Design Improvements in Multi-sectional HFCG using Time-varying Skin Depth

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Abstract. Two-dimensional (2D) modelling procedure uses a filamentary-mesh approach and the dynamic matrix concept which are used in Helical Flux Compression Generator (HFCG) design. Although 2D modelling offer results which have been verified experimentally, yet none of the existing 2D-codes include the effect on skin depth that are caused from the high variation of current in very short duration of time. In most published codes, the formula that is used, provides simply a steady-state approximation of the skin depth. The influence of skin depth is represented approximately by using a static expression where frequency is assumed constant. This paper describes a new tactic by using a dynamic time-varying expression for skin depth, which is implemented for use with an existing 2D model. It is shown that effect of high change in current on skin effect is significant than those predicted by the constant skin depth formula.

1. Introduction

HFCG is a single-pulse high-power device, it converts the chemical energy available in High Explosive (HE) to mechanical energy which then gets converted in electrical energy, due to compression of the magnetic field. HFCG consists of an armature filled with explosive and surrounding it a helical coil which is connected with a load, it is provided with a seed source to generate the initial magnetic field, which get compressed once the armature explodes. HFCG can generate very high power and high current and they are compact in terms of mass and size, due to these advantages HFCG is widely used in many applications. These applications are in field of medical science e.g. tooth, eye and blood vessel treatment and X-ray etc, and in the civil world e.g. oil well exploration, radar etc. As these devices can be used only once, because they are completely destroyed during its operation. So, these are single shot devices in nature and hence they are widely used in defence applications.

The HFCG was independently envisaged by scientists at All-Soviet Scientific Institute of Experimental Physics (VNIIEF) in the Soviet Union and the Los Alamos National Laboratory (LANL) in the United States in early 1950s. To analyse the HFCG design, there are various approaches such as Marx IX Generator [1], which makes use of marxing technique which enables to employ capacitor bank energy of up to 1 MJ as a seed source without completely destroying the HFCG. The essential studies for the design of HFCG have been summarized in [2]. The design factors when HFCG is being energized by the capacitive discharge are summarized in [3]. The design optimization for end cover of the HFCG is presented in [4]. The calculation of losses in HFCG due to turn skipping is presented by Khanzade et al. in [5]. The calculation using 2-D code given in [6,7] which allows the elaborated calculation of
several phenomena. The validity criteria for the different solution models has been proposed by Anischenko et al. in [8]. There are various codes for the modelling and simulation of HFCG presented in various literature. Still only few codes are free to use or available commercially, and most of them are developed by the user himself at universities or research laboratories. These codes can be very complex, though for all codes, the circuit equations of generator are common but the calculation of inductance of generator and its variation with respect to time, are different.

The focus of this paper is on implementing the time varying skin depth with the existing two-dimensional modelling. During the operation of HFCG current varies rapidly in a very short duration. As all the existing two-dimensional models make use of the constant skin depth where frequency is assumed constant, hence the effect of high variation of current with respect to the time is neglected. In time-varying skin depth, the effect of high current variation with respect to time is incorporated, so here it will be implemented in existing two-dimensional model of multi-section HFCG.

2. Design Description
This model of multi-section HFCG has been chosen from the experiment conducted by Swedish Armed Forces, published by Appelgren et al., in 2008 [9]. The HFCG comprises of armature and helical coil and load as shown in Fig. 1. The specifications of the HFCG are given in Table 1.

![Fig. 1. Equivalent circuit of HFCG](image)

Table 1. Specification of HFCG in mm

| Specification                        | Section | 1  | 2  | 3  | 4  |
|--------------------------------------|---------|----|----|----|----|
| Length of each section in mm         |         | 84 | 55 | 48 | 50.1 |
| Number of helical coil turns          |         | 24 | 11 | 8  | 7  |
| Pitch of helical coil in mm          |         | 3.5| 5  | 6  | 7  |
| Width of conductor in mm             |         | 2  | 3  | 4  | 5  |
| Thickness of conductor               |         | 1.5| 1.5| 1.5| 1.5|
| Inner diameter in mm                 |         |    |    |    | 53 |
| Armature length in mm                |         |    |    |    | 330|
| Internal diameter of armature in mm  |         |    |    |    | 20 |
| External diameter of armature in mm  |         |    |    |    | 24 |
| Material of armature and conductor   |         |    |    |    | Copper |
| High explosive                       |         |    |    |    | C-4 |

