Lattice Calculation of the Strangeness Magnetic Moment of the Nucleon

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Abstract

We report on a lattice QCD calculation of the strangeness magnetic moment of the nucleon. Our result is $G_M^s(0) = -0.36 \pm 0.20$. The sea contributions from the u and d quarks are about 80\% larger. However, they cancel to a large extent due to their electric charges, resulting in a smaller net sea contribution of $-0.097 \pm 0.037 \mu_N$ to the nucleon magnetic moment. As far as the neutron to proton magnetic moment ratio is concerned, this sea contribution tends to cancel out the cloud-quark effect from the Z-graphs and result in a ratio of $-0.68 \pm 0.04$ which is close to the SU(6) relation and the experiment. The strangeness Sachs electric mean-square radius $\langle r_s^2 \rangle_E$ is found to be small and negative and the total sea contributes substantially to the neutron electric form factor.

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The strangeness content of the nucleon has been a topic of considerable recent interest for a variety of reasons. The studies of nucleon spin structure functions in polarized deep inelastic scattering experiments at CERN and SLAC \[1\], combined with neutron and hyperon $\beta$ decays, have turned up a surprisingly large and negative polarization from the strange quark. In addition, there is a well-known long-standing discrepancy between the pion-nucleon sigma term extracted from the low energy pion-nucleon scattering \[2\] and that from the octect baryon masses \[3\]. This discrepancy can be reconciled if a significant $\bar{s}s$ content in the nucleon \[1, 3\] is admitted.

This naturally leads to the question that if the strange quarks can contribute substantially in the axial-vector and scalar current matrix elements, how important are they in other matrix elements involving the vector, pseudoscalar, and tensor currents. The case of the vector current matrix element $\langle N | \bar{s} \gamma_{\mu} s | N \rangle$ is especially interesting. If the strange magnetic moment (m.m.) is large, it is likely to spoil the nice SU(6) prediction of the neutron to proton m.m. ratio of $-3/5$ which lends credence to the valence quark picture. On the other hand, if it is small one would like to understand why it should be different from the axial-vector and scalar cases.

To address some of these issues, an experiment to measure the neutral weak magnetic form factor $G^Z_M$ via elastic parity-violating electron scattering at backward angles was recently carried out by the SAMPLE collaboration \[5\]. The strangeness magnetic form factor is then obtained by subtracting out the nucleon magnetic form factors $G^p_M$ and $G^n_M$. The reported value is $G^s_M(Q^2 = 0.1\text{GeV}^2) = +0.23 \pm 0.37 \pm 0.15 \pm 0.19$, where the last error is due to an uncertainty associated with axial radiative corrections. Future experiments have the promise of tightening the errors and isolate the radiative corrections so that we can hope to have a well-determined value and sign for $G^s_M(0)$.

Theoretical predictions of $G^s_M(0)$ vary widely. The values from various models and analyses vary from $-0.75 \pm 0.30$ in a QCD equalities analysis \[6\] to $+0.37$ in an $SU(3)$ chiral bag model\[7\]. While a few give positive values \[4, 8\], most model predictions are negative with a typical range of $-0.25$ to $-0.45$ \[6, 8, 9, 10, 11, 12, 13, 14\]. Summaries of these predictions can be found in Refs. \[6, 14\]. A similar situation exists for the strangeness electric mean-square radius $\langle r^2_s \rangle_E$. A number of the predictions are positive \[5, 13\], while the others are negative \[8, 10, 11, 12, 14\]. Elastic $e^p$ and $e^4He$ parity-violation experiments are currently planned at TJNAF \[15\] to measure the asymmetry $A_{LR}$ at forward angles to extract the strangeness electric mean-square radius. Hopefully, they will settle the issue of the sign of $\langle r^2_s \rangle_E$.

