Neural Network Activation Quantization with Bitwise Information Bottlenecks

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Abstract

Recent researches on information bottleneck shed new light on the continuous attempts to open the black box of neural signal encoding. Inspired by the problem of lossy signal compression for wireless communication, this paper presents a Bitwise Information Bottleneck approach for quantizing and encoding neural network activations. Based on the rate-distortion theory, the Bitwise Information Bottleneck attempts to determine the most significant bits in activation representation by assigning and approximating the sparse coefficient associated with each bit. Given the constraint of a limited average code rate, the information bottleneck minimizes the rate-distortion for optimal activation quantization in a flexible layer-by-layer manner. Experiments over ImageNet and other datasets show that, by minimizing the quantization rate-distortion of each layer, the neural network with information bottlenecks achieves the state-of-the-art accuracy with low-precision activation. Meanwhile, by reducing the code rate, the proposed method can improve the memory and computational efficiency by over six times compared with the deep neural network with standard single-precision representation. Codes will be available on GitHub when the paper is accepted

https://github.com/BitBottleneck/PublicCode

1 Introduction

Research on activation compression for efficient neural computing is an emerging popular topic, which reveals great potential for a variety of edge-computing and computer vision applications [5, 28, 22, 30]. Both stochastic and deterministic approaches were proposed for neural network activation quantization [11, 7, 34, 29], which adopted various quantization functions to reduce the precision of activation representation while training the neural network. Though the loss of activation precision causes notable loss of classification accuracy, yet by reducing the number of bits required, the deep neural networks with quantized activation could improve both memory and time efficiency by orders of magnitude compared with the standard floating-point implementation [7, 14, 8, 24].

One challenge of the researches for activation quantization is the lack of theoretical ground [26]. The performance of existing approaches relies on their choices of quantization strategy, and no minimal loss of representation precision can be determined given the average code rate. Recently, the concept of information bottleneck is attracting attention for its potential to bring a better understanding of the deep learning optimization process [27, 23]. The theory of information bottleneck defines the relevant information in a signal X as being the information that this signal provides about another signal Ÿ. In the context of deep neural network, Ÿ is the signal of quantized activation, and we formalize this problem as that of finding a shortcode for Ÿ that preserves the maximum information about X. That is, we minimize the loss of the information caused by signal quantization through a ‘bottleneck’ in the deep neural network.

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Technically, the proposed Bitwise Information Bottleneck (BIB) approach is based on the rate-distortion theory \[4\]. As a lossy data compression operation, we attempt to determine the most significant bits in the activation that minimize the quantization distortion. To achieve this, the BIB approach directly minimizes the distortion of quantization given the constraint of the maximum code rate of the compressed activations. The BIB optimization is formulated as a sparse convex optimization problem, which estimates the coefficients of the most significant bits in the activation that lead to the minimal squared rate-distortion. Since the nonsignificant bits generally have near-zero coefficients, the activation can be optimally compressed in a bitwise way.

The contributions of this paper are three-fold. First, this paper presents an information-bottleneck-based method for bitwise activation quantization and compression. As far as we know, this is the first attempt to address the challenge of optimal activation quantization by the information-bottleneck-based method. Second, the code rates of different information bottlenecks can be tuned adaptably by a single hyperparameter of the threshold of peak-signal-to-noise-ratio (PSNR) loss, allowing the proposed method to flexibly trade-off between efficiency and accuracy for different applications. Finally, the Bitwise Information Bottleneck minimizes the loss of information caused by activation quantization; therefore, the proposed method suffers almost no loss of classification accuracy while obtaining over six times of memory and computational efficiency improvement.

2 Related Work

Emerging research on neural encoding based on the information bottleneck is an interesting topic which attempts to answer the basic questions about the design principle of deep networks such as the optimal architecture and the optimal quantization scheme. Recently proposed deep neural networks with information bottlenecks usually had an encoder-decoder architecture for feature-wise compression, which treated the neural network as a trade-off between compression and prediction \[27, 9\]. As far as we know, this work is the first attempt for bitwise compression addressing the challenge of minimizing the code rate while optimally preserving activation integrity.

