Inside Flat Event Horizons

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ABSTRACT

Grinberg and Maldacena have recently used certain thermal one-point functions, evaluated outside an AdS black hole, to determine the proper time of fall from the event horizon to the singularity (in the uncharged case) or to the Cauchy horizon (in the charged, unperturbed case). This raises the hope that one can use data outside the black hole to investigate the claim that black holes “have no interiors”, by proving that the time of fall is exceptionally short, perhaps effectively zero. We examine this question for the particularly interesting case of a black hole (with a flat, toroidal event horizon) emitted by a larger black hole, after the manner postulated by the Weak Gravity Conjecture. We find that the proper time of fall to the Cauchy horizon is indeed always very short for these black holes, so they are prime candidates for an investigation of this kind.
1. Inside the Event Horizon

It has long been hoped that it might be possible to probe the interiors of black hole event horizons, using theoretical methods (see for example [1–3], or more recently [4–6]). Ultimately even observational methods may be possible (for example, [7, 8]).

Recently, Grinberg and Maldacena [9] have given a very concrete discussion of a simple yet crucial question of this kind: how is the *proper time of fall* from the event horizon to the singularity of an (AdS) black hole reflected in quantities defined on the exterior spacetime? (In the case of a charged black hole, the corresponding quantity is the time of fall from the event horizon to the Cauchy horizon, which is thought to become singular when perturbations are taken into account.) The suggestion in [9] is that the large-mass behaviour of the thermal one-point function (associated with higher-derivative corrections) of a massive field outside an AdS black hole contains a term involving this time of fall quite explicitly. If, for example, the time of fall is extremely short, it should therefore be possible to prove this by studying the precise nature of fields propagating *outside* the black hole.

In order to explore this very interesting and potentially very important observation, one would like to examine what happens in cases where the time of fall can be constrained in some way; it should then be possible to verify that this constraint is reflected in the relevant one-point functions or similar objects defined on the external spacetime. The purpose of this note is to draw attention to a specific class of black holes which, on the one hand, are of great current interest (in connection with the Weak Gravity Conjecture or WGC [10–12]), and which, on the other, do have strongly constrained times of fall: the time is always very short. In this sense, the event horizons of these black holes effectively “have no interior”.

2. AdS$_5$-Reissner-Nordström Black Holes In General

The principal reason for studying asymptotically AdS spacetimes is of course their role in gauge-gravity duality [13–15]. Here, one studies a strongly-coupled field theory (which, in practice, is usually not a conformal theory) defined on a non-dynamical four-dimensional spacetime with metric $-dt^2 + h_\kappa$, where $h_\kappa$ is a metric on a space of constant curvature $\kappa/X^2$; here $\kappa = 0, \pm 1$, and, when the space is compact, $X$ is the characteristic length scale of the space. For example, when $\kappa = 1$ and the space is simply connected, the circumference of the three-sphere is $2\pi X$. Similarly, when $\kappa = -1$, the injectivity radius of the relevant compact space of constant negative curvature is some multiple of $X$; and, when $\kappa = 0$ and the space is a cubic torus, then we can take $X$ to be the side length of that cube. (In all three cases, there are many possible choices of topology and geometry: see [16] for the flat case. For simplicity, when three-dimensional space is compact and $\kappa = 0$, we assume that the space is a cubic torus.)

In most applications (for example, to the holography of the Quark-Gluon Plasma) one wishes $-dt^2 + h_\kappa$ to be simply the metric of ordinary four-dimensional Minkowski space, so the case $\kappa = 0$, with spatial topology $\mathbb{R}^3$, is the one usually encountered in applications.

However, in that case, the dual AdS$_5$ black hole spacetime has no Hawking-Page phase...
transition [17], and this is a serious drawback for many purposes. Compactifying the spatial dimensions (so that, as above, we have a cubic torus) remedies this [18,19] (see [20] for a recent discussion). This is acceptable, in the sense that it will not interfere with the local description of the physics (such as by allowing matter to circulate around the compactified space, which of course would be unphysical), provided that the compactification length scale \( X \) is very large compared with the largest length scale intrinsic to the problem, which is normally assumed to be the AdS scale \( L \).

