Gravi-Leptogenesis:
Leptogenesis from Gravity Waves in Pseudo-scalar Driven Inflation Models

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In this talk we present a mechanism for leptogenesis which is based on gravity waves produced during inflation. We show that when inflation is driven by a pseudo-scalar field the metric perturbations generated during inflation can become birefringent, therefore giving a non-vanishing contribution to the gravitational triangle anomaly and sourcing lepton anti-lepton asymmetry. As this asymmetry is sourced by the fields which are active during inflation, it is not washed out or diluted by inflation. The amount of matter asymmetry generated in our model can be of realistic size for the parameters within the range of some inflationary scenarios and grand unified theories. This talk is based on [1] which has appeared on the arXiv as hep-th/0403069.

I. INTRODUCTION

One of the puzzles of astroparticle physics and cosmology, which has been around for more than half a century and since the existence of antimatter and anti-particles established in collider experiments, has been to explain if our universe is mostly made out of matter, why and how this has happened during the course of the evolution of the universe starting from a symmetric soup of matter and antimatter soon after the Big Bang.

The fact that in the parts of the universe visible to us there is an excess of matter over antimatter has now been backed by the recent determinations of the cosmological parameters from the cosmic microwave background observations and the WMAP experiment. Quantitatively this asymmetry is usually given through the ratio of excess of baryon density to the photon density [2]

\[
\frac{n_B}{n_\gamma} = (6.5 \pm 0.4) \times 10^{-10},
\]

where \(n_B = n_b - n_{\bar{b}}\) and \(n_\gamma\) is the number density of photons. This ratio has the nice property that is time independent, as the evolution of the \(n_b\) and \(n_\gamma\) with the cosmic Hubble expansion are the same. This is a small number, but at the same time it is large enough to be a puzzle for models of particle physics.

About forty years ago Sakharov stated the three necessary conditions to generate a matter-antimatter asymmetry dynamically from a symmetric initial conditions [3]:

i) our particle physics model should have baryon number violating vertices.

ii) CP should be violated.

iii) CP and baryon number violating interactions should be active at a time when the universe is out of thermal equilibrium.

A baryon excess this large cannot be produced in the early universe within the Standard Model (SM) of particle physics [4]. This is due to the fact that baryon number violating interactions in the Standard Model are loop suppressed and the only source of CP violation in the hadronic sector is in the Dirac phase of the CKM mixing matrix, which is not enough for explaining the baryon asymmetry observed today. Moreover, assuming that the scale of inflation is larger than TeV, i.e. the SM is at work after inflation is ended, the out-of-equilibrium condition can be created at phase transitions or through late decay of massive particles. The most attractive choice for a phase transition is that associated with electroweak symmetry breaking. However, that phase transition is probably not sufficiently strongly first-order.

Although what is observed is a baryon asymmetry, since the 1980’s it has been realized that the standard weak interactions contain processes, mediated by sphalerons (SU(2) instantons), which interconvert baryons and leptons and are thermally activated at temperatures greater than 1 TeV. Thus, we can also create the baryon asymmetry by creating net lepton number at high temperature through out-of-equilibrium and CP-asymmetric processes [2-4]. Scenarios of this type are known as leptoogenesis.

To use the possibility of lepton asymmetry and sphalerons we, however, need to fulfill Sakharov conditions for leptons. Within the usual SM this does not solve the issue and we are hence forced to associate the observed baryon asymmetry of the universe to physics beyond the SM.
Most of the allowed parameter space of the minimal supersymmetric Standard Model has already been excluded and large CP violating phases are strongly constrained in supersymmetric models even though they could still appear in the neutrino Yukawa couplings that are used in the Fukugita-Yanagida scenario for leptogenesis. Models that can explain the baryon excess typically involve exotic nonstandard physics, CP violating couplings in the Higgs or supersymmetry sectors or in the couplings of the heavy neutral leptons associated with neutrino mass. In any event, there is good reason to seek more effective sources of CP-violating out-of-equilibrium physics.

