Measurement of the spatio-temporal distribution of harmonic and transient eddy currents in a liquid metal

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Abstract
Harmonic and transient eddy currents in the eutectic liquid metal alloy GaInSn positioned above an excitation coil are determined by measuring the corresponding voltage drop in an electric potential probe. The resulting spatio-temporal eddy current field is compared with the corresponding analytical expressions for a conducting half-space. Further, a deformation of the eddy current distribution due to a non-conducting torus immersed into the liquid metal is measured and compared with numerical results. The method can be generalized to arbitrary geometries, and might help us to validate numerical models for non-destructive testing and magnetic inductance tomography.

Keywords: eddy currents, magnetic induction tomography, non-destructive testing, liquid metals

1. Introduction
Eddy currents play a crucial role in magnetic induction tomography (MIT) [1], non-destructive testing [2] and different versions of contactless flow measurements for liquid metals such as, for example, in contactless inductive flow tomography (CIFT) [3, 4], induction flow-metering based on phase shift measurements [5] and Lorentz force velocimetry [6]. Usually, the eddy currents in the bulk material are indirectly inferred from non-invasive measurements of external induced magnetic fields. The final inverse problem of inferring the material inhomogeneities or flow velocities from the induced magnetic fields is still a formidable task.

In many cases, it would be desirable to have independent eddy current measurements in order to validate numerical solvers for the forward and inverse problems. For evident reasons, a detailed eddy current measurement in solid materials is hardly possible. Up to the present, only some eddy current measurements at the surface of thin solid aluminum plates have been reported [7] and served for the validation of a numerical benchmark problem.

In contrast to solid metals, liquid metals are perfectly suited for a full scan of the eddy current distribution in the bulk of the conducting material. Actually, various liquid metals (mercury [8], Wood’s metal [9]) have already been used in eddy current testing since they allow easy defect modeling. However, to the best of our knowledge, in this paper we present the first measurement of the full spatio-temporal distribution of eddy currents in a liquid metal. For this purpose, we utilize the eutectic metallic alloy Ga_{67}In_{20.5}Sn_{12.5}. This alloy, which is liquid at room temperatures, has significant handling advantages over mercury or Wood’s metal as it can be used without risk to health providing the safety guidelines for the alloy are followed.

The measurements are carried out both for the case of harmonic excitation and for the case of a pulsed (transient) excitation. Harmonic excitation has long been used in non-destructive testing. Transient excitation, which was already studied in the 1960s and 1970s [10, 11], has experienced a revival in the last decade. Nowadays it is used for non-destructive testing in many areas, including the detection of corrosion and cracks in aging aircraft fleets [12].

In order to start with an accurate reference model, we determine harmonic and transient eddy currents in a cylindrical vessel which is large compared to the radius and the distance of the exciting cylindrical coil. This problem can be reasonably approximated by the eddy current problem in a conducting...
half-space for which analytical solutions are known, both in the harmonic and in the transient case [13].

Another measurement is carried out for the more complicated situation that a non-conducting torus with quadratic cross-section is immersed into the liquid metal. These measurements are then compared with numerical results from the commercial FEM software OPERA (Vector Fields Ltd). The paper closes with a summary and some conclusions.

2. Theory

2.1. Analytical models

Consider a conducting half-space with conductivity \( \sigma \) and a coil of radius \( R \) located at a distance \( a \) from the boundary of the half-space (figure 1). Independent of the time dependence of the exciting current, the eddy currents in the half-space will only have an axi-symmetric azimuthal component \( j_\phi(r, z, t) \).

First, we consider a harmonic current \( I(t) = I_0 \sin(\omega t) \) in the coil with amplitude \( I_0 \) and angular frequency \( \omega = 2\pi f \). In [13], the distribution of the eddy current density \( j_\phi(z, r, t) \) in the conducting half-space was shown to have the form

\[
j_\phi(z, r, t) = -i\mu_0\omega \sigma RI(t)
\times \int_0^\infty e^{-kz} J_1(kR) J_1(kr) \frac{e^{-q(k - a)}}{k + q} \, dk,
\]

where the abbreviation

\[
q = \sqrt{k^2 + i\mu_0\sigma \omega}
\]

is used. \( J_1 \) is the Bessel function of order 1 and \( \mu_0 \) is the magnetic permeability of free space (throughout the paper we assume the relative magnetic permeability \( \mu_{rel} \) to be equal to 1).

