Heterotic Black Horizons

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Abstract

We show that the supersymmetric near horizon geometry of heterotic black holes is either an $AdS_3$ fibration over a 7-dimensional manifold which admits a $G_2$ structure compatible with a connection with skew-symmetric torsion, or it is a product $\mathbb{R}^{1,1} \times S^8$, where $S^8$ is a holonomy $Spin(7)$ manifold, preserving 2 and 1 supersymmetries respectively. Moreover, we demonstrate that the $AdS_3$ class of heterotic horizons can preserve 4, 6 and 8 supersymmetries provided that the geometry of the base space is further restricted. Similarly $\mathbb{R}^{1,1} \times S^8$ horizons with extended supersymmetry are products of $\mathbb{R}^{1,1}$ with special holonomy manifolds. We have also found that the heterotic horizons with 8 supersymmetries are locally isometric to $AdS_3 \times S^3 \times T^4$, $AdS_3 \times S^3 \times K_3$ or $\mathbb{R}^{1,1} \times T^4 \times K_3$, where the radii of $AdS_3$ and $S^3$ are equal and the dilaton is constant.
1 Introduction

It has been known for some time, following Israel’s uniqueness proof for the Schwarzschild black hole [1] and the early results of [2, 3], that the most general rotating asymptotically flat black hole solution in four dimensions is the Kerr solution [4] which is characterized by its mass and angular momentum. These results have also been generalized to black holes with electric and magnetic charges [5, 6]. Under certain assumptions, four-dimensional black holes exhibit spherical horizon geometry; for a recent review of these classic results as well as an extended set of references see [7]. In five dimensions, there are no generic black hole uniqueness theorems, and most of the investigations have focused either on the supersymmetric case, or on static solutions. It has been shown [8] that the near horizon geometries of supersymmetric black holes are either the near-horizon geometry of the BMPV black hole [9], or $AdS_3 \times S^2$, or $\mathbb{R}^{1,1} \times T^3$; and so the horizon is either a (squashed) $S^3$, or $S^1 \times S^2$ or $T^3$, respectively. For the first two cases, the full black hole solutions, and not just the near horizon geometries, are known. In particular, it has been shown in [8] that the only supersymmetric, regular, asymptotically flat black hole which has the near-horizon BMPV solution as its near horizon geometry is the BMPV black hole [9]; when the BMPV black hole is static, the near horizon geometry simplifies to $AdS_2 \times S^3$. It is also known that the supersymmetric black ring found in [10] has near-horizon geometry $AdS_3 \times S^2$ (which is also the near horizon geometry of the black string [11]); however in this case it is not known if the supersymmetric black ring is the unique solution with this near-horizon geometry. No black hole solution has been found with near horizon geometry $\mathbb{R}^{1,1} \times T^3$. Uniqueness theorems have also been constructed in various theories for static black holes in higher dimensions [12, 13], and for black holes that admit a sufficient number of commuting rotational isometries [14, 15, 16, 17, 18]. An effective worldvolume theory for higher dimensional black branes has also been recently proposed in [19].

The main aim of this paper is to investigate the near horizon geometry of supersymmetric heterotic black holes. For this, we shall use the solution of the Killing spinor equations (KSE) of heteroric supergravity presented in [20, 21, 22] and adapt the results to the near horizon geometry of supersymmetric black holes. The latter is described in a Gaussian null co-ordinate system adapted to the stationary Killing vector field of a black hole. The description of this coordinate system as well as the definition of the near horizon geometry are reviewed in section 2. We shall mostly focus on the near horizon geometries for which the 3-form flux $H$ is closed. This is the case whenever the anomalous contribution to the Bianchi identity vanishes. However, we shall also investigate the geometry of the solutions with $dH \neq 0$ and point out the differences between the two cases.

Supersymmetric solutions of heterotic supergravity always admit a null, $\hat{\nabla}$-parallel, and so Killing, vector field $V$, where $\hat{\nabla}$ is the connection whose skew-symmetric torsion is the 3-form flux of the theory. Moreover, the supersymmetric solutions for which the holonomy of $\hat{\nabla}$ is compact admit at least one time-like $\hat{\nabla}$-parallel, and so Killing, vector field. It is clear from this that all heterotic supersymmetric black holes, and so their near

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1 It is likely that there is no black hole solution with $\mathbb{R}^{1,1} \times T^3$ near horizon geometry and this background is just the vacuum of a compactification of 5-dimensional supergravity to two dimensions.
horizon geometries, admit at least one null Killing vector field defined everywhere on the black hole spacetime. The compactness of the horizon imposes additional restrictions on the geometry. In particular, there are near horizon geometries which preserve 2, 4, 6 and 8 supersymmetries. The geometry of spacetime for this class is a principal bundle $P(G, B; \pi)$ with fibre group $G = SL(2, \mathbb{R}) \times H$, where $H = \{1\}$, $U(1)$ and $SU(2)$, and the base space $B$ has structure group $G_2$, $SU(3)$ and $SU(2)$, respectively. The structure group of $B$ is compatible with a metric connection $\tilde{\nabla}$ with skew-symmetric torsion. In the cases with 2, 4 and 6 supersymmetries, the $SL(2, \mathbb{R})$ subgroup is gauged over $B$ with a $U(1)$ connection, and the fibration can be geometrically twisted. The dilaton may not be constant and typically depends on the coordinates of $B$. However, in the cases with 8 supersymmetries, the spacetime is a product $AdS_3 \times M_7$ and the dilaton is constant.

We furthermore show that all other near-horizon geometries have a constant dilaton, and the 3-form flux vanishes. In this case, the near horizon geometries that admit one supersymmetry are isometric to $\mathbb{R}^{1,1} \times S^8$, where $S^8$ is a holonomy $Spin(7)$ manifold. Moreover, there are also solutions $\mathbb{R}^{1,1} \times S^8$ which preserve 2, 3, 4 and 8 supersymmetries provided that $S^8$ has holonomy $SU(4)$ (Calabi-Yau) and $G_2$, $Sp(2)$ (hyper-Kähler), $\times^2Sp(1)$ and $SU(3)$ (Calabi-Yau), and $Sp(1)$ (hyper-Kähler) manifold, respectively. A more detailed exposition which includes the geometry of the horizons can be found in table 1.

The near horizon geometries with 8 supersymmetries are also classified. We prove that they are isometric up to discrete identifications, to $AdS_3 \times S^3 \times T^4$, $AdS_3 \times S^3 \times K_3$ and $\mathbb{R}^{1,1} \times T^4 \times K_3$. In the first two cases, the fibre group $G = SL(2, \mathbb{R}) \times SU(2)$, $SL(2, \mathbb{R}) = AdS_3$ and $SU(2) = S^3$ does not twist over the base space $T^4$ or $K_3$ and the solution is a product. The radii of $AdS_3$ and $S^3$ are equal, the dilaton is constant and the 3-form field strength is the sum of the volume forms of the non-abelian groups in the product. Moreover, we demonstrate that $AdS_3 \times S^3 \times T^4$ does not receive $\alpha'$ corrections.

Our paper is organized as follows. In section 2, we examine the relation between near horizon geometries and supersymmetry, emphasizing some of the silent features. In section 3, we solve the KSEs for heterotic horizons which exhibit one supersymmetry. In section 4, we show heterotic horizons with non-trivial fluxes necessarily exhibit at least 2 supersymmetries, and the spacetime admits a $G_2$ structure. In section 5, we analyze the KSEs for heterotic horizons with extended supersymmetry, and find that the solutions with non-trivial fluxes preserve 2, 4, 6 and 8 supersymmetries. In sections 6, 7 and 8, we solve the KSEs for heterotic horizons with non-trivial fluxes and identify the spacetime geometry. In particular in section 8, we classify all heterotic horizons that preserve 8 supersymmetries. In section 9, we first examine the $\alpha'$ corrections of heterotic horizons. Then we compare our heterotic horizon geometries with those that arise in the context of brane configurations, and discuss the regularity of the dilaton. In section 10, we give our conclusions. In appendices A and B, we have collected some calculations necessary for the analysis of the KSEs.

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2If the dilaton is allowed to be singular on the horizon, $AdS_3 \times S^3 \times S^3 \times S^1$ is also a solution. In particular, the dilaton depends linearly on the angular coordinate of $S^1$ and so it is not periodic.
2 Near horizon geometry and $N = 1$ supersymmetry

2.1 Near horizon limit for extreme black holes

In what follows, we shall focus on black holes for which the event horizon is a Killing horizon, i.e. there is a time-like (stationary) Killing vector field $K$ defined everywhere on the spacetime which becomes null only on the horizon. It has been shown for non-extremal black holes in higher-dimensional Einstein-Maxwell theory that the event horizon is a Killing horizon, and furthermore there must exist at least one rotational Killing vector field. A similar analysis has been carried out for extremal solutions of Einstein-Maxwell-Dilaton theory and modulo certain technical assumptions, the same result holds. However, for the heterotic theory under consideration here, we shall simply assume that the event horizon is a Killing horizon. Therefore $H$ is identified with the hyper-surface given by $g(K, K) = 0$. Under this assumption, one can adapt Gaussian null coordinates to $K$ and the black hole metric near $H$ can be written as

$$ds^2 = 2(dr + rh_I dy^I + rf du) du + \gamma_{IJ} dy^I dy^J,$$

(2.1)

where $K = \partial_u$ and the components of the metric depend on $r, y$. The $r$ coordinate is chosen such that $H$ is located at $r = 0$. Since $g(K, K) = rf$, $K$ becomes null at the horizon as expected. Regularity at the horizon requires that at $r = 0$ the metric is non-singular; typically the components of the metric are taken to be analytic in $r$.

The expression (2.1) for the metric is not unique. If $V$ is any other Killing vector field such that

$$g(V, V)|_H = 0, \quad L_V g(K, K) = 0,$$

(2.2)

one can introduce a Gaussian null co-ordinate system adapted to $V$. The expression for the spacetime metric is as in (2.1) but now $V = \partial_u$. We shall use this arbitrariness in adapting a Gaussian null co-ordinate system in the investigation of supersymmetric black holes later. The spatial horizon section $S^8$ given by $u = \text{const}, r = 0$, with metric $ds^2_S = \gamma_{IJ} dy^I dy^J$, is required to be compact. The above analysis can also be adapted in the presence of fluxes, like a Maxwell field, or supergravity form field strengths.

Since the components of the metric in (2.1) are analytic in $r$, one has an expansion

$$h_I(y, r) = \sum_{n=0}^{\infty} \frac{r^n}{n!} \partial^n_r h_I|_{r=0},$$

$$f(y, r) = \sum_{n=0}^{\infty} \frac{r^n}{n!} \partial^n_r f|_{r=0},$$

$$\gamma_{IJ}(y, r) = \sum_{n=0}^{\infty} \frac{r^n}{n!} \partial^n_r \gamma_{IJ}|_{r=0}.$$

(2.3)

Some black hole properties depend on the first few non-vanishing terms in the above analytic expansions. In particular, calculating the surface gravity, one finds that

$$i_K dK|_{r=0} = -f(y, 0) K|_{r=0},$$

(2.4)
where we have used the same symbol to denote the vector field $K$ and the associated 1-form. Thus if $f(y, 0) \neq 0$, the black hole has temperature. So for extreme black holes, one should take $f(y, 0) = 0$.

