On the Quantum Structure

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Abstract Quantum mechanics is a special kind of description of motion. The concept of wave function itself implies the openness of quantum system. We show that quantum mechanics describes the quantum correlation, i.e., entanglement, and information in a new kind of space, tangnet $T^2$, where exist the basic quantum structure of qubit and the universal out-in symmetry. This work tries to form a new view to the fundamental problems of the foundation of quantum mechanics.

Keywords Quantum Structure · Openness · Tangnet · Out-in Symmetry

1 Introduction

In this work, we address the basic concepts of quantum mechanics (QM) relating to the great development both on the foundation and application these years. Besides the mathematical and experimental aspects, the physical concepts of QM need particular attentions, e.g., there are still lots of confusions of the "weirdness" of QM at present. From the seminal work of EPR [1], Schrödinger [2], Bohm [3], Bell [4] etc, the methods of entanglement and nonlocality have been widely studied. Here, we do not focus on the confusions and differences between the two methods; instead, we study the physical essence of entanglement, quantum information, and further the new views of "quantum". Physically, the generalization from superposition to entanglement is nontrivial. On one hand, it offers new ideas on what superposition is; on the other hand, it leads to the re-consideration of what quantum means. The existence of entanglement has led to the growth of the fields of quantum information and quantum computation (QIQC) [5] and quantum foundation (QF). In the research of QF, roughly speaking, there are mainly two research trends: one is the interpretation of quantum mechanics (IQM) [6], such as the many-world interpretation [7]; the other is the post-quantum mechanics (PQM), such as the general probabilistic theory [8].

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both IQM and PQM, entanglement plays the basic role. In this short work, we do not intend to give the detailed analysis of various theories; instead, we further investigate the meaning of entanglement, information, and quantum from a new point different with the present ones.

In Sec. 2, we begin from the well known concept of “openness”, and we discuss the strict physical meaning of wave function. Then we introduce the method of “quantum structure” in Sec. 3 which is the generalization of quantum state and entanglement. We study the physical role of mutual information under the spirit of openness. Also, we introduce the new space, “tangnet”, where information is commonly shared, which is particularly demonstrated by QM. Last, in Sec. 4 we briefly analyze several related issues and open problems.

2 Openness

In this section we discuss the concept of openness in the study of quantum open system. Many quantum processes are due to the openness of quantum system. For example, the lifetime of micro-particle, the decay of electron from the excited state to ground state etc, those are due to the interaction with the vacuum, which cannot be removed. When the system is coupled with the uncontrolled environment, decoherence will occur, which is described by Zurek as the disturbance of system to environment [9]. The role of openness in QM had been demonstrated a lot, such as the early work of Zeh [10]. Further, if we take a historical view, one will find that early in the formation of the theory of density matrix, the openness had already been addressed, and the concept of mixed state was introduced [11]. We should note that the approach of decoherence relies on density matrix. Below, we analyze the meaning of mixed state. Generally, there are two related views of mixed state, as follows

(I). The “tracing” view: The mixed state is an inner part of a global pure state, tracing out the rest.

(II). The “summing” view: The mixed state is the mixture of several pure states, as \( \rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \) is pure state, \( \sum_i p_i = 1 \).

We should address that there is no standard reason why there should be the two views, also, whether there could be anything more. Mathematically, we can easily show that the two views are equivalent. For the mixed state, according to the tracing view, introducing the parameter \( p_{ij} = \sum_{\mu} a_{ij\mu}^* a_{ij\mu} \), then

\[
\rho = \sum_{i,j} a_{ij}^* a^\dagger_{ji} |i\rangle\langle j| \tag{1}
\]

\[
= \sum_{i,j\mu\nu} |\psi\rangle\langle\nu| a_{ij\mu} a_{ji\nu}^* |i\rangle\langle j|.
\]
\[
= \sum_{\mu \nu} \langle \psi | a_{\mu} a_{\nu}^* | i \rangle \langle j | \psi \rangle = \langle \Psi | \Psi \rangle,
\]
where \( | \Psi \rangle = \sum_{\mu} a_{\mu} | i \rangle | \mu \rangle \) is the global pure state relating to \( \rho \), the normalization rule \( \sum_{\mu} | a_{\mu} |^2 = 1 \).

