Modelling and assessment of carbon fiber reinforced aluminum matrix composites and their laminate squeeze casting fabrication

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Keywords: aluminum matrix composite, carbon fiber reinforcement, laminate squeeze casting, equibiaxial bend behavior, analytical modelling, finite element modelling

Abstract
The equibiaxial bend behavior of laminate carbon fiber fabric reinforced aluminum matrix composites is modelled and assessed. Analytical modelling and finite element analysis are comparatively investigated to study the mechanical properties, with particular focus on the elastic modulus and flexural strength. The investigation allows evaluating how far the experimental results deviate from idealized assumptions of the models, which provides insight into the composite quality and the effectiveness of the used laminate squeeze casting technique. Specifically, discrepancies shed light on the interlaminate and fiber-matrix interface bond as well as on the stability of the laminate layers during fabrication. The two model approaches are in good agreement with differences below 8%. Moreover, the models agree with experimental data in predicting an overall improvement in properties with increasing carbon fiber content up to 4.89 vol%. Overall, the composite samples outperform the model predictions, which indicates good interface bonding. However, microstructure investigations also indicate that the outperformance is partly caused by a shifting of the carbon fibers during squeeze casting closer to the later bend tensile loaded surface due to their lower density compared to aluminum. The result is higher load bearing capacity of the composites than estimated by the models that assume perfectly symmetrical composite structure. The experimental outperformance in ultimate flexural strength vanishes at higher carbon fiber contents. This is due to imperfect interlaminate and fiber-matrix interfaces where some defects such as pores, carbides and oxide particles tend to locate, leading to damage initiation and potentially interface failure.

1. Introduction
The mechanical properties of continuous fiber reinforced composites can be varied using different carbon fiber types and arrangements. While unidirectionally oriented fibers provide the highest strength and stiffness performance in the fiber direction, bidirectionally and multi-directionally oriented fibers provide more balanced performance in the different room directions as required in many structural applications [1–3]. Experimentally, pin-on-ring bend testing is evolving as an efficient biaxial flexural test method used so far mostly for ceramic materials. From the modelling perspective, Kirstein et al derived elastic equations of pin-on-ring bending based on Bassali’s solutions [4]. Hsueh et al also suggested formulas that are suitable for the calculation of multilayer biaxial bending problems [5, 6]. Their approach is based on monolayer solutions and assumes perfect interface bonding between layers. Huang and Hsueh revisited the solutions and drew comparison with finite element modelling and experimental results [7]. They found good agreement for monolayer and multilayer dental ceramics. Moreover, Tsangarakis et al studied the biaxial flexural loading of silicon carbide fiber reinforced aluminum composites [8]. The relationship of displacement and load as well as the failure formulation under monotonic and cyclic loading were assessed.

Despite the considerable advances reported in existing literature, modelling remains a challenging aspect in the development and characterization of composites due to their complexity, particularly at dissimilar materials
interfaces [9, 10]. For continuous fiber fabric reinforcement, the model challenges can increase due to the bidirectional orientation of fibers and the resulting complex anisotropy [11, 12]. Therefore, the current study comparatively investigates two modelling approaches for assessing the stiffness and strength of bidirectional carbon fiber fabric reinforced laminate aluminum matrix composites. On the one hand, the analytical (ANA) model starts from the approach of Hsueh et al who assumed little to no plastic deformation as typical for ceramics. This study then expands the model in the following aspects: (1) woven fabric reinforcement results in anisotropic composite properties. Therefore, an effective elastic modulus is calculated. (2) The large plastic deformation undergone by the aluminum matrix in the high stress region is considered. On the other hand, finite element (FEA) modelling has evolved into a powerful tool for materials characterization and design [13, 14]. In this study, FEA is defined to consider the woven pattern and the non-linear effects in the composites. Pin-on-ring equibiaxial bend testing is conducted to experimentally validate the models. This allows shedding light on the composite quality and the efficiency of the used new laminate squeeze casting technique. Special focus is given to the stability and potential shifting of the carbon fiber laminate layers during fabrication. Moreover, as perfect interfaces are assumed by the models, the discrepancies between the model predictions and experimental results are expected to strongly depend on the level of interlaminate and fiber-matrix interface imperfections or defects.

