Examination of the Gunion-Bertsch formula for soft gluon radiation

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The spectrum of emitted gluons from the process \( g + g \rightarrow g + g + g \) has been evaluated by relaxing some of the approximations used in earlier works. The difference in the results from earlier calculations have been pointed out. The formula obtained in the present work has been applied to estimate physical quantities like equilibration rate of gluons and the energy loss of fast gluon in the gluonic plasma.

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The radiation of soft gluons from the generic partonic (P) processes \( P_1 + P_2 \rightarrow P_3 + P_4 + \text{gluon} \) plays a crucial role in the study of quark gluon plasma (QGP) expected to be formed in heavy ion collisions (HIC) at ultra-relativistic energies. The number nonconserving process, \( g + g \rightarrow g + g + g \) has drawn particular attention in view of its importance for the (i) chemical equilibration in the deconfined phase of quarks and gluons [1–3], (ii) energy loss of fast gluons propagating through gluonic plasma [4, 5], (iii) evaluation of transport coefficients of the gluonic plasma [6–9], etc. The Gunion-Bertsch (GB) [10] formula for the spectrum of the radiated gluons from the processes \( g + g \rightarrow g + g + g \) has been widely used in many of these calculations. Recently, attempts have been made to revisit/generalize the GB formula [11, 12]. The main purpose of the present work is to find the corrections to the GB formula by relaxing some of the approximations previously adopted. Then, we inspect the effects of the correction terms relative to the GB formula on some physical quantities like the equilibration time and the energy loss of fast gluon in a gluonic plasma. The results obtained here can be extended to other partonic processes such as \( q + q \rightarrow q + q + g \), \( q + g \rightarrow q + g + g \), etc., where \( q \) stands for quark.

We consider the process \( g(k_1) + g(k_2) \rightarrow g(k_A) + g(k_4) + g(k_5) \). The square of the invariant amplitude for this reaction can be written elegantly as [13]:

\[
|M_{gg\rightarrow ggg}|^2 = \frac{1}{2} g^6 \frac{\mathcal{N}}{\mathcal{N}_\mathcal{D}} \frac{N_3}{N_2^2 - 1} \cdot D = (k_1.k_2)(k_1.k_3)(k_1.k_4)(k_1.k_5)(k_2.k_3) \\
\times (k_2.k_4)(k_2.k_5)(k_3.k_4)(k_3.k_5)(k_4.k_5),
\]

(3)

and

\[
(ijklm) = (k_i.k_j)(k_j.k_k)(k_k.k_m)(k_m.k_l).
\]

(4)

\( N_c (=3) \) is the number of colors, \( g = \sqrt{4\pi\alpha_s} \) is the color charge, and \( \alpha_s \) is the strong coupling.

The quantity \( |M_{gg\rightarrow ggg}|^2 \) after simplification can be written as (see Appendix):

\[
|M_{gg\rightarrow ggg}|^2 = 12g^2|M_{gg\rightarrow gg}|^2 \frac{1}{k^2} \times \left[ (1 + \frac{t}{2s} + \frac{5t^2}{2s^2} - \frac{t^3}{s^3}) \right.
\]

\[
- \frac{3}{2\sqrt{s}} + \frac{4t}{s\sqrt{s} - \frac{2s^2}{\sqrt{s}}}k^2 \\
\left. + \frac{5}{2s} + \frac{t}{s^2} + \frac{5t^2}{s^3}k^2 \right],
\]

(5)

where \( |M_{gg\rightarrow gg}|^2 = (9/2)g^2s^2/t^2 \), \( s = (k_1 + k_2)^2 \), \( t = (k_1 - k_3)^2 \), \( u = (k_1 - k_4)^2 \), \( k \) is the transverse momentum of the radiated gluon. The \( O(k^4) \) and \( O(k^6) \) terms appearing in Eq. \( 5 \) were absent in Ref. [12]. Henceforth these two terms will be called the correction terms. We will demonstrate that the contributions from these terms are non-negligible and will have crucial importance for the phenomenology of heavy ion collisions at ultra-relativistic energies.

