Delocalization in the Anderson model due to a local measurement

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We study a one-dimensional Anderson model in which one site interacts with a detector monitoring the occupation of that site. We demonstrate that such an interaction, no matter how weak, leads to total delocalization of the Anderson model, and we discuss the experimental consequences.

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Consider an electron in a one-dimensional array of \( N \) coupled wells. The system is described by the Anderson tunneling Hamiltonian

\[
H_A = \sum_{j=1}^{N} E_j c_j^\dagger c_j + \sum_{j=1}^{N-1} (\Omega_j c_j^\dagger c_{j+1} + H.c.),
\]

where the operator \( c_j^\dagger \) (\( c_j \)) corresponds to the creation (annihilation) of an electron in the well \( j \). We assume for simplicity that each of the wells contains one bound electron localization. We demonstrate in this letter the interference inside the entire system and hence, the delocalization. One might suppose there-fore that such a local measurement cannot totally destroy the occupation of the electron state  \( |\Psi_N(t)\rangle \) = \( |\psi_N(0)\rangle \) even for \( \Omega_j \) real without loss of generality.

The electron-wave function in this system can be written as  \(|\Psi(t)\rangle = \sum_j b_j(t)c_j^\dagger |0\rangle\), where \( b_j(t) \) is the probability amplitude of finding the electron in the well \( j \) at time \( t \). These amplitudes are obtained from the time-dependent Schrödinger equation  \( i\hbar \partial_t |\Psi(t)\rangle = H_A |\Psi(t)\rangle\). It is well known that for randomly distributed levels \( E_j \) (or random couplings \( \Omega_j \)) all electronic states in this structure are localized. Hence, if the electron initially occupies the first well, \( b_j(0) = \delta_{j,1} \), the probability of finding it in the last well, \( P_N(t) = \langle 0 |\psi_N(t)\rangle^2 \), is exponentially with \( N \): \( \langle 0 |\psi_N(t \rightarrow \infty)\rangle_{ensemble} \rightarrow \exp(-\alpha N) \).

Anderson localization is usually associated with destructive quantum-mechanical interference between different probability amplitudes \( b_j(t) \). This interference, however, can be affected by measuring the electron’s position in the system due to interaction of the electron with a macroscopic detector. For instance, the continuous monitoring of one of the wells of a double-well system \( (N = 2 \text{ in Eq. } (1)) \) destroys the off-diagonal elements (coherences) of the electron density matrix. As a result, the latter become the statistical mixture: \( \sigma_{jj'}(t) \rightarrow (1/2)\delta_{jj'} \) for \( t \rightarrow \infty \).

In the case of the \( N \)-well structure, however, the monitoring of one of the wells cannot determine the electron’s position in the entire system. One might suppose therefore that such a local measurement cannot totally destroy the interference inside the entire system and hence, the electron localization. We demonstrate in this letter the contrary: any interaction, no matter how weak, of the electron with a macroscopic detector placed on only one well leads to total delocalization of the electron state: \( P_N(t \rightarrow \infty) = 1/N \), even when \( N \rightarrow \infty \).

As a physical realization we consider a mesoscopic system of coupled quantum dots (Fig. 1), where a point contact is placed near the first dot. The point contact is coupled to two reservoirs, emitter and collector, at different chemical potentials, \( \mu_L \) and \( \mu_R \). A current \( I = eT(\mu_L - \mu_R)/(2\pi) \) flows through the point contact, where \( T \) is its transmission coefficient. If the electron occupies the first dot, the transmission coefficient of the point contact decreases, \( T' < T \), due to the electrostatic repulsion generated by the electron. As a result, the current \( I' < I \) (Fig. 1a). The current returns to its previous value \( I \) whenever the electron occupies any other dot, since then it is far away from the contact (Fig. 1b).