In order to avert the electrical connection between the coil and armature throughout the FCG operation, the internal portion of last section of HFCG is shielded with the help of an epoxy layer with 1.25 mm thickness. Hence the last section acts as a load for the generator, as it will not short out. The shape of armature is cylindrical, with 24 mm outer diameter and 2 mm thickness and a length of 330 mm. The helical coil is of rectangular cross section with variable width and pitch for each section, it is composed of copper. The armature is stuffed with the C-4 explosive. Armature contains a conical part which coincides with last section of coil, and has a cone angle of 80 as shown in Fig. 2.
3. Working

In the starting, a capacitor charged with certain voltage initially, is connected across the armature and helical coil via load as illustrated in Fig. 1. The capacitor is charged so it also acts as a source to the circuit, here coil acts as an inductor, and hence a closed LC circuit is formed. Current starts rising due to the oscillation in the circuit and when it attains its peak value, the detonation starts at the one end of armature. The armature starts protruding out and becomes short circuited through the crowbar ring at the front end, this instant is known as crowbar time. Hence, capacitor gets separate from the circuit and flux compression starts. The magnetic flux which is existing amid the armature and the coil, gets compressed because of the reduction of coil turns. Generator’s inductance reduces radically, which leads to the increase of the current to a large value, to maintain the flux constant, the magnetic flux present amid the armature and the coil gets compressed because of the reduction in coil turn and space volume. Hence, there is a drastic reduction in the inductance of the generator which in turn increases the current to a very large value in order to maintain the flux constant.

4. Model Implementation

The 2D model is being applied here on the HFCG with multiple sections. Here, the coil is separated into nc number of rings, and armature is separated into nr no of rings, to denote the currents directed circularly flowing on the armature’s surface as shown in Fig. 3.

![Equivalent circuit for HFCG based on the application of 2d model](image)

The value of self and mutual inductances for these rings are evaluated discretely with the help of analytical expressions which is shown in [7], similarly the resistance value is calculated individually with the help of analytical expressions which is also shown in [7], and are represented in a matrix form as illustrated below.

\[
[L] = \begin{bmatrix}
L_{11}(t) & L_{12}(t) & \cdots & M_{1,n+1}(t) \\
L_{21}(t) & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
M_{n,1}(t) & \cdots & L_{n+1,n}(t)
\end{bmatrix}
\]
With the variation in time, each ring’s radius varies in regard to its axial position from the starting point of the armature. The values of radius are obtained with the help of following expression.

\[ r_{ar}(x,t) = r_{ar0} + (V_{ar0} - x)\tan\theta \quad \text{for} \quad r_{ar0} \leq r_{ar} \leq r_c \]  

Here,
- \( r_c \) is coil radius
- \( r_{ar0} \) is initial radius of armature
- \( x \) is axial position
- \( \theta \) is angle of expansion

The calculation of mutual inductance amid any two rings is done by

\[ M_{ij}(t) = 2\mu_0 \frac{r_i(t)r_j(t)}{k} \left[ 1 - \frac{m^2}{2} \right] K(m^2) - E(m^2) \]  

Here,
- \( r_i(t) \) is radius of \( i^{th} \) ring at time \( t \)
- \( r_j(t) \) is radius of \( j^{th} \) ring at time \( t \)

\[ m^2 = \frac{4r_i(t)r_j(t)}{(r_i(t) + r_j(t))^2 + d_{ij}^2} \]  

Here,
- \( d_{ij} \) is the axial distance between \( i^{th} \) and \( j^{th} \) rings
- \( K(m^2) \) is first type of elliptical integral
- \( E(m^2) \) is second type of elliptical integral

The value of self-inductance of armature’s \( i^{th} \) ring is calculated by

\[ L_i(r_i(t), h) = \mu_0 r_i \left( h \cdot \frac{8r_i(t)}{h} - 0.5 \right) \]  

Here,
- \( h \) is the axial length of the \( i^{th} \) armature ring

The calculation of the \( i^{th} \) coil ring’s self-inductance is done by using the formula as (6) however \( h \) is replaced by \( w \) (coil conductor’s width). Hence the self-inductance of the complete helical coil can be evaluated by (7)

\[ L_{cc}(t) = \sum_{i=1}^{n_c} \left( L_i(t) + \sum_{j=1}^{n_c} M_{ij}(t) \right) \]  

Here,
- \( n_c \) is the number of coil turns

The calculation of \( M_i(t) \) and \( L_i(t) \) is done by using (4) and (6) respectively.
The calculation of mutual inductance between $i^{th}$ armature ring with radius $r_{ari}$ and $j^{th}$ helical coil ring with radius $r_{cj}$ can be done as (8)

$$M_{ari, j}(t) = M_{i,j}(r_{ari}, r_{cj}, d_{acij}, t) \quad i = 1, 2, 3, \ldots, n_i$$

Here,

$d_{acij}$ is the axial distance between $i^{th}$ armature ring and $j^{th}$ coil rings.

