Antenna Models for Electromagnetic Compatibility Applications
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Guest Editors: Dragan Poljak, Khalil El Khamlichi Drissi, Sergey V. Tkachenko, and Andres Peratta
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In recent decades there have been a number of significant advances in EMC modeling which can be carried out within a significantly shorter time than it would be necessary for building and testing the appropriate prototype via experimental procedures. Moreover EMC simulation can predict the system behaviour for a rather wide variety of parameters including different initial and boundary conditions, excitation types, and different configuration of the system itself. EMC-computational models are often classified as

(i) circuit theory models (featuring the concentrated electrical parameters),

(ii) transmission line models (using distributed parameters in which low-frequency electromagnetic field coupling is taken into account), antenna systems, overhead power lines, and buried cables.

(iii) models based on the full-wave (antenna theory) approach (taking into account radiation effects for the treatment of electromagnetic wave propagation problems).

The antenna models represent a vital area of EMC related to the study of electromagnetic field coupling to systems or devices and represent the most rigorous approach to EMC problems. Furthermore, the transmission line (TL) approximation, whose principal advantage is simplicity and relatively low computational cost, fails to account for various radiation effects, such as resonances, presence of a lossy ground, the effects at the wire ends, and so forth. However, the main restriction of the wire antenna model applied to complex structures is related to a rather high computational cost.

Therefore, the Guests Editors enthusiastically accepted the opportunity to edit this special issue on antenna models for Electromagnetic Compatibility Applications by collecting some interesting contributions achieved by some researchers in this area. This special issue deals with various aspects of antenna EMC models covering applications related to both continuous wave (CW) problems and transient problems.

Guest Editors have received 11 submissions for this Special issue. Having completed the peer review process, the Guest Editors have finally accepted 5 papers, while 5 papers have been rejected and 1 paper has been withdrawn.

The issue entitled “Electromagnetic Field Coupling to Overhead Wire Configurations”, by D. Poljak and K. El-Khamlichi Drissi reviews various aspects of wire antenna and transmission line models in both frequency and time domain. The paper by S. Tkachenko et al. entitled “High Frequency Electromagnetic Field Coupling to Small Antenna in a Rectangular Resonator” deals with an efficient analytical approach to analyze the coupling of electrically small antennas in resonators. The paper “Modeling of Coaxial Slot Waveguides Using Analytical and Numerical Approaches—Revisited” written by Y. K. You et al. reviews some analytical and numerical models for coaxial slot waveguides. Paper: “Evaluation of the Inductive Coupling between Equivalent Emission Sources of Components” by M. Ferber et al. presents methodology providing the evaluation of coupling.
parameters of components by using equivalent emission sources. Finally, the paper written by V. Javor and entitled “Modeling of Lightning Strokes using Two-Peaked Channel-Base Currents” is related to a study of lightning induced electromagnetic field by means of engineering models of lightning return strokes and new channel-base currents.

The Guest Editors hope they managed to put together an interesting piece of work regarding the use of antenna models in electromagnetic compatibility applications.

They would also like to thank all contributors for their valuable contributions and to the reviewers for their prompt response and excellent work, as well.

Dragan Poljak
Khalil El Khamlichi Drissi
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Research Article

Modeling of Lightning Strokes Using Two-Peaked Channel-Base Currents

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Lightning electromagnetic field is obtained by using “engineering” models of lightning return strokes and new channel-base current functions and the results are presented in this paper. Experimentally measured channel-base currents are approximated not only with functions having two-peaked waveshapes but also with the one-peaked function so as usually used in the literature. These functions are simple to be applied in any “engineering” or electromagnetic model as well. For the three “engineering” models: transmission line model (without the peak current decay), transmission line model with linear decay, and transmission line model with exponential decay with height, the comparison of electric and magnetic field components at different distances from the lightning channel-base is presented in the case of a perfectly conducting ground. Different heights of lightning channels are also considered. These results enable analysis of advantages/shortages of the used return stroke models according to the electromagnetic field features to be achieved, as obtained by measurements.

1. Introduction

One of the key issues in research of environmental electromagnetic interference with electric systems, electronic devices and equipment inside imperfectly enclosed structures, so as with power and communication lines, is modeling of lightning discharges. An adequate EMC simulation would include wide variety of modeling parameters such as various initial and boundary conditions, excitation types, ground electrical properties and different configurations of the observed systems. Rapid advance in EMC modeling and computation due to development of numerical procedures programs and computers in last few decades partly replaced expensive and cumbersome building and testing of appropriate prototypes, so as some experimental procedures. Although measurements are nowadays carried out for triggered lightning and at instrumented tall towers throughout the world, the most referred and comprehensive measurement results for natural lightning are given in [1–3].

According to [4] lightning stroke models are classified into physical (gas-dynamic) models, electromagnetic models, “engineering” models, and distributed-circuit theory models. Models based on the full-wave approach take into account radiation effects for the treatment of electromagnetic wave propagation problems. “Engineering” models use simplified approach with respect to the current distribution along the lightning channel, but electromagnetic field is determined based on the same relations as for electromagnetic models. In fact, all these models enable approximate analysis of lightning electromagnetic field and its coupling to systems or devices, due to a wide variety of conditions accompanying this natural phenomenon. There are various classifications of models given in the literature, for example [4, 5], and, besides, a certain model can belong to more than one class.

Many researchers are focused on comparison of different “engineering” models using the same channel-base current, but the influence of an implied current waveshape itself on lightning electric and magnetic field components is not investigated, as to the author’s best knowledge. Both the selected model and channel-base current waveshape determine lightning electric and magnetic field results to be compared with experimentally obtained waveshapes at different distances from the channel base. Compliance
with some measurements results, as, for example [1–3], is important for the validation of models.

If the new two-peaked channel-base current function [6] is used, it is possible to obtain some of the features that a certain model would not give with the one-peaked current waveshape as in [7, 8] or with the one-peaked current as usually used in the literature [9, 10], and so forth. If using [10] at least two terms are needed to obtain theoretically assumed current [11] having one initial and a subsidiary peak [12]. However, for experimentally measured currents as at Monte San Salvatore [1, 3] seven terms are needed in the linear combination of Heidler’s functions, as given in [13]. This was used for calculations of lightning induced overvoltages at power transmission lines in [14]. The same number of terms was used in [13] for approximating experimentally measured currents at the Morro do Cachimbo station [15]. The adequate parameters of the new two-peaked channel-base current function are given in this paper for these two, and also for the one-peaked function [7] approximating the first negative stroke channel-base current [9].

A function having similar mathematical expression to given in [6] can be used to include even more peaks in the current waveshape. It is demonstrated that such obtained lightning electric and magnetic fields results at different distances perform some features of experimental results. A review of new functions is given in [16] for representing IEC 62305 standard currents [17] and other typical lightning stroke currents. These functions are suitable for use in both “engineering” and antenna theory models [18] to approximate the excitation at the channel base.

2. First Negative Stroke Channel-Base Currents

A negative first-stroke channel-base current is characterized with emphasized peaks (an initial and a few subsidiary peaks). The function including two peaks in the rising part can be approximated with the following analytical expression [6]:

\[
i(0, t) = \begin{cases} 
I_{m1} \sum_{i=1}^{k} d_i \left[ \left( \frac{t}{t_{m1}} \right) \exp \left( 1 - \frac{t}{t_{m1}} \right) \right]^{a_i}, & 0 \leq t \leq t_{m1}, \\
I_{m1} + I_{m2} \sum_{i=1}^{l} f_i \left[ \left( \frac{t}{t_{m2} - t_{m1}} \right) \exp \left( 1 - \frac{t - t_{m1}}{t_{m2} - t_{m1}} \right) \right]^{b_i}, & t_{m1} \leq t \leq t_{m2}, \\
(I_{m1} + I_{m2}) \sum_{i=1}^{n} g_i \left[ \left( \frac{t}{t_{m2}} \right) \exp \left( 1 - \frac{t}{t_{m2}} \right) \right]^{c_i}, & t_{m2} \leq t < \infty,
\end{cases}
\]

(1)

The number of terms \( k, l, \) and \( n \) is selected according to the desired accuracy of approximation. Parameters of the current function are \( a_i, b_i, c_i \), and the weighting coefficients are \( d_i, f_i, g_i \), so that \( \sum_i d_i = \sum_i f_i = \sum_i g_i = 1 \). The initial peak \( I_{m1} \) is obtained at \( t_{m1} \), and the subsidiary peak \( I_m = I_{m1} + I_{m2} \) at \( t_{m2} \) (Figure 1). For the analytical expression (1) parameters to represent median characteristics of channel-base currents measured at Monte San Salvatore [3] and at the Morro do Cachimbo station [15] are given in Table 1.

In order to compare the results of calculations to results from the literature, parameters of the one-peaked NCBC current function are calculated to represent channel-base current from [9], as used in [4] and a number of papers.

NCBC function [7, 8] is given with

\[
i(0, t) = \begin{cases} 
I_m \left( \frac{t}{t_m} \right)^a \exp \left[ a \left( 1 - \frac{t}{t_m} \right) \right], & 0 \leq t \leq t_m, \\
I_m \sum_{j=1}^{n} \left( \frac{t}{t_m} \right)^{b_j} \exp \left[ b_j \left( 1 - \frac{t}{t_m} \right) \right], & t_m \leq t < \infty,
\end{cases}
\]

(2)
for $a$ and $b_i$ parameters, and $c_i$ weighting coefficients, so that $\sum c_i = 1$. Time $t_m$ is the rise time to the maximum current value $I_m$, for the chosen number of terms $n$ in the decaying part.

NCBC function (2) approximates the waveshape from [9] presented with dots in Figure 2. Parameters for NCBC function, presented with the full line in Figure 2, are calculated as following: $I_m = 11\text{kA}$, $t_m = 0.472\text{\mu s}$, $a = 1.1$, $b_1 = 0.16$, $c_1 = 0.34$, $b_2 = 0.0047$, and $c_2 = 1 - c_1 = 0.66$ [7]. The waveshape from [9] is often used in the literature for lightning negative strokes modeling.

3. "Engineering" Models of Lightning Strokes

Thin wire representation of a lightning channel at the perfectly conducting ground is presented in Figure 3. An “engineering” model assumes an impulse current, propagating along the channel with a current-wave propagation speed $v/f$ and the speed $v$ of the return stroke, presented with

$$i(z', t) = u\left(t - \frac{z'}{v_f}\right) P(z', t) i\left(0, t - \frac{z'}{v}\right),$$

(3)

where $u(t)$ is the Heaviside function and $P(z', t)$ the height and time-dependent current attenuation factor. In the transmission line (TL) model this factor is $P(z', t) = 1$, so the current is propagating along the channel without attenuation. In the transmission line model with linear decay (MTLL), for the height $z'$ of the observed current element above the ground, the attenuation factor is $P(z', t) = 1 - z'/H$ for the assumed channel height $H$. In this paper $H$ is chosen to be 2600 m or 7500 m. In the transmission line model with exponential decay (MTLE) is $P(z', t) = \exp(-z'/\lambda)$, for the decay constant $\lambda = 2000\text{ m}$ chosen in [9], so as in this paper. For all the three models $v = v_f = 1.3 \times 10^6\text{ ms}^{-1}$, as in [4], is taken for obtaining results in this paper.

The current $i(0, t)$ in (3) is the channel-base current to be approximated with one- or two-peaked pulse functions, as presented in Figures 1 and 2. All the results for vertical electric and azimuthal magnetic field are presented for points at the ground surface. There are no other electric or magnetic field components at the ground surface due to $\sigma \to \infty$ for the lower half-space (Figure 3). Above the ground other electromagnetic field components exist, but measurements results are usually given for the points very near to the ground surface.

For MTLL model the current at the end of the channel (for $z' = H$) is equal to zero. The consequence is that the interrupt of the channel does not produce a spike in the waveshapes of far electric and magnetic fields. For MTLE model, the degree of attenuation depends on the value of constant $\lambda$. The higher the value of $\lambda$, the less the current is attenuated, and vice versa.

4. Results for Lightning Electromagnetic Field

In the upper half-space electric field has both vertical and radial component, and magnetic field just azimuthal component, whereas other field components are equal to zero in the case of perfectly conducting ground. Vertical electric field at the field point $M(r, \psi, z)$, as in Figure 3, can be calculated as

$$E_z = \frac{1}{4\pi\varepsilon_0} \int_H^{\infty} \left[ \frac{2(z-z')^2 - r^2}{R^3} \int_{r_0}^{r-t} \frac{i'(z', \tau - \frac{R}{c})}{c} d\tau + \frac{2(z-z')^2 - r^2}{c R^4} \frac{i(z', t - \frac{R}{c})}{c} \right] dz'.

(4)
and radial electric field as

\[ E_r = \frac{1}{4\pi\varepsilon_0} \int_H^{H'} \left[ \frac{3r(z-z')}{R^3} \int_{t=0}^{t=t'} i(z',\tau - \frac{R}{c}) d\tau + \frac{3r(z-z')}{cR^5} i(z', t - \frac{R}{c}) \right. \]
\[ \left. + \frac{r(z-z')}{c^2R^3} \frac{\partial i(z', t - \frac{R}{c})}{\partial t} \right] dz', \tag{5} \]

whereas azimuthal magnetic field is

\[ H_\psi = \frac{1}{4\pi} \int_H^{H'} \left[ \frac{r}{R^3} i(z', t - \frac{R}{c}) + \frac{r}{cR^2} \frac{\partial i(z', t - R/c)}{\partial t} \right] dz', \tag{6} \]

for \( R = \sqrt{r^2 + (z-z')^2} \) the distance from the current element of length \( dz' \), with the current \( i(z', t) \) or its image in the plane mirror replacing the influence of the lower perfectly conducting half-space, to the field point \( M(r, \psi, z) \). In (6), \( c = (\varepsilon_0\mu_0)^{-1/2} \) is the speed of light, \( \varepsilon_0 \) is the permittivity and \( \mu_0 \) the permeability of the air.
For the three “engineering” models (TL, MTLL, and MTLE), vertical electric field results are presented in Figures 4, 5, 6, 7, 8, and 9 for the two channel heights: 2600 m and 7500 m.

For usually used waveshape from [9], approximated with one-peaked NCBC function [7], these results are presented in Figures 4–6 for the radial distances 50 m, 5 km, and 100 km from the channel base.

For MSS_FST current from [3], approximated with [6], and the same three models, vertical electric field results are presented in Figures 7–9, for two different channel heights (2600 m and 7500 m), and for the distances 50 m, 5 km, and 100 km from the channel base.

For MCS_FST current from [15], approximated with [6], lightning electric and magnetic field results are also obtained, but being similar to the waveshapes of MSS_FST, as can be concluded from Table 1, these are not included in this paper.

Azimuthal magnetic field results are presented for the one-peaked NCBC function in Figures 10, 11, and 12, and for the approximation of MSS_FST with the two-peaked current function in Figures 13, 14, and 15, for the distances 50 m,
Some main features of lightning electromagnetic field waveshapes given in [4, 5, 19] are: (1) a sharp initial peak in both electric and magnetic fields beyond a km or so, (2) a slow ramp following the initial peak for electric fields within a few tens of km, (3) a hump following the initial peak in magnetic field within a few tens of km, (4) a zero crossing within tens of microseconds after the initial peak in both electric and magnetic fields at 50–200 km, and (5) a characteristic flattening of vertical electric field at tens to hundreds of meters.

It can be concluded from these results that magnetic field at a few tens of meters follows the channel-base current waveshape. The feature (1) is valid for all the models, and (2) a slow ramp is obtained with TL model after tens of microseconds, so as with other models. A hump mentioned as the feature (3) is obtained at 5 km with TL and MTLE model, whereas for NCBC just with TL model. A zero crossing mentioned as (4) is obtained for far fields with TL model.
model also, so as with others. The characteristic flattening (5) is obtained with TL model calculated for smaller channel heights. Some of these features are not obtained for all the models if a one-peaked channel-base current is used.

5. Conclusion

New functions for approximating one- and two-peaked lightning channel-base current waveshapes are used in this paper. Parameters of these functions are calculated for the measured channel-base currents, so as for the current waveshape often used in the literature. Similarly derived functions can be used to approximate even more peaks in the current waveshapes which are characteristic for the measured negative first-stroke currents.

The three different “engineering” models (transmission line model, transmission line model with linear decay, and transmission line model with exponential decay with height) are used for calculating lightning electromagnetic field. These results enable analysis of the models efficiency and validity. In this analysis transmission line model with linear decay proved to have some advantages over other two models if a double-peaked channel-base current is used.

The new one- and two-peaked channel-base current functions also have analytically obtained derivatives and integrals, so as Fourier transforms which are very useful for electromagnetic modeling and for application in frequency domain [20, 21]. The next step would be including more peaks in the rising part of the channel-base current to comply better with measurements results of channel-base currents and lightning electromagnetic field components.

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Review Article

Modeling of Coaxial Slot Waveguides Using Analytical and Numerical Approaches: Revisited

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Our reviews of analytical methods and numerical methods for coaxial slot waveguides are presented. The theories, background, and physical principles related to frequency-domain electromagnetic equations for coaxial waveguides are reassessed. Comparisons of the accuracies of various types of admittance and impedance equations and numerical simulations are made, and the fringing field at the aperture sensor, which is represented by the lumped capacitance circuit, is evaluated. The accuracy and limitations of the analytical equations are explained in detail. The reasons for the replacement of analytical methods by numerical methods are outlined.

1. Introduction

Since the 1940s, coaxial slot waveguides have been used as antennas and as cables for electric power. Recently, this type of waveguide has been used in dielectric measurements and the treatment of cancer. Hence, a theoretical formula was needed to design and model the waveguide. Many theoretical formulas have been developed and derived in the past 70 years. Indeed, in the past, the analytical formulas were an important tool for waveguide design because few numerical methods were available and advance computer equipment and software were not yet available to reduce the costs associated with experimental design. Until the 1960s, many reliable numerical techniques were developed and applied for solving electromagnetic problems, such as the finite difference time-domain (FDTD) method [1], the moment method (MoM) [2], and the finite element method (FEM) [3]. At that time, 2D numerical techniques were used to analyze coaxial waveguides because of their simplicity and the symmetrical shapes they produced.

