THE EXISTENCE OF STERILE NEUTRINO HALOS IN GALACTIC CENTERS AS AN EXPLANATION OF THE BLACK HOLE MASS–VELOCITY DISPERSION RELATION

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ABSTRACT

If sterile neutrinos exist and form halos in galactic centers, they can give rise to observational consequences. In particular, the sterile neutrinos decay radiatively and heat up the gas in the protogalaxy to achieve hydrostatic equilibrium, and they provide the mass to form supermassive black holes (BHs). A natural correlation between the BH mass and velocity dispersion thus arises: \( \log(M_{\text{BH}, f}/M_\odot) = \alpha \log(\sigma/200 \text{ km s}^{-1}) + \beta \) with \( \alpha \approx 4 \) and \( \beta \approx 8 \).

Key words: Galaxy: general

1. INTRODUCTION

Understanding the nature of dark matter remains a fundamental problem in astrophysics and cosmology. Since the discovery of neutrinos’ nonzero rest mass (Fukuda et al. 1998; Bilenky et al. 1998), the possibility that neutrinos contribute to cosmological dark matter has become a hot topic again. In particular, the sterile neutrinos are a class of candidate dark matter particles with no standard model interaction. Although the recent MiniBooNE result challenges the liquid scintillator neutrino detector (LSND) result that suggests the existence of eV scale sterile neutrinos (Aguilar-Arevalo et al. 2007), more massive sterile neutrinos (Aguilar-Arevalo et al. 2007), more massive sterile neutrinos with rest mass \( m_s \) (Adams et al. 2001; MacMillan & Henriksen 2002; Robertson et al. 2005; Murray et al. 2005; King 2003, 2005; McLaughlin et al. 2006). We assume that a degenerate sterile neutrino halo exists in the center of a protogalaxy. There are two different decay modes for sterile neutrinos \( \nu_s \). The major decaying channel is \( \nu_s \to 3\nu \) with decay rate (Barger et al. 1995; Boyarsky et al. 2008)

\[
\Gamma_{3\nu} = \frac{G_F^2}{384\pi^3} \sin^2 2\theta m_s^5 = 1.77 \times 10^{-20} \sin^2 2\theta \left( \frac{m_s}{1\text{keV}} \right)^5 \text{s}^{-1},
\]

where \( G_F \) and \( \theta \) are the Fermi constant and mixing angle of sterile neutrino with active neutrinos, respectively. The minor decaying channel is \( \nu_s \to \nu + \gamma \) with decay rate (Barger et al. 1995; Boyarsky et al. 2008)

\[
\Gamma = \frac{9\alpha G_F^2}{1024\pi^4} \sin^2 2\theta m_s^5
\]

\[
= 1.38 \times 10^{-22} \sin^2 2\theta \left( \frac{m_s}{1\text{keV}} \right)^5 \text{s}^{-1} \approx \frac{1}{128} \Gamma_{3\nu},
\]

where \( \alpha \) is the fine structure constant. It is quite difficult to detect the active neutrinos produced in the major decaying channel. Therefore, we focus on the observational consequence of the radiative decay of the sterile neutrinos with rest mass \( m_s \geq 10 \text{ keV} \). They emit high energy photons (\( \approx m_s/2 \)) which heat up the surrounding gas so that hydrostatic equilibrium of the latter is maintained. The sterile neutrino halo also provides mass to form a supermassive BH from a small seed BH. Without any further assumption, \( \alpha \approx 4 \) is consistent with the range of the decay rate observed by observational data from cooling flow clusters. In this paper, we first give a brief review of three popular analytic models that explain the \( M_{\text{BH}, f} - \sigma \) relation. Then we give a detailed description of our model and compare it with the existing models.

2. REVIEW ON MODELS OF THE \( M_{\text{BH}, f} - \sigma \) RELATION

2.1. Super-Eddington Accretion Model

King (2003) presented a model to explain the \( M_{\text{BH}, f} - \sigma \) relation. He assumed that the gas density profile of a protogalaxy is isothermal \( (\rho \sim r^{-2}) \) (King 2003, 2005). Therefore the gas mass inside radius \( R \) is

\[
M(R) = 4\pi \int_0^R \rho r^2 dr = \frac{2f_s\sigma^2 R}{G},
\]