Toroidal black holes do have a Hawking-Page phase transition, at a temperature which is inversely related to \( X \). Choosing \( X \) to be large therefore drives down the transition temperature. This is reasonable, since, in real strongly coupled matter, such as the Quark-Gluon Plasma, the phase transition temperature is in fact quite low relative to typical temperatures of such matter. However, in this work we will not try to set up a “realistic” model of strongly-coupled matter: it will be enough for us that the transition temperature does not vanish. In short, we will take it that \( X/L \) can be chosen to be arbitrarily large, but finite.

Motivated by this, we therefore proceed as follows. We take the non-dynamical metric 

\[-dt^2 + h_\kappa,\]

set \( \kappa = 0 \), and interpret \( h_0 \) as a metric on a cubic torus with a side length, \( X \), which is very large relative to \( L \). We then use this flat four-dimensional spacetime as a boundary condition for the Einstein(-Maxwell) equations in an asymptotically AdS\(_5\) bulk.

However, it is instructive to return temporarily to the general case. If we fix 

\[-dt^2 + h_\kappa\]

(or rather the conformal structure defined by it), for any of the three possible values of \( \kappa \), as a boundary condition, then it is possible to construct [21] a matching (AdS\(_5\)-Reissner-Nordström) metric in the bulk which solves the Einstein-Maxwell equations, taking the form

\[g(\text{AdS}_5) = - \left( \kappa + \frac{r^2}{L^2} - \frac{16\pi M^* \ell_5^3}{3r^2} + \frac{4\pi k_5 Q^* \ell_5^3}{3r^4} \right) dt^2 + \frac{dr^2}{\kappa + \frac{r^2}{L^2} - \frac{16\pi M^* \ell_5^3}{3r^2} + \frac{4\pi k_5 Q^* \ell_5^3}{3r^4}} + \frac{r^2}{L^2} h_\kappa. \] (1)

Here \( L \) is the AdS\(_5\) scale as above, \( M^* \) and \( Q^* \) are mass and charge parameters (not equal to the physical mass \( M \) and the physical charge \( Q \), see below), \( k_5 \) is the five-dimensional Coulomb constant (with units of length, unlike its four-dimensional counterpart), and \( \ell_5 \) is the gravitational length scale for AdS\(_5\).

However, this metric only solves the equations in the \( \kappa = \pm 1 \) cases if the spatial length scale \( X \) in \( h_\kappa \) is precisely equal to \( L \). That is: in these cases, the boundary length scale is forced to coincide with the bulk curvature scale.

In sharp contrast, when \( \kappa = 0 \), there is no such requirement, so \( X \) is in principle entirely independent of \( L \) (though, as we saw, there may be physical reasons for requiring, for example, that \( X \) must be large relative to \( L \)). This means, in effect, that there is a new independent parameter in the problem in the \( \kappa = 0 \) case, and we have to deal with this.

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1 We use units in which \( M^* \) has units of inverse length, \( Q^* \) is dimensionless and so is entropy. We never use Planck units.
With this point understood, from this point onwards we only consider the \( \kappa = 0 \) case in equation (1).

3. Censorship and the Weak Gravity Conjecture

We now wish to consider the Weak Gravity Conjecture [10–12], which is closely related to the idea that near-extremal black holes must decay, either by emitting particles/branes, or by emitting a smaller black hole. The latter process, which is the one that concerns us here, is associated [22], at least at the classical level, with the production of a naked singularity; that is, with a violation of Cosmic Censorship.

The status of Censorship, both in asymptotically flat [23] and in asymptotically AdS [24] spacetimes, has been much debated recently: see [25] for an overview. Here we follow the point of view advocated in [27] (see also [28]): naked singularities probably are produced in certain highly dynamical spacetimes, but they do not persist, and so Censorship is a reliable guide to the structure of the final state. That is, we will assume that the emitted black hole is indeed a black hole and not a naked singularity.

Censorship in this case implies the existence of horizons of the black hole with metric given in (1). The outer horizon, located at \( r = r_H \), will (in the \( \kappa = 0 \) case we consider here) be a flat cubic torus of side length \( \Delta \), where, from equation (1), \( \Delta = r_H X/L \), and where \( X \) was defined in the preceding Section.

