Renormalized theory of the ion cyclotron turbulence in magnetic field–aligned plasma shear flow

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Abstract

The analytical treatment of nonlinear evolution of the shear-flow-modified current driven ion cyclotron instability and shear-flow-driven ion cyclotron kinetic instabilities of magnetic field–aligned plasma shear flow is presented. Analysis is performed on the base of the nonlinear dispersion equation, which accounts for a new combined effect of plasma turbulence and shear flow. It consists in turbulent scattering of ions across the shear flow with their convection by shear flow and results in enhanced nonlinear broadening of ion cyclotron resonances. This effect is found to lead to the saturation of ion cyclotron instabilities as well as to the development of nonlinear shear flow driven ion cyclotron instability.

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I. INTRODUCTION.

The important feature of the near-Earth space plasma is its irregularities [1, 2] in density, temperature of ions and electrons, currents and electric field. The existence of inhomogeneous flows and currents along and across Earth’s magnetic field is a fundamental characteristic of space plasmas. One important element of the ionospheric flows appears to be a velocity shear. Compared to the homogeneous plasma case, sheared flows along the magnetic field introduce significant modifications to plasma stability. The presence of magnetic field–aligned plasma shear flows give rise to numerous instabilities of plasmas, which are predicted theoretically and observed experimentally in a broad frequency range with applications to ionospheric and fusion plasmas. The interest in scrutinizing the shear flow driven instabilities, that are still under intense investigation (see, e.g., recent papers [4]-[5] and references therein), lies, in part, in a number of observations which indicate correlation in place and time of broadband, low–frequency waves, and transverse ion energization with sheared flows on the boundaries of plasma structures[6]. Plasma shear flows were often encountered by Prognoz-8 satellite [7] near the high latitude magnetopause which is the boundary between the Earth’s magnetic field and solar wind. It was found[7] that such plasma flows are always associated with strong wave activity at ion cyclotron and lower hybrid frequencies ranges. High time resolution measurements of ion distribution by Fast Auroral Snapshot (FAST) satellite have revealed[2, 3] at an altitude of 4000km narrow ion and electron beams with steep transverse spatial gradients in their energy. These beams were clearly identified as the sources of free energy to drive observed electrostatic and electromagnetic ion cyclotron waves[8, 9] and particles energization in regions containing ion beams and field-aligned currents. Electrostatic IC waves have been invoked in the explanations of the transverse anomalously strong heating of ions in ionosphere, that cannot be accounted for by frictional (Joule) heating [10, 11].

The field-aligned currents[12], ion beams[13] or relative streaming between ion species [14], have been proposed to provide the free energy for the development of IC instabilities. There are some in situ ionospheric observations [15, 16] that support the development of the classical current driven electrostatic IC instability [17] in ionosphere. (Comprehensive review of the theoretical, numerical and laboratory experimental investigations of the current driven electrostatic IC instability was given in Ref. [18]). However, generally the levels of field–aligned current at which IC waves and transverse ion heating were observed in ionosphere were subcritical for the development of this instability, which has the lowest threshold for current density in the ionospheric plasma environment [12]. Kinetic effects, such as finite ion Larmor radius effects, electron Landau damping and ion cyclotron damping, are pronounced for ion cyclotron modes and kinetic theory is requisite for treating these modes. In Ref [19] the linear dispersion equation for plasma with the magnetic field aligned flows with velocities $\mathbf{V}_{0\alpha}(x) || \mathbf{B} || \mathbf{e}_z$ of plasma components of $\alpha$ species ($\alpha = i$ for ions and $\alpha = e$ for electrons) with inhomogeneous number densities $n_{0\alpha}(x)$ was obtained using a kinetic approach in the local approximation, for which $k_x L_n \gg 1$ and $k_x L_v \gg 1$, where $L_n = [d \ln n_0(x)/dx]^{-1}$,
\( L_v = [d \ln V_0(x)/dx]^{-1} \), and \( k_x \) is the projection of the wave vector \( \mathbf{k} \) normal to the flow velocity and along the velocity gradient. That equation may be presented in a form (see, also papers [4],[19]-[24])

\[
\varepsilon(\mathbf{k}, \omega) = 1 + \sum_{\alpha = i, e} \delta \varepsilon_\alpha(\mathbf{k}, \omega) = 1 + \sum_{\alpha = i, e} (\delta \varepsilon_\alpha^{(1)}(\mathbf{k}, \omega) + \delta \varepsilon_\alpha^{(2)}(\mathbf{k}, \omega)) = 0. \tag{1}
\]

In Eq.(1) \( \delta \varepsilon_\alpha^{(1)}(\mathbf{k}, \omega) \) is the conventional dielectric permittivity of the \( \alpha \)-species of the plasma without shear flow and \( \delta \varepsilon_\alpha^{(2)}(\mathbf{k}, \omega) \) is the velocity shear dependent part of the dielectric permittivity \( \delta \varepsilon_\alpha(\mathbf{k}, \omega) \). Here and in what follows \( \mathbf{k} = (k_x, k_y, k_z) = (k_\perp, \theta, k_z) \), where \( k_x = k_\perp \cos \theta \), \( k_y = k_\perp \sin \theta \), and \( k_z \) is the projection of the wave vector on the magnetic field \( \mathbf{B} \). It was assumed in [4],[19]-[24] the equilibrium distribution function \( F_{\alpha 0} \) to be a drifting Maxwellian (see below Eq.(26)), even though this may be somewhat unrealistic for the collisionless ionosphere. The Maxwellian assumption permits calculationally convenient and comprehensive exploration of the IC instabilities. Eq.(1) was investigated for high frequency instabilities[19] with frequencies \( \omega \) well above the IC frequency \( \omega_{ci} \), as well as for low frequency instabilities[20, 21] with frequencies below \( \omega_{ci} \). The investigation of the IC instabilities in magnetic field–aligned plasma shear flows, that is a focus of this paper, was initiated in Ref.[21], where IC instability driven by velocity shear of hot ion beam in plasma with cold ions and electrons was considered. Thereafter it was derived in Ref.[22] that plasma shear flow along the magnetic field may be unstable against the development of an IC instability of the hydrodynamic type with wavelength much less than ion thermal Larmor radius \( \rho_i = v_{Ti}/\omega_{ci} \). That instability was developed even when the velocity of the electron-ion drift along the magnetic field was below the threshold for current driven IC instability.

The comprehensive investigation of the IC instabilities of magnetic-field aligned plasma shear flow was undertaken in Ref.[4]. It was shown analytically that under conditions

\[
|\text{Re } \varepsilon_1^{(1)}(\mathbf{k}, \omega)| \sim |\text{Re } \varepsilon_1^{(2)}(\mathbf{k}, \omega)| > |\text{Im } \varepsilon_1^{(2)}(\mathbf{k}, \omega)| \gtrsim |\text{Im } \varepsilon_1^{(1)}(\mathbf{k}, \omega)| \tag{2}
\]

shear flow along the magnetic field does not only modify the frequency, growth rate and the threshold of the known current driven IC instability, but it is a source of the development of the kinetic and hydrodynamic shear-flow-driven IC instabilities at the levels of field-aligned current which are subcritical for the development of the current driven IC instability. Shear flow along the magnetic field leads to the splitting of the separate IC mode, existing in the plasma without shear flow[17], into two IC modes with frequencies \( \omega_{1,2}(\mathbf{k}) \), which correspond to different kinds of the IC instabilities. In the limiting case of IC waves, which propagate almost across the magnetic field, the frequencies \( \omega_{1,2}(\mathbf{k}) \), which are the solutions of the linear dispersion equation \( 1 + \varepsilon_1^{(1)}(\mathbf{k}, \omega) + \varepsilon_1^{(2)}(\mathbf{k}, \omega) + \varepsilon_e(\mathbf{k}, \omega) = 0 \), may be obtained in the form

\[
\omega_{1,2}(\mathbf{k}) = k_z v_{0i} + n \omega_{ci} + \delta \omega_{1,2}(\mathbf{k}), \tag{3}
\]

with \( |\delta \omega_{1,2}(\mathbf{k})| \ll n \omega_{ci} \), \( |z_{1,2}| = |\delta \omega_{1,2}(\mathbf{k})|/\sqrt{2}|k_z|v_{Ti} \gg 1 \), \( |z_e| = |n \omega_{ci} - k_z (v_{Te} - V_{0i})|/\sqrt{2}|k_z|v_{Te} \lesssim 1 \). In the limiting case of weak flow shear, for which

\[
(n \omega_{ci} - k_y v_{di})^2 A_{in} (k_\perp^2 \rho_i^2) > 4 \frac{k_y v_{01}'}{k_z \omega_{ci}} k_z^2 v_{T_i}^2 (1 - A_{i0} (k_\perp^2 \rho_i^2) + \tau ),
\]
two kinetic IC instabilities are excited. Here and in what follows \( \tau = T_i/T_e \) with \( T_{e,i} \) being the electron and ion temperatures, respectively, \( \lambda_{De}^2 \) is the Debye radius, \( A_{in} (k_\perp \rho_i^2) = I_n (k_\perp \rho_i^2) e^{-k_\perp \rho_i^2} \), and \( v_{di} = (eT_i/eB_0) \ln n_i(x)/dx \). The frequency \( \text{Re} \delta \omega_1 (k) = \delta \omega_{01} (k) \), and the growth rate, \( \gamma_{01} = \text{Im} \delta \omega_1 (k) = \gamma_{i1} + \gamma_{e1} \) of the first instability are equal approximately to

\[
\text{Re} \delta \omega_1 (k) = \delta \omega_{01} (k) \approx \frac{(n_{\omega_{ci}} - k_y v_{di}) A_{in} (k_\perp \rho_i^2)}{(1 - A_{i0} (k_\perp \rho_i^2) + \tau)},
\]

\[
\gamma_{i1} = \frac{(\delta \omega_{01} (k))^2}{\Omega_n} k^2 \lambda_{Di}^2 \text{Im} \delta \varepsilon_i (k, n_{\omega_{ci}}) = -\frac{(\delta \omega_{01} (k))^2}{\Omega_n} A_{in} (k_\perp \rho_i^2) \sqrt{\frac{\pi}{2}} e^{-z_i^2} \left[ (n_{\omega_{ci}} - k_y v_{di}) - \frac{k_y v'_{0i}}{k_z \omega_{ci}} \delta \omega_{01} (k) \right],
\]

\[
\gamma_{e1} = \frac{(\delta \omega_{01} (k))^2}{\Omega_n} \tau \sqrt{\frac{\pi}{2}} \frac{(n_{\omega_{ci}} - k_z (V_{0e} - V_{0i}) - k_y v_{de})}{|k_z| v_T e} \exp (-z_e^2),
\]

where

\[
\Omega_n^2 = (n_{\omega_{ci}} - k_y v_{de})^2 A_{in} (k_\perp \rho_i^2) - 4 \frac{k_y v'_{0i}}{k_z \omega_{ci}} k_z^2 v_T^2 A_{in} (k_\perp \rho_i^2) \left( 1 - A_{i0} (k_\perp \rho_i^2) + \tau \right).
\]

It follows from Eqs. (4)-(6), that \( \delta \omega_1 (k) \), defines the frequency and the growth rate of the current driven IC instability[17], modified by flow shear[4]. This instability develops due to inverse electron Landau damping, when \( k_z (V_{0e} - V_{0i}) + k_y v_{de} > n_{\omega_{ci}} \) under condition \( |z_{i0}| \gg 1 \) of negligible IC damping of the mode \( \omega_1 \).

