Attenuation of the effects produced by the bending modes of a flexible launch vehicle using second order filters

Ioana-Carmen Boglis1,2, Adrian M. Stoica1

1POLITEHNICA "University of Bucharest, Faculty of Aerospace Engineering, Str. Gheorghe Polizu, nr. 1, sector 1, 011061, Bucharest, Romania.
2Military Technical Academy “Ferdinand I”, Center of Excellence in Self-Propelled Systems and Technologies for Defense and Security, 39-49 George Cosbuc Ave., Sector 5, 050141, Bucharest, Romania. Ioana.boglis@mta.ro.

Abstract. This article describes the application of an adaptive control algorithm for the atmospheric phase control of a launch vehicle. A dynamic model representing the pitch motion of the launch vehicle is introduced. As the bending modes can cause instability, the first three bending modes are modelled. A case study on design of the bending filters for a flexible launch vehicle using a model reference adaptive control is conducted to demonstrate its effectiveness.

1. Introduction
The control system of launch vehicles plays an important role in cargo delivery missions, as they influence both the performance and operations of these. It must accurately track the trajectory guidance commands in order to deliver the payload to the target orbit. Problems with the launch vehicle control systems are due to the fact that launch vehicles must be treated as flexible structures as the resulting dynamics is strongly coupled with significant interactions between rigid body dynamics and structural modes. The forces acting on the launch vehicle resulting from atmospheric disturbances or active control of the vehicle excite the structure and cause bending of the body [1].

Because the structure has a low damping capacity, oscillating bending modes of considerable amplitude can be produced, thus subjecting the control sensors to large amplitudes to their particular location. the flexible modes have an important role in the dynamic environment to the vehicle systems as well as vehicle systems control design because if these bending modes are not properly considered, the local sensor data is interpreted as describing the total vehicle behavior that may cause self-excitation and instability of the control system [2].

This problem is addressed in various studies on the attitude control of a rocket. Gain scheduling and bending filter design are presented [3,4]. Also a gain scheduling methodology based on the Youla parameterization and optimization with linear matrix inequality (LMI) constraints is described in [5]. In [6] a notch filter is proposed considering a control algorithm using LQR technique, and in [7] adaptive notch filtering with real-time control successfully stabilizes the flexible launch vehicle model. In [8] a model reference method for estimating the bending frequency of a flexible system designed based on MIT rule shows satisfactory results. A suitable adaptive output feedback controller is proposed in [9], being able to handle unstable flexible plant with large parametric variation. Another method is used in [10] consisting of an observer-based methodology capable to robustly stabilizing a flexible launch vehicle.

The aim of this work is to provide an approach to attenuate the effects of the bending modes for a flexible launch vehicle in order to obtain the stability of the control system, using second-order filters, such as Butterworth and notch filters. This approach is illustrated by a case study for the VEGA launcher.

VEGA is the European Launch Vehicle with the mission to carry payloads from 300 kg to 2500 kg into the Sun Synchronous Orbits (SSOs) and Low Earth Orbits (LEOs). It is a single-body launcher, consisting of four
stages (P80, Zefiro 23, Zefiro 9 and Attitude and Vernier Upper Module-AVUM), controlled by a Thrust Vector Control (TVC). The TVC subsystem must ensure the stability of the guiding commands, while satisfying the demanded performance and strict requirements in the presence of external disturbances.

2. Flexible launch vehicle modelling and control

This section introduces the equations of motion in order to describe the launch vehicle’s dynamics. This consists of two parts: the rigid body motion and the bending motion. The bending modes must be taken into account because of the slender shape of the launch vehicle which causes body bending in the attitude motion, leading to pitch instability.

