Adaptive Control for Quadrotor Trajectory Tracking With Accurate Parametrization

RICARDO PÉREZ-ALCOCER1 AND JAVIER MORENO-VALENZUELA2, (Member, IEEE)

1CONACyT–Instituto Politécnico Nacional–CITEDI, Tijuana 23435, México
2Instituto Politécnico Nacional–CITEDI, Tijuana, Baja California 23435, México
Corresponding author: Ricardo Pérez-Alcocer (rrperez@citedi.mx)

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ABSTRACT In this paper, a novel adaptive controller for quadrotor position and orientation trajectory tracking is introduced. By taking into account the coupling between the position and the orientation dynamics, an adaptive scheme based on an accurate parameterization of the model-based feedforward compensation is presented. The adaptation update laws for the adaptation parameters are designed on Lyapunov’s theory so that the stability of the state space origin of the error dynamics is guaranteed. Barbalat’s lemma ensures convergence of the tracking errors and bounding of the adaptation parameters. The extensive real-time experimental results show the practical viability of the proposed scheme. More specifically, the performance of the proposed controller is compared with an adaptive controller taken from the literature and the non-adaptive version of the proposed controller. Better results are obtained with the novel adaptive approach.

INDEX TERMS Adaptive control, Lyapunov-theory, quadrotor, accurate parameterization.

I. INTRODUCTION

Nowadays, quadrotors are used in a wide range of activities including surveillance, transport and entertainment. The utility of quadrotors has increased by their low cost in comparison to other unmanned aerial vehicles and by their important flight characteristics. However, efficient motion control of quadrotors is still an important scientific challenge because the quadrotors are underactuated systems [1] with highly nonlinear and coupled dynamics. Furthermore, the development of effective quadrotor controllers is essential.

Classical control techniques have been used for quadrotor trajectory tracking control. In [2], an optimal PID controller based on integral of time multiplied by absolute error indexes was presented. The authors added a Kalman filter to predict the change of error and get better performance. A modified version of the PID controller was presented in [3]. The control algorithm was developed to reject wind disturbances, and its performance was validated experimentally. In [4], a PID-type motion controller was introduced. The robustness of this controller was validated theoretically and experimentally when a fault is presented in one of the actuators. The position control performance of a V-tail quadrotor was compared using PD, PID, and sliding-mode control schemes in [5].

Model-based controllers have been used for quadrotor tracking task [6], [7]. However, the parametric inaccuracies and the system disturbances have motivated the development of robust control schemes in closed-loop. Sliding mode controllers have been used in several studies. In [8], a backstepping control approach was combined with sliding mode control for the quadrotor trajectory tracking problem under the presence of several types of disturbances. Similarly, in [9], the sliding-mode control and the backstepping methodology were combined to achieve position and yaw angle trajectory tracking. In particular, Lyapunov’s theory was used in the stability analysis of the overall system. In [10], a robust sliding mode controller was introduced for trajectory tracking of a quadrotor. This controller uses a nonlinear disturbance observer to improve the error performed.

Neural networks have also been used for quadrotor trajectory tracking. In [11], an intelligent adaptive backstepping controller was presented. This controller was designed using a radial basis function neural network and a fuzzy compensator. In the same way, an adaptive radial basis function neural network and an integral sliding mode control were combined in [12]. The controller was theoretically analyzed proving
convergence of the states to desired values in finite time. A nonlinear resilient trajectory controller for a quadrotor was presented in [13]. External disturbances were estimated with a nonlinear observer. In [14], the path following problem was addressed using an integral predictive and robust nonlinear $H_{\infty}$ controller. The theory of systems in cascade was used in [15] to design a hierarchical control scheme which guarantees accurate quadrotor positioning while eliminating the payload swing.

Adaptive schemes have also been widely employed in the control of mechanical systems [16]. The quadrotors are not the exception. A general review of adaptive control applied to quadrotors was presented in [17]. In [18], the trajectory tracking control problem was addressed using direct and indirect model reference adaptive control. In that work, a simplified linear model was considered during the controller design. Decentralized control has been combined with adaptive nonlinear control techniques for quadrotor trajectory tracking. In [19], the quadrotor dynamic was separated into two subsystems, and decentralized adaptive controls were applied to solve the trajectory tracking problem. Similarly, a decentralized adaptive controller was proposed in [20] to stabilize the altitude and attitude of the quadrotor when model uncertainties are present. The given theoretical analysis guarantees global asymptotical stability. In [21], the modeling errors and the disturbances are managed by a PD controller plus an adaptation control term. The tracking convergence of the closed-loop system was validated by using Lyapunov’s theory. An intelligent decentralized controller was presented for quadrotor attitude tracking in [22]. The system was decomposed into three sub-systems, and the chattering was reduced with a fuzzy logic component. In [23], the tracking quadrotor control was addressed using a decentralized PID neural network control scheme for the attitude dynamics when external wind disturbances affect the system. In [24], a robust controller based on the integral of the error signum and the immersion and invariance methodology was presented. Hardware in the loop simulation was used to validate the controller. In [25], the tracking problem was addressed using an adaptive sliding mode approach. The adaptive scheme was designed grouping the uncertainties in one single vector for position dynamics and another for attitude dynamics. The results obtained with numerical simulation were presented and discussed. In [26], [27], an adaptive control approach was proposed. The assumption that the center of mass coincides with the body reference frame was removed in the design. However, the assumption that the attitude dynamics is faster than the position dynamics was kept. In [28], an adaptive controller was designed combining fuzzy and backstepping control techniques with a mass observer for the quadrotor trajectory tracking. In [29], the authors presented an adaptive $L_1$ controller for a quadrotor when actuator faults and parametric uncertainties are presented. In [30], an immersion and invariance–based adaptive controller for quadrotor systems was proposed to deal with uncertain inertial parameters. Let us notice that many of the adaptive control schemes in literature present significant disadvantages. For example, the parameterization is based on simplified versions of the dynamic model [26], [27], [31], the control design results in an over-parameterization problem [32], or the control implementation is complex [33], [34]. It should be noticed that the adaptive control of quadrotors taking into account a precise parameterization of the model-based feedforward compensation is still open. We provide a successful solution to this problem when the six parameters of the inertia tensor are considered, which is theoretically and experimentally supported.

