Design of internal model control-proportional integral derivative controller with improved filter for disturbance rejection

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The internal model control (IMC) proportional integral derivative (PID) controller tuning rules provide an excellent tracking of set-point, but sluggish disturbance rejection, as the conventional IMC filter introduces slow process pole. Disturbance rejection is significant than set-point tracking in many industrial applications. This paper proposes the design of IMC-PID controller, cascaded with lead–lag filter, with an improved IMC filter to provide effective disturbance rejection and robust operation of the first-order processes (FOPs). The simulation study was conducted to show the effectiveness of the proposed method on various structures of the FOPs, calculating the controller parameters, maintaining similar robustness in terms of maximum sensitivity MS. Perturbations were incorporated in the parameters of the plant model simultaneously, for the robustness analysis of the controller. The closed-loop performance was evaluated with the integral error criteria, namely, integral absolute error, integral square error, and integral time absolute error. The proposed IMC filter provided good disturbance rejection. The results of recently published design methods were compared with that obtained from the proposed method, validating the usefulness of the proposed method.

Keywords: IMC; PID; improved filter; disturbance rejection; robustness; sensitivity; integral criteria; first-order process

Introduction

The dynamic elements, usually of the first order, constitute the vast majority of processes in an industry. This leads the process to have a linear model of a very high order (Kurtulan & Goren, 2005). Although these higher-order models are very precise, for control purposes, they are not to be used. In majority of cases, a linear first-order system with dead time element representation is used to model the behavior of the higher-order models (Eris & Kurtulan, 2011; Kurtulan & Goren, 2005).

The widely used controller in the process industries is proportional integral derivative (PID) controller, as it can assure satisfactory performances with a simple algorithm for a wide range of processes (Lee, Shamsuzzoha, & Luan Vu, 2008; Pai, Chang, & Huang, 2010). They can compensate the effect of both the delayed and non-delayed processes. It is important to note that the cost benefit ratio achieved through the PID controller is not easy to achieve with other controllers (Eris & Kurtulan, 2011; Gopi Krishna Rao, Subramanyam, & Satyaprasad, 2013; Lee et al., 2008; Pai et al., 2010; Shamsuzzoha & Lee, 2008a). An observation indicates, 97% of the regulatory controllers use a PID algorithm (Desborough & Miller, 2001). Numerous methods have been projected, including modern control methods for tuning these controllers, but every method has some constraint (Chidambaram, 1998; Cohen & Coon, 1953; Kumanan & Nagaraj, 2013; Shamsuzzoha & Lee, 2008a; Zhang, Shi, & Mehr, 2011; Zhang, Shi, & Mu, 2013). As a result, the design of PID controller remains a challenge for researchers and engineers.

The internal model control (IMC) provides a progressive, effective, natural, generic, unique, powerful, and simple framework for analysis and synthesis of control system performance (Morari & Zafiriou, 1989; Saxena & Yogesh, 2012; Shamsuzzoha & Lee, 2007, 2008b). The easiness and enhanced performance of the IMC-based tuning rule, and the analytically derived IMC-PID tuning techniques have appealed the attention of the industrial users, in the past decade (Shamsuzzoha & Lee, 2007, 2008b). The well-known IMC-PID tuning rule provides a clear compromise in the midst of closed-loop performance and robustness to model uncertainties, and is achieved by only one user-defined tuning parameter, which is directly related to the closed-loop time constant (Horn, Arulandu, Christopher, Van Antwerp, & Braatz, 1996; Lee et al., 2008; Rivera, Morari, & Skogestad, 1986; Saxena & Yogesh, 2012; Shamsuzzoha & Lee, 2007, 2008b). The IMC-PID tuning methods and direct synthesis, achieve the PID controller parameters, computing the controller, which provides the desired closed-loop response (Chen & Seborg, 2002; Eris & Kurtulan, 2011; Horn et al., 1996; Lee, Park, Lee, &
Load disturbance rejection is the important issues in the context of process control. IMC-PID controller delivers good set-point tracking, but the disturbance response is sluggish, especially when \( \theta/t \ll 1 \) (Chen & Seborg, 2002; Liu & Gao, 2012; Morari & Zafiriou, 1989; Seborg et al., 2004). For many single loop controllers, disturbance rejection is important than the set-point tracking, a controller design that emphasizes the disturbance rejection is an important design goal (Morari & Zafiriou, 1989; Seborg et al., 2004; Shamsuzzoha & Lee, 2008b). The goal can be achieved with the design of the controller for disturbance rejection, rather than set-point tracking. PID controller cascaded with a lead–lag filter was suggested in the literature (Horn et al., 1996; Lee et al., 1998; Morari & Zafiriou, 1989; O’Dwyer, 2006; Rivera et al., 1986; Shamsuzzoha & Lee, 2007, 2008a, 2008b, 2008c) for disturbance rejection, the efficiency of the PID controller is based on the structure of the IMC filter. The IMC filter structure was selected to make the IMC controller realizable while satisfying the performance requirements. The efficiency of IMC controller and the close approximation of IMC controller to ideal feedback controller determine the efficiency of the resulting PID controller. Therefore, the suitable IMC filter structure has to be selected based on the performance of the resulting PID controller, than the performance of IMC controller (O’Dwyer, 2006; Shamsuzzoha & Lee, 2007; Toscano, 2005).

