Learning to Resolve Conflicts for Multi-Agent Path Finding with Conflict-Based Search

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Abstract
Conflict-Based Search (CBS) is a state-of-the-art algorithm for multi-agent path finding. At the high level, CBS repeatedly detects conflicts and resolves one of them by splitting the current problem into two subproblems. Previous work chooses the conflict to resolve by categorizing the conflict into three classes and always picking a conflict from the highest-priority class. In this work, we propose an oracle for conflict selection that results in smaller search tree sizes than the one used in previous work. However, the computation of the oracle is slow. Thus, we propose a machine-learning framework for conflict selection that observes the decisions made by the oracle and learns a conflict-selection strategy represented by a linear ranking function that imitates the oracle’s decisions accurately and quickly. Experiments on benchmark maps indicate that our method significantly improves the success rates, the search tree sizes and runtimes over the current state-of-the-art CBS solver.

1 Introduction
Multi-Agent Path Finding (MAPF) is the problem of finding a set of conflict-free paths for a given number of agents on a given graph that minimizes the sum of costs or the makespan. Although MAPF is NP-hard to solve optimally (Yu and LaValle 2013), significant research effort has been devoted to MAPF to support its application in distribution management (Dresner and Stone 2008), airplane taxiing (Morrison et al. 2015; Balakrishnan and Jung 2007) and computer games (Ma et al. 2017b).

Conflict-Based Search (CBS) (Sharon et al. 2015) is one of the leading algorithms for solving MAPF optimally, and a number of enhancements to CBS have been developed (Boyarski et al. 2015; Li et al. 2019a; Felner et al. 2018; Barer et al. 2014). The key idea behind CBS is to use a bi-level search that resolves conflicts by adding constraints at the high level and replans paths for agents respecting these constraints at the low level. The high level of CBS is a best-first search on a binary search tree called constraint tree (CT). To expand a CT node (that consists of a set of paths and a set of constraints on these paths), CBS has to choose a conflict in the current set of paths to resolve and add constraints that prevent this conflict in the child nodes. Picking good conflicts is important, and a good strategy for conflict selection could have a big impact on the efficiency of CBS by reducing both the size of CT and its runtime. (Boyarski et al. 2015) proposes to prioritize conflicts by categorizing them into three classes and always picking one from the top class. Such a strategy has been proven to be efficient (Boyarski et al. 2015) and is commonly used for conflict selection in recent research (Li et al. 2019a; Felner et al. 2018; Li et al. 2019c). In this paper, we propose a new conflict-selection oracle that results in smaller CT sizes than the one used in previous work but is much more computationally expensive since it has to compute 1-step lookahead heuristics for each conflict.

To overcome the high computational cost of the oracle, we leverage insights from studies on variable selection for branching in Mixed Integer Linear Programming (MILP) solving and propose to use machine learning (ML) techniques for designing conflict-selection strategies that imitate the oracle’s decisions to speed up CBS. Variable selection for branching in MILP is analogous to conflict selection in CBS. As part of the branch-and-bound algorithm for MILP (Wolsey and Nemhauser 1978), non-leaf nodes in the CT must be expanded into two child nodes by selecting one of the unassigned variables and splitting its domain by adding new constraints, while CBS chooses and splits on conflicts. Recent studies (Khalil et al. 2016, 2017; He, Daume III, and Eisner 2014) have shown that data-driven ML approaches for MILP solving are competitive with and can even outperform state-of-the-art commercial solvers.

We borrow such ML tools from MILP solving (Khalil et al. 2016) and propose a data-driven framework for designing conflict-selection strategies for CBS. In the first phase of our framework, we observe and record decisions made by the oracle on a set of instances and collect data on features that characterize the conflicts at each CT node. In the second phase, we learn a ranking function for conflicts in a supervised fashion that imitates the oracle but is faster to evaluate. In the last phase, we use the learned ranking function to replace the oracle and select conflicts in CBS to solve unseen instances. Compared to previous work on conflict selection in CBS, our approach is able to discover more efficient rules for conflict selection that significantly improve the success rate and reduce the CT size and the
 runtime of the search. Our method is flexible since we are able to customize the conflict-selection strategies easily for different environments and do not need to hard-code different rules for different scenarios. Different from recent work on ML-guided MILP solving, we utilize problem-specific features which contain essential information about the conflicts, while previous work only takes MILP-level features (e.g., counts and statistics of variables) into account (Khalil et al. 2016, 2017). Another advantage of our offline learning method over training an instance-specific model on-the-fly is that our learned ranking function is able to generalize to instances and graphs unseen during training.

2 MAPF

Given an undirected unweighted underlying graph $G = (V, E)$, the Multi-Agent Path Finding (MAPF) problem is to find a set of conflict-free paths for a set of agents $\{a_1, \ldots, a_k\}$. Each agent $a_i$ is assigned a start vertex $s_i \in V$ and a goal vertex $t_i \in V$. Time is discretized into time steps, and, at each time step, every agent can either move to an adjacent vertex or wait at the same vertex in the graph. The cost of an agent is the number of time steps until it reaches its goal vertex and no longer moves. We consider two types of conflicts: i) a vertex conflict $\langle a_i, a_j, v, t \rangle$ occurs when agents $a_i$ and $a_j$ are at the same vertex $v$ at time step $t$; and ii) an edge conflict $\langle a_i, a_j, u, v, t \rangle$ occurs when agents $a_i$ and $a_j$ traverse the same edge $(u, v) \in E$ in opposite directions between time steps $t$ and $t + 1$. Our objective is to find a set of conflict-free paths that move all agents from their start vertices to their goal vertices with the optimal cost, that is, the minimum sum of all agents’ costs.

