Performance of super-resolution algorithms under applicative noise

S V Savvin and A A Sirota
Faculty of Computer Sciences, Voronezh State University, 1 Universitetskaya pl., Voronezh, 394018, Russia
Email: savvin_s_v@sc.vsu.ru, sir@cs.vsu.ru

Abstract The paper considers the problem of multi-frame super-resolution under applicative noise which generates distributed regions of outlying observations in low resolution images. The analysis of existing solutions is performed. They include algorithms based on spin-glass models and Markov random fields used to remove applicative noise. The authors suggest their own approach, which involves using a recurrent algorithm of quasi-linear optimal filtering of a sequence of low resolution images together with superpixel segmentation performed in order to determine the regions damaged by applicative noise. The considered algorithms are compared as applied to a set of test images. The results of the experiment demonstrate that the suggested approach allows for more accurate recovery of HR images than the existing analogues.

1. Introduction
The resolution of a digital image depends on the physical properties of the imaging system, which produces, registers, and transmits the image via various communication channels. At the same time, information processing systems quite often can only work with high-resolution (HR) images, which have the required level of detail. Such images, however, are hard to acquire directly due to the limitations of the registration techniques. The most common approach to this problem is multi-frame image super-resolution (SR) [1–6], which recovers a HR image from a sequence of low-resolution (LR) images of the same scene with sub-pixel transitions between multiple frames.

Another factor that affects the quality of the registered images is, alongside with additive noise, so-called applicative noise, which generates distributed regions of outlying observations in LR images. In this case, applicative noise may also be considered as additional factor that reduces the image resolution, with the areas of low resolution being randomly located.

At the moment, there are a number of super-resolution algorithms [1, 2]. However, few of them [3–6] aim to compensate for the resolution loss resulting from applicative noise, and at the same time increase the resolution by accumulating and processing sequences of LR images. The aim of this paper is to analyse and compare multi-frame image super-resolution algorithms based on different approaches, namely, algorithms based on spin-glass models [3], algorithms based on Markov random fields [4], and our own algorithm based on quasi-linear optimal filtering [5].
2. Analysis of the existing algorithms

Let us consider a sequence of LR images \( \{ y^{(k)} \} \) of the same scene and assume that each of these images can be marred by noise. Each LR image is presented as a vector \( y^{(k)} \in \mathbb{R}^L \). A superresolution algorithm then recovers a single HR image (vector) \( x \in \mathbb{R}^M \) where \( M > L \).

2.1. Algorithms based on spin-glass models (ASGM)

The algorithms described in [3] are based on spin-glass models [7]. These models are used to simulate the noise distorting the image rather than the image itself. Let the latent vector \( \{ z^{(t)} \} \) be defined as:

\[
\beta_{H}, \quad z^{(t)} = +1, \quad \beta_{L} \quad z^{(t)} = -1.
\]

Prior distribution of latent variables \( z^{(t)} \) is described by the Boltzmann distribution

\[
p(z^{(t)}) = \frac{1}{Z} \exp \left( -E(z^{(t)}) \right).
\]

The energy \( E(z, J) \) is determined as follows:

\[
E(z, J) = -J_{\text{eff}} \sum_i z_i^{(t)} - J_{\text{inter}} \sum_{i \neq j} z_i^{(t)} z_j^{(t)},
\]

where \( i \neq j \) means that \( i \)th and \( j \)th pixels are adjacent, and constants \( J_{\text{eff}} > 0 \) and \( J_{\text{inter}} > 0 \) are determined depending on the type of noise. Since each element \( z^{(t)} \) takes either \(-1\) or \(+1\) (the Ising model), expressions (1) and (2) describe the spin-glass model.

A prior observation model \( y^{(t)} \) is described by Gaussian distribution

\[
p(y^{(t)} | x, z^{(t)}) = N(y^{(t)}, W^{(t)}x, B(z^{(t)})^{-1}),
\]

where \( W^{(t)} \) is the operator that describes the influence of the imaging system, and \( B(z^{(t)}) \) is the precision matrix whose diagonal elements are set as follows:

\[
\beta_l^{(t)} = \begin{cases} 
\beta_{H}, & z^{(t)} = +1, \\
\beta_{L}, & z^{(t)} = -1, 
\end{cases}
\]

Prior distribution \( x \) is also described by Gaussian distribution

\[
p(x) = N(x, 0, (\rho A)^{-1}),
\]

where \( \rho \) is the precision strength.

