We present a calculating method for the quark and lepton mixing angles. After a general discussion in field theoretic models, we present a working model from a string compactification through $\mathbb{Z}_{12}$ orbifold compactification. It is beyond presenting just three families of the standard model but is the first example from string compactification successfully fitting to the observed data. Assuming that all Yukawa couplings from string compactification are real, we also comment on a relation between the $CP$ phases in the Jarlskog determinants obtained from the Cabibbo-Kobayashi-Maskawa and Pontecorvo-Maki-Nakagawa-Sakada matrices. The flipped SU(5) model leads to the doublet-triplet splitting and possible proton decay operators. It is shown that the vacuum expectation values can be tuned such that the proton lifetime is long enough.

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I. INTRODUCTION

“How is the current allocation of flavors realized?” is the most urgent and also interesting theoretical problem in the standard model (SM). Extension of the SM to grand unification (GUT) and string models [1,2] continues to require to solve this flavor problem. Gauge symmetries as family groups should satisfy the anomaly freedom, which can be achieved in extended GUTs [3] and in models without anomaly [4]. Not to worry about the gauge anomalies, sometimes global symmetries are used for the family groups [5–7]. It has been reviewed at several places [8,9].

In the SM, the difference of families is manifested in the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the quark sector [10,11] and in the Pontecorvo-Maki-Nakagawa-Sakada (PMNS) matrix in the leptonic sector [12,13]. To relate the left(L) and right(R) mixing angle parameters, the flavor group $G_f$ has been introduced to obtain more relations between flavor parameters [14–17]. In most cases, a factor flavor group $G_f$ is introduced in addition to the SM or GUT. On the other hand, an attractive mechanism is to unify all the fermion representations in an irreducible set of SU($N$) representations of an extended GUT [3,18–20].

The $E_8 \times E_8$ gauge group can be considered to belong to this class but in ten dimensions. Compactification of six extra dimensions may be the key to the unification of families in ten dimensional superstring models.

A notable difference between the CKM and the PMNS matrices lies in the fact that in the CKM matrix the large elements are located in the diagonal entries while it is not so in the PMNS matrix. So, for the CKM parameters the quark mass ratios were used before [14,15]. On the other hand, for the PMNS parameters non-Abelian discrete groups are used [21,22]. One may say that there is one similarity in the $CP$ phases of the CKM and PMNS matrices. The CKM phase is close to 90 degrees in the Kim-Seo (KS) parametrization [23] and the PMNS phase is $-90$ degrees (but with a large error bars) [24]. Even, there exists an attempt to unify these $CP$ phases [25].

To reduce the number of parameters in the flavor sector, family symmetries can be used. Simple ones are U(1) groups. But, to introduce a hierarchy, vacuum expectation values (VEVs) of the SM singlets are suggested, which is known as the Froggatt-Nielsen (FN) mechanism [26].

In this paper, we study singlet representations beyond the SM based on family symmetry groups. For various reasons in field theoretic models, we consider U(1)$^2$ among which one is anomalous and the other is anomaly free. We attempt to obtain singlets from the orbifold compactification of the $E_8 \times E_8$ heterotic string [27] based on the simplest $\mathbb{Z}_{12-I}$ lattice [28,29]. Fixed points of 13 prime orbifolds listed in [30] shows that the $\mathbb{Z}_{12-I}$ lattice can be...
considered to be the simplest because there are only three fixed points.\footnote{The $\mathbb{Z}_3$ orbifold, seemingly looking very simple, has 27 fixed points. Being simple, $\mathbb{Z}_{12-1}$ may not be general enough, but can present the basic working principles in terms of small number of fields.}

Yet the clearest statement to date is that standardlike models are exceedingly rare \cite{31,32}. The degree of acceptable standard-like models can be guessed from minilandscape studies of $\mathbb{Z}_{6-11}$ models \cite{33,34}. In \cite{34}, it was shown that acceptable model is $O(1)$ out of $O(10^6)$. Even in this case, one has to check all elements of the CKM and PMNS matrices.

In Sec. II, we briefly recapitulate the fermion mass structure: Dirac fermions of charged leptons and quarks, and Majorana fermions for the SM neutrinos. We set up the scheme to use Weyl fermions to express both the Dirac and Majorana fermions for the SM neutrinos. We present the basic working principles in terms of small number of fields: Dirac fermions of charged leptons and quarks, and PMNS matrices.

In Sec. II, we briefly recapitulate the fermion mass structure: Dirac fermions of charged leptons and quarks, and Majorana fermions for the SM neutrinos. We set up the scheme to use Weyl fermions to express both the Dirac and Majorana masses presented in subsequent sections. Those who are familiar to these can skip this section. In Sec. III, we present a beyond-SM with two $U(1)$s toward useful fermion mass textures, where one is $U(1)_{\text{anom}}$, global symmetry for the “invisible” axion and the other is anomaly free gauged $U(1)$. In Sec. IV, we obtain a successful flavor structure from $\mathbb{Z}_{12-1}$ orbifold compactification. The 3rd family is assumed to be the one from the untwisted sector $U$. Section V is a conclusion.

### II. FERMION MASSES

For continuous parameters of transformation, let us begin with the axial-vector currents of fermions

\[ J^\mu_\Gamma = \bar{\Psi} \gamma^\mu \gamma_5 \Gamma \Psi \]  
where $\Gamma$ is a charge operator and $\Psi$ is a column vector of three fermions (families). The divergence of the current is

\[ \partial_\mu J^\mu_\Gamma = \frac{A_\Gamma}{32\pi^2} \mathcal{G}^\mu_\Gamma + 2 \mu J^\mu_\Gamma \]  
where $A_\Gamma$ is the anomaly coefficient of the gauge fields $A^\mu_\Gamma$ for the gauge group $SU(N)_\Gamma$, and $J^\mu_\Gamma$ depends on the masses of fermions,

\[ J^\mu_\Gamma = \frac{1}{\mu} \Psi i \gamma_5 M^\mu_\Gamma \Psi, \]  
where $\mu$ is a mass scale and $M$ is the mass matrix in the flavor basis. The anomaly term is a flavor singlet which can be written in terms of a flavor singlet quark fields in the $\theta$-vacuum, e.g., for two flavors in $SU(3)_c$ \cite{35},

\[ G^{\mu_5}_\Gamma \propto \sqrt{\frac{m_u m_d}{2 \cos \theta + (1 + Z^2)/Z} (\bar{u} i \gamma_5 u + \bar{d} i \gamma_5 d) \sin \theta} \]  
where $Z = m_u / m_d \approx 1/2$. Obviously, the anomalous and anomaly free terms give nonzero trace for the fermion mass matrix. In Ref. \cite{9}, two $U(1)$ symmetries were considered, one anomalous and the other anomaly-free. The anomalous global symmetry is to introduce the so-called “invisible” axion. Since the sum of quark masses is nonzero and large $O(m_t)$, we also attempt to have the anomalous $U(1)$. The anomalous $U(1)$ must be a global symmetry and the anomaly free part can be a gauge symmetry. Let us start using two component fermions to write down mass terms.

### A. Weyl fermions

A four component Dirac spinor, e.g., for the electron field, can be split into two Weyl spinors $\xi$ and $\eta$,

\[ \psi_e = \begin{pmatrix} e_L \\ e_R \end{pmatrix} = \begin{pmatrix} e_{L1} \\ e_{L2} \\ e_{R1} \\ e_{R2} \end{pmatrix} \rightarrow \begin{pmatrix} \xi_L \\ \eta_R \end{pmatrix}. \]  
Gauge interactions do not change the chirality. Quantum field $e_L$ destroys a L-handed electron and creates a R-handed positron. But, $e_L$ has nothing to do with destroying a R-handed electron $e_R$ and creating a L-handed positron. On the other hand, the antiparticle of the L-handed electron $e_L = (e_{L1}, e_{L2}, 0, 0)^T$ is

\[ (e_L)^c = \begin{pmatrix} e_{L1} \\ e_{L2} \\ 0 \end{pmatrix} = \begin{pmatrix} 1 + \gamma_5/2 \end{pmatrix} e^c = i \sigma_2 \begin{pmatrix} 1 + \gamma_5/2 \end{pmatrix} e^c \]  
which is a R-handed field. This R-handed field destroys R-handed positron and creates L-handed electron. With these two Weyl fields, we can destroy L- and R-electrons and create L- and R-positrons, which is done by a four-component Dirac electron. Thus, two Weyl fields are enough at this stage. With the Weyl field $\xi$, let us construct a Lorentz invariant $\epsilon_{ij} e_{Li} e_{Lj}$. It is the mass term but the electron number is broken by this term. So, for charged particles, one Weyl field cannot be massive. For neutrinos, one Weyl field can give a mass term which is known to be
Majorana mass. In this paper, we will use Weyl fields even for expressing Majorana masses,
\[ e_{ij}(\xi^T)^i\xi^j = \xi^T \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \xi = \xi^T i \sigma_2 \xi = \xi^T \gamma_1 \gamma_3 \xi. \] (7)

For a Dirac mass we use the opposite chirality, i.e.,
\[ \tilde{\xi}_L^i = e^{ik\eta_{kr}} \xi_L^i \rightarrow e^{ik\eta_{kr}} \tilde{\eta}_{kr} = \tilde{\eta}_{kr}. \] (8)

so that (7) becomes
\[ \tilde{\eta}_{KR} \tilde{\xi}_L. \] (9)

Assigning the same charge conjugation for \( \xi \) fields in (7), the Majorana mass term breaks C, but the Dirac mass term (9) can preserve C by assigning the same C's for \( \xi \) and \( \eta \).

Discussing both Majorana and Dirac masses, using the Weyl fermion is therefore simple enough.