Here we present a new mechanism for the creation of the matter-antimatter asymmetry, one associated with gravitational fluctuations created during cosmological inflation.

II. OUTLINE OF THE MECHANISM

Let us first spell out how the three Sakharov conditions are realized in our model of matter-antimatter asymmetry, gravi-leptogenesis.

A. Lepton number violation

The lepton number violation in our model comes from triangle anomaly. As it is well-known, the lepton number current, and hence the total fermion number current, has a gravitational anomaly in the Standard Model. Explicitly,

\[ \partial_\mu J_\mu^L = \frac{N}{16\pi^2} R \tilde{R} \]

where

\[ J_\mu^L = \sum_{i=1}^3 \bar{\ell}_i \gamma^\mu \ell_i + \bar{\nu}_i \gamma^\mu \nu_i \]

and \( N = N_L - N_R \), which is three in the Standard Model. The anomaly is a consequence of an imbalance between left- and right-handed leptons. In general when heavy right-handed neutrinos are also added to the Standard Model, as is done in the seesaw mechanism for explaining the smallness of the neutrino mass, will be correct in an effective theory valid below the right-handed neutrino mass scale \( N = 3 \). At higher energies, \( N \) could be anywhere between zero to three, depending on the details of the particle physics model invoked. In the usual seesaw scenarios with three right handed neutrinos, \( \mu \) can be as large as \( 10^{14} \), for energies below \( \mu \n = 3 \), and above that \( N = 0 \). In any case, here we do not restrict ourselves to a specific particle physics model, and keep an open mind on larger values of \( \mu \).

B. CP violation

The need for CP violation manifests itself in our model through the fact that a non-zero lepton number generation can be achieved when \( \langle R \tilde{R} \rangle \) is non-vanishing. As we will show explicitly, \( R \tilde{R} \) receives a contribution with a definite sign from gravitational fluctuations produced during inflation, which is driven by a pseudo-scalar field. In other words, CP-violation in our model arises from the inflaton field with a CP-odd component. The simplest model of this kind is when we have a single field inflation and a pseudo-scalar \( \phi \) as the inflaton, known as natural inflation, however it can be incorporated to have multiple axions such as in N-inflation models. In fact, these models of inflation fit quite nicely into extensions of the standard model and in string inspired inflation. Note that during inflation the expectation value of the inflaton is non-vanishing and is rolling in time.

Such models of inflation can be naturally achieved if the inflaton is a complex modulus field such as one finds in supergravity or superstring models. In order to use these models, however, we need to make sure that they have flat enough potentials required for (slow-roll) inflation.

The imaginary part \( \phi \) of this field (which we henceforth call an ‘axion’) can couple to gravity through an interaction

\[ \Delta \mathcal{L} = F(\phi) R \tilde{R} , \]

where \( F \) is odd in \( \phi \). Under \( P \) and \( CP \)

\[ \phi \rightarrow -\phi \ & \ F(\phi) \rightarrow -F(\phi). \]

Terms like would generically appear once we integrate out heavy fermions axially coupled to \( \phi \) or as a result of the Green-Schwarz mechanism. In the Appendix we will explicitly show how a linear \( F \) of the form

\[ F(\phi) = \frac{N}{(16\pi^2 M_{Pl})} \phi , \]

with \( N \) depending on the details of string compactification, arises from heterotic string theory compactified to four dimensions.

C. Out-of-equilibrium

Out of equilibrium in our model is achieved noting that we apply the interaction in to the dynamics of
metric fluctuations during inflation where due to (exponential) growth of the background space-time, the lepton number production is naturally out of equilibrium.