3 Background and Related Work

In this section, we first provide a brief introduction to CBS and its variants. Then, we summarize other related work using ML in MAPF and MILP solving.

Conflict-Based Search (CBS)

CBS is a bi-level tree search algorithm. It records the following information for each CT node $N$:

1. $N_{\text{Con}}$: There are two types of constraints: i) a vertex constraint $\langle a_i, v, t \rangle$, corresponding to a vertex conflict, prohibits agent $a_i$ from being at vertex $v$ at time step $t$; and ii) an edge constraint $\langle a_i, u, v, t \rangle$, corresponding to an edge conflict, prohibits agent $a_i$ from moving from vertex $u$ to vertex $v$ between time steps $t$ and $t + 1$.

2. $N_{\text{Sol}}$: A solution of $N$ consists of a set of individually cost-minimal paths for all agents respecting the constraints in $N_{\text{Con}}$. An individually cost-minimal path for an agent is the cost-minimal path between its start and goal vertices assuming it is the only agent in the graph.

3. $N_{\text{Cost}}$: the cost of $N$ defined as the sum of costs of the paths in $N_{\text{Sol}}$.

4. $N_{\text{Conf}}$: the set of conflicts in $N_{\text{Sol}}$.

On the high level, CBS starts with a tree node whose set of constraints is empty and expands the CT in a best-first manner by always expanding a tree node with the lowest $N_{\text{Cost}}$. After choosing a tree node to expand, CBS identifies the set of conflicts $N_{\text{Conf}}$ in $N_{\text{Sol}}$. If there are none, CBS terminates and returns $N_{\text{Sol}}$. Otherwise, CBS randomly (by default) picks one of the conflicts to resolve and adds two child nodes of $N$ to the CT by imposing, depending on the type of conflict, an edge or vertex constraint for one of two conflicting agents to $N_{\text{Con}}$ of one of the child node under and for the other conflicting agent to $N_{\text{Con}}$ of the other child node. On the low level, it replans the paths in $N_{\text{Sol}}$ to accommodate the newly-added constraints, if necessary. CBS guarantees completeness by exploring both ways of resolving each conflict and optimality by performing best-first searches on both of its high and low levels.

Variants of CBS

CBS chooses conflicts randomly, but this conflict-selection strategy can be improved. Improved CBS (ICBS) (Boyarski et al. 2015) categorizes conflicts into three types to prioritize them. A conflict is cardinal iff, when CBS uses the conflict to split CT node $N$, the costs of both resulting child nodes are strictly larger than $N_{\text{Cost}}$. A conflict is semi-cardinal otherwise. By first resolving cardinal conflicts, then semi-cardinal conflicts and finally non-cardinal conflicts, CBS is able to improve its efficiency since it increases the lower bound on the optimal cost more quickly by generating child nodes with larger costs. ICBS uses Multi-Valued Decision Diagrams (MDD) to classify conflicts. An MDD for agent $a_i$ is a directed acyclic graph consisting of all cost-minimal paths from $s_i$ to $t_i$ of a given cost that respect the current constraints $N_{\text{Con}}$. The nodes at depth $t$ of the MDD are exactly the nodes that agent $a_i$ could be at when following one of its cost-minimal paths. A vertex (edge) conflict $\langle a_i, a_j, v, t \rangle$ (or $\langle a_i, a_j, u, v, t \rangle$) is cardinal iff vertex $v$ (edge $(u, v)$) is the only vertex at depth $t$ (the only edge from depth $t$ to depth $t + 1$) in the MDDs of both agents. Li et al. (2019b) proposes to add disjoint constraints to two child nodes when expanding a CT node in CBS and prioritize conflicts based on the number of singletons in or the widths of the MDDs of both agents.

Another line of research focuses on speeding up CBS by calculating a tighter lower bound on the optimal cost to guide the high-level search. When expanding a tree node $N$, CBSH (Felner et al. 2018) uses the CG heuristic, which builds a conflict graph (CG) whose vertices represent agents and whose edges represent cardinal conflicts in $N_{\text{Sol}}$. Then, the lower bound on the optimal cost within the subtree rooted at $N$ is guaranteed to increase at least by the size of the minimum vertex cover of this CG. We refer to this increment as the $h$-value of the CT node. Based on CBSH, CBSH2 (Li et al. 2019a) uses the DG and WDG heuristics that generalize CG and compute $h$-values for CT nodes using (weighted) pairwise dependency graphs that take into account semi-cardinal and non-cardinal conflicts besides cardinal ones. CBSH2 with the WDG heuristic is the current state-of-the-art CBS solver for MAPF (Li et al. 2019a).

