Drawing graph of quadratic functions: what affects it?

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Abstract. The quadratic function of the part of mathematics that is dynamic. This gives the meaning that drawing graphs of quadratic functions can use a variety of different ways. The purpose of this study is to find out what underlies a person in solving mathematical problems in different ways. Sampling was done by purposive sampling with two respondents who had learned quadratic function material. The study was conducted qualitatively by analyzing the way respondents in the presentation of graphs of quadratic functions. The results showed that the effectiveness of drawing graphs of quadratic functions is influenced by the ability to think creatively. This implies that it is necessary to develop creative thinking skills in the educational environment.

1. Introduction

One of the challenges in learning mathematics is that someone must function as a problem solver rather than as a formula memorizer. A study has shown that someone who is involved in mathematics learning who is creative in solving non-routine problems has a much better performance than someone who is given a routine task [1]. This shows that someone's way of thinking will affect their ability to provide different ways of solving mathematical problems. The quadratic function is a part of mathematics which generally discusses drawing graphics, symmetry and optimum values, determining quadratic function equations, and solving quadratic function problems in everyday life. Resolving problems related to quadratic functions can certainly be solved in different ways. The way students think about concepts that have been studied before influences them in learning new material [2].

An effective thing is done to find out how someone thinks in drawing graphs of quadratic functions, namely by analyzing a person's answer in solving a quadratic function problem. It is important to analyze the tasks of interpretation and construction related to functions in the form of algebra, tables or graphs [3]. The analysis is one way to find out what cognitive aspects underlie someone in providing answers to the problems given. Seeing and analyzing one's answers in solving algebraic problems is a tool to see the ability to think algebra [4]. Previous research has stated that knowledge of different representations is not independent, but is related to knowledge of various approaches to functions, knowledge of the context of the presentation and knowledge of the underlying ideas [5]. Therefore, this study wants to analyze the different ways in which a person draws a quadratic function and what influences it.

2. Methods

The method of this research is carried out qualitatively by describing the respondents' answers to see various ways of presenting graphs of quadratic functions. The sample technique used was purposive sampling by taking two samples namely the Indonesian Education Institute students. Data collection is
done by giving questions about the quadratic function to the two students and then an analysis of the results of the answers given.

3. Results and discussion

The purpose of this study is to analyze various ways of presenting graphs of quadratic functions. The three respondents were given two questions about how to present graphs of quadratic functions and graph peak points.

The first question is how to form graph \( f(x) = (x - 1)^2 \). The first respondent immediately gave the answer as presented in Figure 1.

![Figure 1. Respondent answers 1.](image)

Figure 1 shows the answer of respondent 1 about the graph of the function squared with the peak point in \((1,0)\). He argued that the answer in figure 1 was obtained from a graph shift \( f(x) = x^2 \) to the right of one unit. While the second respondent gave an answer by outlining the form first \( f(x) = (x - 1)^2 \) becomes \( f(x) = x^2 - 2x + 1 \) then looks for the peak point that is \( x = -\frac{b}{2a} = -\frac{-2}{2} = 1 \) and \( y = -\frac{D}{4a} = -\frac{b^2-4ac}{4.1} = -\frac{(-2)^2-4.1.1}{4.1} = 0 \) so that the peak point of the graph \( f(x) = (x - 1)^2 \) which is \((1,0)\). Then the respondent specifies \( x = 0 \) and \( x = 2 \) as a help point so that the value \( f(0) = (0 - 1)^2 = 1 \) and \( f(2) = (2 - 1)^2 = 1 \). From taking the help point coordinates \((0,1)\) and \((2,1)\) are obtained. After finding the three coordinate points, second respondent draws the same graph as first respondent.

Although the answers from the two respondents were correct, the differences in the thinking process were apparent. Respondent 1 is more effective in providing answers by utilizing functions \( f(x) = x^2 \). While the second respondent is more procedural using the peak point formula.

