Deriving Einstein-Podolsky-Rosen steering inequalities from the few-body Abner Shimony inequalities

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For the Abner Shimony (AS) inequalities, the simplest unified forms of directions attaining the maximum quantum violation are investigated. Based on these directions, a family of Einstein-Podolsky-Rosen (EPR) steering inequalities is derived from the AS inequalities in a systematic manner. For these inequalities, the local hidden state (LHS) bounds are strictly less than the local hidden variable (LHV) bounds. This means that the EPR steering is a form of quantum nonlocality strictly weaker than Bell nonlocality.

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I. INTRODUCTION

Quantum entanglement distinguishes quantum theory from classical theory. With entangled states, the first authors to identify an interesting nonlocal effect were Einstein, Podolsky, and Rosen (EPR) in 1935 [1]. Subsequently, the concept of steering was first introduced in 1935 by Schrödinger [2] as a reply to the Einstein-Podolsky-Rosen [1] paradox. EPR steering reflects a “spooky action” feature that manipulating one object seemingly affects another instantaneously, even it is far away. Wiseman pointed out in [4] that steerability is stronger than entanglement but it is weaker than Bell nonlocality. Different from entanglement and Bell nonlocality, quantum steering is asymmetric between the two parties. In details, It may happen that Alice can steer Bob but Bob can never steer Alice. This distinguished feature would be useful for some one-way quantum information tasks, such as quantum cryptography [3].

In 1964, Bell proposed an famous inequality for local hidden variable (LHV) models [5] to refute EPR paradox. Bell inequalities revealed that quantum mechanics is incompatible with local realism. The more Bell inequalities we know, the more we know about the boundaries between Einstein’s local realism and the genuinely nonclassical areas of quantum physics, which are potentially useful in quantum information applications. For instance, Bell inequalities have gained a utilitarian power in different quantum information tasks, such as quantum key distribution [6], communication complexity [7], and recently random number generation [8].

Similar to Bell inequalities, steering inequalities [9] have been proposed to reveal the EPR steerability of quantum states. It is in principle easier to experimentally observe the violation than Bell inequalities, because one has no concerns about closing the notorious locality loophole as in a Bell test [10]. Therefore, there is an important research significance in theory [11–13] and experiment [9, 14]. Based on the research approaches in the field of Bell’s nonlocality, we have constructed chained steering inequalities from the chained Bell inequalities [15]. Since the more steering inequalities we know, the more steerable states can be detected. In this paper, we focus on deriving EPR steering inequalities from the Abner Shimony (AS) inequalities introduced in Ref. [16].

Without loss of generality, we will take an Alice-to-Bob steering scenario where correlations between classical variables declared by Alice but quantum expectation values found by Bob, in this sense [9]. In fact, Bob’s directions are taken as those that can maximally violate the AS inequalities. Then a family of steering inequalities are constructed. Finally, a comparison between their local hidden state (LHS) bound and their quantum violation is made in a systematical manner. Thus, based on the comparison, we are able to compare the EPR steering with the Bell nonlocality.

The paper is organized as follows. In Sec. II, we shall be briefly reviewing the AS inequalities and research the
corresponding directions taken by Alice and Bob for every \( N \), for which the maximum quantum values can be obtained. In Sec. III, we will derive EPR-steering inequalities from the Abner Shimony inequalities, and compute the LHS bounds. By comparing the LHS bounds and the LHV bounds, we can find that EPR-steering is a form of quantum nonlocality strictly weaker than Bell nonlocality. Conclusions and discussion are put in the end of the paper.

II. THE AS INEQUALITIES

The AS inequalities are a new family of tight bipartite Bell inequalities for any even number \( N \) of inputs and binary outcomes, generalizing a tight inequality introduced in Ref. [17]. For binary inputs, the AS inequality is nothing but the Clauser-Horne-Shimony-Holt (CHSH) inequality [18] (see Eq. (14)). For ternary inputs, there is nothing but the Clauser-Horne-Shimony-Holt (CHSH) inequality [17]. For binary inputs, the AS inequality is the classical bound for LHV models. The bound is obtained straightforwardly by the definition of the LHV models, i.e., by numerating all possible values \( A_i \) (measured on Alice’s particle) and \( B_j \) (measured on Bob’s particle), and

\[
C_{\text{LHV}}^N = \frac{N}{2} \left( \frac{N}{2} + 1 \right) ,
\]

is the classical bound for LHV models. The bound is obtained straightforwardly by the definition of the LHV models, i.e., by numerating all possible values \( A_i, B_j = \pm 1 \) in \( I_N \).