The main contribution of this work is a novel adaptive control scheme designed to guarantee trajectory tracking of quadrotors. In our controller, the dynamic parameters are updated on-line in contrast to others that rely in fixed estimates of the parameters [4], [35]. A parameterization of the model-based feedforward compensation is introduced, which simplifies the regressor computation. Specifically, the regressor matrix is obtained by means of three vectors and a transformation matrix, resulting in a compact representation and a viable implementation. Besides, the given parameterization is also more precise than others found in the literature, in the sense that the body is assumed asymmetric and the six parameters of the inertial tensor are considered. To the best of our knowledge, this method has not been reported for quadrotor adaptive control. The closed-loop system resulting from the proposed adaptive scheme and the quadrotor dynamics is studied by Lyapunov’s theory showing local stability of the state space origin. Besides, Barbalat’s lemma is invoked to show the convergence of the tracking errors and the bounding of the adaptation parameters. In the stability analysis, no assumptions are made on the time-scale separation between the quadrotor position and attitude dynamics. Additionally, the experimental evaluation of the novel controller is presented and compared with a known scheme already reported in the literature and the non-adaptive version of the proposed scheme.

This paper is organized as follows. In Section II the quadrotor dynamic model is given. The novel nonlinear adaptive controller is presented in Section III. The stability analysis for the resulting closed-loop system is given in Section IV. Experimental results of a quadrotor trajectory tracking are presented in Section V. Finally, we present some conclusions in Section VI.

Notation: Throughout this paper, the following notation is used. The norm of vector $\mathbf{x} \in \mathbb{R}^n$ is denoted as $|\mathbf{x}| = \sqrt{\mathbf{x}^T \mathbf{x}}$, $I_3 \in \mathbb{R}^{3 \times 3}$ is the identity matrix, while the minimum and maximum eigenvalues of a symmetric positive definite matrix $A \in \mathbb{R}^{n \times n}$ are represented by $\lambda_{\text{min}}[A]$ and $\lambda_{\text{max}}[A]$, respectively. The trigonometric functions $\sin(x)$, $\cos(x)$ and $\tan(x)$ for all $x \in \mathbb{R}$ are denoted by $s_x$, $c_x$ and $t_x$, respectively.

II. DYNAMIC MODEL

In this Section, the quadrotor motion equations and their properties are presented. In the literature, several works have
introduced accurate quadrotor dynamic models considering that this vehicle is a rigid body which moves freely in space [14], [19], [36], [39], [40], [40], [41]. Figure 1 shows a quadrotor scheme in which the reference frames, the quadrotor motions, and the rotors positions are presented.

Considering $p = [x \ y \ z]^T \in \mathbb{R}^3$ and $\eta = [\phi \ \theta \ \psi]^T \in \mathbb{R}^3$ as the position and attitude vectors of the quadrotor, respectively. It is possible to obtain the quadrotor dynamics with Lagrange properties, which is expressed as follows

$$m \ddot{\mathbf{p}} + mg \mathbf{e}_z = u_T R(\eta) \mathbf{e}_z,$$  

$$H_o(\eta) \ddot{\eta} + C_o(\eta, \dot{\eta}) \dot{\eta} = \tau,$$

where $m$ is the constant vehicle mass, $g$ represents the gravitational constant, $\mathbf{e}_z = [0 \ 0 \ 1]^T \in \mathbb{R}^3$ is the unit vector expressed in the inertial frame, $R(\eta) \in SO(3)$ is an orthogonal rotation matrix defined as

$$R(\eta) = \begin{bmatrix}
  c_\phi c_\psi & -c_\phi s_\psi & s_\phi \\
  s_\phi c_\psi & c_\phi s_\psi & -c_\phi c_\psi \\
  -s_\psi & c_\psi & 0
\end{bmatrix},$$  

$u_T \in \mathbb{R}$ is the total thrust produced by the rotors,

$$H_o(\eta) = W(\eta)^{-1} IW(\eta)^{-1},$$  

$$C_o(\eta, \dot{\eta}) = W(\eta)^{-T} \left[ S(I\omega) - IW(\eta)^{-1} \dot{W}(\eta) \right] W(\eta)^{-1},$$

where $I$ is the constant symmetric matrix defined as

$$I = \begin{bmatrix}
  I_{xx} & I_{xy} & I_{xz} \\
  I_{yx} & I_{yy} & I_{yz} \\
  I_{zx} & I_{zy} & I_{zz}
\end{bmatrix},$$

which represents the inertia tensor, $\omega$ is the angular velocity

$$\omega = \begin{bmatrix}
  \omega_x \\
  \omega_y \\
  \omega_z
\end{bmatrix} = W(\eta)^{-1} \dot{\eta},$$

and $S(\omega)$ is the skew symmetric matrix defined as

$$S(\omega) = \begin{bmatrix}
  0 & -\omega_z & \omega_y \\
  \omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix},$$

the transformation matrix $W(\eta) \in \mathbb{R}^{3 \times 3}$ and its inverse $W(\eta)^{-1} \in \mathbb{R}^{3 \times 3}$ given by

$$W(\eta) = \begin{bmatrix}
  1 & s_\phi t_\theta & c_\phi t_\theta \\
  0 & c_\phi & -s_\phi \\
  0 & s_\phi/c_\theta & c_\phi/c_\theta
\end{bmatrix},$$  

$$W(\eta)^{-1} = \begin{bmatrix}
  1 & 0 & -s_\theta \\
  0 & c_\phi & c_\theta c_\phi \\
  0 & -s_\phi & c_\phi s_\phi
\end{bmatrix},$$

respectively, and $\tau \in \mathbb{R}^3$ represents the torque vector expressed in the inertial reference frame. Note that the matrix $W(\eta)$ is singular at $\theta = \pm \pi/2$. Therefore, the matrix $C(\eta, \dot{\eta})$ is well defined only for pitch angles different from these values.

