The annihilation decays $B_c^- \rightarrow \eta'(\eta, \pi^0)l^- \bar{\nu}^*$

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Abstract

We investigate the simileptonic OZI-forbidden annihilation decays $B_c^- \rightarrow \eta'(\eta, \pi^0)l^- \bar{\nu}$ for $l = \mu, e$ in the perturbative QCD, and carry out a precise calculation without any approximation for the one-loop contributions, which involves integrals of 4- and 5-point loop functions. Our results show that the branching ratios of decays $B_c^- \rightarrow \eta l^- \bar{\nu}$, $B_c^- \rightarrow \eta \bar{l} \nu$ and $B_c^- \rightarrow \pi^0 l^- \bar{\nu}$, turn out to be of orders $10^{-4}$, $10^{-5}$ and $10^{-6}$, respectively, which could be observable in the future experiments at the LHC.

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1 Introduction

Being the lowest bound state of two heavy quarks (charm and bottom) with open (explicit) flavors, the $B_c$ meson provides a unique window into the heavy quark physics, which has caused wide experimental and theoretical investigations. After the report of the CDF collaboration on the observation of the $B_c$ ground state at the Fermilab Tevatron [1], people believe that it is possible to accumulate more $B_c$ meson events in the experiments at the Tevatron Run II[2,3] and the future CERN Large Hadron Collider (LHC). At the CERN LHC with the luminosity of about $L \sim 10^{34} \text{cm}^{-2}\text{s}^{-1}$ one can expect around $5 \times 10^{10}$ $B_c$ events per year[4].

Unlike symmetric heavy quarkonium ($c\bar{c}$ and $b\bar{b}$ bound states), the $B_c$ meson is composed of heavy quarks with different flavors and it lies below the $B\bar{D}$-threshold, so its decays via strong and electromagnetic interactions are forbidden. Therefore, the investigation of the $B_c$ meson decay can offer special information compared to symmetric heavy quarkonium. In the framework of the SM its decays can occur via three mechanisms: (1) the $c$-quark decay with the $b$-quark being a spectator, (2) the $b$-quark decay with the $c$-quark being a spectator, (3) $b$-quark and $c$-quark annihilation. The first two mechanisms are expected to contribute about 90% of the total width, and the remaining 10% is owed to the annihilation process.

There is another decay mode which does not belong to the aforementioned types, and it can only occur via the OZI processes. As we know that the Okubo-Zweig-Iizuka (OZI) rule[5] plays an important role in the processes which occur via strong interaction and in general at the parton level the concerned calculations are carried out in the framework of the perturbative QCD (PQCD). Thus careful studies on these OZI-forbidden processes can deepen our insight to the perturbative QCD. But these processes are very difficult to evaluate, because one not only needs to carry out complicated loop calculations at the parton level, but also has to deal with the non-perturbative effects of the QCD, which is involved in the hadronization of partons. Because of lack of solid knowledge on the non-perturbative QCD, for the whole calculation one has to adopt concrete models which may contaminate the theoretical results. The factorization scheme is just to properly separate calculable perturbative part from the non-perturbative contributions which must be evaluated either in terms of experimental data or using concrete models[6]. In estimating the B decays, model-dependent wave-functions are adopted for the light mesons which are the final decay products. Among most of the commonly used wave-functions, the light-cone wave-function is more favorable, because the finally produced mesons are light and so that should be more relativistic. Obviously the errors brought out by using such model-dependent wave-function are not accurately estimated, but in some sense they are controllable while one uses as much as possible information from available data. In order to reduce the theoretical uncertainties of the non-perturbative QCD, the meson wave functions should be well theoretically investigated and experimentally
tested. To gain more knowledge about the whole picture, one hopes to make the part which can be calculated in the framework of the perturbative QCD, as accurate as possible and to employ more reasonable model-dependent wave-functions which have been tested by fitting data obtained from precise measurements to gain final results. Comparing the results with experimental data, one may obtain information about both governing mechanisms which are calculated in perturbative framework and the wave-functions. In the case, we can also find a trace of new physics.

The OZI-forbidden process $B_c^- \rightarrow \eta' l^- \bar{\nu}$ was studied by Sugamoto and Yang years ago \cite{7}. In their work, an effective Lagrangian was adopted to avoid introducing the $B_c$ meson wave function, meanwhile they dealt with the light meson by using an effective $g^*_a g^*_b \rightarrow \eta'$ coupling \cite{7} \cite{8}, which was obtained in the NRQM (non-relativistic quark model) approximation. The valence quark $q$ and anti-quark $\bar{q}$ in the light meson were assumed to possess equal momenta and be on their mass shells, i.e., $p_q = p_{\bar{q}}$ and $p_q^2 = m_q^2$. With such approximation and kinematic assumption, they tactfully reduced the complicated Feynman four-point and five-point integral functions\cite{9} \cite{10} to be expressed in three-point functions. As for the heavy meson $B_c$, the authors neglected the relative momentum and binding energy of heavy quark constituents. Then they obtained a reasonable branching ratios of $B_c^- \rightarrow \eta' l^- \bar{\nu}$. Furthermore they estimated the branching ratio of $B_c^- \rightarrow \pi^0 l^- \bar{\nu}$ based on $J/\psi$ decay.

