Four-neutrino MS$^2$ mixing

C. Giunti

INFN, Sezione di Torino, and Dipartimento di Fisica Teorica, Università di Torino,
Via P. Giuria 1, I–10125 Torino, Italy

Abstract

We present a simple scheme of mixing of four neutrinos that can accommodate the results of all neutrino oscillation experiments, the observed abundances of primordial elements and the current upper bound for the effective Majorana neutrino mass in neutrinoless double-$\beta$ decay (assuming that massive neutrinos are Majorana particles). The scheme has maximal mixing in the $\nu_\mu, \nu_\tau - \nu_3, \nu_4$ sector and small mixings in the $\nu_e, \nu_\tau - \nu_1, \nu_2$ and $\nu_e, \nu_\mu - \nu_1, \nu_3$ (or $\nu_e, \nu_\mu - \nu_1, \nu_4$) sectors (MS$^2$). We discuss the implications of this scheme for short and long baseline oscillation experiments and for neutrinoless double-$\beta$ decay and tritium $\beta$-decay experiments.

PACS numbers: 14.60.St

Typeset using REVTeX
I. INTRODUCTION

The Super-Kamiokande measurement of atmospheric neutrino oscillations [1] has launched the physics of massive neutrinos [2] into a new era of model-independent evidence. Hopefully other experiments will provide in the near future further model-independent information on neutrino masses and mixing through the observation of solar neutrinos (SNO, Borexino and others, see [3]), atmospheric and long-baseline accelerator neutrinos (see [4]), short-baseline accelerator neutrinos (see [5]) and reactor neutrinos (see [6]).

At present there are three types of experiments that provide indications in favor of neutrino oscillations: solar neutrino experiments (Homestake, Kamiokande, GALLEX, SAGE and Super-Kamiokande [7,8]), atmospheric neutrino experiments (Kamiokande, IMB, Super-Kamiokande, Soudan-2 and MACRO [9,1]) and the accelerator LSND experiment [10]. In order to accommodate these experimental results at least three independent neutrino mass-squared differences are needed (see [11] for a simple proof):

\begin{align}
\Delta m^2_{\text{sun}} &\sim 10^{-10} \text{eV}^2 \text{ (VO)} \quad \text{or} \quad \Delta m^2_{\text{sun}} \sim 10^{-6} - 10^{-4} \text{eV}^2 \text{ (MSW)}, \\
\Delta m^2_{\text{atm}} &\sim 10^{-3} - 10^{-2} \text{eV}^2, \\
\Delta m^2_{\text{LSND}} &\sim 1 \text{eV}^2.
\end{align}

The two possibilities (see [12]) for the solar mass-squared difference \( \Delta m^2_{\text{sun}} \) correspond, respectively, to the vacuum oscillation (VO) solution and to the MSW effect [13].

Three independent neutrino mass-squared differences require the existence of at least four light massive neutrinos. Here we consider the minimal possibility of four light massive neutrinos [14–20,11] that allows to explain all the data of neutrino oscillation experiments. In this case, in the flavor basis the three active neutrinos \( \nu_e, \nu_\mu, \nu_\tau \) are accompanied by a sterile neutrino \( \nu_s \) that does not take part in standard weak interactions (see [21]).

Further constraints on neutrino mixing can be obtained from the measurement of the primordial abundance of light elements produced in Big-Bang Nucleosynthesis (BBN) (see [22]) and from the results of neutrinoless double-\( \beta \) decay experiments (\( \beta\beta^0\nu \)) (see [23]), assuming that massive neutrinos are Majorana particles.

The analysis of recent astrophysical data yields the upper bound [24]

\[ N^\text{BBN}_\nu \leq 3.2 \quad (95\% \text{ CL}) \]

for the effective number of neutrinos in BBN, although the issue is still rather controversial (see [25]). In the framework of four-neutrino mixing, the upper bound \( N^\text{BBN}_\nu < 4 \) implies that atmospheric neutrino oscillations occur in the \( \nu_\mu \rightarrow \nu_\tau \) channel and solar neutrino oscillations occur in the \( \nu_e \rightarrow \nu_s \) channel [16,19]. In this case only the small mixing angle solution of the solar neutrino problem is allowed, with

\[ 3 \times 10^{-6} \text{eV}^2 \lesssim \Delta m^2_{\text{sun}} \lesssim 8 \times 10^{-6} \text{eV}^2, \]
\[ 10^{-3} \lesssim \sin^2 2\vartheta_{\text{sun}} \lesssim 10^{-2}, \]

at 99\% CL [12]. Here \( \vartheta_{\text{sun}} \) is the two-neutrino mixing angle used in the analysis of solar neutrino data. Solar neutrino experiments (SNO, Borexino and others, see [3]) could prove the dominance of the \( \nu_e \rightarrow \nu_s \) channel in solar neutrino oscillations in the near future,
hopefully in a model-independent way \cite{26}. The dominance of $\nu_\mu \to \nu_\tau$ oscillations for atmospheric neutrinos will be checked by future atmospheric and long-baseline experiments (see \cite{4}). Already the fit of Super-Kamiokande data favors the $\nu_\mu \to \nu_\tau$ with respect to the $\nu_\mu \to \nu_s$ channel \cite{27}.

It is important to notice that in the framework of four-neutrino mixing the BBN bound (1.4) implies that solar and atmospheric neutrino oscillations are effectively decoupled and the two-generation analyses of solar and atmospheric neutrino data yield correct information on the $4 \times 4$ four-neutrino mixing matrix \cite{19,28}.

If massive neutrinos are Majorana particles, the matrix element of $\beta\beta_0$ decay is proportional to the effective Majorana mass

$$\langle m \rangle = \left| \sum_k U_{ek}^2 m_k \right|,$$

where $U$ is the mixing matrix that connects the flavor neutrino fields $\nu_{\alpha L}$ ($\alpha = e, \mu, \tau$) to the fields $\nu_{kL}$ of neutrinos with masses $m_k$ through the relation

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL}.$$

The present experimental upper limit for $\langle m \rangle$ is \cite{29}

$$\langle m \rangle \leq 0.2 - 0.4 \text{ eV}. \quad (1.8)$$

The uncertainty of a factor of two for this upper bound stems from the uncertainty of the theoretical calculation of the nuclear matrix element \cite{30}. The next generation of $\beta\beta_0$ decay experiments is expected to be sensitive to values of $\langle m \rangle$ in the range $10^{-2} - 10^{-1}$ eV \cite{31}. Values of $\langle m \rangle$ as small as about $10^{-3}$ eV may be reached not far in the future \cite{32}.

