Vector Manifestation in Hot Matter and Violation of Vector Dominance

Masayasu Harada* and Chihiro Sasaki*

*Department of Physics, Nagoya University, Nagoya, 464-8602, Japan

Abstract. We summarize main mechanisms to realize the vector manifestation (VM), in which the massless vector meson becomes the chiral partner of pion, at the critical temperature in hot QCD within the framework of the model based on the hidden local symmetry. Then, we show a recent analysis on the direct photon-$\pi$-$\pi$ coupling which measures the validity of the vector dominance (VD) of the electromagnetic form factor of the pion: The VM predicts that the VD is largely violated at the critical temperature.

1. Introduction

Spontaneous chiral symmetry breaking is one of the most important properties of QCD in low energy region. This chiral symmetry is expected to be restored in hot and/or dense QCD and properties of hadrons will be changed near the critical temperature of the chiral symmetry restoration [1, 2, 3]. The CERN Super Proton Synchrotron (SPS) observed an enhancement of dielectron ($e^+e^-$) mass spectra below the $\rho/\omega$ resonance [4]. This can be explained by the dropping mass of the $\rho$ meson (see, e.g., Refs. [5, 2, 3]) following the Brown-Rho scaling proposed in Ref. [6]. Furthermore, the Relativistic Heavy Ion Collider (RHIC) has started to measure several physical processes in hot matter which include the dilepton energy spectra. Therefore it is interesting to study the temperature dependence of the vector meson mass which is one of the important quantities in the chiral phase transition.

In Ref. [7], we showed how the vector manifestation (VM) [8], in which the chiral symmetry is restored by the massless degenerate pion (and its flavor partners) and the vector meson (and its flavor partners) as the chiral partner, can be realized in hot matter using the model for pion and vector meson based on the hidden local symmetry (HLS) [9]. There, the intrinsic temperature dependences [7] of the parameters of the HLS Lagrangian, which is introduced by applying the Wilsonian matching [10] at non-zero temperature, played important roles to realize the chiral symmetry restoration consistently: In the framework of the HLS the equality between the axialvector and vector current correlators at critical point can be satisfied only if the intrinsic thermal effects are included.

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So far, several predictions of the VM in hot matter were made: The vector and axial vector susceptibilities are predicted to be equal [11]; The pion velocity becomes the speed of light when we neglect the small Lorentz violating effects in the bare HLS Lagrangian [11]; the vector dominance of the electromagnetic form factor of pion is largely violated [12]. Quite recently in Ref. [13], the Lorentz breaking effects were included in the bare HLS Lagrangian, and it was shown that the pion velocity at the critical temperature receives neither quantum nor hadronic thermal corrections.

In this write-up we show how the VM is realized in hot QCD following Refs. [7, 12], and then we summarize a main result obtained in Ref. [12]: The VM predicts the large violation of the vector dominance at the critical temperature.

This write-up is organized as follows: In section 2, we briefly review the difference between the VM and the conventional picture based on the linear sigma model. In section 3, we briefly introduce the HLS model. Section 4 is devoted to review the way to incorporate the intrinsic temperature dependences of the bare HLS parameters through the Wilsonian matching. Then, in section 5, we summarize how the VM is realized in hot matter. Section 6 is a main part of this write-up where we show that the VM predicts that the vector dominance is largely violated at the critical temperature. Finally in section 7, we give a brief summary.

2. Vector Manifestation

In this section, following Ref. [14], we briefly review the difference between the VM and the conventional picture based on the linear sigma model in terms of the chiral representation of the mesons by extending the analyses done in Refs. [15, 16] for two flavor QCD.

The vector manifestation (VM) was first proposed in Ref. [8] as a novel manifestation of Wigner realization of chiral symmetry where the vector meson $\rho$ becomes massless at the chiral phase transition point. Accordingly, the (longitudinal) $\rho$ becomes the chiral partner of the Nambu-Goldstone boson $\pi$. The VM is characterized by

\begin{equation}
\text{(VM)} \quad F_\pi^2 \to 0, \quad m_\rho^2 \to m_\pi^2 = 0, \quad F_\rho^2/F_\pi^2 \to 1,
\end{equation}

where $F_\rho$ is the decay constant of (longitudinal) $\rho$ at $\rho$ on-shell. This is completely different from the conventional picture based on the linear sigma model where the scalar meson becomes massless degenerate with $\pi$ as the chiral partner:

\begin{equation}
\text{(GL)} \quad F_\pi^2 \to 0, \quad m_S^2 \to m_\pi^2 = 0.
\end{equation}

In Ref. [14] this was called GL manifestation after the effective theory of Ginzburg–Landau or Gell-Mann–Levy.