In reality, no completely analytical solutions exist due to the difficulty of deriving the formulas and the many unknown environmental factors that are not included in the formulation. In the author’s view, analytical formulas referred to equations that were based on theoretical concepts and that had no unknown variables. Although the complicated analytical formulas were solved with the aid of numerical methods, such as series expansion and the trapezium rule, the solutions were still obtained by an analytical approach. When a theoretical formula has a matrix of unknown variables and is solved by using a modern numerical routine, such as FEM and MoM, we will define it as a numerical approach. In fact, modern numerical methods can produce results that are close to practical measurements because they divide the problem into many small segments and solve them one by one.
However, the development of powerful numerical techniques does not mean that analytical formulas are not still useful in predicting waveguide performance, and, in addition, many of numerical models are based on analytical formulations. For instance, many online broad-band dielectric measurements involve the prediction of the dielectric properties of samples based on measurements of electric signals, but, unfortunately, those numerical techniques are not suitable for solving this type of inverse problem. Therefore, until now, analytical formulas have continued to be important tools for the analysis of coaxial sensors. Furthermore, the application of numerical methods to electromagnetic fields is very challenging because the method strongly affects the operational wavelength and the size of the discrete mesh grid.

Most of the origin publications for those analytical formulations do not describe in detail what are the weaknesses and limitations of their equations. However, among are some analytical formulas which are quite reliable and accurate for low-frequency modeling. In this paper, the detailed results of our investigation of the restrictions and limitations of frequency-domain analytical models for coaxial waveguides are presented, and the reasons for using numerical methods to replace analytical methods are discussed. In addition, the strengths and weaknesses of numerical methods are analyzed and discussed.

2. Analytical Methods

2.1. Open-Ended Coaxial Line. The earliest work [4, 5] concerned the homogeneous case, in which an air-filled coaxial line with an infinite conducting flange is radiated into the infinite half-space. Simultaneously, the variation expressions for normalized admittance aperture, \( \tilde{Y} \), are given by

\[
\tilde{Y} = \frac{j k_2}{k_1 \ln(b/a)} \frac{d}{d} \left[ \frac{1}{\zeta (\zeta^2 - k_2^2)^{1/2}} \right] \left[ J_0(\zeta a) - J_0(\zeta b) \right]^2
\]

\[ = \frac{G(0)}{Y_o} + \frac{B(0)}{Y_o}. \tag{1} \]

The real part and the imaginary part in (1) are called normalized conductance, \( G(0)/Y_o \), and susceptance, \( B(0)/Y_o \), respectively [4, 5]:

\[
\frac{G(0)}{Y_o} = \frac{k_2}{k_1 \ln(b/a)} \int_0^\infty \frac{d\theta}{\sin \theta} \left[ J_0(k_2 a \sin \theta) - J_0(k_2 b \sin \theta) \right]^2,
\]

\[
\frac{B(0)}{Y_o} = \frac{k_2}{\pi k_1 \ln(b/a)} \int_0^\pi d\phi \left\{ 2 \text{Si}(k_2 a |u|) - \text{Si} \left( \frac{2k_2 a}{\sin \left( \frac{\phi}{2} \right)} \right) - \text{Si} \left( \frac{2k_2 b}{\sin \left( \frac{\phi}{2} \right)} \right) \right\}, \tag{2}
\]

where \( u^2 = a^2 + b^2 - 2 ab \cos \phi \) and \( \text{Si}(x) = \int_0^x \sin t/t \, dt \). Presently, a simplified version of (1) introduced in [6] is the most commonly used equation to calculate the admittance of the open-ended coaxial sensor

\[
\tilde{Y} = \frac{j k_2^2}{\pi k_1 \ln(b/a)} \int_a^b \int_a^b \frac{\exp(-jk_2 r)}{r} \cos \phi \, d\phi \, dr \, dp. \tag{4}
\]

The discontinuity at the open end of the coaxial line is commonly interpreted using admittance expressions (1) and (4) due to the naturally capacitive properties that dominate at the open-end surface (\( z = 0 \)). Finally, analytical equations (1) and (4) can be solved using several classical numerical approaches, such as the series expansion method and the Gaussian quadrature routines. However, the analytical equations only are suitable for ideal conditions in which the component electric field, \( E_z \), has been neglected in (1) and (4) [4], as shown in Figure 1(a). In actual situations, in addition to components \( E_p \) field, the component \( E_z \) also is present on the surface \( z = 0 \), as shown in Figure 1(b).

Typically, component \( E_z \) is neglected to facilitate solving the problem, since components \( E_z \) involve unknown variables in its formulation [7]. So, to get accurate results from the calculations, modern numerical methods have played an important role in determining the values of unknown variables.

In the early 1980s, the admittance model was simplified by using an equivalent circuit, in which the circuit consists...
of two parallel capacitance terms and one conductance term, \( G_o \) [8]:

\[
\tilde{Y}_m = \frac{1}{Y_0} \left[ j\omega(C_1 + \varepsilon_r C_2) + G_o \omega^{5/2} \right].
\] (5)

The two capacitances, \( C_1 \) and \( C_2 \) in (5), are the capacitive properties inside the coaxial line near the aperture probe and the capacitive properties in an external sample under test, respectively. However, the two capacitance terms are not sufficient to represent the equivalent circuit for the aperture probe, which interacts with lossy materials at high-frequency operation. In this study, we propose that inductance, \( L \), and resistance, \( R \), should be taken into account in the circuit model, as shown in Figure 2. In this work, the composition for the \( C, R, \) and \( L \) elements refer to the impedance before it is converted to function admittance. Thus, the normalized input impedance, \( \tilde{Z}_m \), and admittance, \( \tilde{Y}_m \), are given as

\[
\tilde{Z}_m = Y_0 \left[ \frac{1}{j\omega(C_1 + \varepsilon_r C_2)} + j\omega(L_1 + \varepsilon_r L_2) + R \right],
\] (6a)

\[
\tilde{Y}_m = \frac{1}{\tilde{Z}_m}.
\] (6b)

The capacitances \( C_1 \) and \( C_2 \) in (6a) have the same meaning as in (5). The \( L_1 \) and \( L_2 \) are the inductive properties inside the coaxial line near the aperture probe and the inductive properties in an external sample under test, respectively. The \( R \) is the resistive properties at the aperture probe. The values of components \( C_1, L_1, L_2, \) and \( R \) are obtained by optimization between the normalized input impedance, \( \tilde{Z}_m \), and the finite element simulation results, while the \( C_2 \) values in (5) and (6a) are calculated from \( C_2 = 2.388 \varepsilon_r (b - a) \) [9].

The comparison of calculated normalized conductance, \( G(0)/Y_0 \), and susceptance, \( B(0)/Y_0 \), using several models and the simulation results (using COMSOL Multiphysics [10], a finite element analysis simulation software) and measurements for water at room temperature are shown in Figures 2(a) and 2(b), respectively. The calculations for admittance in (4) were conducted by using a \( 5 \times 5 \times 6 \) order Gaussian triple integral method, while (2) and (3) were solved by using the series expansion method with 25 series terms. The dispersive properties of the water in simulation and all admittance calculations were obtained from the Cole-Cole model [11] with \( \varepsilon_r = 78.6, \varepsilon_\infty = 4.22, \tau = 8.8 \) ps, and \( \alpha = 0.013 \). The results measured over the frequency range from 0.3 GHz to 18 GHz were obtained by using a Teflon-filled coaxial sensor with \( a = 0.65 \) mm and \( b = 2.05 \) mm.

From Figures 2(a) and 2(b), the closed form equation (5) with \( C_1 = 0.1 \) pF, \( C_2 = 0.0293 \) pF, and \( G_o = 9 \times 10^{-29} \) is applicable only up to 6 GHz, while (6a) shows that the calculated values are in good agreement with the corresponding simulation results up to 40 GHz. The values for the components \( C_1, L_1, L_2, \) and \( R \) in (6a) were given as \( C_1 = 0.1 \) pF, \( C_2 = 0.0293 \) pF, \( L_1 = 7 \) pH, \( L_2 = 0.9 \) pH, and \( R = 2.8 \) \( \Omega \). Equations (2), (3), and (4) were solved analytically, but with different approaches. The series expansion solution of (2)-(3) agrees with those corresponding admittance results for frequencies only up to 18 GHz and becomes unstable for higher frequencies. This unstable condition may be due to an inaccuracy of the decimal numeral in the series terms. However, the series expansion solutions can be used efficiently to minimize the run time of the program compared to the Gaussian integral method.

2.2. Monopole Driving from Coaxial Line. For the extended-conductor case, the normalized input impedance, \( Z_{in} \), can be derived using the induced electromagnetic field (EMF) method [12] as

\[
\tilde{Z}_{in} = \frac{j(0.5)k_1}{k_2 \ln(b/a)\sin^2(k_2 h)} \int_{0}^{h} \sin[k_2(h - z)]
\times \left[ \frac{e^{-jk_2 R_1}}{R_1} + \frac{e^{-jk_2 R_2}}{R_2} \right] - 2\cos(kh) \frac{e^{-jk_2 r'}}{r'} dz.
\] (7)

While, in [12], listed out the input impedance, \( \tilde{Z}_{in} \) for monopole driven from coaxial line can be written as

\[
\tilde{Z}_{in} = \frac{j(0.5)k_1}{k_2 \ln(b/a)\sin^2(k_2 h)} \int_{0}^{h} \sin[k_2(h - z)]
\times \left[ \frac{e^{-jk_2 R_1}}{R_1} - \cos(kh) \frac{e^{-jk_2 r'}}{r'} \right]
- z \sin(k_2 h) e^{-jk_2 r'} \left( \frac{j}{r'^2} + \frac{1}{k_2 r'^3} \right) dz,
\] (8)

where \( \varepsilon_r \) is the relative permittivity of lossless material filled in coaxial line (at region \( z \leq 0 \)). Equations (7) and (8) can be solved easily by Simpson’s rules. For extended conductor cases, the energy is released through the extended conductor from the end of the coaxial line, thus the inductive properties are dominant at driving point \( (z = 0) \). Hence, the analysis and the prediction of the performance of an extended conductor waveguide are often referred as input impedance, \( Z_{in} = R_{in} + jX_{in} \), rather than input admittance, \( Y_{in} = G_{in} + jB_{in} \).

Figure 3 shows the calculated normalized resistance, \( R_{in}/Z_o \), and the normalized reactance, \( X_{in}/Z_o \), for air at room temperature using (7) and (8). It was found that (7) and (8) give similar resistance values, \( R_{in}/Z_o \), in the frequency range of 300 kHz to 20 GHz. Meanwhile, both equations give less consistent values of reactance, and this may be due to the fact that the field distribution near the source point was different between the two models. The impedance integral equations are important in analytical calculations and for numerical solutions. Numerical solutions, such as MoM, cannot be used if the problems have no established analytical integral equation. Moreover, the accuracy of the MoM method is
The distribution current, $I_z$, substituted in (7) and (8) was assumed to have a convenient sinusoidal form, such as

$$I_z = \frac{I(0) \sin k_2(h - z)}{\sin(k_2 h)},$$

where $I(0)$ is the amplitude of the driving point current at $z = 0$, and it was canceled out in the derivations in (7) and (8).
(8). Until now, no completely actual distribution current, \( I_z \), formulation has existed for an arbitrarily sized extended conductor. The current calculated using (9) is clearly the real value, but, in an actual case, the current is a complex value due to the azimuthal current, \( I_\phi \), contributed by the size of the radius of conductor \( a \), as shown in Figure 4(b). This means that (9) is valid only for thin extended conductors. In addition, it cannot be guaranteed that the current distribution will always be sinusoidal in all cases of the extended conductor.

Figure 5 shows the variational of distribution current, \( I_z \), along the length, \( z \), of monopole. The unknown complex current, \( I_z \), is determined by using point matching MoM with Pocklington’s integral equation and frill-generator source \([14, 16, 17]\). For cylindrical monopole structure, the magnetic field, \( H_z \), around the surface monopole which is generated by a current, \( I_z \), is in an azimuthal, \( \phi \), direction, and the relationship is given as \( H_\phi = I_z / 2\pi a \phi \). This means that the imaginary part of current, \( I_\phi \), is referring to the current flow in \( z \)-direction (perpendicular to the \( \phi \) direction). While, the real part of \( I_z \) is represented in the azimuthal current. From Figure 5, it is clear that, at low frequencies (\( f < 3 \) GHz), the value of current, \( I_z \), is approaching the real number. When the monopole operated at higher frequencies, the distribution current, \( I_z \), along the length of monopole become complex number.

In addition to the uncertainty of the distribution current, (7) and (8) also have not considered the fringing fields contributed by the electric field, \( E_\rho \), near the driving point (\( z = 0 \)) and the end of the terminate conductor (\( z = h \)). For a short conductor, (\( h < 10 \ a \)), driven from a coaxial line, the input impedance is affected significantly by the fringing field at the end of the coaxial line, especially for high-frequency operation \([18]\). This means that the analytical impedance formulas of (7) and (8) are accurate only for the low-frequency condition. For the reasons stated, the full wave (\( H_\phi, E_\rho, \) and \( E_z \)) analysis of the waveguide must be conducted by using a numerical method. In practice, the effects of fringing fields at the driving point are always empirically corrected by capacitance, \( C_f \), element circuits.

In addition to integration models, the input impedance of the monopole also can be represented by using the lumped-element model \([19]\) and expressed as

\[
Z = j \left( \frac{\omega L_1}{\omega C_1} \right) + \frac{1}{j \omega C_2 + \left( 1/(j \omega L_2 + R_1) \right) \text{ Low frequencies}} + \frac{1}{j \omega C_3 + \left[ 1/(j \omega L_3 + (50 j \omega L_4/(j \omega L_4 + 50))) \right] \text{ Driving fringing fields}}.
\]

From Figures 7(a) and 7(b), the input impedance show that only two peaks of curve line occurred from 300 kHz to 20 GHz, the three terms of (10) are sufficient to model the impedance properties cover the frequency range. The first term in (10) describes the characteristics of the impedance at low frequencies, while the second term contributes to the modeling of impedance for high frequencies (>10 GHz). The third term is used to model the equivalent circuit, which is near the driving point for the monopole. This means that if there are three peaks of impedance curve line (See Figure 14) over 300 kHz to 20 GHz, up to four terms of lumped-element expression are required. The equivalent circuit and all optimized values of resistance, \( R \), capacitance, \( C \), and inductance, \( L \), elements in (10) for air are shown in Figure 6(b), and the values are accurate up to 20 GHz. The \( RLC \) values in (10) are obtained by optimizing (10) to the measurement results. At low frequencies, the capacitance term, \( C_1 \), in (10) has played an important role in the modeling, since the input impedance at driving point (\( z = 0 \)) shows the nature of capacitive properties (\( X_{in}/Z_o \) in negative (-) sign) at a frequency of less than 4 GHz, as shown in Figure 7(b).

For instance, a monopole with \( h = 1.436 \) cm driven from the coaxial line, as shown in Figure 6(a), was tested in this work. The comparison of calculated normalized resistance, \( R_{in}/Z_o \), and reactance, \( X_{in}/Z_o \), using several models, moment method \([14, 16, 17]\), simulation results (using COMSOL Multiphysics \([10]\) and CST Microwave Studio \([20]\)), and measurements for air at room temperature are shown in Figures 7(a) and 7(b), respectively. Different from the coaxial probe, for a monopole, the fringing field capacitance, \( C_f \), elements first refer to the admittance before being converted to the impedance function, as follows:

\[
\tilde{Y} = \frac{1}{\text{Equation (7)}} + j \omega C_f, \quad (11a)
\]

\[
\tilde{Z}_{\text{corrected}} = \frac{1}{\tilde{Y}}. \quad (11b)
\]

In this work, the value of \( C_f = 5.5 \) pF. Figure 7 shows that a 1.436 cm monopole has been matched with a standard, 50 \( \Omega \) cable at a frequency of 4.6 GHz and 14.4 GHz.

### 2.3. Coupling Monopole Driving from Coaxial Line.

When two different lengths of monopoles are placed closed to each other, as shown in Figure 8(a), the combination of two electromagnetic fields occurs. Now, the input impedance of the monopole is required to consider the self-radiation and mutual-radiation effects.

The mutual normalized impedance, \( \tilde{Z}_{12} \), is well expressed as \([21]\):

\[
\tilde{Z}_{12} = \frac{j(0.5) \ k_1}{k_2 \ ln(b/a)} \ sin(k_2 h_1) \ sin(k_2 h_2) \times \int_{0}^{h_2} \ sin(k_2 (h_2 - z)) \times \left[ e^{-jk_2 R_1} - e^{-jk_2 R_2} \right] dz, \quad (12)
\]
Figure 4: (a) Ideal analytical line form current; (b) Actual situations of current distribution on a cylindrical extended conductor.

Figure 5: Simulated distribution current, $I_z$, on the 1.436 cm of monopole driving from coaxial line with $a = 0.65$ mm and $b = 2.05$ mm.
where $R'_1 = \sqrt{D^2 + (z-h_1)^2}$, $R'_2 = \sqrt{D^2 + (z+h_1)^2}$, and $R' = \sqrt{D^2 + z^2}$. Finally, the input impedance, $\tilde{Z}_{in\_port-1}$, of monopole 1 can be calculated as [15, 21, 22]:

$$\tilde{Z}_{in\_port-1} = \tilde{Z}_{11} - \frac{\tilde{Z}_{12}}{\tilde{Z}_{22}},$$  \hspace{1cm} (13)$$

where the term $\tilde{Z}_{11} = (7)$ is assumed to be equal to $\tilde{Z}_{22}$, and $\tilde{Z}_{21}$ is equal to $\tilde{Z}_{12}$. Similarly, the input impedance, $Z_{in\_port-1}$, of the coupling monopole also can be represented by using the lumped-element model and expressed as

$$Z_{in\_port-1} = j\left(\frac{\omega L_1 - \frac{1}{\omega C_1}}{\text{Low frequencies}} + \frac{1}{\frac{j\omega C_2}{\text{High frequencies}} + \frac{1}{(j\omega L_2 + R_1)} + \frac{1}{\frac{j\omega C_3}{\text{Driving fringing fields}} + \frac{1}{(j\omega L_3 + (50j\omega L_4/(j\omega L_4 + 50))}}}}\right).$$  \hspace{1cm} (14)
The equivalent circuit for (14) and its resistance, $R$, capacitance, $C$, and inductance, $L$, elements for air, which are accurate up to 20 GHz, are shown in Figure 9. The $RLC$ values in equation (14) are obtained by optimizing (14) to the measurement results. From the comparison of calculated normalized resistance, $R_{in}/Z_o$, and reactance, $X_{in}/Z_o$, using several models, the CST simulation results and measurements for air at room temperature are shown in Figures 10(a) and 10(b), respectively.

