1. INTRODUCTION

The gravitational potential of a planet surrounded by a protoplanetary disk is known to give rise to non-axisymmetric density waves, which propagate away from the planet-carrying angular momentum. Gravitational coupling with these density perturbations exerts a torque on the planet, which might lead to its migration if the one-sided torques produced by the inner and outer disks do not cancel. Pioneering linear calculations by Goldreich & Tremaine (1980) have shown that, neglecting this possible small asymmetry, the one-sided torque acting on a planet moving on a circular orbit in a razor-thin (2D) disk is

\[ T = q_{2D} (GM_p)^2 \frac{\Sigma_p R_p \Omega_p}{c_s^3}, \quad q_{2D} \approx 0.93, \]

where \( M_p, R_p, \) and \( \Omega_p \) are the planetary mass, semimajor axis, and angular frequency, respectively, \( c_s \equiv (\partial P/\partial \rho)_S \)^{1/2} is the adiabatic sound speed, and \( \Sigma_0 \) is the unperturbed disk surface density. Most of this torque is excited at Lindblad resonances corresponding to the \( m \sim R_p/h \gtrsim 1 \) azimuthal harmonic of the planetary potential and located at a distance \( \sim h = c_s/\Omega_p \) away from the planetary orbit. Closer to the planet, at separations \( h \) the excitation torque density decreases exponentially—the so-called torque cutoff phenomenon.

In three-dimensional (3D) stratified disks a variety of waves can be excited (Lubow & Pringle 1993; Korycansky & Pringle 1995; Lubow & Ogilvie 1998), including buoyancy waves whenever the Brunt–Väisälä frequency

\[ N = \left[ \frac{g}{\gamma} \frac{\partial}{\partial z} \ln \left( \frac{P_0}{\rho_0} \right) \right]^{1/2} \]

is non-zero. Global analytical calculation of the wave properties for all modes is available only for one special case, in which both the vertical disk structure and the equation of state (EOS) are isothermal, and \( N = 0 \). In this case, the result (1) still holds but with \( q_{2D} \) lowered to \( q_{3D} \approx 0.25 q_{2D} = 0.37 \) as a result of averaging the planetary potential over the vertical disk structure (Takeuchi & Miyama 1998). For other combinations of the vertical disk structure and EOS, in particular those characterized by non-zero \( N \), analytical results are limited only to either axisymmetric or low-\( m \) modes excited far from the planet. In that case, Lubow & Ogilvie (1998) found that although buoyancy waves are responsible for a fraction of the total torque excited by the planet, the total torque from all modes is the same as that given by traditional Lindblad torque formulae.

In this Letter, we carry out 3D simulations of stratified disks characterized by non-zero \( N \) with embedded low-mass planets to explore global excitation of buoyancy waves and their role in angular momentum exchange with the planet.

2. NUMERICAL METHOD

The numerical tool we used is Athena (Stone et al. 2008), a grid-based code with a higher-order Godunov scheme, piecewise parabolic method (PPM) for spatial reconstruction, and the corner transport upwind (CTU) method for multidimensional integration. We use the stratified shearing-box setup implemented in Athena (Stone & Gardiner 2010). Since the potential vorticity is zero in the shearing-box, the traditional corotation torque is zero (Goldreich & Tremaine 1979).

A planet is placed at the origin of a Cartesian box in \( x, y, z \) coordinates. Its smoothed potential is approximated as

\[ \Phi_p(d) = -GM_p \frac{d^2 + 3r_s^2/2}{(d^2 + r_s^2)^{3/2}}, \]

where \( r_s \) is the smoothing length and \( d = \sqrt{x^2 + y^2 + z^2} \). This potential converges to the point mass potential as \( (r_s/d)^4 \) for \( d \gg r_s \). In most runs \( r_s = 0.125h \), which is resolved by four grid cells with our normal resolution of \( 32/h \).

To reduce the computation time, we take advantage of the symmetry of the problem and only simulate the disk for
We take PA poly.

... potential and the disk structure are symmetric with (or momenta) in the x-directions are copied with the y-directions are copied with the z-directions are copied with the ghost zones in the active zones of one portion are copied into the ghost zones in the other portion symmetric with respect to z. At z = Z we use an outflow BC but the densities are extrapolated assuming hydrostatic equilibrium into the ghost zones, and to prevent inflow if vz < 0, then we set vz to 0.

