Multi-planner Intervention in Network Games with Community Structures

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Abstract—Network games study the strategic interaction of agents connected through a network. Interventions in such a game – actions a coordinator or planner may take that change the utility of the agents and thus shift the equilibrium action profile – are introduced to improve the planner’s objective. We study the problem of intervention in network games where the network has a group structure with local planners, each associated with a group. The agents play a non-cooperative game while the planners may or may not have the same optimization objective. We model this problem using a sequential move game where planners make interventions followed by agents playing the intervened game. We provide equilibrium analysis and algorithms that find the subgame perfect equilibrium. We also propose a two-level efficiency definition to study the efficiency loss of equilibrium actions in this type of game.

I. INTRODUCTION

Strategic decision making of (physically or logically) connected agents is often studied as a network game, where the utility of an agent depends on its own actions as well as that of those in its neighborhood as defined by an interaction graph or adjacency matrix. This framework can be used to capture different forms of interdependencies between agents’ decisions. Network games and their equilibrium outcomes have been studied in a variety of application areas, including the private provision of public goods [2], [3], [11], security decision making in interconnected cyber-physical systems [8], [12], and shock propagation in financial markets [1].

Within this context, intervention in a network game typically refers to changes in certain game parameters made by a utilitarian welfare maximizer with a budget constraint, who wishes to induce a more socially desirable outcome (in terms of social welfare) under the revised game. A prime example is the study presented in [7], where the intervention takes the form of changing the agents’ standalone marginal benefit terms (in a linear quadratic utility model) and changes are costly; this is done by a central/global planner, who wishes to find the set of interventions that lead to the highest equilibrium social welfare subject to a cost constraint.

Finding optimal interventions could be viewed as a form of mechanism design, because in both cases the design or intervention essentially induces a new game form with desirable properties. But there are a few distinctions between intervention and the standard mechanism design framework. Specifically, mechanism design is often not limited to a specific game form, the latter being the outcome of the design, while intervention typically starts from a specified game form and seeks improvement through local changes. Mechanism design typically has the goal of social optimality (i.e., that the outcome/equilibria of the designed game are social welfare maximizing), while intervention aims to do the best under the constraints of a budget and specified forms of intervention.

In this paper, we are interested in intervention in a network game where the network exhibits a group or community structure and each group or community has its own local group planner. Since group structures are a common phenomenon across networks of all types, be it social, technological, political, or economic, this modeling consideration allows us to investigate a number of interesting features that often arise in realistic strategic and decentralized decision making. For instance, a single global budget may be first divided into separate chunks of local budgets at the local planners’ disposal; these local budgets may or may not be transferred from one community to another, and the local planners’ decisions may or may not take into account the connectivity between themselves and other neighboring communities; local planners may or may not wish to cooperate with each other; and so on.

Of particular interest to our study is the issue of efficiency in this type of decision making systems. A standard notion that measures efficiency loss in a strategic game is the Price of Anarchy (POA); this is defined as the upper bound on the ratio of the maximum social welfare (sum utility) divided by the social welfare attained at a Nash equilibrium (NE) of the game. The numerator is what a social planner aims for, while the denominator is the result of agents optimizing their own utilities and best-responding to each other. POA has been extensively studied in a variety of games, including in interdependent security games such as [13], [15], where agents’ incentive to free-ride or over-consume contributes to the efficiency loss; in routing and congestion games [5], [14]; and in network creation games [6].

It is clear that additional sources of efficiency loss exist in the community intervention problem we are interested in: in addition to agents’ self-interested decision making, local planners’ non-cooperation as well as sub-optimal budget allocation among groups can both results in efficiency loss. The main findings of the paper are summarized as follows:

1) We show that through backward induction the planners can obtain a reduced version of the planners’ game that only depends on each other’s intervention profiles. Regardless of being cooperative or not, the sequential game always has a unique subgame perfect equilibrium. Moreover, this equilibrium can be achieved
through a decentralized algorithm based on the best responses of the planners.

2) We introduce a two-level definition of efficiency loss that allows us to discuss how the planners’ actions influence the outcome of the game separately from the agents’ actions, and we show that the efficiency loss due to the planners’ non-cooperation can be characterized with the budget constraints and shadow prices.

