Security Properties as Nested Causal Statements

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Abstract
Thinking in terms of causality helps us structure how different parts of a system depend on each other, and how interventions on one part of a system may result in changes to other parts. Therefore, formal models of causality are an attractive tool for reasoning about security, which concerns itself with safeguarding properties of a system against interventions that may be malicious. As we show, many security properties are naturally expressed as nested causal statements: not only do we consider what caused a particular undesirable effect, but we also consider what caused this causal relationship itself to hold. We present a natural way to extend the Halpern-Pearl (HP) framework for causality to capture such nested causal statements. This extension adds expressivity, enabling the HP framework to distinguish between causal scenarios that it could not previously naturally tell apart. We moreover revisit some design decisions of the HP framework that were made with non-nested causal statements in mind, such as the choice to treat specific values of causal variables as opposed to the variables themselves as causes, and may no longer be appropriate for nested ones.

1 Introduction
Causality is a common feature of our discourse; indeed, it could be argued that the notion that some circumstance is the cause of another is fundamental to the way we make sense of the world around us, providing both explanations of why things are the way they are and guidance on how we should act in order to influence their course. The standard approach to causality involves counterfactuals: had the cause not occurred or occurred in a different way than it actually did, the effect would not have come to pass. We are typically interested in the effects of interventions, which can be viewed as ways of making the cause occur in a different way.

A system, such as a computer program, mechanism, or collection of real-life entities such as the staff of a commercial business, is considered to be secure against a potentially malicious actor (“attacker”) if no action the attacker can take could cause some desirable property of the system to be violated. By viewing the actions available to the attacker as interventions, this becomes a causal statement: no desirable property can be violated because of something the attacker did or didn’t do. We would like to use a formal account of causality to represent and analyse security properties. This is an attractive approach because, while many formal models of security have been proposed, especially in the programming language research community (Goguen and Meseguer 1982; Haigh and Young 1987; Zdancewic and Myers 2001; Sabelfeld and Myers 2003), these models are typically designed for the purpose of analysing computer programs only, and are therefore tightly coupled to a machine model such as state machines or sequences of memory snapshots. In contrast, general-purpose formalisms for causality can capture and represent a wider range of scenarios, including everyday events and their relations. This means that a causal characterization of a security property can be evaluated with respect to a real-world scenario, rather than only the operation of some computer system. Since real-world scenarios are typically closer to human intuition than computer programs, we expect this to be helpful in understanding what a particular security property “really means”.

A variety of formal models have been proposed for reasoning about causal statements and formally defining what it means to be a cause. For definiteness, we use the Halpern-Pearl (HP) framework (Halpern 2015; Halpern 2016). In this framework, we first represent the relevant features of the world (“a switch, which can be either on or off”, “a lamp, which can be on or off”) as variables, and the rules that govern their interdependency (“the lamp is on exactly when the switch is in the on”) as structural equations to form a causal model, which enables counterfactual reasoning: that is, a comparison between the real world and a hypothetical alternative which differs from the actual world in some relevant aspect.

In the HP framework, potential causes are taken to be conjunctions of atomic propositions about the values taken by variables, while effects are taken to be Boolean combinations of atomic propositions. However, in real-world discourse, we often encounter seemingly more complex statements, including, in particular, ones where the purported effect is itself a causal statement (“because I have paid my electricity bill on time, flipping the light switch on causes the lamp to turn on”). Nested causal statements of this form are particularly common when discussing notions of authorisation, delegation, and endorsement.

For example, consider a government employee who is authorised to publicly confirm a classified piece of information, say, that the government has made contact with an alien civilization. This employee is corrupt, and may release the
information if he is bribed. The release of the information, which itself can be interpreted causally as saying that the fact that contact was made is a cause of the newspaper article saying that it is, is allowed; the employee is authorised to release information. The bribe on its own is also not necessarily bad; nothing prohibits acts of kindness towards strangers. The problem here is that the gift of money caused the alien contact to be a cause of the newspaper article. This corresponds to the notion of robust declassification from the security literature (Zdancewic and Myers 2001; Myers et al. 2004; Cecchetti et al. 2017), which can be interpreted as more generally saying that whenever secret causes have public effects, this must not itself be due to untrustworthy causes. The underlying notion of robustness can be taken more generally to denote that some security-relevant circumstance, which could be primitive or itself involve causality, is not caused by an untrusted party.

