Primordial Trispectrum from Entropy Perturbations in Multifield DBI Model

Xian Gao, Bin Hu

Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100190, China
E-mail: gaoxian@itp.ac.cn, hubin@itp.ac.cn

ABSTRACT: We investigate the primordial trispectra of the general multifield DBI inflationary model. In contrast with the single field model, the entropic modes can source the curvature perturbations on the super horizon scales, so we calculate the contributions from the interaction of four entropic modes mediating one adiabatic mode to the trispectra, at the large transfer limit ($T_{RS} \gg 1$). We obtained the general form of the 4-point correlation functions, plotted the shape diagrams in two specific momenta configurations, “equilateral configuration” and “specialized configuration”. Our figures showed that we can easily distinguish the two different momenta configurations.
1. Introduction

The Cosmic Microwave Background (CMB) provides us with remarkably detailed signatures of the early universe. The observations from large scale structures are consistent with an almost scale invariant, Gaussian primordial density perturbations generated during inflation. Precision measurements of any small deviation from Gaussian distribution enables us to distinguish different cosmological models. In the scalar field(s) inflation case, the non-Gaussian fluctuations can be parametrized by $f_{NL}$ at the leading order and $\tau_{NL}$ at sub-leading order respectively. The current experimental bound for the bispectrum (the three point correlation function of the primordial curvature perturbation $\zeta$) is $-9 < f_{NL}^{\text{local}} < 111$ from WMAP5 [1], and for the trispectrum (four point correlation function) is $|\tau_{NL}| < 10^8$ [2], the next generation of experiments such as PLANCK will increase the sensitivity to about $\tau_{NL} \sim 560$ [3].

On the theoretical aspect, models with non-Gaussianities have been intensively investigated in recent years (see [4] for a review). Standard single-field slow-roll inflationary model predicts an almost Gaussian fluctuation with undetectable non-Gaussianity [5]. 3-point functions, or its Fourier transformation the bispectra for single-field and multifielded inflation models are investigated in [6, 7, 18]. Higher-order correlation function, e.g. the trispectra (Fourier transformation of connected 4-point function) are studied in [8, 9, 10, 11, 12, 13]. For k-inflation models [14], DBI inflation models [18, 24, 25, 27] and curvaton scenario [14], the primordial bispectra and trispectra are calculated in [24, 25, 27, 28, 29, 30, 31] and in [15, 16] respectively. And the loop corrections to the power spectrum and bispectrum [32, 33, 46], the non-Gaussianities originated in non-commutative effect [34], $\alpha$ vacuum [35], thermal fluctuations [38], string gas [37], matter bounce [36] are all investigated recently.
In this paper we will focus on the multiple fields DBI inflation model. For single field DBI models, the effective four-dimensional scalar field corresponds to the radial position of a brane in a higher dimensional warped conical geometry with other angular degree of freedom frozen for simplicity. However, if we consider the brane can also move in the angular directions, more than one effective scalars will turn on. Generally the Lagrangian of the multifield DBI model can be expressed as 

\[ P(X^{IJ}, \phi^K) \]

with 

\[ X^{IJ} = -\partial_\mu \phi^I \partial^\mu \phi^J / 2 \]

for \( I, J, K = 1, 2, \ldots, N \) labels the \( N \) multiple fields. And the trajectory of the background fields can be decomposed into one adiabatic mode which is along the direction of the trajectory and the other \((N-1)\) entropic modes which are orthogonal to the adiabatic direction. In contrast with the single field model, in which the curvature perturbations on uniform energy density hypersurfaces \( \zeta \) are conserved after horizon crossing, the curvature perturbations generally evolve in time even on large scales in the multifield models. The reason of this effect can be interpreted as due to the transfer between the adiabatic and entropic modes \([39, 40, 41]\), so if there exists a large transfer from entropic modes to adiabatic mode, then the final curvature perturbations are mostly of entropic origin. Considering this effect, in this paper we calculate the contributions from the interaction of four entropic perturbations mediating one adiabatic perturbation to the trispectra in the general multifield scenario.

This paper is organized as follows. In Sec. II we firstly present the background setup, solve the equation of motion for the background fields, and secondly do the linear perturbations on the background, derive the second order action, and calculate the adiabatic and entropic power spectra in the massless limit. In first subsection of the Sec. III we calculate the 4-point correlation functions of the entropic perturbations \( Q_s \) from the interaction of four entropic modes mediating one adiabatic mode. In the second subsection we further analyze the shape function of the trispectra in two specific momenta configurations, one is the “equilateral configuration” another is the “specialized planar configuration”. In Sec IV, we conclude our results.

2. Review of generalized multifield model

In this section, we firstly review the setup of the general multifield model and the dynamics of the cosmological background, then concentrate on the linear perturbation theory, including the second order action and power spectra.

2.1 Setup and Background

In this subsection we briefly review the general multifield model which was proposed by Langlois et al. in [23], where the Lagrangian is proposed to be of the form 

\[ P(X^{IJ}, \phi^K) \]

with 

\[ X^{IJ} = -\frac{1}{2} \partial_\mu \phi^I \partial^\mu \phi^J \]

in particular, for the multifield DBI model the Lagrangian can be reduced into 

\[ P(X^{IJ}, \phi^K) = -\frac{1}{f(\phi^I)} (\sqrt{D} - 1) - V(\phi^I), \quad I = 1, 2, \cdots N, \]

where the determinant \( D \equiv \det(\delta^\mu_\nu + fG_{IJ} \partial^\mu \phi^I \partial_\nu \phi^J), \) \( f(\phi^I) \) is the warping factor and \( G_{IJ} \) is the field space metric. As a specific example, such as the standard AdS throat, the warp
factor depends only on one of the fields and takes form of \( f(\phi_1) = \lambda/\phi_1^4 \) (where \( \lambda \) depends on the flux numbers in specific string constructions).

