Effect of Intentional Mistuning on Dynamic Characteristics of a Energy Turbomachine Bladed Disk

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Abstract. In this article the effect intentional mistuning of an axial turbomachine bladed disk has been analyzed in order to reduce forced response due to low-order engine excitation. The maximum value of forced response of turbomachine rotor’s blades with mistuning parameters is usually much more than the value of the tuned rotors. An increase level mistuning of this critical value actually leads to a decrease magnifications of the forced response. Thus, the actual work has been introducing some degree of intentional mistuning in the design to achieve these purposes. The effectiveness of intentional mistuning has been researched at the design stage of the bladed disk in the energy turbomachines, which is introduced into the rotor’s design by changing the nominal mass of the blades in harmonic models.

1. Introduction

The designs of the bladed disks undergo a strong increase in the forced response due to the mistuning parameter in combination with a low level of structural damping. The negative effects of mistuning parameter on the vibration and forced response of the bladed disk have been the subject of intensive research by many authors for more than 50 years [8-9,20].

Mistuning in the design of the bladed disk arises when it has a slight difference between the blades in terms of mass, geometry, material, or non-identical rotor’s sectors, violating the cyclic symmetry of the rotor. Also, the reasons for the mistuning parameter are caused by the inevitable technological tolerances for their manufacture, the heterogeneity of the material, different fit in the locks, the action of various operational factors and damage during operation. The values of the mistuning parameter of the bladed disk are determined in the form:

\[ \Delta f_i = \frac{f_{jj} - \bar{f}_j}{f_j} \]  

where \( \bar{f}_j \) - the average value of the fundamental frequencies of the \( j \)-th mode of vibration; \( f_{jj} \) - the value of the frequency of the \( j \)-th mode of vibration of blades, \( j = 1, \ldots, N \) (\( N \) – the number of blades).

A significant effect in the case of vibration of mistuned system is an increase the amplitude of the vibration and also the stress in comparison with the ideal system. For a quantitative assessment, the
maximum amplitude magnification $\gamma$ is introduced, which connects the maximum amplitude of the mistuned system with the maximum amplitude of the tuned system, and the amplitude is understood as the maximum displacement:

$$\gamma = \frac{u_{\text{mistuned(max)}}}{u_{\text{tuned(max)}}}$$

(2)

Many authors investigate the maximum increase in the forced response, such as Kuehhorn, Chan, Beirow, and others [1-4,14,22]. One of the most famous papers was published by Whitehead, who first formulated a conservative limit on the increase forced response. In his works, Whitehead shows an approach to estimating the maximum amplitude of over control [9]. He establishes the following empirical relationship between the maximum value of $\gamma$ and the number of rotor’s blades:

$$\gamma_{\text{max}} = \frac{1}{2}(1+\sqrt{N})$$

(3)

An improvement to the Whitehead limit was introduced by Martel and Corral, which replace the number of blades with the number of active modes [6]. The conservative limit takes into account different natural frequencies in the same bladed modes and takes into account the modal coupling between the motion of the disk and the blade. More recently, Figaschewsky and Kuehhorn have further improved the calculation of the limit by considering the standard deviation of the mistuning [10].

The idea of optimizing mistuned model was also taken up with regard to reducing the mistuned forced response. Han et al. could prove that certain intentional mistuning is approached for reducing the maximum amplification of the forced response, similar results were achieved by Castanier and Pierre [7,12-13].

The mistuning values are random values. The use of experimental methods to assess the effect of mistuning parameter on the dynamics of the bladed disk is a difficult task, because it is necessary to analyze a large number of mistuning variants during the experiment. In these cases, we can use such numerical analyzes to study random processes, using the complex program ANSYS WORKBENCH, ABAQUS, etc.

Currently, the finite element method (FEM) is widely used by an engineer to solve problems of statics, vibrations and calculating the resource of various technical systems, including rotors of the energy turbomachines [5,11,15,21,22]. FEM is an effective engineering tool, which has a common algorithm and allows performing calculations of various variants of complex structures in a short time. Many works are devoted to the theory and application of this method in calculating the dynamic characteristics of vibrations and predicting the durability of the bladed disks in the energy turbomachines. In particular, we can note the works [16-19]. As finite elements of structures, it is represented by a set of a sufficiently large number of points, the so-called nodes. The coordinates of nodes are set in the general coordinate system OXYZ and determine the geometric shape of structure. A combination of several nodes creates an element that is set by additional geometric characteristics (thickness, etc.) and material properties (modulus of elasticity, Poisson's ratio, density).

