Quantum entanglement and modulation enhancement of free-electron–bound-electron interaction

Zhexin Zhao¹, Xiao-Qi Sun²,³, and Shanhui Fan¹*

1Ginzton Laboratory, 348 Via Pueblo, Stanford University, CA, 94305, USA
2Department of Physics, McCullough Building, Stanford University, Stanford, CA 94305, USA
3Department of Physics, Institute for Condensed Matter Theory, University of Illinois at Urbana-Champaign, IL 61801, USA

*shanhui@stanford.edu

Abstract: We study the modulation enhancement of interaction and entanglement between distant atoms in the interaction between free electron and two-level atom. The interactions of free electrons with matters have provided a number of technologies for the study of material and photonic systems. Recent experimental developments on quantum engineering show that ultra-fast laser technique can modulate a single free-electron wave function [1]. Therefore, it is a timely and important problem to investigate how such quantum engineering can be used to enhance and tailor electron-material interaction and to create new functionalities [2,3].

In this presentation, we apply a full quantum theory to study the dynamics of a free electron passing by a two-level atom (Fig. 1(a)). Our study features a new approach to probe atom coherence from modulation of the free-electron wave function. In presence of modulation of the free-electron wave function, we show that electron energy loss spectrum is greatly enhanced for a coherent initial state of the atom. For a dilute beam of modulated electrons interacting with the atom, the quantum theory demonstrates the enhanced interaction due to resonant modulation and provides physical foundation for the semi-classical results of the Rabi oscillation of the atom. Apart from the enhancement of interaction, our study also shows opportunities to create entanglement between two atoms by measuring the free-electron energy after it interacts with both atoms (Fig. 1(b)) [4].

We consider the scattering problem between a free electron with a two-level system (Fig. 1(a)). The simplified Hamiltonian is

$$H = \sum_{\alpha=1,2} E_{\alpha} c_{\alpha}^\dagger c_{\alpha} + \sum_k E^\text{free}_{k} c_k^\dagger c_k + \sum_q b_q [g_{21}(q) \sigma^+ + g_{12}(q) \sigma^-],$$

(1)

where $\alpha=1,2$ represents the bound states of the two-level system, $c_{\alpha}^\dagger$ and $c_{\alpha}$ are the creation and annihilation operators, $\sigma^+ = c_2^\dagger c_1$, $\sigma^- = c_1^\dagger c_2$, and $b$ is the electron ladder operator $b_q = \sum_{k} c_{2-k}c_2$ [1]. This ladder operator down-shifts the electron momentum by $q$ when acting on a single electron wave function. The coupling between the free electron and the bound state electron ($g_{ij}$) can be generally derived from the Coulomb interaction. The

Fig. 1. (a) Schematic of the interaction between an electron (blue wave packet) and a two-level system (orange circle). The insert shows a schematic of a train of electrons interacting with the two-level atom. (b) Schematic of an electron interacting with two atoms to create entanglement. Spectrum change of the free electron interacting with a two-level system ($|\psi_{\alpha}\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$), for a quasi-monochromatic electron (c-e) or a modulated electron (f-h).
interaction is generically weak and can be treated perturbatively. Thus, only the electron ladder operator with $q = \omega_0 / v_0$ matches the two-level system transition and contributes to the scattering. Here, $\omega_0$ is the transition frequency of the two-level system and $v_0$ is the velocity of the electron wave packet. To second order in the dimensionless coupling coefficient $g$, which is proportional to $g_{21}(\omega_0/v_0)$, the scattering matrix is

$$S \approx (1 - \frac{1}{2} |g|^2) I - i (g b \sigma^+ + g^* b^\dagger \sigma^-),$$

where $I$ is the identity operator, $b = b_q e^{-i \omega_0 t / m}$.

Equipped with the scattering matrix, we first study the perturbation on the two-level system induced by the free-electron scattering. The change in the density matrix of the two-level system is

$$\Delta \rho = -iM \rho_{ab},$$

where $s = (s_1)$ and $s_2 = (s_2^*)$ are parameters determined by the state of the incident electron. We find that the induced change on the bound-electron state is characterized by

$$\langle \Psi_i \rangle \Delta \sigma_n = \langle \rho_s \rangle \sigma_n \langle \Psi_i \rangle.$$

Thus, the free-electron spectrum change is $\Delta \rho_e = (k|\Delta \rho_e|k)$. We find that the energy exchange between the free electron and the two-level system depends on the initial states of the two-level system and the free electron. When $(k|b \rho^a_e|k) \neq 0$, the change in electron energy spectrum is proportional to the off-diagonal elements of the two-level system density matrix to the first order in $g$, which is absent for a quasi-monochromatic electron beam. As an demonstration, we study the free-electron spectrum change of a modulated free electron interacting with a two-level system in a superposition state $|\Psi_s\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. We assume that the electron is modulated at resonant frequency $\omega_0$ with strength $g_m = 3$ and drifts a length $L_F = 3.4$ mm for maximal $|b|^2$ value. Before interacting with the two-level system, the electron spectrum is shown in Fig. 1(f), which is a typical PINEM spectrum [1]. After the interaction, the spectrum change that is proportional to $|g|^2 (|g|^2)$ is plotted in Fig. 1(g) (Fig. 1(h)), where we assume a typical value $g = 1 \times 10^{-3}$. In comparison, we study a Gaussian wave packet with small initial energy spread ($\sigma_\epsilon \ll \omega_0 / v_0$ (Fig. 1((i))) interacting with the same two-level system in state $|\Psi_s\rangle$. After the interaction, the spectrum change is proportional to $|g|^2$, with zero contribution proportional to $|g|$ (Fig. 1(d-e)).

Two separated two-level atoms can be entangled through the interaction with the same free electron. As illustrated in Fig. 1(b), a single electron interacts in sequence with two-level atom 1 and 2, which have the same transition frequency $\omega_0$. Suppose that both atoms are at the ground state and the free electron is quasi-monochromatic before the interaction, and the coupling coefficient between the electron and atom 1 (2) is $g_1 (g_2)$. After the interactions, if certain energy of the electron is measured, the two atoms are in an entangled state. For instance, when the electron has initial energy $E$ and final energy $E - h \omega_0$, the two atoms are in the entangled state

$$\frac{1}{\sqrt{|g_1|^2 + |g_2|^2}} \left[ g_2 |\Psi_1\rangle \left( \sigma^+ |\Psi_2\rangle \right) + g_1 \left( \sigma^- |\Psi_1\rangle \right) |\Psi_2\rangle \right].$$

References

1. Armin Feist, Katharina E Echternkamp, Jakob Schauss, Sergey V Yalunin, Sascha Schäfer, and Claus Ropers. Quantum coherent optical phase modulation in an ultrafast transmission electron microscope. Nature, 521(7551):200–203, 2015.
2. Yiming Pan and Avraham Gover. Spontaneous and stimulated emissions of a preformed quantum free-electron wave function. Physical Review A, 99(5):052107, 2019.
3. Avraham Gover and Amnon Yariv. Free-electron–bound-electron resonant interaction. Physical Review Letters, 124(6):064801, 2020.
4. Zhexin Zhao, Xiao-Qi Sun, and Shanhui Fan. Quantum entanglement and modulation enhancement of free-electron-bound-electron interaction. ArXiv preprint arXiv:2010.11396, 2020.