To the best of our knowledge, other than prioritizing con-
Table 1: Performance of CBSH2 with different oracles and our solver. Oracle time is the total time that the oracle takes per instance. Search time is the runtime minus the oracle time. All entries are averages taken over the instances that are solved by all solvers.

|                  | The Random Map | The Game Map |
|------------------|----------------|--------------|
|                  | Runtime        | CT Size      | Oracle Time | Search Time | Runtime        | CT Size      | Oracle Time | Search Time |
| CBSH2+O₀         | 9.95s          | 2,362 nodes  | 0.00s       | 9.95s       | 19.8min       | 952 nodes     | 0.0min       | 2.3min      |
| CBSH2+O₁         | 24.89s         | 746 nodes    | 21.34s      | 3.55s       | 19.8min       | 565 nodes     | 19.0min      | 0.8min      |
| CBSH2+O₂         | 12.13s         | 632 nodes    | 9.52s       | 2.61s       | 27.4min       | 2,252 nodes   | 23.4min      | 4.0min      |
| Our Solver       | 6.19s          | 998 nodes    | 0.88s       | 5.31s       | 1.6min        | 754 nodes     | 0.2min       | 1.4min      |

4 Oracles for Conflict Selection

Given a MAPF instance, at a particular CT node \( N \) with the set of conflicts \( N_{\text{conf}} \), an oracle for conflict selection is a ranking function that takes \( N_{\text{conf}} \) as input, calculates a real-valued score per conflict and outputs the ranks determined by the scores. We say that CBS follows an oracle for conflict selection iff CBS builds the CT by always resolving the conflict with the highest rank. We define oracle \( O_0 \) to be the one proposed by (BoyarSKI et al. 2015), that uses MDDs to rank conflicts.

Definition 4.1. Given a CT node \( N \), oracle \( O_0 \) ranks the conflicts in \( N_{\text{conf}} \) by cardinality in the order of cardinal conflicts, semi-cardinal conflicts and non-cardinal conflicts, breaking ties in favor of conflicts at the smallest time step and remaining ties randomly.

Next, we define oracles \( O_1 \) and \( O_2 \) that both calculate 1-step lookahead scores by using, for each conflict, the two child nodes of \( N \) that would result if the conflict were resolved at \( N \).

Definition 4.2. Given a CT node \( N \), oracle \( O_1 \) computes the score \( v_c = \min\{g_c^l + h_c^l, g_c^l + h_c^u\} \) for each conflict \( c \in N_{\text{conf}} \), where \( g_c^l \) and \( g_c^u \) would be the costs of the two child nodes of \( N \) and \( h_c^l \) and \( h_c^u \) the \( h \)-values given by the WDG heuristic of the two child nodes of \( N \) if conflict \( c \) were resolved at \( N \). Then, it outputs the ranks determined by the descending order of the scores (i.e., the highest rank for the highest score).

Oracle \( O_1 \) chooses the conflict that results in the tightest lower bound on the optimal cost in the child nodes. We use the WDG heuristic to compute the \( h \)-values since it is the state of the art. The intuition behind using this oracle is that the sum of the cost and the \( h \)-value of a node is a lower bound on the cost of any solution found in the subtree rooted in the node, and, thus, we want CBS to increase the lower bound as much as possible to find a solution quickly.

Definition 4.3. Given a CT node \( N \), oracle \( O_2 \) computes the score \( v_c = \min\{m_c, m_c^l\} \) for each conflict \( c \in N_{\text{conf}} \), where \( m_c \) and \( m_c^l \) would be the number of conflicts in the two child nodes of \( N \) if conflict \( c \) were resolved at \( N \). Then, it outputs the ranks determined by the increasing order of the scores (i.e., the highest rank for the lowest score).

Oracle \( O_2 \) chooses the conflict that results in the least number of conflicts in the child nodes.

We use CBSH2 with the WDG heuristic as our search algorithm and run it with oracles \( O_0, O_1 \) and \( O_2 \) on (1) a random map, a \( 20 \times 20 \) four-neighbor grid map with 25% randomly generated blocked cells, and (2) the game map "lak503d" (Sturtevant 2012), a \( 192 \times 192 \) four-neighbor grid map with 51% blocked cells. The figures of the maps are shown in Table [I]. The experiments are conducted on 2.4 GHz Intel Core i7 CPUs with 16 GB RAM. We set the runtime limit to 20 minutes for the random map and 1 hour for the game map. We set the number of agents to \( k = 18 \) for the random map and \( k = 100 \) for the game map and run the solvers on 50 instances for each map. Following (Stern et al. 2019), the start and goal vertices are randomly paired among all vertices in each map’s largest connected component for each instance throughout the paper. In Table [I] we...
present the performance of the three oracles as well as our solver. All entries are averages taken over the instances that are solved by all solvers. We consider the CT size since a small CT size implies a small runtime and first look at the performance of CBH2 with the three oracles. Oracle $O_2$ is the best for the random map, followed closely by oracle $O_1$. Oracle $O_1$ is the best for the game map. Overall, oracle $O_1$ is the best. Therefore, in the rest of the paper, we mainly focus on learning a ranking function to imitate oracle $O_1$. Table[1] shows that by learning to imitate oracle $O_1$, our solver achieves the best performance in term of the runtime, even though it induces a larger CT than CBH2+$O_1$. We introduce our machine learning methodology in Section 5 and show experimental results in Section 6. We use the solver, ML-S, introduced in Section 6 to generate results of our solver in Table[1].

5 Machine Learning Methodology

We now introduce our framework for learning which conflict to resolve in CBS. The key idea is that, by observing and recording the features and ranks of conflicts determined by the scores given by the oracle, we learn a ranking function that ranks the conflicts as similarly as possible to the oracle without actually probing the oracle. Our framework consists of three phases:

1. Data collection. We obtain two sets of instances, a training dataset $I_{Train}$ and a test dataset $I_{Test}$. For each instance $I \in I_{Train} \cup I_{Test}$, we obtain a dataset $D_I$ by running the oracle.

