A reliable order-statistics-based approximate nearest neighbor search algorithm

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Abstract—We propose a new algorithm for fast approximate nearest neighbor search based on the properties of ordered vectors. Data vectors are classified based on the index and sign of their largest components, thereby partitioning the space in a number of cones centered in the origin. The query is itself classified, and the search starts from the selected cone and proceeds to neighboring ones. Overall, the proposed algorithm corresponds to locality sensitive hashing in the space of directions, with hashing based on the order of components. Thanks to the statistical features emerging through ordering, it deals very well with the challenging case of unstructured data, and is a valuable building block for more complex techniques dealing with structured data. Experiments on both simulated and real-world data prove the proposed algorithm to provide a state-of-the-art performance.

Index Terms—Approximate nearest neighbor search, locality sensitive hashing, vector quantization, order statistics.

I. INTRODUCTION

A large number of applications in computer vision and image processing need to retrieve, in a collection of vectors, the nearest neighbor (NN) to a given query. A well-known example is image retrieval based on compact descriptors, but there are countless more, from patch-based image denoising, to copy-move forgery detection, data compression, and so on. Typically, large sets of points are involved, calling for fast search techniques to guarantee an acceptable processing time. However, for high-dimensional data, no exact NN search algorithm can provide a significant speed-up w.r.t. linear search, so one is forced to settle for some approximate search algorithms, trading off accuracy for efficiency.

In recent years, there has been intense research on techniques that improve this trade-off. These can be classified in two large families according to their focus on memory or time efficiency. A first family [1], [2], [3], [4] addresses the case of very large datasets, that do not fit in memory. This may occur, for example, in image retrieval and other computer vision applications. In this condition, performing an accurate search based on Euclidean vector distances would require data transfers from disk, with an exceedingly large delay. By associating compact codes with original vectors, and approximating exact distances based on such codes, one can drastically reduce memory usage and avoid buffering, with a huge impact on search speed. In this case, therefore, memory efficiency is the main issue and the prevailing measure of performance, while actual search time is of minor interest.

On the contrary, when data and associated structures can fit in memory, processing time becomes the main issue, and performance is measured in terms of accuracy vs speed-up, with memory usage assuming minor importance. This is the case of a large number of image processing application, especially in the patch-based paradigm. Notable examples are nonlocal denoising [5], exemplar-based inpainting [6], copy-move forgery detection [7] and optical flow estimation [8]. The present work fits in the latter family. That is, our aim is to provide a large speed-up with respect to linear search while still guaranteeing a very high accuracy based on Euclidean distance computation on the original vectors.

Most techniques of this family follow a similar path, with many variations: first the search space is partitioned into suitable cells [1] and all vectors are classified based on the cell they belong to. Then, at run time, the query is itself classified, and the NN is searched only among vectors falling in the same cell as the query, or possibly in some neighboring ones. Stated in these terms, approximate NN (ANN) search bears striking similarities with the vector quantization (VQ) problem [9], [10]. In both fields, defining a good partition of the space and a fast classification rule are the key ingredients of success. Hence similar concepts and tools are used [11].

In the quantization literature there has been intense research on optimal (minimum distortion) space partitioning. The k-means algorithm provides a locally optimal solution [12], with cells well packed in the space and adapted to data point density. However, to classify a query, M vector distances must be computed, with M the partition size, making this approach unsuited to fast NN search. Therefore, a number of constrained solutions have been adopted in practice (see Fig. 1). In hierarchical k-means, a tree-structured search is carried out, using partitions of size M′ ≪ M at each node, with a sharp reduction of complexity. The limiting case M′ = 2 corresponds to binary kd-trees search [13]. With binary decisions at the nodes, the dataset is recursively partitioned by k-dimensional hyperplanes, allowing a very fast search. However, complex tree-visiting schemes are needed to obtain an acceptable reliability. In [14], [15] a good performance is obtained using multiple randomized kd-trees.

Another popular approach is product quantization (PQ) [11], with its many state-of-the-art variants, inverted multi index [2]. Cartesian k-means [16], optimized PQ [4], locally optimized PQ [17], composite quantization [18]. In PQ the vector space is decomposed into the Cartesian product of subspaces of lower dimensionality, and quantization is carried out independently in each subspace. Data points are then represented by compact

1Often the cells overlap, so it is not a partition in strict sense; this is an example of the many variations which we will overlook, from now on, for the sake of readability.
codes obtained by stacking the quantization indices of all subspaces. Thanks to these compact codes and to approximate code-based distances, large datasets can be dealt with efficiently. The limiting case of scalar subspaces corresponds to the independent quantization of vector components. This solution is simple but largely suboptimal, and much better regular partitions of the space can be found [19].

Interestingly, one of the most popular ANN search techniques, locality sensitive hashing (LSH) [20], [21], amounts to nothing more than scalar product quantization [11], with data projected on randomly oriented, non-orthogonal, axes. Nonetheless, LSH can achieve a good performance through a number of clever expedients, like the use of multiple hash tables [20], and multi-probe search [22]. Moreover, data-dependent variants of LSH improve largely over basic LSH, by learning projection axes from the data [23], [24], [25], and using non-uniform quantizers [11].

In general, taking advantage of the intrinsic structure of data can greatly help speeding up the search. However, often data have little or no structure, or become unstructured after some preliminary processing. In hierarchical k-means, for example, as well as in IVFADC [1], after the top-level quantization, the points become almost uniformly distributed in their cells. Looking for the NN when data have no structure, it can make sense resorting to product scalar quantization (d). However, lattice VQ (e) provides a better tessellation of the space. Basic LSH corresponds to non-orthogonal product quantization (f), hence it is theoretically worse than both product and Lattice VQ.

In this work we propose a Reliable Order-Statistics based Approximate NN search Algorithm (ROSANNA) suited for unstructured data. Like in the methods described before, we define a suitable partition of the space, and carry out classification and priority search. The main innovation consists in classifying the data based just on the index and sign of their largest components. This simple pre-processing allows us to partition the search space effectively, with negligible complexity and limited impact on memory usage. For each query, only a short list of candidates is then selected for linear search. ROSANNA can be also regarded as a variant of LSH, where the hashing is based on the vector directions. This makes full sense in high dimensions, since data tend to distribute on a spherical shell. ROSANNA produces a uniform partition of the space of directions, and all vectors are automatically classified based on the order of their sorted components. By using multiple hash tables, obtained through random rotations of the basis, and a suitable visiting scheme of the cells, a very high accuracy is obtained, with significant speed-up w.r.t. reference techniques. Experiments on both unstructured and real-world structured data prove ROSANNA to provide a state-of-the-art search performance. We also used ROSANNA to initialize the NN field in copy-move forgery detection, a real-world computation-intensive image processing task, obtaining a significant speed-up.

In the vast literature on ANN search, several papers related with ROSANNA have obviously appeared. For example, sorting has been already exploited by Chavez et al. [26]. This technique, however, is pivot-based rather than partition-based. A number of anchor vectors, or “pivots” are chosen in advance. Then, for each database vector the distances to all pivots are computed and sorted, keeping track of the order in a permutation vector. At search time, a permutation vector is computed also for the query and compared with those of the database points to select a short list of candidate NNs, assuming that close vectors have similar permutation vectors. In summary, sorting is used for very different goals than in the proposed method.

