Universal dynamics of a degenerate unitary Bose gas

P. Makotyn, C. E. Klauss, D. L. Goldberger, E. A. Cornell* and D. S. Jin*

From neutron stars to high-temperature superconductors, strongly interacting many-body systems at or near quantum degeneracy are a rich source of intriguing phenomena. The microscopic structure of the first-discovered quantum fluid, superfluid liquid helium, is difficult to access owing to limited experimental probes. Although an ultracold atomic Bose gas with tunable interactions (characterized by its scattering length, $a$) had been proposed as an alternative strongly interacting Bose system\textsuperscript{1-8}, experimental progress\textsuperscript{9-12} has been limited by its short lifetime. Here we present time-resolved measurements of the momentum distribution of a Bose-condensed gas that is suddenly jumped to unitarity, where $a = \infty$. Contrary to expectation, we observe that the gas lives long enough to permit the momentum to evolve to a quasi-steady-state distribution, consistent with universality, while remaining degenerate. Investigations of the time evolution of this unitary Bose gas may lead to a deeper understanding of quantum many-body physics.

A powerful feature of atom gas experiments that provides access to these new regimes is the ability to change the interaction strength using a magnetic-field Feshbach resonance\textsuperscript{13}. In particular, at the resonance location, $a$ is infinite. For atomic Fermi gases\textsuperscript{14-20}, accessing this regime by adiabatically changing $a$ led to the achievement of superfluids of paired fermions and enabled investigation of the crossover from superfluidity of weakly bound pairs, analogous to the Bardeen–Cooper–Schrieffer theory of superconductors, to Bose–Einstein condensation (BEC) of tightly bound molecules\textsuperscript{21,22}. For bosonic atoms, however, this route to strong interactions is stymied by the fact that three-body inelastic collisions increase as $a$ to the fourth power\textsuperscript{23-25}. This circumstance has limited experimental investigation of Bose gases with increasing interaction strength to studying either non-quantum-degenerate gases\textsuperscript{24,25} or BECs with modest interaction strengths ($na^4 = 0.008$, where $n$ is the atom number density)\textsuperscript{26-28}.

The problem is that the loss rate scales as $na^4$ whereas the equilibration rate scales as $na^3\nu$, where $\nu$ is the average velocity. Thus, it would seem that the losses will always dominate as $a$ is increased to $\infty$. Even if we were to forsake thermal equilibrium and suddenly change $a$ to project a weakly interacting BEC onto strong interactions\textsuperscript{22,26-28}, one might expect that three-body losses would still dominate the ensuing dynamics for large $a$. In this work, however, we use this approach to take a BEC to the unitary gas regime, and we observe dynamics that in fact saturate on a timescale shorter than that set by three-body losses and that exhibit universal scaling with density.

One of the intriguing aspects of the unitary gas is that because $a$ diverges, it can no longer be a physically relevant scale for describing the system and its behaviour. For a gas near zero temperature, such as a BEC, the only physical scale that remains at unitarity is the interparticle spacing. (In principle, the size of the cloud, or, equivalently the trap parameters, can provide a length scale, although one that is not intrinsic to the system. In addition, we are ignoring here any explicit three-body interactions, which could provide an additional length scale.) The gas behaviour should then be universal in the sense that it is characterized only by the density $n$. This means that energies scale as $n^{2/3}$, momenta as $n^{1/3}$, and times as $n^{-2/3}$, which we parameterize respectively by $E_n \equiv \hbar^2(6\pi^2n)^{2/3}/2m$, $k_n \equiv (6\pi^2n)^{1/3}$ and $\tau_n \equiv \hbar/E_n$.

The universality that makes the unitary gas so remarkable also provides a reason to hope that rapid three-body loss will not necessarily be an insurmountable barrier to experimental exploration of bulk (as opposed to lattice-confined) degenerate Bose gases with unitarity-limited interactions. For the degenerate unitary Bose gas, both the loss rate and the equilibration rate must scale as $n^{2/3}$. The comparison of the two rates then hinges on unknown numerical prefactors, and it becomes an experimental question whether losses dominate or a local equilibrium can be reached. In addition, we note that on resonance, the shallow bound state that exists for finite positive $a$ disappears, so that loss requires atoms to decay to deeply bound molecular states\textsuperscript{29}. For $^{85}$Rb atoms, the previous experimental observation of a relatively narrow, and therefore long-lived, Efimov resonance (characterized by a dimensionless width, $\eta = 0.057 \ll 1$; ref. 11) is indicative that atoms close together do not decay instantaneously to deeply bound molecular states.