The solution \( \delta \omega_2 (k) \) defines a new shear flow driven branch of the IC waves, which is absent in shearless plasma flows. The frequency \( \text{Re} \delta \omega_2 (k) \) and growth rate \( \gamma_{02} = \text{Im} \delta \omega_2 (k) = \gamma_{i2} + \gamma_{e2} \) of this second instability are equal to[4]

\[
\text{Re} \delta \omega_2 (k) = \delta \omega_{02} (k) \approx \frac{k_y v'_{0i}}{k_z \omega_{ci}} \frac{k_z^2 v_T^2}{(n_{\omega_{ci}} - k_y v_{di})},
\]

\[
\gamma_{i2} = \frac{(\delta \omega_{02} (k))^2}{\Omega_n} k^2 \lambda_{Di}^2 \text{Im} \delta \varepsilon_i (k, n_{\omega_{ci}}) = \left( \frac{k_y v'_{0i}}{k_z \omega_{ci}} \frac{k_z^2 v_T^2}{(n_{\omega_{ci}} - k_y v_{di})} \right)^2 \times \sqrt{\frac{\pi}{2}} A_{in} (k_\perp \rho_i^2) \frac{e^{-z_i^2}}{|k_z| v_T e} \left[ (n_{\omega_{ci}} - k_y v_{di}) - \left( \frac{k_y v'_{0i}}{k_z \omega_{ci}} \frac{k_z^2 v_T^2}{(n_{\omega_{ci}} - k_y v_{di})} \right)^2 \frac{k_z^2 v_T^2}{(n_{\omega_{ci}} - k_y v_{di})} \right],
\]

\[
\gamma_{e2} = \frac{(\delta \omega_{02} (k))^2}{\Omega_n} \tau \sqrt{\frac{\pi}{2}} \frac{(n_{\omega_{ci}} - k_z (V_{0e} - V_{0i}) - k_y v_{de})}{|k_z| v_T e} \exp (-z_e^2).
\]

As it follows from Eq.(10), this mode becomes unstable for any values of \( k_\perp \rho_i \) due to inverse electron Landau damping, when the velocity of the relative drift between ions and electrons is below the critical value \( V_{0e}^{(c)} \) [4], roughly estimated as \( V_{0e}^{(c)} = V_{0i} + (n_{\omega_{ci}} - k_y v_{de})/k_z \), i.e. under conditions at which the current driven IC instability modified by shear flow does not develop.
In the case of a strong flow shear, for which the condition opposite to the above presented condition (3) is met, but with $k_y V'_0 / k_z \omega_{ci} < 0$, the growth rates $\gamma_1$ and $\gamma_2$ are determined by Eqs.(5), (6) and (9), (10), but with frequencies $\delta \omega_{01,02}$ replaced with

$$\delta \omega_{(+,-)} \approx \pm k_z v_{T_1} \left( \frac{k_y V'_0}{k_z \omega_{ci}} \frac{A_{in} (k^2 \rho_i^2)}{(1 - A_{i0} (k^2 \rho_i^2) + \tau)} \right)^{1/2}.$$  \hfill (11)

In the case of a sufficiently strong flow shear, when $\Omega_n^2 < 0$, the shear flow driven IC instability of the hydrodynamic (reactive) type is excited with frequency $\delta \omega_{(H)}$ and growth rate $\gamma_{(H)}$ approximately equal to

$$\delta \omega_{(H)} \simeq \frac{1}{2} \left( n \omega_{ci} - k_y v_{di} \right) A_{in} \left( k^2 \rho_i^2 \right),$$  \hfill (12)

$$\gamma_{(H)} \simeq \frac{\left( k_y V'_0 \right) k^2 \omega_{ci} A_{in} (k^2 \rho_i^2) (1 + \tau) - \frac{1}{4} \left( n \omega_{ci} - k_y v_{di} \right)^2 A_{in}^2 \left( k^2 \rho_i^2 \right)}{(1 - A_{i0} (k^2 \rho_i^2) + \tau)}.$$  \hfill (13)

This instability is in fact the extension of the hydrodynamic D’Angelo instability [25] onto the IC frequency range.

An important, but still absent, element in the studies of IC instabilities of plasmas with parallel shear flow is an understanding of the processes of their nonlinear evolution and saturation. For the current driven IC instabilities of plasma without flow shear these studies were grounded on the nonlinear theory of the IC resonances broadening, which was developed in Ref. [26]. The saturation level of the current driven IC instability obtained in Refs. [26, 27] appears to agree with IC heating experiments [29, 18]. The presence of shear flows leads to more complicate picture of ions scattering in turbulent electric fields and resonance broadening saturation mechanism. The present work extends the earlier studies of the renormalized theory of the IC turbulence by including combined effect of plasma turbulence and shear flow, which consists in turbulent scattering of ions across the shear flow into the regions with a greater or smaller flow velocity and enhancing by this means transport of ions along shear flow. In Section II of this paper we derive the nonlinear dispersion equation, which accounts for the effect of IC resonances broadening resulted from the random motion of ions in the electric field of the IC turbulence in magnetic field–aligned plasma shear flow. In Section III we present the approximate qualitative analysis of the nonlinear evolution of the shear flow modified current driven IC instability and shear flow driven kinetic IC instabilities, which resulted from the IC resonance broadening effect in the presence of shear flow. Finally, in Section IV we summarize our results.

II. NONLINEAR DISPERSION EQUATION

Our theory is based on the Vlasov-Poisson system of equations. We use leading center coordinates for ions $X = x + (v_{\perp}/\omega_{ci}) \sin \phi$, $Y = y - (v_{\perp}/\omega_{ci}) \cos \phi$, where $x, y, z$ are usual local particle
coordinates with $z$-axis directed along the magnetic field $B$, and where $v_\perp$ is velocity and $\phi$ is gyrophase angle of the gyromotion of ion. We find it suitable to use instead of $z$ and $\phi$ new variables $z_1 = z - \int v_\perp(\tau)\,d\tau, \phi_1 = \phi + \omega_{ci}t$. With these variables the governing Vlasov equation describing the perturbation $f_i$ by the self-consistent electrostatic potential $\Phi$ of the ion distribution function $F_i$, $F_i = F_{i0} + f_i$, where $F_{i0}$ is the equilibrium function of the distribution of ions, takes the form

$$
\frac{\partial f_i}{\partial t} + \frac{e}{m_i\omega_{ci}} \left( \frac{\partial \Phi}{\partial X} \frac{\partial f_i}{\partial Y} - \frac{\partial \Phi}{\partial Y} \frac{\partial f_i}{\partial X} \right) + \frac{e}{m_i} \omega_{ci} \left( \frac{\partial \Phi}{\partial \phi} \frac{\partial f_i}{\partial v_\perp} - \frac{\partial \Phi}{\partial v_\perp} \frac{\partial f_i}{\partial \phi} \right) - \frac{e}{m_i} \frac{\partial \Phi}{\partial z_1} \frac{\partial f_i}{\partial v_z} = e m_i \omega_{ci} \frac{\partial \Phi}{\partial Y} \frac{\partial F_{i0}}{\partial X} - \frac{e}{m_i} \omega_{ci} v_\perp \frac{\partial \Phi}{\partial \phi_1} \frac{\partial F_{i0}}{\partial v_\perp} + \frac{e}{m_i} \frac{\partial \Phi}{\partial z_1} \frac{\partial F_{i0}}{\partial v_z}. 
$$

(14)

This form of the Vlasov equation we consider as the most efficient for the deriving the renormalized solution for $f_i$. Using the system of equations for characteristics for Eq. (14),

$$
dt = \frac{dX}{e \frac{\partial \Phi}{\partial \phi_1}} = \frac{dY}{e \frac{\partial \Phi}{\partial \phi_1}} = \frac{dv_\perp}{e \frac{\partial \Phi}{\partial \phi_1}} = \frac{d\phi_1}{e \frac{\partial \Phi}{\partial \phi_1}} = \frac{dv_z}{e \frac{\partial \Phi}{\partial \phi_1}} = \frac{e}{m_i} \frac{\partial \Phi}{\partial \phi_1} \frac{\partial F_{i0}}{\partial X} - \frac{e}{m_i} \omega_{ci} v_\perp \frac{\partial \Phi}{\partial \phi_1} \frac{\partial F_{i0}}{\partial v_\perp} + \frac{e}{m_i} \frac{\partial \Phi}{\partial \phi_1} \frac{\partial F_{i0}}{\partial v_z}. 
$$

