Abstract

The paper deals with the Fleet Composition Problem (FCP) for a 2-echelon fuel distribution system, composed of a fuel regional warehouse (depot) and 100 gas stations (customers). The customers’ orders include random quantities of 4 types of fuel, which are distributed by a specialized fleet of road tankers. The fleet includes 4- and 8-chamber tankers that are characterized by different load capacities and fixed and variable costs. Since different types of fuel cannot be mixed together during transportation process specific assignment of orders (transportation tasks) to the vehicles (their chambers) is required. The decision problem consists in composing an optimal fleet of tankers, i.e. in defining optimal types of tankers to be used and an optimal number of vehicles in each type. It is considered as a vehicle assignment problem in which different types of vehicles are assigned to customers’ orders and formulated as a single objective mathematical programming problem, where the optimization criterion is a total daily distribution cost. Two alternative formulations of the decision problem are proposed, based on different definitions of the vehicles to be assigned to transportation tasks. To solve the problem, represented by both formulations, specialized alternative heuristic procedures are constructed. They are based on: local search (LS), evolutionary algorithms (EA) and hybrid algorithm (LS + EA). Computational experiments are carried out and their results are presented and compared in the paper.

1. Introduction

The daily operations of any carrier are inseparably linked to fleet management. In many cases the proper fleet management results in the economic efficiency of a whole transportation company or system (Żak et al., [1]; Sawicki and Żak [2]) and influences customer satisfaction. Some of activities within fleet management have
strategic and other operational character. One of the most important issues in strategic fleet management is a fleet sizing problem (FSP) and/or a fleet composition problem (FCP), which are focused on matching supply and demand in a certain transportation company. They consist in defining such a number of vehicles in a fleet that on the one hand satisfies a complete fulfillment of incoming transportation orders (market demand meeting) and on the other hand allows avoiding high fixed costs associated with oversized and as a result underutilized fleet (supply surplus). In fact the FSP is focused on definition of fleet size while vehicle types are fixed (known, predefined). On the other hand, defining both the fleet size and the types of vehicles in a fleet constitute the FCP.

The objective of this paper is to compose an optimal fleet of tankers in a central depot fuel distribution system. The decision problem is considered as a vehicle assignment in which different types of vehicles are assigned to customers’ orders (resulting in the construction of an average / typical routes) and formulated as a single objective mathematical programming model, where the optimization criterion is a total daily distribution cost. The following constraints are defined: maximum capacity of each chamber, maximum working time of drivers and vehicles and assumption that decision variables are integer numbers only. Two alternative formulations of the decision problem are proposed, based on different definition of the vehicles to be assigned to transportation tasks. In the first case, a finite list of specific vehicles is defined, while in the second case, the list of vehicle types is constructed. To solve the decision problem, represented by both formulations, specialized alternative heuristic procedures are constructed. They are based on: local search (LS), evolutionary algorithms (EA) and hybrid algorithms (LS + EA).

2. Literature review

Fleet composition problem in the literature is considered very often as a complex problem, i.e. is combined with other transportation problems. Hoff et al. [3] have presented a wide literature review on combined fleet composition and routing problem in maritime and road transportation industries. They suggested that there is generally a strong dependency between fleet composition and routing, and in their opinion, by ignoring routing aspects, fleet composition decisions may be based on a too simplified view on transportation demand. Conversely, routing decisions strongly depends on the available fleet. Hoff et al. [3] have reviewed a significant number of papers, and investigated an integration of routing aspects in fleet composition decisions. As a result, they classified all the papers into 4 general groups, including:
- fleet size mixed vehicle routing problem (FSMVRP),
- heterogeneous fixed fleet vehicle routing problem (HFFVRP),
- fleet size mixed vehicle routing problem with time windows (FSMVRPTW),
- fleet size mixed vehicle routing problem with multiple depots (FSMVRPMD).

The objective of FSMVRP is to minimize a total cost function that includes fixed costs of managing the vehicles in the fleet and variable routing costs. The first paper published with reference to the FSMVRP is Golden et al. [4]. In this work authors formulate problem using integer programming technique. The problem is then solved with several heuristics; some are based on the savings algorithm for the VRP of Clarke and Wright, others are based on a giant TSP-tour that is partitioned into subtours that fit the capacity of the vehicles. There is also several other papers with reference to FSMVRP which are based on classical operational research techniques, including: integer programming (e.g. Choi and Tcha [5]), mixed integer programming (e.g. Salhi and Rand [6], Osman and Salhi [7], Lai and Lo [8], Baldacci et al. [9]) or linear programming (e.g. Wu et al. [10]). In a vast majority those formulations are solved with constructive heuristics or tabu search. If the problem is formulated with integer programming a column generation technique is applied to solve the model (Choi and Tcha [5]), and the problem expressed by linear programming is solved with Lagrangian relaxation (Wu et al. [10]). A significant number of works is devoted to formulating and solving the FSMVRP using heuristics, including: tabu search (e.g. Wassan and Osman [11], Lee et al. [12], Brandao [13]), genetic algorithms (e.g. Liu
et al. [14]) or hybrid genetic algorithms (e.g. Lima et al. [15]). Most of the papers on the FSMVRP are considered within generic modality.

