \sqrt{m^2_i - m^2_j + \bar{k}^2_i} \hspace{1cm} (17)$$

Using $k^2_i = k^2_j$, $k^2_i = k^2_j$ and (17) we can find that

$$|k^2_j| = \sqrt{m^2_i - m^2_j + (k^2_j)^2} \hspace{1cm} (18)$$

We will concentrate on massive CBNs which are non-relativistic in the present universe. The velocity of massive CBNs at the position of solar system depends on the clustering properties of CBNs. If CBNs are in virial equilibrium in the local galactic halo the velocity of these CBNs is $\sim 200 \text{ km/s} \sim 10^{-3}c$ where $c$ is the speed of light. If CBNs are in virial equilibrium in the local cluster halo the velocity of CBNs is a bit larger and can reach $\sim 1000 \text{ km/s} \sim 10^{-2}c$. According to (6) we can get $m_i \lesssim 0.2 - 0.3 \text{ eV}$. So we can conclude that $|k^2_i| = |m_i \bar{v}_i| \lesssim 10^{-3} \text{ eV}$ and $\bar{k}^2_i \approx |\Delta m^2_{21,32}|$. So using (18) we get $|k^2_j| \approx \sqrt{\Delta m^2_{ij}, \frac{|k^2_j|}{E_j} \text{ in (13)}}$ can be approximated as $|k^2_j|/E_j \approx \sqrt{1 - m^2_j/m^2_i}$.

Apparently the $\nu_i - \nu_j$ conversion can not happen if $\Delta m^2_{ij} < 0$. For $m^2_i - m^2_j > 0$, $\nu_i - \nu_j$ conversion is allowed and it means that non-relativistic $\nu_i$ converts to $\nu_j$ and releases part of its rest energy to kinetic energy of $\nu_j$.

The transferred momentum from neutrino to the target is $\bar{k}_i - \bar{k}_j$. Using (10) we can find that for resonant $\nu_i - \nu_j$ conversion the momentum transfer is $\bar{q}_n$. Using (13) we can find that $|q_n| = |k^2_j - k^2_i| \approx \sqrt{\Delta m^2_{ij}}$ and $\bar{q}_n \approx \pm \sqrt{\Delta m^2_{ij}} \bar{z}$. $q_n$ can be positive or negative which correspond to cases that $\nu_j$ is reflected or is refracted by the target matter.

FIG. 2: Asymmetry of $p'_{+1}$ versus $v_x^2$ for $\nu_2 - \nu_1$ transition of NH: $m_2 = \sqrt{\Delta m^2_{21}}, k_x L_z = 5$. $k_s = m_2 |\bar{v}_s|$. net momentum transfer per unit time from CBNs is

$$P = |q_{\pm 1}| S n_i \int dv_i (p_{+1} - p_{-1}) v_i f(v_i), \hspace{1cm} (19)$$

where $f$ is the velocity distribution of local $\nu_i$ CBNs, $n_i$ the number density of local $\nu_i$ and $\bar{v}_i$ CBNs, $S$ the geometric cross section of target matter. $P$ can be positive or negative which correspond to cases that the momentum transfer to target is of positive or negative $z$ direction. Apparently there will be no net momentum transfer from CBNs to target detector if the probabilities of CBNs being refracted or being reflected by target matter are equal. A periodic structure of target matter can make these two probabilities differ significantly and net momentum transfer from CBNs to target can be obtained.

To illustrate that net momentum transfer can be achieved we consider, as an example, the case that $q_{+1}, -1$ dominate the $\nu_i - \nu_j$ conversion. In such a case the period of the target detector should be arranged to satisfy $\frac{2\pi}{\Delta \nu_{ij}} \approx \sqrt{\Delta m^2_{ij}}$. When neutrino is refracted by the detector $k^2_j = \sqrt{\Delta m^2_{ij} + (k^2_i)^2}$ and $n = -1$. We get

$$\Delta_{-1} = k^2_j + \frac{2\pi}{d} - \sqrt{\Delta m^2_{ij} + (k^2_i)^2} \hspace{1cm} (20)$$

When neutrino is reflected by the detector $k^2_j = -\sqrt{\Delta m^2_{ij} + (k^2_i)^2}$ and $n = +1$. We get

$$\Delta_{+1} = k^2_i - \frac{2\pi}{d} + \sqrt{\Delta m^2_{ij} + (k^2_i)^2} \hspace{1cm} (21)$$

We can see that $p_{+1} - p_{-1}$ would not be zero and net momentum transfer would be obtained if the detector is arranged in such a way that $|\Delta_{+1} L_z| < 1 < |\Delta_{-1} L_z|$ or $|\Delta_{-1} L_z| < 1 < |\Delta_{+1} L_z|$.

