Two-loop Anomalous Dimensions of Heavy Baryon Currents in Heavy Quark Symmetry

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Abstract

This talk presents details of the calculation of two-loop corrections to heavy baryon currents containing one heavy quark in leading order of the $1/m_Q$-expansion, i.e. in the limit of Heavy Quark Symmetry (HQS). The calculations lead to the determination of the anomalous dimension of these currents. I also discuss problems with different $\gamma_5$-schemes and their solution by the finite renormalization procedure.
1 Introduction

Heavy baryons such as Λ_c, Σ_c, Λ_b or Σ_b containing one heavy quark can be successfully described by the Heavy Quark Effective Theory (HQET), especially in leading order of the 1/m_Q-expansion, i.e., in the limit of Heavy Quark Symmetry (HQS) (for a review on HQET see [1], for details on HQS see [2]). For each of the ground-state baryons there are two independent current components, e.g.,

\[ J_{Λ_1} = [q_i^T C τ γ_5 q]^k Q^j ε_{ijk} \]

and

\[ J_{Λ_2} = [q_i^T C τ γ_5 γ_0 q]^k Q^j ε_{ijk}, \]

while the general structure of heavy baryon currents has the form

\[ J = [q_i^T C τ Γ q]^j Γ' Q^k ε_{ijk}, \]

where the index \( T \) means transposition, \( C \) is the charge conjugation matrix, \( τ \) is a matrix in flavour space, and \( i, j \) and \( k \) are colour indices. The effective static field of the heavy quark is denoted by \( Q \), and \( Γ \) and \( Γ' \) are the Dirac structures of the different current parts.

In Fig. 1 are shown some examples of diagrams up to two-loop order which we calculated. The one- and two-loop diagrams can be grouped in three different classes, namely the heavy diquark system (diagrams (b), (c), (e), (f) and (g)), the light diquark system (diagrams (d) and (h)) and the irreducible vertex contribution (diagrams (i) and (j)). The first two can be handled in a manner similar to the meson case, and actually it turns out that the replacement \( C_B \rightarrow C_F \) leads from the diquark to the meson quark-antiquark case (and not vice-versa). So the mesonic case can be seen as a byproduct of our calculations.

2 Algorithmic calculations

The calculation of all these diagrams can be executed by nearly the same procedure. The procedure is strictly algorithmic, so that the calculations could be automatized and programmed in MATHEMATICA. In the following I want to emphasize the main steps of this calculation, and I want to do this for the first two-loop heavy diquark diagram which is displayed in Fig. 2 (the arrows on the gluon lines only indicate the momentum flow).

2.1 Colour structure

Before starting the main calculation, the colour structure can be treated. In general it contains the Gell-Mann matrices, the epsilon tensor for reson of colourless, and the colour couplings of the gluons, which consists of a \( δ_{ab} \) for a single gluon and a factor \( i f_{abc} \) for a three-gluon vertex. In our case we get

\[ q^i q^{i'} (T^a)^{j'}_i (T^b)^{j'}_j (T^c)^{k'}_{j'} (T^d)^{k'}_{j'} (T^d)^{k'}_k ε_{ijk} = \left( \frac{N_C + 1}{2N_C} \right)^2 q^i q^{i'} Q^k ε_{ijk} =: C_B^2 q^i q^{i'} Q^k ε_{ijk}. \]

\( N_C \) is the number of colours, and the results can be expressed by the constants \( C_A = N_C \), \( C_B = (N_C + 1)/2N_C \), \( C_F = (N_C^2 - 1)/2N_C \), \( T_F = 1/2 \) and \( N_F \), which denotes the number of flavours.

\(^2\)For a complete description see [3]
2.2 Other structures

Without the colour structure we get

\[ \tilde{b}_1^{(ht)} = \int \int (-ig_s \gamma_\alpha) \frac{i}{k} (-ig_s \gamma_\beta) \frac{i}{l} \Gamma \left( \frac{-i}{k^2} \right) \left( \frac{-i}{(k-l)^2} \right) \cdot \frac{i}{\omega + kv} (-ig_s v^\beta) \frac{i}{\omega + lv} (-ig_s v^\alpha) \frac{d^n k}{(2\pi)^n} \frac{d^n l}{(2\pi)^n} = \]

\[ = -\frac{g_s^4}{\omega^2} \int \int \psi \gamma_\mu \psi \gamma_\nu \Gamma \left( \frac{-1}{k^2} \right) \left( \frac{-1}{l^2} \right) \left( \frac{-1}{(k-l)^2} \right) \omega \omega \frac{d^n k}{(\omega + kv)} \frac{d^n l}{(\omega + lv)} (2\pi)^n (2\pi)^n. \]

