Imposed pressure driven flow in peristalsis

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Abstract
Peristaltic transport phenomena are of great significance in biological sciences. The physiological transport of fluid takes place under the action of peristalsis generated as a pressure gradient. The peristaltic waves generate a pressure gradient which is responsible for the fluid flow in the forward direction. The further properties of this phenomena can be seen if an imposed pressure gradient is applied in addition to the one appearing due to peristaltic waves. This situation has not been discussed in the literature that needs further attention. The effects of the wavy boundaries and imposed pressure on the velocity of the flow field are analyzed. Here we impose a question: what happens if an imposed pressure gradient is also applied? This question of physical importance has not been addressed; and thus, remains the topic of this study. In previous papers of peristaltic motion, the flow generated by peristaltic waves only has been examined while in this study we will discuss the contribution of imposed pressure gradient on velocity field. The analytical results for the velocity field are obtained using the boundary perturbation method. The study shows that the impact of the wavy boundaries on the flow increases with the increase in corrugation parameter and imposed pressure.

Keywords
Imposed pressure, peristalsis, wavy channel, Navier-stokes

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Introduction
This article is concerned with the fluid mechanics of peristaltic pumping under conditions for which the length of peristaltic wave is large compared with the channel width and the appropriate Reynolds number is sufficiently small for the flow to be considered inertia-free. The basic principle of peristaltic motion is based on the sinusoidal wave boundary which is produced due to the traveling of sinusoidal wave along flexible walls of the channel. There are other muscular structures which are responsible for pumping the fluids by means of peristaltic motion. Technologically, peristaltic devices are commonly used for slurries, pumping of blood through human body, transporting corrosive fluids etc. In bio-mechanics; the peristaltic phenomena have great relevance in arterioles, heart-lung machine, intestines, ureter and many other. The procedures are adopted by pumping of blood, food in small intestines, passing of fluid from kidney to ureter. Peristalsis in human physiological systems have been the reasons of many recent investigations.

Mathematically, the peristaltic flows induced by the waves traveling along a flexible channel have been treated under various geometrical situations and various fluid models. It must be kept in mind that the fluid is transported by the pressure gradient developed in the peristalsis mechanism. Before going further, we would like to refer few papers of major interest.
A literature review gives a general idea of the literature and theories related to the subject of the present problem. With the help of literature review, historical aspects of theories and literature is reached which lays importance forthrightly to the subject of the research. Even though peristalsis existed very well in physiology, its relevance came about mainly through the work of Engelmann.\textsuperscript{1} He was the first to investigate the phenomena of peristaltic waves in human ureter. He performed an analysis on physiological or methodological issues which opened new ways for further investigation of physiological mechanisms. After that, Latham\textsuperscript{2} investigated the mechanism of augmented fluid mixing within a peristaltic pump. After Latham several investigations have been made on the functioning of ureter following the peristaltic motion. Impacts of long wavelengths at low Reynolds number are assumed by Shapiro et al.\textsuperscript{3} He modeled the problem and found all elegant analytical results for channel and tube. A number of physically impact and phenomena of bolus, augmented flow and backward flow are discussed. An analysis of peristaltic transport is addressed by considering nonlinear convective acceleration and no slip condition on wavy wall by Fung.\textsuperscript{4} Hayat et al.\textsuperscript{5} adopted perturbation method to investigate the peristaltic transport in a planar channel. Shapiro et al.\textsuperscript{6} modeled a problem to present the simple physical picture of working of a peristaltic pump by means of peristaltic waves in the laboratory frame of reference. Importance of investigations in medical and industrial sciences based on peristaltic motion cannot be ignored. Duane\textsuperscript{7} examined the impact of peristaltic motion in urinary tract and provide an overview of the basic anatomy and physiology of the urinary tract with an emphasis on their specific roles in host defense. Srivastava et al.\textsuperscript{8} modeled a problem to investigate the attributes of peristaltic fluid suspension in transport of fluid and revealed that the reversal flow increases with increasing particle concentration and Reynolds number. Yin et al.\textsuperscript{9} employed a relation between theory and experiments in peristaltic transport. When theoretical and experimental results are compared, they are in good accord and provide confirmation to the theoretical analysis. Zien and Ostrach\textsuperscript{10} investigated the peristaltic transport for a long wave approximation and indicated that mean pressure gradient is associated with the motion. Leal gave a brief study of fluid flow in curved tubes.\textsuperscript{11} Hanin\textsuperscript{12} investigated the peristalsis in nano-fluid under endoscopic effects.

