On the size of isospin violation in low–energy pion–nucleon scattering #1

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Abstract

We present an analysis of isospin–breaking effects in threshold pion–nucleon scattering due to the light quark mass difference and the dominant virtual photon effects. We discuss the deviation from various relations, which are exact in the isospin limit. The size of the isospin–violating effects in the relations involving the isovector $\pi N$ amplitudes is typically of the order of one percent. We also find a new remarkably large effect (~40%) in an isoscalar triangle relation connecting the charged and neutral pion scattering off protons.

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1. Pion–nucleon ($\pi N$) scattering is one of the prime reactions to test our understanding of the spontaneous and explicit chiral symmetry breaking QCD is supposed to undergo. During the last years, there has been considerable interest in using $\pi N$ scattering data to extract information about the violation of isospin symmetry of the strong interactions \[1, 2\], some analyses indicating effects as large as 7\% \[3, 4\]. In both these analyses, the source of this rather large effect remains mysterious. Microscopically, there are two competing sources of isospin violation, which are generally of the same size, namely the strong effect due to the light quark mass difference $m_d - m_u \simeq m_u$ and the electromagnetic one caused by virtual photons. For neutral pion scattering off nucleons, these effects can be dramatically enhanced due to the smallness of the isoscalar pion–nucleon amplitude \[5, 6\]. This spectacular effect in the difference of the $\pi^0 p$ and $\pi^0 n$ scattering lengths is, however, at present not amenable to a direct experimental verification. It is therefore mandatory to include also the channels with charged pions in any analysis of isospin violation. To do this in a consistent fashion, one has to develop an effective field theory (EFT) of pions, nucleons and virtual photons. The corresponding effective Lagrangian was developed in refs.\[7, 6\] extending the standard $\pi N$ EFT (for a review, see ref.\[8\]). The pertinent power counting of the EFT is based on the observation that besides the pion mass and momenta, the electric charge $e$ should be counted as an additional small parameter, given the fact that $e^2/4\pi \simeq M^2_\pi/(4\pi F_\pi)^2 \simeq 1/100$ (with $M_\pi$ and $F_\pi$ the pion mass and decay constant, respectively). From here on, we collectively denote these small parameters by $q$. Similar information can also be obtained from precise data on pion photoproduction, as detailed in ref.\[10\] (for an overview, see the talks \[11, 12\]). The aim of this paper is to give a first systematic study of the expected size of isospin violation in the $\pi N$ amplitude at threshold based on a set of relations, which are fulfilled in the limit of exact isospin. We stress again that in the framework we are using, a consistent separation of the electromagnetic and the strong effects is possible and to our knowledge this has not been achieved before. Only when a mapping of the method developed here on the commonly used procedures of separating electromagnetic and hadronic mass effects (such as the NORDITA method \[13\]) has been performed, a sensible comparison with the numbers quoted in the literature will be possible.

2. Consider now the process $\pi^a(q_a) + N(p_1) \rightarrow \pi^b(q_b) + N(p_2)$, where $\pi^a$ denotes a pion of (cartesian) isospin $a$ and $N$ the nucleon. In the centre–of–mass system (cms), the four–momentum of the incoming nucleon is $m \cdot v + p_1 = (E_1 = m + v \cdot p_1, -\vec{q}_a)$, the one of the outgoing nucleon is $m \cdot v + p_2 = (E_2 = m + v \cdot p_2, -\vec{q}_b)$, where $m$ denotes the nucleon mass in the chiral limit. Similarly, the incoming pion has $q_a = (\omega_a, \vec{q}_a)$ and the outgoing pion $q_b = (\omega_b, \vec{q}_b)$. The analysis of isospin violation in $\pi N$ scattering proceeds essentially in three steps. First, one ignores all isospin breaking effects, i.e. one sets $e = 0$ and $m_u = m_d$. Only if within this approximation one is able to describe the low $\pi N$ partial waves in the threshold region as given by various partial wave analyses, one can be confident to have a sufficiently accurate starting point.\[^5\] That this is indeed the case was demonstrated in refs.\[9, 14, 15\]. Ref.\[15\] also contains a detailed discussion of the kinematics pertinent to the case considered here. In the second step, one should include the leading isospin breaking terms encoded in the pion and nucleon mass differences. The corresponding terms in the effective Lagrangian read \[7, 6\] (also shown are the terms responsible for the explicit chiral symmetry breaking)

