Coulomb drag between quantum wires with different electron densities

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We study the way back-scattering electron–electron interaction generates Coulomb drag between quantum wires with different densities. At low temperature $T$ the system can undergo a commensurate-incommensurate transition as the potential difference $|W|$ between the two wires passes a critical value $\Delta$, and this transition is reflected in a marked change in the dependence of drag resistivity on $W$ and $T$. At high temperature a density difference between the wires suppresses Coulomb drag induced by back scattering, and we use the Tomonaga–Luttinger model to study this suppression in detail.

PACS

I. INTRODUCTION

A constant electric current in a conductor establishes a stationary non-equilibrium distribution of electron momenta. Relaxation processes with electrons in a second (“passive”) conductor nearby can result in a significant drag force, which leads to a measurable drag current or drag voltage in the passive conductor. This effect is known as Coulomb drag if the momentum transfer is mediated by the Coulomb interaction [1]. The drag effect is very sensitive to electronic correlations and can therefore be used to probe them.

While the Coulomb drag between two-dimensional electron systems has been extensively studied experimentally [2], the observation of this effect in strictly one-dimensional (1D) systems still presents a challenge and experimental work on this subject is quite limited [3–5]. The 1D case is especially interesting since here electronic correlations are much stronger than in two- or three-dimensional systems. This is reflected by the fact that the Fermi-liquid theory, which applies in three dimensions and marginally holds for two dimensional systems, generally fails in one dimension. Instead, essential features of a correlated 1D electronic system are captured by the Tomonaga-Luttinger (TL) model [6].

A theory for the 1D Coulomb drag that is based on the TL model [7,8] predicts a behaviour that qualitatively deviates from the one in higher dimensions: At temperatures $T$ below a certain energy gap $\Delta$ the drag resistivity $\rho_D$ between long wires of equal electron density increases exponentially with decreasing temperature, $\rho_D \propto \exp{\Delta/T}$ [9]. At temperatures well above the gap the drag resistivity shows a distinct power law behaviour, $\rho_D \propto T^x$, where the exponent $x$ is determined by the forward scattering part of intra- and inter-wire interaction. For vanishing forward scattering interaction $x$ equals unity (and the linear $T$-dependence of a Fermi-liquid approach [10] is reproduced), while it decreases with increasing repulsive electron-electron interaction. The gap energy $\Delta$ corresponds to a correlated ground state, where the electrons in the two wires order in a zigzag formation with period equal to the Fermi wavelength $\lambda_F$. The gap energy $\Delta$ therefore strongly depends on the backscattering ($2k_F$) component of the ee-interaction. It is exponentially suppressed if the distance $d$ between the wires is large compared to $\lambda_F$.

Under the strictly linear spectrum assumed by the TL model a drag response is generated only by inter-wire back-scattering. A recent analysis [11] by Pustilnik and collaborators addresses drag induced by forward scattering in a wire with non-linear dispersion. In particular, it is shown in that work that at low temperatures forward scattering gives a contribution $\rho_D^{(f)} \propto T^2$ to the drag resistivity if the difference in the Fermi velocities $\delta v_F$ is small compared to $T/k_F$. For $\delta v_F$ larger than $T/k_F$, the contribution due to forward scattering decays more rapidly, $\rho_D^{(f)} \propto T^5$.

This communication focuses on the backscattering drag between wires of different electron densities. In principle, the difference in densities is expected to suppress inter-wire $2k_F$ scattering, since the momentum $2k_F$ is different in the two layers, and to affect drag also through the difference in Fermi velocities in the two wires. Here, we neglect the difference in Fermi velocities between the wires (a difference which induces corrections of the order of $\mu_1 - \mu_2$, with $\mu_i$ the electro-chemical potential of the $i$th wire), and focus on the effect of the difference between the relevant $2k_F$’s (a difference which induces a much larger correction of the order of $\mu_{11} - \mu_{12}$). We envisage the experimental situation of two identical wires on different electro-static potentials [3,5].