We first consider the representations of the following zero helicity ($\lambda = 0$) states under $\text{SU}(3)_L \times \text{SU}(3)_R$: the $\pi$, the (longitudinal) $\rho$, the (longitudinal) axialvector meson denoted by $A_1$ ($a_1$ meson and its flavor partners) and the scalar meson denoted by $S$. The $\pi$ and the longitudinal $A_1$ are admixture of $(8, 1) \oplus (1, 8)$ and $(3, 3^*) \oplus (3^*, 3)$ since the symmetry is spontaneously broken [16, 15]:

\[ |\pi\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \sin \psi + |(8, 1) \oplus (1, 8)\rangle \cos \psi, \]
\[|A_1(\lambda = 0)\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \cos \psi - |(8, 1) \oplus (1, 8)\rangle \sin \psi, \quad (2.3)\]

where the experimental value of the mixing angle \(\psi\) is given by approximately \(\psi = \pi/4\) \[14, 15\]. On the other hand, the longitudinal \(\rho\) belongs to pure \((8, 1) \oplus (1, 8)\) and the scalar meson to pure \((3, 3^*) \oplus (3^*, 3)\):

\[
|\rho(\lambda = 0)\rangle = |(8, 1) \oplus (1, 8)\rangle, \\
|S\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle. \quad (2.4)
\]

When the chiral symmetry is restored at the phase transition point, it is natural to expect that the chiral representations coincide with the mass eigenstates: The representation mixing is dissolved. From Eq. (2.3), one can easily see \[8\] that there are two ways to express the representations in the Wigner phase of the chiral symmetry: The conventional GL manifestation corresponds to the limit \(\psi \to \pi/2\) in which \(\pi\) is in the representation of pure \((3, 3^*) \oplus (3^*, 3)\) together with the scalar meson, both being the chiral partners:

(GL) \[
\begin{cases}
|\pi\rangle, |S\rangle & \to |(3, 3^*) \oplus (3^*, 3)\rangle, \\
|\rho(\lambda = 0)\rangle, |A_1(\lambda = 0)\rangle & \to |(8, 1) \oplus (1, 8)\rangle.
\end{cases} \quad (2.5)
\]

On the other hand, the VM corresponds to the limit \(\psi \to 0\) in which the \(A_1\) goes to a pure \((3, 3^*) \oplus (3^*, 3)\), now degenerate with the scalar meson in the same representation, but not with \(\rho\) in \((8, 1) \oplus (1, 8)\):

(VM) \[
\begin{cases}
|\pi\rangle, |\rho(\lambda = 0)\rangle & \to |(8, 1) \oplus (1, 8)\rangle, \\
|A_1(\lambda = 0)\rangle, |S\rangle & \to |(3, 3^*) \oplus (3^*, 3)\rangle.
\end{cases} \quad (2.6)
\]

Namely, the degenerate massless \(\pi\) and (longitudinal) \(\rho\) at the phase transition point are the chiral partners in the representation of \((8, 1) \oplus (1, 8)\). \(^2\)

Next, we consider the helicity \(\lambda = \pm 1\). Note that the transverse \(\rho\) can belong to the representation different from the one for the longitudinal \(\rho(\lambda = 0)\) and thus can have the different chiral partners. According to the analysis in Ref. \[15\], the transverse components of \(\rho(\lambda = \pm 1)\) in the broken phase belong to almost pure \((3^*, 3)\) \((\lambda = +1)\) and \((3, 3^*)\) \((\lambda = -1)\) with tiny mixing with \((8, 1) \oplus (1, 8)\). Then, it is natural to consider in VM that they become pure \((3, 3^*)\) and \((3^*, 3)\) in the limit approaching the chiral restoration point \[14\]:

\[|\rho(\lambda = +1)\rangle \to |(3^*, 3)\rangle, \quad |\rho(\lambda = -1)\rangle \to |(3, 3^*)\rangle. \quad (2.7)\]

As a result, the chiral partners of the transverse components of \(\rho\) in the VM will be themselves. Near the critical point the longitudinal \(\rho\) becomes almost \(\pi\), namely the would-be NG boson \(\sigma\) almost becomes a true NG boson and hence a different particle than the transverse \(\rho\).

\(^2\) It should be stressed that the VM is realized only as a limit approaching the critical point from the broken phase but not exactly on the critical point where the light spectrum including the \(\pi\) and the \(\rho\) would disappear altogether.
3. Hidden Local Symmetry

In this section, we briefly review the model based on the hidden local symmetry (HLS) \[9\] in which the vector manifestation is formulated.