Our experiments (Fig. 1a) begin with a $^{85}$Rb BEC of between 5 and $7 \times 10^5$ atoms confined in a 10 Hz spherical magnetic trap\textsuperscript{26}. The magnetic field, $B$, is set approximately 8 G above the $^{85}$Rb Feshbach resonance at $B_0 = 155.04$ G (ref. 31). This sets the initial $a$ to 142 $a_0$, which gives the BEC a Thomas–Fermi density distribution with an average density ($n$) = 5.5(3) $\times 10^{12}$ cm$^{-3}$. With a typical initial temperature $10$ nK, the thermal de Broglie wavelength is large compared with $(n)^{-1/3}$ and is not a relevant length scale in the physics of the ensuing experiment. Starting with this BEC in the extremely dilute limit, with $(n)a^2 < 10^{-5}$, we then decrease $B$ to $B_0$ in 5 ms. During the final 3 ms of the ramp of $B$, $(n)a^2$ goes from an essentially dilute value of $10^{-4}$ to $(n)a^2 > 1$.

After allowing the cloud to evolve at unitarity for a time $t$, we measure the momentum distribution of atoms by ramping, equally rapidly, back to small $a$ and allowing the gas to expand ballistically before imaging the cloud using resonant, high-intensity absorption imaging\textsuperscript{28}. From an azimuthal average of the image, we extract a momentum-space column density $\bar{n}(\hat{k})$ as a function of the component of momentum perpendicular to the line of sight, $\hat{k}$. By imaging at various times of flight (7, 13, 25 ms), we increase the dynamic range of our data and reduce the region of $\hat{k}$ that is obscured by initial-size effects. We repeat this experimental procedure for various $t$ to explore the evolution of the momentum distribution as a function of time at unitarity.

From images of the expanded cloud, we also obtain the number of atoms, $N$, which we show in Fig. 1b as a function of $t$. Fitting an
exponential decay to this early time data yields a time constant of 630 ± 30 µs. In addition, the measured change in the spatial volume of the condensate is (6 ± 9)% during the first 500 µs at unitarity. A fact that is immediately clear from this data is that the density loss at unitarity occurs on a timescale that is much longer than the few microsecond duration of our ramps onto and away from the Feshbach resonance. The ramp duration is also much shorter than the characteristic timeset by the interparticle spacing, \(n_s = 57\) µs.

Equipped with this information regarding the timescales for number loss and for expansion of the trapped gas at unitarity, we now consider the measured momentum distributions. These are shown in Fig. 2 for various \(t\), with the inset showing the same data on a log–linear plot. Given the finite times of flight before imaging, the data at small \(\mathbf{k}\) are strongly affected by the initial size of the BEC and do not accurately reflect \(\tilde{n}(\mathbf{k})\); the grey regions in Fig. 2 indicate where initial-size effects are non-negligible, and we see that a significant fraction of the signal lies within this region. Nevertheless, the data clearly show the emergence of signal at high \(\mathbf{k}\), outside the grey regions. The signal at high \(\mathbf{k}\) grows as a function of \(t\) before saturating in approximately 100 µs. In this time, the gas has not yet lost a significant number of atoms or significantly reduced its density. The fact that the evolution timescale for \(\tilde{n}(\mathbf{k})\) is very different from the loss timescale clearly points to a mechanism for this dynamics that is distinct from three-body loss. Furthermore, the much shorter timescale for saturation of \(\tilde{n}(\mathbf{k})\) suggests the existence of a ‘quasi-equilibrium’ metastable state of a degenerate Bose gas at unitarity.

To look for evidence of universality, we repeated the measurements for a lower initial density of the BEC. The measured \(\tilde{n}(\mathbf{k})\) for lower initial spatial density \(\langle n \rangle\) also shows the emergence of signal at high \(\mathbf{k}\) at unitarity. The distributions are similar to those measured for the higher \(\langle n \rangle\) (Fig. 2), except that the dynamics occur over a longer timescale, with \(\tilde{n}(\mathbf{k})\) saturating in approximately 200 µs. To extract the three-dimensional \(n(k)\), we use an inverse Abel transform. In Fig. 3, we show the saturated momentum distributions as a function of the scaled momentum, \(\kappa = k/k_s\), where

\[
\kappa = \frac{k}{k_s}
\]

\[
\tilde{n}(\mathbf{k}) \sim \frac{1}{\kappa^2}
\]
\( k_n \) is calculated at the average density \( \langle n \rangle \). We find that the shape of the distributions for the two \( \langle n \rangle \) are very similar.

Given that our data are consistent with a universal shape for the saturated \( n(\kappa) \) at high \( \kappa \), we now discuss aspects of this distribution. First, we note that although much of the signal remains at small \( \kappa \) where our data are affected by initial-size effects, the population with \( \kappa > 0.5 \) for the saturated \( n(\kappa) \) is nearly 50% of the initial \( N \).

