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Glitches and precession damping in the framework of 3-component model of neutron star

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Abstract. The evolution of the inclination angle and precession damping is considered. It is assumed that the neutron star consists of 3 freely rotating components: the crust and two core components, one of them contains pinned superfluid vortices. We suppose that each component rotates as a rigid body. Also the influence of the small-scale magnetic field on the star’s breaking process is examined. Within the framework of this model the star simultaneously can have glitch-like events combined with long-period precession (with periods $10^{-10^4}$ years). It is shown that the case of the small quantity of pinned superfluid vortices seems to be more consistent with observations.

1. Introduction
Radio pulsars can be considered as exceptionally stable clock. But sometimes besides gradually braking its rotation suffers some irregularities called glitches \[1, 2\]. Usually it is related to superfluid pinning. Pinned vortices do not allow superfluid to participate in slow pulsar braking and in some moments it leads to the unpinning and to the transfer of the angular momentum from superfluid to the crust \[2\]. That is observed as a glitch. There are some evidences that isolated neutron stars may precess with enough long period $T_p$. Pulsar B1821-11 precesses with period $T_p \sim 500$ days \[3\]. Some other pulsars show periodic variations which may be explained by precession with $T_p \sim 100 - 500$ days \[4\]. The changing of pulse profile of Crab pulsar also may be related to precession with $T_p \sim 10^2$ years \[5\]. The increase of radio luminosity of Geminga pulsar \[6\] may be related to precession with $T_p > 10$ years too. The nonregular pulse variations known as ”red noise” \[1, 2\] with timescales $T \sim 1$ month – $10^4$ years also can be related to precession \[7\]. But presence of pinned vortices leads to precession with periods $T_p \sim P \cdot (I_{tot}/L_g) \sim 10^2 - 10^5 P$ \[8\], where $P$ is pulsar period, $I_{tot}$ is moment of inertia of the star, $L_g$ is angular momentum of pinned superfluid. Such precession seems to be rapidly damped and incompatible with existence of the long-period precession \[8\]. In this paper we consider a model, proposed in \[9\]. It allows the coexistence of long-term precession and quasi-glitch events. We also take into account the influence of the small-scale magnetic field on pulsar braking.

2. Base equations
We assume that neutron star consists of 3 components, named as crust ($c$-component), $g$-component and $r$-component:

\textbf{The crust ($c$-component) }. We suppose that it rotates as a rigid body with angular velocity
\( \bar{\Omega}_c \). It is the outer component so its rotation is observed as a pulsar rotation and hence \( \bar{\Omega}_c \) can be considered as the angular velocity of pulsar \( \bar{\Omega} \). We suppose that

\[ \vec{M}_c = I_c \vec{\Omega}_c \quad \text{and} \quad \dot{\vec{M}}_c = \vec{K}_{ext} + \vec{N}_{gc} + \vec{N}_{rc} \]

(1)

where \( \vec{M}_c \) is the angular momentum of the crust, \( I_c \) is moment of inertia, \( \vec{K}_{ext} \) is the external torque (magnetospheric origin), acting on the crust, \( \vec{N}_{gc} \) and \( \vec{N}_{rc} \) are torques acting on the crust due to its interaction with \( g \) and \( r \) components correspondingly.

g-component. It’s the one of two inner components. We assume that it consists of normal matter rotating as a rigid body with angular velocity \( \bar{\Omega}_g \) and superfluid matter firmly pinned to normal matter so that superfluid vortices rotate together with normal matter with angular velocity \( \vec{\Omega}_g \).

\[ \vec{M}_g = I_g \vec{\Omega}_g + \vec{L}_g \quad \text{and} \quad \dot{\vec{M}}_g = \vec{N}_{cg} + \vec{N}_{rg} \quad \text{,} \quad \dot{\vec{L}}_g = [\vec{\Omega}_g \times \vec{L}_g] \]

(2)

where \( \vec{M}_g \) is total angular momentum of \( g \)-component, \( I_g \) is moment of inertia of its normal matter, \( L_g \) is angular momentum of pinned superfluid, \( \vec{N}_{cg} \) and \( \vec{N}_{rg} \) are torques acting on the crust due to its interaction with the crust and with \( r \)-component correspondingly.

