Airy wavepackets are coherent states

Vivek M. Vyas
Theoretical Physics Group,
Raman Research Institute,
Sadashivnagar, Bengaluru 560 080, INDIA
vivekv@rri.res.in

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Abstract

Accelerating non-spreading wavepackets in nonrelativistic free particle system, with probability distribution having an Airy function profile, were discovered by Berry and Balazs (Am. J. Phys., 47, 264 (1979)), and have been subsequently realised in several optical experiments. It is shown that these wavepackets are actually coherent states. It is found that the Galilean invariance of the Schrödinger equation plays a key role in making these states unique and giving rise to their unusual propagation properties.

1 Introduction

Since last several years, non-diffracting self-accelerating optical Airy beams have been a subject of intense study. Non-spreading uniformly accelerating Airy wavepackets were discovered a long time ago by Berry and Balazs [1] in the nonrelativistic quantum free particle system, and since then have been carefully studied from various aspects [2, 3]. It is well known that, the Helmholtz equation governing the dynamics of the electric field envelope of a plane polarised optical beam, becomes identical to free Schrödinger equation, in the paraxial approximation [5]. This identification naturally provides a route for unhindered exchange of concepts and results between these two different physical systems, which was exploited by Siviloglou and Christodoulides [4], to show the possibility of realisation of the optical beam counterpart of non-spreading accelerating Airy wavepacket. In 2007, such optical Airy beams were experimentally realised by Siviloglou et. al. [6], and since then they have been keenly studied. Apart from being non-diffracting beam, which makes them useful in their own right [7], these beams also display self-healing [8], possess a nontrivial orbital angular momentum behaviour [9] and polarisation property [10]. These unique properties makes such beams suitable for a number of applications, generation of plasma in a dielectric media in a controlled manner [11] and optical manipulation of dielectric microparticles [12] are two notable ones amongst the others.

In an introductory quantum mechanics course, one is introduced to the Schrödinger equation obeyed by the wave function of the system and its probabilistic interpretation.
While discussing the quantum mechanical free particle system, it is presumed that the free particle can not be localised in an arbitrarily small volume at all times. This point is conveyed by showing that the wavefunction \( \psi(x, t) \) at any time \( t \), is related to the same at an earlier time \( t' \) by convolution:

\[
\psi(x, t) = \int_{-\infty}^{\infty} dy K(x, y; t, t') \psi(y, t').
\]

(1)

Here \( K(x, y; t, t') \) is the propagator and it reads:

\[
K(x, y; t, t') = \frac{m}{2\pi i \hbar (t - t')} \exp \left( \frac{i m(x - y)^2}{2 \hbar (t - t')} \right).
\]

(2)

The dispersive character of the wave function is made apparent by considering \( \psi(y, t') = \delta(y - y_0) \), an infinitely narrow wavepacket located at \( y_0 \), and showing that its spreads indefinitely for any time \( t > t' \). Such an exposition leaves an impression that it is not possible to have a wavepacket evolution in the free Schrödinger equation without its dispersion and/or distortion. As a result of this a natural question arises, in what conditions can a wavepacket evolution take place without dispersion and distortion of its shape. This immediately reminds one of the celebrated coherent (wavepacket) states of the harmonic oscillator problem, which are well known to evolve in time without dispersion. Thus one is left with an impression that the existence of a confining potential, such that \( V(x) \to \infty \) as \( x \to \pm \infty \), is required to save a wavepacket from spreading indefinitely.

The existence of non-spreading accelerating solutions to free Schrödinger equation, may appear incomprehensible and contradictory with the wavepacket dynamics in quantum free particle system. One may naively wonder, whether these wavepackets have any connection with the harmonic oscillator coherent states or not. To the best of the author’s knowledge this natural question has not been yet answered. The goal of this letter is to bring to fore an interesting relation between these two wavepackets, and show that the non-spreading Airy wavepackets are actually coherent states.

