Shape invariance method for quintom model in the bent brane background

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Abstract

In the present paper, we study the braneworld scenarios in the presence of quintom dark energy coupled by gravity. The first-order formalism for the bent brane (for both de Sitter and anti-de Sitter geometry), leads us to discuss the shape invariance method in the bent brane systems. So, by using the fluctuations of metric and quintom fields we obtain the Schrodinger equation. Then we factorize the corresponding Hamiltonian in terms of multiplication of the first-order differential operators. These first-order operators lead us to obtain the energy spectrum with the help of shape invariance method.
1 Introduction

It is known that all analytically solvable potentials in quantum mechanics have the property of shape invariance [1]. In fact shape invariance is an integrability condition, however, one should emphasize that shape invariance is not the most general integrability condition as not all exactly solvable potentials seem to be shape invariance to [2, 3]. An interesting feature of supersymmetric quantum mechanics is that for a shape invariant system [4, 5] the entire spectrum can be determined algebraically without ever referring to underlying differential equations.

In the present paper we would like to use this method for an interesting problem in cosmology. Here we consider the quintom model of dark energy [6] in the bent brane background. One of the most important problems of cosmology, is the problem of so-called dark energy. The type Ia supernova observations suggests that the universe is dominated by dark energy with negative pressure which provides the dynamical mechanism of the accelerating expansion of the universe [7, 8, 9]. The strength of this acceleration is presently matter of debate, mainly because it depends on the theoretical model implied when interpreting the data. Most of these models are based on dynamics of a scalar [10, 11, 12, 13] or multi-scalar fields [6]. Primary scalar field candidate for dark energy was quintessence scenario [10, 11], a fluid with the parameter of the equation of state lying in the range, \(-1 < w < \frac{1}{3}\). The analysis of the properties of dark energy from recent observations mildly favor models with \(w\) crossing \(-1\) in the near past. Meanwhile for the phantom model [12] of dark energy which has the opposite sign of the kinetic term compared with the quintessence in the Lagrangian, one always has \(w \leq -1\). Neither the quintessence nor the phantom alone can fulfill the transition from \(w > -1\) to \(w < -1\) and vice versa. But one can show [6] that considering the combination of quintessence and phantom in a joint model, the transition can be fulfilled. This model, dubbed quintom, can produce a better fit to the data than more familiar models with \(w \geq -1\).

This paper is organised as follows. In section 2 we consider the quintom model of dark energy in the background of a five-dimensional space-time with warped geometry. Then we consider the fluctuations of the metric and quintom scalar fields. In section 3 we review the supersymmetry algebra with the central charge and shape invariance method. In section 4 we obtain the factorized Hamiltonian for the bent brane, which leads us to investigate the shape invariance method with considering the central extended algebra. Finally by taking advantage of shape invariance method we obtain energy spectrum in our interesting geometry.

2 The quintom model in the bent brane background

The quintom model of dark energy [6] is of new models proposed to explain the new astrophysical data, due to transition from \(w > -1\) to \(w < -1\), i.e. transition from quintessence dominated universe to phantom dominated universe. Containing the normal scalar field \(\phi\) and negative kinetic scalar field \(\chi\), the action which describes the quintom model is expressed
as the following form
\[ S = \int d^4x dy \sqrt{|g|} \left( -\frac{1}{4}R + \frac{1}{2} \partial_a \phi \partial^a \phi - \frac{1}{2} \partial_a \chi \partial^a \chi - V(\phi, \chi) \right). \] (1)

where we have not considered the lagrangian density of matter field, and we take \(4\pi G = 1\). The line element of the five-dimensional space-time can be written
\[ ds^2_5 = g_{ab} dx^a dx^b = e^{2A} ds^2_4 - dy^2 \] (2)

where \(a, b = 0, 1, 2, 3, 4\), and \(e^{2A}\) is the warp factor. \(ds^2_4\) represent the four-dimensional metric:
\[ ds^2_4 = dt^2 - e^{2\sqrt{\Lambda}}(dx_1^2 + dx_2^2 + dx_3^2) \] (3)

where \(\Lambda\) is four dimensional cosmological constant. We note that constant \(\Lambda\) is positive for de Sitter (\(dS_4\)) spacetime, negative for anti-de Sitter (\(AdS_4\)) spacetime and zero for Minkowski (\(M_4\)) spacetime.

At first we consider the interacting case with \(\Lambda = 0\), also the functions \(A\), \(\phi\), \(\chi\) are \(A(y), \phi(y), \chi(y)\).

