Magnetic field imaging of a model electric motor using polarized pulsed neutrons at J-PARC/MLF

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Abstract. We have been developing a magnetic field imaging method using polarized pulsed neutrons for quantitative evaluation of magnetic fields in industrial equipment. To reduce the power loss of these products, it is important to quantitatively evaluate the distribution of not only strength but also direction of the internal field. In this study, we attempted to visualize the directional distribution of a magnetic field in a model electric motor. Because of the limited wavelength range that we could use in the present analysis, the polarization distribution image of the model motor was confirmed to reflect the directional distribution of the magnetic field outside the motor. It was also clarified that usage of shorter wavelength neutrons would enable us to visualize the field inside the motor. Then polarized neutron imaging was found to be a potentially powerful method for observing the directional distribution of the driving field of a rotating motor.

1.  Introduction

Electric motors, which are widely used in many electric machines to produce mechanical force from electric power, consume large amounts of the world’s electricity. Thus, improving the energy efficiency of electric motors is very important for the reduction of power loss. Evaluating the magnetic field in a motor under driving state and comparing it with the design value are necessary for the development of motors with high energy efficiency. However, there is no appropriate technique to observe the field distribution in a practical motor, and hence, the actual field distribution in a motor under driving state has not been evaluated before now. Neutron imaging [1] is a promising technique to solve such problems because it has been used for the non-destructive visualization of internal structures of bulk objects. Recently this technique was applied to a novel magnetic field imaging method using neutron spin analysis [2-3]. In our previous work, we successfully demonstrated quantitative magnetic field imaging using a polarized pulsed neutron beam [4].

Here, we briefly explain the principal of our magnetic field imaging method. Neutrons experience Larmor precession when they travel in a field whose direction is not parallel to their spin vector, and its precession angle \(\varphi\) can be expressed as

\[
\varphi = \omega_L t = \frac{\gamma}{v} \int_{\text{path}} B \cdot ds = \frac{\gamma m \lambda}{h} \int_{\text{path}} B \cdot ds,
\]

(1-1)
where $B$ is a given magnetic field, $\omega_B$ is an angular velocity of the precession proportional to $B$, $t$ is the time a neutron travels through the field, $\gamma$ is the gyromagnetic ratio of a neutron, $h$ is Plank’s constant, and $m$, $v$, and $\lambda$ are the mass, velocity, and wavelength of the neutron, respectively. According to this equation, $\varphi$ is proportional to the neutron wavelength and strength of the field integrated along the neutron flight path. Then, by measuring the wavelength dependence of the neutron polarization, which is a projection of the neutron spin vector onto a quantized axis of neutron polarization, we can observe the precession of the neutron spin as an oscillatory behavior in the wavelength dependent polarization. Assuming that the magnetic field along the neutron beam trajectory is uniform, the expression of such oscillations was described by Rekveldt [5] as

$$P = D \cdot P_0 = \begin{bmatrix}
1 - (1 - \cos \varphi) \cdot (1 - n_z^2) & (1 - \cos \varphi) n_y n_z - n_z \sin \varphi & (1 - \cos \varphi) n_y n_z + n_z \sin \varphi \\
(1 - \cos \varphi) n_x n_z - n_z \sin \varphi & 1 - (1 - \cos \varphi) \cdot (1 - n_x^2) & (1 - \cos \varphi) n_x n_y - n_y \sin \varphi \\
(1 - \cos \varphi) n_x n_y + n_y \sin \varphi & (1 - \cos \varphi) n_y n_z - n_z \sin \varphi & 1 - (1 - \cos \varphi) \cdot (1 - n_y^2)
\end{bmatrix} \cdot P_0,$$

(1-2)

where $P_0$ and $P$ are the polarization vectors of the incident and transmitted neutron beam, and $n_i$ is a directional cosine of each component of the field. By choosing the direction of $P_0$ and $P$ appropriately, each component of depolarization matrix $D$ can be measured, and then the integrated field strength and direction can be determined using these relations. Furthermore, the analysis of the wavelength dependent polarization with spatial resolution enables us to obtain a quantitative field image of a bulk object and the surrounding space. This method is thought to be suitable for observing the field distribution in moving products like rotating motors, because we can perform the measurement nondestructively with a large field of view up to several centimeters. However, there are some anticipated difficulties in applying this method to the measurement of actual products, e.g., the field cannot be assumed as homogeneous along the beam trajectory neither in direction nor in strength, the strength of the field integral is too strong to be analyzed, etc. In this study, we performed polarized neutron imaging of a model electric motor using a pulsed neutron beam and investigated the relationship between observed polarization images and the field of the motor using simulations.

