Asymmetric preheating

Seishi Enomoto
Department of Physics, University of Florida, Gainesville, Florida 32611, USA

Tomohiro Matsuda
Laboratory of Physics, Saitama Institute of Technology, Fukaya, Saitama 369-0293, Japan

The Bogoliubov transformation is an important idea in modern physics. It was developed for finding solutions of BCS theory and is also used to explain the Unruh effect, Hawking radiation and many other topics. In this paper, we study CP violation and generation of matter-antimatter asymmetry during preheating when particles are generated due to the Bogoliubov transformation. We also discuss a new baryogenesis scenario, in which the asymmetry is generated without introducing loop corrections.

PACS numbers: 98.80Cq

I. INTRODUCTION

Violation of CP symmetry is important in particle physics and cosmology. The idea of CP violation is crucial in the attempts to explain the dominance of matter over antimatter in the present Universe, and is also important in the study of the weak interactions of the Standard Model (SM). If cosmological inflation explains the origin of the large-scale structure of the cosmos, the temperature after inflation is very low. Therefore, when inflation ends, the Universe must be thermalized by the decay of potential energy of the inflaton field. Although this process is still not well understood, it is believed that reheating takes place through a process called “preheating.” However, asymmetric particle production that works during preheating is still unexplored. Indeed, some secondary effects (e.g., later decay of a heavy particle or a false vacuum) have been considered by many authors, but few discussions have been given for direct production of the asymmetry. In this paper, we analyze conditions for the asymmetric particle production in preheating. Since preheating uses the Bogoliubov transformation, what is needed in this work is the asymmetric form of the Bogoliubov transformation and the evolution equations when CP is violated. Initially, we have suspected that the difference appears in the multiplicity parameter of the particle production. This is true for the case with an asymmetric initial condition, but in other cases the source of the asymmetry is different. We consider effective chemical potential, violation of CP in an initial condition, and CP violation in a kaon-like system, to understand the origin of the asymmetry. The asymmetry in the kaon-like model is similar to the wave function mixing in leptogenesis. Then we show that in a simple multi-field model, kinetic terms and a time-dependent transformation matrix can introduce the asymmetry without introducing loop corrections. Using the asymmetric preheating scenario, we show a simple baryogenesis scenario in Appendix A.

II. REVIEW OF BOSONIC PREHEATING

First we briefly explain the traditional bosonic preheating scenario and fix our notations. Note that matter and antimatter are always discriminated explicitly in this paper. We start with the action given by

\[ S_0 = \int d^4x \sqrt{-g} \left[ \partial_\mu \phi^* \partial^\mu \phi - m^2 |\phi|^2 + \xi R |\phi|^2 \right]. \]  

Using conformal time \( \eta \), we write the metric \( g_{\mu\nu} = a^2(\eta) \text{diag}(1, -1, -1, -1) \) and \( R = -6a / a^3 \), where \( a \) is the cosmological scale factor and the dot denotes time-derivative with respect to the conformal time. It is convenient to define a new field \( \chi \equiv a \phi \) and rewrite the action

\[ S_0 = \int d^4x \left[ |\chi|^2 - \omega^2 |\chi|^2 \right], \]

where

\[ \omega^2 \equiv a^2 m^2 + \left(-\Delta + \frac{\dot{a}}{a} (6\xi - 1) \right). \]  

Here \( \Delta \) is the Laplacian.

We perform decompositions using annihilation \((a, b)\) and creation \((a^\dagger, b^\dagger)\) operators of “particle” and “antiparticle”;

\[ \chi = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ h(\eta) a(k) e^{i k \cdot x} + g^*(\eta) b^\dagger(k) e^{-i k \cdot x} \right]. \]  

Here \( a(k) \) is the annihilation operator of the positive energy state. \( b^\dagger(k) \) is the creation operator of the antiparticle.

For our calculation we introduce conjugate momenta \( \Pi^\dagger \equiv \dot{\chi} \), which can be decomposed as

\[ \Pi^\dagger = \int \frac{d^3k}{(2\pi)^{3/2}} \left[ \dot{h}(\eta) a(k) e^{i k \cdot x} + \dot{g}^*(\eta) b^\dagger(k) e^{-i k \cdot x} \right]. \]  

1 See also Appendix B for previous works.
To quantize the system, we impose relations
\[ [\chi(x), \Pi(y)] = i\delta^3(x-y), \] (6)
and
\[ [a(k), a^\dagger(p)] = \delta^3(k-p) \] (7)
\[ [b(k), b^\dagger(p)] = \delta^3(k-p). \] (8)
Using \( \Pi \), the second order equation of motion can be written in the first order equations \[ \Pi^I = 0, \]
\[ \Pi^I + \omega^2 \chi = 0. \] (9)

Following Ref. [7], we expand \( h, \tilde{h} \) and \( g, \tilde{g} \) in the following way:
\[ h = \frac{e^{-i\int^t\omega dt'}}{\sqrt{2\omega}} A_h + \frac{e^{i\int^t\omega dt'}}{\sqrt{2\omega}} B_h, \]
\[ \tilde{h} = -i\omega e^{-i\int^t\omega dt'} \frac{1}{\sqrt{2\omega}} A_h + i\omega e^{i\int^t\omega dt'} \frac{1}{\sqrt{2\omega}} B_h, \] (10)
and
\[ g = \frac{e^{-i\int^t\omega dt'}}{\sqrt{2\omega}} A_g + \frac{e^{i\int^t\omega dt'}}{\sqrt{2\omega}} B_g, \]
\[ \tilde{g} = -i\omega e^{-i\int^t\omega dt'} \frac{1}{\sqrt{2\omega}} A_g + i\omega e^{i\int^t\omega dt'} \frac{1}{\sqrt{2\omega}} B_g, \] (11)
where \( A \) and \( B \) are known as the Bogoliubov coefficients. For further simplification we introduce \( \alpha \) and \( \beta \), which are defined as
\[ \alpha_{h,g} \equiv \frac{e^{-i\int^t\omega dt'}}{\sqrt{2\omega}} A_{h,g}, \]
\[ \beta_{h,g} \equiv \frac{e^{i\int^t\omega dt'}}{\sqrt{2\omega}} B_{h,g}. \] (12)

