Weak gravitational interaction of fermions: quantum viewpoint

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Abstract

Using the quantum theory of linearized gravity, gravitational interaction differential cross sections of one fermion by another fermion, a photon and a scalar particle are calculated in the fermion rest-frame. Then, according to the obtained results, it is shown that in the lab frame, the gravitational interaction depends on the spin of the moving particle and is independent of the spin of the rest particle. After that, on the dependency of the gravitational interaction of fermion-photon upon the various states of photons polarization is discussed.

1 Introduction

In recent decades, many attempts have been made to unify gravity with other gauge fields. Generally speaking, it can be said that the main research in the fundamental physics is in this scope. Due to the fundamental differences between classical and quantum nature of a typical theory, the assumption that the Universe can undergo a combined classical and quantum description is implausible. Since most of fundamental theories are successfully formulated in the context of quantum mechanics, one believes that the fundamental theory of gravity should be reformulated in a quantum mechanical form. For that reason, the quantum mechanical expression of general relativity is one of the most important issues of modern theoretical physics.

Based on the interesting consequences of the LIGO experiment [1], in which confirmed one of the challenging predictions of Einstein gravity, it is logical to respect Einstein general relativity

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in the gravitational interactions. Moreover, since most of experimental tests of gravity are in the weak field limit, the pioneer efforts are implemented on the quantizing gravity, by linearizing the Einstein theory of gravity. However, another non-perturbative and background independent quantum theory of gravity is based on the so-called Loop quantum gravity \cite{2,3} and its covariant approach which is known as the spinfoam formulation \cite{4–7}. In this paper, we focus on the perturbative linearized method.

One of the systematic methods for the quantization of Einstein gravitational field in the weak field limit, has been established in a pioneering work by Gupta \cite{8}. This theory predicts a massless spin-two particle (the so-called graviton) as an intermediate particle of gravitational interaction. In this regard, one can describe the gravitational interaction of elementary particles in the same way that employs for electromagnetic interactions in the quantum electrodynamics. However, the study of gravitational interactions requires extensive algebraic calculations.

Following the mentioned method, the Lagrangian that describes the gravitational interaction of a given field can be obtained by weak field expansion of the general covariant Lagrangian of this field around the Minkowski spacetime. Subsequently, the conventional methods of the quantum field theory for linearized gravity are used to obtain the graviton vertices. Moreover, following Donoghue’s procedure, one finds that the problem of renormalizability can be solved by regarding the theory of general relativity as an effective field theory in the low-energy regime \cite{9,10}.

So far, some gravitational interactions of elementary particles have been investigated using the quantum linearized gravity theory \cite{11–21}. But differential cross sections of the gravitational interaction of a fermion with another fermion, photon, and a scalar particle in the rest frame of the fermion particle have not been calculated yet. The purpose of this paper is to calculate the differential cross sections for this interactions and compare their results with the previously obtained results for the gravitational interaction of a scalar particle with another scalar particle, photon, and a fermion in the rest frame of scalar particle. Thereafter, dependency of differential cross section on the spin of both moving and rest particles in gravitational interactions will be discussed. In the next section, we briefly discuss how to extract the Feynman rules for graviton vertices. After that, by using these rules, the mentioned interactions is studied with details. The discussion on the results of the differential cross sections has also been made in the last section of the paper. Hereafter, our conventions are: $\eta_{\mu\nu} = (+,−,−,−)$, and $\hbar = c = 1$.

### 2 Interaction Lagrangian

In the weak gravitational field regime, the line element of the Riemann spacetime ($g_{\mu\nu}$) is defined as the Minkowski metric tensor $\eta_{\mu\nu}$ accompanying with a symmetric perturbation tensor $h_{\mu\nu}$ as follows \cite{8}

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu},$$

(1)
where $\kappa^2 = 32\pi G_N$, and $G_N$ is the Newton gravitational constant. In this regime and following the Gupta procedure, one is able to quantize the gravitational field by applying the harmonic gauge defined as [8]

$$\partial_\lambda h^\lambda_\mu - \frac{1}{2} \partial_\mu h^\lambda_\lambda = 0. \quad (2)$$

