Unsteady stokes flow of dusty fluid between two parallel plates through porous medium in the presence of magnetic field

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Abstract: This paper focus on the result of dust particle between two parallel plates through porous medium in the presence of magnetic field with constant suction in the upper plate and constant injection in the lower plate. The partial differential equations governing the flow are solved by similarity transformation. The velocity of the fluid and the dust particle decreases when there is an increase in the Hartmann number.

1. Introduction
The necessities of modern machinery is motivated the interest in fluid flow studies, which involves the interaction of several phenomena. The presented of S.Ganesh, S.Krishnambal [1] have studied the unsteady flow of an incompressible viscous fluid between two parallel porous plates. C.S.Bagewadi and B.J.Gireesha [2] have studied of two-dimensional dusty fluid flow under varying temperature and time dependent pressure gradients. The work deals with the study of laminar flow of an unsteady dusty fluid in porous medium through uniform pipe with sector of a circle as cross-section. The fluid and dust particle are assume to be a rest. The motion of fluid is due to influence of time dependent pressure gradient. The analytical expressions are obtained for velocities of fluid and dust particles. For each case the skin friction at boundaries are obtained. Joseph and Tao [3] in the field of water in river beds, in the petroleum technology to study the movement of natural gas, oil and water through oil reservoirs, in chemical engineering for filtration and purification process.

The author H.A.Attia [4] has studied of the effect suction and injection on the unsteady flow between two parallel plates with variable properties. I.A.Hassaninen,M.A.Mansour [5] discussed the unsteady magnetic flow through a porous medium between two infinite parallel plates. I.C.Bagchi [6] discussed the unsteady flow of viscoelastic maxwell fluid with heat transfer considering varying temperature.

H.A.Attia [7] has considered the Unsteady Hartmann flow with heat transfer of a visco-elastic fluid considering the Hall Effect. M.Ezzat [8] studied the problem of micro polar Magneto hydrodynamic boundary layer flow. The author Soundalgkar, A.Uplekar [9] discussed the steady magnetic hydro dynamic of couette flow with heat transfer considering varying temperature. S.Krishnambal and S.Ganesh [10] discussed the Unsteady magnetohydrodynamic stokes flow of viscous fluid between two parallel porous plates. T.M.Nabil [11] studied the MHD flow of non-Newtonian visco-elastic fluid through a porous medium.
near an accelerated plate. E.A.Hamza [12] the suction and injection effects on the similar flow between parallel plates.

In this paper, we consider the unsteady stokes flow of dusty fluid between two parallel plates through porous medium in the presence of magnetic field with the velocity of the fluid and the dusty particle decreases when there is an increase in the Hartmann number.

2. Nomenclature

\( \rho \) - Density of the fluid

\( h \) - Height of the channel

\( K^* \) - Stokes constant

\( K \) - Porous medium

\( N \) - No. of Dust Particles

\( \mu \) - Coefficient of viscosity

\( \psi \) - Stream function

\( u \) - Axial component of the velocity

\( v \) - Transverse Component of the velocity

\( \eta \) - Dimensionless distance

\( \sigma \) - Electrical conductivity of the fluid

\( B_0 \) - Electromagnetic induction

\( M \) - Hartmann number

3. Mathematical formulation

The flow of an incompressible viscous fluid between two parallel porous plates in the presence of magnetic field \( y = -h \) and \( y = h \) in a parallel plate channel bounded by a loosely packed porous medium. The fluid is driven by a uniform magnetic pressure gradient parallel to the channel plates. Let \( u \) and \( v \) be the velocity components in the \( x \) and \( y \) directions respectively at time \( t \) in the flow field and \( u_p \) is the velocity of the dust particles.

The equation of continuity is:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

Navier Stoke Equations are:

\[ \rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - k^* N \left( u - u_p \right) - \frac{\mu u}{k} - \frac{\sigma B_0^2}{\rho} u \]

(1)

\[ \rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \]

(2)

The Motion of dust particles is governed by Newton’s second law

\[ m_p \frac{\partial u_p}{\partial t} = k^* (u - u_p) \]

(3)

Let us consider the solutions of the equations (1)-(3) respectively as

\[ u(x,y) = u(x,y)e^{-nt} \]

\[ v(x,y) = v(x,y)e^{-nt} \]

\[ p(x,y) = p(x,y)e^{-nt} \]

\[ u_p(x,y) = u_p(x,y)e^{-nt} \]

(4)

With the boundary conditions are,

\[ u(x, -h) = 0 \]

\[ u(x, h) = 0 \]

\[ v(x, -h) = v_1 \]

\[ v(x, h) = v_2 \]

(5)

Using (5) in (4), the velocity of dust particle becomes

\[ u_p = \frac{k^*}{K^* - \rho m_p} u \]

(6)

Let the stream functions are
\[ u(x, y) = \frac{\partial v}{\partial y} \quad u(x, y) = \frac{\partial v}{\partial x} \quad (7) \]

From equations (2), (3) and (8), we have

\[ -n \rho \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \frac{\mu}{\partial (\nabla^2 \psi)} - k^* N (1 - c^2) \frac{\partial u}{\partial y} - \frac{\mu}{k} \frac{\partial v}{\partial y} - \frac{\rho}{\rho} \frac{\partial w}{\partial y} \quad (8) \]

Where,

\[ c^2 = \frac{k^*}{k^* - n m} \rho \]
\[ n \rho \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial y} - \frac{\partial u}{\partial x} \quad (9) \]