In practice, the differences between the two views are seldom noticed, which can be explained by the mathematical equivalence. However, for the physical meaning, the two views are different, and we emphasize that the summing view (II) is wrong. From the tracing view (I), we know that in reality we often cannot decide the wave function of a system, as the system is often correlated with environment (or other systems). Thus, as assumption, we use the density matrix to describe the state of the system; that is, density matrix is only the “fragment”, which is not a complete characterization of a state. In other words, there does not exist several pure states, or “relative state” using Everett’s terminology, to mix up, that is, the summing view is in conflict with the tracing view, physically. Further, taking from another point, the summing view (II) of the mixed state actually comes from the method of classical statistical physics. Classically, we can say, e.g., the gas is the mixture of molecules. Yet, according to QM, the concept of “mixture”, which relates to the concept of “classicality”, does not capture the feature of coherence, which leads to, e.g., interference. So, it is not proper to introduce the concept of mixture to QM, instead, we should study quantum process from pure quantum methods, e.g., decoherence, and to avoid any confusion with classicality.

Further, relating to the tracing view (I), we discuss the concept of wave function. According to the standard QM, the property of a system can be wholly described by its wave function. This is definitely right, yet, not complete. This statement relies on the assumption that there exists the wave function of the system. Yet, according to the concept of openness we studied above, quantum system basically is correlated with its environment, thus, the wave function should also include the environment even when the environment is insignificant. Along this logic, we will eventually get that the whole universe as a whole should be a pure state, which, in fact, is also one assumption. Below, we study the widely concerned model of the pure universe. Let universe \( \mathcal{U} \) be composed with system \( \mathcal{S} \) and environment \( \mathcal{E} \), labeled as \( \mathcal{U} = \mathcal{S} + \mathcal{E} \). We note that there is no need to specialize the interaction in between. The model is depicted in Fig. 1(a). Thus,

\[
| \psi \rangle_\mathcal{S} \approx \text{tr}_\mathcal{E} \rho_\mathcal{U}, \tag{2}
\]
on the left-hand-side (LHS), \( | \psi \rangle_\mathcal{S} \) is the pure state of the system; on the RHS, \( \text{tr}_\mathcal{E} \rho_\mathcal{U} \) is a decohered state. When the role (effect) of environment \( \mathcal{E} \) is trivial, we can let them equal in mathematics, also in physics we assume they are the same. This kind of approximation should be nontrivial for the understanding of QM. So, for the method of wave function, we should make clear the conjecture (or assumption) as follows:

**Conjecture 1** In the pure universe, there exists system which has one wave function.
This conjecture addresses that the concept of wave function itself indicates the concept of openness in QM. Thus, pure quantum mechanically, when we study some quantum theory, the starting point of the theory should be the open system, which is just opposite to the classical mechanics (CM). In CM, often the individual behavior of a certain object is described, instead of the correlation with the rest of the universe. Conjecture 1 is one of the main differences between QM and CM. We will address later in the last section the differences between QM and CM in detail. In addition, we note that the similar problem has also been studied mathematically from a statistical view, e.g., in Ref. [12].

3 Quantum Structure

In our study we view entanglement and information as different quantities, as the quantity “discord” indicates that there could exist quantum information without entanglement [13,14]. Entanglement describes the quantum correlation of a system, and by information, we mean Von Neumann entropy. Here, we do not intend to make mathematical study. For the indication of entanglement and information to QF, there are many progresses. For instance, Mermin stated that QM describes the correlation without “correlata” [15], which just demonstrated the openness of quantum theory. Bub studied QM from a broader view, i.e., comparing to the theory of Relativity, and he claimed that QM is the “principle” theory, the CBH theorem tried to serve as the principle of QM, and QM is about quantum information instead of wave or particle [16], which arouses great interests [17]. Gisin viewed nature as nonlocal fundamentally, and nonlocality does not exist in space-time [18], this observation indicates that there may exist another kind of fundamental symmetry behind the standard QM. Relating to decoherence, Zurek systematically studied the relation between quantum and classical. He introduced the symmetry envvariance, and viewed it as the fundamental symmetry [2]. With all these explorations, yet, there are still primary problems remain, e.g., the physical meaning of entanglement is not clear, one expression of this confusion is that there are too many quantities to characterize entanglement.
at present. Below, we present a new kind of picture to understand entanglement and information, i.e., we introduce a new kind of space based on entanglement and information via the $\mathcal{U} = S + E$ bi-party model above.

