On the synthesis of a geared adjustable stroke mechanism

Volkan PARLAKTAŞ*
*Department of Mechanical Engineering, Hacettepe University, 06800 Ankara, Turkey
E-mail: volkan@hacettepe.edu.tr

Received: 15 January 2018; Revised: 12 March 2018; Accepted 10 June 2018

Abstract
This study presents novel analysis and synthesis methods for a certain type of geared adjustable stroke mechanism. The mechanism has significance in mechanical design, as it has been used in practice. However, to the best of our knowledge, any theory on its analysis or synthesis is not available in the literature. The problem is divided into four parts and each part is generalized. Novel synthesis procedures for possible largest stroke are developed. Design charts are prepared for maximum stroke configuration. Stroke variation is an important concern in mechanism design. By adopting the introduced method, the output stroke can simply be adjusted even during the operation. The output stroke of the mechanism can be decreased more than 50% with proper transmission, velocity and acceleration characteristics. Use of non-integer gear ratio is also investigated. That property leads a variable stroke mechanism inherently. The relationship between the gear ratio and the number of cycles needed to return to the initial stroke value is determined. As a result, all possible design methods are investigated and generalized for this mechanism. Therefore, significant flexibility is introduced for adapting this mechanism to various practical applications.

Keywords: Geared linkages, Adjustable stroke mechanisms, Stroke adjustment, Variable stroke, Mechanism synthesis

1. Introduction

Adjustable linkages can provide flexibility that is required in many industrial applications by using the same hardware. The advantages of such linkages are high operational speeds, high load-bearing, and high precision capabilities, easy adjustability, faster response, easy manufacture with low manufacturing and maintenance costs. Several authors have made significant contributions in the field of adjustable mechanism synthesis and analysis. Optimum synthesis of adjustable four-bar path generators is carried out (Shimojima et al., 1983). Adjustable six link function generators are studied (Shimojima and Ogawa, 1984). Crank length adjusting mechanisms are proposed in order to change input-output relations of arbitrary planar spherical and spatial mechanisms and also to stop output motions of crank-and rocker mechanisms during rotations of their crank shafts (Funabashi et al., 1986). Adjustable mechanisms with multiple degree-of-freedom (DOF) are studied from the standpoints of both hardware and software (Shimojima et al., 1989). Synthesizing adjustable planar 6-link mechanisms for function generation with dwells at the edges of rocking is presented (Shimojima et al., 1986). A method is presented to design planar five-bar mechanisms to achieve multiple phases of prescribed rigid body path points (Russel and Sodhi, 2004). They also presented methods for designing slider-crank mechanisms to achieve multiphase motion generation, path, and function generation (Russel and Sodhi, 2005a, 2005b). A technique is presented for synthesizing adjustable RSSR-SC mechanisms to achieve phases of prescribed rigid body positions (Russel and Sodhi, 2005c). A design method is proposed to improve or obtain the desired output motion characteristics of single DOF four-bar mechanisms by varying the speed trajectory and length of the input link (Soong, 2009). A novel design procedure for seven link two DOF variable oscillation mechanisms is proposed (Tanık and Söylemez, 2015). A kinematic analysis and synthesis of an adjustable six bar linkage that is proposed as a variable speed transmission mechanism is presented (Pennock and Israr, 2009). Geared linkages can be used to carry out path or function generation (Mundo et al., 2009, Soong, 2015, Dooner, 1999).

This study presents analysis and synthesis methods for a certain type of geared adjustable stroke mechanism.
In adjustable stroke mechanisms, generally large variation in the stroke is desirable. For the mechanism considered, part of the problem. In this study, an analysis method is developed, and transmission angles are defined for the GASM. A synthesis procedure is developed to obtain GFBM with optimum transmission angle, that is considered as the first uses a certain type of geared four-bar mechanism (GFBM) in its structure. In the previous work has to be altered to meet different stress characteristics, however lacks flexibility. Beiner (2001) proposed a three gear four-bar linkage that transforms a uniform input crank rotation into a forward-dwell motion of the output gear. GASM is also convenient as a quick return mechanism. A quick return mechanism converts rotary motion into reciprocating motion at different rate for two strokes. If the time required for the working stroke is greater than that of the return stroke, it is called a quick return mechanism. This property yields a significant improvement in machining productivity. Currently, it is widely used in machine tools, for instance, shaping machines, power-driven saws, flow metering pumps, and other applications requiring a working stroke with intensive loading, and a return stroke with non-intensive loading (Hsieh et al., 2009, Dwivedi, 1984, Chang, 2007, Fung and Chen, 1997). GASM can also be used to control ram’s motion of mechanical press that can be applied in forming such as shearing, deep drawing, coining, bending, precision forming, and cutting. The mechanical press is the most widely used among all types of presses because of its low price and maintenance. Moreover, the mechanical press has high speed characteristics, however lacks flexibility. In general, the ram’s motion of a mechanical press cannot be adjusted. However, in certain metal forming, e.g., deep drawing or precision cutting, the ram’s motion has to be controlled to meet different motion requirements. Nowadays, demands on the motion flexibility of mechanical press are increasing. For instance, the stroke has to be changed according to different thicknesses in sheet metal forming, the ram velocity has to be altered to meet different stress–strain relationships in precision forming, and it has to be flexible to produce diverse products quickly to meet market demands (Hsieh and Tsai, 2011, Soong, 2010, Yossifon et al., 1991). As it is a very complex mechanism, the problem is divided into four parts and each part is generalized. The GASM uses a certain type of geared four-bar mechanism (GFBM) in its structure. In the previous work (Parlaktaş et al., 2010), a synthesis procedure is developed to obtain GFBM with optimum transmission angle, that is considered as the first part of the problem. In this study, an analysis method is developed, and transmission angles are defined for the GASM. In adjustable stroke mechanisms, generally large variation in the stroke is desirable. For the mechanism considered, rotation of the input link produces a large-stroke reciprocating motion of the output link. As the stroke increases, movability and the transmission angles of the mechanism becomes more critical. Therefore, in the second part of the problem, the adjustment link is fixed at a certain position, gear ratios are chosen as unity and synthesis procedures are developed for possible largest stroke. Here, three methods have been developed to obtain mechanisms that can produce possible largest stroke. An optimum synthesis procedure for maximum possible stroke is developed and corresponding design charts are prepared. By using these design charts, the optimum values can be obtained and a GASM for maximum stroke can be designed easily. In the third part of the problem, a novel method for decreasing the stroke is developed. By adopting the introduced method, the output stroke can simply be adjusted by changing the position of the adjustment link, even during the operation. The fourth part deals with use of non-integer gear ratio. It is observed that this property leads a variable stroke mechanism inherently.