Using these equations one can solve \( \alpha \) and \( \beta \). Now the equation of motion can be written as
\[ \dot{h} - \tilde{h} = 0 \] (14)
\[ \dot{\tilde{h}} + \omega^2 h = 0, \] (15)
which can be solved for \( \dot{\alpha} \) and \( \dot{\beta} \) as
\[ \dot{\alpha}_h = -i\omega \alpha_h + \frac{\dot{\omega}}{2\omega} \beta_h \]
\[ \dot{\beta}_h = i\omega \beta_h + \frac{\dot{\omega}}{2\omega} \alpha_h. \] (16)

To understand particle production, it is useful to calculate time-derivatives of \( |\alpha| \) and \( |\beta| \). For real \( \omega \), one immediately finds
\[ \frac{d}{dt} |\alpha|^2 = \dot{\alpha}^* \alpha + \alpha^* \dot{\alpha} \]
\[ = \left( -i\omega \alpha + \frac{\dot{\omega}}{2\omega} \beta \right) \alpha^* + \alpha \left( i\omega \alpha^* + \frac{\dot{\omega}}{2\omega} \beta^* \right) \]
\[ = \frac{\dot{\omega}}{2\omega} (\alpha \beta^* + \alpha^* \beta) \]
\[ = \frac{\dot{\omega}}{\omega} |\alpha||\beta| \cos (\theta_\alpha - \theta_\beta), \] (17)
where we defined the phase parameters as \( \alpha \equiv |\alpha| e^{i\theta_\alpha} \) and \( \beta \equiv |\beta| e^{i\theta_\beta} \). Here the subscripts \( h \) and \( g \) are omitted. The same calculation shows
\[ \frac{d}{dt} |\beta|^2 = \frac{\dot{\omega}}{\omega} |\alpha||\beta| \cos (\theta_\alpha - \theta_\beta). \] (18)

Comparing these equations one can see that the evolution equations of \( |\alpha| \) and \( |\beta| \) are identical. Therefore \( |\alpha|^2 - |\beta|^2 = 1 \) does not change during preheating, as required from the definition of these parameters [7].

If particle production is not symmetric, \( |\beta|_h \) and \( |\beta|_g \) must be discriminated. Since matter and antimatter are sharing the same \( \omega(t) \), the only possible way is to find \( \cos (\theta_{\alpha_h} - \theta_{\beta_h}) \neq \cos (\theta_{\alpha_g} - \theta_{\beta_g}) \). Note that \( h \) and \( g \) are obeying the same equation of motion as long as there is no complex parameter. The complex parameter, if exists, could or could not be the source of the asymmetry. One has to examine if such parameter generates the asymmetry during preheating.

In the next section we study asymmetric preheating in several cases. For the first example we show that CP violation in the initial condition can introduce the asymmetry via \( \cos (\theta_{\alpha_h} - \theta_{\beta_h}) \neq \cos (\theta_{\alpha_g} - \theta_{\beta_g}) \). For the second example we see that the complex parameter appears when an effective chemical potential is introduced to the model. The chemical potential can indeed split \( \alpha_h, \beta_h \) and \( \alpha_g, \beta_g \), but the absolute values of these parameters remain identical. Therefore, in this model the chemical potential cannot generate the asymmetry. For the third example we consider a kaon-like model. At the lowest order of the perturbation, eigenstates of the equation of motion are the CP eigenstates and also they are symmetric combination of matter and antimatter. Therefore the asymmetry is not generated at the lowest order. However, in this model, loop corrections introduce \( \Gamma \), which mixes the CP eigenstates and causes the asymmetry between matter and antimatter. The last example uses multi-field preheating. In this model, although eigenstates are not the CP eigenstates, they are still symmetric as far as the transformation matrix is constant. Using numerical calculation, we show that a time-dependent transformation matrix can introduce the required matter-antimatter asymmetry.

### III. ASYMMETRY IN PREHEATING

#### A. CP violation in the initial condition

From Eq. (17) and (18), one can read out the evolution equations of matter \( (\alpha_h, \beta_h) \) and antimatter \( (\alpha_g, \beta_g) \). Since matter and antimatter are sharing the same \( \dot{\omega}/\omega \), matter and antimatter evolve simultaneously when \( \delta_{h-g} \equiv \cos (\theta_{\alpha_h} - \theta_{\beta_h}) - \cos (\theta_{\alpha_g} - \theta_{\beta_g}) = 0 \). The relation \( \delta_{h-g} = 0 \) is rigorous when CP is conserved. Interestingly, the relation could be violated when CP is not conserved (temporally) in the early Universe. In that case, CP violation may appear in the initial condition, even if CP
violation does not appear (or hidden) in the low-energy effective action. Fig.1 shows our result of numerical calculation. In this scenario, \( n_\beta = n_\gamma \) is possible in the initial condition, and no explicit CP-violating interaction is needed in the effective low-energy action. In that sense, the asymmetry production is simply the consequence of the initial condition \( \delta_{h-g} \neq 0 \), which discriminates the later evolution of matter and antimatter densities. Although the mechanism is quite different, this scenario reminds us of Affleck-Dine baryogenesis\[^2\] or oscillons\[^3\], in which baryogenesis is triggered by an initial phase-shift. Unfortunately, at this moment we have no idea how this condition is realized in a realistic cosmological scenario. Nevertheless, the idea of violating CP by the initial condition is fascinating, since preheating always starts with a non-equilibrium state and the initial condition \( \delta_{h-g=0} \) will not be valid when CP is not the symmetry of the system.

### B. Chemical potential

There are vast variety of interactions that could violate CP. Among all, chemical potential would be the simplest. Using chemical potential we can see how preheating works with such interaction.\[^2\]

To understand more about the matter-antimatter asymmetry during preheating, we consider a new scalar field \( \varphi \) and its derivative coupling to the current\[^3\] \( J^\mu \equiv -i(\chi \partial^\mu \chi^* - \chi^* \partial^\mu \chi) \),

\[
\mathcal{L}_c = -\frac{\partial \varphi}{M_*} J^\mu.
\]  

This term is commonly used in spontaneous baryogenesis\[^6\]. In this paper, we are not arguing the source of this term. Assuming a homogeneous background, we have

\[
\mathcal{L}_c = -\frac{\dot{\varphi}}{M_*} J^0 
= \mu_\chi (n_\chi - \bar{n}_\chi).
\]

This term introduces chemical potential \( \mu_\chi \equiv -\dot{\varphi}/M_* \), which is expected to bias the matter and the antimatter densities.