We start by analyzing Coulomb drag in the high temperature regime, where inter-wire back-scattering may be treated perturbatively. In Sec. III we reinvestigate the drag in the incommensurate phase with different electron densities and present detailed results for the non-linear...
drag at finite temperatures. Parts of these results are already published in the review [4]. Following that analysis, we address the low temperature regime. For equal densities, the low temperature regime is that of an interlocked crystal, characterized by a drag resistance that diverges at zero temperature. As an electro-chemical potential difference $W$ is turned on between the wires, the relative density between the wires is initially incompressible, until an incommensurate-commensurate (IC) transition [12,13] takes place [14]. This transition markedly influences the drag. The transition from the incommensurate to the commensurate phase is, roughly speaking, a readjustment of charges into an energetically more favorable zigzag configuration.

We employ the theory of the IC transition [12,13] and use a simple Drude model for solitonic charge transport to investigate the drag at low temperatures $T \ll \Delta$. We conclude that for $|W| < \Delta$ the drag resistivity $\rho_D$ is exponentially large, $\rho_D \propto \exp(\Delta - |W|)/T$. Above the transition point the drag shows an inverse square root behaviour, $\rho_D \propto (W^2 - \Delta^2)^{-1/2}$, which changes to an $1/|W|$ dependence for large $|W| \gg \Delta$ (c.f. Fig. 4). This is studied in Sec. IV.

We hope that the present work may help to better understand present and near-future drag experiments. In particular, together with the recent results of Pustilnik et al. [11] it may help to clarify the role of forward and backward scattering in the drag effect.

II. COULOMB COUPLED DOUBLE WIRE

We consider electrons in two parallel, strictly 1D wires of length $L$ separated by a distance $d$. We assume the wires to be identical, and assume that a voltage $W/e$ is imposed between them, leading to a difference between their electro-chemical potentials $W = \mu_2 - \mu_1$. It is assumed that $L$ is much larger than all other relevant length scales of the problem, i.e. the wires are practically of infinite length. For the beginning we neglect the electron spin.

Using the same notations as in [8] and assuming that the electro-static potential is constant along each wire, kinetic and potential energy of the electrons are given by

$$H_0 = \sum_{r w k} v_F (r k - k_F) a_{r w}^\dagger (k) a_{r w} (k) + \frac{1}{2} W (N_2 - N_1)$$

The index $r$ corresponds to left ($r = -$) and right ($r = +$) moving electrons, $w$ refers to active ($w = 1$) and passive ($w = 2$) wire. The electron number in wire $w$ is $N_w = N_{w+} + N_{w-}$, where $N_{w+} = \sum_k a_{r w}^\dagger (k) a_{r w} (k)$. The Fermi wave-number $k_F$ is defined via the identical equilibrium density in both wires for vanishing external potentials.

The expressions for forward scattering and backward scattering electron-electron interactions $H_f$ and $H_b$, resp., are precisely as in Ref. [8] Sec. III B, and are therefore not repeated here.

We proceed as in [8] with a standard transformation [6] to bosonic fields

$$\phi_w(x) = -\frac{i \pi}{L} \sum_q e^{-iqx-\frac{aq}{2}} [\varphi_{w+}(q) + \varphi_{w-}(q)]$$

with their conjugated

$$\Pi_w(x) = \frac{1}{L} \sum_q e^{-iqx+\frac{aq}{2}} [\varphi_{w+}(q) - \varphi_{w-}(q)]$$

$$+ \frac{1}{L} J_w ,$$

where $J_w = N_{w+} - N_{w-}$, and $\varphi_w$ is the electron density in wire $w$. Note that in the present case the particle number $N_w$ may change with variation of the external potentials and therefore the zero-mode terms must be included here. In the following, the inverse momentum cut-off $\alpha$ is set to the Fermi wavelength $2\pi/k_F$. Then, transformation to symmetric ($+$) and antisymmetric ($-$) charge modes $\phi_{c\pm} = 2^{-1/2}(\phi_1 \pm \phi_2)$ decouples the Hamiltonian into independent parts, $H = H_{c+} + H_{c-}$, with

