Let $\mathcal{F}^{cu}$ be the family of critically periodic unimodal continuous maps of the interval. Thurston’s Master Teapot, denoted by $\Upsilon^{cu}_2$, is defined as the closure of $\{(z, \lambda) \subseteq \mathbb{C} \times \mathbb{R}\}$ where $\lambda$ is the growth rate of some map in $\mathcal{F}^{cu}$ and $z$ is a Galois conjugate of $\lambda$.

One of the authors’ main results is that $(z, \lambda) \in \Upsilon^{cu}_2$ implies $(z) \times [\lambda, 2] \subseteq \Upsilon^{cu}_2$, which is a type of vertical persistence. The proof uses a combinatorial approach of G. Tiozzo [Int. Math. Res. Not. 2020, No. 2, 607–640 (2020; Zbl 1444.37015)].

The authors also prove that the Master Teapot is connected and contains the unit cylinder. Furthermore they show that the Thurston set, which is the projection of $\Upsilon^{cu}_2$ onto $\mathbb{C}$, is not equal to the Thurston set for postcritically finite tent maps. A gap theorem is proven for the Thurston set.

Reviewer: Steve Pederson (Atlanta)

MSC:

37E05 Dynamical systems involving maps of the interval
37E15 Combinatorial dynamics (types of periodic orbits)
37F20 Combinatorics and topology in relation with holomorphic dynamical systems
37F10 Dynamics of complex polynomials, rational maps, entire and meromorphic functions; Fatou and Julia sets

Keywords:

entropy; interval map; kneading theory; Galois conjugates; $\beta$-expansion; master teapot

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