In this paper, we will investigate the OZI forbidden $B_c^- \rightarrow \eta' l^- \bar{\nu}$ in the perturbative QCD without any effective coupling approximation and kinematic assumption in one-loop Feynman diagram evaluation. While calculating the amplitudes of these decay processes, we have to properly deal with the dynamics of bound states. As for the $B_c$ meson, we adopt the way similar to that shown in Ref.\cite{6}, namely, we ignore the relative momentum of the two heavy constituents and their binding energy in comparison with their masses. Furthermore, it is commonly assumed that the constituents of $B_c$ are on mass shell and move together with the same velocity for simplicity. In dealing with the final light meson states, we keep an arbitrary relative momentum for the light quarks $q$ and $\bar{q}$ and take the valence quarks (anti-quarks) of the light mesons to be on their mass shells. Due to the mass difference between up, down and strange quarks, the decay widths of processes $B_c^- \rightarrow \eta' (\eta, \pi^0) l^- \bar{\nu}$ would receive non-zero contributions. By this method, obviously, the advantages of simplifying the loop integral calculations vanish. Namely, one cannot reduce the five-point and four-point loop integral functions into simple expressions involving three-point integral functions. However, with the progress in the technique of calculating loop diagrams, we can directly calculate the five-point and four-point one-loop integrals numerically in general case without any additional approximation and kinematic assumption\cite{9}. Then we can obtain non-zero rates for the decay modes which obviously violate isospin.

The paper is organized as follows: In Section 2, we derive the amplitudes. The numerical results of the decay rates for the $B_c^- \rightarrow \eta' (\eta, \pi^0) l^- \bar{\nu}$ processes are pre-
sented in Section 3, along with all the necessary parameters being listed explicitly. Finally, the conclusion is drawn in the last section.

2 Calculation

2.1 The amplitude at parton level

Being OZI-forbidden processes, there is no contribution at the tree-level for the decay channel \( B_c^- \rightarrow P l^- \bar{\nu} \), and at one-loop level six diagrams which contribute to the decay width, are shown in Fig.1. Here \( P \) stands for a pseudoscalar meson \( \eta' (\eta, \pi^0) \), and \( l = \mu \) or \( e \) respectively. The six diagrams can be divided into three parts as Fig.1 (a,d),(b,e) and (c,f). It is clear that the figures in Fig.1(d),(e) and (f) can be obtained by exchanging two internal gluons of the corresponding diagrams in Fig.1(a),(b) and (c), respectively.

![Diagrams](image)

Figure 1: The diagrams at one-loop level for the process \( B_c^- \rightarrow P l^- \bar{\nu} \), where \( P \) stands for a pseudoscalar meson \( \eta' (\eta, \pi^0) \)

According to the Feynman rules, the amplitudes at parton level corresponding to Fig.1(a), (b) and (c) can be written in the forms as shown in Appendix A. By introducing necessary notations (see Appendix B) and carrying out tedious, but straightforward derivations, we obtain the explicit expressions of the amplitudes for Fig.1(a)-(c). For Fig.1(a), it is

\[
\mathcal{M}_a = -\frac{i\pi^2 g_s^4 g^2 V_{cb} T^a T^b \otimes T^a}{8[(p_1 + p_2 - p_3 - p_4)^2 - M_W^2][(p_2 - p_3 - p_4)^2 - m_c^2](2\pi)^4} \times \epsilon_{\alpha\beta\theta\delta}[-m_c^2(X^{\beta\alpha\mu\rho} + i\epsilon^{\beta\alpha\mu\rho})D^\alpha(u, m_q) - g_{\nu\xi}(Y^{\beta\nu\alpha\delta\theta\rho} + Z^{\beta\nu\alpha\delta\theta\rho})] \\
\times (p_3 + p_4 - p_2)_{\theta} D_{a}^{\nu\xi}(u, m_q) \bar{u}_q(p_3)\gamma^\delta \gamma^5 v_q(p_4) v_c(p_2) \gamma^\alpha \gamma^5 u_b(p_1) \\
\times i\gamma_\mu (1 - \gamma_5) \nu_l. \tag{1}
\]
The amplitude for Fig.1(b) is expressed as

\[ \mathcal{M}_b = -\frac{i\pi^2 g_s^4 g_s^2 V_{cb} T^a T^b T^a}{8[(p_1 + p_2 - p_3 - p_4)^2 - m_W^2][(p_1 - p_3 - p_4)^2 - m_W^2]} \]

