RESEARCH PAPER

Modeling of electromagnetic fields of pipelines cathodic protection systems in horizontally layered medium

by Vladimir N. Krizsky*, Pavel N. Aleksandrov 2, Alexey A. Kovalskii 1, Sergey V. Viktorov 1

1 Sterlitamak Branch of Bashkir State University, Sterlitamak, Russian Federation
2 Centre for Geoelectromagnetic Research – Branch of Schmidt Institute of Physics of the Earth of the Russian Academy of Sciences, Moscow, Russian Federation

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ABSTRACT

Design of cathodic protection systems of the trunk pipeline is regulated by current standards, based on the condition of uniformity and constancy of the electric conductivity of the multilayered half-space surrounding the pipeline. The current mathematical models of such systems also use an average value of the medium electric conductivity, which does not fully reflect the actual characteristics of the soil, in which the pipeline is laid. The authors present a method that accounts for the thickness and electrical conductivity of individual beds in a vertically-inhomogeneous, horizontally layered medium (the most practically appropriate case). Using method of computational experiment, the authors showed the importance of accounting for the effect of the medium layers structure and electrical resistivity on the protective voltage of the electric current in the cathodic protection system for underground trunk pipeline and studied the magnetic field sensitivity dependence on the insulation resistance of the pipeline defect-containing segments and on the altitude of data acquisition.

Key words: piecewise homogeneous space, horizontally layered space, cathodic protection, mathematic modeling, electromagnetic field.

INTRODUCTION

Corrosion protection of pipelines is provided by a combination of passive (insulation coatings) and active (electrochemical) protection. Affected by various factors (mechanical damage when laying pipes, freezing and thawing of the soil, shear deformations of the soil, etc.), insulation coating defects occur, which in combination with an aggressive environment activate the process of metal corrosion. To obtain data on the state of the insulation coating, the pipeline right-of-way is monitored using unmanned aerial vehicles (UAV) that host the diagnostic instrumentation. Currently, UAV overflights involving photo and video shooting, thermal-imaging or gas analysis of facilities are carried out in order to detect leaks. At that, it is a challenging task to design hardware and software systems for UAV airborne magnetic measurements that enable detecting stress-strained sections of the pipeline and sections with a critical state of the insulation coating thus preventing possible incidents. The analytical component of such systems is the software designed on the basis of adequate mathematical models, up-to-date numerical methods and high-performance computation algorithms.

The mathematical model of the problem to determine the insulation resistance of a cathodic-polarized underground pipeline is an inverse boundary problem of mathematical physics, where the sought-for function is the coefficient function of the third-order boundary condition at the ‘soil-pipe’ boundary. The complex underground distribution of electric currents generated by the cathodic protection station (CPS) is caused by the spatial geometry of the system of interdependent pipelines and anode earthing.
devices, and by the spatial distribution of the electrical and magnetic properties of the surrounding soil. The search for a solution to the inverse problem as an extremal of the regularizing functional of A. N. Tikhonov is based on the fast and acceptable in accuracy computer generation of a set of solutions to direct problems based on adequate mathematical models of the studied fields [1, 2].

The paper deals with the direct problem of electric and magnetic fields distribution that occur in electrochemical cathodic protection systems of trunk pipelines. GOST standards and guidelines of pipeline companies regulate the parameters calculating of cathodic protection system to the trunk pipeline, based on the condition of uniformity and constancy of the electric conductivity of the half-space surrounding the pipeline. Up-to-date mathematical models of such systems [3, 4] also rely on this assumption, using an average value of the electric conductivity of the soil, which does not fully reflect the actual characteristics of the soil, in which the pipeline is laid. The authors consider the most appropriate case: the presented calculation method considers the thickness and electric conductivity of layers in a vertically inhomogeneous, horizontally layered medium.