The entire system can be described by the tunneling Hamiltonian \( H = H_A + H_{PC} + H_{int} \), where \( H_A \) is given by Eq. (1) and

\[
H_{PC} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} (\Omega_{lr} a_l^\dagger a_r + H.c.),
\]

\[
H_{int} = \sum_{l,r} \delta_{lr} c_l^\dagger c_l (a_r^\dagger a_l + H.c.),
\]

where \( a_l^\dagger \) (\( a_l \)) and \( a_r^\dagger \) (\( a_r \)) are the creation (annihilation) operators in the left and the right reservoirs, and \( \Omega_{lr} \) is the hopping amplitude between the states \( l \) and \( r \) of the reservoirs.
Consider an initial state where all the levels in the emitter and the collector are filled up to the Fermi energies \( \mu_L \) and \( \mu_R \), respectively, and the electron occupies the first well. The many-body wave function describing the entire system can be written in the occupation number representation as

\[
|\Psi(t)\rangle = \sum_{j=1}^{N} \left[ b_j(t)c_j^\dagger + \sum_{l,r} b_{jl}(t)c_{jl}^\dagger a_{l} \right] + \sum_{l<r} b_{jl,r}(t)c_{jl}^\dagger a_{jl}a_{r} + \cdots |0\rangle ,
\]

where \( b(t) \) are the probability amplitudes of finding the system in the states defined by the corresponding creation and annihilation operators. Using these amplitudes one defines the reduced density matrices \( \sigma_{jj'}^{(m)}(t) \) that describe the electron and the detector,

\[
\sigma_{jj'}^{(0)}(t) = b_j(t)b_{j'}^\dagger(t), \quad \sigma_{jj'}^{(1)}(t) = \left( \sum_{l,r} b_{jl}(t)b_{jl}^\dagger(t) \right), \quad \sigma_{jj'}^{(2)}(t) = \left( \sum_{l'<r'} b_{jl,r'}(t)b_{jl,r'}(t) \right), \quad \cdots
\]

Here \( j,j' = \{1,2,\ldots,N\} \) denote the occupation states of the \( N \)-dot system. The index \( m \) denotes the number of electrons that have reached the right-hand reservoir by time \( t \). The total probability for the electron to occupy the dot \( j \) is \( \sigma_{jj}(t) = \sum_m \sigma_{jj}^{(m)}(t) \). The off-diagonal density-matrix element \( \sigma_{jj'}(t) = \sum_m \sigma_{jj'}^{(m)}(t) \) describes interference between the states \( E_j \) and \( E_{j'} \).

In order to find the amplitudes \( b(t) \), we substitute Eq. (3) into the time-dependent Schrödinger equation \( i\hbar \frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle \), and use the Laplace transform \( \tilde{b}(E) = \int_0^\infty b(t) \exp(iEt)dt \). Then we find an infinite set of algebraic equations for the amplitudes \( \tilde{b}(E) \), given by

\[
\begin{align}
(E - E_1)\tilde{b}_1 - \Omega_{11}\tilde{b}_1 - \sum_{l,r} \Omega_{1l} \tilde{b}_{l1r} &= i \\
(E - E_2)\tilde{b}_2 - \Omega_{12}\tilde{b}_2 - \Omega_{12}\tilde{b}_3 - \sum_{l,r} \Omega_{l2} \tilde{b}_{2l1r} &= 0 \\
(E + E_l - E_1 - E_r)\tilde{b}_{1l1r} - \Omega_{1l} \tilde{b}_1 - \Omega_{1l} \tilde{b}_{1l1r} - \sum_{l',r'} \Omega_{l'l'} \tilde{b}_{1l'r'r'} &= 0 \\
(E + E_l - E_2 - E_r)\tilde{b}_{2l1r} - \Omega_{l2} \tilde{b}_2 - \Omega_{l2} \tilde{b}_{2l1r} - \Omega_{2l} \tilde{b}_{2l1r} - \sum_{l',r'} \Omega_{l'l'} \tilde{b}_{2l'1r'} &= 0
\end{align}
\]

This technique has been derived in [4]. We explain below only the main points of this procedure and the conditions for its validity.

Consider, for example, Eq. (3). In order to perform the summation in the term \( \sum_{l,r} \Omega_{lr} \tilde{b}_{l1r} \), we solve for \( \tilde{b}_{l1r} \) in Eq. (3). Then substituting the result into the sum, we can rewrite Eq. (3) as

\[
\left(E - E_1 - \int \frac{\Omega_{1l}^2 \rho_L(E_l) \rho_R(E_r) dE_l dE_r}{E - E_1 - E_r} \right) \tilde{b}_1 - \Omega_{12} \tilde{b}_2 + F = i ,
\]

where we have replaced the sum in Eq. (3) by an integral \( \sum_{l,r} \rightarrow \int \rho_L(E_l) \rho_R(E_r) dE_l dE_r \), with \( \rho_{LR} \) the density of states in the emitter and collector. We split this integral into its principle value and singular part. The singular part yields \( iD'/2 \), where \( D' = 2\pi l^2 \rho_L(\mu_L - \mu_R) \), and the principal part is zero, providing \( \Omega_L \) and \( \rho_{LR} \) are weakly dependent on the energies \( E_{l,r} \). Note that \( 2\pi l^2 \rho_L \rho_R = T \), where \( T \) is the tunneling transmission coefficient of the point contact. Thus, \( \epsilon D' = I' \) is the current flowing through the point contact [3] whenever the electron occupies the first dot.