\[ \times \varepsilon_{\alpha \beta} \left[-m_W^2 (X^{\mu \beta \rho} + \varepsilon^{\mu \beta \rho}) D_0^\delta (u, m_q) - g_{\rho \theta} (Y^{\nu \xi \lambda \alpha \rho} + Z^{\nu \xi \lambda \alpha \rho}) \right] \]

\[ \times \left[(p_1 - p_3 - p_4) \xi D_0^\theta (u, m_q) \right] \bar{u}_q(p_3)\gamma^\delta \gamma_5 v_q(p_4) \bar{v}_c(p_2) \gamma_\mu \gamma_5 u_b(p_1) \]

\[ \times \bar{\gamma}_\mu(1 - \gamma_5)\nu, \]

and the amplitude corresponding to Fig.1(c) reads

\[ \mathcal{M}_c = -\frac{i\pi^2 g_s^4 g_s^2 V_{cb} T^a T^b T^a}{8[(p_1 + p_2 - p_3 - p_4)^2 - m_W^2]} \]

\[ \times \varepsilon_{\alpha \beta} \left[-m_W^2 (X^{\mu \beta \rho} - i\varepsilon^{\mu \beta \rho}) E_0^a (u, m_q) - g_{\rho \theta} (Y^{\nu \xi \lambda \alpha \rho} + Z^{\nu \xi \lambda \alpha \rho}) \right] \]

\[ \times E_0^e (u, m_q) \bar{u}_q(p_3)\gamma^\delta \gamma_5 v_q(p_4) \bar{v}_c(p_2) \gamma_\mu \gamma_5 u_b(p_1) \]

\[ \times \bar{\gamma}_\mu(1 - \gamma_5)\nu. \]

The notations \( D_0^a (u, m_q) \), \( D_0^e (u, m_q) \), \( D_0^a (u, m_q) \), \( D_0^e (u, m_q) \), \( E_0^a (u, m_q) \) and \( E_0^e (u, m_q) \), are defined as the one-loop vector/tensor integrals of four- and five-point functions \( g \) \cite{2} \cite{10}, integrating over the loop internal momentum \( k \) and their explicit expressions can be found in Appendix B.

The definitions of the variables \( X \), \( Y \) and \( Z \) can be known from the following identities,

\[ \gamma^a \gamma^b \gamma^c = \gamma^a g_{bc} - \gamma^b g_{ac} + \gamma^c g_{ab} - i\varepsilon^{abce} \gamma_\mu = (g^{a\mu} g_{bc} - g^{b\mu} g_{ac} + g^{c\mu} g_{ab}) \gamma_\mu - i\varepsilon^{abce} \gamma_\mu \gamma_5 \]

\[ = X^{abce} \gamma_\mu - i\varepsilon^{abce} \gamma_\mu \gamma_5 \]

\[ \gamma^a \gamma^b \gamma^c \gamma^d \gamma^e = \left(\varepsilon^{cde\mu} g_{ab} - \varepsilon^{bde\mu} g_{ac} + \varepsilon^{ade\mu} g_{bc} - \varepsilon^{abc\mu} g_{de} - \varepsilon^{bcde\mu} g_{a} + \varepsilon^{abcd\mu} g_{e}\right)i\gamma_5 \]

\[ + g^{ab} g^{cd} g^{e\mu} - g^{ac} g^{bd} g^{e\mu} + g^{ad} g^{bc} g^{e\mu} - g^{ab} g^{ce} g^{d\mu} + g^{ac} g^{be} g^{d\mu} \]

\[ - g^{ad} g^{bc} g^{e\mu} + g^{ab} g^{de} g^{c\mu} - g^{ac} g^{de} g^{b\mu} + g^{bc} g^{de} g^{a\mu} - g^{ab} g^{de} g^{c\mu} + g^{ac} g^{be} g^{d\mu} \]

\[ + g^{ae} g^{bd} g^{c\mu} - g^{ad} g^{be} g^{c\mu} - g^{ae} g^{bd} g^{e\mu} + g^{ad} g^{be} g^{c\mu} ] \gamma_\mu = Y^{abde\mu} \gamma_5 \gamma_\mu + Z^{abce\mu} \gamma_\mu \]

The expressions of the contributions from other three diagrams (Fig.1(d)-(f)) are similar to that of the first three (for Fig.1(a)-(c)). For convenience, we omit their explicit expressions in the text. It is noted that the contribution of Fig.1(c) involves five-point tensor integration functions. We follow the approach in Ref. \cite{9} to calculate directly the five-point scalar and tensor integrals.
2.2 The hadronic matrix elements

In above subsection we derive the amplitudes corresponding to the Feynman diagrams shown in Fig.1 at the parton level. In order to obtain the decay rates, one has to evaluate the hadronic matrix elements. Hadronization happens at the energy scale of $\Lambda_{QCD}$ which is the region governed by the non-perturbative QCD. So far, there is no any reliable way to evaluate the hadronic matrix elements from any underlying theory. Instead, to do this job, one needs to invoke concrete models. The initial meson $B_c$ is composed of only heavy quarks, so that we can suppose the two valence quarks $b$ and $\bar{c}$ in $B_c$ meson to be on mass shell approximately and the composition of $B_c$ can be well described by its wavefunction at origin. On the contrary, the produced pseudoscalar meson is composed of light quark and antiquark whose three-momentum are much larger than $\Lambda_{QCD}$, so that they are very relativistic, in this case, the light-cone wavefunctions seem to be applicable for the hadronization of light mesons\cite{11, 12, 13}.

