Construction and reconstruction concept in mathematics instruction

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Abstract. The purpose of this paper is to describe two learning activities undertaken by lecturers, so that students can understand a mathematical concept. The mathematical concept studied in this research is the Vector Space in Linear Algebra instruction. Classroom Action Research used as a research method with pre-service mathematics teacher at University of Papua as the research subject. Student participants are divided into two parallel classes, 24 students in regular class, and remedial class consist of 18 students. Both approaches, construct and reconstruction concept, are implemented on both classes. The result shows that concept construction can only be done in regular class while in remedial class, learning with concept construction approach is not able to increase students' understanding on the concept taught. Understanding the concept of a student in a remedial class can only be carried out using the concept reconstruction approach.

1. Introduction

In education system there are various methods and approaches of instruction used by teachers in order to create their teaching more effective [1, 2]. In mathematics instruction, teachers generally use two methods, deductive and inductive methods. Deductive is an instruction method of reasoning by which concrete applications are deducted from general principles, theorems are deduced from definitions and postulates. A deductive approach involves the learners being given a general rule, which is then applied to specific examples and practice exercises [2]. In mathematics instruction, teachers start lesson by writing definitions, followed by presenting examples and counter examples [3]. This method causes students to tend to be inactive in learning.

In contrast to the deductive method, inductive instruction makes use of student “noticing”. Instead of explaining a given concept and following this explanation with examples, the teacher presents students with many examples showing how the concept is used [4]. An inductive approach involves the learners detecting, or noticing, patterns and working out a 'rule' for themselves before they practise the concept [5]. This method causes students to tend to be more active in learning. However, not all mathematical concepts can be understood by studying patterns. In an abstract concept, conceptual understanding cannot be done by studying patterns, because patterns cannot be found.

Furthermore, mathematics learning in university should be different from mathematics learning in schools. Mathematics learning in university is more difficult because the concepts studied are abstract. Mathematics learning in school is also abstract but can still be given a real example. One of the
abstract mathematics courses in the university is the linear algebra. This is in accordance with the statement of Adams and Elliott [6], which states that one mathematics topic that the majority of preparatory students fear is linear algebra, which is often due to it being perceived as more abstract and thus irrelevant to the ‘real world’. Moreover some experts stated that the abstract and formal nature of linear algebra generate two sources of difficulty in its understanding. The nature of linear algebra itself (conceptual difficulties) and the kind of thinking required for the understanding of linear algebra (cognitive difficulties) [7, 8].

One of the most abstract university mathematical concepts in linear algebra course is concept of vector space. On the other hand, Vector Space is one of the important concepts to be well understood by mathematics students. Linear algebra is essentially based on the theory of vector spaces [9]. This concept, as well as the concept of linear independence (or linear dependence), is initially quite challenging for students [10]. Understanding the concept of vector space is an important part in understanding other concepts in other abstract mathematics courses such as algebraic structures and real analysis. Nevertheless vector space learning has been done with a less accurate approach.

However, unlike calculus, that often emphasizes manipulation of symbols in order to solve problems, the focus in linear algebra is on the description of concepts, often through word definitions, and derivation of further concepts from these [11]. Lecturers who teach linear algebra generally start the lesson by defining the vector space and then discuss the example and counter example of the vector space. The learning approach of lecturer causes the students to not understand the concept of vector space comprehensively. Vector Space concepts in the form of definitions containing vector space axioms are only memorized by the students without knowing the process of obtaining them.

Various authors have investigated the learning and teaching of linear algebra from an educational perspective showed that students tend to think about the concepts as the application of techniques [8]. They do not usually understand the meaning of definitions, and they are unable to apply them, even in simple problems. In fact, their findings show that most students manipulate the symbols algebraically without understanding the concepts they refer to Thomas and Stewart [12]. The situation affects the difficulties when the student tries to solve a problem somewhat different from the given example. This situation is different when students understand the concept through a process of constructivism.

Constructivism is a learning theory describing the process of knowledge construction. Knowledge construction is an active, rather than a passive process. Constructivists believe that knowledge should not be just deposited into the learners’ minds; instead it should be constructed by the learners through active involvement in the learning process. In the constructivist perspective, knowledge is constructed by the individual through his/her interactions with the environment. Unlike the traditional mode of learning whereby the teacher plays an active role in the teaching/learning environment, and learners passively receive the content, constructivists believe the learning should be centred on the learner [13].

Constructivism in learning mathematics can be done through analogy. Analogy is a sort of similarity [14]. Analogies show how one thing is similar to another. Therefore using analogies in thinking refers to reasoning by using parallels [15]. Analogies are an excellent way to teach mathematics to adults. They enable connections to be forged and assist in removing fear and increase confidence that may have been lost due to boring rote learning. Analogies enable mathematical concepts to be conveyed in a form that students can relate to, thus, increasing their understanding and confidence. The use of analogies enables adult students to form connections between mathematics and real world situations. Connection with the familiar makes mathematical concepts easier to comprehend [6]. Furthermore Mofidi et al [16] stated that one of the methods that teachers apply to assist students with building conceptual knowledge is to use analogies.