To define the near horizon geometry of an extreme black hole, we perform the coordinate transformation

$$r \rightarrow \epsilon r \, , \quad u \rightarrow \epsilon^{-1} u \, , \quad y^I \rightarrow y^I$$

and in the resulting metric, we take the limit $\epsilon \rightarrow 0$. After taking this limit, the metric reads as

$$ds^2 = 2(dr + rh_Idy^I + r^2\Delta du)du + \gamma_{IJ}dy^Idy^J \, ,$$

where now $h_I$ and $\gamma_{IJ}$ are evaluated at $r = 0$ and $\Delta = \partial_r f|_{r=0}$. Observe that if $f(y, 0) \neq 0$, the above limit does not exist. Thus the near horizon geometry can only be defined for extreme black holes.

The near horizon geometry (2.6) of an extreme black hole admits, in addition to $K = \partial_u$, an extra Killing vector field

$$D = -r\partial_r + u\partial_u \, ,$$

associated with the scale symmetry $r \rightarrow \ell r \, , \quad u \rightarrow \ell^{-1} u$ which does not commute with $K$, i.e.

$$[K, D] = K \, .$$

In the presence of other fields, like Maxwell or supergravity form field strengths, one can extend the definition of the above limit. In particular for heterotic supergravity, the theory admits a 2-form gauge potential $B$. So one has

$$B = bdu \wedge dr + b_I dr \wedge dy^I + c_I du \wedge dy^I + b_{IJ}dy^I \wedge dy^J \, ,$$

where all components depend on $y^I$ and $r$ coordinates. Assuming analyticity in the components of $B$ in the $r$ coordinate, one can define the near horizon gauge potential by taking the limit (2.5) provided that $c_I(y, 0) = 0$. This condition is similar to the extremality restriction for the metric. After taking the limit, the gauge potential can be rearranged as

$$B = r du \wedge N + S du \wedge (dr + rh_Idy^I) + W \, ,$$

where now $S$ is a scalar function, and $N$ and $W$ are 1- and 2-forms on $S^8$, respectively. The $b_I$ component of $B$ vanishes in the limit. Observe that $B$ is also invariant under the scale symmetry generated by Killing vector (2.7). One therefore concludes that the scale symmetry is a generic feature of the near horizon geometries. The dilaton $\Phi$ in the near horizon limit depends only on the $y$ coordinates.

For later use, we collect the heterotic fields in the near horizon limit as

$$ds^2 = 2e^+e^- + \delta_{ij}e^ie^j \, ,$$
\[ H = e^+ \wedge e^- \wedge (dS - N - Sh) + re^+ \wedge (h \wedge N - dN - Shd) + dW , \]
\[ \Phi = \Phi(y) , \]

where \( H := dB \),
\[ e^+ = du , \quad e^- = dr + rh + r^2 \Delta du , \quad e^i = e^i dy^j , \]
and \( \gamma_{IJ} = \delta_{ij} e^i I e^j J \).

### 2.2 Supersymmetry

The supersymmetric heterotic backgrounds have been classified in [20, 21]. There are two large classes of supersymmetric heterotic backgrounds depending on whether the holonomy of the connection, \( \nabla \), with skew-symmetric torsion \( H \), is a subgroup of a compact or non-compact isotropy group of \( \nabla \)-parallel spinors. We shall first focus on the non-compact case and in particular on the backgrounds which preserve one supersymmetry. For these backgrounds the holonomy of \( \nabla \) is contained in \( \text{Spin}(7) \ltimes \mathbb{R}^8 \) and admit a local frame \((e^+, e^-, e^i)\) such that
\[ \nabla e^- = 0 , \quad \nabla (e^- \wedge \phi) = 0 , \]
where
\[ \phi = \frac{1}{4!} \phi_{i_1...i_4} e^{i_1} \wedge \cdots \wedge e^{i_4} , \]
is the self-dual fundamental form of \( \text{Spin}(7) \). This is the full content of the gravitino KSE. The dilatino KSE implies that
\[ \partial_+ \Phi = 0 , \quad d e^- \in \text{spin}(7) \oplus \mathbb{R}^8 , \quad 2 \partial_+ \Phi = (\theta_\phi)_{ij} + H_{++i} , \]
where \( \theta_\phi = -\frac{1}{6} \ast (\ast \tilde{d} \phi \wedge \phi) \) is the Lee form of \( \phi \) and \( \tilde{d} \) is the exterior derivative projected along the directions transverse to the light-cone. For a detailed explanation of these results and for our notation, see [20, 21].

The metric and 3-form field strength can then be expressed as
\[ ds^2 = 2 e^- e^+ + \delta_{ij} e^i e^j \]
\[ H = d(e^- \wedge e^+) + e^- \wedge L + \tilde{H} , \quad \tilde{H} = \frac{1}{3!} H_{i_1 i_2 i_3} e^{i_1} \wedge e^{i_2} \wedge e^{i_3} , \]
where \( L \in \text{spin}(7) \) is not determined by the KSEs and \( \tilde{H} \) can be expressed in terms of \( \phi \) and its first derivatives as
\[ \tilde{H} = - \ast \tilde{d} \phi + \ast (\theta_\phi \wedge \phi) . \]

The expression for \( \tilde{H} \) is as that for 8-dimensional manifolds with \( \text{Spin}(7) \)-structure and compatible connection with skew-torsion [26]. This is the full content of the KSEs.
2.3 Supersymmetric heterotic black holes

Supersymmetric black holes are those black holes of supergravity theories that also satisfy
the KSE, and the Killing spinor vector bilinear(s) are well-defined everywhere on the
spacetime, and in particular analytic in $r$ near the horizon.

Suppose that $V$ is a Killing vector field constructed as spinor bi-linear. If $g(V, V) = 0$
at $H$ and the black hole is extreme, using $g(K, K) = 0$, one can show that

$$\mathcal{L}_V g(K, K)|_H = 0,$$

i.e. a Gaussian null coordinate system can be adapted to $V$ as well and the metric can
be written as in (2.1).

Now let us turn to the heterotic case. Supersymmetric backgrounds in heterotic theory
admit always a null $\nabla$-parallel, and so Killing, vector field constructed as a Killing spinor
bi-linear. Depending on the number of supersymmetries and the holonomy of $\nabla$, they
may admit a time-like and several space-like $\nabla$-parallel, and so Killing, Killing spinor
vector bi-linears. The time-like and spacelike $\nabla$-parallel vector fields cannot be used to
adapt a null Gaussian coordinate system for a black hole. This is because their length is
constant and so they do not vanish anywhere on the spacetime. So it remains to consider
the null vector bilinears $V$. Since $V$ is null, $g(V, V) = 0$ everywhere on the spacetime
and so on the horizon as well. Moreover for extreme black holes one also has (2.18) and so a
Gaussian null coordinate system can be adapted to $V$. For such a system $V = \partial_u$ is null
and so one has $f = 0$. If in addition, we take the near horizon limit now adapted to $V$,
the heterotic fields are given in (2.16) and (2.17) but now with $\Delta = 0$.

It is not apparent what kind of supersymmetric black holes one should expect to be
present in heterotic supergravity. Asymptotically flat black holes with fluxes flowing over
an 8-sphere at infinity may be ruled out because of the presence of an everywhere null
Killing vector field which suppresses the dependence on a radial direction that it is needed
for the appropriate decay of the fields.

The class of supersymmetric black holes that is expected to be present consists of
the Kaluza-Klein (KK) black holes. It may seem worrying that if KK black holes are
solutions of heterotic supergravity, there is not a time-like Killing vector field constructed
from Killing spinor bilinears which becomes null at the horizon. However, this is not
necessary. Some cases are known for which, after lifting a black hole solution from a
lower-dimensional theory to 10 or 11 dimensions, the stationary time-like Killing vector
field becomes null everywhere on the spacetime. This happens, for example, when we lift
the asymptotically $AdS_5$ black hole of 5-dimensional supergravity \cite{27} to IIB supergravity.
Note though that our assumptions in 10 dimensions require that the spacetime admits a
time-like Killing vector field $K$ which becomes null at the horizon. However $K$ may not
be written as a Killing spinor bilinear. Nevertheless it is an additional restriction on the
geometry of such black hole spacetimes.

We shall solve the heterotic KSEs at the near horizon limit by adapting a Gaussian
null coordinate system to the null Killing vector field constructed from a Killing spinor bi-
linear. We shall refer to all these solutions as heterotic black horizons or simply horizons.
However, it is not apparent that all these geometries can be extended to a black hole

\footnote{This is an assumption and it may not follow from the KSEs of supergravity theories.}
spacetime. It is likely that some of them are simply the Kaluza-Klein vacua of compactifications of heterotic supergravity to two dimensions. Nevertheless the heterotic horizons include all the near horizon geometries of extreme supersymmetric heterotic black holes.

3 \textbf{N}=1 heterotic horizons

3.1 \textbf{N}=1 supersymmetry

As we have explained in the previous section, we adapt Gaussian null coordinates to the vector field constructed as a bilinear of the Killing spinor of these backgrounds. In general, the natural frame adapted to supersymmetric backgrounds \((e^+, e^-, e^i)\) is distinct to that associated with the Gaussian null coordinates \((e^+, e^-, e^i)\). But, as we have explained, the Gaussian null coordinates are taken with respect to the null Killing spinor bilinear, \(e^- = e^-\). Moreover, we shall show in appendix A that without loss of generality, one can set

\[
e^- = e^- , \quad e^+ = e^+ , \quad e^i = e^i ; \quad \Delta = 0 ,
\]

where \(\Delta\) vanishes because the Killing vector field is null. Comparing the expression for \(H\) in (2.16) and (2.11), one finds that

\[
ds^2 = 2e^-e^+ + ds^2 , \quad d\tilde{s}^2 = \delta_{ij}e^i e^j ,
\]

\[
H = d(e^- \wedge e^+) + \tilde{H} , \quad \tilde{H} = dW ,
\]

\[
\Phi = \Phi(y) ,
\]

where

\[
e^- = dr + rh e^i , \quad e^+ = du , \quad e^i = e^i_I dy^I ,
\]

see also (2.12), and the Killing spinor is

\[
\epsilon = 1 + e_{1234} .
\]

It is essential in what follows to notice that \(h\) is a \textit{globally} defined 1-form on the horizon section \(S^8\). Observe that the term involving \(L\) in (2.16) vanishes in the near horizon limit.

The conditions that arise in the dilatino KSE (2.15) can be rewritten as

\[
dh \in \text{spin}(7) , \quad 2\partial_I \Phi + h_I = (\theta_\phi)_I .
\]

The only condition that the gravitino KSE imposes is that the submanifold \(S^8\) given by \(r = u = \text{const}\) has a \(\text{Spin}(7)\) structure. This is because every manifold with a \(\text{Spin}(7)\) structure admits a compatible connection with skew-symmetric torsion given in (2.17).

