SUPERCOMMUTATOR ALGEBRAS OF RIGHT (HOM-)ALTERNATIVE SUPERALGEBRAS

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Abstract. The supercommutator algebra of a right alternative superalgebra is a Bol superalgebra. Hom-Bol superalgebras are defined and it is shown that they are closed under even self-morphisms. Any Bol superalgebra along with any even self-morphism is twisted into a Hom-Bol superalgebra. The supercommutator algebra of a right Hom-alternative superalgebra has a natural Hom-Bol superalgebra structure.

Keywords: Right alternative algebra, Superalgebra, Bol algebra, Hom-algebra.
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1. Introduction

A right alternative algebra is an algebra satisfying the right alternative identity \((xy)y = x(yy)\). These algebras were first considered in [2]. For further studies on right alternative algebras, one may refer to [12], [23], [24].

It turns out that right alternative algebras have close relations with a type of binary-ternary algebras called Bol algebras which were introduced in [17] (see also references therein). In fact, it is proved [18] that any right alternative algebra has a natural Bol algebra structure.

The general theory of superalgebras started with the introduction of \(Z_2\)-graded Lie algebras (i.e. Lie superalgebras) coming from physics (see [10], [20] and references therein for basics on Lie superalgebras). The \(Z_2\)-graded generalization of algebras is first extended to Jordan algebras in [11]. Next, alternative superalgebras were introduced in [28] whereas Maltsev superalgebras were introduced in [21]. A \(Z_2\)-graded generalization of Bol algebras is considered in [19].

With the introduction of Hom-Lie algebras (see [8], [13], [14]) began studies of Hom-type generalizations of usual algebras. Apart from Hom-Lie algebras, first Hom-type algebras were defined in [16] while Hom-alternative and Hom-Jordan algebras were defined in [15] (see also [27] where Maltsev algebras were defined) and the Hom-type generalization of right alternative algebras was considered in [26]. It should be observed that, in general, the twisting map in a Hom-algebra is neither injective nor surjective and when the twisting map is the identity map, then one recovers the ordinary (untwisted) algebraic structure. So ordinary algebras are viewed as Hom-algebras with the identity map as twisting map. Moving further in the theory of Hom-algebras, the twisting principle of algebras is extended to binary-ternary algebras in [9] and next Hom-Bol algebras were defined in [5]. As in the case of right alternative algebras, it is shown in [6] that a Hom-Bol algebra structure can be defined on any multiplicative right Hom-alternative algebra.
In this paper we extend, but with a different approach, the result in [18] to the cases of right alternative superalgebras and right Hom-alternative superalgebras. Some basics on superalgebras are reminded in section 2. In section 3 it is proved that any right alternative superalgebra has a natural Bol superalgebra structure. In section 4, in order to deal with the Hom-version of the results from section 3, we first define Hom-Bol superalgebras and next, in section 5, prove that right Hom-alternative superalgebras are in fact Hom-Bol superalgebras.

All vector spaces and algebras are considered over a fixed ground field of characteristic not 2 or 3.

2. Preliminaries

A superspace (or a $\mathbb{Z}_2$-graded space) $V$ is a direct sum $V = V_0 \oplus V_1$, where $V_i$ are vector spaces. An element $x \in V_i$ ($i \in \mathbb{Z}_2$) is said to be homogeneous of degree $i$ and the degree of $x$ will be denoted by $\overline{x}$.

Definition 2.1 (i) Let $f : A \to A'$ be a linear map, where $A = A_0 \oplus A_1$ and $A' = A'_0 \oplus A'_1$ are superspaces. The map $f$ is said to be even (resp. odd) if $f(A_i) \subset A'_i$ (resp. $f(A_i) \subset A'_{i+1}$) for $i = 0, 1$.

