Optimal strategy analysis of N-policy Two-Phase M/E\(k\)/1 Vacation Queueing system with Server Start-Up, Time-Out and Breakdown

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Abstract: This article presents optimal strategy N-Policy M/E\(k\)/1 vacation queueing system when the server is in Start-up, Time-out and Breakdown for two-phase. The individual arrivals considered to follow Poisson process and receive batch service of k-stages in the first phase, individual service in the second phase. During the service arrivals are allowed to enter the batch. On completion of the second phase service to all customers in the second phase, the server come back to the first phase and do the service to the customers who have arrived by providing them first phase service of k-stages followed by second phase service. If there is no customer waiting in the phase one service, the server waits for a fixed time ‘C’ is called server Time-out. If units arrived during this fixed time, then the server provides first phase followed by second phase service, otherwise after expiration of fixed time he takes a vacation. After N-customers accumulate in the system the server returns from vacation. Before going to first phase service, the server spends a random startup period for pre-service after coming back from vacation. During individual service the server is susceptible to random breakdown according to a Poisson process and the repair time follows an exponential distribution. After repair the server resumes individual service. Exact notations for steady state distribution of the number of customers in the system are derived. Developed a cost model for determining the optimal operating policy at a minimum cost. By using numerical illustrations the sensitivity analysis is also presented.

Keywords: Two-phase, Vacation, N-Policy, Start-Up, Time-Out and Server Breakdown.

1. Introduction

Queueing theory is the mathematical study of waiting lines or queue. The theory enables mathematical analysis of several related processes, including arriving at the queue, waiting in the queue and being served by the server(s) at the front of the queue. The theory permits the derivation and calculation of several performance measures, including the average waiting time in the queue or the system, the expected number of customers (waiting or receiving service) and the probability of encountering the system in certain states, such as empty, full, having an available server or having to wait for a certain time to be served.
In queues the situation where a server is unavailable for primary customers in occasional intervals of time is known as a vacation. Queues with vacations or simply called vacation models attracted great attention of queueing researchers and became an active research area. Miller[4] (1964) was the first to study a queueing system in which the server becomes idle and is unavailable during some random length of time for the M/G/1 queueing system. J.R. Murray and W.D. Kelton[2] (1988) have derived transition probabilities for the transient, discrete time M/Ek/2 queueing system. Levy and Yechiali[8] (1975) also studied Queueing models of similar nature which include several types of generalizations of the classical M/G/1 queueing system.

A wide class of controlling policies for governing the vacation mechanism has been discussed in the literature viz., single vacation policy, multiple vacation policy, N-policy, (M,N)-policy, T-policy, D-policy, (p,T)-policy, Q-policy etc. Under the N-policy vacation models, the server leaves the service channel when it becomes empty and returns when the queue length reaches to a desired level N (≥ 1) and begins service with startup or without startup time. The N-policy was first introduced by Yadin and Naor[5] (1963). Baker[3] (1973) first proposed the N-policy M/M/1 queueing system with exponential startups.

Vacation queueing systems with server timeout are studied with some of the authors. Oliver C.Ibe [6] discussed an M/ G/1 vacation queueing system with server timeout. E.Ramesh Kumar and Y. PrabyLoit [1] studied Vacation Bulk Queueing Model with setup time and server timeout. V.Vasantakumar and K.Chandan[7] have given cost analysis for an N-policy M/Ek/1 queueing system with server breakdowns and two-phases of service. However, to the best of our knowledge, for Two-phase M/Ek/1 queueing systems with N-Policy, server timeout and breakdown there is no literature which takes cost function into consideration. This motivates us to present a cost analysis of a M/Ek/1 system with N-policy, server start-up and Timeouts in Transient State. Thus, in this present paper, we consider Cost analysis of the M/Ek/1 queueing system with server Start-up, N-Policy and server timeouts.

