Computation of charged current neutrino-Te reactions cross sections

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Abstract. Neutrino-nucleus reactions, involving both neutral current (NC) and charged current (CC) interactions are important probes in modern neutrino physics searches. In the present work, we study the concrete CC reactions $^{130}$Te$(\nu_\ell, \ell^-)^{130}$I and $^{130}$Te$(\overline{\nu}_\ell, \ell^+)^{130}$Sb which are of current experimental interest for the CUORE and COBRA experiments operating at Gran Sasso underground laboratory in Italy. The nuclear wave functions for the required initial and final nuclear states are derived by employing the proton-neutron (p-n) quasi-particle random phase approximation (QRPA) which has been previously tested in our neutral-current $\nu$-nucleus studies for Te isotopes.

1. Introduction
The last few decades, charged current (CC) neutrino-nucleus scattering became a quite interesting research topic in astro-nuclear physics, particle physics and cosmology [1, 2]. Neutrinos are extremely sensitive probes for investigating the fundamental electroweak interactions and the neutrino properties (via their interactions with nucleons and nuclei) as well as the stellar evolution (several neutrino processes take place in core-collapse supernova, in the sun’s interior and in other stellar systems) [3]. Thus, solar neutrinos, for example, provide significant information for the thermonuclear reactions (p-p and CNO-cycle processes) occurring inside the sun, while supernova neutrino reactions are important for modelling supernova explosions, etc. [3, 4, 5].

On the other hand, promising nuclear detectors have extensively employed as micro-laboratories in neutrino searches through their conventional NC and CC processes with neutrinos that involve vector and axial-vector interactions. Nuclear detector responses to neutrinos are, furthermore, useful tools for investigating exotic (non-standard) interactions at low and intermediate neutrino energies [6, 7, 8, 9]. In particular, many terrestrial experiments, being in operation or designed to operate the next years for detection of laboratory neutrinos (beta-beam neutrinos, spallation neutron source neutrinos, etc.) and astrophysical neutrinos (solar, supernova, geo-neutrinos, etc.), are good sources of information for the exotic properties of neutrinos ($\nu$-oscillations, neutrino magnetic moments, etc.) [8, 9].

In the present work, we examine the role of $^{130}$Te isotope as neutrino detector by studying its conventional charged-current neutrino-induced reactions at low and intermediate neutrino energies ($0 \leq \epsilon_i \leq 100$ MeV). This work is an extension of our previous studies [3, 4, 5] in
which we have comprehensively evaluated the NC reactions of this isotope with neutrinos and anti-neutrinos. Tellurium isotopes are contents of the detectors of the COBRA and CUORE experiments at Gran Sasso.

2. Brief description of the formalism

Within the context of the current-current interaction theory, the standard model effective Hamiltonian for conventional charged changing reactions is written as [1]

\[ \mathcal{H} = \frac{G \cos \theta_c}{\sqrt{2}} j_\mu(x) J^\mu(x), \]  

where \( G = 1.1664 \times 10^{-5} \text{ GeV}^{-2} \) is the weak coupling constant and \( \theta_c \) is the Cabibo angle. \( j_\mu \) and \( J^\mu \) stand for the leptonic and hadronic (nucleonic) currents, respectively. The leptonic current \( j^\mu \) has the form (V-A theory)

\[ j^\mu = \bar{u}_\ell(k_\ell) \gamma^\mu (1 - \gamma_5) u_\nu(k_\nu), \]  

where \( u_\nu, \ u_\ell \) (with the normalization \( \bar{u}u = 1 \)), are Dirac spinors for the neutrino and lepton having four-momenta \( k_i = (\varepsilon_i, k_i) \), \( i = \nu_\ell, \ell \), respectively. The corresponding Feynman diagrams describing the CC neutrino nucleus interactions is shown in Fig. 1.

