A comparative analysis of wave properties of the finite and infinite doubly periodic arrays of volumetric and thin defects

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Abstract. A two-dimensional problem on wave propagation through doubly periodic arrays of defects located in an elastic material is studied. The incident wave is longitudinal and the defects may be thin (cracks) or volumetric (voids). For both the types of defects the main aim is to compare the wave properties of the structure whose geometry may be either finite or infinite in the transversal periodic direction with respect to the direction of the incident wave. The physical parameters under consideration are the reflection and the transmission coefficients, which are studied versus frequency parameter in the one-mode regime.

1. Introduction
In the present paper we continue to study the properties of metamaterials with application to mechanical, electromagnetic and acoustic problems possessing some specific periodic internal structure [1, 2]. The most part of the theoretical methods are based on numerical treatment such as Finite element method or Boundary element method. In recent years, the experimental base devoted to this topic is being actively developed. There are also some semi-analytical methods used for infinite or semi-infinite periodic structures, which are based on some asymptotic (low-frequency or high-frequency) estimates, being valid only in the far zone of the wave field [3–8]. In [5, 9–11], the analytical formulas for the coefficients of reflection and transmission in the low frequency range of the acoustic, electromagnetic and elastic waves, penetrating through the periodic system of holes of arbitrary shape and three-dimensional obstacles, are presented. Two-dimensional problems of wave propagation through a periodic screen lattice in elastic bodies with a single-periodic system of cracks are studied in [12, 13], and in [14–18] there is considered a doubly-periodic system. In [19–21] the problems of diffraction by a plane lattice of cylindrical cavities are solved. It should be noted that the wave properties of elastic media containing periodic structures of more complex physical nature — pores, inclusions, et al., have been analyzed in [22–25]. The problems discussed in this paper are related to the theory of the so-called “acoustic metamaterials,” which, due to their specific internal structure, have the properties of acoustic filters. This means that such a material is able to pass the passing wave over certain frequency intervals and lock the wave channel for other frequencies. The property was experimentally discovered and presented in [26]. Some fundamental aspects related to the acoustic metamaterials are discussed in [27–29] and some other publications. It should be noted
that a semi-analytical method in wave dynamics of periodic structures has also been proposed in [30].

In the present paper we consider an arbitrary finite number of vertical rows for both the types of defects, when the structure of the system has geometry either finite or infinite in the transversal periodic direction, with respect to the direction of the incident wave.

The main goal of the paper is to give a comparison between wave properties of infinite array of defects in one direction and analogous array of defects finite in both directions. In order to perform a detailed comparative analysis of the question, a numerical solution of both cases is made based on the semi-analytical method and Boundary Element method. The influence of the number of rows on the reflection and transmission coefficients is also extensively studied.

2. Thin defects problem

![Figure 1](image-url)

**Figure 1.** Incidence of a plane wave on a periodic array of linear obstacles

To study the filtration properties, let us consider the normal incidence of a plane longitudinal wave, propagating in an unbounded elastic medium \( p^{inc} = e^{ik_1x_1} \), on a doubly-periodic system of finite number \( M > 2 \) of identical vertical arrays, which are finite or infinite along \( x_2 \) and finite in the direction \( x_1 \). Each of them is an ordinary periodic system of coplanar linear cracks located at \( x_1 = 0, d, 2d, \ldots, (M - 1)d \). In the infinite case, under the natural symmetry, the problem is reduced to the consideration of a plane waveguide of the width \( 2a \), which includes \( M \) cracks (figure 1). For the finite case, for \( M \) vertical arrays with \( N \) cracks in each vertical one, it is necessary to solve the corresponding boundary integral equation over all available contours of the crack system. It is assumed that with the normal wave incidence \( e^{i(k_1x_1 - \omega t)} \) there is a regime of one-mode propagation with \( k_1 a < \pi \), where \( k_1 \) is the wave number of the longitudinal wave, \( \lambda = 2\pi/k_1 \) is the longitudinal wave length, \( 2a \) and \( d \) are the period of the system in the vertical and horizontal directions, respectively. The semi-analytical method is used when the distance between the adjacent parallel arrays \( d \) is such that the condition \( d/a \gg 1 \) is satisfied. The analysis of the properties of the scattering coefficients depending on the physical parameters for the three diffraction problems: a finite periodic system of thin defects in a scalar formulation, an infinite periodic system in a scalar formulation, an infinite periodic system in a plane problem of the elasticity theory, was carried out in [5, 7, 17, 33]. Let us for the sake of brevity cite here
only the main properties of numerical analysis of the solutions for the periodic systems by the
developed semi-analytical method [33].

