Double resummation for Higgs production

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We present the first double-resummed prediction of the inclusive cross section for the main Higgs production channel in proton-proton collisions, namely gluon fusion. Our calculation incorporates to all orders in perturbation theory two distinct towers of logarithmic corrections which are enhanced, respectively, at threshold, i.e. large $x$, and in the high-energy limit, i.e. small $x$. Large-$x$ logarithms are resummed to next-to-next-to-leading logarithmic accuracy, while small-$x$ ones to leading logarithmic accuracy. The double-resummed cross section is furthermore matched to the state-of-the-art fixed-order prediction at next-to-next-to-leading accuracy. We find that double resummation corrects the Higgs production rate by 2% at the currently explored center-of-mass energy of 13 TeV and its impact reaches 10% at future circular colliders at 100 TeV.

The major achievement of the first run of the CERN Large Hadron Collider (LHC) was the discovery of the Higgs boson [1–2], thus confirming the Brout-Englert-Higgs mechanism [3–6] for the electroweak symmetry breaking. The current and future runs of the LHC are rightly considered the Higgs precision era. The ATLAS and CMS collaborations are continuously producing experimental analyses of ever increasing sophistication, which allow for a more detailed inspection of the Higgs sector. Examples include measurements of fiducial cross sections in different decay channels, as well as kinematic distributions of the Higgs boson [7–17]. In order to perform meaningful comparisons, it is imperative for the theoretical physics community to deliver calculations with uncertainties that match in magnitude those quoted by the experimental collaborations. Therefore, it does not come as a surprise that perturbative calculations in QCD have reached the astonishing next-to-next-to-next-to-leading accuracy [18, 42–48], and leads to a significantly improved convergence of the perturbative expansion [19]. In the opposite kinematic limit, i.e. at high energy (or small $x$), resummation [50–54] is also possible. In particular, small-$x$ resummation of splitting functions [55], which govern the evolution of the parton distribution functions (PDFs), was obtained in [50] (see also [66–80]). Furthermore, high-energy resummation of partonic coefficient functions can be achieved in the framework of $k_t$-factorization [51, 52], and improved with the inclusion of subleading (but important) running corrections [63]. All-order calculations in this framework have been exploited to assess and improve the aforementioned EFT, which is known to fail in the high-energy limit [53, 55]. However, thus far, investigations in the high-energy regime were performed at fixed-order, i.e. by expanding out the all-order results. All-order phenomenological results at small $x$ will be presented in this work for the first time.

In this letter, we combine the above all-order approaches, together with the state-of-the-art fixed-order calculation, to obtain the most accurate prediction for Higgs production at the LHC. To our knowledge, it is the first time that double (large- and small-$x$) resummation is achieved. This breakthrough is possible because of two distinct advancements in the field. On the one hand, a general framework to combine the two resummations has been developed in [86] and implemented in public codes TROLL [45, 49] and HELL [87, 88] so that numerical results can be easily obtained. On the other hand, recently, all-order calculations have been considered in the context of PDF determination, both at large-$x$ [89]
and at small-$x$ [90, 91]. This opens up the possibility of achieving fully consistent resummed results. While we presently concentrate on the Higgs production cross section, our technique is fully general and can be applied to other important processes, such as the Drell-Yan process or heavy-quark production. We leave further phenomenological analyses to future work.

Let us start our discussion by introducing the factorized Higgs production cross section

$$\sigma(\tau, \mu_R^2) = \sigma_0(m_H^2, \alpha_s(\mu_R^2)) \times \sum_{ij} \int_\tau^1 \frac{dx}{x} L_{ij}(\frac{x}{\tau}, \mu_F^2) C_{ij}(x, \alpha_s(\mu_F^2), \frac{m_H^2}{\mu_F^2}, \frac{m_Z^2}{\mu_R^2}),$$

where $\sigma_0$ is the lowest-order partonic cross section, $L_{ij}$ are parton luminosities (convolutions of PDFs), $C_{ij}$ are the perturbative partonic coefficient functions, $\tau = m_H^2/s$ is the squared ratio between the Higgs mass and the collider center-of-mass energy, and the sum runs over all parton flavors. Henceforth, we suppress the dependence on renormalization and factorization scales $\mu_R, \mu_F$. Moreover, because the Higgs couples to the gluon via a heavy-flavor loop, (1) also implicitly depends on any heavy virtual particle mass.

