Graviweak Unification, Invisible Universe and Dark Energy

C.R. Das\(^1\), L.V. Laperashvili \(^2\), A. Tureanu \(^3\)

\(^1\) Centre for Theoretical Particle Physics, CFTP (IST), Avenida Rovisco Pais, 1 1049-001 Lisbon, Portugal
\(^2\) The Institute of Theoretical and Experimental Physics, Bolshaya Cheremushkinskaya, 25, 117218 Moscow, Russia
\(^3\) Department of Physics, University of Helsinki, P.O. Box 64, FIN-00014 Helsinki, Finland

Abstract

We consider a graviweak unification model with the assumption of the existence of a hidden (invisible) sector of our Universe, parallel to the visible world. This Hidden World (HW) is assumed to be a Mirror World (MW) with broken mirror parity. We start with a diffeomorphism invariant theory of a gauge field valued in a Lie algebra \(g\), which is broken spontaneously to the direct sum of the spacetime Lorentz algebra and the Yang–Mills algebra: \(\tilde{g} = su(2)_{grav}^{(grav)} \oplus su(2)_L\) – in the ordinary world, and \(\tilde{g}' = su(2)_{grav}^{(grav)} \oplus su(2)_R\) – in the hidden world. Using an extension of the Plebanski action for general relativity, we recover the actions for gravity, \(SU(2)\) Yang–Mills and Higgs fields in both (visible and invisible) sectors of the Universe, and also the total action. After symmetry breaking, all physical constants, including the Newton’s constants, cosmological constants, Yang–Mills couplings, and other parameters, are determined by a single parameter \(g\) present in the initial action, and by the Higgs VEVs. The dark energy problem of this model predicts a too large supersymmetric breaking scale \((M_{SUSY} \sim 10^{10} \text{ GeV})\), which is not within the reach of the LHC experiments.

Keywords: unification, gravity, mirror world, cosmological constant, dark energy

PACS: 04.50.Kd, 98.80.Cq, 12.10.-g, 95.35.+d, 95.36.+x

\(^*\)crdas@cftp.ist.utl.pt
\(^†\)laper@itep.ru
\(^‡\)anca.tureanu@helsinki.fi
1 Introduction

In this paper, developing the ideas of Ref. [1], we construct a model of unification of gravity and the $SU(2)$ gauge and Higgs fields, using only the graviweak part of the model proposed in Ref. [2]. In contrast to Ref. [2], we assume the existence of the hidden sector of the Universe [3–7], when this Hidden World (HW) is a Mirror World (MW) with broken mirror parity (see Refs. [8–13] and reviews [14–17]). We also discuss the problem of the Dark Energy (DE), and give the prediction for the supersymmetry breaking scale.

Considering a diffeomorphism invariant theory of a gauge field with a Lie algebra broken to the direct sum of the spacetime Lorentz algebra, Yang-Mills algebra and their complement, we assume that the action, obtained in the graviweak model [2], has a modified duplication for the hidden sector of the Universe, in accordance with ideas proposed in Ref. [1].

The paper is organized as follows. In Section 1 we discuss the graviweak unification model in the ordinary and hidden worlds of the Universe. An extension of Plebanski’s action for general relativity was used in this unification model, and in Section 2 we introduce the main ideas of the Plebanski’s theory of gravity [18]. In Section 3 we review the theory of the Mirror World (MW) parallel to the visible Ordinary World (OW), and also the model of the Mirror World with broken mirror parity (MP), which assumes that the Higgs VEVs in the O- and M-worlds are not equal $\langle \phi \rangle = v$, $\langle \phi' \rangle = v'$ and $v \neq v'$. In Section 4, we build on the proposal made in Ref. [1], according to which the OW-gravity is described by the self-dual left-handed Plebanski’s gravitational action, while the gravity in the MW is described by the anti-self-dual right-handed gravitational action. Constructing the graviweak unification model in both sectors of the Universe [2], we start with a $g = \text{spin}(4, 4)$-invariant extended Plebanski’s action. We show that the action for gravity and $SU(2)$ Yang-Mills and Higgs fields, constructed in the ordinary world, has a modified duplication for the hidden sector of the Universe. After symmetry breaking of the graviweak unification, we obtain Newton’s constants and the cosmological constants, which are not equal in the OW and MW. The mirror cosmological constant is larger than the ordinary cosmological constant, while the OW and MW Yang-Mills coupling constants are equal. In Section 5 we discuss the problems of communications between visible and invisible worlds. These communications are given by $\mathcal{L}_{(\text{mix})}$-term of the total Lagrangian of the Universe, which describes extremely small contributions of communication processes. We also discuss the experimental tests of the parity violation in gravity. Section 6 is devoted to the problem of the DE. We show that in the framework of the broken graviweak unification model, the tiny value of the DE density, given by the recent astrophysical and cosmological measurements, leads to the cut-off scale of the theory being less than the Planck scale, and equal to the large supersymmetry breaking scale $M_{\text{SUSY}} \sim 10^{10}$ GeV, which is not within the reach of the LHC experiments. However, we discuss a possibility to reduce the theoretically predicted value of $M_{\text{SUSY}}$.

---

1In this paper the superscript ‘prime’ denotes the M- or hidden H-world.
2 Unification models and Plebanski’s theory of gravity

General relativity (GR), originally formulated by Einstein as the dynamics of a metric, \(g_{\mu\nu}\), was presented by Plebanski [18], Ashtekar [19, 20] and others [21–23] in a self-dual approach, when the true configuration variable of GR is a connection corresponding to the gauging of the local Lorentz group, \(SO(1,3)\), and the spin group, \(Spin(1,3)\) (or its chiral subgroup).

The idea that interactions are described by a gauge connection common to both Yang–Mills theory and GR, leads to the unification of gravity with the other forces. In the unification model [2], the fundamental variable is a connection, \(A\), valued in a Lie algebra, \(\mathfrak{g}\), that includes a subalgebra \(\tilde{\mathfrak{g}}\):

\[
\tilde{\mathfrak{g}} = \mathfrak{g}^{(\text{spacetime})} \oplus \mathfrak{g}_{YM},
\]

which is the direct sum of the Lorentz algebra (or a chiral subalgebra of it) and a Yang–Mills gauge algebra.

Previously, graviweak unification models were presented in Refs. [24–26]. The gravi-GUT unification was suggested in [27–29] and discussed in [30]. As it was shown in Ref. [30], these unifications are based on the formulation of a fully \(\mathfrak{g}\)-invariant theory via an extension of the covariant Plebanski action of gravity [18–23]. This kind of the extension previously had been studied in Refs. [31–39] (see also review [40] and Ref. [1]), especially for the self-dual Plebanski action, where \(\mathfrak{g}^{(\text{spacetime})} = \mathfrak{su}(2)\).