Differentiating equations (8) & (9) with respect to \( y \) & \( x \) partially, we get,

\[ \frac{\partial^2 p}{\partial x \partial y} = - \mu \frac{\partial^2 \psi}{\partial y^2} + n \rho \frac{\partial^2 \psi}{\partial x^2} - k^* N (1 - c^2) \frac{\partial^2 \psi}{\partial y^2} - \frac{\mu}{k} \frac{\partial^2 \psi}{\partial y^2} - M^2 \frac{\partial^2 \psi}{\partial y^2} \quad (10) \]

The equation of continuity can be satisfied by a stream function of the form

\[ \psi(x, \eta) = h \left[ \frac{u}{a} - \frac{v}{h} \right] (\eta) \quad (13) \]

Dimensionless distance \( \eta \) defined by

\[ \eta = \frac{y}{h}, \quad \alpha = 1 - \frac{v_1}{v_2}, \leq v_1 \leq v_2 \]

And \( u_0 \) is the average entrance velocity.

Substituting (13) in (10), we have

\[ f''(\eta) - \alpha^2 h^2 f(\eta) = 0 \quad (14) \]

Where

\[ \alpha = \sqrt{k^* N (1 - c^2) - n \rho + \mu \frac{\mu}{k} M^2} \]

Equation (15) reduces to the form

\[ (D^4 - \alpha^2 h^2 D^2) f(\eta) = 0 \quad (15) \]

\[ f(\eta) = (A + B \eta) + Ce^{ah \eta} + De^{-ah \eta} \quad (16) \]
\[ f'(\eta) = B + ah[Ce^{ah \eta} - De^{-ah \eta}] \quad (17) \]

With the boundary conditions

\[ f(-1) = \frac{v_1}{v_2}, \quad f(1) = 1 \]
\[ f'(-1) = 0, \quad f'(1) = 0 \quad (18) \]

Hence the solution of (15) subjecting to the boundary condition (18) is

\[ f(\eta) = (2 - a)[\sinh(ah) - \cosh(ah)] - aahcosh(ah) + asinh(ah) \quad (19) \]

Substituting the value of \( f(\eta) \) in the stream function

\[ \psi(x, \eta) = h \left[ \frac{u}{a} - \frac{v}{h} \right] f(\eta) \]

Hence the Axial Velocity of the Fluid

\[ u = u(x, y)e^{-nt} \]
\[ \frac{\partial u}{\partial y} e^{-nt} \]
\[ u = \left[ \frac{u_0}{a} - \frac{v}{h} \right] \left[ \frac{aahcosh(ah) - cosh(ah)}{2[\sinh(ah) - \cosh(ah)]} \right] e^{-nt} \]
The Transverse Velocity of the Fluid

\[ v = v(x, y)e^{-nt} \]

\[ = -\frac{\partial v}{\partial x}e^{-nt} \]

\[ v = v_2 \left( \frac{(2-a)[\sinh(ah) - a\cosh(ah)] - aahcosh(ah) + asinh(ah)}{2[\sinh(ah) - a\cosh(ah)]} \right) e^{-nt} \]

4. Results and discussions

The analytical solutions of the problem are performed and the results are illustrated graphically. Figs. 1-9 shows the important physical parameters on the velocity, density, time, dust particles. Throughout the values are \( u_0=0.5, v_2=-1; x=3; y=-1 \) to 1, \( h=1, a=0.3, K^*=3, N=10, \rho=2, K=3, \mu =0.5, n=0.5, t=0.5, m_p =2 \) are considered as input value for the graph. Fig 1-9, present the effect of axial velocity of fluid, velocity of fluid and dust particles, transverse velocity of fluid and pressure respectively.

Figure 1: Axial velocity of fluid when \( t \) increases
Figure 2: velocity of fluid and dust particles when $t$ increases

Figure 3: Axial velocity of fluid when $\rho$ increases
Figure 4: velocity of fluid and dust particles when $\rho$ increases

Figure 5: Axial velocity of fluid when $N$ increases
Figure 6: velocity of fluid and dust particles when N increases

Figure 7: Transverse velocity of fluid when t increases
Fig 1 shows that the velocity of the fluid decreases as time increases when $|y|<0.5$ and increases when $0.5<|y|<1$. Fig 2 shows that velocity of fluid and dust particle decreases as time increases. Fig 3 and 5 shows that the velocity of the fluid decreases as density and number of dust particles increases when $|y|<0.4$ and velocity of the fluid increases as density and number of dust particles increases when $0.4<|y|<1$. Fig 4 and
6 shows that velocity of fluid and dust particle decreases as density and number of dust particles increases. Table show that the velocity of the fluid is proportional to the velocity of the dust Fig.7-10 shows that the variation of transverse velocity with respect to the variations in the parameters time, density and number of dust particles. Fig. 7, 8 and 9 shows the magnitude of the upper and lower plates are same. Fig. 8 and 9 shows that transverse velocity of fluid are intermingled as density and number of dust particle increases.

5. Conclusion
Analytical solutions are obtained for the Unsteady Stokes flow of dusty fluid between two parallel plates through porous medium in the presence of magnetic field. The Similarity transformation method is used to solve the problem and the results are evaluated analytically and displayed in graphically. Following conclusions can be drawn:

- The velocity of dust particle is higher than that velocity the fluid for all the parameters of this problem.
- The velocity of the fluid and dust particle decreases as density and number of dust particles increase.
- The Axial velocity of the fluid decreases as time increases and transverse velocity of the fluid increases as time increases.
- Transverse velocities of fluid are intermingled as density and number of dust particle increases.

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