Solid, liquid and gas structure of light nuclei

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Abstract. Nuclear systems are usually considered to be in the liquid phase, however, here we discuss the competition between the shell (liquid phase) and cluster structure in light neutron-rich nuclei. The appearance of molecular structure of $\alpha$ clusters with a geometric (equilateral triangular) shape is stabilized by the valence neutrons, and the nuclear systems with specific shapes are considered to be in the solid state. Also, $\alpha$-condensed states originally suggested in $^{12}$C and $^{16}$O are extended to heavier nuclei with core nucleus, which are in the gas phase.

1. Cluster-shell competition in C isotopes

Nuclear systems show various structure as a function of excitation energy. The shell-model is a standard model, where a self-consistent mean-field is created by the nucleons themselves. Here, the spin-orbit interaction is very strong contrary to the atomic systems, that makes it possible to explain the observed magic numbers. The other important aspect of the nuclear structure is cluster, where strongly correlated subsystems are spatially localized. The $\alpha$-particle is a typical example, which corresponds to the lowest closed shell of the shell model. In an $\alpha$-cluster, four nucleons strongly interact with each other, however, since the $\alpha$-$\alpha$ interaction is weak, the $\alpha$-particles can be considered as subunits in nuclei.

Although the shell and cluster models have quite different feature, some of the light nuclei have both components. If we start with the cluster model, the effect of dissolution of the $\alpha$-cluster(s) due to the spin-orbit interaction should be taken into account \cite{1}. It is important to generalize this idea and propose a simplified method to express the transition from the cluster to the shell structure. In Ref. \cite{2}, this transition is expressed by introducing an order parameter $\Lambda$. Now, this method is applied to the neutron-rich C isotopes to discuss how the valence neutrons affect the cluster-shell competition \cite{3}.

In the $\alpha$-cluster models, four nucleons in each $\alpha$-cluster are described as Gaussian wave packets centered at the same position. If the Gaussian center parameters ($R_i$) are real numbers, the wave function corresponds to the Brink-Bloch wave function, and the spin-orbit interaction vanishes for the $N\alpha$-systems. However, in the present case, they are allowed to be complex numbers. The real and imaginary parts of $R_i$ represent the expectation values of the position and momentum of the i-th particle ($< r_i > = \text{Re}[ R_i ]$, $< p_i > = 2 \sqrt{\hbar} \text{Im}[ R_i ]$). We introduce a `general rule" to take into account the spin-orbit interaction, where the dissociation of one of (Brink-Bloch) clusters to a "quasi cluster" is expressed by introducing a parameter $\Lambda$. Suppose that a nucleus consists of a quasi-cluster $C'_1$ and $\alpha$-clusters \{C\textsubscript{2}, C\textsubscript{3},..., C\textsubscript{m}\}. The Gaussian-center parameters \{R\textsubscript{i}\} for nucleons in $\alpha$-clusters are real numbers. However, for the nucleons in the quasi-cluster, in addition to the real part of Re[ R\textsubscript{i} ] = S\textsubscript{i} (i in C\textsubscript{1}), imaginary part is introduced, which expresses the momentum components of the nucleons. The direction of the imaginary part of the Gaussian-center parameter is introduced for each nucleon as $R_i = S_i + i \Lambda e^{(\text{spin})} S_i$.
where e, (spin) is the unit vector for the intrinsic-spin orientation, \( \Lambda \) is an order parameter of the dissolution of the cluster and * shows the vector product. The contribution of the spin-orbit interaction vanishes at \( \Lambda = 0 \), and it acts attractively or repulsively, if \( \Lambda \) is positive or negative, respectively. This can be understood in the following way: the spin-orbit interaction is intuitively interpreted as \( (\mathbf{r} \times \mathbf{p}) \cdot \mathbf{s} \) and this is equal to \( (\mathbf{s} \times \mathbf{r}) \cdot \mathbf{p} \), where \( \mathbf{r} \), \( \mathbf{p} \), and \( \mathbf{s} \) represent the position, momentum, and spin of the nucleon, and * and \( \cdot \) show the vector and scalar product, respectively. Therefore, if the nucleons in the quasi-cluster have the momentum components parallel to \( (\mathbf{s} \times \mathbf{r}) \), the spin-orbit interaction acts attractively, and if they have momentum components anti-parallel to \( (\mathbf{s} \times \mathbf{r}) \), it acts repulsively, although the spin-orbit interaction is a two-body operator in the actual calculation.

