**T-odd quark fragmentation function and transverse spin asymmetries in the pion production**

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We study the time-reversal odd quark fragmentation function and its consequences on the hard processes. T-odd quark fragmentation function may arise, although QCD is T-invariant, from the non-perturbative dynamics in the fragmentation process. Assuming the factorization, we extract the T-odd fragmentation function from the inclusive pion production in the transversely polarized proton-proton collision. We then estimate the single spin asymmetry $A_{OT}$ of the semi-inclusive deep inelastic scattering with unpolarized lepton and the transversely polarized proton. We also discuss the meson cloud effects on the transversity distribution $h_1(x)$ of the nucleon. Asymmetry for the $\pi^-$ production is found to be much smaller than the naive expectation, since the $d$-quark transversity is considerably suppressed by the pion cloud effects.

§1. Introduction

Recently much attention is paid on the transverse spin phenomena at high energies. Large single spin asymmetries of the pion production cross section are found in the transversely polarized proton-proton collision $\vec{p} + p \rightarrow \pi^a + X^{1)}$. Similar results are obtained for the kaon and eta meson productions. Significant polarizations of hyperons are also measured in the unpolarized hadron-hadron collision. Since the hard scattering process calculated by perturbative QCD predicts negligible asymmetry $\sim \frac{\alpha m Q^2}{2}$, we expect some non-perturbative mechanism as the origin of such transverse spin effects.

Collins pointed out that the non-trivial azimuthal angle dependence of the distribution of produced particles exists in the fragmentation of the transversely polarized quark $a^{3), 4), 5)}$. Let us consider the pion production as an example. The ‘standard’ unpolarized quark fragmentation function for the pion is defined by

$$D_{\pi/a}(z,k_\perp) = \sum_X \int \frac{dy^- d^2 y_\perp}{12(2\pi)^3} e^{ik^+ y^- - i k_\perp \cdot y_\perp} \times \text{tr} \gamma^+ \langle 0 | \bar{\psi}_{a}(0, y^-, y_\perp) | h, X \rangle \langle h, X | \bar{\psi}_{a}(0) | 0 \rangle$$

(1.1)

where a pion with momentum $p$ and $z = p^+/k^+$ is created by the quark $a$ with momentum $k$. When the quark is transversely polarized, it is possible to consider the following one,

$$H_{\perp \pi/a}(z,k_\perp,s_\perp) = \sum_X \int \frac{dy^- d^2 y_\perp}{12(2\pi)^3} e^{ik^+ y^- - i k_\perp \cdot y_\perp} \times \text{tr} \gamma^+ \gamma_5 \gamma_\perp \cdot s_\perp \langle 0 | \bar{\psi}_{a}(0, y^-, y_\perp) | h, X \rangle \langle h, X | \bar{\psi}_{a}(0) | 0 \rangle$$

(1.2)

where $s_\perp$ is the transverse spin of the quark. This fragmentation function is propor-
\( \epsilon_{\kappa \lambda \mu \nu} S^\kappa P^\lambda \eta^\mu n^\nu \)

where \( n \) is the light-cone vector. Time-reversal invariance prohibits existence of such a quantity, unless there are non-trivial phase differences of the amplitudes from the final state interaction in the hadronization process. However, non-perturbative hadronization process may generate such phase differences and allow a T-odd quark fragmentation. We will demonstrate how the T-odd fragmentation process is generated using a toy model in section 2.

If this is the case, one can use the T-odd quark fragmentation process in order to probe the chiral-odd transversity distribution function \( h_1(x) \) of the nucleon. The transversity distribution \( h_1(x) \) has not been measured so far, because it can not be probed by the deep inelastic scattering due to its chiral-odd nature. If we assume the existence of the T-odd fragmentation function and the factorization, we may write the single spin asymmetry of the hard process e.g. semi-inclusive deep inelastic scattering with the transversely polarized nucleon and unpolarized lepton as

\[ \sim h_1(x) \otimes \sigma_H \otimes H^\perp. \]

This unable us to measure the transversity distribution \( h_1(x) \) by the hard processes. Here, we focus on the single pion production in the proton-proton collision and the semi-inclusive deep inelastic scattering off the proton. In both cases a target proton is transversely polarized. In section 3 we try to extract the T-odd fragmentation function from the existing data of the transversely polarized proton-proton collision in which significant analyzing powers are observed. After fixing the fragmentation functions, we analyze the semi-inclusive DIS with an unpolarized lepton and a transversely polarized proton \( \ell + \bar{p} \rightarrow \ell' + \pi^a + X \) in section 4. We shall estimate the transverse spin asymmetry which will be accessed by future experiments.

