String–Like Description of Gravity
and Possible Applications for $F$–theory.

Devoted to the memory of
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Abstract

The Lorentz harmonic formulation of $D$–dimensional bosonic $p$–brane theory with $D \geq (p+1)(p+2)/2$ coupled to an antisymmetric tensor field of rank $d = (p+1)$ provides the dynamical ground for the description of $d = (p+1)$ dimensional Gravity. It hence realizes the idea of Regge and Teitelboim on a 'string–like' description of gravity. The simplest nontrivial models of such a kind are provided by free $D$– dimensional $p$–branes in which world volumes are embedded as minimal surfaces. Possible applications of such a model with $d = 2+2$ and $D = 2+10$ for studying a geometry of bosonic sector of $F$–theory are considered. Some speculations inspired by the proposed model are presented.

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In this short note we report briefly the main result of our investigation of $D$–dimensional $p$–brane theory coupled to an antisymmetric tensor field of rank $d = (p + 1)$ (referred further as a generalized Kalb–Ramond, or GKR field): For $D \geq (p + 1)(p + 2)/2$ this theory describes a general type of $d = (p + 1)$ dimensional gravity. Hence it realizes the idea by Regge and Teitelboim [1] about 'string–like' description of gravity and provides the dynamical ground for the description of gravity in the frame of the so–called isometric embedding formalism [4]. The details of the derivation of this result and its relation with Poisson–sigma–model approach [3] will be the subject of the forthcoming paper [4].

As one of the possible applications of the proposed approach we consider briefly a minimal self–dual embedding which seems to be related to $D = 2 + 10$ dimensional $F$–theory [5, 6, 7], a topic of intensive recent studies [8, 9].

1. The action for $D$ dimensional bosonic $p$–brane interacting with generalized Kalb–Ramond (GKR) field $B_{m_1 \ldots m_{p+1}} \equiv B_{[m_1 \ldots m_{p+1}]}(X)$

$$S = S_{D,p}^0 + S_{D,p}^{int}$$

is the sum of the free $p$–brane action [10, 11]

$$S_{D,p}^0 = \int_{\mathcal{M}^{p+1}} \mathcal{L}_{D,p}^0 = \int_{\mathcal{M}^{p+1}} \left( -\frac{(-1)^p}{p!} (E^a E^{a_1} \ldots E^{a_p} - \frac{p}{p+1} e^{a_1} \ldots e^{a_p}) \varepsilon_{a_1 \ldots a_p} \right)$$

and of the interaction term

$$S_{D,p}^{int} = -q \int_{\mathcal{M}^{p+1}} B_{p+1}$$

In (3) $q$ is GKR charge of $p$–brane and the GKR form

$$B_{p+1} = dX^{m_{p+1}} \ldots dX^{m_1} B_{m_1 \ldots m_{p+1}}(X)$$

is pulled back and integrated over the $p$–brane world volume

$$\mathcal{M}^{p+1} = \{(\xi^m)\} = \{ (\tau, \sigma^1, \ldots, \sigma^p) \}, \quad (m = 0, 1, \ldots, p)$$

In (2) the Lagrangian two–form $\mathcal{L}_{D,p}^0$ is constructed from some of the basic one–forms of target space–time $R^D$.

$$E^a = (E^a, E^i) = (dX^m u^a_m, dX^m u^i_m) \quad a = 0, \ldots, (D - 1); \quad a = 0, \ldots, p$$

and world volume

$$e^a = d\xi^m e^a_m(\xi)$$

1 In the first four sections, for definiteness, one can suppose that the world volume $\mathcal{M}^{p+1} = \mathcal{M}^{1,p}$ and target space $R^D = R^{1,D-1}$ (both with only one space–like direction) are considered. However, the proposed model permits the straightforward generalization for the case with any number of time–like directions, i.e. for $\mathcal{M}^d = \mathcal{M}^{d-1}$ and $R^D = R^{d-1}$. This shall be used in the section 5 to consider an embedding related to $F$–theory [3, 4].
using the external products of the forms only \[\text{3}\]. The vielbein of flat target space–time \(E^a\) (3) differs from the holonomic basis \(\{dX^m\}\) of cotangent space by a Lorentz rotation whose vector representation is given by the matrix \(u^{\alpha}_m\)

\[\begin{align*}
(u^{\alpha}_m) &= (u^a_m, u^i_m) \\
&\in SO(1, D - 1) \\
&\iff u^{\alpha}_m \eta^{mn} u^{\beta}_n = \eta^{\alpha\beta}
\end{align*}\]

(7)

