New possibilities of hybrid texture of neutrino mass matrix

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Abstract

In this paper, we investigate the novel possibilities of hybrid textures comprising a vanishing minor (or element) and two equal elements (or cofactors) in light neutrino mass matrix $M_\nu$. Such type of texture structures lead to sixty phenomenological cases each, out of which only fifty six are viable with texture containing a vanishing minor and an equality between the elements in $M_\nu$, while fifty are found to be viable with texture containing a vanishing element and an equality of cofactors in $M_\nu$ under the current experimental test at $3\sigma$ confidence level. Detailed numerical analysis of all the possible cases have been presented.

1 Introduction

During the last two decades, our knowledge regarding the neutrino sector has enriched to a great extent. Thanks to solar, atmospheric, reactor and accelerator based experiments which convincingly reveal that neutrinos have non-zero and non-degenerate masses and can convert from one flavor to another. While the developments over the past two decades have brought out a coherent picture of neutrino mixing, there are still several intriguing issues without which our understanding of neutrino physics remains incomplete. For instance, the present available data does not throw any light on the neutrino mass spectrum, which may be normal/inverted and may even be degenerate. In addition, nature of neutrino mass whether Dirac or Majorana particle, determination of absolute neutrino mass, leptonic CP violation and Dirac CP phase $\delta$ are still open issues. Also the information regarding the lightest neutrino mass has to be sharpened further to pin point the specific possibility of neutrino mass spectrum.

After the precise measurement of reactor mixing angle $\theta_{13}$ in T2K, MINOS, Double Chooz, Daya Bay and RENO experiments [1-5], five parameters in the neutrino
sector have been well measured by neutrino oscillation experiments. In general, there are nine parameters in the lightest neutrinos mass matrix. The remaining four unknown parameters may be taken as the lightest neutrino mass, the Dirac CP violating phase and two Majorana phases. The Dirac CP violating phase is expected to be measured in future long baseline neutrino experiments, and the lightest mass can be determined from beta decay and cosmological experiments. If neutrinoless double beta decay ($0\nu\beta\beta$) is detected, a combination of the two Majorana phases can also be probed. Clearly, the currently available data on neutrino masses and mixing are insufficient for an unambiguous reconstruction of neutrino mass matrices.

In the lack of a convincing fermion flavor theory, several phenomenological ansatze have been proposed in the literature such as some elements of neutrino mass matrix are considered to be zero or equal \cite{6,11} or some co-factors of neutrino mass matrix to be either zero or equal \cite{5,11,12}. The main motivation for invoking different mass matrix ansatze is to relate fermion masses and mixing angles in a testable manner which reduces the number of free parameters in the neutrino mass matrix. In particular, mass matrices with zero textures (or cofactors) have been extensively studied \cite{6,11} due to their connections to flavor symmetries. In addition, texture specific mass matrices with one zero element (or minor) and an equality between two independent elements (or cofactors) have also been studied in the literature \cite{7,9,10,12}. Out of sixty possibilities, only fifty four are found to be compatible with the neutrino oscillation data \cite{10} for texture structures having one zero element and an equal matrix elements in the neutrino mass matrix (also known as hybrid texture), while for texture with one vanishing minor and an equal cofactors in the neutrino mass matrix (also known as inverse hybrid texture) only fifty two cases are able to survive the data \cite{12}.

In the present paper, we propose the noval possibilities of hybrid textures where we assume one texture zero and an equality between the cofactors (referred as type X) or one zero minor and an equality between the elements (referred as type Y) in the Majorana neutrino mass matrix $M_\nu$. Such type of texture structures sets two conditions on the parameter space and hence reduces the number of free parameters to seven. Therefore the proposed texture structures are as predictive as texture two zeros and any other hybrid textures. There are total sixty such possibilities in each case which have been summarized in Table 1.

In Ref. \cite{8}, it is demonstrated that an equality between the elements of $M_\nu$ can be realized through type-II seesaw mechanism \cite{13} while an equality between cofactors of $M_\nu$ can be generated from type-I seesaw mechanism \cite{14}. The zeros element(or minor) in $M_\nu$ can be obtained using $Z_n$ flavor symmetry \cite{11,15}. Therefore the viable cases of proposed hybrid texture can be realized within the framework of seesaw mechanism.

In the present work, we have systematically, investigated all the of sixty possible cases belonging to type X and type Y structures, respectively. We have studied the implication of these textures for Dirac CP violating phase ($\delta$) and two Majorana phases ($\rho, \sigma$). We, also, calculate the effective Majorana mass and lowest neutrino mass for all viable hybrid textures belonging to type X and type Y structures. In addition, we present the correlation plots between different parameters of the hybrid
textures of neutrinos for $3\sigma$ allowed ranges of the known parameters.

The layout of the paper is planned as follows: In Section 2, we shall discuss the methodology to obtain the constraint equations. Section 3 is devoted to numerical analysis. Section 4 will summarize our result.

2 Methodology

Before proceeding further, we briefly underline the methodology relating the elements of the mass matrices to those of the mixing matrix. In the flavor basis, where the charged lepton mass matrix is diagonal, the Majorana neutrino mass matrix can be expressed as,

$$M_\nu = P_l U P_\nu M^{\text{diag}}_\nu U^T P_l^T,$$

where $M^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$ is the diagonal matrix of neutrino masses and $U$ is the flavor mixing matrix, and

$$P_\nu = \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad P_l = \begin{pmatrix} e^{i\phi_e} & 0 & 0 \\ 0 & e^{i\phi_\mu} & 0 \\ 0 & 0 & e^{i\phi_\tau} \end{pmatrix}; \quad (2)$$

where $P_\nu$ is diagonal phase matrix containing Majorana neutrinos $\rho, \sigma$. $P_l$ is unobservable phase matrix and depends on phase convention. Eq. (1) can be re-written as

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = P_l U \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} U^T P_l^T,$$

where $\lambda_1 = m_1 e^{2i\rho}, \lambda_2 = m_2 e^{2i\sigma}, \lambda_3 = m_3$. For the present analysis, we consider the following parameterization of $U$ [16]:

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} \\ -c_{12} s_{23} s_{13} - s_{12} c_{23} e^{-i\delta} & -s_{12} s_{23} s_{13} + c_{12} c_{23} e^{-i\delta} & s_{23} c_{13} \\ -c_{12} s_{23} s_{13} + s_{12} c_{23} e^{-i\delta} & -s_{12} s_{23} s_{13} - c_{12} c_{23} e^{-i\delta} & c_{23} c_{13} \end{pmatrix}, \quad (4)$$

where, $c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}$. Here, $U$ is a $3 \times 3$ unitary matrix consisting of three flavor mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and one Dirac CP-violating phase $\delta$.

