CORRELATED NEUTRINO OSCILLATIONS

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ABSTRACT

In solar neutrino oscillations, if $\nu_e$ has a significant third massive component, the allowed parameter space in $\Delta m^2$ and $\sin^2 2\theta$ for the first two components is shown to be greatly increased. This third component may be correlated to atmospheric neutrino oscillations, as shown in a specific predictive seesaw model of the $3 \times 3$ neutrino mass matrix. Possible variations to include the recent LSND results are briefly discussed.

1. Introduction

There are now three categories of data which show evidence of neutrino oscillations. (a) In experiments which detect neutrinos from the sun, there appears to be a deficit. Hence $\nu_e$ has apparently disappeared. (b) In experiments which measure the ratio $\nu_\mu/\nu_e$ in the atmosphere, there also appears to be a deficit. Hence a combination of $\nu_e$ and $\nu_\mu$ disappearance and appearance may have also occurred. (c) The recent results of the LSND (Liquid Scintillator Neutrino Detector) experiment seem to indicate that $\nu_e$ has appeared where there is originally only $\nu_\mu$.

Conventional interpretations of the above as neutrino oscillations always plot $\Delta m^2$ versus $\sin^2 2\theta$, assuming implicitly that only two neutrinos are involved in each case. As long as all mixing angles are small, this is a good approximation because the $\Delta m^2$ in each case is very different from one another. However, the atmospheric data are strongly indicative of a large mixing between $\nu_\mu$ and $\nu_e$ or $\nu_\tau$ or both. Hence $\nu_e$ may well be composed of three mass eigenstates with a massive third component to account for all or part of the atmospheric oscillations, whereas the solar oscillations are explained by the first two components with a small $\Delta m^2$ together with a nonnegligible contribution from the massive third component. In the following it will be shown that this has the important consequence of enlarging the parameter space of $\Delta m^2$ and $\sin^2 2\theta$ for the first two components that is allowed by the present solar data.

Since the LSND results are indicative of a much larger $\Delta m^2$, of order eV$^2$, a fourth neutrino is required if all of the data are to be explained by neutrino oscillations. This possibility will also be discussed.
2. Three-Neutrino Analysis of Solar Data

The work that I will describe in this section was done in collaboration with J. Pantaleone, who has been considering neutrino oscillations of all three flavors for many years. Other authors are now beginning to follow suit.

Let the electron neutrino be a linear combination of three mass eigenstates:

\[ \nu_e = \cos \theta \nu_1 - \sin \phi \sin \theta \nu_2 + \cos \phi \sin \theta \nu_3, \]  

(1)

and assume\(^*\)

\[ \Delta m_{13}^2 \simeq \Delta m_{23}^2 \sim 10^{-2} \text{ eV}^2. \]  

(2)

Then for a given value of \( \theta \), one may use the solar data to find the allowed region in \( \Delta m_{12}^2 \) and \( \sin^2 2\theta_{e2} \equiv 4|U_{e2}|^2(1 - |U_{e2}|^2) \). The results for \( \sin^2 2\theta = 0.35 \) and 0.75 are shown below.

On the left is the allowed region for \( \sin^2 2\theta = 0.35 \) which differs from that of the two-neutrino analysis, i.e. \( \theta = 0 \), by only a little. However, there is already a firm indication that it has enlarged. On the right is the allowed region for \( \sin^2 2\theta = 0.75 \) which shows dramatically that it has greatly increased and that the adiabatic branch of the solution at around \( \Delta m^2 = 10^{-4} \text{ eV}^2 \) is now allowed. The dashed lines are theoretical predictions to be discussed in the next section.

\(^*\)Note that if \( m_3 \) were of order a few eV, reactor data would require that \( \nu_3 \) overlaps very little with \( \nu_e \). See the talk by M. C. Gonzalez-Garcia, these proceedings.
3. Seesaw Structure Revealed

Recall that the well-known empirical relationship for the Cabbibo angle in terms of the ratio of the $d$ and $s$ quarks, i.e. $\sin^2 \theta_C \simeq m_d/m_s$, has led to the suggestion that

$$M_{ds} = \begin{bmatrix} 0 & a \\ a & b \end{bmatrix}.$$  \hspace{1cm} (3)

This simple observation has generated over the years an enormous literature on quark mass matrices. It is an especially active field of research in the past two or three years. Consider now a trivial extension of this seesaw structure and apply it to the neutrino mass matrix, namely

$$M_\nu = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & a \\ 0 & a & b \end{bmatrix},$$  \hspace{1cm} (4)

but in the basis $\cos \theta \nu_e - \sin \theta \nu_\mu, \nu_\tau$, and $\cos \theta \nu_\mu + \sin \theta \nu_e$. For small $a/b$, the mass eigenvalues are simply $0, -a^2/b, \text{and } b$. The usual three neutrinos are related to the mass eigenstates by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \phi \sin \theta & \cos \phi \sin \theta \\ -\sin \theta & -\sin \phi \cos \theta & \cos \phi \cos \theta \\ 0 & \cos \phi & \sin \phi \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},$$  \hspace{1cm} (5)

where $\sin \phi \simeq a/b \simeq \sqrt{m_2/m_3}$.