While the details for the derivation of the Eq. \( 5 \) is given in the Appendix, we would like to check the effects of the correction terms in Eq. \( 5 \) to physical quantities like equilibration time of gluons and energy loss of fast gluons propagating through a gluonic fluid.

Let us first discuss the role of the correction terms in the equilibrium time scale of gluons. The processes \( g + g \leftrightarrow g + g, g + g \leftrightarrow g + g + g, g + g \leftrightarrow g + g + g + g \) etc are responsible for maintaining equilibration in the system. In the present work we would like to estimate the contribution from the correction terms of the process \( g + g \leftrightarrow g + g + g \) as mentioned above. We evaluate the equilibration rate for this process (with \( s = 18T^2 \) as in [1]) with and without the correction terms. The equilibration rates obtained from the spectra of Refs. [11, 12]...
and present work normalized by the GB spectra \((\Gamma_R)\) are displayed in Fig. 1. We observe that the equilibration rate obtained with the correction terms is smaller compared to the scenario when the corrections are neglected for the entire range of temperature under consideration. This indicates that the contribution from the correction terms will enhance the equilibration in the gluonic system.

Before going into estimating the energy loss of fast partons moving through a gluonic fluid, some clarification are in order here. The energy loss of fast partons in QGP has been evaluated rigorously (see [14, 15] for review). In the present work the aim is not to achieve the same level of rigor but to check the effects of the correction terms discussed before on the energy loss of fast partons.

The energy loss of high-energy partons propagating through QGP is a field of high contemporary interest. Experimentally the energy dissipation has been measured through the suppression of the transverse momentum \((p_T)\) distribution of hadrons produced in Au+Au collision relative to the binary scaled p+p interaction at the same center of mass energy. The nature of the suppression may be used as a tool for diagnosis of QGP formation in nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC). The two most common mechanisms for the energy loss are elastic and inelastic or radiative processes. Among various in-elastic processes involving quarks and gluons, the process: \(g + g \to g + g\) plays a major role.

In the hard processes the gluons produced with virtuality \(\sim Q^2\) is highly off-shell because during the collision process the color field of the gluon is stripped off. Therefore, the highly virtual gluon will develop a dead cone - which will results in radiative suppression. Following the procedure outlined in Ref. [16] we evaluate the energy loss of gluon, \(\Delta E(L)\) as a function of length, \(L\). The results obtained with GB and present gluon spectra are displayed in Fig. 2. We observe that the difference in \(\Delta E(L)\) between the GB spectra and the spectra derived in the present work (with corrections described above) is small at lower \(L\) but grows up with increase in \(L\).

The spectrum of the radiated gluon in medium, derived by using the ratio of the amplitude square of the radiative process \(gg \to g\) to that of the elastic process, \(gg \to gg\) is given by,

\[
\frac{dn_g}{d^2k_\perp d\eta} = \frac{\left[ \frac{dn_g}{d^2k_\perp d\eta} \right]_{GB}}{\left( 1 + \frac{t}{2s} + \frac{5t^2}{2s^2} - \frac{t^3}{3s^3} \right)_{GB}} \left( \frac{3}{2\sqrt{s}} + \frac{4t}{s\sqrt{s}} - \frac{3t^2}{2s^2\sqrt{s}} \right)k_\perp^2 \left[ \frac{1}{2s} + \frac{t}{2s^2} + \frac{5t^2}{8s^2}k_\perp^2 \right],
\]

where \(\eta\) is the rapidity of the radiated gluon, the subscript GB has been used to indicate the gluon spectrum obtained using the approximation considered in [16] (see also [17]) which is given by,

\[
\left[ \frac{dn_g}{d^2k_\perp d\eta} \right]_{GB} = \frac{C_A\alpha_s}{\pi^2} \frac{q_\perp^2}{k_\perp^2 \left[ (k_\perp - q_\perp)^2 + m_D^2 \right]},
\]

where \(m_D = \sqrt{2\pi\alpha_s(T)(C_A + \frac{N_c}{2})T/3}\), is the thermal mass of the gluon, \(N_F\) is the number of flavors contributing in the gluon self-energy loop, \(C_A = 3\) is the Casimir invariant for the SU(3) adjoint representation, \(\alpha_s\) is the temperature-dependent strong coupling and \(q_\perp\) is the transverse momentum transfer. The thermal mass in the denominator of Eq. 7 has been introduced to shield the infrared divergence arising from the massless intermediary gluon exchange.

