Fully decoupled fixed-time cascade control of three-phase voltage source converters with LC output filter

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Abstract. This paper presents a fully decoupled fixed-time cascade control system of three-phase voltage source converters (VSCs) with LC output filter. The proposed controller has a cascade structure and it is designed in the synchronous reference frame d-q. The outer loop is for voltage control and the inner loop is for current control. A fixed-time controller is developed for voltage regulation and reference tracking before the desired pre-fixed time despite the unknown disturbances. The current controller uses PI controllers for current regulation and feedforward terms to decouple the direct and quadrature of voltage and current control loops. Unlike the conventional PI-based synchronous reference frame VSC control, the proposed approach provides better control performance with smaller and smoother control signal and enables the d-q components of voltage and current to be fully decoupled. A comparative simulation study with the conventional control system is provided to highlight the effectiveness of the proposed approach.

1. Introduction

Voltage source converters (VSCs) are among the main building blocks of power generation systems and they are commonly used for interfacing renewable energy sources and energy storage systems with the primary grid [1, 2]. VSCs have many applications including voltage and current regulation [2-5], active power filtering [6-8] and power factor control [9, 10]. VSCs require proper control to achieve the desired objectives in presence of the system nonlinearities and the unknown disturbances. Several works addressed VSCs output voltage and current control in the last decade. The main idea is to design a cascade control system where the inner loop is for current control and the outer loop is for voltage control. In inverter-based distributed generators control, synchronous (direct-quadrature d-q) reference frame is commonly used as the manipulated variables are not sinusoidal which facilitate the control. The conventional VSC control system uses PI controllers for voltage and current control [3]. In this paper a fully decoupled fixed-time cascade control system is designed for voltage and current control of VSCs with LC output filter in the d-q reference frame. The proposed controller has the following advantages compared to the conventional one:
- Voltage regulation and reference tracking before the desired pre-fixed time despite the unknown disturbances.
- Ensuring current stability by maintaining control of the current reference.
- Fully decoupled control of voltage and current direct and quadratic components.
The rest of the paper is organized as follows. In Section 1 the VSC model is presented. Section 2 is devoted to the design of the proposed control system. In Section 3, a comparative simulation is provided to highlight the controller performance. Finally, the study is summarized and concluded in Section 4.

2. Mathematical Preliminaries

Lemma 1. [11][12] Consider the simple-integrator system
\[ \dot{x} = g(t, x), x(0) = 0 \]  
(1)

Where \( g : \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n \) is a nonlinear function. If \( V(x) \) exists a continuous radially unbounded and positive definite function \( V(x) \) such that
\[ V(x) \leq -\alpha V^{1+\frac{1}{\mu}} - \beta V^{1-\frac{1}{\mu}} \]  
(2)

Where \( \alpha, \beta > 0 \) and \( \mu > 1 \) then the origin of system (1) is globally fixed-time stable with the settling-time \( T \) verifying
\[ T \leq T_{\text{max}} = \frac{n\mu}{\sqrt{\alpha\beta}} \]  
(3)

3. VSCs Mathematical Model

Three-phase VSC with LC output filter can be modeled, based on the abc/dq transformation, in two single-phase systems [13] as depicted in Fig. 1. The model contains the following elements:
- The voltage \( V_{dc} \) of DC voltage source.
- The PWM transfer function \( G_{PWM}(s) \) given as follows [14]
\[ G_{PWM}(s) = \frac{1}{1+1.5Ts} \]  
(4)

Where \( T_s \) is the sampling time.
- The filter inductance \( L \) and capacity \( C \).
- \( V_{inv,d} \) and \( V_{inv,q} \) are the direct and quadratic components of the control inputs of the VSC. \( I_{L,d} \) and \( I_{L,q} \) are the inductance current \( I_L \) direct and quadratic components. \( I_{o,d} \) and \( I_{o,q} \) are the VSC output current \( I_o \) direct and quadratic components. \( V_{o,d} \) and \( V_{o,q} \) are the direct and quadratic components of the output voltage \( V_o \). \( \omega \) is the rotation pulsation of the reference frame.