If ROSANNA is regarded as LSH in the space of directions, then the same goal is pursued by the Spherical LSH (SLSH) proposed in [27], (not to be confused with the unrelated Spherical Hashing [28]), where a regular Voronoi partition of the unit hypersphere is built based on the vertices of an inscribed polytope. For example, working in a 3d space, the unit sphere can be partitioned in 8 cells corresponding to the 8 vertices of the inscribed cube. Therefore, in SLSH, a regular partition is obtained with a low-complexity hashing rule [27]. Unfortunately, in high-dimensional spaces ($K \geq 5$) there are only three kinds of regular polytopes, simplex, with $K+1$ vertices, orthoplex, with $2K$ vertices, and hypercube with $2^K$ vertices. SLSH uses eventually only the orthoplex polytope, which corresponds to a strongly constrained version of ROSANNA. Likewise, Iterative quantization (ITQ), proposed in [3] for ANN search through compact codes, makes reference to hyper-octants, and hence corresponds to another (opposite) constrained version of ROSANNA. By removing such constraints, ROSANNA is able to provide much better results.

In Concomitant LSH [29], instead, a Voronoi partition of the space of directions is built based on a set of $M$ points taken at random on the unit sphere. Although originally proposed for cosine distance, it presents some similarities with ROSANNA, the use of directions and sorting, and can be easily adapted to deal with the Euclidean distance. However, to classify the query, $M$ distances must be computed at search-time, before inspecting the candidates, which is a severe overhead for
Fig. 2. Two different organization of the same dataset. Left: only the largest component is used for classification, \( G=1 \). Profile \( p\{1\} \) includes all vectors where the first component, \( x_1 \), is largest. The profile is divided in two cones, \( c=0 \), including vectors with \( x_1 < 0 \), and \( c=1 \), including the others. Right: the two largest components are used for classification, \( G=2 \). Profile \( p\{1, 2\} \) includes all vectors where \( x_1 \) and \( x_2 \) are the largest components. The profile is divided in four cones, according to the sign of these components. The largest \( G \) components are shown in bold and color.

| p  | c   | \( x_1 \) | \( x_2 \) | \( x_3 \) |
|----|-----|--------|--------|--------|
| 1  | -23 | 12     | 5      |        |
| 2  | -19 | -12    |        |        |
| 3  | 29  | 24     | -13    |        |
| 4  | 44  | 17     | -4     |        |
| 5  | 49  | -6     | 5      |        |
| 6  | 57  | 8      | -2     |        |

| p  | c   | \( x_1 \) | \( x_2 \) | \( x_3 \) |
|----|-----|--------|--------|--------|
| 1-2| -21 | -19    | -12    |        |
| 2  | -13 | 0      |        |        |
| 3  | 24  | -13    |        |        |
| 4  | 11  | 14     | -3     |        |
| 5  | 29  | 24     | -13    |        |
| 6  | 44  | 17     | -4     |        |
| 7  | 57  | 8      | -2     |        |

Table showing data points and their classification.

Fig. 3. Atomic orbitals \( p_x, p_y \) and \( p_z \) (top row) resemble (loosely) the three profiles arising in a 3d space with \( G=1 \), including two cones each. Likewise orbitals \( d_{xy}, d_{xz} \) and \( d_{yz} \) (bottom row) resemble the three four-cone profiles of the case \( G=2 \). The same color coding as figure 2 is used.

II. ANN SEARCH BASED ON ORDER STATISTICS

Given a set of \( N \) vectors \( \mathbf{x}_n \in \mathbb{R}^K, n = 1, \ldots, N \), drawn from a common source, we look for the nearest neighbor to query \( \mathbf{y} \) drawn from the same source, according to the Euclidean distance,

\[
||\mathbf{x}_n - \mathbf{y}||^2 \leq ||\mathbf{x}_m - \mathbf{y}||^2, \quad n = 1, \ldots, N
\]

We organize in advance the dataset in disjoint sets, called profiles, based on the index of the \( G \) vector components that are largest in absolute value\(^2\). For example, taking \( G=1 \), we define \( K \) disjoint profiles, with profile \( j \) including only the vectors for which the \( j \)-th component is the largest

\[
p_j = \{ \mathbf{x}_n : |x_{nj}| \geq |x_{ni}|, \quad i = 1, \ldots, K \}
\]

Within each profile, we further divide the vectors in subsets, called cones, according to the sign of the largest components, for example only two cones if \( G=1 \). Fig. 2 shows two alternative organizations of a toy dataset, composed by 16 3d vectors, in the cases \( G=1 \) and \( G=2 \). Taking advantage of some well-known pictures of atomic orbitals, Fig. 3 provides an approximate representation of profiles and cones in the 3d space, again for \( G=1 \) and \( G=2 \).

At run-time, the query \( \mathbf{y} \) is itself classified, based on index and sign of its \( G \) largest components, searching for the nearest neighbor only in the corresponding profile and cone. In our classification, we tell apart the largest \( G \) components from the \( K-G \) smallest ones, but do not sort components within these groups. Therefore, there are \( \binom{K}{G} \) distinct profiles and, \( N_C = \left( \binom{K}{G} \right)^2 \), \( 2^G \) cones, counting \( 2^G \) cones for each profile. In conditions of perfect symmetry for the source, \( N_C \) represents also the average speed-up, measured as the ratio between dataset size \( N \) and number of vectors searched. For the example dataset of Fig. 2, \( N_C \) equals 6 when \( G=1 \), and 12 when \( G=2 \).

This brief description elicits some natural questions: is there potential for significant speed-up with this approach? Is the NN really likely to belong to the same cone as the query?\(^2\)

\(^2\)The angular distance is also appropriate for ROSANNA, but we focus on the Euclidean distance for its higher relevance in real-world problems.

\(^2\)In the following, we omit “in absolute value” when obvious.
Both questions may have a positive answer when we move to high-dimensional spaces, $K \gg 1$, thanks to the properties of order statistics.

Concerning speed-up, it is important to note that the number of cones grows very quickly with $K$ and $G$. For example, it is almost 30000 for $K=16$ and $G=4$. Note that $K=16$ is considered a relatively small dimensionality for NN search problems. By increasing the number of cones, one can reduce at will the average number of vectors per cone, hence the search time. Needless to say, in doing so, one should always guarantee a high probability that the NN is actually found in the searched cone (or cones).

To analyze this point, let us focus on the specific case of vector components modeled as independent identically distributed (i.i.d.) random variables with standard Gaussian distribution, $X_i \sim \mathcal{N}(0, 1)$. This is arguably a worst case for the NN search problem, as there is no structure in the data to take advantage of. It can be appreciated, for example, in the left scatter plot of Fig.4 showing the first two components of 512 8-dimensional vectors with i.i.d. standard Gaussian components. In the same plot we also show, in red, the first two components of a query drawn from the same distribution. Clearly, there is not much structure in the data to help speeding up the NN search. The right scatter plot of Fig.4 is obtained as the left one, except that we now include only vectors such that the first two components are also the largest ones. The difference between the two scatter plots is striking: sorting the components has created a structure in the data, which can be exploited to speed-up the search. In particular, we can safely restrict attention to only one of the four emerging clusters (our cones), comprising vectors where the sign of the two largest components is the same as in the query, gaining therefore a factor four in speed. Still, it is not impossible that the true NN belongs to a different cluster (it depends on the remaining factors).