Considering \( \mathcal{N} \) scalar fields coupled with gravity minimally, the action can be written as

\[
S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R + 2P(X^{IJ}, \phi^K) \right],
\]

with \( 8\pi G = 1 \).

The energy-momentum tensor can be derived by varying \( P(X^{IJ}, \phi^K) \) with respect to the metric \( g_{\mu\nu} \)

\[
T^\mu{}^\nu = P g^\mu{}^\nu + P_{(IJ)} \partial^\mu \phi^I \partial^\nu \phi^J,
\]

where we define

\[
P_{(IJ)} \equiv \frac{1}{2} \left( \frac{\partial P}{\partial X^{IJ}} + \frac{\partial P}{\partial X^{JI}} \right) = P_{(JI)}.
\]

In the last part of this section, we will investigate the background dynamics in the homogeneous and isotropic universe with the flat Friedmann-Robertson-Walker metric

\[
ds^2 = -dt^2 + a^2(t) dx^i dx^j,
\]

where \( a(t) \) is the scale factor and \( H = \dot{a}/a \) is the Hubble parameter. Thus, from (2.3) we can see that the pressure is simply \( P \) and the energy density can be expressed as

\[
\rho = 2P_{(IJ)}X^{IJ} - P,
\]

with \( X^{IJ} = \frac{1}{2} \dot{\phi}^I \dot{\phi}^J \) and \( \dot{} = d/dt \).

Following the assumption of metric (2.5), the equation of motion for scalar fields, the Friedmann equation and the continuity equation will be reduced to

\[
0 = (P_{(IJ)} + P_{(IL), (JK)} \dot{\phi}^L \dot{\phi}^K) \dot{\phi}^J + (3H P_{(IJ)} + P_{(IJ), K} \dot{\phi}^K) \dot{\phi}^J - P_I ,
\]

\[
H^2 = \frac{1}{3} (2P_{(IJ)}X^{IJ} - P) ,
\]

\[
\dot{H} = -X^{IJ} P_{(IJ)} .
\]

2.2 Linear perturbation: Second order action and Power spectra

In this subsection, we will briefly mention the derivation of the second order action and derive the power spectra for the two fields DBI model. Following the standard approach which was proposed in \[5\], one can get the rigorous second order action of the general multifield inflation model

\[
S^{(2)} = \frac{1}{2} \int d^3x \ a^3 \left[ \left( P_{(IJ)} + 2P_{(ML), (NK)} X^{MK} \right) \dot{Q}^I \dot{Q}^J - P_{(IJ)} h^{ij} \partial_i Q^I \partial_j Q^J - \mathcal{M}_{KL} Q^K Q^L + 2\Omega_{KL} Q^K \dot{Q}^L \right],
\]

where the explicit form of mass matrix \( \mathcal{M}_{KL} \) and mixing matrix \( \Omega_{KL} \) read

\[
\mathcal{M}_{KL} = -P_{KL} + 3X^{MN} P_{(NK) L} X^{ML} + \frac{a^3}{H} P_{(NL)} \dot{\phi}^N \left[ 2P_{(IJ), K} X^{IJ} - P_K \right] - \frac{1}{H^2} X^{MN} \times P_{(NK) L} \left[ X^{IJ} P_{(IJ), AB} X^{IJ} X^{AB} \right] - \frac{1}{a^3} \frac{d}{dt} \left( \frac{a^3}{H} P_{(AK) L} P_{(IJ), AB} X^{AD} \right) .
\]
\[ \Omega_{KL} = \dot{\phi}^{J} P_{(IJ),K} - \frac{2}{H} P_{(LK)} P_{(MJ)} X^{LN} X^{MJ}. \]  

(2.12)

In the derivation of (2.10), we used the ADM metric \[ ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \]  

(2.13)

and chose the spatially flat gauge \( h_{ij} = a^2 \delta_{ij} \). In this gauge, the spatial part of the metric remains unperturbed, thus the physical degree of freedom are the perturbations of the multiple scalar fields which are denoted as \( Q^I \). Generally the matrix (2.11) is not diagonal, so \( Q^I \) are not the canonical quantities which can be promoted to operators through canonical quantization procedure, so we must construct the canonical quantities in the next step. Fortunately, we can achieve our aim through decomposing \( Q^I \) into one adiabatic mode which is along the background trajectory in field space and \((N-1)\) entropic modes which are orthogonal to the trajectory, i.e.

\[ Q^I = Q^\sigma e^I_{\sigma} + Q^s e^I_{s}, \quad s = 2, \cdots, N, \]  

(2.14)

where \( e^I_{\sigma} \) is the unit base vector which is along the trajectory, \( e^I_{s} \) are the \((N-1)\) unit base vectors which are orthogonal to the trajectory, and they satisfy the normalized and orthogonal relation

\[ e^I_{\sigma} e^I_{\sigma} = e^I_{s} e^I_{s} = 1, \quad e^I_{\sigma} e^I_{s} = 0, \quad e^I_{\sigma} = G_{IJ} e^I_{\sigma} . \]  

(2.15)

In order to further simplify our calculation, we will assume the straight line background trajectory and flat field space metric, i.e., \( \dot{e}^I_{\sigma} = \dot{e}^I_{s} = 0 \), and \( G_{IJ} = \delta_{IJ} \).