The general method to consistently combine large- and small-$x$ resummation of partonic coefficient functions $C_{ij}(x, \alpha_s)$ was developed in [86]. The basic principle is the definition of each resummation such that they do not interfere with each other. This statement can be made more precise by considering Mellin ($N$) moments of (1). The key observation is that while in momentum ($x$) space coefficient functions are distributions, their Mellin moments are analytic functions of the complex variable $N$ and therefore, they (in principle) fully determined by the knowledge of their singularities. Thus, high-energy and threshold resummations are consistently combined if they mutually respect their singularity structure. In [86], where an approximate $N^3$LO result for $C_{ij}$ was obtained by expanding both resummations to $O(\alpha_s^3)$, the definition of the large-$x$ logarithms from threshold resummation was improved in order to satisfy the desired behavior, and later this improvement was extended to all orders in [145], leading to the so-called $\psi$-soft resummation scheme. Thanks to these developments, double-resummed partonic coefficient functions can be simply written as the sum of three terms [92]

$$C_{ij}(x, \alpha_s) = C_{ij}^{\text{f.o.}}(x, \alpha_s) + \Delta C_{ij}^{\text{LLX}}(x, \alpha_s) + \Delta C_{ij}^{\text{ex}}(x, \alpha_s),$$

where the first term is the fixed-order calculation, the second one is the threshold-resummed $\psi$-soft contribution minus its expansion (to avoid double counting with the fixed-order), and the third one is the resummation of small-$x$ contributions, again minus its expansion. Note that not all partonic channels contribute to all terms in (2). For instance, the $qg$ contribution is power-suppressed at threshold but it does exhibit logarithmic enhancement at small $x$.

Our result brings together the highest possible accuracy in all three contributions. The fixed-order piece is $N^3$LO [18–22], supplemented with the correct small-$x$ behavior, as implemented in the public code $\text{ggHiggs}$ [49, 86, 93]. Threshold-enhanced contributions are accounted for to next-to-next-to-next-to-leading logarithmic accuracy ($N^3\text{LL}$) in the $\psi$-soft scheme, as implemented in the public code $\text{TROLL}$ [145, 49]. Finally, for high-energy resummation we consider the resummation of the leading non-vanishing tower of logarithms (here LLX) to the coefficient functions [63, 84], which we have now implemented in the code $\text{HELL}$ [87, 88]. The technical details of the implementation will be presented elsewhere [94]. Additionally, on top of scale variations, subleading terms can be varied in both resummed contributions, thus al-
Having determined the resummation of the partonic coefficient functions, we now discuss the role of the parton luminosities \( L_{ij} \) that enter (1). Ideally, we would like to use PDFs that have been fitted using a double-resummed theory. However, this is clearly not possible. Indeed, this is the first study that aims to combine threshold and high-energy resummation, so a PDF fit with this theory will only appear in the future. Therefore, we have to find an acceptable compromise. Within the NNPDF framework [95], PDFs with threshold resummation were obtained in [89], while small-\( x \) resummation was considered in [90]. We note that the inclusion of the latter was a challenging enterprise because small-\( x \) logarithms appear both in coefficient functions and PDF evolution, while in the \( N\bar{S} \) scheme large-\( x \) resummation only affects coefficient functions [96, 97]. In order to make an informed decision, we separately consider in Fig. 1 the impact of small-\( x \) resummation (on the left) and large-\( x \) resummation (on the right) on the GF cross section, as a function of the center-of-mass energy of the colliding protons.