Conversely, nested causes may also render normally unacceptable causal relationships acceptable. For example, if A performs an action that infringes on B’s possessions, such as redecorating their office, shredding papers, or changing the settings of B’s computer, and B finds this objectionable, a common defense that A might invoke is to say that they would have stopped if B had told them to (and B had the opportunity to do so). In other words, whatever causal relationship there was between A’s intentions and B’s property held only because B didn’t voice an objection, and thus implicitly endorsed the act. We can view this line of reasoning, where an unauthorised cause has a certain effect because of an authority’s implicit or explicit go-ahead, as a form of authorisation. The construction can easily be nested, obtaining examples where one authority holds a veto over another authority’s ability to hold a veto over a causal relationship, and so on. There is extensive literature on reasoning about an authority’s ability to hold a veto over a causal relationship, including the grower’s state of origin, were subject to catastrophic floods instead. The grower now makes the following statement: “Because we moved to Texas, the weather caused our crop to fail.” This is arguably false: had the grower not moved to Texas, the flooding would have led to crop failure all the same. How would we make sense of this in the HP framework? What value of the variable weather are we referring to here? Naively extending upon the previous observation, the first thought would be to use the value of weather in the actual context, so it becomes “Because we moved to Texas, the weather being dry caused our crop to fail.” But this statement is true: had the grower stayed in New York, the weather would not have been dry, and so dry weather could not have caused crop failure. Plugging in another constant value does not work either; while “Because we moved to Texas, the weather being very wet caused our crop to fail” is false, due to the statement “the weather being very wet caused our crop to fail” itself being false. This statement can’t be the intended meaning of “the weather caused our crop to fail”, because we would normally take the latter statement to be true!

We claim that the most reasonable interpretation of this type of statement is instead as a form of causality that is independent of the concrete value that the weather takes. This can be interpreted in terms of the HP notion in terms of existential quantification: “there exists a state v of the weather such that the weather being v caused our crop to fail”. We return to this point in Section 4.

## 2 Review of the HP framework

We first review the Halpern-Pearl notion of causality. The first step is to define causal models.

**Definition 2.1.** A causal model is a pair \((S, F)\), consisting of a signature \(S\) and a collection of structural equations \(F\) for this signature. The signature \(S\) is a triple \((U, V, R)\); \(U\) is a nonempty finite set of exogenous variables, to be thought of as external inputs to the model, or features of the world whose values are determined outside the model; \(V\) is a nonempty finite set of endogenous variables, whose causal dependencies on each other and on the inputs we wish to analyse; each variable \(W \in U \cup V\) can take values from a finite range \(R(W)\); \(F\) associates with each endogenous variable \(V \in V\) a function denoted \(F_V\) that determines the value of \(V\) in terms of the values of all the other variables in \(U \cup V\); thus, \(F_V : \prod_{W \in (U \cup V) \setminus V} R(W) \to R(V)\). We typically write, say, \(V = U + X\) rather than \(F_V(u, x) = u + x\) for all \(u \in R(U)\) and \(x \in R(X)\).

A variable \(V\) depends on \(W\) if the structural equation for \(V\) nontrivially depends on the value taken by \(W\): that is, there are some settings \(\mathcal{z}\) and \(\mathcal{z}'\) of the variables in \(U\) and \(V\) other than \(V\) that only differ in the entry corresponding to
that on. In particular, in this model, whether the lamp is on does
not depend on the state of either of the switches. We also have
\[ M^{\land \text{lamp}} = \text{LAMP} = \text{on} \land [\text{SWITCH1} \leftarrow \text{off}] \text{LAMP} = \text{off} : \]
the lamp is on, and if the first switch were set to be switched off, the lamp would be off.

As is standard, we define \( M \models \varphi \) to mean that \((M, \vec{u}) \models \varphi \) for all contexts \( \vec{u} \). In particular, if there is only a single possible context \( u \) (as is the case in many of our examples), then it is equivalent to \((M, u) \models \varphi \). Moreover, since the ranges of all variables are finite, we can take \( \exists v \in R(X). \varphi \) to be syntactic sugar for the disjunction \( \bigvee_{x \in R(X)} \varphi[x/v] \), that is, the formula that is true iff it is true with some value from the range of \( X \) substituted for \( v \). When the range is clear from the context, we may simply write \( \exists v. \varphi \).

We can now state the HP definition of causality. There are actually three definitions (see Halpern and Pearl 2001; Halpern and Pearl 2005; Halpern 2015; Halpern 2016), called the modified HP definition, since it is simplest and seems most robust.

**Definition 2.3.** \( \vec{X} = \vec{x} \) is an actual cause of \( \varphi \) in \((M, \vec{u})\) if

- **AC1.** \((M, \vec{u}) \models \varphi \) and \((M, \vec{u}) \models \vec{X} = \vec{x} ; \)
- **AC2.** There is a set \( \vec{W} \) of variables in \( \mathcal{V} \) and a set of alternative values \( \vec{x}' \) for the variables in \( \vec{X} \) such that if \((M, \vec{u}) \models \vec{W} = \vec{w} \), then \((M, \vec{u}) \models [\vec{X} \leftarrow \vec{x}'], \vec{W} \leftarrow \vec{w} \] \( \models \neg \varphi \).
- **AC3.** \( \vec{X} \) is minimal; there is no strict subset \( \vec{X}' \) of \( \vec{X} \) for which AC2 holds.

We write \((\vec{X} = \vec{x}) \rightarrow \varphi \) to represent \( \vec{X} = \vec{x} \) is a cause of \( \varphi \), so that \((M, \vec{u}) \models (\vec{X} = \vec{x}) \rightarrow \varphi \) if AC1–3 hold. Note that conditions AC1–3 are all expressible in the language LI.

