RADIATION OF ANGULAR MOMENTUM BY NEUTRINOS FROM MERGED BINARY NEUTRON STARS

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ABSTRACT
We study neutrino emission from the remnant of an inspiraling binary neutron star following coalescence. The mass of the merged remnant is likely to exceed the stability limit of a cold, rotating neutron star. However, the angular momentum of the remnant may also approach or even exceed the Kerr limit, $J/M^2 = 1$, so that total collapse may not be possible unless some angular momentum is dissipated. We find that neutrino emission is very inefficient in decreasing the angular momentum of these merged objects and may even lead to a small increase in $J/M^2$. We illustrate these findings with a post-Newtonian, ellipsoidal model calculation. Simple arguments suggest that the remnant may form a bar mode instability on a timescale similar to or shorter than the neutrino emission timescale, in which case the evolution of the remnant will be dominated by the emission of gravitational waves.

Subject headings: binaries: close --- elementary particles --- gravitation --- instabilities

1. INTRODUCTION
Considerable theoretical effort is currently directed toward understanding the coalescence of binary neutron stars. The interest stems partly from the promise such coalescence holds for generating gravitational waves that can be detected by laser interferometers now under construction, like LIGO, VIRGO, and GEO. Additional interest is triggered by the idea that neutrinos generated during the merger and coalescence might be the source of gamma rays bursts (see, e.g., Paczyński 1986; Eichler et al. 1989; Janka & Ruffert 1996). Most of the theoretical work has, to date, been performed in a Newtonian framework (see, e.g., Rasio & Shapiro 1994; Ruffert, Janka, & Schäfer 1996; Zhuge, Centrella, & McMillan 1996). Only a few hydrodynamical calculations have taken into account post-Newtonian corrections (Oohara & Nakamura 1997; Shibata, Oohara, & Nakamura 1997), while results from calculations in full general relativity are very preliminary (Oohara & Nakamura 1997; Wilson & Mathews 1995; Wilson, Mathews, & Marronetti 1996).

There are a number of issues, however, that cannot be addressed even qualitatively in Newtonian gravitation. Such issues include the collapse of the remnant to a black hole. Such a collapse is quite likely because the mass of the remnant will probably exceed the maximum allowed mass for a single neutron star. However, the final fate of the collapse is completely unknown if the remnant's angular momentum parameter $J/M^2$ exceeds unity. In this case, the remnant cannot form a Kerr black hole without first losing some of its angular momentum. In principle, angular momentum could be radiated away by the emission of gravitational waves, neutrinos, and/or electromagnetic (e.g., magnetic dipole) radiation.

During the coalescence of the two neutron stars, some fraction of the kinetic energy will be converted into thermal energy. Therefore, the remnant is likely to be hot and emit thermal neutrinos. During the rapid plunge of the binary once it reaches the innermost stable circular orbit (ISCO), the angular momentum of the system will be conserved approximately, so the remnant will be rotating very rapidly. In their fully relativistic treatment of corotating, polytropic binaries in quasi-equilibrium circular orbit, Baumgarte et al. (1997, 1998) found that, for a polytropic equation of state with polytropic index $n = 1$, the total $J/M^2$ of the binary system at the ISCO exceeds unity for all binaries in which each member has a rest mass $\lesssim 0.94 M_{\odot}^{\text{max}}$. Here $M_{\odot}^{\text{max}}$ is the maximum allowed rest mass of a nonrotating, isolated, cold neutron star, and $M_0$, $M$, and $J$ denote the total rest mass, energy, and angular momentum of the binary system. For more realistic, nonsynchronous binaries, $J/M^2$ is likely to be somewhat smaller but may still be appreciable. For many of these binaries the rest mass of the merged remnant also exceeds $M_{\odot}^{\text{max}}$. Immediately following merger, these objects may be temporarily stabilized by thermal pressure and angular momentum. Only after some of the thermal energy and angular momentum have been radiated away can the star later collapse to a black hole. The ultimate fate of such an object can only be established by a three-dimensional, relativistic hydrodynamic numerical simulation that employs a hot nuclear equation or state and full neutrino radiation transport. While the development of such a code is underway by several groups, it is still very far from complete.

Our goal in this paper is to isolate the role of the neutrino emission in the early evolution following merger and to estimate its effect on the angular momentum $J$ and the angular momentum parameter $J/M^2$. To do so, we consider a simplified scenario for the postmerger neutron star remnant and imagine that the remnant evolves to an axisymmetric, quasi-equilibrium state on a dynamical (orbital) timescale immediately following merger. Since the merger follows a rapid plunge from the ISCO, the merged remnant forms with the same total mass $M$, the rest mass $M_0$, and the angular momentum $J$ of the progenitor binary at the ISCO (here we neglect the small loss of mass and...
angular momentum via gravitational waves and gas ejection during the dynamical plunge). Accordingly, $J/M^2$ will typically be near unity. We assume that the merged configuration quickly settles into uniformly rotating, dynamically stable quasi-equilibrium state and focus on its quasi-static evolution. We show that the emission of neutrinos is very inefficient in carrying off $J$. Depending on the stiffness of the equation of state, this emission may lead to either an increase or a decrease of $J/M^2$. Also, we argue that these objects typically develop a bar instability on a timescale comparable to or shorter than the neutrino emission timescale. We therefore conclude that while neutrino emission may be important for the cooling of the remnant, it is negligible for the emission of angular momentum.

The paper is organized as follows: In § 2 we estimate the rate of angular momentum loss by neutrino emission. In § 3 we incorporate our estimate in a post-Newtonian, ellipsoidal model calculation, which follows the postmerger evolution of the remnant. The equations and formalism for this approximation are presented in Appendix A. In § 4 we compare the relevant timescales for angular momentum loss by competing processes, and we summarize our conclusions in § 5.