**B. \( m_b \approx m_e \) and Georgi-Jarlskog relation**

The observed ratio of the third family masses \( m_b/m_e \approx 4.5/1.5 \approx 3 \) hints that \( m_b \approx m_e \) at the unification scale. The factor 3 arises by renormalization group evolution [36]. In the Georgi-Glashow (GG) SU(5) [37], \( \mathbf{10}_1, \mathbf{5}_1, \mathbf{5}_H \) gives the same mass to \( b \) and \( r \) by \( \langle \mathbf{5}_H \rangle \) and it is considered to be a success of the GUT [38]. For the muon and strange quark, however, there is a big problem in the GG SU(5) model. The low energy mass ratio at 100 MeV is \( m_s/m_\mu \approx 1 \) while the renormalization group evolution expects it to be 3 if \( m_s \approx m_\mu \) at the unification scale. If \( m_s/m_\mu \approx _3^1 \) at the unification scale, then the low energy mass ratio is understandable. But, this is a big problem with Higgs quintets only. One way out is the Georgi-Jarlskog relation introducing a big Higgs representation \( 4\mathbf{5}_H \) [39]. If \( \langle 4\mathbf{5}_H \rangle \) is the leading contribution to the second family fermions in the GG model, then \( m_s/m_\mu \approx _3^1 \) is obtained at the unification scale.

To present a rationale for \( 4\mathbf{5}_H \) for the needed mass matrix texture, two U(1)'s were suggested long time ago [9].

**C. Flipped SU(5)**

Our terminology of flipped SU(5) is a rank 5 gauge group \( SU(5)_{\text{flip}} = SU(5) \times U(1)_X \). Representations will be denoted as \( SU(5)_{U(1)} \). In the flipped SU(5) [40,41], masses of charged leptons and \( d \)-type quarks are not related, which is considered to be a merit in relating masses. In string compactification, reasonable supersymmetric SM's are obtained from compactification of heterotic string. The reason is the following. For \( N = 1 \) supersymmetric (SUSY) massless fields, only the completely antisymmetric representations are allowed with one compactification scale from heterotic string [28]. If the Higgs fields breaking a GUT group appear as massless spectra, then there is no adjoint representation at the GUT scale which is needed for breaking the GG SU(5) or SO(10) GUT [42,43] or some Pati-Salam (PS) [44] gauge groups. In SU(5)\(_{\text{flip}}\), the representation \( \mathbf{10}_{r+1} \oplus \mathbf{\overline{10}}_{r-1} \) can break the rank-5 SU(5)\(_{\text{flip}}\) down to the rank-4 SM gauge group. At the GUT level, therefore only the flipped SU(5) is actually realized in several string compactifications [1,29,46,47].

**III. U(1)\(_{\text{anom}}\) \times U(1)\(_{B-L}\) FAMILY SYMMETRY**

We introduce supersymmetry and two U(1) gauge symmetries, \( U(1)_{\text{anom}} \times U(1)_{B-L} \), where \( U(1)_{\text{anom}} \) is anomalous and \( U(1)_{B-L} \) is free of gauge anomalies. Dangerous dimension-4 superpotential of the 1st family members triggering proton decay is
\[ q_1 q_1 l_1 \] (10)

where the subscript 1 denotes the first family, \( U(1)_{B-L} \) allows the above superpotential but \( U(1)_{\text{anom}} \) or \( U(1)_{B-L} \) may not allow it. Thus, the extra U(1)'s may be useful forbidding some unwanted proton decay operators. In string compactification, one has to check the \( U(1)_{\text{anom}} \times U(1)_{B-L} \) quantum numbers of the first family members to see if the unwanted proton decay operators are forbidden. If the proton decay problem is safe, one can consider the superpoten-tials generating fermion masses.

The mass eigenstates of quarks, \( q^m \), are related to the weak eigenstates by L- and R-unitary matrices, \( U \) and \( V \),
\[ q^m_{d,uL} = U_{d,u} q^m_{d,uL}, \]
\[ q^m_{d,uR} = V_{d,u} q^m_{d,uR}, \] (11)

and the charged \( W^\pm \) coupling for the L-handed quark doublets is
\[ W = U^*_a U_d, \] (12)

which is the CKM matrix.

**A. Effects of U(1)\(_{\text{anom}}\) on the texture of mass matrix**

To see the essence, let us consider two families of quarks. Let us choose the basis where \( Q^0 = \frac{1}{2} \) quarks are already mass eigenstates. Then, the mass matrices of weak and mass eigenstates of \( Q^0 = \frac{1}{2} \) quarks are related by \( M^a_d = V^a_d M^0_d U_d \). Parametrizing the unitary matrices as

\[ ^2\text{The electroweak PS gauge group SU}(2)_L \times SU(2)_R \times U(1)_{B-L} \text{ is broken by a GUT scale VEV to SM} \times U(1)_{B-L} \text{, needing an adjoint representation not to reduce the rank. Usually, it is denoted as } \Delta = (1,3,0). \text{ A VEV of an adjoint representation does not reduce the rank of the gauge group. But, note that an adjoint representation is possible in some scenarios in } \mathbf{Z}_{\text{PS}} \text{ by introducing two compactification scales for } N = 2 \text{ SUSY in an interim effective 5 dimensions [45].} \]
where $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$ for $i = 1, 2$. Thus, $M_{\nu}^\eta$ is given by

$$M_{\nu}^\eta = \begin{pmatrix} c_1c_2m_d + s_1s_2m_s, & s_1c_2m_d - c_1s_2m_s, \\ c_1s_2m_d - s_1c_2m_s, & s_1s_2m_d + c_1c_2m_s \end{pmatrix},$$

(14)

where $m_d$ and $m_s$ are eigenvalues of the mass matrix. If any one element of Eq. (14) is zero, then $\theta_1$ and $\theta_2$ are related. Weinberg’s choice [14] is $m_d \rightarrow -m_d$ and $(M_{\nu}^\eta)_{11} = 0$, leading to $s_1s_2/c_1c_2 = m_d/m_s$. Since $\sin \theta_c \approx \sqrt{m_d/m_s}$ numerically, we use the freedom in $V$ and choose $s_2/c_2 = s_1/c_1$, which means that the R-handed fields transform in the same way as the L-handed fields. This implies that under any extra U(1) gauge group the gauge transformations of the L- and R-handed fields are identical. Thus, extra U(1)'s should be free of gauge anomalies. Therefore, if we do not consider extra quarks beyond the SM the Fritzsch texture [15], following Ref. [14], is not valid with the U(1)_{anom} gauge group. Namely, in the presence of U(1)_{anom} gauge group, we must choose $V$ differently from $U$ even for three families.

Similarly, let us consider two families of leptons where charged lepton mass matrix is already diagonalized. Then, the mass matrices of weak and mass eigenstates of Majorana neutrinos are related by $M_{\nu}^\eta = U_L^T M_{\nu} U_\nu$. Thus, $M_{\nu}^\eta$ is given by

$$M_{\nu}^\eta = \begin{pmatrix} C_1^2 m_{\nu_\mu} + S_1^2 m_{\nu_\tau}, & -C_1 S_1 (m_{\nu_\mu} - m_{\nu_\tau}), \\ -C_1 S_1 (m_{\nu_\mu} - m_{\nu_\tau}), & S_1^2 m_{\nu_\mu} + C_1^2 m_{\nu_\tau} \end{pmatrix},$$

(15)

where $C_1 = \cos \Theta_1$ and $S_1 = \sin \Theta_1$. Since the mixing angle of the second and the third neutrinos is large, we can approximate $C_1 \approx S_1 = 1/\sqrt{2}$. In this case, the mass matrix is of the form\(^3\)

$$M_{\nu}^\eta = \begin{pmatrix} A, & -B \\ -B, & A \end{pmatrix},$$

(16)

\(^3\)This case with two parameters is including the possibility of family indices carried by Higgs fields. If family indices of Higgs fields are independent from the family index of quark and leptons, then there must be one parameter.

where $A = (m_{\nu_\mu} + m_{\nu_\tau})/2$ and $B = (m_{\nu_\mu} - m_{\nu_\tau})/2$, which has the permutation symmetry $S_2$ between the second and the third family indices. The useful discrete symmetries of [21,22] contain this $S_2$ as a subgroup. In this case of introducing U(1)$_{anom}$ where we introduced only L-handed neutrinos, the anomaly freedom must be satisfied by the quantum numbers of the first family leptons or by heavy leptons.

### B. Quark mass matrices

Let us begin with the diagonalized Dirac masses of the form (9) for $Q_{em} = +\frac{1}{3}$ quarks,

$$M_{u}^{\text{diag}} = \begin{pmatrix} \xi_{1L} \\ \xi_{2L} \\ \xi_{3L} \end{pmatrix} = \begin{pmatrix} \eta_{1R} \\ \eta_{2R} \\ \eta_{3R} \end{pmatrix} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_e & 0 \\ 0 & 0 & m_\tau \end{pmatrix},$$

(17)

where $\eta_{1R} = \bar{q}_{uR}$ and $\xi_{L} = q^{m}_{uL}$. The diagonal form (17) with the needed hierarchy can be obtained by the U(1) charges of Table I,

$$M_u \propto \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(18)

where $\sigma$ is a SM singlet field carrying $Q = -3$. The mass term for up-type quarks is

$$\bar{q}_{uR} M_{u}^{\text{diag}} q_{uL}^{m} = \bar{q}_{uR} V_u M_{u}^{\text{diag}} U_u q_{uL}^{m},$$

(19)

Of course, $V_u = U_u = \mathbf{1}$.