Before doing the orthogonal decomposition, we need to stop here and introduce the sound speed of perturbations in multifield DBI model. As illustrated in [23], the remarkable result of the action (2.1) is that all perturbations propagate at the same sound speed \( c_s = \sqrt{1 - f G_{IJ} \dot{\phi}^I \dot{\phi}^J} \).

After introducing the sound speed in multifield DBI model, we can define a new field space metric as

\[ \tilde{G}_{IJ} = \perp_{IJ} + \frac{1}{c_s^2} e^I_{\sigma} e^I_{\sigma} , \]  

\[ \perp_{IJ} = G_{IJ} - e^I_{\sigma} e^I_{\sigma} , \]  

(2.16)

(2.17)

where \( \perp_{IJ} \) represents the projection to the entropy direction and \( e^I_{\sigma} e^I_{\sigma} / c_s^2 \) represents the projection to the adiabatic direction. Here we emphasize that the tilde metric is diagonal, and using this metric we can reduce (2.10) into

\[ S_{(2)} = \frac{1}{2} \int dtd^3 x \ a^3 \left[ \frac{1}{e_s} \left( \tilde{G}^{IJ} \dot{Q}^I \dot{Q}^J - c_s^2 \tilde{G}^{IJ}_{IJ} \partial_i Q^I \partial_i Q^J \right) - \tilde{M}_{IJ} Q^I Q^J + 2 f_{IJ} X \dot{\phi}_I \dot{Q}^I \dot{Q}^J \right]. \]  

(2.18)

In order to gain some intuition, in what follows, we will restrict ourselves to a two field model \((\phi_1, \phi_2)\), and it is straightforward to generalize our analysis to any number of fields.
By virtue of (2.14), \((\phi_1, \phi_2)\) are decomposed into \((\sigma, s)\) with \(\dot{\sigma} = \sqrt{2X}\) and \(\dot{s} = 0\). After introducing three "slow variation parameters" as in standard slow roll inflation

\[
\epsilon = -\frac{\dot{H}}{H^2} = \frac{X}{c_s H^2}, \tag{2.19}
\]

\[
\tilde{\eta} = \frac{\dot{\epsilon}}{\epsilon H}, \tag{2.20}
\]

\[
\tilde{s} = \frac{\dot{c}_s}{c_s H}, \tag{2.21}
\]

one can reduce (2.18) further

\[
S_{(2)} \simeq \frac{1}{2} \int d\eta d^3x \frac{1}{c_s H^2 \eta^2} \left\{ \frac{1}{c_s^2} \left[ (Q'_\sigma)^2 - c_s^2 (\partial_i Q^\sigma)^2 \right] + \left[ (Q'_s)^2 - c_s^2 (\partial_i Q^s)^2 \right] \right\}, \tag{2.22}
\]

where we change the cosmic time \(t\) into comoving time \(\eta = -1/aH\) (\(= d/d\eta\)) and drop the last two sub-leading terms in (2.18) as in [23], because these two terms are suppressed by the slow roll parameters which are defined in (2.19). From (2.22), we can see that \(Q_\sigma\) and \(Q_s\) are the canonical quantities, up to a normalization factor, which should be quantized, and the propagation speed of both the adiabatic mode and the entropy mode equal to \(c_s\).

Then we go to momentum space to do quantization, the Fourier mode of \(Q_\sigma\) and \(Q_s\) can be quantized as

\[
Q_\sigma(\eta, k) = a_k u_k(\eta) + a^{\dagger}_k u^*_k(\eta), \tag{2.23}
\]

\[
Q_s(\eta, k) = b_k v_k(\eta) + b^{\dagger}_k v^*_k(\eta), \tag{2.24}
\]

where the creation and annihilation operators satisfy the standard communication relation

\[ [a(k), a^{\dagger}(k')] = [b(k), b^{\dagger}(k')] = (2\pi)^3 \delta^3(k - k'), \]

and we choose the Bunch-Davies vacuum

\[
u_k = \frac{H}{\sqrt{2k^3 c_s}} \left( 1 + i k c_s \eta \right) e^{-ikc_s\eta}, \tag{2.25}
\]

\[
v_k = \frac{H}{\sqrt{2k^3 c_s}} \left( 1 + i k c_s \eta \right) e^{-ikc_s\eta}. \tag{2.26}
\]

