Abstract
In this article, a method of centralised distribution of university e-learning electronic educational resources is proposed. E-learning is seen as a complex organisational hierarchy. The method applied in this article is based on n structured analysis of the problems and objectives of the system. The method considers electronic educational resources for e-learning. The advantages of the proposed method include the timely completion of multi-session e-learning and the availability of a reserve of electronic educational resources.

Keywords: e-learning, electronic educational resources, hierarchical structure

Streszczenie
W niniejszym artykule zaproponowano metodę scentralizowanej dystrybucji elektronicznych zasobów edukacyjnych akademickiego e-learningu. E-learning jest postrzegany jako złożona hierarchia organizacyjna. Metoda zastosowana w tym artykule opiera się na analizie struktury problemów i celów systemu. Metoda uwzględnia elektroniczne zasoby edukacyjne dla e-learningu. Zaletami proponowanej metody są między innymi terminowe ukończenie wielosesyjnego e-learningu i dostępność względnej rezerwy elektronicznych zasobów edukacyjnych.

Słowa kluczowe: e-learning, elektroniczne zasoby edukacyjne, struktura hierarchiczna
1. Introduction

Electronic educational resources (EERs) and open educational resources (OERs) in particular are becoming more and more popular in the modern university [12]. There is a trend for the transition of university e-learning support systems to hyper-convergent platforms. This significantly reduces maintenance costs.

E-learning on the hyper-convergent platform can be considered as a complex organisational hierarchical system. In scientific development, much attention is paid to the research of properties of hierarchical systems and their functional processes [1, 2]. In these operations, it is shown that the process of framing decisions in different complex organisational hierarchical systems has some important features. These features should be considered in the case of creation of appropriate systems. The problem arises of determining the corresponding singularities. The most effective way to solve this problem is to model the functioning processes [3–5]. The organisational system is considered as a mathematical object. Research on such an object is best performed by investigating its properties. Therefore, it is necessary to develop mathematical models of the functioning of e-learning within the framework of the system approach. On the basis of this model, it is possible to offer a method for the distribution of system resources. This method has to consider the features of the hyper-convergent platform, in other words, it needs to provide centralised distribution.

The purpose of this article is to present the development of a method of centralised distribution of electronic educational resources in university e-learning.

2. Analysis of existing approaches

The systems that have a hierarchical structure are researched into [1, 2]. In [3], the approach to the description of the modelled system is proposed. Here, the organisational system is perceived as a set of interacting elements. The nature of this interaction depends on the objects or tasks the system needs to perform. It is assumed that the set of elements is fixed. The level of description of such systems is determined by the degree of detail of the processes under consideration.

In [4], the possibility of the interaction of the elements of the system with each other is considered. Information channels of communication between elements are analysed. The type of channel is associated with the type of information. The general list of information channels is determined by the level of the system description. Each problem is treated as a system transformation operator.

In [5], it was shown that the structure of the system can be interpreted as the state of the system at a particular point in time. In hierarchical systems, all states are hierarchies. The definition of a complete hierarchical system is proposed.

In [6–9], the main problems that arise in the modelling of complex organisational hierarchical systems are shown; these are:
- identification of systems and specification of model components and selecting a method for determining a parameter model,
classification of organisational systems,
- the choice of the subsystem and its communication operations,
- the study of structures within the framework of one system under study.

In [10, 11], the basis of the model is the structure of the goals and objectives of the complex organisational hierarchical system. The system is represented by the GX graph tree [5]:

\[ G_X = (\overline{X}, R) \]  \hspace{1cm} (1)

where

\[ \overline{X} = \left( X^0, \overline{X}^1, ..., \overline{X}^{m-1} \right) \] – a cortege consisting of a set of controls of various ranks,

\[ X^0 \] – the main governing body,

\[ \overline{X}^i = (X_i^1, X_i^2, ..., X_i^l_i) \] , \( 0 \leq i \leq m - 1 \) – set of controls of various ‘i-th’ ranks,

\[ R = \{ r^i_{j} \}; \quad 0 \leq i \leq m - 2; \quad 1 \leq j \leq l_i; \quad 1 \leq v \leq l_i + 1 \] – communication subordination between government bodies,

\( i \) – the rank of the control ‘j’ from which the link leaves,

\( v \) – the number of the vertex (i + 1)-th rank into which the connection enters.

We use this approach to construct a graph model of the functioning of the academic e-learning on a hyper-convergent platform.

3. Graph model of e-learning

We will deliver in compliance to a graph GX isomorphic to it graph \( G_C(\overline{C}, H) \), when \( \overline{C} \) – the set of the purposes; \( H = \{ h^i_j \} \) – set of graph edges. In the process of achieving the main object of the system, \( C_0 \) external disturbances arise. They are of a mostly situational, non-stochastic character. Before governing bodies (m – 1)-th rank \( \{ X_{m-1}^v \}, 1 \leq v \leq m - 1 \), there is a set of the purposes and tasks. They lead to omission of the appropriate objects \( \{ C_{m-1}^v \} \).