It has been shown \cite{17,20,11} that only one mass spectrum with four neutrinos can accommodate the results of all neutrino oscillation experiments, the BBN bound \cite{14} and the neutrinoless double-$\beta$ decay limit \cite{18} (assuming that massive neutrinos are Majorana particles):

$$\begin{aligned}
\begin{array}{cc}
\text{sun} & \text{atm} \\
\text{LSND} & \\
\hline
m_1 < m_2 & \ll m_3 < m_4
\end{array}
\end{aligned}. \quad (1.9)$$

This mass spectrum is characterized by having

$$\Delta m^2_{\text{LSND}} = \Delta m^2_{41}, \quad \Delta m^2_{\text{atm}} = \Delta m^2_{43}, \quad \Delta m^2_{\text{sun}} = \Delta m^2_{21}. \quad (1.10)$$

In the framework of the mass spectrum (1.9) the BBN bound (1.4) gives a strong constraint on the mixing of the sterile neutrino with the two “heavy mass” eigenstates $\nu_3$ and $\nu_4$ \cite{19}:

$$|U_{s3}|^2 + |U_{s4}|^2 \lesssim 10^{-5}, \quad (1.11)$$

or $|U_{s3}|^2 + |U_{s4}|^2 \lesssim 10^{-4}$ if the weaker bound $N^\text{BBN}_\nu < 4$ is correct.
In this paper we consider the mass spectrum (1.9) and we present, in Section II, a simple mixing scheme that can accommodate the results of all neutrino oscillation experiments (the solar, atmospheric and LSND indications in favor of neutrino oscillations and the negative results of all the other experiments), the BBN bound (1.4) and the neutrinoless double-$\beta$ decay bound (1.8). In Sections III, IV, V and VI we discuss the predictions of the scheme under consideration for short-baseline oscillation experiments, long-baseline oscillation experiments, neutrinoless double-$\beta$ decay experiments and tritium $\beta$-decay experiments, respectively. Conclusions are drawn in Section VII.

II. THE MS$^2$ MIXING SCHEME

As shown in [19,28], the BBN bound (1.4) implies that the $\nu_e$, $\nu_s$, $\nu_1$, $\nu_2$, $\nu_\mu$, $\nu_\tau$, $\nu_3$, $\nu_4$ sectors of the $4 \times 4$ neutrino mixing matrix are approximately decoupled: in the ($\nu_e$, $\nu_s$, $\nu_\mu$, $\nu_\tau$) basis we have

$$U \simeq \begin{pmatrix} \cos \vartheta_{\text{sun}} & \sin \vartheta_{\text{sun}} & 0 & 0 \\ -\sin \vartheta_{\text{sun}} & \cos \vartheta_{\text{sun}} & 0 & 0 \\ 0 & 0 & \cos \vartheta_{\text{atm}} & \sin \vartheta_{\text{atm}} \\ 0 & 0 & -\sin \vartheta_{\text{atm}} & \cos \vartheta_{\text{atm}} \end{pmatrix}. \quad (2.1)$$

Here $\vartheta_{\text{sun}}$ is the solar mixing angle whose allowed range is given in Eq. (1.5) and $\vartheta_{\text{atm}}$ is the atmospheric mixing angle whose $90\%$ CL allowed range is

$$0.9 \lesssim \sin^2 2\vartheta_{\text{atm}} \leq 1. \quad (2.2)$$

Since the best fit of the Super-Kamiokande data is obtained in the case of maximal mixing, $\vartheta_{\text{atm}} = \pi/4$, and theoretically maximal or small mixing angles are preferred in comparison with intermediate configurations (the large mixing angle in the $\nu_\mu$, $\nu_\tau$, $\nu_3$, $\nu_4$ sector could be related to the almost degeneracy of $\nu_3$ and $\nu_4$, whereas the small mixing angle in the $\nu_e$, $\nu_s$, $\nu_1$, $\nu_2$ sector could be related to the hierarchy $m_1 \ll m_2$), in the following we consider the case of maximal mixing in the $\nu_\mu$, $\nu_\tau$, $\nu_3$, $\nu_4$ sector (corrections to this scheme due to $\vartheta_{\text{atm}} \neq \pi/4$ can be easily computed):

$$U \simeq \begin{pmatrix} \cos \vartheta_{\text{sun}} & \sin \vartheta_{\text{sun}} & 0 & 0 \\ -\sin \vartheta_{\text{sun}} & \cos \vartheta_{\text{sun}} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \quad (2.3)$$

The mixing matrix (2.3) cannot be exact, because it cannot accommodate the $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ oscillations observed in the LSND experiment. Indeed, the probability of $\nu_\alpha \to \nu_\beta$ transitions for neutrinos with energy $E$ in a short-baseline experiment with a source-detector distance $L$ is [17]

$$P^{(\text{SBL})}_{\nu_\alpha \to \nu_\beta} = A_{\alpha\beta} \sin^2 \left( \frac{\Delta m^2_{\text{LSND}} L}{4E} \right), \quad (2.4)$$

with the oscillation amplitude

4
where

\[ A_{\alpha\beta} = 4 \left| \sum_{k=3,4} U_{\alpha k}^* U_{\beta k} \right|^2. \]  

(2.5)

One can easily see that if the mixing matrix \( (2.3) \) were exact the amplitude \( A_{\nu e} \) of short-baseline \( \nu_\mu \rightarrow \nu_e \) oscillations would vanish, in contrast with the LSND result. (The expression \( (2.3) \) implies that \( P^{(\text{SBL})}_{\nu_\alpha \rightarrow \nu_\beta} = P^{(\text{SBL})}_{\nu_\alpha \rightarrow \nu_\beta} \) and CP or T violation effects are not observable in short-baseline experiments independently from the value of the neutrino mixing matrix \( [3] \).)