2 Harmonic oscillator coherent state and its generalisation

The coherent states were first studied by Schrödinger, and came into prominence from the works of Sudarshan and Glauber in quantum optics [13, 14]. It is worth recalling that, in terms of annihilation operator: \( \hat{a} = \sqrt{\frac{m \omega}{2 \hbar}} \hat{x} + i \frac{1}{\sqrt{2 m \omega \hbar}} \hat{p} \), and creation operator \( \hat{a}^\dagger \), such that \( [\hat{a}, \hat{a}^\dagger] = \hat{I} \); the harmonic oscillator Hamiltonian reads: \( \hat{H} = \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \hat{I} \right) \). The coherent state in this system can be constructed from the ground state \( |0\rangle \), which is annihilated by \( \hat{a} \): \( \hat{a} |0\rangle = 0 \), from the application of a unitary operator, which is constructed using \( \hat{a}^\dagger \) and \( \hat{I} \), on it [15]:

\[
|\alpha\rangle = \exp \left( \alpha \hat{a}^\dagger - \frac{|\alpha|^2}{2} \hat{I} \right) |0\rangle.
\]

(3)
There are several properties that make this family of coherent states special [13, 14]. An important property is its temporal stability, that is, a coherent state is stable under time evolution, which only changes the value of $\alpha$: $e^{-i\frac{\alpha}{\hbar}t}|\alpha\rangle = |\alpha e^{-i\omega t}\rangle$. One of the most important properties possessed by these states is that the functional form of the probability density remains the same under time evolution, except that the peak of the function performs periodic classical motion around the origin:

$$P_\alpha(x, t) = \frac{1}{x_0\sqrt{\pi}} \exp \left(-\frac{1}{x_0^2}(x - \langle x(t) \rangle_\alpha)^2\right),$$

where $\langle x(t) \rangle_\alpha = \sqrt{2x_0}|\alpha\rangle \cos(\omega t)$, and $x_0^2 = \frac{\hbar}{m\omega}$. This shows that this coherent wave packet does not disperse or distort as it evolves.

One naturally wonders whether there exists other quantum systems which possess such coherent states or not. It turns out that there exist coherent states in several other quantum systems, but they do not obey all the properties possessed by the harmonic oscillator coherent states. In this scenario, one has to contend working with different generalisations of harmonic oscillator coherent states, each of them possessing distinct trait and character. One of such well studied generalisations are the so called Perelomov coherent states [16, 17], which possess a rich mathematical structure. In a given system, say if the operators $\hat{X}$, $\hat{Y}$ and $\hat{Z}$ form a closed algebra, so that the commutator of any two operators is equal to the third one (modulo a c-number factor), then the Perelomov coherent states can be straightforwardly constructed in such a case. From the state $|x\rangle$, which is an eigenstate of $\hat{X}$: $\hat{X}|x\rangle = x|x\rangle$, the coherent state $|a, b\rangle$ can be constructed as:

$$|a, b\rangle = \exp \left(a\hat{Y} + b\hat{Z}\right) |x\rangle. \quad (4)$$

Analogously one can construct coherent states from the eigenstates of other operators as well, which would provide with a different family of coherent states\(^\dagger\). The initial state, which ought to be an eigenstate of an operator which forms the algebra, is often called the fiducial state in the literature. One notes that, in essence, the Perelomov definition of coherent states aims at giving a group theoretic generalisation of harmonic oscillator coherent states. In certain cases, it has been shown that these coherent states show classical behaviour, and saturate the Heisenberg uncertainty relation as well [16].

