The wave function of the universe and spontaneous breaking of supersymmetry

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(April 5, 2017)

Abstract

In this work we define a scalar product “weighted” with the scalar factor $R$ and show how to find a normalized wave function for the supersymmetric quantum FRW cosmological model using the idea of supersymmetry breaking selection rules under local $n=2$ conformal supersymmetry. We also calculate the expectation value of the scalar factor $R$ in this model and its corresponding behaviour.

PACS numbers: 11.30.Pb, 12.60.Jv, 98.80.Hw

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Our universe as a whole can hardly be called a usual quantum system, however, if we want to apply the laws of quantum mechanics to such exiting processes as the birth of the universe and its subsequent evolution we have to treat it as a quantum object obeying some fundamental principles of the usual quantum mechanics including unitarity, and hence the normalizability of the wave function of the universe and other characteristics that only appear in the quantum level. In the usual Hamiltonian formulation the wave function of the universe $\Psi(g_{ij}, \phi)$ is a functional only of the metric field $g_{ij}(x^i)$ and other fields $\phi(x^i)$ defined on three-space, so the relation in classical general relativity $H\Psi = 0$, known as the Wheeler-DeWitt (WD) equation, seems to indicate that theories no causality in quantum cosmology, no dynamics and no obvious analogue of a conserved probability distribution. Quantum cosmology has revived a very old issue in quantum theory, i.e. the relation between wave function and probability distributions [1], but it is well known, that the Hartle-Hawking wave function is not normalizable in a semiclassical approximation [1,2], and also it does not allow a conserved current with a positive-definite probability density. A partial answer to this subject was introduced, for first time in [3], where a “weight” function is used in the definition of a positive-definite probability density $\rho$, using the Dirac quantization for FRW cosmologies. Here we will give other answer to this subject, using supersymmetric quantum approach n=2, where we also introduce one “weight” function in the definition for the inner product of two supersymmetry states, that permits us to avoid the ordinary real parameter “p” that measure the ambiguity in the factor ordering (here is fixed!).

Other intrinsic problem is selection of the appropriate boundary conditions on the wave function. To solve this problem, some proposals have been suggested [3,4,5,6]. Also at the quantum level the problem of the ambiguity in the factor ordering, when the WD equation contains the product of non-commuting operators $\xi$ and $\pi_\xi$, where $\xi$ is any canonical “coordinate” field and $\pi_\xi$ is the canonical conjugate momentum to this “coordinate”. In the usual way, this ambiguity has been “measured” with the parameter “p” [5,6], where p is a real constant. For some particular “p” values it was possible to solve the WD equation [9,10].

In this work we present an approach to solve this last problem, using the ideas of supersymmetry breaking selection rules under local n=2 conformal supersymmetry. We present a solution for the wave function of the universe for the FRW cosmological model, that is normalizable, and also we obtain the behaviour of the spectation value for the scalar factor in the usual way, giving us proportional to $\ell_{pl}^{2/3}$. The existence of normalizable solution of the system, that appears in this approach means in its turn, that supersymmetry is unbroken quantum mechanically.

To solve this problem in our supersymmetric approach the “coordinate” field corresponds to the scalar factor of the universe for the FRW cosmological model, when we do the integration with measure $R^{1/2}dR$ in the inner product of two states, thus the momenta hermitian-conjugate to $\pi_R$ is non-hermitian with $\pi_R^\dagger = R^{-1/2}\pi_R R^{1/2}$. However, we can construct one hermitian operator and by means of this avoid the use of the real parameter “p”.

This work is organized in the following way. In Sec. II we present the lagrangian, including the auxiliary fields and with them we give the supersymmetry breaking selection
rules. In Sec. III we give the super-algebra, that satisfies the different operators in the theory. Sec. IV is devoted to the solution of the normalized wave function state with zero energy, besides, we present the speculation value for the scalar factor. Finally, Sec. V is for conclusions and remarks.