The exponential term, $e^{-(\alpha+\beta)d'}$, of the coated monopole at the drive point $(z = 0)$ [23–25] is expressed as:

$$ T = \left( 1 - \frac{\text{Equation (14)} - Z_o}{\text{Equation (14)} + Z_o} \right) e^{-(\alpha+\beta)d'}, $$ \hspace{1cm} (15a)

$$ Z_{\text{in, port-2}} = 2Z_o \left( 1 - \frac{T}{T'} \right), $$ \hspace{1cm} (15b)

where $T$ is the transmission coefficient at parasitic port-2. The exponential term, $e^{-(\alpha+\beta)d'}$, is the transmission factor of the transmitted waves from port-1 to port-2. The symbols $\alpha$ and $\beta$ are the attenuation constant and phase constant, respectively. The equation $d' = h_1 + h_2 + D + \Delta$ gives the length of the transmission wave from port-1 to port-2. The normalized input impedances, $Z_{\text{in, port-2}} = R_{in} + jX_{in}$, calculated by using (15b), are plotted and compared with measurement and CST simulation results, as shown in Figure 11.

The normalized resistance, $R_{in}/Z_o$, and reactance, $X_{in}/Z_o$, in Figures 11(a) and 11(b) are calculated with $\alpha = 3.18 \times 10^{-5}$ neper/m, $\beta = 2\pi f/c$ rad/m, $d' = 0.044$ m and $Z_o = 50\Omega$.

2.4. Coated Conductor Driving from Coaxial Line. A coated monopole is a bare monopole enclosed by a thin cylindrical low-loss dielectric material with an outer radius, $b$, as shown in Figure 12.

In general, the transmission line formulas are adapted easily to the analysis of a coated antenna. The input impedance, $\tilde{Z}_{\text{in}}$, of the coated monopole at the drive point $(z = 0)$ is expressed as:

$$ \tilde{Z}_{\text{in}} = -j\frac{Z_o}{Z_o} \cot(k_h l), $$ \hspace{1cm} (16)

where $Z_o$ and $Z_c$ are the characteristic impedance in the coaxial line and the characteristic impedance for the coated monopole transmission line, respectively, which can be given as:

$$ Z_c = \frac{y k_L}{2\pi k_e} \left[ \ln \left( \frac{b}{a} \right) + \left( \frac{k_z}{k_h} \right)^2 \frac{H_0^{(2)}(k_z b)}{k_z^2 H_1^{(2)}(k_z b)} \right]. $$ \hspace{1cm} (17)

The complex propagation constant, $k_L$, is expressed as:

$$ k_L = k_z \left[ \frac{H_0^{(2)}(k_z b) + k_z b \ln(b/a)H_1^{(2)}(k_z b)}{k_z^2 H_0^{(2)}(k_z b) + k_z^2 b \ln(b/a)H_1^{(2)}(k_z b)} \right]^{1/2}. $$ \hspace{1cm} (18)
The transmission line model requires that the propagation constant, $k_2$, of the external medium be much larger than the propagation constant, $k_e$, of the insulation [23–25]. Thus, the coated monopole is almost always used in an ambient medium with higher dielectric properties than its insulation, such as soil, seawater, or biological tissue. Moreover, coating material, such as Teflon, functions as a hygienic protector when the monopole is immersed in a biological sample. However, for the case in Figure 12, the fringing effects at the top end of the monopole are required to consider when its input impedance is calculated using (16). In this study, the effective length, $h'$, of the monopole was used to correct the fringing effects, and it is given as

$$h' = h + \frac{k}{\sqrt{f}},$$  \hspace{1cm} (19)
where $h$ and $f$ are the actual length of the coated monopole and the operational frequency, respectively. The symbol $\kappa$ is a coefficient value which depends on the dimensions of the monopole. The fringing effects are assumed to be inversely proportional to the square root of frequency. Similarly, the equivalent circuit can be used to represent the input impedance properties of the coated monopole, as shown in Figure 12, and the corresponding formulations are expressed as (20): 

$$Z_{in} = j\left(\omega L_T - \frac{1}{\omega C_T}\right) + \sum_{n=1}^{m} \left[ j\omega C_n + \frac{1}{1/(j\omega L_n + R_n)} \right]$$

$$+ \frac{1}{j\omega C_B + [1/(j\omega L_B + (50j\omega L'_B/(j\omega L'_B + 50)))]}.$$  

(20)

Table 1: $RCL$ component values in (20) for air and water samples.

| $RCL$ components | Air    | Water   |
|------------------|--------|---------|
| $C_B$            | 0.33 pF| 0.70 pF |
| $C_T$            | 0.40 pF| 2.50 pF |
| $C_1$            | 0.54 pF| 0.65 pF |
| $C_2$            | —      | 0.45 pF |
| $L_B$            | 1 nH   | 1 nH    |
| $L'_B$           | 1.1 nH | 2.4 nH  |
| $L_T$            | 0.5 nH | 0.35 nH |
| $L_1$            | 0.2 nH | 0.32 nH |
| $L_2$            | —      | 0.16 nH |
| $R_1$            | 2.9 \Omega | 5.56 \Omega |
| $R_2$            | —      | 4.0 \Omega |
the frequency range from 300 kHz to 20 GHz. The results were measured using a Teflon-coated monopole driven from coaxial line with \( a = 0.65 \) mm, \( c = 2.05 \) mm, and \( h = 13.92 \) mm. The values for component resistor, \( R \), reactance, \( L \), and capacitor, \( C \), in (20) for air and water are listed in Table 1, which provides the accurately calculated impedance up to 20 GHz. The RLC values in (20) are obtained by optimizing (20) to the measurement results. As expected, compared to the water sample, the calculated normalized resistance, \( R_{in}/Z_o \), and reactance, \( X_{in}/Z_o \), for air using (16) do not agree well with the measurements and simulation results.

3. Conclusions

In this work, the coaxial waveguide was used as an example of the problem of linking electromagnetic theory with practical modeling, since the coaxial slot waveguides have been used as antennas over the past of 70 years. Recently, many scientific applications have involved this kind of waveguide. For instance, an open-ended coaxial probe was applied as a dielectric probe to measure the dielectric properties of the material being tested. In addition, the dielectric-coated monopole was used for hyperthermia treatment, and the coupler monopole was designed to be an array antenna. Hence, many models have been developed for the coaxial waveguide, and those models have been modified for use in modeling other devices. In particular, the sinusoidal current model is also used for planar waveguides. In this study, the accuracy of frequency-domain analytical models was tested by acquiring measurements and numerical simulation results. We found that the semiempirical equivalent circuit modeling worked successfully, covered a wide frequency range, and was very useful in the design of circuits that matched the waveguides. Although the analytical models are less accurate compare to numerical method, it provides significant rapid and economize computation. Implicitly, the analytical models still retain the academy valuable, especially for who has preliminary study of the antenna modeling.

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Review Article

Electromagnetic Field Coupling to Overhead Wire Configurations: Antenna Model versus Transmission Line Approach

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The paper deals with two different approaches for the analysis of electromagnetic field coupling to finite length overhead wire: the wire antenna theory (AT) and the transmission line (TL) method. The analysis is carried out in the frequency and time domain, respectively. Within the frequency domain analysis the wire antenna formulation deals with the corresponding set of Pocklington integrodifferential equation, while the transmission line model uses the telegrapher’s equations. The set of Pocklington equations is solved via the Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM), while the telegrapher’s equations are treated using the chain matrix method and the modal equation to derive per-unit-length parameters. For the case of the time domain analysis AT model uses the space-time Hallen integral equation set, while TL approach deals with the time domain version of the telegrapher’s equations. Hallen equations are handled via time domain version of GB-IBEM, while time domain telegrapher’s equations are solved by using Finite Difference Time Domain (FDTD) method. Many illustrative computational examples for the frequency and time domain response, respectively, for several configurations of overhead wires, obtained via different approaches, are given in this paper.

1. Introduction

The electromagnetic field coupling to overhead wires is of great practical interest for many EMC applications [1–11], such as transient excitation of antennas, power, or communications cables. The electromagnetic field coupling to finite length overhead wires can be determined by means of the transmission line model or the thin wire antenna theory in either frequency or time domain [1]. In particular, the transient response of a wire configuration of interest can be computed directly, by solving the related time domain equations or by the indirect approach, that is, by solving their frequency domain counterpart. When the indirect approach is used the frequency spectrum has to be calculated, and then the transient response is computed by means of the Inverse Fourier Transform (IFT).

Many practical engineering problems dealing with electromagnetic field coupling to thin wires can be analyzed by using the Transmission Line (TL) models [1–6]. These models include the analysis of incident electromagnetic field exciting the line and the propagation of induced currents and voltages along the line.

The TL models yield valid results if the line length is significantly larger than the separation between the wires and also larger than the actual height above ground [8].

On the other hand the TL approximation cannot provide a complete solution for the excitation of a given wire configuration by an incident field if the wavelength of the electromagnetic field exciting a wire structure is comparable to or less than the transverse electrical dimensions of the structure. Namely, the TL model fails to predict resonances and accounts for the presence of a lossy ground only
approximately [1]. One of the serious problems with TL approach occurs due to the fact that current grows to infinity at resonant points as there are no losses and radiation resistance to limit its flow [8]. The full wave approach, based on the wire antenna theory and related integral equations, is more rigorous and should be used whenever the above-ground transmission lines of the finite length are considered. However, a serious drawback of AT approach is rather long computational time required for the calculations pertaining to long lines.

This paper deals with the analysis of electromagnetic field coupling to overhead wires in either frequency or time domain by using both antenna model and transmission line approach, respectively.

A number of illustrative computational examples regarding electromagnetic coupling to overhead wires are given in the paper.

The aboveground wires are subjected to electromagnetic fields arriving from a distant source and inducing current to flow along the wires. The key to understanding the behaviour of induced fields is the knowledge of current distribution induced along the wires. These currents generate scattered fields propagating away from the equipment.

The paper is organized as follows: Section 2 deals with the frequency domain analysis followed by related numerical solution methods for overhead wires. Section 2 ends up with many illustrative examples related to the aboveground lines and PLC (power line communications) systems.

Section 3 outlines the time domain analysis and related method of solutions of governing equations. Some computational examples pertaining to the multiconductor above-ground lines are given. Finally, the conclusion summarizes what has been discussed throughout this work.

2. Frequency Domain Models and Methods

This section deals with the wire antenna theory and transmission line (TL) approximation, respectively, for the analysis of electromagnetic field coupling to overhead lines of finite length in the frequency domain. The formulation arising from the wire antenna theory is based on the set of coupled Pocklington integrodifferential equation for half-space problems. The effect of a two-media configuration is taken into account by means of the reflection coefficient approximation [12]. The resulting integro-differential expressions are numerically handled via the frequency domain Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM) [8].

Transmission line model in the frequency domain is based on the corresponding telegrapher’s equations which are handled by using the chain matrix method [10].

2.1. Antenna Theory Approach: Set of Coupled Pocklington Equations. Modeling of arbitrarily shaped wires located at different heights above a lossy ground is an important task in both antenna and electromagnetic compatibility (EMC) studies [1].

This section firstly deals with an assessment of the current induced along multiple wire configurations above a lossy ground. Once the currents along the wire array have been obtained, the radiated field components could be determined.

The set of Pocklington equations for a configuration of overhead wires can be obtained as an extension of the Pocklington integro-differential equation for a single wire of arbitrary shape. The Pocklington equation for a single wire above a lossy ground can be derived by enforcing the continuity conditions for the tangential components of the electric field along the perfectly conducting (PEC) wire surface. First, a single wire of arbitrary shape, insulated in free space, as shown in Figure 1 is considered.

For the PEC wire the total field composed from the excitation field $\vec{E}^{\text{exc}}$ and scattered field $\vec{E}^{\text{scat}}$ vanishes [1, 8]:

$$\vec{e}_x \cdot (\vec{E}^{\text{exc}} + \vec{E}^{\text{scat}}) = 0 \quad \text{on the wire surface.} \quad (1)$$

Starting from Maxwell’s equations and Lorentz gauge the scattered electric field can be expressed in terms of the vector potential $A$:

$$\vec{E}^{\text{scat}} = -j \omega \vec{A} + \frac{1}{j \omega \mu \epsilon} \nabla \left( \nabla \cdot \vec{A} \right). \quad (2)$$

The vector potential is defined by the particular integral over a given path $C$ (considered conductive wire structure):

$$\vec{A}(s) = \frac{H}{4\pi} \int_C I(s') g_0(s, s') s' ds', \quad (3)$$

where $I(s')$ is the induced current along the line and $g_0(s, s')$ denotes the lossless medium Green function:

$$g_0(s, s') = \frac{e^{-jkR}}{R}, \quad (4)$$

and $R$ is the distance from the source point to the observation point, respectively, while the propagation constant of the homogeneous medium is given by

$$k^2 = \omega^2 \mu \epsilon_0. \quad (5)$$

![Figure 1: Single wire of arbitrary shape in free space.](image-url)
Inserting (3) into (2) gives the relation for the scattered electric field:

\[ \mathbf{E}^{\text{sc}} = \frac{1}{j4\pi\omega \epsilon_0} \int_C I(s') \cdot \hat{s'} \cdot [k^2 + \nabla \nabla] \mathbf{g}_0(s, s') ds'. \]  

(6)

Combining (6) and (1) results in the Pocklington integral equation for the unknown current distribution along the wire of arbitrary shape insulated in free space:

\[ E_{\text{tan}}^{\text{exc}}(s) = -\frac{1}{j4\pi\omega \epsilon_0} \int_C I(s') \cdot \hat{s'} \cdot [k^2 + \nabla \nabla] \mathbf{g}_0(s, s') ds'. \]  

(7)

where \( E_{\text{tan}}^{\text{exc}} \) denotes the tangential component of the electric field illuminating the wire.

Now the case of curved wire located above an imperfectly conducting ground can be analyzed by extending integro-differential equation (7) using the reflection coefficient approach [12]. The geometry of an arbitrary wire and its image, respectively, is shown in Figure 2.

The excitation function \( E^{\text{exc}} \) is now composed from incident and reflected field, respectively,

\[ E^{\text{exc}} = E^{\text{inc}} + E^{\text{ref}}. \]  

(8)

Performing certain mathematical manipulations the Pocklington integro-differential equation for a curved wire above a lossy ground becomes [12]

\[ E_{\text{tan}}^{\text{exc}}(s) = \frac{j}{4\pi\omega \epsilon_0} \int_0^L \left[ k^2 \mathbf{e}_p \mathbf{e}_p - \frac{\partial^2}{\partial s \partial s'} \right] \mathbf{g}_0(s, s') ds' \]
\[ + R_{\text{TM}} \left[ k^2 \mathbf{e}_n \mathbf{e}_n - \frac{\partial^2}{\partial s \partial s'} \right] g(s, s^*) ds' \]
\[ + (R_{\text{TE}} - R_{\text{TM}}) \mathbf{e}_n \mathbf{e}_n \]
\[ \cdot \left[ k^2 \mathbf{e}_p \mathbf{e}_p - \frac{\partial^2}{\partial p \partial s'} \right] g_i(s, s^*) ds', \]  

(9)

where \( \mathbf{e}_p \) is the unit vector normal to the incident plane, while \( g_i(s, s^*) \) arises from the image theory and is given by

\[ g_i(s, s^*) = \frac{e^{-jkr^*}}{R^*}, \]  

(10)

and \( R^* \) is the distance from the image source point to the observation point, respectively.

An extension to the case of multiple curved wires is straightforward, that is, it follows [12]

\[ E_{\text{tan}}^{\text{exc}}(s) = \frac{j}{4\pi\omega \epsilon_0} \sum_{n=1}^{N_w} \int_0^L \left[ k^2 \mathbf{e}_n \mathbf{e}_n - \frac{\partial^2}{\partial s \partial s'} \right] \mathbf{g}_0(s, s') ds' \]
\[ + R_{\text{TM}} \left[ k^2 \mathbf{e}_n \mathbf{e}_n - \frac{\partial^2}{\partial s \partial s'} \right] g_i(s, s^*) ds' \]
\[ + (R_{\text{TE}} - R_{\text{TM}}) \mathbf{e}_n \mathbf{e}_n \]
\[ \cdot \left[ k^2 \mathbf{e}_p \mathbf{e}_p - \frac{\partial^2}{\partial p \partial s'} \right] g_i(s, s^*) ds', \]  

(11)

where \( N_w \) is the total number of wires and \( I_i(s_i') \) is the unknown current distribution induced on the wire. Furthermore, \( g_{0mn}(s, s') \) and \( g_{imn}(s, s') \) are the Green functions of the form

\[ g_{0mn}(s, s') = \frac{e^{-jkr_{1mn}}}{R_{1mn}}, \]
\[ g_{imn}(s, s') = \frac{e^{-jkr_{2mn}}}{R_{2mn}}, \]  

(12)

where \( R_{1mn} \) and \( R_{2mn} \) are distances from the source point and from the corresponding image, respectively, to the observation point of interest.

The influence of a lossy half-space is taken into account via the Fresnel plane wave reflection coefficient (RC) for TM and TE polarization, respectively [12],

\[ R_{\text{TM}} = \frac{n \cos \theta' - \sqrt{n - \sin^2 \theta'}}{n \cos \theta' + \sqrt{n - \sin^2 \theta'}}, \]  

(13)
\[ R_{\text{TE}} = \frac{\cos \theta' - \sqrt{n - \sin^2 \theta'}}{\cos \theta' + \sqrt{n - \sin^2 \theta'}}, \]  

(14)

where \( \theta' \) is the angle of incidence and \( n \) is given by

\[ n = \frac{\epsilon_{\text{eff}}}{\epsilon_0}, \quad \epsilon_{\text{eff}} = \epsilon_r \epsilon_0 - j \frac{\sigma}{\omega}, \]  

(15)

and \( \epsilon_{\text{eff}} \) is the complex permittivity of the ground.

For the special case of single horizontal straight wire above a lossy half-space, Figure 3 integro-differential equation (9) simplifies into

\[ E_{\text{tan}}^{\text{exc}} = \frac{j \omega \mu}{4\pi} \int_0^L g_0(x', x') dx' \]
\[ - \frac{1}{j4\pi\omega \epsilon_0} \int_0^L \frac{\partial}{\partial x'} g(x', x') dx', \]  

(16)
where $I(x')$ is the induced current along the horizontal wire and $g(x,x')$ denotes the Green’s function given by

$$g(x,x') = g_0(x,x') - R_{TM} g(x,x'). \quad (17)$$

Furthermore, if an array of multiple horizontal wires is considered (Figure 4), system of (11) becomes

$$E_{\text{exc}} = -\frac{1}{j4\pi\omega\varepsilon_0} \sum_{n=1}^{Nw} \int_0^L \left[ \frac{\partial^2}{\partial x'\partial x} + k_1^2 \right] g_{mn}(x,x') I_n(x') dx', \quad m = 1, 2, \ldots, M, \quad (18)$$

where $I_n(x')$ is the unknown current distribution induced along nth wire, $E_{\text{exc}}$ is the known excitation field tangential to the jth wire surface, and $g_{mn}$ is the corresponding Green function:

$$g_{mn}(x,x') = g_{0mn}(x,x') - R_{TM} g_{mn}(x,x'). \quad (19)$$

It is worth noting that a trade-off between the rigorous Sommerfeld integral approach and approximate RC approach is presented in [12]. Although reflection coefficient approximation causes certain error (up to 10%) it takes a significantly less computational effort than a rigorous Sommerfeld approach [8].