After adding chemical potential to \( S_0 \), field equations are changed. Using

\[
\frac{d}{d\eta} \left( \frac{\partial \mathcal{L}}{\partial \dot{\chi}^*} \right) - \frac{\partial \mathcal{L}}{\partial \chi^*} = 0
\]

we find

\[
\dot{\chi} - 2i\mu_\chi \chi + (\omega^2 - i\dot{\mu}_\chi) \chi = 0.
\]

There are two terms which might cause differences. One is \(-2i\mu_\chi \chi \), and the other is \(-i\dot{\mu}_\chi \chi \). If one assumes that the chemical potential is not changing with time, one can simply assume \( \dot{\mu}_\chi \approx 0 \). Then the equation of motion can be written as

\[
\begin{align*}
\dot{h} - \bar{h} - i\mu_\chi h &= 0 \\
\dot{\bar{h}} + \omega^2 h - i\mu_\chi \bar{h} &= 0,
\end{align*}
\]

where a complex parameter (\( \sim i\mu_\chi \)) appears. The imaginary part discriminates matter and antimatter equations.

The above equations can be written using \( \alpha \) and \( \beta \). One can solve these equations for \( \dot{\alpha} \) and \( \dot{\beta} \) to find

\[
\begin{align*}
\dot{\alpha}_h &= -i(\omega - \mu_\chi)\alpha_h + \frac{\omega}{2\omega}\beta_h \\
\dot{\beta}_h &= \frac{\omega}{2\omega}\alpha_h + i(\omega + \mu_\chi)\beta_h,
\end{align*}
\]

and

\[
\begin{align*}
\dot{\alpha}_g &= -i(\omega + \mu_\chi)\alpha_g + \frac{\omega}{2\omega}\beta_g \\
\dot{\beta}_g &= \frac{\omega}{2\omega}\alpha_g + i(\omega - \mu_\chi)\beta_g.
\end{align*}
\]

Although the equations are discriminated, it is still not clear if the physical quantities (e.g. \( |\beta|^2 \)) are discriminated. The point is that the chemical potential can be absorbed by redefining the field as

\[
\tilde{\chi} = \chi e^{-i\int^n_0 \mu_\chi(n') \, dn'},
\]

\(^4\) This assumption is made for a single particle production process of preheating, which has a much shorter timescale compared with \( \dot{\varphi} \). Within the cosmological timescale, \( \mu \) is not constant because \( \dot{\varphi} \) must vary in the expanding Universe. However, since Eq. (20) is not restricted to \( \dot{\mu} = 0 \), the chemical potential can be rotated away and it does not cause asymmetry in the above model.
and after this transformation one can erase $\mu_\chi$ from the equation of motion. The Lagrangian density for the “new field” $\hat{\chi}$ can be written as

$$\mathcal{L} = \dot{\hat{\chi}}^* \hat{\chi} - W^2 |\hat{\chi}|^2,$$  

(29)

where $W \equiv \sqrt{\omega^2 + \mu_\chi^2}$. The new field recovers the original ($\mu_\chi = 0$) calculation. Obviously, for the new field the particle and the antiparticle are not distinguishable and therefore their $\beta$ and $\alpha$ are not distinguishable as well. Since the original and the new fields are related by the phase rotation defined by Eq.(28), there is no asymmetry production in this case.

As we have seen in Sec III A violation of $\delta_{h-g} = 0$ is crucial for the asymmetric evolution. Although chemical potential differentiates $\alpha$ and $\beta$, it does not violate $\delta_{h-g} = 0$. We show our numerical calculation in Fig.2, which shows that the chemical potential does not violate $\delta_{h-g} = 0$.

Our result suggests that the “asymmetric preheating” requires more complex setups for CP violation. Reflecting on the current status of CP violation in the SM, realistic CP violation could not be a simple story. Therefore, it is useful to start with a model in which the mechanism of CP violation is already established and its consequences are widely known. For this purpose, we choose a K meson (kaon) for our discussion.

C. CP-violating interaction

Perhaps the most famous CP violation can be found for a kaon, which denotes a group of mesons distinguished by strangeness. In the quark model of the SM, they are bound states of a strange quark and an up or down antiquark. We consider $K^0 = ds, \bar{K}^0 = d\bar{s}$, which are neutrally charged and distinguishes matter and antimatter. Since CP symmetry exchanges matter and antimatter, eigenstates of CP are $K_1 = (|K^0\rangle + |\bar{K}^0\rangle)/\sqrt{2}$ (CP even) and $K_2 = (|K^0\rangle - |\bar{K}^0\rangle)/\sqrt{2}$ (CP odd). If these were the eigenstates of weak interaction, CP will be conserved. Of course in reality they are not weak eigenstates and therefore CP is not conserved. Note that in this model the statement “CP is conserved during evolution” is equivalent to “CP eigenstates $K_1$ and $K_2$ do not mix during evolution”. In other words, $K_1$ and $K_2$ can never be the eigenstates of the equation if the evolution equation violates CP. This point is crucial for later argument.

Naively, the Schrödinger equation for the state $\psi_0^T \equiv (K^0, \bar{K}^0)$ can be given by

$$i \frac{d}{dt} \psi_0 = H \psi_0,$$

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} M & \Delta_M \\ \Delta_M^* & M \end{pmatrix}.$$  

(30)

However, since a kaon decays during evolution, one has to introduce $\Gamma$ as $H \rightarrow H - i\Gamma$, where

$$\Gamma = \begin{pmatrix} \Gamma_d & \Gamma_{\Delta} \\ (\Gamma_{\Delta})^* & \Gamma_d \end{pmatrix}.$$  

(31)

CPT transformation ensures $(H - i\Gamma)_{11} = (H - i\Gamma)_{22}$. Note that $i\Gamma$ is anti-Hermite.

