On Structure and History of Space-time with Variable Speed of Light

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We apply the variable speed of light into general relativity in order to solve the problems we met in the standard cosmology. We’re surprised to find that, the results from the general relativity in cosmology are exactly the same as those we got from Newtonian dynamics. The relation between the Newtonian dynamics and the relativistic dynamics can be demonstrated with the variable speed of light. With this approach, some problems in the standard cosmology such as the flatness problem and the horizon problem doesn’t arise any more. All the cosmological results and the physical results are reasonable and natural. There are no any difficulties in the standard cosmology.

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I. INTRODUCTIONS

There’re two serious problems in the standard cosmology, flatness problem and horizon problem [1]. The cosmology has gone too far before these two problems were solved. In an early paper [2], I introduced the variable speed of light to the Newtonian dynamics, and successfully used it to solve these two problems. Here in this paper, I further introduce the same speed of light to the general relativity in order to solve the problems in cosmology under the general relativity. Surprisingly, we found that, the results from the general relativity are exactly the same as those from the Newtonian dynamics. These encourage us to believe that the relations between Newtonian dynamics and relativity can be demonstrated. And the speed of light is changed with the age of universe. But this variable of speed of light doesn’t violate the invariable principle of the speed of light under relativity. Because the former is the change in the magnitude of the speed, and the latter is the independence of speed to the observers.

II. BASIC STRUCTURE OF COSMOS

With the spherical coordinates system, the metric of cosmos has the simple form as

\[ ds^2 = -e^\nu dt^2 + e^\mu dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \] (2.1)

The Schwarzschild’s exterior solution of the cosmos system is

\[ ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \] (2.2)

M and r are mass and radius of the cosmos respectively.

In the cosmos system, the gravitational constant G and light speed C have the following relations [2]

\[ G = ZC \] (2.3)

\[ C = \frac{2ZM}{R} \] (2.4)

R is the radius of cosmos system. Z is a constant, \( Z = 2.224 \times 10^{-19} m^2 s^{-1} kg^{-1} \).

From (2.2), we can see that the surface of the cosmos is a special interface. The characteristic of the metric tensor is

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\[ g_{00} = 0, g^{00} = \infty \]
\[ g_{11} = \infty, g^{11} = 0 \]  \hfill (2.5)

The observer outside this interface can never get any information from inside the interface. So, the whole cosmos system is a giant black hole to the outside world. But for the variable of speed of light with matter distribution inside the cosmos, there is no any special area in the cosmos [2]. It’s to say, motion of matter in any direction inside the cosmos is possible, but matter needs an infinite time to reach this special interface. The Black Hole Paradox about the whole universe system was solved.

The Schwarzschild’s interior solution is

\[ e^{\nu/2} = \frac{3}{2} \left( 1 - \frac{2GM}{R} \right)^{\frac{1}{2}} - \frac{1}{2} \left( 1 - \frac{2GMr^2}{R^3} \right)^{\frac{1}{2}} \]  \hfill (2.6)

\[ e^{-\mu} = 1 - \frac{2GMr^2}{R^3} r \leq R \]  \hfill (2.7)

The distribution of pressure inside the cosmos system is given as [3]

\[ p(r) = \frac{\rho (1 - \frac{2GM}{R})^{\frac{3}{2}} - (1 - \frac{2GM}{R})^{\frac{3}{2}}}{3(1 - \frac{2GM}{R})^{\frac{3}{2}} - (1 - \frac{2GM}{R})^{\frac{3}{2}}} \]  \hfill (2.8)

Notice that the medium of the cosmos is extremely relativistic, the thermal motion speed of most of the matter in cosmos included the photons and neutrinos, and other fields is exactly the speed of light, so the pressure inside the cosmos is isotropy. And

\[ p = \rho/3 \]  \hfill (2.9)

### III. THE STANDARD MODEL OF COSMOS

The standard model of the cosmos was described with the Robertson-Walker metric [3]

\[ ds^2 = -dt^2 + R^2(t)(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \]  \hfill (3.1)

The energy-momentum tensor for idea fluids of the cosmos is given by

\[ T^{\mu\nu} = (p + \rho)U^\mu U^\nu + pg^{\mu\nu} \]  \hfill (3.2)

\[ U^\mu = (1, 0, 0, 0). \] Apply (3.1) and (3.2) to the following field equation

\[ R_{\mu\nu} = -8\pi G(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) \]  \hfill (3.3)

