Theory of a Novel Superstring in Four Dimensions

B. B. Deo
Physics Department, Utkal University, Bhubaneswar-751004, India.

An open string in four dimensions is supplemented by forty four Majorana fermions. The fermions are grouped in such a way that the resulting action is supersymmetric. The super-Virasoro algebra is constructed and closed by the use of Jacobi identity. The tachyonic ground state decouples from the physical states. After a GSO projection, the resulting physical mass spectrum is shown to be $\alpha' M_n^2 = n$ where $n = 0, 1, 2, \cdots$. There are fermions and bosons in each mass level. The internal symmetry group of the string breaks to $SU(3) \times SU(2) \times U(1) \times U(1)$.

PACS : 11.17.+y

String theory was invented \[1\] as a sequel to dual resonance models \[2\] to explain the properties of strongly interacting particles in four dimensions. Assuming the string to live in a background gravitational field and demanding Weyl invariance, the Einstein equations of general relativity could be deduced. It was believed that about these classical solutions one can expand and find the quantum corrections. But difficulties arose at the quantum level. Eventhough the strong interaction amplitude obeyed crossing, it was no longer unitary. There were anomalies and ghosts. Therefore it was necessary for the open string to live in 26 dimensions \[3\]. in this letter we report that fermionising the 22 additional coordinates for a four dimensional open string and regrouping them, we can obtain a novel superstring with an unusual but correct mass spectrum containing fermions and bosons.

There are forty four Majorana fermions representing the 22 bosonic coordinates \[11\]. We divide them into four groups or ‘generations’ (one time-like and three space-like). They are labelled by $\mu = 0, 1, 2, 3$ like the space time indices and each contain 11 fermions. These 11 fermions are again divided into two groups, one containing six and the other five. For convenience, in one group we have $j = 1, 3, 5, 7, 9, 11$ six odd numbers, and in other, the five even numbers $k = 2, 4, 6, 8, 10$.

The string action is

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[ \partial_\sigma X^\mu \partial^\sigma X_\mu - i \bar{\psi}_{\mu,j} \rho^\sigma \partial_\sigma \psi_{\mu,j} - i \phi_{\mu,k} \rho^\sigma \partial_\sigma \phi_{\mu,k} \right],$$

(1)

$\rho^\sigma$ are the two dimensional Dirac matrices

$$\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

(2)

and obey

$$\{\rho^\alpha, \rho^\beta\} = -2\eta_{\alpha\beta}.$$

(3)

In general we follow the notations and conventions of reference \[8\] whenever omitted by us. $X^\mu(\sigma, \tau)$ are the string coordinates. The $\psi$’s are the odd indexed and $\phi$’s the even indexed Majorana fermions decomposed in the basis

$$\psi = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}, \quad \text{and} \quad \phi = \begin{pmatrix} \phi_- \\ \phi_+ \end{pmatrix}.$$ 

(4)

The nonvanishing commutation and anticommutations are

$$[\dot{X}^\mu(\sigma, \tau), X^\nu(\sigma', \tau)] = -i \delta(\sigma - \sigma')\eta^\mu\nu$$

(5)

$$\{\psi^A_+ (\sigma, \tau), \psi^B_+ (\sigma', \tau)\} = \pi \eta^{AB} \delta(\sigma - \sigma')$$

(6)

$$\{\phi^A_+ (\sigma, \tau), \phi^B_+ (\sigma', \tau)\} = \pi \eta^{AB} \delta(\sigma - \sigma')$$

(7)

The action is invariant under infinitesimal transformations

$$\delta X^\mu = \dot{\epsilon} \left( \sum_j \psi^j_{\mu,j} + i \sum_k \phi^j_{\mu,k} \right)$$

(8)
where $\epsilon$ is an infinitesimally constant anticommuting Majorana spinor. The commutator of the two supersymmetry transformation gives a spatial translation, namely

$$[\delta_1, \delta_2]X^\mu = a^\alpha \partial_\alpha X^\mu$$

(11)

and

$$[\delta_1, \delta_2]\Psi^\mu = a^\alpha \partial_\alpha \Psi^\mu$$

(12)

where

$$a^\alpha = 2i \epsilon_1 \rho^\alpha \epsilon_2$$

(13)

and

$$\Psi^\mu = \sum_j \psi^{\mu, j} + \sum_k \phi^{\mu, k}$$

(14)