Let us choose the definition in which kaon has real $\Delta_M$. Without $\Gamma$, the two eigenstates of the equation are $K_1$ and $K_2$. If $\Gamma$ is not real, one can decompose it as $\Gamma = \Gamma_d + i\Gamma_{\Delta}$, which gives

$$(H - i\Gamma)_{12} = \Delta_M - i(\Gamma_{\Delta} + \Gamma_d),$$

$$(H - i\Gamma)_{21} = \Delta_M - i(\Gamma_{\Delta} - \Gamma_d).$$  

(32)

The Schrödinger equation for $\hat{\psi}^T \equiv (K_1, K_2)$ becomes

$$i \frac{d}{dt} \hat{\psi} = \hat{H} \hat{\psi},$$

$$\hat{H} = \begin{pmatrix} H_{11} - \Delta & -\Gamma_{\Delta} \\ -\Gamma_{\Delta}^* & H_{11} + \Delta \end{pmatrix}.$$
where $\hat{\Delta} \equiv \Delta_M - i \Gamma^R M$. The off-diagonal element $\sim \Gamma^I \Delta$ measures CP violation, since it introduces mixing between the two CP eigenstates.

Comparing diagonal elements, one will find a difference proportional to $\Delta$. Therefore, evolution of $K_1$ and $K_2$ are distinguishable. Although this means that production of $K_1$ and $K_2$ are not equivalent during preheating, it does not lead to production of the asymmetry. There are two major problems, which have to be solved. First, particle production during preheating has to be considered for the eigenstates of the equation. Therefore, what is produced (directly) during preheating is neither $K_1$ nor $K_2$. Second, even if $K_1$ and $K_2$ are produced and their number densities are different, both $K_1$ and $K_2$ are one-to-one mixed states of matter ($K_0$) and antimatter ($\bar{K}_0$). In this paper, these states are called “symmetric eigenstates”, because they do not generate asymmetry as far as their decay rates are symmetric.

Sometimes a kaon is analyzed using another pair, the two eigenstates of the above equation, $K_L$ and $K_S$. They are called weak eigenstates. Although they do not form orthogonal eigenstates, preheating must be calculated for the weak eigenstates, since they are the eigenstates of the equation of motion.

The above argument for a kaon is very useful for our calculation. To see more details of the model in the light of preheating, we first reconsider complex off-diagonal element ($\sim \Delta_M$) and then introduce complex $\Gamma^I$.

1. Complex off-diagonal elements with $\Gamma = 0$

Now consider complex $\Delta (= \Delta_M)$ and find eigenstates of the equation. We have

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} M & \Delta \\ \Delta^* & M \end{pmatrix}. \quad (33)$$

The eigevectors of the matrix is given by

$$\begin{pmatrix} e^{i\theta_\Delta} & 1 \\ \sqrt{2} & \sqrt{2} \end{pmatrix}, \quad \begin{pmatrix} e^{i\theta_\Delta} & 1 \\ \sqrt{2} & \sqrt{2} \end{pmatrix}^*, \quad (34)$$

where $\Delta = |\Delta| e^{i\theta_\Delta}$. Their eigenvalues are

$$M - |\Delta|, M + |\Delta|. \quad (35)$$

Of course the two eigevectors are orthogonal. In this case the eigenstates are symmetric eigenstates. This means that there is no asymmetry in particle production. Obviously, this is due to the CPT transformation, which strictly decides the diagonal elements of the original matrix to be $H_{11} = H_{22}$. For the system with only matter and antimatter, this dilemma is quite serious.

2. Off-diagonal $\Gamma$

Now we introduce $\Gamma$ to the above model. Since we are seeing matter-antimatter bias in the eigenstates, we focus on the off-diagonal elements of $\Gamma$ (and disregard diagonal elements of $\Gamma$). We have

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} M & \Delta^* \\Delta + i\Gamma^I M \end{pmatrix}. \quad (36)$$

Eigenvectors are

$$\begin{pmatrix} \pm \frac{r}{\sqrt{1 + r^2}} \pm \frac{1}{\sqrt{1 + r^2}} \end{pmatrix}, \quad (37)$$

where $r = \sqrt{\frac{\Delta + i\Gamma^I}{\Delta - i\Gamma^I}}$. This parameter is commonly used to measure the CP-violation in a kaon.

Their eigenvalues are

$$M \pm \sqrt{\Delta + i\Gamma^I)(\Delta + i\Gamma^I)}. \quad (38)$$

Now we have eigenstates which is a set of biased mixed states of matter and antimatter (asymmetric states).

Note that in this model $r = 1$ (symmetric eigenstate) is realized when both $\Gamma^I$ and $\Delta$ are real, or either $\Gamma^I$ or $\Delta$ vanishes. On the other hand, $r \neq 1$ (asymmetric eigenstate) can be realized if either $\Gamma^I$ or $\Delta$ are complex and both $\Gamma^I$ and $\Delta$ do not vanish.

During preheating, the two eigenstates of the equation are produced. They are expected to have different number densities. However, since the matter-to-antimatter ratio ($r$) is identical between these eigenstates, the ratio between matter and antimatter does not depend on their number densities.

Application of the above scenario can be found in leptogenesis. The CP violation in the decays of heavy singlet neutrinos, which arises from the wave function mixing, has been calculated in Ref.\[10, 11\]. In their analyses, CP violation in the wave function mixing is found to generate a similar contribution (bias) to the eigenstates of the heavy singlet neutrinos\[5\]. Therefore, if one of the heavy singlet neutrinos has a field-dependent mass and is generated during preheating, it causes the bias $r \neq 0$. Calculation of the bias is given in Ref.\[10, 11\]. To avoid thermalization after preheating, which washes out the lepton number and resets the initial condition, one has to assume that the reheating temperature $T_R$ is low enough to avoid thermalization of the singlet fermions. However, even if the asymmetry is generated directly during preheating, the result is not so much different from the old scenario, in which the initial number densities are symmetric but the decay is not, since the same bias appears both in the asymmetric number densities and in the decay width.

\[5\] Since the contribution appears for the “mixing”, one has to introduce at least two singlet neutrinos for the model.
D. Distinguishing sources of the asymmetry

The above arguments are based on a system with a kaon. Before moving forward, we need to show how these arguments are applied to the preheating scenario.

