T-odd quark-gluon-quark correlation function in the diquark model

Zhun Lu\textsuperscript{a,}\textsuperscript{*}, Ivan Schmidt\textsuperscript{b}

\textsuperscript{a}Department of Physics, Southeast University, Nanjing 211189, China

\textsuperscript{b}Departamento de Física, Universidad Técnica Federico Santa María, and Centro Científico-Tecnológico de Valparaíso Casilla 110-V, Valparaíso, Chile

Abstract

We study the transverse momentum dependent quark-gluon-quark correlation function. Using a spectator diquark model, we calculate the eight time-reversal-odd interaction-dependent twist-3 quark distributions appearing in the decomposition of the transverse momentum dependent quark-gluon-quark correlator. In order to obtain finite results, we assume a dipole form factor for the nucleon-quark-diquark coupling, instead of a point-like coupling. The results are compared with the time-reversal-odd interaction-independent twist-3 TMDs calculated in the same model.

Key words: transverse momentum distributions, twist-3, spectator diquark model, time-reversal-odd

1 Introduction

In the theoretical description of high energy (semi-)inclusive processes involving hadrons, the cross-sections are usually expanded in powers of $1/Q$, where $Q$ is the large momentum transfer of the collision. The contribution at the leading power can be expressed as a convolution of the leading-twist (or twist-two) distribution/fragmentation functions and the hard scattering coefficients. In the first subleading power of the $1/Q$ expansion, the twist-3 distribution and/or fragmentation functions contribute to the cross-section [1]. Unlike the twist-two distribution functions that describe the parton densities inside the

\textsuperscript{*} Corresponding author at: Department of Physics, Southeast University, Nanjing 211189, China.

Email address: zhunlu@seu.edu.cn (Zhun Lu).
nucleon, there is no probability interpretation for the twist-3 distribution functions. However, they provide a wealth of information about the nucleon parton structure [2], especially when the parton transverse momenta are present. The interest on the twist-3 contributions also comes from the fact that they are related to the multi-parton correlation inside the nucleon [3,4].

In this letter we apply a phenomenological model to study twist-3 quark distributions that are encoded in the quark-gluon-quark correlation. We pay special attention on the time-reversal-odd (T-odd) transverse momentum dependent (TMD) distributions. In the TMD factorization approach, the leading twist T-odd TMD [5] play important roles [6,7,8] in the single-spin asymmetries (SSAs) measured in semi-inclusive deeply inelastic scattering (SIDIS) [9,10,11]. At the twist-3 level, much richer phenomena arise, i.e. there are eight T-odd distributions that can contribute to various azimuthal asymmetries in the SIDIS [2] and Drell-Yan [12] processes. Although the twist-3 contributions are suppressed by 1/Q, they are potential experimental observables and may be accessible in the kinematical regime where Q is not so large. The experiments at Jefferson Lab [13,14] and PAX [15] are ideal for exploring this kinematical region.

2 Interaction-dependent T-odd twist-3 distributions

Our starting point is the transverse momentum dependent quark-gluon-quark correlation function, which is defined as [16]

\[
\left( \tilde{\Phi}^{|\pm\alpha}_A \right)_{ij}(x, p_T) \equiv \int \frac{d^2\xi_T d\xi^-}{(2\pi)^3} e^{i p \cdot \xi} \\
\times \langle P, S | \bar{\psi}_i(0) g \int_{\pm\infty}^{\xi^-} d\eta^- \mathcal{L}^{[\pm]}(0, \eta^-) F^{\alpha\beta}(\eta) \mathcal{L}^{\xi_T} \xi^+(\eta^-, \xi^-) \psi_j(\xi) | P, S \rangle_c \bigg|_{\eta^-=0, \eta^+=0, p^-=xP^-}
\]

