Interference effects in the Coulomb dissociation of $^{15,17,19}$C.

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Abstract

In this work the semiclassical model of pure Coulomb excitation was applied to the breakup of $^{15,17,19}$C. The ground state wave functions were calculated in the particle-rotor model including core excitation. The importance of interference terms in the dipole strength arising after including core degrees of freedom is analyzed for each isotope. It is shown that Coulomb interference effects are important for the case of $^{17}$C.

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I. INTRODUCTION

The study of neutron rich light exotic nuclei has been the subject of extensive experimental and theoretical work for more than two decades already. One of the tools to probe the structure of these nuclei are the experiments on the Coulomb excitation and breakup whose reaction mechanism is very well known.

Several experiments have been recently performed to study carbon isotopes $^{15,17,19}$C. It was found [1] that $^{19}$C is a candidate to have one-neutron halo. Ground state with spin-parity $^{1+}$ and one-neutron separation energy less than 1 MeV [1, 2, 3] favors the formation of the halo in this nucleus, which is also confirmed by narrow momentum distributions of $^{18}$C following $^{19}$C breakup [1].

There were several experiments to measure separation energy of $^{19}$C. The experiments using time-of-flight techniques suggest small separation energy, that is, weighted average yields $242\pm95$ keV [1]. The Coulomb dissociation of $^{19}$C was studied by Nakamura in [2]. The analysis of angular distributions of breakup products suggests the value $0.53\pm0.13$ MeV. Using this value in the simple cluster model calculation of the dipole strength gives good agreement with the data. But the analysis of recent experiment of Maddalena et al. [3] on nuclear breakup of $^{19}$C yields $0.65\pm0.15$ MeV and $0.8\pm0.3$ MeV. The adopted value of one-neutron separation energy of $^{19}$C is given in Ref. [4] and $S_n = 0.58$ MeV.

Unlike $^{19}$C, the spin-parity of $^{17}$C was found to be $^{3+}$ [3], with binding energy also smaller then 1 MeV [3]. However the spin $\frac{3}{2}^+$ is an indication of a $d$-wave single particle structure and accordingly the existence of centrifugal barrier in this case does not favor the halo.

$^{15}$C again has a $\frac{1}{2}^+$ ground state [6] with a large amount of $s$-wave in the wave function with the separation energy of around 1.2 MeV [4]. Although this isotope is not generally recognized as a halo-nucleus, there is an evidence that this nucleus has a halo-like structure (see [7] and [8]).

Coulomb breakup of $^{15}$C and $^{17}$C at relativistic energies has recently been studied in Ref. [9]. The main ground state configuration of $^{15}$C is found to be $^{14}$C($0^+$) $\otimes \nu_s$ with spectroscopic factor consistent with earlier studies. The predominant ground state configuration is of $^{17}$C is found to be $^{16}$C($2^+$) $\otimes \nu_{s,d}$.

A number of theoretical studies of Coulomb and nuclear breakup of heavy carbon isotopes were performed in the framework of post-form distorted wave Born approximation [8, 10, 11].
The study of Ref. [8] confirmed the existence of one-neutron halo in $^{11}\text{Be},^{19}\text{C}$ and $^{15}\text{C}$ but not in $^{17}\text{C}$. In Ref. [10] Coulomb breakup of $^{11}\text{Be}$ and $^{19}\text{C}$ on $^{208}\text{Pb}$ was analyzed, where the transitions to excited states of the projectiles core were calculated. It was found that the contributions of the core excited states are very small. In Ref. [11] the respective roles of the first order and higher order effects for different beam energies were investigated. It was found that higher order effects are small in case of higher beam energies and forward scattering, but important at incident energies $\leq 30\text{ MeV/nucleon}$. In Ref. [13] a semiclassical dynamical model of projectile excitation was applied to nuclear and Coulomb breakup of $^{11}\text{Be}$ and $^{19}\text{C}$ on $^{208}\text{Pb}$. It was shown that Coulomb breakup dominates relative energy spectra around the peak region while the nuclear breakup is important at higher relative energy. The higher order effects are found to be generally small and dependent on the theoretical model.

