Reconstruction of the optical potential in the inverse quantum scattering. Application to the relativistic inelastic NN scattering

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Abstract

The numerical algorithm of the inverse quantum scattering is developed. This algorithm is based on the Marchenko theory, and includes three steps. The first one is the algebraic Padé approximation of the unitary S-matrix, what is realized by solving a system of linear equations. Second step is the exact solution of the Marchenko equation. The used approximant reduces it to another system of linear equations. At this step we get the real-valued potential. It is shown numerically that the developed algorithm is able not only to generate the given S-matrix dependence, but converges to the initial potential. At third step we construct the optical complex-valued potential which gives the needed S-matrix. It is shown that the modern phase shift analysis data allow to construct the nucleon-nucleon optical potentials of two kinds. These potentials describe the deuteron properties and the phase shift analysis data up to 3 GeV and they have different behavior at short distances. One is a repulsive core potential and another is a Moscow attractive potential with forbidden states.

1 Introduction

Quantum inversion has many applications in nuclear physics. Most of the potential descriptions of the few-body nuclear scattering base on the inversion of the scattering data. Though nucleon-nucleon interaction is used as input for all nuclear calculations, here we always have some fitted parameters and this fitting is, of course inversion. Moreover all modern high-precision nucleon-nucleon potentials are fitted to scattering data and are now perceived as phenomenology. The problem here is "that quantitative models for the nuclear force have only a poor theoretical background, while theory based models yield only poor results"[1]. At the same time the nucleon-nucleon phase shift analysis data are smooth in all investigated energy region up to 3 GeV[2]. This fact justifies the potential description without explicit internal degrees of freedom. It is commonly supposed that nucleon-nucleon potential is nonlocal. However local configuration space potentials simplify nuclear calculations greatly. Nonlocality effects may be treated as corrections due to the internal degrees of freedom. For these reasons construction of the high-precision local NN potential describing at least NN scattering data is necessary for exact calculations in nuclear theory. Recently such results appeared in the literature[3]. To simplify the following investigations we worked out the presented simple algorithm which allows to invert scattering data above inelasticity limit and to get the corresponding optical potential.

In Sect. 2 we describe our inversion algorithm, that allows to get the configuration space potential from phase shift analysis data neglecting inelasticity. In Sect. 3 we show how to get an optical complex-valued potential from the real-valued one. This optical potential describes phase shift analysis data and loss of flux due to inelastic processes. In Sect. 4 we apply the worked out method to the nucleon-nucleon scattering data. We extract two different optical potentials corresponding to the different asymptotic behavior of the scattering data.

2 Inversion algorithm

The input data of the Marchenko inversion are

\[ \{ S(q), (0 < q < +\infty), \ q_j, \ M_j, \ j = 1, ..., n_b \}, \]  

where \( S(q) \) – is the scattering matrix dependance on the momentum \( q \), \( q^2 = Em, q_j^2 = mE_j \leq 0 \), \( E_j \) is the energy of the j-th bound state, so that \( uq_j \geq 0 \), \( m \) – is the particle (reduced) mass. The \( M_j \) matrices
give the asymptotic behavior of the corresponding normalized bound states. These are output data of the partial scattering Schrödinger equation

\[ \left[ -\frac{\frac{d^2}{dr^2}}{r^2} + \frac{l(l+1)}{r^2} + V(r) \right] \psi(r, q) = q^2 \psi(r, q) \]  

(2)

To illustrate the worked out algorithm first we consider the one channel case. We proceed from the Marchenko equation

\[ F(x, y) + L(x, y) + \int_x^{+\infty} L(x, t) F(t, y) dt = 0, \]

(3)

where

\[ F(x, y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} h_i^+ (qx) (I - S(q)) h_i^+ (qy) dq + \sum_{j=1}^{n_h} M_j^2 h_i^+ (iq_j x) h_j^+ (iq_j y), \]

(4)

\( h_i^+ (z) \) are the Riccati-Hankel functions.

