Abstract

We explore the possibility of achieving one–step unification of the standard model coupling constants within non supersymmetric and supersymmetric gauge models, which at low energies have only the standard particle content. The constraints are the experimental values of $\alpha_{em}$, $\alpha_s$ and $\sin^2\theta_W$ at $10^2 GeV$, and the lower bounds for FCNC and proton decay rates. The analysis is done in a model independent way.

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It has been known for more than a decade that if we let the three gauge couplings $\alpha_i$ run through the “desert” from low to high energies, they do not merge together into a single point, as it is shown in Fig.1 ($\alpha_i = g_i^2/4\pi, i = 1, 2, 3$ are the gauge couplings for $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ respectively, the subgroups of the standard model (SM) gauge group $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$).

This result, which emerges from the accuracy measurements of the several parameters in the SM done at the LEP machine at CERN, claims either for new physics at intermediate energy scales, or for new approaches to the unification problem. Conspicuously among the solutions calling for new physics are those which introduce the minimal supersymmetric
(SUSY) partners of the SM fields at an energy scale of 1 TeV [3].

The unification of the SM gauge couplings $\alpha_i, \ i = 1, 2, 3$ is properly achieved if the three values meet together into a common value $\alpha = g^2/4\pi$ at a certain energy scale $M >> 10^2$ GeV, where $g$ is the gauge coupling constant of the unifying group. Since $G \supset G_{SM}$, the normalization of the generators corresponding to the subgroups $U(1)_Y, \ SU(2)_L$ and $SU(3)_c$ is in general different for each particular group $G$, and therefore the SM coupling constants $\alpha_i$ differ at the unification scale from $\alpha$ by numerical factors $c_i (\alpha_i = c_i\alpha)$ which are pure rational numbers satisfying $0 < c_i \leq 1$ [4]. As a matter of fact, in Fig.1 the coupling constants have been normalized to the values $\{c_1, c_2, c_3\} = \{\frac{3}{5}, 1, 1\}$, which are the normalization constants corresponding to the most popular grand unified theories (GUT) like SU(5), SO(10), E$_6$, etc. [5].

But, can we normalize $\alpha_1$ to a different $c_1$ value in such a way that $\alpha_1^{-1}(\mu)$ merge into the intersection of $\alpha_2^{-1}(\mu)$ and $\alpha_3^{-1}(\mu)$ in the $\alpha_i^{-1}$-$\mu$ plane? The answer is yes. A value $c_1 = 39/50$ will produce one-step unification of the three gauge couplings (see the dotted line in Fig. 1), at an energy scale of the order of $10^{17}$ GeV, for the standard contain of particles. Unfortunately, the values $\{c_1, c_2, c_3\} = \{\frac{39}{50}, 1, 1\}$ do not correspond to any GUT known so far.

Our approach is to analyze in a model independent way the solutions to the renormalization group equations using $\{c_1, c_2, c_3\}$ as free parameters, in order to look for non SUSY and SUSY GUT models able to achieve one-step unification of the SM gauge coupling constants, consistent with the low energy phenomenology. As we already know, the value of the SM coupling constants at the $m_Z$ scale and the bounds on the proton life time, rule out models like minimal SU(5), and other models that contain minimal SU(5) as an intermediate stage in their symmetry braking chain (sbc). To simplify matters we use for $c_3$ only the values 1 and $\frac{1}{2}$. ($c_3 = 1$ for models which contain $SU(3)_c$ embedded into a simple subgroup of $G$, and $c_3 = \frac{1}{2}$ for models which contain $SU(3)_c$ embedded into the chiral color extension $SU(3)_{cL} \otimes SU(3)_{cR} \subset G$ [6]).