Second, for two-body short-range interactions, such as those that give rise to the \( s \)-wave scattering length for atoms, one expects a \( 1/\kappa^2 \) tail at high momentum for an equilibrium gas, where the amplitude of this tail is the thermodynamic parameter known as the contact\(^{33} \).

We do not find evidence for a \( 1/\kappa^2 \) tail at high momentum, which would appear as a flat line for large \( \kappa \) in Fig. 2 (inset); however, a \( 1/\kappa^4 \) tail may exist below our detection limit at large \( \kappa \) where the signal-to-noise ratio is poor. In addition, three-body interactions could modify the high-momentum tail at unitarity\(^ {34} \).

Finally, we consider the low-\( \kappa \) part of the momentum distribution and the question of whether or not the gas remains degenerate after the rapid sweep to unitarity. At low \( \kappa \), initial-size effects can play a non-negligible role. However, this effect is such that we can obtain a lower limit on the fraction of atoms that have \( \kappa < \kappa_{\text{max}} \) by integrating our \( n(\kappa) \) data up to \( \kappa_{\text{max}} \). This allows us to extract a lower limit for the density of atoms in phase space. Specifically, we calculate the average occupancy per state at low \( \kappa \), which is given by the number of atoms divided by the number of states in phase space:

\[
\langle \rho_{\text{occ}} \rangle = \left( \frac{N}{8\pi} \int_0^{\kappa_{\text{max}}} n(\kappa) 4\pi\kappa^2 d\kappa \right) / \left( \frac{V}{3} \frac{4\pi}{h^3} \left( \frac{h\kappa}{\varepsilon} \right)^3 \right)
\]

where we conservatively use for the effective coordinate-space volume, \( V = (4\pi/3)R_{TF}^3 \), where \( R_{TF} \) is the Thomas–Fermi radius of the initial weakly interacting BEC. For the higher \( \langle n \rangle \) data, where the effects of the initial size are smaller, choosing for example \( \kappa_{\text{max}} = 0.26 \) gives 23% of the atoms and \( \langle \rho_{\text{occ}} \rangle = 7.1 \) for \( t = 170 \mu s = 3\tau_c \).

The fact that this lower limit for the density in phase space is much larger than 1 for a significant fraction of the atoms indicates that the gas is degenerate.

In conclusion, we have projected initially weakly interacting BECs onto unitarity-limited interactions and measured the resulting momentum-space dynamics. Three key findings of this work are as follows: the momentum distribution of the unitary gas evolves and then saturates on a timescale that is significantly shorter than the timescale for three-body loss; both the shape of the saturated momentum distribution and the timescale for the dynamics seem to be universal; the low-momentum part of the momentum distribution indicates that the density of atoms in phase space exceeds 1. This work raises some interesting questions: to what extent can the gas locally be described by a temperature, and is this temperature below the critical temperature for a Bose gas with unitarity-limited short-range interactions? What is this critical temperature in units of the critical temperature for an ideal Bose gas? At high momentum, what is expected for the contact, and does a high-momentum tail whose amplitude corresponds to the contact exist beyond the range of our data, or below our detection limit? Finally, what does the observed momentum dependence of the distribution tell us about the evolution of the system at unitarity?

**Methods**

**Magnetic-field control.** To rapidly change the magnetic field, we use an additional pair of coils, each with ten turns and a diameter of 1.0, 2.8 cm apart. The step response of the system has a 10–90% rise time of 2.1 \( \mu s \); thus, the 5 \( \mu s \) magnetic-field
sweep used in the measurements is below the maximum bandwidth of the system. We characterize and pre-correct for induced currents from mutual inductances between these coils and the magnetic trap coils as well as eddy currents in surrounding conductors. Taking into account roughly equal contributions from uncertainty in our magnetic field and the uncertainty in the Feshbach resonance location \(B_0\) (ref. 31), we estimate that our experiments are within ±50 mg of the Feshbach resonance, which corresponds to \(a|>95,000a_0\).

**Loss rate at unitarity.** Using the initial loss rate implied by the exponential fit to the data shown in Fig. 1, and using \(\Delta n/\Delta t = -L_j n(r)\delta V^4\), we extract \(L_j = 5.1(1) \times 10^{-21} \text{ cm}^2 \text{s}^{-1}\). Unitarity-limited three-body loss rates for a non-degenerate Bose gas have recently been investigated\(^4\). Using Eqn. 5 from ref. 24 and the Efimov resonance width, \(\eta\), from ref. 11, the predicted \(L_j\) for \(^{87}\)Rb atoms at a temperature of 10 nK is \(3 \times 10^{-21} \text{ cm}^2 \text{s}^{-1}\), which is two and a half orders of magnitude larger than what we measure. On the other hand, after the jump to unitarity, universality suggests that we should use an energy scale that is determined by the interparticle spacing. Replacing \(k_F T\) with \(E_F\), where \(E_F\) is the Boltzmann constant, gives an estimate for \(L_j\) of 1.7 \(\times 10^{-22} \text{ cm}^2 \text{s}^{-1}\), which is within a factor of 4 of our measurement. For the low \((n)\) data, \(L_j\) is a factor of 6.2(5) larger than for the high \((n)\) data. Using \(E_F\) for the two different densities, we would expect this ratio to be 5.2(6).