For the illustration of type X and Y structures, we consider a case $A_1$, satisfying following conditions

$$C_{11} = M_{\mu\mu} M_{\tau\tau} - M_{\mu\tau} M_{\mu\tau} = 0,$$

and

$$M_{e\mu} - M_{e\tau} = 0,$$

for type X, while in case of type Y, it contains

$$M_{ee} = 0,$$

and

$$C_{12} - C_{13} = 0,$$

for type X and Y, respectively.
or
\[-1 \cdot (M_{e\tau} M_{\tau\tau} - M_{\mu\tau} M_{\tau\tau}) - (M_{e\mu} M_{\mu\tau} - M_{\mu\mu} M_{\tau\tau}) = 0, \quad (9)\]
where \( C_{ij} \) denotes cofactor corresponding to \( i^{th} \) row and \( j^{th} \) column. Then \( A_1 \) can be denoted in a matrix form as
\[
\begin{pmatrix}
0 & \Delta & \Delta \\
\Delta & \times & \times \\
\Delta & \times & \times 
\end{pmatrix},
\quad (10)
\]
where "\( \Delta \)" stands for nonzero and equal elements (or cofactors), while "0" stands for vanishing element (or minor) in neutrino mass matrix. The "\( \times \)" stands for arbitrary elements.

2.1 One Vanishing minor with Two Equal Elements of \( M_\nu \)

Using Eq. 1, any element \( M_{pq} \) in the neutrino mass matrix can be expressed in terms of mixing matrix elements as
\[
M_{pq} = e^{i(\phi_p + \phi_q)} \sum_{i=1,2,3} U_{pi} U_{qi} \lambda_i, \quad (11)
\]
where \( p, q \) run over e, \( \mu \) and \( \tau \), and \( e^{i(\phi_p + \phi_q)} \) is phase factor.

The existence of a zero minor in the Majorana neutrino mass matrix implies
\[
M_{pq} M_{rs} - M_{tu} M_{vw} = 0 \quad (12)
\]
The above condition yields a complex equation as
\[
\sum_{i,j=1,2,3} (e^{i(\phi_p + \phi_q + \phi_r + \phi_s)} U_{pi} U_{qi} U_{rj} U_{sj} - e^{i(\phi_t + \phi_u + \phi_v + \phi_w)} U_{ti} U_{ui} U_{vj} U_{wj}) \lambda_i \lambda_j = 0, \quad (13)
\]
It is observed that for any cofactor there is an inherent property as \( \phi_p + \phi_q + \phi_r + \phi_s = \phi_t + \phi_u + \phi_v + \phi_w \). Thus we can extract this total phase factor from the bracket in Eq.13.

Hence Eq.13 can be rewritten as
\[
X_3 \lambda_1 \lambda_2 + X_1 \lambda_2 \lambda_3 + X_2 \lambda_3 \lambda_1 = 0, \quad (14)
\]
where
\[
X_k = (U_{pi} U_{qi} U_{rj} U_{sj} - U_{ti} U_{ui} U_{vj} U_{wj}) + (i \leftrightarrow j), \quad (15)
\]
with \((i,j,k)\) as the cyclic permutation of \((1, 2, 3)\).

On the other hand, the condition of two equal elements in \( M_\nu \) yields following equation
\[
M_{ab} - M_{cd} = 0. \quad (16)
\]
Eq. 16 yields a following complex equation

\[ \sum_{i=1,2,3} (P_1 U_{ai} U_{bi} - P_2 U_{ci} U_{di}) \lambda_i = 0, \]  

(17)

where \( P_1 = e^{i(\phi_a + \phi_b)} \) and \( P_2 = e^{i(\phi_c + \phi_d)} \).

or

\[ \sum_{i=1,2,3} (P U_{ai} U_{bi} - U_{ci} U_{di}) \lambda_i = 0 \]  

(18)

where \( P \equiv P_1 P_2 = e^{i(a+b-c-d)} \) and \( a, b, c, d \) run over \( e, \mu \) and \( \tau \).

Eq. 18 can be rewritten as

\[ Y_1 \lambda_1 + Y_2 \lambda_2 + Y_3 \lambda_3 = 0 \]  

(19)

where

\[ Y_1 = U_{ai} U_{bi}, \quad Y_2 = P U_{ai} U_{bi} - U_{ci} U_{di}, \quad Y_3 = (P U_{ai} U_{bi} - U_{ci} U_{di}). \]

Solving Eqs. 14 and 19 simultaneously lead to the following complex mass ratio in terms of \( (\lambda_{13})_{\pm} \)

\[ (\lambda_{13})_{\pm} = \frac{-(Y_1 X_1 - Y_2 X_2 + Y_3 X_3 + \sqrt{C})}{2Y_1 X_3}, \]  

(20)

and

\[ (\lambda_{13})_{-} = \frac{-(Y_1 X_1 - Y_2 X_2 + Y_3 X_3 - \sqrt{C})}{2Y_1 X_3}. \]  

(21)

Using Eqs. 14, 20 and 21 we obtain the relations for complex mass ratio in terms of \( (\lambda_{23})_{\pm} \)

\[ (\lambda_{23})_{\pm} = -\frac{X_2}{X_3} \times \frac{Y_1 X_1 - Y_2 X_2 + Y_3 X_3 + \sqrt{C}}{-Y_1 X_1 - Y_2 X_2 + Y_3 X_3 + \sqrt{C}}, \]  

(22)

and

\[ (\lambda_{23})_{-} = -\frac{X_2}{X_3} \times \frac{Y_1 X_1 - Y_2 X_2 + Y_3 X_3 - \sqrt{C}}{-Y_1 X_1 - Y_2 X_2 + Y_3 X_3 - \sqrt{C}}, \]  

