Implementation of a Joint System for Waves and Currents in the Black Sea

Robert Toderascu 1* and Eugen Rusu 1

1 Department of Applied Mechanics, Dunarea de Jos University of Galati, Romania

(Manuscript Received December 12 2013; Revised January 15, 2014; Accepted February 9, 2014)

Abstract

The objective of this paper is to present the implementation of a joint modeling system able to evaluate the propagation of the polluting agents in the marine environment. The system is composed by circulation model (Mohid) and a spectral wave model (SWAN). The results coming from the circulation model are provided as input to the SWAN simulations. Following this target the Mohid water circulation model was implemented and calibrated in the Black Sea basin. The current simulations were run for one year (2010) with a time step of 24 hours, using wind fields from ECMWF. The results concerning the current fields were introduced into SWAN, and the difference between the results of the SWAN simulations with and without the current input from Mohid was assessed. In this regard, 10 points where the significant wave height difference is higher were considered and analyzed. The conclusion of the work is that such a joint system provides more reliable results concerning the wave and current conditions in the Black Sea as it is very useful in providing the support in the case of environmental alerts that may occur in marine environments.

Keywords: Numerical models, Black Sea, Mohid, SWAN, currents and waves

1. Introduction

In the last decades the constant increase of the economic activities in the nearshore enhanced the importance of a better prediction of the wave and current conditions in such areas. Moreover, the global changes in the climate often induce unusual patterns in the coastal environment and various new scenarios should be considered. The aim of the present work is to establish a methodology, based on numerical models (SWAN and MOHID), for predicting the wave and current climate in the Black Sea basin. Numerical wave models are currently used in the Black Sea basin to assess waves. They present the advantage that can cover large coastal zones and provide operational forecast products. Presently, the most accurate estimation of the wave parameters is given by the spectral phase averaging models and, among them, SWAN is being considered the state of the art. Nevertheless, the capacities of the model were substantially extended in the last years both in offshore and near shore directions. As regards its offshore extension, the main improvements are the high order propagation schemes, almost free of numerical diffusion that is associated to large scale propagations, and the parameterization to counteract the Garden Sprinkler effect that may show up due to this small numerical diffusion associated with a reduced resolution in the spectral space. These new features allow the implementation of the SWAN model for the entire Black Sea basin and then to focus the system on nearshore areas in a multilevel wave prediction system. Such approach presents the major advantage that one single model covers the full scale

*Corresponding author. Tel.: +40-753391217, Fax.: +40-336 130 283, E-mail address: Robert.Toderascu@ugal.ro
Copyright © KSOE 2014.
Figure 1 presents the structure of the proposed system. Wind fields are provided by NCEP or ECMWF. Alternatively the implementation of the WRF atmospheric model is also considered. The offshore module consists of the models SWAN for wave generation and MOHID for currents that are run in an interactive manner. For the nearshore module high resolution SWAN computational domains such as SWAN HR or SURF-ISSM are considered together with the SHORECIRC system [1, 2].

2. Theoretical Background of the Mohid Circulation Model

Mohid is a three-dimensional water modeling system, developed by MARETEC (Marine and Environmental Technology and Research Center) at IST (Instituto Superior Tecnico) that belongs to the Technical University of Lisbon. [3]

Mohid evolved from a sequential FORTRAN 77 model to an object oriented model programmed in FORTRAN 95. The structure of the system is divided into several FORTRAN modules. Each of the modules has the functionality of an object class. The modules are combined by geometric requirements and groups of state variables. There is a central module (Module Model) that is in charge of controlling the whole system, which is formed by key modules as WaterProperties, Hydrodynamics, Geometry, AdvectionDiffusion, Atmosphere and Benthos that are used to simulate free surface flows. The Mohid model uses HDF files as input of the different properties, and also for writing the results.

Most Mohid applications use parallelepiped control volumes with orthogonal horizontal axes to simplify the calculations; however Mohid can also use curvilinear grids to compute calculate flows in horizontal anisotropic systems.