The gravitational interaction with a given gauge field is described by the coupling energy-momentum tensor of that field with the gravitational field, and is expressed by the following interaction Lagrangian

$$L_{int} = -\frac{k}{2} h_{\mu\nu} T^{\mu\nu}. \quad (3)$$

The interaction Lagrangian of gravitational field with an arbitrary gauge field is obtained by the expansion of the general covariant Lagrangian of that field by using Eq. (1). The gravitational interaction Lagrangian for massive scalar particles is expressed as

$$L_{int} = \frac{1}{2} h \left[ (\partial^\mu \phi)^* (\partial_\mu \phi) - m^2 (\phi^* \phi) \right] - h_{\mu\nu} (\partial^\mu \phi)^* (\partial^\nu \phi), \quad (4)$$

where $h = h^\mu_\mu$ and $\phi^*$ is the complex conjugate of $\phi$. The same Lagrangian for photons is expressed as [18]

$$L_{int} = -\frac{1}{8} h F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} h^\tau_\nu F^{\mu\nu} F_{\mu\tau}, \quad (5)$$

where the Faraday anti-symmetric tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. In the case of massive spinor fields, in order to obtain the gravitational interaction Lagrangian, the tetrad formalism [22] is also used, the result for this Lagrangian is given by [16]

$$L_{int} = \frac{1}{4} h \left[ i (\bar{\Psi} \gamma^\mu \partial_\mu \Psi - \partial_\mu \bar{\Psi} \gamma^\mu \Psi) - 2 m \bar{\Psi} \Psi \right] - \frac{i}{4} h_{\mu\nu} (\bar{\Psi} \gamma^\mu \partial^\nu \Psi - \partial^\nu \bar{\Psi} \gamma^\mu \Psi), \quad (6)$$

where $\gamma_\mu$ are the ordinary Dirac matrices. The Feynman rules for graviton vertices that are needed in this paper, can be derived From this interaction Lagrangians. The related Feynman rules are listed in the appendix.

### 3 Gravitational interaction of fermions

Here, we investigate the gravitational interactions of non-similar fermions, fermion-photon and fermion-scalar particle in the fermion rest frame. In this regard, we use the Feynman rules for graviton propagator and vertices that are presented in the appendix.

First of all, the gravitational interaction of a fermion particle with mass $m$ and a scalar particle with mass $m'$ is considered. The Feynman diagram for this interaction is shown in Fig. 1(a). According to this diagram, the matrix element for this interaction is written as
\[ i\mathcal{M} = \tau_{1}^{\alpha\beta}(p_1, p_3, m') \frac{iP_{\rho \sigma \alpha \beta}}{q^2} \bar{u}(p_4) \tau_{2}^{\rho \sigma}(p_2, p_4, m) u(p_2) \]

\[ = -\frac{ik^2}{32q^2} \bar{u}(p_4) \left[ -32m'^2 m + 12m^2 (\hat{p}_2 + \hat{p}_4) + 16m(p_1 \cdot p_3) \right. \]

\[ \left. + 4(\hat{p}_3 p_1 + \hat{p}_1 p_3) \cdot (p_2 + p_4) - 8(p_1 \cdot p_3)(\hat{p}_2 + \hat{p}_4) \right] u(p_2), \quad (7) \]

where \( q = p_3 - p_1 = p_2 - p_4 \) and the initial and final four-momenta of the scalar particle are denoted by \( p_1 \) and \( p_3 \) and those for the fermion particle are indicated by \( p_2 \) and \( p_4 \).

We can use the following equation to obtain the differential cross section in the rest frame of fermion particle [23]

\[ \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 m^2 E_1^2} |\mathcal{M}|^2, \quad (8) \]

where \( E_1 \) and \( E_3 \) are the initial and final energies of the scalar particle and

\[ |\mathcal{M}|^2 = \frac{1}{1 + 2s_f} \sum_{\text{spins}} |\mathcal{M}|^2, \quad (9) \]

in which \( s_f \) indicates the spin of fermion. Using completeness relation

\[ \sum_{\text{spins}} u^{(s)}(p) \bar{u}^{(s)}(p) = \hat{p} + m, \quad (10) \]
with the following equation

\[
\sum_{\gamma} \bar{u}_{\alpha}(s') (p') \gamma_{\alpha \beta} \sum_{s} u_{\beta}(p) \bar{u}_{\gamma}(s) (p) \gamma_{\gamma \delta} u_{\delta}(s') (p') = \sum_{\gamma} \bar{u}_{\alpha}(s') (p') \gamma_{\alpha \beta} \sum_{s} u_{\beta}(p) \bar{u}_{\gamma}(s) (p) \gamma_{\gamma \delta} (\bar{p} + m') \gamma_{\beta \gamma} \gamma_{\gamma \delta} \nonumber
\]

\[
= \text{Tr}[(\bar{p} + m') \gamma^\mu (\bar{p} + m) \gamma^\nu].
\]