Solution of the eq. (3) is the function \( L(x, y) \), which gives the reconstructed potential \( V(r) \) for the eq. (2)

\[ V(r) = -\frac{dL(r, r)}{dr}. \]

(5)

Contrary to the algorithm used in [4] we use the algebraic approximant not for the phase shifts \( \delta(q) \) but for the \( S \)-matrix

\[ S(q) = e^{i2\delta} = \frac{f_2(q) - if_1(q)}{f_2(q) + if_1(q)} \]  

(6)

or

\[ S(q) = e^{i2\delta} = \left( \frac{f_2(q) - if_1(q)}{f_2(q) + if_1(q)} \right)^2. \]  

(7)

In the approximant [6] \( f_1(q) \) and \( f_2(q) \) are an odd and even polynomials of \( q \), which do not turn to zero at the real axis simultaneously (for \( l = 0 \), in the case then there is a bound state with \( q_1 = 0 \), conversely \( f_1(q) \) and \( f_2(q) \) are even and odd polynomials). In the approximant [7] \( f_1(q) \) and \( f_2(q) \) must be of different parity, but we cannot describe the case with zero energy bound state. The important feature of approximants [6] and [7] is that these are the most common Padé approximants for the \( S \)-matrix consistent with its properties. Both approximants lead to the finite-dimensional kernel \( F(x, y) \) of the equation (3). The function \( L(x, y) \) and potential \( V(r) \) are expressed through the elementary functions (\( \sin(r) \), \( \cos(r) \) and powers of \( r \)), so we get Bargmann potentials. Choice [6] or [7] is fixed for needed (phenomenological) dependance \( S(q) \) by numerical experiment. We choose the relation which approximates \( S(q) \) better with less number of \( S(q) \) poles (taking into account their multiplicity). The less number of these poles gives the more simple potential.

The properties of the Padé approximants are well known, and there are broad enough functional classes, for which this approximation converges everywhere besides poles of these functions. In particular, the consequence of the Padé-hypothesis [4] is that if a function is analytical at point \( q = 0 \) and meromorphic at circle \( D \), containing this point, then its diagonal Padé approximations \( [M/M] \) (where \( M \) is the number of the approximation poles) converges to this function on compact subsets of \( D \), which do not contain the poles of this function. Obviously for the major part of nuclear physics problems the \( S \)-matrix does satisfy these conditions. Particularly for the short range potentials the \( S \)-matrix is meromorphic at all complex plane.

Another remark concerns the \( S \)-matrix poles positions. Our approximants for \( S \)-matrix [6], [7] suggest that \( S \)-matrix poles may be positioned everywhere besides the real axis. Previously at numerical approaches the following \( S \)-matrix approximant was used [6]:
with real values \( a_\nu \), what assumes that \( S \)-matrix poles are positioned only at the imaginary axis. It is obvious, our approximants are more common and allow to reconstruct the potentials of more complex form.

The approximants \( \ref{6} \) and \( \ref{7} \) lead to the following expressions for the phase shifts \( \delta (q) \)

\[
\tan (-\delta (q)) = \frac{f_1 (q)}{f_2 (q)}
\]

or

\[
\tan \left( -\frac{1}{2} \delta (q) \right) = \frac{f_1 (q)}{f_2 (q)}.
\]

Having the scattering data set \( \delta (q_i) = \delta_i (i = 1, ..., N) \), we get system of \( N \) linear equations which define \( N \) unknown coefficients of \( f_2 (q) + if_1 (q) \) from \( \ref{8} \) and \( \ref{9} \). The number of experimental phase shifts differs generally from the chosen number \( N \). Therefore the phenomenological dependance \( \delta (q) \) must be approximated by appropriate spline. The \( N \) needed values \( \delta (q_i) = \delta_i \) are defined by some selection from this spline.

The increase in number of precise experimental phase shifts would make approximants \( \ref{6} \), \( \ref{7} \) more close to the true \( S \)-matrix. The scattering data are defined experimentally and they have errors. The arbitrary selection of \( \delta (q_i) = \delta_i (i = 1, ..., N) \) from the experimentally defined allowable region may lead to close roots of the polynomials \( f_1 (q) \) and \( f_2 (q) \), i.e. to false \( S \)-matrix poles. Therefore \( f_1 (q) \) and \( f_2 (q) \) must be factored and close roots particularly those situated near the real axis must be eliminated. This elimination corresponds to elimination of the Padé approximant defects.