II. SUPERSYMMETRIC LAGRANGIAN AND SUSY BREAKING

The most general superfield action for a homogeneous scalar supermultiplet interacting with the scalar factor in the supersymmetric FRW model \[11,12\] has the form

\[
S = S_{\text{FRW}} + S_{\text{mat}},
\]

\[
S_{\text{FRW}} = \int 6 \left[ -\frac{1}{2k^2} \frac{R}{N} \mathcal{D}_\eta R \mathcal{D}_\eta R + \frac{\sqrt{k}}{2k^2} R^2 \right] d\eta d\bar{\eta} dt,
\]

\[
S_{\text{mat}} = \int \left[ \frac{1}{2} \frac{R^3}{N} \mathcal{D}_\eta \Phi \mathcal{D}_\eta \Phi - 2R^3 g(\Phi) \right] d\eta d\bar{\eta} dt,
\]

with \( k = 0,1 \) stands for flat and closed space, and \( \kappa^2 = 8\pi G_N \), where \( G_N \) is Newton’s constant of gravity, \( (\hbar = c = 1) \). The units for the constants and fields in this work are the following: \( |\kappa^2| = \ell^2 \), \( |N| = \ell^0 \), \( |R| = \ell^1 \), \( |\Phi| = \ell^{-1} \) and superpotential \( |g(\Phi)| = \ell^{-3} \), where \( \ell \) correspond to units of length.

For the one-dimensional gravity superfield \( N(t, \eta, \bar{\eta}) \) \( (\mathcal{N}^\dagger = \mathcal{N}) \) we have the following series expansion

\[
\mathcal{N}(t, \eta, \bar{\eta}) = N(t) + i\eta \bar{\psi}'(t) + i\bar{\eta}\psi'(t) + \eta\bar{\eta}\mathcal{V}'(t),
\]

where \( N(t) \) is the lapse function and also we have introduced the reparametrization \( \psi'(t) = N^{1/2}(t)\psi(t) \) and \( \mathcal{V}'(t) = \mathcal{N}(t)\mathcal{V}(t) + \bar{\psi}(t)\psi(t) \).

The Taylor series expansion for the superfield \( \mathcal{R}(t, \eta, \bar{\eta}) \) with the scalar factor \( R(t) \) has the similar form

\[
\mathcal{R}(t, \eta, \bar{\eta}) = R(t) + i\eta \bar{\chi}'(t) + i\bar{\eta}\chi'(t) + \eta\bar{\eta}\mathcal{B}'(t),
\]

where \( \chi'(t) = \kappa N^{1/2}(t)\chi(t) \) and \( \mathcal{B}'(t) = \kappa N(t)\mathcal{B}(t) + \frac{\kappa}{2}(\bar{\psi}\chi - \psi\bar{\chi}) \).

The component of the (scalar) matter superfields \( \Phi(t, \eta, \bar{\eta}) \) may be written as \( (\Phi^+ = \Phi) \)

\[
\Phi = \varphi(t) + i\eta \bar{\chi}'(t) + i\bar{\eta}\chi'(t) + F'(t)\eta\bar{\eta}
\]

where \( \chi'(t) = N^{1/2}(t)\chi(t) \) and \( F'(t) = NF + \frac{1}{2}(\bar{\psi}\chi - \psi\bar{\chi}) \), besides, \( \mathcal{D}_\eta = \frac{\partial}{\partial \eta} + i\bar{\eta}\frac{\partial}{\partial \bar{\eta}} \) and \( \mathcal{D}_{\bar{\eta}} = -\frac{\partial}{\partial \eta} - i\eta\frac{\partial}{\partial \bar{\eta}} \) are the supercovariant derivatives of the conformal supersymmetry \( n = 2 \), which has dimension \( [\mathcal{D}_\eta] = [\mathcal{D}_{\bar{\eta}}] = \ell^{-\frac{1}{2}} \).

After the integration over Grassmann complex coordinate \( \eta \) and \( \bar{\eta} \), and making the following redefinition of the “fermion” fields (Grassmann variables) \( \lambda(t) \rightarrow \frac{1}{\sqrt{3}} R^{-1/2}(t)\lambda(t) \) and \( \chi(t) \rightarrow R^{-3/2}(t)\chi(t) \), we find the Lagrangian in which the fields \( \mathcal{B}(t) \) and \( F'(t) \) are auxiliary and they can be eliminated with the help of their equations of motion.

