Large-$N_c$ Regge models and the $\langle A^2 \rangle$ condensate

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Abstract

We explore the role of the $\langle A^2 \rangle$ gluon condensate in matching Regge models to the operator product expansion of meson correlators.

This talk is based on Ref. [1], where the details may be found. The idea of implementing the principle of parton-hadron duality in Regge models has been discussed in Refs. [2–8]. Here we carry out this analysis with the dimension-2 gluon condensate present. The dimension-two gluon condensate, $\langle A^2 \rangle$, was originally proposed by Celenza and Shakin [9] more than twenty years ago. Chetyrkin, Narison and Zakharov [10] pointed out its sound phenomenological as well as theoretical [11–15] consequences. Its value can be estimated by matching to results of lattice calculations in the Landau gauge [16, 17], and their significance for non-perturbative signatures above the deconfinement phase transition was analyzed in [18]. Chiral quark-model calculations were made in [19] where $\langle A^2 \rangle$ seems related to constituent quark masses.

In spite of all this flagrant need for these unconventional condensates the dynamical origin of $\langle A^2 \rangle$ remains still somewhat unclear; for recent reviews see, e.g., [20, 21].

For large $Q^2$ and fixed $N_c$ the modified OPE (with the $1/Q^2$ term present) for the chiral combinations of the transverse parts of the vector and axial currents is

$$\Pi^T_{V+A}(Q^2) = \frac{1}{4\pi^2} \left\{ -\frac{N_c}{3} \log \frac{Q^2}{\mu^2} \frac{\alpha_S}{\pi} \frac{\lambda^2}{Q^2} + \frac{\pi}{3} \langle \alpha_S G^2 \rangle \frac{Q^4}{Q^2} + \ldots \right\}$$

$$\Pi^T_{V-A}(Q^2) = \frac{32\pi}{9} \frac{\alpha_S \langle \bar{q}q \rangle^2}{Q^6} + \ldots$$

On the other hand, at large-$N_c$ and any $Q^2$ these correlators may be saturated by infinitely many mesonic states,

$$\Pi^T_V(Q^2) = \sum_{n=0}^\infty \frac{F^2_{V,n}}{M_{V,n}^2 + Q^2} + c.t., \quad \Pi^T_A(Q^2) = \frac{f^2}{Q^2} + \sum_{n=0}^\infty \frac{F^2_{A,n}}{M_{A,n}^2 + Q^2} + c.t. \quad (2)$$

The basic idea of parton-hadron duality is to match Eq. (1) and (2) for both large $Q^2$ and $N_c$ (assuming that both limits commute). We use the radial Regge spectra, which are well supported experimentally [22]

$$M_{V,n}^2 = M_{V}^2 + a_V n, \quad M_{A,n}^2 = M_{A}^2 + a_A n, \quad n = 0, 1, \ldots \quad (3)$$

The vector part, $\Pi^T_V$, satisfies the once-subtracted dispersion relation

$$\Pi^T_V(Q^2) = \sum_{n=0}^\infty \left( \frac{F^2_{V,n}}{M_{V}^2 + a_V n + Q^2} - \frac{F^2_{V,n}}{M_{V}^2 + a_V n} \right). \quad (4)$$
We need to reproduce the \( \log Q^2 \) in OPE, for which only the asymptotic part of the meson spectrum matters. This leads to the condition that at large \( n \) the residues become independent of \( n \), \( F_{V,n} \approx F_V \) and \( F_{A,n} \approx F_A \). Thus all the highly-excited radial states are coupled to the current with equal strength! Or: asymptotic dependence of \( F_{V,n} \) or \( F_{A,n} \) on \( n \) would damage OPE. Next, we carry out the sum explicitly (the dilog function is \( \psi(z) = \Gamma'(z)/\Gamma(z) \))

\[
\sum_{n=0}^{\infty} \left( \frac{F^2_i}{M^2_i + a_i n + Q^2} - \frac{F^2_i}{M^2_i + a_i n} \right) = \frac{F^2_i}{a_i} \left[ -\log \left( \frac{Q^2}{a_i} \right) + \psi \left( \frac{M^2_i}{a_i} \right) + \frac{a_i - 2M^2_i}{2Q^2} + \frac{6M^4_i - 6a_i M^2_i + a_i^2}{12Q^4} + \ldots \right],
\]

where \( i = V, A \). \( \Pi_{V-A} \) satisfies the unsubtracted dispersion relation (no \( \log Q^2 \) term), hence

\[ F^2_i/V_A = F^2_i/a_A. \]

This Compliance to the chiral symmetry restoration in the high-lying spectra [23, 24]. Further, we assume \( a_V = a_A = a \), or \( F_V = F_A = F \), which is well-founded experimentally, as \( \sqrt{a} = 464\text{MeV}, \sqrt{a} = 470\text{MeV} \) [22].

