X-ray afterglows of gamma-ray bursts in the synchrotron self-Compton dominated regime

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ABSTRACT

We consider in this paper the effect of synchrotron self-Compton process on X-ray afterglows of gamma-ray bursts. We find that for a wide range of parameter values, especially for the standard values which imply the energy in the electrons behind the afterglow shock is tens times as that in the magnetic field, the electron cooling is dominated by Compton cooling rather than synchrotron one. This leads to a different evolution of cooling frequency in the synchrotron emission component, and hence a different (flatter) light curve slope in the X-ray range. This effect should be taken into account when estimating the afterglow parameters by X-ray observational data. For somewhat higher ambient density, the synchrotron self-Compton emission may be directly detected in X-ray range, showing varying spectral slopes and a quite steep light curve slope.

Key words: gamma-rays: bursts — radiation mechanisms: nonthermal — relativity

1 INTRODUCTION

The current model (see reviews of Cheng & Lu 2001 and Meszaros 2002) of gamma-ray burst (GRB) afterglows is that a sideways expanding jet (Rhoads 1999) drives a blast wave propagating into the circum-burst medium, and the shock-accelerated electrons give rise to the afterglow emission. The main radiation process is believed to be synchrotron emission by electrons (e.g., Meszaros & Rees 1997; Sari, Piran & Narayan 1998), which is consistent with the afterglow spectra (e.g., Galama et al. 1998). The polarization detections in afterglows have also implicated the synchrotron mechanism (e.g., Covino et al. 1999; Wijers et al. 1999). The synchrotron self-Compton (SSC) emission is also important if the energy density of radiated synchrotron photons exceeds that of the magnetic field in the shock. This may be always the case since the ratio between the post-shock energies in electrons and in the magnetic field is commonly larger than \( \sim 10 \), and a significant fraction of the shock-heated electron energy is radiated away. The SSC emission has been studied by previous works based on the spherical afterglow model (Panaitescu & Meszaros 1998; Wei & Lu 1998; Totani 1998; Chiang & Dermer 1999; Dermer, Chiang & Mitman 2000; Dermer, Böttcher & Chiang 2000; Panaitescu & Kumar 2000; Sari & Esin 2001; Zhang & Meszaros 2001). Two cases of X-ray excess in the afterglow spectra have been explained to be the inverse-Compton components (Harrison et al. 2001; Yost et al. 2002).

The observed X-ray emission from GRB afterglows usually comes from synchrotron by fast cooling electrons, those electrons with energy-lose times less than the dynamical time of the system. If these electrons lose energy mainly by SSC rather than synchrotron, more shock-heated electrons would cool rapidly, thus the distribution of electrons and hence the X-ray light curve index would be different from the synchrotron-dominated case (Panaitescu & Kumar 2001; Li, Dai & Lu 2002). Furthermore, the SSC component is possible to be directly detected in the X-rays in some cases. Thus the SSC effects should be taken into account when modelling the afterglow observational data. Panaitescu & Kumar (2002) had incorporated numerically the SSC mechanism in the modelling of many GRB afterglows, in order to give out the physical condition of relativistic jets in GRB afterglows. However, most people still tend to use the simple asymptotic relation of light curve, rather than the numerical modelling, to fit the observational data. In many cases, though the afterglows are in the SSC-dominated regime, the asymptotic relation in synchrotron-dominated regime are still used in the fitting. Therefore it is necessary to derive the parameter range in which the afterglow are SSC-dominated, and then the analytical asymptotic relation of afterglow light curve in this regime.

In this paper, we make more detailed study on the inverse-Compton processes in GRB afterglows, especially the X-ray afterglow emission from jets in SSC-dominated regimes. We first introduce in section 2 the whole dynamical evolution of a beaming afterglow. In section 3 we calculate the light curve of synchrotron emission and the model...
parameter constraint on the SSC-dominated case. We then discuss in section 4 the case when SSC emission emerges directly in X-rays. Section 5 is a brief summary and discussion.

