Recent work in light flavour hadron physics is reviewed. In particular, I discuss the significance of the progress achieved with light dynamical quarks on the lattice for the effective low energy theory of QCD. Also, I draw attention to some puzzling results from NA48 and KTeV concerning the scalar form factor relevant for $K_{\mu3}$ decay – taken at face value, these indicate physics beyond the Standard Model.

Keywords: Quantum chromodynamics, light quarks

1 Introduction

At low energies, the most important characteristic of QCD is that the energy gap is very small – a consequence of the fact that the $u$- and $d$-quarks happen to be very light. In the theoretical limit where the quark masses $m_u$ and $m_d$ are set equal to zero, the Hamiltonian of QCD acquires an exact, spontaneously broken symmetry. The spectrum of the theory does then not have an energy gap at all: in the limit, the pions become massless particles, playing the role of the Goldstone bosons which necessarily occur if a continuous symmetry spontaneously breaks down.

The remarkable theoretical progress made in light flavour hadron physics in recent years relies on the fact that these properties can be used to construct an effective theory (“chiral perturbation theory”, referred to as $\chi$PT), that allows us to analyze the low energy structure of QCD in a controlled manner. In this framework, all of the Green functions formed with the quark currents can be calculated in terms of the coupling constants occurring in the effective Lagrangian, order by order in the chiral expansion. For recent reviews of $\chi$PT, I refer to Scherer $^1$, Bijnens $^2$ and Colangelo.$^3$ An up-to-date account of our knowledge of the effective coupling constants can be found in a recent conference report by Ecker.$^4$

In my talk, I focused on a few selected aspects of this development. In particular, I emphasized the fact that the progress achieved in the numerical simulation of QCD on a lattice made it possible to reach sufficiently light quarks, so that the extrapolation to the quark masses of physical interest can be done in a controlled manner, using $\chi$PT. I will discuss this in some detail below. Another recent development which I will briefly report on, concerns the semileptonic decay $K \rightarrow \pi \mu \nu$. Both KTeV and NA48 have recently published new results on the scalar form factor of this decay, which are in flat conflict with a venerable low energy theorem, established by Callan and Treiman in the sixties.$^5$ If these results are confirmed, then the Standard Model is in conflict with observation in one of those reactions which we thought are best understood.

For lack of space, I cannot cover the third topic which I dealt with at these Rencontres: the progress made in understanding the properties of the interaction among the pions. This inter-
action plays a crucial role in many contexts – the Standard Model prediction for the magnetic moment of the muon is perhaps the most prominent example. To close this introductory section, I briefly list the corresponding keywords and indicate where more information about this can be found. The $\pi\pi$ scattering amplitude has been calculated to NNL in $\chi$PT. The resulting representation is very accurate in the interior of the Mandelstam triangle, but in the physical region, the convergence of the series is rather slow. The range of energies where the chiral representation yields meaningful results can be extended with the inverse amplitude trick, but one is then leaving the territory where model independent statements can be made. There is a general method that does not suffer from such shortcomings: in a limited region of the complex plane, dispersion theory imposes a set of exact relations between the real and imaginary parts of the partial wave amplitudes, the Roy equations. The region where these equations are valid includes the poles on the second sheet generated by the lowest resonances of QCD: $\sigma$, $\rho$, $\omega$, $f_0(980)$, $a_0(980)$. The crucial parameters that control the low energy properties of the scattering amplitude in this framework are the two subtraction constants. It is convenient to identify these with the two S-wave scattering lengths, because the low energy theorems of $\chi$PT make very sharp predictions for these. Together with the low energy theorems, the Roy equations pin the scattering amplitude down within remarkably small uncertainties. The angular momentum barrier ensures that the S- and P-waves dominate at low energies, but the framework also yields very accurate predictions for partial waves with higher angular momenta. On the basis of this method, it can be demonstrated beyond reasonable doubt that the lowest resonance of QCD carries the quantum numbers of the vacuum and the position of the corresponding pole can be worked out rather accurately. For an overview of these developments, I refer to the proceedings of Chiral Dynamics 2006 and the references quoted therein.