2. Model learning. The training dataset is fed into a machine learning algorithm to learn a ranking function that maximizes the prediction accuracy.

3. ML-guided search. We replace the oracle with the learned ranking function to rank conflicts in the CBH2 solver. We run the new solver on randomly generated instances on the same graphs seen during training or unseen graphs.

Data Collection

The first task in our pipeline is to construct a training dataset from which we can learn a model that imitates the oracle’s output. We first fix the graph underlying the instances that we want to solve and the number of agents. The number of agents is only fixed during the data collection and model learning phases. We obtain two sets of instances, $I_{Train}$ for training and $I_{Test}$ for testing. A dataset $D_I$ is obtained for each instance $I \in I_{Train} \cup I_{Test}$, and the final training (test) dataset is obtained by concatenating these datasets. To obtain dataset $D_I$, oracle $O_1$ is run for each CT node $N$ to produce the ranking for $N_{Conf}$. The data consists of: (i) a set of CT nodes $N$; (ii) a set of conflicts $N_{Conf}$ for a given $N \in N$; (iii) binary labels $y_N \in \{0, 1\}^{|N_{Conf}|}$ for all $N \in N$ transformed from the oracle’s ranking of the conflicts; and (iv) a feature map $\Phi_N : N_{Conf} \to \{0, 1\}^p$ for all $N \in N$ that describes conflict $c \in N_{Conf}$ at each CT node with $p$ features. The test dataset is used to evaluate the prediction accuracy of the learned model.

Features

We collect a $p$-dimensional feature vector $\Phi_N(c)$ that describes a conflict $c \in N_{Conf}$ with respect to CT node $N$. The $p = 67$ features of a conflict $\langle a_i, a_j, v, t \rangle$ in our implementation are summarized in Table[2]. They consist of: (1) the properties of the conflict, (2) statistics of CT node $N$, the conflicting agents $a_i$ and $a_j$ and the contested vertex or edge w.r.t. the current solution, (3) the frequency of a conflict being resolved for a vertex or an agent, and (4) features of the MDD and the weighted dependency graph. For each feature, we normalize its value to the range $[0, 1]$ across all conflicts in $N_{Conf}$. All features of a given conflict $c \in N_{Conf}$ can be computed in $O(|N_{Conf}| + k)$ time.

Labels

We aim to label each conflict in $N_{Conf}$ such that conflicts with higher ranks determined by the oracle have larger labels. Instead of using the full ranking provided by oracle $O_1$, we use a binary labeling scheme similar to the one proposed by [Khalil et al. 2016]. We assign label 1 to each conflict strictly among the top 20% of the full ranking and label 0 to the rest, with one exception. When more than 20% of the conflicts have the same highest $O_1$ score, we assign label 1 to those conflicts and label 0 to the rest. By doing so, we ensure that at least one conflict is labeled 1 and conflicts with the same score have the same label. This labeling scheme relaxes the definition of “top” conflicts that allows the learning algorithm to focus on only high-ranking conflicts and helps avoid the irrelevant task of learning the correct ranking of conflicts with low scores.

Model Learning

We learn a linear ranking function with parameters $w \in \mathbb{R}^p$

$$f : \mathbb{R}^p \to \mathbb{R} : f(\Phi_N(c)) = w^T \Phi_N(c)$$

that minimizes the loss function

$$L(w) = \sum_{N \in N} l(y_N, \hat{y}_N) + \frac{C}{2} ||w||_2^2,$$

where $y_N$ is the ground-truth label vector, $\hat{y}_N$ is the vector of predicted scores resulting from applying $f$ to the feature vectors of every conflict in $N_{Conf}$, $l(\cdot, \cdot)$ is a loss function measuring the difference between the ground truth labels and the predicted scores, and $C > 0$ is a regularization parameter. The loss function $l(\cdot, \cdot)$ is based on a pairwise loss that has been used in the literature [Joachims 2002]. Specifically, we consider the set of pairs $P_N = \{(c_i, c_j) : c_i, c_j \in N_{Conf} \land y_N(c_i) > y_N(c_j)\}$, where $y_N(c)$ is the ground-truth label of conflict $c$ in label vector $y_N$. The loss function $l(\cdot, \cdot)$ is the fraction of swapped pairs, defined as

$$l(y_N, \hat{y}_N) = \frac{1}{|P_N|} |\{(c_i, c_j) \in P_N : \hat{y}_N(c_i) \leq \hat{y}_N(c_j)\}|.$$

We use an open-source package made available by [Joachims 2006] that implements a Support Vector Machine (SVM) approach [Joachims 2002] that minimizes an upper bound on the loss, which is NP-hard to minimize.
Table 2: Features of a conflict $c = (a_i, a_j, u, t) \in \mathcal{C}$ of a CT node $N$. Given the underlying graph $G = (V, E)$, let $V_T = \{(v, t) : v \in V, t \in \mathbb{Z}_{\geq 0}\}$, $E_T = \{(u, t), (v, t+1) : t \in \mathbb{Z}_{\geq 0} \land (u = v \lor (v, u) \in E)\}$ and define the time-expanded graph as an unweighted graph $G_T = (V_T, E_T)$. Let $d_{u,v, t} \in \mathbb{R}$ be the cost of the cost-minimal path between vertices $u$ and $v$ in $G$ and $d_{(u', v'), (u, t)}$ be the minimum distance between vertices $(u', t')$ and $(u, t)$. Let $v_{u,t} = \{(u, t), (v, t+1) : t \in \mathbb{Z}_{\geq 0} \land (u = v \lor (v, u) \in E)\}$ and define $V_T^c = \{(u', t') \in \mathbb{Z}_{\geq 0} \land (u' = v' \lor (v', u') \in E)\}$. For an agent $a$, define $V_a = \{(u, t) : a \text{ is at } u \text{ at } t\}$.