According to Hoff et al. [3], in the heterogeneous fixed fleet vehicle routing problem (HFFVRP), the fleet size is fixed or bounded by a maximum size, but the vehicles can be of different capacity and have different fixed and variable costs. The first work within this area of research was published in by Taillard [16] using the heuristic column generation to solve the problem. The other papers within this group mostly utilize heuristics, including tabu search (Tarantilis and Kiranoudis [17]) or simulated annealing (Tavakkoli-Moghaddam et al. [18]) and metaheuristic approaches, including threshold accepting metaheuristic (Tarantilis et al. [19]). Almost a half of papers within the HFFVRP are considered within generic modality, and a half in road transportation.

Another extension of the FSMVRP is based on the introduction of time windows associated with each customer, defining time interval wherein customer service has to start. Thus, this kind of the FCP, a fleet size mixed vehicle routing problem with time windows is denoted as a FSMVRPTW. The first work within this area of research was published by Liu and Shen [20], and the problem was solved based on constructive heuristic. Among other works in the area of the FSMVRPTW most of them formulate and solve the problem with the application of heuristics, including: hybrid simulated annealing (Tavakkoli-Moghaddam et al. [21]), hybrid local search (Yepes and Medina [22]), reactive variable neighborhood tabu search (Paraskevopoulos et al. [23]) or metaheuristics, including deterministic annealing (Braysy et al. [24]) or guided local search (Braysy et al. [25]). Authors of two another papers applied integer programming combined with simulation (Vis et al. [26]) and mixed integer goal programming combined with enumeration-followed-by-optimization technique (Calvete et al. [27]) for modeling and solving the problem. Almost all papers on the FSMVRPTW are considered within generic modality of transportation.

Another group of papers is devoted to fleet size mixed vehicle routing problem with multiple depots, denoted by FSMVRPMD. This problem is to determine which customers have to be serviced from the particular depots in addition to find the optimal composition of the fleet and the best possible routes for the vehicles. The first works in this area of research were published by Salhi and Fraser [28] and Salhi and Sari [29] and authors have formulated the problem and solved it with multi-level composite heuristic. In two other existing works, network modeling and set covering heuristics (e.g. Irnich [30]), and a mixed-integer linear programming and two specialized heuristics for clustering and scheduling (Dondo and Cerda [31]) are proposed for modeling and solving the considered problem. All papers on the FSMVRPMD are devoted to generic modality of transportation.

Additionally to classification proposed by Hoff et al. [3], the authors of this paper would suggest another general group of papers devoted to the FCP. Within this group of works the FCP problem is treated as independent from the vehicle routing. One of the first works in this area was published by Etezadi and Beasley [32], where mixed-integer linear programming, including continuous and discrete variables, with simulation technique are applied to model and solve the problem, respectively. In other work (Wu et al. [10]) authors address the FCP in the context of a truck-rental industry. A two-phase solution procedure is developed where in the phase one a Benders decomposition technique is used to allocate customers’ demand among assets and in the phase two the Lagrangian relaxation is applied to further improvement of the solution convergence. Two of the current works in this area use a large scale simulation technique (Petering [33] 2011) and fuzzy analytic hierarchy process (Bojovic et al. [34]) to express demand variability. In the first case a discrete event simulation model applied to vessel-to-vessel transshipment terminal is constructed. In the second one, the optimal fleet composition of rail freight cars is determined using the fuzzy multi objective linear programming approach.

3. Problem formulation

The problem considered in the paper consists in constructing the adequate set of types and number of vehicles in each type to service all customers in the analyzed time horizon. The analyzed FCP concerns fleet of road
tankers to transport fuels from the base to the finite set of gas stations. It is considered on the real-life example of the PETRI Company (name is changed), operating on the Polish fuel market. The main warehouse of PETRI is located in Poznan and it supplies in fuel regional gas stations. Fuel is transported to the company’s own and other gas stations using tractors and trailer tankers. The fuel distribution network of the PETRI Company is composed of about 160 gas stations, located all around Greater Poland region in Poland.

When fuel is distributed some key restrictions have to be respected, i.e.:

- mixing various types of fuel in one chamber is not allowed,
- the demand for fuel has to be fully satisfied, which means that each customer receives the ordered fuel types in the required quantities,
- the required quantities of fuel must be matched with the available capacity of specific chambers of the considered vehicles.