The Mohid modeling system was successfully implemented in various coastal areas and estuaries like Minho, Lima, Douro, Mondego, Tejo, Sado, Mira, Arade and Guadiana along the Portugal coast, Rias de Vigo by Taboada [4, 5], Ria de Pontevedra by Taboada and Villarreal [5] and some European estuaries – West Scheldt Holland, Giorde France and Carlingford Ireland by Leitao [6, 7]. Regarding the open sea the model was implemented and validated for the northeast Atlantic area by Neves and Coelho [8].

The model solves the three-dimensional incompressible primitive equations. Hydrostatic equilibrium is assumed as well as Boussinesq and Reynolds approximations. All the equations below have been derived taken into account these approximations. The momentum balance equations for mean flow horizontal velocities are, in Cartesian form
\[ \partial_t v = -\partial_x(uv) - \partial_y(vv) - \partial_z(uw) - f - \frac{1}{\rho_0} \partial_x p + \partial_x \left((v_H + v)\partial_x v + \partial_y \left((v_H + v)\partial_y v + \partial_z \left((v_H + v)\partial_z v\right)\right)\right) \]

\[ \partial_t u = -\partial_x(uu) - \partial_y(uv) - \partial_z(uw) + f v - \frac{1}{\rho_0} \partial_x p + \partial_x \left((v_H + v)\partial_x u + \partial_y \left((v_H + v)\partial_y u + \partial_z \left((v_H + v)\partial_z u\right)\right)\right) \]

(1)

(2)

Where \( u, v \) and \( w \) are the components of the velocity vector in the \( x, y \) and \( z \) directions respectively, \( f \) the Coriolis parameter, \( v_H \) and \( v_t \) the turbulent viscosities in the horizontal and vertical directions, \( v \) is the molecular kinematic viscosity (equal to \( 1.310^{-6} \text{m}^2 \text{s}^{-1} \)), \( p \) is the pressure. The temporal evolution of velocities (term on the left hand side) is the balance of advective transports (first three terms on the right hand side), Coriolis force (fourth term), pressure gradient (next three terms) and turbulent diffusion (last three terms).

The vertical velocity is calculated from the incompressible continuity equation (mass balance equation):

\[ \partial_x u + \partial_y v + \partial_z w = 0 \]

(3)

By integrating between the bottom and the depth \( z \) where \( w \) is to be calculated:

\[ w(z) = \partial_x \int \frac{1}{\rho} \text{d}x + \partial_y \int \frac{1}{\rho} \text{d}y \]

(4)

The free surface equation is obtained by integrating the equation of continuity over the whole water column (between the free surface elevation \( \eta(x, y) \) and the bottom -h):

\[ \partial_t \eta = -\partial_x \int \frac{1}{\rho} \text{d}x - \partial_y \int \frac{1}{\rho} \text{d}y \]

(5)

The hydrostatic approximation is assumed with:

\[ \partial_z p + g \rho = 0 \]

(6)

where \( g \) is gravity and \( \rho \) is density. If the atmospheric pressure \( p_{atm} \) is subtracted from \( p \), and density \( \rho \) is divided into a constant reference density \( p_0 \) and a deviation \( \rho' \) from that constant reference density, after integrating from the free surface to the depth \( z \) where pressure is calculated, we arrive to:

\[ p(z) = p_{atm} + g \rho_0 \eta(z) + g \int \eta \rho' \text{d}z \]

(7)

Equation 7 relates pressure at any depth with the atmospheric pressure at the sea surface, the sea level and the anomalous pressure integrated between that level and the surface. By using this expression and the Boussinesq approximation, the horizontal pressure gradient in the direction \( x \) can be divided in three contributions:

\[ \partial_x p = \partial_x p_{atm} - g \rho_0 \partial_x \eta - g \int \eta \rho' \text{d}z \]

(8)

The total pressure gradient is the sum of the gradients of atmospheric pressure, of sea surface elevation (barotropic pressure gradient) and of the density distribution (baroclinic pressure gradient). This decomposition of the pressure gradient is substituted in Equations 1 and 2. The density is obtained from the salinity and from the temperature, which are transported by the water properties module [9].