(15)

the following nonlinear solution for the function $f_i$ with known $F_{i0}$ is obtained:

$$
f_i = e m_i \int_0^t \left[ \frac{1}{\omega_{ci}} \left( \frac{\partial \Phi}{\partial \phi_1} \frac{\partial F_{i0}}{\partial X} - \frac{\partial \Phi}{\partial \phi_1} \frac{\partial F_{i0}}{\partial Y} \frac{\partial \Phi}{\partial \phi_1} \frac{\partial F_{i0}}{\partial v_\perp} \frac{\partial \Phi}{\partial \phi_1} \frac{\partial F_{i0}}{\partial v_z} \right) \right] dt'.
$$

(16)

Supposing that the particle orbit disturbance $\delta X$ due to the electrostatic plasma turbulence is sufficiently small, we find the solution for system (15). From the first equation of that system we obtain

$$
X = \bar{X} + \delta X, \quad \delta X = -\frac{e}{m_i \omega_{ci}} \int_0^t \frac{\partial \Phi}{\partial \phi_1} dt_1,
$$

(17)

where $\bar{X}$ and $\bar{Y}$ are the guiding center coordinates averaged over the turbulent pulsations. All other equations of system (14) have the following approximate solutions:

$$
Y = \bar{Y} + \delta Y, \quad \delta Y = \frac{e}{m_i \omega_{ci}} \int_0^t \frac{\partial \Phi}{\partial X} dt_1,
$$

(18)

$$
v_\perp = \bar{v}_\perp + \delta v_\perp, \quad \delta v_\perp = \frac{e \omega_{ci}}{m_i v_\perp} \int_0^t \frac{\partial \Phi}{\partial \phi} dt_1,
$$

(19)

$$
\phi = \bar{\phi} + \delta \phi, \quad \delta \phi = -\frac{e \omega_{ci}}{m_i v_\perp} \int_0^t \frac{\partial \Phi}{\partial v_\perp} dt_1.
$$

(20)
\[ v_z = \bar{v}_z + \delta v_z, \quad \delta v_z = -\frac{e}{m} \int_0^t \frac{\partial \Phi}{\partial \bar{z}} dt_1. \]  

(21)

In Eqs.(17)-(21) the perturbed electrostatic potential \( \Phi \) is defined in variables \( \bar{X}, \bar{Y}, \bar{v}_\perp, \bar{\phi}, \bar{v}_z \) and \( \delta X, \delta Y, \delta v_\perp, \delta \phi, \delta v_z \) as

\[
\Phi (\mathbf{r}, t) = \sum_{n=-\infty}^{\infty} \int dk J_n \left( \frac{k \bar{v}_\perp}{\omega_{ci}} \right) \exp \left[ ik_x \bar{X} + ik_y \bar{Y} + ik_z \bar{z} \right. \\
+ \left. ik_z (\bar{v}_z + V_{i0} (\bar{X})) t - in (\bar{\phi} - \omega_{ci} t - \theta) \right] \exp (i k \delta \mathbf{r} (t)) \int d\omega \Phi (k, \omega) \exp (-i\omega t),
\]

(22)

where the perturbations of the ions orbits due to wave-ion interactions,

\[
k \delta \mathbf{r} (t) = k_x \delta X (t) + k_y \delta Y (t) + k_z \delta z (t) - \frac{k \bar{v}_\perp (t)}{\omega_{ci}} \sin (\phi - \theta) - \frac{k \bar{v}_\perp}{\omega_{ci}} \cos (\phi - \theta) \delta \phi (t),
\]

(23)

are included. In Eq.(24) the first term corresponds to the scattering of ions along the magnetic field by the \( E_z = -ik_x \Phi \) projection of the turbulent electric field. The second term corresponds to the combined effect of turbulence and shear flow. This effect consists in turbulent scattering of ions across the magnetic field along the velocity shear from the flow layer with velocity \( V_0 (\bar{X}) \) into the layer with velocity \( V_0 (\bar{X}) + V'_0 (\bar{X}) \delta X \) and enhancing by this means the transport of ions along the shear flow. The order of value of the ratio of the second term in Eq.(24) to the first one is \( k_y V'_0 / k_z \omega_{ci} \).

In laboratory and ionospheric plasmas[6] this parameter may exceed unity considerably. In Eq.(24) the terms of the second order in \( \delta X, \delta Y, \delta v_\perp, \delta v_z \) and \( \delta \phi \) are omitted. The Fourier transformed Poisson’s equation,

\[
\Phi (k, \omega) = -\frac{4\pi}{k^2} \sum_{\alpha=i,e} e_\alpha \delta n_\alpha (k, \omega) = -\frac{4\pi}{k^2} \sum_{\alpha=i,e} e_\alpha \int f_\alpha d\mathbf{v},
\]

(25)

and Eq.(16) in which the potential \( \Phi (\mathbf{r}, t) \) is related with \( \Phi (k, \omega) \) by Eq.(22) with \( \delta \mathbf{r} \) determined by Eqs.(17)-(21), compose the system of nonlinear integral equations for \( \Phi (k, \omega) \) and \( f_\alpha \). We obtain from this system the nonlinear dispersion equation which accounted for the scattering of ions by turbulence in shear flow. We assume the equilibrium distribution function \( F_{i0} \) to be a drifting Maxwellian

\[
F_{i0} = \frac{n_{i0} (\bar{X})}{(2\pi v_{T_i}^2)^{3/2}} \exp \left( -\frac{v_z^2}{v_{T_i}^2} - \frac{(v_z - V_{i0} (\bar{X}))^2}{v_{T_i}^2} \right).
\]

(26)
Then the ion density perturbation will be equal to

\[ \delta n_i (k, \omega) = \int d\vec{v}_i (\omega, k, \vec{v}_\perp, \vec{\phi}, \vec{v}_z) \]

\[ = 2\pi i e \frac{e}{T_i} \sum_{n=-\infty}^{\infty} \int dv_z \int dv_{\perp} \int d\omega_1 \Phi (k, \omega_1) \int d\tau e^{i(\omega-\omega_1)\tau} \int dt_0 F_{i0} (\vec{X}, \vec{v}) \times \exp \left[ i\omega_1 (t - t_1) - ik_z (\vec{v}_z + V_{i0} (\vec{X})) (t - t_1) - in\omega_{ci} (t - t_1) \right] \times \exp \left[ -ik\Delta r (t, t_1) \right] J_n^2 \left( \frac{k_\perp \vec{v}_\perp}{\omega_{ci}} \right) \left[ k_y v_{yi} + \frac{V_{i0}}{\omega_{ci}} k_y \vec{v}_z - n\omega_{ci} - k_z \vec{v}_z \right] \]

(27)

where \( \Delta r (t, t_1) = \delta r (t) - \delta r (t_1) \). As in conventional renormalized theory [26, 27] we account for the average effect of the perturbations of ions orbits in Eq.(15). For this we can use the cumulant expansion [28],

\[ \langle \exp (-ik\Delta r (t, t_1)) \rangle = \exp \left( \sum_{n=1}^{\infty} \frac{1}{n!} \left[ -ik\Delta r (t, t_1) \right]^n \right) \]

(28)

where \( \langle \ldots \rangle_c \) is the cumulant. In the following we use the simplified approximation that the particles scattering by plasma turbulence is a Gaussian process, for which expansion (28) under Markovian approximation is reduced to a single term [26],

\[ \langle \exp (-ik\Delta r (t, t_1)) \rangle \approx \exp \left( -\frac{1}{2} \left[ \frac{\langle (k\delta r (t - t_1) \rangle} \right] \right) \]

(29)

With these approximations the ion density perturbation will be equal to

\[ \delta n_i (k, \omega) = \int d\vec{v}_i (\omega, k, \vec{v}_\perp, \vec{\phi}, \vec{v}_z) \]

\[ = i\frac{e}{T_i} n_{i0} (\vec{X}) \sum_{n=-\infty}^{\infty} A_{in} \left( \frac{k_\perp^2}{} \right) \left( k_y v_{yi} - n\omega_{ci} \right) \Phi (k, \omega) R_1 (k, \omega) \]

\[ - e \frac{e}{T_i} n_{i0} (\vec{X}) \sum_{n=-\infty}^{\infty} A_{in} \left( \frac{k_\perp^2}{} \right) k_z^2 \frac{v_z^2}{T_i} \left( 1 - \frac{k_y V_{i0}}{k_z \omega_{ci}} \right) \Phi (k, \omega) R_2 (k, \omega), \]

(30)

where

\[ R_1 (k, \omega) = \int_0^\infty d\tau \exp \left[ i\left( \omega - n\omega_{ci} - k_z V_{i0} (\vec{X}) \right) \tau - \frac{1}{2} k_z^2 \frac{v_z^2}{T_i} \tau^2 - \frac{1}{2} \left( \frac{k\delta r (\tau)} \right)^2 \right] \]

(31)

\[ R_2 (k, \omega) = \int_0^\infty d\tau \tau \exp \left[ i\left( \omega - n\omega_{ci} - k_z V_{i0} (\vec{X}) \right) \tau - \frac{1}{2} k_z^2 \frac{v_z^2}{T_i} \tau^2 - \frac{1}{2} \left( \frac{k\delta r (\tau)} \right)^2 \right] \]

(32)

with \( \tau = t - t_1 \) instead of \( t_1 \), are resonance functions which replace the familiar resonant denominators of the linear theory. Using Eq.(23) and omitting the terms which oscillate with frequency \( n\omega_{ci} \) \( (n = \pm 1, \pm 2, \ldots) \) we can write the mean square displacement as

\[ \frac{1}{2} \langle (k \cdot \delta r)^2 \rangle = \frac{1}{2} \langle (k \cdot \delta r)^2 \rangle_0 + k_z k_z \langle \delta X \delta z \rangle + k_y k_z \langle \delta Y \delta z \rangle + \frac{1}{2} k_z^2 \langle (\delta z)^2 \rangle, \]