\( r \)-component. It is inner component too. We assume that it rotates as a rigid body with angular velocity \( \vec{\Omega}_r \).

\[ \vec{M}_r = I_r \vec{\Omega}_r \quad \text{and} \quad \dot{\vec{M}}_r = \vec{N}_{cr} + \vec{N}_{gr} \]

(3)

where \( \vec{M}_r \) is angular momentum of \( r \)-component, \( I_r \) is moment of inertia, \( \vec{N}_{cr} \) and \( \vec{N}_{gr} \) are torques acting on the crust due to its interaction with the crust and with \( g \)-component correspondingly. In the frame of references, related to the crust, the equation of rotation (1)-(3) can be rewritten as

\[ \dot{\vec{\Omega}} = \vec{S}_{ext} + \vec{R}_{gc} + \vec{R}_{rc} \]

(4)

\[ \dot{\vec{\mu}}_{cg} + [\vec{\Omega} \times \vec{\mu}_{cg}] + [\vec{\Omega} \times \vec{\omega}_g] + [\vec{\mu}_{cg} \times \vec{\omega}_g] = \vec{R}_{cg} + \vec{R}_{rg} - \vec{R}_{gc} - \vec{R}_{rg} - \vec{S}_{ext} \]

(5)

\[ \dot{\vec{\mu}}_{cr} + [\vec{\Omega} \times \vec{\mu}_{cr}] = \vec{R}_{cr} + \vec{R}_{gr} - \vec{R}_{rc} - \vec{R}_{gc} - \vec{S}_{ext} \]

(6)

\[ \dot{\vec{\omega}}_g = [\vec{\mu}_{cg} \times \vec{\omega}_g] \]

(7)

where \( \vec{\mu}_{ij} \) is \( \vec{\Omega}_j - \vec{\Omega}_i \), \( \vec{N}_{ij} = -\vec{N}_{ji}, \vec{R}_{ij} = \vec{N}_{ij}/I_j \) \( (i, j = c, g, r) \), \( \vec{S}_{ext} = \vec{K}_{ext}/I_c \) and \( \vec{\omega}_g = \vec{L}_g/I_g \).

In this paper we consider only small perturbation of the equilibrium state \( \vec{\Omega}_c = \vec{\Omega}_g = \vec{\Omega}_r \), \( \vec{L}_g = L_g \vec{e}_\Omega \), where \( \vec{e}_\Omega = \vec{\Omega}/\Omega \) and we neglect any terms quadratic in \( \vec{\mu}_{ij} \). For simplicity we suppose that

\[ \vec{N}_{ij} = -I_j \left( \alpha_{ij} \vec{\mu}_{ij} - \beta_{ij} \vec{\mu}_{ij} \right) \]

(8)

where \( \alpha_{ij}, \beta_{ij}, \gamma_{ij} \) are some constants and we introduce \( A^\parallel \vec{A} \), \( A^\perp \vec{A} = \vec{A} - A^\parallel \vec{e}_\Omega \) for any value \( \vec{A} \).