In Ref. [2], \(\mathfrak{g}^{(\text{spacetime})} = \mathfrak{spin}(1,3)\) is the gravitational gauge algebra, and the Yang–Mills gauge algebra is a spin algebra, \(\mathfrak{g}_{YM} = \mathfrak{spin}(N)\). Finally, the model of unification of gravity, the \(SU(N)\) or \(SO(N)\) gauge fields and Higgs bosons is based on the full initial gauge algebra \(\mathfrak{g} = \mathfrak{spin}(p,q)\).

The main idea of Plebanski’s formulation of the 4-dimensional theory of gravity [18] is the construction of the gravitational action from the product of two 2-forms (see [18,23] and [31,40]). These 2-forms are constructed using the connection \(A^{IJ}\) and tetrads, or frames, \(e^I\) as independent dynamical variables. Both \(A^{IJ}\) and \(e^I\), also \(A\), are 1-forms:

\[
A^{IJ} = A_{\mu}^{IJ}dx^\mu \quad \text{and} \quad e^I = e_{\mu}^I dx^\mu,
\]

\[
A = \frac{1}{2} A^{IJ} \gamma_{IJ}.
\]

Here the bivector generators \(\gamma_{IJ}\) can be understood as the product of \(Cl(1,3)\) Clifford algebra basis vectors: \(\gamma_{IJ} = \gamma_I \gamma_J\).

The indices \(I,J = 0,1,2,3\) refer to the spacetime with Minkowski metric \(\eta_{IJ}\): \(\eta^{IJ} = \text{diag}(1, -1, -1, -1)\). This is a flat space which is tangential to the curved space with the metric \(g_{\mu\nu}\). The world interval is represented as \(ds^2 = \eta_{IJ} e^I \otimes e^J\), i.e.

\[
g_{\mu\nu} = \eta_{IJ} e^I_{\mu} \otimes e^J_{\nu}.
\]
Considering the case of the Minkowski flat spacetime with the group of symmetry $SO(1,3)$, we have the capital Latin indices $I, J, ... = 0, 1, 2, 3$, which are vector indices under the rotation group $SO(1,3)$.

In the general case of the gauge symmetry $\mathfrak{g}$ with the Lie algebra $\mathfrak{g} = spin(p,q)$, we have $I, J = 0, 1, 2, ..., p + q - 1$.

The 2-forms $B^{IJ}$ and $F^{IJ}$ are defined as:

$$B^{IJ} = e^I \wedge e^J = \frac{1}{2} e^I_\mu e^J_\nu dx^\mu \wedge dx^\nu, \quad (5)$$

$$F^{IJ} = \frac{1}{2} F^{IJ}_\mu dx^\mu \wedge dx^\nu. \quad (6)$$

Here the tensor $F^{IJ}_\mu$ is the field strength of the spin connection $A^{IJ}_\mu$:

$$F^{IJ}_\mu = \partial_\mu A^{IJ}_\nu - \partial_\nu A^{IJ}_\mu - [A^\mu_\mu, A^\nu_\nu], \quad (7)$$

which determines the Riemann–Cartan curvature:

$$R^{\kappa \lambda \mu \nu} = e^I_\kappa e^J_\lambda F^{IJ}_\mu. \quad (8)$$

We also consider the 2-forms $B$ and $F$:

$$B = \frac{1}{2} B^{IJ} \gamma_{IJ} \quad \text{and} \quad F = \frac{1}{2} F^{IJ} \gamma_{IJ}, \quad (9)$$

which are spin(1,3) valued 2-form fields, and:

$$F = dA + \frac{1}{2} [A, A]. \quad (10)$$

In the Plebanski BF-theory, the gravitational action with nonzero cosmological constant $\Lambda$ is given by the integral:

$$I_{GR} = \frac{1}{\kappa^2} \int \epsilon^{IJKL} \left( B^{IJ} \wedge F^{KL} + \Lambda \frac{1}{4} B^{IJ} \wedge B^{KL} \right), \quad (11)$$

where $\kappa^2 = 8\pi G_N$, $G_N$ is the gravitational constant, $M_{Pl}^2 = 1/\sqrt{8\pi G_N}$.

For any antisymmetric tensors $F^{\mu \nu}$, there exist dual tensors given by the Hodge star dual operation:

$$F^{\mu \nu}_{*} \equiv \frac{1}{2\sqrt{-g}} \epsilon^{\rho \sigma} F_{\mu \nu}^\rho \sigma, \quad (12)$$

and for any antisymmetric tensors $A^{IJ}$ there exists dual operation:

$$A^{*IJ} = \frac{1}{2} \epsilon^{IJKL} A^{KL}. \quad (13)$$
Here $\epsilon$ is the completely antisymmetric tensor with $\epsilon^{0123} = 1$.

We can define the algebraic self-dual and anti-self-dual components of $A^{IJ}$:

$$A^{(\pm)IJ} = (\mathcal{P}^{\pm} A)^{IJ} = \frac{1}{2}(A^{IJ} \pm iA^{*IJ}).$$  \hfill (14)

The two projectors $\mathcal{P}^{\pm} = \frac{1}{2}(\delta^{IJ}_{KL} \pm \epsilon^{IJ}_{KL})$ realize explicitly the familiar homomorphism:

$$\mathfrak{so}(1,3) = \mathfrak{su}(2)_R \oplus \mathfrak{su}(2)_L,$$  \hfill (15)

which rather than self-dual and anti-self-dual are more commonly dubbed right-handed and left-handed.

To make the mapping more explicit, it is convenient to pick out the time direction $I = 0$, and define

$$A^{(\pm)i} = A^{(\pm)0i},$$  \hfill (16)

with $i = 1, 2, 3$ as an $SU(2)^{(grav)}_L$ adjoint index.

(Anti)self-duality then means:

$$A^{(\pm)0i} = \pm \frac{i}{2} \epsilon^{i}_{jk} A^{(\pm)jk}.$$  \hfill (17)

The correct gauge was chosen by Plebanski, when he introduced in the gravitational action the Lagrange multipliers $\psi_{ij}$ – an auxiliary fields, symmetric and traceless. These auxiliary fields $\psi_{ij}$ provide the correct number of constraints.

Including the constraints, we obtain the following gravitational action:

$$I(\Sigma, A, \psi) = \frac{1}{\kappa^2} \int [\Sigma^i \wedge F^i + (\Psi^{-1})_{ij} \Sigma^i \wedge \Sigma^j]$$  \hfill (18)

with

$$(\Psi^{-1})_{ij} = \psi_{ij} - \Lambda \delta_{ij}.$$  \hfill (19)

Using the simpler self-dual variables instead of the full Lorentz group, Plebanski and the authors of Refs. [19–23] suggested to consider the left-handed $\mathfrak{su}(2)_L$-invariant gravitational action (18) with self-dual $F = F^{(+)}i$ and $\Sigma = \Sigma^{(+)}i$. In general, we have:

$$\Sigma^{(\pm)i} = e^0 \wedge e^i \pm \frac{1}{2} \epsilon^i_{jk} e^j \wedge e^k.$$  \hfill (20)

The self-dual action (18) is equivalent to the Einstein-Hilbert action for general relativity with cosmological constant $\Lambda$. 