The differentials of the moving frame variables \(u^{\alpha}_m\)

\[du^{\alpha}_m = u^{\beta}_m \Omega^\alpha_m (d) \iff \begin{cases} du^a_m = u^{\beta}_m \Omega^a_m (d) + u^i_m \Omega^ai_m (d), & \alpha = a; \\
du^i_m = -u^j_m \Omega^{ji} + u^m_u \Omega^ai_m (d), & \alpha = i
\end{cases}\]

are expressed in terms of the \(so(1, D - 1)\) valued Cartan 1–form

\[\Omega^{ab} = -\Omega^{ba} = \begin{pmatrix} \Omega^{ab} & \Omega^{aj} \\
-\Omega^{ai} & \Omega^{ij} \end{pmatrix} = u^{\alpha}_m du^m_{\alpha}\]

(9)

Let us note that all the variables in the functional (1)–(3) shall be considered as world volume fields

\[X^m = X^m(\xi), \quad u^a_m = u^a_m(\xi), \quad e^a_m = e^a_m(\xi).\]

The detailed consideration of the properties of the action (2) can be found in Refs. \[\text{10, 11}\]. The moving frame variables (7) can be regarded as \(SO(1, D - 1)\)\(SO(1,p) \times SO(D-p-1)\) Lorentz harmonics \[\text{12, 10, 11}\] (and references in \[\text{11}\]).

2. The equations of motion following from the action (1)–(3) naturally split into rheotropic conditions (in the terminology of [13])

\[E^a = e^a, \quad E^i = 0, \quad \Rightarrow \quad dX^m = e^a u^m_a,\]

(10)

which have the same form as for the free \(p\)–brane case \[\text{10, 11}\], and the proper dynamical equation \(u^{m_0} \delta S/\delta X^m = 0\). The latter can be written in terms of the pull–back \(\Omega^{ai} = d\xi^m \Omega^m_{ai}\) of the covariant Cartan form \(\Omega^{ai} = u^a_m du^m_{ai}\)

\[\Omega^{ai}(\nabla_a) = e^a \Omega^{ai} = q \frac{1}{(p + 1)!} \epsilon^{a_0...a_p} u^{a_0m_0}...u^{a_pm_p} u^{m_{p+1}} H_{m_0...m_{p+1}}(X(\xi)),\]

(11)

and involves the \(p\)–brane charge \(q\) and GKR field strength

\[H_{m_0...m_{p+1}}(X(\xi)) = (p + 1) \partial_{[m_0} B_{m_1...m_{p+1}]}(X(\xi))\]

in the r.h.s.

For the case of a free \(p\)–brane (or uncharged one: \(q = 0\)), (11) acquires the form

\[\Omega^{ai}(\nabla_a) \equiv e^m_a \Omega^{ai}_m = 0,\]

(12)

\[2\text{Here and below } \Omega_r \Omega_q = (-1)^q \Omega_q \Omega_r, \quad d(\Omega_r \Omega_q) = \Omega_r d\Omega_q + (-1)^q d\Omega_r \Omega_q \text{ are assumed for product of any } r \text{– and } q \text{–forms } \Omega_q = dx^{m_0}...dx^{m_q} \Omega_{m_1...m_q}(x)\]
Passing from Eqs. (10) to their selfconsistency (integrability) conditions \((ddX = 0 = d(e^{au})\)) we can exclude the embedding functions \(X(\xi)\) as well as the moving frame fields \(u(\xi)\) from our consideration (of course, for \(q \neq 0\) the embedding functions remain, in general, in the r.h.s. of Eq. (11)) and get the equations

\[
e_a \Omega^{ai} = 0, \quad T^a \equiv D e^a \equiv de^a - e_b \Omega^{ba} = 0,
\]

written in terms of intrinsic vielbeins \(e^a = d\xi^m e^a_m\) and Cartan forms \((\Omega)\) only. The latter, by definition \((\Omega)\), satisfy the Maurer–Cartan equations

\[
\begin{align*}
d\Omega^{ab} - \Omega^a_c \Omega^{cb} &= 0, \\
R^{ab}(d, d) &= d\Omega^{ab} - \Omega^a_c \Omega^{cb} = \Omega^{ai} \Omega^{bi}, \\
R^{ij}(d, d) &= d\Omega^{ij} + \Omega^{ij} \Omega^{ji} = -\Omega^{ai} \Omega^{aj},
\end{align*}
\]

The equations for the forms \(\Omega^{ai}, \Omega^{ab}, \Omega^{ij}\) give rise to the Peterson–Codazzi, Gauss and Ricci equations of the surface theory \([14, 15]\), respectively.