$$L_{\pi\pi} = \partial_\mu \pi^a \partial^\mu \pi^a + B_0(m_u + m_d)\pi^2 + 2Ce^2\pi^+\pi^- + \ldots$$

\[^5\]Note that in the available partial–wave analyses, electromagnetic and some hadronic mass effects are generally removed by some methods like e.g. the one from NORDITA \[13\].

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where \( B_0 \) is related to the scalar quark condensate, \( B_0 = \langle 0 | \bar{q}q | 0 \rangle / F_\pi^2 \), and \( F \) is the pion decay constant in the chiral limit, \( F_\pi = F[1 + \mathcal{O}(q^2)] \). The pion mass difference \( M_{\pi^+} - M_{\pi^0} \) is entirely determined by the low–energy constant (LEC) \( C \) [16],\(^{#6}\) while to third order in small momenta the strong (electromagnetic) proton–neutron mass splitting is given by the LECs \( c_3 \) (\( f_2 \)) [6]. Note that the operator \( \sim c_5 \) does not only contribute to the strong np mass splitting but also has a contribution \( \sim \bar{N}N\pi\pi \) to the two–pion vertex which will be of relevance later. In terms of the operators defined in eq.(1), retaining only the terms leading to the strong and electromagnetic (em) hadron mass splitting is achieved by setting

\[
C\epsilon^2 \neq 0, f_2\epsilon^2 \neq 0, m_u - m_d \neq 0, \text{ but } \epsilon^2 = 0.
\]

This is the approximation which we will consider here. In fact, in neutral pion photoproduction off nucleons, to third order in small momenta, this approximation leads to the only isospin breaking effect, which reveals itself in the large cusp effect at the secondary threshold (i.e. at the \( \pi^+n \) threshold in the case of \( \gamma p \to \pi^0p \)). In the third step, which goes beyond the scope of this paper, one has to account for all virtual photon effects, in particular soft photon emission from charged particle legs and the Coulomb poles due to the ladder exchange of (hard) virtual photons between charged external particles. In that case, the notion of partial waves becomes doubtful and one better compares directly to the available cross section and polarization data.

We believe, however, that the essential effects of isospin violation are captured in the calculation presented here.

3. In the presence of isospin violation, i.e. isovector symmetry breaking terms such as \( (m_u - m_d)(\bar{u}u - \bar{d}d) \), one has to generalize the standard form of the \( \pi N \) scattering amplitude to

\[
T^{ab}(\omega, t) = \delta^{ab}T^{3+}_{ab}(\omega, t) + \delta^{ab}\tau^3 T^{3+}_{ab}(\omega, t) + i\epsilon^{bac}\tau^c T^{3-}_{ab}(\omega, t) + i\epsilon^{bac}\tau^c \tau^3 T^{3-}_{ab}(\omega, t),
\]

in terms of two isoscalar \((T^{3+}_{ab})\) and two isovector amplitudes \((T^{3-}_{ab})\). These are functions of two variables, here we choose the pion energy \( \omega \) and the invariant momentum transfer squared \( t \). More precisely, \( \omega \) can be chosen to be either the energy of the in–coming or out–going pion, since these are no longer equal

\[
\Delta\omega = \omega_b - \omega_a = \frac{(M_b^2 - M_a^2) - (m_b^2 - m_a^2)}{2\sqrt{s}} = \frac{(M_b^2 - M_a^2) - (m_2^2 - m_1^2)}{2m_1} \left[ 1 - \frac{w_a}{m_1} + \mathcal{O}(q^2) \right],
\]

