Finite element analysis of an actively controlled heavy rotor using a PVDF piezoelectric layer as a sensor and actuator

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Abstract
Reduction in the torsional vibration of heavy rotors (eg, turbo-generator rotors) is important for the safe and efficient functioning of the power plant. In this paper, a theoretical study is performed to control the torsional vibration of a turbo-generator rotor using a piezoelectric material, namely a polyvinylidene fluoride (PVDF) layer, as a sensor and actuator. Proportional and velocity feedback is used as a control law. The variation in the electromagnetic torque of the synchronous generator during various electrical faults is evaluated using a dq0 model. The finite element (FE) method is used to model the rotor elements. The coupled equations are solved in MATLAB using the Newmark-beta integration method. The coupling elements of the turbine and generator are most susceptible to shear failure. Hence, the torsional vibration of the coupled rotor on coupling elements is compared for the controlled and uncontrolled scenarios. The simulation results show that, for an actively controlled rotor, a significant reduction in the amplitude of the torsional vibrations is observed.

KEYWORDS
active control, piezoelectric material, rotor vibrations, sensor and actuator, torsional vibrations

1 | INTRODUCTION

Large rotors such as coupled turbo-generator rotors are susceptible to the torsional vibrations due to variation in electromagnetic torque of the synchronous generator. The variation in the electromagnetic torque of generator exists due to various electrical faults such as line to ground fault, line to line fault, three phase fault and mal-synchronization fault.1-3 Due to variation in the electromagnetic torque the torsional vibrations originate in the coupled rotor of the power plant that can result in complete shutdown of the power plant or sometimes these vibrations in the rotor become fatal also.4,5 So, the control of torsional vibrations of heavy rotor up to certain limit is required for continuous and safe operation of the power plant.6,7 The schematic diagram of power distribution system is shown in Figure 1.

To apply input torque to analyze vibrations of the coupled turbo-generator rotor, different researcher calculates the electromagnetic torque in generator using analytical methods.8-11 Reference 12 described Increment Transfer Matrix Method (ITM) to calculate the response of the torsional vibrations in generator torque. Reference 13 described a comprehensive process model for simulation of short circuit faults of initially unloaded synchronous
generator, using the generalized dq0 mathematical model. Most of the previous studies are performed using analytical relations and electromagnetic torque is simulated for unloaded state of synchronous generator. Further, various researchers use lumped mass model to analyze the torsional vibration in big turbo-generator rotor under unloaded state of the generator. The variation in the electromagnetic torque of generator cause torsional vibrations in turbo-generator rotor. Therefore, it is essential to measure these torsional vibrations so that further effective control mechanisms can be developed.

In the beginning, passive vibration control of engineering structures was the only option available. In passive vibration control tuned vibrations absorbers (TVAs) and elastic support isolators (ESIs) are used to mitigate the structural vibrations. With the development of different types of new materials such as self-sensing and self-actuation materials (eg, piezoelectric materials) a new term smart material was first used in 1980s. Subsequently with the advances in technology a new field of active vibration control emerged in 1989 and References further comprehensively described the methods of active vibration control of structure.

Nowadays, extensive research is going on active vibration control of structures. The area of active vibration control of structures is gigantic and divided into various subfields. Various researchers suggest different schemes and studies to control the torsional vibrations of the turbo-generator rotor. The majority of research is performed to control the transverse vibrations on the heavy rotors. Very limited literature is present to control the torsional vibrations of heavy rotors like turbo-generator rotor. Reference 27 analyze shear mode magnetorheological (MR) fluid damper for rotor vibration control. Reference 28 develop a new electrorheological dynamic torsional absorber, called the Smart Electrorheological Dynamic Absorber (SERDA) to be used in reducing torsional rotor vibrations. Reference 29 studied the application of Electrorheological Fluids (ERF) to the reduction of torsional vibrations of a rotor by controlling damping and stiffness of a rotational dynamic absorber.

Similarly, method of the semi-active control of torsional vibrations in a working machine drive system by means of rotary actuators with a magneto-rheological fluid is presented by Reference 30. Reference 31 investigates the reduction of rotor shaft torsional vibrations through active control of the generator torque. Reference 32 presented the theoretical study to actively control the torsional vibrations of turbo-generator rotor due to electromagnetic disturbances using lumped mass model. The various previous studies associated with the turbo-generator rotor are only restricted to the measurement of torsional vibrations on the different sections of the rotor. Further, limited literature is present to control the torsional vibrations produced in the turbo-generator rotor during any particular electrical disturbance. The absence of comprehensive study on effect of various electrical faults on the turbo-generator rotor makes it difficult to design an effective control mechanism. Nowadays power plants use heavy mechanical and electrical components to control the torsional vibration produced during the electrical faults. But with the compact size and minimum moving parts, the usability of the piezoelectrical material as sensor and actuator in control system designing cannot be neglected.