- Fig. 4. Scatter plots of the first two components of 512 8d Gaussian vectors. Left: randomly selected vectors. Right: vectors where the first two components are the largest ones.

- Fig. 5. pdf of the components of an 8-dimensional vector of i.i.d. standard Gaussians after sorting for decreasing magnitude. The largest components (top) have a high variance and are clearly bi-modal. The first component alone holds 44% of the vector energy, the first four almost 90% of it.

$$A_{(1)}$$ is smaller than a given value $x$ only if all the $A_i$'s are. Likewise, $A_{(i)}$ is smaller than $x$ only if at least $K-i+1$ of the $A_i$'s are. Based on such observations, and due to the independence of the $A_i$'s, we can compute the marginal pdf of the sorted absolute values (see [3], Appendix A)

$$f_{A_{(i)}}(x) = \frac{K!}{(K-i)!(i-1)!} \times [F_A(x)]^{K-i} [1-F_A(x)]^{i-1} f_A(x) \quad (3)$$

where $F(.)$ denotes cumulative distribution function (CDF).

In our example, the components are standard Gaussian, with CDF expressed in terms of the $Q$-function [22] as $F_X(x) = 1 - Q(x)$. For the case $K=8$, Fig.5 shows the pdf of all components, which are very different from one another. The first components (top) have a much larger variance than the last ones, holding most of the vector energy. Therefore, they impact heavily in the computation of the Euclidean distance w.r.t. a given query, while the last ones are almost negligible. Moreover, the largest components are markedly bimodal, with modes growing farther apart as $K$ grows.

This figure provides, therefore, some more insight into the rationale of our approach. We are trying to classify vectors beforehand, in a sensible way, to reduce the search space. Doing this by taking into account all components with equal importance would be impractical (or infeasible) as $K$ grows large, and not much reliable, because most components are scarcely informative. Therefore, we focus only on the largest components, those holding most of the energy and of the information content, obtaining a much smaller (and tunable) number of classes and, eventually, some stronger guarantee that the NN will indeed belong to the same profile as the query.

As a matter of fact, we chose the name “profile” for analogy with the actions naturally taken to identify a person based on a summary description, focusing on the most prominent
features, “...he had the most unusual nose...”, “...she had a curious accent...”, to reduce the search space while preserving accuracy. Given the profile, and assuming the NN is actually found in that class, the analysis of signs restricts the search very reliably on the cone of interest. In fact, since the largest components have such a strongly bimodal distribution, it is very unlikely that the smallest components cause a cone switch.

Taking a different point of view, ROSANNA can be seen as a form of locality sensitive hashing. Component sorting becomes just a means to determine algorithmically a partition of the space based on vector direction. Given the identity of the $G$ out of $K$ largest components, a data vector is automatically associated with one of the cells of the partition, and the same happens with the query. With $G=1$, the space in divided in $2K$ cells, our cones, which become $2K(K-1)$ with $G=2$, and so on, up to $2^K$ cells for $G=K$. It is worth underlining that the space partition is, by definition, completely symmetric, and induces a partition of the unit hyper-sphere with the same property. In Appendix B we characterize the proposed approach in terms of collision probability.

III. IMPLEMENTATION

We now turn the naive basic algorithm into a reliable ANN search tool. The weak point in the basic version is the assumption that the NN is found exactly in the cone singled out by the query. This is quite likely if the dataset has a high density of points,

$$\rho = \log_2 N/K$$

as in the 2d example on the left of Fig. 6, much less so in the case of lower density, shown on the right. This latter plot shows how the NN may happen not to be in the query’s cone, especially if the query lies near the boundary of the cone and not in the very center. This might look as a rare unfortunate case. However, in high-dimensional spaces, this is actually quite likely, especially at low density. As a matter of facts, even in the right plot the point density is actually quite high, while in most real-world applications densities in the order of $\rho=1$ or even lower are to be expected, in which case the NN may easily happen to be far from the query. Fig. 6, however, suggests also possible countermeasures, amounting basically in considering alternative bases (see the dash-dot lines on the right), obtained through rotation, or including also neighboring cones in the search.

To make our OS-based search reliable we resort therefore to some typical expedients of LSH methods, enlarging the set of candidate points and exploring them with suitable priority.

A. Using multiple bases

To increase the reliability of our search algorithm we deal first with the boundary problem. Although this is not obvious in the 2d case, in higher-dimensional spaces it is quite likely that the query lies far from the center of its cone. When this happens, the probability that the NN belongs to a different cone is quite large, exceeding 1/2 when the query lies exactly on a boundary. To address this problem, we consider multiple reference systems, obtained from one another through random rotations, like in [14], and look for the NN in the union of all the cones where the query belongs. This solution corresponds to the use of multiple hash tables in LSH algorithms, and presents the same pros and cons. The probability of finding the NN in the enlarged cone is much higher than before, but there is a processing cost, since the query is projected on multiple bases and more points are checked, and a memory overhead, due to the need to store multiple classifications.

B. Checking neighboring cones

Using multiple bases increases the probability of finding the NN in the query’s enlarged cone, but there is still a non-negligible probability of missing it, especially in the low-density case. Therefore, it can make sense to extend the search to some close cones, as far as a positive time-accuracy trade-off is kept, which is the multiprobe search used in LSH methods [22]. Rather than computing the actual Euclidean distances between the query and candidate cones, we exploit the intrinsic structure of profiles defined for various values of $G$. Let $i_1, i_2, \ldots, i_K$ be the indexes of the query coordinates sorted by decreasing magnitude. Therefore, for a given value $G$, the query belongs to the profile identified by \{ $i_1, i_2, \ldots, i_G$ \}. The most likely reason why the NN may not belong to the same profile is that its $G$-th coordinate differs

\begin{algorithm}
\caption{ROSANNA (NN search)}
\begin{algorithmic}
\Require $y$ \Comment{query}
\Ensure $NN$ \Comment{index of approximate nearest neighbor}
\For{$r = 1 : R$} \Comment{for each rotation}
\State compute $y^{(r)}$ \Comment{projection of $y$ on $r$-th basis}
\State find $\{ c_1^{(r)}, \ldots, c_G^{(r)} \}$ \Comment{ordered list of cones to be visited}
\EndFor
\For{$l = 1 : C$} \Comment{for $C$ cones visited for each basis}
\For{$r = 1 : R$} \Comment{do}
\For{each $x \in c_l^{(r)}$} \Comment{for each cone $c_l$}
\If{$x$ not analyzed} \Comment{boolean side information}
\State compute $\| x - y \|^2$ \Comment{with partial distance elimination}
\State update NN \Comment{mark $x$ as analyzed}
\EndIf
\EndFor
\EndFor
\EndFor
\end{algorithmic}
\end{algorithm}
from the query’s, so we should begin by looking in all profiles sharing the first $G-1$ indexes with the query’s profile but differing in the last one. Note that all such level-$G$ profiles are “children” of the same level-$(G-1)$ profile, identified by $\{i_1, i_2, \ldots, i_{G-1}\}$. Therefore, we define a distance between two level-$G$ profiles, $p_1$ and $p_2$, as $G$ minus the level of their closest common ancestor. Based on such a distance, for each rotation an ordered list of cones to be visited is established.

