An Integrated Navigation Algorithm for AUV Based on Pseudo-range Measurements and Error Estimation

Wang Yiqun, Xu Chunhui, Xu Huixi, Zhao Hongyu and Liu Jian

Abstract—The Long Baseline (LBL) system is very important to precisely localize autonomous underwater vehicles (AUVs) close to the sea bottom in deep water. But the calibration error of the LBL beacons and the pseudo-range measurement caused by the motivation of the AUV, both them dilute the accuracy of LBL acoustic position. This paper proposes a quantitative description of the calibration error of beacons and pseudo-range how they reduce the LBL acoustic position precise. And then this paper proposes an integrated navigation algorithm for AUV, which integrates inertial navigation and LBL acoustic location to reduce the influence of the calibration errors of beacons and pseudo-range by online calibrating the beacon location errors and compensating the motivation of AUV to pseudo-range model. We name this algorithm as Pseudo-range and SLAM of beacons EKF (P-SLAM EKF). Through the sea trials of Qianlong-1, our algorithm proves very reliable and effective.

I. INTRODUCTION

Recently, the autonomous underwater vehicles are widely used in ocean missions as they have become more cost-effective, safer and have been improved at performing difficult tasks that are unfeasible for humans. AUVs do routinely perform tasks such as bathymetric mapping, pipeline inspection, scientific data collection or geological surveys. Underwater navigation is still one of the pillars in successful AUV missions\(^{(1)}\). Nowadays, many AUVs rely on dead reckoning estimate which inevitably dilutes accuracy with mission time, while acoustic navigation is introduced to solve the problem\(^{(2)}\). Acoustic navigation has no accumulative error, and is exemplified by a Long Baseline (LBL) and Ultra Short Baseline (USBL). The USBL system on surface vessel extracts the arrival phase of an acoustic ping transmitted by a single acoustic transponder on AUV, using a transducer with multiple acoustic receivers on USBL to calculate the bearing and distance of AUV respect to the USBL\(^{(2)}\). With the aid of GPS, USBL can get absolute position of the AUV. USBL positions are seldom fed to navigation through acoustic modems which will introduce substantial delays in locations, and therefore USBL positions are commonly used for supervising vehicles. LBL acoustic system consists of a grid of pre-surveyed beacons on the seabed, and triangulate AUV’s position with range measurements from beacons. Many oceanographic AUVs integrate LBL with dead reckoning to bound position estimate error to a few meters\(^{(4)}\).

For example, with the aid of LBL, the Autonomous Benthic Explorer (ABE) has conducted 191 benthic surveys\(^{(3)}\) at Mid-ocean ridge sites (See Fig.1), and Qianlong-1 (See Fig.2) has conducted 31 missions at Pacific Ocean, whose navigation precision is on the order of a few meters.

Existing LBL navigation system is supposed that a transducer installed on AUV sends interrogation signal and a grid of pre-surveyed beacons respond after receiving the interrogation signal. The range estimation between AUV and beacons is typically obtained through the two-way Time-of-Flights (ToF's) of exchanged signals, and then triangulate the location of AUV as in \([6]\) by neglecting the arrival of responding signal is asynchronous and assuming that the calibration error of beacons can be neglected.

![ABE AUV](image1)

Figure 1. ABE AUV\(^{(2)}\)

![Qianlong-1 AUV.](image2)

Figure 2. Qianlong-1 AUV.

Indeed, different factors can influence the actual performance of LBL, mainly attributable to: (1) the similar level of pseudo-range and real range between AUV and
beacons; (2) the calibration of the beacon after its deployment; (3) the technical specifications of the beacons.

The first factor can affect navigation precision whose order of magnitude is meters and even decameters. The arrival of responding signal is asynchronous, so the range obtained by ToFs is not real distance between AUV and responders which is called pseudo-range.

The second factor is that actual beacon location is different from the assumed ones, which results that estimate errors of triangulation evidently arise from meters to hundred meters.

The last factor is related with time-resolution measurements, ToFs. This aspect is not just with the technology level of the acoustic modem and the propagation of the acoustic signal in the considered ocean medium, which affects navigation precision on the level of meters.

The three above factors are also considered in this literature. Firstly, the paper proposes a quantitative description of the calibration error of beacons how it influences the LBL acoustic location precise. Secondly, the paper proposes a function of the motivation and actual position of AUV, which characterizes the pseudo-range measurement how it dilutes the LBL acoustic location precise. Thirdly, propose an integrated navigation algorithm which integrates inertial navigation with the LBL acoustic position to solve the above problems. The algorithm is named Pseudo-range and SLAM of beacons EKF (P-SLAM EKF) that online calibrates the location error of beacons and compensates the motivation of AUV to observe model to improve the navigation precise.