Furthermore, using the approximant \( \ref{6} \) (in case of \( \ref{7} \) the solution is a particular case of the considered below solution for two bound channels) and calculating the integral in the eq. \( \ref{11} \) using the residue theorem, we get the following expression for the kernel \( F (x, y) \) of eq. \( \ref{11} \)

\[
F (x, y) = \sum_{i=1}^{n_h} M_i^2 h_i^+ (q_i x) h_i^+ (q_i y) + \sum_{i=1}^{n_{pos}} b_i h_i^+ (\beta_i x) h_i^+ (\beta_i y) = \sum_{i=1}^{n} b_i h_i^+ (\beta_i x) h_i^+ (\beta_i y),
\]

where \( \beta_i (i = 1, ..., n_{pos}) \) are the \( S \)-matrix poles. Summing is taken over all the \( S \)-matrix poles \( n_{pos} \) positioned above the real axis and over bound states \( n_h \): \( n = n_h + n_{pos} \).

So in this case the kernel of the eq. \( \ref{8} \) is finite-dimensional. It is known that solution of such equations reduces to the linear equations solving. We present the specific solution of this equation for our case.

We search the solution of eq. \( \ref{11} \) in the form

\[
L (x, y) = \sum_{i=1}^{n} P_i (x) h_i^+ (\beta_i y),
\]

where \( P_i (x) \) are unknown coefficients. Substituting \( \ref{10} \) and \( \ref{11} \) into \( \ref{5} \) we get the following system of linear equations

\[
\sum_{k=1}^{n} A_{ik} (x) P_k (x) = D_i (x) \quad (i = 1, ..., n),
\]

where

\[
A_{ik} = \delta_{ik} - b_i \frac{\beta_i h_{i-1}^+ (\beta_i x) h_i^+ (\beta_i x) - \beta_k h_i^+ (\beta_i x) h_{i-1}^+ (\beta_k x)}{\beta_i^2 - \beta_k^2}, \quad D_i (x) = -b_i h_i^+ (\beta_i x).
\]

The functional coefficients \( P_i (x) \) are defined by \( \ref{12} \)

\[
P_i (x) = (A^{-1}D)_i,
\]

then \( L (x, y) \) and the potential \( V (r) \) from \( \ref{11} \) and \( \ref{5} \).

The worked out technique of the inversion problem was approved on restoration of the square pit potentials. The accuracy of the results is illustrated by initial and restored potentials in fig.1. The convergence of the restoration algorithm with increasing of power \( N \) of the \([N/N]\) Padé approximant of the \( S \)-matrix is also shown here.
In case of two bound channels the system of the partial Schrödinger equations is

\[
\left( \frac{d^2}{dr^2} + V (r) + \begin{pmatrix} \frac{l_1(l_1+1)}{r^2} & 0 \\ 0 & \frac{l_2(l_2+1)}{r^2} \end{pmatrix} \right) \begin{pmatrix} \chi_1(r) \\ \chi_2(r) \end{pmatrix} = q^2 \begin{pmatrix} \chi_1(r) \\ \chi_2(r) \end{pmatrix},
\]

(15)

\[
V (r) = \begin{pmatrix} V_1 (r) & V_T (r) \\ V_T (r) & V_2 (r) \end{pmatrix},
\]

(16)

where \( V_1 (r) \), \( V_2 (r) \) are potentials in channels 1 and 2, \( V_T (r) \) is potential bounding them, \( \chi_1(r) \) and \( \chi_2(r) \) are channel wave functions.