(11)

the following expression is obtained for \( |\mathcal{M}|^2 \) in fermion-scalar particle interaction

\[
|\mathcal{M}|^2 = \frac{2k^4}{(64\pi^2)^2} \text{Tr}\{[(\bar{p} + m)[16m^2(\bar{p} + p_4 - 2m) + 16(p_1 \cdot p_3) - 4m^2(\bar{p} + \bar{p}_4) + 4p_1 \cdot (p_2 + p_4)p_3] + 4p_1 \cdot (p_2 + p_4)p_3] + 4p_3 \cdot (p_2 + p_4)p_1 - 8(p_1 \cdot p_3)(\bar{p} + \bar{p}_4)\}\}
\]

(12)

Using Tracer package in Mathematica environment, we can compute the traces.

In the fermion rest frame and assuming that the mass of the scalar particle is negligible compared to the fermion mass, we can write

\[
p_1 \cdot p_4 = p_1 \cdot (p_1 + p_2 - p_3) \implies p_1 \cdot p_4 \simeq p_1 \cdot p_2 - p_1 \cdot p_3, \quad \text{(13)}
\]

\[
p_2 \cdot p_4 = p_2 \cdot (p_1 + p_2 - p_3) \implies p_2 \cdot p_4 = p_2 \cdot p_1 + m^2 - p_2 \cdot p_3, \quad \text{(14)}
\]

\[
p_3 \cdot p_4 = p_3 \cdot (p_1 + p_2 - p_3) \implies p_3 \cdot p_4 \simeq p_3 \cdot p_1 + p_3 \cdot p_2, \quad \text{(15)}
\]

\[
q^2 = (p_1 - p_3)^2 = p_1^2 + p_3^2 - 2p_1 \cdot p_3 \implies q^2 \simeq -2p_1 \cdot p_3, \quad \text{(16)}
\]

\[
p_1 \cdot p_2 = mE_1, \quad \text{(17)}
\]

\[
p_2 \cdot p_3 = mE_3, \quad \text{(18)}
\]

\[
p_1 \cdot p_3 = E_1E_3 - \bar{p}_1 \cdot \bar{p}_3 \simeq E_1E_3 - E_1E_3 \cos \theta = 2E_1E_3 \sin^2 \frac{\theta}{2}, \quad \text{(19)}
\]

\[
q + p_2 = p_4 \implies q^2 = -2p_2 \cdot q = -2(E_1 - E_3)m. \quad \text{(20)}
\]

Using Eqs. (13), (19) and (20), we get

\[
E_1 - E_3 = \frac{2E_1E_3 \sin^2 \frac{\theta}{2}}{m}. \quad \text{(21)}
\]

After trace completion and inserting Eqs. (13)–(19) in Eq. (12), we obtain

\[
|\mathcal{M}|^2 = \frac{k^4}{64} \left[ 2mE_1E_3 \sin^2 \frac{\theta}{2}(E_3 - E_1) + m^2(E_1^2 + E_3^2 - 10E_1E_3) + \frac{6m^3}{\sin^2 \frac{\theta}{2}}(E_1 - E_3) 
\right.
\]

\[
+ \left. \frac{4m^4}{\sin^4 \frac{\theta}{2}} \right]. \quad \text{(22)}
\]
Using Eq. (21), we can simplify Eq. (22) as

\[ |\mathcal{M}|^2 = \frac{k^4 m^2}{16} \left[ \frac{(E_1 - E_3)m}{2 \sin^2 \frac{\theta}{2}} + \frac{m^2}{\sin^4 \frac{\theta}{2}} \right]. \tag{23} \]

Assuming that the collision is elastic, the initial and final energies of the scalar particle are the same. With this condition, and inserting Eq. (23) in Eq. (8), one can obtain the differential cross section for the gravitational interaction of fermion-scalar particle as

\[ \frac{d\sigma}{d\Omega} = \frac{k^4 m^2}{(32\pi)^2} \frac{1}{\sin^4 \frac{\theta}{2}}. \tag{24} \]

According this relation, one finds decreasing the scattering angle leads to increasing the differential cross section.