Let us start by illustrating the situation concerning small-\( x \) resummation (left-hand plot). The plot shows the ratio of resummed results to the fixed order one, computed at \( N^3\bar{L}O \) with the fixed-order NNLO set of [90]. We include resummation in two steps. First (dashed blue), we compute the \( N^3\bar{L}O \) cross section using the "resummed PDFs" of [90], i.e. those fitted including resummation and evolving with NNLO+NLLx theory. Then (solid red), we add the LLx resummation to the Higgs coefficient functions, which provides the consistent resummed result. In all cases, the bands correspond to PDF uncertainties. The plot clearly shows that small-\( x \) resummation has a modest effect at current LHC energies, but its impact grows substantially with the energy, reaching the 10% level at 100 TeV, heralding the fact that electroweak physics at 100 TeV is small-\( x \) physics. The plot also shows that the bulk of the effect comes from the resummed PDFs and their resummed evolution, while small-\( x \) resummation of the Higgs coefficient functions is only a small correction. This perhaps surprising result can be understood by noting that, while the high-energy behavior of the PDFs is essentially determined by deep-inelastic scattering data at small \( x \) and low \( Q^2 \), the Higgs cross section is characterized by a much higher value of \( Q^2 \), and it is dominated by soft emissions [88]. Furthermore, the large discrepancy between resummed and NNLO PDFs at large \( \sqrt{s} \) is a manifestation of the perturbative instability of the latter. Indeed, as discussed at length in [90], resummed PDFs are close to the NLO ones, while the NNLO set significantly deviates at small \( x \).

The situation is rather different if we analyze large-\( x \) resummation (right-hand plot). Here we use the PDFs of [89], obtained with either NNLO and NNLO+NNLL theory, which however suffer from a larger uncertainty compared to standard global fits because of the reduced dataset used in their determination. In this case the impact of the resummation on the \( N^3\bar{L}O \) cross section is smaller and fairly constant in the whole energy range considered here. The plot shows that about half of the 2% effect originates from the resummation in the PDFs (dashed blue), which is however not significant due to the large PDF uncertainties, and the other half by the resummation in the coefficient functions (solid red).

Therefore, by comparing the two plots in Fig. 1 we...
conclude that, lacking double-resummed PDFs, the use of small-$x$ resummed PDFs is preferred for the fairly large energy range considered here, because threshold-resolution effects in PDFs have a much smaller impact on the Higgs cross section. From the plots one may wonder whether double resummation of the coefficient functions is at all needed for phenomenology. Certainly its impact is numerically modest but we argue that its inclusion brings significant advantages both at small- and large-$x$. Firstly, it allows for a fully consistent treatment at small-$x$. Furthermore, the inclusion of large-$x$ resummation, although being a small correction to the N$^3$LO results, allows for a more robust estimate of the theoretical uncertainty [15] [19].

We present double-resummed results for the Higgs cross section in GF in Fig. 2 where we show three plots at representative center-of-mass energies of the colliding protons. We consider the current energy of the LHC, $\sqrt{s} = 13$ TeV, and two possible energies for future colliders, namely $\sqrt{s} = 27$ TeV (HE-LHC) and $\sqrt{s} = 100$ TeV (FCC). We choose as central scale $\mu_F = \mu_R = m_t/2$. Numerical results are presented in Tab. 1 where we also report for completeness the correction $\Delta\sigma_{b,c}$ to the fixed-order calculation due to the presence of massive bottom and charm quarks running in the loop, following the recommendation of [99]. Furthermore, electroweak corrections in the factorized approach, when included, amount to an extra 5% increase [99].