1.1 GASM and GFBM

GASM is displayed in Fig.1 (a), (b), and (c). DOF of this mechanism is equal to two. Link 1 is the fixed link; i.e.
the ground. The slider; link 6 is the stroke adjustment link. From a kinematic point of view, this adjustment can be accomplished by moving this slider in a translational slot in any direction or in a circular slot as displayed in Fig.1. $\delta$ is the angle between X-axis and $A_oB_o$, that is called “adjustment angle”, when the adjustment angle is changed, position of link 6 and thus the output stroke of link 9 changes. When link 6 is fixed, the mechanism is at the state of working and possesses one DOF. The rotational input is at the bottom gear; link 2, and the translational output is at the slider; link 9. Link 2 and the arm (link 3) are pivoted to the ground by a ternary revolute joint (two independent kinematic pairs) at fixed point $A_o$. Link 5 is pivoted to link 6 by a revolute joint at fixed point $B_o$ (link 6 is fixed to the frame). Links 4 and 5 are connected by a revolute joint at point $B$. The arm carries the floating gears; links 4 and 7 are connected to link 3 by revolute joints at $A$ and $D$ respectively. Links 7 and 8 are connected by a revolute joint at point $E$. Links 8 and 9 are connected by a revolute joint at point $F$. The position variables and dimensions of the links are displayed in Fig. 1(b) and (c). The radius of the gears 2, 4, and 7 are $r_2$, $r_4$ and $r_7$ respectively. Link length dimensions are: $|A_A| = a_2$, $|A_D| = d_3$, $|AB| = a_4$, $|B_5B_6| = a_5$, $|A_1B_0| = a_1$, $|DE| = a_7$, $|EF| = a_8$. $s_{19}$ is the position variable of link 9, measured from the origin $A_o$. $c_9$ is the fixed distance in Y direction from the origin to the slider axis, $\theta_{12}$, $\theta_{13}$, $\theta_{14}$, $\theta_{15}$, $\theta_{17}$, and $\theta_{18}$ are the position variables of the links 2, 3, 4, 5, 7, and 8 respectively.

Fig. 1 (a) Geared adjustable stroke mechanism 3-D view, (b) Solid model planar view, c) Schematic planar view
The GASM uses a certain type of geared four bar mechanism (GFBM) in its structure. If links 7, 8, and 9 are neglected, then another mechanism is considered that is formed by the links 1, 2, 3, 4, 5, and 6. When link 6 is fixed at a certain position, this mechanism becomes a geared four bar mechanism (GFBM), as shown in Fig. 2. The input is at link 2, and the output is at link 3.

The GFBM was studied extensively (Parlaktaş et al., 2010): A novel analysis method was devised, expressions for the transmission angle were derived and a synthesis procedure was developed for the optimum design of the GFBM. It was observed that the GFBM considered is inherently a quick-return mechanism. Direction of rotation of the input link affects the force transmission characteristics; according to the direction of rotation of the input, there are two different optimum GFBM that have the same output swing angle and corresponding input rotation.

2. Analysis of the GASM

Note that, \( \theta_{13} \), \( \theta_{14} \), and \( \theta_{15} \) were obtained as a function of the input, \( \theta_{12} \) in the previous work (Parlaktaş et al., 2010). Analysis of the GASM involves the relationship between \( s_{19} \), \( \theta_{17} \), and \( \theta_{18} \) as a function of \( \theta_{12} \). When link 6 is fixed at a certain position, the mechanism has single DOF. \( \delta \) is the angle (fixed for the analysis) between the X-axis and \( A_0B_0 \), that is called as “adjustment angle”.

The velocity ratio between the gears 4, 7, and the arm (link 3) is:

\[
\frac{r_7}{r_4} = \frac{\dot{\omega}_{14} - \dot{\omega}_{13}}{\dot{\omega}_{17} - \dot{\omega}_{13}} = -R_2
\]  

Integrating Eq. 1, \( \theta_{17} \) can be determined as:

\[
\theta_{17} = \frac{(R_2+1)(\theta_{13}+\delta) - \theta_{14} + k_2}{R_2}
\]

where, \( k_2 \) is the integration constant determined by the initial relative positions of links 4 and 7 with respect to link 3 (\( \theta_{14} \) and \( \theta_{17} \) w.r.t \( \theta_{13} \)), that is defined the “phase angle”.

The position of the output link, \( s_{19} \), can be obtained from the loop closure equation as:

\[
s_{19} = \frac{-D \pm \sqrt{D^2 - 4E}}{2}
\]

where, \( D = -2d_3 \cos(\theta_{13} + \delta) - 2a_7 \cos(\theta_{17}) \)

\[
E = d_3^2 + a_7^2 + c_9^2 + 2d_3a_7 \cos(\theta_{12} + \delta - \theta_{17}) - 2c_9d_3 \sin(\theta_{13} + \delta) - 2c_9 a_7 \sin(\theta_{17}) - a_9^2
\]

\( \theta_{18} \) can be determined from the loop closure equation after \( s_{19} \) is solved.

The velocity and acceleration of output link can be determined as:
For the GASM considered, there are two critical transmission angles (Alt, 1932), (Balli and Chand 2002). The first transmission angle is defined for the GFBM (Parlaktaş et al., 2010). The second transmission angle is between the output link 9 and link 8, similar to that of the slider crank mechanism: \( \mu_2 = \theta_{18} - \pi/2 \).