The simplest way to generalize the mixing matrix \( (2.3) \) in order to accommodate the LSND oscillations is to rotate its \( \nu_e - \nu_\mu \) sector by a small angle. There are four rotations that can be performed in the \( \nu_e, \nu_\mu, \nu_4, \nu_4 \) sectors, with \( i = 1, 2 \) and \( j = 3, 4 \). However, small rotations in the \( \nu_e, \nu_\mu - \nu_2, \nu_3 \) and/or \( \nu_e, \nu_\mu - \nu_2, \nu_4 \) sectors are not effective in generating \( \nu_\mu \rightarrow \nu_e \) transitions because \( \nu_e \) has large mixing with \( \nu_1 \) and small mixing with \( \nu_2 \), in order to accommodate the small mixing angle solution \( (1.3) \) of the solar neutrino problem. Moreover, such rotations generate relatively large elements \( U_{34} \) and/or \( U_{44} \), that are incompatible with the BBN bound \( (1.11) \). Hence, small rotations in the \( \nu_\mu, \nu_4 - \nu_2, \nu_3 \) and/or \( \nu_\mu, \nu_4 - \nu_2, \nu_4 \) sectors are excluded. On the other hand, small rotations in the \( \nu_e, \nu_\mu - \nu_1, \nu_3 \) and/or \( \nu_e, \nu_\mu - \nu_1, \nu_4 \) sectors are effective in generating the \( \nu_\mu \rightarrow \nu_e \) oscillations observed in the LSND experiment and the resulting mixing matrix is compatible with the BBN constraint \( (1.11) \). Unfortunately, if both rotations are allowed the resulting mixing matrix is quite complicated and difficult to handle. Therefore, we assume that one of the mixings in the \( \nu_e, \nu_\mu - \nu_1, \nu_3 \) and/or \( \nu_e, \nu_\mu - \nu_1, \nu_4 \) dominates over the other. Which one dominates is irrelevant for the resulting phenomenology, because the two possibilities give equivalent predictions for neutrino oscillations experiments, neutrinoless double-\( \beta \) decay experiments and tritium \( \beta \)-decay experiments. In the following we will consider explicitly the case of dominance of the mixing in the \( \nu_e, \nu_\mu - \nu_1, \nu_3 \) sector. Hence, we rotate the \( \nu_e, \nu_\mu - \nu_1, \nu_3 \) sector by a small angle \( \sqrt{2} \theta_{\text{LSND}} \) [the coefficient \( \sqrt{2} \) is introduced in order to allow the interpretation of \( \theta_{\text{LSND}} \) as the usual mixing angle measured in the LSND experiment; see Eq. \( (2.3) \)]:

\[
U = \begin{pmatrix}
\cos \theta_{\text{sun}} & \sin \theta_{\text{sun}} & 0 & 0 \\
-\sin \theta_{\text{sun}} & \cos \theta_{\text{sun}} & 0 & 0 \\
0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\
0 & 0 & -1/\sqrt{2} & 1/\sqrt{2}
\end{pmatrix}
\begin{pmatrix}
\cos \sqrt{2} \theta_{\text{LSND}} & 0 & \sin \sqrt{2} \theta_{\text{LSND}} & 0 \\
0 & 1 & 0 & 0 \\
-\sin \sqrt{2} \theta_{\text{LSND}} & 0 & \cos \sqrt{2} \theta_{\text{LSND}} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
\]

(2.6)

that yields

\[
U = \begin{pmatrix}
\cos \theta_{\text{sun}} \cos \sqrt{2} \theta_{\text{LSND}} & \sin \theta_{\text{sun}} \sin \sqrt{2} \theta_{\text{LSND}} & 0 & 0 \\
-\sin \theta_{\text{sun}} \cos \sqrt{2} \theta_{\text{LSND}} & \cos \theta_{\text{sun}} \sin \sqrt{2} \theta_{\text{LSND}} & 0 & 0 \\
-\sin \sqrt{2} \theta_{\text{LSND}} / \sqrt{2} & 0 & \cos \sqrt{2} \theta_{\text{LSND}} / \sqrt{2} & 1/\sqrt{2} \\
\sin \sqrt{2} \theta_{\text{LSND}} / \sqrt{2} & 0 & -\cos \sqrt{2} \theta_{\text{LSND}} / \sqrt{2} & 1/\sqrt{2}
\end{pmatrix}.
\]

(2.7)

We call MS\(^2\) the mixing scheme represented by this matrix, which has maximal mixing in the \( \nu_\mu, \nu_\mu - \nu_3, \nu_4 \) sector and small mixings in the \( \nu_e, \nu_\mu - \nu_1, \nu_2 \) and \( \nu_e, \nu_\mu - \nu_1, \nu_3 \) sectors. For simplicity, we did not introduce any CP-violating phase or different CP parities for the massive neutrinos (see [4]), because there is no information at present on these quantities. Hence, in the following we will not make any distinction between the oscillations of neutrinos and antineutrinos.
The amplitude of short-baseline $\nu_\mu \rightarrow \nu_e$ oscillations that follows from the MS$^2$ mixing matrix (2.7) is

$$A_{\mu e} = \frac{1}{2} \cos^2 \vartheta_{\odot} \sin^2 2\sqrt{2} \vartheta_{\mathrm{LSND}}. \quad (2.8)$$

Taking into account the fact that the mixings $\nu_e, \nu_s - \nu_1, \nu_2$ and $\nu_e, \nu_\mu - \nu_1, \nu_3$ sectors are small ($\cos^2 \vartheta_{\odot} \simeq 1, \sin^2 2\sqrt{2} \vartheta_{\mathrm{LSND}} \simeq 8 \vartheta_{\mathrm{LSND}}^2$), we have

$$A_{\mu e} \simeq 4 \vartheta_{\mathrm{LSND}}^2 \simeq \sin^2 2 \vartheta_{\mathrm{LSND}}. \quad (2.9)$$

Hence, $\vartheta_{\mathrm{LSND}}$ can be identified with the mixing angle measured in the LSND experiment. From the results of the LSND experiment it follows that the preferred range for $\sin^2 2 \vartheta_{\mathrm{LSND}}$ (90% likelihood region) is

$$2 \times 10^{-3} \lesssim \sin^2 2 \vartheta_{\mathrm{LSND}} \lesssim 4 \times 10^{-2}. \quad (2.10)$$

The MS$^2$ mixing matrix is compatible with the BBN bound (1.11). Indeed, in the MS$^2$ mixing scheme we have

$$|U_{s3}|^2 + |U_{s4}|^2 = \sin^2 \vartheta_{\odot} \sin^2 2\sqrt{2} \vartheta_{\mathrm{LSND}} \simeq \frac{1}{8} \sin^2 2 \vartheta_{\odot} \sin^2 2 \vartheta_{\mathrm{LSND}}, \quad (2.11)$$

that is doubly suppressed by the smallness of $\sin^2 \vartheta_{\odot}$ and $\sin^2 2 \vartheta_{\mathrm{LSND}}$. From the limits (1.5) and (2.10) we obtain

$$2 \times 10^{-7} \lesssim |U_{s3}|^2 + |U_{s4}|^2 \lesssim 5 \times 10^{-5}, \quad (2.12)$$

that is well compatible with the bound (1.11).

