Influence of the strain dependent material behaviour under plane strain on the yield locus modelling

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Abstract. Modern lightweight materials mostly exhibit a low formability especially in the plane strain state. Having in mind that over 80% of the occurred failures in the press shop in the automotive sector can be related to this strain state, the characterization under plane strain load should be further intensified. The evaluation of the plane strain flow curve is commonly done by notched tensile tests or biaxial tensile tests with optimized strain path. Both tests have some disadvantages in the flow curve evaluation especially at a high degree of deformation. With an elliptic hydraulic bulge test, the material behaviour under near plane strain condition can be evaluated up to fracture. The aim in this investigation is the analysis of the resulting material behaviour under plane strain of commonly used alloys AA5182, DC06 and DP600 with respect to the material anisotropy. Therefore, hydraulic bulge tests with a plane strain load path are performed in rolling direction and transversal direction. The resulting flow curves are used to analyse the material hardening with respect to the anisotropy. The results show differing hardening mechanisms for the investigated materials which will lead to different strategies for the material modelling.

1. Introduction

Efficient and versatile manufacturing processes are nowadays based on a precise numerical process design. In order to guarantee a reliable numerical process layout, material models with a high mapping accuracy are required. Due to the improved possibilities in material characterization, models with an increasing number of degrees of freedom can be identified. Especially the use of optical strain measurement systems, which make it possible to characterize even inhomogeneous strain fields without contact, has rapidly enriched the possibilities for the determination of material parameters. But also the growing computing power enables the use of modern material models.

Especially the plane strain state is recently in the focus of research, since the material flow in this loading condition is purely in the direction of the sheet thickness. As a result, the sheet metal thins extensively even at a comparatively low strain and the forming limit is at a minimum. Investigations by Xavier et al. [1] attributed 80% of the failures in the forming of sheet metal components in the automotive industry to the state of plane strain or near plane strain. Due to this fact it is desirable to be able to represent this strain state and the associated flow stress as precisely as possible in the numerical FE-model. The predominant number of material models, which are available in commercial finite element (FE) software, cannot represent this point directly. Instead, the point of plane strain is interpolated in the yield locus curve between two support points and is thus subject to a large scatter depending on the quality of the input parameters. In order to increase the accuracy of the model, Safei
et al. [2] used a polynomial function to vary the eight model parameters, which are normally identified at the beginning of the plastic deformation, in dependence on the true strain. With this extension, the distortional and kinematic hardening of materials could be modeled more accurate. A further possibility for the representation of extended material parameters is the approach of Küsters and Brosius [3]. They varied the yield locus exponent in dependency of the true strain with a Bézier curve interpolation in order to be able to map the distortional hardening. Kuwabara et al. [4] was also able to demonstrate an improvement in the mapping accuracy by varying the yield locus exponent. Conventionally, the choice of the yield locus exponent is purely heuristic and does not relate to the laws of metal physics. Lenzen and Merklein [5] showed that the numerical mapping accuracy could be improved by optimizing the yield locus exponent by using experimental data.

For the characterization of flow curves under plane strain, there are two testing methods in the literature, the notched tensile test and the biaxial tensile test. When testing with notched specimens, Wagoner [6] or Flores et al. [7] developed evaluation methods to derive flow curves from the measured data. The greatest limitation of these approaches is the exclusive characterization of the first principal stress. For a precise modeling of yield loci, however, the first and second principal stresses are required. Kuwabara [8] was able to counter this with a biaxial tensile test. The limitation in these tests was the maximum strain of 0.09 for an IF steel. Another test setup suitable for material characterization at plane strain is the hydraulic bulge test with elliptical die. With formulas from membrane theory, yield stresses can be determined in elliptical hydraulic bulge tests. An advantage of this test method is the absence of friction or notching effects, which allows characterization up to high values of true strain. Conventionally used materials in the automotive sector, such as deep-drawing steel, dual-phase steel, but also aluminum alloys, exhibit characteristic material and hardening behavior, respectively. Therefore, a uniform recommendation for the choice of required tests for material characterization and further the material modelling is not reasonable.

The objective of the investigation is therefore to characterize the plastic material behavior under plane strain and to identify material-typical properties. In addition, standardized tensile tests and layer compression tests are carried out as a basis for the analysis. Based on these tests, recommendations for efficient material characterization and modeling are derived. The aim is to be able to transfer the real material behavior into a model without loss of relevant information and with the smallest possible number of required tests.

