Scalar resonances in leptophilic multi-Higgs models

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Abstract

We show that the Higgs resonance can be amplified in a 3-3-1 model with a multi-Higgs “leptophilic” scalar sector. This would allow the observation of the Higgs particle in muon colliders even for Higgs masses considerably higher than the ones expected to be seen in the electroweak standard model framework.

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The study of the electroweak symmetry breaking is one of the main goals of future colliders. The Higgs coupling to fermions are proportional to the fermion masses and hence the \( s \)-channel Higgs contribution to \( e^+e^- \rightarrow f \bar{f} \) is highly suppressed in electron-positron colliders. However, since muons are nearly 200 times heavier than electrons it is usually considered in literature that the \( s \)-channel production of the Higgs boson in the \( \mu^+\mu^- \) collider is one of the main features of muon colliders. The main process is also \( \mu^+\mu^- \rightarrow f \bar{f} \) \(^1\).

We would like to point out that there exist a kind of models in which a scalar multiplet couples only with leptons (not with quarks). If this is the case the vacuum expectation value (VEV) of the neutral component of such multiplet can be naturally small since it is the only responsible for generating the leptons masses. An example of this kind of models are the multi-Higgs extensions of the electroweak standard model (ESM) with three Higgs doublets in such a way (say, by imposing an appropriate discrete symmetry) that one of them couples only with leptons in the fermion sector.

There are models in which such a situation arises in a more natural way than in the several doublets extensions of the ESM. These are models based on \( SU(3)_C \otimes SU(3)_L \otimes U(1)_N \) gauge symmetry (3-3-1 model by short) \(^2\). In the minimal version of this model a sextet \((6,0)\) is necessary for giving mass to the charged leptons and it does not couple to quarks indeed, so, its VEV does not have to be necessarily a large VEV, say it can be of the order of a few GeVs. A scalar sector having this particular characteristic will be called “leptophilic”. Hence, leptophilic models can have a lepton-Higgs coupling stronger than the ESM one. This characteristic will be the key for having a Higgs resonance–enhancing–effect in processes involving leptons in the initial and/or final state. Here we will consider a \( \mu^+\mu^- \) collider to study the Higgs-resonance in the reactions \( \mu^+\mu^- \rightarrow W^+W^-, \mu^+\mu^-, \bar{b}b, (WW, \mu\mu, bb, by short) \).

Throughout this work, for each process, we will refer to the pure Higgs contribution at \( \sqrt{s} = m_h \) as signal \((S = \sigma_h)\), and to the non-Higgs ones as background \((B = \sigma_B)\). Here we are also assuming that the pure 3-3-1 contributions to each reaction, \textit{i.e.}, the ones that are not contained in the ESM, are mediated by fields which are massive enough to make
these contributions negligible. In this way the only difference between the 3-3-1 model and the ESM lies in the scalar sector so that the background will be the same for both models for each process considered. The background itself depends on the process: it is formed by the \( \nu, \gamma \) and \( Z \) contributions for \( WW \) and by the \( \gamma \) and \( Z \) ones for \( \mu\mu \) and \( bb \). Once the ratio \( N_S / \sqrt{N_B} = \mathcal{L}_{\mu\mu}^3 \sigma_h / \sqrt{\sigma_B} \) and \( \mathcal{L}_{\mu\mu} \), the muon-collider luminosity, depends only on the machine, we focus our attention on the model dependent dimensional ratio \( \delta = S / \sqrt{B} = \sigma_h / \sqrt{\sigma_B} \) (cm). When necessary the expected First Muon Collider (FMC) luminosity is assumed: \( \mathcal{L}_{\mu\mu} = 10^{33} \text{ cm}^{-2} \text{s}^{-1} = 10 \text{ fb}^{-1} \text{yr}^{-1} \). Although large polarization can enhance the ratio \( S/B \) \[3\], here we are not considering muon-beam polarization.

The total Higgs-width is model dependent, however, we are assuming that for the Higgs-mass range considered here (100-200 GeV) no new 3-3-1 decay-modes are allowed living the total \( \Gamma_h \) equal to the one of the ESM. In order to study the scalar resonance when comparing the 3-3-1 model with the ESM we will introduce the Higgs-signal-enhancing-factor (HSEF) \( f \) defined for each process by \( S^{331}/S^{ESM} \).

