New physics interpretation of $W$-boson mass anomaly

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Abstract

The CDF collaboration has recently reported an updated result on the $W$-boson mass measurement, showing a $7\sigma$ deviation from the standard model prediction. The discrepancy may indicate new contributions to the Fermi coupling constant. We study simple extensions of the standard model by introducing an extra scalar, fermion or vector field. It is found that the tension implies the new physics existing in multi-TeV scales if the new coupling to the electron and/or muon is of order unity.
1 Introduction

The CDF collaboration has very recently reported an updated result of the W-boson mass \([1]\),

\[
M_W = (80.4335 \pm 0.0064_{\text{stat}} \pm 0.0069_{\text{syst}}) \text{ GeV} = 80.4335 \pm 0.0094 \text{ GeV}. \tag{1}
\]

The result shows a deviation at more than 7\(\sigma\) level from the Standard Model (SM) prediction,

\[
M_W = (80.3500 \pm 0.0056) \text{ GeV} \tag{2}
\]

By combining the experimental results of \(M_W\) from LEP 2, Tevatron \([1]\), LHC ATLAS \([3]\) and LHCb \([4]\), the averaged value is (see, e.g., Ref. \([2]\))

\[
M_W = 80.4133 \pm 0.0080 \text{ GeV},
\]

and the deviation becomes 6.5\(\sigma\). Although the CDF result has a tension with the previous experimental data as well as the SM prediction, if this discrepancy would be confirmed in future, it might be a sign of new physics beyond the SM. Implications of the discrepancy have been studied in Refs. \([2,5–32]\).

The SM prediction of the W-boson mass is determined by the electroweak precision observables (EWPO). The CDF discrepancy implies the following three possibilities in terms of the SM effective field theory (SMEFT \([33]\)); i) new physics contributions arising in the operator \((\phi^\dagger \sigma^a \phi W^a_{\mu \nu} B^\mu_{\nu})\), ii) those appearing in \((\phi^\dagger D^\mu \phi)((D^\mu \phi)\dagger \phi)\) and iii) those via the Fermi coupling constant. Here, \(\phi\) is the SM Higgs boson, \(W^a_{\mu \nu}\) \((B^\mu_{\nu})\) is the field strength of the SU(2)\(_L\) (U(1)\(_Y\)) gauge boson, and \(D^\mu\) is the covariant derivative. In other words, the contributions to the first and second operators are understood as new physics effects on the oblique \(S\) and \(T\) parameters, respectively. See, e.g., Refs. \([2,5,7–10,12,14,18–20,23–25,27,28,30,32]\) for such studies in light of the CDF result. Alternatively, we study the third possibility in this paper \([\#1]\).

The Fermi coupling constant \(G_F\) is determined precisely by measuring the muon decay to the electron, and receives new physics corrections of \((\phi^\dagger i \overline{\ell}^\gamma_{\mu} \ell^a_j)(\overline{\ell}^\gamma_{\mu} \ell^a_i)\) and \((\overline{\ell}^\gamma_{\mu} \ell^a_j)(\overline{\ell}^\gamma_{\mu} \ell^a_i)\), where \(\ell^a_i\) is a left-handed lepton in the \(i\)-th generation.

In this paper, we discuss new physics scenarios that affect \(G_F\). In particular, we consider simple extensions of the SM by introducing an extra scalar, fermion or vector field. Among them, scalar fields with a hypercharge 1 and with couplings to the SM leptons can contribute to \(G_F\) via the four-Fermi interactions \((\overline{\ell}^\gamma_{\mu} \ell^a_j)(\overline{\ell}^\gamma_{\mu} \ell^a_i)\). Also, extra vector bosons which have charged-current interactions with the SM leptons may mimic the SM W boson and contribute to \(G_F\). On the other hand, extra leptons which couple to the SM Higgs boson as well as the SM leptons induce SMEFT operators including \((\phi^\dagger i \overline{\ell}^\gamma_{\mu} \ell^a_j)\). Thus, these fields

\#1In Ref. \([15]\), the W-boson mass discrepancy is resolved by a shift in the Fermi constant caused by right-handed neutrinos.
may change the $W$-boson mass via $G_F$. In this paper, we also examine flavor-dependent contributions of new physics.

This paper is organized as follows. In Sections 2 we explain constraints from the EWPO and study the updated result on the $W$-boson mass measurement in the SMEFT framework. In Section 3 we investigate single-field extensions of the SM. Finally our conclusions are drawn in Section 4.

2 Electroweak precision observables

The EWPO including the $W$-boson mass $M_W$ receive contributions of new physics. If its energy scale is higher than the electroweak scale, they are represented in terms of higher dimensional operators of the SMEFT,

$$\mathcal{L}_{d>4} = \sum_i C_i \mathcal{O}_i,$$

where $C_i$ is a Wilson coefficient and $\mathcal{O}_i$ is a higher dimensional operator. The dimension-six operators relevant for $M_W$ in our scenarios are

$$(\mathcal{O}_{\phi}\phi^{(3)})_{ij} = (\phi^\dagger iD_\mu \phi)(\bar{\ell}_i \gamma^\mu \sigma^a \ell_j),$$

$$(\mathcal{O}_{\ell\ell})_{ijkl} = (\bar{\ell}_i \gamma_\mu \ell_j)(\bar{\ell}_k \gamma_\mu \ell_l),$$

where $\ell_i (e_{Rj})$ denotes the SU(2)$_L$ doublet (singlet) lepton in the $i$-th generation, and the derivatives mean

$$\phi^\dagger \overset{\leftrightarrow}{D_\mu} \phi = \phi^\dagger (D_\mu \phi) - (D_\mu \phi)^\dagger \phi, \quad \phi^\dagger \overset{\leftrightarrow}{\sigma^a} D_\mu \phi = \phi^\dagger \sigma^a (D_\mu \phi) - (D_\mu \phi)^\dagger \sigma^a \phi,$$

with the Pauli matrix $\sigma^a$. In addition, when fermion extensions of the SM are discussed in the next section, the EWPO and the leptonic decay of the Higgs boson are affected via the operators,