3. Theoretical Background of the SWAN

Wave Generation Model

SWAN is a third generation wave model, developed at Delft University of Technology, which computes random, short crested wind-generated waves in coastal regions and inland waters.

Although initially designed especially for nearshore areas, in the last few years, the capabilities of SWAN were substantially extended both in offshore and nearshore directions. As regards its offshore extension the main improvements are: the
high order propagation scheme S&L [10], almost free of numerical diffusion, that is associated to large scale propagations in the non-stationary mode, and the parameterization to counteract the Garden Sprinkler effect [11], that may show up due to small numerical diffusion associated with a reduced resolution in the spectral space. To extend the performance of the model in the nearshore direction, the most recent major improvement concerns designing a phase decoupled approach to account for the diffraction effect [12].

Hence, although probably not so efficient from a computational point of view as WW3 or WAM at oceanic scales, at sub oceanic scales, SWAN seems to be now the most appropriate operational wave model. The main reason is its greater flexibility since it includes various alternatives for modeling and tuning physical processes, either in shallow or deep water. By calibrating the model, an appropriate combination of physical processes and parameters can be identified for a particular geographic site. This leads to better quality predictions of the main wave parameters. Another advantage is that a single model can be employed for the full range of the wave modeling processes and the focusing of the wave prediction system in the nearshore direction is thus straightforward. For this reason a wave prediction system, SWAN based, that covers the entire Black Sea was implemented and validated with both in situ and remotely sensed data [13, 14].

As a third generation wave model, SWAN solves the spectral energy balance equation that describes the evolution of the wave spectrum in time, geographical and spectral spaces [15]. The spectrum that is considered is the action density spectrum (N), rather than the energy density spectrum (E), since in the presence of currents, action density is conserved whereas energy density is not. The action density is equal to the energy density divided by the relative frequency (σ). Hence, the spectral action balance equation is given by:

\[
\frac{\partial N}{\partial t} + \nabla \left[ (\vec{c}_g + \vec{U}) N \right] + \frac{\partial}{\partial \sigma} c_\sigma N + \frac{\partial}{\partial \theta} c_\theta N = \frac{S}{\sigma}.
\]  

(8)

where \( k \) is the wave number related to the relative frequency through the dispersion relationship, \( \theta \) is the wave direction and \( \vec{U} \) the velocity of the ambient current which is considered uniform with respect to the vertical coordinate. The propagation velocities of the wave energy are the group velocity \( \vec{c}_g \) in physical space \( \left( \vec{c}_g = \frac{\partial \sigma}{\partial k} \right) \) and \( c_\sigma = \sigma \) and \( c_\theta = \theta \) in spectral space. For large scale applications this equation relates to the spherical coordinates defined by longitude \( \lambda \) and latitude \( \phi \):

\[
\frac{\partial N}{\partial t} + \frac{\partial}{\partial \lambda} c_\lambda N + \frac{1}{\cos \phi} \frac{\partial}{\partial \phi} c_\phi N \cos \phi + \frac{\partial}{\partial \sigma} c_\sigma N + \frac{\partial}{\partial \theta} c_\theta N = \frac{S}{\sigma}.
\]  

(9)

On the right side of the action balance equation is the source expressed in terms of energy density. In deep water, the source comprises three primary components: the atmospheric input \( (S_{in}) \), whitecapping dissipation \( (S_{w}) \) and nonlinear quadruplet interactions \( (S_{nl}) \). In shallow water, additional phenomena like bottom friction, depth induced wave breaking and triad non-linear wave-wave interactions induced by the finite depth effects \( (S_{fd}) \) may play an important role. Hence the total source becomes:

\[
S_{total} = S_{in} + S_{dis} + S_{nl} + S_{fd}.
\]  

