CHIRAL EFFECTIVE THEORY FOR HEAVY MESONS

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ABSTRACT

I review recent developments in the description of the interactions between light and heavy mesons by an effective chiral lagrangian having the symmetries of the Heavy Quark Effective Theory. In particular the problem of the determination of the strong coupling constants \( g_{P^*P^*\pi} \) and \( g_{P^*P^*\rho} \) (\( P, P^* \) = heavy mesons) is addressed.

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1 Introduction

A recent and interesting development of the Heavy Quark Effective Theory (HQET) \[32\] is represented by the study of the chiral effective lagrangian containing heavy mesons \[2\], \[3\], \[4\], \[5\]. The idea is to generalize chiral effective theory by terms containing heavy mesons, described by effective field operators, as well as their interactions with the light mesons.

A consistent way to implement this program should not only define the heavy meson field operators, but also respect all the symmetries of HQET, i.e.,

a) spin symmetry; a well known consequence of it is the fact that heavy mesons can be organized in spin multiplets, e.g. the $0^-$ and the $1^-$ mesons;

b) heavy flavour symmetry; this symmetry holds provided $m_Q >> \Lambda_{QCD}$ and applies to quantities that remain finite in the limit $m_Q \rightarrow \infty$;

c) velocity superselection rule, which implies that the effective lagrangian describing strong interactions should be written as a sum of terms that are diagonal in the velocity dependent heavy meson field operators.

Moreover the lagrangian should respect chiral symmetry not only in the terms containing light fields, but also in the part containing the heavy meson operators.

An important step of this program is the determination of the coupling constants of the lagrangian. To this issue I shall devote most of this talk. This determination can render chiral effective theory for heavy mesons a useful tool to deal with different aspects of the interactions between light and heavy mesons as well as with their weak and electromagnetic interactions.

2 The chiral effective lagrangian for light and heavy mesons

To begin with, we describe the effective field operators appearing in the chiral lagrangian. Negative parity heavy $Q\bar{q}_a$ mesons are represented by field operators in the form of a $4 \times 4$ Dirac matrix

\begin{align*}
H_a &= \frac{(1 + \not{v})}{2}[P^*_a \gamma^\mu - P_a \gamma^5] \\
\bar{H}_a &= \gamma_0 H_a^\dagger \gamma_0
\end{align*}

Here $v$ is the heavy meson velocity, $a = 1, 2, 3$ (for $u, d$ and $s$ respectively), $P^*_a$ and $P_a$ are annihilation operators normalized as follows

\begin{align*}
\langle 0 | P_a | Q\bar{q}_a(0^-) \rangle &= \sqrt{M_H} \\
\langle 0 | P^*_a | Q\bar{q}_a(1^-) \rangle &= \epsilon^\mu \sqrt{M_H}
\end{align*}

with $\nu^\mu P^*_a = 0$ and $M_H = M_P = M_{P^*}$, is the heavy meson mass. The pseudoscalar light mesons are described, as usual in chiral effective theory, by

$$\xi = \exp \frac{i M}{f_\pi}$$
where

\[ M = \begin{pmatrix}
                                                                 \sqrt{\frac{\pi^0}{2} + \sqrt{\frac{\pi^0}{6}}} & \pi^+ & K^+ \\
                                                                 \pi^- & -\sqrt{\frac{\pi^0}{2} + \sqrt{\frac{\pi^0}{6}}} & K^0 \\
                                                                 K^- & K^0 & -\sqrt{\frac{\pi}{3}}
\end{pmatrix} \tag{2.6}
\]

where \( f_\pi \simeq 132 \text{MeV} \) is the pion leptonic decay constant in the chiral limit. Under the chiral symmetry the fields transform as follows

\[ \xi \rightarrow g_L \xi U^\dagger = U \xi g_R^\dagger \tag{2.7} \]
\[ \Sigma \rightarrow g_L \Sigma g_R^\dagger \tag{2.8} \]
\[ H \rightarrow H U^\dagger \tag{2.9} \]
\[ \bar{H} \rightarrow U \bar{H} \tag{2.10} \]

where \( \Sigma = \xi^2, g_L, g_R \) are global \( SU(3) \) transformations and \( U \) is a function of \( x \), of the fields and of \( g_L, g_R \).

The lagrangian describing the fields \( H \) and \( \xi \) and their interactions, under the hypothesis of chiral and spin-flavour symmetry and at the lowest order in light mesons derivatives is \[2,3,4,5\] :

\[ \mathcal{L}_0 = \frac{f_\pi^2}{8} \left< \partial^\mu \Sigma \partial_\mu \Sigma^\dagger \right> + i < H b \nu^\mu D_{\mu ba} \bar{H}_a > + i g < H b^\gamma_\mu \gamma_5 A_{\mu ba} \bar{H}_a > \tag{2.11} \]

where \( \left< \ldots \right> \) means the trace, and

\[ D_{\mu ba} = \delta_{ba} \partial_\mu + \nu_{\mu ba} = \delta_{ba} \partial_\mu + \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right)_{ba} \tag{2.12} \]
\[ A_{\mu ba} = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right)_{ba} \tag{2.13} \]

Besides chiral symmetry, which is obvious, since, under chiral transformations,

\[ D_\mu H \rightarrow U D_\mu H \]
\[ A_\mu \rightarrow U A_\mu U^\dagger \tag{2.14} \]

the lagrangian (2.11) possesses the heavy quark spin symmetry \( SU(2)_v \), which acts as

\[ H_a \rightarrow \hat{S} H_a \tag{2.15} \]
\[ \bar{H}_a \rightarrow \bar{H}_a \hat{S}^\dagger \tag{2.16} \]

with \( \hat{S} \hat{S}^\dagger = 1 \) and \( [\hat{\xi}, \hat{S}] = 0 \), and a heavy quark flavour symmetry arising from the absence of terms containing \( m_Q \).