2 DYNAMICAL EVOLUTION

Consider a beaming outflow from the GRB source, so called a jet, which decelerates as sweeping up the ambient medium and sideways expanding in the local sound speed. If the radiation energy is negligible compared to the jet kinetic energy, the jet can be regarded as adiabatic when considering its dynamic evolution. This is always the case provided the energy fraction that goes into shocked electrons is $\epsilon_e \lesssim 0.1$, which is the common value from model fit to observational data. For a higher $\epsilon_e$, the jet will undergo first an early radiative stage, in which the afterglow light curve index is relevant to $\epsilon_e$ (Böttcher & Dermer 2000; Li, Dai & Lu 2002). We consider only adiabatic dynamics here.

A jet, with initial isotropic energy $E$, coasts first with initial Lorentz factor $\gamma_0$ until it sweeps up enough material at a deceleration time $t_0 = (3E/32\pi m_n c^3)^{1/3}$, with $n$ the ambient medium density. After $t_0$ the jet begins to decelerate. The deceleration of the jet includes three stages: First, when the sideways expansion is not significant compared to the initial jet open angle $\theta_0$, the jet undergoes a spherical-like phase where the jet Lorentz factor decreases as $\gamma \propto t^{-3/5}$ (Blandford & McKee 1976), with $t$ the observer’s time, and the jet open angle is $\theta \approx \theta_0$. Secondly, when the sideways expansion begins to dominate the dynamical evolution at $t_j = t_0(\gamma_0\theta_0)^{9/5}$, we have $\theta \approx 1/\gamma$, and the jet turns into a spreading phase where $\gamma \propto t^{-1/2}$ (Rhoads 1999).

Here we have assumed that the sound speed in the relativistic stage is comparable to light speed, $c_s \sim c$. Finally, the jet becomes non-relativistic, $\gamma \approx 1$, at $t_n = t_j^{5/4}t_0^{-1/5}$ and $\theta \approx 1$. In the non-relativistic phase the sideways expansion is not important and the jets evolve as $\gamma \propto t^{-3/5}$, its radius $r \propto t^{2/5}$.

The three special times are calculated in the following:

$$t_0 = 90(E_{52}/n)^{1/3} \gamma_0^{-8/3} \text{s}, \quad (1)$$

$$t_j = 4.2 \times 10^4(E_{52}/n)^{1/3} \gamma_0^{-8/3} \text{s}, \quad (2)$$

$$t_n = 4.2 \times 10^6(E_{52}/n)^{1/3} \gamma_0^{2/3} \text{s}, \quad (3)$$

where we have used the convention $U = 10^9 U_x$ and c.g.s units. Hereafter, by “sphere” we mean the $t_0 < t < t_j$ phase, by “jet” the $t_j < t < t_n$ phase and by “NR” the $t > t_n$ phase.

3 SYNCHROTRON EMISSION

The shock accelerates the ambient electrons to high energies, with electron Lorentz factors described by a power-law distribution: $dN_e/d\gamma \propto \gamma^{-\gamma_m}$ for $\gamma > \gamma_m$. The typical Lorentz factor of electrons is proportional to the internal energy density of the shock as $\gamma_m \propto \gamma - 1$. At the beginning it is approximated as $\gamma_m \approx 610\epsilon_e \gamma_0$ at $t_0$, and evolves as $\gamma_m \propto \gamma$ in the relativistic regime since $\gamma - 1 \approx \gamma$, while in the NR phase it becomes $\gamma_m \propto \gamma^2 \propto t^{-6/5}$ since $\gamma - 1 \approx \gamma^2$ for NR. The magnetic field is also created by the shock, commonly assumed to carry a fraction $\epsilon_B$ of the total internal energy behind the shock front. Thus the energy density of magnetic field, $B^2/4\pi$, is also proportional to $\gamma - 1$. At the deceleration time $t_0$, the magnetic field is $B = (32\pi \epsilon_e m_n c^2)^{1/2}/t_0$, later on it evolves as $B \propto \gamma$ in the relativistic regime $\propto t^{-3/8}$ (sphere) and $\propto t^{-1/2}$ (jet), while $B \propto v \propto t^{-3/5}$ in NR phase.

Under these conditions the synchrotron radiation is produced, with the instantaneous spectrum described as power-law segments (Sari, Piran & Narayan 1998). The typical frequency of synchrotron photons is relevant to the typical electron energy,

$$\nu_m = \frac{x_p e}{\pi m_e c} B_j^2 \gamma^2 \propto \begin{cases} t^{-3/2} & \text{sphere}, \\ t^{-2} & \text{jet}, \\ t^{-3} & \text{NR}, \end{cases} \quad (4)$$

where $x_p$ is defined by Wijers & Galama (1999) and of order of unity.