In view of the large spread of theoretical predictions for both $G^s_M(0)$ and $\langle r^2_s \rangle_E$ and in view of the fact that the experimental errors on $G^s_M(0)$ are still large, it is clearly important to perform a lattice calculation of this quantity since this is a first principles theoretical approach. We present our calculation from lattice QCD in the hope that this will shed some light on these quantities. Our previous results on flavor-singlet quantities which involve the so-called “disconnected insertions” (DI) for the sea quarks in addition to the “connected insertions” (CI) for the valence and cloud quarks \[16, 17\] reveal that the sea quark contribution to the flavor-singlet $g_A^0$ from
the DI is negative and the magnitude large enough (e.g. the strangeness polarization $\Delta s = 0.12 \pm 0.01$) to cancel the positive CI contribution to a large extent. This results in a small $g_0^A$ at $0.25 \pm 0.12$, which is in agreement with the experimental results [1]. Similarly, the calculated ratio $y = \langle N|ss|N \rangle / \langle N|\bar{u}\bar{u} + \bar{d}d|N \rangle = 0.36 \pm 0.03$ [17] gives the right amount of strangeness content to resolve the $\pi N\sigma$ puzzle we alluded to earlier. Given these reasonably successful estimates of strangeness in the axial-vector and scalar channels, we feel that it should yield meaningful results in the vector current as well. In particular, we would like to understand why the SU(6) valence quark picture fails badly in the flavor-singlet axial-vector and scalar cases and yet gives an apparently good prediction in the neutron to proton m.m. ratio – a yet unresolved puzzle in low-energy hadron physics.

The lattice formulation of the electromagnetic form factors has been given in detail in the past [18, 19]. Here, we shall concentrate on the DI contribution, where the strangeness current contributes. In the Euclidean formulation, the Sachs electromagnetic form factors can be obtained by the combination of two- and three-point functions

$$G_{\alpha\alpha}^{NN}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \langle 0 | \bar{\chi}^\alpha(x) \chi^\alpha(0) | 0 \rangle,$$

$$G_{\alpha\beta}^{NV}(t_f, \vec{p}, t, \vec{q}) = \sum_{\vec{x}_f, \vec{x}} e^{-i\vec{p} \cdot \vec{x}_f + i\vec{q} \cdot \vec{x}} \langle 0 | \bar{\chi}^\alpha(x_f) V_\mu(x) \chi^\beta(0) | 0 \rangle,$$

where $\chi^\alpha$ is the nucleon interpolating field and $V_\mu(x)$ the vector current. With large Euclidean time separation, i.e. $t_f - t >> a$ and $t >> a$, where $a$ is the lattice spacing,

$$\frac{\Gamma^\beta_i G_{\alpha\beta}^{NN}(t_f, \vec{0}, t, \vec{q}) G_{\alpha\alpha}^{NN}(t, \vec{0})}{G_{\alpha\alpha}^{NN}(t_f, \vec{0})} \rightarrow \frac{\varepsilon_{ijk} q_k}{E_q + m} G_M(q^2),$$

$$\frac{\Gamma_E^\beta G_{\alpha\beta}^{VN}(t_f, \vec{0}, t, \vec{q}) G_{\alpha\alpha}^{NN}(t, \vec{0})}{G_{\alpha\alpha}^{NN}(t_f, \vec{0})} \rightarrow G_E(q^2),$$

where $\Gamma_i = \left( \begin{array}{cc} \sigma_i & 0 \\ 0 & 0 \end{array} \right)$, and $\Gamma_E = \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right)$.

We shall use the conserved current from the Wilson action which, being point-split, yields slight variations on the above forms and these are given in Ref. [18, 19]. Our 50 quenched gauge configurations were generated on a $16^3 \times 24$ lattice at $\beta = 6.0$. In the time direction, Dirichlet boundary conditions were imposed on the quarks to provide larger time separations than available with periodic boundary conditions. We also averaged over the directions of equivalent lattice momenta in each configuration; this has the desirable effect of reducing error bars. Numerical details of this procedure are given in Refs. [19, 21]. The dimensionless nucleon masses $M_N a$ for $\kappa = 0.154, 0.152, \text{and } 0.148$ are $0.738(16), 0.882(12),$ and $1.15(1)$ respectively. The corresponding dimensionless pion masses $m_\pi a$ are $0.376(6), 0.486(5),$ and $0.679(4)$. Extrapolating the nucleon and pion masses to the chiral limit we determine $\kappa_c = 0.1567(1)$ and the dimensionless nucleon mass at the chiral limit to be $0.547(14)$. Using the nucleon mass to set the scale, which we believe to be appropriate for studying nucleon properties [21].
The lattice spacing \( a^{-1} = 1.72(4) \) GeV is determined. The three \( \kappa' \)'s then correspond to quark masses of about 120, 200, and 360 MeV respectively.