Explicit symmetry breaking terms can also be introduced, by adding to \( \mathcal{L}_0 \) an extra piece \( \mathcal{L}_1 \) containing corrections at the lowest order in \( m_q \) and \( 1/m_Q \). I do not write down it here \[2, 3\] but it is worth stressing that it is precisely this term which is responsible for the light pseudoscalar meson masses and for the mass difference \( \delta m_H \) between the particles \( P \) and \( P^* \) contained in the field \( H \), i.e.:

\[ M_P = M_H \]
\[ M_{P^*} = M_H + \delta m_H \tag{2.17} \]
The vector meson resonances belonging to the low-lying $SU(3)$ octet can be introduced by using the hidden gauge symmetry approach [6], [5] (for another approach see [7]). The new lagrangian containing these particles, to be added to $L_0 + L_1$, is as follows:

\[
L_2 = -\frac{f_v^2}{2} \mu <(V_\mu - \rho_\mu)^2> + \frac{1}{2g_v^2} <F_{\mu\nu}(\rho)F^{\mu\nu}(\rho)>
+ i\beta <H_b \nu^{\mu} (V_\mu - \rho_\mu)_{ba} \tilde{H}_a>
+ \frac{\beta^2}{2f_v^2 a} <\tilde{H}_b H_a \tilde{H}_a H_b > + i\lambda <H_b \sigma^{\mu\nu} F_{\mu\nu}(\rho)_{ba} \tilde{H}_a>
\]

(2.18)

where $F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu]$, and $\rho_\mu$ is defined as

\[
\rho_\mu = i\frac{g_v}{\sqrt{2}} \hat{\rho}_\mu
\]

(2.19)

$\hat{\rho}$ is a hermitian $3 \times 3$ matrix analogous to (2.6) containing the light vector mesons $\rho^{0,\pm}$, $K^*$, $\omega$, $g_v$, $\beta$, $\lambda$ and $a$ are coupling constants; by imposing the two KSRF relations [3] one obtains

\[
a = 2 \quad g_v \approx 5.8
\]

(2.20)

The resulting effective lagrangian $L = L_0 + L_1 + L_2$ can be generalized to include low-lying positive parity $Q\bar{q}_a$ heavy meson states. For $p$ waves ($l = 1$), the heavy quark effective theory predicts two distinct multiplets, one containing a $0^+$ and a $1^+$ degenerate states, the other one comprising a $1^+$ and a $2^+$ state [8], [9] [10]. Their inclusion is needed if one wishes to describe heavy mesons semileptonic decays into a final state containing a light vector meson. We refer to the literature for such an analysis [5]. In the remaining part of this paper I will address the problem of the determination of the unknown strong coupling constants $g$ and $\lambda$ (no determination of $\beta$ is available yet).

### 3 Semileptonic decays

I now wish to show that semileptonic decays of the heavy pseudoscalar mesons into final states containing light mesons can be used to determine the strong coupling constants $g$ and $\lambda$. In doing that, however one has to make a further assumption concerning the $q^2$ behaviour of the semileptonic form factors.

At the lowest order in derivatives of the pseudoscalar couplings and in the symmetry limit, weak interactions between light pseudoscalars and a heavy meson are described by the weak current [2]:

\[
L_\mu^a = \frac{i\hat{F}}{2} <\gamma^\mu(1-\gamma_5)H_b \xi_{ba}^\dagger>
\]

(3.1)

where $\hat{F}$ is related to the pseudoscalar heavy meson decay constant $f_P$, defined by

\[
<0|\bar{q}_a \gamma^\mu \gamma_5 Q|P_b(p)> = i\rho^\mu f_P \delta_{ab}
\]

(3.2)

as follows:

\[
\hat{F} = f_P \sqrt{M_H}.
\]

(3.3)
\( \hat{F} \) is finite in the \( m_Q \to \infty \) limit and is independent of the heavy quark mass, but for logarithmic corrections that are however tiny. Its numerical value can be inferred by QCD sum rules analyses \([1]\):

\[
\hat{F} = 0.41 \pm 0.04 \text{ GeV}^{3/2}
\]

This result is obtained including radiative \( O(\alpha_s) \) corrections.

Let us first consider the semileptonic decay of the heavy pseudoscalar meson into a light pseudoscalar meson. To be definite we consider the decay:

\[ B \to \pi \ell \nu_\ell \]

The hadronic matrix element can be written in terms of the form factors \( F_0, F_1 \) as follows

\[
<\pi(p')|V^\mu|B(p)> = [(p + p')^\mu + \frac{M_\pi^2 - M_B^2}{q^2} q^\mu] F_1(q^2) - \frac{M_\pi^2 - M_B^2}{q^2} q^\mu F_0(q^2)
\]

where \( q^\mu = (p - p')^\mu \), \( F_0(0) = F_1(0) \). The form factors \( F_0 \) and \( F_1 \) take contributions, in a dispersion relation, from the 0+ and 1− meson states respectively.

Using the chiral lagrangian and the current (3.1), one obtains, at the leading order in \( 1/m_Q \) and at \( q^2 = q^2_{\text{max}} \), the following result

\[
F_1(q^2_{\text{max}}) = \frac{g \sqrt{M_B} \hat{F}}{2 f_\pi (v \cdot k - \delta m_H)}
\]

whereas \( F_0(q^2_{\text{max}}) \) is found to satisfy the analogous of the Callan Treiman relation in the chiral limit \([3]\). It is worth stressing that this result arises from a polar diagram with a \( B^* \) exchange; \( k^\mu \) is the residual momentum related to the physical momenta by \( k^\mu = q^\mu - M_{B^*} v^\mu \) (and \( p^\mu = M_B v^\mu \)).