Despite information bottleneck motivated researches, the research on deterministic or stochastic model-based activation quantization is also attracting more attention lately. Lee quantized neural networks according to channel-level distribution \[19\]. Zhao used outlier channel splitting to eliminate the effect of outliers caused by quantization \[31\]. Minimum mean squared error \[16\] and complementary approach \[11\] were both used for reducing quantization error. Meanwhile, the research on data-driven activation quantization was also booming. Qiu presented a Fisher vector method to encode activation with a fixed-point number using a deep generative model \[23\]. Jacob quantized both the activations and weights as 8-bit integers using an iterative approach \[13\]. Li proposed an entropy-based method for interpretable quantization \[21\]. Compared with information-bottleneck-based methods, these approaches were generally based on different assumptions on the quantization function, many researchers used non-derivative quantization operation \[23\] \[13\] \[21\], which might lead to uninterpretable and sub-optimal results.

3 The Bitwise Information Bottleneck Method

Recently, there has been growing interest in applying information theory for deep neural network activation compression. By interpreting the deep neural network as a lossy data compression approach, the black box of the deep neural network could be opened and its performance can be optimized by the tool of rate-distortion theory, which is widely applied in the area of telecommunications \[25\].

3.1 Rate-distortion Theory

Assume \( \mathbf{X}^{(l)}_{i} \in \mathbb{R}^{P \cdot Q \cdot K} \) is the floating-point neural network activation tensor associated with the \( i \)th sample output by \( l \)th layer, where \( P, Q, K \) are respectively the height, width and number of feature maps at the \( l \)th layer. The common quantization function \( Q(\cdot) \) can be written as

\[
\hat{\mathbf{X}}^{(l)}_{i} = Q(\mathbf{X}^{(l)}_{i})
\]
Figure 1: Illustration of the Bitwise Information Bottleneck approach. The BIB operation can be inserted in the convolutional neural network for activation quantization. The idea of the proposed method is to substitute the constant levels of the standard quantization, which are the power of two in this example, for variable coefficients. By exploiting the sparsity of the bitwise coefficients, the information bottleneck could reduce the number of bits required for activation representation.

where \( \hat{X}_i^{(l)} \) is the fixed-point representation of the floating-point activation. The goal of lossy data compression is to achieve minimal rate-distortion given the constraint of the maximum code rate. Assume \( g(\cdot) \) is the function that indicates the number of bits of the given data. According to rate-distortion theory, the typical lossy data compression approach attempts to minimize the distortion function \( d(\cdot) \) given the maximum number of bits \( \eta \) over \( N \) training samples as

\[
\min_{Q(\cdot)} \sum_{i=1}^{N} d(\hat{X}_i^{(l)}, Q(X_i^{(l)})) \quad \text{s.t.} \quad g(\hat{X}_i^{(l)}) \leq \eta
\]  

In practice, the quantization function \( Q(\cdot) \) is nondifferentiable and nonconvex due to its integer output, which makes Eq. 2 difficult to solve. Different from typical rounding-based quantization approaches that settle for suboptimal solutions, this paper attempts to find the optimal solution by reformulating Eq. 2 as a sparse coding problem.

3.2 Activation Quantization via Bitwise Bottleneck Encoding

Fig. 1 illustrates the Bitwise Information Bottleneck (BIB) approach. Formally speaking, assume \( \hat{X}_i^{(l)} \) is the \( D \)-bit fixed-point approximation of \( X_i^{(l)} \) as defined in Eq. 3, where \( \hat{X}_{ij}^{(l)} \in \{0, 1\}^{P \times Q \times K} \) is a three-dimensional binary tensor representing the \( j \)th bit of \( \hat{X}_i^{(l)} \).

\[
\hat{X}_i^{(l)} = Q(X_i^{(l)}) = 2^0 \hat{X}_{i1}^{(l)} + 2^1 \hat{X}_{i2}^{(l)} + ... + 2^{D-1} \hat{X}_{iD}^{(l)}
\]  

where each bitwise data matrix is assigned a constant coefficient of \( 2^0, ..., 2^{D-1} \). In practice, this bitwise activation representation allows the computation of fixed-point data to be implemented in a bitwise way. The computational and memory complexity are proportional to the number of bits of different representations.

