Meso-scale analysis of angle-ply laminates

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Abstract

In this paper, a meso-scale repeated unit cell model for a general \([\pm \theta]_n\) angle-ply laminate is developed and the implementation of periodic boundary conditions in a finite element analysis for this 3-D model is presented. A nonlinear viscoelastic constitutive model and a post-damage constitutive model based on the concept of a smeared crack are also incorporated in the finite element analysis. The stress-strain response and the damage initiation and propagation for different values of \(\theta\) are predicted.

Keywords: Composites; damage; finite element method; meso-mechanics; periodic boundary conditions; viscoelasticity.

1. Introduction

For angle-ply laminates, matrix cracking and interface debonding may exist at load levels much lower than the final failure load or even during the material curing process. However, these damage manifestations may be confined to certain laminae, thus the entire laminate can still carry loads until the final fracture occurs, as seen in a \([\pm 45^\circ]\) angle-ply laminate under tensile loading. In a glass fibre reinforced epoxy laminate, for instance, damage is observed in the regime with low tangent modulus. However, the specimen does not fail until the global strain reached a large value of approximately 15% \([1, 2]\). Thus, in the analyses of stress-strain behaviour of angle-ply laminates, it is imperative to consider the prevailing damage mechanisms involved.

In this paper, a meso-scale repeated unit cell (RUC) model for a general \([\pm \theta]_n\) angle-ply laminate is developed and the implementation of periodic boundary conditions in a finite element analysis (FEA) for
this 3-D model is presented [3]. For the polymer matrix, a nonlinear viscoelastic constitutive model in differential form is employed [4-6]. A post-damage constitutive model based on the concept of a smeared crack has been introduced to simulate the response after damage initiation [7]. It permits a crack description in terms of stress-strain relations and stiffness reduction in the direction normal to the crack plane, instead of the common element-death method used in FEA codes. Finally, by combining the three essential features mentioned above into an FEA code, the stress-strain response and the damage initiation and propagation in laminates with five different values of $\theta$ are predicted. The results are compared with available experimental data and are found to be in fair agreement.

2. Meso-scale repeated unit cell model for angle-ply laminates

2.1 A meso-scale RUC model

For an angle-ply $[\pm \theta]_n$ laminate, each ply is represented by a rhombohedron containing a single fibre in $+\theta$ or $-\theta$ directions. In the in-plane directions X and Y, the laminate can then be seen as a periodic structure of a stack of n rhombohedrons, each with a single fibre in the direction $+\theta$ or $-\theta$, alternatively, see Fig. 1(a) and 1(b). Furthermore, if there are large numbers of plies in the thickness direction, considering the periodicity in thickness direction, a repeated unit cell (RUC) can be cut as shown in Fig. 1(c). Due to the symmetry of the RUC, only half of the full RUC is required in the analysis, Fig. 1(d).

2.2 Periodic boundary conditions for angle-ply RUC model under uniaxial loading

As shown in Fig. 2, we study the case in which only the global stress component $\bar{\sigma}_{11} \neq 0$ by applying the global strain component $\bar{e}_{11}$ at a specified strain rate of $\bar{\epsilon}_{11}$. The periodic boundary conditions for applying the global strain $\bar{e}_{11}$, see Fig. 2, can be described [3] in the following Eq. 1 through Eq. 4.

\begin{align}
&u'_1 - u'_3 = u_1(A) - u_1(D), \quad u'_2 - u'_3 = u_2(A) - u_2(D), \quad u'_3 - u'_3 = 0 \quad (1) \\
&u'_1 - u'_1 = u_1(C) - u_1(D), \quad u'_2 - u'_2 = u_2(C) - u_2(D), \quad u'_3 - u'_3 = 0 \quad (2)
\end{align}

where A, B, C and D are four corner points on the bottom plane of the RUC, and $\Gamma_i$ represents the boundary surfaces, see Fig. 2(b). In the Z direction, symmetric conditions are applied:
Moreover, the application of the global strain $\varepsilon_{11}'$ can be realized by applying a nodal displacement at the corner point A,

$$u_1(A) = \varepsilon_{11}'[x_1(A) - x_1(C)]$$  \hspace{1cm} (4)

Fig. 2. Meso-scale repeated unit cell: (a) 3-D representation and notation; (b) uniaxial loading

3. Constitutive and damage modelling of polymer

3.1 Formulation of nonlinear viscoelastic constitutive model

A rheological model in a differential form, developed by Xia, Ellyin and co-investigators, is employed for the description of the matrix constituent [4-6]. In this model, the total strain rate is assumed to be the sum of the elastic and the creep strain rates, which are expressed as:

$$\dot{E}_e = (1/E)A\dot{\sigma}$$  \hspace{1cm} (5)

$$\dot{E}_c = \sum_{i=1}^{N} \dot{E}_{ci} = \sum_{i=1}^{N} \left( \frac{A}{E_i \tau_i} \sigma - \frac{1}{\tau_i} \dot{\varepsilon}_{ci} \right)$$  \hspace{1cm} (6)

In the above, a single underscore denotes a 6 component vector. $E$ is the elastic modulus and $A$ (a square matrix) consists of various functions of the Poisson’s ratio, $\nu$, defined in [5]. Moreover, $\tau_i$ is the relaxation time and $E_i$ is the spring stiffness of the $i$th ‘Kelvin-Voigt type’ element. $\tau_i$ is chosen to be an exponential function with a time scale factor $\alpha$ and $E_i$ is assumed to be a modulus function of the Stassi equivalent stress [8],

$$\tau_i = \tau_1 \alpha^{i-1}$$  \hspace{1cm} (7)

$$E_i = E_i(\sigma_{eq}), \quad \sigma_{eq} = [(R-1)I_1 + \sqrt{(R-1)^2 I_1^2 + 12RJ_2}] / 2R$$  \hspace{1cm} (8)

Note that $R$ is the ratio of the uniaxial compressive to tensile stress at a given octahedral strain and strain rate value. $I_1$ is the first invariant of the stress tensor and $J_2$ is the second invariants of the stress deviator.