2. The system and assumptions
Arriving customers are assumed to follow a Poisson process with mean arrival rate N and join the first phase batch queue. The server will spend a random startup time t for pre-service when the batch size reaches N (≥1) and is assumed to follow an exponential distribution with mean 1/θ. The server begins batch service k-stages in the first phase immediately after completion of startup period. Then go to the individual service in a second phase to serve all customers. Individual queue is served in FIFO mode. The batch size is independent to batch service time, batch service time and individual service times are assumed to follow exponentially distributed with mean 1/β and 1/μ respectively. The server may breakdown at any time with a Poisson breakdown rate α during the service of individual queue. The server immediately repaired at a repair rate ϑ which is assumed to be exponentially distributed when it fails and then resumes service in individual queue. After serving the individual queue the server returns to the batch queue for serving the customers who have arrived by providing them first phase service k-stages followed by second phase service. If no customers are waiting in the batch queue then the server waits for a fixed time ‘C’ is called server Timeout. If units arrived during this fixed time he does the service to that unit as a first phase service followed by second phase service. If no units arrived during this fixed time, then he takes a vacation and after N customers accumulate in the first phase queue it starts pre-service work.
3. Analysis of the model

The following notations are used to represent probabilities for the system at various states in steady-state.

\[ P_{0,i,0} = P(\text{batch queue} i=0,k,2k,3k,\ldots) \] the server on vacation.

\[ P_{1,i,0} = P(\text{batch queue} i=Nk,(N+1)k,(N+2)k,\ldots) \] the server on startup.

\[ P_{2,i,0} = P(\text{batch queue} i=0,k,2k,3k,\ldots) \] the server on time-out.

\[ P_{3,i,0} = P(\text{batch queue} i=k,2k,3k,\ldots) \] the server on batch service.

\[ P_{4,i,j} = P(\text{batch queue} i=0,k,2k,3k,\ldots \text{ and individual queue } j=1,2,3\ldots) \] the server on individual service.

\[ P_{5,i,j} = P(\text{batch queue} i=0,k,2k,3k,\ldots \text{ and individual queue } j=1,2,3\ldots) \] the server on breakdown.

The following are the steady state equations; those are satisfied with the system sizes:

\[ \lambda P_{0,0,0} = \lambda P_{0,0,0} \]  
\[ \lambda P_{0,i,0} = \lambda P_{0,i-k,0} \]  
\[ (\lambda + C) P_{2,0,0} = k \mu P_{4,0,1} \]  
\[ (\lambda + \beta) P_{3,k,0} = \lambda P_{2,0,0} + k \mu P_{4,k,0} \]  
\[ (\lambda + \beta) P_{3,i,0} = \lambda P_{2,i,0} + k \mu P_{4,i,0} \]  
\[ (\lambda + \beta) P_{3,i,0} = \lambda P_{2,i,0} + k \mu P_{4,i,0} + \theta P_{5,i,0} \]  
\[ (\lambda + \beta) P_{3,i,0} = \alpha P_{4,i,0} \]  
\[ (\lambda + \beta) P_{3,i,0} = \alpha P_{4,i,0} + \lambda P_{5,i,0} \]  
\[ (\lambda + \beta) P_{3,i,0} = \alpha P_{4,i,0} + \lambda P_{5,i,0} + \theta P_{5,i,0} \]  
\[ (\lambda + \beta) P_{3,i,0} = \alpha P_{4,i,0} + \lambda P_{5,i,0} \]  

The p.g.f.'s are defined as follows.

\[ G_i(z) = \sum_{i=0}^{\infty} P_{0,i,0} z^i, \quad |z| \leq 1 \]
\[ G_1(z) = \sum_{i=1}^{Nk} P_{1,i} z^i, \quad |z| \leq 1 \]

\[ G_2(z) = P_{2,0,0}. \]

\[ G_3(z) = \sum_{i=1}^{Nk} P_{3,i} z^i, \quad |z| \leq 1 \]

\[ G_4(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} P_{4,i,j} z^i y^j, \quad |z| \leq 1 \text{ and } |y| \leq 1 \]

\[ G_5(z,y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} P_{5,i,j} z^i y^j, \quad |z| \leq 1 \text{ and } |y| \leq 1 \]

\[ R_j = \sum_{i=0}^{\infty} P_{4,i,j} z^i y^j, \quad |z| \leq 1 \]

And \[ S_j = \sum_{i=0}^{\infty} P_{5,i,j} z^i y^j, \quad |z| \leq 1 \]

From equation (1)

\[ G_2(z) = \frac{\lambda}{C} P_{0,0,0} \]