This event horizon “radius” is related to the mass and charge of the black hole in a way which may be unfamiliar, so let us review it.

For later convenience we define a dimensionless quantity \( W \) by \( W \equiv (X/L)^{3/2} \); according to our discussion above, it is to be regarded as an arbitrarily large (but finite) dimensionless parameter. The side length of the horizon is then

\[
\Delta = W^{1/3} r_H. \tag{2}
\]

Notice that, even if \( \Delta \) is large, it by no means follows that the same is true of \( r_H \). Unlike their more familiar spherical counterparts, these black holes can have large event horizons (and therefore entropies) whether the “radius” be small or large.

It must be stressed that \( \Delta \) should be regarded as an observational parameter: an observer in the spacetime who is willing to venture sufficiently near to the event horizon can measure its side length directly. (In addition, the entropy of the black hole is proportional to \( \Delta^3 \), again underlining the “physicality” of this quantity.) One should think of it as a new physical parameter describing the black hole, like (but independent of) the mass and the charge.

In fact, a measurement of \( \Delta \) will be essential in order to determine \( r_H \), since, in contrast to the \( \kappa = \pm 1 \) cases, the latter is not fixed even if the physical mass \( M \) and physical charge \( Q \) of the black hole are known. Instead, \( r_H \) is a definite function of \( M \), \( Q \), and \( \Delta \), given in the following manner.

\[2\] If we do not compactify the three-dimensional space, taking it to have the topology of flat simply-connected \( \mathbb{R}^3 \), then the situation is radically different. The metric (1) is of course still a valid solution of the Einstein equation, but now we have nothing analogous to \( \Delta \), \( M \), and \( Q \) (all of which are now formally infinite) to help us to fix \( M^* \) and \( Q^* \). In this case, one has to rely on holography itself to do this, by relating these parameters to well-defined physical parameters (chemical potential, enthalpy density and so on) describing the boundary field theory.
First, it turns out (see [29] for an elementary derivation) that the parameters $M^*$ and $Q^*$ are related to the physical mass and charge through $M^* = M/W$, and $Q^* = Q/W$. We therefore have

$$\frac{r_H^2}{L^2} - \frac{16\pi M \ell_5^3}{3W r_H^2} + \frac{4\pi k_5 Q^2 \ell_5^3}{3W^2 r_H^4} = 0.$$  

(3)

We can then solve for $r_H$ by eliminating $W$ between equations (2) and (3). We will return to this, below.

Black holes with flat event horizons do not exist in the asymptotically flat case, and so the black holes we are considering here are not well-behaved in the limit $L \to \infty$. They form a disjoint branch of solutions, and can therefore behave in unexpected ways. An important example [29] is the explicit condition for (classical) Cosmic Censorship to hold here:

$$\frac{M \ell_5}{Q} = \frac{M^* \ell_5}{Q^*} \geq \frac{3}{16} \left( \frac{12 k_5^2}{\pi L^2} \right)^{\frac{1}{3}} Q^{*\frac{4}{3}}.$$  

(4)

This is very different from the asymptotically flat case, where of course the mass to charge ratio is bounded below by a fixed constant if Cosmic Censorship is to hold. Notice in particular that, if $Q^*$ is small (which in fact it normally is here, since $W$ is so large), then $M \ell_5$ can be small relative to $Q^*$ violating classical Censorship.

The Weak Gravity Conjecture also takes an unusual form here [29]. Let us suppose that we have a black hole of the above kind, with mass $M$ and charge $Q$. We assume, in accordance with the ideas associated with “Weak Gravity” [10–12], that it emits a toroidal black hole with smaller parameters $m < M$ and $q < Q$. (We assume throughout, however, that neither $Q$ nor $q$ vanishes.) Then it is possible to show [29] that, if the original black hole continues to satisfy Censorship after this bifurcation, then we must have

$$\frac{m \ell_5}{q} = \frac{m^* \ell_5}{q^*} < \frac{1}{4} \left( \frac{12 k_5^2}{\pi L^2} \right)^{\frac{1}{4}} Q^{*\frac{4}{3}}.$$  

(5)

As is well known, the analogue of this relation in the asymptotically flat case [11] is simply the statement that classical Censorship must be violated by the smaller black hole, which motivates the claim that the bifurcation can only be understood with the aid of a quantum-gravitational modification of Censorship itself. But it was shown in [22] that this is not true in the toroidal case: the replacement of the 3/16 factor on the right side of (4) by the 1/4 factor in (5) means that it is possible (though not compulsory) for the emitted black hole to satisfy classical Censorship. Because it can be analysed very simply, this is the case on which we focus here.