5
3 Invisible Universe

3.1 Mirror World

Mirror matter is a new form of matter which is predicted to exist if mirror symmetry is respected by Nature. At the present time, evidence that mirror matter actually exists in the Universe is in abundance, coming from a range of observations and experiments in astronomy, particle physics, stars and planetary science.

The results of Refs. [3–17] are based on the hypothesis of the existence in Nature of the invisible mirror parallel to the visible ordinary world. The Standard Model (SM) group of symmetry $G_{SM}$ was enlarged to $G_{SM} \times G'_{SM'}$, where $G_{SM}$ stands for the observable SM, while $G'_{SM'}$ is its mirror gauge counterpart. The M-particles are singlets of $G_{SM}$ and the O-particles are singlets of $G'_{SM'}$. These different O- and M-worlds are coupled only by gravity, or possibly by another very weak interaction [3–7].

The M-world is a mirror copy of the O-world and contains the same particles and types of interactions as our visible world. The observable elementary particles of our O-world have the left-handed (V-A) weak interactions which violate P-parity. If a hidden M-world exists, then mirror particles participate in the right-handed (V+A) weak interactions and have the opposite chirality. Lee and Yang were the first [3] to suggest such a duplication of the worlds, which restores the left-right symmetry of Nature. They introduced a concept of right-handed particles, but their R-world was not hidden. The term ”Mirror Matter” was introduced by Kobzarev, Okun and Pomeranchuk [4]. They first suggested the MW as the hidden (invisible) sector of our Universe, which interacts with the ordinary (visible) world only via gravity, or another very weak interaction. They have investigated a variety of phenomenological implications of such parallel worlds (see reviews [14, 15]).

Superstring theory also predicts that there may exist in the Universe another form of matter – hidden (‘shadow’) matter, which only interacts with ordinary matter via gravity or gravitational-strength interactions [5–7]. According to the superstring theory, the two worlds, ordinary and shadow, can be viewed as parallel branes in a higher dimensional space, where O-particles are localized on one brane and hidden particles – on another brane, and gravity propagates in the bulk. In Refs. [41–46] we considered the theory of the superstring-inspired $E_6$ unification with different types of breaking in the visible (O) and hidden (M) worlds.

3.2 Mirror world with broken mirror parity

If the ordinary and mirror worlds are identical, then O- and M-particles should have the same cosmological densities. But this is immediately in conflict with recent astrophysical measurements.

Astrophysical and cosmological observations (see, for example, [47–49]) have re-
vealed the existence of Dark Matter (DM) which constitutes about 23% of the total energy density of the present Universe. This is five times larger than all the visible matter, $\Omega_{DM} : \Omega_{M} \simeq 5 : 1$. In parallel to the visible world, the mirror world conserves mirror baryon number and thus protects the stability of the lightest mirror nucleon. Mirror particles have been suggested as candidates for the inferred dark matter in the Universe (see Refs. [8–17]). This theory explains the right amount of dark matter, which is generated via the mirror leptogenesis [50–54], just like the visible matter is generated via ordinary leptogenesis [55,56].

Therefore, mirror parity (MP) is not conserved, and the O- and M-worlds are not identical. In Refs. [8–12] it was suggested that the VEVs of the Higgs doublets $\phi$ and $\phi'$ are not equal:

$$\langle \phi \rangle = v, \quad \langle \phi' \rangle = v' \quad \text{and} \quad v \neq v'. \quad (21)$$

Introducing the parameter characterizing the violation of MP,

$$\zeta = \frac{v'}{v} \gg 1, \quad (22)$$

we have the estimates of Refs. [8–12] and [57–60]:

$$\zeta > 30, \quad \zeta \sim 100. \quad (23)$$

Then the masses of mirror fermions and massive bosons are scaled up by the factor $\zeta$ with respect to the masses of their counterparts in the OW:

$$m'_{q',l'} = \zeta m_{q,l}, \quad (24)$$

and

$$M'_{W',Z',\phi'} = \zeta M_{W,Z,\phi}, \quad (25)$$

while photons and gluons remain massless in both worlds.

In the language of neutrino physics, the O-neutrinos $\nu_e, \nu_\mu, \nu_\tau$ are active neutrinos, and the M-neutrinos $\nu'_e, \nu'_\mu, \nu'_\tau$ are sterile neutrinos [61]. If MP is conserved ($\zeta = 1$), then the neutrinos of the two sectors are strongly mixed (see Refs. [8–12]). However, the present experimental and cosmological limits on the active-sterile neutrino mixing do not confirm this result.

In Refs. [62,63] the exact parity symmetry explains the solar neutrino deficit, the atmospheric neutrino anomaly and the LSND experiment.

In the context of the SM, in addition to the fermions with non-zero gauge charges, one introduces also the gauge singlets, the so-called right-handed neutrinos $N_a$ with large Majorana mass terms. They have equal masses in the O- and M-worlds [8,9]:

$$M'_{\nu,a} = M_{\nu,a}. \quad (26)$$
According to the usual seesaw mechanism \cite{55, 56}, heavy right-handed neutrinos are created at the seesaw scale $M_R$ in the O-world and $M'_R$ in the M-world. From the Lagrangians, considering the identical O- and M-Yukawa couplings, we obtain:

$$m_\nu^{(i)} = \frac{v'^{2}}{M_R^{(i)}}. \quad (27)$$

The equality of the seesaw scales, $M'_R = M_R$, immediately leads to the relation between the masses of light neutrinos:

$$m'_\nu = \zeta^2 m_\nu. \quad (28)$$

Here sterile neutrinos are $\sim 10^4$ times heavier than their O-partners. Eqs. (27) and (28) were first obtained in Ref. \cite{57}.

