Quark liberation and coalescence at CERN SPS

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Abstract

The mischievous linear coalescence approach to hadronization of quark matter is shown to violate strangeness conservation in strong interactions. The simplest correct quark counting is shown to coincide with the non-linear algebraic coalescence rehadronization model, ALCOR. The non-linearity of the ALCOR model is shown to cancel from its simple predictions for the relative yields of (multi-)strange baryons. We prove, model independently, that quark degrees of freedom are liberated before hadron formation in 158 AGeV central Pb + Pb collisions at CERN SPS.

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1 Introduction

It is a deep rooted desire to explain complicated experimental observations with simple and transparent models, understandable in laymen’s terms. This very acceptable ambition inspired a recent publication [1], where an attempt was made to explain the relations between the multiplicities of different strange baryons produced in heavy ion reactions with the help of linear coalescence rehadronization model. This model was based on simple quark counting and elementary probability estimates. However, the considerations in Ref. [1] lead to a clear violation of strangeness conservation in strong interactions: In order to explain the data in laymen’s terms, Bialas had to assume in heavy ion reactions the number produced of strange quarks, \( s \), is not equal with the number of produced anti-strange quarks, \( \bar{s} \).

We show here that this unacceptable assumption was related to the neglected requirement that all quarks have to hadronize, as no free quarks are observed in nature. This necessity leads to a non-linearity even in the simplest algebraic rehadronization model (see Ref. [2]). Properly taking into account the non-linear competition for quarks by the various hadron forming coalescence channels is a natural way to correct the linear coalescence treatment in the simplest possible manner. This leads to the normalized, non-linear algebraic coalescence rehadronization model, ALCOR [3], which automatically takes care of the conservation of strangeness in strong interactions as well.

2 Linear coalescence in rehadronization?

In the linear coalescence rehadronization model Bialas assumed, that the number of produced particles is proportional to the product of the numbers of constituent particles within the reaction volume. In the following we denote the number of particles by the particle symbols, in particular, \( q \) stands for the number of light (up and down) quarks. Linear coalescence yields the following relations:

\[
\begin{align*}
p &= a_p q^3, \\
\Lambda|\Sigma &= a_\Lambda q^2 s, \\
\Xi &= a_\Xi q s^2, \\
\Omega &= a_\Omega s^3,
\end{align*}
\]

where \( \Lambda|\Sigma = \Lambda + \Sigma \) stands for the total number of strange baryons that contain a single strange quark.
Similarly, for the anti-baryons one obtains

\[
\overline{p} = a_{p} \overline{p}^3, \tag{5}
\]

\[
\overline{\Lambda}/\Sigma = a_{\Lambda} \overline{\Lambda}^2 \Sigma, \tag{6}
\]

\[
\overline{\Xi} = a_{\Xi} \overline{\Xi} \Sigma^2, \tag{7}
\]

\[
\overline{\Omega} = a_{\Omega} \overline{\Omega}^3. \tag{8}
\]

Furthermore, Ref. [1] also assumed, that the coefficients of proportionality for particles and their antiparticles are equal: \( a_{\Xi} = a_{\Omega} \), etc. As the coefficients \( a_{\Xi} \) describe effectively the (particle anti-particle symmetric) likelihood that a multi-quark bound state is formed once the constituents are given, this seems to be a very reasonable model at first sight.

