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Analysis of Necessity of FDA Selecting Optimal Frequency Interval

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Abstract. The relationship between the output signal to jamming and noise ratio (SJNR) of frequency diverse array (FDA) and the correlation coefficient between the jamming and signal steering vector are deduced. The influence of the change of signal and jamming position on the output SINR is obtained. When jamming is located in mainlobe, the improper frequency interval will sharply reduce the output SINR and form a null. The detection ability of the FDA radar will also drop sharply and the null will change periodically with the selection of frequency interval. Both the theories and simulations verified this periodic null and demonstrated the necessity for selecting FDA's optimal frequency interval.

1. Introduction
In 2006, Antonik[1] pointed out that compared with phased arrays, frequency diverse array (FDA) inserts a small frequency interval between adjacent elements and could form a range-angle-dependent beam, which is periodic in the distance, angle and time[2-3]. It can point to different distances from same angle, which makes FDA have the ability of mainlobe jamming suppression[4-5].

In a complex and changing environment, the traditional FDA uses a fixed frequency interval, which does not make the FDA radar always have the best performance of target detection. As a result, a lot of research has been conducted on the frequency interval of FDA. Based on the calculation of the frequency interval of cognitive radar, Basit proposed a method to make the FDA adaptively control beam to point to the target location[6]. Khan pointed out that frequency interval can be adaptively selected according to the time variation and the period of time can be eliminated[7]. Reference [8] and [9] point out that the frequency interval corresponding to the maximum output SINR can be used as optimal frequency interval to improve the anti-interference ability in different environments. In addition, by selecting the best frequency interval for sub-arrays with different carrier frequencies, the accuracy of target position estimation is improved by minimizing the Cramér-Rao lower bound (CRLB) in [10]. In Reference [11-12], Gao separately analyzed the effects of the frequency interval error on beam forming and target location estimation based on FDA radar and FDA-MIMO radar.

However, although the above methods of selecting the FDA optimal frequency interval are given, neither of them analyzes the necessity of selecting the optimal frequency interval. In light of this, this paper in depth analyzes the impact of jamming and target location on output SINR. When the jamming is located in the mainlobe, the output SINR appears deep null with the increase of the frequency interval and the null exhibit periodic changes. If the frequency interval is not properly selected, it will severely affect the anti-interference performance of FDA radar. Therefore, the selection of optimal frequency interval of FDA is of great significance.
2. Basic model

In a standard uniform linear array FDA with \( N \) elements, the distance between two adjacent array elements is \( d \). The signals radiated by the array are far-field narrow-band signals and the first element is selected as the reference element. The array model is shown in Figure 1.

![Figure 1. FDA model](image)

The radiated frequency from the \( m \)th element is

\[
f_m = f + (m-1)\Delta f \quad m = 1, 2, \ldots, N
\]  

(1)

Where, \( f \) is the FDA carrier frequency and \( \Delta f \) is the frequency interval between the adjacent elements. Signal radiated by the \( m \)th element arriving at the position of target with angle \( \theta \) and range \( r \) (that could express as \((\theta, r)\)) can be expressed as

\[
s_m(t; \theta, r) = w_m \exp \left\{-j2\pi f_m(t - \frac{r_m}{c_o})\right\}
\]  

(2)

Where, \( c_o \) is the speed of light. The distance between the target and the \( m \)th element is \( r_m \) and \( r_m = r - (m-1)d \sin \theta \). \( w_m \) represents the amplitude information of the signal.

The phase difference between the \( m \)th element and first element is given by

\[
\Delta \phi_m = \phi_m - \phi_1 = -2\pi \Delta f \frac{r}{c_o} (m-1) + 2\pi f \frac{d(m-1)\sin \theta}{c_o} + 2\pi \Delta f \frac{d(m-1)^2 \sin \theta}{c_o}
\]  

(3)

Noticed that \( f \gg N\Delta f \), the item 3 is negligible compared with the phase difference brought by the first two terms. Therefore, regardless of amplitude information, the FDA’s transmit pattern can be expressed as

\[
P(t; \theta, r) = \frac{\sin \left[N\pi \left(\Delta ft - \frac{\Delta f}{c_o} r + \frac{d}{\lambda_0} \sin \theta\right)\right]}{\sin \left[N\pi \left(\Delta ft - \frac{\Delta f}{c_o} r + \frac{d}{\lambda_0} \sin \theta\right)\right]} \exp \left\{j\pi(N-1) \left(\Delta ft - \frac{\Delta f}{c_o} r + \frac{d}{\lambda_0} \sin \theta\right)\right\}
\]  

(4)

From (4), it is obvious that FDA’s beam direction is related to the angle and distance of the target and array, which is called range-angle coupling.