Mathematical model of the electric field

Let there be given a horizontally layered piecewise homogeneous space separated by flat boundaries $y_i (i = 0, N - 1)$ into horizontal beds $\Omega_y, \ldots, \Omega_y$, where $\Omega_0$ means air, $\Omega_1, \ldots, \Omega_n$ mean soil beds (Fig. 1). The physical properties of the substances that fill the areas $\Omega_y, \ldots, \Omega_n$, are described by constant electrical conductivities $\sigma_i = \text{const}$ (S/m) $\left( i = 0, N \right)$. The electrical conductivity of the air is assumed to be zero ($\sigma_0 = 0$ S/m).

Let the pipeline $l_i$ (extended circular cylindrical cathode grounding electrode) be located in the layer $\Omega_y$. The geometry of the pipe axis is described by a parametric line $l_i(x_1(s), y_1(s), z_1(s))$, $s \in [0, L_i]$, where $L_i$ – pipe length (m). Values of $s$ parameter equal to 0 and $L_i$ correspond to the points with no electric current through them along the pipe. Radius $R_p$ pipe metal electric conductivity $\sigma_m$ and metal cross-section area $s_m$ of the pipe are known.

The pipeline is serviced by $N_{\text{CPS}}$ (ea.) cathode stations, each of which injects a protective direct electric current into the soil via the anode grounded – the extended anodes $I_{an}^k (n = 1, N_{\text{an}}^k)$ and/or point anodes $A_{an}^k (x_{an}, y_{an}, z_{an}) (n = 1, N_{an}^k)$ ($k$ – CPS number).

The axial lines of extended anode grounding electrodes are described parametrically – $l_i^a(x(s), y(s), z(s))$, $s \in [0, L_{an}^k]$ (here $L_{an}^k$ means the length of $n$th extended anode of $k$th CPS). Current with amperage $I_{an}^k$ is supplied to the extended anodes in the points $A_{an}^k (x_{an}, y_{an}, z_{an})$. Radii $R_{an}^k$ of extended circular cylindrical anode grounding electrodes, electric conductivity of their metal rods $\sigma_{an}^k$, metal cross-section area $s_{an}^k$, electric conductivity of anode cushioning layer $\sigma_{cush}^k (s) = \rho_{cush}^k = 1/\sigma_{cush}^k$ are known. Currents with amperage $I_{an}^k$ are supplied into the system via point anodes located in the points $A_{an}^k (x_{an}, y_{an}, z_{an}) (n = 1, N_{an}^k)$.

Electric current with amperage $I_{an}^k$ sinks from the pipeline surface to the $k$th CPS in points $B_i^k (x_i^k, y_i^k, z_i^k), k = 1, N_{\text{CPS}}$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure1.png}
\caption{Principle of the pipeline cathodic protection in a horizontally layered medium.}
\end{figure}
The current amperage generated by $k^{th}$ CPS is known and is equal to $I_0 \, A$. For parallel energized anode grounding electrodes, the Kirchhoff rule shall be used:

$$I_0 = \sum_{i=1}^{n_{Ea}} I_{Ea}^{i} + \sum_{i=1}^{n_{Pa}} I_{Pa}^{i} = -\left( I_{Pa}^{i} + I_{Ea}^{i} \right).$$

Let's select a system of Cartesian coordinates $(x, y, z)$ with a datum point on the ‘daylight’ surface (air-soil surface) and 0, axis directed downward. The potential function at a point $P = (x, y, z)$ of sub-area $\Omega$ shall be denoted by $U(P)$ - in the pipe, $U_a^{i}(P)$ - in the extended anode grounding electrode.