Each plot shows the perturbative progression of the cross section as obtained in different approximations: fixed-order, fixed-order and threshold, fixed-order and double resummation. We also show the three main contributions to the theoretical uncertainty, namely PDFs, subleading logarithms at small-$x$ and scale variation. The latter also includes an estimate of subleading corrections at large $x$, resulting in 42 variations, as detailed in [19]. The uncertainty due to subleading logarithms at small-$x$ has been determined by taking the envelope of two variants of the coefficient-function resummation, which take as input resummed splitting functions either at LL$^x$ (to be precise, it is a modification of LL$^x$ resummation which was called LL$'$ in [87] [88]) or at NLL$^x$ [90] [94]. We note that the PDFs are, in principle, affected by analogous uncertainty, which however is not currently included in their determination. Thus, the overall small-$x$ uncertainty might be underestimated. A qualitative assessment of this uncertainty was performed in [90] and its impact on the Higgs cross section will be investigated in [93].

We note that double resummation, mostly because of its threshold component, has a much more stable perturbative progression than its fixed-order counterpart: convergence is faster and uncertainty bands always cover the next perturbative order and shrink as higher orders are included [19]. While double resummation is a small (2%) correction to the N$^3$LO at current LHC energies, because of its small-$x$ component its impact grows with $\sqrt{s}$, becoming 5% at 27 TeV, before reaching approximately 10% at 100 TeV. Furthermore, we point out that a large contribution to the theoretical uncertainty originates from unknown subleading logarithms at small $x$. As a consequence, our double-resummed prediction exhibits larger uncertainties than the N$^3$LO one. On the one hand this highlights the importance of pushing the resummation of coefficient functions at small $x$ one order higher. On the other hand, this also implies that the uncertainty from missing higher orders is likely underestimated in a purely fixed-order approach, mostly due to the fact that PDF uncertainty does not fully account for it. Thus, even at LHC energies where its impact is modest, double-resummation provides a more reliable estimate of the theoretical uncertainty affecting the Higgs cross section.

In this letter we have presented, for the first time, results in perturbative QCD that supplement a fixed-order calculation with both threshold and high-energy resummation. We have applied our double-resummed framework to calculate the inclusive cross section for Higgs production in gluon fusion. Our result features the state-of-the-art accuracy N$^3$LO+N$^3$LL+LL$^x$ and crucially, it makes use of recently determined resummed parton distributions. The method presented here is rather general and it can be applied to a variety of processes currently studied at the LHC, such as electroweak-boson produc-

| $\sqrt{s}$ (TeV) | $\sigma_{N^3LO}$ (pb) | $\delta_{scale}$ | $\delta_{PDFs}$ | $\sigma_{N^3LO+N^3LL+LL^x}$ (pb) | $\delta_{scale}$ | $\delta_{PDFs}$ | $\delta_{subllogs}$ | $\Delta\sigma_{b,c}$ (pb) |
|------------------|----------------------|----------------|----------------|-------------------------------|----------------|----------------|-----------------|------------------|
| 7                | 16.76 ±0.77% ±1.7%   |                |                | 16.83 ±4.28% ±1.5% ±1.3% ±1.01 pb |
| 8                | 21.32 ±0.77% ±1.6%   |                |                | 21.47 ±4.0% ±1.4% ±1.4% ±1.26 pb |
| 13               | 48.28 ±0.9% ±1.4%    |                |                | 49.26 ±4.0% ±1.2% ±1.8% ±2.66 pb |
| 14               | 54.32 ±0.9% ±1.3%    |                |                | 55.56 ±4.0% ±1.2% ±1.9% ±2.96 pb |
| 27               | 144.7 ±0.9% ±1.1%    |                |                | 151.6 ±4.0% ±1.0% ±2.3% ±7.2 pb |
| 100              | 786.7 ±1.9% ±1.1%    |                |                | 873.9 ±4.0% ±1.2% ±3.0% ±32.0 pb |

TABLE I. Values of the N$^3$LO and N$^3$LO+N$^3$LL+LL$^x$ GF cross section for selected values of the collider energy and Higgs mass $m_t = 125$ GeV. We use the NNPDF31xs PDFs with $\alpha_s(m_Z^2) = 0.118$, $m_t = 173$ GeV, $m_b = 4.92$ GeV and $m_c = 1.51$ GeV.
tion or top-quark production. Furthermore, we anticipate that its generalization to differential distributions, such as rapidity and transverse momentum, is possible and we look forward to future work in this direction.

