Evidence of Bose–Einstein condensation of alpha clusters in $^{16}\text{O}$ in field theoretical superfluid cluster model

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Observed well-developed $\alpha$ cluster states in $^{16}\text{O}$, located above the four $\alpha$ threshold, are investigated from the viewpoint of Bose–Einstein condensation of $\alpha$ clusters by using a field-theoretical superfluid cluster model in which the order parameter is defined. The experimental energy levels are reproduced well for the first time by calculation. In particular, the observed 16.7 MeV $0^+_2$ and 18.8 MeV $0^+_3$ states with low-excitation energies from the threshold are found to be understood as a manifestation of the states of the Nambu–Goldstone zero-mode operators, associated with the spontaneous symmetry breaking of the global phase, which is caused by the Bose–Einstein condensation of the vacuum 15.1 MeV $0^+_0$ state with a dilute well-developed $\alpha$ cluster structure just above the threshold. This gives evidence of the existence of the Bose–Einstein condensate of $\alpha$ clusters in $^{16}\text{O}$. It is found that the emergence of the energy level structure with a well-developed $\alpha$ cluster structure above the threshold is robust, almost independently of the condensation rate of $\alpha$ clusters under significant condensation rate. The finding of the mechanism why the level structure that is similar to $^{12}\text{C}$ emerges above the four $\alpha$ threshold in $^{16}\text{O}$ reinforces the concept of Bose–Einstein condensation of $\alpha$ clusters in addition to $^{12}\text{C}$.

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In this paper, we present evidence of Bose–Einstein condensation (BEC) of $\alpha$ clusters in $^{16}\text{O}$. The idea of cluster structure of nuclei was originally proposed in Refs. 1, 2 as the first nuclear structure model and since the proposal of the Ikeda diagram in $^{8}\text{Be}–^{24}\text{Mg}$ 3 and $^{8}\text{Be}–^{32}\text{S}$ 4, the last century witnessed a fruitful existence of cluster structure in light nuclei 5, 6. The cluster structure has been shown to persist also in medium-weight and heavy mass region 7, where the spin-orbit force becomes important. The diagram was extended to the $fp$-shell region, $^{44}\text{Ti}$ region in Ref. 8 and $^{60}\text{Zn}$ region in Ref. 9. The discoveries of the excitations of the inter-cluster relative (vibrational) motion (the higher nodal state) typically in $^{20}\text{Ne}$ 11 and $^{44}\text{Ti}$ 12, which is not expected from the mean field picture of nuclei 15, 14, showed a diversity of collective motion caused by clustering.

As a new diversity of collective motion, BEC of $\alpha$ clusters has been attracting much attention in the $N\alpha$ systems in $4N$ nuclei, such as three $\alpha$ clusters in $^{12}\text{C}$ and four $\alpha$ clusters in $^{16}\text{O}$, which are the highest order clustering in the Ikeda diagram 3, 4. This is a new collective motion caused by the spontaneous symmetry breaking (SSB) of the global phase in the gauge space and different from the collective motions in configuration space such as rotation and vibration with a geometrical intrinsic structure, which are also widely seen in the mean field picture 13, 14.

In fact, three $\alpha$ clusters in $^{12}\text{C}(0^+_2, 7.654\text{ MeV})$, the Hoyle state, has been shown to be in a gas phase by Uegaki et al. 15 in contrast to the preceding picture of a rigid three $\alpha$ chain structure 16. Matsumura et al. 17 showed that about 70% of the $\alpha$ clusters in the Hoyle state are sitting in the $0s$ state. Many theoretical and experimental studies were devoted to investigate whether the Hoyle state is a Bose–Einstein condensate 17, 30.

The most intriguing nucleus beyond $^{12}\text{C}$ is $^{16}\text{O}$, for which systematic experimental data have been accumulated 31, 36. Despite theoretical studies 21, 37, 39 whether the observed energy level structure of the $\alpha$ cluster states just above the four $\alpha$ threshold in $^{16}\text{O}$ is due to BEC of $\alpha$ clusters has not been confirmed. Full microscopic four $\alpha$ cluster model calculations and $ab$ initio calculations to understand the $\alpha$ cluster level structure are presently formidable difficult for $^{16}\text{O}$. Also we note that whatever the calculated rate of $\alpha$ clusters sitting in the $0s$ state may be, any theory without the order parameter is unable to conclude whether the system is in the Nambu–Goldstone (NG) phase of BEC or in the normal Wigner phase.