For a numerical analysis of the problems considered, an example of the medium with the
longitudinal wave speed \( c = 6000 \text{ m/s} \) (steel) and the ratio of the longitudinal and the transverse
wave speeds \( c_1/c_2 = 1.87 \) is performed. To begin with, let us compare the amplitudes of reflection
and transmission coefficients versus frequency parameter between the three studied cases for a
single vertical array (figures 2 and 3). So, we assume, that the longitudinal wave speed in the
problem 2 is equal to the transverse wave speed of the problems 1 and 3.

\[ R \]

\[ 1 \]

\[ 0.5 \]

\[ 0 \]

\[ 0 \]

\[ 1 \]

\[ 1.5 \]

\[ ka \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

\[ 1 \]

\[ 2 \]

\[ 3 \]

Figure 2. Comparison of three different periodic models: one vertical row \((M = 1)\), period
of the lattice is \(2a = 0.02 \text{ m} \), size of each crack is \(2b = 0.015 \text{ m} \); line 1 — infinite array, elastic
theory; line 2 — infinite array, scalar theory; line 3 — finite array with \(M_1 = 7\) vertical cracks, scalar theory

This condition shortens the one-mode frequency interval, whose limit from the right becomes
\(\pi/1.87 = 1.680\) (figures 2 and 3). In figures 2 and 3 the comparative numerical analysis of the
scalar problems 1 and 3 has been performed for the transverse incident wave. For all the cases
the filtration interval can be seen in the upper part of the one-mode frequency range. It is shown,
that lines 2 and 3 in figures 2 and 3, related to the scalar problems, are practically coinciding
that takes place even for \(N = 10\) cracks in each vertical array. It should also be noted, that line
1 related to the elastic problem, shows a significant domination of the filtration property, when
compared with both infinite and finite scalar problems. Let us also notice, that for two vertical
arrays in the elastic problem a perfect filtration takes place for \(ak_1 \geq 0.7\), but for one vertical
row this property is valid only for \(ak_1 \geq 1.5\); this also confirms the evident property, that with
the growth of the vertical rows the filtration becomes stronger.

Let us pass to the analysis of the grid size to the precision of the obtained results. It is stated,
that in the case of a single obstacle it is sufficient to take 10 grid nodes per each wavelength,
to provide reliable results. Thus, for the frequency 0.16 MHz in this formulation the wavelength
is 0.0375 m, hence on the obstacle of the length 0.015 m it is sufficient to take only 5 nodes.
However, the complex geometry of the diffraction lattice requires greater number of nodes. It
can be seen from figure 4, which represents the results for the array of 10 vertical rows \((M = 10)\),
Figure 3. Comparison of three different periodic models: two vertical rows ($M = 2$), period of the lattice is $2a = 0.02\text{ m}$, size of each crack is $2b = 0.015\text{ m}$, distance between the rows is $d = 0.02\text{ m}$; line 1 — infinite array, elastic theory; line 2 — infinite array, scalar theory; line 3 — finite array with $M_1 = 7$ vertical cracks, scalar theory.

Figure 4. Comparison of two scalar models: ten vertical rows ($M = 10$), period of the lattice is $2a = 0.02\text{ m}$, size of each crack is $2b = 0.015\text{ m}$, distance between the rows is $d = 0.02\text{ m}$; problem 1 — infinite array; problem 3 — finite array with $M_1 = 10$ vertical cracks; $n$ is the number of the numerical grid nodes on each crack.
Figure 5. Comparison of two scalar models: five vertical rows \((M = 5)\), period of the lattice is \(2a = 0.02\) m, size of each crack is \(2b = 0.018\) m, distance between the rows is \(d = 0.02\) m; problem 1 — infinite array; problem 3 — finite array with various number of vertical cracks \(M\) each containing 10 obstacles \((M_1 = 10)\), that with 10 nodes \((n = 10)\) over each obstacle the calculations are correct only in the low-frequency case \((ka < 1)\). The analysis shows, that for 5 vertical arrays \((M = 5)\) with the obstacles of the length 1.8 cm, 10 obstacles in each vertical row \((M_1 = 10)\) are sufficient to get finite case quite close to the infinite one (see figure 5). It is stated that with the growing number of obstacles in a single vertical row, so with the growth of parameter \(M_1\), keeping all other parameters unchanged, the interval of the frequency cutoff varies insignificantly. The reflection inside this frequency interval is almost constant, being equal to unit value. This property takes place also for infinite arrays, where \(M\) is the number of such arrays.