The total electric field irradiated by configuration of multiple wires of arbitrary shape is given by [13, 14]

$$\vec{E} = \sum_{n=1}^{Nw} \left[ \vec{E}_0 + R_{TM} \vec{E}_n + (R_{TM} - R_{TE})(\vec{E}_n \cdot \vec{e}_p) \vec{e}_p \right], \quad (20)$$

where

$$\vec{E}_0 = \frac{1}{j4\pi\omega\varepsilon_0} \int_0^L \left[ k_1^2 \int_0^L \vec{e}_p I_n(s_n) g_0(\vec{r}, \vec{r}') ds_n \right. + \left. \int_0^L \frac{\partial I_n(s_n)}{\partial s_n} \nabla g_0(\vec{r}, \vec{r}') ds_n \right], \quad (21)$$

$$\vec{E}_n = \frac{1}{j4\pi\omega\varepsilon_0} \int_0^L \left[ k_1^2 \int_0^L \vec{e}_n I_n(s_n) g_n(\vec{r}, \vec{r}') dw' \right. - \left. \int_0^L \frac{\partial I_n(s_n)}{\partial s_n} \nabla g_n(\vec{r}, \vec{r}') ds' \right]. \quad (22)$$

Note that index 0 and i are related to the source and image wire, respectively.

For the special case of single horizontal straight wire above a lossy half-space (Figure 3), it follows [15]

$$E_x = \frac{1}{j4\pi\omega\varepsilon_0} \int_0^L \left[ \frac{\partial I(\vec{x}')}{\partial x} \frac{\partial g(x,\vec{x}')}{\partial x} \, dx' \right. + \left. k^2 \int_0^L g(x,\vec{x}') I(x') \, dx' \right], \quad (22)$$

$$E_y = \frac{1}{j4\pi\omega\varepsilon_0} \int_0^L \frac{\partial I(\vec{x}')}{\partial y} \frac{\partial g(x,\vec{x}')}{\partial y} \, dx', \quad (23)$$

$$E_z = \frac{1}{j4\pi\omega\varepsilon_0} \int_0^L \frac{\partial I(\vec{x}')}{\partial z} \frac{\partial g(x,\vec{x}')}{\partial z} \, dx'. \quad (24)$$

For the case of multiple horizontal wires the expressions for electric field are given by [15]

$$E_x = \frac{1}{j4\pi\omega\varepsilon_0} \sum_{n=1}^{Nw} \int_0^L \left[ \frac{\partial I_n(\vec{x}')}{\partial x} \frac{\partial g_{mn}(x,\vec{x}')}{\partial x} \, dx' \right. + \left. k^2 \int_0^L g_{mn}(x,\vec{x}') I_n(x') \, dx' \right], \quad (25)$$

$$E_y = \frac{1}{j4\pi\omega\varepsilon_0} \sum_{n=1}^{Nw} \int_0^L \frac{\partial I_n(\vec{x}')}{\partial y} \frac{\partial g_{mn}(x,\vec{x}')}{\partial y} \, dx', \quad (26)$$

$$E_z = \frac{1}{j4\pi\omega\varepsilon_0} \sum_{n=1}^{Nw} \int_0^L \frac{\partial I_n(\vec{x}')}{\partial z} \frac{\partial g_{mn}(x,\vec{x}')}{\partial z} \, dx'. \quad (27)$$

The radiated magnetic field of the curved wire system can be written as follows [13, 14]:

$$\vec{H} = \sum_{n=1}^{Nw} \left[ \vec{H}_0 + R_{TE} \vec{H}_n + (R_{TM} - R_{TE})(\vec{H}_n \cdot \vec{e}_p) \vec{e}_p \right], \quad (28)$$
where
\[
\begin{align*}
\tilde{H}_{Sn} &= -\frac{1}{4\pi} \int_{0}^{L} I_s(r) \frac{\partial \vec{E}_n}{\partial \xi} \times \nabla g_{ln}(\tilde{r}, \tilde{r}') \, ds', \\
\tilde{H}_{jn} &= -\frac{1}{4\pi} \int_{0}^{L} I_j(r) \frac{\partial \vec{E}_n}{\partial \xi} \times \nabla g_{jn}(\tilde{r}, \tilde{r}^\prime) \, ds'.
\end{align*}
\] (29)

The reduction to the case of a single straight wire or straight wire array is straightforward, as in the case of electric field given by (22)–(27).

2.2. Numerical Solution. The set of Pocklington integrodifferential equations (11) has been solved by using the Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM). An outline of the method is given here, for the sake of completeness while the method has been presented in detail elsewhere, for example, in [8].

Performing the Galerkin-Bubnov scheme of (GB-IBEM) in the frequency domain the set of coupled integrodifferential equations (11) is transformed into the following matrix equation [13]

\[
\sum_{n=1}^{M} \sum_{n=1}^{N} [Z]_{ij}^f \{V\}_j^f = \{V\}_j^f,
\] (30)

where the mutual impedance matrix is given by [13]:

\[
[Z]_{ij}^f = -\int_{-1}^{1} \left\{ D\right\}_j^f \left\{ D\right\}_j^f \frac{\partial d_m}{\partial \xi} \frac{d_m}{d\xi} \, d\xi + k_{j}^f \frac{\partial \vec{E}_n}{\partial \xi} \frac{\partial d_m}{\partial \xi} \frac{d_m}{d\xi} \, d\xi - R_{TM} \int_{-1}^{1} \left\{ D\right\}_j^f \left\{ D\right\}_j^f \frac{\partial d_m}{\partial \xi} \frac{d_m}{d\xi} \, d\xi + R_{TM} \frac{\partial \vec{E}_n}{\partial \xi} \frac{\partial d_m}{\partial \xi} \frac{d_m}{d\xi} \, d\xi \times \int_{-1}^{1} \left\{ f\right\}_j^f \left\{ f\right\}_j^f \frac{\partial \vec{E}_n}{\partial \xi} \frac{\partial d_m}{\partial \xi} \frac{d_m}{d\xi} \, d\xi + \frac{j}{4\pi \omega \epsilon_0} \int_{-1}^{1} \left\{ f\right\}_j^f \left\{ f\right\}_j^f \frac{\partial d_m}{\partial \xi} \frac{d_m}{d\xi} \, d\xi,
\] (31)

while the voltage vector is given by [13]

\[
\{V\}_j^f = -j4\pi \omega \epsilon_0 \int_{-1}^{1} E_{sc}^f(s_n) \frac{d_m}{d\xi} \frac{d_m}{d\xi} \, d\xi.
\] (32)

Once the current distribution is obtained, the radiated field can be obtained applying the similar BEM formalism [13]. Thus, the total field is given by

\[
\tilde{E} = \sum_{k=1}^{N} \left[ \tilde{E}_{Sk} + R_{TM} \tilde{E}_{jk} + (R_{TE} - R_{TM}) \left( \tilde{E}_{jk} \cdot \vec{e}_p \right) \tilde{e}_p \right],
\] (33)

where the field components due to a wire segment radiation are given by

\[
\begin{align*}
\tilde{E}_{Sk} &= \frac{1}{j4\pi \omega \epsilon_0} \sum_{i=1}^{N} k_i^f \int_{-1}^{1} I_{ik} \frac{\partial f_i(\xi)}{\partial \xi} \frac{d_m}{d\xi} \frac{d_m}{d\xi} \, d\xi, \\
\tilde{E}_{jk} &= \frac{1}{j4\pi \omega \epsilon_0} \sum_{i=1}^{N} k_i^f \int_{-1}^{1} I_{ik} \frac{\partial f_i(\xi)}{\partial \xi} \frac{d_m}{d\xi} \frac{d_m}{d\xi} \, d\xi. 
\end{align*}
\] (34)

The total magnetic field is given by [13]

\[
\tilde{H} = \sum_{k=1}^{N} \left[ \tilde{H}_{Sk} + R_{TM} \tilde{H}_{jk} + (R_{TE} - R_{TM}) \left( \tilde{H}_{jk} \cdot \tilde{e}_p \right) \tilde{e}_p \right],
\] (35)

while the magnetic field components are given by [13]

\[
\begin{align*}
\tilde{H}_{Sk} &= -\frac{1}{4\pi} \sum_{i=1}^{N} I_{ik} f_i(\xi) \frac{\partial \vec{E}_n}{\partial \xi} \frac{\partial d_m}{\partial \xi} \frac{d_m}{d\xi} \, d\xi, \\
\tilde{H}_{jk} &= -\frac{1}{4\pi} \sum_{i=1}^{N} I_{ik} f_i(\xi) \frac{\partial \vec{E}_n}{\partial \xi} \frac{\partial d_m}{\partial \xi} \frac{d_m}{d\xi} \, d\xi.
\end{align*}
\] (36)

The reduction to the case of a single straight wire or straight wire array is straightforward and can be found elsewhere, for example, in [8].

2.3. Transmission Line Approximation: Telegrapher’s Equations in the Frequency Domain. Voltages and currents along the multiconductor transmission line shown in Figure 4 induced by an external field excitation can be obtained using the field-to-transmission line matrix equations in the frequency domain [10]:

\[
\begin{align*}
\frac{d}{dx} \left[ \tilde{V}(x) \right] + \left[ \tilde{Z} \right] \cdot \left[ \tilde{I}(x) \right] &= -j\omega \mu_0 \int_{0}^{h} \left[ \tilde{H}_{yc}^e(x, z) \right] dz, \\
\frac{d}{dx} \left[ \tilde{I}(x) \right] + \left[ \tilde{Y} \right] \cdot \left[ \tilde{V}(x) \right] &= -j\omega \mu_0 \int_{0}^{h} \left[ \tilde{E}_{zc}^e(x, z) \right] dz,
\end{align*}
\] (37)

where the longitudinal impedance matrix is given by

\[
\left[ \tilde{Z} \right] = j\omega [L] + [Z_w] + [Z_e],
\] (38)

and the transversal admittance matrix can be written as

\[
\left[ \tilde{Y} \right] = j\omega [C] + [G],
\] (39)

where [L] is the per-unit-length longitudinal inductance matrix for a perfect soil [C] and [G] are the per-unit length transverse capacitance and conductance matrix of the multiconductor line, respectively. Furthermore, [Z_w] is the per-unit-length internal impedance matrix of the conductors, and [Z_e] is the per-unit-length ground impedance matrix. Finally, \[ \tilde{H}_{yc}^e(x, z) \] and \[ \tilde{E}_{zc}^e(x, z) \] are sources vectors expressed in terms of the incident magnetic and electric field, respectively [1, 8].
2.4. Computational Examples. The first computational example is related to the analysis of an overhead wire (Figure 3) of length $L = 20$ m, radius $a = 0.005$ m located at height $h = 1$ m above PEC ground and illuminated by the plane wave. The amplitude of the electric field is $E_0 = 1$ V/m and it is parallel to $x$-axis. Figure 5 shows the frequency response at the center of the line. The results computed via GB-IBEM and TL are compared to the results obtained via NEC using RC and Sommerfeld integral approach, respectively, to account for the presence of a lossy half-space. The agreement between the results obtained via the different approaches is found to be satisfactory.

Figure 6 shows the frequency response for the same line located above an imperfectly conducting half-space for various values of ground conductivity $\sigma = 1$ mS/m. The results calculated via different approaches agree satisfactorily again.

Next computational example is related to a simple Power Line Communications (PLCs) system. PLC technology aims to provide users with necessary communication means by using the already existing and widely distributed power line network and electrical installations in houses and buildings. However, one of the principal drawbacks of this technology is related to electromagnetic interference (EMI) problems, as overhead power lines at the PLC frequency range (1 MHz to 30 MHz) act as transmitting or receiving antennas, respectively [13].

Figure 7 shows the geometry of a simple PLC system consisting of two conductors placed in parallel above each other at the distance $d$. The conductors are suspended between two poles of equal height, thus heaving the shape of the catenary.

The geometry of a catenary is fully defined by such parameters as the distance between the points of suspension, $L$, the sag of the conductor, $s$, and the height of the suspension point, $h$, as shown in Figure 7. The imperfectly conducting ground is characterized with electrical permeability $\varepsilon_r$ and conductivity $\sigma$.

The conductors are modeled as thin wire antennas excited by the voltage generator $V_g$ at one end and terminated by the load impedance $Z_L$ at the other end.

The influence of the load impedance is taken into account by modifying continuity condition for the tangential components of the electric field at the wire surface:

$$E_{\text{inc}}^s + E_{\text{ext}}^s = Z_L' I(s),$$

(40)

where $Z_L'$ is the corresponding conductor per length impedance of the conductor.

The modified Pocklington equation for the wire containing the load impedance is now given by

$$E_{\text{inc}}^s = -\frac{1}{j4\pi\varepsilon_0} \int_0^L \left\{ \left[ k^2 - \frac{\partial^2}{\partial s'^2} \right] g_0(s, s') \right. + R_T \left[ k^2 - \frac{\partial^2}{\partial s'^2} \right] g_1(s, s') \bigg\} \times I(s') ds' + Z_L' I(s).$$

(41)

Set of integral equations (41) is numerically solved using via GB-IBEM.
The actual example is related to the simple PLC circuit shown in Figure 7. The distance between poles is \( L = 200 \, \text{m} \), with the radii of wires \( a = 6.35 \, \text{mm} \). The wires are suspended on the poles at heights \( h_1 = 10 \, \text{m} \) and \( h_2 = 11 \, \text{m} \). The maximum sag of the conductor is assumed to be \( s = 2 \, \text{m} \). Ground parameters are \( \varepsilon_r = 13 \) and \( \sigma = 0.005 \, \text{S/m} \). The power of the applied voltage generator is 2.5 \( \mu \text{W} \) (minimum power required for the PLC system operation) and operating frequency is chosen to be 14 MHz. The value of the terminating load \( Z_L \) is 500 \( \Omega \). Figure 8 shows the current distribution along the simple PLC system for different values of sag.

Radiated electric and magnetic fields at the distance of 30 m from the wires and 10 m above ground are shown at the Figures 9 and 10, respectively.

Analysis of the radiated field distributions shows that the conductor sag does not influence the far-field region significantly while the near-field distribution is mainly determined by the conductor geometry. Finally, the power of the applied voltage generator is changed to 1 mW (average power used at the actual PLC systems) and operating frequency is varied between 1 and 30 MHz. The values of the terminating load \( Z_L \) are chosen to be 50 \( \Omega \), 500 \( \Omega \), 5000 \( \Omega \), thus simulating different conditions within the power grid. The maximum values of the radiated electric field at the distance of 30 m for different arrangements are shown in Table 1.

According to the available international standards [16, 17], radiated electric fields should not exceed level of 30 \( \mu \text{V/m} \) at the distance of 30 m. Obviously, the radiated field levels are at best case more than 10 times higher than the proposed limit. The spatial distributions of the radiated electric field have been calculated for the number of frequencies in the frequency range from 1 to 30 MHz. Maximum levels of the calculated electric fields values are shown to exceed the limits defined by the standard for the disturbances caused by information technology equipment.

### Table 1: Maximum values of the radiated electric field at the 30 m distance.

| Frequency | \( Z_L (\Omega) \) | \( |E|_{\text{max}} \) (mV/m) |
|-----------|------------------|------------------------|
| 7 MHz     | 50               | 0.459                  |
|           | 500              | 0.341                  |
|           | 5000             | 0.380                  |
| 14 MHz    | 50               | 0.477                  |
|           | 500              | 0.458                  |
|           | 5000             | 0.541                  |
| 28 MHz    | 50               | 2.394                  |
|           | 500              | 0.853                  |
|           | 5000             | 2.043                  |

### 3. Time Domain Models and Methods

This section deals with direct time domain analysis of transient electromagnetic field coupling to straight overhead wires using the wire antenna theory and the transmission line method, respectively. The time domain antenna theory formulation is based on a set of the space-time Hallen integral equations. The transmission line approximation is based on the corresponding time domain Telegrapher’s equations. The space-time integral equations arising from the wire antenna theory are handled by the time domain scheme of GB-IBEM. The time domain Telegrapher’s equations are solved using the Finite Difference Time Domain (FDTD) method. Time domain numerical results obtained with both approaches are compared to the results computed via NEC.2 code combined with Inverse Fourier Transform procedure. Some illustrative
comparisons of results obtained by means of antenna theory and transmission line approach are presented in this section.

It is worth mentioning that, for the sake of simplicity, only straight wires are analyzed in this paper.

3.1. The Antenna Theory Model. Generally, a direct time-domain analysis of thin wire in the presence of a lossy half-space can be carried out via the appropriate space-time integral equations of either Pocklington or Hallen type [1, 8]. When applied to the solution of the Hallen integral equation the Galerkin-Bubnov Indirect Boundary Element Method (GB-IBEM) [8] results in relatively complex procedures compared to various procedures for the solution of Pocklington equations, but, at the same time, it is proven to be highly efficient, accurate, and unconditionally stable [8, 18, 19]. On the other hand, the implementation of GB-IBEM to the solution of the Pocklington-type equation is relatively simple, but suffers from serious numerical instabilities. The origin of these instabilities is the discretization of space-time differential operator [8]. The GB-IBEM solution of the Pocklington equation in free space for certain values of time domain integration parameters has been presented elsewhere, for example, in [19], while the Hallen integral equation solution by means of GB-IBEM has been obtained for thin wire structures in the presence of a dielectric half-space, for example, in [11]. In both cases, the influence of imperfect ground has been taken into account via the corresponding reflection coefficient. The numerical solution was mostly limited to scenarios in which the finite conductivity of the ground could be ignored. This approximation involves cases where the wires are sufficiently far from the two-media interface or where the ground conductivity is appreciably low or very high, that is, where the approximation of pure dielectric medium or perfect ground is applied. Through these approximations the time-dependent part of the reflection coefficient function vanishes, and the resulting matrix equation simplifies significantly.

However, for the cases where these approximations are not valid, modifications to the original methods are required in order to include the ground conductivity [8]. Namely, the related reflection coefficient is space-time dependent, and the resulting convolution integrals have to be included in the matrix system and numerically computed. This leads to a significant increase in the overall computational cost of the method and consequently requires several modifications.