In a field theory, the coupling constants are defined as effective values including loop
corrections of the particle propagators according to the renormalization group equations. They are therefore energy scale dependent. In the modified minimal substration scheme ($\mathcal{MS}$) [9], which we adopt in what follows, the one–loop renormalization group equations (rge) are

$$ \mu \frac{d\alpha_i}{d\mu} \simeq -b_i\alpha_i^2, \quad (1) $$

where $\mu$ is the energy at which the coupling constants $\alpha_i$ are evaluated. The constants $b_i$ are completely deterninated by the particle content in the model by

$$ 4\pi b_i = \frac{11}{3}C_i(vectors) - \frac{2}{3}C_i(fermions) - \frac{1}{3}C_i(scalars), \quad (2) $$

being $C_i(\cdots)$ the index of the representation to which the $\cdots$ particles are assigned, and where we are considering Weyl fermion and complex scalar fields [8]. The boundary conditions at the scale $m_Z \simeq 10^2 GeV$ for these equations are determined by the relationships

$$ \alpha_{em}^{-1} = \alpha_1^{-1} + \alpha_2^{-1}, \quad \text{and} \quad \tan^2 \theta_W = \frac{\alpha_1}{\alpha_2}, \quad (3) $$

where $\alpha_{em} = e^2/4\pi$ ($e$ the electric charge), and by the experimental values

$$ \alpha_{em}^{-1}(m_Z) = 127.90 \pm 0.09 \ [2,9,10], $$

$$ \sin^2 \theta_W(m_Z) = 0.2312 \pm 0.00017 \ [4,9] \quad \text{and} \quad (4) $$

$$ \alpha_3(m_Z) = \alpha_s = 0.1191 \pm 0.0018 \ [9]. $$

which are the updated world average of all current data.

From eq.(3), which are valid at all energy scales, it follows that at the unification scale $M$, the value $\sin^2 \theta_W$ is

$$ \sin^2 \theta_W(M) = \frac{\alpha_{em}(M)}{\alpha_2(M)} = \frac{c_1}{c_1 + c_2}. \quad (5) $$

For the non SUSY case, under the assumptions that only the three standard families of particles are light, and using the decoupling theorem [11], the solution to the rge can be written as
$$\alpha^{-1}(m_Z) = \frac{1}{c_i} \alpha^{-1} - b_i(F, H) \ln \left( \frac{M}{m_Z} \right), \quad (6)$$

where

$$2\pi \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{22}{3} \\ 11 \end{pmatrix} - \begin{pmatrix} \frac{20}{9} \\ \frac{4}{3} \\ \frac{4}{9} \end{pmatrix} F - \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix} H. \quad (7)$$

$F = 3$ is the number of light families and $H = 1$ is the number of low energy complex Higgs doublets. (Notice that we are not including in Eq.(7) the normalization factor $\frac{3}{7}$ into $b_1$ coming from the $SU(5)$ theory, and wrongly included in some general discussions.) Once the set $\{c_1, c_2, c_3\}$ is provided for a particular group $G$, the former equations constitute a system of three equations with two unknowns: $\alpha$ and $M$ ($\alpha_i(m_Z), i = 1, 2, 3$ are obtained from the values presented in (4)). So, a consistent check of the GUT hypothesis is in principle possible.

Our approach now is the following [12]: we consider the system of three equations (6) with the three unknowns $\alpha, M$ and $H$, each one of the unknowns a function of the parameters $\{c_1, c_2, c_3\}$; we solve for the three unknowns as functions of $c_i, i = 1, 2, 3$ and draw curves for physical values of $M$ in a 3 dimensional cartesian space, where the coordinate axis are provided by the set $\{c_1, c_2, c_3\}$, (actually, since $c_3 = 1, \frac{1}{2}$ we considered only two dimensional spaces with axis $\{c_1, c_2\}$, projected into the planes $c_3 = 1$ and $c_3 = \frac{1}{2}$).

