Gravity from Quantum Information

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The Einstein equation express a relation between matter and the spacetime geometry it disturbs. The origin of this relation remains a mystery since its discovery by Einstein. In 1995 Jacobson linked the laws of thermodynamics to the Einstein equation [1, 2]. Now, one can ask why there is such a surprising relationship between thermodynamics, or entropy and gravity? We try to answer to these fundamental questions by considering information erasure in a curved spacetime.

Recently, Verlinde brought us a remarkable new idea [3] linking gravitational force to entropic force and derived the Newton’s equation and the Einstein equation from this relation. Padmanabhan also proposed a related idea [4].

In this paper, we present a slightly different, but related study connecting gravity to information, performed independently by the authors [5, 6]. In a series of works [6–8] we have emphasized the quantum informational nature of gravity. For example, a few years ago we suggested that dark energy responsible for the cosmic accelerating expansion is related to quantum entanglement of the vacuum fluctuation [6] or erasure of quantum information at a cosmic horizon [7, 8]. Using a similar approach we also derived the first law of black hole thermodynamics from the second law of thermodynamics [3] and obtained a discrete black hole mass formula. These works are based on the Landauer’s principle in quantum information theory and the holographic principle.

As a variant of the second law of thermodynamics, the Landauer’s principle states that to erase N bits of information of a system irreversibly at least $k_B N$ entropy of a bath should be increased and at least $k_B T N$ energy should be consumed, where $k_B$ is the Boltzmann’s constant and $T$ is the temperature of the thermal bath contacting with the system. For a black hole and the universe their causal horizons play roles of the bath and an information barrier. We suggested that there is energy $E_h$ related to the information erasing at the horizons given by

$$dE_h = k_B T_h dS_h,$$

where $T_h$ is a horizon temperature and $dS_h$ is a horizon entropy change due to the information erasing. We identified this energy to be the origin of dark energy and black hole mass. Thus, one can see that this energy is very similar to the equipartition energy for the entropic force [3, 4].

All our results above indicate that there is an intrinsic relationship between quantum information and gravity. This can be seen as another realization of a famous slogan in quantum information community: “It from Bit!”

Along this line we now suggest that the Einstein equation can be derived using quantum information theory applied at a causal horizon of a given spacetime. Our work is also based on the intriguing work by Jacobson linking the first law of thermodynamics to the Einstein equation.

Let us start by briefly reviewing our previous works related to this subject. First, in [6], it was pointed out that a cosmic horizon with a radius $R_h \sim O(H^{-1})$ has a kind of thermal energy $E_h \propto T_h S_h \propto R_h$ associated with its holographic entropy $S_h \propto R_h^2$, and this thermal energy has an information theoretic origin. Here $H$ is the Hubble parameter. (Recently, there appear similar suggestions based on the Verlinde’s idea [11, 12].)

To be specific we identified $S_h$ as an entanglement entropy $S_{Ent}$ associated with the erased vacuum information outside the horizon, and $T_h \propto 1/R_h$ as the Hawking-Gibbons temperature of the horizon. Then, it was straightforward to get a horizon energy density $\rho_h \sim E_h/R_h^3 \sim N_c M_P^3/R_h^2$ which can be interpreted as a holographic dark energy density [13]. Note that in principle one can explicitly obtain $S_{Ent}$ for the vacuum state.
using quantum field theory, not just a heuristic estimation. In [16], $S_{Ent}$ for a spherical region is obtained by partial tracing a ground state of discretized quantum fields. With reasonable input parameters such as Planck mass $M_P$ as a UV-cutoff and the number of spin degree of freedom $N_s$ of quantum fields, our dark energy model gives energy density and the equation of state for dark energy comparable to the observational data [8]. (This dark energy could be also regarded as the energy of cosmic Hawking radiation [10].)