4. Just Under the Event Horizon

It is well known that exact Reissner-Nordström spacetimes have Cauchy horizons in their interiors. The fate of these objects when perturbations, both classical and quantum-mechanical, are taken into account, remains a matter of very current debate: see for

\[3\]More concretely: because $W$ is very large, we take it that, as mentioned earlier, $Q^* = Q/W$ is extremely small. The inequality (5) then means that $m \ell_5$ is also very small relative to $q$. However, as we stressed earlier, in the toroidal case this does not necessarily violate classical Censorship.
example \[30,31\], and references therein. The simplest possibility is that the Cauchy horizon becomes singular in some way \[32\]. In order to make contact with the work of Grinberg and Maldacena \[9\], we will work with this assumption: that is, we interpret the “proper time of fall to the singularity” to mean the time to fall from the outer horizon to the idealised location of the Cauchy horizon.

In any case, it seems unlikely that our conclusions would have to be materially revised if some more exotic possibility proves to be correct, but this remains to be investigated\(^4\).

Marolf \[34\] has pointed out that, in the case of near-extremal black holes, a singularity located at the erstwhile Cauchy horizon is extremely close to the outer horizon, in the sense that an object (with an energy per unit mass which is not very small) falling through the event horizon will reach the singularity almost instantly. (See \[35\] for a recent discussion of this and related claims.) This is interesting for a variety of reasons. One such reason is that the spacetime curvature at and just outside the outer horizon can of course be very small; so “closeness” to a spacetime singularity, even if it be a strong curvature singularity, need not imply the presence of intense gravitational fields. Quantum effects of the singularity may nevertheless be detectable outside the horizon, and of course this would be of the utmost interest.

Here we wish to study this question for a toroidal black hole emitted, in accordance with the WGC, by a near-extremal toroidal black hole.

As mentioned earlier, equations (2) and (3) can be combined to eliminate \(W\), and the result (for the emitted black hole, indicated by the superscript) is

\[
 r_E^E(\Delta^E) = \frac{16\pi m \ell_5^3}{3 (\frac{\Delta^E}{L^2} + \frac{4\pi k_5 q^2 \ell_5^2}{3(\Delta^E)})}. 
\]  

(6)

For fixed \(m\) and \(q\), then, \(r_E^E\) is a simple function of \(\Delta^E\). Assuming henceforth that \(q \neq 0\), one finds that this function is bounded above; an elementary computation shows that, for all \(\Delta^E\),

\[
 r_E^E \leq \frac{4\sqrt{\pi m L \ell_5^2}}{\sqrt{3\ell_5 k_5 q}}. 
\]  

(7)

Combining this with the inequality (5), we now have

\[
 \frac{r_E^E}{L} \leq \left(\frac{16\pi k_5}{3L}\right)^{\frac{1}{6}} Q^*^{\frac{1}{3}},
\]  

(8)

where, as above, \(Q^*\) is the charge parameter of the original toroidal black hole.

Under the physical assumptions we are making here, it follows that the “radius” of the emitted black hole must be very small (relative to \(L\)), in the sense that (see above) \(Q^*\) is very small in these circumstances.

Intuition suggests that the proper time of fall to the Cauchy horizon must be very small for these black holes. We can confirm this intuition in an elementary way as follows.

Following \[34\], we consider a particle of non-zero energy per unit mass \(\gamma\) falling into the emitted black hole. Here we have to consider the simple fact that the “proper time of fall
from the event horizon to the Cauchy horizon” is of course not uniquely defined: it depends on $\gamma$. However, we can fix $\gamma$ at some typical value (bearing in mind that, generically, the energy per unit mass of particles falling across event horizons is not small), and study the time of fall for the specific black hole in which we are interested here, as compared with other metrics in some general class.