Pressure driven flows has demonstrated the ability to control fluids pressure in modern investigation like peristaltic pumping, cell sorting and microfluidic injections. In microfluidics, it is often desirable to minimize the circulating volume of fluid. Imposed pressure can regulate the desired volume of fluid flow. Pressure driven flows has been analyzed by.\textsuperscript{1,6,17,18} We observe that in these cases, the flow is generated by the peristaltic waves only through the pressure gradient developed in the channel or tube. However, the question remains; what happens if an external pressure gradient is imposed in addition to the one generated by the peristalsis. We believe that such situation is of much interest both from mathematical and physical points of view that has not been discussed so far.

The imposed pressure gradient is superimposed, and the peristalsis problem is remodeled. The solution is obtained using perturbation technique. The effects and the consequences of the imposed pressure gradient are investigated analytically. Hopefully, this will be a step further in the discussion of peristalsis flows.

### Mathematical formulation

Consider a steady two-dimensional boundary layer flow of a viscous and in compressible fluid with imposed pressure gradient along a horizontal channel bounded by two periodic sinusoidal boundaries having separation $2h$. The fluid is of density $\rho$, dynamic viscosity $\mu$ and with imposed pressure gradient $G$. The walls of the channel are flexible, on which are traveling, sinusoidal wave of small amplitude. In figure 1, $y_t = 1 + \varepsilon \sin(2\pi x)$ represents the top wavy boundary and $y_b = -(1 + \varepsilon \sin(2\pi x))$ represents bottom wavy boundary.

The above figure shows a long channel in which a peristaltic wave of contraction is produced by moving walls to the right at speed $c$. In peristalsis the sinusoidal wave form is assumed that propagates in one direction with a constant velocity. It is important to mention that the variable boundary give rise to an additional dimension in the modeling of the problem. For the present case, this makes the problem two dimensional instead of one dimensional.

We assume that the pressure gradient $G$ is a nonzero constant and that the upper boundary moves in the same direction as the pressure gradient with a constant velocity $c$. As the consequence of the variations in the channel width with $x$, the fluid must accelerate and decelerate, and the pressure gradient will therefore be a function of position. However, we can still impose a mean pressure gradient in the $x$ direction, which we denote as $-G$. We can then write

$$\frac{\partial p}{\partial x} = -G + \frac{\partial p'}{\partial x}, \quad (1)$$

$$\frac{\partial p}{\partial y} = \frac{\partial p'}{\partial y}. \quad (2)$$

**Law of Mass conservation:**

$$\frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial y} = 0, \quad (3)$$
Law of Momentum:

Momentum equation with imposed pressure gradient $G$ is presented as

In x-direction

$$
\rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = G - \frac{\partial p'}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),
$$

in y-direction

$$
\rho \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p'}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right).
$$

In the above equations $u$ and $v$ are velocities in $x$ and $y$ directions respectively and $2h$ is the channel’s width. Here the channel (of plane geometry) has been taken to be infinite in the $y$ plane. The boundaries are located at $y = 1/2$ and $y = -1/2$.

For the present model the relevant boundary conditions are given as

$$
\frac{\partial u}{\partial y} = 0 \text{ at } y = 0, \quad (6)
$$

$$
u = -c, v = 0 \text{ at } y = \pm h. \quad (7)
$$

We introduce the following dimensionless variable transformations

$$
\bar{u} = \frac{u}{c_x}, \bar{v} = \frac{v}{c_y}, \quad (8)
$$

$$
\bar{x} = \frac{x}{L}, \bar{y} = \frac{y}{d}, \quad (9)
$$

$$
c_y = c_x \frac{d}{L}, \bar{p} = \frac{p'}{GL}. \quad (10)
$$

Using the above transformation, the equation of continuity is comparatively satisfied and the remaining equations (4) and (5) along with boundary conditions (6) and (7) are reduced into the following non-dimensional partial differential equations.

$$
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \quad (11)
$$

$$
\rho' c_x d \frac{d^2 \bar{u}}{L} \left( \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = \frac{Gd^2}{\mu c_x} \left( 1 - \frac{\partial \bar{v}}{\partial \bar{x}} \right)
$$

$$+ \left( \frac{d^2 \bar{v}}{L^2 \partial^2 \bar{x}} + \frac{\partial^2 \bar{v}}{\partial^2 \bar{y}} \right), \quad (12)
$$

$$
\rho' c_x d \frac{d^2 \bar{v}}{L} \left( \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} \right) = -\frac{GL^2}{\mu c_x} \frac{\partial \bar{v}}{\partial \bar{y}} + \frac{d^2 \bar{v}}{L^2 \partial^2 \bar{x}} + \frac{\partial^2 \bar{v}}{\partial^2 \bar{y}}. \quad (13)
$$

The notation $\bar{u}, \bar{v}$ of the right-hand side of the (12) and (13) emphasize that this is a dimensionless parameter known as Reynolds number, multiplies by the geometric parameter $d/L$.