The mass matrix for $Q_{em} = -\frac{2}{3}$ quarks is

$$\bar{q}_{dR} M_{d}^{\text{diag}} q_{dL}^{m} = \bar{q}_{dR} V_d M_{d}^{\text{diag}} U_d q_{dL}^{m},$$

(20)

with

$$M_d = V_d^\dagger \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} U_d,$$

(21)
In this paper, we use the KS parametrization [23] of the CKM matrix $W_{KS}$ where $\delta_{CKM} = \frac{\pi}{2}$ is simple,\(^4\)

$$W_{KS} = U_d = \begin{pmatrix}
    c_1, & +s_1c_3, & +s_1s_3 \\
    -c_2s_1, & +c_1c_2c_3 + s_2s_3e^{-i\theta}, & +c_1c_2s_3 - s_2c_3e^{-i\theta} \\
    s_1s_2e^{i\theta}, & -c_2s_3 + c_1s_2c_3e^{i\theta}, & +c_2c_3 + c_1s_2s_3e^{i\theta}
\end{pmatrix}, \tag{22}$$

with the unitary matrix for R-handed fields in the diagonalization process parametrized by another 4 parameters

$$V_d^\dagger = \begin{pmatrix}
    c_4, & -c_5s_4, & -s_4s_5e^{-i\Delta} \\
    +s_4c_6, & +c_4c_5c_6 + s_5s_6e^{i\Delta}, & -s_5s_6 + c_4s_5c_6e^{-i\Delta} \\
    +s_4s_6, & +c_4c_5s_6 - s_5c_6e^{i\Delta}, & +c_5c_6 + c_4s_5s_6e^{-i\Delta}
\end{pmatrix}. \tag{23}$$

Then, we obtain $V_d^\dagger M_d^{diag} U_d$ as

$$\begin{pmatrix}
    c_4c_1m_d + c_5s_4c_2s_1m_s + s_4s_5s_1s_2e^{-i\Delta + i\theta}m_b \\
    -c_4s_1c_3m_d - c_5s_4c_2c_3m_s \\
    -c_4s_5c_6c_3m_d - c_5s_4c_2s_3m_s
\end{pmatrix},$$

$$\begin{pmatrix}
    +c_4c_5c_6c_2c_3m_s + s_4c_5s_6c_1c_3m_s \\
    +c_4c_5s_6c_2c_3e^{i\theta}m_s \\
    -c_4c_5s_6c_2c_3e^{i\Delta}m_s
\end{pmatrix},$$

$$\begin{pmatrix}
    +c_4c_5s_6c_1c_3m_s \\
    +c_4c_5s_6c_2c_3e^{i\theta}m_s \\
    +c_4c_5s_6c_1c_3e^{i\Delta}m_s
\end{pmatrix},$$

$$\begin{pmatrix}
    c_4c_1m_d + c_5s_4c_2s_1m_s + s_4s_5s_1s_2e^{-i\Delta + i\theta}m_b \\
    -c_4s_1c_3m_d - c_5s_4c_2c_3m_s \\
    -c_4s_5c_6c_3m_d - c_5s_4c_2s_3m_s
\end{pmatrix},$$

Then, we obtain $V_d^\dagger M_d^{diag} U_d$ as

Change the sign $m_d \to -m_d$, and to reduce the number of parameters let us choose parameters of R-fields as

$$\frac{s_4}{c_4} = \frac{s_1}{c_1}, \quad \frac{s_5}{c_5} = \frac{m_s}{m_b} \frac{s_2}{c_2}, \quad s_6 = 0, \quad \Delta = \delta. \tag{25}$$

Note that $s_4 \gg s_1s_2^2$. Then, keeping the largest terms in the weak basis mass matrix,

$$M_d^y \simeq \begin{pmatrix}
    0, & -s_1c_4c_1c_2^{-1}c_3m_s, & -c_5c_1s_4c_2^{-1}s_3m_s, \\
    -c_1c_5s_1c_2^{-1}m_s, & +c_1c_5s_1c_2^{-1}c_3m_s, & +c_1c_5s_1c_2^{-1}s_3m_s, \\
    -c_5s_1s_2e^{i\theta}m_b, & -c_5s_2c_3m_b + c_5s_1c_2c_3e^{i\theta}m_b, & +c_5c_2c_3m_b
\end{pmatrix},$$

$$\simeq \frac{1}{c_5c_2c_3m_b} \begin{pmatrix}
    0, & -4.43 \times 10^{-3}, & -0.690 \times 10^{-4} \\
    -4.43 \times 10^{-3}, & 1.918 \times 10^{-2}, & 2.99 \times 10^{-4} \\
    -0.9008 \times 10^{-2}e^{i\theta}, & -1.557 \times 10^{-2} + 3.90 \times 10^{-2}e^{i\theta}, & 1
\end{pmatrix} \tag{26}$$

\(^4\)As stressed in [23], the CP phase in the CKM matrix is close to 90 degrees if we parametrize it by $W_{KS}$. The phase $\delta$ is the phase in the Jarlskog determinant [48].
where we used
\[ m_s = 93.8 \text{ MeV}, \quad m_b = 4.65 \text{ GeV}, \quad \frac{m_s}{m_b} = 0.0202, \]
\[ s_1 = 0.2252, \quad c_1 = 0.9743, \quad s_2 = 0.0400, \quad c_2 = 0.9992, \quad s_3 = 0.01557, \quad c_3 = 0.9999, \]
\[ \frac{s_5}{c_5} = 0.809 \times 10^{-5}, \quad \frac{s_4}{c_4} = \frac{s_1}{c_1}. \]

In Eq. (26), the \( (32) \) element can be \( 4.2 \times 10^{-2} e^{i\delta} \) where \( \tan(\pi - \delta') = -0.9286 \sin \delta \). The Yukawa couplings run from the compactification scale down to the electroweak scale in which the case dimensionless Yukawa couplings cannot be used directly for assigning the input mass parameters. But all Yukawa couplings leading to parameters in Eq. (26) are arising from the VEVs of FN singlet fields and we may use those given in Eq. (26) as the input parameters determining the CKM matrix.

### 1. Mass matrix in field theory

Let us present a possibility of obtaining a mass matrix similar to Eq. (26) in field theory. Let the \( U \) - and \( V \) -parameters in Eq. (26) be arising from the VEVs of FN singlet \( Q \) -quantum numbers are shown as \( Q \). After diagonalizing the \( Q_{em} = \frac{2}{3} \) quark masses, the \( L \)- and \( R \)-fields of \( Q_{em} = \frac{1}{3} \) quarks, \( \xi \) and \( \eta \), quantum numbers of \( \bar{\eta}_q \) are

\[
\begin{align*}
\xi_{1L}(2, -1) & \quad \xi_{2L}(0, 1) & \quad \xi_{3L}(1, 0) \\
\bar{\eta}_{1R}(1, -1) & \quad (3, -2) & \quad (1, 0) & \quad (2, -1) \\
\bar{\eta}_{2R}(2, 1) & \quad (4, 0) & \quad (2, 2) & \quad (3, 1) \\
\bar{\eta}_{3R}(-1, 0) & \quad (1, -1) & \quad (-1, 1) & \quad (0, 0)
\end{align*}
\]

Thus, the quantum numbers of Higgs fields appearing in the mass matrix are

\[
Q(M^d) = \begin{pmatrix}
(\frac{-3, +2}{-4, 0}) & (\frac{-1, 0}{-2, -2}) & (\frac{-3, -1}{-1, +1}) \\
(\frac{+1, -1}{0, 0}) & (\frac{+1}{0}) & (\frac{1}{0})
\end{pmatrix}
\]

To mimic the order appearing in Eq. (26), let us introduce small parameters via the FN SM singlet fields, \( \delta_1, \delta_2, \delta_3, \Delta_1, \Delta_2, \varepsilon_1 \), and \( e_2 \) whose quantum numbers are shown in Table II.

Let us assume that only \( \delta_1 \) and \( \Delta_1 \) have complex VEV’s, \( \delta_1 e^{i\delta} \) and \( \Delta_1 e^{i\Delta} \) while all the other FN fields have real VEV’s. Thus, \( M^d \) can be written as

\[
M^d = m_b \begin{pmatrix}
\delta_1^3 e^{i\delta} \delta_1^2 e_1 & -\delta_1 |e^{i\delta}| & \delta_2 e_1, \Delta_2 e_1^2 \\
-\Delta_1 |e^{i\Delta}| & |\Delta_1| e^{i\Delta} \delta_2, \Delta_2 \delta_1 |e^{i\delta}| e_1 \\
-\delta_1 |e^{i\delta}| & (|\delta_1| e^{i\delta} + \delta_3) e_2 & 1
\end{pmatrix},
\]

where the overall constant is \( m_b \) and for simplicity we do not write group theoretic numbers of O(1). The element \( M^d_{23} \) can have \( \delta_3^2 e^{3i\delta} e_2 \) which we neglected because it is much smaller than the other terms. A negative signed phase in \( M^d_{23} \), of Eq. (30) may need a complex conjugate field, but we do not introduce complex conjugate fields in the mass matrix for a SUSY extension.