### 3 Galilean invariance of Schrödinger equation

It is a well known fact that, the Newton’s laws of motion remain the same for any two observers moving inertially with respect to one another. This is a manifestation of (Galilean) principle of relativity. This principle also holds in nonrelativistic quantum mechanics, where it manifests as the form invariance of the Schrödinger equation for the two observers. An observer observing a particle of mass $m$ moving in one dimensional

\(^*\)A familiar example of these coherent states is given by angular momentum coherent states, obtained by the action of $SO(3)$ operators on $\hat{L}_z$ eigenstate: $\exp(i\theta \hat{L}_x + i\theta \hat{L}_y)|m\rangle$, where $\hat{L}_z|m\rangle = m|m\rangle$.

\(^\dagger\)What is presented here is the essence of the coherent state construction due to Perelomov. For a mathematically rigorous and nuanced treatment, the reader is referred to Refs. [14] and [16].
space, finds that its motion is described by the wavefunction $\psi(x, t)$, which solves the Schrödinger equation:

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = i\hbar \frac{\partial \psi}{\partial t}. \quad (5)$$

The same motion is observed by some other observer, who is moving relative to the former with a constant velocity $v$, using coordinates $x' = x - vt$ and $t' = t$. He finds that the motion is being described by the wavefunction $\psi'(x', t')$ which solves the Schrödinger equation:

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi'}{\partial x'^2} = i\hbar \frac{\partial \psi'}{\partial t'}, \quad (6)$$

albeit in his coordinate frame. It can be easily checked that the two wavefunctions $\psi$ and $\psi'$ are connected via a unitary transformation [15]:

$$\psi'(x', t') = \exp \left[ i \frac{\hbar}{mv^2} \frac{mv^2 t^2}{2} - i \frac{\hbar}{mv} \frac{mv^2}{2} \hat{p} \right] \psi(x, t), \quad (7)$$

which does not alter their normalisations. This unitary transformation which connects the two inertial observers can be simply seen to be expressed in terms of the operator $\hat{K}(t) = t\hat{p} - m\hat{x}$ called the **Galilean boost (generator)** [15]. This can be seen by writing it as:

$$\langle x' | \psi'(t') \rangle = e^{-i \frac{\hbar}{mv^2} \frac{mv^2 t^2}{2} - i \frac{\hbar}{mv} \frac{mv^2}{2} \hat{p}} \langle x | \psi(t) \rangle$$

$$= \langle x' | e^{i \frac{\hbar}{mv^2} \hat{K}(t)} | \psi(t) \rangle, \quad (8)$$

($|x\rangle$ is a position eigenstate with eigenvalue $x$) which shows that any given state $|\psi(t)\rangle$ for the former observer corresponds to the transformed state (called the **boosted** state):

$$|\psi'(t')\rangle = e^{i \frac{\hbar}{mv^2} \hat{K}(t)} |\psi(t)\rangle \quad (10)$$

to the latter. In particular, $|x'\rangle = e^{i \frac{\hbar}{mv} \hat{K}(t)} |x\rangle = \exp \left( i \frac{\hbar}{mv^2} \frac{mv^2 t^2}{2} - i \frac{\hbar}{mv} \frac{mv^2}{2} \right) |x - vt\rangle$. Interestingly both the observers agree that the average of $\hat{K}(t)$ itself is a constant of motion: $\frac{d}{dt} \langle \psi(t)|\hat{K}(t)|\psi(t)\rangle = 0$ and $\frac{d}{dt} \langle \psi'(t')|\hat{K}(t')|\psi'(t')\rangle = 0$. This is a consequence of Schrödinger equation and can be easily derived using relations: $m \frac{d(x)}{dt} = \langle p \rangle$ and $\frac{d(p)}{dt} = 0$.