When the formula (5) is satisfied, the pattern takes the extreme value.

\[
\pi \left(\Delta ft - \frac{\Delta f}{c_o} r + \frac{d}{\lambda_0} \sin \theta\right) = \pm i\pi \quad i = 1, 2, \ldots, n
\]  

(5)

It can be found that FDA’s beam is periodic in angle, distance and time.

3. Output SINR null theoretical analysis

The third one received the highest target gain of the three FDA radar acceptance methods, so the third acceptance method is adopted for analysis. When the FDA is receiving, it needs to extract the signals of \( N \) frequency received by each element and rearrange them and then do signal processing. So the FDA’s elements from the original \( N \) virtual expand into \( N \times N \). For ease of mathematical analysis, suppose the location of one goal is \((\theta_s, r_s)\). One jamming is located at \((\theta_j, r_j)\). The noise is a Gaussian
white noise with a mean of 0 and a variance of $\delta^2_n$, which has no correlation with signal and jamming. Then the jamming plus noise covariance matrix is given by

$$R_{\delta n} = (\delta^2_v n + n) \times (\delta^2_v n + n)^H = \delta^2_v v_n^H + \delta^2 I$$

(6)

Where, $\delta^2_v$ is the jamming power, $v_n$ is the jamming steering vector and $I$ is the unit matrix.

$$R_{\delta n} = \frac{1}{\delta^2_v} I - \frac{\delta^2_v v_n^H}{\delta^2_v + N^2 \delta^2_v}$$

(7)

Filtered by the optimal beam former, the output SINR is

$$SINR_{out} = \delta^2_v v_n^H R_{\delta n}^{-1} v = \frac{\delta^2_v}{\delta^2_v + N^2 \delta^2_v} \left( v_n^H v_j \right)^2$$

(8)

When the noise power, signal power, jamming power and number of elements are fixed, the output SINR is only related to $v_n^H v_j$. The correlation coefficient between the target and jamming vector is defined as $\rho_{n} = \frac{v_n^H v_j}{\|v_n\|\|v_j\|}$, It is a constant value because of $\|v_n\|\|v_j\| = N^2$. The output SINR is

$$SINR_{out} = \frac{\delta^2_v}{\delta^2_v + N^2 \delta^2_v} \rho_{n}^2$$

(9)

When the target and jamming position are fixed, the parameters of the variation are only the frequency interval $\Delta f$. Therefore, by minimizing the correlation between the target vector and jamming vector, the output SINR can be maximum. That is

$$\min \|v_n^H v_j\|$$

(10)

On the contrary, if the correlation between the target and the jamming vector is stronger, $\rho_{n}^2$ will be larger and the output SINR will be smaller. In particular, under the circumstances that the jamming power is much larger than the noise power, when the target vector is perturbed with jamming vector, the output SINR will drop to 0.

The impact of the target and jamming location on the output SINR is further analyzed below.

$$v_j = \left[ e^{j 2\pi \frac{d}{\lambda_{0}} \sin \theta_j}, \cdots, e^{j 2\pi \frac{d}{\lambda_{0}} (N-1) \sin \theta_j} \right]^T$$

(11)

Let $B_j = \left[ e^{j 2\pi \frac{d}{\lambda_{0}} \sin \theta_j}, \cdots, e^{j 2\pi \frac{d}{\lambda_{0}} (N-1) \sin \theta_j} \right]^T$, $x_j = \frac{d}{\lambda_{0}} \sin \theta_j$, $y_j = 2 \frac{\Delta f}{c_0} r_j$. So

$$v_j^H = [B_j^H, B_j^H e^{j 2\pi \frac{d}{\lambda_{0}} (y_j - 1)}, \cdots, B_j^H e^{j 2\pi \frac{d}{\lambda_{0}} (N-1)(y_j - 1)}]$$

(12)

Similarly,

$$v_j = \left[ B_j e^{j 2\pi (x_j - 1)}, \cdots, B_j e^{j 2\pi (N-1)(x_j - 1)} \right]^T$$

(13)

Where, $B_j = \left[ e^{j 2\pi \frac{d}{\lambda_{0}} \sin \theta_j}, \cdots, e^{j 2\pi \frac{d}{\lambda_{0}} (N-1) \sin \theta_j} \right]^T$, $x_j = \frac{d}{\lambda_{0}} \sin \theta_j$, $y_j = 2 \frac{\Delta f}{c_0} r_j$. There is

$$\|v_n^H v_j\| = \left| \sin \left( \pi \Delta f \left( \frac{2r_n - 2r_j}{c_0} \right) + \frac{\pi d}{\lambda} N \left( \sin \theta_j - \sin \theta_n \right) \right) \right|$$