Therefore, a complete study of big turbo-generator rotor under line to ground, line to line, three phase and mal-synchronization faults is performed using finite element method. The dq0 model is used to analyze the torque due to various electrical faults in the loaded generator. The rotor is modeled using solid, hollow and tapered cylindrical elements. The piezoelectric material layer is used as sensor and actuator with proportional control and velocity feedback. The coupled equations are solved numerically in MATLAB using Newmark-beta integration scheme to theoretically analyze the control system for turbo-generator rotor.
2 | MATHEMATICAL MODELING

2.1 | Torque modeling of synchronous generator

The electromagnetic torque in synchronous generator during various electrical faults is determined using dq0 model. The variation in the electromagnetic torque of synchronous generator connected to the infinite load generally arises due to fault on line. The dq0 representation of synchronous generator is shown in Figure 2. The generator torque for line to ground and line to line fault is calculated using Equations (1), (3), (4) and (5). whereas for three phase fault and mal-synchronization fault torque is calculated using Equations (2) to (5). The electromagnetic torque in synchronous generator during various faults is calculated using Equation (5). The coupled equations are solved numerically using fourth order Adam’s predictor-corrector scheme with startup by fourth order Runge-Kutta method in MATLAB. The detailed method to determine the electromagnetic torque is given in References 13 and 33.

\[
L \cdot \frac{d}{dt} \begin{bmatrix} i_{phase} \\ i_{d} \\ i_{q} \\ i_{D} \\ i_{Q} \end{bmatrix} + X \begin{bmatrix} i_{phase} \\ i_{d} \\ i_{q} \\ i_{D} \\ i_{Q} \end{bmatrix} = \begin{bmatrix} u_{fd} \\ 0 \\ 0 \\ 0 \end{bmatrix},
\]

(1)

\[
L \cdot \frac{d}{dt} \begin{bmatrix} i_{phase} \\ i_{d} \\ i_{q} \\ i_{D} \\ i_{Q} \end{bmatrix} + X \begin{bmatrix} i_{phase} \\ i_{d} \\ i_{q} \\ i_{D} \\ i_{Q} \end{bmatrix} = \begin{bmatrix} u_{d} \\ u_{q} \\ 0 \\ 0 \end{bmatrix},
\]

(2)

\[
\frac{d\omega}{dt} = -\frac{P}{J} T_{em} + T_{drive},
\]

(3)

\[
\frac{dy}{dt} = -\omega,
\]

(4)

\[
T_{em} = \frac{3}{2} P (-L_{md}i_{md}i_{q} + L_{mq}i_{dq}),
\]

(5)

where,

**FIGURE 2** Vector representation of synchronous generator on a-b-c and d-q axis
For line to ground fault.

\[
L = \begin{bmatrix}
0 & L_{fd} & -L_{D} & 0 \\
L_{md} & L_{D} & 0 & 0 \\
c \cdot L_{md} \cos \gamma & 0 & 0 & L_{Q} \\
\frac{L_{md} + L_{q}}{3} & c \cdot L_{md} \cos \gamma & c \cdot L_{md} \sin \gamma & L_{md} \cos \gamma \quad L_{md} \cos \gamma & L_{mq} \sin \gamma
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
0 & R_{fd} & -R_{D} & 0 \\
L_{md} & L_{D} & 0 & 0 \\
\frac{L_{md} + L_{q}}{3} & -c \cdot L_{md} \cos \gamma & -c \cdot L_{md} \sin \gamma & -c \cdot L_{md} \cos \gamma \\
0 & 0 & 0 & L_{Q} \\
\omega \cdot L_{md} \cos \gamma & \omega \cdot L_{md} \sin \gamma & -\omega \cdot L_{mq} \cos \gamma & -\omega \cdot L_{mq} \sin \gamma
\end{bmatrix}
\]

\[G = c \cdot \omega [(L_{d} - L_{q}) \cos \gamma \sin \gamma + (L_{d} - L_{q}) \sin \gamma \cos \gamma] + R + R_{e}\]

\[c = \frac{2}{3}\]

