Characteristics between the meshing pairs with different envelope profile in single screw compressors

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Abstract. Single screw compressors have been used in various industrial fields. However, because the star-wheel teeth are easy to wear, the market for the development of single screw compressors is limited. In order to extend the service life of the star-wheel, researchers have developed different kinds of star-wheel tooth profile, such as single line envelope profile, single column envelope profile, and multi-column envelope profile. These profiles greatly affect the lubrication characteristics between the star-wheel teeth and the screw grooves. In this article, the lubrication characteristics between the meshing pairs with different envelope profiles are analyzed. Results show that the pressure peak of the single line envelope profile, single column envelope profile, and multi-column envelope profile are $3.23 \times 10^5$ Pa, $3.38 \times 10^5$ Pa, and $4.31 \times 10^5$ Pa, respectively. This means that the multi-column enveloped meshing pair can resist the biggest external impact load. The deviation angle ($\gamma$) of the single line envelope profile, single column envelope profile, and multi-column envelope profile are $0.0139^\circ$~$0.0286^\circ$, $0.0225^\circ$~$0.0306^\circ$ and $0.0122^\circ$~$0.0262^\circ$, respectively. Thus, the self-balancing ability of the multi-column enveloped meshing pair is the strongest, and the oil film thickness on both sides of the multi-column enveloped star-wheel tooth is the most reasonable, which indicates a good lubrication state during operation, that is, longer operation life of the star-wheel teeth.

1. Introduction

The single screw compressor (SSC) is a positive displacement rotary compressor used in various industrial fields, such as refrigeration, air/gas compression, and chemical engineering systems. A typical structure of a SSC is shown in figure 1, and the main components include two star-wheels, a screw rotor and a casing. As it can be seen from the figure 1, the two star-wheel rotors are symmetrical about the center axis of the screw. Because of the symmetrical structure, SSC has the advantages of rotor force balance, no clearance volume, high speed with small loading, simple structure. One star-wheel and the screw constitute a couple of meshing pair. The key technologies of a SSC are the meshing pair’s profile design and manufacturing. If the design of the profile is not reasonable, the star-wheel tooth will be under high relative sliding speed between the tooth flank and the screw groove flank. At present, various of the meshing pair profiles have been published [1], but because of the restrictions of machining method, the widely used profiles are still single line envelope profile, single column envelope profile, and multi-column envelope profile [1] which was proposed in 2011 and...
developed with its manufacturing method in recent years. The three types of profiles are shown in figure 2.

![Figure 1. Typical structure of a single screw compressor.](image1)

![Figure 2. Three types of the meshing pair profiles.](image2)
According to the hydrodynamic lubrication theory, the geometry between the star-wheel tooth flank and the screw groove flank is directly related to the lubrication state between the meshing pair, and the lubrication state can greatly affect the wear resistance of tooth flank [2-4]. Therefore, it is very important and urgent to carry out the research on the lubrication characteristics between the meshing pairs with different envelope profile in SSCs.

In 1982, Jin Guangxi [5] recognized the importance of the hydrodynamic lubrication and used the Matin equation to calculate the minimum oil film thickness between the meshing pair. In 1986, Post and Ziwaan [6] studied the lubrication characteristics in single screw compressor, and they proposed that the fluid inertia must be considered in the oil film force calculation because the meshing pair has a high relative velocity. Post and Ziwaan used and recommended B.E Lauder sliding bearing lubrication calculation model (including the inertia term) [7] in their paper. Liu Yu [8] calculated the lubrication angle in a meshing period. He proposed that the lubrication angle varies from 80° to 90°, and the minimum value appears when the tooth of the star-wheel just engaged with the groove, the maximum value appears at the high pressure side of the compression chamber. The calculated results show that the hydrodynamic lubrication state of the high pressure side is always superior to the low-pressure side in SSCs. In recent years, some researchers mentioned that study on the hydrodynamic lubrication state can improve the meshing pair profile [9], but the work is not carried out in depth.

In this paper, B.E Lauder bearing lubrication calculation model is adopted, and the lubrication characteristic between the star-wheel tooth flank and the screw groove flank with different envelope profiles are compared and analyzed.