The energy loss of a gluon passing through a gluonic medium can now be calculated using the gluon spectrum of Eq. 6. The energy loss per collision can now be estimated as:

\[
e = \int \frac{d^2k_\perp dq_\perp}{d^2k_\perp d\eta}k_\perp\theta(\Lambda^{-1} - \tau_F) \times \theta(E - k_\perp \cosh\eta),
\]

where \(k_0 = k_\perp \cosh\eta\) is the energy of the radiated gluon and \(\tau_F\) is the formation time of the gluon. The first \(\theta\)-function in Eq. 8 involving \(\Lambda^{-1}\) or interaction time, is introduced for the Landau-Pomeranchuk-Migdal (LPM) effect. The LPM effect imposes restriction on the phase-space of the radiated gluon, it must have \(\tau_F = \cosh\eta/k_\perp > \Lambda^{-1}\). The gluon can be emitted for time scales larger than \(\tau_F\). The second \(\theta\)-function sets the upper limit for the energy of the radiated gluon.

To proceed further, we replace \(q_\perp^2\) by its average value evaluated as follows:

\[
\langle q_\perp^2 \rangle = \frac{1}{\sigma_{el}} \int_{m_D^2}^{\infty} dq_\perp^2 \frac{d\sigma_{el}}{dq_\perp^2} q_\perp^2, \tag{9}
\]

where

\[
\sigma_{el} = \int_{m_D^2}^{\infty} dq_\perp^2 \frac{d\sigma_{el}}{dq_\perp^2}. \tag{10}
\]

For dominant small-angle scattering \((t \to 0)\),

\[
\frac{d\sigma_{el}}{dq_\perp^2} = C_t \frac{2\pi\alpha_s^2}{q_\perp^4}. \tag{11}
\]

\(C_t\) is 9/4, 1 and 4/9 for gg, qg, and qq scattering. \(\langle q_\perp^2 \rangle\) is then obtained as,

\[
\langle q_\perp^2 \rangle = \frac{sm_D^2}{s - 4m_D^2} \ln\left( \frac{s}{4m_D^2} \right). \tag{12}
\]

For \(\sqrt{s} \to \infty\), i.e. in the high-energy limit one can make the replacement \(t \sim -q_\perp^2\). In contrast to previous
works allowing the possibility for the incident gluon to remain out of thermal equilibrium. With all the above ingredients we are now ready to evaluate energy loss ($dE/dx$) of a fast gluon in a gluonic medium as follows:

$$-\frac{dE}{dx} = \epsilon \cdot \Lambda.$$  \hspace{1cm} (13)

The interaction rate, $\Lambda$ has been evaluated by using the procedure similar to Ref. [21]. We need to implement now the radiative suppression of gluons due to its possible off-shellness (Fig. 2). The energy loss ($\Delta E$) of off-shell gluons of energy 15 GeV moving through a gluonic fluid of dimension 4 fm for the gluon spectrum derived in Refs. [11], [12] and in the present work obtained from Eqs. 6, 8 and 13 normalized to that resulting from GB approximations ($\Delta E_R$) are displayed in Fig. 3. We observe that with the correction terms the value of $\Delta E$ is enhanced by about 40% and 20% for $T_{300}$ MeV and $T_{400}$ MeV, respectively, compared to the $\Delta E$ obtained from the spectra of Refs. [11] and [12]. Such differences may have important consequences on the heavy-ion phenomenology at RHIC and LHC collision energies.