with \( M_{a,b} \) \((m_{1,2})\) the mass of the in–coming, out–going pion (nucleon) and \( \sqrt{s} \) the total cms energy. It is important to note that while the pion energies \( \omega_{a,b} \) are of order \( q \), their difference only starts out at second order in the chiral expansion. This has important consequences as will be discussed later. The \( T \)–amplitudes split, of course, into a spin non–flip and a spin–flip term, denoted by \( g \) and \( h \), respectively (for more precise definitions, see e.g. ref.[15]). At threshold, only the spin non–flip amplitudes can contribute and eq.(3) simplifies to

\[
T_{\pi N, \text{thr}}^{ab} = \mathcal{N}_1\mathcal{N}_2 \left\{ \delta^{ab} g_{ab}^+ + \delta^{ab}\tau^3 g_{ab}^+ + i\epsilon^{bac}\tau^c g_{ab}^- + i\epsilon^{bac}\tau^c \tau^3 g_{ab}^- \right\},
\]

\(^{#6}\)Note that we work in the \( \sigma \)–model gauge so that the term \( C(QUQU^\dagger) \) only contributes to two–point functions.
with $N_i = \sqrt{(E_i + m_i)/2m_i}$ ($i = 1, 2$) the standard spinor normalization (its relevance is discussed in detail in ref.[17]). In what follows, we do not consider these normalization factors since they are related to the external kinematics. Consequently, the isospin violating effects are entirely confined to happen within the given Feynman graphs we consider. The $g_{ab}^{\pm,3\pm}$ are, of course, proportional to the corresponding $S$–wave scattering lengths. Note that in the presence of isospin violation, these amplitudes can become complex even at threshold (since the mass of the incoming two–particle system is no longer equal to the mass of the out–going one). It is also important to realize that the $g_{ab}^{\pm}$ are exclusively sensitive to the neutral to charged pion mass difference (i.e. the LEC $C$) whereas the $g_{ab}^{3\pm}$ are given by the operators $\sim f_2, \sim c_5$, i.e the strong and em proton–neutron mass difference.

4. Isospin violation is best characterized in terms of quantities which are exactly zero in the isospin limit of equal quark masses and vanishing em coupling. With the three pion ($\pi^\pm, \pi^0$) and two nucleon $(p, n)$ fields, we have ten reaction channels, which in the case of isospin symmetry are entirely described in terms of two amplitudes. One thus can write down eight isospin relations (see also ref.[18] for a general analysis)

\[
R_1 = \frac{T_{\pi^+ p \to \pi^+ p} + T_{\pi^- p \to \pi^- p} - 2T_{\pi^0 p \to \pi^0 p}}{T_{\pi^+ p \to \pi^- p} + T_{\pi^- p \to \pi^+ p} + 2T_{\pi^0 p \to \pi^0 p}} = \frac{g_{11}^+ + g_{22}^+ - 2g_{33}^+ + g_{11}^3 + g_{22}^3 - 2g_{33}^3}{g_{11}^1 + g_{22}^1 + 2g_{33}^1 + g_{11}^2 + g_{22}^2 + 2g_{33}^2}, \tag{6}
\]

\[
R_2 = \frac{T_{\pi^+ p \to \pi^- p} - T_{\pi^- p \to \pi^+ p} - \sqrt{2}T_{\pi^0 p \to \pi^0 n}}{T_{\pi^+ p \to \pi^- p} - T_{\pi^- p \to \pi^+ p} + \sqrt{2}T_{\pi^0 p \to \pi^0 n}} = \frac{g_{12}^+ + g_{21}^+ - g_{13}^- - g_{23}^- + g_{12}^3 - g_{21}^3 - g_{13}^3 + g_{23}^3}{g_{12}^- + g_{21}^- + g_{13}^+ + g_{23}^+ + g_{12}^3 + g_{21}^3 + g_{13}^3 + g_{23}^3}, \tag{7}
\]

\[
R_3 = \frac{T_{\pi^0 p \to \pi^+ n} - T_{\pi^- p \to \pi^0 n}}{T_{\pi^0 p \to \pi^+ n} + T_{\pi^- p \to \pi^0 n}} = \frac{g_{31}^+ + g_{32}^+ - g_{33}^+ - g_{31}^- + g_{32}^- - g_{33}^-}{g_{31}^3 + g_{32}^3 + g_{33}^3 + g_{31}^- + g_{32}^- + g_{33}^-}, \tag{8}
\]