Why are we considering $S_{Ent}$ for $S_h$? One of the key concepts of quantum information theory allowing useful quantum information processes such as quantum key distribution is the quantum entanglement. The entanglement is a quantum nonlocal correlation which can not be described by a classical correlation. The entanglement entropy $S_{Ent}$ is a good measure of entanglement for pure states such as the vacuum. It is the von Neumann entropy $S_{Ent} = - Tr(\rho_A \ln \rho_A)$ associated with the reduced density matrix $\rho_A \equiv Tr_B \rho_{AB}$ of a bipartite system $AB$ described by a full density matrix $\rho_{AB}$. For a causal horizon playing a role of information barrier, it is very natural to divide the system into two subsystems $A$ and $B$ - inside and outside the horizon - and to trace over one of the two regions to obtain the entanglement entropy of the horizon. Thus, $S_{Ent}$ is ideal for $S_h$, when there is a causal horizon.

Second, in [7], we calculated black hole mass increase due to absorption of a test particle with energy $\delta E$ in the context of quantum information theory. To the observer outside the black hole, this corresponds to the erasure of $\delta S$ bits of information by the thermal bath of the event horizon. In this case the Landauer’s principle demands that the entropy of the black hole increases at least by $\delta S$ during this process. Assuming this erasing process is optimal, we obtained the increase of the black hole mass

$$\delta M_{BH} = \delta E = k_B T \delta S,$$

which looks like the first law. Here, $T$ is the Hawking temperature for the black hole. This implies that the first law of black hole thermodynamics is just the second law disguised and information plays a crucial role in gravity.

In the work, to calculate the lost information of the particle crossing the horizon we assumed an information erasing process similar to that considered by Song and Winstanley [17]. They derived a generalized second law for black hole thermodynamics from the view point of quantum information theory, especially by applying the Landauer’s principle. They considered a small quantum system falling into the black hole with Hawking temperature $T$. The system has Hamiltonian $H$, density matrix $\rho_1$, and energy $dE_{sys} = Tr(\rho_1 H)$. Around the horizon it comes into thermal equilibrium with the black hole and its final state becomes $\rho_f = e^{-H/k_B T}/Z$. They showed that the total change in entropy can be written as

$$\delta S \equiv \delta S_{sys} + \delta S_{BH} = Tr[\rho_1 \ln \rho_1 - \rho_f \ln \rho_f] \simeq dE_{sys}/k_B T \geq 0.$$  

Recently, a more adequate derivation of the relation for causal horizons is given by Padmanabhan [18], and we adopt his result in this paper. Consider a quantum density matrix $\rho_1$ of the particle (excited state) crossing the horizon. The density matrix can be obtained by tracing out unobservable modes outside the horizon. This state has an entanglement entropy $S_1 = -Tr(\rho_1 \ln \rho_1)$. Similarly defined vacuum state $\rho_0$ has an entanglement entropy $S_0 = -Tr(\rho_0 \ln \rho_0)$. Padmanabhan showed that the entropy difference representing information loss is just

$$\delta S \equiv S_1 - S_0 \simeq dE_{sys}/k_B T$$

in the strong gravity limit. Here, $dE_{sys} = Tr((\rho_1 - \rho_0)H)$ and $\rho_0 = e^{-H/k_B T}/Z$. Thus, we can generally use Eq. (1) to calculate the horizon entropy change due to the particle crossing a causal horizon. It is also reasonable to say that the first law applied to causal horizons has a quantum informational origin and lost information can be quantified in terms of the entanglement entropy $S_{Ent}$.

![FIG. 1. To calculate the gravitational field at $P$ by a massive object $M$ at the center, consider an accelerating observer $\Theta$ relative to the local inertial frame at $P$. It is possible that for the observer the object $M$ appears to cross a local Rindler horizon (represented by the dotted line) of the observer. This results in the increase of the horizon entropy.](image)

Putting it altogether, it is natural to imagine that gravity itself has a quantum informational origin. We need to go one more step to confirm that. In 1995, Jacobson showed that one can derive the Einstein equation by demanding the first law of thermodynamics,

$$\delta Q = k_B T_U dS_h,$$  

hold at Rindler horizons. Here,

$$T_U = \frac{\hbar a}{2\pi k_B c}$$

is the Unruh temperature of the horizon seen by an observer, $S_h$ is the entropy of the horizon, $a$ is acceleration, and $dQ$ is the heat flux crossing the horizon. He also assumed the area law for the horizon entropy. By demanding that the first law holds at local Rindler horizons for
each spacetime point, one can derive the Einstein equation. Below we follow his derivation keeping in mind that $dS_h$ in Eq. (1) and Eq. (3) represents the lost information of matter crossing the horizon.