2 Size of the energy gap

In order to illustrate the progress achieved on the lattice, I consider one of the key issues in QCD: understanding the size of the energy gap. The quark masses $m_u$ and $m_d$ are very small, but they are different from zero. Accordingly, the Hamiltonian of QCD is not invariant under chiral rotations – the quark mass term breaks chiral symmetry and there is an energy gap: the symmetry breaking equips the Goldstone bosons with a mass. The quark masses $m_u, m_d$ represent a quantitative measure of the strength of the symmetry breaking. We know that the symmetry breaking is very small, but, unfortunately, the Standard Model does not offer an understanding of why that is so – the entire fermion mass pattern looks bizarre and yet remains to be understood.

As pointed out by Gell-Mann, Oakes and Renner, the square of the pion mass is proportional to the strength of the symmetry breaking, $M_\pi^2 \propto (m_u + m_d)$. This property can now be checked on the lattice, where – in principle – the quark masses can be varied at will. Fig. 1 shows the result for two recent lattice simulations of QCD with two flavours. In view of the fact that in these calculations, the quarks are treated dynamically, the quality of the data is impressive. The masses are sufficiently light for $\chi$PT to allow a meaningful extrapolation to the quark mass values of physical interest. The results indicate that the ratio $M_\pi^2/(m_u + m_d)$ is nearly constant out to values of $m_u, m_d$ that are about an order of magnitude larger than in nature.

3 Lattice determinations of the effective coupling constants $\bar{\ell}_3$ and $\bar{\ell}_4$

The Gell-Mann-Oakes-Renner relation represents the leading term in the expansion in powers of the quark masses. At next-to-leading order, this expansion contains a logarithm:

$$M_\pi^2 = M^2 \left\{ 1 + \frac{M^2}{32\pi^2 F^2} \ln \frac{M^2}{\Lambda^2} + O(M^4) \right\}, \quad M^2 \equiv B(m_u + m_d). \quad (1)$$
Chiral symmetry fixes the coefficient of the logarithm in terms of the pion decay constant $F$, but does not determine the scale $\Lambda_3$ of the logarithm. A crude estimate was obtained more than 20 years ago,\textsuperscript{14} on the basis of the SU(3) mass formulae for the pseudoscalar octet:

\[
0.18 \text{GeV} < \Lambda_3 < 2 \text{GeV} \iff \bar{\ell}_3 \equiv \ln \frac{\Lambda_3^2}{M_\pi^2} = 2.9 \pm 2.4 .
\]

The logarithmic term implies that the lines in Fig. 1 cannot be straight. For the central value, $\bar{\ell}_3 = 2.9$, the formula (1) yields only little curvature, but if $\bar{\ell}_3$ was at the lower (upper) end of the quoted range, the plot of $M_\pi^2$ versus $m_u = m_d$ would be strongly bent upwards (downwards), visibly departing from the lattice results shown in Fig. 1, already at the lowest quark mass value. Evidently, these results strongly constrain the value of the coupling constant $\bar{\ell}_3$.

| Table 1: Determinations of the effective coupling constants $\bar{\ell}_3$ and $\bar{\ell}_4$ |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| \(\chi PT\) \textsuperscript{9,14} | \(\text{MILC}\) \textsuperscript{17} | \(\text{Del Debbio et al.}\) \textsuperscript{18} | \(\text{ETM}\) \textsuperscript{12} |
| $\bar{\ell}_3$ | $2.9 \pm 2.4$ | $0.6 \pm 1.2$ | $3.0 \pm 0.5$ | $3.62 \pm 0.12$ |
| $\bar{\ell}_4$ | $4.4 \pm 0.2$ | $3.9 \pm 0.5$ | ____ | $4.52 \pm 0.06$ |