(33)
The term \( \langle (k \cdot \delta r)^2 \rangle_0 \) exists in plasma even without shear flow\[26\]. The first three terms in Eq.(33) describe the diffusion of the ion guiding center coordinates \( X \) and \( Y \) and the last two terms describe the random changes of the IC radius \( \nu_\perp/\omega_{ci} \) and phase angle \( \phi \) of the Larmor orbit. In the asymptotic limit of large time compared with the correlation time \( \tau_{corr} \) of the turbulent electric field along the particle orbit, and after averaging over ions velocity assuming a Maxwellian distribution we obtain that the term \( \langle (k \cdot \delta r)^2 \rangle_0 \) is equal to

\[
\frac{1}{2} \langle (k \cdot \delta r)^2 \rangle_0 = C_1 t = \frac{e^2}{2m_i^2 \omega_{ci}} \text{Re} \sum_{n_i = -\infty}^{\infty} \int dk_x |\Phi(k_x)|^2 e^{-k_x^2 \rho_i^2} [2 (k_x k_{1y} - k_{1x} k_y)^2 I_{n_1} (k_{1\perp} \rho_i^2)] + \frac{k_x^2 k_{1\perp}^2}{2} \left( I_{n_1+1} (k_{1\perp} \rho_i^2) + I_{n_1-1} (k_{1\perp} \rho_i^2) \right) \\
\times \int_0^\infty d\tau \exp \left[ -i (\omega - n \omega_c - k_z V_{i0} (X)) \tau - \frac{1}{2} \left\langle (k \delta r (\tau))^2 \right\rangle \right],
\]

Last three terms in Eq.(33), which are due to scattering of ions along the magnetic field, are equal to

\[
k_x k_z \langle \delta X \delta z \rangle + k_y k_z \langle \delta Y \delta z \rangle + \frac{1}{2} k_z^2 \langle (\delta z)^2 \rangle = C_2 t^2 + C_3 t^3,
\]

with

\[
C_2 = \frac{e^2}{2m_i^2 \omega_{ci}} \text{Re} \sum_{n_i = -\infty}^{\infty} \int dk_x |\Phi(k_x)|^2 e^{-k_x^2 \rho_i^2} k_z k_{1z} \left( 1 + \frac{k_{1y} V_{i0}}{k_{1z} \omega_{ci}} \right) (k_x k_{1y} - k_{1x} k_y) \\
\times I_{n_1} (k_{1\perp} \rho_i^2) \int_0^\infty d\tau \exp \left[ -i (\omega - n \omega_c - k_z V_{i0} (X)) \tau - \frac{1}{2} \left\langle (k \delta r (\tau))^2 \right\rangle \right],
\]

\[
C_3 = \frac{e^2}{3m_i^2 \omega_{ci}} \text{Re} \sum_{n_i = -\infty}^{\infty} \int dk_x |\Phi(k_x)|^2 e^{-k_x^2 \rho_i^2} k_z^2 k_{1z}^2 \left( 1 + \frac{k_{1y} V_{i0}'}{k_{1z} \omega_{ci}} \right)^2 \\
\times I_{n_1} (k_{1\perp} \rho_i^2) \int_0^\infty d\tau \exp \left[ -i (\omega - n \omega_c - k_z V_{i0} (X)) \tau - \frac{1}{2} \left\langle (k \delta r (\tau))^2 \right\rangle \right].
\]

where the term with \( C_2 \) originates from the first two terms on the left of Eq.(36), whereas the term with \( C_3 \) resulted from the last left term of Eq.(36). Due to the term \( V_{i0}' (X) \int_0^t \delta X (t_1) dt_1 \) in Eq.(24), term \( C_3 \) includes the effect of the anomalous viscosity, resulted from the turbulent redistribution of the shear flow moment.

The frequencies and growth rates of ion cyclotron instabilities, which were considered in Ref.[4], are determined under condition of weak IC damping, i.e. for \( |\omega - k_z V_{i0} (X) - n \omega_c|/k_z V_{Ti} \gg 1 \).
Under this condition the asymptotics of integrals (31) and (32) are determined by the contributions over the time interval $0 < \tau \ll |\nu - k_z V_{i0} (X) - n \omega_{ci}|^{-1}$ and contributions from the stationary phase point. Our calculations of the integrals $R_1$ and $R_2$ give that the contribution from the stationary phase point is negligible for both integrals and main contributions proceed from the vicinity of $\tau = 0$ point and are equal to

$$R_1 = \int_0^\infty d\tau \exp \left( i \delta \omega \tau - C_2 \tau^2 - C_3 \tau^3 \right) \approx \frac{i}{\delta \omega} + \frac{2iC_2}{(\delta \omega)^2} - \frac{6C_3}{(\delta \omega)^4} ,$$

$$R_2 = \int_0^\infty d\tau \exp \left( i \delta \omega \tau - C_2 \tau^2 - C_3 \tau^3 \right) \approx - \frac{1}{(\delta \omega)^2} - \frac{6C_2}{(\delta \omega)^4} - \frac{24iC_3}{(\delta \omega)^6} ,$$

where now $\delta \omega (k)$ is the renormalized version of the linear $\delta \omega (k)$, presented in Introduction,

$$\delta \omega (k) = \nu - k_z V_{i0} (X) - n \omega_{ci} + iC_1 ,$$

and $C_2 = C_2 - \frac{1}{2} k_z v_T^2$. In Eq.(39) the first term $i/\delta \omega$ includes known [26] ion cyclotron resonance broadening effect; the following two ones, which are proportional to $C_2$ and $C_3$, reflect the combined effect of turbulent scattering and shear flow. Using above obtained results for $R_1$ and $R_2$ in Eq.(30) we obtain the renormalized version of the equation for perturbed ion density, $\delta n_i (k, \omega)$, which accounted for the the combined effect of shear flow and ions scattering by electrostatic turbulence,

$$\delta n_i (k, \omega) = - \frac{e_i n_{0i}}{T_i} \Phi (k, \omega) \left[ 1 - \sum_{n=-\infty}^\infty \left( \frac{\omega - k_z V_0 (X) - k_y v_{di} + iC_1}{\delta \omega} \right) \right] \times \left( 1 + \frac{k_x^2 v_T^2}{(\delta \omega)^2} \right) A_{in} \left( k_{\perp 1}^2 \rho_i^2 \right) - \frac{e_i n_{0i}}{T_i} \Phi (k, \omega) \frac{k_y V_0'}{k_z \omega_{ci}} \sum_{n=-\infty}^\infty \frac{k_x^2 v_T^2}{(\delta \omega)^2} A_{in} \left( k_{\perp 1}^2 \rho_i^2 \right) - \frac{2e_i n_{0i}}{T_i} \Phi (k, \omega) \sum_{n=-\infty}^\infty \frac{(-n \omega_{ci} + k_y v_{di})}{(\delta \omega)^3} A_{in} \left( k_{\perp 1}^2 \rho_i^2 \right) \left( C_2 + 3i \frac{C_3}{\delta \omega} \right) + 6 \frac{e_i n_{0i}}{T_i} \Phi (k, \omega) \left( 1 - \frac{k_y V_0'}{k_z \omega_{ci}} \right) \sum_{n=-\infty}^\infty \frac{k_x^2 v_T^2}{(\delta \omega)^4} A_{in} \left( k_{\perp 1}^2 \rho_i^2 \right) \left( C_2 + 4i \frac{C_3}{\delta \omega} \right) .$$

Inserting Eq.(42) to Poisson’s equation (25), we obtain the nonlinear dispersion equation

$$1 + \varepsilon_{0i} (k, \omega) + \varepsilon_e (k, \omega) + \varepsilon_{sh} (k, \omega) = 0 ,$$

which together with Eqs.(35)-(38) comprises the system of nonlinear integral equations for spectral intensity $|\Phi (k_1)|^2$ and frequency $\omega$. This system is derived under condition that the dominant nonlinear effect is a randomization of ion orbits. This effect enters the dispersion equation only
The solution is through resonance functions (39) and (40). In Eq.(43) we introduce the notations

\[\varepsilon_{0i}(\mathbf{k}, \omega) = \text{Re} \varepsilon^{(1)}_{i}(\mathbf{k}, \omega) + \text{Re} \varepsilon^{(2)}_{i}(\mathbf{k}, \omega)\]

\[= \frac{1}{k^2 \lambda^2_{Di}} \left\{ 1 - \sum_{n=-\infty}^{\infty} \left( \frac{\omega - k_z V_0(\tilde{X}) - k_y v_{di} + i C_1}{\delta \omega} \right) \left( 1 + \frac{k^2 v_{T_i}^2}{(\delta \omega)^2} A_{in} \left( k^2 \rho_i^2 \right) \right) \right\}

+ \frac{1}{k^2 \lambda^2_{Di}} \frac{k_y V_0'}{k_z \omega_{ci}} \sum_{n=-\infty}^{\infty} \frac{k^2 v_{T_i}^2}{(\delta \omega)^2} A_{in} \left( k^2 \rho_i^2 \right), \quad (44)\]

\[\varepsilon_e(\mathbf{k}, \omega) = \frac{1}{k^2 \lambda^2_{De}} \left[ 1 + i \sqrt{\frac{\pi}{2}} z_{\alpha} \exp(-z_{\alpha}^2) \right] = \frac{1}{k^2 \lambda^2_{De}} + \Delta \varepsilon_e(\mathbf{k}, \omega), \quad (45)\]

where \(z_{\alpha} = (\omega - k_z V_{\alpha}) / \sqrt{2 k_z v_T e}\) and

\[\varepsilon_{sh}(\mathbf{k}, \omega) = \frac{2}{k^2 \lambda^2_{Di}} \sum_{n=-\infty}^{\infty} \frac{(-n \omega_{ci} + k_y v_{di})}{(\delta \omega)^3} A_{in} \left( k^2 \rho_i^2 \right) \left( C_2 + 3 i \frac{C_3}{\delta \omega} \right)\]

\[- \frac{6}{k^2 \lambda^2_{Di}} \left( 1 - \frac{k_y V_0'}{k_z \omega_{ci}} \right) \sum_{n=-\infty}^{\infty} \frac{k^2 v_{T_i}^2}{(\delta \omega)^3} A_{in} \left( k^2 \rho_i^2 \right) \left( C_2 + 4 i \frac{C_3}{\delta \omega} \right). \quad (46)\]