$$H_{c+} = \frac{u}{2\pi} \int \left[ K \pi^2 \Pi^2_{c+} + \frac{1}{R} (\partial_x \phi_{c+})^2 \right] dx$$

$$+ \frac{\lambda E_0}{\pi \alpha} \int \cos \sqrt{8} \phi_{c+} dx - \frac{W}{2\pi} \int \partial_x \sqrt{8} \phi_{c-} dx,$$

and a similar expression without cos-term for $H_{c-}$. The parameters $u \equiv u_{c-}$ and $K \equiv K_{c-}$ are determined by the interwire and intrawire couplings $g_i$ and $\bar{g}_i$ as explained in [8]. $\lambda = g_1/2\pi u$ denotes the dimensionless interwire backscattering coupling, $E_0 = u/\alpha$ is of order the Fermi energy.

III. HIGH TEMPERATURE DRAG

At high temperatures the two wires are only weakly correlated, and drag may be calculated perturbatively, in a method similar to that employed in [7,8]. Here we will focus on the case of wires with different densities.

In the weakly coupled regime it is appropriate to switch to a statistical ensemble with fixed electron numbers $N_w = L n_w$ ($w = 1, 2$). This enables us to define periodic bosonic fields by

$$\tilde{\phi}_{c-} (x) = \phi_{c-} (x) + qx ,$$

where $q = 2^{-1/2}(n_1 - n_2) = 2^{-1/2}(k_{F1} - k_{F2})$. The dynamics of $\tilde{\phi}$ follows from Eq. (1) to be given by the Hamiltonian
\[
H_{e-} = \frac{u}{2\pi} \int K \pi^2 \Pi_{e-}^2 + \frac{1}{K} (\partial_x \tilde{\phi}_{e-})^2 dx \\
+ \frac{\lambda E_0}{\pi \alpha} \int \cos \sqrt{8} (\tilde{\phi}_{e-} - qx) dx + \text{const.} \quad (2)
\]

We calculate the drag using the formalism of Refs. [7] and [8], whereby the drag voltage is

\[
e\frac{V_D}{L} = -\frac{1}{2\kappa L} \int_0^L (\partial_x n_-)_I = \frac{1}{\sqrt{8\pi \kappa}} (\partial_x^2 \phi_{e-})_I. \quad (3)
\]

Here, \(n_- = (n_1 - n_2)/2\) and \(\kappa = K/2\pi u\) are density and compressibility of the anti-symmetric channel, and the averaging is taken with respect to a state carrying a current \(I\) in the active wire (1).

Using the condition of stationarity and the equation of motion for \(\tilde{\phi}\) we find that to second order in the backward scattering the drag \(\varepsilon_D = eV_D/L\) is

\[
\varepsilon_D = \frac{4}{\pi} \left( \frac{\lambda E_0}{\alpha} \right)^2 \times \left( \frac{\pi T}{E_0} \right)^{4K-2} \times \left( N_{-\omega+\omega,K} N_{-\omega-\omega,K} - N_{-\omega+\omega,K} N_{\omega-\omega,K} \right),
\]

where the function \(N_{r,K}\) is given by

\[
N_{r,K} = \lim_{\delta \to 0} \int ds e^{-isr/\pi} \left( \frac{\delta}{8} + i \sinh s \right)^{-2K} \quad (6)
\]

\[
= 2^{2K} \Gamma(1 - 2K) \text{Re} \left[ \frac{e^{i\pi K} \Gamma(K - i \frac{r}{2\pi})}{\Gamma(1 - K + i \frac{r}{2\pi})} \right]. \quad (7)
\]

The result (5) is limited to the weakly coupled regime, which requires \(\Delta \ll T\). Further, we have assumed that all three relevant energies \(T, |\omega|, |Q|\) are small compared to \(E_0\). Boundary effects are also neglected, which is allowed as long as the wire length \(L\) exceeds the thermal wavelength \(u/T\).