\[
R_4 = \frac{T_{\pi^+ p \to \pi^0 n} - T_{\pi^- n \to \pi^- n}}{T_{\pi^+ p \to \pi^- n} + T_{\pi^- n \to \pi^+ n}} = \frac{g_{11}^+ + g_{22}^+ - g_{12}^- - g_{21}^-}{g_{11}^- + g_{22}^- + g_{12}^+ + g_{21}^+}, \tag{9}
\]

\[
R_5 = \frac{T_{\pi^- p \to \pi^0 n} - T_{\pi^0 n \to \pi^+ n}}{T_{\pi^- p \to \pi^+ n} + T_{\pi^0 n \to \pi^- n}} = \frac{g_{11}^+ + g_{22}^+ - g_{12}^- - g_{21}^-}{g_{11}^- + g_{22}^- + g_{12}^+ + g_{21}^+}, \tag{10}
\]

\[
R_6 = \frac{T_{\pi^0 p \to \pi^0 p} - T_{\pi^0 n \to \pi^0 n}}{T_{\pi^0 p \to \pi^0 p} + T_{\pi^0 n \to \pi^0 n}} = \frac{g_{33}^+}{g_{33}^+}, \tag{11}
\]

\[
R_7 = \frac{T_{\pi^- p \to \pi^0 n} - T_{\pi^+ n \to \pi^0 p}}{T_{\pi^- p \to \pi^0 n} + T_{\pi^+ n \to \pi^0 p}} = R_3, \tag{12}
\]

\[
R_8 = \frac{T_{\pi^0 p \to \pi^+ n} - T_{\pi^0 n \to \pi^- p}}{T_{\pi^0 p \to \pi^+ n} + T_{\pi^0 n \to \pi^- p}} = -R_3. \tag{13}
\]

The first two, the so–called triangle relations, are based on the observation that in the isospin conserving case, the elastic scattering channels involving charged pions are trivially linked to the corresponding neutral pion elastic scattering or the corresponding charge–exchange amplitude. To be precise, these ratios are to be formed with the real parts of the corresponding amplitudes.
evaluated at the pertinent threshold, symbolically $T_{\pi^0 N \rightarrow \pi^0 N}$ should read $\text{Re} \, T_{\pi^0 N \rightarrow \pi^0 N}^{\text{thr}}$. The imaginary parts of some of the amplitudes will be discussed later. Of particular interest is the second ratio, which is often referred to as the triangle relation. Only in this case all three channels have been measured (for pion kinetic energies as low as 30 MeV in the cm system) and the 7% strong isospin violation reported in refs.[3, 4] refers to this ratio. We stress again that it is difficult to compare this number with the one obtained in our calculation since a very different method of separating the em effects is used. The ratio $R_6$ parametrizes the large isospin violation effect for $\pi^0$ scattering off nucleons first found by Weinberg [5] and sharpened in ref.[6], $R_6 \simeq 25\%$. Note that in $R_1$ the isovector terms drop out completely and one thus expects also a large isospin violation in this ratio (since the isoscalar parts are strongly suppressed and are of the same size as the symmetry breaking terms). To our knowledge, this is the first time that this particular ratio has been called attention to. From an experimental point of view, it has the advantage of avoiding the almost unmeasurable $n\pi^0$ amplitude appearing in $R_6$. However, both $R_1$ and $R_6$ are sensitive to the precise values of the combination of LECs $c_2 + c_3 - 2c_1$ since the strong contribution to the isoscalar scattering length is not even known in sign at present. The predictions for the other ratios are more stable since they involve the larger (and better determined) isovector quantities. Note that the relations $R_7 = R_3 = -R_8$ follow from time reversal invariance. In what follows, we will calculate the six ratios $R_i$ to leading one loop accuracy, i.e. to third order in small momenta. For that, we have to consider tree graphs, some with fixed coefficients and some with LECs, and the leading one loop graphs involving lowest order couplings only.