3.1 Mutual Information

We first study the property of information via the well known mutual information. It is direct to introduce another system or environment, or to divide $E$ ($S$) into different parts. In Fig. 1(b) and (c), we show the two basic models. Panel (b) shows the multi-environment model (multi-$E$), and panel (c) shows the multi-world model (multi-$W$). Here, we should demonstrate that one world (or universe) should contain at least one system and one environment, i.e., one system coexists with at least one environment, this is the result of openness. In panel (c), there are two systems $S_1$ and $S_2$, then in the whole universe there are multi-world, three kinds of environment, $E_0$, $E_1$, and $E_2$, coexist.

For the multi-$E$ model, the wave function of the system $S$ is

$$|\psi\rangle_S \approx \text{tr}_{E_0}\rho_{SE_1},$$

(3)

For the multi-$W$ model, the wave function of the system $S_1 (S_2)$ is

$$|\psi\rangle_{S_{1(2)}} \approx \text{tr}_{E_0+E_2}\text{tr}_{E_1}\rho_{SE_{1(2)}},$$

(4)

We should note again, the LHS and RHS of both equation (3) and (4) are made equal in both mathematics and physics.

To be more precise, we analyze the mutual information, which is viewed as the total correlation, for the different models. The mutual information $I$ is widely involved in the research of QIQC, such as the discord [13][14], squashed entanglement [19,20,21], conditional entanglement of mutual information [22] with the operational meaning of partial state merging [23], etc. In Fig. 1(a), the basic $\mathcal{U} = S + E$ model is quite simple, as $\rho_{\mathcal{U}}$ is pure, there always exists the bi-party Schmidt decomposition of the pure state of the universe $|\Psi\rangle_{\mathcal{U}} = \sum_i \lambda_i |S_i\rangle|E_i\rangle$, where $|S_i\rangle$ and $|E_i\rangle$ are local basis, that is, the entanglement can always be realized by the rotation of the basis. The mutual information is

(a) : $I = S_S + S_E - S_{SE} = 2S_E$.

(5)

And the classical information is $S_E$, then the quantum information is just $S_E$, the von Neumann entropy, which is the well known result.

For the multi-$E$ model in Fig. 1(b), the density matrix for the party $S + E_1$ is $\rho_{SE_1} = \text{tr}_{E_0}\rho_{\mathcal{U}}$, and the mutual information is

(b) : $I = S_S + S_{E_1} - S_{SE_1}$.

(6)

When $E_0 = \emptyset$ or $E_0 = E_1$, the multi-$E$ model reduces to the model (a).

For the multi-$W$ model, here we aim to quantify the mutual information between systems $S_1$ and $S_2$, with environments $E_1$ and $E_2$, respectively. There can be mutual
information among any two of the four parties. From the Venn diagram, which we do not show here, it is direct to get the mutual information between the two systems

\[ I = I(S_1E_1 : S_2E_2) - I(E_1 : E_2) - I(E_1 : S_2 | E_2) - I(E_2 : S_1 | E_1), \]

where, e.g., \( I(E_1 : S_2 | E_2) \) is the conditional mutual information. This expression is general, and it can be reduced to special forms under the particular cases as below:

1. When \( E_1 = E_2 = E_0 \), the multi-W model reduces to the analogy of the multi-E model in Fig. 1(b).
2. When \( E_1 = E_2 = E \neq E_0 \) (or \( E_1(E_2) = \emptyset \)), the mutual information reduces to

\[ I = I(S_1 : S_2 | E), \]

which is the same with the squashed entanglement physically \([19,20,21]\).
3. When \( E_0 = \emptyset \), that is, the state \( \rho_{S_1E_1S_2E_2} \) is pure, then the three-party state \( \rho_{S_1S_2E_1} \) is mixed, and the problem becomes the same with case (2).
4. When \( I(E_1 : S_2 | E_2) \) and \( I(E_2 : S_1 | E_1) \) are zero, which can be easily depicted via the Venn diagram, the mutual information in equation (7) reduces to

\[ I = I(S_1E_1 : S_2E_2) - I(E_1 : E_2), \]

which is physically the same with the conditional entanglement of mutual information defined in Ref. [22]. This indicates that our definition is more general.

From the above study, we can see that we can use the mutual information to characterize the information within the different models. And from the concept of openness, the mutual information for the multi-W model in equation (7) is the most general one, i.e., it demonstrates that if we intend to extract the mutual information between two systems, we need to consider the corresponding environments of the two systems. In reality, the three environments \( E_0, E_1, \) and \( E_2 \) can be the same, which can simplify the complexity of the correlation.

3.2 Tangnet

We now turn to another aspect of QM. According to the standard QM, the state vector exists in the Hilbert space. The mathematical element in QM is operator or algebra, instead of number, that is, QM describes the logical structure of the state. In the Heisenberg picture, the commutation relation of the operator and the related group can manifest the algebraic structure better than the Schrödinger picture. Referring to openness, we can say that QM describes the correlation and information of a certain dynamics. Along the logic of the study in the above subsection, the whole universe can be eventually depicted as a kind of "lattice", with entanglement and information within. We can name this kind of space as informet (information-net) or tangnet (tangle-net), shown as the lattice in Fig. 2. Mathematically, the tangnet is the topological two-dimensional complex lattice space \( T^2 \). We note that it is easy to put the multi-E and multi-W models on the tangnet.

Tangnet is different with other spaces we are familiar with. For instance, the configuration space or Cartesian space \( \mathbb{R}^3 \) in classical physics describes the possible
Fig. 2 The tangnet space $\mathbb{T}^2$ (black lattice) and the quantum structure of qubit (blue dots, circles, and lines). The two parties are $A$ and $B$ (dashed-dot elliptical circles), the two basis are $|1\rangle$ and $|0\rangle$.

places of the object, which is static without time. The Minkowski space $\mathbb{M}^4$ is the generalization of $\mathbb{R}^3$ as the result of Relativity (we do not study the relation between QM and Relativity in this paper). $\mathbb{M}^4$ can enclose the motion of field, which is exotic for $\mathbb{R}^3$, by putting time and space on the equal footing. The phase space is primarily different with $\mathbb{R}^3$ and $\mathbb{M}^4$. Phase space combines the object (its place $\mathbf{r}$ and its movement (the momentum $\mathbf{p} = m \frac{d \mathbf{r}}{dt}$ and time) together, thus it can describe the motion more systematically. The Hilbert space $\mathbb{H}$ is the space of state vector, it defines the operation of operator and vector. The tangnet $\mathbb{T}^2$ is not the differential manifold, which is the central feature of this space. It should be interesting that this feature can be viewed as the origin of “quantisation”, with manifold as the classical limit. Tangnet $\mathbb{T}^2$ is not the same with the Hilbert space $\mathbb{H}$. In $\mathbb{T}^2$, the state is described as the “node” on the lattice instead of a kind of vector, and the lines between nodes cannot be described in $\mathbb{H}$.

From the new concept of tangnet, it is not enough to say that quantum system is open or closed, instead, the basic object becomes quantum structure, which is formed by the states of the system. QM describes the quantum structure in tangnet. Information is stored and shared in the unique and holistic quantum structure, which indicates that the primary feature of information is sharing.