2.1. Physical Setup

In our calculations, we use both an isothermal (γ = 1 in p ∝ ργ), and an adiabatic EOS (γ = 5/3 or 7/5). The unit of length in our simulations is defined via the adiabatic scale height h = cs,iso/Ω, where cs,iso = (kT/μ)1/2 and cs is the adiabatic sound speed calculated using the midplane temperature. Obviously, cs = cs,iso for an isothermal EOS (γ = 1). This choice keeps sound waves traveling the same distance in a given amount of time in different models so that the wake position is always the same.

The vertical structure of a protoplanetary disk is typically determined by irradiation from the central star so that it is vertically isothermal:

\[
\frac{\rho_0(z)}{\rho_{00}} = \exp\left(-\frac{z^2\Omega^2}{2c_{s,iso}^2}\right),
\]

where \(\rho_{00}\) is the midplane density. The value of \(c_{s,iso}\) is determined by the central irradiation flux (Kenyon & Hartmann 1987; Chiang & Goldreich 1997).

We also consider thermally stratified disks in which the disk midplane is hotter than its atmosphere, which is expected if viscous heating is important. Including the fact that such disks will eventually become isothermal at height z, the vertical structure of the disk is better represented as

\[
\rho_0(z) = c_{s,iso}^2 \rho_0 z \left[\left(\frac{\Gamma - 1}{\Gamma}\right) \left(\frac{\rho_0(z)}{\rho_z}\right)^{\Gamma - 1} + 1\right],
\]

where \(c_{s,iso}\) and \(\rho_z\) (set as 0.01 \(\rho_0\)) represent the sound speed and density at the transition where the disk becomes isothermal (Lin et al. 1990; Bate et al. 2002). However, unlike Bate et al. (2002), we do not necessarily set \(\Gamma\) in Equation (5) the same as \(\gamma\) in the EOS. We derive \(\rho_0(z)\) by solving the equation of vertical hydrostatic equilibrium with Equation (5) using the Newton–Raphson method. When \(\Gamma\) → 1 the isothermal disk structure is recovered.

We run a set of five models up to 10 orbits with different disk structure and EOS, summarized below and in Table 1.

| Case | Structure | EOS | γ | y-direc. Boundary | Brunt–Väisälä N | Torque Coefficient (q) | Z Domain Size | Resolution X × Y × Z |
|------|-----------|-----|---|-------------------|-----------------|----------------------|---------------|----------------------|
| II   | isothermal | 1   | in/outflow | 0 | 0.34 | [0, 5h] | 160 x 1280 x 160 |
| AA   | poly, Γ = 5/3 | 5/3 | in/outflow | 0 | 0.44 | [0, 2h] | 160 x 1280 x 64 |
| Aaper | poly, Γ = 5/3 | 5/3 | periodic | 0 | 0.44 | [0, 2h] | 160 x 1280 x 64 |
| IA   | isothermal | 5/3 | in/outflow | ≠0 | 0.48 | [0, 3.873h] | 160 x 1280 x 160 |
| Iaper | isothermal | 5/3 | periodic | ≠0 | 0.48 | [0, 3.873h] | 160 x 1280 x 160 |
| IA (64/h) | isothermal | 5/3 | in/outflow | ≠0 | 0.48 | [0, 3.873h] | 320 x 2560 x 320 |
| IAss (64/h, r = 1/16h) | isothermal | 5/3 | in/outflow | ≠0 | 0.48 | [0, 3.873h] | 320 x 2560 x 320 |
| IA2  | isothermal | 7/5 | in/outflow | ≠0 | 0.41 | [0, 3.873h] | 160 x 1280 x 160 |
| PA   | poly, Γ = 7/5 | 5/3 | in/outflow | ≠0 | 0.45 | [0, 2.5h] | 160 x 1280 x 80 |

Notes: * q as in \( T = q(GM_p)^2 \Sigma_0 R_p \Omega^2/r_\gamma^3 \).
2. Disk structure (5) with adiabatic EOS and $\Gamma = \gamma = 5/3$ (model AA).
3. Isothermal disk structure with adiabatic EOS having $\gamma = 5/3$ (model IA). This setup is typical for a centrally irradiated disk in which density perturbations are adiabatic.
4. Analogous to (4) but with $\gamma = 7/5$ (model IA2), designed to illustrate the effect of adiabatic index on the buoyancy wave coupling.
5. Disk structure (5) with $\Gamma = 5/3$ and adiabatic EOS with $\gamma = 7/5$ (model PA), to illustrate viscously heated accretion disks.