However, we do assume that the Reynolds number is vanishingly small compared even with the geometric amplitude parameter $\varepsilon$, which we shall also assume is small $\varepsilon \ll 1$, showing that the inertial effects can be neglected and that the flow is dominated by viscous and pressure gradient effects. The resulting problem is still of some practical importance.

For $Re \ll 1$

$$
0 = \frac{Gd^2}{\mu c_x} \left( 1 - \frac{\partial \bar{v}}{\partial \bar{x}} \right) + \left( \frac{d^2 \bar{v}}{L^2 \partial^2 \bar{x}} + \frac{\partial^2 \bar{v}}{\partial^2 \bar{y}} \right), \quad (14)
$$

$$0 = -\frac{GL^2}{\mu c_x} \frac{\partial \bar{v}}{\partial \bar{y}} + \frac{d^2 \bar{v}}{L^2 \partial^2 \bar{x}} + \frac{\partial^2 \bar{v}}{\partial^2 \bar{y}}. \quad (15)
$$

The transformed boundary conditions are

$$
\frac{\partial \bar{u}}{\partial \bar{y}} = 0 \text{ at } \bar{y} = 0, \quad (16)
$$

$$
\bar{u} = -1, \bar{v} = 0 \text{ at } \bar{y} = \pm h. \quad (17)
$$

Where $h = 1 + \varepsilon \sin(2\pi \bar{x})$.

We see that the dimensionless problem (and its solution) depends on a single dimensionless parameter $\frac{GL^2}{\mu c_x}$.

This parameter is just the ratio of the two possible velocity scales $\frac{\varepsilon d}{\mu}$ and $c_x $, one characterized by the magnitude of the pressure gradient and the other by the magnitude of the boundary velocity. The form of the velocity distribution (usually called the velocity profile) depends on the magnitude of the dimensionless ratio of velocities. When $\frac{\varepsilon d}{\mu} \ll 1$, in this case, the fluid motion is dominated by the motion of the boundary and the velocity profile reduces to a linear (simple) shear flow. When $\frac{\varepsilon d}{\mu} \gg 1$, on the other hand, that the quadratic contribution dominates the velocity profile.

Here we consider the domain perturbation method. In this section we seek the solution for limiting case $\varepsilon \ll 1$ by means of asymptotic expansions of the form

$$
\bar{u} = u_0 + \varepsilon u_1 + O(\varepsilon^2), \quad (18a)
$$

$$
\bar{v} = v_0 + \varepsilon v_1 + O(\varepsilon^2), \quad (18b)
$$
\( \nabla \rho = \varepsilon [G_1(\{ x \}, \{ y \})] + G_2(\{ x \}, \{ y \})] + O(\varepsilon^2). \)  \hfill (18c)

For this limiting case, the boundary conditions, can also be approximated in terms of asymptotically equivalent boundary conditions applied at \( y = \pm 1/2. \) Applying Taylor’s expansion on the boundary we have:

\[
u_{|y - 1 + \sin(2\pi x)} = (u_0 + \varepsilon u_1 + \varepsilon^2 u_2)_{y - \frac{1}{2}} + \varepsilon \sin(2\pi x) \frac{\partial}{\partial y} (u_0 + \varepsilon u_1)_{y - \frac{1}{2}} + \frac{1}{2} \varepsilon \sin(2\pi x)^2 \left( \frac{\partial^2 u_0}{\partial y^2} \right)_{y - \frac{1}{2}} + \ldots
\]

(19)

Using the above relation in equations and comparing the coefficients of \( \varepsilon \) up to first order, we obtain following set of equations:

**Zeroth order equations**

\[
\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0, \quad \frac{-Gd^2}{\mu c_x} = \frac{\partial^2 v_0}{\partial x^2}, \quad 0 = \frac{\partial^2 v_0}{\partial y^2} + \frac{d^2 \partial^2 v_0}{L^2 \partial x^2},
\]

(20)

\[
u_0 = -1, v_0 = 0 \text{ at } y = \pm \frac{1}{2}.
\]

