Square-Wave Generation in Vertical External-Cavity Kerr-Gires-Tournois Interferometers

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We study theoretically the mechanisms of square-wave (SW) formation in a monomode micro-cavity, containing a nonlinear Kerr medium coupled to a long external feedback cavity under continuous wave injection (see Fig. 1). The resulting so-called Kerr-Gires-Tournois interferometer can be described by the first-principles model

\[
\dot{E}(t) = (i [E(t)]^2 - \delta) - 1) E + hY(t),
\]

\[
Y(t) = \eta e^{i\phi} (E(t-\tau) - Y(t-\tau)) + \sqrt{1 - \eta^2}Y_0,
\]

which is delay algebraic equation (DAE) system for the slowly evolving envelope field dynamics of \(E\) and \(Y\) under injection with amplitude \(Y_0\) and frequency \(\omega_0\). Here, \(h = h(r_1, r_2)\) is the light coupling efficiency where usually \(h(r_1, 1) = 2\) is used for perfectly reflective bottom mirror. \(\phi = \omega_0 + \phi\) is the feedback phase consisting of a phase jump \(\phi\) and an accumulated roundtrip phase, while \(\delta = \omega_0 - \omega_0\) is the detuning with respect to micro-cavity resonance. We investigate dynamics in the dispersive Gires-Tournois interferometer regime, which corresponds to \(\eta \to 1, h = 2\) [1]. In [2, 3] it was shown that for \(\phi = 0\) the Kerr nonlinearity causes a bistable continuous wave (CW) response in a certain \(Y_0\) and \(\delta\) range. However, the bistability can be lost, e.g. for the antiresonant situation \(\phi = \pi\), which is also a regime where SWs can be observed. Using a two-plateau approximation, we provide a simple analytical model for the calculation of SW’s plateau intensities and the bifurcation points limiting the range of existence of the SWs. Using a combination of path-continuation techniques and direct numerical simulations, we demonstrate that beyond the well-established supercritical scenario for their emergence, SWs can also exhibit emerge in a homoclinic snaking scenario leading to the formation of complex-shaped multistable SW solutions (see Fig. 2). Beyond that, more complex SW dynamics can be identified, including a period doubling route to chaos. The results obtained from the full DAE model and the simple analytical approximation are in excellent agreement. Finally, the presence of the highly oscillatory tails of SWs opens up the possibility to build more complex patterns. Hence, we demonstrate that SWs can be used as a platform to host other structures and we show that robust multiple bound states consisting of localized pulses can be formed on the SW plateaus [4].

References
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