$$(\mathcal{O}_{\phi\ell^{(1)}})_{ij} = (\phi^\dagger i\overset{\leftrightarrow}{D_\mu} \phi)(\bar{\ell}_i \gamma^\mu \ell_j),$$

$$(\mathcal{O}_{e\phi})_{ij} = (\phi^\dagger \phi)(\bar{\ell}_i \phi e_{Rj}).$$

Let us define dimensionless coefficients as

$$\tilde{C}_i = v^2 C_i.$$ 

Here $v$ denotes the vacuum expectation value (VEV) of the Higgs field, $\phi = [0, (v + h)/\sqrt{2}]^T$, where the Nambu–Goldstone bosons are ignored.
The operator $O_{Φℓ}^{(3)}$ alters the charged-current interactions of leptons after the electroweak symmetry breaking, and the four-Fermi operator $O_{ℓℓ}$ contributes directly to the muon decay to the electron and neutrinos. Therefore, the measured value of $G_F$ from the decay is shifted from the SM prediction as

$$G_F = \frac{1}{\sqrt{2} v^2} (1 + \delta_{G_F}), \quad \delta_{G_F} = (\hat{C}_{Φℓ}^{(3)})_{11} + (\hat{C}_{Φℓ}^{(3)})_{22} - (\hat{C}_{ℓℓ})_{1221}. \quad (10)$$

Note that $(\hat{C}_{ℓℓ})_{1221} = (\hat{C}_{ℓℓ})_{2112}$. Then, the modification of $G_F$ affects the $W$-boson mass as

$$M_W = (M_W)_{SM} \left[ 1 - \frac{s_W^2}{2(c_W^2 - s_W^2)} \delta_{G_F} \right], \quad (11)$$

where $s_W$ and $c_W$ are the sine and cosine of the weak mixing angle. A quantity with the subscript “SM” denotes the SM prediction, which is calculated with the measured values of the input parameters $G_F$, $\alpha$, $M_Z$, etc. The $W$-boson partial widths also receive the corrections to $M_W$ and those to the charged-current couplings as

$$\Gamma(W^+ \to ℓ^+ γ) = \Gamma(W^+ \to ℓ^+ γ)_{SM} \left[ 1 - \frac{1 + c_W^2}{2(c_W^2 - s_W^2)} \delta_{G_F} + 2 (\hat{C}_{Φℓ}^{(3)})_{ii} \right],$$

$$\Gamma(W^+ \to ij) = \Gamma(W^+ \to ij)_{SM} \left[ 1 - \frac{1 + c_W^2}{2(c_W^2 - s_W^2)} \delta_{G_F} \right]. \quad (13)$$

where $ij$ in the second equation represents quark final states such as $d u$ and $s c$.

The operators $O_{Φℓ}^{(1)}$ and $O_{Φℓ}^{(3)}$ contribute to the neutral-current interactions of left-handed leptons. The $Z$-boson couplings to the SM fermions $f$ are written as

$$\mathcal{L}_Z = \frac{g}{c_W} \bar{f} γ^μ \left[ (T_{L}^{3} - Q s_W^2 + δ g_L) P_L + (T_{R}^{3} - Q s_W^2 + δ g_R) P_R \right] f Z^μ, \quad (14)$$

where $T_{L,R}^3$ and $Q$ are the weak isospin and the electric charge of $f$. The new physics contributions are obtained as

$$δ g_L = \begin{cases} -\frac{1}{2} \left[ T_L^3 + \frac{Q s_W^2}{c_W^2 - s_W^2} \right] δ_{G_F} - \frac{1}{2} (\hat{C}_{Φℓ}^{(1)})_{ii} + (\hat{C}_{Φℓ}^{(3)})_{ii} & \text{for } f = ℓ, ν, \\
-\frac{1}{2} \left[ T_L^3 + \frac{Q s_W^2}{c_W^2 - s_W^2} \right] δ_{G_F} & \text{otherwise,} \end{cases} \quad (15)$$

$$δ g_R = -\frac{Q s_W^2}{2(c_W^2 - s_W^2)} δ_{G_F}. \quad (16)$$

#2 Here, we omitted contributions from $O_{ΦWB} = (Φ^+ σ^σ_Φ$ $W_{μν} B^{νµ}$) and $O_{ΦD} = (Φ^+ D_μ φ)(D^µ φ)^†$, which can be taken into account by replacing $δ_{G_F} \to δ_{G_F} + \frac{s_W^2}{c_W^2 - s_W^2} \hat{C}_{ΦWB} + \frac{c_W^2}{2 s_W^2} \hat{C}_{ΦD}$ in Eq. (11).
Table 1: Experimental measurement of the SM input parameters and EWPO.

| Measurement | Ref. | Measurement | Ref. |
|-------------|------|-------------|------|
| $\alpha_s(M_Z^2)$ | 0.1177 ± 0.0010 | $M_Z$ [GeV] | 91.1876 ± 0.0021 |
| $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ | 0.02766 ± 0.00010 | $\Gamma_Z$ [GeV] | 2.4955 ± 0.0023 |
| $m_t$ [GeV] | 171.79 ± 0.38 | $\sigma_h^0$ [nb] | 41.4807 ± 0.0325 |
| $m_h$ [GeV] | 125.21 ± 0.12 | $R^0_\mu$ | 20.8038 ± 0.0497 |
| $M_W$ [GeV] | 80.4133 ± 0.0080 | $R^0_\tau$ | 20.7842 ± 0.0335 |
| $\mathcal{B}(W \to e\nu)$ | 0.1071 ± 0.0016 | $A^0_{FB}$ | 0.0145 ± 0.0025 |
| $\mathcal{B}(W \to \mu\nu)$ | 0.1063 ± 0.0015 | $A^0_{FB}$ | 0.0169 ± 0.0013 |
| $\mathcal{B}(W \to \tau\nu)$ | 0.1138 ± 0.002 | $A^{0,\tau}_{FB}$ | 0.0188 ± 0.0017 |
| $R(\tau/\mu)$ | 0.992 ± 0.013 | $R^0_\mu$ | 0.21629 ± 0.00066 |
| $A_e$ (SLD) | 0.1516 ± 0.0021 | $R^0_\tau$ | 0.1721 ± 0.0030 |
| $A_\mu$ (SLD) | 0.142 ± 0.015 | $A^{0,b}_{FB}$ | 0.0996 ± 0.0016 |
| $A_\tau$ (SLD) | 0.136 ± 0.015 | $A^{0,c}_{FB}$ | 0.0707 ± 0.0035 |
| $A_e$ (LEP) | 0.1498 ± 0.0049 | $A_6$ | 0.923 ± 0.020 |
| $A_\tau$ (LEP) | 0.1439 ± 0.0043 | $A_c$ | 0.670 ± 0.027 |