For line to line fault

\[
L = \begin{bmatrix}
0 & L_{fd} & -L_{D} & 0 \\
L_{md} & L_{D} & 0 & 0 \\
c \cdot L_{md} \sin \gamma & 0 & 0 & L_{Q} \\
\frac{L_{md} + L_{q}}{3} & c \cdot L_{md} \cos \gamma & c \cdot L_{md} \sin \gamma + 2L_{e} & L_{md} \sin \gamma & L_{md} \sin \gamma & -L_{mq} \cos \gamma
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
0 & R_{fd} & -R_{D} & 0 \\
L_{md} & L_{D} & 0 & 0 \\
\frac{L_{md} + L_{q}}{3} & -c \cdot L_{md} \cos \gamma & -c \cdot L_{md} \sin \gamma & -c \cdot L_{md} \cos \gamma \\
0 & 0 & 0 & L_{Q} \\
\omega \cdot L_{md} \cos \gamma & \omega \cdot L_{md} \sin \gamma & -\omega \cdot L_{mq} \cos \gamma & -\omega \cdot L_{mq} \sin \gamma
\end{bmatrix}
\]

\[G = c \cdot \omega [(-L_{d} + L_{q}) \cos \gamma \sin \gamma + (-L_{d} + L_{q}) \sin \gamma \cos \gamma] + c \cdot R + 2R_{e}\]

\[c = -\frac{2}{\sqrt{3}}\]

For three phase and mal-synchronization fault

\[
L = \begin{bmatrix}
L_{d} + L_{e} & 0 & L_{md} & L_{md} & 0 \\
0 & L_{q} + L_{e} & 0 & 0 & L_{mq} \\
L_{md} & 0 & L_{fd} & 0 & L_{md} \\
L_{md} & 0 & L_{md} & L_{D} & 0 \\
0 & L_{md} & 0 & 0 & L_{Q}
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
R + R_{e} & -\omega \cdot L_{q} & 0 & 0 & -\omega \cdot L_{mq} \\
\omega \cdot L_{d} & R + R_{e} & \omega \cdot L_{md} & \omega \cdot L_{md} & 0 \\
0 & 0 & R_{fd} & 0 & 0 \\
0 & 0 & 0 & R_{D} & 0 \\
0 & 0 & 0 & 0 & R_{Q}
\end{bmatrix}
\]

where,
$u_a$, $u_b$, $u_c$ are a-b-c phase voltages, $i_{\text{phase}}$ is phase current, $i_{fd}$ is field winding current, $i_D$ and $i_Q$ are damper winding currents, $u_d$ is d-axis voltage, $u_q$ is q-axis voltage, $u_{fd}$ is field winding voltage, $T_{\text{em}}$ is electromagnetic torque of generator, $T_{\text{drive}}$ is torque of turbines, $P$ is number of pairs of magnetic poles of rotor, $\omega$ is angular velocity, $\gamma$ is rotor angle, $L_{md}$ is mutual inductance of d-axis, $L_{mq}$ is mutual inductance of q-axis, $L_0$ is 0-axis inductance, $L_d$ is d-axis inductance, $L_q$ is q-axis inductance, $L_D$ and $L_Q$ are damper winding inductances, $L_{fd}$ is field winding leakage inductances, $LD_\sigma$ and $LQ_\sigma$ are damper winding leakage inductances, $R$ is stator resistance, $RD$ and $RQ$ are damper winding resistances, $Le$ is line inductance and $Re$ is line resistance.

2.2 | Finite element modeling of rotor

Heavy turbo-generator rotor is modeled using finite element method (FEM). Three types of elements viz solid, hollow and tapered elements are used to analyze the coupled rotor using finite element method. Each element holds two degrees of freedom that is, one rotational degree of freedom at each node. The various components of the turbo-generator rotor (High pressure, intermediate pressure, low pressure turbines [HP, IP, LP], generator and exciter) are shown in Figure 3 along with the node distribution along the length of the rotor. Since torsional vibrations are independent from translational and axial vibrations so other DOFs are neglected to keep the formulation simple. The effect of blade mass on the inertia and stiffness is taken into consideration by assuming equivalent increase in radius of the rotor. The effect of blade vibrations due to high speed rotation and steam velocity is neglected due to very high inertia of the rotor. The effect of high temperatures due to superheated steam on rotor elements are taken in to consideration by calculating the modified shear modulus as given in Equation (7). Hamilton’s principle is used to formulate the global finite element equations. The elemental finite element equation for a particular element is given by Equation (6). The elemental equations are combined to form global finite element equations for rotor dynamics as shown in Equation (8).

$$
[I]^{(e)} \{\dot{\theta}\}^{(e)} + [K]^{(e)} \{\theta\}^{(e)} = [T_p] + [T_{\text{unif}}].
$$

where,

$$
[K]^{(e)} = \int_0^1 G^{(e)} f^{(e)} \begin{bmatrix} \left(\frac{dN_1}{dx}\right)^2 & \frac{dN_1}{dx} \frac{dN_2}{dx} \\ \frac{dN_1}{dx} \frac{dN_2}{dx} & \left(\frac{dN_2}{dx}\right)^2 \end{bmatrix} dx
$$

$$
[I]^{(e)} = \int_0^1 \rho^{(e)} f^{(e)} \begin{bmatrix} N_1^2 & N_1N_2 \\ N_1N_2 & N_2^2 \end{bmatrix} dx
$$