3. Synthesis of the GASM

First, gear ratios are considered: If \( R_1 (r_4/r_2) \) is increased, larger swing angle of the arm can be determined with permissible transmission characteristics. On the other hand, radius of floating gears, and length of the arm increase, resulting in very large mechanisms, that is not a desired situation. Therefore, \( R_1 \) is taken as unity. Use of the gear ratio, \( R_2 \), as a design parameter for variable stroke can be considered. However, it is observed that if \( R_2 = 0.5 \), a pilgrim step motion is obtained. If \( R_2 \) is not an integer and not equal to 0.5, then the output motion completes a full cycle after several cycles of the input link rotation. For every cycle of the input, the output stroke decreases a little bit, up to a minimum value, afterwards for every cycle of the input, output stroke increases a little bit, up to the initial stroke (or vice versa), therefore the mechanism completes a full cycle after several cycles of the input depending on the value of \( R_2 \). This property; continuously varying length of stroke, can also be used for stroke adjustment and it is investigated in Section 5. Further, if \( R_2 \) is both an integer and greater than one (\( R_2 = n \), where \( n > 1 \)), then the output completes a full cycle after \( n \) cycles of the input link, and a pilgrim step motion is obtained.

In section 3, for the proper output reciprocating motion, \( R_2 \) is chosen as unity. In an adjustable stroke mechanism usually large variation in the stroke is required. Further, as the stroke increases movability and the transmission angle of the mechanism become more critical. For this purpose, first the aim is to design mechanisms that have largest stroke. Once the largest stroke is obtained, by changing the position of the adjustment link, the stroke is decreased. Three methods are developed to obtain mechanisms that can produce largest strokes.

3.1 Dead center synthesis of the GASM

A synthesis procedure is developed by use of the dead-centers of the GASM. During the synthesis of the GFBM, it was defined that \( \phi \) is the rotation of link 3 from the folded position to the extended position, and \( \beta \) is the corresponding rotation of the input link 2. When \( \phi \) and \( \beta \) are selected, \( \lambda_{opt} \) and the link lengths of the GFBM were obtained (Parlaktaş et al., 2010). It can easily be observed that, if links 7 and 8 are at the extended position when the GFBM is at the folded position, then the output link 9 is at the forward position (Fig. 3(a)). Moreover, if links 7 and 8 are at the folded position when the GFBM is at the extended position, then the output link 9 is at the fully withdrawn position (Fig. 3(b)). These positions are the dead-center positions of the GASM.

Fig. 3 (a) Designed GASM at the dead centers, output link is at the forward position (b) output link is at the fully withdrawn position.
For both gear ratios of unity ($R_1 = R_2 = 1$), it can be determined that the rotation of link 7 is equal to the rotation of the input link 2. Therefore, the rotation of link 7 between the dead-center positions ($\Delta \theta_{17}$) can be obtained as; $\Delta \theta_{17} = \beta$. The length of the arm can be obtained as; $d_1 = 4r_2$, as the radius of the gears are equal. The adjustment angle (orientation of the fixed link), $\delta$, is a free parameter. Another free parameter is defined as: $\lambda_2 = a_7/a_8$. The output stroke of the mechanism is the difference between the values of $s_{19}$ at the extended and the folded positions: $\Delta s = s_8 - s_6$. Hence, for a given stroke $\Delta s$, the extended and folded positions of link 7 ($\theta_{17e} = \theta_{17f} - \Delta \theta_{17}$) is obtained from the geometry of the mechanism at the dead centers (Eq.6). Therefore, the phase angle, $k_2$, can be determined from Eq. (2).

Finally, the link lengths $a_6$, $a_7$, and $c_9$ can be obtained from the geometry at the dead centers as in Eqs. (7) and (8).

$$\theta_{17f} = \tan^{-1} \left[ \frac{K_2 \cos(\Delta \theta_{17}) - K_3 - K_4 \sin(\Delta \theta_{17})}{K_1 K_3 - K_1 K_2 \cos(\Delta \theta_{17}) - K_2 \sin(\Delta \theta_{17})} \right]$$  \hspace{1cm} (6)

where, $K_1 = \frac{\Delta s - d_3 \left( \cos(\theta_{13f} + \delta) - \cos(\theta_{13e} + \delta) \right)}{d_3 \left( \sin(\theta_{13f} + \delta) - \sin(\theta_{13e} + \delta) \right)}$, $K_2 = \lambda_2 + 1$, $K_3 = \lambda_2 - 1$

$$a_8 = \frac{\Delta s - d_3 \left( \cos(\theta_{13f} + \delta) - \cos(\theta_{13e} + \delta) \right)}{K_2 \cos(\theta_{17e}) - K_3 \cos(\theta_{17f})}$$ \hspace{1cm} (7)

$$c_{19} = d_3 \sin(\theta_{13f} + \delta) + (a_7 + a_8) \sin \theta_{17e}$$ \hspace{1cm} (8)

By this approach, a closed-form solution for the link length proportions is obtained. However, solutions with permissible transmission characteristics are obtained in a limited region. The reason of this situation: the rotation of link 7 is equal to the rotation of the input link. It was shown that the use of centric (time ratio = 1) GFBM was not feasible because of low transmission angle values (Parlaktaş et al., 2010). Therefore, in order to obtain proper values for the transmission angle of the GFBM (first transmission angle of the GASM), rotation of the input link between the dead centers must be far away from $180^\circ$. On the other hand, as the rotation of the input link and thus link 7 between the dead-centers is closer to $180^\circ$ the second transmission angle of the GASM improves (similar to slider-crank mechanisms). Therefore, obtaining solutions satisfying both of the dead-center positions with acceptable transmission characteristics is possible in a limited region. However, obtaining solutions satisfying one of the dead-center positions with acceptable transmission characteristics can be also useful for applications where a high mechanical advantage is required.