For the pseudoscalar mesons, the SU(3) flavor wavefunctions are

$$\pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}, \quad \eta_0 = \frac{d\bar{d} + u\bar{u} + s\bar{s}}{\sqrt{3}} \quad \text{and} \quad \eta_8 = \frac{d\bar{d} + u\bar{u} - 2s\bar{s}}{\sqrt{6}},$$  \hspace{1cm} (6)

$\eta$ and $\eta'$ are mixtures of $\eta_0$ and $\eta_8$, and

$$\eta = \cos \theta \eta_8 - \sin \theta \eta_0, \quad \eta' = \sin \theta \eta_8 + \cos \theta \eta_0.$$ \hspace{1cm} (7)

Among the final produced light mesons concerned in the decays of $B_c \rightarrow \pi^0(\eta, \eta') + l\bar{\nu}$, only $\eta_0$ is SU(3) singlet. Therefore, the decays $B_c \rightarrow \pi^0 + l\bar{\nu}$ and $B_c \rightarrow \eta_8 + l\bar{\nu}$ are related to isospin or SU(3) violation. In general, there are two possibilities which can induce isospin or SU(3) violation. The first is photon emission and the second is due to mass splitting of u and d quarks. In our case, the violation comes from the quark mass splitting in the effective Hamiltonian. There are two sources which are related to the quark mass and can contribute to the amplitude. One is from the Feynman diagrams where the quark propagators contain quark masses, and another is from the higher-twist parts of the wavefunction of meson. But normally we calculate the hadron matrix elements with only the leading-twist part of light pseudoscalar wave function with valence quarks on-shell, since it can give enough accuracy and simplify our calculations.

The hadronic matrix element part for process $B_c^- \rightarrow Pl^-\nu$ can be written as

$$\left\langle P(p_p) | \bar{q}_\alpha \gamma_\gamma \gamma_5 q \sum \int_0^1 du \phi_p(u, \mu) \sum_i C_i^{\mu \alpha \beta}(u, m_q) \bar{c}_\beta \gamma_5 b \right| B_c(p_{B_c}) \right\rangle \Bigg|_{p_{B_c}} \right\rangle$$

$$= -if_{p_p} \int_0^1 du \phi_p(u, \mu) \sum_i C_i^{\mu \alpha \beta}(u, m_q) i f_{B_c} p_{B_c \beta},$$ \hspace{1cm} (8)
where $P$ represents $\pi^0, \eta$ and $\eta'$, $C_i$ are the coefficients, the summation is over the diagrams shown in last subsection, and in Eq. (8) we used the expression of the leading-twist light-cone distribution amplitudes for pseudoscalar mesons($\pi^0, \eta$ and $\eta'$) with flavor content($\bar{q}q$) written as $[12]$

$$<\not{P}(p_P)\not{q}(y)_a q(x)_\beta|0>_{(x-y)^2=0} = \frac{i f_p}{4} (\not{p}_P \gamma_5)_{\beta \alpha} \int_0^1 du e^{i(\bar{u}q_x + uq_y)} \phi_p(u, \mu). \quad (9)$$

where the "bar"-notation over $u$ is defined as $\bar{u} \equiv 1 - u$, and parameter $\mu$ is the renormalization scale of the light-cone operators on the left-hand side. Also the light-cone wavefunction of the leading twist is normalized to unity:

$$\int_0^1 du \phi_p(u, \mu) = 1, \quad (P = \pi^0, \eta, \eta'). \quad (10)$$

The asymptotic distribution amplitude is defined as the limit in which the renormalization scale goes to infinity. The explicit asymptotic forms of the leading twist light-cone wavefunctions of the light pseudoscalar mesons can be different. There are several typical leading twist light-cone distribution amplitudes for the light pseudoscalar mesons, so far, one cannot determine which one is the most suitable. In our following calculation, we take three different types of the leading twist light-cone wavefunctions for light meson, which are frequently adopted $[12, 14, 15, 16]$. They are expressed as

$$\phi_{P,1}(u, \mu \to \infty) = 6u(1 - u), \quad (11)$$
$$\phi_{P,2}(u, \mu \to \infty) = 30u^2(1 - u)^2, \quad (12)$$
$$\phi_{P,3}(u, \mu \to \infty) = \frac{15}{2}(1 - 2u)^2[1 - (1 - 2u)^2]. \quad (13)$$