The equations (12)–(14) describe the minimal embedding of the free \(p\)–brane world volume into the flat target space–time and are referred to as geometric approach equations \([15, 11, 16]\).

For the case of \(D = 3\) string \((p = 1)\) these equations can be reduced to the nonlinear Liouville equation \([15]\). Thus the geometric approach relates string theory to exactly solvable nonlinear systems. This can be useful both for extended object theory and for the investigation of exactly solvable nonlinear equation (see, for example, \([16]\)).

The supersymmetric generalization of the geometric approach for the cases of superstrings and supermembranes was performed in \([11]\) starting from the geometrodynamic condition. In \([13, 17, 18]\) a generalized action \([13]\) has been used for this purpose. Its pure bosonic limit is given by the functional \((2)\). Some results concerning the investigation of the geometric approach equations for the simplest cases of free \(N = 1\) and 2 superstrings in \(D = 3\) can be found in Ref. \([17, 18, 19]\).

3. Our main point is that the \(p\)–brane in \(D\) dimensional GKR background provides a model for the description of a general type of \(d = p + 1\) dimensional gravity for \(D \geq d(d + 1)/2\).

Indeed, Eqs. (13), (14) describe the embedding of arbitrary \(d\)–dimensional surface into flat \(D\)–dimensional space–time \([14, 12]\). To describe the embedding of the definite surface

\[3\] It is natural to use spin connections and \(SO(D - p - 1)\) gauge fields induced by the embedding, i.e. ones coinciding with the pull–back of the Cartan form \(\Omega^{ab}\) and \(\Omega^{ij}\) \(\([\Omega]\)\).

\[4\] The same name is used also for more general systems of equations, e.g. for the set \((11), (13), (14)\) considered before for the case of \(D = 4\) string by Lund and Regge \([\Omega]\), and for the system describing string in \(D = 3\) de Sitter space, considered by Barbashov and Nesterenko \([\Omega]\).

\[5\] Recently the results of investigations of \(D = 11\) supersymmetric five–brane and type \(II\) super–\(p\)–branes were reported in Ref. \([20]\), where the technique, which can be regarded as a linearized version of one developed in Ref. \([11]\), was used.
one has to specify the expression for main extrinsic curvatures $h^i = \epsilon_a^m \Omega^i_m$. So, as it was noted above, the vanishing of the main curvatures (12) defines the minimal surface.

The key observation is that the interaction with the GKR background does not change eqs. (13), (14), but replaces (12) by (11). This means that the world volume of $p$–branes interacting with the GKR field is embedded as a nonminimal surface and its main curvature is defined by the field strength of the background field which can be considered as arbitrary function of the embedding coordinates $X(\xi)$.

Hence, choosing an appropriate GKR field we can, in principle, describe an arbitrary $d$–dimensional surface in the flat $D$–dimensional space time as a world volume of a charged ($q \neq 0$) $p$–brane.

The general theorem about local isometric embedding (see [14, 2] and refs. therein) guarantees that, if the dimension of target space time is $D \geq (p+1)(p+2)/2$, then we can describe arbitrary curved $d = (p+1)$–dimensional manifold as a surface in such space–time (at least locally). Thus, for such a case the arbitrary $d$ dimensional manifold can be described by the model under consideration. On the other hand, arbitrary curved manifold can be described by the gravity theory with an appropriately chosen matter fields.

Hence, one can conclude that the proposed model describes a general type of $d = (p+1)$–dimensional gravity and provides the dynamical ground for the embedding approach used before for the investigation of General Relativity [2]. So, it realizes the idea of Regge and Teitelboim [1] about a string–like description of gravity.

4. Some speculations are inspired by the proposed model.

The model for $d = 4$ gravity is provided by a 3–brane in $D = 10$ dimensional space–time with the 4–th rank antisymmetric tensor (GKR) background. Thus also our Universe could be considered as such 3–brane. with the 4–th rank antisymmetric tensor background. The matter in the Universe appears as a manifestation of that $D = 10$ GKR field. In this connection it is interesting that the number $D = 10$ of space time dimensions is distinguished by superstring theory, and that a 3–brane supersymmetric soliton exists in $D = 10$ type IIB superstring theory [21]. Moreover, this soliton is exceptional for several reasons [22].

In accordance with the Mantonen–Olive conjecture [23], the dual theory, where solitons become fundamental objects, should exist. Such a dual theory is just one of a (type IIB super–) 3–brane. The 4–form GKR gauge field can be coupled naturally to this 3–brane. If we will not try to solve the GKR field equations together with 3–brane equations of motion and suppose this field being arbitrary function, the embedding of the 3–brane into flat 10–dimensional Minkowski space–time should be nonminimal and should describe arbitrary curved 4–dimensional (Einstein) space–time, in particular a model for the Universe.