For any \( N \geq 2 \) and \( i = 1, 2, \cdots, N \), if Alice and Bob choose the following directions

\[
a_i = (\sin \theta a_i, \cos \phi a_i, \sin \theta a_i, \sin \phi a_i, \cos \theta a_i)
\]

and

\[
b_i = (\sin \theta b_i, \cos \phi b_i, \sin \theta b_i, \sin \phi b_i, \cos \theta b_i)
\]

respectively, then the observables that Alice and Bob choose are

\[
A_i = \vec{a} \cdot a_i = \sin \theta a_i \cos \phi a_i + \sin \theta a_i \sin \phi a_i + \cos \theta a_i \sigma z
\]

and

\[
B_i = \vec{b} \cdot b_i = \sin \theta b_i \cos \phi b_i + \sin \theta b_i \sin \phi b_i + \cos \theta b_i \sigma z
\]

respectively, where \( \sigma x, \sigma y, \sigma z \) are the Pauli matrices. Thus, the outcomes of \( A_i \) and \( B_j \) are either 1 or -1.

The AS inequalities are a new family of tight bipartite Bell inequalities, and the LHV bounds, we can find that EPR-steering is a form of quantum nonlocality strictly weaker than Bell nonlocality. Conclusions and discussion are put in the end of the paper.
Assume that the initial quantum state of the compound system $\mathbb{C}^2 \otimes \mathbb{C}^2$ is in the Spin singlet state
\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle), \tag{9} \]
here $|0\rangle$ and $|1\rangle$ are eigenstates of $\sigma_z$ with eigenvalues 1 and $-1$, respectively. Then
\[ \langle A_i B_j \rangle = \text{tr}(A_i B_j |\psi\rangle \langle \psi|). \tag{10} \]
By Ref. [17], the maximum quantum value of $I_N$ is
\[ I_{QM}^{\text{max}} = \frac{(N + 1) \sqrt{N(N + 2)}}{3}. \tag{11} \]
Since the directions that Alice and Bob choose to attain the maximum quantum value are not unique, we assume that Alice and Bob choose the measurement directions with the following forms:
\[ a_i = \begin{cases} (Y, \sqrt{1 - Y^2} \sin \phi_0, \sqrt{1 - Y^2} \cos \phi_0) & i = 1, \\ (-Y, \sqrt{1 - Y^2} \sin \phi_0, \sqrt{1 - Y^2} \cos \phi_0) & i = 2, \\ (0, \sin \theta_{i-2}, \cos \theta_{i-2}) & 3 < i < N - 1, \\ (1, 0, 0) & i = N. \end{cases} \tag{12} \]
\[ b_i = \begin{cases} (Y, \sqrt{1 - Y^2} \sin \theta_0, \sqrt{1 - Y^2} \cos \theta_0) & i = 1, \\ (-Y, \sqrt{1 - Y^2} \sin \theta_0, \sqrt{1 - Y^2} \cos \theta_0) & i = 2, \\ (0, \sin \theta_{i-2}, \cos \theta_{i-2}) & 3 < i < N - 1, \\ (1, 0, 0) & i = N. \end{cases} \tag{13} \]
here
\[ Y = \frac{1}{\sqrt{\frac{N}{2} (\frac{N}{2} + 1)}}. \]
To the best of our knowledge, directions (12) and (13) are the simplest unified forms for $N = 4, 6, 8, 10$ to attain the maximum quantum value.