### C. With SUSY

Not to introduce complex conjugate fields in the mass matrix, let us consider the fields presented in Table III,

\[
M^d = m_b \begin{pmatrix}
\delta_1^3 e^{3i\delta} \delta_1^2 e_1 & -\delta_1 |e^{i\delta}| & \delta_1 \Delta_3, \Delta_2 e_1^3 \\
-\Delta_1 |e^{i\Delta}| & |\Delta_1| e^{i\Delta} \delta_2, \Delta_2 \delta_1 |e^{i\delta}| e_1 \\
-\delta_1 |e^{i\delta}| & -|\Delta_1| e^{i\Delta} \delta_2, -\delta_3 e_2 & 1
\end{pmatrix},
\]

For \( \delta_3 = O(1) \) and small \( \Delta_3 \) and \( e_2 \), and redefine \( \xi_2 \rightarrow e^{i\delta} \xi_2, \bar{\eta}_2 \rightarrow e^{-i\Delta} \bar{\eta}_2 \). By choosing \( \Delta_{ph1} = \Delta_{ph3} = \delta \), and \( \delta = \xi \), we obtain

\[
M^d \simeq m_b \begin{pmatrix}
\mp \delta_1^3 e_1^2 i & -|\delta_1| & -a_1 |\Delta_2| e_1^3 \\
-|\Delta_1| & |\Delta_2| & a_2 |\Delta_1| \delta_2 e_2 + a_3 \Delta_2 \delta_1 |e_1 + a_4 \Delta_2 |\Delta_3| \\
-\delta_1 |e_1 | & -|\Delta_3| + \delta_2 e_2 i & 1
\end{pmatrix},
\]

where we introduced O(1) numbers \( a_{1,2,3,4} \). Firstly, \( |\Delta_1| = |\delta_1| = 4.432 \times 10^{-3} \) and require \( a_1 \Delta_2 e_1^3 = 0.690 \times 10^{-4} \) (with \( a_i \approx 1 \)). Let \( |\Delta_2| = 0.01918 \) and \( |\delta_1| e_1 = 0.9008 \times 10^{-2} \). Then, we have \( e_2 = 2.032, \quad |\Delta_3| = 1.557 \times 10^{-2}, \)
\[ \delta_1 e_2 \equiv A = 3.90 \times 10^{-2}. \]  

Requiring \( a_3 |\Delta_1| \delta_1 e_2 + a_4 |\Delta_2| \delta_1 e_1 = 2.99 \times 10^{-4} \), where all terms are \( \mathcal{O}(10^{-4}) \). To obtain the relations between phases, \( \Delta_{ph1} = \Delta_{ph3} = \delta \), we can consider the following superpotential,

\[
W_{CP} = -i \mu_1 \delta_1 \delta_3 + \frac{1}{\mu_1} \delta_4^2 + i M_0^2 \Delta_1 + M_1 \Delta_1^2 + M_2 \Delta_2^2 \\
+ i \lambda_1 \Delta_1 \Delta_2^2 + i \lambda_2 \Delta_2 \delta_2 e_2 + \lambda_3 \Delta_2 \Delta_3^2 + \frac{1}{\mu_2} \delta_3 \Delta_3^2 e_1,
\]

where parameters are real numbers. The following SUSY conditions lead to the desired relations:

\[
U_{PMNS} = \begin{pmatrix}
C_1, & +S_1 C_3, & +S_1 S_3 \\
-S_2 S_1, & +C_1 C_2 C_3 + S_2 S_3 e^{-i \delta}, & +C_1 C_2 S_3 - S_2 C_3 e^{-i \delta} \\
-S_1 S_2 e^{i \delta}, & -C_1 C_3 + S_1 C_3 e^{i \delta}, & +C_2 C_3 + C_1 S_2 S_3 e^{i \delta}
\end{pmatrix},
\]

where the parameters are the leptonic parameters, \( \Theta_{1,2,3} \) and \( \delta_L \). Since the PMNS matrix elements are not known as accurately as the CKM matrix elements, we do not present a detailed study of the leptonic sector. But note that the phase \( \delta_L \) in Eq. (35) is the PMNS phase \( \delta_{PMNS} \).

**IV. FROM E_8 × E_8 HETEROPTIC STRING**

In this section, we attempt to realize the texture of quark mass matrix discussed in Subsection III B. We will not discuss the texture of neutrino mass matrix since the PMNS matrix elements are not known very accurately. Nevertheless, we will comment on the relation of \( CP \) phases in the quark and lepton sectors in this section.

**A. Z_{12-1} orbifold compactification**

Note that the SM mass matrix

\[
\Psi_L^c C^{-1} \Psi_L M^{IJ} + \text{H.c.}
\]

(36)

gives in general non-symmetric mass matrix of \( M \) because \( \Psi_L^c \) and \( \Psi_L \) transform differently under \( SU(2)_L \times U(1)_Y \). In the GUT model, Majorana neutrinos in the \( SU(2)_L \) doublets are embedded in \( 5_0 \) of \( SU(5) \) in the GG model and \( 5_{+3} \) in the \( SU(5)_{\text{flip}} \). Then, the effective neutrino mass matrix in these simple GUTs are symmetric. For the quark mass matrix, \( \mathbf{10}_f, \mathbf{5}_{Higgs} \) is the up-type quark mass matrix in the GG model, which is symmetric. In the GG model, we usually use diagonalized up-type quark mass matrix, and consider non-symmetric \( 5_f/\mathbf{10}_f, \mathbf{5}_{Higgs} \) for the down-type quark mass matrix. On the other hand, in the \( SU(5)_{\text{flip}} \) the down-type quark mass matrix, \( \mathbf{10}_f, \mathbf{5}_{Higgs} \) is symmetric and the up-type quark mass matrix \( 5_f/\mathbf{10}_f, \mathbf{5}_{Higgs} \) is non-symmetric. So, we prefer to consider a symmetric down-type quark mass matrix in \( SU(5)_{\text{flip}} \). The up-type quark mass matrix is non-symmetric, and we can assign different coefficients for \( M_{(u)IJ} \) and \( M_{(u)JI} \).

\[
\text{down type quark mass matrix} = \text{symmetric} \\
\text{up type quark mass matrix} = \text{asymmetric}
\]

(37)

The \( SU(5)_{\text{flip}} \) GUT gauge group presented in Ref. [2] is

\[
SU(5) \times U(1)_X \times SU(5)' \times U(1)'^6,
\]

(38)

where, in the notation of [28],

\[
X = (-2, -2, -2, -2, 0, 0, 0, 0) (0^8)'.
\]

(39)
TABLE IV. The SU(5)_{\text{flip}} fields. Fields needed in the SM are on the four left columns and SM singlet components needed at the GUT scale toward the FN mechanism are on the four right columns. Both neutrino components in C_{11} and C_{12} develop a GUT scale VEV to break SU(5)_{\text{flip}} down to the SM.

| SU(5)_{\text{flip}} (Symbol) | Sect. | U(1)_{\text{anom}} (Q_{\text{anom}}) | Q_{1}/2 | SU(5)_{\text{flip}} (Symbol) | Sect. | U(1)_{\text{anom}} (Q_{\text{anom}}) | Q_{1}/2 |
|-------------------------------|-------|-------------------------------------|--------|-------------------------------|-------|-------------------------------------|--------|
| 1_{-5} (S_1)                 | U     | +5                                  | −3     | 1_{0} (\sigma_1)              | T_0^6 | −12                                 | −4     |
| 1_{-5} (S_2)_{\text{Sf}}    | T_3^0 | −3                                  | −1     | 1_{0} (\sigma_2)              | T_3^0 | −2                                  | −4     |
| 1_{-5} (S_3)_{\text{Sf}}    | T_3^0 | −3                                  | −1     | 1_{0} (\sigma_3)              | T_3^0 | −8                                  | +2     |
| 1_{0} (C_2)                  | U     | T_3                                 | −13    | 1_{0} (\sigma_4)              | T_3^0 | +10                                 | +2     |
| 5_{+3} (C_1)                 | U     | +1                                  | +3     | 1_{0} (\sigma_5)              | T_6   | +14                                 | 0      |
| 5_{+3} (C_{3a})              | T_3^0 | −3                                  | −1     | 1_{0} (\sigma_6)              | T_6   | −4                                  | 0      |
| 5_{+3} (C_{3b})              | T_3^0 | −3                                  | −1     | 1_{0} (\sigma_9)              | T_9   | −6                                  | −2     |
| 1_{0} (C_{4a})               | T_3^0 | −3                                  | −1     | 1_{0} (\sigma_{10})            | T_9   | −6                                  | −2     |
| 1_{0} (C_{4b})               | T_3^0 | −3                                  | −1     | 1_{0} (\sigma_{11})            | T_3   | +12                                 | +3     |
| 5_{-2} (H_u)                 | T_6   | 0                                   | 0      | 1_{0} (\sigma_{13})            | T_9   | +12                                 | +3     |
| 5_{-2} (H_d)                 | T_6   | 0                                   | 0      | 1_{0} (\sigma_{21})            | T_9   | +12                                 | +3     |

and six U(1) directions of Ref. [2] are

\[Q_1 = (0^6, 12, 0, 0)(0^8)',\]
\[Q_2 = (0^6, 0, 12, 0)(0^8)',\]
\[Q_3 = (0^8, 0, 0, 12)(0^8)',\]
\[Q_4 = (0^8)(0^4, 0, 12, −12, 0)',\]
\[Q_5 = (0^8)(0^4, 12, −6, −6, 12)',\]
\[Q_6 = (0^8)(−6, −6, −6, 18, 0, 0, 6)'.\]

The second equality is for vanishing D-term at the GUT scale. The renormalizable coupling, including the Higgs quintet \(10_{+1}5_{-2}10_{+1}1\) \(\Phi^{ab}\Phi'\Phi^{de}\epsilon_{abcde}\) might give the GUT scale mass term to colored scalars by \(\{de\} = \{45\}\), but \(C_{12}H_dC_{12}\) coupling is not allowed by the nonvanishing \(Q_{\text{anom}}\). A possible higher dimensional operator consistent with the orbifold selection rules and \(U(1)_{\text{anom}} \times U(1)_{\text{3S}}\) gauge symmetry is

\[
\frac{1}{M^2}C_{111}[\overline{10}_{-1}(T_3)]H_d[5_{+2}(T_6)]C_{111}[\overline{10}_{-1}(T_3)]\sigma_5[1(T_3^0)]
\times \sigma_5[1(T_6^0)]\sigma_{21}[1(T_1^0)]\sigma_{21}[1(T_1^0)]
\]