(23)

where \( C = (-Y_1 X_1 + Y_2 X_2 + Y_3 X_3)^2 - 4X_2 X_3 Y_2 Y_3 \), and \( (\lambda_{13})_{\pm} \equiv \left( \frac{\lambda_{13}}{\lambda_3} \right)_\pm \). The magnitudes of the two neutrino mass ratios in Eqs. 20, 21, 22 and 23 are given by \( \xi_{\pm} = |(\lambda_{13})_{\pm}|, \zeta_{\pm} = |(\lambda_{23})_{\pm}|, \) while the Majorana CP-violating phases \( \rho \) and \( \sigma \) can be given as \( \rho = \frac{1}{2} arg(\lambda_{13})_{\pm}, \sigma = \frac{1}{2} arg(\lambda_{23})_{\pm}. \)

### 2.2 One Vanishing element with Two Equal Cofactors of \( M_\nu \)

If one of the elements of \( M_\nu \) is considered zero, [e.g. \( M_{\alpha\beta} = 0 \)], we obtain the following constraint equation

\[ \sum_{i=1,2,3} U_{ai} U_{\beta i} \lambda_i = 0, \]  

(24)
or

$$
\lambda_1 A_1 + \lambda_2 A_2 + \lambda_3 A_3 = 0,
$$

(25)

where $A_1 = U_{\alpha 1} U_{\beta 1}$, $A_2 = U_{\alpha 2} U_{\beta 2}$ and $A_3 = U_{\alpha 3} U_{\beta 3}$.

The condition for two equal cofactors [e.g., $C_{mn} = C_{m'n'}$] in neutrino mass matrix implies

$$
(-1)^{m+n}(M_{ab} M_{cd} - M_{af} M_{gh}) - (-1)^{m'+n'}(M_{a' b'} M_{c'd'} - M_{a' f'} M_{g'h'}) = 0,
$$

(26)
or

$$
\sum_{i,j=1,2,3} \left\{ (-1)^{m+n} (Q_3 U_{ai} U_{bi} U_{cj} U_{dj} - Q_4 U_{ei} U_{fi} U_{gj} U_{hj}) 
- (-1)^{m'+n'} (Q_5 U_{a'i} U_{b'i} U_{c'j} U_{d'j} - Q_6 U_{e'i} U_{f'i} U_{g'j} U_{h'j}) \right\} \lambda_i \lambda_j = 0,
$$

(27)

where $Q_3 = Q_4$ and $Q_5 = Q_6$ due to inherent property of any cofactor. Thus we can write

$$
\sum_{i,j=1,2,3} \left\{ (-1)^{m+n} Q_3 (U_{ai} U_{bi} U_{cj} U_{dj} - U_{ei} U_{fi} U_{gj} U_{hj}) 
- (-1)^{m'+n'} Q_5 (U_{a'i} U_{b'i} U_{c'j} U_{d'j} - U_{e'i} U_{f'i} U_{g'j} U_{h'j}) \right\} \lambda_i \lambda_j = 0,
$$

(28)
or

$$
\sum_{i,j=1,2,3} \left\{ (-1)^{m+n} Q (U_{ai} U_{bi} U_{cj} U_{dj} - U_{ei} U_{fi} U_{gj} U_{hj}) 
- (-1)^{m'+n'} (U_{a'i} U_{b'i} U_{c'j} U_{d'j} - U_{e'i} U_{f'i} U_{g'j} U_{h'j}) \right\} \lambda_i \lambda_j = 0,
$$

(29)

where $Q \equiv \frac{Q_3}{Q_5} = e^{i(\phi_a + \phi_b + \phi_c + \phi_d - \phi_{a'} - \phi_{b'} - \phi_{c'} - \phi_{d'})}$.

Eq. 29 can be rewritten as

$$
\lambda_1 \lambda_2 B_3 + \lambda_2 \lambda_3 B_1 + \lambda_3 \lambda_1 B_2 = 0,
$$

(30)

where

$$
B_k = (-1)^{m+n} Q (U_{ai} U_{bi} U_{cj} U_{dj} - U_{ei} U_{fi} U_{gj} U_{hj}),
$$

$$
- (-1)^{m'+n'} (U_{a'i} U_{b'i} U_{c'j} U_{d'j} - U_{e'i} U_{f'i} U_{g'j} U_{h'j}) + (i \leftrightarrow j),
$$

(31)

with $(i, j, k)$ a cyclic permutation of $(1, 2, 3)$.

Solving Eqs. 25 and 30 simultaneously we obtain the analytical expressions of $(\lambda_{13})_{\pm}$

$$
(\lambda_{13})_{\pm} = \frac{-(B_1 A_1 - B_2 A_2 + B_3 A_3 + \sqrt{D})}{2B_1 A_3},
$$

(32)
Using Eqs. 30, 32 and 33, we get the relations for complex mass ratio in terms of $(\lambda_{23})_{\pm}$

$$(\lambda_{23})_{+} = -\frac{B_2}{B_3} \times \frac{(B_1A_1 - B_2A_2 + B_3A_3 + \sqrt{D})}{(-B_1A_1 - B_2A_2 + B_3A_3 + \sqrt{D})},$$

and

$$(\lambda_{23})_{-} = -\frac{B_2}{B_3} \times \frac{(B_1A_1 - B_2A_2 + B_3A_3 - \sqrt{D})}{(-B_1A_1 - B_2A_2 + B_3A_3 - \sqrt{D})},$$

where, $D = (-B_1A_1 + B_2A_2 + B_3A_3)^2 - 4A_2A_3B_2B_3$.

The magnitudes of the two neutrino mass ratios are given by $\xi_{\pm} = |(\lambda_{13})_{\pm}|$, $\zeta_{\pm} = |(\lambda_{23})_{\pm}|$, while the Majorana CP-violating phases $\rho$ and $\sigma$ can be given as $\rho = \frac{1}{3}arg(\lambda_{13})_{\pm}, \sigma = \frac{1}{3}arg(\lambda_{23})_{\pm}$.