2. Materials and methods

The models are applied on plain weave AS4 Hexcel carbon fiber fabric reinforced aluminum matrix composites fabricated by the laminate squeeze casting technique [15]. In this method, aluminum sheets and carbon fiber fabric are assembled in an alternating layered configuration. The composite laminate is then heated to 800 °C above the melting point of aluminum, and subjected to squeeze pressure during complete cooling to room temperature. The sample fabrication set-up is illustrated in figure 1. The fibers are PAN-based with 94.0% carbon content and 3000 (3K) filament count tows [1, 16]. Cylindrical disk samples with r3 = 38 mm radius and t = 4 mm total thickness are used. The composites are modelled as a sandwich material composed of two carbon fiber reinforced layers (CF) and three unreinforced aluminum layers (Al) in Al/CF/2Al/CF/Al alternating configuration as shown in figure 2(a). A monolayer sample with only the matrix is also modelled to calibrate the applied load by matching with the flexural bend strength of aluminum 6061 T4. For model validation, composite quality assessment and evaluation of the laminate squeeze casting technique, mechanical property data are measured experimentally using equibiaxial pin-on-ring bend tests.

The tests simulate two-dimensional loading of the composites as illustrated in figure 2(b). They are conducted and evaluated according to the ISO 6872 standard [17–19]. The pin (r1) and ring (r2) radii are 15.7 mm and 31.75 mm, respectively. A loading pin displacement speed of 10 mm min⁻¹ is used. The sample laminate configuration is modelled symmetric around the neutral bending plane located midway along the thickness, with the fiber fabric layers placed closer to top and bottom surfaces to optimize the bend resistance of the composites. This leaves the area close to the neutral plane practically unreinforced. Considering the sample bottom surface as the origin of the y axis (y = 0), the location yᵢ in the thickness direction of the top surface of each ith layer is given as in equation (1):

![Figure 1. Set-up of the laminate squeeze casting process for composite sample fabrication.](image-url)
Where \( t_i \) is the thickness of each individual layer, with: \( t_2 = t_4 = 0.2 \text{ mm} \) is the thickness of the two fiber fabric reinforced layers; \( t_3 = t_5 = 1 \text{ mm} \) is the thickness of the two external aluminum matrix layers; and \( t_3 = 2t_4 = 2 \text{ mm} \) is the thickness of the central aluminum matrix layer around the neutral plane.

The plain weave fiber fabric pattern unit within the reinforced layers is defined as previously suggested by Lee et al [12]. It is considered to be formed by interlacing warp and weft yarns with 1:1 ratio as seen in figures 3(a) and (b). For modelling, the cross-section of individual yarns is defined by the intersection of two identical circles of radius \( r \) equal to 14.17 mm as shown in figure 3(c). Further model geometry parameters are a yarn thickness \( t \) of 0.17 mm; a yarn width \( w \) of 3.10 mm; and a unit length \( L \) of 6.20 mm equal to 2 yarn widths. Different carbon fiber volume fractions are obtained by varying the spacing of parallel yarns in the 2 orthogonal fiber elongation directions. 0 yarn spacing corresponds to the original plain weave fabric with no gap between neighboring parallel yarns. This corresponds to the maximum investigated carbon fiber content as purchased from the supplier. Yarn spacings of up to 1 yarn width are achieved by removing warp and weft yarns at the edges of original fabric sections, and then spreading the remaining yarns to create lower yarn packing.

Within the carbon fiber reinforced layers (CF), there is a relatively high effective carbon fiber volume fraction \( V_{cf} \) of up to 63.31 vol% as approximated using equation (2) [12]:

\[
V_{cf} = \frac{V_y}{V_u^\kappa}
\]

Where \( V_y, V_u \) and \( \kappa \) are the total yarn volume in the carbon fiber reinforced layers, the layer volume and the yarn packing fraction, respectively. The yarn packing fraction \( \kappa \) represents the effective volume the fibers occupy within the yarn and is primarily related to the cross-section shape of the individual fibers [12]. In this study, \( \kappa \) is determined to be 0.43. This value is calculated by estimating the carbon fiber volume fraction in the entire composite \( V_f \) according to equation (3), and setting it equal to the experimentally measured carbon fiber volume fraction calculated using the respective masses of fibers and aluminum matrix within the fabricated composite samples, respectively.
Where $t_{cf}$ and $t_s$ are the carbon fiber reinforced layer thickness and total composite sample thickness, respectively.