From the Einstein and Euler-Lagrange equations we obtain,
\[
\begin{align*}
A'' &= -\frac{2}{3}(\phi'^2 - \chi'^2), \\
A'^2 &= \frac{1}{6}(\phi'^2 - \chi'^2) - \frac{1}{3} V(\phi, \chi), \\
V_{\phi} &= \phi'' + 4A'\phi', \\
V_{\chi} &= -\chi'' - 4A'\chi'
\end{align*}
\] (4)

where a prime denotes a derivative with respect to \(y\), and
\[
V_{\phi} = \frac{dV}{d\phi}, \quad V_{\chi} = \frac{dV}{d\chi}.
\] (5)

In order to obtain the first-order equation, we use \([14]\)
\[
\begin{align*}
A' &= -\frac{1}{3} W, \\
\phi' &= \frac{1}{2} W_{\phi}, \\
\chi' &= -\frac{1}{2} W_{\chi}
\end{align*}
\] (6)

From (4) and (6) the explicit form of the potential is
\[
V(\phi, \chi) = \frac{1}{8}(W_{\phi}^2 - W_{\chi}^2) - \frac{1}{3} W^2
\] (7)

Next we consider the general case with \(\Lambda \neq 0\) and we obtain
The cosmological constant leads us to define the function which corresponds to the scalar fields φ and χ. It means that this function is completely coupled and generally responsible for the cosmological constant. Thus we gain

\[
\begin{align*}
A'' + \Lambda e^{-2A} &= -\frac{2}{3}(\phi'^2 - \chi'^2), \\
A'^2 - \Lambda e^{-2A} &= \frac{1}{6}(\phi'^2 - \chi'^2) - \frac{1}{3}V(\phi, \chi)
\end{align*}
\]  

(8)

where \(Z = Z(\phi, \chi)\) is a new and arbitrary function of the scalar fields and respond for the presence of the cosmological constant. \(\alpha, \beta\) and \(\gamma\) are constants. The corresponding potential is,

\[
V(\phi, \chi) = \frac{1}{8}(W_\phi + \Lambda(\alpha + \gamma)Z_\phi)(W_\phi + \Lambda(\gamma - 3\alpha)Z_\phi) - \frac{1}{3}(W + \Lambda\gamma Z)^2 
- \frac{1}{8}(W_\chi + \Lambda(\beta + \gamma)Z_\chi)(W_\chi + \Lambda(\gamma - 3\beta)Z_\chi)
\]  

(10)

Here we assume that

\[W_{\phi\chi} = 0\]  

(11)

So, by inserting this potential in the equations of motion one can obtain the following constraint,

\[
\begin{align*}
\alpha W_{\phi\phi}Z_\phi + \alpha W_\phi Z_{\phi\phi} + 2\Lambda\alpha(\alpha + \gamma)Z_\phi Z_{\phi\phi} - \frac{4}{3}\alpha Z_\phi(W + \Lambda\gamma Z) &= 0 \\
\beta W_{\chi\chi}Z_\chi + \beta W_\chi Z_{\chi\chi} + 2\Lambda\beta(\beta + \gamma)Z_\chi Z_{\chi\chi} + \frac{4}{3}\beta Z_\chi(W + \Lambda\gamma Z) &= 0
\end{align*}
\]  

(12)

For simplicity, we consider \(Z(\phi, \chi) = W(\phi, \chi)\). This possibility leads to equations,

\[
\begin{align*}
\frac{3}{2}dW_{\phi\phi} - W &= 0, \\
-\frac{3}{2}d'W_{\chi\chi} + W &= 0
\end{align*}
\]  

(14)

(15)

where \(d = \frac{1+\Lambda(\gamma+\alpha)}{1+\Lambda\gamma}\) and \(d' = -\frac{(1+\Lambda(\gamma+\beta))}{1+\Lambda\gamma}\). If we consider the combination of Eqs. (14, 15) we obtain:

\[dW_{\phi\phi} - d'W_{\chi\chi} = 0\]  

