Description and prediction of even-A nuclear masses based on residual proton-neutron interactions

JIAO BaoBao
Department of Physics, University of Shanghai for Science and Technology, Shanghai 200093, People’s Republic of China
E-mail: baobaojiao91@126.com

June 2017

Abstract. The odd-even staggering of neighboring nuclei masses is very useful in calculating local mass relations and nucleon-pair correlations. Based on the odd-even staggering of the proton-neutron interactions between the last proton and the last neutron, we obtain the odd-even features of the mass relations and related quantities exhibited in masses of neighboring nuclei. In recent years, many papers have a large root-mean-squared deviation (RMSD) in their descriptions and evaluations of even-A (A is the mass number) nuclear masses. In this work, we empirically obtained a residual proton-neutron interactions formula of even-A nuclei based on studying the neighboring nuclei (After choosing a nuclear, we made an analysis of its neighboring nuclei on the upper left corner and the lower right corner respectively). We then calculated the even-A nuclear masses. The differences between our calculated values and the AME2012 databases show that the RMSDs are small (for even-A nuclei: $A \geq 42$, RMSD $\approx 162$ keV; $A \geq 100$, RMSD $\approx 125$ keV), while for heavy nuclei, our calculated values can reach an accuracy of a few tens of keV. With our residual proton-neutron interactions formula including one parameter, we have successfully predicted some unknown masses. Some of our predicted values have good accuracy and compared well with experimental values (AME2016). In addition, the accuracy and simplicity of our predicted masses for medium and heavy nuclei are comparable to those of the AME2012 (AME2016) extrapolations.

Keywords: residual proton-neutron interactions, nuclear masses, binding energies

1. Introduction

Nuclear masses [1][5] and energy levels are important issues in the field of nuclear physics. For a given nucleus with $Z$ protons and $N$ neutrons, the relationship between binding energy $B(Z, N)$ [16][19] and nuclear mass $M(Z, N)$ ($B(Z, N) = ZM_p + NM_n - M(Z, N)$, where the $M_p$ and the $M_n$ are the mass of a free proton and a free neutron) is of great importance in areas of physics, such as nuclear structure and fundamental interactions. It is also applicable to nuclear astrophysics, which thus indicates the significance of studying nuclear mass. The atomic mass evaluation (AME) was published
in 2012 and 2017 (AME2012 [20] and AME2016 [21]), in which approximately two hundred additional nuclei were listed than in AME2003 [22].

The description and evaluation of the nuclear masses are one of the focuses in nuclear structure physics. There is a significant quantity of research in this direction. In nuclear physics, there are many mass models and mass formulas. Generally, mass formulas are divided into two major categories: global mass relations and local mass relations. The first one is global mass relations. Earlier studies are as follows: the famous Weizsäcker formula [1] and the finite range droplet model [2]. For the past few years, the BCS theory [3] based on the relativistic mean field model has reached an accuracy of root-mean-squared deviation RMSD $\approx 2\text{MeV}$, the Skyrme-Hartree-Fock-Bogoliubov theory [4,5] RMSD $\approx 581\text{keV}$, the finite range droplet model [2,6] RMSD $\approx 570\text{keV}$, the macroscopic-microscopic mass formula [7,8] RMSD $\approx 441\text{keV}$, the Duflo-Zuker model [9] RMSD $\approx 380\text{keV}$. For a comprehensive review, see [13]. The second is local mass relations. Local mass relations have also proved to be useful for the application of Coulomb displacement energies of mirror nuclei in mass predictions. Such as Audi-Wapstra systematics, the Garvey-Kelson (G-K) mass relations [10] (for even-$A$ nuclei with $A \geq 100$, RMSD $\approx 170\text{keV}$), the nucleon-pair correlations mass relations [23] (for even-$A$ nuclei with $A \geq 100$, RMSD $\approx 168\text{keV}$). Our relation including one parameter is more precise than other relations (for even-$A$ nuclei: $A \geq 42$, RMSD $\approx 162\text{keV}$; $A \geq 100$, RMSD $\approx 125\text{keV}$).