This section deals with the transient analysis of multiple horizontal wires above a lossy ground using the Hallen integral equation approach.

The set of space-time Hallen’s integral equations can be derived as an extension of the single wire case. First, a single wire insulated in free space is considered.

Thin wire antenna or scatterer of length $L$ and radius $a$, oriented along the $x$-axis, is considered. The wire is assumed to be perfectly conducting and excited by a plane wave electric field. For the sake of simplicity, the analysis is restricted to the case of a normally incident electric field.

The tangential component of the total field vanishes on the PEC wire surface, that is,

$$E^\text{inc}_x + E^\text{scat}_x = 0,$$

where $E^\text{inc}_x$ is the incident and $E^\text{scat}_x$ scattered field on the metallic wire surface. Starting from Maxwell equations and obeying the Lorentz gauge one obtains a time domain version of (2):

$$\left(\frac{\partial^2 A}{\partial t^2} - \frac{1}{\mu_0} \nabla \left(\nabla \cdot A\right)\right)_{\text{tan}} = \frac{\partial E^\text{inc}_x}{\partial t}_{\text{tan}},$$

where $\vec{A}$ is the space-time-dependent vector potential.

According to the thin wire approximation, only the axial component of the vector potential exists, that is, (43) becomes

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{1}{c^2} \frac{\partial^2 A_x}{\partial t^2} = -\frac{1}{c^2} \frac{\partial E^\text{inc}_x}{\partial t},$$

where $c$ denotes the velocity of light.

The corresponding solution of (44) can be expressed in terms of a sum of the general solution of the homogeneous equation and the particular solution of the inhomogeneous equation:

$$A_x(x,t) = A^h_x(x,t) + A^f_x(x,t).$$

The solution of the homogeneous wave equation is given as a superposition of incident and reflected wave [8]:

$$A^h_x(x,t) = F_1 \left( t - \frac{x}{c} \right) + F_2 \left( t + \frac{x}{c} \right),$$

while the particular solution is given by the integral [8]:

$$A^f_x(x,t) = \frac{1}{2\pi \mu_0} \int_0^L E^\text{inc}_x \left( x', t - \frac{|x - x'|}{c} \right) dx',$$

where $L$ denotes the total antenna length.

On the other hand, the magnetic vector potential on the PEC wire surface is given by the particular integral:

$$A_s(x,t) = \frac{\mu}{4\pi} \int_S \frac{I(x',t - R/c)}{R} dx'. $$

Combining (45)–(48) yields the space-time Hallen equation:

$$\int_0^L \frac{I(x',t - R/c)}{4\pi R} \frac{dx'}{dx} = F_0 \left( t - \frac{x}{c} \right) + F_L \left( t - \frac{L - x}{c} \right) + \frac{1}{2\pi \mu_0} \int_0^L E^\text{inc}_x \left( x', t - \frac{|x - x'|}{c} \right) dx' ,$$

where $I(x')$ is the equivalent axial current to be determined, $E^\text{inc}_x$ is the known tangential incident field, $R = [(x - x')^2 + a^2]^{1/2}$ is the distance from the source point (the equivalent current in the antenna axis) to the observation point, and $Z_0$ is the wave impedance of a free space.
The unknown functions $F_0(t)$ and $F_1(t)$ account for the multiple reflections of the current at the free ends of the wire.

A direct time formulation for a straight thin wire above a dissipative half-space can be obtained as the extension of the free-space Hallen equation (49).

The free space Hallen equation (49) is first transferred into the Laplace frequency domain:

$$\int_0^L \frac{I(x', s) e^{-sR/c}}{4\pi R} \, dx' = F_0(s) e^{-sR/c} + F_L(s) e^{-s(L-x)/c} + \frac{1}{2Z_0} \int_0^L E^{\text{inc}}(x', s) e^{-s|x-x'|/c} \, dx',$$

(50)

where $s = j\omega$ is the Laplace variable.

According to the theory of images the free space integral equation (50) is extended by an additional term multiplying the Green function of the image source by space-frequency-dependent reflection coefficient $R_{TM}(\theta', s)$ for TM polarization. The integral equation in the frequency domain is given by

$$\int_0^L \frac{I(x, s) e^{-sR/c}}{4\pi R} \, dx' - \int_0^L \frac{R_{TM}(\theta, s) I(x, s) e^{-sR/c}}{4\pi R} \, dx' = F_0(s) e^{-sR/c} + F_L(s) e^{-s(L-x)/c} + \frac{1}{2Z_0} \int_0^L E^{\text{inc}}(x', s) e^{-s|x-x'|/c} \, dx',$$

(51)

where $R^* = \sqrt{(x-x')^2 + 4h^2}$ and $R_{TM}(\theta', s)$ is determined by the expression [1]

$$R_{TM}(\theta', s) = \frac{\epsilon_r (1 + \sigma/\epsilon_s) \cos \theta' - \sqrt{\epsilon_r (1 + \sigma/\epsilon_s) - \sin^2 \theta'}}{\epsilon_r (1 + \sigma/\epsilon_s) \cos \theta' + \sqrt{\epsilon_r (1 + \sigma/\epsilon_s) - \sin^2 \theta'}},$$

(52)

where $\sigma$ and $\epsilon$ are the lossy medium conductivity and permittivity, respectively, and $\theta' = \arctan |x - x'|/2h$.

The reflection coefficient (RC) approach is a satisfactory approximation in half-space calculations, as long as the field is calculated far away from the source, and the imperfect ground, respectively, to ensure $\theta' \approx \pi/2$ [8].

Performing the convolution, the time domain counterpart of (52) is obtained in the form

$$\int_0^L \frac{I(x', t - R/c)}{4\pi R} \, dx' - \int_{-\infty}^t \int_0^L \frac{r(\theta, \tau) I(x', t - R^*/c - \tau)}{4\pi R^*} \, dx' \, d\tau = F_0(t) - \frac{1}{2Z_0} \int_0^L E^{\text{inc}}(x', t - |x-x'|/c) \, dx' + \frac{1}{2Z_0} \int_0^L E^{\text{inc}}(x', t - |x-x'|/c) \, dx' + F_0(t - x/c) + F_L(t - L-x/c),$$

(53)

where $r(\theta, \tau)$ is the space-time reflection coefficient which, for convenience, can be written in the form [18]

$$r(\theta, \tau) = r'(\theta, \tau) + r''(\theta, \tau),$$

(54)

where

$$r'(\theta, t) = K \delta(t),$$

$$r''(\theta, t) = \frac{4\beta}{1 - \beta^2} e^{-\frac{at}{1 - \beta^2}} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} n K^n I_n(at)}{t},$$

$$\tau = \frac{\sigma}{\epsilon_0 \epsilon_s}, \quad \beta = \frac{\sqrt{\epsilon_r - \sin^2 \theta}}{\epsilon_r \cos \theta}, \quad \gamma = \frac{\tau}{1 - (\sin^2 \theta/\epsilon_r)},$$

$$\theta = \arctan \left(\frac{|x-x'|}{2h}\right),$$

(55)

Note that $I_n$ is the modified Bessel function of the first order and $n$th degree.

For the case of normal incidence, which is considered for the sake of simplicity, the excitation term is given by

$$E^{\text{inc}}_x(t) = E^{\text{inc}}_x(t) - E^{\text{ref}}_x(t^*),$$

(56)

where $t^* = t - R^*/c$.

The transient ground-reflected field is obtained as the convolution of the incident field and the space-time reflection coefficient for the angle of incidence $\theta = 0$ (in accordance with the parallel incidence of the electric field), as is proposed in [20]

$$E^{\text{ref}}_x(t) = \int_{-\infty}^t E^{\text{inc}}_x(t - \tau) r(\theta = 0, \tau) d\tau$$

(57)

and the integral equation (53) becomes

$$\int_0^L \frac{I(x', t - R/c)}{4\pi R} \, dx' - \int_{-\infty}^t \int_0^L \frac{R_{TM}(\theta, s) I(x, s) e^{-sR/c}}{4\pi R} \, dx' \, d\tau = F_0(t) - \frac{1}{2Z_0} \int_0^L E^{\text{inc}}(x', t - |x-x'|/c) \, dx' + \frac{1}{2Z_0} \int_0^L E^{\text{inc}}(x', t - |x-x'|/c) \, dx' + F_0(t - x/c) + F_L(t - L-x/c),$$

(58)

The unknown time functions $F_0(t), F_L(t), F_0(t - (L/c))$, and $F_L(t - (L/c))$ can be obtained in the same manner, as in the case of free space, in terms of as auxiliary functions $K_0(t)$ and $K_L(t)$ [8]:

$$F_0(t) = \sum_{n=0}^{\infty} K_0(t - \frac{2nL}{c}) - \sum_{n=0}^{\infty} K_L(t - \frac{(2n+1)L}{c}),$$

$$F_L(t) = \sum_{n=0}^{\infty} K_L(t - \frac{2nL}{c}) - \sum_{n=0}^{\infty} K_0(t - \frac{(2n+1)L}{c}),$$

(59)
where

\[
K_L(t) = \int_0^t \frac{I(x', t - R_0/c)}{4\pi R_0} dx' - \int_{-\infty}^t \int_0^L r(\theta, \tau) \frac{I(x', t - R_0^s/c - \tau)}{4\pi R_{0s}^p} dx' d\tau \\
- \frac{1}{2Z_0} \int_0^L E_{x'} \left( x', t - \frac{x' - x}{c} \right) dx',
\]

(60)

while \( R_0 \) and \( R_1 \) are the distances from the wire ends to the source point and \( R_0^s, R_1^s \) are the distances from the image wire ends to the source point.

If the case of a perfect (ideal) dielectric half-space is considered, the Hallen equation (53) simplifies into

\[
\int_0^t \frac{I(x', t - R/c)}{4\pi R} dx' - \int_{-\infty}^t \int_0^L r(\theta, \tau) \frac{I(x', t - R^s/c - \tau)}{4\pi R^p} dx' d\tau \\
= F_0 \left( t - \frac{x}{c} \right) + F_t \left( t - \frac{L - x}{c} \right) + \frac{1}{2Z_0} \int_0^L E_{x'} \left( x', t - \frac{|x' - x|}{c} \right) dx'.
\]

(61)

Space-time integral equation (53) or (61), respectively, can be solved assuming the zero current at the free ends of the wire and with the initial conditions requiring the wire not to be excited before the certain instant \( t = t_0 \) [8].

The transient behavior of \( M \) straight horizontal thin wires located at different heights above an infinite ground plane is determined by a set of the coupled space-time integral equations of the Hallen type [11]:

\[
\sum_{s=1}^M \int_{x_0}^{x_{s1}} \frac{I_s(x', t - R_{ss}/c)}{4\pi R_{ss}} dx' \\
- \sum_{s=1}^M \int_{-\infty}^t \int_{x_0}^{x_{s1}} r_{ss}(\theta, \tau) \frac{I_s(x', t - R_{ss}^p/c - \tau)}{4\pi R_{ss}^p} dx' d\tau \\
= F_{0s} \left( t - \frac{x - x_{0s}}{c} \right) + F_{ts} \left( t - \frac{x_{1s} - x}{c} \right) + \frac{1}{2Z_{0s}} \int_{x_0}^{x_{s1}} E_{x'} \left( x', t - \frac{|x' - x|}{c} \right) dx',
\]

(62)

where \( x_{0s}, x_{1s} \) and \( x_{ss}, x_{ts} \) are distances from considered source point on each wire \( s \) to a corresponding observation point at the ends of the wire \( v \):

\[
R_{0s}^{(0)} = R_{ss} \mid_{x=x_{0s}}, \quad R_{ts}^{(0)} = R_{ss} \mid_{x=x_{ts}},
\]

(63)

while \( R_{ss}^{(0)} \) and \( R_{ss}^{(L)} \) are distances between the source point at the image of the wire \( s \) and observation point located at the ends of the wire \( v \):

\[
R_{ss}^{(0)} = R_{ss} \mid_{x=x_{0s}}, \quad R_{ss}^{(L)} = R_{ss} \mid_{x=x_{ts}},
\]

(64)

The space-time reflection coefficient \( r_{ss}(\theta, t) \) accounts for the influence of the interface and is given by [11]

\[
r_{ss}(\theta', t) = A\delta(t),
\]

(65)

where

\[
A = \frac{1 - \beta}{1 + \beta}, \quad \beta = \frac{\sqrt{\varepsilon_r - \sin^2\theta'}}{\varepsilon_r \cos \theta'},
\]

\[
\theta' = \text{Arctg} \left( \frac{\sqrt{(x' - x)^2 + (y' - y)^2}}{z' + z} \right).
\]

(66)

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The angle \( \theta \) is the angle between the source point on the image of the wire \( s(x', y', z') \) and the observation point \( (x, y, z) \) on wire \( v \).

Substituting (70) into (62) yields

\[
\frac{M}{\pi} \int_{x_0}^{x_0} \frac{I_s(x', t - R_{sv}/c)}{4\pi R_{sv}} \, dx' - \frac{M}{\pi} \int_{x_0}^{x_0} r_{sv}^{(\theta)} \frac{I_s(x', t - R_{sv}/c)}{4\pi R_{sv}} \, dx' = F_{iv} \left( t - \frac{x - x_0}{c} \right) + F_{lv} \left( t - \frac{x_{lv} - x}{c} \right) + \frac{1}{2Z_0} \int_{x_0}^{x_0} E_{sv}^{ex} \left( x', t - \frac{|x - x'|}{c} \right) \, dx',
\]

(72)

where \( E_{sv}^{ex} \) is the space-time-dependent tangential electric field on the \( v \)th wire.

For the case of normal incidence the total excitation field \( E_{sv}^{ex}(x', t) \) is given as the sum of the incident field \( E_{sv}^{inc}(x', t) \) and the field reflected from the interface \( E_{sv}^{ref}(x', t) \) [11],

\[
E_{sv}^{ex}(x', t) = E_{sv}^{inc}(x', t - T) + E_{sv}^{ref}(x', t - T).
\]

(73)

The time shift \( T \) represents the time required for the wave to travel from the highest wire to the height \( z \) of the observed \( v \)th wire. Assigning the highest wire with index \( U \), it can be written

\[
T = \frac{z_U - z}{c}, \quad z_U = \max(z_1, z_2, \ldots, z_L, \ldots, z_M).
\]

(74)

The field reflected from the interface for the case of normal incidence is given by

\[
E_{sv}^{ref}(x', t - T) = r(\theta = 0) \cdot E_{sv}^{inc}(x', t - T - \frac{2z}{c}),
\]

(75)

where \( t - T - 2z/c \) is the time needed for the wave to travel from observed \( v \)th wire to the interface.

For the case of PEC ground plane, the space-time reflection coefficient (54) simply becomes

\[
r_{sv}(\theta, t) = 1.
\]

(76)

Thus, the set of (72) simplifies into

\[
\frac{\pi}{M} \int_{x_0}^{x_0} \frac{I_s(x', t - R_{sv}/c)}{4\pi R_{sv}} \, dx' - \frac{\pi}{M} \int_{x_0}^{x_0} r_{sv}^{(\theta)} \frac{I_s(x', t - R_{sv}/c)}{4\pi R_{sv}} \, dx' = F_{iv} \left( t - \frac{x - x_0}{c} \right) + F_{lv} \left( t - \frac{x_{lv} - x}{c} \right) + \frac{1}{2Z_0} \int_{x_0}^{x_0} E_{sv}^{ex} \left( x', t - \frac{|x - x'|}{c} \right) \, dx',
\]

(77)

and the field reflected from PEC ground is simply given by

\[
E_{sv}^{ref}(x', t - T) = E_{sv}^{inc}(x', t - T - 2z/c).
\]

(78)

3.2. The Numerical Solution. First, numerical procedure for single wire Hallen equation is outlined. Applying the weighted residual approach in the spatial domain and GB-IBEM procedure [8], the following local matrix system is obtained:

\[
[A] [I]_{1-2R/c} - [A^*] [I]_{1-2R/c} - \{A\} \bigg|_{1-2R/c} + [B] \{E\} \bigg|_{1-2R/c} - [C] \bigg|_{1-2R/c} + [C^*] \bigg|_{1-2R/c}
\]

\[
= [B] \{E\} \bigg|_{1-2R/c} - [C] \bigg|_{1-2R/c} + [C^*] \bigg|_{1-2R/c}.
\]
Assembling the local matrices and vectors into the global ones the following global matrix system is formed:

\[
[A] \{I\} \big|_{t=R/c} = \{g\} \big| \text{previous time} + \{\hat{g}\} \big| \text{previous time},
\]

where

\[
\{g\} = [A^*] \{I\} \big|_{t=R^*/c} + [B] \{E\} \big|_{t-(x-x')/c},
\]

\[
\{\hat{g}\} = [C] \left\{ \sum_{n=0}^{\infty} E^n \right\} \big|_{t-(R_0/c) - (2nL/c) - (x/c)} + [C^*] \left\{ \sum_{n=0}^{\infty} I^n \right\} \big|_{t-(R_0/c) - (2nL/c) - (x/c)}.
\]

The space-dependent local matrices representing the interaction between \(i\)th source and \(j\)th observation element are defined as follows:

\[
\begin{align*}
[A] &= \int_{\Delta_{ij}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T \frac{1}{4\pi R} \, dx' \, dx, \\
[B] &= \int_{\frac{\Delta_{ij}}{2}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T \frac{1}{4\pi R_0} \, dx' \, dx, \\
[C] &= \int_{\frac{\Delta_{ij}}{2}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T \frac{1}{4\pi R_L} \, dx' \, dx, \\
[D] &= \int_{\frac{\Delta_{ij}}{2}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T \frac{r(\theta)}{4\pi R_L} \, dx' \, dx, \\
[A^*] &= \int_{\Delta_{ij}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T \frac{r(\theta)}{4\pi R} \, dx' \, dx, \\
[C^*] &= \int_{\frac{\Delta_{ij}}{2}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T \frac{r(\theta)}{4\pi R_L} \, dx' \, dx, \\
[D^*] &= \int_{\frac{\Delta_{ij}}{2}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T \frac{r(\theta)}{4\pi R} \, dx' \, dx,
\end{align*}
\]

where \(\{f\}\) stands for the shape functions, while additional time dependent vectors are given by

\[
\begin{align*}
\{\hat{A}\} &= \int_0^{t-R^*/c} \int_{\Delta_{ij}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T H_1 \, dx' \, dx \{I(\tau)\}, \, d\tau \\
\{\hat{C}^n\} &= \int_0^{t-(R_0^*/c) - (2nL/c) - (x/c)} \int_{\Delta_{ij}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T H_2 \, dx' \, dx \{I(\tau)\}, \, d\tau \\
\{\hat{D}^n\} &= \int_0^{t-(R_L^*/c) - ((2n+1)L/c) - (x/c)} \int_{\Delta_{ij}} \int_{\Delta_{lj}} \{f\}_i \{f\}_j^T H_3 \, dx' \, dx \{I(\tau)\}, \, d\tau,
\end{align*}
\]

where

\[
\begin{align*}
H_1 &= \frac{r''(\theta, t - (R_0^*/c) - \tau)}{4\pi R_0^*}, \\
H_2 &= \frac{r''(\theta, t - (R_0^*/c) - (2nL/c) - (x/c) - \tau)}{4\pi R_0^*}, \\
H_3 &= \frac{r''(\theta, t - (R_L^*/c) - ((2n+1)L/c) - (x/c) - \tau)}{4\pi R_L^*}.
\end{align*}
\]
\[
\{ \hat{g} \} = \left\{ \hat{A} \right\} \left|_{t=-(R\gamma/c)} \right. - \left\{ \sum_{n=0}^{\infty} \hat{C}^n \right\} \left|_{t=-(R\gamma/c) - (2nL/c) - (x/c)} \right.
\]
\[
+ \left\{ \sum_{n=0}^{\infty} \hat{D}^n \right\} \left|_{t=-(R\gamma/c) - ((2n+1)L/c) - (x/c)} \right.
\]
\[
+ \left\{ \sum_{n=0}^{\infty} \hat{D}^n \right\} \left|_{t=-(R\gamma/c) - (2nL/c) - (L-x'/c)} \right.
\]
\[
- \left\{ \sum_{n=0}^{\infty} \hat{C}^n \right\} \left|_{t=-(R\gamma/c) - ((2n+1)L/c) - (L-x'/c)} \right.
\]
\[
\] (85)

Applying the weighted residual approach in the time domain and using the Dirac impulses as weight functions, the time sampling is provided, and the following recurrent formula is obtained:

\[
I_j \left|_{t_k} \right. = \frac{\sum_{n=1}^{N'} a_{ji} I_j \left|_{t_k-R_n/c} - \hat{g}_j \right|_{\text{previous time instants}} - \hat{g}_j \left|_{\text{previous time instants}} \right.}{a_{jj}},
\] (86)

where \( I_j \left|_{t_k} \right. \) is current for the \( j \)th space node at \( k \)th time instant, \( N \) is total number of space segments, while the overbar indicates the absence of diagonal members.