Finally, we obtain the hadronic matrix elements of $M_a$, $M_b$ and $M_c$ as follows

$$\langle Pl\bar{\nu}|M_a|B_c\rangle = -\frac{i \pi^2 g_A^4 g^2 V_{cb} T^a T^b \otimes T^{\mu\nu} \int_0^1 du \phi_p(u, \mu \to \infty)}{8 (2\pi)^4 N_c^2 [(p_{Bc} - p_p)^2 - M_W^2][(p_{Bc} - p_p)^2 - m_c^2]} \int_0^1 du \phi_p(u, \mu \to \infty)$$
$$ \times \{i f_p f_{Bc} p^\delta p_{Bc} \varepsilon_{\alpha\beta\delta}[ - \epsilon^2(X^{\beta\alpha\mu\rho} + i \epsilon^{\beta\alpha\mu\rho}) \sum_{q=u,d,s} D^\sigma_a(u, m_q)$$
$$ - g_{\rho\epsilon}(Y^{\beta\alpha\mu\rho} + Z^{\beta\alpha\mu\rho}) \left(p_p - \frac{m_c}{M_{Bc}} p_{Bc}\right) \sum_{q=u,d,s} D^\xi_a(u, m_q) \}$$
$$ \times \bar{l}_\gamma (1 - \gamma_5) l_\mu, \quad (14)$$
\[ \langle P \bar{\nu} | M_b | B_c \rangle = -\frac{i \pi^2 g_s^4 g^2 V_{cb} T^a T^b \otimes T^b T^a}{8(2\pi)^4 N_c^2 (p_{B_c} - p_p)^2 - M_W^2} \int_0^1 du \phi_p(u, \mu \to \infty) \]
\[ \times \{ i f_p f_{B_c} b \bar{b} p_{B_c} \sigma_{a b c d} [ m_b m_c ( X^{a b c d} - i \rho^{a b c d} ) \sum_{q=u,d,s} D_b (u, m_q) ] \}
\[ - g_{\nu \theta} Y^{a b c d} + Z^{a b c d} \sum_{q=u,d,s} D_b (u, m_q) \} \]
\[ \times \bar{l} \gamma \mu (1 - \gamma_5) \nu, \]
\[ (15) \]

\[ \langle P \bar{\nu} | M_c | B_c \rangle = -\frac{i \pi^2 g_s^4 g^2 V_{cb} T^a T^b \otimes T^b T^a}{8(2\pi)^4 N_c^2 (p_{B_c} - p_p)^2 - M_W^2} \int_0^1 du \phi_p(u, \mu \to \infty) \]
\[ \times \{ i f_p f_{B_c} b \bar{b} p_{B_c} \sigma_{a b c d} [ m_b m_c ( X^{a b c d} - i \rho^{a b c d} ) \sum_{q=u,d,s} D_b (u, m_q) ] \]
\[ - g_{\nu \theta} Y^{a b c d} + Z^{a b c d} \sum_{q=u,d,s} D_b (u, m_q) \} \]
\[ \times \bar{l} \gamma \mu (1 - \gamma_5) \nu, \]
\[ (16) \]

Then we can get
\[ \frac{dBr(B_c^- \to P l^- \nu)}{ds dt} = \frac{1}{(2\pi)^3 32 M_{B_c}^2} |M|^2 \tau_{B_c} \]
\[ (17) \]

where \( M \) is the total amplitude for the process \( B_c^- \to P l^- \nu \) at one-loop level and the Mandelstam variables \( s = (p_P + p_l)^2, t = (p_l + p_\nu)^2. \)

3 Numerical results

In our calculation, no ultraviolet(UV) divergence appears, but there is a mild superficial infrared(IR) divergence problem. Our method to check the cancellation of the IR divergence is standard. Namely, we assign a small gluon mass as a regulator and vary it to check if the result is stable. In practical calculations, we set the small gluon mass varying in the range between \( 10^{-4} \text{ MeV} \) to \( 10^{-2} \text{ MeV} \) and find that the result changes only with a negligible small fraction. Therefore, we can trust the obtained result which is free of IR problem. In following numerical calculation we set the gluon mass being \( 10^{-3} \text{ MeV} \).