The system of equations (35), (37), (38) and (43) is the main result of this theory. To obtain the solution \(\omega(\mathbf{k})\) of Eq.(43) we expand it about the solution \(\omega_0(\mathbf{k})\) of the linear dispersion equation,

\[1 + \varepsilon_{0i}(\mathbf{k}, \omega_0(\mathbf{k})) + 1/k^2 \lambda^2_{De} = 0,\]

\[1 + \varepsilon_{0i}(\mathbf{k}, \omega_0(\mathbf{k})) + \frac{\partial \varepsilon_{0i}(\mathbf{k}, \omega_0)}{\partial \omega_0} (\omega + i C_1 - \omega_0(\mathbf{k})) + \Delta \varepsilon_e(\mathbf{k}, \omega) + \varepsilon_{sh}(\mathbf{k}, \omega_0(\mathbf{k})) = 0. \quad (47)\]

The solution is

\[\omega(\mathbf{k}) = \omega_0(\mathbf{k}) - i C_1 - \frac{\Delta \varepsilon_e(\mathbf{k}, \omega) + \varepsilon_{sh}(\mathbf{k}, \omega_0(\mathbf{k}))}{\frac{\partial \varepsilon_{0i}(\mathbf{k}, \omega_0)}{\partial \omega_0}}. \quad (48)\]

Thus, the nonlinear growth/damping rate, which accounts for the scattering of ions by plasma turbulence in along-field shear flow is equal to

\[\gamma(\mathbf{k}) = \text{Im} \omega(\mathbf{k}) = \gamma_0(\mathbf{k}) - C_1 + \gamma_{sh}(\mathbf{k}), \quad (49)\]

where

\[\gamma_0(\mathbf{k}) = - \frac{\text{Im} \Delta \varepsilon_e(\mathbf{k}, \omega_0(\mathbf{k}))}{\frac{\partial \varepsilon_{0i}(\mathbf{k}, \omega_0)}{\partial \omega_0}}, \quad \gamma_{sh}(\mathbf{k}) = - \frac{\text{Im} \varepsilon_{sh}(\mathbf{k}, \omega_0(\mathbf{k}))}{\frac{\partial \varepsilon_{0i}(\mathbf{k}, \omega_0)}{\partial \omega_0}}. \quad (50)\]

In the case of the shearless plasma flow IC resonance broadening effect, determined by \(C_1\), leads to the saturation of the current driven IC instability[26]. Below we examine the role of the shear flow driven term \(\gamma_{sh}(\mathbf{k})\) in the nonlinear evolution of the shear flow modified and shear flow driven kinetic IC instabilities. It follows from Eq.(50) that when \(\gamma_{sh}(\mathbf{k}) < 0\) the combined effect of the shear flow and IC resonance broadening leads to the steady state, \(\gamma(\mathbf{k}) = 0\,\), and the saturation level of the IC turbulence is estimated from the equation

\[\gamma_0(\mathbf{k}) = C_1 - \gamma_{sh}(\mathbf{k}). \quad (51)\]

However, when \(\gamma_{sh}(\mathbf{k}) > 0\), a nonlinear instability may occur.
III. NONLINEAR STAGE AND SATURATION OF THE ION CYCLOTRON INSTABILITIES OF MAGNETIC FIELD-ALIGNED PLASMA SHEAR FLOW

In this section we apply the renormalized dispersion equation (43) to the qualitative analysis of the nonlinear evolution of the kinetic IC instabilities presented in Introduction, which are developed due to inverse electron Landau damping. It follows from Eqs.(6) and (10), that growth rates \( \gamma_{0(1,2)} \simeq \gamma_{e(1,2)} \) of the investigated instabilities, attain their maximal values at \( |z_e| = |n\omega_{ci} - k_z (V_{e0} - V_{i0})| / \sqrt{2}|k_z|v_{Te} \lesssim 1 \). For \( n\omega_{ci} \sim |k_z (V_{e0} - V_{i0})| \) that gives the estimate

\[
\frac{k_z}{k_\perp} \sim \frac{v_{Ti}}{v_{Te} k_\perp \rho_i},
\]

which links possible values of \( k_z/k_\perp \) and \( k_\perp \rho_i \) for which the maximum values of the growth rates may be developed. Applying this estimate to condition (3) of weak flow shear, and to the condition of strong flow shear, opposite to (3), we come to the following estimates:

\[
\left| \frac{V_0'}{\omega_{ci}} \right| < \frac{v_{Te}}{v_{Ti} k_\perp \rho_i} \frac{A_{in}(k_\perp \rho_i^2)}{1 - A_{i0}(k_\perp \rho_i^2) + \tau}
\]

for weak flow shear, and

\[
\left| \frac{V_0'}{\omega_{ci}} \right| > \frac{v_{Te}}{v_{Ti} k_\perp \rho_i} \frac{A_{in}(k_\perp \rho_i^2)}{1 - A_{i0}(k_\perp \rho_i^2) + \tau}
\]

for strong flow shear, which determine the possible values for \( k_\perp \rho_i \) in the cases of weak or strong flow shear for which maximal values of the growth rates may be attained. Accounting for these estimates, as well as the results of the linear theory presented in Introduction, we consider the cases of plasma flows with weak and strong flow shear separately.

A. Weak flow shear

In this subsection we consider the case of weak flow shear, limited by condition (3), for which two kinetic IC instabilities may develop [4]. The first one is the shear flow modified current driven IC instability, which is excited when \( k_z (V_{e0} - V_{i0}) + k_y v_{de} > n\omega_{ci} \) with frequency \( \omega_{01} = n\omega_{ci} - k_z V_{e0} + \delta \omega_{01} \) and with growth rate \( \gamma_{01} = \gamma_{ii} + \gamma_{ie} \), where \( \delta \omega_{01}, \gamma_{ii} \) and \( \gamma_{ie} \) are determined by Eqs.(4), (5), (6), respectively. The second one is the shear flow driven IC instability[4]. It is excited with frequency \( \omega_{02} = n\omega_{ci} - k_z V_{e0} + \delta \omega_{02} \) and with growth rate \( \gamma_{02} = \gamma_{ii} + \gamma_{ie} \), where \( \delta \omega_{02}, \gamma_{ii} \) and \( \gamma_{ie} \) are determined by Eqs.(8), (9), (10), respectively, when the velocity of the relative drift between ions and electrons is below the critical value \( V_{0e}^{(c)} \), roughly estimated as \( V_{0e}^{(c)} = V_{0i} + (n\omega_{ci} - k_y v_{de}) / k_z \), i.e. under conditions at which modified by shear flow current driven ion cyclotron instability does not develop. In the case of shearless current the dominant nonlinear effect responsible for the saturation of the current driven IC instability is the turbulent broadening of the ion cyclotron resonances[26, 27],
determined by the term $C_1$ in Eq.(49). The shear flow introduces new combined effect of shear flow and turbulent scattering, which is incorporated in $\gamma_{sh}$ term in Eq.(49). Here we evaluate the joint effect of shear flow and IC turbulence, determined by the terms $C_1$ and $\gamma_{sh}$ in (49) on the nonlinear evolution of the above mentioned shear flow modified and shear flow driven kinetic IC instabilities.

1. Shear flow modified ion cyclotron current driven instability

Under the condition of the weak flow shear (3), the ratio $|\gamma_{01}C_2/C_3|$ of the $C_2$ and $C_3$ contained terms in $\gamma_{sh}$ is

$$\left|\frac{\gamma_{01}C_2}{C_3}\right| \sim \frac{\gamma_{01} k_i^2}{\omega_{ci} k^2} \frac{1}{\left(1 + \frac{k_v V_{i0}}{k_i \omega_{ci}}\right)} > \frac{k_i^2 \rho_i^2}{A_{in} (k_i^2 \rho_i^2)} \left(1 - A_{i0} (k_i^2 \rho_i^2) + \tau\right)^2. \tag{55}$$

It follows from Eq.(55) that for $k_i \rho_i \gg 1$ $|\gamma_{01}C_2/C_3| > k_i^4 \rho_i^4 \gg 1$ and for $k_i \rho_i \ll 1$ $|\gamma_{01}C_2/C_3| > (k_i \rho_i)^2 \ll 1$. For this reason, for $k_i \rho_i \gg 1$ as well as for $k_i \rho_i \ll 1$, term with $C_3$ in Eq.(45) may be omitted and $\gamma_{sh}$ is determined by $C_2$ term in the first line of Eq.(46) as

$$\gamma_{sh(1)} (k) \simeq -\frac{6}{(1 - A_{i0} (k_i^2 \rho_i^2) + \tau)} \frac{n \omega_{ci} \gamma_0 C_2}{(\delta \omega_{01})^3 A_{in} (k_i^2 \rho_i^2)}. \tag{56}$$

Note, that because of $k_i v_{Ti} \ll \delta \omega_{01}$, the second line in Eq.(46) may be neglected. Now we estimate the ratio $C_1/\gamma_{sh(1)}$ under condition of weak flow shear. We have that

$$\frac{C_1}{\gamma_{sh(1)}} > \frac{k_i^2 \rho_i^2}{A_{in} (k_i^2 \rho_i^2)} \left(1 - A_{i0} (k_i^2 \rho_i^2) + \tau\right)^2 \gg 1 \tag{57}$$

for $k_i \rho_i \gg 1$ as well as for $k_i \rho_i \ll 1$. Therefore the effect of shear flow on the saturation of the shear flow modified IC current driven instability is subdominant. The level of the IC turbulence in the steady state, $e\Phi/T_e$, where $\Phi = \left(\int |\Phi (k_1)|^2 dk_1\right)^{1/2}$ is the root-mean-square (rms) magnitude of the perturbed electrostatic potential, is estimated from the balance equation $\gamma_0 (k) = C_1$. Using the mean value theorem for the integral over $k_1$ in Eq.(35) we have

$$\frac{e\Phi}{T_i} \sim \frac{1}{(k_i \rho_i)^{5/2}} \gtrsim \left(\frac{v_{Ti} V_{i0}'}{v_{Te} \omega_{ci}}\right)^{5/4}. \tag{58}$$

in the short wavelength, $k_i \rho_i \gg 1$, part of the spectrum, and

$$\frac{e\Phi}{T_i} \sim 2^{-\frac{n+1}{2}} (k_i \rho_i)^{n-1} \gtrsim 2^{-\frac{n+1}{2}} \left(\frac{v_{Ti} V_{i0}'}{v_{Te} \omega_{ci}}\right)^{\frac{n+1}{4}}. \tag{59}$$

in the long wavelength part, $k_i \rho_i \ll 1$, where we have used Eqs.(52) and (53) in limits $k_i \rho_i \gg 1$ and $k_i \rho_i \ll 1$, respectively. It follows from estimates (58) and (59) that $\Phi$ has a very low level (58) for IC turbulence in the short wavelength part of spectrum and attains its maximal value for $n = 1$ and $k_i \rho_i \lesssim 1$ [26], which is equal to

$$\frac{e\Phi}{T_i} \sim \frac{1}{2}. \tag{60}$$
2. Shear flow driven ion cyclotron instability