This relation has been derived by recent theoretical models (Adams et al. 2001; MacMillan & Henriksen 2002; Robertson et al. 2005; Murray et al. 2005; King 2003, 2005; McLaughlin et al. 2006). We assume that a degenerate sterile neutrino halo exists in the center of a protogalaxy. There are two different decay modes for sterile neutrinos \( \nu_s \). The major decaying channel is \( \nu_s \to 3\nu \) with decay rate (Barger et al. 1995; Boyarsky et al. 2008)
where \( f_g \approx 0.16 \) is the cosmological ratio of baryon to total mass, assumed to be the same throughout a galaxy, and the Virial theorem is used. Consider a super-Eddington accretion onto a seed BH. The accretion feedback produces a momentum-driven superbubble that sweeps ambient gas into a thin shell which expands to the galaxy. The equation of motion is

\[
\frac{d}{dt}[M(R)\mathbf{R}] + \frac{GM(R)[M_{BH}(t) + M(R)]}{R^2} = \frac{L_{\text{edd}}}{c},
\]

where \( L_{\text{edd}} = 4\pi GM_{BH}(t)c/\kappa \), with \( \kappa \) the opacity and \( M_{BH}(t) \) the mass of the central BH at time \( t \). Integrating twice and assuming \( R \gg GM_{BH,f}/\sigma^2 \), one gets

\[
R^2 = R_0^2 + 2R_0\dot{R}_0t - \sigma^2 \left(1 - \frac{M_{BH}(t)}{M_\sigma}\right)t^2,
\]

(5)

where \( \dot{R}_0 = \ddot{R} \) at \( R = R_0 \), with \( R_0 \) some large radius (\( \gg GM_{BH,f}/\sigma^2 \)), and \( M_\sigma \equiv f_g\kappa\sigma^4/\pi G^2 \). Therefore, the maximum radius \( R_{\text{max}} \) is given by

\[
\frac{R_{\text{max}}^2}{R_0^2} = 1 + \frac{\ddot{R}_0^2}{2\sigma^2(1 - M_{BH}(t)/M_\sigma)}.
\]

(6)

When \( M_{BH}(t) \) approaches \( M_\sigma \), \( R_{\text{max}} \) becomes very large, because such that the cooling of the shocked wind is inefficient because the cooling time \( t_{\text{cooling}} \propto R^2 \) and the accretion is stopped because the shell can escape the galaxy entirely by gas pressure (King 2003). Therefore, given an adequate mass supply (such as in a merger), we get (King 2005)

\[
M_{BH,f} = \frac{f_g \kappa}{\pi G^2} \sigma^4.
\]

(7)

Here, the proportionality constant \( f_g \kappa/\pi G^2 \) lies within the observational constraints. To summarize, the \( M_{BH,f} - \sigma \) relation is obtained with three important assumptions: (1) isothermal gas density distribution throughout the galaxy formation, (2) super-Eddington accretion, and (3) an adequate mass supply.

2.2. The Self-similar Model

MacMillan & Henriksen (2002) obtained a relation between \( M_{BH,f} \) and \( \sigma \) by assuming that the density and velocity distributions of matter are self-similar. They assumed that the galaxy is formed by the extended collapse of a halo composed of collisionless matter. The central BH is grown proportionally to the halo as matter continues to fall in. The relation is given by MacMillan & Henriksen (2002)

\[
\log M_{BH,f} \propto \left(\frac{3\delta/\alpha - 2}{\delta/\alpha - 1}\right) \log \sigma.
\]

(8)

where \( \delta \) and \( \alpha \) are scales in space and time, respectively, and their ratio is related to the power-law index of the initial density perturbation \( \epsilon \) in the spherical infall model of halo growth (Henriksen & Widrow 1999):

\[
\frac{\delta}{\alpha} = \frac{2}{3} \left(1 + \frac{1}{\epsilon}\right).
\]

(9)

The power-law index \( \epsilon = (n + 3)/2 \), where \( n \) is the index of the primordial matter power spectrum \( P(k) \propto k^n \). If \( n = -2 \), Equation (8) agrees with the observation \( M_{BH,f} \propto \sigma^4 \). This model involves a relation (Equation (9)) which is quite model dependent.