where \( F^{\mu\nu} \) is the antisymmetric field strength tensor of the gluon, \( \mathcal{L}^{[\pm]} \) and \( \mathcal{L}^{\xi_T} \) are the gauge-links ensuring the gauge-invariance of the definition. The sign “±” in the superscript or subscript indicates that the gauge-link between the quark and the gluon is future/past-pointing [7], corresponding to the SIDIS/Drell-Yan processes, respectively.
The correlator in Eq. (1) can be rewritten further to

\[
\left( \tilde{\Phi}^{[\pm]}_{A\alpha} \right)_{ij}(x, p_T) = ig \int \frac{d^2 \xi d\eta}{(2\pi)^4} \int dx' e^{ix'p^+\eta-} e^{i[(x-x')p^+\xi^-]} \times \langle P, S| \bar{\psi}_j(0) F^{+\alpha}(\eta) \gamma^\sigma(\eta^-) \psi_i(\xi^-)| P, S \rangle \bigg|_{\eta^+ = \xi^+ = 0},
\]

(2)

If the parton transverse momentum is integrated over, one can define the collinear quark-gluon correlator, the so-called Efremov-Teryaev-Qiu-Sterman (ETQS) function \[1,17\], as

\[
T_{q,F}(x, x) = \int d\xi d\eta e^{ixp^+\xi^-} \times \langle P, S| \bar{\psi}(0) \gamma^\sigma(0^-) \gamma^\rho(\eta^-) \psi(x^-)| P, S \rangle,
\]

(3)

where \(\epsilon^{\alpha ST} = \epsilon^{\alpha T} S_T\), with \(S_T\) the transverse polarization vector of the nucleon. In the twist-3 collinear factorization approach \[1\], a nonzero ETQS function has been applied to explain the large single transverse spin asymmetries (SSAs) observed in \(p^+p \rightarrow \pi X\) processes \[18\]. According to Eq. (2), the ETQS function can be obtained from the transverse momentum dependent correlator \(\tilde{\Phi}^{[\pm]}_{A\alpha}\) by

\[
g T_{q,F}(x, x) = \epsilon^{\alpha ST} \int \frac{d^2 p_T Tr}{4\pi} \left( \gamma^+ \tilde{\Phi}^{[+]}_{A\alpha}|_{G.P.}(x, p_T) \right),
\]

(4)

where \(\tilde{\Phi}^{[+]}_{A\alpha}|_{G.P.}\) denotes the gluonic pole part of \(\tilde{\Phi}^{[+]}_{A\alpha}\), which can be obtained by taking the imaginary component of the factor \(1/(x' \mp i\epsilon)\) in Eq. (2):

\[
\frac{1}{(x' \mp i\epsilon)} = P \left( \frac{1}{x'} \right) \pm i\delta(x').
\]

(5)

The quark-gluon-quark correlator can be decomposed as \[2\]

\[
\tilde{\Phi}^\alpha_A(x, p_T) = \frac{x M}{2} \left[ \left( \tilde{f}^\perp - i \tilde{g}^\perp \right) \frac{p_T \rho}{M} - \left( \tilde{f}^\perp_T + i \tilde{g}^\perp_T \right) \epsilon_{T \rho \sigma} S_T^\sigma \right.
\]

\[
- \left( \tilde{f}^\perp_s + i \tilde{g}^\perp_s \right) \frac{\epsilon_{T \rho \sigma} p_T^\sigma}{M} \left( g_T^{\alpha \rho} - i\epsilon_T^{\alpha \rho \gamma_5} \right) \left( \tilde{h}_s + i \tilde{e}_s \right) \gamma_T^\alpha \gamma_5
\]

\[
+ \left( \tilde{h} + i \tilde{e} \right) \epsilon_T^{\alpha \rho} p_T^\rho S_T^\sigma \left[ \left( \tilde{h}_T^\perp - i \tilde{e}_T^\perp \right) \frac{\epsilon_T^{\alpha \rho} p_T^\rho S_T^\sigma}{M} \right] \left[ \frac{\epsilon_T^{\alpha \rho}}{2} \right].
\]