We start with

\[ L = |\bar{\chi}|^2 - \omega^2|\bar{\chi}|^2 + [g\chi^2 + h.c.] , \]

which leads to the equation of motion;

\[ \ddot{\chi} + \omega^2 \chi + g^* \chi^* = 0. \]

If one considers the expansion

\[ \chi = \int \frac{d^3k}{(2\pi)^3/2} [u(\eta, k)a(k) + v^*(\eta, k)b^\dagger(k)] \]

and defines states by \(|\chi, \eta\rangle \equiv u^*a^\dagger|0\rangle\) and \(|\bar{\chi}, \eta\rangle \equiv v^*b^\dagger|0\rangle\), the equation of motion for the state \(X^\dagger \equiv \langle |\chi\rangle, |\bar{\chi}\rangle\) is

\[ \frac{d^2}{dt^2} X = -\left( \begin{array}{cc} \omega^2 & g \\ g^* & \omega_2 \end{array} \right) X. \]

Assuming interaction that introduces imaginary parts for two-point correlation functions, one can modify the matrix of the equation as

\[ \left( \begin{array}{cc} \omega^2 & g \\ g^* & \omega_2 \end{array} \right) \rightarrow \left( \begin{array}{cc} \omega^2 & g + i\Gamma_\Delta \\ g^* + i\Gamma_\Delta & \omega_2 \end{array} \right), \]

where the diagonal elements of \(\Gamma\) are neglected.

Following a kaon, two eigenstates of the equation are

\[ |\chi_{\pm}\rangle \equiv \pm \frac{1}{\sqrt{1 + |r|^2}} |\chi\rangle + \frac{r}{\sqrt{1 + |r|^2}} |\bar{\chi}\rangle, \]

where \(r \equiv \sqrt{\frac{g + i\Gamma_\Delta}{g^* + i\Gamma_\Delta}}\). These eigenstates are not orthogonal but linearly independent. Their eigenvalues are

\[ \omega^2 \pm \sqrt{(g + i\Gamma_\Delta)(g^* + i\Gamma_\Delta)}. \]

If we define that \(\hat{X}^\dagger\) gives the new eigenstates, we have

\[ \frac{d^2}{dt^2} \hat{X} = -\left( \begin{array}{cc} \omega_+ & \omega_- \\ \omega_- & \omega_+ \end{array} \right) \hat{X}, \]

where \(\omega_\pm \equiv \omega^2 \pm \sqrt{(g + i\Gamma_\Delta)(g^* + i\Gamma_\Delta)}.\)

During preheating these particles are produced independently. Their number densities are different because of \(\omega_+ \neq \omega_-\). On the other hand, since these eigenstates are sharing the same \(r\), the asymmetry is uniquely determined by \(r\). This property is very important. In a model where the particle number density grows exponentially during preheating, one can see that the \(j\)-th scattering gives

\[ n_k^{j+1} = e^{-\pi \kappa^2} + (1 + 2e^{-\pi \kappa^2})n_k^j, \]

where \(\kappa^2 \equiv \frac{\kappa^2}{2} \equiv \frac{k^2}{2}\) and \(\theta^j\) is the total phase accumulated by the moment \(t = t_j\). Here \(k_s\) gives the typical width of the Gaussian function \(e^{-\pi k^2}/2\). The first term gives the particle production without resonance. The second term is the source of resonant amplification of the number density. The third term may lead to stochastic behavior. Eq. (44) shows that a bias in the second term (in the multiplication factor \(1+2e^{-\pi \kappa^2}\)) gives a growing asymmetry. On the other hand, our model has a bias \((r)\) in the eigenstates, which leads to a constant asymmetry. These are crucially discriminating in the numerical calculation. Namely, if the numerical calculation shows that the asymmetry is nearly constant from the beginning, one can predict that the asymmetry is mostly controlled by the bias in the eigenstates. On the other hand, if the asymmetry grows significantly during preheating, one can predict that the multiplication factor \(\sim e^{-\pi \kappa^2}\) is the source of the asymmetry. If the asymmetry behaves somewhat randomly, there could be a mismatch in the phase (see Section IIIA). Since preheating is a highly non-linear process, despite the progress in numerical calculations, it seems very difficult to understand the origin of the asymmetry when many fields and various interactions are introduced. In that case, the above properties are the lead to the origin of the asymmetry.

IV. MULTI-FIELD EXTENSION : ASYMMETRY WITHOUT LOOP CORRECTIONS

In the previous section we have seen how the asymmetry appears in a system of a complex scalar field. In the kaon-like model, \(\Gamma \neq 0\) is essential for the asymmetry. Because of \(\Gamma\), the Hamiltonian becomes effectively non-Hermitite. We have also seen that complex parameters do not contribute the asymmetry if they can be rotated away by redefining the fields. For our purpose, we are going to introduce a complex parameter by introducing another field and interaction.

In this section, we try to construct a model of asymmetric preheating when the Hamiltonian is Hermite. In
this model, loop correction is not the source of the asymmetry. Our idea is very simple. In the kaon-like model, the asymmetry appears when interaction introduces loop corrections. Then, it is very natural to think that kinetic terms, instead of loop corrections, may contribute the asymmetry as well. Note that we are not considering non-minimal kinetic terms for our story. Suppose that the interaction is diagonalized using a transformation matrix $U$. If one considers conventional (minimal) kinetic terms, the kinetic terms are already diagonalized for the original fields. Normally $U$ is not time dependent and thus nothing will happen: kinetic terms are (usually) diagonal for both the original and the new fields. However, during preheating the matrix can be time-dependent. In that case, the minimal kinetic term can generate additional (effective) CP-violating interaction and mixing between states.

In this section, we show a simple model in which asymmetric preheating is realized by the time-dependent matrix $U$.

The simplest model can be given by the following Lagrangian:

$$L = |\partial_{\mu}\phi|^{2} - m_{\phi}^{2}\phi|^{2} + \frac{1}{2}(\partial_{\mu}\eta)^{2} - \frac{1}{2}m_{\eta}^{2}\eta^{2} - \frac{1}{2}(e\phi^{2} + h.c.) - (g\phi\eta + h.c.),$$

(48)

where $\phi$ is a complex scalar and $\eta$ is a real scalar and $\epsilon, g$ are complex coupling constants. Note that the complex phases of $\epsilon$ and $g$ are not removed simultaneously. Here, we choose the definition that makes $\epsilon$ to be real. Furthermore, we assume that $\\phi$’s mass $m_{\phi}^{2}$ depends on time in order to take into account the situation of preheating. The equations of motion are given by

$$\dot{\Psi} + \Omega^{2}\Psi = 0 \quad (49)$$

where

$$\Psi = \begin{pmatrix} \phi \\ \eta \end{pmatrix}, \quad \Omega^{2} = \begin{pmatrix} \omega_{\phi}^{2} & \epsilon \\ \epsilon & \omega_{\phi}^{2} \end{pmatrix}. \quad (50)$$