Now, let us proceed our paper with the consideration of the gravitational interaction of two different fermions with masses \( m \) and \( m' \). Considering Fig. 1(b), we obtain the following expression for matrix element of the interaction

\[
i\mathcal{M} = \bar{u}(p_3)\tau_2^{\alpha\beta}(p_1, p_3, m')u(p_1)i\frac{p_{\rho\sigma\alpha\beta}}{q^2}\bar{u}(p_4)\tau_2^{\rho\sigma}(p_2, p_4, m)u(p_2)
\]

\[ = \frac{-ik^2}{64q^2} \bar{u}(p_3)[2\eta^{\alpha\beta}(\not{p}_1 + \not{p}_3 - 2m') - (p_1 + p_3)^\alpha\gamma^\beta - (p_1 + p_3)^\beta\gamma^\alpha]u(p_1)\bar{u}(p_4)
\]

\[ \times \left[-\eta_{\alpha\beta}(\not{p}_2 + \not{p}_4 - 4m) - (p_2 + p_4)^\alpha\gamma^\beta - (p_2 + p_4)^\beta\gamma^\alpha\right]u(p_2), \tag{25} \]

where \( p_1 \) and \( p_3 \) are the initial and final four-momenta of the fermion particle with mass \( m' \), \( p_2 \) and \( p_4 \) are those for the fermion particle with mass \( m \) and \( q = p_3 - p_1 = p_2 - p_4 \).

If we investigate this interaction in the rest frame of fermion particle with mass \( m \), the differential cross section can be obtained from Eq. (8), and \(|\mathcal{M}|^2\) is given by

\[ |\mathcal{M}|^2 = \frac{1}{1 + 2s_f} \frac{1}{1 + 2s_{f'}} \sum_{f_{\text{spins}}} \sum_{f'_{\text{spins}}} |\mathcal{M}|^2. \tag{26} \]

Using Eqs. (11) and (12), we can rewrite Eq. (26) as

\[ |\mathcal{M}|^2 = \frac{k^4}{4(64q^2)^2} A^{\alpha\beta\rho\sigma} B_{\alpha\beta\rho\sigma}, \tag{27} \]

where

\[
A^{\alpha\beta\rho\sigma} = \text{Tr} \{ [2\eta^{\alpha\beta}(\not{p}_1 + \not{p}_3 - 2m') - (p_1 + p_3)^\alpha\gamma^\beta - \gamma^\alpha(p_1 + p_3)^\beta](\not{p}_1 + m')
\]

\[ \times [2\eta^{\rho\sigma}(\not{p}_2 + \not{p}_4 - 2m') - (p_2 + p_4)^\rho\gamma^\sigma - \gamma^\rho(p_2 + p_4)^\sigma](\not{p}_2 + m') \}, \tag{28} \]
The cross section for gravitational interaction of non-similar fermions is obtained as

\[
B_{\alpha\beta\rho\sigma} = \text{Tr}\{[-\eta_{\alpha\beta}(p_2 + p_1 - 4m) - (p_2 + p_4)\alpha\gamma_\beta - \gamma_\alpha(p_2 + p_4)\beta](p_2 + m) \\
\times[-\eta_{\rho\sigma}(p_2 + p_1 - 4m) - (p_2 + p_4)\rho\gamma_\sigma - \gamma_\rho(p_2 + p_4)\sigma](p_4 + m)\}.
\]  

(29)

Using the approximation that the rest fermion is so heavy, the mass of another fermion in the interaction can be neglected and Eqs. (13)–(21) are satisfied in this interaction. By calculating Eqs. (28) and (29) by using \textit{Tracer} package and inserting Eqs. (13)–(19) and (21) in the obtained expression for Eq. (27), we obtain

\[
|\mathcal{M}|^2 = \frac{2k^4}{(32)^2} \left[ 4m^2(E_2 - E_4)^2 + 4m^3(E_2 - E_4) + \frac{4m^3}{\sin^2\frac{\theta}{2}}(E_2 - E_4) \right. \\
\left. + \frac{8m^4\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} \frac{(E_2 + E_4)^2}{E_2E_4} \right].
\]  

(30)

With the assumption that the collision is elastic and inserting Eq. (30) in Eq. (8), the differential cross section for gravitational interaction of non-similar fermions is obtained as

\[
\frac{d\sigma}{d\Omega} = \frac{k^4m^2\cos^2\frac{\theta}{2}}{(32\pi)^2\sin^4\frac{\theta}{2}}
\]  

(31)

It is clear that when \(\theta\) goes to zero, the differential cross section diverges. In addition, for the case \(\theta = \pi\), unlike Eq. (21), the cross section for fermion-fermion gravitational interaction vanishes.