A similar analysis can be performed for the semileptonic decay process with a light vector in the final state. For definiteness we examine

\[ B \to \rho \ell \nu_\ell \]

and we consider only the vector current matrix element

\[
<\rho(\epsilon,p')|V^\mu|B(p)> = \frac{2 V(q^2)}{M_B + M_\rho} \epsilon^{\mu \alpha \beta} \epsilon^*_{\alpha \rho \beta} p'_{\rho}
\]

In a dispersion relation the form factor \( V(q^2) \) takes contribution from the 1− pole, i.e. the \( B^* \) particle.

Using the chiral lagrangian and the current (3.1) one gets, at \( q^2 = q^2_{\text{max}} \) and at leading order in \( 1/m_Q \) the result

\[
V(q^2_{\text{max}}) = -\frac{g V}{\sqrt{2}} \frac{M_B + M_\rho}{\sqrt{M_B (v \cdot k - \delta m_B)}}
\]

These results are obtained in the chiral limit and for \( m_Q \to \infty \); in these limits they apply not only to the decays \( B \to \pi \ell \nu_\ell \) or \( B \to \rho \ell \nu_\ell \), but also, using the symmetries of the effective current and lagrangian, to e.g. \( D \to \pi \ell \nu_\ell \), \( D \to \rho \ell \nu_\ell \), \( D \to K \ell \nu_\ell \), \( D \to K^* \ell \nu_\ell \) etc.
Therefore we could use experimental data on these decays to fix the unknown quantities \( g \) and \( \lambda \). In order to make contact with the experimental data, however, we have to make an ansatz on the \( q^2 \) behaviour of the form factors. The contributions we have written down arise from polar diagrams, which suggests a simple pole behaviour. This is also hinted by the QCD sum rules analysis contained in Ref. \[12\]. Therefore we assume for the form factors \( F_1(q^2) \) and \( V(q^2) \) the generic formula

\[
F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m^2}}
\]  

(3.11)

For the pole masses we use the masses indicated by a pole dominated dispersion relation \[13\].

For the \( D \rightarrow \pi \) semileptonic decay one gets:

\[
F_1(0) = -\frac{g\hat{F}}{2f_\pi}\sqrt{M_D} \frac{M_{D^*} + M_D - M_\pi}{M_{D^*}^2}.
\]  

(3.12)

From the experimental value \(|F_1(0)| = 0.79 \pm 0.20\), one gets:

\[
|g| = 0.40 \pm 0.10
\]  

(3.13)

This result agrees, within the errors, with the result obtained using as an input \( D \rightarrow K \) semileptonic decay \[14\].

Let us now turn to semileptonic decays into vector mesons. The experimental input we can use from \( D \rightarrow K^*\ell\nu_\ell \) is as follows:

\[
V(0) = 0.95 \pm 0.20
\]  

(3.14)

This is an average among the data from the different Fermilab experiments \[15\], \[16\]. More recent data from the E-687 Collaboration \[17\] agree, within the errors, with \[3.14\].

The calculated weak coupling at \( q^2 = 0 \) is:

\[
V(0) = \frac{g_Y \lambda (M_D + M_{K^*})(M_{D^*} + M_D - M_{K^*})}{\sqrt{2}} \frac{\hat{F}}{M_{D^*}} \frac{\sqrt{M_D}}{M_D}
\]  

(3.15)

and one numerically obtains for \( \lambda \):

\[
|\lambda| = 0.40 \pm 0.10 \text{ GeV}^{-1},
\]  

(3.16)

It is interesting to compute, by the values of \( g \) and \( \lambda \) in Eqs. \(3.13\), \(3.16\) and by using previous results for \( F_1(0) \) and \( V(0) \) (adapted to \( B \) case) the values of the form factors for the \( B \rightarrow \pi\ell\nu_\ell \) and \( B \rightarrow \rho\ell\nu_\ell \) decays. One obtains

\[
|F_{1B\rightarrow\pi}(0)| = 0.53 \pm 0.13
\]  

(3.17)

\[
|V_{B\rightarrow\rho}(0)| = 0.61 \pm 0.15
\]  

(3.18)

To conclude, Eqs. \(3.13\), \(3.16\) are the results for the strong coupling constants of the chiral effective lagrangian obtained by the analysis of semileptonic decays. Since they are based on an extra assumption (polar \( q^2 \) behaviour of the form factors \( F_1 \) and \( V \)) and on the neglect of the heavy mass corrections, it is worth looking for different theoretical and/or phenomenological determinations.
4 Radiative decays

In this section we shall show that the results of Eqs. (3.13) and (3.16) are compatible with the experimental data on the radiative decay

\[ D^* \rightarrow D \gamma. \]  

(4.1)

The matrix element for this decay can be written as follows:

\[ \mathcal{M}(D^* \rightarrow D \gamma) = e e^\mu J_{\mu} \]  

(4.2)

with:

\[ J_{\mu} = \langle D(p')|J_{\mu}^m|D^*(p, \eta)\rangle = \langle D(p')|e_Q \bar{Q}\gamma^\mu Q + e_q \bar{q}\gamma^\mu q|D^*(p, \eta)\rangle = e_Q J_{\mu}^Q + e_q J_{\mu}^q, \]  

(4.3)

where \( e_Q = \frac{2}{3} \) is the heavy quark (\( Q = c \)) charge and \( e_q \) is the light quark charge (\( e_q = \frac{2}{3} \) for \( D^{*0} \) and \( e_q = -\frac{1}{3} \) for \( D^{*+} \) and \( D_s^* \)). Let us consider the two currents appearing in (4.3) separately. \( J_{\mu}^Q \) can be expressed in terms of the Isgur-Wise universal form factor \([32]\) as follows:

\[ <D(p')|\bar{c}\gamma^\mu c|D^*(p, \eta)> = i \sqrt{M_D M_{D^*}} \xi(v \cdot v') \epsilon_{\mu\nu\alpha\beta} \eta_\nu v_\alpha v'_\beta, \]  

(4.4)

where \( p' = M_D v' \), \( p = M_{D^*} v \) and \( v \cdot v' \approx 1 \) at \( q^2 = 0 \).