The electron neutrino is then as given by Eq. (1) and the discussion of the previous section applies. However, $\Delta m^2_{12} = m_2^2$ is now correlated with $\sin^2 2\theta_{e2}$ for a given choice of $m_3$ which is of course constrained by atmospheric data. In the figures above, the dashed lines represent the predictions of Eq. (4) for $m_3 \simeq 0.17 \text{ eV (left)}$ and $0.063 \text{ eV (right)}$. They do indeed intersect the allowed regions.

The atmospheric neutrino oscillations are given by

$$P(\nu_\mu \to \nu_e) = \frac{1}{2} \cos^4 \phi \sin^2 2\theta \left(1 - \cos \frac{t\Delta m^2_{23}}{2p} \right),$$  \hspace{1cm} (6)

$$P(\nu_\mu \to \nu_\tau) = \frac{1}{2} \sin^2 2\phi \cos^2 \theta \left(1 - \cos \frac{t\Delta m^2_{23}}{2p} \right).$$  \hspace{1cm} (7)

Since the angle $\phi$ is small, $\nu_\mu$ oscillates mainly into $\nu_e$ in this simplest realization of the seesaw ansatz. On the other hand, the $\nu_\mu - \nu_\tau$ submatrix may be rotated without affecting $\nu_e$, in which case a better fit to the atmospheric data can be obtained.

4. A Specific Model

To obtain Eq. (4), start with $\nu_e, \nu_\mu, \nu_\tau$, and four singlets: $\nu_S, N_1, N_2, N_3$. Assume a discrete $Z_6$ symmetry [$\omega^6 = 1$] and assign

$$(\nu_e, \nu_\mu, \nu_\tau) \sim (\omega, \omega^{-2}, 1); \quad (\nu_S, N_1, N_2, N_3) \sim (\omega, 1, \omega^2, \omega^{-2}).$$  \hspace{1cm} (8)
The Higgs sector is taken to consist of two doublets ($\Phi_1, \Phi_2 \sim (1, \omega^{-3})$) and one singlet $\chi \sim \omega$. The resulting $7 \times 7$ mass matrix is then given by

$$
M_7 = 
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & m_1 & 0 \\
0 & 0 & 0 & 0 & 0 & m_2 & 0 \\
0 & 0 & 0 & 0 & m_3 & 0 & 0 \\
0 & 0 & 0 & m_4 & 0 & m_5 & 0 \\
m_1 & m_2 & 0 & 0 & 0 & 0 & M_1 \\
0 & 0 & m_5 & 0 & M_2 & 0 & 0 \\
\end{pmatrix},
$$

(9)

where $m_1$ comes from $\langle \phi^0_2 \rangle$, $m_{2,3}$ from $\langle \phi^0_1 \rangle$, and $m_{4,5}$ from $\langle \chi \rangle$. Large $M_{1,2}$ reduce the above to a $4 \times 4$ mass matrix

$$
M_4 = 
\begin{pmatrix}
0 & 0 & 0 & 0 & m_1 m_5/M_2 \\
0 & 0 & 0 & 0 & m_3 m_5/M_2 \\
0 & 0 & m_2^2/M_1 & m_3 m_4/M_1 & m_3^2/M_1 \\
m_1 m_5/M_2 & m_2 m_5/M_2 & m_3 m_4/M_1 & m_3^2/M_1 & 0
\end{pmatrix}.
$$

(10)

Assume now that $m_2^2/M_1$ dominates, then

$$
M_3 = 
\begin{pmatrix}
bs^2 & bsc & as \\
bsc & bc^2 & ac \\
as & ac & 0
\end{pmatrix}
= 
\begin{pmatrix}
c & 0 & s \\
-s & 0 & c \\
0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & a \\
0 & a & b
\end{pmatrix}
\begin{pmatrix}
c & -s & 0 \\
s & c & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(11)

where $b = (m_1^2 + m_2^2)m_5^2M_1/m_2^2M_2^2$, $a = m_3 m_5 \sqrt{m_1^2 + m_2^2}/m_4 M_2$, $s = m_1/\sqrt{m_1^2 + m_2^2}$, $c = m_2/\sqrt{m_1^2 + m_2^2}$. The desired seesaw structure is thus obtained.