For each point \( r, z \) integral (1) can be easily evaluated. Typically, we have used discretizations with 50–200 points in \( k \) space, and carefully checked the convergence of the integration procedure.

Now let us assume that the current in the excitation coil is suddenly switched on so that it corresponds to a step function \( I(t) = I_0\Theta(t) \). Quite similar to the harmonic case, the induced transient eddy current density \( j_\phi \) can be represented as an integral over the wave number \( k \):

\[
j_\phi(z, r, t) = -\mu_0\sigma RI_0
\times \int_0^\infty e^{-kz} J_1(kR) J_1(kr) L(k, z, t)k \, dk,
\]

with \( J_1 \) being again the Bessel function of order 1 and \( L(k, z, t) \) being defined as

\[
L(k, z, t) = \left[ \frac{1}{\sqrt{\pi t}} e^{-\frac{z^2}{4t}} - \frac{k}{\sqrt{\mu_0 \sigma}} e^{i\frac{k_0 z^2}{4t}} \text{erfc} \left( \frac{k\sqrt{t} + \sqrt{\mu_0 \sigma z}}{\sqrt{4t}} \right) \right] e^{-\frac{k_0 z^2}{4t}}.
\]

The results of equation (1) for the harmonic case and equations (3) and (4) for the transient case will be visualized in the following section where we will compare them with the measured current distribution.

2.2. Finite-element models

In addition to the analytical expressions (1) and (3), we also determine the eddy current by the commercial FEM package OPERA. OPERA is finite-element analysis software for time-varying electromagnetic fields. It includes a module for solving eddy current problems in which the exciting currents can vary sinusoidally or in another predetermined way in time.

2.3. The measurement

The measurement principle for the eddy currents relies on Ohm’s law in non-moving conductors which connects the electric current density \( j \) with the electric field \( E \) via \( j = \sigma E \). The measured voltage \( U_{12} \) between two points \( P_1 \) and \( P_2 \) can be expressed by the line integral over the electric field and hence by the line integral over the electric current density:

\[
U_{12} = \int_{P_1}^{P_2} E \cdot ds = \frac{1}{2} \int_{P_1}^{P_2} j \cdot ds.
\]

In a straight channel with a nearly homogenous and parallel current field, the current density can be easily determined by measuring the voltage between the two points \( P_1 \) and \( P_2 \) (see figure 2(a)). Actually, we have validated the measurement principle in such a set-up. In our particular eddy current problem, the only current...
component is the azimuthal one which can be determined by the voltage between two electrodes whose difference vector points roughly in the azimuthal direction. At every instant $t$ and every position $r, z$, this azimuthal current density $j_0(z, r, t)$ can be approximated by

$$j_0(z, r, t) \approx \frac{U_{12}(z, r, t)\sigma}{d_{12}}, \tag{5}$$

where $d_{12}$ stands for the distance between the electrodes. However, when the radius $r$ becomes comparable to the distance $d_{12}$, we get an increasing (with $1/r$) measuring error since then the direction of $E$ deviates more and more from the distance vector between $P_1$ and $P_2$ (see figure 2(b)). In our experiment, we will have typical current densities of 1 A cm$^{-2}$, the value of $d_{12} = 5.8$ mm chosen in the experiment results as a compromise between maximizing the measurable voltage (around 20 $\mu$V for a current density 1 A cm$^{-2}$) and minimizing the inaccuracies which appear in particular for small radii $r$.

### 3. Experimental set-up

The experimental set-up is shown in figure 3. The central part is a cylindrical glass vessel (1) filled with the eutectic alloy Ga$_{67}$In$_{20}$Sn$_{12.5}$, which has the advantage of being liquid down to temperatures of about 10$^\circ$ C. The physical properties of GaInSn at 25 $^\circ$C are: density $\rho = 6.36 \times 10^3$ kg m$^{-3}$, kinematic viscosity $\nu = 3.40 \times 10^{-7}$ m$^2$ s$^{-1}$ and electrical conductivity $\sigma = 3.27 \times 10^6$ (Ω m)$^{-1}$. The non-conducting ring (3) is immersed into the liquid only in the second part of the experiment and is held by one stainless steel rod ($d = 3$ mm, $\sigma = 1.3 \times 10^6$ S m$^{-1}$) which we assume to be non-essential for the current distribution.

