The exact quantization of the CLEO and BELLE data for $D$ mass differences by the harmonic quarks and oscillators

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Abstract

The harmonic quarks and their complete oscillators are successfully used for the exact quantitative results and an explanation of the $D$ meson mass differences. For the first time it is shown that the mass differences of the charmed mesons with the same flavors are strictly quantized by the rest masses of harmonic quarks in both "free" state and complete oscillator state. The $D^*(2007)^0 \rightarrow D^0$ transition has the following neutral quark group: $d$-oscillator + $u^0$-quark with rest energy 142.124 MeV. It is argued that it is the unique model solution within the $4\sigma$ experimental interval. The $D_s^{*+} - D_s^+$ mass difference is quantized by the simplest quark reaction with the difference energy 143.756 MeV. The energy of the $D_{sJ}(2460)^+ \rightarrow D_s^+$ transition explains by the decay of the one complete $s$-oscillator with the energy 491.33 MeV. The full agreement with experimental data of CLEO and BELLE is obtained. The problems of a quark shells and the quantization by quark rest masses are discussed.

1 Introduction

In this article the harmonic quarks and their complete oscillators are used for a decoding of the mass differences of the $D$ mesons.

The articles [1, 2, 3] have shown that the harmonic quarks are powerful and effective tool for study of hadronic structures. This tool works equally well with both ground energy levels, and resonances. The harmonic quarks and oscillators allow decrypt the mass spectrum of charmed mesons [4] completely and with great precision. Simultaneously, there was established that the model harmonic spectrum is not sensitive to the isospin. In other words, the model harmonic spectrum is a degenerate spectrum in relation to this property. At the same time, the levels of harmonic spectrum of charmed mesons are only quantized by masses of the harmonic quarks and their oscillators of accessible flavors. All levels are the sums of the rest masses of the harmonic quarks and oscillators. Therefore we may suppose that both real masses of charmed hadrons and the mass differences between
them can also be quantized by similar combinations. The investigation of this assumption can help us resolve the open questions of the hadronic physics, such as the hadronization stage, or the electromagnetic split of a levels.

The present paper can be seen as the continuation of the previous work \cite{1}, with the only difference, that it will be oriented to a study of the energy structure of the real transitions in charmed mesons. With the help of the most precise experimental data \cite{5,7,8,9} we shall discover the groups of the harmonic quarks and oscillators which will be in full agreement with experience data.

Here we shall use the same labels as in \cite{1} with the standard notation for the harmonic quarks \((d, \, u, \, s, \, c)\), and circle-enclosed quarks of the complete harmonic oscillators \((\bar{d}, \, \bar{u}, \, \bar{s}, \, \bar{c})\).

The masses of harmonic quarks and complete harmonic oscillators are given in the table 1.

| Quark | Quark mass, MeV | Harmonic oscillator | Oscillator energy, MeV |
|-------|----------------|---------------------|-----------------------|
| \(d\) | 28.8106        | \(\bar{d}(\bar{d})\) | 36.683                |
| \(u\) | 105.441         | \(\bar{u}(\bar{u})\) | 134.251               |
| \(s\) | 385.891         | \(\bar{s}(\bar{s})\) | 491.332               |
| \(c\) | 1412.28         | \(\bar{c}(\bar{c})\) | 1798.17               |
| \(b\) | 5168.7          | \(\bar{b}(\bar{b})\) | 6581.0                |

2 Transitions between \(D\) mesons

The most precise experimental data about mass differences of the charmed mesons are given in the table 2. In general we shall use the consistent data of PDG \cite{6}, and also the most precise data of CLEO collaboration for transitions \(D_{s}^{*+}\) to \(D_{s}^{+}\) \cite{9} and, especially, \(D^{*0}\) to \(D^{0}\) and \(D^{*+}\) to \(D^{0}\) \cite{5,8}. The data for 5 transitions in table 2 are given in decreasing order of precision. All transitions occur without change of the strangeness with formation of stable \(D\) mesons at decays of the resonances.