Now we estimate the ratio \( \gamma_{02} C_2 / C_3 \) which determines relative importance of the \( C_2 \) and \( C_3 \) contained terms in \( \text{Im} \, \varepsilon_{sh} (k, \omega_{02} (k)) \). It follows from Eqs. (7), (9), (37) and (38) that

\[
\left| \frac{\gamma_{02} C_2}{C_3} \right| \sim \tau \left| \frac{k_y V_{i0}'}{k_z \omega_{ci}} \right| \frac{k_z^4 \rho_i^4}{A_{in} (k_z^2 \rho_i^2) k_z^2} \frac{k_y^2}{(1 - A_{i0} (k_z^2 \rho_i^2) + \tau + k^2 \lambda_{ci}^2)},
\]

where constraint (3) was used. Therefore for long wavelength, \( k_z \rho_i < 1 \), IC mode \( \omega_2 (k) \) the term with \( C_3 \) is dominant in \( \text{Im} \, \varepsilon_{sh} \). In this case \( \gamma_{sh}^{(2)} \) is equal to

\[
\gamma_{sh}^{(2)} \simeq A_{in} (k_{1z}^2 \rho_i^2) C_3 \left( \frac{k_y V_{i0}'}{k_z \omega_{ci}} \right)^2 \frac{n \omega_{ci} + 24 \frac{k_z^2 \rho_i^2}{\delta \omega_{02}} \left( 1 - \frac{k_y V_{i0}'}{k_z \omega_{ci}} \right)}{(1 + \tau) (\delta \omega_{02})^2},
\]

where the approximation of the separate IC mode was used. In Eq. (62) \( C_3 \) is determined by the relation

\[
C_3 + \frac{2 e^2}{m_i} \sum_{n_1 = -\infty}^{\infty} \int d k_1 |\Phi (k_1)|^2 A_{in} (k_{1z}^2 \rho_i^2) k_z^2 k_y^2 \left( \frac{k_{1y} V_{i0}'}{k_z \omega_{ci}} \right)^2 \frac{C_3}{(\delta \omega_{02})^4}
\]

\[
= \frac{e^2}{3 m_i} \sum_{n_1 = -\infty}^{\infty} \int d k_1 |\Phi (k_1)|^2 A_{in} (k_{1z}^2 \rho_i^2) k_z^2 k_y^2 \left( \frac{k_{1y} V_{i0}'}{k_z \omega_{ci}} \right)^2 \frac{\gamma_{02}}{(\delta \omega_{02})^2},
\]

which follows from (38) and (39). In Eq. (63) the first term on the left side, \( C_3 \), is dominant over the second one when

\[
\frac{e \tilde{\Phi}}{T_i} \lesssim \frac{k_z^2 \rho_i^2}{\omega_{ci}^2} \frac{k_z \omega_{ci}}{A_{in} (k_z^2 \rho_i^2)} \frac{1}{k_y V_{i0}'}.
\]

However at level (64) \( \gamma_{sh}^{(2)} \simeq 24 \left( k_{1y} V_{i0}' / k_z \omega_{ci} \right)^2 \gamma_{02} \gg \gamma_{02} \), i.e. the saturation occurs before the second term in left hand side of Eq. (63) becomes comparable with the first one. In the case \( k_z \rho_i < 1 \)

\[
\frac{C_1}{\gamma_{sh}^{(2)}} \sim k_z \omega_{ci} \frac{k_z \omega_{ci}}{A_{in} (k_z^2 \rho_i^2)} \frac{1 + \tau}{\tau} \frac{\left( k_z \omega_{ci} \right)^2}{k_y V_{i0}'} \ll 1.
\]

Thus the nonlinear damping rate \( \gamma_{sh}^{(2)} \) (62), which for \( k_z \rho_i < 1 \) may be approximated as

\[
\gamma_{sh}^{(2)} \simeq -\frac{24 (\tau + k_z^2 \rho_i^2) C_3 \omega_{ci}^2}{1 + \tau} \frac{k_z \omega_{ci}}{k_y V_{i0}'} \frac{A_{in}^{1/2} (k_z^2 \rho_i^2)}{k_z \rho_i},
\]

dominates over \( C_1 \) in this case. The level of the IC turbulence in this case will be determined by the balance equation \( \gamma_{sh}^{(2)} \simeq \gamma_{02} \) with \( C_3 \) term determined by Eq. (63) without the second term. This balance equation gives the level

\[
\frac{e \tilde{\Phi}}{T_i} \sim \frac{k_z^2 \rho_i^2}{\omega_{ci}^2} \frac{1}{A_{in} (k_z^2 \rho_i^2)} \frac{k_z \omega_{ci}}{A_{in}^{1/2} (k_z^2 \rho_i^2)} \sim \frac{k_z \omega_{ci}}{A_{in} (k_z^2 \rho_i^2)} \frac{k_z \omega_{ci}}{k_y V_{i0}'} \frac{A_{in}^{1/2} (k_z^2 \rho_i^2)}{k_z \rho_i}.
\]

where Eq. (3) was used. Level (67) gives the estimate for the amplitudes of the perturbed potential for different values of the wave number \( k \) for long wavelength IC waves. This level is determined by
balancing linear growth rate with nonlinear damping rate, which we consider in our estimations as formed by perturbations of the electrostatic potential with spectrum width $\Delta k \sim k$ about $k$. The ultimate saturation level at which a steady state occurs over all wave number space is determined as a level which is sufficient for the stabilization of the wave number region of the modes with the maximal growth rate. It follows from Eqs.(10) that the growth rate $\gamma_{02} \approx \gamma_{e2}$ attains its maximal value at $z_{2e} \sim 1$. Thus the ultimate saturation level is determined by Eq.(67) with $k_z/k_\perp$ determined by Eq.(52) and and it is equal to

$$e\tilde{\Phi}/T_i \sim v_{Ti} \omega_{ci} A_{in}^{1/2} (k_{\perp 0}^2 \rho_i^2) / v_{Te} V_{i0}' k_{\perp 0}^2 \rho_i^2.$$  \hspace{1cm} (68)$$

In Eq.(52) $k_{\perp 0} \rho_i$ is determined by Eq.(53) and it is equal to

$$k_{0\perp} \rho_i = \sqrt{2} \left( n! \sqrt{2} v_{Ti} V_{i0}' / v_{Te} \omega_{ci} \right)^{1/(2n-1)}.$$ \hspace{1cm} (69)$$

Level (68) is in maximum for the fundamental IC mode with $n = 1$, for which

$$1 > k_{0\perp} \rho_i \geq v_{Ti} V_{i0}' / v_{Te} \omega_{ci}.$$ \hspace{1cm} (70)$$

The condition $|z_i| > 1$ of the weak ion cyclotron damping, under which Eq.(42) was obtained, imposes other restriction on the admissible values for $k_{0\perp} \rho_i$, which for mode $\delta \omega_2$ is

$$1 > k_{0\perp} \rho_i \geq \omega_{ci} V_{i0}'. $$ \hspace{1cm} (71)$$

The maximal level of IC turbulence, for which conditions (70) and (71) are met both, is of the order of

$$e\tilde{\Phi}/T_i \sim \left( \omega_{ci} / V_{i0}' \right)^2 $$ \hspace{1cm} (72)$$

for

$$\omega_{ci} / V_{i0}' < \left( v_{Ti} / v_{Te} \right)^{1/2},$$ \hspace{1cm} (73)$$

and

$$e\tilde{\Phi}/T_i \sim v_{Ti} / v_{Te}.$$ \hspace{1cm} (74)$$

when the condition opposite to (73) met. Note, that levels (72) and (74) appear to be much lower then level (60) for modified by shear flow current driven IC instability.