The four-momentum of the heavy quark is given by \( q = m_Q v + p \), and we have \( \omega = pv \). This vertex correction can be split up into a Dirac structure \( \psi \gamma_\mu \psi \gamma_\nu \Gamma \), a momentum structure \( k^\mu l^\nu \) and a scalar integral (also called “race integral”)

\[ \tilde{b}_1^{(nt)} = -\frac{g_s^4}{\omega^2} \int \int \left( \frac{-1}{k^2} \right) \left( \frac{-1}{l^2} \right) \left( \frac{-1}{(k-l)^2} \right) \omega \omega \frac{d^n k}{(\omega + kv)} \frac{d^n l}{(\omega + lv)} (2\pi)^n (2\pi)^n = \]

\[ = \frac{g_s^4}{(4\pi)^n \omega^2} (-2\omega)^{2(n-4)} I_5(2, 1, 1, 1, 1). \]

2.3 One- and two-loop functions

With the help of the recursion formulas developed in [4, 5, 6] it is possible to reduce each two-loop function \( I_5 \) of the HQET and each two-loop function \( G_5 \) of the massless QCD to corresponding one-loop functions \( I_2 \) and \( G_2 \), which dissolve into Euler’s Gamma functions. The recursion formulas are one of the main parts of the developed program package, and we would like to thank David Broadhurst for providing us with a REDUCE implementation of these formulas.

2.4 Momentum structure, covariant evaluation and operators

By extracting the Dirac structure, the remaining integral receives corresponding Lorentz indices. It can be evaluated covariantly, in this case into

\[ Ag^{\mu \nu} + Bv^\mu v^\nu. \]

The parts \( A \) and \( B \) can be obtained by contracting the integral with the corresponding dual tensors. This contraction pairs the momentum structure to the scalar products

\[ kl = \frac{1}{2}(k^2 + l^2 - (k-l)^2) \quad \text{and} \quad (kv)(lv), \quad \text{resp.} \]

The scalar products “operate” on the entries of the two-loop function (e.g., \( k^2 \) decreases the first entry by one unit), and can therefore be identified with operators on the different entries of the two-loop functions.
2.5 Dirac structure and its reduction

The covariant evaluation in combination with the Dirac structure gives

\[
\bar{b}_1^{(hl)} = \psi \gamma_\mu \psi \gamma_\nu \Gamma(Ag_{\mu\nu} + Bv^\mu v^\nu) = A\psi \gamma_\mu \psi \gamma_\mu \Gamma + B\psi \psi \psi \psi \Gamma = ((2 - n)A + B)\Gamma.
\] (8)

For the heavy diquark system the matrix \( \Gamma \) will always remain on the edge of the structure. In contrast to this, for the light diquark system the Dirac structure can be reduced to one of the main structures

\[
\Gamma_0 := \Gamma, \quad \Gamma_1 := \gamma_\mu \gamma_\nu \Gamma, \quad \Gamma_2 := \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \Gamma \quad \text{and} \quad \Gamma_4 := \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_\mu.
\] (9)

3 Result of the perturbation series

The result of the perturbation series to second order for the different ground-state hadron currents can be written in quite a compact form, if we make some slight assumption for \( \Gamma \). In this case, we can split off a scalar valued vertex function \( V \).

3.1 Introducing \( n_\gamma \) and \( s_\gamma \)

If \( \Gamma \) is totally antisymmetrized, \( \Gamma \in \{1, \gamma_\mu, \gamma_\mu \gamma_\nu, \ldots\} \), we get

\[
\gamma_\mu \Gamma \gamma_\mu' = (-1)^{n_\gamma} (n - 2n_\gamma) \Gamma \quad \text{and} \quad \gamma_0 \Gamma \gamma_0 = (-1)^{n_\gamma} s_\gamma \Gamma,
\] (10)

where \( n \) is the space-time dimension, \( n_\gamma \) the number of Dirac matrices and \( s_\gamma \) denotes the number of \( \gamma_0 \)'s in \( \Gamma \), i.e. being +1 for an even and −1 for an odd number of \( \gamma_0 \)'s. Problems will emerge from the appearance of \( \gamma_5 \), but I will postpone the discussion of this to the end of the talk.