The mathematical model of the direct electric current field in the system is as follows:

$$\nabla \cdot (\sigma_i \nabla U_i(P)) = -\sum_{i=1}^{N} \sum_{i=1}^{N} \delta_i \cdot \delta_j \cdot I_{Pa}^{i} \cdot (P - A_{Pa}^{j}),$$

$$P \in \Omega, \quad \chi = \frac{1}{k}, N_{s}, \delta_i = \begin{cases} 1, A_{Pa}^{i} \in \Omega_i, \\ 0, A_{Pa}^{i} \notin \Omega_i \end{cases}. \tag{1}$$

$$\left( \sigma \nabla U_i, \nabla U_{i, P} \right) \gamma^0 \cdot 0 = \left( U_i, \gamma^i \right), \quad \gamma \cdot \gamma = U_{i, Pa}^{i} \tag{2}$$

$$\left( \sigma \nabla U_i, \nabla U_{i, P} \right) \gamma^i = \left( \sigma_{i, n}, \nabla U_{i, n} \right), \quad \chi = 1, N_{s} - 1; \tag{3}$$

$$\sigma_{\alpha, \varepsilon} \frac{\partial U_i}{\partial I_i} \biggl|_{i = 0}, L_i = 0, U_i - c_{\alpha, \varepsilon}(s) \left( \sigma \nabla U_i, \nabla U_{i, P} \right) \biggl|_{s_{\alpha}, i} = U_i; \tag{4}$$

$$U_{\varepsilon, a}^{i} + c_{\alpha, \varepsilon}(s) \left( \sigma_{\alpha, \varepsilon} \nabla U_{\varepsilon, a}^{i}, \nabla U_{\varepsilon, a}^{i} \right) \biggl|_{s_{\alpha}} = U_{a}^{i}, \quad k = 1, N_{s}, n = 1, N_{s} \tag{5}$$

$$\left[ \frac{\partial U_i}{\partial s} \right] \biggl|_{B_i} = \frac{I_{Ea}^{i}}{\sigma_{\alpha, \varepsilon}} \biggl|_{s_{\alpha}}, \quad k = 1, N_{s}, n = 1, N_{s} \tag{6}$$

$$U_i(P) \rightarrow 0, P \rightarrow \infty, i = 1, N_{s}. \tag{7}$$

Where:

1. equations of electric current propagation in the soil
2. equations of current in the pipe metal and in the extended anode grounding electrodes
3. interface conditions at the contact boundaries of soil layers
4. the boundary conditions of electric current at the $s_{\alpha}$ ‘soil-pipe’ boundary, where the factor function $c_{\alpha, \varepsilon}(s)$ is the resistance of the pipe insulation
5. describe the electric current conditions at the ‘extended anode – soil’ boundary $s_{\alpha, \varepsilon}$, where the factor function $c_{\alpha, \varepsilon}(s)$ is the electric resistance of the anode cushioning layer
6. conditions for connecting the cathode station to pipelines and to the extended anode grounding electrodes
7. the regularity condition of the solution at infinity.

The solution of the problem (1) – (7) can be obtained using the method of dummy sources [3, 4]. Let’s break down the pipeline $I_i$ into $M_i$ equal segments, and each extended anode grounding electrode $I_{Ea}^{i}$ into $M_{Ea}^{i}$ equal segments, assuming that for each of them the considered electrical parameters are constant and equal to a certain average value (its own for each segment).

Each segment on the extended anodes will be considered as a dummy point source of current, and each segment of the pipe – as a current sink point. Let’s take the coordinates of the segments geometric centers as the coordinates of electric current dummy sources $A_{Ea}^{i}$ / sinks $B_{Ea}^{i}$. Let’s also assume that the coordinates of the electric current supply points $A_{Ea}^{i}$ to anode grounding electrode, and current sink points $B_{Ea}^{i}$ from the surface of the pipe coincide with coordinates of one of the introduced dummy sources /sinks.

For each dummy source on an extended anode grounding electrode, let’s consider the following averaged values of unknown variables: $U_{Ea}^{i}$ – voltage in the anode metal rod; $U_{Ea}^{i}$ – voltage on the ‘extended anode-soil’ boundary; $I_{Ea}^{i}$ – amperage of electric current sinking into the ground via the side surface of the anode; $I_{Ea}^{i}$ – amperage of axial current in the anode metal between adjacent dummy sources.