4. Problem modeling

4.1. Decision variables

There are two decision variables in the proposed model. \( x_i \) which represents an assignment of the available resources (vehicles \( v = 1, 2, 3, \ldots, V \)) to the customers (gas stations \( i = 1, 2, 3, \ldots, I \)) to service them, delivering demanded fuel. The second variable is \( x_{vc} \) which represents an assignment of the particular types of fuel \( (f = 1, 2, 3, \ldots, F) \) to particular vehicles and their chambers \( (c = 1, 2, 3, \ldots, C) \). The decision variables have an integer character and are defined as follows:

\[
x_i \in \{1, V\} - \text{its value denominates the number of a one particular vehicle } v \text{ assigned to service customer } i. \text{ As a result each customer can be serviced by one and only one vehicle.}
\]

\[
x_{vc} \in \{1, F\} - \text{its value denominates the particular fuel type } f \text{ assigned to be transported by the vehicle number } v \text{ in its chamber number } c. \text{ As a result fuel of the same type can’t be mixed with other types of fuel; however, it can be transported together with other types of fuel if those are allocated into other chambers in a given vehicle.}
\]

The combination of the decision variables \( x_i \) and \( x_{vc} \) strictly defines in which chamber or chambers \( c \) of a certain vehicle \( v \) the fuel of type \( f \) ordered by customer \( i \) will be transported. However, if the same fuel type will be assigned to more than one chamber of the given vehicle it is not strictly pointed out in exactly which chamber fuel for a given customer will be transported.

4.2. Criteria

There are two criterion functions considered in the analyzed FCP, i.e.:

- \( F_1 \) - total daily operating cost; is the total daily cost of fuel distribution, which is the sum of both fixed and variable costs associated with assuring complete availability of vehicles to carry out transportation tasks. The cost of drivers is also included. The criterion is minimized and expressed in monetary units – polish currency [PLN**] per day, as:

\[
\text{Min } F_1 = \sum_{v=1}^{V} \{ VC_v[AD_{avg} + DD_{avg}(NC_v - 1) + BD_{avg}] + FC_v \} \text{ [PLN/day]} \tag{1}
\]

** PLN – Polish New “Zloty” (golden) that is a Polish currency (1PLN = 0.24EUR / 1EUR = 4.18PLN and 1PLN = 0.31$ / 1$ = 3.18PLN) on 17th of April 2012
where:

- \( V_{Cv} \) - variable unit cost of vehicle \( v \) exploitation per kilometer; expressed in [PLN/km],
- \( AD^{av} \) - average distance from the base (depot) to the first customer on the average route (approaching distance); expressed in [km],
- \( DD^{av} \) - average distance between customers on the average route (delivering distance); expressed in [km],
- \( BD^{av} \) - average distance from the last customer on the average route to the base (distance back); expressed in [km],
- \( FC_v \) - daily fixed cost of maintaining vehicle \( v \) in the fleet; expressed in [PLN/day]; added if at least one \( x_i = v (NC_v > 0) \).
- \( NC_v \) - total number of customers (unloading points = gas stations) on the average route of vehicle \( v \) [-], calculated as follows:

\[
NC_v = \text{card}\{i : x_i = v\} \tag{2}
\]

- \( F_2 \) – average fleet capacity utilization index; is an average utilization factor of a vehicles’ capacity. It is calculated globally for a whole fleet. The criterion is maximized and expressed in [%].

The heuristic procedure described in the next section is based on the optimization of the first criterion function \( (F_1) \), which is the principal measure of the evaluation of generated solutions. The second criterion \( (F_2) \) is not used in the optimization process and it is considered as an additional measure of merit that helps to assess the generated results.

4.3. Constraints

Three constraints are formulated in the proposed model, i.e.:

- **The capacity of each chamber is limited** – the main idea of this constraint is to prevent specific chambers \( c \) and vehicles \( v \) from overloading.

As a result, this constraint assures that all the demand is met (ordered fuel is delivered to customers) and transported according to technical restrictions of vehicles. In combination with the decision variable \( x_{vc} \) it assures that different fuel types are not mixed in one chamber, as described above.

- **Working time of drivers and vehicles is constrained** – the maximum working time \( T_{max} \) of each vehicle resulting from the drivers’ working hours regulated by the law (AETR) and the number of drivers in the crew \( ND_v \) can not be exceeded.

The time limit has been converted into the maximum mileage that vehicle \( v \) can drive during one working day. This computation is based on the average operating speed of each vehicle \( - V_v \) (including riding time, loading and unloading activities). Finally, the comparison of a mileage that vehicle \( v \) must travel to visit all customers on its average route and its maximum daily mileage is carried out.

- **Decision variables are integer numbers only** – all the decision variables are both integer and positive numbers.

4.4. Alternative problem formulations

The FCP considered in this paper is formulated alternatively – in two variants:

- **Variant 1 (V1)** – the decision variables denote a specific vehicle; thus the customers’ demand is assigned to a specific vehicle; in this case it is necessary to predefine the list of all vehicles – the finite set of unique vehicles taken into account during the optimization process.
Variant 2 (V2) – the decision variables denote a type of vehicles in the fleet only; thus the assignment of customers’ demand is carried out on the subsequent vehicles from each type. As a result, it is necessary to predefine the list of vehicle types only and the maximum number of subsequent vehicles in each type. The maximum number of vehicles in each type should be high enough to service all the customers by this type of vehicles (to allow for homogeneous, not only heterogeneous fleet).