The input parameters which we are going to use in the numerical computations are taken as follows [7, 11, 17, 18, 19, 20, 21, 22]: \( f_{B_c} = 500 \text{MeV}, f_x = 131 \text{MeV}, f_\eta = 157 \text{MeV}, m_\pi = 134.9766 \text{MeV}, m_\eta = 547.75 \text{MeV}, m_{\eta'} = 957.78 \text{MeV}, m_b = 4800 \text{MeV}, \alpha_s(m_{B_c}) = 0.20, m_c = 1500 \text{MeV}, M_{B_c} = 6300 \text{MeV}, V_{cb} = 0.04, \)
$\tau_{B_c} = 0.46\, ps$, the mixing angle of $\eta, \eta'\theta = -11^\circ$, and three possible leading twist light-cone wavefunctions of pseudoscalar light meson are given in Eqs. (11,12,13). During the calculation, we keep an arbitrary relative momentum for the light valence quark(antiquark) $q(\bar{q})$, and moreover, $q(\bar{q})$ is on mass shell, while dealing with the light meson.
| Process | $m_u$ | $m_d$ | $m_s$ | $10^6 \times BR(\phi_{P,1})$ | $10^6 \times BR(\phi_{P,2})$ | $10^6 \times BR(\phi_{P,3})$ |
|--------|------|------|------|----------------|----------------|----------------|
| $B_c^- \to \pi^0 l^- \nu$ | 1.5  | 4    | /    | 0.92818       | 0.37502        | 9.2459         |
|        | 2    | 4    | /    | 0.50312       | 0.21671        | 6.2409         |
|        | 3    | 5    | /    | 0.61314       | 0.28907        | 5.6131         |
|        | 3    | 7    | /    | 1.42393       | 0.55988        | 10.793         |
|        | 4    | 6    | /    | 0.63533       | 0.30333        | 5.4237         |
| $B_c^- \to \eta l^- \nu$ | $m_u$ | $m_d$ | $m_s$ | $10^5 \times BR(\phi_{P,1})$ | $10^5 \times BR(\phi_{P,2})$ | $10^5 \times BR(\phi_{P,3})$ |
|        | 2    | 4    | 80   | 0.39826       | 0.21839        | 6.6085         |
|        | 2    | 5    | 90   | 0.41058       | 0.23082        | 6.7120         |
|        | 3    | 5    | 100  | 0.42533       | 0.24725        | 6.7201         |
|        | 3    | 6    | 110  | 0.42835       | 0.25345        | 6.96826        |
|        | 4    | 6    | 120  | 0.43766       | 0.27107        | 7.17638        |
|        | 3    | 7    | 130  | 0.44001       | 0.29839        | 7.3408         |
| $B_c^- \to \eta' l^- \nu$ | $m_u$ | $m_d$ | $m_s$ | $10^4 \times BR(\phi_{P,1})$ | $10^4 \times BR(\phi_{P,2})$ | $10^4 \times BR(\phi_{P,3})$ |
|        | 2    | 4    | 80   | 0.52462       | 0.41698        | 2.3586         |
|        | 2    | 5    | 90   | 0.52065       | 0.40817        | 2.3007         |
|        | 3    | 5    | 100  | 0.51300       | 0.40567        | 2.2829         |
|        | 3    | 6    | 110  | 0.50178       | 0.39619        | 2.2713         |
|        | 4    | 6    | 120  | 0.49432       | 0.39048        | 2.2498         |
|        | 3    | 7    | 130  | 0.49133       | 0.38767        | 2.2233         |

Table 1: The branching ratios of the decays $B_c^- \to \pi^0 l^- \nu$, $B_c^- \to \eta l^- \nu$ and $B_c^- \to \eta' l^- \nu$ in the rest frame of $B_c$ are listed, and the three columns correspond to the three different leading twist light-cone wavefunctions of the produced pseudoscalar light mesons ($\pi^0$, $\eta$, $\eta'$). The masses for the light quarks (u, d, s) are in MeV.

We carry out the integration of the scalar and tensor four- and five-point integral functions precisely. We adopted the FF package\cite{23} in the calculation of two-, three- and four-point integral functions, while the implementations of the scalar and the tensor five-point integrals are done exactly by using the Fortran programs as we used in our previous works on $e^+ e^- \to t\bar{t} H^0$ and $e^+ e^- \to Z^0 H^0 H^0$ processes\cite{24,25} by using the approach presented in Ref.\cite{9}.

In order to model the light hadronic effects, we use three different types of wavefunctions for light mesons in our calculation. We present the theoretical predictions on the decay widths of $B_c^- \to \pi^0 l^- \nu$, $B_c^- \to \eta l^- \nu$ and $B_c^- \to \eta' l^- \nu$ in the rest frame of $B_c$, corresponding to the three different leading twist light-cone wavefunctions of light mesons respectively in Table 1.
4 Conclusion