The application of constructivism in learning mathematics depends on the initial knowledge that students have. Students who learn a concept should construct their own concepts so that students can understand them well. The process is called a construct. However, in students who have studied the concept, the construction process is different. Those who have already studied a concept, generally have a misconception about the concept. Subanji and Nusantara [17] stated that the error was due to an error in the thinking process, known as pseudo thinking. Pseudo thinking is a thinking process that
results in an answer to a problem or construction to a concept "that is not true". The construction concept does not represent the actual thinking. So the analogy process on a student who has studied and new students learn a different concept of analogy approach. Thus in order to improve the understanding of students who have studied a particular concept requires a process called reconstruction.

Reconstruction is an attempt to make corrections to students' misconceptions of the concepts they have learned. Like concept construction, concept reconstruction also uses analogies. The difference between the two lies in the type of analogy used. In reconstruction, the analogy used is based on students' misconception of the concept. Misconceptions can occur in both capable and weak students. According to Harel [18], teacher tends to associate misconception and missing conceptions only with mathematically weak students. But in fact, all students, the weak and the able, in desire to understand and make sense of mathematical concepts, encounter difficulty, and demonstrate as a result behaviors that in many cases are difficult to explain [18].

Based on the previous statement, the following questions can be raised. How do students who study mathematics develop their understanding of a mathematical concept based on the analogy process? Is there a difference in the process of constructing a mathematical concept between students on regular classes and students on remedial classes? Is it true that the construction process is more effectively applied to students in regular classes than applied to students in remedial classes? Is it true that the reconstruction process needs to be done to students in the remedial class? How is the reconstruction process done?

2. Method
The study was conducted using classroom action research. Classroom action research is a wide variety of evaluative, investigative, and analytical research methods designed to diagnose problems in classroom [19]. Furthermore McNiff and Whitehead [20] stated that classroom action research is a method of finding out what works best in classroom so that teacher can improve student learning. This method is a form of enquiry that enable teacher to investigate and evaluate their work [21]. Classroom action research aims to improve the quality of instruction, teaching quality of teacher and learning quality of students [19, 22]. Thus, in this study was conducted evaluation, investigation and analysis of two learning activities, construction and reconstruction concept in order to obtain the appropriate learning activities so that the students' learning achievement who study the concept of Vector Space in Linear Algebra instruction will be better.

Subjects for this research were 42 students of mathematics education from University of Papua, Manokwari West Papua Province, Indonesia. The students are those who contract linear algebra lesson in the 2017/2018 academic year. In this research the subjects, students who participants are divided into two parallel classes, 24 students in regular class, and remedial class consist of 18 students. The remedial class is a class consisting of students who have taken the course, but did not pass the exam.

The grouping of students into two classes aims to acquire groups of students with similar ability to each class. According to Slota [23] teaching a group of students with similar abilities allowed teachers to adjust the pace of instruction to best reach students' needs. A teacher would instruct at a slower pace which provided more repetition and reinforcement with a group of low achieving students than they would have a group of high achievers.

2.1. Instruction plan
In instruction planning, researchers have to considering the kind of data they will need to collect and the processes involved in the data collection [3, 22]. Instruction plan is prepared through discussion among researchers. The result of the planning establishes that the researcher divided into two responsibilities, as a lecturer and as an observer. There are three instruments produced, namely observation guide, student activity sheet (SAS) and questionnaire. The observation guide is intended to observe student activity during the lesson. SAS consist of some questions used by used to help
students understand the concept of vector Space, whereas the questioner was used to measure students’ responses to the learning process.

2.2. Action and observation
Observations were implemented during the lesson. The classroom action research would be more effective if instruction and observations were hold simultaneously [3, 22]. The learning is carried out separately on two parallel classes, regular class and remedial class. Learning is done by constructing and reconstructing concepts, respectively in each class, through analogy process. Observations were made to students' activities and expressions in learning. Observations are recorded for discussion at the reflection stage.

2.3. Reflection
Reflection was carried out after completion of an instruction. Based on some records of observations, conducted by lecturer and observers, reflection is carried out through discussion. Discussion aims to find problems in instruction and tried to find solutions. Criticism and suggestions as a result of reflection are used to rearrange the next instruction plan.

3. Results and discussion
The lecture on Linear Vector Space Concepts was first performed on a regular class. The results of reflection on the regular class are used to rearrange the lesson plans that have been prepared for the remedial class. Based on observations and reflections performed on the regular and remedial classes, various problems were found, as well as suggestions that could be used to improve the learning process in the remedial class.