In the first two models $N = 0$ so that buoyancy waves are absent, while the last three models feature non-zero $N$ and support buoyancy waves. All models were run at a resolution of $32/h$, but we also run model (3) at higher resolution ($64/h$) to verify numerical convergence.

The planet mass is $M_p = 0.0058 M_{\text{th}}$, where $M_{\text{th}}$ is the thermal mass

$$M_{\text{th}} \equiv \frac{c^3}{G \Omega_p}.$$  

Thus, disk–planet coupling is well inside the linear regime. Density wave generated by such a planet in a 2D disk would shock at $|x| = 6 h$ (Goodman & Rafikov 2001), which is outside our box.

3. RESULTS

3.1. Without Buoyancy Waves

First we show the results for models with no buoyancy waves (II and AA). The spatial distributions of the excitation torque density $dT/dx$ (the amount of torque excited per unit radial distance) for these two models are shown in Figures 2(a) and (b) as green and orange curves. One can see that $dT/dx$ rapidly goes to zero for $|x| \lesssim h$, clearly exhibiting the “torque cutoff” phenomenon (Goldreich & Tremaine 1980) in both models.

For model II, our simulation agrees with the semi-analytical calculation by Takeuchi & Miyama (1998) remarkably well, and we confirm that Equation (1) holds in this case with $q_{3D}(II) = 0.25 q_{2D}$. Although wave structures are different in model AA (Lubow & Pringle 1993), the distribution of $dT/dx$ for this model is quite similar in shape to that of model II, but the amplitude is slightly different.

3.2. With Buoyancy Waves

We next look at the behavior of $dT/dx$ in models with non-zero $N$. The most important feature of these models, evident in Figure 2, is the weak torque cutoff near the planet—all show a significant torque contribution near $x = 0$, including model IA2, which provides the best description for an irradiated protoplanetary disk. Since higher-$m$ modes are excited at Lindblad resonances closer to the planet, the strong torque close to the planet also suggests the traditional torque cutoff in Fourier space (Ward 1997) is weaker when buoyancy waves are present.

The one-sided torque $T$ (Figures 2(c) and (d)) in models IA and IA2 is characterized by $q_{3D}(IA) = 0.48$ and $q_{3D}(IA2) = 0.41$, respectively, which should be compared with $q_{3D}(II) = 0.34$ for model II having the same vertical structure as IA and IA2 but different EOS.

In order to identify the origin of this excess torque near the planet, we integrate the volume density of the torque along the $y$ direction

$$d^2T = \int_{-\infty}^{\infty} dy \delta \rho(x, y, z) \frac{\partial \phi}{\partial y}. \quad (7)$$
Figure 3. Upper panels: (volume) density of the planet-induced torque integrated over $y$-coordinate for models II (left panel) and IA (right panel). Strong excess torque due to buoyancy waves close to the planet is clearly visible at $z \sim h$ in model IA, which features non-zero Brunt–Väisälä frequency. Lower panels: density contours in the $xy$ plane at $z$ where $\rho_0$ equal to 0.8755 $\rho_0$ (indicated by the dashed line at $z \sim 0.5h$ in (a) and $z \sim 0.4h$ in (b), $h$ are different in these two cases). Geometric locations corresponding to the resonance condition for buoyancy waves (Equation (11)) are shown by dotted lines in model IA, and agree quite well with the pattern of density fluctuations derived from simulations.

This quantity is shown in Figure 3(a) for models II and IA. We see that the excess torque in model IA comes from the region $z \sim 0.4h$ and $|x| < h$. In Figure 3(d) we plot the density contours in the $xy$ plane at $z = 0.4h$. In clear contrast with model II (Figure 3(c)), the density distribution in model IA exhibits density fluctuations/ridges close to the planet which are different from the usual wake structure. They extend to $y > 0$ and look like rays emanating from the origin. It is these density fluctuations that contribute to the excess torque.

These fluctuations also have vertical structure, as shown in Figure 4 where we plot $\delta \rho = \rho - \rho_0$ and $v_z$ in the $yz$ plane at $x = 0.5h$ for both models II and IA. Again, significant density fluctuations close to the planet exist only in case IA.