The main goal of this paper is to show the possible existence of a Higgs-signal-enhancing effect due to the manifestation of the leptophilic quality of a given model, hence as we are not considering the experimental detection aspects in detail we assume that the detector efficiency is 1.

Although we will consider explicitly a 3-3-1 model, we would like to stress that our results will be valid in multi-Higgs extensions of the ESM if they are implemented in such a way that there is only a doublet coupling to leptons as we said above.

In the 3-3-1 model considered here the lepton mass term transforms as \((1, 3, 0) \otimes (1, 3, 0) = (1, 3^*, 0)_A \oplus (1, 6, 0)_S\), then we can introduce a triplet \( \eta = (\eta^0, \eta^-, \eta^+)^T \sim (1, 3, 0) \) and a symmetric sextet \( S \sim (1, 6^*, 0) \). With the \( \eta \)- triplet only, one of the charged leptons remains massless and the two other are mass degenerate. Hence, the sextet \( S \) at least has to be introduced in order to give arbitrary masses to all charged leptons. This scalar multiplet has the following charge attribution:
In the model, in order to give mass to the quarks, it is necessary to have two more triplets: 
\[ \rho = (\rho^+, \rho^0, \rho^{++})^T \sim (1, 3, 1) \text{ and } \chi = (\chi^-, \chi^{--}, \chi^0)^T \sim (1, 3, -1). \]

Denoting the respective VEVs by \( v_a \) \((a = \eta, \rho, \chi, \sigma_2)\) and also assuming \( v_{\sigma_1} = 0 \), in such a way that neutrinos remain massless (assuming that the total lepton number is conserved), we can write the masses of the vector bosons

\[ M_{W}^2 = \frac{g^2}{4} v_1^2 + v_2^2 + v_3^2, \quad M_{Z}^2 \approx \frac{g^2}{4v_W^2} \left( v_1^2 + v_2^2 + v_3^2 \right). \]

Notice that \( v_\chi \) does not contribute to the vector boson masses (it only contributes to the \( Z \) boson mass but in such a way that it is proportional to \( v/v_\chi \), with \( v \) being any of the other VEVs. Since the \( v_\chi \) is the VEV which is in control of the \( SU(3)_L \) breaking, it is larger than the other ones and we can neglect its contribution to \( M_Z \). In this case the physical neutral Higgs-scalars relevant at low energies are only those related to \( \eta, \rho \) and \( S \) with the decoupling of the \( \chi \) field \[4\]. On the other hand, since the sextet \( S \) couples only with leptons its VEV can be of the order of a few GeVs; its contribution to the vector boson masses can be smaller than the other VEVs contributions. Hereafter we will change the notation \( v_\eta \rightarrow v_1, v_\rho \rightarrow v_2 \) and \( v_{\sigma_2} \rightarrow v_3 \) and \( H_i^0 \) \((i = 1, 2, 3)\) denote the real part of the scalar fields. These are related to the mass eigenstates \( h_i^0 \) by the orthogonal matrix \( O \):

\[ H_i^0 = O_{ij} h_j^0. \]

The interaction of the \( W \) boson with the scalars is

\[ \mathcal{L}_{hWW} = \frac{g^2}{2} v_i O_{ij} h_j^0 W^+ W^-, \]

while the Yukawa interaction in the lepton sector is given by

\[ \mathcal{L}^Y = -\frac{m_l}{v_3} O_{3j} h_j^0 \bar{l} l, \]
since only the sextet \( \sigma_2^0 = O_{3j} h_j \) couples with them. Notice that this scalar interaction with leptons are not necessarily negligible since \( \nu_3 \) can be of the order of some GeVs as commented earlier.

We will study the \( \mu^+ \mu^- \rightarrow W^+ W^- \) process in some detail and for the other ones we will omit some of the technicalities for shortness. There are four contributions to the \( WW \) process being three of them in the \( s \)-channel with \( Z^0 \)-, \( A \) (photon)- and \( h^0 \)'s exchanges and one in the \( t \)-channel with a neutrino exchange.