### 4 Accelerating free particle coherent states

In a given system the Hamiltonian, which in our case is $\hat{H} = \frac{\hat{p}^2}{2m}$, generates the time evolution. The stationary states of the Schrödinger equation, being the eigenstates of Hamiltonian, evolve trivially under time evolution. One wonders whether there exists states which get trivially transformed under a boost transformation (10). Using the analogy one finds that such states would be the eigenstates of $\hat{K}$:

$$\hat{K}(t)|\xi; t\rangle = \xi |\xi; t\rangle, \quad (11)$$
with real eigenvalue $\xi$. Note that the set of operators $\{\hat{I}, \hat{x}, \hat{p}, \frac{\hat{p}^2}{2}\}$ with the commutation relations:

$$\begin{align*}
[\hat{x}, \hat{p}] &= i\hbar \hat{I}, \\
[\hat{x}, \frac{\hat{p}^2}{2}] &= i\hbar \hat{p}, \\
[\hat{p}, \frac{\hat{p}^2}{2}] &= 0,
\end{align*}$$

(12)

constitute a closed algebra. This algebra can be thought of as a generalisation of harmonic oscillator algebra of $\{\hat{a}, \hat{a}^\dagger, \hat{I}\}$, which can also be expressed using the operators $\{\hat{I}, \hat{x}, \hat{p}\}$.

Considering $|\Phi\rangle = |x = 0; t = 0\rangle$ as the fiducial state\(^\dagger\), one can now construct Perelomov coherent states from it by the application of unitary operators constructed using $\hat{p}$ and $\frac{\hat{p}^2}{2}$. Surprisingly one finds that the coherent state so generated is actually the state $|\xi; t\rangle$:

$$|\xi; t\rangle = \exp \left(-\frac{it}{2\hbar m} p^2 + \frac{i\xi}{\hbar m} \hat{p}\right) |\Phi\rangle,$$

(13)

for any real $\xi$. Being eigenstates of a Hermitean operator $\hat{K}$, these states form an orthonormal complete basis set:

$$\int_{-\infty}^{\infty} d\xi |\xi; t\rangle \langle \xi; t| = 1 \quad \text{and} \quad \langle \xi; t|\xi'; t\rangle = \delta(\xi - \xi').$$

(14)

These relations can be explicitly checked using their $x$-representation:

$$\psi_\xi(x, t) = \langle x|\xi; t\rangle = \frac{1}{\sqrt{2\pi\hbar t}} \exp \left(i \frac{m x^2}{2t} - \frac{\xi x}{t}\right).$$

(15)

It is worth noting that, though the wavepacket states $\psi_\xi(x, t)$ are Perelomov coherent states, they do suffer dispersion and for large times they get indefinitely spread out.

Note that the set of operators $\{\hat{I}, \hat{x}, \hat{p}, \frac{\hat{p}^2}{2}, \frac{\hat{p}^3}{6}\}$, with nontrivial commutation relations:

$$\begin{align*}
[\hat{x}, \hat{p}] &= i\hbar \hat{I}, \\
[\hat{x}, \frac{\hat{p}^2}{2}] &= i\hbar \hat{p}, \\
[\hat{x}, \frac{\hat{p}^3}{6}] &= i\hbar \frac{\hat{p}^2}{2},
\end{align*}$$

(16)

form a closed algebra. This motivates one to construct a generalisation of coherent state $|\xi; t\rangle$, from the fiducial state $|\Phi\rangle = |x = 0; t = 0\rangle$, albeit using the operators $\{\hat{p}, \frac{\hat{p}^2}{2}, \frac{\hat{p}^3}{6}\}$. Such a coherent state $|\varepsilon, \xi; t\rangle$ is given by:

$$\begin{align*}
|\varepsilon, \xi; t\rangle &= \mathcal{U}(\varepsilon, t, \xi)|\Phi\rangle \\
&= \exp \left(-\frac{i\varepsilon}{6\hbar m^2} \hat{p}^3 - \frac{it}{2\hbar m} p^2 + \frac{i\xi}{\hbar m} \hat{p}\right) |\Phi\rangle,
\end{align*}$$

(17)

(18)

for any real $\varepsilon$ and $\xi$. Interestingly it turns out that this state is an eigenstate of $\hat{K}(t) + \varepsilon \hat{H}$:

$$\left(\hat{K}(t) + \varepsilon \hat{H}\right) |\varepsilon, \xi; t\rangle = \xi |\varepsilon, \xi; t\rangle.$$

