An improved centroid localization algorithm based on acoustic for proton range verification

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Abstract. The proton beam generates a unique acoustic signal, which can be used for range verification. It is a crucial issue to use the acoustic signal for accurate positioning. The propagation of sound waves emitted by a proton pulse (20 MeV, 1×10⁷ protons) is simulated, and acoustic signals are recorded. The distance between sensors and Bragg Peak is calculated by the time of arrival and speed of sound. An improved centroid localization algorithm based on trilateration is presented to locate the Bragg Peak with the distance. After evaluating the verification accuracy of different methods, the results show that this algorithm significantly improves the verification accuracy.

1. Introduction
Compared with conventional photon radiotherapy, proton therapy has unique dose deposition characteristics. Due to these unique dose characteristics, the tumor at Bragg Peak (BP) can receive the maximum radiation dose, and the surrounding tissues will be protected[1]. In recent years, proton therapy technology has been extensively developed. As of May 2020, there are 86 proton therapy centers worldwide. However, because of the uncertainties of the proton range, the accuracy of proton therapy is affected. Several solutions that have been proposed and passed clinical tests to reduce the uncertainty of the proton range are based on positron emission tomography (PET) and prompt gamma (PG)[2, 3]. These methods require expensive verification equipment, which increases the cost of proton treatment. Contrary to traditional verification methods, proton-induced acoustic (protoacoustic) is a verification method that measures the range of protons based on measuring the time of flight (TOF) of sound waves, which emitted due to local pressure changes caused by heat.

In the late 1970s, the pioneering work of Sulak et al. had discovered the acoustic effect of the proton beam[4]. In recent years, many groups conducted experiments and simulations on the protoacoustic field. However, most of the research focuses on the characteristics of acoustic signals, and there is no in-depth study on the algorithm of positioning BP.

The paper is organized as follows: The propagation of sound waves induced by proton is simulated. According to the recorded acoustic signals, the required distance between the sensors and BP is calculated by TOF. Then, an improved centroid localization algorithm based on trilateration is presented for range verification. This algorithm is aimed at how to obtain the highest verification accuracy. Finally, the accuracy of this method is evaluated.
2. Methods

During the proton irradiation process, almost all the energy deposited in the medium will be converted into heat. It causes the temperature to rise, which creates expansion that emits pressure waves. The initial dose distribution can be converted to initial pressure by:

\[ p_d(r, 0) = \rho(r)\Gamma(r)D_{\delta}(r, 0) \]  

(1)

where \( \Gamma = \frac{\varepsilon c^2}{C_p} \) is the dimensionless coefficient. And the wave equation that describes the pressure, \( p \), at a time, \( t \), and position, \( r \), is:

\[
\left( \nabla^2 + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) p(r, t) = -\frac{\varepsilon c^2}{C_p} \frac{\partial^2 T(r, t)}{\partial t^2}
\]  

(2)

Where \( c \) is the speed of sound (m s\(^{-1}\)), \( \alpha \) is the thermal expansion coefficient (K\(^{-1}\)), \( C_p \) is the specific heat capacity (J kg\(^{-1}\) K\(^{-1}\)), \( T \) is the rate of input energy absorption that is converted to heat (J s\(^{-1}\) m\(^{-3}\)).

In this work, a mono-energetic proton pencil beam (20 MeV, \( 1 \times 10^7 \) protons) is modeled to treat a liver tumor in the water phantom. The proton beam is assumed to have a Gaussian distribution of 0.5 mm FWHM and 10 µs in width. The dose calculation is done through Monte Carlo (MC) code FLUKA. The calculated energy deposition is then converted to pressure distribution, which is used to simulate the acoustic propagation with the k-Wave MATLAB toolbox\[6\]. The simulation environment is a 40×100(x,y) rectangular plane. The grid is set to 1 mm along the x-axis and y-axis. The medium is water at 29 °C. The speed of sound is 1507 m s\(^{-1}\). Sensors are uniformly arranged along the propagation axis of the proton beam. The sensors are spaced at 5 gp in the axial direction and 2 gp in the radial direction. There are 6 rows of 120 sensors in total. In order to simplify the simulation, the directivity of the sensor is ignored.

2.1. Acoustic wave propagation simulation

The propagation process of sound waves generated by protons is shown in Fig 1. As can be seen from the figure, the pre-Bragg region generates \( \alpha \) acoustic peak while the BP region generates the \( \gamma \) acoustic peak. The \( \alpha \) acoustic peak is a cylindrical wave that propagates laterally to the proton beam direction while the \( \gamma \) acoustic peak is a spherical wave\[7\]. According to these findings, the arrival time of \( \alpha \) acoustic peak reveals the distance between sensors and the proton beam axis while the arrival time of \( \gamma \) acoustic peak reveals the distance between sensors and BP.