The external magnetic field is generated by a circular coil (2) with radius $R = 17.3$ mm, centered below the vessel (1). This coil consists of 63 turns of 0.6 mm Cu wire and is wound in such a way that the coil has a nearly circular cross section. The coil is fed by a precise U-I-converter (HERO PA2024C) controlled by an arbitrary-waveform generator (AD-win-Pro). In the case of harmonic excitation, a current of 1 A amplitude and a frequency of 480 Hz was applied to the coil (see figure 4(a)). In the case of transient excitation, the limited bandwidth of the amplifier requires that the pulse-edges have to be approximated by a continuous function. For this purpose, the first quarter-period of $I(t) = I_0 \sin(2\pi f t)$ with $I_0 = 5$ A and $f = 2$ kHz was chosen. The resulting rise time (see figure 5) of $t_r = 125$ $\mu$s is small compared to the typical diffusion time of the magnetic field $t_{diff} = \mu_0 \sigma R^2 = 1.23$ ms. The pulse duration was set to $t_{on} = 12$ ms, to avoid interactions between the eddy currents resulting from the rising and the falling edges. The pause between the pulses was set to $t_{off} = 100$ ms. This rather long time is necessary to avoid significant heating of the coil.

The spatio-temporal distribution of the induced eddy current is determined by pointwise measuring the voltage drop $U_{12}(t)$ between the blank ends of two coated wires (0.1 mm Cu-wire) which are twisted to avoid inductive pick-up (see figure 3). The connecting line between the points $P_1, P_2$ points in the azimuthal direction. The wires are guided into the medium through a glass pipette (4) which is as thin as possible (3 mm diameter) in order to minimize the deformation of

![Figure 3. Experimental set-up (not to scale): (a) side view and (b) bird’s eye view. (1) Glass vessel, (2) excitation coil, (3) PVC torus (only in the second part of the experiment), (4) scanning potential probe and (5) scan region. All given dimensions are in mm.](image)
the current field. In the top view of the probe (figure 3(b))
the electrode distance \( d = 5.8 \text{ mm} \) is indicated. The wires
are connected to a tunable active filter (Krohn-Hite Corp,
model 3382) which provides differential input performance
(1 M\(2\), 25 pF). After differential amplification (\( g_d = +60 \text{ dB} \))
and low-pass filtering (\( f_c = 200 \text{ kHz} \)), the signal is
routed directly into a digital storage oscilloscope (Tektronix
TDS3034B), which records the response with a sample-rate
of 2.5 Megasamples/s over 10 kilosamples. To improve
the signal-to-noise ratio, the oscilloscope averages over eight
trigger events before the waveform is captured from the central
computer. A slowly drifting offset voltage is compensated
by averaging over 500 pre-trigger samples and subtracting
this value from the data. Due to the signal decrease with
respect to the coil–probe distance, the vertical sensitivity of
the oscilloscope is adapted to exploit its dynamic range as
well as possible. The entire signal path is realized as a dc-
measurement to get correct transients. To specify the exact
position \((r_p, \phi_p, z_p)\) of the potential probe, the pipette is
mounted onto a three-axis traversing robot. The \( r \)-axis is
located 1 mm above the boundary of the half-
space. In order not to deform or crush the probe, a safety distance of 1 mm to
the vessel and to the ring has been chosen. The extension of the
scan-region (5), as well as the spatial resolution are the same
for all arrangements to get comparable results. In the present
case, a step size of \( \delta r = \delta z = 1.25 \text{ mm} \) is applied which
leads to a raster-size of 65 \times 33 pixels. Thus, a measurement
time of about 2 h is necessary to traverse through the whole
scan region. The eddy current signal is logged pointwise to a
PC which also controls the traversing robot and the waveform
generator.