This linear model is attractive not only because of its simple formulation but also because of its simple predictions, namely that the unknown coefficients cancel from the anti-baryon to baryon ratios. This leads to the following equations:

\[
\frac{\overline{p}}{p} = \left[ \frac{\overline{\Xi}}{\Xi} \right]^3, \tag{9}
\]

\[
\frac{\overline{\Lambda}/\Sigma}{\Lambda}/\Sigma = \left[ \frac{\overline{\Omega}/\Omega}{\Omega} \right]^2 \left[ \frac{\Xi}{\Sigma} \right], \tag{10}
\]

\[
\frac{\overline{\Xi}}{\Xi} = \left[ \frac{\overline{\Omega}/\Omega}{\Omega} \right] \left[ \frac{\Xi}{\Sigma} \right]^2, \tag{11}
\]

\[
\frac{\overline{\Omega}/\Omega}{\Omega} = \left[ \frac{\Xi}{\Sigma} \right]^3. \tag{12}
\]

In order to explain the value of

\[
\frac{\overline{\Omega}/\Omega}{\Omega} = 0.383 \pm 0.081, \tag{13}
\]

as measured in Pb +Pb reactions at CERN SPS energies, Bialas had to assume \( s \neq \overline{s} \) as a consequence of eq. (12) of his linear model, which is a clear violation of strangeness conservation in strong interactions.

What is the origin of this contradiction?

The linear coalescence model, as defined in eqs. (3) and (4) is a good approximation only if the composite particles use up a small fraction of the constituents. This is the case e.g. in the coalescence treatment of the deuteron \((pn)\) or triton \((pnn)\) formation from the gas of protons and neutrons. In that situation most particles will remain in nucleon \((p \text{ or } n)\) state. Thus the competition is negligible among the tritons and the deuterons for the building block nucleons in that case.

In the case of hadronization of a quark matter all of the constituent particles (the quarks) have to be placed into composite particles, namely into colorless hadrons. No free quarks are observed in Nature. This is the very essence of color confinement. Thus, during the hadronization process, a fixed number of strange quarks must be distributed among hyperons and anti-K mesons. The anti-strange quarks have to be distributed among anti-hyperons and K mesons. [Note that K meson is a \((q\overline{s})\) bound state, while the anti-K meson is a \((\overline{q}s)\) bound state.] However, in heavy ion collisions we have incoming nuclei in the initial state, that contain nucleons, formed by constituent quarks, only. As quarks are produced in quark antiquark pairs, the number of quarks in the final state is larger than the number of anti-quarks. Hence we must have more K mesons than anti-K mesons.

The processes creating different hadrons are not independent, they compete with each other, contrary to the basic assumption in the linear coalescence model.

The redistribution can be counted for by introducing normalization factors, \( b_{k}, b_{\Xi}, b_{\Omega}, b_{\overline{\Xi}}, b_{\overline{\Omega}} \). These \( b_{k} \) normalization factors are not free parameters! They are determined from the requirement that all quarks must be recombined into hadrons during the hadronization process, as described first in the ALCOR model, Ref. [3].
3 The ALCOR approach

The non-linear coalescence equations for the formation of quark-antiquark clusters during rehadronization read as:

\[ p = C_p b_q^3 q^3, \]  
\[ \Lambda|\Sigma = C_\Lambda b_q^2 b_s q^2 s, \]  
\[ \Xi = C_\Xi b_q b_s^2 q s^2, \]  
\[ \Omega = C_\Omega b_s^3, \]  
\[ \bar{p} = C_{\bar{p}} b_q^3 q^3, \]  
\[ \bar{\Lambda}|\bar{\Sigma} = C_{\bar{\Lambda}} b_q^2 b_s q^2 s, \]  
\[ \bar{\Xi} = C_{\bar{\Xi}} b_q b_s^2 q s^2, \]  
\[ \bar{\Omega} = C_{\bar{\Omega}} b_s^3. \]

The meson yields are determined similarly,

\[ \pi^d = C_\pi b_q b_s q \bar{q}, \]  
\[ K = C_K b_q b_s q \bar{s}, \]  
\[ \bar{K} = C_{\bar{K}} b_q b_s q \bar{s}, \]  
\[ \eta = C_\eta b_s b_s \bar{s}. \]