In summary we have evaluated the spectrum of the emitted gluon from the processes $g + g \rightarrow g + g + g$ by relaxing some of the approximations used in recent calculations. The results derived in the present calculation has been applied to evaluate some physical quantities, e.g. energy loss of energetic partons in HIC and time scale for equilibration in gluonic plasma. Results obtained in the present work have been compared with the earlier calculations [11, 12]. We find that the contributions from the previously neglected terms $O(k_1^{-1})$ and $O(k_1^0)$ in the matrix element play crucial role in heavy-ion phenomenology. The gluon spectrum obtained in Eq. 13 can be used for other $2 \rightarrow 2 + g$ partonic processes also.

$\Delta E_R$ normalized by the corresponding value obtained from GB approximation and the resulting quantity is called $\Delta E_R$.

**Appendix**

In this appendix we derive Eq. 5 for the square of the invariant amplitude for the radiative process, $gg \rightarrow ggg$ up to orders $O(k_1^{-1})$ and $O(k_1^0)$. Consider the reaction

$$g(k_1) + g(k_2) \rightarrow g(k_3) + g(k_4) + g(k_5),$$  \hspace{1cm} (14)

where $k_5$ is the four-momentum of the radiated gluon. The Mandelstam variables for the above process are defined as

$$s = (k_1 + k_2)^2, \quad t = (k_1 - k_3)^2,$$

$$u = (k_1 - k_4)^2, \quad s' = (k_3 + k_4)^2,$$

$$t' = (k_2 - k_4)^2, \quad u' = (k_2 - k_3)^2.$$

\hspace{1cm} (15)

**FIG. 1:** (Color online) Temperature variation of the ratio of the equilibration rate (inverse of the time scale) obtained in the present work (solid line), Ref. [11] (dashed line), and [12] (dot-dashed) normalized by the GB value for the process $gg \rightarrow ggg$.

**FIG. 2:** Energy loss of a 15 GeV gluon in vacuum as a function of path length.

**FIG. 3:** (Color online) Temperature variation of $\Delta E_R$ of a 15 GeV gluon moving through a gluonic heat bath of dimension 4 fm. Solid (dashed) line indicates result for the gluon spectrum obtained in the present work ([11]). The dot-dashed line stands for the results for the gluon spectrum of [12]. The $\Delta E$ normalized by the corresponding value obtained from GB approximation and the resulting quantity is called $\Delta E_R$. 
Because gluons massless we can write

\[
\begin{align*}
    k_1.k_2 &= \frac{s}{2}, \quad k_1.k_3 = \frac{-u}{2}, \\
    k_1.k_4 &= \frac{-u'}{2}, \quad k_3.k_4 = \frac{s'}{2}, \\
    k_2.k_4 &= \frac{-u'}{2}, \quad k_2.k_3 = \frac{-u'}{2}.
\end{align*}
\]

(16)

We also have the relations

\[
\begin{align*}
    k_1.k_5 &= \frac{s + t + u}{2}, \quad k_2.k_5 = \frac{s + t + u'}{2}, \\
    k_3.k_5 &= \frac{s + t + u}{2}, \quad k_4.k_5 = \frac{s + t + u'}{2}.
\end{align*}
\]

(17)

For soft gluon emission,

\[
s + t + u + s' + t' + u' = 0.
\]

(18)

The matrix element square of the radiative process \(gg \rightarrow ggg\) is given by 

\[
|M_{gg \rightarrow ggg}|^2 = \frac{1}{2} g^6 \frac{N_c^3}{N_c^2 - 1} \mathcal{N} \times \left[ (12345) + (12354) + (12453) + (12453) + (13245) + (13254) + (13425) + (13524) + (14235) + (14325) \right],
\]

(19)

where \(N_c\) is the number of colors, \(g = \sqrt{4 \pi \alpha_s}\) is the strong coupling,

\[
\begin{align*}
    \mathcal{N} &= (k_1.k_2)^4 + (k_1.k_3)^4 + (k_1.k_4)^4 \\
    &\quad + (k_2.k_3)^4 + (k_2.k_4)^4 + (k_2.k_5)^4 \\
    &\quad + (k_3.k_4)^4 + (k_3.k_5)^4 + (k_4.k_5)^4,
\end{align*}
\]