Decay \(D_{s}^{+}\) to \(D\) will not be considered, since it arises from a weak interaction.
Table 2. The experimental data for transitions between $D$ mesons with the same flavors.

| Transition | Energy of transition, MeV | Main channels of decay | Probability of decay, % |
|------------|---------------------------|------------------------|-------------------------|
| $(1^-)D(2010)^+\rightarrow(0^-)D^0$ | $145.412 \pm 0.002 \pm 0.012$ [8] | $D^{*+} \rightarrow D^{0}\pi^+$ | 68 |
| | $145.421 \pm 0.010$ [6] | | |
| | $142.12 \pm 0.05$ [5] | $D^{*0} \rightarrow D^{0}\pi^0$ | 62 |
| | | $D^{*0} \rightarrow D^{0}\gamma$ | 38 |
| $(1^-)D(2010)^+\rightarrow(0^-)D^+$ | $140.64 \pm 0.08 \pm 0.06$ [5] | $D^{*+} \rightarrow D^{+}\pi^0$ | 31 |
| | $140.64 \pm 0.10$ [6] | $D^{*+} \rightarrow D^{+}\gamma$ | 1.6 |
| $(1^-)D_s(2112)^+\rightarrow(0^-)D_s^+$ | $143.76 \pm 0.39 \pm 0.4$ [9] | $D_s^{*+} \rightarrow D_s^{+}\pi^0$ | 94 |
| | $143.8 \pm 0.4$ [6] | $D_s^{*+} \rightarrow D_s^{+}\gamma$ | 6 |
| $(1^+)D_{sJ}(2460)^+\rightarrow(0^-)D_s^+$ | $491.4 \pm 0.9 \pm 1.5$ [7] | $D_{sJ}^{+}\rightarrow D_s^{+}\gamma$ | 94 |
| | $491.0 \pm 1.2$ [8](FIT) | $D_{sJ}^{+}\rightarrow D_s^{+}\pi^0$ | 6 |
| | $491.3 \pm 1.4$ [6](AVR) | $D_{sJ}^{+}\rightarrow D_s^{+}\pi^+\pi^-$ | seen |

The most precise data was obtained by collaboration CLEO [5, 8, 9]. We can calculate the energies of the same transitions (see table 3), using the model harmonic spectrum for the charmed mesons [1] with the aforementioned model restriction of not considering electromagnetic split.

Table 3. The energy differences of harmonic levels and their quark representation.

| Transition | Difference between harmonic levels, MeV [11] | Quark representation of difference |
|------------|---------------------------------------------|----------------------------------|
| $(1^-)D(2010)^+\rightarrow(0^-)D^0$ | $143.756 \pm 0.007$ | $u\bar{u} \rightarrow (\bar{u})^0$ |
| $(1^-)D(2007)^0\rightarrow(0^-)D^0$ | $143.756 \pm 0.007$ | $u\bar{u} \rightarrow (\bar{u})^0$ |
| $(1^-)D(2010)^+\rightarrow(0^-)D^+$ | $143.756 \pm 0.007$ | $u\bar{u} \rightarrow (\bar{u})^0$ |
| $(1^-)D_s(2112)^+\rightarrow(0^-)D_s^+$ | $141.86 \pm 0.007$ | $1/2(\bar{s}s + u\bar{u}) \rightarrow (\bar{u}u) + (d\bar{d})$ |
| $(1^+)D_{sJ}(2460)^+\rightarrow(0^-)D_s^+$ | $491.33 \pm 0.025$ | $(s\bar{s}) \rightarrow 0$ |

The data in table 3 is clearly showing to us the limits of the model spectrum with respect to isospin. Only transitions between $D_s$ singlet states are simple and have single meanings. There are two important coincidences that should be noted when comparing the experimental and model data in tables 2 and 3.