Let us finish this section by emphasizing that the MS$^2$ mixing matrix (2.7) is a valid approximation of the real neutrino mixing matrix if the mixing in the $\nu_e, \nu_\mu - \nu_1, \nu_3$ sector dominates over the mixing in the $\nu_e, \nu_\mu - \nu_1, \nu_4$ sector. Both mixings are allowed by the data. The opposite situation in which the mixing in the $\nu_e, \nu_\mu - \nu_1, \nu_4$ sector dominates over the mixing in the $\nu_e, \nu_\mu - \nu_1, \nu_3$ sector produces a mixing matrix that can be obtained from the one in Eq. (2.7) by exchanging the third and fourth columns and leads to the same phenomenology.

### III. SHORT-BASELINE EXPERIMENTS

The MS$^2$ mixing scheme provides precise predictions for the amplitudes of $\nu_\mu \rightarrow \nu_\tau, \nu_e \rightarrow \nu_\tau, \nu_e \rightarrow \nu_s, \nu_\mu \rightarrow \nu_s$ and $\nu_\tau \rightarrow \nu_s$ in short-baseline experiments:

$$A_{\mu \tau} = \sin^4 \sqrt{2} \vartheta_{\mathrm{LSND}} \simeq \frac{1}{4} A_{\mu e}^2, \quad (3.1)$$

$$A_{e \tau} = \frac{1}{2} \cos^2 \vartheta_{\odot} \sin^2 2\sqrt{2} \vartheta_{\mathrm{LSND}} = A_{\mu e}, \quad (3.2)$$

$$A_{\mu s} = A_{\tau s} = \frac{1}{2} \sin^2 \vartheta_{\odot} \sin^2 2\sqrt{2} \vartheta_{\mathrm{LSND}} \simeq \frac{1}{4} \sin^2 2 \vartheta_{\odot} \sin^2 2 \vartheta_{\mathrm{LSND}}, \quad (3.3)$$

$$A_{e s} = \sin^2 2 \vartheta_{\odot} \sin^4 \sqrt{2} \vartheta_{\mathrm{LSND}} \simeq A_{\mu e} A_{\mu s}. \quad (3.4)$$
The approximations in Eqs. (3.1) and (3.4) are valid because of \( \vartheta_{\text{sun}} \ll 1 \) and \( \vartheta_{\text{LSND}} \ll 1 \). Notice that the amplitudes \( A_{\mu\tau} \) and \( A_{\mu s} = A_{\tau s} \) are quadratically suppressed by the smallness of \( \sin^2 2\vartheta_{\text{sun}} \) and \( \sin^2 2\vartheta_{\text{LSND}} \) and the amplitude \( A_{\epsilon s} \) is even cubically suppressed. Hence the oscillations in the corresponding channels will be very difficult to observe. On the other hand, the equality of the amplitudes \( A_{e\tau} \) and \( A_{\mu e} \), that are suppressed only linearly by the smallness of \( \sin^2 2\vartheta_{\text{LSND}} \), could be checked in a not too far future, perhaps with neutrino beams produced in muon storage rings \([33]\). Let us emphasize that (obviously) in the MS\(^2\) mixing scheme (as well as in any four-neutrino mixing scheme compatible with the present neutrino oscillation data) short-baseline oscillations in all channels are generated by the same mass-squared difference \( \Delta m_{\text{LSND}}^2 \) and this prediction can and must be checked experimentally.

The survival probability of \( \nu_\alpha \)s in short baseline experiments is given by \([17]\)

\[
P^{(\text{SBL})}_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - B_{\alpha\alpha} \sin^2 \left( \frac{\Delta m_{\text{LSND}}^2 L}{4E} \right),
\]

with the oscillation amplitudes

\[
B_{\alpha\alpha} = \sum_{\beta \neq \alpha} A_{\alpha\beta}.
\]

Since in the MS\(^2\) mixing scheme \( A_{e\tau} = A_{\mu e} \) and \( A_{\epsilon s} \) is negligible, for the survival probability of \( \nu_e \)s in short-baseline experiments we have

\[
B_{ee} = 2 A_{\mu e}.
\]

In general the oscillation amplitude \( A_{\mu e} \) is constrained by the unitarity inequality \( A_{\mu e} \leq B_{ee} \) and this bound has been used in order to restrict the LSND-preferred region in the \( A_{\mu e} - \Delta m_{\text{LSND}}^2 \) plane \([10]\) using the exclusion curve obtained in the Bugey \([34]\) \( \nu_e \) disappearance experiment. From Eq. (3.7) it follows that in the MS\(^2\) scheme the constraint on \( A_{\mu e} \) coming from the Bugey exclusion curve is stronger:

\[
A_{\mu e} \leq \frac{1}{2} B_{ee}^{\text{Bugey}},
\]

where \( B_{ee}^{\text{Bugey}} \) is the upper limit for \( B_{ee} \) obtained in the Bugey experiment \( (B_{ee} \) coincides with the parameter \( \sin^2 2\vartheta \) used in the two-generation analysis of the Bugey data). It is clear that this stronger bound is due to the fact that in the MS\(^2\) mixing scheme \( \nu_e \rightarrow \nu_\mu \) and \( \nu_e \rightarrow \nu_\tau \) transitions contribute equally to the disappearance of \( \nu_e \)s in short-baseline experiments.

The LSND favored region in the \( \sin^2 2\vartheta_{\text{LSND}} - \Delta m_{\text{LSND}}^2 \) plane (90\% likelihood region) \([10]\) that takes into account the constraint (3.8) is shown in Fig. 1 as the dark shadowed region [remember that \( \sin^2 2\vartheta_{\text{LSND}} \) is practically equivalent to \( A_{\mu e} \); see Eq. (2.9)]. The thin solid line represents the bound (3.8), whereas the thick solid line represents the general bound \( A_{\mu e} \leq B_{ee}^{\text{Bugey}} \) and the light plus dark shadowed areas represent the usual LSND favored region \([10]\). In the MS\(^2\) mixing scheme the preferred range (2.10) for \( \sin^2 2\vartheta_{\text{LSND}} \) must be restricted to

