The mechanisms of initiative projects stimulating

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Abstract. The mechanisms of stimulating initiative projects are considered. In terms of content, these can be mechanisms for stimulating social projects, volunteer movements, etc. Each project has an assessment of value obtained by an expert. To stimulate projects allocated a certain fund. This fund is distributed in proportion to the values of the projects implemented. Each project performer has an estimate of minimum remuneration, depending on the value of the projects. Various models of project performers' behavior are investigated. In the first model, the performer performs all projects total remuneration for which is not less than the minimum value. In the second model, the performer minimizes the difference between the received remuneration and its minimum value.

1. Introduction

Initiatives are projects that people or organizations carry out on their own initiative, while receiving certain remuneration (sometimes purely moral). Such projects include many social and youth projects, the work of volunteers, the publication of scientific works, etc. Naturally, financial support for such projects increases their number. Consider an organization (research institute, university) interested in increasing the number and value of initiative projects (for example, the management of research institutes are interested in increasing the number and significance of scientific publications of employees). To this end, an incentive fund for initiative projects (FIP) is created and a certain mechanism for distributing this fund among the implementers of initiative projects (IP). Further, the executives of IP will be called agents, and the body that distributes the FIP fund - the center. The task of distributing a CIP fund belongs to the class of resource allocation tasks. Various resource allocation mechanisms are considered in many papers [1-10]. This article takes into account various types of agent behavior. We assume that for each agent there is a minimum amount of remuneration of at least which he wants to receive, performing projects of a certain value. Consider two types of agent behavior. Agents of the first type perform an IP of the maximum total value; provided that the remuneration is no less than the minimum (we call such agents workaholics). Agents of the second type maximize the difference between the received remuneration and its minimum value (let's call such agents rationalists). The article assesses the effectiveness of the mechanism of distribution of the CIP fund directly proportional to the total value of IPs performed by agents with various types of their behavior.
2. Formulation of the problem
There are \( n \)-agents performing of IPs. We denote \( Z_i \) - the total value of the IP, performed by the agent \( i \), \( \phi_i(Z_i) \) - the minimum amount of the agent’s remuneration in the implementation of projects of total value \( Z_i \), \( \Phi \) – the value of the FIP fund.

We assume that the FIP fund is distributed directly proportional of \( Z_i \), that is, the remuneration of the \( i \)-th agent is

\[
\sigma_i(Z_i) = \frac{Z_i}{\sum_j Z_j} \cdot \Phi
\]

Next, we will assume the hypothesis of weak influence [2], according to which agents do not take into account their influence on the value \( \lambda(Z) = \frac{\Phi}{\sum_j Z_j} \), considering \( \lambda(Z) \) simply the parameter \( \lambda \).

The behavior of agents of the first type (workaholics) is determined by the desire to carry out projects of the maximum total value, provided that

\[
\sigma_i(Z_i) = \lambda Z_i \geq \phi_i(Z_i)
\]

Point A in figure 1 corresponds to this type of behavior. Figure 2 illustrates the case of linear dependencies. Point B in figure 1 corresponds to this type of behavior.

The behavior of agents of the second type (rationalists) is determined by the desire to maximize the difference

\[
\lambda Z_i - \phi_i(Z_i)
\]

The task is to assess the effectiveness of mechanism (1) in the situation of Nash equilibrium with different types of agent behavior.

3. Assessment of the effectiveness of the fund distribution mechanism
We begin with the study of the case when all agents are rationalists. We assume that \( \phi_i(Z_i) \) - are convex, increasing, continuously differentiable functions, and \( \phi_i'(0) = 0, i = 1, n \).

In this case, the maximum condition (3) has the form

\[
\phi_i'(Z_i) = \lambda, Z_i = \zeta_i(\lambda), i = 1, n
\]
where \( \zeta_i \) is an inverse function.

Out of limitation \[ \sum \lambda z_i = \sum \lambda \zeta_i(\lambda) = \Phi \] we define \( \lambda \). It is easy to show that if all agents are rationalists, then mechanism (1) does not provide the maximum possible value from the IP. Consider the case when all agents are workaholics. We have a condition for project selection \( \lambda z_i = \varphi_i(z_i) \) and

\[
\frac{\varphi_i(z_i)}{z_i} = \lambda
\]

This condition differs from the optimality condition (4). Therefore, in the case of workaholic agents in the general case, it is also not possible to ensure the maximum effect from mechanism (1).

We obtain the conditions under which (4) implies (4).

**Theorem 1.** In order condition (5) follows from condition (5), it is necessary and sufficient that the functions \( \varphi_i(z_i) \) be power functions of the form:

\[
\varphi_i(z_i) = b_i x_i^\alpha, \quad 0 < \alpha < 8, \quad i = 1, n.
\]

**Proof:** Necessity, albeit from the fact that

\[
\frac{\varphi_i(z_i)}{z_i} = \lambda, \quad i = 1, n
\]

follows \( \varphi_i'(z_i) = \mu, \quad i = 1, n \).

We have

\[
\frac{d\varphi_i(x_i)}{dx_i} = \mu = \lambda \frac{\mu}{\lambda} = \frac{\varphi_i(x_i)}{x_i} \alpha,
\]

where \( \alpha = \frac{\mu}{\lambda} \). Consequently \( \frac{d\varphi_i}{\varphi_i} = \alpha \frac{dx_i}{x_i} \). Solving this equation, we get

\[
\varphi_i(x_i) = c_i x_i^\alpha, \quad \text{where } c_i > 0, \quad \alpha > 1, \quad i = 1, n.
\]

Adequacy. Let be

\[
\varphi_i(x_i) = c_i x_i^\alpha, \quad i = 1, n
\]

We have \( \varphi_i'(x_i) = c_i \alpha x_i^{\alpha-1} = \alpha \frac{c_i x_i^\alpha}{x_i} = \alpha \frac{\varphi_i(x_i)}{x_i} \).