(16)
To explain this new result, we take super-potential as,

$$ W(\phi, \chi) = 3a \sinh (b\phi + c\chi) \quad (17) $$

where $b = \sqrt{\frac{2}{3d}}$ and $c = \sqrt{\frac{2}{3d'}}$, and $a$ is a constant. By substituting super-potential (17) into Eqs.(10), (9) one can obtain following equations respectively for potential, scalar fields, and $A(y)$

$$ V(\phi, \chi) = -\frac{3}{4}a^2(1 + \Lambda\gamma)(2 + \Lambda(2\gamma + 3\alpha + 3\beta)) \cosh^2(b\phi + c\chi) + 3a^2(1 + \Lambda\gamma)^2 \quad (18) $$

also the $\phi(y)$, $\chi(y)$ and $A(y)$ as follow,

$$ \phi(y) = \pm \sqrt{\frac{3d}{8}} \ln \left[ \tan(a(1 + \Lambda\gamma)y) \right] \quad (19) $$

$$ \chi(y) = \pm \sqrt{\frac{3d'}{8}} \ln \left[ \tan(a(1 + \Lambda\gamma)y) \right] \quad (20) $$

$$ A(y) = \frac{1}{2} \ln \left[ \frac{1}{2} \sin(2a(1 + \Lambda\gamma)y) \right] \quad (21) $$

Now we are going to discuss shape invariance condition. This condition help us to investigate the stability of system. So, first we consider the fluctuations of the metric and scalar fields. The perturbed metric is,

$$ ds^2 = e^{2A}(g_{\mu\nu} + \epsilon h_{\mu\nu})dx^\mu dx^\nu - dy^2 \quad (22) $$

We use the coordinate $z$ which is defined by following expression,

$$ dz = e^{-A(y)}dy \quad (23) $$

So, one can obtain the coordinate $z$ as a follow,

$$ z = \frac{1}{a(1 + \Lambda\gamma)} \ln \left[ \tan(a(1 + \Lambda\gamma)y) \right] \quad (24) $$

and $y$ is

$$ y = \frac{1}{a(1 + \Lambda\gamma)} \arctan(e^{a(1+\Lambda\gamma)z}) \quad (25) $$

Also, $A(z)$ be in the form,

$$ A(z) = \frac{1}{2} \ln \left[ \frac{1}{2} \text{sech}(\eta z) \right] \quad (26) $$

where for simply, $a = 1$ and $\eta = (1 + \Lambda\gamma)$. 

3 Shape Invariance Method

If the ground state energy is zero, we can factorize the Hamiltonian as,

$$H_1(g) = B^\dagger(g)B(g)$$  
(27)

where $g$ is (are) the real parameter(s), which give(s) us the potential, and $B(g)$ is a first order differential operator. The ground state of $H_1$ is annihilated by $B(g)$. The partner Hamiltonian of $H_1$ will be obtain with reversing the order of $B$ and $B^\dagger$,

$$H_2(g) = B(g)B^\dagger(g).$$  
(28)

The spectrum of $H_1$ and $H_2$ is degenerate. The only difference is that $H_1$ has a zero-energy state and $H_2$ in general does not, so we have,

$$H_2B = BH_1.$$  
(29)

If we had for $n \geq 0$,

$$H_1\Psi_n^{(1)} = E_n^{(1)}\Psi_n^{(1)}$$  
(30)

this implies that,

$$H_2(B\Psi_n^{(1)}) = E_n^{(1)}(B\Psi_n^{(1)}).$$  
(31)

So, the relation between the eigenvalues and eigenfunctions of the two hamiltonians $H_1$ and $H_2$ are,

$$E_n^{(2)} = E_{n+1}^{(1)}, \quad E_0^{(1)} = 0,$$

$$\Psi_n^{(2)} \propto A\Psi_{n+1}^{(1)},$$  
(32)

where the ground state wavefunction for $H_1$ (or $H_2$) can be obtained as,

$$B\Psi_0^{(1)}(x) = 0 \Rightarrow \Psi_0^{(1)}(x) = N \exp(-\int x W(y)d(y))$$

$$B^\dagger\Psi_0^{(2)}(x) = 0 \Rightarrow \Psi_0^{(2)}(x) = N \exp(+\int x W(y)d(y)).$$  
(33)

We know that, supersymmetry give relationships between Hamiltonian $H_1$ and $H_2$, where $H_1$ and $H_2$ are partner of each other.

$$H = \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix}. $$  
(34)

This matrix can be obtained from anticommutator $H = \{Q,Q^\dagger\}$, where $Q$ and $Q^\dagger$ are supercharges, given by,

$$Q = \begin{pmatrix} 0 & 0 \\ B & 0 \end{pmatrix}, \quad Q^\dagger = \begin{pmatrix} 0 & B^\dagger \\ 0 & 0 \end{pmatrix}$$  
(35)
In this algebra we have

\[
[H, Q] = [H, Q^\dagger] = 0 \\
\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0
\] (36)