Regarding disturbance rejection of lag time dominant processes, Ziegler and Nichols’ (1942) established design method (ZN) shows better performance than IMC-PID design methods based on the IMC filter \( G_f(s) = 1/(\lambda s + 1)^\nu \). Lee et al. (2008), Horn et al. (1996), and Liu and Gao (2010), proposed IMC filter; the resulting controller exhibited the advantages over those based on the conventional IMC filter.

This paper proposes a new type of IMC filter structure that includes a lead term to cancel process-dominant poles. The IMC-PID tuning rules, for the first-order systems, lead to the structure of a PID controller cascaded with a lead–lag filter, developed based on this filter. The resulting controller shows advantages, than the once based on the conventional IMC filter.

The objectives of the present work are

- Consider IMC-PID cascaded with lead–lag filter and determine the IMC filter structure to optimize the performance of the resulting PID controller for disturbance rejection.
- Analyze robustness, closed-loop system performance, incorporating perturbations in the plant (predictive) model parameters, and integral performance criteria, respectively.

**IMC-PID controller design**

Garcia and Morari (1982) introduced IMC it is characterized as a controller where the process model is explicitly an integral part of the controller. The design process of IMC involves factorizing the predictive plant model \( G_M(s) \) as invertible \( G_{M_+}(s) \) and non-invertible \( G_{M_-}(s) \) parts depicted in Equation (1) by simple factorization or all pass factorization (Horn et al., 1996; Lee et al., 1998; Rivera et al., 1986; Saxena & Yogesh, 2012; Shamsuzzoha & Lee, 2008c). The internal model controller in Equation (2) is the inverse of the invertible \( G_{M_+}(s) \) portion of the plant model \( G_M(s) \), IMC filter \( G_f(s) \) is used to realize the controller.

\[
G_{M_+}(s) = G_{M_-}(s) G_{M_+}(s).
\]  

The design of the IMC controller is

\[
Q(s) = G_{M_+}^{-1}(s) G_f(s).
\]  

The IMC controller can take the form of ideal feedback controller of Figure 1, expressed mathematically in terms of \( Q(s) \) and \( G_M(s) \) as Equation (3), by making small modifications to Figure 2

\[
G_C(s) = \frac{Q(s)}{1 - Q(s) G_M(s)}.
\]  

The ideal feedback controller of Equation (3) lacks the standard PID form; the PID parameters are achieved by reducing the controller form to that of a PID controller cascaded with a lead–lag filter of Equation (4), with clever approximation of the dead time in the process model if any

\[
G_C(s) = K_p \left( 1 + \frac{1}{T_1 s} + T_d s \right) \left( \frac{ds^3 + ds^2 + cs + 1}{hs^3 + hs^2 + as + 1} \right).
\]  