In order to understand the above equations, we first discuss the eigenvalues and eigenvectors of the matrix $\Omega^{2}$. Although the formula becomes cumbersome, since their eigenvalues are given by the solutions of a cubic equation, fortunately it is easy to find that the eigenvectors are expressed as $\propto (a_{i}, a_{i}^{*}, 1)$ for $i = 1, 2, 3$. Here $a_{i}$ is a complex number. This means that the eigenstates are $\psi_{i} \equiv b_{i}(a_{i}\phi + a_{i}^{*}\phi^{*} + \eta)$, where $b_{i}$ is a normalization factor. Although these “eigenstates” are not CP eigenstates (except for the special points in the parameter space), they are symmetric in the sense that they are the one-to-one mixing states of $\phi$ and $\phi^{*}$. The point is that since $\omega_{\phi}(t) \neq 0$ is considered, the above “eigenstates” do not diagonalize the equation of motion during preheating. Assume that the mass matrix $\Omega^{2}$ is diagonalized by a unitary matrix $U$ given by

$$\omega^{2} \equiv \begin{pmatrix} \omega_{1}^{2} & \omega_{2}^{2} \\ \omega_{2}^{2} & \omega_{3}^{2} \end{pmatrix} = U^{\dagger}\Omega^{2}U, \quad (51)$$

where we defined

$$U^{\dagger} = \begin{pmatrix} a_{1}b_{1} & a_{3}b_{1} & b_{1} \\ a_{2}b_{2} & a_{3}b_{2} & b_{2} \\ a_{3}b_{3} & a_{3}b_{3} & b_{3} \end{pmatrix} = U^{-1}. \quad (52)$$

Then the equations of motion $\Psi$ becomes

$$(U^{\dagger}\Psi)^{\dagger} + 2\gamma(U^{\dagger}\Psi) + (\omega^{2} + \gamma^{2} + \dot{\gamma})(U^{\dagger}\Psi) = 0, \quad (53)$$

where we defined

$$\gamma \equiv U^{\dagger}\dot{U}. \quad (54)$$

which is anti-Hermite. Because of $\gamma$, the equation of motion is not diagonalized by the “symmetric states”. Here $U$ depends on time as long as $\dot{\omega}_{\phi} \neq 0$. This fact indicates that the asymmetry may appear when $\gamma \neq 0$. If the speculation is correct, the asymmetry between particles and anti-particles would be enhanced in the non-adiabatic regime, where time-dependence becomes significant. This corresponds to the area where particle production becomes significant.

Since the asymmetry is evaluated by

$$n_{\phi} - \bar{n}_{\phi} = \frac{1}{V} \int d^{3}x \left( \langle \phi^{\dagger}\phi \rangle - \langle \phi^{\dagger}\phi^{\dagger} \rangle \right), \quad (55)$$

where $V$ is a volume of the system, in principle one can follow the evolution of the asymmetry by solving eq.(10). The problem is that it is not an easy task to obtain an analytic solution of eq.(10). Instead, we show the results of our numerical calculation in Fig.13 which clearly shows that the asymmetry is generated but the ratio does not grow when the total number density grows. This result reminds us of the kaon-like model, in which the ratio is expected to be constant during preheating.

---

8 In this paper, we have assumed that higher interactions are very small and negligible. Preheating with higher interactions is partially analyzed in Ref.[13,14].

9 The situation reminds us of Cabibbo-Kobayashi-Maskawa(CKM) Matrix in the SM. Unless $U$ is given by a real orthogonal matrix, a complex phase will remain and it becomes the source of the asymmetry.

10 Remember that these are not the eigenstates of the equation of motion when $U$ is time-dependent.

11 Note that $\dot{\omega}_{\phi} \neq 0$ does not always mean $\dot{U} \neq 0$. A typical example would be $\omega_{\phi}(t) = \omega_{\eta}(t)$, in which $U$ does not depend on $\omega_{\phi}$. In that case production of the asymmetry is impossible.
FIG. 3: Time evolution of the net number (upper) and the asymmetry (bottom) of $\phi$. We choose parameters as $m_\phi^2 = 0.15^2 + 4 \cos^2 0.03 t, m_\eta^2 = 0.1^2, \epsilon = 10^{-4}, g = 10^{-2}i$. From the second plot we can see that the ratio behaves like a constant from the beginning. This clearly shows that the bias in the eigenstates is controlling the asymmetry.

V. CONCLUSIONS AND DISCUSSIONS

In this paper we studied the possibility of generating asymmetry between matter and anti-matter with the CP violating models due to the parametric resonance. Effective chemical potential, CP violation in the initial condition, CP violation in a kaon-like system (loop corrections) and a time-dependent transformation matrix are examined.

Violation of the CP initial condition is fascinating since inflation may start with a non-equilibrium state, and the CP initial condition is useless when CP is not the symmetry of the high energy theory. However, at this moment appearance of such initial condition is an optimistic option.

If the Hamiltonian is Hermite, the unremovable phases and the time-dependent transformation matrix play crucial role. We considered a multi-field model to show how this idea works. In this model, the eigenstates are not the CP eigenstates, but without $\dot{U} \neq 0$, eigenstates of the equation of motion are still symmetric. Therefore the matter-antimatter symmetry still remains when $U$ is constant. To achieve $\dot{U} \neq 0$, we considered $\dot{\omega}_\phi \neq 0$. The mass of the additional scalar field does not have to be time-dependent but must not be identical to the complex field mass, since in that case $U$ does not depend on time. In this story, no special structure is needed for the kinetic term. In this scenario, one can say that violation of the matter-antimatter symmetry appears not from an interaction term but from a kinetic term.

In the latter half of this paper, we have tried to reveal the origin of the asymmetry focusing on the bias in the eigenstates. Such bias may appear either from the loop corrections or from the dynamical effects. We have seen that in the former case the interference between corrections gives $\Gamma \neq 0$, which is crucial for the asymmetry. In the latter case, $\gamma \neq 0$ plays the role.

Usually in baryogenesis and leptogenesis, one has to consider interference in loop corrections, but our study indicates that a time-dependent background can introduce mixing between states and can play similar role during preheating.

