On the Observability of Quantum Information Radiated from a Black Hole

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Abstract

We propose a resolution to the black-hole information-loss paradox: in one formulation of physical theory, information is preserved and macroscopic causality is violated; in another, causality is preserved and pure states evolve to mixed states. However, no experiments can be performed that would distinguish these two descriptions. We explain how this could work in practice; a key ingredient is the suggested quantum-chaotic nature of black holes.

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I. INTRODUCTION

Twenty-seven years ago, Hawking proposed that black-hole evaporation required a modification of quantum mechanics: pure states had to be able to evolve into mixed states [1]. Otherwise, there would have to be nonlocal effects operating on macroscopically large scales, in regions of low space-time curvature [2]; this was (and remains) an anathema to devotees of Einstein’s theory. On the other hand, the evolution of pure to mixed states was equally abhorrent to devotees of the theory of Schrödinger, Heisenberg, and Dirac. Consequently one’s position on this key issue of fundamental physics has tended to be sociologically determined, though there are some notable exceptions.

The discovery that string theory allowed one to count black-hole microstates [3], and get a number that agreed with the one predicted by the Bekenstein-Hawking entropy [4–6], gave a strong boost to those on the side of quantum mechanics: string theory is explicitly a quantum mechanical theory, and it clearly violates field theoretic notions of locality. More recently, the discrete light-cone quantization scheme (DLCQ, also known as the “matrix model” [7]), and the equivalence of string theory in anti-de-Sitter space to a conformal field theory in a lower dimension (AdS/CFT [8]), give us, in principle, explicit hamiltonian descriptions of black-hole formation and decay. This would seem to settle the issue decisively in favor of quantum mechanics, at least for string-theoretic descriptions of quantum gravity.

However, in all the years of debate, it has been implicitly assumed that the question of whether pure states evolve to mixed states could, in principle, be answered experimentally by performing appropriate measurements.

In this paper, I wish to call this into question. I will suggest a physical picture of the quantum states of black holes such that, even if the string-theoretic scenario outlined above is correct, experiments would still show apparent information loss during black hole evaporation. This then further suggests that there may be a dual or complementary description of this process in which pure states evolve to mixed states, but macroscopic causality is preserved. These two descriptions would be experimentally indistinguishable.

II. BLACK-HOLE RESONANCES IN THE S-MATRIX

Black holes are unstable, and decay by emitting Hawking radiation [5]. If quantum mechanics is unmodified, black holes will show up as resonances in the S-matrix [9] for scattering of stable particles [10]. In the complex s plane (where the Mandelstam variable s is the total center-of-mass energy squared), there will be poles at

\[ s_n = M_n^2 - i\Gamma_n, \]

with

\[ M_{n+1} - M_n = C_n \exp(-4\pi M_n^2). \]

Here \( \Gamma_n \) and \( C_n \) are numbers of order one in Planck units (\( \hbar = c = G = k = 1 \)). These formulas are appropriate for Schwarzschild black holes in 3+1 dimensional spacetime. Eq. (2) follows from the density of black-hole states \( e^S \), where \( S = 4\pi M^2 \) is the Bekenstein-Hawking entropy. Eq. (3) follows from the Hawking decay rate.
\[ \frac{dM}{dt} = -4\pi R^2 \sigma T^4, \quad (3) \]

where \( T = 1/8\pi M \) is the Hawking temperature, \( R = 2M \) is the Schwarzschild radius, and \( \sigma = g_s \pi^2/30 \) is the Stefan-Boltzmann constant (\( g_s \) is the number of effectively massless particle species at temperature \( T \)). Dividing \( dM/dt \) by the typical energy \( T \) of an emitted quantum gives us the width of the resonance in the energy plane, and multiplying by \( M \) gives us the width \( \Gamma_n \) in the \( s \) (energy-squared) plane (assuming \( \Gamma_n \ll M_n \)). We conjecture that the numbers \( \Gamma_n \) and \( C_n \) vary erratically with \( n \); we will elaborate on this in Section III.

Eq. (2) is a highly unusual behavior for a set of resonances. Not only does their density increase rapidly with \( n \), but the series never terminates. Other known systems (such as complex atoms or nuclei) have dense resonances, but eventually there is a threshold (for, e.g., ionization or fission), and above this threshold the poles are replaced by a cut. This is not supposed to happen for black holes, which can be arbitrarily large. This makes the very definition of the S-matrix problematic. To define the S-matrix, one typically considers widely separated (and therefore noninteracting) wave packets for individual particles that come together in an interaction region that is bounded in space and time. It is assumed that this can be done for arbitrarily large energy, but here large energy implies a large black hole, and thus the size of the interaction region grows with energy.

To regularize this problem, let us compactify the three dimensions of space onto a torus. We will consider this system at all energies, and try to deduce the properties of the S-matrix that could be extracted from it.