As for vector currents containing light quarks \( J_{\mu}^q \), one can assume Vector Meson Dominance \([18]\) and write:

\[ J_{\mu}^q = \sum_{V, \lambda} \langle D(p')|V(q, \lambda)|D^*(p, \eta)\rangle \frac{i}{q^2 - M_V^2} \times \langle 0|\bar{q}\gamma^\mu q|V(q, \lambda)\rangle, \]  

(4.5)

where \( q^2 = 0 \) and the sum is over the vector meson resonances \( V = \omega, \rho^0, \phi \) and over the \( V \) helicities. The vacuum-to-meson current matrix element appearing in (4.5) is given by:

\[ <0|\bar{q}T^i\gamma^\mu q|V(q, \lambda)> = \epsilon_1^\mu f_V, \]  

(4.6)

From \( \omega \rightarrow e^+e^- \) and \( \rho^0 \rightarrow e^+e^- \) decays \([19]\) we get \( f_\omega = f_\rho = 0.17 \text{ GeV}^2 \); from \( \phi \rightarrow e^+e^- \) we have \( f_\phi = 0.25 \text{ GeV}^2 \). Using (4.5) and the strong lagrangian \( L_2 \) we can easily compute \( J_{\mu}^q \) and therefore (4.3). The results are \([18]\):

\[ \mathcal{M}(D^* \rightarrow D \gamma) = i \epsilon^{\mu\alpha\beta\epsilon}_{\mu\eta\nu \alpha \nu \beta} \sqrt{M_D M_{D^*}} \left[ e_Q - e_q 2\sqrt{2} g_V \lambda M_D^* \frac{f_V}{M_\omega^2} \right], \]  

(4.7)

\[ \mathcal{M}(D_{s}^* \rightarrow D_{s} \gamma) = i \epsilon^{\mu\alpha\beta\epsilon}_{\mu\eta\nu \alpha \nu \beta} \sqrt{M_{D_s} M_{D_{s}^*}} \left[ e_Q + \frac{1}{3} 2\sqrt{2} g_V \lambda M_{D_s}^* \frac{f_\phi}{M_\phi^2} \right], \]  

(4.8)
where \( e_Q = e_c = \frac{2}{3} \). Eq.(4.7) holds for both \( D^{*+} \to D^{+}\gamma \) and \( D^{*0} \to D^{0}\gamma \) (with \( e_q = -\frac{1}{3} \) and \( \frac{2}{3} \) respectively). We can now use the determination of \( \lambda \) contained in the previous section to obtain the radiative widths. Since we have only obtained the absolute value of \( \lambda \), we have to fix the sign, which can be done by imposing that the relative sign between the two contributions is identical to the one given by the constituent quark model \[20\], i.e. we take

\[
\lambda = -0.40 \pm 0.10.
\]  (4.9)

It is clear that Eqs.(4.7), (4.8) describe with obvious changes also \( B^{*} \) radiative decays.

From the amplitudes (4.7), (4.8) we can compute radiative decay rates for \( D^{*} \) and \( B^{*} \). Moreover one can compute the decay width for the process

\[
D^{*} \to D\pi ,
\] (4.10)

that can be written in terms of the matrix element

\[
<\pi^{-}(q) \ D^{\alpha}(q_2)|D^{*^{-}}(q_1, \epsilon) > = g_{D^{*}D\pi} \ \epsilon^{\mu} \cdot q_{\mu}
\] (4.11)

The strong coupling constant \( g_{D^{*}D\pi} \) is related to the scaled constant \( g \) of the effective chiral lagrangian by the formula

\[
g_{D^{*}D\pi} = \frac{2M_D}{f_{\pi}} g .
\] (4.12)

which is valid in the infinite heavy quark mass limit.

The numerical results for the \( D^{*} \) and \( B^{*} \) decay widths are reported in Table I together with the CLEO data \[21\] on radiative \( D^{*} \) decays. We observe an overall agreement between theoretical results and experiment, which we interpret as a corroboration of the numerical values indicated by the semileptonic decays. We point out however that if we use semileptonic \( D \) decays to predict radiative and strong \( D \) decays we do not actually test the heavy flavour symmetry and, in particular, the results for \( g \) and \( \lambda \) that have been obtained could be effective values, containing a heavy quark mass dependence. This is the reason to get independent determinations of these constants, as we will see in the next sections.