\[
2 \times 10^{-3} \lesssim \sin^2 2\vartheta_{\text{LSND}} \lesssim 2 \times 10^{-2}.
\]
Let us consider now short-baseline $\nu_\mu \to \nu_\tau$ oscillations. The region in the $\sin^2 2\theta_{\mu\tau} - \Delta m^2_{\text{LSND}}$ plane obtained from the LSND favored region in Fig. 1 through Eq. (3.1) is shown in Fig. 2 as the shadowed area. Here $\sin^2 2\theta_{\mu\tau} = A_{\mu\tau}$ is the amplitude of $\nu_\mu \to \nu_\tau$ oscillations measured in short-baseline experiment, i.e. it coincides with the parameter $\sin^2 2\theta$ used in the usual two-generation analyses of the data of these experiments. The thin solid line in Fig. 2 represents the upper bound for $\sin^2 2\theta_{\mu\tau}$ obtained in [19] for the mass spectrum (1.9) assuming the validity of the BBN bound $N_{\text{BBN}}^\nu < 4$. The thick solid line on the right in Fig. 2 shows the final sensitivity of the CHORUS and NOMAD experiments [35]. One can see that unfortunately the sensitivity region of the CHORUS and NOMAD experiments is rather far from the region predicted in the MS$^2$ mixing scheme and, if this scheme is correct, it will be very difficult to reveal $\nu_\mu \to \nu_\tau$ oscillations in short-baseline experiments.

IV. LONG-BASELINE EXPERIMENTS

The probability of $\nu_\alpha \to \nu_\beta$ transitions in vacuum in long-baseline (LBL) experiments is given by [18]

$$P^{(\text{LBL})}_{\nu_\alpha \to \nu_\beta} = \left| \sum_{k=1,2} U_{\alpha k}^* U_{\beta k} \right|^2 + \left| U_{\alpha 3}^* U_{\beta 3} + U_{\alpha 4}^* U_{\beta 4} \exp \left( -i \frac{\Delta m^2_{\text{atm}} L}{2E} \right) \right|^2 ,$$  \hspace{1cm} (4.1)

where $L$ is the source-detector distance and $E$ is the neutrino energy. For the different channels we obtain

$$P^{(\text{LBL})}_{\nu_\mu \to \nu_e} = P^{(\text{LBL})}_{\nu_e \to \nu_\tau} = \frac{1}{4} \cos^2 \vartheta_{\text{sun}} \sin^2 2\sqrt{2} \vartheta_{\text{LSND}} \simeq \frac{1}{2} \sin^2 2\vartheta_{\text{LSND}} ,$$

$$P^{(\text{LBL})}_{\nu_\mu \to \nu_\tau} = \frac{1}{2} \left[ 1 - \cos^2 \sqrt{2} \vartheta_{\text{LSND}} \cos \left( \frac{\Delta m^2_{\text{atm}} L}{2E} \right) - \frac{1}{4} \sin^2 2\sqrt{2} \vartheta_{\text{LSND}} \right] \simeq \sin^2 \left( \frac{\Delta m^2_{\text{atm}} L}{4E} \right) \left[ 1 - \frac{1}{2} \sin^2 2\vartheta_{\text{LSND}} \right] ,$$  \hspace{1cm} (4.3)

$$P^{(\text{LBL})}_{\nu_\mu \to \nu_\nu} = P^{(\text{LBL})}_{\nu_\tau \to \nu_\nu} = \frac{1}{4} \sin^2 \vartheta_{\text{sun}} \sin^2 2\sqrt{2} \vartheta_{\text{LSND}} \simeq \frac{1}{8} \sin^2 2\vartheta_{\text{LSND}} \sin^2 2\vartheta_{\text{LSND}} ,$$

$$P^{(\text{LBL})}_{\nu_e \to \nu_\nu} = \frac{1}{2} \sin^2 2\vartheta_{\text{sun}} \sin^4 \sqrt{2} \vartheta_{\text{LSND}} \simeq 2 P^{(\text{LBL})}_{\nu_\mu \to \nu_e} P^{(\text{LBL})}_{\nu_\mu \to \nu_\nu} .$$  \hspace{1cm} (4.5)

As expected, the probability of $\nu_\mu \to \nu_\tau$ oscillations is given with a good approximation by the standard two-generation formula in the case of maximal mixing. This is the only probability that is predicted to have an oscillatory behavior as a function of $E/L$ in long baseline experiments. The corrections to these oscillation probabilities due to matter effects will be discussed elsewhere [36].

From Eq. (1.2) one can see that the probability of $\nu_\mu \to \nu_e$ oscillations in long-baseline experiments is given by the average of the oscillation probability measured in the LSND experiment. Hence, the two-generation mixing angle used in the analyses of long-baseline $\nu_\mu \to \nu_e$ experiments coincides with the LSND mixing angle and has the allowed range given in Eq. (3.3). MINOS, ICARUS and other experiments [16] will be sensitive to this range of the mixing angle.
In the framework of the MS\textsuperscript{2} scheme the probability of $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\tau$ long-baseline oscillations are equal [as the corresponding probabilities in short-baseline experiments, see Eq. (3.2)]. This equality could be checked by future experiments with neutrino beams produced in muon storage rings \cite{33}.

\section{V. NEUTRINOLESS DOUBLE-$\beta$ DECAY}

Let us consider now neutrinoless double-$\beta$ decay. In the MS\textsuperscript{2} mixing scheme the effective Majorana mass is given by

$$|\langle m \rangle| = \cos^2 \vartheta_{\text{sun}} \cos^2 \sqrt{2} \vartheta_{\text{LSND}} m_1 + \sin^2 \vartheta_{\text{sun}} m_2 + \cos^2 \vartheta_{\text{sun}} \sin^2 \sqrt{2} \vartheta_{\text{LSND}} m_3. \quad (5.1)$$

Assuming that the contribution of $m_1$ is negligible ($m_1$ could even be zero), since the contribution of $m_2$ is suppressed by the small $\sin^2 \vartheta_{\text{sun}}$ factor, the dominant contribution is given by $m_3 \simeq \sqrt{\Delta m^2_{\text{LSND}}}$:

$$|\langle m \rangle| \simeq \cos^2 \vartheta_{\text{sun}} \sin^2 \sqrt{2} \vartheta_{\text{LSND}} m_3 \simeq \frac{1}{2} \sin^2 2 \vartheta_{\text{LSND}} \sqrt{\Delta m^2_{\text{LSND}}}. \quad (5.2)$$

Hence, the MS\textsuperscript{2} mixing scheme predicts a connection between the value of the effective neutrino mass in $\beta\beta_{0\nu}$ decay and the quantities $\sin^2 2 \vartheta_{\text{LSND}}$ and $\Delta m^2_{\text{LSND}}$ measured in short-baseline $\nu_\mu \rightarrow \nu_e$ oscillation experiments.