The HLS model is based on the $G_{\text{global}} \times H_{\text{local}}$ symmetry, where $G = SU(N_f)_L \times SU(N_f)_R$ is the chiral symmetry and $H = SU(N_f)_V$ is the HLS. The basic quantities are the HLS gauge boson $V_\mu$ and two matrix valued variables $\xi_L(x)$ and $\xi_R(x)$ which transform as

$$
\xi_{L,R}(x) \to h(x)\xi'_{L,R}(x)g^{L,R}_\mu \sigma(x) F_{L,R} \, ,
$$

(3.8)

where $h(x) \in H_{\text{local}}$ and $g_{L,R} \in [SU(N_f)_{L,R}]_{\text{global}}$. These variables are parameterized as

$$
\xi_{L,R}(x) = e^{i\sigma(x)/F_\sigma} e^{-i\pi(x)/F_\pi} \, ,
$$

(3.9)

where $\pi = \pi^a T^a$ denotes the pseudoscalar Nambu-Goldstone bosons associated with the spontaneous symmetry breaking of $G_{\text{global}}$ chiral symmetry, and $\sigma = \sigma^a T^a$ denotes the Nambu-Goldstone bosons associated with the spontaneous breaking of $H_{\text{local}}$. This $\sigma$ is absorbed into the HLS gauge boson through the Higgs mechanism. $F_\pi$ and $F_\sigma$ are the decay constants of the associated particles. The phenomenologically important parameter $a$ is defined as

$$
a = \frac{F_\sigma^2}{F_\pi^2} \, .
$$

(3.10)

The covariant derivatives of $\xi_{L,R}$ are given by

$$
D_\mu \xi_L = \partial_\mu \xi_L - iV_\mu \xi_L + i\xi_L \cdot L_\mu \, ,
$$

$$
D_\mu \xi_R = \partial_\mu \xi_R - iV_\mu \xi_R + i\xi_R \cdot R_\mu \, ,
$$

(3.11)

where $V_\mu$ is the gauge field of $H_{\text{local}}$, and $L_\mu$ and $R_\mu$ are the external gauge fields introduced by gauging $G_{\text{global}}$ symmetry.

The HLS Lagrangian with lowest derivative terms at the chiral limit is given by

$$
\mathcal{L}_{(2)} = F_\pi^2 \text{tr}[\hat{\alpha}_L \cdot \hat{\alpha}_L] + F_\sigma^2 \text{tr}[\hat{\alpha}_L \cdot \hat{\alpha}_R] - \frac{1}{2g^2} \text{tr}[V_\mu V^{\mu\nu}] \, ,
$$

(3.12)

where $g$ is the HLS gauge coupling, $V_{\mu\nu}$ is the field strength of $V_\mu$ and

$$
\hat{\alpha}_L^{\mu} = \frac{1}{2i} \left[ D_\mu \xi_L \cdot \xi_R + D^{\mu}_{\xi_R} \cdot \xi_L \right] \, ,
$$

$$
\hat{\alpha}_R^{\mu} = \frac{1}{2i} \left[ D_\mu \xi_R \cdot \xi_R - D^{\mu}_{\xi_R} \cdot \xi_L \right] \, .
$$

(3.13)

4. Intrinsic Thermal Effects

In this section we briefly review how to extend the Wilsonian matching to the version at non-zero temperature in order to incorporate the intrinsic thermal effect into the bare parameters of the HLS Lagrangian.
We extend the Wilsonian matching proposed at $T = 0$ \cite{10} to the one at non-zero temperature. It should be noticed that there is no longer Lorentz symmetry in hot matter, and the Lorentz non-scalar operators such as $\bar{q} \gamma_\mu D^\nu q$ may exist in the form of the current correlators derived by the OPE \cite{17}. This leads to, e.g., a difference between the temporal and spatial bare pion decay constants. However, we neglect the contributions from these operators since they give a small correction compared with the main term $1 + \frac{\alpha_s}{\pi}$. This implies that the Lorentz symmetry breaking effect in the bare pion decay constant is small, $F^t_{\pi,\text{bare}} \approx F^s_{\pi,\text{bare}}$ \cite{11}. Thus it is a good approximation that we determine the pion decay constant at non-zero temperature through the matching condition obtained at $T = 0$ in Ref. \cite{10} with putting possible temperature dependences on the gluonic and quark condensates \cite{7, 11}:

$$ F^2_{\pi}(\Lambda; T) \Lambda^2 = \frac{1 + \alpha_s}{8 \pi^2} \left[ 1 + \frac{2 \pi^2}{3} \frac{\langle \bar{q} q \rangle}{\Lambda^4} + \frac{\langle \bar{q} q \rangle}{\Lambda^6} \right]. \tag{4.1} $$

Through this condition the temperature dependences of the quark and gluonic condensates determine the intrinsic temperature dependences of the bare parameter $F_{\pi}(\Lambda; T)$, which is then converted into those of the on-shell parameter $F_{\pi}(\mu = 0; T)$ through the Wilsonian RGEs.