The electrons lose energy through both synchrotron and SSC, and the Compton parameter $\gamma$, i.e., the ratio between the inverse-Compton to synchrotron luminosity, is calculated as (Sari & Esin 2001)

$$Y = \frac{1 + \sqrt{1 + 4\eta_e \epsilon_B}}{2} \approx \begin{cases} \eta_e/\epsilon_B, & \text{if } \eta_e/\epsilon_B \ll 1, \\ \sqrt{\eta_e/\epsilon_B}, & \text{if } \eta_e/\epsilon_B \gg 1, \end{cases} \quad (5)$$

where $\eta$ is the fraction of electron energy that is radiated away (by both synchrotron and SSC). The synchrotron cooling frequency, i.e. the frequency of the synchrotron photons radiated by those electrons which cool on the dynamical time of the shock, is given by

$$\nu_c = \frac{36 \pi c m_e c}{4\pi^2 B^3 \gamma^2 (1 + Y)^2}. \quad (6)$$

Since the electrons responsible to synchrotron frequencies above $\nu_c$ lose energy quickly, the radiated fraction of electron energy is therefore

$$\eta = \begin{cases} 1 & \text{for fast cooling, } \nu_c < \nu_m, \\ (\nu_c/\nu_m)^{2-p} & \text{for slow cooling, } \nu_c > \nu_m. \end{cases} \quad (7)$$

The equations (5)-(7) show that $Y$ and $\nu_c$ are correlated, and these three equations should be combined to solve the time evolutions of both $Y$ and $\nu_c$, especially for the IC-dominated case, $Y > 1$, which we focus on in this paper. The $Y$ and $\nu_c$ should be solved by numerical calculation, while for extreme case $Y \gg 1$ we can reach an analytical result (see also Li, Dai & Lu 2002).

$$\nu_c \propto \begin{cases} t^{-3+2/(4-p)} & \text{sphere}, \\ t^{-2+4/(4-p)} & \text{jet}, \\ t^{-3+28/(5(4-p))} & \text{NR}. \end{cases} \quad (8)$$

The flux peaks at the lower one of the two frequencies $\nu_m$ and $\nu_c$. The swept-up electron number is approximated by $N_e \approx \pi \theta^2 r^3/3$, and the power per unit time per unit frequency emitted by single electron is (in the comoving frame)

$$P_e = (3/2) \phi_p c^3/m_e c^2 B,$$

where $\phi_p$ is calculated by Wijers & Galama (1999) and of order of unity. Furthermore, the energy emitted by total electrons is distributed over an area of $\Delta S \sim \pi \theta^2 D^2$ at a luminosity distance $D$ from the source, the observed peak flux density is therefore
\[ F_{v,\text{max}} \simeq \frac{N \gamma P_e}{\Delta S} \propto \gamma^2 B \propto \begin{cases} \text{const. sphere,} & t^{-1} \text{ jet,} \end{cases} t^{3/5} \text{ NR.} \] (9)

Except for the very early times (see equation 11), the afterglow is generally in slow cooling regime with \( \nu_c \gg \nu_m \).

We focus on the highest radiation energy range of afterglows, i.e., the X-ray band, which usually corresponds to the \( \nu > \nu_c \) flux,

\[ F_{\nu > \nu_c} = F_{\nu,\text{max}}(\nu/\nu_m)^{-\frac{p-1}{2}}(\nu/\nu_c)^{\frac{p}{2}} \]

\[ \propto \begin{cases} \text{t}^{-3p/4} & \text{sphere,} \end{cases} t^{-p+1} \text{ jet,} \]

\[ t^{-66p-15p^2-52}/[10(4-p)] \text{ NR.} \] (10)

This above equation expresses the light curve of synchrotron emission in the IC-dominated case (\( Y > 1 \)).