2. Experimental procedures

Magnetic field imaging experiments using a polarized pulsed neutron beam were performed at beamline BL10 “NOBORU” [6] in the Materials and Life Science Experimental Facility (MLF) of J-PARC. Figure 1 shows a schematic illustration of our experimental setup. In this system, the $z$ direction was taken along the path direction of the neutron beam, and we obtained polarization distribution images in the $xy$ plane. The pulsed neutrons were polarized and analyzed using a polarizer and analyzer that consisted of stacked bend magnetic super-mirrors [7]. The quantized axis of neutron polarization for the polarizer and analyzer was chosen as the $y$ (vertical) direction. We set collimators behind both the polarizer and analyzer to remove neutrons reflected from the super-mirrors. To obtain the neutron polarization, $P$, we flipped the neutron spin polarity of the incident beam using an adiabatic fast passage (AFP) spin flipper [8] that was placed behind the polarizer and measured the neutron transmission intensity with the spin flipper switched off ($I_{off}$) and on ($I_{on}$). The neutron polarization was then obtained by calculating $P = (I_{on} - I_{off})/(I_{on} + I_{off})$ [9]. Additionally, spin rotators, composed of two pairs of orthogonally arranged spin-precession coils which are used to control the polarization direction of the incident and transmitted beam, were located together with the object in the magnetic shield chamber to reduce the effect of any environmental magnetic fields [10]. A damping current is applied to the precession coils in inverse proportion to time and synchronized with the generation of a neutron pulse to give a constant spin rotation for a wide wavelength range of neutrons. Since the rotation angle of the neutron spin is proportional to the amplitude of the field applied by the coils, it is possible to adjust the rotation angle of the polarization vector arbitrarily by controlling the applied electric currents. The $\mu$PIC-based Neutron Imaging Detector ($\mu$NID) [11] was
used as a two-dimensional detector. Distance between the neutron source and the object was 14.18 m, and the distance between the source and the detector was 14.87 m.

Figure 2(a) shows a photograph of a small model motor that we used as an object. This motor is a brushless-type that consists of a central rotor, which is a quadrupole magnet of samarium cobalt (SmCo), and a stator with driving coils around the rotor. The gap distance between the rotor and the stator is about 2 mm. In this experiment, we are especially interested in the quantification of the field in this gap because it is an important characteristic for determining the performance of this type of motor. As a pilot study to visualize the magnetic field in the moving motor, we performed an imaging experiment with a model motor in a static state and attempted to evaluate its magnetic field. The integrated field strength in the gap was roughly estimated to be several T·mm from the magnetization of the rotor magnet and width of the motor. The expected period of polarization oscillation that will appear in the analysis of the field in the gap was shorter than 0.035 Å from equation (1-1). Analyzing such short-period oscillation seems to be difficult due to the restriction of our polarization analysis equipment, which has a minimum bin width of wavelength $\delta \lambda$ longer than 0.01 Å originating from the intrinsic pulse width of our neutron source. Therefore, quantification of the field strength in the gap by analyzing the frequency of polarization oscillation was difficult. On the other hand, focusing on a diagonal term of equation (1-2), the polarization degree $P$ of neutrons that pass through a strong magnetic field can be expressed as follows,

$$P / P_0 = 1 - \left( 1 - \langle \cos \varphi \rangle \right) \cdot \sin^2 \theta = \cos^2 \theta \quad (2-1)$$

where $\theta$ is the angle between the magnetic field and the polarization direction of the neutron. While the oscillation term, which depends on the Larmor precession angle $\varphi$, is smeared out due to the coarse $\delta \lambda$, information about field direction, which corresponds with $\theta$, is still observable in the polarization distribution images. Accordingly, by analyzing the change in polarization depending on the polarization direction using spin rotators, the field direction can be evaluated independent of the strength of the field. In addition, this analysis method is also applicable to continuous sources because the energy-resolved analysis is not necessary.