(10)

4. Implementation of the Proposed System

The water movement is computed using Mohid water modeling system by taking into account salinity and temperature fields, wind field, coastlines and bathmetry. The water density fields were obtained from MyOcean website [16] with a geographic coverage of 27.4°E-41.9°E and 40.9°N-46.9°N. The data included in this product are water temperature and salinity with a spatial resolution of about 5km, on 35 depth levels, starting from 2.5m to 2100m. The water density fields are gridded in a 238x132x35 mesh. The wind fields are provided by ECMWF (European Centre for Medium-Range Weather Forecasts). Wind fields are interpolated to fit the water density grid. The initial bathmetry is provided by ETOPO2 Global Gridded 2 minute Database from NOAA. Prior to the analysis a number of Matlab scripts were created to convert the available water density and wind fields into HDF5 format required by Mohid.
Fig. 2. Significant wave height for 15 January, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.

Fig. 3. Significant wave height for 15 February, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.

Fig. 4. Significant wave height for 15 March, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.
Fig. 5. Significant wave height for 15 April, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.

Fig. 6. Significant wave height for 15 May, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.

Fig. 7. Significant wave height for 15 June, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.
Fig. 8. Significant wave height for 15 July, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.

Fig. 9. Significant wave height for 15 August, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.

Fig. 10. Significant wave height for 15 September, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.
Fig. 11. Significant wave height for 15 October, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.

Fig. 12. Significant wave height for 15 November, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.

Figure 13. Significant wave height for 15 December, hour 12: a) simulations performed using the results from Mohid model, b) simulations performed without the current input from Mohid.
The simulations are carried out with a time step of 24 hours, on a 238x132x35 mesh using the Cartesian domain, for a period of 365 days starting 1st of January 2010.

Alternatively, simulations for the same time period are performed with SWAN. The SWAN simulations are performed in bi-dimensional non-stationary regime with a time step of 1 hour, on a 175x75 grid covering the entire basin of the Black Sea. The wind data for SWAN simulations are also obtained from ECMWF and converted to fit the grid. For every time step the model performs a number of maximum 50 iterations or until it reached a convergence of at least 98%. This set of simulations performed with SWAN, without considering the results from Mohid circulation model, will serve as a base of comparison.

The results obtained with Mohid are converted and inserted into the SWAN model. The different time step used for the two simulations (Mohid and SWAN) did not pose a problem since SWAN is able to interpolate data between the previous and next time steps.

Figures 2-13 present the significant wave height $H_s$ (with black vectors) for the days of 15 of each month of the year 2010, with the influence of currents from Mohid circulation model considered (a) and without (b).

As it can be observed from Figures 2-13, the presence of the currents has an important influence on the significant wave height value and waves vector orientation. In most cases the presence of the currents increases the value of $H_s$, especially in the winter time when current velocities can reach a maximum of 2ms$^{-1}$.

