Magnetic Metamaterials: A comparative study of resonator geometry and metal conductivity

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Abstract. In this work, split ring resonators based metamaterials are studied for microwave, terahertz and infrared frequency regimes. Two different geometries, circular and rectangular split ring resonators based metamaterials are investigated numerically for different frequency regimes. Our study indicates that the effect of metal conductivity and resonator geometry shows very little impact on the fundamental resonance mode. However the higher order modes go through significant frequency tuning because of the change in resonator geometry. We have further shown that the metal conductivity is an important parameter for the metamaterials employed in infrared domains.

1. Introduction

Metamaterials (MM) are artificial composite materials with user defined electromagnetic (EM) properties, typically made from dielectrics and arrays of structured metallic particles/features much smaller than the wavelength of the propagating electromagnetic waves [1]. These structured particles are generally termed as split ring resonators (SRRs). Metamaterials can strongly interact with EM waves and capable of controlling EM waves unlike any natural materials. Practically MMs can be employed to control waves with unprecedented flexibility. This has been indicated in the large number of MM based unique device/physical concepts including electromagnetic cloaking [2, 3], perfect imaging [4, 5], optical magnetism [6], near field coupling [7 - 10], sensing [11, 12], electromagnetic wave modulations [13 - 17], polarization rotation [18 - 20] etc. Metamaterials are also scalable in size making its resonances tuneable for use at different wavelengths of the entire electromagnetic spectrum [1]. In this work, we have investigated the effect of split ring resonator geometry on the different resonance modes for microwave (MW), Terahertz (THz) and Infrared (IR) domain metamaterials (MMs). Particularly rectangular and circular split ring resonators are studied extensively through numerical simulations. Additionally we have included effects of using different metals with versatile conductivity in our study.

For both the geometries, we have purposefully kept resonator size along with the split gap dimension constant. These two conditions allowed us to keep the intrinsic inductance and capacitance of the individual resonator fixed. Our investigations reveal that the first order or inductive-capacitive (LC) resonance remains almost unperturbed with respect to variable resonator geometry as well as metal conductivity. This result is somewhat expected since the SRR inductance and capacitance played the major role for the first order resonance and in agreement with previous work [14]. However higher order resonances, such as, 2nd order and 3rd order resonances go through large frequency shift. We
attribute this resonance tuning to the relative variation of the resonator arm length for the SRRs with different geometry. Additionally we have observed a reduction in resonance strength particularly for the IR MMs. Such reduction in resonance strength is attributed to the increasing loss due to the electron scattering effect at the higher operating frequencies.

2. Simulations and Discussions

In this work we have studied two geometrical configurations of metamaterials, one circular ring based MM and the other square ring based MM. In both cases resonators are composed of gold, Silver or bad silver. Bad silver is defined as metal with reduced conductivity compared to Silver. Additionally we have studied these samples at the different regions of the electromagnetic wavelengths, namely, Microwaves, Terahertz and Infrared. We have intentionally kept the split gap geometry constant for both the configurations for particular frequency regime. Lossless silicon is used as substrate for all the studied MM. Please see the figure below for detailed geometry of the structures.

![Figure 1](image_url)

**Figure 1.** The periodicity of the unit cell (a) is 4500um (microwave frequency), 60 um (THz frequency) and 1.35 um (Infrared frequency). Similarly, the length of the pattern (lx) is 4500 um, 60 um, 1.35 um; the breadth of the pattern (ly) is 1515 um, 20 um, 0.496 um; the radius of the circular ring (r) is 1709.5 um, 22.57 um, 0.56 um; gap in the pattern (g) is 500 um, 4 um, 0.1 um; thickness of the pattern (t) is 3000 um, 25 um, 10 um; the thickness of the substrate (tsub) is 3000um, 25um, 10um; the width of the pattern (w) is 400um, 4um, 0.1um; respectively for microwave, terahertz and infrared regions. The frequency ranges used are: 0.5GHz to 20Hz (microwave region), 300GHz to 1800GHz (THz region) and 30THz to 80THz (Infrared region).