We use Volkov No.2 interaction for the central part, where \( W=1-M \), \( M=0.61 \), and \( B=H=0.125 \). For the spin-orbit term, we use the G3RS interaction with \( V_0=2000 \) MeV. All of the parameters of the interaction are determined from \( \alpha+n \) and \( \alpha+\alpha \) scattering phase shifts and the binding energy of the deuteron. However, \( M \) is slightly modified from the original value (0.6) to 0.61 for the analysis of the carbon isotopes, and furthermore, the bound state for the \( n+n \) system is eliminated by introducing \( B \) and \( H \) parameters.

Using this model, structure of C isotopes is calculated. The \( ^{12}\text{C} \) core is described as three \( \alpha \) clusters with an equilateral triangle shape. The parameter \( R_1 \) is introduced, which stands for the distance between the Gaussian center parameters of the \( \alpha \)-clusters. In addition to this parameter, to describe the dissolution of \( \alpha \), we transform one of the \( \alpha \) clusters to four independent particles by giving the imaginary parts for the Gaussian center parameters. The optimum values of \( \alpha-\alpha \) distance \( (R_i) \) and \( \Lambda \) are much different each other in these nuclei. As shown in Fig. 1, in \( ^{12}\text{C} \), the energy of \( R_1 = 2.5 \) fm gives the lowest than that energy at \( \Lambda = 0 \), where the spin-orbit interaction does not work. However, as \( \Lambda \) increases, the energy curve of \( R_1 = 1.5 \) fm goes down, and the minimum point appears at around \( \Lambda = 0.4 \). This shows that the contribution of the spin-orbit interaction makes the \( R_1 \) value smaller, and both the cluster and shell-model components mix in the ground state. In \( ^{14}\text{C} \) and \( ^{16}\text{C} \), even smaller \( R_1 \) value gives the lowest energy \( (R_1=0.5 \) fm). Due to the presence of valence neutrons, the nucleus is strongly bound and the wave function approaches to the shell-model picture. Thus the optimal \( \Lambda \) value increases to 0.8 in these nuclei. When the valence neutrons are added, the optimum value of \( R_1 \) becomes smaller, on the contrary, the optimum \( \Lambda \) becomes larger. Therefore, it can be concluded that the \( 3 \alpha \)-structure of \( ^{12}\text{C} \) is dissolved as the increase of the number of the valence neutrons. In \( ^{14}\text{C} \), the \( ^{12}\text{C} \) core is completely different from the "free" \( ^{12}\text{C} \) judging from the small \( R_1 \) and large \( \Lambda \) values, which is consistent with the anomalously small B(E2) value recently observed.

Although the shell-model-like component is dominant in the ground state of \( ^{16}\text{C} \), cluster state appears in the excited states. We superpose the wave functions with the optimum values \( (R_1=0.5 \) fm and \( \Lambda = 0.8 \) and cluster-like ones \( (R_1=2.5 \) fm and \( \Lambda = 0.0 \) and diagonalize the Hamiltonian. We found that the energy of the \( 0^+ \) state significantly decreases, whereas the energies of the ground and second \( 0^+ \) states do not change drastically. Therefore, it is considered that \( 3 \alpha \) cluster structure with a geometric shape appears in \( E_x = 10 \) MeV region of \( ^{16}\text{C} \), in analogy with the crystallization of the clusters in \( ^{14}\text{C} \)[4]. The cluster structure with geometric shape is stabilized in the excited state of neutron-rich nuclei, which is considered to be in the solid phase. Other kind of cluster structure with geometric configuration is the linear-chain structure of \( 3 \alpha \) clusters stabilized by the valence neutrons.

The valence neutrons which occupy the \( \pi \) orbit increase the binding energy and stabilize the linear chain of \( 3 \alpha \) against the breathing-like breakup. However, \( ^{14}\text{C} \) with the \( \pi \) orbit does not show a clear energy minimum against the bending-like path. The combination of the valence neutrons in the \( \pi \) and \( \sigma \) orbits is promising to stabilize the linear-chain state against the breathing and bending modes, and it is found that the excited states of \( ^{16}\text{C} \) with the \( (3/2 \pi)^2 (1/2 \sigma)^2 \) configuration for the four valence neutrons is one of the most promising candidates for such a structure[5].
2. \(\alpha\) condensed state around core nucleus