In particular, one has that

\[
\hat{\nabla}_I \phi_{j_1 \ldots j_4} = 0
\]

where \(\hat{\nabla}\) is the connection with skew-symmetric torsion constructed from the data \((d\tilde{s}^2, \tilde{H})\) of \(S^8\), and \(\phi\) is the fundamental self-dual 4-form of \(\text{Spin}(7)\). However, the closure of \(H\) given in (2.17) is not implied by the KSEs and has to be imposed as an additional constraint.
3.2 Field equations

It is known that the KSEs, the field equations of the 3-form flux, the $E_{-\cdot}$ component of the Einstein equations and $dH = 0$ imply all the rest of the field equations of heterotic supergravity $[20, 21]$. So to find the heterotic horizons, we have to solve in addition to the KSEs some of the field equations of the theory. Using the special geometry of heterotic horizons (3.2), we decompose the field equation of the 3-form flux

$$d \ast (e^{-2\Phi} H) = 0$$

(3.7)

in terms of the various forms defined on $S^8$ as

$$\tilde{\nabla}_i (h^i e^{-2\Phi}) = 0,$$

(3.8)

$$e^{2\Phi} \tilde{\nabla}_j (e^{-2\Phi} (dh)^{ji}) + \frac{1}{2} (dh)_{jk} \tilde{H}^{jki} + h_j (dh)^{ji} = 0,$$

(3.9)

$$e^{2\Phi} \tilde{\nabla}_k (e^{-2\Phi} \tilde{H}^{kij}) + (dh)^{ij} - h_k \tilde{H}^{kij} = 0,$$

(3.10)

where here the frame indices are those of $S^8$, and $\tilde{\nabla}$ is the Levi-Civita connection of $d\tilde{s}^2$. In what follows, we shall also use the field equation of the dilaton

$$\tilde{\nabla}^2 \Phi - 2\tilde{\nabla}^i \Phi \tilde{\nabla}_i \Phi + \frac{1}{12} \tilde{H}_{ijk} \tilde{H}^{jki} - \frac{1}{2} h_i h^i = 0,$$

(3.11)

and the $E_{ij}$ component of the Einstein equations

$$\tilde{R}_{ij} = \frac{1}{4} \tilde{H}_{imn} \tilde{H}_j^{mn} - 2\tilde{\nabla}_i \tilde{\nabla}_j \Phi - \tilde{\nabla}_{(i} h_{j)} ,$$

(3.12)

where $\tilde{R}_{ij}$ denotes the Ricci tensor of $S^8$.

3.3 Solutions

3.3.1 $h = 0$

We first consider solutions for which $h = 0$. In this case, the dilaton field equation (3.11) can be written as

$$\tilde{\nabla}^2 e^{-2\Phi} = \frac{1}{6} e^{-2\Phi} \tilde{H}_{ijk} \tilde{H}^{jki} .$$

(3.13)

Hence, the maximum principle implies that $\Phi$ is constant and $\tilde{H} = 0$. It follows that $H = 0$ and the spacetime metric is

$$ds^2 = ds^2(\mathbb{R}^{1,1}) + ds^2(S^8) .$$

(3.14)

Moreover $S^8$ is a compact holonomy $Spin(7)$ manifold [28]. Examples of such manifolds can be found in [29]. Such geometries are also the vacua of heterotic compactifications to two dimensions.
Before we proceed to examine the remaining cases, it is worth mentioning that the dilaton field equation together with compactness impose strong restrictions on the existence of solutions. This is the case irrespective of whether the solution is supersymmetric or not, but it is dependent on the couplings. For this consider solutions with metric

$$ds^2 = ds^2(\mathbb{R}^{n,1}) + ds^2(X),$$

for which $H$ is either purely magnetic or purely electric, and all fields depend only on the coordinates of $X$. If $X$ is compact, then it is clear that the dilaton field equation implies that only solutions are those that have constant dilaton, $H = 0$ and $X$ Ricci-flat. This is in agreement with the more general results in [30]. A similar conclusion can be reached using the KSEs [31].

### 3.3.2 $\tilde{H} = 0$

Suppose that $\tilde{H} = 0$. In this case, (3.6) implies that $S^8$ is a holonomy $Spin(7)$ manifold. As a result the Lee form $\theta$ vanishes and the dilatino KSE implies that

$$h = -2d\Phi.$$  \hspace{1cm} (3.16)

Substituting this condition into the dilaton equation (3.11), one obtains

$$\hat{\nabla}^2 e^{-4\Phi} = 0.$$  \hspace{1cm} (3.17)

Again compactness of $S^8$ and the maximum principle implies that $\Phi$ is constant. In turn, (3.16) gives $h = 0$. Thus the solutions we find are identical to those of the previous section.

### 3.3.3 $h \neq 0$ and $\tilde{H} \neq 0$

It remains to investigate the heterotic horizons for which both $h$ and $\tilde{H}$ are non-vanishing. It turns out that such solutions always preserve at least two supersymmetries, and both the spacetime $M$ and $S^8$ admit a $G_2$ structure. We examine this case in the following section.

## 4 The $G_2$ structure of heterotic horizons

Let us assume that $h$ and $\tilde{H}$ do not vanish and that the heterotic horizons admit one supersymmetry. To prove that such heterotic horizons admit two supersymmetries and that the holonomy of the connection $\hat{\nabla}$ reduces to $G_2$, we shall show first that the $Spin(7)$ holonomy of $\hat{\nabla}$ reduces to $G_2$.

### 4.1 $\hat{\nabla}$ has $G_2$ holonomy

To proceed with the analysis, we shall first demonstrate that

$$h^i \partial_i \Phi = 0.$$  \hspace{1cm} (4.1)
For this first compute \( h^2 \) by contracting (3.5) with \( h \) and substitute the resulting expression into the dilatino field equation (3.11) to obtain
\[
\bar{\nabla}^2 \Phi - 4 \bar{\nabla}_i \Phi \bar{\nabla}^i \Phi + \frac{1}{3} \phi^{i_1 i_2 i_3} \bar{\nabla}_i \Phi \bar{H}_{n_1 n_2 n_3} + \frac{1}{8} \phi^{n_1 n_2 m_1 m_2} \bar{H}_{n_1 n_2 \ell} \bar{H}_{m_1 m_2} = 0 \, .
\] (4.2)

Next taking the covariant derivative of (3.5) to compute the divergence of \( h \), and using (3.6) to eliminate covariant derivatives of \( \phi \) and the field equation (3.8), one obtains
\[
\bar{\nabla}^2 \Phi - 2 \bar{\nabla}_i \Phi \bar{\nabla}^i \Phi + \frac{1}{6} \phi^{i_1 i_2 i_3} \bar{\nabla}_i \Phi \bar{H}_{n_1 n_2 n_3} + \frac{1}{8} \phi^{n_1 n_2 m_1 m_2} \bar{H}_{n_1 n_2 \ell} \bar{H}_{m_1 m_2} = 0 \, .
\] (4.3)

Moreover comparing (4.2) with (4.3), one finds that
\[
\bar{\nabla}^2 \Phi + \frac{1}{6} \phi^{i_1 i_2 i_3} \bar{\nabla}_i \Phi \bar{H}_{n_1 n_2 n_3} + \frac{1}{8} \phi^{n_1 n_2 m_1 m_2} \bar{H}_{n_1 n_2 \ell} \bar{H}_{m_1 m_2} = 0 \, .
\] (4.4)

Taking the inner product of (3.5) with \( d \Phi \) and using (4.5), one derives (4.1).

The next step is to prove the identity
\[
\bar{\nabla}^2 h^2 + (\bar{\nabla} \Phi - h^i \bar{\nabla}^i) h^2 = 2 \bar{\nabla}_i (h^i) \bar{\nabla}^i (h^j) + \frac{1}{2} (dh - i_h \bar{H})_{ij} (dh - i_h \bar{H})^{ij} \, ,
\] (4.6)
where \( h^2 = h_i h^i \). For this write
\[
\bar{\nabla}^2 h^2 = 2 \bar{\nabla}_i h^i \bar{\nabla}^i h^2 + 2 h^j \bar{\nabla}^2 h^j = 2 \bar{\nabla}_i h^i \bar{\nabla}^i h^2 + 2 \bar{\nabla}^i (dh - i_h \bar{H})_{ij} h^j + 2 \bar{\nabla}_i (dh - i_h \bar{H})^{ij} \, .
\] (4.7)

Next using the field equations (3.9) and (3.12), and (4.1) and after some re-arrangement of terms, one obtains (4.6).

Applying the maximum principle on (4.6) using the compactness of \( S^8 \), we find that \( h^2 \) must be constant, and hence the RHS of (4.6) must vanish identically. Therefore, we have that
\[
\bar{\nabla}_i (h^i) = 0 \, , \quad dh = i_h \bar{H} \, .
\] (4.8)

These two conditions are equivalent to requiring that
\[
\hat{\nabla} h = 0 \, .
\] (4.9)

Thus \( h \) is a \( \hat{\nabla} \)-parallel vector of \( S^8 \). Since the isotropy group of a non-vanishing element in the spinor representation of \( Spin(7) \) holonomy is \( G_2 \), the holonomy of \( \hat{\nabla} \) is contained in \( G_2 \).

Using the above results, one can show the identities
\[
i_h dh = 0 \, ,
\] (4.10)
\[
\mathcal{L}_h \bar{H} = 0 \, ,
\] (4.11)
and
\[
\mathcal{L}_h \phi = 0 \, .
\] (4.12)

The first identity follows from the properties that \( h^2 \) is constant and \( h \) is Killing, the second follows from the second condition in (4.8) and \( d \bar{H} = 0 \), and the third follows from the first equation in (3.5) and (3.6).
4.2 Killing spinor equations revisited

Before we proceed to prove that the spacetime admits an additional supersymmetry, it is convenient to reexamine the KSEs of the backgrounds assuming that they admit at least one supersymmetry, using the results of the previous section. Suppose that $\epsilon$ is a Killing spinor. The gravitino KSE implies that $\epsilon$ does not depend on $r$. Moreover

$$\eta_{\pm} = \frac{u}{2} h_i \Gamma^{-i} \eta_- + \eta_+ + \eta_- , \quad \Gamma_{\pm} \eta_{\pm} = 0 ,$$

(4.13)

where $\eta_{\pm} = \eta_{\pm}(y)$, solves the gravitino KSE, iff

$$\tilde{\nabla} \eta_{\pm} = 0 ,$$

(4.14)

$$dh_{ij} \Gamma_{ij} \eta_{\pm} = 0 .$$

(4.15)

In addition $\epsilon$ solves the dilatino KSE, iff

$$\left( (2d \Phi \mp h) i \Gamma^i - \frac{1}{6} \bar{H}_{ijk} \Gamma^{ijk} \right) \eta_{\pm} = 0 .$$

(4.16)

Furthermore, it suffices to solve (4.14)-(4.16) for either $\eta_- \text{ or } \eta_+$. Notice that if there is a solution $\eta_+$, then there is another solution with $\eta_- = \Gamma_i h_i \eta_+$, and vice versa. This is because $\Gamma^{-i} h_i$ and $\Gamma^{-i} h_i$ commute with (4.14) and (4.15), and anti-commute with (4.16) up to a change of sign in the $h$ term. One can demonstrate this by using the relations (4.8), (4.9) and (4.10) of the previous section. This will simplify the analysis for all heterotic horizons that admit more than one supersymmetry.

4.3 $N=2$ supersymmetry and $G_2$ holonomy

To construct the second Killing spinor, we set $\eta_+ = 1 + e_{1234}$. This spinor satisfies the KSEs (4.14)-(4.16) because these are the conditions that arise on the geometry from the requirement that the solutions admit one supersymmetry. Moreover, we set $\eta_- = h_i \Gamma^i \eta_+$ and substitute this into (4.13) to find that the two linearly independent Killing spinors are

$$\epsilon^1 = 1 + e_{1234} , \quad \epsilon^2 = -k^2 u (1 + e_{1234}) + h_i \Gamma^i (1 + e_{1234}) ,$$

(4.17)

where $k^2 = k^2$ is the constant length of $h$. It is easy to see from the results of [20] that the isotropy group of both Killing spinors in $Spin(9,1)$ is $G_2$. Therefore the holonomy of $\tilde{\nabla}$ reduces to a subgroup of $G_2$.