The quantity \( F \) in Eq. (3) denotes the terms in which the amplitudes \( \tilde{b} \) cannot be factored out of the integrals. These terms vanish in the large-bias limit, \( (\mu_L - \mu_R) \gg \Omega_L^2 \rho \). Indeed, all the singularities of the amplitude \( \tilde{b}(E, E_l, E_r, E_{l,r}) \) in the \( E_l, E_r \) variables lie below the real axis. This can be seen directly from Eqs. (3) by noting that \( E \) lies above the real axis in the Laplace transform. Assuming that the transition amplitudes \( \Omega \) as well as the densities of states \( \rho_{LR} \) are independent of \( E_{l,r} \), one can close the integration contour in the upper \( E_{l,r} \)-plane. Since the integrand decreases faster than \( 1/E_{l,r} \), the resulting integrals are zero.

Applying analogous considerations to the other equations of the system [3] we convert Eqs. (3) directly into rate equations via the inverse Laplace transform. The details can be found in [3,4]. Here we present only the final equations for the electron density matrix \( \sigma_{jj'}(t) \):

\[
\dot{\sigma}_{jj} = i\Omega_{j-1}(\sigma_{jj-1} - \sigma_{jj+1}) + i\Omega_j(\sigma_{jj+1} - \sigma_{jj+1}), \quad (7a)
\]

\[
\dot{\sigma}_{jj'} = i\epsilon_{jj'}\sigma_{jj'} + i\Omega_{j-1,1}(\sigma_{jj-1} + \sigma_{jj+1}) - i\Omega_{j-1,1}(\sigma_{jj+1} + \sigma_{jj+1}) - \sigma_{jj'} - \Omega_{jj'} - \sum_{jj'} \sigma_{jj'}(\delta_{jj} + \delta_{jj'}), \quad (7b)
\]

where \( \epsilon_{jj'} = E_{jj'} - E_j - \Gamma = (\sqrt{T/e} - \sqrt{T'/e})^2 \) is the decoherence rate, generated by interaction with the detector. Note that these equations have been obtained from the many-body Schrödinger equation for the entire system. No stochastic assumptions have been made in their derivation.

Eqs. (7) are analogous to the well-known optical Bloch equations used to describe a multilevel atom interacting with the quantized electromagnetic field [3]. To our
knowledge, this is their first appearance in connection with the Anderson model. The equations can be rewritten in Lindblad form as

$$\dot{\sigma} = -i[H_A, \sigma] - \frac{\Gamma}{2}(Q\sigma + \sigma Q - 2Q\sigma Q^\dagger),$$ (8)

where $H_A$ is given by Eq. (4) and $Q_{j,j'} = \hat{Q}_{j,j'} = \delta_{jj'}\sigma_{jj'}$. If $\Gamma = 0$, Eq. (8) is equivalent to the Schrödinger equation $i\partial_t|\Psi(t)\rangle = H_A|\Psi(t)\rangle$. In this case the electron density matrix $\sigma(t)$ displays Anderson localization, i.e., $\sigma_{NN}(t \to \infty) \sim \exp(-\Omega N)$. If $\Gamma \neq 0$, however, the asymptotic behavior of the reduced density-matrix, $\sigma_{jj'}(t \to \infty)$, changes dramatically: all eigenfrequencies (except for the zero mode) obtain an imaginary part due to the second (damping) term in Eq. (8), so that only the stationary terms survive in the limit $t \to \infty$. This damping is illustrated in Fig. 2 which displays the numerical solution of Eqs. (8) for $N = 4$, $\Omega_j = \Omega = \text{const}$, and $E_j/\Omega = \{0, 2, 4, 1\}$. The occupation of the first dot, $P_1(t) = \sigma_{11}(t)$, and the last dot, $P_4(t) = \sigma_{44}(t)$, is shown in Fig. 2 for $\Gamma = 0$ by the dashed lines, and for $\Gamma/\Omega = 1$ by the solid lines. One can clearly see that all oscillations decay for $\Gamma \neq 0$, so that the density matrix reaches a stationary limit. Then we see the opposite of localization, as the probability of finding the electron in the last dot, $P_4(t)$, becomes the same as the probability of finding it in the first dot, $P_1(t)$.

