De Sitter Space from Warped Supergravity Solutions

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ABSTRACT

The solutions of 10 and 11 dimensional supergravity that are warped products of de Sitter space with a non-compact ‘internal’ space are investigated. A convenient form of the metric is found and it is shown that in each case the internal space is asymptotic to a cone over a product of spheres. A consistent truncation gives gauged supergravities with non-compact gauge groups. The BPS domain wall solutions of the non-compact gauged supergravities are lifted to warped solutions in 10 or 11 dimensions.
1. Non-Compact Gaugings and Higher Dimensional Solutions

Gauged supergravities with compact gauge groups typically arise from dimensional reduction of higher dimensional supergravities with compact internal spaces. In [1], it was shown that the supergravities with non-compact gauge groups are associated with higher dimensional supergravity solutions that have a non-compact ‘internal’ space. In particular, de Sitter space solutions in \( D = 4 \) arise in this way. The no-go theorems of [2,3] imply that de Sitter space cannot arise form a compactification of a higher-dimensional supergravity theory, and the solutions of [1] get around this by having a non-compact internal space. Our purpose here is to analyse the solutions of [1], and to generalise them to obtain the 11-dimensional origin of the solutions of [4].

In \( D = 4 \) the Cremmer-Julia \( N = 8 \) supergravity theory [5], with scalars in the coset space \( E_7/SU(8) \), can be gauged by promoting a subgroup of the rigid \( E_7 \) symmetry to a local symmetry. The gauge group is necessarily 28-dimensional (as there are 28 vector fields in the \( N = 8 \) supermultiplet) and is always contained in the \( SL(8,\mathbb{R}) \) subgroup of \( E_7 \) which is a symmetry of the ungauged action. The \( SO(8) \) gauging of [6] has a maximally supersymmetric anti-de Sitter vacuum. In [7,8], gaugings with gauge group \( CSO(p, q, r) \) were obtained for all non-negative integers \( p, q, r \) with \( p + q + r = 8 \), where \( CSO(p, q, r) \) is the group contraction of \( SO(p + r, q) \) preserving a symmetric metric with \( p \) positive eigenvalues, \( q \) negative ones and \( r \) zero eigenvalues. Then \( CSO(p, q, 0) = SO(p, q) \) and \( CSO(p, q, 1) = ISO(p, q) \), and \( CSO(p, q, r) \) is not semi-simple if \( r > 0 \). In [12], it was argued that these are the only possible gauge groups. Note that despite the non-compact gauge groups, these are unitary theories. Of these theories, the ones with gauge groups \( SO(4, 4) \) and \( SO(5, 3) \) have de Sitter vacua arising at local maxima of the potentials [8,9]. The \( CSO(2, 0, 6) \) gauging has a Minkowski space solution and the potential has flat directions [8]. The structure and potentials of these models were analysed further in [10]; no other critical points are known. In \( D = 5 \), the gauged \( N = 8 \) supergravities include those with gauge groups \( SO(p, 6 - p) \) [11,13] and of
these the \( SO(3,3) \) gauged theory has a de Sitter vacuum. In \( D = 7 \), there are supergravities with gauge groups \( SO(p,5-p) \) [16].

The higher-dimensional origin of these theories was found in [1]. Whereas the internal spaces for the compact gaugings are spheres, for the non-compact gaugings, they are typically non-compact spaces of negative curvature. Consider a solution of a \( D \)-dimensional gauged supergravity in which the only non-vanishing fields are a metric \( \bar{g}_{\mu\nu}(x) \) on a space-time \( M \) and certain scalar fields \( \phi(x) \). Then these lift to solutions of \( d \)-dimensional supergravity where \( d = 11 \) for \( D = 4, 7 \), while for \( D = 5 \) the solution lifts to a solution of \( d = 10 \) IIB supergravity. In each of these cases, the metric is of the form

\[
ds^2 = V^a \bar{g}_{\mu\nu}(x)dx^\mu dx^\nu + V^b g_{mn}(x,y)dy^m dy^n \tag{1.1}
\]

where \( g_{mn}(x,y) \) is a metric on an ‘internal’ space \( N \) with coordinates \( y^m \) and dimension \( d - D \), \( V(x,y) \) is a warp factor and \( a, b \) are constants. For solutions with constant scalars \( \phi(x) \) at a critical point of the scalar potential, the anti-symmetric tensor field strength is given in terms of the volume forms on \( M \) or \( N \).