In section 5 we emphasize roles of the meson clouds in the transversity distribution function. Physical nucleon state involves meson clouds, \( \pi, K \ldots \) as higher Fock components. Existence of such meson clouds is strongly suggested by the large isospin symmetry breaking in the nucleon sea. In ref. we have found that the pion cloud gives quite different contributions to the transversity distribution \( h_1(x) \) and the helicity one \( g_1(x) \). Such interesting differences will be checked by forthcoming experiments, and provides new insight of the roles of the meson clouds.

§2. Model of T-odd fragmentation function

Let us consider how T-odd fragmentation function appears by using a toy model. For this purpose, we take Nambu–Jona-Lasinio model in which non-perturbative gluon degrees of freedom are frozen into the effective 4-quark interaction. This model provides a schematic picture of the dynamical chiral symmetry breaking in QCD, and describes the Goldstone pion as a highly collective \( q\bar{q} \) state.

Evaluating the matrix element in eq. (1-1), one finds the unpolarized quark
T-odd quark fragmentation function

\[ p(k, s_\perp) \]

Fig. 1. Tree diagram for \( q \to \pi \) process. Solid and dashed lines represent the quark and pion, respectively.

\[ q(k, s_\perp) \]

Fig. 2. Typical diagrams contributing to T-odd process. Thick line denotes the \( \sigma \) meson

The fragmentation function for the pion at tree level (Fig.1) as

\[ D(z, k_\perp) = g^2 \int \frac{dk^-}{16\pi^4 (k^2 - m^2)^2} 2\pi \delta \left[(k - p)^2 - m^2\right] \]

\[ \times \frac{1}{2} \text{tr} \left[ \gamma^+ (k + m) \gamma^5 (k - p + m) \gamma^5 (k + m) \right] \]

\[ = g^2 \frac{z}{8\pi^3 (m^2 - k^2)} \left[ 1 + \frac{1}{z} \left( \frac{1}{m^2 - k^2} \right) \right] \]

(2.1)

where \( m \) is the constituent quark mass and \( k^2 = z/(1 - z)k_\perp^2 + m^2/(1 - z) + m_\pi^2/z \).

On the other hand, it is obscure whether or not T-odd fragmentation function exists in the hadronization process. Tree diagram calculation of eq. (1.2) yields

\[ H_1^\perp(z, k_\perp, s_\perp) = g^2 \int \frac{dk^-}{16\pi^4 (k^2 - m^2)^2} 2\pi \delta \left[(k - p)^2 - m^2\right] \]

\[ \times \frac{1}{2} \text{tr} \left[ \gamma^+ \gamma^5 s_\perp \cdot s_\perp (k + m) \gamma^5 (k - p + m) \gamma^5 (k + m) \right] \]

\[ = 0 \] (2.2)

This is because we use the plane wave propagator in this model. To obtain a finite contribution to the T-odd fragmentation function, we must calculate the interference of several diagrams shown in Fig.2 which produces the non-trivial phase difference. For example, self-energy diagram produces an imaginary part in the present kinematics, and thus contributes to the T-odd fragmentation function. We find such a contribution from the imaginary part is about 10% of the real part; \( H_1^\perp / D \sim 0.1 \). The lower diagram of Fig.2 represents the interference of the two-pion fragmentation process considered in ref.\(^7\). All these diagrams give non-vanishing contributions to the T-odd quark fragmentation process. Here, instead of doing explicit calculations, we rewrite the quark propagator\(^4\) based on the above argument;