The physical values of $M$ are provided by experimental and theoretical bounds in the following way: first, the unification scale $M$ must be lower than the Plank scale $M_P \sim G_N^{-1/2} \sim 10^{19} GeV$; second, it must be greater than $10^5 GeV$ in order to cope with experimental bounds on FCNC [4]. Finally, since some models predict proton decay, and the experimental bound for the proton life time $\tau_p$ is $\tau_{p \rightarrow e\pi} \sim M^4 > 10^{33}$ Yrs [13], then $M$ must be greater than $10^{16}$ GeV if the proton is unstable in the model under consideration. Hence, in the analysis we have to consider two different zones in the $c_1 - c_2$ plane, given by $10^{16} GeV < M < M_P$ and $10^5 GeV \leq M \leq 10^{16} GeV$, which admit and does not admit proton
decay respectively. Also, since \( b_3 > 0 \) and \( b_1 < 0 \) always, \( \alpha_1(m_Z) < \alpha < \alpha_s(m_Z)/c_3 \) then \( \alpha, \ln(M/m_Z) \) and \( H \) should be finite, and there is an upper bound \( H_{\text{max}} \) which represents the maximum number of low energy Higgs doublets allowed. Therefore, \( 0 \leq H \leq H_{\text{max}} \).

These bounds limit the region in the \( \{c_1, c_2\} \) plane where the coupling constant one-step unification is possible and consistent with the experimental data and theoretical constraints.

The solutions to Eqs. (6) for \( \alpha, H \) and \( M \) as functions of \( c_i \) are:

\[
\alpha^{-1} = c_1 c_2 c_3 \cdot \frac{(\alpha_1^{-1} - \alpha_2^{-1})(99 - 12F) + \alpha_3^{-1}(8F + 66)}{c_1 c_2 (8F + 66) + c_1 (c_1 - c_2)(12F - 99)},
\]

\[
H = \frac{2}{3} \cdot c_2 (\alpha_1^{-1} c_1 - \alpha_3^{-1} c_3)(66 - 12F) + c_3 (\alpha_1^{-1} c_1 - \alpha_2^{-1} c_2)(12F - 99) + 20c_1 (\alpha_2^{-1} c_2 - \alpha_3^{-1} c_3),
\]

\[
\ln \left( \frac{M}{m_Z} \right) = 18\pi \cdot \frac{c_1 c_2 (\alpha_1^{-1} - \alpha_2^{-1}) + \alpha_3^{-1} c_3 (c_1 - c_2)}{c_1 c_2 (8F + 66) + c_1 (c_1 - c_2)(12F - 99)}.
\]

From these expressions, the limited region obtained for values of \( c_1 \) and \( c_2 \) that give unification is plotted in figure 2 for \( c_3 = 1 \) and in figure 3 for \( c_3 = \frac{1}{2} \), where we used \( F = 3 \) for three light families, and central values for \( \alpha_s(m_Z), \alpha_{\text{em}}(m_Z) \) and \( \sin^2 \theta_W(m_Z) \). Let us analyze those graphs:

**Analysis of Fig. 2:** It corresponds to GUT groups with vector-like color symmetries. The allowed region of parameters \( \{c_1, c_2\} \) lies inside the lines \( M = 10^5 \text{ GeV}, \; H = 0, \; \text{and} \; c_2 = 1 \).

There is a maximum unification mass scale given by \( M \leq 10^{17.5} \text{ GeV} < M_P \) and the number of Higgs field doublets allowed is such that \( 0 < H \leq 91 \) in general, but if the proton does decay in the context of the GUT model then \( 0 < H \leq 2 \). The implications are:

1. For \( SU(5) \) \[14\], \( SO(10) \) \[15\], \( E_6 \) \[16\], and \( SO(18) \) \[17\], \( \{c_1, c_2\} = \{\frac{3}{5}, 1\} \) and proton decay is always present. The point lies inside the allowed zone, but in a region where \( M \simeq 10^{13}\text{GeV} \) in conflict with the bounds for proton decay. Since \( SU(5) \) allows only the one step sbc \( SU(5) \xrightarrow{M} SM \), \( SU(5) \) is ruled out in general (not only minimal \( SU(5) \) but also all the possible extensions which include arbitrary representations of Higgs field multiplets). The one-step sbc for \( SO(10), E_6 \) and \( SO(18) \) are also ruled out by the same reason.
2: For $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ \cite{18}, and $[SU(3)]^3 \times Z_3$ \cite{19}, $\{c_1, c_2\} = \{\frac{2}{3}, 1\}$ again. In those models the proton can not decay via leptoquark gauge bosons (see the first paper in \cite{18} and the last paper in \cite{19}), but it can decay via Higgs field scalars. So, the one stage breaking of those models is not ruled out as long as one can break the symmetry using scalars which do not break spontaneously the baryon quantum number. The GUT scale for those models is $M \simeq 10^{13}$ GeV and the number of Higgs field doublets is $H = 7 \pm 1$.