Table 3: Numbers of agents in instances for data collection, test losses and accuracies. The swapped pairs (%) are the fractions of swapped pairs averaged over all test CT nodes and the top pick accuracies are the accuracies of the ranking function picking the conflicted labled as $1$ in the test dataset.

ML-Guided Search

After offline data collection and ranking function $f(\cdot)$ learning, we replace the oracle for conflict selection in CBS with the learned function. At each CT node $N$, we first compute the feature vector $\Phi_N(c)$ for each conflict $c \in \mathcal{C}$ and pick the conflict with the maximum score $c^* = \underset{c \in \mathcal{C}}{\arg\max} F(\Phi_N(c))$. The overall time complexity for conflict selection at node $N$ is $O(\mathcal{C} |\mathcal{C}| + k)$. Even though the complexity of conflict selection with oracle $O_0$ is only $O(\mathcal{C})$, we will show in our experiments that we are able to outperform CBSH2+O0 in terms of both the CT size and the runtime.
| Map      | k  | Success Rate (%) | Runtime (min) | CT Size (nodes) | PAR10 Score (min) |
|----------|----|-----------------|---------------|-----------------|-------------------|
|          | CBSH2 | ML-S | ML-O | CBSH2 | ML-S | ML-O | CBSH2 | ML-S | ML-O | CBSH2 | ML-S | ML-O |
| Warehouse| 30  | 93   | 96 (93) | 96 (93) | 0.20 | 0.06 | 0.07 | 1,154 | 294 | 378 | 7.18 | 4.14 | 4.25 |
|          | 36  | 72   | 86 (71) | 88 (71) | 0.54 | 0.24 | 0.19 | 3110 | 980 | 797 | 28.46 | 14.56 | 12.81 |
|          | 42  | 55   | 68 (55) | 70 (55) | 1.27 | 0.65 | 0.38 | 6,834 | 2,874 | 1,781 | 45.70 | 32.61 | 30.56 |
|          | 48  | 17   | 32 (17) | 32 (17) | 1.99 | 1.12 | 0.56 | 9,646 | 5,357 | 2,221 | 83.34 | 68.84 | 68.48 |
|          | 54  | 6    | 16 (6) | 15 (6) | 2.82 | 1.70 | 1.23 | 12,816 | 8,886 | 6,427 | 94.17 | 84.42 | 85.36 |
|          |      |      |      |      | 0      | 0.99% | 0.08% | 0      | 0.06% | 0.05% | 0      | 0.00% | 0.00% |

Table 4: Success rates, average runtimes and CT sizes of instances solved by all solvers and PAR10 scores for different number of agents $k$ in 6 maps. For the success rates of ML-S and ML-O, the fractions of instances solved by both our solver and CBSH2 are given in parentheses (bolded if it solves all instances that CBSH2 solves). For each map, we report the percentage of improvement of our solvers over CBSH2 on the runtime and CT size on instances solved by all solvers and PAR10 score.

### 6 Experimental Results

In this section, we demonstrate the efficiency and effectiveness of our solver, ML-guided CBS, through extensive experiments. We use the C++ code for CBSH2 with the WDG heuristic made available by (Li et al. 2019a) as our CBS version. We compare against CBSH2+$O_h$ as baseline since $O_h$ is the most commonly used conflict-selection oracle. The reason why we choose CBSH2 with the WDG heuristic over CBS, ICBS or CBSH2 with the CG or DG heuristics is that it performs best, as demonstrated in (Li et al. 2019a). All reported results are averaged over 100 randomly generated instances.

Our experiments provide answers to the following questions: i) If the graph underlying the instances is known in advance, can we learn a model that performs well on unseen instances on the same graph with different numbers of agents? ii) If the graph underlying the instances is unknown, can we learn a model from other graphs that performs well on instances on that graph?

We use a set of six four-neighbor grid maps $\mathcal{M}$ of different sizes and structures as the graphs underlying the instances and evaluate our algorithms on them. $\mathcal{M}$ includes: (1) a warehouse map (Li et al. 2020), a $79 \times 31$ grid map with 100 $6 \times 2$ rectangle obstacles; (2) the room map “room-32-32-4” (Stern et al. 2019), a $32 \times 32$ grid map with 64 $3 \times 3$ rooms connected by single-cell doors; (3) the maze map “maze-128-128-2” (Stern et al. 2019), a $128 \times 128$ grid map with two-cell-wide corridors; (4) the random map; (5) the city map “Paris-1,256” (Stern et al. 2019), a $256 \times 256$ grid map of Paris; (6) the game map. The figures of the maps are shown in Table [4]. For each grid map $M \in \mathcal{M}$, we collect data from randomly generated training instances.
We use CBSH2, ML-S and ML-O on randomly generated instances on each of the six maps and vary the number of agents. The runtime limits are set to 60 minutes for the two largest maps (the city and game maps) and 10 minutes for the others. In Table 3.4 we report the success rates, the average runtimes and CT sizes of instances solved by all solvers and the PAR10 scores (a commonly used metric to score solvers where we count the runs that exceed the given runtime limit as 10 times the limit when computing the average runtimes) for some numbers of agents on each map and defer the full table to Appendix A. We plot the success rates on the warehouse and city maps in Figure 1 and the rest in Appendix A. ML-S and ML-O perform similarly. We test the learned ranking functions on the test dataset collected by solving \( t_{test}^{(M)} \). The numbers of agents in the instances used for data collection, the test losses and the test accuracies of picking the conflicts labeled as 1 are reported in Table 4. We varied the numbers of agents for data collection and found that they led to similar performances. In general, the losses of the ranking functions for ML-O are larger and their accuracies of picking “good” conflicts are lower than those for ML-S.