As a forward to a discussion of the drag (5) as a function of the parameters \(T, Q\) and \(\omega\), we first analyze the function \(N_{r,K}\). An asymptotic expansion of Eq. (7) shows that \(N_{r,K}\) decays exponentially \(\sim e^{-r}\) for large positive argument \(r\), and that \(N_{r,K} \sim r^{2K-1}\) for large negative \(r\). More precisely, we find

\[
N_{r,K} \approx 2 \sin(2\pi K) \Gamma(1 - 2K) \left| \frac{r}{\pi} \right|^{2K-1} \times \left\{ \begin{array}{ll}
1, & r \gg 1 \\
- e^{-r}, & r \gg 1
\end{array} \right.
\]

For the particular values \(K = 1\), corresponding to non-interacting electrons, and \(K = 1/2\) the expression (7) simplifies to

\[
N_{r,1} = \frac{2r}{e^{r} - 1}, \quad N_{r,1/2} = \frac{2\pi}{e^{r} + 1}.
\]

While \(N_{r,1}\) bears some resemblance to the Bose distribution, \(N_{r,1/2}/2\pi\) is the Fermi function (cf. Fig. 1).

### A. Non-linear drag

For \(T \ll |\omega| \) and \(T \ll |Q| \) (but still \(T \gg \Delta\) and \(T \gg u/L\)) we can approximate \(N_{r,K}\) in Eq. (5) by the asymptotic expression for negative \(r\), and \(N_{r,K} = 0\) for positive argument \(r\). This shows that for \(|\omega| - |Q| \gg T\)

\[
\varepsilon_D = \frac{2E_0^2}{\pi e u} \chi^2 \sin^2(2\pi K) \Gamma^2(1 - 2K) \left| \frac{\omega^2 - Q^2}{E_0^2} \right|^{2K-1} \times \text{sign}(\omega),
\]

whereas the drag becomes exponentially suppressed for \(|Q| - |\omega| \gg T\). This result has been already derived by Nazarov and Averin [7]. The seeming \(|\omega^2 - Q^2|^{2K-1}\) singularity it exhibits near the threshold \(|\omega| = |Q|\) is smeared over a regime where \(|\omega| - |Q| \lesssim T\), which can be quantitatively described by Eq. (5). For large currents, \(|\omega| \gg |Q|\), the drag goes as \(\varepsilon_D \propto \omega^{4K-2}\). Fig. 2 illustrates this behaviour.

### B. Linear drag resistance

From Eq. (5) one easily finds the linear drag resistivity \(\rho_D = \frac{d\varepsilon_D}{dT} \big|_{T=0}\) as
3, where the normalized drag resistance

\[ \rho_D = \rho_0 \left( \frac{T}{E_0} \right)^{4K-3} \left( N_{\phi,K} N_{\phi,K'} + N_{\phi,K'} N_{\phi,K} \right), \]

(8)

with \( \rho_0 = \sqrt{8\pi} E_0 \lambda^2/e^2 u \) and \( N_{\phi,K} \equiv d N_{\phi,K}/dr \).

The drag resistivity is a symmetric function of \( Q \) with its maximum at \( Q = 0 \), where \( \rho_D \propto T^{4K-3} \). For \( |Q| \) larger than temperature the linear drag resistance is exponentially suppressed, due to the exponential suppression of \( N_{\phi,K} \) for large and positive \( r \). Thus, the peak of \( \rho_D(Q) \) has height \( \propto T^{4K-3} \) and its width is directly proportional to temperature. More details can be extracted from Fig. 3, where the normalized drag resistance

\[ \tilde{\rho}_D(Q/T) = \frac{\rho_D(Q,T)}{\rho_D(Q=0,T)} \]

(9)

is plotted against \( Q/T \) for \( K = 1, 0.75, 0.5 \) and \( 0.25 \).