Using equation (2) by multiplying \( z \) and \( i \) takes over \( k \leq i \leq (N-1)k \) yields

\[ G_0(z) = \frac{(1 - z^{Nk}) P_{0,0,0}}{1 - z^k} \]

Using equation (3) and (4) by multiplying \( z \) and \( i \) takes over \( i \geq Nk \) yields

\[ G_i(z) = \frac{\lambda z^{Nk} P_{i,0,0}}{[(1 - z^k) \lambda + \theta]} \]

From equation (5)

\[ P_{4,0,i} = \frac{\lambda (\lambda + C)}{k \mu C} P_{0,0,0} \]

Using equation (6) , (7) and (8) by multiplying \( z \) and \( i \) takes over \( i \geq k \) yields

\[ (\lambda (1 - z^k) + \beta) G_i(z) = k \mu R_i(z) + \theta G_i(z) + \frac{\lambda}{C} \left[ (z^{k+1} - 1) \lambda - C \right] P_{0,0,0} \]

Using equation (10) by multiplying \( z \) and \( i \) takes over \( i \geq k \) and using (9) yields

\[ \left[ (1 - z^k) \lambda + k \mu + \alpha \right] R_i(z) = k \mu R_{i,1}(z) + \theta S_j(z) + \beta P_{3,i,0} \]

This equation multiplying by \( y \) and \( j \) takes over \( j \geq 1 \) yields

\[ \left[ (1 - z^k) \lambda y + (y-1) k \mu + \alpha y \right] G_i(z,y) = \theta y G_i(z,y) + \beta y G_i(y) - k \mu y R_i(z) \]

From equation (12) by multiplying \( z \) and \( i \) takes over \( i \geq k \) and using (11) yields

\[ (\lambda + \theta) S_j(z) = \alpha R_j(z) + \lambda z^k S_j(z) \]

This equation multiplying by \( y \) and \( j \) takes over \( j \geq 1 \) yields

\[ \left[ (1 - z^k) \lambda + \theta \right] G_i(z,y) = \alpha G_i(z,y) \]

The total p.g.f \( G(z,y) \) is given by
$G(z,y)= G_0(z)+ G_1(z)+ G_2(z)+ G_3(z)+ G_4(z,y)+ G_5(z,y)$

The normalizing condition is

$G(1,1)= G_0(1)+ G_1(1)+ G_2(1)+ G_3(1)+ G_4(1,1)+ G_5(1,1) =1 \quad (20)$

From equations (13) to (19)

$G_0(1) = NP_{0,0,0}$ \hspace{1cm} (21)

$G_1(1) = \frac{\lambda}{\theta} P_{0,0,0}$ \hspace{1cm} (22)

$G_2(1) = \frac{\lambda}{C} P_{0,0,0}$ \hspace{1cm} (23)

$G_3(1) = \frac{k\mu}{\beta} R_1(1)$ \hspace{1cm} (24)

$G_4(1,1) = \frac{k^2 \lambda \mu \theta R_1(1) + \beta \theta \varphi G_4^1(1)}{\beta k \left(\mu \varphi - \lambda (\alpha + \varphi)\right)} + \frac{\varphi \lambda^2}{C \left[\mu \varphi - \lambda (\alpha + \varphi)\right]} P_{0,0,0}$ \hspace{1cm} (25)

$G_5(1,1) = \frac{\alpha}{\varphi} G_4(1,1)$ \hspace{1cm} (26)

Where $P_{0,0,0} = \frac{1}{N + \frac{\lambda}{\theta} + \frac{\lambda}{C}}$ \hspace{1cm} (27)

Normalizing condition (20) yields

$R_1(1) = \frac{\lambda \beta (\alpha + \varphi)(1-k) + \lambda \mu k \varphi}{k^2 \mu^2 \varphi}$

Substituting the value of $R_1(1)$ in (24), (25) and (26) yields

$G_5(1) = \frac{\lambda \beta (\alpha + \varphi)(1-k) + \lambda \mu k \varphi}{\beta k \mu \varphi}$

$G_4(1,1) = \frac{\lambda}{\mu}$

And

$G_3(1,1) = \frac{\lambda \alpha}{\varphi \mu}$

By steady state conditions, the following are the probabilities when the server on different states respectively. Then
4. Number of expected customers in the system

Using the p.g.f’s the number of expected customers in the system at different states is presented below. Let \( L_0 \) (idle), \( L_1 \) (startup), \( L_2 \) (timeout), \( L_3 \) (batch service), \( L_4 \) (individual service) and \( L_5 \) (Breakdown) be the expected number of customers in the system when the server at different states respectively.