2. Experimental setup and approach

The objective of this investigation is to gain a better understanding for the hardening behavior of different materials under plane strain. Derived from this, recommendations for numerical modeling with these parameters are formulated. The investigations are done for three different materials, one aluminum alloy and two steel grades. The aluminum alloy AA5182 is characterized by high strength and good formability. Due to its excellent corrosion resistance and weldability, this material is used for interior body panels in automotive engineering. The first steel grade is the deep drawing steel DC06. This alloy shows a distortional hardening behavior and an anisotropy concerning the flow stress. Due to the excellent formability, the DC06 is used for complex parts for example oil sumps. The DP600 is a dual phase steel grade. This alloy is widely used for parts under high cyclic loading like wheel rims. Material characterization is done under uniaxial tensile, equi-biaxial and near plane strain load. The uniaxial tests are performed according to DIN EN ISO 6892-1, the hydraulic bulge tests are done according to the standard DIN EN ISO 16808. The characterization under near plane strain loading condition is done with a hydraulic bulge test with elliptical die. The geometry of the die is 200 mm in length and 75 mm in width. Detailed information about the testing setup and flow curve evaluation can be found in [5]. For all tests, a testing velocity of 0.004 s⁻¹ for steel and 0.0067 s⁻¹ is set.

Based on the resulting flow curves, the plastic material behavior is analyzed. Especially the plastic anisotropy of the flow stress between rolling direction and transversal direction and the evolution of
this flow stress anisotropy are focused. For this purpose stress differences for the first and second principal stress are defined:

\[ \Delta \sigma_1 = |\sigma_{1,WR0} - \sigma_{1,WR90}| \] (1)

\[ \Delta \sigma_2 = |\sigma_{2,WR0} - \sigma_{2,WR90}| \] (2)

With these two parameters, the hardening evolution can be discussed for each material separately. Based on the gained knowledge of the material characteristics under plane strain, material dependent modelling strategies are derived in order to improve the mapping accuracy and simultaneously minimize the testing effort to a necessary minimum.

3. The Investigation of the hardening behaviour

For a complete understanding of the hardening mechanisms of the investigated alloys the resulting plastic material behavior, characterized in standardized tests, is presented and discussed. The flow curves out of uniaxial tensile tests and equi-biaxial hydraulic bulge tests for all three investigated materials are shown in Figure 1. The uniaxial tensile tests are carried out according to DIN EN ISO 6892-1 and the hydraulic bulge tests with circular die are performed according to DIN EN ISO 16808. For comparability, the equivalent strain \( \varepsilon_{VM} \) and equivalent stress \( \sigma_{VM} \) according to von Mises is used in Figure 1.

![Flow curves out of uniaxial tensile tests and hydraulic bulge tests of the materials DC06, DP600 and AA5182.](image)

**Figure 1.** Flow curves out of uniaxial tensile tests and hydraulic bulge tests of the materials DC06, DP600 and AA5182.

For the DC06 a significant higher stress in the flow curve under equi-biaxial loading than under uniaxial tensile load can be seen. This is relied to the stress state dependent hardening of this material [9] with a more pronounced hardening under equi-biaxial load. The flow stress of tensile tests in rolling direction is on a slightly higher level than in transversal direction with a rising difference
between the two directions with rising plastic deformation. This can be relied to the distortional hardening mechanism of the material which leads to a different hardening in the two directions. It has to be mentioned that the initial yield strength is higher in transversal direction (see Table 1). When material modeling is done with parameters at the onset of the plastic deformation, the stress ratio between rolling direction and transversal direction is $\sigma_{RD} < \sigma_{TD}$. This would lead to a poor mapping of the model in the simulation. At a true strain of 0.025 the stress ratio changes to $\sigma_{RD} > \sigma_{TD}$. Beside the different hardening for all three load directions, the wrong stress ratio will lead to poor mapping accuracy in the area of plane strain in the modelled yield surface when using a conventional material model like Yld2000-2d [10] or BBC05 [11]. Flow curves of DP600 reveal an equal hardening behavior for all tests with a slightly higher progression for the uniaxial yield stress in transversal direction. The flow stress under equi-biaxial load is on a lower level than under uniaxial tensile load. Considering this material behavior, an anisotropic material model with isotropic hardening should be able to map the experimental flow curves accurate. The AA5182 exhibits an isotropic hardening behavior. However, there is an anisotropy in the flow stress between rolling direction and transversal direction with higher values in rolling direction. For this alloy, an anisotropic material model with isotropic hardening will lead to accurate results. For completeness, the resulting mean Lankford coefficients and the initial yield strengths are given in Table 1.