2. ANGULAR MOMENTUM LOSS BY NEUTRINO EMISSION

Consider a neutrino $\nu$ with four-momentum $p^\nu$ emitted from the surface of a uniformly rotating, axisymmetric equilibrium star into an arbitrary direction. A local observer with four-velocity $u^\nu$ comoving with the surface will measure the neutrino’s energy to be

$$\delta W^{(e)} = -u^a p_a = -u^t p_t - u^\phi p_\phi.$$  \hspace{1cm} (1)

Here and throughout the paper we adopt geometrized units, $c \equiv 1 \equiv G$. Since the spacetime is stationary and axisymmetric, the components $p_t$ and $p_\phi$ are conserved along the neutrino’s path. Therefore, a distant observer at rest with respect to the star’s center of mass will measure the neutrino’s energy to be

$$\delta W^{(r)} = -p_t = -dM,$$  \hspace{1cm} (2)

and a distant observer will identify the neutrino’s angular momentum to be

$$dJ = -p_\phi.$$  \hspace{1cm} (3)

Combining the last three equations yields the familiar result

$$dM = \Omega dJ - \frac{1}{u^r} \delta W^{(r)}$$  \hspace{1cm} (4)

(e.g., Thorne 1971), where $\Omega = u^r/u^t$ is the angular velocity of the rotation. The luminosity as measured by a local, comoving observer can be defined as

$$L = \frac{\delta W^{(e)}}{d\tau},$$  \hspace{1cm} (5)

where $\tau$ is the observer’s proper time. Equation (4) can then be rewritten as

$$\dot{M} = \Omega \dot{J} - \frac{1}{u^r} L,$$  \hspace{1cm} (6)

where the dot denotes a derivative with respect to $\tau$.

Equation (6) is fully relativistic. The relation between $L$ and $J$ depends on the characteristics of the emission of the neutrinos from the star’s surface, as well as the relativistic structure of the star. Establishing this relationship therefore requires detailed numerical models of radiating, relativistic, rotating stars. Instead, we adopt a simple, first-order Newtonian description to find the approximate relationship given by equation (14) below. We assume that the neutrinos are emitted isotropically in the rest frame of a local stationary observer comoving with the surface. An observer in a nonrotating static frame will therefore find that, on average, each neutrino carries off some angular momentum. This angular momentum can be found by Lorentz transforming from the stationary frame into the static frame. To lowest order in $v$ we find that, on average, each neutrino carries off a linear momentum $p_\phi = v E_\phi = \Omega \sigma E_\phi$, where $\Omega$ is the angular velocity of the star, $\sigma$ is the distance of the surface element from the axis of rotation, and $E_\phi$ is the energy of the neutrino in the comoving frame (eq. [1]). The corresponding angular momentum is

$$p_\phi = \sigma^2 \Omega E_\phi.$$  \hspace{1cm} (7)

The total rate of loss of angular momentum is therefore (see also Kazanas 1977)

$$J = - \langle \sigma^2 \rangle \Omega L,$$  \hspace{1cm} (8)

where $L$ is the neutrino luminosity and $\langle \sigma^2 \rangle$ denotes the average of $\sigma^2$ over the surface of the star. We can get a reasonable estimate for this average by adopting a rotating, compressible ellipsoid model for the star (see Lai, Rasio, & Shapiro 1993, hereafter LRS). If we denote the principal axes by $a_1$, $a_2$, and $a_3$ (where $a_1 = a_2$ is measured in the equatorial plane and $a_3$ is along the rotation axis), we find

$$\langle \sigma^2 \rangle = \frac{2}{3} a_1^2 f(e),$$  \hspace{1cm} (9)

where

$$f(e) = \frac{6\pi}{a_1^4 \mathcal{A}} \int_0^{a_1} \frac{\sigma^4}{\sigma^3 - \lambda^2} \left( \frac{a_1^2}{\sigma^2} \right)^{1/2} d\sigma = 1 - \frac{1}{12} e^2 + O(e^4).$$  \hspace{1cm} (10)

Here

$$e^2 \equiv 1 - (a_3/a_1)^2$$  \hspace{1cm} (11)

is the eccentricity, and

$$\mathcal{A} = 4\pi \int_0^{a_1} \frac{\sigma^4}{\sigma^3 - \lambda^2} \left( \frac{a_1^2}{\sigma^2} \right)^{1/2} d\sigma$$  \hspace{1cm} (12)

is the surface area of the ellipsoid. Note that $f(e) \sim 1$ even for large eccentricities. It is convenient to rewrite equation (8) in terms of the angular momentum $J$ and the moment of inertia $I$. The latter is $I = \frac{3}{5} \kappa_n M a_1^2$, where $\kappa_n$ is a dimensionless structure constant of order unity that depends on the star’s density profile. For a polytropic equation of state

$$P = K \rho^{1 + 1/n},$$  \hspace{1cm} (13)

where $P$ is the pressure, $\rho$ is the rest mass density, $n$ is the polytropic index, and $K$ is the polytropic constant, $\kappa_n$ can
be derived from the Lane-Emden function (see, e.g., LRS). Equation (8) can then be written as

\[ J = -f(e) \frac{5}{3\kappa_n} \frac{J}{M} L . \] (14)

From equation (14), we find that the angular momentum \( J \) changes at most by

\[ \frac{\Delta J}{J} \sim -\frac{5}{3\kappa_n} \frac{U_{\text{hot}}}{M} \]

(15)

(P. Huet, 1996, private communication), assuming that the merged remnant radiates away all of its thermal energy \( U_{\text{hot}} \sim L\Delta r \) prior to collapse. The ratio \( U_{\text{hot}}/M \) following merger can be estimated by taking the progenitor stars to be cold, with \( U_{\text{hot}} = 0 \), prior to reaching the ISCO. During the subsequent plunge and merger, a part of the kinetic energy will be converted into thermal energy by contact shocks & Shapiro et al. In the (Rasio 1994; Ruffert 1996). In the extreme case of a head-on collision, a very large part of the kinetic energy will be converted into heat. The ISCO, however, usually occurs at a very small separation of the stars, and therefore we expect that only a small part of the kinetic energy will be converted into heat.