4.5. Data

4.5.1. Demand
In the considered problem the following data concerning demand \( D_{ij} \) (daily demand of customer \( i \) for fuel type \( f \)) is taken into account:

- Four types of fuel \((F = 4)\) are ordered by the operated gas stations, including: diesel fuel, denominated by ON \((f = 1)\); regular unleaded fuel, called Euro Super, denominated by E95 \((f = 2)\); regular fuel, called Universal, denominated by U95 \((f = 3)\); supreme unleaded fuel, called Euro Super Plus, denominated by E98 \((f = 4)\).
- The average daily number of customers = orders is \( I = 100 \).
- The daily orders are in various quantities ranging from \( D_{i1} = 232 \) liters (for customer \( i = 97 \) and fuel type \( f = 4) \) to \( D_{i4} = 2850 \) liters.
- The total daily demand for fuel of respective types is as follows: ON – 122114 liters; E95 – 89847 liters; U95 – 54501 liters; E98 – 51323 liters, which sums up into an average total daily fuel demand of 317785 liters.
- The minimum and maximum demand for fuel of respective types is as follows: ON \((i = 1)\) – 412 liters \((i = 91)\) and 2850 liters \((i = 84); \) E95 \((i = 2)\) – 446 liters \((i = 95)\) and 1616 liters \((i = 48); \) U95 \((i = 3)\) – 254 liters \((i = 91)\) and 1058 liters \((i = 88)\); E98 \((i = 4)\) – 232 liters \((i = 97)\) and 886 liters \((i = 96)\).
- Each customer represents a gas station at a certain location and is characterized by an order defined as the average daily demand for each fuel type in liters; these orders, while being served constitute transportation tasks that must be carried out on average day.

4.5.2. Supply
The supply in this problem is represented by the fleet of vehicles and its capability to carry transportation tasks. Thus, there are following list of characteristics that are taken into account:

- Each vehicle can be equipped in a certain number of fuel chambers \( c \), equal to 4 or 8. The capacities of fuel chambers, expressed in liters, range from 1500 liters (e.g. vehicle \( v = 22 \); chamber \( c = 8 \)) to 6000 liters (e.g. vehicle \( v = 8 \); chamber \( c = 1 \)). Thus, the overall transportation capacity of road fuel tankers differs substantially. Vehicle 3 \((v = 3)\) is equipped in 8 fuel chambers and their overall transportation capacity equals to 27500 liters; the smallest chamber in this vehicle has a capacity of 2500 liters, while the largest one chamber can carry 4000 liters. At the same time vehicle 8 \((v = 8)\) has 4 chambers and the overall transportation capacity of 20700 liters. Chamber capacities in this vehicle range between 4650 and 6000 liters. Four vehicles \((v = 7, 23, 39 \) and 55\), are characterized by the smallest overall capacity of 13700 liters. They are have 4 chambers with capacities of: 5000, 4200, 2500 and 2000 liters. On the other hand, 3 vehicles in the fleet \((v = 3, 35 \) and 51\), each one having 8 chambers, are characterized by the largest capacity of 27500 liters.
- A daily fixed cost \( FC_v \) of vehicles exploitation is 48 or 58 [PLN/day], according to vehicle type, 4 or 8 chambers.
- A variable cost \( VC_v \) of fuel distribution per one kilometer is between 4.78 and 5.46 [PLN/km] for 4- and 8-chamber vehicles, respectively.
- The average operating speed \( V_v \) of delivering fuel equals to 35 [km/h] (and in the analyzed case is assumed to be constant for all vehicles \( v)\).
4.5.3. Other data

There is a list of other important characteristics of the considered FCP, i.e.:

- A daily working time $T^{\text{max}}$ of vehicles / drivers cannot exceed 8.5 hours per driver a day (taking into account roughly 2.6 drivers per vehicle it gives around 22 working hours of vehicle per day).
- The base (a warehouse) and each station operate 24 [h/day] (it implies 22 vehicle’s working time a day, excluding 2 hours organizational stopover).
- The average distance from the base to the first customer $AD^{\text{avg}}$ on the average route (approaching distance) is 105.1 [km].
- The average distance between customers $DD^{\text{avg}}$ on the average route (delivering distance) is 79.3 [km].
- The average distance from the last customer to the base $BD^{\text{avg}}$ on the average route (distance back) is 105.1 [km].

5. Implementation

Since finding exact solutions for the real-world instances of the FCP is computationally demanding, the authors decided to use metaheuristic approach that is based on evolutionary algorithms augmented by a local search procedure and a heuristic method for assigning fuel types to chambers at a single vehicle level. The algorithm was implemented for the Java platform and was run on the computer with the Intel Pentium M 1.70 GHz processor and 1 GB RAM.