(6)

The functions appearing with a tilde in the above expression are the interaction-dependent twist-3 quark distributions, which depend also on the longitudinal momentum fraction \(x\) and the transverse momentum \(p_T\). Among them, \(\tilde{e}, \tilde{f}^\perp,\)
\( \tilde{g}_T \) (or \( \tilde{g}'_T \)), \( \tilde{g}^\perp_T \), \( \tilde{g}^\perp_L \), \( \tilde{h}_L \) and \( \tilde{h}^\perp_T \) are T-even; and \( \tilde{e}_L \), \( \tilde{e}_T \), \( \tilde{e}^\perp_T \), \( \tilde{f}_T \) (or \( \tilde{f}'_T \)), \( \tilde{f}^\perp_L \), \( \tilde{g}^\perp \) and \( \tilde{h} \) are T-odd. All the TMDs can be projected out by different Dirac matrices:

\[
\frac{1}{2M_x} \text{Tr} \left[ \Phi_{A\alpha} \sigma^\alpha+ \right] = \tilde{h} + i \tilde{e} + \frac{e_{T\rho}^\alpha p_{T\rho}^\alpha}{M} \left( \tilde{h}^\perp_T - i \tilde{e}^\perp_T \right), \tag{7}
\]

\[
\frac{1}{2M_x} \text{Tr} \left[ \Phi_{A\alpha} i\sigma^\alpha+ \gamma_5 \right] = S_L \left( \tilde{h}_L + i \tilde{e}_L \right) - \frac{p_{T\cdot S_T}}{M} \left( \tilde{h}_T + i \tilde{e}_T \right), \tag{8}
\]

\[
\frac{1}{2M_x} \text{Tr} \left[ \Phi_{A\rho} (g_{T\rho}^\alpha + i\epsilon_{T\rho}^\alpha \gamma_5) \gamma^+ \right] = \frac{p_{T\cdot S_T}}{M} \left( \tilde{f}^\perp_L - i \tilde{g}^\perp \right) - \epsilon_{T\rho}^\alpha S_{T\rho} \left( \tilde{f}_T + i \tilde{g}_T \right)
- S_L \frac{e_{T\rho}^\alpha p_{T\rho}^\alpha}{M} \left( \tilde{f}^\perp_L + i \tilde{g}^\perp_L \right) - \frac{p_{T\cdot S_T}^2}{M^2} \epsilon_{T\rho}^\alpha S_{T\rho} \left( \tilde{f}_T^\perp + i \tilde{g}_T^\perp \right). \tag{9}
\]

Taking the real part of the right hand side of Eq. (5), one can deduce from the traces the T-even TMDs; if one takes the imaginary part of the right hand side of Eq. (5) instead, one can obtain the T-odd TMDs. In this work we will study the possibility to calculate these T-odd TMDs from specific models. In a realistic calculation we ignore all the gauge-links in the correlator (1), as a lowest order approximation. Also, we choose the diquark model \([22,32]\), which has been widely applied in the calculation of TMD distributions, and it is the first model that shows that the T-odd distributions are non-vanishing. For simplicity we only consider the case that the diquark is a scalar, since the extension to the spectator model with (axial-)vector diquarks is straightforward. In the following we will calculate the T-odd TMDs appearing in the DIS process, since those in Drell-Yan process are related to them by a minus sign, as indicated by Eq. (5).