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[1] Georges Aad et al. (ATLAS Collaboration), “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” Phys.Lett. B716, 1–29 (2012) arXiv:1207.7214 [hep-ex]

[2] Serguei Chatrchyan et al. (CMS Collaboration), “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,” Phys.Lett. B716, 30–61 (2012) arXiv:1207.7235 [hep-ex]

[3] F. Englert and R. Brout, “Broken Symmetry and the Mass of Gauge Vector Mesons,” Phys.Rev. Lett. 13, 321–323 (1964)

[4] Peter W. Higgs, “Broken symmetries, massless particles and gauge fields,” Phys.Lett. B12, 132–133 (1964)

[5] Peter W. Higgs, “Broken Symmetries and the Masses of Gauge Bosons,” Phys.Rev.Lett. 13, 508–509 (1964)

[6] Peter W. Higgs, “Spontaneous Symmetry Breakdown without Massless Bosons,” Phys.Rev. 145, 1156–1163 (1966)

[7] Morad Aaboud et al. (ATLAS), “Measurement of inclusive and differential cross sections in the $H \rightarrow ZZ^* \rightarrow 4\ell$ decay channel in $pp$ collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector,” JHEP 10, 132 (2017) arXiv:1708.02810 [hep-ex]

[8] Morad Aaboud et al. (ATLAS), “Measurement of the Higgs boson coupling properties in the $H \rightarrow ZZ^* \rightarrow 4\ell$ decay channel at $\sqrt{s} = 13$ TeV with the ATLAS detector,” (2017) arXiv:1712.02304 [hep-ex]

[9] Georges Aad et al. (ATLAS, CMS), “Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC $pp$ collision data at $\sqrt{s} = 7$ and 8 TeV,” JHEP 08, 045 (2016) arXiv:1606.02266 [hep-ex]

[10] Georges Aad et al. (ATLAS), “Measurement of fiducial differential cross sections of gluon-fusion production of Higgs bosons decaying to $WW^* \rightarrow e\nu\mu\nu$ with the ATLAS detector at $\sqrt{s} = 8$ TeV,” JHEP 08, 104 (2016) arXiv:1604.02997 [hep-ex]

[11] Georges Aad et al. (ATLAS), “Measurements of the Higgs boson production and decay rates and coupling strengths using pp collision data at $\sqrt{s} = 7$ and 8 TeV in the ATLAS experiment,” Eur. Phys. J. C76, 6 (2016) arXiv:1507.04548 [hep-ex]

[12] Georges Aad et al. (ATLAS), “Measurements of the Total and Differential Higgs Boson Production Cross Sections Combining the $H \rightarrow \gamma\gamma$ and $H \rightarrow ZZ^* \rightarrow 4\ell$ Decay Channels at $\sqrt{s}=8$ TeV with the ATLAS Detector,” Phys. Rev. Lett. 115, 091801 (2015) arXiv:1504.05833 [hep-ex]

[13] Albert M Sirunyan et al. (CMS), “Measurement of the $Z\gamma^* \rightarrow \tau\tau$ cross section in pp collisions at $\sqrt{s} = 13$ TeV and validation of $\tau$ lepton analysis techniques,” (2018) arXiv:1801.03535 [hep-ex]

[14] Albert M Sirunyan et al. (CMS), “Measurements of properties of the Higgs boson decaying into the four-lepton final state in $pp$ collisions at $\sqrt{s} = 13$ TeV,” JHEP 11, 047 (2017) arXiv:1706.09936 [hep-ex]

[15] Vardan Khachatryan et al. (CMS), “Measurement of the transverse momentum spectrum of the Higgs boson produced in $pp$ collisions at $\sqrt{s} = 8$ TeV using $H \rightarrow WW$ decays,” JHEP 03, 032 (2017) arXiv:1606.01522 [hep-ex]