3: For $[SU(6)]^3 \times Z_3$ \cite{20}, $\{c_1, c_2\} = \{\frac{3}{14}, \frac{1}{3}\}$ which lies outside the allowed zone. The one stage sbc is ruled out for this model (the two stage sbc is presented in some papers of Ref. \cite{20}).

4: For $E_7$ \cite{21}, $\{c_1, c_2\} = \{\frac{3}{2}, \frac{1}{2}\}$ \cite{4}, values way out the allowed region. The one stage sbc is also ruled out for this model.

**Analysis of Fig. 3:** It corresponds to GUT groups with chiral color symmetries. The allowed region of parameters $\{c_1, c_2\}$ lies inside the lines $M = 10^5$ GeV, $H = 0$, $c_2 = 1$, and $M = M_P = 10^{19}$ GeV. There is not maximum bound for a unification mass scale, and the allowed number of Higgs field doublets is $0 < H \leq 136$ in general, but if the proton does decay in the context of the GUT model then $0 < H \leq 28$. The implications for some specific models are:

1: For $SU(5) \otimes SU(5)$ \cite{22}, $\{c_1, c_2\} = \{\frac{3}{13}, 1\}$ which lies inside the allowed zone but in a region where $M < < 10^{16}$ GeV, in serious conflict with bounds for proton decay. The one step sbc for this model is ruled out.

2: For $[SU(6)]^4 \times Z_4$ \cite{23}, $\{c_1, c_2\} = \{\frac{5}{19}, \frac{1}{3}\}$ which lies inside the allowed zone (the proton is stable in the context of this model). So, the one stage sbc for this model is allowed (it is presented in Ref. \cite{23}), the unification scale is $M \sim 10^7$ GeV, and the number of low energy Higgs field doublets is $H = 2 \pm 1$.

**NOTE** Comparing Fig. 2 with Fig. 3 we conclude that one-step family unification is more likely achieved if $SU(3)_c$ is embedded into the chiral color group $SU(3)_{cL} \otimes SU(3)_{cR}$.

If SUSY plays a role in our low energy world, its most likely mass scale is consider to be $M_S \sim 1$ TeV \cite{24} (the inclusion of the low energy SUSY threshold correction and others,
could be important, in such a way that the effective mass scale $M_S$ should be taken lower than 1 TeV, may be as low as $m_Z$ \[^{24}\]). In what follows we assume that below $M_S$ we have only the SM physics and above $M_S$ the minimal supersymmetric standard model (MSSM) manifest itself, up to the unification scale $M$. In this context, the solution to the rge with the mass hierarchy $m_Z < M_S < M$ is given by

$$
\alpha_i^{-1}(m_Z) = \alpha_i^{-1}(M) - b_i(F,H)\ln\left(\frac{M_S}{m_Z}\right) - b_i^{SS}(F,H)\ln\left(\frac{M}{M_S}\right)
$$

(11)

where $b_i^{SS}$ are the contributions to the beta function of the MSSM, and $b_i$ given by Eq. (7), the contributions of the SM. Again $b_i$ and $b_i^{SS}$ depend on the number of low energy families $F$ and Higgs field doublets $H$, decoupling in each case the extra massive particles according to the decoupling theorem \[^{11}\]. The analysis now produces

$$
2\pi \begin{pmatrix}
 b_1^{SS} \\
 b_2^{SS} \\
 b_3^{SS}
\end{pmatrix} = \begin{pmatrix}
 0 \\
 6 \\
 9
\end{pmatrix} - \begin{pmatrix}
 \frac{10}{3} \\
 2 \\
 2
\end{pmatrix} F - \begin{pmatrix}
 \frac{1}{2} \\
 \frac{1}{2} \\
 0
\end{pmatrix} H,
$$

(12)

where again the $\frac{3}{5}$ normalization factor coming from SUSY $SU(5)$ has not been included in $b_1^{SS}$.