The Lagrangian for the scalar diquark model with Abelian gauge field (electromagnetism) reads:

\[
\mathcal{L}(x) = \bar{\Psi}(x) (i\gamma^\mu \partial_\mu - M) \Psi(x) + \bar{\psi}(x) (i\gamma^\mu D_{q,\mu} - m_q) \psi(x)
+ \varphi^*(x) \left( \bar{D}_s^{\mu*} \bar{D}_s_{\mu} - m_s^2 \right) \varphi(x) - \frac{1}{4} F^{\mu\nu}(x) F_{\mu\nu}(x)
+ \lambda \left[ \psi(x) \Psi(x) \varphi^*(x) + \bar{\Psi}(x) \psi(x) \varphi(x) \right], \tag{10}
\]

where \( \lambda \) denotes the nucleon-quark-diquark coupling, and

\[
D^{\mu} q \psi(x) = \left[ \partial^\mu + i e_q A^\mu(x) \right] \psi(x), \quad D^{\mu} s \varphi(x) = \left[ \partial^\mu + i e_s A^\mu(x) \right] \varphi(x), \tag{11}
\]

here \( e_q \) and \( e_s \) are the charges for the quark and the spectator diquark, and \( M, m \) and \( m_s \) denote the masses for the nucleon, the quark and the scalar diquark, respectively. Our calculation was performed in the Feynman gauge. We also checked the calculation in the light-cone gauge (corresponding to the gluon polarization sum \( d^{\mu\nu} = -g^{\mu\nu} + (q^{\mu}n^{\nu} + q^{\nu}n^{\mu})/q^{+} \)), and found
that it leads to the same results as in the Feynman gauge. The Feynman diagram for calculating the correlator defined in the scalar diquark is shown in Fig. 1 in the right panel of which we list explicitly the Feynman rules of the vertices, propagators and external lines derived from the Lagrangian in (10). The vertical dashed line cutting the external diquark imposes the onshell condition $\delta((P - q)^2 - m_s^2)$. When the calculation for the quark-gluon-quark correlator is transformed from the real QCD to an Abelian theory, one should use the replacement $g \rightarrow -e_q$. Also a specific Feynman rule [19,20] (depicted by the open circle at the end of the gluon line in Fig. 1) for the field strength tensor $F^{+\alpha}$ has been applied: $-i(q^+ g^{\alpha \rho} - q^\alpha g^{+\rho})$.

In the simplest version of the diquark model [6], the coupling $\lambda$ is treated as a constant. However, we find that in this simplest model there are divergences emerging for some T-odd TMDs when one performs the integrations over the transverse momentum. Similar divergences have already appeared explicitly in the calculation of interaction-independent twist-3 T-odd TMDs in the same model [21]. We conclude that this is a common feature for the T-odd twist-3 distributions when one applies the point-like coupling for the nucleon-quark-diquark interaction vertex. In order to obtain finite results, instead of a point-like coupling constant $\lambda$, we choose a dipole form factor for the nucleon-quark-diquark coupling [22]:

$$\lambda \rightarrow \lambda(p^2) = \frac{N_s(p^2 - m^2)}{(p^2 - \Lambda^2)^2}, \quad (12)$$

where $N_s$ is the normalization constant, $m$ is the mass for the quark, and $\Lambda$ is the cut-off parameter for the quark momentum. The same choice has been applied to calculate the leading-twist T-odd TMDs for the nucleon [24] and the pion [25].

Applying the scalar diquark model in Eq. (10) to the quark-gluon-quark cor-
relator \([2]\) and using the traces in Eqs. \([7,8,9]\), we obtain non-zero results for the eight T-odd interaction dependent twist-3 quark distributions. Here we refrain from giving further details on our calculation, and only write down the final expressions for these TMDs:

\[
\tilde{\epsilon}_T(x,p_T^2) = -\frac{A L - (x M + m)^2}{2x L(L + \vec{p}_T^2)^3} + \frac{A}{2x \vec{p}_T^2(L + \vec{p}_T^2)^2} \frac{1}{L} \ln \frac{\vec{p}_T^2 + L}{L},
\]

\[(13)\]

\[
\tilde{\epsilon}_L(x,p_T^2) = -\frac{A}{2x} \frac{(x M + m)}{L - \vec{p}_T^2} \frac{1}{L(L + \vec{p}_T^2)^3},
\]

\[(14)\]