\[ S(p) = \frac{A \cdot \slashed{p} + B}{p^2 - m^2} \] (2.3)

with \( A, B \) are complex numbers. In this case, we obtain

\[ H_1^\perp(z, k_\perp, s_\perp) = g^2 \frac{z}{2} \frac{2m}{8\pi^3 (1 - z)} \text{Im}[A^* B] \left( k_\perp \times s_\perp \right) z \] (2.4)
This clearly shows non-trivial dependence on the transverse spin and momentum of the quark. Calculations presented here are of course model-dependent, but the \((k_\perp \times s_\perp)_z\) dependence is expected from the T-odd nature of this process.

§3. Single spin asymmetry in inclusive pion production

In the inclusive pion production in the transversely polarized collision \(\vec{p} + p \rightarrow \pi^a + X\), significant analyzing powers are observed at high \(x_F\) and rather low \(p_T \sim 1 \sim 2\)GeV. They are also measured for the kaon and eta productions. The asymmetry grows as \(p_T\) increases up to about 2GeV. Experimental data at higher \(p_T\) will be available at RHIC or HERA in near future.

Assuming the factorization one can express the inclusive meson production cross section as

\[
E_h \frac{d\sigma}{d^3p_h} = \sum_{ab} \int dx_a dx_b \frac{dz}{z} q_{a/p}(x_a) \rho_{\alpha\alpha'} q_{b/p}(x_a) .
\]

\[
\times H_{\alpha\alpha'\beta\beta'}(a + b \rightarrow c + d) \ D_{\pi/c}^{\beta\beta'}(z, k_\perp).
\]

where \(q_{a/p}(x_a)\) and \(D_{\pi/c}^{\beta\beta'}(z, k_\perp)\) are quark distribution and fragmentation functions, and \(\rho_{\alpha\alpha'}\) the helicity density matrix. Hard scattering part \(H_{\alpha\alpha'\beta\beta'}(a + b \rightarrow c + d)\) are calculated within perturbative QCD. Using the standard helicity basis, spin transfer coefficient is given by

\[
\frac{M_{++}M_{+-}^* + M_{+-}M_{++}^*}{|M_{++}|^2 + |M_{+-}|^2} = \frac{4(1 + \cos \theta)}{4 + (1 + \cos \theta)^2}
\]

where \(\theta\) is the scattering angle of partons. It is easily seen that the spin transfer coefficient is large in the forward direction. The single spin asymmetry is defined by

\[
A_N = (d\sigma(\uparrow) - d\sigma(\downarrow))/(d\sigma(\uparrow) + d\sigma(\downarrow))
\]

We are in the position to fix the non-perturbative parts, distribution and fragmentation functions. For the transversity distribution of the nucleon, we use the results of ref. [12] by the quark model as inputs. We then try to determine the T-odd fragmentation function. Here, we simply assume the following form for the T-odd fragmentation function;

\[
H_{\perp}^{\uparrow}(z, k_\perp, s_\perp) = C \frac{z}{1 - z} \frac{k^2}{(k^2 - m^2)^2} (k_\perp \times s_\perp)_z
\]

which is the simple parametrization motivated by eq. (2.4) in section 2. The constant parameter \(C\) is fixed to reproduce the \(\pi^+\) asymmetry.

We show in Fig.3 results of the single spin asymmetries for \(\pi^+\) and \(\pi^-\) with the experiments. Here, Feynman-\(x\) is roughly approximated by \(x_F \sim x_a z\). It is worth noting that high-\(x_F\) behavior of the analyzing power is sensitive to a ratio of transversity to spin-averaged distributions \(h_1(x)/f_1(x)\) at the large \(x\) [13]. Experimental data \(A_N(\pi^+)/A_N(\pi^-) \sim -1\) at \(x_F \sim 1\) suggests \(h_1^\uparrow(x)/f_1^\uparrow(x) \sim -h_1^\downarrow(x)/f_1^\downarrow(x) \sim 1\) in our approach. Results for \(\pi^0\) as well as \(K, \eta\) mesons are also consistent with experiments. In Fig.4 we show pion \(p_T\) dependence of the single spin asymmetry.
§4. Semi-inclusive pion production in DIS