In Definition 2.3, HP assume that \( \varphi \) is a Boolean combination of atomic propositions. In particular, \( \varphi \) may not itself be a formula of the form \((\vec{X} = \vec{x}) \rightarrow \varphi \). Since we want to reason about the expressive power of this notion of causality, it is useful to explicitly define the language obtained by augmenting propositional logic with it.

**Definition 2.4.** A formula \( \varphi \) is simple if it is a Boolean combination of atomic propositions of the form \( X = x \).

A formula \( \psi \) is a simple causal formula if it is a Boolean combination of atomic propositions of the form \( X = x \) and causal statements of the form \((\vec{X} = \vec{x}) \rightarrow \varphi \), where \( \varphi \) is simple. The language of all simple causal formulae is called LC1.

We give semantics to formulae in LC1 by converting them to formulae in LI and using Definition 2.3.

### 3 Nested causal statements

Our goal is to investigate the role of nested causal statements such as \( A = \vec{u} \) is a cause of \( B = \vec{b} \) being a cause of \( \varphi \). These statements have no formal counterpart in LC1, but we can define a language which includes them.
Definition 3.1. The language $LC_\infty$ of nested causal formulæ is defined recursively as follows:

- Simple formulæ $\varphi$ are in $LC_\infty$.
- If $\varphi$ is in $LC_\infty$, then so is $(X = \bar{x}) \rhd \varphi$.
- Boolean combinations of formulæ in $LC_\infty$ are in $LC_\infty$.

Since Definition 2.3 does not depend on the structure of $\varphi$ in $(X = \bar{x}) \rhd \varphi$, we can once again use it to give a semantics to $LC_\infty$ by evaluating the corresponding formula in $L$. As we now show, the inclusion of nested causal statements results in $LC_\infty$ being more expressive than $LC_1$ once we exclude a particular set of undesirable formulæ.

Let $M^{\neg \square \text{LAMP}}$ be the same as the model $M^{\square \text{LAMP}}$ from Example 2.2, except that

$$F_\text{LAMP} = \begin{cases} \text{on} & \text{if SWITCH1=SWITCH2} \\ \text{off} & \text{otherwise,} \end{cases}$$

so the lamp is on if both or neither switch is. Intuitively, the two models $M^{\square \text{LAMP}}$ and $M^{\neg \square \text{LAMP}}$ are quite distinct. If we think of each model as representing different setups in which two people each control a light switch, then in $M^{\square \text{LAMP}}$, each participant has a veto on whether the light is on: if they choose to keep their switch “off”, then nothing the other person can do has any bearing on whether the light is on or not. On the other hand, in $M^{\neg \square \text{LAMP}}$, by flipping their own switch, each person can only temporarily toggle the light, perhaps to mess with the other participant, but has no way of ensuring that the light will permanently remain in any particular state. In security parlance, we could think of this as saying that in $M^{\square \text{LAMP}}$, each person has to independently authorise the other to be able to influence the light.

We want to show that natural simple causal formulæ are not sufficiently expressive to capture the difference between $M^{\square \text{LAMP}}$ and $M^{\neg \square \text{LAMP}}$. The qualification natural, however, does some work here: in order to make this statement precise, we need to restrict the set of causal formulæ that we consider. Specifically, the intuition above does not necessarily hold for some causal statements where cause and effect refer to the same thing (such as “it is raining because it is warm and raining”). We typically do not make such statements; it seems strange to say “$X = x$ is a cause of $X = x$” or “$X = x$ is a cause of $X = x$ and $Y = y$”.

Definition 3.2. For a given formula $\varphi$ let $\text{Vars}(\varphi)$ denote the set of variables (each $X$ in the atom $X = x$) in $\varphi$. The formula $\varphi \rhd \psi$ is circular if $\text{Vars}(\varphi) \cap \text{Vars}(\psi) \neq \emptyset$. Let $LC^\text{nc}_1$ and $LC^\text{nc}_\infty$ denote the non-circular fragments of $LC_1$ and $LC_\infty$ respectively.

As we now show, we cannot distinguish $M^{\square \text{LAMP}}$ and $M^{\neg \square \text{LAMP}}$ using non-nested non-circular formulæ, but with nested non-circular formulæ, we can.

Theorem 3.3. For all $\varphi$ in $LC^\text{nc}_1$, $M^{\square \text{LAMP}} \models \varphi$ if and only if $M^{\neg \square \text{LAMP}} \models \varphi$, but there exists a nested non-circular causal statement $\psi \in LC^\text{nc}_\infty$ such that $M^{\square \text{LAMP}} \models \psi$ and $M^{\neg \square \text{LAMP}} \models \neg \psi$.

Proof. We start by showing that non-circular formulæ cannot distinguish the models. By exhaustive checking, we can confirm that $M^{\square \text{LAMP}} \models X = x$ if and only if $M^{\neg \square \text{LAMP}} \models X = x$ for all $X \in \mathcal{V}$ and $x \in \mathcal{R}(X)$. It easily follows that $M^{\square \text{LAMP}} \models \varphi$ if and only if $M^{\neg \square \text{LAMP}} \models \varphi$ for simple formulæ.