5. The pertinent Born graphs calculated to first, second and third order are depicted in fig.2 of ref.[15]. The ones contributing here are 1b, 2a, 2b, 2d, 3a-3f and the additional tree graphs $\sim f_2, \sim c_5$ are shown in fig.1. Before giving the results for the sum of all Born graphs (tree graphs with or without LECs), some important remarks concerning the chiral power counting are in order. Although the so-called Weinberg–Tomozawa $\bar{N}N\pi\pi$ contact graph gives a first order contribution to $g_{\pi\pi}$, its effect is always proportional to $\Delta\omega$, which is of second order, cf. eq.(4). Consequently, isospin violation only starts at second order in the chiral expansion. Furthermore, in some tree graphs with intermediate nucleon lines the pion energy difference enters since $v \cdot p_1 - v \cdot p_2 = \omega_b - \omega_a$, which is thus also of order $q^2$. We have only accounted for this difference whenever it was necessary and consequently neglected it when it would only lead to a fourth (or higher) order contribution. This applies in particular also to the loop graphs given below. Therefore, the final results depend on the choice of taking either the energy of the in–coming or the out–going pion as reference energy. This difference is, however, beyond the accuracy we are working and thus gives us the possibility to estimate some higher order effects. After mass and coupling constant renormalization, the sum of all Born graphs gives the following contributions (for notations, see ref.[15])

\[
F_\pi^2 g_{ab}^+ = -4c_1M_0^2 + 2(c_2 - \frac{g_\pi^2}{8m})\omega^2 + 2c_3\omega^2, \quad (14)
\]
\[
F_\pi^2 g_{ab}^0 = -2B_0(m_u - m_d)c_5\delta^{a3} + \frac{1}{2}e^2f_2F_\pi^2(\delta^{a3} - 1), \quad (15)
\]
\[
F_\pi^2 g_{ab}^- = \frac{\omega_a + \omega_b}{4} + \frac{|q_a|^2 + |q_b|^2}{8m}(1 - 2g_\pi^2) + \frac{4}{3}e^2f_2F_\pi^2(d_1 + d_2 + d_3) + 2M_0^2d_5 + M_0^2d_28
\]
\[
\frac{\omega(1 - 4g_\pi^2)}{16m^2}\left(|q_a|^2 + |q_b|^2\right) + \frac{g_\pi^2\omega^3}{8m^2} - \frac{\omega}{2m}(c_4 + \frac{1}{8m})\left(|q_a|^2 + |q_b|^2\right), \quad (16)
\]
\[ F_\pi^2 g_{ab}^{3-} = \left\{ -B_0(m_u - m_d)c_5 + \frac{1}{4}e^2 f_2 F_\pi^2 \right\} (\delta^{a3} - \delta^{b3}) \]
\[ + \frac{g_A^2}{2m\omega} (|q_a|^2 + |q_b|^2) \left\{ 2B_0(m_u - m_d)c_5 + \frac{1}{2}e^2 f_2 F_\pi^2 \right\} (\delta^{a3} - \delta^{b3}) . \]  

At third order, we have to consider the set of one loop graphs shown in fig.2. These give