We summarize the results together with previous works for synchrotron-dominated case (\( Y < 1 \) in table 1. Since the synchrotron emission in the \( \nu_m < \nu < \nu_c \) range, \( F_{\nu < \nu_c} = F_{\nu,\text{max}}(\nu/\nu_m)^{-\frac{p-1}{2}} \), is irrelevant to the evolution of \( \nu_c \), the light curve index in this frequency range is the same as the synchrotron-dominated case. The scaling relations for synchrotron-dominated case have not been included in table 1 and can be found in Sari, Piran & Halpern (1999) and Dai & Lu (1999, 2000).

### 3.1 Parameter range for strong Compton cooling

With different values of physical parameters, e.g., \( \epsilon_e \) and \( \epsilon_B \), the system may correspond to different cases of whether synchrotron- or IC-dominated, therefore we discuss the parameter range now. In general, the afterglow is initially in the fast cooling regime, with \( \nu_c < \nu_m \) and \( \eta = \frac{1}{3} \), and then the Compton parameter is a constant, \( Y_0 \approx \sqrt{\epsilon_e/\epsilon_B} \), provided commonly \( \epsilon_e > \epsilon_B \). It is not until a time,

\[ t_{cm} = 1.0 \times 10^3 E_{52} n^{-2} \epsilon_{e,-1} \theta_{1/2}^{-2} \left( 1 + \sqrt{\frac{\epsilon_{e,-1}}{\epsilon_{B,-2}}} \right)^2, \] (11)

that the afterglow becomes slow cooling and the Compton parameter decreases as \( Y \propto t^{-\frac{p-2}{2}[4-4p]} \). For the cooling of electrons to be still dominated by SSC process, the Compton parameter at the point of jet break should be larger than unity: \( Y(t_j) > 1 \). This, with help of equation (2), leads to

\[ \epsilon_{e,-1} > 0.24 \theta_{1/2}^{-2} \frac{t_{\nu_c}^{2/3} E_{52}^{1/18}}{n^{1/21} \epsilon_{B,-2}^{4/21}} \] (12)

for \( p = 2.2 \) and

\[ \epsilon_{e,-1} > 0.45 \theta_{1/2}^{-2} \frac{t_{\nu_c}^{2/3} E_{52}^{1/18}}{n^{1/21} \epsilon_{B,-2}^{4/21}} \] (13)

for \( p = 2.4 \). Thus, with the commonly taken parameters, such as \( \epsilon_e \sim 0.1 \) and \( \epsilon_B \sim 0.01 \), the afterglow is still Compton-dominated when the jet break in the light curve appears.

After the jet break point, the Compton parameter turns to drop faster as \( Y \propto t^{-\frac{p-2}{2}[4-4p]} \). If we require that the \( Y \) value is still larger than unity when the jet goes into NR phase, i.e., \( Y(t_n) > 1 \), the condition is

\[ \epsilon_{e,-1} > 0.52 \theta_{1/2}^{-2} \frac{t_{\nu_c}^{2/3} E_{52}^{1/18}}{n^{1/21} \epsilon_{B,-2}^{4/21}} \] (14)

for \( p = 2.2 \) and

\[ \epsilon_{e,-1} > 1.7 \theta_{1/2}^{-2} \frac{t_{\nu_c}^{2/3} E_{52}^{1/18}}{n^{1/21} \epsilon_{B,-2}^{4/21}} \] (15)

for \( p = 2.4 \). Therefore the Compton cooling may dominate synchrotron cooling even in the NR phase for the common parameter values. So in the whole period of X-ray observation, Compton cooling is strong. These above inequalities are insensitive to the initial condition of afterglows, like the total (isotropic) energy \( E \), the ambient density \( n \) and the jet open angle \( \theta_0 \), but sensitive to shock physics. We show the parameter ranges in figure 1.

### 4 DIRECT DETECTION OF INVERSE-COMPTON COMPONENT

The SSC component dominates the synchrotron one in high enough energy range, and its spectral shape can also be approximated by broken power laws as synchrotron one (Panaitescu & Kumar 2000; Sari & Esin 2001): \( F_{\nu} \propto \nu^{\frac{p-1}{2}}/\nu_m \) for \( \nu < \min(\nu_m, \nu_c) \); \( F_{\nu} \propto \nu^{\frac{p-1}{2}}/\nu_c \) for \( \nu_c < \nu < \nu_c \) (or \( F_{\nu} \propto \nu^{\frac{p-1}{2}}/\nu_c \) for \( \nu_c < \nu < \nu_m \); and \( F_{\nu} \propto \nu^{\frac{p-1}{2}} \) for \( \nu_m < \nu < \nu_c \). For the synchrotron cooling frequency \( \nu_c \), where \( \nu_m \approx 210\nu_c \) and \( \nu_c \approx 2.7\gamma_c \), with \( \gamma_l \) being the electron Lorentz factor corresponding to synchrotron frequency \( \nu_c \).