3. Volumetric defects problem
Here the normal incidence of the same plane wave \(p^{inc} = e^{i k x_1}\) in an unbounded acoustic medium on a doubly periodic array of cylindrical obstacles, which is strictly finite in \(x_1\) direction, being either infinite or finite in \(x_2\) direction, is considered. Here \(k = \omega/c\) is the wave number, \(c\) is the wave speed.

According to [3, 21], in the case of an infinite array of holes along the \(x_2\), the reduction of the problem to a solution in a layer is possible only for a boundary contour symmetric with respect to \(x_1\) (the cross-section of each cylindrical hole must have a symmetric form with respect to \(x_1\)). Note that the cross-sections of different holes inside the selected horizontal layer may differ from each other. Let us designate by \(M\) the number of vertical rows of holes in the array and by \(N\) the number of holes in one row, i.e. in the direction \(x_2\). Then, the case of an infinite array along \(x_2\) is described as \(N = \infty\). In the case of a finite \(N\) the problem is solved on all holes in the array, which is expressed through the following expression for the boundary contour \(l = \sum_{j=1}^{N M} l_j\), where \(l_j\) is the boundary contour of \(j\)th hole. Thus, for \(N = \infty\), by the evident symmetry the problem reduces to a solution in a layer and only the holes located in this layer.
(0 < x_2 < d) will be considered and therefore \( l = \sum_{j=1}^{M} l_j \), where in figure 6 \( M = 4 \).

\[
\frac{1}{2} \varphi(x) - \int_{l} \varphi(y) \frac{\partial \Phi(y, x)}{\partial n_y} dy = e^{ikx_1}, \quad x \in l, \tag{1}
\]

where \( \varphi(x) \) is a full wave field at point \( x = (x_1, x_2) \), \( n \) is a unit normal vector to the boundary contour, \( l = \sum_{j=1}^{M} l_j \) is the sum of the boundaries of each hole, \( k \) is the wave number.

The Green function \( \Phi \) is presented by the exact mode expansion [6]:

\[
\Phi(y, x) = \frac{i e^{ik|y_1-x_1|}}{2kd} + \sum_{n=1}^{+\infty} e^{-\beta_n|y_1-x_1|} \frac{\cos[(2\pi n/d)(y_2-x_2)]}{\beta_n d}, \tag{2}
\]

where the basic assumption \( k < 2\pi/d \) (i.e. \( \lambda = 2\pi/k > d \)) is accepted, providing that the square-root \( \beta_n = \sqrt{(2\pi n/d)^2 - k^2} \) is always positive that physically means the one-mode wave process.

In order to compare this method with a direct numerical approach in the case of periodic arrays with finite number of holes along the vertical direction, the same integral equation (1) with some modification was solved numerically. It is well-known that in this case the kernel is simply represented by the Hankel function [36]:

\[
\frac{\partial \Phi(y, x)}{\partial n_y} = -\frac{i k}{4} H_1^{(1)}(k|y-y|) \frac{(r, n_y)}{r} \tag{3}
\]

and the total boundary line is represented as the sum \( l = \sum_{j=1}^{N} l_j \), where \( N \) is a number of the horizontal rows.

**Figure 6.** Propagation of the plane wave through the medium with a doubly-periodic array of symmetric holes
The formulas obtained in the previous section make it possible to construct a numerical solution of the described problem. So, we use the Boundary Element Method using the collocation technique. According to the approach, the contour of each hole is divided into a finite number of \( n \) small sub-contours. It allows us to replace the integral in Eq. (3) by a sum of \( n \) integrals over the introduced sub-contours. For a doubly periodic array of holes, the integral is replaced by a double sum, where the first summation is applied along the contours of all the holes

\[
\int_{l} p(y) \frac{\partial \Phi(y; x)}{\partial n_y} \, dy = \sum_{j=1}^{NM} \sum_{m=1}^{n} \int_{l_{jm}} p(y) \frac{\partial \Phi(y; x)}{\partial n_y} \, dl_{jm}
\]

and in the case of an infinite along the \( x_2 \)-axis array the value \( N = 1 \), since the problem is reduced to a solution in one layer.

Next, the nodes of the computational grid are arranged at the center of each small contour \( l_{jm} \). Following the collocation method, both the “external” \( x \)- and the “internal” \( y \)-points in Eq. (13) will run through the same set of nodes. Assuming that the introduced contours are small enough, the integrand can be replaced by its value at the node, corresponding to the current sub-contour. Since the integral of a constant value is known, this leads to the following discrete form of the basic integral equation of the problem

\[
\frac{1}{2} p(x_{uw}) - \sum_{j=1}^{NM} \sum_{m=1}^{n} p(y^{jm}) \frac{\partial \Phi(y^{jm}; x_{uw})}{\partial n^{jm}} \Delta l_{jm} = e^{ikx_1}, \quad x_{uw} \in l_{uw},
\]

Note, that when the points of both variables are at same computational nodes \( x_{jm} = y_{jm} \), a singularity arises in the kernel of the integral equation. This feature can be calculated accurately by taking the integral over a small-contour. It is known, that the contribution of this term is small in comparison with the contribution \( 1/2 \) from the first term of the BIE (14), so it can be neglected without affecting the numerical precision. The resulting system of linear algebraic equations (14) can be solved by any suitable numerical method, for example, the classical Gauss-elimination technique.