## 4 Graviweak action in the visible and invisible sectors of the Universe

In Ref. \cite{1} we suggested to describe the gravity in the visible Universe by the self-dual left-handed Plebanski’s gravitational action, while the gravity in the invisible Universe – by the anti-self-dual right-handed gravitational action:

$$I^{(\text{gravity})}_L(A, B, \Phi) = \frac{1}{g} \int_{\mathbb{M}} [BF + B\Phi B + \frac{1}{3}B\Phi\Phi B]. \quad (30)$$

The wedge product $\langle ... \rangle$ is assumed between the forms. In this action with a parameter $g$, the connection, $A = A^{IJ}\gamma_{IJ}$, is the independent physical variable describing the geometry of the spacetime, while $\Phi$, or $\Phi_{IJKL}$, are auxiliary fields. Here, $F = dA + \frac{1}{2}[A, A]$ is the curvature and $B = B^{IJ}\gamma_{IJ}$ is a $\text{spin}(4, 4)$-valued 2-form auxiliary field. The generators $\gamma_{IJ} = \gamma_I\gamma_J$ of the $\text{spin}(4, 4)$-group have indices running over all $8 \times 8$ values: $I, J = 0, 1, 2, ..., 6, 7$.

Now we distinguish the following two actions:

1) the $\text{spin}(4, 4)_L$-invariant action $I_{\text{left}}(A, B, \Phi)$ with self-dual left-handed fields $A = A^{(+)}$, $B = B^{(+)}$ and $\Phi_{IJKL}$ – in the ordinary (visible) world OW, and
2) the spin$(4,4)_R$-invariant action $I_{\text{right}}(A', B', \Phi')$ with anti-self-dual right-handed fields $A' = A(\perp)$, $B' = B(\perp)$ and $\Phi'_{IJKL}$ in the hidden (invisible) world MW.

The action (30) is a modification of Plebanski’s action, which allows the symmetry breaking to a nontrivial vacuum expectation value (VEV).

Varying the fields $A, B$ and $\Phi$, one obtains the field equations:

$$DB = dB + [A, B] = 0,$$

where $D$ is the covariant derivative, $D^I_\mu = \delta^I J_\mu - A^I_\mu$, and

$$F = -2 \left( \Phi + \frac{1}{3} \Phi \Phi \right) B,$$

$$B^{IJ} B^{KL} = -\frac{1}{16} B^{IJ} \Phi^{KL} \Phi^{MN} B^{PQ}.$$  

The first equation (31) describes the dynamics, while (32) and (33) determine $B$ and $\Phi$.

Solving the equations of motion for $B$ in terms of $F$, we obtain the following $g$-invariant gravitational, gauge and Higgs field action:

$$I(e, A) = \frac{3}{8g} \int_M \langle FF^* \rangle.$$  

where $F = dA + \frac{1}{2}[A, A]$ and $A = A^I J_\mu$ with $I, J = 0, 1, 2, \ldots, 6, 7$.

Eqs. (31)-(34) are valid in the OW, and similar equations hold in MW for $A', B', F'$ and $\Phi'$.

For completeness, we briefly present, according to Ref. [2], the spontaneous symmetry breaking of the $g$-invariant action (34) that produces the dynamics of the SU$(2)_L$-gravity, and the SU$(2)_L$ gauge and Higgs fields with subalgebra

$$\tilde{g} = \mathfrak{su}(2)^{(\text{grav})}_L \oplus \mathfrak{su}(2)_L.$$  

The indices $a, b \in \{0, 1, 2, 3\}$ are used to sum over a subset of $I, J \in 0, 1, 2, \ldots, 7$, and thereby select a spin$(1,3)$ subalgebra of spin$(4,4)$. The indices $m, n \in \{4, 5, 6, 7\}$ sum over the rest. We also consider $i, j \in \{1, 2, 3\}$, thus selecting a su$(2)^{\text{grav}}_L$ subalgebra of spin$(4,4)_L$.

Analogous equations are valid in the MW with the initial spin$(4,4)_R$-algebra, and with a subalgebra:

$$\tilde{g}' = \mathfrak{su}(2)^{(\text{grav})}_R \oplus \mathfrak{su}(2)'_R.$$  

The spontaneous symmetry breaking of the graviweak unification in this model gives separate parts of the connection in terms of the following 2-forms:

$$A = \frac{1}{2} \omega + \frac{1}{4} E + A_W.$$  

9
Here the gravitational spin connection is:
\[ \omega = \omega^{ab} \gamma_{ab}, \tag{37} \]
or
\[ \omega = \omega^i \sigma_i, \tag{38} \]
which is valued in \( \mathfrak{su}(2)^{\text{grav}}_L \), and \( \sigma_i \) are Pauli matrices, \( i = 1, 2, 3 \).

The frame-Higgs connection
\[ E = E^{am} \gamma_{am}, \tag{39} \]
which is valued in the off-diagonal complement of \( \mathfrak{spin}(4,4) \), is assumed to have the expression:
\[ E = e \phi = e^a \sigma_a \phi^i \sigma_i dx^\mu. \tag{40} \]
The field \( \phi = \phi^i \sigma_i \) is the scalar Higgs doublet of \( \mathfrak{su}(2)_L \).

The gauge field:
\[ A_W = \frac{1}{2} A^{mn} \gamma_{mn}, \tag{41} \]
or
\[ A_W = \frac{1}{2} A^i_W \tau_i, \tag{42} \]
is valued in \( \mathfrak{su}(2)_L \).

Analogous equations exist in MW for \( A', \omega', E', A'_W \).

Finally, using the results of Refs. [1] and [2], we have the following actions for gravitational, the \( SU(2)_L \) and \( SU(2)'_R \) gauge and Higgs fields in the ordinary and hidden sectors of the Universe, respectively:
\[ I^{(\prime)}(e^{(\prime)}, \phi^{(\prime)}, A^{(\prime)}, A'^{(\prime)}_W) = \frac{3}{8g} \int_{M} d^4 x |e^{(\prime)}| \left( -\frac{1}{16} |\phi^{(\prime)}|^2 R^{(\prime)} + \frac{3}{32} |\phi^{(\prime)}|^4 + \frac{1}{16} R^{(\prime)}_{ab} \right) \right), \tag{43} \]

Here \( g = g', R^{(\prime)} \) is the Riemann curvature scalar, \( |\phi|^2 = \phi^\dagger \phi \) is the squared magnitude of the Higgs field, \( D \phi = d \phi + [A_W, \phi] \) is the covariant derivative of the Higgs, and \( F_W = dA_W + [A_W, A_W] \) is the curvature of the gauge field. Similar notations are made for \( A', \phi', A'_W \). The third term of (43) is a modification to the standard gravitation related to the Gauss–Bonnet topological action [64, 65].

The nontrivial vacuum solutions to the actions (43) give the non-vanishing Higgs vacuum expectation values (VEVs): \( \langle \phi^{(\prime)} \rangle = \phi^{(\prime)}_0 \), at which the standard Higgs potentials have an extremum corresponding to a de Sitter spacetime background solution:
\[ \langle \phi^{(\prime)} \rangle^2 = \frac{R^{(\prime)}_0}{3} \tag{44} \]
Here $R_0^{(i)} \neq 0$ is a constant background scalar curvature. We see that in this model the spacetime geometry determines the Higgs VEVs, e.g. the masses of the SM and SM’ particles.