![Figure 1. d-q model of a three-phase VSC with LC output filter.](image)

4. The Proposed VSCs Control System

In this work we propose a cascade-based fixed-time controller of three-phase VSCs. A schematic of the proposed controller is depicted in Fig. 2.
Figure 2. The proposed controller of three-phase VSC with LC output filter.

$V_{ref,d}$ and $V_{ref,q}$ are the references for the output voltage $V_o$ direct and quadrature components respectively. $V_{ref,d}$ and $V_{ref,q}$ Since output voltage magnitude verifies $V_{o,magn} = \sqrt{V_{o,d}^2 + V_{o,q}^2}$, the voltage references will be set as follows: $V_{ref,d} = V_{ref}$ and $V_{ref,q} = 0$ where $V_{ref}$ is the desired voltage magnitude. $V_{ref}$ is supposed constant as in practice voltage magnitude in power networks is standardized and fixed.

Current feedforward terms are introduced to decouple the control of voltage and current direct and quadrature components. PI controllers are used for current regulation. For voltage control we propose the following fixed-time controller:

$$
\begin{align*}
(5) \quad & f(V_{ref,d}, V_d) = K \int \text{sign}(V_{ref,d} - V_d) |V_{ref,d} - V_d|^{1+\frac{\mu}{2}} + \text{sign}(V_{ref,d} - V_d) |V_{ref,d} - V_d|^{1-\frac{\mu}{2}} dt \\
& f(V_{ref,q}, V_q) = K \int \text{sign}(V_{ref,q} - V_q) |V_{ref,q} - V_q|^{1+\frac{\mu}{2}} + \text{sign}(V_{ref,q} - V_q) |V_{ref,q} - V_q|^{1-\frac{\mu}{2}} dt
\end{align*}
$$

Where $K > 0$ and $\mu > 1$ are the control gains.

**Theorem 1.** The control protocol (5) drives the voltage direct and quadrature components $V_d$ and $V_q$ to the desired references $V_{ref,d}$ and $V_{ref,q}$ before the pre-fixed settling-time given as follows

$$
T \leq \frac{\pi \mu}{4z_{load}}
$$

Where $z_{load}$ the load impedance.

**Proof.** Since the demonstration is the same for $V_d$ and $V_q$, to avoid redundancy we will present only the proof for $V_d$. Consider the positive definite Lyapunov function defined as follows

$$
V = (V_{ref,d} - V_d)^2
$$

The derivative of the Lyapunov function (7) is given as follows

$$
\dot{V} = 2(V_{ref,d} - V_d)(\dot{V}_{ref,d} - \dot{V}_d)
$$

Since $V_{ref,d}$ is supposed constant, then $\dot{V}_{ref,d} = 0$. Thus,

$$
\dot{V} = -2(V_{ref,d} - V_d)\dot{V}_d
$$

Denote $I_{L,d,ref}$ inductance current direct component reference. One can write $I_{L,d,ref} = f(V_{ref,d}, V_d)$.

In cascade control structure the inner loop is faster than the outer loop, therefore the current steady state is reached faster than voltage steady state. Since the controller is designed in the $d-q$ frame, the manipulated variables are static and thus, using PI for current control, two scenarios are considered: Either the current tracking error is canceled in which case we have $I_{L,d,ref} = I_{d,ref}$ or the PI controller
provides a constant steady state error $E \neq 0$, $I_{L,d,ref} = I_{L,d} + E$. In both cases we, have $\dot{I}_{L,d,ref} = \dot{I}_{L,d}$. One also has using VSC model

$$
(I_{L,d} - I_{o,d}) \frac{1}{c_s} = V_{o,d}
$$

where $I_{o,d} = \frac{V_{o,d}}{Z_{load}}$ and the load impedance $Z_{load} = \sqrt{R_{load}^2 + (L_{load}\omega)^2}$. This yields

$$
V_{o,d} = \frac{Z_{load}}{Z_{load}c_s + 1} I_{L,d}
$$

System (11) has a first order system dynamics thus $V_{o,d}$ converge exponentially to its final value $Z_{load}I_{L,d}$ in the settling-time $T_{setting} \approx 3Z_{load}C$. Since $C \ll 1$ and $L_{load} \ll 1$, then $T_{setting} \approx 3Z_{load}C \ll 1$. Therefore system (11) has fast dynamics and $V_{o,d}$ converge exponentially in a negligible time to its final value $Z_{load}I_{L,d}$. Thus, the convergence time can be safely neglected and one can write $V_{o,d} = Z_{load}I_{L,d}$. This yield