\[
\tilde{\epsilon}_T(x,p_T^2) = \frac{A}{2x} \frac{(m + x M)^2 + 2L + \vec{p}_T^2}{L(L + \vec{p}_T^2)^3} - \frac{A}{4x \vec{p}_T^2(L + \vec{p}_T^2)^2} \ln \frac{\vec{p}_T^2 + L}{L},
\]

\[(15)\]

\[
\tilde{h}(x,p_T^2) = -\frac{A}{2M x} \frac{x M + m}{L(L + \vec{p}_T^2)^2},
\]

\[(16)\]

\[
\tilde{g}^\perp(x,p_T^2) = \frac{A}{2x} \frac{(m + x M)^2 + \vec{p}_T^2}{L(L + \vec{p}_T^2)^3},
\]

\[(17)\]

\[
\tilde{f}_L^+(x,p_T^2) = -\frac{A}{2x} \frac{(x M + m)^2 - \vec{p}_T^2}{L(L + \vec{p}_T^2)^3},
\]

\[(18)\]

\[
\tilde{f}_T^+(x,p_T^2) = -\frac{A}{2x} \frac{x M + m}{L(L + \vec{p}_T^2)^2},
\]

\[(19)\]

\[
\tilde{f}_L^-(x,p_T^2) = \frac{A}{x} \frac{M(x M + m)}{L(L + \vec{p}_T^2)^3},
\]

\[(20)\]

\[
\tilde{f}_T^-(x,p_T^2) = -\frac{A}{2x} \frac{x M + m}{L(L + \vec{p}_T^2)^3},
\]

\[(21)\]

where \(L = (1 - x)\Lambda^2 + x m_s^2 - x(1 - x)M^2\), and \(A = (1 - x)^3 e_q e_s N_s^2 / (4(2\pi)^4)\).

In the following we point out several comments about our results.

(i) Our calculation shows that one can obtain non-zero results for the T-odd interaction-dependent TMDs, even without the presence of the gauge-links connecting the quark and gluon fields.

(ii) Among the eight T-odd TMDs, \(\tilde{g}^\perp\) and \(\tilde{f}_L^+\) are finite when one chooses a point-like nucleon-quark-diquark coupling\(^1\) which are different to their interaction dependent partners \(g^\perp\) and \(f_L^+\). The other six TMDs are divergent in this case. For consistency, all the TMDs are calculated by adopting a dipole form factor for the nucleon-quark-diquark coupling. The need of form factor to regularize the light-cone divergence in certain twist-3 T-odd correlator has also been pointed out in Ref. \([23]\).

(iii) The results we present here are at the order \(\mathcal{O}(e_q e_s)\). We recall that

\(^1\) we find that in the integrands of calculating \(\tilde{g}^\perp\) and \(\tilde{f}_L^+\), the power of \(q_T\) appearing is one, after integrating over \(q^+\) and \(q^-\). Therefore, they are finite in the case of pointlike coupling.
in the diquark model the calculation of leading-twist T-odd TMDs (i.e., the Sivers function and the Boer-Mulders function \cite{26}) in the one-gluon exchange approximation, the results are at the same order. Therefore, our calculation can be compared with the one for the Sivers function. To do this, we evaluate the ETQS function from the transverse momentum dependent quark-gluon-quark correlator \cite{11} in the scalar diquark model:

\[ e_q T_{q,F}(x,x) = -\tilde{S}_T^2 \pi A(xM + m) \frac{\pi A(xM + m)}{2L^2}. \] (22)

The first transverse-moment of the Sivers function in SIDIS in the scalar diquark model is \cite{24}:

\[ f_{\perp}^{T(1)}(x) = \int d^2 p_T \frac{\vec{p}_T^2}{2M^2} f_{\perp}^{T}(x,\vec{p}_T^2) = \frac{\pi A(xM + m)}{4ML^2}. \] (23)

Therefore, in the diquark model at the order \( O(e_q e_s) \), we verify the famous identity \[16,27,28,29\] between \( T_{q,F}(x,x) \) and \( f_{\perp}^{T(1)}(x) \) (assuming \( \tilde{S}_T^2 = 1 \)):

\[ e_q T_{q,F}(x,x) = -2M f_{\perp}^{T(1)}(x), \] (24)

which indicates the unification \[28\] of two mechanisms for SSAs in hard processes. We point out that the relation in Eq. (24) has also been verified in Ref. \[30\] in diquark model calculation.