5.1. Solution representation and the fuel-type-to-chamber assignment procedure

Each individual genotype consists of a set of variables $\{x_1, x_2, \ldots, x_i, \ldots, x_I\}$ where variable $x_i$ represents the assignment of vehicle $v$ ($1 \leq x_i \leq V$) to a customer $i$. However, this vehicle-to-customer assignment is not sufficient for representing the complete solution in case of the analyzed problem. Another information indispensable for having a fully defined solution is a fuel-type-to-chamber assignment $\{x_{v1}, x_{v2}, \ldots, x_{vc}, \ldots, x_{vC}\}$ defined for each vehicle $v$ separately, which states the type of a fuel to be transported in chamber $c$ of a given vehicle $v$. The fuel-type-to-chamber assignment is not explicitly provided in the genotype but computed for each vehicle $v$ according to the following method:

1. For each fuel type $f$ calculate the total demand $d_{vf}$ for fuel $f$ to be delivered to all the customers assigned to vehicle (average route) $v$:

$$d_{vf} = \sum_{i: x_i = v} D_{if}$$

where:

- $D_{if}$ - daily demand of customer $i$ for fuel type $f$; expressed in [liter/day].
2. Create a sorted list of chambers $c$ in descending order of their capacities $CC_{vc}$.
3. Create a sorted list of fuel types $f$ in descending order of the total demand $d_{vf}$.
4. Choose the first chamber $c$ from the list of chambers.
5. Choose the first fuel type $f$ from the list of fuel types.
6. Set $x_{vc} = f$.
7. If $CC_{vc} < d_{vf}$ (some demand for fuel $f$ remains unmet) reduce $d_{vf}$ by $CC_{vc}$ and reorder list of fuel types $f$ according to the updated total demand $d_{vf}$. Otherwise (the demand for fuel $f$ is satisfied), remove fuel type $f$ from list of fuel types $f$.
8. Remove chamber $c$ from list of chambers.
9. If lists of chambers and fuel types are both not empty go to step 4.
10. If list of fuel types is empty (all the demand has been satisfied), for each chamber remaining in the list of chambers, set \( x_{vc} = 0 \).

As a result of running this procedure for each vehicle \( v \), all \( x_{vc} \) values are determined. This procedure is a greedy one since it attempts first to satisfy the highest demand by allocating the corresponding fuel type to the largest chamber. Towards the end of the procedure, smaller fuel amounts (small demand) are assigned to smaller chambers. In most cases this approach gives optimal results, however one can easily find cases this behavior is not optimal. Nonetheless, the speed of this heuristic compensates that small probability of obtaining a non-optimal fuel-type-to-chamber assignment.

The validity of the output of the fuel-type-to-chamber assignment (values of the variables \( x_{vc} \)) depends significantly on the input vehicle-to-customer assignment (values of the variables \( x_i \)). Depending on the input assignment, there may or may not exist a fuel-type-to-chamber assignment that satisfies all the constraints (e.g. maximal chamber capacity utilization). In rare cases the heuristic procedure may be not able to find a valid fuel-type-to-chamber assignment.

5.2. Optimization approach

The evolutionary algorithm used in the research consists of nine steps, where steps 3-9 are executed in a cycle until the termination condition (step 9) is met.

1. **Generation of the initial population** - At this step the initial population of \( n \) individuals is generated. Each individual’s genotype is created by drawing a random value between 1 and \( V \) for each gene \( x_i \). Then for each individual and for each vehicle the fuel-type-to-chamber assignment is run, resulting in determination of all \( x_{vc} \) values.

2. **Evaluation of the initial population** - Each individual is evaluated according to the criterion of the total operation cost \( F_1 \). The fitness is then reduced by the sum of penalties for exceeding maximum chamber capacity (4) and maximum driving time (5) constraints.

3. **Selection for reproduction** - The roulette selection was used to select \( n/2 \) pairs of parent individuals. It is possible to have unique (drawing without returning) or non-unique (drawing with returning) selection. A scaling function is used to convert negative fitness values (all individuals have negative fitness) into positive share values for the drawing purposes. The weakest individuals (with fitness below a given threshold value) are given a zero share.

4. **Crossover** - The uniform crossover is used. Based on two parent individuals a pair of new offspring individuals is generated. The crossover concerns only variables \( x_i \) (the vehicle-to-customer assignment) whereas values of the variables \( x_{vc} \) are determined in step 7. The degree of uniformity of parent’s genes distribution among the offspring can be adjusted allowing for generating offspring individuals that are expected to be more similar to one of the parents or equally similar to both parents.

5. **Mutation** - Each gene \( (x_i) \) is subject to mutation according to a predefined probability. Each gene selected for mutation is set to a random value between 1 and \( V \).

6. **Local search** - A steepest descend local search procedure is run for each of the offspring solutions. The local search neighborhood is defined as a set of solutions that differs from the current one only at one gene. Since the algorithm is greedy, all the neighboring solutions are searched and evaluated before shifting from the current solution to the best neighbor. The procedure ends when the current solution is not worse than any other solution in its neighborhood.