(14)
When above formula taking the maximum value, the target and the jamming vector are also the most relevant and the output SINR is also the smallest. So there is

\[
\begin{aligned}
\pi d \left( \frac{2r_i - 2r_j}{c_o} \right) + \frac{\pi d}{\lambda} (\sin \theta_j - \sin \theta_i) &= k\pi \\
\pi d \left( \sin \theta_j - \sin \theta_i \right) &= p\pi
\end{aligned}
\]

(15)

Where \(k, p\) is respectively any integer. The jamming distance corresponding to maximum the \(\rho_o\) is

\[r_j = r_i - \frac{c_o}{2\Delta f} q\]

(16)

Where \(q\) is any integer. The period of jamming distance is \(\frac{c_o}{2\Delta f}\). There is

\[\Delta f = \frac{c_o}{2(r_i - r_j) q}\]

(17)

It can be seen that the maximum correlation between the jamming and target steering vector will show periodic changes. There will be a periodic null in the graph of the change of output SINR with frequency interval and the period is \(\frac{c_o}{2(r_i - r_j)}\).

It should be noted here in (15) that \(\frac{\pi d}{\lambda} (\sin \theta_j - \sin \theta_i) \in [-1, 1]\) because \(\frac{d}{\lambda} = \frac{1}{2}\). So if the jamming angle is slightly different from the target, it is difficult to satisfy the condition \(\frac{\pi d}{\lambda} (\sin \theta_j - \sin \theta_i) = p\pi\) and the correlation between the target and the jamming vector is greatly reduced. Therefore, it can be found that the angle difference between the target and jamming has a greater influence on the correlation of the steering vectors than distance difference. The correlation between the target and jamming vector is high only when the jamming is located in the mainlobe and the phenomenon of periodic null appears.

When there is only sidelobe jamming, the correlation between the target and the steering vector is very low and the output SINR is always high. At this moment, it is not meaningful to select the optimal frequency interval. Therefore, the selection of the optimal frequency interval is mainly carried out in the presence of mainlobe jamming and the anti-interference performance can also be greatly improved.

4. Simulation and analysis

Suppose the number of FDA elements is 20 and the element spacing is half of a wavelength. The carrier frequency and frequency interval respectively is 3GHz and 5kHz. The position of target is located at \((0, 50\text{km})\) and the jamming is located anywhere in the angle range of \((-90^\circ, 90^\circ)\) and the distance range of \((0, 100\text{km})\). Figure.2 is a two-dimensional map of the correlation coefficient between the jamming and the target steering vector. It is obvious that the period is 30km from Figure.3 and it is consistent with \(\frac{c_o}{2\Delta f} = \frac{3 \times 10^8}{2 \times 5 \times 10^7} = 30\text{km}\). From Figure.2, it can be seen that when the angle difference between the jamming and the target is greater than one beam width (that is not in the mainlobe), the correlation between the two vectors is greatly reduced, which is consistent with the theoretical analysis.
The following simulation of the target and jamming location on the output SINR is further analyzed. The target position is located at $(0, 50km)$. Assuming the one jamming is located at $(0^\circ, 100km)$, which made the jamming is in the mainlobe. The frequency interval selection range is $[0: 200Hz:20kHz]$. Figure 4 is the simulation diagram of the correlation coefficient between the jamming and target steering vector and Figure 5 is the change of the output SINR with the frequency interval.

It can be seen from the Figure 4 and Figure 5 that the frequency interval period of the peak of correlation interval and zero-collapse of output SINR is $3kHz$, which is consistent with the theoretical calculation:

$$\frac{c_0}{2(r_j - r_t)} = \frac{3 \times 10^8}{2 \times (100 - 50) \times 10^9} = 3kHz.$$ 

The jamming location is changed to $(10^\circ, 80km)$, which made the jamming is not in the mainlobe. Simulation figures are as follows.
It can be found from Figure 6 and Figure 7 that although the correlation coefficients show periodic changes when the jamming is not in the mainlobe, it is clear that the output SINR no longer has obvious periodic null due to the sharp decrease of the correlation. In this case, it is found that no matter how the frequency interval changes, the output SINR is always high. Therefore, it is of little significance to select the optimal frequency interval under the condition of only the sidelobe jamming.

5. Conclusions
In this paper, the necessity of choosing the optimal frequency interval of FDA radar is mainly and deeply studied. The relationship between output SINR and the correlation coefficient between target and jamming vector are derived and the concrete impact of the relative position of target and jamming on output SINR are obtained. FDA radar by using the fixed frequency interval can get high SINR and need not to choose the optimal frequency interval when only the sidelobe jamming is present. In the case of mainlobe jamming, selecting an improper frequency interval will cause the output SINR to drastically decrease, which will seriously affect the anti-interference performance of FDA radar. So it is necessary to choose FDA optimal frequency interval.

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