An imaging experiment was performed to confirm the validity of this method in analyzing the motor field. The polarization vector of the incident beam, which is along the $y$ direction, is rotated into the desired direction by the first pair of precession coils located in front of the object. After transmission through the object, the polarized neutrons enter the second pair of precession coils behind the object where the initial rotation is exactly reversed. In this way, we controlled the neutron spin orientation of the incident and transmitted beam and measure the depolarization component along any arbitrary direction.

![Figure 1 A schematic illustration of the experimental setup at BL10](image)

### 3. Results and Discussion

#### 3.1. Evaluation of the field direction in the motor using polarized neutron imaging

We positioned the model motor with the rotation axis parallel to the $z$ direction in order to observe the field in the gap. In this arrangement, the field in the gap was mainly directed in the $xy$ plane, and then
we measured polarization distribution images while changing the direction of $P_0$ in the $xy$ plane. Figure 2(d) shows polarization distribution images of the model motor in a static state obtained for various polarization directions. As can be seen from this figure, the polarization distribution in the gap gradually changes as the polarization direction is turned, rotating 90 degrees when polarization direction was turned from the $y$ to the $x$ direction. This result indicates that the directional information of the field in the gap was preserved in the observed polarization images. Figure 2(b) presents the wavelength dependence of $P$ and $P_0$ at the gap position for the polarization direction along the $y$ axis. The oscillatory behavior that is indicative of Larmor precession was not observed in the wavelength dependence of $P$, which was expected from the above discussion. On the other hand, the value of $P$ decreased compared to that of $P_0$. This could be explained by the equation (2-1): the reduction of the polarization degree occurred due to the disagreement between the direction of field in the motor and the polarization direction of $P_0$. We analyzed this polarization change by fitting with the equation (2-1) revised as

$$P/P_0 \propto \cos^2 (\theta' + \alpha),$$

where $\theta'$ is the angle of the polarization direction from the $y$ axis and $\alpha$ corresponds to the angle of the motor field from $y$ axis. It is worthwhile mentioning that we can extract the information of field direction in the $xy$ plane from this analysis even if the motor field has a $z$ component, because it does not affect the phase component of the sinusoidal form in the $\theta'$ dependence of $P/P_0$ but affects the amplitude of the waveform according to the equation (1-2). Figure 2(c) shows the $\theta'$ dependence of $P/P_0$ at the gap position of the motor. We measured several polarization images changing the polarization direction from the $y$ ($0^\circ$) to the $x$ direction ($90^\circ$). The number of directional steps was five: $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, $90^\circ$, which was adequate to fit the experimental results to equation (3-1). In this figure, the averaged value of $P/P_0$ over a wavelength range from 4 to 8 Å was used because $P/P_0$ was almost independent from the neutron wavelength. $P/P_0$ plotted against $\theta'$ clearly exhibited a sinusoidal shape and could be well fitted by the equation (3-1). We performed this analysis position by position and obtained the two-dimensional distribution image of the field direction of the motor by plotting $\alpha$ at each position (Figure 3). From this image, it was found that the field in the left and right side of the rotor was aligned along the $y$ direction and the top and bottom side was along the $x$ direction.

![Figure 2](image)

Figure 2 (a) Photograph of the model motor. The area surrounded by the dashed line indicates the field of view for the polarization images. (b) Wavelength dependence of polarization at the gap position of $(X, Y) = (40 \text{ [ch]}, 20\text{[ch]})$ in the polarization image and that of the incident beam $P_0$ (the polarization
axis is in the $y$ direction). (c) $\theta'$ dependence of $P/P_0$ at the same position as (b). The solid line indicates the result of fitting using equation (3-1). (d) Polarization distribution images obtained with various incident polarization directions (1 [ch]= 0.4 [mm]).

Figure 3 Distribution image of the field direction around the rotor obtained by the analysis of the depolarization as a function of incident polarization direction.