**Figure 1.** Feedback control structure.

**Figure 2.** Basic IMC structure.
The output response $Y(s)$ of the closed-loop system for set-point input $R(s)$, load disturbance input $L(s)$, and output load disturbance input $D(s)$ is shown in the following equation:

$$Y(s) = \frac{G_C(s)G_P(s)}{1 + G_C(s)G_P(s)}R(s) + \frac{1}{1 + G_C(s)G_P(s)}D(s) + \frac{G_P(s)}{1 + G_C(s)G_P(s)}L(s).$$  

(5)

The conventional IMC filter structure, in Equation (6) proposed by Rivera et al., introduces a slow process pole $s = -1/\tau$, in the dynamic relation between controlled output $Y(s)$ and load disturbance $L(s)$ with $n = 1$, represented in Equation (7). This results in sluggish disturbance rejection by the controller. To enhance the disturbance rejection efficiency of the PID controller, an optimum IMC filter of the form Equation (8) is suggested for the first-order process (FOP)

$$G_f(s) = \frac{1}{(\lambda s + 1)^n},$$  

(6)

$$Y(s) = \frac{K[\theta/2\lambda s^2 + (\theta + \lambda)s][1 - (\theta/2)s]}{(1 + \tau s)(1 + \lambda s)(1 + (\theta/2)s)}.$$  

(7)

Proposed of IMC filter

$$G_f(s) = \frac{(\alpha s + 1)^n}{(\lambda s + 1)^{n+1}}, \text{ where } n = 0 \text{ to } 2.$$  

(8)

The optimum IMC filter for the FOP with time delay (FOPTD) system is obtained from Equation (8) with $n = 2$. Based on integral performance criteria’s integral absolute error (IAE), integral square error (ISE), and integral time absolute error (ITAE), it is observed, that the higher-order IMC filter structure provides a better performance of the PID controller, than the lower-order IMC filter structure

$$G_f(s) = \frac{(\alpha s + 1)^2}{(\lambda s + 1)^3}.$$  

(9)

Additional degree of freedom provided by $\alpha$, eliminates the slow process pole $s = -1/\tau$, of Equation (7), improving the disturbance rejection efficiency of the controller, this criteria require $[1 - G_M(s)Q(s)]_{s=\infty} = 0$.

**IMC-PID tuning rules for typical process models**

**First-order process with time delay**

The widely used approximate or predictive models of the process, more specifically chemical processes are the FOPTD, Equation (10). The plant model $G_M(s)$ is factored into invertible and non-invertible portions; the delay term $e^{-\theta s}$ forms the non-invertible portion, as it becomes $e^{\theta s}$ a predictive term after inversion

$$G_M(s) = \frac{K e^{\theta s}}{\tau s + 1}.$$  

(10)

$$G_{\bar{M}}(s) = \frac{K}{\tau s + 1}, \quad G_M(s) = e^{-\theta s}.$$  

(11)

IMC controller $Q(s)$ is

$$Q(s) = \frac{(1 + \tau s)(\alpha s + 1)^2}{K(\lambda s + 1)^3(1 + (\theta/2)s)(\alpha s + 1)^2(1 - (\theta/2)s)}.$$  

(12)

The ideal feedback controller, equivalent of IMC controller is

$$G_C(s) = \frac{(1 + \tau s)(\alpha s + 1)^2}{K[\lambda s + 1]^3 - (\alpha s + 1)^2 e^{-\theta s}].}.$$  

(13)

The ideal feedback controller in Equation (13) lacks the delay term $e^{-\theta s}$ with $1/\tau$ padé expansion

$$e^{-\theta s} = \frac{1 - (\theta/2)s}{1 + (\theta/2)s},$$  

(14)

results in $G_C(s)$ as

$$G_C(s) = \frac{(1 + \tau s)(1 + (\theta/2)s)(\alpha s + 1)^2}{K[\lambda s + 1]^3(1 + (\theta/2)s)(\alpha s + 1)^2(1 - (\theta/2)s)}.$$  

(15)