### 3.2 Synthesis of the GASM by use of the inflection circle of link 7

It is observed that the path of point $E$ is very important, because the shape of this path is the major parameter for the output stroke. Point $E$ is the attachment point of the connecting rod (link 8) to link 7. As seen in Fig. 4, the distance between the maximum and minimum $X$ coordinate of this path that is defined as $s$, is the major parameter that assesses the length of the output stroke. Similarly, for the slider-crank mechanisms, the length of the crank (traces a circle) is the major parameter that defines the stroke. The distance between the maximum and the minimum $Y$ coordinate of this path is defined as $h$, and it significantly affects the second transmission angle. For a large stroke, $s$ must be increased, and for a better transmission angle, $h$ should be decreased. Therefore, $s/d_1$ and $s/h$ should be increased ($d_1$ is the arm). For the same link lengths, by assigning different phase angles ($k_2$) from 0 to $2\pi$, different paths of point $E$ can be obtained from the same mechanism, as shown in Fig. 4(a). After many trials it is observed that as the lower portion of the path of point $E$ converges to an approximate straight line, $s$ increases. Hence, for a large output stroke, it is aimed to obtain an approximate straight line for the lower portion of the path of point $E$ as in Fig. 4 (red curves). For this purpose, a novel a procedure is developed. First, the motion is canonically represented and the equations of moving and fixed centrodes of link 7 are obtained. After calculating radius and center of curvatures of moving and fixed centrodes, the inflection circle of link 7 throughout a full cycle is determined. Finally, the symmetric position (midpoint of the oscillation of the arm) of the GASM is located and the inflection pole at that position is chosen as point $E$.  

[DOI: 10.1299/jamdsm.2018jadm0062] © 2018 The Japan Society of Mechanical Engineers
Initial design step is to choose an optimum GFBM from the related design charts (Parlaktaş et al., 2010). Consequently, the link lengths \(a_1, a_3, a_4, a_5\) are determined. When the gear ratios are specified, the length of the arm can be determined as well. The link lengths \(a_1, a_6\), phase angle \(k_2\), adjustment angle \(\delta\), and height of slider axis \(c_9\) have to be determined. The arm performs an oscillating motion. The determined part of the mechanism can be analyzed; path of point \(D\) and the midpoint of this oscillation can be obtained. Point \(D\) passes from this midpoint two times, but the position during the return stroke is chosen, because the desired approximate straight line occurs in the return stroke. Note that this midpoint is approximately a symmetric position for the path of point \(E\) (Fig. 4(b)).

The motion is canonically represented and the equations of moving centrode (\(\vec{Z}_p\)) and fixed centrode (\(\vec{Z}_p\)) of link 7 are evaluated throughout the whole cycle as in Eqns. (9) and (10). In order to obtain the diameter and center of the inflection circle at each position, radius and center of curvatures of the moving and fixed centrodes of link 7 throughout the cycle are evaluated. For example, in Fig. 5(a) the fixed centrode, the moving centrode, and their centers of curvatures are displayed throughout the cycle. The point of tangency of the moving and the fixed centrodes is the pole at the position located (midpoint of the oscillation of the arm).

\[
\vec{Z}_p = -d_3 \left( \frac{d\theta_{13}}{d\theta_{17}} \right) e^{i(\theta_{13}-\theta_{17})} \\
\vec{Z}_p = d_3 \left( 1 - \frac{d\theta_{13}}{d\theta_{17}} \right) e^{i\theta_{13}}
\]

where,
\[
\frac{d\theta_{13}}{d\theta_{17}} = \frac{K_2 \sin(-\theta_{13}+\theta_{17}-k_2)+K_4 \sin(2\theta_{13}-\theta_{17}+k_2)}{K_2 \sin(-\theta_{13}+\theta_{17}-k_2)+K_3 \sin(\theta_{13})+2K_4 \sin(2\theta_{13}-\theta_{17}+k_2)} , \quad K_1 = a_1^2 + a_3^2 + a_4^2 - a_5^2 \\
K_2 = 2a_3a_4 , \quad K_3 = 2a_1a_3 , \quad K_4 = 2a_1a_4
\]
At the position located, the diameter of the inflection circle, $\gamma$, is determined from the radius of curvatures of the fixed and moving centrodes. The center of the inflection circle is on the pole normal, and its radius is $\gamma/2$. The slope of the centrodes is $\zeta$ and it can be evaluated throughout the cycle after equations of centrodes are obtained. $\nu$ is the angle between the positive X-axis and pole normal. The center of the inflection circle, and then the inflection circle can be determined as (Fig. 5(b)):

$$ I_c^* = Z_p + \frac{\gamma}{2} e^{i\nu} $$

(11)

$$ I = I_c^* + \frac{\gamma}{2} e^{i(0\cdots2\pi)} $$

(12)

After obtaining the inflection circle, the inflection pole is chosen as point $E$ on link 7. The inflection pole is the intersection of the inflection circle and the pole normal (Fig. 5(b)), and it can be determined as:

$$ I_p = Z_p + \gamma e^{i\nu} $$

(13)

![Fig. 6 In order to calculate the phase angle and link length 7, $DE$ is determined by difference of the position vectors of points $E$ and $D$](image)

The vector from the origin ($A_0$) to the center of gear 7 (point $D$) can be written as: $D = d_3 e^{i\theta_13}$. Therefore, the vector $DE$ can be determined as in Fig. 6:

$$ DE = I_p - D $$

(14)

The magnitude of this vector is the link length $a_7$, and the angle of this vector is $\theta_17$. Therefore, the phase angle $k_2$ can be obtained from Eq.(2). As the inflection pole is chosen as point $E$, the approximate straight line is parallel to the pole tangent. The angle between the X-axis and the pole tangent is determined as, $\zeta$ (Fig. 5(b)). Therefore, the fixed link of the mechanism must be rotated $\pi - \zeta$ radians (CCW) in order to obtain an approximate straight line parallel to the X-axis. Hence, the adjustment angle is determined as: $\delta = \pi - \zeta$.

From in-line slider-crank mechanisms it is known that the length of the connecting rod does not affect the output stroke and for the other type of slider-crank mechanisms its effect to the output stroke is negligible. But, as the length of the connecting rod increases, the transmission characteristic improves. After several trials it is observed that the effect of length of the connecting rod to the output stroke is also negligible for the GASM. Therefore, the length of the connecting rod, $a_8$, is not a design parameter to be calculated. It can be chosen as long as possible for better transmission characteristics (up to a link length ratio of 1/10). The remaining parameter is the height of the slider axis, $c_9$, and its effect to the output stroke within the feasible design region is also negligible. However, $c_9$ has an important
effect on the second transmission angle, and it should be chosen considering the transmission angle. If it is chosen around \(h/2\) (Fig. 4), the transmission angle deviation from 90° would be equal during the work and reverse stroke. If it is chosen around the apex of the path of point \(E\), the transmission angle deviation from 90° would be very low during the work stroke. Note that, the lower part (approximate straight line) of the path of point \(E\) is the quick return stroke.