Following the earlier treatment, we consider the fermion-photon gravitational interaction where its Feynman diagram is shown in Fig. 1(c). We find that the matrix element of this diagram is

\[
i\mathcal{M} = \bar{u}(p_4)\epsilon^{\alpha\beta\rho\sigma}(p_2, p_4)u(p_2)\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma  \\
\times\frac{\epsilon_{1\gamma}\epsilon_{3\delta}(p_1, p_3)\epsilon_{3\delta}^*}{q^2} \\
= -\frac{ik^2}{16q^2} \bar{u}(p_4)[2\eta^{\alpha\sigma}(p_2 + p_1 - 2m) - (p_2 + p_4)^\rho\gamma^\sigma - \gamma^\rho(p_2 + p_4)^\sigma]u(p_2) \\
\times [p_1 \cdot p_3(\epsilon_{1\sigma}\epsilon_{3\rho} + \epsilon_{1\rho}\epsilon_{3\sigma}) - \epsilon_1 \cdot p_3(\epsilon_{3\sigma}p_{1\rho} + \epsilon_{3\rho}p_{1\sigma}) - \epsilon_3 \cdot p_1(\epsilon_{1\sigma}p_{3\rho} + \epsilon_{1\rho}p_{3\sigma}) \\
+ \epsilon_1 \cdot \epsilon_3(p_{1\sigma}p_{3\rho} + p_{1\rho}p_{3\sigma}) + \epsilon_1 \cdot \epsilon_3 \cdot p_1\eta_{\rho\sigma} - \epsilon_1 \cdot \epsilon_3 p_1 \cdot p_3\eta_{\rho\sigma}],
\]  

(32)

where \(q = p_3 - p_1 = p_2 - p_4\), \(p_1\) and \(p_3\) are the initial and final propagation four-vectors of photon and \(p_2\) and \(p_4\) are those for fermion particle, \(\epsilon_{1\gamma}\) and \(\epsilon_{3\delta}\) are the polarization vectors of photon and \(m\) is the fermion mass.

We study this interaction in the rest frame of the fermion and regard the following definition for \(p_1\), \(p_2\) and \(p_3\)

\[
p_1 = (E_1, 0, 0, E_1), \quad \text{(33)}
\]

\[
p_2 = (m, 0, 0, 0), \quad \text{(34)}
\]

\[
p_3 = (E_3, E_3\sin\theta, 0, E_3\cos\theta), \quad \text{(35)}
\]
and also the circular polarization vectors of photon which are defined as

\[
\epsilon_{1+} = -\frac{1}{\sqrt{2}}(0, 1, i, 0),
\]

\[
\epsilon_{1-} = \frac{1}{\sqrt{2}}(0, 1, -i, 0),
\]

\[
\epsilon_{3+} = \frac{1}{\sqrt{2}}(0, -\cos \theta, -i, \sin \theta),
\]

\[
\epsilon_{3-} = \frac{1}{\sqrt{2}}(0, \cos \theta, -i, -\sin \theta).
\]

Taking into account the recent equations, one finds that there are four different combinations of polarization vectors of photon for matrix element.

We can calculate the differential cross section with use of Eq. (8), where \(|\mathcal{M}|^2\) is defined with Eq. (9). If the polarized photons participate in the interaction, the following expression is obtained for \(|\mathcal{M}|^2\)

\[
|\mathcal{M}|^2 = \frac{k^4}{2(16\ell^2)^2} A^{\rho\sigma\mu\nu} B_{\rho\sigma\mu\nu},
\]

where

\[
A^{\rho\sigma\mu\nu} = \text{Tr}\{[2\eta^{\rho\sigma}(p_2 + p_4 - 2m) - (p_2 + p_4)^\rho\eta^\sigma - \eta^\rho(p_2 + p_4)^\sigma](p_2 + m)
\]