It can be finally observed that the general structure of the matrix elements (4.7) and (4.8) of \[18\] coincides with analyses of other authors \[20\], \[22\], \[23\], \[24\], \[25\], but there are some numerical differences mainly arising from the light quark current that is not provided by the heavy quark effective theory and is treated by different authors in different manners.
Table I

| Decay rate/ BR | theory  | experiment |
|----------------|---------|------------|
| $\Gamma(D^{*+})$ | 46.1 ± 14.2 KeV | < 131 KeV [20] |
| $BR(D^{*+} \rightarrow D^0\pi^0)$ | 31.2 ± 17.4% | 30.8 ± 0.4 ± 0.8 |
| $BR(D^{*+} \rightarrow D^0\pi^+)$ | 67.7 ± 34.2% | 68.1 ± 1.0 ± 1.3 |
| $BR(D^{*+} \rightarrow D^+\gamma)$ | 1.1 ± 0.9% | 1.1 ± 1.4 ± 1.6 |

| $\Gamma(D^0)$ | 36.7 ± 9.7 KeV |
| $BR(D^0 \rightarrow D^0\pi^0)$ | 56.4 ± 27.1% | 63.6 ± 2.3 ± 3.3 |
| $BR(D^0 \rightarrow D^0\pi)$ | 43.6 ± 17.8% | 36.4 ± 2.3 ± 3.3 |

$\Gamma(D^*_s) = \Gamma(D^*_s \rightarrow D_s\gamma)$ (0.24 ± 0.24) KeV

$\Gamma(B^{*+}) = \Gamma(B^{*+} \rightarrow B^+\gamma)$ (0.22 ± 0.09) KeV

$\Gamma(B^{*0}) = \Gamma(B^{*0} \rightarrow B^{*0}\gamma)$ (0.075 ± 0.027) KeV

5 Quark model determination of the strong coupling constant $g$

The previous analyses, based on semileptonic and radiative decays of heavy mesons point to a rather small value of the strong coupling constant $g$ appearing in the effective chiral lagrangian. In the literature one can find the value $g \simeq 1$, [25] as given by the nonrelativistic potential model. It is interesting to show that the quark model result for $g$ can be reconciled with our previous finding $g \simeq 0.40$ provided one takes into account the relativistic motion of the light quark inside the heavy meson [27]. This analysis is based on a QCD inspired relativistic potential model where the heavy hadrons $D_a$ and $D^*_a$, made up by the quark $Q$ and the antiquark $\bar{q}_a$, are described by a relativistic wavefunction \( \psi(\vec{k} + x\vec{p}, \bar{\vec{k}} + (1 - x)\vec{p}) \). This wavefunction satisfies the Salpeter equation [28]

\[
\left\{ \sqrt{(\vec{k} + x\vec{p})^2 + m_Q^2} + \sqrt{[-\vec{k} + (1 - x)\vec{p}]^2 + m_{\bar{q}_a}^2 - \sqrt{M_D^2 + \vec{p}^2}} \right\} \psi(\vec{k} + x\vec{p}, \bar{\vec{k}} + (1 - x)\vec{p}) + \int d\vec{k}' V(\vec{p}, \vec{k}, \vec{k}') \psi(\vec{k}', \vec{p} - \vec{k}') = 0
\]

(5.1)

that arises from the bound-state Bethe Salpeter equation by considering the instantaneous time approximation and restricting the Fock space to the $Q\bar{q}$ pairs (for more details see [29]). The Salpeter equation includes relativistic effects due to the kinematics explicitly and is valid in a moving frame where the meson $D$ (or $D^*$), having mass $M_D$, has momentum $\vec{p}$; the wave function $\psi$ is normalized as follows:

\[
\frac{1}{(2\pi)^3} \int d\vec{k} |\psi|^2 = 2\sqrt{M_D^2 + \vec{p}^2},
\]

(5.2)

Note that the quark $Q$ and the antiquark $\bar{q}_a$, carry momenta $\vec{k} + x\vec{p}$ and $-\vec{k} + (1 - x)\vec{p}$ respectively.
The instantaneous potential $V$ coincides, in the meson rest frame, with the Richardson potential [30]; in the $r$-space it grows linearly when $r \to \infty$ and follows QCD predictions for small $r$. In order to avoid unphysical singularities [31], one assumes that $V(r)$, near the origin, is constant:

$$V(r) = V(r_M) \quad (r \leq r_M = \frac{\lambda' 4\pi}{3M_D} ) .$$

(5.3)

The values of the parameters, as obtained by fits to meson masses, are as follows: $m_u = m_d = 38 \text{ MeV}$; $m_s = 115 \text{ MeV}$, $m_c = 1452 \text{ MeV}$, $m_b = 4890 \text{ MeV}$, $\lambda' = 0.6$.

In order to compute the strong coupling constant $g$ one expresses the axial current $A_\mu$ containing the light quarks in terms of quark operators. Taking the derivative of $A_\mu$, one obtains ($J_5^\prime = i\bar{d}\gamma_5 u$):

$$(m_u + m_d) < D^0(k)|J_5|D^*(p, \epsilon) >= -i(\epsilon \cdot q) 2M_D^* A_0(q^2) .$$

(5.4)

where $A_0(q^2)$ is a form factor that, for small $q^2$, is dominated by the $\pi$ pole. One therefore obtains, for $q^2$ small:

$$g_{D^*D\pi} = \frac{M_D^2 - q^2}{M_D^2} 2M_D^* A_0(q^2) ,$$

(5.5)

which shows that, in the chiral limit ($q^2 = 0$),

$$g = A_0(0) .$$

(5.6)

If $E_q = \sqrt{k^2 + m_q^2}$, $m_u = m_d = m_q$ and $\tilde{u}(k)$ is related to the wave function $\psi$ with $\tilde{p} = 0$ by the equation:

$$\tilde{u}(k) = \frac{k \psi(k)}{\sqrt{2\pi}} ,$$

(5.7)

one obtains

$$g = A_0(0) = \frac{1}{4M_D} \int_0^\infty dk |\tilde{u}(k)|^2 \frac{E_q + m_q}{E_q} \left[ 1 - \frac{k^2}{3(E_q + m_q)^2} \right] .$$

(5.8)

It is interesting to consider immediately the non-relativistic limit, where: $E_q \simeq m_q \gg k$. In this limit one obtains:

$$g = \frac{1}{2M_D} \int_0^\infty dk |\tilde{u}(k)|^2 = 1$$

(5.9)

because of the normalization of the wavefunction. Eq. (5.9) reproduces the well known constituent quark model result [1], [32].
Let us now take in (5.8) the limit \( m_q \rightarrow 0 \), which is possible since we work in the chiral limit and there is no restriction to the values of \( m_q \) in the Salpeter equation. In this case, we obtain:

\[
g = \frac{1}{3}. \tag{5.10}
\]

It is worth to stress that the strong reduction of the value of \( g \) from the naive non relativistic quark constituent model value \( (g = 1) \) to the result (5.10) has a simple explanation in the effect of the relativistic kinematics taken into account by the Salpeter equation.