3.2 Field simulation of the model motor and comparison with the experimental result

To confirm the validity of this experimental result, we performed a simulation study of the magnetic field inside and outside the model motor and compared the result with the field distribution obtained from the experiment. Magnetic fields inside and outside the model motor were calculated using the commercial simulation software JMAG [12]. In this simulation, we calculated the $x$, $y$ and $z$ components of the internal field and the leaked field outside the model motor. Then, we determined the field direction in the $xy$ plane by calculating $\tan^{-1}(B_x/B_y)$ where $B_x$ and $B_y$ are the $x$ and $y$ components of the field. Figure 4(a-c) show the distributions of field direction at different positions along the neutron beam trajectory. The distribution of the leaked field outside the motor (Figure 4(c)) was clearly different from that inside the motor (Figure 4(a)). Comparing these figures with the experimental result shown in Figure 3, the distribution for the leaked field (Figure 4(c)) appears to best reproduce the experimental results. The reason why the experimentally obtained distribution image resembles that outside the motor, not inside the motor, can be explained by the adiabatic transition of neutrons in the motor. We calculated the adiabaticity parameter $E$ along the beam path, which was defined as

$$E = \frac{\omega_m}{\omega_B} = \frac{\gamma B}{\frac{d\theta_B}{dz}}$$

(3-2)

where $\omega_m$ was the rotation speed of the magnetic field in the transmission path and $d\theta_B/dz$ was the change of field direction along the path. If the change of field direction is much slower than the Larmor precession ($\omega_m >> \omega_B$), the component of the polarization vector parallel to the field is conserved. Such an adiabatic rotation occurs when $E > 10$ [13]. Figure 4(d) shows the calculated adiabaticity parameters in some positions at the gap of the motor. In this calculation, we considered neutrons with the wavelength of 4 Å. $E$ exhibits its maximum at the center of the motor whose value is larger than 500. The value of $E$ decreases to about 100 at the edge of the motor and becomes smaller than 10 at the position 8~9 mm away from the edge. According to these results, we surmised the motion of neutron spin in the motor as follows: (i) Polarized neutrons enter the upstream leaked field region where $E > 10$, maintaining its polarization vector along the controlled direction by the spin rotator since the magnetic field in the region where $E < 10$ is very small. (ii) Then neutron spins experience Larmor precession around the field in the region where $E > 10$. The direction of the component of the polarization vector parallel to the field follows the direction of the field by adiabatic rotation. (iii) After passing through the motor field, neutrons travel through the region where $E < 10$ again, and then the component of the polarization vector selected by the spin rotator is projected to the
polarization axis of the analyzer. The directional information of the component of the polarization vector parallel to the field is a key information in estimating field direction. However, this information is changed by the adiabatic rotation in the beam path, and then information of the directional distribution in the motor is masked by that of the leaked field. In this way, the distribution image of the field direction obtained by this analysis is thought to become that of the leaked field where $E \sim 10$. In order to check the correctness of this supposition, we simulated the polarization distribution image by calculating the Larmor precession of the neutron spin inside and outside the motor using the magnetic field simulation results.

![Figure 4 Distribution images of the field direction](image)

**3.3 Simulation of the polarization distribution images**

We calculated the Larmor precession of a neutron spin in the motor based on the magnetic field simulation result and simulated the polarization distribution image of the model motor. The motion of the neutron spin in a given magnetic field can be expressed as

$$\frac{d\sigma}{dt} = \gamma (\sigma \times B),$$  \hspace{1cm} (3-3)

where $\sigma_n$ is a unit vector parallel to the neutron spin. We calculated a change of neutron spin vector along the beam path by numerically solving the above equation, and the neutron polarization $P$ was obtained by projecting the final state of $\sigma_n$ onto the polarization vector of the analyzer $A$ by calculating

$$P = \frac{\sigma_n A_x + \sigma_y A_y + \sigma_z A_z}{||\sigma_n||A},$$  \hspace{1cm} (3-4)

and averaging over the same wavelength range as used in the experiment (i.e., from 4 Å to 8Å). By performing these calculations position by position, we simulated the polarization distribution images using the results of the motor field in Section 3.2. We took into account the approximate beam divergence for the experimental condition at BL10 and assumed that polarized neutrons were only precessed by the magnetic field of the motor. Figure 5(a) shows the polarization images calculated...
from the simulation results for several polarization directions (corresponding to those measured during the experiment). The analysis of section 3.1 was performed on these simulated polarization images, and the distribution image of the field direction obtained by this calculation is shown in Figure 5(b). The simulated polarization distribution images and corresponding directional distribution image exhibit a similar tendency to that obtained from the experimental data. These results justify the correctness of our experimental results and the supposition about the reason why the obtained image of field direction was similar to that outside the motor.