The simplest model we consider has strictly linear trajectories all the way down,

\[
\Pi_{V-A}(Q^2) = \frac{F^2}{a} \left[ -\psi \left( \frac{M^2_V}{a} \right) + \psi \left( \frac{M^2_A}{a} \right) \right] = \frac{F^2}{Q^2} \left( a - M^2_V \right)^2.
\]

Matching to OPE yields the two Weinberg sum rules:

\[
\begin{align*}
f^2 &= \frac{F^2}{a} (M^2_A - M^2_V), & (\text{WSR I}) \\
0 &= (M^2_A - M^2_V)(a - M^2_A - M^2_V). & (\text{WSR II})
\end{align*}
\]

The \( V + A \) channel needs regularization. We proceed as follows: carry \( d/dQ^2 \), compute the convergent sum, and integrate back over \( Q^2 \). The result is

\[
\Pi_{V+A}(Q^2) = \frac{F^2}{a} \left[ -\psi \left( \frac{M^2_V}{a} \right) - \psi \left( \frac{M^2_A}{a} \right) \right] + \frac{F^2}{Q^2} + \text{const.} = -\frac{2F^2}{a} \log \frac{Q^2}{\mu^2}
\]

\[
+ \left( f^2 + F^2 - \frac{F^2}{a} (M^2_A + M^2_V) \right) \frac{1}{Q^2} + \frac{F^2}{6a} (a^2 - 3a(M^2_A + M^2_V) + 3(M^2_A + M^2_V)) \frac{1}{Q^4} + \ldots
\]

Matching of the coefficient of \( \log Q^2 \) to OPE gives the relation

\[ a = 2\pi\sigma = \frac{24\pi^2 F^2}{N_c}, \]

where \( \sigma \) denotes the (long-distance) string tension. From the \( \rho \to 2\pi \) decay one extracts \( F = 154 \text{MeV} \) [25] which gives \( \sqrt{\sigma} = 546 \text{MeV} \), compatible to the value obtained in lattice simulations: \( \sqrt{\sigma} = 420 \text{MeV} \) [26]. Moreover, from the Weinberg sum rules

\[ M^2_A = M^2_V + \frac{24\pi^2 F^2}{N_c}, \quad a = M^2_A + M^2_V = 2M^2_V + \frac{24\pi^2 F^2}{N_c}. \]

Matching higher twists fixes the dimension-2 and 4 gluon condensates:

\[
-\frac{\alpha_s \lambda^2}{4\pi^3} = f^2, \quad \frac{\alpha_s (Q^2)}{12\pi} = \frac{M^2_A - 4M^2_V M^2_A + M^2_V}{48\pi^2}.
\]

Numerically, it gives \( -\frac{\alpha_s \lambda^2}{4\pi^3} = 0.3 \text{GeV}^2 \) as compared to 0.12GeV² from Ref. [10, 20]. The short-distance string tension is \( \sigma_0 = -2\alpha_s \lambda^2/N_c = 782 \text{MeV} \), which is twice as much as \( \sigma \). The major problem of the strictly linear model is that the dimension-4 gluon condensate is
negative for $M_V \geq 0.46$ GeV. Actually, it never reaches the QCD sum-rules value. Thus, the strictly linear radial Regge model is too restrictive!

We therefore consider a modified Regge model where for low-lying states both their residues and positions may depart from the linear trajectories. The OPE condensates are expressed in terms of the parameters of the spectra. A very simple modification moves only the position of the lowest vector state, the $\rho$ meson.

$$M_{V,0} = m_\rho, \quad M_{V,n}^2 = M_V^2 + an, \quad n \geq 1$$
$$M_{A,n}^2 = M_A^2 + an, \quad n \geq 0. \quad (10)$$

For the Weinberg sum rules (we use $N_c = 3$ from now on)

$$M_A^2 = M_V^2 + 8\pi^2 f^2, \quad a = 8\pi^2 f^2 = \frac{8\pi^2 f^2 (4\pi^2 f^2 + M_V^2)}{4\pi^2 f^2 - m_\rho^2 + M_V^2}. \quad (11)$$