In this work, we studied the OZI forbidden quark-level sub-processes of $B_c^- \rightarrow \eta', \pi^0 l^{-} \bar{\nu}$ in the framework of the perturbative QCD. For $B_c^- \rightarrow \pi^0 l^{-} \bar{\nu}$ process, within reasonable ranges of the masses of $u$ and $d$ quarks, the branching ratio with the distribution amplitude $\phi_{P,3}$ can be about one order larger than other two distribution amplitudes ($\phi_{P,1,2}$) and can reach the order of $10^{-5}$ with specific light quark masses. For $B_c^- \rightarrow \eta l^{-} \bar{\nu}$ process, the branching ratios turn out to be of the order $10^{-6}$ for distribution amplitudes $\phi_{P,1,2}$, and about $10^{-5}$ for distribution amplitude $\phi_{P,3}$. For $B_c^- \rightarrow \eta' l^{-} \bar{\nu}$ decay, the $\phi_{P,3}$ can give the branch ratio values of the order $10^{-4}$. These results seem to be at the reach of future experiments at the LHC where it is expected to produce $5 \times 10^{10} B_c$ meson events per year with $\sqrt{s} = 14$ TeV and luminosity $\mathcal{L} = 10^{34}$cm$^{-2}$s$^{-1}$. We conclude that the decays $B_c^- \rightarrow \eta'(\eta, \pi^0) l^{-} \bar{\nu}$ could be investigated experimentally at the LHC, and the study on these decays can deepen our understanding on both perturbative and non-perturbative QCD.

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Appendix A

The amplitudes at parton level corresponding to Fig.1 (a), (b) and (c) are explicitly expressed as

$$\mathcal{M}_a = \int \frac{d^4 k}{(2\pi)^4} \bar{u}_q(p_3) (-ig_s T^a \gamma_\alpha) \frac{i}{-\slashed{k} - m_q} (-ig_s T^b \gamma_\beta) v_q(p_4) \bar{v}_c(p_2) (-ig_s T^c \gamma_\gamma)$$
$$\times \frac{i}{\slashed{p}_4 - \slashed{p}_2 - \slashed{k} - m_c} (-ig_s T^a \gamma_\alpha) \frac{i}{\slashed{p}_3 + \slashed{p}_4 - \slashed{p}_2 - m_c 2\sqrt{2}} \gamma^\mu (1 - \gamma_5) V_{cb} u_b(p_1)$$
$$\times \frac{-ig}{2\sqrt{2}} \gamma^\nu (1 - \gamma_5) \nu_{\ell} \times \frac{-i}{(p_3 + k)^2 (p_4 - k)^2 (p_1 + p_2 - p_3 - p_4)^2 - M_W^2}.$$  
$$= -g_s^4 g^2 V_{cb} T^a T^b T^c \frac{1}{(p_1 + p_2 - p_3 - p_4)^2 - M_W^2} \int \frac{d^4 k}{(2\pi)^4} \bar{u}_q(p_3) \gamma_\alpha (-\slashed{k} + m_q) \gamma_\beta$$
$$\times v_q(p_4) \bar{v}_c(p_2) \gamma_\beta (\slashed{p}_4 - \slashed{p}_2 - \slashed{k} + m_c) \gamma_\alpha (\slashed{p}_3 + \slashed{p}_4 - \slashed{p}_2 + m_c) \gamma_\mu (1 - \gamma_5) u_b(p_1) \bar{\nu}_{\ell} \gamma_\mu (1 - \gamma_5) \nu_{\ell}$$
$$\times \frac{1}{(k^2 - m_\nu^2) (p_3 + k)^2 (p_4 - k)^2 ((p_2 - p_4 + k)^2 - m_\nu^2) ((p_2 - p_3 - p_4)^2 - m_c^2)}, \quad (18)$$
\[ \mathcal{M}_b = \int \frac{d^4k}{(2\pi)^4} \bar{u}_q(p_3)(-ig_s T^a \gamma_{\alpha}) \frac{i}{-\not{k} - m_q} (-ig_s T^b \gamma_{\beta}) v_q(p_4) \bar{v}_c(p_2) \frac{-ig}{2\sqrt{2}} \gamma^\mu (1 - \gamma_5) V_{cb} \]

\[ \times \frac{i}{\not{p}_1 - \not{p}_3 - \not{p}_4 - m_b} (-ig_s T^b \gamma_{\beta}) \]

\[ \times \frac{i}{\not{p}_1 - \not{p}_3 - \not{f} - m_b} (-ig_s T^a \gamma_{\alpha}) u_b(p_1) \]

\[ \times \frac{i}{2\sqrt{2}} \gamma^\nu (1 - \gamma_5) \nu \times \frac{1}{(p_3 + k)^2 (p_4 - k)^2 (p_1 + p_2 - p_3 - p_4)^2 - M_W^2} \]

\[ = -g_s^4 g^2 V_{cb} T^a T^b \frac{1}{(p_1 + p_2 - p_3 - p_4)^2 - M_W^2} \int \frac{d^4k}{(2\pi)^4} \bar{u}_q(p_3) \gamma_{\alpha} (-\not{k} + m_q) \gamma_{\beta} \]

\[ \times v_q(p_4) \bar{v}_c(p_2) \gamma^\mu (1 - \gamma_5) (\not{p}_1 - \not{p}_3 - \not{f} + m_b) \gamma^\beta (\not{p}_1 - \not{p}_3 - \not{k} + m_b) \gamma^\alpha u_b(p_1) \bar{f}_\gamma (1 - \gamma_5) \nu \]

\[ \times \frac{1}{(k^2 - m_q^2)(p_3 + k)^2(p_4 - k)^2((p_1 - p_3 - k)^2 - m_b^2)((p_1 - p_3 - p_4)^2 - m_b^2)}. \]  