Now consider the short wavelength, $k_\perp \rho_i > 1$, part of the spectrum of IC waves. Accounting for Eqs.(52) and (53) we obtain that

$$\left| \frac{\gamma_{02} C_2}{C_3} \right| \sim \tau \left| \frac{k_y V_{i0}'}{k_z \omega_{ci}} \right| \frac{k_z^4 \rho_i^4}{A_{in} (k_{\perp 0}^2 \rho_i^2)^2 k_y^2} \gtrsim \frac{v_{Ti} \omega_{ci}}{v_{Te} V_{i0}'} \gg 1.$$ \hspace{1cm} (75)$$
which furnishes the dominance of the $C_2$ terms in $\gamma_{sh}$ for that part of the spectrum. Nonlinear damping rate $\gamma_{sh}$ is equal now to

$$\gamma_{sh} \sim \frac{6\gamma_0 C_2}{(\delta \omega_0)^2}. \quad (76)$$

The estimation of the ratio $C_1/\gamma_{sh}$ with $\gamma_{sh}$ determined by (76) gives, that

$$\frac{C_1}{\gamma_{sh}} \sim \frac{1}{k_{\perp} \rho_i} \frac{k_y \omega_{ci}}{k_z V_0'} \sim \frac{v_{Te} \omega_{ci}}{v_{Ti} V_0'} \gg 1. \quad (77)$$

Thus, the term $\gamma_{sh}$ in Eq.(49) may be omitted. The amplitude of the electrostatic potential $\tilde{\Phi}$ in the steady state may be determined in this case from the balance equation $\gamma_0 (k) = C_1$, which yields the estimate for the rms amplitudes of the perturbed potential for different values of $k_{\perp} \rho_i$ for short wavelength IC waves,

$$\frac{e \tilde{\Phi}}{T_i} \sim \frac{v_{Ti} V_0'}{v_{Te} \omega_{ci}} \frac{1}{(k_{\perp} \rho_i)^{1/2}}, \quad (78)$$

where the Eq.(52) was used. Eq.(53) and condition of weak IC damping of IC waves impose on $k_{\perp} \rho_i$ the restriction

$$\left(\frac{v_{Te} \omega_{ci}}{v_{Ti} V_0'}\right)^{1/2} \gtrsim k_{\perp} \rho_i \gtrsim \frac{\omega_{ci}}{V_0'}. \quad (79)$$

For that range of $k_{\perp} \rho_i$ level (78) is in the range

$$\frac{v_{Ti}}{v_{Te}} \left(\frac{V_0'}{\omega_{ci}}\right)^{3/2} \gtrsim \frac{e \tilde{\Phi}}{T_i} \gtrsim \left(\frac{v_{Ti} V_0'}{v_{Te} \omega_{ci}}\right)^{5/4}, \quad (80)$$

and the electric field strength $\tilde{E}_\perp = -k_{\perp} \tilde{\Phi}$ is

$$\tilde{E}_\perp \lesssim \frac{T_i}{e \rho_i} \left(\frac{v_{Ti} V_0'}{v_{Te} \omega_{ci}}\right)^{3/4}. \quad (81)$$

As a demonstration of level (81) we take the data detected in magnetopause by satellite Prognoz-8 [7, 22], $B = 2 \cdot 10^{-3} G$, $V_0'/\omega_{ci} = 0.5$, $T_e \sim T_i = 100 eV$, and obtain the estimate $\tilde{E}_\perp \lesssim 0.4 \cdot 10^{-3} V/m$ at fundamental cyclotron mode, which is in good agreement with measured value[22] $\tilde{E}_\perp \simeq 0.5 \cdot 10^{-3} V/m$.

**B. Strong flow shear. Shear flow driven ion cyclotron instabilities**

Under conditions of sufficiently strong flow shear, for which condition opposite to (3) is met, two shear flow driven IC kinetic instabilities may be developed for $k_y V_0'/k_z \omega_{ci} < 0$ with frequency determined by Eq.(11). These instabilities are excited with a growth rate determined by (5) and (6) when $k_z (V_{0e} - V_{0i}) > n \omega_{ci}$, and with growth rate (9) and (10), when $k_z (V_{0e} - V_{0i}) < n \omega_{ci}$. For short wavelength, $k_{\perp} \rho_i > 1$, ion cyclotron waves the estimate

$$\frac{\gamma_{01.02} C_2}{C_3} \sim \tau (k_{\perp} \rho_i)^{1/2} \left|\frac{k_y \omega_{ci}}{k_z V_0'}\right|^{1/2} > 1 \quad (82)$$
is valid, which defines the dominance of the $C_2$ terms in $\gamma_{sh}$ for that part of the spectrum. Nonlinear damping rate $\gamma_{sh}$ in this case is equal to

$$
\gamma_{sh} \simeq -24 \frac{\gamma_{01} C_2 A_{in} (k_{\perp}^2 \rho_i^2) k_z^2 v_T^2}{(1 + \tau) (\delta \omega_+)^4} \left( 1 + \frac{k_y V_{0i}'}{k_z \omega_{ci}} \right). \tag{83}
$$

The estimation of the ratio $C_1/\gamma_{sh}$ with $\gamma_{sh}$ determined by (83) gives that

$$
\frac{C_1}{\gamma_{sh}} \simeq \frac{(1 + \tau)}{24 \tau} \left( k_{\perp} \rho_i \right)^{1/2} \left( \frac{k_y \omega_{ci}}{k_z V_{0i}'} \right) \gg 1.
$$

Therefore the term $\gamma_{sh}$ in Eq.(49) may be omitted. The amplitude of the electrostatic potential $\tilde{\Phi}$ in the steady state may be determined in this case from the balance equation $\gamma_0 (k) = C_1$, which gives the following estimate:

$$
e \tilde{\Phi} \simeq \frac{1}{k_{\perp} \rho_i} \left( \frac{k_z V_{0i}'}{k_{\perp} \omega_{ci}} \right)^{1/2} \tag{85}.$$

By using Eqs.(52) and (54) we obtain from Eq.(85) the ultimate estimate for the value of the perturbed potential in the short wavelength part of the spectrum,

$$
e \tilde{\Phi} \simeq \left( \frac{v_{Ti} V_{0i}'}{v_T \omega_{ci}} \right)^{7/4} \tag{86}.$$

Now we consider the long wavelength, $k_{\perp} \rho_i < 1$, spectrum subrange of IC waves. Comparing the contributions of $C_2$ and $C_3$ to $\gamma_{sh}$ under condition of strong flow shear we obtain that

$$
\frac{\gamma_{01,02} C_2}{C_3} \sim \frac{k_{\perp} v_{Ti}}{\omega_{ci}} \left| \frac{k_z \omega_{ci}}{k_z V_{0i}'} \right|^{1/2} \left( A_{in} \left( k_{\perp}^2 \rho_i^2 \right) \right)^{1/2} (1 - A_{i0} \left( k_{\perp}^2 \rho_i^2 \right) + \tau + k^2 \lambda_B^2) \left( 1 \right)^{1/2}
\lesssim \tau k_{\perp}^2 \rho_i^2 \left( 1 - A_{i0} \left( k_{\perp}^2 \rho_i^2 \right) + \tau + k^2 \lambda_B^2 \right) \left( 1 \right)^{1/2}. \tag{87}
$$

Thus for long wavelength, $k_{\perp} \rho_i < 1$, ion cyclotron waves the nonlinear growth rate $\gamma_{sh}$ in the case of strong flow shear is dominated by the $C_3$ term and is approximated as

$$
\gamma_{sh} (k) \simeq \frac{24}{1 + \tau} C_3 A_{in} \left( k_{\perp}^2 \rho_i^2 \right) \frac{k_z^2 v_{Ti}^2}{(\delta \omega_+)^4} \left| \frac{k_y V_{0i}'}{k_z \omega_{ci}} \right|, \tag{88}
$$

where the coefficient $C_3$ is determined from the equation

$$
C_3 + \frac{2 e^2}{m_i^2} \sum_{n_1=-\infty}^{\infty} \int d\mathbf{k}_1 |\Phi (k_1)|^2 A_{in} \left( k_{\perp}^2 \rho_i^2 \right) k_z^2 k_1 \left( k_{\perp}^2 \rho_i^2 \right) \left( k_{\perp}^2 \rho_i^2 \right) \left( \frac{k_y V_{0i}'}{k_z \omega_{ci}} \right)^2 \left( \frac{C_3}{(\delta \omega_+)^4} \right) = \frac{e^2}{3 m_i^2} \sum_{n_1=-\infty}^{\infty} \int d\mathbf{k}_1 |\Phi (k_1)|^2 A_{in} \left( k_{\perp}^2 \rho_i^2 \right) k_z^2 k_1 \left( k_{\perp}^2 \rho_i^2 \right) \left( \frac{k_y V_{0i}'}{k_z \omega_{ci}} \right)^2 \left( \frac{\gamma_0}{(\delta \omega_+)^2} \right). \tag{89}
$$

Remarkably, $\gamma_{sh} (k)$ determined by Eq.(88) is positive and it may be responsible for the development of the nonlinear instability. From the estimate of the ratio $|C_1/\gamma_{sh}|$ under condition of strong flow shear (54),

$$
\left| \frac{C_1}{\gamma_{sh}} \right| \sim A_{in} \left( k_{\perp}^2 \rho_i^2 \right) \frac{\tau^2 (1 + \tau)}{24} \left| \frac{k_z \omega_{ci}}{k_y V_{0i}'} \right| < \frac{\tau^2 (1 + \tau) k_z^2 v_{Ti}^2}{24 n^2 \omega_{ci}^2} \ll 1. \tag{90}
$$
it follows, that the dominant nonlinear term in Eq.(49) in the long wavelength part of the spectrum of IC waves in the case of strong flow shear is $\gamma_{sh}$, determined by Eq.(88) with coefficient $C_3$, determined by Eq.(89). Thus the combined effect of strong shear flow and turbulent scattering of ions, determined by the $\gamma_{sh}$, leads to the nonlinear instability with growth rate (88). Note, that the first term of the left-hand side of Eq.(89) dominates over the second when

$$\frac{e\tilde{\Phi}}{T_i} \lesssim \tau A_{in}^{1/2}$$  \quad (91)

and it becomes less than the second term at the level of turbulence above (91). At the level

$$\frac{e\tilde{\Phi}}{T_i} \gtrsim \tau^{3/2} A_{in}^{1/2}$$  \quad (92)

the nonlinear growth rate becomes greater than the linear growth rate $\gamma_0(k)$.

IV. DISCUSSION AND CONCLUSION.

In this study, we have presented the renormalized theory of the IC turbulence in the magnetic field-aligned plasma shear flow. The developed theory extends the earlier studies [26, 27] of the renormalized theory of the IC turbulence by including a new combined effect of plasma turbulence and shear flow, determined in nonlinear dispersion equation (43) by the term $\varepsilon_{sh}(k, \omega)$. This effect consists in turbulent scattering of ions by the IC turbulence across the shear flow into the regions with a greater or smaller flow velocity and is the manifestation of the anomalous viscosity due to ion cyclotron turbulence. Analytically it manifests in nonlinear broadening of IC resonances. We have derived the approximate solution (49) of that equation and use it for qualitative analysis of the role of the discovered effect in nonlinear evolution of the shear flow modified current driven IC instability and shear flow driven kinetic IC instabilities. We have considered the limits of weak and strong flow shear for short wavelength and long wavelength parts of the spectrum of IC waves separately and arrived at the following conclusions.