In deriving this, the Dirac equation for the spinors have been used. The Noether super-current is

$$J_{\alpha} = \frac{1}{2} \rho^\beta \rho_{\alpha} \Psi^\mu \partial_\beta X^\mu$$

(15)

We now follow the standard procedure. The light cone components of the current and energy momentum tensors are

$$J_+ = \partial_+ X^\mu \Psi_+^\mu$$

(16)

$$J_- = \partial_- X^\mu \Psi_-^\mu$$

(17)

$$T_{++} = \partial_+ X^\mu \partial_+ X^\mu + \frac{i}{2} \psi^{\mu, j}_{+} \partial_+ \psi^{\mu, j}_{+} + \frac{i}{2} \phi^{\mu, k}_{+} \partial_+ \phi^{\mu, k}_{+}$$

(18)

$$T_{--} = \partial_- X^\mu \partial_- X^\mu + \frac{i}{2} \psi^{\mu, j}_{-} \partial_- \psi^{\mu, j}_{-} + \frac{i}{2} \phi^{\mu, k}_{-} \partial_- \phi^{\mu, k}_{-}$$

(19)

where $\partial_{\pm} = \frac{1}{2} (\partial_\tau \pm \partial_\sigma)$.

The theory is quantised with

$$\partial_{\pm} X^\mu = \frac{1}{2} \sum_{n=\infty}^{+\infty} \alpha_n^\mu e^{-in(\tau \pm \sigma)}$$

(20)

$$[\alpha_m^\mu, \alpha_n^\nu] = m \delta_{m+n} \eta^{\mu\nu}$$

(21)

In this work, we always choose the Neveu-Schwarz (NS) [4] boundary condition. Then the mode expansions of the fermions are

$$\psi^{\mu, j}_{\pm}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in Z_{\pm+}} b_{r}^{\mu, j} e^{-ir(\tau \pm \sigma)}$$

(22)

$$\phi^{\mu, k}_{\pm}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in Z_{\pm+}} d_{r}^{\mu, k} e^{-ir(\tau \pm \sigma)}$$

(23)
The sum is over all the half-integer modes.

\[ \{ b^{\mu,j}, b^{\nu,j'} \} = \eta^{\mu\nu} \delta_{j,j'} \delta_{r+s} \]  

(24)

\[ \{ d^{\mu,k}, b^{\nu,k'} \} = \eta^{\mu\nu} \delta_{k,k'} \delta_{r+s} \]  

(25)

Virasoro generators are given by the modes of the energy momentum tensor \( T_{++} \) and Noether current \( J_+ \),

\[ L^M_m = \frac{1}{\pi} \int^{+\pi}_{-\pi} d\sigma \, e^{im\sigma} T_{++} \]  

(26)

\[ G_r = \frac{\sqrt{2}}{\pi} \int^{+\pi}_{-\pi} d\sigma \, e^{ir\sigma} J_+ \]  

(27)

‘M’ stands for matter. In terms of creation and annihilation operators

\[ L^M_m = L^{(\alpha)}_m + L^{(b)}_m + L^{(d)}_m \]  

(28)

where

\[ L^{(\alpha)}_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} : \alpha_{-n} \cdot \alpha_{m+n} : \]  

(29)

\[ L^{(b)}_m = \frac{1}{2} \sum_{r=-\infty}^{\infty} (r + \frac{1}{2}m) : b_{-r} \cdot b_{m+r} : \]  

(30)

\[ L^{(d)}_m = \frac{1}{2} \sum_{r=-\infty}^{\infty} (r + \frac{1}{2}m) : d_{-r} \cdot d_{m+r} : \]  

(31)

In each case normal ordering is required. The single dot implies the sum over all qualifying indices. The fermionic generator is

\[ G_r = \sum_{n=-\infty}^{\infty} \alpha_{-n} \cdot (b_{r+n} + i d_{r+n}) \]  

(32)

The Virasoro algebra in this NS sector is

\[ [L^M_m, L^M_n] = (m-n) L^M_{m+n} + A(m) \delta_{m+n} \]  