For each dummy sink at the pipeline, let’s also consider the average values of unknown variables: $U_{Pa}^{i}$ – voltage in the pipe metal; $U_{Pa}^{i}$ – voltage on the ‘soil-pipe’ boundary; $I_{Pa}^{i}$ – amperage of electric current ingress from the soil via the side surface of the pipe; $I_{Pa}^{i}$ – amperage of axial current in the anode metal between adjacent dummy sinks.

Let’s write down the Kirchhoff laws for each current source/sink, describing inflow/outflow electric currents:

$$I_{Ea}^{i} = I_{Pa}^{i} + I_{Pa}^{j} + I_{Pa}^{i} - I_{Pa}^{j} - \delta \left( B_{Ea}^{i}, B_{Ea}^{j} \right) = 0 \tag{8}$$

$$I_{Pa}^{i} = I_{Pa}^{i, 1} + I_{Pa}^{j} - I_{Pa}^{j} - \delta \left( B_{Pa}^{i}, B_{Pa}^{j} \right) = 0 \tag{9}$$
For electric currents along extended anodes and pipeline, let’s write down discrete analogues of the Ohm’s law between all pairs of adjacent dummy sources/sinks:

\[
U_{\text{in},i} - U_{\text{in},j} = -R_i I_{\text{in},i} \left( i = 1, M_a - 1, n = 1, N_a, k = 1, N_c \right),
\]

(10)

\[
U_{\text{m},i} - U_{\text{m},j} = -R_m I_{\text{m},j} \left( j = 1, M_t - 1 \right),
\]

(11)

Where:

- \( R_i \) – axial resistance of the anode grounding electrodes metal between adjacent dummy sources;
- \( R_m \) – axial resistance of the pipe metal between adjacent dummy sinks.

For each section \( S_{\text{ag},i} \) of the ‘extended anode-soil’ boundary, corresponding to \( i \) \( \left( i = 1, M_a \right) \) dummy source,

\[
U_{\text{ag},i} + c_{\text{ag},i} I_{\text{ag},i} / S_{\text{ag},i} = U_{\text{in},i},
\]

(12)

\[
(i = 1, M_a - 1, n = 1, N_a, k = 1, N_c)
\]

Where:

- \( c_{\text{ag},i} \) – average value of the transition electric resistance of the anode grounding electrode cushioning layer corresponding to a dummy source \( \left( i = 1, M_a \right) \);
- \( S_{\text{ag},i} \) – area of the external surface of the anode grounding conductor section corresponding to one dummy source.

Similarly, for each section \( S_{\text{ag},j} \) of the ‘soil-pipe’ boundary corresponding to \( j \) \( \left( j = 1, M_t \right) \) dummy sink, we obtain the discrete analogue of the boundary conditions of the third kind (4):

\[
U_{\text{ag},j} - c_{\text{ag},j} I_{\text{ag},j} / S_{\text{ag},j} = U_{\text{m},j},
\]

(13)

where:

- \( c_{\text{ag},j} \) – the average value of the pipe insulation resistance corresponding to the dummy sink \( \left( j = 1, M_t \right) \);
- \( S_{\text{ag},j} \) – the external surface area of the pipe section corresponding to one dummy sink.

The values of the direct current (DC) voltage at any point in the enclosing horizontally layered half-space are given by the formula:

\[
U(P) = \sum_{k=1}^{N_p} \sum_{n=1}^{N_{ag}} I_{\text{ag},n} G(P, A_{\text{ag},n}) + \sum_{k=1}^{N_p} \sum_{n=1}^{N_{ag}} \sum_{j=1}^{M_t} I_{\text{ag},n} G(P, B_{\text{m},j}).
\]

(14)

Here, \( G(P, Q) \) is the Green’s function of the enclosing medium that enables to determine the voltage value at a point \( P(x_p, y_p, z_p) \) in the soil, provided that the unit-intensity electric current source is located at the point \( Q(x_q, y_q, z_q) \) of the plane-parallel horizontally layered medium. The function \( G(P, Q) \) is [2]:

\[
G(P, Q) = \int_{\alpha} g(\alpha, z_p, z_q) \cdot \alpha \cdot J_0 \left( \alpha \sqrt{(x_p - x_q)^2 + (y_p - y_q)^2} \right) d\alpha,
\]

(15)

where:

- \( g(\alpha, z_p, z_q) \) is defined by recurrent formulas [5];
- \( J_0(\alpha) \) – the zeroth-order Bessel function.