For \( D = 4 \), the \( d = 11 \) field strength \( F_4 \) is proportional to the volume form on \( M \), for \( D = 5 \) the IIB \( F_5 \) is a self-dual combination of the volume forms on \( M \) and \( N \), and for \( D = 7 \), \( \ast F_4 \) is proportional to the volume form on \( M \). For solutions with varying scalars, the ansatz for the field strength is a little more complicated.

The compact \( SO(p) \) gaugings arise when the internal space \( N \) is a sphere \( S^{p-1} \). If all the scalars vanish, then \( g_{mn} \) is the round metric and the warp factor is \( V = 1 \), but non-vanishing scalars lead to a squashed metric on the sphere and a non-trivial warp factor [17,18]. For the \( CSO(p,q,r) \) gauging, the space \( N \) is \( \mathcal{H}^{p,q,r} \) where \( \mathcal{H}^{p,q,r} \) is a hypersurface of \( \mathbb{R}^{p+q+r} \), with a positive definite metric induced from the Euclidean metric in \( \mathbb{R}^{p+q+r} \). The hypersurface in \( \mathbb{R}^{p+q+r} \) is that in which the real Cartesian coordinates \( z^A \) of \( \mathbb{R}^{p+q+r} \) satisfy

\[
\eta_{AB} z^A z^B = R^2, \tag{1.2}
\]

where \( \eta_{AB} \) is a metric with \( p \) positive eigenvalues, \( q \) negative ones and \( r \) zero
eigenvalues. The scale $R$ is determined by the flux of an antisymmetric tensor gauge field strength. For example, for $D = 4, d = 11$, the 4-form field strength of 11-dimensional supergravity is proportional to $R$ times the volume form on $M$. Then $\mathcal{H}^{p,0,0}$ is a sphere $S^{p-1}$, $\mathcal{H}^{p,1,0}$ is the hyperboloid $H^p$, which is the coset space $SO(p,1)/SO(p)$, and $\mathcal{H}^{p,q,0}$ is a hyperbolic space (a non-symmetric space with negative curvature) and $\mathcal{H}^{p,q,r} = \mathcal{H}^{p,q,0} \times \mathbb{R}^r$ is a generalised cylinder with cross-section $\mathcal{H}^{p,q} \equiv \mathcal{H}^{p,q,0}$. For the cylinders, the flat directions can be compactified to give $\mathcal{H}^{p,q,0} \times T^r$.

It will be useful to consider metrics of the form

$$\eta_{AB} = \begin{pmatrix} 1_{p \times p} & 0 & 0 \\ 0 & \xi \lambda 1_{q \times q} & 0 \\ 0 & 0 & \zeta 1_{r \times r} \end{pmatrix}, \quad (1.3)$$

where we will usually take $\zeta = 0$. Then the hypersurface (1.2) is a cylinder $\mathbb{R}^r \times K$ (or $T^r \times K$) with $p+q-1$ dimensional cross-section $K$. If $\lambda = 1$ and $\xi = 1$, then $K$ is a sphere $K = S^{p+q-1}$ with a ‘round’ metric, while if $\lambda = 1$ and $\xi = -1$, then $K$ is the hyperbolic surface $\mathcal{H}^{p,q}$. If $\lambda \neq 1$, then the resulting metric on $S^{p+q-1}$ or $\mathcal{H}^{p,q}$ is ‘squashed’. The warp factor (with $\zeta = 0$) is given by

$$V^2 = R^{-2} \left[ \sum_{i=1}^{p} (z^i)^2 + \xi^2 \lambda^2 \sum_{i=p+1}^{p+q} (z^i)^2 \right]. \quad (1.4)$$

The simplest case is that in which all scalars $\phi$ are constant and $(M, \tilde{g})$ is an Einstein space with cosmological constant $\Lambda$. Then the metric $\eta_{AB}$ is a constant matrix, $R$ is also a constant and the warp factor is a function of $y^m$ only, $V(x,y) = V(y)$. The more general case in which some of the scalars are not constant will be considered in section 4, giving rise to a metric $g_{mn}(x,y)$ on $N$ which varies with the coordinates $x$ on $M$.