3) We present numerical results on welfare and efficiency in several commonly seen types of interaction graphs and commonly used budget allocation rules.

For the remainder of the paper, Section II introduces the intervention game model; Section III presents the analysis and characterization of the subgame perfect equilibrium of the intervention game; in Section IV we study the Level-1 and Level-2 efficiencies of the subgame perfect equilibrium; numerical experiments are presented in Section V; and Section VI concludes the paper. All proofs can be found in [10].

II. GAME MODEL

We consider a network game among $N$ agents, denoted by $a_1, \ldots, a_N$, represented by a directed graph $G = (N, E)$, where $N$ is the set of nodes/agents and $E \subseteq N \times N$ the set of edges. Let $G = (g_{ij})_{i,j}$ denote the adjacency matrix, assumed to be symmetric and as a convention $g_{ii} = 0$; $g_{ij} \neq 0$ implies dependence between $a_i$ and $a_j$, $i \neq j$.

Agents are divided into $M$ disjoint communities, the $k$th community denoted as $S_k$ with size $N_k$. Agent $a_i$ takes an action $x_i \in \mathbb{R}$. Let $x_{-i} = [x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N]^T$ denote the action profile of all except $a_i$, $x_{S_k} = (x_i)_{a_i \in S_k}$ the action profile of members in community $S_k$, and $x_{-S_k}$ the action profile of all agents other than members of $S_k$.

We consider a family of games with utility:

$$u_i(x_i, x_{-i}, y_i) = (b_i + y_i)x_i - \frac{1}{2}x_i^2 + x_i \left( \sum_{j \neq i} g_{ij}x_j \right),$$  \hspace{1cm} (1)

which depends on the action profile and a real valued parameter $y_i$ controlled by a planner. This utility function with intervention is studied in [4], [7]. The $-\frac{1}{2}x_i^2$ is the individual cost for $a_i$, the $b_i x_i$ term is the initial individual marginal benefit, and $x_i \left( \sum_{j \neq i} g_{ij}x_j \right)$ models the network influence. The intervention component can be seen as a linear subsidiary (discount) term if $y_i > 0$ and a linear penalty (price) term if $y_i < 0$. In this non-cooperative game, the optimization problem of agent $a_i$ for given intervention is

maximize \hspace{1cm} \[ u_i(x_i, x_{-i}, y_i). \]  \hspace{1cm} (2)

The NE of the game $\mathbf{x}^*$, is the action profile where no agent has an incentive to unilaterally deviate, i.e.,

$$x_i^* = \text{argmax}_{x_i} u_i(x_i, \mathbf{x}_{-i}^*).$$  \hspace{1cm} (3)

We denote the planner for $S_k$ as $p_k$, which has a budget constraint $C_k > 0$: $\sum_{a_i \in S_k} y_i^2 \leq C_k$. We denote $y_{S_k} = (y_i)_{a_i \in S_k}$ as the intervention profile of $p_k$ and $y_{-S_k}$ the intervention profile of planners other than $p_k$. Denote $Q_k = \{y_{S_k} \mid \sum_{a_i \in S_k} y_i^2 \leq C_k \}$; thus $Q_k$ is nonempty, convex and compact. Finally, $Q = \prod_{i=1}^{M} Q_i$.

We consider two cases. In the first, planner $p_k$ is a group-welfare maximizer, whose objective is to maximize the sum of its members’ utilities at the NE, formally

$$\text{maximize} \hspace{1cm} y_{S_k} \in Q_k \hspace{1cm} \sum_{a_i \in S_k} u_i(x^*, y_i),$$  \hspace{1cm} (4)

and we denote $y_{S_k}^* = \text{argmax}_{y_{S_k} \in Q_k} \sum_{a_i \in S_k} u_i(x^*, y_i)$. When all planners are group-welfare maximizers, we say they are non-cooperative. In the second case, planner $p_k$ is a social-welfare maximizer, whose objective is to maximize the sum of all agents’ utilities at the NE, formally

$$\text{maximize} \hspace{1cm} y_{S_k} \in Q_k \hspace{1cm} \sum_{i=1}^{N} u_i(x^*, y_i),$$  \hspace{1cm} (5)

and we denote $y_{S_k}^* = \text{argmax}_{y_{S_k} \in Q_k} \sum_{i=1}^{N} u_i(x^*, y_i)$. When all planners are social-welfare maximizers, we say they are cooperative.

