Mass Bounds for the Neutral Higgs Bosons in the Next–To–Minimal Supersymmetric Standard Model

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Abstract
In the Next–To–Minimal Supersymmetric Standard Model (NMSSM), the Higgs and neutralino/chargino sectors are strongly correlated by four common parameters at tree level. Therefore we analyze the experimental data from both the search for Higgs bosons as well as for neutralinos and charginos at LEP 100 in order to constrain the parameter space and the masses of the neutral Higgs particles in the NMSSM. We find that small singlet vacuum expectation values are ruled out, but a massless neutral Higgs scalar and pseudoscalar is not excluded for most of the parameter space of the NMSSM. Improved limits from the neutralino/chargino search at LEP 200, however, may lead to nonvanishing lower Higgs mass bounds.

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1 Introduction

The search for Higgs bosons is one of the most exciting challenges at the present and future high energy colliders. Suppose that a Higgs particle will be discovered then the next question to be answered is whether it belongs to the Standard Model (SM) or to a model with an enlarged Higgs sector.

Supersymmetric models are the most attractive candidates for such extended models. The Minimal Supersymmetric Standard Model (MSSM), which technically solves the hierarchy problem of the SM, contains two Higgs doublets with vacuum expectation values \( v_1 \) and \( v_2 \) (\( \tan \beta = v_2/v_1 \)). So the Higgs sector consists of five physical Higgs bosons, two CP-even and one CP-odd neutral scalars, and a pair of charged scalars. There is only a weak connection with the neutralino/chargino sector by one common parameter \( \tan \beta \).

In the context of superstring and GUT theories \([1, 2, 3]\) often the simplest extension of the MSSM by a Higgs singlet superfield is favored. This Next-To-Minimal Supersymmetric Standard Model (NMSSM) may also provide a natural solution for the \( \mu \)-problem of the MSSM \([4]\). It contains five physical neutral Higgs bosons, three Higgs scalars \( S_a \) (\( a = 1, 2, 3 \)) and two pseudoscalars \( P_b \) (\( b = 1, 2 \)) \([5, 6]\). As a further crucial difference to the MSSM, the Higgs- and neutralino/chargino sectors of the NMSSM are strongly correlated: Once the parameters of the Higgs sector are fixed, also the masses and mixings of the neutralinos and charginos are determined by only one further parameter, the gaugino mass \( M \).

For both the SM and MSSM lower limits for the Higgs masses have been derived from the LEP experiments. For a SM Higgs boson, there exists a lower mass bound of 63.5 GeV \([7]\). In the MSSM, the situation is more complex due to the larger particle content. Here the lower mass bound is 44 GeV for the lightest scalar Higgs and 21 GeV for the pseudoscalar Higgs particle \([8]\).

The purpose of this letter is to derive Higgs mass bounds and to constrain the parameter space of the NMSSM by the results of the LEP experiments. Due to the strong correlation between the Higgs and neutralino sectors, constraints originate not only from the unsuccessful direct Higgs search via \( e^+e^- \rightarrow ZZ S_a \), \( S_aP_b \) and the contribution \( Z \rightarrow S_aP_b \) to the total \( Z \)-width, but also from the unsuccessful neutralino and chargino search. We therefore make use of the LEP constraints for the neutralino sector of the NMSSM previously obtained in ref. \([9]\).

In our analysis the one-loop radiative corrections of the Higgs masses due to stop and top loops are included \([10]\). With respect to CPU time we did not take into account corrections from Higgs and Higgssino loops which, however, are not expected to change our results significantly.

The Higgs mass spectrum of the NMSSM has been the topic of several previous studies. Kim et al. \([11]\) analyzed the masses of the scalar and pseudoscalar Higgs particles at tree level and obtained excluded regions for certain of the NMSSM parameters by assuming a rough discovery limit of 10 fb for the process \( e^+e^- \rightarrow b\bar{b} S_1 \). Elliott et al. \([10]\) computed the radiative corrections and presented the scalar and pseudoscalar masses in dependence of the mass of the charged Higgs for some parameters. But both authors did not address the question in detail which parameters and masses are already excluded by the present experimental results from LEP. Ellwanger et al. \([12]\) studied the particle spectrum of the NMSSM under the assumption of universal soft symmetry breaking terms at the GUT scale. They included the LEP bound for the \( ZZ S_a \) coupling, but did not discuss neither...
the parameter dependence of the Higgs masses nor the constraints of the parameter space.

This letter is organized as follows: In Section 2 we briefly describe the Higgs-Sector of the NMSSM. With the experimental constraints described in Sec.3, we analyze in Sec.4 the restrictions for the parameter space of the NMSSM and discuss the resulting Higgs mass bounds and their dependence on the model parameters.