We have proposed a field theoretical superfluid cluster model in which the order parameter is defined 29, 30 and showed in $^{12}\text{C}$ that the emergence of the peculiar energy levels with a well-developed gas-like $\alpha$ cluster structure built on the Hoyle state is a manifestation of the NG phase caused by the BEC of the vacuum Hoyle state.

The purpose of this paper is to show for the first time that the observed peculiar $\alpha$ cluster level structure just above the four $\alpha$ threshold in $^{16}\text{O}$ can be understood in a superfluid $\alpha$ cluster model and that in particular the emergence of the ground and excited states of the NG operators is a manifestation of the evidence for the BEC of $\alpha$ clusters.

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From the theoretical side, Tohsaki et al. [21] considered that the 0\(^+_2\) (14.0 MeV) state in \(^{16}\)O, located 0.44 MeV only just below the four \(\alpha\) threshold energy, is a BEC state of \(\alpha\) clusters. On the other hand, Funaki et al. [37] discussed, using a four \(\alpha\) semi-microscopic orthogonality condition model (OCM), that the 0\(^+_3\) (15.1 MeV) state located just above the four \(\alpha\) threshold [40] is a condensate of four \(\alpha\) clusters. Ohkubo et al. [39] suggested, in the unified description of nuclear rainbows in \(\alpha+^{12}\)C scattering and \(\alpha\) cluster structure in the bound and quasi-bound energy region of \(^{16}\)O, that the observed 0\(^+_6\) (15.1 MeV) state could be a superfluid state of four \(\alpha\) clusters. As for the excited states above the 0\(^+_6\) (15.1 MeV) state, i.e., 2\(^+\) (16.95 MeV), 2\(^+\) (17.15 MeV), 4\(^+\) (18.05 MeV) and 6\(^+\) (19.35 MeV) states, observed by Chevallier et al. [32], which had been considered to have a four \(\alpha\) linear chain structure [32, 41], Ohkubo et al. [38] showed that they can be understood to have the local condensate \(\alpha+^{12}\)C (0\(^+_3\)) structure. As for the four \(\alpha\) linear chain in \(^{16}\)O, Ichikawa et al. [32] and others [43, 44] showed that the excitation energy of the band head state is much higher, above 30 MeV. A search to observe such a high lying state has been attempted [45].

On the other hand, from the experimental side, very important developments in searching for BEC of \(\alpha\) clusters in \(^{16}\)O were recently reported by Itoh et al. [31], who observed two broad resonant 0\(^+\) states, i.e., 0\(^+_2\) at 16.7 MeV with the \(\alpha+^{12}\)C(0\(^+_3\)) structure and 0\(^+_4\) at 18.8 MeV with the \(^{8}\)Be+\(^{8}\)Be structure just above the 0\(^+_6\) (15.1 MeV) state. We note that this energy 16.7 MeV of 0\(^+_2\) exactly agrees within the width with 16.6 MeV predicted by Ohkubo et al. [38] from the viewpoint of BEC and superfluidity of \(^{16}\)O. In Ref. [40], Ohkubo suggested that the 0\(^+_2\) state (16.7 MeV) may be a NG state due to the spontaneous symmetry breaking of the global phase caused by the BEC of the 0\(^+_6\) (15.1 MeV) state. In addition to the experimental data by Chevallier et al. [32] and Freer et al. [33, 35, 36], Curtis et al. [34] very recently observed in the \(^{12}\)C(\(^{4}\)He, 4\(\alpha\)) breakup reaction the 2\(^+\) (17.3 MeV), 4\(^+\) (18.0 MeV), 2\(^+\) or 4\(^+\) (19.4 MeV), and 4\(^+\) or 6\(^+\) (21.0 MeV) states, which decay into the \(^{8}\)Be+\(^{8}\)Be channel. These observations seem to be consistent with the previous results by Freer et al. [33], the 2\(^+\) (17.1 MeV), 2\(^+\) (17.5 MeV), 4\(^+\) (19.5 MeV) and 6\(^+\) (21.4 MeV) states in the \(^{12}\)C(\(^{4}\)He, 4\(\alpha\))\(^{12}\)C reaction, and by Freer et al. [35], the 6\(^+\) states at 20.0 and 21.2 MeV in the \(^{8}\)Be+\(^{8}\)Be decay channel of \(^{16}\)O. These states may be considered to be fragmented from the centroid energy of each spin as suggested in Ref. [38].