\[ F_\pi^4 g_{ab}^{+} = -\frac{\omega^2}{4} (J_0^a(\omega) + J_0^b(-\omega)) (\delta^{ac} - \delta^{cc}) - \frac{g_A^2}{4} \delta^{cc} M_0^2 (J_0^c(0) - M_c^2 J_0^c(0)) \]
\[ - \frac{g_A^2}{2} (M_a^2 (2M_a^2 - M_b^2) \gamma_0^{aa}(0) + (M_b^2 - 3M_a^2) J_0^a(0)) \]  
\[ + \frac{\omega^2}{16} (4\omega (J_0^a(\omega) - J_0^c(-\omega)) + 6\Delta^c_\omega) \]
\[ + \frac{\omega}{12t} \left( t^2 - 2t(M_a^2 + M_b^2) + (M_a^2 - M_b^2)^2 I^{ab}_0(t) \right) \]
\[ + t(\Delta^a_\omega + \Delta^b_\omega) + \Delta^{a}_\omega - \Delta^{b}_\omega + \frac{t^2}{24\pi^2} \right) \]
\[ + \frac{g_A^2}{48} \left\{ 4\omega [5t - 4(M_a^2 + M_b^2) - (M_a^2 - M_b^2)^2] I^{ab}_0(t) + 20\omega (\Delta^a_\omega + \Delta^b_\omega) \right\} \]
\[ - \frac{4\omega}{t} (M_a^2 - M_b^2) (\Delta^a_\omega - \Delta^b_\omega) - \frac{\omega}{6\pi^2} (t - 3(M_a^2 + M_b^2)) \right) \]
\[ + \frac{\delta^{0d}}{8} (\delta^{dd} \Delta^a_\omega + \Delta^c_\omega) - \frac{g_A^2}{8} \omega \left( M_d^2 J_0^d(0) - \Delta^d_\omega \right) (2\delta^{de} - \delta^{dd}) \]
\[ + \frac{\omega_a + \omega_b}{4} \left( \frac{g_A^2}{F_\pi^2} \right)^2 (M_c^2 J_0^c(0) - \Delta^c_\omega) + 8M_d^2 d_{28}(\lambda) + \frac{1}{F_\pi^2} \delta^{ce} \Delta^c_\omega ) , \]

where one has to generalize the standard loop functions \( I_0, \Delta_\pi, \gamma_0, J_0 \) (see the appendices in refs.[8, 19]) to the unequal mass case as indicated by the superscripts ‘a, b, c’ referring to the pion isospin indices. The loop contributions to \( g_{ab}^- \) are divergent. These divergences are cancelled by the appropriate dimension three operators from \( \mathcal{L}_{\pi N}^{(3),em} \) as constructed in ref.[6]. Note that in the approximation we are using, the finite parts of these terms are set to zero. We remark that the imaginary parts generated by the loops are very small for the threshold kinematics considered here. This is made more precise in the following.

6. We are now in the position to analyze the ratios \( R_i \) as defined in eqs.(6-13). We use the standard masses as given in the PDG tables for \( m_{\pi,N}, M_{\pi^0,\pm} \) [20]. For the mass parameter \( m \) we can use \( m = (m_\pi + m_N)/2 \) to the order we are working and we identify the leading term in the quark mass expansion of the pion mass with the neutral pion mass, \( M_0 = M_{\pi^0} \). We also use \( F_\pi = 92.4 \text{ MeV} \)\(^7\) and \( g_A = 1.26 \). The LECs \( c_{1,2,3,4} \) and \( d_i \) are taken from fit 1,2,3\(^8\) of ref.[15] (we refer to that paper as “FMS”), \( c_5 \) from ref.[9] and \( f_2 \) from ref.[6].

In table 1, we give the results for the ratios \( R_i \) that are not entirely given by isoscalar quantities. These numbers should be more stable than the ones for the isoscalar ratios \( R_{1,6} \).

\(^7\)In principle, we would have to differentiate between \( F_{\pi^+} \) and \( F_{\pi^0} \). This difference is of second order and would therefore show up as a third order contribution due to the Weinberg-Tomozawa term. At present, the empirical determinations do not allow to differentiate between these two values and we thus work with one value given by the charged decay constant.

\(^8\)Fit 1,2 and 3 refers to the Karlsruhe [21], the Matsinos [22] and the VPI [23] partial wave analysis, in order.
This is indeed the case, there are no large variations between the three parameter sets given in FMS. For the pion energy $\omega$, we have used the arithmetic mean of the in–coming and out–going energies (this is also the most natural choice since it preserves the time reversal invariance between $\pi^+n \leftrightarrow \pi^0p$ and $\pi^-p \leftrightarrow \pi^0n$). For parameter set 1, we have also varied the pion reference energy and used either $\omega_a$ or $\omega_b$. The $R_{1,4,5,6}$ are insensitive to this choice, whereas $R_{2,3}$ can vary between approximately zero and 1.5%. This points towards the necessity of a fourth order calculation.

| Fit  | $R_2$ [%] | $R_3$ [%] | $R_4$ [%] | $R_5$ [%] |
|------|-----------|-----------|-----------|-----------|
| 1    | 0.9       | -0.5      | -0.7      | 1.1       |
| 2    | 1.1       | -0.6      | -0.9      | 1.1       |
| 3    | 0.9       | -0.5      | -0.8      | 1.0       |

Table 1: Values of the ratios $R_i$ ($i = 2, 3, 4, 5$) for the various parameter sets as given by the fits of FMS.

