SSP- Structure of k-Sun Flower, Friendship and Jahangir Graphs

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Abstract. A graph is SSP (Super Strongly Perfect) if all of its (induced) subgraph H in G obsesses a (minimal dominating set) MDS that link up all of its cliques (maximal) in H. It is offered the cyclic structure of k-sun flower graph, friendship and Jahangir graphs and later explored their SSP parameters like counting of colourability, MDS (minimal dominating set) and cliques (maximal) of these graphs.

1. Introduction
SSP graph was demarcated by B. D. Acharya and its Depiction has been set as an open problem in 2006. The class of perfect graphs include countless significant families of graphs, and help to merge results relating colorings, cliques in those families. For example, in every perfect graphs, the graph coloring problem (i.e., scheduling, frequency assignment, register allocation, pattern matching, the recreational puzzle sudoku), maximum clique problem (social network, electronics, computer science (i.e., compiler design), bioinformatics and computational chemistry, chemical information, and computational biology) and maximum independent set problem (computer vision/pattern recognition, information/coding theory, map labeling, molecular biology, scheduling) all could be solved in polynomial time. Likewise, several complicated problems in practice which are obstinate in general can be resolved powerfully when constrained to the sub class of SSP graphs [2].

The selected graphs here, are simple and connected. In G, a clique is a collection of mutually connected vertices. M⊆V (G) is a dominating set if every single vertex in V - M is coupled with at least one vertex of M. N ⊆ V is a minimal (MDS) dominating set if N - {v} fails to be a dominating set for all v ∈ N.

2. Overview of the Paper
It has been categorized the structure of SSP Graphs in some classes of graphs like Trees, Complete graphs, Complete Bipartite graphs etc., [1, 3, 4]. In this paper, it is deliberated some other graph classes like k-sun flower graph, friendship and Jahangir graphs. Using this investigation, it is got the count of cliques (maximal), colouring and MDS.

2.1. Super Strongly Perfect (SSP) Graph
A graph is SSP (Super Strongly Perfect) if all of its (induced) subgraph H in G obsesses a (minimal dominating set) MDS that link up all of its cliques (maximal) in H. Figure 1, 2 illustrate SSP and non-SSP graphs. Every SSP graph with maximal clique K2 is isomorphic to any bipartite graph [2].
2.1.1. Illustration

Figure 1. SSP Graph

2.1.2. Illustration

Figure 2. Non-SSP Graph

2.1.3. Theorem [1]

G is SSP iff G does not induce a cycle odd length at least five as a subgraph (induced).

2.1.4. Theorem [3]

A graph G with at least one maximal clique $K_n$, $n = 2, 3, \ldots$, is n-colourable $\iff$ if it is SSP.

3. k-Sun Flower Graph

A k-sun flower graph $S_k$, where $k \geq 4$ is the graph obtained from k-cycle $C_k$ by including a triangle on each edge so that one vertex of the triangle has degree 2. Figure 3 illuminates 4-sun flower graph.

3.1. Illustration

Figure 3. 4-sun flower graph
3.1.1. Theorem
Every $k$-sunflower graph, $k \geq 4$, $k$ is even, is SSP.

Proof:
Let $G$ be a $k$-sunflower graph, $k \geq 4$, $k$ is even.
$\Rightarrow$ $G$ is formed from $C_k$ by including a triangle on each edge and it $G$ does not surround $C_{2k+1}$, $k \geq 2$, as a subgraph (induced).
$\Rightarrow$ $G$ is SSP (theorem 2.1.3).

3.1.2. Theorem
Every $k$-sunflower graph, $k \geq 5$, $k$ is odd, is non-SSP.

Proof:
Let $G$ be a $k$-sunflower graph, $k \geq 5$, $k$ is odd.
$\Rightarrow$ $G$ is formed from $C_k$ by including a triangle on each edge and it $G$ surrounds $C_{2k+1}$, $k \geq 2$, as a subgraph (induced).
$\Rightarrow$ $G$ is non-SSP (theorem 2.1.3).

3.1.3. Proposition
Let $G$ be a $k$-sunflower graph, $k \geq 4$, $k$ is even, then
1) $G$ consumes $k$-cliques (maximal) $K_3$.
2) $G$ consumes colourability 3.
3) $G$ consumes a count of MDS $\frac{k}{2}$ vertices.

Proof:
Consider a $k$-sunflower graph, $k \geq 4$, $k$ is even.
$\Rightarrow$ $G$ is formed from $C_k$ by including a triangle on each edge.
$\Rightarrow$ the inclusion of $k$ triangles on each $k$-edges of $C_k$, provide $k$ cliques (maximal) $K_3$.
$\Rightarrow$ $G$ consumes $k$-cliques (maximal) $K_3$.
$\Rightarrow$ $G$ is 3-colourable (theorem 2.1.4).