The material constants obtained for the resin studied in this paper (Epon 826) are: $E = 3400$ MPa, $\nu = 0.42$, $\tau_1 = 6.116$ s, $R = 1.15$, $N = 6$ and $E_i(\sigma_{eq}) = 1.055 \times 10^5 \exp(1.26 - \sigma_{eq}/18.0)$ MPa.
3.2 Modelling of matrix damage

To model matrix cracking, a maximum principal stress criterion is adopted to define the initiation of micro-cracking of the polymer matrix [7]. A local (crack) coordinate system O-1-2-3 is established in which the three axes are along the directions of the three principal stresses (\( \sigma_1 \geq \sigma_2 \geq \sigma_3 \)). Once a crack is formed, it is assumed that it cannot transfer normal and shear stresses across the crack plane, i.e., \( \sigma_{12} = \sigma_{13} = 0 \). The post-damage constitutive model in the crack coordinate system is written in the incremental form as [5]:

\[
\Delta \sigma_{cr} = E_{cr} D \Delta \epsilon_{cr} - \chi B \sigma_{cr}
\]

In the above, \( E_{cr} \) and \( \chi \) are constants determined by test or numerical trials. \( B \) and \( D \) are square matrices. Note that \( D \) is determined by the Poisson’s ratio and a numerical constant, \( \beta \), see Ref. [7].

4. Results and discussions

4.1 Material constants

The composite material studied in this paper is an E-glass/epoxy system from the 3M Company with a fibre volume fraction of 52.5%. The tensile strength of the epoxy is 75 MPa. The E-glass fibre is modelled as an isotropic material. The modulus, Poisson’s ratio, and tensile strength are 72.4 GPa, 0.22, and 2000 MPa, respectively [7]. For constants expressed in Eq. (9), a test value of \( E_{cr} = 284 \) MPa is employed from Ref. [4]. Constants \( \beta \) and \( \chi \) in Eq. (9) are determined by numerical tests. \( \beta = 0.0001 \) and \( \chi = 0.025 \) were used for all the computations in this study.

The nonlinear constitutive model and the damage model are inserted as a user-defined material model in the finite element code ANSYS.

4.2 Prediction of stress-strain curves

Figure 3 displays the stress-strain curves of five angle-ply laminates with \( \theta = 15^\circ, 30^\circ, 45^\circ, 60^\circ \) and \( 75^\circ \). Figure 4 compares the predicted stress-strain curve of \( \theta = 45^\circ \) with test results.
As shown in Fig. 3, different stress-strain behaviours are predicted using the same set of material and numerical constants. For instance, the stress-strain curve of \( \theta = 15^\circ \) appears to be a straight line, implying that the fibre dominates the load carrying behaviour of the laminate. The stress-strain curves of \( \theta = 60^\circ \), \( 75^\circ \) show that the maximum global stresses occur at about 0.5% strain. All these behaviours are in agreement with test results [1].

In Fig. 4 the stress-strain curves of \( \theta = 45^\circ \) with and without the damage modelling are compared with the test results [1]. It is seen that the test curves lies in between the predicted curves with and without the damage modelling. It is observed from the stress-strain curve that the damage initiates at a global strain of 1.2% and following this the laminate can still carry load up to a large strain.

4.3 Prediction of damage evolution

For the sake of illustration, the first principal stress (in the RUC of \( \theta = 15^\circ \) and \( 75^\circ \)) approximately at the instants of the damage initiation and the final failure are plotted in Figs. 5 and 6, respectively.

As shown in Fig. 5(a) and 5(b), for both laminates, the first principal stress in the matrix portion of the RUC takes its maximum value at the fibre/matrix interface, but at different locations for each type of laminate. For the \( 15^\circ \) laminates it is near the interlaminar zone of the two layers (the maximum value is
indicated by a red colour); while for the 75° laminates it is at the edge of the RUC in a band form. Note that at $\delta = 2.3\%$ and $0.32\%$, the maximum stress value in both cases is 74 MPa, which is close to the tensile strength of the epoxy resin (75MPa). Upon further loading damage will initiate at the expected locations. Therefore, different damage behaviour of the two laminates can be predicted by the model.

As depicted by Figs. 6(a) and 6(b), at final stages, the locations of the maximum values of the first principal stress occurs at different constituents for the two laminates. For the 15° laminates it is in the fibre while for the 75° laminates it is in the matrix (around the interlaminar zone of the model, progressing from the edge of the RUC, as shown in Fig. 5b). Upon further loading, the two laminates will have fibre failure and matrix failure, respectively and both laminates will cease to carry further loads.

5. Conclusions

Angle-ply laminates with five different angle orientations have been analyzed by a meso-scale model. The predicted global stress-strain curves of the five types of angle-ply laminates studied herein are in agreement with the experimental observations. The results indicate that after damage, the load carrying capacity is decreased only in directions perpendicular to the fibre, and thus explains the varying stress-strain responses of laminates with different types of fibre architecture.

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