### 2.1. Analytical (ANA) modelling

#### 2.1.1. Modelling the effective elastic properties of the carbon fiber reinforced layers

For the analytical (ANA) model, a unidirectional fiber reinforcement approach is applied on the two orthogonal $x$ and $z$ fabric fiber elongation directions. The carbon fibers are considered uniformly distributed within the reinforced layers. Transverse isotropy is assumed, meaning sections through the thickness normal to either the $x$ or the $z$ direction feature a homogeneous distribution of circular fiber cross-sections as shown in Figure 4 (a). In the illustrated case, the section normal to the $x$ direction shows isotropic properties in the transverse $yz$ plane with $y$ being the thickness direction of the composite. An identical configuration is obtained for sections normal to the $z$ direction by switching the roles of the $x$ and $z$ directions. Furthermore, each carbon fiber layer is taken as thin ply assuming plane stress conditions. As a result, the stresses along the sample thickness $y$ are set equal to 0 and neglected as in equation (4):

$$
\sigma_y = \tau_{yz} = \tau_{xy} = 0
$$

Considering the stress components in Figure 4 (b), the Hooke’s law of stiffness can then be characterized in terms of the following stress-strain relations:

$$
\varepsilon = [S] \sigma
$$

or

$$
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_z \\
\gamma_{xz}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_{fx}} & -\nu_{fxz} & 0 \\
-\nu_{fxz} & \frac{1}{E_{fz}} & 0 \\
0 & 0 & \frac{1}{G_{fz}}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_z \\
\tau_{xz}
\end{bmatrix}
$$

Where $[S]$ is the compliance matrix; $E_{fx}$ and $E_{fz}$ are the young’s moduli of the reinforced layers along the $x$ and $z$ axes, respectively; $\nu_{fxz}$ and $G_{fz}$ are the poisson’s ratio and shear modulus of the reinforced layers in the $xz$ plane, respectively; $\varepsilon_x, \varepsilon_z, \sigma_x, \sigma_z$ are the normal strains and stresses in the $x$ and $z$ directions, respectively; $\gamma_{xz}$ and $\tau_{xz}$ are the shear strain and stress in the $xz$ plane, respectively. The values of all elastic constants are estimated according to the rule of mixture [20] using the properties of the unreinforced aluminum matrix and carbon fiber as summarized in table 1 [16, 21, 22].

In the case of loading with the primary tensile stress in the $x$ fiber elongation direction as illustrated in Figure 4, the Voigt model is used for the longitudinal $x$ direction with uniform strain as described in equations (7) and (8); and the Reuss model is applied to the $z$ transverse direction with uniform stress according to equations (9) and (10).
Table 1. Mechanical properties of aluminum matrix and carbon fiber [16, 21, 22].

| Property          | Aluminum 6061 | AS4 Hexcel Carbon fiber |
|-------------------|---------------|-------------------------|
| $E_m$ (GPa)       | 68.9          | 231                     |
| $G_m$ (GPa)       | 26            | 40                      |
| $\nu_m$           | 0.33          | 0.2                     |
| $\sigma_{my}$ (MPa) | 145           | 4619                    |

Where $V_{cf}$ is the carbon fiber volume fraction within the reinforced layers; $E_f$ and $E_m$ are the longitudinal and radial elastic moduli of the carbon fibers, respectively; $\nu_{fxz}$ and $G_{fxz}$ are the poisson’s ratio and shear modulus of the fibers in the xz plane, respectively; $E_m$, $\nu_m$ and $G_m$ are the elastic modulus, poisson’s ratio and shear modulus of the aluminum matrix that is assumed isotropic.

The fiber geometry effect in the longitudinal fiber orientation direction is minimal as uniform strain is assumed. However, the volume occupied by the fibers, as well as stress concentrations due to their circular cross-section, can weaken the composites in the transverse direction. Therefore, they can considerably alter the transverse properties. For this reason, effective transverse modulus values are estimated for the fiber reinforced layers considering cylindrical fibers in square arrays using the Hopkins-Chamis model [23, 24] as illustrated in figure 5. Here, the transverse section of each basic unit is composed of two different regions: the unreinforced aluminum matrix with an elastic modulus $E_m$; and the central reinforced region containing the fiber cross-section with an effective elastic modulus $E_{T2}$. The circular fiber cross-section is then assumed to be equivalent to an effective square of equal area. Accordingly, the transverse modulus ($E_{T2}$) and the shear modulus ($G_{T2}$) of the fiber reinforced layers can be calculated in two steps: first, fiber and matrix in the central section are modelled as serial elements assuming uniform stress conditions as per the Reuss model. Second, the properties of the entire basic unit are obtained by adding up sections 1 and 2 as parallel elements under uniform strain conditions as per the Voigt model. The final relations relating the fiber volume fraction to the transversal elastic modulus and the shear modulus of the reinforced layers are then given by [23, 24]:

$$E_{T2} = E_m \left( 1 - \sqrt{V_f} + \frac{\sqrt{V_f}}{1 - (1 - E_m/E_f) \sqrt{V_f}} \right)$$

$$G_{T2} = G_m \left( 1 - \sqrt{V_f} + \frac{\sqrt{V_f}}{1 - (1 - G_m/G_{fxz}) \sqrt{V_f}} \right)$$