* The notation for various quantities $V, \lambda, R$ used here are related to that in [1] as follows: $V = \mu$, $\lambda = c^2$, $R = r$. 

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Such solutions with $\Lambda < 0$ arise for the compact gaugings in $D = 4, 5, 7$, while solutions with $\Lambda > 0$ arise for the $D = 4$ gaugings with gauge groups $G = SO(4, 4)$ and $SO(5, 3)$, and for the $D = 5$ gauging with $G = SO(3, 3)$. There are also solutions with $\Lambda = 0$, for example in the $D = 4$ gauging with $G = SO(2, 0, 6)$, but these will not be discussed here. For the other gaugings, there are no Einstein space solutions with constant scalars, but there are BPS domain wall solutions with one or more non-constant scalars [4], and these will be considered in section 4.

The isometry group of the internal space is the unbroken gauge symmetry in the corresponding gauged supergravity solution. In a non-compact gauging, the gauge group is spontaneously broken to a compact subgroup, and in each of the cases considered here, the isometry group of the internal space is compact. This in particular implies that these spaces cannot be compactified by identifying under the action of a discrete isometry group, although new non-compact solutions can be obtained in some cases by such discrete identifications. The de Sitter solutions necessarily have no Killing spinors and so completely break supersymmetry. The isometry group of $H^{p,q}$ is $SO(p) \times SO(q)$, and for the de Sitter solutions with $(p, q) = (4, 4), (5, 3)$ or $(3, 3)$, the $SO(p, q)$ gauge group is broken to $SO(p) \times SO(q)$. Similarly, for the inhomogeneously squashed $n$-sphere compactifications of [15] discussed below, the isometry group is $SO(n)$ and the corresponding gauged supergravity solutions have scalar expectation values that break the $SO(n+1)$ gauge symmetry to $SO(n)$.

The solutions of the form (1.1) are as follows [1]. In $D = 4$, the solutions all have

\[
a = \frac{2}{3}, \quad b = -\frac{1}{3}, \quad \Lambda = \frac{4\lambda}{R^2}. \tag{1.5}
\]

The de Sitter solutions with $\Lambda > 0$ and $\xi = -1$ of the $SO(4, 4)$ and $SO(5, 3)$ gauged theories have

\[
p = 4, \quad q = 4, \quad \xi = -1, \quad \lambda = 1 \tag{1.6}
\]
and

\[ p = 5, \quad q = 3, \quad \xi = -1, \quad \lambda = 3, \quad (1.7) \]

respectively. There are also two anti-de Sitter solutions of this form with \( \Lambda < 0 \) arising for the compact \( SO(8) \) gauging with \( \xi = 1 \); these are the round seven sphere solution \( AdS_4 \times S^7 \) and the inhomogeneously squashed 7-sphere compactification of [15], which is the solution with

\[ p = 7, \quad q = 1, \quad \xi = 1, \quad \lambda = -5. \quad (1.8) \]

In \( D = 5 \), the solutions all have

\[ a = 1/2, \quad b = -1/2, \quad \Lambda = \frac{2\lambda}{R^2}. \quad (1.9) \]

The de Sitter solution of the \( SO(3,3) \) gauged theory in \( D = 5 \) arises from the solution of IIB supergravity with \( D = 5 \) and

\[ p = 3, \quad q = 3, \quad \lambda = 1, \quad (1.10) \]

and with the self-dual 5-form field strength given in terms of the volume forms on \( dS_5 \) and the internal space. There are also two \( \Lambda < 0 \) solutions for the compact \( SO(6) \) gauging, the round \( S^5 \) solution and the inhomogeneously squashed 5-sphere compactification of [15], which is the solution with

\[ p = 5, \quad q = 1, \quad \xi = 1, \quad \lambda = -3 \quad (1.11) \]

In \( D = 7 \), the solutions all have

\[ a = 1/3, \quad b = -2/3, \quad \Lambda = \frac{\lambda}{R^2}. \quad (1.12) \]

There are no \( \Lambda > 0 \) solutions, but there are two \( \Lambda < 0 \) solutions for the compact gauging, the round 4-sphere solution and the inhomogeneously squashed 4-sphere compactification of [15], which is the solution with

\[ p = 4, \quad q = 1, \quad \xi = 1, \quad \lambda = -2. \quad (1.13) \]
2. The Metric on the Internal Space