By this approach, mechanisms that have very large stroke can be obtained. However, in order to increase the output stroke, oscillation of the arm should be increased. This yields an increase in the diameter of the inflection circle, therefore larger lengths of link 7 are obtained. Consequently, as the stroke increases, point \(E\) is determined to be outside the gear (link 7). An optimum configuration for the maximum stroke is investigated in the next section.

### 3.3 Optimum synthesis of the GASM

Initial design step is to choose an optimum GFBM from the related design charts (Parlaktaş et al., 2010). The link lengths \(a_1, a_2, a_4, a_5\) are determined, accordingly. When the gear ratios are specified, the length of the arm can be determined as well. The link lengths \(a_7, a_4\), phase angle \(\delta\), adjustment angle \(\delta\), and height of slider axis \(c_9\) have to be determined. The arm passes from the midpoint two times, but the position that the arm passes from the midpoint during the return stroke should be chosen, as this midpoint is approximately a symmetric position for the path of point \(E\) (Fig. 4(b)). As it is experienced from the previous sections, this is the key position that leads to a very large stroke. At that position, the angle between X-axis and the arm, \(\theta_{3avg}\) is determined and the fixed link of the mechanism is rotated ccw until the arm comes to the vertical position. Hence, the initial value of the adjustment angle, \(\delta\) can be determined as: \(\delta = 90° - \theta_{3avg}\). After that, a point on link 7, but in line with the arm is selected as point \(E\); that leads \(\theta_{71} = 270°\). Consequently, the phase angle, \(k_2\), can be determined from Eq. (2). Note that, the determined values for parameters \(\delta\) and \(k_2\) are not the optimum values, but initial feasible solutions for a parametric optimization routine.

It is observed that as the link length \(a_7\) increases, larger paths of point \(E\) are obtained. Hence, \(s\) and \(h\) increases; the stroke increases, but the transmission characteristic gets worse. Consequently, the effect of link length \(a_7\) is clear and it is a free design parameter. It is chosen on the gear (link 7), and the value of \(a_7\) is close to the radius of the gear. The determination of \(a_7\) and \(c_9\) is explained in the previous section. Therefore, the optimization parameters for the maximum stroke are the adjustment angle, \(\delta\), and the phase angle, \(k_2\). For a given \(\phi\) (swing angle of the arm; link 3) and \(\beta\) (corresponding rotation of the input link 2) the optimum \(\delta\) and \(k_2\) can be determined by searching around the initial feasible solutions. In order to generalize the design procedure, a parametric optimization routine for maximum stroke is developed to obtain the optimum \(\delta\) and \(k_2\) for every given \(\phi\) and \(\beta\). In this routine, optimum \(\delta\) and \(k_2\) values that results in maximum \(s\) is searched. It is observed that, when \(s\) is maximum \(h\) is approximately minimum (Fig. 9(b)). Initial step of this routine is to apply the method explained in the first paragraph of this section and to determine preliminary feasible solutions. Note that, according to the direction of rotation of the input there are two different optimum GFBM (Parlaktaş et al., 2010). Therefore, according to the direction of rotation of the input there are also two different optimum GASM for the same \(\phi\) and \(\beta\). Hence, two design charts are determined for the GASM according to the input rotation direction. \(\delta_{opt}\) is searched in a region \(±8°\) around \(\delta_4\), and \(k_{2opt}\) is searched in a region \(±30°\) around \(k_{2i}\), because after numerous trials it is observed that the optimum values are located in these regions.

The resulting design charts are shown in Figs. 7 and 8, here the X-axis corresponds to \(\beta\), Y-axis corresponds to \(\phi\). Full lines represent \(\delta_{opt}\) and dotted lines represent \(k_{2opt}\). For example, if \(\phi = 50°\), and \(\beta = 90°\) and rotation of the input link is CW, from Fig. 7 the optimum values for maximum stroke can be determined as: \(\delta_{opt} = 43.5°\), \(k_{2opt} = -22°\). Therefore, for every given \(\phi\) and \(\beta\), the optimum \(\delta\) and \(k_2\) can be determined from these design charts and the GASM for maximum stroke can be designed easily. A design example is presented in Section 4.1.
After synthesizing the mechanisms for a large stroke, by using one of the methods explained in previous sections, the effect of changing the position of the adjustment link on the length of stroke is investigated. In adjustable stroke mechanisms, generally maximum variation in the stroke is desirable. Hence, a novel method is developed for decreasing the stroke.

4. Stroke adjustment

Fig. 7 Design chart for maximum stroke (input cw), X-axis corresponds to $\beta$; rotation of the input link, Y-axis corresponds to $\phi$; swing angle of link 3. Full lines represent $\delta_{opt}$ and dotted lines represent $k_{2opt}$ for maximum stroke.

Fig. 8 Design chart for maximum stroke (input ccw), X-axis corresponds to $\beta$; rotation of the input link, Y-axis corresponds to $\phi$; swing angle of link 3. Full lines represent $\delta_{opt}$ and dotted lines represent $k_{2opt}$ for maximum stroke.
The slider, link 6 can be moved by an actuator in a translational slot in any direction, then the orientation (δ) and the length of fixed link (A₂B₂) changes. If the length of A₁B₁ is altered, then the oscillation of the arm changes. First, the oscillation change of the arm is investigated. However, it is observed that any possible change in oscillation of the arm can be limited. The reason is; the length of A₁B₁ can be altered slightly, because the transmission angle of the GFBM is sensitive to any change in the length of the fixed link. If the length of the fixed link is changed, first transmission angle of the GASM (transmission angle of the GFBM) deteriorates. Therefore, it is observed that the output stroke of the GASM cannot be changed significantly by decreasing the oscillation of the arm. Hence, it is clear that, if only the orientation of A₁B₁ (δ) is changed, maximum change in the stroke can be obtained. This adjustment can simply be accomplished by rotating the adjustment link in a circular slot even during the operation. The center of this circle is at A₁ and the radius of the circle is equal to a₁ (Fig.1). The stroke change can be revealed again by the path of point E. As the adjustment angle δ is changed, s decreases and h increases drastically as seen in Fig. 9.