First of all, the experimental and model data for transition $D_{sJ}(2460)^+ \rightarrow D_s^+$ agree completely: $491.3 \pm 1.4$ (PDG, AVERAGE) and $491.33 \pm 0.025$ MeV.

This can only mean one thing: there’s an annihilation of a neutral harmonic oscillator $s\bar{s}$ in the decay $D_{sJ}(2460)^+\rightarrow D_s^+$. Apparently, $D_{sJ}(2460)^+$ just loses one neutral shell in its structure. With the annihilation of a neutral strange oscillator $491.33$ MeV
is released and it is subsequently used to form the neutral $\gamma$ (or a group $\pi^+\pi^-$) and the some part goes to kinetic energy.

Secondly, the energy of the model quark reaction $u\bar{u} \rightarrow \bar{u}^0$ completely agrees with the experimental energy of the transition $D_s^*(2112)^+ \rightarrow D_s^+$: $143.756 \pm 0.007$ and $143.76 \pm 0.39 \pm 0.4$ MeV (CLEO) respectively. Hence, the quark reaction $u\bar{u} \rightarrow \bar{u}^0$ really occurs at decay $D_s^*(2112)^+$, instead of the model reaction with almost equal energy which is given in table 3. Quark reaction of this kind was considered earlier in [1].

These two facts support our guess about a quantization of the $D$ meson transitions by the quark masses.

These transitions have a lot in common: they both happen between singlet states ($D_s$), the spin decreases on 1 in both of them and they both generate $D_s^+$. Therefore it can be said that the transitions $D_s^+$ into excited states (inverse transitions are experimentally observed) are strictly quantized by harmonic quark masses.

Let’s get to the next problem. Whether the most precise experimental data in table 2 can be similarly represented by the rest masses of quark groups?

All unconsidered mass differences relate to the transitions between the $D$ and the $D^*$ doublet states. The three of them are experimentally observed, and the fourth transition $D^{*0} \rightarrow D^+$ can only be caused by weak coupling and is not considered here. Only one of them – $D^*(2007)^0 \rightarrow D^0$ (see tab.2) with energy 142.12 MeV is similar to the transitions examined above. The decay of the $D^*(2007)^0$, moreover, with a high probability (38%) has the mode $D^0\gamma$, while the decay $D^{*+}$ has the mode $D^{+}\gamma$ with a much smaller probability (1.6%).

The energy of transition $D^*(2007)^0 \rightarrow D^0$ is quantizing excellently. One just has to subtract from it the energy of the oscillator $\bar{d}d^0$ from it: $142.12 - 36.68 = 105.44$ (MeV) to obtain a rest mass of $u$ quark with 5 decimal digits precision! Thus the mass difference $\Delta M \equiv M_{D^*(2007)^0} - M_{D^0}$ can be written as:

$$\Delta M = M_u + M_{\bar{d}d^0} = 142.124 \pm 0.007(MeV)$$

(1)

There is only two neutral quark groups in the energy range from the oscillator $\bar{d}d^0$ to 145 MeV: $\bar{d}d^0 + 2\bar{d}\bar{d}$ and $u^0 + \bar{d}\bar{d}$, with respective energies of 140.49 and 142.12 MeV. (The first value agrees with mass difference $D^*(2010)^+ - D^+$: 140.64 \pm 0.10 [6].)

Thus, the quark representation of the $D^*(2007)^0 - D^0$ mass difference can only be $u^0 + \bar{d}\bar{d}$, if we are considering simple neutral quark groups. However, it’s possible to imagine rather complicated quark groups to account for the difference and therefore all combinations were computed up to the maximum energy (488.98 MeV). This energy is the mass of $D^*(2007)^0$ without the masses of valence quarks ($c\bar{u}$), and, above this energy,
difference quark groups can be formed only with additional energy (vacuum fluctuations), which we shall not consider. The calculations were done for $4\sigma$ probability interval, which is equal to $\pm 0.28\text{ MeV}$ \cite{6}. It is necessary to exclude the groups with one $s$ quark from analysis. The groups found are given in table 4.