[16] Vardan Khachatryan et al. (CMS), “Measurement of differential and integrated fiducial cross sections for Higgs boson production in the four-lepton decay channel in $pp$ collisions at $\sqrt{s} = 7$ and 8 TeV,” JHEP 04, 005 (2016) arXiv:1512.08377 [hep-ex]

[17] Vardan Khachatryan et al. (CMS), “Measurement of differential cross sections for Higgs boson production in the diphoton decay channel in $pp$ collisions at $\sqrt{s} = 8$ TeV,” Eur. Phys. J. C76, 13 (2016) arXiv:1508.07819 [hep-ex]

[18] Charalampos Anastasiou, Claude Duhr, Falko Dulat, Elisabetta Furlan, Thomas Gehrmann, et al., “Higgs boson gluon fusion production at threshold in N$^3$LO QCD,” Phys.Lett. B737, 325–329 (2014) arXiv:1403.4616 [hep-ph]

[19] Charalampos Anastasiou, Claude Duhr, Falko Dulat, Elisabetta Furlan, Thomas Gehrmann, Franz Herzog, and Bernhard Mistlberger, “Higgs Boson Gluon Fusion Production Beyond Threshold in N$^3$LO QCD,” JHEP 03, 091 (2015) arXiv:1411.4584 [hep-ph]

[20] Charalampos Anastasiou, Claude Duhr, Falko Dulat, Franz Herzog, and Bernhard Mistlberger, “Higgs Boson Gluon-Fusion Production in QCD at Three Loops,” Phys. Rev. Lett. 114, 212001 (2015) arXiv:1503.06056 [hep-ph]

[21] Charalampos Anastasiou, Claude Duhr, Falko Dulat, Elisabetta Furlan, Thomas Gehrmann, Franz Herzog, Achilleas Lazopoulos, and Bernhard Mistlberger, “High precision determination of the gluon fusion Higgs boson cross-section at the LHC,” JHEP 05, 058 (2016) arXiv:1602.00695 [hep-ph]

[22] Bernhard Mistlberger, “Higgs Boson Production at Hadron Colliders at N$^3$LO in QCD,” Phys. Rev. D91, 036008 (2015) arXiv:1412.2771 [hep-ph]

[23] Ye Li, Andreas von Manteuffel, Robert M. Schabinger, and Hua Xing Zhu, “Soft-virtual corrections to Higgs production at N$^3$LO,” Phys. Rev. D90, 054029 (2014) arXiv:1404.5839 [hep-ph]

[24] Chihaya Anzai, Alexander Hasselhuhn, Maik H. oschele, Jens Hof, William Kilgore, Matthias Steinhauser, and Takahiro Ueda, “Exact N$^3$LO results for $q\bar{q} \rightarrow H + X$,” JHEP 07, 140 (2015) arXiv:1506.02674 [hep-ph]

[25] Fredric A. Dreyer and Alexander Karlberg, “Vector-Boson Fusion Higgs Production at Three Loops in QCD,” Phys. Rev. Lett. 117, 072001 (2016) arXiv:1606.00840 [hep-ph]

[26] H. M. Georgi, S. L. Glashow, M. E. Machacek, and Dimitri V. Nanopoulos, “Higgs Bosons from Two Gluon An-
Cross-sections at High Energy,” Nucl.Phys. B796, 137–183 (2008) [arXiv:0708.1277 [hep-ph]].

[64] Guido Altarelli, Richard D. Ball, and Stefano Forte, “Structure Function Resummation in small-x QCD,” Applications of quantum field theory to phenomenology. Proceedings, 8th International Symposium on Radiative Corrections, RADCOR 2007, Florence, Italy, October 1-5, 2007, PoS RADCOR2007, 028 (2007) [arXiv:0802.0968 [hep-ph]].

[65] Guido Altarelli, Richard D. Ball, and Stefano Forte, “Small x Resummation with Quarks: Deep-Inelastic Scattering,” Nucl.Phys. B799, 199–240 (2008) [arXiv:0802.0332 [hep-ph]].