## 2 The Higgs sector of the NMSSM

The superpotential of the NMSSM is given by

\[
W = \lambda \varepsilon_{ij} H_1^i H_2^j N - \frac{1}{3} k N^3 \\
+ h_U \varepsilon_{ij} \tilde{Q}^i \tilde{U} H_2^j - h_D \varepsilon_{ij} \tilde{Q}^i \tilde{D} H_1^j - h_E \varepsilon_{ij} \tilde{L}^i \tilde{R} H_1^j ,
\]

(1)

where \( H_1 = (H_1^0, H^-) \) and \( H_2 = (H^+, H_2^0) \) are the \( SU(2) \) Higgs doublets with hypercharge \(-1/2\) and \(1/2\), respectively, \( N \) is the Higgs singlet with hypercharge 0, and \( \varepsilon_{ij} \) is totally antisymmetric with \( \varepsilon_{12} = -\varepsilon_{21} = 1 \). The notation of the squark/slepton doublets and singlets is conventional, generation indices are understood. In the following we neglect all quark and lepton couplings apart from that of the top quark.

The superpotential leads to the tree-level Higgs potential at the supersymmetric mass scale

\[
V_{\text{Higgs}} = \frac{g^2 + g'^2}{8} (|H_1|^4 + |H_2|^4) - \frac{g^2 + g'^2}{4} |H_1|^2 |H_2|^2 \\
+ \frac{g^2}{2} |H_1^i H_2^j|^2 + \lambda^2 [(|H_1|^2 + |H_2|^2)|N|^2 + |\varepsilon_{ij} H_1^i H_2^j|^2] \\
+ k^2 |N|^4 - \lambda k (\varepsilon_{ij} H_1^i H_2^j N + \text{h.c.}) + V_{\text{soft}},
\]

(2)

where \( g', g \) are the usual \( U(1) \) and \( SU(2) \) gauge couplings, and the soft symmetry breaking potential is

\[
V_{\text{soft}} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 |N|^2 \\
+ m_Q^2 |\tilde{Q}|^2 + m_U^2 |\tilde{U}|^2 + m_D^2 |\tilde{D}|^2 \\
+ m_L^2 |\tilde{L}|^2 + m_E^2 |\tilde{E}|^2 \\
- (\lambda A_\lambda \varepsilon_{ij} H_1^i H_2^j N + \text{h.c.}) - \frac{1}{3} k A_k N^3 + \text{h.c.} \\
+ (h_t A_t \varepsilon_{ij} \tilde{Q}^i \tilde{U} H_2^j + \text{h.c.}) .
\]

(3)

After eliminating the ”soft” Higgs masses \( m_1, m_2, m_3 \) by minimizing the Higgs potential with respect to the vacuum expectation values the Higgs sector can be parametrized in terms of six free parameters: the ratio \( \tan \beta = v_2/v_1 \) of the vacuum expectation values of the Higgs doublets, the vacuum expectation value \( x \) of the Higgs singlet, the trilinear couplings in the superpotential \( \lambda \) and \( k \) and the ”soft” scalar masses \( A_\lambda \) and \( A_k \). In addition, the radiative corrections to the scalar potential due to top and stop loops \([10]\) depend on the ”soft” top mass \( A_t \) and the mass eigenvalues of the scalar top quarks \( m_{\tilde{t}_1} \) and \( m_{\tilde{t}_2} \), so that in total there are nine parameters which determine the masses and couplings of the five neutral Higgs bosons.
Decomposing the Higgs fields into their real and imaginary parts, the $3 \times 3$ mass matrices for the scalar and pseudoscalar Higgses decouple with one of the CP-odd eigenstates being a massless Goldstone boson.

Due to theoretical considerations we restrict the parameter range as follows \cite{1,10,13}:

1. As a sufficient condition for no explicit CP violation in the scalar sector we choose $\lambda, k, A_\lambda, A_k$ to be real and positive.

2. The vacuum state can be chosen such that $v_1, v_2, x > 0$.

3. Assuming that perturbative physics remains valid up to the unification scale we bound the couplings $\lambda$ and $k$ by their fixed-points

   $$\lambda \leq 0.87, \quad k \leq 0.63.$$  \hspace{1cm} (4)

4. The eigenvalues of the Higgs mass squared matrices are positive. For the charged Higgs masses this condition is equivalent to the condition that the vacuum does not break QED in the charged Higgs sector.