Now, let us consider the Wilsonian matching near the chiral symmetry restoration point with assuming that the quark condensate becomes zero continuously for $T \to T_c$. First, note that the Wilsonian matching condition (4.1) provides

$$ F^2_{\pi}(\Lambda; T_c) \Lambda^2 = \frac{1 + \alpha_s}{8 \pi^2} \left[ 1 + \frac{2 \pi^2}{3} \frac{\langle \bar{q} q \rangle}{\Lambda^4} \right] \neq 0, \tag{4.2} $$

which implies that the matching with QCD dictates

$$ F^2_{\pi}(\Lambda; T_c) \neq 0 \tag{4.3} $$

even at the critical temperature where the on-shell pion decay constant vanishes by adding the quantum corrections through the RGE including the quadratic divergence \cite{10} and hadronic thermal corrections \cite{7, 12}. As was shown in Ref. \cite{18} for the VM in dense matter, Lorentz non-invariant version of the VM conditions for the bare parameters are obtained by the requirement of the equality between the axialvector and vector current correlators in the HLS, which should be valid also in hot matter \cite{11}:

$$ a^t_{\text{bare}} \equiv \left( \frac{F^t_{\sigma,\text{bare}}}{F^t_{\pi,\text{bare}}} \right)^2 \to T \to T_c \to 1, \quad a^s_{\text{bare}} \equiv \left( \frac{F^s_{\sigma,\text{bare}}}{F^s_{\pi,\text{bare}}} \right)^2 \to T \to T_c \to 1, \tag{4.4} $$

$$ g^T_{\text{bare}} \to T \to T_c \to 0, \quad g^L_{\text{bare}} \to T \to T_c \to 0, \tag{4.5} $$

where $a^t_{\text{bare}}, a^s_{\text{bare}}, g^T_{\text{bare}}$ and $g^L_{\text{bare}}$ are the extensions of the parameters $a_{\text{bare}}$ and $g_{\text{bare}}$ in the bare Lagrangian with the Lorentz symmetry breaking effect included as in Appendix A of Ref. \cite{18}.

When we use the conditions for the parameters $a^{t,s}$ in Eq. (4.4) and the above result that the Lorentz symmetry violation between the bare pion decay constants $F^t/s_{\pi,\text{bare}}$ is
small, we can easily show that the Lorentz symmetry breaking effect between the temporal and spatial bare $\sigma$ decay constants is also small, $F_{t,\text{bare}}^{s,\text{bare}} \approx F_{s,\text{bare}}^{s,\text{bare}}$. While we cannot determine the ratio $g_{L,\text{bare}}/g_{T,\text{bare}}$ through the Wilsonian matching since the transverse mode of vector meson decouples near the critical temperature. However this implies that the transverse mode is irrelevant to the quantities studied in this paper. Therefore in the present analysis, we set $g_{L,\text{bare}} = g_{T,\text{bare}}$ for simplicity and use the Lorentz invariant Lagrangian at bare level. In the low temperature region, the intrinsic temperature dependences are negligible, so that we also use the Lorentz invariant Lagrangian at bare level as in the analysis by the ordinary chiral Lagrangian in Ref. [19].

5. Vector Manifestation in Hot Matter

In this section, we briefly summarize how the vector manifestation (VM) is realized in hot matter following Refs. [7, 12].

As we discussed in the previous section, we start from the Lorentz invariant bare Lagrangian even in hot matter, and then the axial vector and the vector current correlators $G_A^{(\text{HLS})}$ and $G_V^{(\text{HLS})}$ are expressed by the same forms as those at zero temperature with the bare parameters having the intrinsic temperature dependences [7]:

$$
G_A^{(\text{HLS})}(Q^2) = \frac{F_\pi^2(\Lambda;T)}{Q^2} - 2z_2(\Lambda;T),
$$

$$
G_V^{(\text{HLS})}(Q^2) = \frac{F_\sigma^2(\Lambda;T)[1 - 2g^2(\Lambda;T)z_3(\Lambda;T)]}{M_\rho^2(\Lambda;T) + Q^2} - 2z_1(\Lambda;T). \tag{5.1}
$$

At the critical temperature, the axial vector and vector current correlators derived in the OPE agree with each other for any value of $Q^2$. Thus we require that these current correlators in the HLS are equal at the critical temperature for any value of $Q^2$ around $\Lambda^2$. By taking account of the fact $F_\pi^2(\Lambda;T_c) \neq 0$ derived from the Wilsonian matching condition given in Eq. (4.2), the requirement $G_A^{(\text{HLS})} = G_V^{(\text{HLS})}$ is satisfied only if the following conditions are met [7]:

$$
g(\Lambda;T) \rightarrow 0, \quad a(\Lambda;T) \rightarrow 1, \quad z_1(\Lambda;T) - z_2(\Lambda;T) \rightarrow 0. \tag{5.2}
$$