II. PROBLEM STATEMENT AND ERROR ANALYZE

A. Problem Model

Four transponders are deployed on the seafloor, and the calibration position of i-th beacon is \((x, y, z)_i\). The most used algorithm for localizing AUV with LBL is trilateration. According to that, define \((x, y, z)\) the coordinate of the AUV at time \(t_i\) when it receives the ToFs signal and is translated by the AUV into an estimation of its distance \(r_i\) in 2D from the \(i\)-th beacon (See Fig.3). With regard to a given Cartesian frame, the relationship between AUV and beacons can be formulated as \((1)\).

\[(x-x_i)^2 + (y-y_i)^2 = r_i^2; (1 <= i <= 4)\]  

\[F_i(x_i, r_i) = \begin{bmatrix} F_{1i} \end{bmatrix} = \left( A^T A \right)^{-1} A^T B \]  

where

\[A = \begin{bmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \\ x_3 - x_4 & y_3 - y_4 \\ x_4 - x_1 & y_4 - y_1 \end{bmatrix} \]

\[B = \frac{1}{2} \begin{bmatrix} r_1^2 - r_2^2 & (x_2^2 + y_2^2 - x_1^2 - y_1^2) \\ r_2^2 - r_3^2 & (x_3^2 + y_3^2 - x_2^2 - y_2^2) \\ r_3^2 - r_4^2 & (x_4^2 + y_4^2 - x_3^2 - y_3^2) \\ r_4^2 - r_1^2 & (x_1^2 + y_1^2 - x_4^2 - y_4^2) \end{bmatrix} \]  

B. Calibration Error of Beacons upon Acoustic Positions

The first factor that may induce acoustic position error is the calibration error of beacons. To simply characterize how the calibration errors affect acoustic positions, assume that beacons are located in a square configuration with LxL, and the origin of the coordinate system is in the center of the square (see Fig.4).
Define $\Delta X = [\Delta x_i, \Delta y_i]^T$ the vector representing the calibration error of the $i$-th beacon, and $X_i = [x_i, y_i]^T$ the vector represents the nominal position of $i$-th beacon. With regard to the error transfer function, obtain the estimated acoustic position error $e_x$ and $e_y$ which represent the error in X-coordinate and Y-coordinate in (4), and $\Omega(\rho^2)$ is Peano term which can be neglected.

$$
\begin{align*}
  e_x &= < F_1, \Delta X > + \frac{1}{2} \Delta X^T \hat{H}_{\rho_1}(X) \Delta X + O(\rho^2) \\
  e_y &= < F_2, \Delta X > + \frac{1}{2} \Delta X^T \hat{H}_{\rho_2}(X) \Delta X + O(\rho^2)
\end{align*}
$$

(4)

where

$$
< F_1, \Delta X > = \begin{bmatrix}
  \frac{y}{2L} - \frac{1}{4} & \frac{y}{2L} - \frac{1}{4} & \frac{y}{2L} - \frac{1}{4} & \frac{y}{2L} - \frac{1}{4} \\
  - \frac{x}{2L} - \frac{1}{4} & - \frac{x}{2L} - \frac{1}{4} & - \frac{x}{2L} - \frac{1}{4} & - \frac{x}{2L} - \frac{1}{4}
\end{bmatrix} \Delta X,
$$

$$
\hat{H}_{\rho_1}(X) = \frac{1}{2L} \text{diag}(-1,-1,1,1,-1,-1,1,1),
$$

$$
< F_2, \Delta X > = \begin{bmatrix}
  - \frac{x}{2L} + \frac{1}{4} & \frac{x}{2L} - \frac{1}{4} & \frac{x}{2L} - \frac{1}{4} & \frac{x}{2L} - \frac{1}{4} \\
  - \frac{y}{2L} - \frac{1}{4} & \frac{y}{2L} - \frac{1}{4} & \frac{y}{2L} - \frac{1}{4} & \frac{y}{2L} - \frac{1}{4}
\end{bmatrix},
$$

$$
\hat{H}_{\rho_2}(X) = \frac{1}{2L} \text{diag}(1,-1,1,-1,1,1,-1,-1,1),
$$

$$
\Delta X = [\Delta x_1, \Delta y_1, \Delta x_2, \Delta y_2, \Delta x_3, \Delta y_3, \Delta x_4, \Delta y_4] \\
\rho = \sqrt{\sum_{i=1}^{8} \Delta x_i^2 + \sum_{i=1}^{8} \Delta y_i^2}
$$

Equation (4) characterizes how calibration error affects the precise of the acoustic position.