The strangeness current \( \bar{s}\gamma_\mu s \) contribution appears in the DI only. In this case, we sum up the current insertion time slice \( t \) from the nucleon source to the sink in Eqs. (3) and (4) to gain statistics \([16, 17]\). This leads to \( \text{const} + t_f G_{E, \text{dis}}(q^2) \) for Eq. (4). For Eq. (3), we average over the three spatial components \( \bar{s}\gamma_\mu s \) and obtain \( \text{const} + t_f G_{E, \text{dis}}(q^2) \). Similar to our previous studies of \( \Delta s \) \([16]\) and \( \langle N|\bar{s}s|N \rangle \) \([17]\), we use 300 complex \( Z_2 \) noises \( [22] \) and 100 gauge configurations to calculate the sea quark contribution (DI) with \( \kappa = 0.148, 0.152 \) and 0.154. In calculating the strange current, we have considered the correlation between the quark loop with \( \kappa_s = 0.154 \) and the valence quarks at \( \kappa_v = 0.148, 0.152 \), and 0.154. The ratio in Eq. (3) with the sum in \( t \) and average in \( V_i \), which leads to the expression \( \text{const} + t_f G_{M, \text{dis}}(q^2) \), is plotted in Fig. 1 as a function of \( t_f \) for \( |\vec{q}| = 2\pi/La \). Then \( G_{M, \text{dis}}(q^2) \) from the DI is obtained from fitting the slopes in the region \( t_f \geq 8 \) where the nucleon is isolated from its excited states with the correlation among the time slices taken into account \([16]\). The resultant straight-line fits covering the ranges of \( t_f \) with the minimum \( \chi^2 \) are plotted in Fig. 1. Finally, the errors on the fit, also shown in the figure, are obtained by jackknifing the procedure. To obtain the physical \( G_M^*(q^2) \), we extrapolate the valence quarks to the chiral limit while keeping the sea quark in the loop at the strange quark mass (i.e. \( \kappa_s = 0.154 \)). It has been shown in the chiral perturbation theory with a kaon loop that \( G_M^*(0) \) is proportional to \( m_K \), the kaon mass \([23]\). Thus, we extrapolate with the form \( C + D\sqrt{\hat{m}} + m_s \) where \( \hat{m} \) is the average u and d quark mass and \( m_s \) the strange quark mass to reflect the \( m_K \) dependence. This is the same form adopted for extracting \( \langle N|\bar{s}s|N \rangle \) in Ref. \([17]\), which also involves a kaon loop in the chiral perturbation theory.

![Figure 1](image-url)  

**Figure 1:** The ratio of Eq. (3) as a function of \( t_f \) so that the slope is \( G_{M, \text{dis}}(q^2) \) at \( |\vec{q}| = 2\pi/La \). The sea quark is fixed at \( \kappa_s = 0.154 \), the strange quark mass, and the valence quark masses are at \( \kappa_v = 0.148, 0.152, \) and 0.154. \( M \) is the fitted slope.

Plotted in Fig. 2(a) is the extrapolated \( G_M^*(q^2) \) at 4 nonzero \( q^2 \) values. The errors are again obtained by jackknifing the extrapolation procedure with the covariance.
matrix used to include the correlation among the three valence $\kappa$’s. In view of the fact that the scalar current exhibits a very soft form factor for the sea quark (i.e. $g_{S,\text{dis}}(q^2)$) which has been fitted well with a monopole form [17], we shall similarly use a monopole form to extrapolate $G_M^s(q^2)$ with nonzero $q^2$ to $G_M^s(0)$. Indicated as $\diamond$ in Fig. 2(a), we find $G_M^s(0) = -0.36 \pm 0.20$. Again, the correlation among the $q^2$ are taken into account and the error is from jackknifing the fitting procedure. This is consistent with the recent experimental value within errors (see Table 1). The monopole mass is found to be $0.58 \pm 0.16 m_N$. To explore the uncertainty of the $q^2$ dependence, we also fitted $G_M^s(q^2)$ with a dipole form and found $G_M^s(0) = -0.27 \pm 0.12$ with a dipole mass $m_D/m_N = 1.19 \pm 0.22$. Similar results are obtained for u and d quarks with monopole fits. They turn out to be $G_u/d_M,\text{dis}(0) = -0.65 \pm 0.30$, which is about 1.8 times the size of $G_M^s(0)$. These are tabulated in Table 1.