By giving GUT to Planck scale VEVs to \(C_{11}, \sigma_3, \sigma_5,\) and \(\sigma_{21},\) we obtain a GUT scale mass term for colored scalars,

\[
M_T\epsilon_{\alpha\beta\gamma}\Phi'^\alpha\Phi'^\beta\Phi'^\gamma
\]

where \(\alpha, \beta, \gamma\) are the color indices. Thus, the color antitriplet in \(\overline{10}\) combines with the color triplet in the Higgs quintet \(\overline{5}\). The colored scalar in the Higgs quintet \(H_d\) is removed at the GUT scale, and there remains just the Higgs doublet from \(H_d\). For this doublet-triplet splitting, we need

\[
\langle \sigma_3 \rangle \neq 0, \quad \langle \sigma_5 \rangle \neq 0, \quad \langle \sigma_{21} \rangle \neq 0.
\]

and the color triplet mass is estimated as

\[
M_T \approx V \frac{\langle \sigma_3 \rangle \langle \sigma_5 \rangle \langle \sigma_{21} \rangle^2}{M^4}.
\]
Suppose $V \sim M$, $\sigma_{3,5} \sim 10^{-2.5}M$ and $\langle \sigma_{21} \rangle^2 \sim 10^{-1}M$. Then, we obtain $M_T \sim 10^{-6}M \sim 0.6 \times 10^{12}$ GeV for $M \sim 6 \times 10^{17}$ GeV. $10^{12}$ GeV colored scalar in small Yukawa couplings of the first family is acceptable. Similarly, considering $C_{12}H_uC_{12} \sim \mathbf{10}_{+1}\mathbf{5}_{-2}\mathbf{10}_{+1}$, 

$$
\frac{1}{M^6}C_{12}[\mathbf{10}_{+1}(T)\mathbf{10}_{-1}](\mathbf{5}_{-2}(T_6))C_{12}[\mathbf{10}_{+1}(T)\mathbf{10}_{-1}(T_3)]\sigma_1[1(T_5^0)]
\times \sigma_0[1(T_5^0)]\sigma_{15}[1(T_0^0)]\sigma_{15}[1(T_0^0)]
$$

(47)

the colored scalar in the Higgs quintet $H_u$ is removed at the GUT scale and there remains just the Higgs doublet from $H_u$. For this, we further require

$$
\langle \sigma_1 \rangle \neq 0, \quad \langle \sigma_0 \rangle \neq 0, \quad \langle \sigma_{15} \rangle \neq 0.
$$

(48)

### C. Proton decay problem

One may consider another gauge symmetry to obtain a Z2 discrete group by breaking U(1)$_1$ by some VEVs of singlet fields carrying even quantum numbers of $\mathbf{Q}_1/2$ in Table IV. It can serve as a kind of matter parity since SU(5)$_{\text{flip}}$ matter fields carry odd $\mathbf{Q}_1/2$. But this discrete symmetry does not work because $\langle \sigma_{21} \rangle$ of Eq. (45) and $\langle \sigma_{15} \rangle$ of Eq. (48) carry the odd quantum number of $\mathbf{Q}_1/2$. We do not have any mechanism for matter parity. The proton decay amplitude must be estimated in detail.$^5$

In SUSY models, the dimension 5 proton decay operator must be sufficiently suppressed [50]. The dimension 5 proton decay operators to electronic and muonic leptons are from the superpotential $q^i q^j q^k l^{i,j,k}$, i.e., $C_{15} C_{13} C_{17}$ and $C_{15} C_{15} C_{15} C_{16}$. Note that $C_{15}$, $C_{17}$, and $C_{16}$ are allowed from the sector $T_{\nu}^0$. Therefore, the $Z_{12-1}$ orbifold selection rules forbid the product of these four fields from $T_{\nu}^0$, and hence there is no serious proton decay problem from the above dimension 5 operator multiplied by FN singlet (σ’s) appear at least at dimension 7 level in our $Z_{12-1}$ model.

If it were the GG SU(5), the cubic superpotential

$$
\mathbf{10}_0 \mathbf{5}_0 \mathbf{5}_0
$$

triggers proton decay as shown in Fig. 1 [51]. In the SU(5)$_{\text{flip}}$ also arise dangerous proton decay operators

$$\mathbf{10}_m^m, \mathbf{10}_m^{-1}, \mathbf{5}_m^{m}, \mathbf{10}_m^{H}, \mathbf{5}_m^{m}, \mathbf{5}_m^{m}, \mathbf{5}_m^{m}, \mathbf{1}_m^{m}, \mathbf{1}_m^{H}, \mathbf{1}_m^{H},$$

(49)

where fields with superscript $m$ are matter fields and $\mathbf{10}_m^H$ is the field breaking SU(5)$_{\text{flip}}$ to the SM. The above operators trigger proton decay in our model by products of FN singlets (σ’s) appear at dimension 10 level.

$^5$If an R parity is introduced [49], the proton decay problem is automatically solved.

---

**FIG. 1.** A diagram for $\Delta B \neq 0$.
$U$ sector. Then, there are three possibilities of choosing two remaining quark doublets: (1) the antisymmetric combination of $\mathbf{10}_{-1}$’s from the $T^0_4$ sector and $\mathbf{10}_{-1}$ from the $T_3$ sector, (2) two $\mathbf{10}_{-1}$’s from the $T^0_4$ sector, and (3) a linear combination of $\mathbf{10}_{-1}$ of $T_3$ and antisymmetric $\mathbf{10}_{-1}$ from $T^0_4$, and a linear combination of $\mathbf{10}_{-1}$ of $T_3$ and symmetric $\mathbf{10}_{-1}$ from $T^0_4$. All these are considered by mixing three $\mathbf{10}_{-1}$’s, introducing three angles $\alpha$, $\beta$, and $\gamma$,

\begin{align}
C_{13}[\mathbf{10}_{-1}(T^0_4), \mathbf{10}_{-1}(T_3)] &= +C_{11}c_{\beta} + C_{4a}c_{\alpha}s_{\beta} - C_{4b}s_{\alpha}s_{\beta}, \\
C_{14}[\mathbf{10}_{-1}(T^0_4), \mathbf{10}_{-1}(T_3)] &= -C_{11}s_{\beta}c_{\gamma} + C_{4a}(c_{\alpha}c_{\gamma} - s_{\alpha}c_{\gamma}) \\
&\quad - C_{4b}(s_{\alpha}c_{\gamma} + c_{\alpha}s_{\gamma}), \\
C_{15}[\mathbf{10}_{-1}(T^0_4), \mathbf{10}_{-1}(T_3)] &= -C_{11}s_{\beta}s_{\gamma} + C_{4a}(s_{\alpha}c_{\beta} - c_{\alpha}s_{\beta}) \\
&\quad + C_{4b}(c_{\alpha}c_{\gamma} - s_{\alpha}s_{\gamma}).
\end{align}

(52)

where $s_{\alpha,\beta,\gamma} = \sin\alpha,\beta,\gamma$ and $c_{\alpha,\beta,\gamma} = \cos\alpha,\beta,\gamma$. We choose two out of the above three combinations. Similarly, we define

\begin{align}
C_{16}[\mathbf{5}_{+3}(T^0_4)] &= \frac{1}{\sqrt{2}} (+C_{3a} + C_{3b}), \\
C_{17}[\mathbf{5}_{+3}(T^0_4)] &= \frac{1}{\sqrt{2}} (-C_{3a} + C_{3b}).
\end{align}

(53)

Now, let us identify $\mathbf{10}_{+1}$ and $\mathbf{10}_{-1}$’s of Table I as $C_{12}[\mathbf{10}_{+1}] \oplus C_{14}[\mathbf{10}_{-1}]$:

The Higgs set for breaking $SU(5) \times U(1)_X$, (54)

and

\begin{align}
C_{15} &\quad : 1\text{st family}, \\
C_{13} &\quad : 2\text{nd family}, \\
C_{2} &\quad : 3\text{rd family},
\end{align}

(55)

and $\mathbf{5}_{+3}$’s of Table I as

\begin{align}
C_{17} &\quad : 1\text{st family}, \\
C_{16} &\quad : 2\text{nd family}, \\
C_{1} &\quad : 3\text{rd family},
\end{align}

(56)

In this paper, it is outside the scope of current analysis to see the details of superpotential. So, we choose the needed VEVs by hand.

1. Down-type quarks

Let us scale scalar fields and mass matrices such that they are made dimensionless by dividing with a mass parameter, e.g., by $M$.

The down-type quark masses are

\begin{align}
M^w_{d(11)} &= C_{15}[\mathbf{10}_{T_3}]C_{15}[\mathbf{10}_{T_3}]H_d(\tilde{\mathbf{5}}_{T_3})\{\sigma_1(1_{T_3})\sigma_6(1_{T_3})\sigma_9(1_{T_3})\sigma_{13}(1_{T_3})\sigma_{13}(1_{T_3})\}, \\
M^w_{d(22)} &= C_{13}[\mathbf{10}_{T_3}]C_{13}[\mathbf{10}_{T_3}]H_d(\tilde{\mathbf{5}}_{T_3})\{\sigma_4(1_{T_3})\sigma_6(1_{T_3})\sigma_{15}(1_{T_3})\sigma_{21}(1_{T_3})\}, \\
M^w_{d(33)} &= C_{2}[\mathbf{10}_U]C_{2}[\mathbf{10}_U]H_d(\tilde{\mathbf{5}}_{T_3})\sigma_5(1_{T_3})\sigma_7(1_{T_3})\sigma_7(1_{T_3})\sigma_4(1_{T_3})\sigma_4(1_{T_3}),
\end{align}