We study these \(\alpha\) cluster states from the viewpoint of BEC of \(\alpha\) clusters, using the field-theoretical superfluid \(\alpha\) cluster model [29, 30]. We briefly recapitulate the formulation. The model Hamiltonian for a bosonic field \(\psi(x)\ (x = (x, t))\) representing the \(\alpha\) cluster is given as follows:

\[
\hat{H} = \int d^3x \hat{\psi}^\dagger(x) \left( -\frac{\nabla^2}{2m} + V_{ex}(x) - \mu \right) \hat{\psi}(x) + \frac{1}{2} \int d^3x \int d^3x' \hat{\psi}^\dagger(x) \hat{\psi}(x')U(|x-x'|)\hat{\psi}(x')\hat{\psi}(x). \tag{1}
\]

Here, the potential \(V_{ex}\) is introduced phenomenologically to trap the \(\alpha\) clusters inside the nucleus, and is taken to have a harmonic form, \(V_{ex}(r) = m\Omega^2 r^2/2\), and the \(\alpha-\alpha\) interaction is given by the Ali–Bodmer potential [37],

\[U(|x-x'|) = V_{ex}e^{-m^2|x-x'|^2} - V_{ex}e^{-m^2|x-x'|^2}.\]

We set \(\hbar = c = 1\) throughout this paper.

When BEC of \(\alpha\) clusters occurs, i.e. the global phase symmetry of \(\hat{\psi}\) is spontaneously broken, we decompose \(\hat{\psi}(x) = \xi(r) + \hat{\varphi}(x)\), where the c-number \(\xi(r) = \langle 0 | \hat{\psi}(x) | 0 \rangle\) is an order parameter and is assumed to be real and isotropic. To obtain the excitation spectrum, we need to solve three coupled equations, which are the Gross–Pitaevskii (GP) equation, Bogoliubov–de Gennes (BdG) equations, and zero-mode equation [29, 30]. The GP equation that determines the order parameter is given by

\[
\left\{ \begin{array}{l}
-\frac{\nabla^2}{2m} + V_{ex}(r) - \mu + V_H(r) \xi(r) = 0,
\end{array} \right.
\]

where \(V_H(r) = \int d^3x' U(|x-x'|)\xi^2(r')\). The repulsive Coulomb potential affects numerical results little as in the case of \(^{12}\)C [29, 30] and is suppressed in the present work. The order parameter \(\xi\) is normalized with the condensed particle number \(N_0\) as

\[
\int d^3x |\xi(r)|^2 = N_0. \tag{3}
\]

The BdG equations that describe the collective oscillations on the condensate are given by

\[
\int d^3x' \left( \begin{array}{c}
\mathcal{L} & M
\\
-M^* & -\mathcal{L}^*
\end{array} \right) \left( \begin{array}{c}
u_n
\\v_n
\end{array} \right) = \omega_n \left( \begin{array}{c}
u_n
\\v_n
\end{array} \right), \tag{4}
\]

where \(M(x, x') = U(|x-x'|)\xi(r)\xi(r')\), \(\mathcal{L}(x, x') = \delta(x-x')\left\{ \frac{\nabla^2}{2m} + V_{ex}(r) - \mu + V_H(r) \right\} + M(x, x')\). The index \(n = (n, \ell, m)\) stands for the main, azimuthal and magnetic quantum numbers. The eigenvalue \(\omega_n\) is the excitation energy of the Bogoliubov mode. For isotropic \(\xi\), the BdG eigenfunctions can be take to have separable forms, \(u_n(x) = U_{n\ell}(r)Y_{\ell m}(\theta, \phi)\), \(v_n(x) = V_{n\ell}(r)Y_{\ell m}(\theta, \phi)\). We necessarily have an eigenfunction belonging to zero eigenvalue, explicitly \((\xi(r), -\xi(r))\), and its adjoint function \((\eta(r), \eta(r))\) is obtained as

\[
\eta(r) = \frac{\partial}{\partial N_0} \xi(r). \tag{5}
\]

The field operator is expanded as

\[
\hat{\varphi}(x) = -i\hat{Q}(t)\xi(r) + \hat{P}\eta(r) + \sum_n \left\{ \hat{a}_n(x) + \hat{a}^\dagger_n^*(x) \right\}, \tag{6}
\]
with the commutation relations $[\hat{Q}, \hat{P}] = i$ and $[\hat{a}_n, \hat{a}^\dagger_{n'}] = \delta_{nn'}$. The operator $\hat{a}_n$ is an annihilation operator of the Bogoliubov mode, and the pair of canonical operators $\hat{Q}$ and $\hat{P}$ originate from the SSB of the global phase and are called the NG or zero-mode operators.