2. Oil film force analysis

The geometry between the meshing pair, shown in figure 3, is a long narrow channel. The narrow channel shows geometrical convergence at the oil inlet section, and shows geometric divergence at the oil outlet section. It is generally believed that the oil film pressure only appears in the convergence area of the channel, while in the divergent region the oil film collapsed [9]. Therefore, oil film pressure on the star-wheel tooth side is produced only in the convergence area, and the divergent region only bears the suction pressure of the SSC, because the divergent area of the tooth flank is exposed in the suction chamber. Besides the oil film pressure, the oil friction force, which caused by the shearing action of the lubricating oil, also acts on the tooth side. However, according to the literature [3], the frictional force is much smaller than the oil film force, so the friction force is ignored in the calculation process in this paper.

![Figure 3. Cross-sectional view of the clearance between a tooth and a groove.](image)

![Figure 4. Deviation angle.](image)

During meshing, both the front side flank and the back side flank of the tooth bear the oil film forces. Hence, theoretically, the total oil film torque acting on the tooth is the sum of the torques produced by the oil film pressure in the two sides narrow channels. Under the action of the oil film torque, the star-wheel tooth will be pushed to the direction which the oil film force is smaller, and then
the tooth deviation occurs. The deviation of the tooth changes the geometry between the meshing pair, and then changes the oil film forces. When the sum of the oil film torques is equal to zero, the star-wheel tooth will stay in a balanced position. The balanced position is called deviation position in figure 4. Therefore, the deviation angle \( \gamma \), which between the tooth centerline when the tooth is in the middle of the groove (design position) and the centerline when the tooth is in the deviation position, is used to investigated the unbalanced degree of the oil film torques, shown in figure 4.

3. Lubrication model

B.E. Lauder sliding bearing lubrication model [7] is used in this paper, and the coordinate system is established in figure 5. Due to the fact that the tooth flank thickness is much smaller than the screw groove length, the cross-sectional curve of the screw groove can be set as the tangent line of the envelope column at the meshing point. As shown in the figure 5, the upper surface represents the star-wheel tooth flank, the lower surface represents the screw groove flank.

Because of the complex geometry shape between the meshing pair, the continuous variation of the sliding velocity and the inclusion of the inertial item, it is very difficult to solve the two-dimensional B.E. Lauder lubrication model. Model should be reasonably simplified. Compared the characteristic width \( B \) of the lubricating oil flow path, the tooth length is much longer. So, the tooth flank is divided into differential elements along the tooth length direction, and in any differential element, \( dP/dz \) is much smaller than \( dP/dx \). It follows that the oil film pressure gradients in z direction are neglected. The distribution of oil film pressure in each differential elements is solved, then the pressure distribution over the entire tooth flank can be obtained.

![Coordinate system in 3D geometry](image1)

![Geometric parameters in the coordinate system](image2)

**Figure 5.** Geometrical configuration and coordinate system.

The one-dimensional B.E. Lauder lubrication model is expressed in the following equation:

\[
\frac{dP(x)}{dx} = \frac{12\mu}{h(x)^2} \left( \frac{U}{2} - u_m \right) + \frac{\rho}{h(x)} \left( \alpha u_m^2 - \beta U^2 \right) \frac{dh(x)}{dx}
\]

(1)

where, \( \alpha = 1.2; \ \gamma = 0.133; \ u_m = \frac{1}{\rho h} \int_0^h u dx; \ h(x) \) is a function of the minimum film thickness \( \delta \) and the profile design parameters. \( \delta \) can be derived from the deviation angle \( \gamma \) (refer to the equation (11) and equation (12)).
The continuity equation is expressed as follows:

$$\frac{d}{dx} [\rho u_m h(x)] = 0$$  \hspace{1cm} (2)

where the term of $\rho u_m h(x)$ is not varying with $x$, so it was set to be a constant value $M$.