Our purpose in this paper is to describe a residual proton-neutron interactions that can be useful in describing and predicting some of the unknown even-$A$ nuclear masses. We obtained a residual proton-neutron interactions formula of even-$A$ nuclei based on studying the neighboring nuclei (After choosing a nuclear, we made an analysis of its neighboring nuclei on the upper left corner and the lower right corner respectively). There is comparatively good agreement between the calculated and experimental values [20] (for even-$A$ nuclei with $A \geq 42$, RMSD $\approx 162\text{keV}$), while for medium-mass and heavy nuclei, they are in good agreement with the AME2012 databases (for even-$A$ nuclei with $A \geq 100$, RMSD $\approx 125\text{keV}$; $A \geq 192$, RMSD $\approx 88\text{keV}$). The study of proton-neutron interactions is very helpful in describing known nuclear masses, which demonstrates that our approach is feasible and can be used to predict unknown masses. The focus is that we can use one parameter of the proton-neutron interactions formula to describe and predict the even-$A$ nuclear masses. In section 2, we obtain our formula based on the proton-neutron interactions between the last proton and the last neutron [23]. In addition, we introduce and explain two corrections: the Coulomb correction and the symmetry energy correction. The contributions from two corrections are much smaller. We then discuss the RMSDs of known even-$A$ nuclear masses. In section 3, by applying our proton-neutron interactions formula, we successfully predict some unknown masses and discuss their deviations. We note that our predicted masses are very close to those predicted in the AME2012 database, in particular, those with $A \geq 42$. The result demonstrates that some of our predicted values and the experimental values in AME2016 [21] have good accuracy and agree well. In this paper, our results
are compared with the AME2012 and AME2016 databases. In section 4, we discuss and summarize the results of our work.

2. Residual proton-neutron interactions

Residual proton-neutron interactions play an important role in nuclear physics. For the past few years, they have attracted more and more attention [24–36]. The study of proton-neutron interactions is very helpful in studying shell model theory and phase transitions [37–39]. In this section, we study the residual proton-neutron interactions. We empirically obtained a residual proton-neutron interactions formula of even-\(A\) nuclei based on studying the neighboring nuclei (After choosing a nuclear, we made an analysis of its neighboring nuclei on the upper left corner and the lower right corner respectively). Then, we use our residual proton-neutron interactions formula to describe and predict the even-\(A\) nuclear masses. The residual proton-neutron interactions between the last \(i\) protons and \(j\) neutrons is given by

\[
\delta V_{ip-jn}(Z, N) = B(Z, N + j) + B(Z - i, N) - B(Z, N) - B(Z - i, N + j).
\]

So the residual proton-neutron interactions between the last proton and the last neutron is defined as

\[
\delta V_{1p-1n}(Z, N) = B(Z, N + 1) + B(Z - 1, N) - B(Z, N) - B(Z - 1, N + 1)
= M(Z, N) + M(Z - 1, N + 1) - M(Z, N + 1) - M(Z - 1, N).
\]

In recent years, many papers have a large RMSD in their descriptions and evaluations of even-\(A\) nuclear masses. In this work, we empirically obtained a residual proton-neutron interactions formula of even-\(A\) nuclei based on studying the neighboring nuclei. Then, we use our residual proton-neutron interactions formula to describe and predict the even-\(A\) nuclear masses.

2.1. Residual proton-neutron interactions

We empirically obtained the residual proton-neutron interactions formula of even-\(A\) nuclei based on the above study. We successfully describe and predict some even-\(A\) nuclear masses from some experimentally known nuclear masses and the residual proton-neutron interactions formula. We calculate \(\delta V_{1p-1n}\) by using these equations and some experimentally known nuclear masses compiled in AME2012 [20]. Residual proton-neutron interactions for nuclei with mass number \(A \geq 42\). Our calculated \(\delta V_{1p-1n}\) are presented in Figure 1. As shown by using the smoothed curve in Figure 1, we empirically have
Figure 1. Circles show that the residual proton-neutron interactions $\delta V_{1p-1n}$. The curve is plotted by using the average values of $\delta V_{1p-1n}$ for nuclei with the same mass number $A$, expressed as $\overline{\delta V_{1p-1n}}$. In Figure 1, the smoothed curve are plotted in terms of equation (3): $\overline{\delta V_{1p-1n}}(A) = \frac{13000}{A}$ keV for even-$A$ nuclei with $A \geq 42$.