Next, we discuss the gas-like states of nuclei: \(\alpha\) condensed states. The \(\alpha\)-condensation in atomic nuclei has attracted increased interest these days. Here, it is considered that all \(\alpha\)-clusters occupy the same 0s orbital except for the correction due to the antisymmetrization effect. This situation is well expressed by introducing the so-called Schuck wave function, and there, the oscillator parameter is large, which is completely different from the normal 0s-orbital for each nucleon. The most plausible candidates for this kind of state is the second 0\(^+\) state of \(^{12}\text{C}\) at \(E_x = 7.65\) MeV around the 3\(\alpha\)-threshold energy. The squared overlap between the wave function of a microscopic cluster model and the Schuck wave function is more than 70% suggesting that the single Schuck wave function is a good approximation for the description of the condensed state. Furthermore, a candidate for the 4\(\alpha\) condensed state is discussed in \(^{16}\text{O}\) from both theoretical and experimental sides. Recently, coherent emission of these condensed states from the compound nucleus has been reported, which suggests the possibility of condensation around the core in heavier nuclei [6,7]. Using the \(^{28}\text{Si} + ^{24}\text{Mg}\) reaction, the compound states of \(^{52}\text{Fe}\) have been populated, and the \(^3\text{Be}(0^+_{1^-})\) and \(^{10}\text{C}(0^+_{2^-})\) emissions from this state have been observed, which are much enhanced compared with the sequential \(\alpha\) emission. From statistical model point of view, it is natural to consider that the emitted second 0\(^+\) state of \(^{12}\text{C}\) (or the ground state of \(^8\text{Be}\)) is formed inside the Coulomb barrier of the compound nucleus. Therefore, these experimental evidences lead us to a hypothesis that \(\alpha\) condensed states can be formed not only in \(^8\text{Be}\) and \(^{12}\text{C}\) but also in heavier nuclei with some core. The enhancement of the emission of the condensed states can be interpreted to be due to an effective Coulomb barrier, which is lower for the condensed states, because they spread widely in space with low density, which can be proven by calculating the folding potential [8]. From the experimental side, the kinetic energy of the emitted \(^{12}\text{C}\) in coincidence with \(\gamma\) emission has been observed to be much smaller than the sum of three \(\alpha\)'s in the sequential 3\(\alpha\) emission. The \(\alpha\)-condensed state around core nucleus raises the possibility of multi-\(\alpha\)-condensed state in heavier nuclei. Theoretically, it has been pointed out that ten is the maximum number of \(\alpha\)-clusters for the multi-\(\alpha\)-condensed state, since the state becomes unstable due to the increase of the Coulomb energy. However, with the presence of the core nucleus, which provides attractive potential
for the $\alpha$ clusters, it becomes possible to overcome this instability and multi-$\alpha$ condensed states could be realized in heavier systems. As a first step for the clarification of this hypothesis, in this study, we demonstrate that the $\alpha$ condensed states appear not only in light nuclei such as $^{12}$C and $^{16}$O but also in heavier nuclei with an inner core at the corresponding $\alpha$ threshold. For this purpose, we introduce a microscopic $\alpha$ cluster model. To extend the study of normal $\alpha$ condensed states to the cases of heavier nuclei with a core, we introduce a Monte Carlo technique for the description of the Schuck wave function, which is called "virtual Schuck" wave function[9]. Here, the integration over Gaussian center parameters in the original Schuck wave function is performed by using a Monte Carlo technique. The total wave function $\Psi$ is expressed as the summation of many Slater determinants $\Psi_k$. Here, the Gaussian center parameters $\{R_i\}$ are randomly generated, however, the random numbers are generated by the weight function $W$ with a Gaussian shape:

$$\Psi = \sum_{k=1}^{m} P^x P^y P^z \Psi_k,$$

$$\Psi_k = |A G_1(R_1)G_2(R_2)G_3(R_3)\cdots G_n(R_n)|_k.$$

With increasing ensemble number, the distribution of $\{R_i\}$ approaches the Gaussian with the width parameter of $\sigma$.

$$W(R_i) \propto \exp[-\frac{R_i^2}{\sigma^2}].$$

and the wave function agrees with the original Schuck wave function when the number of Slater determinants superposed increases. This treatment has an advantage that there is no difficulty for adding a core nucleus. This approach is introduced for $^{24}$Mg (Fig. 3) and $^{28}$S (Fig. 4) around the $^{16}$O core.