In (5) the precision matrix \( A \) is defined as

\[
A_{ij} = \begin{cases} 
[N(i)] & i = j, \\
-1, & i \sim j, \\
0, & \text{otherwise}
\end{cases}
\]

where \( N(i) = \{ j | i \sim j \} \) is the number of neighbours of the \( i \)th pixel. This regularization provides for the smoothing constraint of HR images.

The HR image \( x \) can be defined as the mean of the posterior probability distribution \( p(x|y) \), but it cannot be calculated analytically due to latent variables. To solve this problem, [3] suggests using variational Bayesian Inference, where the posterior probability distribution over a set of unobserved variables is approximated by another distribution \( p(z|x) \approx q(z) \), called variational.

The optimal approximate solution to \( x \) is the mean \( \mu \) of its variational Gaussian distribution.
The diagonal elements of the expected precision matrix $B(z^{(t)})$ are set specifically to demonstrate their dependence on the posterior probability of the latent variables:

$$\langle \beta(z^{(t)}) \rangle = q(z^{(t)} = 1)\beta_H + q(z^{(t)} = -1)\beta_L. \quad (8)$$

A large size of the covariance matrix $\Sigma_z$ makes it difficult to obtain this matrix directly. However, it follows from (7) that

$$\Sigma^{-1} = \rho A + \sum_{i=1}^{T} W_i^{oT} B(z^{(i)}) W_i^{oT}^{-1} , \quad b = \sum_{i=1}^{T} W_i^{oT} B(z^{(i)}) y^{(i)}.$$  

Therefore, to determine $\mu$ we need to solve the linear equation (9), which can be done numerically by means of the conjugate gradient method [8].

The optimum variational distribution of $z^{(t)}_i$ is described by the Bernoulli distribution:

$$q^*(z^{(t)}_i) = \text{Ber}(z^{(t)}_i \mid v_n) = \frac{1}{1 + \exp(-2\lambda_n)} ,$$

$$v_n = \text{sig}(2\lambda_n) = \frac{1}{1 + \exp(-2\lambda_n)}, \quad (10)$$

where $e_n = y_n - [W_i x_i]$ is the error in the $i$th pixel of the $t$th observation (HR image).

Thus, the HR image recovery algorithm can be presented as follows.

1. $l \leftarrow 0$.
2. $l \leftarrow l + 1$.
3. Calculate $\mu^{(l)}$ according to (9), using the conjugate gradient method.
4. Recalculate $v^{(l)}$ according to (10).
5. Repeat stages 2–4 until $\| \mu^{(l)} - \mu^{(l-1)} \| / \| \mu^{(l-1)} \| < \epsilon$.

2.2. Algorithms based on Markov random fields (AMRF) [4] describes an approach based on Markov random fields [9]. The observation model is described by the following relation

$$y_t = O_t D H_t x + \omega_t , \quad (11)$$

where operator $O_t$ removes the pixels damaged by applicative noise, operator $D$ decimates the HR image, operator $H_t$ describes the influence of the imaging system, and $\omega_t$ is Gaussian noise. Similar operators for LR images are either already known, or can be easily estimated. The estimation of $O_t$ involves sequence-independent segmentation of each LR image $y_t$ into true and spurious observations.
The HR image $x$ is viewed as a discontinuity adaptive Markov random field (DAMRF) [13], which helps to preserve all discontinuities and details. The joint density $x$ is determined as follows:

$$p(x) = \frac{1}{Z} \exp \left( - \sum_{c \in C} V_c(x) \right), \quad (12)$$

where $Z$ is the normalization constant, $C$ is the set of all combinations, and $V_c(x)$ are such clique potential functions that $\sum_{c \in C} V_c(x) = \sum_{c \in C} g(d, x)$.

The choice of the model is important, as it gives the information about the smoothness of the image by analysing the local spatial variations $d, x$. DAMRF model performs adaptive evaluation of the degree of similarity between pixels in order to preserve the inhomogeneities

$$g(\eta) = -\gamma \cdot \exp \left( - \frac{\eta^2}{\gamma} \right), \quad (13)$$

where $\eta$ is the difference between the values of adjacent pixels.