Finally, the Lagrangian in terms of components of the superfields \( \mathcal{R}, \mathcal{N}, \Phi \) takes the form
\[ L = -\frac{3}{\kappa^2} \frac{R(DR)^2}{N} + \frac{2}{3} i\tilde{\lambda}D\lambda + \sqrt{k} \frac{\kappa}{R} (\tilde{\psi}\lambda - \psi\tilde{\lambda}) \\
+ \frac{1}{3} N R^{-1} \sqrt{k} \tilde{\lambda}\lambda + \frac{3k}{\kappa^2} N R + \frac{R^3 (D\varphi)^2}{2 \kappa^2} N - i\tilde{\chi}D\chi \\
- \frac{3}{N} k N R^{-1} \tilde{\chi}\chi - \kappa^2 N g(\varphi) \tilde{\lambda}\lambda - 6\sqrt{k} N g(\varphi) R^2 \\
- N R^3 V(\varphi) + \frac{3}{2} \kappa^2 N g(\varphi) \tilde{\chi}\chi + \frac{i\kappa}{2} D\varphi(\tilde{\lambda}\chi + \lambda\tilde{\chi}) \\
- 2N \frac{\partial^2 g(\varphi)}{\partial\varphi^2} \tilde{\chi}\chi - \kappa N \frac{\partial g(\varphi)}{\partial\varphi} (\tilde{\lambda}\chi - \lambda\tilde{\chi}) + \kappa^2 \frac{R^{-3/2}}{4} (\psi\tilde{\lambda} - \tilde{\psi}\lambda) \tilde{\chi}\chi \\
- \kappa R^{3/2} (\psi\lambda - \tilde{\psi}\tilde{\lambda}) g(\varphi) + R^{3/2} \frac{\partial g(\varphi)}{\partial\varphi} (\tilde{\psi}\chi - \psi\tilde{\chi}) , \tag{5} \]

where \( DR = \dot{R} - \frac{i\kappa}{6} R^{-\frac{3}{2}} (\psi\tilde{\lambda} + \tilde{\psi}\lambda) \) and \( D\varphi = \dot{\varphi} - \frac{i}{2} R^{-\frac{3}{2}} (\psi\tilde{\chi} + \tilde{\psi}\chi) \) are the supercovariant derivatives, and \( D\lambda = \dot{\lambda} - \frac{i\kappa}{2} V\lambda, D\chi = \dot{\chi} - \frac{i\kappa}{2} V\chi \) are the \( U(1) \) covariant derivatives.

The potential for the homogeneous scalar fields

\[ V(\varphi) = 2 \left( \frac{\partial g(\varphi)}{\partial\varphi} \right)^2 - 3\kappa^2 g^2(\varphi), \tag{6} \]

consists of two terms, one of them is the potential for the scalar field in the case of global supersymmetry. The potential \( (5) \) is not positive semi-definite in contrast with the standard supersymmetric quantum mechanics. Unlike the standard supersymmetric quantum mechanics, our model, describing the minisuperspace approach to supergravity coupled to matter, allows the supersymmetry breaking when the vacuum energy is equal to zero \( V(\varphi) = 0 \).

In order to give some implications of spontaneous supersymmetry breaking we display the potential \( V(\varphi) (6) \) in terms of the auxiliary fields \( F(t) \) and \( B(t) \),

\[ V(\varphi) = \frac{1}{2} F^2 - \frac{3}{\kappa^2 R} B^2, \tag{7} \]

where the bosonic \( F \) and \( B \) are

\[ F = 2 \frac{\partial g(\varphi)}{\partial\varphi}, \quad B = -\kappa^2 R g(\varphi). \tag{8} \]