Technically, the binary quantization of Eq. 3 inherently assumes that each of the \( D \) bits in the activation representation is needed, although different bits contain different but fixed amounts of information. By removing this assumption, the proposed method substitutes the fixed coefficient for a variable \( \alpha_i^{(l)} \in \mathbb{R}^D \) as

\[
\hat{X}_i^{(l)} = Q(X_i^{(l)}) = \alpha_1^{(l)} \hat{X}_{i1}^{(l)} + \alpha_2^{(l)} \hat{X}_{i2}^{(l)} + ... + \alpha_D^{(l)} \hat{X}_{iD}^{(l)}
\]  

The Bitwise Information Bottleneck approach treats the design of neural networks as a trade-off between compression and prediction, which assumes that the bitwise bottlenecks can exploit the sparsity in the activation representation so that one can reduce the precision of activation representation without hurting classification accuracy. Fig. 2 illustrates the activation distributions of the first 4 layers of ResNet50 [12] over the CIFAR10 dataset. It seems that activations of different layers are
Figure 2: Feature-wise vs. bitwise activation sparsity. (a) The distributions of real-valued (upper) and quantized (lower) activations of the first four layers of ResNet50 over CIFAR10. (b) The average rate of code one of each bit of the quantized activation (upper), and the estimated coefficient $\alpha^{(l)}$ (lower). It seems that the optimal code rates of different layers depend on the level of activation sparsity, and the Bitwise Information Bottleneck can adaptively remove the near-zero high-end bits and the less-informative low-end bits of the activation representation.

sparse, but the sparsity varies from layer to layer. Fig. 2(b) illustrates the average rate of code one in each bit of the activation representation (upper graph). Despite the change of feature-wise sparsity as shown in Fig. 2(a), the bitwise sparsity over the high-end bits of activation representation is easier to detect. Therefore, one might be able to estimate the optimal and sparse coefficients $\alpha^{(l)}$ associated with the most significant bits.

According to the rate-distortion theory (Eq. 2), the Bitwise Information Bottleneck attempts to find the optimal quantization scheme by minimizing the standard squared distortion rate over $N$ training samples given that less than $\eta$ bitwise coefficients are nonzero,

$$\arg\min_{\alpha^{(l)}} \sum_{i=1}^{N} \|X^{(l)}_i - \sum_{j=1}^{D} \alpha^{(l)}_j \hat{X}^{(l)}_{ij}\|^2_2 \quad \text{s.t. } \|\alpha^{(l)}\|_0 \leq \eta$$

(5)

where $\hat{X}^{(l)}_{ij}$ calculated by initial quantization operation are usually known as the quantization codebook. In practice, different initial quantization operations can be applied, and Eq. 5 is a simple example of the rounding quantization approach. It is worth noting that since the number of nonzero coefficients in $\alpha^{(l)}$ equals the number of bits in the fixed-point representation, the constraint function of Eq. 5 actually limits the maximum number of bits in the quantized representation as required by the rate-distortion theory. Recent research shows that Eq. 5 is equivalent to the following L1-norm-based problem, which leads to a sparse solution [2].

$$\arg\min_{\alpha^{(l)}} \sum_{i=1}^{N} \|X^{(l)}_i - \sum_{j=1}^{D} \alpha^{(l)}_j \hat{X}^{(l)}_{ij}\|^2_2 + \lambda \|\alpha^{(l)}\|_1 \quad \text{s.t. } \|\alpha^{(l)}\|_1 \leq \eta$$

(6)

The BIB operation solves Eq. 6 to determine the sparse significant bits that lead to the minimal rate distortion. In practice, one usually calculates Eq. 6 by solving its dual form as

$$\arg\min_{\alpha^{(l)}} \sum_{i=1}^{N} \|X^{(l)}_i - \sum_{j=1}^{D} \alpha^{(l)}_j \hat{X}^{(l)}_{ij}\|^2_2 + \lambda \|\alpha^{(l)}\|_1$$

where $\lambda$ is the hyperparameter for controlling the trade-off between the optimized error rate and the code rate. Eq. 6 generally leads to a sparse solution of the coefficients $\alpha^{(l)}$, so the activation bits associated with zero coefficients are removed during the inference stage, and the computational efficiency can be improved significantly.