The following state space equation \[1\], assuming that the launch vehicle is a rigid body, describe the model for the pitch motion:

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\epsilon} \\
\dot{z}
\end{bmatrix} = 
\begin{bmatrix}
a_n & 0 & 0 \\
0 & a_g / v & \theta \\
-\alpha_g & 0 & \dot{z}
\end{bmatrix} 
\begin{bmatrix}
\theta \\
\epsilon \\
\dot{z}
\end{bmatrix} + 
\begin{bmatrix}
0 \\
-\delta \\
\alpha_a
\end{bmatrix}
\]

where \(\theta\) is the pitch angle attitude with respect to the vertical inertial reference axis, its derivative \(\dot{\theta}\) and \(\dot{z}\) stands for the inertial drift velocity, \(\delta\) is the gimbal deflection angle and \(\alpha_a\) the wind incidence, and \(v\) is the launch vehicle velocity, \(T\) the thrust, \(D\) the drag force, \(L_\alpha\) the aerodynamic force acting on the centre of pressure, \(I_y\) the pitch moment of inertia, \(l_{GA}\) the distance from the centre of gravity to the aerodynamic centre of pressure and \(l_{CG}\) the distance from centre of gravity to the nozzle pivot point.

The bending modes can be modeled as a second-order systems with natural frequency \(\omega_i\) and damping \(\xi_i\) as it follows:

\[
G_i(s) = \frac{K_i}{s^2 + 2\xi_i \omega_i s + \omega^2_i}
\]

where \(K_i\) is the DC gain of the \(i^{th}\) bending mode.

Because mode shapes can exist in all six directions, the number of these and their corresponding frequencies can be infinite. Typically, the first three bending modes are considered (figure 1), as these are considered the most relevant for the design and analysis of the control system.

The operational modal analysis uses natural excitation sources and it is studied and implemented to obtain the modal parameters of the excited Vega launcher during the flight by A. De Vivo [12]. The analysis gives the following results: the first flexible mode has the frequency of 4.416 Hz and the damping ratio is 1.10%, the second mode has the frequency of 11.01 Hz and the damping ratio equivalent to 0.89%, and the third mode has frequency of 16.71 Hz and damping ratio of 0.54%. The damping coefficient decreases from 1.10% for the first bending mode to 0.54% for the third bending mode due to the fact that the fuel inside the rocket engines has more depreciation values than the external launcher structure. The first mode generates the movement of a larger quantity of fuel so that the value related to the equivalent damping ratio is greater than the one for the other modes. The
values obtained are those of the first flight made by the Vega launcher, on February 13, 2012, with a load of 680kg due to the transport of nine satellites into orbit: LARES and ALMA Sat-1, of the Italian Space Agency, together with seven CubeSats standard CubeSat nanosatellites from European universities.

The control system is composed of a PD regulator and a Model Reference Adaptive Algorithm and it is shown in figure 2. The algorithm and the values of the parameters used are described in [13]. As the uncertainties and errors in modeling are inevitable and the parameters may vary over time, the control systems must satisfy a certain degree of robustness and adaptability. The bending motion is modeled through the sum of the contributions of each of the three first bending modes (figure 3). For the current study, the frequencies of the flexible bending modes mentioned above are used.

All the simulations are run considering a worst case scenario such as the high dynamic pressure region (at time t=56s) and all the parameters are considered as time invariant. Also, the effects of sloshing are neglected and the wind disturbance is 0.

Firstly, the algorithm control is considered without the bending filter. As it can be seen from figure 4, the control system provides an unstable response because of the influence of the flexible modes. This is mainly due to the appearance of the second flexible mode, and the very small value of the damping coefficient \( \gamma = 0.0089 \) which does not allow the mitigation of their effect by the control architecture.

The primary objective of the VEGA launch control system design is to provide sufficient stability margins in the presence of various parameter uncertainties while maintaining adequate system response. Studies have shown that the following design objectives are adequate [14]: both nominal and perturbed closed-loop Vega control systems must be stable, at least 6 dB / 100 ms rigid gain/phase margin is required for nominal control systems, at least 0.5 dB / 40 ms rigid gain/phase margin is required for perturbed control systems, no more than -3 dB nominal gain margin is required for gain stabilized bending modes and at least 50 ms nominal phase margin is required for phase stabilized bending modes. In the above specifications the phase is expressed as the ratio between the phase [rad] and the frequency [rad/sec].
3. Filters design

In order to stabilize the system, a bending filter is added. Two possible choices were considered: to add a filter for each bending mode, and the second to add a filter on the feedback of $\theta$ (figure 5). The latter was considered because adding for each mode a bending filter could interfere with the overall system’s performance.