Lowering the point \( P \) in the formula (14) on the boundaries

\[
S_{\text{ag},n} \left( n = 1, N_{ag} \right) \rightarrow ‘\text{anode grounding electrode-soil’}
\]

and \( S_{\text{g},j} \) – ‘soil-pipe’, we can calculate the voltage values:

\[
U_{\text{ag},n} G(P, A_{\text{ag},n}) = \sum_{k=1}^{N_p} \sum_{n=1}^{N_{ag}} \sum_{j=1}^{M_t} I_{\text{ag},n} G(P, B_{\text{m},j}),
\]

(16)
Equations (8) – (13), (16) – (17) – represent a system of linear algebraic equations for unknown parameters of electric current dummy sources and sinks

\[ U_{in}^{m}, U_{ek}^{m}, I_{ag}^{m}, I_{aq}^{m}, U_{m}, I_{aq}^{m}, I_{aq}^{m}, U_{aq}^{m}, I_{aq}^{m}, I_{aq}^{m}, \text{with dimension} \]

\[ \sum_{k=1}^{N} \sum_{m=1}^{N} (4M_n^{2} - 1) \left( 4M_i^{2} - 1 \right). \]

By defining these variables, it is possible to calculate the operation parameters of electrochemical protection that are of interest:

- protection voltage \( U_{pr, i} = U_{aq, i} - U_{aq, i} \) (17)
- density of current via a side surface of the pipe:
  \[ j_{aq, i} = j_{aq, i} / n_{aq, i} \] (18)
- density of current via a side surface of the anode grounding electrodes:
  \[ j_{aq, i}^{m} = I_{aq}^{m} / S_{aq}^{m} \] (19)
- density of current in metal core of the extended anode grounding electrodes:
  \[ j_{aq, i}^{m} = I_{aq}^{m} / S_{aq}^{m} \] (20)

The Ohm’s law in differential form

\[ j_{aq, i} \left( P \right) = - \sigma_{aq} \nabla U \left( P \right), P \in \Omega_{aq} \]

enables using formula (14) to find the electric current density at any point in the pipe metal and in the horizontally layered half-space enclosing the pipeline. Here, definition of the vector \( \nabla U \left( P \right) \) components is reduced to definition of the vector \( \nabla_{P} G \left( P, Q \right) \) using the formula (15).

### Magnetic field at cathodic protection of the pipeline

Upon the availability of sedimentary horizontally layered rocks that in most cases are present in the areas of pipelines construction, regarding the small difference between relative magnetic permeability of soil layers and relative magnetic permeability of air \( \mu \), the medium enclosing the pipeline can be considered as homogeneous in terms of magnetic properties. Then, the magnetic induction vector \( B(\mathbf{r}) \) at any spatial point defined by the radius-vector \( \mathbf{r} \) (including the air in the zone above the pipeline) can be calculated using the Biot-Savart-Laplace formula [6] and known DC density \( j(r) \) distribution in the vector field domain:

\[ B(\mathbf{r}) = \frac{\mu_{0}}{4\pi} \int_{V} j(r) dV, \]

where \[ j(r) dV, \mathbf{r} - \mathbf{r} \] – vector product; \( \mu_{0} = 4\pi \times 10^{-7} \text{H/m} \) – magnetic constant.

In the integration area \( V \) of the subdomain with high (relative to soil) modulus values of the electric current density vector in metal, two zones shall be highlighted: \( V_{a} \) – pipeline area and \( V_{aq} \) – areas of extended anode grounding electrodes.