The amplitude \( \mathcal{M}_{WW} \) is:

\[
\mathcal{M}_{WW} = \mathcal{M}^\nu + \mathcal{M}^A + \mathcal{M}^Z + \mathcal{M}^{h_j}. 
\]  

In the 3-3-1 model all the \( W \)-couplings have the same form as in the standard electroweak model. The neutrino \( \mathcal{M}^\nu \) and the vector boson contributions \( (\mathcal{M}^\gamma, \mathcal{M}^Z) \) are exactly the same as in the ESM and are given by

\[
\mathcal{M}^\nu = \frac{-4iG_F m_W^2}{\sqrt{2} t} \bar{v}(p_2, r_2) H_R \gamma^\rho \gamma_\mu k_1^\rho \gamma^\sigma u(p_1, r_1) \epsilon^\rho \epsilon^\sigma, 
\]

\[
\mathcal{M}^\gamma = \frac{8iG_F m_W^2 \sin^2 \theta_W}{\sqrt{2} s} \bar{v}(p_2, r_2) \gamma_\nu u(p_1, r_1) \epsilon^\rho \epsilon^\sigma \cdot [(p_4 - p_3)^\nu g^{\rho\sigma} \\
- (k_2 + p_4)^\sigma g^{\rho\nu} + (k_2 + p_3)^\rho g^{\nu\sigma}],
\]

\[
\mathcal{M}^Z = \frac{-4iG_F m_Z m_W^2 \cos \theta_W (s - m_Z^2 - im_Z \Gamma_Z)}{\sqrt{2}[(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2]} \\
\cdot \bar{v}(p_2, r_2) [R H_R + L H_L] \gamma_\mu u(p_1, r_1) (g_{\mu\nu} - \frac{k_2 \mu k_2 \nu}{m_Z^2}) \epsilon^\rho \epsilon^\sigma \\
\cdot [(p_4 - p_3)^\nu g^{\rho\sigma} - (k_2 + p_4)^\sigma g^{\rho\nu} + (k_2 + p_3)^\rho g^{\nu\sigma}],
\]

where \( H_{R,L} = \frac{1}{2}(1 \pm \gamma^5) \); \( R = (2/3) \sin^2 \theta_W \), \( L = -1 + (2/3) \sin^2 \theta_W \); \( p_1, p_2, p_3, p_4 \) are the momenta of the electron, positron, \( W^- \), and \( W^+ \), respectively; \( k_2 = p_3 + p_4 \) is the momentum of the photon and \( Z \).

However, the scalar contributions are different in 3-3-1 models. Each \( s \)-channel exchange of a physical Higgs \( h_j \) gives the contribution:
\[ \mathcal{M}^{h_j} = \frac{4iG_F m_{\mu} m_W^2 O_{3j}}{\sqrt{2} v_3 \left( (s - m_j^2)^2 + m_j^2 \Gamma_j^2 \right)} \sum_i O_{ij} v_i \left( (s - m_j^2) - i m_j \Gamma_j \right) \bar{v}(p_2, r_2) u(p_1, r_1) g_{\rho\sigma} e^\rho e^\sigma, \]  

(9)

where \( m_j \) and \( \Gamma_j \) are the mass and total width of the scalar \( j \) (with no sum in \( j \)).

In the ESM a scalar resonance can only be revealed in a \( \mu \)-collider for a relatively light Higgs. This can be understood by the following simple reasoning. At the Higgs pole \( (\sqrt{s} = m_h) \) the squared-Higgs-amplitude is proportional to \( 1/m_h^2 \Gamma_h^2 \) being the total Higgs width \( \Gamma_h \) a fast growing function of the Higgs mass: it goes from \( \sim 3.1 \times 10^{-3} \) GeV for \( m_h \sim 110 \) GeV to \( \sim 1.6 \) GeV for \( m_h \sim 200 \) GeV. Hence the total Higgs contribution is suppressed for relatively large values of the Higgs mass. This argument is also valid for \( \mu\mu \) and \( bb \).

On the other hand for a 3-3-1 model, as it can be seen from Eq.(9), the squared-Higgs-amplitude at \( \sqrt{s} = m_j \) is proportional to

\[ \frac{1}{m_j^2 \Gamma_j^2} \left[ \frac{O_{3j}^2}{v_3^2} (O_{1j} v_1 + O_{2j} v_2 + O_{3j} v_3)^2 \right]. \]

(10)

Where the mixing-angles must obey the unitary condition \( \sum_j O_{3j}^2 = 1 \) and the VEVs are related by the constraint \( v_1^2 + v_2^2 + v_3^2 = v_{ESM}^2 = (246 \text{ GeV})^2 \). As we said before once the VEV \( v_3 \) is only needed to give mass to the massive leptons, it can take small values so that the VEV-mixing-angles relation between the square brackets above can enhance the total Higgs contribution in such way that it can be still significant even for large values of the Higgs mass.