We run CBSH2, ML-S and ML-O on randomly generated instances on each of the six maps and train a ranking function that is trained using 5,000 CT nodes i.i.d. sampled from the training dataset collected by solving instances \( t_{train}^{(M)} \) on the same map and another that is trained using 5,000 CT nodes sampled from the training dataset collected by solving instances \( \cup_{M \in M} I_{train}^{(M)} \setminus I_{test}^{(M)} \) on the other maps, 1,000 i.i.d. CT nodes sampled for each of the five other maps. For each \( M \in M \), we denote our solver that uses the ranking function trained on the same map as ML-S and the solver that uses the one trained on the other maps as ML-O. We set \( C = 1/100 \) to train an \( SVM_{rank} \) (Joachims 2002) with a linear kernel to obtain each of the ranking functions. We varied \( C \in \{1/10, 1/100, 1/1000\} \) and found that ML-S and ML-O perform similarly. We test the learned ranking functions on the test dataset collected by solving \( t_{test}^{(M)} \). The numbers of agents in the instances used for data collection, the test losses and the test accuracies of picking the conflicts labeled as 1 are reported in Table 3. We varied the numbers of agents for data collection and found that they led to similar performances. In general, the losses of the ranking functions for ML-O are larger and their accuracies of picking “good” conflicts are lower than those for ML-S.

In this paper, we proposed the first ML framework for conflict selection in CBS. The extensive experimental results showed that our learned ranking function generalizes across different numbers of agents on a fixed graph (map) or unseen graphs. Our objective was to imitate the decisions made by the oracle that picks the conflict that produces the tightest lower bound on the optimal cost in its child nodes. We are also interested in discovering a better oracle for conflict selection from which we can learn. We expect our method to work well with other newly-developed techniques, such as symmetry breaking techniques (Li et al. 2020), and it remains future work to incorporate those techniques into the framework of CBSH2 to work with our ML-guided conflict selection.


Acknowledgments

We thank Jiaoyang Li, Peter J. Stuckey and Danial Harabor for helpful discussions. The research at the University of Southern California was supported by the National Science Foundation (NSF) under grant numbers 1409987, 1724392, 1817189, 1837779 and 1935712 as well as a gift from Amazon.

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Appendix:
Learning to Resolve Conflicts for Multi-Agent Path Finding with Conflict-Based Search

A Additional Experimental Results
The success rates on the room, maze, random and game maps are shown in Figure 3, where we can see that the success rates of ML-S and ML-O are both marginally higher than CBSH2. Table 5 includes the results on all data points in Figures 1 and 3. The success rates for different runtime limits varying the number of agents on the warehouse, room, maze, random, city and game maps are shown in Figures 4, 5, 6, 7, 8 and 9 respectively.

B Feature Importance
For ML-S, we plot the weights of the features in decreasing order of their absolute values for each of the six ranking functions in Figures 10 and 11. For ML-O, we take the average weight of each feature over the six ranking functions and sort them in decreasing order of their absolute values. The plot is shown in Figure 12.

The top five features for the warehouse map are: (1) the minimum of the numbers of conflicts involving agent \( a_i (a_j) \) that have been selected and resolved; (2) the maximum of the differences of the cost of the path of agent \( a_i (a_j) \) and time step \( t \); (3) the sum of the numbers of conflicts involving agent \( a_i (a_j) \) that have been selected and resolved; (4) the minimum of the differences of the cost of the path of agent \( a_i (a_j) \) and time step \( t \); (5) the binary indicator for non-cardinal conflicts.

The top five features for the room map are: (1) the binary indicator for non-cardinal conflicts; (2) the weight of the edge between agents \( a_i \) and \( a_j \) in the weighted dependency graph; (3) the binary indicator for cardinal conflicts; (4) number of empty cells that are two steps away from where the conflicts occur; (5) the minimum of the numbers of conflicts involving agent \( a_i (a_j) \) that have been selected and resolved.

The top five features for the maze map are: (1) the maximum of the widths of level \( t - 2 \) of the MDD for agent \( a_i (a_j) \); (2) the weight of the edge between agents \( a_i \) and \( a_j \) in the weighted dependency graph; (3) the maximum of the differences of the cost of the path of agent \( a_i (a_j) \) and time step \( t \); (4) the binary indicator for semi-cardinal conflicts; (5) the maximum of the widths of level \( t \) of the MDD for agent \( a_i (a_j) \).

The top five features for the random map are: (1) the binary indicator for non-cardinal conflicts; (2) the weight of the edge between agents \( a_i \) and \( a_j \) in the weighted dependency graph; (3) the binary indicator for cardinal conflicts; (4) the maximum of the widths of level \( t \) of the MDD for agent \( a_i (a_j) \); (5) the maximum of the differences of the cost of the path of agent \( a_i (a_j) \) and time step \( t \).