Formulæ in $LC^\text{nc}_1$ are Boolean combinations of atomic propositions of the form $X = x$ and non-circular causal formulæ of the form $(X = \bar{x}) \rhd \varphi$, where $\varphi$ is a simple formulæ. Hence, if we can also establish that causal formulæ are valid in $M^{\square \text{LAMP}}$ if and only if they are valid in $M^{\neg \square \text{LAMP}}$, the result easily follows. We do this by considering a number of cases.

If the variable LAMP does not occur in $\varphi$, then the causal formulæ are false in both $M^{\square \text{LAMP}}$ and $M^{\neg \square \text{LAMP}}$. To see this, note that by non-circularity, $\varphi$ can mention only SWITCH1 and SWITCH2, but neither of these variables depends on any other variable in either model. By non-circularity, $X$ does not mention the variables in $\varphi$, so no change to the value of $X$ change the value of SWITCH1 or SWITCH2 (even if some variables $\bar{W}$ are fixed to their actual values). Thus, AC2 cannot hold in either model, so the causal formulæ is false in both models.

Now suppose that LAMP occurs in $\varphi$, and hence LAMP $\not\models X$. Consider the possible cases for $X$. If $X = \bar{x}$ contains either SWITCH1 = off or SWITCH2 = off, then $X = \bar{x}$ is false in both models, so AC1 fails, and the causal formulæ is false in both models. If $X = \emptyset$, then AC2 must fail in both models (since no change in the value of a variable in $\bar{X}$ can cause a change in the truth value of $\varphi$. If $X = \bar{x}$ is Switch1 = on, then changing Switch1 to off (while possibly keeping some variables fixed at their actual values) has the same effect on the truth value of all the variables in both models, so AC2 will either hold in both models or in neither, and AC3 trivially holds in both. A similar argument works if $X = \bar{x}$ is Switch1 = on. Finally, if $X = \bar{x}$ is Switch1 = on $\land$ Switch2 = on, then the causal formulæ must be false in both models. Either at least one of AC1 and AC2 is violated, or we must have $\varphi \equiv \text{LAMP = on}$ in both models as AC1 necessitates $M^{\square \text{LAMP}}, M^{\neg \square \text{LAMP}} \models \varphi$. AC2 necessitates $M^{\square \text{LAMP}}, M^{\neg \square \text{LAMP}} \models [\text{SWITCH1} \leftarrow v, \text{SWITCH2} \leftarrow w] \models \varphi$, and by non-circularity, $\varphi$ can mention only LAMP. But in both models, $[\text{SWITCHi} \leftarrow \text{off}] \models \text{LAMP = on}$ for $i \in \{1, 2\}$, so Switch1 = on is already a cause of LAMP = on and AC3 (minimality) is violated.

We now show that the models can be distinguished by nested non-circular formulæ. We claim that $M^{\square \text{LAMP}} \models \text{SWITCH1 = on}$

$$\rhd (\exists v. \text{SWITCH2 = on} \rhd \text{LAMP = v}),$$

but

$$M^{\neg \square \text{LAMP}} \not\models \text{SWITCH1 = on} \rhd (\exists v. \text{SWITCH2 = on} \rhd \text{LAMP = v}).$$

In both models, both SWITCH1 = on and SWITCH2 = on $\rhd$ LAMP = on are valid, as intervening to set SWITCH2 $\leftarrow$ off results in LAMP = off. However, if we intervene to set SWITCH1 $\leftarrow$ off, we have $M^{\square \text{LAMP}} \not\models \text{SWITCH1} \leftarrow \text{off}$.
\( \text{SWITCH2} = \text{on} \leadsto \text{LAMP} = v \) for any \( v \), as \( \text{LAMP} = \text{off} \) regardless of the setting of \( \text{SWITCH2} \). This is not the case in \( M^{\sim \text{LAMP}} \), as there we have
\[
M^{\sim \text{LAMP}} = [\text{SWITCH1} \leftarrow \text{off}](\text{SWITCH2} = \text{on} \leadsto \text{LAMP} = \text{off}).
\]
Intervening further to set \( \text{SWITCH2} \leftarrow \text{off} \) results in the lamp turning back on, as both switches are in the same position again.

**Remark 3.4.** It is worth noting that non-circularity is really a necessary condition here. Let \( \varphi \) be the formula \( \text{SWITCH1} = \text{on} \land \text{SWITCH2} = \text{on} \land \text{LAMP} = \text{on} \); let \( \psi \) be the formula \( \text{SWITCH1} = \text{on} \lor \text{SWITCH2} = \text{on} \lor \text{LAMP} = \text{on} \). Note that the formula \( \varphi \leadsto \psi \) is circular. Moreover, \( M^{\sim \text{LAMP}} \models \varphi \leadsto \psi \), as we can intervene to set all the variables to \( \text{off} \), but \( M^{\text{LAMP}} \not\models \varphi \leadsto \psi \), because \( \text{AC3} \) is violated: we have \( M^{\text{LAMP}} \models \text{SWITCH1} = \text{on} \land \text{SWITCH2} = \text{on} \leadsto \psi \). The corresponding causal statement does not hold in \( M^{\sim \text{LAMP}} \) because \( \text{LAMP} = \text{on} \) after setting both switches \( \text{off} \).