To estimate \( U_{\text{hot}} \), we construct post-Newtonian, compressible ellipsoidal models of hot, uniformly rotating stars, generalizing the cold, post-Newtonian models of Lombardi, Rasio, & Shapiro (1997). Matching these configurations to binary models at the ISCO calculated by Baumgarte et al. (1998), we typically find \( U_{\text{hot}}/U_{\text{cold}} \sim 0.2-0.3 \), where \( U_{\text{cold}} \) is the total internal energy excluding the thermal energy (i.e., kinetic energy of degenerate nucleons; see § 3 and Appendix A). Since for typical neutron stars \( U_{\text{cold}}/M \sim 0.1 \), we have \( U_{\text{hot}}/M \sim U_{\text{cold}}/M \sim 0.1 \) percent following merger. Hence, from equation (15), we expect that the relative change of the angular momentum due to neutrino emission can be at most a few percent. We conclude that neutrinos are very inefficient in reducing the merged remnant’s angular momentum.

We now focus on the change in \( J/M^2 \) associated with the neutrino luminosity \( L \):

\[ \frac{M^2}{J} \frac{dJ}{dt} = -2 \frac{M}{J} \frac{dL}{dt} = -J \left( \frac{1 - 4T_{\text{rot}}}{M} - \frac{6}{5} \frac{\kappa_n}{u'f(e)} \right). \] (16)

Here we have used equations (6) and (14) as well as \( T_{\text{rot}} = \Omega J/2 \). The magnitude of \( J/M^2 \) therefore increases if

\[ 1 - 4T_{\text{rot}} - \frac{6}{5} \frac{\kappa_n}{u'f(e)} < 0 . \]

(17)

For a dynamically stable rotating ellipsoid with rotational velocity \( v \ll 1 \) and \( T_{\text{rot}}/|W| \lesssim 0.27 \) (see below), we have \( T_{\text{rot}}/M \lesssim 1 \), \( u' \sim 1 \), and \( f(e) \sim 1 \), so that this criterion reduces to

\[ \kappa_n > \frac{5}{6} . \]

(18)

The critical value of \( \kappa_n = \frac{5}{6} \) corresponds to a polytropic index of \( n_{\text{crit}} = 0.45 \) (see, e.g., LRS, their Table 1). Neutrino radiation thus leads to an increase of the angular momentum parameter \( J/M^2 \) for all equations of state with \( n < n_{\text{crit}} \).

Interestingly, the value of \( n_{\text{crit}} = 0.45 \) is very close to typical values that are expected in realistic neutron star equations of state. Note that we have derived this numerical value in the limit of slow rotation. For rapid rotation, as in the numerical example of § 3, we find that the value for \( n_{\text{crit}} \) increases in the ellipsoidal approximation.

For values of \( n \) close to \( n_{\text{crit}} \), the terms on the left-hand side of equation (17) will always be close to zero. According to equation (16), the fractional change of \( J/M^2 \) will therefore be much smaller than the fractional change of \( J \), which we have estimated to be in the order of a few percent at most. Nevertheless, it is interesting to note that the emission of neutrinos can lead to an increase in \( J/M^2 \), and that for nearly Newtonian configurations the sign of the change is determined essentially by the stiffness of the equation of state.

3. A NUMERICAL EXAMPLE

To illustrate these effects, we dynamically evolve axisymmetric, post-Newtonian, compressible ellipsoid models of hot neutron star merger remnants. We employ the formalism of Lai et al. (1994), supplemented by post-Newtonian corrections (Lombardi et al. 1997) and thermal contributions to the internal energy. The thermal corrections are taken into account by naively decomposing the polytropic constant \( K \) in equation (13) into a linear sum of a cold and a hot contribution, \( K = K_{\text{cold}} + K_{\text{hot}} \). The complete formalism and equations can be found in Appendix A.

For initial data we construct equilibrium configurations as described in Appendix A.3. These models of hot, rotating neutron stars form a three parameter family, which can be uniquely determined by the central density, the eccentricity, and the thermal heat content \( (K_{\text{hot}}/K_{\text{cold}}) \). We can choose these three parameters in such a way that the stars’ rest mass \( M_0 \), total energy \( E_0 \), and angular momentum \( J_0 \) match those of binary configurations at the ISCO as calculated by Baumgarte et al. (1997, 1998). Here, we summarize results for two different progenitor binary models with \( n = 1 \) and \( J/M^2 \gtrsim 1 \) (see lines 5 and 6 of Table 2 in Baumgarte et al. 1998 for the binary parameters). The two models differ by how close the stellar masses are to the maximum allowed mass of an isolated star in spherical equilibrium. We pick these two particular cases since their evolutions turn out to be qualitatively different. For both models, we fix \( K_{\text{cold}} \) in such a way that the rest mass of the remnant is \( M_0 = 2 \times 1.5 M_\odot \), since observed binary neutron stars all have gravitational masses close to 1.4 \( M_\odot \), corresponding to rest masses \( \sim 1.5 M_\odot \). The value \( K_{\text{cold}} \) is determined by the relation \( K_{\text{cold}} = (M_0/M_\odot)^2/14, \) where the nondimensional quantity \( M_0 \) is given in Table 2 in Baumgarte et al. (1998).