We get the time-time component as

\[ 3\ddot{R} = -4\pi G(\rho + 3p)R \]  \hfill (3.4)

and space-space component as

\[ R\ddot{R} + \dot{R}^2 + 2k = 4\pi G(\rho - p)R^2 \]  \hfill (3.5)

Then we get from the last two equations the following differential equation

\[ \dot{R}^2 + k = \frac{8\pi G}{3}\rho R^2 \]  \hfill (3.6)

The conservation law of energy and momentum requires the energy momentum tensor in (3.2) satisfies with

\[ T^{\mu\nu}_{\;\;\nu} = 0 \]  \hfill (3.7)

Its time component gives

\[ \frac{d\rho}{dt} + 3\frac{\dot{R}}{R}(\rho + p) = 0 \]  \hfill (3.8)

Together with (2.9), we have all the equations needed to get the space, pressure and density relation \( R(t), p(t) \) and \( \rho(t) \).
IV. THE CURVATURE OF COSMOS

We reasonable assume that the interface of cosmos expands with the speed of light. The Hubble parameter can be defined as

\[ H = \frac{\dot{R}}{R} = \frac{C}{R} = \frac{2ZM}{R^2} \quad \text{(4.1)} \]

The basic equation (3.6) can be changed into

\[ \rho = \rho_c + \frac{3}{8\pi G} \frac{k}{R^2} \quad \text{(4.2)} \]

\[ \rho_c \equiv \frac{3H^2}{8\pi G} \quad \text{(4.3)} \]

From (2.3) and (2.4), we get

\[ \rho_c \equiv \frac{3H^2}{8\pi G} = \frac{3M}{4\pi R^3} = \rho \quad \text{(4.4)} \]

(4.2) gives

\[ k \equiv 0 \quad \text{(4.5)} \]

It’s to say, in any period of the cosmos, the space is always flat. We can prove this conclusion by the followings.

The matter and entropy densities of the cosmos from the Planck formula are given as

\[ \rho = \frac{\pi^2}{30} NT^4 \quad \text{(4.6)} \]

\[ s = \frac{2\pi^2}{45} NT^3 \quad \text{(4.7)} \]

N is the total degree of freedom. The particle physics gives \( N = 10^2 \)

The expansion of cosmos is entropy conservative. It’s to say

\[ S = sR^3 = \text{const.} \quad \text{(4.8)} \]

(4.7) and (4.8) show that

\[ TR = \text{const.} \quad \text{(4.9)} \]

(4.2) can be changed into

\[ 1 - \frac{\rho_c}{\rho} = \frac{3k}{8\pi G} \frac{1}{\rho R^2} \quad \text{(4.10)} \]

Together with (4.6), (4.7) and (4.8), (4.10) can be further changed into

\[ 1 - \frac{\rho_c}{\rho} = \frac{3k}{8\pi G} \left( \frac{160}{3\pi^2 N} \right)^{\frac{1}{2}} S^{-\frac{1}{2}} T^{-2} \quad \text{(4.11)} \]

We use the entropy from the background radiations to substitute the entropy of cosmos

\[ S \approx S_\gamma = s_\gamma R^3 = \frac{2\pi^2}{45} T_\gamma^3 R^3 \quad \text{(4.12)} \]

Substitute the data of today’s cosmos to (4.12), we get

\[ S \geq 10^{87} \quad \text{(4.13)} \]

In the early cosmos when \( t = 10^{-20} s, T = 10^5 GeV, G = 6.71 \times 10^{-21} Gev^{-2} \), so

\[ 1 - \frac{\rho_c}{\rho} \leq 10^{-61} \quad \text{(4.14)} \]

This proves that the space is strictly flat even in the early cosmos when the gravitation is much stronger. This solved the flatness problem.
V. THE SPACE-TIME AND LIGHT SPEED OF COSMOS

The decelerate parameter of the cosmos is defined as

$$ q = -\frac{\ddot{R}R}{R^2} \quad (5.1) $$

The expansion of the cosmos with the light speed is extremely relativistic, so

$$ p = \rho/3 \quad (5.2) $$

From (3.4) we obtain

$$ q = \frac{\rho}{\rho_c} \quad (5.3) $$

From (3.4) and (3.6), we obtain

$$ \frac{k}{R^2} = H^2(q - 1) \quad (5.4) $$

Equation (3.8) has an equivalent form as

$$ \frac{d}{dR}(\rho R^3) = -3pR^2 \quad (5.5) $$

From equation (5.2) and (5.5) we obtain

$$ \rho R^4 = \text{const.} \quad (5.6) $$

Rewrite the basic equation (3.6) with (5.4) and (5.6), we obtain

$$ \dot{R}^2 = \frac{8\pi G\rho_0 R_0^4}{3R^2} - R_0^2 H_0^2(q_0 - 1) \quad (5.7) $$