The resistance of the $i^{th}$ ring of the armature or coil is obtained by

$$R_i(r_i, h) = \frac{2\pi \rho r_i}{\delta h}$$

4.1. Skin Effect

During the flow of an alternating current (AC) through a conductor, more current will be flowing within the conductor’s outer layer and less current will be flowing through the inner layer by nature. It is because flux linkages are less on conductor’s outer part and more in the inner part of conductor. This effect is characterised as Skin Effect. The depth at which current density is high is called as Skin Depth.

As the simulation has already been done using the constant skin depth for multi-section HFCG in [10]. Here to include the effect of high variation in current with variation in time, on the skin depth, time-varying skin depth is being used. It can be evaluated by using (11) as given in [11].

The constant skin depth $\delta$ is calculated by (10) as given in [12]

$$\delta = \frac{\rho}{\sqrt{\mu_0 \pi f}}$$

Here,

$\mu_0$ is magnetic permeability for free space

$\rho$ is resistivity of the conductor

$f$ is frequency

In the above formula, the average frequency obtained by applying FFT technique is taken as frequency.

For exponentially increasing field diffusion. The calculation of skin depth $\delta_{skin}$ can be done by (11) as given in [11].

$$\delta_{skin} = \sqrt{\frac{I}{\mu_0 \sigma_0 \frac{dI(t)}{dt}}}$$

Here,

$\mu_0$ is magnetic permeability for free space

$\sigma_0$ is conductor’s conductivity

$I(t)$ is instantaneous current

$\frac{dI(t)}{dt}$ is time derivative of instantaneous current

The calculation of helical coil’s resistance is done by (12) as illustrated below

$$R_{n+1} = \sum_{i=1}^{n} R_i(r_i, d)$$

Here,

$R_i$ is the resistance of helical coil’s $i^{th}$ ring and is evaluated by (9).
5. Simulation Method

Simulation starts once the capacitor bank has been isolated from the generator circuit at crowbar time. As shown in Fig. 3, the switch S will be closed at the crowbar time. Due to initial voltage of capacitor, there will be some initial current in the helical coil at the crowbar time, as capacitor is used as a seed current source. Here capacitor is charged with some initial voltage and it discharges through the inductor (coil), so some current starts flowing through this LC circuit, switch will be closed when that current has attained its peak, so that maximum seed current can be given as input.

Now with the help of Kirchhoff’s voltage law in the circuit as shown in Fig. 3, following equations can be obtained.

\[
\frac{dL_{11}I_1}{dt} + \frac{dM_{12}I_2}{dt} + \cdots + \frac{dM_{1n+1}I_{n+1}}{dt} + RI_1 = 0 \quad (13)
\]

\[
\frac{dM_{21}I_1}{dt} + \frac{dL_{22}I_2}{dt} + \cdots + \frac{dM_{2n+1}I_{n+1}}{dt} + RI_2 = 0 \quad (14)
\]

And in the end

\[
\frac{dM_{n+1,1}I_1}{dt} + \frac{dM_{n+1,2}I_2}{dt} + \cdots + \frac{dL_{n+1,n+1}I_{n+1}}{dt} + RI_{n+1} = 0 \quad (15)
\]

Above equations can be represented in matrix form as shown below.

\[
\frac{d[L][I]}{dt} + [R][I] = 0 \quad (16)
\]

Here,

- \([L]\) is the inductance matrix, diagonal elements of this matrix represent self-inductance values and the off-diagonal elements represent the mutual inductance values.
- \([R]\) is the resistance matrix, it contains only diagonal elements which represents the resistance of each ring.
- \([I]\) is the current matrix, it is a column matrix which contains the value of current in each ring.