(33)

\[ [L^M_m, G_r] = \left( \frac{1}{2} m - r \right) G_{m+r} \]  

(34)

The anticommutator \( \{ G_r, G_s \} \) is not directly obtainable as there are mixed terms. Instead we use the Jacobi identity

\[ \{ G_r, G_s \}, L^M_m \} + \{ [ L^M_m, G_r ] , G_s \} + \{ [ L^M_m , G_s ] , G_r \} = 0 \]  

(35)

which implies, consistent with equations (34) and (35),

\[ \{ G_r , G_s \} = 2 L^M_{r+s} + B(r) \delta_{r+s} \]  

(36)

\( A(m) \) and \( B(r) \) are normal ordering anomalies. Taking the vacuum expectation value in the Fock ground state \(|0,0\rangle\) with four momentum \( p^\mu = 0 \) of the commutator \([ L_1 , L_{-1} ]\) and \([ L_2 , L_{-2} ]\), it is easily found that
\[ A(m) = \frac{26}{8} (m^3 - m) \]  

(37)

and using the Jacobi identity for \( L_0, G_{1/2}, G_{-1/2} \) and \( L_{-2}, G_{3/2}, G_{1/2} \) for the Fock ground state, one obtains

\[ B(r) = \frac{26}{2} \left( r^2 - \frac{1}{4} \right) \]  

(38)

The central charge \( c = 26 \). This is what is expected. Each bosonic coordinate contribute 1 and each fermionic ones contribute \( \frac{1}{2} \), so that the total central charge is +26.

A physical state satisfies

\[ L_M |\text{phys}\rangle = 0 \quad n > 0 \]  

(39)

For obtaining a zero central charge so that the anomalies cancel out and also ghosts are isolated, Faddeev-Popov (FP) ghosts \( \bar{G} \) are introduced. The FP ghost action is

\[ S_{FP} = \frac{1}{\pi} \int (c^+ \partial_- b_{++} + c^- \partial_+ b_{--}) d^2 \sigma \]  

(40)

where the ghost fields \( b \) and \( c \) satisfy the anticommutator relations.

\[ \{ b_{++} (\sigma, \tau), c^+(\sigma', \tau) \} = 2\pi \delta(\sigma - \sigma') \]  

(41)

\[ \{ b_{--} (\sigma, \tau), c^-(\sigma', \tau) \} = 2\pi \delta(\sigma - \sigma') \]  

(42)

and are quantized with the mode expansions

\[ c^\pm = \sum_{-\infty}^{\infty} c_n e^{-in(\tau \pm \sigma)} \]  

(43)

\[ b_{\pm\pm} = \sum_{-\infty}^{\infty} b_n e^{-in(\tau \pm \sigma)} \]  

(44)

The canonical anticommutator relations for \( c_n \)'s and \( b_n \)'s are

\[ \{ c_m, b_n \} = \delta_{m+n} \]  

(45)

\[ \{ c_m, c_n \} = \{ b_m, b_n \} = 0 \]  

(46)

Deriving the energy momentum tensor from the action and making the mode expansion, the Virasoro generators for the ghosts (G) are

\[ L_m^G = \sum_{n=-\infty}^{\infty} (m - n) b_{m+n} c_{-n} - a \delta_m \]  

(47)

where \( a \) is the normal ordering constant. These generators satisfy the algebra

\[ [L_m^G, L_n^G] = (m - n) L_{m+n}^G + A^G(m) \delta_{m+n} \]  

(48)

The anomaly term is deduced as before and give

\[ A^G(m) = \frac{1}{6} (m - 13m^3) + 2a m \]  

(49)

With \( a = 1 \), this anomaly term becomes

\[ A^G(m) = -\frac{26}{12} (m^3 - m) \]  

(50)
The central charge is $-26$ and cancels the $A(m)$ term of the $L_m^M$'s.

The BRST charge operator is

$$Q_{BRST} = \sum_{-\infty}^{\infty} L_m^M c_m - \frac{1}{2} \sum_{-\infty}^{\infty} (m-n) : c_{-m} c_{-n} b_{m+n} : -ac_0$$

(51)

and the nilpotent for $a = 1$. The physical states are such that $Q_{BRST} |_{phys} = 0$.