This property can also be observed in the stroke charts. These charts can be prepared by analyzing the mechanism that is determined for the large stroke, and plotting path of point E corresponding to a range of δ and k₂. The stroke chart in Fig. 9(b) is prepared for φ = 50° and β = 90°, where δ₁ optimal ≈ 43.5°, k₂opt ≈ -22°, corresponding to the maximum stroke of s = 2.14. Here, if only δ is changed (k₂opt is fixed), then s decreases (s can be decreased from 2.14 to 0.9 units) as shown in Fig. 9(b). However, as s decreases, h increases. Consequently, the adjustment angle (δ) is changed to decrease the stroke, however the second transmission angle should be checked in the low stroke position.

In order to obtain appropriate transmission characteristics for the first and second positions (large stroke and short stroke), some conditions should be considered. It was mentioned in section 3.3 that, as a₁ is increased, larger path of point E is determined and a larger stroke is obtained. However, at the second position, s approaches to h₁ (s₁→h₁) (Fig. 9(a)) thus, a very large stroke at the first position may lead to an unacceptable transmission angle at the second position. Also, h₁ approaches to s₂ (h₁→s₂) thus, a small h₁ leads a short stroke at the second position. Consequently, in order to obtain a short stroke with permissible transmission characteristics at the second position, a₁ should not be chosen very large; i.e. not outside the gear. As a conclusion, it is observed that more stroke change can be accomplished if point E is chosen on the gear (link 7), and the value of a₁ should be close to r₁.

The adjustment angle δ can be changed in cw or ccw direction as in Fig. 9(a). However, the output reciprocating motion differs according to direction of rotation of δ. When δ is changed in cw direction, pilgrim step motion is obtained at the second position.

If the height of the slider axis c₂ is selected just considering the first position, then deviation of the transmission angle from 90° becomes unacceptable at the second position. Therefore, c₂ must be chosen below the path of point E at the first position, for an acceptable transmission angle during both of the positions as presented in Fig. 10.

As a result, value of δ for a required stroke change can be determined from the stroke charts that can be prepared for any φ and β as in Fig. 9(b). However, the conditions listed above should be considered. By this approach, stroke can be decreased more than 50%, with a proper transmission angle.
4.1. Design Example

First, an optimum GFBM is chosen from the related design chart (cw input, gear ratios are equal to 1) for the oscillation of the arm $\phi = 50^\circ$ and the corresponding rotation of the input link $\beta = 90^\circ$. Corresponding link lengths can be determined as: $a_1 = 1, \quad a_3 = 0.873, \quad a_4 = 0.365, \quad a_5 = 0.722$ (Parlaktaş et al., 2010). The gear ratios are equal to 1, so radius of all gears are $r = a_3/2 = 0.436$. The lengths of the arm and link 7 are calculated as $d_3 = 1.746$, and $a_7 = 0.9 \times r_7 = 0.39$. From Fig. 7, the optimum values for maximum stroke can be determined as: $\delta_{opt} \approx 43.5^\circ$ and $k_{2opt} \approx -22^\circ$. The height of the slider axis is selected considering both of the positions as: $c_9 = 0.5$. The connecting rod is selected as: $a_8 = 2.6$ (in order to obtain a better transmission angle, it can be increased up to 3.6, because the length of the smallest link is equal to 0.365). The GASM is presented at the first position in Fig. 11(a). The output stroke of the mechanism is calculated as 2.18 units, as shown in Fig. 11(b). In the normalized form the output stroke is equal to 1. The second transmission angle deviates maximum 39$^\circ$ from 90$^\circ$ at the first position.

![Diagram](image)

**Fig. 10** Height of the slider axis $c_9$ should be chosen below the path of point $E$ at the first position for an acceptable transmission angle during both of the positions

If the adjustment angle is changed ccw, the value of $\delta$ for maximum decrease in the stroke is obtained from Fig.9 (b) as $\delta = 138^\circ$. In this case, GASM at the second position and the output reciprocating motion of the mechanism are displayed in Fig.12 (a), (b). The second transmission angle deviation is maximum 40$^\circ$ at the second position. The output stroke at the second position is 0.99 units. In the normalized form, the output stroke is equal to: 0.99/2.18 = 0.45 units, that is less than half of the maximum stroke achieved at the first position. Note that, the calculated results are also verified with simulations in MathCAD (Figs. 11(a) and 12(a)) and Catia (Appendix A).

GASM can be used to control ram’s motion of a mechanical press that can be applied in forming such as shearing, deep drawing, coining, bending, precision forming, forging, and cutting. A forming tool such as a punch or die can be attached to the output slider. An open die press process is displayed as an example. At the maximum stroke position, a workpiece can be formed in a very small size as shown in Fig 11(a). Whereas, at the minimum stroke position, a same or different size workpiece can be formed into a shape larger than the one obtained at the maximum stroke.
position, with the same machine, as presented in Fig.12 (a). Note that, output stroke may also be decreased to any intermediate stroke position due to the value of the adjustment angle, as explained in Section 4, thus different size workpieces can be formed into various dimensions. As a conclusion, by changing the position of the adjustment link, output stroke can be adjusted and stock materials with various dimensions can be formed into wide range of sizes with the same machine. This is an important flexibility for the design of such kind of machines.