A potential is said to be shape invariant, if its supersymmetry partner potential has the same spatial dependence as the original potential, with possibly altered parameters. Suppose that, these Hamiltonian are linked by the condition,

\[
B(g_1)B^\dagger(g_1) = B^\dagger(g_2)B(g_2) + c(g_2),
\] (37)

where the real parameters \( g_1 \) and \( g_2 \) are related by a mapping \( f : g_1 \rightarrow g_2 \), and \( c(g) \) is a \( c \)-number that depends on the parameter(s) of the Hamiltonian, so we have,

\[
H_k = B^\dagger(g_k)B(g_k) + c(g_k) + \cdots + c(g_2),
\] (38)

where

\[
g_{j+1} = f(g_j) \\
B^\dagger(g_k)H_{k+1} = H_kB^\dagger(g_k)
\] (39)

The ground state of each of these sectors satisfies a first-order equation, namely

\[
B(g_k)\Psi_1(x; g_k) = 0.
\]

Now, we study supersymmetry with central charge. Supersymmetric quantum mechanics [10] can be formulated as a one-dimensional supersymmetric quantum field theory. A bosonic field is, then, a real-valued function of time, and a fermionic field is a Grassman-valued function of time. The \( d = 1, N = 1 \) superalgebra with a central charge is specified by the following relations,

\[
\{Q, Q^\dagger\} = H \\
[H, Q] = [H, Q^\dagger] = 0 \\
\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = C,
\] (40)

where \( Q \) and \( C \) are supercharge and central charge respectively. The above algebra implies \([Q, C] = [Q^\dagger, C] = 0\)

To realize the algebra (40), we represent the supercharges as matrices,

\[
Q = \begin{pmatrix} \lambda & 0 \\ B & -\lambda \end{pmatrix} \quad Q^\dagger = \begin{pmatrix} \lambda & B^\dagger \\ 0 & -\lambda \end{pmatrix}
\] (41)

where \( \lambda \) is a real \( c \)-number. This approach is first to present an implementation of this algebra in a two-sector model, and then to generalize this construction to an arbitrary
number of sectors.
The corresponding Hamiltonian for two-sector is,

$$H = \begin{pmatrix} B^\dagger B + 2\lambda^2 & 0 \\ 0 & BB^\dagger + 2\lambda^2 \end{pmatrix}, \quad C = \begin{pmatrix} 2\lambda^2 & 0 \\ 0 & 2\lambda^2 \end{pmatrix}, \quad C \geq 0 \quad (42)$$

To construct a model with 4 sectors, one can concentrate 2 two-sector model. It has supercharges

$$Q = \begin{pmatrix} -\lambda_1 & 0 & 0 & 0 \\ B_1 & \lambda_1 & 0 & 0 \\ 0 & 0 & -\lambda_3 & 0 \\ 0 & 0 & B_3 & \lambda_3 \end{pmatrix}, \quad Q^\dagger = \begin{pmatrix} -\lambda_1 & B_1^\dagger & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & -\lambda_3 & B_3^\dagger \\ 0 & 0 & 0 & \lambda_3 \end{pmatrix} \quad (43)$$

we obtain,

$$H = \begin{pmatrix} B_1^\dagger B_1 + 2\lambda_1^2 & 0 & 0 & 0 \\ 0 & B_1 B_1^\dagger + 2\lambda_1^2 & 0 & 0 \\ 0 & 0 & B_3^\dagger B_3 + 2\lambda_3^2 & 0 \\ 0 & 0 & 0 & B_3 B_3^\dagger + 2\lambda_3^2 \end{pmatrix} \quad (44)$$

and

$$C = \begin{pmatrix} 2\lambda_1^2 & 0 & 0 & 0 \\ 0 & 2\lambda_1^2 & 0 & 0 \\ 0 & 0 & 2\lambda_3^2 & 0 \\ 0 & 0 & 0 & 2\lambda_3^2 \end{pmatrix} \quad (45)$$

As we see the sectors 1 and 2 are degenerate, with energies bounded from below by $2\lambda_1^2$, and sectors 3 and 4 are degenerate, with energies bounded from below by $2\lambda_3^2$. The only exceptions are that sectors 1 and 3 each have states that saturate their respective energy bounds while the even sectors do not.

