LIGHT-CONE FORM OF FIELD DYNAMICS
IN ADS SPACE-TIME

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Light-cone approach to field dynamics in AdS space-time is discussed.

1. Introduction
In spite of its Lorentz noncovariance, the light-cone formalism offers conceptual and technical simplifications of approaches to various problems of modern quantum field theory. For example, one can mention the construction of first quantized light-cone string action and manifestly supersymmetric formulation for superfield theories of superstrings. Another attractive application of the light-cone formalism is a construction of interaction vertices in the theory of higher spin massless fields. Note that sometimes, a theory formulated within this formalism turns out to be a good starting point for deriving a Lorentz covariant formulation.

Motivated by a desire to solve the problem of AdS superstring, a light-cone gauge form of field dynamics in AdS space-time was recently developed. Application of light-cone formalism to IIB supergravity in AdS$_5 \times S^5$ background and discussion of various related issues may be found in Refs. Study of AdS superstring in the framework of light-cone gauge may be found in Refs. In the particle theory limit, the string Hamiltonian reduces to the light-cone Hamiltonian for a superparticle in AdS$_5 \times S^5$. This implies that the “massless” (zero-mode) spectrum of the superstring light-cone gauge action coincides indeed with the spectrum of type IIB supergravity compactified on $S^5$. Discussion of alternative gauges for the string in AdS$_5 \times S^5$ background may be found in Refs. Here we restrict our attention to light-cone gauge field dynamics in AdS space-time.

2. Light-cone gauge action
Let $\phi$ be arbitrary spin field propagating in AdS space-time. Light-cone gauge action for this field can be cast into the following “nonrelativistic form”:

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Another problem which triggered our investigation of AdS light-cone formalism is a construction of action for the equations of motion of higher spin massless field theory.

We use parametrization of AdS$_d$ space in which $ds^2 = R^2(-dx_0^2 + dx_1^2 + \ldots + dx_{d-1}^2)/z^2$, where $R$ is radius of AdS geometry. Light-cone coordinates in $\pm$ directions are defined as $x^\pm = (x^d \pm x^0)/\sqrt{2}$.
where $P^-$ is the (minus) Hamiltonian density and $A$ is some operator does not depending on space-time coordinates and their derivatives. This operator acts only on spin indices of field $\phi$. From the expressions above it is clear that the action can be rewritten in the following 'covariant form'

$$S_{l.c.} = \frac{1}{2} \int d^d x \phi (\Box - \frac{1}{2} A) \phi,$$

where $\Box$ is the flat D'Alembertian operator. We shall call the operator $A$ the $\text{AdS}$ mass operator. By now this operator is known for the following cases

- massive fields of arbitrary spin and arbitrary type of Young symmetry
- massless fields of arbitrary spin corresponding to totally symmetric and totally antisymmetric representations of $\text{so}(d-2)$ algebra
- type IIB supergravity in $\text{AdS}_5 \times S^5$ background

For the case of massless mixed symmetry fields the $\text{AdS}$ mass operator is still to be found. Let us discuss $\text{AdS}$ mass operator for various fields.

**Massive scalar field.** In this case starting with the standard action

$$S = \frac{1}{2} \int d^d x \sqrt{g} \Phi \left( \frac{1}{\sqrt{g}} g^{\mu \nu} \partial_\mu \partial_\nu - m^2 \right) \Phi$$

and making rescaling $\Phi = z^{(d-2)/2} \phi$ one finds an action similar to (2) where operator $A$ takes the form

$$A = (mR)^2 + \frac{d(d-2)}{4}.$$

For conformal invariant scalar field one has $m^2 = -\frac{d(d-2)}{4R^2}$ and this gives $A = 0$.

**Massless spin 1 field.** In light-cone gauge physical d.o.f. of spin one massless field are described by vector field $\phi^I$. It turns out that the field $\phi^I$ is not eigenvector of the operator $A$. However if we decompose the field $\phi^I$, which is $\text{so}(d-2)$ vector, into $\text{so}(d-3)$ vector $\phi^I_\lambda \equiv \phi^I$ and $\text{so}(d-3)$ scalar $\phi_0 \equiv \phi^z$ then one has

and we adopt the following conventions: $I, J = 1, \ldots, d-2; i, j, k, l = 1, \ldots, d-3$. $\partial^I = \partial_I \equiv \partial/\partial x^I$, $\partial^z = \partial_z \equiv \partial/\partial x^z$, $z \equiv x^{d-2}$. The coordinate $x^+$ is taken to be a light-cone time.

d Note that the $\text{AdS}$ mass operator $A$ for massless fields does not equal to zero in general. The operator $A$ is equal to zero only for massless representations which can be realized as irreducible representations of conformal algebra, which for the case of $d$-dimensional $\text{AdS}$ space-time is the $\text{so}(d,2)$ algebra.

e Note that all tensor fields are defined in tangent space.
Light-cone form of field dynamics

\[ A\phi^i_1 = \frac{(d-2)(d-4)}{4} \phi^i_1, \quad A\phi_0 = \frac{(d-4)(d-6)}{4} \phi_0, \quad (5) \]

and the light-cone gauge action \( S_{l.c.} \) takes the form

\[ S_{l.c.} = \frac{1}{2} \int d^d x \left( \phi^I \Box \phi_I - \frac{(d-2)(d-4)}{4 z^2} |\phi_1|^2 - \frac{(d-4)(d-6)}{4 z^2} |\phi_0|^2 \right). \quad (6) \]