\[
	imes[2\eta^{\mu\nu}(p_2 + p_4 - 2m) - (p_2 + p_4)^\mu\eta^\nu - \eta^\mu(p_2 + p_4)^\nu](p_4 + m)\},
\]

\[
B_{\rho\sigma\mu\nu} = [p_1 \cdot p_3(\epsilon_{3\mu}\epsilon_{3\rho} + \epsilon_{3\rho}\epsilon_{3\mu}) + \epsilon_1 \cdot p_3(p_1\epsilon_{3\rho} + \epsilon_{1\rho}\epsilon_{3\mu} + \epsilon_{1\mu}\epsilon_{3\rho})]
\]

\[
+ \epsilon_1 \cdot p_3(p_1\epsilon_{3\rho} p_{3\mu} + p_{1\rho} p_{3\mu}) + \epsilon_1 \cdot p_3(\epsilon_{3\mu} p_{1\rho} + \epsilon_{1\mu} p_{3\rho} - \epsilon_1 \cdot p_3(\epsilon_{3\mu} p_{1\rho} + \epsilon_{1\mu} p_{3\rho}).
\]

After calculating the trace in Eq. (41), developing the products in Eq. (42) and inserting Eqs. (33)–(39) in Eq. (40), we can obtain the following expressions for the differential cross sections of gravitational interaction for the fermion-polarized photon

\[
\frac{d\sigma}{d\Omega_{++}} = \frac{d\sigma}{d\Omega_{--}} = \frac{E_3^2}{E_1^2} \frac{k^4 m^2}{(2\pi)^2} \frac{\cos^4 \theta}{\sin^4 \frac{\theta}{2}},
\]

\[
\frac{d\sigma}{d\Omega_{+-}} = \frac{d\sigma}{d\Omega_{-+}} = 0,
\]

where we have used the notation \(\frac{d\sigma}{d\Omega_{\lambda_1\lambda_2}}\), in which \(\lambda_1 = \pm\) means \(\epsilon_{1\pm}\) and \(\lambda_2 = \pm\) means \(\epsilon_{2\pm}\).
Eventually, the differential cross section for the gravitational interaction of fermion-photon with unpolarized photons is given by

\[
\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{\text{pol.}} \frac{d\sigma}{d\Omega_{\lambda_1 \lambda_3}} = \frac{k^4 m^2 \cos^4 \frac{\theta}{2}}{(32\pi)^2 \sin^4 \frac{\theta}{2}},
\]

where we use the assumption that the target is so heavy and its recoil momentum is neglected. It is clear that the gravitational interaction of fermion-photon with unpolarized photons is a quartic function of \(\cos \frac{\theta}{2}\) which is different comparing to former incoming particles.

Table 1: The differential cross sections for the gravitational interaction of one fermion by another fermion, photon and scalar particle, in the fermion rest frame. Also the differential cross sections for the gravitational interaction of one scalar particle by another scalar particle, photon and fermion, in the rest frame of scalar particle. \(m\) is the mass of rest fermion particle and \(m'\) is the mass of rest scalar particle.

| Interaction |
|-------------|
| \(\frac{k^4 m^2}{(32\pi)^2 \sin^4 \frac{\theta}{2}}\) |
| \(ss' \rightarrow ss'\) |
| \(\frac{k^4 m^2 \cos^2 \frac{\theta}{2}}{(32\pi)^2 \sin^4 \frac{\theta}{2}}\) |
| \(sf \rightarrow sf\) (in the rest frame of scalar particle) |
| \(\frac{k^4 m^2 \cos^4 \frac{\theta}{2}}{(32\pi)^2 \sin^4 \frac{\theta}{2}}\) |
| \(s\gamma \rightarrow s\gamma\) |
| \(\frac{k^4 m^2}{(32\pi)^2 \sin^4 \frac{\theta}{2}}\) |
| \(sf \rightarrow sf\) (in the rest frame of fermion particle) |
| \(\frac{k^4 m^2 \cos^2 \frac{\theta}{2}}{(32\pi)^2 \sin^4 \frac{\theta}{2}}\) |
| \(ff' \rightarrow ff'\) |
| \(\frac{k^4 m^2 \cos^4 \frac{\theta}{2}}{(32\pi)^2 \sin^4 \frac{\theta}{2}}\) |
| \(f\gamma \rightarrow f\gamma\) |