We are now ready to compute the amount of the lepton number generated during inflation. In our analysis, we assume that we have a given successful model of inflation which involves an axion field and do not elaborate on the details of the inflationary model. The axion driven models of inflation are known as natural inflation and are preferred from a particle physics perspective. Natural inflation gives a small self-coupling for the inflaton field without fine tuning [10]. Throughout this note we use notations and conventions of [13] and in particular, we use the reduced Planck mass $M_{Pl} = 2.44 \times 10^{16}$ GeV.

### III. GRAVITY WAVE EVOLUTION

To begin, we must compute the production of gravitational waves during inflation under the influence of (4). The action which describes the gravitational waves during inflation and is invariant to use a helicity basis is

$$\mathcal{L} = \frac{1}{2}M_{Pl}^2 \sqrt{-\det g} R + F(\phi) R \tilde{R}.$$  \hspace{1cm} (6)

In general metric perturbations about an FRW universe can be parameterized as

$$ds^2 = -(1 + 2\varphi)dt^2 + w_i dt dx^i + a^2(t) \left[(1 + 2\psi)\delta_{ij} + h_{ij}\right] dx^i dx^j$$  \hspace{1cm} (7)

where $\varphi$, $\psi$, $w_i$ and $h_{ij}$ respectively parameterize the scalar, vector, and tensor fluctuations of the metric. It is straightforward to show that the scalar and vector perturbations do not contribute to $R \tilde{R}$, and so we ignore these fluctuations in the following discussion. We can also fix a gauge so that the tensor fluctuations are parameterized by the two physical transverse traceless elements of $h_{ij}$. For such physical gravity waves which are moving in the $z$ direction, the metric takes the form

$$ds^2 = -dt^2 + a^2(t) \left[(1 + h_+)dx^2 + (1 + h_+)dy^2 + 2h_x dx dy + dz^2\right]$$  \hspace{1cm} (8)

where $a(t) = e^{Ht}$ during inflation and $h_+ , h_x$ are functions of $t, z$.

To see the CP violation more explicitly, it is convenient to use a helicity basis

$$h_L = \frac{1}{\sqrt{2}}(h_+ - i h_x) , \quad h_R = \frac{1}{\sqrt{2}}(h_+ + i h_x).$$  \hspace{1cm} (9)

Here $h_L$ and $h_R$ are complex conjugate scalar fields. To be very explicit, the negative frequency part of $h_L$ is the conjugate of the positive frequency part of $h_R$, and both are built from wavefunctions for left-handed gravitons.

#### A. The equations of motion

Plugging (8) into the action (6), up to second order in $h_L$ and $h_R$, we obtain

$$\mathcal{L} = -(h_L \Box h_R + h_R \Box h_L)$$

$$+ 16i F(\phi) \left[\left(\frac{\partial^2}{\partial z^2} h_R \frac{\partial^2}{\partial t \partial z} h_L - \frac{\partial^2}{\partial z^2} h_L \frac{\partial^2}{\partial t \partial z} h_R\right)\right.$$

$$+ a^2 \left(\frac{\partial^2}{\partial t^2} h_R \frac{\partial^2}{\partial t \partial z} h_L - \frac{\partial}{\partial t} h_L \frac{\partial^2}{\partial t \partial z} h_R\right) + H a^2 \left(\frac{\partial}{\partial t} h_R \frac{\partial^2}{\partial t \partial z} h_L - \frac{\partial}{\partial t} h_L \frac{\partial^2}{\partial t \partial z} h_R\right)\right] + O(h^4)$$  \hspace{1cm} (10)

where

$$\Box = \frac{\partial^2}{\partial t^2} + 3H \frac{\partial}{\partial t} - \frac{1}{a^2} \frac{\partial^2}{\partial z^2}.$$  

As it is explicitly seen from (10), if $h_L$ and $h_R$ have the same dispersion relation, $R \tilde{R}$ vanishes. Thus, nonzero $R \tilde{R} \Box$ requires “cosmological birefringence” during inflation. Such an effect is induced by the addition of (4) to the gravitational equations. Lue, Wang, and Kamionkowski (LWK) [18] and Alexander and Martin [19] have studied the effects of such an interaction in generating observable parity-violation in the cosmic microwave background. In a future work, this coupling will be related to an observable to detect birefringence in binary systems for the LISA and Advanced LIGO gravitational wave detectors [17].