We now turn to the two isoscalar ratios. First, we consider $R_6$, which was first discussed by Weinberg [5]. For fit 1, we find $R_6 = 19\%$, which is somewhat smaller than the 25% reported in ref.[6]. Note, however, that the value for $a^+$ based on the KA85 phase shifts is larger (in magnitude) than the one used in ref.[6] (based on the LECs as determined in ref.[9]) and thus the isospin violating effect is indeed expected to be smaller. On the other hand, for the parameters of fits 2 and 3, $R_6$ gets much larger, but not because the isospin–violating function $g_3^{3+}$ changes (it is indeed stable up to fourth order as argued in ref.[6]), but rather the isospin–conserving function $g_3^{3}$ varies considerably. Thus, to sharpen the prediction for $R_6$, one has to go to next order in the isospin–conserving case. Interestingly, for the same parameter set (fit 1 of FMS), the prediction for $R_1$ is even larger,

$$R_1 = 36.7\%,$$

which is again a huge isospin violating effect in an isoscalar quantity. The same remarks as made for $R_1$ apply here. However, we stress again that this novel ratio could be accessible experimentally if the proposal of Bernstein [10] to extract the $\pi^0p$ scattering by precise pion photoproduction experiments could be carried out. Again, we remark that the value for $R_1$ given in eq.(20) should be considered as a lower limit since for the other parameter sets the isospin–conserving isoscalar amplitude is smaller (in magnitude) which enhances the very stable isospin–breaking difference in the ratio. It is thus mandatory from theory and experiment to get a more precise value for the isoscalar amplitude.

It is most interesting to separate the hadronic isospin violation encoded in the operator $\sim c_5(m_u - m_d)$ from the virtual photon effects. One could set $c_5 = 0$, i.e. all strong isovector terms would vanish. This is not quite what one wants since then the proton is heavier than the neutron. In that case, the masses of the external particles would also change and a meaningful comparison becomes difficult. We can, however, keep $c_5 \neq 0$ for the nucleon mass insertions and set it to zero in the $\bar{N}N\pi\pi$ terms $\sim c_5$, cf. eq.(1), which we denote by $c_5(\pi\pi) = 0$ in table 2. This splitting of the strong isospin violating terms is similar to what is called “static” and “dynamical” isospin breaking in ref.[10]. We see that while the em isospin breaking is generally dominant (with the exception of $R_6$), there is also some sizeable strong isospin breaking.
Table 2: Values of the ratios $R_i$ ($i = 1, 2, 3, 4, 5, 6$) for the parameters of fit 1 from FMS including the full contribution from the strong isospin breaking ($c_5 \neq 0$) and with the strong isospin violation only contributing to the proton–neutron mass difference ($c_5(\pi\pi) = 0$).

|        | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $R_6$ |
|--------|-------|-------|-------|-------|-------|-------|
| $c_5 \neq 0$ | 36.7  | 0.9   | -0.5  | -0.7  | 1.1   | 19.3  |
| $c_5(\pi\pi) = 0$ | 45.4  | 1.6   | 0.8   | -0.7  | 1.1   | 0     |