Table 4. The difference quark groups within the $142.12 \pm 0.28\text{ MeV}$ interval for $D^{*}(2007)^0$.

| Quark group     | Energy group, MeV | Deviation from $142.12$, MeV |
|-----------------|-------------------|-----------------------------|
| $u\bar{d}(\bar{d}\bar{d})$ | 142.124           | 0.0036                      |
| $7(\bar{d}\bar{d}) + 7d - 3u$ | 142.131           | 0.0113                      |

It's possible to construct only one more difference quark group within the $4\sigma$ interval for the $D^{*}(2007)^0 - D^0$ experimental error ($\sigma = 0.07\text{ MeV}$ \cite{5}) – but there are many reasons not to consider it seriously. Here are some of them: color suppression, charges of reaction and phase volume suppression. For example, there can not be 7 harmonic coupled quarks ($\bar{d}\bar{d}$ or $\bar{d}\bar{d}$) without two of them having the same color and spin. Therefore, we have the unique solution in the $4\sigma$ interval. The existence of a neutral mass combination $u^0$ was assumed earlier in \cite{1}. The same approach can be used for study of other mass differences.

Precise equality of the experimental and model quantities is the real success of the harmonic quark model.

There are two more mesons in the $D$ meson spectrum with similar mass difference: $D_{sJ}^*(2317)^+$ and $D_{sJ}(2460)^+$, which have harmonic levels with energies 2318 and 2460 MeV \cite{1}. Quark representations for these levels are $c + \bar{s}s + \bar{u}\bar{u}$ and $\bar{c}\bar{c} + \bar{d}\bar{d} + \bar{u}\bar{u} + \bar{d}\bar{d}$. The energy difference of these levels are also equal to $\bar{d}\bar{d} + u^0$. This result can be easily achieved using first proposition \cite{4}. The experimental mass difference $D_{sJ}^*(2460)^+$ and $D^*_{sJ}(2317)^+$ is equal to $141.9 \pm 1.6$ \cite{6}, which is also agrees well with the energy of the model quark group. It should be noted that $D_{sJ}(2460)^+$ contains the decay mode $D_{sJ}^*(2317)^+$ with photon. It is necessary to mark, that this annihilation group is not the single solution because of large experimental uncertainty.

Thus, these 4 transitions can be simply interpreted by means of the rest masses of harmonic quarks and oscillators, and this interpretation is in the full accordance with the best experimental data. All these transitions are accompanied by change of $J$ on 1, and they have the $\gamma$ mode in the main channels of the decay.

The obtained data are presented in the final table 5.
Table 5. The summed data for the mass differences of the $D$ mesons.

| Transition                  | Quark group of transition | Energy of group or reaction, MeV |
|-----------------------------|---------------------------|----------------------------------|
| $D^*(2007)^0 \rightarrow D^0$ | $(d)u\bar{d}$            | $142.124 \pm 0.007$             |
| $D_s^+(2112)^+ \rightarrow D_s^+$ | $u\bar{u}$ - $\bar{c}c^0$ | $143.756 \pm 0.007$             |
| $D_{sJ}(2460)^+ \rightarrow D_s^+$ | $(\bar{s}s)$             | $491.33 \pm 0.024$             |
| $D_{sJ}(2460)^+ \rightarrow D_{sJ}(2317)^+$ | $(d)\bar{u}^0\bar{d}$    | $142.124 \pm 0.007$             |
| ONE OF                      | A POSSIBLE                | WAY                              |
| $D^*(2010)^+ \rightarrow D^+$   | $(d)\bar{c}^0\bar{d}$     | $140.49 \pm 0.007$              |

3 Discussion of Results

We established that several transitions in the charmed meson spectrum are strictly quantized. That wasn’t unexpected, it should be so. However, quantization of mass differences by the harmonic quark rest masses is a new unexplored phenomenon. Both the ”free” harmonic quarks and the harmoniously bound quarks both participate in this quantization. What does it mean? Why are there only rest masses of quarks? It means that there is no active motion inside the group of the annihilating quarks. The quark group manifests itself as a something which is whole and annihilates as a whole to the $\gamma$ or $\pi^0$.