Below, we introduce some symbols and rules of the quantum structure:

1. Label the eigntate as dots “•” or circles “◦” with each party the same symbol;

2. Label the entanglement as string “-”, the length of string relates to the coefficients in the entangled state;

3. There is no restriction on the spatial orientation of $|0\rangle$, $|1\rangle$, $\cdots |n\rangle$ of each party;
(4). The states of different parties correlate with each other one-to-one;

(5). The phase among the branches is defined via the relative spatial orientation.

We should note that for the concept of quantum structure, we introduce “string” which may reflect more information than just “state”. Next we discuss the basic quantum structure in QM. Fig. 2 shows the quantum structure of the qubit, the state is set as

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A |1\rangle_B - i|1\rangle_A |0\rangle_B),$$

where the two parties are A and B, there are two branches $|0\rangle_A |1\rangle_B$ and $|1\rangle_A |0\rangle_B$, which have the same length, thus, equally weighted as $\frac{1}{\sqrt{2}}$. It takes the branch $|1\rangle_A |0\rangle_B$ anticlockwise to rotate to the same orientation with the other branch $|0\rangle_A |1\rangle_B$, thus, the relative phase between them is $-i$.

We note that the study here can also be generalized to the general basis

$$|\psi\rangle_A^A = \cos \alpha |0\rangle_A + \sin \alpha e^{i\theta} |1\rangle_A,$$

$$|\psi\rangle_A^S = \cos \alpha |0\rangle_A + \sin \alpha e^{i\theta} |1\rangle_A,$$

where $|\psi\rangle_A^A$ is asymmetric, and $|\psi\rangle_A^S$ is symmetric. Here, e.g., symmetric means $|0(1)\rangle$ relates to $|0(1)\rangle$. When $\alpha = 45^\circ$, and $\theta = 0^\circ, 180^\circ$, the Bell basis is realized. When $\alpha = 45^\circ$, $\theta = 270^\circ$, $|\psi\rangle_A^S$ reduces to the state in equation (10). Different states of qubit have different quantum structure. It is easy to verify that there are totally eight kinds of structures relating to qubit (we do not present them here). In addition, for more complicated entangled state, there are more parties and more branches.

In QIQC, qubit is viewed as the element of quantum information, “ebit”, which equals to one qubit plus one bit. Here, we demonstrate that qubit is the basic quantum structure, not just from the information-theoretic view.

Next, we study the property of the structure of qubit. Actually, this problem has been studied quite widely. For instance, Zurek introduced “enviarance” [9], from which he studied the Born’s rule, we do not analyze this subject in detail. This symmetry states that the local unitary transformation $U_A$ and $U_B$ cannot change the global property of the entangled state $|\psi\rangle_{AB}$, which is

$$U_A U_B |\psi\rangle_{AB} = |\psi\rangle_{AB}.$$  

For another line of research, this problem is often mentioned as exchange/permutation symmetry, which is studied mainly mathematically, e.g., in Ref. [24]. Also, we should pay attention that the permutation symmetry has already been studied well in quantum field theory (QFT) decades ago yet without entanglement. Here again, we focus on the physical implication of this quantum structure. There exist two classes of basic operations:

a. Local base rotation (e.g., flipping).

For instance, for A, if $|0\rangle \rightarrow |0\rangle + i|1\rangle$, $|1\rangle \rightarrow |0\rangle - i|1\rangle$, then for B, $|0\rangle \rightarrow |0\rangle + i|1\rangle$, $|1\rangle \rightarrow |0\rangle - i|1\rangle$. 
b. Permutation (or mirror/specular reflection).

This transformation, i.e., exchange the states of $A$ and $B$ correspondingly, causes nothing or a global phase change.

We can draw the conclusion that the quantum structure is invariant under the unitary transformation.