3.2 Vertex function for the heavy baryon currents

Without looking at the parts from different classes of diagrams, the vertex function for the heavy baryon current in its generalized form is given by

\[
V = 1 + 2\Delta V^{(hl)} + \Delta V^{(ll)} + \Delta V^{(ir)}
\] (11)

This expression can be evaluated in powers of \( 1/\varepsilon \) with \( \varepsilon = (4 - n)/2 \) and shows singularities which can be canceled by means of the minimal subtraction scheme (MS). Indeed, the evaluation in

\[
\frac{1}{\varepsilon'} := \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi) = \frac{1}{\varepsilon} \ln(4\pi e^{\gamma_E})
\] (12)

is rather more simple. This evaluation allows for the application of the modified minimal subtraction scheme (\( \text{MS} \)). By using a general gauge parameter \( a \) (\( a = 1 \) means Feynman gauge) for the one-loop contributions, we get

\[
V = \sum_{m=0}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^m V_m = \sum_{m=0}^{\infty} \sum_{k=0}^{m} \left( \frac{\alpha_s}{4\pi} \right)^m \frac{1}{\varepsilon'^k} V_m^k \quad \text{with}
\] (13)
\( V_1' = C_B (n_\gamma - 2)^2 + 3a - 1, \)
\( V_0' = C_B (2n_\gamma^2 - 6n_\gamma + 3 + a + (n_\gamma - 2)s_\gamma), \)
\( V_2' = \frac{C_B}{6} (3C_B((n_\gamma - 2)^2 + 2)^2 - C_A(11n_\gamma^2 - 44n_\gamma + 51) + \)
\( + 4N_F T_F(n_\gamma - 3)(n_\gamma - 1)) \) und
\( V_2^0 = \frac{C_B}{36} (9C_B((n_\gamma - 2)(13n_\gamma^3 - 70n_\gamma^2 + 126n_\gamma - 60 + 4(n_\gamma^2 - 4n_\gamma + 7)s_\gamma) + 32\zeta(2) + \)
\( - C_A(18n_\gamma^4 - 144n_\gamma^3 + 289n_\gamma^2 + 128n_\gamma - 516 + 72\zeta(2)) + \)
\( - 72C_F(n_\gamma - 3)(n_\gamma - 1) - 4N_F T_F(n_\gamma^2 - 16n_\gamma + 24)). \)

\( V_2^0 \) is omitted, because it has no significance for the following calculations.

### 4 The renormalization

The vertex function still contains UV-singularities. It is a bare quantity, from now on denoted by \( V^0 \), which has to be renormalized. This renormalization is done with the help of a renormalization factor. Dimensional regularization is used from the beginning, what remains is the subtraction with the aid of the modified minimal subtraction scheme, as indicated above. In the following we simplify the notation by using the term \( \varepsilon \) instead of \( \varepsilon' \), but we keep in mind that we have used the \( \overline{\text{MS}} \)-scheme.

#### 4.1 From the perturbation series to the renormalization factor

The divergencies of \( V^0 \) will be absorbed by the renormalization factor \( Z_V \). We have to state therefore that \( Z_V^{-1} V^0 = V \) is a finite quantity. By expanding the renormalization factor \( Z_V \) in a double series

\[
Z_V = 1 + \sum_{m=1}^{\infty} \sum_{k=1}^{m} \left( \frac{\alpha_s}{4\pi} \right)^k \frac{1}{\varepsilon^k} Z_m = \sum_{k=0}^{\infty} \frac{1}{\varepsilon^k} Z^k \quad \text{with} \quad Z^0 = 1,
\]

a comparison of the coefficients up to second order gives

\[
Z_1^1 = V_1^1, \quad Z_2^2 = V_2^2 \quad \text{and} \quad Z_2^1 = V_2^1 - V_1^0 V_1^1.
\]

#### 4.2 Renormalization of coupling and gauge parameter

Up to now we have left out the renormalization of the coupling constant \( \alpha_s \) and the gauge parameter \( a \),

\[
\alpha_s^0 = \alpha_s \left( 1 + \frac{\alpha_s}{4\pi \varepsilon} \delta_\alpha + O(\alpha_s^2) \right), \quad a^0 = a \left( 1 + \frac{\alpha_s}{4\pi \varepsilon} \delta_\alpha + O(\alpha_s^2) \right).
\]

Looking at terms up to second order in perturbation theory, we only need to correct the terms of first order. In this way we get corrections for the coefficient \( Z_2^1 \) of the renormalization factor \( Z_V \), namely

\[
Z_2^1 = V_2^1 - V_1^0 V_1^1 + \delta_\alpha V_1^0 + \delta_a \tilde{V}_1^0,
\]

where \( \tilde{V}_1^0 \) is the gauge dependent part of \( V_1^0 \).
4.3 From the renormalization factor to the anomalous dimension