To obtain a more useful form of the metric on the hyperboloid $\mathcal{H}^{p,q}$, introduce first coordinates $\sigma, \theta_i, \tilde{\sigma}, \tilde{\theta}_i$ on $\mathbb{R}^{p+q}$, where $\theta_i$ are coordinates on $S^{p-1}$, $\tilde{\theta}_i$ are coordinates on $S^{q-1}$ and $\sigma, \tilde{\sigma}$ are radial coordinates on $\mathbb{R}^p$ and $\mathbb{R}^q$, respectively. The Euclidean metric on $\mathbb{R}^{p+q}$ is then

$$ds^2 = d\sigma^2 + \sigma^2 d\Omega^2_{p-1} + d\tilde{\sigma}^2 + \tilde{\sigma}^2 d\Omega^2_{q-1}$$

(2.1)

where $d\Omega^2_n$ is the round metric on $S^n$. Next, we introduce coordinates $r, \rho$ with

$$\tilde{\sigma} = \rho r, \quad \sigma = \rho (1 + \lambda r^2)^{1/2},$$

(2.2)

so that

$$\sigma^2 - \lambda \tilde{\sigma}^2 = \rho^2.$$  

(2.3)

Then the hypersurface $\mathcal{H}^{p,q}$ is defined by $\rho = R$ and $r, \theta_i, \tilde{\theta}_i$ are intrinsic coordinates on the space, and the metric on $\mathcal{H}^{p,q}$ induced by (2.1) is given by transforming to coordinates $\rho, r, \theta_i, \tilde{\theta}_i$ and setting $\rho = R$. This gives

$$ds^2 = R^2 \left( \frac{1 + (\lambda + \lambda^2) r^2}{1 + \lambda r^2} dr^2 + (1 + \lambda r^2) d\Omega^2_{p-1} + r^2 d\Omega^2_{q-1} \right).$$

(2.4)

The warp factor is

$$V = R^{-2} \left( 1 + (1 + \lambda^2) r^2 \right)$$

(2.5)

so that the metric on $N$ is (2.4) times $V^b$.

At large $r$, the metric takes the asymptotic form

$$ds^2 = (1 + \lambda) R^2 \left( dr^2 + \alpha r^2 d\Omega^2_{p-1} + \beta r^2 d\Omega^2_{q-1} \right)$$

(2.6)

where

$$\alpha = \frac{\lambda}{1 + \lambda}, \quad \beta = \frac{1}{1 + \lambda},$$

(2.7)

so that asymptotically this is a cone over $S^{p-1} \times S^{q-1}$, with a product metric that
is not Einstein for \( q > 1 \). Asymptotically, the warp factor becomes

\[
V = R^{-2}(1 + \lambda^2)r^2. \tag{2.8}
\]

The distance from an interior point to the boundary at \( r \to \infty \) is infinite for the metric (2.4), and remains infinite for the metric warped by \( V^b \) if

\[
b > -1 \tag{2.9}
\]

For all the solutions considered here, \( b > -1 \) and the internal space has infinite volume.

For \( \lambda = 1 \), the metric (2.4) is

\[
ds^2 = R^2 \left( \frac{1 + 2r^2}{1 + r^2} dr^2 + (1 + r^2) d\Omega_{p-1}^2 + r^2 d\Omega_{q-1}^2 \right) \tag{2.10}
\]

and the warp factor is

\[
V = R^{-2} (1 + 2r^2). \tag{2.11}
\]

This is the geometry of the \( D = 4, \ SO(4,4) \) gauging with \( p = q = 4 \) and the \( D = 5, \ SO(3,3) \) gauging with \( p = q = 3 \), while that for the \( D = 4, \ SO(5,3) \) gauging is a squashed version of this with \( \lambda = 3 \).

It is interesting to note that the radius of the \( S^{p-1} \) never vanishes. This means that one may identify points under the action of a discrete subgroup \( \Gamma \subset SO(p) \) and as long as \( \Gamma \) acts freely on \( S^{p-1} \), i.e. has no fixed points, the quotient will be smooth. In general this will break the \( SO(p) \) factor of the gauge group down to that continuous subgroup of \( SO(p) \) which commutes with \( \Gamma \). In some cases this may break \( SO(p) \times SO(q) \) to \( SO(q) \). In this way we could eliminate some Klauza-Klein gauge states. However since the radius of the \( S^{q-1} \) vanishes at the origin \( r = 0 \) we cannot use this mechanism to break the \( SO(p) \) factor.
3. A Lower-Dimensional Interpretation?