Substituting equation (2) into equation (1) and simplifying:

$$\frac{dP(x)}{dx} = \frac{6\mu U}{h(x)^2} - \frac{12\mu M}{\rho h(x)^3} + \frac{M^2}{\rho h(x)^3} \frac{dh(x)}{dx} - \frac{\rho}{h(x)} \beta U^2 \frac{dh(x)}{dx}$$  \hspace{1cm} (3)

The boundary condition is expressed as:

$$\begin{cases} x = x_i, & P(x) = P_{in} \\ x = x_o, & P(x) = P_{out} \end{cases}$$ \hspace{1cm} (4)

where $P_{in}$ is equal to the instantaneous gas pressure $P(\theta_2)$ in the compressing chamber, $P_{out}$ is the suction pressure in the suction chamber, because the divergent area of the tooth flank is exposed in the suction chamber.

The pressure distribution along the groove surface is derived by integrating equation (3):

$$P(x) = P_{out} + \int_{x_i}^{x} \frac{6\mu U}{h(x)^2} dx - \frac{12\mu M}{\rho} \int_{x_i}^{x} \frac{1}{h(x)^3} dx - \frac{M^2}{2\rho} \left( \frac{1}{h(x)^2} - \frac{1}{h(x_o)^2} \right)$$

$$- \rho \beta U^2 \left[ \ln h(x) - \ln h(x_o) \right]$$ \hspace{1cm} (5)

where,

$$M = \frac{12\mu}{\rho} \int_{x_i}^{x_o} \frac{1}{h(x)^3} dx - \sqrt{\left( \frac{12\mu}{\rho} \int_{x_i}^{x_o} \frac{1}{h(x)^3} dx \right)^2 + \frac{2\alpha}{\rho} \left( \frac{1}{h(x_o)^2} - \frac{1}{h(x)^2} \right)^2}$$

$$N = \int_{x_i}^{x_o} \frac{6\mu U}{h(x)^2} dx - \rho \beta U^2 \left[ \ln h(x) - \ln h(x_o) \right] + P_{out} - P(\theta_2)$$

The oil film pressure $P(x)$ calculated by equation (5) is perpendicular to the star wheel tooth flank. However, the calculation of oil film moment only need to consider the pressure component (refer to figure 3, $P_f'$ or $P_b'$) which parallel to the upper tooth surface. In this paper, $P(x)$ is decomposed, and the component pressure is integrated along the tooth thickness, then the oil film forces can be obtained:

$$F_{oil}' = \cos \alpha \cdot \int_{x_i}^{x_o} P(x) dx$$ \hspace{1cm} (6)

where $F_{oil}'$ is the oil film forces acting on a differential element of the tooth flank.

It can be seen from figure 6, in most cases, only part of the tooth flank involved in the engagement during meshing, while the remaining tooth flank is exposed in the suction chamber. Therefore, the forces acting on the tooth flank can be divided into three parts: the oil film pressure caused by the lubricant film in the convergent region, the suction pressure on the engaged tooth flank in the
divergent region, and the suction pressure on the unengaged segment flank. Thus, the torque on a tooth is the sum of the torques produced by the pressures mentioned above.

The torques acting on the front flank and the back flank of a tooth can be expressed respectively:

\[ T_{fi}(\phi_2) = \int_{l_2(\phi_2)}^{L} F_{foij} \cdot r_{2j}(l)dl + \int_{l_2(\phi_2)}^{l_0(\phi_2)} F_{fgi} \cdot r_{2j}(l)dl \]  

\[ T_{bi}(\phi_2) = \int_{l_0(\phi_2)}^{L} F_{oijb} \cdot r_{2j}(l)dl + \int_{l_0(\phi_2)}^{l_0(\phi_2)} F_{hgi} \cdot r_{2j}(l)dl \]  

where \( l_0(\phi_2) \) and \( l_0(\phi_2) \) is the unengaged segment length.

![Figure 6. Geometric relations between star-wheel teeth and screw rotor.](image)

The total torque on a tooth is indicated as follows:

\[ T_i(\phi_2) = T_{fi}(\phi_2) + T_{bi}(\phi_2) \]  

(9)

In this article, the torque acting on the front tooth flank is set to be negative, and the torque acting on the back tooth flank is set to be positive.

During the operation, three or four teeth of the star-wheel simultaneously participate in the engaging. Therefore, the sum of \( T_i \) (composite torque) should be obtained as follows:

\[ T(\phi_2) = \sum_{i=1}^{n} T_i(\phi_2) \]  

(10)

where, \( n = 3\sim 4 \).