The solar and atmospheric mass squared differences ($\delta m^2, \Delta m^2$), where $\delta m^2$ corresponds to solar mass-squared difference and $\Delta m^2$ corresponds to atmospheric mass-squared difference, can be defined as [9]

$$\delta m^2 = (m_2^2 - m_1^2),$$

$$\Delta m^2 = m_3^2 - \frac{1}{2}(m_1^2 + m_2^2).$$

The sign of $\Delta m^2$ is still unknown: $\Delta m^2 > 0$ or $\Delta m^2 < 0$ implies normal mass spectrum (NS) or inverted mass spectrum (IS). The lowest neutrino mass ($m_0$) is $m_1$ for NS and $m_3$ for IS. The experimentally determined solar and atmospheric neutrino mass-squared differences can be related to $\xi$ and $\zeta$ as

$$R_\nu \equiv \frac{\delta m^2}{|\Delta m^2|} = \frac{2(\zeta^2 - \xi^2)}{|2 - (\zeta^2 + \xi^2)|},$$

and the three neutrino masses can be determined using following relations

$$m_3 = \sqrt{\frac{\delta m^2}{\zeta^2 - \xi^2}}, \quad m_2 = m_3\zeta, \quad m_1 = m_3\xi.$$  

From the analysis, it is found that cases belonging to type X (or type Y) exhibit the identical phenomenological implications and are related through permutation symmetry [13,16]. This corresponds to permutation of the 2-3 rows and 2-3 columns of $M_\nu$. The corresponding permutation matrix can be given by

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. $$
The effective Majorana mass relevant for neutrinoless double beta ($0\nu\beta\beta$) decay is given by

$$|M|_{ee} = |m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2|.$$  \hspace{1cm} (42)

This effective mass is just the absolute value of $M_{ee}$ component of the neutrino mass matrix. The observation of $0\nu\beta\beta$ particles would establish neutrinos to be Majorana particles. For recent reviews see Refs. [18, 19]. Several running and forthcoming neutrinoless double decay experiments such as CUORICINO [20], CUORE [21], GERDA [22], MAJORANA [23], SuperNEMO [24], EXO [25], GENIUS [26] target to achieve a sensitivity up to 0.01eV for $|M|_{ee}$. In the present analysis, we consider

| Parameter | Best Fit | 1$\sigma$ | 2$\sigma$ | 3$\sigma$ |
|-----------|----------|-----------|-----------|-----------|
| $\delta m^2$ [10$^{-5}$eV$^2$] | 7.60 | 7.42 - 7.79 | 7.26 - 7.99 | 7.11 - 8.18 |
| $|\Delta m^2_{21}|$ [10$^{-3}$eV$^2$] (NS) | 2.48 | 2.41 - 2.53 | 2.35 - 2.59 | 2.30 - 2.65 |
| $|\Delta m^2_{31}|$ [10$^{-3}$eV$^2$] (IS) | 2.38 | 2.32 - 2.43 | 2.26 - 2.48 | 2.20 - 2.54 |
| $\theta_{12}$ | 34.0° | 33.6° - 35.6° | 32.7° - 36.7° | 31.8° - 37.8° |
| $\theta_{23}$ (NS) | 48.9° | 41.7° - 50.7° | 40.0° - 52.1° | 38.8° - 53.3° |
| $\theta_{23}$ (IS) | 49.2° | 46.9° - 50.7° | 41.3° - 52.0° | 39.4° - 54.1° |
| $\theta_{13}$ (NS) | 8.6° | 8.4° - 8.9° | 8.2° - 9.1° | 7.9° - 9.3° |
| $\theta_{13}$ (IS) | 8.7° | 8.5° - 8.9° | 8.2° - 9.1° | 8.0° - 9.4° |
| $\delta$ (NS) | 254° | 182° - 333° | 0° - 360° | 0° - 360° |
| $\delta$ (IS) | 266° | 210° - 322° | 0° - 16° + 155° - 360° | 0° - 360° |

Table 1: Current neutrino oscillation parameters from global fits at 1$\sigma$, 2$\sigma$ and 3$\sigma$ confidence level (CL) [17]. NS(IS) refers to normal (inverted) neutrino mass spectrum.

With the help of permutation symmetry, one obtains the following relations among the neutrino oscillation parameters

$$\theta_{12}^l = \theta_{12}^m, \quad \theta_{23}^l = 90° - \theta_{23}^m, \quad \theta_{13}^l = \theta_{13}^m, \quad \delta^l = \delta^m - 180°,$$  \hspace{1cm} (41)

where $l$ and $m$ denote the cases related by 2-3 permutation. The following pairs among sixty possibilities of type X (or type Y) are related via permutation symmetry

$(A_1, A_1)$; $(A_2, A_8)$; $(A_3, A_7)$; $(A_4, A_6)$; $(A_5, A_5)$; $(A_9, A_{10})$; $(B_1, C_1)$; $(B_2, C_7)$; $(B_3, C_6)$; $(B_4, C_5)$; $(B_5, C_4)$; $(B_6, C_3)$; $(B_7, C_2)$; $(B_8, C_{10})$; $(B_9, C_9)$; $(B_{10}, C_8)$; $(D_1, F_2)$; $(D_2, F_1)$; $(D_3, F_4)$; $(D_4, F_3)$; $(D_5, F_3)$; $(D_6, F_9)$; $(D_7, F_8)$; $(D_8, F_7)$; $(D_9, F_6)$; $(D_{10}, F_{10})$; $(E_1, E_2)$; $(E_3, E_4)$; $(E_5, E_5)$; $(E_6, E_9)$; $(E_7, E_8)$; $(E_{10}, E_{12})$.

Clearly we are left with only thirty two independent cases. It is worthwhile to mention that $A_1, A_5, E_5$ and $E_{10}$ are invariant under the permutations of 2- and 3-rows and columns.

3 Numerical analysis

The effective Majorana mass relevant for neutrinoless double beta ($0\nu\beta\beta$) decay is given by

$$|M|_{ee} = |m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2|.$$
more conservative upper bound on $|M|_{ee}$ i.e. $|M|_{ee} < 0.5$eV at 3 $\sigma$ CL \cite{19}. We span the parameter space of input neutrino oscillation parameters ($\theta_{12}$, $\theta_{23}$, $\theta_{13}$, $\Delta m^2$, $\Delta m^2$) lying in their 3$\sigma$ ranges by randomly generating points of the order of $10^7$. Since the Dirac CP-violating phase $\delta$ is experimentally unconstrained at 3$\sigma$ level, therefore, we vary $\delta$ within its full possible range [0°, 360°]. Using Eq. 38 and the experimental inputs on neutrino mixing angles and mass-squared differences, the parameter space of $\delta$, $\rho$, $\sigma$, and $|M|_{ee}$ and $m_0$ can be subsequently constrained.

In Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 we demonstrate the correlations for $A_1$, $B_2$, $D_7$ and $E_1$ cases. Since there are large number of viable cases, therefore it is not practically possible to show all the plots. We have simply taken arbitrary independent cases from each category for the purpose of illustration of our results. The predictions regarding three CP-violating phases ($\rho$, $\sigma$, $\delta$), effective neutrino mass $|M|_{ee}$ and lowest neutrino mass $m_0$ for all the allowed cases of type X and type Y textures have been encapsulated in Table 3, 4, 5, 6. Before proceeding further, it is worth pointing out that the phenomenological results for $\rho$, $\sigma$, $\delta$, $|M|_{ee}$ and $m_0$ have been obtained using the two possible solutions of $\lambda_{13}$ and $\lambda_{23}$ respectively\cite{20,21,22,23}. All the sixty phenomenologically possible cases have been divided into six categories A, B, C, D, E, F. Among them large number of cases are found to overlap in their predictions regarding $\delta$, $\rho$, $\sigma$, $|M|_{ee}$ and $m_0$ and are related via permutation symmetry as pointed out earlier. The main results and the discussion

Figure 1: Correlation plots for texture $A_1$ (IS) for type X at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta$, $\rho$, $\sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.
are summarized as follows:

**Category A**: In Category A, all the ten cases $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}$ are found to be viable with the data at 3σ CL for type X structure, and normal mass spectrum (NS) remain ruled out for all these cases [Table 3]. On the other hand, only four $A_1, A_4, A_5, A_6$ seem to be viable with current oscillation data for type Y, while inverted mass spectrum (IS) is ruled out for these cases.

For both type X and Y, no noticeable constraint has been found on the parameter space of CP violating phases ($\rho, \sigma, \delta$). For type X, all the viable cases predict the value of $|M|_{ee}$ in the range of 0.01eV to 0.05eV. This prediction lies well within the sensitivity limit of neutrinoless double beta decay experiments as mentioned above. On the other hand, for type Y, $|M|_{ee}$ is predicted to be zero implying that neutrinoless double beta decay is forbidden. Also the lower bound on lowest neutrino mass ($m_0$) is found to be extremely small ($\sim 10^{-3}$ or less) for all the viable cases of type X and type Y structure [Table 5]. For the purpose of illustration, we have presented the correlation plots for $A_1$ indicating the parameter space of $\rho, \sigma, \delta$, $|M|_{ee}$ and lowest neutrino mass ($m_0$) [Figs. 12].

**Category B (C)**: In Category B, all the ten possible cases are allowed for both type X and type Y structure, respectively at 3σ CL [Table 4]. Cases $B_{2,3,4,5,8,9,10}$
allow both NS as well as IS for type X, while cases $B_{1,2,3,4,5,8,9,10}$ allow both NS and IS for type Y. As mentioned earlier, cases of Category B are related to cases belonging to Category C via permutation symmetry, therefore we can obtain the results for Category C from B by using Eq.41.

![Figure 3: Correlation plots for texture $B_2$ (NS) for type X at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.](image)

Type X cases $B_1$ (IS), $B_2$ (IS), $B_3$ (NS, IS), $B_4$ (IS), $B_5$ (NS, IS), $B_7$ (NS), $B_9$ (NS, IS), $B_{10}$ (IS), $C_1$ (IS), $C_2$ (IS), $C_4$ (IS), $C_5$ (NS, IS), $C_6$ (NS, IS), $C_7$ (NS), $C_8$ (IS), $C_9$ (NS, IS), $C_{10}$ (IS) cover literally the complete range of $\delta$. However, for $B_2$ (NS), $B_4$ (NS), $B_6$ (NS), $B_8$ (NS), $B_{10}$ (NS), $C_3$ (NS), $C_5$ (NS), $C_7$ (NS), $C_8$ (NS) and $C_{10}$ (NS) the parameter space of $\delta$ is found to be reduced to an appreciable extent [Table 4].

On the other hand, type Y cases $B_1$ (NS), $B_2$ (NS), $B_3$ (NS, IS), $B_4$ (NS), $B_5$ (NS, IS), $B_7$ (IS), $B_8$ (NS), $B_9$ (NS, IS), $B_{10}$ (NS), $C_1$ (NS), $C_2$ (NS), $C_4$ (NS), $C_5$ (NS, IS), $C_6$ (NS, IS), $C_7$ (IS), $C_8$ (NS), $C_9$ (NS, IS), $C_{10}$ (NS) cover approximately the complete range of $\delta$. $B_1$ (IS), $B_2$ (IS), $B_3$ (IS), $B_4$ (IS), $B_6$ (IS), $B_8$ (IS), $B_{10}$ (IS), $C_1$ (IS), $C_3$ (IS), $C_5$ (IS), $C_7$ (IS), $C_8$ (IS) and $C_{10}$ (IS) the $\delta$ is found to be reduced appreciably [Table 4].

From the analysis, it is found that textures $B_2, B_4, C_5$ and $C_7$ belonging to type X predict near maximal Dirac type CP violation (i.e. $\delta \approx 90^0$ and $270^0$) for NS. In addition, the Majorana phases $\rho$ and $\sigma$ are found to be very close to $0^0$ for these
Figure 4: Correlation plots for texture $B_2$ (IS) for type X at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.

cases. On the other hand, in case of type Y, $B_1, B_2, B_4, B_6, C_1, C_4, C_5$ and $C_7$ show almost similar constraints on the parameter space for $\delta$ however for opposite mass spectrum [Table 4]. In Figs. 3(a, b) and 6(a, b), $\delta \approx 90^0$ and $270^0$, while $\rho, \sigma \approx 0^0$. The correlation plots between $|M|_{ee}$ and $m_0$ have been encapsulated in Figs. 3(c), 4(c), 5(c), 6(c). The plots indicate the strong linear relation correlation between these parameters and in addition, the lower bound on both the parameters is somewhere in the range from 0.001 to 0.01 eV. The prediction for the allowed space of $|M|_{ee}$ for all the cases of category B is given in Table 4.