By analogy with (7) we approximate the \( S \)-matrix by the following expression

\[
S(x) = \begin{pmatrix} \exp (2i\delta_1) \cos 2\varepsilon & \exp (i(\delta_1 + \delta_2)) \sin 2\varepsilon \\ i \exp (i((\delta_1 + \delta_2)) \sin 2\varepsilon & \exp (2i\delta_2) \cos 2\varepsilon \end{pmatrix}
\]

\[
= \begin{pmatrix} \frac{f_1^{(1)}(q)}{f_2^{(1)}(q)} & \frac{f_1^{(1)}(q)}{f_2^{(1)}(q)} \\ \frac{f_1^{(2)}(q)}{f_2^{(2)}(q)} & \frac{f_1^{(2)}(q)}{f_2^{(2)}(q)} \end{pmatrix} + \sum_{j=1,2} \left( \frac{f_1^{(j)}(q)}{f_2^{(j)}(q)} - \frac{f_1^{(j)}(q)}{f_2^{(j)}(q)} \right) \prod_{j=1,2} \left( \frac{f_1^{(j)}(q)}{f_2^{(j)}(q)} - \frac{f_1^{(j)}(q)}{f_2^{(j)}(q)} \right)
\]

(17)

Here we again choose the most general form of Padé approximant in contrast to the form used in [13]. The positions of \( S \)-matrix poles are determined from the equations analogous to (9), but with additional equations for the mixing parameter

\[
tg \varepsilon (q) = \frac{f_1(q)}{f_2(q)},
\]

(18)

with scattering data \( \varepsilon (q_i) = \varepsilon_i \).

In case of bound channels and \( l_1 \neq 0 \), in the original Marchenko theory it was proposed to use transformation of the initial eqns. (13) with \( l_1 = l_2 = 0 \). The same approach was used in [5]. Nevertheless, from the numerical point of view this approach is less effective than the direct solution of the inverse problem with generalized Marchenko equation [6]. Formally it has the former view

\[
L (x,y) + F (x,y) + \int_x^{+\infty} L (x,t) F (t,y) \, dt = 0,
\]

(19)

but functions involved are matrices \((2 \times 2)\)

\[
F (x,y) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H (qxy) [1 - S (q)] H (qy) \, dq + \sum_{i=1}^{n_b} H (\beta_i x) M_i H (\beta_i y),
\]

(20)

where

\[
H (x) = \begin{pmatrix} h_{11}^+(x) & 0 \\ 0 & h_{22}^+(x) \end{pmatrix}.
\]

(21)

Using the chosen \( S \)-matrix approximant and the residue theorem we get the following expression for \( F (x,y) \)

\[
F (x,y) = i \sum_{l_1, l_2 > 0} Res H (qx) (I - S (q)) H (qy) + \sum_{i=1}^{n_b} H (\beta_i x) M_i H (\beta_i y) =
\]

\[
= \sum_{l_1, l_2 > 0} H (\beta_i x) Q_i^1 H (\beta_i y) + \sum_{\beta_i \in A} xH' (\beta_i x) Q_i^2 H (\beta_i y) +
\]

\[
+ \sum_{\beta_i \in A} H (\beta_i x) Q_i^3 H' (\beta_i y) x
\]

(22)

here \( Q_i^j \) \((j = 1, 2)\) are constant matrices.
\[ H'(x) = \begin{pmatrix} dh_{1i}^- (x)/dx & 0 \\ 0 & dh_{1i}^+ (x)/dx \end{pmatrix}, \]

\( \beta_i \) are the \( S \)-matrix poles, values \( q_j \) correspond to bound states, \( \mathbf{A} \) is the set of \( S \)-matrix poles of the second order positioned above the real axis.

We solve eq. (19) using substitution

\[ L(x,y) = \sum_{i=1}^{n} P_i (x) H(\beta_i y) + \sum_{i=1}^{n} N_i (x) y H'(\beta_i y), \]

where summing is taken over all the \( S \)-matrix poles \( n_{\text{pos}} \) positioned above the real axis and over all values \( q_j \). This construction leads to the system of linear equations for the functional \((2 \times 2)\) matrix-coefficients \( P_i (x), N_i (x) \)

\[ \sum_i P_i (x) Q^3_{ij} (x) + \sum_i N_i (x) Q^5_{ij} (x) = H(\beta_j x) Q^1_j + x H'(\beta_j x) Q^2_j \]