The normalization coefficients \( b_q, b_s \) and \( b_\eta \) are determined uniquely by the requirement, that the number of the constituent quarks do not change during the hadronization — which is the basic assumption for all quark counting methods:

\[ s = 3 \Omega + 2 \Xi + \Lambda|\Sigma + \bar{\Xi} + \bar{\Lambda} + \eta, \]  
\[ \bar{s} = 3 \Omega + 2 \Xi + \bar{\Lambda}|\bar{\Sigma} + K + \eta, \]  
\[ q = 3 p + 2 \Lambda|\Sigma + \Xi + K + \pi^d, \]  
\[ \bar{q} = 3 \bar{p} + 2 \bar{\Lambda}|\bar{\Sigma} + K + \bar{\Xi} + \bar{\Xi} + \eta. \]

Here \( \pi^d \) is the number of directly produced (\( q\bar{q} \)) states. (Note that other particle symbols also stand for the number of directly produced particles of a given type, and care must be taken when comparing the ALCOR predictions with data, especially regarding the corrections of particle numbers for the feed-down of resonance decay contributions.)

Similar, non-linear coalescence equations with constraints of using up all the coalescing particles were proposed in a simpler form first in Ref. [2] in the so called combinatoric break-up model, assuming a constant coalescence coefficient for all the baryons and another one for all the mesons. Later the ALCOR model was formulated as presented above in a simple form. ALCOR equations were solved numerically in Refs. [3, 4, 5], using, as input values to the coalescence equations, the number of quarks as predicted by Monte Carlo event generators. The ALCOR equations were derived from a set of rate equations in Ref. [6], in the sudden approximation.

Substituting eqs. (14 - 25) into the constraints given by eqs. (26-29) one obtains a set of non-linear equations for the \( b_i \) normalization constants. However, one can predict some relations even without solving these set of nonlinear equations. It turns out that the particle-antiparticle ratios can be expressed in terms of the effective number of quarks even in the non-linear ALCOR model, similarly as it was done by Bialas in the linear coalescence model.

Let us denote by upper case \( Q, S \) the reduced, effective number of up+down and strange quarks, in contrast to the lower case \( q \) and \( s \), that stand for the total number of (up+down) and strange quarks before the hadronization:

\[ Q = b_q q, \]  
\[ \frac{Q}{Q} = b_s q, \]  
\[ S = b_s s, \]  
\[ \bar{S} = b_{\bar{s}} \bar{s}. \]
Furthermore, one could make the usual assumption that the $C$ coalescence coefficients for baryons are equal to that of the corresponding antibaryons,

\[ C_p = C_{\bar{p}}, \quad (34) \]

\[ C_\Lambda = C_{\bar{\Lambda}}, \quad (35) \]

\[ C_\Xi = C_{\bar{\Xi}}, \quad (36) \]

\[ C_\Omega = C_{\bar{\Omega}}. \quad (37) \]

With these notations, the following relations are obtained in the non-linear ALCOR model:

\[ \frac{\bar{p}}{p} = \left[ \frac{Q}{Q} \right]^3, \quad (38) \]

\[ \frac{\Lambda|\Sigma}{\Lambda|\Sigma} = \left[ \frac{Q}{Q} \right]^2 \left[ \frac{S}{S} \right], \quad (39) \]

\[ \frac{\Xi}{\Xi} = \left[ \frac{Q}{Q} \right] \left[ \frac{S}{S} \right]^2, \quad (40) \]

\[ \frac{\Omega}{\Omega} = \left[ \frac{S}{S} \right]^3, \quad (41) \]

\[ \frac{K}{K} = \frac{Q}{Q} \left[ \frac{S}{S} \right]. \quad (42) \]

These equations are formally similar to the equations of the linear treatment displayed in eqs. (12), however, they are given in terms of the effective number of quarks, $Q = b_q q$, that are complicated, non-linear functions of the number of quarks $q, s$ available before the hadronization. Thus one easily gets the following interesting relations:

\[ \frac{\Lambda|\Sigma}{\Lambda|\Sigma} = \frac{\bar{p}}{p} \left[ \frac{K}{K} \right], \quad (43) \]

\[ \frac{\Xi}{\Xi} = \left[ \frac{Q}{Q} \right] \left[ \frac{K}{K} \right]^2, \quad (44) \]

\[ \frac{\Omega}{\Omega} = \left[ \frac{S}{S} \right]^3 \left[ \frac{K}{K} \right]. \quad (45) \]