If one introduces light quark masses as given by the fit of the meson masses \( (m_q = 38 \text{ MeV}) \), one has to consider Eq. (5.8). \( \tilde{u}(k) \) is obtained by solving the Salpeter equation numerically by the Multhopp method. In this case one obtains

\[
g \simeq 0.39, \tag{5.11}
\]

a result in agreement with the previous determination based on the semileptonic \( D \) decays.

### 6 QCD sum rule calculation of \( g \)

Finally I report on a recent calculation based on QCD sum rules (for other similar calculations see [22], [35]).

Let us consider the off-shell process

\[
B^{*-}(\epsilon, q_1) \rightarrow \bar{B}^0(q_2) + \pi^-(q). \tag{6.1}
\]

One considers the correlator

\[
A_\mu(P, q) = i \int dx < \pi^-(q) | T(V_\mu(x) j_5(0)) | 0 > e^{-i q_1 x} = A_\mu + B P_\mu \tag{6.2}
\]

where \( V_\mu = \overline{u} \gamma_\mu b, j_5 = i \overline{d} \gamma_5 d, P = q_1 + q_2 \) and \( A, B \) are scalar functions of \( q_1^2, q_2^2, q^2 \).

Both \( A \) and \( B \) satisfy dispersion relations and are computed, according to the QCD sum rules method, in two ways: either by saturating the dispersion relation by physical hadronic states or by means of the operator product expansion (OPE). Considering the invariant function \( A \) in the soft pion limit \( (q \rightarrow 0) \) and for large Euclidean momenta \( (q_1^2 = q_2^2 \rightarrow -\infty) \) and performing the OPE one has the following result

\[
A = A^{(0)} + A^{(1)} + A^{(2)} + A^{(3)} + A^{(4)} + A^{(5)} \tag{6.3}
\]

with

\[
A^{(0)} = \frac{-1}{q_1^2 - m_b^2} \left[ m_b f_\pi + \frac{< \overline{u} u >}{f_\pi} \right],
\]

\[
A^{(1)} = \frac{-2}{3} \frac{1}{q_1^2 - m_b^2} \frac{< \overline{u} u >}{f_\pi} \left[ \frac{m_b^2}{q_1^2 - m_b^2} - 2 \right],
\]

\[
A^{(2)} = \frac{m_b f_\pi m_1^2}{9(q_1^2 - m_b^2)^2} \left[ 1 + 10 \frac{m_b^2}{q_1^2 - m_b^2} \right] - \frac{m_o^2 < \overline{u} u >}{4 f_\pi (q_1^2 - m_b^2)^2} \left[ 1 - \frac{2 m_b^2}{q_1^2 - m_b^2} \right]
\]

and

\[
A^{(3)} + A^{(4)} + A^{(5)}.
\]
\[ A^{(3)} = \frac{m_0^2 < \bar{u} u >}{6f_\pi} \left[ \frac{1}{(q_1^2 - m_b^2)^2} - \frac{2m_b^2}{(q_1^2 - m_b^2)^3} + \frac{6m_b^4}{(q_1^2 - m_b^2)^4} \right] \]
\[ A^{(4)} = \frac{1}{(q_1^2 - m_b^2)^2} \left[ \frac{m_0^2 < \bar{u} u >}{4f_\pi} + m_b f_m m_i^2 \right] \]
\[ A^{(5)} = \frac{m_0^2 < \bar{u} u >}{6f_\pi} \left[ \frac{1}{(q_1^2 - m_b^2)^2} - \frac{2m_b^2}{(q_1^2 - m_b^2)^3} \right]. \] (6.4)

In eqs. (6.4) \( \bar{w} u > \) is the quark condensate \( \langle \bar{w} u >= -(240 MeV)^3 \rangle \), \( m_0 \) and \( m_1 \) are defined by the equations

\[ \langle \bar{w} g_q \sigma \cdot G u > = m_0^2 < \bar{w} u > \] (6.5)

and

\[ \langle \pi(q) | \bar{w} B^2 \gamma_\mu \gamma_5 d(0) > = -i f_\pi m_1^2 q_\mu \] (6.6)

and their numerical values are: \( m_0^2 = 0.8 \text{ GeV}^2 \), \( m_1^2 = 0.2 \text{ GeV}^2 \) \[36, 37\].