The region in the $\Delta m^2_{\text{LSND}}$–$|\langle m \rangle|$ plane obtained from the LSND favored region in Fig. 1 through Eq. (5.2) is shown in Fig. 3 as the dark shadowed area. This figure shows that the predicted range for the effective Majorana mass in $\beta\beta_{0\nu}$ decay is

$$1.3 \times 10^{-3} \text{ eV} \lesssim |\langle m \rangle| \lesssim 7 \times 10^{-3} \text{ eV}. \quad (5.3)$$

Such small values of $|\langle m \rangle|$ may be reached by $\beta\beta_{0\nu}$ decay experiments in a not too far future \cite{32}.

The dark shadowed region in Fig. 3 obtained in the MS\textsuperscript{2} mixing scheme is more restrictive than the light plus dark shadowed region in the same figure that has been obtained in \cite{11} from the LSND favored region in a general scheme with the mass spectrum (1.9) under the natural assumption that massive neutrinos are Majorana particles and there are no unlikely fine-tuned cancellations among the contributions of the different neutrino masses. Indeed, in the MS\textsuperscript{2} mixing scheme there is no cancellation among the contributions of the different neutrino masses, because the dominant contribution is given by $m_3$. The thin solid line in Fig. 3 represents the upper bound for $|\langle m \rangle|$ derived in \cite{37} for the mass spectrum (1.9) and the thick solid line represents the unitarity limit $|\langle m \rangle| \leq \sqrt{\Delta m^2_{\text{LSND}}}$. Notice that the prediction for the effective Majorana mass in $\beta\beta_{0\nu}$ decay in the MS\textsuperscript{2} mixing scheme is independent from a possible introduction of CP-violating phases or of different CP parities of the mass eigenstates, because there is only one dominant contribution coming from $m_3$. 

9
VI. TRITIUM $\beta$-DECAY EXPERIMENTS

In the case of neutrino mixing, the Kurie function in tritium $\beta$-decay experiments \cite{38} is given by \cite{39}

$$K(T) = \sqrt{Q - T} \left[ \sum_k |U_{ek}|^2 \sqrt{(Q - T)^2 - m_k^2} \right]^{1/2}. \tag{6.1}$$

In the MS$^2$ mixing scheme we have

$$K(T) = \sqrt{Q - T} \left[ \cos^2 \vartheta_{\text{sun}} \cos^2 \sqrt{2} \vartheta_{\text{LSND}} \sqrt{(Q - T)^2 - m_1^2} + \sin^2 \vartheta_{\text{sun}} \sqrt{(Q - T)^2 - m_2^2} + \cos^2 \vartheta_{\text{sun}} \sin^2 \sqrt{2} \vartheta_{\text{LSND}} \sqrt{(Q - T)^2 - m_3^2} \right]^{1/2}. \tag{6.2}$$

The maximum deviation of this Kurie function from the massless one $K_0(T) = Q - T$ is obtained for $T = Q - m_3$ and is given by

$$\Delta K_{\text{max}} \simeq \frac{1}{4} \sin^2 2 \vartheta_{\text{LSND}} m_3. \tag{6.3}$$

For example, the maximum value of $\sin^2 2 \vartheta_{\text{LSND}}$ allowed by the LSND favored region in Fig. \ref{fig:1} for $m_3 \simeq \sqrt{\Delta m^2_{\text{LSND}}} = 1$ eV is $9 \times 10^{-3}$, that yields

$$\Delta K_{\text{max}}(m_3 = 1 \text{ eV}) \simeq 2 \times 10^{-3} \text{ eV}. \tag{6.4}$$

The possibility to measure such small deviations is well beyond the present technology. Hence, if the MS$^2$ mixing scheme is correct, tritium $\beta$-decay experiments \cite{38} will not observe any effect of neutrino mass in any foreseeable future. This obviously means that these important experiments have the potentiality to rule out the MS$^2$ mixing scheme by measuring a positive effect.

VII. CONCLUSIONS

We have presented the MS$^2$ neutrino mixing scheme, that is a simple scheme of mixing of four neutrinos that can accommodate the results of all neutrino oscillation experiments, (the solar, atmospheric and LSND indications in favor of neutrino oscillations and the negative results of all the other experiments), the measurement of the primordial abundance of light elements produced in Big-Bang Nucleosynthesis and the current upper bound for the effective Majorana neutrino mass in neutrinoless double-$\beta$ decay. The MS$^2$ mixing scheme has maximal mixing in the $\nu_\mu, \nu_\tau - \nu_3, \nu_4$ sector and small mixings in the $\nu_e, \nu_\mu - \nu_1, \nu_2$ and $\nu_e, \nu_\mu - \nu_1, \nu_3$ (or $\nu_e, \nu_\mu - \nu_1, \nu_4$) sectors. This scheme follows naturally from the experimental information, with the only assumption that the mixing in the $\nu_e, \nu_\mu - \nu_1, \nu_4$ dominates over the mixing in the $\nu_e, \nu_\mu - \nu_1, \nu_3$ sector (or vice-versa, see the discussion in Section \ref{sec:II}).
As we have shown, the MS$^2$ mixing scheme has rather precise predictions for the oscillation probabilities in short-baseline and long-baseline experiments, for neutrinoless double-$\beta$ decay and for tritium $\beta$-decay experiments. Hence, it is rather easily falsified by future experiments, if wrong. On the other hand, its confirmation is difficult because it predicts small signals for new oscillation channels, for neutrinoless double-$\beta$ decay and in tritium $\beta$-decay experiments. The prediction that probably can be verified in the near future is the energy-independence of long-baseline $\nu_\mu \rightarrow \nu_e$ transitions with a probability given by the average of the oscillation probability measured in the LSND experiment [see Eq. (4.2)]. Other predictions that could be confirmed in a not too far future are the equality of the probabilities of $\nu_e \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations in both short-baseline and long-baseline experiments [see Eqs. (3.2) and (4.2)] and the range (5.3) for the effective Majorana neutrino mass in neutrinoless double-$\beta$. 
REFERENCES

[1] Y. Fukuda et al., Phys. Rev. Lett. 81, 1562 (1998), hep-ex/9807003; ibid 82, 2644 (1999), hep-ex/9812014; A. Habig (Super-Kamiokande Coll.), hep-ex/9903047; K. Scholberg (Super-Kamiokande Coll.), hep-ex/9905016.