We will consider a set of the purposes and tasks facing governing bodies \( \{ X_{m-1}^v \} \). We will present this set in the form of a graphs set \( G_{C_0}^{m-1} = \{ G_{C_0}^{v_{m-1}} \} \) purposes and tasks of system (Fig. 1):

\[ G_{C_0}^{v_{m-1}} = (\overline{C}_{0}^{v_{m-1}}, h) \]  \hspace{1cm} (2)

where:

\[ \overline{C}_{0}^{v_{m-1}} = \left( C_{0}^{v_{m-1},0}, C_{0}^{v_{m-1},1}, ..., C_{0}^{v_{m-1},n-1} \right) \] – cortege, consisting of a number of operational management objectives for different ranks,

\( C_0^{v_{m-1},0} \) – the main objective of operational management \( v \)-th governing body (m – 1)-th ranks,
\[
\overline{C}^{m_{n-1}} = \left( C^{m_{n-1},f_1}, ..., C^{m_{n-1},f_f} \right); \quad 0 \leq f \leq n - 1; f - \text{the rank identifier in the graph } G_{m_0}^{y_{m_{n-1}}},
\]

\( l_j \) – number of objects \( f \)-th ranks,

\( h = \left\{ f \right\}, \quad 0 \leq f \leq n - 2; \quad 1 \leq j \leq l_j; \quad 1 \leq g \leq l_j + 1 - \text{set of the graph edges.} \)

Peaks in the graph \( G_{m_0}^{y_{m_{n-1}}} \) connect edges \( h_{jg}^{m_{n-1}} \), when \( 0 \leq f \leq n(v) - 1; \quad 0 \leq Z \leq n(\alpha) - 1; \quad 1 \leq j \leq l_j; \quad 1 \leq g \leq l_j \), with one or several peaks in the graph \( G_{m_0}^{y_{m_{n-1}}} \); \( \alpha \neq v; \quad 0 \leq \alpha, v \leq \ell' \) (Fig. 2).

Edge \( h_{jg}^{m_{n-1}} \) defines two subgraphs: \( G \left( X^{y_{m_{n-1}}} \right) \) and \( G \left( X^{x_{m_{n-1}}} \right) \).

In this way, a graph of coordinating objects and tasks can be constructed:

\[
G_{CK} = \left( \overline{C}_K, S_K \right)
\]

where:

\( \overline{C}_K = (\overline{C}^0_K, \overline{C}^1_K, ..., \overline{C}^{m_{n-2}}_K) \) – the vector consisting of a set of the coordinating objects of governing bodies of different ranks,

\( \overline{C}^0_K = (C^{0}_{K_1}, C^{0}_{K_2}, ..., C^{0}_{K_{l_0}}) \) – set of the coordinating purposes of governing body \( X_0 \) in graph \( G_{x_0} \),

\( \overline{C}^{ij}_K = (C^{ij}_{K_1}, C^{ij}_{K_2}, ..., C^{ij}_{K_{l_t}}), \quad 1 \leq j \leq l_j; \quad 0 \leq I \leq m - 2; \quad 1 \leq t \leq l_t \) – set \( l_t \) the coordinating control objects \( X^{i_j}_t \)

\[
S_K = S_K^T \cap S_K^{TA}, \quad S_K^T \cap S_K^{TA} = \emptyset
\]
where:

\[ S_k^T = \left\{ s_{k0}^{T_{ij}} \right\}, \quad 1 \leq t, \omega \leq l; \quad 0 \leq i, \tau \leq m - 2; \quad 1 \leq j \leq l; \quad 1 \leq \gamma \leq l; \quad 1 \leq \nu \leq l_{m - 1} - 1 \] - set of the non-oriented relations between \( t \)-th and \( \omega \)-th the coordinating purposes, \( O \).

\[ S_k^{TA} = \left\{ s_{k0}^{T_{ij}} \right\} \] - set of the oriented (transitive and antisymmetric) relations between the appropriate coordinating objects.

The creation of the coordinating object \( C_k^{ij} \) communications \( h_{ij}^{V_{m-1} Z_{m-1}} \) in the graph \( G_{C_0}^{V_{m-1}} \) is performed as follows: in the graph \( G_{X} \), the governing body decides on the smallest value of a rank \( i \), with which organs \( \alpha \) and \( \nu \) rank \( (m - 1) \) are transitively connected by the relations \( R_{ij} \).

Let \( C_0^{V_{m-1}} \) - quantity of the resources selected for the achievement of the operational management \( \nu \)-th goals. Governing body \( X_{ij} \) is transitively connected to \( \nu \) relations \( R_{ij} \).