There are two possible explanations of such a phenomenon: either the quark group is forming just before annihilation, or the group exists as a shell inside the meson. In the table 4 the second option the $\bar{s}s$ item may be the shell, though it is still an open question. The transitions $\bar{s}s \rightarrow 0$ and $u\bar{u} \rightarrow c^0$ give us a certain understanding of quark configuration of the corresponding mesons. With the 142.12 MeV group there is no such clarity.

All four transitions between doublet states $D^*$ and $D$ cannot be exactly quantized by quark masses since not more than 3 of them can be independent. If an electromagnetic split in a doublet state is determined by the other circumstances then only one transition may be exactly quantized by quark masses, and will define the mutual position of the doublets on a mass scale.

We will name this quantized transitions as the basic transitions, and other transitions as additional.

The three of these transitions are experimentally observed, an the fourth transition $D^{*0} \rightarrow D^+$ can only be caused by a weak interaction and not considered here. One basic transition is the quark annihilation reaction $(d)u^0\bar{d} \rightarrow 0$ agrees precisely with CLEO [5] experimental data. The other basic transitions, if they exists, are not detected. Although
the one quark group of the mass difference $D^*(2010)^+ - D^+$ agree with experimental data 140.64 [6].

Transitions for singlet states are unambiguous and therefore we observe 3 simply interpreted transitions between the strange charmed mesons. Two of them agree with a model spectrum [1]. In the third transition we observe the quark reaction $(u\bar{u} \rightarrow \Xi^0)$ which is simpler than it follows from model spectrum [1].

For three transitions $D_{sJ}(2460)^+ \rightarrow D_{sJ}(2317)^+$, $D_{sJ}(2460)^+ \rightarrow D_s^+$ and $D_{sJ}(2317)^+ \rightarrow D_s^+$ repeat the situation in which are analogy with the transitions between the doublet states. Only two transitions can be the basic transitions, the third mass difference is by their consequence. The author considers the $D_{sJ}(2460)^+ \rightarrow D_s^+$ is the basic transition. The second basic transition are perhaps the $D_{sJ}(2460)^+ \rightarrow D_{sJ}(2317)^+$, but it is preliminary conclusion.

The results of this work can use for the correction of some $D_s$ mass data. The relative error for quark masses is $\approx 5$ times less than the experimental error for the masses of $D$ mesons. With the help of $D_s^+$ experimental mass and precise energies of the harmonic groups (table 5), this makes possible to redefine the masses of several excited $D_s$ states and, moreover, to decrease their mass errors up to $\pm 0.5$ MeV.

The obtained results are given in the table 6.

Table 6. The calculated masses of strange charmed mesons.

| Meson                  | Experimental meson mass, MeV [6] | Calculated meson mass, MeV (this work) |
|------------------------|----------------------------------|----------------------------------------|
| $D_s^+$                | 1968.3 ± 0.5                     | -                                      |
| $D_s^{*+}$             | 2112.1 ± 0.7                     | 2112.1 ± 0.5                           |
| $D_{sJ}^*(2317)^+$     | 2317.4 ± 0.9                     | 2317.5 ± 0.5                           |
| $D_{sJ}(2460)^+$       | 2459.3 ± 1.3                     | 2459.6 ± 0.5                           |

The author implies, that all obtained results can be directly carried over to the corresponding antiparticles. The energy of most precisely measured transition $D^+$ in $D^0$ can also be calculated from the model, but this calculation and explanation of the results are beyond the scope of this work.

4 Conclusion

It’s discovered that the mass differences of the $D$ mesons are rigorously quantized by the rest masses of harmonic quarks.

The harmonic quarks and their derivatives are the main building blocks for the hadrons.
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