The delocalization phenomenon, illustrated by Fig. 2, can be proven analytically for any $N$. Indeed, let us consider Eqs. (8) in the asymptotic limit $t \to \infty$, where the electron density matrix reaches its stationary limit: $\sigma_{jj'}(t \to \infty) = u_{jj'} + i v_{jj'}$. Since for the stationary solution $\partial_t \sigma_{jj'} \to 0$, Eqs. (8) become

$$0 = \epsilon_{jj'}v_{jj'} + \Omega_{jj'}v_{jj'-1} + \Omega_{jj}v_{jj'+1} - \Omega_{j-1,j}v_{j-1,j'} - \Omega_{j+1,j'} + \frac{\Gamma}{2}u_{jj'}(\delta_{jj} + \delta_{jj'}) (1 - \delta_{jj'}) \quad (9a)$$

$$0 = \epsilon_{j,j'}u_{jj'} + \Omega_{jj'}u_{jj'-1} + \Omega_{jj}u_{jj'+1} - \Omega_{j-1,j}u_{j-1,j'} - \Omega_{j+1,j'} - \frac{\Gamma}{2}v_{jj'}(\delta_{jj} + \delta_{jj'}) (1 - \delta_{jj'}) \quad (9b)$$

Eqs. (8) have the unique solution $v_{jj'} = 0$ and $u_{jj'} = (1/N)\delta_{jj'}$. This can be obtained by solving these equations sequentially, starting with $j, j' = N$, and then continuing for $j, j' = N - 1, N - 2, \ldots$. Since $u_{jj} \equiv \sigma_{jj}(t \to \infty)$, we finally obtain that

$$\sigma_{jj'}(t \to \infty) = \frac{1}{N} \delta_{jj'}.$$ (10)

This corresponds to the totally delocalized electron state. Since Eq. (10) represents the unique solution of Eqs. (8), it implies that the asymptotic behavior of the electron density matrix is always given by Eq. (10) for any initial conditions. Note that this result is true only for $\Gamma \neq 0$. Otherwise the solution of Eqs. (8) is not unique.

Eq. (10) tells us that an arbitrarily weak interaction with the environment (detector) leads to delocalization in the Anderson model, even though this interaction affects only one of the sites. In other words, Anderson localization is unstable under infinitely small decoherence. One aspect of this instability is the importance of the order of limits $t \to \infty$ and $N \to \infty$. Taking $t \to \infty$ first, as above, gives delocalization, while taking $N \to \infty$ first would preserve localization. In the non-interacting model, $\Gamma = 0$, the order of limits is immaterial and the electron is localized.

Even though a local interaction with the environment destroys the localization, the latter should affect the time-dependence of the observed system. We expect the delocalization time to increase exponentially with $N$ and to be dependent on both the decoherence rate and the localization length. This matter deserves further investigation.

We would like to stress that our result is not an effect of finite temperature, as is so called the hopping conductivity $\overline{\Im}$. In the latter case, each site of the Anderson model interacts with the thermal bath; in our case, only one site is coupled to the detector (environment). If we were to let all the sites interact equally with the detector ($I = I'$, Fig. 1), we would obtain no delocalization in our model, since $\Gamma = 0$ in Eqs. (8). Indeed, in this case Eq. (8) is equivalent to the Schrödinger equation $i\partial_t|\Psi(t)\rangle = H_A|\Psi(t)\rangle$ leading to Anderson localization. Note that there is no measurement when $I = I'$. The origin of delocalization in our case is therefore the break of coherence due to the measurement process.

Delocalization of the Anderson model due to measurement has been studied previously [10, 11]. Yet the limit of a local and weak measurement has not been achieved. In
the present work we include the detector in the quantum mechanical description, avoiding the use of the projection postulate in the course of measurement. This enables us to study delocalization due to local measurement and also in the limit of weak coupling with the measurement device.