Simplification of Equation (15), results in

$$G_C(s) = \left[ \frac{(\alpha s + 1)^2}{Ks(3\lambda + \theta - 2a)} \right] \left[ \frac{\left(\frac{\theta}{2}\right)^2 s^2 + \left(\frac{\theta}{2} + \tau\right)s + 1}{\frac{1}{3\lambda + \theta - 2a} \frac{\theta^2}{2} s^3 + \frac{2}{3\lambda + \theta - 2a} \frac{\theta^2}{2} s^2 + \frac{1}{3\lambda + \theta - 2a} s + 1} \right].$$  

(16)

Comparing Equations (16) and (4), the PID tuning parameters and lead–lag filter coefficients are obtain as

$$K_p = \frac{\theta/2 + \tau}{K(3\lambda + \theta - 2a)}, \quad T_i = \tau + \frac{\theta}{2}T_d = \frac{(\theta/2)\tau}{\theta/2 + \tau},$$  

(17)

$$\hat{d} = 0, \quad d = 2\alpha, \quad \hat{\theta} = \alpha^2,$$

$$\alpha = \frac{(3\lambda^2 + (3/2)\theta \lambda + \theta \alpha - \alpha^2)}{(3\lambda + \theta - 2a)},$$

$$b = \frac{(\lambda^2 + (3/2)\theta \lambda^2 + (\theta/2)\alpha^2)}{(3\lambda + \theta - 2a)},$$

$$c = \frac{(\theta/2)\lambda^3}{(3\lambda + \theta - 2a)}.$$  

(18)

The extra degree of freedom provided by $\alpha$, cancels the slow process pole $s = -1/\tau$, which requires $[1 -$
Table 1. IMC tuned PID controller parameter formulas.

| Process/parameters | $K e^{-\alpha \tau} / (\tau s + 1)$ | $K/(\tau s + 1)$ | $K(-s + z)/(\tau s + 1)$ |
|--------------------|-------------------------------------|-----------------|--------------------------|
| $K_P$              | $\frac{\tau + \theta/2}{K(3\lambda + \theta - 2\alpha)}$ | $\frac{\tau + \alpha}{K(3\lambda - 2\alpha)}$ | $\frac{\tau + \alpha}{K(3\lambda z - 2\alpha + 2)}$ |
| $T_i$              | $\tau + \frac{\theta}{2}$ | $\frac{\tau \alpha}{\tau + \alpha}$ | $\frac{\tau \alpha}{\tau + \alpha}$ |
| $T_d$              | $\frac{2\tau + \theta}{(\theta/2)\lambda}$ | $\frac{\tau \alpha}{\tau + \alpha}$ | $\frac{\tau \alpha}{\tau + \alpha}$ |
| $c$                | $\frac{(3\lambda + \theta - 2\alpha)}{2\alpha}$ | $\lambda^2$ | $\lambda^2$ |
| $d$                | $\frac{(3\lambda^2 + (3/2)\theta \lambda + \theta \alpha - \alpha^2)}{(3\lambda - 2\alpha)}$ | $\frac{(3\lambda^2 - \alpha^2)}{(3\lambda - 2\alpha)}$ | $\frac{3\lambda^2 Z - 2\alpha Z + 2\alpha^2}{(3\lambda - 2\alpha)}$ |
| $d^*$              | $\frac{(3\lambda^2 Z - 2\alpha Z + 2\alpha^2)}{(3\lambda - 2\alpha)}$ | $\frac{(3\lambda^2 - \alpha^2)}{(3\lambda - 2\alpha)}$ | $\frac{3\lambda^2 Z - 2\alpha Z + 2\alpha^2}{(3\lambda - 2\alpha)}$ |
| $\alpha$           | $\alpha$ | $\frac{1 - \sqrt{1 - \frac{\lambda^3}{\tau}}}{\tau}$ | $\frac{1 - \sqrt{1 - \frac{\lambda^3}{\tau}}}{\tau}$ |

$G_M(s)Q(s)\big|_{s=-1/\tau} = 0$, simplification yields

$$\alpha = \tau \left[ 1 - \sqrt{1 - \frac{\lambda^3}{\tau}} e^{-\theta/\tau} \right], \quad (19)$$

$\alpha$ is a function of $\lambda$, sufficing only one user-defined tuning parameter $\lambda$, to achieve controller parameters. Assumptions $\alpha > 0$, $\tau > \lambda$ are made to prevent $\alpha$ introducing undesired zeros in the right-half plane. The optimum IMC filter form obtained for FOPTD extends to FOP with no delay and FOP with right-half plane zeros (FOPRHZ).