On the other hand, comparing the distribution image of the field direction obtained by this analysis (Figure 5(b)) and that predicted by the field simulation (Figure 4(c)), it is seen that the two results do not perfectly coincide with each other. The disagreement originates not only from image blurring due to the beam divergence but mainly from the incorrect assumption of a uniform field distribution along the $z$ direction in the conversion from polarization distribution images to the directional distribution image using the equation (3-1). This interpretation means that a uniform field along the $z$ direction should be exactly quantified. To check the validity of this method, we performed another simulation where we removed the leaked field component from the result of the field simulation of Section 3.2 and performed the polarization simulation without beam divergence. Since the field distribution in the motor is almost uniform along the beam path direction, the directional distribution of the field should be precisely visualized in this condition. Figure 6 shows the polarization images for several polarization directions and the distribution image of the field direction obtained by the analysis. The directional distribution obtained by this simulation agrees well with that in the motor (Figure 4(a)). This result shows that in certain cases of restricted geometries (e.g. in-plane) and a uniform field along the flight path, our analysis method can quantify the orientation distribution of the field vector with respect to two dimensions. While such a suitable target is limited, this method is useful to quantify the field direction no matter how strong its magnitude is.

In the case of a field direction whose distribution changes along the beam path, the influence from such field variation should be carefully considered, and the precise determination of such a field is difficult using the present analysis method. One idea for discussing the field distribution of the motor is to use the polarization distribution images themselves. Although it was difficult to straightforwardly quantify the field direction of the motor, we confirmed that the polarization distribution image could be predicted from a prior knowledge of the field distribution. Using polarization distribution images, we can directly compare the experimental results with predictions of simulation without any assumptions and discuss where actual field distribution deviates from the designed one. This measurement will be helpful to inspect the distribution of real magnetic fields of industrial products. It remains to be investigated, however, whether the differences of model simulations and measurements would originate in variations of reality with respect to the idealized model or from certain issues of the measurement technique.

In our analysis method, the main reason why we can only obtain the directional information outside the motor comes from the limitation in wavelength range of the available polarized neutron beam, which was dictated by the polarizer performance. Since the adiabaticity parameter $E$ decreases inversely proportional to the velocity of the neutron, utilization of polarized neutrons with shorter wavelengths $\lambda < 1$ Å will enable us to visualize inner field information of the motor. Moreover, it will become possible to evaluate the strength of the field in the motor with such short-wavelength polarized neutrons, because analysis of shorter-period oscillation in wavelength dependent polarization becomes possible by means of a narrower $\delta \lambda$ in such a wavelength range. Finally, we mention about the possible application of this technique. An attractive subject is to observe an actual driving motor. The polarization distribution measurements will be a help to identify the critical problem of an energy loss of the motor, which cannot be found only from the design work based on the numerical simulations. So as to utilize this technique, it is necessary to combine with the AC field observation method, which we have already established in our previous work [14]. Then this technique enables us to visualize the time-dependent change of the polarization distribution, and we can investigate the movement of the field of the rotating motor. Thus, our magnetic field imaging method
using polarized pulsed neutrons is regarded to contribute to the development of high efficiency motors as one of the useful tools for the characterization of driving motors.

Figure 5 (a) Polarization distribution images of the motor field obtained by the simulation of Section 3-2. The white arrows indicate polarization direction. (b) Distribution image of the field direction obtained by the analysis.

Figure 6 (a) Polarization distribution images of the motor without leaked field. The white arrows indicate polarization direction. (b) Distribution image of the field direction obtained by the analysis.

4. Conclusion
We attempted to quantify the directional distribution of a magnetic field in a model electric motor using polarized pulsed neutron imaging. We analyzed the change in the polarization distribution image depending on the incident polarization direction and obtained the distribution image of the field direction. Using simulation, the field distribution image was confirmed to be similar to that of the leaked field from the motor, and the information of the field inside the motor could not be extracted due to the limitation of the present analysis method. However, the polarization distribution image is considered to be useful to investigate the field distribution of a motor because the polarization distribution image directly reflects the directional distribution of the field outside the motor and we can directly compare experimental results with simulations. In addition, utilization of shorter-wavelength polarized neutrons in imaging experiments will enable to investigate inner field distribution of the motor.

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