7. **Evaluation** - Each offspring individual is evaluated according to the rules described in step 2.

8. **Succession** - The elitism succession (with a predefined elitism rate) is used to construct the next generation’s population based on the current population and the offspring population.

9. **Termination criterion** - By default, a maximum number of generations is used as the termination condition.
6. Analysis of generated results

The above described metaheuristic and heuristic procedures have been applied to solve the real-life road tankers fleet composition problem. They have been tested and validated in a series of computational experiments taking into account two variants of the problem formulation (V1 – decision variable $x_i$ of type 1 based on the particular vehicles and V2 – decision variable $x_i$ of type 2 based on the types of vehicles) and three different options of the solution algorithm:

- Option 1 – O1: homogenous procedure based explicitly on evolutionary algorithms only,
- Option 2 – O2: homogenous algorithm based explicitly on local search only,
- Option 3 – O3: hybrid algorithm based on a combination of local search (LS) and evolutionary algorithms (EA).

As a result of combining variants of problem formulation V1 and V2 with alternative options of applied algorithms the following computational versions have been generated:

- V1O1 – based on decision variable of type 1 and evolutionary algorithms (EA);
- V1O2 – based on decision variable of type 1 and local search (LS);
- V1O3 – based on decision variable of type 1 and hybrid algorithm (LS + EA);
- V2O1 – based on decision variable of type 2 and evolutionary algorithms (EA);
- V2O2 – based on decision variable of type 2 and local search (LS);
- V2O3 – based on decision variable of type 2 and hybrid algorithm (LS + EA).

The analysis of computational results has been carried out in two dimensions, i.e.: in the criteria space and in the decision variables space. The results concerning the best solutions are presented in table 1.

Table 1. Major features of the best solutions generated by different computational versions (different character of the decision variables and different solution procedures applied)

| Variants (V) and options (O) | $F_1$ [PLN] | $F_2$ [%] | Number of vehicles in the fleet | Computational time [sec.] |
|-----------------------------|-------------|----------|-------------------------------|--------------------------|
|                             | Total       | 4-chambers | 8-chambers                   |
| V1O1                        | 37791       | 72.0      | 26 | 21 | 5 | 175 |
| V1O2                        | 37625       | 73.2      | 27 | 26 | 1 | 37  |
| V1O3                        | 35962       | 79.2      | 25 | 24 | 1 | 58  |
| V2O1                        | 47698       | 45.9      | 35 | 22 | 13 | 1506 |
| V2O2                        | 38552       | 73.6      | 28 | 28 | - | 112 |
| V2O3                        | 39478       | 73.2      | 29 | 29 | - | 737 |

The generated results lead to the following conclusions:

- The algorithms utilizing the formulation of the decision problem based on type 1 decision variable – variant V1 are computationally more efficient than those based on type 2 decision variable – variant V2. All computational procedures – options O1, O2 and O3 generate, using decision variables of type 1, better results in the criteria space in a shorter computational time. The comparison between particular solutions generated within respective variants (V1O1 and V2O1; V1O2 and V2O2; V1O3 and V2O3) shows that the average improvement on criterion $F_1$ reaches the level of 11% generated with average saving of computational time at the level of 82%.

- The best solution, in terms of both single ($F_1$) and bi-criterion ($F_1$ and $F_2$) optimization has been generated by version V1O3, which is a hybrid algorithm based on local search and evolutionary algorithms with type 1 decision variable. This solution outperforms all other best solutions on both criteria. It is better cost-wise (criterion $F_1$) by 11% from the average of best costs generated by all remaining versions and by 4% from the second best result obtained by version V1O2 (local search with type 1 decision variable). As far as fleet utilization is concerned (criterion $F_2$) the best solution generates a 17% improvement with respect to the average value of this criterion generated by the remaining versions and an 8% improvement with respect to the
second best result generated by version V2O2. In a multiple-criteria sense the solution generated by version V1O3 is a Pareto optimal solution with respect to the best solutions generated by other versions of problem formulation and solution procedure.

- Interpreting the results in the decision variables space it is worth mentioning that version V1O3 guarantees a solution with a minimal number of 25 vehicles in the fleet. Other versions generate solutions with larger fleets by 1 to 10 units. Again, the algorithms utilizing the formulation of the decision problem based on type 1 decision variable (variant V1) guarantee smaller fleets than computational procedures based on type 2 decision variable – variant V2. In two cases (versions V2O2 and V2O3) the proposed fleets are homogenous as opposed to the remaining solutions with heterogeneous fleets and dominant number of smaller, 4-chamber road tankers. The fleet in the winning solution is close to homogeneity with only 1 vehicle different from the dominant majority of 24.