(20)

\[
\begin{align*}
    \mathcal{D} &= (k_1.k_2)(k_1.k_3)(k_1.k_4)(k_1.k_5)(k_2.k_3) \\
    &\quad \times (k_2.k_4)(k_2.k_5)(k_3.k_4)(k_3.k_5)(k_4.k_5),
\end{align*}
\]

(21)

and

\[
(ijklm) = (k_i.k_j)(k_j.k_k)(k_k.k_l)(k_l.k_m)(k_m.k_i).
\]

(22)

Simplifying Eq. (19) we get,

\[
|M_{gg \rightarrow ggg}|^2 = 16 g^6 \frac{N_c^3}{N_c^2 - 1} \mathcal{N} \times \left[ \frac{1}{s'(s + u + t)(s + u' + t')} \left( \frac{1}{tt'} + \frac{1}{uu'} \right) \right.
\]

\[
\left. + \frac{1}{s(s + u + t)(s + u' + t')} \left( \frac{1}{tt'} + \frac{1}{uu'} \right) \right]
\]

\[
\left. - \frac{1}{t'(s + u + t)(s + u' + t')} \left( \frac{1}{uu'} + \frac{1}{ss'} \right) \right]
\]

\[
\left. - \frac{1}{u'(s + u + t)(s + u' + t')} \left( \frac{1}{tt'} + \frac{1}{ss'} \right) \right]
\]

(23)

and \(\mathcal{N}\) can now be written as

\[
\begin{align*}
    \mathcal{N} &= \frac{1}{16} \left[ s^4 + t^4 + u^4 + s'^4 + t'^4 + u'^4 \right. \\
    &\quad + (s + t + u)^4 + (s + t' + u')^4 + (s + t' + u)^4 \\
    &\quad + (s + t + u')^4].
\end{align*}
\]

(24)

For a soft gluon emission \((k_5 \rightarrow 0)\) \(s \rightarrow s', t \rightarrow t', u \rightarrow u'\). We can express the transverse momentum of the emitted gluon as

\[
\begin{align*}
    k_5^2 &= 4(k_1.k_5)(k_2.k_5)/s \\
    &= (s + t + u)(s + t' + u') \\
    &= (s + t + u)^2/s.
\end{align*}
\]

(25)

Using Eqs. (23), (24) and (25), the square of the matrix element can be written as

\[
|M|^2_{gg \rightarrow ggg} = \frac{27}{2} g^6 (s^4 + t^4 + u^4 + 2s^2k_\perp^2) \frac{1}{sk_\perp^2}
\]

\[
\times \left[ \frac{1}{s} \left( \frac{1}{t^2} + \frac{1}{u^2} \right) - \frac{1}{t} \left( \frac{1}{s^2} + \frac{1}{u^2} \right) - \frac{1}{u} \left( \frac{1}{s^2} + \frac{1}{t^2} \right) \right]
\]

\[
= g^2 \left( \frac{27}{2} g^4 s^4 \right) \left( 1 + \frac{t^4}{s^4} + \frac{u^4}{s^4} + 2 \frac{k_\perp^4}{s^4} \right)
\]

\[
\times \frac{1}{s^2 k_\perp^2 t^2} \left[ 1 + t^2 u^2 - t s - s t^2 - s u - u t^2 \right]
\]

\[
= g^2 \left( \frac{9}{2} s^2 t^2 \right) \left( 3 + \frac{3t^4}{s^4} + 3 \frac{u^4}{s^4} + 6 \frac{k_\perp^4}{s^4} \right)
\]

\[
\times \frac{1}{s^2 k_\perp^2} \left[ 1 + t^2 u^2 - t s - s t^2 - s u - u t^2 \right]
\]

\[
= g^2 \left( \frac{9}{2} s^2 t^2 \right) \left( 3 + \frac{3t^4}{s^4} + 3 \frac{u^4}{s^4} + 6 \frac{k_\perp^4}{s^4} \right)
\]

\[
\times \frac{1}{s^2 k_\perp^2} \left[ 1 + t^2 u^2 - t s - s t^2 - s u - u t^2 \right]
\]