The treatment of the zero-mode operators is a chief feature of our approach. The naive choice of the unperturbed bilinear Hamiltonian with respect to $\hat{Q}$ and $\hat{P}$ fails due to their large quantum fluctuations. Instead, we gather all the terms consisting only of $\hat{Q}$ and $\hat{P}$ in the total Hamiltonian to construct the unperturbed nonlinear Hamiltonian of $\hat{Q}$ and $\hat{P}$, denoted by $H_{QP}^0$. The coefficients in $H_{QP}^0$ are $I = \frac{\partial \mu}{\partial N}$ and the integrations involving $\xi$, $\eta$ and $U$, whose explicit forms are seen in the Ref. [30]. We set up the eigenequation for $H_{QP}^0$, called the zero-mode equation,

$$\hat{H}_{QP}^0 |\Psi_\nu\rangle = E_\nu |\Psi_\nu\rangle \quad (\nu = 0, 1, \cdots). \quad (7)$$

This equation is similar to a one-dimensional Schrödinger equation for a bound problem.

The total unperturbed Hamiltonian $\hat{H}_u$ is $\hat{H}_u = H_{QP}^0 + \sum_n \omega_n \hat{a}_n^\dagger \hat{a}_n$. The ground state energy is set to zero, $E_0 = 0$. The states that we consider are $|\Psi_0\rangle |0\rangle_{\text{ex}}$ with energy $E_\nu$, called the zero-mode state, and $|\Psi_\nu\rangle \hat{a}_n^\dagger |0\rangle_{\text{ex}}$ with energy $\omega_n$, called the BdG state, where $\hat{a}_n |0\rangle_{\text{ex}} = 0$.

As in Refs. [29, 30], we adjust the strength parameter $V_\nu$ of the short range repulsive potential of the Ali–Bodmer potential (set d0) [47], because it stabilizes the condensate under the trapping potential and is the most sensitive in our analysis. The chemical potential is fixed by the input $N_0$.

We identify $0^+_2$ (15.10 MeV) state, considered to be a Hoyle analog-state [37, 38], as the vacuum $|\Psi_0\rangle |0\rangle_{\text{ex}}$. In fact, according to Ref. [37], 61% of the $\alpha$ clusters in this state are sitting in the 0s state, while the rate is less than 25% for all the other $0^+$ states below, including $0^+_1$ (14.0 MeV). The two parameters, $\Omega = 1.57$ MeV/$\hbar$ and $V_r = 514$ MeV, which are determined by inputting $N_0 = 0.61 \times 4$ and $E_1 = 16.7-15.1-1.6$ MeV when the experimental $0^+_1$ (16.7 MeV) state is identified as $|\Psi_1\rangle |0\rangle_{\text{ex}}$. $0^+_1$ (ZM) and also by minimizing the mean square errors between the experimental and calculated energies of the states above it, and to reproduce well the excitation energy of the experimental $0^+_2$ (16.7 MeV) state from the vacuum $0^+_0$ (15.10 MeV), i.e., $E_1 = 16.7-15.1-1.6$ MeV under the condensation rate 61%, are used in the calculations below.

In Fig. 1 the energy levels calculated using the obtained parameters ($\Omega$, $V_r$) are displayed. We notice that the calculated second zero-mode state $0^+_2$ (ZM) appears at low excitation energy from the vacuum $0^+_0$ (15.10 MeV) and agrees well with the broad $0^+_7$ (18.8 MeV) state observed by Itoh et al. [31]. Our calculation locates also the $0^+_1$ (BdG) near the zero-mode $0^+_2$ (ZM) state, which seems to be within the broad width of the experimental $0^+_5$ (18.8 MeV) state. From the excitation function of Fig. 4(b) in Ref. [31], it is difficult to know whether the broad $0^+_5$ (18.8 MeV) state is fragmented. The calculated BdG excited states $4^+_2$ (BdG), $4^+_3$ (BdG) and $4^+_4$ (BdG) states agree with the experimental energy levels observed in [33, 34]. Our calculation predicts a $8^+_1$ (BdG) at 23.7 MeV and a $10^+_1$ (BdG) at 26.4 MeV. The calculated $8^+$ state may correspond to the $8^+$ state at 23.6 MeV observed in Ref. [35].