In order to calculate the magnetospheric torque \( \vec{K}_{ext} \) acting on the crust we use the model proposed in [13]. It is assumed that neutron star is braking simultaneously by both magnetodipolar and current losses. Hence

\[ \vec{K}_{ext} = -\frac{I_{\text{tot}}}{\tau_0} \left( \vec{e}_\Omega - (1 - \alpha) \cos \chi \vec{e}_m - R_{\text{eff}} [\vec{e}_\Omega \times \vec{e}_m] \right) \]

(9)

where \( m \vec{e}_m \) is the dipolar magnetic moment of neutron star, \( \chi \) is the inclination angle (angle between \( \vec{e}_\Omega \) and \( \vec{e}_m \), see fig.3), \( \tau_0 = \frac{3}{2} R_{\text{ns}}^3 I_{\text{tot}} \), the coefficient \( R_{\text{eff}} \) is related to inertia of the magnetic field [14, 15]. In this paper we accept that \( R_{\text{eff}} = \frac{\mu_0}{m_e} \frac{e_m}{G_{\text{ns}}} \sim 5 \times 10^3 \left( \frac{\mu}{G_{\text{ns}}} \right) \) [16], where
$r_{ns}$ is neutron star radius. The coefficient $\alpha$ is related to the value of the current flowing through the pulsar tubes. In this paper we assume that there are only "inner gaps" with free electron emission from neutron star surface in pulsar tube. Hence the current depends on the value of surface small-scale field, see fig. 1. The value of $\alpha$ averaged over precession angle $\phi_\Omega$, see fig 3, $\langle \alpha \rangle (\chi) = \frac{1}{2\pi} \int_0^{2\pi} \alpha(\chi, \phi_\Omega) d\phi_\Omega$ calculated in [13] is shown in fig 2. Here $\nu = B_{sc}/B_{dip}$, $B_{sc}$ is induction of small-scale magnetic field, $B_{dip} = 2m/r_{ns}^3$ induction dipolar field on magnetic pole of neutron star.

\begin{figure}[h]
\centering
\includegraphics[width=0.3\textwidth]{fig1.png}
\caption{It is shown the direction of small scale magnetic field $\vec{B}_{sc}$ and the direction of dipolar magnetic moment $\vec{m}$ in model considered by [13]. "Inner gap" is shown by brown area, neutron star is shown by grey area.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.3\textwidth]{fig2.png}
\caption{The dependence of the parameter $\langle \alpha \rangle$ on the inclination angle $\chi$ taken from [13]}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.3\textwidth]{fig3.png}
\caption{Definition of angles $\chi$ and $\phi_\Omega$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.3\textwidth]{fig4.png}
\caption{The evolution of angular velocity $\Omega(t)$ during quasi-glitch event. The pulsar braking is neglected.}
\end{figure}

3. Quasi-glitch events
Taking into account (5) and (6) the equation (7) can be rewritten as
\begin{equation}
\dot{\omega}^g_{||} = 0 \quad \text{and} \quad \frac{d}{dt} (\omega^g_{\perp}) = -\frac{\omega^g_{\perp}}{\Omega} \left( \vec{S}_{ext} + \vec{R}_{gc} + \vec{R}_{rc} \right) - \omega^g_{\perp} [\vec{e} \Omega \times \vec{\mu}^g_{\perp}] \tag{10}
\end{equation}
Hence, during precession the direction of $\vec{\omega}_g$ is adjusted so that pinned superfluid vortices will be directed almost along $\vec{e}_Q$. But its value $\omega_g$ does not change at all and due to pulsar braking the difference between $\omega_g$ and $\Omega$ will grow. So at some moment the glitch must occur in $g$-component. Assume that during the glitch the angular momentum $\Delta\vec{L}_g = \Delta L_g \vec{e}_\Omega$ is transferred from superfluid part to the normal part of $g$-component. If we neglect pulsar braking term $\vec{S}_{ext}$ and assume that before glitch neutron star rotates as a rigid body then the solution of (4)-(7) gives us, see fig. 4

$$\Omega(t) = \Delta\Omega \left(1 - e^{-p+\gamma} - Q(1 - e^{-p-\gamma})\right)$$

where $\Delta\Omega = \frac{\Delta\Omega_{\infty}}{1 - Q}$ and $\Delta\Omega_{\infty} = \frac{\Delta L_g}{I_{tot}}$ (11)

the coefficients $p_+$ and $p_-$ ($p_+ > p_-$) are roots of the equation

$$p^2 - (\alpha_{cg} + \alpha_{rg} + \alpha_{gr} + \alpha_{gc} + \alpha_{rc} + \alpha_{rc})\, p +$$

$$+ (\alpha_{gc} + \alpha_{rg} + \alpha_{cg}) \cdot (\alpha_{cr} + \alpha_{gr} + \alpha_{rc}) + (\alpha_{rc} - \alpha_{rg}) \cdot (\alpha_{gr} - \alpha_{gc}) = 0$$