3 Interaction-independent T-odd twist-3 distributions

As a comparison, we also calculate the interaction-independent twist-3 T-odd TMD distributions in the same model. As shown in Ref. \[2\], the interaction-dependent and interaction-independent twist-3 distributions satisfy model-independent relations provided by the equation of motion (EOM) for the quark field. It is interesting to check whether these relations (see Eqs.(3.64-3.72) in Ref. \[2\]) for T-odd distributions hold in the diquark model.

Up to order \( 1/Q \), the quark-quark correlator can be decomposed as \[2,31\]:

\[
\Phi(x,p_T) = \ldots + \frac{M}{2P_+} \left\{ e - i e_s \gamma_5 - e_T^\perp \frac{e_T^{\rho\sigma} p_T^{\rho} S_T^{\sigma}}{M} \\
+ f_{\perp}^T \frac{\vec{p}_T}{M} - f_T^{\rho\sigma} \gamma_5 S_T^{\rho} - f_{\perp}^{\rho\sigma} \gamma_5 P_T^{\rho} S_T^{\sigma} \\
+ g_T^\sigma \gamma_5 S_T^\sigma + g_5^\rho \gamma_5 \frac{\vec{p}_T}{M} - g_{\perp}^\rho \gamma_5 \frac{e_T^{\rho\sigma} P_T^{\sigma}}{M} \right\}.
\]
where we only write the terms containing twist-3 distributions, and we use ellipse to denote the twist-two terms that have not been considered in this letter. The leading-twist T-odd and T-even TMDs have also been calculated in the diquark model, and a complete calculation can be found in [32]. The distribution functions in (25) can be deduced from the traces of the correlator \( \Phi(x, p_T) \):

\[
\begin{align*}
\Phi^{[1]} &= M P^+ \left[ e - \frac{\epsilon_T^{\rho\sigma} p_T \rho S_T \sigma}{M} \epsilon_T^+ \right], \\
\Phi^{[\gamma_5]} &= M P^+ \left[ S_L e_L - \frac{p_T \cdot S_T}{M} e_T \right], \\
\Phi^{[\gamma_\alpha]} &= M P^+ \left[ - \epsilon_T^{\alpha\rho} S_T \rho f_T - S_L \frac{\epsilon_T^{\rho\sigma} p_T \rho}{M} f_L^+ \right. \\
& \quad - \frac{p_T^2}{M^2} \left. \epsilon_T^{\rho\sigma} S_T^\sigma f_T^+ + \frac{p_T^2}{M} f_T^+ \right], \\
\Phi^{[\gamma_\alpha_\gamma_5]} &= M P^+ \left[ S_T^\alpha g_T + S_L \frac{p_T^2}{M} g_L^+ \right. \\
& \quad - \frac{p_T^2}{M^2} \left. S_T \rho g_T^+ - \epsilon_T^{\rho\sigma} p_T \rho g_T^+ \right], \\
\Phi^{[\sigma^{\alpha_\beta}_5]} &= M P^+ \left[ S_T^\alpha p_T^\beta - p_T^\alpha S_T^\beta \right. \\
& \quad - \epsilon_T^{\alpha_\beta} h_T \left. - \epsilon_T^{\alpha_\beta} h_T \right], \\
\Phi^{[\sigma^{+}_5]} &= M P^+ \left[ S_L h_L - \frac{p_T \cdot S_T}{M} h_T \right].
\end{align*}
\]