The maximum posterior probability $x$ is determined using the graduated non-convex optimization (GNC) [9].

$$\hat{x} = \arg \min_x \left\{ \|y - O_iDHx\|^2 + \beta \sum_{c \in C} V_c(x) \right\}, \quad (14)$$

where $\beta$ is the regularization parameter. Variable $\gamma$ changes during each iteration according to $\gamma^{(i+1)} = k\gamma^{(i)}$, $0 < k < 1$.

2.3. Algorithms based on quasi-linear optimal filtering (AQOF)

This approach involves recurrent processing of a sequence of LR images. Each processed LR image is interpreted as a realization of a random vector, which describes the observations of the HR image which is being recovered under additive and applicative noise. Algorithms of this type require creating a state model of the original HR image and an observation model of the sequence of LR frames.

The state model is represented by the recurrence equation which describes the unobserved HR images

$$x_{k+1} = F_kx_k + u_k, \quad (15)$$

where $F_k$ is the operator which determines the sub-pixel shift between images; $u_k$ is Gaussian random vector with the covariance matrix $Q_k$ which describes random interframe changes.

The observation model considers the resolution loss occurring during the image registration process, and the likelihood of applicative noise artefacts occurring in the acquired images [6]. The model’s equation is presented as follows

$$y_k = A_k(h_kx_k + v_k) + B_k(\tilde{y}_{k-1} + w_k), \quad (16)$$

$$A_k + B_k = I_k, \quad A_k = \begin{pmatrix} a_{k1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & a_{kM} \end{pmatrix}, \quad B_k = \begin{pmatrix} b_{k1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & b_{kM} \end{pmatrix},$$

where $y_k$ is the vector corresponding to the observed LR image; $v_k$ is the vector of additive measurement noise with the covariance matrix $R_k$; $H_k$ is the operator that characterises the influence of the imaging system; $A_k$ and $B_k$ are diagonal matrices with random elements which take on the values of 0 or 1, when the primary element sends true ($a_{k1} = 1, b_{k1} = 0$) or spurious ($a_{k1} = 0, b_{k1} = 1$)
information; \( \tilde{y}_{t,k} \) is the vector of the estimate of the observed LR image extrapolated to the moment \( t_k \), which estimate is based on the a priori information or the results of processing of a set of \( t_{k-1} \) previous LR images; \( w_k \) is the vector with zero mean and the covariance matrix \( S_k \) which describes the deviation of the spurious observations from the vector \( \hat{z}_{t,k} \).

To synthesise an optimal filter, we should define matrices \( P_{\alpha\alpha} = M[A_k] \), \( P_{\beta\beta} = M[B_k] \), \( P_{\alpha\beta} + P_{\beta\alpha} = I \) (diagonal elements of these matrices denote the probability of values \( p_{\alpha\alpha} = M[a_{kl}] \) and \( p_{\beta\beta} = M[b_{kl}] \), as well as matrices of pairwise probabilities of true \( p_{\alpha\beta} \) and spurious \( p_{\beta\alpha} \) observations occurring together in different pixels of the image, where

\[
p_{\alpha\beta} = P(a_{kl} = 1, a_{km} = 1), \quad p_{\beta\alpha} = P(b_{kl} = 1, b_{km} = 1).
\]

There are two possible super-resolution algorithms, if spurious observations are present:

- using optimal linear filter (i.e. only the a priori information about spurious observations is used);
- using conditionally linear filter (i.e. a posteriori information about spurious observations for each of the LR images is used).

It was demonstrated in [6] that the conditionally linear filter allows for significantly better recovery of HR images than the optimal linear filter, which justifies the need for additional calculations for every new observation \( y_k \).

The conditionally linear filter uses additional information component corresponding to each of the observed images \( y_k \). The information component is presented as a binary vector \( \theta_k \) and contains data about spurious observations in all the pixels of the LR image. Elements \( \theta_k \) are denoted 0 (when spurious observations are present) and 1 (when there are no spurious observations). They are determined following the sequence independent segmentation of applicative noise in each LR image.