Category D (F): In Category D, only nine cases are acceptable with neutrino oscillation data at $3\sigma$ CL for both type X and type Y structures respectively, while case $D_8$ is excluded for both of them [Table 4]. Cases $D_1, D_2, D_4, D_5, D_6, D_7, D_9$ show both NS and IS for type X and type Y respectively, while $D_3$ and $D_10$ are acceptable for IS (NS) and NS(IS) respectively in case of type X (type Y) structure. Similarly, the results for cases belonging to Category F can be obtained from Category D since both are related via permutation symmetry. It is found that only nine cases are allowed with data in category F, while $F_7$ is excluded at $3\sigma$ CL. Cases $D_1$(NS), $D_2$ (NS, IS), $D_3$(IS), $D_4$(NS), $D_5$(NS, IS), $D_6$(NS), $D_7$(NS), $D_9$(NS), $D_{10}$(NS), $F_1$(NS), $F_2$ (NS, IS), $F_3$(IS), $F_4$(NS), $F_5$(NS, IS), $F_6$(NS), $F_7$(NS), $F_9$(NS),
Figure 5: Correlation plots for texture $B_2$ (NS) for type Y at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.

$F_{10}$ (NS) predict literally no constraints on $\delta$ for type X texture. These cases give identical predictions for type Y as well, however for opposite mass ordering. On the other hand, for cases $D_6$ (IS), $D_4$ (IS), $D_7$ (IS), $D_9$ (IS), $F_3$ (IS), $F_6$ (IS), $F_8$ (IS), $F_9$ (IS) $\delta$ is notably constrained for type X, and similar observation have been found for these cases in type Y, however for opposite mass ordering [Table 5].

It is found that textures $D_7$ (IS), $D_9$ (IS), $F_6$ (IS) and $F_8$ (IS) belonging to type X predict near maximal Dirac CP violation (i.e. $\delta \approx 90^0$ and $270^0$). In addition, the Majorana phases $\rho$ and $\sigma$ are found to be very close to $0^0$ for these cases. The similar predictions hold for these cases belonging to type Y structure however for opposite mass spectrum.

The prediction on the allowed range of $|M|_{ee}$ for all the cases of category D is provided in Table 5. As an illustration, in Figs. 7, 8, 9, 10 we have complied the correlation plots for case $D_7$ for type X and type Y structures. Figs. 7(a, b), 10(a, b)) indicate no constraint on $\delta, \rho, \sigma$ for NS(IS) corresponding to type X (type Y) structure at 3$\sigma$ CL. On the other hand, $\delta \approx 90^0$ and $270^0$, while $\rho$ and $\sigma$ approach to $0^0$ for IS in case of type X structure [Fig. 8]. However, similar predictions for $\delta, \rho, \sigma$ have been observed for type Y, however for NS [Fig 9]. In Figs. 7 (c), 9 (c), 10 (c), 11 (c), we have presented the correlation plots between $|M|_{ee}$ and $m_0$ indicating the linear correlation.

**Category E:** In Category E, only eight out of ten cases are allowed with exper-
Figure 6: Correlation plots for texture $B_2$ (IS) for type Y at $3\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.
Figure 7: Correlation plots for texture $D_7$ (NS) for type X at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.

4 Summary and Conclusion

To summarize, we have discussed the novel possibilities of hybrid textures in the flavor basis wherein the assumption of either one zero minor and an equality between the elements or one zero element and an equality between the cofactors in the Majorana neutrino mass matrix is considered. Out of sixty phenomenologically possible cases, only 56 are found to be viable for type X, while only 50 are viable with the present data for type Y at 3$\sigma$ CL. Therefore, out of 120 only 106 cases are found to be viable with the existing data. However only 38 seems to restrict the parameteric space of CP violating phases $\delta, \rho, \sigma$, while 16 out of these predict near maximal Dirac CP violation i.e. $\delta \simeq 90^0, 270^0$. The allowed parameter space for effective mass term $|M|_{ee}$ related to neutrinoless double beta decay as well as lowest neutrino mass term for all viable cases have been carefully studied. The present viable cases may be derived from the discrete symmetry. However the symmetry realization for each case in a systematic and self consistent way deserve fine-grained research. The viability of these cases suggests that there are still rich unexplored structures of the neutrino mass matrix from both the phenomenological and theoretical points of view.

To conclude our discussion, we would like add that the hybrid textures comprising either one zero element and an equality between the elements or one zero
Figure 8: Correlation plots for texture $D_7$ (IS) for type X at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.

minor and an equality between the cofactors lead to 106 viable cases, therefore there are now total 212 viable cases pertaining to the hybrid textures of $M_{\nu}$ in the flavor basis. Since most of these cases overlap in their predictions regarding the experimentally undetermined parameters, therefore we expect that only the future longbaseline experiments, neutrinoless double beta decay experiments and cosmological observations could help us to select the appropriate structure of mass texture.

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References

[1] K. Abe et al., [T2K Collaboration], *Phys. Rev. Lett.* **107**, 041801 (2011), arXiv: 1106.2822 [hep-ex].
Figure 9: Correlation plots for texture $D_7$ (NS) for type Y at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.

[2] P. Adamson et al., [MINOS Collaboration], \textit{Phys. Rev. Lett.} \textbf{107}, 181802 (2011), arXiv: 1108.0015 [hep-ex].

[3] Y. Abe et al., [Double Chooz Collaboration], \textit{Phys. Rev. Lett.} \textbf{108}, 131801 (2012), arXiv: 1112.6353 [hep-ex].

[4] F. P. An et al., [Daya Bay Collaboration], \textit{Phys. Rev. Lett.} \textbf{108}, 171803 (2012), arXiv: 1203.1669 [hep-ex].

[5] J. K. Ahn et al., [RENO Collaboration], \textit{Phys. Rev. Lett.} \textbf{108}, 191802 (2012), arXiv: 1204.0626 [hep-ex].

[6] Paul H. Frampton, Sheldon L. Glashow and Danny Marfatia, \textit{Phys. Lett.} B \textbf{536}, 79 (2002), hep-ph/0201008; Zhi-zhong Xing, \textit{Phys. Lett.} B \textbf{530}, 159 (2002), hep-ph/0201151. Bipin R. Desai, D. P. Roy and Alexander R. Vaucher, \textit{Mod. Phys. Lett.} A \textbf{18}, 1355 (2003), hep-ph/0209035; S. Dev, Sanjeev Kumar, S. Verma and S. Gupta, \textit{Nucl. Phys.} B \textbf{784}, 103 (2007), hep-ph/0611313; G. Ahuja, S. Kumar, M. Randhawa, M. Gupta, S. Dev, \textit{Phys. Rev.} D \textbf{76}, 013006 (2007), hep-ph/0703005; S. Kumar, \textit{Phys. Rev.} D \textbf{84}, 077301 (2011), arXiv:1108.2137 [hep-ph]; G. Blankenburg, D. Meloni, \textit{Nucl. Phys.} B \textbf{867}, 749 (2013), arXiv:1204.2706 [hep-ph]; W. Grimus, P. O. Ludl, \textit{J. Phys.} G \textbf{40},
Figure 10: Correlation plots for texture $D_7$ (IS) for type Y at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.

