Dynamic deflection of concrete plate in semi-rigid supports and various damping condition

Yenny Untari Liucius1*, Sofia W. Alisjahbana2

1Civil Engineering Department, Faculty of Engineering, Universitas Tarumanagara, Jakarta, Indonesia
2Faculty of Engineering and Informatics, Bakrie University, Jakarta, Indonesia

*yenny@ft.untar.ac.id

Abstract. There are numerous researches about the dynamic behaviour of concrete plate. Most of them are done with the assumptions of simply supported supports along its edge. This study will calculate the dynamic deflection of isotropic concrete plate in semi-rigid support condition along the edges. The damping ratio will be varied to analyse how much it affects the dynamic deflection. The free vibration solution will be done with Modified Bolotin Method to solve the transcendental equations. The load subjected to plate, as the forced vibration, is the dynamic moving load considered as vehicle’s load on the road, moving along the mid-side of the plate with constant velocity. The result shows that damping ratio can reduce the dynamic deflection up to 48% at lower velocity (around 20-50 km/h); up to 30% at mid velocity (around 60-180 km/h); and up to 8% at upper velocity (around 190-200 km/h). Critical velocity is also analysed in this study. The critical velocity of this system is on 112 km/h which will conduct the maximum dynamic deflection on the plate. This result can be used as a basic for designing the plate efficiently yet safety for the users.

1. Introduction
This study aims to calculate the dynamic deflection with variation in damping ratio of an isotropic concrete plate subjected to moving load with constant velocity. Such a load represents the vehicle load along the road. According to Alisjahbana and Wangsadinata, the dynamic moving vehicle load can be represented by a single concentrated harmonic loading, moving along the mid-side of plate with constant velocity [1]. Considering the load over the mid-side of plate, so the deflections of the plate are maximum about the middle of the plate [2]. The assumption for the boundary supports of the plate is semi-rigid with rotational restraint. This is a very realistic assumption, particularly for concrete plates, because the rotational deformations exist along the joints [3]. This assumption is due to the tie bars and steel dowels, providing plate with rotational restraints [4]. Beside the assumption of the supports, damping is also one of the main factor to the dynamic behavior of the plate. Earlier study shows that damping plays a very significant role in the vibration of solid structure which affects the deflection profile of the structure [5, 6]. In this study, the damping is used as a viscous damping which provides force in the opposite direction to transverse deflection and proportional to the velocity of the system [7]. The solution of this structure will be done with Modified Bolotin Method to solve the
transcendental equation. This method is used to solve the shell and plate problems numerically with the trigonometry function [8]. The study will use the variation of the damping ratio to study on its effect to the deflection of plate. Variation of the velocity is also given to obtain the critical velocity of the system. This is crucial too as the moving vehicle load reach its critical velocity, the dynamic response (plate displacement) will be maximum [9].

2. Methods and formulations

Numeric calculation and analytical approach is used as method in this study. The calculation will be done with help of Wolfram Mathematica to solve the equation of the plate. The work for this analysis is divided by two steps. First is obtaining the solution for free vibration analysis done with help of Wolfram Mathematica to solve the equation of the plate. The work for this analysis will give the result of dynamic deflection of plate. Next step is obtaining the solution for forced vibration analysis with Duhamel’s integration. Forced vibration analysis depends on the dynamic load in function of time that working on the system. Completing this forced vibration analysis based on the solution of the free vibration analysis will give the result of dynamic deflection of plate.

Based on the work of Alisjahbana et.al [2], based on a classical theory of thin plate, the governing equation for the plate subjected to the lateral load is given by:

\[
D \left( \frac{\partial^4 w}{\partial x^2} + \frac{2}{\partial x^2} + \frac{\partial^4 w}{\partial y^2} \right) + \left( \frac{\partial^2 w}{\partial t^2} \right) + \gamma h \frac{\partial^2 w}{\partial t} \equiv p_z(x, y, t)
\]

(1)

where \( w \) is the transversal displacement of plate; \( D \) is the flexural rigidity which determined by \( \frac{Eh^3}{12(1-\nu^2)} \); \( E \) is Young’s Modulus; \( \nu \) is the Poisson’s ratio; \( \rho \) is the density of concrete plate; \( h \) is the thickness of plate; \( \gamma \) is the damping ratio; and \( p_z(x, y, t) \) is the lateral dynamic moving load.