(26)
where the subscript GB stands for the approximation used by Gunion and Bertsch \[10\]. For the elastic process,
\[
|M_{gg \rightarrow gg}|^2 = \frac{9}{2} g^4 s^2 \frac{t^2}{u^2}.
\] (27)

On simplifying Eq. 26 we obtain,
\[
|M|^2_{gg \rightarrow gg} = g^2 |M_{gg \rightarrow gg}|^2
\]
\[
\times \left[ \frac{1}{k_{\perp}^2} \left( 3 \frac{3 s t}{u} + 3 \frac{u^4}{s^4} - 3 \frac{u^3 t}{s^3} - \left( \frac{t}{s} + \frac{3 t s}{u^2} \right) + 3 \frac{u^4 t}{s^3} + 3 \frac{u^2 t}{s^3} \right) + \left( \frac{3 t^2}{u^2} - \frac{3 t^2}{u s} + 3 \frac{u^2 t^2}{s^4} - 3 \frac{u^3 t^2}{s^5} \right) + \left( \frac{3 t^4}{s^4} - \frac{3 t^4}{u s^3} \right) - \left( \frac{3 t^6}{s^6} + \frac{3 t^6}{u^2 s^4} + 3 \frac{u^6 t^2}{s^7} \right) + \left( 6 \frac{k_{\perp}^4}{s^2} - 6 \frac{k_{\perp}^4}{u s^2} \right) - \left( 6 \frac{k_{\perp}^4}{u^2 s^2} - 6 \frac{k_{\perp}^4}{u^3 s^3} \right) \right].
\] (28)

In the proposed kinematic limit we set terms which are linear in \(k_{\perp}^2\) to zero and keep terms \(O(k_{\perp}^0), O(k_{\perp}^{-1})\), and \(O(k_{\perp}^{-2})\) in \(|M|^2_{gg \rightarrow gg}\). We also neglect terms \(O(1/s)\) and higher order in the matrix element. To proceed further one needs to express the Mandelstam variable \(u\) in terms of \(s, t\), and \(k_{\perp}\) by the following relation:
\[
k_{\perp}^2 = \frac{(s + t + u)^2}{s}
\]
\[
\Rightarrow u = \sqrt{s}k_{\perp} - s - t
\]
\[
\Rightarrow \frac{1}{u} = \frac{1}{\sqrt{s}k_{\perp} - s - t}
\]
\[
\Rightarrow \frac{1}{u} = -\frac{1}{s} \left[ 1 - \left( \frac{k_{\perp}}{\sqrt{s}} - \frac{t}{s} \right) \right]^{-1}
\]
\[
\Rightarrow \frac{1}{u} \approx -\frac{1}{s} \left[ 1 + \left( \frac{k_{\perp}}{\sqrt{s}} - \frac{t}{s} \right) \right]^2
\]
\[
+ \left( \frac{k_{\perp}}{\sqrt{s}} - \frac{t}{s} \right)^3
\]
\[
+ \left( \frac{k_{\perp}}{\sqrt{s}} - \frac{t}{s} \right)^4
\]
\[
+ \left( \frac{k_{\perp}}{\sqrt{s}} - \frac{t}{s} \right)^5 + \ldots.
\] (29)