It is worth noting that the numerical calculation of convolution integrals is rather tedious task leading to tremendously large computational time of the overall method. The main advantage of the method, on the other hand, is its unconditional stability.

Time domain GB-IBEM procedure for the set of Hallen equations is undertaken in a similar manner as in the case of a single wire.

The solution of (72) and (77), respectively, is also carried out using the GB-IBEM technique.

Applying the boundary element discretisation to (72) and (77), respectively, leads to a local system of linear equations for the \( v \)th observed wire:

\[
M \sum_{s=1}^{M} \left[ A_{vs} \right] \left\{ I_s \right\}_{t=R_{ss}/c} - \sum_{s=1}^{M} \left[ A_{vs}^s \right] \left\{ I_s \right\}_{t=R_{ss}/c} = \left[ B_v \right] \left\{ E_v \right\}_{t=-(x-x_s)/c}
\]
\[
+ \left[ \int_{\Delta l} F_0 \left( t - \frac{x-x_{0v}}{c} \right) \left\{ f \right\}_j \right] dx
\]
\[
+ \left[ \int_{\Delta l} F_1 \left( t - \frac{x_{1v} - x}{c} \right) \left\{ f \right\}_j \right] dx,
\] (87)

where \( i, j = 1, 2, \ldots, N \) denotes the index of the elements located on the \( s \)th source wire and the \( v \)th observed wire, respectively, with \( N \) as the total number of space segments while \( M \) is the actual number of wires.

Finally, substituting (64)–(67) into (87), the following local matrix system is obtained:

\[
\]
where \( \{ E \} \) denotes excitation vector and space-dependent matrices are of the form

\[
\begin{align*}
[A_{vy}] &= \sum_{i=1}^{M} \int_{\Delta l} \frac{1}{4\pi R_{vy}} \{ f \} \{ f \}^T dx' dx,

[A_{vn}] &= \sum_{i=1}^{M} \int_{\Delta l} \int_{\Delta l} \frac{r_{vn} (\theta)}{4\pi R_{vy}} \{ f \} \{ f \}^T dx' dx,

[B_{vn}] &= \frac{1}{2Z_{o}} \sum_{i=1}^{M} \int_{\Delta l} \int_{\Delta l} \{ f \} \{ f \}^T dx' dx,

[C_{vy}] &= \sum_{i=1}^{M} \int_{\Delta l} \frac{1}{4\pi R_{vy}} \{ f \} \{ f \}^T dx' dx,

[C_{vn}] &= \sum_{i=1}^{M} \int_{\Delta l} \int_{\Delta l} \frac{r_{vn} (\theta)}{4\pi R_{vy}} \{ f \} \{ f \}^T dx' dx,

[D_{vn}] &= \frac{1}{2Z_{o}} \sum_{i=1}^{M} \int_{\Delta l} \int_{\Delta l} \{ f \} \{ f \}^T dx' dx,

[E_{vy}] &= \sum_{i=1}^{M} \int_{\Delta l} \frac{1}{4\pi R_{vy}} \{ f \} \{ f \}^T dx' dx,

[E_{vn}] &= \sum_{i=1}^{M} \int_{\Delta l} \int_{\Delta l} \frac{r_{vn} (\theta)}{4\pi R_{vy}} \{ f \} \{ f \}^T dx' dx.
\end{align*}
\]  

(89).

Relations containing summation from \( n = 0 \) to infinity pertain to the reflections of transient current from the wire ends. Note as the observed time interval is always finite, only a finite number of reflections occurs within a given observation interval. A shorter observed interval requires smaller number of summands and vice versa.

According to GB-IBEM, a global matrix system is assembled from the local matrix systems for all wires \( v = 1, 2, \ldots, M \). Finally, the resulting global matrix system can be written as follows:

\[
[A] \{ I \} \big|_{t-R_{vn}/c} - [A^*] \{ I \} \big|_{t-R_{vn}/c} = \{ g \}.
\]  

(90)

The time-domain solution on the \( j \)th boundary element is given by

\[
I_j(t) = \sum_{k=1}^{N_t} I_j^k T_k (t'),
\]  

(91)

where \( I_j^k \) are unknown coefficients, \( T_k \) are the linear time-domain shape functions, and \( N_t \) is the total number of time samples.

Applying the weighted residual approach to (90) leads to the expression

\[
\int_{t_k}^{t_k+\Delta t} (A \{ I \})_{t-R_{vn}/c} - [A^*] \{ I \} \big|_{t-R_{vn}/c} - \{ g \} \theta_k dt = 0,
\]  

(92)

where \( \theta_k \) denotes the set of time-domain weights.

Using the set of Dirac impulses for the test functions, time sampling is ensured and (92) becomes

\[
[A] \{ I \} \big|_{t_k-R_{vn}/c} - [A^*] \{ I \} \big|_{t_k-R_{vn}/c} = \{ g \} \text{ all previous discrete instants},
\]  

(93)

If the space-time discretization is performed by satisfying the Courant condition, \( \Delta x \geq c\Delta t \), the transient current for a \( j \)th space node and \( k \)th time node can be obtained from a recurrence formula. Separating the terms relating to the current induced at the instant \( t_k \) in (93) yields

\[
A_{jj} I_j \big|_{t_k} + \left[ A \right] \{ I \} \big|_{t_k-R_{vn}/c} - [A^*] \{ I \} \big|_{t_k-R_{vn}/c} = \{ g \} \text{ all previous discrete instants},
\]  

(94)

where overbar indicates the absence of diagonal terms.

The first term in (94) pertains to the current at the \( j \)th space node and \( k \)th time node, that is, the present instant. Other terms are related to all previous instants. Finally, the recurrence formula for the transient current at \( j \)th space node and \( k \)th time node is obtained in the forms

\[
I_j \big|_{t_k} = \sum_{i=1}^{N} \left( \frac{A_{jj} I_j \big|_{t_k-R_{vn}/c} + A_{jj}^* I_j \big|_{t_k-R_{vn}/c}}{A_{jj}} \right) + \{ g \} \text{ all previous discrete instants},
\]  

(95)

\[
\frac{\partial }{\partial x} \left[ V(x,t) \right] + \left[ R \right] \left[ I(x,t) \right] + \left[ L \right] \cdot \frac{\partial }{\partial t} \left[ I(x,t) \right]
\]  

\[
= \left[ E_F(x,t) \right] - \left[ z'(t) \right] \ast \left[ I(x,t) \right],
\]  

(96)

\[
\frac{\partial }{\partial x} \left[ I(x,t) \right] + \left[ G \right] \cdot \left[ V(x,t) \right] + \left[ C \right] \cdot \frac{\partial }{\partial t} \left[ V(x,t) \right]
\]  

\[
= \left[ H_F(x,t) \right],
\]  

(97)

3.3. The Transmission Line Model. The time-domain field-to-transmission line coupling equations can be written in the matrix form [11]
where "⊗" stands for the convolution product, \([z'(t)]\) is the transient inverse Fourier transform of the ground, conductors matrix \([Z_w(s) + Z_d(s)]\), and \(s = j\omega\) is the Laplace variable.

\([E_F(x,t)]\) and \([H_F(x,t)]\) are the excitation terms, given by [11]

\[
\begin{align*}
[E_F(x,t)] &= -\frac{\partial}{\partial x}[V_T(x,t)] + [E_L(x,t)], \\
[H_F(x,t)] &= -[G][V_T(x,t)] - [C] \frac{\partial}{\partial t}[V_T(x,t)],
\end{align*}
\]

where \([V_T(x,t)]\) is the transverse voltage derived from the transverse incident field excitation [11], and \([E_L(x,t)]\) represents the longitudinal electrical field excitation.

The classical per-unit-length inductances of \([L]\) matrix have been used:

\[
L_{ii} = \frac{\mu_0}{2\pi} \ln \left( \frac{2h_i}{a_i} \right), \quad L_{ij} = \frac{\mu_0}{2\pi} \ln \left( \frac{D_{ij}}{d_{ij}} \right),
\]

where \(h_i,a_i\) are, respectively, the height and the radius of the \(i\)th conductor. \(D_{ij}\) is the distance between the \(i\)th conductor and the image of \(j\)th conductor and \(d_{ij}\) is the distance between the \(i\)th conductor and the \(j\)th conductor.

The capacitance matrix \([C]\) is derived from the inductance matrix \([L]\) by \([C] = \varepsilon_0 \mu_0 [L]^{-1}\).

In case of PEC ground, the transient ground matrix \([z'(t)]\) is equal to \([Z_{np}(t)]\) while if a dielectric half-space is of interest, the ground matrix \([Z_d(s)]\) is given by Carson integral [11].

It is worth emphasizing that, for both cases, the conductivity matrix is neglected.

### 3.4. The Solution of Transmission Line Equations via FDTD

Discretizing each conductor of the multiconductor transmission line (MTL) into \(N_x\) sections of length \(\Delta x\) and discretizing the entire time interval into increments of duration \(\Delta t\), the FDTD method is applied to (88) and (97).

The solutions of (91) and (99) for lossy MTL by FDTD are given by

\[
\begin{align*}
[Z^+] \left[ I_{k}^{n+1/2} \right] &= -[Z^-] \left[ I_{k}^{n-1/2} \right] + \left[ V_{k}^{n} \right] \\
&\quad \quad + \left[ V_{T,k}^{n} \right] - \left[ V_{T,k+1/2}^{n} \right] \frac{\Delta x}{12} \\
&\quad \quad - \left[ E_{T,k}^{n} \right] \left[ S_{T,k}^{n} \right] \frac{\Delta x}{12}
\end{align*}
\]

for \(0 \leq k \leq N_x - 1\), \(n \geq 1\),

where

\[
\begin{align*}
[R] &= \frac{\varepsilon_0 \mu_0}{2} \left[ \frac{[L]}{[C]} \right] \frac{\Delta t}{\Delta x}, \\
[Y^+] &= \left[ \frac{V_{k+1/2}^{n+1}}{V_{T,k+1/2}^{n+1}} \right] + \left[ V_{k+1/2}^{n+1} \right] - \left[ V_{T,k+1/2}^{n+1} \right] \\
[Y^-] &= -\left[ \frac{V_{k+1/2}^{n-1}}{V_{T,k+1/2}^{n-1}} \right] + \left[ V_{k+1/2}^{n-1} \right] - \left[ V_{T,k+1/2}^{n-1} \right] \\
[Y^\pm] &= \frac{[G]}{2} \pm \frac{[C]}{[L]} \frac{\Delta t}{\Delta x}.
\end{align*}
\]

The expressions for \([V], [I], [V_T], [E_T], [S]\) are given for time \(n \geq 1\) and space \(k \geq 0\):

\[
\begin{align*}
\left[ V_{k+1/2}^{n} \right] &= \left[ V((k + 1/2)\Delta x, n\Delta t) \right], \quad 0 \leq k \leq N_x - 1.
\end{align*}
\]

Voltage and adjacent current are interlaced in time and space, respectively, by \(\Delta t/2\) and \(\Delta x/2\), then

\[
\begin{align*}
\left[ I_{k}^{n-1/2} \right] &= \left[ I(\Delta x, n - \frac{1}{2}\Delta t) \right], \\
\left[ V_{T,k}^{n+1/2} \right] &= \left[ V_T((k + 1/2)\Delta x, n\Delta t) \right], \\
\left[ E_{T,k}^{n} \right] &= \left[ E_T(k\Delta x, n\Delta t) \right], \\
\left[ S_{T,k}^{n} \right] &= \left[ S(k\Delta x, n\Delta t) \right], \quad 0 \leq k \leq N_x.
\end{align*}
\]

The convolution product \([S_{T,k}^{n}]\) appearing in (100) can be written as follows:

\[
\left[ S_{T,k}^{n} \right] = \left[ Z_0(2)(t) \right] \left[ I_{k}^{n-1/2} \right] + \sum_{l=1}^{n-1} \left[ Z_0(l + 1) - Z_0(l) \right] \left[ I_{k}^{n-l+1/2} \right] - \left[ Z_0(n) \right] \left[ I_{k}^{n+1/2} \right],
\]

where

\[
\begin{align*}
[Z_0(l)] &= \left[ Z_{0w}(l) \right] + \left[ Z_{0g}(l) \right], \\
&= \int_{t-1}^{l} \left[ Z_w + Z_s \right](u\Delta t) du, \quad 1 \leq l \leq n, \\
t = n\Delta t, \quad N_x \text{ is the number of space steps, and } n \text{ is the number of time steps.}
\end{align*}
\]

The corresponding components of the \(M \times 1\) vectors \([E_{T,k}^{n}], [E_{L,k}^{n}]\) are as follows:

\[
\begin{align*}
\left[ V_{T,k}^{n} \right] &= \left[ y_i E_{T}^{inc}(x_i = k\Delta x, y_i, z_i, n\Delta t) \right] \\
&\quad + \left[ y_i E_{T}^{inc}(x_i = k\Delta x, y_i, z_i, n\Delta t) \right], \\
\left[ E_{L,k}^{n} \right] &= \left[ E_{L}^{inc}(x_i = k\Delta x, y_i, z_i, n\Delta t) \right] \\
&\quad - \left[ E_{L}^{inc}(x_i = k\Delta x, 0, 0, n\Delta t) \right].
\end{align*}
\]

Equations (107) and (108) are valid for \(1 \leq i \leq M\), where \(M\) is the number of conductors, and \(y_i, z_i\) are the positions of \(i\)th conductor. \(E_{x}^{inc}, E_{y}^{inc}, E_{z}^{inc}\) are the components of the incident electromagnetic field evaluated in the absence of conductors. \([R], [L], [G]\), and \([C]\) are, respectively, the per-unit-length resistance, inductance, conductance, and capacitance matrices of dimension \(M \times M\).

\([Z_g(t)]\) is the transient ground resistance and is equal to the inverse Fourier of \([Z_g(s)]/s\):

\[
Z_{gij}(t) = F^{-1}\left( \frac{Z_{gij}(s)}{s} \right),
\]

where the ground impedance in frequency domain is given by Carson formula:

\[
Z_{gij}(s) = \frac{\mu_0}{\pi} \int_{0}^{\infty} \frac{e^{-(b+ch)} \delta}{\sqrt{\lambda^2 + (b+ch)^2}} \cdot \cos(d_{ij}\lambda) \lambda.\]
plane wave with illuminated by the tangential electromagnetic pulse (EMP)

\[ Z_{wi}(t) = \frac{1}{\pi \sigma_{wi} a_i^2} \sum_{m=1}^{\infty} e^{-\frac{1}{2} \alpha_m^2 (t / r_{wi})} , \quad \text{where } r_{wi} = \mu_0 \sigma_{wi} a_i^2, \]

and \( h_i, h_j, d_{ij} \) are the corresponding heights of the two conductors \( i, j \) and the distance between the two conductors in the horizontal plane.

Terms \( x_m \) stand for the zeros of \( J_1 \), Bessel function of first kind.

Finally, \( y_g \) is the propagation constant defined as

\[ y_g^2 = 4 \mu_0 (\sigma_g + s e_x e_{rg}), \]

where \( \sigma_g \) and \( e_{rg} \) are, respectively, ground conductivity and permittivity.

For the cases considered in this paper, all conductors are in open circuit at both ends, so the currents vanishes at near and far ends.

By using the boundary formulation [11], voltages at both ends are simply expressed as follows:

\[
\begin{bmatrix}
V_0^n \\
V_{N_r}^n
\end{bmatrix} = 
\begin{bmatrix}
V_0^n (1/2) \\
V_{N_r}^n (1/2)
\end{bmatrix},
\]

\[
\begin{bmatrix}
I_0^n \\
I_{N_r}^n
\end{bmatrix} = [0],
\]

\[
\begin{bmatrix}
I_0^n \\
I_{N_r}^n
\end{bmatrix} = [0].
\]

4. Numerical Results

Figure 11 shows the transient response at the centre of the straight wire \( L = 20 \text{ m}, a = 0.005 \text{ m}, \) located at height \( h = 1 \text{ m} \) above a dielectric half-space \( (\varepsilon_r = 10) \) excited by the electromagnetic pulse (EMP):

\[ E_{\text{inc}} = E_0 (e^{-at} - e^{-bt}), \]

with \( E_0 = 1.1 \text{ V/m}, a = 7.92 \times 10^4 \text{ s}^{-1}, b = 4 \times 10^4 \text{ s}^{-1}. \)

The next example is related to a transient scattering from a straight thin wire of length \( L = 1 \text{ m}, \) radius \( a = 2 \text{ mm}, \) located at height \( h = 0.25 \text{ m} \) above ground with permittivity \( \varepsilon_r = 10, \) while the conductivity is varied. The wire is illuminated by the tangential electromagnetic pulse (EMP) plane wave with \( E_0 = 1 \text{ V/m}, a = 4 \times 10^7 \text{ s}^{-1}, b = 6 \times 10^8 \text{ s}^{-1}. \)

Figure 12 shows the transient current induced at the wire center for different ground conductivities.

