The “ab initio” approach to the nuclear equation of state: review and
discussion

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Abstract

We review the main components of our microscopic model of nuclear matter, which we have recently extended to incorporate isospin-asymmetry. Some frequently discussed issues concerning nuclear many-body approaches are revisited and critically analysed.

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1 Introduction

Issues related to the nuclear equation of state (EoS), in particular the one describing isospin-asymmetric nuclear matter, are gathering increased interest within the nuclear physics community. Heavy-ion (HI) collision observables which are sensitive to the symmetry energy are being identified and used to constrain this fundamentally important quantity [1, 2, 3, 4, 5]. The neutron skin in neutron-rich nuclei is also related to some features of the EoS, specifically the difference between the pressure in neutron matter and in symmetric matter, that is, the pressure gradient that pushes neutrons outwards to form the skin. Therefore, empirical information on the structure of these nuclei can help constrain the shape of the symmetry energy. Parity-violating electron scattering experiments [6] appear to be the most promising way to obtain information on neutron densities in the near future. Vice versa, independent reliable constraints on the density dependence of the symmetry energy would facilitate predictions of neutron skins.

In this paper, we like to suggest that “ab initio” calculations are the best way to complement such rich and intense experimental and phenomenological efforts. “Ab initio” means that the starting point are realistic free-space nucleon-nucleon (NN) interactions (potentially complemented by many-body forces) which are then applied in the nuclear many-body system. Phenomenological interactions, such as Skyrme forces (see Ref. [7] and references therein), or various parametrizations of relativistic mean field models (see Ref. [8] and references therein), may not be able to provide sufficient physical insight. Of course, phenomenological models are practical and very useful as a testing tool, for instance, when setting empirical boundaries to the EoS through analyses of HI collisions. But, ultimately, comparison between constraints and theoretical predictions is important for an understanding on a more fundamental
level. Such comparison will help identify strengths and weaknesses of the theoretical models and at the same time provide insight into the physical relevance of the “observable” under consideration.

In the next sections, we review the main aspects of our microscopic approach, starting from the two-body input and proceeding into our nuclear matter calculations, which are based on the Dirac-Brueckner-Hartree-Fock (DBHF) method. At each step, we will discuss and motivate our choices.

2 The “ab initio” approach

2.1 The two-body framework

Our present knowledge of the nuclear force is the result of decades of struggle [9]. After the development of QCD and the understanding of its symmetries, chiral effective theories [13] became popular as a way to respect the symmetries of QCD while keeping the degrees of freedom (nucleons and pions) typical of low-energy nuclear physics. However, chiral perturbation theory (ChPT) has definite limitations as far as the range of allowed momenta is concerned. For the purpose of applications in dense matter, where higher and higher momenta become involved with increasing Fermi momentum, ChPT is inappropriate. A relativistic, meson-theoretic model is the better choice.

The one-boson-exchange (OBE) model has proven very successful in describing NN data in free space and has a good theoretical foundation. Among the many available OBE potentials (some being part of the “high-precision generation” [10, 11, 12]), we seek a momentum-space potential developed within a relativistic scattering equation, such as the one obtained through the Thompson three-dimensional reduction of the Bethe-Salpeter equation. Furthermore, we require a potential that uses the pseudovector coupling for the interaction of nucleons with pseudoscalar mesons. With this in mind, as well as the requirement of a good description of NN data, Bonn B [9] has been our standard choice. As is well known, the NN potential model dependence of nuclear matter predictions is not negligible. The saturation points obtained with different NN potentials move along the famous “Coester band” depending on the strength of the tensor force, with the weakest tensor force corresponding to the largest attraction. For the same reason (that is, the role of the tensor force in nuclear matter), the potential model dependence is strongly reduced in pure (or nearly pure) neutron matter, due to the absence of isospin-zero partial waves.

Already when QCD (and its symmetries) were unknown, it was observed that the contribution from the nucleon-antinucleon pair diagram, Fig. 1, is unreasonably large when the pseudoscalar (ps) coupling is used, leading to very large pion-nucleon scattering lengths [14]. We recall that the Lagrangian density for pseudoscalar coupling
of the nucleon field \( \psi \) with the pseudoscalar meson field \( \phi \) is

\[
\mathcal{L}_{ps} = -ig_{ps} \bar{\psi} \gamma_5 \psi \phi.
\] (1)

On the other hand, the same contribution (Fig. 1) is heavily suppressed by the pseudovector (pv) coupling (a mechanism which became known as “pair suppression”). The reason for the suppression is the presence of the covariant derivative (that is, a four-momentum dependence) at the pseudovector vertex,

\[
\mathcal{L}_{pv} = \frac{f_{ps}}{m_{ps}} \bar{\psi} \gamma_5 \gamma^\mu \psi \partial_\mu \phi.
\] (2)

which reduces the contribution of the diagram. Because \( \partial_\mu \) is equivalent to the momentum \( q_\mu \) (in momentum space), the equation above explains the small values of the pion-nucleon scattering length at threshold \[14\]. Considerations based on chiral symmetry \[14\] can further motivate the choice of the pseudovector coupling. We will come back to this point in the next section.

The most important aspect of the “ab initio” approach is that the only free parameters of the model (namely, the parameters of the NN potential) are determined by the fit to the free-space data and never readjusted in the medium. In other words, the model parameters are tightly constrained and the calculation in the medium is parameter free. The presence of free parameters in the medium would generate effects and sensitivities which can be very large and hard to control.