| Point | Jan | Feb | Mar | Apr | May | Jun | July | Aug | Sept | Oct | Nov | Dec |
|-------|-----|-----|-----|-----|-----|-----|------|-----|------|-----|-----|-----|
| H1    | 0.56| 0.65| 0.29| 0.16| 0.07| 0.29| 0.5  | 0.21| 0.15 | 0.16| 0.00| 0.07|
| H2    | 0.34| 0.34| 0.21| 0.11| 0.05| 0.2  | 0.31 | 0.16| 0.14 | 0.14| 0.00| 0.03|
| H3    | 0.3 | 0.82| 0.17| 0.23| 0.11| 0.42| 0.84 | 0.68| 0.29 | 0.19| 0.06| 0.24|
| H4    | 0.36| 0.82| 0.23|-0.04| 0.1  | 0.18| 0.51 | 0.07| 0.15 | 0.21|-0.01| 0.23|
| H5    | 0.1 | 0.65| 0.31| 0.00|-0.01| 0.54| 0.12 | 0.01| 0.04 | 0.12| 0.00| 0.07|
| H6    | 0.25| 0.69| 0.22| 0.38| 0.12| 0.35| 0.98 | 0.12| 0.27 | 0.31| 0.05| 0.27|
| H7    | 0.38| 0.87| 0.36|-0.01| 0.13| 0.28| 0.63 | 0.07| 0.12 | 0.33|-0.02| 0.31|
| H8    | 0.25| 0.46| 0.16|-0.04| 0.05| 0.23| 0.36 | 0.04| 0.04 | 0.58|-0.02| 0.21|
| H9    | 0.09| 0.1 | 0.1 | 0.01| 0.07| 0.27| 0.18 | 0.02| 0.01 | 0.19| 0.05| 0.62|
| H10   | 0.38| 0.64| 0.18| 0.22| 0.11| 0.54| 0.54 | 0.11| 0.28 | 0.21|-0.01| 0.18|
Table 2. Coordinates of the 10 considered points and some statistical analyses of significant wave height difference

| Point | Coordinates       | Minimum (m) | Maximum (m) | Mean (m) | Median (m) | Standard deviation (m) | Skewness | Kurtosis |
|-------|-------------------|-------------|-------------|----------|------------|------------------------|----------|----------|
| H1    | 41.56N, 38.7E     | -0.26       | 6.15        | 0.26     | 0.1        | 0.53                   | 4.85     | 36.58    |
| H2    | 41.96N, 9.42E     | -0.39       | 5.92        | 0.17     | 0.07       | 0.39                   | 6.72     | 74.44    |
| H3    | 41.56N, 7.18E     | -0.37       | 7.15        | 0.36     | 0.11       | 0.64                   | 4.05     | 31.9     |
| H4    | 43.16N, 8.86E     | -0.49       | 6.04        | 0.23     | 0.02       | 0.62                   | 3.72     | 22.71    |
| H5    | 45.48N, 1.98E     | -0.32       | 5.94        | 0.16     | 0.009      | 0.55                   | 5.51     | 42.5     |
| H6    | 41.96N, 5.82E     | -0.48       | 5.25        | 0.33     | 0.08       | 0.69                   | 3.32     | 16.75    |
| H7    | 43N, 28.86E       | -0.47       | 5.36        | 0.28     | 0.03       | 0.7                    | 3.01     | 14.31    |
| H8    | 43.48N, 9.34E     | -0.72       | 6.15        | 0.19     | 0.03       | 0.62                   | 5.05     | 36.95    |
| H9    | 44.44N, 4.78E     | -0.61       | 5.43        | 0.14     | 0.03       | 0.48                   | 5.74     | 49.32    |
| H10   | 42.2N, 36.54E     | -0.33       | 4.85        | 0.28     | 0.08       | 0.57                   | 3.52     | 19.41    |

Fig. 14. The bathymetry of the Black Sea and the position of the 10 considered points.

4. Discussions

In order to obtain a better understanding of the influence that the water movement have on the significant wave height, 10 points where higher differences between the significant wave height with and without currents appear were chosen for a closer look. Figure 14 presents the bathymetry of the Black Sea basin along with the 10 considered points. For each one, $\Delta H_s$ was computed as follows:

$$\Delta H_s = H_s^{\text{currents+wind}} - H_s^{\text{wind}}$$  \hspace{1cm} (11)

Table 1 shows the monthly medium averaged values for each of the considered points. The month of November presents the smallest medium averaged values, ranging from -0.02m for points H7 and H8 to 0.06 for H3. The highest medium averaged values are registered for the months of July, with 0.98m and 0.84m for H6 and H3, and February with 0.87m for H7 and 0.82m for H3 and
H4. The water movement simulations performed with Mohid show higher values for current velocities for the months of January, February and July, and their influence can be clearly observed in the monthly averaged values table. Furthermore, during the water movement simulations it was also observed that the current velocities presents smaller values for the months of November and December. Table 2 presents the coordinates of the 10 considered points along with some statistical analyses performed.