**FIG. 2.** Drag in arbitrary units as a function of \( \omega \propto \frac{u}{T} \) for interaction parameters \( K = 0.25 \) and \( 0.75 \) at temperatures \( T = 0.25Q \) (smooth curves) and \( T \ll Q \). For large currents \((|\omega| \gg |Q|)\) the drag goes as \( \omega^4K-2 \). At low temperatures the drag shows a singularity near the threshold \( |\omega| \sim |Q| \), which is smeared over a regime where \( |\omega| - |Q| \sim T \).

**FIG. 3.** Normalized drag resistivity \( \tilde{\rho}_D \) (cf. Eq. (9)) as function of \( Q/T = u\delta k_F/T \) for interaction parameter \( K = 1, 0.75, 0.5 \) and \( 0.25 \), respectively.

expressions for drag \( \varepsilon_D \) and drag resistance \( \rho_D \) are basically identical to (5) and (8), except of one difference, which is essential: The interaction-determined constant \( K \) is replaced by an effective \( \tilde{K} = 0.5(K_s + K_c) \), where \( K_s \) is the interaction parameter of the spin modes (cf. [8]). For weak backscattering coupling \( K_s \approx 1 \), such that \( \tilde{K} = 0.5(1 + K) \). I.e. the effective parameter \( \tilde{K} \) of the spin-full system is closer to the non-interacting (Fermi-liquid) value \( K_{FL} = 1 \), which reflects the moderating effect of the spin-modes.

**IV. INCOMMENSURATE-COMMENSURATE TRANSITION**

Eq. (1) is the Hamiltonian of the sine-Gordon model with an additional coupling of the soliton density \( n_s = \int dx \partial_x \sqrt{\delta \phi_c}/2\pi L \) to the potential difference \( W \). It is this coupling which gives rise to the IC transition [12,13]: At large \( \lambda E_0 \) and small \( |W| \) the cos-term dominates and suppresses the soliton density \( n_s \) to zero, which is the commensurate phase. In the opposite situation, the cos-term is negligible and the ground state density of the solitons is adjusted to \( n_s = \pi KW/u \). This value of \( n_s \) corresponds to electron densities \( n_w = \langle \phi_w \rangle \) in the two wires that are independently tuned by the respective electro-statical potentials \( V_w \). This is the incommensurate phase.

In the classical model, for \( W \) less than the energy \( E_s \) of a single soliton the density \( n_s \) vanishes strictly at zero temperature. The transition to the incommensurate phase sets in at \( |W| = E_s \), where for small \( |W| - E_s > 0 \) the soliton density increases logarithmically with \( n_s \sim |\ln(|W| - E_s)|^{-1} \), corresponding to a critical exponent \( \beta = 0 \) [12,13]. At finite temperatures
the transition is smeared out, as usual for 1D statistical models.

Quantizing the model modifies the transition in two aspects: the transition energy increases to a renormalized soliton energy $E_s = \Delta$ and the critical exponent changes to $\beta = 1/2$ [12,13].

Obviously, the IC transition is accompanied by a drastic change in the inter-wire electronic correlations. It can be therefore expected that the Coulomb drag varies strongly near the transition, and we now address this variation.

In the commensurate phase the electron densities in the two wires are inter-locked to one another. The commensurate phase as a gapped phase has no low-energetic excitations and therefore shows no response to a small anti-symmetric field $\epsilon_{\text{c}}$ at strictly vanishing temperature. Hence, at linear response the resistivity to the flow of non-identical currents in the two wires is infinite, and so is the drag resistivity.