The selection rules for the occurrence of spontaneous supersymmetry breaking are

i) \[ \frac{\partial V(\varphi)}{\partial\varphi} = 4 \frac{\partial g(\varphi)}{\partial\varphi} \left[ \frac{\partial^2 g}{\partial\varphi^2} - \frac{3}{2} \kappa^2 g \right] = 0, \text{ at } \varphi = \varphi_0 \tag{9} \]

ii) \[ V(\varphi_0) = 0 \Rightarrow \left[ \left( \frac{\partial g}{\partial\varphi} \right)^2 - \frac{3}{2} \kappa^2 g^2 \right] = 0, \tag{10} \]

iii) \[ F = 2 \frac{\partial g(\varphi)}{\partial\varphi} \neq 0, \text{ at } \varphi = \varphi_0 , \tag{11} \]

The first condition implies the existence of a minimum, in the scalar field; the second condition is the absence of the cosmological constant, the third condition is for the breaking of supersymmetry. The measure of this breakdown is the term \( -\kappa^2 g(\varphi) N\tilde{\lambda}\lambda \) in the lagrangian \( (5) \). Furthermore, we can identify
\[ m_{3/2} = \kappa^2 g(\varphi_0) = \frac{g(\varphi_0)}{M_{pl}^2} \]

as the gravitino mass in the effective supergravity theory, and \( M_{pl} = \frac{1}{\kappa} = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}\text{GeV} \) is the reduced Planck mass.

The factor \( R \) in the kinetic terms of the scalar factor \(-\frac{3}{\kappa^2 N} R(\dot{R})^2 \) plays the role of a “metric” tensor in the Lagrangian, the kinetic energy for the scalar factor is negative definite, this is due to the fact, that the particle-like fluctuations do not correspond to the scalar factor, and the kinetic energy of the scalar fields \( \frac{\kappa^4}{2N} \varphi^2 \) is positive.

### III. THE CORRESPONDING SUPERSYMMETRIC QUANTUM MECHANICS

The Hamiltonian can be calculated in the usual way. We have the classical canonical Hamiltonian

\[ H_{\text{can}} = NH + \frac{1}{2} \bar{\psi} S - \frac{1}{2} \psi \bar{S} + \frac{1}{2} \mathcal{V} F, \]

where \( H \) is the Hamiltonian of the system, \( S, \bar{S} \) are supercharges and \( F \) is the \( \mathcal{U}(1) \) rotation generator. The form of the canonical Hamiltonian \([13]\) explains the fact, that \( N, \psi, \bar{\psi} \) and \( \mathcal{V} \) are Lagrange multipliers which enforce only the first class constraints \( H = 0, S = 0, \bar{S} = 0 \) and \( F = 0 \), which express the invariance of the conformal \( n = 2 \) supersymmetric transformations. As usual with the Grassmann variables, we have the second-class constraints which can be eliminated by the Dirac procedure, as a result only the following non-zero Dirac brackets \( \{ , \} \) remain. For Grassmann variables \( \lambda, \bar{\lambda}, \chi \) and \( \bar{\chi} \)

\[ \{ \lambda, \bar{\lambda} \}^* = \frac{3}{2} i, \quad \{ \chi, \bar{\chi} \}^* = -i. \]

The canonical Poisson brackets for the \( R, \pi_R \) and \( \varphi, \pi_\varphi \), have the following form

\[ \{ R, \pi_R \}_{pb} = -1, \quad \{ \varphi, \pi_\varphi \}_{pb} = -1. \]

In a quantum theory the brackets \([14]\) and \([13]\) must be replaced by anticommutator

\[ \{ \lambda, \bar{\lambda} \} = -\frac{3}{2}, \quad \{ \chi, \bar{\chi} \} = 1, \]

and can be considered as generators of the Clifford algebra, as well as the commutators

\[ [ R, \pi_R ] = -i, \quad [ \varphi, \pi_\varphi ] = -i. \]