We apply the T-odd fragmentation function to the deep inelastic scattering off a transversely polarized nucleon with a pion in the final state, $\ell + \vec{p} \to \ell' + \pi^a + X$. General analysis of the semi-inclusive process with inclusion of T-odd distribution and fragmentation functions is done in ref. 4 at tree level. With the unpolarized lepton and the transversely polarized nucleon target, one can access the transversity distribution of the nucleon and the T-odd quark fragmentation.

Angle $\sin \phi$ weighted single spin asymmetry of the pion production cross section in ref. 4 is given by

$$A_{OT}^a \equiv \frac{\int d^2 P_+ \frac{P_z}{zMT_x} \sin \phi (d\sigma \uparrow - d\sigma \downarrow)}{\int d^2 P_\perp (d\sigma \uparrow + d\sigma \downarrow)} = -|S_T| \frac{2(1-y)}{1 + (1-y)^2} \frac{h_1^q(x) H_{1}^+(z)}{f_1^q(x) D^a(z)}$$

(4.1)

Because we have already determined the T-odd fragmentation function in the previous section, we can evaluate this asymmetry shown in Fig. 5. Asymmetry is order of about 10% which is much smaller than the $pp$ collision case. This is simply due to the difference of kinematical condition. In the $pp$ case, the asymmetry at $x_F \sim 1$ is given by the quark distributions at $x \sim 1$, in which $h_1(x)/f_1(x) \sim 1$. On the other hand, in the semi-inclusive DIS, we observe rather middle or small-$x$ behavior of the distribution functions, where $h_1(x)/f_1(x)$ is much smaller than unity.

Recently, measurements of the single spin asymmetries were done by 14 and 15. The experimental data actually show desired Collins-angle $\sin \phi$ dependence 5. Further experiments will clarify the existence of the T-odd quark fragmentation process.

Here, we assume that the pion production is dominated by the ‘favored’ process like $u \to \pi^+$, and neglect the ‘unfavored’ contribution such as $d \to \pi^+$. Recently, Schäfer and Teryaev have derived a sum rule for the T-odd fragmentation function, 16, and concluded that the unfavored process is severely suppressed, which supports our assumption adopted here.
§5. Meson cloud effects on the transversity $h_1(x)$

As seen in eq. (1.4), the existence of the T-odd fragmentation function makes it possible to measure the chiral-odd transversity distribution function $h_1(x)$ directly. In the non-relativistic limit, the transversity distribution coincides with the helicity one $g_1(x)$, however, $h_1(x)$ should differ from $g_1(x)$ in the real world. $h_1(x)$ can be understood as the difference of the numbers of quarks with eigenvalues +1 and −1 of the transverse Pauli-Lubanski operator $S_\perp\gamma_5$ in the transversely polarized nucleon. Unlike the helicity distribution $g_1(x)$, the gluon operators do not mix quark ones.

Simple estimates of the quark model, e.g. MIT bag model, yield almost similar behavior of the transversity and helicity distribution functions, although there is a small difference. Major source of this difference comes from the relativistic effect on the quark wave function in the nucleon. However, interesting observation made in ref. is that the meson cloud model gives very different contributions to $h_1(x)$ and $g_1(x)$. One can write the Fock-state of the physical nucleon as

$$|p\rangle = \sqrt{Z} |p_0\rangle + a_\pi |n\pi^+\rangle + \frac{a_\pi}{2} |p\pi^0\rangle + \cdots$$

where the renormalization constant $Z$ is found to be about 0.6 with the standard meson-baryon interaction. The quark distribution function of the physical nucleon is given by the convolution integral;

$$q_j(x) = \int_x^1 \frac{dy}{y} P_j\pi/\pi(y) q_i \left(\frac{x}{y}\right) + \cdots$$

where $P_j\pi/\pi(y)$ represents the splitting function which gives the probability of the meson-baryon fluctuation with the light-cone momentum fraction $y$. We calculate the splitting functions in the Infinite Momentum Frame with the time-ordered perturbation theory.