**Sample volume.** The spherical aspect ratio of the trap was chosen to maximize the time before the cloud radius changes significantly. *A priori*, it is not obvious whether the BEC will expand or collapse after the jump to unitarity. With *in situ* images of the gas at unitarity, we find that the cloud volume remains unchanged to within our measurement precision for \(\sim 500\) ms and then slowly increases. In experiments that require a lower initial coordinate space density, we begin the experimental cycle by imaging the atoms after a 4 ms imaging pulse. The direction of the imaging beam and the magnetic-field constant. As the trap turns off in a time that is much shorter than the trap period, it has a negligible effect on the momenta of the atoms. We image the atoms surrounding conductors. Taking into account roughly equal contributions from the magnetic field constant, \(\eta\), from ref. 11, the predicted \(L_j\) for \(^{87}\)Rb atoms at a temperature of 10 nK is \(3 \times 10^{-21} \text{ cm}^2 \text{s}^{-1}\), which is two and a half orders of magnitude larger than what we measure. On the other hand, after the jump to unitarity, universality suggests that we should use an energy scale that is determined by the interparticle spacing. Replacing \(k_F T\) with \(E_F\), where \(E_F\) is the Boltzmann constant, gives an estimate for \(L_j\) of 1.7 \(\times 10^{-22} \text{ cm}^2 \text{s}^{-1}\), which is within a factor of 4 of our measurement. For the low \((n)\) data, \(L_j\) is a factor of 6.2(5) larger than for the high \((n)\) data. Using \(E_F\) for the two different densities, we would expect this ratio to be 5.2(6).

**Momentum distributions.** For the time-of-flight expansion, the 10 Hz spherical magnetic trap is turned off over 2 ms, while keeping the magnitude of the total magnetic field constant. As the trap turns off in a time that is much shorter than the trap period, it has a negligible effect on the momenta of the atoms. We image the atoms using a 5 \(\mu\)s imaging pulse. The direction of the imaging beam and the magnetic-field direction are shown in Fig. 1b. For each hold time at unitarity, we repeat the experiment four times for each of three different times of flight, \(t_{\text{exp}}\): 25, 13 and 7 ms. Each image is azimuthally averaged, and the curves for the same time of flight are averaged together. We then combine the averaged curves into a single momentum distribution, \(n(k)\), using the largest \(t_{\text{exp}}\) data at the smallest \(k\) and the smallest \(t_{\text{exp}}\) data at the largest \(k\). This initializes the size of effects at small \(k\), while improving the signal-to-noise ratio at larger \(k\). In combining the curves, we enforce agreement in the overlap regions by applying a multiplicative factor to the data for shorter \(t_{\text{exp}}\). This additional scaling factor, which ranges from 1.07 to 1.26 for the \(t_{\text{exp}}\) = 13 ms data and from 1.5 to 2.1 for the \(t_{\text{exp}}\) = 7 ms data, reflects systematic uncertainties that become increasingly important as \(n(k)\) decreases by orders of magnitudes.

At small \(k\), the measured \(n(k)\) is distorted by the initial size of the BEC (the Thomas–Fermi radius is 16 \(\mu\)m for the higher \((n)\) data and 22 \(\mu\)m for the lower \((n)\) data) and by our imaging resolution (characterized by a Gaussian width of approximately 6 \(\mu\)m). The grey regions in Fig. 2, and the corresponding regions where the data are shown as dashed lines in Fig. 3, are bounded by a radius of 58 \(\mu\)m in the expanded cloud image. In the absence of the jump to unitarity, the BEC with \((n)\) \(= (23)(3) \times 10^3\) is 99.7% of the atoms. In this cloud radius after an expansion time of 25 ms. We note that all the effects discussed here cause low-momentum atoms to appear at larger radii than one would expect from the product of velocity and \(t_{\text{exp}}\). Therefore, integrating the signal up to a particular momentum gives a lower limit to the number of atoms that have momenta below that value. We use this fact in extracting a lower bound for the density in phase space.

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**Author contributions**

All the authors contributed to the experimental research described in the paper and to the writing of the manuscript.

**Additional information**

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**Competing financial interests**

The authors declare no competing financial interests.