For model 1 (line 5 of Table 2 in Baumgarte et al. 1998), we have \( K_{\text{cold}} = 208 \text{ km}^2 \). In our post-Newtonian approximation, this yields a maximum allowed rest mass of \( M_\text{max}^{0.5} = 1.92 M_\odot \) (corresponding to a maximum allowed gravitational mass of \( M_\text{max} = 1.71 M_\odot \) for isolated, non-rotating cold stars. These values are consistent with those obtained from recent, realistic nuclear equations of state (see, e.g., Pandharipande 1998). The initial gravitational energy of model 1 is \( M = 2.75 M_\odot \) and the initial angular momentum parameter is \( J/M^2 = 1.05 \). The remnant has an eccentricity of \( e = 0.935 \), a thermal heat content \( K_{\text{hot}}/K_{\text{cold}} = 0.25 \), and \( a_1 = 28.0 \text{ km} \). The thermal energy in this model corresponds to a maximum temperature of \( \sim 14 \text{ MeV} \).
For model 2 (line 6), we have \( K_{\text{cold}} = 173 \text{ km}^2 \), corresponding to \( M_{\text{max}}^0 = 1.75 M_\odot \) and \( M_{\text{max}} = 1.56 M_\odot \), so that this model’s rest mass exceeds the maximum allowed rest mass by a larger amount than model 1. It has an initial gravitational mass of \( M = 2.72 M_\odot \) and \( J/M^2 = 1.0 \), corresponding to a remnant of \( e = 0.94 \), \( K_{\text{hot}}/K_{\text{cold}} = 0.33 \), and \( a_1 = 23.3 \text{ km} \). Here the maximum temperature is \( T_{\text{max}} = 28 \text{ MeV} \), which is similar to typical values found by Ruffert et al. (1996).

We show the dynamical evolution of these two models in Figures 1 and 2. Even though the rest mass of model 1 greatly exceeds \( M_{\text{max}}^0 \), it is dynamically stabilized within our approximations by its high angular momentum, even after all the thermal energy has been radiated away. By contrast, model 2 collapses after it has emitted part of its thermal energy. In model 1, the angular momentum decreases by \( \sim 1.5\% \), while \( J/M^2 \) increases by \( \sim 0.3\% \). This shows that post-Newtonian corrections in an ellipsoid approximation, as well as rotation and deviations from sphericity, cause the critical polytropic index \( n_{\text{crit}} \) to increase to a value greater than one. In model 2, the changes in \( J \) and \( J/M^2 \) show the same trends but are smaller, because the star emits only a small part of the thermal energy before it collapses. In a fully relativistic treatment, the collapse would not proceed directly to a black hole, because the Kerr limit on \( J/M^2 \) would be exceeded. Binaries with masses closer to the maximum nonrotating mass have \( J/M^2 < 1 \) (see Table 2 in Baumgarte et al. 1998). Their remnants could collapse directly to Kerr black holes.

Note also that the neutrino luminosity increases as model 2 undergoes “delayed” collapse. This is in contrast to spherically symmetric results for delayed collapse (Baumgarte, Shapiro, & Teukolsky 1996b; Baumgarte et al. 1996a), in which the luminosity always decreases during...
collapse. This difference can be understood from a simple geometrical scaling argument. As explained in detail in Appendix A, we assume for simplicity that we can write the thermal energy in the polytropic form $e_{\text{hot}} = nK_{\text{hot}}\rho^{1+1/n}$ (eq. [28]). In the high temperature limit, $e_{\text{hot}} \sim e_{\text{rad}}$, so that $T^4 \sim \rho^{1+1/n} \sim (a_1^2 a_s)^{-1(1+1/n)}$. Taking the surface area of the ellipsoid to be $\sim 4\pi a_1^2$, we find from equation [57] that in the diffusion approximation the luminosity scales as

$$L \sim a_1^{3-1/n} a_s^{-1(1+1/n)/2}.$$  (19)

For spherically symmetric collapse, $a_1 = a_s$ and $L \sim a_1^{3-3/n)/2}$. For all $n > \frac{5}{3}$ (which accommodates most realistic equations of state), spherically symmetric collapse therefore leads to a decrease of the luminosity: the neutrino optical thickness increases faster than the thermal energy increases. If, however, the star collapses to a pancake, as in Figure 2, $a_1$ will remain finite, and $L \sim a_s^{-(1+1/n)/2}$. In this case the increase of the thermal energy overcomes the increase of the optical thickness, and the luminosity increases. Whenever the surface of a collapsing star approaches a newly formed event horizon, the luminosity will always be suppressed by the increasing redshift and black hole capture. These effects are absent in our model calculations. Nevertheless, our results suggest that the collapse of a rapidly rotating, hot neutron star following merger may leave a very distinct signature in the neutrino signal.

4. ESTIMATE OF TIMESCALES

So far we have focused on the emission of neutrinos and their role in the evolution of the merged remnant. To estimate whether this evolution is indeed governed by neutrinos, we now compare the timescale for the neutrino emission with those associated with electromagnetic and gravitational radiation.

The timescale for the emission of the neutrinos is given by the diffusion time, $\tau_\nu$. Taking $\kappa$ to be Rosseland mean
opportunity due to scattering off nondegenerate neutrons and protons (see, e.g., Baumgarte et al. 1996b), we find
\[
\tau_s \sim \kappa \rho R^2 \sim \kappa \left( \frac{M}{R} \right) \frac{4}{20 \text{MeV}} \left( \frac{kT}{2 M_\odot} \right)^2 \left( \frac{R}{15 \text{ km}} \right)^{-1} \text{s}.
\]

(20)

Assuming that electromagnetic emission is dominated by a magnetic dipole radiation, its timescale can be estimated from
\[
\tau_{EM} \sim \frac{T_{rot}}{E} \sim \frac{M}{3B^2 \sin^2 \alpha R^2 \Omega^2} \sim 7 \times 10^3 \left( \frac{M}{2 M_\odot} \right) \times \left( \frac{B \sin \alpha}{10^{12} \text{G}} \right)^{-2} \left( \frac{R}{15 \text{ km}} \right)^{-4} \left( \frac{\Omega}{10^4 \text{s}^{-1}} \right)^{-2} \text{s}.
\]

(21)

(see, e.g., Shapiro & Teukolsky 1983).

Clearly, \( \tau_{EM} \) is by many orders of magnitudes larger than \( \tau_s \). Electromagnetic radiation is therefore negligible in the early evolution of hot neutron stars, unless the magnetic fields are extreme (\( B \gtrsim 10^{15} \text{G} \)).