4.4 Geometry

4.4.1 Geometry of spacetime

To investigate the geometry of spacetime, we first compute the 1-form bi-linears $\lambda$ associated with the Killing spinors (4.17) to find

$$\lambda^- = e^- , \quad \lambda^+ = e^+ - \frac{1}{2} k^2 u^2 e^- - uh , \quad \lambda^1 = k^{-1} (h + k^2 u e^-) .$$

(4.18)
Moreover, the associated vector fields $\xi_a$, $a = -, +1$, satisfy the Lie bracket algebra

$$[\xi_+, \xi_-] = -k\xi_1, \quad [\xi_+, \xi_1] = k\xi_+, \quad [\xi_-, \xi_1] = -k\xi_-, \quad (4.19)$$

which is isomorphic to $\mathfrak{sl}(2, \mathbb{R})$. It is then a consequence of the classification results of [20] that the spacetime is a principal bundle $M = P(SL(2, \mathbb{R}), B^7; \pi)$ with fibre group $SL(2, \mathbb{R})$, base space $B^7$ and principal bundle connection $\lambda$. Moreover the spacetime metric and 3-form flux can be written as

$$ds^2 = \eta_{ab}\lambda^a \lambda^b + d\tilde{s}^2_{(7)}, \quad H = CS(\lambda) + \tilde{H}_{(7)}, \quad (4.20)$$

where $d\tilde{s}^2_{(7)}$ and $\tilde{H}_{(7)}$ is a metric and 3-form flux of $B^7$, respectively, ie $d\tilde{s}^2_{(7)}$ and $\tilde{H}_{(7)}$ are orthogonal to the directions $\lambda^a$. Moreover

$$CS(\lambda) = \frac{1}{3}\eta_{ab}\lambda^a \wedge d\lambda^b + \frac{2}{3}\eta_{ab}\lambda^a \wedge F^b, \quad (4.21)$$

is the Chern-Simons form of $\lambda$, where

$$F^a = d\lambda^a - \frac{1}{2}H^a_{b_1b_2}\lambda^{b_1} \wedge \lambda^{b_2}, \quad (4.22)$$

is the curvature of $\lambda$. The dilaton $\Phi$ depends only on the coordinates of $B^7$.

The curvature of $\lambda$ is non-vanishing and so the $SL(2, \mathbb{R})$ fibre twists over the base space $B^7$. In particular, one finds that

$$F^+ = -u(1 + \frac{1}{2}k^2ru)dh, \quad F^- = rdh, \quad F^1 = k^{-1}(1 + k^2ru)dh. \quad (4.23)$$

Moreover a straightforward calculation reveals that

$$CS(\lambda) = du \wedge dr \wedge h + rdu \wedge dh + k^{-2}h \wedge dh. \quad (4.24)$$

Since $F$ has only one independent component determined by $dh$, it is clear that only an abelian subgroup of the $SL(2, \mathbb{R})$ fibre is gauged. As expected $F^a$ is a 2-form over $B^7$, because $i_h dh = 0$, and a $G_2$-instanton, i.e.

$$dh \in \mathfrak{g}_2. \quad (4.25)$$

This can been seen from (4.15). The $G_2$ fundamental form is

$$\varphi = k^{-1}i_h \phi, \quad (4.26)$$

where $\phi$ is the fundamental $Spin(7)$ form. The conditions that arise from the dilatino KSE (4.16) are

$$k - \frac{1}{6}H_{ijk}\varphi^{ijk} = 0, \quad \theta_\varphi = 2d\Phi, \quad (4.27)$$

12
where now $i, j, k = 2, 3, 4, 6, 7, 8, 9$.

The geometric data $(\tilde{d}s^2_7, \tilde{H}(7))$ of $B^7$ are compatible with a $G_2$ structure, i.e. the metric connection, $\hat{\nabla}(7)$, on $B^7$ with skew-symmetric torsion $\tilde{H}(7)$ has holonomy contained in $G_2$. This in turn determines $\tilde{H}(7)$ in terms of the fundamental $G_2$ form $\varphi$ as

$$\tilde{H}(7) = k\varphi + e^{2\Phi} \ast_7 d(e^{-2\Phi} \varphi) .$$

(4.28)

In contrast to the $Spin(7)$ case, not all 7-dimensional manifolds with a $G_2$ structure admit a compatible connection with skew-symmetric torsion $\tilde{H}(7)$ has holonomy contained in $G_2$. For this to hold, one must in addition have $d[e^{-2\Phi} \ast_7 \varphi] = 0$.

Furthermore, in the $G_2$ holonomy case all the field equations of the heterotic supergravity are implied provided that $dH = 0$. Therefore, the only remaining equations that one has to solve are

$$d[e^{-2\Phi} \ast_7 \varphi] = 0 , \quad k^{-2} dh \wedge dh + d\tilde{H}(7) = 0 , \quad (dh)_{ij} = \frac{1}{2} \ast_7 \varphi_{ij}^k (dh)_{kl} .$$

(4.29)

The first is the geometric condition on the $G_2$ structure of $B^7$, the second arises from $dH = 0$ and the last is equivalent to $dh \in g_2$ (4.25). The geometric condition implies that the manifold $B^7$ must be conformally co-calibrated. The $dH = 0$ condition is more involved and it is reminiscent of the equations that one solves for heterotic supergravity after taking into account the one-loop anomalous contribution. Of course, we have assumed that the anomaly cancels since we have taken $dH = 0$. But from the perspective of the base space $B$, the equation that $\tilde{H}$ obeys is similar to that which would hold if there were an anomalous contribution. We have not been able to prove that it admits non-trivial solutions. To summarize, the spacetime metric and 3-form field strength can be written as

$$ds^2 = \eta_{ab} \lambda^a \lambda^b + d\tilde{s}_7^2 ,

H = du \wedge dr \wedge h + rdu \wedge dh + k^{-2} h \wedge dh + k\varphi + e^{2\Phi} \ast_7 d(e^{-2\Phi} \varphi) ,$$

(4.30)

subject to the conditions (4.29).

### 4.4.2 Geometry of $S^8$

The geometry of $S^8$ can be investigated separately from that of the spacetime. This is because the geometry of the KSEs (4.14)-(4.16) can be analyzed without reference to the original 10-dimensional spacetime. To proceed from now on we shall reserve the Latin indices $i, j, k$ for the base space $B$ in each case and denote the indices of directions transverse to the light-cone, and so also those of the horizon section $S^8$, with $\underline{i}, \underline{j}, \underline{k}$. In this notation $\hat{\nabla}$-parallel spinors are

$$\eta_+^\underline{i} = 1 + e_{1234} , \quad \eta_-^\underline{i} = \Gamma^+ h_+^\underline{i}(1 + e_{1234}) .$$

(4.31)

The geometric condition can also be written as $d \ast_7 \varphi = \theta_\varphi \wedge \varphi$ and this corrects a sign in [20] [21].
Table 1. Some geometric data of the horizon geometries with \( h = 0 \) are described. In the first column, we give the different spacetime geometries that occur. In the second column, we present the holonomy groups of the associated spacetime Levi-Civita connection. In the third and fourth column, we describe the number of parallel spinors and representatives of the parallel spinors, respectively. HK and CY stand for hyper-Kähler and Calabi-Yau manifolds, respectively. \( T^n \) is the \( n \)-dimensional torus.

The isotropy group of both spinors in \( \text{Spin}(8) \) is \( G_2 \) and so the holonomy of \( \hat{\nabla} \) is contained in \( G_2 \). The associated \( \hat{\nabla} \)-parallel bilinears are

\[
h, \varphi, \quad (4.32)
\]

where \( \varphi = k^{-1}i_h\phi \) and \( \phi \) is the fundamental \( \text{Spin}(7) \) form. As in the spacetime case, \( h \) can be viewed as the connection of a \( S^1 \) bundle over a base space \( B^7 \). The conditions that arise from the dilatino KSE are given in (4.27). The metric and torsion of \( S^8 \) can be written as

\[
d\tilde{s}^2 = k^{-2}h \otimes h + ds^2(7), \quad \tilde{H} = k^{-2}h \wedge dh + \tilde{H}(7), \quad (4.33)
\]

where \( ds^2(7) \) and \( \tilde{H}(7) \) are given in previous section, see eg (4.28). Moreover, they satisfy (4.29).

5 Extended supersymmetry

5.1 \( h=0 \)

There are two cases to consider depending on whether \( h \) vanishes. If \( h = 0 \), we have seen that the spacetime is a product \( \mathbb{R}^{1,1} \times S^8 \), where \( S^8 \) is a holonomy \( \text{Spin}(7) \) manifold. The flux \( H \) vanishes and the dilaton is constant. Substituting these into the KSEs, they reduce to a parallel transport equation for the Levi-Civita connection \( \nabla \) of \( S^8 \). As a result, the solutions with more than one supersymmetry are products, up to discrete identifications, of Minkowski space \( \mathbb{R}^{1,1} \) with those Berger type of manifolds which admit parallel spinors. The results are tabulated in table 1.
Table 2. Some of the geometric data used to solving the gravitino KSE are described. In the first column, we give the isotropy groups, Iso(η+), of \{η+\} spinors in Spin(9,1). In the second column we state the holonomy of the connection with torsion \(\tilde{\nabla}\). The holonomy of \(\tilde{\nabla}\) is identical to that of \(\nabla\). In the third column, we present the number of \(\tilde{\nabla}\)-parallel spinors and in the last column we give representatives of the \{η+\} spinors.

| Iso(η+)             | hol(∇)             | N   | η+             |
|---------------------|--------------------|-----|----------------|
| Spin(7) × R^8       | G_2                | 2   | 1 + e_1234     |
| SU(4) × R^8         | SU(3)              | 4   | 1              |
| Sp(2) × R^8         | SU(2)              | 6   | 1, i(e_{12} + e_{34}) |
| ×^2 Sp(1) × R^8     | SU(2)              | 8   | 1, e_{12}      |

5.2 h̸=0

If \(h \neq 0\), it suffices to investigate the KSEs (4.14)-(4.16) for the η+ spinors. This is because as we have already mentioned the solutions for the η− spinors are given by \(η− = h_2 Γ^+ η_+\). As a result, the heterotic horizons with \(h \neq 0\) always preserve an even number of supersymmetries.

5.2.1 Gravitino

To solve the gravitino KSE for η+, i.e. the parallel transport equation for \(\tilde{\nabla}\) (4.14), it suffices to find the subgroups of Spin(8) which leave invariant spinors in the even chirality Majorana representation of Spin(8). These are

\[\text{Spin}(7) \ (1), \ SU(4) \ (2), \ Sp(2) \ (3), \ ×^2 Sp(1) \ (4), \ Sp(1) \ (5), \ U(1) \ (6), \ \{1\} \ (8)\]

and have been stated in [28, 33, 34], where the number \(N^+\) of invariant η+ spinors is in parentheses. The number of parallel spinors of spacetime is \(N = 2N^+\). In particular in the Sp(1) and U(1) cases, the number of Killing spinors is 10 and 12, respectively. All backgrounds with 10 and 12 supersymmetries are plane waves which in addition are group manifolds [33, 21]. Such solutions do not have AdS_3 as a submanifold and so they must be excluded.

Since both \(η_+\) and \(η− = h_2 Γ^+ η_+\) solve the parallel transport equation for \(\tilde{\nabla}\) (4.14), the holonomy of \(\tilde{\nabla}\) reduces to the isotropy group of both η+ and η− spinors. In particular, it reduces to the subgroups of (5.1) which in addition preserve the parallel vector \(h\). It is straightforward to find all these groups and the results are tabulated in table 2.

5.2.2 Dilatino

As in the general heterotic case, only some of solutions of the gravitino KSE (4.14) are also solutions of the dilatino one (4.16). To find the solutions of the dilatino KSE given a solution of the gravitino KSE, it suffices to focus on the η+ parallel spinors. Since the isotropy groups of η+ spinors in Spin(9,1), given in table 2, are non-compact, the analysis of the dilatino KSE (4.16) is similar to that in [21] for backgrounds for which
hol(\(\nabla\)) is a non-compact group. In particular, one can find the \(\Sigma\)-groups in each case and investigate the orbits of these on the space of \(\{\eta_+\}\) spinors. The end result is that it suffices to investigate only those cases for which all \(\{\eta_+\}\) \(\nabla\)-parallel spinors also solve the dilatino KSE. This is because all the rest are just special cases. For example, in the \(SU(4) \ltimes \mathbb{R}^8\) case in table 2, there are two \(\nabla\)-parallel spinors. It is possible that only a linear combination of them solves the dilatino KSE, i.e. there is one Killing spinor. If this is the case, then such backgrounds will be included in those of \(Spin(7) \ltimes \mathbb{R}^8\), table 2, for which the \(\nabla\)-parallel spinor also solves the dilatino KSE. A similar conclusion holds for all the other cases.