It is straightforward to obtain equations of motion for $h_L$ and $h_R$:

$$\Box h_L = -2i\frac{\Theta}{a} h_L', \quad \Box h_R = +2i\frac{\Theta}{a} h_R'$$  \hspace{1cm} (11)

where

$$\Theta = \frac{4}{a^2} \frac{d}{dt} \left(F a^2 / M_{Pl}^2\right)$$

$$\simeq 4(F'' \dot{\phi}^2 + 2F' \dot{\phi} \ddot{\phi}) / M_{Pl}^2$$  \hspace{1cm} (12)

dots denote time derivatives, and primes denote differentiation of $F$ with respect to $\phi$. To obtain the above equations we have used the fact that the inflaton field is only a function of time $t$. In the second line of the expression for $\Theta$, assuming the slow-roll inflation, we have dropped the terms proportional to $\dot{\phi}$ (explicitly we have dropped $4F' \dot{\phi} / M_{Pl}^2$).

Note that with a constant $\phi$, (11) is the Gauss-Bonnet term and being a total divergence (in four dimensions), cannot affect the equations of motion; thus, all terms in $\Theta$ involve derivatives of $\phi$. These equations should be compared to those for evolution in flat space given by LWK [18]. The new term proportional to $H \dot{\phi}$ leads to a substantial enhancement.
in the size of $\Theta$. With this simplification, and the approximate form (13),
\[ \Theta = \sqrt{\frac{2\epsilon}{\pi}} \left( \frac{H}{M_{Pl}} \right)^2 N, \]  
(13)
where $\epsilon = \frac{1}{2} (\dot{\phi})^2 / (HM_{Pl})^2$ is the slow-roll parameter of inflation [15].

B. Gravitational birefringence

To see gravitational birefringence we need to solve the equations of motion explicitly. Let us focus on the horizon modes and at early times. This corresponds to attenuation of $h_L$ and amplification of $h_R$ in the early universe. This is the anticipated cosmological birefringence which was also discussed by LWK [18].

We now look for solutions to (15) with $h_L \sim e^{ikz}$. To do this, let
\[ h_L = e^{ikz} \cdot (-i k \eta) \Theta \, g(\eta), \]  
(18)
where $g(\eta)$ is a Coulomb wave function, i.e.
\[ \frac{d^2}{d\eta^2} g + \left[ k^2 (1 - \Theta^2) - \frac{2}{\eta^2} - \frac{2k\Theta}{\eta} \right] g = 0. \]  
(19)
This is the equation of a Schrödinger particle with $\ell = 1$ in a weak Coulomb potential. When $\Theta = 0$, the Coulomb term vanishes and we find the spherical Bessel function (17). For $h_L$, the Coulomb term is repulsive; for $h_R$, with the opposite sign of the $\Theta$ term, the Coulomb potential is attractive. This leads to attenuation of $h_L$ and amplification of $h_R$ in the early universe. This is the anticipated cosmological birefringence which was also discussed by LWK [18].

As we will see generation of the matter asymmetry is dominated by modes at short distances (subhorizon modes) and at early times. This corresponds to the limit $k \eta \gg 1$. In this region, we can ignore the potential terms in (19) and take the solution to be approximately a plane wave. More explicitly,
\[ g(\eta) = \exp[i k (1 - \Theta^2)^{1/2} \eta (1 + \alpha(\eta))], \]  
(20)
where $\alpha(\eta) \sim \log \eta/\eta$.