It is now straightforward to calculate the two point functions

\[
\left\langle Q_\sigma(\eta, k_1)Q_\sigma(\eta', k_2) \right\rangle = (2\pi)^3 \delta^3(k_1 + k_2) F^>_k(\eta, \eta'), \tag{2.27}
\]

\[
\left\langle Q_\sigma(\eta', k_2)Q_\sigma(\eta, k_1) \right\rangle = (2\pi)^3 \delta^3(k_1 + k_2) F^<_k(\eta, \eta'), \tag{2.28}
\]

\[
\left\langle Q_s(\eta, k_1)Q_s(\eta', k_2) \right\rangle = (2\pi)^3 \delta^3(k_1 + k_2) G^>_k(\eta, \eta'), \tag{2.29}
\]

\[
\left\langle Q_s(\eta', k_2)Q_s(\eta, k_1) \right\rangle = (2\pi)^3 \delta^3(k_1 + k_2) G^<_k(\eta, \eta'), \tag{2.30}
\]

where we set \(\eta > \eta'\), and the Wightman functions for adiabatic and entropy modes read

\[
F^>_k(\eta, \eta') = u_k(\eta)u^*_k(\eta'), \quad F^<_k(\eta, \eta') = u^*_k(\eta)u_k(\eta'), \tag{2.31}
\]

\[
G^>_k(\eta, \eta') = v_k(\eta)v^*_k(\eta'), \quad G^<_k(\eta, \eta') = v^*_k(\eta)v_k(\eta'). \tag{2.32}
\]
Then the power spectra of $Q_\sigma$ and $Q_s$ are

\begin{align}
P_k^\sigma &= |Q_\sigma^*|^2 = \frac{H_*^2}{2k^3}, \quad (2.33) \\
P_k^s &= |Q_s^*|^2 = \frac{H_*^2}{2k^3c_s^2}, \quad (2.34)
\end{align}

where the subscript * indicates that the corresponding quantity is evaluated at the sound horizon crossing $kc_s = aH$.

In single field model, the curvature perturbation $R$ ($R = -\zeta$) remains constant in the large scale limit due to the local energy conservation. However, in multifield model, the adiabatic perturbations can not produce the entropic perturbations, but the entropic perturbations can source the curvature perturbations on the large scales. Generally, the time dependence of the adiabatic and entropic perturbations on the superhorizon scales are always described by

\begin{align}
\dot{R} &= \alpha H S, \\
\dot{S} &= \beta H S, \quad (2.35)
\end{align}

with

\begin{align}
R &\equiv \frac{H}{\dot{\sigma}} Q_\sigma, \\
S &\equiv c_s \frac{H}{\dot{\sigma}} Q_s, \quad (2.36)
\end{align}

and $\alpha$, $\beta$ are time dependent dimensionless functions. After performing the time integration for (2.35), one can get the general form of the transfer matrix which relate the curvature and entropic perturbations generated when a given mode crosses the sound horizon at time $t_*$ to those at some later time $t$ (on superhorizon scales)\cite{23, 39, 45}

\begin{align}
\begin{pmatrix} R \\ S \end{pmatrix} = \begin{pmatrix} 1 & T_{RS} \\ 0 & T_{SS} \end{pmatrix} \begin{pmatrix} R_0 \\ S_0 \end{pmatrix}, \quad (2.37)
\end{align}

where

\begin{align}
T_{SS}(t, t_*) = \exp \left\{ \int_{t_*}^{t} dt' \beta(t') H(t') \right\}, \\
T_{RS}(t, t_*) = \int_{t_*}^{t} dt' \alpha(t') T_{SS}(t', t_*) H(t'). \quad (2.38)
\end{align}

Substitute (2.36) into (2.37), one can obtain the relationship between the $\zeta(t)$ and $Q_i(t_*)$ with $i = \sigma, s$

\begin{align}
\zeta(t) = -A_\sigma Q_\sigma(t_*) - A_s Q_s(t_*) , \quad (2.39)
\end{align}

where

\begin{align}
A_\sigma = \left( \frac{H}{\dot{\sigma}} \right)_*, \\
A_s = T_{RS}(t, t_*) \left( c_s \frac{H}{\dot{\sigma}} \right)_*, \quad (2.40)
\end{align}

and $T_{RS}(t, t_*)$ is the transfer coefficient which reflects the transfer between the adiabatic and entropic modes.

If $T_{RS}(t, t_*) \gg 1$, $\zeta(t) \simeq -A_s(t, t_*)Q_s(t_*)$, i.e., the curvature perturbations on super-horizon scales are mainly transferred from the entropic perturbations. Thus the late time power spectrum of $\zeta(t)$ becomes

\begin{align}
P_k^\zeta(t) \simeq T_{RS}^2(t, t_*) \frac{c_s}{2\epsilon} |Q_s|^2 = T_{RS}^2(t, t_*) \frac{H_*^2}{4\epsilon c_s k^3}. \quad (2.41)
\end{align}
3. Trispectra from “scalar-exchanging” interaction

In this section, we will derive the general form of the 4-point correlation functions and plot the shape function (momentum dependence) for two special momenta configurations.