6 \(N=4\) horizons

6.1 Geometry of spacetime

The first two Killing spinors are those of the \(G_2\) case (4.17). The additional two Killing spinors are given by

\[
\epsilon^3 = i(1 - e_{1234}) , \quad \epsilon^4 = -ik^2u(1 - e_{1234}) + ih\Gamma^+\epsilon(1 - e_{1234}) .
\] (6.1)

The isotropy group of all these spinors is in \(SU(3)\). To proceed, we find that a basis for the 1-form bi-linears is

\[
\lambda^+ = e^+ - \frac{1}{2}k^2u^2e^- - uh , \quad \lambda^1 = k^{-1}(h + k^2ue^-) , \quad \lambda^6 = k^{-1}\ell_2e^\perp , \quad \ell_2 = h\Gamma^2 .
\] (6.2)

where the hermitian form of \(I\) is

\[
\omega^{(8)}_1 = -(e^1 \wedge e^6 + e^2 \wedge e^7 + e^3 \wedge e^8 + e^4 \wedge e^9) .
\] (6.3)

All 1-form bi-linears \(\lambda^a, a = -, +, 1, 6\), are \(\nabla\)-parallel.

As in the \(G_2\) case, the associated vector fields \(\xi^a, a = -, +, 1\) to the 1-forms \(\lambda^a\) span a \(\mathfrak{sl}(2, \mathbb{R})\) Lie algebra. It remains to find the commutator of \(\xi_6\) with the other three vector fields. For this observe that (4.15) implies that \(dh\) is orthogonal to all \(\xi^a\) directions and

\[
dh \in \mathfrak{su}(3) .
\] (6.4)

In particular,

\[
dh_{ij}\ell_2 = 0 .
\] (6.5)

Moreover since all \(\xi^a\) are \(\nabla\)-parallel,

\[
[\xi_6, \xi^a] = -i\xi_6i\xi^a H = i\xi_6F_a = 0 , \quad a = -, +, 1 .
\] (6.6)

The last equality follows from (6.5) and (4.23). Therefore, the Lie algebra of the vector fields is

\[
\mathfrak{sl}(2, \mathbb{R}) \oplus \mathfrak{u}(1) .
\] (6.7)
The heterotic horizons with Killing spinors that have isotropy group $SU(3)$ are principal bundles $M = P(SL(2, \mathbb{R}) \times U(1), B^6)$ with fibre group $SL(2, \mathbb{R}) \times U(1)$ equipped with the connection $\lambda$ (6.2). From the general results of [20], the spacetime metric and 3-form flux can be written as

$$ds^2 = \eta_{ab}\lambda^a\lambda^b + d\tilde{s}_6^2 , \quad H = CS(\lambda) + \tilde{H}(6) ,$$

where $d\tilde{s}_6^2$ and $\tilde{H}(6)$ are orthogonal to the $\lambda^a$ directions.

To continue with the investigation of the geometry of spacetime, it remains to determine the geometry of $B^6$. From the results of [20], the spacetime admits a $\hat{\nabla}$-parallel Hermitian 2-form $\omega$ and a $(3,0)$-form $\chi$ which are orthogonal to the $\lambda^a$ directions and are constructed as Killing spinor bi-linears. Moreover, the dilatino KSE implies that

$$\partial_a \Phi = 0 , \quad \theta_\omega = 2d\Phi , \quad F^6_{ij}\omega^{ij} = -2k , \quad (F^6)^{2,0} = 0 , \quad \hat{N}(I) = 0 ,$$

where $I$ is the almost complex structure associated to $\omega$ and $ds^2_6$, the superscript in $F^6$ denotes the $(2,0)$ part of the curvature in a decomposition under $I$ in holomorphic and anti-holomorphic indices and $N$ is the Nijenhuis tensor of $I$. Furthermore, (6.4) implies that

$$F^a_{ij}\omega^{ij} = 0 , \quad (F^a)^{2,0} = 0 , \quad a = -, +, 1 .$$

The base space $B^6$ is a complex manifold. It admits a Hermitian form because

$$i_{\xi_a}\omega = 0 , \quad L_{\xi_a}\omega = 0$$

and so $\omega$ descends to a Hermitian form on $B^6$. The last equation holds because $\hat{\nabla}\omega = 0$, and the $(2,0)$-part of all $F$ curvatures vanishes. The integrability of the almost complex structures follows from the Nijenhuis condition in (6.9). This complex structure is compatible with geometric data $d\tilde{s}_6^2$ and $\tilde{H}(6)$. Furthermore observe that

$$i_{\xi_a}\chi = 0 , \quad a = -, +, 1, 6; \quad L_{\xi_a}\chi = i\kappa \chi , \quad L_{\xi_a}\chi = 0 , \quad a = -, +, 1 .$$

So although $\chi$ is orthogonal to the $\lambda^a$ directions, it is not invariant under the action of $\xi_6$ and so it descends to $B^6$ as a line bundle valued $(3,0)$-form. As a result $B^6$ does not have an $SU(3)$ structure but rather a $U(3)$ one. In particular, the connection with skew-symmetric torsion, $\hat{\nabla}(6)$, on $B^6$ has holonomy contained in $U(3)$. The torsion can be expressed as

$$\tilde{H}(6) = -i_1d\omega = e^{2\Phi} \ast_6 d[ e^{2\Phi}\omega] ,$$

see eg [36, 37, 38, 39, 40, 41, 42, 32]. The equation that remains to be solved is the closure of $H$. In particular, one finds that

$$dH = \eta_{ab}F^a \wedge F^b + d\tilde{H}(6) = k^{-2}dh \wedge dh + k^{-2}d\ell \wedge d\ell + d\left( e^{2\Phi} \ast_6 d[ e^{-2\Phi}\omega] \right) = 0 .$$

As in the $G_2$ case, this is reminiscent of the equations that arise in the heterotic theory in the presence of anomalies. However, there are some differences. The connections
that contribute to the anomaly term are abelian and one of them does not satisfy the Hermitian-Einstein condition but rather it satisfies the Hermitian-Einstein condition with cosmological constant. In addition $B^6$ does not have a $SU(3)$ structure but rather a $U(3)$ one. Moreover there is no relative minus sign between the two terms which depend on the $h$ and $\ell$ in the “anomaly” term. Nevertheless there are sufficient similarities between the above equations and the anomalous Bianchi identity which appears in the anomaly cancelation mechanism to suggest that there may exist solutions analogous to those found for the latter in [43, 44, 45].

6.1.1 Geometry of $S^8$

The first two $\hat{\nabla}$-parallel spinors are as in (4.31). The additional two are

$$\eta^3_+ = i(1 - e_{1234}), \quad \eta^4_+ = i\Gamma^+ h_2(1 - e_{1234}) .$$  

(6.15)

The isotropy group of both spinors in $Spin(8)$ is $SU(3)$. Therefore the holonomy of $\hat{\nabla}$ is contained in $SU(3)$. The associated $\hat{\nabla}$-parallel bilinears are

$$h, \ell, \omega, \chi ,$$  

(6.16)

where $\ell, \omega$ and $\chi$ have the properties mentioned in the previous section. Both $h$ and $\ell$ can be viewed as the connections of a $T^2$ bundle over a base space $B^6$. In fact $S^8$ is a holomorphic $T^2$ fibration over $B^6$. The complex structure on $S^8$ is associated with the Hermitian form

$$\omega(8) = k^{-2} h \wedge \ell + \omega .$$  

(6.17)

The metric and torsion of $S^8$ can be written as

$$ds^2 = k^{-2}(h \otimes h + \ell \otimes \ell) + d \tilde{s}^2_6, \quad \tilde{H} = k^{-2}(h \wedge dh + \ell \wedge d\ell) + \tilde{H}_6 ,$$  

(6.18)

where $d \tilde{s}^2_6$ and $\tilde{H}_6$ are given in previous section. Since $h$ and $\ell$ commute, one can adapt coordinates such that

$$h = d\tau + p_i e^i, \quad \ell = d\sigma + q_i e^i ,$$  

(6.19)

where $p, q$, and all the other components of the fields, depend only on the coordinates of $B^6$.

7 N=6 horizons

7.1 Geometry of Spacetime

The first four Killing spinors are the same as those given for the $SU(3)$ case in (4.17) and (6.1). For the additional two Killing spinors, it can be shown that they can be expressed as

$$\epsilon^5 = i(e_{12} + e_{34}) , \quad \epsilon^6 = -ik^2 u(e_{12} + e_{34}) + ih_2\Gamma^+(e_{12} + e_{34}) .$$  

(7.1)
The isotropy group of all these spinors is in $SU(2)$. To proceed, we find that a basis in the space of 1-form bilinears is

$$\lambda^- = e^- , \quad \lambda^+ = e^+ - \frac{1}{2} k^2 u e^- - u h , \quad \lambda^1 = k^{-1} (h + k^2 u e^-) ,$$

$$\lambda'^r = k^{-1} (\ell'^r) h , \quad (\ell'^r)_h = -(I'^r)_h , \quad r' = 2, 6, 7 \quad (7.2)$$

where $I'^r$ is a triplet of almost complex structures satisfying the algebra of the imaginary unit quaternions

$$I'^r I'^s = -\delta^{rs'} I_{18} + \epsilon^{rs'} t I'^r , \quad \epsilon_{627} = 1 . \quad (7.3)$$

The associated Hermitian forms can be written as

$$\omega^{(8)}_r = e^1 \wedge e^2 + e^3 \wedge e^4 + e^1 \wedge e^2 + e^3 \wedge e^4$$

$$\omega^{(8)}_s = -i (e^1 \wedge e^1 + e^2 \wedge e^2 + e^3 \wedge e^3 + e^4 \wedge e^4)$$

$$\omega^{(8)}_t = -i (e^1 \wedge e^2 + e^3 \wedge e^4 - e^1 \wedge e^2 - e^3 \wedge e^4) , \quad (7.4)$$

in terms of the standard holomorphic frame basis. These three hermitian forms also arise as spinor bilinears, which can be constructed explicitly from the Killing spinors $\epsilon^1, \epsilon^3, \epsilon^5$, as described in Appendix A of [20]. Notice that the isotropy group of $\epsilon^1, \epsilon^3, \epsilon^5$ is $Sp(2) \ltimes \mathbb{R}^8$ and so this case here is closely related to the backgrounds with 3 supersymmetries in [20].

All 1-form bi-linears $\lambda^a$, $a = +, -, 1, 2, 6, 7$ are $\nabla$-parallel, and so in particular their associated vector fields $\xi^a$ are all Killing. As in the $G_2$ case, the vector fields $\xi^a$, for $(a = -, +, 1)$ close under Lie brackets to a $\mathfrak{sl}(2, \mathbb{R})$ Lie algebra. To find the rest of the commutators first observe that (4.15) implies that

$$dh \in \mathfrak{su}(2) . \quad (7.5)$$

As a result, $dh$ is orthogonal to all $\xi^a$ directions. Then an argument similar to that which we have used in equation (6.6) gives

$$[\xi^a, \xi^r] = 0 , \quad a = -, +, 1 . \quad (7.6)$$

It remains to find the commutators $[\xi^r, \xi^s]$. If $[\xi^r, \xi^s]$ cannot be expressed in terms of $\xi^a$, then one can show there will be additional linearly independent $\nabla$-parallel vector fields on the spacetime and therefore the holonomy of $\nabla$ will be reduced to $\{1\}$. The only such solutions are group manifolds and preserve 8 supersymmetries. Thus, we shall take

$$[\xi^r, \xi^s] = -c k \epsilon r s t \xi^t , \quad (7.7)$$

for some constant $c$. The above commutator cannot close in the $-, +$ and 1 directions because of (7.6), and the structure constants are skew.