IV. THE GREEN’S FUNCTION

We would now like to use (18) to compute the expectation value of $\langle R \tilde{R} \rangle$ in the inflationary space-time. It turns out that it will be dominated by the subhorizon, quantum part of the gravity-wave evolution. Hence, to compute $\langle R \tilde{R} \rangle$ we only need the two point (Green’s) function $\langle h_L h_R \rangle$:
\[ G(x, t; x', t') = \langle h_L(x,t) h_R(x',t') \rangle = \int \frac{d^3 k}{(2\pi)^3} e^{ik(x-x')} G_k(\eta, \eta'). \]  
(21)
For $k$ parallel to $z$, the Fourier component $G_k$ satisfies (15) with a delta-function source
\[ \left[ \frac{d^2}{d\eta^2} - \frac{2}{\eta^2} + k^2 \right] G_k(\eta, \eta') = i \frac{(H\eta)^2}{M_{Pl}^2} \delta(\eta - \eta'). \]  
(22)
For $\Theta = 0$, the solution of this equation is
\[ G_{k0}(\eta, \eta') = \begin{cases} R h_L^+(k, \eta) h_R^-(k, \eta') \quad & \eta < \eta' \\ R h_L^-(k, \eta) h_R^+(k, \eta') \quad & \eta' < \eta \end{cases}, \]  
(23)
where
\[ R \equiv \left( \frac{H}{M_{Pl}} \right)^2 \frac{1}{2k^3} \]  
(24)
and $h_L^-$ is the complex conjugate of (17), and $h_R^+$, $h_R^-$ are the corresponding solutions of the $h_R$ equation. For $\Theta = 0$, these solutions are the same as for $h_L$, but the structure of (23) will be preserved when we go to the case $\Theta \neq 0$. The leading effect of $\Theta$ is to introduce the exponential dependence from (18).

\[ G_k = e^{-i \theta(\eta')} e^{i \theta(\eta')} G_{k0} \]  
(25)
for both $\eta > \eta'$ and $\eta < \eta'$. The prefactor is modified in order $\Theta^2$, and the wavefunctions acquire additional corrections that are subleading for $k \eta \gg 1$. Neither of these effects will be important for our result.

The Green’s function (25) can now be used to contract $h_L$ and $h_R$ to evaluate the quantum expectation value of $R \tilde{R}$. The result is
\[ \langle R \tilde{R} \rangle = \frac{16}{a^2} \int \frac{d^3 k}{(2\pi)^3} \frac{H^2}{2k^3 M_{Pl}^2} k^4 \Theta + O(\Theta^3) \]  
(26)
where we have picked up only the leading behavior for $k\eta \gg 1$.

We would like to emphasize that our expression for $(RR)$ is nonzero because of the effect of inflation in producing a CP asymmetry out of equilibrium. The original quantum state for the inflaton might have had nonzero amplitude for a range of values of $\phi$ and might even have been CP-invariant. However, inflation collapses the wavefunction onto a particular value of $\phi$ that is caught up in the local expansion of the universe. This value gives us a classical background that is CP-asymmetric.

The above result and computations seems to be crucially dependent on the form of the Green’s function or the vacuum state we have used. To resolve the possible ambiguity in this regards, one may perform the above computation using a different method, the fermion level crossing, e.g. following [21]. This computation confirms the above results [21].

V. COMPUTING THE RATIO OF LEPTON TO PHOTON DENSITIES

To complete our leptogenesis model, there are two remaining steps. First, we need to compute the net lepton anti-lepton asymmetry generated. This, in our model, should be computed during inflation. We then need to have a reheating model through which we can compute the temperature and hence the entropy of the Universe after the inflation ended.