![Figure 5. Model of the composite transverse elastic modulus by the serial-parallel element approach.](image-url)
For the off-axis in-plane elastic moduli ($E'_{cfx}$ and $E'_{cfz}$) and Poisson’s ratio ($\nu'_{cfxz}$) in different $x'$ and $z'$ directions with varying orientation angles $\theta$ from the $x$ and $z$ fiber elongation directions, as seen in figure 6(c), the procedure described by Udhayaraman et al [24] is used. First, off-axis normal strain ($\varepsilon'_{x}$, $\varepsilon'_{z}$) and stress ($\sigma'_{x}$, $\sigma'_{z}$) tensors are calculated using coordinate transformation matrices as shown in equations (13) and (14). Second, $E'_{cfx}, E'_{cfz}$ and $\nu'_{cfxz}$ are calculated by substituting $\varepsilon'_{x}$, $\varepsilon'_{z}$, $\sigma'_{x}$ and $\sigma'_{z}$ for $\varepsilon_{x}$, $\varepsilon_{z}$, $\sigma_{x}$ and $\sigma_{z}$ in equation (6) derived earlier.

Assuming uniform deformation of the composites in all directions as well as constant shear modulus $G_{cfxz}$ within the $xz$ plane, and considering that the $x$ and $z$ orthogonal fiber elongation directions are equivalent in the plain weave fiber fabric, the effective elastic modulus and Poisson’s ratio of the fiber reinforced layers are then calculated by integrating over $\theta$ from $0$ to $\pi/2$, and then averaging as:

$$E_{cfx} = E_{cfx} = \frac{\int_{0}^{\pi/2} E'_{cfx}(\theta) d\theta + \int_{0}^{\pi/2} E'_{cfx}(\pi/2 - \theta) d\theta}{\pi}$$

(15)

$$\nu_{cfxz} = \frac{\int_{0}^{\pi/2} \nu'_{cfxz}(\theta) d\theta + \int_{0}^{\pi/2} \nu'_{cfxz}(\pi/2 - \theta) d\theta}{\pi}$$

(16)

2.2. Modelling of the composites as multilayer disks under pin-on-ring equibiaxial bending

Following the approach by Hsueh et al [5, 7, 25], the composite samples are modelled as material disks made of multiple layers of different properties. Assuming perfect interfaces, the stress ($\sigma$) -moment (M) relation is written as:

$$\sigma_i = \frac{E_i(y - y^*)M}{(1 - \nu_i)(1 + \nu_{ave})D^*}$$

(17)

Where $\sigma_i$, $E_i$, and $\nu_i$ are flexural stress, elastic modulus and Poisson’s ratio of the $i$th layer, respectively; $y$ is the layer’s position in the direction of the composite thickness and bend loading pin displacement direction; $D^*$ and $y^*$ are the flexural rigidity and neutral plane position, respectively; and $\nu_{ave}$ is the average Poisson’s ratio of the entire composite. $y'$, $D'$, $\nu_{ave}$ and $M$ are calculated as in equations (18)–(21), respectively.
Where $t_i$ and $y_i$ are the thickness and position of the $i$th layer as defined earlier, respectively. $P$ is the maximum bend force applied through the loading pin; $r_1$, $r_2$, and $r_3$ are the radii of loading pin, base ring, and specimen, respectively. It is noted that in the current model set-up of a composite disk with a total of 5 layers as defined earlier in table 1, and the elastic constants for layers 2 and 4 are as calculated earlier for the carbon reinforced layers. The effective Young's modulus $E_{eff}$ of the entire multilayer composite is then estimated as [5, 7, 25]:

$$E_{eff} = \frac{12(1 - \nu_{ave}^2)D^*}{t_s^3}$$  \hspace{1cm} (22)

Where $t_s$ is the sample total thickness.

For the flexural strength modelling, the aluminum matrix is assumed to exhibit ideal elastic-plastic behavior. Therefore, it is considered to be subjected to plastic deformation at constant yield strength ($\sigma_{my}$) across the entire composite thickness along the disk axis at failure point as illustrated in figure 7(a). The matrix yield strength is taken equal to that of aluminum alloy 6061 T4 as defined in table 1.