An effective action for such a ”solitonic” Universe is just the one presented here in eqs. (1)–(3). It may provide a quasiclassical description of the 4–dimensional Universe in
string theory.

From this point of view the supersymmetric generalization becomes interesting, because it could select the models for the Universe. Indeed, the interaction with super-\( p \)-branes leads to restrictions on the background even on the classical level (see [24] and refs. therein).

5. The simplest nontrivial case of induced gravity corresponds to the \textit{minimal} embedding of the world volume, i.e. to the vanishing GKR background or uncharged \( p \)-brane. To demonstrate the power of our approach, let us discuss the simplest possible applications of the proposed model related to such case which seems to be of interest for modern directions of development of Superstring theory.

Recently the attempts to reach a progress in understanding the nonperturbative aspects of string theory have resulted in the significant comprehension in the duality symmetries [25]. The unification of \( T \)- and \( S \)-dualities [26] and the discovery of the duality transformations related the types of (super)string theories which had seemed to be completely different [27] indicate the existence of more general theories which include all the previously considered superstring models [28]–[32], [4]–[9].

Some set of \textit{self–dual embeddings} are related to the most general theory of such a type, which is \( D = 2 + 10 \) dimensional \( F \)-theory 6. The key property of all of them is self–duality of the world volume field theories. An example of such type embeddings was known few years from \( n = 2 \) (spinning) string theory whose quantum state spectrum contains self–dual gravity only [33] .

The string–like description of gravity gives a natural basis for the description of such embeddings. Let us consider an embedding of \( d = 2 + 2 \) dimensional manifold into flat \( D = 2 + 10 \) dimensional ”space–time” with two time–like directions. It is natural to represent a \( 2 + 2 \) dimensional vector index as a set of two spinor indices of different \( SL(2, R) \) groups (\( SO(2, 2) = SL(2, R) \times SL(2, R) \)) using relativistic \( d = 2 + 2 \) Pauli matrices

\[
(\tilde{\sigma}^a)_{\dot{\alpha}\alpha} = \begin{pmatrix} -i\tau^2, I, -\tau^1, -\tau^3 \end{pmatrix}, \quad \sigma^a_{\dot{\alpha}\alpha} = \begin{pmatrix} -i\tau^2, I, \tau^1, \tau^3 \end{pmatrix},
\]

\[
\sigma^a\tilde{\sigma}^b = \eta^{ab} + 1/2\varepsilon^{abcd}\sigma_c\tilde{\sigma}_d \quad \tilde{\sigma}^a\sigma^b = \eta^{ab} - 1/2\varepsilon^{abcd}\sigma_c\sigma_d
\]

In such a way we get

\[
e^a \rightarrow e^a\tilde{\sigma}^\dot{\alpha}\alpha = e^{\dot{\alpha}\alpha}, \quad u^a_\alpha \rightarrow u^a_\alpha\tilde{\sigma}^\dot{\alpha}\alpha = u^\dot{\alpha}_\alpha, \quad E^a = (E^{\dot{\alpha}\alpha}, E^\alpha) = dX^aw_\dot{\alpha}^\alpha
\]

\footnote{They are the embeddings of \( d = 1 + 1 \) and \( d = 1 + 2 \) dimensional manifolds into \( D=2+2 \) and of \( d = 2 + 2 \) dimensional one into \( D = 2 + 10 \) [4]. It should be stressed that in ref. [4] specific reasons for the investigation of \( D = 12 + 4 \) dimensional theories are given.}

\footnote{We shall note that the embeddings of string theories into hyperbolic string theory living in the space–times with two time like directions were considered in Refs. [34].}
Henceforth, the induced spin connection $\Omega_{a}^{b} \rightarrow \Omega_{a}^{\beta\dot{\beta}}$ and Riemannian curvature two-form $R_{a}^{b} \rightarrow R_{a}^{\beta\dot{\beta}}$ split naturally into self–dual and anti–self–dual parts

$$
\Omega_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} \equiv \Omega^{ab}(\sigma_{a})_{\alpha\dot{\alpha}}\tilde{\sigma}_{b}^{\beta\dot{\beta}} = \delta_{\alpha}^{\beta} \Omega_{\dot{\alpha}}^{\dot{\beta}} + \delta_{\alpha}^{\dot{\beta}} \tilde{\Omega}_{\dot{\alpha}}^{\beta}, \quad R_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} = \delta_{\alpha}^{\dot{\beta}} R_{\alpha}^{\beta} + \delta_{\alpha}^{\beta} \tilde{R}_{\dot{\alpha}}^{\dot{\beta}}
$$