C. Pseudo-range Error of Acoustic Position

The second factor that may induce acoustic position error is the Pseudo-range error caused by the motivation of AUV. By substituting heading ($\psi$) and velocity ($\nu$) of AUV to the error transfer function, obtain the estimated acoustic position error $e_x$ and $e_y$ which represent the error in X-coordinate and Y-coordinate in (6), and $\Omega(\rho^2)$ is Peano term which can be neglected.

$$
\begin{align*}
  e_x &= < F_1, T > + \frac{1}{2} \Delta T^T \hat{H}_{\rho_1}(T) T + O(\rho^2) \\
  e_y &= < F_2, T > + \frac{1}{2} \Delta T^T \hat{H}_{\rho_2}(T) T + O(\rho^2)
\end{align*}
$$

(6)

Where:

$$
< F_1, T > = \begin{bmatrix}
  - \frac{v \sin \psi}{2L} & - \frac{v \sin \psi}{2L} & - \frac{v \cos \psi}{4} & - \frac{v \cos \psi}{4} \\
  - \frac{v \sin \psi}{2L} & - \frac{v \sin \psi}{2L} & - \frac{v \cos \psi}{4} & - \frac{v \cos \psi}{4}
\end{bmatrix},
$$

$$
< F_2, T > = \begin{bmatrix}
  - \frac{v \sin \psi}{2L} & - \frac{v \sin \psi}{2L} & - \frac{v \cos \psi}{4} & - \frac{v \cos \psi}{4} \\
  - \frac{v \sin \psi}{2L} & - \frac{v \sin \psi}{2L} & - \frac{v \cos \psi}{4} & - \frac{v \cos \psi}{4}
\end{bmatrix},
$$

$$
\hat{H}_{\rho_1}(T) = \frac{v^2}{2L} \text{diag}(-1,-1,1,1),
$$

$$
\hat{H}_{\rho_2}(T) = \frac{v^2}{2L} \text{diag}(1,-1,1,-1),
$$

$$
T = [t_1, t_2, t_3, t_4]^T
$$

$$
\rho = \sqrt{\sum_{i=1}^{4} \Delta \psi_i^2}
$$

Equation (6) characterizes how pseudo-range error affects the precise of the acoustic position. The navigation precise level of magnitude of the pseudo-range is related with the motivation and the actual position of AUVs from (6).

III. SOLUTION MODEL

To improve the navigation of AUVs, an integrated navigation algorithm based pseudo-range measurements and calibration error estimation of beacons (P-SLAM EKF) is proposed. The integrated algorithm combines inertial navigation with LBL, and reduce the effect of the calibration.
error of beacons and the pseudo-range upon the navigation precise. The algorithm can provide the navigation position of the AUV, and also online adjust the calibration error of beacons. The algorithm consists of state model and Observe model. The solution of the algorithm is as [8], which is not described in the chapter.

A. State Model

Define AUV 2D position vector \( X = [x, y]^T \), the estimate calibration error vector of beacons \( \Delta X = [\Delta x, \Delta y]^T \), and obtain augmented matrix \( X_b \) by combining \( X \) with \( \Delta X \).

\[
X_b = \begin{bmatrix} X \\ \Delta X \end{bmatrix}
\]

The state model can be described as (9). The symbol \( u(k) \) represents the input of the model, which contains heading and velocity of the AUV. The symbol \( T \) is a period time.

\[
X_b(k+1) = F(X_b(k), u(k), W(k))
\]

Where:

\[
F(X_b(k), u(k), W(k)) = \begin{bmatrix} X + Tv(k) \times \begin{bmatrix} \sin \psi \\ \cos \psi \end{bmatrix} + W(k) \\ \Delta X \end{bmatrix}
\]

\[
u(k) = [v(k), \psi(k)]^T,
W = [w_v(k), w_\psi(k)]^T.
\]

Note that symbol \( W(k) \) is a input Gaussian noise of the state model, which is induced by heading and velocity measurement. Obviously, the location of beacons is fixed, and so the transition matrix of the beacons is identity matrix.

B. Observe Model

The observe model is the distance between the AUV and the \( i \)-th beacon in 2D as follows (11).

\[
z_i(k) = R_i(X_b(k)) + W_i(k)
\]

\[
R_i(X_b(k)) = \sqrt{(x + v(k)T \sin \psi - \Delta x_i - x_i)^2 + (y + v(k)T \cos \psi - \Delta y_i - y_i)^2}
\]

1 \leq i \leq 4

Note that symbol \( W(k) \) is a measure noise of the \( i \)-th beacon, which is white Gaussian noise process. The observe model introduces the pseudo-range and the calibration errors of beacons to improve the navigation precise. Now the algorithm model can be solved as [8].