In fact, as we show in the appendix, similar circular formulae in \( \text{LC}_1 \) can distinguish any two distinct causal models. \( \square \)

While Theorem 3.3 formalizes our claim that \( M^{\text{LAMP}} \) and \( M^{\sim \text{LAMP}} \) can be distinguished by nested causal formulae, the existential quantification in the distinguishing statement makes it somewhat difficult to understand exactly what it is about the models that is different. The formula seems to be saying that the first switch being on is a cause of the second switch being on causing *something* about the lamp.

This *something* can not be expressed as \( \text{LAMP} \) taking a particular value: if we took it to be \( \text{LAMP} = \text{on} \), then the resulting nested causal statement would be valid in both \( M^{\text{LAMP}} \) and \( M^{\sim \text{LAMP}} \) by \( \text{AC2} \), as after intervening to set \( \text{SWITCH1} \leftarrow \text{off} \), \( \text{LAMP} \) is not \( \text{on} \) anymore, and so the effect \( \text{SWITCH2} = \text{on} \leadsto \text{LAMP} = \text{on} \) is becomes false by \( \text{AC1} \). On the other hand, if we took it to be \( \text{LAMP} = \text{off} \), then the nested causal statement would be invalid in both, as \( \text{AC1} \) requires that the effect \( \text{SWITCH2} = \text{on} \leadsto \text{LAMP} = \text{off} \) is valid in the real world, and another application of \( \text{AC1} \) necessitates \( \text{LAMP} = \text{off} \), but the lamp is really on.

In the next section, we argue that the deliberately fuzzy wording ("...causing something about the lamp") actually captures the existential quantification, which sidesteps the issue of not being able to choose a fixed value, and that in the presence of nested causality, it turns out to sometimes make sense to avoid committing to particular values.

### 4 Variables, rather than facts, as causes

We previously observed that out of all the possible causal statements of the form \((A = a) \leadsto (C = c)\), where \( a \in R(A) \), \( c \in R(C) \), only one is potentially true, namely the one where \( a \) and \( c \) are the actual values that \( A \) and \( C \), respectively, take in the context. There is therefore a sense in which specifying \( a \) and \( c \) is redundant: we could unambiguously interpret a formula like \( A \leadsto C \) as meaning \((A = a) \leadsto (C = c)\) in each context \( u \) with \( a \), \( c \) such that \((M, u) \vdash A = a \land C = c \). Things are not so simple in the case of nested causality. Suppose that, in the causal formula, \((C = \bar{c}) \leadsto \varphi \), \( \varphi \) itself is a causal formula of the form \((A = \bar{a}) \leadsto (B = \bar{b})\). Now the values of \( a \) and \( b \) for which the formula is true depend on the value \( c \) to which \( C \) is set when evaluating the counterfactual. As the following example shows, this can play a critical role.

**Example 4.1.** Let \( M \) be the following model of the farmer story from the introduction. A farmer may relocate to Texas \((R = 1)\) or stay in New York \((R = 0)\), and this will impact the level of drought his crops are exposed to \((W = \text{standing for drought,} W = 1 \text{ standing for normalcy, and} W = 2 \text{ standing for flooding})\). If he relocates, his crops will suffer dry weather; otherwise, they will be flooded:
\[
W = \begin{cases} 2 & \text{if} \ R = 0 \\ 0 & \text{otherwise.} \end{cases}
\]

The crops, however, can survive \((C = 1)\) only if the weather is fair \((W = 1)\):
\[
C = \begin{cases} 1 & \text{if} \ W = 1 \\ 0 & \text{otherwise.} \end{cases}
\]

Intuitively, assuming the farmer relocated, we hold that the statement "the weather caused the crops to fail" is true; but the statement "because the farmer relocated to Texas, the weather caused the crops to fail" is false. Finally, we add a single exogenous variable \( U \) with singleton range. In this context, \( R = 1 \).

What should the formal interpretation of the nested statement be? If we take the interpretation of "the weather caused the crops to fail" to be \((W = 0) \leadsto (C = 0)\), then this formula is indeed true (as \( M \models [W \leftarrow 1](C = 1) \)). If we then interpret the nested statement as
\[
(R = 1) \leadsto ((W = 0) \leadsto (C = 0)),
\]
then this is in fact true as well: \( M \models [R \leftarrow 0](W = 2) \), and hence \( M \models [R \leftarrow 0]¬((W = 0) \leadsto (C = 0)) \). It does not help to interpret the causal formula as
\[
(R = 1) \leadsto ((W = 2) \leadsto (C = 0)).
\]

This statement is false for vacuous reasons: \( R = 1 \) is not a cause because \((W = 2) \leadsto C = 0 \) is false, as \( W \) is not 2. What seems to capture this example best is to use of existential quantification:
\[
(R = 1) \leadsto (∃w. (W = w) \leadsto (C = 0)).
\]