A state vector is taken for solving these equations, and it is solved with the help of numerical methods. Here Runge-kutta method is being used. The product of \([I]\) and \([L]\) will provide flux and it will be taken as state vector and solved as shown below.

\[
[L][I] = [\phi] \quad (17)
\]

\[
\frac{d[\phi]}{dt} + [R][L]^{-1}[\phi] = 0 \quad (18)
\]

\[
\frac{d[\phi]}{dt} = -[R][L]^{-1}[\phi] \quad (19)
\]

The above state equations will be solved, with the help of ODE45 solver, available in MATLAB. The initialization of state vector \([\phi]\) will be done with the product \([L(0)][I(0)]\). Here \([I(0)]\) contains the seed current value, which is the current value at the crowbar time. It consists the seed current value as \((n+1)^{th}\) element, where as other values will be zero.

During the simulation, for every time step, the state equation will be solved. The time-step is given as duration of the explosion of single ring of armature. As the armature ring explodes, it gets short circuited with the helical coil, so based on the number of armature rings assumed, some turns of helical coil also get exploded. When the armature ring explodes, it will be discarded from the calculation of resistance and inductance matrix. The coil turns which explodes with the armature, will not be comprised in resistance and inductance evaluation.
Hence the first row and column of the resistance and inductance matrix will be removed at every time step, and these matrices will be dynamic in nature. The calculation of time step will be done by using the formula shown below

$$t_i = \frac{1}{V_{det}} \left( \frac{r_i - r_{ax0}}{\tan \theta} + x_i \right) \quad (20)$$

Here,
- $t_i$ is the time when the armature’s $i^{th}$ ring and the coil comes in contact.
- $x_i$ is the axial distance of the $i^{th}$ ring from the initial point of the armature.
- The calculation of time step is done by finding the difference between the $t_i$ and $t_{i-1}$.

6. Results

During the simulation, the instantaneous resistance and inductance matrices were found by using (1)-(8) and these matrices were solved in MATLAB using the ode45 solver for each time. The armature’s enlargement is presumed to be linear and in the shape of a cone with a $10^0$ cone angle. In order to include the effect of high variation of current with variation in time, on the calculation of skin depth, time-varying skin depth is being used as given by (11), instead of the constant skin depth given by (10), in which frequency is assumed constant but in the range of 100 kHz. The crowbar time is found to be $56.5 \mu$s and the seed current given to helical coil is found to be $11.2 \text{ kA}$. It is observed that the peak current found during simulation is almost equal to the experimental value.

Here the comparison between results obtained from constant skin depth and from time-varying skin depth has been shown in plots for various parameters of HFCG. It can be observed that the peak current value is $435.6 \text{ kA}$ when calculated using the constant skin depth and it is $480.4 \text{ kA}$ when calculated using the time-varying skin depth, as shown in Fig 4.

The $dI/dt$ is a very crucial parameter for the design consideration, it also affects the skin depth and other parameters of HFCG and determines the performance of the generator as shown in Fig 5.

The flux profile for the coil for constant and time-varying skin depth is shown in the Fig 6, the Inductance profile is similar for constant and time-varying skin depth as it is shown in Fig 7, and the Resistance profile of the coil for constant and time-varying skin depth are shown in the Fig 8.

![Fig. 4. The current profile of the Helical coil](image-url)
Fig. 5. The time derivative of the helical coil current

Fig. 6. The flux profile of the helical coil
Fig. 7. The Inductance profile of the helical coil

The flux value in the coil, when calculated using constant skin depth, varies from a maximum value of 0.3236 Wb and it decreases due to losses to 0.0928 Wb. While the flux value in the coil, when calculated using time-varying skin depth, varies from a maximum value of 0.3236 Wb and it decreases to 0.0977 Wb as shown in Fig 6. The flux loss occurs because of linear magnetic diffusion in the coil.

The time-varying skin depth has no effect on inductance as it varies from a maximum value of 28.8 \( \mu \)H to a minimum value of 2 \( \mu \)H as shown in Fig 7.

The total resistance of generator due to constant skin effect varies from a maximum value of 450.8 m\( \Omega \) to a minimum value of 183.7 m\( \Omega \), while the total resistance of generator due to time-varying skin effect varies from 515.9 m\( \Omega \) to 177.8 m\( \Omega \) as shown in Fig 8.