Instruction activities, both in the regular and remedial classes are carried out in three stages of instruction, i.e. preliminary activities, main activities and closing activities. In the preliminary activities, lecturers provide illustrations related to learning objectives and the importance of learning the concept of Vector Space as well as illustration of Vector Space application. Illustrations are also given use a simple example of what analogies are meant in Mathematics. Lecturer then managed the students to sit in groups according to the group that has been prepared. Each group consists of 4 members and is managed by a leader who is a student with a Grade Point Average (GPA) greater than 3.5. Furthermore, the lecturer asks questions to explore students’ memories related to the integer system $Z$, as well as vector operations in the two-dimensional space ($R^2$) and three-dimensional space ($R^3$).

The next stage, the lecturer distributes Student Activity Sheets (SAS) for students in groups. Using SAS, students tried to construct the concept of Vector Space. At this stage, the lecturer guides the group in completing SAS. After the appointed time, the lecturer invites each group to write down the results of their investigations on the board. The results were responded by another group, looking for similarities and differences of investigation results. Lecturers also provide opportunities for students who have GPA about 2.00 the challenge to write down the results of his group's investigation. In closing activities, the lecturer guides the students to construct the definition of the Vector Space by reconstructing the results of the appropriate investigation. Thus there is a process experienced by students in understanding the concept of Vector Space. Conceptual knowledge is part of an active process [24]. Conceptual knowledge puts the focus on relationships [25].

The learning activities conducted by students and teachers is known as the process of constructivism. Constructivism focuses upon individuals building up representations of student knowledge, which is tested against experience. When concepts are introduced in school, they are not transmitted to students, but students will attempt to fit them to the models or concepts they currently have [24]. Furthermore, at the end of the lesson, lecturer distributed the questionnaire of smiley face, collected the SAS and the smiley face questionnaire, and gave the personal task.

Some notes about student activities on regular class are described as follows. Some students look to have really learned. This can be seen from their liveliness in groups, they make notes by listening without always looking forward. Indeed, at first some of the students looked confused but long ago they shared a
smile with the group members as a sign they understood the results of their group discussion. They are able to ask questions and statements to assure their understanding. This shows that they have understood the concept well. Hopkins [26] states that students asked better quality questions when they had prior knowledge of the study area and had already participated in classroom activities relating to it.

In contrast to the regular class, in remedial class students look confused, which appears in their eyes. They are not able to ask questions well. Their SAS are not done right. This initially confuses the lecturer, but ultimately the lecturer realizes that they need concept reconstruction. This is due to several things, namely students in the remedial class have a low GPA. In addition, students make mistakes in understanding the concepts learned. The mistake made by students in doing mathematics requires attention [27, 28]. The mistake would seriously affect the subsequent understanding of their mathematical concepts [27]. In order to minimize the impact of the mistake in building the next concept, it is important to track the sources and causes of the mistake [28]. The sources can possibly be found in the formation of student’s thinking scheme called the construction process of student’s concept.

Based on the results of the reflection of remedial class learning, the SAS reconstructed so-called SAS reconstruction. The SAS reconstruction is used for both regular and remedial classes in the second lecture. SAS reconstruction was prepared after the research team studied what the students knew and did not know in each class, with the aim of reconstructing the students' knowledge of the concepts taught.

Secondary learning activities in regular classes by applying concept reconstruction is not as difficult as teaching in remedial classes. This is because concept errors occur only in a few students. In the remedial class, reconstruction requires considerable activity. Nevertheless both yield satisfactory results. Most students can understand the concept well at the end of the course.

4. Conclusion
Based on the results of the research, it can be concluded that there are significant differences in learning approaches in regular class and remedial class. In regular class students whom construction concept is applied are more active in learning activities than students in remedial class. Students in regular class are more willing to express their opinions, ask questions, answer questions, and working on their SAS. As a result, more percentages of students in regular class who understand the concept of vector space correctly, compared with the percentage of remedial class students who understand the concept of vector space in the first lecture.

Students in regular and remedial classes who have not understood the concept of vector space can be overcome by implementing concept reconstruction. The number of students who did not understand the concept was greatly reduced after analyzing their misconceptions. Based on the results of the analysis is done reconstruction concept to improve understanding of their concept. Thus it can be stated that concept construction is more effectively implemented in the regular class only, while in the remedial class, learning is more effective when implemented by reconstruction of concept. In other words, concept construction can only be done in regular class while in remedial class, learning with concept construction approach is not able to increase students' understanding on the concept taught. Understanding the concept of a student in a remedial class can only be carried out using the concept reconstruction approach. The reconstruction of the concept in the remedial class is preceded by learning what is known and what the student does not know.

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