VI. ACKNOWLEDGEMENTS

SE is supported by the Heising-Simons Foundation grant No 2015-109.

Appendix A: Baryogenesis

In this appendix, we are going to discuss how baryogenesis can be realized using $\gamma \neq 0$. For simplicity, it would be useful to follow the scenario considered in Ref. [2]. The original setup, simple chaotic inflation, could not be explaining the current cosmological parameters, but the inflaton oscillation after inflation is still generic. Here, the significant difference from the traditional scenario is in the source of the asymmetry. In the traditional scenario of Grand Unified Theories (GUT) baryogenesis, no asymmetry is assumed for the initial number densities of the heavy fields. Instead, the asymmetry is generated by the asymmetric decay width, which is due to the loop corrections. In our scenario of asymmetric preheating, the number densities of the heavy fields are not symmetric, while no asymmetry is assumed for the decay rates.

We start our discussion with the traditional Sakharov conditions [15]: B violation, out of thermal equilibrium and C, CP violation.

1. B violation

In GUT, there are heavy gauge bosons $X^\mu$ and Heavy Higgs bosons $Y$ with couplings to quarks and leptons. The couplings are schematically written as $X_{qq}, X_{\bar{q}\bar{q}}$ (and similar for $Y$), which implies that baryon number cannot be assigned consistently to these heavy bosons.
In supersymmetric models, one could avoid R-parity and introduce dimension-four baryon number violating operators, such as
\[ \bar{q}_a q_b \phi^A \epsilon_{abc}, \]  
where \( \bar{q} \) is the bosonic partner of the top quark (squark). This interaction could exist without violating the constraint from the proton decay, if lepton number was independently conserved for some reason (i.e., \( p \to \pi^+ \nu \) and \( p \to \pi^0 e^+ \) are forbidden by the lepton number conservation.) Another way of realizing a similar scenario is to consider asymmetric generation of a singlet sneutrino. Note that, in this simple model, asymmetric particle production is realized by the time-dependent mass \( m^2 \phi = m^2_0 + \lambda \phi^2 \). One can assume that the potential of \( \phi \) is effectively quadratic during oscillation. Since the heavy fields \( \phi \) and \( \eta \) are violating the baryon number, these fields must not be in thermal equilibrium. Therefore, we assume for the reheating temperature \( T_R \ll m_0 < m_\eta \), which does not constitute any additional limiting factor.\(^1\)

2. Out of thermal equilibrium

If a baryon number violating process is still in thermal equilibrium, the inverse process destroys baryon as fast as it is created. If the heavy fields are generated in a thermal plasma, this condition is very important. On the other hand, if the heavy fields are generated during preheating, the Universe is already far away from thermal equilibrium. Therefore, as far as the baryon number violating process does not come into thermal equilibrium after preheating, washout process is not important. Of course the sphaleron process is important when it is activated in the thermal plasma. However, it cannot wash out \( B - L \), where \( B \) and \( L \) are the baryon and the lepton numbers of the Universe, respectively. This condition is rather trivial in our scenario.

3. C,CP violation

Let us consider the Lagrangian given by Eq.(48). CP is violated when both \( \epsilon \) and \( g \) are complex. These CP phases can be changed by using the phase rotation of \( \phi \). After rotation, one can find the Lagrangian written with real \( \epsilon \) and complex \( g \). This is the starting point of our baryogenesis scenario. Obviously, the asymmetry disappears when all these parameters become real after the phase rotation.

In our simple scenario, the complex scalar field \( \phi \) is the heavy field that is responsible for baryogenesis. One might think that tuning parameters, it is possible to make the real field a candidate of the dark matter. Such scenario could be fascinating, but in this paper, we are simply assuming that the real field is heavier than the complex scalar field and it decays fast. Then, since the real field does not generate asymmetry, its decay dilutes the baryon number of the Universe. However, since our scenario can expect asymmetry \( \epsilon_\phi \equiv \frac{\pi_\phi}{n_\phi + n_\bar{\phi}} \) up to \( O(0.1) \) for the maximum CP violation, and the initial density of the real field can be the same order as the heavy field, it is easy to generate the required baryon number of the Universe using the mechanism.

Appendix B: Comments for related studies

As we have shortly mentioned in the introduction, there has been a few papers in which direct production scenarios of matter-antimatter asymmetry has been discussed using preheating.

\(^1\) A similar condition has been used in Ref.\(^2\).
First, a scenario for multi-field bosonic preheating has been discussed by Funakubo et al. in Ref. [17]. The model discussed in this paper partially overlaps with our study. However, unlike our analysis, they did not introduce eigenstates for the calculation and concluded that at least two complex fields and time dependence of “not only mass terms but also phases” in the mass matrix parameters are needed in order to generate charge asymmetry. In the light of cosmology, these conditions are very stringent.

Our discussion is different from Ref. [17]. We noticed that the complexity of preheating in a multi-field system makes the calculation quite unclear and it disturbs finding the required condition for the asymmetry. To make the condition for the asymmetry clear in multi-field preheating, we have used the eigenstates of the equation to find that the dynamical mixing between eigenstates causes interference when CP is violated, and it can lead to the bias between matter and antimatter. To realize this scenario, one has to introduce at least one complex and one real scalar fields, which couple via complex interaction. Also, the complex scalar should have CP-violating interaction $\sim \lambda \phi^0 + h.c.$ With these simple setups, we have shown that asymmetry can be generated by a time-dependent mass of the complex scalar field. Our model does not need any time-dependent parameter other than the scalar mass, which can make the scenario of asymmetry preheating very simple.