\[ \sum_i N_i (x) Q^6_{ij} (x) + \sum_i P_i (x) Q^4_{ij} (x) = H(\beta_j x) Q^2_j \]

where

\[ Q^3_{ij} (x) = I \delta_{ij} + \int_{x}^{+\infty} H(\beta_i t) H(\beta_j t) dt \times Q^1_j + \int_{x}^{+\infty} t H(\beta_i t) H'(\beta_j t) dt \times Q^2_j \]

\[ Q^4_{ij} (x) = \int_{x}^{+\infty} H(\beta_i t) H(\beta_j t) dt \times Q^2_j \]

\[ Q^5_{ij} (x) = \int_{x}^{+\infty} t H'(\beta_i t) H(\beta_j t) dt \times Q^1_j + \int_{x}^{+\infty} t^2 H'(\beta_i t) H'(\beta_j t) dt \times Q^2_j \]

\[ Q^6_{ij} (x) = I \delta_{ij} + \int_{x}^{+\infty} t H'(\beta_i t) H(\beta_j t) dt \times Q^2_j, \]

\( I \) is the unit matrix. The integrals in expressions (25) are easily calculated analytically, but are cumbersome so we do not present them.

Having solved this linear equation system we get the sought-for potential from (23) and (5).

The multichannel generalization is made analogously.

### 3 The optical potential

The Marchenko inversion does not give the needed optical potential. But found potential may serve as the initial potential for the iteration procedure that converts it into the optical (complex-valued) potential that describes inelastic processes.

First we consider the one channel problem.

The phase equation [7] for the initial potential \( V^0 (r) \) obtained by some inversion procedure (say from Marchenko equation) is

\[ \delta_l^{(0)} = -\frac{1}{k} \int_{0}^{\infty} V^{(0)} (r) D^2 \left( qr \sin^2 \left( qr + \delta_l^{(0)} (r) \right) \right) dr, \]
where $D_l(z)$ is Riccati-Bessel amplitude [7].

Let us consider the complex-valued potential $V^{(1)}(r)$ obtained from $V^{(0)}(r)$ by transformation

$$V^{(1)}(r) = (1 + i\alpha) V^{(0)}(r),$$ 

where $\alpha$ is some real parameter. Evidently the phase equation for this potential is

$$\delta^{(1)} = -\frac{1}{k} (1 + i\alpha) \int_0^\infty V^{(0)}(r) D_l^2(kr) \sin^2\left(qr + \delta^{(1)}(r)\right) dr. \tag{28}$$

From eqs. (26) and (28) we get

$$\delta^{(1)} - (1 + i\alpha) \delta^{(0)} = -\frac{1+i\alpha}{k} \int_0^\infty V^{(0)}(r) D_l^2(kr) \left(\sin^2\left(qr + \delta^{(1)}(r)\right) - \sin^2\left(qr + \delta^{(0)}(r)\right)\right) dr = \tag{29}$$

$$= -\frac{1+i\alpha}{k} \int_0^\infty V^{(0)}(r) D_l^2(kr) \sin\left(2qr + \delta^{(1)}(r) + \delta^{(0)}(r)\right) \sin\left(\delta^{(1)}(r) - \delta^{(0)}(r)\right) dr$$

For smooth enough potentials and $\alpha \equiv \alpha(q)$ not rapidly increasing with increasing of $q$ the right side of eq. (29) rapidly decreases comparing with $\delta^{(0)}$ and $\delta^{(1)}$, because under the integral in (29) there is more rapidly oscillating function than in (26) and (28). Then as a first approximation we may take

$$\delta^{(1)} \approx (1 + i\alpha) \delta^{(0)} = \delta_R + i\delta_I. \tag{30}$$

For inelastic scattering the $S$-matrix is expressed through the real inelastic parameter $\rho$ and the real phase shift $\delta$

$$S = \cos^2(\rho) e^{2i\delta} = e^{2i(\delta_R + i\delta_I)}, \tag{31}$$

so it is easily arrived at

$$\delta \approx \delta^{(0)}, \tag{32}$$

$$\cos^2(\rho) \approx e^{-2\alpha\delta^{(0)}}, \tag{33}$$

whence it follows that $\alpha\delta \geq 0$.