The first row in Table 1 compares the number in equation (2) with recent determinations of $\bar{\ell}_3$ on the lattice. The second row lists the analogous results for the coupling constant $\bar{\ell}_4$, which controls the quark mass dependence of the pion decay constant. To my knowledge, the first lattice calculation of effective coupling constants based on dynamical quarks was carried out by the MILC collaboration.\textsuperscript{15} In that project, the coupling constants $L_4, L_5, L_6, L_8$, which occur in the effective chiral SU(3)$\times$SU(3) Lagrangian at first nonleading order, were determined by analyzing the quark mass dependence of $M_\pi, M_K, F_\pi$ and $F_K$ by means of $\chi PT$. The corresponding values of $\bar{\ell}_3$ and $\bar{\ell}_4$ are readily worked out, using standard one loop formulae.\textsuperscript{16} The numbers quoted in the third column are obtained from a recent update of the MILC analysis, which relies on staggered quarks.\textsuperscript{17} The results obtained by Del Debbio et al.\textsuperscript{18} are based on two flavours of Wilson quarks, while the European Twisted Mass Collaboration\textsuperscript{12} uses two flavours of twisted mass Wilson quarks. All of the numbers are consistent with the values found in $\chi PT$, but some of them are more accurate. Unfortunately, the results obtained with staggered quarks
do not agree well with those based on Wilson quarks. Possibly, this is related to the fact that the latter calculations treat all quarks except $u$ and $d$ as infinitely heavy – as emphasized by Stern and collaborators, the strange quark may play a significant role here. Lattice results for 3 flavours of light Wilson quarks should become available soon, so that it should be possible to identify the origin of the difference.

The values of the low energy constants $\ell_3$ and $\ell_4$ also enter the theoretical prediction for the $\pi\pi$ scattering lengths. This is illustrated in Fig. 2, where theory is compared with experiment (the lattice results for $\ell_3$, $\ell_4$ are converted into corresponding values for $a_0^0, a_0^2$ using $\chi$PT and dispersion theory). While the lattice results, DIRAC and the NA48 data on the cusp in $K \rightarrow 3\pi$ confirm our predictions for $a_0^0, a_0^2$, the $K_{e4}$ data of NA48/2 give rise to a puzzle: the phase extracted from the transition amplitude deviates from the theoretical prediction for $\delta_0^0 - \delta_1^1$. The discrepancy originates in the fact that neutral kaons may first decay into a pair of neutral pions, which then undergoes scattering and winds up as a charged pair. As pointed out by Colangelo, Gasser and Rusetsky, the mass difference between the charged and neutral pions affects this process in a pronounced manner: it pushes the phase of the transition amplitude up by about half a degree – an isospin breaking effect, due almost exclusively to the electromagnetic interaction. The dash-dotted line in Fig. 2, which is taken from the talk of B. Bloch-Devaux at KAON 2007, shows the likelihood contour ($\chi^2 = \chi_{\text{min}}^2 + 2.3$) of the so corrected, preliminary NA48/2 data. The intersection with the region allowed by the low energy theorem for the scalar radius yields $a_0^0 = 0.220(9)$. However, there is a discrepancy with the E865 data, for which the likelihood contour is shown as a dashed line (kindly provided by G. Colangelo): the isospin correction spoils the good agreement between these data and the prediction. The discrepancy only concerns the region $M_{\pi\pi} > 350$ MeV. While E865 collects all events in this region in a single bin, the resolution of NA48/2 is better. The fit to all $K_{e4}$ data is therefore dominated by NA48/2. For a detailed discussion of these issues, I refer to the talks by B. Bloch-Devaux, G. Colangelo and J. Gasser at KAON 2007. I conclude that the puzzle is gone: $K_{e4}$ confirms the theory to remarkable precision.
4 Puzzling results in $K_{\mu3}$ decay