When the star-wheel tooth is operated at the design position, the calculation flow chart of \( P(x) \) and \( T(\phi_2) \) is shown in figure 7.

If the composite torque \( T(\phi_2) \) is not equal to zero, the star-wheel tooth will deviate from the design position until the composite torque is balanced. The calculation flow chart of the deviation angle \( \gamma \) is shown in figure 8.
When the deviation angle $\gamma$ is calculated, the oil film minimum thickness of each elements along the tooth flank can be obtained as follows:

$$
\delta_j = \cos \alpha_j \left( \frac{\sigma - \gamma \cdot \pi}{180} \cdot r_j \right)
$$

(11)

$$
\delta_b = \cos \alpha_b \left( \frac{\sigma + \gamma \cdot \pi}{180} \cdot r_j \right)
$$

(12)

**Figure 7.** Calculation flow chart of $P(x)$ and $T(\phi)$.  

**Figure 8.** Calculation flow chart of deviation angle $\gamma$.

### 4. Calculation results

In this paper, the meshing pairs with different profiles in an oil-flooded SSC with 2.6 m$^3$/min displacement was selected as the study object. The major design parameters of the SCC are:

| Parameter                        | Value | Parameter                        | Value |
|----------------------------------|-------|----------------------------------|-------|
| Diameter of screw rotor $d_1$/mm| 136   | Tooth length $L$/mm              | 35    |
| Diameter of star wheel $d_2$/mm  | 148   | Central distance of the compressor $A$/mm | 108   |
| Tooth width $b$/mm               | 20    | Discharge pressure $P$/10$^5$ Pa | 8     |
| Design clearance width $\sigma$/mm| 0.04  | Motor rotation speed $/r$·min$^{-1}$| 2970  |
In this compressor, when the star-wheel tooth rotation angle $\phi$ is 12.7°, the compression chamber instantaneous pressure is $2.88 \times 10^5\text{Pa}$, and star-wheel tooth is in the middle of the screw groove, the oil film pressure distribution along the front tooth flank ($t$ denotes the tooth thickness direction, and $l$ denotes the tooth length direction) with different envelope profiles are shown in figure 7, where $l=0$ mm represents the tooth root, $l=35$ mm represents the tooth tip, $r=0$ mm represents the lower surface of the tooth, $t=8$ mm represents the upper surface of the tooth.

When the star-wheel tooth rotation angle $\phi$ is 12.7°, the unengaged segment of the tooth is 0–4.4 mm, and the engaged segment is 4.4–35 mm. The convergent region of the engaged part of the tooth flank bears oil film pressure, the remaining part of the tooth flank (both engaged part and unengaged part) bears suction pressure. It can be seen from figure 9, the pressure peak of the single line envelope profile, single column envelope profile, and multi-column envelope profile are $3.23 \times 10^5\text{Pa}$, $3.38 \times 10^5\text{Pa}$, and $4.31 \times 10^5\text{Pa}$, respectively. It shows that the multi-column envelope profile can establish larger hydrodynamic oil film pressure peak than the other two profiles.

![Figure 9. Oil film pressure distribution along the tooth flanks with different profiles. (a) single line envelope profile; (b) single column envelope profile; (c) multi-column envelope profile.](image)

In addition, the single line envelope profile have the smallest hydrodynamic oil film action area, while the multi-column envelope profile have the largest oil film action area. Therefore, the oil film at the multi-column envelope profile flank can generate the largest bearing capacity.

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1 In this compressor, when the star-wheel tooth rotation angle is about 12.7°, the meshing points of the front tooth flank with multi-column envelope profile is on the surface of column 3, then the difference of the oil film pressure distribution of the three kinds of profiles are most obviously.
It can be seen from figure 10, the composite oil film torque of the single line envelope profile, single column envelope profile, and multi-column envelope profile are 0.22 N·m–0.77 N·m, 2.92 N·m–4.5 N·m, 1.3 N·m–3.7 N·m, respectively. The composite oil film torques of the single line envelope profile is the most balanced, and that of the single column envelope profile is the most unbalanced. At the beginning of the multi-column envelope profile's design, measures to enhance the oil film torques on the front tooth side were taken into account. Two columns surfaces were added to the front side flank of the tooth to increase the oil film action area, which effectively reduce the composite oil film torque's imbalance.