A. Lepton number density

We are now ready to evaluate the lepton density that arises through the gravitational anomaly [2]. Inserting (26) into (2) and integrating over the time period of inflation, we obtain

\[
N = \int_0^{H^{-1}} d\eta \int \frac{d^3k}{(2\pi)^3} \frac{3}{16\pi^2} \frac{16H^2k\Theta}{M_{Pl}},
\]

where $N$ is the lepton number density. The integral over $k$ runs over all of momentum space, up to the scale $\mu$ at which our effective Lagrangian description breaks down; that is the momentum integral range is $1 < k\eta < \mu/H$. The dominant contribution comes not from the modes which have become classical and left the horizon by the end of inflation (the super-horizon modes), $k/(aH) < 1$, but rather from very short distances compared to these scales; the main contribution comes from subhorizon “quantum modes.” As the integrand also shows, the dominant contribution to the right-hand side comes from $1 \ll k\eta < \mu/H$, and as we had anticipated.

We can now perform the integrals to find the lepton number density produced by the end of inflation [1]

\[
n = \frac{N}{48\pi^4} \left( \frac{H}{M_{Pl}} \right)^2 \Theta H^3 \left( \frac{\mu}{H} \right)^4.
\]

Let us now analyze each factor in $n$:

- The factor $(H/M_{Pl})^2$ is the magnitude of the gravity wave power spectrum. We should stress that the usual gravity wave power spectrum is coming from the super-horizon modes, while the main contribution to $n$ has come from the sub-horizon modes.
- The factor $\Theta$ is a measure of the effective CP violation caused by birefringent gravity waves.
- The factor $H^3$ is the inverse horizon size at inflation; this gives the density $n$ appropriate units.
- Finally, the factor $(\mu/H)^4$ gives the enhancement over one’s first guess due to our use of strongly quantum, short distance fluctuations to generate $RR$, rather than the super-horizon modes which effectively behave classically. It is intriguing to note that the vacuum energy of gravity mode also grows like $(\mu/H)^4$.

B. Photon number density

To understand the significance of the lepton number density $n$, we should compare it to the entropy density of the Universe just after reheating, or to the photon number density. (Recall that almost all of the entropy of the Universe is generated during the reheating time and is carried in the massless degrees of freedom, i.e. photons.) In order this we need a reheating model.

To illustrate the physical relevance of the result of our model, we take the simplest (and at the same time very naive) reheating model, the instant reheating model. That is, we assume that all of the energy of the inflationary phase has been converted to the heat of a gas of massless particles instantaneously. The reheating converts the energy density of the inflaton field $\phi$

\[
\rho = 3H^2 M_{Pl}^2
\]

to radiation with

\[
\rho = \frac{\pi^2 g_* T^4}{30}
\]

and to the entropy density $s$, $s = 2\pi^2 g_* T^4/45$, where $T$ is the reheating temperature, $g_*$ is the effective number of massless degrees of freedom and $s = 1.8g_* n_{\gamma}$ [15]. (Here we have assumed that the

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1. In the original version of this paper, due to a computational mistake, we had $(\mu/H)^8$ rather than $(\mu/H)^4$. We thank Azadeh Maleknejad for pointing out this mistake.
evolution of the Universe after the reheating era has been adiabatic.) This gives
\[ n_\gamma = 1.28 g_*^{-3/4} (H M_{Pl})^{3/2}. \]

Recalling that the ratio of the present baryon number to the lepton number originally generated in leptogenesis is approximately \( n_B/n_L = 4/11 \)[5], in our model we obtain
\[ \frac{n_B}{n_\gamma} = 1.8 \times 10^{-4} g_*^{3/4} \left( \frac{H}{M_{Pl}} \right)^{7/2} \Theta \left( \frac{\mu}{H} \right)^4. \]  
(31)

If we are less naive, we might follow the dilution of \( n \) and \( \rho \) with the expansion of the universe to the end of reheating. The final result is the same (see, however, [23] for a comment on this point). With the adiabatic expansion assumption, (31) can be compared directly to the present value of \( n_B/n_\gamma \) given in (1).