To achieve net momentum transfer from neutrino background we note that the solar system is

\[\Delta_n = k^2_{ij} - k^2_{ij} - q_n,\]  \hspace{1cm} (14)
moving in the local galactic halo or cluster halo. The momentum of CBNs in the rest frame of solar system can be written as

$$\vec{k}_i = \vec{k}_s + \Delta \vec{k}_i,$$  \hspace{1cm} (22)

where $\vec{k}_s$ is the momentum caused by motion of solar system relative to CBN halo. If the structure of detector is arranged such that $|q_{\pm 1}| = 2\pi = k_s^z + \sqrt{\Delta m_{ij}^2 + \delta}$ we can find that

$$\Delta_{-1} = 2k_s^z + \Delta k_i^z + \delta - \frac{1}{2} \frac{(k_i^z)^2}{\Delta m_{ij}}$$  \hspace{1cm} (23)

$$\Delta_{+1} = \Delta k_i^z - \delta + \frac{1}{2} \frac{(k_i^z)^2}{\Delta m_{ij}^2},$$  \hspace{1cm} (24)

where $\delta$ is a possible small mismatch between $2\pi$ and $k_s^z + \sqrt{\Delta m_{ij}^2}$. Since $|\vec{k}_i|/\sqrt{\Delta m_{ij}^2} \lesssim 10^{-1} - 10^{-2}$ as observed in previous discussions the last terms in (23) and (24) can be neglected. If $|2k_s^z L_z| > 1$ it’s easy to see that a difference between $p_{-1}$ and $p_{+1}$ can be achieved. Apparently if $k_s^z = 0$ two probabilities should be equal. In Fig. 2 we can see the asymmetry of $p_{\pm 1}$ clearly when $v_s^z$ approaches 250 km/s.

In Fig 2 we compute $\Delta p'$:

$$\Delta p' = \int dv_s^z f(v_s^z) (p_{+1} - p_{-1}),$$  \hspace{1cm} (25)

where $p_n' = \sin^2(\Delta_n L_z)/(\Delta_n L_z)^2$. The net momentum transfer per unit time from CBNs, $P$, can be expressed using $\Delta p'$ as

$$P = |q_{\pm 1}| Sn_i c \sqrt{-\frac{m_i^2}{m_s^2}} |V_{\pm 1} L_z U_{n_i}^* U_{n_i}|^2 \Delta p',$$  \hspace{1cm} (26)

where $|M|^2 \approx E_i E_j$ has been used. As an estimation we use the Maxwellian distribution

$$f(v) = \frac{1}{\pi^{1/2} a} e^{-(v^2 - v_s^z)^2/a^2}.$$  \hspace{1cm} (27)

In galactic halo $v_s^z$ can reach $|v_s^z| = 250$ km/s when $z$ direction follows the direction of motion of the solar system in Milky Way. $a$ is the velocity dispersion of CBNs which is taken as $a = 150$ km/s in galactic halo. In computing Fig. 2 we have used $2\pi/d = m_2|v_s^z| + \sqrt{\Delta m_{ij}^2}$ and $\delta = 0$. We can see in Fig. 2 that the asymmetry of $p_{\pm 1}$ disappears when the $v_s^z$ approaches zero and it’s maximal when $v_s^z = |v_s^z|$. $\Delta p'$ does not reach 1 partly due to the cancellation of $p_{\pm 1}$. For $|k_s L_z| < 1$ the cancellation can be significant. For $|k_s L_z| \sim 1$ we can have $\Delta p'$ of order one. The other factor which reduces $\Delta p'$ is that for large $L_z$ only parts of CBNs with $|\Delta k_i^z L_z| \lesssim 1$ contribute to the resonant conversion. The larger $L_z$ is, the smaller the fraction of CBNs contributing to resonant conversion. $|\Delta p' / k_s L_z|$ actually increases as $k_s L_z$ increases and it approaches a fixed value for $k_s L_z \gg 1$. In Fig. 3 we can see this effect. We can find that for $k_s L_z = 80$ and 40 the two curves of $\Delta p' / k_s L_z$ converge. In this case $\Delta p = p_{+1} - p_{-1}$ approaches to a value which is approximately proportional to $3L_z / k_s$.