Here \( \Phi^{[\Gamma]} = \frac{1}{2} \text{Tr}[\Phi \Gamma] \). The eight T-even twist-3 TMDs have been calculated in the diquark model [22] and the Bag model [33]. The T-odd twist-3 TMDs has been studied in Ref. [21], where an explicit calculation for \( g^+ \) in the scalar diquark model showing that the T-odd TMD distributions are divergent when one chooses a point-like nucleon-quark-diquark coupling. In order to compare to the interaction-dependent twist-3 distributions listed in Eqs. (13) to (21), again we use a dipole form factor for the nucleon-quark-diquark coupling to obtain finite results to order \( O(e_q e_s) \), as also suggested in Ref. [21].

The expressions for the eight T-odd distributions in the scalar diquark model are listed as follows

\[
\begin{align*}
\epsilon_T^+(x, p_T^2) &= - \frac{A}{2(1-x)} \frac{(1-x)\Lambda^2 + (1+x)M_s^2 - (1-x)(1-2x)M^2}{L(L + p_T^2)^3}, \\
\epsilon_L(x, p_T^2) &= - \frac{A}{2(1-x)} \frac{xM + m}{M} \frac{L - p_T^2}{L(L + p_T^2)^3},
\end{align*}
\]
Among the above results, the calculation of \( g_{L} \) is already given in Ref. [21], while the others are new results. In the scalar diquark model, we find that \( f_{T} \) is zero. We have checked that the inclusion of an (axial-)diquark yields non-zero results for \( f_{T} \).

Combining the results for the Sivers function and the Boer-Mulders function in the same model [24], we find that the interaction-dependent and interaction-independent twist-3 TMDs in the diquark model do not satisfy the EOM relations. The reason for the disagreement between the diquark model calculations and the EOM relations is that our calculation is model dependent, while EOM relations in Ref. [2] is derived from the genuine QCD; and we have only considered the lowest order contribution to the T-odd TMDs.

Although our calculation is model dependent, it is interesting to point out that our results show that the following relations are satisfied:

\[
\int d^2 \vec{p}_T f_T(x, \vec{p}_T^2) = 0, \quad \int d^2 \vec{p}_T e_L(x, \vec{p}_T^2) = 0, \quad \int d^2 \vec{p}_T h(x, \vec{p}_T^2) = 0 . \tag{40}
\]

This can be seen from the fact that the following integration yields zero result:

\[
\int d^2 \vec{p}_T \frac{L - \vec{p}_T^2}{L(L + \vec{p}_T^2)^3} = 0 . \tag{41}
\]

The results showing in Eq. (40) agree with the general constraint of time-reversal invariance on the integrated quark correlator \( \Phi(x) \), that is, that the integrated twist-3 T-odd distributions should vanish [231].
4 Summary

In summary, we have studied the possibility to calculate the T-odd interaction-dependent twist-3 quark distributions in the spectator diquark model. We find that in the lowest order approximation (neglecting the gauge-links in the quark-gluon-quark correlator), we can obtain non-zero results for the eight T-odd interaction-dependent twist-3 quark TMDs. In the calculation we apply a dipole form factor for the nucleon-quark-diquark coupling, instead of a point-like coupling, to avoid the light-cone divergences. We also calculate the interaction-independent T-odd twist-3 TMDs in the same model for comparison. In the order $O(e_q e_s)$ in the diquark model, we verify the identity between the ETQS function $T_{q,F}(x,x)$ and the first transverse moments of the Sivers function $f_{T}^{\perp\perp}(1)$. However, we find that in the same model, the interaction-dependent and interaction-independent T-odd twist-3 TMDs do not satisfy the relations provided by the equation of motion for the quark field. The diquark calculation also shows that integrated T-odd distributions $f_{T}(x)$, $e_{L}(x)$ and $h(x)$ vanish, as required by time reversal-invariance for the integrated distributions. Our study demonstrates the applicability and the limitations of the diquark model in the understanding of the parton structure of the nucleon at the twist-3 level.

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