Recent progress constraining the nuclear equation of state from astrophysics and heavy ion reactions

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Abstract. The quest for the nuclear equation of state (EoS) at high densities and/or extreme isospin is one of the longstanding problems of nuclear physics. Ab initio calculations for the nuclear many-body problem make predictions for the density and isospin dependence of the EoS far away from the saturation point of nuclear matter. On the other hand, in recent years substantial progress has been made to constrain the EoS both, from the astrophysical side and from accelerator based experiments. Heavy ion experiments support a soft EoS at moderate densities while recent neutron star observations require a “stiff” high density behavior. Both constraints are discussed and shown to be in agreement with the predictions from many-body theory.

1. Introduction

The isospin dependence of the nuclear forces which is at present only little constrained by data will be explored by the forthcoming radioactive beam facilities at FAIR/GSI, SPIRAL2/GANIL and RIA. Since the knowledge of the nuclear equation-of-state (EoS) at supra-normal densities and extreme isospin is essential for our understanding of the nuclear forces as well as for astrophysical purposes, the determination of the EoS was already one of the primary goals when first relativistic heavy ion beams started to operate. A major result of the SIS100 program at the GSI is the observation of a soft EoS for symmetric matter in the explored density range up to 2-3 times saturation density. These accelerator based experiments are complemented by astrophysical observations. The recently observed most massive neutron star with $2.1^{+0.4}_{-0.3} \, M_\odot$ excludes exotic phases of high density matter and requires a relatively stiff EoS. Contrary to a naive expectation, the astrophysical observations do, however, not stand in contradiction with those from heavy ion reactions. Moreover, we are in the fortunate situation that ab initio calculations of the nuclear many-body problem predict a density and isospin behavior of the EoS which is in agreement with both observations.

2. The EoS from ab initio calculations

In *ab initio* calculations based on many-body techniques one derives the EoS from first principles, i.e. treating short-range and many-body correlations explicitly. This allows
Recent progress constraining the nuclear equation of state to make prediction for the high density behavior, at least in a range where hadrons are still the relevant degrees of freedom. A typical example for a successful many-body approach is Brueckner theory (for a recent review see [1]). In the following we consider non-relativistic Brueckner and variational calculations [2] as well as relativistic Brueckner calculations [3]. It is a well known fact that non-relativistic approaches require the inclusion of - in net repulsive - three-body forces in order to obtain reasonable saturation properties. In relativistic treatments part of such diagrams, e.g. virtual excitations of nucleon-antinucleon pairs are already effectively included. Fig. 1 compares now the predictions for nuclear and neutron matter from microscopic many-body calculations – DBHF [5] and the 'best' variational calculation with 3-BFs and boost corrections [2] – to phenomenological approaches (NL3 and DD-TW from [6]) and an approach based on chiral pion-nucleon dynamics [7] (ChPT+corr.). As expected the phenomenological functionals agree well at and below saturation density where they are constrained by finite nuclei, but start to deviate substantially at supra-normal densities. In neutron matter the situation is even worse since the isospin dependence of the phenomenological functionals is less constrained. The predictive power of such density functionals at supra-normal densities is restricted. Ab initio calculations predict throughout a soft EoS in the density range relevant for heavy ion reactions at intermediate and low energies, i.e. up to about \(3 \rho_0\). Since the \(nn\) scattering length is large, neutron matter at subnuclear densities is less model dependent. The microscopic calculations (BHF/DBHF, variational) agree well and results are consistent with 'exact' Quantum-Monte-Carlo calculations [8].

Fig. 2 compares the symmetry energy predicted from the DBHF and variational

\[\text{Figure 1. EoS in nuclear matter and neutron matter. BHF/DBHF and variational calculations are compared to phenomenological density functionals (NL3, DD-TW) and ChPT+corr.. The left panel zooms the low density range. The Figure is taken from Ref. [4].}\]

\[\text{Figure 2.}\]
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In addition the relativistic DD-\(\rho\delta\) RMF functional is included. Two Skyrme functionals, SkM* and the more recent Skyrme-Lyon force SkLya represent non-relativistic models. The left panel zooms the low density region while the right panel shows the high density behavior of \(E_{\text{sym}}\).