At finite but small temperatures $T \ll \Delta - |W|$ solitons and antisolitons are thermally activated with a density $n_{s/a} \propto \exp(-E_{s/a}/T)$, where $E_{s/a} = \Delta \pm W$ denotes the energy of soliton and antisoliton in the external potential. In the presence of an anti-symmetric field $\epsilon_{\text{c}} = \epsilon_1 = -\epsilon_2$ both solitons and antisolitons contribute to an anti-symmetric current $j_{\text{c}}$. A detailed calculation of this current response to the field $\epsilon_{\text{c}}$, taking into account the damping of the soliton motion is beyond our present scope. However, we are able to conjecture that the temperature dependence of the anti-symmetric conductance is exponentially activated, following the temperature dependence of the soliton density. The conductance is the product of the soliton density by their mobility. As long as the soliton density is small, we expect the mobility to be limited by the interaction of each soliton with its environment, rather than by the interaction with other solitons. In that limit the mobility will not depend on the soliton density, and thus will not be thermally activated. In that limit, then, we expect an exponentially large drag resistivity

$$\rho_D \sim ae^{(\Delta - |W|)/T}$$

valid at temperatures $T \ll |W| - \Delta$. Also the coefficient $a$ may exhibit a significant temperature dependence. The transition from the exponential behaviour (10) to the inverse square root dependence takes place as $W$ passes through an energy window of width of the order of $T$ at $\pm \Delta$.

At large external potentials, $|W| \gg \Delta$, the (anti)soliton density is nearly linear in $|W|$, such that, assuming that the soliton mobility is still independent of its density, the drag resistance goes as $\rho_D \propto 1/|W|$. The low-temperature behaviour of the drag resistivity is summarized in Fig. 4.

FIG. 4. The inverse drag resistivity of a pair of wires at low temperature, $T \ll \Delta$, as a function of the potential difference $W$ (schematic). For $|W| < \Delta$ (shaded) the drag resistance decays linearly with $|W|$ (I). For $|W| \geq \Delta$ (II) the drag resistance decreases as $(|W| - \Delta)^{-1/2}$ with increasing $W$ (II). At large $W \gg \Delta$ the drag resistance decays linearly with $W$ (III).

2. Influence of electron spin

So far the discussion ignores electron spin and is therefore limited to a spin-polarized system. What will happen for unpolarized electrons? Referring to previous work [8] the following can be said: The spin fluctuations moderate correlations of the charge modes, such that $\Delta$ assumes a much lower value than in a spin-polarized system. If $T \ll \Delta$ can nevertheless be realized, we expect qualitatively the same characteristics of the drag as in the spin-less case. This can be seen by the renormalization group equations of a spin-full system, which suggest that both spin and relative charge modes become gapped at low temperatures (cf. [8]). Inspection of the backscattering hamiltonian (see Eqs. (41-43) in [8]) reveals that also in this case solitonic excitations in the $c$- mode are possible. This leads to the same phenomenology of the low temperature drag as just discussed for the spin-less case.
V. DISCUSSION

We have seen that the incommensurate-commensurate transition of a strongly coupled double wire is accompanied by a marked change of the drag resistance (cf. Eqs. (10) and (11)). Observation of this effect requires that the temperature is of order of or below the energy gap $\Delta$. Taking the estimate of $\Delta$ of order $mK$ for a spin-unpolarized double wire with $E_0 \approx 10K$ [8] we conclude that the strongly coupled regime might be elusive for these systems. However, in case of spin-polarized electrons the situation looks much better, where for the same parameters one obtains $\Delta$ of order 100$mK$.

The perturbative results derived in Sec. III should be applicable to existing experimental data [3–5]. In fact, both groups observe a peak of the drag resistivity as the gate voltages are varied. While the qualitative agreement is obvious, a quantitative comparison is difficult because the relation between electron densities in the wires and gate voltages is not trivial. Eq. (5) suggests that measurement of the non-linear drag might be an insightful probe in future experiments.

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