Since the Killing spinors have isotropy group $SU(2)$, the holonomy of $\nabla$ is contained in $SU(2)$. It is then a consequence of the results of [20, 21] that the spacetime is a principal bundle with fibre the 6-dimensional Lorentzian group $G$ over a 4-manifold $B^4$ such that the spacetime metric and 3-form can be written as

$$ds^2 = \eta_{ab} \lambda^a \lambda^b + e^{2\Phi} d\bar{s}_{(4)}^2$$

19
\[ H = CS(\lambda) + \tilde{H}(4) , \] (7.8)

where we have re-scaled the 4-dimensional metric of the base space with the dilaton \( \Phi \) for later convenience. Moreover the spacetime admits three \( \nabla \)-parallel Hermitian forms \( \omega^r \) which are orthogonal to all \( \lambda^a \) 1-forms and they are compatible with the metric \( d\tilde{s}_{(4)}^2 \). The associated almost complex structures \( I_r \) satisfy

\[ I_r I_s = -\delta_{rs}1_{4\times4} + \epsilon_{rst}I_t . \] (7.9)

This is the full content of the gravitino KSE.

To solve the dilatino KSE, first observe that another consequence of (7.5) is that \( F^a, a = -, +, 1 \) does not contribute. This is because the Killing spinors are \( SU(2) \) invariant and \( F^a, a = -, +, 1 \) is proportional to \( dh \). A direct computation of the remaining dilatino KSE using the \( Sp(2) \times \mathbb{R}^8 \) results of [20], or a comparison with the backgrounds that preserve 6 supersymmetries in [22], reveals that

\[ \partial_a \Phi = 0 , \quad (F^{sd})^r' = \nu \tilde{\omega}^r' , \quad \nu = \frac{1}{2}(k + ck) , \quad \tilde{H}(4) = -\tilde{\ast}_4 de^{2\Phi} , \] (7.10)

where

\[ (F^{sd})^r' = \frac{1}{2}(F^{r'} + \ast_4 F^{r'}) , \quad (F^{ad})^r' = \frac{1}{2}(F^{r'} - \ast_4 F^{r'}) , \] (7.11)

are the self-dual and anti-self-dual components of \( F \), respectively, and \( \omega^6 \equiv \omega^1, \omega^2 \equiv \omega^2 \) and \( \omega^7 \equiv \omega^3 \). \( F^{ad} \) is not restricted by the KSE.

There is an additional condition on the parameters of the solution. To see this first use \( \tilde{\nabla} \omega^r = 0 \) and (7.10) to show that

\[ \mathcal{L}_{r'} \omega_{s'} = 2\nu \epsilon_{r's't'} \omega_{t'} . \] (7.12)

Then either by comparing \( dH = 0 \) with the dilatino field equation or evaluating the identity \( [\mathcal{L}_{\xi'}, \mathcal{L}_{\xi'}] = \mathcal{L}_{[\xi', \xi']} \) on \( \omega^r \) using (7.12) and (7.7), one finds that

\[ (1 + c)(2c + 1) = 0 \] (7.13)

and so either \( c = -1 \) or \( c = -\frac{1}{2} \). If \( c = -1 \), \( \nu = 0 \) and the supersymmetry enhances to \( N = 8 \). These solutions will be investigated later.

Now, if \( c = -\frac{1}{2} \) these solutions are special cases of those given in [22] that preserve 6 supersymmetries. The only remaining equation that has to be solved is

\[ \tilde{\nabla}^2 e^{2\Phi} = -\frac{1}{2}(F^{sd})^{r'}(F^{sd})^{ij}_{r'} - \frac{k^{-2}}{2}dh_{ij}dh^{ij} + \frac{3}{8}k^2 e^{4\Phi} , \] (7.14)

where the inner products in the rhs have been taken with respect to the \( d\tilde{s}_{(4)}^2 \) metric. The sign of the rhs is indefinite. As a result, there may be solutions which preserve strictly 6 supersymmetries. As the \( N = 6 \) solutions are included in the \( N = 2 \) and \( N = 4 \) heterotic horizons, this sign is also significant for the existence of solutions in the \( G_2 \) and \( SU(3) \) cases.

The geometry of \( B^4 \) can be investigated as in [22]. In particular, the self-dual component of the Weyl tensor of \( B^4 \) vanishes but it may not be Einstein with cosmological constant. For the latter, the anti-self-dual part of the Weyl tensor must vanish as well. Because of (7.12), the three hermitian forms \( \omega^r \) do not descend on \( B^4 \) as hermitian forms but they are rather twisted with an \( SU(2) \) bundle. More details on the geometry of \( B^4 \) can be found in [22].
7.1.1 Geometry of $S^8$

The first four $\hat{\nabla}$-parallel spinors are as in the $SU(3)$ case (4.31) and (6.15). The additional two Killing spinors can be written as

$$\eta^5_+ = i(e_{12} + e_{34}) , \quad \eta^6_+ = i\Gamma^+ h_2 (e_{12} + e_{34}) .$$

(7.15)

The isotropy group of all 6 spinors in $Spin(8)$ is $SU(2)$. Therefore the holonomy of $\hat{\nabla}$ is contained in $SU(2)$. The associated $\hat{\nabla}$-parallel bilinears are

$$h , \quad \ell^r' , \quad \omega_r ,$$

(7.16)

where $\ell^r'$ and $\omega_r$ have the properties mentioned in the previous section. $S^8$ is a $S^1 \times S^3$ fibration over $B^4$. Both $h$ and $\ell^r'$ can be viewed as the connections of a $U(1) \times SU(2)$ bundle over a base space $B^4$. The metric and torsion of $S^8$ can be written as

$$d\tilde{s}^2 = k^{-2} (h \otimes h + \sum_{\nu'} \ell^r' \otimes \ell^r') + e^{2\Phi} d\tilde{s}^2_{(4)} , \quad \tilde{H} = k^{-2} (h \wedge dh) + CS(\lambda^r') + \tilde{H}_{(4)} ,$$

(7.17)

where $d\tilde{s}^2_{(4)}$ and $\tilde{H}_{(4)}$ are given in the previous section.

8 N=8 horizons

The heterotic horizons that preserve 8 supersymmetries are special cases of the half supersymmetric backgrounds classified in [46]. We shall demonstrate that within the class of $AdS_3$ horizons, there are two heterotic horizon geometries with 8 supersymmetries which, up to discrete identifications, are isometric to either $AdS_3 \times S^3 \times K_3$ or $AdS_3 \times S^3 \times T^4$.

The first six Killing spinors are as those of the solutions with 6 supersymmetries given in (4.17), (6.1) and (7.1). The remaining two can be chosen as

$$\epsilon^7 = e_{12} - e_{34} , \quad \epsilon^8 = -k^2 u (e_{12} - e_{34}) + h_2 \Gamma^{+4} (e_{12} - e_{34}) .$$

(8.1)

Since the first two Killing spinors are those that we have found in the $G_2$ case (4.17), the 1-form bilinears include those of (4.18) for which the associated vector fields span the $\mathfrak{sl}(2, \mathbb{R})$ algebra. As a result the fibre group of the spacetime has a $SL(2, \mathbb{R})$ subalgebra. It is then a consequence of the general classification results of [20] that the spacetime is a principal bundle, $M = P(SL(2, \mathbb{R}) \times SU(2), B^4)$, with fibre group $AdS^3 \times S^3$ and base space $B^4$ which is conformal to a 4-dimensional hyper-Kähler manifold. Moreover the $AdS^3 \times S^3$ fibre twists over the base space with the connection $\lambda$ which is an anti-self dual instanton. The dilaton $\Phi$ is function of the base space $B^4$ and independent from all the coordinates along the fibre directions.

The metric and 3-form flux can be written as

$$ds^2 = \eta_{ab} \lambda^a \lambda^b + e^{2\Phi} d\tilde{s}^2_{(4)} , \quad H = CS(\lambda^a) + CS(\lambda^{su}) + \tilde{H}_{(4)} , \quad \tilde{H}_{(4)} = -\kappa_4 de^{2\Phi} ,$$

(8.2)
where \( ds_{(4)}^2 \) is the hyper-Kähler metric on \( B^4 \), the Hodge operation in \( \tilde{H}_{(4)} \) is taken with respect to the hyper-Kähler metric and \( a, b = 0, 5, 1, 6, 2, 7 \). Moreover, we have split the Chern-Simons form of \( \lambda \) into the \( \mathfrak{sl}(2, \mathbb{R}) \) and \( \mathfrak{su}(2) \) parts.

The only equation that remains to be solved is the closure of \( H, dH = 0 \). This in turn implies, using the anti-self duality of the curvature \( F \) of \( \lambda \), that
\[
\hat{\nabla}^2 e^{2\Phi} = -\frac{1}{2} \eta_{ab} F^a_{ij} F^{bij},
\]
where \( i, j = 3, 8, 4, 9 \) are now \( B^4 \) indices and the indices in the right-hand-side have been raised with respect to the hyper-Kähler metric. The right-hand-side of (8.3) has a definite sign. The possibility of an indefinite sign only arises from the \( \mathfrak{sl}(2, \mathbb{R}) \) component of the curvature \( F \). But a straightforward calculation reveals, using (4.23), that
\[
\eta_{ab} F^a_{ij} F^{bij} |_{\mathfrak{sl}(2, \mathbb{R})} = k^{-2} (dh)_{ij} (dh)^{ij} \geq 0.
\]
Thus the right-hand-side of (8.3) is strictly negative and so a partial integration argument over the compact base space \( B^4 \) implies that it should vanish. Thus we conclude that for the horizon geometries
\[
F^a = 0,
\]
and the dilaton \( \Phi \) is constant. Since the curvature of the principal bundle vanishes, the spacetime is a product \( AdS_3 \times S^3 \times B^4 \), where \( B^4 \) is a compact 4-dimensional hyper-Kähler manifold and so it is either \( T^4 \) or \( K_3 \).

\section{9 Non-vanishing anomaly and near brane horizons}

So far in our analysis, we have not included the heterotic anomaly contribution. In particular, we have assumed that the gravitational anomaly cancels the gauge sector anomaly, and so the 3-form flux is closed. If the anomaly contributes, our analysis is modified. There are two ways of thinking about this. One is in the context of perturbation theory where the KSEs, the field equations and \( dH \) receive higher order curvature corrections expressed as a Taylor series expansion in \( \alpha' \). Therefore one expects that the spacetime metric and 2-form gauge potential for generic backgrounds get corrected to all orders in perturbation theory. Alternatively, one can take the anomaly contribution which appears at order \( \alpha' \) as exact. Such an assumption has led to applications in differential geometry \cite{43}. These two different ways of thinking give distinct results which we shall explain. We shall mostly focus on perturbation theory and then we shall comment how our analysis is altered in the exact case.

\subsection{9.1 Perturbation theory}

We shall not explore the theory to all orders in \( \alpha' \). Instead we shall follow \cite{47} and consider the correction up to and including two loops in sigma model perturbation theory.