After the symmetry breaking of the graviweak unification in both worlds, we obtain:

1) the Newton’s constants in the OW and MW of our Universe, respectively, are equal to

$$G_N^{(i)} = \frac{128g}{3v^{(i)}},$$

(45)

2) the cosmological constants $\Lambda^{(i)}$ (given by the second term of (43)) are equal to

$$\Lambda^{(i)} = \frac{3}{4}v^{(i)}^2,$$

(46)

3) the weak coupling constants:

$$g_{W}^{(i)} = \frac{8}{3}.$$  

(47)

Finally, we have the following relations:

$$G_N^{ \prime} = \frac{G_N}{\zeta^2}, \quad \Lambda^{ \prime} = \zeta^2\Lambda, \quad M_{Pl}^{\prime} = \zeta M_{Pl}, \quad g_{W}^{\prime} = g_{W}.$$  

(48)

All physical constants are determined by a parameter $g$ and the Higgs VEVs $v^{(i)}$. Here we assume that the equality $g^{\prime} = g$ is a consequence of the existence of the Grand Unification at the early stage of the Universe when the mirror parity was unbroken.

Of course, the present model is too simple to have the correct phenomenological applications. It is necessary to note that all parameters – Newton’s constants, the cosmological constants, the gauge couplings $g_{YM} = g_{W}$, etc., considered in (43) – are the bare parameters, which refer to the Planck scale (see, for example, the Planck scale physics in Refs. [66–68]).

Using the experimentally known values of $G_N$, where $M_{Pl}^{red.} = 1/\sqrt{8\pi G_N} \approx 2.43 \cdot 10^{18}$ GeV, and $v \approx 246$ GeV, we can obtain the value of $g$ from Eq. (45). However, we cannot relate the value $g_{W}^{\prime} = g_{W}^2/3$ with the value of $g_{W}^2$ obtained by the extrapolation of experimental values of running $\alpha_2 = g_{2}^2/4\pi$ from the electroweak scale to the Planck scale. The reason is that such a running $\alpha_2$ includes the electroweak breaking symmetry and also is determined by the fermion (quark) contributions, which are absent in the present model. Nevertheless, the explicit relation between the gravitational and Yang–Mills couplings is a consequence of a genuine unification of gravity and Yang–Mills theory.

5 Communications between Visible and Hidden worlds

The broken MP means that parity violation applies not only for the weak interaction, but also in the gravitational sector. The parity violation of gravity was considered in...
Ref. [69], where the authors suggested to find the effect of birefringence amplitude of gravitational waves, whereby left and right circularly-polarized waves propagate at the same speed but with different amplitude evolution. A test of this effect was proposed through coincident observations of gravitational waves and short gamma-ray bursts from binary mergers involving neutron stars. Such gravitational waves are highly left or right circularly-polarized due to the geometry of the merger. All sky gamma-ray telescopes can be sensitive to the propagating sector of gravitational parity violation.

The dynamics of the two worlds of our Universe, visible and hidden, is governed by the following action (see [1]):

\[
I = \int [L_{(\text{grav})} + L'_{(\text{grav})} + L_{\text{SM}} + L'_{\text{SM}} + L_{(\text{mix})}] |\epsilon| d^4x, \quad (49)
\]

where \( L_{(\text{grav})} \) is the gravitational (left-handed) Lagrangian in the visible world, and \( L'_{(\text{grav})} \) is the gravitational right-handed Lagrangian in the hidden world, \( L_{\text{SM}} \) and \( L'_{\text{SM}} \) are the Standard Model Lagrangians in the O- and M-worlds, respectively, \( L_{(\text{mix})} \) is the Lagrangian describing all mixing terms (see [8–14]) giving small contributions to physical processes: mirror particles have not been seen so far, and the communication between visible and hidden worlds is hard.

There are several fundamental ways by which the hidden world can communicate with our visible world. The Lagrangian \( L_{(\text{mix})} \) describes all possible mixing terms which are consistent with the symmetries of the theory:

\[
L_{\text{mix}} = \alpha F^i \wedge F'^i + \frac{\epsilon_Y}{2} F_{Y,\mu\nu} F'^{\mu\nu}_Y + \lambda_2 \phi^\dagger \phi \phi'^\dagger \phi', \quad (50)
\]

where the first term is the gravitational mixing term (see [40]), \( F_{Y,\mu\nu} \) (\( F'^{\mu\nu}_Y \)) is the \( U(1)_Y \) (\( U(1)'_Y \)) field strength tensor and \( \phi \) (\( \phi' \)) is the ordinary (mirror) Higgs doublet (see [70]). The last term of Eq. (50) provides the ‘Higgs-mirror Higgs’ mass mixing that can lead to non-standard Higgs boson physics at the LHC [70].

The left-handed gravity interacts not only with visible matter, but also with mirror matter. The right-handed gravity also interacts with matter and mirror matter. We assume that a fraction of the mirror matter exists in the form of mirror galaxies, mirror stars, mirror planets etc., (see, for example, Ref. [71]). These objects can be detected using gravitational microlensing [72].

There exists the kinetic mixing between the electromagnetic field strength tensors for visible and mirror photons (see Refs. [73,74] and [75]):

\[
L_{\text{mix}}^\gamma = \frac{\epsilon_\gamma}{2} F_{\gamma,\mu\nu} F^{\mu\nu}_\gamma. \quad (51)
\]

The ‘photon–mirror photon’ mixing induces the ‘orthopositronium–mirror orthopositronium’ oscillations [73,74]. Future experiments with orthopositronium were suggested in Ref. [76].
Interactions between visible and mirror quarks and leptons are expected to take place. Mirror neutrons can oscillate to ordinary neutrons giving 'neutron–mirror neutron' oscillations (Refs. [77–80]).

Mass mixings between visible and mirror (sterile) neutrinos lead to the 'neutrino–mirror neutrino' oscillations what was suggested by Ref. [57] (see also [81, 82] and [14]).

Heavy Majorana neutrinos $N_a$ are singlets of $G_{SM}$ and $G'_{SM}$, and they can be messengers between visible and hidden worlds (see Refs. [50–54]).

Also any weakly interacting singlet scalar field can be a messenger between OW and MW.

The search for mirror particles at the LHC is discussed in Ref. [83].

6 Dark Energy of the Universe

In both worlds, O and M, the Einstein equations

$$R^\nu_\mu - \frac{1}{2}R^\nu = 8\pi G_N T^\nu_\mu - \Lambda_N^\nu$$

(52)

contain the energy momentum tensor of matter $T^\nu_\mu$.