The renormalized coupling is an object in $n$ space-time dimensions. Therefore, it gets a double dependence in the renormalization scale $\mu$,

$$\alpha_s = \alpha_s(\mu, \varepsilon) = \alpha_s(\mu)\mu^{2\varepsilon} \quad (\alpha_s(\mu) \text{ is the coupling in 4 space-time dimensions}). \quad (19)$$

The beta-function for the coupling and the gauge, two essential tools for the renormalization group equation, can then be calculated as

$$\beta = \frac{d \ln \alpha_s(\mu, \varepsilon)}{d \ln \mu} = \frac{d \ln \alpha_s(\mu)}{d \ln \mu} + 2\varepsilon, \quad \beta_a = \frac{d \ln a(\mu)}{d \ln \mu}. \quad (20)$$

This can be used for the calculation of the anomalous dimension $\gamma$,

$$\gamma Z = \frac{dZ(\alpha_s(\mu), a(\mu); \varepsilon)}{d \ln \mu} = \frac{\partial Z}{\partial \ln \alpha_s} \frac{d \ln \alpha_s(\mu)}{d \ln \mu} + \frac{\partial Z}{\partial \ln a} \frac{d \ln a(\mu)}{d \ln \mu} = \frac{\partial Z}{\partial \ln \alpha_s}(\beta - 2\varepsilon) + \frac{\partial Z}{\partial \ln a} \beta_a. \quad (21)$$

By evaluating the finite quantities $\beta$, $\beta_a$ and $\gamma$ in a perturbation series, starting at first order, the comparison of the coefficients leads to a determining equation for the coefficients of the anomalous dimension,

$$\gamma_m = -2mZ_m^1, \quad (22)$$

and on the other hand to a class of consistency checks, which can be revolved to the coefficients of the perturbation series, e.g.

$$2V_2^2 = V_1^1V_1^1 + \delta_a V_1^1 + \delta_a \tilde{V}_1^1 \quad (23)$$

where $\tilde{V}_1^1$ again denotes the gauge dependent part of $V_1^1$. For the derivation we should note that

$$\delta_a = \frac{1}{2} \beta_1 = -\frac{11}{3} C_A + \frac{4}{3} N_F T_F \quad \text{and} \quad \delta_a = \frac{1}{2} (\beta_a) = \frac{13 - 3a}{6} C_A - \frac{4}{3} N_F T_F. \quad (24)$$

5 The anomalous dimension of the generalized heavy baryon current

To renormalize the current, we also have to include the renormalization of the outer legs, because we have

$$J^0 = [\eta^{0T} C \tau V^0 \Gamma_q^0] \Gamma' Q^0 = Z_q Z_q^{1/2} Z_V J = Z_J J \quad \Rightarrow \quad \gamma_J = 2\gamma_q + \gamma_Q + \gamma_V. \quad (25)$$

By taking this into account $[3, 4]$, to first order we get

$$\gamma_J = \frac{\alpha_s}{4\pi} \left( -2C_B((n_\gamma - 2)^2 + 3a - 1) + 3C_F(a - 1) \right) + O(\alpha_s^2). \quad (26)$$

For the values $C_B = 2/3$ and $C_F = 4/3$ of $SU(3)_C$, this expression is gauge independent. So we can use these values, $C_A = 3$ and $T_F = 1/2$ and restrict ourselves to the Feynman gauge.
5.1 Different γ\textsubscript{5}-schemes

The problem in presenting the final result consists in the occurrence of different schemes for γ\textsubscript{5}.

In the scheme due to 't Hooft, Veltman, Breitenlohner and Maison [8, 9], an additional γ\textsubscript{5} in the vertex chances \( n_\gamma \) to 4 - \( n_\gamma \) and \( s_\gamma \) to -\( s_\gamma \). For the naively anticommuting γ\textsubscript{5}-scheme [10], both defining equations for \( n_\gamma \) and \( s_\gamma \) change their general sign, but this will cancel for all main structures \( \Gamma_i \).