(19)

\(^\dagger\)One can consider any other fiducial state say $|x = x_0; t = t_0\rangle$, without affecting subsequent conclusions about Perelomov coherent states. The state $|x = 0; t = 0\rangle$ only chosen for simplicity.
The coherent states \(|\xi, t\rangle\) are a special case of these general coherent states corresponding to \(\varepsilon = 0\). It immediately follows from (19) that for the same value of \(\varepsilon\), the states \(|\varepsilon, \xi; t\rangle\) with different \(\xi\) span the Hilbert space and provide with a complete orthonormal basis set:

\[
\int_{-\infty}^{\infty} d\xi |\varepsilon, \xi; t\rangle \langle \varepsilon, \xi; t| = 1 \quad \text{and} \quad \langle \varepsilon, \xi; t|\varepsilon, \xi'; t\rangle = \delta(\xi - \xi').
\]

(20)

The states with same \(\xi\) but different \(\varepsilon\) are however not orthogonal:

\[
\langle \varepsilon, \xi; t|\varepsilon', \xi; t\rangle = \left(\frac{2\hbar m^2}{\varepsilon - \varepsilon'}\right)^{\frac{1}{3}} \text{Ai}(0),
\]

(21)

which can be easily deduced by working with the \(p\)-representation \(\tilde{\psi}_{\varepsilon, \xi}(p, t)\) of state \(|\varepsilon, \xi; t\rangle\):

\[
\tilde{\psi}_{\varepsilon, \xi}(p, t) = \langle p|\varepsilon, \xi; t\rangle = e^{i\frac{p^2 \xi^2}{2m} - \frac{ip\varepsilon}{\hbar \varepsilon}}.
\]

(22)

Equation (19) would lead one to believe that the parameters \(\xi\) and \(\varepsilon\) are not independent, since:

\[
\xi = \langle \hat{K}\rangle_{\varepsilon, \xi, t} + \varepsilon \langle \hat{H}\rangle_{\varepsilon, \xi, t},
\]

(23)

where \(\langle \hat{A}\rangle_{\varepsilon, \xi, t}\) stands for \(\langle \varepsilon, \xi; t|\hat{A}|\varepsilon, \xi; t\rangle\). However by an explicit calculation one indeed finds that \(\frac{\partial \xi}{\partial \varepsilon} = 0\), depicting their independence. This follows by noting that: \(\frac{\partial}{\partial \varepsilon} \langle \hat{H}\rangle_{\varepsilon, \xi, t} = 0\) and \(\frac{\partial}{\partial \varepsilon} \langle \hat{K}\rangle_{\varepsilon, \xi, t} + \langle \hat{H}\rangle_{\varepsilon, \xi, t} = 0\), both of which arise as a consequence of (18). This also shows that the coherent states \(|\varepsilon, \xi; t\rangle\) are temporally stable, the coherent character of state \(|\varepsilon, \xi; t\rangle\) remains intact under time evolution due to independence of \(t\), \(\xi\) and \(\varepsilon\). By an explicit calculation, using (18) or (22), one finds that the averages \(\langle \hat{p}\rangle_{\varepsilon, \xi, t}, \langle \hat{H}\rangle_{\varepsilon, \xi, t}\) and \(\langle \hat{K}\rangle_{\varepsilon, \xi, t}\) are constants of motion, but are divergent. Interestingly one can rewrite (23) so that it reads:

\[
m\langle \hat{x}\rangle_{\varepsilon, \xi, t} = -\xi + t\langle \hat{p}\rangle_{\varepsilon, \xi} + m\varepsilon \langle \hat{H}\rangle_{\varepsilon, \xi},
\]

(24)

which shows that the constant parameters \(\xi\) and \(\varepsilon\) contribute to the initial average displacement.