Figure 11 shows the variations of $\gamma$ when the composite torques are balanced in a meshing period. The $\gamma$ of the single line envelope profile, single column envelope profile, and multi-column envelope profile are $0.0139^\circ$–$0.0286^\circ$, $0.0225^\circ$–$0.0306^\circ$, $0.0122^\circ$–$0.0262^\circ$, respectively. It shows that the self-balancing ability of the multi-column envelope profile is the strongest, and it can easily balance the composite torques through adjusting the tooth deviation angle $\gamma$. The self-balancing ability of the single line envelope profile and single column envelope profile is relatively poor, and it requires a large adjustment of the tooth deviation angle to make the torques balanced.

5. Conclusions
In this paper, B.E. Lauder bearing lubrication model is used to analyze the lubrication characteristics of meshing pairs with single line envelope profile, single column envelope profile and multi-column envelope profile, and the following conclusions can be obtained.
1) Compared with the single line envelope profile and the single column envelope profile, the oil film pressure action area and the peak pressure distribution of the multi-column envelope profile
is the largest, which indicates that the oil film torque is the largest too. In other words, the oil film bearing capacity of the multi-column enveloped tooth is the biggest, which can resist more stronger impact load from the screw rotor to the star wheel teeth.

2) The calculation results of the composite oil film torques and the deviation angle \( \gamma \) show that the single line enveloped tooth has the smallest composite oil film torques and weak self-balancing ability, the single column enveloped tooth has the largest composite oil film torques and the weakest self-balancing ability, while the multi-column enveloped tooth has middle composite oil film torques and the strongest self-balancing ability. It is illustrated that the meshing pair with multi-column enveloped profile can make both side of the star-wheel teeth operated under a good hydrodynamic lubrication state, thus reducing the wear of the star-wheel teeth and prolonging the teeth operation life.

**Appendix**

**Nomenclature**

\[
\begin{align*}
P & \quad \text{Pressure (Pa)} \\
P_{in} & \quad \text{Lubricating oil inlet pressure (Pa)} \\
P_{out} & \quad \text{Lubricating oil outlet pressure (Pa)} \\
P(\phi_2) & \quad \text{Instant gas pressure in compressing chamber (Pa)} \\
U & \quad \text{Relative sliding velocity (m/s)} \\
T & \quad \text{Torque (N·m)} \\
u_m & \quad \text{Axial mass-mean velocity (m/s)} \\
F_o & \quad \text{Oil film force (N)} \\
F_g & \quad \text{Gas force (N)} \\
L & \quad \text{Length of the star-wheel tooth} \\
B & \quad \text{Distance from the lubricating oil inlet to the meshing point (mm)} \\
R_1 & \quad \text{Radius of the screw rotor (mm)} \\
R_2 & \quad \text{Radius of the star-wheel (mm)} \\
p & \quad \text{Instant meshing point on arbitrary cross section along the length of the tooth} \\
r_{2j} & \quad \text{Distance from a meshing point } p_r (p_b) \text{ to the star-wheel rotation centre} \\
h & \quad \text{Oil film thickness (mm)} \\
r & \quad \text{Radius (mm)} \\
b & \quad \text{Tooth width (mm)} \\
t & \quad \text{Tooth thickness (mm)}
\end{align*}
\]

**Subscripts**

\[
\begin{align*}
i & \quad \text{Different tooth} \\
j & \quad \text{Meshing point in every element of the tooth flank} \\
b & \quad \text{Back side of the tooth} \\
f & \quad \text{Front side of the tooth} \\
o & \quad \text{Lubricant} \\
g & \quad \text{gas}
\end{align*}
\]

**Greek symbols**

\[
\begin{align*}
\gamma & \quad \text{Deviation angle (°)} \\
\mu & \quad \text{Dynamic viscosity (Pa·s)} \\
\rho & \quad \text{Density of the lubricant (kg/m}^3) \\
\phi & \quad \text{Rotation angle (°)} \\
\alpha & \quad \text{Groove inclined angle (rad)} \\
\delta & \quad \text{Minimum oil film thickness (mm)}
\end{align*}
\]
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