A compact simplified pseudo-code of ROSANNA (only the search phase) is shown in Algorithm 1.

C. Preliminary assessment of reliability

We ran a few experiments to gain insight into the importance of using multiple bases and searching neighboring cones, and to assess the reliability of the proposed search algorithm.

We consider a running example with i.i.d. Gaussian components, with $K=16$ and $\log_2 N=16$, hence density $\rho=1$, and report in Fig 7 (top-left) the probability (MonteCarlo estimates) that the first $G$ components of query and NN have the same sign. Using a single basis, this probability is very large only for small values of $G$: for example, while it is almost certain that the first 3 components have the same sign, the probability drops to about 60% for $G=8$. Note that for $G=16$, the probability is almost 0, showing that using signs for classification without prior sorting, like in ITQ, is prone to errors. Using multiple bases guarantees a significant improvement, and already with 8 rotations the first 8 components of query and NN have almost certainly the same sign.

The top-right plot of Fig 7 reports the probability that the largest component of the query ($F=1$) is among the $G$ largest components of the NN. The probability is just above 40% for $G=1$, that is, even in this relatively high-density case, we cannot fully trust the largest component for reliable classification. If we look also in profiles where the largest component of the query is only the second, third, ..., $G$-th largest, the probability of finding the NN grows, but not as quickly as we might hope. However, by using multiple bases, the whole curve drifts rapidly towards 1, becoming almost flat for 8 rotations. To increase search efficiency, however, classification should be carried out based on more components, $F>1$. The bottom plots report, therefore, the same data with reference to the two and three largest components of the query, respectively. In both cases, the classification is very reliable for small values of $G$ if 8-16 rotations are used.

In Fig 8 we explore the dependence on dataset density, going from 2 (high) to 1/2 (quite low), with 16 rotations. The density is modified by keeping $\log_2 N \approx 16$ and varying the vector length $K$. In any case, results are shown for $G \leq 16$, for clarity. The top-left plot shows that sign agreement between query and NN is always guaranteed, at least for the first 8 components. The other plots report the probability that the largest 1, 2, 3 components, respectively, of the query are among the $G$ largest of the NN. This probability is very high for $F=1$ at any density, and for any $F$ at high density. At low density, instead, classification becomes unreliable for $F>2$ unless a large value of $G$ is used, which is impractical, however, as it would require visiting a huge number of profiles. Therefore, we cannot expect the search to be much reliable in this case. On the other hand, the same holds (to the best of our knowledge) for all other ANN search methods at such low densities. Indeed, most large-dimensionality sources considered in the literature present strong dependencies which reduce significantly the intrinsic data dimensionality [33, 3], allowing for an effective ANN search.
D. Assessment of complexity

We can now provide a theoretical assessment of computational complexity, keeping in mind, however, that some processing steps include random components that may affect results significantly. To this end, Tab. I lists the main parameters of the algorithm and the associated symbols, while Tab. II reports the complexity assessment as a function of these quantities.

The dataset preparation phase is normally of minor interest since it is carried out off-line once and for all. Considering that $N$ is much larger than $K$, the dominant term of this phase is the rotation of the dataset points along the $R$ bases. We are considering the use of structured orthonormal matrices, like DCT or Walsh-Hadamard, in place of pseudo-random matrices, which may reduce this cost. Hashing, instead, has more limited cost, related to vector sorting.

For on-line NN search the most critical item is typically the linear search of the candidate short-list where the distance from query to all candidates must be computed. However, the corresponding entry in Tab. II is only an approximation, based on the assumptions that dataset points are uniformly distributed among the cones, and that the visited cones include disjoint sets of points. The first assumption is pretty reasonable, the second much less. When multiple rotations are used, it is very likely that some points are visited more than once, in which case the distance is not computed anew. Therefore, our estimate is a bit pessimistic, but how much so depends on many parameters. We can however single out best and worst cases. In the best case, the shortlist includes only one candidate, and search complexity is dominated by the cost of query rotation $O(RK^2)$. In the worst case, the shortlist includes all dataset points, coming down to linear search, with cost $NK$.

In next Section, ROSANNA will be applied also to long vectors (e.g., 128 components) reduced to shorter unstructured vectors through PCA and random rotation. Taking into account also the estimation of covariance matrix and the PCA, and the use of partial distance elimination techniques, the above analysis holds with minor adjustments also in such a case.

IV. EXPERIMENTAL ANALYSIS

We carried out a number of experiments to assess the performance of the proposed method. Results are given in terms of accuracy-efficiency plots, as in [15], [14] with accuracy meant as the probability that the selected point is the actual NN (also called precision and recall@1), and efficiency measured in terms of speed-up w.r.t. linear search. The software is written in C++ using open libraries and some routines of the FLANN package [1] and is published online [2] to guarantee full reproducibility of results. There are only a few parameters to set: the number of components used for classification, $G$, the number of rotations, $R$, used for multiple-basis search, and the number of visited cones per basis, $C$. In the preparation phase, for each rotation, all dataset points are projected on the new basis and classified according to the index and sign of their $G$ largest components. Therefore we need $R$ hash tables, with a relative memory overhead of $R/K$. A second-level hash table

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
$N$ & number of dataset vectors \\
$K$ & vector length \\
$G$ & number of components used for query \\
$N_C$ & number of cones, $N_C = \binom{K}{G}$ \\
$R$ & number of rotations (hash tables) \\
$C$ & number of cones visited for each rotation \\
\hline
\end{tabular}
\caption{Main parameters of the algorithm.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{phase} & \textbf{action} & \textbf{complexity} \\
\hline
dataset preparation & generate rotation matrices & $O(RK^2)$ \\
& rotate dataset points & $O(NRK^2)$ \\
& hash dataset points & $O(NRK \log(K))$ \\
\hline
NN search & rotate query & $O(RK^2)$ \\
& hash query & $O(RK \log(K))$ \\
& search short-list & $O(RCK(N/N_C))$ \\
\hline
\end{tabular}
\caption{Complexity assessment.}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig9.png}
\caption{Experimental results at $\rho=1$ with Gaussian (left), Uniform (center), and Laplace (right) i.i.d. data. In all cases ROSANNA, outperforms all references, sometimes by an order of magnitude, and results are only weakly affected by the data pdf.}
\end{figure}
is used to manage the tables when the number of cones grows very large.

We compare results with a number of relevant state-of-the-art references: i) plain Euclidean LSH (E2LSH) with the implementation available online, including the automatic setting of most parameters; ii) randomized kd-trees (RKDT) and, iii) hierarchical k-means (HKM), both implemented in the FLANN package, together with iv) FLANN itself, which is always inferior to the the best of RKDT and HKM but sets automatically all parameters; v) the IVFADC algorithm, based on product quantization, and implemented by us starting from the Matlab code published by the authors; and finally vi) Inverted Multi-Index (IMI) developed by the authors except for the final linear search among candidates, which we carried out as in ROSANNA. All these techniques are implemented in C++ language. We also implemented and run optimized PQ, both parametric and non-parametric, but did not include results, generally worse than those of the PQ-based IMI and IVFADC, in order not to clutter further the figures. Curves are obtained (except for FLANN) as the upper envelope, in the accuracy-time plane, of points corresponding to different parameter settings. For ROSANNA, we consider $G \in \{1, 2, \ldots, K/2\}$, $R \in \{1, 2, 4, 8, 16\}$, $C \in \{1, 2, 4, \ldots, 128\}$, and similar wide grids are explored for the main parameters of all other techniques. To focus on the more interesting high-accuracy range, in all graphs we use a logarithmic scale for both accuracy and speed-up.