We assume the validity of this symmetry without making any further proof mathematically. Instead, we discuss what this symmetry may means to the foundation of QM. From the concept of openness, every system should has the inside and outside also the surface. For example, for the party $A$ of qubit, there are two states in it, and $B$ is the outside. With the permutation, the qubit remains. We note that this property relates to the identity principle, here the party $A$ and $B$ are identical. Relating to the entangled structure, this symmetry is a kind of “out-in” symmetry, that is, there is actually no distinction between outside and inside. $A$ and $B$ connect with each other in such a coherent way that they become one unique entity without boundary, i.e., the quantum structure, and the information is shared commonly. This out-in symmetry is the universal and elementary symmetry in the tangnet space which has never been demonstrated before. Below, we set the theorem of this symmetry.

**Theorem 1** In quantum mechanics, there exists the out-in symmetry in the tangnet, under which the entangled quantum structure is invariant.

In addition to this theorem, we need to quantify the quantum structure via entropy, entanglement, etc, which we do not study here. This theorem relates to the Conjecture 1 above. There seems a kind of confliction with the superposition for the one-body system. However, relating to the Conjecture 1 above, there is no one-body problem in QM, the simplest case should be two-party system, that is, the out-in symmetry demonstrated by Theorem 1 acts always. Thus physically, there is no confliction between the one-body superposition and multi-party entanglement. We give two simple examples to illustrate this point.

The first one is the double-slit interference of electron. As we know, the state of electron is the superposed state of the two slits, labeled as $|r\rangle$ and $|l\rangle$. As the existence of measurement, we need to include the apparatus. When detecting at the slit $|r\rangle$ ($|l\rangle$), the state of the apparatus is $|R\rangle$ ($|L\rangle$), the global state of electron and apparatus is the entangled state. By tracing out the apparatus (when $|R\rangle$ and $|L\rangle$ are orthogonal), we get the statistical results. Also, we can apply the “weak measurement” to get both the wave and particle properties of the electron. The interruption of different measurements can get different information of the entangled state, and the entangled quantum structure exists always and the out-in symmetry acts always, too.

The second example is the Rabi oscillation of the two-level atom in cavity. When the atom emits one photon, the atom evolves from the excited state $|e\rangle$ to the ground state $|g\rangle$, and the vacuum from the $|n-1\rangle$ state to the $|n\rangle$ state in the Fock space. The global state of the atom and vacuum is entangled. We can view the atom as in the superposed state, and the out-in symmetry still acts.

From the above analysis, we can know that the generalization from superposition to entanglement is nontrivial, most importantly, it brings out the new out-in symmetry underlying QM demonstrated by Theorem 1.
In conclusion, in this work, we briefly discussed the basic concepts in QM due to the development of entanglement and information. We stated that QM indicates another kind of space, tangnet (or infornet), where exists the universal out-in symmetry and quantum structure, e.g., the most basic one, qubit. We add that further work should be carried out on the mathematical properties of tangnet. We also constructed the general form of the mutual information between two systems, i.e., equation (7) in section 3.1.

For the theorem [1] we conjectured, a mathematical study is needed, particularly, the unique definition of entanglement is necessary. Since there are too many quantities at present, such as, concurrence, robustness etc, we need to compare them in detail. Here we addressed that entanglement is not information physically; instead, it forms the element of the quantum structure.

We need to discuss a little about the differences between QM and CM. At present, there is no standard answer to this problem. According to the orthodox interpretation, QM and CM are connected by the "corresponding principle", now we often relate to decoherence and measurement. From our study, we can infer that QM is a special kind of description of motion different with CM, they describe motion in different ways without referring to special scales. For QM, we showed that it describes the information and entanglement of the motion of a certain system. For a systematic study of various descriptions of motion, we will present in the future.

Another point is about the identity principle and the quantum statistics. According to the standard QM, spin is viewed as the pure quantum quantity without classical analogy, also there exists spin only for micro-particles. The phenomenon of superposition is also believed forbidden on the macroscopic scale. However, we have known that the superposition can act on the macroscopic scale, e.g., the Schrödinger cat. Following the method in this work, we may have new view of spin. There is no reason to restrict spin in the micro-world and it is possible that there exists spin on the macroscopic scale, one of the possibilities comes from that we should find more physical meaning of spin different with the traditional one.