Note that the Earth orbits the Sun with a speed $v_o = 30$ km/s and $v_E$, the speed of the Earth relative to the local halo, varies in the range $v_s - v_o, v_s + v_o$. As a consequence, $\Delta_{\pm 1}$ are modulated by $v_o$ if considering CBNs interacting with a detector on the Earth. In the case $k_s L_z \gg 1$ it’s easy to figure out that the range of CBN distribution relevant to resonant conversion, which gives $|\Delta k_i^z L_z| \lesssim 1$, is shifted by velocity up to $\pm v_s$ in modulation. Using Eq. (27) with $v_s^z$ replaced by $v_E$, one can find that for $2\pi/d = m_i v_s + \sqrt{\Delta m_{ij}^2}$
the probability is changed by a factor in a range $[e^{-\Delta m^2_{ij}/\alpha^2}, 1]$ which corresponds to modulation of probability around 4% for $\alpha = 150$ km/s. One can also see that if the detector is designed such that $2\pi/d = m_i(v_s+v_o) + \sqrt{\Delta m^2_{ij}}$, the probability is changed by a factor in a range $[e^{-(2\nu \alpha)^2/\alpha^2}, 1]$ which corresponds to modulation of probability around 15%. The amplitude of modulation depends on $2\pi/d$ and varies from about 4% to about 15%. Result of numerical computations confirms this estimate. When $k_sL_z \sim 1$ the range of $\nu$ contributing to resonant conversion is broad and a shift in velocity of order $v_o$ does not change much the result. In this case the modulation is weak.

We note that to achieve maximal $\Delta p'$ sufficiently good matching between $|q_{\pm 1}|$ and $\sqrt{\Delta m^2_{ij} \pm m_i|\vec{v}_s|}$ is needed. Unfortunately we do not have very precise knowledge of $\Delta m^2_{ij}$. With $QD$ it’s hard to achieve a matching with precision to one of a hundred or one of a thousand. To overcome this problem one can use a number of detectors which makes a scan of the range of $\Delta m^2_{ij}$. For example one can use a hundred copies of detectors with identical $L_z$ and slightly different $|q_{\pm 1}|$. The values of $|q_{\pm 1}|$ are evenly distributed in the uncertain range of $\Delta m^2_{ij}$. In Fig. [4] we give an example for $\nu_3 - \nu_1$ conversion and QD: $m_3 = 3 \times \sqrt{\Delta m^2_{31}}$, $\nu_3$ is considered heavier than $\nu_1$ in this example. The range of $|q_{\pm 1}|$ in this figure corresponds to $\Delta m^2_{31}$ in the uncertain range $[2.08, 2.68] \times 10^{-3}$ eV$^2$. We see that the values of $|q_{\pm 1}|$ which give resonant conversion are different for different values of $\Delta m^2_{31}$ and for each $|\Delta m^2_{31}|$ there are 6-8 values of $|q_{\pm 1}|$ of total one hundred which give resonant enhancement. Positive or negative $\Delta p'$ correspond to the cases that the momentum transfer to target is of the positive or negative z direction. We note that by carefully adjusting the period of target matter the momentum transfer from CBNs can be of the positive or negative direction of the CBN wind.

The number of $|q_{\pm 1}|$ which gives resonant conversion depends on the neutrino mass pattern. For QD mass pattern the neutrino mass is larger than that of NH and IH. The initial momentum of CBNs are larger too. So it’s easier for QD to achieve a matching of $|q_{\pm 1}|$ and $\sqrt{\Delta m^2_{ij} \pm k_s}$ to the precision of $k_s$. In Fig. [5] we can see this clearly in $\nu_2 - \nu_1$ conversion. The range of $|q_{\pm 1}|$ in this figure corresponds to $\Delta m^2_{21}$ in the uncertain range $[6.90, 8.17] \times 10^{-5}$ eV$^2$. We see that for QD the resonant region is broad. But for NH the resonant conversion happens in a narrow region of $|q_{\pm 1}|$.

For matter on Earth we find that $V_e \sim 10^{-13}$ eV. A detector can be designed to have $V_{\pm 1}$ of the same order of magnitude of $V_e$. For $L_z = 1$ cm we have $|V_{\pm 1}| \sim 10^{-8}$. When sufficiently large $\Delta p'$ is achieved we can have a rough estimate of the net

![Diagram](image_url)

**FIG. 5:** Asymmetry of $p_{\pm 1}$ versus $|q_{\pm 1}|$ for $\nu_2 - \nu_1$ transition. $k_sL_z = 10$. $k_s^1 = k_s = m_2|\vec{v}_s|$. NH: $m_2 = \sqrt{\Delta m^2_{31}}$; QD1: $m_2 = 2 \times \sqrt{\Delta m^2_{31}}$; QD2: $m_2 = 3 \times \sqrt{\Delta m^2_{31}}$.