(57)

where we presented only the antisymmetric part of the mass matrices in $M^w_{d(22)}$ and only the component from $T_3$ in $M^w_{d(11)}$. For the down-type quarks, it is enough to show nonzero $M^w_{d(33)}$ and $M^w_{d(22)}$ and the conditions for making the off-diagonal elements vanish,

\begin{align}
M^w_{d(12)} &= C_{15}[\mathbf{10}_{T_3}]C_{15}[\mathbf{10}_{T_3}]H_d(\tilde{\mathbf{5}}_{T_3})\{\sigma_2(1_{T_3})\sigma_3(1_{T_3})\sigma_5(1_{T_3})\} = 0, \\
M^w_{d(21)} &= C_{13}[\mathbf{10}_{T_3}]C_{15}[\mathbf{10}_{T_3}]H_d(\tilde{\mathbf{5}}_{T_3})\{\sigma_2(1_{T_3})\sigma_3(1_{T_3})\sigma_{13}(1_{T_3})\} = 0, \\
M^w_{d(13)} &= C_{15}[\mathbf{10}_{T_3}]C_{2}[\mathbf{10}_U]H_d(\tilde{\mathbf{5}}_{T_3})\{\sigma_4(1_{T_3})\sigma_4(1_{T_3})\sigma_6(1_{T_3}) + c^\prime\sigma_4(1_{T_3})\sigma_{15}(1_{T_3})\sigma_{21}(1_{T_3})\} = 0, \\
M^w_{d(31)} &= C_{2}[\mathbf{10}_U]C_{13}[\mathbf{10}_{T_3}]H_d(\tilde{\mathbf{5}}_{T_3})\{\sigma_4(1_{T_3})\sigma_6(1_{T_3}) + c^\prime\sigma_4(1_{T_3})\sigma_{15}(1_{T_3})\sigma_{21}(1_{T_3})\} = 0, \\
M^w_{d(23)} &= C_{13}[\mathbf{10}_{T_3}]C_{2}[\mathbf{10}_U]H_d(\tilde{\mathbf{5}}_{T_3})\sigma_{13}(1_{T_3}) = 0, \\
M^w_{d(32)} &= C_{2}[\mathbf{10}_U]C_{13}[\mathbf{10}_{T_3}]H_d(\tilde{\mathbf{5}}_{T_3})\sigma_{13}(1_{T_3}) = 0.
\end{align}

(58)

To satisfy the conditions of Eq. (58), let us choose

\begin{align}
\langle \sigma_{13} \rangle &= 0.
\end{align}

(59)
and
\[ c\langle \sigma_4 \sigma_6 \rangle + c' \langle \sigma_{15} \sigma_{21} \rangle = 0. \]  \hfill (60)

\( M^w_{d(13)} \) and \( M^w_{d(31)} \) can be made to vanish.

2. Up-type quarks

Therefore, we consider the \( W^-_u \) coupling instead of \( W^+_u \) coupling of Eq. (24), \( V^\dagger_u M^\text{diag}_u U_u \) as

\[
\begin{pmatrix}
  c_4 c_1 m_u + c_5 s_4 c_2 s_1 m_c & c_4 s_1 c_3 m_u - c_5 s_4 c_2 c_3 m_c & c_4 s_1 s_3 m_u - c_5 s_4 c_2 c_3 m_c \\
  + s_4 s_5 s_1 s_2 e^{-i\Delta + i\delta} m_t & - c_5 s_4 s_2 s_3 e^{-i\Delta} m_c + s_4 s_5 c_2 s_3 e^{-i\Delta} m_t & + c_5 s_4 s_2 c_3 e^{-i\Delta} m_c - s_4 s_5 c_2 c_3 e^{-i\Delta} m_t \\
  s_4 c_6 c_1 m_u - c_4 c_5 c_6 c_2 s_1 m_c & s_4 s_6 c_1 c_3 m_u + c_4 c_5 c_6 c_1 c_3 m_c & s_4 c_6 c_1 s_3 m_u + c_4 c_5 c_6 c_1 s_3 m_c \\
  - s_4 s_6 c_2 s_1 e^{-i\Delta} m_c & + c_4 c_5 s_6 c_2 s_3 e^{-i\Delta} m_c & - c_4 c_5 s_6 c_2 c_3 e^{-i\Delta} m_c \\
  + c_5 s_6 s_1 s_2 e^{-i\Delta} m_t & + s_5 s_6 c_1 c_2 c_3 e^{-i\Delta} m_c + s_5 s_6 s_2 s_3 e^{-i\Delta} m_c & + s_5 s_6 c_1 s_2 e^{-i\Delta} m_c - s_5 s_6 s_2 c_3 e^{-i\Delta} m_c \\
  - c_4 s_5 s_6 s_1 s_2 e^{-i\Delta + i\delta} m_t & + s_5 s_6 c_1 s_2 c_3 e^{-i\Delta} m_c & + s_5 s_6 c_1 s_2 c_3 e^{-i\Delta + i\delta} m_t \\
  s_4 s_6 c_1 m_u - c_4 c_5 s_6 c_2 s_1 m_c & s_4 s_6 s_1 c_3 m_u + c_4 c_5 c_6 s_1 c_3 m_c & s_4 s_6 s_1 s_3 m_u + c_4 c_5 c_6 s_1 s_3 m_c \\
  + s_4 s_6 c_2 s_1 e^{-i\Delta} m_c & + c_4 c_5 s_6 c_2 s_3 e^{-i\Delta} m_c & + c_4 c_5 s_6 c_2 c_3 e^{-i\Delta} m_c \\
  - c_5 s_6 s_1 s_2 e^{-i\Delta} m_t & - s_5 s_6 c_1 c_2 c_3 e^{-i\Delta} m_c - s_5 s_6 s_2 s_3 e^{-i\Delta} m_c & - s_5 s_6 c_1 s_2 e^{-i\Delta} m_c + s_5 s_6 s_2 c_3 e^{-i\Delta} m_c \\
  - c_5 s_6 s_1 s_2 e^{-i\Delta + i\delta} m_t & - c_4 s_5 s_6 c_1 s_2 c_3 e^{-i\Delta} m_c & - c_4 s_5 s_6 c_1 s_2 c_3 e^{-i\Delta + i\delta} m_t \\
  ,
\end{pmatrix}
\]

Change the sign \( m_u \to -m_u \), and to reduce the number of parameters let us choose parameters of R-fields as

\[
\frac{s_4}{c_4} = \frac{s_1}{c_1}, \quad \frac{s_5}{c_5} = \frac{m_c}{m_t} c_2, \quad s_6 = 0, \quad \Delta = \delta.
\]

Then, we obtain

\[
V^\dagger_u M^\text{diag}_u U_u = \begin{pmatrix}
  - c_4 c_1 m_u + c_5 s_4 c_2^{-1} s_1 m_c & - c_4 s_1 c_3 m_u - c_5 s_4 c_2^{-1} c_3 m_c & - c_4 s_1 s_3 m_u - c_5 c_4 s_1 c_2^{-1} s_3 m_c \\
  - s_4 c_1 c_3 m_u & - s_4 s_1 c_3 m_u & - s_4 s_1 s_3 m_u \\
  - c_4 c_5 s_1 c_2^{-1} m_c & + c_4 c_5 c_1 c_3 c_2^{-1} m_c & + c_4 c_5 c_1 s_3 c_2^{-1} m_c \\
  + s_5 c_2 s_1 e^{i\Delta} m_c & - s_5 c_1 c_2 s_1 e^{i\Delta} m_c - s_5 s_2 s_3 m_c & - s_5 c_1 s_2 s_3 e^{i\Delta} m_c + s_5 s_2 c_3 m_c \\
  - c_5 c_2 s_1 e^{i\Delta} m_c & - c_5 c_2 s_3 m_c + c_5 c_1 s_2 e^{i\Delta} m_c & + c_5 c_2 s_3 m_c + c_5 c_1 s_2 e^{i\Delta} m_c \\
  ,
\end{pmatrix}
\]

where we require \( c_2, c_3, c_5 \approx O(1) \). Also, \( s_5 \) can be \( O(1) \). Thus, we consider,

\[
V^\dagger_u M^\text{diag}_u U_u = m_t \begin{pmatrix}
  - c_4 c_1 m_u c_{cm} + s_4 s_1 m_c m_{cm} \s\frac{m_m}{m_t} & - c_4 s_1 m_u m_{cm} - s_4 c_1 m_c m_{cm} \s\frac{m_m}{m_t} & - c_4 s_1 s_3 m_u m_{cm} - c_4 c_1 c_2^{-1} s_3 m_c m_{cm} \\
  - s_4 c_1 m_u m_{cm} & - s_4 s_1 m_u m_{cm} & - s_4 s_1 s_3 m_u m_{cm} \\
  - c_4 c_5 m_u m_{cm} - s_5 c_1 m_c m_{cm} & + c_4 c_1 m_c m_{cm} & + c_4 c_5 s_1 m_c m_{cm} \\
  - s_5 c_2 s_1 e^{i\Delta} m_c & - s_5 c_1 c_2 e^{i\Delta} m_c - s_5 s_2 e^{i\Delta} m_c - s_5 s_2 s_3 m_c & - s_5 c_1 s_2 e^{i\Delta} m_c + s_5 s_2 c_3 m_c \\
  - s_5 c_2 e^{i\Delta} m_c & - c_5 c_2 s_3 m_c + c_5 c_1 s_2 e^{i\Delta} m_c & + c_5 c_2 s_3 m_c + c_5 c_1 s_2 e^{i\Delta} m_c \\
  \s\frac{m_m}{m_t} & \frac{m_m}{m_t} & \frac{m_m}{m_t}
\end{pmatrix}
\]

where we neglected \( m_t s_2 s_3, m_c s_2, m_c s_3 \).