C. Numerical results

We should now estimate the numerical value of \( n_B/n_\gamma \) in our model to see if we have a viable leptogenesis model. In our final result [31] we have five dimensionless parameters, \( g_* \), \( H/M_{Pl} \), \( \mu/M_{Pl} \) and the slow-roll parameter \( \epsilon \) and \( N \) (the latter two appear through the CP violation parameter \( \Theta \)). Inserting the expression for \( \Theta \) we have
\[ \frac{n_B}{n_\gamma} \simeq 1.3 \times 10^{-5} g_*^{3/4} \sqrt{\epsilon} N \left( \frac{H}{M_{Pl}} \right)^{3/2} \left( \frac{\mu}{M_{Pl}} \right)^4. \]  
(32)

Within the usual supersymmetric particle physics models \( g_* \sim 1000 \) is a reasonable choice. The WMAP data, through the density perturbation ratio \( \delta \rho/\rho \) (for a single field inflation) leads to an upper bound on \( H/M_{Pl} \) ratio as [22]
\[ \frac{H}{M_{Pl}} \lesssim 10^{-4}, \quad \epsilon \lesssim 0.01, \]  
(33)

or \( H \lesssim 10^{14} \text{ GeV}. \)

The factor \( N \) in (31) is inferred from the string theory compactification and is proportional to the square of the four dimensional \( M_{Pl} \) to ten dimensional (fundamental) Planck mass [14] (see also the Appendix). Within string theory,
\[ N \simeq 10^2 - 10^{10} \]
could be a reasonable range. Therefore, assuming that the scale of inflation \( H \) saturates its current bound \( H/M_{Pl} \simeq 10^{-4} \),
\[ \Theta \simeq 10^{-8} - 10^0. \]  
(34)

The physically viable range for the parameter \( \mu \) depends on the details of the underlying particle physics model on which our gravi-leptogenesis is based. For example within the Standard Model + three heavy right handed neutrinos (and the seesaw mechanism) \( \mu \) could be of order of the right handed neutrino mass, in which case \( \mu \lesssim 10^{12}\text{GeV} \), which is too low for our model to produce the observed matter-antimatter asymmetry. If we do not restrict ourselves to the seesaw mechanism, \( \mu \) can be larger. The upper bound on \( \mu \) is then only coming from the range of validity of our effective field theory analysis. For example, we can accommodate \( \mu \)'s as large as the string scale where quantum gravity effects become important. Therefore, depending on the details of the model
\[ 10^{12}\text{GeV} \lesssim \mu \lesssim 10^{16} - 10^{17}\text{GeV} \]
or
\[ 10^{-6} \lesssim \frac{\mu}{M_{Pl}} \lesssim 10^{-2} - 10^{-1} \]  
(35)

is a reasonable range for \( \mu \) from the particle physics viewpoint. The observed value of matter-antimatter asymmetry [11], with the value of other parameters as discussed above implies that
\[ 3 \times 10^{-3} \lesssim \frac{\mu}{M_{Pl}} \sim N^{-1/4} \lesssim 3, \]
which is well within the particle physics acceptable range [35].

In sum, assuming that scale of inflation \( H \) saturates its current bound \( H \simeq 10^{14}\text{GeV} \), we have
\[ \frac{n_B}{n_\gamma} \simeq 3.2 \Theta \left( \frac{\mu}{M_{Pl}} \right)^4. \]  
(36)

Noting (34) and (35), it is readily seen that there is big parameter space for obtaining the observed baryon asymmetry of the Universe within our gravi-leptogenesis model.

VI. SUMMARY AND OUTLOOK

Here we have constructed a leptogenesis model in which the lepton asymmetry is a result of gravitational chiral anomaly. Indeed our model is a “module,” which uses minimal ingredients and could be fit into a successful inflation model in which the inflation field(s) has a pseudo-scalar component. From the particle physics side, it is again like a module and many different particle physics models could be invoked. Here we did not discuss the details of neither the inflationary nor the underlying particle physics models. It is of course very important to explicitly fit our module in viable physical models.