![Figure 5: Block diagram of Classical Adaptive Control including the bending filter](image)

The filter must be stable and minimal phase to guarantee stability and performance. The bandwidth of the bending filter should be greater than that of the PD controller to avoid rigid performance degradation. These constraints are used to set the upper and lower bounds for the bending filter design. As it was needed to attenuate a particular frequency component, two types of filters were considered: a notch filter and a maximally flat filter. Their frequency responses are shown in figure 6.

![Figure 6: Frequency responses of the notch filter and Butterworth filter](image)

1. Notch filter

A notch filter is a very narrow bandwidth band stop filter. The transfer function the notch filter is given by:
\[ H_{notch}(s) = \frac{s^2 + \omega^2}{s^2 + 2\xi\omega s + \omega^2} \]  

where \( \omega = 69.08 \text{ rad/s} \) and \( \xi = 0.1 \), tuned at the known resonant frequency of the flexible launch vehicle.

The step response shows an acceptable percentage overshoot of 60\% and the final asymptotic value is 0.82 (for a step of 1). The stability is guaranteed, and the system provides sufficient performance.

2. Butterworth filter

The main advantage of this particular family of filters is that they have very flat stop- and passbands. The major disadvantage is that the initial roll-off rate is fairly slow. The Butterworth filter is designed using Matlab. Its transfer function is:

\[ H_{Butter} = \frac{4772.04}{s^3 + 97.4s + 4772.04} \]  

As shown in figure 8, the designed filter significantly reduces high-frequency components, which are greater than 10.99 Hz, maintaining the stability of the system.

The next cases are tested, in order to test the robustness of the control system, denoting by \( f_1 \) [rad/sec] the frequency of the first bending mode, \( f_2 \) [rad/sec] the frequency of the second bending mode and \( f_3 \) [rad/sec] the frequency of the third bending mode:

1. Payload of 300kg and \( a_u = 3.21 \), \( k_1 = 6.74 \), \( f_1 = 27.08 \), \( f_2 = 60.06 \) and \( f_3 = 82.14 \);
2. Payload of 1200kg and \( a_u = 3.23 \), \( k_1 = 7.07 \), \( f_1 = 24.66 \), \( f_2 = 53.93 \) and \( f_3 = 74.15 \);
3. Payload of 2200kg and \( a_u = 3.26 \), \( k_1 = 7.42 \), \( f_1 = 22.85 \), \( f_2 = 50.43 \) and \( f_3 = 70.06 \).

and \( \xi = 0.008 \) for all cases.
The step response for the control algorithm containing the Butterworth filter (figure 9) shows good results in all three cases, having an overshoot of 60% and tracking error of 0.19.

Considering the notch filter as the bending filter for the adaptive algorithm, it leads to complete loss of stability of the launch vehicle (figure 10). This is caused by the large dispersion of the frequency for the tested cases. It is necessary to use different notch filters for all the different payloads.

In order to check if the performance of the control system is deteriorated when considering the bending modes and the bending filter, a comparison between the responses of the adaptive control for the rigid body and the flexible body is made.
As it can be seen from figure 11, the overshoot and the rise time are approximately the same for both cases, validating the model reference adaptive control algorithm for a flexible launch vehicle.

4. Conclusions
Because of the slender shape of a launch vehicle, body bending occurs in the attitude motion, leading to pitch instability. In order to regain the stability of the system, two types of low pass filters of second order are proposed, a notch filter and a Butterworth filter. Simulation results show that the model reference adaptive control system containing a bending filter for the Vega launcher can guarantee stability, without affecting the performance of the system.

Several cases were tested during the atmospheric phase of the flight, proving the robustness of the control system containing the Butterworth filter. In order to maintain the stability for the control system using the notch filter, further tuning of the filters’ coefficients is needed.

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