In this paper we use the semiclassical model of pure Coulomb breakup. In this model the Coulomb breakup cross-section is calculated using the virtual photon numbers formalism and the dipole strength function $dB(E1)/dE$ which requires the knowledge of the ground state and excited states wave functions. For the ground state we use wave functions obtained in the particle-rotor model with core excitation. For the continuum states we use plane waves. The inclusion of core degrees of freedom gives rise to the interference terms in the dipole strength function. The model employed here was used before in Ref. [14], where the relative energy spectra were calculated for the Coulomb breakup of $^{19}\text{C}$ and integrated cross-sections for the Coulomb breakup of $^{15,17,19}\text{C}$ were presented for different ground state scenarios of these carbon isotopes. The lack of experimental data made it difficult to draw final conclusions about structure of these heavy carbon isotopes.

The purpose of this paper is to give a detailed analysis of the particle-core result. In particular we assess the importance of the interference terms in the dipole strength function, linear in the core deformation, on the shape and value of the cross section. In our approach we shall analyze the nuclear-corrected data, namely the ones where the nuclear contribution has been removed through the commonly used extrapolation procedure. For a recent overview of this scaling method see Ref. [15].
II. CORE-PARTICLE MODEL WITH CORE EXCITATION

The inclusion of core degrees of freedom as it is done in the core-particle model with core excitation is simple and physically transparent. This method was described in [16]. It was employed in [17, 18, 19] to study the properties of such weakly-bound and unbound two-body systems as $^{11}$Be, $^{13}$C, $^{10}$Li and three-body $^{12}$Be. The inclusion of core excitation made it possible to describe $^{11}$Be as an s-intruder nucleus. Esbensen et al. [20] also applied this model to positive parity states in $^{11}$Be and $^{13}$C. In the work by Ridikas et al. [14] different scenarios were proposed for the g.s. structure of $^{15,17,19}$C. Recently this model was applied to the simultaneous description of borromean nucleus $^{14}$Be and its binary constituent $^{13}$Be [21]. $^{11}$Be was also studied in the particle-vibrator model by Vinh Mau in [22].

The basic idea of this method is that deformation of the core can lead to couplings with excited states of the core. The total Hamiltonian $\hat{H}$ of the two-body system (core + n) can be written in the following way:

$$\hat{H} = \hat{T} + \hat{H}_{rot} + \hat{V},$$

where $\hat{T}$ is kinetic energy of the relative motion of core and valence neutron, $\hat{H}_{rot}$ is the Hamiltonian of a deformed axially symmetric rigid rotor, $\hat{V}$ is the interaction between the core and the neutron. The wave function of the system with total spin $J$ has the following form:

$$\Psi^{JM} = \sum_{ijI} \sum_{m_im_jm} \frac{\chi_{ijI}^J(r)}{r} \langle lm_s sm | jm \rangle \times$$

$$\langle jm_j Im_j | JM \rangle Y_{lm}^{m}(\hat{r}) X_{sm}^{m}(\hat{\sigma}) \phi_{m_j0}^{J} (\hat{\xi}),$$

where $\chi_{ijI}^J(r)$ are the radial wave functions, $Y_{lm}^{m}(\hat{r})$ are the spherical harmonics, $X_{sm}^{m}(\hat{\sigma})$ are the spin functions. The core states $\phi_{m_j0}^{J} (\hat{\xi})$ are eigenvalues of the $\hat{H}_{rot}$ and are proportional to the rotational matrices:

$$\phi_{m_j0}^{J} (\hat{\xi}) = \frac{\hat{I}}{\sqrt{8\pi^2}} D_{m_j0}^{J} (\hat{\xi}),$$

where $\hat{\xi}$ are the Euler angles, characterizing the orientation of the core in the laboratory system and $\hat{I} \equiv \sqrt{(2I + 1)}$.