**Figure 7.** The reflection coefficients for infinite (line 1) and finite (\( N = 7 \) — line 2) structures: \( M = 4 \)

The analysis of the properties of the scattering coefficients for diffraction problems with volumetric defects, for finite and infinite periodic systems in the scalar formulation, was carried
out in [11, 21, 34, 35, 37, 38]. Let us also cite only the main properties of numerical analysis of the solutions [34]. At least 20–30 nodes should be chosen per each wavelength, which taking into account very regular shape of the obstacles can guarantee sufficient precision. Since some methods were used with the discretization [32, 36] and since the maximal size of each obstacle in our examples is equal to the wavelength, we chose 64 grid nodes per each cylindrical obstacle, to provide three right significant digits. The precision was checked by calculation with 128 nodes. Only round shape of the obstacles, with diameter 0.015 m was considered for the numerical experiments. The distance between two neighbor obstacles is taken the same in the both directions, and it is equal to the thickness of the strip \(d = 0.02\) m, which simultaneously is the period of the grating. The steel material with the longitudinal wave velocity \(c = 6000\) m/s is used as the modeled medium. It is easily seen that the one-mode range \(kd < 2\pi\) makes it possible to compare the infinite and finite structures in the LF one-mode ultrasonic regime up to the frequency \(f = 300\) KHz. Note, that quantities \(R\) and \(T\) can be defined only in the infinite case. For a finite number of obstacles, as an analogue of the reflection coefficient \(R\), the average value of the reflected wave field calculated on the vertical interval just before the first vertical row to the left, i.e. a certain small \(x_1 < 0\), is used. The size of the interval along axis \(x_2\) is equal to \(Md\). In the same way, the average value of the transmission coefficient \(T\) is calculated for the total wave field just immediately after the last right obstacle.

Figure 8. The transmission coefficients for different number of vertical columns \(M\): the infinite array (the left figure); the finite array with \(N = 31\) (the right figure)

An example of such a comparison between the arrays with four vertical rows \((M = 4)\) is presented in figure 7. It is evident, that even with a small number of obstacles in the vertical direction \((N = 7)\), both the methods are close to each other. For all the cases the filtering takes place over the frequency range \(1.0 \leq kd/2 \leq 2.0\), which is however more pronounced in the case of infinite array. The influence of the number of vertical rows \(M\) on the above-mentioned filtration property is displayed in figure 8. The analysis of the curves of the left figure shows, that for the infinite array the increase in the number of rows \(M\) leads to the clear filtration properties of the periodic structure. Qualitatively the finite structure possesses the same properties, however in the latter case this behavior is less pronounced and has a smoother character, see the curves of the right figure. For both the geometries one can see the two opposite types of filtration, with a rapid passage from the cut off frequency interval \(1.0 \leq kd/2 \leq 2.0\) to the almost full transmission frequency interval \(2.0 \leq kd/2 \leq 2.5\) in both the cases.

4. Conclusions
1) Virtually, any frequency interval with the wave channel locking can be created by controlling the relative crack size or diameter of the round obstacle, the number of vertical arrays and the lattice period in the horizontal direction.
2) Numerical analysis of the proposed approach for the doubly-periodic array of thin and volumetric obstacles infinite in one direction can precisely predict the scattering parameters property of the array finite in the both directions. Thus, increasing the number of the cracks and the holes in the array in the direction, where the number of holes is infinite, increases the accuracy of the prediction of the method.

3) The figures illustrate the difference in the properties of the scattering coefficients between thin and volumetric defects. Doubly periodic array of round holes works well as an acoustic filter in the central part of the single-mode frequency range, while the doubly periodic structure of thin screens is more efficient in this sense, for higher frequencies of the considered frequency interval.

4) If the number of defects in vertical row increases, the locking interval changes slightly. Even 7–10 obstacles sufficiently well approximate the case of the infinite system. The increasing number of vertical rows in the doubly-periodic array emphasize the filtering property of the considered structure.

5) The enhancement of the filtering property occurs when considering infinite doubly-periodic systems of thin defects in the context of the elastic model in comparison with the infinite and finite scalar analogue.

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