The obtained expressions for the cross sections in the present work are displayed in Table 1. In order to compare these results with the differential cross sections of gravitational scattering of different quantum particles by a scalar particle, the cross sections for these interactions in the approximation of no recoil of the rest particle is also shown in the Table 1 [15, 25, 26].
4 Conclusions

In this paper, we have used the weak field limit of gravitational interaction by linearized gravity as an effective theory with quantum gravity motivation. We have obtained the cross sections of our favorite interactions by using a process that involves specifying graviton vertices by evaluation of the interaction Lagrangians of the gravitational field with the other gauge fields. We have calculated the matrix elements and eventually obtained the cross sections. These results can now be compared with the differential cross sections of gravitational scattering of particles with different spins by a scalar particle; According to the analytical results in Table 1, it can be concluded that the differential cross section for the gravitational interaction of elementary particles is independent of the spin of the rest particle but depends on the spin of the moving particle. As a result, the fermion-scalar particle gravitational interaction in the fermion rest frame and in the scalar particle rest frame are completely different.

Furthermore, according to Eqs. (43) and (44) we have found that the gravitational interaction of fermion-photon depends on the polarization of photons, as in this interaction the polarizations of the incoming and the outgoing photons are different, the differential cross section becomes zero, which means that the polarization of the photons in this interaction remains unchanged.

Moreover, the differential cross sections for small scattering angle is considered. According to Table 1, the differential cross sections obtained in this note are large in the forward scattering. This indicates significant gravitational effects for very small scattering angle.

As a final remark, we should note that the interactions in the presented table indicate that the differential cross section for a gravitational interaction is a function of $(\cos \frac{\theta}{2})^{4s}$, where $s$ is the spin of moving particle in the interaction. In other word, the differential cross section for a gravitational interaction does not depend on the spin of target in the target rest frame.

A Graviton propagator and vertices

A.1 Graviton propagator

\[ p_{\mu\nu} = \frac{iP_{\mu\nu\rho\sigma}}{p^2 + i\epsilon} \]

\[ P_{\mu\nu\rho\sigma} \equiv \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}). \]

(46)
A.2 Scalar-scalar-graviton vertex

\[
\begin{align*}
\mu\nu \quad q & \quad m & = \tau_{1}^{\mu\nu}(p_1, p_2, m) \\
p_1 & \\
p_2 \\
p_1 \\
p_2
\end{align*}
\]

\[
= \frac{i}{2}k[\eta^{\mu\nu}(p_1 \cdot p_2 - m^2) - p_1^\mu p_2^\nu - p_1^\nu p_2^\mu].
\tag{47}
\]

A.3 Fermion-fermion-graviton vertex

\[
\begin{align*}
\mu\nu \quad q & \quad m & = \tau_{2}^{\mu\nu}(p_1, p_2, m) \\
p_1 & \\
p_2 \\
p_1 & \\
p_2
\end{align*}
\]

\[
= \frac{i}{8}\kappa [2\eta^{\mu\nu}(p_1 + p_2 - 2m) - (p_1 + p_2)^\mu\gamma^\nu - (p_1 + p_2)^\nu\gamma^\mu].
\tag{48}
\]

A.4 Photon-photon-graviton vertex

\[
\begin{align*}
\mu\nu \quad q & \quad m & = \tau^{\mu\nu}(p_1, p_2, m) \\
p_1 & \\
p_2 \\
p_1 & \\
p_2
\end{align*}
\]

\[
= \frac{i}{2}k[2\mathcal{P}^{\mu\nu(\gamma\delta)}(p_1 \cdot p_2) + \eta^{\mu\nu}p_1^\delta p_2^\gamma + \eta^{\gamma\delta}p_1^\mu p_2^\nu + \eta^{\nu\delta}p_1^\mu p_2^\nu + \eta^{\nu\gamma}p_1^\delta p_2^\mu + \eta^{\mu\gamma}p_1^\delta p_2^\mu]
\]

\[
- (\eta^{\delta\nu}p_1^\nu p_2^\gamma + \eta^{\gamma\nu}p_1^\delta p_2^\delta + \eta^{\nu\gamma}p_1^\mu p_2^\mu + \eta^{\mu\gamma}p_1^\mu p_2^\nu + \eta^{\mu\gamma}p_1^\delta p_2^\nu)].
\tag{49}
\]
References

[1] LIGO Scientific Collaboration and Virgo Collaboration, *Observation of Gravitational Waves from a Binary Black Hole Merger*, Phys. Rev. Lett. 116 (2016) 061102.