The interaction term in the Hamiltonian consists of the deformed Woods-Saxon potential
and standard undeformed spin-orbit part:

\[ \hat{V} = V^{ws}(r, \theta') + 1 \cdot sV^{so}(r) \]

\[ V^{ws}(r, \theta') = \frac{V_0^{ws}}{1 + \exp \frac{r - R(\theta')}{a}} \]  \hspace{1cm} (3)

\[ V^{so}(r) = 2\left(\frac{\hbar^2}{m_pc}\right)^2 \frac{d}{dr} \left\{ 1 + \exp \frac{r - R_0}{a} \right\}^{-1}. \]  \hspace{1cm} (4)

We include only quadrupole term in the expansion of the radius:

\[ R(\theta') = R_0 \left[ 1 + \beta Y_{20}(\cos \theta') \right], \]  \hspace{1cm} (5)

where \( \beta \) is the quadrupole deformation parameter and \( \theta' \) is the polar angle of the particle in the body-fixed coordinate system.

To solve the problem we substitute the expansion (2) into the total Schrödinger equation to obtain the set of coupled equations:

\[ (T + V^{J}(r) - E + \epsilon_I) \chi^{J}(r) = -\sum_{\gamma' \neq \gamma} V_{\gamma\gamma'}^{J}(r) \chi^{J}(r), \]  \hspace{1cm} (6)

where \( \gamma = \{l, j, I\} \), kinetic energy \( T = -\hbar^2/2\mu [d^2/dr^2 - l(l + 1)/r^2] \) and matrix elements \( V_{\gamma\gamma'}(r) = \langle l, j, I; JM | \hat{V}(r, \theta') | l', j', I', JM \rangle \). \( \epsilon_I \) is the excitation energy of the core state with spin \( I \).

The deformed part of the interaction is then expanded in terms of the Legendre polynomials in order to separate the angular and radial part in the calculation of the matrix elements:

\[ V(r, \theta') = \sum_Q V_Q(r)P_Q(\cos \theta') \]  \hspace{1cm} (7)

The corresponding angular part of the matrix elements could be found in [16].

The alternative and further simplified way to calculate matrix elements is to expand the deformed Woods-Saxon potential in Taylor series over deformation parameter \( \beta \) keeping only the leading term:

\[ V(r, \theta') = V_0 f(r) + \beta v(r)Y_{20}(\cos \theta') \]  \hspace{1cm} (8)

\[ f(r) = \frac{1}{1 + \exp \frac{r - R_0}{a}} \]  \hspace{1cm} (9)

where \( v(r) = V_0 f(r)^2 \exp \left(\frac{r - R_0}{a}\right) \frac{R_0}{a} \) is the radial part of the coupling matrix element. This method allows the analytical calculation of the radial part of the coupling matrix elements.
TABLE I: Angular matrix elements $c_{ii'}$ for the ground state $\frac{1}{2}^+$ ($^{19}$C,$^{15}$C) and $\frac{3}{2}^+$ ($^{17}$C)

|      | $\frac{1}{2}^+$ |      | $\frac{3}{2}^+$ |
|------|----------------|------|----------------|
| $i$  | 1             | 2    | 3             |
|      |               |      |               |
| 1    | 0             | 0.218| 0.178         |
| 2    | 0.218         | 0.14 | 0.044         |
| 3    | 0.178         | 0.044| 0.12          |

In our analysis we will employ particle-core method with core excitation to describe $^{19}$C and $^{17}$C. We also use this model to describe $^{15}$C, although the core $^{14}$C is not a rotor. It is already established that the ground state of the $^{19}$C is $1/2^+$ state. Thus, for the ground state of $^{19}$C and $^{15}$C we will have to couple following three channels:

$$|\frac{1}{2}^+\rangle = a_1|2s_{1/2} \otimes 0^+\rangle + a_2|1d_{5/2} \otimes 2^+\rangle + a_3|1d_{3/2} \otimes 2^+\rangle,$$

For $\frac{3}{2}^+$ ground state in $^{17}$C we also couple three channels for simplicity:

$$|\frac{3}{2}^+\rangle = b_1|1d_{3/2} \otimes 0^+\rangle + b_2|1d_{3/2} \otimes 2^+\rangle + b_3|2s_{1/2} \otimes 2^+\rangle.$$