\[
\overline{\delta V_{1p-1n}}(A) \simeq B(Z, N + 1) + B(Z - 1, N) - B(Z, N) - B(Z - 1, N + 1) \simeq \frac{13000}{A} \text{ keV},
\]

where $N + Z = A$ is even and $A \geq 42$. The systematicness of the proton-neutron interactions is better in the heavy nuclei region, and poor in the light nuclei region. Our empirical formula can be approximated to the laws of the proton-neutron interactions. Our relation including one parameter is more precise than other relations (for even-$A$ nuclei: $A \geq 42$, RMSD $\approx 162$ keV; $A \geq 100$, RMSD $\approx 125$ keV). The main advantage of our formula given in equation (3) is that it involves masses of only four neighboring nuclei, while the number of neighboring nuclei involved in the Garvey-Kelson mass relation is six. The local mass relation, which work very accurately for masses of four neighboring nuclei. This is important for reliable predictions in the process of iterative extrapolations. The smaller the number of nuclei involved in local mass relations, the more reliable the predictions in iterative extrapolations, and the smaller the deviations are in the extrapolation \cite{40} process. The focus is that we can use one parameter of the residual proton-neutron interactions formula to describe and predict the even-$A$ nuclear masses.
2.2. Corrections and RMSDs

Based on the known nuclear masses and the residual proton-neutron interactions, we get the binding-energy formula and the mass equation:

\[
B(Z, N) = B(Z, N + 1) + B(Z - 1, N) - B(Z - 1, N + 1) - \delta V_{1p-1n}(A). \tag{4}
\]

\[
M(Z, N) = M(Z, N + 1) + M(Z - 1, N) - M(Z - 1, N + 1) - \delta V_{1p-1n}(A). \tag{5}
\]

The residual proton-neutron interactions can be improved by some corrections. We obtained our empirical corrections from studying the residual proton-neutron interactions between the last proton and the last neutron. In this work, we focus only on the Coulomb energies and symmetry energies enlightened by [23, 41]. The first item is the Coulomb correction. We find that light nuclei are not useful for our purpose, while for the nuclei with \( A \geq 164 \), the contributions from Coulomb energies are much smaller (\( \approx 0.1-0.9 \text{ keV} \)). The Coulomb correction is denoted by \( \Delta_C \):

\[
\Delta_C(Z, N) \approx a_C(-\frac{4}{9}Z^{4/3}A^{-7/3} - \frac{2}{3}Z^{-4/3}A^{-7/3} + \frac{4}{9}Z^2A^{-7/3} + \frac{4}{9}Z^{1/3}A^{-4/3}). \tag{6}
\]

Where \( a_C \) is a parameter to be determined.

The second item is the symmetry energy correction. We find that light nuclei are not useful for our purpose, while for the nuclei with \( A \geq 164 \), the contributions from symmetry energy correction are much smaller (\( \approx 0.3-2.8 \text{ keV} \)). The symmetry energy correction is denoted by \( \Delta_{\text{sym}} \):

\[
\Delta_{\text{sym}}(Z, N) = a_{\text{sym}}\frac{1}{A(2 + |IA|^3)} + b_{\text{sym}}A. \tag{7}
\]

Where \( I = (N - Z)/A \). The \( a_{\text{sym}} \) and \( b_{\text{sym}} \) are parameters to be determined.

Our predicted \( \delta V_{1p-1n}(Z, N) \) are summarized as follows:

\[
\delta V_{1p-1n}^{\text{dcal}}(Z, N) = \delta V_{1p-1n}(A) - \Delta_C(Z, N) - \Delta_{\text{sym}}(Z, N). \tag{8}
\]

The parameters [23] used in equations (6)-(8) \( (a_C = -34.80 \text{ keV}, a_{\text{sym}} = 12007 \text{ keV}, b_{\text{sym}} = -179.7 \text{ keV}) \). The modified mass equation:

\[
M^{\text{dcal}}(Z, N) = M(Z, N + 1) + M(Z - 1, N) - M(Z - 1, N + 1) + \delta V_{1p-1n}^{\text{dcal}}(Z, N). \tag{9}
\]

The contributions from two corrections are much smaller. Although the corrections are small in the present work, we believe that one will achieve important improvements along this line in future.
To illustrate the significance of the calculation masses, we calculate the root-mean-squared deviation. The formula is as follows:

\[ \sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (M_{i}^{\text{exp}} - M_{i}^{\text{cal}})^2}. \]  

(10)

Figure 2. Shows the RMSDs of even-\(A\) nuclei. We obtain the even-\(A\) nuclear masses from some experimentally known nuclear masses and the residual proton-neutron interactions formula: \(\delta V_{p-n}(A) \simeq \frac{13000}{A} \text{ keV}\). Comparing calculated values with the AME2012 databases obtain the RMSDs. The solid triangles are plotted by using the RMSDs of our calculated values (each point represents ten nuclei). We obtain the five-pointed stars by using the formula [23].