The common feature of this example and the distinguishing formula in the proof of Theorem 3.3 is that we do not know in advance what value is appropriate to impute to variables in the inner causal statement: when considering the outer statement, we need to consider all possible counterfactual scenarios concerning its cause, but the values of the variables mentioned in the inner statement may be different in each of those scenarios. Therefore, we make the following definition, which allows us to capture the notion that \( A \) taking the situationally appropriate value, whatever it is in the scenario that we might be considering for a particular causal (sub)formula, is the cause of the \( B \) taking whatever value it happens to take. In the next section, we will see examples of several security properties that are expressed more succinctly with this notation, and in some cases cannot be expressed adequately without the existential quantification at all.
Definition 4.2. Suppose \( \overrightarrow{A} \) and \( \overrightarrow{B} \) are lists of variables of an appropriate causal model. Let \( \overrightarrow{A} \rightsquigarrow \overrightarrow{B} \) denote the formula

\[
\exists \overrightarrow{a} \in R(\overrightarrow{A}), \overrightarrow{b} \in R(\overrightarrow{B}). (\overrightarrow{A} = \overrightarrow{a}) \rightsquigarrow (\overrightarrow{B} = \overrightarrow{b}).
\]

We can define \( (\overrightarrow{A} = \overrightarrow{a}) \rightsquigarrow \overrightarrow{B} \) and \( \overrightarrow{A} \rightsquigarrow (\overrightarrow{B} = \overrightarrow{b}) \) analogously. \( \square \)

Using this definition, we can now state the causal statement of Example 4.1 in a way that mirrors the natural-language version, by saying \( \overrightarrow{R} \equiv 1 \rightsquigarrow (W \rightsquigarrow (C = 0)) \), or even more succinctly as \( R \rightsquigarrow (W \rightsquigarrow C) \). Likewise, the distinguishing formula from the proof of Theorem 3.3 can now be stated as \( \text{SWITCH1} = \text{on} \rightsquigarrow (\text{SWITCH2} = \text{on} \rightsquigarrow LAMP) \), or more compactly as \( \text{SWITCH1} \rightsquigarrow (\text{SWITCH2} \rightsquigarrow LAMP) \).

5 Examples of causal security

Now that we have given the definitions, we are ready to revisit several examples of propositions about security that we want to argue are naturally viewed as nested causal statements.

Example 5.1. A government employee has the authority to declassify government secrets and release them to the press. The employee turns out to be corrupt: if someone pays him a sufficient amount of money, he will declassify a secret and have it published in the press. As it happens, a UFO enthusiast community scrapes together a bribe and pays the employee, who subsequently publishes the announcement that the government has been in contact with aliens.

In what sense can we say that something inappropriate occurred? By assertion, we considered it permissible for the employee to declassify and release the secret (and thus for the truth about aliens to be a cause of the press release). In a free market economy, tax regulations not withstanding, people are free to give money to whomever they please. Lastly, the UFO enthusiasts instead paid a struggling newspaper directly to announce that the government found UFOs, this would also not be problematic. The problem here is that that whether the information was released depended on whether the bribe was paid. In other words, the problem is that the bribe was a cause of the government being in contact with aliens being a cause of the press release.

Formally, we can represent the example as a causal model \( M_{\text{aliens}} \) with a single exogenous variable \( U \) whose range is a singleton, and three endogenous binary variables \( S, P, \) and \( B \), representing whether the government is secretly in contact with aliens, whether there is a press article to the effect, and whether the UFO enthusiasts paid a bribe respectively. Due to the employee’s corruption, we have

\[
F_P = \begin{cases} 
S & \text{if } B=1 \\
0 & \text{otherwise}.
\end{cases}
\]

The undesirable causal relationship then is represented by the formula \( B \rightsquigarrow (S \rightsquigarrow P) \).

The security property being violated here is an instance of Zdancewic and Myers’s notion of robust declassification (Zdancewic and Myers 2001; Myers et al. 2004; Cecchetti et al. 2017). Roughly speaking, a declassification (release of a secret) is considered robust if whether the declassification occurred was not up to an untrusted actor. What is considered a secret and what actors are trusted (to declassify the secret) has to be specified as part of the security policy. In causal terms, we can say that a system satisfies robust declassification if there is no instance of an untrusted variable (such as the UFO enthusiasts’ decision to pay) being a cause of a secret variable being a cause of a public variable. Which variables belong to each of the three classes has to be specified as part of the security policy. Intuitively, secret variables are those for which we would consider it a priori unacceptable for parties unaffiliated with the principal that the security policy seeks to protect to learn their value, unless this was explicitly desired by the system designer. We can assume that their value is not directly visible to outsiders, for otherwise the system would be trivially insecure. Public variables are all those that are assumed to be visible to outside observers. Trusted variables are those whose value is taken to be under the control of the principal; untrusted variables may have had their value influenced by outsiders whose interests may not align with those of the principal.

The presence of an untrusted variable as a cause can turn an otherwise acceptable causal relationship unacceptable. Conversely, the presence of a trusted cause can turn an otherwise unacceptable causal relationship acceptable.