We now write down the hadronic side of the sum rule. In the dispersion relation

\[ A(0, q_1^2, q_2^2) = \frac{1}{\pi^2} \int dsds' \frac{\rho(s, s')}{(s-q_1^2)(s'-q_2^2)}. \] (6.7)

one divides the integration region into three parts. The first region (I) is the square given by \( m_b^2 \leq s \leq s_0, \) \( m_b^2 \leq s' \leq s_0 \); for \( s_0 \) small enough, (I) contains only the \( B \) and \( B^* \) poles, whose contribution is

\[ A_I(0, q_1^2, q_2^2) = \frac{f_B f_{B^*} M_B^2}{4m_b M_{B^*}} \left[ \frac{g_{B^* B \pi}(3M_{B^*}^2 + M_B^2)}{(q_1^2 - M_{B^*}^2)(q_2^2 - M_B^2)} + \frac{g_{B^* B \pi}}{q_1^2 - M_{B^*}^2} + \frac{3f_+ - f_-}{q_2^2 - M_B^2} \right] \] (6.8)

where \( f_B, f_{B^*} \) are the usual leptonic decay constants, while \( f_+ \) and \( f_- \) are defined by

\[ \langle \pi^-(q) | \bar{B}^0(q_2) | B^{*-}(q_1) > = (f_+ P_\mu - f_- q_\mu) e^\mu. \] (6.9)

The remaining integration regions in (6.7) contain new unknown couplings. However one can get rid of them as well as of the term containing \( 3f_+ - f_- \) in (6.8). Indeed the unwanted terms (the so called ”parasitic terms”) for \( q_1^2 = q_2^2 \) are proportional, after the Borel transformation, to \( 1/M^2 \), while the contribution we are interested in, i.e. the term containing the factor \( g_{B^* B \pi}(3M_{B^*}^2 + M_B^2) \), after the Borel transform, gives rise to a contribution proportional to \( 1/M^4 \), if one neglects the tiny mass difference between \( M_{B^*} \) and \( M_B \). Therefore one can exploit the different \( M^2 \) behaviour to isolate the relevant contribution [34]. Without going into details, I report here the result of this analysis for the case \( m_Q \to \infty \).

The infinite heavy quark mass limit \( (m_b \to \infty) \) is performed according to the usual procedure \[38, 11, 39\]. In terms of low energy parameters the quantities appearing in the finite mass sum rule are written as follows:

\[ M_B = m_b + \omega \]
\[ M_{B^*} - M_B = O \left( \frac{1}{m_b} \right) \]
\[ f_B = f_{B^*} = \frac{\hat{F}}{\sqrt{m_b}} \] (6.10)
\( \omega \) represents the binding energy of the meson, which is finite in the limit \( m_b \to \infty \); \( \hat{F} \) has been computed by QCD sum rules: for \( \omega = 0.625 \, \text{GeV} \) and the threshold \( y_0 = 2m_b^2/s_0 \) in the range \( 1.1 - 1.4 \, \text{GeV} \) the result is \( \hat{F} = 0.30 \pm 0.05 \, \text{GeV}^{3/2} \) (at the order \( \alpha_s = 0 \)) and \( \hat{F} = 0.41 \pm 0.04 \, \text{GeV}^{3/2} \) (including radiative corrections), as we have stressed already. [11].

The sum rule for \( g \) is derived after having expressed the Borel parameter \( M^2 \) in terms of the low energy parameter \( E \):

\[
M^2 = 2m_b E.
\]

One readily obtains:

\[
g = \frac{f^2 \pi^2}{F^2} e^{\omega/E} \left\{ \frac{1}{y_0 - \omega} \left[ \omega \left( 1 - \frac{<\bar{u}u>}{3f^2} - \frac{2m^2}{36E^2} + \frac{m^2 - <\bar{u}u>}{48E^2} \right) \right] + \right. \\
- 2\omega \left( \frac{<\bar{u}u>}{3f^2} + \frac{5m^2}{18E} - \frac{m^2 - <\bar{u}u>}{16E^2f^2} \right) - \frac{m^2}{18} + \frac{m^2 - <\bar{u}u>}{8E^2f^2} \right) + \\
+ \omega \left( 1 - \frac{<\bar{u}u>}{3f^2} - \frac{2m^2}{36E^2} + \frac{m^2 - <\bar{u}u>}{48E^2f^2} \right) - \frac{<\bar{u}u>}{3f^2} + \\
- \frac{5m^2}{18E} + \frac{m^2 - <\bar{u}u>}{16E^2f^2} \right\} 
\tag{6.11}
\]

We observe that the sum rule only gives the combination \( \hat{F}^2 g \); therefore the result has a strong dependence on \( \hat{F} \). This sum rule must be studied in the region of the external parameter \( E \) where the OPE is assumed to converge and where the contribution of higher resonances is small (“duality” region); moreover the various terms of the OPE should display a hierarchical structure, according to their dimension. The corresponding result is, without inclusion of \( O(\alpha_s) \) corrections:

\[
g = 0.44 \pm 0.10 \, \text{GeV}^3. 
\tag{6.12}
\]

One may have a hint on the possible role of the \( O(\alpha_s) \) corrections, by considering only those induced by \( \hat{F} \), that are available and should represent the largest part of such corrections [34]; in this case one would get

\[
g \simeq 0.24 
\tag{6.13}
\]

The difference between (6.12) and (6.13) reflects the well known important role of radiative corrections in the determination of \( f_B \) by QCD sum rules in the \( m_Q \to \infty \) limit [40]. Results compatible, within the theoretical uncertainties, with (6.12) have been obtained by light cone sum rules in [11].

### 7 Conclusions

The strong couplings \( g \) and \( \lambda \) play an important role in heavy meson phenomenology. They are relevant in the strong and radiative decays of the heavy mesons and are expected to be important for their semileptonic decays into final states containing light mesons. They are also important inputs in the effective chiral lagrangians for heavy mesons. I have presented several ways to determine these constants: by semileptonic decays [3], using analysis of radiative transitions [18], a relativistic potential model approach [27] and QCD sum rules [34]. These results can be summarized as follows:

\[
\lambda = -0.40 \pm 0.10 
\tag{7.1}
\]
\[ |g| = 0.25 - 0.50 , \tag{7.2} \]

where the interval of values for \( |g| \) represent a realistic range of values as derived from previous analyses.