Figure 1 shows the structure of the intervention game described in this section. It’s easy to see that with a single planner ($M = 1$), the above may be viewed as a two-stage game: the first mover the planner chooses the intervention actions $y$ in anticipation of the (simultaneous) second movers the agents playing the induced game with actions $x$. There is a similar two-stage sequentiality in the case of $M$ local planners as shown in Figure 2: the local planners are simultaneous first movers in choosing interventions for their
respective communities, in anticipation of interventions by other local planners and actions by the simultaneous movers the agents. For this reason, the solution concept we employ in this study is the subgame perfect equilibrium.

III. THE SUBGAME PERFECT EQUILIBRIUM

In this section, we characterize the subgame perfect equilibrium of the system and introduce an algorithm to compute it. We assume the following holds throughout this paper:

Assumption 1: Matrix $I - G$ is positive definite.

We start with computing the NE under an arbitrary intervention, we can compute the first order derivatives as follows

$$\frac{\partial u_i}{\partial x_i} = -x_i + \sum_{j \neq i} g_{ij}x_j + (b_i + y_i), \quad (6)$$

and the unique NE of the game is

$$\mathbf{x}^* = [I - G]^{-1}(\mathbf{b} + \mathbf{y}), \quad (7)$$

which is the fixed point of the best responses. Denote $A = [I - G]^{-2}$ for simplicity of notation. This NE is known to all planners through backward induction.

A. Finding the subgame perfect equilibrium

We denote $G_{S_k,S_{-k}}$ as the block of $G$ corresponding to the rows in $S_k$ and columns in $S_{-k}$, and $G_{S_k}$ as the block of $G$ corresponding to the rows in $S_k$ and all columns. It’s worth noting that given the representation of $\mathbf{x}^*$ in Eqn (7), the objective of a group welfare maximizer $p_k$ is (See Appendix)

$$U_k(\mathbf{x}^*, \mathbf{y}_{S_k}) = \frac{1}{2} \mathbf{x}_{S_k}^T \mathbf{x}_{S_k}. \quad (8)$$

We can then rewrite the objective of $p_k, \forall k$, and the non-cooperative optimization problem (P-NC) as

$$\text{maximize} \quad W_k(\mathbf{y}) = \frac{1}{2} ||(A^{1/2}(\mathbf{y} + \mathbf{b}))_{S_k}||_2^2$$

subject to $\sum_{i:a_i \in S_k} (y_i)^2 \leq C_k. \quad (9)$

It’s worth noting that this doesn’t imply the planners’ optimization problems are independent, since we can write $\mathbf{x}_{S_k}^* \equiv (A_{S_k}^{1/2}) (\mathbf{y} + \mathbf{b})_k$, which depends on $\mathbf{y}_{-S_k}$ unless $S_k$ is isolated. Similarly, we can rewrite the cooperative optimization problem (P-C) as

$$\text{maximize} \quad W(\mathbf{y}) = \frac{1}{2} (\mathbf{y} + \mathbf{b})^T \mathbf{A}(\mathbf{y} + \mathbf{b})$$

subject to $\sum_{i:a_i \in S_k} (y_i)^2 \leq C_k, \forall k \quad (10)$

We can also write out the decentralized version of (P-C) where each planner $p_k$ has its own optimization problem (P-C$k$) given other planners’ intervention profile $\mathbf{y}_{-S_k}$

$$\text{maximize} \quad W(\mathbf{y}) = \frac{1}{2} (\mathbf{y} + \mathbf{b})^T \mathbf{A}(\mathbf{y} + \mathbf{b})$$

subject to $\sum_{i:a_i \in S_k} (y_i)^2 \leq C_k. \quad (11)$

Theorem 1: Either planners are all cooperative or all non-cooperative, there is a unique optimal intervention, i.e., unique subgame perfect equilibrium, and under the optimal intervention, the budget constraints are tight.