[66] G.P. Salam, “A Resummation of large subleading corrections at small x,” JHEP 9807, 019 (1998) [arXiv:hep-ph/9806482 [hep-ph]].

[67] M. Ciafaloni and D. Colferai, “The BFKL equation at next-to-leading level and beyond,” Phys. Lett. B452, 372–378 (1999) [arXiv:hep-ph/9812366 [hep-ph]].

[68] M. Ciafaloni, D. Colferai, and G.P. Salam, “Renormalization group improved small x equation,” Phys.Rev. D60, 114036 (1999) [arXiv:hep-ph/9905566 [hep-ph]].

[69] M. Ciafaloni, D. Colferai, and G.P. Salam, “A collinear model for small x physics,” JHEP 10, 017 (1999) [arXiv:hep-ph/9907109 [hep-ph]].

[70] Marcello Ciafaloni, Dimitri Colferai, and Gavin P. Salam, “On factorization at small x,” JHEP 07, 054 (2000) [arXiv:hep-ph/0007240 [hep-ph]].

[71] M. Ciafaloni, D. Colferai, G. P. Salam, and A.M. Stasto, “Expanding running coupling effects in the hard pomeron,” Phys. Rev. D66, 054014 (2002) [arXiv:hep-ph/0204282 [hep-ph]].

[72] M. Ciafaloni, D. Colferai, G.P. Salam, and A.M. Stasto, “Renormalization group improved small x Green’s function,” Phys.Rev. D68, 114003 (2003) [arXiv:hep-ph/0307188 [hep-ph]].

[73] Marcello Ciafaloni, Dimitri Colferai, Gavin P. Salam, and Anna M. Stasto, “The Gluon splitting function at moderately small x,” Phys. Lett. B587, 87–94 (2004) [arXiv:hep-ph/0311325 [hep-ph]].

[74] M. Ciafaloni and D. Colferai, “Dimensional regularisation and factorisation schemes in the BFKL equation at subleading level,” JHEP 09, 069 (2005) [arXiv:hep-ph/0507160 [hep-ph]].

[75] M. Ciafaloni, D. Colferai, G. P. Salam, and A. M. Stasto, “Minimal subtraction vs. physical factorisation schemes in small-x QCD,” Phys. Lett. B635, 320–329 (2006) [arXiv:hep-ph/0601200 [hep-ph]].

[76] M. Ciafaloni, D. Colferai, G.P. Salam, and A.M. Stasto, “A Matrix formulation for small-x singlet evolution,” JHEP 0708, 046 (2007) [arXiv:0707.1453 [hep-ph]].

[77] Robert S. Thorne, “Explicit calculation of the running coupling BFKL anomalous dimension,” Phys. Lett. B474, 372–384 (2000) [arXiv:hep-ph/9912284 [hep-ph]].

[78] Robert S. Thorne, “NLO BFKL equation, running coupling and renormalization scales,” Phys. Rev. D60, 054031 (1999) [arXiv:hep-ph/9901331 [hep-ph]].

[79] Robert S. Thorne, “The Running coupling BFKL anomalous dimensions and splitting functions,” Phys. Rev. D64, 074005 (2001) [arXiv:hep-ph/0103210 [hep-ph]].

[80] C. D. White and R. S. Thorne, “A Global Fit to Scattering Data with NLL BFKL Resummations,” Phys. Rev. D75, 034005 (2007) [arXiv:hep-ph/0611204 [hep-ph]].

[81] Stefano Catani, M. Ciafaloni, and F. Hautmann, “High energy factorization and small-x heavy flavour production,” Nucl. Phys. B366, 135–188 (1991) [arXiv:hep-ph/9405388 [hep-ph]].

[82] F. Hautmann, “Heavy top limit and double logarithmic contributions to Higgs production at m_H/s much less than 1,” Phys.Lett. B535, 159–162 (2002) [arXiv:hep-ph/0203140 [hep-ph]].