The top five features for the city map are: (1) the binary indicator for non-cardinal conflicts; (2) the maximum of the numbers of conflicts involving agent \( a_i (a_j) \) that have been selected and resolved; (3) the minimum of the numbers of conflicts involving agent \( a_i (a_j) \) that have been selected and resolved; (4) the binary indicator for cardinal conflicts; (5) the sum of the numbers of conflicts involving agent \( a_i (a_j) \) that have been selected and resolved.

The top five features for the game map are: (1) the minimum of the differences of the cost of the path of agent \( a_i (a_j) \) and time step \( t \); (2) the maximum of the differences of the cost of the path of agent \( a_i (a_j) \) and time step \( t \); (3) the maximum of the numbers of conflicts involving agent \( a_i (a_j) \) that have been selected and resolved; (4) the minimum of the numbers of conflicts involving agent \( a_i (a_j) \) that have been selected and resolved; (5) the sum of the numbers of conflicts involving agent \( a_i (a_j) \) that have been selected and resolved.

Results on Feature Selection
In this subsection, we present preliminary results on feature selection. We select five categories of features: (1) features related to the cardinal conflicts (features 3, 4 and 5); (2) features related to the frequency of a conflict being resolved for an agent (features 6, 7 and 8); (3) features related to the numbers of conflicts that the agent is involved in (features 12 and 13); (4) features related to the difference of the cost of the path of the agent and its individually cost-minimal path; (5) features related to the MDDs and the WDG (features 62, 63 and 67). The features selected cover the top ten features for ML-O (as shown in Figure 12) that can be computed in constant time and four features among the top five features for each individual map (as shown in Figures 11 and 10). We train a ranking function with all five categories of features and denote the corresponding solver as ML-S (5). We then hold
out each of the five categories and train a ranking functions with the rest of four categories. We denote the solver that is trained without the $i$-th category as ML-S ($-i$). Since now we use only 9 to 12 features for training, we can afford to train a SVM\textsuperscript{rank} using a polynomial kernel of degree 2 for each solver while keeping the other parameters the same. We show the success rates on the warehouse map in Figure 2. ML-S (5) performs similarly to ML-S, implying that when using only the selected features, we are still able to achieve performance as good as ML-S which uses all features. ML-S (-1) trained without using the 1-st category (features related to the cardinal conflicts) performs the worse among our solvers, only slightly better than CBSH2. ML-S (-3) is the best among our solvers, which dominates ML-S.

C Code and Data for Reproducibility

We provide the core of our code in the supplementary material, which is based on the open-source code from [Li et al. 2019a]. We also include the random and warehouse maps used in experiments, while other maps could be found at [Stern et al. 2019] and [Sturtevant 2012]. We do not include the training and test data due to the file size limit.
Figure 3: Success rates within the runtime limit.

Figure 4: The warehouse map: Percentage of solved instances under problem parameters.

Figure 5: The room map: Percentage of solved instances under problem parameters.
Figure 6: The maze map: Percentage of solved instances under problem parameters.

Figure 7: The random map: Percentage of solved instances under problem parameters.
Figure 8: The city map: Percentage of solved instances under problem parameters.

Figure 9: The game map: Percentage of solved instances under problem parameters.
Figure 10: Feature importance plots for the warehouse, room, maze and random maps.
Figure 11: Feature importance plots for the city and game maps.