2.1. Simulation procedure

In this work, all the metamaterials are simulated using finite difference frequency domain (FDFD) solver using numerical computations. The pattern along with the substrate is defined as a unit cell, see figure 1 for elaboration. This is periodically repeated to form the metamaterial. In simulations we have employed electric field either parallel or perpendicular to the split gap of the resonator. Simulation results for the metamaterials for different frequency ranges are shown in the figure below.
As evident from figure 2, the fundamental resonance remains almost unperturbed whereas the third order resonance goes through maximum tuning due to the change in resonator shape. Fundamental resonance broadly depends on inductance and capacitance of the individual resonators [21, 22]. The inductance and capacitance of the metamaterials for the particular frequency regime does not alter much because of the two guiding rules we have followed in this work. First the split gap is kept constant and second the total area of the split resonator is also kept constant. Because of the constant geometry of the split gap the capacitance remains same for both the structures (rectangular and circular). Similarly size of the resonators (circular or rectangular) for the particular frequency domain is also kept constant which effectively makes the inductance of the resonators constant. Therefore these two constant parameters do not allow the fundamental resonance to shift considerably. Note that resonance frequency for the fundamental mode depends strongly on these two geometrical parameters of the resonators [21, 22].

However, the higher order resonances are more dependent on arm lengths [23]. This effectively results into shifting of the higher order resonances significantly, see Fig 2. For microwave frequency range, the resonance strengths remain almost same for all the resonance modes. This is because at this relatively low frequency range the metal conductivity is significantly high and therefore does not affect the resonance positions significantly, hence the resonances are not affected significantly even though different metals are employed (Figure 2). We have further studied the similar circular and rectangular resonators based metamaterials for THz frequency regime.

**Figure 2.** Simulation of metamaterials for bad silver, Silver and gold for Microwave frequency range.

**Figure 3.** Simulation of metamaterials for bad silver, Silver and gold for Terahertz frequency range.
The simulation outcomes are shown in figure 3 for THz metamaterials. Because of the similar reasons as explained in the previous paragraph, the fundamental resonances remain almost intact. However the second order and third order resonances shift with change in resonator geometry. Resonance shifts toward the high frequency side in case of the circular resonator based MM because of the shorter resonator arm in the circular resonators compared to the rectangular resonator. However both the resonators are equal in size. This effect is even more prominent in case of third order resonance. The similar trend is demonstrated by the Infrared (IR) metamaterials as well as shown in figure 4. However the resonance strength reduces in a constant manner when we move from microwave to IR metamaterials through THz frequency regimes as demonstrated in figures 2, 3 and 4. The reduction in resonance strength is because of the electron-electron scattering losses. Such loss is actually increases as operating frequency increases. Our extensive numerical study comprising of circular and rectangular resonator based metamaterials for three important frequency regime clearly demonstrate that the metal conductivity does not play a significant role as long as it is higher than a critical value, for example silver conductivity. However, resonator shape can lead to significant shift in the higher order resonance modes while keeping the fundamental resonance position almost unperturbed.

**Table 1.** Relative shift of different resonance modes for circular and rectangular resonators as extracted from the individual simulations and depicted in figures 2, 3 and 4. Bad silver is modeled as conductor with conductivity poorer than silver and of the order of $1 \times 10^7$ S/m.

|          | Gold     | Bad Silver | Silver   |
|----------|----------|------------|----------|
| **Mode 1** | 0.156    | 0.156      | 0.156    |
| **Mode 2** | 0.312    | 0.312      | 0.312    |
| **Mode 3** | 1.3455   | 1.326      | 1.3455   |

The above table demonstrates the relative shift in resonance frequency after extracting from the spectrums of the metamaterials studied numerically. The 1st order mode or fundamental mode resonance frequency remains almost unperturbed when compared for different resonator geometry. Precisely the frequency shift is less than even 2 percent with respect to the individual resonances. However the second and third order resonance shifts are relatively more prominent of the order of 4 percent and 7 percent compared to the corresponding individual resonances, respectively.
3. Conclusions

In this work, we have studied the influence of different geometry and constitutive metals on split ring resonators based magnetic metamaterials through extensive numerical simulations. We have extended our study to metamaterials operating at different wavelength regimes of the electromagnetic spectrum, termed as microwave, terahertz and infrared. Our investigations show that resonator geometry does not have much impact on the fundamental resonances irrespective of the regime of operations. However resonator shape has strong influence on the higher order resonances. We attribute this resonance shift to the change in the metal arm length. For the metamaterials studied in the infrared domains, the resonances lose strength when compared to the microwave and terahertz metamaterials for the similar modes. We attribute such loss in resonance strength to electron scattering which is dominant at higher frequencies. Overall, this study of metamaterials with respect to different geometry, metals and operational wavelength regimes can be useful for designing MM for a particular application in different operational frequency regimes.