(ii) A (multiplicative) $n$-ary Hom-superalgebra is a triple $(A, \{\cdot, \cdot, \cdots, \cdot\}, \alpha)$ consisting of a superspace $A = A_0 \oplus A_1$, an $n$-linear map $\{\cdot, \cdot, \cdots, \cdot\} : A^\otimes n \to A$ such that $\{A_i, \cdots, A_s\} \subset A_{i+s+1}$, and an even linear map $\alpha : A \to A$ such that $\alpha(\{x_1, \cdots, x_n\}) = \{\alpha(x_1), \cdots, \alpha(x_n)\}$ (multiplicativity).

One observes that if $\alpha = Id$ (the identity map), we get the corresponding definition of an $n$-ary superalgebra. One also notes that $\overline{\alpha(x)} = \overline{x}$ for all homogeneous $x \in A$.

We will be interested in binary ($n = 2$), ternary ($n = 3$) and binary-ternary Hom-superalgebras (i.e. Hom-superalgebras with binary and ternary operations). For convenience, throughout this paper we assume that all Hom-(super)algebras are multiplicative.

Definition 2.2 ([14]). Let $A := (A, *, \alpha)$ be a binary Hom-superalgebra. The supercommutator Hom-algebra (or the minus Hom-superalgebra) of $A$ is the Hom-superalgebra $A^- := (A, [\cdot, \cdot], \alpha)$, where $[x, y] := \frac{1}{2}(xy - (-1)^{xy}yx) + \frac{1}{2}(xy + (-1)^{xy}yx)$ for all homogeneous $x, y \in A$. The product $[\cdot, \cdot]$ is called the supercommutator bracket.

In the sequel, we will also denote by $\\{\\cdot, \cdot\\}$ the binary operation in (Hom-)superalgebras. Besides the supercommutator of elements in Hom-superalgebras, one also considers the super-Jordan product

$$x \circ y := \frac{1}{2}(xy + (-1)^{xy}yx),$$

the Hom-Jordan associator

$$as_J^\alpha(x, y, z) := xy \cdot \alpha(z) + \alpha(x) \cdot yz$$

and the Hom-associator $as_\alpha(x, y, z)$ defined as

$$as_\alpha(x, y, z) := xy \cdot \alpha(z) - \alpha(x) \cdot yz$$
for \(x, y, z\) in the given Hom-superalgebra. When \(\alpha = \text{Id}\) we recover the usual Jordan associator \(as^\alpha(x, y, z)\) and associator \(as(x, y, z)\) respectively in usual algebras. The Hom-superalgebra \(A^+ := (A, \circ, \alpha)\) is usually called the plus Hom-superalgebra of \(A := (A, \ast, \alpha)\).

**Definition 2.3** A Hom-superalgebra \(A\) is said to be **right Hom-alternative** if

\[
as_{\alpha}(x, y, z) = -(-1)^{\bar{y} \bar{z}} as_{\alpha}(x, z, y) \quad \text{(right superalternativity)}
\]

for all \(x, y, z \in A\).

Likewise is defined a left Hom-alternative superalgebra. A Hom-superalgebra that is both right and left Hom-alternative is said to be **Hom-alternative**. Of course, for \(\alpha = \text{Id}\), one gets the definition of an alternative superalgebra given in [28].

From (2.1), expanding Hom-associators, it is easily seen that (2.1) is equivalent to

\[
\alpha(x)(yz + (-1)^{\bar{y} \bar{z}} zy) = (xy)\alpha(z) + (-1)^{\bar{y} \bar{z}} (xz)\alpha(y).
\]

**3. Right alternative superalgebras and Bol superalgebras**

In this section we prove that the commutator algebra of a right alternative superalgebra is a Bol superalgebra. First we recall the following