This suggest an enhanced algebraic structure. In four sector case, we define the shift operator $S$ by

$$S \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ B_1 & 0 & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & B_3 & 0 \end{pmatrix} \quad (46)$$

We can choose $D$ such that shape invariance and $[H, S] = 0$ satisfy to corresponding case. For this, we suppose a unitary transformation which is represented by an operator $\Omega$ such that $B_3 = \Omega^\dagger B_1 \Omega$. And also, we use a unitary operator $U$ such that $U^2 = \Omega$, and the conserved shift operator takes the form,

$$S \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ B_1 & 0 & 0 & 0 \\ 0 & U B_1 U & 0 & 0 \\ 0 & 0 & U^2 B_1 U^2 & 0 \end{pmatrix} \quad (47)$$
From conservation of $S$, one can obtain the shape invariance relation,

$$B_1 B_1^\dagger - U^\dagger B_1 B_1 U = k,$$

$$2 \lambda_3^2 = 2 \lambda_1^2 + k + U^\dagger k U. \quad (48)$$

Therefore, the Hamiltonian of the four sector model is related to $S$ in an especially simple way. In particular

$$H = S^\dagger S + F, \quad (49)$$

where

$$F = \begin{pmatrix} 2 \lambda_1^2 & 0 & 0 & 0 \\ 0 & 2 \lambda_1^2 + k & 0 & 0 \\ 0 & 0 & 2 \lambda_1^2 + k + U^\dagger k U & 0 \\ 0 & 0 & 0 & H_4 \end{pmatrix}. \quad (50)$$

In the first three sectors, the energies are constrained by a Bogmol’nyi bound, $H_k \geq (F)_{kk}$, because each of the first sector has to be degenerate with the Bogmol’nyi - saturating ground state of one of the first three sectors. The constants in $F$ represent not only the Bogmol’nyi bounds of the various sectors, but also the first three energy eigenvalues of the original Hamiltonian.

Next section, we take advantage from above information, and also apply to bent brane with quintom dark energy.

### 4 The Stability of System with Shape Invariance Method

In order to discuss the shape invariance condition we have to make the corresponding Schrödinger equation in terms of coordinate $z$

$$- \frac{d^2 \psi(z)}{dz^2} + V(z) \psi(z) = n^2 \psi(z) \quad (51)$$

where

$$V(z) = - \frac{9}{4} \Lambda + \frac{3}{2} A''(z) + \frac{9}{4} A'^2(z) \quad (52)$$

Now we can factorize the corresponding Schrödinger equation in the form

$$\left[ \frac{d}{dz} + \frac{3}{2} A'(z) \right] \left[ - \frac{d}{dz} + \frac{3}{2} A'(z) \right] \psi(z) = (n^2 + \frac{9}{4} \Lambda) \psi(z) \quad (53)$$

The corresponding potential in term of $z$ is

$$V(z) = s^2 \eta^2 - s(s + 1) \eta^2 sech^2(\eta z) \quad (54)$$

where $s = \frac{3}{4}$.

By using the new variable $x = \lambda z$ the Schrödinger equation can be written as,

$$- \frac{d^2 \psi(x)}{dx^2} + \left[ s^2 - s(s + 1) sech^2(x) \right] \psi(x) = \left( \frac{n^2}{\eta^2} + \frac{9 \Lambda}{4 \eta^2} \right) \psi(x) \quad (55)$$
and we have
\[ H = -\frac{d^2}{dx^2} - s(s + 1) \sec h^2(x) + s^2 \]  
(56)

Now we are going to factorize \( H \) in terms of lowering and raising operators, respectively,

\[ B = -\frac{d}{dx} - s \tanh(x) \]
\[ B^\dagger = \frac{d}{dx} - s \tanh(x), \]  
(57)

and one can obtain the paired Hamiltonians

\[ H_1 = B^\dagger B = -\frac{d^2}{dx^2} - s(s + 1) \sec h^2(x) + s^2 \]
\[ H_2 = BB^\dagger = -\frac{d^2}{dx^2} - s(s - 1) \sec h^2(x) + s^2, \]  
(58)

where
\[ H_2(s) = H_1(s - 1) + c(s). \]  
(59)

This relation shows us there is a shape invariance condition with \( c(s) = 2s - 1 \).