With a non-compact internal space, an important issue is whether the theory is intrinsically higher dimensional, or whether the physics can be consistently interpreted as $D$ dimensional, perhaps through a brane-world scenario [19] or a Kaluza-Klein-like dimensional reduction with non-compact space, as in [20,21]. In particular, because the internal space is non-compact there appears to be a difficulty in obtaining conventional 4-dimensional gravity for sources which are located at fixed values of the internal coordinate $y$. For instance one could imagine sources localized near the centre $r = 0$. The conventional criterion for getting gravity in this way is obtained by computing from the action the effective $D$-dimensional Newton’s constant $G_D$, giving $G_D = G_d/V$, where $G_d$ is the $d$-dimensional Newton’s constant and $V$ is the volume of the internal manifold $N$ with respect to the metric $V^b g_{mn}$. For instance in the case $p = q = 4, b = -\frac{1}{3}, \lambda = 1$ the integral is

\[ V = \int d^7 y \sqrt{g} V^{\frac{2b}{3}} = \int d^7 y \sqrt{g} V^{-\frac{2}{3}}. \quad (3.1) \]

Using the formulae for the metric and warp factor this integral is easily seen to diverge, and it also does so for the other cases [1], with the result that the effective gravitational coupling $G_D = 0$ in each case. This indicates that the Randall-Sundrum mechanism [19] should not apply to this case, and gravity is not localised in the large extra dimensions of the internal space; indeed, we have checked that there are no graviton modes that are normalizable with respect to the internal space.

An alternative viewpoint might be to consider sources which are in some sense delocalized over the internal space. Modes that are constant over the internal space would be described by the consistent truncation of the higher dimensional supergravity equations to the lower dimensional gauged supergravity equations. This would be to discard all solutions with non-trivial dependence on the internal coordinates. While mathematically consistent it is physically more appealing to consider
non-trivial dependence and to seek boundary conditions on the internal manifold which would allow a lower dimensional interpretation. Note that despite the vanishing of the effective Newton’s constant $G_D$ and the difficulty of reducing the action, dimensionally reducing the field equations is straightforward and involves no divergent integrals [9].

To see what is involved in more detail, we consider those graviton modes in $d$ dimensions which take the form $\psi(y)h_{\mu\nu}(x)$ where $h_{\mu\nu}$ are $D$-dimensional graviton modes and $\psi$ satisfies the scalar wave equation with respect to the higher dimensional metric, with the warp factor taken into account. Assuming only dependence on $r$ we find for the $D = 4$, $SO(4, 4)$ case that

$$\frac{(1 + 2r^2)^{\frac{3}{2}}}{(1 + r^2)r^3} \frac{d}{dr} \left( \frac{(1 + r^2)r^3}{(1 + r^2)^{\frac{3}{2}}} \frac{d\psi}{dr} \right) = -\Lambda \psi,$$

where $\Lambda$ is a separation constant. The equation is self-adjoint with respect to the inner product

$$\int |\psi|^2 dr r^3 (1 + r^2)(1 + 2r^2)^{-\frac{3}{2}}.$$

Note that the norm for constant $\psi$, (with $\Lambda = 0$) is proportional to the volume integral giving $V$.

The problem now is whether one can find boundary conditions to fix the values of the separation constant $\Lambda$ to give a discrete spectrum. By looking at the behaviour of the solutions near infinity this looks doubtful. If $\Lambda$ is negative we get solutions which either blow up or decay exponentially. If we pick the latter, then multiplying the equation (3.2) by $\psi$ and integrating by parts, dropping a surface term which should vanish with the exponential fall-off, then we obtain a contradiction, as a manifestly positive integral is set equal to one that is manifestly negative for any solutions which fall off sufficiently fast at infinity to allow the dropping of the surface term. We conclude that $\Lambda$ cannot be negative for such boundary conditions. If $\Lambda$ is positive the solutions oscillate near infinity and are not normalizable; in this regime, we expect a continuous spectrum for $\Lambda$. It seems
therefore that one obtains similar difficulties to those encountered in [30, 29]. This is a pity since, as remarked in [31] a picture of this sort could explain in natural way the dimensionality of our spacetime.