The fact that \(|\varepsilon, \xi; t\rangle\) is an eigenstate of \(\hat{K}(t) + \varepsilon \hat{H}\), gives it a very unique dynamical behaviour. Using the Zassenhaus formula:

\[
e^{t(X+Y)} = e^{tX}e^{tY}e^{-\frac{t^2}{4}[X,Y]}e^{\frac{t^2}{6}(2[Y,[X,Y]]+[X,[X,Y]])} \ldots
\]

(25)

one can decompose exponential of \(\hat{K}(t) + \varepsilon \hat{H}\) as:

\[
e^{\frac{it}{\hbar}(K(t)+\varepsilon H)} = e^{-\frac{imv^3}{3\hbar^3}}e^{\frac{imv^2}{2\hbar^2}\hat{H}}e^{\frac{imv^3}{3\hbar^3}\hat{K}(t)}e^{-\frac{imv^2}{2\hbar^2}\hat{p}}.
\]

(26)

Noting that the product \(\varepsilon v\) has dimensions of time, and denoting it as \(\tau(= \varepsilon v)\), this equation on state \(|\varepsilon, \xi; t\rangle\) at \(t = 0\) leads to a very interesting relation:

\[
e^{-\frac{im\tau}{\hbar}\hat{H}}|\varepsilon, \xi; 0\rangle = \left(e^{-\frac{im\tau}{\hbar}}e^{-\frac{imv^3}{3\hbar^3}}\right)e^{\frac{imv^3}{3\hbar^3}\hat{K}(0)}e^{\frac{imv^2}{2\hbar^2}\hat{p}}|\varepsilon, \xi; 0\rangle.
\]

(27)
It essentially shows that effect of time evolution operator on such a state is same as that of action of spatial translation operator, albeit with time dependent translation parameter $\tau$. Followed by a Galilean boost, albeit with a time dependent velocity $\dot{\tau}$. A little reflection will convince the reader that these two operations actually lead to a constant acceleration $\frac{1}{\epsilon}$. This shows the unique feature of dynamics of these coherent states: time evolution is (modulo a time dependent phase) is same as being transformed with a constant acceleration.

Above inference about the constant accelerating nature of the coherent states was done by inspecting the effect of time evolution operator on them. However a sceptical reader might wonder whether this acceleration physically manifests in any aspect or whether is just a mathematical artefact. This can be best addressed by working in $x$-representation, were $\psi_{\varepsilon,\xi}(x,t) = \langle x|\varepsilon,\xi; t \rangle$ stand for the wavefunction corresponding to state $|\varepsilon,\xi; t \rangle$. The relation (27) in this representation reads:

$$\psi_{\varepsilon,\xi}(x,\tau) = e^{-\frac{i\tau\xi}{\hbar\epsilon}}e^{-\frac{i}{\hbar}(\frac{m\varepsilon^2}{2}\dot{\tau}^2 + \frac{mx}{\epsilon})}\psi_{\varepsilon,\xi}(x + \frac{\tau^2}{2\varepsilon},0). \quad (28)$$

This elegant relation depicts that the shape of probability density for such (wave packet) states does not change under time evolution, and they accelerate with a constant acceleration $\frac{1}{\epsilon}$:

$$|\psi_{\varepsilon,\xi}(x,\tau)|^2 = |\psi_{\varepsilon,\xi}(x + \frac{\tau^2}{2\varepsilon},0)|^2. \quad (29)$$