The logical structure that the energy levels of the two $0^+$ zero-mode states appear at low energies from the vacuum followed by the $2^+$ (BdG) and $4^+$ (BdG) states in agreement with experiment is the same as that of the excited $\alpha$ cluster states above the Hoyle state in $^{12}$C. The level structure is a manifestation of the emergence of the zero-mode and BdG states due to BEC of the $\alpha$ clusters. The appearance of the zero-mode states confirms the conjecture by Ohkubo in Ref. [46].

In Fig. 2 the density distribution of the order parameter $\xi(r)$, which is related to the superfluidity density $\rho(r) = |\xi(r)|^2/N_0$, is displayed. The rms radius of the Hoyle-analog $0^+_6$ (15.1 MeV) state $\bar{r}$ is calculated to be 5.22 fm as $\bar{r}^2 = \int d^3x \ r^2 |\xi(r)|^2/N_0$. Although the experimental value is not available, this large size compared with the experimental 2.71 fm of the ground state of $^{16}$O supports that the Hoyle-analog state is a dilute $\alpha$ cluster state. This value is also consistent with 5 fm in Ref. [37]. In Fig. 2 (a) the superfluid density is the largest in the center of the condensate and decreases gradually toward the surface. We note that the density extends considerably beyond the rms radius 5.2 fm up to 10 fm. In Fig. 2 (b) the calculated $\eta$, which is the derivative of $\xi$ with respect to the number of $\alpha$ clusters, represents the number fluctuation of the superfluid condensate. We note that the fluctuation is the largest not in the central region but at around 6 fm in the surface region. This behavior is similar to the $^{12}$C case in Ref. [30]. The number fluctuation is visible even beyond 10 fm up to 12 fm where
FIG. 2. (a) the calculated order parameter $\xi(r)$ and (b) its adjoint eigenfunction $\eta(r)$ with 61% condensation rate.

FIG. 3. The calculated BdG wave functions $U_{\ell}(r)$ and $V_{\ell}(r)$ for the states (a) $0^+$, (b) $2^+$, (c) $4^+$ and (d) $6^+$ with 61% condensation rate.

The superfluid density is very low. This feature does not change if the condensation rate is significantly increased (for example 100%) or decreased from the present 61%.

In Fig. 3, the wave functions $U_{\ell}(r)$ and $V_{\ell}(r)$ of the BdG states ($n = 1$) with $\ell = 0, 2, 4$ and 6 are displayed. We note that $|V_{\ell}(r)|$ is much smaller than $|U_{\ell}(r)|$, which becomes more notable as $\ell$ increases. The peak of $U_{\ell}(r)$ for $\ell \neq 0$ is in the surface region because of the repulsive force of the central condensate and moves outward with increasing $\ell$ due to the centrifugal force. For $\ell = 0$, the orthogonality of $U_{00}(r)$ and $V_{00}(r)$ to the nodeless $\xi$ and $\eta$ makes the node at around 6 fm and the energy level is raised above the $\ell = 2$ state. The amplitude of $V_{00}(r)$ is not small even in the central region. These features are similar to the BEC states built on the Hoyle state in $^{12}$C.