![Diagram of mechanism](image)

Fig. 12 (a) Minimum stroke position of the same mechanism, as an example it is forming a workpiece into a different size; (b) Output reciprocating motion $s_{19}$, corresponding to input link rotation $\theta_{12}$, at the minimum stroke position

It is observed that GASM is inherently a quick return mechanism. Note that, for the GASM analyzed, the time ratio is $\approx 2$ at the maximum stroke position (Fig 11 (b)), and at the minimum stroke position it is $\approx 1.6$ (Fig 12 (b)). Therefore, GASM is also convenient as a quick return mechanism; where the time required for the working stroke is greater than that of the return stroke. Quick return mechanism is widely used in shaping machines where single point cutting tool is mounted on the front of the slider or ram. As another example, a quick return shaping machine is presented in Fig 13. The tool cuts on the slow forward stroke and returns on the quick reverse stroke.

![Diagram of quick return shaping machine](image)

Fig. 13 Example of an application of GASM as a quick return shaping machine

5. **Use of non-integer gear ratio**

It was mentioned in Section 3 that, if $R_2$ is not an integer and not equal to 0.5, then the output motion completes a full cycle (returns to the initial stroke) after several cycles of the input link rotation. For every cycle of the input, the output stroke decreases, up to a minimum value, afterwards for every cycle of the input, the output stroke increases, up to the initial stroke (or vice versa). Therefore, the mechanism completes a full cycle after several cycles of the input, depending on the value of $R_2$. A variable stroke mechanism; continuously varying length of stroke, can be achieved by
the aid of this property. However, the associated number of cycles of the input should be calculated. In this section, the number of these cycles is determined by a novel method.

In Section 3, it was explained that, \( R_1 \) should be equal to unity. Here, let the gear ratio between gears 7 and 4, \( R_2 \) is not an integer: \( R_2 = 1.t \) (e.g 1.3, where \( t \) is an integer). Therefore, the velocity ratio between the gears 2, 7 and the arm (link 3) can be written as:

\[
\frac{\omega_{17} - \omega_{13}}{\omega_{12} - \omega_{13}} = \frac{r_2}{r_7} = \frac{1}{1.t}
\]

Integration Eq. (15) by assuming initially \( \theta_{12i} = \theta_{13i} = \theta_{17i} = 90^\circ \):

\[
(1.t)\theta_{17} = \theta_{12} + (0.t)\theta_{13}
\]

At the end of each cycle of the input, the arm comes to its initial position, so at the end of each cycle \( \theta_{13} = \theta_{12i} \). After \( n \) cycles, \( \theta_{17} \) can be determined from Eq. (16) as:

\[
\theta_{17} = \theta_{12i} + \frac{n}{1.t} 2\pi
\]

From Eq. (17), it is clear that if \( n/1.t \) is an integer, then \( \theta_{17} = \theta_{12i} \). Therefore, link 7 comes to its initial position; the mechanism completes a full cycle.

Similarly, if \( R_2 = 0.t \) (e.g 0.9, where \( t \) is an integer), \( \theta_{17} \) can be determined after \( n \) cycles as:

\[
\theta_{17} = \theta_{12i} + \frac{n}{0.t} 2\pi
\]

From Eq. (18), it is clear that if \( n/0.t \) is an integer, then \( \theta_{17} = \theta_{12i} \); the mechanism completes a full cycle.

The number of cycles of the input where the output link returns to its initial value can be calculated from Eqns. (17) and (18). In these equations there exists a general rule: if \( t \) is an odd number, then the number of cycles is calculated as: \( n = 10R_2 \), if \( t \) is an even number, then the number of cycles is calculated as: \( n = 5R_2 \).

Example 4.1 is reconsidered in the first position (\( \delta = 43.5^\circ \)), with an only difference of \( R_2 = 1.1 \) (\( d_i \) becomes 1.789). The resulting output reciprocating motion is displayed in Fig. 14. Here, the cycle is completed at 3960\(^\circ\), therefore, \( n = 11 \) (3960/360). Note that, large amount of stroke change and continuously varying length of stroke is obtained by this method. However, if \( R_2 \) is not an integer, some pilgrim step motions may also occur at the output link. Due to the value of \( R_2 \), the number of pilgrim step motions can be few or much (or rapid) and their magnitude can be small or large.

Fig. 14 Output reciprocating motion of the GASM if \( R_2 = 1.1 \), full cycle is completed after 11 (3960/360) cycles of the input link.
6. Conclusion

This study presents an analysis and synthesis method for a certain type of geared adjustable stroke mechanism (GASM). Although it has been used in practice, to the best of our knowledge, any theory on its analysis and synthesis is not available in the literature. Three methods have been devised for designing mechanisms with large strokes. Gear ratios are chosen equal to unity. With the first two methods very large strokes can be determined. However, a parametric optimization routine for maximum stroke is developed. According to the direction of rotation of the input, there exist two different optimum GASM, therefore two design charts are adopted. By these design charts, optimum parameters can be obtained for every possible mechanism and GASM for maximum stroke can be designed easily.

A method for decreasing the stroke is developed. It is observed that the output stroke of the GASM can be decreased more than 50% (that is a significant value) with proper transmission, velocity and acceleration characteristics. Moreover, this adjustment can be achieved during operation that is one of the most important advantages of such mechanisms. The oscillation change of the arm has a limited effect on the output stroke. If the length of the fixed link is changed, first transmission angle of the mechanism gets worse, therefore only the orientation of the fixed link is changed in order to minimize the stroke.

Different gear ratios are also investigated. If $R_1$ is increased, larger strokes can be determined. On the other hand, radius of floating gears and length of the arm increase, resulting in larger mechanisms, that is not a desired situation. It is observed that, if $R_2 = 0.5$, then pilgrim step motion is obtained. Also, if $R_2$ is not an integer and not equal to 0.5, then the mechanism completes a full cycle after several cycles of the input link rotation. A variable stroke mechanism; continuously varying length of stroke, can be achieved by the aid of this property. The number of cycles of the input where the stroke returns to its initial value is determined by a novel method. Further, if $R_2$ is an integer greater than unity ($R_2 = n$, where $n>1$), then the output completes a cycle after $n$ cycles of the input link, and a pilgrim step motion is obtained.

In this study, a specific type of GASM is investigated in detail and capabilities of the mechanism is determined and explained. The results of the analytical approaches are checked and verified with simulations in MathCAD and Catia. Feasibility of this mechanism is increased for various practical applications. It is believed that the analysis-synthesis procedures and the design charts developed can be very useful for numerous applications of this type of mechanism.