1. The shear flow modified current driven IC instability, which develops under condition of weak flow shear (3), saturates as the ordinary IC instability driven by shearless current on the high level (60) in the long wavelength, $k_\perp \rho_i \ll 1$, part of the spectrum and on a very low level (58) in the short wavelength, $k_\perp \rho_i \gg 1$, part. The effect of the shear flow, determined by the term $\gamma_{sh}(k)$, is negligible on the saturation of this instability.

2. The long wavelength part of the spectrum of the shear flow driven IC instability under condition of weak flow shear saturates due to combined effect of shear flow and turbulent scattering of ions, determined by the term $\gamma_{sh}(k)$ in Eq.(48), on levels (72) or (74) (depending on the condition (73)), which appear to be much lower than the corresponding level (60) for current driven IC instability modified by shear flow.
3. The saturation of the short wavelength spectrum subrange of the shear flow driven instability under conditions of weak (49), as well as strong (49) flow shear arises from the turbulent scattering of ions by IC turbulence as in shearless plasma and determined by term $C_1$ in Eq.(49). It occurs at very low levels (78) and (86), respectively, which are comparable with corresponding level (58) for shear flow modified IC current driven instability.

4. Nonlinear evolution of the long wavelength part of the IC turbulence spectrum developed by the shear flow driven instability under conditions of strong flow shear is determined by the $\gamma_{sh}(k)$ term in Eq.(49). In this case combined effect of shear flow and turbulent scattering of ions introduces a principally new effect into the nonlinear development of the IC turbulence, which is absent in shearless plasma flows. It is the shear flow driven nonlinear instability with growth rate (88), which above level (91) becomes greater than the corresponding linear growth rate $\gamma_{02}(k)$, determined by Eqs.(9) and (10). This effect resembles the effect of the negative viscosity. It contrasts dramatically with effect of ion scattering by IC turbulence in shearless plasma flows[26, 27], where it leads only to saturation of IC instability.

It has to be noted here that we consider the model of the collisionless plasma. However even weak ion-neutral collisions, roughly $2 \cdot 10^{-2}$ of the IC frequency, are stabilizing. Ion collision frequency may be absorbed into the linear growth rate, leading to reducing the maximal growth rate. When ion-ion or ion-neutral collision frequencies are even marginally greater than an ion cyclotron instability growth rate, such instability does not develop [31]. The effect of ion collisions on the evolution of the considered ion cyclotron instabilities in near-Earth space plasmas determines the threshold altitude for the instabilities development. Our collision-free analysis is completely applicable to shear flows formed by solar wind around the Earth magnetosphere, to auroral acceleration region where at an altitude of 4000km the NASA’s FAST (Fast Auroral SnapshoT) satellite observed quite narrow ion beams[2] with $V'_0/\omega_{ci}$ ratio of the order of[6] 0.3 for hydrogen ions and of the order of 10 for oxygen ions. The collision-free analysis of our paper is pertinent to the F-region of the ionosphere for altitudes above 180km. In this region ion-neutral collision frequency $\nu_{in}$ is roughly $4 \cdot 10^{-3}$ of the ion cyclotron frequency and this value diminishes with altitude. That value may become less than the linear growth rate $\gamma_{e2}$ (Eq.(10)), $\gamma_{e2} \sim (v_{Ti}/v_{Te})^2 (V'_0/\omega_{ci})^2 (k^2 \rho_i^2 / A_{in} (k^2 \rho_i^2)) \omega_{ci}$, and is much less then $\gamma_{e2} \sim (v_{Ti}/v_{Te})^{1/2} (V'_0/\omega_{ci})^{1/2} (k\rho_i A_{in} (k^2 \rho_i^2))^{1/2} \omega_{ci}$ (see Eqs.(10) and (11)) for $\tau \sim 1$, $k\rho_i \sim 1$ for shear flows with $V'_0 \gtrsim \omega_{ci}$.

This paper is the first attempt of the analysis of the nonlinear evolution of the IC turbulence of magnetic field–aligned plasma shear flow. Here we have considered only the effect of scattering of ions by the IC turbulence in shear flow on the nonlinear evolution of kinetic IC instabilities. It is well known that other nonlinear effects, such as quasilinear flattening of the electron distribution function in wave-particle resonance region[17], and weak turbulence effects[32] may change the ultimate picture of the nonlinear stage of the IC turbulence. Real process of the nonlinear evolution of the instabilities considered will include the interplay of all these processes. That defines the need in the development of additional nonlinear theories for the plasma shear flows, which, however, are absent today. These
are IC weak turbulence theory for plasma shear flow, which has to include the processes of the induced
scattering of IC waves on ions and electrons and processes of the decay of IC waves, ordinary and
renormalized quasilinear theories for ion and electron distribution functions. Adequate coverage of
such a broad area of nonlinear plasma physics is difficult in a limited-page article and much more
have to be done for the development of a comprehensive nonlinear theory of the turbulent state of
plasma shear flows.

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References

[1] B.G.Fejer, and M.C.Kelly, Rev. Geophys., 18, 401 (1980)
[2] J.P.McFadden, C.W.Carlson, R.E.Ergun, F.S.Mozer, M.Temerin, W.Peria, D.M.Klumpar,
E.G.Shelley, W.K.Peterson, E.Moebius, L.Kistler, R.Elphic, R.Strangeway, C.Cattell, and
R.Pfaff, Geophys. Res. Lett., 25, 2021 (1998)
[3] C.W.Carlson, J.P.McFadden, R.E.Ergun, M.Temerin, W.Peria, F.S.Mozer, D.M.Kumpar,
E.G.Shelley, W.K.Peterson, E.Moebius, R.Elphic, R.Strangeway, C.Cattell, and R.Pfaff, Geophys. Res. Lett., 25, 2017 (1998)
[4] V.S.Mikhailenko, D.V.Chibisov, and V.V.Mikhailenko, Phys. Plasmas, 13, 102105 (2006)
[5] V.S.Mikhailenko, D.V.Chibisov, Phys. Plasmas, 14, 082109 (2007)
[6] W.E.Amatucci, Journ. Geophys.Res. 104, 14481 (1999)
[7] J.Bleski, K.Kossacki, B.Popielawska, S.I.Klimov, S.A.Romanov, S.P.Slavin, L.M.Zeleny, Physica
Scripta, 37, 623 (1988)
[8] J.P.McFadden, C.W.Carlson, R.E.Ergun, C.C.Chaston, F.S.Mozer, M.Temerin, D.M.Klumpar,
E.G.Shelley, W.K.Peterson, E.Moebius, L.Kistler, R.Elphic, R.Strangeway, C.Cattell., Geophys.
Res. Lett., 25, 2045 (1998)
[9] C.Cattell, R.Bergmann, K.Sigsbee, C.Carlson, C.Chaston, R.Ergun, J.MacFadden, F.C.Mozer,
M.Temerin, R.Strangeway, R.Elphic, L.Kistler, E.Moebius, L.Tang, D.Klumpar, R.Pfaff, Geophys.
Res. Lett., 25, 2053 (1998)
[10] T.H.Okuda, M.Ashour-Abdala, Geophys. Res. Lett., 8, 811, (1981)

[11] P.M.Kintner, J.Bonnell, R.Arnoldy, K.Lynch, C.Pollock, and T.Moor, Geophys. Res. Lett., 23, 1873 (1996)

[12] J.M.Kindel and C.F.Kennel, J. Geophys. Res., 76, 3055 (1971)

[13] R.Bergmann, J. Geophys. Res., 89, 953 (1984)

[14] R.Bergmann, I.Roth, and M.K.Hudson, J. Geophys. Res., 93, 4005 (1988)

[15] E.A.Bering, M.C.Kelley, and F.S.Mozer, J. Geophys. Res., 80, 4612 (1975)

[16] P.M.Kintner, W.Sales, J.Vago, R.Arnoldy, G.Garbe, and T. Moore, Geophys. Res. Lett., 16, 739 (1989)

[17] W.E.Drummond and M.N.Rosenbluth, Phys. Fluids, 5, 1507 (1962)

[18] J.J.Rasmussen, R.W.Schrittwieser, IEEE Transaction on Plasma Science, 19, 457 (1991)

[19] A.Hirose, I.Alexeff, Phys. Fluids, 16, 1087 (1973)

[20] S.P.Gary, S.J.Schwartz, Journ. Geophys.Res. 85, 2978 (1980)

[21] G.S.Lakhina, Journ. Geophys.Res. 92, 12161 (1987)

[22] E.V.Belova, Ya.Blensky, M.Denis, L.M.Zelenyy, S.P.Savin, Sov. Plasma Physics, 17, 401 (1991)

[23] V.V.Gavrishchaka, G.I.Ganguli, W.A.Scales, S.P.Slinker, C.C.Chaston, J.P.McFadden, R.E.Ergun, and C.W.Carlson, Phys. Rev. Lett., 85, 4285 (2000)

[24] G.Ganguli, S.Slinker, V.Gavrishchaka, W.A.Scales, Phys. Plasmas, 9, 2321 (2002)

[25] N.D’Angelo, Phys. Fluids, 8, 1748 (1965)

[26] C.T. Dum, T.H.Dupree, Phys. Fluids, 13, 2064 (1971)

[27] G.Benford, J. Plasma Phys., 15, 431 (1976)

[28] J.Weinstock, Phys. Fluids, 11, 1977 (1968)

[29] D.R.Dakin, T.Tajima, G.Benford and N.Rynn, J. Plasma Phys., 15, 175 (1976)

[30] P.K.Chaturvedi, J. Geophys. Res., 81, 6169 (1976)

[31] P.Satyanarayana, P.K.Chaturvedi, M.J.Keskinen, J.D.Huba, and S.L.Ossakow, J. Geophys. Res., 90, 12209 (1985)

[32] V.S. Mikhailenko, K.N.Stepanov, Plasma Physics, 33, 1165 (1981)