After a time \( t_m \), the system becomes slow cooling, with \( \nu_m < \nu_c \) for synchrotron component and \( \nu_m < \nu_c \) for SSC component. If taken \( \epsilon_e \sim 0.1 \) and \( \epsilon_B \sim 0.01 \) typically, the system is in the SSC-dominated regime. Therefore here we limit our discussion to the SSC-dominated (\( Y > 1 \)) and slow cooling (\( t_j > t_{cm} \)) case, during which for typical parameters the crossing point between the synchrotron and the SSC spectral components, \( \nu_c \), generally lies above the synchrotron cooling frequency \( \nu_c \) and below SSC cooling frequency \( \nu_c \). For the SSC emission to be detected directly in X-rays, we need \( \nu_c \lesssim 10^{18} \) Hz. This condition places a lower limit on the ambient density (Sari & Esin 2001). We numerically calculate the emission by both synchrotron and SSC and then the evolution of the crossing frequency \( \nu_c \) with time for different ambient densities, as show in figure 2. In general, the lower limit is \( n > 1 \) cm\(^{-3} \).

In general, the \( \nu_c \) moves into the X-ray band in the jet spreading phase (\( t_j < t < t_m \)). For fixed X-ray frequency \( \nu x = 10^{18} \) Hz, the crossing time is \( t_m \approx 5 \times 10^4 \epsilon_{e,-1}^{1/18} \epsilon_{B,-2}^{1/21} \theta_{1/2}^{-1} n^{-1/6} \nu_{18}^{-1/3} \) s. Note that we have assumed the slow cooling case which requires \( \nu_c > t_j > t_{cm} \). Around \( t_m \) the observed flux evolves as

\[ F_{\nu_c} \propto \left\{ \begin{array}{ll} \nu_c^{1/3} t_{\nu_c}^{-1/3} & t_{j} < t < t_{\nu_c}, \\ \nu_c^{1/3} t_{\nu_c}^{-2/3} & t_{\nu_c} < t < t_n. \end{array} \right. \]

(16)

The spectral slope changes gradually from \( 1/3 \) to \( -(p-1)/2 \), which is different from the \( -p/2 \) slope in the high energy tail of synchrotron component, and in the same time the light curve index changes from zero to a steep decline. The relation between the steep light curve index \( \alpha_{\text{IC}} \) (\( F_{\nu_c} \propto \epsilon_{e,-1}^{1/18} \epsilon_{B,-2}^{1/21} \theta_{1/2}^{-1} n^{-1/6} \nu_{18}^{-1/3} \)).
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t^{−α_{IC}} \) and the spectral index β_{IC} \left(F_{ν}^{IC} \propto ν^{−β_{IC}}\right) in the rapid decline is:

\[ α_{IC} - 3β_{IC} - 1 = 0. \] (17)

We emphasize that a steep light curve together with a shallow spectral slope in X-ray band \( \left(F_{ν}^{IC} \propto ν^{−β_{IC}}\right) \) may imply the direct detection of SSC-dominated emission component. Figure 3 has shown a case when SSC dominates the X-ray emission in jetted afterglows.

5 SUMMARY AND DISCUSSION

We have discussed in this paper the effect of SSC process on the X-ray afterglow. For a wide range of parameter values (see figure 1), including the commonly taken ones \( \epsilon_e \simeq 0.1 \) and \( \epsilon_B \simeq 0.01 \), the electron cooling is dominated by IC cooling rather than synchrotron one. This leads to a different evolution of cooling frequency \( ν_c \) in the synchrotron emission component, and hence a different (flatter) synchrotron light curve slope above \( ν_c \), say, the X-ray range. The light curve index of jet-spreading phase in SSC-dominated \( (Y > 1) \) case is flatter by a factor of \( (p - 2)/(4 - p) \) than synchrotron-dominated case. This SSC effect should be taken into account when modelling in detail the X-ray observational data. It should be noticed that in many case we should use the SSC-dominated \( α - β \) relations (in table 1) rather than the synchrotron-dominated ones to fit the observation.