In (5.7), all the parameters are those for today’s cosmos. Notice that $\rho_0$, $H_0$, and $q_0$ have the relations shown in equation (4.3) and (5.3). Substitute $\rho_0$ from (5.7), we obtain

$$ \left( \frac{\dot{R}}{R} \right)^2 = H_0^2 \left( 1 - q_0 + q_0 \frac{R_0^2}{R^2} \right) \quad (5.8) $$

Let $X = R/R_0$, then $dR/R_0 = dX$, (5.8) can be changed into

$$ dt = \frac{1}{H_0} (1 - q_0 + q_0 X^{-2})^{-\frac{1}{2}} dX \quad (5.9) $$

From (4.4) and (5.3), we can see that $q_0 = 1$, equation (5.9) can be changed into

$$ dt = \frac{X}{H_0} dX \quad (5.10) $$

Notice that $H_0 = 2ZM/R_0^2$, so

$$ t = \frac{R^2}{2H_0} \bigg| \frac{R}{R_0} \bigg| 0 \quad (5.11) $$

or

$$ R(t) = 2Z^\frac{3}{2} M^\frac{1}{2} t^\frac{1}{2} \quad (5.13) $$

From (4.1), the Hubble parameter is given as

$$ H = \frac{1}{2} t^{-1} \quad (5.14) $$

And the speed of light is given as

$$ C = Z^\frac{3}{2} M^\frac{1}{4} t^{-\frac{3}{4}} \quad (5.15) $$

From above we can see that the Hubble parameter and the speed of light are changed with the age of universe.
VI. THE THERMODYNAMICS OF COSMOS

The space is strictly flat, so the basic equation (3.6) can be rewrite as

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} \rho$$

(6.1)

Apply (4.6) and (4.9) to (6.1), the temperature equation of cosmos can be obtained as

$$\frac{dT}{dt} = -\left(\frac{4\pi^3 NG}{45}\right)^{\frac{1}{2}} T^3$$

(6.2)

$$T(t) = \left(\frac{45}{16\pi^3 NG}\right)^{\frac{1}{4}} t^{-\frac{1}{2}}$$

(6.3)

The density equation can be obtained from (4.6) and (6.3) as

$$\rho(t) = \frac{3t^{-2}}{32\pi G}$$

(6.4)

VII. THE HORIZON OF COSMOS

The motion of photons satisfies $ds = 0$, this can be obtained from equation (3.1) as

$$\frac{dr}{\sqrt{1 - kr^2}} = -\frac{dt}{R(t)}$$

(7.1)

The horizon radius of the cosmos is defined as

$$\int_0^{r(H)} \frac{dr}{\sqrt{1 - kr^2}} = \int_0^t \frac{dt}{R(t)}$$

(7.2)

The horizon radius is in fact the instantaneous distance $l_{H(t)}$ from $r = 0$ to $r = R(H)$

$$l_{H(t)} = R(t) \int_0^{r(H)} \frac{dr}{\sqrt{1 - kr^2}} = R(t) \int_0^t \frac{dt'}{R(t')}$$

(7.3)

(5.13) shows that $R$ is in direct proportion to $t^{\frac{1}{2}}$. Solve the equation (7.3) we get

$$l_{H(t)} = 2t$$

(7.4)

Notice the nature unit system and speed of light (5.15), the horizon of the cosmos (7.4) is exactly the radius of the cosmos (5.15) in any moment of the cosmos. We can also prove this conclusion with the observations by the followings.

In the moment of $10^{-20}s$, the horizon of cosmos is about $10^8 m$ from (7.4). And the temperature of the cosmos is about $10^5 Gev$ from (6.3). Now let’s calculate the scale of our observed 10 billion light-year ($10^{26} m$) cosmos with a temperature of about 2.8K. From (4.9) , we obtain

$$R = \frac{T_0 R_0}{T} = \frac{2.8K \times 10^{26} m}{10^5 \times 10^{13} K} \approx 10^8 m$$

(7.5)

It’s exactly the horizon at that moment. So, there is no any abnormity in the whole history of cosmos. This solved the horizon problem.
VIII. THE HISTORY OF COSMOS

All the parameters of cosmos in any moment can be calculated accurately with equation (5.13), (5.14), (5.15), (6.3) and (6.4), included the radius $R$, light velocity $C$, gravitational constant $G$, temperature $T$ and density $\rho$ of the cosmos. Table 8.1 shows these parameters in four important epochs in the history of cosmos.