2.3. The Ballistic Model

Adams et al. (2001) assume the dark matter and baryons to be unsegregated and the isothermal initial mass density distribution \( M(r) \propto r \). The specific orbital energy is conserved when the particles fall into the small seed BH:

\[
E = \frac{1}{2} v^2 + j^2/2r^2 - GM(r)/r,
\]

(10)

where \( v \) and \( j \) are the radial velocity and angular momentum per unit mass. When the particles fall into the equatorial plane, their pericenters are

\[
p = j^2/(2GM(r)) = (GM(r)/\Omega^2)^2\sigma^8/2c^2.
\]

(11)

where \( \Omega \) is the average angular speed of the particles. In the early stage, all the particles fall into the BH until the BH mass reaches a critical point that corresponds to \( p = 4R_\sigma \), with \( M_{BH}(t) = M(R_\sigma) \), with \( R_\sigma = 2GM_{BH}(t)/c^2 \) the Schwarzschild radius. This gives the final relation (Adams et al. 2001)

\[
M_{BH,f} = 4\sigma^4/Gc^2\Omega.
\]

(12)

This model is based on the assumption of the isothermal distribution of matter, and there is a free parameter \( \Omega \), which is assumed to have the same value for all galaxies.

3. DECAYING STERILE NEUTRINO HALO MODEL

Suppose the sterile neutrino halo dominates the mass in the protogalactic center, most of the mass in the BH comes from the sterile neutrino halo with radius \( \bar{R} \), which was formed in the very early universe \( t \sim 0 \) (Munyanzea & Biermann 2005; Chan & Chu 2007). The total mass of the degenerate sterile neutrino halo at time \( t_0 \) is

\[
M_s(t_0) = 4\pi \int_0^\bar{R} \rho_s r^2 dr = M_{s,0} e^{-T_{s,0}/t_0},
\]

(13)

where \( \rho_s \) is the mass density of the sterile neutrino halo. Assume that a seed BH with mass \( M_{BH,0} \) of order solar mass is formed at \( t_0 \), long after the formation of the sterile neutrino halo. It would grow by accreting mass of the sterile neutrino halo to mass \( M_{BH}(t) \) at time \( t \). As some sterile neutrinos would be accreted by the seed BH, the degenerate pressure is decreased and more sterile neutrinos will fall into the BH as their Fermi speed is less than their escape speed (Munyanzea & Biermann 2005). The falling timescale at distance \( r \) from the BH in a free falling model is given by (Phillips 1994)

\[
t_{ff} = \frac{\pi}{2} \sqrt{\frac{r^3}{2G[M_{BH,0} + M_s(r, t)]}}.
\]

(14)

For \( M_s(t) \geq 10^6 M_\odot, m_s \geq 10 \text{ keV and } \bar{R} \leq 0.04 \text{ pc, } t_{ff} \leq 160 \text{ years for } r \leq \bar{R}, \text{ which is much shorter than the Hubble time. Therefore, we do not need any intermediate mass BHs since the small seed BH can grow to a } 10^6-10^7 M_\odot \text{ supermassive BH rapidly as long as there is enough mass in the initial sterile neutrino halo. In the following, we assume that all sterile neutrinos fall into the BH and decay into active neutrinos and photons, and so } M_{BH,f} = M_s(t_0) + M_{BH,0} \approx M_{s,0} e^{-T_{s,0}/t_0}. \]
The photons emitted by the original decaying sterile neutrinos provide energy to the gas in the protogalaxies by Compton scattering. The optical depth of a decayed photon in the bulge is \( \tau = n_e \sigma_T R_e \), where \( R_e \) is the J-band effective bulge radius (Marconi & Hunt 2003), \( \sigma_T \) is the Compton scattering cross section, and \( n_e \) is the mean number density of the gas in the protogalaxies. In equilibrium, the heating rate is equal to the cooling rate by bremsstrahlung radiation \( \Lambda_B \), recombination \( \Lambda_R \), and adiabatic expansion \( \Lambda_a \). We have (Katz et al. 1996)

\[
L(1 - e^{-\tau}) = \Lambda_B + \Lambda_R + \Lambda_a \\
= \left[ \Lambda_{B0} n_e^2 T^{0.5} + \Lambda_{R0} n_e^2 T^{0.3} (1 + T/10^6K)^{-1} \right] V + PV^{2/3}c_s, \tag{15}
\]

where \( \Lambda_{B0} = 1.4 \times 10^{-27} \text{ erg s}^{-1} \), \( \Lambda_{R0} = 3.5 \times 10^{-26} \text{ erg s}^{-1} \), \( c_s \), \( P \), and \( V \) are the sound speed, pressure, and total volume of the gas within \( R_e \), respectively. The \( M_{BH, f} - \sigma \) relation can be obtained by using Equation (15) and the Virial theorem numerically. Nevertheless, we first illustrate the idea by obtaining analytic relations in two different regimes. Suppose \( M_f(t_b) \gg 10^6 M_B \); if \( \tau \gg 1 \), the resulting temperature is above \( 10^6 \) K and the total cooling rate is dominated by \( \Lambda_a \). For \( \tau \lesssim 1 \), the resulting temperature is lower and the total cooling rate is dominated by \( \Lambda_B \) and \( \Lambda_R \) (see Figure 1).