It is possible to apply our strategy to a system with a Majorana neutrino and an effective chemical potential. This model has been studied in Ref. [18]. Using the original notations, the starting point is Eq.(42) of Ref. [18], which is given by

$$\mathcal{L}_{e f f} = i \hat{\nu}_L\tilde{\sigma}^\mu \partial_\mu \hat{\nu}_L - \frac{i}{2} \tilde{M}_L \left[ \hat{\nu}_L^\dagger \tilde{\sigma}_2 \hat{\nu}_L - \hat{\nu}_L^\dagger \tilde{\sigma}_2 \hat{\nu}_L \right] + \tilde{\mu}_{e f f} \hat{\nu}_L^\dagger \hat{\nu}_L. \quad (B1)$$

where $\tilde{\mu}_{e f f}$ comes from the effective chemical potential and the field $\hat{\nu}_L$ is the SM neutrino. If we apply our method used for the bosonic preheating, the mass term and the effective potential in the above effective Lagrangian can be expressed using a matrix given by

$$\mathcal{L}_{m a s s} = -\frac{1}{2} \left( \begin{array}{c} \hat{\nu}_L \\ i\tilde{\sigma}_2 \hat{\nu}_L^* \end{array} \right)^\dagger \left( \begin{array}{c} -\tilde{\mu}_{e f f} \tilde{M}_L \\ \hat{\nu}_L \end{array} \right) \left( \begin{array}{c} \hat{\nu}_L \\ i\tilde{\sigma}_2 \hat{\nu}_L^* \end{array} \right). \quad (B2)$$

Omitting the normalization factors, eigenstates are given by

$$\left( 1, \frac{\tilde{\mu}_{e f f}}{\tilde{M}_L} \pm \sqrt{1 + \left( \frac{\tilde{\mu}_{e f f}}{\tilde{M}_L} \right)^2} \right)^T. \quad (B3)$$

The eigenstates of the equation are biased when $\tilde{\mu}_{e f f} \neq 0$, but in total we find the identity

$$\frac{1}{1 + |p_+|} + \frac{1}{1 + |p_-|} = 1, \quad (B4)$$

where we defined $p_\pm = \tilde{\mu}_{e f f} / \tilde{M}_L \pm \sqrt{1 + \left( \tilde{\mu}_{e f f} / \tilde{M}_L \right)^2}$. In this case, the bias in the eigenstates may cancel each other. When the number densities of these states are equal, and they decay simultaneously, the cancellation becomes exact.

[1] J. H. Traschen and R. H. Brandenberger, “Particle Production During Out-of-equilibrium Phase Transitions,” Phys. Rev. D 42, 2491 (1990); L. Kofman, A. D. Linde and A. A. Starobinsky, “Reheating after inflation,” Phys. Rev. Lett. 73, 3195 (1994) doi:10.1103/PhysRevLett.73.3195 [hep-th/9405187]; L. Kofman, A. D. Linde and A. A. Starobinsky, “Towards the theory of reheating after inflation,” Phys. Rev. D 56, 3258 (1997) [hep-ph/9704452].

[2] E. W. Kolb, A. D. Linde and A. Riotto, “GUT baryogenesis after preheating,” Phys. Rev. Lett. 77 (1996) 4290, [hep-ph/9606260].

[3] G. W. Anderson, A. D. Linde and A. Riotto, “Preheating, supersymmetry breaking and baryogenesis,” Phys. Rev. Lett. 77, 3716 (1996), [hep-ph/9606416]; J. Garcia-Bellido, D. Y. Grigoriev, A. Kusenko and M. E. Shaposhnikov, “Nonequilibrium electroweak baryogenesis from preheating after inflation,” Phys. Rev. D 60, 123504 (1999) [hep-ph/9902449].

[4] R. Allahverdi, R. Brandenberger, F. Y. Cyr-Racine and A. Mazumdar, “Reheating in Inflationary Cosmology: Theory and Applications,” Ann. Rev. Nucl. Part. Sci. 60, 27 (2010) [arXiv:1001.2600 [hep-th]].

[5] A. G. Cohen and D. B. Kaplan, “Thermodynamic Generation of the Baryon Asymmetry,” Phys. Lett. B 199, 251 (1987). doi:10.1016/0370-2693(87)91369-4.

[6] A. G. Cohen and D. B. Kaplan, “Spontaneous Baryogenesis,” Nucl. Phys. B 308, 913 (1988).

[7] Y. B. Zeldovich and A. A. Starobinsky, Particle production and vacuum polarization in an anisotropic gravitational field, Sov. Phys. JETP 34 (1972) 1159.

[8] I. Affleck and M. Dine, “A New Mechanism for Baryogenesis,” Nucl. Phys. B 249, 361 (1985).

[9] K. D. Lozanov and M. A. Amin, “End of inflation, oscillons, and matter-antimatter asymmetry,” Phys. Rev. D 90 (2014) no.8, 083528 [arXiv:1408.1811 [hep-ph]].

[10] L. Covi, E. Roulet and F. Vissani, “CP violating decays in leptogenesis scenarios,” Phys. Lett. B 384 (1996) 169 [hep-ph/9605319].

[11] M. Flanz, E. A. Paschos, U. Sarkar and J. Weiss, “Baryo-
genesis through mixing of heavy Majorana neutrinos,” Phys. Lett. B 389 (1996) 693 [hep-ph/9607310].

[12] S. Enomoto, S. Iida, N. Maekawa and T. Matsuda, “Beauty is more attractive: particle production and moduli trapping with higher dimensional interaction,” JHEP 1401 (2014) 141 doi:10.1007/JHEP01(2014)141 [arXiv:1310.4751 [hep-ph]].

[13] S. Enomoto, N. Maekawa and T. Matsuda, “Preheating with higher dimensional interaction,” Phys. Rev. D 91 (2015) no.10, 103504 doi:10.1103/PhysRevD.91.103504 [arXiv:1405.3012 [hep-ph]].

[14] S. Enomoto, O. Fuksinska and Z. Lalak, “Influence of interactions on particle production induced by time-varying mass terms,” JHEP 1503 (2015) 113 doi:10.1007/JHEP03(2015)113 [arXiv:1412.7442 [hep-ph]].

[15] A. D. Sakharov, ”Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe”, Journal of Experimental and Theoretical Physics Letters. 5 (1967) 24-27

[16] A. Dolgov and K. Freese, “Calculation of particle production by Nambu Goldstone bosons with application to inflation reheating and baryogenesis,” Phys. Rev. D 51 (1995) 2693 [hep-ph/9410346].

[17] K. Funakubo, A. Kakuto, S. Otsuki and F. Toyoda, “Charge generation in the oscillating background,” Prog. Theor. Phys. 105, 773 (2001) [hep-ph/0010266].

[18] L. Pearce, L. Yang, A. Kusenko and M. Peloso, “Leptogenesis via neutrino production during Higgs condensate relaxation,” Phys. Rev. D 92, no. 2, 023509 (2015) doi:10.1103/PhysRevD.92.023509 [arXiv:1505.02461 [hep-ph]].