The non-compact gauged supergravities can then be obtained from a consistent truncation of the higher dimensional theory to those modes that are constant on the internal space. Without the truncation, the \( D \)-dimensional spectrum appears to consist of the gauged supergravity together with a continuous spectrum of massive states arising from non-normalizable modes on \( N \), and there does not seem to be any natural way to impose boundary conditions or otherwise restrict the theory in such a way that a \( D \)-dimensional spectrum with a mass gap emerges. In this sense, the theory is more naturally viewed from the \( d \)-dimensional viewpoint.

4. Domain Wall Solutions from Higher Dimensions

Any solution of a gauged supergravity should have a lifting to \( d = 10 \) or \( d = 11 \) of the kind discussed above, although the ansatz will be more complicated for solutions with more non-vanishing fields. Most of the gauged supergravity theories do not have de Sitter or anti-de Sitter solutions, but most have half-supersymmetric domain wall solutions with \( D - 1 \) dimensional Poincaré symmetry [4,22-28]. In these, the metric has the form

\[
ds^2 = e^{2A(u)} ds^2 (\mathbb{R}^{(1,D-2)}) + e^{2B(u)} du^2
\]

(4.1)

and the functions \( A, B \) and the scalars \( \phi(u) \) are functions of the transverse coordinate \( u \) only. We will now consider the lifting of these to \( d = 10 \) or \( d = 11 \) solutions.

Consider the subset \( SL(n, \mathbb{R})/SO(n) \) of the scalar coset space, where \( n = 8 \) for \( D = 4 \), \( n = 6 \) for \( D = 5 \) and \( n = 5 \) for \( D = 7 \). (For \( D = 7 \), this is the whole scalar coset space, while for \( d = 4, 5 \) it is a subspace.) A configuration of scalar fields from this subspace is then represented by an \( SL(n, \mathbb{R}) \) matrix \( S_A^B(x) \). In
the compact $SO(n)$ gauging, such a scalar configuration leads to a squashing of
the $n-1$ sphere to the surface (1.2) in $\mathbb{R}^n$ with

$$\eta_{AB} = S_A^C S_B^D \delta_{CD} \quad (4.2)$$

and a warp factor

$$V = \eta^{AC} \eta^{AD} z^C z^D. \quad (4.3)$$

The ansatz for the 4-form [17,18] or 5-form field strength $F$ is also given in terms
of $S(x)$.

This can then be analytically continued to the non-compact gaugings [1]. For
any $p, q$ with $p + q = n$ there is an $SO(p) \times SO(q)$ invariant direction in the scalar
coset space represented by matrices of the form

$$S_{p,q}(t) = \begin{pmatrix}
e^{t/2} & 0 \\
0 & e^{-\beta t/2}
\end{pmatrix} \quad (4.4)$$

where $\beta = p/q$ and $t$ is a parameter. For any finite $t$, acting with the $SL(n, \mathbb{R})$
transformation $S_{p,q}(t)$ and rescaling the coupling constant

$$g \rightarrow ge^{-t} \quad (4.5)$$

defines a one-parameter family of theories, all equivalent to the compact gauging
through a field redefinition. Let

$$\xi = \exp[-(1 + p/q)t]. \quad (4.6)$$

For the $D = 4$ theory, continuing to $t = \infty$, $\xi = 0$ or to $t = -iq\pi/(p + q), \xi = -1$
define new theories, the $CSO(p, 0, q)$ gauging at $\xi = 0$ and the $SO(p, q)$ gauging
at $\xi = -1$. For $D = 5, 7$, non-compact semi-simple gaugings arise in a similar way,
or the $SO(p, q)$ gauging is associated with the continuation to $\xi = -1.$
It was seen in [1] that this continuation could be applied to the $d$-dimensional theory, with the $SL(n, \mathbb{R})$ acting naturally on the space $\mathbb{R}^n$ in which the sphere $S^{n-1}$ or hypersurface $\mathcal{H}^{p,q,r}$ (with $p+q+r = n-1$) is embedded. The ansatz for the 4-form or 5-form field strength is also transformed by the $SL(n, \mathbb{R})$ transformation, but we will only display the transformed metric here. Note that the continuation takes solutions of the complexified field equations of the compact gauging to solutions of the complexified field equations of the non-compact gaugings, but does not in general take real solutions to real solutions.