In the optically thick regime, \( \tau \gg 1 \) and \( \Lambda_a \gg \Lambda_B + \Lambda_R \), and we get

\[
kT = \left( \frac{m_g}{\gamma} \right)^{1/3} V^{-4/9} L^{2/3} n_e^{-2/3}, \tag{16}
\]

where \( m_g \) is the mean mass of a gas particle. By using the Virial theorem \( kT = f_1 G M_B m_g / 3 R_e \), where \( M_B \) is the effective bulge mass of the protogalaxy within \( R_e \), and substituting \( L \sim M_x \Gamma c^2 / 2 \), we get

\[
M_x = 2 \gamma f_1^{3/2} G^{3/2} e^{-\tau_{b,0}} \left( \frac{M_B}{R_e} \right)^{5/2}, \tag{17}
\]

where \( \gamma \) is the adiabatic index of the gas and \( f_1 \sim 1 \) is a constant that depends on the density distribution of the protogalaxy. As time passes, the energy gained by the gas would decrease gradually and the mass distribution at the center would also change slightly. If the supermassive BH was formed when the galaxy formation was nearly completed \( (t_b = 10^{16} - 10^{17} \) s), the ratio \( M_B / R_e \) and the velocity dispersion do not change significantly. According to Equation (17), the ratio \( M_B / R_e \) is fixed by \( M_x \) and \( \Gamma \). By using the Virial theorem again and assuming spherical symmetry, one can relate this ratio to the final bulge velocity dispersion after the supermassive BH is formed,\n
\[
\sigma^2 = f_2 \frac{G M_B}{R_e}, \tag{18}
\]

where \( f_2 \sim 1 \) is a constant that depends on the mass distribution at present. Combining Equations (17) and (18), we get

\[
M_{BH, f} = M_x (t_b) = 2 \gamma f_1^{3/2} e^{-\tau_{b,0}} f_2^{5/2} G \Gamma c^2 \sigma. \tag{19}
\]

In the optically thin regime, \( \tau \lesssim 1 \) and \( \Lambda_B + \Lambda_R \gg \Lambda_a \), the total power absorbed by the gas in the protogalaxies within \( R_e \) is approximately \( L_{B0} \sigma_T R_e \). If the cooling rate is dominated by bremsstrahlung radiation, in equilibrium, we get

\[
kT = k \left( \frac{L \sigma_T R_e}{\Lambda_0 n_e V} \right)^2. \tag{20}
\]
By using the Virial theorem and Equation (18), we obtain

$$M_{\mathrm{BH},f} = \frac{2 \Lambda R_0 e^{-\Gamma_{\nu} \beta}}{\sigma T f_2^{3/2} G \Gamma c^2} \left( \frac{f_1}{3 km_g} \right)^{\frac{1}{2}} \sigma^3. \quad (21)$$

If the cooling rate is dominated by recombination, in equilibrium and for $T \lesssim 10^6$ K, we get

$$kT = k \left( \frac{L \sigma T R_e}{\Lambda_0 n_e V} \right)^{10/3} \quad (22)$$

and

$$M_{\mathrm{BH},f} = \frac{2 \Lambda R_0 e^{-\Gamma_{\nu} \beta}}{m_g^{7/10} \sigma T f_2^{13/10} G \Gamma c^2} \left( \frac{f_1}{3 km_g} \right)^{\frac{3}{10}} \sigma^{2.6}. \quad (23)$$

Therefore, $M_{\mathrm{BH},f}$ and $\sigma$ are closely related in both optically thick and thin regimes.

We use 500 random data in the ranges of $\tau = 0.01$–10.000, $f_1 = 0.6$–3, $f_2 = 0.6$–3, $R_e = 10^{10}$–$10^{12}$ s, $10^9 M_\odot \lesssim M_B \lesssim 10^{12} M_\odot$, and 0.1 kpc $\leq R_e \leq 10$ kpc to generate the values of $M_{\mathrm{BH},f}$ and $\sigma$ by using Equations (15) and (18) (see Figure 2).