The nucleonic current \( J_\mu \) in Eq. (1) (neglecting, as usually, contributions of the second class currents) is given by

\[ J_\mu = \bar{u}_p(p_p) \left[ F_1(q^2) \gamma_\mu + F_2(q^2) \frac{i\sigma_\mu \times q^\lambda}{2M} + F_A(q^2) \gamma_\mu \gamma_5 + F_P(q^2) q_\mu \gamma_5 \right] u_n(p_n), \]  

\( (M \) denotes the nucleon mass). The functions of the four momentum transfer \( q^2 \) (with

![Figure 1. Feynman diagrams describing the neutrino nucleus interactions at nuclear level via the exchange of intermediate vector bosons (W±) for charged changing currents.](image)

\( q = p_n - p_p \): \( F_1, F_2, F_A, \) and \( F_P, \) are the well known Dirac, Pauli, axial vector, and pseudo-scalar form factors, respectively. In the convention used in the present work, \( q^2 \), is written as

\[ q^2 = q^\mu q_\mu \equiv q_0^2 - q^2 = (\varepsilon_\nu_\ell - \varepsilon_\ell)^2 - (k_\nu_\ell - k_\ell)^2 \]  

In the case of the neutrino reaction the total energies of the neutron \( (E_n) \) and proton \( (E_p) \) are written as

\[ E_n = \sqrt{p_n^2 + M_n^2}, \quad E_p = \sqrt{(p_n + q)^2 + M_p^2} \approx \sqrt{p_n^2 + q^2 + M_p^2}. \]  

(5)
where $M_n \approx M_p \approx M$ (the approximation in $E_p$ holds if $\mathbf{q} \cdot \mathbf{p}_n \approx 0$). For the anti-neutrino reaction, the leptonic and hadronic currents are the complex conjugates of Eqs. (2) and (3).

The differential cross section for CC neutrino-nucleus scattering, after applying a multipole decomposition of the weak hadronic current (method of Donnelly-Walecka) [4], is written as

$$
\frac{d^2\sigma_{i\to f}}{d\Omega d\omega} = \frac{G^2}{\pi} \frac{|k_i|\varepsilon_i}{(2J_i + 1)} F(Z, \varepsilon_i) \left( \sum_{j=0}^{\infty} \sigma^j_{CL} + \sum_{j=1}^{\infty} \sigma^j_{TM} \right),
$$

where the summations in the right hand side contain the contributions $\sigma^j_{CL}$, for the Coulomb $\mathcal{M}_J$ and longitudinal $\mathcal{L}_J$, and $\sigma^j_{TM}$, for the transverse electric $\mathcal{T}^j_{el}$ and magnetic $\mathcal{T}^j_{mag}$ multipole operators (see Ref. [10, 11] for definitions of these operators and other notations). They include both polar-vector and axial-vector weak interaction components. The function $F(Z, \varepsilon_i)$ (known as Fermi function) takes into account the Coulomb-final-state interaction between the final nucleus and the outgoing lepton (electron in the case of $\nu_e$ scattering). In Eq. (6) $i(f)$ denote the initial (final) state and $\Omega (\omega)$ the solid angle and excitation energy of the daughter nucleus.

### 3. The QRPA for charged current reactions

For CC neutrino-nucleus reactions considered in the present work, low or intermediate energy neutrinos (or anti-neutrinos) are inelastically scattered from a target nucleus (A,Z). There are only incoherent channels open in these processes. The initial and final nuclear isotopes are assumed to be spherically symmetric. The neutrino-detector, $^{130}\text{Te}$ (initial nucleus), has a ground state with spin equal to $|J^\pi_0| = |0^+\rangle$ (even-even nucleus). The ground state spin of the final nuclei, $^{130}\text{I}$ and $^{130}\text{Sb}$, are listed in Table 1 together with the model space chosen and $Q$–values of the studied processes.

| Isotope | $Z$, $N$ | Model Space | $Q$–value (MeV) | $J_{gs}$ |
|---------|---------|-------------|----------------|--------|
| $^{130}\text{Te}$ | 52, 78 | 1p-0f-2s-1d-0g-2p-1f-0h | -0.4173 | 0$^+$ |
| $^{130}\text{Sb}$ | 51, 79 | 1p-0f-2s-1d-0g-2p-1f-0h | 5.0671 | 5$^-$ |
| $^{130}\text{I}$ | 53, 77 | 1p-0f-2s-1d-0g-2p-1f-0h | 2.9440 | 5$^+$ |

The required wave functions for the initial and final nuclear states for charged current neutrino-nucleus induced reactions, can be derived in the context of the proton-neutron (p-n) quasi-particle random phase approximation (QRPA) [3, 4, 5]. In this method, the $m^{th}$ excited state with total angular momentum $J$ projection $M$ and parity $\pi$, denoted by $|J^\pi_m, M\rangle$, is created by acting on the QRPA ground state with the phonon operator

$$
|J^\pi_M\rangle = \sum_{m, J} \left[ X^m(kl, J)A^\dagger(kl, JM) + Y^m(kl, J)\tilde{A}(kl, JM) \right]\n,$$

where $X$ and $Y$ are the forward and backward going amplitudes which can be determined from the common QRPA matrix equations (see, e.g., Ref. [4]).

