Random Access to Grammar Compressed Strings

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Random Access to Compressed Strings

text
DNA
XML
Random Access to Compressed Strings

• What is the $i$th character?

• What is the substring at $[i,j]$?

• Does pattern $P$ appear in text? (perhaps with $k$ errors?)
Random Access to Grammar Compressed Strings

AGTAGTAG  \( N = 8 \)

- Grammar based compression captures many popular compression schemes with no or little blowup in space [Charikar et al. 2002, Rytter 2003].
- Lempel-Ziv family, Sequitur, Run-Length Encoding, Re-Pair, ...

\[ X_7 \rightarrow X_6 X_3 \]
\[ X_6 \rightarrow X_5 X_5 \]
\[ X_5 \rightarrow X_3 X_4 \]
\[ X_4 \rightarrow T \]
\[ X_3 \rightarrow X_1 X_2 \]
\[ X_2 \rightarrow G \]
\[ X_1 \rightarrow A \]

\( n = 7 \)

\( \leq n \)

\( N \)

1  2  3  4  5  6  7  8
Tradeoffs and Results

- What is the $i$th character?
  - O(N) space
  - O(1) query

- What is the substring at $[i,j]$?
  - O(n) space
  - O(log N + j - i) query
Application: Black-Box Compressed String Matching

• Does “AGGA” appear in the text (perhaps with k errors)?
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Extension: Compressed Trees

- Linear space in compressed tree.
- Fast navigation operations (select, access, parent, depth, height, subtree_size, first_child, next_sibling, level_ancestor, nca).
Heavy Path Decomposition
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Random Access Query

• The path from root to $i$ goes through $O(\log N)$ heavy paths

• Query: Binary search all heavy paths on the way
  
  $O(\log n) \cdot O(\log N)$
Random Access Query

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  \[ O(\log n) \cdot O(\log N) \]
The path from root to $i$ goes through $O(\log N)$ heavy paths.

Query: Binary search all heavy paths on the way

$O(\log n) \cdot O(\log N) = O(\log N/x)$ Telescopes to $O(\log N)$

Space: Each IBSTs uses linear space => total $O(n^2)$ space for all heavy paths.
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$O(n^{\alpha(n)})$
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- $O(n)$ with bittricks
Summary

• Random access and substring decompression.
  • $O(n)$ space and $O(\log N + \text{length of substring})$ time.

• Black compressed (approximate) string matching.

• Random access in compressed trees.