Since the only difference between case IA and case II is that the case IA has non-zero Brunt–Väisälä frequency $N$, it is natural to relate these density fluctuations to buoyancy waves. To confirm this hypothesis, we note that a mode with wavenumber $k_y$ has the buoyancy frequency $N$ whenever the condition

$$2A k_y x = N$$

is fulfilled. For comparison, the Lindblad resonance condition is $2A k_y x = \pm \kappa$. In the disk with isothermal structure and adiabatic EOS (models IA and IA2), we have

$$N = \frac{\Omega_z}{h} \sqrt{\frac{\gamma - 1}{\gamma}}.$$  \hspace{1cm} (9)

With $\gamma = 5/3$ and $\lambda_y = 2\pi/k_y$, Equations (8) and (9) can be combined to derive

$$\frac{\lambda_y}{h} = 11.543 \frac{x}{z}.$$ \hspace{1cm} (10)

If we assume the phase of buoyancy waves is 0 at $x = 0$, the geometric location of the constant phase $2n\pi$ ($n$ is integer) is given by

$$\frac{y}{h} = 11.543 \frac{x}{z} + n = 1, 2, 3, \ldots.$$ \hspace{1cm} (11)

Curves corresponding to Equation (11) are drawn in the lower right panels of Figures 3 and 4. They agree with the pattern of the density perturbation and $v_z$ quite well, strongly suggesting that these fluctuations and the excess torque are related to buoyancy waves (their dissipation or excitation).

The torque density and integrated torque for model IA2 (isothermal disk with adiabatic EOS and $\gamma = 1.4$) plotted as the cyan curves in Figure 2 also exhibit excess torque density due to buoyancy waves close to the planet. However, the integrated torque in this case is characterized by $q_{3D}(IA2) \approx 0.41$, which is lower than $q_{3D}(IA)$, and the buoyancy wave coupling to the planetary potential is weaker for smaller $\gamma$ as seen from $dT/dx$ curve.

Blue curves in the right panels of Figure 2 show $dT/dx$ and $T(x)$ for model PA. There is again an apparent torque excess near the planet, which is not surprising since $N \neq 0$ in this model, allowing buoyancy waves to be excited.
4. DISCUSSION

The phenomenon of torque cutoff in 2D disks is due to the fact that high-$k_y$ modes ($k_y \gtrsim h^{-1}$, excited close to the planet) of rotation-modified sound waves couple poorly to the planetary potential (even though it is strongest there). However, the dispersion relation for buoyancy waves is very different from that of the waves in 2D disks. Based on Equation (10), the pattern in our simulations should have $k_y \approx h^{-1}$ at $x \sim 0.05 h$ and $z = 0.1 h$, resulting in strong coupling between buoyancy waves and the planetary potential. This may explain the lack of torque cutoff in models with non-zero $N$. A further investigation will be presented in Z. Zhu et al. (in preparation).

Since the buoyancy torque is present very close to the planet, its strength could be sensitive to the potential softening length $r_s$ in the simulation. Thus, we re-ran model IA with a smaller smoothing length ($r_s = 1/16 h$) but obtained essentially the same result (purple curves in Figure 2), suggesting that our numerical results are robust.

4.1. Comparison with Previous Studies

Previous analytical and semi-analytical studies of buoyancy waves (Lubow & Pringle 1993; Korycansky & Pringle 1995; Lubow & Ogilvie 1998) have been limited to axisymmetric waves or low-$m$ modes excited far away from the planet, at $|x| \gg h$. These studies have demonstrated such modes to play rather insignificant role in planet–disk angular momentum exchange.

The main result of this work is that coupling of the planetary potential to the locally excited ($|x| \lesssim h$) buoyancy waves channels a considerable amount of angular momentum into these modes. As a result, the buoyancy torque strongly contributes to the planetary torque, at the level of tens of percent. This result is different from analytical expectations extrapolated from low-$m$ buoyancy modes (Lubow & Ogilvie 1998).

Most previous 3D simulations of planet–disk interaction explored isothermal disk structure with an isothermal EOS (e.g., Bate et al. 2003; D’Angelo & Lubow 2010). An adiabatic EOS has been explored in Paardekooper & Mellema (2006). However, because of the global geometry used in their work, the buoyancy torque may have been hard to distinguish from the corotation torque; see Section 4.3.