As is vividly depicted in the figure 2, the differences between our calculated values and the AME2012 databases [20] show that the RMSDs are small (for even-\(A\) nuclei with \(A \geq 42\), RMSD \(\approx 162\) keV), while for medium-mass and heavy nuclei, our calculated values can reach an accuracy of a few tens of keV (for even-\(A\) nuclei with \(A \geq 100\), RMSD \(\approx 125\) keV; \(A \geq 192\), RMSD \(\approx 88\) keV).

We calculate some average deviations in order to fully understand and evaluate our results. The formula is as follows:

\[ \varepsilon = \frac{1}{n} \sum_{i=1}^{n} |M_{i}^{\text{exp}} - M_{i}^{\text{cal}}|. \]  

(11)

We obtain some average deviations:

\[ \varepsilon_1 = \frac{1}{911} \sum_{i=1}^{911} |M_{i}^{\text{exp}} - M_{i}^{\text{cal}}| = 120\ \text{keV}(A \geq 42) \]

\[ \varepsilon_2 = \frac{1}{680} \sum_{i=1}^{680} |M_{i}^{\text{exp}} - M_{i}^{\text{cal}}| = 96 \text{ keV}(A \geq 100) \]
\[ \varepsilon_3 = \frac{1}{253} \sum_{i=1}^{253} |M_i^{\text{exp}} - M_i^{\text{cal}}| = 80 \text{ keV} (A \geq 180). \] (12)

Based on results so far, our method of studying the neighboring nuclei on the upper left corner or the lower right corner respectively is better than others.

3. Prediction of nuclear masses

The study of proton-neutron interactions is very helpful in describing known nuclear masses, which demonstrates that our approach is feasible and can be used to predict unknown masses. In this section, we predict nuclear masses which are not experimentally accessible by using local mass relations and the residual proton-neutron interactions. Based on equation (5) we can easily obtain:

\[ M(Z, N) = M(Z+1, N) + M(Z, N-1) - M(Z+1, N-1) + \delta V_{1p-1n}(A). \] (13)

We predict unknown masses with equation (5) and equation (13). We calculate the average value if equation (5) and equation (13) obtain the same nuclear mass. At the same time, we find the average binding energies of our predicted masses are in good agreement with the curve of specific binding energy [42], while for the heavy nuclei, they are in reasonable agreement.

We predict the residual proton-neutron interactions and binding energies of unknown masses based on equation (3) and equation (4); we can then get the mass excess (ME\text{pred}). In Table 1 we present a set of selected data of our predicted results (in units of keV). Obviously, our predicted values have good accuracy and compared well with AME2012 databases. Note that our predicted values are very close to the AME2012 extrapolations. Some nuclei that are important either in astrophysics or in nuclear structure will be measured in the near future.

Now let us focus on a few examples of our predicted values. Table 1 shows that \(^{190}\text{At}\), \(^{192}\text{Rn}\) and \(^{198}\text{Fr}\) are not predicted in the AME2003 and AME2012 databases. Very interestingly, for \(^{198}\text{Fr}\) the deviation of our predicted masses from the experimental results [21] is only \(\sim 37 \text{ keV}\). Three additional nuclei are \(^{200}\text{Ir}\), \(^{224}\text{Np}\) and \(^{230}\text{Rn}\). Their values cannot be predicted in the AME2003 databases, but the differences between our predicted values and others [20] are approximately 100 keV.

In Table 2 (in units of keV), we list some of our predicted nuclear mass excesses (ME\text{pred}) and new experimental values (ME\text{exp}), where some new experimental values are released in 2013. It is easy to observe that a comparison of our predicted values and the experimental values shows that the difference is hundreds of keV, while for some nuclei, our calculation values can reach an accuracy of a few tens of keV. Our predicted method is feasible in even-\(A\) nuclear mass calculations.