**First-order process with no delay**

The most frequently used approximate models of chemical processes are the FOP model given below

$$G_M(s) = \frac{K}{\tau s + 1}. \quad (20)$$

Factorizing invertible and non-invertible portions of the plant model $G_M(s)$

$$G_{M-}(s) = \frac{K}{\tau s + 1}, \quad G_{M+}(s) = 1. \quad (21)$$

Extending the optimum IMC filter of Equation (9), obtained for FOPTD system to the FOP system, the IMC controller $Q(s)$ is obtained as

$$Q(s) = \frac{(1 + \tau s)(\alpha s + 1)^2}{K(\lambda s + 1)^3}. \quad (22)$$

The ideal feedback controller, equivalent of IMC controller is

$$G_C(s) = \frac{\frac{(\tau \alpha s^2 + (\tau + \alpha)s + 1)}{(\lambda^3/(3\lambda - 2\alpha)s + 1)}}{\frac{\frac{(\lambda^3/(3\lambda - 2\alpha))^2 + (3\lambda^2 - \alpha^2)}{(3\lambda - 2\alpha)s + 1}}{Ks(3\lambda - 2\alpha)}}. \quad (23)$$

The resulting PID tuning rules and lead–lag filter coefficients are given in Table 1.

**First-order process with right half plane zeros**

The structure of the FOPRHZ model is

$$G_M(s) = \frac{K(-s + Z)}{(\tau s + 1)}. \quad (24)$$

Since $(-s + Z)$ fall in the non-invertible part, $G_M(s)$ in Equation (24), is multiplied and divided with $(s + Z)$ to have the significance of the zero preserved.

$$G_M(s) = \frac{K(-s + Z)(s + Z)}{(\tau s + 1)(s + Z)}. \quad (25)$$

Separating invertible and non-invertible portions of $G_M(s)$, with all pass factorization

$$G_{M-}(s) = \frac{K(s + Z)}{(\tau s + 1)}, \quad G_{M+}(s) = \frac{(-s + Z)}{(s + Z)}. \quad (26)$$

Extending the optimum IMC filter of Equation (9), obtained for FOPTD system to the FOPRHZ system, the
IMC controller $Q(s)$ is obtained as

$$Q(s) = \frac{(1 + \tau s)(\alpha s + 1)^2}{K(s + Z)(\lambda s + 1)^3}. \quad (27)$$

The ideal feedback controller, equivalent of IMC controller is

$$G_C(s) = \begin{cases} 
\frac{(\tau \alpha^2 + (\tau + \alpha)s + 1)}{K(3\lambda Z - 2\alpha Z + 2)} \left( \frac{\alpha s + 1}{(\lambda^3/(3\lambda Z - 2\alpha Z + 2))s^2 + ((3\lambda + 3\lambda^2 Z + 2\alpha - Z\alpha^2)/(3\lambda Z - 2\alpha Z + 2))s + 1} \right) 
\end{cases}. \quad (28)$$

Table 1 depicts the PID tuning rules and lead–lag filter coefficients obtained by comparison of Equation (4).

**Performance assessment**

It is known that a well-designed control system should meet the requirements of disturbance attenuation, set-point tracking and, robust stability and/or robust performance, besides nominal stability. The first two requirements traditionally referred to as ‘Performance’ and the third ‘Robustness’ of a control system (Garcia & Morari, 1982; Rivera et al., 1986; Shamsuzzoha & Lee, 2008d; Tan, Marquez, & Chen, 2004).