The system of coupled channel equation using potential expansion of Eq.(9) after calculation of angular matrix elements $\langle ljI; JM|Y_{20}(\cos \theta')|l'j'I'; JM\rangle$ has the following form:

$$(T_i + V_i^{so}(r) + V_0f(r) + c_{ii'}\beta v(r) + \epsilon_i - E)\chi_i(r) = -\beta v(r) \sum_{i' \neq i} c_{ii'}\chi_{i'}(r),$$

where index $i$ numbers the definite channel $|ljI\rangle$, $V_i^{so}(r)$ is a spin orbit potential in channel $i$ and $c_{ii'}$ is array of angular matrix elements (see Table I).

The solution of the system of coupled channel equations Eq.(6) or Eq.(10) gives the bound state energies and the corresponding radial functions of the components with their relative strengths. In the present calculation we used the R-matrix method on Lagrange mesh (see Ref. [23] and references therein) to solve the system of coupled channel equations (9).
III. COULOMB BREAKUP CROSS SECTIONS AND DIPOLE STRENGTH FUNCTION

In the semiclassical model of Coulomb excitation and breakup the process of absorption of radiation is treated quantum mechanically but the projectile is assumed to move in a straight line (see Refs. [24, 25]). In the first order perturbation theory the process of Coulomb excitation can be described as emission and absorption of virtual photons. Thus, Coulomb dissociation spectrum is related to the dipole strength function \( dB(E_1) / dE \) through

\[
\frac{d\sigma_C}{dE} = \frac{16\pi^3}{9hc} \frac{dB(E_1)}{dE} \frac{1}{E} \int_{b_0}^{\infty} 2\pi bdbN_{E_1}(E^*, b),
\]

where \( E \) is the relative energy between the core and the neutron, \( E^* \) is an excitation energy, \( N_{E_1}(\omega, b) \) is the number of virtual photons, \( \omega = E^* / h \). \( b_0 \) is the cutoff parameter, which is approximated by the sum of projectile and target radii. Here we assumed sharp cutoff theory for Coulomb breakup. Within the framework of a direct breakup mechanism, dipole strength function is given by the transition matrix element

\[
\frac{dB(E_1)}{dE} = \sum_M \left| <\mathbf{q}|\hat{d}|\Phi(r)> \right|^2, \quad \hat{d} = \frac{Ze}{A} rY_{1M},
\]

where \( <\mathbf{q}| \) represents the scattering state, \( \Phi(r) \) is the ground state wave function of the nucleus and \( \hat{d} \) is the electric dipole operator.

The cluster model or Yukawa+plane wave approximation [26] is usually used to calculate \( dB(E_1)/dE \) distributions. In this approximation one assumes Yukawa wave function for the ground state and a plane wave for a continuum state and dipole strength function takes the form

\[
\frac{dB(E_1)}{dE} = S N_0 \left( \frac{Z_c e}{A_c + 1} \right)^2 \frac{\sqrt{E_B E^{3/2}}}{(E + E_B)^4}
\]

where \( S \) is the spectroscopic factor for the \( 2s_{1/2} \) state and \( N_0 \) is the normalization factor (see Refs. [27, 28, 29]). \( E_B \) is the binding energy of the neutron in the ground state, \( \mu \) is the reduced mass of core+neutron and \( A_c \) is the mass of the core. It is seen that the shape of the distribution depends only on the binding energy \( E_B \) and has a peak at \( E = \frac{2}{3} E_B \).

The integral, \( B(E1) \) is given by

\[
B(E1) = \frac{3h^2 e^2}{16\pi E_B} S N_0
\]

The validity of the Yukawa+plane waves approximation is based on the fact that for low excitation energy \( dB(E1)/dE \) is determined by the outer part of the wave function.
When allowing the core to be excited (deformed or vibrational), then the cross section can be expressed as the incoherent sum of components $d\sigma(I)/dE$ corresponding to different core states with spin $I$ populated after one neutron removal. Furthermore for each core state the cross section is further decomposed into an incoherent sum over contributions from different angular momenta $j$ of the valence neutron in its initial state. Accordingly one has the general expression for a final plane wave (n+core), $\langle q \rangle$, 

$$
\frac{d\sigma(I)}{dE} \propto N_{E1}(E) \sum_j C^2 S(I, nlj) \times \sum_m \left| \langle q \otimes I| Z \frac{r}{A} Y_{Im}(r) |\psi_{nlj}(r) \otimes I \rangle \right|^2 ,
$$

where the value of $q$ depends on the energy of the core state.