The $Z$-boson observables in Table 1 are represented in terms of these effective $Zfff$ couplings (see, e.g., Ref. [34]).

We perform a Bayesian fit of the SMEFT operators to the experimental data of the EWPO [35][37]. The analysis utilizes the HEPfit v1.0 package [38], which is based on the Markov Chain Monte Carlo provided by the Bayesian Analysis Toolkit (BAT) [39]. The full two-loop electroweak corrections are included for the SM contributions to $M_W$ and the $Z$-boson observables [40][42], while the $W$-boson widths are calculated at one-loop level [43][44]. Additionally new physics contributions are implemented to the package for the current work. Theoretical uncertainties from missing higher-order corrections in the SM are included only for the $W$ mass as $\delta_{\text{th}}M_W = 0 \pm 4$ MeV [40] assuming the Gaussian distribution, while those for the other observables are not significant [45] and are neglected from the fit. The input values necessary for this study are summarized in Table 1 where the the choice of $\alpha_s(M_Z^2)$, $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ and $m_t$ is followed by those in Refs. [2][46]. In the following analysis, the parameters $G_F$, $\alpha$ and the light fermion masses are fixed to be constants.

As shown in Eq. (10), the Fermi coupling constant is affected by $(C_{\phi\ell}^{(3)})_{11,22}$ and $(C_{\phi\ell}^{(3)})_{1221}$. In Figs. 1 and 2, we show the probability distributions for $(C_{\phi\ell}^{(3)})_{1221}$ and $(C_{\phi\ell}^{(3)})_{ii}$, respectively.
Figure 1: Probability distribution for \((C_{\ell\ell})_{1221}\) in units of TeV\(^{-2}\) obtained from a fit to the EWPO, where the darker (lighter) region corresponds to the 68\% (95\%) probability.

The horizontal axes are shown in units of TeV\(^{-2}\). It is noticed from Eq. (10) that \(G_F\) depends on \(C^{(3)}_{\phi \ell}\) in a combination of \((C^{(3)}_{\phi \ell})_{11} + (C^{(3)}_{\phi \ell})_{22}\). According to Eqs. (12) and (15), these Wilson coefficients also affect the \(Z\)-observables and leptonic decays of the \(W\)-boson directly. Hence we performed the analysis in a flavor-dependent way. For the top two plots in Fig. 2, only a single operator for \((C^{(3)}_{\phi \ell})_{11}\) or \((C^{(3)}_{\phi \ell})_{22}\) is switched on, while the lepton-flavor universality is assumed for the bottom-left plot: \((C^{(3)}_{\phi \ell})_{\text{univ}} \equiv (C^{(3)}_{\phi \ell})_{11} = (C^{(3)}_{\phi \ell})_{22} = (C^{(3)}_{\phi \ell})_{33}\). In the bottom-right plot, the two coefficients \((C^{(3)}_{\phi \ell})_{11}\) and \((C^{(3)}_{\phi \ell})_{22}\) are fitted simultaneously.

From the figures it is found that the Wilson coefficient is implied to take a positive value for \((C_{\ell\ell})_{1221}\), while \((C^{(3)}_{\phi \ell})_{\text{univ}} < 0\) is favored to relax the \(W\)-boson mass discrepancy. The numerical values of the 68\% and 95\% probability ranges are summarized in Tables 2 and 3. As will be discussed in the next section, these results are useful to discriminate the models.

Table 4 shows pull values for the individual parameters and observables in each fit together with the values of the information criterion (IC) defined by

\[
IC = -2 \ln \mathbb{L} + 4 \sigma_{\ln \mathbb{L}}^2,
\]

where \(\ln \mathbb{L}\) and \(\sigma_{\ln \mathbb{L}}^2\) are the posterior mean and the variance of the log-likelihood distribution, respectively. The IC is a measure of relative goodness of fit; preferred scenarios give smaller IC values. It is found that all the SMEFT scenarios considered in Figs. 1 and 2 give smaller IC values compared to the SM, and therefore we conclude that they are preferred over the SM. The quality of each SMEFT fit is similar to each other. In those scenarios the \(W\)-boson mass discrepancy is relaxed by the new physics contributions to \((C^{(3)}_{\phi \ell})_{11,22}\) or \((C_{\ell\ell})_{1221}\), but the pull values for \(M_W\) cannot be smaller than the two sigma level due to the increases of the pull for other observables. Especially, in all the scenarios, the larger pulls
for $M_W$ and $A_e$ (SLD) observed in the SM fit are reduced by the new physics contributions, but that for $A_{FB}^{0,b}$ is worsened at the same time.

### 3 New physics interpretation

Let us investigate new physics scenarios in light of the $W$-boson mass discrepancy. The CDF result may indicate extra contributions to the Fermi coupling constant via $(C_{\phi\ell}^{(3)})_{11,22}$ and/or $(C_{\ell\ell})_{1221}$. We consider simple extensions of the SM, i.e., introduce an extra single scalar, fermion, or vector field at a time. We assume that their masses are much larger than the Higgs VEV, $v$. The complete list of the Wilson coefficients of the dimension-six SMEFT operators induced by general single scalar, fermion and vector field is provided at
regions the W

In this section, we will show by which fields and in which parameter
can contribute to (the tree level in Ref. [51]. According to the reference, it is found that there are six types
are likely to generate too large neutrino masses [52–56].