In statistical analysis, the standard deviation measures the data dispersion from the mean value [17]:

\[ \text{Std} = \sqrt{E\left[ (X - \mu)^2 \right]} , \]  \hspace{1cm} (12)

with \( \mu = E[X] \) representing the mean value, where \( E \) is the expectation operator. \( X \) represents a discrete random variable with the probability mass function \( p(x) \). Then the expected value will be:

\[ E\left(X\right) = \sum x_i p\left(x_i\right) . \]  \hspace{1cm} (13)

In probability theory and statistics, skewness is a measure of the symmetry distribution in a certain data set. The skewness value can be positive, negative or undefined [18]. The skewness of a variable \( X \) is defined as the third standardized moment:

\[ \text{Skew} = \frac{\mu_3}{\sigma^3} , \]  \hspace{1cm} (14)

Where \( \mu_3 \) is the third moment above the mean and the \( k^{th} \) moment about the mean is defined as:

\[ \mu_k = E\left[ (X - E[X])^k \right] . \]  \hspace{1cm} (15)

Kurtosis represents the relative concentration of the data in the centre versus in the tails of a frequency distribution when is compared with the normal distribution (which has a kurtosis value of 3). This is equal to the fourth moment around the mean divided by the square of the variance (or the fourth power of the standard deviation) of the distribution minus 3 [19].

\[ Kurt = \frac{\mu_4}{\sigma^4} - 3 . \]  \hspace{1cm} (16)

Figures 15-19 present the variation in time of the \( \Delta H_s \) (m) parameter for each of the 10 considered points.

Table 2 shows that the maximum values of the \( \Delta H_s \) term can reach values of 7.15m as it is the case for H3 point, followed closely by the points marked as H1 and H8 with 6.15m. The mean values range from 0.14m in the point designated as H9 and 0.36 for H3. The median values range from 0.009m for H5 to 0.11 for H3.

The maximum dispersion related to the mean value, or standard deviation, contains values that range from 0.39m for H2 to 0.7 for the point designated H7. Skewness or the measure of the distribution of symmetry of data contain values that vary from 3.01 for H7 to 6.72 for H2.

Kurtosis, which represents the relative concentration of the data in the center versus the frequency distribution when compared with the normal distribution, presents values that range from 14.31 for H7 to 42.39 for H9, with the exception of the point designated as H2, where the kurtosis value reaches a staggering value of 74.44.

Fig. 15. Variation in time of \( \Delta H_s \) (m) in points H1 and H2.
Fig. 16. Variation in time of $\Delta H_s$ (m) in points H3 and H4.

Fig. 17. Variation in time of $\Delta H_s$ (m) in points H5 and H6.

Figure 18. Variation in time of $\Delta H_s$ (m) in points H7 and H8.

Figure 19. Variation in time of $\Delta H_s$ (m) in points H9 and H10.
5. Conclusions

The presence of currents in the simulations modifies not only the value of the Hs term (significant wave height), but also the general direction of the wave vectors, fact proven by figures 2 to 13.

A general increase of the significant wave height is observed when current results from Mohid model are included in SWAN simulations, even if the current velocities have smaller values, as is the case for the Black Sea basin.

Therefore, the present paper validates the importance of the implementation of a joint system that uses current results from a specialized circulation model (Mohid) with a wave generation model (SWAN). The proposed model can be used not only to study the physical phenomena that occur in a marine environment, but also for different purposes. For example, with the addition of a coastal generation model it can be used to predict the oil spill movements in the unfortunate cases of marine or drilling accidents. Such models, like SURF-ISSM, SWAN HR and Shorecirc are considered for future implementations and coupling with the existent system.

6. Acknowledgments

The work of the first author has been made in the scope of the project EFICIENT (Management System for the Fellowships Granted to the PhD Students) supported by the Project SOP HRD - EFICIENT 61445/2009.