Figure 12: Feature importance plots for ML-O.
Table 5: Success rates, average runtimes and CT sizes of instances solved by all solvers, and PAR10 scores (calculated using the runtimes in minutes) for different number of agents $k$. For the success rates of ML-S and ML-O, the fractions of instances solved by both the solver and the baseline are given in parentheses (bolded if the solver solves all instances that CBSH2 solves).
| Index | Feature |
|-------|---------|
| 1     | Binary indicators for edge conflicts. |
| 2     | Binary indicators for vertex conflicts. |
| 3     | Binary indicators for semi-cardinal conflicts. |
| 4     | Binary indicators for non-cardinal conflicts. |
| 5     | Number of conflicts involving agent \(a_i\), \(a_j\) that have been selected and resolved. |
| 6     | Maximum of the numbers of conflicts involving agent \(a_i\), \(a_j\) that have been selected and resolved. |
| 7     | Sum of the numbers of conflicts involving agent \(a_i\), \(a_j\) that have been selected and resolved. |
| 8     | Minimum of the numbers of conflicts at vertex \(v\) that have been selected and resolved. |
| 9     | Maximum of the numbers of conflicts at vertex \(v\) that have been selected and resolved. |
| 10    | Sum of the numbers of conflicts at vertex \(v\) that have been selected and resolved. |
| 11    | Minimum of the numbers of conflicts that agent \(a_i\), \(a_j\) is involved in. |
| 12    | Maximum of the numbers of conflicts that agent \(a_i\), \(a_j\) is involved in. |
| 13    | Sum of the numbers that conflicts that agent \(a_i\), \(a_j\) is involved in. |
| 14    | Time step 1 of the conflict. |
| 15    | Rate of \(t\) and the makespan of the solution. |
| 16    | Minimum of the costs of the path of agent \(a_i\), \(a_j\). |
| 17    | Maximum of the costs of the path of agent \(a_i\), \(a_j\). |
| 18    | Sum of the costs of the path of agent \(a_i\), \(a_j\). |
| 19    | Absolute difference of the costs of the path of agent \(a_i\), \(a_j\). |
| 20    | Minimum of the ratios of the cost of the path of agent \(a_i\), \(a_j\). |
| 21    | Minimum of the ratios of the cost of the path of agent \(a_i\), \(a_j\). |
| 22    | Maximum of the costs of the cost of the path of agent \(a_i\), \(a_j\) and its individually cost-minimal path. |
| 23    | Maximum of the ratios of the cost of the path of agent \(a_i\), \(a_j\) and its individually cost-minimal path. |
| 24    | Minimum of the ratios of the cost of the path of agent \(a_i\), \(a_j\) and \(N_{Conf}\). |
| 25    | Maximum of the ratios of the cost of the path of agent \(a_i\), \(a_j\) and \(N_{Conf}\). |
| 26    | Minimum of the ratios of the cost of the path of agent \(a_i\), \(a_j\) and \(N_{Conf}\). |
| 27    | Maximum of the ratios of the cost of the path of agent \(a_i\), \(a_j\) and \(N_{Conf}\). |
| 28    | Binary indicator whether none of agents \(a_i\) and \(a_j\) has reached its goal by time step \(t\). |
| 29    | Binary indicator whether at least one of agents \(a_i\) and \(a_j\) has reached its goal by time step \(t\). |
| 30    | Minimum of the differences of the cost of the path of agent \(a_i\), \(a_j\) and \(t\). |
| 31    | Maximum of the differences of the cost of the path of agent \(a_i\), \(a_j\) and \(t\). |
| 32    | Minimum of the ratios of the cost of the path of agent \(a_i\), \(a_j\) and \(t\). |
| 33    | Maximum of the ratios of the cost of the path of agent \(a_i\), \(a_j\) and \(t\). |
| 34    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V', q' \in V'\} = 0\). |
| 35    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V', q' \in V'\} = 1\). |
| 36    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V', q' \in V'\} = 2\). |
| 37    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V', q' \in V'\} = 3\). |
| 38    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V', q' \in V'\} = 4\). |
| 39    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V', q' \in V'\} = 5\). |
| 40    | Number of agents \(a\) such that there exists \(q' \in V'\) and \(q \in V'\) such that \(d_{q,q'} = 0\). |
| 41    | Number of agents \(a\) such that there exists \(q' \in V'\) and \(q \in V'\) such that \(d_{q,q'} = 1\). |
| 42    | Number of agents \(a\) such that there exists \(q' \in V'\) and \(q \in V'\) such that \(d_{q,q'} = 2\). |
| 43    | Number of agents \(a\) such that there exists \(q' \in V'\) and \(q \in V'\) such that \(d_{q,q'} = 3\). |
| 44    | Number of agents \(a\) such that there exists \(q' \in V'\) and \(q \in V'\) such that \(d_{q,q'} = 4\). |
| 45    | Number of agents \(a\) such that there exists \(q' \in V'\) and \(q \in V'\) such that \(d_{q,q'} = 5\). |
| 46    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V, q' \in V\} = 0\). |
| 47    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V, q' \in V\} = 1\). |
| 48    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V, q' \in V\} = 2\). |
| 49    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V, q' \in V\} = 3\). |
| 50    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V, q' \in V\} = 4\). |
| 51    | Number of conflicts \(c'\) \(\in\) \(N_{Conf}\) such that \(\min\{d_{q,q'} : q \in V, q' \in V\} = 5\). |
| 52    | Minimum of the widths of level \(t-2\) of the MDDs for agent \(a_i\), \(a_j\). |
| 53    | Maximum of the widths of level \(t-2\) of the MDDs for agent \(a_i\), \(a_j\). |
| 54    | Minimum of the widths of level \(t-1\) of the MDDs for agent \(a_i\), \(a_j\). |
| 55    | Maximum of the widths of level \(t-1\) of the MDDs for agent \(a_i\), \(a_j\). |
| 56    | Minimum of the widths of level \(t\) of the MDDs for agent \(a_i\), \(a_j\). |
| 57    | Maximum of the widths of level \(t\) of the MDDs for agent \(a_i\), \(a_j\). |
| 58    | Minimum of the widths of level \(t+1\) of the MDDs for agent \(a_i\), \(a_j\). |
| 59    | Maximum of the widths of level \(t+1\) of the MDDs for agent \(a_i\), \(a_j\). |
| 60    | Minimum of the widths of level \(t+2\) of the MDDs for agent \(a_i\), \(a_j\). |
| 61    | Maximum of the widths of level \(t+2\) of the MDDs for agent \(a_i\), \(a_j\). |
| 62    | Number of vertices \(q\) in graph \(G\) such that \(\min\{d_{q,q'} : q \in V\} = 1\). |
| 63    | Number of vertices \(q\) in graph \(G\) such that \(\min\{d_{q,q'} : q \in V\} = 2\). |
| 64    | Number of vertices \(q\) in graph \(G\) such that \(\min\{d_{q,q'} : q \in V\} = 3\). |
| 65    | Number of vertices \(q\) in graph \(G\) such that \(\min\{d_{q,q'} : q \in V\} = 4\). |
| 66    | Number of vertices \(q\) in graph \(G\) such that \(\min\{d_{q,q'} : q \in V\} = 5\). |
| 67    | Weight of the edge between agents \(a_i\) and \(a_j\) in the weighted dependency graph. |

Table 6: Features with their indices.