Besides, the total thrust and the torque vector, are related to the thrust of each rotor $u_i$ as follows

$$u_T = \sum_{i=1}^{4} u_i,$$  

$$\tau = W(\eta)^{-T} \begin{bmatrix}
  l(u_2 - u_4) \\
  l(u_3 - u_1) \\
  k_d \sum_{i=1}^{4} (-1)^i \dot{u}_i
\end{bmatrix},$$

where $l$ is the distance from the quadrotor center of mass to the rotor axis and $k_d$ is a coefficient associated with the drag force. Our design relies in the assumption that $u_T$ and $\tau$ are the control inputs.

The quadrotor dynamic model (1)-(2) exhibits some properties useful in the coming analysis.

**Property 1:** The matrix $H_o(\eta) \in \mathbb{R}^{3 \times 3}$ is symmetric and positive definite for all $\eta$ such that $|\theta| < \pi/2$, $\phi, \psi \in \mathbb{R}$. □

**Property 2:** The matrix $H_o(\eta) - 2 C_o(\eta, \dot{\eta})$ is a skew symmetric matrix, i.e.,

$$x^T \left[ \frac{1}{2} H_o(\eta) - C_o(\eta, \dot{\eta}) \right] x = 0, \quad \forall \eta, \dot{\eta}, x \in \mathbb{R}^3.$$

III. ADAPTIVE CONTROL DESIGN

Control schemes can be developed using the quadrotor dynamics presented in the previous Section. However, the dynamic parameters are frequently inaccessible, they change when a payload is added, or the estimates are inaccurate. Therefore, adaptive schemes provide better performance in a real-time implementation [41], [42]. In this Section, a novel controller for quadrotor trajectory tracking is presented. This controller is developed based on a new parameterization of the model-based feedforward compensation where no simplifications are required. It is worth mentioning that in this work, the dynamic parameters of the quadrotor model in (1)-(2) are constant for all flying time, although they could be different between one flying mission and another. Besides, aggressive maneuvers are not considered, and consequently, the roll $\phi(t)$ and pitch $\theta(t)$ angles are assumed to
be bounded as follows:
\[ |φ(t)| < \pi/2, \quad |θ(t)| < \pi/2, \quad \forall t ≥ 0. \]

**A. CONTROL OBJECTIVE**

The quadrotor is an underactuated system because only has four actuators to control their six degrees of freedom [1]. These four actuators produce the total thrust \( u_T(t) \) and the torque vector \( τ(t) \), which are used as control input for the quadrotor dynamic model. Using these inputs, the altitude and orientation of the quadrotor are directly controlled; however, the displacements in the horizontal plane are not controlled directly. The proposed controller for the position trajectory tracking task is structured as shown in Figure 2. In this scheme, the position controller calculates in real-time the desired roll \( φ_d(t) \) and pitch \( θ_d(t) \) angles and then uses them as references for the attitude controller.

Being the desired position \( p_d(t) \) and desired yaw angle \( ψ_d(t) \) established by the user as at least four times differentiable and bounded signals, the position error vector \( \hat{p}(t) = \begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \end{bmatrix} \) and the attitude error vector \( \hat{η}(t) = \begin{bmatrix} \dot{φ}(t) \\ \dot{θ}(t) \\ \dot{ψ}(t) \end{bmatrix} \) are given by

\[ \hat{p}(t) = p_d(t) - p(t) \quad (10) \]

and

\[ \hat{η}(t) = η_d(t) - η(t). \quad (11) \]

respectively. Thus, the control problem is to design the total thrust \( u_T(t) \), the torque vector \( τ(t) \), and the desired angles \( φ_d(t) \) and \( θ_d(t) \) so that the limit

\[ \lim_{t \to \infty} \begin{bmatrix} \hat{p}(t) \\ \hat{η}(t) \end{bmatrix} = 0 \quad (12) \]

is satisfied.

**B. POSITION CONTROLLER**

The position trajectory tracking of the quadrotor is achieved by establishing the total thrust \( u_T(t) \) and the desired angles \( φ_d(t) \) and \( θ_d(t) \). Then, the proposed position controller is given by

\[ u_T = \frac{f_z}{r_{33}(η)}, \quad (13) \]

\[ \dot{θ}_d = \tan^{-1} \left( \frac{1}{f_z} \left[ f_x \sin(ψ_d) + f_z \cos(ψ_d) \right] \right), \quad (14) \]

\[ ϕ_d = \tan^{-1} \left( \frac{\cos(θ_d)}{f_z} \left[ f_z \sin(ψ_d) - f_x \cos(ψ_d) \right] \right). \quad (15) \]

where \( f = [f_x, f_y, f_z]^T \in \mathbb{R}^3 \) is the vector defined as

\[ f = \dot{m} \hat{p}_d + mg_e + K_p \hat{p} + K_d \dot{p}. \quad (16) \]

\( r_{33}(η) \) is the (3, 3) element of the rotation matrix \( R(η) \) defined in (3), \( \dot{m} \) is a dynamic estimate for the mass of the quadrotor \( m \), and \( K_p \in \mathbb{R}^{3 \times 3} \) and \( K_d \in \mathbb{R}^{3 \times 3} \) are diagonal positive definite matrices.