This work was also supported by a grant of the Romanian Ministry of National Education, CNCS – UEFISCDI PN-II-ID-PCE-2012-4-0089 (project DAMWAVE).

7. References

[1] Rusu, E., Conley, D.C. and Coelho, E.F., 2008: “A Hybrid Framework for Predicting Waves and Longshore Currents.” Journal of Marine Systems, Volume 69, Issues 1-2, pp 59–73
[2] http://chinacat.coastal.udel.edu/programs/shorecirc/shorecirc.html
[3] http://www.mohid.com/what_is_mohid.htm
[4] J. J. Taboada, M. Ruíz-Villarreal, M. Gómez-Gesteira, P. Montero, A. P. Santos, V. Pérez-Villar and R. Prego, “Estudio del transporte en la Ría de Pontevedra (NOEspaña) mediante un modelo 3D: Resultados preliminares, In: Estudios de Biogeocquímica na zona costeira ibérica.” Servicio de Publicaciones da Universidade de Aveiro in press, 2000.
[5] P. Montero, M. Gómez-Gesteira, J. J. Taboada, M. Ruíz-Villarreal, A. P. Santos, R. J. J. Neves, R. Prego and V. Pérez-Villar, “On residual circulation of Vigo Ría using a 3D baroclinic model”, Boletín Instituto Español de Oceanografía, no 15., Suplemento 1, 1999.
[6] L. Cancino, and R. Neves “Hydrodynamic and sediment suspension modelling in estuarine systems. Part II: Application to the Western Scheldt and Gironde estuaries”, Journal of Marine Systems, vol. 22, pp. 117-131, 1999.
[7] P. C. Leitão, “Modelo de Dispersão Lagrangeano Tridimensional.” Ms. Sc. Thesis, Universidade Técnica de Lisboa, Instituto Superior Técnico, 1996.
[8] R. Neves, H. Coelho, P. Leitão, H. Martins, and A. Santos, “A numerical investigation of the slope current along the western European margin.”, Computational Methods in Water Resources XII, vol. 2, pp. 369-376, 1998.
[9] Chippada S., C. Dawson, M. Wheeler, (1998) - Agdonov-type finite volume method for the system of shallow water equations, Computer methods in applied mechanics and engineering, 151(01): 105-130
[10] Stelling, G.S. and Leendertse, J.J., 1992. Approximation of convective processes by cyclic AOI methods, Proceeding 2nd international conference on estuarine and coastal modeling, ASCE Tampa, Florida, 771-782.
[11] Booij, N., and Holthuijsen, L.H., 1987. Propagation of ocean waves in discrete spectral wave models, Journal of Computational Physics, vol. 68, 307-326.
[12] Holthuijsen, L.H., Herman, A. and Booij, N., 2003. Phase-decoupled refraction/diffraction for spectral wave models, Coastal Engineering, 49, 291-305
[13] Rusu, E., Rusu, L. and Guedes Soares, C., 2006: Assessing of extreme wave conditions in the Black Sea with numerical models, the 9th International Workshop on Wave Hindcasting and Forecasting, Victoria, Canada, September, 2006.
[14] Rusu, E, 2009: Wave energy assessments in the Black Sea, Journal of Marine Science and Technology, Vol. 14, pp. 359–372.
[15] Holthuijsen, H., 2007. Waves in Oceanic and Coastal Waters, Cambridge University Press, pp. 387.

[16] http://www.myocean.eu/

[17] Ghahramani, Saeed (2000). Fundamentals of Probability (2nd Edition). Prentice Hall: New Jersey. p. 438.

[18] Szekely, G.J. and Mori, T.F. (2001) "A characteristic measure of asymmetry and its application for testing diagonal symmetry", Communications in Statistics - Theory and Methods 30/8&9, 1633–1639.

[19] Hosking, J.R.M. (2006). "On the characterization of distributions by their L-moments". Journal of Statistical Planning and Inference 136: 193–198