For somewhat higher ambient density, \( n \gtrsim 3 \text{ cm}^{-3} \), the SSC emission dominates the synchrotron in X-ray range and can be detected directly (see also Sari & Esin 2001). The SSC light curve shows a slope of \( α_{IC} = 2.5 - 3.4 \) for \( p = 2 - 2.6 \), quite steeper than the synchrotron one. When the SSC component emerges, the X-ray spectral slope varies, which may be detected by observation.

The upcoming Swift satellite is due to launch at the end of 2003, which is expected to catch more than 200 afterglows per year. Owing to its rapid response, many afterglows may be rapidly observed in O/UV and X-rays within one minute. The current operating X-ray satellites, Chandra and XMM-Newton, have high sensitive and spectral resolution. So many more detailed X-ray observations of GRB afterglows are expected. We emphasize that the X-ray observation of afterglows may help to follow the cooling of electrons and help to investigate the SSC characteristics of afterglows.

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Figure 1. Regions in the $\epsilon_e$, $\epsilon_B$ parameter space in which synchrotron or SSC dominates when the jet break occurs. The critical cases of $Y(t_j) = 1$ are shown for $p = 2.2$ and $p = 2.4$. The upper-left region is still SSC-dominated after jet break at $t_j$, while the bottom-right region has become synchrotron-dominated before $t_j$.

Figure 2. The frequency above which the emission is dominated by SSC, as function of time, for $n = 0.3$, $1$ and $10$ cm$^{-3}$, using $E = 10^{53}$ ergs, $\theta_0 = 0.1$, $\gamma_0 = 150$, $p = 2.4$, $\epsilon_e = 0.1$, $\epsilon_B = 10^{-3}$ and $D = 10^{28}$ cm. The horizon line shows a X-ray frequency $\nu_X = 10^{18}$ Hz. Only for the cases of $n > 1$ cm$^{-1}$ can $\nu^{IC}$ drops below the X-ray band. For the case of $n = 10$ cm$^{-3}$ the special times of $\nu^{IC} = \nu_X$ and $\nu^{IC} = \nu^{IC}_m$ are marked.

Figure 3. X-ray ($\nu = 10^{18}$ Hz) light curve in the case of $n = 10$ cm$^{-3}$. The other parameters are taken as: $E = 10^{53}$ ergs, $\theta_0 = 0.1$, $\gamma_0 = 150$, $p = 2.4$, $\epsilon_e = 0.1$, $\epsilon_B = 10^{-3}$ and $D = 10^{28}$ cm. The total flux (thick solid) consists of synchrotron (dashed) and SSC (dashed-dot) components. At early times the X-ray flux is dominated by synchrotron with spectral slope of $\nu^{-p/2}$. Later, when $\nu^{IC}$ moves into the X-ray band and the emission is dominated by SSC, the flux rises/flattens and has a spectral form of $\nu^{1/3}$ and then, when the $\nu^{IC}_m$ drops into the X-ray band, the flux decays fast and has a spectral form of $\nu^{-(p-1)/2}$.
Table 1. The synchrotron light-curve index $\alpha$ ($F_\nu \propto t^{-\alpha}$) as function of $p$ in the range of $\nu > \nu_c$. The parameter-free relation between $\alpha$ and the spectral index $\beta$ ($F_\nu \propto \nu^{-\beta}$) is given for each case by substituting $p = 2\beta$ as for $\nu > \nu_c$. The numerical factors in the bracket correspond to $p = 2.4$.

| $\nu > \nu_c$, $Y < 1$ | sphere | jet | non-relativistic |
|--------------------------|--------|-----|------------------|
| $\nu > \nu_c$, $Y > 1$  | $\alpha = 3(p - 1)/4$ | $\alpha = p$ | $\alpha = (3p - 4)/2$ |
|                          | (1.05) | (2.4) | (1.6)            |
| $\nu > \nu_c$, $Y > 1$  | $\alpha = 3\beta/2 - 1/2$ | $\alpha = 2\beta$ | $\alpha = (6\beta - 4)/2$ |
|                          | (1.18) | (2.15) | (1.25)           |