In details, we list as follows some case studies:

- **N=2**: The coefficient matrix $A_{S_2}$ is
\[ A_{S_2} = \begin{pmatrix} A_1 & A_2 \\ B_1 & 1 \\ B_2 & 1 \end{pmatrix}, \tag{14} \]
and so the AS inequality is nothing but the CHSH inequality, i.e.,
\begin{align*}
I_2 &= \sum_{i,j} A_{S_2}[i,j] \langle A_i B_j \rangle \\
&\equiv \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle \\
&\leq 2 = C_{LHJV}^2.
\end{align*}

If Alice and Bob fix their directions as
\[ a_1 = (0, \sin \phi_0, \cos \phi_0), \]
\[ a_2 = (0, \sin \phi_1, \cos \phi_1), \tag{15} \]
and
\[ b_1 = (0, \sin \theta_0, \cos \theta_0), \]
\[ b_2 = (0, \sin \theta_1, \cos \theta_1), \tag{16} \]
respectively, where
\[ \theta_0 = 0, \theta_1 = \pi, \phi_0 = -\phi_1 = -\frac{\pi}{2} - \arccos \frac{1}{\sqrt{2}}, \tag{17} \]
then the maximum quantum value $I_{2_{\text{max}}}^{QM} = 2\sqrt{2}$ is obtained.

- **N=4**: The coefficient matrix $A_{S_4}$ is
\[ A_{S_4} = \begin{pmatrix} A_1 & A_2 & A_3 & A_4 \\ B_1 & 1 & 1 & 1 \\ B_2 & 1 & 1 & -1 \\ B_3 & 1 & 1 & -2 \\ B_4 & 1 & -1 & 0 \end{pmatrix}, \tag{18} \]
and so the Bell expression $I_4$ is
\[
I_4 = \sum_{i,j} A_{S_4}[i,j] \langle A_i B_j \rangle \\
\equiv \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_1 B_3 \rangle + \langle A_1 B_4 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle + \langle A_2 B_3 \rangle + \langle A_2 B_4 \rangle + \langle A_3 B_1 \rangle + \langle A_3 B_2 \rangle + \langle A_3 B_3 \rangle + \langle A_3 B_4 \rangle + \langle A_4 B_1 \rangle + \langle A_4 B_2 \rangle + \langle A_4 B_3 \rangle + \langle A_4 B_4 \rangle \\
\leq 6 = C_{LHJV}^4.
\]
If Alice and Bob choose the measurement directions with forms as (12) and (13), with
\[ Y = \frac{1}{\sqrt{\frac{N}{2} (\frac{N}{2} + 1)}} = \frac{1}{\sqrt{6}}, \]
and
\[ \theta_1 = \frac{1}{2} \arccos \frac{-5}{3\sqrt{6}}, \theta_0 = \arccos \frac{4}{3\sqrt{6}} - \theta_1, \]
\[ \phi_0 = \arccos \frac{-4}{3\sqrt{6}} + \theta_1, \phi_1 = \arccos \frac{5}{3\sqrt{6}} + \theta_1, \tag{19} \]
then the maximum quantum value $I_{2_{\text{max}}}^{QM} = 10\sqrt{\frac{2}{3}}$ is obtained.

- **N=6**: The coefficient matrix $A_{S_6}$ is
\[ A_{S_6} = \begin{pmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ B_1 & 1 & 1 & 1 & 1 & 1 \\ B_2 & 1 & 1 & 1 & 1 & -1 \\ B_3 & 1 & 1 & 1 & -2 & 0 \\ B_4 & 1 & 1 & -3 & 0 & 0 \\ B_5 & 1 & -2 & 0 & 0 & 0 \\ B_6 & 1 & -1 & 0 & 0 & 0 \end{pmatrix}, \tag{20} \]
and so the Bell expression $I_6$ is

$$I_6 = \sum_{i,j}^6 A_{S_6[i,j]}(A_i B_j)$$

$$\equiv \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_1 B_3 \rangle + \langle A_1 B_4 \rangle + \langle A_1 B_5 \rangle$$
$$+ \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle + \langle A_2 B_3 \rangle + \langle A_2 B_4 \rangle$$
$$+ \langle A_3 B_1 \rangle - \langle A_3 B_2 \rangle + \langle A_3 B_3 \rangle - 2 \langle A_3 B_4 \rangle$$
$$+ \langle A_4 B_3 \rangle - 3 \langle A_4 B_4 \rangle + \langle A_5 B_1 \rangle + \langle A_5 B_2 \rangle - 2 \langle A_5 B_3 \rangle + \langle A_6 B_1 \rangle$$
$$- \langle A_6 B_2 \rangle \leq 12 = C_{LHV}^6.$$