The influence of the ground conductivity to the transient response is particularly visible from around 0.1 S/m to 1 S/m.

The time domain results obtained via different approaches are found to agree satisfactorily.

Next set of examples is related to a two-wire array above a PEC ground Figure 13, (Geometry No. 1) and dielectric-half space \( (\varepsilon_r = 10), \) Figure 14 (Geometry No. 2), respectively.

Figures 15 and 16 show the transient current induced at the center of wire 2 for the case of Geometry No. 1 and No. 2, respectively, obtained via TD GB-IBEM, TL, and NEC 2 combined with inverse fast Fourier transform (IFFT).

Generally, the results calculated via different approaches are in relatively acceptable agreement. Nevertheless, some discrepancies can be noticed, in particular for the case of PEC ground. In this analysis, the applied TL model accounts not only for classical propagation effect but also for skin effects and for a correction resistance representing the radiation effect.

In order to include the radiation effect in TL model, a small DC resistance \( (1 \Omega/\text{m}) \) has been used to to represent the attenuation effect.
It is known that TL model accounts for coupling between transverse cells only while AT takes into account mutual effects. This phenomenon is assumed to be the source of the differences in propagation velocities which are observed.

5. Conclusion

The paper reviews the models and methods used for the analysis of electromagnetic field coupling to overhead wires in the frequency and time domain, respectively, using the wire antenna theory and the transmission line approximation, respectively. The frequency domain wire antenna model is based on the space-time Pocklington integral equations, while the transmission line model is based on the frequency domain Telegrapher’s equations. The time domain wire antenna model is based on a set of the space-time Hallen integral equations, while the transmission line model is based on the time domain Telegrapher’s equations. The set of Pocklington equations is solved via the Galerkin-Bubnov variant of the Indirect Boundary Element Method (GB-IBEM), while the frequency-domain transmission line equations are treated using the chain matrix method and modal equation to derive per-unit-length parameters. A number of illustrative computational examples for the frequency response of several configurations of overhead wires, obtained via different approaches, are given in this paper.

The coupled space-time Hallen integral equations are numerically solved via the time domain Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM), while the time-domain transmission line equations are solved by the finite difference time domain (FDTD) method.

Some numerical results pertaining to the transient behaviour of overhead wires, obtained via different approaches, are given in this paper.

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Research Article

High-Frequency Electromagnetic Field Coupling to Small Antennae in a Rectangular Resonator

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The integral-differential equation for the current of an electrically small antenna, inside a resonator, which is induced by given sources, is approximately solved by the so-called “Method of Small Antenna,” both for dipole and loop antennas. The current induced in the antenna is evaluated using the scattering characteristics of small antennas in free space and regularized Green’s function of resonator. As example of application of the theory, a transfer function (“external field → induced voltage”) for the coupling through aperture is calculated.

1. Introduction

Investigation of the coupling of high-frequency electromagnetic fields caused by intentional electromagnetic interferences to linear structures becomes an actual topic. Usually the corresponding test experiments and simulation models are applied to devices in free space [1]. However, in reality, electronic equipment is enclosed in different kinds of resonator-like shells: cabinets of computers, airframes, frames of cars, and so forth. These enclosures change the interaction of electromagnetic fields with the scatterers remarkably due to rereflections of electromagnetic fields inside the resonator [2].

It was shown in many experiments that the main mechanism of such interaction in free space is electromagnetic coupling to interconnections of different scales. Often these interconnections are electrically small (printed circuit boards, chips, etc.) but can have own resonances. Currents and voltages induced in such objects in free space can be evaluated by a method, which includes two simple models: a model of a small near-resonance linear antenna to describe the common mode and a model of a small loop to describe the differential mode [3].

In our papers, [4, 5], we proposed a method to analyze the coupling to an electrically short dipole (or monopole) antenna in a resonator (Method of Small Antenna, MSA) by consequently using the scattering theory. The MSA is based on the analysis of the integro-differential equation describing the induced current in the neighborhood of the antenna. This approach gives the possibility to analytically express the solution for the scattered current in the small antenna inside the resonator from the free space solution and the regularized cavity Green’s function of resonator. One can also investigate the input impedance of the small antenna, the current transfer ratio for two small antennas, and so forth.

In the present work we expand our method for the case of electromagnetic coupling to an electrically small loop in the resonator. Again, using an approximate solution for the induced current in free space and a regularized cavity Green’s function, we derive an equation for the induced current in an electrically small loop in a resonator. The solution looks like the one for free space but contains a so-called “resonator impedance,” instead the radiation resistance of free space. This “resonator impedance” depends on the parameters of the antenna, the parameters of the resonator, and also the coordinates of the loop.

The developed theory was applied to investigate an important practical problem: the calculation of the transfer function (external electromagnetic field → scattered current) for electrically small wiring objects (printed circuit, chip, etc.) inside a rectangular resonator. It is assumed that electromagnetic waves penetrate into the cavity through...
In the short conclusion, we outline directions of future investigations.

In the Section 4, we calculate transfer functions for both types of scatterers and compare our results with numerical results of MSA for the small loop inside the resonator.

For a thin symmetric dipole antenna [3],

\[ \omega_a \approx \frac{\pi c}{L}, \]

\[ C_a = -\frac{\pi \varepsilon_0 L/2}{\ln(2L/d)} - 2, \]

\[ L_a = \frac{1}{C_a \omega_a^2}. \]

Here the quantities \( L, d \) are the antenna length and diameter, respectively, \( \omega_a \) is the first resonance frequency of the antenna, \( C_a \) and \( L_a \) are the antenna capacitance and inductance, respectively, and \( R_a \) is the antennaload.

The effective electromotive force \( \varepsilon_{E, \text{eff}} \) and the current distribution function \( f(l) \) for a passive antenna with a distributed uniform excitation \( E_z(R_a \ll \sqrt{L_a/C_a}) \), see Figure 2(a) are given by (3) and (4), respectively [3–5],

\[ \varepsilon_{E, \text{eff}} = \frac{E_z}{2}, \]

\[ f(l) \approx 1 - \frac{4l^2}{L^2}. \]

By the analysis of the EFIE for the small antenna in a resonator, it can be shown that the induced current is described by a simple equation like (1), which, however, contains together with the load resistance \( R_a \) also the “impedance of the resonator” \( Z_{\text{RES}} \). This value is calculated using the resonator Green’s function, regularized for coinciding arguments, \( \tilde{Z}_{\text{RES}} \), and it depends on both the antenna and resonator parameters. For the passive dipole (distributed excitation), one obtains (detailed derivation and explanation of expression (5) the reader can find in [3, 4])

\[ \tilde{J}_E(l) \approx E_z \tilde{K}_E(j\omega) f(l) = 0.5E_z L \tilde{Y}_E(j\omega) f(l), \]

\[ \tilde{Y}_E = \frac{1}{1/j\omega C_a + j\omega L_a + R_{\text{load}} + Z_{\text{RES}}(j\omega)}, \]

\[ Z_{\text{RES}}(j\omega) = -\left( \tilde{Z}_{\text{RES}} \right)_{ZZ} F^{d.e./2}, \]

\[ F^{d.e.} := \int_{-L/2}^{L/2} f^{d.e.}(l) dl = \frac{2}{3L}, \]
I. Introductio

II. Theoretical Analysis

III. Experimental Results

IV. Conclusion
method clearer, we begin from the case in free space. The $zz$-component of the Green’s function for the magnetic field from (8) and (9) is approximated by
\[
\left( \frac{G_{H,M}}{G_{f,sp}} \right)_{zz} \bigg|_{\begin{array}{l} x=x_1 \\ y=y_1 \\ \rho \to 0 \end{array}} \approx \frac{1}{4\pi j\omega} \left[ \frac{2}{|z-z'|^2} + \frac{k_0^2}{|z-z'|^2} - \frac{\gamma_v}{3} \right] \tag{11}
\]

As one can see, the Green’s function is decomposed into two parts. The first, singular part (first and second term in the brackets) represents the near field and contains Electrostatic and magnetostatic energy. The second, regular part (third term in the bracket) is constant in the neighborhood of the scatterer and is responsible for the far field (radiation for the case of free space, see below).

In order to extract the divergence of the Green’s function inside the resonator in explicit form, we divide the summation domain of (9) into two parts [4] (see Figure 4): one corresponding to the values of wave number $y_i$ (less than some value $\gamma_{\text{max}}$) and the other part contains those values, which exceed $\gamma_{\text{max}}$. Then the summation may be approximated by integration (practical numerical calculation has shown that a good choice for $\gamma_{\text{max}}$ is $\sim 2.5k$). As a result, we obtain the following expression for $(G_{\text{RES}})_{zz}$ in the vicinity of the scatterer:
\[
\left( \frac{G_{H,M}}{G_{\text{RES}}} \right)_{zz} \bigg|_{\begin{array}{l} x=x_1 \\ y=y_1 \\ \rho \to 0 \end{array}} \approx \frac{\mu_0}{ab} \sum_{n_1 n_2=0}^{y_{\text{max}}} \frac{\mu_0}{4\pi} \exp\left(-\gamma_{\text{max}}|z-z'|\right) \tag{12}
\]

Now, substituting (12) into (7b), going to the limit $k|z-z'| \to 0$ and keeping main terms, including constant ones, we obtain the desired decomposition of the resonator Green’s function for the magnetic field:
\[
\left( \frac{G_{H,M}}{G_{\text{RES}}} \right)_{zz} \bigg|_{\begin{array}{l} x=x_1 \\ y=y_1 \\ \rho \to 0 \end{array}} \approx \frac{1}{j\omega ab} \sum_{n_1 n_2=0}^{y_{\text{max}}} \varepsilon_{\epsilon_{n_1 \epsilon_{n_2}}} \left( (k_r^2)^2 + (k_{\gamma}^2)^2 \right) \times \cos^2(k_r^2 x) \cos^2(k_{\gamma}^2 y) \times \text{sinh}(y_r z) \sinh(y_{\gamma} (h - z)) \times \frac{1}{y_r \sin(y_{\gamma} h)} + \frac{1}{4\pi j\omega} \left[ 2 \frac{k_0^2}{|z-z'|^2} - \frac{\gamma_{\text{max}}^3}{3} - k_1^2 \gamma_{\text{max}} \right] \tag{13}
\]

Again, as for the case of free space, we can extract from (13) the singular near field part (the first and the second terms in the square brackets), which look like that one in free space and the regular far field part (the sum and the third and fourth terms in the square brackets, which take into account reflection of the signal from the cavity walls and contain all information about system resonances).

The solution of the interaction problem with the singular part of the Green’s function can be obtained by the model of a small loop [3] (see Figures 3(a) and 3(b))
\[
J_H(j\omega) \approx \varepsilon_{H,\text{eff}} Y_H(j\omega) = K_H(j\omega)H_z, \tag{14a}
\]
\[
\varepsilon_{H,\text{eff}} = -j\omega H_z \mu_0 S, \tag{14b}
\]
\[
Y_H(j\omega) = \frac{1}{j\omega L_a + R_a}. \tag{14c}
\]

Here the quantities $S$, $L_a$ are the antenna area and inductance, and $R_a$ is the antenna load. For a thin circular loop antenna with radius $R$ and diameter of the wire $d$ [3],
\[
L_a = \mu_0 R \left( \ln\left( \frac{16R}{d} \right) - 2 \right). \tag{15}
\]
account the regular part of the Green’s function, we have to add to the magnetic field in (14a) an additional part $\vec{H}_z$
\begin{align}
\vec{H}_z &= \left( \frac{\omega H_{\text{M}}}{G} \right)_{zz} \int f_z^m(\vec{r}) dV' = \left( \frac{\omega H_{\text{M}}}{G} \right)_{zz} \int f_z^m(\vec{r}) dV' \\
&= \left( \frac{\omega H_{\text{M}}}{G} \right)_{zz} j\omega M_z = \left( \frac{\omega H_{\text{M}}}{G} \right)_{zz} j\omega SF
\end{align}
(16)

Here we take into account (11), (6) and the connection of the magnetic moment with the electrical current. Then, from (14a), we have
\begin{align}
\vec{J}_H &= \left( H_z + \left( \frac{\omega H_{\text{M}}}{G} \right)_{zz} j\omega SF \right) \cdot K_H(j\omega).
\end{align}
(17)

This is a linear algebraic equation for the induced current $\vec{J}_H$ whose solution yields
\begin{align}
\vec{J}_H &= \frac{K_H(j\omega)}{1 - \left( \frac{\omega H_{\text{M}}}{G} \right)_{zz} j\omega SF} H_z \\
&= \frac{-j\omega \mu_0 S}{j\omega L_a + R_a - \mu_0 \omega^2 S^2 \left( \frac{G H_{\text{M}}}{G} \right)_{zz}} H_z \\
&= \vec{J}_H(j\omega) H_z.
\end{align}
(18)

By this way, taking into account the regular part of the Green’s function yields the additional impedance $Z_a(j\omega)$ to the antenna resistance $R_a$ in (14b)
\begin{align}
Z_a(j\omega) &= -\mu_0 \omega^2 S^2 \left( \frac{G H_{\text{M}}}{G} \right)_{zz}.
\end{align}
(19)

For free space, with $\left( \frac{G H_{\text{M}}}{G} \right)_{zz}$ from (11), (19) yields a well-known equation for the radiation resistance of the electrically small loop [3]:
\begin{align}
Z_a^{\text{free space}}(j\omega) &= \frac{\eta_0 k^4 S^2}{6\pi}.
\end{align}
(20)

This agreement shows the correctness of our method for a small loop.

For the loop inside the resonator, (19) gives an impedance value for the antenna (“impedance of loop-resonator system”) $Z_a^{\text{RES}}$, similar to the case of a small near-resonance dipole in Section 2. This value depends on both, the loop antenna and resonator parameters, and describes all resonances of the following system:
\begin{align}
Z_a^{\text{RES}}(j\omega) &\approx j\eta_0 k^4 S^2 \left\{ \frac{1}{a^b} \sum_{n_x = 0}^{n_{\text{max}}} e_{n_x} e_{n_y} \left( (k_x')^2 + (k_y')^2 \right) \\
&\times \cos^2(k_x' x) \cos^2(k_y' y) \\
&\times \sinh(y_x z) \sinh(y_y (h - z)) \\
&\gamma_z \sinh(y_x h) \\
&- \frac{1}{4\pi} \left[ \frac{\gamma_z^3}{3} - k^2 y_{\text{max}}^3 \right] \right\}.
\end{align}
(21)

Note, that the (18)–(21) in the present section describe, namely, the differential current mode in the loop. This mode prevails for small frequencies, much smaller than the frequency of the first resonance (for the circular loop $\omega_1 \approx c/R$). For the higher frequencies to evaluate the total current, one has to take into account the common current mode, which can be obtained using the approach of small electrical dipole antenna described in previous section. Combination of these two modes (depending from the orientation of the loop) gives a total current, which can be compared, for example, with numerical calculations. Such comparison will be a subject of nearest investigations.

4. Application of the Developed Theory: Transfer Function “External Field-Induced Current”

Now it is assumed that an exterior field penetrates through a rectangular aperture into the cavity and excites a small dipole (see Figure 5). The dimensions of the resonator are $a = h = 0.79 \text{ m}$, $b = 0.534 \text{ m}$. The center of the $z$-directed unloaded dipole antenna with length $L = 0.25 \text{ m}$ and radius $r = 1 \text{ mm}$ is placed in the point with coordinates $x = 0.395 \text{ m}$, $y = 0.3 \text{ m}$, and $z = 0.295 \text{ m}$. The cavity is excited through the rectangular slit by the normally incident plane wave with amplitude $E_0 = 100 \text{ V/m}$. The position of the center of the slit is $x_1 = 0.395 \text{ cm}$, $z_1 = 5 \text{ mm}$; the dimensions of the slit are $d_x = 100 \text{ mm}$, $d_z = 5 \text{ mm}$.

The aperture is modeled by two equivalent electric and magnetic dipoles placed in the center of the aperture [7]. For the considered case, only the $xx$ component of the magnetic dipole moment $M_x = -\beta_{xx} H_{z}^{\text{sc}}$ of the aperture is important, which can be calculated using the magnetic polarisability $\beta_{xx} = \pi/24d_x^3(\ln(4d_x/d_z) - 1)^{-1}$. Here $H_{z}^{\text{sc}}$ is a short-circuit magnetic field because the plane of the aperture is assumed to be perfectly conducting when solving the external problem of diffraction. Now using the numerical data for the $H_{z}^{\text{sc}}$, we can calculate the magnetic moment of the aperture. Then we calculate the internal electrical field using (7a) and (7c), and (9) and, after that, calculate the current amplitude in the dipole antenna using the results of Section 2. Comparison
of the MSA results with those of the well-known numerical code “CONCEPT” is presented in Figure 6. One can see a satisfactory agreement of the numerical and analytical results, especially in the frequency region up to the first resonance of the small antenna. The observed differences (especially the additional peaks near the main resonance) are due to the fact that in this example, taken from a practical HPM coupling problem, the conditions of applicability of the method of small antennas are not well satisfied: the size of the antenna is comparable with the wavelength and the cavity dimensions. Moreover, there may be additional resonances in the penetration of radiation through the slit.

Note, that the calculation of MSA requires about one minute of calculation by the PC notebook (Processor T7250 2 GHz); at the same time direct CONCEPT calculation requires approximately one week of calculations by the PC cluster (6 PCs).

5. Conclusion

Interactions of high-frequency electromagnetic fields with small scatterers inside rectangular (resonators taking into account both common and differential modes) were investigated using the Method of Small Antenna. The results are important for the study of natural and intentional electromagnetic interferences with printed circuits, chips, and so forth inside racks and housings. Comparison of the results with numerical ones has shown a good agreement. In future, we intend to investigate effective damping of the interferences inside cavities using the generalization of the developed model for the multiscatterer case.