The formula (33) allows to calculate the parameter $\alpha$ from the known values $\rho$ and $\delta^{(0)} \approx \delta$.

This approximation works well enough for many quantum mechanical problems. For example in case of nucleon-nucleon scattering $\rho = 0$ below the inelasticity limit which is high enough and $\rho$ grows slowly enough above this limit.

Nevertheless in some instances the initial potential $V^{(0)}(r)$ must be corrected. In this case the following iteration procedure may be efficient. First from the formula (33) values $\alpha(q)$ are calculated from $\rho(q)$ and $\delta(q)$. Then with the potential $V^{(1)}(r)$, defined by formula (27) new phase shifts $\delta^{(1)}$ are calculated from (28). If they do not satisfy phase shift analysis data $\delta^{(0)} \pm \Delta\delta$ then changing the input data to $\delta^{(1)} = 2\delta^{(0)} - \delta^{(1)}$ we repeat inversion (eqs. (1)-(14)) and determine new $\alpha$ from formula (33).

The case of two channels is considered in a like manner, though the final expressions are more complex. By analogy with the one channel case the following generalization for the optical potential is derived (the mixing parameter $\varepsilon$ is considered small):

$$V^{(1)}(r) = \begin{pmatrix}
(1 - i\alpha) V^{(0)}_{11} & (1 - i(\alpha + \beta)/2) V^{(0)}_{12} \\
(1 - i(\alpha + \beta)/2) V^{(0)}_{12} & (1 - i\beta) V^{(0)}_{22}
\end{pmatrix} \tag{34}$$

The first $S$-matrix approximation is
The optical nucleon-nucleon potential

We applied the described algorithm of inversion to reconstruction of the nucleon-nucleon potential. As input data for this reconstruction we used modern phase shift analysis data up to 1100 MeV for $^{3}S_{1} - ^{3}D_{1}$ state and up to 3 GeV for $^{1}S_{0}$ state of nucleon-nucleon system [2]. The deuteron properties were taken from [3].

The relativistic effects were taken into account in frames of relativistic quantum mechanics of systems with a fixed number of particles (point form dynamics). The review of this approach can be found in [4].

This approach is based on the assumption that at not high energies we may consider the number of particles fixed, but the invariance group is the Poincare group. A system of two particles is described by the wave function, which is an eigenfunction of the mass operator or the mass squared operator. In this case we may represent this wave function as a product of the external and internal wave functions [10, 11].

The internal wave function is also an eigenfunction of the mass operator or the mass squared operator. It is shown that the mass squared method is consistent with conventional fitting of the Lorentz invariant cross section as a function of laboratory energy [13]. We consider system of two particles (nucleons) with equal masses. Then the internal wave function $\chi(\mathbf{q})$ satisfies the following equation

$$[4(m^2 + \mathbf{q}^2) + V] \chi = M^2 \chi$$

or

$$\left(\frac{\mathbf{q}^2}{m} + \frac{V}{4m}\right) \chi = E \chi,$$

where

$$E = \frac{M^2 - 4m^2}{4m} = \frac{\kappa^2}{m}.$$

The eq. 37 formally coincides with the Schrödinger equation. In eqs. 36, 37 $M^2$ is the mass squared operator, $m$ is the mass of nucleon, $V$ is the nucleon-nucleon potential, \(\mathbf{q}\) is the momentum operator of one of the nucleons in the center of masses system. We use system $\hbar = c = 1$. The quasicoordinate representation corresponds to the realization $\mathbf{q} = -i \frac{\partial}{\partial r}$, $V = V(\mathbf{r})$. In [12] we showed that this formalism can be easily generalized to the case of inelastic channels, particularly it allows to take into account the isobar channels in NN scattering.