Alternate vertices from the induced cycle $C_n$ (i.e.,) $\frac{k}{2}$ vertices will provide a required count of MDS.

$\Rightarrow$ $G$ consumes a count of MDS of $\frac{k}{2}$ vertices.

Proposition 3.1.3 is illustrated below in figure 4.

3.2. Illustration

![6-sunflower graph](image-url)
In figure 4,
1) $G$ consumes 6 times $K_3$ (maximal cliques).
2) $G$ is 3-colourable.
3) $G$ consumes a count of MDS 3 vertices.

4. Friendship Graph
A Friendship graph on $2k + 1$ vertices is constructed by considering $k$ prints of $C_3$ together with a single vertex in public and it is denoted by $F_k$. Figure 4 illuminates Friendship graph $F_5$.

4.1. Illustration

4.1.1. Theorem
$F_k$ is SSP.

Proof:
Consider a graph $F_k$.
⇒ It consists of $k$ triangles interconnecting at a vertex, say $v$.
⇒ $v$ intersects all cliques (maximal) of $G$.
⇒ $G$ is SSP.

4.1.2. Proposition
Let $G$ be a friendship graph $F_k$, then
1) $G$ consumes $k$-cliques (maximal) $K_3$.
2) $G$ is 3-colourable.
3) $G$ consumes a count of MDS 1 vertex.

Proof:
Consider a $k$-sun flower graph, $k \geq 4$, $k$ is even.
⇒ $G$ is formed from $C_k$ by including a triangle on each edge.
⇒ the inclusion of $k$ triangles on each $k$-edges of $C_k$, provide $k$ cliques (maximal) $K_3$.
⇒ $G$ consumes $k$- cliques (maximal) $K_3$.
⇒ $G$ is 3-colourable (theorem 2.1.4).
As $G$ is the graph of $k$ triangles intersecting in a single vertex, $G$ consumes a count of MDS 1 vertex.
Proposition 4.1.2 is exemplified below in figure 6

![Figure 6. F_6](image)

In figure 6,
1) G consumes 6 cliques (maximal), each of which is a $K_3$.
2) G is 3-colourable.
3) G consumes a count of MDS one vertex.

5. Jahangir Graph
Jahangir Graph $J_{n,m}$ ($m \geq 3$) is a graph with $nm + 1$ vertices. It is containing $C_{nm}$ with one additional vertex that is linked to $m$ vertices which are at distance $n$ to each other of $C_{nm}$. Figure 7 illuminates Jahangir Graph $J_{2,4}$.

5.1. Illustration

![Figure 7. J_{2,4}](image)

5.1.1. Theorem
Every $J_{2,m}$ is SSP.

Proof:
Consider $J_{2,m}$.
⇒ It is containing $C_{2m}$ with one additional vertex that is linked to $m$ vertices which are at distance $n$ to each other of $C_{2m}$.
⇒ the central vertex along with the $m$ vertices of $C_{2m}$ interconnect all (maximal) cliques in G.
⇒ G is SSP.

5.1.2. Proposition
Let G be $J_{2,m}$, then
1) G consumes 3m times $K_2$ (maximal cliques).
2) G is 2-colourable.
3) \( G \) consumes a count of MDS \( m+1 \) vertices.

**Proof:**
Consider a Jahangir graph \( J_{2,m} \).
\( \Rightarrow \) \( G \) consists of a cycle \( C_{2m} \) with one additional vertex which is adjacent to \( m \) vertices of \( C_{2m} \) at distance \( n \) to each other on \( C_{2m} \).
\( \Rightarrow \) Along with \( 2m \) times \( K_2 \), the accumulation of one more vertex which is linked to \( m \) vertex in \( C_{2m} \) (with distance \( n \) to all other vertex on \( C_{2m} \)), will provide additional \( m \) times \( K_2 \).
\( \Rightarrow \) \( G \) consumes \( 2m+m = 3m \) times \( K_2 \).
\( \Rightarrow \) \( G \) is 2-colourable (theorem 2.1.4).
Also, the central vertex and \( m \) vertices with the required distance property of \( C_{2m} \) will provide \( 1+m \) vertices which interconnect all \( K_2 \) of \( G \).
\( \Rightarrow \) \( G \) consumes a count of MDS of \( m+1 \) vertices.

Proposition 5.1.2 is exemplified below in figure 8

![Figure 8. J_{2,6}](image)

In figure 8,
1) \( G \) consumes 18 times \( K_2 \) (maximal cliques).
2) \( G \) is 2-colourable.
3) \( G \) consumes a count of MDS 7 vertices.

6. Conclusion
It is detailed the structural analysation of SSP graphs in some subclasses of graphs like k-sun flower, friendship and Jahangir along with its SSP parameters. It can be stretched out to general graph classes also.

7. References
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