This is obvious when all planners are cooperative from Eqn (10); for the other case see Appendix. We also propose the following decentralized algorithm based on best response dynamics (BRD) that computes the subgame perfect equilibrium in both cases. Note that the planners’ best-response computation utilizes Eqn (9) and (10).

Algorithm 1 Planners’ BRD

Initialize: $\mathbf{y}(0) = \mathbf{y}_0, t = 0$

while $\mathbf{y}$ not converged do

for all $k = 1:M$ do

$\mathbf{y}_{S_k}(t+1) = \text{argmax}_{\mathbf{y}_{S_k}} U_k(\mathbf{y}_{S_k}, \mathbf{y}_{-S_k}(t))$

$\triangleright$ best response w.r.t objective $U_k$, which can be either group or social welfare

t $\leftarrow t + 1$
end for

end while

Set optimal intervention profile as $\mathbf{y}^* = \mathbf{y}(t)$
Compute the agents’ Nash equilibrium in the intervened game $\mathbf{x}^* = (I - G)^{-1}(\mathbf{b} + \mathbf{y}^*)$

Theorem 2: Either planners are all cooperative or all non-cooperative, Algorithm 1 converges to the unique subgame perfect equilibrium.

Proposition 1: The following cooperative optimization problem (P-NC-alt) has the same optimal intervention outcome as the original non-cooperative problem (P-NC$k$) where all planners are group-welfare maximizers:

$$\text{maximize} \quad W(\mathbf{y}) = \frac{1}{2} (\mathbf{y} + \mathbf{b})^T \hat{\mathbf{A}}(\mathbf{y} + \mathbf{b})$$

subject to $\sum_{i:a_i \in S_k} (y_i)^2 \leq C_k, \forall k \quad (12)$

where $\hat{\mathbf{A}} = \begin{bmatrix} A_{S_1,S_1} & \frac{1}{2} A_{S_1,S_2} & \cdots & \frac{1}{2} A_{S_1,S_M} \\ \frac{1}{2} A_{S_1,S_2} & A_{S_2,S_2} & \cdots & \frac{1}{2} A_{S_2,S_M} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{2} A_{S_M,S_1} & \frac{1}{2} A_{S_M,S_2} & \cdots & A_{S_M,S_M} \end{bmatrix}$

Proposition 1 is obtained by studying the gradient of each planner’s objective function. We also characterize the direction of the optimal intervention profile in the appendix.

B. Lagrangian Dual and Shadow Prices

Next we introduce some concepts related to the Lagrangian dual variables and shadow prices, which is related to the efficiency of the equilibrium in the next section.

Since $Q$ clearly satisfies Slater’s Constraint Qualification, the planners’ optimization problems are convex regardless of whether they are group or social welfare maximizers. Then based on the KKT condition, we know that strong duality holds for both cooperative and non-cooperative planners’ optimization problems. If we define the Lagrangian as

$$L(\mathbf{y}, \lambda) = W(\mathbf{y}) + \sum_{k=1}^M \lambda_k (C_k - ||\mathbf{y}_{S_k}||^2),$$

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and $W(y)$ is either social or group welfare, then we can obtain an optimal dual $\lambda_k^*$, the shadow price for $p_k$. We can then equivalently think of $p_k$’s problem as maximizing the above Lagrangian with a cost of intervention $\lambda_k \sum_{i \in S_k} y_i^2$. We use $\lambda_k^*$ (resp. $\tilde{\lambda}_k$) to denote the dual variable corresponding to group (resp. social) welfare maximization problem in the (P-NCK) (resp. (P-C)) problem for the planners.

IV. EFFICIENCY AND THE BUDGET ALLOCATION

In this section, we discuss the efficiency of the subgame perfect equilibria under a fixed budget allocation and then study the impact of different budget allocations on the equilibrium and its efficiency.

A. Efficiency of the subgame perfect equilibrium

For conventional single-planner multi-agent systems, the efficiency of an NE is characterized as the ratio of the social objective value in the NE divided by the socially optimal outcome, formally $e(x^*) = U(x^*)/\max_{x \geq 0} U(x)$, and an upper bound on its reciprocal is referred to as the price of anarchy (PoA) if the objective $U(x) = \sum_{i=1}^N u_i(x)$. This maxima is achievable if the agents’ utility functions are strictly individually concave and always have a zero point in the first order derivative.