If Alice and Bob choose the measurement directions with forms as (12) and (13), with

$$Y = \frac{1}{\sqrt{\frac{N}{2} \left( \frac{N}{2} + 1 \right)}} = \frac{1}{2\sqrt{3}},$$

and

$$\theta_3 = -\arcsin \left[ \frac{4}{3\sqrt{11}} \right], \phi_1 = \theta_3 + \arccos \left[ \frac{5}{6\sqrt{3}} \right],$$
$$\theta_2 = -\arccos \left[ \frac{5}{2\sqrt{21}} \right] + \arccos \left[ \frac{\sqrt{83}}{2\sqrt{231}} \right],$$
$$\phi_2 = \theta_2 + \arccos \left[ \frac{7}{6\sqrt{3}} \right], \theta_1 = \theta_2 - \phi_1 + \phi_2,$$
$$\phi_3 = \theta_1 + \arccos \left[ \frac{5}{6\sqrt{3}} \right], \theta_0 = \phi_3 - \arccos \left[ \frac{4}{3\sqrt{11}} \right],$$

then the maximum quantum value $I_{6\text{ max}}^{QM} = \frac{28}{\sqrt{3}}$ is obtained.

- $N=8$: The coefficient matrix $A_{S_8}$ is

$$A_{S_8} = \begin{pmatrix}
A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 \\
B_1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
B_2 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\
B_3 & 1 & 1 & 1 & 1 & 1 & -2 & 0 \\
B_4 & 1 & 1 & 1 & 1 & -3 & 0 & 0 \\
B_5 & 1 & 1 & 1 & -4 & 0 & 0 & 0 \\
B_6 & 1 & 1 & -3 & 0 & 0 & 0 & 0 \\
B_7 & 1 & -2 & 0 & 0 & 0 & 0 & 0 \\
B_8 & 1 & -1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

and so the Bell expression $I_8$ is

$$I_8 = \sum_{i,j}^8 A_{S_8[i,j]}(A_i B_j) \leq 20 = C_{LHV}^8.$$

If Alice and Bob choose the measurement directions with forms as (12) and (13), with

$$Y = \frac{1}{\sqrt{\frac{N}{2} \left( \frac{N}{2} + 1 \right)}} = \frac{1}{2\sqrt{5}},$$

and

$$\theta_5 = -\arcsin \left[ \frac{4}{3\sqrt{19}} \right], \phi_1 = \theta_5 + \arccos \left[ \frac{5}{6\sqrt{5}} \right],$$
$$\theta_4 = -\arccos \left[ \frac{5}{2\sqrt{39}} \right] + \arccos \left[ \frac{\sqrt{155}}{2\sqrt{741}} \right],$$
$$\theta_3 = -\arccos \left[ \frac{4}{15} \right] + \arccos \left[ \frac{235\sqrt{589} + 53\sqrt{12445}}{7410\sqrt{12}} \right],$$
$$\phi_2 = \theta_4 + \arccos \left[ \frac{7}{6\sqrt{5}} \right], \phi_3 = \theta_3 + \arccos \left[ \frac{3}{2\sqrt{5}} \right],$$
$$\phi_4 = \theta_3 - \phi_2 + \phi_3, \phi_5 = \theta_2 - \phi_4, \theta_1 = \theta_2 - \phi_1 + \phi_2,$$
$$\phi_4 = \theta_1 - \theta_4 + \phi_1, \theta_0 = \phi_5 - \arccos \left[ \frac{4}{3\sqrt{19}} \right],$$

then the maximum quantum value $I_{8\text{ max}}^{QM} = 12\sqrt{5}$ is obtained.

In Figure 1, we plot Bob’s directions (13) attaining the maximum quantum value $I_{8\text{ max}}^{QM}$ for $N = 4, 6, 8$ with parameters $\theta_i$ being fixed as (19), (21) and (23) respectively.

### III. STEERING INEQUALITIES AND COMPARISON

In this section, we derive Einstein-Podolsky-Rosen steering inequality from the Abner Shimony inequality.