Table 1. Lankford coefficients and Yield strength of DC06, DP600 and AA5182.

| Material | r-value RD | r-value TD | Yield strength RD in MPa | Yield strength TD in MPa |
|----------|------------|------------|--------------------------|-------------------------|
| DC06     | 2.195 ± 0.040 | 2.644 ± 0.030 | 168.4 ± 0.5             | 174.5 ± 0.4             |
| DP600    | 0.779 ± 0.074 | 0.931 ± 0.042 | 401.4 ± 3.2             | 421.7 ± 1.8             |
| AA5182   | 0.773 ± 0.008 | 0.629 ± 0.011 | 122.4 ± 0.7             | 120.4 ± 0.3             |

Based on the material characterization, that is conventionally done for material modelling with the models Yld2000-2d [10] or BBC05 [11], elliptic hydraulic bulge tests are performed to characterize the resulting flow stress under near plane strain conditions for both principal stress directions. For an analysis of the material anisotropy and the hardening evolution, tests are performed in rolling direction and transversal direction. The resulting flow curves are shown in Figure 2. For a comparability with the results above, the equivalent strain $\varepsilon_{eq}$ according to von Mises is calculated. The first principal stress of the tests is always in the direction of loading with plane strain. The second principal stress is the stress perpendicular to the first principal stress and can be relied to the restraining force that is necessary for the hold back of the material. The displayed flow curves are average values from three repeated tests.

For the DC06 in Figure 2 on the left the flow curves in both principal stress directions are on a higher level in transversal direction with a rising difference with rising strain. The higher values under plane strain in transversal direction can be verified with the initial yield stress under uniaxial loading, see Table 1. But under uniaxial loading, there is a change of the stress ratio at 0.025 true strain between rolling direction and transversal direction with higher values in transversal direction. This is opposite to the resulting flow curves under plane strain load and will be further discussed later. The dual phase steel in Figure 2 in the middle reveals a higher stress progression for the flow curves characterized in transversal direction. The similar relationship was identified in the analysis of the flow curves with uniaxial load in Figure 1. The aluminum alloy reveals a higher flow stress in rolling direction for both principal stress progression. Again, this correlates with the characterized yield strength in the uniaxial tensile test, see Table 1. In General, the stress values are on a higher level in transversal direction for the steel grades in the tensile tests and the aluminum alloy has a higher yield strength in rolling direction. Therefore, the conclusion can be drawn that the direction dependent stress anisotropy, that is well known in the uniaxial yield stress is also present under plane strain loading.
But there is a limitation in the validity for the DC06. In this case the correlation between uniaxial tensile load and load under plane strain is only correct for the initial yielding up to 0.025 true strain.

For a further analysis of the stress differences between the flow curves under plane strain load in rolling direction and transversal direction, Equation 1 and Equation 2 are used. By observing the stress differences over the strain, the hardening mechanisms such as isotropic or distortional hardening can be illustrated. Additionally, the material anisotropy can be discussed. Isotropic hardening leads to a constant (linear) rise of the stress difference due to the work hardening. When the difference is zero in the beginning, it is an isotropic material. If the difference is unequal zero, it is an anisotropic material. In contrast to a linear progression the distortional hardening leads to inhomogeneous progression of the stress difference. The resulting stress differences $\Delta \sigma_1$ and $\Delta \sigma_2$ of the elliptic hydraulic bulge tests for all three materials are presented in Figure 3.

When analyzing the resulting stress differences, two basic effects can be identified, the material anisotropy and the anisotropy of hardening. If the value is close to zero, the materials are isotropic with respect to the yield stress. For example, the steel DC06, at a low strain of 0.05, reveals only a very small difference in the flow stresses in tests at rolling direction and transversal direction. This applies to both principal stresses. When the true equivalent strain increases, the stresses of tests at rolling direction and transversal direction deviate more. This is due to the anisotropic or distortional strain hardening of this alloy [9]. The modelled yield locus should be distorted depending on the strain. However, this behavior cannot be considered in isotropic material laws such as Yld2000-2d or BBC05. When using these models, a rising discrepancy between material behavior and material model will lead to poor results in simulations, especially in areas under plane strain load. Thus, a strain dependent modelling is necessary for the accurate mapping of this material in numerical simulations. The materials DP600 and AA5182 show pronounced differences in the yield stress between the rolling
direction and transversal direction at low degrees of deformation which can be relied to the material anisotropy that is also visible in the difference of the yield strength between rolling and transversal direction under uniaxial load, see Table 1. The differences in the flow stress of the DP600 material become smaller as the true equivalent strain increases. This can be explained with an anisotropic hardening of this material with a distortion of the yield surface to a more isotropic behavior. For the mapping of this material behavior an extended material law would be necessary. The application of an isotropic material law inevitably leads to an inaccurate numerical mapping with high strains.