Figure 3 shows that our predicted values and the experimental values in AME2016 coincide well. It is easy to observe that a comparison of our predicted values and the experimental values shows that the difference is hundreds of keV. Some of our
Table 1. Mass excess of some unknown mass nuclei with us and others. (keV)

| Nucleus | AME2003 | AME2012 | MEpred | Nucleus | AME2003 | AME2012 | MEpred |
|---------|---------|---------|--------|---------|---------|---------|--------|
| 42V     | -8170   | -7620   | -8200  | 150La   | -57040  | -56383  | -56511 |
| 46Mn    | -12370  | -12957  | -12324 | 158Nd   | -54400  | -54055  | -53844 |
| 62Ge    | -42240  | -41899  | -41860 | 190At   | 7074    |          |        |
| 62V     | -24420  | -25476  | -25124 | 191Tl   | -24330  | -24379  | -24412 |
| 62 Mn   | -48040  | -48481  | -48264 | 192Rn   | 10039   |          |        |
| 64As    | -39520  | -39652  | -39340 | 193W    | -29650  | -29649  | -29755 |
| 64Cr    | -33150  | -33459  | -33497 | 194Re   | -27550  | -27237  | -27243 |
| 66Sc    | -41720  | -41368  | -41295 | 194Bi   | -15990  | -16036  | -15904 |
| 68Br    | -38640  | -38441  | -38311 | 198Fr   | 9533    |          |        |
| 70Kr    | -41680  | -40948  | -40827 | 198Ir   | -25820  | -25821  | -25685 |
| 72Co    | -39300  | -39784  | -40071 | 198At   | -6670   | -6721   | -6596  |
| 74Ni    | -48370  | -48456  | -48398 | 200Ir   | -21611  | -21475  |        |
| 82Zn    | -42460  | -42607  | -42368 | 204Au   | -20750  | -20650  | -20677 |
| 86Tc    | -53210  | -51297  | -50938 | 222Pa   | 22120   | 22155   | 22115  |
| 86Ge    | -49840  | -49760  | -49628 | 224Np   | 31876   | 31771   |        |
| 88As    | -51290  | -50720  | -50578 | 226Np   | 32740   | 32777   | 32699  |
| 100Rb   | -46700  | -46547  | -46807 | 230Rn   | 42048   | 41973   |        |
| 142Dy   | -49960  | -50120  | -50059 | 258Db   | 101750  | 101799  | 101567 |
| 150Tm   | -46610  | -46491  | -46601 | 260Db   | 103680  | 103673  | 103499 |

Table 2. Mass excess of experimental and predicted. (keV)

| Nucleus | ME\textsuperscript{exp} | ME\textsuperscript{pred} | Ref |
|---------|--------------------------|--------------------------|-----|
| 82Zn    | -42314                   | -42368                   | 43  |
| 100Rb   | -46247                   | -46807                   | 44  |
| 198Fr   | 9570                     | 9533                     | 21  |

predicted values can reach an accuracy of a few tens of keV, while for some nuclei, our predicted values can reach an accuracy of several keV. Very interestingly, the predicted values calculated with the formula \[23\] are larger than the experimental values, but our predicted values are both large and small. In addition, the deviation of the relation \[23\] is large for 42 \( \leq A < 100 \), because the paper studies the nucleus with \( A \geq 100 \). The systematicness of the proton-neutron interactions is better in the heavy nuclei region, and poor in the light nuclei region. So the deviation of some predicted masses (in the light nucleus region) from the experimental results is large. We empirically obtained the residual proton-neutron interactions formula by using the average values of \( \delta V_{1p-1n} \) for nuclei with the same mass number \( A \), which result in some of the calculated values deviates from the experimental values. Very interestingly, large deviation in predicted values are odd-odd nuclei (\( ^{56}\text{Sc} \), \( ^{84}\text{Nb} \), \( ^{100}\text{Rb} \), \( ^{110}\text{Nb} \) and \( ^{150}\text{La} \)). There exists an additional binding energy in odd-odd nuclei, which is one of the reasons why the large deviation in the predicted values. The weak form of odd-even feature in both even-A (even-even and odd-odd nuclei) and odd-A (even-odd and odd-even nuclei) nuclei. More accurate predictions could be readily made if the odd-even features were more accurate. Based on results so far, our method of studying the neighboring nuclei on the upper left
corner or the lower right corner respectively is better than others. Our predicted values of some unknown masses can provide useful references for experimental physicists in planning experiments. This is a major benefit of our approach.