This formal coincidence allows us to apply our inversion algorithm. The modern phase shift analysis data [2] allow to construct nucleon-nucleon potentials of two different kinds, depending on the asymptotic behavior of the phase shifts above investigated energy region of 3 GeV. We constructed the nucleon-nucleon optical potentials of two kinds for $^{1}S_{0}$ and $^{3}S_{1} - ^{3}D_{1}$ partial waves. These potentials describe the deuteron properties and the phase shift analysis data up to 3 GeV and they have different behavior at short distances. One is a repulsive core potential and another is a Moscow attractive potential with forbidden states. The Moscow potential was introduced in [14]. These potentials are not phase equivalent and their $S$-matrices differ even below 3 GeV, but within the experimental errors. The $^{3}S_{0}$ phase shift of Moscow potential begins from $\pi$. $^{3}S_{0}$ phase shifts of Moscow potential begin from $2\pi$. The mixing parameter $\epsilon_1$ of Moscow potential differs from this of repulsive core potential by sign. Above 3 GeV these potentials give different $S$-matrices.
The real parts of the constructed partial potentials are presented in fig. 2 and fig. 3. They may be downloaded from www.physics.khstu.ru in numerical form. Standard notation for central and tensor parts is used, so for $^3S_1 - ^3D_1$

$$V_{CS}(r) = V_1(r), \quad V_{CD}(r) = V_2(r) + V_T(r)/2\sqrt{2}, \quad V_{tens}(r) = V_T(r)/\sqrt{2}$$

The imaginary parts of potentials are defined by eqs. (27, 34), where parameters $\alpha$ and $\beta$ can be easily calculated from the phase shift analysis data [2]. The phase shifts and mixing parameter are compared with the phase shift analysis data [2] in fig. 4. In table 1 the results of our deuteron properties calculations are compared with the experimental data [8]. Both kinds of the constructed partial potentials describe the data well in the limits of the experimental errors.

We conclude that the available data of the NN scattering and the deuteron properties are not discriminative with respect to the kind of the NN potential. To discriminate between the considered two kinds of nucleon-nucleon potential we need careful experimental examination of other inelastic processes such as $pp \rightarrow pp\gamma$ [11] or $^2H + \gamma \rightarrow n + p$.

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Table 1. The deuteron properties

| Property                  | Exp. a | Calculation with repulsive core potential | Calculation with Moscow potential |
|---------------------------|--------|-------------------------------------------|----------------------------------|
| Energy (MeV)              | $2.22458900(22)$ | $2.2246^a$ | $2.2246^a$ |
| $Q$ (Fm$^2$)              | $0.2859(3)$ | $0.2639^c$ | $0.277^c$ |
| $A_S$ (Fm$^{-1/2}$)       | $0.8802(20)$ | $0.8802$ | $0.8802$ |
| $r_d$ (Fm)                | $1.9627(38)$ | $1.951$ | $1.956$ |
| $\eta_d/s$               | $0.02714$ | $0.02714$ | $0.02714$ |
| $\mu_d$                  | $0.857406(1)$ | $0.8497^c$ | $0.859^c$ |

$a$ relativistic correction included; $b$ Data are from [8]; $c$ Meson exchange currents are not included.

Figure 1: Reconstructed square pit potential. Solid line N=31, dotted line N=23. N is power of the $[N/N]$ Padé approximation.

Figure 2: Real parts of optic repulsive core potentials. Solid line $^1S_0$ -one channel. Two bound channels long dashed line $V_{CS}(r)$ ($^3S_1$), short dashed line $V_{CD}(r)$ ($^3D_1$), dotted line $V_{tens}(r)$. 
Figure 3: Same as in Fig. 2 but for the Moscow potentials.

Figure 4: Phase shifts and mixing parameter. Solid line for reconstructed Moscow potential, dashed lines for reconstructed repulsive core potential. The phase shift analysis data are from [2]. For $S$ waves there are two sets of the phase shifts. First one is the original data set from [2] - small symbols. These data are described by the repulsive core potential. Second one are the same phase shifts raised 180 degrees up - big symbols. These data are described by the Moscow potential. To leave the $S$-matrix unchanged we must then change the sign of the mixing parameter $\epsilon_1$ for the Moscow potential. The mixing parameters for both kinds of potentials differ only by sign in our calculation and corresponding curves coincide in this figure.