In our theory the dark energy density of the Universe is:

$$\rho_{DE} = \rho_{vac} = \frac{\Lambda_{eff}}{8\pi G_N} + \frac{\Lambda'_{eff}}{8\pi G'_N}$$

(53)

where

$$\frac{\Lambda_{eff}'}{8\pi G'_N} = \frac{\Lambda'}{8\pi G'_N} + \rho_{vac}^{(SM')}$$

(54)

and $\Lambda'$ are bare cosmological constants (46) of our theory.

Astrophysical measurements [47–49] give:

$$\rho_{DE} \approx 0.73\rho_{tot} \approx (2.3 \times 10^{-3} \text{ eV})^4.$$  

(55)

Recalling that $M_{Pl}^{red} = (8\pi G_N)^{-1} \text{ GeV}$ and $v' = \zeta v$, we obtain:

$$\rho_{DE} = \rho_{vac} = \frac{3}{4}v^2(1 + \zeta^4)(M_{Pl}^{red,2})^2 + \rho_{vac}^{(SM)} + \rho_{vac}^{(SM')}.$$

(56)

All quantum fluctuations of the matter contribute to the vacuum energy density $\rho_{vac}$ of the Universe. We expect that quantum field theory is valid up to some cut-off scale $M_{(cut-off)}$, and the vacuum energy density in the SM and SM' is evaluated by the sum of zero-point energies of quantum fields. Then we have:

$$\rho_{DE} = \rho_{vac} = \frac{3}{4}v^2(1 + \zeta^4)(M_{Pl}^{red,2})^2 + C(M_{(cut-off)})^4 + C'(M'_{(cut-off)})^4.$$

(57)
The tiny value of the dark energy density $\rho_{DE} \approx (2.3 \times 10^{-3} \text{ eV})^4$, verified by astronomical and cosmological observations, leads to the conclusion that

$$\frac{3}{4} v^2 (1 + \zeta^4)(M_{Pl}^{\text{red.}})^2 \sim -C M_{(\text{cut-off})}^4 - C' M'_{(\text{cut-off})}.$$  \hspace{1cm} (58)$$

This means that all matter quantum fluctuations in SM and SM’ must be almost compensated by the contribution of cosmological constants $\Lambda$ and $\Lambda'$. This compensation is possible if $C, C' < 0$, what means the dominance of degrees of freedom (DOF) of fermions in the the sum of zero-point energies of quantum fields.

Assuming $C_1^{(i)} = | - C^{(i)} |^{1/4} \sim 1$, we have:

$$\frac{3}{4} v^2 (1 + \zeta^4)(M_{Pl}^{\text{red.}})^2 \sim M_{(\text{cut-off})}^4 + M'_{(\text{cut-off})}.$$  \hspace{1cm} (59)$$

According to Eq. (58), we have:

$$M_{(\text{cut-off})} = \zeta M_{(\text{cut-off})},$$  \hspace{1cm} (60)$$

what means that

$$\frac{3}{4} v^2 (1 + \zeta^4)(M_{Pl}^{\text{red.}})^2 \sim (1 + \zeta^4)M_{(\text{cut-off})}^4.$$  \hspace{1cm} (61)$$

Using $v \approx 246 \text{ GeV}$ and $M_{Pl}^{\text{red.}} \approx 2.43 \cdot 10^{18} \text{ GeV}$, we obtain:

$$M_{(\text{cut-off})} \sim \sqrt{v M_{Pl}^{\text{red.}}} \sim 2.3 \cdot 10^{10} \text{ GeV.}$$  \hspace{1cm} (62)$$

This result means that in the framework of graviweak unification, the cut-off scale is less than Planck scale.

The existence of supersymmetry can explain the tiny value of $\rho_{DE}$. If the supersymmetry breaking scale $M_{SUSY}$ coincides with the cut-off scale (62), we obtain:

$$M_{SUSY} \sim 10^{10} \text{ GeV.}$$  \hspace{1cm} (63)$$

We see that the supersymmetry breaking scale is essentially large, and not within the reach of the LHC experiments. The next possibility to reduce the value $M_{SUSY}$, given by Eq. (63), is to assume the existence of large negative contributions to $\Lambda^{(i)}$, or to assume that $C_1^{(i)} \gg 1$. Unfortunately, both these assumptions are doubtful, and we hope that the future LHC result for $M_{SUSY}$ will shed light on these issues.

7 Summary and Conclusions

In this paper we developed a graviweak unification of gravity and the SU(2) gauge and Higgs fields, assuming the existence of the hidden sector of the Universe.
Developing ideas of Refs. [1, 2], we presented a graviweak unification model in the visible and invisible parts of the Universe. We started with an extended $g = \text{spin}(4, 4)_L$-invariant Plebanski action in the OW, and $g = \text{spin}(4, 4)_R$-invariant Plebanski action in the MW.

We showed that the graviweak symmetry breaking leads to the following subalgebras: $\tilde{g} = \text{su}(2)^{(\text{grav})}_L \oplus \text{su}(2)_L$ – in the ordinary world, and $\tilde{g}' = \text{su}(2)^{(\text{grav})}_R \oplus \text{su}(2)'_R$ – in the hidden world. These subalgebras contain the self-dual left-handed gravity in the OW, and the anti-self-dual right-handed gravity in the MW.

The nontrivial vacuum solutions corresponding to the obtained actions are non-vanishing Higgs vacuum expectation values (VEVs): $v(\phi^{(i)}) = \langle \phi(\phi') \rangle = \phi(\phi')_0$.

Using recent astrophysical and cosmological measurements, we considered a model of the Mirror World with broken mirror parity (MP), which assumes that the Higgs VEVs in the visible and invisible worlds are not equal: $\langle \phi \rangle = v$, $\langle \phi' \rangle = v'$ and $v \neq v'$. Introducing the parameter characterizing the violation of MP, $\zeta = \frac{v}{v'} \gg 1$, we have used the estimate $\zeta \sim 100$.

In this model, we showed that the action for gravity, and the $SU(2)$ Yang–Mills and Higgs fields, constructed in the ordinary world, has a modified duplication for the hidden sector of the Universe. Considering the graviweak unification in the both worlds of the Universe, we obtained, after symmetry breaking, Newton’s constants $G^{(\phi)}_N = \frac{128g}{3607}$, and the cosmological constants $\Lambda^{(\phi)} = \frac{4}{7}v^{(\phi)^2}$, which are not equal in the OW and MW. The mirror cosmological constant $\Lambda' = \zeta^2\Lambda$ is larger than the ordinary cosmological constant, while the OW and MW Yang–Mills coupling constants are equal: $g^{(\phi)}_{YM} = g_Y^2 = \frac{3}{7}$. We also discussed the problems of communications between visible and invisible worlds. Mirror particles have not been seen so far in the visible world, and the communication between visible and hidden worlds is hard. This communication is given by the $L_{(\text{mix})}$-term of the total Lagrangian of the Universe, and the contributions of processes described by $L_{(\text{mix})}$ are extremely small.