Then

\[
L_0 = \sum_{i=0}^{\infty} P_{0,i,0} G_0(1) = \frac{Nk(N-1)}{2} P_{0,0,0}
\]

\[
L_1 = \sum_{i=0}^{\infty} P_{1,i,0} G_1(1) = \frac{\lambda k(\lambda + N\theta)}{\theta^2} P_{0,0,0}
\]

\[
L_2 = G_2(1) = 0
\]

\[
R_1(1) = \frac{\lambda^2 (1 + \frac{\alpha}{\theta})}{\mu^2}
\]

\[
L_3 = \frac{\lambda k}{\beta}
\]

\[
L_4 = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (i+j) P_{4,i,j} = G_4(1,1) = \frac{2\lambda^3 k \alpha + \lambda^2 \beta (\alpha + \theta) + \lambda^2 \mu k \theta^2 + \mu \theta^2 (\lambda \beta + 2\lambda^2 k)}{2\beta \mu \theta [\mu \theta - \lambda (\alpha + \theta)]}
\]

\[
+ \frac{\lambda [(N \theta + C)(N \theta + \lambda) + \lambda (\lambda C + \theta C^2 + \theta^2 C)][\mu \theta (\beta - \lambda) - \lambda \beta (\alpha + \theta)]}{2\beta \mu \theta [\mu \theta - \lambda (\alpha + \theta)][N \theta + \lambda C + \lambda \theta]}
\]

\[
L_5 = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (i+j) P_{5,i,j} = G_5(1,1) = \frac{\lambda k \alpha}{\theta^2} G_4(1,1) + \frac{\alpha}{\theta} G_4(1,1)
\]

The total number of expected units in the system is

\[
L(N) = L_0 + L_1 + L_2 + L_3 + L_4 + L_5
\]

5. Using other system characteristics
Let \( L_0 \) (idle), \( L_1 \) (startup), \( L_2 \) (timeout), \( L_3 \) (batch service), \( L_4 \) (individual service) and \( L_5 \) (Breakdown) denote the expected lengths of different states and long run fractions are given below. Also the expected length of a cycle is given by

\[
E_c = E_{0} + E_{1} + E_{2} + E_{3} + E_{4} \quad (35)
\]

\[
\frac{E_0}{E_c} = P_0 = G_0 (1) = NP_{0,0,0} \quad (36)
\]

\[
\frac{E_1}{E_c} = P_1 = G_1 (1) = \frac{\lambda}{\theta} P_{0,0,0} \quad (37)
\]

\[
\frac{E_2}{E_c} = P_2 = G_2 (1) = \frac{\lambda}{C} P_{0,0,0} \quad (38)
\]

\[
\frac{E_3}{E_c} = P_3 = G_3 (1) = \frac{\lambda (\beta + \theta) (1 - k) + \lambda \mu k \theta}{\beta k \mu \theta} \quad (39)
\]

\[
\frac{E_4}{E_c} = P_4 = G_4 (1,1) = \frac{\lambda}{\mu} \quad (40)
\]

And

\[
\frac{E_5}{E_c} = P_5 = G_5 (1,1) = \frac{\lambda \alpha}{\partial \mu} \quad (41)
\]

Expected length of idle period is

\[
E_0 = \frac{N}{\lambda}
\]

Substituting this in equation (36)

\[
E_c = \frac{N \lambda + \lambda^2 \theta + \lambda^3 C}{\lambda \left( 1 - \frac{\lambda}{\mu k (1 + \theta \beta) - \frac{\lambda}{\beta}} \right)} \quad (42)
\]

6. DETERMINATION OF OPTIMAL N-POLICY (N*)

For the N-policy two-phase M/E\( _k /1 \) vacation queueing system with server startup, timeout and breakdown, we develop a steady state total expected cost function per unit time in which N is decision variable. With the cost structure being constructed, the objective is to determine the optimal operating N-policy so as to minimize this function.

The following are the different costs per unit time when the system is in different states.