For scalar configurations $S(x)$ such that $S(x)$ commutes with $S_{p,q}(t)$, it is straightforward to calculate the effect of such continuations on the potential [8] or the $d$-dimensional geometry [1]. (For more general configurations, see [10].) The hypersurface becomes (1.2) with

$$\eta_{AB} = \hat{S}_A^C \hat{S}_B^D \delta_{CD}$$

where

$$\hat{S} = e^{-t/2} S_{p,q}(t) S(x).$$

Consider the case in which $S(x)$ is $SO(p) \times SO(q)$ invariant

$$S(x) = \begin{pmatrix} e^{\phi/2} 1_{p \times p} & 0 \\ 0 & e^{-\beta \phi/2} 1_{q \times q} \end{pmatrix}.$$  

where $\phi(x)$ is a scalar field. Configurations with constant $\phi$ were considered explicitly in [1], but the generalisation to non-constant fields is straightforward. The resulting geometry of the internal space $N$ now depends on the position in $M$, so that the internal metric $g_{mn}$ depends on both $x$ and $y$. The hypersurface is now (1.2) with

$$\eta_{AB} = e^{\phi} \begin{pmatrix} 1_{p \times p} & 0 \\ 0 & \xi e^{-(1+\beta)\phi} 1_{q \times q} \end{pmatrix},$$
The surface can then be written as
\[ e^{-\phi} R^2 = \sum_{i=1}^{p} (z^i)^2 + \xi e^{-(1+\beta)\phi} \sum_{i=p+1}^{p+q} (z^i)^2, \]
which is of the form (1.2) with
\[ \eta_{ab} = \begin{pmatrix} 1_{p\times p} & 0 \\ 0 & \xi 1_{q\times q} \end{pmatrix}, \]
and where now the moduli \( R, c \) are replaced with functions of \( x \):
\[ R(x) = e^{-\phi/2} R, \quad \lambda(x) = e^{-(1+\beta)\phi} \]
for some constant \( R \). The metric on this hypersurface is now (2.10) with these \( x \)-dependent \( R, c \), so that the radius \( R \) and the squashing parameter \( \lambda \) both vary with \( x \). The warp factor is (2.5), but with \( R, \lambda \) replaced by \( R(x), \lambda(x) \) given by (4.13).

For \( D = 4 \), the \( CSO(p, q, r) \) gaugings can be obtained from the \( SO(p, q + r) \) gaugings by a further field continuation in the \( SO(p+q) \times SO(r) \) invariant direction, using \( S'(s) = S_{p+q,r}(s) \). Then the hypersurface becomes (1.2),(4.7) with
\[ \hat{S} = e^{-(t+s)/2} S_{p,q+r}(t) S_{p+q,r}(s) S(x), \]
giving a 2-parameter set of theories depending on
\[ \xi = \exp \left[ - \left( 1 + \frac{p}{q + r} \right) t \right], \quad \zeta = \exp \left[ - \left( 1 + \frac{p + q}{r} \right) s \right]. \]
The \( CSO(p, q, r) \) gauging arises at \( \xi = -1, \zeta = 0 \), where
\[ \eta_{AB} = e^{\phi + \chi} \begin{pmatrix} 1_{p\times p} & 0 & 0 \\ 0 & -e^{-(1+\beta)\phi} 1_{q\times q} & 0 \\ 0 & 0 & 0_{r\times r} \end{pmatrix}, \]
so that the metric is the metric (2.10) on $\mathcal{H}^{p,q,r}$ with

\[ R(x) = e^{-(\phi + \chi)} R, \quad \lambda(x) = e^{-(1+\beta)\phi} \]

(4.17)

with warp factor given in terms of $R(x), \lambda(x)$ by (2.5).

The domain wall solutions of [4], with metric (4.1) for particular $A(u), B(u)$ and scalar $\phi(u)$, are then lifted to $d = 10$ or $d = 11$ dimensional solutions with metric (1.1),(2.10) and $R(u), \lambda(u)$ given by (4.13) or (4.17), and warp factor (2.5). It is straightforward to obtain the relevant antisymmetric gauge field for these solutions by transforming the standard ansatz.

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