Another experimental setup for delocalization due to a local measurement is shown schematically in Fig. 3. It can be realized in atomic systems, for instance, in experiments with Rydberg atoms [13]. For $N = 2$ this setup is similar to a V-level system used for investigation of the quantum Zeno effect [14]. The occupation of $E_1$ is

$$\sigma_{jj} = i\Omega_{j-1}(\sigma_{j,j-1} - \sigma_{j-1,j})$$

$$+ i\Omega_{j}(\sigma_{j,j+1} - \sigma_{j+1,j}) - \Gamma(\delta_{j1} - \delta_{j0})\sigma_{11} .$$

The last term in Eq. (11) describes the rates due to spontaneous photon emission, Fig. 3. Here again the Bloch equations for the electron density matrix can be rewritten in Lindblad form, Eq. (8), with $Q_{jj'} = \delta_{jj'}$ and $Q_{jj'} = \delta_{j1}\delta_{j'1}$. For $N = 2$ Eq. (8) coincides with the optical Bloch equations used for analysis of a V-level system [15]. Similar to the previous case, Fig. 1, Anderson localization is destroyed for any value of $\Gamma$, and the asymptotic electron distribution, $\sigma_{jj}(t \to \infty)$, does not depend on the initial electron state. Here, however, the electron density matrix in the asymptotic state is not a pure mixture, $\sigma_{jj}(t \to \infty) \not= 0$, and the probabilities $\sigma_{jj}(t \to \infty)$ are not equally distributed between different wells (c.f. Eq. (10)).

The delocalization of the Anderson model should also affect its transport properties. Indeed, by connecting the first and the last dot in Fig. 1 to leads (reservoirs) one can expect current to flow through the dot array whenever any of the dots is monitored. Indeed, the stationary current through coupled dots is proportional to the occupation probability of the last dot, attached to the collector $E_1$. The current should appear with a delay after a voltage bias to the leads is switched on. This time delay is precisely the relaxation time needed for the electron to be delocalized.

Anderson localization appears not only in quantum mechanics, but also in classical wave mechanics. Therefore the described delocalization due to local interaction with an environment should have a classical analogy. It can appear, for instance, in propagation of waves through coupled cavities with randomly distributed resonant frequencies. A wave cannot ordinarily penetrate through such a system due to the Anderson localization. Random vibration of one of the cavities, however, should destroy the localization, so that waves begin to penetrate through the system after some time delay, corresponding to the delocalization time. Such an experiment can also be done using the system of transparent plates with randomly varying thicknesses, described in [10].

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[1] N.F. Mott and W.D. Twose, Adv. Phys. 10, 107 (1961);
P.W. Anderson, D.J. Thouless, E. Abrahams and D.S. Fisher, Phys. Rev. B22, 3519 (1980).
[2] S.A. Gurvitz, Phys. Rev. B56, 15215 (1997).
[3] R. Landauer, IBM J. Res. Dev. 1, 223 (1957); R. Landauer, J. Phys. Condens. Matter 1, 8099 (1989).
[4] S.A. Gurvitz and Ya.S. Prager, Phys. Rev. B53 (1996), 15932; S.A. Gurvitz, Phys. Rev. B57 (1998) 6602.
[5] J. Bardeen, Phys. Rev. Lett. 6, 57 (1961).
[6] B. Elattari and S.A. Gurvitz, Phys. Rev. Lett. 84, 2047 (2000).
[7] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, Atom-Photon Interactions: Basic Processes and Applications (Wiley, New York, 1992).
[8] G. Lindblad, Commun. Math. Phys. 48, 119 (1976).
[9] N.F. Mott, Metal-Insulator transitions (Taylor & Francis, London, 1974).
[10] T. Dittrich and R. Graham, Europhys. Lett. 11, 589 (1990); Phys. Rev. A 42, 4647 (1990).
[11] P. Facchi, S. Pascazio, and A. Scardicchio, Phys. Rev. Lett. 83, 61 (1999).
[12] J.C Flores, Phys. Rev. B60, 30 (1999).
[13] R. Blümel, A. Buchleitner, R. Graham, L. Sirko, U. Smilansky, and H. Walther, Phys. Rev. A44, 4521 (1991).
[14] W.M. Itano, D.J. Heinzen, J.J. Bollinger, and D.J. Wineland, Phys. Rev. A 41, 2295 (1990).
[15] V. Frerichs and A. Schenzle, Phys. Rev. A44, 1962 (1991).
[16] M. V. Berry and S. Klein, Eur. J. Phys. 18, 222 (1997).