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so that $M''_{u}/m_{t}$ is approximately given by

$$
\begin{pmatrix}
+1.67 \times 10^{-3}s_{4}, & -0.721 \times 10^{-2}s_{4}, & -2.59 \times 10^{-5}c_{4} \\
-1.67 \times 10^{-3}c_{4}, & +0.721 \times 10^{-2}c_{4}, & +1.12 \times 10^{-4}c_{4}
\end{pmatrix}
\begin{pmatrix}
+0.67 \times 10^{-6} + 1.24 \times 10^{-5}t_{5}e^{i\delta}, & -1.15 \times 10^{-3} + (2.88 \times 10^{-4} - 0.534 \times 10^{-2}t_{5})e^{i\delta}, & 1
\end{pmatrix}
$$

The up-type quark masses are

$$
\begin{align*}
M''_{u(11)} & = 0, \\
M''_{u(22)} & = C_{16}(\bar{5}_{L})C_{13}(\bar{T}_{L})H_{u}(\bar{5}_{R})\{\sigma_{4}(T_{4}^{0})\sigma_{6}(T_{6}), \sigma_{15}(T_{9})\sigma_{21}(T_{1}^{0}); \sigma_{4}(T_{4})\sigma_{6}(T_{6})\sigma_{21}(T_{1}^{0})\}, \\
M''_{u(33)} & = C_{1}(\bar{5}_{L})C_{2}(\bar{T}_{L})H_{u}(\bar{5}_{R})\sigma_{5}(1_{R}), \\
M''_{u(12)} & = C_{17}(\bar{5}_{L})C_{13}(\bar{T}_{L})H_{u}(\bar{5}_{R})\{\sigma_{1}(T_{1}^{0})\sigma_{5}(T_{6}), \sigma_{15}(T_{9})\sigma_{21}(T_{1}^{0}); \\
& \quad \sigma_{2}(1_{R}), \sigma_{3}(1_{R})\sigma_{5}(1_{R}), \sigma_{6}(1_{R})\sigma_{9}(1_{R}), \sigma_{13}(1_{R}), \sigma_{15}(1_{R}), \sigma_{21}(T_{0}^{3})\sigma_{5}(T_{6})\}, \\
M''_{u(21)} & = C_{16}(\bar{5}_{L})C_{15}(\bar{T}_{L})H_{u}(\bar{5}_{R})\{\sigma_{4}(T_{4}^{0})\sigma_{5}(T_{6}), \sigma_{15}(T_{9})\sigma_{21}(T_{1}^{0}); \\
& \quad \sigma_{2}(1_{R}), \sigma_{3}(1_{R})\sigma_{5}(1_{R}), \sigma_{6}(1_{R})\sigma_{9}(1_{R}), \sigma_{13}(1_{R}), \sigma_{15}(1_{R}), \sigma_{21}(T_{0}^{3})\sigma_{5}(T_{6})\}, \\
M''_{u(23)} & = C_{16}(\bar{5}_{L})C_{2}(\bar{T}_{L})H_{u}(\bar{5}_{R})\sigma_{5}(1_{R})\sigma_{4}(1_{R})\sigma_{5}(1_{R}), \\
M''_{u(32)} & = C_{1}(\bar{5}_{L})C_{13}(\bar{T}_{L})H_{u}(\bar{5}_{R})\{\sigma_{1}(T_{1}^{0})\sigma_{5}(T_{6}), \sigma_{15}(T_{9})\sigma_{21}(T_{1}^{0}); \\
& \quad \sigma_{13}(1_{R}), \sigma_{2}(1_{R})\sigma_{6}(1_{R})\sigma_{9}(1_{R}), \sigma_{13}(1_{R}), \sigma_{15}(1_{R}), \sigma_{21}(T_{0}^{3})\sigma_{5}(T_{6})\}, \\
& \quad \sigma_{21}(1_{R})\sigma_{5}(1_{R})\sigma_{6}(1_{R})\sigma_{9}(1_{R}), \sigma_{21}(1_{R})\sigma_{5}(1_{R})\sigma_{15}(1_{R})\sigma_{21}(1_{R})^{2}\}, \\
M''_{u(13)} & = 0, \\
M''_{u(31)} & = C_{1}(\bar{5}_{L})C_{13}(\bar{T}_{L})H_{u}(\bar{5}_{R})\{\sigma_{5}(T_{5}), \sigma_{6}(1_{R})\sigma_{9}(1_{R}), \sigma_{2}(1_{R})\sigma_{4}(1_{R})\sigma_{6}(1_{R}); \\
& \quad \sigma_{13}(1_{R}), \sigma_{2}(1_{R})\sigma_{6}(1_{R})\sigma_{9}(1_{R}), \sigma_{13}(1_{R}), \sigma_{15}(1_{R}), \sigma_{21}(T_{0}^{3})\sigma_{5}(T_{6})\}, \\
& \quad \sigma_{21}(1_{R})\sigma_{5}(1_{R})\sigma_{6}(1_{R})\sigma_{9}(1_{R}), \sigma_{21}(1_{R})\sigma_{5}(1_{R})\sigma_{15}(1_{R})\sigma_{21}(1_{R})^{2}\}, \\
M''_{u(33)} & = 0,$$
\end{align*}

$M''_{u(33)}$ is the largest value, and we set $\langle \sigma_{3} \rangle = O(1)$, and automatically we have $M''_{u(11)} = M''_{u(13)} = 0$ by the unavoidable antisymmetric property among $\bar{5}_{-3}(T_{4}^{0})$, viz. $C_{17}$ in Eq. (53).

The following example is just showing a possibility. We have chosen $\sigma_{13} = 0$ in Eq. (59) to make down-type quark masses diagonal. Let us further simplify by setting $\langle \sigma_{2} \rangle = 0$,

$$
\begin{align*}
M''_{u(11)} & = 0, \\
M''_{u(22)} & = C_{16}(\bar{5}_{L})C_{13}(\bar{T}_{L})H_{u}(\bar{5}_{R})\left\{ \frac{1}{\sigma_{5}(T_{6})}\sigma_{4}(T_{4}^{0})\sigma_{6}(T_{6}), \frac{\sigma_{15}(T_{9})}{\sigma_{5}(T_{6})}\sigma_{21}(T_{1}^{0}) \right\}; \\
M''_{u(33)} & = C_{1}(\bar{5}_{L})C_{2}(\bar{T}_{L})H_{u}(\bar{5}_{R})\sigma_{5}(1_{R})
\end{align*}
$$
\[ M^{w}_{u(12)} = C_{13} (5^w_{12}) C_{15} (\overline{10}^w_{12}, \overline{10}^w_{11}) H_u (S_{11}) \left\{ \sigma_3(T^0_4), \frac{1}{\sigma_5(T_6)} \sigma_{15}(T_9) \sigma_{21}(T^0_4), 0 \right\} \sigma_5(T_6), \]
\[ M^{w}_{u(21)} = C_{14} (5^w_{12}) C_{15} (\overline{10}^w_{12}, \overline{10}^w_{11}) H_u (S_{11}) \left\{ \sigma_3(T^0_4), \frac{1}{\sigma_5(T_6)} \sigma_{15}(T_9) \sigma_{21}(T^0_4); 0; \sigma_2(1^w_{12}) \sigma_9(1^w_{12}) \sigma_{21}(1^w_{12}) \right\} \sigma_5(T_6), \]
\[ M^{w}_{u(23)} = C_{14} (5^w_{12}) C_{22} (\overline{10}^w_{12}) H_u (S_{11}) \left\{ \sigma_3(1^w_{12}) \sigma_4(1^w_{12}) \right\} \sigma_3(T_9), \]
\[ M^{w}_{u(32)} = C_{14} (5^w_{12}) C_{13} (\overline{10}^w_{12}, \overline{10}^w_{11}) H_u (5_{11}) \left\{ 0; \sigma_6(1^w_{12}) \sigma_9(1^w_{12}) \sigma_9(1^w_{12}) \sigma_{21}(1^w_{12}) \right\} \sigma_5(T_6), \]
\[ M^{w}_{u(21)} = 0, \]
\[ M^{w}_{u(31)} = C_{14} (5^w_{12}) C_{15} (\overline{10}^w_{12}, \overline{10}^w_{11}) H_u (5_{11}) \left\{ \sigma_6(1^w_{12}) \sigma_9(1^w_{12}) \sigma_6(1^w_{12}) \sigma_9(1^w_{12}) \right\} \sigma_5(T_6). \]

where the antisymmetric combination of \( \overline{10}_{-1} \)'s from \( T^0_4 \) is written before the semicolon and the symmetric combinations of \( \overline{10}_{-1} \)'s from \( T^0_4 \) is written after the semicolon. Zeros indicate this symmetry properties.

\[
\begin{pmatrix}
0, & (p \sigma_3 + q \sigma_{15} \sigma_{21}), & 0 \\
(c \sigma_3 + d \sigma_{15} \sigma_{21}) + e \sigma_3 \sigma_{21}, & f \sigma_3 \sigma_{21} + g \sigma_{15} \sigma_{21} + h \sigma_3 \sigma_{21}, & \ell \sigma_3 \sigma_{21} \\
(a \sigma_6 \sigma_9(1 + r \sigma_{21}), & b \sigma_6 \sigma_9 \sigma_2 \sigma_{21}, & 1)
\end{pmatrix}
\]

(71)

To present a simple numerics, let us neglect the \( \sigma_{15} \) terms. So, consider

\[
\begin{pmatrix}
0, & p \sigma_3, & 0 \\
c \sigma_3 + e \sigma_3 \sigma_{21}, & f \sigma_3 \sigma_{21} + k \sigma_3 \sigma_{21}, & \ell \sigma_3 \sigma_{21} \\
(a \sigma_6 \sigma_9(1 + r \sigma_{21}), & b \sigma_6 \sigma_9 \sigma_2 \sigma_{21}, & 1)
\end{pmatrix}
\]

(72)

Assuming hierarchies of VEVs with O(1) coefficients,

\[
\sigma_4 \ll 1,
-p \sigma_3 = f \frac{\sigma_6}{\sigma_5} + k \sigma_3 \sigma_{21} \approx O(10^{-3}),
\]
\[
c \sigma_3 + e \sigma_3 \sigma_{21} \approx O(10^{-3}),
\]
\[
a \sigma_6 \sigma_9 \approx O(10^{-5}),
\]
\[
\sigma_{21} \approx O \left( \frac{a}{b} \right) \ll 1,
\]
\[
r \sigma_{21} \approx O(1),
\]
\[
p \sim c \sim e \sim f \sim k \sim \ell \sim r \approx O(1),
\]

we estimate

\[
\frac{1}{a \sigma_6 \sigma_9} \approx \left( \begin{array}{ccc}
0, & p \sigma_3, & 0 \\
c \sigma_3, & f \frac{\sigma_6}{\sigma_5}, & \ell \sigma_3 \sigma_{21} \\
(a \sigma_6 \sigma_9, & b \sigma_6 \sigma_9 \sigma_2 \sigma_{21}, & 1)
\end{array} \right)
\]

(74)

which can be close to Eq. (66). Let all singlet VEVs are real except \( \sigma_9 \) and \( \sigma_{21} \) [23].