- The best solution generated by computational version V1O3 is presented in table 2 in the decision variables space as an assignment of vehicles to customers. The fleet is composed of 25 vehicles, including the following numbers: 1, 2, 7, 8, 16, 17, 18, 20, 21, 26, 27, 29, 32, 33, 34, 36, 39, 40, 43, 45, 49, 50, 52, 53, 57 among 64 vehicles taken into consideration. Each of these vehicles services 3 to 6 customers. The only vehicle in the fleet that delivers fuel to 6 customers is a 4-chamber road tanker (No. 40) with a capacity of 20700 liters. The only 8-chamber road tanker in the fleet (No. 57) has a capacity of 27000 liters and delivers fuel to 3 customers, only. The overall assignment of the fleet to customers is as follows: the majority of vehicles (14 units), all 4-chamber road tankers with a capacity ranging between 13700 and 16900 liters service 4 customers; 6 vehicles, including the above mentioned 8-chamber unit and 5 4-chamber road tankers which capacity varies between 14200 and 16900 liters service 3 customers; 4 vehicles, all 4-chamber units with capacity between 13300 and 20700 liters deliver fuel to 5 customers and, as mentioned above, 1 vehicle services 6 customers.

Table 2. The best solution generated by computational version V1O3 in the decision variables space representing the composition of the fleet and assignment of vehicles to particular customers

| Customer No. | Vehicle No. |
|--------------|-------------|
| 1            | 16          |
| 2            | 18          |
| 3            | 26          |
| 4            | 28          |
| 5            | 29          |
| 6            | 31          |
| 7            | 32          |
| 8            | 33          |
| 9            | 34          |
| 10           | 36          |
| 11           | 39          |
| 12           | 40          |
| 13           | 45          |
| 14           | 46          |
| 15           | 47          |
| 16           | 48          |
| 17           | 49          |
| 18           | 50          |
| 19           | 51          |
| 20           | 52          |
| 21           | 53          |
| 22           | 54          |
| 23           | 55          |
| 24           | 56          |
| 25           | 57          |
| 26           | 58          |
| 27           | 59          |
| 28           | 60          |
| 29           | 61          |
| 30           | 62          |
| 31           | 63          |
| 32           | 64          |
| 33           | 65          |
| 34           | 66          |
| 35           | 67          |
| 36           | 68          |
| 37           | 69          |
| 38           | 70          |
| 39           | 71          |
| 40           | 72          |
| 41           | 73          |
| 42           | 74          |
| 43           | 75          |
| 44           | 76          |
| 45           | 77          |
| 46           | 78          |
| 47           | 79          |
| 48           | 80          |
| 49           | 81          |
| 50           | 82          |
| 51           | 83          |
| 52           | 84          |
| 53           | 85          |
| 54           | 86          |
| 55           | 87          |
| 56           | 88          |
| 57           | 89          |
| 58           | 90          |
| 59           | 91          |
| 60           | 92          |
| 61           | 93          |
| 62           | 94          |
| 63           | 95          |
| 64           | 96          |
| 65           | 97          |
| 66           | 98          |
| 67           | 99          |
| 68           | 100         |

The overall composition of the fleet is as follows:

- 1 vehicle – 4-chamber unit with capacity of 13300 liters (No. 21);
- 1 vehicle – 4-chamber unit with capacity of 13700 liters (No. 7);
13 vehicles – 4-chamber units with capacity of 14200 liters (No. 1, 16, 17, 20, 26, 29, 32, 33, 36, 39, 45, 49, 52);
7 vehicles – 4-chamber units with capacity of 16900 liters (No. 2, 18, 27, 34, 43, 50, 53);
2 vehicles – 4-chamber units with capacity of 20700 liters (No. 8, 40);
1 vehicle – 8-chamber unit with capacity of 27000 liters (No. 57).

Using the data for all computational procedures based on evolutionary algorithms a chart (see fig. 1) presenting the changes of the values of the objective function $F_1$ in consecutive generations have been plotted. This chart lets us draw some conclusions about convergence and quality of the proposed evolutionary algorithms. In the chart the numbers of generations have been replaced by their relative shares (in %) to guarantee the comparability of results. In computational versions V1O1 and V2O1 roughly 65000 generations have been obtained, while in computational versions V1O3 and V2O3 15 and 25 generations have been reached, respectively. The charts are plotted based on the discrete points representing the average values of criterion $F_1$ generated in each generation. Due to the random character of the algorithms these values vary. Thus, their average values and dispersions (standard deviations) have been reported.