3.3 Neural Network with Bitwise Information Bottlenecks

As shown in Eq. 6, the BIB operation calculates the optimal coefficient associated with each bit of the compressed activation representation so that a minimal distortion rate can be achieved given the maximum code rate. This section shows how the Bitwise Information Bottleneck works in the deep neural network.
Figure 3: Deep neural network with Bitwise Information Bottleneck layers. The second macroblock is extended and shown in detail. By setting a single hyperparameter of the threshold of PSNR loss, the Bitwise Information Bottlenecks in different macroblocks can be trained to quantize the normalized activations flexibly with different optimal code rates.

Fig. 3 shows an example of efficient neural computing with Bitwise Information Bottlenecks. The whole network is built based on the classic ResNet, although the BIB operation can be easily integrated into different networks. In practice, the BIB operation can be inserted in the macroblock of the deep neural network. A typical macroblock contains a BIB layer, a convolution layer, a pooling layer (optional), a batch-norm layer and an activation layer. Thanks to the BIB layer, which transforms the normalized floating-point activation to compressed fixed-point activation, the convolution layer can substitute the computationally expensive floating-point multiplications for efficient fixed-point bitwise multiplications.

The benefits of inserting the Bitwise Information Bottlenecks in deep neural networks are twofold. From the perspective of memory efficiency, compared with the standard deep neural networks using 32-bit single-precision activation representation, the Bitwise Information Bottlenecks can compress the activation into arbitrary low-precision (1 to 8 bit), obtaining an improvement of memory efficiency by 4 to 32 times. From the perspective of computational efficiency, single-precision floating-point multiplication generally requires considerably more hardware resources and calculation time compared with fixed-point multiplication. By employing the BIB layers before the convolution layers (as shown in Fig. 1), the inference latency of deep neural networks may be reduced by over 90%, as indicated in early research [35].

3.4 Training Sparse Bitwise Information Bottlenecks

In practice, the sparsity level of the activation may vary from layer to layer in the neural network. It is desired that different macroblocks should use different quantization precisions. As one of the advantages of the proposed method, the optimal code rate of different Bitwise Information Bottlenecks can be approximated by tuning a single hyperparameter, which is the threshold of peak-signal-to-noise-ratio (PSNR) loss $T$. The measurement of the PSNR is defined as $PSNR = 10 \cdot \log_{10} \left( \frac{(2^D-1)^2}{\text{MSE}} \right)$, where $\text{MSE} = \frac{1}{N \cdot P \cdot Q \cdot K} \sum_{i=1}^{N} \left\| \mathbf{X}(l,i) - \sum_{j=1}^{D} a(j)(l) \mathbf{X}(l,j) \right\|_2^2$. By increasing the hyperparameter of $\lambda$ and comparing the PSNR loss with the threshold, each layer can approximate the respective minimal code rate acceptable independently. More detailed information about the training algorithm can be found in Algorithm [1].

An illustrative example of training the neural network with multiple Bitwise Information Bottlenecks is shown in Fig. 2. A closer look at the image reveals three interesting observations. First, the statistical phenomenon of bitwise sparsity exists. Specifically, the high-end bits of the activation representation are statistically near-zero for different layers of the neural network, which validates our assumption. Second, the optimal code rates of different layers depend on the level of activation sparsity. As an example, the activations of the first macroblock of ResNet50 are less sparse than the next three macroblocks, resulting in a higher minimal code rate acceptable. Third, it seems that the Bitwise Information Bottleneck can adaptively reduce the coefficients $a(j)(l)$ of both near-zero high-end bits and less-informative low-end bits of the activation representation.
Algorithm 1: Training Algorithm

Input: Pre-trained floating-point CNN model $\Theta$, threshold of PSNR loss $T$, number of layers $L$, number of training samples $N$, number of bits for initial quantization $D$;

Output: An Optimized Quantized model $\hat{\Theta}$;