### Results of computation experiments

The computation method described above is implemented in software. Options of the parameter’s values in the performed computation experiments are shown in the Table.

Parameters of the pipe and CPS are the same for all options. In options 2 and 3, the half-space surrounding the pipeline is piecewise homogeneous, flat-parallel, horizontally layered with layer lower boundaries 0.2; 0.4; 5.0; ∞ m and with electric conductivities 40 (black soil), 150 (wet sand, loam), 60 (wet loam) and 500 (gravel + clay) Ohm·m. The half-space surrounding the pipeline in option 1 is homogeneous with average medium electric conductivity. Averaging was performed using the data in option 2 and formula \[ \rho = \sum_{i} \rho_{i} / \sum_{i} \] down to 5 m depth and was equal to 62.8 Ohm·m. In options 1 and 2, the pipe is broken down into 99 segments, in option 3 – into 1355. In options 1 and 2, the segment #65 with its center coordinate \( x = 10593.6 \) is taken as defect-containing; in option 3 – the nearest to that segment #883 with its center coordinate \( x = 10590.0 \).

Comparison of the calculated current distribution parameters along the pipeline route for all data options is shown in Fig. 2, where the number of the computational experiment data option corresponds to the curve’s number indicated in the legend. Here, the electric resistance of insulation on the defect-containing segment was equal to 5874.24 Ohm·m², or to 40% of the insulation resistance (basic value) \( c_{aq} \) of the rest pipe segments.

There was a significant difference for options 1 and 2 in the curves for the current density through the side surface of the pipe, and for the protective voltage in areas close to CPS and on the defect-containing segment (Figs. 2a and 2c, respectively). A significant difference in the protective voltage values (the maximum difference by modulus was 0.227 V) enables to conclude that it is necessary to take into account the structure of the medium (its layering) and the electric resistivity of the lower soil layers (usually not defined by soil surveys using four-electrode symmetrical Schlumberger units, especially in the presence of high-resistivity underlying rocks, such as permafrost, as well as using deep anode grounding electrodes).

Increasing the number of dummy current sinks on the pipeline (the number of pipeline segments) refines the calculation results (comparison of curves 2 and 3 in Fig. 2),
| Parameter                                      | Parameter value |
|-----------------------------------------------|-----------------|
| **Pipeline**                                  |                 |
| Length of protected section $L_t$, m          | 16260.0         |
| Outer diameter $D_t$, m                       | 0.53            |
| Wall thickness $h_{wt}$, mm                    | 8.0             |
| Electric conductivity of steel $\sigma_{sw}$, Ohm·m | $2.45 \cdot 10^{-7}$ |
| Pipeline burial depth (centerline) $h_t$, m    | 1.7             |
| Insulation resistance (basic value) $c_{gr}$, Ohm·m$^2$ | 14685.6         |
| **Enclosing medium**                          |                 |
| Number of layers (including air), ea.         | 2               |
| Lower boundary of layers, m                   | $z_0 = 0.0$     |
|                                               | $z_1 = 2.0$     |
|                                               | $z_2 = 0.4$     |
|                                               | $z_3 = 9.0$     |
|                                               | $z_4 = \infty$  |
| Electric conductivity of layer, S/m           | $\sigma_0 = 0.0000$ |
|                                               | $\sigma_1 = 0.01592$ |
| **Cathodic protection units**                 |                 |
| Number of service stations, ea.               | 2               |
| **CPS 1**                                     |                 |
| CPS work current, A                           | 1.2             |
| Coordinates of the sink point $(x_t, y_t, z_t)$, m | (4819.0; 0.0; 1.435) |
| Coordinates of the point anode grounding electrode $(x_{pa}, y_{pa}, z_{pa})$, m | (4819.0; 240.0; 22.5) |
| **CPS 2**                                     |                 |
| CPS work current, A                           | 0.8             |
| Coordinates of the sink point $(x_t, y_t, z_t)$, m | (13477.0; 0.0; 1.435) |
| Coordinates of the point anode grounding electrode $(x_{pa}, y_{pa}, z_{pa})$, m | (13477.0; 345.0; 22.5) |
| **Pipeline break-down parameters**            |                 |
| Number of segments, ea.                       | 99              |
| Section length, m                             | 164.24          |

*Table. Computation experiments data.*
leads to a decrease in the value of the protective voltage on the defect-containing section by reducing the length of the pipe segment and, consequently, its surface area, but at the same time increases the processor running time.

