The moment of inertia of V-shaped internal combustion engines

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Abstract. This work contains a solution to the problem of developing a mathematical model and the results of numerical simulation of the change in the inertia of the moving mass V-shaped piston internal combustion engines (ICE) reduced to the axis of rotation of the crankshaft depending on the angle of rotation of the crankshaft during one revolution. The solution of this problem is an important step in the mathematical description of the dependence of the torque developed by the V-shaped piston ICE, depending on the angle of rotation of the crankshaft. Knowing the dependence of the torque developed by the V-shaped piston ICE, depending on the angle of rotation of the crankshaft, is a prerequisite for designing and optimizing the design of non-linearly loaded processing equipment using V-shaped piston ICEs as the drive. The results of mathematical modeling show that the moment of inertia of the moving masses of the V-shaped piston ICE reduced to the axis of rotation of the crankshaft is constantly changing during one revolution of the crankshaft. The greatest change in the moment of inertia of the moving masses reduced to the axis of rotation of the crankshaft takes place in 2- and 4-cylinder V-shaped piston internal combustion engines.

1. Introduction

As an analysis of open sources of information shows, currently work in the field of modeling V-shaped piston ICEs does not stand out as a separate unit, but is carried out as a whole for piston ICEs in the field of:

- modeling of fuel combustion processes [1-3];
- simulation of gas-dynamic processes both inside the cylinder and in the intake and exhaust systems [4-14];
- modeling of heat transfer processes inside the cylinder [15-18];
- modeling of friction processes in the details of a cylinder-piston group [19];
- modeling in order to optimize the design [20-23];
- simulation of the impact interaction of the piston [24];
- simulation of engine cycles [25];
- simulation of an engine control system [26].
To solve the problems of optimal aggregation of non-linearly loaded technological equipment using V-shaped piston ICEs as a drive, it is necessary to know the law of change of the moment of inertia of the moving masses of the engine brought to the axis of rotation of the crankshaft. Knowledge of this law is also required to solve the problems of developing methods and means of dynamic control of the mechanical parameters of rotary motors and drives based on them [27-28].

2. Materials and methods
The most common to date schemes of crank mechanisms (CM) are V-shaped axial (two, four, six, eight and twelve cylinder).

According to [29], the accuracy of determining power and torque should be at least 99%. Therefore, it is important to evaluate the nature of the change in the moment of inertia of the crankshaft during one revolution of the crankshaft in order to achieve the required accuracy indicator.

To do this, we will carry out a qualitative analysis of the nature of the change in the moment of inertia of the V-shaped axial CShM.

Figure 1 shows a diagram of an axial crankshaft V-shaped ICE with one pair of cylinders. In figure 1, the following designations are adopted: \( r \) - radius of the crank; \( l \) - is the length of the connecting rod; \( l_0 \) - is the distance from the axis of the connecting rod journal to the center of gravity of the connecting rod; \( b_{10} \) - is the distance from the center of gravity of the left connecting rod to the axis of rotation of the crankshaft; \( b_{11} \) - is the distance from the center of gravity of the left piston pin to the axis of rotation of the crankshaft; \( b_{20} \) - is the distance from the center of gravity of the right connecting rod to the axis of rotation of the crankshaft; \( b_{21} \) - is the distance from the center of gravity of the right piston pin to the axis of rotation of the crankshaft; \( c_0 \) - is the distance from the center of gravity of the piston pin to the center of gravity of the piston; \( \alpha \) - is the angle of deviation of the axis of the left connecting rod from the axis of the cylinder; \( \alpha_1 \) - is the angle of deviation of the axis of the right connecting rod from the axis of the cylinder; \( \beta \) - is the angle between the crank and the left connecting rod; \( \beta_1 \) - is the angle between the right crank and the left connecting rod; \( \varphi \) - is the angle of rotation of the crank, counted in the direction of rotation of the crank from its position at which the piston is at top dead center; \( \gamma \) - is the angle between the axes of the left and right cylinders.

![Figure 1](image.png)

**Figure 1.** Scheme of axial CShM V-shaped ICE of a V-shaped internal combustion engine with one pair of cylinders.

The angle \( \alpha \) is found from the relation \( r \sin \varphi = l \sin \alpha \)
\[ \alpha = \arcsin \left( \frac{r}{l} \sin \varphi \right). \]  

(1)

Angle

\[ \beta = 180 - \alpha - \varphi. \]  

(2)

The distances \( b_{10} \) and \( b_{11} \) are determined by the cosine theorem:

\[ b_{10} = \sqrt{r^2 + l_0^2 - 2rl_0 \cos \beta}; \]  

(3)

\[ b_{11} = \sqrt{r^2 + l^2 - 2rl \cos \beta}. \]  

(4)