3.1 Four point correlation functions

In the first subsection, we will calculate the 4-point correlation functions for the entropic perturbations $Q_s$ through the “scalar-exchanging” interaction. For this purpose, we firstly listed the third order action which has be derived in [23]

$$S_{(3)}^\text{main} = \int d\eta d^3 x \left\{ \frac{1}{2H^2 \eta^2 c_3^2 \sigma'} \left[ (Q'_\sigma)^3 - c_s^2 Q'_\sigma (\nabla Q_\sigma)^2 \right] + \frac{1}{2H^2 \eta^2 c_3^2 \sigma'} \left[ Q'_\sigma (Q'_\sigma)^2 + c_s^2 Q'_\sigma (\nabla Q_\sigma)^2 - 2c_s^2 Q'_\sigma \nabla Q_\sigma \nabla Q_\sigma \right] \right\}. \quad (3.1)$$

As proved in [30], up to the third order, the Hamiltonian equals to the opposite Lagrangian $H^I_{(3)} = -I^I_{(3)}$, where the supper script “$I$” denotes for the interaction picture

$$H^I_1(\eta) = -\frac{1}{2H^2 \eta^2 c_3^2 \sigma'} \left[ \prod_{i=1}^3 \int \frac{d^3 k_i}{(2\pi)^3} \right] (2\pi)^3 \delta^3(\mathbf{k}_{123}) Q'_\sigma(\eta, \mathbf{k}_1) Q'_\sigma(\eta, \mathbf{k}_2) Q'_\sigma(\eta, \mathbf{k}_3), \quad (3.2)$$

$$H^I_2(\eta) = -\frac{1}{2H^2 \eta^2 c_3^2 \sigma'} \left[ \prod_{i=1}^3 \int \frac{d^3 k_i}{(2\pi)^3} \right] (2\pi)^3 \delta^3(\mathbf{k}_{123}) (\mathbf{k}_2 \cdot \mathbf{k}_3) Q'_\sigma(\eta, \mathbf{k}_1) Q'_\sigma(\eta, \mathbf{k}_2) Q'_\sigma(\eta, \mathbf{k}_3), \quad (3.3)$$

$$H^I_3(\eta) = -\frac{1}{2H^2 \eta^2 c_3^2 \sigma'} \left[ \prod_{i=1}^3 \int \frac{d^3 k_i}{(2\pi)^3} \right] (2\pi)^3 \delta^3(\mathbf{k}_{123}) Q'_\sigma(\eta, \mathbf{k}_1) Q'_\sigma(\eta, \mathbf{k}_2) Q'_\sigma(\eta, \mathbf{k}_3), \quad (3.4)$$

$$H^I_4(\eta) = -\frac{1}{2H^2 \eta^2 c_3^2 \sigma'} \left[ \prod_{i=1}^3 \int \frac{d^3 k_i}{(2\pi)^3} \right] (2\pi)^3 \delta^3(\mathbf{k}_{123}) (\mathbf{k}_2 \cdot \mathbf{k}_3) Q'_\sigma(\eta, \mathbf{k}_1) Q'_\sigma(\eta, \mathbf{k}_2) Q'_\sigma(\eta, \mathbf{k}_3), \quad (3.5)$$

$$H^I_5(\eta) = -\frac{1}{2H^2 \eta^2 c_3^2 \sigma'} \left[ \prod_{i=1}^3 \int \frac{d^3 k_i}{(2\pi)^3} \right] (2\pi)^3 \delta^3(\mathbf{k}_{123}) (\mathbf{k}_2 \cdot \mathbf{k}_3) Q'_\sigma(\eta, \mathbf{k}_1) Q'_\sigma(\eta, \mathbf{k}_2) Q'_\sigma(\eta, \mathbf{k}_3). \quad (3.6)$$

In what follows, we will still concentrate on the $T_{R S} \gg 1$ case. For this one, the main contributions to trispectra comes from the vertices with two external entropic legs $H^I_1$, $H^I_4$, and $H^I_5$ (see Fig. 1), because on the large scales the entropic perturbations source the curvature perturbations, i.e., we can approximately take $H^I \simeq H^I_3 + H^I_4 + H^I_5$.

**Figure 1:** Diagrammatic representation of the 3-point vertices: dashed line denotes for the entropic mode $Q_s$, solid line for the adiabatic mode $Q_\sigma$, and vertex $a$ presents for the interaction $H^I_1$, vertex $b$ for $H^I_4$, vertex $c$ for $H^I_5$. 

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[17]
Now, we are ready for calculating the 4-point correlation functions of the entropic perturbations \( Q_s \). The “scalar-exchanging” interaction can be illustrated diagrammatically in Fig. 2, in which we merely plot one of the nine similar diagrams. Since there exist three different 3-point vertices, as shown in Fig. 1, the 4-point correlation functions of the entropic modes can be expressed as

\[
\langle Q_s^4(\eta_*) \rangle = \sum_{i,j=3}^5 \langle Q_s^4(\eta_*) \rangle_{ij},
\]

with

\[
\langle Q_s^4(\eta_*) \rangle_{ij} = \langle 0 | \left[ T e^{-i \int_{\eta_0}^{\eta_*} d\eta' H_i(\eta')} \right] Q_s(p_1, \eta_*) Q_s(p_2, \eta_*) Q_s(p_3, \eta_*) Q_s(p_4, \eta_*) \left[ T e^{-i \int_{\eta_0}^{\eta_*} d\eta' H_j(\eta')} \right] | 0 \rangle
\]

\[
\simeq \int_{\eta_0}^{\eta_*} d\eta' \int_{\eta_0}^{\eta_*} d\eta'' \langle 0 | H_i(\eta') Q_s^4(\eta_*) H_j(\eta'') | 0 \rangle
\]

\[
- \int_{\eta_0}^{\eta_*} d\eta' \int_{\eta_0}^{\eta_*} d\eta'' \langle 0 | H_j(\eta'') Q_s^4(\eta_*) H_i(\eta') | 0 \rangle
\]

\[
- \int_{\eta_0}^{\eta_*} d\eta' \int_{\eta_0}^{\eta_*} d\eta'' \langle 0 | Q_s^4(\eta_*) H_i(\eta') H_j(\eta'') | 0 \rangle.
\]

Figure 2: This figure illustrates the interaction between four entropic modes through mediating one adiabatic mode.