(12) and $Q = \left(I_{tot}\alpha_{cg} - I_c p_+ / I_{tot}\alpha_{cg} - I_c p_-.\right)$ In the case of $\alpha_{cg} \gg \left(1 + \frac{L_c}{I_c}\right)\, \alpha_{rg},\, p_- \approx \frac{I_{tot}}{I_c + I_c} (\alpha_{cg} + \alpha_{gr})$ and $1 - Q \approx \frac{L_c + I_c}{I_{tot}}$. If we try to relate these quasi-glitch events to observed glitches then $1/p_-$ determine the glitch growth time and $1/p_-$ is related to glitch relaxation time we receive restrictions $1/p_- \leq 1$ min [10, 2] and $1/p_- ~ 1 - 10^2$ days [1]. Unfortunately, in our model $1 - Q \sim 10^{-2} - 10^{-1}$ that may be not so bad for glitches in some pulsar like Grab $Q \geq 0.8$ [2] and J0205+6446 $Q \approx 0.77$ [11], but obviously contradicts glitches in majority of pulsars $Q \ll 1$ [11, 12]. In particular, our model does not describe glitches in Vela pulsar $Q \leq 0.2$ [2].

4. Quasistatic approximation

Firstly we take into account that $\tau_{rel} \ll \tau_0$, where $\tau_{rel} \sim \max(1/\alpha_{ij}, 1/\beta_{ij}, 1/\gamma_{ij}) \sim 1 - 10^7 s$ is relaxation time [2]. Hence we can consider the rotation of neutron star under small slowly varying torque $\vec{R}_{ext}$. In this case it’s possible to neglect terms $\dot{\mu}_{ij}$ in (4)-(7). And consequently we have

$$\dot{\Omega} = -\frac{\Omega}{\tau_0} \left(\sin^2 \chi + \alpha \cos^2 \chi\right)$$

(13)

$$\dot{\chi} = \frac{I_{tot}}{I_c \tau_0} \sin \chi \cos \chi \cdot (B (1 - \alpha) - \Gamma R_{eff})$$

(14)

$$\dot{\phi}_\Omega = -\frac{I_{tot}}{I_c \tau_0} \cos \chi \cdot (B R_{eff} + \Gamma (1 - \alpha))$$

(15)

where coefficients $B$ and $\Gamma$ are defined as

$$B + i\Gamma = \frac{\Omega(\Omega - i(\xi_{cr} + \xi_{gr}))}{\Omega^2 - i\Omega(\xi_{cr} + \xi_{gr} + \xi_{rc} + \xi_{gc}) - (\xi_{gr}\xi_{gc} + \xi_{gr}\xi_{rc} + \xi_{cr}\xi_{gc})}$$

(16)

where $\xi_{pq} = \beta_{pq} + i\gamma_{pq}$. In the case of weak viscosity $|\xi_{pq}| \ll \Omega$ we have $B \approx 1 - \frac{\gamma_{cr} + \gamma_{gc}}{\Omega}$ and $\Gamma \approx \frac{\beta_{cr} + \beta_{gc}}{\Omega}$. It’s worth to note that equation (13) has the same form as in the case of absolutely rigid star. According to (15) as long as inclination angle $\chi$ differs from 0° or 90° the star will precess with period $T_p \sim \left(\tau_0/R_{eff}\right) \cdot \left(I_c/I_{tot}\right) \sim 10 - 10^4$ years. We suppose that the period of precession $T_p \ll (I_c/I_{tot}) \cdot \tau_0 \approx \tau_0$ hence we can average (13) and (14) over precession.