\footnote{If \( B^4 \) is not simply connected, there is the possibility that the fibre twists with a flat but not trivial connection \( \lambda \). We shall not investigate this further here.}
Now taking $dH \neq 0$, it is known that the KSEs of supergravity theory do not change up to this correction \[49\]. However field equations do.

We are working in perturbation theory and so a background gets corrected order by order in $\alpha'$. Since the anomaly contributes in the first order in $\alpha'$, we must begin from a configuration for which the 3-form field strength is closed. As a result, the analysis we have performed is valid at the zeroth order in $\alpha'$. Moreover since the Killing spinor equations remain the same up to the order we consider, the only alteration in the analysis is to set

$$dH = -\frac{\alpha'}{4} [\text{tr} \hat{R} \wedge \hat{R} - \text{tr}(F \wedge F)] + \mathcal{O}(\alpha'^2) ,$$

(9.1)

instead of taking $dH = 0$ as in previous sections, where the contribution in the rhs is due to the anomaly. $\hat{R}$ is the curvature of the connection $\hat{\nabla} = \nabla - \frac{1}{2} H$ and $F$ is the curvature of the gauge sector. Observe that since the rhs is first order in $\alpha'$, the contribution in $\hat{R}$ comes from the zeroth order metric and 2-form gauge potential and so

$$\hat{R}_{AB,CD} = \hat{R}_{CD,AB} + \mathcal{O}(\alpha') ,$$

(9.2)

as a consequence of a Bianchi identity and $dH = 0+\mathcal{O}(\alpha')$. Therefore, $\hat{\nabla}$ is an $\mathfrak{Lie}(\text{hol}(\hat{\nabla}))$-instanton connection with gauge group contained in $SO(9,1)$, ie

$$\hat{R} \in \mathfrak{Lie}(\text{hol}(\hat{\nabla})) ,$$

(9.3)

where $\mathfrak{Lie}(\text{hol}(\hat{\nabla})) = G_2$, $SU(3)$ or $SU(2)$. The gaugino KSE also implies that $F$ is a $\mathfrak{Lie}(\text{hol}(\hat{\nabla}))$-instanton and the gauge group must be a subgroup of $E_8 \times E_8$ or Spin(32)/$\mathbb{Z}_2$. In (9.1), $\hat{R}$ can be replaced with any other curvature of the spacetime because of the scheme dependence of perturbation theory. However to preserve extended worldvolume supersymmetry in perturbation theory, $\hat{R}$ is a natural choice \[50\].

First suppose that we are considering the heterotic horizons $\mathbb{R}^{1,1} \times S^8$ tabulated in table 1. Since the zeroth order contribution for $H$ vanishes, $\hat{R} = R$. The solutions, although they begin with vanishing torsion, can develop non-vanishing torsion which is proportional to $\alpha'$. This is the case for all backgrounds preserving up to and including 8 supersymmetries as demonstrated in \[47\]. Moreover, it is expected that there will be corrections to all order in $\alpha'$. The precise corrections that they receive depend on a case by case analysis.

Next let us consider heterotic horizons associated with $AdS_3$. In this case the anomalous Bianchi identity (9.1) can be rewritten as

$$d\hat{H}_{(n)} = -\eta_{abc} \mathcal{F}^a \wedge \mathcal{F}^b - \frac{\alpha'}{4} [\text{tr} \hat{R} \wedge \hat{R} - \text{tr}(F \wedge F)] + \mathcal{O}(\alpha'^2) ,$$

(9.4)

where $\hat{H}_{(n)}$ is the 3-form field strength of the base space and so $n = 7, 6$ or 4. However since the zeroth order in $\alpha'$ term in $H$ does not vanish, $\hat{R}$ is different from $R$. It is expected that there will be $\alpha'$ corrections to the fields for all backgrounds that preserve less than 8 supersymmetries.

\[6\] There is the possibility that the anomaly correction is not compatible with the Einstein equation when the two loop correction is included. This has been examined in detail in \[17\].
To investigate the case with 8 supersymmetries first observe that the first term in the rhs of (9.4) vanishes because $F = 0$. Then (9.4), following (8.3), can be rewritten as

$$\hat{\nabla}^2 e^{2\phi} = -\frac{\alpha'}{8} \text{tr} \hat{R}_{ij} \hat{R}^{ij} + \frac{\alpha'}{8} \text{tr} F_{ij} F^{ij} + O(\alpha'^2) .$$

(9.5)

The gaugino KSE requires that $F$ is an anti-self-dual instanton on $B^4$. It is significant that the inclusion of the anomaly makes the sign of the rhs indefinite. Therefore solutions with a non-trivial dilaton cannot be ruled out. In the $AdS_3 \times S^3 \times K_3$ background, $\hat{R} = R \neq 0$ and so the rhs of (9.5) may not vanish. The existence of $\alpha'$ corrections depends on the choice of $F$. However for the $AdS_3 \times S^3 \times T^4$ background $\hat{R} = 0$, then using the compactness of $T^4$ one concludes that $F = 0$ and $\Phi$ is constant at order $\alpha'$. Therefore $AdS_3 \times S^3 \times T^4$ is an exact solution up to and including 2 loops in the sigma model perturbation theory. Since it is a group manifold solution, it must be exact to all orders in perturbation theory.

### 9.2 Exact modification

Next suppose that the $\alpha'$ correction to $dH$ is exact. The Bianchi identity of $H$ is different from that of (9.1) and reads

$$dH = -\frac{\alpha'}{4} [\text{tr} \hat{R} \wedge \hat{R} - \text{tr}(F \wedge F)] ,$$

(9.6)

where now $\hat{R}$ is a curvature of the spacetime which is a $\text{Lie}(\text{hol}(\nabla))$-instanton with gauge group $SO(9,1)$ and similarly for $F$. Moreover $\alpha'$ is any positive number. Since $dH \neq 0$, $\hat{R}$ is not a $\text{Lie}(\text{hol}(\nabla))$-instanton and so it cannot be identified with $R$.

Consistency requires that the field equations are also modified. In particular both the Einstein and dilaton field equations alter. In particular the dilaton field equation now reads

$$\hat{\nabla}^2 \Phi - 2\hat{\nabla}^A \Phi \hat{\nabla}_A \Phi + \frac{1}{12} H_{ABC} H^{ABC} + \frac{\alpha'}{8} \text{tr} \hat{R}_{AB} \hat{R}^{AB} - \frac{\alpha'}{8} \text{tr} F_{AB} F^{AB} = 0 .$$

(9.7)

Note that the sign of the curvature terms associated with the anomaly is indefinite, and so the arguments presented in sections 3 and 4 for the heterotic horizons do not generalize. Therefore a new investigation is required. It is likely that many more solutions exist in this case as the gauge connection can be used to lessen the restrictions on the differential system imposed by the compactness of $S^8$.

### 9.3 Brane horizons and dilaton singularities

Some of the heterotic horizon geometries we have found can be identified as the near horizon geometries of brane configurations. To distinguish between the two, we shall refer to the latter as "near brane geometries". In particular the near horizon geometry $AdS_3 \times S^3 \times T^4$ arises as the near brane geometry of a fundamental string [51] on a 5-brane [52]. In this identification, $AdS_3$ is spanned by the worldvolume directions of the string and the overall radial transverse direction, $S^3$ is the 3-sphere of the overall transverse
sphere and $T^4$ are the relative transverse directions of the string on the worldvolume of the 5-brane. This is also the case with the $AdS_3 \times S^3 \times K_3$ solution. Again one should consider a fundamental string on a 5-brane but replace the $T^4$ relative transverse directions of the string on the 5-brane with $K_3$. So both heterotic horizons that preserve 8 supersymmetries can arise as near brane geometries.

Some brane configurations in heterotic supergravity exhibit near brane geometries which are not included in the analysis we have done for the heterotic horizons. For example the near brane geometry of the 5-brane is $\mathbb{R}^{1,1} \times T^4 \times S^3 \times S^1$ and has a linear dilaton depending on the angular coordinate of $S^1$ [52]. Another similar example is $\mathbb{R}^{1,1} \times T^2 \times S^3 \times S^3$ with linear dilaton depending on the angular coordinates of $T^2$, and arises as the near brane geometry of the 5-brane configurations of [53].

There are two main differences between the near brane geometries mentioned above and the near horizon geometries that we have investigated. First, both $\mathbb{R}^{1,1} \times T^4 \times S^3 \times S^1$ and $\mathbb{R}^{1,1} \times T^2 \times S^3 \times S^3$ near brane geometries are not horizons but rather other asymptotic regions. They are located at infinite affine distance away from any interior point of the brane spacetime. Another difference is that the dilaton is not well defined on the near brane spacetime because it is not periodic in an angular coordinate(s). As a result, it cannot be thought of as a well-defined function of the compact section of the near brane geometry. It is clear from both these points that the KSEs do not guarantee either analyticity in the radial direction $r$ or regularity of the fields on the horizon section. However, enforcing one or the other will rule both these near brane geometries out. As a result, it may be that enforcing one of them, together with the Killing spinor and field equations, will imply the other.

10 Concluding Remarks

We have found that there are two classes of heterotic horizons. One class is $\mathbb{R}^{1,1} \times S^8$, where $S^8$ is a product of special holonomy manifolds which admit parallel spinors, the dilaton is constant and the 3-form flux vanishes. A more detailed description is given in table 1. The other class of solutions contains $AdS_3$ as subspace and preserves 2, 4, 6 and 8 supersymmetries. In particular, the spacetime is a fibration with fibre that contains $AdS_3$, and $AdS_3$ twists over the base space with a suitable $U(1)$ connection. The solutions with 8 supersymmetries are isometric up to discrete identifications with $AdS_3 \times S^3 \times K_3$, $AdS_3 \times S^3 \times T^4$ or $\mathbb{R}^{1,1} \times T^4 \times K_3$, the radii of $AdS_3$ and $S^3$ are equal and the dilaton is constant. Clearly the first two heterotic horizons are of the $AdS_3$ class while the third is of the $\mathbb{R}^{1,1} \times S^8$ class. Moreover, we have shown that $AdS_3 \times S^3 \times T^4$ does not receive $\alpha'$ corrections.

Throughout most of our analysis, we have taken $dH = 0$, so our results are automatically extended to the common sector of type II supergravities. Clearly all the heterotic horizons of table 1 can be interpreted as solutions of the common sector preserving twice as many supersymmetries. The $AdS_3$ class of heterotic horizons, without the anomaly correction, can also be embedded in the common sector of type II supergravities but it is not a priori apparent that there will be a doubling of supersymmetry. An exception to this is the heterotic horizons which preserve 8 supersymmetries. It can be easily seen that as
common sector solutions they preserve 16 supersymmetries. Moreover these are the only common sector horizons which preserve 16 supersymmetries. This is because if the left or right sector KSEs preserve more than 8 supersymmetries, then the solutions are plane waves \cite{21} and in particular they do not contain $AdS_3$ as a subspace. So for the common sector horizons to preserve 16 supersymmetries, the left and right sector KSEs must preserve precisely 8 supersymmetries each. This proves that the common sector horizon geometries that preserve 16 supersymmetries are locally isometric to $AdS_3 \times S^3 \times K_3$, $AdS_3 \times S^3 \times T^4$ or $\mathbb{R}^{1,1} \times T^4 \times K_3$.

Having determined the geometry of heterotic horizons, it is natural to wonder whether there are extreme black holes which exhibit such near horizon geometries. It is not a priori clear that this is the case for all heterotic horizons that we have found. However the solutions which contain an $AdS_3$ subspace can be trivially identified with a Kaluza-Klein black hole. This is because they can be seen as an embedding of the 3-dimensional Kaluza-Klein hole \cite{54} in the heterotic supergravity. It is also known that the near horizon geometries of black rings have an $AdS_3$ subspace. So alternatively, it may be possible to view this type of heterotic horizon as the near horizon geometry of a Kaluza-Klein black ring.