The control law is complemented with the following adaptation law for the estimated mass \( \dot{m} \)

\[ \dot{m} = γ_p Y_p(\hat{p}_d)^T \hat{p} + γ_p \epsilon Y_p(\hat{p}_d)^T \hat{p}, \quad (17) \]

where \( γ_p \) and \( ε \) are positive constants, and \( Y_p(\hat{p}_d) \in \mathbb{R}^{3 \times 1} \) is the position regression matrix explicitly given by

\[ Y_p(\hat{p}_d) = \hat{p}_d + ge_z. \quad (18) \]

In order to obtain the closed-loop position error dynamics, the right-hand side of (1) can be expressed as

\[ R(η)e_u_T = r_c(η_d)r_{33}(η)u_T + \left[ r_c(η) - r_c(η_d) \right] r_{33}(η)u_T. \quad (19) \]
where
\[
\begin{bmatrix}
  r_1(\eta) \\
  r_2(\eta) \\
  r_3(\eta)
\end{bmatrix} = \begin{bmatrix}
  1 \\
  t_2(\eta) \\
  t_3(\eta)
\end{bmatrix} \in \mathbb{R}^3.
\] (20)

Besides, considering the following relationship
\[
r_e(\eta_d)r_33(\eta)u_T = f,
\] (21)
we obtain
\[
R(\eta)e_T = \hat{m}Y_p(\hat{\eta}_d) + K_p\hat{p} + K_n\hat{p} + [r_e(\eta) - r_e(\eta_d)]f_z.
\] (22)

Note that the control inputs in (13)-(15) are the solution of the equation system presented in (21).
Then, the position error dynamics are given as
\[
m\ddot{\hat{p}} = Y_p(\hat{\eta}_d)\hat{m} - K_p\hat{p} - K_n\hat{p} - [r_e(\eta) - r_e(\eta_d)]f_z,
\] (23)
with the mass estimation error defined as
\[
\hat{m} = m - \hat{m}.
\] (24)

The following facts are important for the future analysis.

**Fact 1:** The function \(r_e(\eta)\) in (20) is locally Lipschitz, i.e.,
\[
\|r_e(\eta_d) - r_e(\eta_d - \eta)\| \leq k_r\|\eta\|,
\] (25)
for all \(\eta\) and \(\eta_d\), such that
\[
|\phi|, |\theta|, |\phi_d|, |\theta_d| < \frac{\pi}{2}, \text{ and } \psi, \psi_d \in \mathbb{R}.
\]**

**Fact 2:** The auxiliary control signal \(f_z\) in (13) is upper bounded by
\[
|f_z| \leq k_{f_z},
\] (26)
where
\[
k_{f_z} = \max_{\mathbf{w} \in B_{f_z}} |m\ddot{\hat{p}}_d + mg + K_p\dot{w}_1 + K_n\dot{w}_2|,
\]
with the set
\[
B_{f_z} = \{\mathbf{w} \in \mathbb{R}^2 : \|\mathbf{w}\| \leq r_{f_z}\}.
\]

**C. ATTITUDE CONTROLLER**

Let us define the vectors \(\omega_r = [\omega_{rx} \omega_{ry} \omega_{rz}]^T \in \mathbb{R}^3\) and \(\dot{\omega}_r = [\dot{\omega}_{rx} \dot{\omega}_{ry} \dot{\omega}_{rz}]^T \in \mathbb{R}^3\) as
\[
\omega_r = W(\eta)^{-1}\dot{\hat{\eta}}_r,
\] (27)
\[
\dot{\omega}_r = W(\eta)^{-1}\dot{\tilde{\eta}}_r - \dot{W}(\eta)\omega_r,
\] (28)
with the vector \(\dot{\hat{\eta}}_r \in \mathbb{R}^3\) defined as
\[
\dot{\hat{\eta}}_r = \dot{\tilde{\eta}}_r + \Lambda \hat{\eta},
\] (29)
and \(\Lambda \in \mathbb{R}^{3 \times 3}\) a diagonal positive definite matrix. Besides, the filtered attitude error is defined as
\[
s = \hat{\eta} + \Lambda \hat{\eta}.
\] (30)
Therefore, the open-loop filtered attitude error dynamics are expressed as follows
\[
H_o(\eta)\ddot{s} = H_o(\eta)\dot{\hat{\eta}}_r + C_o(\eta, \dot{\hat{\eta}}_r)\dot{\hat{\eta}}_r - C_o(\eta, \dot{\hat{\eta}})s - \tau.
\] (31)

The first two elements of the right-hand side of the attitude error dynamics in (31) can be linearly parameterized as follows
\[
H_o(\eta)\dot{\hat{\eta}}_r + C_o(\eta, \dot{\hat{\eta}})\dot{\hat{\eta}}_r = Y(\eta, \omega, \dot{\omega}_r, \dot{\omega})\chi,\] (32)
where \(\chi = [I_{x\eta} I_{y\eta} I_{z\eta}]^T \in \mathbb{R}^6\) is the parameters vector of the attitude dynamics, and \(Y(\eta, \omega, \dot{\omega}_r, \dot{\omega}) \in \mathbb{R}^{3 \times 6}\) is the regression matrix, which is explicitly expressed as
\[
Y(\eta, \omega, \dot{\omega}_r, \dot{\omega}) = W(\eta)^{-T}[Y_1(\dot{\omega}_r) + Y_2(\omega, \dot{\omega})],
\] (33)
with \(Y_{11}, Y_{12} \in \mathbb{R}^{3 \times 6}\) defined as
\[
\begin{bmatrix}
  \omega_{rx} & \omega_{ry} & \omega_{rz} & 0 & 0 & 0 \\
  0 & \omega_{rx} & \omega_{ry} & \omega_{rz} & 0 & 0 \\
  0 & 0 & \omega_{rx} & \omega_{ry} & \omega_{rz} & 0
\end{bmatrix},
\] (34)
\[
\begin{bmatrix}
  w_3 & w_2 & -w_6 & w_4 & w_9 \\
  -w_2 & w_1 & -w_9 & w_5 & -w_8 \\
  w_3 & w_6 & w_7 & -w_5 & -w_8
\end{bmatrix},
\] (35)
and
\[
w_1 = \omega_0 \omega_{rx} - \omega_4 \omega_{ry},
\quad w_2 = \omega_0 \omega_{ry},
\quad w_3 = \omega_0 \omega_{rz},
\quad w_4 = \omega_0 \omega_{ry} - \omega_2 \omega_{rz},
\quad w_5 = \omega_0 \omega_{rx},
\quad w_6 = \omega_0 \omega_{rz},
\quad w_7 = \omega_0 \omega_{rz} - \omega_2 \omega_{rx},
\quad w_8 = \omega_0 \omega_{rx},
\quad w_9 = \omega_0 \omega_{ry}.
\] (36)