To do this consider a massive object with mass $M$ at the center and an accelerating observer $\Theta$ with arbitrary large acceleration $a$ relative to the local inertial frame at a point $P$ (See Fig. 1). Thanks to the equivalence principle one can choose an approximately flat patch for every spacetime point $P$ with boost Killing vector field $\xi^\alpha = -\kappa \lambda k^\alpha$ generating a local Rindler horizon for the observer $\Theta$ where the object crosses. Here, $k^\alpha$ is the tangent vector to the horizon generators with an affine parameter $\lambda$ and $\kappa$ is the acceleration of the Killing orbit satisfying $T_U = h \kappa / 2 \pi k_B c$.

Then, the situation in Fig.1 at $P$ can be approximately described using the Rindler coordinate chart in Fig. 2:

\[
ct = \left( \frac{r^2}{a} + X \right) \sinh \left( \frac{a \tau}{c} \right), \\
x = \left( \frac{r^2}{a} + X \right) \cosh \left( \frac{a \tau}{c} \right)
\]

where $(t, x)$ is the Minkowski coordinate and $(\tau, X)$ is the Rindler coordinate of the observer. Each point in the figure represents a 2-dimensional spatial plane.

The bottom line of our new idea is as follows. For the observer, the object with $Mc^2 \equiv dE$ may seem to cross the Rindler horizon resulting in the information erasing and decrease of the total entropy. According to the Landauer’s principle, to save the second law of thermodynamics, the horizon entropy should increase by at least the information loss of the matter crossing the horizon. Due to the energy conservation $dE_h = dE$, and the amount of the entropy change is given by Eq. (1)

\[
dS_h = \frac{dE_h}{k_B T^h} = \frac{dE}{k_B T_U},
\]

hence, $dE = k_B T^h dS_h$.

Following Jacobson we can genericize this relation by defining the energy flow across the horizon $\Sigma$

\[
dE = -\kappa \lambda \int T_{\alpha \beta}^\Sigma \xi^\alpha dS^\beta
\]

where $dS^\beta = \xi^\beta d\lambda dA$, $dA$ is the spatial area element, and $T_{\alpha \beta}$ is the energy momentum tensor of matter distribution. Using the Raychaudhuri equation one can denote the horizon area expansion $\delta A \propto dS_h$ and the increase of the entropy as

\[
dS_h = \eta \delta A = -\eta \kappa \lambda \int R_{\alpha \beta} \xi^\alpha dS^\beta,
\]

with some constant $\eta \parallel$. If $S_h$ saturates the Bekenstein bound, $\eta = c^3/4 \hbar G$.

At this moment, if we identify the horizon entropy $S_h$ to be the entanglement entropy $S_{ent}$, we can avoid the use of the holographic principle and some circular logics.

\[
\text{FIG. 2. To calculate the metric evolution at } P \text{ consider the approximate Rindler chart for the observer } \Theta, \text{ who would see the matter with energy } dE \text{ crosses the local Rindler horizon. The horizon divides the spacetime into two causally disconnected regions } F \text{ and } R, \text{ and should expand appropriately to satisfy the Landauer’s principle. This leads to the Einstein equation.}
\]

Furthermore, in principle one can calculate $\eta$ using the quantum field theory by adding up contributions from all fields $\parallel$. If there are $N_j$ spin degrees of freedom of the $j$-th field, this implies that for a spherical region with radius $r$

\[
S_{ent} = \sum_j \beta_j N_j \frac{r^2}{L_P^2} = 4 \pi \eta r^2.
\]

where $\beta_j$ is an $O(1)$ numerical constant and $L_P = \sqrt{\frac{8 \pi \hbar G}{c^3}}$ is the reduced Planck length.