![Chart](chart.png)

**Figure 3.** Comparison between mass excess (in units of keV) obtained on experimental values, those deviations (the absolute value of deviation) in [23] extrapolations (solid circles), and our approach (solid triangles).

4. Discussion and Conclusions

In this work, we empirically obtained the residual proton-neutron interactions formula to describe and predict even-\(A\) nuclear masses. In order to improve the accuracy of the residual proton-neutron interactions \(\delta V_{ip-jn}\), we using the average values of \(\delta V_{ip-jn}\) for nuclei with even-\(A\) (expressed as \(\delta \overline{V}_{ip-jn}\)) and introduces two modifications (The contributions from two corrections are much smaller).

We study nuclear masses origin of the odd-even difference in terms of residual proton-neutron interactions \(\delta V_{ip-jn}\). The systematicness of the proton-neutron interactions is better in the heavy nuclei region, and poor in the light nuclei region. Our empirical formula can be approximated to the laws of the proton-neutron interactions. We find a useful formula based on \(\delta V_{ip-1n}\) for even-\(A\) nuclei with \(A \geq 42\)

\[
B(Z, N + 1) + B(Z - 1, N) - B(Z, N) - B(Z - 1, N + 1) \approx \frac{13000}{A} \text{ keV.}
\]

We calculate the nuclear masses by this formula. We then obtain the RMSDs by comparing the calculative values with the experimental values [20] (for even-\(A\) nuclei: \(A \geq 42\), RMSD \(\approx 162 \text{ keV}\); \(A \geq 100\), RMSD \(\approx 125 \text{ keV}\)). However, others' research on the masses: the Garvey-Kelson(G-K) mass relations [10] (for even-\(A\) nuclei with \(A \geq 100\), RMSD \(\approx 170 \text{ keV}\)); the pairing interactions mass relations [23] (for even-\(A\) nuclei with \(A \geq 100\), RMSD \(\approx 168 \text{ keV}\)). Figure 2 demonstrates that though we use the formula [23] in AME2012 [20],
the contribution for reducing the RMSD is small. The differences between our calculated values and the AME2012 databases have small RMSDs; for the medium-mass and heavy nuclei, calculated values can reach an accuracy of a few tens of keV. Comparing our predicted values with the AME2012 databases shows that the deviations are small. The study of proton-neutron interactions is very helpful in describing known nuclear masses, which demonstrates that our approach is feasible and can be used to predict unknown masses. Additionally, some of our predicted values and experimental values (AME2016) are in good agreement. Besides that, we predict the $^{198}$Fr mass that cannot predicted in AME2003 databases and AME2012 databases, and the deviation of our predicted masses from the experimental results [21] is only $\sim 37$ keV. Therefore, our accurate and simple predictions of masses for medium and heavy nuclei are comparable with those of the AME2012 extrapolations. Additionally, we need one parameter of the residual proton-neutron interactions formula to describe and predict the even-$A$ nuclear masses. Based on results so far, our method of studying the neighboring nuclei on the upper left corner or the lower right corner respectively is better than others. We can predict other unknown masses by using our empirical formula and the predicted masses; they are not detailed here.

Our purpose here is to describe a new empirical residual proton-neutron interactions formula that can be useful in describing and predicting masses of even-$A$ nuclei. In predicting the unknown masses, the Garvey-Kelson mass relations require five nuclei, but our formula requires only three. Obviously, the smaller the number of nuclei involved in local mass relations, the more reliable the predictions in iterative extrapolations, and the smaller the deviations are in the extrapolation [40] process. In addition, our residual proton-neutron interactions formula including one parameter. This is another advantage of our mass relation. We study the residual proton-neutron interactions and make use of these results in evaluating nuclear masses and predicting the unknown masses. Further, our predicted values of unknown masses can provide useful reference points for experimental physics. More accurate predictions could be readily made if the predicted proton-neutron interactions were more accurate.

Acknowledgements

The author would like to thank L. Y. Jia for reading and commenting of this paper. Support is acknowledged from the National Natural Science Foundation of China, Grant No. 11405109 and the Shanghai Key Lab of Modern Optical System.