**First order equations**

\[
\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \quad \frac{Gd^2}{\mu c_x} G_1(\tilde{x}, \tilde{y}) = \frac{\partial^2 u_1}{\partial y^2} + \frac{d^2 \partial^2 u_1}{L^2 \partial x^2}, \quad \frac{Gd^2}{\mu c_x} G_2(\tilde{x}, \tilde{y}) = \frac{\partial^2 v_1}{\partial y^2} + \frac{d^2 \partial^2 v_1}{L^2 \partial x^2},
\]

(21)

\[
u_1 = - \left( \frac{\partial u_0}{\partial y} \right) \frac{1}{2} \sin(2\pi x), \quad |v_1| = - \left( \frac{\partial v_0}{\partial y} \right) \frac{1}{2} \sin(2\pi x) \text{ at } y = \pm \frac{1}{2}.
\]

(22)

The Reynolds number is \( Re = \rho' \frac{u_0}{v_0}. \)

For the \( O(1) \) problem, the boundaries are flat; hence \( v_0 = 0. \)

### Perturbative solution

In this article we provide the solution of the formulated mathematical model by perturbation method. The focus has been on velocity along the channel. This method applies to problems in which the geometry of the flow or transport domain is irregular in the sense that the boundaries do not correspond to coordinate surfaces of any known analytic coordinate system but are nevertheless near to such coordinate surfaces. The basic idea is to replace the exact boundary condition with the approximate boundary condition that is asymptotically equivalent for \( \varepsilon \ll 1 \) but now applied at the coordinate surface \( y = \pm \frac{1}{2}. \)

**Zeroth order solution**

\[
u_0 = -1 + \frac{1}{2} \frac{Gd^2}{\mu c_x} \left( -y^2 + \frac{1}{4} \right).
\]

(28)

**First order solution**

This solution at \( O(\varepsilon) \) is the first correction for the geometry of sinusoidal wave and \( G_1 \) must be

\[
G_1 = C \sin(2\pi \tilde{x}),
\]

(29)

To solve this problem, we note that the boundary condition imposes a specific \( \tilde{x} \) dependence on the solution, which will be preserved throughout the domain. This suggests that we should a solution in the form

\[
u_1 = F(\tilde{y}) \sin(2\pi \tilde{x}).
\]

(30)

It is clear from (28) that the \( x \) component of the pressure gradient must have the same dependence on \( \tilde{x} \), where we have assumed that the coefficient \( C \) is a constant to be determined. We will, of course, need to verify the assumption \( C = \text{const} \) is sufficiently general by showing that all of the equations and the boundary conditions for \( u_1 \) and \( v_1 \) can be satisfied. The function \( F(\tilde{y}) \) must then satisfy the ODE

\[
\frac{\partial^2 F(\tilde{y}) \sin(2\pi \tilde{x})}{\partial y^2} + \frac{d^2 \partial^2 F(\tilde{y}) \sin(2\pi \tilde{x})}{L^2 \partial x^2} = \frac{Gd^2}{\mu c_x} C \sin(2\pi \tilde{x}),
\]

(31)

\[
\sin(2\pi \tilde{x}) \frac{\partial^2 F(\tilde{y})}{\partial y^2} + 2\pi d^2 \frac{\partial F(\tilde{y}) \cos(2\pi \tilde{x})}{\partial x} = \frac{Gd^2}{\mu c_x} C \sin(2\pi \tilde{x}).
\]

(32)

Simplifying equation (31) we obtain
\[ \frac{\partial^2 F(y)}{\partial y^2} - \left(2\pi\right)^2 \frac{d^2}{L^2} F(y) = \frac{Gd^2}{\mu c_x} C, \]  
where \( C \) is independent of \( \bar{x} \).

The general solution that satisfies the symmetry is
\[ F(y) = A \exp\left(\frac{2\pi d\bar{y}}{L}\right) + B \exp\left(-\frac{2\pi d\bar{y}}{L}\right) + C^* \frac{Gd^2}{\mu c_x}, \]  
where \( C^* = \frac{CL^2}{(2\pi d)^2} \).