**Definition 3.1** ([19]). A **Bol superalgebra** is a triple \((A, [\cdot, \cdot], \{\cdot, \cdot, \cdot\})\) in which \(A\) is a superspace, \([\cdot, \cdot]\) and \(\{\cdot, \cdot, \cdot\}\) are binary and ternary operations on \(A\) such that

\[
\begin{align*}
\text{(SB1)} \quad [x, y] &= -(-1)^{\bar{x} \bar{y}} [y, x], \\
\text{(SB2)} \quad \{x, y, z\} &= -(-1)^{\bar{x} \bar{y}} \{y, x, z\}, \\
\text{(SB3)} \quad \{x, y, z\} + (-1)^{\bar{x}(\bar{y} + \bar{z})} \{y, z, x\} + (-1)^{\bar{x}(\bar{y} + \bar{z})} \{z, x, y\} &= 0, \\
\text{(SB4)} \quad \{x, y, [u, v]\} &= \{[x, y, u], v\} + (-1)^{\bar{x}(\bar{y} + \bar{u})}\{u, \{x, y, v\}\} \\
&\quad + (-1)^{\bar{x}(\bar{y} + \bar{u})}(\{u, v, [x, y]\} - \{[u, v], [x, y]\})}, \\
\text{(SB5)} \quad \{x, y, \{u, v, w\}\} &= \{\{x, y, u\}, v, w\} + (-1)^{\bar{x}(\bar{y} + \bar{u})}\{u, \{x, y, v\}, w\} \\
&\quad + (-1)^{\bar{x}(\bar{y} + \bar{u})}(\{u, v, \{x, y, w\}\})
\end{align*}
\]

for all homogeneous \(u, v, w, x, y, z \in A\).

Clearly, any Bol superalgebra with zero odd part is a (left) Bol algebra. If \([x, y] = 0\) for all homogeneous \(x, y \in A\), then \((A, [\cdot, \cdot], \{\cdot, \cdot, \cdot\})\) reduces to a **Lie supertriple system** \((A, \{\cdot, \cdot, \cdot\})\).

**Example 3.1** Let \(A = A_0 \oplus A_1\) be a superspace where \(A_0\) is a 2-dimensional vector space with basis \(\{i, j\}\) and \(A_1\) a 1-dimensional vector space with basis \(\{k\}\). Define on \(A\) the following binary and ternary nonzero products:

\[
\begin{align*}
[i, j] &= j, & [i, k] &= k, \\
[j, i] &= -j,
\end{align*}
\]
Then it could be checked that \((A, \cdot, \{\cdot, \cdot, \cdot\})\), with the multiplication table as above, is a (3-dimensional) Bol superalgebra. Observe that \((A, \cdot, \{\cdot, \cdot\})\) is a 3-dimensional Maltsev superalgebra [17] and the table for the ternary product as above is obtained using the \(\mathbb{Z}_2\)-graded version of the ternary product that produces a Bol algebra from a Maltsev algebra [17].

In [18] it is proved that on any right alternative algebra one may define a Bol algebra structure. The \(\mathbb{Z}_2\)-graded version of this result is given by the following

**Theorem 3.1** The supercommutator algebra of any right alternative superalgebra is a Bol superalgebra.

**Proof** Let \(A\) be a right alternative superalgebra. Then \((A, \circ)\) is a Jordan superalgebra [22]. Now define on \((A, \circ)\) a ternary product
\[
[x, y, z] := 2(-1)^{y+z}as^2(y, z, x).
\]
Then one checks that \((A, \cdot, \{\cdot, \cdot, \cdot\})\) is a Lie supertriple system. So is \((A, \{\cdot, \cdot, \cdot\})\), where
\[
\{x, y, z\} = (-1)^{x+y+z}as^2(y, z, x).
\]
Now, using specific properties of right alternative superalgebras, one gets that \((A, \cdot, \{\cdot, \cdot, \cdot\})\) is a Bol superalgebra, where \([\cdot, \cdot]\) is the supercommutator operation on \(A\). \(\square\)

4. **Hom-Bol superalgebras. Construction theorems and example**

In [5] Hom-Bol algebras are defined. In this section we define Hom-Bol superalgebras as a generalization both of Bol superalgebras [19] and Hom-Bol algebras [5]. Next we point out some construction theorems.