$$\frac{d\chi}{dP} = -\frac{1}{P} \cdot \frac{I_{tot}}{I_c} \cdot \sin \chi \cos \chi \cdot \frac{B (1 - <\alpha>) - \Gamma R_{eff}}{\sin^2 \chi <\alpha > + \cos^2 \chi}$$

(17)
We consider only one model of components interaction, when the crust and \(g\)-component interact with \(r\)-component like a normal matter with superfluid and there are normal viscosity between the crust and \(g\)-component \([9, 2]\)

\[
\beta_{cr} = \beta_{gr} = \Omega \frac{\sigma}{1 + \sigma^2}, \quad \alpha_{cr} = \alpha_{gr} = 2\beta_{gc}, \quad \gamma_{cr} = \gamma_{gr} = -\sigma \beta_{gc}, \quad \beta_{cg} = \alpha_{cg} \quad \text{and} \quad \gamma_{cg} = 0 \quad (18)
\]

The evolution of the inclination angle \(\chi\) for different initial inclination angle \(\chi_{start}\) in the case of \(\sigma = 10^{-6}\), \(\alpha_{cg} = 10^{-1}s^{-1}\) and \(\nu = 0.5\) is shown in fig 5. We can see that in the majority of cases inclination angle became equal to its equilibrium value \(B(1 - <\alpha>) - \Gamma R_{eff} \approx 0\) very rapidly and all subsequent evolution is related with the slow changing of \(R_{eff}\) value due to pulsar braking. The evolution of the inclination angle \(\chi\) for various values \(\nu\) in the case of starting angle \(\chi_{start} = 45^\circ\) and \(\sigma = 10^{-6}\), \(\alpha_{cg} = 10^{-1}s^{-1}\) is shown in fig 6. Left graph corresponds to \(I_c = 10^{-1}I_{tot}, I_g = 10^{-5}I_{tot}\) right graph to \(I_c = 10^{-2}I_{tot}, I_g = 10^{-3}I_{tot}\). Observed values of inclination angles \(\beta_2\) taken from \([17]\) are shown by red dots \((C > 0)\) and blue dots \((C < 0)\). Pulsar parameters are taken from \([18]\). It seems that the case of small \(g\)-component is in the better agreement with observations.

![Figure 5](image)

**Figure 5.** The evolution of inclination angle \(\chi\) in case of \(\sigma = 10^{-6}\), \(\alpha_{cg} = 10^{-1}s^{-1}\), \(\nu = 0.5\) for different initial inclination angles \(\chi_{start}\). Left graph corresponds to \(I_c = 10^{-1}I_{tot}, I_g = 10^{-5}I_{tot}\), right corresponds to \(I_c = 10^{-2}I_{tot}, I_g = 10^{-3}I_{tot}\). The observed values of inclination angles \(\beta_2\) taken from \([17]\) are shown by red dots \((C > 0)\) and blue dots \((C < 0)\). Pulsar parameters are taken from \([18]\).

5. Conclusion
We consider a model, proposed in \([9]\), that allows simultaneously the long-period precession and quasi-glitch events with taking into account the influence of the small-scale magnetic field on pulsar braking. For simplicity we consider only the case of axial symmetric precession and don’t take into account that the presence of the small scale magnetic field makes precession triaxial \([15]\). The main problem of this model is the nature of \(g\)-component. It may be related with existence of rigid star core where superfluid is pinned \([19]\) or with long living tangle of closed fluxoids flowing inside liquid core on which superfluid vortices are pinned, compare with \([20]\).

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Figure 6. The same as in fig. 5 but for initial angle $\chi_{\text{start}} = 45^\circ$ and different values for parameter $\nu$.

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