An axisymmetric, stationary star does not emit gravitational waves. However, if the ratio of the kinetic and potential energy \( t = T_{rot}/|W| \) is large enough, the star will develop a bar instability and will then emit gravitational radiation. More specifically, the star is secularly unstable to the formation of a bar mode if and dynamical radiation. More specifcally, the star is secularly unstable to develop a bar instability and will then emit gravitational waves. We then see that the evolution of these merged neutron stars will be faster and more efficient in reducing \( J/M^2 \) than neutrinos. Together, these arguments may have important consequences for gravitational wave detectors, since they suggest that the evolution of merged binary remnants will be dominated by the emission of gravitational waves. In particular, should \( J/M^2 \) be larger than unity upon merger, gravitational waves seem to be the only means by which \( J/M^2 \) can be efficiently reduced to a value smaller than unity, so that the entire merged configuration can collapse to a black hole. Otherwise, hydrodynamic stresses will have to support or expel some of the matter to allow the interior regions to collapse.

Since relativistic configurations typically have \( M/R \gtrsim 0.1 \), this suggests that all neutron stars rotating with \( J/M^2 \gtrsim 1 \) have \( t \gtrsim 0.2 \) and are at least secularly unstable to the formation of a bar mode. This is in agreement with the general relativistic, numerical models of Cook, Shapiro, & Teukolsky (1992). For \( t_{sec} < t < t_{dyn} \), the timescale for the formation of a bar mode driven by gravitational radiation can be approximated by
\[
\tau_{bar} \sim 3 \left( \frac{M}{2 M_\odot} \right) \left( \frac{R}{0.2 M_\odot} \right)^{-4} \left( \frac{t - t_{sec}}{0.1} \right)^{-5} \text{s}.
\]

(23)

(see, e.g., Friedman & Schutz 1975; Lai & Shapiro 1995).

Note that, although viscosity can also drive a bar mode instability, but it is inefficient in hot neutron stars with \( T \sim 10 \text{ MeV} \), as is the case here (see Bonazzola, Frieben, & Gourgoulhon 1996 and references therein). Once a bar has fully developed, the gravitational radiation dissipation timescale can be estimated from the quadrupole emission formula
\[
\tau_{grav} \sim \frac{T_{rot}}{E} \sim \frac{I \Omega}{(M/R)(\Omega/5)} \sim \frac{M}{(M/R)(\Omega/5)^2} \sim 4 \times 10^{-3} \left( \frac{M}{2 M_\odot} \right) \left( \frac{R}{0.2 M_\odot} \right)^{-4} \left( \frac{\Omega}{0.5} \right)^{-4} \text{s}.
\]

(24)

For \( t < t_{dyn} \), the total gravitational radiation timescale \( \tau_{GW} \sim \tau_{bar} + \tau_{grav} \) is therefore dominated by the initial growth time \( \tau_{bar} \). Since \( \tau_{bar} \) strongly depends on \( t \), this timescale is quite uncertain. However, the estimate (eq. [22]) suggests that for compact stars with \( J/M^2 \gtrsim 1 \), we have \( \tau_{GW} \lesssim 3 \text{ s} \). We then see that the evolution of these merged stars is characterized by the formation of a bar mode (see Lai & Shapiro 1995) and emission of gravitational waves, which may then carry off angular momentum very efficiently. In some cases, \( t > t_{dyn} \) and a bar may develop on a dynamical timescale (\( \sim \Omega^{-1} \approx \text{a few ms} \); see Rasio & Shapiro 1994 for a numerical demonstration).

5. CONCLUSIONS

We conclude that neutrinos are very inefficient in carrying off angular momentum from hot, massive remnants of neutron star binary mergers, even if these are rapidly rotating and have an angular momentum parameter \( J/M^2 \gtrsim 1 \). Curiously, neutrino emission may even increase \( J/M^2 \) by a small amount. We find that the timescale for the formation of a bar mode may, for such objects, be much shorter than the neutrino emission timescale. For pure gravitational radiation, the third term in equations (16) and (17) vanishes, so that \( J/M^2 \) always decreases. This suggests that gravitational waves will be faster and more efficient in reducing \( J/M^2 \) than neutrinos. Together, these arguments may have important consequences for gravitational wave detectors, since they suggest that the evolution of merged binary remnants will be dominated by the emission of gravitational waves. In particular, should \( J/M^2 \) be larger than unity upon merger, gravitational waves seem to be the only means by which \( J/M^2 \) can be efficiently reduced to a value smaller than unity, so that the entire merged configuration can collapse to a black hole. Otherwise, hydrodynamic stresses will have to support or expel some of the matter to allow the interior regions to collapse.

These arguments imply that, except for cooling and deleptonizing, neutrino emission plays a very minor role in determining the mass and spin of the final configuration (black hole or rotating neutron star). However, because of the role in inducing delayed collapse in some cases, the neutrinos will have to be included in calculations of the late stages of coalescence. Gravitational radiation will play a very important role in determining the final mass and spin of the black hole or neutron star. This result strengthens the conclusion that a fully general relativistic description of binary neutron star mergers is essential for full understanding.

Note that our arguments apply to massive, hot, rotating neutron stars in general and not only to the remnants of binary neutron stars. For example, they apply equally to newly formed, hot neutron stars in supernovae. It has recently been suggested that these stars may be stable initially but may later deleptonize, undergo a phase transition, and collapse to a black hole (see, e.g., Brown & Bethe 1994). Alternatively, such a delayed collapse might result from the loss of angular momentum from a nascent, rapidly rotating neutron star that is stabilized initially by its high spin. Neutrino emission could, in principle, reduce this angular momentum and hence induce collapse. While for type II supernovae \( U_{b,	ext{b}}/M \) is slightly larger than in the examples presented in § 3, our arguments show that the neutrino emission is still very inefficient in carrying off angular momentum. This fact makes this scenario very unlikely (P. Huet, 1996, private communication). Similarly, our analysis suggests that neutrino emission plays a negligible role in determining the quasi-steady spin rate of accreting neutron stars at the center of Thorne-Zytkow objects, which have
recently been suggested to be likely sources of gravitational waves for detection by GEO (Schutz 1998).