In Table V, we list \( \theta + \phi \) for a few \( t_5 \). For \( \delta_{\text{CKM}} = \frac{\pi}{2} \) and \( t_5 \approx 5.5 \), we obtain \( \phi \approx -\theta \). Irrespective of the value of \( \phi \), the \( CP \) phase in the Jarlskog determinant, \( \delta_{\text{CKM}} \), is the phase in \( M^{w}_{u(31)} \) with the KS parametrization given in Eq. (22).

### E. \( CP \) phases in the quark and lepton sectors

As done before, let us diagonalize the symmetric fermion masses first. In the flipped SU(5) model, therefore, we diagonalize down-type quark masses and neutrino masses. Then, we consider up-type quarks and charged leptons. Then, the (3,1) elements of the mass matrices are the key.

For the third family members from \( U \), masses of \( t \) quark and \( \tau \) lepton arise from

| \( t_5 \) | \( \frac{-0.288 \pm 0.0534 t_5}{1 + t_5} \) | \( \theta + \phi \) |
|----|-------------------------------|------------------|
| 0  | -0.250                        | -14.04°, -0.244π |
| 5.5| 0.0054                        | 0.286°, ~0        |
| 10 | 0.214                         | 12.0°, 0.209π    |
The phenomenologically determined leptonic mass element $M_{e(31)}^w$ can be obtained from Eq. (61) by changing the quark parameters $\theta$, $\delta$, $\Delta$, $m_u$, $m_d$, $m_t$ to leptonic parameters of Eq. (35): $\Theta$, $\delta_L$, $\Delta_L$, $m_e$, $m_\mu$, $m_\tau$. Choose the leptonic $V$ matrix elements such that

$$\begin{aligned}
S_4 &= S_1, \\
C_4 &= C_1, \\
S_5 &= S_2, \\
C_5 &= C_2, \\
S_6 &= 0, \\
\Delta_L &= \delta_L.
\end{aligned}$$

(78)

Then, $M_{e(31)}^w/m_t \sim -\sin \Theta \sin \Theta e^{i\delta_L}$ where $\delta_L$ is the PMNS phase.

In our model, Table IV, there are three $e^c$ fields in the leptonic case (instead of four $u^c$ fields in the quark case), and we can choose $S_{24}$ which is the antisymmetric combination of $S_{24a}$ and $S_{24b}$ in $T^0$. So, the leptonic mass matrix has four zero entries with the antisymmetric 1st row and antisymmetric 2nd column,

$$\begin{pmatrix}
0, & M_{e(12)}, & 0 \\
M_{e(21)}^w, & 0, & M_{e(23)}^w \\
M_{e(31)}^w, & 0, & M_{e(33)}^w
\end{pmatrix}$$

(79)

For the lepton phase, we need $M_{e(31)}^w$ whose phase is $\delta_L$.

$$\begin{aligned}
\frac{1}{M^0} & C_3[S_{+3}(T^0)]C_3[S_{+3}(T^0)]H_u[S_{+2}(T_6)]H_u[S_{+2}(T_6)]C_{11}[\overline{10}_{-1}(T_3)]C_{11}[\overline{10}_{-1}(T_3)] \\
& \cdot \sigma_3[1(T^0)]\sigma_3[1(T^0)]\sigma_5[1(T_6)]\sigma_5[1(T_6)]\sigma_{21}[1(T^0)]\sigma_{21}[1(T^0)], \\
\frac{1}{M^2} & C_3[S_{+3}(T^0)]C_3[S_{+3}(U)]H_u[S_{+2}(T_6)]H_u[S_{+2}(T_6)]C_{11}[\overline{10}_{-1}(T_3)]C_{11}[\overline{10}_{-1}(T_3)] \\
& \cdot \sigma_5[1(T_6)]\sigma_6[1(T_6)]\sigma_{21}[1(T^0)]\sigma_{21}[1(T^0)], \\
\frac{1}{M^2} & C_1[S_{+3}(U)]C_1[S_{+3}(U)]H_u[S_{+2}(T_6)]H_u[S_{+2}(T_6)]C_{11}[\overline{10}_{-1}(T_3)]C_{11}[\overline{10}_{-1}(T_3)] \\
& \cdot \sigma_2[1(T^0)]\sigma_5[1(T_6)]\sigma_6[1(T_6)]\sigma_{21}[1(T^0)]\sigma_{21}[1(T^0)].
\end{aligned}$$

(82)

where $\sigma_0$ does not appear. So, the phase in $\sigma_0$ is the PMNS phase. The generic magnitudes of masses from the above couplings are $(v_{\text{ew}}/M)(V/M)^{6,7}$ where $V$ and $M$ are some scales around/above the GUT scale, and we can obtain reasonable strength for neutrino masses.

Equation (77) shows that the L-handed up-type quarks, appearing in $\overline{10}_{-1}$, use charge lowering operators to couple to $W^w_\mu$ and the L-handed charged leptons, appearing in $S_{+3}$.
use charge raising operators to couple to $W'_\mu$. So, we must consider the same charge charged-gauge boson $W'_\mu$ to compare the signs of $\delta_{\text{CKM}}$ and $\delta_{\text{PMNS}}$. Also, we must specify the signs of the effective Yukawa couplings in $M'_{u(31)}$ and $M'_{e(31)}$ dictated by string compactification. At this stage, we allow any sign for $M'_{u(31)}$ and $M'_{e(31)}$ since we considered only the selection rules. If the signs of $M'_{u(31)}$ and $M'_{e(31)}$ are the same (opposite), then we conclude that $\delta_{\text{CKM}}$ and $\delta_{\text{PMNS}}$ have the opposite (same) signs.\(^8\) The case of opposite signs is consistent with the currently favored phases of $\delta_{\text{CKM}}$ [23] and $\delta_{\text{PMNS}}$ [24].

In the PS type standard model SU(4) $\times$ SU(2)$_L \times$ SU(2)$_R$, we would have fermion matter spectra, containing quark and lepton doublets,

\[(4, 2, 1)_L \oplus (4, 1, 2)_R + \cdots \] (83)

Suppose that the Yukawa coupling $(4, 2, 1)_L \times (4', 1, 2)_L \times (1, 2, 2)_h$ via Higgs $(1, 2, 2)_h$ is present from the orbifold compactification. Then, the Yukawa coupling arises from the L-handed Higgs field doublets $c^{ij}(1, 2, (ij))_h = (1, 2, (12))_h - (1, 2, (21))_h$ where the R-hand index (12) gives the Higgs doublet coupling to quark doublets and the R-hand index (21) gives the Higgs doublet coupling to lepton doublets. We use the same charge $W$, i.e., $W_{\mu}$, for coupling to down-type quarks and charged leptons. So, the relative signs of $M'_{u(31)}$ and $M'_{e(31)}$ are opposite if the product with FN singlet contributions give the same sign. If we use the mass matrices of $M'_{u(31)}$ and $M'_{e(31)}$ for asymmetric mass matrices as in the GG model, then $\delta_{\text{CKM}}$ and $\delta_{\text{PMNS}}$ have the opposite signs. But, here one needs an example for breaking SO(10) down to SU(4) $\times$ SU(2)$_L \times$ SU(2)$_R$, where the rank is not reduced, from the spectra of orbifold compactification. One may use the bulk fields for an adjoint representation as pointed out for $\mathbb{Z}_{6-11}$ in Ref. [45] and for $\mathbb{Z}_2 \times \mathbb{Z}_2$ in Ref. [52] where the $N = 2$ gauge multiplet in an effective 5-dimensional SUSY model allows an adjoint representation of spin-0 fields.

V. CONCLUSION

In this paper, we presented a theory toward understanding the quark and lepton mixing angles. Specifically, we presented a working example obtained from a string compactification [47] with $Q_{\text{anom}}$ charge presented in [2]. Explicit presentations were given for the CKM matrix. The (3,3) element of quark mass matrix in the weak basis, is assumed to be close to the t-quark mass. Because there are only three L-handed quark doublets in the model, the up-type quark mass matrix is antisymmetric under the exchange of $a \leftrightarrow b$ among R-handed flavor indices (or $u'$ fields) obtained from $T_4^0$. This is because the multiplicity 2 for $s_{-3}$ from $T_4^0$ is generic and there is no way to distinguish these two. The antisymmetric combination of $a$ and $b$ is named for the 1st family member of $s_{+3}$’s. But, there are four L-handed up-type quark doublets and the up-type quark families have a freedom to choose from these four. We used the freedom of choosing the unitary matrix for the R-handed quarks to fit to the data, and showed that this model predicts reasonable mixing angles within experimental error bounds. Also, we studied the relation between $\delta_{\text{CKM}}$ and $\delta_{\text{PMNS}}$ by the phases of some SM singlet scalar fields, assuming that all Yukawa coupling constants from string compactification are real. For the proton decay problem, a $\mathbb{Z}_2$ matter parity cannot be introduced consistently with the solution of the doublet-triplet splitting problem by the GUT scale VEVs, $(\overline{10}_{-1}(T_3))$ and $(\overline{10}_{+1}(T_0))$. But, we showed that the proton decay operator appears at a dimension 10 level, which can be made small enough while achieving the doublet-triplet splitting. It will be interesting if a kind of R parity is found within the scheme, which will be published soon [53].

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\[^{8}\] In the GG model [37], the symmetric quark mass matrices are for neutrinos and up-type quarks. The asymmetric quark mass matrices are for the down-type quarks and charged leptons via the same coupling $\overline{10}, 5_0 H_0$, and if we had tried the strategy we chose here then we would have obtained the same sign for $\delta_{\text{CKM}}$ and $\delta_{\text{PMNS}}$ irrespective of the signs of $M'_{u(31)}$ and $M'_{e(31)}$. But this idea is not workable in the GG model because we lack an adjoint representation for breaking SU(5) down to the SM.

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