The low density part of the symmetry energy is in the meantime relatively well constraint by data. Recent NSCL-MSU heavy ion data in combination with transport calculations are consistent with a value of \(E_{\text{sym}} \approx 31\) at \(\rho_0\) and rule out extremely ”stiff” and ”soft” density dependences of the symmetry energy. The same value has been extracted from low energy elastic and \((p,n)\) charge exchange reactions on isobaric analog states, i.e. \(p(^{6}\text{He}, ^{6}\text{Li})n\) measured at the HMI. At sub-normal densities \(\rho_0\) recent data points have been extracted from the isoscaling behavior of fragment formation in low-energy heavy ion reactions where the corresponding experiments have been carried out at Texas A&M and NSCL-MSU.

However, theoretical extrapolations to supra-normal densities diverge dramatically. This is crucial since the high density behavior of \(E_{\text{sym}}\) is essential for the structure and the stability of neutron stars. The microscopic models show a density dependence which can still be considered as “asy-stiff”. DBHF is thereby stiffer than the variational results of Ref. [2]. The density dependence is generally more complex than in RMF theory, in particular at high densities where \(E_{\text{sym}}\) shows a non-linear and more pronounced increase. Fig. 2 clearly demonstrates the necessity to constrain the symmetry energy at supra-normal densities with the help of heavy ion reactions.
3. Constraints from heavy ion reactions

![Excitation function of the $K^+$ multiplicities in $Au+Au$ and $C+C$ reactions.](image)

**Figure 3.** Excitation function of the $K^+$ multiplicities in $Au+Au$ and $C+C$ reactions. RQMD [13] and IQMD [14] with in-medium kaon potential and using a hard/soft nuclear EoS are compared to data from the KaoS Collaboration [15].

Experimental data which put constraints on the symmetry energy have already been shown in Fig. [2]. The problem of multi-fragmentation data from low and intermediate energy reactions is that they are restricted to sub-normal densities up to maximally saturation density. However, from low energetic isospin diffusion measurements at least the slope of the symmetry around saturation density could be extracted [16]. This puts already an important constraint on the models when extrapolated to higher densities. It is important to notice that the slopes predicted by the ab initio approaches (variational, DBHF) shown in Fig. [2] are consistent with the empirical values. Further going attempts to constrain the symmetry energy at supra-normal densities from particle production in relativistic heavy ion reactions [11, 17, 18] have so far not yet led to firm conclusions since the corresponding signals are too small, e.g. the isospin dependence of kaon production [19].

Firm conclusions could only be drawn on the symmetric part of the nuclear bulk properties. To explore supra-normal densities one has to increase the bombarding energy up to relativistic energies. This was one of the major motivation of the SIS100 project at the GSI where - according to transport calculation - densities between $1 \div 3 \rho_0$ are reached at bombarding energies between $0.1 \div 2 \text{AGeV}$. Sensitive observables are the collective nucleon flow and subthreshold $K^+$ meson production. In contrast to the flow signal which can be biased by surface effects and the momentum dependence of the optical potential, $K^+$ mesons turned out to an excellent probe for the high density
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At subthreshold energies the necessary energy has to be provided by multiple scattering processes which are highly collective effects. This ensures that the majority of the $K^+$ mesons is indeed produced at supra-normal densities. In the following I will concentrate on the kaon observable.

Subthreshold particles are rare probes. However, within the last decade the KaoS Collaboration has performed systematic high statistics measurements of the $K^+$ production far below threshold [15, 21]. Based on this data situation, in Ref. [13] the question if valuable information on the nuclear EoS can be extracted has been revisited and it has been shown that subthreshold $K^+$ production provides indeed a suitable and reliable tool for this purpose. In subsequent investigations the stability of the EoS dependence has been proven [20, 14].

Excitation functions from KaoS [15, 22] are shown in Fig. 3 and compared to RQMD [13, 20] and IQMD [14] calculations. In both cases a soft (K=200 MeV) and a hard (K=380 MeV) EoS have been used within the transport approaches. The forces where Skyrme type forces supplemented with an empirical momentum dependence. As expected the EoS dependence is pronounced in the heavy Au+Au system while the light C+C system serves as a calibration. The effects become even more evident when the ratio $R$ of the kaon multiplicities obtained in Au+Au over C+C reactions (normalized to the corresponding mass numbers) is built [13, 15]. Such a ratio has the advantage that possible uncertainties which might still exist in the theoretical calculations should cancel out to large extent. This ratio is shown in Fig. 4. The comparison to the experimental data from KaoS [15], where the increase of $R$ is even more pronounced, strongly favors a soft equation of state. This conclusion is in agreement with the conclusion drawn from
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the alternative flow observable $^{23, 24, 25, 26}$.