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References

[1] J.B. Pendry et al. IEEE Trans. Microwave Theory Tech 47, 2075 (1999)
[2] D Schurig, J Mock, B J Justice, S A Cummer, J B Pendry, A F Starr, D R Smith, Science 314, 977 (2006)
[3] S A Cummer, B I Popa, D Schurig, D R Smith, J Pendry, Physical Review E, 74, 036621 (2006)
[4] J. B. Pendry, Phys. Rev. Lett. 85, 3966 (2000)
[5] D. R. Smith, J. B. Pendry, M. C. K. Wiltshire, Science, 305, 788 (2004)
[6] A I Kuznetsov, A E Miroshnichenko, Y Hsing Fu, J B Zhang, B Luk’yanchuk, Scientific Report, 2, 492 (2012)
[7] Na Liu, Hui Liu, Shining Zhu & Harald Giessen, Nature Photonics, 3, 157 (2012)
[8] D Roy Chowdhury, J F O’Hara, A J Taylor, and A K Azad, Applied Physics Letters 104, 101105 (2014)
[9] D Roy Chowdhury, N Xu, W Zhang, R Singh, Journal of Applied Physics 118, 023104 (2015)
[10] D A Powell, M Lapine, M V Gorkunov, I V Shadrivov, Y S Kivshar, Physical Review B, 82, 155128 (2010)
[11] J F O’Hara, R Singh, I Brener, E Smirnova, J Han, A J Taylor, W Zhang Optics Express 16, 1786 (2008)
[12] W Withayachumnankul, H Lin, K Serita, C M Shah, S Sriram, M Bhaskaran, M Tonouchi, C Fumeaux, and D Abbott 20, 3345 2012()
[13] HT Chen, WJ Padilla, JMO Zide, AC Gossard, AJ Taylor, RD Averitt, Nature 444, 597 (2006)
[14] D Roy Chowdhury, R Singh, A J Taylor, H T Chen, and A K Azad, Applied Physics Letters 102, 011122 (2013)
[15] M Liu, H Y Hwang, H Tao, A C Strikwerda, K Fan, G R Keiser, A J Sternbach, K G West, S Kittiwatanakul, J Lu, S A Wolf, F G Omenetto, X Zhang, K A Nelson, and R D Averitt, Nature, 487, 345 (2012)
[16] D Roy Chowdhury, A K Azad, W Zhang, and R Singh, IEEE Transactions on Terahertz Science and Technology, 3, 783 (2013)
[17] W Withayachumnankul, and D Abbott, IEEE Photonics Journal, 1, 99 (2009)
[18] D-H Kwon, P. L. Werner, and D. H. Werner, Optics Express, 16, 11802 (2008)
[19] N. K. Grady, J. E. Heyes, D. Roy Chowdhury, Y. Zeng, M. T. Reiten, A. K. Azad, A. J. Taylor, D. Dalvit, and H.-T. Chen Science, 340, 1304 (2013)
[20] L Cong, W Cao, X Zhang, Z Tian, J Gu, R Singh, J Han, W Zhang, *Applied Physics Letters*, 103, 171107 (2013)

[21] C Rockstuhl, F Lederer, C Etrich, T Zentgraf, J Kuhl, H Giessen, *Optics express* 14, 8827 (2006)

[22] D Roy Chowdhury, R Singh, J F O’Hara, H T Chen, A J Taylor, A K Azad, *Applied Physics Letters* 99, 231101 (2011)

[23] A K Azad, A J Taylor, E Smirnova, J F O’Hara, *Applied Physics Letters*, 92, 011119 (2008)