1. Obtain the floating-point activation tensors $X^{(1)}_i, ..., X^{(L)}_i$ of $\Theta$ for each sample;
2. for $l = 1, ..., L$ do
3.     while $t^{(l)} < T$ do
4.         for $i = 1, ..., N$ do
5.             Obtain the codebook $\hat{X}^{(l)}_{i1}, \hat{X}^{(l)}_{i2}, ..., \hat{X}^{(l)}_{iD}$ by initial quantization of $X^{(l)}_i$;
6.             Obtain the coefficient vector $\alpha^{(l)}_i$ via Eq. 6 and restore $\hat{X}^{(l)}_i$ via Eq. 4;
7.             Compute the PSNR loss between $\hat{X}^{(l)}_i$ and $X^{(l)}_i$ as $t^{(l)}_i$;
8.         $t^{(l)} = \max(t^{(l)}_i)$ for each $i = 1, ..., N$;
9.         Increase $\lambda$;
10. Insert the Bitwise Information Bottleneck layers into $\Theta$ as a new model $\hat{\Theta}$;
11. Refine and quantize the weight parameters of $\hat{\Theta}$ by backpropagation until reaching convergence;

4 Experiments

4.1 Experimental Setting

Our experiments were performed on three standard benchmarks: the MNIST\cite{18}, CIFAR10\cite{17} and ImageNet (ILSVRC2012)\cite{10} datasets. The MNIST dataset of $28 \times 28$ monochrome images contained 60 thousand training samples and 10 thousand test samples. The CIFAR10 dataset of $32 \times 32$ color images contained 50 thousand training samples and 10 thousand test samples, which had 10 classes. The ImageNet dataset contained 1.28 million training samples, 50 thousand verification samples, 100 thousand test samples and 1,000 classes.

The proposed BIB approach required an initial quantization operation to calculate the binary activation codebook at the first step. Two different initial quantization methods were adopted, including the iterative rounding method \cite{6} and the DoReFa method \cite{34}. To validate our method, we used the standard average classification accuracy over the test dataset as our evaluation criterion. The results of the proposed method were achieved based on the ResNet50, and all the compared approaches were also based on ResNet50 for a fair comparison.

Similar to other studies, a backpropagation-based training process was adopted in the proposed method to refine and quantize the weight parameters of the deep neural network. In our experiment, the ADAM\cite{15} algorithm was applied to implement the backpropagation process, where the learning rate was 0.0001, and the batch size was 64. We trained 4,000 iterations on the MNIST and CIFAR10 datasets and 5 epochs on the ImageNet dataset.

4.2 Visualization and Analysis

A group of experiments was performed over the MNIST dataset to analyze and visualize the performance of the proposed method compared with the initial and other quantization approaches. The Bitwise Information Bottleneck reduced the code rate of the fixed-point representation from the initial quantization length of $D$ bits to the effective code rate $d$, which was the number of nonzero coefficients in vector $\alpha^{(l)}_i$ (Eq. 6). Fig. 4(a) shows the relation between the effective code rate $d$ and the average PSNR achieved over the MNIST dataset. Overall, the proposed BIB approach achieved 1.98 dB to 12.44 dB higher PSNR compared with the classic rounding-based quantization and sensitivity-based quantization methods \cite{20}. The superiority of PSNR was consistent when different numbers of nonzero coefficients were chosen, which can be controlled by the hyperparameter of $\lambda$, as shown in the figure.

Similar to other activation quantization approaches, a backpropagation retraining process was applied after the activation quantization process to refine the weight parameters in the proposed method. Fig.
Figure 4: (a) The relation between the PSNR and the effective code rate, which equals the number of nonzero coefficients in $\alpha^{(l)}$ and the relation between the effective code rate and the value of $\lambda$. (b) Accuracy of the deep neural network with Bitwise Information Bottlenecks over the MNIST (upper) and CIFAR10 (lower) datasets, which can compress the activation of the deep neural network with different effective code rates $d$. (c) The change in accuracy and average effective code rate of different layers when using the Bitwise Information Bottleneck to compress the 8-bit fixed-point activation according to different PSNR loss thresholds. (d) The ratio of $\alpha^{(l)}_j/2^j$ with different threshold of PSNR loss ($T = 26$ upper, $T = 30$ lower).