The introduction of group planners in our intervention problem means there are now multiple sources of efficiency loss. Accordingly, we will decompose this into a level-1 (L1) component and a level-2 (L2) component, caused by the non-cooperation of agents and planners, respectively. Following the notation of $y^*$ and $\bar{y}$ in Eqn (4) and (5), we formally define the two efficiency loss measures as

$$ e_{L1}(y) = \frac{U(x^*, y)}{\max_{x \geq 0, y \in G} U(x, y)}, \quad e_{L2} = \frac{W(x^*, y^*)}{W(x^*, \bar{y})} . \quad (13) $$

Thus the overall efficiency, which resembles the conventional definition, can be written as

$$ e(x^*, y^*) = \frac{U(x^*, y^*)}{\max_{x \geq 0, y \in G} U(x, y)} = e_{L2} \cdot e_{L1}(\bar{y}) . $$

The L1 efficiency has been well studied in the literature, e.g., [9]. For an arbitrary intervention profile $y$, if $I - 2G > 0$, then L1 efficiency can be written as

$$ e_{L1}(y) = \frac{(b + y)^T (I - G)^{-2} (b + y)}{(b + y)^T (I - 2G)^{-2} (b + y)} , $$

since

$$ \frac{\partial \sum_{i=1}^N u_i}{\partial x_i} = -x_i + 2 \sum_{j \neq i} g_{ij} x_j + (b_i + y_i) , $$

and by computing the fixed point we know the action profile maximizing the social welfare is $x = (I - 2G)^{-1} (b + y)$.

We have the following result on the L2 efficiency.

**Theorem 3:** When $b = 0$ or $C_k \gg ||b_{S_k}||_2^2, \forall k$, the welfare in (P-C) can be computed by $W = \sum_{k=1}^M \tilde{\lambda}_k C_k$, and a lower bound on the L2 efficiency is

$$ e_{L2} \geq \frac{\sum_{k=1}^M (2\tilde{\lambda}_k - 1/2 \rho_k) C_k}{\sum_{k=1}^M \tilde{\lambda}_k C_k} , \quad (14) $$

where $\rho_k$ denotes the spectral radius of $A_{S_k}$.
may interact more frequently with buyers than with another seller. In the extreme case where sellers (resp. buyers) only interact with buyers but not with other sellers (resp. buyers), a multipartite graph can be used to capture their interactions.

**Type 3:** evenly distributed connections. Here groups become a rather arbitrarily constructed concept that may not correspond to agent interactions in a game.

We also consider three types of budget allocation.

**Proportional:** each group is assigned a budget proportional to its size, i.e., \( C_k = \frac{N_k}{N}C \).

**Identical:** each group is assigned an equal share of the total budget, i.e., \( C_k = C/M \).

**Cooperative socially optimal:** the allocation in the optimal solution of the cooperative optimization problem (Eqn (15)), where the shadow prices \( \lambda_k \) are the same for all \( k \).

Sample games used in the numerical experiments are generated as follows. In generating a random symmetric \( G \), the diagonal elements are set to 0 as previously described in Section II. The off-diagonal elements in the diagonal blocks are generated using a Bernoulli distribution with parameter \( p_{\text{exist}} \), the probability for an edge (non-zero element) to exist between a pair of agents. The absolute value of a non-zero element (strength of a connection) \( |g_{ij}| \) is drawn from a uniform distribution on the interval \([s_{\text{low}}^\text{in}, s_{\text{high}}^\text{in}]\). The off-diagonal blocks of \( G \) are similarly generated using the same approach, with parameters \( p_{\text{exist}}^\text{out} \) and \([s_{\text{low}}^\text{out}, s_{\text{high}}^\text{out}]\), respectively. The signs of the connections are assigned to yield the following two types of games. In the first, within-group connections and between-group connections have the same sign (all positive); in the second, they have opposite signs (positive within-group, negative between-group; this is also referred to as conflicting groups below). The \( b \) vector is generated by sampling every element uniformly from an interval \([b_{\text{low}}, b_{\text{high}}]\).