Adopting the non-dimensional coordinates for solving the free vibration analysis, where \( \xi = x/a \), \( \eta = y/b \), \( s = a/b \), equation (1) becomes:

\[
\left( \frac{\partial^4 w}{\partial \xi^2} \right) + 2s^2 \left( \frac{\partial^4 w}{\partial \eta^2} \right) + \left( \frac{\partial^2 w}{\partial \eta^2} \right) + \gamma \left( \frac{\partial^2 w}{\partial t^2} \right) = 0
\]

(2)

The vehicle load which is moving along the surface of the concrete plate is assumed as the bus’ load \( p_z(x, y, t) \) and can be written as:

\[
P_z(x, y, t) = P(t) \delta[x - x(t)] \delta[y - y(t)]
\]

(3)

The \( P(t) \) is the vehicle load which is function of time can be expressed as \( P(t) = P_0 + P(t) = P_0 (1 + \alpha \cos(\omega t)) \) where:

- \( P_0 \) = equivalent single axle load (ESAL)
- \( \alpha \) = load coefficient, as a function of wheel suspension and toughness of the surface of road
- \( \omega \) = load frequencies
- \( x(t) \) = position of load in x direction
- \( y(t) \) = position of load in y direction
- \( \delta[] \) = Dirac-delta function

The dynamic load function in equation (3) can be solved with Duhamel’s integration to obtain forced vibration solution and can be written as:

\[
T_{pq}(t) + 2s^2 \omega_{pq} T_{pq}(t) + \omega_{pq}^2 T_{pq}(t) = \frac{1}{\rho h Q_{pq}} \int_{x=0}^{s} \int_{y=0}^{b} X_{pq}(x) Y_{pq}(y) P_z(x, y, t) dx dy
\]

(4)
where $Q_{pq}$ is the normalization factor of eigenvector:

$$Q_{pq} = \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} X^{2}(x)Y^{2}(y) \right) dx dy$$

(5)

3. Results and discussion

The data for isotropic damped plate subjected to lateral moving load is given in Table 1.

| Notation | Implication | Value | Units |
|----------|-------------|-------|-------|
| a        | Length of plate | 5     | m     |
| s        | Length to width ratio | 1     |       |
| h        | Plate thickness  | 0.15  | m     |
| $\rho$   | Concrete density | 2400  | kg/m$^3$ |
| $E_x$    | Young’s modulus in x direction | 2.35x10$^{10}$ | N/m$^2$ |
| $E_y$    | Young’s modulus in y direction | 2.35x10$^{10}$ | N/m$^2$ |
| $\nu_x$  | Poisson Ratio in x direction | 0.3   |       |
| $\nu_y$  | Poisson Ratio in y direction | 0.3   |       |
| $D_x$    | Flexural rigidity in x direction | 7.26305x10$^6$ | Nm |
| $D_y$    | Flexural rigidity in y direction | 7.26305x10$^6$ | Nm |
| $P_0$    | Vehicle load | 100000 | N     |

Based on data given above, with Modified Bolotin Method, we can calculate the natural frequencies (free vibration solution) of the plate for the first 5 modes as below:

| Table 2. Natural frequencies |
|-------------------------------|
| n   | m = 1  | m = 2  | m = 3  | m = 4  | m = 5  |
|     | $\omega$ (rad/s) | $\omega$ (rad/s) | $\omega$ (rad/s) | $\omega$ (rad/s) | $\omega$ (rad/s) |
| 1   | 111,752 | 280,143 | 560,594 | 953,157 | 1457,850 |
| 2   | 280,143 | 448,403 | 728,823 | 1121,380 | 1626,080 |
| 3   | 560,594 | 728,823 | 1009,220 | 1401,760 | 1906,450 |
| 4   | 953,157 | 1121,380 | 1401,760 | 1794,300 | 2298,980 |
| 5   | 1457,850 | 1626,080 | 1906,450 | 2298,980 | 2803,660 |

The force vibration solution is done afterward to obtain the deflection of the concrete plate. Variation of damping ratio is used to compare the deflection between its values.
Based on the result above, plate without damping will deflect greater than plate with damping ratio. The damping affects more significant in the lower velocity and tends to be smaller as the velocity reaches 190 km/h. Damping can reduce the dynamic deflection for up to 48% at lower velocity condition (around 20-50 km/h); up to 30% at mid velocity (around 60-180 km/h); and up to 8% at upper velocity (around 190-200 km/h). Furthermore, maximum deflection occurs at 112 km/h for all three kinds of damping ratio which is called as the critical velocity of the system.

4. Conclusion
A numerical analysis has been done to obtain the dynamic deflection of an isotropic concrete plate with semi-rigid support along the edges and variation of damping ratio. The analysis starts from obtaining the solution of free vibration analysis and then the force vibration solution as completion for the dynamic deflection of plate. Result shows that damping ratio significantly reduces the dynamic deflection of plate for up to 48% at lower velocity condition (around 20-50 km/h). The effect tends to be smaller as the increasing of velocity. Usually the bus will travel at this rate of velocity. So the damping ratio is very important to reduce the dynamic deflection of system. The critical velocity of system is at 112 km/h and causes the maximum deflection to the plate. So the dynamic behaviour at this rate of velocity must be considered when designing the plate.

5. References
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