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References

[1] N.Isgur and M.B.Wise, Phys. Lett. B232 (1989) 113; ibidem B237 (1990) 527; M.B.Voloshin and M.A.Shifman, Sov.J.Nucl.Phys. 45 (1987) 292; ibidem 47 (1988) 511; H.D.Politzer and M.B. Wise, Phys. Lett. 206B (1988) 681; ibidem 208B (1988) 504; E.Eichten and B.Hill, Phys. Lett. 234B (1990) 511; H.Georgi, Phys.Lett. 240B (1990) 447; B.Grinstein, Nucl. Phys. B339 (1990) 253; A.F.Falk, H.Georgi, B.Grinstein and M.B.Wise, Nucl. Phys. B343 (1990) 1.

[2] M.B.Wise, Phys. Rev. D45 (1992) R2188.

[3] G.Burdman and J.F.Donoghue, Phys. Lett. B280 (1992) 287; P.Cho Harvard preprint HUTP-92/A014 (1992).

[4] T.-M. Yan, H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Lin and H.-L. Yu, Phys. Rev. D46 (1992) 1148.

[5] R.Casalbuoni, A.Deandrea, N.Di Bartolomeo, R.Gatto, F.Feruglio and G.Nardulli, Phys. Lett. B299 (1993) 139.

[6] R.Casalbuoni, A.Deandrea, N.Di Bartolomeo, R.Gatto, F.Feruglio and G.Nardulli, Phys. Lett. B292 (1992) 371.

[7] J.Schechter and A.Subbaraman, Preprint SU-4240-519, September 1992.

[8] N.Isgur and M.B.Wise, Phys. Rev. Lett. 66 (1991) 1130; Phys. Rev. D43 (1991) 651.

[9] J.Rosner, Comm. Nucl. Part. Phys. 16 (1986) 109.

[10] A.F.Falk and M.Luke, Phys. Lett B292 (1992) 119; U.Kilian, J.C. Körner and D. Pirjol, Phys. Lett. B288 (1992) 360.

[11] M.Neubert, Phys. Rev. D 45 (1992) 2451.

[12] P.Ball, Phys. Rev. D 48 (1993) 3190.

[13] J.G.Körner, K.Schilcher, M.Wirbel and Y.L.Wu, Zeit. Phys. C48 (1990) 663.

[14] S.Stone, Syracuse University Report No. HEPSY-1-92.

[15] K.Kodama et al., E-653 Collaboration, Phys. Rev. Lett. 66 (1991)1819; Phys. Lett. B263 (1991) 573; Phys. Lett. B286 (1992) 187.

[16] J.C.Anjos et al., E-691 Collaboration, Phys. Rev. Lett. 62 (1989) 1587; Phys. Rev. Lett. 65 (1990) 2630; Phys. Rev. Lett. 67 (1991) 1507.

[17] P. L. Frabetti et al., E-687 Collaboration, Phys. Lett. B 307 (1993) 262.

[18] P. Colangelo, F. De Fazio and G. Nardulli, Phys. Lett. B 316 (1993) 555.

[19] Particle Data Group, Review of Particle Properties, Phys. Rev. D45 (1992) S1.

[20] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D21 (1980) 203.
[21] F. Butler et al., CLEO Collaboration Phys. Rev. Lett. 69 (1992) 2041.

[22] V. L. Eletsky and Ya. I. Kogan, Zeit. fur Phys. C 28 (1985) 155.

[23] J. F. Amundson, C. G. Boyd, E. Jenkins, M. Luke, A. V. Manohar, J. L. Rosner, M. J. Savage and M. B. Wise, Phys. Lett. B296 (1992) 415.

[24] P. Cho and H. Georgi, Phys. Lett. B296 (1992) 408; Phys. Lett. B300 (1993) 410 (E).

[25] H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Lin, T.-M. Yan, H.-L. Yu, Phys. Rev. D47 (1993) 1030.

[26] The ACCMOR collaboration (S. Barlag et al.), Phys.Lett. B278 (1992) 480.

[27] P. Colangelo, F. De Fazio and G. Nardulli, Phys. Lett. B 334 (1994) 175.

[28] E. E. Salpeter, Phys. Rev. 87 (1952) 328.

[29] P. Colangelo, G. Nardulli and M. Pietroni, Phys. Rev. D43 (1991) 3002.

[30] J. L. Richardson, Phys. Lett. B 82 (1979) 272.

[31] P. Cea and G. Nardulli, Phys. Rev. D34 (1986) 1863.

[32] N. Isgur and M. B. Wise, Phys. Rev. D41 (1990) 151.

[33] K. Karamcheti, Principles of ideal fluid aerodynamics (Wiley, New York, 1966).

[34] P. Colangelo, G. Nardulli, A. Deandrea, N. Di Bartolomeo, R. Gatto and F. Feruglio, Phys. Lett. B 339 (1994) 151.

[35] A.G. Grozin and O.I. Yakovlev, preprint BUDKERINP-94-3 (hep-ph/9401267).

[36] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, Nucl. Phys. B 237 (1984) 525.

[37] A.R. Zhitnitskii, I.R. Zhitnitskii and V.L. Chernyak, Sov. J. Nucl. Phys. 38 (1983) 645.

[38] E. Shuryak, Nucl. Phys. B 198 (1982) 83.

[39] P. Colangelo, G. Nardulli and N. Paver, Proceedings of the ECFA Workshop on a European B-Meson Factory, R. Aleksan and A. Ali Eds., ECFA 93/151, pag.155.

[40] D.J. Broadhurst and A.G. Grozin, Phys. Lett. B 274 (1992) 421.

[41] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, preprint MPI-PhT/94-62, CEBAF-TH-94-22, LMU 15/94 (September 1994).