![Figure 2](image-url)

Figure 2: (a) Strange magnetic form factor $G_M^s(q^2)$. $G_M^s(0)$, indicated by $\diamond$, is obtained from a monopole fit. (b) Strangeness electric form factor $G_E^s(q^2)$. The solid line is a fit with the monopole form shown in the figure and the dashed line is obtained with the monopole mass $m_M$ from $G_M^s(q^2)$ in (a).

Now, we are ready to address the question of why the SU(6) relation is badly broken in the scalar current (e.g. $F_S$, $D_S$) and axial current (e.g. $g_A^0$) and yet is so good for the neutron-proton magnetic moment ratio $\mu_n/\mu_p$. The lattice calculations for the scalar [17] and axial [18] currents reveal the fact that the SU(6) breaking comes from both the sea quarks in the DI and the cloud quarks in the CI. We shall see how these degrees of freedom play out in the case of the m.m. We first plot in Fig. 3 the ratio $\mu_n/\mu_p$ for the CI part (shown as $\diamond$) as a function of the valence quark mass. We see that when the quark mass is near the charm region ($m_q a$ at 0.55 corresponds to $m_q \sim 1$ GeV), the ratio is close to the SU(6) prediction of -2/3. This is quite reasonable as this is in the non-relativistic regime where one expects SU(6) to work well. As the quark mass comes down to the strange region ($m_q a = 0.07$), the ratio becomes less negative. Extrapolated to the chiral limit, the ratio is $-0.616 \pm 0.022$ which deviates from the SU(6) prediction by 8%. We understand this deviation as
mainly due to the cloud quark effect in the Z-graphs. As we switch off these Z-graphs in a valence approximation \[16, 17\], the ratio (plotted as ◆ in Fig. 3) becomes closer to the SU(6) value which resembles the non-relativistic case. Similar behaviors were observed for the scalar and axial matrix elements \[17, 16\]. Now we add the sea quark contribution from the DI to give $\mu_{\text{dis}} = \left( \frac{2}{3} G^u_M(0) - \frac{1}{3} G^d_M(0) - \frac{1}{3} G^s_M(0) \right) \mu_N$ to the CI and find that it tends to cancel the cloud effect and bring the ratio back to be similar to what the valence approximation predicts. For the $G^s_M(0)$ at various $\kappa_v$, we use the $G^u_M(0)/G^u_{M,\text{dis}}(0)$ ratio from the chiral limit to obtain it from the $G^u_{M,\text{dis}}(0)$ at each $\kappa_v$. At the chiral limit, when the total sea contribution $\mu_{\text{dis}} = -0.097 \pm 0.037 \mu_N$ is added to the CI, the $\mu_n/\mu_p$ ratio then comes down to $-0.68 \pm 0.04$ which is consistent with the experimental value of 0.685. We note that the $\mu_n/\mu_p$ ratio for the full result (●) is more negative at the chiral limit compared with those at other $m_qa$. This has to do with the fact that the CI employs the linear quark mass extrapolation, as do other observables for the CI \[16, 17, 21\], whereas the DI uses the $\sqrt{m_q}$ dependence for the chiral extrapolation as mentioned above. From this analysis, we see that although the individual $G^u_{M,\text{dis}}(0), G^d_{M,\text{dis}}(0),$ and $G^s_{M}(0)$ are large, their net contribution $\mu_{\text{dis}} = -0.097 \pm 0.037 \mu_N$ to $\mu_n$ and $\mu_p$ is much smaller because of the partial cancellation due to the quark charges of u, d, and s. The sea contribution turns out to be further canceled by the cloud effect to bring the $\mu_n/\mu_p$ ratio close to the experimental value and the SU(6) relation. Barring any known symmetry principle yet to surface, this cancellation is probably accidental and in stark contrast with the $\pi N\sigma$ term and flavor-singlet $g^0_A$ where the cloud and sea effects add up to enhance the SU(6) breaking \[17, 16\].