4.3 Compared with the State-of-the-Art

We evaluated the proposed method with the state-of-the-art activation quantization approaches over the ImageNet dataset. The original floating-point model and the proposed BIB approach were compared with the ACIQ [1], Focused compression [32], Integer-only [13], UNIQ [3], INQ [33], DoReFa-Net [34] and Iterative rounding [6] approaches. A threshold of PSNR loss of 16 dB and 8 dB was adopted to quantize the activation of the deep neural network, which resulted in an average...
| Method                                      | Year | Weights | Activations | Top-1 Acc. | Top-5 Acc. |
|---------------------------------------------|------|---------|-------------|------------|------------|
| Floating-point Model                        | 2016 | 32      | 32          | 75.6%      | 92.8%      |
| Bitwise Information Bottleneck              | –    | 8       | 5           | 75.1%      | 92.5%      |
| Focused compression                         | 2019 | 5       | 32          | 74.9%      | 92.6%      |
| Integer-only                                | 2018 | 8       | 8           | 74.9%      | –          |
| INQ                                         | 2017 | 5       | 32          | 74.8%      | 92.5%      |
| DoReFa-Net (initial quantization)           | 2016 | 8       | 5           | 73.8%      | 91.7%      |
| Iterative rounding                          | 2014 | 8       | 5           | 72.1%      | 90.4%      |
| Bitwise Information Bottleneck              | 2014 | 4       | 4           | 73.7%      | 91.7%      |
| UNIQ                                        | 2018 | 4       | 8           | 73.4%      | –          |
| ACIQ                                        | 2019 | 8       | 4           | 71.8%      | 89.3%      |
| DoReFa-Net (initial quantization)           | 2016 | 8       | 4           | 70.1%      | 89.3%      |
| Iterative rounding                          | 2014 | 8       | 4           | 70.0%      | 89.4%      |

Table 2: The memory and computational efficiency improvement achieved by the BIB operations with different effective code rates. In the table, M indicates megabytes, B indicates billion.

| Number of Bits | 1 bit | 2 bit | 3 bit | 4 bit | 5 bit | 6 bit | 7 bit | 8 bit | 32 bit |
|----------------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| Operation      | 8.9B  | 17.8B | 26.7B | 35.6B | 44.5B | 53.4B | 62.3B | 71.3B | 285.0B |
| Memory         | 1.1M  | 2.1M  | 3.2M  | 4.3M  | 5.3M  | 6.4M  | 7.4M  | 8.5M  | 34.0M  |
| Improvement    | 30.9× | 16.2× | 10.6× | 7.9×  | 6.4×  | 5.3×  | 4.6×  | 4.0×  | 1.0×   |

effective code rate of 4.0(±1) and 5.0(±1) bits. The quantization method of DoReFa-Net[34] was adopted for initial quantization. The standard TensorFlow quantization tool was used to quantize the weight to 8 bits.

Table 1 summarizes the results of classification accuracies over the ImageNet dataset. In summary, the proposed method achieved the highest classification accuracy when five bits were adopted for activation representation. The BIB approach with 5-bit quantization suffered almost no loss of accuracy (less than 1%) compared with the floating-point model. The BIB approach with 5-bit activation representation achieved similar accuracy with the Integer-only approach with 8-bit activation and achieved similar results with the INQ and Focused compression approach with 32-bit floating-point activation. When using 4-bit fixed-point representation, the performance of the BIB approach was still competitive with the state-of-the-art.

4.4 Efficiency Improvement

The BIB approach attempts to reduce the complexity of the inference stage for embedded computer vision applications. As shown in Table 2 compared with the 32-bit activation representation, the Bitwise Information Bottleneck could improve the computational efficiency by over 6.4 times without hurting the performance. From the perspective of reducing memory occupation, the running memory can be reduced by up to 84%.

5 Conclusion and Discussion

This paper presents a Bitwise Information Bottleneck approach for neural network activation quantization. Based on the rate-distortion theory, the Bitwise Information Bottlenecks inserted in the deep neural network attempt to minimize the quantization error rate and code rate. Experiments over different datasets show that the proposed method could reduce the activation of the deep neural network to one to five bits without hurting the performance, which could improve the memory efficiency as well as the computational efficiency of the neural-network inference by over 6 times.

The proposed Bitwise Information Bottleneck method assumes that the activation of deep neural networks has a certain level of sparsity; however, in practice, the activation of different layers has a different level of sparsity. Although applying flexible quantization as adopted by different Bitwise Information Bottlenecks could reduce the quantization PSNR loss, the proposed method is subject to the constraint of activation sparsity. In the future, we plan to incorporate sparse regularization to intentionally introduce sparsity in the activation, so that the Bitwise Information Bottlenecks could achieve higher activation integrity with a lower code rate.
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