4. Constraints from neutron stars

![Figure 5](image_url)

**Figure 5.** Mass versus central density for compact star configurations obtained for various relativistic hadronic EoS. Crosses denote the maximum mass configurations, filled dots mark the critical mass and central density values where the DU cooling process becomes possible. According to the DU constraint, it should not occur in “typical NSs” for which masses are expected from population synthesis $^{27}$ to lie in the lower grey horizontal band. The dark and light grey horizontal bands around 2.1 $M_\odot$ denote the 1σ and 2σ confidence levels, respectively, for the mass measurement of PSR J0751+1807 $^{28}$. Figure is taken from $^{29}$.

Measurements of “extreme” values, like large masses or radii, huge luminosities etc. as provided by compact stars offer good opportunities to gain deeper insight into the physics of matter under extreme conditions. There has been substantial progress in recent time from the astrophysical side.

The most spectacular observation was probably the recent measurement $^{28}$ on PSR J0751+1807, a millisecond pulsar in a binary system with a helium white dwarf secondary, which implies a pulsar mass of $2.1 \pm 0.2 \left( ^{+0.4}_{-0.5} \right) M_\odot$ with 1σ (2σ) confidence. Therefore, a reliable EoS has to describe neutron star (NS) masses of at least 1.9 $M_\odot$ (1σ) in a strong, or 1.6 $M_\odot$ (2σ) in a weak interpretation. This condition limits the softness of the EoS in neutron star (NS) matter. One might therefore be worried about an apparent contradiction between the constraints derived from neutron stars and those from heavy ion reactions. While heavy ion reactions favor a soft EoS, PSR J0751+1807 requires a stiff EoS. The corresponding constraints are, however, complementary rather than contradictory. Intermediate energy heavy-ion reactions, e.g. subthreshold kaon production, constrains the EoS at densities up to $2 \div 3 \rho_0$ while the maximum NS
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mass is more sensitive to the high density behavior of the EoS. Combining the two constraints implies that the EoS should be soft at moderate densities and stiff at high densities. Such a behavior is predicted by microscopic many-body calculations (see Fig. 5). DBHF, BHF or variational calculations, typically, lead to maximum NS masses between $2.1 \div 2.3 \, M_{\odot}$ and are therefore in accordance with PSR J0751+1807, as can be seen from Fig. 5 and Fig. 6 which combines the results from heavy ion collisions and the maximal mass constraint.

![Figure 6](image.png)

**Figure 6.** Combination of the constraints on the EoS derived from the maximal neutron star mass criterium and the heavy ion collisions constraining the compression modulus. Values of various microscopic BHF and DBHF many-body calculations are shown.

There exist several other constraints on the nuclear EoS which can be derived from observations of compact stars, see e.g. Refs. [29, 30]. Among these, the most promising one is the Direct Urca (DU) process which is essentially driven by the proton fraction inside the NS [31]. DU processes, e.g. the neutron $\beta$-decay $n \rightarrow p + e^{-} + \bar{\nu}_{e}$, are very efficient regarding their neutrino production, even in super-fluid NM and cool NSs too fast to be in accordance with data from thermally observable NSs. Therefore, one can suppose that no DU processes should occur below the upper mass limit for “typical” NSs, i.e. $M_{DU} \geq 1.5 \, M_{\odot}$ ($1.35 \, M_{\odot}$ in a weak interpretation). These limits come from a population synthesis of young, nearby NSs [27] and masses of NS binaries [28]. While the present DBHF EoS leads to too fast neutrino cooling this behavior can be avoided if a phase transition to quark matter is assumed [32]. Thus a quark phase is not ruled out by the maximum NS mass. However, corresponding quark EoS have to be almost as stiff as typical hadronic EoS [32].
5. Summary

Heavy ion reactions provide in the meantime reliable constraints on the isospin dependence of the nuclear EoS at sub-normal densities up to saturation density and for the symmetric part up to - as a conservative estimate - two times saturation density. These are complemented by astrophysical constraints at high densities. The present situation is in fair agreement with the predictions from nuclear many-body theory.

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