5. There exists no alternative lower minimum of the Higgs potential with vanishing vacuum expectation values.

For the analysis of the experimental constraints the couplings of the scalar Higgs to two $Z$ bosons and to a pseudoscalar Higgs and a $Z$ boson are crucial. They are displayed in Fig. 1 where $U^S$ and $U^P$ are the transformation matrices between the interaction and mass eigenstates of the CP-even and CP-odd Higgs bosons, respectively:

$$\begin{pmatrix}
S_1 \\
S_2 \\
S_3
\end{pmatrix} = \frac{1}{\sqrt{2}} U^S \begin{pmatrix}
\text{Re} H^0_1 \\
\text{Re} H^0_2 \\
\text{Re} N
\end{pmatrix} - \begin{pmatrix}
v_1 \\
v_2 \\
x
\end{pmatrix},$$  \hspace{1cm} (5)

$$\begin{pmatrix}
P_1 \\
P_2
\end{pmatrix} = \frac{1}{\sqrt{2}} U^P \begin{pmatrix}
\text{Im} H^0_1 \\
\text{Im} H^0_2 \\
\text{Im} N
\end{pmatrix}.$$  \hspace{1cm} (6)

### 3 Experimental constraints

Constraints of the NMSSM parameter space arise from searches for both Higgs bosons and neutralinos at LEP. Due to the unsuccessful direct and indirect Higgs search the masses of the Higgs particles and the parameter space of the NMSSM are constrained by

1. the limit for the factor $\xi^2_a = (U^a_{a1} \cos \beta + U^a_{a2} \sin \beta)^2$ by which the production rate for $e^+e^- \rightarrow Z S_a \ (a = 1, 2, 3)$ in the NMSSM is reduced compared to the SM. The ALEPH collaboration performed a detailed analysis of the lower Higgs mass bound compatible with a model-dependent $\xi^2$ \cite{3}. The 95 % c.l. limits can be found in Fig. 2. Here one has to distinguish between three different scenarios, where $S_a$ decays invisibly into two LSP’s or into two pseudoscalars with masses smaller than $2m_b$ or like a SM-Higgs, respectively.
2. limits from the direct search for the pseudoscalar Higgs in the processes

\[ e^+e^- \rightarrow S_1P_1 \rightarrow b\bar{b}b\bar{b}, \tau\bar{\tau}\tau\bar{\tau}, bb, \tau\bar{\tau}. \]  \( (7) \)

They are given by the L3 collaboration as a function of the masses of the scalar and pseudoscalar Higgs bosons \([13]\).

3. the upper limit \([14]\)

\[ \Delta \Gamma_Z \leq 35.1 \text{ MeV} \]  \( (8) \)

of Higgs physics contributing to the total Z-width via

\[ Z \rightarrow S_aP_b \ (a = 1, 2, 3; \ b = 1, 2). \]  \( (9) \)

The relevant cross sections and decay rates can be easily derived with the Feynman rules in Fig. 1.

The constraints from direct and indirect neutralino search at LEP were presented in \([9]\) and included in our analysis as far as they also constrain the parameters of the Higgs sector.

## 4 Higgs mass bounds in the NMSSM

From the experimental limits of Sec. 3 we extract mass bounds for the neutral Higgs bosons in the following way: We scan for fixed values of \(x, \lambda, k, \tan \beta\) as well as \(A_t\) and the stop masses the parameters \(A_\lambda\) and \(A_k\) over the theoretically allowed range. For each set of parameters the masses and mixings of the neutral Higgs bosons, the \(\xi^2\) factors and the decay rates \((7)\) and \((9)\) are computed and compared to the LEP results in order to find the lower mass bound. Since in addition to the nine parameters of the Higgs sector the gaugino mass parameter \(M\) is sufficient to determine also the masses and couplings of the neutralinos we have to respect constraints of the parameter space from neutralino search illustrated in Fig. 3.

Fig. 2 shows a typical example for a scenario with a light LSP of about 10 GeV, which is well below the current lower bound for a MSSM neutralino. Depicted is the limit for \(\xi^2\) from LEP and the theoretical range as a function of the mass of the lightest scalar Higgs \(S_1\). The lower mass bound for \(S_1\) lies between 37 GeV (SM decay) and 43 GeV (invisible decay). For the large singlet vacuum expectation value \(x = 1000\) GeV this bound is not affected by the limits \((7)\) and \((9)\) because in this case the lighter pseudoscalar Higgs is almost a pure singlet. Since for \(m_{S_1} > 20\) GeV the invisible decay into two neutralinos is kinematically possible the dominant decay channel depends on the choice for \(A_\lambda\) and \(A_k\).

For such scenarios with a light, singlet-like neutralino a very light neutral scalar Higgs of some GeV is excluded by the present LEP data whereas for the pseudoscalar Higgs no such restriction exists.