The quasi-particle pair-creation and pair-annihilation operators, $A^\dagger$ and $\tilde{A}$, are defined as

$$
A^\dagger(kl, JM) \equiv \left[ a^\dagger_m a^\dagger_l \right]^J_M = \sum_{m, m} \langle j_k m_j m_l |JM\rangle a^\dagger_{km} a^\dagger_{lm},
$$

$$
\tilde{A}(kl, JM) = (-1)^{J-M} A(kl, J-M).
$$

In the latter definitions, the square brackets denote angular-momentum coupling while $a^\dagger$ and $a$ denote quasi-particle creation and annihilation operators, respectively.
Table 2. Harmonic oscillator size parameter \( b \) and pairing interaction parameters for protons (\( g^p_{\text{pair}} \)) and neutrons (\( g^n_{\text{pair}} \)). The relevant theoretical values of proton/neutron energy gaps (\( \Delta_{th}^{p/n} \)) that reproduce well the corresponding empirical gaps, \( \Delta_{exp}^{p/n} \) for \(^{130}\text{Te} \), are also listed.

| Isotope | Abundance (\%) | \( b_{\text{HO}} \) (fm) | \( g^p_{\text{pair}} \) (MeV) | \( g^n_{\text{pair}} \) (MeV) | \( \Delta_{exp}^{p} \) (MeV) | \( \Delta_{th}^{p} \) (MeV) | \( \Delta_{exp}^{n} \) (MeV) | \( \Delta_{th}^{n} \) (MeV) |
|---------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(^{130}\text{Te} \) | 33.8           | 2.257           | 1.037           | 0.843           | 1.017           | 1.214           | 1.0170          | 1.2069          |

4. Results and Discussion

For the present cross section calculations of CC (anti)neutrino-nucleus reactions, we have chosen a set of interesting neutrino detectors employed in current underground experiments, i.e. the COBRA and CUORE experiments operating at Gran Sasso, Italy. We focus on the \(^{130}\text{Te} \) target isotope content of the CdTe and CdZnTe semiconductors, detector materials of COBRA experiment, and the \( \text{TeO}_2 \) detector material of the CUORE experiment.

We study the two charged-current (anti)neutrino-nucleus processes:

\[
^{130}\text{Te} + \nu_\ell \rightarrow ^{130}I + \ell^-, \\
^{130}\text{Te} + \bar{\nu}_\ell \rightarrow ^{130}\text{Sb} + \ell^+
\]  

(9)

The first is the charged current reaction of \(^{130}\text{Te} \) with neutrinos (producing a lepton \( \ell^- \)), and the second with anti-neutrinos (producing an anti-lepton \( \ell^+ \)). The corresponding neutral current processes of both of them have previously been studied theoretically by using the method of Ref. [4].

At first, the strong pairing interaction between the nucleons was treated in the framework of the BCS theory. As usually, we adjusted the pairing force and obtain realistic quasi-particle energies separately for protons and neutrons as shown in Table 2. The fit was performed in such a way that the lowest quasi-particle energy to approach the phenomenological pairing gaps \( \Delta_{n/p} \) (for neutrons and protons). The latter are also given from empirical expressions in terms of the proton/neutron separation energies \( S_{n(p)} \) [4, 5].

For the derivation of the nuclear states of the final nuclei we used the proton-neutron quasi-particle RPA (p-n QRPA) based on the Bonn C-D two-nucleon strong interaction. The reliability of this QRPA method was checked through the reproducibility of the experimental spectra for the daughter isotopes \(^{130}\text{I} \) and \(^{130}\text{Sb} \). The theoretical and experimental spectra were constructed and compared as shown in Ref. [2]. Results of the cross sections, obtained for realistic state-by-state calculations in inelastic neutrino-Te scattering processes will be published elsewhere.

References

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