We fix $m_\rho = 0.77$ GeV, and $\sigma$ is the only free parameter of the model. Then

$$M_V^2 = \frac{-16\pi^3 f^4 + 4\pi^2 f^2 - m_\rho^2 \sigma}{4f^2 \pi - \sigma}, \quad \frac{\alpha_s \lambda^2}{12\pi} = \frac{16\pi^3 f^4 - \sigma^2 + m_\rho^2 \sigma}{16f^2 \pi - \sigma}, \quad \frac{\alpha_s (G^2)}{12\pi} = 2\pi^2 f^4 - \pi \sigma f^2 + \frac{3\sigma (\frac{m_\rho^2 \sigma}{\sigma - 4f^2 \pi^2} - 2\pi) m_\rho^2}{8\pi^2} + \frac{\sigma^2}{12}. \quad (12)$$

The window for which both condensates are positive yields very acceptable values of $\sigma$. The consistency check of near equality of the long- and short-distance string tensions, $\sigma \approx \sigma_0$, holds for $\sqrt{\sigma} \approx 500$MeV. The magnitude of the condensates is in the ballpark of the “physical” values. The value of $M_V$ in the “fiducial” range is around 820 MeV. The experimental spectrum in the $\rho$ channel is has states at 770, 1450, 1700, 1900$, and 2150$ MeV, while the model gives 770, 1355, 1700, 2147 MeV (for $\sigma = (0.47 \text{ GeV})^2$). In the $a_1$ channel the experimental states are at 1260 and 1640 MeV, whereas the model yields 1015 and 1555 MeV.

The value of $M_V$ and the $\rho$ state are tuned to reproduce the two lowest states.

We note that the $V-A$ channel well reproduced with radial Regge models. The Das-Mathur-Okubo sum rule gives the Gasser-Leutwyler constant $L_{10}$, while the Das-Guralnik-Mathur-Low-Yueng sum rule yields the pion electromagnetic mass splitting. In the strictly linear model with $M_A^2 = 2M_V^2$ and $M_V = \sqrt{24\pi^2 / N_c} f = 764$ MeV we have $\sqrt{\sigma} = \sqrt{3/2\pi} M_V = 532$ MeV, $F = \sqrt{3} f = 150$ MeV, $L_{10} = -N_c / (96\sqrt{3}\pi) = -5.74 \times 10^{-3}(-5.5 \pm 0.7 \times 10^{-3})_{\exp}$, $m_{\pi^\pm}^2 - m_{\pi^0}^2 = (31.4 \text{ MeV})^2(35.5 \text{ MeV})^2_{\exp}$. In our second model with $\sigma = (0.48 \text{ GeV})^2$ we find $L_{10} = -5.2 \times 10^{-3}$ and $m_{\pi^\pm}^2 - m_{\pi^0}^2 = (34.4 \text{ MeV})^2$.

To conclude, let us summarize our results and list some further related studies.

- Matching OPE to the radial Regge models produces in a natural way the $1/Q^2$ correction to the $V$ and $A$ correlators. Appropriate conditions are satisfied by the asymptotic spectra, while the parameters of the low-lying states are tuned to reproduce the values of the condensates.
• In principle, these parameters of the spectra are measurable, hence the information encoded in the low-lying states is the same as the information in the condensates.
• Yet, sensitivity of the values of the condensates to the parameters of the spectra, as seen by comparing the two explicit models considered in this paper, makes such a study difficult or impossible at a more precise level.
• Regge models work very well in the $V-A$ channel. In [28] it is shown how the spectral (in fact chiral) asymmetry between vector and axial channel is generated via the use of $\zeta$-function regularization for each channel separately.
• We comment that effective low-energy chiral models produce $1/Q^2$ corrections (i.e. provide a scale of dimension 2), e.g., the instanton-based chiral quark model gives [19]

$$-\frac{\alpha_S}{\pi} \lambda^2 = -2N_c \int du \frac{uu}{u + M(u)^2} M(u) M'(u) \simeq 0.2 \text{ GeV}^2.$$ 

(13)
• In the presented Regge approach the pion distribution amplitude is constant, $\phi(x) = 1$, at the low-energy hadronic scale, similarly as in chiral quark models [27].

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