Figure 3 shows the energy convergence of $^{24}$Mg measured from the $^{16}$O+$\alpha+\alpha$ threshold, where two $\alpha$-clusters occupy the $\alpha$-condensed state around the $^{16}$O core. The solid- and dotted-lines ($\alpha = 2, 3$ fm) converge to the energies below the threshold, however, the dashed- and dash-dotted-lines ($\sigma = 4, 5$ fm) converge almost around the threshold energy. Here, the converged energies of these lines show that they are rather insensitive to the $\sigma$-value. Thus, the $\alpha$-condensed state is considered to be realized in $^{24}$Mg around the $^{16}$O+$\alpha+\alpha$ threshold energy. The energy convergence for $^{28}$Si ($^{16}$O+$\alpha+\alpha+\alpha$) measured from the $^{16}$O+$\alpha+\alpha+\alpha$ threshold is shown in Fig. 4. The axes and lines are the same.
as in Fig. 3. The solid-line ($\sigma = 2$ fm) converges at an energy much below the corresponding threshold, however, the dotted- and dashed-lines ($\sigma = 3, 4$ fm), which are proper $\sigma$-values for the $\alpha$-condensed states, converge around the threshold energy. These obtained energies around the threshold are rather insensitive to the $\sigma$-value, and it is considered that the $\alpha$-condensed state is realized.

3. Comparison with group theoretical studies

The question of phases and phase-transitions of cluster systems has recently been addressed from the algebraic viewpoint, too [10,11]. In what follows we compare the basic features of the two methods, the microscopical (M) one of the present contribution with the algebraic (A) one of References [10,11], and discuss some further possibilities for continuing this kind of studies.

First we should note some remarkable differences: i) contrary to the present microscopical calculations, the algebraic method was applied to semi-microscopical [12] and phenomenological models, ii) M is applied to real nuclear states, while A is a schematic study, iii) here we discussed three phases, while in [10] only two ones are investigated, iv) the key interaction responsible for the phase transition is the spin-orbit one microscopically, and the (competition between the quadrupole and the) dipole one(s) in semi-microscopic and phenomenological models. On the other side the advantage of the algebraic method is that v) for such models the methodology is well-defined [13]; the control parameter(s) correspond to the relative weights of the Hamiltonians of different dynamical symmetry, the phase-transitions and their order can be calculated analytically in the limit of large particle number, and it also turned out that the phases can be determined in terms of quasi-dynamical symmetries [10,14]. In particular in [10,11] it turned out that both the phenomenological and the semi-microscopical cluster model has a first order phase transition at a critical point of the control parameter. Furthermore, the whole phase between the endpoint of the real U(3) dynamical symmetry and the critical point is characterized by a quasi-dynamical U(3) symmetry. On the other hand, in the microscopical method the analogy with thermodynamical phases are based on some qualitative arguments, and no quantitative analysis are available at present for the phase-transitions.

In spite of these differences there are some signs which seem to indicate that the underlying physics of the two methods may have something in common. The correspondence between two phases seem to be straightforward. Here we called a phase solid, when the clusters form some sort of cristallic structure, which seem to be in line with the O(4) phase of the algebraic models, corresponding to a rigid dipole rotor from the macroscopic, and to localized clusters from the microscopical viewpoint. Our liquid phase is associated with the shell-model-like states, and algebraically the U(3) limit corresponds macroscopically to a soft vibrator with spherical equilibrium shape, microscopically to shell-model-like clusterization (and the quasi-dynamical U(3) symmetry which defines the phase is obtained as a superposition of such states). The algebraic counterpart of the gas-phase is an open question, but it is interesting to note that those models also have a third limit, corresponding to the O(3) dynamical symmetry, which describes week-coupling, i.e. loosely bound cluster systems.

In light of the differences of the two methods a couple of investigations seem to be promising for converging them, and capitalizing from their relative advantages. From the algebraic side it would be most interesting: i) To extend the calculations of phase transitions by including the third i.e. O(3) dynamical symmetry. This could give an answer to the question if this phase has any similarity to that of the gas-phase. ii) To perform not only schematic, but also realistic calculations in order to see, how much the specific nuclear states correspond to the different phases. i)ii) Try to derive the phenomenological cluster-cluster interactions from the effective two-nucleon forces (e.g. by the extension of the method of [15] to the U(3) symmetry-breaking interactions). In this way we could see how the spin-orbit (and maybe other) two nucleon forces map to the cluster-cluster interaction; i.e. whether or not the above mentioned two forces, which are responsible for the phase-transition in the different models are related to each other. iv) On the other hand it could also be very conclusive to reformulate the basic problem of the present microscopic studies (e.g. the role of the spin-orbit interaction) in terms of an algebraic framework, and investigate what one obtains with the application of the algebraic technology.
Hopefully these investigations could contribute to our understanding on the phases and phase-transitions in cluster systems.

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