**Figure 3** Various components and node distribution along the length of the turbo-generator rotor with PVDF actuator and sensor under PD control
\( \theta \) is angular displacement of the node, \( \rho \) is density, \( J \) is polar moment of inertia, \( l \) is length, \( G \) is shear modulus, \( T_p \) is point torque and \( T_{\text{unif}} \) is uniform torque on the element. \( N_1 \) and \( N_2 \) are shape functions. Linear shape function with single degree of freedom is considered to analyze the vibrations at nodes because torsional vibrations of the rotor are independent of the lateral and transverse vibrations.

\[
N_1 = 1 - \frac{x}{l} \quad \text{and} \quad N_2 = \frac{x}{l}.
\]

Most of the elements of turbo-generator rotor perceive high temperatures. The effect of temperature on the stiffness of rotor steel is taken into consideration using modified shear modulus as given in Equation (7).

\[
E' = E' \left(1 - 0.193562388 \times T + 837.81238 \times T^2 + 2119625.94 \right) \times 98066.5
\]

\[
G = \frac{E' \times (1 + \nu)}{2},
\]

where \( E' \) is elastic modulus of element in Pa and \( T \) is temperature of the section in \( ^\circ \text{C} \).

On combining the elemental matrices, we get global finite element formulation.

\[
[I]_{(g)} \{\ddot{\theta}\}_{(g)} + [K]_{(g)} \{\dot{\theta}\}_{(g)} = [T_p]_{(g)} + [T_{\text{unif}}]_{(g)}
\]

Appropriate damping matrix can be introduced to get final global finite element formulation.

\[
[I]_{(g)} \{\ddot{\theta}\}_{(g)} + [C]_{(g)} \{\dot{\theta}\}_{(g)} + [K]_{(g)} \{\theta\}_{(g)} = [T_p]_{(g)} + [T_{\text{unif}}]_{(g)}.
\]

where,

\[
[I]_{(g)} = \text{Global moment of inertia matrix}
[C]_{(g)} = \text{Global damping matrix}
[K]_{(g)} = \text{Global stiffness matrix}
[T_p]_{(g)} = \text{Global point torque matrix}
[T_{\text{unif}}]_{(g)} = \text{Global distributed torque matrix}
\]

### 2.3 Active control of turbo-generator rotor

The coupled turbo-generator rotor dynamics is solved using Newmark-beta method. The electromagnetic torque calculated in Equation (5) is applied as uniform torque on generator elements after multiplying it with the base torque.\(^\text{13}\) The rotor at the start of the fault is assumed to be rotating at 3000 RPM. The initial torsional displacement of rotor is assumed to be zero at the time of start of electrical fault. Due to various electrical faults, the electromagnetic torque change in synchronous generator that produces torsional vibrations in the coupled turbo-generator rotor. To control the vibrations in the rotor an equivalent counter torque is applied using piezoelectric material. The piezoelectric material is used as sensor and actuator to actively control the rotor vibrations. The sensor material is placed at specific elements of the rotor. The conductive continuous electrodes are considered to have negligible thickness. However, the PVDF layer is assumed to be bounded perfectly with the rotor using a high strength epoxy to sustain the centrifugal force due to the rotational speed of the heavy turbo-generator rotor which can otherwise limit its applicability and effectiveness in the practical power plant applications. The shear mode voltage generated in the piezoelectric material due to rotor vibrations is calculated using Equation (9).
\[ U_s = \frac{G_{pe} d_{15} R_r}{\varepsilon_0 \varepsilon_{pe}} [\theta_{pe}(x_{s_2}, t) - \theta_{pe}(x_{s_1}, t)]. \]  

(9)

Closed loop Feed-back control law is used to give feedback voltage to the piezoelectric actuator as shown in Figure 3. Proportional control plus velocity feedback is used to keep the system simple and stable at high speed. The PD control law is given by Equation (10). Torque generated by the actuator in closed loop plant is given by Equation (11).

\[ U_A = k_p U_s + k_d \frac{dU_s}{dt}. \]  

(10)

\[ T_{act} = \frac{4d_{15}}{3h_{pe}} U_A [(R_r + t_s + h_{pe})^3 - (R_r + t_s)^3] G_{pe}. \]  

(11)

where,

\( \theta_{pe} \) is angular displacement of the piezoelectric layer at position \( x_{s_2} \) and \( x_{s_1} \) along length of the rotor and at time \( t \), \( U_s \) is sensor voltage, \( U_A \) is actuator voltage, \( G_{pe} \) is shear modulus of piezoelectric material, \( d_{15} \) is piezoelectric charge constant, \( R_r \) is radius of rotor element, \( \varepsilon_{pe} \) is relative permittivity of piezoelectric material, \( \varepsilon_0 \) is permittivity of free space, \( t_s \) is glue layer thickness, \( h_{pe} \) is width of actuator, \( k_p \) is proportional gain and \( k_d \) is velocity gain.