Example 5.2. Alice’s computer-illiterate boss, Bob, has asked Alice to fix his computer. While she is at it, she realises that his desktop background is the default colour (say, white). She decides to set the desktop background to her favourite colour. (For simplicity, in the remainder of the discussion, we assume that there are only two possible colours.)

Consider two scenarios:

1. Alice is sensitive to the circumstance that she is working on somebody else’s machine. Her understanding with Bob is that has she is entitled to change some setting (like background colour) unless Bob explicitly tells her not to.
2. Alice is quite fed up with Bob’s lack of taste and clueless management. If Bob were to tell her to leave the desktop background alone, she would just get spiteful and instead set it to the opposite of her favourite colour. \( \square \)

Our intuition says that in the first case, the colour change was (implicitly) authorised by Bob. Were he to complain about it, Alice could rightly respond that she wouldn’t have done it if he had told her not to, and he had ample opportunity to. On the other hand, Bob would not be wrong to complain about her meddling and insubordination in the second case. This is not just a matter of control; knowing Alice’s behaviour, Bob can make her set the background to any colour he prefers by tactically choosing whether to tell her to back off.

Formally, we could represent the cases as causal models \( M_{\text{obedient}} \) and \( M_{\text{defiant}} \) with a single exogenous variable whose range is a singleton, and three binary endogenous variables, \( A \) representing Alice’s favourite colour, \( B \) representing whether Bob tells Alice that it is okay to change the colour, and \( C \) representing the resulting background colour.
In the first, “obedient” case, we have

\[ F_C^{\text{obedient}} = \begin{cases} A & \text{if } B = 1 \\ 1 & \text{otherwise.} \end{cases} \]

On the other hand, in the “defiant” case,

\[ F_C^{\text{defiant}} = \begin{cases} A & \text{if } B = 1 \\ 1 - A & \text{otherwise.} \end{cases} \]

\((F_A \text{ and } F_B \text{ just set } A \text{ and } B \text{ to Alice and Bob’s actual actions in each case.})\)

It is easy to check that these two models are just relabellings of the models \(M_A^{\text{lamp}} \text{ and } M_B^{\text{lamp}}\) from earlier, respectively; thus, we have \(M_C^{\text{obedient}} \models B \Rightarrow (A \Rightarrow C)\), but \(M_C^{\text{defiant}} \not\models B \Rightarrow (A \Rightarrow C)\). More generally, we could consider this an instance of a security policy that we could call \textit{authorisation}: the untrusted variable \(A\) is only a cause of the privileged outcome \(C\) if this causal relationship itself had a trusted cause \(B\), interpreted as the causation happening at \(B\)’s pleasure, with \(B\) having the option to prevent it and choosing to not making use of it.

The “authorisation” construction that we have just described can be easily iterated to generate more complex meaningful examples.

\textbf{Example 5.3.} Suppose Bob is not present during Alice’s fixing of his computer, and instead has told Alice to let Bob’s secretary Dylan supervise her. Would Alice listen to Dylan if he were to tell her to leave Bob’s desktop background unchanged (which, in fact, he doesn’t)? Once again, consider two cases:

1. Alice respects Bob’s delegation of authority, and sets the desktop to her preferred background colour only if Dylan doesn’t tell her to leave it alone. If Bob had instead told her not to listen to Dylan, she would have strictly acted according to her own best judgement, and set the desktop background to her preferred colour no matter what he said.

2. Alice thinks much more highly of Dylan than the boss they work for, and will listen to him even if Bob tells her not to. As it happens, Bob trusts Alice’s artistic judgement much more than Dylan’s, and will be quite displeased to hear that his overbearing underling stopped Alice from setting him up with an artfully chosen background.

\[\square\]

In both scenarios, the final setting of the desktop background is caused by Alice’s preferred colour, and this causal relationship in turn is caused by Dylan’s acquiescence. What intuitively distinguishes the two scenarios (and would continue distinguishing them if Alice’s defiance were to come to the fore under some combinations of Bob’s and Dylan’s instructions) is whether Dylan’s control over this itself was “at Bob’s pleasure”, that is, could have been vetoed by Bob, or Bob’s authority was usurped. Formally, we can capture them as two causal models, both with a single exogenous variable whose range is a singleton, four endogenous variables, and

\[ F_C^1 = \begin{cases} A & \text{if } B = 1 \text{ and } D = 1 \\ 1 & \text{else} \end{cases} \]

\[ F_C^2 = \begin{cases} A & \text{if } D = 1 \\ 1 & \text{else} \end{cases} \]

We then find while \(M_1 \models B \Rightarrow (D \Rightarrow (A \Rightarrow C))\), we have \(M_2 \not\models B \Rightarrow (D \Rightarrow (A \Rightarrow C))\).

This construction can be iterated further in a straightforward manner, allowing us to express any number of steps of delegation. Such chains of delegation are often considered in \textit{authorisation logics} (see e.g. Abadi’s survey (Abadi 2003)), but rarely given a formal semantics (Hirsch and Clarkson 2013), let alone one that can be applied in a setting as general as causal models.