Finally, we considered the problem of the DE. The recent astrophysical and cosmological measurements give a tiny value of the dark energy density $\rho_{DE} = \rho_{\text{vac}} \simeq (2.3 \times 10^{-3} \text{ eV})^4$, what means that the sum $\frac{\Lambda}{8\pi G_N} + \frac{\Lambda'}{8\pi G'_N}$ are almost compensated by the sum $\rho_{\text{vac}} + \rho_{\text{vac}}^{(SM')}$. This compensation is possible if we have the dominance of DOF of fermions in the sum of zero-point energies of all quantum field fluctuations. In the framework of the present graviweak unification model with broken symmetry, the estimate gives a cut-off scale, which is less than the Planck scale. If this scale is equal to the supersymmetry breaking scale, then it is extremely large: $M_{SUSY} \sim 10^{10}$ GeV, and not within the reach of the LHC experiments. A possibility to reduce this value of $M_{SUSY}$ is also discussed.
8 Acknowledgments

We thank Masud Chaichian and Tiberiu Harko for fruitful discussions. L.V. Laperashvili deeply thanks A. Garrett Lisi, also O.V. Kancheli, R.B. Nevzorov, V.A. Novikov, M.A. Trusov and all members of the ITEP Theoretical Seminar (Moscow, May, 16, 2013) for the interesting discussion and advices. The support of the Academy of Finland under the Projects No. 136539 and 140886 is gratefully acknowledged. CRD acknowledges a scholarship from the Fundação para a Ciência e a Tecnologia (FCT, Portugal) (ref. SFRH/BPD/41091/2007). This work was partially supported by FCT through the projects CERN/FP/123580/2011 PTDC/FIS/ 098188/2008 and CFTP-FCT Unit 777 which are partially funded through POCTI (FEDER).

References

[1] D.L. Bennett, L.V. Laperashvili, H.B. Nielsen and A. Tureanu, Int. J. Mod. Phys. A 28, 1350035 (2013). arXiv:1206.3497.

[2] A. Garrett Lisi, L. Smolin and S. Speziale, J. Phys. A 43, 445401 (2010), arXiv:1004.4866.

[3] T.D. Lee and C.N. Yang, Phys. Rev. 104, 254 (1956).

[4] I.Yu. Kobzarev, L.B. Okun and I.Ya. Pomeranchuk, Yad. Fiz. 3, 1154 (1966) [Sov. J. Nucl. Phys. 3, 837 (1966)].

[5] K. Nishijima and M.H. Saffouri, Phys. Rev. Lett. 14, 205 (1965).

[6] E.W. Kolb, D. Seckel and M.S. Turner, Nature 314, 415 (1985).

[7] E.W. Kolb, D. Seckel and M.S. Turner, Report Fermilab-Pub-85/16-A (1985).

[8] Z. Berezhiani, A. Dolgov and R.N. Mohapatra, Phys.Lett. B 375, 26 (1996), hep-ph/9511221.

[9] Z. Berezhiani, Through the looking-glass: Alice’s adventures in mirror world, in: Ian Kogan Memorial Collection “From Fields to Strings: Circumnavigating Theoretical Physics”, Eds. M. Shifman et al., World Scientific, Singapore, Vol. 3, pp. 2147-2195, 2005, hep-ph/0508233.

[10] R. Foot, H. Lew, and R.R. Volkas, Phys. Lett. B 272, 67 (1991).

[11] R. Foot, Mod. Phys. Lett. A 9, 169 (1994), hep-ph/9402241.

[12] R. Foot, Int. J. Mod. Phys. D 13, 2161 (2004), astro-ph/0407623.
[13] S.I. Blinnikov and M.Yu. Khlopov, Sov. Astron. 27, 371 (1983) [Astron. Zh. 60, 632 (1983)].

[14] L.B. Okun, Phys. Usp. 50, 380 (2007), hep-ph/0606202.

[15] S.I. Blinnikov, Phys. Atom. Nucl. 73, 593 (2010), arXiv:0904.3609.

[16] P. Ciarcelluti, Int. J. Mod. Phys. D 19, 2151 (2010), arXiv:1102.5530 and references therein.

[17] Jian-Wei Cui, Hong-Jian He, Lan-Chun Lu and Fu-Rong Yin, Phys. Rev. D 85, 096003 (2012), arXiv:1110.6893 [hep-ph].

[18] J.F. Plebanski, J. Math. Phys. 18, 2511 (1977).

[19] A. Ashtekar, Phys. Rev. Lett. 57, 2244 (1986).

[20] A. Ashtekar, Phys. Rev. D 36, 1587 (1987).

[21] T. Jacobson and L. Smolin, Phys. Let. B 196, 39 (1987).

[22] R. Capovilla, T. Jacobson, J. Dell and L.J. Mason, Class. Quant. Grav. 8, 41 (1991).

[23] R. Capovilla, T. Jacobson and J. Dell, Class. Quant. Grav. 8, 59 (1991).

[24] S. Alexander, Isogravity: Toward an Electroweak and Gravitational Unification, arXiv:0706.4481.

[25] F. Nesti, Eur. Phys. J. C 59, 723 (2009), arXiv:0706.3304.

[26] F. Nesti and R. Percacci, J. Phys. A 41, 075405 (2008), arXiv:0706.3307.

[27] A.G. Lisi, An Exceptionally Simple Theory of Everything, arXiv:0711.0770.

[28] F. Nesti and R. Percacci, Phys. Rev. D 81, 025010 (2010), arXiv:0909.4537.

[29] A. Torres-Gomez and K. Krasnov, Phys. Rev. D 81, 085003 (2010), arXiv:0911.3793.

[30] L. Smolin, Phys. Rev. D 80, 124017 (2009), arXiv:0712.0977.

[31] K. Krasnov, Gen. Rel. Grav. 43, 1 (2011), arXiv:0904.0423.

[32] K. Krasnov, Class. Quant. Grav. 26, 055002 (2009), arXiv:0811.3147.

[33] K. Krasnov, Class. Quant. Grav. 25, 025001 (2008), gr-qc/0703002.

[34] K. Krasnov, Mod. Phys. Lett. A 22, 3013 (2007), arXiv:0711.0697.
[35] M.P. Reisenberger, Class. Quant. Grav. 16, 1357 (1999), gr-qc/9804061.

[36] Eyo Eyo Ita III, CDJ formulation from the instanton representation of Plebanski gravity, arXiv:0911.0604

[37] E. Buffenoir, M. Henneaux, K. Noui and Ph. Roche, Class. Quant. Grav. 21, 5203 (2004), gr-qc/0404041.