In Fig. 4, the squares of the calculated zero-mode wave functions $|\Psi_{\nu}(q)|^2$ for the first three ($\nu = 0, 1$ and 2) states are displayed. The wave function of the $\nu = 0$ zero-mode state or the vacuum has no node in the $q$-space, as in Fig. 4 (a). It is important to emphasize that in the present theory we obtain a series of $0^+$ states, which are the zero-mode excited states with $\nu \neq 0$. These excited states are realized by increasing the number of the nodes of the wave function in the $q$-space, as seen in Fig. 4 (b) and (c). The first excited state with one node ($\nu = 1$) in Fig. 4 (b) and the second excited state ($\nu = 2$) with two nodes in Fig. 4 (c) are assigned to the experimental $0^+_1$ state at 16.7 MeV and $0^+_2$ state at 18.8 MeV, respectively. The reason why we have the two zero-mode $0^+$ excited states is because we treat the quantum fluctuations of $\hat{Q}$ and $\hat{P}$ properly and are naturally led to the zero-mode equation (7). The logic that more than two $0^+$ states appear at low excitation energies from the Hoyle-analog state is shared by both the cases of $^{12}$C and $^{12}$O. This logic is quite similar to the case of a deformed nucleus, for which the rotational symmetry is spontaneously broken and the zero-mode states $2^+, 4^+, 6^+, \cdots$ are the members of the rotational band by increasing the angular momentum. In the case of the zero-mode excited states for the BEC, the quantum number counting the number of nodes in the $q$-space plays the role of the angular momentum. Both of the emergences of the zero-mode excited states for the BEC of $\alpha$ clusters and the rotational band for the deformed nucleus are due to the finiteness of the systems. We note that in the infinite limit of the system size of the BEC of $\alpha$ clusters, all the zero-mode excited states become degenerate to zero energy.

To see the robustness of the emergence of energy level structure of the zero-mode and BdG states, the calculated energy levels for different condensation rates, 40, 60, 80 and 100%, are displayed in Fig. 5. While the parameter $\Omega$ is kept to be 1.57 MeV/$\hbar$, which was obtained...
The energy levels in $^{16}\text{O}$ calculated for different condensation rates and the experimental $\alpha$ cluster states.

For the case of 61%, the parameter $V_r$ is determined so as to minimize the mean square errors between the experimental and calculated energies of the state $0^+$ and the states above it. The energy level structure is little changed by the different condensation rates. Especially the excitation energies of the two $0^+$ states above the vacuum are almost independent of the condensation rate as long as the BEC is formed. Although the condensation rate has not been known experimentally, it is clear that the level structure due to BEC in Fig. 1 is robust.

In Table I the calculated $E0$ and $E2$ transition probabilities are listed. Although no experimental data are available, the transitions from the $2^+_1$ (BdG) state to the $0^+_2$ (ZM) and $0^+$ (Vac) states are strong. Experimental measurements, which may serve to check the condensation rate and the models, are desired.

The above discussions and conclusions are reconfirmed also by the other method of fitting the parameters, in which the rms radius of the Hoyle-analog state is constrained to 5 fm of Ref. [37], which gives the best values of $\Omega=1.67 \text{MeV}/h$ and $V_r=497 \text{MeV}$ to fit the 16.7 MeV $0^+_7$ state.

To summarize, we have studied the observed well-developed $\alpha$ cluster states above the four $\alpha$ threshold from the viewpoint of Bose–Einstein condensation of $\alpha$ clusters using a field theoretical superfluid cluster model in which the order parameter is defined. We could reproduce the observed level structure of the $\alpha$ cluster states above the threshold for the first time. It is found that the emergence of the level structure that the two $0^+$ states with a well-developed $\alpha$ cluster structure at very low excitation energies from the threshold is a manifestation of the Nambu–Goldstone zero-mode states due to the BEC of the vacuum $0^+_2$ (15.1 MeV). This mechanism is the same to the $\alpha$ cluster structure above the three $\alpha$ threshold in $^{12}\text{C}$. It is found that the obtained level structure is robust and changes little once the $\alpha$ cluster condensation is realized with a significant condensation rate. The present results give evidence of the existence of Bose–Einstein condensate of $\alpha$ cluster in $^{16}\text{O}$.

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Table I. Calculated reduced $E0$ and $E2$ transition probabilities, $B(E2)$ and $M(E0)$, in $^{16}\text{O}$, in units of $e^2 \text{fm}^4$ and $\mu^2$, respectively, for 61% and 100% condensations.

| Transition | 61% | 100% |
|------------|-----|------|
| $M(E0) : 0^+_{\nu=0}(\text{ZM}) \rightarrow 0^+_{\nu=0}(\text{ZM})$ | 4.2 | 7.6 |
| $M(E0) : 0^+_{\nu=1}(\text{ZM}) \rightarrow 0^+_{\nu=0}(\text{ZM})$ | 0.24 | 0.62 |
| $B(E2) : 2^+_{\nu=1}(\text{BdG}) \rightarrow 0^+_{\nu=0}(\text{ZM})$ | 369 | 606 |
| $B(E2) : 2^+_{\nu=2}(\text{BdG}) \rightarrow 0^+_{\nu=1}(\text{ZM})$ | 1644 | 1724 |

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