We will ignore renormalization-group corrections to the SMEFT operators. Their effects are expected
to be insignificant, because the particles do not carry color charges.

| Coefficient | 68% prob. range | 95% prob. range |
|-------------|-----------------|-----------------|
| \( (C_{\ell\ell})_{121} \) | [0.018, 0.030] | [0.013, 0.036] |
| \( (C_{\ell\ell})_{11} \) | [-0.027, -0.017] | [-0.032, -0.012] |
| \( (C_{\ell\ell})_{22} \) | [-0.028, -0.018] | [-0.034, -0.012] |
| \( (C_{\ell\ell})_{\text{univ}} \) | [-0.021, -0.014] | [-0.024, -0.011] |

\[ |g_{\Xi_1}\Phi_{\ell}^{(3)}| \leq 0.14, 0.17 \]
\[ |g_{W}\Phi_{\ell}^{(3)}| \leq 0.38, 0.49 \]
\[ |(\lambda_E)_{11}| \leq 0.13, 0.20 \]
\[ |(\lambda_E)_{22}| \leq 0.23, 0.30 \]
\[ |(\lambda_E)_{\text{univ}}| \geq 0.17, 0.22 \]

Table 2: Fit results for the SMEFT coefficients in units of TeV$^{-2}$ and for the new couplings
at 68% and 95% probability ranges. Here \( \mathcal{G}_{W} = \sqrt{-g_{WW}(g_{WW})_{22}} \) and \( M_{\Sigma_{1,WW}} = 1 \) TeV.

In the SMEFT fits, only a single operator is switched on at a time, while the lepton-flavor
universality is assumed for \( (C_{\ell\ell'})_{\text{univ}} \) and \( (\lambda_E)_{\text{univ}} \), i.e., \( (C_{\ell\ell'})_{\text{univ}} \equiv (C_{\ell\ell'})_{11} = (C_{\ell\ell'})_{22} = (C_{\ell\ell'})_{33} \) and \( (\lambda_E)_{\text{univ}} \equiv (\lambda_E)_1 = (\lambda_E)_2 = (\lambda_E)_3 \).

| Coefficient | 68% prob. range | 95% prob. range |
|-------------|-----------------|-----------------|
| \( (C_{\ell\ell})_{11} \) | [-0.022, -0.011] | [-0.027, -0.005] |
| \( (C_{\ell\ell})_{22} \) | [-0.023, -0.011] | [-0.028, -0.005] |
| \( |(\lambda_E)_{1}| \) | [0.13, 0.20] | [0.07, 0.23] |
| \( |(\lambda_E)_{2}| \) | [0.23, 0.29] | [0.18, 0.32] |

Table 3: Same as Table 2, but for the fits with the two coefficients \( (C_{\ell\ell})_{11} \) and \( (C_{\ell\ell})_{22} \), or \( |(\lambda_E)_{1}| \) and \( |(\lambda_E)_{2}| \).

the tree level in Ref. [51]. According to the reference, it is found that there are six types
of fields which can contribute to \( (C_{\ell\ell})_{11,22} \) and/or \( (C_{\ell\ell})_{1221} \). Their quantum numbers are summarized in Table 5. In this section, we will show by which fields and in which parameter
regions the W-boson discrepancy is relaxed.

#3 We will not consider a gauge singlet \( N \sim (1, 1)_0 \) and an SU(2)$_L$ adjoint lepton \( \Sigma \sim (1, 3)_0 \), because they are likely to generate too large neutrino masses [52, 56].