Now, we are in position to introduce the attitude controller as
\[
\tau = Y(\eta, \omega, \dot{\omega}_r, \dot{\omega}_r)\dot{\tilde{\eta}}_r + K_s s,
\] (37)
where \(\dot{\tilde{\eta}}_r \in \mathbb{R}^6\) is the vector of estimated parameters and \(K_s \in \mathbb{R}^{3 \times 3}\) is a diagonal positive definite matrix. Besides, the update law for the vector of estimated parameters \(\dot{\tilde{\eta}}_r\) is defined as follows
\[
\dot{\tilde{\eta}}_r = \Gamma_\eta Y_\eta(\eta, \omega, \dot{\omega}_r, \dot{\omega}_r)^T s,
\] (38)
where \(\Gamma_\eta \in \mathbb{R}^{6 \times 6}\) is a positive definite matrix.

Thus, from (31), (32) and (37), the closed-loop filtered attitude error dynamics can be written as
\[
H_o(\eta)\ddot{s} = Y(\eta, \omega, \dot{\omega}_r, \dot{\omega}_r)\dot{\tilde{\eta}}_r - C_o(\eta, \dot{\tilde{\eta}})s - K_s s,
\] (39)
where \(\dot{\tilde{\eta}}_r = \dot{\tilde{\eta}} - \dot{\tilde{\eta}}_r \in \mathbb{R}^6\) is the parameter estimation error.

**IV. STABILITY ANALYSIS**

The overall closed-loop system obtained with the quadrotor dynamics in (1)-(2), the controller in (13)-(15) and (37) and the adaptation laws (17) and (38) is given by
\[
d\dot{\hat{\eta}}_r = \dot{\tilde{\eta}}_r + \Lambda \hat{\eta},
\]
\[
d\dot{\eta} = Y(\eta, \omega, \dot{\omega}_r, \dot{\omega}_r)\dot{\tilde{\eta}}_r - C_o(\eta, \dot{\tilde{\eta}})s - K_s s,
\]
\[
d\dot{s} = Y(\eta, \omega, \dot{\omega}_r, \dot{\omega}_r)\dot{\tilde{\eta}}_r - C_o(\eta, \dot{\tilde{\eta}})s - K_s s,
\]
\[
d\dot{\tilde{\eta}}_r = \Gamma_\eta Y_\eta(\eta, \omega, \dot{\omega}_r, \dot{\omega}_r)^T s,
\] (38)
where \(\Gamma_\eta \in \mathbb{R}^{6 \times 6}\) is a positive definite matrix.

Thus, from (31), (32) and (37), the closed-loop filtered attitude error dynamics can be written as
\[
H_o(\eta)\ddot{s} = Y(\eta, \omega, \dot{\omega}_r, \dot{\omega}_r)\dot{\tilde{\eta}}_r - C_o(\eta, \dot{\tilde{\eta}})s - K_s s,
\] (39)
where \(\dot{\tilde{\eta}}_r = \dot{\tilde{\eta}} - \dot{\tilde{\eta}}_r \in \mathbb{R}^6\) is the parameter estimation error.
\[
\frac{d}{dt}\ddot{m} = -\gamma_p Y_\rho(t)^T \ddot{p} - \gamma_p \epsilon Y_\rho(t)^T \dot{p},
\]
\[
\frac{d}{dt} \ddot{\eta} = -\Gamma_\eta \eta_\rho(\eta, \omega, \omega_r, \omega_r)^T s.
\]

(40)

Note that the an equilibrium point of the closed-loop system (40) is given by
\[
x(t) = \left[ p(t)^T \dot{p}(t)^T \dot{\eta}(t)^T s(t)^T \ddot{m}(t)^T \ddot{\eta}(t)^T \right]^T = 0 \in \mathbb{R}^{19}.
\]

**Proposition 1**: Suppose that control and adaptation gains are selected such that
\[
0 < \epsilon < \frac{\lambda_{\min}(K_d)}{m},
\]
(41)
\[
\gamma_p > 0, \Gamma_\eta \text{ is a } 6 \times 6 \text{ positive definite matrix, and } K_p, K_d, \Lambda \text{ and } K_\eta \text{ are diagonal positive definite matrices. Then, the equilibrium point } x = 0 \text{ of the closed-loop system (40) is locally stable.}
\]

In addition, the limit
\[
\lim_{t \to \infty} \begin{bmatrix}
\ddot{p}(t) \\
\ddot{\eta}(t) \\
s(t)
\end{bmatrix} = 0
\]
(42)
is satisfied.

**Proof**: Consider the following Lyapunov function candidate
\[
V = \alpha_1 V_1 + \alpha_2 V_2,
\]
where
\[
V_1 = \frac{1}{2} m \dddot{p}^T \dddot{p} + \frac{1}{2} p^T K_d \dddot{p}
\]
\[
+ \frac{1}{2} \epsilon p^T K_d \dddot{p} + \epsilon m \dddot{p}^T \dddot{p} + \frac{1}{2} \eta^T \dddot{m}^2,
\]
and
\[
V_2 = \frac{1}{2} \eta^T H_\eta(\eta) s + \frac{1}{2} \alpha_2 \eta^T \ddot{\eta} + \frac{1}{2} \eta^T \Gamma_\eta s \eta
\]
with \( \alpha_1, \alpha_2, \alpha_3 \) strictly positive constants. Note that
\[
V_1 = \frac{1}{2} \begin{bmatrix}
\dddot{p} \\
\dddot{\eta} \\
s
\end{bmatrix}^T \begin{bmatrix}
K_p + \epsilon K_d & \epsilon m \eta \\
\epsilon m \eta & m \eta
\end{bmatrix} \begin{bmatrix}
\dddot{p} \\
\dddot{\eta} \\
s
\end{bmatrix}
+ \frac{1}{2} \eta^T \dddot{m}^2
\]
is a positive definite function if \( \epsilon \) is selected such that (41) is satisfied.