The steps for the algorithm of Newmark-beta method are shown below. The coupled system is numerically solved in MATLAB.

Steps for solving forced vibration equation \( M \ddot{u} + C \dot{u} + Ku = F \) using Newmark-beta method are as given below.

1. Input mass, damping and stiffness matrices
2. Initialize the state of system \((u_0, \dot{u}_0, \ddot{u}_0)\) and choose \( \alpha, \beta \) parameters and the time step.
3. Precompute the \( \tilde{K} \) matrix. \( \tilde{K} = \{M + \alpha h C + \beta h^2 K\} \)
4. For each time step \( h \):

   (a) Compute \( \Delta \tilde{F}_i = \Delta F_i + \left[ \frac{1}{\beta h^2} M + \frac{2}{\beta} C \right] \dot{u}_i + \left[ \frac{1}{2 \beta} M + h \left( \frac{a}{2 \beta} - 1 \right) C \right] \ddot{u}_i. \)

   (b) Solve for \( \Delta u_i \) in \( \tilde{K} \Delta u_i = \Delta \tilde{F}_i. \)

   (c) Then, \( u_{i+1} = u_i + \Delta u_i. \)

   (d) Compute the new velocity and acceleration vectors with equations

   (i) \( \Delta \dot{u}_i = \frac{a}{3 h} \Delta u_i - \frac{2}{\beta} \dot{u}_i + h \left( 1 - \frac{a}{2 \beta} \right) \ddot{u}_i \)

   (ii) \( \Delta \ddot{u}_i = \frac{1}{3 h^2} \Delta u_i - \frac{a}{\beta h} \dot{u}_i - \frac{1}{2 \beta} \ddot{u}_i \)

3 | VALIDATION

The dynamic model for synchronous generator as described in Section 2.1 is validated by the results in Reference 13. The values of inductance \( L_e \) and resistance \( R_e \) are assumed to be zero for validation purpose only. Since the dynamic model is same for line to ground and line to line short circuit fault, the results are validated for generator torque during line to ground short circuit for the most unfavorable conditions given in Reference 13 as shown in Figure 4.

Similarly, the dynamic model is the same for three phase short circuit and phase synchronization fault. So, results are validated for generator torque during three phase short circuit for most unfavorable condition as given in Reference 13 and shown in Figure 5.

To validate the code in all circumstances, the numerical results obtained from MATLAB are validated with peak value of generator torque during line to ground short circuit over a range of rotor lag angle as given in Reference 13 and shown in Figure 6. From Figures 4 to 6, it is clear that numerical results are matching well with the results of Reference 13.

Further the finite element model for the rotor is validated with the results in Reference 34. The first three natural frequencies are calculated to validate the global mass and stiffness matrices. The first three natural frequencies are compared...
with the\textsuperscript{34} and convergence of the code is established in Table 1. The first three natural frequencies of the tube as given in Reference 34 show close resemblance to the simulated results.

4 | RESULTS AND DISCUSSION

A coupled turbo-generator rotor with 187 elements are chosen for the vibration analysis. The synchronous generator rated 588 MVA power is connected to rotors of Low pressure, intermediate pressure and high-pressure turbines through couplings. The moment of inertia of the generator is 10 244 kg.m\textsuperscript{2}. The diameter plot along length of the big turbo-generator rotor is shown in Figure 7. The synchronous generator is connected to infinite load with inductance $L_e$ and resistance $R_e$ due to grid and distribution line. The power is given to the generator using steam turbines. Energy contained in the steam is extracted by allowing the steam to expand and cool in the blades of the turbine. This steam energy rotates the synchronous generator at 3000RPM to produce electricity at 50 Hz.
### Table 1
Comparison of first three torsional natural frequencies from MATLAB and reference [34]