A. Unstructured data

Fig.9 (left) shows results for our running example, Gaussian i.i.d. data, $k=16$, $\rho=1$. ROSANNA guarantees uniformly the best performance, being almost twice as fast as the second best, HKM, at all levels of accuracy, and much faster than all the other references, gaining a full order of magnitude w.r.t. RKDT and E2LSH. The FLANN curve lies somewhat below HKM, since the parameters are selected in advance and may turn out not to be the best possible. However, we could not compute results for FLANN at high accuracy due to the large time needed to optimize the parameters. In Fig.2, we also shows results obtained in the same conditions as before but using Uniform (left) and Laplace (right) random variables in place of Gaussian. The general behavior is the same as before, with slight improvements observed in the Uniform case, probably due to the smaller entropy.

With Fig.10 we go back to the Gaussian case, and change the dataset density considering $\rho = 1.5$ (left), $\rho = 1.0$ (center) as before, for ease of comparison, and $\rho = 0.75$ (right). As expected, ROSANNA works especially well at high density, while its performance becomes very close to the best reference, HKM, at lower density. In this latter case, however, a significant speed-up can be obtained only at pretty low accuracy, whatever the technique used.

B. Structured data

Previous experiments confirm that ROSANNA works very well with unstructured data. It can be argued, however, that most real-world datasets are highly structured, and often have large dimensionality. Therefore, we now consider some popular structured sources, SIFT descriptors, and MNIST images of handwritten digits, often used to test the performance of ANN search algorithms. In particular we will use the 100K-vector UBC SIFT dataset, the 1M-vector IRISA SIFT dataset, and the 60K-vector MNIST database with the train/test split coming with each one. SIFT vectors have length 128, while MNIST images comprise 784 pixels. In both cases, we search the datasets based on reduced-dimensionality vectors. First, we compute the PCA and project the points on the new basis, and then classify the data based only on the first 16 components, which account for a large fraction of the energy (about 70% for the SIFT datasets and 60% for the MNIST database). In any case, the NN is searched among the selected candidates by computing distances over

http://www.mit.edu/~andoni/LSH/
http://people.rennes.inria.fr/Herve.Jegou/projects/ann.html
http://arababenko.github.io/MultiIndex/
http://people.cs.ubc.ca/~mariusm/uploads/FLANN/datasets/sift100K.h5
http://corpus-texmex.irisa.fr/
http://fann.lecun.com/express/nnist/
The intrinsic data dimensionality is much smaller than 128. On both the UBC SIFT and MNIST datasets, ROSANNA outperforms all reference techniques in the medium-accuracy and especially high-accuracy range. In the 0.9–0.99 accuracy range, it is about twice as fast as the best competitors. The situation changes with the IRISA SIFT dataset, where all techniques, including ROSANNA (and except E2LSH), provide a comparable performance. The reason lies in the much stronger structure of the IRISA data, where the first PCA component accounts for almost 33% of the total energy, as opposed to just above 10% for UBC data (the dataset size is, instead, immaterial, as the same behavior is observed with 100K vectors). This is not surprising, since ROSANNA is not designed to exploit data dependencies. In this case, the best performance is provided by IVFADC, which performs a preliminary k-means clustering, thus exploiting major data dependencies, before resorting to product quantization within a restricted number of clusters. We resorted therefore to a similar solution to adapt ROSANNA to the case of highly dependent data. Data are clustered off-line by k-means. At search time, the query is compared with the cluster centroids, and only the nearest clusters are analyzed with ROSANNA, collecting the candidates that are eventually searched for the NN. The overall search time is roughly halved w.r.t. the basic version, providing much better results than all references, including IVFADC, especially at high accuracies. Preliminary k-means clustering is instead ineffective with the less structured UBC SIFT and MNIST data.

We conclude this analysis by showing, in Tab.III inspired to Tab.I of [15], some numerical performance figures of ROSANNA for the 100K UBC-SIFT dataset as a function of its main parameters. In the first line we consider a pivot configuration with speed-up 100, while the following six rows (labeled $G^-$, $G^+$, $R^-$, $R^+$, $C^-$, $C^+$) provide some insight into the effect of increasing/decreasing only one of the parameters at a time w.r.t. the pivot. By increasing $G$, narrower cones are generated, leading to faster search (remember that the other parameters are fixed) but lower accuracy, while the opposite happens when $G$ decreases. Operating on $R$ and $C$ produces similar effects, slower search and higher accuracy.
when the parameter grows, and the opposite when it decreases. In all these cases, there is a nearly linear relationship between accuracy and speed-up, when one grows the other decreases. Memory overhead, instead, exhibits a more varied behavior. It remains constant when operating on C, since the data structure does not change. It is positively correlated with speed when operating on G, because faster search is obtained by defining more cones. On the contrary, it is negatively correlated with speed when operating on R, because faster search is obtained by using less rotations. Therefore, one can keep memory low both when high-speed is required (reducing R) and when high accuracy is required (reducing G). This is reflected in the next two configurations, selected to guarantee high speed, and high accuracy, respectively, where memory overhead is always relatively low. The last configuration instead has almost no memory overhead and still a good performance.

### C. Fast copy-move forgery detection

With the diffusion of powerful image editing tools, image manipulation, often with malicious and dangerous aims, has become easy and widespread. Copy-move is one of the most common attacks, where a piece of the image is cut and paste somewhere else in the same image to hide some undesired objects. Using material drawn from the same target image, in fact, raises the likelihood to escape the scrutiny of a casual observer.

The most effective copy-move detectors \[38\], \[39\], \[7\] are based on dense feature matching. A feature is associated with each block, and the most similar feature is searched for over the whole image. Eventually, a dense field of offsets linking couple of pixels is obtained which, after some suitable post-processing, may reveal the presence of near-duplicate regions. By using scale/rotation invariant features, copy-moves can be effectively detected even in the presence of geometrical distortions.

Feature matching is the most computation-intensive phase of copy-move detection algorithms. In \[7\], this task is carried out by resorting to PatchMatch \[40\], a fast randomized iterative algorithm which outputs a smooth and accurate offset field. Nonetheless, this process turns out to be relatively slow, since several iterations of PatchMatch are needed for convergence. This may be a serious problem when a large number of images must be analyzed in search of possible manipulations, especially with today’s multi-million pixel images. To speed-up this phase, one can improve the initialization of PatchMatch (random in the original algorithm) by means of approximate NN search tools, as done for example in \[41\]. This may be effective, indeed, if the ANN algorithm is itself fast and reasonably accurate. We show next that ROSANNA, with a suitable choice of the parameters, may accomplish very well this task.

We use the copy-move detection algorithm proposed in \[7\], referring the reader to the original paper for all details. For each 16×16-pixel block we compute the first 16 Zernike moments, which represent the associated rotation-invariant feature. Based on these features, we then build the offset field by means of PatchMatch, and finally use a suitable post-processing to extract possible near-duplicates.