After some straightforward but lengthy calculations we can obtain the analytic expressions of the 4-point correlation functions of the entropic modes

\[
\langle Q_s^4(\eta_*) \rangle_{33} = \frac{(2\pi)^3 \delta^3(\sum_{i=1}^{4} p_i) H^6}{\prod_{i=1}^{4} p_i^2} \frac{H^6}{2^8 e^g_s}
\]

\[
\times \left\{ p_{12} \left[ \prod_{i=1}^{4} p_i^2 \right] \left[ \frac{2(10q_1^2 + 5q_1q_2 + q_2^2)}{q_1^3 K^5} + \frac{1}{q_1^3 q_2^3} \right] + 23 \text{ perm.} \right\},
\]

\[
\langle Q_s^4(\eta_*) \rangle_{44} = \frac{(2\pi)^3 \delta^3(\sum_{i=1}^{4} p_i) H^6}{\prod_{i=1}^{4} p_i^2} p_{12}(p_1 \cdot p_2)(p_3 \cdot p_4) \times \{ F(p_1, p_2, q_3)F(p_3, p_4, q_1)
\]

\[
+ 2G(p_1, p_2, p_4, p_3, q_1) \} + 23 \text{ perm.},
\]

The in-in formalism \cite{43} which is often used in the literature in terms of a commutator form, is equivalent to the form \cite{53} presented here.
\[ \langle Q_s^4(q_s) \rangle_{55} = \frac{(2\pi)^3 \delta^3(\sum_{i=1}^4 p_i) H^6 - p_1 p_2^2 (p_2 \cdot p_{12}) (p_4 \cdot p_{12})}{\prod_{i=1}^4 p_i^3 2^6 c_s^6 p_{12}^3} \times \{ \text{F}(p_{12}, p_4, q_1) \text{F}(p_{12}, p_2, q_3) + 2G(-p_{12}, p_2, p_4, p_{12}, q_1) \} + 23 \text{ perm.} \right) \times (3.11) \\
\langle Q_s^4(q_s) \rangle_{34} = \frac{(2\pi)^3 \delta^3(\sum_{i=1}^4 p_i) H^6}{\prod_{i=1}^4 p_i^3 2^7 c_s^6 p_{12}^3 p_2^2 (p_3 \cdot p_4)} \left\{ \frac{\text{F}(p_3, p_4, q_1)}{q_3^3} + E(p_3, p_4, q_1) \right\} + 23 \text{ perm.} \right) \times (3.12) \\
\langle Q_s^4(q_s) \rangle_{43} = \frac{(2\pi)^3 \delta^3(\sum_{i=1}^4 p_i) H^6}{\prod_{i=1}^4 p_i^3 2^7 c_s^6 p_{12}^3 p_2^2 (p_1 \cdot p_2)} \left\{ \frac{\text{F}(p_1, p_2, q_3)}{q_1^3} + H(p_1, p_2, q_1) \right\} + 23 \text{ perm.} \right) \times (3.13) \\
\langle Q_s^4(q_s) \rangle_{35} = \frac{(2\pi)^3 \delta^3(\sum_{i=1}^4 p_i) H^6}{\prod_{i=1}^4 p_i^3 2^6 c_s^6 p_{12}^3 p_2^2 (p_4 \cdot p_{12})} \left\{ \frac{\text{F}(p_{12}, p_4, q_1)}{q_3^3} + E(p_{12}, p_4, q_1) \right\} + 23 \text{ perm.} \right) \times (3.14) \\
\langle Q_s^4(q_s) \rangle_{53} = \frac{(2\pi)^3 \delta^3(\sum_{i=1}^4 p_i) H^6 p_3 p_2^2 p_1^2 (p_2 \cdot p_{12})}{\prod_{i=1}^4 p_i^3 2^6 c_s^6 p_{12}^3} \left\{ \frac{\text{F}(p_{12}, p_2, q_3)}{q_1^3} + H(-p_{12}, p_2, q_1) \right\} + 23 \text{ perm.} \right) \times (3.15) \\
\langle Q_s^4(q_s) \rangle_{45} = \frac{(2\pi)^3 \delta^3(\sum_{i=1}^4 p_i) H^6 p_3 (p_1 \cdot p_2) (p_4 \cdot p_{12})}{\prod_{i=1}^4 p_i^3 2^7 c_s^6 p_{12}^3} \times \{ \text{F}(p_1, p_2, q_3) \text{F}(p_{12}, p_4, q_1) + 2G(p_1, p_2, p_4, p_{12}, q_1) \} + 23 \text{ perm.} \right) \times (3.16) \\
\langle Q_s^4(q_s) \rangle_{54} = \frac{(2\pi)^3 \delta^3(\sum_{i=1}^4 p_i) H^6 p_2^2 (p_3 \cdot p_4) (p_2 \cdot p_{12})}{\prod_{i=1}^4 p_i^3 2^7 c_s^6 p_{12}^3} \times \{ \text{F}(p_3, p_4, q_1) \text{F}(p_{12}, p_2, q_3) + 2G(p_2, -p_{12}, p_4, p_3, q_1) \} + 23 \text{ perm.} \right) \times (3.17) \\
\text{with} \quad q_1 \equiv p_3 + p_4 + p_{12} , \quad q_2 \equiv p_1 + p_2 - p_{12} , \quad q_3 \equiv p_1 + p_2 + p_{12} , \quad K \equiv \sum_{i=1}^4 p_i , \quad (3.18) \\
F(p_i, p_j, q_k) = \frac{2p_i p_j + (p_i + p_j)q_k + q_k^2}{q_k^3} \times (3.19) \\
E(p_i, p_j, q_k) = \frac{4p_i p_j}{K^3 q_k^3} + \frac{2(p_i + p_j)}{K^3 q_k^2} + \frac{12p_i p_j}{K^4 q_k} + \frac{2}{K^5 q_k} + \frac{6(p_i + p_j)}{K^3 q_k} + \frac{24p_i p_j}{K^5 q_k} \times (3.20) \\
H(p_i, p_j, q_k) = \frac{4p_i p_j}{K^3 q_k^3} + \frac{2}{K^3 q_k^2} + \frac{2}{K^2 q_k} + \frac{6p_i + p_j}{K^2 q_k} + \frac{4p_i + p_j}{K^3 q_k} + \frac{12p_i p_j}{K^4 q_k} + \frac{6(p_i + p_j)}{K^4 q_k} + \frac{24p_i p_j}{K^5 q_k} \times (3.21)
\[ G(p_i, p_j, p_l, p_m, q_n) \equiv \]
\[
\frac{2p_mp_l}{K q_n^3} + \frac{2p_mp_l(p_i + p_j)}{K^2 q_n^3} + \frac{4p_ip_j p_mp_l}{K^2 q_n^3} + \frac{p_m + p_l}{K q_n^3} + \frac{(p_i + p_j)(p_l + p_m) + 2p_l p_m}{K^2 q_n^3} \\
+ \frac{2p_ip_j(p_m + p_l) + 4p_mp_l(p_i + p_j)}{K^2 q_n^3} + \frac{12p_ip_j p_mp_l}{K^4 q_n^3} + \frac{1}{K q_n} + \frac{p_i + p_j + p_l + p_m}{K^2 q_n} \\
+ \frac{2p_ip_m + 2(p_i + p_j)(p_l + p_m) + 2p_j p_l}{K^3 q_n} + \frac{6[p_i p_j (p_l + p_m) + p_l p_m (p_i + p_j)]}{K^4 q_n} \\
+ \frac{24p_ip_j p_mp_l}{K^5 q_n},
\]