$$
\dot{V}_{o,d} = Z_{load} \dot{I}_{L,d,ref}
$$

$$
= Z_{load}K \left[ \text{sign}(V_{ref,d} - V_d)\left| V_{ref,d} - V_d \right|^{1+\frac{2}{\mu}} + \text{sign}(V_{ref,d} - V_d)\left| V_{ref,d} - V_d \right|^{1-\frac{2}{\mu}} \right]
$$

Therefore

$$
\dot{V} = -2Z_{load}K (V_{ref,d} - V_d) \left[ \text{sign}(V_{ref,d} - V_d)\left| V_{ref,d} - V_d \right|^{1+\frac{2}{\mu}} + \text{sign}(V_{ref,d} - V_d)\left| V_{ref,d} - V_d \right|^{1-\frac{2}{\mu}} \right]
$$

$$
= -2Z_{load}K \left[ \left| V_{ref,d} - V_d \right|^{1+\frac{1}{\mu}} + \left| V_{ref,d} - V_d \right|^{1-\frac{1}{\mu}} \right]
$$

In virtue of Lemma 1, the error $V_{ref,d} - V_d$ converge to zero, and thus $V_d$ converge to $V_{ref,d}$, before the desired pre-fixed settling-time $T$ verifying

$$
T \leq \frac{\pi \mu}{4Z_{load}K}
$$

5. Comparative Simulation Study

The effectiveness of the proposed control strategy is verified using a comparative study with the conventional cascade control developed in [3] where PI controllers are used for both voltage and current control. The test system is 400 V RMS electrical network containing a DC voltage source $V_{dc} = 700 \text{ V}$, a VSC, an LC filter with $L = 2 \text{ mH}$ and $C = 29.8 \text{ µF}$ and a resistive charge with impedance $Z_{load} = 53 \Omega$. The control gains used in the simulations are as follows $K = 10$, $\mu = 20$, $K_p = 15$ and $K_i = 1 \times 10^{-5}$ where $K_p$ and $K_i$ are the proportional and integral gains of the current PI controllers. The pre-fixed settling-time using these control gains verify $T \leq 0.0296 \text{ s}$. The control gains of the proposed control system were chosen to highlight the ability of the proposed control system to provide better performance with smaller control signal.

The results of the simulations are shown in Fig. 3 and Fig. 4. As seen in Fig. 3, The RMS of the output voltage direct component reach the reference value 400 V before the pre-fixed time 0.0296 s as designed and thus exhibits faster dynamics using the proposed approach. The quadrature component of the output voltage is canceled also faster using the fixed-time reference tracking before the desired pre-fixed time $T \leq 0.0296 \text{ s}$. In addition, the proposed approach provides a less fluctuating output.
voltage compared to the conventional PI based voltage control. As for the control signals, Fig. 4, shows that voltage control requires a control signal with lower values using the proposed control system. Fixed-time voltage control signal is also smoother and less fluctuating compared to conventional.

![Graph showing voltage comparison](image)

**Figure 3.** Output direct and quadrature components using the proposed and the conventional control methods

![Graph showing control signals comparison](image)

**Figure 4.** Control signals using the proposed and the conventional control methods
6. Conclusion
In this paper, a fully decoupled fixed-time cascade control system of three-phase voltage source converters (VSCs) with LC output filter is designed. For the outer loop, a fixed-time controller is developed for voltage regulation and reference tracking before the desired pre-fixed time despite the unknown disturbances. The inner loop uses PI controllers for current regulation and feedforward terms to decouple the control of direct and quadrature voltage and current components. A comparative simulation study with the conventional PI-based controller confirmed the theoretical results regarding voltage regulation and smoother control signals. The simulations confirmed the theoretical results regarding voltage regulation and reference tracking before the desired pre-fixed time. The simulations showed also that the proposed control strategy provides a smaller and less fluctuating control signal compared to conventional control.

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