In the case of a central charge, we choose unitary operator \( U \) as follows

\[ U = \exp(\frac{\partial}{\partial s}), \quad U^\dagger = \exp(-\frac{\partial}{\partial s}), \]  
(60)

where
\[ U^\dagger f(s)U \rightarrow f(s - 1) \]

From Eqs. (46) and (47) we have,

\[ S^\dagger S = \begin{pmatrix} B_1^\dagger B_1 & 0 & 0 & 0 \\ 0 & U^\dagger B_1^\dagger B_1 U & 0 & 0 \\ 0 & 0 & \Omega^\dagger B_1^\dagger B_1 \Omega & 0 \\ 0 & 0 & 0 & H_4 \end{pmatrix}, \]  
(61)

with
\[ B_1^\dagger B_1 = -\frac{d^2}{dx^2} - s(s + 1) \sec h^2(x) + (s - 1)^2 \]
\[ U^\dagger B_1^\dagger B_1 U = -\frac{d^2}{dx^2} - (g - 1)(g) \sec h^2(x) + (s - 2)^2 \]
\[ \Omega^\dagger B_1^\dagger B_1 \Omega = -\frac{d^2}{dx^2} - (s - 2)(s - 1) \sec h^2(x) + (s - 3)^2. \]  
(62)

Also, by using Eqs.(49),(50), one can obtain \( F \) as follows:

\[ F = \begin{pmatrix} -\frac{9\Lambda}{4\eta^2} & 0 & 0 & 0 \\ 0 & -\frac{9\Lambda}{4\eta^2} + 2s - 1 & 0 & 0 \\ 0 & 0 & -\frac{9\Lambda}{4\eta^2} + 4s - 4 & 0 \\ 0 & 0 & 0 & H_4 \end{pmatrix} \]  
(63)
Therefore, the energy spectrum of $H_1$ is

$$E_0^{(1)} = (-\frac{9\Lambda}{4})$$
$$E_1^{(1)} = (-\frac{9\Lambda}{4} + \frac{1}{2}\eta^2)$$
$$E_2^{(1)} = (-\frac{9\Lambda}{4} - \eta^2)$$
$$E_3^{(1)} = H_4$$

(64)

So, here we can discuss three cases as $\Lambda = 0$, $\Lambda < 0$ and $\Lambda > 0$ which are corresponding to flat, AdS and dS space respectively. The energy spectrum of $H_1$ for flat, AdS and dS spaces are as following respectively. In the another term for $\Lambda = 0$ we have

$$E_0^{(1)} = 0$$
$$E_1^{(1)} = \frac{1}{2}\eta^2 > 0$$
$$E_2^{(1)} = -\eta^2 < 0$$
$$E_3^{(1)} = H_4$$

(65)

for $\Lambda < 0$ we have

$$E_0^{(1)} = (-\frac{9\Lambda}{4}) > 0$$
$$E_1^{(1)} = (-\frac{9\Lambda}{4} + \frac{1}{2}\eta^2) > 0$$
$$E_2^{(1)} = (-\frac{9\Lambda}{4} - (1 + \Lambda\gamma)^2) \leq 0 \text{or} \geq 0$$
$$E_3^{(1)} = H_4 > 0$$

(66)

and finally for $\Lambda > 0$ we have

$$E_0^{(1)} = (-\frac{9\Lambda}{4}) < 0$$
$$E_1^{(1)} = (-\frac{9\Lambda}{4} + \frac{1}{2}(1 + \Lambda\gamma)^2) \leq 0 \text{or} \geq 0$$
$$E_2^{(1)} = (-\frac{9\Lambda}{4} - \eta^2) < 0$$
$$E_3^{(1)} = H_4 > 0$$

(67)

We note that in the all above cases we have some tachyonic states with negative energy. From (66) one can see, if $|\frac{9\Lambda}{4}| > (1 + \Lambda\gamma)^2$ then for the $AdS_4$ case, all states have positive eigenvalues. In this case the transition from $AdS_4$ to $M_4$ and $dS_4$ geometry is not stable.
5 Conclusion

In the present paper we have described the algebra which gives a natural framework for understanding the origins of shape invariance in our interesting problem. The study of shape invariance solutions can be done by the factorization method. Our aim was to solve and discuss the stability of a bent brane in the presence of quintom dark energy and in different geometries with a non-zero cosmological constant. We have done the perturbation to the metric and fields and achieved the corresponding Schrödinger equation which was the second-order equation. Then we have factorized the equation to the first-order equations which are raising and lowering operators and have generated the algebra. From first order equations we easily discussed the energy spectrum and also the stability of the system in the transition to different geometries.

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