The binomial expansion of \(1 - \left( \frac{k_{\perp}}{\sqrt{s}} - \frac{t}{s} \right)^{-1}\) converges if \((\frac{k_{\perp}}{\sqrt{s}} - \frac{t}{s}) < 1.\) For the kinematic limit mentioned above \(i.e.\) for \(k_{\perp} \to 0\) and keeping terms up to \(O(\frac{1}{s})\), the inequality \((\frac{k_{\perp}}{\sqrt{s}} - \frac{t}{s}) < 1\) is satisfied. We have checked that terms beyond \((\frac{k_{\perp}}{\sqrt{s}} - \frac{t}{s})^3\) in the expression of \(\frac{1}{u}\) are not required for the kinematic limit under consideration. With all these we get,
\[
\frac{1}{u} = -\frac{1}{s} \left( 1 - \frac{t}{s} + \frac{t^2}{s^2} - \frac{t^3}{s^3} \right)
\]
\[
+ \left( \frac{1}{\sqrt{s}} - \frac{2t}{s\sqrt{s}} + \frac{3t^2}{s^2\sqrt{s}} \right) k_{\perp}
\]
\[
+ \left( \frac{1}{s} - \frac{3t}{s^2} + \frac{6t^2}{s^3} \right) k_{\perp}^2 \right].
\] (30)

Similarly \(1/u^2\) can be written as
\[
\frac{1}{u^2} = \frac{1}{s^2} \left( 1 - \frac{2t}{s} + \frac{3t^2}{s^2} - \frac{4t^3}{s^3} \right)
\]
\[
+ \left( \frac{2}{\sqrt{s}} - \frac{6t}{s\sqrt{s}} + \frac{12t^2}{s^2\sqrt{s}} \right) k_{\perp}
\]
\[
+ \left( \frac{3}{s} - \frac{12t}{s^2} + \frac{30t^2}{s^3} \right) k_{\perp}^2 \right].
\] (31)

For the assumed kinematic conditions \(u^4/s^4\) can be expressed as follows:
\[
\frac{u^4}{s^4} = \left[ (1 + \frac{4t}{s} + \frac{6t^2}{s^2} + \frac{4t^3}{s^3} \right)
\]
\[
- \left( \frac{4}{\sqrt{s}} + \frac{12t}{s\sqrt{s}} + \frac{12t^2}{s^2\sqrt{s}} \right) k_{\perp}
\]
\[
+ \left( \frac{6}{s} + \frac{12t}{s^2} + \frac{6t^2}{s^3} \right) k_{\perp}^2 \right].
\] (32)

Similarly,
\[
\frac{u^3}{s^3} = -\left[ (1 + \frac{3t}{s} + \frac{3t^2}{s^2} + \frac{t^3}{s^3} \right)
\]
\[
- \left( \frac{3}{\sqrt{s}} + \frac{6t}{s\sqrt{s}} + \frac{3t^2}{s^2\sqrt{s}} \right) k_{\perp}
\]
\[
+ \left( \frac{3}{s} + \frac{3t^2}{s^2} \right) k_{\perp}^2 \right];
\] (33)

and
\[
\frac{u^2}{s^2} = \left[ (1 + \frac{2t}{s} + \frac{t^2}{s^2} \right) - \left( \frac{2}{\sqrt{s}} + \frac{2t}{s\sqrt{s}} \right) k_{\perp}
\]
\[
+ \frac{1}{s} k_{\perp}^2 \right].
\] (34)

Putting Eqs. 30 to 34 in 28 we get,
\[
|M|^2_{gg \rightarrow gg} = 12g^2 |M_{gg \rightarrow gg}|^2 \frac{1}{k_{\perp}^2}
\]
\[
\times \left[ (1 + \frac{t}{2s} + \frac{5t^2}{2s^2} - \frac{t^3}{s^3} \right)
\]
\[
- \left( \frac{3}{2s\sqrt{s}} + \frac{4t}{s\sqrt{s}} - \frac{3t^2}{2s^2\sqrt{s}} \right) k_{\perp}
\]
\[
+ \left( \frac{5}{2s} + \frac{t}{2s^2} + \frac{5t^2}{s^3} \right) k_{\perp}^2 \right].
\] (35)
The terms $\mathcal{O}(k^{-1})$ and $\mathcal{O}(k^0)$ contribute to the energy loss of the gluons in a gluonic plasma and hence are important for heavy-ion phenomenology at RHIC and LHC energies. These terms were absent in the previous work [12] (also in [11]).

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