where “23 perm.” denotes for the 23 permutations between the momenta \( p_1, p_2, p_3 \) and \( p_4 \).

Finally, we can get the general form of trispectra for the curvature perturbations \( \zeta \), by virtue of (2.39) and (2.40)

\[
\langle \zeta^4(\eta, p_1, p_2, p_3, p_4) \rangle \simeq A_s^4(\eta) \langle Q_s^4(\eta) \rangle = T^{4}_{RS}(\eta, \eta^*) \left( \frac{c_s H}{\dot{\sigma}} \right)^4 \langle Q_s^4(\eta^*) \rangle \\
\simeq \frac{T^{4}_{RS}(\eta, \eta^*) c_s^2}{4 \epsilon^2} \langle Q_s^4(\eta^*) \rangle \\
= \frac{(2\pi)^3 H^6 T^{4}_{RS}(\eta, \eta^*) \epsilon^3 (\sum_{i=1}^{4} p_i) A(p_1, p_2, p_3, p_4)}{2^8 \epsilon^4 c_s^7}, \tag{3.23}
\]

where in the second line we used the “slow variation parameters” defined in (2.19), and the function \( A(p_1, p_2, p_3, p_4) \) defined in the third line is called shape function, which we will analyze numerically in the next subsection.

3.2 Shapes of the trispectra

As we have obtained the general form of the trispectra, in this subsection, we will turn to plot the shape diagrams for the equilateral configuration with \( p_1 = p_2 = p_3 = p_4 \) and the “specialized planar” configuration with \( p_3 = p_4 = p_{12} \).

Before the discussion of the shape functions, we note that the number of the independent arguments for the trispectra are six. In this paper, we choose six independent momenta \( p_1, p_2, p_3, p_4, p_{12}, p_{14} \), and one can also choose four independent momenta and two angles. In order for these momenta to form a tetrahedron (see Fig. 3), two conditions must be satisfied:

First, we define three angles at one vertex (see Fig. 3)

\[
\cos(\alpha) = \frac{p_1^2 + p_2^2 - p_{12}^2}{2p_1 p_2}, \tag{3.24}
\]
\[
\cos(\beta) = \frac{p_2^2 + p_{14}^2 - p_3^2}{2p_2 p_{14}}, \tag{3.25}
\]
\[
\cos(\gamma) = \frac{p_1^2 + p_{14}^2 - p_{12}^2}{2p_1 p_{14}}, \tag{3.26}
\]

where these three angles should satisfy \( \cos(\alpha - \beta) \geq \cos(\gamma) \geq \cos(\alpha + \beta) \). This inequality is equivalent to

\[
1 - \cos^2(\alpha) - \cos^2(\beta) - \cos^2(\gamma) + 2 \cos(\alpha) \cos(\beta) \cos(\gamma) \geq 0, \tag{3.27}
\]

\[ -10 - \]
where we can take the equal sign when the tetrahedron reduces to a planar quadrangle.