**Performance**

The optimum settings of PID minimize the integral error criterion. The integral error is a good measure, to evaluate the set-point and disturbance responses (Jin, Liu, Wang, Tian, & Wang, 2013; Stephanopoulos, 1984). The popular integral error criteria, for evaluating the performance of the controller, for set-point tracking or disturbance rejection are

$$IAE = \int_0^\infty |e(t)| \, dt, \quad (29)$$

$$ISE = \int_0^\infty e(t)^2 \, dt, \quad (30)$$

$$ITAE = \int_0^\infty t|e(t)| \, dt. \quad (31)$$

IAE penalizes small errors; ISE criterion penalizes large errors, and ITAE criterion penalizes the errors that persist for a long period. ISE criterion provides most aggressive

![Figure 3. Response of nominal model for unit step disturbance of FOPTD.](image)

**Table 2. Performance of IMC-PID controller for FOPTD.**

| Tuning method | $\lambda$ | $K_P$ | $T_i$ | $T_d$ | $M_S$ | Peak | IAE | ISE | ITAE |
|---------------|----------|-------|-------|-------|-------|------|-----|-----|------|
| Proposed      | 3.155    | 8.081 | 100.5 | 0.498 | 1.59  | 1.607| 10.93| 12.74| 64.25|
| Horn et al    | 2.62     | 8.035 | 100.5 | 0.498 | 1.59  | 1.751| 12.46| 14.62| 80.54|
| Rivera et al  | 0.874    | 0.536 | 100.5 | 0.498 | 1.59  | 1.779| 38.25| 62.6 | 502  |

Notes: Lead–lag filter in series with PID controller: Proposed: $(26.73s^2 + 10.34s + 1)/(126.2s^3 + 480s^2 + 88.58s + 1); a = 88.58, b = 480, \hat{b} = 126.2, c = 10.34, d = 26.73, \hat{d} = 0; Horn et al.: (6.115s + 1)/(27.44s^2 + 100s + 1); Rivera et al.: 1/(0.2332s + 1).
Table 3. Robustness analysis of IMC-PID controller for FOPTD.

| Tuning method  | $\lambda$ | Peak | IAE  | ISE  | ITAE |
|----------------|-----------|------|------|------|------|
| Proposed       | 3.155     | 1.497| 10.95| 11.88| 64.98|
| Horn et al.    | 2.62      | 1.573| 12.49| 13.61| 81.48|
| Rivera et al.  | 0.874     | 1.75 | 38.05| 61.31| 498.5|

settings, while the IAE criterion tends to produce controller settings that are between those for ITAE and ISE criteria (O’Dwyer, 2006; Rivera et al., 1986; Tan et al., 2004).

Robustness analysis

Robustness is the ability of the closed-loop system to be insensitive to component variations. It is one of the most useful properties of feedback (Jin et al., 2013).

Robustness is also what makes it possible to design feedback system based on strongly simplified models. It is necessary to have quantitative ways to express how well a feedback system performs. Measures of performance and robustness are closely related. In closed-loop system, the robust performance is computed by the sensitivity function ($S$) which relates to disturbance rejection performance, while the complementary sensitivity function ($T$) provides a measure of set-point tracking performance (Arputha Vijaya Selvi, Radhakrishnan, & Sundaram, 2007; Jin et al., 2013; Saxena & Hote, 2013)

\[
S \triangleq \frac{1}{1 + G_C G_P}, \quad (32)
\]

\[
T \triangleq \frac{G_C G_P}{1 + G_C G_P}. \quad (33)
\]
Table 4. Performance of IMC-PID controller for FOP.

| Tuning method     | $\lambda$ | $K_P$  | $T_i$  | $T_d$  | $M_S$ | Peak   | IAE    | ISE    | ITAE   |
|-------------------|-----------|--------|--------|--------|-------|--------|--------|--------|--------|
| Proposed          | 0.9442    | 2.313  | 1.987  | 0.497  | 1.155 | 0.332  | 0.8596 | 0.191  | 2.507  |
| Horn et al.       | 1.999     | 0.251  | 1.002  | 0.002  | 1.155 | 0.503  | 3.992  | 1.997  | 19.9   |
| Rivera et al.     | 0.7       | 0.714  | 1      | 0      | 1.155 | 0.735  | 1.4    | 0.487  | 4.27   |

Notes: Lead–lag filter in series with PID controller: Proposed: \( \frac{(0.9868s + 1)}{0.98s^2 + 1.98s + 1} \); \( \hat{a} = 1.98, \hat{b} = 0.98, \hat{c} = 0.9868, \hat{d} = 0 \); Horn et al.: \( \frac{1}{s + 1} \); Rivera et al.: \( \frac{1}{0.35s + 1} \).