In contrast, the carbon fibers within the reinforced layers are assumed to deform purely linear elastically with varying stress ($\sigma_{fc}$) during complete bend loading to composite failure as seen in figure 7(b). As such, the elastic behavior of the composites at high bend loads is primarily contributed by the carbon fiber layers yielding a stress ($\sigma_{c}$) distribution across the composite thickness as illustrated in figure 7(c). The plasticity and deformation to fracture of aluminum are much larger than those of carbon fiber. Therefore, the failure criterion of the composites is based on carbon fiber fracture in the current case where substantial plastic deformations are observed during pin-on-ring bend testing. The failure stress in the reinforced layers ($\sigma_{ffail}$) is then calculated with the fibers subjected to their ultimate tensile strength ($\sigma_{UTS}$) while the surrounding aluminum matrix ideallastically deforms at its yield strength ($\sigma_{my}$). For this purpose, the rule of mixture is used as:

$$\sigma_{ffail} = V_f \sigma_{UTS} + (1 - V_f) \sigma_{my}$$  \hspace{1cm} (23)

Where $\sigma_{my} = 145$ MPa and $\sigma_{UTS} = 4619$ MPa as defined earlier in table 1.

To be able to validate the models by comparing them with experimental data, an effective flexural strength of the entire composites needs to be calculated using the same equations as for the conducted pin-on-ring flexural bend tests following applicable standards [17–19]. The approach is based on the linear elastic theory assuming purely elastic bending of a homogeneous material with the maximum bend stresses acting at the tensile loaded composite surface as follows: first, the failure bending moment of the multilayer composite ($M_{fail}$) upon fracture of the fibers under their ultimate tensile strength is assessed as:

$$M_{fail} = \frac{-P}{8\pi} \left\{ 1 + \nu_{ave} \left( 1 + 2 \ln \left( \frac{r_2}{r_1} \right) \right) \right\} + (1 - \nu_{ave}) \left\{ 1 - \frac{\eta^2}{2r_2^2} \right\} \frac{r_2^2}{r_1^2}$$  \hspace{1cm} (21)

$$D^* = \frac{\sum_{i=1}^{n} \left( \frac{E_i t_i}{1 - V_i^2} \right) \left( y_{i-1}^2 + y_{i-1} t_i + \frac{t_i^2}{3} - \left( y_{i-1}^2 + \frac{t_i^2}{2} \right) y^* \right)}{\sum_{i=1}^{n} \frac{E_i t_i}{1 - V_i^2} \left( y_{i-1}^2 + y_{i-1} t_i + \frac{t_i^2}{3} - \left( y_{i-1}^2 + \frac{t_i^2}{2} \right) y^* \right)}$$  \hspace{1cm} (19)

$$y^* = \sum_{i=1}^{n} \left( \frac{E_i t_i}{1 - V_i^2} \right) \left( y_{i-1}^2 + y_{i-1} t_i + \frac{t_i^2}{3} - \left( y_{i-1}^2 + \frac{t_i^2}{2} \right) y^* \right) \frac{v_{ave}}{Y_{i-1}^2}$$  \hspace{1cm} (18)
\[ M_{\text{par}} = M_m + M_{cf} \]  

(24)

Where \( M_m \) is the bending moment contribution of the aluminum matrix by plastic deformation at constant \( \sigma_{\text{my}} \); and \( M_{cf} \) is the bending moment contribution of carbon fibers subject to \( \sigma_{\text{UTS}} \). The bending moment equations (21) and (24) are then set equal and solved for the effective bending load \( P \) as:

\[
P = -8\pi M_{\text{par}} / \left\{ (1 + \nu_{\text{ave}}) \left( 1 + 2 \ln \left( \frac{r_2}{r_1} \right) \right) + (1 - \nu_{\text{ave}}) \left[ 1 - \frac{r_1^2}{2r_2^2} \left( \frac{r_2}{r_1} \right) \right] \right\}
\]

(25)

Finally, the effective flexural strength of the composites under pin-on-ring bend testing (\( \sigma_{\text{por}} \)) is calculated as per the ISO 6872 standard \[17\] as:

\[
\sigma_{\text{por}} = -0.2387P(X - Y) / t u^2
\]

(26)

Where:

\[
X = (1 + \nu) \ln \left( \frac{r_1}{r_3} \right)^2 + \left( 1 - \nu \right) / 2 \left( \frac{r_1}{r_3} \right)^2
\]