The final eddy current distribution is determined in the
post-processing. In the case of harmonic excitation, the measured voltages \( U_{12}(r_p, z_p, t) \) are fitted by a sine function
with the amplitude \( a_t(r_p, z_p) \) and the phase \( \phi_t(r_p, z_p) \) (see
figure 4). For transient excitation, a moving average with a
length of 100 samples is applied before time-slicing through
the acquired data results in \( u_t(r_p, z_p) \) for each instant (see
figure 5). Both fields \( a_t, u_t \) are converted from voltage
to current by equation (5). To illustrate and compare the
measured versus calculated distributions the computation of isolines was performed. Furthermore, spatial averaging over
3 \times 3 pixels leads to smoother isolines. The required software
was developed under the usage of MATLAB (Mathworks Inc).

4. Results

In this section, we compare the measured eddy currents with
the analytical results (in the case of homogeneous fluid) and
with numerical results (in the case of an immersed torus). It
should be noted that the specified lengths in the \( r, z \) directions
refer to \( r_p, z_p \), respectively (cf figure 3). Hence, the shown
\( r \)-axis is located 1 mm above the boundary of the half-
space.

4.1. Homogeneous fluid

We start with the case of a homogeneous fluid for which
analytical solutions for the harmonic and transient excitations
were given in section 2.

4.1.1. Harmonic excitation. In the case of harmonic
oscillation, we show both the amplitude and the phase shift
of the induced currents for a frequency of 480 Hz (figure 6).

In addition to the measured values, we show also the
results of equation (1). Actually, we have also computed
the eddy current by the commercial FEM code OPERA.
However, in the adopted spatial resolution the results are
indistinguishable from the analytical ones. In general, we
see a good correspondence of experimental and theoretical data which expectedly gets worse for smaller radii where the finite distance (5.8 mm) of the two electrodes of the potential probe makes a precise measurement of the azimuthal currents impossible.

4.1.2. Transient excitation. In figure 7, we present the measured and theoretical eddy current distribution for the case of pulsed excitation for the time instants 200, 600, 1000 and 1400 µs. These instants were already indicated as circles in figure 5(b). Again, we observe a good correspondence, despite the approximations used with respect to geometry and to the pulse shape.

It might also be instructive to compare the time evolution of the total currents which is the integral in the $r$ and $z$ directions of the eddy current density (figure 8). In a wide range of the time evolution, the misfit between the two data is of the order of a few per cent. Only at the very beginning and at the end (when the measured signal is already very small) do we get a discrepancy of around 10%. Note that in addition to the switch-on version of the pulsed excitation shown in figure 5, we have also tested the switch-off version. Expectedly, no significant differences (apart from the sign) were observed between the induced eddy currents in both versions.

4.2. Fluid with an immersed torus

In order to study a slightly more complicated problem which is not solvable anymore by an analytical expression, we have immersed a PVC torus into the liquid metal. This problem has been solved again by means of the commercial FEM solver OPERA which, in the case of a homogeneous fluid, had provided results more or less identical to the analytical
Figure 10. Case of an inserted PVC ring with quadratic cross-section: contour lines of the amplitude of the azimuthal eddy current in the case of pulsed excitation with an excitation current of 5 A. The full lines are the measured data, the dotted lines are the numerical ones. (a)–(d) correspond to time instants 200, 600, 1000 and 1400 µs, respectively.

Figure 11. Time evolution of the total current for the pulsed current excitation in the case with an immersed PVC torus.

Figure 12. Movement of the maximum of the induced eddy currents for the case of a homogeneous liquid and for the case of an immersed PVC torus.

5. Conclusions

We have measured the two-dimensional, axi-symmetric distribution of eddy currents in the liquid metallic alloy GaInSn arising from harmonic or pulsed excitations in a nearby coil. In both cases, the measured values show satisfactory agreement with the analytical solution of the corresponding problem for a conducting half-space. Expectedly, a non-conducting torus immersed into the fluid disturbs the eddy current distribution, which was also confirmed by means of a commercial numerical solver (OPERA). This is illustrated again in figure 12 which shows the movement of the maximum of the induced current for the case of a homogeneous liquid and for the case of an inserted PVC ring.

Although we have restricted our interest to axi-symmetric problems in which the only relevant current component is the azimuthal one, the method can easily be generalized to fully three-dimensional scans of all three eddy current components. This way, it might help to validate numerical models for non-destructive testing and magnetic inductance tomography.
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