These conditions (“VM conditions in hot matter”) for the bare parameters are converted into the conditions for the on-shell parameters through the Wilsonian RGEs. Since $g = 0$ and $a = 1$ are separately the fixed points of the RGEs for $g$ and $a$ [20], the on-shell parameters also satisfy $(g, a) = (0, 1)$, and thus the parametric $\rho$ mass satisfies $M_\rho = 0$.

Now, let us include the hadronic thermal effects to obtain the $\rho$ pole mass near the critical temperature. As we explained above, the intrinsic temperature dependences imply that $M_\rho/T \rightarrow 0$ for $T \rightarrow T_c$, so that the $\rho$ pole mass near the critical temperature
is expressed as \[7, 12\]

\[
m^2_\rho(T) = M^2_\rho + g^2 N_f \frac{15 - a^2}{144} T^2.
\] (5.3)

Since \( a \approx 1 \) near the restoration point, the second term is positive. Then the \( \rho \) pole mass \( m_\rho \) is bigger than the parametric \( M_\rho \) due to the hadronic thermal corrections. Nevertheless, the intrinsic temperature dependence determined by the Wilsonian matching requires that the \( \rho \) becomes massless at the critical temperature:

\[
m^2_\rho(T) \to 0 \quad \text{for } T \to T_c,
\] (5.4)

since the first term in Eq. (5.3) vanishes as \( M_\rho \to 0 \), and the second term also vanishes since \( g \to 0 \) for \( T \to T_c \). This implies that the vector manifestation (VM) actually occurs at the critical temperature [7].

6. Parameter \( a \) and Violation of Vector Dominance

In this section we study the validity of vector dominance (VD) of electromagnetic form factor of the pion in hot matter. In Ref. [21] it has been shown that VD is accidentally satisfied in \( N_f = 3 \) QCD at zero temperature and zero density, and that it is largely violated in large \( N_f \) QCD when the VM occurs. At non-zero temperature there exists the hadronic thermal correction to the parameters. Thus it is nontrivial whether or not the VD is realized in hot matter, especially near the critical temperature. Here we will show that the intrinsic temperature dependences of the parameters of the HLS Lagrangian play essential roles, and then the VD is largely violated near the critical temperature.

6.1. Parameter \( a \) at \( T = 0 \)

We first study the direct \( \gamma\pi\pi \) interaction at zero temperature. At the leading order of the derivative expansion in the HLS, the form of the direct \( \gamma\pi\pi \) interaction is given by

\[
\Gamma_{\gamma\pi\pi(\text{tree})}^\mu = e (q - k)\mu (1 - a) \frac{1}{2},
\] (6.1)

where \( e \) is the electromagnetic coupling constant and \( q \) and \( k \) denote outgoing momenta of the pions. For \( a = 2 \) the direct \( \gamma\pi\pi \) coupling vanishes, which leads to the vector dominance of the electromagnetic form factor of the pion [9].

At the next order there exist quantum corrections which we calculated in the background field gauge in Ref. [12]. The resultant direct \( \gamma\pi\pi \) interaction in the low-energy limit is read from the two-point functions of \( \mathcal{V}_\mu - \mathcal{V}_\nu \) and \( \mathcal{A}_\mu - \mathcal{A}_\nu \):

\[
\Gamma_{\gamma\pi\pi}^\mu = e \frac{1}{F_\pi^2(0)} \left[ q_\nu \Pi_{\perp}^{\mu\nu}(q) - k_\nu \Pi_{\perp}^{\mu\nu}(k) - \frac{1}{2} (q - k)_\nu \Pi_{\parallel}^{\mu\nu}(p) \right],
\] (6.2)
where \( q \) and \( k \) denote the momenta of outgoing pions and \( p_\nu = (q + k)_\nu \) is the photon momentum.