The constraints for the parameters \(A_\lambda\) and \(A_k\) and the Higgs masses for fixed \(\lambda = 0.4, k = 0.02, \tan \beta = 2\) and various values of the singlet vacuum expectation value \(x\), the stop masses and \(A_t\) are studied in Figs. 4 and 5. The \(\lambda\)-dependence of the lower bound of the scalar Higgs boson mass is shown in Fig. 6 for \(|M| = 400\) GeV, \(\tan \beta = 2\) and two values \(x = 100\) GeV and \(x = 1000\) GeV of the singlet vacuum expectation value. The results do
not substantially change for other choices of \( \tan \beta \) as long as the bottom contributions to the Higgs mixing matrix can be neglected.

In order to combine all LEP constraints from neutralino and Higgs search we first present in Fig. 3 the excluded parameter space in the \( M - x \) plane from unsuccessful neutralino search at LEP (for details see \([4]\)). Fig. 3 shows that for \( \lambda = 0.4, k = 0.02, \tan \beta = 2 \) all \( x \) values are allowed. Contrary to this scenario, however, our analysis in \([4]\) resulted in lower \( x \) bounds of about 50 – 100 GeV for most values of \( \lambda, k \), and \( \tan \beta \). This gap of small allowed \( x \) values is expected to be closed by LEP 200 unless a chargino will be found. But as we will show in the following, combination of all present experimental constraints from the gaugino/higgsino and Higgs sectors already leads to the exclusion of very small \( x \) values (\( x \lesssim 14 \) GeV for \( \tan \beta = 2 \)).

In Fig. 4 the excluded \( A_\lambda - A_k \) region and the allowed Higgs masses are shown for three values of \( x \) with \( A_t = 0 \) GeV and the stop masses \( m_{\tilde{t}_1} = 150 \) GeV and \( m_{\tilde{t}_2} = 500 \) GeV fixed, whereas in Fig. 5 these parameters are varied with \( x = 1000 \) GeV. In the \( A_\lambda - A_k \) plane (Fig. 4 (a), (c), (e)), also plotted are the contour lines for the masses of the Higgs bosons. There, the region above the \( m_{S_1} = 0 \) contour line is forbidden because the mass squared would become negative. The domain beyond the dashed line is excluded since there exists an alternative lower minimum of the Higgs potential with vanishing vacuum expectation values. The shaded region is forbidden due to the experimental constraints given in Sec. 3. The allowed mass region is shown in the \( m_{S_1} - m_{P_1} \) plane (Fig. 4 (b), (d), (f)), where the solid line encloses the theoretical Higgs mass spectrum.

The dependence of the allowed range of parameters and Higgs masses on the stop masses is rather weak. Larger stop masses as in Fig. 5 (a) and (b) only lead to an insignificant increase of the allowed domain. On the other hand, the mass bounds are very sensitive to the choice of the parameter \( A_t \). As shown in Fig. 5 (c) and (d), large \( A_t \) values combined with relatively small stop masses may substantially restrict the parameter space and therefore cause very strong mass bounds.

Before continuing the discussion of the experimental constraints let us shortly describe the theoretical Higgs mass range for different parameters \( x, \lambda \) and \( k \). Generally, one always finds values for \( A_\lambda \) and \( A_k \) leading to a massless scalar Higgs boson. But, as Fig. 4 shows, for the lightest pseudoscalar Higgs particle the lower mass bound increases with increasing values for \( x, \lambda \) and \( k \). Also the upper mass bounds depend on the parameters \( x, \lambda, k \) in the same way: both the scalar and pseudoscalar Higgs bosons can be heavier for larger values of \( x, \lambda \) and \( k \).

For large \( x \) values (\( x \gg m_Z \)) the light pseudoscalar becomes approximately a pure singlet \([5]\) so that there are only weak restrictions on the pseudoscalar Higgs mass by experimental results in Fig. 4 (b) and (d).

Since for smaller \( x \) values the singlet components of the lightest scalar and pseudoscalar Higgs bosons decrease, the lower mass bounds approach the respective values of the MSSM. The smaller the singlet vacuum expectation value \( x \) becomes the larger the coupling \( \lambda \) must be in order to respect the constraints from neutralino search and to allow for heavy Higgs bosons \( S_1 \) and \( P_1 \) which are not excluded despite their MSSM-like mixing type. Keeping the maximal \( \lambda \) value restricted as suggested by eq. (4), the upper mass bound for the lightest scalar becomes so small that values \( x \lesssim 14 \) GeV are excluded for \( \tan \beta = 2 \). For \( x \geq 14 \) GeV, however, very light Higgs particles are generally allowed provided the other parameters are properly chosen.