- \( C_h \) = holding cost for each customer present in the system
- \( C_o \) = cost for keeping the server on and in operation
- \( C_m \) = startup cost per cycle
- \( C_t \) = timeout cost per per cycle
- \( C_s \) = setup cost per cycle
- \( C_b \) = breakdown cost per unit time
- \( C_r \) = rewards for the server doing secondary work during the vacation period.
The following is the total expected cost function per unit time

\[ T(N) = C_h L(N) + C_o \left[ \frac{E_s + E_k}{E_c} \right] + C_m \left[ \frac{E_t}{E_c} \right] + C_s \left[ \frac{E_s}{E_c} \right] + C_r \left[ \frac{1}{E_c} \right] - C_c \left[ \frac{E_o}{E_c} \right] \] (43)

For obtaining the optimal value \( N^* \) of \( N \), we are differentiating \( T(N) \) with respect to \( N \) and setting the result to zero. Hence,

\[ N^* = \frac{[\mu \vartheta - \lambda(\alpha + \vartheta)] \left[ C_s(C + \vartheta) + \vartheta C_m \left( \frac{C_m}{\vartheta} + \frac{C_c}{C_s} \right) \right]}{C_m k \vartheta (C + \vartheta)} + \frac{\mu \vartheta [C - 2\lambda + \vartheta] - \lambda(\alpha + \vartheta)(C + \vartheta)(C + 1)}{2\mu \vartheta (C + \vartheta)} \] (44)

7. Sensitivity analysis

We perform numerical experiments for the sensitivity of our analytical results. Different parameters (both non-monetary and monetary) variety on the \( N^* \) (Optimal threshold), \( L(N^*) \) (Mean number of customers in the system) and minimizes \( T(N^*) \) (expected cost) are shown. By fixing Non-monetary parameters for sensitivity analysis as \( \lambda = 0.5, \mu = 4, \beta = 2, \Theta = 2, \vartheta = 2, \alpha = 0.1, C = 1.0, k = 3 \) and monetary parameters as \( C_h = 5, C_o = 100, C_b = 100, C_m = 100, C_r = 40, C_s = 500, C_t = 30 \) is performed.

Table 1: The variation of Non-monetary parameters \( \lambda, \mu, \beta, \Theta, \vartheta, \alpha, C \) and k effect on \( N^*, \) excepted system length (\( L(N^*) \)) and optimum cost (\( T(N^*) \))

| Parameter | Values | \( N^* \) | \( L(N^*) \) | \( T(N^*) \) |
|-----------|--------|-----------|-------------|-------------|
| \( \lambda \) | 0.4 | 25.719444 | 32.38412 | 162.0841 |
| | 0.8 | 22.494444 | 23.37712 | 153.9707 |
| | 1.2 | 19.269444 | 15.10523 | 144.9485 |
| | 1.6 | 16.044444 | 7.909044 | 134.5185 |
| | 2 | 12.819444 | 2.384127 | 121.7777 |
| | 4 | 24.91319 | 30.07647 | 160.1225 |
| | 6 | 26.20139 | 30.62937 | 160.599 |
| \( \mu \) | 8 | 26.84549 | 30.90905 | 160.8621 |
| | 10 | 27.23194 | 31.07779 | 161.0272 |
| | 12 | 27.48958 | 31.19066 | 161.1401 |
| | 2 | 24.91319 | 30.07647 | 160.1225 |
| | 3 | 24.91319 | 33.19932 | 165.0866 |
| \( \beta \) | 4 | 24.91319 | 34.76074 | 167.5686 |
| | 5 | 24.91319 | 35.6976 | 169.0578 |
| | 6 | 24.91319 | 36.32216 | 170.0507 |
| \( \Theta \) | 2 | 24.91319 | 30.07647 | 160.1225 |
From Table 1 we observe that
• As λ, α are increasing, N*, expected system length (L(N*)) and minimum expected cost (T(N*)) are decreasing.
• As μ, θ, ϑ, C are increasing, N*, expected system length (L(N*)) and minimum expected cost (T(N*)) are increasing.
• As β increasing, no effect on N* and expected system length (L(N*)), minimum expected cost (T(N*)) are increasing.
• As k increasing, N* decreasing and expected system length (L(N*)), minimum expected cost (T(N*)) are increasing.