Substituting the decomposition of the two-point function given by
\[
\Pi^\mu\nu_{\perp,\parallel}(p) = \Pi^S_{\perp,\parallel}(p^2)g^\mu\nu + \Pi^T_{\perp,\parallel}(p^2)(g^\mu\nu p^2 - p^\mu p^\nu),
\]
and taking the low-energy limit \( q^2 = k^2 = p^2 = 0 \), we obtain
\[
\Gamma^\mu_{\gamma\pi\pi} = e(q - k)^\mu \left[ 1 - \frac{1}{2} \frac{\Pi^S(p^2 = 0)}{F_\pi^2(0)} \right],
\]
where we used \( \Pi^S(p^2 = 0) = F_\pi^2(0) \). Comparing the above expression with the one in Eq. (6.1), we define the parameter \( a(0) \) at one-loop level as
\[
a(0) = \frac{\Pi^S(p^2 = 0)}{F_\pi^2(0)}. \tag{6.5}
\]
We note that, in Ref. [10], \( a(0) \) is defined by the ratio \( F_\sigma^2(M_\rho)/F_\pi^2(0) \) with neglecting the finite renormalization effect which depends on the details of the renormalization condition. Here we adopt
\[
\text{Re} \left[ \Pi^S(p^2 = M_\rho^2) \right] = F_\sigma^2(\mu = M_\rho), \tag{6.6}
\]
as the renormalization condition for the parametric \( F_\sigma^2 \). From this the parameter \( a(0) \) is expressed as [12]
\[
a(0) = \frac{F_\sigma^2(M_\rho)}{F_\pi^2(0)} + \frac{N_f}{(4\pi)^2} M_\rho^2 F_\pi^2(0)^2 \left( 2 - \sqrt{3}\tan^{-1}\sqrt{3} \right). \tag{6.7}
\]
Using \( M_\rho = 771.1 \text{ MeV}, F_\pi(\mu = 0) = 86.4 \text{ MeV} \) estimated in the chiral limit [22] and \( F_\sigma^2(M_\rho)/F_\pi^2(0) = 2.03 \) predicted by the Wilsonian matching for \( \Lambda_{\text{QCD}} = 400 \text{ MeV} \) and the matching scale \( \Lambda = 1.1 \text{ GeV} \) in Ref. [14], we estimate the value of \( a(0) \) at zero temperature as [12]
\[
a(0) \simeq 2.31. \tag{6.8}
\]
This implies that the VD is well satisfied at \( T = 0 \) even though the value of the parameter \( a \) at the scale \( M_\rho \) is close to one. It should be noticed that the above result is the prediction of the Wilsonian matching.

### 6.2. Parameter \( a \) for \( T \to T_c \) and violation of VD

Now, let us study the direct \( \gamma\pi\pi \) coupling in hot matter. In general, the electric mode and the magnetic mode of the photon couple to the pions differently in hot matter, so

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3 Note that we adopt \( \Pi^S(p^2 = 0) = F_\pi^2(0) \) as the renormalization condition for the parametric \( F_\pi^2 \).
that there are two parameters as extensions of the parameter $a$. Furthermore, there are four polarization tensors to decompose the $\mathcal{M}_\mu G_\nu$ and $\mathcal{V}_\mu G_\nu$ two-point functions. Here we adopt

$$
\Pi^{\mu\nu}_{\perp\perp} = u^\mu u^\nu \Pi^t_{\perp\perp} + (g^{\mu\nu} - u^\mu u^\nu) \Pi^L_{\perp\perp} + P^{\mu\nu}_L \Pi^L_{\perp\perp} + P^{\mu\nu}_T \Pi^T_{\perp\perp},
$$

where $u^\mu = (1, \vec{0})$, and $P^{\mu\nu}_L$ and $P^{\mu\nu}_T$ are the polarization tensors. Similarly to the one obtained at $T = 0$ in Eq. (6.2), at low-energy limit the direct $\gamma\pi\pi$ interaction derived from $\mathcal{M}_\mu G_\nu$ and $\mathcal{V}_\mu G_\nu$ two-point functions is expressed as

$$
\Gamma^{\mu}_{\gamma\pi\pi}(p; q, k) = \frac{1}{F(\vec{q}; T) F(\vec{k}; T)} \left[ q_v \Pi^{\mu\nu}_L(q_0, \vec{q}; T) - k_v \Pi^{\mu\nu}_T(k_0, \vec{k}; T) - \frac{1}{2} (q - k)_v \Pi^{\mu\nu}_T(p_0, \vec{p}; T) \right],
$$

where $\vec{q} = |\vec{q}|$ and $\vec{k} = |\vec{k}|$. $F$ is the wave function renormalization of the $\pi$ field given by

$$
F^2(\vec{p}; T) = F_\pi^2(0) + \text{Re} \Pi^L_{\perp\perp}(\vec{p}; \vec{p}; T),
$$