These relations define tasks \( \Pi \) on the redistribution of resources between governing bodies with indexes \( \nu \) and \( \alpha \) \((1 \leq \nu, \alpha \leq \ell_{m-1}, \nu \neq \alpha)\) \((m - 1)\)-th rank.

We will construct the graph of the purposes and tasks of operational redistribution of resources:

\[ G_{\Pi} = (\overline{C}_{\Pi}, S_{\Pi}), \quad (5) \]

where:

\[ \overline{C}_{\Pi} = (\overline{C}_{\Pi}^0, \overline{C}_{\Pi^1}, ..., \overline{C}_{\Pi^{m-2}}) \] - the tuple purposes set; \( \overline{C}_{\Pi}^0 = (C_{\Pi1}^0, C_{\Pi2}^1, ..., C_{\Pi\ell-j}^{m-2}), \]

\[ 1 \leq t \leq \ell t \] - the tuple purposes set for \( X_0^0 \).

For governing bodies \( X_{ij} \):

\[ \overline{C}_{\Pi}^{ij} = (C_{\Pi1}^{ij}, C_{\Pi2}^{ij}, ..., C_{\Pi\ell-j}^{ij}) \]; \[ S_{\Pi} = S_{\Pi}^T \cup S_{\Pi}^{TA}, \quad S_{\Pi}^T \cap S_{\Pi}^{TA} = \emptyset \quad (6) \]

where:

\[ S_{\Pi}^T = \left\{ s_{\Pi0}^{T_{ij}} \right\}, \quad 1 \leq t, \omega \leq l; \quad 0 \leq i, \tau \leq m - 2; \quad 1 \leq j \leq l; \quad 1 \leq \gamma \leq l; \quad 1 \leq \nu \leq l_{m - 1} - 1 \] - set of.

They provide the decision tasks of operational management \( \nu_{m-1} = 1 \). \( S_{\Pi}^{TA} = \left\{ s_{\Pi0}^{T_{ij}} \right\} \) - the ratio \( s \).

We will call elements with indexes \( \nu \) and \( \alpha \) \((1 \leq \nu_{m-1}, \alpha \leq \ell_{m-1})\) \((m - 1)\)-th rank, which have isomorphic graphs \( G_{C_0}^{V_{m-1}} \) and \( G_{C_{\alpha}}^{V_{m-1}} \).

On the set \( \overline{X}^{m-1} = \{ X_{\nu_{m-1}} \}, \quad 1 \leq \nu \leq \ell_{m-1} \) on types \( (y = I, \ell_y \) - set of types of governing bodies \((m - 1)\)-th rank).

Thus, it is enough to define:

- graphs of the objects and tasks of operational management of each of type of governing bodies \((m - 1)\)-th rank,

- set of edges \( h_{ij}^{V_{m-1} Z_{m-1}} \) between the vertices \( X_{\alpha_{m-1}} \) and \( X_{\nu_{m-1}} \),
set of pointers $d = \{d_{ij}^{m-1,0}_{n-1}\}$ — displays:

$$F_{\Pi}: d \rightarrow \overline{C}_{\Pi}, \quad F_{K}: h \rightarrow \overline{C}_{K}. \quad (7)$$

The structure $W$ defined by a tuple

$$M = \{G_{X}, G_{C_{0}}^{m-1}, G_{C_{K}}, G_{C_{\Pi}}, F_{\Pi}, F_{K}\}. \quad (8)$$

It is a union of substructures $W = \bigcup_{v=1}^{n} W_{v}.$

4. **Method of electronic educational resource allocation**

We decompose each graph from the set $G_{c_{0}}^{m-1}$. To this end, we construct a family of embedded partitions:

$$K = \{K_{1}, ..., K_{n-2}\} \quad (9)$$

on graphs $\{G_{c_{0}}^{m-1}\}$:

$$K_{f} = \{K_{1}^{f}, ..., K_{n-2}^{f}\} \quad (10)$$

and:

$$\bigcup_{j=1}^{\ell_{f}} K_{f}^{j} = G_{c_{0}}^{m-1}, K_{f}^{j} \cap K_{p}^{j} = \emptyset, \quad j \neq p, \quad 1 \leq f \leq n-2, \quad 1 \leq \ell_{f} \leq \ell_{f+1}. \quad (11)$$

For every element $K_{f}^{j}$ right: $K_{f}^{j} = K_{f+1}^{j} \cup ... \cup K_{\ell_{f+1}}^{j}.$

The partition is carried out as follows. In the graph $G_{c_{0},f}^{m-1}$ (root $G_{c_{0},f}^{m-1}$) select subgraphs with peaks $\{c_{0,f+1}\}$. These are related by relations $K = \{K_{1}, ..., K_{n-2}\} \subset \{h_{f}^{j}\}$.