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Appendix A  Supersymmetric horizons

The main purpose of the appendix is to show that if we take Gaussian null coordinates with respect to the null Killing vector field constructed as a Killing spinor bi-linear, then, without loss of generality, we can make the identification (3.1) leading to (3.2).

Before proceeding with the analysis of the KSEs, note that the non-vanishing components of the spin connection are given by

$$
\begin{align*}
\Omega_{+,i} &= -\frac{1}{2} h_i \\
\Omega_{+,ij} &= -\frac{1}{2} r(dh)_{ij} \\
\Omega_{-,i} &= -\frac{1}{2} h_i \\
\Omega_{i,-} &= \frac{1}{2} h_i \\
\Omega_{i,j} &= -\frac{1}{2} r(dh)_{ij} \\
\Omega_{i,jk} &= \tilde{\Omega}_{i,jk} ,
\end{align*}
$$

\hspace{1cm} (A.1)

where $\tilde{\Omega}$ is the spin connection of the horizon section $S^8$.

A.1  Gravitino

The gravitino equation $\hat{\nabla} \epsilon = 0$ can be written explicitly as

$$
\partial_A \epsilon + \frac{1}{4} \Omega_{A,B_1,B_2} \Gamma^{B_1,B_2} \epsilon - \frac{1}{8} H_{AB_1,B_2} \Gamma^{B_1,B_2} \epsilon = 0 ,
$$

\hspace{1cm} (A.2)

where the spin connection is given in (A.1) and $H$ in (2.11). To proceed, we write

$$
\epsilon = \epsilon_+ + \epsilon_- , \quad \Gamma_\pm \epsilon_\pm = 0 .
$$

\hspace{1cm} (A.3)
After some straightforward computation, it is possible to solve the + and − components of the KSE to find

\[ \epsilon_+ = \eta_+ + \frac{1}{4} r ((S - 1)h - dS + N) \Gamma^i \Gamma_+ \eta_-, \]
\[ \epsilon_- = \eta_- - \frac{1}{4} r ((S + 1)h - dS + N) \Gamma^i \Gamma_- \eta_+ - \frac{1}{8} i r ((S - 1)dh - h \wedge N + dN) \Gamma^{ij} \eta_- , \]  
(A.4)

where \( \eta_\pm = \eta_\pm(y) \) do not depend on \( r \) and \( u \). These expressions are sufficient to prove (3.1) but we shall state the rest of the equations arising from the KSEs for completeness.

In addition, the + and − components of the KSE give

\[ - (Sh - dS + N)^2 + h^2 + d((S - 1)h + N)_{ij} \Gamma^{ij} \eta_+ = 0 , \]  
(A.5)
\[ ((S - 1)dh - h \wedge N + dN)_{ij} \Gamma^{ij} ((S + 1)h - dS + N) \Gamma^\ell \eta_+ = 0 , \]  
(A.6)
\[ ((S - 1)h - dS + N)^2 - h^2 + d((S - 1)h + N)_{ij} \Gamma^{ij} \eta_- = 0 , \]  
(A.7)
\[ ((S - 1)h - dS + N) \wedge ((S - 1)dh - h \wedge N + dN) \Gamma^{\ell_1 \ell_2 \ell_3} \eta_- = 0 , \]  
(A.8)
\[ - ((S - 1)dh - h \wedge N + dN)_{ij} ((S - 1)dh - h \wedge N + dN)^{ij} + \frac{1}{2} ((S - 1)dh - h \wedge N + dN)_{\ell_1 \ell_2} ((S - 1)dh - h \wedge N + dN)_{\ell_3 \ell_4} \Gamma^{\ell_1 \ell_2 \ell_3 \ell_4} \eta_- = 0 . \]  
(A.9)

The remaining components of the KSE then imply that \( \eta_\pm \) satisfy

\[ \hat{\nabla}_i \eta_\pm + \left( - \frac{1}{8} (dW)_{ijk} \Gamma^{ijk} \pm \frac{1}{4} ((S + 1)h + dS - N) \right) \eta_\pm = 0 , \]  
(A.10)

together with the following algebraic constraints

\[ \left( - \hat{\nabla}_i ((S + 1)h - dS + N)_{ij} - \frac{1}{2} ((S + 1)h - dS + N)_{ij} ((S - 1)h - dS + N) \right) + ((S + 1)dh - h \wedge N + dN)_{ij} + \frac{1}{2} ((S + 1)h - dS + N)^{\ell} (dW)_{ij\ell} \right) \Gamma^\ell \eta_+ = 0(A.11) \]
\[ \left( - \hat{\nabla}_i ((S - 1)dh - h \wedge N + dN)_{\ell_1 \ell_2} - ((S - 1)dh - h \wedge N + dN)_{\ell_1 \ell_2} ((S - 1)dh - h \wedge N + dN)^{\ell_1 \ell_2} + h_i ((S - 1)dh - h \wedge N + dN)_{\ell_1 \ell_2} \right) \Gamma^{\ell_1 \ell_2} \eta_- = 0 \]  
(A.12)
\[
\left( \tilde{\nabla}_i((S - 1)h - dS + N)_{j} + \frac{1}{2}((S - 1)h - dS + N)_j(- (S + 1)h + dS - N)_i \right. \\
\left. - \frac{1}{2}((S - 1)h - dS + N)^\ell(dW)_{ij\ell} \right) \Gamma^{ij}_- = 0 .
\] (A.13)

### A.2 Dilatino

To proceed further, we consider the dilatino KSE

\[
\left( \partial_A \Phi \Gamma^A - \frac{1}{12} H_{B_1 B_2 B_3} \Gamma^{B_1 B_2 B_3} \right) \epsilon = 0 .
\] (A.14)

On substituting (A.4) into (A.14), one finds the following algebraic conditions

\[
\left( 2d\Phi + dS - N - Sh \right)_{i} \Gamma^{i} \left. - \frac{1}{6} (dW)_{i i_{1} i_{2} i_{3}} \Gamma^{i_{1} i_{2} i_{3}} \right) \eta^+ = 0 ,
\] (A.15)

\[
\left( h \wedge N - dN - Sdh \right)_{\ell_{1} \ell_{2}} \Gamma^{\ell_{1} \ell_{2}} \eta^+ = 0 ,
\] (A.16)

\[
\left. \left( h \wedge N - dN - Sdh \right)_{\ell_{1} \ell_{2}} \Gamma^{\ell_{1} \ell_{2}} ( (S + 1)h - dS + N )_{i} \Gamma^{i} \eta^+ = 0 , \right. \quad (A.17)
\]

\[
\left( 2d\Phi + dS - N - Sh \right)_{i} \Gamma^{i} \left. - \frac{1}{6} (dW)_{i i_{1} i_{2} i_{3}} \Gamma^{i_{1} i_{2} i_{3}} \right) \left. ((S - 1)h - dS + N)_{\ell} \Gamma^{\ell} \eta^- = 0 \right. \quad (A.18)
\]

\[
\left. \left( h \wedge N - dN - Sdh \right)_{\ell_{1} \ell_{2}} \Gamma^{\ell_{1} \ell_{2}} ( (S - 1)dh - h \wedge N + dN )_{i} \Gamma^{i} \eta^- = 0 \right. , \quad (A.19)
\]

\[
\left( 2d\Phi - dS + N + Sh \right)_{i} \Gamma^{i} \left. - \frac{1}{6} (dW)_{i i_{1} i_{2} i_{3}} \Gamma^{i_{1} i_{2} i_{3}} \right) \eta^- = 0 ,
\] (A.20)

\[
\left( 2d\Phi - dS + N + Sh \right)_{i} \Gamma^{i} \left. - \frac{1}{6} (dW)_{i i_{1} i_{2} i_{3}} \Gamma^{i_{1} i_{2} i_{3}} \right) \left. ((S + 1)h - dS + N)_{j} \Gamma^{j} \eta^+ = 0 \right. \quad (A.21)
\]

\[
\left. \left( 2d\Phi - dS + N + Sh \right)_{i} \Gamma^{i} \left. - \frac{1}{6} (dW)_{i i_{1} i_{2} i_{3}} \Gamma^{i_{1} i_{2} i_{3}} \right) \times \left( (S - 1)dh - h \wedge N + dN \right)_{q_{1} q_{2}} \Gamma^{q_{1} q_{2}} \eta^- = 0 \right. .
\] (A.22)
A.3 N=1 Supersymmetry

Now let us focus on the solutions that preserve one supersymmetry for which $\partial_u$ is identified with the Killing spinor bi-linear. The components of the associated 1-form bilinear $X \equiv e^-$ in the basis (2.12) are

$$X_+ = 0, \quad X_- = 1, \quad X_i = 0. \quad (A.23)$$

It is clear that the data, including the KSEs, are invariant under locally dependent $Spin(8)$ gauge transformations. As $\eta_\pm$ depend only on $y$, there is a gauge transformation such that

$$\eta_+ = \alpha(y)(1 + e_{1234}), \quad \eta_- = \beta(y)(e_{15} + e_{2345}). \quad (A.24)$$

This follows from the fact that $Spin(8)$ has a single type of orbit in the Majorana-Weyl 8-dimensional representation with isotropy group $Spin(7)$, and $Spin(7)$ has also a single type of orbit in the anti-Majorana-Weyl 8-dimensional representation with isotropy group $G_2$.

Next, consider the spinor bilinear

$$Y = Y_A e^A \equiv \langle B \epsilon^*, \Gamma_A \epsilon \rangle e^A, \quad (A.25)$$

using (A.4) and (A.24). By comparing the components of $Y$ with those of $X$ in the basis (2.12), we require that

$$Y_+|_{r=0} = 0 \quad (A.26)$$

which imposes the condition $\beta = 0$, i.e.

$$\eta_- = 0. \quad (A.27)$$

Next, we require that the $O(r)$ term in the spinor bilinear $Y$ should vanish. This imposes the condition

$$\langle B(1 + e_{1234}), \Gamma_M((S + 1)h - dS + N), \Gamma^i \Gamma_-(1 + e_{1234}) \rangle = 0, \quad (A.28)$$

for all $M$. This implies that

$$(S + 1)h - dS + N = 0. \quad (A.29)$$

Hence

$$\epsilon = \eta_+ = \alpha(1 + e_{1234}). \quad (A.30)$$

Next, observe that

$$Y_- = -2\sqrt{2}\alpha^2. \quad (A.31)$$

On comparing with $X_- = 1$, we require that $\alpha$ be constant; without loss of generality take $\alpha = 1$, so

$$\epsilon = \eta_+ = 1 + e_{1234}. \quad (A.32)$$
Having obtained this simplification, it is straightforward to summarize the conditions for the \( N = 1 \) heterotic horizon geometries as

\[
\tilde{\nabla}_i \eta_+ - \frac{1}{8} (dW)_{ijk} \Gamma^{jk} \eta_+ = 0 , \tag{A.33}
\]

\[
(S + 1) h - dS + N = 0 , \tag{A.34}
\]

\[
(dh)_{ij} \Gamma^{ij} \eta_+ = 0 , \tag{A.35}
\]

\[
\left( (2d\Phi + h) \Gamma^i - \frac{1}{6} \Gamma^{ijk} (dW)_{ijk} \right) \eta_+ = 0 . \tag{A.36}
\]

Note that (A.34) can be used to eliminate \( S \) and \( N \) from the 3-form field strength \( H \) to obtain

\[
H = e^+ \wedge e^- \wedge h + re^+ \wedge dh + dW . \tag{A.37}
\]

Thus we have justified (3.1) and derived (3.2).

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