2.2 The many-body framework: Brueckner theory, three-body forces, and relativity

Excellent reviews of Brueckner theory have been written which we can refer the reader to (see \[9\] and references therein). Here, we begin by defining the contributions that are retained in our calculation. Those are the lowest order contribution to the Brueckner series (two-hole lines) and the corresponding exchange diagram. With the G-matrix as the effective interaction, this amounts to including particle-particle (that is, short-range) correlations, which are absolutely essential to even approach a realistic
The issue of three-nucleon forces (3NF), of course, remains to be discussed. In Fig. 2 we show a 3NF originating from virtual excitation of a nucleon-antinucleon pair, known as the “Z-diagram”. Notice that the observations from the previous section ensures that the corresponding diagram at the two-body level, Fig. 1, is small with pv coupling. At this point, it is useful to recall the main feature of the Dirac-Brueckner-Hartree-Fock (DBHF) method, as that turns out to be closely related to the 3NF depicted in Fig. 2. In the DBHF approach, one describes the positive energy solutions of the Dirac equation in the medium as

\[ u^*(p, \lambda) = \left( \frac{E^*_p + m^*}{2m^*} \right)^{1/2} \left( \frac{1}{E^*_p + m^*} \right) \chi_\lambda, \tag{3} \]

where the effective mass is given by \( m^* = m + U_S \), with \( U_S \) an attractive scalar potential. It turns out that both the description of single-nucleon propagation via Eq. (3) and the evaluation of the Z-diagram, Fig. 2, generate a repulsive effect on the energy/particle in symmetric nuclear matter which depends on the density approximately as

\[ \Delta E \propto \left( \frac{\rho}{\rho_0} \right)^{8/3}, \tag{4} \]

and provides the saturating mechanism missing from conventional Brueckner calculations. Brown showed that the bulk of this effect can be obtained as a lowest order (in \( p^2/m \)) relativistic correction to the single-particle propagation \[16\].

The approximate equivalence of the effective-mass spinor description and the contribution from the Z-diagram has a simple intuitive explanation in the observation that Eq. (3), like any other solution of the Dirac equation, can be written as a combination of positive and negative energy solutions. On the other hand, the “nucleon” in the middle of the Z-diagram, Fig. 2, is precisely a superposition of positive and negative energy states. In summary, the DBHF method effectively takes into account a particular class of 3NF, which are crucial for nuclear matter saturation. Notice that the effective mass \( ansatz \) just outlined is extended to deal with protons and neutrons in different concentrations in the case of asymmetric matter \[17\].
Other, more popular, three-body forces need to be examined as well. Figure 3 shows the 3NF that is included in essentially all 3NF models, regardless other components; it is the Fujita-Miyazawa 3NF \[18\]. With the addition of contributions from $\pi N$ S-waves, one ends up with the well-known Tucson-Melbourne 3NF \[19\]. The microscopic 3NF of Ref. \[20\] include contributions from excitations of the Roper resonance (P\(_{11}\) isobar) as well. Now, if diagrams such as the one shown on the left-hand side of Fig. 3 are included, consistency requires that medium modifications at the corresponding two-body level are also included, that is, the diagram on the right-hand side of Fig. 3 should be present and properly medium modified. Large cancellations then take place, a fact that was brought up a long time ago \[21\] but perhaps not fully appreciated. When the two-body sector is handled via OBE diagrams, the two-pion exchange is effectively incorporated through the $\sigma$ “meson”, which certainly cannot generate the (large) medium effects (dispersion and Pauli blocking on $\Delta$ intermediate states) required by the consistency arguments above.

Finally, one may wonder whether other 3N forces are being overlooked that might change the above scenario in a significant way. Although a definite answer to the question of which 3NF should be included can only come from chiral perturbation theory (at each order), in meson theory one can find guidance from considerations of range. Given that there are form factors at the vertices, inclusion of 3NF diagrams such as the one shown in Fig. 3, with both $\pi$ and $\rho$, should be able to account for the major 3NF contribution. There are, of course, other 3NF, such as the three-pion rings included in the Illinois 3NF \[22\]. Those have been found to be helpful in subtleties of the spin dependence when calculating spectra of light nuclei, but it seems unlikely that they would have a major impact on the properties of nuclear matter.

In summary, our calculation does not include 3NF, except for those from virtual pair excitations, which are accounted for indirectly. Collecting all the considerations above, we conclude that the DBHF method may be a reliable, yet practical many-body framework, and, possibly, more internally consistent than other microscopic approaches which include explicit 3NF.
3 Conclusions

It is the purpose of this note to underline the importance of “ab initio” calculations of the EoS to complement on-going experimental efforts. Several microscopic models are available which include either microscopic [20] or phenomenological [23] 3NF or are based on the DBHF scheme [24, 25] (limiting ourselves to models that have recently been concerned with asymmetric matter). We have re-examined some issues frequently encountered in the literature concerning “popular” 3NF. Such 3NF are absent from our (DBHF) calculation for reasons of consistency with the two-body sector, whereas those originating from virtual nucleon-antinucleon excitation are inherent to the DBHF scheme. It is in fact remarkable that a relativistic effect can be shown to be essentially equivalent to a many-body force.

In conclusion, microscopic calculations of the EoS and stringent constraints from EoS-sensitive observables can reveal information about the nature of the underlying nuclear force and its behavior in the medium. With the wealth of experiments/analyses presently going on or planned for the near future, and coherent effort from experiment and theory, the prospects of a significant improvement in our knowledge of the equation of state, particularly its isospin asymmetries, are very good.

Acknowledgments

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