Secondly, the six momenta should also satisfy all the triangle inequalities

\[
p_1 + p_4 > p_{14}, \quad p_1 + p_2 > p_{12}, \quad p_2 + p_3 > p_{14},
\]
\[
p_1 + p_{14} > p_4, \quad p_1 + p_{12} > p_2, \quad p_2 + p_{14} > p_3,
\]
\[
p_4 + p_{14} > p_1, \quad p_2 + p_{12} > p_1, \quad p_3 + p_{14} > p_2,
\]

and the last triangle inequality involving \((p_3, p_4, p_{12})\) is always satisfied given \((3.27)\) and \((3.28)\).

After discussing the two conditions which should be satisfied, now we will plot the shape function \(A(p_1, p_2, p_3, p_4, p_{12}, p_{14})\) which is defined in \((3.23)\). The first momenta configuration which we are interested in is called the “equilateral configuration” \((p_1 = p_2 = p_3 = p_4 = 1)\). In this configuration, we plot the shape function \(A(p_{12}, p_{14})\) versus \(p_{14}\) and \(p_{12}\) (see Fig. 3). And one can easily see from Fig. 3 that the amplitude of the shape function blows up on the circle with radius \(2p_1\), which represents for the limit with \(p_1 \cdot p_{13} = 0\) \((p_{13} = 0)\).

In the second case, we consider a specialized planar momenta configuration, i.e., the quadrangle with \(p_3 = p_4 = p_{12} = 1\) (see Fig. 4). As we have said, in the planar limit, \((3.27)\) takes the equal sign, so we can obtain \(p_{14}\) by solving \((3.27)\)

\[
p_{14} = \frac{\sqrt{p_1^2 (-p_1^2 + p_3^2 + p_{12}^2) \pm p_{s1}^2 p_{s2}^2 + p_{12}^2 p_2^2 + p_{12}^2 p_3^2 + p_2^2 p_3^2 - p_2^2 p_3^2 - p_{12}^4 + p_{12}^2 p_{14}^2}}{\sqrt{2} p_{12}},
\]

where \(p_{s1}\) and \(p_{s2}\) are defined as

\[
p_{s1}^2 \equiv 2 \sqrt{(p_1 p_{12} + p_1 \cdot p_{12}) (p_1 p_{12} - p_1 \cdot p_{12})},
\]
\[
p_{s2}^2 \equiv 2 \sqrt{(p_3 p_{12} + p_3 \cdot p_{12}) (p_3 p_{12} - p_3 \cdot p_{12})}.
\]

As pointed out in \([31]\), the \(-\) solution (blue) and \(+\) solution (orange) in fact dual with each other (see Fig. 6), so we can choose arbitrary one to discuss, in the following we take the \(+\) solution.
Figure 5: Shape of the equilateral configuration $A(p_{12}, p_{14})$: in this configuration we set $(p_1 = p_2 = p_3 = p_4 = 1)$.

Figure 6: Shape of the “specialized planar” configuration $A(p_1, p_2)$: in this configuration we set $(p_3 = p_4 = p_{12} = 1)$.

After setting $p_3 = p_4 = p_{12} = 1$ and solving $p_{14}$, the two independent arguments of the shape function are $p_1$ and $p_2$, so, in this “specialized planar configuration”, we plot the shape function $A(p_1, p_2)$ versus $p_1$ and $p_2$ (see Fig. 6). From Fig. 6, one can see that the shape function was highly-peaked at the “squeezed limit” $(p_1, p_2 \rightarrow 0)$.

Figure 7: This figure illustrates the two dual quadrangles with the same absolute value of momenta $(p_1, p_2, p_3, p_4, p_{12})$: the orange one corresponds to the $+$ solution and the blue one to the $-$ solution.

4. Conclusion

In this paper, we investigated the primordial trispectra produced by the “scalar-exchanging” interaction of the general multifield DBI inflationary model. In section 2, we studied the power spectra of the entropic modes and adiabatic modes without considering the mass term and the mixing term $\langle Q_s Q_\sigma \rangle$. In contrast with the single field model, the curvature perturbations generally evolve in time on the large scales in the multifield scenario, because the entropic modes can source the curvature perturbations on the super horizon scales, which can be described by the transfer coefficient $T_{RS}$. So, if the transfer process is strong ($T_{RS} \gg 1$), the late time curvature perturbations will be mostly of the entropic origin.

Given that reason, in section 3, we calculate the contributions from the interaction of four entropic modes mediating one adiabatic mode to the trispectra. In subsection 3.1, 

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we derived the general form of all the 4-point correlation functions, and in subsection 3.2 we further analyzed the shape function and plotted the shape diagrams for two specific momenta configuration “equilateral configuration” and “specialized planar configuration”. And our figures showed that one can easily distinguish the two types of configurations, because in the “equilateral configuration” the shape function blows up when \( p_1 \) was perpendicular to \( p_{13} \), or equivalent to say when \( p_{13} \rightarrow 0 \), however, in the “specialized planar configuration” it was highly-peaked in the “squeezed limit” \( (p_1, p_2 \rightarrow 0) \).

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