### III. BLACK HOLES IN A BOX

Consider putting energy \( E \) into our toroidal box of linear size \( L \). We will consider two possible generic forms for this energy: a gas of massless particles, and a Schwarzschild black hole. A gas of massless particles with energy \( E_{gas} \) in a volume \( L^3 \) has entropy \( S_{gas} \sim (LE_{gas})^{3/4} \). A Schwarzschild black hole of mass \( M \) has entropy \( S_{BH} = 4\pi M^2 \). If we maximize the total entropy \( S_{gas} + S_{BH} \) subject to the constraint \( E_{gas} + M = E \) (this is equivalent to requiring the gas and the black hole to have the same temperature), we find \( M^4 E_{gas} \sim L^3 \).

Dividing both sides by \( E^5 \), we see that the left hand-side is always less than one, which means there is a solution only if \( E > L^{3/5} \). In this case we find \( M = (1 - \varepsilon)E \) and \( E_{gas} = \varepsilon E \), where \( \varepsilon \sim L^3/E^5 \). If \( E < L^{3/5} \), then the entropy is maximized by putting all of the energy into the gas. In this regime, the probability of finding a black hole of mass \( M \) is governed by the Boltzmann factor \( \exp(-M/T_{gas}) \), where \( T_{gas} = (\partial S_{gas}/\partial E)^{-1} \) is the gas temperature.

We now assume that some sort of hamiltonian description of this black-hole-in-a-box is available, analogous to the hamiltonians provided by DLCQ or AdS/CFT. The thermodynamic analysis in the previous paragraph made implicit use of the microcanonical ensemble: the defining thermodynamic relation is entropy as a function of energy and volume. For classical systems, microcanonical averages involve integrating over a constant-energy surface in phase space. For quantum systems, one traditionally sums over energy eigenstates in some “small” energy range. A natural question is, how small can this energy range be? Or, equivalently, how many energy eigenstates must we use for the microcanonical averaging? For interacting many-body quantum systems that behave chaotically, the answer is now...
believed to be one [1]. That is, individual energy eigenstates have the properties of a state of thermal equilibrium. (To motivate this for the reader who does not want to go through the quantum-chaos literature, note that energy eigenstates are time independent; thus a system in one of them does not evolve. If an isolated complex system of many interacting particles does not evolve, then, according to standard thermodynamic reasoning, it must be in a state of thermal equilibrium. This argument was first made for black holes in [2].) In the context of black-hole thermodynamics, thermal means classical. Thus, for example, if we can identify an operator $\hat{R}^{\mu \nu \rho \sigma}(x)$ corresponding to the curvature tensor, and $|\alpha\rangle$ is an energy eigenstate in the black-hole regime $E \gg L^{3/5}$, then $\langle \alpha | \hat{R}^{\mu \nu \rho \sigma}(x) | \alpha \rangle$ should be equal to the Schwarzschild curvature, up to corrections of order $\varepsilon \sim L^3/E^5$.

This means that if we put energy $E \gg L^{3/5}$ into a box of linear size $L$, we should find energy eigenstates that look very much like black holes of mass $E$. Note that we must have $R = 2E \ll L$ for the black hole to fit into the box, but this is not a problem for large $L$. If we take $E = L^{4/5}$, for example, then we obviously have $L^{3/5} \ll E \ll L$ when $L$ is large. In this case, for an energy equivalent to one solar mass, the Schwarzschild radius is 3 km, and $L$ is nearly a billion times larger.

The emergence of black holes out of a gas in a box at high energy is due to the negative specific heat of the black hole. Consider what would happen if we put, say, a uranium nucleus in the box instead. A nucleus with energy $E$ (we work in units where $E = 0$ is the energy of the ground state, $E = 1$ is the energy of the first excited state, and $\hbar = c = 1$) has entropy (logarithm of the density of quantum states) $S_{\text{nuc}} \sim \sqrt{E}$. Suppose we give our nucleus energy $E$; it will decay by emission of various particles, which we will suppose ultimately form a gas of entropy $S_{\text{gas}} \sim (LE_{\text{gas}})^{3/4}$; the nucleus will then have energy $E_{\text{nuc}} = E - E_{\text{gas}}$.

To determine $E_{\text{nuc}}$ and $E_{\text{gas}}$, we again minimize the total entropy at constant total energy. This time we find that the system is dominated by the nucleus only at low energy, $E < L^{-3}$. In our units, the size of the nucleus is roughly one, so we cannot have a small ($L < 1$) box. For a large ($L \gg 1$) box, we are limited to such low energies that we can see only the ground state of the nucleus ($E \ll 1$). Thus, unlike a black hole, we cannot study the quantum states of a nucleus by putting it in a box and letting it come to thermal equilibrium with its decay products.

We now make note another result from quantum chaos theory. While individual energy eigenstates give thermal expectation values to simple observables, the wave functions of these states are very complex and easily disturbed by small perturbations. Correspondingly, the precise energy eigenvalues are also highly sensitive to perturbations. Thus, if we were to change the shape of our box (or, equivalently, the modular parameters of the compactification torus), we would expect the thermal and classical aspects of these states to be unchanged (so that $\langle \alpha | \hat{R}^{\mu \nu \rho \sigma}(x) | \alpha \rangle$ would still have the Schwarzschild value), but the detailed energy eigenvalues and eigenfunctionals would be different. This is true even though the walls of the box can be arbitrarily many Schwarzschild radii away.