Srednicki obtained a value $\beta_j = 0.3$ for the massless scalar field by performing numerical calculations on a sphere lattice. A similar value was obtained for a massless scalar field in the Friedmann universe in $\parallel$. Thus, the Bekenstein bound gives a constrain $\Sigma_j \beta_j N_j = 8 \pi^2$ on the number and characteristics of quantum fields in the universe $\parallel$.

Inserting Eqs. (7) and (3) into $dE = k_B T^h dS_h = h \kappa dS_h / 2 \pi c$ one can see $2 \pi c T_{\alpha \beta} \xi^\alpha dS^\beta = \eta R_{\alpha \beta} \xi^\alpha dS^\beta$.

For all local Rindler horizons this equation should hold. Then, this condition and Bianchi identity lead to the Einstein equation

\[
R_{\alpha \beta} - \frac{R g_{\alpha \beta}}{2} + \Lambda g_{\alpha \beta} = \frac{2 \pi}{\eta c} T_{\alpha \beta}
\]

with the cosmological constant $\Lambda$ as shown in his paper. Strangely, information theoretic aspects of Jacobson’s theory were not discussed widely so far, though the entropy $S_h$ clearly has an information theoretic meaning in
modern physics. Now, we can interprete the first law in Eq. (3) in terms of the Landauer’s principle. We assumed the second law of thermodynamics is more fundamental than the first one and the first law should be satisfied to hold the second law for all causal horizons blocking information.

How can one reconcile the irreversibility of the entropy and reversibility of gravity in our interpretation? As in black hole cases, free falling observer comoving with free falling matter would not see the Rindler horizons, while fixed observers (accelerating against the matter frame relatively) can see the horizons. Thus, irreversibility in a gravitational system is an observer dependent phenomena in general. Nonetheless, since the Einstein equation derived above is covariant, the equation should hold for every frame, once it is satisfied in a specific frame.

Summarizing all these facts, we can say that the Einstein equation simply states that total entropy of matter and horizon should not decrease and this is the bottom line of all gravitational phenomena. The causal structure of the spacetime should be automatically arranged so that the area of the Rindler horizons appropriately increase to compensate the information loss of the gravitating matter which crosses the horizons for some observers.

Therefore, our theory is in concordance with Verlinde’s proposal in a general sense. However, there are some differences between our theory and Verlinde’s theory too. First, in our theory we assumed neither the proportionality of entropy on the distance, nor the entropic force. The equipartition condition is not necessary too. Second, we suggested the horizon entropy is originated from quantum information erasing at a horizon rather than coarse graining of microscopic degree of freedom. This explain why the derivation of classical gravity is involved with \( \hbar \) and why gravity has something to do with entropy or information. The Newton’s gravity could arise, of course, from the non-relativistic limit of the Einstein equation. Third, our theory does not demand the generalized holographic principle for equipotential surface. Ordinary quantum field theory is enough to calculate the entanglement entropy of the horizons. In principle, with reasonable assumptions, one may even explicitly calculate some relevant physical quantities such as dark energy. Note that we did not assume that the spacetime is emergent but assume the existence of spacetime and its geometry a priori. Since our theory links quantum mechanics to classical gravity, it might provide us a new way to quantum gravity.

Considering the second law, we expect the causal horizon area of the universe has a strong tendency to be extended. That is, matter in the universe distribute themselves so that the horizon entropy of the universe to be maximized. This might be the origin of the gravitational force as an entropic force considered by Verlinde.

In short, the Einstein equation links matter to gravity and his famous formula \( E = mc^2 \) links matter to energy. We know also that the Landauer’s principle links information to energy. Thus, now we have a relation between information and gravity, the Einstein equation with the quantum informational interpretation. Our theory implies that the Einstein equation is more about information rather than energy or equation of state. In other words, information might be more profound physical entity than matter or field.

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