Repeating the analysis done for the non SUSY case we get the results plotted in Figs. 4 and 5. From the graphs we see that the existence of SUSY partners not only changes the shape of the allowed regions but has deeper consequences, as can be seen from Fig. 4 where the point $\{c_1, c_2\} = \{\frac{3}{5}, 1\}$ associated with $SU(5)$ and related models now fits into the allowed region, a well known result from a related analysis \[^{3}\]. For the several models we have studied, our conclusions for the SUSY case (from Figs. 4 and 5) are:

1- SUSY models with one step sbc allowed: $SU(5)$, $SO(10)$, $E(6)$, $SU(4)\otimes SU(2)_L\otimes SU(2)_R$, $[SU(3)]^3 \times Z_3$, and $[SU(6)]^4 \times Z_4$.

2- SUSY models with one step sbc ruled out: $[SU(6)]^3 \times Z_3$, $E_7$, and $SU(5)\otimes SU(5)$.

Our conclusion is that it is always possible for a certain class of GUT models to achieve a one–step unification, both in the non SUSY and in the SUSY cases. For some SUSY
models this result was known before [3], but for the non SUSY cases the result is new and not trivial.

Our analysis has been done including only the one-loop beta function. If one uses a two-loop beta function, then one-loop threshold corrections (which are model dependent) must be included, and the experimental errors of the SM group coupling constants (specially for $\alpha_s$) must be taken into account since all of them are of the same order of magnitude (typically of the order of 2 to 8%). The inclusion of those contributions to the gauge coupling constants do not change our conclusions. (In Ref. [25] such analysis is presented for the SM, the MSSM and GUT SU(5)).

Finally notice that the gauge coupling constants $g_i$ of the three SM interactions are related at the GUT scale by the relationship $c^{-1}_1 g_1^2 = c^{-1}_2 g_2^2 = c^{-1}_3 g_3^2$, a result which resemble the string gauge coupling unification. Indeed, defining $c^{-1}_i = \kappa_i$, the affine level (or Kac-Moody level) at which the group factor $G_i$ is realized in the effective four dimensional string, we get the string coupling relation $\kappa_1 g_1^2 = \kappa_2 g_2^2 = \kappa_3 g_3^2$. This analogy, together with the results presented here, should provide further insight into the String-GUT problem [27], for the SUSY and the non SUSY string unification. As a matter of fact, Fig. 1 above suggests a string unification without supersymmetry for the Kac-Moody levels $\kappa_1 = \kappa_Y = 1.28$ and $\kappa_2 = \kappa_3 = 1$ (see also Refs. [28]).

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FIG. 1. One-loop evolution of the gauge couplings with the (non-susy) Standard Model. Here \( \alpha_1 \equiv (5/3)\alpha_Y \) for the solid line and \( \alpha'_1 \equiv (50/39)\alpha_Y \) for the dotted line, where \( \alpha_Y \) is the hypercharge coupling.

FIG. 2. Plots for some values of H and M for the non chiral color models. The bounds in \( c_1 \) and \( c_2 \), impose at once for \( \alpha \) the bounds 16.0921 < \( \alpha^{-1} \) < 48.0186.
FIG. 3. Plots for some values of $H$ and $M$ for GUT containing the chiral color extension. In this case we have $8.0461 < \alpha^{-1} < 26.003$.

FIG. 4. Comparing between the fits for $H$ and $M$ in SUSY and non SUSY unification, for the non chiral color models. The dashed lines stand for the SUSY case. Now, $13.16 \leq \alpha^{-1} \leq 28.552$. 
FIG. 5. Comparing between the bounded surface for SUSY and non SUSY simple unification, containing the chiral color extension. Again, the dashed lines correspond to the SUSY fits. In this case $6.58 \leq \alpha^{-1} \leq 14.276$. 