The combination of these two results (dominance of the black-hole states at high energy, and sensitivity of the energy eigenvalues to large-scale boundary conditions) leads us to conjecture that the locations of the black-hole resonance poles in the $S$-Matrix, eq. (1), will also be sensitive to the large-scale boundary conditions.

Another way to see how this sensitivity could arise is to consider small changes in local fields (such as the metric). In the language of Feynman diagrams, these changes would
be represented by insertions on the internal black-hole propagator. These insertions would couple together different black-hole states, so that

\[
\frac{\delta_{mn}}{s - M_n^2 + i\Gamma_m} \to \sum_k \frac{\delta_{mk}}{s - M_k^2 + i\Gamma_k} \lambda_{kl}(s) \frac{\delta_{ln}}{s - M_l^2 + i\Gamma_l} + \ldots
\]

\[= \left( \frac{1}{s - M^2 - \lambda(s) + i\Gamma} \right)_{mn}, \tag{4} \]

where \(\lambda_{mn}(s)\) is a schematic representation of how a small change in the metric (or other external field) couples to the black-hole states. Since these states are separated by an exponentially small mass gaps, even very small \(\lambda\)'s have a significant effect at large enough \(s\). Note that this effect is different from that of external fields that simply shift the value of \(s\) itself (such as tidal effects of the moon at the LEP accelerator\[13\]); in order to understand and compensate for eq. (4), we would have to be able to calculate the \(\lambda_{mn}(s)\) matrices from first principles, which in turn would require a detailed knowledge of the black hole states.

IV. IMPLICATIONS FOR EXPERIMENTAL MEASUREMENTS

We begin by noting the obvious fact that nothing can be learned about information loss by watching the formation and evaporation of a single black hole, even if it were possible to have perfect measurements of the energies and spins of every incoming and outgoing particle. As always in quantum mechanics, we must do repeated experiments with “identically prepared” initial states in order to measure cross sections, and from these infer results about the \(S\)-matrix itself. Thus our goal would be to measure scattering cross sections precisely enough to discover the presence and locations of the black-hole resonance poles, eq. (1). This would require extraordinarily precise measurements of both energy and event rate, but there does not seem to be any obvious obstacle to making these exacting measurements in principle.

However, in the previous section we conjectured that the precise resonance locations would be highly sensitive to large-scale boundary conditions, and therefore to the entire history of the past light-cone at the spacetime location of the scattering event. This history necessarily differs from event to event, and so (we conjecture) do the locations of the resonance poles. Thus, while any single scattering event could be described by a unitary \(S\)-matrix that maps an initial density matrix \(\rho_{in}\) to a final density matrix \(\rho_{out}\),

\[\rho_{out} = S\rho_{in}S^\dagger, \tag{5}\]

experimentally measured cross sections (which necessarily require many scattering events) would instead yield

\[\rho_{out} = \sum_\gamma p_\gamma S_\gamma \rho_{in} S_\gamma^\dagger. \tag{6}\]

Here \(\gamma\) stands schematically for the values of the \(C_n\)'s and \(\Gamma_n\)'s (which change from event to event), and \(p_\gamma\) is a suitable probability distribution. The theory of random scattering matrices of this type is well developed in mesoscopic physics, and is routinely used to treat scatterers with chaotic properties (see, e.g., [14]).
In the present context, eq. (6) is Hawking’s proposed “dollar” matrix that turns a pure initial state into a mixed final state \[ \Pi \]. Thus, although the time evolution of the quantum state of the universe is unitary, this fact cannot be determined by making a series of repeated measurements that are sensitive to the black-hole poles in the \( S \)-matrix.

If unitarity cannot be verified experimentally, then it may be possible to devise a different, physically equivalent, mathematical description in which density matrices are the fundamental objects, the theory has manifest field-theoretic locality, and non-unitary evolution consistent with eq. (6). This theory, if it exists, would be experimentally indistinguishable from the non-local, unitary version of physics that results in eq. (4).

V. CONCLUSIONS

This paper has consisted of a series of increasingly bold speculations, with the goal of producing a resolution, acceptable to all parties, of the debate of the last quarter-century concerning the possible loss of quantum information in the evaporation of black holes: everyone is right.

A precise conjecture is that the locations of black-hole resonances [that is, the poles in the \( S \)-matrix that follow the general pattern of eq. (1)] will depend sensitively on large-scale boundary conditions, such as the moduli of a spatial compactification torus. We may hope that such calculations will become feasible in the near future.

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[10] This statement actually should be reviewed with some care. Formally, a theory with massless particles has no S-matrix, because the probability for scattering with any finite number of particles (in either the initial or the final state) is zero. What can be observed, in general, is scattering of jets of energy that include an arbitrary number of soft photons and gravitons. The infinite number of unobservable particles in these jets leaves plenty of room for hiding the information stored in the formation of the black hole. On the other hand, the physics of these soft particles is a well-understood low energy phenomenon; see S. Weinberg, Phys. Rev. 140, B516 (1965). Thus, while hiding information in unobserved quanta is a logical possibility, the fact that we can in principle make arbitrarily good (though never perfect) detectors is a strong argument against this idea.
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