Let us consider the dominant pion contributions. It is well known that the pion cloud provides the $d$ quark excess in the nucleon sea, which could account for the violation of Gottfried sum rule. For the spin properties of the nucleon, the pion fluctuation also gives substantial depolarization effects by emitting the pion into the relative $P$-wave state. Define the nucleon-pion splitting functions which contribute to the spin-averaged, helicity, and transversity distributions as $P(y)_{N\pi}$, $\Delta P_{N\pi}(y)$ and $\delta P_{N\pi}(y)$, respectively. With the axial-vector pion-nucleon interaction, we find a
simple exact relation,

\[ P(y)_{N\pi} + \Delta P_{N\pi}(y) = 2\delta P_{N\pi}(y). \]  

Since \( P \geq |\Delta P|, |\delta P| \), the pion cloud contributions calculated by eq. (5.2) are quite different between the transversity and helicity distributions. With the standard parameters of the meson-baryon interaction, \( \Delta P \) is very small and thus \( \delta P \) is about a half of \( P \) which causes the large effects.

Such a difference is clearly seen in Fig.6. Dashed curves show distribution of the bare nucleon (as inputs) and the solid ones denote the results with the meson cloud contributions. The \( d \)-quark transversity distribution is suppressed very much compared with the helicity one. The 1st moment of the transversity, tensor charge, of \( d \)-quark is found to be reduced by about 40\% by the pion depolarization effect, while the corresponding axial charge is almost unchanged. In fact, such a tendency is consistent with the lattice QCD calculation of the tensor charge.

In order to clarify the effects of the pion clouds, we take a ratio of the \( d \)-quark distribution to the \( u \)-quark one. In Fig.7 we show our result \( h^u_1(x)/h^d_1(x) \) (solid) with \( g^u_1(x)/g^d_1(x) \) with the pion cloud, and thin dashed one is calculated by the parametrization of ref. 19. The ratio of the transversity distributions is very small at small-\( x, x < 0.3 \), due to the suppression of the \( d \)-quark transversity distribution.

This ratio can be extracted from the experiments directly. Measuring the transverse single spin asymmetries of eq. (4.1) for charged pions, \( \pi^+ \) and \( \pi^- \), and taking a ratio, one can obtain

\[ \frac{A^+_{OT}}{A^-_{OT}} = \frac{h^u_1(x) f^d_1(x)}{h^d_1(x) f^u_1(x)} \]  

where the unknown T-odd fragmentation functions are canceled out. Hence, one can extract the ratio of transversity \( h^u_1(x)/h^d_1(x) \) without ambiguities of the T-odd fragmentation functions. Our calculation indicates that the \( A_{OT} \) of \( \pi^- \) is much

* For \( u \)-quark, corrections from the meson clouds are small, and essentially negligible.
suppressed compared with $A_{OT}$ of $\pi^+$ and the naive estimate. Measurement of the transverse single spin asymmetries for charged pions could provide severe constraints on the validity of the meson cloud model.

§6. Conclusions

In summary, we have studied the transverse spin phenomena at high energies, namely, T-odd fragmentation function and transversity distribution $h_1(x)$. The T-odd quark fragmentation function depends on the transverse spin of the quark and thus provides an opportunity to observe $h_1(x)$. We have estimated the T-odd quark fragmentation function from the analyzing powers in $\vec{p} + p \rightarrow \pi^a + X$ experiments, assuming the factorization. We then calculate the single spin asymmetry of the semi-inclusive deep inelastic scattering off the transversely polarized nucleon $\ell + \vec{p} \rightarrow \ell' + \pi^a + X$, which will be measured in future experiments. We have also discussed the pion cloud effects on the transversity distribution $h_1(x)$. The pion cloud model predicts the strong suppression of the $d$-quark transversity distribution of the proton.

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Fig. 7. Modifications of $g_1(x)$ and $h_1(x)$ of $d$-quark