[2] See: S.M. Bilenky and B. Pontecorvo, Phys. Rep. 41, 225 (1978); S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59, 671 (1987); C.W. Kim and A. Pevsner, Neutrinos in Physics and Astrophysics, Contemporary Concepts in Physics, Vol. 8, Harwood Academic Press, Chur, Switzerland, 1993; K. Zuber, Phys. Rep. 305, 295 (1999), hep-ph/9811267; S.M. Bilenky, C. Giunti and W. Grimus, hep-ph/9812360; G. Raffelt, hep-ph/9902271; E. Torrente-Lujan, hep-ph/9902339; A.B. Balantekin and W.C. Haxton, nucl-th/9903038; W.C. Haxton and B.R. Holstein, hep-ph/9905257; P. Fisher, B. Kayser, K.S. McFarland, hep-ph/9906244; R.D. Peccei, hep-ph/9906509.

[3] G. Jonkmans (SNO Coll.), Nucl. Phys. B (Proc. Suppl.) 70, 329 (1999); F. von Feilitzsch, ibid 70, 292 (1999); R.E. Lanou, hep-ex/9808033.

[4] M. Campanelli, hep-ex/9905035; P. Picchi and F. Pietropaolo, hep-ph/9812222; K. Zuber, hep-ex/9810022; P. Adamson et al., MINOS Coll., NuMI-L-476 (March 1999); P. Cennini et al., ICARUS Coll., LNGS-94/99-I (1994).

[5] Booster Neutrino Experiment (BooNE), http://nu1.lampf.lanl.gov/BooNE; I-216 νµ → νe proposal at CERN, http://chorus01.cern.ch/~pzucchel/loi/; Oak Ridge Large Neutrino Detector (ORLaND), http://www.phys.subr.edu/orland/; Neutrinos at the European Spallation Source (NESS), http://www.isis.rl.ac.uk/ess/neut%5Fess.htm.

[6] G. Gratta, hep-ex/9905011.

[7] B.T. Cleveland et al., Homestake Coll., Astrophys. J. 496, 505 (1998); K.S. Hirata et al., Kamiokande Coll., Phys. Rev. Lett. 77, 1683 (1996); W. Hampel et al., GALLEX Coll., Phys. Lett. B 447, 127 (1999); D.N. Abdurashitov et al., SAGE Coll., Phys. Rev. Lett. 77, 4708 (1996).

[8] Y. Fukuda, Phys. Rev. Lett. 81, 1158 (1998), Erratum ibid 81, 4279 (1998), hep-ex/9805021; ibid 82, 2430 (1999), hep-ex/9812011; M.B. Smy, hep-ex/9903034.

[9] Y. Fukuda et al., Kamiokande Coll., Phys. Lett. B 335, 237 (1994); R. Becker-Szendy et al., IMB Coll., Nucl. Phys. B (Proc. Suppl.) 38, 331 (1995); W.W.M. Allison et al., Soudan Coll., Phys. Lett. B 449, 137 (1999); M. Ambrosio et al., MACRO Coll., Phys. Lett. B 434, 451 (1998).

[10] C. Athanassopoulos et al., Phys. Rev. Lett. 75, 2650 (1995); ibid 77, 3082 (1996); ibid 81, 1774 (1998).

[11] C. Giunti, hep-ph/9906275.

[12] J.N. Bahcall, P.I. Krastev and A.Yu. Smirnov, Phys. Rev. D 58, 096016 (1998), hep-ph/9807216; Y. Fukuda et al., Phys. Rev. Lett. 82, 1810 (1999), hep-ex/9812009; V. Barger and K. Whisnant, Phys. Lett. B 456, 54 (1999), hep-ph/9903262; M.C. Gonzalez-Garcia et al., hep-ph/9906469.

[13] S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. 42, 1441 (1985) [Sov. J. Nucl. Phys. 42, 913 (1985)]; Il Nuovo Cimento C 9, 17 (1986); L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); ibid. 20, 2634 (1979).

[14] J.T. Peltoniemi, D. Tommasini and J.W.F. Valle, Phys. Lett. B 298, 383 (1993); E.J. Chun et al., ibid 357, 608 (1995); S.C. Gibbons et al., ibid 430, 296 (1998); B. Brah-
machari and R.N. Mohapatra, *ibid* 437, 100 (1998); S. Mohanty, D.P. Roy and U. Sarkar, *ibid* 445, 185 (1998); J.T. Peltoniemi and J.W.F. Valle, Nucl. Phys. B 406, 409 (1993); Q.Y. Liu and A.Yu. Smirnov, *ibid* 524, 505 (1998); D.O. Caldwell and R.N. Mohapatra, Phys. Rev. D 48, 3259 (1993); E. Ma and P. Roy, *ibid* 52, R4780 (1995); S. Goswami, *ibid* 55, 2931 (1997); A.Yu. Smirnov and M. Tanimoto, *ibid* 55, 1665 (1997); N. Gaur *et al.*, *ibid* 58, 071301 (1998); E.J. Chun, C.W. Kim and U.W. Lee, *ibid* 58, 093003 (1998); K. Benakli and A.Yu. Smirnov, Phys. Rev. Lett. 79, 4314 (1997); Y. Chikira, N. Haba and Y. Mimura, hep-ph/9808254; C. Liu and J. Song, hep-ph/9812381; W. Grimus, R. Pfeiffer and T. Schwetz, hep-ph/9905320.

[15] J.J. Gomez-Cadenas and M.C. Gonzalez-Garcia, Z. Phys. C 71, 443 (1996), hep-ph/9504246; S.M. Bilenky, C. Giunti, C.W. Kim and S.T. Petcov, Phys. Rev. D 54, 4432 (1996), hep-ph/9604364; S.M. Bilenky, C. Giunti and W. Grimus, *ibid* 58, 033001 (1998), hep-ph/9712537; V. Barger, Y.B. Dai, K. Whisnant and B.L. Young, *ibid* 59, 113010 (1999), hep-ph/9901385; V. Barger, T.J. Weiler and K. Whisnant, Phys. Lett. B 427, 97 (1998), hep-ph/9712493.

[16] N. Okada and O. Yasuda, Int. J. Mod. Phys. A 12, 3669 (1997), hep-ph/9606411.

[17] S.M. Bilenky, C. Giunti and W. Grimus, Proc. of Neutrino ’96, Helsinki, June 1996, edited by K. Enqvist *et al.*, p. 174, World Scientific, 1997, hep-ph/9609343; Eur. Phys. J. C 1, 247 (1998), hep-ph/9607372; S.M. Bilenky, C. Giunti, W. Grimus and T. Schwetz, hep-ph/9903454 [to be published in Phys. Rev. D].