Simulation results

The simulation was conducted to demonstrate effectiveness of the proposed IMC filter, resulting in a PID controller cascaded with lead–lag filter for disturbance rejection, on various processes. The process models used by other researchers were considered for simulation. The closed-loop performance was evaluated using integral criteria IAE, ISE, ITAE and the robustness was evaluated with the maximum sensitivity $M_S$ for each process for unit step load disturbance input. The IMC-PID tuning parameters were calculated to have same the robustness in terms of maximum sensitivity $M_S$ to ensure uniform comparability by varying $\lambda$. The performances of the IMC-PID tuning rules obtained with proposed IMC filter were compared for compiseness with that of IMC filter suggested by Rivera et al. (1986), Horn et al. (1996), Liu and Gao (2010), and Lee, Masum Jujuly, and Shamsuzzoha (2009).

First-order process with time delay

The lag time dominant FOPTD model $G(s) = 100e^{-1s}/(100s + 1)$ (Chen & Seborg, 2002), with $\theta/\tau = 0.01$ was used for study. Unit step change in load disturbance input was applied at $t = 0$. The IMC-PID tuning techniques with different IMC filter structures considered for comparison were designed to have the same robustness of $M_S = 1.59$, with adjustment of single tuning parameter $\lambda$.

Table 5. Robustness analysis of IMC-PID controller for FOP.

| Tuning method     | $\lambda$ | Peak   | IAE    | ISE    | ITAE   |
|-------------------|-----------|--------|--------|--------|--------|
| Proposed          | 0.9442    | 0.349  | 0.8625 | 0.2062 | 2.416  |
| Horn et al.       | 1.999     | 0.544  | 4.064  | 2.396  | 18.38  |
| Rivera et al.     | 0.7       | 0.826  | 1.417  | 0.552  | 4.095  |

$|S(j\omega)|$ and $|T(j\omega)|$ are the amplitude ratios of $S$ and $T$, respectively. The maximum values of amplitude ratios provide a useful measure of robustness and serve as design criteria for the control system (Arputha Vijaya Selvi et al., 2007). The maximum sensitivity $M_S \triangleq \max_{\omega} |S(j\omega)|$ is the inverse of the shortest distance from Nyquist plot to the critical point (Arputha Vijaya Selvi et al., 2007; Seborg et al., 2004; Yesil, Zuzelkaya, Eksin, & Tekin, 2008). The robustness of the closed-loop systems increases, as $M_S$ decreases (Liu & Gao, 2012; Rivera et al., 1986). The second robustness measure is $M_T \triangleq \max_{\omega} |T(j\omega)|$, referred as resonant peak. For a satisfactory control system $M_S$ should be in the range of 1.2–2.0 and $M_T$ should be in the range of 1.0–1.5 (Arputha Vijaya Selvi et al., 2007; Seborg et al., 2004; Singhal & Salsbury, 2005).
Figure 7. Response of nominal model for unit step disturbance of FOPRHZ.

Table 6. Performance of IMC-PID controller for FOPRHZ.

| Tuning method | $\lambda$ | $K_P$ | $T_I$ | $T_d$ | $M_S$ | Peak | IAE | ISE | ITAE |
|---------------|-----------|-------|-------|-------|-------|------|-----|-----|------|
| Proposed      | 0.69      | 3.5585| 7.4049| 1.1384| 1.95  | 0.6367| 2.302| 0.779| 9.86 |
| Horn et al.   | 9         | 1.2434| 14.5385| 3.5238| 1.95  | 0.639 | 11.87| 5.133| 153.9|
| Lee et al.    | 0.625     | 3.0547| 7.9257| 1.4578| 1.95  | 0.608 | 2.809| 0.896| 14.07|

Notes: Lead–lag filter in series with PID controller: Proposed: $(1.405s + 1)/(0.6315s^3 + 7.803s^2 + 7.283s + 1)$; $a = 7.283, b = 7.803, \hat{b} = 0.6315, c = 1.405, d = 0, \hat{d} = 0$; Horn et al.: $1/(6s + 1)$; Rivera et al.: $1/(6.1004s + 1)$.