**Definition 4.1** A *Hom-Bol superalgebra* is a quadruple \(A_{\alpha} := (A, [\cdot, \cdot, \cdot], \{\cdot, \cdot, \cdot\}, \alpha)\) where \(A\) is a superspace, \([\cdot, \cdot]\) (resp. \(\{\cdot, \cdot, \cdot\}\)) is a binary (resp. ternary) operation on \(A\) such that

\[
\begin{align*}
(SBH1) & \quad \alpha([x, y]) = [\alpha(x), \alpha(y)], \\
(SBH2) & \quad \alpha([x, y, z]) = \{\alpha(x), \alpha(y), \alpha(z)\}, \\
(SBH3) & \quad [x, y] = (-1)^{y}y[x, y], \\
(SBH4) & \quad \{x, y, z\} = (-1)^{y}y\{x, y, z\}, \\
(SBH5) & \quad \{x, y, z\} + (-1)^{x+y+z}\{y, z, x\} + (-1)^{y+z}\{z, x, y\} = 0, \\
(SBH6) & \quad \{\alpha(x), \alpha(y), [u, v]\} = \{\alpha(x), \alpha(y), \{[\alpha(u), \alpha(v)], \alpha(x), \alpha(y)\}\}, \\
(SBH7) & \quad \{\alpha^2(x), \alpha^2(y), \{u, v, w\}\} = \{\{x, y, u\}, \alpha^2(v), \alpha^2(w)\} + (-1)^{y+z}\{\alpha^2(u), \{x, y, v\}, \alpha^2(w)\} + (-1)^{x+y+z}\{\alpha^2(u), \alpha^2(v), \{x, y, w\}\}.
\end{align*}
\]
for all homogeneous \( u, v, w, x, y, z \in A \).

We observe that for \( \alpha = \text{Id} \), any Hom-Bol superalgebra reduces to a Bol superalgebra and a Hom-Bol superalgebra with a zero odd part is a Hom-Bol algebra. If \([x, y] = 0\) for all homogeneous \( x, y \in A \), one gets a Hom-Lie supertriple system \((A, \{\cdot, \cdot\}, \alpha^2)\).

**Theorem 4.1** Let \( \mathcal{A}_\alpha := (A, [\cdot, \cdot], \{\cdot, \cdot\}, \alpha) \) be a Hom-Bol superalgebra and \( \beta : A \to A \) an even self-morphism of \( \mathcal{A}_\alpha \) such that \( \alpha \circ \beta = \beta \circ \alpha \). Let \( \beta^0 = \text{Id} \) and \( \beta^n = \beta \circ \beta^{n-1} \) for any integer \( n \geq 0 \) and define on \( A \) a binary operation \([\cdot, \cdot]_{\beta^n}\) and a ternary operation \(\{\cdot, \cdot, \cdot\}_{\beta^n}\) by

\[
[x, y]_{\beta^n} := \beta^n([x, y]),
\{x, y, z\}_{\beta^n} := \beta^{2n}(\{x, y, z\}).
\]

Then \( \mathcal{A}_{\beta^n} := (A, [\cdot, \cdot]_{\beta^n}, \{\cdot, \cdot, \cdot\}_{\beta^n}, \beta^n \circ \alpha) \) is a Hom-Bol superalgebra.

**Proof** The proof is similar to that of Theorem 3.2 in [5]. \(\square\)

From Theorem 4.1 we get the following extension of the Yau’s twisting principle [25] giving a construction of Hom-Bol superalgebras from Bol superalgebras.

**Corollary 4.1** Let \((A, [\cdot, \cdot], \{\cdot, \cdot\})\) be a Bol superalgebra and \( \beta \) an even self-morphism of \((A, [\cdot, \cdot], \{\cdot, \cdot\})\). Define on \( A \) a binary operation \([\cdot, \cdot]_{\beta}\) and a ternary operation \(\{\cdot, \cdot, \cdot\}_{\beta}\) by

\[
[x, y]_{\beta} := \beta([x, y]),
\{x, y, z\}_{\beta} := \beta^2(\{x, y, z\}).
\]