[83] Simone Marzani, Richard D. Ball, Vittorio Del Duca, Stefano Forte, and Alessandro Vicini, “Higgs production via gluon-gluon fusion with finite top mass beyond next-to-leading order,” Nucl.Phys. B800, 127–145 (2008) [arXiv:0801.2544 [hep-ph]].

[84] R. S. Pasechnik, O. V. Teryaev, and A. Szzurek, “Scalar Higgs boson production in a fusion of two off-shell gluons,” Eur. Phys. J. C47, 429–435 (2006) [arXiv:hep-ph/0603258 [hep-ph]].

[85] Richard D. Ball, Marco Bonvini, Stefano Forte, Simone Marzani, and Giovanni Ridolfi, “Higgs production in gluon fusion beyond NNLO,” Nucl.Phys. B874, 746–772 (2013) [arXiv:1303.3590 [hep-ph]].

[86] Marco Bonvini, Simone Marzani, and Tiziano Peraro, “Small-x resummation from HELL,” Eur. Phys. J. C76, 597 (2016) [arXiv:1607.02150 [hep-ph]].

[87] Marco Bonvini, Simone Marzani, and Claudio Muselli, “Towards parton distribution functions with small-x resummation: HELL 2.0,” JHEP 12, 117 (2017) [arXiv:1708.07510 [hep-ph]].

[88] Marco Bonvini, Simone Marzani, Juan Rojo, Luca Rotoli, Maria Ubiali, Richard D. Ball, Valerio Bertone, Stefano Carrazza, and Nathan P. Hartland, “Parton distributions with threshold resummation,” JHEP 09, 191 (2015) [arXiv:1507.01006 [hep-ph]].

[89] Richard D. Ball, Valerio Bertone, Marco Bonvini, Simone Marzani, Juan Rojo, and Luca Rotoli, “Parton distributions with small-x resummation: evidence for BFKL dynamics in HERA data,” (2017) [arXiv:1710.05935 [hep-ph]].

[90] Hamed Abdolmaleki et al. (xFitter Developers’ Team), “Impact of low-x resummation on QCD analysis of HERA data,” (2018) [arXiv:1802.00064 [hep-ph]].

[91] The fixed-order and small-x contributions are written in this form, while the large-x one is written in N space and its Mellin inversion is performed together with the parton luminosity using the so-called minimal prescription [93].

[92] Marco Bonvini, Richard D. Ball, Stefano Forte, Simone Marzani, and Giovanni Ridolfi, “Updated Higgs cross section at approximate N^3LO,” J. Phys. G41, 095002 (2014) [arXiv:1404.3204 [hep-ph]].

[93] Marco Bonvini, “Small-x phenomenology at the LHC and beyond: HELL 3.0 and the case of the Higgs cross section,” (2018) [arXiv:1805.08785 [hep-ph]].

[94] Richard D. Ball et al. (NNPDF), “Parton distributions from high-precision collider data,” Eur. Phys. J. C77, 663 (2017) [arXiv:1706.00428 [hep-ph]].

[95] G. P. Korchemsky, “Asymptotics of the Altarelli-Parisi-Lipatov Evolution Kernels of Parton Distributions,” Mod. Phys. Lett. A4, 1257–1276 (1989).

[96] Simon Albino and Richard D. Ball, “Soft resummation of quark anomalous dimensions and coefficient functions in MS-bar factorization,” Phys. Lett. B513, 93–102 (2001) [arXiv:hep-ph/0011133 [hep-ph]].
[98] Marco Bonvini, Stefano Forte, and Giovanni Ridolfi, “The Threshold region for Higgs production in gluon fusion,” Phys. Rev. Lett. 109, 102002 (2012), arXiv:1204.5473 [hep-ph].

[99] D. de Florian et al. (LHC Higgs Cross Section Working Group), “Handbook of LHC Higgs Cross Sections: 4. Deciphering the Nature of the Higgs Sector,” (2016), 10.23731/CYRM-2017-002 arXiv:1610.07922 [hep-ph].