![Figure 3: Neutron to proton m.m. ratio $\mu_n/\mu_p$ as a function of the dimensionless quark mass $m_qa$. The ◆ indicates the valence result. The ○ is the result for the CI and ● indicates the full result with the inclusion of the sea from DI.](image-url)
The sea contribution from the u, d, and s quarks \(G^u,d_M(q^2)\) and \(G^s_M(q^2)\) are added to the valence and cloud part in the CI, \(G_{M,\text{con}}(q^2)\), to give the full \(G^p_M(q^2)\) and \(G^n_M(q^2)\). They are plotted in Fig. 4(a) and 4(b) and indicated by \(\bullet\). Also plotted are the \(G_{M,\text{con}}(q^2)\) (denoted by \(\circ\)) and the experimental fits (in solid line). We see from Fig. 4 and Table 1 that \(\mu_p\) and \(\mu_n\) are smaller than the experimental results by \(\sim 6\%\) in absolute values. This is presumably due to the systematic errors of the finite volume, finite lattice spacing, and the quenched approximation. We should point out that in the earlier discussion of the neutron to proton m. m. ratio \(\mu_n/\mu_p\), the systematic errors are expected to be cancelled out in the ratio to a large extent. Our conclusion of the ratio \(\mu_n/\mu_p\) in the preceding paragraph is thus based on this assumption.

![Graph](image)

**Figure 4:** (a) Proton magnetic form factor \(G^p_M(q^2)\). \(\circ\) indicates the result from the CI. They are shifted slightly to the left in \(-q^2\) to avoid overlap with the full result which is shown as \(\bullet\). The solid line is the fit to the experiment [24]. (b) the same as in (a) for the neutron form factor \(G^n_M(q^2)\).

A similar analysis is done for the strange Sachs electric form factor \(G^s_E(q^2)\). This is plotted in Fig. 2 (b). We see that \(G^s_E(0)\) is consistent with zero as it should be. This serves as a test of the stochastic noise estimation with the \(Z_2\) noise. We fitted \(G^s_E(q^2)\) with the form \(G^s_E(q^2) = f q^2/m^2_N/(1 - q^2/m^2_M)\) (solid line in Fig. 2(b)). The resultant electric mean-square radius \(\langle r^2_s \rangle_E = 6 \frac{dG^s_E(q^2)}{dq^2}|_{q^2=0} = -0.061 \pm 0.003\,\text{fm}^2\). This is shown in Table 1.

In view of the large errors, we also plot the above form for \(G^s_E(q^2)\) with the monopole mass \(m_M\) taken from \(G^s_M(q^2)\) which is shown by the dashed line. This gives \(\langle r^2_s \rangle_E = -0.16 \pm 0.06\,\text{fm}^2\) with \(\chi^2/N_{DF} = 0.24\). This shows that that the uncertainty in the fitting can be as large as a factor of two. Nevertheless, \(\langle r^2_s \rangle_E\) is relatively small. This small negative value in \(\langle r^2_s \rangle_E\) and large negative \(G^s_M(0)\) are consistent with the kaon loop picture [11] and VMD [12] but is inconsistent with most of the other model predictions [3, 14].
Since the DI of $u$ and $d$ quarks are slightly larger than that of the $s$ quark, the total sea contribution $G_{E,\text{dis}}(q^2) = 2/3 G_{E,\text{dis}}^u(q^2) - 1/3 G_{E,\text{dis}}^d(q^2) - 1/3 G_{E,\text{dis}}^s(q^2)$ adds a small positive value to the valence and cloud part $G_{E,\text{con}}(q^2)$ in the CI. The proton $G_{E}^p(q^2)$ and neutron $G_{E}^n(q^2)$ are plotted in Figs. 5(a) and 5(b) respectively. We see that the CI part $G_{E,\text{con}}^p(q^2)$ (shown as ○ in Fig. 5(a)) gives the main contribution in proton. $G_{E,\text{dis}}^p(q^2)$ adds only a little change to it. The resultant dipole fit gives a dipole mass of $0.857 \pm 0.031$ GeV (Table 1). This is consistent with the experimental dipole mass of 0.842 GeV. In the case of the neutron, since $G_{E,\text{con}}^n(q^2)$ itself (○ in Fig. 5(b)) is small, the sea contribution $G_{E,\text{dis}}^n(q^2)$ becomes a sizable part of the total $G_{E}^n(q^2)$ (● in Fig. 5(b)). We see that when the sea is included we have a reasonably good match with the experimental results (solid line in Fig. 5(b)). The total mean square charge radius of $-0.123 \pm 0.019$ fm$^2$ is obtained from fitting with the form $G_{E}^n(q^2) = f \frac{q^2}{4m_N^2}(1 - q^2/M_N^2)^2/(1 - 5.6q^2/4M_N^2)$ which has been used to fit the experimental results [24]. This is consistent with the experimentally fitted result of $-0.127$ fm$^2$.