The boundary condition then requires that
\[ F = \frac{1}{4} \text{ at } \bar{y} = \frac{1}{2}. \]

Hence, the boundary condition then requires that
\[ F = \frac{1}{4} \text{ at } \bar{y} = \frac{1}{2}. \]

Hence,
\[ F(y) = \frac{Gd^2}{\mu c_x} \left[ \left(\frac{1}{4} - C^*\right) \frac{\cosh\left(\frac{2\pi d\bar{y}}{L}\right)}{\cosh\left(\frac{2\pi d\bar{y}}{L}\right)} + C^* \right]. \]  

The constant \( C^* \) can be determined from the “mass conservation” constraint that the volume flux must be independent of \( \bar{x} \). This means that
\[ \int_0^1 12 \left[ \bar{u}_0 + \epsilon \bar{u}_1 + O(\epsilon^2) \right] d\bar{y} = \text{const}, \]  
that is independent of \( \bar{x} \), because \( \bar{u}_1 = F(y) \sin\left[\frac{2\pi y}{L}\right] \bar{x} \), this implies
\[ \int_0^1 F(y) d\bar{y} = 0. \]  

To satisfy this condition, we must determine a specific value for \( C^* \) such that
\[ \bar{u} = \bar{u}_0 + \epsilon \bar{u}_1 + O(\epsilon^2), \]  
\[ \bar{u} = -1 + \frac{1}{2} \frac{Gd^2}{\mu c_x} \left( -\bar{y}^2 + \frac{1}{4} \right) + \epsilon(\sin(2\pi \bar{x})) \]
\[ - \frac{Gd^2}{\mu c_x} \left(\frac{1}{4} - C^*\right) \frac{\cosh\left(\frac{2\pi d\bar{y}}{L}\right)}{\cosh\left(\frac{2\pi d\bar{y}}{L}\right)} + C^* \].

The only remaining issue is to determine the \( O(\epsilon) \) contribution to \( \bar{v} \) to satisfy the continuity equation. To solve for \( \bar{v}_1 \) we first note from (23) that
\[ \bar{v}_1 = \frac{Gd^2}{\mu c_x} 2\pi \cos(2\pi \bar{x}) \]
\[ \int_0^1 \left[ \frac{1}{4} - C^* \right] \frac{\cosh\left(\frac{2\pi d\bar{y}}{L}\right)}{\cosh\left(\frac{2\pi d\bar{y}}{L}\right)} + C^* \]  

Hence

\[ \bar{v}_1 = \frac{Gd^2}{\mu c_x} 2\pi \cos(2\pi \bar{x}) \]
\[ \int_0^1 \left[ \frac{1}{4} - C^* \right] \frac{\cosh\left(\frac{2\pi d\bar{y}}{L}\right)}{\cosh\left(\frac{2\pi d\bar{y}}{L}\right)} + C^* \]  
\[ \bar{v} \]
Results

This section deals with the physical interpretation of the analytical results obtained in Section 3. The velocity of the fluid flow is analyzed. The influence of the imposed pressure on the viscous fluid flow in the corrugated channel is particularly of interest. The 3D-graphical illustration of the modification of the axial velocity in the channel is provided in Figures 2 to 8. These graphs describe the behavior of imposed pressure gradient and the velocity profile of peristaltic flow of a Newtonian fluid with different values of $\varepsilon$. Figures 2 and 3 depicts the velocity $u_0$ and $u_1$. Figures 4 to 8 represents velocity component $u$ in $x$-direction with different values of perturbation parameter $\varepsilon$. Figure 9 presents velocity $v_1$ of fluid in $y$ direction. Whereas $v_0$ is zero.

The solution that we have generated is quite complicated. The fluid velocity in the $x$-direction, $u_0 + \epsilon u_1$, decreases where the channel expands and increases where it contracts to maintain constant-volume flux along the channel. For $\varepsilon = 0$, $v_0 = 0$. The flow resistance increases with the corrugation wavenumber and consequently decreases the flow. There is an increase in velocity by increasing external pressure. In general, these results could have a potential application in

Figure 4. Velocity profile $u$ for $\varepsilon = 0$.

Figure 5. Velocity profile $u$ for $\varepsilon = 0.025$.

Figure 6. Velocity profile $u$ for $\varepsilon = 0.05$.

Figure 7. Velocity profile $u$ for $\varepsilon = 0.075$.
enhancing fluid transport in physical, biological and engineering processes.

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Appendix

Nomenclature

\( \mu \) \quad \text{dynamic viscosity}

\( G \) \quad \text{imposed pressure gradient}

\( p \) \quad \text{pressure}

\( L \) \quad \text{length of channel}

\( \rho \) \quad \text{fluid density}

\( Re \) \quad \text{Reynolds number}

\( d \) \quad \text{width of channel}

\( c \) \quad \text{characteristic velocity}