The above form for the exclusive Coulomb dissociation has an analogous, one-neutron removal cross section (nuclear) obtained within the eikonal approximation by Tostevin [30, 31] and used extensively for spectroscopic study of radioactive nuclei.

When the inclusive Coulomb dissociation is required one, namely the final state of the core is not observed, then the cross-section becomes

$$
\frac{d\sigma}{dE} = \sum_I \frac{d\sigma(I)}{dE}
$$

(16)

The question we ask here is how large are the interference terms present in the individual $d\sigma(I)/dE$ and in the sum, Eq (16)? These genuine quantal interference terms arise naturally when calculating the matrix elements $\langle q| \frac{Z}{A} r Y_{Im} |\psi_{nlj}\rangle$ using the well known Bauer’s expansion of the plain wave into partial waves

$$
\langle q|r \rangle = \frac{4\pi}{kr} \sum_{l,m} Y_{lm}(\hat{q}) Y_{lm}(\hat{r})(-i)^{l+j}(kr).
$$

(17)

For each core state $I$ the dipole strength wave function is calculated using the wave functions of Eq.(2) for the ground states (with spin $J$) and plane waves for continuum states (with spin $J'$) and has the following form [17]:

$$
\frac{dB(E1, I, J \rightarrow J')}{dE} = \frac{j^2}{j'f^2} \left| \sum_{l,j,j'} \mathcal{M}_{lj,l'j'}^{I} R_{lj,l'j'} \right|^2 ,
$$

(18)
where angular part of the matrix element is given by:

\[
\mathcal{M}^{I}_{ij,l_j,l_j'} = \frac{Z_e e}{A_c + 1} \sqrt{\frac{3}{4\pi}} \hat{J} \hat{j} \hat{j'} \times \\
(-)^{I+j+I+j'-\frac{1}{2}} \left\{ \begin{array}{ccc} J' & J & 1 \\ j & j' & I \end{array} \right\} \left( \begin{array}{ccc} j' & j & 1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right)
\]

(19)

and the radial part of the matrix element:

\[
\mathcal{R}^{I}_{ij,l_j,l_j'} = \sqrt{\frac{2\mu k}{\hbar^2 \pi}} \int_0^{\infty} \chi^{J}_{ij I}(r) r j_{\nu}(kr) dr,
\]

(20)

where \(\chi^{j}_{ij I}(r)\) are g.s. radial wave functions of Eq.(2) and \(j_{\nu}(kr)\) are the spherical Bessel functions.

The expression for the dipole strength function of Eq.(18) contains the coherent sum of contributions from different channels (but with the same core state), which appear because of the expansion of the total wave function of the ground state in terms of the core states. This gives rise to the interference terms in the dipole strength and consequently in the cross section.

To compare with experimental data, one needs to sum the expression (18) over allowed final angular momentum \(J'\). The spin of the ground state of \(^{19}\text{C}\) and \(^{15}\text{C}\) is \(J = \frac{1}{2}^+\) and for the final state we have \(J' = \{\frac{1}{2}^-, \frac{3}{2}^-\}\). For the g.s. of \(^{17}\text{C}\) with \(J = \frac{3}{2}^+\) one also has \(J' = \{\frac{1}{2}^-, \frac{3}{2}^-\}\).

The contributions of different terms entering Eq.(18) will be analyzed in the next Section.

IV. APPLICATION

In this section we present our calculations for the Coulomb dissociation of the carbon isotopes \(^{19}\text{C}\), \(^{17}\text{C}\) and \(^{15}\text{C}\) in the field of \(^{208}\text{Pb}\). The parameters of particle-core model used here for each of the carbon isotopes are given in Table II.