In EPR-steering one considers correlations between classical variables declared by Alice but quantum expectation values found by Bob, in this sense, we call it Alice’s steering Bob’s particle. The steering inequality can be written as:

$$I_N^{steer \text{ LHS}} \leq C_{LHS}^N,$$

where

$$I_N^{steer} = \sum_{i,j}^N A_{S_8[i,j]}(A_i B_j) = \sum_{i,j}^N A_{S_8[i,j]}(\hat{\sigma} \cdot b_j),$$

is the EPR steering expression with $A_i \in \{1, -1\}$, and $C_{LHS}^N$ is the classical bound for LHS model, which we shall determine.

If Bob fixes his directions as (13) which maximally violate the AS inequalities, then $C_{LHS}^N$ is a function of $\theta_0, \theta_1, \cdots, \theta_{N-3}$. Namely,

$$C_{LHS}^N(\theta_0, \theta_1, \cdots, \theta_{N-3}) = \max \left\{ I_{i}^{QM}, A_i \in \{1, -1\} \right\}.$$ (26)

We list as follows:
Figure 1. Bob’s directions (13) with parameters \( \theta_i \) being fixed as (19), (21) and (23) for \( N = 4, 6, 8 \), respectively.

- \( N=2 \): If Bob fixes his directions as (16) with \( \theta_0, \theta_1 \) being listed in (17), then
  \[
  C_{\text{LHS}}^2(\theta_0, \theta_1) = 2 = C_{\text{LHV}}^2.
  \]  
(27)

- \( N=4 \): If Bob fixes his directions as (13) with \( \theta_0, \theta_1, \theta_2 \) being listed in (19), then
  \[
  C_{\text{LHS}}^4(\theta_0, \theta_1) = 2\sqrt{\frac{23}{3}} \simeq 5.5377 < 6 = C_{\text{LHV}}^4.
  \]  
(28)

- \( N=6 \): If Bob fixes his directions as (13) with \( \theta_0, \theta_1, \theta_2, \theta_3 \) being listed in (21), then
  \[
  C_{\text{LHS}}^6(\theta_0, \theta_1, \theta_2, \theta_3) = \sqrt{\frac{358}{3}} \simeq 10.924 < 12 = C_{\text{LHV}}^6.
  \]  
(29)

- \( N=8 \): If Bob fixes his directions as (13) with \( \theta_0, \ldots, \theta_5 \) being listed in (23), then
  \[
  C_{\text{LHS}}^8(\theta_0, \ldots, \theta_5) = \sqrt{\frac{2(10444 + \sqrt{20305})}{65}} \approx 18.0482 < 20 = C_{\text{LHV}}^8.
  \]  
(30)

- \( N=10 \): If Bob fixes his directions as (13) with \( \theta_0, \ldots, \theta_7 \) being listed in the following:
  \[
  \begin{align*}
  \theta_0 &= -2.5496, \theta_1 = 3.1742, \theta_2 = 1.9715, \\
  \theta_3 &= -1.5541, \theta_4 = -1.0945, \\
  \theta_5 &= -0.7886, \theta_6 = -0.5108, \theta_7 = -0.2502,
  \end{align*}
  \]  
(31)
  then
  \[
  I_{10}^{\text{QM}} \leq 22\sqrt{\frac{10}{3}}
  \]  
(32)
  and so
  \[
  C_{\text{LHS}}^{10}(\theta_0, \ldots, \theta_7) \simeq 27.0955 < 30 = C_{\text{LHV}}^{10}.
  \]  
(33)

Table I. \( C_{\text{LHV}}^N \) and \( C_{\text{LHS}}^N \) for \( N = 2, 4, 6, 8, 10 \).

| \( N \) | \( C_{\text{LHV}}^N \) | \( C_{\text{LHS}}^N \) |
|---|---|---|
| 2 | 2 | 2√\( \frac{23}{3} \) |
| 4 | 12 | \( \sqrt{\frac{358}{3}} \) |
| 6 | 20 | \( \sqrt{\frac{2(10444 + \sqrt{20305})}{65}} \) |
| 8 | 30 | 27.0955 |

Figure 2. The \( C_{\text{LHV}}^N \) and \( C_{\text{LHS}}^N \) for \( N = 2, 4, 6, 8, 10 \). The blue dot-lines is the relationship between \( C_{\text{LHV}}^N \) and \( N \), and the orange dot-lines is the relationship between \( C_{\text{LHS}}^N \) and \( N \).