These are the relations among the ratios of the observable number of particles that should be satisfied if the particle production in some reaction proceeds via algebraic recombination of the independent valence quarks. This way a direct relation is obtained in a self-consistent manner, that connects the presence/absence of independent, initial quarks with the observable yields of hadrons. This relation is to a large extent model-independent (i.e. independent of the initial quark content, independent from the values of the coalescence coefficients $C$ and independent from the values of the non-linear renormalization factors $b_i$). Only two physical assumptions enter these model-independent eqs. (43-45): that the rehadronization process is sudden and that the valence quarks are available in unbound states before the hadronization.

One may evaluate the following measurable numbers:

\[ d_\Lambda = \frac{\Lambda|\Sigma}{\Lambda|\Sigma} \frac{p}{\bar{p}}, \quad (46) \]

\[ d_\Xi = \left[ \frac{\Xi}{\Xi} \frac{p}{\bar{p}} \right]^{1/2}, \quad (47) \]

\[ d_\Omega = \left[ \frac{\Omega}{\Omega} \frac{p}{\bar{p}} \right]^{1/3}, \quad (48) \]

\[ d = \left[ \frac{K}{K} \right]. \quad (49) \]
If hadronization proceeds via a sudden recombination of quarks, then all these \( d \) numbers should be equal, \( d = d_\Lambda = d_\Xi = d_\Omega \).

The experimental values for the anti-baryon to baryon number ratios are determined in Refs. \[7, 8, 9\] as follows:

\[
\begin{align*}
\frac{N}{N} &= 0.070 \pm 0.010, \\
\frac{\Lambda|\Sigma}{\Lambda|\Sigma} &= 0.133 \pm 0.007, \\
\frac{\Xi}{\Xi} &= 0.249 \pm 0.019, \\
\frac{\Omega}{\Omega} &= 0.383 \pm 0.081, \\
\frac{K^+}{K^-} &= d = 1.80 \pm 0.2
\end{align*}
\]

which yields the following values for the \( d \) factors of strange baryons:

\[
\begin{align*}
d_\Lambda &= 1.9 \pm 0.3, \\
d_\Xi &= 1.89 \pm 0.15, \\
d_\Omega &= 1.76 \pm 0.15.
\end{align*}
\]

These numbers show a very good agreement with \( d \), a statistically acceptable, good agreement between the ALCOR model and the experimental data in case of Pb + Pb reaction at CERN SPS. This indicates, regardless of the details of the confinement mechanism, that hadron production in Pb +Pb reaction at CERN SPS proceeds via a sudden and complete coalescence of constituent quarks to hadrons.

Taking into account the conservation of strangeness in strong interactions, which demands \( \overline{s} = s \), from the \( \Omega/K \) ratio we arrive at

\[
\overline{s}/s = \frac{b_\Omega}{b_s} = 0.75 \pm 0.06.
\]

Thus we obtained agreement with the experimental data without assuming \( s \neq \overline{s} \). The deviation of \( \Omega/K \) from unity is caused by the difference in the normalization factors \( b_\Lambda \) and \( b_\Xi \) which can easily be understood: there are more quarks than antiquarks in the initial system, and thus more \( \overline{s} \) quarks are used up in the \( K^+ \) production than \( s \) quarks in the \( K^- \) meson creation. Thus less \( \overline{s} \) remains for the \( \Omega \) production.