#4 We will ignore renormalization-group corrections to the SMEFT operators. Their effects are expected
to be insignificant, because the particles do not carry color charges.
| \( IC \) | \( C_{\ell\ell} \) | \( C_{\ell1}^{(3)} \) | \( |\lambda_E| \) |
|-------|-----------------|-----------------|----------------|
|       | SM              | 1221            | 11 22          | 11 22          | 1 2 | 1 2 |
| \( \alpha_c(M^2_Z) \) | –0.1            | 0.1             | 0.5 0.2        | 0.5 0.5        | 0.3 | 0.3 | 0.6 | 0.6 |
| \( \Delta \alpha_{\text{had}}(M^2_Z) \) | 0.9             | 0.2             | 0.4 0.2        | 0.0 0.1        | 0.8 | 0.4 | 0.5 | 0.3 |
| \( m_t \) | –1.1            | –0.5            | –0.7 –0.6      | –0.4 –0.4      | –1.0 | –0.7 | –0.8 | –0.6 |
| \( m_h \) | 0.0             | 0.0             | 0.0 0.0        | 0.0 0.0        | 0.0 | 0.0 | 0.0 | 0.0 |
| \( M_W \) | 4.6             | 2.9             | 2.9 3.0        | 2.1 2.2        | 4.0 | 3.4 | 3.1 | 2.9 |
| \( \delta_{\text{had}} M_W \) | –2.0            | –1.3            | –1.3 –1.3      | –1.0 –1.0      | –1.7 | –1.5 | –1.4 | –1.3 |
| \( \Gamma_W \) | –0.1            | –0.2            | –0.2 –0.2      | –0.2 –0.2      | –0.1 | –0.2 | –0.1 | –0.2 |
| \( B(W \to e\nu) \) | –0.8            | –0.8            | –0.7 –0.8      | –0.7 –0.7      | –0.8 | –0.8 | –0.8 | –0.8 |
| \( B(W \to \mu\nu) \) | –1.4            | –1.4            | –1.4 –1.2      | –1.3 –1.3      | –1.4 | –1.3 | –1.3 | –1.3 |
| \( B(W \to \tau\nu) \) | 2.6             | 2.6             | 2.6 2.6        | 2.6 2.6        | 2.6 | 2.6 | 2.6 | 2.6 |
| \( R(\tau/\mu) \) | –0.6            | –0.6            | –0.6 –0.8      | –0.6 –0.8      | –0.6 | –0.8 | –0.6 | –0.8 |
| \( \mathcal{A}_c \) (SLD) | 2.0             | 0.4             | 1.7 0.5        | 0.6 0.7        | 2.2 | 0.9 | 1.7 | 1.1 |
| \( \mathcal{A}_\mu \) (SLD) | –0.4            | –0.6            | –0.6 –0.4      | –0.5 –0.5      | –0.4 | –0.3 | –0.4 | –0.3 |
| \( \mathcal{A}_\tau \) (SLD) | –0.8            | –1.0            | –1.0 –1.0      | –0.9 –1.1      | –0.8 | –0.9 | –0.8 | –1.0 |
| \( \mathcal{A}_c \) (LEP) | 0.5             | –0.2            | 0.4 –0.1       | –0.1 –0.1      | 0.6 | 0.0 | 0.4 | 0.1 |
| \( \mathcal{A}_\tau \) (LEP) | –0.8            | –1.5            | –1.5 –1.5      | –1.5 –1.8      | –1.0 | –1.3 | –0.9 | –1.5 |
| \( M_Z \) | –1.2            | –0.4            | –0.7 –0.5      | –0.3 –0.3      | –1.1 | –0.6 | –0.8 | –0.6 |
| \( \Gamma_Z \) | 0.4             | –1.4            | –0.9 –1.0      | –1.4 –1.5      | –0.0 | –0.7 | –0.7 | –1.0 |
| \( \sigma^0_h \) | –0.2            | –0.3            | 2.2 –1.0       | 0.8 1.0        | 1.4 | –0.7 | 1.8 | 0.9 |
| \( R_e^0 \) | 1.4             | 1.3             | 0.2 1.3        | 0.4 0.5        | 0.7 | 1.4 | 0.4 | 0.7 |
| \( R_\mu^0 \) | 1.5             | 1.3             | 1.4 –0.3       | 0.0 0.1        | 1.5 | –0.9 | 0.1 | –0.9 |
| \( R_\tau^0 \) | –0.3            | –0.5            | –0.4 –0.4      | –1.4 –0.5      | –0.3 | –0.4 | –1.4 | –0.4 |
| \( \mathcal{A}_{FB}^{e} \) | –0.7            | –1.0            | –0.8 –1.0      | –1.0 –1.0      | –0.7 | –0.9 | –0.8 | –0.9 |
| \( \mathcal{A}_{FB}^{\mu} \) | 0.5             | –0.1            | 0.1 0.2        | –0.0 0.0       | 0.4 | 0.4 | 0.4 | 0.4 |
| \( \mathcal{A}_{FB}^{\tau} \) | 1.5             | 1.0             | 1.2 1.1        | 1.1 1.0        | 1.4 | 1.2 | 1.4 | 1.2 |
| \( R_b^0 \) | 0.6             | 0.6             | 0.6 0.6        | 0.6 0.6        | 0.6 | 0.6 | 0.6 | 0.6 |
| \( R_\ell^0 \) | –0.0            | –0.0            | –0.0 –0.0      | –0.0 –0.0      | –0.0 | –0.0 | –0.0 | –0.0 |
| \( \mathcal{A}_{FB}^{b} \) | –2.3            | –3.5            | –2.6 –3.5      | –3.5 –3.4      | –2.1 | –3.2 | –2.6 | –3.0 |
| \( \mathcal{A}_{FB}^{c} \) | –0.9            | –1.4            | –1.0 –1.4      | –1.4 –1.3      | –0.8 | –1.2 | –1.0 | –1.2 |
| \( \mathcal{A}_b \) | –0.6            | –0.6            | –0.6 –0.6      | –0.6 –0.6      | –0.6 | –0.6 | –0.6 | –0.6 |
| \( \mathcal{A}_c \) | 0.1             | 0.0             | 0.0 0.0        | –0.0 0.0       | 0.1 | 0.0 | 0.0 | 0.0 |

Table 4: \( IC \) value in each fit and pull for the difference between the measurement and the fit result in units of standard deviation.
### Table 5: Quantum numbers of the particles considered in this study, where the hypercharge is normalized as $Y = Q - T^3_L$.

|       | $S_1$ | $\Xi_1$ | $E$ | $\Sigma_1$ | $B$ | $W$ |
|-------|-------|---------|-----|-------------|-----|-----|
| Spin  | 0     | 0       | 1/2 | 1/2         | 1   | 1   |
| $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ | $(1, 1)_1$ | $(1, 3)_1$ | $(1, 1)_{-1}$ | $(1, 3)_{-1}$ | $(1, 1)_0$ | $(1, 3)_0$ |

#### 3.1 Scalar extension

The $W$-boson mass can be modified by complex scalar fields, $S_1$ and $\Xi_1$, via $(C_{\ell\ell})_{1221}$. They have Yukawa interactions with the SM (left-handed) leptons as

$$-L_{\text{int}} = (y_{S_1})_{ij} S^*_1 \tilde{\ell}_i \sigma^2 \ell^c_j + (y_{\Xi_1})_{ij} \Xi^*_1 \tilde{\ell}_i \sigma^3 \ell^c_j + \text{h.c.}, \quad (18)$$

where $c$ denotes the charge conjugation. Here, $(y_{S_1})_{ij}$ and $(y_{\Xi_1})_{ij}$ is anti-symmetric (symmetric) under $i \leftrightarrow j$ [57]. Since we are interested in the single-field extension of the SM, $(y_{S_1})_{ij} = 0$ is set when we focus on $S_1$ ($\Xi_1$). The SMEFT operators receive corrections by exchanging the scalar bosons. If we focus on the above Yukawa interactions, only the Wilson coefficient $C_{\ell\ell}$ among the SMEFT operators is shifted as [51][58]

$$\left( C_{\ell\ell} \right)_{ijkl} = \frac{(y_{S_1})^*_{ij} (y_{S_1})_{ik}}{M^2_{S_1}} + \frac{(y_{\Xi_1})^*_{kj} (y_{\Xi_1})_{lj}}{M^2_{\Xi_1}}, \quad (19)$$

at the tree level. Here $M_{S_1}$ and $M_{\Xi_1}$ are the masses of $S_1$ and $\Xi_1$, respectively. Although $\Xi_1$ can also have interactions with the SM Higgs boson, they are generically independent of the above Yukawa interaction (18) and irrelevant for $G_F$. Hence, we neglect them in the following analysis.