Then \( \mathcal{A}_\beta := (A, [\cdot, \cdot]_{\beta}, \{\cdot, \cdot, \cdot\}_{\beta}, \beta) \) is a Hom-Bol superalgebra. Moreover, if \((A', [\cdot, \cdot]', \{\cdot, \cdot, \cdot\}')\) is another Bol superalgebra, \( \beta' \) an even self-morphism of \((A', [\cdot, \cdot]', \{\cdot, \cdot, \cdot\}')\) and if \( f : A \to A' \) is a Bol superalgebra even morphism satisfying \( f \circ \beta = \beta' \circ f \), then \( f : \mathcal{A}_\beta \to \mathcal{A}'_{\beta'} \) is a morphism of Hom-Bol superalgebras, where \( \mathcal{A}'_{\beta'} := (A', [\cdot, \cdot]', \{\cdot, \cdot, \cdot\}', \beta') \).

**Proof** The first part of the corollary comes from Theorem 4.1 when \( n = 1 \). The second part is proved in a similar way as Corollary 4.5 in [9]. \(\square\)

**Example 4.1** Let \( A = A_0 \oplus A_1 \) be a superspace over a field of characteristic not 2 where \( A_0 \) is a 1-dimensional vector space with basis \( \{i\} \) and \( A_1 \) a 2-dimensional vector space with basis \( \{j, k\} \). Define on \( A \) the following only nonzero products on basis elements:

\[
i \ast j = k; 
\]

\[
j \ast i = k, \; j \ast k = 2i; 
\]

\[
k \ast j = 4i.
\]

Then \((A, \ast)\) is a right alternative superalgebra [22]. Now consider on \((A, \ast)\) the supercommutator \([\cdot, \cdot]\) and the ternary operation defined as

\[
\{x, y, z\} := (-1)^{\ell(y) + \ell(z)} \alpha(x' \ast y \ast z', y, z).
\]

Then it could be checked that \((A, [\cdot, \cdot], \{\cdot, \cdot, \cdot\})\) is a Bol superalgebra, where the only nonzero products are:

\[
[j, k] = 6i; 
\]

\[
[k, j] = 6i; 
\]
\{i, j, j\} = 4i;
\{j, i, j\} = -4i, \{j, j, k\} = -8i, \{j, j, j\} = 4k;
\{k, j, j\} = 4k.

Next define a linear map \(\beta : A \to A\) by setting
\(\beta(i) = ai\), \(\beta(j) = j + bk\), \(\beta(k) = ak\)
with \(a \neq 0\). Then it is easily seen that \(\beta\) is an even self-morphism of \((A, [\cdot, \cdot], \{\cdot, \cdot, \cdot\})\) and Corollary 4.1 implies that \(A_\beta := (A, [\cdot, \cdot]_\beta, \{\cdot, \cdot, \cdot\}_\beta, \beta)\) is a Hom-Bol superalgebra with products given as
\([j, k]_\beta = 6ai;\)
\([k, j]_\beta = 6ai;\)
\{i, j, j\}_\beta = 4a^2i;
\{j, i, j\}_\beta = -4a^2i, \{j, j, k\}_\beta = -8a^2k, \{j, k, j\}_\beta = 4a^2k;
\{k, j, j\}_\beta = 4a^2k.

The notion of an \(n\)th derived (binary) Hom-algebra of a given Hom-algebra is first introduced in [27] and the closure of a given type of Hom-algebras under taking \(n\)th derived Hom-algebras is a property that is characteristic of the variety of Hom-algebras. Later on, this notion is extended to binary-ternary Hom-superalgebras [5] (for binary-ternary Hom-superalgebras [7], the notion is the same as in the case of binary-ternary Hom-algebras).