4.2. Existence and Signatures of Buoyancy Waves

The vertical structure of protoplanetary disks around T Tauri stars should be close to isothermal given the dominant role of stellar irradiation in their thermal balance. Existence of buoyancy waves in such disks depends on the thermodynamics of the fluid perturbations on timescales $\sim N^{-1}$. Density perturbations induced by planets result in temperature fluctuations which tend to be erased by radiative diffusion. If the cooling time $t_c$ of such perturbations is longer than $N^{-1}$ then modes are adiabatic with $\gamma > 1$ (as in models IA and IA2) and buoyancy waves should be present. In the opposite case of $t_c N \lesssim 1$ fluid perturbations behave as isothermal ($\gamma \approx 1$) and buoyancy waves are not excited, as in model II.

We expect that the separation between the two regimes should occur between several to several tens of AU, depending on the disk properties, and that buoyancy waves should be efficiently
excited by relatively close-in planets. Further out, $t_e N \lesssim 1$ and excitation of buoyancy waves should be suppressed. Note that both $t_e$ and $N$ are in general functions of $z$; see Equation (9). This makes cooling considerations dependent not only on the radius but also on the height in the disk.

Whenever buoyancy waves are efficiently excited by planets they can reveal themselves via associated vertical motions (see Figure 1) which should give rise to corrugations of the disk surface aligned with density ridges seen in Figure 3. Near-IR imaging of stellar light scattered by such perturbations at very high resolution (on scales $\sim h$) can reveal even low-mass planets embedded in protoplanetary disks.

### 4.3. Migration

The rate at which a planet migrates is determined by the imbalance of one-sided torques exerted on it by the inner and outer parts of the disk, caused by the global radial gradients of the disk density and temperature (Ward 1986; Tanaka et al. 2002). The net torque can also be caused by the local asymmetries in the disk properties caused by the presence of the planet itself, such as the temperature perturbations due to shadowing near the planet (Jang-Condell & Sasselov 2005), or the planet’s own heat output caused by, e.g., the accretion of planetesimals. Such asymmetries generated very close to the planet may not strongly affect the net standard Lindblad torque because of the torque cutoff at such separations. However, their effect on the net buoyancy torque excited primarily at small $x$ may be disproportionately large and affect the planetary migration considerably.

This statement is only true if the buoyancy waves observed in this work can propagate and deposit their angular momentum far from the corotation region. Otherwise, their angular momentum accumulates in the narrow annulus around the planetary orbit and the corresponding torque on the planet may be subject to saturation, like the standard corotation torque (Balmforth & Korycansky 2001; Masset 2001, 2002). This would eliminate the effect of the buoyancy waves on the planetary migration. We will investigate these possibilities in Zhu et al. (in preparation).

Direct measurement of the buoyancy torque effect on the migration speed would require global disk–planet calculations, in which corotation torque appears as well. It may then be difficult to separate the effects of the buoyancy and corotation torques in global simulations. However, the strength of the corotation torque depends on the radial gradients of specific vortensity and entropy across the horseshoe region (Paardekooper & Mellema 2006; Baruteau & Masset 2008; Paardekooper & Papaloizou 2008). By properly setting the disk properties to nullify these gradients one can eliminate the corotation torque in global simulations, thus isolating the contribution due to the buoyancy torque.

### 4.4. Wave Dissipation and Gap Opening

Gap opening by massive planets depends not only on the wave excitation but also on wave damping as a means of transferring angular momentum to the disk fluid (Lunine & Stevenson 1982; Rafikov 2002). The global density wave excited by planet is thought to dissipate primarily via shock damping (Goodman & Rafikov 2001; Dong et al. 2011), but buoyancy waves are very distinct from rotation-modified sound waves and should dissipate differently (Lubow & Ogilvie 1998; Bate et al. 2002). Channeling of the wave action (Lubow & Ogilvie 1998) may considerably speed up their nonlinear evolution resulting in more efficient damping. Since we showed that buoyancy waves carry good fraction of the total angular momentum flux, understanding their damping mechanism and spatial pattern of dissipation may be important for clarifying the issue of gap opening by planets (Zhu et al. in prep).

Our work suggests buoyancy waves can play an important role in planet migration and gap opening which demands further studies.

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