The corrections to the Fermi coupling constant correspond to $(C_{\ell\ell})_{ijkl}$ with $\{i, j, k, l\} = \{1, 2, 2, 1\}$ (see Eq. [10]). Because of the anti-symmetric or symmetric structure of the Yukawa coupling, the Wilson coefficient becomes

$$\left( C_{\ell\ell} \right)_{1221} = -\frac{|(y_{S_1})_{12}|^2}{M^2_{S_1}} + \frac{|(y_{\Xi_1})_{12}|^2}{M^2_{\Xi_1}}, \quad (20)$$

From Fig. 1, it is found that the $W$-boson mass discrepancy favors a positive value for $(C_{\ell\ell})_{1221}$. Hence, only $\Xi_1$ can be a source of the anomaly. As a result, the couplings and masses are favored to be within the range,

$$0.14 < \frac{|(y_{\Xi_1})_{12}|}{M_{\Xi_1}} < 0.17 \text{ TeV}^{-1}. \quad (68\%) \quad (21)$$
This result implies that the new physics scale is around 6–7 TeV for \((y_{\Xi_1})_{12} \sim 1\). The pull of \(M_W\) is the same as that for \((C_{\ell\ell})_{1221}\) because the scalar bosons contribute to the EWPO only via this Wilson coefficient.

### 3.2 Vector extension

The \(W\)-boson mass can be modified by massive vector bosons, \(B\) and \(W\), via \((C_{\ell\ell})_{1221}\). Here, we do not assume any mechanism to generate the vector boson mass or any UV realization of the model, but consider a low-energy effective framework. The vector interactions with the SM (left-handed) leptons are represented as

\[
-\mathcal{L}_{\text{int}} = (g_{B})_{ij} B_\mu \bar{\ell}_i \gamma^\mu \ell_j + \frac{1}{2} (g_{W})_{ij} W^a_\mu \bar{\ell}_i \sigma^a_\mu \ell_j. \tag{22}
\]

Here, \((g_{B,W})_{ij}\) are hermitian matrices. When we focus on \(B \ (W)\), only \((g_{B,W})_{ij}\) is turned on.

By exchanging the vector bosons, the Wilson coefficient \((C_{\ell\ell})_{ijkl}\) becomes

\[
(C_{\ell\ell})_{ijkl} = -\frac{(g_{B})_{kl}(g_{B})_{ij}}{2M_B^2} - \frac{(g_{W})_{kj}(g_{W})_{il}}{4M_W^2} + \frac{(g_{W})_{kl}(g_{W})_{ij}}{8M_W^2}, \tag{23}
\]

at the tree level, where \(M_B\) and \(M_W\) are the masses of \(B\) and \(W\), respectively. Similar to the scalar case, there are no contributions to other SMEFT operators as long as only the interactions [22] are considered. Although the vector bosons may also have interactions with the SM right-handed leptons, quarks or Higgs boson, they are not always correlated with \((g_{B,W})_{ij}\). Since they are irrelevant for \(G_F\), we neglect them in the following analysis.

The Fermi coupling constant receives corrections from \((C_{\ell\ell})_{ijkl}\) with \(\{i,j,k,l\} = \{1,2,2,1\}\) (see Eq. (10)). Then, the above Wilson coefficient is shown as

\[
(C_{\ell\ell})_{1221} = -\frac{|(g_{B})_{12}|^2}{2M_B^2} - \frac{|(g_{W})_{11}(g_{W})_{22}|}{4M_W^2} + \frac{|(g_{W})_{12}|^2}{8M_W^2}. \tag{24}
\]

The sign of the second term on the right-hand side can be flipped if the product \((g_{W})_{11}(g_{W})_{22}\) is negative. On the other hand, the first and third terms do not have such a degree of freedom.

Since the \(W\)-boson mass discrepancy favors a positive value for \((C_{\ell\ell})_{1221}\) as shown in Fig. 1, only the vector boson \(W\) can be a source of the anomaly. Consequently, we obtain

\[
0.27 < \frac{\mathcal{G}_W}{M_W} < 0.35 \text{ TeV}^{-1}, \quad 0.38 < \frac{(g_{W})_{12}}{M_W} < 0.49 \text{ TeV}^{-1}, \tag{25}
\]

at the 68% probability. Here \(\mathcal{G}_W = \sqrt{-(g_{W})_{11}(g_{W})_{22}}\). The results imply that the mass of \(W\) is around 2–4 TeV for \(g_{W}, \mathcal{G}_W \sim 1\). Similar to the scalar case, the pull of \(M_W\) is the same as that for \((C_{\ell\ell})_{1221}\).
In Eq. (23), the first and third terms on the right-handed side are understood as neutral-current contributions, while the second term is a charged-current contribution. In the presence of \((g_W)_{12}\), the muonium-antimuonium oscillation can be induced by exchanging the neutral vector boson, because \(|(C_{\ell\ell})_{1212}| = |(C_{\ell\ell})_{1221}| \) due to \((g_W)_{12} = (g_W)_{21}^*\). Currently, the experimental constraint is \(|(C_{\ell\ell})_{1212}| < 0.1 \text{ TeV}^{-2}\) at 90% C.L. \([59]\). Thus, the limit is weaker than the parameter required to relax the W-boson mass discrepancy.