This is a clear proof of the non-spreading nature of these wave packet states: the form of the density remains the same, the time evolution only transports it quadratically as a function of time. This treatment essentially shows that the accelerating motion and the non-spreading nature of this coherent state arise out of its definition from (18). The explicit form of $\psi_{\varepsilon,\xi}(x,t)$ is given by:

$$\psi_{\varepsilon,\xi}(x,t) = \frac{1}{\sqrt{\hbar m}} \left( \frac{2\hbar m^2}{\varepsilon} \right)^{\frac{1}{4}} \exp \left( \frac{i}{\hbar} \frac{(\xi - mx)t}{\varepsilon} - \frac{i}{\hbar} \frac{mt^3}{3\varepsilon^2} \right) \times \text{Ai} \left( -\frac{1}{\hbar} \left( \frac{2\hbar m^2}{\varepsilon} \right)^{\frac{1}{4}} \left( x - \frac{\xi}{m} + \frac{t^2}{2\varepsilon} \right) \right), \quad (30)$$

which can be used to check the aforementioned results. The accelerating motion of these coherent states does not conflict with the Ehrenfest theorem, as is clear from (24).

However note that unlike harmonic oscillator coherent states, the peak of the probability density of these coherent states does not traverse a classical trajectory, which in this system is the one without any acceleration. It is worth emphasizing that, from this treatment it becomes apparent that the Galilean invariance of free Schrödinger equation has played a vital role in realisation of these coherent states in the system. In the presence of a potential, the Galilean and translational invariance of the Schrödinger equation is lost, and as a result such non-spreading accelerating coherent states can not be constructed in any other system with a potential.

From the above discussion the physical significance of the parameter $\varepsilon$ and the nature of unitary transformation (17) which generates it, becomes clear. The action of $U(\delta,0,0)$ on a state accelerating with acceleration $\frac{1}{\kappa}$ transforms it to the one with acceleration $\frac{1}{\kappa+\delta}$. 


The state of zero acceleration can be obtained by taking the limit $\varepsilon \to \infty$ with a fixed finite value of $\xi$. The relation (27) in this limit yields:

$$e^{-i\frac{\varepsilon}{\hbar}H}|\varepsilon \to \infty, \xi; 0\rangle = |\varepsilon \to \infty, \xi; 0\rangle$$

which shows that the state $|\varepsilon \to \infty, \xi; \tau\rangle$ is actually the ground state of the system with zero energy $|p = 0\rangle$, since it is the only state that does not evolve in time. On the other hand, in the limit $\varepsilon \to 0$, the state $|\varepsilon, \xi; 0\rangle$ goes over to become the eigenstate of $\hat{K}$, implying that the eigenstates of $\hat{K}$ are the ones moving with infinite acceleration. Thus the accelerating coherent states $|\varepsilon, \xi; \tau\rangle$ are seen to interpolate smoothly and continuously from the ground state $|p = 0\rangle$, which has no acceleration; to state $|\varepsilon = 0, \xi; \tau\rangle$ which has an infinite acceleration.

5 Conclusion

In this letter, it is shown that the accelerating wavepacket solutions of free Schrödinger equation, discovered by Berry and Balazs, are indeed coherent states. The harmonic oscillator coherent states are known to arise out of the algebra generated by $\{\hat{I}, \hat{x}, \hat{p}\}$. By considering a larger algebra, consisting of $\{\hat{I}, \hat{x}, \hat{p}, \hat{p}^2, \hat{p}^3\}$, it is found that the accelerating Airy wavepackets arise out of it as coherent states. Moreover it is found that these coherent states solve the eigenvalue problem for the linear combination of boost operator $\hat{K}$ and Hamiltonian $\hat{H}$, which is responsible for their seemingly intriguing non-spreading accelerating nature. This provides one with a representation independent understanding of the origin and the nature of such accelerating coherent states. To the best of the author’s knowledge, all the other works concerning origin of these accelerating wavepacket states commit to some or the other representation. Moreover the present treatment is naturally suitable for a realisation and identification of such coherent states in systems involving statistical mixtures and in open systems, wherein the quantum free particle is interacting with a larger reservoir system, while being in or away from thermal equilibrium. It also allows one to straightforwardly construct such coherent states in systems involving quantised fields. Such systems naturally appear while dealing with indefinite and large number of particles, and are often encountered in quantum optics and condensed matter experiments.

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