In order to make such a comparison, let us consider any charged black hole metric which is such that, in Reissner-Nordström-like coordinates, the $tt$ and $rr$ components satisfy $g_{tt}g_{rr} = -1$ (see [36]); obviously the metrics in (1) are of this kind. Then the trajectory of a (not necessarily free) particle with energy per unit mass $\gamma$ falling through the region between the event and Cauchy horizons satisfies $(dr/d\tau)^2 = g_{tt} + \gamma^2$, where $\tau$ is proper time along the worldline and where $g_{tt} \geq 0$ in that region. Consequently, if $r^E_C$ denotes the location of the Cauchy horizon, then the proper time of fall $\tau$ satisfies (in the case of free fall, in which $\gamma$ is constant)

$$\tau \leq \frac{r^E_H - r^E_C}{\gamma},$$

and so $\tau < r^E_C/\gamma$, whatever the value of $r^E_C$ may be. This is useful, because perturbations will convert the Cauchy horizon to a singularity and may well cause it to shift position (see for example [30, 31]). Thus finally we have

$$\frac{\tau}{L} < \frac{1}{\gamma} \left( \frac{16\pi k_5}{3L} \right)^\frac{1}{4} Q^* \frac{1}{3}.$$  

Again, the generic smallness of $Q^* = Q/W$, which holds for all but the most highly charged black holes, means that $\tau$ is small relative to $L$.

There is no reason to expect the “radius” of the event horizon of a general black hole to be particularly small, and so the proper time of free fall fall can be large even if $\gamma$ is large. But in the case of the black holes we are studying here, we know that $r^E_H$ is indeed small; so the proper time of free fall from event horizon to Cauchy horizon (or whatever replaces it in the perturbed case) must likewise be small, unless $\gamma$ is fine-tuned to be very small. (That is, for a given value of $\gamma$, $\tau$ is unusually small for this variety of black hole, compared to other black holes.)

Notice that, in contrast to the situation considered by Marolf [34], we are not assuming here that the emitted black hole is extremal or even near-extrema. (Near-extremality is measured not by the size of $r^E_H$ but rather by the dimensionless ratio $(r^E_H - r^E_C)/r^E_H$.) Despite this, we arrive at a similar conclusion: for the (classical) toroidal black holes emitted by larger toroidal black holes in accordance with the WGC, the singularity lies “just under” the event horizon: a generic object falling through the event horizon would meet the singularity almost instantly.

The curvature near to the event horizon can be small here, despite the fact that $r^E_H$ is extremely small. Again, this is due to the fact that the event horizon side length, $\Delta^E_H$, is an independent parameter. If it is large, then the Kretschmann invariant at the event...
the geometry at the event horizon differs little from the asymptotic AdS\(_5\) geometry, even if \(r_{E}^{H}\) happens to be small, as it is here. As in Marolf’s case, then, the nearness of the singularity does not mean that the curvature is large just outside the event horizon. There is no classical indication of its proximity: one needs effects like those studied in \[9\] to demonstrate this.

5. Conclusion

Recently it has been argued, from several different points of view (see for example \[37,38\]), that the interiors of black hole event horizons differ very radically from the structure suggested by classical General Relativity. Perhaps the most extreme, yet simple, proposal is that, in some sense, the interior “does not exist.”

The simplest way to approach this idea is to examine the singularities which are thought to replace the Cauchy horizons of charged black holes. If these singularities can be shown to lie just below the event horizon, then one has a simple model of a black hole with essentially “no interior”.

The question then, of course, is how this absence of an interior can be detected from outside the black hole. As we have seen here, the curvature just outside the event horizon is not always sensitive to the presence of unusual behaviour just underneath it. However, the work of Grinberg and Maldacena offers hope that a sufficiently close examination of (for example) the large-mass behaviour of the one-point function of a massive field could detect such behaviour, because it might reveal that the proper time of fall from the event horizon is so short that it is negligible.

Marolf \[34\] showed that this kind of scenario does arise for near-extremal Reissner-Nordström black holes. Here we have shown that the same is true of toroidal black holes emitted, in accordance with the WGC, by larger toroidal black holes, even if the emitted black hole is \textit{not} extremal. Thus, black holes with “no interiors” arise naturally, indeed inevitably, in the WGC context.

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