Fig. 3 shows the curves of the electric current voltage and the electric current density modulus on the ‘daylight surface’ – the air-ground boundary (Fig. 3a and 3b, respectively). Calculations were performed for the case of a layered medium (Table, option 2). A defect-containing section with the insulation resistance of 5874.24 Ohm·m² (40% of the basic value) is indicated by a peak spike in the field data.

To calculate the components of the magnetic induction vector \( B(\mathbf{r}_0) \) (Fig. 4 a-c) using the Biot-Savart-Laplace formula (18), the magnetic induction vector modulus (Fig. 4 d), and the gradient of the magnetic induction vector modulus with respects to the variable \( x \) (Fig. 4 e), the parallelepiped \([0…16260; -50…50; 0…22\,\text{m}]\) with faces parallel to the coordinate axes is taken as the integration domain \( v \), where a uniform in \( OX, OY \), and non-uniform in \( OZ \) grid area is formed (with an increasing increment in the last layer) with segmentation \([99; 20; 42]\) along the axes, respectively.

The given surfaces for the magnetic field are calculated using the data of option 2 (see Table) with insulation resistance of the defect-containing segment equal to 5874.24 Ohm·m² (40% of basic value).

The measured components of the magnetic induction vector (for example, using the fluxgate three-component magnetometer), contrary to the measurements of the vector modulus only, contain additional information about the test object, which can be used to control the UAV during the data acquisition flight.

Thus, the \( B_x \) component indicates the position of the sink points and the defect-containing section on the pipeline (Fig. 4 a). The sign change of the component \( B_x \) indicates a change in the current direction via the pipe: either because of crossing the sink point (in case of high-amplitude variations and the sign change from ‘+’ to ‘–’), or because of a transit to the next CPS reach (in case of a smooth transition through zero and the sign change from ‘–’ to ‘+’). The pipeline route can be zoned according to the \( B_x \) component sign.

Deviation of the UAV flight path from the pipeline route (from the projection of the pipe center line on the ‘daylight’
Figure 4. Magnetic field surfaces and its characteristics at the altitude of 10 m: a) x-coordinate of the magnetic induction vector (nT); b) y-coordinate of the magnetic induction vector (nT); c) z-coordinate of the magnetic induction vector (nT); d) the magnetic induction vector modulus (nT); e) gradient (with respect to x) of the magnetic induction vector modulus (nT/m); f) ‘impedance’ relationship.

Findings

A mathematical model is presented for the DC electric
| Insulation resistance of the defect-containing segment (% of the basic value) | Data options |
|-------------------------------------------------|-------------|
| **Option 2** | **Option 3** |
| 0 % | ![Graph](image1) | ![Graph](image2) |
| 20 % | ![Graph](image3) | ![Graph](image4) |
| 40 % | ![Graph](image5) | ![Graph](image6) |
| 60 % | ![Graph](image7) | ![Graph](image8) |
| 80 % | ![Graph](image9) | ![Graph](image10) |

*Figure 5. The x-gradient of the magnetic induction vector modulus over the defect-containing section versus the altitude of UAVs data acquisition flight and insulation resistance.*
field distribution in the cathodic protection system of a trunk pipeline located in a horizontally layered piecewise homogeneous medium. The magnetic induction vector field over the pipeline with one insulation defect-containing section is plotted and analyzed using the estimated distribution of the electric current volume density in the pipe and soil.

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Competing interests
The authors declare that there is no competing interest regarding the publication of this paper.

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