Let \( Y_k = \{y_{kl} | \theta_{kl} = 1\}, \quad l = 1, L \) be a set of all the pixels of image \( y_k \). The purpose of the segmentation is to divide this set into two disjoint subsets: pixels with true content (\( O_k \subset Y_k \)) and pixels with spurious observations (\( H_k \subset Y_k \)). Therefore, the values of the vector \( \theta_k \) are calculated as follows:

\[
\theta_{kl} = \begin{cases} 1, & y_{kl} \in O_k, \\ 0, & y_{kl} \in H_k. \end{cases}
\]  

(17)

Vector \( \theta_k \), obtained as a result of the segmentation, is used to calculate the posterior probabilities of the presence of true information in the pixels of the image:

\[
p_{\alpha\beta}(\theta_k) = \begin{cases} \frac{p_{\alpha\alpha}(\theta_{kl} = 1 | a_{kl})}{p_{\alpha\alpha}(\theta_{kl} = 1 | a_{kl}) + p_{\beta\alpha}(\theta_{kl} = 1 | b_{kl})}, & \theta_{kl} = 1, \\ \frac{p_{\beta\alpha}(\theta_{kl} = 0 | a_{kl})}{p_{\alpha\alpha}(\theta_{kl} = 0 | a_{kl}) + p_{\beta\alpha}(\theta_{kl} = 0 | b_{kl})}, & \theta_{kl} = 0, \end{cases}
\]

(18)

\[
p_{\beta\alpha}(\theta_k) = \begin{cases} \frac{p_{\beta\alpha}(\theta_{kl} = 1 | b_{kl})}{p_{\beta\alpha}(\theta_{kl} = 1 | b_{kl}) + p_{\alpha\beta}(\theta_{kl} = 1 | a_{kl})}, & \theta_{kl} = 1, \\ \frac{p_{\alpha\beta}(\theta_{kl} = 0 | b_{kl})}{p_{\beta\alpha}(\theta_{kl} = 0 | b_{kl}) + p_{\alpha\beta}(\theta_{kl} = 0 | a_{kl})}, & \theta_{kl} = 0, \end{cases}
\]
where probabilities $p(\theta_a = 0.1 | a_k)$ and $p(\theta_a = 0.1 | b_k)$ are calculated using the values of the parameters corresponding to true and spurious observations which in turn can be calculated using the results of the segmentation.

Correction equations for quasi-linear filtering are then presented as follows

$$\bar{x}_{k+1|k} = F x_{k|k} = \bar{x}_{k|k-1} + W_k (\theta^k) (y_k - \bar{y}_{k|k-1}),$$

$$\bar{y}_{k|k-1} = H \bar{x}_{k|k-1}, \quad F_k = I, \quad k = 1, K,$$

$$V_{k|0} = P_{\theta_{k-1}} (\theta^{k-1}) H_k P_{\theta_{k-1}}^T (\theta_k), \quad W_k (\theta^k) = V_{k|0} U_{k|0}^{-1},$$

(19)

$$U_{k|0} = P_{\theta_{k|0}} = \left( H_k P_{\theta_{k-1}} (\theta^{k-1}) H_k^T + R_k \right) + P_{\theta_{k|0}} \circ S_k,$$

$$P_{k+1|k} (\theta^k) = F_k P_{k|k} (\theta^k) F_k^T + Q_k = P_{k|k-1} (\theta^{k-1}) - W_k (\theta^k) U_{k|0} W_k^T (\theta^k) + Q_k,$$

where vectors $\bar{x}_{k|k}$ and $\bar{x}_{k+1|k}$ denote the estimate of the LR image and the expected estimate of the following frame; $P_{k|k}$, $P_{k+1|k}$ are covariance matrices of the estimate; $\theta^k = \{ \theta_1, ..., \theta_k \}$ is the sequence of binary vectors resulting from the segmentation; operation $A \circ B$ denotes element-by-element multiplication of the operators $A$ and $B$.

Matrices of posterior probabilities of the presence of true and spurious observations $P_{\theta_{k} \theta_{k}}$ and $P_{\theta_{k} \theta_{k}}$ are calculated using the results of the segmentation. Matrices of posterior pairwise probabilities of true and spurious observations $P_{\theta_{k} \theta_{k}}$ and $P_{\theta_{k} \theta_{k}}$ can be calculated numerically by averaging the elements $\theta_k$, corresponding to the neighbouring pixels of every element of the LR image for which $\theta_k$ takes on values 1 and 0 respectively.