So far, we have discussed the ratios $R_i$ from the real parts of the complex–valued $\pi N$ amplitudes evaluated at the pertinent threshold kinematics. It is also instructive to give the corresponding scattering lengths. We define these as the amplitude at threshold (i.e. in some cases we have complex numbers) including the normalization factors, symbolically $a = \sqrt{m_1m_2T_{th}/(4\pi\sqrt{s})}$. For scattering pions off protons, we get the numbers given in table 3. The imaginary part in $a(\pi^- p \rightarrow \pi^- p)$ is due to the intermediate $\pi^0 n$ state, whereas for $a(\pi^0 p \rightarrow \pi^+ n)$ the energy of the initial two–body system is smaller than the one of the final two–body state. In both cases, these imaginary parts are fairly small. As a check on the numerics, we find that $a(\pi^+ n \rightarrow \pi^0 p) = a(\pi^0 p \rightarrow \pi^+ n)$ and $a(\pi^- p \rightarrow \pi^0 n) = a(\pi^0 n \rightarrow \pi^- p)$ as demanded by time reversal invariance. For comparison, the scattering lengths in the isospin limit (using the charged pion and the proton mass) can be obtained from table 2 of FMS by use of the relations $a(\pi^\pm p \rightarrow \pi^\pm p) = a_{0+}^+ \mp a_{0-}^+$, $a(\pi^0 p \rightarrow \pi^0 p) = a_{0+}^0$ and $a(\pi^- p \rightarrow \pi^0 n) = a(\pi^0 p \rightarrow \pi^+ n) = -\sqrt{2}a_{0+}^-$. 

|        | $a(\pi^\pm p \rightarrow \pi^\pm p)$ | $a(\pi^- p \rightarrow \pi^- p)$ | $a(\pi^0 p \rightarrow \pi^0 p)$ | $a(\pi^- p \rightarrow \pi^0 n)$ | $a(\pi^0 p \rightarrow \pi^+ n)$ |
|--------|------------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| Fit 1  | -108.7                             | 70.2 + 3.65                      | -13.4                            | -125.5                           | -124.7 - 0.63                    |
| Fit 2  | -83.8                              | 71.3 + 3.65                      | -0.1                             | -108.6                           | -107.9 - 0.63                    |
| Fit 3  | -94.9                              | 77.7 + 3.65                      | -1.8                             | -121.1                           | -120.2 - 0.63                    |

Table 3: Values of the scattering lengths for pion scattering of protons in units of $10^{-3}/M_{\pi^+}$ for the various parameter sets as given by the fits of FMS.

7. In this paper, we have considered isospin violation in low energy pion–nucleon scattering in the framework of heavy baryon chiral perturbation theory to third order in small momenta. We have taken into account all operators related to strong isospin breaking and the electromagnetic ones, which lead to the pion and nucleon mass differences. Stated differently, the finite parts of some of the virtual photon operators contributing at this order have been set to zero. This allows in particular to isolate the contribution of the strong dimension two isovector operator first considered by Weinberg. We have considered a set of six ratios $R_i$, which vanish in the limit of isospin conservation. From these, six involve isovector and isoscalar amplitudes ($R_{2,3,4,5,7,8}$) and the two others are purely of isoscalar type ($R_{1,6}$). While in the first case, isospin violation is typically of the order of one percent, more sizeable effects are found in $R_6$ [5, 6] and, as for the first time noted here, in $R_1$. These results strongly motivate efforts to measure more precisely the isoscalar scattering length $a^-$ and try to determine the $\pi^0 p$ scattering length e.g. from accurate
threshold pion photoproduction experiments. We also stress again that within the framework presented here, a unique and unambiguous separation of all different isospin violating effects is possible. To access the size of isospin violation encoded in the presently available pion–nucleon scattering data, the extension of this scheme to include Coulomb (hard) and soft photons is mandatory. Once this is done, it will be possible to analyze directly the cross section data without recourse to any model for separating electromagnetic or hadronic mass effects thus, avoiding any mismatch by combining different approaches or models. Work along such lines is underway.

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Figures

Figure 1: Tree graphs contributing to isospin violation in $\pi N$ scattering. Solid and dashed lines denote nucleons and pions, in order. The circle–cross (heavy dot) refers to a dimension two insertions $\sim f_2$ or $\sim c_5$ ($\sim 1/m$).

Figure 2: One loop graphs contributing to isospin violation in $\pi N$ scattering. Solid and dashed lines denote nucleons and pions, in order.