2. Fundamentals

In this article, TET10 finite element from the ANSYS WORKBENCH software package was used to study the effect of intentional mistuning on the dynamic characteristics of the rotor’s blades. The TET10 element has 10 nodes with 3 degrees of freedom in a node (Fig. 1). Thus, the three-dimensional triangular element TET10 has 30 nodal parameters.
To study the vibration frequencies of the academic disk’s blades, the measuring system of a laser scanning vibrometer that manufactured at Brandenburg University of Technology (BTU) is used as an experiment. The measurement system for analyzing the excitation of vibrations of a disk with 10 blades (laser scanning vibrometer) is shown in Fig. 2.

Fig. 2 shows an experimental setup of the laser scanning vibrometer system: 1 - foam pad, 2 - additional mass, 3 - control device, 4 - laser vibrometer, 5 - modal hammer, 6 - bladed disk. The calculations of the vibration frequencies of one sector in comparison with the experiment and in the software package ANSYS WORKBENCH and ABAQUS are shown in Table 1.

The bladed model was rigidly fixed to the rim of the disk. The finite element TET10 with 3 degrees of freedom in one node is used as a finite element model in the ANSYS WORKBENCH software package, and the ABAQUS software package uses a triangular finite element C3D10, which has 3 degrees of freedom in one node.

Table 1. Calculation of vibration frequencies of an academic disk’s one sector, Hz.

| Modes | ABAQUS  | ANSYS  | Author's experiment |
|-------|---------|--------|---------------------|
| 1     | 264.49  | 261.43 | -                   |
| 2     | 923.46  | 901.97 | 919.69              |
| 3     | 1361.5  | 1415.0 | -                   |
| 4     | 1958.0  | 2724.7 | -                   |
| 5     | 2857.1  | 3538.2 | 2752.50             |
| 6     | 3745.7  | 3758.5 | -                   |
| 7     | 4957.6  | 4467.1 | 4489.84             |
| 8     | 5422.9  | 5307.4 | 5319.30             |
| 9     | 5628.6  | 6893.6 | -                   |
| 10    | 7204.2  | 7387.7 | 6914.84             |
In this article, we consider the object of research - an academic disk with 10 blades, studied at BTU, Germany (Fig. 3a). As a finite element model in this work, the finite element TET10 of the commercial program ANSYS WORKBENCH is used with 3 degrees of freedom in one node and with a total number of finite elements - 38830 and 76280 node points. The number of degrees of freedom is 228840 (Fig. 3b).

![Academic disk with 10 blades](image)

**Figure 3.** Academic disk with 10 blades (a, 3D disk; b, FEM).

Fig. 4 shows the values of vibration frequencies of the bladed disk without intentional mistuning, see formula (1).

![Frequency mistuning](image)

**Figure 4.** Frequency mistuning of an academic bladed disk without intentional mistuning.

Table 2 shows the values of the natural vibration frequencies of the academic disk with 10 blades.

**Table 2.** Natural vibration frequencies of the academic disk with 10 blades, Hz.

| Blades | Mode 1  | Mode 2  | Mode 3  | Mode 4  | Mode 5  |
|--------|---------|---------|---------|---------|---------|
| 1      | 261.43  | 901.97  | 1415.0  | 2724.7  | 3538.2  |
| 2      | 261.43  | 901.97  | 1415.6  | 2724.7  | 3538.2  |
| 3      | 261.44  | 901.99  | 1415.6  | 2724.7  | 3537.5  |
| 4      | 261.46  | 901.99  | 1416.8  | 2724.7  | 3538.2  |
| 5      | 261.46  | 902.06  | 1416.8  | 2725.1  | 3540.9  |
| 6      | 261.49  | 902.06  | 1417.4  | 2725.1  | 3538.2  |
| 7      | 261.49  | 902.09  | 1417.4  | 2725.3  | 3538.2  |
| 8      | 261.49  | 902.13  | 1417.5  | 2725.4  | 3540.3  |
| 9      | 261.49  | 902.13  | 1417.5  | 2725.4  | 3540.3  |
| 10     | 261.50  | 902.16  | 1417.6  | 2725.5  | 3544.0  |
| Blades | Mode 6   | Mode 7   | Mode 8   | Mode 9   | Mode 10  |
|--------|----------|----------|----------|----------|----------|
| 1      | 3758.5   | 4467.1   | 5307.4   | 6893.6   | 7387.7   |
| 2      | 3758.5   | 4467.1   | 5307.4   | 6894.9   | 7387.7   |
| 3      | 3763.6   | 4467.5   | 5307.4   | 6894.9   | 7391.4   |
| 4      | 3767.7   | 4468.2   | 5307.4   | 6895.9   | 7390.2   |
| 5      | 3767.7   | 4468.2   | 5308.7   | 6895.9   | 7390.2   |
| 6      | 3773.3   | 4468.6   | 5308.7   | 6896.2   | 7387.7   |
| 7      | 3773.3   | 4468.6   | 5309.6   | 6896.2   | 7387.7   |
| 8      | 3775.2   | 4468.7   | 5309.6   | 6896.3   | 7391.4   |
| 9      | 3775.2   | 4468.7   | 5309.6   | 6896.3   | 7391.4   |
| 10     | 3775.7   | 4468.7   | 5309.9   | 6896.3   | 7392.2   |