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APPENDIX

POST-NEWTONIAN, ELLIPSOIDAL MODELS OF HOT, ROTATING NEUTRON STARS

A1. THE EULER-LAGRANGE EQUATIONS

The total energy of an axisymmetric, rotating, hot neutron star, in a post-Newtonian, ellipsoidal approximation, can be written as the sum

\[ M = U + U_{PN} + W + W_{PN} + T, \]

where \( U \) is the internal energy, \( W \) the potential energy, \( T \) the kinetic energy, and \( U_{PN} \) and \( W_{PN} \) are post-Newtonian corrections to the internal and potential energy.

Including the post-Newtonian terms not only improves the solution for relativistic stars quantitatively, but also, more importantly, it changes the solution qualitatively. Without the post-Newtonian corrections, a graph of the rest mass as a function of the central density exhibits no turning point. By including the post-Newtonian terms, we find turning points reasonably close to the turning points of the corresponding, fully relativistic (Tolman-Oppenheimer-Volkov) solution (see Lombardi et al. 1997). Since these turning points give the maximum allowed mass configuration, the post-Newtonian corrections allow us to approximate the onset of radial instability and to mimic the delayed collapse of hot, rotating neutron stars to black holes.

We assume a polytropic equation of state

\[ P = K \rho^{1 + 1/n}, \]

and crudely account for thermal pressure by taking the constant \( K \) to have a cold contribution due to degenerate nucleons, \( K_{cold} \), and a hot contribution due to thermal heating, \( K_{hot} \):

\[ K = K_{cold} + K_{hot}. \]

The internal thermal energy density \( \epsilon_{hot} \) is therefore taken to be of the form

\[ \epsilon_{hot} = nK_{hot} \rho^{1 + 1/n}. \]

While \( K_{cold} \) is assumed to be constant in both space and time, we allow \( K_{hot} \) to depend on time, keeping it constant throughout the star. Integrating the energy density over the star allows us to write the total internal energy in terms of the central density \( \rho_c \) as a sum of a cold and a hot contribution

\[ U = U_{cold} + U_{hot} = k_1(K_{cold} + K_{hot}) \rho_c^{1/n} M_0, \]

where \( k_1 \) is a dimensionless structure constant that depends on the density profile, i.e., the polytropic index \( n \). Numerical values for \( k_1 \) can be found, for example, in LRS (see their Table 1). In terms of the mean radius

\[ R \equiv (a_1^2 a_3)^{1/3}, \]

the central density can be written as

\[ \rho_c = \beta \frac{M_0}{R^3}, \]

where we have defined

\[ \beta \equiv \frac{\xi}{4\pi |\theta'(\xi)|}, \]

and where \( \theta \) and \( \xi \) are the usual Lane-Emden parameters for a polytrope (see, e.g., Chandrasekhar 1939). The internal energy can therefore be rewritten as

\[ U = k_1 K \beta^{1/n} \frac{M_0^{1+1/n}}{R^{3/n}}. \]

The potential energy \( W \) is given by

\[ W = -\frac{3}{5 - n} \frac{M_0}{R} \sin^{-1} \frac{e}{e} (1 - e^2)^{1/6} = -\frac{3}{5 - n} \beta^{-1/3} M_0^{5/3} \rho_c^{1/3} \sin^{-1} \frac{e}{e} (1 - e^2)^{1/6}. \]
Introducing the dimensionless coefficients $A_i$,

$$A_1 = (1 - e^2)^{1/2} \frac{\sin^{-1} e}{e^3} - \frac{1 - e^2}{e^2}$$  \hspace{1cm} (35)$$

(see §17 in Chandrasekhar 1969), and

$$A_3 = 2 - 2A_1$$  \hspace{1cm} (36)$$

(for $a_1 > a_3$), the potential energy can be rewritten as

$$W = -\frac{3}{5 - n} \frac{M_0^2}{R} \frac{J}{2R^2},$$  \hspace{1cm} (37)$$

where

$$J = 2A_1 a_1^2 + A_3 a_3^2.$$  \hspace{1cm} (38)$$

The last term $J/(2R^2)$ takes into account corrections to the potential energy due to deviations from sphericity. Note that partial derivatives of $J$ take the form

$$\frac{\partial J}{\partial a_i} = \frac{1}{a_i} (J - a_i^2 A_i),$$  \hspace{1cm} (39)$$

so that

$$\frac{\partial W}{\partial a_i} = \frac{3}{5 - n} \frac{M_0^2}{2R^2} a_i A_i$$  \hspace{1cm} (40)$$

(see Chandrasekhar 1969).

Assuming the star to be uniformly rotating, the kinetic energy $T$ can written as a sum of a contribution due to expansion,

$$T_{\text{exp}} = \frac{1}{10} \kappa_n M_0 (2\dot{a}_1^2 + \dot{a}_3^2),$$  \hspace{1cm} (41)$$

and a contribution due to spin,

$$T_{\text{rot}} = \frac{1}{2} \Omega^2 = \frac{5}{4\kappa_n} \beta^{-2/3} J^2 M^{-5/3} \rho_c^{2/3} (1 - e^2)^{1/3}.$$  \hspace{1cm} (42)$$

Here the moment of inertia is given by

$$I = \frac{2}{5} \kappa_n M_0 a_1^2,$$  \hspace{1cm} (43)$$

and $\kappa_n$ is a structure constant of order unity that depends only on the polytropic index $n$ (see, e.g., LRS).

Following Lombardi et al. (1997), the post-Newtonian corrections can be written as

$$U_{\text{PN}} = -l_1 K \rho_c^{1/n+1/3} M_0^{5/3} = -l_1 K \beta^{1/n+1/3} \frac{M_0^{2+1/n}}{R^{1+3/n}}$$  \hspace{1cm} (44)$$

and

$$W_{\text{PN}} = -l_2 \rho_c^{2/3} M_0^{1/3} = -l_2 \beta^{2/3} \frac{M_0^3}{R^2}.$$  \hspace{1cm} (45)$$

Numerical values for the structure constants $l_1$ and $l_2$ are calculated in Lombardi et al. (1997). Both post-Newtonian terms are correct in this form only for spherical configurations. However, we shall assume the slow rotation limit ($T_{\text{rot}}/|W| \ll 1$), for which corrections arising from deviations from sphericity are higher order and will be neglected.