The variation of time-varying skin depth value and its comparison with constant skin depth value has been shown in Fig 9.
Fig. 8. The Resistance profile of the HFCG

Fig. 9. The skin depth of the helical coil
Table 2. Comparison of experimental current and current derivative of HFCG with simulated (constant and time-varying skin depth) results obtained by using 2D method

|                        | Experimental Value [9] | Simulated Value | Simulated Value |
|------------------------|------------------------|-----------------|-----------------|
|                        |                        | State equation approach (Constant skin depth [9]) | State equation approach (Time-varying skin depth) |
| Maximum current        | 436 kA, 87.7 μs        | 435.64 kA, 80.1 μs | 480.4 kA, 80.1 μs |
| Value of time derivative of maximum current | 174 GA/s, 87.1 μs | 130.36 GA/s, 78.8 μs | 147.3 GA/s, 78.8 μs |

Table 2 illustrates, the final results of simulation for constant skin depth and time-varying skin depth, a comparison with the experimental results for this multisectional HFCG, and it has been observed that the simulated result for time-varying skin depth have a bit high value when compare to the experimental values.

7. Conclusion
The implementation of time-varying skin depth within the existing 2D model has been done productively for a HFCG with multiple sections. Here state equations approach was used for simulation of HFCG, In the state equation approach, a state vector is assumed for a set of differential equations, which is solved with the help of ODE solver available in MATLAB for each time step. In this method the reduction of differential equations after each time step is incorporated by reducing the dimension of the resistance and inductance matrices. Magnetic diffusion was assumed to be linear, hence the resistivity is assumed to be constant. It is spotted that the simulation’s results by using constant and time-varying skin depth are close, but there is also a significantly small difference. Leakage flux, turn skipping losses and contact point resistance are not considered throughout the simulation. Certain important parameters for proper designing and material selection in HFCG, like resistance and inductance profile of coil and coil current derivative with respect to time, has been discussed.

References
[1] Fowler C M and Caird R S August 2002 The Mark IX Generator Proc. 7th IEEE Conf. on Pulse Power, Los Alamos National Laboratory (New Mexico) pp 475-478.
[2] Neuber A, Dickens J and Giesselmann M June 2000 Fundamental studies of a Helical Magnetic Flux Compression Generator 13th Int. Conf. on High Power Particle Beams (BEAMS 2000) (Nagaoka Japan) pp 329-332.
[3] Young A, Neuber A and Kristiansen M June 2011 Design Consideration for Flux-Trapping Helical Flux Compression Generators energized by Capacitive Discharge Proc. IEEE Conf. on Pulsed Power (Chicago IL USA) pp 1-5.
[4] Kumar V, Soni B and Gupta H K April 2014 Design Optimization of End Cover of an Explosive Driven Helical Flux Compression Generator through Explicit Numerical Simulations IEEE Conf. on Recent Advances in Engineering and Computational Sciences (RAECS) (Chandigarh Punjab India) pp 1-6.
[5] Khanzade M H, Beromi Y A and Shoulaie A February 2012 Calculation of Turn Skipping Losses in Helical Flux Compression Generators IEEE Transactions on Plasma Science (vol 40 no 2) pp 505-510.
[6] Anischenko S V, Bogdanovich P T and Gurinovich A A May 2017 2D Simulation of Helical Flux Compression Generator (Minsk Belarus : arXiv Publications) pp 1-6.
[7] Khanzade M H, Beromi Y A and Shoulaie A February 2010 Solution of the State Equations of a Helical Flux Compression Generator using Dynamics Matrix Concept Proc. 1st IEEE Conf. on Power Electronic and Drive Systems and Technologies pp 1-6.
[8] Anischenko S V, Bogdanovich P T and Gurinovich A A May 2018 Simulation of Helical Flux Compression Generator *IEEE Transactions on Plasma Science* (vol 46 no 5) pp 1859- 1863.

[9] Appelgren P 2008 Experiments with and modelling of Explosively Driven Magnetic Flux Compression Generators *Licentiate Dissertation Swedish Defence Research Agency* (Stockholm, Sweden) pp 68-92.

[10] Korikar H N, Chanana S and Bhatia R S June 2018 Two Dimensional Modelling of Multi-sectional Helical Flux Compression Generator using State equations approach 2nd Int. *IEEE Conf. on Energy, Power and Environment* (Shillong Meghalaya India).

[11] Brenning N, Hurtig T, Appelgren P and Novac B M October 2008 Modeling of a Small Helical Magnetic Flux-Compression Generator *IEEE Transactions on Plasma Science* (vol 36 no 5) pp 2662-2672.

[12] Altgilbers L L, Grishnaev I, Smith I R, Tkach Y, Brown M D J, Novac B M, and Tkach I 2000 *Magnetocumulative Generators* (New York NY: SpringerVerlag).