From the Property 1 and condition (41), we can conclude that \( V \) in (43) is locally positive definite.

The time derivative of \( V \) in (43) is obtained in two steps, which consist in computing the time derivative of \( V_1 \) and \( V_2 \) separately.

Firstly, \( V_1 \) is computed and upper bounded:
\[
\dot{V}_1 = \dddot{p}^T m \dddot{p} + \dddot{p}^T K_d \dddot{p} + \epsilon \dddot{p}^T K_d \dddot{p}
+ \epsilon \dddot{p}^T m \dddot{p} + \epsilon \dddot{p}^T \dddot{p} + \frac{1}{2} \dddot{m}^T \dddot{m}
+ \frac{1}{\gamma_p} \dddot{p}^T K_d \dddot{p} + \frac{1}{\gamma_p} \dddot{p}^T K_d \dddot{p}
+ \epsilon \dddot{p}^T \dddot{p} + \frac{1}{\gamma_p} \dddot{p}^T K_d \dddot{p}
\]
\[
\leq -\lambda_{\min}(K_d) \| \dddot{p} \|^2 - \epsilon \lambda_{\min}(K_d) \| \dddot{p} \|^2
+ k \left( \| \dddot{p} \| + \epsilon \| \dddot{p} \| \right) \| \dddot{\eta} \|,
\]
where Facts 1 and 2 were used to define
\[
k = k_r k_{\varepsilon},
\]
where \( k_r \) is related to the inequality (25), and \( k_{\varepsilon} \) is defined in (26). Additionally, by grouping terms, the upper bound of the derivative of \( V_1 \) can be written as
\[
\dot{V}_1 \leq -\begin{bmatrix}
\dddot{p} \\
\dddot{\eta} \\
s
\end{bmatrix}^T Q_1 \begin{bmatrix}
\dddot{p} \\
\dddot{\eta} \\
s
\end{bmatrix}
+ \alpha_1 k \left( \| \dddot{p} \| + \epsilon \| \dddot{p} \| \right) \| \dddot{\eta} \|,
\]
with
\[
Q_1 = \begin{bmatrix}
\epsilon \lambda_{\min}(K_p) & 0 \\
0 & \lambda_{\min}(K_d) - \epsilon m
\end{bmatrix}.
\]

Note that \( Q_1 \) is a positive definite matrix if \( \epsilon \) is selected as in the condition (41).

Secondly, the function \( V_2 \) is differentiated with respect to time and upper bounded as follows:
\[
\dot{V}_2 = s^T H_\eta(\eta) s + \frac{1}{2} \eta^T H_\eta(\eta) s + \alpha_3 \dddot{\eta}^T \dddot{\eta}
- s^T K_d s - \alpha_3 \dddot{\eta}^T \Lambda \dddot{\eta} + \alpha_3 \dddot{\eta}^T s + \dddot{\eta}^T \Gamma_\eta s \dddot{\eta}
\]
\[
\leq -\lambda_{\min}(K_d) \| s \|^2 - \alpha_3 \lambda_{\min}(\Lambda) \| \dddot{\eta} \|^2
+ \alpha_3 \| \dddot{\eta} \| \| s \|
\]
\[
- \begin{bmatrix}
\| \dddot{\eta} \| \\
\| s \|
\end{bmatrix}^T Q_2 \begin{bmatrix}
\| \dddot{\eta} \| \\
\| s \|
\end{bmatrix},
\]
where
\[
Q_2 = \begin{bmatrix}
\alpha_3 \lambda_{\min}(\Lambda) & -\frac{1}{2} \alpha_3 \\
-\frac{1}{2} \alpha_3 & \lambda_{\min}(K_d)
\end{bmatrix},
\]
which is a positive definite matrix when
\[
0 < \alpha_3 < 4 \lambda_{\min}(\Lambda) \lambda_{\min}(K_d).
\]

Finally, the time derivative of \( V \) is obtained and upper bounded as
\[
\dot{V} = \alpha_1 \dot{V}_1 + \alpha_2 \dot{V}_2
\]
\[
\leq -\alpha_1 \lambda_{\min}(Q_1) \| \dddot{p} \|^2 - \alpha_2 \lambda_{\min}(Q_2) \| \dddot{\eta} \|^2
+ \alpha_1 k \sqrt{1 + \epsilon^2} \| \dddot{p} \| \| \dddot{\eta} \| \| s \|
\]
By defining
\[
\sigma_1 = \frac{\| \dddot{p} \|}{\| \dddot{\eta} \|}, \quad \text{and} \quad \sigma_2 = \frac{\| s \|}{\| \dddot{\eta} \|},
\]
the upper bound of \( \dot{V} \) can be rewritten as
\[
\dot{V} \leq -\begin{bmatrix}
\sigma_1 \\
\sigma_2
\end{bmatrix}^T Q \begin{bmatrix}
\sigma_1 \\
\sigma_2
\end{bmatrix},
\]
(44)
where
\[
Q = \begin{bmatrix}
\alpha_1 \lambda_{\text{min}}(Q_1) & -\frac{1}{2} \alpha_1 k \sqrt{1 + \epsilon^2} \\
-\frac{1}{2} \alpha_1 k \sqrt{1 + \epsilon^2} & \alpha_2 \lambda_{\text{min}}(Q_2)
\end{bmatrix}.
\]

Thus, we can conclude that $\dot{V}$ is a locally negative definite function if $Q$ is a positive definite matrix, which is always satisfied for
\[
\alpha_2 > \frac{\alpha_1 k^2(1 + \epsilon^2)}{4\lambda_{\text{min}}(Q_1)\lambda_{\text{min}}(Q_2)}.
\]