Figure 8. Response of perturbed model for unit step disturbance of FOPRHZ.

The simulation results of Figure 3 and Table 2 show that the IMC-PID controller, with improved IMC filter structure provides better disturbance rejection in comparison with the IMC filter proposed by Rivera et al., Horn et al., and Liu and Gao. The robustness evaluation of the controllers was performed for model mismatch by incorporating perturbation of 25% in the three parameters of the FOPTD model simultaneously for the worst-case scenario, which results in the form of $G(s) = 75 e^{-0.75s} / (75s + 1)$. The simulation results of Figure 4 and Table 3 dictate the efficiency and robustness of the IMC-PID controller with improved IMC filter structure.
Table 7. Robustness analysis of IMC-PID controller for FOPRHZ.

| Tuning method | $\lambda$ | Peak | IAE   | ISE   | ITAE |
|---------------|----------|------|-------|-------|------|
| Proposed      | 0.69     | 0.6386 | 2.257 | 0.7292 | 9.768 |
| Horn et al.   | 9        | 0.653 | 11.83 | 5.242 | 148.9 |
| Lee et al.    | 0.625    | 0.6127 | 2.766 | 0.8573 | 13.9 |

**FOP with no delay**

The FOP model $G(s) = 1/(s + 1)$ (Lee et al., 2009) was used for study. The robustness with $M_S = 1.155$ was used to calculate the controller parameters. The response for a unit step load disturbance for nominal model is shown in Figure 5 and Table 4. The robustness evaluation of the controllers was performed for model mismatch by incorporating perturbation of 25% in the three parameters of the FOP model that has the form $G(s) = 1.25/(1.25s + 1)$. The simulation results of Figure 6 and Table 5 dictate the robustness of the controller with improved IMC filter structure.

**First-order process with right half plane zeros**

The FOPRHZ model $G(s) = 4(-s + 2)/(6s + 1)$ (Lee et al., 2009) was used for study. The robustness of $M_S = 1.95$ was used for calculating the controller parameters. The responses for a unit step load disturbance of the nominal model and 25% perturbed model $G(s) = 5(-s + 2.5)/(7.5s + 1)$ are shown in Figure 7, Table 6 and Figure 8, Table 7, respectively. The PID controller with proposed IMC filter demonstrates better disturbance rejection efficiency.

**Conclusions**

A design method for IMC-PID controller cascaded with lead–lag filter, with improved IMC filter structure was suggested for disturbance rejection. The suggested method provided an excellent performance for disturbance rejection for FOPs. The simulation study was conducted on different FOPs, with tuning of the PID controller with different IMC filter structures to have the same robustness in the form of $M_S$ for uniform comparison. The robustness test for model mismatch was conducted by incorporating 25% variation in the process model parameters simultaneously for the worst-case scenario. The suggested IMC filter has proved to provide an excellent disturbance rejection for lag dominant processes compared to other methods. The optimum IMC filter structure obtained for FOPTD was extended to FOP and FOPRHZ systems, which provided better response in comparison with other filter forms, justifying the proposed filter can be used for a class of the first-order systems. It is suggested that for processes with $\theta/\tau = 1$ or $\theta/\tau > 1$ the single tuning parameter should be $0.8r < \lambda < r$. The proposed IMC filter provides an excellent closed-loop performance which was evaluated with integral performance criteria IAE, ISE, and ITAE. The suggested method provides satisfactory responses for both nominal and perturbed models. A clear compromise was achieved with only one tuning parameter $\lambda$ amid closed-loop performance and robustness to model inaccuracies.

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