Last, we relate to the fundamental Holographic principle [25], which deals with entropy of black hole. This principle states that the information in a region bounded by a causal horizon is finite in bits and proportional to the area of the horizon. Here, in the context of the quantum structure and out-in symmetry, there is no definite boundary, or, there can be boundary everywhere. This physical picture should also be quite interesting.

References
1. Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 47, 777-780 (1935)
2. Schrödinger, E.: An undulatory theory of the mechanics of atoms and molecules. Phys. Rev. 28, 1049-1070 (1926)
3. Bohm, D.: A suggested interpretation of the quantum theory in terms of “hidden variables”, I and II. Phys. Rev. 85, 166-193 (1952)
4. Bell, J. S.: Speakable and Unspeakable in Quantum Mechanics. Cambridge University Press (1993)
5. Nielsen, M. A., Chuang, I. L.: Quantum Computation and Quantum Information. Cambridge University Press, Cambridge, England (2000)
6. Schlosshauer, M.: Decoherence, the measurement problem, and interpretations of quantum mechanics. Rev. Mod. Phys. 76, 1267-1305 (2004)
7. Everett, H., III: “Relative State” Formulation of Quantum Mechanics. Rev. Mod. Phys. 29, 454-462 (1957)
8. Barrett, J.: Information processing in general probabilistic theories. Phys. Rev. A 75, 032304 (2007)
9. Zurek, W. H.: Decoherence, einselection, and the quantum origins of the classical. Rev. Mod. Phys. 75, 715-775 (2003)
10. Zeh, H. D.: On the interpretation of measurement in quantum theory. Found. Phys. 1, 69-76 (1970); Toward a quantum theory of observation. Found. Phys. 3, 109-116 (1973)
11. Greenberger, D., Hentschel, K., Weinert, F.: Compendium of Quantum Physics: Concepts, Experiments, History and Philosophy. Springer-Verlag, Berlin, Heidelberg (2009)
12. Popescu, S., Short, A. J., Winter, A.: The foundations of statistical mechanics from entanglement: Individual states vs. averages. e-print arXiv:quant-ph/0511225 (2006)
13. Ollivier, H., Zurek, W. H.: Quantum discord: A measure of the quantumness of correlations. Phys. Rev. Lett. 88, 017901 (2002)
14. Henderson, L., Vedral, V.: Classical, quantum and total correlations. J. Phys. A 34, 6899 (2001)
15. Mermin, N. D.: The Ithaca interpretation of quantum mechanics. Pramana 51, 549-565 (1998)
16. Clifton, R., Bub, J., Halvorson, H.: Characterizing quantum theory in terms of information-theoretic constraints. Found. Phys. 33, 1561-1591 (2003)
17. Brassard, G.: Is information the key? Nature Physics 1, 2 (2005)
18. Gisin, N.: Quantum nonlocality: How does nature do it? Science 326, 1357 (2009)
19. Tucci, R. R.: Quantum Entanglement and Conditional Information Transmission. e-print arXiv:quant-ph/9909041; Entanglement of Distillation and Conditional Mutual Information. e-print arXiv:quant-ph/0202144
20. Christandl, M., Winter, A.: “Squashed entanglement”: An additive entanglement measure. J. Math. Phys.(N.Y.) 45, 829 (2004)
21. Oppenheim, J.: A paradigm for entanglement theory based on quantum communication. e-print arXiv:quant-ph/0801.0458
22. Yang, D., Horodecki, M., Wang, Z. D.: An Additive and Operational Entanglement Measure: Conditional Entanglement of Mutual Information. Phys. Rev. Lett. 101 140501 (2008); Conditional Entanglement. e-print arXiv:quant-ph/0701149
23. Horodecki, M., Oppenheim, J., Winter, A.: Partial quantum information. Nature (London) 436, 673 (2005)
24. Tóth, G., Gühne, O.: Entanglement and permutational symmetry. Phys. Rev. Lett. 102, 170503 (2009)
25. Bousso, R.: The holographic principle. Rev. Mod. Phys. 74, 825-874 (2002)