**Definition 4.2** ([7]) Let \(A := (A, * , \{\cdot, \cdot\}, \alpha)\) be a binary-ternary Hom-superalgebra and \(n \geq 0\) an integer. Define on \(A\) the \(n\)th derived binary operation \(*^{(n)}\) and the \(n\)th derived ternary operation \(\{\cdot, \cdot, \cdot\}^{(n)}\) by
\(x *^{(n)} y := (x*y)^{\alpha^{2n-1}}\)
\(\{x, y, z\}^{(n)} := \alpha^{2n+1-2}(\{x, y, z\})\),
for all homogeneous \(x, y, z\) in \(A\). Then \(A^{(n)} := (A, *^{(n)}, \{\cdot, \cdot, \cdot\}^{(n)}, \alpha^{2n})\) is called the \(n\)th derived (binary-ternary) Hom-superalgebra of \(A\).

As for Hom-Bol algebras, the category of Hom-Bol superalgebras is closed under taking derived Hom-superalgebras as stated in the following

**Theorem 4.2** Let \(A := (A, [\cdot, \cdot], \{\cdot, \cdot, \cdot\}, \alpha)\) be a Hom-Bol superalgebra. Then, for each \(n \geq 0\), the \(n\)th derived Hom-superalgebra \(A^{(n)}\) is a Hom-Bol superalgebra. In particular, the \(n\)th derived Hom-superalgebra of a Hom-Lie supertriple system is a Hom-Lie supertriple system.

**Proof** The proof of the first part of the theorem is similar to that of Theorem 3.5 in [5], and the second part follows immediately. \(\square\)
5. Hom-Bol superalgebra structures on right Hom-alternative superalgebras

In this section we prove a $\mathbb{Z}_2$-graded generalization of results connecting right Hom-alternative algebras and Hom-Bol algebras [6].

**Lemma 5.1** Let $\mathcal{A} := (A, \ast, \alpha)$ be a multiplicative right Hom-alternative superalgebra. Then $\mathcal{A}$ is Hom-Jordan-admissible, i.e. $A^+$ is a Hom-Jordan superalgebra.

**Proof** A proof comes from a generalization to right alternative case of the proof of Theorem 6.1 in [1].

**Lemma 5.2** Let $(A, \circ, \alpha)$ be a multiplicative Hom-Jordan superalgebra. If define on $A$ a ternary product as

$$[x, y, z] := 2(-1)^{\frac{g}{2} + \frac{y}{2}} a_{\alpha}^J(y, z, x)$$

for all homogeneous $x, y, z \in A$, then $(A, [\cdot, \cdot, \cdot], \alpha^2)$ is a Hom-Lie supertriple system.

**Proof** If define on $(A, \circ, \alpha)$ a ternary product

$$(x, y, z) := (x \circ y) \circ \alpha(z) + (-1)^{\frac{g}{2} + \frac{y}{2} + \frac{z}{2}} \circ \alpha(y) \circ (x \circ z),$$

then it is verified that $(A, (\cdot, \cdot, \cdot), \alpha^2)$ is a Hom-Jordan superalgebra. Next, defining on $A$ the ternary product

$$[x, y, z] := (x, y, z) - (-1)^{\frac{g}{2}} (y, x, z),$$

one checks that $[x, y, z]$ expresses as (5.1) and $(A, [\cdot, \cdot, \cdot], \alpha^2)$ turns out to be a Hom-Lie supertriple system.

We can now prove the main result of this section.

**Theorem 5.1** The supercommutator Hom-algebra of any multiplicative right Hom-alternative superalgebra is a Hom-Bol superalgebra.

**Proof** Let $\mathcal{A} := (A, \ast, \alpha)$ be a multiplicative right Hom-alternative superalgebra. Then, by Lemma 5.1, $A^+$ is a Hom-Jordan superalgebra and Lemma 5.2 says that $(A, [\cdot, \cdot, \cdot], \alpha^2)$ is a Hom-Lie supertriple system, where $[\cdot, \cdot, \cdot]$ is defined by (5.1). Now define on $A$ the ternary product

$$\{x, y, z\} := (-1)^{\frac{g}{2} + \frac{y}{2}} a_{\alpha}^J(y, z, x).$$

Then, using properties of right Hom-alternative superalgebras, we get that $(A, [\cdot, \cdot, \cdot], \{\cdot, \cdot, \cdot\}, \alpha^2)$ is a Hom-Bol superalgebra.

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