\section{Conclusions}

We have shown that a variety of security properties can be expressed as nested causal statements. To give a formal account of such statements, we extended the Halpern-Pearl framework of causality to allow formulae that may themselves refer to causal relationships as effects of causal statements. As we have shown, the language of nested causal statements thus obtained is more expressive than the language of simple causal statements that the HP framework normally deals in. This extension also led us to revisit a particular design assumption of the HP framework, namely that it is always appropriate to have causal statements refer to variables taking particular values as causes and effects. We have argued that in nested causal statements, it is often more natural to implicitly existentially quantify over values, using just the variables as causes and effects (interpreted as meaning that whatever value the variable actually takes is the cause or effect).

We view this paper as laying the groundwork for the characterization of security properties using (nested) causality. In future work, we plan to formalize various security properties using causality, show how to represent arbitrary programs as causal models, and use these causal models to provide efficient sound procedures for verifying that the programs satisfy these security properties.

\section*{Appendix: The power of circular causal statements}

\begin{proposition}

Let \((M_1, \overline{u}_1)\) and \((M_2, \overline{u}_2)\) be two contexts of recursive models with the same signature \((\mathcal{U}, \mathcal{V}, \mathcal{R})\), same immediate dependency ordering, same unique consistent assignment \(\overline{r}\) and different structural equations. Then there is a simple causal formula \((X = \overline{x}) \Rightarrow \varphi \in \mathbb{C}_1\) that is valid in \((M_1, \overline{u}_1)\) but not in \((M_2, \overline{u}_2)\).

\end{proposition}

\begin{proof}

Let \(D\) be a minimal variable in the dependency ordering (which is the same for both models) whose structural equation takes different values in \((M_1, \overline{u}_1)\) and \((M_2, \overline{u}_2)\) on some input (assignment to endogenous variables that the structural equation depends on) \(\overline{v}ʼ\), say \(F^1_D(\overline{u}_1, \overline{v}ʼ) \neq F^2_D(\overline{u}_2, \overline{v}ʼ)\), and take \(\overline{X} \subseteq \mathcal{V}\) to be the strict descendants of

\end{proof}
$D$ in the dependency ordering. Let $\vec{x}$ be the corresponding entries of $\vec{u}$, and let
\[ \varphi = \neg((D = F_1^D(\vec{u}_1, \vec{v}')) \lor \bigvee_{Y \in \vec{x}} \neg(Y = \vec{v}'(Y))). \]

That is, $\varphi$ says that $D$ or one of its descendants has a different value than it would have on input $\vec{v}'$.

Then $(M^1, \vec{u}_1) \models X = \vec{x}$ and $(M^2, \vec{u}_2) \models X = \vec{x}$ by construction, and $(M^1, \vec{u}_1) \models \varphi$, $(M^2, \vec{u}_2) \models \varphi$ because $\vec{v} \neq \vec{v}'$ (recall we are assuming that the two models have the same unique consistent assignment, so $F_1^D(\vec{v}) = F_1^D(\vec{v}) = \vec{v}(D)$) and so at least one of the disjuncts must be true in both models (which are only consistent with the assignment $\vec{v}$). Moreover, $(M^1, \vec{u}_1) \models [\vec{x} \leftarrow \vec{x}'] \neg \varphi$, where $\vec{x}'$ are the entries of $\vec{v}'$ corresponding to $X$: the clauses of the disjunction are satisfied by the intervention directly setting $Y = \vec{v}'(Y)$ for all $Y \in \vec{x}$, and $D = F_1^D(\vec{v}')$ as a consequence of the structural equations of $M^1$. So either $(M^1, \vec{u}_1) \models (\vec{x} = \vec{x}) \hookrightarrow \varphi$, or this only fails AC3 and so there is some proper subvector $\vec{x}'$ such that $(M^1, \vec{u}_1) \models (\vec{x}' = \vec{x}) \hookrightarrow \varphi$ for which this is true. However, for all $\vec{x}''$, $(M^2, \vec{u}_2) \models [\vec{x} \leftarrow \vec{x}''] \varphi$: either $\vec{x}'' \neq \vec{x}'$, in which case one of the disjuncts about $Y \in \vec{x}$ is true in $(M^2, \vec{x} \leftarrow \vec{x}'', \vec{u}_2)$, or $\vec{x}'' = \vec{x}'$, in which case we have $(M^2, \vec{x} \leftarrow \vec{x}'', \vec{u}_2) \models D = F_1^D(\vec{v}')$, and hence $\neg((D = F_1^D(\vec{v}')))$. Since this is true for all values we could assign to $X$, restricting to a subvector $\vec{x}'$ of $X$ will not help, and $(M^2, \vec{u}_2) \not\models (\vec{x}' = \vec{x}) \hookrightarrow \varphi$ for all $\vec{x}' \subseteq \vec{x}$.

Note that if the two models do not have the same unique consistent assignment $\vec{v}$, we don’t even need causal statements to distinguish them: just use a single atomic proposition about a variable on which their consistent assignments differ.

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