The quantization procedure takes into account the dependence of the Lagrangian on the metric “R”, but at the quantum level we must consider the nature of the Grassmann variables \( \lambda, \bar{\lambda}, \chi \) and \( \bar{\chi} \), and with respect to these ones we perform the antisymmetrizations, then we can write the bilinear combination in the form of the commutators e.g. \( \bar{\chi} \chi \to \frac{1}{2} [ \bar{\chi}, \chi ] \) and this leads to the following quantum Hamiltonian \( H \)
\[ H = -\frac{k^2}{12} R^{-\frac{3}{2}} \pi_R R^{-\frac{3}{2}} \pi_R - \frac{3kR}{\kappa^2} - \frac{1}{6} \frac{\sqrt{k}}{R} [\lambda, \lambda] + \frac{\pi_2^2}{2R^3} \]

\[-\frac{i\kappa}{4R^2} \pi_\varphi \left( [\bar{\lambda}, \lambda] + [\lambda, \bar{\chi}] \right) - \frac{\kappa^2}{16R^3} [\lambda, \lambda] [\bar{\chi}, \chi] + \frac{3\sqrt{k}}{4R} [\bar{\chi}, \chi] \]

\[ + \frac{\kappa^2}{2} g(\varphi) [\bar{\lambda}, \lambda] + 6\sqrt{k} g(\varphi) R^2 + R^3 V(\varphi) + \frac{3}{4} \kappa^2 g(\varphi) [\bar{\chi}, \chi] \]

\[ + \frac{\partial^2 g(\varphi)}{\partial \varphi^2} [\bar{\chi}, \chi] + \frac{1}{2} \kappa \frac{\partial g(\varphi)}{\partial \varphi} \left( [\bar{\lambda}, \lambda] - [\lambda, \bar{\chi}] \right), \]

(18)

where \( \pi_R = i \frac{\partial}{\partial R} \) and \( \pi_\varphi = i \frac{\partial}{\partial \varphi} \). The supercharges \( S, S^\dagger \) and the fermion number operator \( \mathcal{F} \) have the following structures

\[ S = A \lambda + B \chi, \quad S^\dagger = A^\dagger \lambda^\dagger + B^\dagger \chi^\dagger, \]

(19)

where

\[ A = i R^{-\frac{3}{2}} \pi_R - \frac{2\sqrt{k}}{\kappa} R^\frac{3}{2} + 2\kappa R^2 g(\varphi) + \frac{1}{4} \kappa^2 R^{-\frac{3}{2}} [\bar{\chi}, \chi], \]

\[ B = i R^{-\frac{3}{2}} \pi_\varphi + 2R^2 \frac{\partial g(\varphi)}{\partial \varphi}, \]

(20)

with \( A^\dagger \) and \( B^\dagger \) are hermitian to \( A \) and \( B \) respectively, and

\[ \mathcal{F} = -\frac{1}{3} [\bar{\lambda}, \lambda] + \frac{1}{2} [\bar{\chi}, \chi]. \]

(21)

An ambiguity exist in the factor ordering of these operators, such ambiguities always arise, when the operator expression contains the product of non-commuting operators \( R \) and \( \pi_R \) as in our case. It is then necessary to find some criteria to know which factor ordering should be selected. We propose the following: to integrate with measure \( R^\frac{3}{2} dR \) in the inner product of two states \([13]\). In this measure the conjugate momentum \( \pi_R \) is non-hermitian with \( \pi_R^\dagger = R^{-\frac{3}{2}} \pi_R R^\frac{3}{2} \), however, the combination \( (R^{-\frac{3}{2}} \pi_R)^\dagger = \pi_R^\dagger R^{-\frac{3}{2}} = R^{-\frac{3}{2}} \pi_R \) is a hermitian one, and \( (R^{-\frac{3}{2}} \pi_R R^{-\frac{3}{2}} \pi_R)^\dagger = R^{-\frac{3}{2}} \pi_R R^{-\frac{3}{2}} \pi_R \) is hermitian too, with this the parameter \( p = 1/2 \) is fixed.