[18] S.M. Bilenky, C. Giunti and W. Grimus, Phys. Rev. D 57, 1920 (1998), hep-ph/9710209.

[19] S.M. Bilenky, C. Giunti, W. Grimus and T. Schwetz, hep-ph/9804421 [to be published in Astropart. Phys.].

[20] V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, Phys. Rev. D 58, 093016 (1998), hep-ph/9806328.

[21] A. Yu. Smirnov, hep-ph/9901208; R.N. Mohapatra, hep-ph/9903261.

[22] D.N. Schramm and M.S. Turner, Rev. Mod. Phys. 70, 303 (1998), astro-ph/9706069.

[23] M. Moe and P. Vogel, Annu. Rev. Nucl. Part. Sci. 44, 247 (1994); A. Morales, hep-ph/9809540. Talk presented at Neutrino ’98, Takayama, Japan, 4-9 June 1998.

[24] S. Burles, K.M. Nollett, J.N. Truran and M.S. Turner, Phys. Rev. Lett. 82, 4176 (1999), astro-ph/9901157.

[25] K.A. Olive, astro-ph/9903308; E. Lisi, S. Sarkar and F.L. Villante, Phys. Rev. D 59, 123520 (1999), hep-ph/9914044; S. Sarkar, astro-ph/9903183.

[26] S.M. Bilenky and C. Giunti, Phys. Lett. B 320, 323 (1994), hep-ph/9310314; Astrop. Phys. 2, 353 (1994), hep-ph/9403343; Z. Phys. C 68, 495 (1995), hep-ph/9502263.

[27] J. Learned, Talk presented at the 23rd Johns Hopkins Workshop on Current Problems in Particle Theory “Neutrinos in the Next Millennium”, Baltimore, 10–12 June 1999.

[28] S.M. Bilenky, C. Giunti, W. Grimus and T. Schwetz, hep-ph/9807569. Talk presented at the Ringberg Euroconference “New Trends in Neutrino Physics”, 24–29 May 1998; in *New Era in Neutrino Physics*, p. 179, edited by H. Minakata and O. Yasuda, Universal Academy Press, 1999, hep-ph/9809466.

[29] L. Baudis *et al.*, hep-ex/9902014.

[30] A. Staudt, K. Muto and H.V. Klapdor-Kleingrothaus, Europhys. Lett. 13, 31 (1990); F. Simkovic *et al.*, Phys. Lett. B 393, 267 (1997).

[31] C.E. Aalseth *et al.*, Nucl. Phys. B (Proc. Suppl.) 70, 236 (1999); X. Sarazin (NEMO
Coll.), *ibid.* 70, 239 (1999); V.D. Ashitkov et al., *ibid.* 70, 233 (1999); E. Fiorini, Phys. Rep. 307, 309 (1998); A. Alessandrello et al., in *New Era in Neutrino Physics*, p. 271, edited by H. Minakata and O. Yasuda, Universal Academy Press, 1999.

[32] J. Hellmig and H.V. Klapdor-Kleingrothaus, Z. Phys. A 359, 351 (1997); H.V. Klapdor-Kleingrothaus and M. Hirsch, Z. Phys. A 359, 361 (1997); H.V. Klapdor-Kleingrothaus, J. Hellmig and M. Hirsch, J. Phys. G 24, 483 (1998).

[33] S. Geer, Phys. Rev. D 57, 6989 (1998), Erratum *ibid.* 59, 039903 (1999), hep-ph/9712290; R.N. Mohapatra, hep-ph/9711444; B. Autin et al., CERN-SPSC/98-30; A. De Rujula, M.B. Gavela and P. Hernandez, Nucl. Phys. B 547, 21 (1999), hep-ph/9811390; S. Dutta, R. Gandhi and B. Mukhopadhyaya, hep-ph/9905473; V. Barger, S. Geer and K. Whisnant, hep-ph/9906487.

[34] B. Achkar et al., Nucl. Phys. B 434, 503 (1995).

[35] CHORUS Coll., hep-ex/9807024; J. Altegoer et al., NOMAD Coll., Phys. Lett. B 431, 219 (1998).

[36] C. Giunti, work in progress.

[37] S.M. Bilenky, C. Giunti, C.W. Kim and M. Monteno, Phys. Rev. D 57, 6981 (1998), hep-ph/9711400; S.M. Bilenky, C. Giunti and W. Grimus, hep-ph/9809368. Talk presented at Neutrino ’98, Takayama, Japan, 4-9 June 1998; S.M. Bilenky and C. Giunti, hep-ph/9904328. Talk presented at WIN99, Cape Town, South Africa, 24-30 January 1999.

[38] A.I. Belesiev, Phys. Lett. B 350, 263 (1995); H. Barth et al., Prog. Part. Nucl. Phys. 40, 353 (1998).

[39] R.E. Shrock, Phys. Lett. B 96, 159 (1980); I.Yu. Kobzarev et al., Yad. Fiz. 32, 1590 (1980) [Sov. J. Nucl. Phys. 32, 823 (1980)].
FIG. 1. Dark shadowed area: LSND favored region in the $\sin^2 2\theta_{\text{LSND}} - \Delta m^2_{\text{LSND}}$ plane (90% likelihood region) that takes into account the constraint (3.3) in the MS$^2$ scheme. The thin solid line represents the bound (3.8), whereas the thick solid line represents the exclusion curve of the Bugey experiment [34] and the light plus dark shadowed areas represent the usual LSND favored region [10].
FIG. 2. Shadowed area: the allowed region in the \( \sin^2 2\theta_{\mu\tau} - \Delta m^2_{\text{LSND}} \) plane in the MS\(^2\) scheme. The thin solid line represents the upper bound for \( \sin^2 2\theta_{\mu\tau} \) that follows from the BBN bound \( N^\text{BBN}_\nu < 4 \). The thick solid line on the right shows the final sensitivity of the CHORUS and NOMAD experiments. 

16
FIG. 3. Dark shadowed area: the allowed region in the $\Delta m^2_{\text{LSND}}-|\langle m \rangle|$ plane in the MS$^2$ scheme. The light plus dark shadowed region is allowed in a general scheme with the mass spectrum (1.9) under the natural assumption that massive neutrinos are Majorana particles and there are no unlikely fine-tuned cancellations among the contributions of the different neutrino masses [11]. The thin solid line represents the upper bound for $|\langle m \rangle|$ following from the mass spectrum (1.9) without additional assumptions [37] and the thick solid line represents the unitarity limit $|\langle m \rangle| \leq \sqrt{\Delta m^2_{\text{LSND}}}$. 