As our construction is a leptogenesis model, we need a mechanism to convert the lepton asymmetry to baryon asymmetry. With the Standard Model this is
usually performed via the thermally activated electroweak sphalerons, which in our model should be active after inflation. This happens for reheating temperature \( T_r \gtrsim 1 \) TeV. Within our naive “instant reheating” model that translates to \( H \gtrsim 10^{-3} \) eV, which is not a serious constraint on the inflationary models used.

According to recent WMAP observations, the scalar metric perturbations generated during inflation have a size that gives density fluctuations with \( \delta \rho/\rho \sim 10^{-5} \). On the other hand, noting (1), one has

\[
\frac{n_B}{n_\gamma} \sim \left( \frac{\delta \rho}{\rho} \right)^2.
\]

(37)

The above could be a simple numerical accident or there might be some underlying deeper physics. In any case, our model which invokes gravity waves as the source for the baryon asymmetry and operates during inflation, could be an interesting framework to uncover the possible deeper physics behind (37).

We would also like to comment on the dependence of the lepton asymmetry produced on the cutoff \( \mu \). The \( (\mu/H)^4 \) dependence in (27) is intriguingly the same as the cutoff dependence of gravity mode vacuum energy.

\[ S = M_1^8 \int d^4 y \int d^4 x \ e^{-\Phi} \left( dB + \alpha' \Omega_3(\omega) \right)^2 \]

\[ = M_1^8 \frac{V_6}{g_s} \int d^4 x \left( [dB]^2 + 2\alpha' (\ast dB) \wedge \Omega_3(\omega) + \cdots \right) \]

\[ = \frac{M_{pl}^2}{g_s} \int d^4 x \left( (\partial \phi)^2 - 2\alpha' \phi R \right) \]

where the pseudoscalar \( \phi \) (our inflaton field) is dual to the two form \( B_2 \) and the 4d Planck length, \( M_{pl} \) is defined as

\[ M_{pl}^2 = M_1^8 V_6. \]

And hence, finally:

\[ N = 8\pi^2 \frac{M_{pl}^2 \alpha'}{g_s} = 8\pi^2 \left( \frac{M_{pl}}{M_1} \right)^2 \left( \frac{1}{\sqrt{g_s}} \right). \]

\[ \]

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Appendix A: Realization of the model within string theory [14]

To see how this term appears in string theory, consider e.g. the Heterotic SUGRA action:

\[
S = M_1^8 \int d^4 x \sqrt{\det g_{10}} \left( R + \frac{1}{2} (\partial \Phi)^2 \right)
+ \frac{1}{12} e^{-\Phi} H_{ABC}^2 + \frac{1}{4} e^{-\Phi/2} (F_{AB})^2
\]

where

\[
H_3 = dB_2 - \frac{1}{4} (\Omega_3(A) + \alpha' \Omega_3(\omega))
\]

\( \Omega_3(A) \) and \( \Omega_3(\omega) \) are the gauge and gravitational Chern-Simons three-forms, respectively. Explicitly,

\[ \Omega_3(\omega) = Tr(\omega \wedge d\omega + \frac{2}{3} \omega \wedge \omega \wedge \omega) \]

In particular note that

\[ \ast (d\Omega_3(\omega)) = R \ast R. \]

Upon compactification to four dimensions, the \( H^2 \) term leads to

\[
S \sim M_1^8 \int d^6 y \int d^4 x \ e^{-\Phi} \left( dB + \alpha' \Omega_3(\omega) \right)^2
\]

\[ = M_1^8 \frac{V_6}{g_s} \int d^4 x \left( [dB]^2 + 2\alpha' (\ast dB) \wedge \Omega_3(\omega) + \cdots \right) \]

\[ = \frac{M_{pl}^2}{g_s} \int d^4 x \left( (\partial \phi)^2 - 2\alpha' \phi R \right) \]

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