The anticommutator value \( \{\lambda, \bar{\lambda}\} = -\frac{3}{2} \) of the superpartners \( \lambda \) and \( \bar{\lambda} \) of the scalar factor \( R \) is negative, unlike the anticommutation relation for \( \chi \) and \( \bar{\chi} \) in \([16]\), which is positive. They can be redefined and one becomes the following conjugate operation for the operators \( \lambda \) and \( \chi \)

\[ \bar{\lambda} = \xi^{-1} \lambda^\dagger \xi = -\lambda^\dagger, \quad \bar{\chi} = \xi^{-1} \chi^\dagger \xi = \chi^\dagger, \]

(22)

where \( \{\lambda, \lambda^\dagger\} = \frac{3}{2} \), and the operator \( \xi \) possesses the following properties

\[ \lambda^\dagger \xi = -\xi \lambda^\dagger, \quad \chi^\dagger \xi = \xi \chi^\dagger \quad \text{and} \quad \xi^\dagger = \xi. \]

(23)

So, for the supercharge operator \( \bar{S} \) we have the following equation

\[ \bar{S} = \xi^{-1} S^\dagger \xi. \]

(24)
For the quantum generators $H, S, \bar{S}$ and $\mathcal{F}$ we obtain the following superalgebra

$$\{S, \bar{S}\} = 2H, \quad S^2 = \bar{S}^2 = 0, \quad [S, H] = 0,$$

$$[\bar{S}, H] = 0, \quad [\mathcal{F}, S] = -S, \quad [\mathcal{F}, \bar{S}] = \bar{S}. \quad (25)$$

We can see, that the anticommutator of supercharges $S$ and their conjugate $\bar{S}$ under our conjugate operation has the form

$$\overline{\{S, \bar{S}\}} = \xi^{-1}\{S, \bar{S}\}^\dagger \xi = \{S, \bar{S}\}, \quad (26)$$

and the Hamiltonian operator is self-conjugate under the operation $\bar{H} = \xi^{-1}H^\dagger\xi$. In the case of standard supersymmetric quantum mechanics we would have $\bar{\lambda} = \lambda^\dagger$ and $\bar{S} = S^\dagger$, and the Hamiltonian would be positive. In our case, the algebra (25) does not define positive-definite Hamiltonian in a full agreement with the circumstance that the potential $V(\varphi)$ of the scalar field is not positive semi-definite in general, in contrast with the standard supersymmetric quantum mechanics.

We can choose the following matrix representation for the operators $\lambda, \bar{\lambda}, \chi, \bar{\chi}$ and $\xi$ in the form of the tensorial product of matrices $2 \times 2$

$$\lambda = \sqrt{\frac{3}{2}}\sigma(-) \otimes 1, \quad \bar{\lambda} = -\sqrt{\frac{3}{2}}\sigma(+) \otimes 1,$$

$$\chi = \sigma_3 \otimes \sigma(-), \quad \bar{\chi} = \sigma_3 \otimes \sigma(+), \quad \xi = \sigma_3 \otimes 1, \quad (27)$$

where $\sigma_\pm = \frac{\sigma_1 \pm i\sigma_2}{2}, \sigma_1, \sigma_2$ and $\sigma_3$ are the Pauli matrices.

**IV. THE WAVE FUNCTION FOR THE ZERO ENERGY STATE**

In the quantum theory the first-class constraints $H = 0, S = 0, \bar{S} = 0$ and $\mathcal{F} = 0$ associated with the invariant action (1) under the local $n = 2$ conformal supersymmetry become conditions on the wave function $\Psi$. So that any physical state must obey the following quantum constrains

$$\hat{H} \Psi = 0, \quad \hat{S} \Psi = 0, \quad \hat{\bar{S}} \Psi = 0 \quad \text{and} \quad \hat{\mathcal{F}} \Psi = 0, \quad (28)$$

where the first equation in (28) is the Wheeler-DeWitt equation for the minisuperspace model. The eigenstates of the Hamiltonian (18) have four-components

$$\Psi(R, \varphi) = \begin{bmatrix} \psi_1(R, \varphi) \\ \psi_2(R, \varphi) \\ \psi_3(R, \varphi) \\ \psi_4(R, \varphi) \end{bmatrix}. \quad (29)$$