A. \(^{19}\text{C}\)

The role of the transitions to the core excited states in the Coulomb breakup of \(^{19}\text{C}\) (and \(^{11}\text{Be}\)) on \(^{208}\text{Pb}\) was analyzed before in Ref. [10], where the integrated partial cross sections and momentum distributions for the ground state as well as excited bound states of core
TABLE II: Parameters of the model used to describe $^{15}$C,$^{17}$C and $^{19}$C and the corresponding structure of the ground state wave functions. The values of the radius $R_0$, diffuseness $a$, spin-orbit depth $V_0^{so}$ and deformation parameter $\beta$ are the same as in Ref. [14].

| parameter     | $^{19}$C | $^{17}$C | $^{15}$C |
|---------------|---------|---------|---------|
| potential     |         |         |         |
| $R_0$ (fm)    | 3.00    | 2.82    | 2.45    |
| $a$ (fm)      | 0.65    | 0.65    | 0.65    |
| $V_0^{so}$ (MeV) | 6.5    | 6.5     | 6.5     |
| $V_0^{ws}$ (MeV) | 42.95  | 59.33   | 71.12   |
| $\beta$       | 0.5     | 0.55    | 0.42    |
| $\epsilon_{2+}$ (MeV) | 1.2    | 1.776   | 7.1     |
| $S_n$ (MeV)   | 0.65    | 0.73    | 1.22    |

results

| g.s. structure (%) | $^{1/2+}$ | $^{3/2+}$ | $^{1/2+}$ |
|--------------------|-----------|-----------|-----------|
| $s_{1/2} \otimes 0^+$ | 71.5      | 14.5      | 87.2      |
| $d_{5/2} \otimes 2^+$ | 25.3      | 76.5      | 10.9      |
| $d_{3/2} \otimes 2^+$ | 3.2       | 9.0       | 1.9       |

nuclei were calculated within the finite-range DWBA as well as within the adiabatic model of the Coulomb breakup. It was found that the transitions to excited states of the core are quite weak and the interference effects are suppressed.

In our calculation, there are 7 coherent contributions to the $^{1/2+} \rightarrow ^{3/2-}$ transition and 4 coherent contributions to the $^{1/2+} \rightarrow ^{1/2-}$ transition. In fact, in the $^{1/2+} \rightarrow ^{3/2-}$ case, we found out that the dominant contribution is the direct $s_{1/2} \rightarrow p_{3/2}$ one (see Fig.1a). All the other 6 contributions add up to a 6% of the total, confirming the result of Ref. [10]. The contributions of core excited state $2^+$ are very small and in order to make them visible they are multiplied by 100 in Fig.1a and Fig.1b and by 10 in Fig.1c. In the $^{1/2+} \rightarrow ^{1/2-}$ transition, besides the dominant $s_{1/2} \rightarrow p_{1/2}$ one, there is only one interference term which add up destructively, namely $2\langle 1d_{5/2}, 2^+ | \hat{d}| p_{3/2}, 2^+; k \rangle \langle 1d_{3/2}, 2^+ | \hat{d}| p_{3/2}, 2^+; k \rangle$. The other 2 contributions, add up to about few percents, as Fig.1b shows. Total contributions of all
FIG. 1: Contributions to the dipole strength function for $^{19}$C. (a) Dominant contributions to the dipole strength for $\frac{1}{2}^+ \rightarrow \frac{3}{2}^-$ transition; (b) All contributions to the dipole strength for $\frac{3}{2}^+ \rightarrow \frac{1}{2}^-$; (c) Comparison of overall contribution of squared terms (contribution of $0^+$ is not shown here) and interference terms; (d) Overall comparison of contribution of $\frac{1}{2}^+ \rightarrow \frac{3}{2}^-$ and $\frac{3}{2}^+ \rightarrow \frac{1}{2}^-$ and their sum for $0^+$ contributions (thin lines) and $2^+$ contributions (thick lines);

squared terms (core excited components only) and interference terms are compared in Fig.1 (c).