The low energy theorem of Callan and Treiman\textsuperscript{5} predicts the size of the scalar form factor of the decay $K \rightarrow \pi\mu\nu$ at one particular value of the momentum transfer, namely $t = M_K^2 - M_\pi^2$:

$$f_0(M_K^2 - M_\pi^2) = \frac{F_K}{F_\pi} + O(m_u, m_d).$$

(3)

Within QCD, the relation becomes exact if the quark masses $m_u$ and $m_d$ are set equal to zero. The corrections of first nonleading order, which have been evaluated long ago\textsuperscript{20} are tiny: they lower the right hand side by $3.5 \times 10^{-3}$. In the meantime, the chiral perturbation series of $f_0(t)$ has been worked out to NNL\textsuperscript{21}. As pointed out by Jamin, Oller and Pich\textsuperscript{22} the curvature of the form factor can be calculated with dispersion theory, so that the prediction for the value at $t = M_K^2 - M_\pi^2$ can be converted into a rather accurate prediction for the slope: $\lambda_0 = 0.0157(10)$. The dispersive representations of Jamin et al\textsuperscript{22} and Bernard et al\textsuperscript{23} agree very well: theory reliably determines the curvature of the form factor.

Very recently, the NA48 collaboration published their analysis of the $K^0_{\mu3}$ form factors\textsuperscript{24}. Their result for the scalar slope, $\lambda_0 = 0.0117(7)(1)$, is in flat conflict with the prediction of Jamin, Oller and Pich. Using the parametrization proposed by Bernard et al.\textsuperscript{23} NA48 obtains $f_0(M_K^2 - M_\pi^2) = 1.155(8)(13) \times f_+(0)$. The value of $F_K/F_\pi$ is sensitive to the theoretical input used for $f_+(0)$. Accurately measured branching ratios\textsuperscript{25} and the value of $V_{ud}$ (which has been determined to high precision on the basis of nuclear $\beta$ decays) imply $F_K/F_\pi = 1.241(4) \times f_+(0)$. So, if the NA48 results are correct, then the second term on the right hand side of (3) must amount to a contribution of $0.086(17) \times f_+(0)$. For a correction of $O(m_u, m_d)$, this size is unheard of (as mentioned above, the one loop approximation for this term amounts to -0.0035 and is thus smaller by a factor of about 20). At the current experimental accuracy, the radiative corrections may play a significant role. For $K_{\ell3}$ decay, these are known to one loop of $\chi$PT\textsuperscript{26} but the data yet remain to be analyzed in this framework.

The NA48 experiment is not the first to measure the slope of the scalar form factor. The first results were obtained in the seventies. In particular, the high statistics experiment of Donaldson et al.\textsuperscript{27} had confirmed the theoretical expectations with a slope of $\lambda_0 = 0.019(4)$. More recent experiments, however, came up with quite different results. In particular, three years ago, the KTeV collaboration at Fermilab arrived at a remarkably small scalar slope: $\lambda_0 = 0.01372(131)$. Analyzing 0.54 million charged kaon decays, ISTRA, on the other hand, obtained a much higher value: $\lambda_0 = 0.0196(12)(6)$.\textsuperscript{29} If the results of NA48 as well as those of ISTRA were correct, then the dependence of the form factors on the momentum transfer would have to show very strong isospin breaking – that would be extremely interesting in itself.

My conclusion is that the experimental situation calls for clarification. In particular, an analysis of the charged kaon decays collected by NA48 might help removing the dust. There are not many places where the Standard Model fails. Hints at such failures deserve particular attention.\textsuperscript{8}

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\textsuperscript{8}In the meantime, a preliminary analysis of the KLOE data on the scalar $K^0_{\mu3}$ form factor became available.\textsuperscript{25} The result for the slope, $\lambda_0 = 0.0156(26)$, is in excellent agreement with the theoretical prediction, thus removing the puzzle ... at the price of generating a new one: KLOE is in conflict with NA48.
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