3.3 Fermion extension

As the third scenario, let us consider extra leptons, \(E\) and \(\Sigma_1\). The Yukawa interactions with the SM (left-handed) leptons are shown as

\[
-\mathcal{L}_{\text{int}} = (\lambda_E)_i \bar{E}_R \phi^i \ell_i + \frac{1}{2} (\lambda_{\Sigma_1})_i \bar{\Sigma}_a^{1R} \phi^i \sigma^a \ell_i + \text{h.c.}
\]  

(26)

In addition, there are vectorlike mass terms, \(-\mathcal{L}_{\text{mass}} = M_{E}' \bar{E}_L E_R + M_{\Sigma_1}' \bar{\Sigma}_1 \Sigma_1 + \text{h.c.}\), with \(M_{E}', M_{\Sigma_1}' \gg v\). Hereafter, let us denote the mass eigenvalues of the extra leptons as
$M_E (\simeq M_E^\prime)$ and $M_{\Sigma_1} (\simeq M_{\Sigma_1}^\prime)$. Also, only $\lambda_E$ ($\lambda_{\Sigma_1}$) is turned on in the $E$ ($\Sigma_1$) scenario. Then, the Wilson coefficients are obtained as $[51, 58]$

\[
(C_{e\phi})_{ij} = (y_\ell)^*_{j} \left[ \frac{(\lambda_E)_{ik}(\lambda_E)^*_k}{2M_E^2} + \frac{(\lambda_{\Sigma_1})_{ik}(\lambda_{\Sigma_1})^*_k}{8M_E^2} \right],
\]

\[
(C_{\phi\ell}^{(1)})_{ij} = -\frac{(\lambda_E)_{ij}(\lambda_E)^*_j}{4M_E^2} - \frac{3(\lambda_{\Sigma_1})_{ij}(\lambda_{\Sigma_1})^*_j}{16M_E^2},
\]

\[
(C_{\phi\ell}^{(2)})_{ij} = -\frac{(\lambda_E)_{ij}(\lambda_E)^*_j}{4M_E^2} + \frac{(\lambda_{\Sigma_1})_{ij}(\lambda_{\Sigma_1})^*_j}{16M_E^2},
\]

at the tree level \#5 Here, $(y_\ell)_{ij}$ is the lepton Yukawa coupling. The $W$-boson mass receives corrections from $(C_{\phi\ell}^{(3)})_{11,22}$ via the Fermi coupling constant, while the $Z$-boson observables are additionally affected by $(C_{\phi\ell}^{(1,3)})_{ii}$ directly, and the $W$-boson partial decay width $\Gamma (W^+ \to \ell^+_i \nu_\ell) = (C_{\phi\ell}^{(3)})_{ii}$ (see Sec. 2).

For the Fermi coupling constant, the Wilson coefficient in Eq. (29) is shown as

\[
(C_{\phi\ell}^{(3)})_{11,22} = -\frac{|(\lambda_E)_{1,2}|^2}{4M_E^2} + \frac{|(\lambda_{\Sigma_1})_{1,2}|^2}{16M_E^2}.
\]

Since the contribution from the extra lepton $E$ ($\Sigma_1$) is negative (positive), it is found from Fig. 2 that $E$ can be a source of the $W$-boson mass discrepancy. Therefore, we focus on $E$ in the following.

The extra lepton $E$ induces the SMEFT operators $O_{e\phi}$, $O_{\phi\ell}^{(1)}$ and $O_{\phi\ell}^{(3)}$, where the EWPO fit of the model parameters with a lepton-flavor dependent or universal assumption. The results are shown in Fig. 3. In the top two plots, either $(\lambda_E)_{1}$ or $(\lambda_E)_{2}$ is switched on, while in the bottom-left plot, we assume $(\lambda_E)_{1} = (\lambda_E)_{2} = (\lambda_E)_{3} \equiv (\lambda_E)_{\text{univ}}$ and in the bottom-right plot, $(\lambda_E)_{1}$ and $(\lambda_E)_{2}$ are fitted simultaneously with $(\lambda_E)_{3} = 0$.

From the figure, it is found that the muonic interaction $(\lambda_E)_{2}$ is required to be larger than $(\lambda_E)_{1}$; the $W$-boson mass discrepancy implies that the extra lepton $E$ may exist in 5–7 TeV for $(\lambda_E)_{1} \sim 1$ and around 3–4 TeV for $(\lambda_E)_{2} \sim 1$. For the universal case, the extra lepton mass is favored to be 5–6 TeV for $(\lambda_E)_{\text{univ}} \sim 1$. From Table 4, we also found that the pull of $M_W$ is slightly worse than the result of $C_{\phi\ell}^{(3)}$. This is because of the additional

\#5 Lepton flavors are violated generally if the extra leptons couple to multiple SM leptons, e.g., the electron and muon, simultaneously. Such violations are avoided by assuming that each extra lepton couples to only one of the electron, muon and tau-flavor leptons. Hence, $(C_{e\phi})_{ij} = (C_{\phi\ell}^{(1)})_{ij} = (C_{\phi\ell}^{(3)})_{ij} = 0$ for $i \neq j$. Then, multiple extra leptons are introduced when multiple $(\lambda_E)_{i}$ are turned on. In the following discussions, we assume that their masses are common.

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contributions to the EWPO via $C_{\phi\ell}^{(1)}$. However, the degradation is milder when $(\lambda_E)_1$ and $(\lambda_E)_2$ are fitted simultaneously.