The total $\frac{1}{2}^+ \rightarrow \frac{3}{2}^-$ and $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ transitions are shown in Fig.1 (d) together with their sum, to be compared with the data.

When compared to the data, the major low-energy peak is quite nicely reproduced, with
FIG. 2: $^{19}$C+$^{208}$Pb at 67A MeV Coulomb breakup cross sections calculated in different models. Solid line stands for the cluster model calculation with $S=0.67$; dashed line – present calculation with $\epsilon_{2^+}=1.62$ MeV; dotted line – present calculation with $\epsilon_{2^+}=1.2$ MeV; dot-dashed line – toy model calculation (see text). Data from [2].

70% $s$-contribution and 30% $d$-contribution. The data also exhibit some structure at higher excitation energies, which can not be accounted for by the particle-rotor model. In fact, we have already taken the liberty of reducing the energy of the $2^+$ state of the core (since the rotor is found inside the halo nucleus and changes may ensue) to see whether this, with accompanying change in the relative energy of the halo neutron, can reproduce better the data. We found out that the combination $\epsilon_{2^+} = 1.2$ MeV and $S_n = 0.65$ MeV gives the best account, but still misses the strength at higher energies. Since the total contribution to the cross section involves the incoherent sum of the $\frac{1}{2}^+ \rightarrow \frac{3}{2}^-$ and $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$, and the former exhibits a significant structure besides the main peak, we were tempted to consider a two-cluster model for the cross-section, one peaks at $E = \frac{3}{5}E_B$, while the other at $E = \frac{3}{5}(X-E_B)$, with $X$ related to the excitation energy of the $2^+$ state in the deformed core. The result of our calculation is shown in Fig.2. The “spectroscopic factors” attached to the two cluster response are, respectively, 0.7 and 0.3. The above toy model is instructive, as it points to ways of improving the particle-excited core model. Of course, the Coulomb dissociation, being, what is known as, elastic breakup (no target excitation) may have to be corrected owing to possible contributions of nuclear-Coulomb interference [13] and inelastic breakup.
\textbf{B. $^{17}\text{C}$}

In the dissociation of $^{17}\text{C}$, whose ground state spin is $\frac{3}{2}^+$, we again sum incoherently two transitions $\frac{3}{2}^+ \rightarrow \frac{3}{2}^−$ and $\frac{3}{2}^+ \rightarrow \frac{1}{2}^−$. Unlike the case of breakup of $^{19}\text{C}$, here we have $^{17}\text{C}$ mainly with the core excited contribution in the ground state wave function (see Table II). Therefore terms $d_{3/2} \rightarrow p_{3/2}$ and $d_{3/2} \rightarrow p_{1/2}$, which corresponds to the contribution of the $0^+$ in the core, are now small.

For $\frac{3}{2}^+ \rightarrow \frac{3}{2}^−$ there are all together 10 contributions of which 6 are interference terms (see Fig.3). For $\frac{3}{2}^+ \rightarrow \frac{3}{2}^−$ transition, the dominant terms $\frac{3}{2}^+ \rightarrow \frac{1}{2}^−$ and $2\langle 2s_{1/2}, 2^+|\hat{d}|p_{3/2}, 2^+; k\rangle \langle 1d_{3/2}, 2^+|\hat{d}|p_{1/2}, 2^+; k\rangle$ cancel each other. As Fig.3 shows, the overall strength is predominantly due to the $2^+$ state in the core and is rather small because of the cancellation of the two dominant terms above. The contribution of the $0^+$ in the core, namely $|\langle 1d_{3/2}, 0^+|\hat{d}|p_{3/2}, 0^+; k\rangle|^2$ is but a few percent. This shows clearly that in the $\frac{3}{2}^+ \rightarrow \frac{3}{2}^−$ transition quantum interference and core excitation are very important.