(27)

\[
Y = (1 + \nu) \left[ 1 + \ln \left( \frac{r_2}{r_3} \right)^2 \right] + (1 - \nu) \left( \frac{r_1}{r_3} \right)^2
\]

(28)

2.3. Finite element modelling

Finite element (FEA) modelling is conducted using the Abaqus/CAE software. In the first step, the elastic constants of the carbon fiber reinforced layers are modelled by representing them as periodic element units of the carbon fiber and their surrounding matrix similar to the analytical model as shown earlier in figure 3(b). 3D analysis is conducted using hexahedral mesh elements. A mesh size of 0.2 mm is used to fit one element within the carbon fiber strengthened layer thickness, which provided the best combination of result accuracy and computing time. In order to obtain the effective elastic properties, a uniform strain is applied to the structure with a specific value of 1 in different directions. As the plain weave fabric provides identical properties in the two orthogonal x and z fiber elongation directions, the determination of the elastic constants can be simplified by considering only 4 independent loading states as shown in figure 8. The loading and boundary conditions are specified by confining periodic element unit surfaces with no displacement in any other direction, except:

(a) For the determination of the elastic modulus \( E_{cfx} = E_{cfy} \) in the two orthogonal fiber elongation directions:

\[
X_{(x- \text{surface})} = \frac{X}{a}
\]

(29)

(b) For the determination of the transverse elastic modulus \( E_{cfy} \) in the composite thickness direction normal to the fiber axes:

\[
Y_{(y- \text{surface})} = \frac{Y}{b}
\]

(30)

(c) For the determination of the shear modulus \( G_{cfxy} = G_{cfxz} \) in the xy and yz planes:

\[
X_{(y- \text{surface})} = \frac{X}{a}
\]

(31)

(d) And for the determination of the shear modulus \( G_{cfxz} \) in the xz plane:

\[
X_{(z- \text{surface})} = \frac{X}{a}
\]

(32)

Where a, b and c are the periodic element unit size in the x, y and z directions, respectively; (a00), (0b0) and (00c) are the coordinates of the x-surface, y-surface and z-surface, respectively; X, Y and Z are the displacements of the respective surfaces along the x, y and z axes, respectively.

In the second FEA step, the aluminum matrix layers are added to the analysis and modelled using an element size of 1 mm. Hexahedral meshes and sweep meshing technique are used. Symmetric conditions are applied to optimize the computing efficiency. Thus, only a quarter of the sample set-up is modelled as shown in figure 9. In a third step, pin-on-ring bend loading is applied. For this purpose, partitions are created for the loading pin and...
the supporting ring. The loading force is assumed uniformly distributed on the contact surface between the sample and the loading pin, and the supporting ring is defined to restrict the sample displacement in the y direction only. The FEA results are analysed by tracking displacement, deformation and stress components at relevant locations.

Specifically, the bend displacement is taken at the center of the disk samples in the neutral plane as shown in figure 9(a), and the Von Mises stress distribution along the sample disk central axis is analysed along the thickness as illustrated in figure 9(b). To allow validation with experimental data, the effective flexural stresses in the composites are also estimated assuming purely elastic bending of a homogeneous material according to the
linear elastic theory as per equations (24) to (28). The obtained stress data are then used to develop stress-strain curves. Similar to the analytical (ANA) model, failure of the fiber reinforced layers upon reaching their ultimate tensile strength ($\sigma_{\text{cffail}}$), as defined earlier in equation (23), is set as FEA failure criterion of the composites, at which point the analysis is stopped and the corresponding maximum effective stress taken as the effective flexural strength ($\sigma_{\text{por}}$).

### 3. Results

Table 2 shows the elastic property predictions for the carbon fiber strengthened layers. In this table, the out-of-plane shear moduli ($G_{\text{cfxy}}, G_{\text{cfyz}}$) and poisson’s ratios ($\nu_{\text{cfxy}}, \nu_{\text{cfyz}}$) are not provided for the ANA model because it is based on plane stress conditions. Analytical and finite element models are in good agreement for the elastic ($E_{\text{cfx}} = E_{\text{cfz}}$) and shear ($G_{\text{cfxz}}$) moduli, with discrepancies below 7.3%. However, differences in in-plane poisson’s ratio ($\nu_{\text{cfxz}}$) are substantial and vary between 25% and 49%. This can be rationalized by simplifications introduced in the analytical model, primarily the assumption of plane stress conditions and traverse isotropy.