In this case the HSEF is given by

\[ f_{WW} = \frac{O_{33}^2}{v_3^2} (O_{13} v_1 + O_{23} v_2 + O_{33} v_3)^2. \]

(11)

The behaviour of \( f_{WW} \) is showed in Fig. (1) as a function of \( v_3 \) when a choice for the others parameters is made. In Fig.(2) we show S and B for the \( WW \) process for the ESM and the 3-3-1 model. There we can see that due to the HSEF, S is considerably enlarged for the 3-3-1 model providing a better ratio \( S/\sqrt{B} \) when comparing with the ESM. We have built these
figures by considering the s-channel exchange of a single Higgs in the 3-3-1 amplitudes. Here we have taken $j = 3$, however, in order to unify the notation we use $m_3 = m_h$ and $\Gamma_3 = \Gamma_h$.

For very large values of $\sqrt{s}$ we certainly must add all others scalar contributions of the 3-3-1 model (and the boson $Z'$ as well) in order to ensure unitarity. However, as we said before, for the values of $\sqrt{s}$ we are considering here we are assuming that the other scalars $h_1, h_2$ (and also the $Z'$) are massive enough to not affect this picture. Here we have considered only the case of a real $W$-pair production to illustrate that even for a relatively massive scalar a resonance can occur in the $WW$ process. However, the enhancing factor that we have pointed out here will also occur with a virtual pair production since it is introduced by a fundamental vertex of the model.

Next we consider the $\mu^+\mu^- \rightarrow \mu^+\mu^-$ process. As it is well known the Higgs signal is very small in the ESM framework provided that each vertex introduces a $m_\mu/v_{ESM}$ factor, where $m_\mu$ is the muon mass and $v_{ESM} = 246$ GeV. For this process we have the contribution of the $t$ and the $s$-channel, however, the last one is largely dominant at the resonance. Hence the main $s$-channel Higgs contribution to $S_{ESM}$ is proportional to $(m_\mu/v_{ESM})^4/(m_3^2\Gamma_3^2)$ at the Higgs peak. Since $m_\mu/v_{ESM}$ is an additional suppression factor and following the same argument we used before, a ESM Higgs-resonance are not expected to be seen in this process. On the other hand in the 3-3-1 model, due to the Yukawa coupling given by Eq. (4), the Higgs signal $S^{331}$ is proportional to $(O_{33})^4(m_\mu/v_3)^4/(m_h^2\Gamma_h^2)$ at the resonance. As we said early, once $v_3$ can be chosen relatively small ($10 - 50$ GeV), the factor $(O_{33})^4(m_\mu/v_3)^4$ can considerably enhance the Higgs signal when comparing with the ESM one. In fact the HSEF $f_{\mu\mu}$ is given by

$$f_{\mu\mu} = (O_{33})^4(v_{ESM}/v_3)^4,$$

and this quantity can vary from $10^2$ for $v_3 = 50$ GeV to $10^5$ for $v_3 = 10$ GeV assuming that $O_{33} \sim \mathcal{O}(1)$ as showed in Fig. (1). In Fig. (3) we show $S$ and $B$ as a function of $\sqrt{s} = m_h$ and $\sqrt{s}$ respectively. We do not quote $\sigma_h^{ESM}$ because it is negligible. There we can see that for relatively light Higgs $\sigma_h^{331}$ dominates the cross section, and that for $m_h > 135$
GeV $S^{331}$ goes under the $B$ and falls rapidly to zero, however the resulting HSEF is still enough to allow for a Higgs-resonance detection in the range $m_h \sim 150 – 160$ GeV in the $\mu^+\mu^- \rightarrow \mu^+\mu^-$ process. Although that enhancing–factor can provide a very pronounced peak in the total cross section at the Higgs resonance for relatively light Higgs, we must remember that for this sort of masses the Higgs-width is very small and this will require a very high-resolution energy scan. As it was shown in Fig. (2), this is not the case for the $\mu^+\mu^- \rightarrow W^+W^-$ process: $S^{331}$ remains appreciable even for $m_h > 170$ GeV. In this case a resonance detection is much easier since for masses in this range the Higgs-width is considerably large and so the energy-resolution requirements can be less stringent.