In agreement with Lyapunov’s direct method [43], $x = 0$ is a stable equilibrium point of the closed-loop (40). This at the same time implies that $x(t) \in L_\infty^4$. Also, from (14) and (15) and the assumption that $p_d(t)$ and $\psi_d(t)$ are at least four times differentiable and bounded, we can show that $\eta_d(t), \dot{\eta}_d(t), \ddot{\eta}_d(t) \in L_\infty^3$. Therefore, the above facts and the quadrotor closed-loop dynamics in (40) can be used to prove that $\dot{x}(t) \in L_\infty^1$, and the solutions of the closed-loop system $x(t)$ are uniformly bounded. Integrating both sides of (44), the following inequality is given
\[
\frac{V(t_0)}{\lambda_{\text{min}}(Q)} \geq \int_{t_0}^{t} \left\| \left[ \| \dot{p}(t) \| \| \dot{\eta}(t) \| \| s(t) \| \right]^T \right\|^2 dt.
\]

Therefore, there are sufficient conditions to invoke Barbalat’s lemma [43] to conclude that the limit in (42) is satisfied. \qed

V. EXPERIMENTAL RESULTS

In this Section, we present a set of real-time experiments in order to assess the performance of the proposed scheme. The Quanser QBall 2 was used as experimental platform. This quadrotor is equipped with four 10-inch propellers attached to brushless motors. The Qball 2 is provided with a wireless Gumstix DuoVero embedded computer which executes the programmed controllers. Besides, this quadrotor utilizes an onboard data acquisition card (DAQ) with a high-resolution inertial measurement unit (IMU) used to obtain the attitude and angular velocity vectors $\eta(t)$ and $\omega(t)$, respectively. On the other hand, the position is estimated with the vision system Optitrack which is integrated by six Flex 3 cameras and the optical motion capture software Motive. The controllers are implemented using QUARC, which is the Quanser’s real-time control software, and the Matlab–Simulink interface on the host ground station computer. These controllers are automatically compiled into executable files and downloaded in the Gumstix embedded computer using QUARC software. Figure 3 shows the set-up of the experimental platform.

Two control schemes are used to compare the performance of the proposed controller. The collected results in two real-time experimental case studies, which consist to the tracking of circular and lemniscate path, are analyzed.

A. ACAGCF ADAPTIVE CONTROLLER

The controller presented by Antonelli et al. [26], [27], is used to compare the performance of the novel controller.
The gains for the ACAGCF controller in equations (45)-(52) were selected by using a trial and error process resulting in the following values:

\[ \lambda_x = 3.66, \quad \lambda_y = 3.66, \quad \lambda_z = 1.50, \]
\[ k_{xx} = 3.00, \quad k_{yy} = 3.00, \quad k_{zz} = 6.00, \]
\[ k_{yx} = 0.04, \quad k_{xy} = 0.04, \quad k_{yz} = 6.2, \]
\[ K_{p\eta} = \text{diag}(1.80 1.80 4.50), \]
\[ K_{d\eta} = \text{diag}(0.45 0.45 1.00), \]
\[ k_{iq} = 0.05. \]

**B. NON-ADAPTIVE VERSION OF THE PROPOSED CONTROLLER**

Additionally to the ACAGCF controller, a non-adaptive version of the proposed controller in (13)-(15) and (37) was used for the performance comparison. Therefore, the controller is structured as a model-based scheme which uses fixed estimates of the dynamic parameters. In the remaining of this document, this controller will be denoted by CWA (controller without adaptation). The implementation gains for this controller were established as follows

\[ k_{px} = 11.00, \quad k_{py} = 11.00, \quad k_{pc} = 14.00, \]
\[ k_{dx} = 2.50, \quad k_{dy} = 2.50, \quad k_{dc} = 3.50, \]
\[ \Lambda = \text{diag}(4.40 4.40 4.50), \]
\[ K_s = \text{diag}(0.41 0.41 0.90). \]

The fixed estimates of the dynamic parameters used in the two implementations of the CWA controller were set as

\[ \hat{m} = 1.61, \]
\[ \hat{x}_\eta = [0.024 0.0 0.0 0.024 0.0 0.028]^T. \]

**C. PROPOSED ADAPTIVE CONTROLLER**

The proposed adaptive controller in (13)-(15) and (37), and the parameter update laws in (17) and (38) were implemented using the same gains that were established for CWA controller. Besides, adaptation gains were selected as

\[ \gamma_p = 0.0125, \quad \epsilon = 1.39, \]
\[ \Gamma_\eta = \text{diag}(0.004 0.004 0.004 0.004 0.004 0.004), \]

while the adaptation parameters were initialized as

\[ \hat{m}(0) = 1.61, \]
\[ \hat{x}_\eta(0) = [0.024 0.0 0.0 0.024 0.0 0.028]^T. \]

The manufacturer provides the following dynamic parameters for the Qball2 quadrotor [44]:

\[ m = 1.79, \]
\[ \chi_\eta = [0.03 0.0 0.0 0.03 0.0 0.04]^T. \]

Note that the nominal values of the dynamic parameters differ from those used in both the CWA and the proposed adaptive controller. These values are estimates provided by the manufacturer that are not necessarily accurate. Therefore, it is reasonable to consider that all the elements of the inertial matrix are not null. Besides, the discrepancy was intentionally introduced with the objective of evaluating the benefits of the proposed adaptive controller when parametric inaccuracies are presented.