The second question asked is about how the procedure looks for the peak point of the graph \( f(x) = x^2 + 2x \). First Respondent provides answers, namely \( x = -\frac{b}{2a} = -\frac{2}{2} = -1 \) and \( f(-1) = (-1)^2 + 2(-1) = 1 - 2 = -1 \). So the peak point of the graph is \((-1,-1)\). While second respondent gives answers \( x = -\frac{b}{2a} = -\frac{2}{2} = -1 \) and \( y = -\frac{D}{4a} = -\frac{b^2-4ac}{4.1} = -\frac{(-2)^2-4.1.0}{4.1} = -1 \). Both provide correct answers but provide different procedures.

Based on the results of the answers to the first question, different processing processes were obtained between the two respondents. First Respondent gives a short and effective answer compared to the second respondent. Respondent first uses more graphics \( f(x) = x^2 \) to draw graphics \( f(x) = (x - 1)^2 \) while the second respondent is more procedural using the peak point formula. The first process of thinking of respondents shows their ability to develop ideas (elaboration) in connecting the concepts of graphs \( f(x) = x^2 \) to graphic form \( f(x) = (x - 1)^2 \). The Respondent is also flexible in answering questions by looking at problems from different perspectives. The originality of the procedure provided
by the respondent is seen from the ability to provide unusual answers such as the answers given by the second respondent. In addition, the answer of the first respondent shows fluency in answering the problem in the form of ability to provide many ideas in drawing graphs of quadratic functions other than using the peak formula. Elaboration, Flexibility, Originality, and fluency shown by the first respondent show that respondents have the ability to think creatively [6].

The second respondent's answer is more procedural in using the existing formula. However, the second respondent has carried out cognitive activities in the form of understanding procedures in drawing graphs of quadratic functions. A person's cognitive activity consists of: remembering, understanding, applying, analyzing, synthesizing, and evaluating [7]. The second understanding of respondents was shown by their ability to repeat information that had been taught which was then understood to be able to apply procedures in drawing graphs of quadratic functions. Furthermore, the respondents build ideas and recognize needs as additional information to solve problems.

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The same thing is indicated by the second answer given by the two respondents. Respondent one is more flexible in providing the answer to the peak point for the y value by substituting directly to the initial function. While the second respondent is still procedural using the peak point formula. Flexibility shown by respondents shows the ability to think creatively in solving mathematical problems. While the second respondent was still fixated on the existing formula.

Based on the characteristics of respondents' answers in solving quadratic function questions, it can be concluded that one's creative thinking ability can influence one's differences in solving quadratic function problems. In addition, a person's habits in solving mathematical problems are influential in triggering the ability to think creatively. Characteristics of tasks given to someone will trigger the ability to think creatively [7].

The ability to think creatively is interpreted as a whole series of cognitive activities used by individuals in dealing with a particular problem in accordance with their capacity [8]. Everyone has different abilities, and of course they have different creative thinking abilities. The ability to think creatively is one of the high-level thinking skills that is important in mathematics education. A person's ability to solve mathematical problems depends on the ability to think creatively. Creativity can be understood in two different models, namely connectionist and eloquent-flexible-original contextual [9]. The connectionist contextual model means that each person's response to a mathematical task can produce different ideas and not only interrelated between concepts but also related to the initial problem with various ideas. The fluent-original-flexible model means that a person's final response to a mathematical task may deviate from the original idea which will bring up a different set of ideas or solutions.

The ability to think creatively is another form of one's intelligence. Intelligence consists of analytical, creative, and practical [10]. Someone said to be intelligent is not seen from his ability to answer routine questions correctly, but someone who can find diverse and appropriate ways to solve mathematical problems. The ability to think creatively means that someone is able to solve mathematical problems appropriately using a variety of unique ways. The ability to think creatively is owned by everyone and can be developed.

4. Conclusion
A person's cognitive abilities influence his ability to solve mathematical problems. Elaboration, flexibility, originality, and fluency aspects become one of the factors that influence a person's thinking. These four aspects are indicators of the ability to think creatively. The results of the study show that one's creative thinking ability can influence differences in one's way of solving quadratic function problems. Therefore, educational institutions need to make efforts to develop students' creative thinking abilities.

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