For the aluminum alloy, the stress difference further increases linear with the true equivalent strain. This behavior can be mapped by an isotropic hardening law. Basically, the trends of the stress differences between 0° and 90° for the DP600 and the aluminum AA5182 are on an equal level to flow curves from uniaxial tensile tests. When comparing the three materials, it is obvious that the DC06 has a rising stress difference with increasing true equivalent strain while the DP600 shows a reduction of the stress difference with rising strain. This difference can be explained by the change of the biaxial r-value. Figure 4 shows the biaxial r-value for the DC06 and DP600, characterized in layer compression tests with three repetitions n = 3 for each material [12]. In contrast to this, the aluminum AA5182 reveals a constant biaxial anisotropy over the true strain after a transient zone at the beginning of the plastic deformation. In this case, only the material hardening leads to an increase of the stress difference.

As seen in Figure 4, the biaxial anisotropy decreases for the DC06 with a final value of 0.82 at 0.5 true strain. With rising deformation the r-value deviates more from 1.0. A biaxial anisotropy of 1.0 refers to an isotropic material which has the same yield locus progression in \( \sigma_1 \) and \( \sigma_2 \) direction. The resulting angle therefore is 45° of the yield locus normal. Due to the increasing deviation relied to the decreasing biaxial r-value, the stress differences between \( \sigma_1 \) and \( \sigma_2 \) are more pronounced in the yield

Figure 3. Stress difference of the principal flow stresses from plane strain hydraulic bulge tests at rolling direction and transversal direction for the materials DC06, DP600 and AA5182.
stress under plane strain. Contrary to the DC06 is the biaxial r-value progression for DP600. In this case, the biaxial anisotropy increases to 0.9 at 0.4 true equivalent strain. Therefore, the stress difference in the plane strain area is reduced with rising plastic deformation. Due to the fact that the biaxial r-values for both materials are below 1.0 it is expected that the flow stress in transversal direction is higher than the flow stress in rolling direction under plane strain loading, because of the distortion of the yield locus to higher values for $\sigma_2$ than for $\sigma_1$. This can be seen in Figure 2 for the flow stress under plane strain of the two steel grades in transversal direction (green lines). The biaxial anisotropy progression of the aluminum alloy is constant at a value of about 1.25. Since the value is above 1.0, a higher stress value in rolling direction should be the consequence. Looking at Figure 1 and Figure 2, the corresponding flow stress under uniaxial tensile load and under plane strain load are higher in rolling direction than in transversal direction.

![Figure 4. Biaxial r-value of DC06 and DP600 characterized in layer compression tests [12].](image)

Based on the findings, recommendations for the material modelling can be derived. The aluminum AA5182 can be modeled with an isotropic hardening law. In this case it is strongly recommended to take the plane strain yield stress into account to achieve good results [5]. For the steel grades, an isotropic hardening is not suitable due to the anisotropic hardening of both materials. When the determination of the flow stress is not possible, the integration of the strain dependent biaxial anisotropy lead to a better mapping accuracy because of the distortion of the yield locus geometry in the plane strain area due to the angular change within deformation, see Figure 5. An increased biaxial r-value leads to a higher stress value in the plane strain area in rolling direction and a reduced yield stress in the plane strain area in transversal direction. A lower biaxial anisotropy leads to a reversed stress prognosis with a higher yield stress in transversal direction and a reduced stress in rolling direction, when looking at the area of plane strain deformation.

![Figure 5. Influence on the modeled yield locus geometry.](image)
4. Conclusions
Regarding the results of the experimental analysis in the presented investigations the following conclusions can be drawn:

- The material anisotropy of the flow stress is also present under plane strain
- The progression of the stress difference between rolling direction and transversal direction can be relied to the change of the biaxial anisotropy
- Due to the distortional hardening of DP600 and DC06, a strain dependent hardening rule should be used for the material modelling

Further investigations should focus the influence on the numerical results using an optimized material model compared to a conventional flow rule.

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