The transition $\frac{3}{2}^+ \rightarrow \frac{1}{2}^−$ contains altogether three squared and one interference terms, the dominant squared one is $|\langle 2s_{1/2}, 2^+|\hat{d}|p_{3/2}, 2^+; k\rangle|^2$ and the only one interference term $2\langle 2s_{1/2}, 2^+|\hat{d}|p_{3/2}, 2^+; k\rangle \langle 1d_{3/2}, 2^+|\hat{d}|p_{3/2}, 2^+; k\rangle$ positive and almost equal the direct term $d_{3/2} \rightarrow p_{1/2}$.

Transitions $\frac{3}{2}^+ \rightarrow \frac{1}{2}^−$ and $\frac{3}{2}^+ \rightarrow \frac{3}{2}^−$ contribute almost equally into total. If one considers the full contribution of the interference terms, namely from the $\frac{3}{2}^+ \rightarrow \frac{3}{2}^−$ and $\frac{3}{2}^+ \rightarrow \frac{1}{2}^−$ transitions, one get just about 30% of the total squared terms. Note, that in total interference terms contribute destructively. To within error bar accuracy, one may conclude that the $dB(E1)/dE$ distribution for $^{17}\text{C}$ is composed of an incoherent sum of several squared terms that peak at about the same energy, corresponding, to the $2s_{1/2} \otimes 2^+$ component of the g.s. configuration. This attests to the validity of the simple cluster model even in this subtle quantal system.

In Ref. [9], the final channel was clearly identified as $^{16}\text{C}(2^+)+n$. Therefore, to be correct, one has to compare the data with the exclusive Coulomb dissociation cross-section, Eq.(15). However, as we have seen, the core $0^+$ state contribution is at most 10% of the peak value of the $2^+$ contribution. Further, it is mostly concentrated in the relative energy range $0 < E < 2$ MeV. Thus one may safely calculate the inclusive cross-section and ignore the low energy part. The comparison with the data is shown in Fig.4. It can be seen, that
FIG. 3: Contributions to the dipole strength function for $^{17}$C. (a) All squared contributions to the dipole strength for $\frac{3}{2}^+ \rightarrow \frac{3}{2}^-$; (b) Dominant interference contributions for $\frac{3}{2}^+ \rightarrow \frac{3}{2}^-$; (c) All contributions to the dipole strength for $\frac{3}{2}^+ \rightarrow \frac{1}{2}^-$; (d) Comparison of overall contribution of squared terms (contribution of $0^+$ is not shown here) and interference terms.

the present calculation reproduces the data.
FIG. 4: $^{17}$C+$^{208}$Pb at 495A MeV Coulomb breakup cross sections calculated in different models. Data from [9]. $E^*$ is an excitation energy. (Here the theoretical cross sections were convoluted with the experimental response functions.)

C. $^{15}$C

The last example is the calculation of the Coulomb breakup of $^{15}$C. Strictly speaking, we cannot use the rotor model for the core, as it was experimentally shown, that the first excited state of the core is not $2^+$ but $1^-$ at around 7 MeV [32]. But, as it was shown in Ref. [14], particle-rotor model reproduces the spectroscopic number rather well. This calculation gives 87% of the core inert state. Because of this the contribution of core excited components is very small and we do not include it here, that is we only include contributions $2s_{1/2} \rightarrow p_{3/2}$ and $d_{3/2} \rightarrow p_{1/2}$ to the dipole strength function. The resulting cross section is shown in Fig. 5 along with the result of the cluster model Eq. (13). Again the simple cluster picture works nicely for this system.

V. CONCLUSIONS

We have used the particle-deformed core model wave functions to calculate the Coulomb dissociation of $^{15}$C, $^{17}$C and $^{19}$C in the semiclassical model. We have shown that whereas interference effects are important in the non-halo isotope $^{17}$C, they are practically insignificant in $^{15}$C and $^{19}$C. This fact points to the validity of the cluster picture in the latter cases.
FIG. 5: $^{15}$C+$^{208}$Pb at 605A MeV Coulomb breakup cross sections calculated in different models. Data from [9]. $E^*$ is an excitation energy. (Here the theoretical cross sections were convoluted with the experimental response functions.)

where it emphasizes the subtle quantal nature of the former, mostly core-excited projectile.

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