Illustratively, figure 10 shows the FEA Von Mises stress distribution within the strengthened layers for the carbon fiber concentration $V_{\text{cf}}$ of 63.13 vol%, corresponding to $V_1 = 4.89$ vol% in the overall composite.

![Figure 10. FEA Von Mises stress distribution in the strengthened layers illustrated at maximum pin-on-ring bend force for the highest investigated carbon fiber content $V_{\text{cf}} = 63.13$ vol%, corresponding to $V_1 = 4.89$ vol% in the overall composite.](image)

Table 2. Elastic properties of the strengthened layers calculated by the ANA and FEA models for different carbon fiber volume fractions.

| Properties | 31.53 vol% | 42.06 vol% | 63.13 vol% |
|------------|------------|------------|------------|
|            | ANA result | FEA result | ANA result | FEA result | ANA result | FEA result |
| $E_{\text{cfx}}, E_{\text{cfz}}$ (GPa) | 88.93 | 91.10 | 95.91 | 97.51 | 110.1 | 111.1 |
| $G_{\text{cfxy}}, G_{\text{cfyz}}$ (GPa) | — | 23.04 | — | 22.12 | — | 20.25 |
| $G_{\text{cfxz}}$ (GPa) | 25.94 | 24.05 | 25.94 | 23.42 | 22.00 | 22.18 |
| $\nu_{\text{cfxy}}, \nu_{\text{cfyz}}$ | — | 0.3424 | — | 0.3416 | — | 0.3060 |
| $\nu_{\text{cfxz}}$ | 0.2793 | 0.2108 | 0.2687 | 0.1660 | 0.2501 | 0.1273 |

High stresses can be seen to act on the tensile loaded, radially elongated fiber yarns, with the maximum stresses located at the intersection with orthogonal tangential yarns. In contrast, the tangentially oriented transverse yarns carry the least stresses, which can be expected from equibiaxial bend loading of circular disks.

Figure 11 shows the flexural stress-strain curves constructed using the finite element model results with the values at the top ends indicating the effective flexural strength for the different overall composite fiber contents of 2.44 vol%, 3.26 vol% and 4.89 vol%, corresponding to strengthened layer carbon fiber concentrations of 31.53 vol%, 42.06 vol% and 63.13 vol%, respectively. A significant strengthening effect of the fibers can be seen with the substantially higher strength of the composites compared to unreinforced aluminum.
However, the stress-strain curves also show similar elastic modulus and yield strength for unreinforced aluminum and all composites, with little dependence on the specific carbon fiber contents within the investigated range. This indicates that the aluminum matrix determines the mechanical behavior in the elastic region. Figure 12(b) shows the FEA stress distribution in the composite under maximum bend load illustrated on the composite sample with the overall carbon fiber concentration $V_f = 3.26\, \text{vol}\%$ in the composite, corresponding to $V_{cf} = 42.06\, \text{vol}\%$ in the reinforced layer. The chart in figure 12(a) shows the stress variation across the thickness along the disk sample central axis for both the ANA and the FEA models. Note that absolute values are used. Therefore, stresses on both the tensile and the compressive loaded composite sides are represented positive. It can be seen that Finite element (FEA) and analytical (ANA) models are in good agreement. The discrepancies between the two approaches are small below 8% and are primarily due to the three-dimensional strain state considered in the FEA model, while the ANA model is based on plane stress with no consideration of deformation effects. The constant maximum stress value of the FEA model in the carbon fiber strengthened layers is due to the mesh element size that is about equal to the layer thickness. In contrast, the local stresses continuously increase towards the bend sample surfaces across the strengthened layer thickness in the ANA model in accordance with continuum mechanics.

4. Validation and discussion

Figures 13(a) and (b) comparatively show the analytical and finite element modelling results together with experimental pin-on-ring bend test data for the effective flexural modulus and ultimate flexural strength, respectively. Overall, both models agree with experimental data by predicting a continuous increase in
mechanical properties with increasing carbon fiber content in the investigated range. For the flexural modulus however, the models consistently underestimate experimental results.