In Tab.\[IV\] we report the average results (CPU-time and accuracy) observed on a small set of test images when using PatchMatch with random initialization (the original algorithm) and with the initialization provided by ROSANNA. Images were drawn from the FAU database\[15\] used in \[38\], and have an average size of 6 Mpixel. All of them have been manipulated by copy-moving some parts, either to replicate or to hide some objects. An example is show in Fig.\[13\].

![Fig. 13. An example of copy-move forgery detection results. Top: original and forged images. Bottom: color-coded masks output by the original algorithm (left) and its modified version with ANN initialization (right). In both cases, despite rotation, all copied regions are detected (green) with very high accuracy, and no false alarm (red) is present.](http://www5.cs.fau.de/)

### Table III

| configuration | parameters (G, R, C) | accuracy | speed-up | memory overhead | build time |
|---------------|----------------------|----------|----------|-----------------|------------|
| pivot        | 4 8 4                | 0.905    | 100      | 0.36            | 0.36       |
| G−           | 3 8 4                | 0.961    | 37       | 0.16            | 0.35       |
| G+           | 5 8 4                | 0.814    | 168      | 0.69            | 0.36       |
| R−           | 4 4 4                | 0.788    | 180      | 0.18            | 0.26       |
| R+           | 4 16 4               | 0.966    | 54       | 0.71            | 0.57       |
| C−           | 4 8 2                | 0.841    | 145      | 0.36            | 0.35       |
| C+           | 4 8 8                | 0.946    | 66       | 0.36            | 0.36       |
| high speed   | 6 2 16               | 0.595    | 404      | 0.24            | 0.20       |
| high acc.    | 3 16 8               | 0.999    | 14       | 0.30            | 0.56       |
| low memory   | 3 1 128              | 0.901    | 18       | 0.03            | 0.17       |
| Task       | Random initialization | ANN initialization |
|------------|-----------------------|--------------------|
| Feature extraction | 6.19 (4.8%)              | 6.19 (11.8%)        |
| Initialization   | –                      | 27.43 (52.2%)       |
| PatchMatch      | 100.19 (84.9%)          | 7.66 (14.6%)        |
| Post-processing | 13.31 (10.3%)           | 11.22 (21.4%)       |
| TOTAL CPU-time  | 129.69                 | 52.50               |
| Average accuracy (FM) | 0.977                 | 0.971              |

**TABLE IV**

AVERAGE CPU-TIME AND ACCURACY FOR COPY-MOVE FORGERY DETECTION ON 6 MIPixel IMAGES.

11, R = 1, C = 1 and by moderate subsampling (1 : 3) × (1 : 3). Although only a subset of the offsets are initialized, due to subsampling, this is more than enough to ensure convergence to a good offset field with only two iterations of PatchMatch. In addition, the random search phase of PatchMatch is skipped altogether, reducing the cost of a single iteration. Eventually, the extra cost of initialization is more than compensated by the saving in matching, leading to an overall 60% cut in CPU-time for the same accuracy. Fig.13 shows in the bottom the output masks provided by the two version of the copy-move detector, which are barely distinguishable.

V. CONCLUSIONS AND FUTURE WORK

We developed ROSANNA starting from the analysis of Fig.4, and noting that order statistics allow one to induce structure in otherwise unstructured data, so as to speed up ANN search. Under a different point of view, ROSANNA is just spherical LSH where, however, multiple alternative partitions of the space of directions are available. Experiments show ROSANNA to provide a state-of-the-art performance on unstructured data. Such good results are confirmed also on real-world SIFT and MNIST data. However, when data are highly structured, some suitable data-dependent preprocessing is needed. In this work we provided a simple solution to this problem, but there is certainly much room for further improvements. Finally, we illustrated ROSANNA’s potential to address real-world image processing problems by considering the copy-move forgery detection problem.

Due to its simple conception and implementation, ROSANNA may represent a precious tool in a wide range of image processing problems where nearest neighbor search is involved. Research is under way to declinate the same basic concepts in the context of large database image retrieval, defining reliable proxy distances based on compact codes derived from order-statistics.

APPENDIX A

BASIC RESULTS ON ORDER STATISTICS

Let

\[ X = \{X_1, \ldots, X_K\} \]

be a vector of i.i.d. zero-mean random variables with marginal pdf \( f_X(x) \). The order statistics \( X_{(1)}, \ldots, X_{(K)} \) are the random variables obtained by sorting the values of \( X \) in descending order. For example

\[ X_{(1)} = \max(X_1, \ldots, X_K) \]

and

\[ X_{(K)} = \min(X_1, \ldots, X_K) \]

To obtain the marginal pdf of the \( i \)-th order statistics let us compute the probability

\[ \Pr(X_{(i)} \in [x, x + dx]) = f_{X_{(i)}}(x)dx \]

where \( dx \to 0 \). For this event to happen, \( i - 1 \) components of vector \( X \) must be larger than \( x + dx \), \( K - i \) of them must be smaller than \( x \), and exactly one of them must fall in the interval \([x, x + dx]\) (see Fig.14 top). Since the components are i.i.d., by taking into account the different combinations through a multinomial coefficient, and neglecting \( O(dx^2) \) terms, it results

\[ f_{X_{(i)}}(x) = \frac{K!}{(K-i)!(i-1)!} \times [F_X(x)]^{K-i}[1 - F_X(x)]^{i-1}f_X(x) \]  \hspace{1cm} (6)

All joint statistics can be computed in the same way. In particular, for the second-order joint pdf, taking \( i < j \), it results (see Fig.14 bottom)

\[ f_{X_{(i)}, X_{(j)}}(x, y) = \frac{K!}{(K-j)!(j-1)!!(i-1)!} \times [F_X(x)]^{K-j}[F_X(x) - F_X(y)]^{j-1} \times [1 - F_X(x)]^{i-1}f_X(x)f_X(y)u(x-y) \]  \hspace{1cm} (7)

APPENDIX B

THEORETICAL BOUNDS

We want to characterize ROSANNA in terms of its collision probability for unstructured data.

Let

\[ X = \{X_1, \ldots, X_K\} \]

be a vector of i.i.d. zero-mean symmetric random variables with marginal pdf \( f_X(x) \). Then, let

\[ Y = \{Y_1, \ldots, Y_K\} \]

More often, ascending order is used, but the two choices are equivalent and we prefer to remain coherent with ROSANNA’s functioning.
be a second vector which, given $X = x$, is uniformly distributed on the hypersphere of radius $r$ centered on $x$, that is

$$f_Y(y|x) = \frac{1}{S^K(r)} \delta(\|y - x\| - r)$$

where $S^K(r)$ is the measure of the $K$-d hypersphere of radius $r$, and $\delta(\cdot)$ is the Dirac delta function. We want to compute, for any radius $r$, the probability that $X$ and $Y$ belong to the same cone

$$p_{\text{coll}}(r) = P_r[h(X) = h(Y)]$$

where the hash function $h(\cdot)$ associates a vector to a given cone based on the index and sign of its $G$ largest components. Rather than the probability of collision, we will consider its complement

$$p_{\text{cross}}(r) = 1 - p_{\text{coll}}(r)$$

which is the probability that point $Y$ lies across any of the boundaries of $X$’s cone.