It should be noted that, when applied to large images, the algorithms based on optimal filtering require block processing. This means that the image is divided into overlaying blocks, with sequences of corresponding blocks being processed separately and the results being fused at the end [10]. For instance, this is required to reduce the dimensionality of the processed and inverted matrices in (19). However, we do not focus on the block processing procedure in this paper in order not to deviate from the description of the analysed algorithms.

The described above algorithm of processing sequences of LR images is presented in figure 1.

![Figure 1](image_url)

**Figure 1.** The algorithm of LR image processing by means of quasi-linear filtering under applicative noise.
2.4. Applicative noise segmentation
The algorithms described in [4] and [5], require independent segmentation of applicative noise for each of the input LR images. We suggest using a two-stage segmentation algorithm based on super-pixel image representation [11].
- The first stage includes dividing the original image into a large number of small homogeneous regions (superpixels) by means of a corresponding algorithm.
- The second stage includes dividing the obtained superpixels into subclasses of true and spurious observations by means of the expectation–maximization algorithm. The resulting posterior probabilities are then used in the optimal filtering algorithm [5] (probabilities $p(\theta_d = 0,1|a_i)$ and $p(\theta_d = 0,1|b_i)$).

The clustering of superpixels can be based on such parameters as mean values and dispersion of the RGB components, correlation coefficient between the RGB components, etc. The criteria are selected based on a priori information about the noise distorting the original LR images.

3. Results and discussion
The comparison of the described super-resolution algorithms was performed on 12 sets of LR 128x128 colour images each containing 20 frames. The shifts and changes in camera position in the neighbouring frames were random. The regions initially marred by applicative noise were also randomly located. The location of the damaged regions in each subsequent frame depended on their location in the previous one. Sample LR images with randomly located applicative noise regions are given in figure 2.

![Figure 2. Original LR images.](image_url)

Each of the compared algorithms – ASGM, AMRF, and AQOF – was implemented in Matlab, taking into account block processing of the images [10]. Superpixel segmentation was performed using the SLIC algorithm [12].

During the experiment the resolution of the original images increased by two. Figures 3 and 4 demonstrate sample HR images obtained by means of the compared algorithms. A qualitative analysis of the results demonstrates that the suggested algorithm based on quasi-linear optimal filtering [5] recovers HR images better than all the other algorithms:
- the algorithm based on spin-glass model [3] removes applicative noise worst of all (figure 4b);
- the algorithm based on Markov random fields [4] yields images with higher level of blurring, and consequently less detailed (figure 4c,d).
Figure 3. HR images obtained by means of super-resolution (a) the original HR image, (b) ASGM [3], (c) AMRF [4], (d) AQOF [5].

Figure 4. A scaled-up fragment of HR images presented in Fig.3: (a) the original HR image, (b) ASGM [3], (c) AMRF [4], (d) AQOF [5].
The quantitative comparison of the results was performed using
- peak signal-to-noise ratio (PSNR);
- structural similarity (SSIM) index [13].

The greater the values of these parameters, the smaller the difference between the input and output HR images, i.e. the better the performance of the super-resolution algorithm.

Mean values obtained in the experiment are given in Table 1. The results of the quantitative comparison prove that the suggested algorithms based on quasi-linear filtering [5] allow for more accurate recovery of the initial HR image.

Table 1. Accuracy of the recovery of the original HR image.

|          | ASGM [3] | AMRF [4] | AQOF [5] |
|----------|----------|----------|----------|
| PSNR     | 19.17    | 21.39    | 26.18    |
| SSIM     | 0.78     | 0.75     | 0.90     |

4. Conclusion

The paper considers the problem of multi-frame image super-resolution under applicative noise. The following algorithms were analysed and compared: algorithms based on spin-glass models, algorithms based on Markov random fields, and our own the algorithm based on synthesising a conditionally linear filter for a sequence of low resolution images. The algorithms were compared as applied to a set of test images. The comparison demonstrated the advantages of the algorithm based on quasi-linear filtering and two-stage HR image processing procedure employing superpixel segmentation.

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