[2] T. Thiemann, *Modern Canonical Quantum General Relativity*, Cambridge University Press (2007).

[3] M. Han, W. Huang, and Y. Ma, *Fundamental structure of loop quantum gravity*, Int. J. Mod. Phys. D 16 (2007) 1397.

[4] A. Ashtekar and J. Lewandowski, *Background independent quantum gravity: A Status report*, Class. Quant. Grav. 21 (2004) R53.

[5] C. Rovelli and F. Vidotto, *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (2014).

[6] A. Perez, *The Spin Foam Approach to Quantum Gravity*, Living Rev. Rel. 16 (2013) 3.

[7] M. Han, *Einstein Equation from Covariant Loop Quantum Gravity and Semiclassical Continuum Limit*, [arXiv:gr-qc/1705.09030].

[8] S. N. Gupta, *Quantization of Einstein's Gravitational Field: Linear Approximation*, Proc. Phys. Soc. A 65 (1952) 161.

[9] J. F. Donoghue, *General Relativity as an Effective Field Theory: The Leading Quantum Corrections*, Phys. Rev. D 50 (1994) 3874.

[10] J. F. Donoghue, *Leading Quantum Correction to The Newtonian Potential*, Phys. Rev. Lett. 72 (1994) 2996.

[11] B. M. Barker, M. S. Bhatia and S. N. Gupta, *Gravitational Scattering of Light by Light*, Phys. Rev. 158 (1967) 1498.

[12] D. Boccaletti, V. De Sabbata, C. Gualdi and P. Fortini, *Photon-Photon Scattering and Photon-Scalar Particle Scattering via Gravitational Interaction (One-Graviton Exchange) and Comparison of the Processes between Classical (General-Relativistic) Theory and the Quantum Linearized Field Theory*, Nuov. Cim. B 64 (1969) 419.

[13] F. G. Peet, *Cross Sections for the Scattering of Massless Particles by Graviton Exchange*, Can. J. Phys. 48 (1970) 923.
[14] D. Boccaletti, V. De Sabbata, P. Fortini and C. Gualdi, *Gravitational Scattering of Two- and Four-Component Neutrinos. Some Remarks in the One-Graviton-Exchange Approximation*, Nuov. Cim. B 11 (1972) 289.

[15] J. K. Lawrence, *Gravitational Fermion Interactions*, Gen. Rel. Grav. 2 (1971) 215.

[16] N. A. Voronov, *Gravitational Compton Effect and Photoproduction of Gravitons by Electrons*, Zh. Eksp. Teor. Fiz. 64 (1973) 1889.

[17] V. V. Skobelev, *Graviton-Photon Interaction*, Sov. Phys. J. 18 (1975) 62.

[18] S. Y. Choi, J. S. Shim and H. S. Song, *Factorization and Polarization in Linearized Gravity*, Phys. Rev. D 51 (1995) 2751.

[19] B. R. Holstein, *Graviton Physics*, Am. J. Phys. 74 (2006) 1002.

[20] N. E. J. Bjerrum-Bohr, B. R. Holstein, L. Plante and P. Vanhove, *Graviton-Photon Scattering*, Phys. Rev. D 91 (2015) 064008.

[21] F. Ravndal and M. Sundberg, *Graviton-Photon Conversion on Spin 0 and 1/2 Particles*, Int. J. Mod. Phys. A 17 (2002) 3963.

[22] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, John Wiley and Sons (1972).

[23] F. Halzen and A. D. Martin, *Quarks and Leptons: An Introduction Course in Modern Particle Physics*, John Wiley and Sons (1984).

[24] M. Jamin and M. E. Lautenbacher, *TRACER Version 1.1: A MATHEMATICA Package for γ-Algebra in Arbitrary Dimensions*, Comput. Phys. Commun. 74 (1993) 288.

[25] S. R Huggins and D. J Toms, *One-Graviton Exchange Interaction of Nonminimally Coupled Scalar Fields*, Class. Quantum Grav. 4 (1987) 1509.

[26] S. R Huggins, *Cross Sections from Tree-Level Gravitational Scattering from a Nonminimally Coupled Scalar Field*, Class. Quantum Grav. 4 (1987) 1515.