Appendix A. Supplementary data

Simulation via Catia.

http://yunus.hacettepe.edu.tr/~volkan/GASM/

References

Alt Von H., Der ubertragungswinkel und seine bedeutung fur dar konstruieren periodischer getriebe, Werkstattechnik, Vol.26 (Heft4) (1932), pp.61-65.
Balli, S. S. and Chand, S., Transmission angle in mechanisms (Triangle in mech), Mechanism and Machine Theory, Vol.37, No.2 (2002), pp.175-195.
Beiner, L., A three-gear linkage for generating dwell motions, Journal of Mechanical Design, Vol.123, No.4 (2001), pp.637-640.
Chironis, N., P., Mechanisms, Linkages, and Mechanical Controls (1965), McGraw-Hill.
Chang, J. R., Coupling effect of flexible geared rotor on quick-return mechanism undergoing three-dimensional vibration, Journal of Sound and Vibration, Vol.300, No.1–2 (2007), pp.139-159.
Dooner, D. B., A Geared 2-dof Mechanical Function Generator, Journal of Mechanical Design, Vol.121, (1999) pp.65-70.
Dwivedi, S. N. , Application of whitworth quick return mechanism for high velocity impacting press, Mechanism and Machine Theory, Vol.19, No.1 (1984), pp.51-59.
Funabashi, H., Iwatsuki, N. and Yokoyama, Y., Synthesis of crank-length adjusting mechanisms, Bull. JSME, Vol.29, No.252 (1986), pp.1946-1951.
Fung, R. F. and Chen, K. W., Constant speed control of the quick return mechanism, driven by a DC motor, JSME
International journal, Series C, Vol.40, No.3 (1997), pp.454-461.
Hsieh, W. H. and Tsai, C. H., On a novel press system with six links for precision deep drawing, Mechanism and Machine Theory, Vol.46, No.2 (2011), pp.239-252.
Hsieh, W. H. and Tsai C. H., A study on a novel quick return mechanism, Transactions of the Canadian Society for Mechanical Engineering, Vol.33, No.3 (2009), pp.139-152.
Modler, K. H., Lovasz, E. C., Bar, G. F., Neumann, R., Perju, D., Perner, M. and Margineanu, D., General method for the synthesis of geared linkages with non-circular gears, Mechanisms and Machine Theory, Vol.44, No.4 (2009), pp.726-738.
Mundo, D., Gatti, G. and Dooner, D. B., Optimized five-bar linkages with non-circular gears for exact path generation, Mechanism and Machine Theory, Vol.44, No.4 (2009), pp.751-760.
Parlaktaş, V., Söylemez, E. and Tanik, E., On the synthesis of a geared four-bar mechanism, Mechanism and Machine Theory, Vol.45, No.8 (2010), pp.1142–1152.
Pennock, G.R., Israr, A., Kinematic analysis and synthesis of an adjustable six bar linkage, Mechanism and Machine Theory, Vol.44, No.2 (2009), pp.306-323.
Russell, K. and Sodhi, R.S., Kinematic synthesis of planar five-bar mechanisms for multi-phase motion generation, JSME International Journal, Series C: Mechanical Systems, Machine Elements and Manufacturing, Vol.47, No.1 (2004), pp.345-349.
Russell, K. and Sodhi, R.S., On the design of slider-crank mechanisms. Part I: Multi-phase motion generation, Mechanism and Machine Theory, Vol.40, No.3 (2005a), pp.285-299.
Russell, K. and Sodhi, R.S., On the design of slider-crank mechanisms. Part II: Multi-phase path and function generation, Mechanism and Machine Theory, Vol.40, No.3 (2005b), pp.301-317.
Russell, K. and Sodhi, R.S., Design of adjustable RSSR-SC mechanisms for multi-phase motion generation, JSME International Journal, Series C: Mechanical Systems, Machine Elements and Manufacturing, Vol.48, No.4 (2005c), pp.668-673.
Sakamoto, Y., Ogawa, K., Shimojima, H., Sato, O. and Kajikawa, K., A dynamic analysis of an adjustable conveying machine, JSME International Journal, Vol.30, No.266 (1987), pp.1332-1339.
Shimojima, H., Ogawa, K., Fujwara, A. and Soto, O., Kinematic synthesis of adjustable mechanisms (Part 1, Path-Generators), Bull. JSME., Vol.26, No.214 (1983), pp.627-632.
Shimojima, H. and Ogawa, K., Kinematic synthesis of adjustable mechanisms (Part 2, Function-Generators), Bull. JSME., Vol.27, No.227 (1984), pp.1025-1030.
Shimojima, H., Satoh, O. and Kuwabara, M., A study of adjustable mechanisms with multiple-degree of freedom: number and dimensional syntheses of mechanisms, JSME International Journal. Ser. 3, Vol.32, No.3 (1989), pp.436-441.
Shimojima, H., Iida, K. and Kowabara, M., Kinematic synthesis of adjustable mechanisms : 3rd Report, 6-link dwell mechanisms, Bull. JSME, Vol.29, No.254 (1986), pp.2718-2723.
Soong, R.C., A design method for four-bar mechanisms with variable speeds and length-adjustable driving links, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.3, No.4 (2009), pp.312-323.
Soong, R. C., A new design method for variable-speed single-DOF mechanical presses with adjustable-length driving links, Mechanism and Machine Theory, Vol.45, No.3 (2010), pp.496-510.
Soong, R.C., Analysis of novel geared linkage mechanisms, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.8, No.3 (2014), pp.JAMDSM0030.
Soong, R.C., A new cam geared mechanism for exact path generation, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.9, No.2 (2015), pp.JAMDSM0020
Tanik, E. and Söylemez, E, A novel design method for underactuated variable oscillation mechanisms, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.9, No.1 (2015), pp.JAMDSM0009.
Yossifon, S., Messerly, D., Kropp, E., Shivpuri, R. and Altan, T., A servo motor driven multi-action press for sheet metal forming, International Journal of Machine Tools and Manufacture, Vol. 31, No.3 (1991), pp. 345–359.