### A. The 2d case

Let us begin by considering the simplest non-trivial case of $K = 2$ and $G = 1$. Fig. 15 provides a pictorial description of our problem in this setting. In the example, point $x$ belongs to cone $c_1$ where the first component is the largest, and it is positive. In addition, we focus on the half-cone $\Omega$ where also the second component of $x$ is positive

$$\Omega = \{x \in \mathbb{R}^2 : x_1 \geq x_2 \geq 0\}$$

However, thanks to symmetry, all following arguments apply equally well to all other cones and half-cones with obvious modifications. Therefore

$$p_{\text{cross}}(r) = p_{\text{cross}}(r | \Omega) = \int_{\Omega} f_X(x|\Omega)(x)p_{\text{cross}}(r | x)dx$$

Cone $c_1$ has two boundaries, the lines with equations $y_2 = y_1$ and $y_2 = -y_1$. For each $x$ in $c_1$ let us label the boundaries in order of increasing distance from $x$, so that

$$p_{\text{cross},1}(r | x)$$

is the probability of crossing boundary number 1, the nearest one. Let $\Delta$ be the distance of $x$ from this boundary. Since we restrict attention to $\Omega$, this is

$$\Delta = (x_1 - x_2)/\sqrt{2}$$

while more in general it holds

$$\Delta = \max(|x_1|, |x_2|) - \min(|x_1|, |x_2|)$$

$$\sqrt{2}$$

If $\Delta < r$, part of the circumference of radius $r$ centered on $x$ will cross the nearest boundary. For our hypothesis of uniform distribution of $Y|x$, the fraction of the circumference past the boundary represents the probability of crossing it

$$p_{\text{cross},1}(r | x) = \left\{ \begin{array}{ll} \theta(r, \Delta)/\pi & \Delta < r \\ 0 & \text{otherwise} \end{array} \right.$$

where

$$\theta(r, \Delta) = \arccos \left( \frac{\Delta}{r} \right)$$

Of course, other points on the circumference may cross the second boundary, and some may cross both. Therefore

$$p_{\text{cross}}(r | x) = 2p_{\text{cross},1}(r | x) + p_{\text{cross},2}(r | x)$$

which may be difficult to compute. Simpler but looser bounds are

$$2p_{\text{cross},1}(r | x) + 1 - 1/N_C$$

with $N_C$ the total number of cones. In summary

$$p_{\text{cross},1}(r | x) \leq p_{\text{cross}}(r | x) \leq \min \left( 2p_{\text{cross},1}(r | x), \frac{N_C - 1}{N_C} \right)$$

and by averaging over $x \in \Omega$ we obtain lower and upper bounds for $p_{\text{cross}}(r)$.

In Fig. 16 we plot these upper and lower bounds, together with the actual crossing probability estimated through Monte-Carlo simulation, when the $X_i$’s are standard Gaussian RV’s. As expected, for small values of $r$ the Monte-Carlo estimate is very close to the theoretical lower bound (see also the log-scale plot on the right), while the gap grows when the distance between the two points, $X$ and $Y$, becomes comparable with their own norm. When $r \to \infty$, of course, $Y$ belongs to any cone with the same probability, and the curve approaches the theoretical upper bound of $3/4$.

### B. The general case

We now consider the general high-dimensional case, proceeding in the very same way as for the 2d case, except for some suitable modifications. The most important difference with respect to the previous case is that we will resort to
order statistics to reduce the final $K$-dimensional integral to a numerically tractable 2d integral.

In this case we have $N_C = \binom{K}{2} G^2$ cones, statistically indistinguishable from one another. Let us consider the cone $c_1$ where the first $G$ components are also the largest, and they are all positive. Furthermore, let us restrict attention to the subregion of $c_1$, call it again $\Omega$, where also the smallest $K-G$ components are all positive

$$\Omega = \{ x \in \mathbb{R}^K : \min(x_1, \ldots, x_G) \geq \max(x_{G+1}, \ldots, x_K), x_i \geq 0, \ i = 1, \ldots, K \} \quad (12)$$

Again, because of symmetry, this restriction is immaterial, and \[ still holds. So, we will provide lower and upper bounds for $p_{\text{cross}}(r|x)$, and then integrate over $\Omega$.

Cones $c_1$ is now delimited by $2G(K-G)$ hyperplanes, the hyper-planes with equations $y_i = y_j$ and $y_i = -y_j$, for all $i = 1, \ldots, G$ and $j = G + 1, \ldots, K$, and the point $Y$ at distance $r$ from $x$ leaves the cone only if it crosses at least one of such boundaries.

Let again $\Delta$ be the distance of $x$ from the closest boundary. If $\Delta > r$ the probability of crossing that boundary (or any other) is 0. Otherwise, it can be computed as the ratio between the measure $S^K,\Delta(r)$ of the hyper-spherical cap intercepted by a hyperplane at distance $\Delta$ from the center, and the measure $S^K(r)$ of the whole hyper-sphere. It is known that

$$S^K(r) = \frac{2\pi^{K/2}}{\Gamma(K/2)} r^{K-1}$$

with $\Gamma(\cdot)$ the Gamma function, while the measure of the cap can be computed by integrating over $\theta$ the $(K-1)$-dimensional hyper-spheres of radius $r \sin(\theta)$

$$S^K,\Delta(r) = \int_0^{\theta(r, \Delta)} S^{K-1}(r \sin(\theta)) r d\theta$$

where $\theta(r, \Delta)$ is still given by \[. As for $\Delta$, note that the closest boundary to $x$ is the hyperplane of equation $y_m = y_M$ where $m$ is the index of the smallest component among the $G$ largest, and $M$ is the index of the largest component among the $K - G$ smallest. Consequently

$$\Delta = \frac{x_m - x_M}{\sqrt{2}}$$

In summary it results

$$p_{\text{cross},1}(r|x) = \begin{cases} S^{K,\Delta}(r)/S^K(r) & \Delta < r \\ 0 & \text{otherwise} \end{cases}$$

Again, $p_{\text{cross},1}(r|x)$ is a lower bound for $p_{\text{cross}}(r|x)$. An upper bound can be easily obtained as

$$p_{\text{cross}}(r|x) \leq \sum_{i=1}^{2G(K-G)} p_{\text{cross},i}(r|x)$$

$$\leq \min \left( 2G(K-G)p_{\text{cross},1}(r|x), \frac{N_C-1}{N_C} \right)$$

As the last step of our development, we must compute the integral \[ over $X \in \Omega$ which, for $K \gg 1$, is computationally intractable. However, we are not really interested in the $K$-

![Fig. 17. Theoretical bounds and Monte Carlo estimate for the crossing probability in the case of i.i.d. standard Gaussian RV’s, with $K=16$, $G=4$.](image)

**REFERENCES**

[1] H. Jégou, M. Douze, and C. Schmid, “Product quantization for nearest neighbor search,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 33, no. 1, pp. 117–128, January 2011.

[2] A. Babenko and V. Lempitsky, “The inverted multi-index,” in IEEE Conference on Computer Vision and Pattern Recognition, 2012, pp. 3069–3076.