From these energy contributions we can now construct the Lagrangian

$$L(q_i, \dot{q}_i) = T - U - U_{\text{PN}} - W - W_{\text{PN}},$$  \hspace{1cm} (46)$$

where the $q_i$ are the independent variables $a_i$ and $\phi$ (with $\dot{\phi} = \Omega$). The dynamics of the system is then governed by the Euler-Lagrange equations

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0.$$  \hspace{1cm} (47)$$

The angular momentum can be defined as

$$J \equiv \frac{\partial L}{\partial \Omega}.$$  \hspace{1cm} (48)$$
Since $\phi$ is an ignorable coordinate of the Lagrangian (eq. [46]), the Euler-Lagrange equations imply that $J$ is conserved on a dynamical timescale. However, we want to allow for the radiation of angular momentum on a secular timescale by the emission of neutrinos, so we will therefore allow for an external dissipation term on the right hand side of equation (47) when $q_i = \phi$:

$$\frac{dJ}{dt} = -(\dot{J})_n.$$ (49)

Note also that when we allow $K_{\text{hot}}$ to decrease with time as the star cools (see eq. [63] below), the system will no longer conserve energy.

We now find that the Euler-Lagrange equations yield

$$\ddot{a}_1 = \Omega^2 a_1 + K\beta^{1/n} \frac{5k_1}{m_a n} \frac{M_{1/n} b}{R^{3/n} a_1} - \frac{5}{2k_a} \frac{3}{5 - n} \frac{M_0}{R^3} a_1 A_1 - K \left( \frac{1}{n} + \frac{1}{3} \right) \frac{5l_1}{\kappa_n} \frac{M_{1/n+1}}{R^{3/n+1} a_1} - \frac{10}{3} \frac{\beta^{2/3}}{\kappa_n} \frac{l_2}{R^2 a_1} M_0^2,$$ (50)

$$\ddot{a}_3 = K\beta^{1/n} \frac{5k_1}{m_a n} \frac{M_{1/n} b}{R^{3/n} a_3} - \frac{5}{2k_a} \frac{3}{5 - n} \frac{M_0}{R^3} a_3 A_3 - K \left( \frac{1}{n} + \frac{1}{3} \right) \frac{5l_1}{\kappa_n} \frac{M_{1/n+1}}{R^{3/n+1} a_3} - \frac{10}{3} \frac{\beta^{2/3}}{\kappa_n} \frac{l_2}{R^2 a_3} M_0^2,$$ (51)

and

$$\dot{\Omega} = -\frac{1}{I} (\dot{J})_n - 2 \frac{a_1 \dot{a}_1}{a_1^2} \Omega.$$ (52)

In the absence of neutrino emission [(\dot{J})_n = 0], these ordinary differential equations completely describe the adiabatic evolution of a uniformly rotating, axisymmetric neutron star in a post-Newtonian ellipsoid approximation. They represent in this limit the ellipsoidal analog of the post-Newtonian equations of hydrodynamics for an adiabatic gas. In order to allow for cooling and loss of angular momentum, we have to supplement this system of equations with expressions for $K_{\text{hot}}$ and $(\dot{J})_n$.

### A2. Neutrino Cooling and Angular Momentum Loss

We approximate the thermal energy contribution from hot nucleons by

$$\epsilon_{\text{nuc}} = \frac{3}{2} n k T$$ (53)

(strictly true in the limit of high temperatures) and add to it the thermal radiation of photons, electron-positron pairs, and $N_v = 3$ flavors of (nondegenerate) neutrinos:

$$\epsilon_{\text{rad}} = (3 + 7N_v/8)a T^4.$$ (54)

The total thermal energy density is then given by

$$\epsilon_{\text{hot}} = \epsilon_{\text{rad}} + \epsilon_{\text{nuc}}.$$ (55)

This expression can be used to estimate the temperature $T$. Adopting the Rosseland mean opacity arising from scattering off nondegenerate neutrons and protons, the neutrino opacity takes the form

$$\kappa = \frac{7(2\pi)^2}{20} \frac{\sigma_0}{4m_B} \left( \frac{k T}{m_e} \right)^2,$$ (56)

where $m_B$ is the baryon mass, $m_e$ the electron mass, and $\sigma_0 = 1.76 \times 10^{-44}$ cm$^2$. The neutrino luminosity can then be estimated via the diffusion formula

$$L = \frac{\dot{\epsilon}}{3\rho_e \kappa} |\nabla \epsilon_{\text{rad}}| \sim \frac{\dot{\epsilon}}{3\rho_e \kappa a_3} \epsilon_{\text{rad}},$$ (57)

where $\rho_e = 3M_0/(4\pi R^3)$ is the average density. Since $a_3 < a_1$, we expect that the “preferred” direction of diffusion is along the rotation axis and therefore estimate the gradient of $\epsilon_{\text{rad}}$ by dividing $\epsilon_{\text{rad}}$ by $a_3$. Using equation (8), the angular momentum loss can now be written as

$$(\dot{J})_n = \langle \hat{m}^2 \rangle \Omega L.$$ (58)

Next we have to relate the luminosity $L$ to $K_{\text{hot}}$. From equation (5) we have

$$L = \frac{\delta W^{(e)}}{d\tau} = -\left( T \frac{dS}{d\tau} \right),$$ (59)

where the brackets denote an average over the star and $S$ is the total entropy of the star. From the first law of thermodynamics we have

$$T \, dS = a \left( \frac{\epsilon}{\rho} \right) + P \frac{d}{d\tau} \left( \frac{1}{\rho} \right).$$ (60)
Here $\epsilon = nK_0^{1+1/n}$ is the total internal energy density, and $s$ is the specific entropy (per rest mass). For adiabatic changes, $K$ is constant and $T\,ds = 0$. Here, $K = K_{\text{cold}} + K_{\text{hot}}$ is not constant, however, and we find

$$T\,ds = n\rho^{1/n}\,dK \, .$$

(61)

In a Newtonian treatment, integration over the whole star now yields

$$\langle T\,ds \rangle = \int (T\,ds)\,dm = dK \int n\rho^{1/n}\,dm = d\ln K \int \epsilon\,dm = U\,d\ln K \, .$$

(62)

In keeping with our earlier assumption, we take $dK$ to be constant throughout the star. To incorporate the post-Newtonian correction consistently, we simply replace $U$ with $U_{\text{PN}}$. Inserting this result into equation (59) then yields

$$K_{\text{hot}} = \frac{K}{U + U_{\text{PN}}} \, L \, .$$

(63)

Equations (50) to (52) together with equations (10) and (15) now approximate the evolution of a cooling, rotating neutron star. At each time step, the luminosity $L$ can be estimated from equation (57).