The values for the parameters for different heavy baryons are displayed in the enclosed table.

| \( \Gamma \)   | \( n_\gamma \) | \( s_\gamma \) | baryon |
|----------------|----------------|----------------|--------|
| \( \gamma_{5AC} \) | 0              | +1             | \( \Lambda_1 \) |
| \( \gamma_{5AC,\gamma_0} \) | 1              | -1             | \( \Lambda_2 \) |
| \( \gamma_7 \) | 1              | +1             | \( \Sigma_1, \Sigma_1^* \) |
| \( \gamma_7\gamma_0 \) | 2              | -1             | \( \Sigma_2, \Sigma_2^* \) |
| \( \gamma_{5HV} \) | 4              | -1             | \( \Lambda_1 \) |
| \( \gamma_{5HV,\gamma_0} \) | 3              | +1             | \( \Lambda_2 \) |

5.2 Values for the anomalous dimension

For the anomalous dimension of different baryon currents and different schemes we get

\[
\begin{align*}
\gamma_{\Lambda_1}^{AC} &= -8 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9}(16\zeta(2) + 40N_F - 796) \left( \frac{\alpha_s}{4\pi} \right)^2, \\
\gamma_{\Lambda_2}^{AC} &= -4 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9}(16\zeta(2) + 20N_F - 322) \left( \frac{\alpha_s}{4\pi} \right)^2, \\
\gamma_{\Sigma_1} &= -4 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9}(16\zeta(2) + 20N_F - 290) \left( \frac{\alpha_s}{4\pi} \right)^2, \\
\gamma_{\Sigma_2} &= -\frac{8}{3} \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{27}(48\zeta(2) + 8N_F + 324) \left( \frac{\alpha_s}{4\pi} \right)^2, \\
\gamma_{\Lambda_1}^{HV} &= -8 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9}(16\zeta(2) - 24N_F + 260) \left( \frac{\alpha_s}{4\pi} \right)^2, \\
\gamma_{\Lambda_2}^{HV} &= -4 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9}(16\zeta(2) - 12N_F + 206) \left( \frac{\alpha_s}{4\pi} \right)^2.
\end{align*}
\]

The “naive non-Abelianization” recipe, the estimate of the total result by calculating the quark-loop diagram and replacing \( N_F \) by \( N_F - 33/2 \), does work for most of the cases with an accuracy of 15 to 30%. Only in the case of the current \( J_{\Sigma_2} \) it seems to fail. Note, however, that in this case the anomalous dimension is one order of magnitude smaller than in all other cases.

5.3 The finite renormalization

The difference of the anomalous dimension for the two different γ\textsubscript{5}-schemes turns out to be a correction of second order which is proportional to the coefficient \( \beta_1 \) of the beta-function of the coupling. So we might think about connecting the already renormalized currents by some finite renormalization factor [11], namely

\[
J^{AC}(\mu) = Z_\Gamma J^{HV}(\mu).
\]

So we get

\[
\gamma_\Gamma = \frac{d \ln Z_\Gamma}{d \ln \mu} = \gamma_\Gamma^{HV} - \gamma_\Gamma^{AC} =: O_\Gamma \left( \frac{\alpha_s}{4\pi} \right)^2.
\]
Indeed, the ansatz $Z = A_{\Gamma}(\alpha_s(\mu)/4\pi)$ leads to $A_{\Gamma}\beta_1 = O_{\Gamma}$ with

$$O_{\gamma_5} = \frac{16}{3} C_B(11C_A - 4N_FT_F),$$

$$O_{\gamma_5\gamma_i} = O_{\gamma_5\gamma_0} = \frac{8}{3} C_B(11C_A - 4N_FT_F) \quad \text{and} \quad (30)$$

and therefore

$$Z_{\gamma_5} = 1 - 8\frac{\alpha_s}{4\pi} C_B, \quad Z_{\gamma_5\gamma_i} = Z_{\gamma_5\gamma_0} = 1 - 4\frac{\alpha_s}{4\pi} C_B \quad \text{and} \quad Z_{\gamma_5\gamma_i\gamma_j} = Z_{\gamma_5\gamma_i\gamma_0} = 0. \quad (31)$$

The vertex structures $\Gamma = \gamma_5\gamma_i, \gamma_5\gamma_i\gamma_j$ and $\gamma_5\gamma_i\gamma_0$ are included for the sake of completeness, although they are not needed for the baryonic s-wave states discussed in this talk. The meson case, already treated by Trueman and Larin [11], can again be obtained by the replacement $C_B \to C_F$.

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**Figure Captions**

Fig. 1: Examples for corrections up to two-loop order to the baryonic vertex. Double lines indicate heavy quarks, single lines indicate massless quarks, the springs stand for gluons.

Fig. 2: The first heavy diquark diagram (Fig. 1(e)) with momentum labels.
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Figure 2