| S.No | No of Elements considered in MATLAB | First 3 Natural Freqs. From MATLAB ($\times 10^3$Hz) $f_i$ $i \rightarrow 1,2,3$ | First 3 Natural Freqs. of Ref. [34] ($\times 10^3$Hz) $f_i$ $i \rightarrow 1,2,3$ | % Error $\left(\frac{f_i - f_i^{\text{Ref}}}{f_i^{\text{Ref}}}\right) \times 100$ | Avg. % Error $\frac{1}{3}\left(\sum_{i=1}^{3} \left(\frac{f_i - f_i^{\text{Ref}}}{f_i^{\text{Ref}}}\right)\right) \times 100$ |
|------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1    | 5                               | 1.682                           | 1.534                           | 9.647                           | 174.46                          |
|      |                                 | 7.737                           | 4.911                           | 57.544                          |                                 |
|      |                                 | 46.247                          | 8.315                           | 456.187                         |                                 |
| 2    | 6                               | 1.576                           | 1.534                           | 2.737                           | 14.294                          |
|      |                                 | 5.553                           | 4.911                           | 13.072                          |                                 |
|      |                                 | 10.566                          | 8.315                           | 27.071                          |                                 |
| 3    | 7                               | 1.557                           | 1.534                           | 1.499                           | 6.047                           |
|      |                                 | 5.045                           | 4.911                           | 2.728                           |                                 |
|      |                                 | 9.472                           | 8.315                           | 13.91                           |                                 |
| 4    | 8                               | 1.551                           | 1.534                           | 1.108                           | 2.610                           |
|      |                                 | 4.899                           | 4.911                           | 1.058                           |                                 |
|      |                                 | 8.786                           | 8.315                           | 5.664                           |                                 |
| 5    | 9                               | 1.547                           | 1.534                           | 0.847                           | 1.556                           |
|      |                                 | 4.774                           | 4.911                           | 2.789                           |                                 |
|      |                                 | 8.401                           | 8.315                           | 1.034                           |                                 |
| 6    | 10                              | 1.547                           | 1.534                           | 0.847                           | 1.901                           |
|      |                                 | 4.773                           | 4.911                           | 2.810                           |                                 |
|      |                                 | 8.487                           | 8.315                           | 2.068                           |                                 |
| 7    | 12                              | 1.545                           | 1.534                           | 0.717                           | 0.698                           |
|      |                                 | 4.874                           | 4.911                           | 0.753                           |                                 |
|      |                                 | 8.367                           | 8.315                           | 0.625                           |                                 |
The generator is modelled using dq0 formulation. The complete system is connected with the infinite load through grid with resistance $R_e$ and inductance $L_e$. The variation in the generator torque appears due to various electrical faults. This variation in the electromagnetic torque is simulated using the Equations (1) to (5). The coupled equations are solved numerically using fourth order Adams predictor-corrector scheme with startup by fourth order Runge-Kutta method in MATLAB. The generator torque vibrations are plotted in Figure 8 for Line to ground, Line to Line, Three Phase and mal-synchronization faults. Per unit (pu) system is used to calculate the peak values of torque. The peak values of pu torque for line to ground, line to line, three phase and mal-synchronization faults are 12, 7.9, 17.1 and 8.1 pu respectively. The actual value of electromagnetic torque is calculated by multiplying per unit value by torque base $3PUnIn\omega_n^2$. The various parameters of generator are given in Table 2.

First five natural frequencies of the rotor are 0, 20.189, 55.098, 61.692, 105.596 Hz and corresponding mode shapes are plotted in Figure 9. The initial speed of rotor is assumed to be 3000 RPM and initial torsional displacement is assumed zero. From natural frequencies it is established that during electrical fault vibrations the rotor vibrate near third mode of the vibration.

Polyvinylidene Fluoride (PVDF) piezoelectric material is used as sensor and actuator on rotor due to its low brittleness in between $-30°C$ and $140°C$ temperatures. Shear mode operation ($d_{15}$ mode) of piezoelectric sensor and actuator is used to measure torsional displacement. The sensor and actuator can be placed on certain specific elements (near the
TABLE 2 Various parameters used in generator modeling

| Parameter                      | Symbol | Value       |
|--------------------------------|--------|-------------|
| Parameters related to generator|        |             |
| Stator winding parameters      |        |             |
| R                             |        | 0.00193 p.u.|
| L                             |        | 0.1256 p.u. |
| Rotor windings parameters     |        |             |
| \( R_{fd} \)                  |        | 0.0009 p.u. |
| \( R_D \)                     |        | 0.0093 p.u. |
| \( R_Q \)                     |        | 0.008 p.u.  |
| \( L_{fd} \)                  |        | 0.02721 p.u.|
| \( L_{De} \)                  |        | 0.33 p.u.   |
| \( L_{Qe} \)                  |        | 0.08 p.u.   |
| \( d-q \) axis magnetizing inductances | \( L_{md} \) | 1.9539 p.u. |
| \( L_{mq} \)                  |        | 1.8504 p.u. |
| Line resistance               | \( R_e \) | 0.1235 p.u. |
| Line inductance               | \( L_e \) | 6.773 p.u.  |
| Infinite Voltage              | \( U_{\infty} \) | 8.7639 p.u. |

FIGURE 9 First five mode (top to bottom) shapes of coupled turbo-generator rotor

couplings) of the rotor because most of the elements of rotor are concealed and bear high temperatures. The PVDF layer glued along the complete periphery of the element No 118 on the rotor as sensor and on element No 111 acts as actuator. The effect of the glue layer is neglected for simplification. The sensor is placed just near the generator so that it can sense the vibrations in shortest time span and the actuator is placed near the sensor element so that it can respond according to sensor output in minimum time. The hollow cylindrical shaped PVDF layer is used to maintain the balancing of the rotor. The various parameters for controlled system are given in Table 3.