The theorem is proved.

Thus, for power functions, mechanism (1) ensures maximum efficiency if all agents are workaholics. If there is a mixed composition of agents, that is, some of the agents are workaholics, and some of the agents are rationalists, then the maximum efficiency cannot be achieved, even for power functions.

4. **Linear case**

Consider the case of linear dependencies (figure 2)

\[
\varphi_i(z_i) = k_i z_i, \quad z_i \leq a_i
\]

In this case, for a given \( \lambda \) three options are possible. If \( \lambda = \lambda_1 < k_i \), then \( z_i = 0 \). If \( \lambda = \lambda_2 > k_i \), then \( z_i = a_i \). Finally, if \( \lambda = k_i \), then \( z_i \) can take any value on the interval \([0; a_i]\). Note that for the case of linear dependencies, both workaholics and rationalists behave the same way.

5. **Discrete model**
Consider a discrete model in which each agent has a certain set of $R_i$ projects, each of which is characterized by the value of $z_{ij}$ and the value of the minimum reward $c_{ij}$, $j \in R_i$.

Consider the problems solved by agents of the first and second type for a given value of $\lambda$.

The task of the agents of the first type

Denote $x_j = 1$, if agent 2 executes project $j$, $x_j = 0$ otherwise (release agent number).

**Problem.** Determine $x$, maximizing

$$\sum_j x_j z_j$$

while limiting

$$\sum_j (c_j - \lambda z_j)x_j \leq 0$$

This knapsack problem is effectively solved by the dichotomic programming method [3]. Note for projects such that $c_j \leq \lambda z_j$, obviously $x_j = 1$. Denote by $S$ many such projects $A = \sum_{j \in S} (\lambda z_j - c_j)$.

Inequality (8) takes the form

$$\sum_{j \in S} x_j u_j \leq A$$

where $u_j = c_j - \lambda z_j$.

The task of agents of the second type

Determine $x$, maximizing

$$\sum_j (\lambda z_j - c_j)x_j$$

The solution to this problem is obvious: $x_j = 1$ for all $j$, such that $u_j < 0$, $x_j = 0$ for all $j$, such that $u_j > 0$. If $u_j = 0$, then $x_j$ then it can be equal to 1 or 0. If the agent is sympathetic to the center, then $x_j = 1$ for all $u_j = 0$.

For the data from example, if the agent is of the second type, then he performs only projects 1, 2 and 3 with a total value of 24.

Let the CIP fund be 78. Then, if the agent from Example 1 is of the first type, then an equilibrium is obtained, since $\lambda = 78/39 = 2$. If he belongs to the second type, then $\lambda = 78/24 = 3 \frac{1}{4}$. With this value of $\lambda$ it becomes beneficial for the agent to carry out all projects. First of all, he will naturally add project 5 with maximum efficiency. The total value will be 32, and the value $\lambda = 78/32 = 2 \frac{8}{35}$. Further implementation of projects is no longer profitable. An equilibrium situation is obtained.

The study of the existence of a situation of equilibrium, convergence to a situation of equilibrium, the applicability of the weak influence hypothesis for a discrete model is a rather complicated task. For an experimental study, we give a description of the business game.

6. Business game "Stimulating initiative projects" (SIP)

Game Description. There are $n$ of agents (teams). Each team is given a list of initiative projects. If the game is held for competitive purposes, all teams receive the same lists. Either teams are divided into several groups and teams of the same group receive the same lists. Each team is informed of its type of behavior (workaholics or rationalists). The moderator explains the game and the procedure for summing up, and also announces the value of the CIP funds.
Holding the game.

- Each team reports a list of $R_i$, projects that they will execute with an indication of their value $z_{ij}$, $j \in R_i$.
- The leader determines the standard

$$\lambda = \frac{\Phi}{\sum A_i}, \text{ where } A_i = \sum_{j \in R_i} z_{ij}$$

and the teams wins.

In this case, the workaholic team wins are equal to the total value of the IP, provided that

$$\lambda A_1 \geq \sum_{j \in R_i} c_{ij}, \text{ or } \lambda A_1 < \sum_{j \in R_i} c_{ij}.$$ The rationalist team’s win is equal $\lambda A_1 - \sum_{j \in R_i} c_{ij}$.

The game is held in several games (usually 6-8).

- Summarizing. After the end of the game the leader determines the winning teams as the sum of the winnings for all games. At the same time, winners are determined separately for “workaholics” and for “rationalists”, as well as separately for each group.
- Analysis of the results of the game. The analysis is carried out by several indicators.

Firstly, the total value of the IP is determined for all batches. It is compared to the maximum value. Secondly, the strategy of the teams - the winners and the defeated teams is analyzed. Thirdly, the proximity to the equilibrium situation is estimated by the variation of the standard $\lambda$ for the last 2-3 batches.

7. Conclusion

The article describes the mechanism for stimulating initiative projects with two types of agents' behavior (workaholics and rationalists). The problem of convergence of the described procedures and the problem of the existence of an equilibrium for a discrete model require further research.

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The results can be used, for example, when designing software for the following tasks: big data analytics for IoT platform [11], data flows in an IP-based networks [12], information security risk estimation for cloud infrastructure [13].

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