Fig. 6 combines all constraints from neutralino and Higgs search and shows the de-
dependence of the lower mass bounds for the scalar Higgs boson on the coupling $\lambda$. For this plot with the singlet vacuum expectation values $x = 100$ GeV and $x = 1000$ GeV we fixed the gaugino mass parameter at $|M| = 400$ GeV. This value minimizes the neutralino constraints but leads to a gluino mass not much larger than 1 TeV as suggested by naturalness arguments, by the hierarchy problem [15], and by fine tuning constraints [16].

The graphs in Fig. 6 start with the minimal $\lambda$ value compatible with the constraints from the neutralino sector ($\lambda_{\text{min}} \approx 0.04$ for $x = 1000$ GeV and $\lambda_{\text{min}} \approx 0.35$ for $x = 100$ GeV). Within a small region of this starting point, the lower mass bounds vanish, but finally rise sharply. For smaller $x$ values, the curves move to larger couplings $\lambda$ until for $x \lesssim 14$ GeV the whole parameter space is excluded due to the bound $\lambda \leq 0.87$ of eq. (4).

Improved constraints in the gaugino/higgsino sector by LEP 200 will possibly raise the minimal allowed $\lambda$ value for a given parameter $x$. According to the results of Fig. 6 this may cause nonvanishing lower bounds for the mass of the scalar Higgs boson. Suppose that LEP 200 sets a lower chargino mass bound of 80 GeV. Then for $x = 100$ GeV and $\tan \beta = 2$ the minimal $\lambda$ value becomes about 0.7 leading to a mass bound for the scalar Higgs of approximately 40 GeV even if improved constraints from Higgs search are not considered. Also the lower bound for the singlet vacuum expectation value $x$ may be raised up to 80 GeV. That makes clear how in the NMSSM Higgs constraints may be combined with limits from unsuccessful neutralino search in order to constrain the parameter space very effectively and obtain mass bounds.

So the experimental constraints from the unsuccessful Higgs and neutralino search at LEP 100 lead to significant restrictions of the parameter space of the NMSSM especially in scenarios with a light neutralino but are not yet powerful enough to set general lower bounds for the neutral scalar or pseudoscalar Higgs masses. Small singlet vacuum expectation values, however, are already ruled out. When the bounds from chargino and neutralino search will be improved by LEP 200, it may be possible to exclude $x$-values up to about 80 GeV and to impose mass bounds for the neutral Higgs bosons at least for some further singlet vacuum expectation values.
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Figure Captions

1. Feynman rules for the \( ZZ_a \) and \( ZS_aP_b \) couplings \((a = 1, 2, 3; b = 1, 2)\) in the NMSSM.

2. As a function of \( m_{S_1} \), 95\% c.l. upper limits on \( \xi^2 \) from \[ \] (solid lines) compared to the range in the NMSSM (dashed line). Curve (A) applies, if \( S_1 \) decays like a SM Higgs, curve (B), if it decays invisibly. If \( S_1 \) decays into two pseudoscalars, limit (A) is degraded by less than 10 \%. The corresponding mass bounds are indicated.

3. The excluded parameter space in the \( M-x \) plane from total \( Z \) width measurements (bright shaded) and direct neutralino search (dark shaded).

4. The excluded parameter space in the \( A_\lambda - A_k \) plane (shaded in (a), (c), (e)) and the allowed Higgs mass spectrum (shaded in (b), (d), (f)) for various parameters. In (a), (c), (e), the solid lines denote the contour lines for the mass of the lightest scalar Higgs, the dotted lines for the mass of the light pseudoscalar Higgs. The parameter region beyond the \( m_{S_1} = 0 \) GeV contour line and the dashed line is theoretically excluded as explained in the text. In (b), (d), (f) the solid line encloses the theoretically allowed domain.

5. The excluded parameter space in the \( A_\lambda - A_k \) plane (shaded in (a), (c)) and the allowed Higgs mass spectrum (shaded in (b), (d)) for various parameters. In (a), (c) the solid lines denote the contour lines for the mass of the lightest scalar Higgs, the dotted lines for the mass of the light pseudoscalar Higgs. The parameter region beyond the \( m_{S_1} = 0 \) GeV contour line and the dashed line is theoretically excluded as explained in the text. In (b), (d) the solid line encloses the theoretically allowed domain.

6. Lower bound on the mass of the scalar Higgs boson as a function of the coupling parameter \( \lambda \) for the singlet vacuum expectation values \( x = 1000 \) GeV (solid) and \( x = 100 \) GeV (dashed).