The proportional and velocity feedback control (PD control law) is used to keep the formulation simple and to increase the stability. The variation in electromagnetic torque of synchronous generator is equally applied at elements 132 to 140. On applying proportional gain and velocity gain the voltage of the PVDF sensor layer is applied to the PVDF actuator. The maximum electric field applied to the PVDF actuator to control the torsional vibrations produced during the various electrical disturbances is in the range from 75 to 200 Vμm\(^{-1}\). The breakdown electric field for the actuator is 700 Vμm\(^{-1}\) and width of the actuator is 355 mm. One percent internal damping is assumed for the rotor. Multi degree of freedom (MDOF) finite element equations are solved using Newmark-beta integration method in MATLAB.
### TABLE 3 Various parameters for active vibration control of rotor

| Parameter                              | Symbol | Value            |
|----------------------------------------|--------|------------------|
| Piezoelectric coefficient             | \(d_{15}\) | \(-27\text{pCN}^{-1}\) |
| Shear Modulus of PVDF                 | \(G_{pe}\) | 3.5GPa          |
| Relative dielectric permittivity of PVDF | \(\epsilon_{pe}\) | 12           |
| Glue layer thickness                  | \(t_{s}\) | \(1 \times 10^{-6}\) m |
| Piezoelectric layer thickness         | \(h_{pe}\) | \(1 \times 10^{-2}\) m |
| Piezoelectric layer width             | \(l_{sen}\) | 235 mm          |
|                                       | \(l_{act}\) | 355 mm          |
| Amplification coefficients            | \(k_{p}\) | \(10^2\)        |
|                                       | \(k_{d}\) | \(10^{-3}\)     |

**FIGURE 10** A, Angular velocity of all nodes during line to ground fault. B, Angular velocity of all nodes during line to line fault. C, Angular velocity of all nodes during three phase fault. D) Angular velocity of all nodes during mal-synchronization fault

The variation of angular velocity of all the nodes of rotor in actively controlled system is shown in Figure 10. From Figure 10 it is clear that during the fault time the different sections of the rotor rotate with different angular velocity. Due to this, different sections of the rotor receive different peak value of stresses.

Power from the high-pressure turbine, intermediate pressure turbine and low-pressure turbine is transferred to the synchronous generator through couplings as shown in Figure 3. The rotor of the high pressure turbine is coupled with the rotor of the intermediate pressure turbine at coupling element 35, the rotor of the intermediate pressure turbine is coupled with the rotor of the low pressure turbine at coupling element 74 and the rotor of the low pressure turbine is coupled with the rotor of the synchronous generator at coupling element 110. To protect the expensive machinery from damage, the couplings between the different rotors are designed for a particular value of the stress amplitude. The torsional vibrations come on the turbo-generator rotor during the electrical disturbances produce local stress at the coupling sections that can exceeds the yield limit of the material. The coupling joint between various turbines and generator are most probable sections to be failed during the faults. So, under actively controlled system, the vibrations produced during various electrical faults in coupling elements 35, 74 and 110 are simulated numerically.
To establish the stability of the system, the relative angular displacement vs relative angular velocity plots of elements 35, 74 and 110 are shown in Figure 11. The phase plane analysis of elements 35, 74 and 110 are shown for controlled and uncontrolled system. It is clear from Figure 11 that nonlinear system of equations is stable during the simulation time and for four different electrical faults under consideration.

The torque in element 35 for controlled and uncontrolled system during line to ground, line to line, three phase and mal-synchronization fault are compared in Figure 12A-D. The peak torque for uncontrolled system during the line to ground, line to line, three phase and mal-synchronization fault is 0.684, 0.483, 0.981 and 0.516 MNm respectively. The peak torque for actively controlled system during the line to ground, line to line, three phase and mal-synchronization fault is 0.464, 0.314, 0.639 and 0.349 MNm respectively. It is clear from Figure 12 that the most severe fault is the three phase faults, followed by the line to ground fault, followed by the mal-synchronization fault, followed by the line to line fault. Further, in comparison to the uncontrolled system for actively controlled system the peak amplitude of the vibrations in element 35 is approximately reduced by 32%, 35%, 35% and 32% during line to ground, line to line, three phase and mal-synchronization faults respectively.