**Table 8.1 Four Important Epochs in History of Cosmos**

| $t$ (s) | $R$ (m) | $\rho$ (kgm$^{-3}$) | $T$ | Events               |
|--------|---------|---------------------|-----|---------------------|
| $10^{-20}$ | $10^8$  | $10^{-9}$            | $10^5$ Gev | Big Nucleus epoch.  |
| $10^{-14}$ | $10^{10}$ | $10^{-12}$          | $1$ Gev    | Big Atom epoch. Hadrons formed. |
| $10^{-4}$ | $10^{16}$ | $10^{-22}$          | $0.1$ mev  | Big Star epoch. Nuclei formed.   |
| $10^{10}$ | $10^{22}$ | $10^{-15}$          | $1$ ev     | Big Galaxy epoch. Atom formed. |

All the results calculated in table 8.1 are accordant with the conditions for events in every period of cosmos. It enables us to study the early cosmos with quantitative analysis.

IX. THE ABUNDANCE OF HELIUM

We can see from table 8.1 that in the early cosmos of $10^{-4}$s, the density is about $10^5 kgm^{-3}$, and the temperature is about 0.1mev. These are exactly the conditions for the combination of helium in stars. The helium in cosmos is mainly formed in this period.

The combination of helium began from deuteron $^2D$. The binding energy of deuteron is 2.2mev. The combination of deuteron happened in the time when the temperature of cosmos dropped to 0.1mev, or $10^9K$. In the stars, when the temperature reaches $10^7K$ and the density reaches $10^5 kgm^{-3}$, the combination of helium can be started. In the Big Star, the temperature was higher and the interaction was much stronger. So the combination completed very thoroughly. All of the neutrons were combined into helium with protons.

In a definite condition, the neutrons and protons are in a reversible equilibrium. They can be transformed into each other with the following reactions

$$
p + l^- = n + v_l \\
n + l^+ = p + \bar{v}_l
$$

(9.1)

The numerical density of neutrons $N_n$ and protons $N_p$ were given by the Boltzmann formula as

$$
\frac{N_n}{N_p} = e^{-\Delta m/T}
$$

(9.2)

$\Delta m = m_n - m_p = 1.29 mev$ is the difference of the mass between neutron and proton.

From (9.2) we can see that, when the temperature of the cosmos dropped to 0.1mev for the combination of deuterons, $N_n$ is apparently less than $N_p$. From the weak interaction theory in particle physics, we know that when the density dropped to $10^{10} kgm^{-3}$, the transformation between neutrons and protons stopped. The density of neutrons was fixed at this density limit. From the above equations we can see that this density was happened in about $10^{-5}s$, when the radius was about $10^{14}m$. From (6.4) we can see that the corresponded temperature is 0.8mev. It was just before the combination of deuteron and helium. When the temperature dropped to 0.1mev. The combination of helium was started. All the neutrons were combined into helium. The remained protons are the hydrogen. At that time, the density of helium and hydrogen has the following relations

$$
N_{He} = \frac{1}{2}N_n \\
N_H = N_p - N_n
$$

(9.3)

The abundance of helium was so defined as

$$
Y_{He} = \frac{2N_{He}}{N_H + 4N_{He}} = \frac{2}{1 + N_p/N_n} = \frac{2}{1 + e^{\Delta m/T}}
$$

(9.4)

Apply the temperature 0.8mev at which the transformation between neutrons and protons stopped to (9.4) yields

$$
Y_{He} = 0.28
$$

(9.5)
X. CONCLUSIONS

The critical density is exactly the density of cosmos. The space of cosmos is strictly flat anytime and anywhere in the cosmos. The horizon of cosmos is exactly the radius of cosmos. The physical events in any period of cosmos are accordant with the matter conditions calculated by equations presented in this paper. There is no any abnormality in any period or area of the cosmos. All the cosmological results and the physical results are reasonable and natural. There are no any difficulties in the standard cosmology under the general relativity. An experiment is necessary to test the variable speed of light. It’s not difficult, but it still needs someone to finish it. It must be significant to approach the unification of Newtonian and relativistic dynamics.

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