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Research Article

Evaluation of the Inductive Coupling between Equivalent Emission Sources of Components

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The electromagnetic interference between electronic systems or between their components influences the overall performance. It is important thus to model these interferences in order to optimize the position of the components of an electronic system. In this paper, a methodology to construct the equivalent model of magnetic field sources is proposed. It is based on the multipole expansion, and it represents the radiated emission of generic structures in a spherical reference frame. Experimental results for different kinds of sources are presented illustrating our method.

1. Introduction

The development of semiconductor technology in the last decades has greatly increased the use of power electronics in various applications, such as computer power supplies, voltage converters, electronic ballasts, and variable-speed drives [1]. Recently, new applications of power electronics have also appeared in the vehicle industry, such as electric cars and airplanes. However, the commutation of the switches (rectifiers, SCRs and triacs, BJTs, MOSFETs, and IGBTs) generates high currents with high \( \frac{di}{dt} \), and, thus, a wide bandwidth of unwanted electromagnetic interference (EMI) pollutes the electromagnetic environment [2].

The electromagnetic compatibility (EMC) is an engineering domain responsible to ensure that systems, equipment, and devices can coexist satisfactorily in the same electromagnetic environment [3]. Electric cars, for instance, may encounter malfunction in its electronic systems (ESP, ABS, ALS, etc) if special care is not taken. The EMI between the cables of the power electronics and the cables carrying electronic signals, if they are too close to each other without proper shielding, may prevent the correct operation of certain systems [4].

There are not many reliable methods to predict the EMC of a complex system in the design phase [5], and, thus, in practice, the EMC design is still carried out by trial and error [3] causing high development cost in case of malfunctioning due to EMI, when the prototype is tested.

To ensure the compatibility of cables, equipment, and systems at the design phase, EMC predictive tools must be improved [5]. In order to achieve this requirement, frequency domain simulations can be performed utilizing equivalent models for the EMI sources. For instance, in power electronics, the range of frequency analyzed can be restricted from 10 kHz to 50 MHz, which comprises the common operating range of semiconductor switches utilized in power converters and frequency harmonics produced by them.

The EMI is usually established in different ways, for instance, the near-field coupling between filter components [6–9] or the coupling between wires [10]. Each coupling phenomenon is thus best modeled by different mathematical models.

The near-field coupling between filter components can be well modeled by a methodology based on the multipole
expansion, which represents the radiation emission in a spherical reference system \((r, \theta, \phi)\) [6–9], whereas the coupling between wires is usually well modeled by the PEEC method [10].

This paper presents a methodology to determine the first two coefficients of the multipole expansion \((Q_{10} \text{ and } Q_{20})\) of a generic magnetic field source, by a numerical or an experimental approach, depending upon the complexity of the source. The numerical approach is rather limited to simple sources, but the experimental approach has no limitations over the geometrical complexity of the source.

The experimental approach utilizes an antenna consisting of four loops around the magnetic field source. The mutual coupling between the loops must be taken into account when modeling the source, in order to avoid a significant error, which can be up to 40%.

Finally, the methodology is validated by comparing the calculated and measured mutual inductance of a modeled power transformer and a well-known loop.

2. Theory of Multipole Expansion

The multipole expansion can be used to represent electromagnetic fields in 3D, assuming that the field is computed outside a sphere of a given radius that contains the equivalent source. Figure 1 shows the reference sphere considered [11].

In the case of outgoing radiated emission sources, the multipole expansion allows expressing the electric and magnetic fields as [12]

\[
E(r, \theta, \phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} Q_{nm}^{TE} F_{1nm}(r, \theta, \phi) + Q_{nm}^{TM} F_{2nm}(r, \theta, \phi),
\]

\[
H(r, \theta, \phi) = \frac{j}{\eta} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} Q_{nm}^{TM} F_{1nm}(r, \theta, \phi) + Q_{nm}^{TE} F_{2nm}(r, \theta, \phi),
\]

where

(i) \(\eta = \sqrt{\mu/\varepsilon}\) is the intrinsic impedance of the considered environment;

(ii) \(Q_{nm}^{TE}\) and \(Q_{nm}^{TM}\) are the magnetic and electric coefficients, respectively. The coefficients \(Q_{nm}^{TE}\) describe the strength of the transverse-electric (TE) components of the radiated field, while coefficients \(Q_{nm}^{TM}\) describe the strength of the transverse-magnetic (TM) components. Each of them corresponds to the equivalent radiated source. Thus, these coefficients are the parameters to be identified that characterize the equivalent model of the radiated field components;

(iii) \(F_{1nm}\) and \(F_{2nm}\) are the vector spherical harmonics which are solutions of Maxwell's equations in free space, excluding the sphere that involves the sources;

(iv) \(n\) is the degree, and \(m\) is the azimuthal order.

In our study, only the magnetic source in the near-field is considered. That is \(Q_{nm}^{TM} = 0\), and it is assumed that the electric field component is low when compared with the magnetic field. Thus, the computation of the \(Q_{nm}^{TE}\), wrote as \(Q_{nm}\) in (2), is carried out by the radial component \(H_r\), in near field [12, 13]:

\[
H_r = -\frac{1}{4\pi} \sum_{n=1}^{N_{max}} \sum_{m=-n}^{n} Q_{nm} \frac{\partial}{\partial r} \left( \frac{1}{r^{n+1}} \right) Y_{nm}(\theta, \phi),
\]

where \(Y_{nm}\) are the normalized spherical harmonics given by the following expression:

\[
Y_{nm}(\theta, \phi) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}} P_n^{m}(\cos \theta)e^{im\phi}.
\]

One of the main properties of the multipole expansion to be emphasized is the decrease of the terms of order \(n\) with \(r^{-(n+1)}\). This ensures a hierarchy between each order of the decomposition. The larger is the distance to the source, the fewer are the terms required to reconstruct the field. Thus, the accuracy of the mutual inductance computation is related to the choice of the maximum order description, noted \(N_{max}\). It should be observed that there are \((2n+1)\) components for each \(n\) order. For an order source equal to \(N_{max}\), it will correspond to \(N_{max}(N_{max} + 2)\) components, but due to the previously mentioned property (hierarchy between each order), \(N_{max}\) can be limited to 5, based on the present experience of the authors.

3. Multipole Identification

3.1. Numerical Approach. This approach consists in identifying the source utilizing the software Flux2D based on the finite element method. The software calculates the radial component of the magnetic induction on a measurement sphere \(S_M\), which contains the source, as shown in Figure 2. The computation of the \(Q_{nm}\) coefficients is achieved by integrating these components on \(S_M\).
The coefficients of the multipole expansion can be deduced from (2), based on the following expression:

\[ Q_{nm} = 4\pi \frac{r_0^n}{(n + 1)} \int_0^{2\pi} \int_0^\pi H_r(r_0, \theta, \varphi) Y_{nm}(\theta, \varphi) \, d\theta \, d\varphi, \tag{4} \]

where \( H_r \) corresponds to the radial component of the magnetic field on the sphere \( S_M \) of a radius of \( r_0 \). This result is due to the orthogonal property of \( Y_{nm} \) base:

\[ \int_{S_M} Y_{nm} Y_{nm'} \, dS = \begin{cases} r_0^2, & \text{if } (n, m) = (n', m'), \\ 0, & \text{otherwise}. \end{cases} \tag{5} \]

The order of the approximation is not limited for this identification method. However, the computational time increases with the order. Moreover, the discretization of the sphere surface must respect the Shannon theorem in order to avoid spatial aliasing. For instance, with \( n = 1 \), the axes theta and phi require at least two points each, whereas, for \( n = 2 \), four points are required. For \( n = N_{max} \), \( 2N_{max} \) points are necessary for each axis.

The numerical approach can be excessively time consuming or require too much memory, if the modeled source is geometrically complex. The experimental approach is thus an alternative, and it is suitable for practically any source.

### 3.2. Experimental Approach

This approach consists in identifying the source utilizing an antenna and a measurement equipment. Figure 3 shows the prototype antenna with its loop sensors corresponding to the dipole (2 loops for the dipole component \( Q_{10} \)), the quadrupole (2 loops for the quadrupole \( Q_{20} \)), and the loop from the standard CISPR16-1. All mentioned loops were built initially only in the \( z \)-direction. The complete measurement setup is surrounded by a sphere of radius \( r_M \) equal to 0.225 m. The short-circuited loops were proposed as sensors with a flat response within the 9 kHz–30 MHz frequency range. Although the use of short-circuited loops corresponds to high values of currents and thus high sensitivity, the magnetic coupling between them imposes some constraints to the measurement and calibration methodology.

Based on the multipole expansion of the magnetic field, the relationship between the fluxes across the surface delimited by the “sensors set” and the \( Q_{nm} \) components of the expansion can be directly obtained [14, 15]. The quasi-static approximation was adopted, and, for the maximum frequency of 30 MHz, it is valid for a \( r_M \leq 1.7 \) m. In our case, assuming the expansion limited to the second order and in the \( z \)-direction (\( m = 0 \)), we have [15]

\[ Q_{10} = \frac{10^6 r_M}{32\pi} (\varphi_{10} + \varphi_{10}), \tag{6} \]

\[ Q_{20} = \frac{6125 \times 10^4 r_M^3}{3\pi \sqrt{21}} (\varphi_{20} - \varphi_{20}). \tag{7} \]

The correction of the magnetic coupling between the loop sensors, which can be considered as a postprocessing in the identification procedure, is treated as follows: the total concatenated magnetic flux in each loop can be expressed as the sum of the flux produced by the source of interest and the fluxes produced by all the other antenna loops (undesired). The measured current in loop \( n \) is denoted as \( i_{MES}^{(n)} \) and can be obtained by the following expression:

\[ i_{MES}^{(n)} = i_{DUT}^{(n)} - \sum_{k \neq n}^{5} j\omega M_{kn} r_n + j\omega L_n, \tag{8} \]

where \( r_n \) is the resistance and \( L_n \) is the self-inductance of the loop \( n \), \( M_{kn} \) is the mutual inductance between loops \( k \) and \( n \), \( \omega \) is the angular frequency, and \( i_{DUT}^{(n)} \) is the current in loop \( n \) due to the multipole source only.
Thus, considering now the measured currents for all the five loops, one can write (8) in matrix form, solved for \( i_{n}^{\text{DUT}} \):

\[
[i_{n}^{\text{DUT}}]_{n} = [M]_{n} [i_{n}^{\text{MES}}].
\]

(9)

The elements of \([M]_{n,n}\) are unitary in the diagonal and given by \((j\omega Mkn/(r_{n} + j\omega L_{n}))\) otherwise. The coefficients \(Q_{10}\) and \(Q_{20}\) can then be determined by (6), and (7) utilizing the set of currents in (9). This procedure was validated numerically and experimentally, and the results are presented in the following section.

4. Multipole Identification Results

A vector network analyzer (VNA) and large bandwidth current probes were utilized to measure the current ratio of each sensor loop relative to the source, in dB. The frequency range of all experiments was from 20 kHz to 10 MHz.

Three different magnetic field sources were studied: a dipole, a quadrupole, and a generic power transformer. The accuracy of the methodology can be easily verified for the first two sources by utilizing the following expressions:

\[
\text{Dipole: } Q_{10} = \pi r^{2}i; \quad Q_{20} = 0,
\]

\[
\text{Quadrupole: } Q_{10} = 0; \quad Q_{20} = \pi r^{2}h_{0i},
\]

(10)

where \(r\) is the radius of the loop and \(h_{0}\) is the distance between the loops in the quadrupole source. Moreover, for these 2 sources, it is only necessary to present the results for the upper (or lower) loop antennas due to the symmetry on the \(z\)-axis with respect to the origin.

The accuracy of the coefficients \(Q_{10}\) and \(Q_{20}\) of the power transformer can be verified indirectly by calculating [16] and measuring its mutual inductance with a known circular loop and then comparing these results. The VNA and the probes are again used for the measurement.

4.1. Dipole. The measured current ratios loop/source for a dipole of radius 5 cm aligned in the \(z\)-axis for the loop sensors \(Q_{10,1}\) (loop 2) and \(Q_{20,1}\) (loop 1) are presented in Figures 4 and 5, respectively. For each figure, there are 6 curves, in which the 3 upper ones correspond to the lower ones after applying the postprocessing described previously.

Supposing a current of \(1\) A rms in this dipole and the symmetry in the \(z\)-axis and utilizing the plots in Figures 4 and 5, we can determine the 4 currents of (7) and finally the components \(Q_{10}\) and \(Q_{20}\) with (6).

4.2. Quadrupole. The measured current ratios for a quadrupole of parameters \(r\) and \(h_{0}\) both equal to 5 cm are presented in Figures 6 and 7 in a similar fashion done for the dipole.

4.3. Power Transformer. The measured current ratios loop/source for a power transformer rated 220 V—20 A are presented in Figure 8. The experiment was conducted in a similar manner to the previous ones, although there is no longer symmetry in the \(z\)-axis.

The components of the multipole expansion of the dipole, the quadrupole, and the transformer and the errors of the first two sources relatively to (10) are presented in Table 1. The frequency considered for these results was 200 kHz, located on the flat part of the measured curves.

| Component   | \(Q_{10}\) (m·Am\(^{-2}\)) | \(Q_{20}\) (m·Am\(^{-2}\)) | Error (%) |
|-------------|--------------------------|--------------------------|-----------|
| Dipole      | 7.3                      | 0                        | 7.6 (\(Q_{10}\)) |
| Quadrupole  | 0                        | 0.63                     | 20 (\(Q_{20}\))  |
| Transformer | 66.5                     | 0.82                     | —          |
It should be noted that there is a $-4$ dB and $-3.5$ dB difference between the current ratios in Figure 4 and in Figure 5, respectively. Therefore, the effect of the mutual inductances in the antenna would correspond to a difference of 37% and 33% in the calculation of $Q_{10}$ and $Q_{20}$, respectively, for the dipole.

The mutual inductance between this transformer and a loop can be determined by the following expression:

$$M_{12}i_1 = L_2i_2,$$  \hspace{1cm} (11)

where $M_{12}$ is the mutual inductance, $i_1$ the current in the transformer, $L_2$ the self-inductance of the loop, and $i_2$ the current in the loop.

This was done by measuring the ratio of currents using the VNA and applying an analytical formula for the self-inductance of a loop [16] as shown in (11). Figure 9 shows the measurement setup for this case.

5. Computing the Mutual Inductance

Using the equivalent radiated field source model, we can determine the coupling between two equivalent sources through the computation of the mutual inductance. Figure 10 illustrates the configurations regarding the representation of two radiating sources (models 1 and 2).

The computation of the mutual impedance between source 1 and source 2 can be expressed in terms of the electrical field $\mathbf{E}$ and magnetic field $\mathbf{H}$ for each source [12]:

$$Z_{12} = -\frac{1}{k_1k_2} \oint_{\Sigma_1} (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1).$$ \hspace{1cm} (12)
When the spheres that contain the sources do not intersect each other, the mutual impedance can be expressed according to the coefficients of the multipole expansion:

\[
Z_{12} = \frac{1}{i\omega i_1 i_2} \frac{1}{k^2} \frac{1}{\mu_0} \sum_{n=1}^{N_{\text{max}}} \sum_{m=-n}^{n} (-1)^m (Q_{1n,-m} \ast Q_{2n,m}).
\]

(13)

The expression of the mutual inductance is

\[
M_{12} = \frac{1}{j\omega i_1 i_2} \frac{1}{k^2} \frac{1}{\mu_0} \sum_{n=1}^{N_{\text{max}}} \sum_{m=-n}^{n} (-1)^m (Q_{1n,-m} \ast Q_{2n,m}),
\]

(14)

where \(i_1\) and \(i_2\) are, respectively, the current that flows in sources 1 and 2, and \(k\) is the phase constant.

The coefficients associated to the magnetic transverse modes of the multipole expansion of sources 1 and 2 must be expressed in the same reference: a translation is required, for example, the coefficients of the source 2 can be expressed in the reference of the source 1.

The rotation of the coefficients \(Q_{nm}\) is expressed by using Euler angles. It should be mentioned that only two angles are necessary because of the spherical symmetry. The details of the methodology for determining the rotation matrices for the spherical basis are identified. It can be obtained by a numerical or an experimental approach. Both of them were discussed in [17, 18].

The translation is based on the “Addition Theorem for Vector Spherical Harmonics” [18].

The addition theorem links the harmonics evaluated on \(r\) to those evaluated on \(r'\), where \(r\) is measured from the origin of the first spherical basis, whose axes are parallel to the first as shown in Figure 11. The origin of the second spherical basis is located in the first by \(r''\). These 3 vectors are connected by the relation \(r = r' + r''\).

The expression of the translation coefficients \(Q_{nm}\) are:

\[
Q_{n'm'}^{TE} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} Q_{nm}^{TE} A_{n',m',n,m} + Q_{nm}^{TM} B_{n',m',n,m},
\]

\[
Q_{n'm'}^{TM} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} Q_{nm}^{TM} A_{n',m',n,m} + Q_{nm}^{TE} B_{n',m',n,m}.
\]

(15)

The coefficients \(A_{n',m',n,m}\) and \(B_{n',m',n,m}\) involve computing of the Wigner 3j symbol according to quantum mechanics [19].

Utilizing the \(Q_{10}\) and \(Q_{20}\) of the transformer, it is also possible to estimate its mutual inductance with the loop in a simpler manner. This is achieved by using an analytical expression for the mutual inductance between 2 loops [16], by considering that \(Q_{10}\) is represented by a loop and \(Q_{20}\) is represented by 2 loops, both in the \(z\) axis. These results are presented in Table 2.

### Table 2: Comparison between mutual inductances.

| Height (cm) | Measured (nH) | Estimated (nH) | Error (%) |
|------------|---------------|----------------|-----------|
| 29.3       | 3.65          | 3.17           | 13        |
| 35.8       | 1.98          | 1.92           | 3         |

6. Conclusion

The presented methodology enables the evaluation of coupling parameters of components by using equivalent emission sources. This method is composed by two steps. At first, the equivalent sources which represent the radiated field component using the multipole expansion representation are identified. It can be obtained by a numerical or an experimental approach. Both of them were discussed in the paper. Secondly, the equivalent sources will be used to compute the coupling between them, which was represented by a mutual inductance as a function of the distance that separates them.

Other kind of multipole expansions like the cylindrical one can be more suitable for modeling components such as tracks or cables, and it will also be considered. For example, in the case of the coupling between a track and a component, the spherical harmonics method is not very adequate and other harmonics method should be used.

The method proposed could be helpful when used together with other circuit simulator methods in the evaluation of equivalent circuit of power electronics devices (R-L-M-C). This will give us a considerable gain of memory space.
concerning the full model configuration used in EMC filter numerical simulations.

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