momentum transfer per unit time from CBNs:

$$P \sim \frac{|q_{\pm 1}|}{1 - m_i^2/m_i^2} \frac{S}{10^3 \text{ cm}^3} \times \left(\frac{L_z}{1 \text{ cm}}\right)^2 \left(\frac{2}{k_sL_z}\right) \text{s}^{-1}. \quad (28)$$

In (28), $\Delta p' \sim 2/(k_sL_z)$ for $k_sL_z > 1$ has been used. This momentum transfer increases as $L_z$ increases and as can be seen in (28) it approaches to a value proportional to the volume of the target when $k_sL_z > 1$. The momentum transfer to target matter gives rise to a mechanical force exerted on the target detector. The acceleration due to this force is $a_P = P$, where $M$ is the total mass of the target. Taking $M = \rho SL_z$ where $\rho$ is the average density of target and using (28) we can find that $a_P$ approaches to a constant value when $L_z > 1/k_s$:

$$a_P \sim \frac{|q_{\pm 1}|}{\rho \times 10^4 \text{ cm}^3} \frac{1 - m_i^2/m_i^2}{100 \text{ cm}^3}. \quad (29)$$

For example, for $\nu_3 - \nu_1$ transition of NH we can find that $m_3 \approx 0.05$ eV and $k_s \times 1 \text{ cm} \approx 2$. In this case $|q_{\pm 1}| \approx 0.05$ eV/c and we find that the acceleration can reach $10^{-28}$ cm s$^{-2}$ for $\rho = a \text{ few g/cm}^3$. For other neutrino mass pattern the estimate of the momentum transfer and the acceleration is similar except that for $\nu_2 - \nu_1$ transition of NH $L_z \gtrsim 2 \text{ cm}$ should be taken to make $k_sL_z > 1$.

As can be seen the net momentum transfer to target matter is very small. The mechanical force exerted on the target is also very small. The acceleration due to this force is far less than the acceleration that can be detected in modern technology, that is about $10^{-12}$ cm s$^{-2}$. Moreover, it might be much smaller than the possible mechanical force
exerted by dark matter [6]. Detecting this momentum transfer from CBNs using mechanical force is not possible at the moment. We should think about other mechanisms to detect the momentum transfer from the wind of CBNs. We note that the mechanism considered in this article is only sensitive to the charged interaction of neutrino with electrons which is non-universal in neutrino flavors. The momentum is transferred from CBNs to electrons in matter through coherent scattering process. Considering electrons in conductor or superconductor might lead to a better way to detect momentum transfer from the wind of CBNs. The research of this topic is out of the scope of the present article.

We note that the mechanism discussed in this article make uses of the fact that massive neutrinos can convert from one mass state to another in interaction with electrons in matter. Momentum is transferred from CBNs to electrons in coherent scattering process with target matter. This is different from previous works [6–8] which considered the coherent scattering of neutrino with nuclei in matter. Another difference from previous works is that the momentum transfer from CBNs to target matter discussed in this article can be of positive or negative direction of the CBN wind, depending on the period of the target matter. Momentum recoil given by CBN wind to target matter, discussed in [6], is always of the same direction of the CBN wind.

We note that if \( n_i \) in galaxy is much larger than the average density \( \langle n_i \rangle \) the event rate can be enhanced. However, due to Pauli blocking [9] it’s difficult for the density of CBNs in our galaxy to be much larger than the value in [11] unless the neutrino mass reach \( \sim 0.5 \text{ eV} \) [10]. Numerical simulation of clustering of CBNs does not support \( n_i \) in galaxy much larger than the average value either [11]. If considering CBNs clustered in local cluster halo the number density is allowed to be much larger and the signal of CBNs is larger. The mechanism considered in this article can also be applied to eV scale sterile neutrino or keV scale sterile neutrino dark matter when the period of the target matter is designed to be the scale of \( \mu \)m or nm.

In conclusion we have studied the coherent scattering of CBNs with a detector of periodic matter structure. Massive CBNs which are non-relativistic today can convert from one mass state to another in interaction with electrons in matter. Energy of neutrino is released in this scattering process and momentum can be transferred from CBNs to target matter. We show that a periodic structure of matter can enhance the scattering probability when the period is matched to the scale of the mass square difference of neutrinos. A good arrangement of the periodic structure can also select the CBNs to be reflected or be refracted by the target matter and lead to net momentum transfer to the target matter from the wind of CBNs. If a smart way to detect this small momentum transfer can be found the result found in this article might be useful for designing a realistic detector for detecting CBNs in laboratory.

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