The ghosts are not coupled to the physical states. Therefore the latter must be of the form (up to null state).

$$|\{n\} p\rangle_M \otimes c_1|0\rangle_G$$

(52)

$|\{n\} p\rangle_M$ denotes the occupation numbers and momentum of the physical matter states. The operator $c_1$ lowers the energy of the state by one unit and is necessary for BRST invariance. The ghost excitation is responsible for lowering the ground state energy which produces the tachyon.

$$(L_0^M - 1) |_{phys} = 0$$

(53)

Therefore, the mass shell condition is

$$\alpha'M^2 = N^B + N^F - 1$$

(54)

where

$$N^B = \sum_{m=1}^{\infty} \alpha_{-m} \alpha_m$$

(55)

$$N^F = \sum_{r=1/2}^{\infty} r (b_{-r} b_r + d_{-r} d_r) .$$

(56)

$\alpha'M^2$ takes the values $-1, -1/2, 0, 1/2, 1, 3/2, 2$, and so on. The tachyonic ground state $\alpha'M^2 = -1$, belongs to the ghost sector. The remaining states fall into two categories. Defining G-parity as $(-1)^F$ where $F$ is the fermionic number, the half integral spin states of the above spectrum have odd parity. We make the GSO projection by demanding that the physical states have $G = +1$ i.e., even G-parity. Then the physical state mass spectrum can be described by

$$\alpha'M^2 = n$$

(57)

where $n$ takes values $0, 1, 2, \cdots$. This is the Regge conjecture. The zero mass state can be a vector boson or a pair of fermions.

This supersymmetry is natural with no SUSY particles. We now discuss the internal symmetry of the group. Three space-like generations each with $SO(11)$ symmetry are clearly discernible. It is well known that $SO(11) \supset SU(5) \times U(1) \times SU_C(3)$ and that $SO(5)$ can break to $SU(2) \times U(1)$ by the adjoint representation. The resulting symmetry group is $SU_C(3) \times SU(2) \times U(1) \times U(1)$. The additional $U(1)$ makes no difference to the predictions of the standard model. However there can be another $Z$-boson which may be observable.

We have presented the complete theory of a novel superstring in four dimensions. We hope that this string theory will encompass the standard model on one hand and general theory of relativity on the other.

The Library, Computer and other facilities extended by Institute of Physics, Bhubaneswar is thankfully acknowledged.

[1] Y. Nambu, in *Symmetries and quark models*, ed. R. Chand (Gordon and Breach), (1970); H. B. Nicolson, 15th International Conference on High Energy Physics, Kiev (1970); L. Susskind, Phys. Rev. D1, 1182 (1970).
[2] R. Dolen, D. Horn and C. Schmid, Phys. Rev. Lett. 19, 402 (1967); Phys. Rev. 166, 1768 (1968); G. Veneziano, Nuovo. Cim 57A, 190 (1968).
[3] C. Lovelace, Phys. Lett. 34B, 500 (1971).
[4] A. Neveu and J. H. Schwarz, Nucl Phys. B31, 86 (1971); A. Neveu, J. H. Schwarz and C. B. Thorn, ibid 35B, 529 (1971).
[5] M. B. Green, J. H. Schwarz and E. Witten, Superstring Theory Vol. 1, Cambridge University Press, Cambridge, England (1987), references to the materials presented in this work can be found this book.
[6] M. A. Virasoro, Phys. Rev. D1, 2933 (1970).
[7] L. D. Faddeev and V. N. Popov, Phys. Lett 25B, 29 (1967); A. M. Polyakov, Phys. Lett. 103B, 207 (1981), ibid 103B, 211 (1981).
[8] C. Becchi, A. Rouet and R. Stora, Phys. Lett. 52B, 344 (1974); Ann. Phys. 98, 287 (1976); I. V. Tyupin, Lebedev Preprint FIAN No. 39, (1975).
[9] D. Friedman, E. Martinee and S. Shanker, Nucl. Phys. B271, 93 (1986).
[10] F. Gliozzi, J. Scherk and D. Olive, Phys. Lett. 65B, 282 (1976); Nucl. Phys. B122, 253 (1977).
[11] T. H. R. Skyrme, Proc. Roy. Soc. A262, 237 (1961).
[12] R. Slansky, Phys. Rep. 79, 1 (1981).