At the next stage of the analysis, we will consider the block mistuned model No. 2 with an additional mass 5.0 grams, as 2 mistuned blades in one group (Fig. 5). Here, one group includes two tuned blades and two mistuned blades. The third group consists of two mistuned blades.

**Figure 5.** Block mistuned model No. 2 with additional masses (2 mistuned blades in one group).

**Figure 6.** The mistuned values of the vibration frequencies of the block mistuned model No. 2.

**Figure 7.** Calculation of the maximum displacement magnification for the block mistuned model No. 2.

Fig. 6 shows the mistuning value of the vibration frequencies of the disk blades of the block mistuned model No. 2. Fig. 7 shows that almost the same values of the maximum displacement
magnification for all the blades of the academic bladed disk are obtained in the modes 4 and 7, and the vibrational mode 8 has the largest value of the maximum displacement magnification $\gamma_{\text{max}} = 2.0$ (reduce 4\% compared to the result of the Whitehead’s maximum displacement magnification). The maximum displacement magnification of the mode 7 ($\gamma_{\text{max}} = 1.29$) shows that the most effective result for the block mistuned model No. 2 is obtained (reduce 38\% compared to the result of the Whitehead’s maximum displacement magnification).

Next, we consider the block mistuned model No. 5 with the additional mass 5.0 grams, as 5 mistuned blades in one group (Fig. 8). Here, one group includes five mistuned blades. The second group consists of five tuned blades. Fig. 9 shows the mistuning value of the vibration frequencies of the disk blades of the block mistuned model No. 5.

![Block mistuned model No. 5 with additional masses (5 mistuned blades in one group).](image)

**Figure 8.** Block mistuned model No. 5 with additional masses (5 mistuned blades in one group).

![The mistuned values of the vibration frequencies of the block mistuned model No. 5.](image)

**Figure 9.** The mistuned values of the vibration frequencies of the block mistuned model No. 5.

![Calculation of the maximum displacement magnification for the block mistuned model No. 5.](image)

**Figure 10.** Calculation of the maximum displacement magnification for the block mistuned model No. 5.

Fig. 10 shows that almost the same values of the maximum displacement magnification for all the blades of the academic bladed disk are obtained in the modes 1, 6 and 7, the vibrational mode 8 has the largest value of the maximum displacement magnification $\gamma_{\text{max}} = 1.82$ (reduce 12.5\% compared to the result of the Whitehead’s maximum displacement magnification). The maximum displacement magnification of the mode 1 ($\gamma_{\text{max}} = 1.38$) shows that the most effective result for the block mistuned
model No. 5 is obtained (reduce 33.7% compared to the result of the Whitehead’s maximum displacement magnification).

3. Conclusions
This paper presents the numerical researches of the effect of intentional mistuning in order to reduce the maximum blades displacement magnification of a bladed disks turbomachine. The intentional mistuning is obtained by optimizing the algorithms and realized by using small geometric changes on the blades, in detail with additional masses 5.0 grams at the periphery of the blade’s feather.

The results of the study in this paper show the reliability and effectiveness using intentional mistuning in models of a bladed disk, in order to reduce the maximum magnification of the forced response by 38% using different variants of the block model of the disk’s mistuned blades.

4. References
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