where $\Pi^L_{\perp\perp}$ expresses the hadronic thermal correction. In Eq. (6.10) each pion is on its mass shell, so that $q_0 = v_\pi(q) \vec{q}$ and $k_0 = v_\pi(\vec{k}) \vec{k}$. To define extensions of the parameter $a$, we consider the soft limit of the photon: $p_0 \to 0$ and $\vec{p} \to 0$. Then the pion momenta become $q_0 = -k_0$ and $\vec{q} = -\vec{k}$. Note that while only two components $\Pi^L_{\perp\perp}$ and $\Pi^s_{\perp\perp}$ appear in $q_v \Pi^{\mu\nu}_L$ or $k_v \Pi^{\mu\nu}_T$, $(q - k)_v \Pi^{\mu\nu}_T$ includes all four components $\Pi^t_{\perp\perp}$, $\Pi^t_{\parallel\parallel}$, $\Pi^L_{\perp\perp}$ and $\Pi^T_{\parallel\parallel}$. Since the tree part of $\Pi^L_{\parallel\parallel}$ and $\Pi^T_{\parallel\parallel}$ is $-2z_2 p^2$ which vanishes at $p^2 = 0$, it is natural to use only $\Pi^t_{\perp\perp}$ and $\Pi^s_{\perp\perp}$ to define the extensions of the parameter $a$. With including these two parts only, the temporal and the spatial components of $\Gamma^\mu_{\gamma\pi\pi}$ are given by

$$
\begin{align*}
\Gamma^0_{\gamma\pi\pi}(0; q, -q) &= \frac{2q_0}{F^2(\vec{q}; T)} \left[ \Pi^t_{\perp\perp}(q_0, \vec{q}; T) - \frac{1}{2} \Pi^s_{\perp\perp}(0, 0; T) \right], \\
\Gamma^i_{\gamma\pi\pi}(0; q, -q) &= \frac{-2q_i}{F^2(\vec{q}; T)} \left[ \Pi^s_{\perp\perp}(q_0, \vec{q}; T) - \frac{1}{2} \Pi^t_{\perp\perp}(0, 0; T) \right].
\end{align*}
$$

Thus we define $a^t(T)$ and $a^s(T)$ as

$$
a^t(\vec{q}; T) = \frac{\Pi^t_{\perp\perp}(0, 0; T)}{\Pi^t_{\perp\perp}(q_0, \vec{q}; T)} , \quad a^s(\vec{q}; T) = \frac{\Pi^s_{\perp\perp}(0, 0; T)}{\Pi^s_{\perp\perp}(q_0, \vec{q}; T)}. \quad (6.13)
$$

Here we should stress again that the pion momentum $q_\mu$ is on mass-shell: $q_0 = v_\pi(q) \vec{q}$.

In the HLS at one-loop level the above $a^t(\vec{q}; T)$ and $a^s(\vec{q}; T)$ are expressed as

$$
a^t(\vec{q}; T) = a(0) \left[ 1 + \frac{\Pi^t_{\perp\perp}(0, 0; T) - a(0) \Pi^t_{\perp\perp}(\vec{q}, \vec{q}; T)}{a(0) F_\pi^2(0; T)} \right], \quad (6.14)
$$

$$
= a(0) \left[ 1 + \frac{\Pi^s_{\perp\perp}(0, 0; T) - a(0) \Pi^s_{\perp\perp}(\vec{q}, \vec{q}; T)}{a(0) F_\pi^2(0; T)} \right],
$$

where $a(0)$ is the value of $a$ at zero energy.
\[ a^s(\bar{q};T) = a(0) \left[ 1 + \frac{\tilde{\Pi}^s_{\perp}(0,0;T) - a(0)\tilde{\Pi}^s_{\parallel}(\bar{q},\bar{q};T)}{a(0)F^2_\pi(0;T)} \right], \quad (6.15) \]

where \( a(0) \) is defined in Eq. (6.5) and \( \Pi^{s,\parallel} \) and \( \Pi^{s,\perp} \) express the hadronic thermal corrections.

Before going to the analysis near the critical temperature, let us study the temperature dependence of the parameters \( a'(\bar{q};T) \) and \( a^s(\bar{q};T) \) in the low temperature region. At low temperature \( T \ll M_\rho \), the hadronic thermal correction is dominated by the contribution from thermal pions, and \( a' \) and \( a^s \) are expressed as \[ a' \simeq a^s \simeq a(0) \left[ 1 + \frac{N_f}{12} \left( 1 - \frac{a^2}{4a(0)} \right) \frac{T^2}{F^2_\pi(0;T)} \right], \quad (6.16) \]

where \( a \) is the parameter renormalized at the scale \( \mu = M_\rho \), while \( a(0) \) is defined in Eq. (6.5). We think that the intrinsic temperature dependences are small in the low temperature region, so that we use the values of parameters at \( T = 0 \) to estimate the temperature dependent correction to the above parameters. By using \( F_\pi(0) = 86.4 \text{MeV} \), \( a(0) \simeq 2.31 \) given in Eq. (6.8) and \( a(M_\rho) = 1.38 \) obtained through the Wilsonian matching for \( (\Lambda_{\text{QCD}}, \Lambda) = (0.4, 1.1) \text{GeV} \) and \( N_f = 3 \) \[ (14) \], \( a' \) and \( a^s \) in Eq. (6.16) are evaluated as

\[ a' \simeq a^s \simeq a(0) \left[ 1 + 0.066 \left( \frac{T}{50 \text{MeV}} \right)^2 \right]. \quad (6.17) \]

This implies that the parameters \( a' \) and \( a^s \) increase with temperature in the low temperature region. However, since the correction is small, we conclude that the vector dominance is well satisfied in the low temperature region.