Finally we will examine the $\mu^+\mu^- \rightarrow b\bar{b}$ process. In the ESM the $b\bar{b}$ final state considerably increases the $S^{ESM}$ due to the $b$-quark mass as it is shown in Fig. (4). There we can see that, differently from the $\mu\mu$ process, the Higgs signal dominates over the background up to $m_h < 145$ GeV. Although the total cross section for $b\bar{b}$ production is much lower than for $\mu^+\mu^-$, the higher ratio $S/\sqrt{B}$ provided by $b\bar{b}$ make this one the process to be studied at muon colliders in order to detect a relatively light Higgs-resonance [1].

In the 3-3-1 model the couplings lepton–scalar and quark–scalar are different. Once $\sigma_2^0$ couples only with leptons, the lepton–scalar vertex is provided by Eq. (4), while for $b$-like quarks the main flavor-conserving quark–scalar interactions are given by

$$L^Y_{d-like} = -\frac{m_b}{v_\eta} O_{nj} h_j^0 \bar{b} b,$$  

(13)

Hence, the pure $s$-channel Higgs contribution to $b\bar{b}$ at the resonance is proportional to

$$\frac{1}{m_h^2 \Gamma_h^2} (O_{33} O_{13})^2 \left( \frac{m_\mu m_b}{v_3 v_1} \right)^2,$$  

(14)

where we have used the previous redefinition: $v_\eta \rightarrow v_1$, $v_\rho \rightarrow v_2$ and $v_\sigma_2 \rightarrow v_3$. In this case, for $\sqrt{s} = m_h$, the HSEF $f_{b\bar{b}}$ is given by

$$f_{b\bar{b}} = (O_{33} O_{13})^2 \left( \frac{v_{ESM}^2}{v_3 v_1} \right)^2.$$  

(15)

As before this factor allows the possibility of having an enhancement in the pure Higgs amplitude as showed in Fig. (1).
In Fig. (5) we show the quantity $\delta$ defined above for the three processes and the two model here considered. We omit $\delta_{\mu\mu}$ for the ESM because it is negligible. From Fig. (5) we can see that in the ESM framework, and assuming that $N_S/\sqrt{N_B} \geq 5$ is necessary for detection, the $bb$ process is able to detect Higgs-resonances for $m_h$ up to $\sim 162$ GeV. Beyond this Higgs-mass, only the $WW$ process provide a $\delta_{WW}$ large enough to allow for detections for $m_h$ up to $\sim 180$ GeV. In the same way, Fig. (5) also shows that in the 3-3-1 model framework the $\mu\mu$ process is sensitive to detect Higgs-resonances for $m_h$ up to $\sim 167$ GeV, the $bb$ process up to $\sim 180$ GeV, and that the $WW$ is the only process able to investigate Higgs-resonances for $m_h$ in the heavy-mass range $m_h > 200$ GeV. Here we are using the FMC luminosity $L_{\mu\mu} = 10^4 \text{pb}^{-1} \text{yr}^{-1}$ so that in order to be compatible with the $N_S/\sqrt{N_B} \geq 5$ requirement, we must have $\delta \geq 5 \times 10^{-2} \text{pb}^{-1/2}$. This limit line $\delta_l$ is also showed in Fig. (5).

The values given above concerning the 3-3-1 model should be taken as indicative only once they depend on a set of parameters that are a priori free. In Tab. 1 we summarize the above results given $S$ and $B$, and the corresponding $N_S$ and $N_B$, and accuracies, for these representative Higgs-masses.

The existence of a Higgs-resonance far above the ESM-limit will indicate at least that that scalar no longer belongs to the minimal ESM and that new physics must be brought in. Although this resonance may exist in a multi-Higgs $S(U2)_L \otimes U(1)_Y$ model, this is only possible by extending the symmetry with an discrete one. On the other hand, in the 3-3-1 model the fact that the sextet couples only to leptons is just a consequence of the representation content in the lepton sector in the minimal model. Hence, the scalar resonance seems more natural in the last model than in the multi-Higgs extensions of the electroweak standard model.
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REFERENCES

[1] V. Barger, M. S. Berger, J. F. Gunion, and T. Han, Phys. Rev. Lett. 75, 1462 (1995); and Phys. Rep. 286, 1 (1997).