As one can see the behavior of computational versions V1O1 and V2O1 is different from the behavior of two remaining versions: V1O3 and V2O3. The former, based exclusively on evolutionary algorithms initiate their search with a relatively poor solution and then improve its quality substantially. In both versions V1O1 and V2O1 the improvement of the initial solution is quite dynamic in the first 1000 generations (1.5%) and especially steep in the first 100 generations (0.15%). The minimized objective function $F_1$ drops by 28% in version V1O1 and by 21% in version V2O1 in the first 1000 generations (1.5%). After 3000-6000 generations (5-9%) the level of the objective function $F_1$ stabilizes and fluctuates around 43000 [PLN] for version V1O1 and 55000 [PLN] for version V2O1. Due to the random character of the algorithms some solutions generate decrease (improvement) of the objective function while others its increase (deterioration). Further improvement of the objective function $F_1$ between 6500 (10%) and 30000 (46%) generations is slower and has a step-wise character. After the threshold of 32500 generations the objective function stabilizes and its improvement is marginal. The minimum value of the objective function $F_1$ is reached in generation 59287 (91%) at the level of 37800 [PLN] for version V1O1 and in generation 49979 (77%) at the level of 47700 [PLN] in version V2O1. The total relative improvement of the objective function $F_1$ is equal 41% for version V1O1 and 34% for version V2O1. The vast majority of this improvement (roughly 90%) is obtained in the first 20 and 15% of the generations for versions V1O1 and V1O2, respectively while further computation (next 52000-55000 generations) produces only 10% of the additional improvement.

The other two computational versions: V1O3 and V2O3, based on the combination of local search and evolutionary algorithms behave differently. They produce initially relatively good solutions and then improve them marginally, only. In case of version V1O3 this improvement has not been even noticed, while for version V2O3 it has reached the level of 4%. It is worth mentioning that these results have been generated in a relatively small numbers of generations due to the application of the local search procedures within these versions of computations.

It is also noticeable, while comparing versions V1O1 and V2O3, that the character of the decision variable (variant) has a stronger impact on the improvement of the objective function than the structure and composition of the solution algorithm (option). Version V1O1 based on the type 1 decision variable (variant V1) and evolutionary algorithms, only (option 1) generates better results than computational version V2O3, which is featured by type 2 decision variable (variant V2) and a combination of local search and evolutionary algorithms (option 3). The best solution cost-wise (criterion $F_1$) generated by computational version V1O1 is 5% better than the best solution generated by version V2O3.
7. Conclusions

The paper presents the methodology of solving the fleet composition problem (FCP) for a set of road tankers operating in a medium-size distribution network. The vehicles carry out specific transportation tasks while delivering fuel to a large number of customers (gas stations). The authors recognize the decision situation and construct a mathematical model of the FCP. They formulate the decision problem as a single objective mathematical programming problem and suggest its extension to a multiple criteria form (using criterion \( F_2 \)). Different heuristic procedures are applied to solve the FCP, including: local search (LS), evolutionary algorithms (EA) and a combination of both LS + EA. The original output of this research is a formulation and construction of the mathematical model for the FCP with its specific characteristics. The authors have designed and implemented the specialized heuristic procedures, customized to original features of the FCP. The utilization and original adaptation of the concepts of such metaheuristic algorithms as: local search, evolutionary algorithms and their combination is also an important output of this paper. The proposed computational procedures have been tested and validated and finally the original conclusions and comments based on the detailed analysis of the computational results generated in a series of experiments have been formulated.

Based on the carried out research one can conclude that the way of formulating the decision problem may have a strong impact on its ability to be solved. In the analyzed case of the FCP the type and character of the applied decision variable strongly influenced on computational efficiency of the proposed solution algorithms. It has been revealed that type 1 decision variable, assigning certain, fully defined vehicles to the customers, assures better computational results. It leads to the 11% improvement of the value of the optimization criterion and 82% reduction of the computational effort in comparison to the utilization of decision variable assigning the predefined types of vehicles to the customers only. The most efficient procedure for solving the FCP is a hybrid algorithm based on LS and EA with a type 1 decision variable – computational version V103. Its application results in solving a complex FCP with 64 road tankers delivering fuel to 100 customers, solving in a relatively...
short time of less than 1 minute (58 seconds). This method generates the best result and optimizes the total distribution cost at the level of roughly 36000 [PLN].

The best generated solution corresponds to a construction of an optimal fleet composed of 25 road tankers. This fleet has an almost homogeneous character and includes 24 4-chamber vehicles with a fuel transportation capacity ranging between 13300 and 20700 liters and only 1 8-chamber vehicle with a capacity of 27000 liters. The behavior of hybrid algorithms (combination of LS + EA) is different from the behavior of their single-scope counterparts (heuristics based on EA exclusively). In the analyzed case of the FCP computational versions V1O1 and V2O1, based exclusively on evolutionary algorithms initiate their search with a relatively poor solution and then improve its quality substantially. Versions V1O3 and V2O3, based on LS + EA produce initially relatively good solutions and then improve them marginally, only.

The authors would suggest that further research should be carried out with the focus on tunings of the decision model with special emphasis on the redefinition of the decision variables as well as adjustments in the hybrid algorithms based on LS + EA. The next step in the research should also be concentrated on development of an alternative computational procedure based on tabu search (TS) and its modifications and reformulation of the proposed model and its extension accommodating other original features of the FCP that refers to other types of vehicles and characteristics of the different transportation systems.

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