At higher temperature the intrinsic thermal effects are important. As we have shown in section (5) the parameters \((g,a)\) approach (0,1) for \( T \to T_c \) by the intrinsic temperature dependences, and then the parametric vector meson mass \( M_\rho \) vanishes. Near the critical temperature \( \Pi^s_{\perp} \) and \( \Pi^s_{\parallel} \) in Eqs. (6.14) and (6.15) approach the following expressions \[ (12) \]:

\[ \Pi^s_{\perp}(\bar{q},\bar{q};T) \to T_c \to \frac{N_f}{24} T^2, \]
\[ \Pi^s_{\parallel}(\bar{q},\bar{q};T) \to T_c \to \frac{N_f}{24} T^2. \quad (6.18) \]

On the other hand, the functions \( \Pi^s_{\parallel} \) and \( \Pi^s_{\perp} \) at the limit of \( M_\rho/T \to 0 \) and \( a \to 1 \) become

\[ \tilde{\Pi}^s_{\parallel}(0,0;T) = \tilde{\Pi}^s_{\parallel}(0,0;T) \to -\frac{N_f}{2} \tilde{I}_2(T) = -\frac{N_f}{24} T^2. \quad (6.19) \]

Furthermore, from Eq. (6.17), the parameter \( a(0) \) approaches 1 for \( M_\rho \to 0 \) and \( F^2_0(M_\rho)/F^2_\pi(0) \to 1 \):

\[ a(0) \to 1. \quad (6.20) \]
From the above limits in Eqs. (6.18), (6.19) and (6.20), the numerators of \( a^t(\bar{q}; T) \) and \( a^s(\bar{q}; T) \) in Eqs. (6.14) and (6.15) behave as

\[
\bar{\Pi}^t(0, 0; T) - a(0)\bar{\Pi}^t(\bar{q}, \bar{q}; T) \to 0, \\
\bar{\Pi}^s(0, 0; T) - a(0)\bar{\Pi}^s(\bar{q}, \bar{q}; T) \to 0. 
\] (6.21)

Thus we obtain

\[
a^t(\bar{q}; T), a^s(\bar{q}; T) \overset{T \to T_c}{\to} 1. 
\] (6.22)

This implies that the vector dominance is largely violated near the critical temperature.

## 7. Summary

In this write-up we summarized main results which obtained in Refs. [7, 12].

In section 5 we showed how the VM is realized in hot QCD. It should be stressed that the VM conditions in hot matter [Eq. (5.2)] realized by the intrinsic thermal effects are essential for the VM to take place in hot matter.

In section 6, we summarized main points to obtain a new prediction of the VM made in Ref. [12] on the validity of vector dominance (VD) in hot matter. In the HLS at zero temperature, the Wilsonian matching predicts \( a \simeq 2 \) [10, 14] which guarantees the VD of the electromagnetic form factor of the pion. Even at non-zero temperature, this is valid as long as we consider the thermal effects in the low temperature region where the intrinsic temperature dependences are negligible. We showed that, as a consequence of including the intrinsic effect, the VD is largely violated at the critical temperature:

\[
a^t(\bar{p}; T) \overset{T \to T_c}{\to} 1, \quad a^s(\bar{p}; T) \overset{T \to T_c}{\to} 1.
\]

In general, full temperature dependences include both hadronic and intrinsic thermal effects. Then there exists the violations of VD at generic temperature, although at low temperature the VD is approximately satisfied.

In several analyses such as the one on the dilepton spectra in hot matter done in Ref [3], the VD is assumed to be held even in the high temperature region. We should note that the analysis in Ref. [23] shows that, if the VD holds, the thermal vector meson mass goes up. Then the assumption of the VD, from the beginning, seems to exclude the possibility of the dropping mass of the vector meson such as the one predicted by the Brown-Rho scaling [6]. Our result, which is consistent with the result in Ref. [23] in some sense, indicates that the assumption of the VD may need to be weakened, at least in some amounts, for consistently including the effect of the dropping mass of the vector meson into the analysis.

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