**D. REAL-TIME EXPERIMENTS**

1) **CASE I: CIRCULAR PATH TRACKING**

The first set of real-time experiments concern to the tracking of an inclined elliptical path. The desired yaw angle \( \psi_d(t) \) was defined to be null. The reference signals which describe this motion task are given by

\[ x_d(t) = 0.8 \sin(\omega_n t)(1 - e^{-0.1t^2}) \, [m], \]
\[ y_d(t) = 0.8 \cos(\omega_n t)(1 - e^{-0.1t^2}) \, [m], \]
\[ z_d(t) = \begin{cases} 
0.3 + a_1 t^3 + a_2 t^4 + a_3 t^5 & \text{if } t \leq 5, \\
0.9 + 0.2 \sin(\omega_n t) & \text{if } t > 5, 
\end{cases} \]
\[ \psi_d(t) = 0 \, [\text{rad}]. \quad (53) \]

where \( \omega_n = 2\pi/10 \, [\text{rad/s}], a_1 = 2.7894 \times 10^{-02} \, [m/s^3], \]
\[ a_2 = -7.3628 \times 10^{-03} \, [m/s^4], \text{ and } a_3 = 5.4881 \times 10^{-04} \, [m/s^5]. \]

![Figure 4.](image)

**FIGURE 4.** Experimental case I: circle path drawn by the quadrotor for the tested controllers.

The resulting paths described by the quadrotor as result of using the three controllers tested are depicted in Figure 4. The black line shows the desired path. The obtained paths with the ACAGCF controller and the CWA controller are depicted by the blue and green lines, respectively. In addition, the resulting from implementing the proposed controller is given by the red line. As can be seen, the quadrotor remains the closest to the reference path when using the proposed scheme. Figure 5 shows the time evolution of the position signals \( x(t), y(t), z(t) \), and the yaw angle signal \( \psi(t) \).
The respective desired signals are also shown there. Note that the CWA algorithm performs with a considerable error in altitude $z(t)$ caused by the underestimation of thrust which is obtained with a smaller value of the mass than the nominal. On the other hand, the proposed controller presents a good tracking performance.

Finally, for each implemented algorithm, Figure 6 shows the time history of the thrust forces generated by each rotor of the quadrotor. The control inputs computed by the controllers tested are similar to each other.

The RMS (root mean square) values for the position and attitude error signals are presented in Table 1. These values were computed in the time interval $15 [s] \leq t \leq 40 [s]$ after the steady state was reached. These results show congruence with the behavior observed in Figure 4. In other words, the RMS values of the tracking errors when using the new adaptive scheme are the smallest.

2) CASE II: LEMNISCATE PATH TRACKING

In the second set of real-time experiments, a lemniscate curve in the space was defined for the tracking task, while the yaw angle remained null throughout the experiment. The lemniscate period is equal to $20 [s]$. Thus, the references signals were set as

$$
\begin{align*}
    x_d(t) &= 0.5 \sin(\omega_n t)(1 - e^{-0.1t^3}) \ [m], \\
    y_d(t) &= 0.8 \cos \left( \frac{\omega_n}{2} t \right) (1 - e^{-0.1t^3}) \ [m], \\
    z_d(t) &= \begin{cases} 
        0.3 + a_1 t^3 + a_2 t^4 + a_3 t^5 \ [m], & t \leq 5, \\
        0.9 + 0.2 \sin(\omega_n t) \ [m], & t > 5, 
    \end{cases} \\
    \psi_d(t) &= 0 \ [rad],
\end{align*}
$$

(54)

where the frequency $\omega_n$ and the coefficients $a_1$, $a_2$ and $a_3$ were set equal to those used in the circular path (53).

The path described by the quadrotor with each one of the controllers is depicted in Figure 7. Similarly to the circle experiment, it is possible to observe that the path described by the quadrotor with the proposed controller is closer to the reference than the resulting ones with the other controllers. Using the proposed and the ACAGCF controller, the quadrotor altitude is close to the reference signal. However, for the
non-adaptive scheme CWA, the quadrotor does not reach the desired value.

Figure 8 shows the position and yaw angle signals by using the three controllers evaluated. It can be observed that the results obtained with the proposed controller are closer to the reference signals than the other schemes. The control action computed by each controller is shown in Figure 9. After the transient period, the magnitude of the control inputs generated by the controllers tested is similar to each other.

The RMS values of the tracking errors obtained when the lemniscate tracking is specified are presented in Table 2. As in the case of the circular path, the lowest RMS value for each one of the error signals is obtained with the proposed adaptive controller.

VI. CONCLUSIONS

In this paper, we presented an adaptive controller for quadrotor trajectory tracking in presence of parametric inaccuracies, which can be caused by the uncertainty of the nominal values or changes in the system during the flight. We introduced an accurate parameterization of the model-based feed-forward compensation without simplifications. The closed
loop system was analyzed by using Lyapunov’s theory and local stability was proven. Convergence of the tracking errors and boundedness of the adaption parameters errors were also shown. Two real-time experimental case studies were carried out to validate the novel controller. Furthermore, the performance of the proposed controller was compared with an existing controller in the literature and with the non-adaptive version of the novel scheme. The proposed adaptive controller showed the best tracking performance.

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RICARDO PÉREZ-ALCOCER was born in Mérida, México, in 1981. He received the B.Sc. degree in computer sciences and the M.Sc. degree in mathematics from the University of Yucatán, México, in 2004 and 2007, respectively, and the Ph.D. degree in robotics and advanced manufacturing from CINVESTAV Research Center, Saltillo, México, in 2013. He is currently a Research Fellow with the CONACYT–Instituto Politécnico Nacional-CITEDI. His research interests include unmanned vehicles (aerial, aquatic, and wheeled), linear and nonlinear control, multi-agent systems, computer vision, and intelligent systems.

JAVIER MORENO-VALENZUELA received the Ph.D. degree in automatic control from CICESE Research Center, Ensenada, México, in 2002. He was a Postdoctoral Fellow with the Université de Liège, Belgium, from 2004 to 2005. He is currently with the Instituto Politécnico Nacional-CITEDI, Tijuana, México. He has authored many peer reviewed journal and international conference papers and the book entitled: Motion Control of Underactuated Mechanical Systems (Springer-Verlag, 2018). His research interests include nonlinear systems, mechatronics, and intelligent systems. He has served as a reviewer of a number of prestigious scientific journals. He is an Associate Editor of the IEEE Latin America Transactions and Mathematical Problems in Engineering.

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