We have rewritten the equations $S \Psi = 0$ and $\bar{S} \Psi = 0$ in the following form
\[ (\lambda S - \bar{\lambda} S) |\Psi\rangle = \left[ -\left( \frac{A - A^\dagger}{2} \right) \lambda, \lambda \right] - \left( \frac{A + A^\dagger}{2} \right) \bar{\lambda}, \lambda \]
\[ - \left( \frac{B - B^\dagger}{2} \right) \left( \bar{\lambda} \chi + \lambda \bar{\chi} \right) + \left( \frac{B + B^\dagger}{2} \right) \left( \lambda \bar{\chi} - \bar{\lambda} \chi \right) |\Psi\rangle = 0, \]  
\( (\chi S - \chi \bar{S}) |\Psi\rangle = \left[ \left( \frac{A - A^\dagger}{2} \right) \left( \bar{\lambda} \chi + \chi \bar{\lambda} \right) + \left( \frac{A + A^\dagger}{2} \right) \left( \lambda \bar{\chi} - \bar{\lambda} \chi \right) \right] |\Psi\rangle = 0. \]  

Using a matrix representation for \( \lambda, \bar{\lambda}, \chi \) and \( \bar{\chi} \), we found that only \( \psi_4 \) have the right behaviour when \( R \to \infty \) because \( \psi_4 \to 0 \), and due that the others components \( \psi_1, \psi_2 \) and \( \psi_3 \) for the wave function at \( R \to \infty \) are infinite, we consider these components as no physical. Thus, there is a normalizable component \( \psi_4 \) for \( H \) such that \( S\Psi = S\Psi = 0 \), and this eigenstate corresponds to the ground state with eigenvalue \( E = 0 \). The wave function \( \Psi \) has also the non-normalizable components \( \psi_1, \psi_2 \) and \( \psi_3 \) for \( E = 0 \), but for them the eigenvalue of \( H \) is non-zero, for this case all components are normalizable.

So, the partial differential equations for \( \psi_4(R, \varphi) \) have the following forms

\[ -R^{-\frac{3}{2}} \frac{\partial}{\partial R} - 6g(\varphi) R^{3/2} + 6\sqrt{k} M_{pl}^2 R^2 + \frac{3}{4} R^{-3/2} \psi_4 = 0, \]  
\[ \left[ \frac{\partial}{\partial \varphi} + 2R^3 \frac{\partial g(\varphi)}{\partial \varphi} \right] \psi_4 = 0. \]  

We get as solution for (32) and (33)

\[ \Psi \equiv \psi_4(R, \varphi) = C_o R^{3/4} e^{(-2g(\varphi) R^3 + 3 \sqrt{k} M_{pl}^2 R^2)} . \]  

The scalar product for the solution (34) is normalizable with the measure \( R^2 dR d\varphi \) and for the superpotential \( g(\varphi \to \pm \infty) \to \infty \).

Consequently the solution (34) is the eigenstate of the Hamiltonian (18) with zero energy and also with zero fermionic number.

The superpotential for the fluctuating scalar fields \( \varphi = \varphi_0 + \tilde{\varphi} \) near the minimum of the potential \( V(\varphi_0) = 0 \) has the following form (see [11, 12]).

\[ g(\tilde{\varphi}) = g(\varphi_0) + \frac{\partial g(\varphi_0)}{\partial \varphi} \tilde{\varphi} + \frac{1}{2} \frac{\partial^2 g(\varphi_0)}{\partial \varphi^2} \tilde{\varphi}^2 \]
\[ = m_{3/2} M_{pl}^2 \left[ 1 + \sqrt{\frac{3}{2} M_{pl}^2} + \frac{3}{4} \frac{\tilde{\varphi}^2}{M_{pl}^2} \right] = m_{3/2} M_{pl}^2 f(x) , \]  

where \( f(x) = 1 + \sqrt{\frac{3}{2} x + \frac{3}{4} x^2} \), with \( x = \frac{\tilde{\varphi}}{M_{pl}} \).