In the time-dependent solution of confining rate equations for the hadronization \[11\], conservation laws are ensured by the structure of the coupled system of differential equations. The ALCOR model, as presented above, can be reobtained from a set of non-linear rate equations in the sudden approximation \[6\], in the limit when the time of hadronization is very short. Indeed, a detailed analysis of single particle spectra and two-particle correlation data indicates a short duration, \( \Delta \tau \approx 1.5 \text{ fm/c} \), for the production of final state hadrons at CERN SPS Pb + Pb reactions \[10\]. This result excludes a long-lived, evaporative mixture of quark-gluon plasma and hadronic phase, that would produce pions over an order of magnitude larger period. Thus the experimental data prefer a sudden production of hadrons, a process that may happen to be out of local thermal and chemical equilibrium.

Finally, let us point out, that the strangeness conservation leads to the following relation:

\[
s = 3 \frac{\Omega}{\Xi} - 2 \frac{\Xi}{\Lambda|\Sigma} + 1 = 3 \frac{\Omega}{\Xi} + \frac{\Xi}{\Lambda|\Sigma} + 1 = 3 \frac{\Xi}{\Lambda|\Sigma} + 2 \frac{\Xi}{\Lambda|\Sigma} + \frac{K}{\Lambda|\Sigma} + 1.
\]

With the SPS Pb+Pb data \[7, 8, 9\] the left hand side of this equation is 2.57, while the right hand side is 2.66. Thus the strangeness conservation equation is also well fulfilled for this case. The validity of eq. \[60\] is an absolute measure of strangeness conservation in strong interactions. If the experimental values do not satisfy eq. \[60\], then the detectors miss important parts of the particle momentum distributions, or they have a serious systematic error.

Further observations of Ref. \[1\], namely \( d_\Lambda = d_\Xi \) for the SPS \( ^{32}S +^{23}S \) reactions, but \( d_\Lambda \neq d_\Xi \) for the SPS p+Pb reaction remain unchanged with our analysis.
4 Conclusions

We clarified the reason why the linear coalescence model should not be applied to the hadronization of the quark matter: it leads to a violation of strangeness conservation.

We have shown, however, that this shortcoming can be corrected if a nonlinear coalescence model is introduced, that takes into account the conservation of strangeness in strong interactions and the fact that no free quarks are observed in Nature. In particular, we have shown that the solution of the ALCOR hadronization model yields simple, parameter independent relations in a consistent manner. These relations were surprisingly simple and similar to those obtained from the linear coalescence picture.

Data on strange particle production in Pb+Pb reactions at CERN SPS satisfy the coalescence model predictions, while they are known to be not satisfied by data from p + Pb reactions at CERN SPS. Based on considerations of the production ratios of strange hadrons, we have obtained an elementary, self-consistent and model independent proof, that quark degrees of freedom are liberated in Pb + Pb reactions at CERN SPS before the onset of hadron formation.

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References

[1] A. Bialas, Phys. Lett. B442 (1998) 449.
[2] T. S. Biró and J. Zimányi, Nucl. Phys. A395 (1983) 525.
[3] T.S. Biró, P. Lévai, and J. Zimányi, Phys. Lett. B347 (1995) 6.
[4] T.S. Biró, P. Lévai, and J. Zimányi, J. Phys. G23 (1997) 1941.
[5] T.S. Biró, P. Lévai, and J. Zimányi, J. Phys. G25 (1999) 311.
[6] J. Zimányi, T.S. Biró, T. Csörgő, and P. Lévai, Heavy Ion Phys. 4 (1996) 15.
[7] M. Kaneta et al., NA44 collaboration, J. Phys. G23 (1997) 1865.
[8] R. Caliandro et al., WA97 collaboration, J. Phys. G25 (1999) 171.
[9] C. Bormann for the NA49 Coll., J. Phys. G23 (1997) 1817.
[10] A. Ster, T. Csörgő and B. Lóránt, hep-ph/9907338, Nucl. Phys. A (1999) in press.
[11] T.S. Biró, P. Lévai, J. Zimányi, Phys. Rev. C59 (1999) 1574.