055003 (2013) arXiv:1208.4515 [hep-ph]; Manmohan Gupta, Gulshee Ahuja, Int. J. Mod. Phys. A, 27, 1230033 (2012), arXiv:1302.4823 [hep-ph]; J. Liao, D. Marfatia, K. Whisnant, arXiv:1311.2639 [hep-ph]; D. Meloni, A. Meroni, E. Peinado, Phys. Rev. D 89 (2014) 053009, arXiv:1401.3207 [hep-ph]; P. O. Ludl, W. Grimus, JHEP 07, 090 (2014), arXiv: 1406.3546 [hep-ph]; P. O. Ludl, W. Grimus, arXiv: 1501.04942 [hep-ph]; M. Borah, D. Borah, M. K. Das, arXiv: 1503.03431 [hep-ph]; H. Fritzsch, Zhi-zhong Xing, S. Zhou, JHEP 1109, 083 (2011), arXiv: 1108.4534 [hep-ph].

[7] S. Dev, S. Verma, and S. Gupta, Phys. Lett. B 687, 53 (2010).
[8] S. Dev, Radha Raman Gautam and Lal Singh, Phys. Rev. D 87, 073011 (2013), arXiv: 1303.3092 [hep-ph].
[9] J. Y. Liu and S. Zhou, Phys. Rev. 87, 093010 (2013), arXiv:1304.2334 [hep-ph].
[10] S. Kaneko, H. Sawanaka and M. Tanimoto, JHEP 08, 073 (2005), arXiv: hep-ph/0511251.
[11] L. Lavoura, Phys. Lett. B 609, 317 (2005), hep-ph/0411232; E. I. Lashin and N. Chamoun, Phys. Rev. D 78, 073002 (2008), arXiv:0708.2423 [hep-ph]; E. I.
Figure 11: Correlation plots for texture $E_1$ (NS) for type X at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units..

Lashin, N. Chamoun, *Phys. Rev.* D 80, 093004 (2009), arXiv:0909.2669 [hep-ph]; S. Dev, Shivani Gupta, Radha Raman Gautam and Lal Singh, *Phys. Lett.* B 706, 168 (2011), arXiv: 1111.1300 [hep-ph]; T. Araki, J. Heeck and J. Kubo, *JHEP* 1207, 083 (2012), arXiv:1203.4951 [hep-ph].

[12] Weijian Wang, *Eur. Phys. J.* C 73, 2551 (2013), arXiv:1306.3556 [hep-ph]; S. Dev, R. R. Gautam and Lal Singh, *Phys. Rev.* D 88, 033008 (2013), arXiv:1306.4281 [hep-ph].

[13] W. Konetschny and W. Kummer, *Phys. Lett.* B 70, 433 (1977); T. P. Cheng and L. F. Li, *Phys. Rev.* D 22, 2860 (1980); J. Schechter and J. W. F. Valle, *Phys. Rev.* D 22, 2227 (1980); G. Lazarides Q. Shafi and C. Wetterich, *Nucl. Phys.* B 181, 287 (1981); R. N. Mohapatra and G. Senjanovic, *Phys. Rev.* D 23, 165 (1981).

[14] P. Minkowski, *Phys. Lett.* B 67, 421 (1977); T. Yanagida, *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe* (O. Sawada and A. Sugamoto, eds.), KEK, Tsukuba, Japan, 1979, p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, *Complex spinors and unified theories in*
Figure 12: Correlation plots for texture $E_1$ (IS) for type Y at 3 $\sigma$ CL. The symbols have their usual meaning. The $\delta, \rho, \sigma$ are measured in degrees, while $|M|_{ee}$ and $m_0$ are in eV units.

supergravity (P. Van Nieuwenhuizen and D. Z. Freedman, eds.), North Holland, Amsterdam, 1979, p.315; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[15] W. Grimus, A. S. Joshipura, L. Lavoura, and M. Tanimoto, Eur. Phys. J. C 36, 227 (2004).

[16] H. Fritzsch, Z. Z. Xing, Phys. Lett. B 517 (2001) 363-368, arXiv: hep-ph/0103242.

[17] D. V. Forero, M. Trtola, J. W. F. Valle, Phys. Rev. D 90, 093006 (2014), arXiv:1405.7540 [hep-ph].

[18] F. T. Avignone III, S. R. Elliott, J. Engel, Rev. Mod. Phys. 80, 481 (2008), arXiv:0708.1033 [nucl-ex]; J. J. Gomez-Cadenas, J. Martin-Albo, M. Mezzetto, F. Monrabal, M. Sorel, Riv. Nuovo Cim. 35, 29 (2012), arXiv:1109.5515 [hep-ex]; S. M. Bilenky, C. Giunti, Mod. Phys. Lett. A 27, 1230015, arXiv:1203.5250 [hep-ph].
[19] W. Rodejohann, *Int. J. Mod. Phys.* **E**, 20, 1833 (2011), arXiv:1106.1334 [hep-ph].

[20] C. Arnaboldi *et al.*, [CUORICINO collaboration], *Phys. Lett.* **B** 584, 260 (2004).

[21] C. Arnaboldi *et al.*, *Nucl. Instrum. Methods Phys. Res., Sect. A* 518, 775 (2004).

[22] I. Abt *et al.*, [GERDA collaboration] hep-ex/0404039.

[23] R. Gaitskell *et al.* [Majorana Collaboration] nucl-ex/0311013.

[24] A. S. Barabash [NEMO Collaboration], *Czech. J. Phys.*, **52**, 567 (2002), nucl-ex/0203001.

[25] M. Danilov *et al.*, *Phys. Lett.* **B** 480, 12 (2000), hep-ex/0002003.