In Eq. (17)

$$
\Omega_{\alpha}^{\beta} \propto \Omega^{ab}\sigma_{ab}^{\alpha}, \quad \tilde{\Omega}_{\dot{\alpha}}^{\beta} \propto \Omega^{\dot{ab}}\tilde{\sigma}_{\dot{ab}}^{\beta},
$$

and, by definition (15),

$$
\sigma_{ab}^{\alpha} = \sigma^{[a\dot{a}]} = +1/2\varepsilon_{abcd}\sigma_{cd} \quad \tilde{\sigma}_{ab}^{\dot{a}} = \tilde{\sigma}^{[a\dot{a}]} = -1/2\varepsilon_{abcd}\tilde{\sigma}_{cd}
$$

The part of the Maurer–Cartan equation (14) giving rise to the Ricci equation naturally splits in self–dual and anti–self dual parts too

$$
R_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} \equiv R_{a}^{b}\sigma_{a}^{\alpha}\tilde{\sigma}_{b}^{\beta\dot{\beta}} = 1/2\Omega_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} \quad \rightarrow \begin{cases} R_{a}^{\beta} = 1/2\Omega_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} \Omega_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} \\ \tilde{R}_{\dot{\alpha}}^{\beta} = 1/2\Omega_{\dot{\alpha}}^{\beta\dot{\beta}} \Omega_{\dot{\alpha}}^{\beta\dot{\beta}} \end{cases}
$$

For the case of minimal embedding (12) $\Omega_{ai}(\nabla_{a}) \equiv e_{a}^{m}\Omega_{ai}^{m} = 0$, the self duality condition for the induced world volume gravity

$$
\tilde{R}_{\dot{\alpha}}^{\beta} = 0 \quad \Leftrightarrow \quad \Omega_{\alpha\dot{\alpha}}^{\beta\dot{\beta}} = 0
$$

has a solution

$$
\Omega_{\alpha\dot{\alpha}}^{i} = \lambda^{i} e_{\beta\dot{\beta}} \lambda^{\alpha} k_{\dot{\alpha}i}^{\beta} \quad k_{\dot{\alpha}i}^{\beta} = k_{\dot{\alpha}i}^{\beta}
$$

It involves a bosonic spinor field $\lambda^{\alpha}$ and only half of $d = 2 + 2$ dimensional vielbein one–forms $\lambda^{i} e_{\beta\dot{\beta}}$. The bosonic spinor can be regarded as the one related to a $2 + 2$ dimensional null vector $n_{a\dot{a}} = \lambda_{a} \mu_{a} \quad (\Leftrightarrow \quad n_{a\dot{a}}n^{a\dot{a}} = 0)$ appearing in the models related to $F$–theory [33, 34, 35, 36].

The self dual part of Riemannian curvature two–form (18)

$$
R_{a}^{\beta} = \lambda_{a}^{i} \lambda^{\beta} \mathcal{F}
$$

has only one nontrivial component

$$
\mathcal{F} = 1/4\varepsilon^{\gamma\delta} l_{\gamma} e_{\nu} \nu_{\delta} \mathcal{R}, \quad \mathcal{R} = k_{\dot{\alpha}i}^{\beta} k^{\dot{\alpha}i}
$$

So, the self–dual embedding under consideration is nontrivial if the world volume field $k_{\dot{\alpha}i}^{\beta}$ involved in Eq. (20) has nonzero norm

$$
k_{\dot{\alpha}i}^{\beta} k_{\dot{\alpha}i}^{\beta} \neq 0
$$
Substituting Eq. (20) into the part of Maurer–Cartan Equation (14) giving rise to the Ricci equation, one finds that the field strength of the $SO(8)$ gauge field vanishes for the embedding under consideration:

$$ R^{ij} = -1/2 \Omega^i_{\alpha \dot{\alpha}} \Omega^{\alpha \dot{\alpha} j} = \propto \lambda_\alpha \lambda^\alpha = 0 $$

So, the described embedding of $d = 2 + 2$ dimensional world volume into the flat $D = 2 + 10$ dimensional flat space time is characterized by nontrivial self–dual spin connections and vanishing gauge field.

It is interesting to investigate such embedding following the line realized in refs. [16, 17] for the minimal embedding of a $1 + 1$ dimensional world sheet into $D = 1 + 2$. Such an investigation seems to be useful for understanding of the geometry in $F$–theory. In this respect, let us note that the appearance of Liouville and Toda equations in $M$–theory was considered in [32].

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