The angle \( \alpha_i \) is found from the relation \( r \sin(\gamma - \varphi) = l \sin \alpha_i \)

\[ \alpha_i = \arcsin \left( \frac{r}{l} \sin(\gamma - \varphi) \right). \]  

(5)

Angle

\[ \beta_i = 180 - \alpha_i - (\gamma - \varphi). \]  

(6)

The distances \( b_{20} \) and \( b_{21} \) are determined by the cosine theorem:

\[ b_{20} = \sqrt{r^2 + l_0^2 - 2rl_0 \cos \beta_1}; \]  

(7)

\[ b_{21} = \sqrt{r^2 + l^2 - 2rl \cos \beta_1}. \]  

(8)

The moment of inertia of the left piston of mass \( m_p \) relative to the axis of rotation of the crankshaft is

\[ J_{lp} = m_p \left( b_{11} + c_0 \right)^2 + J_{lp0}, \]  

(9)

where \( J_{lp0} \) - is the moment of inertia of the left piston relative to its central axis.

The moment of inertia of the left piston pin \( m_{pp} \) relative to the axis of rotation of the crankshaft has the expression

\[ J_{lpp} = m_{pp} b_{11}^2 + J_{lpp0}, \]  

(10)

where \( J_{lpp0} \) - is the moment of inertia of the left piston pin relative to its central axis.

For the moment of inertia of the left connecting rod \( m_{cr} \) relative to the axis of rotation of the crankshaft, we find

\[ J_{lcr} = m_{cr} b_{10}^2 + J_{lcr0}, \]  

(11)

where \( J_{lcr0} \) - is the moment of inertia of the left connecting rod with respect to its central axis.

The moment of inertia of the right piston of mass \( m_p \) relative to the axis of rotation of the crankshaft will be
\[ J_{pp} = m_p (b_{21} + c_0)^2 + J_{pp0}, \]  

(12)

where \( J_{pp0} \) - is the moment of inertia of the right piston relative to its central axis.

The moment of inertia of the right piston pin \( m_{pp} \) relative to the axis of rotation of the crankshaft will be

\[ J_{ppp} = m_{pp} b_{21}^2 + J_{pp0}, \]  

(13)

where \( J_{ppp} \) - is the moment of inertia of the right piston pin relative to its central axis.

The moment of inertia of the right connecting rod \( m_{cr} \) relative to the axis of rotation of the crankshaft is expressed as

\[ J_{cr} = m_{cr} b_{20}^2 + J_{cr0}, \]  

(14)

where \( J_{cr} \) - is the moment of inertia of the right connecting rod with respect to its central axis.

Assuming the moment of inertia of the crankshaft \( J_c \) to be constant and independent of the position of the crankshaft, we determine the moment of inertia of the axial crankshaft in-line piston ICE:

\[ J_{CShM} = J_{lp} + J_{pp} + J_{lcr} + J_{pp} + J_{ppp} + J_{cr} + J_{c}. \]  

(15)

To determine the dependence \( J_{CShM} \) of four, six, eight, ten and twelve cylinder V-shaped ICEs on the angle of rotation of the crank \( \varphi \), we perform mathematical modeling at \( r/l = 0.3 \), \( m_p / m_{cr} = 0.5 \) [30]. The simulation results are summarized in table 1.

**Table 1.** The simulation results.

| №   | Number of cylinders | The angle between the rows of cylinders, ° | The angle between the cranks, ° | The dependence of \( J_{CShM} \) on the angle of rotation of the crank \( \varphi \) |
|-----|---------------------|------------------------------------------|--------------------------------|-----------------------------------------------|
| 1   | Two                 | 90                                       | 180                            | ![Graph](image.png) Angle of rotation of the crankshaft, degrees |
| 2   | Four                | 90                                       | 180                            | ![Graph](image.png) Angle of rotation of the crankshaft, degrees |
6

90

120

3 Six 120

120

90

90

4 Eight 180

120 90
The nature of the change in the crankshaft moment of inertia of the crankshaft reduced to the axis of rotation of the crankshaft during one revolution of the crankshaft does not exceed 2% for a 6-cylinder internal combustion engine (angle between cylinder axes - 90°, angle between cranks - 120°) and 1% for all other cases.

The use of flywheels on V-shaped ICEs is connected, first of all, with the need to accumulate kinetic energy to ensure the beginning of the operation of technological equipment, and only then - to ensure uniform operation of the ICE itself.

This distinguishes V-engines, for example, from in-line piston ICEs, in which the crankshaft inertia moment for the crankshaft rotation axis during one revolution of the crankshaft can vary significantly.

3. Conclusion
In the programming language Borland Delphi 7.0, an autonomous program has been developed for calculating the crank shaft inertia moment of the crankshaft V-shaped internal combustion engine. The program allows you to determine the moment of inertia of the CABG V-shaped piston machines with the number of cylinders from 4 to 12.

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