By using $\Gamma_{\nu} = (5 \pm 1) \times 10^{-17}$ s$^{-1}$ which solves the cooling flow problem (Chan & Chu 2007)$^1$ and $\Gamma_{\nu} = 128\Gamma$, we get

$$\log \left( \frac{M_{\mathrm{BH},f}}{M_\odot} \right) = (3.98 \pm 0.29) \log \left( \frac{\sigma}{200 \, \text{km s}^{-1}} \right) + (8.04 \pm 0.20), \quad (24)$$

which agrees with the recent observation: $\alpha = 4.02 \pm 0.32$ and $\beta = 8.13 \pm 0.06$ (Tremaine et al. 2002).

As an order of magnitude estimate, $\sin \theta \sim m_D/m_\nu$, where $m_D$ is the neutrino Dirac mass. In the standard seesaw mechanism, $m_D$ is the geometric mean of the light neutrino mass-scale $m_1$ and $m_2$ (Mohapatra & Senjanović 1980).$^2$ Therefore, $\sin \theta \sim \sqrt{m_D/m_\nu}$. From Equation (1), we get

$$\Gamma_{\nu} \sim 10^{-23} \left( \frac{m_\nu}{1\text{eV}} \right) \left( \frac{m_D}{1\text{keV}} \right)^4 \text{s}^{-1}. \quad (25)$$

For $\Gamma_{\nu} \sim 10^{-17}$ s$^{-1}$, $m_\nu \sim 30$ keV for $m_\nu \sim 1$ eV, which is consistent with our assumption ($m_\nu \geq 10$ keV).

4. DISCUSSION AND SUMMARY

We assume the existence of a degenerate neutrino halo ($m_\nu \sim$ keV) at the protogalactic center with a parameter, $\Gamma_{\nu}$, which is universal and can be inferred from observation of cluster hot gas (Chan & Chu 2007). Without any assumptions of the protogalaxy and existence of any intermediate mass BH, $\alpha$ $\approx$ 4 and $\beta$ $\approx$ 8 require the total decay rate to be $\Gamma_{\nu} = (5 \pm 1) \times 10^{-17}$ s$^{-1}$, which is consistent with the observational data from cooling flow clusters (Chan & Chu 2007). We also reviewed several models to explain the $M_{\mathrm{BH},f}/\sigma$ relation. These models require several assumptions or free parameters which may not be true for all galaxies. For example, King’s model assumes the isothermal density profile ($\rho \sim r^{-2}$) for all galaxies during all time of the BH formation. If the density profile changes into the form $\rho \sim r^{-4}$, then $\sigma \sim \sqrt{r}$ which is not a constant. This problem also exists in the ballistic model which is based on the isothermal distribution of matter. However, in the

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$^1$ If we consider the major decaying channel is $\nu_e \rightarrow 3\nu$, the decay rate obtained in this paper should correspond to $\Gamma_{\nu}$.

$^2$ Mohapatra & Senjanović thank the referee for suggesting the estimate of $\theta$ using the seesaw mechanism.
self-similar model, there are several free parameters which are model dependent. Also, one cannot obtain the proportionality constant $\beta$ of the $M_{\text{BH}}/f$ relation.

We have considered a wide range of $\tau = 0.1-10,000$, $f_1 = 0.6-3$, and $f_2 = 0.6-3$ encompassing almost all possibilities in galaxies. In our model, we assume that the supermassive BH was formed at the epoch when the galaxy formation was nearly completed ($t_b = 10^{16-10^{17}}$ s) so that the velocity dispersion does not change significantly between $t_b$ and present. Therefore our result is valid only for supermassive BHs formed nearly at the end of the galaxy formation, as in the super-Eddington accretion and ballistic model. The assumed existence of a decaying sterile neutrino halo inside each galactic center provides enough mass to form the supermassive BH. It can also solve the cooling flow problem in clusters (Chan & Chu 2007) and explain the re-ionization of the universe (Hansen & Haiman 2004), all with the same decay rate $\Gamma_3 = (5 \pm 1) \times 10^{-17}$ s$^{-1}$ and $m_\nu \geq 10$ keV, which are consistent with the standard see-saw mechanism.

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