The extra leptons can also affect the Higgs interactions via the SMEFT operator $O_{e\phi}$. The lepton Yukawa coupling is shifted as

$$y_{\ell i} = \sqrt{2} \frac{m_{\ell i}}{v} + \frac{1}{2} (\hat{C}_{e\phi})_{ii} + \frac{1}{2} (\bar{C}_{e\phi})_{ii},$$

after the electroweak symmetry breaking. Then, the signal strength of the Higgs decay into lepton pair is modified from the SM prediction as

$$\Delta \mu_{\ell i \ell i} \equiv \frac{\Gamma(h \rightarrow \ell_i \ell_i)}{\Gamma(h \rightarrow \ell_i \ell_i)_{SM}} = \left| 1 - \frac{1}{2} \delta_{GF} - \frac{1}{(y_{\ell i})_{SM}} (\bar{C}_{e\phi})_{ii} \right|^2.$$  (32)

In Fig. 4, the theoretical predictions of $\Delta \mu_{\mu \mu}$ and $\Delta \mu_{\tau \tau}$ are shown for the cases when only $(\lambda_E)_2$ is switched on (left) and the universal coupling $(\lambda_E)_{univ}$ is varied (right). Here $\mu^{ee} = \mu^{\mu \mu} = \mu^{\tau \tau}$ in the right plot. The vertical axis is the deviation of the signal strength from the SM prediction, $\Delta \mu_{\ell i \ell i} = \mu_{\ell i \ell i} - 1$. It is found that the signal strengths are changed by $O(0.1)\%$ by the contributions of $E$. The effect is sufficiently weaker than the current experimental sensitivities; the experimental results are $\mu^{\mu \mu} = 1.2 \pm 0.6$ [60], $\mu^{\tau \tau} = 1.14 \pm 0.32$ [61] from ATLAS, and $\mu^{e\mu} = 1.19^{+0.40}_{-0.39} (\text{stat})^{+0.15}_{-0.14} (\text{syst})$ [62], $\mu^{\tau \tau} = 1.02^{+0.26}_{-0.24}$ [63] from CMS. In future, the HL-LHC experiment may achieve $\delta \mu^{\mu \mu}/\mu^{\mu \mu} = 9\%$ and $\delta \mu^{\tau \tau}/\mu^{\tau \tau} = 4\%$ at $\sqrt{s} = 14$ TeV with the integrated luminosity $L = 6$ ab$^{-1}$, and the sensitivities could reach $\delta \mu^{\mu \mu}/\mu^{\mu \mu} = 0.8\%$ and $\delta \mu^{\tau \tau}/\mu^{\tau \tau} = 0.9\%$ at FCC-ee/eh/hh [64]. Therefore, further improvement is necessary to detect the effects of the extra lepton.

4 Conclusions and discussion

Motivated by the tensions reported in the updated measurement of the $W$-boson mass by the CDF collaboration, we studied the new physics scenarios that affect $G_F$. We investigated single-field extensions of the SM, i.e., introducing one of the extra scalars ($S_1$ and $\Xi_1$), leptons ($E$ and $\Sigma_1$), and vector fields ($B$ and $W$). It was found that the models with $\Xi_1$, $E$ and $W$ can relax the discrepancy if the new particles exist in multi-TeV scales when the new coupling to the SM leptons is of order unity. In particular, the scalars and vectors affect $M_W$ via the lepton four-Fermi operator $O_{4\ell}$; its Wilson coefficient is favored to be $\sim 0.02$–0.03 TeV$^{-2}$. On the other hand, the extra leptons contribute to the Fermi coupling constant via $O_{\phi\ell}^{(3)}$, and the $M_W$ discrepancy implies $(C_{\phi\ell}^{(3)})_{11}$ and/or $(C_{\phi\ell}^{(3)})_{22}$ around $-0.01$–0.03 TeV$^{-2}$, leading to $(\lambda_E)_1$ and/or $(\lambda_E)_2$ around 0.1–0.3 for $M_E = 1$ TeV. We also evaluated the pull of EWPO
including $M_W$ in each fit. Although any of the scenarios cannot explain the discrepancy perfectly, the tension is shown to be relaxed. In particular, the lepton-flavor universal contribution of $C_{\phi \ell}^{(3)}$ provides the best result.

If new scalar and vector bosons exist in multi-TeV scales, they could be detected via resonance searches of $pp \rightarrow S, V \rightarrow \ell\ell$, if they couple to the SM quarks. However, such interactions are not necessary for the $W$-boson discrepancy. The required mass scales decrease if the couplings to the SM leptons are weaker. Then, the new particles could be produced via the gauge interactions (except for $B$). They decay into the SM leptons and might be discovered in searching for multi-lepton signatures. Such a study has been performed by ATLAS with the full Run 2 dataset of 139 fb$^{-1}$ [65], though the result has not been applied to the current setup. On the other hand, the extra fermions decay into the SM bosons as well as the SM leptons. ATLAS and CMS have performed such studies with the full Run 2 dataset [66,67] and provided limits on the extra-lepton mass as $M \gtrsim 1$ TeV. The sensitivity might be able to reach multi-TeV scales in future experiments such as HL-LHC or higher energy colliders [68]. Thus, the extra-lepton model could be tested in future.

Let us comment on connections with other anomalies. Currently, the experimental results of the anomalous magnetic moment of muon [69,72] show a 4.2$\sigma$ discrepancy from the SM prediction [73]. This tension can be solved by introducing the vectorlike leptons at the one-loop level [74,89]. In fact, the extra contributions can be as large as $O(10^{-9})$ even for $M_i = O(1)$ TeV by introducing another extra lepton, $\Delta_1 \sim (1, 2)_{-\frac{1}{2}}$ or $\Delta_3 \sim (1, 2)_{-\frac{3}{2}}$, in addition to $E$, because interactions among the extra leptons can enhance a chirality flip.

Figure 4: Signal strength of the Higgs boson decays for $\lambda_E$ with $M_E = 1$ TeV. Only $(\lambda_E)_2$ is switched on in the left plot, while the coupling is universal in the right. The darker (lighter) region corresponds to the 68% (95%) probability provided in Fig. 3.
Note that $\Delta_1$ and $\Delta_3$ do not contribute to $G_F$, while they affect EWPO via the SMEFT operator $(\phi^i D_\mu \phi)(\bar{e}_i \gamma^\mu e_j)$. Hence, further analyses are required for detailed studies.

Note added

When we were finalizing this paper, Ref. [24] appeared on arXiv, which discusses single field extensions of the SM. In contrast to that paper, we investigated here in details the scenarios where the $W$-boson mass anomaly is relaxed by the new physics affecting the Fermi constant. In particular, we studied the effects of the flavor non-universal couplings in addition to the flavor universal ones. Furthermore, we considered the scalar field $\Xi_1$, which was not studied in Ref. [24].

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