5. Addition of a Fourth Neutrino

Since Eq. (10) contains a fourth neutrino which couples to both $\nu_e$ and $\nu_\mu$, it may be considered as a candidate for explaining the recent LSND results. However, the required mass and mixing of this singlet neutrino with $\nu_e$ are then too large to be consistent with the nucleosynthesis bound on the number of light neutrinos. To avoid this problem, the most natural thing to do is to use the singlet neutrino to explain the solar data in the matter-enhanced small-angle nonadiabatic solution, as has been pointed out by many authors. In that case, $\nu_\mu$ and $\nu_\tau$ may be assumed to have masses of a few eV, but a small enough mass difference and large enough mixing to account for the atmospheric data. A small mixing between $\nu_e$ and $\nu_\mu$ may then be invoked to explain the LSND results. A recently proposed model uses a discrete $Z_5$ symmetry and the seesaw reduction of a $7 \times 7$ mass matrix to obtain four approximate
light neutrino mass eigenstates \( \cos \theta \nu_e - \sin \theta \nu_S, \cos \theta \nu_S + \sin \theta \nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2}, \)
and \((\nu_\mu - \nu_\tau)/\sqrt{2}, \) with eigenvalues 0, \( m_1, m_2, \) and \(-m_2\) respectively. In addition, mixing occurs between \( \nu_e \) and \( \nu_\mu, \) as well as between \( \nu_S \) and \( \nu_\tau. \) Note that \( \nu_\mu \) and \( \nu_\tau \) are pseudo-Dirac partners, hence \( \sin^2 2\theta = 1 \) is required for atmospheric neutrino oscillations.

To accommodate a fourth neutrino in the present context, a possible variation is to double Eq. (3) and consider the \(4 \times 4\) mass matrix

\[
\mathcal{M}_\nu' = \begin{bmatrix}
0 & a & 0 & 0 \\
 a & b & 0 & 0 \\
0 & 0 & c & 0 \\
0 & 0 & c & d
\end{bmatrix}
\]  
(12)

in the basis \( \cos \theta \nu_e - \sin \theta \nu_\mu, \nu_S, \cos \theta \nu_\mu + \sin \theta \nu_e, \) and \( \nu_\tau. \) Solar neutrino oscillations are as given before, but now the second mass eigenstate is mostly inert and there is no phenomenological constraint on the ratio \( a/b \) as in the case of Eq. (4). Atmospheric neutrino oscillations are mostly between \( \nu_\mu \) and \( \nu_e \), whereas the LSND results are explained by the fact that both \( \nu_e \) and \( \nu_\mu \) mix with \( \nu_\tau. \) However, because of the seesaw ansatz, the latter is correlated with the former. Numerically, they are indeed consistent with both sets of data, although the value of \( \Delta m^2 \) in the LSND case is required to be less than about 3 eV\(^2\).

6. Conclusions

As more neutrino experiments accumulate more data, there are two important messages for phenomenologists and model builders. First, the naive assumption that each case of neutrino oscillations is to be interpreted as between only two mass eigenstates must be abandoned. Atmospheric data tell us that a large mixing angle exists between \( \nu_\mu \) and \( \nu_e \) or \( \nu_\tau \) or both. In this talk it has been shown that if \( \nu_e \) has a significant third massive component, the analysis of solar data allows a much larger parameter space in \( \Delta m^2 \) and \( \sin^2 2\theta \) for the first two components.

Second, the structure of the neutrino mass matrix is beginning to reveal itself. It is time to look for possible empirical relationships such as the well-known \( \sin^2 \theta_C \simeq m_d/m_s \) for quarks which may give us a glimpse of the underlying theory of the origin of masses. In this talk a first attempt, i.e. Eqs. (4) and (12), has been noted.

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8. References

1. C. Athanassopoulos et al., Los Alamos National Laboratory Report No. LA-UR-95-1238 (April 1995).
2. Y. Fukuda et al., Phys. Lett. B335, 237 (1994).
3. For a review, see for example A. Yu. Smirnov, these proceedings.
4. E. Ma and J. Pantaleone, UCRHEP-T140 (March 1995).
5. T. K. Kuo and J. Pantaleone, Rev. Mod. Phys. 61, 937 (1989); D. Harley, T. K. Kuo, and J. Pantaleone, Phys. Rev. D 47, 4059 (1993); J. Pantaleone, Phys. Rev. D 49, R2152 (1994).
6. A. S. Joshipura and P. I. Krastev, Phys. Rev. D 50, 3484 (1994); M. Narayan, M. V. N. Murthy, G. Rajasekaran, and S. Uma Sankar, hep-ph/9505281; S. M. Bilenkii, C. Giunti, and C. W. Kim, hep-ph/9505301.
7. S. Weinberg, Ann. N. Y. Acad. Sci. 38, 185 (1977).
8. See for example X. Shi et al., Phys. Rev. D 48, 2563 (1993); K. Enqvist et al., Nucl. Phys. B373, 498 (1992).
9. J. T. Peltoniemi and J. W. F. Valle, Nucl. Phys. B406, 409 (1993); D. O. Caldwell and R. N. Mohapatra, Phys. Rev. D 48, 3259 (1993); E. J. Chun, A. S. Joshipura, and A. Yu. Smirnov, hep-ph/9505273 and these proceedings; Z. G. Berezhiani and R. N. Mohapatra, hep-ph/9505388 and these proceedings.
10. E. Ma and P. Roy, UCRHEP-T145 (April 1995).