The torque in element 74 for controlled and uncontrolled system during line to ground, line to line, three phase and mal-synchronization fault are compared in Figure 13A-D. The peak torque for uncontrolled system during the line to ground, line to line, three phase and mal-synchronization fault is 2.171, 1.629, 3.273 and 1.787 MNm respectively. The peak torque for actively controlled system during the line to ground, line to line, three phase and mal-synchronization fault is 1.474, 1.093, 2.202 and 1.224 MNm respectively. It is clear from Figure 13 that the most severe fault is the three phase faults, followed by the line to ground fault, followed by the mal-synchronization fault, followed by the line to line fault.
**FIGURE 12** A, Torque on element no 35 during line to ground fault, B, Torque on element no 35 during line to line fault, C, Torque on element no 35 during three phase fault and D, Torque on element no 35 during mal-synchronization fault for uncontrolled and actively controlled system

**FIGURE 13** A, Torque on element no 74 during line to ground fault, B, Torque on element no 74 during line to line fault, C, Torque on element no 74 during three phase fault and D, Torque on element no 74 during mal-synchronization fault for uncontrolled and actively controlled system
line fault. Further, in comparison to the uncontrolled system for actively controlled system the peak amplitude of the vibrations in element 74 is approximately reduced by 32%, 33%, 33% and 31% during line to ground, line to line, three phase and mal-synchronization faults respectively.

The torque in element 110 for controlled and uncontrolled system during line to ground, line to line, three phase and mal-synchronization fault are compared in Figure 14A-D. The peak torque for uncontrolled system during the line to ground, line to line, three phase and mal-synchronization fault is 7.559, 4.941, 13.69 and 9.531 MNm respectively. The peak torque for actively controlled system during the line to ground, line to line, three phase and mal-synchronization fault is 5.112, 3.385, 9.315 and 6.468 MNm respectively. It is clear from Figure 14 that the most severe fault is the three phase faults, followed by the line to ground fault, followed by the mal-synchronization fault, followed by the line to line fault. The frequency of torque is maximum in case of mal-synchronization fault as compared to the three-phase fault. So, during mal-synchronization fault, rotor can lead to fatigue failure earlier. Further, in comparison to the uncontrolled system for actively controlled system the peak amplitude of the vibrations in element 110 is reduced by 32%, 31%, 32% and 32% during line to ground, line to line, three phase and mal-synchronization faults respectively.

The relative reduction in the peak torque in the closed loop system at coupling elements 35, 74 and 110 during various electrical faults is shown in Figure 15. From Figure 15 it is clear that vibrations are significantly reduced in the actively controlled heavy rotor using PD feedback control.
5 | CONCLUSIONS

This paper presents theoretical analysis of torsional vibration control of the heavy rotor using active control. The variation in the electromagnetic torque of loaded synchronous generator under four types of electric faults are analyzed. The evolution of electromagnetic torque with time is plotted and compared under the most unfavorable conditions. The most severe torque is evolved during three phase fault, followed by the line to ground, followed by mal-synchronization fault and line to line fault. The rotor is modeled using FEM and the coupled electromechanical dynamic system is numerically simulated using Newmark-beta integration method in MATLAB. The PVDF piezoelectric layer is used as a sensor and actuator. Proportional and velocity feedback control law is used. The stability of the simulated results is established using displacement vs velocity plot. The comparative torque on the couplings of the coupled rotor are plotted for the controlled and uncontrolled scenario. The peak amplitude of torsional vibrations in element No 35 is reduced by 32% to 35% for various electrical faults in controlled case. The peak amplitude of torsional vibrations in element No 74 is reduced by 31% to 33% for various electrical faults in controlled case. The peak amplitude of torsional vibrations in element No 110 is reduced by 31% to 32% for various electrical faults in controlled case. Therefore, this study concluded that control of the torsional vibrations of the turbo-generator rotor using piezoelectric material layer as sensor and actuator can effectively reduce the undesirable vibrations produced during the electrical disturbances and can be used to control torsional vibrations of the rotor in the power plant for compact system with minimum moving parts. The PD control law used to give feedback voltage to the actuator is computationally efficient and therefore is suitable for closed loop control of the turbo-generator rotor using PVDF transducers. In future study can be performed to experimentally established the applicability of the present study. Further, genetic algorithm can be used to determine the optimal location of the sensors and actuators. Also effect of the intelligent control like fuzzy logic on the torsional vibration control using piezoelectric material layer can be scope of the future study.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest.

AUTHOR CONTRIBUTIONS

Tarun Kumar: Conceptualization; data curation; formal analysis; funding acquisition; investigation; methodology; resources; supervision; validation; visualization; writing-original draft; writing-review and editing. Rajeev Kumar: Conceptualization; data curation; formal analysis; funding acquisition; investigation; methodology; resources; supervision; validation; visualization; writing-original draft; writing-review and editing. Satish Jain: Conceptualization; data curation; formal analysis; funding acquisition; investigation; methodology; resources; supervision; validation; visualization; writing-original draft; writing-review and editing.

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