The comparison of \( C_{\text{LHV}}^N \) and \( C_{\text{LHS}}^N \) is listed in Table I and plotted in Figure 2.

From Figure 2, we can see that
(i) \( C_{\text{LHS}}^N < C_{\text{LHV}}^N \) for any \( N > 2 \),
(ii) for \( 2 \leq N \leq 8 \), not only both \( C_{\text{LHV}}^N \) and \( C_{\text{LHS}}^N \) increase with the increase of \( N \), but also the difference between \( C_{\text{LHV}}^N \) and \( C_{\text{LHS}}^N \) increases as \( N \) does.

If the initial quantum state of the compound system \( \mathbb{C}^2 \otimes \mathbb{C}^2 \) is in the Werner state
\[
\rho = V |\psi\rangle \langle \psi| + (1 - V) I_4 \frac{I_4}{4},
\]  
(34)
where \( |\psi\rangle \) denotes the singlet state (9), \( I_4 \) is the identity, and \( V \in [0, 1] \), then we use \( V_{\text{LHV}}^N \) to denote the critical value, above which the state cannot be described by local hidden variables, and \( V_{\text{LHS}}^N \) to denote the critical value,
Table II. We list $V_{LHV}^N$ and $V_{LHS}^N$ for the chained (Bell and steering) inequalities with $n = 2, 4, 6, 8, 10$. Here $V_{LHV}^N = 4\sqrt{N(2+\sqrt{N})}/(4+3N)$.

| $N$ | $V_{LHV}^N$ | $V_{LHS}^N$ |
|-----|-------------|-------------|
| 2   | $\sqrt{2}/2$ | $\sqrt{2}/2$ |
| 4   | $2\sqrt{2}/5$ | $2\sqrt{2}/5$ |
| 6   | $3\sqrt{2}/7$ | $3\sqrt{2}/7$ |
| 8   | $\sqrt{2}/3$ | $\sqrt{2}/3$ |
| 10  | $4\sqrt{2}/11$ | $4\sqrt{2}/11$ |

Figure 3. The $V_{LHV}^N$ and $V_{LHS}^N$ for $N = 2, 4, 6, 8, 10$. The blue dot-lines is the relationship between $V_{LHV}^N$ and $N$, and the red dot-lines is the relationship between $V_{LHS}^N$ and $N$.

above which the state cannot be described by local hidden states.

By AS Bell inequalities (1), AS steering inequalities (24) and the initial quantum state (34), we get

$$V_{LHV}^N = c_{LHV}^N/I_N^{QM}, V_{LHS}^N = c_{LHS}^N/I_N^{QM}.$$ (35)

The comparison of $V_{LHV}^N$ and $V_{LHS}^N$ is listed in Table II and plotted in Figure 3. From Figure 3, we can see that

(i) $V_{LHS}^N < V_{LHV}^N$ for any $N > 2$,

(ii) for $2 \leq N \leq 8$, $V_{LHV}^N$ increases and $V_{LHS}^N$ decreases with the increase of $N$, $V_{LHV}^N \rightarrow 0.75$ as $N \rightarrow \infty$, and $V_{LHS}^N \leq 0.7$ for any $N$.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we have researched the simplest unified forms of directions attaining the maximum quantum value of the AS inequalities. Then we have derived EPR-steering inequalities from the AS inequalities, and computed their LHS bounds. Finally, by comparing the two thresholds $V_{LHV}^N$ and $V_{LHS}^N$, we have shown $V_{LHS}^N < V_{LHV}^N$ for any $N > 2$. This means that EPR-steering is a form of quantum nonlocality weaker than Bell-nonlocality, in the sense that some quantum states exist so that they violate the EPR-steering inequality but satisfy the AS-typed Bell inequality. The results are in agreement with the hierarchical structure of quantum nonlocality presented in [4].

To date, we have no idea whether the directions attaining the maximum quantum value of the AS inequalities are optimal in detecting the steerability of Werner states or not. In the future, we shall investigate the optimization directions to detect the steerability of Werner states.

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