The discrepancies are relatively small, but increase with carbon fiber content from 1.5% to 8.5% and 0% to 6.9% for the analytical (ANA) model and finite element (FEA) model as the fiber volume fraction varies from 2.44 vol% to 4.89 vol%, respectively. For the flexural strength, the model underestimation is 7% and 16.14% at the lowest investigated composite carbon fiber content of 2.44 vol% for FEA and ANA, respectively. On the one hand, this experimental outperformance of the fabricated composites suggests that the interlaminate and fiber-matrix interfaces are of considerably good quality and do not constitute major weak points in the composites, which is a basic assumption of the models. On the other hand, the discrepancies can be rationalized by two main factors. First, microscopy cross-section analysis of the samples revealed the main reason for the discrepancy between models and experiment as the inaccurate positions of laminate layers within the fabricated composite disks. Similar shifts of carbon fibers were observed earlier by Li et al [26]. In the case of the current study, the carbon fibers float towards the top surface of the samples during squeeze casting due to their lower density (1.79 g cm$^{-3}$) compared to their surrounding molten aluminum (2.7 g cm$^{-3}$). Therefore, the strengthened layers are closer to the tensile loaded surface of the composites on the ring side where they can bear higher stresses during equibiaxial bend testing. This causes a shift in stress distribution and neutral plane position with the fibers contributing stronger to the composite strength. The result is higher than predicted flexural stiffness and strength as compared to the ideally symmetrical composite structure considered in the models. Second, the prediction models assume ideal elastic-plastic material behavior of the aluminum matrix that, however, shows considerable strain hardening as seen in figure 11. This strength contribution due to strain hardening of the matrix during plastic deformation is ignored by the models, partly causing their underestimation of the mechanical properties. For the ultimate flexural strength (UFS), the underestimation by the models vanishes towards higher carbon fiber contents where a larger scatter is observed in measured experimental data.

Microscopy analysis of the fabricated composite samples suggests remaining imperfections at interlaminate and
fiber-matrix interfaces as potential causes for both the UFS scatter and the discrepancies between models and experiment at higher carbon fiber volume fractions. In fact, good infiltration and interdiffusion between carbon fibers and aluminum matrix can result in strong interfaces\[27–30\].

In that ideal case, the interlaminate and fiber-matrix interface bond is close to the ideal assumption of the models, and crack growth or composite failure occur predominantly across both the matrix and the reinforcement fibers like in the absence of interfaces. However, no perfect interfaces are achieved in the fabricated composites. The use of solid-state aluminum sheets for the composite laminate fabrication as well as the low wettability and high temperature reactions between molten aluminum and carbon fibers are potential causes of interface defects. The tendency of the interface quality to deteriorate at higher fiber contents is due to increased friction and liquid flow resistance, leading to poorer infiltration, wetting and fiber-matrix bonding. Imperfections can include manufacturing defects such as oxides, porosity, carbide formation, residual stresses and other reaction products as shown in the scanning electron micrograph in figure 14 and reported in earlier literature\[27, 31, 32\]. Therefore, the interlaminate and fiber-matrix interfaces seem to constitute potential weak points\[33\] in the fabricated composites where damage and failure can initiate and concentrate, particularly at high loads levels where the ultimate flexural strength is measured. Further limitations of the models include: (a) the fiber distribution is assumed homogeneous within the reinforced layers, which may not reflect the experimental reality with anomalies such as fiber agglomerations; (b) the models consider the contributions of the orthogonal x and z fiber elongation directions to be fully independent, which does not capture their mutual interplay, particularly at fiber yarn intersections; (c) the accuracy of the used rule of mixture decreases at higher fiber contents; and (d) the proposed models introduce an improvement by considering plastic deformation in the overall inhomogeneous laminate composites composed of fiber and aluminum layers of different properties. In contrast, applicable standards\[17–19\] used for the calculation of the effective properties are based on purely elastic bending of a homogeneous material.

5. Conclusion

A good agreement is found between the analytical model (ANA), the finite element analysis (FEA) and experiment data. The following main conclusions can be drawn:

(1) The models moderately underestimate both flexural modulus and strength at low carbon fiber volume fractions and low loads. This is primarily due to the assumption of ideal elastic-plastic behavior of the aluminum matrix and floating of the carbon fibers during squeeze casting towards the later tensile bend loaded side of the composite disks. The result is a shift in neutral plane and a higher load bearing capacity than estimated with the ideal symmetrical structure assumed in the models.

(2) The experimental outperformance of the fabricated samples also suggests overall good quality and strength of interlaminate and fiber-matrix interfaces.
(3) The experimentally measured flexural strength shows a larger scatter around the model predictions at higher carbon fiber contents. This can be rationalised by remaining imperfections at interlaminate and fiber-matrix interfaces.

Acknowledgments

This research has been financially supported by Maxion Wheels Inc. [project number: 580199-180599-2112], and the National Science and Engineering Research Council of Canada [NSERC Discovery Grant number: 210487-180599-2001].

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