IV. RESULT ANALYZE

To prove the efficiency of the algorithm, P-SLAM EKF algorithm is online used by Qianlong-1 in 31-th sea mission. Qianlong-1 has traveled for 6 hours and 21.6 kilometers in 31-th sea mission. The navigation result of P-SLAM EKF is obtained by real trial of Qianlong-1. The tradition EKF (EKF) algorithm results for comparing is obtained by post-processed method based on the attitude and velocity of Qianlong-1. The tradition EKF is not considering the pseudo-range error and the calibration error of the beacons. The attitude of the AUV is measured by compass, and the velocity of the AUV is obtained by Doppler Velocity Logger (DVL) installed on AUV.

The navigation map of our algorithm is shown in Fig.5. The blue point denotes the trajectory of P-SLAM algorithm, the green point denotes the trajectory of standard EKF algorithm, and the red point denotes the LBL acoustic locations. The trajectory of P-SLAM is better than the trajectory of the standard EKF, by taking LBL acoustic locations as benchmark.

![Figure 5. Trajectory of P-SLAM EKF and EKF](image)

Fig.6 is the error comparison of P-SLAM EKF and EKF, comparing with LBL acoustic locations. The blue point denotes the distance between the P-SLAM point and the LBL acoustic location, and the green point denotes the distance between the standard EKF point and the LBL acoustic location. The navigation error of P-SLAM is bounded to less than 5 meters. The navigation error of the standard EKF is more than 10 meters, and sometime is even more than 15-20 meters. Show as Fig.6, it indicates that the navigation precise of the P-SLAM algorithm is higher than that of the standard EKF algorithm.

![Figure 6. Error comparison of P-SLAM EKF and EKF in the 31-th trial mission.](image)
Fig. 7 is the northern error comparison of P-SLAM EKF and EKF, taking LBL acoustic locations as benchmark. The blue point denotes the northern deviation between the P-SLAM point and the LBL acoustic location, and the green point denotes the northern deviation between the standard EKF point and the LBL acoustic location. The trajectory of AUV is mostly on the eastern orientation, so the northern error is the crossed-trajectory error. As shown in Fig. 7, the northern deviation of P-SLAM EKF is bounded to less than 5 meters. And however, northern deviation of the standard EKF is on average of -10 to 10 meters, and sometime is even more than 15 or -15 meters. Shown as Fig. 7, the crossed-trajectory error of the P-SLAM EKF algorithm is lower than that of the standard EKF algorithm.

![Figure 7](image_url)

**Figure 7.** Northern error comparison of P-SLAM EKF and EKF in the 31-th trial mission

Fig. 8 is the eastern deviation comparison of P-SLAM EKF algorithm and EKF algorithm, comparing with LBL acoustic locations. The blue point denotes the eastern deviation between the P-SLAM EKF point and the LBL acoustic location, and the green point denotes the eastern deviation between the standard EKF point and the LBL acoustic location. The trajectory of AUV is mostly on the east orientation, so the eastern deviation is the trajectory-orientation error. As shown in Fig. 8, the eastern deviation of P-SLAM EKF is bounded to less than 5 meters. The eastern deviation of the standard EKF is on average of 10 meters, and sometime is even more than 15 meters. So the trajectory-orientation error of the P-SLAM EKF algorithm is lower than that of the standard EKF algorithm.

![Figure 8](image_url)

**Figure 8.** Eastern error comparison of P-SLAM EKF and EKF in the 31-th trial mission.

As shown in Fig. 5, the trajectory of P-SLAM EKF is better than that of the standard EKF. And from Fig. 6 to Fig. 8, the different orientation deviation of P-SLAM EKF is lower than that of the standard EKF. Obviously, the P-SLAM EKF algorithm has higher navigation precise than that of EKF in every orientation.

### V. Conclusion

Our contributions in this paper are as follows:
- Firstly, propose a quantitative description of the calibration error of beacons how it influences the LBL acoustic location precise.
- Secondly, propose a method which characterizes the pseudo-range measurement caused by the motivation of AUV, how it dilutes the LBL acoustic location precise.
- Thirdly, propose a Pseudo-range EKF algorithm that integrates inertial navigation with the LBL acoustic position based on pseudo-range and online adjusting the calibration error of beacons. This algorithm online calibrates the location error of beacons, and compensates the motivation of AUV to observe model to reduce the pseudo-range influence on the navigation precise.

From the sea trial navigation data of Qianlong-1, the P-SLAM EKF algorithm performs better than the standard EKF algorithm. Through 32 ocean missions of Qianlong-1, the integrated navigation algorithm has been proved very reliable and effective.

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