[3] Y. Gong, S. Lazebnik, A. Gordo, and F. Perronnin, “Iterative quantization: A procrustean approach to learning binary codes for large-scale image retrieval,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 35, no. 12, pp. 2916–2929, December 2013.

[4] T. Ge, K. He, Q. Ke, and J. Sun, “Optimized product quantization,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 36, no. 4, pp. 744–755, April 2014.
A review of image denoising algorithms, with a new one,” Multiscale Modeling and Simulation, vol. 4, no. 2, pp. 490–530, 2005.

C. Guillenmot and O. Le Meur, “Image inpainting: overview and recent advances,” IEEE Signal Processing Magazine, vol. 31, no. 1, pp. 127–144, 2014.

D. Cozzi, G. Poggi, and L. Verdoliva, “Efficient dense-field copy-move forgery detection,” IEEE Transactions on Information Forensics and Security, vol. 10, no. 11, pp. 2284–2297, November 2015.

L. Bao, Q. Yang, and H. Jin, “Fast edge-preserving patchmatch for large displacement optical flow,” IEEE Transactions on Image Processing, vol. 23, no. 12, pp. 4996–5006, December 2014.

R. M. Gray and D. L. Neuhoff, “Quantization,” IEEE Transactions on Information Theory, vol. 44, no. 6, pp. 2325–2383, 1998.

A. Gersho and R. M. Gray, Vector Quantization and Signal Compression, Kluwer Academic Publishers, 1992.

L. Paulevé, H. Jégou, and L. Amsaleg, “Locality sensitive hashing: a comparison of hash function types and querying mechanisms,” Pattern Recognition Letters, vol. 33, no. 1, pp. 1348–1358, August 2010.

S. P. Lloyd, “Least squares quantization in pcm,” IEEE Transactions on Information Theory, vol. 28, no. 2, pp. 129–137, 1982.

J. L. Bentley, “Multidimensional binary search trees used for associative searching,” Communications of the ACM, vol. 18, no. 9, pp. 590–597, 1975.

C. Silpa-Anan and R. Hartley, “Optimized kd-trees for fast image descriptor matching,” in IEEE Conference on Computer Vision and Pattern Recognition, 2008, pp. 1–8.

M. Muja and D. G. Lowe, “Scalable nearest neighbor algorithms for high dimensional data,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 36, no. 11, pp. 2227–2240, November 2014.

M. Norouzi and D. J. Fleet, “Cartesian k-means,” in IEEE Conference on Computer Vision and Pattern Recognition, 2013, pp. 3017–3024.

Y. Kalantidis and Y. Avrithis, “Locally optimized product quantization for approximate nearest neighbor search,” in IEEE Conference on Computer Vision and Pattern Recognition, 2014, pp. 2329–2336.

T. Zhang, C. Du, and J. Wang, “Composite quantization for approximate nearest neighbor search,” in International Conference on Machine Learning, 2014, pp. 838–846.

J. H. Conway and N. J. A. Sloane, Sphere Packings, Lattices and Groups, Springer-Verlag, Ny, 3rd ed., 1998.

M. Datar, N. Immorlica, P. Indyk, and V. Mirrokni, “Locality-sensitive hashing scheme based on p-stable distributions,” in Proc. of the Symposium on Computational Geometry, 2004, pp. 253–262.

J. Wang, H. T. Shen, J. Song, and J. Ji, “Hashing for similarity search: A survey,” in arXiv preprint arXiv:1408.2927, 2014, pp. 1–29.

Q. Lv, W. Josephson, Z. Wang, M. Charikar, and K. Li, “Multiprobe LSH: Efficient indexing for high-dimensional similarity search,” in Proc. of Int. Conf. Very Large Data Bases, 2007, pp. 950–961.

Y. Weiss, A. Torralba, and R. Fergus, “Spectral hashing,” in Proc. Neural Information Processing Systems (NIPS), 2008, pp. 1753–1760.

J. Brandt, “Transform coding for fast approximate nearest neighbor search in high dimensions,” in IEEE Conference on Computer Vision and Pattern Recognition, 2010, pp. 1815–1822.

W. Kong and W.-J. Li, “Isotropic hashing,” in Proc. Advances in Neural Information Processing Systems (NIPS), 2012, pp. 1655–1663.

E. Chuve, K. Figueroa, and G. Navarro, “Effective proximity retrieval by ordering permutations,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 30, no. 9, pp. 1647–1658, 2008.

K. Terasawa and Y. Tanaka, “Spherical LSH for approximate nearest neighbor search on unit hypersphere,” in Algorithms and Data Structures, LNCS, 2007, vol. 4619, pp. 27–38.

J.-P. Heo, Y. Lee, J. He, S.-F. Chang, and S.-E. Yoon, “Spherical hashing,” in IEEE Conference on Computer Vision and Pattern Recognition, 2012, pp. 2957 – 2964.

K. Eshghi and S. Rajaram, “Locality Sensitive Hash Functions Based on Concomitant Rank Order Statistics,” in ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, 2008, pp. 221–229.

A. Cherian, S. Sra, V. Morellas, and N. Papanikolopoulos, “Efficient nearest neighbors via robust sparse hashing,” IEEE Transactions on Image Processing, vol. 23, no. 8, pp. 3646–3655, August 2014.

A. Cherian, “Nearest neighbors using compact sparse codes,” in International Conference on Machine Learning, 2014, pp. 1053–1061.

A. Papoulis and S. Unnikrishna Pillai, Probability, Random Variables, and Stochastic Processes, International Edition, 2002.

H. Jégou, F. Perronnin, M. Douze, I. Sánchez, P. Pérez, and C. Schmid, “Aggregating local image descriptors into compact codes,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 34, no. 9, pp. 1704–1716, September 2012.

M. Iwamura, T. Sato, and K. Kise, “What is the most efficient way to select nearest neighbor candidates for fast approximate nearest neighbor search?” in International Conference on Computer Vision, 2013, pp. 3535–3542.

A. Andoni, M. Datar, N. Immorlica, P. Indyk, and V. Mirrokni, “Locality-sensitive hashing scheme based on p-stable distributions,” in Nearest Neighbor Methods in Learning and Vision: Theory and Practice, T. Darrell, P. Indyk, and V. Shakhnarovich (eds.), Eds. MIT Press, 2006.

G. Lowe, “Distinctive image features from scale-invariant keypoints,” International Journal of Computer Vision, vol. 60, no. 2, pp. 91–110, 2004.

B. Chang-Da and R. M. Gray, “An improvement of the minimum distortion encoding algorithm for vector quantization,” IEEE Transactions on Information Forensics and Security, vol. 7, no. 6, pp. 1841–1854, 2012.

S.-J. Ryu, M. Kirchner, M.-J. Lee, and H.-K. Lee, “Rotation invariant localization of duplicated image regions based on Zernike moments,” IEEE Transactions on Information Forensics and Security, vol. 8, no. 8, pp. 1355–1370, August 2013.

C. Barnes, E. Shechtman, A. Finkelstein, and D.B. Goldman, “Patch-Match: a randomized correspondence algorithm for structural image editing,” ACM Transactions on Graphics (Proc. SIGGRAPH), vol. 28, no. 3, 2009.

S. Korman and S. Avidan, “Coherence sensitive hashing,” in International Conference on Computer Vision, 2011, pp. 1607–1614.

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