[38] F. Tennie and M.N.R. Wohlfarth, Phys. Rev. D 82, 104052 (2010), arXiv:1009.5595 [gr-qc].

[39] D.L. Bennett, C.R. Das, L.V. Laperashvili and H.B. Nielsen, Int. J. Mod. Phys. A 28, 1350044 (2013), arXiv:1209.2155.

[40] S. Alexander, A. Marciano and L. Smolin, Gravitational origin of the weak interaction’s chirality, arXiv:1212.5246.

[41] C.R. Das and L.V. Laperashvili, Phys. Atom. Nucl. 72, 377 (2009), [Yad.Fiz. 72, 407 (2009)].

[42] C.R. Das and L.V. Laperashvili, Int. J. Mod. Phys. A 23, 1863 (2008), arXiv:0712.1326.

[43] C.R. Das, L.V. Laperashvili and A. Tureanu, Eur. Phys. J. C 66, 307 (2010), arXiv:0902.4874.

[44] C.R. Das, L.V. Laperashvili and A. Tureanu, AIP Conf. Proc. 1241, 639 (2010), arXiv:0910.1669.

[45] C.R. Das, L.V. Laperashvili and A. Tureanu, Phys. Part. Nucl. 41, 965 (2010), arXiv:1012.0624.

[46] C.R. Das, L.V. Laperashvili, H.B. Nielsen and A. Tureanu, Phys. Rev. D 84, 063510 (2011), arXiv:1101.4558.

[47] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).

[48] A. Riess et al., Astrophys. J. Suppl. 183, 109 (2009), arXiv:0905.0697.

[49] W.L. Freedman et al., Astrophys. J. 704, 1036 (2009), arXiv:0907.4524.

[50] Z. Berezhiani, Int. J. Mod. Phys. A 19, 3775 (2004), hep-ph/0312335.

[51] L. Bento and Z. Berezhiani, Phys. Rev. Lett. 87, 231304 (2001), hep-ph/0107281.

[52] L. Bento and Z. Berezhiani, Baryogenesis: The Lepton leaking mechanism, hep-ph/0111116.
[53] L. Bento and Z. Berezhiani, Fortsch. Phys. 50, 489 (2002).

[54] C.R. Das, L.V. Laperashvili, H.B. Nielsen and A. Tureanu, Phys. Lett. B 696, 138 (2011), arXiv:1010.2744

[55] M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).

[56] W. Buchmuller, R.D. Peccei and T. Yanagida, Ann. Rev. Nucl. Part. Sci. 55, 311 (2005), arXiv:hep-ph/0502169.

[57] E.K. Akhmedov, Z.G. Berezhiani and G. Senjanovic, Phys. Rev. Lett. 69, 3013 (1992), hep-ph/9205230.

[58] Z. Berezhiani, D. Comelli and N. Tetradis, Phys. Lett. B 431, 286 (1998), hep-ph/9803498.

[59] Z. Berezhiani, P. Ciarcelluti, D. Comelli and F.L. Villante, Int. J. Mod. Phys. D 14, 107 (2005), astro-ph/0312605.

[60] Z. Berezhiani, L. Kaufmann, P. Panci, N. Rossi, A. Rubbia and A. Sakharov, Strongly interacting mirror dark matter, CERN-PH-TH-2008-108 (May 2008).

[61] Z. Berezhiani and R. N. Mohapatra, Phys. Rev. D 52, 6607 (1995), hep-ph/9505385.

[62] R. Foot, H. Lew and R.R. Volkas, Mod. Phys. Lett. A 7, 2567 (1992).

[63] R. Foot and R.R. Volkas, Phys. Rev. D 52, 6595 (1995), hep-ph/9505359.

[64] E.W. Mielke, Phys. Rev. D 77, 084020 (2008), arXiv:0707.3466.

[65] G. de Berredo-Peixoto and I.L. Shapiro, Phys. Rev. D 70, 044024 (2004), hep-th/0307030.

[66] L.V. Laperashvili, Phys. Atom. Nucl. 57, 471 (1994) [Yad.Fiz. 57, 501 (1994)]

[67] D.L. Bennett, L.V. Laperashvili and H.B. Nielsen, What Comes Beyond the Standard Models? Proceedings, 9th Workshop, Bled, Slovenia, September 16-27, 2006 (DMFA, Zaloznistvo, Ljubljana, 2006), hep-ph/0612250.

[68] D.L. Bennett, L.V. Laperashvili and H.B. Nielsen, What Comes Beyond the Standard Models?, Proceedings, 10th Workshop, Bled, Slovenia, July 17-27, 2007 (DMFA, Zaloznistvo, Ljubljana, 2007), arXiv:0711.4681.

[69] N. Yunes, R.O’ Shaughnessy, B.J. Owen and S. Alexander, Phys. Rev. D 82, 064017 (2010), arXiv:1005.3310.
[70] R. Foot, A. Kobakhidze and R.R. Volkas, Phys. Rev. D 84, 09503 (2011), arXiv:1109.0919.

[71] Z. Berezhiani, L. Pilo and N. Rossi, Eur. Phys. J. C 70, 305 (2010), arXiv:0902.0146.

[72] R.N. Mohapatra and V.L. Teplitz, Phys. Lett. B 462, 302 (1999), astro-ph/9902085.

[73] S.L. Glashow, Phys. Lett. B 167, 35 (1986).

[74] A. Badertscher et al., Int. J. Mod. Phys. A 19, 3833 (2004), hep-ex/0311031.

[75] Z. Berezhiani and A. Lepidi, Phys. Lett. B 681, 276 (2009), arXiv:0810.1317.

[76] S.V. Demidov, D.S. Gorbunov and A.A. Tokareva, Phys. Rev. D 85, 015022 (2012), arXiv:1111.1072.

[77] Z. Berezhiani and L. Bento, Phys. Rev. Lett. 96, 081801 (2006), hep-ph/0507031.

[78] Z. Berezhiani and L. Bento, Phys. Lett. B 635, 253 (2006), hep-ph/0602227.

[79] R.N. Mohapatra, S. Nasri and S. Nussinov, Phys. Lett. B 627, 124 (2005), hep-ph/0508109.

[80] Yu.N. Pokotilovski, Phys. Lett. B 639, 214 (2006), nucl-ex/0601017.

[81] Z.K. Silagadze, Phys. Atom. Nucl. 60, 272 (1997) [Yad. Fiz. 60, 336 (1997)], hep-ph/9503481.

[82] V. Berezinsky and A. Vilenkin, Phys. Rev. D 62, 083512 (2000), hep-ph/9908257.

[83] A.Yu. Ignatiev and R.R. Volkas, Phys. Lett. B 487, 294 (2000), hep-ph/0005238.