(19)

\[ \mathcal{M}_c = \int \frac{d^4k}{(2\pi)^4} \bar{u}_q(p_3)(-ig_s T^a \gamma_{\alpha}) \frac{i}{-\not{k} - m_q} (-ig_s T^b \gamma_{\beta}) v_q(p_4) \bar{v}_c(p_2) (-ig_s T^b \gamma_{\beta}) \]

\[ \times \frac{i}{\not{p}_1 - \not{p}_2 - \not{f} - m_c} \frac{-ig}{2\sqrt{2}} \gamma^\mu (1 - \gamma_5) V_{cb} \]

\[ \times \frac{i}{\not{p}_1 - \not{p}_3 - \not{k} - m_b} (-ig_s T^a \gamma_{\alpha}) u_b(p_1) \]

\[ \times \frac{i}{2\sqrt{2}} \gamma^\nu (1 - \gamma_5) \nu \times \frac{1}{(p_3 + k)^2 (p_4 - k)^2 (p_1 + p_2 - p_3 - p_4)^2 - M_W^2} \]

\[ = -g_s^4 g^2 V_{cb} T^a T^b \frac{1}{(p_1 + p_2 - p_3 - p_4)^2 - M_W^2} \int \frac{d^4k}{(2\pi)^4} \bar{u}_q(p_3) \gamma_{\alpha} (-\not{k} + m_q) \gamma_{\beta} \]

\[ \times v_q(p_4) \bar{v}_c(p_2) \gamma^\beta (\not{p}_4 - \not{p}_2 - \not{k} + m_c) \gamma^\alpha (1 - \gamma_5) (\not{p}_1 - \not{p}_3 - \not{k} + m_b) \gamma^\nu u_b(p_1) \bar{f}_\gamma (1 - \gamma_5) \nu \]

\[ \times \frac{1}{(k^2 - m_q^2)(p_3 + k)^2(p_4 - k)^2((p_1 - p_3 - k)^2 - m_b^2)((p_1 - p_3 - p_4)^2 - m_b^2)}. \]  

(20)

Appendix B

The notations of the four-point and five-point one-loop functions in our text are
defined as

\[
D^\sigma_a(u, m_q) = \frac{1}{i\pi^2} \int d^4k \frac{k^\sigma}{[k^2 - m_q^2](p_3 + k)(p_4 - k)^2[(p_2 - p_4 + k)^2 - m_c^2]},
\]

(21)

\[
D^\sigma_\xi_a(u, m_q) = \frac{1}{i\pi^2} \int d^4k \frac{k^\sigma(p_4 - p_2 - k)^\xi}{[k^2 - m_q^2](p_3 + k)(p_4 - k)^2[(p_2 - p_4 + k)^2 - m_c^2]},
\]

(22)

\[
D^\sigma_b(u, m_q) = \frac{1}{i\pi^2} \int d^4k \frac{k^\sigma}{[k^2 - m_q^2](p_3 + k)(p_4 - k)^2[(p_2 - p_4 - k)^2 - m_b^2]},
\]

(23)

\[
D^\sigma_\xi_b(u, m_q) = \frac{1}{i\pi^2} \int d^4k \frac{k^\sigma(p_1 - p_3 - k)^\xi}{[k^2 - m_q^2](p_3 + k)(p_4 - k)^2[(p_1 - p_3 - k)^2 - m_b^2]},
\]

(24)

\[
E^\sigma_c(u, m_q) = \frac{1}{i\pi^2} \int d^4k \frac{k^\sigma}{[k^2 - m_q^2](p_3 + k)(p_4 - k)^2[(p_1 - p_3 - k)^2 - m_c^2][(p_1 - p_3 - k) - m_b^2]},
\]

(25)

\[
E^\sigma_\theta_\xi_c(u, m_q) = \frac{1}{i\pi^2} \int d^4k \frac{k^\sigma(p_4 - p_2 - k)^\xi(p_1 - p_3 - k)^\theta}{[k^2 - m_q^2](p_3 + k)(p_4 - k)^2[(p_1 - p_3 - k)^2 - m_c^2][(p_1 - p_3 - k) - m_b^2]},
\]

(26)

with

\[
p_1 = \frac{m_b}{M_{B_c}} p_{B_c}, \quad p_2 = \frac{m_c}{M_{B_c}} p_{B_c}, \quad p_3 = p_\rho u, \quad p_4 = p_\rho (1 - u).
\]

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