[26] H. V. Klapdor-Kleingrothaus, *et al.*, *Eur. Phys. J.* **A** 12, 147 (2001), hep-ph/0103062.
| Cases | X         | P         | Y         | Q         |
|-------|-----------|-----------|-----------|-----------|
| A₁    | C_{11} = 0, M_{12} = M_{13} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{13} | e^{i(\phi_1 - \phi_2)} |
| A₂    | C_{11} = 0, M_{12} = M_{23} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{23} | e^{i(\phi_1 - \phi_2)} |
| A₃    | C_{11} = 0, M_{12} = M_{33} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{33} | e^{i(\phi_1 - \phi_2)} |
| A₄    | C_{11} = 0, M_{12} = M_{43} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{43} | e^{i(\phi_1 - \phi_2)} |
| A₅    | C_{11} = 0, M_{12} = M_{53} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{53} | e^{i(\phi_1 - \phi_2)} |
| A₆    | C_{11} = 0, M_{12} = M_{63} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{63} | e^{i(\phi_1 - \phi_2)} |
| A₇    | C_{11} = 0, M_{12} = M_{73} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{73} | e^{i(\phi_1 - \phi_2)} |
| A₈    | C_{11} = 0, M_{12} = M_{83} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{83} | e^{i(\phi_1 - \phi_2)} |
| A₉    | C_{11} = 0, M_{12} = M_{93} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{93} | e^{i(\phi_1 - \phi_2)} |
| B₁    | C_{11} = 0, M_{12} = M_{13} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{13} | e^{i(\phi_1 - \phi_2)} |
| B₂    | C_{11} = 0, M_{12} = M_{23} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{23} | e^{i(\phi_1 - \phi_2)} |
| B₃    | C_{11} = 0, M_{12} = M_{33} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{33} | e^{i(\phi_1 - \phi_2)} |
| B₄    | C_{11} = 0, M_{12} = M_{43} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{43} | e^{i(\phi_1 - \phi_2)} |
| B₅    | C_{11} = 0, M_{12} = M_{53} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{53} | e^{i(\phi_1 - \phi_2)} |
| B₆    | C_{11} = 0, M_{12} = M_{63} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{63} | e^{i(\phi_1 - \phi_2)} |
| B₇    | C_{11} = 0, M_{12} = M_{73} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{73} | e^{i(\phi_1 - \phi_2)} |
| B₈    | C_{11} = 0, M_{12} = M_{83} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{83} | e^{i(\phi_1 - \phi_2)} |
| B₉    | C_{11} = 0, M_{12} = M_{93} | e^{i(\phi_1 - \phi_2)} | M_{11} = 0, C_{12} = C_{93} | e^{i(\phi_1 - \phi_2)} |

Table 2: All the sixty phenomenological possible cases belonging to type X and Y respectively have been shown. P and Q are unobservable phases associated with type X and Y respectively.
Table 3: The allowed ranges of Dirac CP-violating phase $\delta$, the Majorana phases $\rho, \sigma$, effective neutrino mass $|M|_{ee}$, and lowest neutrino mass $m_0$ for the experimentally allowed cases of Category A at 3$\sigma$ CL. The predictions corresponding to $(\lambda_{13})_-$ and $(\lambda_{23})_-$ neutrino mass ratios have been put into brackets.
Table 4: The allowed ranges of Dirac CP-violating phase $\delta$, the Majorana phases $\rho, \sigma$, effective neutrino mass $|M_{ee}|$, and lowest neutrino mass $m_0$ for the experimentally allowed cases of Category B(C) at 3$\sigma$ CL. The predictions corresponding to $(\lambda_{13})_-$ and $(\lambda_{23})_-$ neutrino mass ratios have been put into brackets.
Table 5: The allowed ranges of Dirac CP-violating phase $\delta$, the Majorana phases $\beta, \gamma$, and (A2) - neutrino mass ratios have been put into brackets.
| Case | \(\lambda_{13}^+\) | \(\lambda_{23}^+\) | \(\lambda_{13}^-\) | \(\lambda_{23}^-\) |
|------|---------------|---------------|---------------|---------------|
| 1(4) | \((-39^\circ - 60^\circ)\) | \((-39^\circ - 60^\circ)\) | \((-39^\circ - 60^\circ)\) | \((-39^\circ - 60^\circ)\) |
| \(\rho, \sigma\) | \((0^\circ - 36^\circ)\) | \((0^\circ - 36^\circ)\) | \((0^\circ - 36^\circ)\) | \((0^\circ - 36^\circ)\) |
| \((\text{M}_e)\) | \((0.029 - 0.031)\) | \((0.029 - 0.031)\) | \((0.029 - 0.031)\) | \((0.029 - 0.031)\) |
| 2(4) | \((-18^\circ - 90^\circ)\) | \((-18^\circ - 90^\circ)\) | \((-18^\circ - 90^\circ)\) | \((-18^\circ - 90^\circ)\) |
| \(\rho, \sigma\) | \((-18^\circ - 90^\circ)\) | \((-18^\circ - 90^\circ)\) | \((-18^\circ - 90^\circ)\) | \((-18^\circ - 90^\circ)\) |
| \((\text{M}_e)\) | \((0.029 - 0.031)\) | \((0.029 - 0.031)\) | \((0.029 - 0.031)\) | \((0.029 - 0.031)\) |
| 3(4) | \((-90^\circ - 180^\circ)\) | \((-90^\circ - 180^\circ)\) | \((-90^\circ - 180^\circ)\) | \((-90^\circ - 180^\circ)\) |
| \(\rho, \sigma\) | \((-90^\circ - 180^\circ)\) | \((-90^\circ - 180^\circ)\) | \((-90^\circ - 180^\circ)\) | \((-90^\circ - 180^\circ)\) |
| \((\text{M}_e)\) | \((-90^\circ - 180^\circ)\) | \((-90^\circ - 180^\circ)\) | \((-90^\circ - 180^\circ)\) | \((-90^\circ - 180^\circ)\) |

Table 6: The allowed ranges of Dirac CP-violating phase \(\delta\), the Majorana phases \(\rho, \sigma\), effective neutrino mass \(|M|_{ee}\), and lowest neutrino mass \(m_0\) for the experimentally allowed cases of Category E at 3\(\sigma\) CL. The predictions corresponding to \(\langle\lambda_{13}\rangle^+\) and \(\langle\lambda_{23}\rangle^+\) neutrino mass ratios have been put into brackets.