**Massless spin 2 field - graviton.** Physical d.o.f. of spin two field are described by symmetric tracesless \( so(d-2) \) tensor field \( \phi^{IJ} \). As before this field is not eigenvector of \( A \). Decomposing \( \phi^{IJ} \) into \( so(d-3) \) traceless tensor field \( \phi^{ij}_{2} = \phi^{ij} - \delta^{ij} \phi^{kk}/(d-3), \) vector field \( \hat{\phi}_1 = \hat{\phi}^{\ast i} \) and scalar field \( \phi_0 = \hat{\phi}^{zz} \) we get

\[ A\phi^i_2 = \frac{d(d-2)}{4} \phi^i_2, \quad A\phi^i_1 = \frac{(d-2)(d-4)}{4} \phi^i_1, \quad A\phi_0 = \frac{(d-4)(d-6)}{4} \phi_0, \quad (7) \]

and the following light-cone gauge action

\[ S_{l.c.} = \frac{1}{2} \int d^d x \sum_{s' = 0}^{s} (\phi^{1\ldots s}_{s'} \Box \phi^{1\ldots s}_{s'} - 1 \frac{d-5}{2} A_{s'} |\phi_{s'}|^2). \quad (10) \]

Note that the above formulas for AdS mass operator are valid for \( d > 4 \). The representation for operator \( A \) which is valid also for the case of \( d = 4 \) is given by

\[ A = \frac{1}{2} M^{ij} M^{ij} + \frac{(d-4)(d-6)}{4}, \quad M^{ij} = -M^{ji}, \quad (11) \]

where \( M^{ij} \) is a spin part of \( so(d-3) \) angular momentum. From these formulas we see that for \( d = 4 \) the AdS mass operator is equal to zero. This reflects the fact that all massless fields in \( d = 4 \) can be realized as irreducible representations of conformal algebra \( so(4,2) \).

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References

1. P. A. Dirac, Rev. Mod. Phys. 21, 392 (1949).
2. P. Goddard, J. Goldstone, C. Rebbi and C.B. Thorn, Nucl. Phys. B56, 109 (1973).
3. M. B. Green, J. H. Schwarz and L. Brink, Nucl. Phys. B219, 437 (1983).
4. M. B. Green and J. H. Schwarz, Nucl. Phys. B243, 475 (1984).
5. A. K. Bengtsson, I. Bengtsson and L. Brink, Nucl. Phys. B227, 31 (1983).
6. R. R. Metsaev, Mod. Phys. Lett. A6, 359 (1991). Mod. Phys. Lett. A8, 2413 (1993).
7. R. R. Metsaev, Class. Quant. Grav. 10, L39 (1993). Phys. Lett. B309, 39 (1993).
8. E. S. Fradkin and R. R. Metsaev, Class. Quant. Grav. 8, L89 (1991). Phys. Rev. D52, 4660 (1995).
9. R. R. Metsaev, “Cubic interaction vertices for higher spin fields,” Talk given at 2nd International Sakharov Conference on Physics, Moscow, Russia, 20-23 May 1996. hep-th/9705048.
10. W. Siegel, “Introduction To String Field Theory,” Singapore: World Scientific (1988) 244 p. (Advanced series in mathematical physics, 8).
11. M.A. Vasiliev, Phys. Lett. B243, 378 (1990); Int. J. Mod. Phys. D5, 763 (1996) hep-th/9611024.
12. R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. B533, 109 (1998) hep-th/9805025.
13. R. Kallosh, J. Rahmfeld and A. Rajaraman, JHEP 9809, 002 (1998) hep-th/9805217.
14. R. R. Metsaev, Nucl. Phys. B563, 295 (1999) hep-th/9906217.
15. R. R. Metsaev, “Light cone formalism in AdS spacetime,” in “Proc. XIV Int. Workshop on High Energy Physics and Quantum Field Theory” (QFTHEP’99, Moscow, 27 May - 2 June, 1999), Eds. B.B.Levchenko and V.I.Savrin, MSU-Press 2000, hep-th/9911016.
16. R. R. Metsaev, Phys. Lett. B468, 65 (1999) hep-th/9908114.
17. R. R. Metsaev, “IB supergravity and various aspects of light-cone formalism in AdS space-time,” Talk given at International Workshop on Supersymmetries and Quantum Symmetries (SQS 99), Moscow, Russia, 27-31 Jul 1999. hep-th/0002008.
18. R. R. Metsaev and A. A. Tseytlin, hep-th/0007036.
19. R. R. Metsaev, C. B. Thorn and A. A. Tseytlin, hep-th/0009171.
20. R. Kallosh, “Superconformal actions in Killing gauge,” hep-th/9807206.
21. I. Pesando, JHEP 9811, 002 (1998) hep-th/9808020. Phys. Lett. B485, 246 (2000) hep-th/9912284.
22. R. Kallosh and J. Rahmfeld, Phys. Lett. B443, 143 (1998) hep-th/9808038.
23. R. Kallosh and A. A. Tseytlin, JHEP 9810, 016 (1998) hep-th/9808088.
24. R. Roiban and W. Siegel, “Superstrings on AdS5 x S5 super-twistor space,” hep-th/0001014.
25. R. R. Metsaev, Mod. Phys. Lett. A10, 1719 (1995).
26. R. R. Metsaev, Phys. Lett. B354, 78 (1995). Phys. Lett. B419, 49 (1998) hep-th/9802097.
27. R. R. Metsaev, “Arbitrary spin massless bosonic fields in d-dimensional anti-de Sitter space,” Talk given at International Seminar on Supersymmetries and Quantum Symmetries, Dubna, Russia, 22-26 Jul 1997. hep-th/9810231.
28. L. Brink, R. R. Metsaev and M. A. Vasiliev, Nucl. Phys. B586, 183 (2000) hep-th/0005136.