[2] F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992); P. Frampton, Phys. Rev. Lett. 69, 2889 (1992).

[3] B. Kamal, W. J. Marciano, and Z. Parsa, [hep-ph9712270].

[4] M. D. Tonasse, Phys. Lett. B381, 191 (1996).
Figure Captions

**Fig. 1** The HSEF $f_{WW}$ (continuous line), $f_{\mu\mu}$ (dashed line), and $f_{bb}$ (doted line) as a function of $v_3$ for $O_{31} = 0.18$, $O_{32} = 0.085$, $O_{33} = 0.98$, and for $v_1 = v_2 = \sqrt{(v_{ESM}^2 - v_3^2)/2}$.

**Fig. 2** $S$ for the 3-3-1 model (solid line) and for the ESM (dashed line) as a function of $\sqrt{s} = m_h$, and $B$ (doted line) as a function of $\sqrt{s}$ for the $WW$ process with $v_1 = v_2 = 173.8$ GeV, $v_3 = 10$ GeV, $O_{31} = 0.18$, $O_{32} = 0.085$, and $O_{33} = 0.98$.

**Fig. 3** $S$ for the 3-3-1 model (solid line) as a function of $\sqrt{s} = m_h$ and $B$ (doted line) as a function of $\sqrt{s}$ for the $\mu\mu$ process with $v_3 = 10$ GeV.

**Fig. 4** $S$ for the 3-3-1 model (continuous line), and for the ESM (dashed line) as a function of $\sqrt{s} = m_h$ and $B$ (doted line) as a function of $\sqrt{s}$ for the $bb$ process with $v_3 = 10$ GeV, $v_1 = 173.8$ GeV, $O_{33} = 0.98$, and $O_{13} = 0.18$.

**Fig. 5** The quantity $\delta = S/\sqrt{B}$ as a function of $m_h$ for the $WW$, $\mu\mu$, and $bb$ processes for the 3-3-1 model and the ESM, as indicated on the lines, for the same parameters we have used in Figs. 2–4. The limit-line $\delta_{\mu\mu} = 0.05$ pb$^{-1/2}$ for $N_S/\sqrt{N_B} = 5$ for $L_{\mu\mu} = 10^4$ pb$^{-1}$ yr$^{-1}$.
Fig. 1

$V_3$ (GeV)

$\sqrt{s}$ (GeV)

$\text{HSEF}$
Fig. 3

$\sigma \text{ (pb)}$

$m_h, \sqrt{s}$ (GeV)

S331

B

$10^{-2}$ $10^{-1}$ $10^0$ $10^1$ $10^2$ $10^3$ $10^4$
Fig. 4

$m_h$ vs $\sigma$ (GeV)

$\sigma$ (pb)

$10^{-3}$ $10^{-2}$ $10^{-1}$ $10^0$ $10^1$ $10^2$ $10^3$ $10^4$
TABLES

TABLE I. The quantities $S$, $B$, $N_S$, $N_B$, and the accuracies $N_S/\sqrt{N_B}$ and $\sqrt{N_S+N_B}/N_S$ for $\sqrt{s} = m_h$, and for the FMC luminosity.

| process  | $m_h$ (GeV) | $S$ (pb) | $N_S$ (events) | $B$ (pb) | $N_B$ (events) | $N_S/\sqrt{N_B}$ | $\sqrt{N_S+N_B}/N_S$ |
|----------|-------------|----------|----------------|----------|----------------|------------------|----------------------|
| bb-ESM   | 162         | 0.138    | 1380           | 6.219    | 62190          | 5.5              | 0.18                 |
| WW-ESM   | 180         | 0.265    | 2650           | 19.364   | 193640         | 6.0              | 0.16                 |
| $\mu\mu$-331 | 167     | 2.334    | 23340          | 1984.943 | 19849430       | 5.2              | 0.19                 |
| bb-331   | 180         | 0.123    | 1230           | 4.431    | 44310          | 5.8              | 0.17                 |
| WW-331   | 200         | 1.153    | 11530          | 19.263   | 192630         | 26.3             | 0.04                 |