Thus, the partition $K = \{K_{1}, ..., K_{n-2}\}$ defines a set of independent relations $\{h_{f}^{j}\}$ of subgraphs on the graph $G_{c_{0}}^{m-1}$.

We represent them in the form:

$$G_{c_{0}}^{m-1} = \bigcup_{\beta \neq 1} G_{c_{0},\beta}^{m-1,\beta}. \quad (11)$$

We will set external influences as:

$$\tilde{\alpha}: \{G_{c_{0}}^{m-1}\} \rightarrow \{G_{c_{0}}^{m-1,*}\} \quad (12).$$
where:
\[ \left\{ G_{C_0}^{y_{m-1}} \right\} \] – set of the subgraphs, objects and tasks of operational management subject to external influence.

Let
\[ \tilde{\beta}_t: \left\{ G_{C_0}^{y_{m-1}} \right\} \xrightarrow{K} \left\{ G_{C_0}^{y_{m-1}} \right\}. \] (13)

We will define a set of subgraphs of the objects of operational management:
\[ \left\{ G_{C_0}^{y_{m-1}} \right\} = \bigcup_{\beta' = 1}^U G_{\beta', \beta}^{y_{m-1}} \bigcup_{\beta' = 1}^U \left( \bigcup_{\beta' = 1}^U G_{\beta', \beta}^{y_{m-1}} \cap \left\{ \left\{ G_{C_0}^{y_{m-1}} \right\} \times \cdots \times \left\{ G_{C_0}^{y_{m-1}} \right\} \right\} \right). \] (14)

The structure allocation of resource.

\[ S_{pr} = \left( \left\{ G_{C_0}^{y_{m-1}} \right\}, \left\{ G_{C_0}^{y_{m-1}} \right\}, \left\{ G_{C_0}^{y_{m-1}} \right\}, R^5 \right) \] (15)

Peaks in \( S_{pr} \) – subgraphs \( \left\{ G_{C_0}^{y_{m-1}} \right\} \) aren’t to each peak from \( \left\{ G_{C_0}^{y_{m-1}} \right\} \), the vector of required resources is defined:
\[ e_B = \left( e_B^1, e_B^2, \ldots, e_B^s \right) \] (16)

\( R^5 \) – set of arcs from \( \left\{ G_{C_0}^{y_{m-1}} \right\}, \left\{ G_{C_0}^{y_{m-1}} \right\} \) in \( \left\{ G_{C_0}^{y_{m-1}} \right\} \). These are defined on the elements of the set \( \left\{ G_{C_0}^{y_{m-1}} \right\} \).

Each arc has an incident vector of resources:
\[ e_B = \left( e_B^1, e_B^2, \ldots, e_B^s \right) \] (17)

and simultaneously:
\[ \sum_{a \in \alpha} e_{D_a} = e_B \] (18)

Thus, on a set \( \left\{ G_{C_0}^{y_{m-1}} \right\} \) it is possible to define a set of structures \( S_{pr} \), setting different \( R^5 \). Each of the structures \( \left\{ S_{pr}^{y_{m-2}} \right\} \) determines the distribution of resources between the direct descendants of governing body \( v_{m-2} \).

5. Analysis of results

For the assessment of the efficiency of the proposed method of electronic educational resource distribution, two criteria have been chosen, namely the timely completion of multisession e-learning and the existence of a relative reserve of electronic educational
resources. Calculations were performed with use of the developed functioning model of e-learning in V.N. Karazin Kharkiv National University (Ukraine). Each of multi-session was planned to be selected classes lasting 80 minutes.

Schedules of dependences of values of indicators for these criteria are shown in Figs. 2, 3. Apparently from the results of modelling, the method proposed in this article is more effective than the standard method, especially with regard to increases in parallel educational processes.

Fig. 2. Dependence of probability of timely completion of a multisession e-learning \( (P) \) from the number of parallel educational processes \( (N) \): 1 – standard distribution method of electronic educational resources, 2 – the proposed central distribution method of electronic educational resources

Fig. 3. Dependence of reserve of electronic educational resources \( (R_{rel}) \) from the number of parallel educational processes \( (N) \) during the session and selecting the distribution method of electronic educational resources: 1 – standard; 2 – combined; 3 – proposed

6. Conclusion and recommendations for further research

In this article, a method for the centralised distribution of electronic educational resources of academic e-learning is proposed. In this method, e-learning is considered as a difficult organisational hierarchical system. The method is based on an analysis of the structure of the given object and the purpose of the system.
In the method described in the article, the electronic educational resources for e-learning are distributed using a centralised distribution method. In further investigations, we’ll plan to include hyper-convergent features of the basic system of support.

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