So, as an example we can consider the case \( k = 0 \)

\[ 1 = C_o^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R^2 e^{-4g(\tilde{\varphi}) R^3} R^2 dR d\tilde{\varphi} \]
\[ = \frac{C_o^2}{12} \int_{-\infty}^{\infty} d\tilde{\varphi} = \frac{C_o^2}{12 m_{3/2} M_{pl}} \int_{-\infty}^{\infty} dx = \frac{C_o^2 \sqrt{2\pi}}{12 \sqrt{3} m_{3/2} M_{pl}} , \]  

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thus, the normalization constant has the following value

$$C_0 = \left( \frac{3}{2} \right)^\frac{1}{4} \sqrt{\frac{6m_{3/2} M_{pl}}{\pi}}. \tag{37}$$

The behaviour for the wave-function $\Psi$ in the $k = 0$ case is shown in Figure 1.

The expectation value for the scalar factor $R$ with the chosen measure is

$$\overline{R} = \langle \Psi | R | \Psi \rangle = C_o^2 \int_{-\infty}^{\infty} d\tilde{\varphi} \int_0^{\infty} \left[ R^3 e^{-4g(\tilde{\varphi}) R^3} dR \right]
= \left( \frac{\sqrt{3}}{4\pi} \right)^{1/6} \int_{-\infty}^{\infty} \frac{dx}{[f(x)]^{4/3}} \left( \frac{M_{pl}}{m_{3/2}} \right)^{\frac{1}{3}} \frac{1}{M_{pl}}, \tag{38}$$

where $\Gamma\left(\frac{4}{3}\right)$ is the Gamma function.

The size of the universe in the supersymmetry breaking state is of the order of

$$\overline{R} = C_1 \left( \frac{M_{pl}}{m_{3/2}} \right)^{\frac{1}{3}} \ell_{pl}, \tag{39}$$

where $\ell_{pl} = \frac{1}{M_{pl}} = \sqrt{8\pi G_N}$ is the Planck length, and $C_1$ has the following value

$$C_1 = \frac{2^{1/6} \sqrt{3} \left( \frac{3}{4} \right)^{\frac{1}{3}} \Gamma \left( \frac{4}{3} \right) \left[ \Gamma \left( \frac{2}{3} \right) \right]^2}{2 \left( \frac{3}{8} \right)^{\frac{1}{3}} \Gamma \left( \frac{5}{3} \right) \left[ \Gamma \left( \frac{1}{3} \right) \right]^2} \approx 0.505468. \tag{40}$$

V. CONCLUSION

Using the ideas of the supersymmetry breaking selection rules under local n=2 conformal supersymmetry, we have presented a solution to the wave function $\Psi$ of the universe for the FRW cosmological model ($k = 0$), that is normalizable. For this proposal it was necessary to define one “weighted” inner product with the factor $R^{1/2}$ and we constructed the corresponding hermitian operators too. Therefore, it was straightforward to find the behaviour of the expectation value for the scalar factor like $\overline{R} = C_1 \left( M_{pl}/m_{3/2} \right)^{1/3} \ell_{pl}$, giving us the size of our universe.

In the proposed framework it is also possible to include the potential of hybrid inflation scenario \cite{14,15}, since the corresponding potential term can be easily introduced in the supersymmetric case. This subject will be reported elsewhere.

VI. ACKNOWLEDGMENTS

Thanks to I. Lyanzuridi, E. Ivanov, S. Krivonos, L. Marsheva and A. Pashnev for their interest in this paper. This work was supported in part by CONACyT grant 3898P-E9608.
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Fig. 1

The more detailed plot of the region of the definition \( \{R, x\} \rightarrow \{[0, \infty), [-\infty, \infty]\} \) for the normalized wave-function \( \Psi \) for \( k=0 \) case. Here it can be seen the asymptotic behaviour when \( x = \tilde{\phi}/M_{pl} \to \pm\infty \). In this plot for simplicity we have taken units in where \( m_{3/2} = M_{pl} = 1 \).