![Figure 1. Snapshot of pressure wave propagation (a) t=0 µs (b) t=19.6 µs](image)

The sensors are divided into two groups. Sensors in the pre-Bragg peak region are called A group, and the sensors in the post-Bragg peak region are called B group. From the third-row sensors data, it can be found that the sensor in group A can record two acoustic peaks, while the sensor in group B can only record one, as Fig 2 shows.

Based on the above findings, we use the arrival time of the \( \gamma \) acoustic peak to calculate the distance between sensors and BP. The recorded arrival time of \( \alpha \) acoustic peak in group A can be used to locate the position of the beam axis.
2.2. centroid localization algorithm

According to the distance between three sensors and the BP center, the traditional trilateration needs to solve three linear equations (equation 3), which significantly increases the simulation time. The position of BP \( P(x_0, y_0) \) is given by:

\[
\begin{align*}
(x_1 - x_0)^2 + (y_1 - y_0)^2 &= d_1^2 \\
(x_2 - x_0)^2 + (y_2 - y_0)^2 &= d_2^2 \\
&\vdots \\
(x_n - x_0)^2 + (y_n - y_0)^2 &= d_n^2
\end{align*}
\]

(3)

Where \((x_n, y_n)\) is the coordinate of sensors, and \(d_1, d_2, \ldots, d_n\) are the distance between the sensors and sound sources. These nonlinear equations are linearized as \(AX = b\).

\[
A = \begin{bmatrix}
2(x_1 - x_0) & 2(y_1 - y_0) \\
2(x_2 - x_0) & 2(y_2 - y_0) \\
\vdots & \vdots \\
2(x_n - x_0) & 2(y_n - y_0)
\end{bmatrix}
\]  
\[
b = \begin{bmatrix}
x_1^2 - x_0^2 + y_0^2 - y_1^2 - d_1^2 + d_1^2 \\
x_2^2 - x_0^2 + y_0^2 - y_2^2 - d_2^2 + d_2^2 \\
\vdots \\
x_n^2 - x_0^2 + y_0^2 - y_n^2 - d_n^2 + d_n^2
\end{bmatrix}
\]

(4)

The coordinate of BP is given by:

\[
X = (A^T A)^{-1} - A^T b
\]

(5)

The least squares method is used to calculate the solution of the equations. However, the above equations are not always solvable. The circles represented by the three equations do not intersect perfectly at the same point but form a region. The intersection of three circles with the radius of distance \(d_1, d_2, d_3\) are \(A(x_a, y_a), B(x_b, y_b), C(x_c, y_c)\), which form a triangle like Fig 3 shows.

So, in the centroid algorithm based on trilateration, the centroid of the \(\triangle ABC\): \(P'(x'_0, y'_0)\) can be regarded as the estimated value of BP position, which is given as:

\[
\begin{align*}
x'_0 &= \frac{x_a + x_b + x_c}{3} \\
y'_0 &= \frac{y_a + y_b + y_c}{3}
\end{align*}
\]

(6)

2.3. Double centroid localization algorithm

In order to improve the positioning accuracy of the centroid algorithm, a double centroid localization algorithm based on trilateration is proposed to estimate the position of BP. When the number of sensors \(m\) are more than 3, the intersection of circles can form \(C_m^3\) triangles. In this method, the centroid of the polygon composed of the centroids of \(C_m^3\) triangles is taken as an estimated value of the BP position. When there are 4 sensors, the value is given as:

\[
\begin{align*}
x'_0 &= \frac{x_a + x_b + x_c + x_d}{4} \\
y'_0 &= \frac{y_a + y_b + y_c + y_d}{4}
\end{align*}
\]

(7)
3. Results

According to the process of sound propagation in Fig 1, a 100×100 coordinate system is created. In this study, the true position of proton BP is (42,21). Four sensors at position (45,29), (40,29), (45,12), (40,12) are selected as reference node to locate the BP position. The results of selecting 3 sensors to use the centroid algorithm and selecting 4 sensors to use the double centroid algorithm are shown in Fig 4.

![Figure 4.](image)

**Figure 4.** (a) result of the centroid algorithm  (b) result of the double centroid localization algorithm

The root mean square error (RMSE) is used as the judgment standard of accuracy. The $RMSE = \sqrt{(x_0 - x)^2 + (y_0 - y)^2}$ is shown in Table 1:

| Number of sensors | RMSE(mm) |
|-------------------|----------|
| 3                 | 4.073    |
| 4                 | 0.620    |

4. Discussion

The results show that the double centroid algorithm's error is only 15.2% of that of the centroid algorithm under the same conditions, effectively improving the positioning accuracy. Moreover, the real error is less than 1 mm when using 4 sensors. Compared with previous studies, the double centroid algorithm has higher accuracy and does not require many sensors, which reduce the costs. Besides, due to the simplicity of the acoustic method, this method is expected to be widely used in proton therapy range verification.

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