For strong connections, \( P_{\text{exist}} = 0.8 \) and \( S_{\text{low}} = 0.7, S_{\text{high}} = 0.9 \). For weak connections, \( P_{\text{exist}} = 0.2 \) and \( S_{\text{low}} = 0.1, S_{\text{high}} = 0.3 \). For evenly distributed networks, \( p_{\text{exist}} = p_{\text{exist}}^\text{out} = 0.5 \) and \( S_{\text{low}}^\text{in} = S_{\text{low}}^\text{out} = 0.4, S_{\text{high}}^\text{in} = S_{\text{high}}^\text{out} = 0.6 \). We then normalize the generated \( G \) by the total number of agents in the game to make sure that Assumption 1 holds.

We also choose \( b_{\text{low}} = 0.1, b_{\text{high}} = 0.5 \) to make sure that agents will have an initial incentive to take action above 0 and the budget can easily achieve \( C_k \geq \|b_{S_k}\|^2 \).

These sample games contain two groups, \( S_1 \) with 40 agents and \( S_2 \) with 10 agents; we obtained very similar results with more groups and thus will focus on this setting for brevity.

**B. Social Welfare and L2 Efficiency**

For each network type, we show the social welfare with non-cooperative planners and the L2 efficiency on example games with different types of budget allocation rules.

In general, the L2 efficiency is fairly high in all cases except for Type 2 networks with conflicting groups. Therefore, we only show the welfare results in the (P-C) problem. The main reason for this phenomenon is Assumption 1, where we require the elements of \( G \) to have relatively small values compared to 1 and thus in all except for Type 2 with conflicting groups, the difference between matrices \( A \) and \( \tilde{A} \) is small. The major cause of welfare differences comes from budget allocation rules.

Figure 4 and 6 show the welfare for non-cooperative planners and the L2 efficiency in Type 1 network with all positive connections. In this case, the socially optimal budget allocation yields the highest cooperative and non-cooperative welfare assigns almost all budget to \( S_1 \).

Figure 5 and 7 show another case of Type 1 network where between-group connections are all negative, but within-group connections remain positive. This can model that the types of interactions between members in the same group are different from agents in different groups. In a special case of this type of network where every agent is taking a positive action level, an increase in an agent’s action level can increase (resp. decrease) the agents' utilities in the same group (resp. other groups). In this case, the proportional allocation rule is almost socially optimal.

In the Type 2 network, where all connections are positive, Figure 8 and 10 show the welfare with non-cooperative planners. Interestingly, the identical budget allocation rule is actually closer to the socially optimal allocation. For the same Type 2 network but with negative between-group connections, results shown in Figure 9 and 11 are very different from other network types since the efficiency is now significantly below 1 when we have small budgets.

Figure 12 and 13 shows the results in the Type 3 network with all positive connections. In fact, for all combinations of connection signs, the trends are very similar, but the social welfare is significantly lower when we have negative connections. We also see that the proportional allocation rule is almost socially optimal.
Empirically, we observe that for all types of network that when the budget grows larger, under any type of budget allocation rule the welfare grows approximately linearly and the efficiency approximately converges to a fixed value.

We also measured the tightness of the theoretical lower bound on the L2 efficiency. For all the above-introduced network types except for Type 1 with conflicting groups, the gaps between the lower bounds and the actual L2 efficiencies are less than 0.006, for all three budget allocation rules. For Type 1 networks with conflicting groups, the gap is around 0.07 for all three budget allocation rules. All gaps are almost invariant in the total budget.

VI. CONCLUSIONS

We studied an intervention problem in network games with community structures and multiple planners. We showed that given any intervention action, the agents will always have a unique NE. The planners can thus use backward induction and design (locally) optimal interventions. Regardless of whether the planners are cooperative or non-cooperative, the system always has a unique subgame perfect equilibrium that fully spends the budget and is Pareto efficient. We also studied the efficiency of the outcomes under different settings. Our analysis shows that we can use the Lagrangian dual optimal variable values to characterize the efficiency, and planners have incentives to share budgets even when they are non-cooperative. The budget transferability also enables uniformly better outcomes than the non-transferable case. Empirically, we observe that the type of network determines which type of (commonly used) budget allocation rule is the most efficient.

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