![Figure 5](image)

Figure 5: (a) Proton electric form factor $G_{E}^p(q^2)$. ○ shows the result from the CI. They are shifted slighted to the left in $-q^2$ to avoid overlap with the full result which is shown as ●. The solid line is the fit to the experiment [24]. (b) the same as (a) for the neutron form factor $G_{E}^n(q^2)$.

In summary, we have calculated the $s$ and $u$, $d$ contributions to the electric and magnetic form factors of the nucleon. The individual m.m. and electric form factors from the different flavors in the sea are not small, however there are large cancellations among themselves due to the electric charges of the $u$, $d$, and $s$ quarks. We find that a negative $G_{M}^s(0)$ leads to a total negative sea contribution to the nucleon m.m. which cancels the cloud effect to make the $\mu_n/\mu_p$ ratio consistent with the experiment. We also find $G_{E}^n(q^2)$ positive and leads to a positive total sea contribution to the neutron electric form factor $G_{E}^n(q^2)$. Future calculations are needed to investigate the systematic errors associated with the finite volume and lattice spacing as well as the quenched approximation.
Table 1: Strangeness and proton-neutron m.m. and charge radii in comparison with experiments.

|                          | Lattice | Experiments                                      |
|--------------------------|---------|--------------------------------------------------|
| $G^s_M(0)$               | $-0.36 \pm 0.20$ | $G^s_M(Q^2 = 0.1\text{GeV}^2) = 0.23 \pm 0.37 \pm 0.15 \pm 0.19$ [5] |
| $G^s_{M,\text{dis}}(0)$ | $-0.65 \pm 0.30$ |                                                 |
| $\mu_{\text{dis}}$      | $-0.097 \pm 0.037 \mu_N$ |                                                 |
| $\mu_p$                 | $2.62 \pm 0.07 \mu_N$ | $2.79 \mu_N$                                    |
| $\mu_n$                 | $-1.81 \pm 0.07 \mu_N$ | $-1.91 \mu_N$                                   |
| $\mu_n/\mu_p$           | $-0.68 \pm 0.04$ | $-0.685$                                         |
| $\langle r_s^2 \rangle_E$ | $-0.061(3) \ldots -0.16(6) \text{fm}^2$ | $0.659 \text{fm}^2$ [24]                        |
| $\langle r_p^2 \rangle_E$ | $0.636 \pm 0.046 \text{fm}^2$ |                                                 |
| $\langle r_n^2 \rangle_E$ | $-0.123 \pm 0.019 \text{fm}^2$ | $-0.127 \text{fm}^2$ [24]                        |

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$\kappa_v = 0.154$

\[ \chi^2 = 0.02 \]
$G_M^s(q^2) = \frac{G_M^s(0)}{1-q^2/m_M^2}$

$G_M^s(0) = -0.36 \pm 0.20$

$m_M/m_N = 0.58 \pm 0.16$

$\chi^2 = 0.03$
$G_M^p(q^2)$

--- Expt: $m_D$=0.842 GeV

• Sea + CI

○ Connected Insertion ($q^2$ shifted)

\[ G_M^p(q^2) \]
\[ G_E^p(q^2) \]

--- Expt: \( m_D = 0.842 \text{GeV} \)

- Sea + CI
- Connected Insertion (\( q^2 \) shifted)

\[ -q^2 \text{ (GeV}^2) \]
\[ G_E^s(q^2) = \frac{f q^2/m_N^2}{1-q^2/m_M^2} \]
$G_M^n(q^2)$ --- Expt: $m_D = 0.842\text{GeV}$

$\bigcirc$ Connected Insertion ($q^2$ shifted)

$\bullet$ Sea + CI
$G_E^n(q^2)$

--- Expt.

$\bullet$ Sea + CI

$\circ$ Connected Insertion ($q^2$ shifted)
$\kappa_v = 0.148$

\[\chi^2 = 0.1\]
\( \frac{\mu_n}{\mu_p} \)

- Connected Insertion
- Valence
- Expt. \(-0.685\)
- Sea + CI