References

[1] Von Weizsäcker C F 1935 Z. Phys 96 431
[2] Möller P, Nix J R, Myers W D et al 1995 At. Data Nucl. Data Tables 59 185
[3] Geng L, Toki H and Meng J 2005 Prog. Theor. Phys. 113 785
[4] Goriely S, Tondeur F and Pearson J M 2001 At. Data Nucl. Data Tables 77 311
[5] Goriely S, Chamel N and Pearson J M 2009 Phys. Rev. Lett. 102 152503
[6] Möller P, Myers W D, Sagawa H et al 2012 Phys. Rev. Lett. 108 052501
[7] Möller P and Nix J M 1988 At. Data Nucl. Data Tables 39 213
[8] Wang N, Liang Z, Liu M et al 2010 Phys. Rev. C 82 044304
[9] Duffo J and Zuker A P 1995 Phys. Rev. C 52 R23
[10] Garvey G T and Kelso I 1966 Phys. Rev. Lett. 16 197
[11] Bao M, He Z, Lu Y et al 2013 Phys. Rev. C 88 064325
[12] Jiang H, Fu G J, Sun B et al 2012 Phys. Rev. C 85 054303
[13] Lunney D, Pearson J M and Thibault C 2003 Rev. Mod. Phys. 75 1021
[14] Strutinsky V M 1967 Nucl. Phys. A 95 420
[15] Myers W D and Swiatecki W J 1966 Nucl. Phys. 81 1
[16] Haustein P E 1988 At. Data Nucl. Data Tables 39 185
[17] Ren Z 2002 Phys. Rev. C 65 051304
[18] Ren Z, Tai F and Chen D H 2002 Phys. Rev. C 66 064306
[19] Bethe H A and Bacher R F 1936 Rev. Mod. Phys. 8 82
[20] Wang M, Audi G, Wapstra A H et al 2012 Chin. Phys. C 36 1603
[21] Audi G, Kondev F G, Wang M et al 2017 Chin. Phys. C 41 030001
[22] Audi G, Wapstra A H and Thibault C 2003 Nucl. Phys. A 729 337
[23] Fu G J, Lei Y, Jiang H et al 2011 Phys. Rev. C 84 034311
[24] Basu M K and Banerjee D 1971 Phys. Rev. C 4 652
[25] Jünecke J 1972 Phys. Rev. C 6 467
[26] Brenner D S, Wesselborg C, Casten R F et al 1990 Phys. Lett. B 243 1
[27] Monze G and Ythier C 1990 Il Nuovo Cimento A (1965-1970) 103 131
[28] Van Isacker P, Warner D D and Brenner D S 1995 Phys. Rev. Lett. 74 4607
[29] Zaochun G and Chen Y S 1999 Phys. Rev. C 59 735
[30] Cakirli R B, Brenner D S, Casten R F et al 2005 Phys. Rev. Lett. 94 092501
[31] Cakirli R B and Casten R F 2006 Phys. Rev. Lett. 96 132501
[32] Brenner D S, Cakirli R B and Casten R F 2006 Phys. Rev. C 73 034315
[33] Oktem Y, Cakirli R B, Casten R F et al 2006 Phys. Rev. C 74 027304
[34] Stoitsov M, Cakirli R B, Casten R F, Nazarewicz W et al 2007 Phys. Rev. Lett. 98 132502
[35] Breitenfeldt M, Borgmann C, Audi G et al 2010 Phys. Rev. C 81 034313
[36] Cakirli R B, Blaum K and Casten R F 2010 Phys. Rev. C 82 061304
[37] Zamfir N V and Casten R F 1991 Phys. Rev. C 43 2879
[38] Gelberg A, Sakurai H, Kirson M W et al 2009 Phys. Rev. C 80 024307
[39] Casten R F and Zamfir N V 1996 J. Phys. G: Nucl. Part. Phys. 22 1521
[40] Morales I O and Frank A 2011 Phys. Rev. C 83 054309
[41] Mendoza-Temis J, Hirsch J G and Zuker A P 2010 Nucl. Phys. A 843 14
[42] Evans R D and Noyau A 1955 The atomic nucleus (New York: McGraw-Hill)
[43] Wolf R N, Beck D, Blaum K et al 2013 Phys. Rev. Lett. 110 041101
[44] Manea V, Atanasov D, Beck D et al 2013 Phys. Rev. C 88 054322