Alternatively, we could have derived equation (62) from observing that, for quasi-static changes of constant rest mass, the mass energy of an equilibrium configuration changes according to

$$dM = \Omega\,dJ + \Lambda\,d\theta + \langle T\,ds \rangle \, ,$$

(64)

where we have set $u^r \sim 1$, and where for completeness we have allowed a vorticity term $\Lambda\,d\theta$. Here $\theta$ is the circulation and $\Lambda$ is the vorticity of the configuration. Following Appendix D in LRS, this change can also be written as

$$dM = \frac{\partial M}{\partial J} \, dz + \frac{\partial M}{\partial \theta} \, d\theta + \frac{\partial M}{\partial \alpha_i} \, d\alpha_i \, .$$

(65)

Here the $\alpha_i$ are the independent parameters of the configuration, which we can take to be $\rho_c$, $e$ and $K_{\text{hot}}$. Note that

$$\frac{\partial M}{\partial J} \bigg|_{z,\theta} = \Omega \, ,$$

(66)

and

$$\frac{\partial M}{\partial \theta} \bigg|_{z,J} = \Lambda \, ,$$

(67)

(see LRS). Furthermore, since we are considering quasi-static changes between equilibrium configurations, first-order variations with respect to the dynamical parameters vanish, so

$$\frac{\partial M}{\partial \rho_c} \bigg|_{e,K_{\text{hot}},J,\theta} = \frac{\partial M}{\partial e} \bigg|_{\rho_c,K_{\text{hot}},J,\theta} = 0 \, .$$

(68)

(see § A.3). Finally, from equation (25),

$$\frac{\partial M}{\partial K_{\text{hot}}} \bigg|_{\rho_c,e,J,\theta} = \frac{U + U_{\text{PN}}}{K} \, .$$

(69)

Putting terms together, we therefore find

$$dM = \Omega\,dJ + \Lambda\,d\theta + (U + U_{\text{PN}})d\ln K \, .$$

(70)

Comparing this expression with equation (64) shows that we can identify

$$\langle T\,ds \rangle = (U + U_{\text{PN}})d\ln K \, ,$$

(71)

which agrees with equation (62) (with $U$ replaced by $U + U_{\text{PN}}$).

A3. EQUILIBRIUM CONFIGURATIONS

A configuration in equilibrium has $T_{\text{exp}} = 0$, and the equilibrium relations can then be found, for example, by setting the derivatives of the energy functional (eq. [25]) with respect to the central density $\rho_c$ and the eccentricity $e$ to zero:

$$\frac{\partial M}{\partial \rho_c} = \frac{\partial M}{\partial e} = 0 \, .$$

(72)

The first condition yields the virial relation

$$\frac{1}{n} U + \frac{3 + n}{3n} U_{\text{PN}} + \frac{1}{3} W + \frac{2}{3} W_{\text{PN}} + \frac{2}{3} T_{\text{rot}} = 0 \, .$$

(73)
Since only $W$ and $T_{\text{rot}}$ depend on the eccentricity $e$, the second condition yields in this approximation the same result as a purely Newtonian treatment (see eq. [3.21] in LRS):

$$ t = \frac{T}{|W|} = \frac{3}{2e^2} \left[ 1 - \frac{e(1 - e^2)^{1/2}}{\sin^{-1} e} \right] - 1 . $$

(74)

Note that $t$ only depends on the eccentricity and is independent of the mass or the polytropic index $n$, for example.

For a given central density $\rho_c$, eccentricity $e$, and thermal contribution $K_{\text{hot}}$, equilibrium conditions can now be constructed by first finding $t$ from equation (74). This value can then be inserted into equation (73) to eliminate $T_{\text{rot}}$:

$$ \frac{1}{n} U + \frac{3 + n}{3n} U_{\text{PN}} + \frac{1}{3} W(1 - 2t) + \frac{2}{3} W_{\text{PN}} = 0 . $$

(75)

Writing the individual energies in terms of the rest mass $M_o$ and the central density $\rho_c$, we obtain a quadratic equation for $M_0^{2/3}$:

$$ AM_0^{4/3} + BM_0^{2/3} - C = 0 , $$

(76)

or

$$ M_0^{2/3} = - \frac{B}{2A} \pm \left( \frac{B^2}{4A^2} + \frac{C}{A} \right)^{1/2} , $$

(77)

where

$$ A = \frac{2}{3} l_2 \rho_c^{-1/3} , $$

$$ B = \frac{1}{5 - n} \beta^{-1/3} \rho_c^{-2/3} \sin^{-1} e \left( 1 - e^2 \right)^{1/2} (1 - 2t) + \frac{3 + n}{3n} l_1 K \rho_c^{1/n-2/3} , $$

(78)

$$ C = \frac{1}{n} k_1 K \rho_c^{1/n-1} . $$

In equation (77), the smallest positive solution is the physically relevant solution. In the Newtonian limit, $A = 0$ and $M_0$ is given by $(C/B)^{1/2}$ (see eq [3.22] in LRS). Once $M_0$ has been determined as a function of $\rho_c$, $e$, and $K_{\text{hot}}$, the individual energy terms as well as the total energy can be calculated.

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