Table 2: The variation of monetary parameters Ch, C₀, Cₘ, Cₙ, Cₗ, Cₛ and Cₚ, effect on N*, excepted system length (L(N*)) and optimum cost (T(N*)).

| Parameter | Values | N*   | L(N*)   | T(N*)   |
|-----------|--------|------|---------|---------|
| Ch        | 5      | 24.91319 | 30.07647 | 160.1225 |
|           | 10     | 12.55764 | 15.05668 | 168.4588 |
|           | 15     | 8.43912  | 10.0781  | 176.7885 |

From Table 1 we observe that
• As λ, α are increasing, N*, expected system length (L(N*)) and minimum expected cost (T(N*)) are decreasing.
• As μ, θ, ϑ, C are increasing, N*, expected system length (L(N*)) and minimum expected cost (T(N*)) are increasing.
• As β increasing, no effect on N* and expected system length (L(N*)), minimum expected cost (T(N*)) are increasing.
• As k increasing, N* decreasing and expected system length (L(N*)), minimum expected cost (T(N*)) are increasing.
From Table 2 we observe that

- As $C_0$ increasing, $N^*$, expected system length($L(N^*)$) are decreasing and minimum expected cost($T(N^*)$) is increasing.
- As $C_m$, $C_s$, $C_t$ increasing, $N^*$, expected system length($L(N^*)$) and minimum expected cost($T(N^*)$) are increasing.
- As $C_0$, $C_i$ are increasing, no effect on $N^*$, expected system length($L(N^*)$) and increasing minimum expected cost($T(N^*)$).
- As $C_r$ increasing, $N^*$, expected system length($L(N^*)$) are increasing and minimum expected cost($T(N^*)$) is decreasing.

|   |   |   |
|---|---|---|
| 20 | 6.379861 | 7.607707 | 185.1021 |
| 25 | 5.144306 | 6.139133 | 193.3924 |
| 100 | 24.91319 | 30.07647 | 160.1225 |
| 200 | 24.91319 | 30.07647 | 188.8725 |
| $C_0$ | 300 | 24.91319 | 30.07647 | 217.6225 |
|   | 400 | 24.91319 | 30.07647 | 246.3725 |
|   | 500 | 24.91319 | 30.07647 | 275.1225 |
|   | 100 | 24.91319 | 30.07647 | 160.1225 |
|   | 200 | 24.91319 | 30.07647 | 160.7475 |
| $C_b$ | 300 | 24.91319 | 30.07647 | 161.3725 |
|   | 400 | 24.91319 | 30.07647 | 161.9975 |
|   | 500 | 24.91319 | 30.07647 | 162.6225 |
|   | 100 | 24.91319 | 30.07647 | 160.1225 |
|   | 200 | 26.84375 | 32.42711 | 171.9131 |
| $C_m$ | 300 | 28.77431 | 34.7782 | 183.7011 |
|   | 400 | 30.70486 | 37.12968 | 195.487 |
|   | 500 | 32.63542 | 39.48146 | 207.2711 |
|   | 40 | 24.91319 | 30.07647 | 160.1225 |
|   | 50 | 25.49236 | 30.78161 | 156.5296 |
| $C_i$ | 60 | 26.07153 | 31.48679 | 152.9393 |
|   | 70 | 26.56069 | 32.19202 | 149.3516 |
|   | 80 | 27.22986 | 32.89729 | 145.7662 |
|   | 500 | 24.91319 | 30.07647 | 160.1225 |
|   | 1000 | 44.21875 | 53.59647 | 277.9515 |
| $C_s$ | 1500 | 63.52431 | 77.12948 | 395.7075 |
|   | 2000 | 82.82986 | 100.6665 | 513.4413 |
|   | 2500 | 102.1354 | 124.2052 | 631.1655 |
|   | 30 | 24.91319 | 30.07647 | 160.1225 |
|   | 40 | 25.49236 | 30.78161 | 156.5296 |
| $C_t$ | 50 | 25.68542 | 31.01667 | 164.8391 |
|   | 60 | 26.07153 | 31.48679 | 167.1972 |
|   | 70 | 26.45764 | 31.95694 | 169.5552 |

From Table 2 we observe that

- As $C_0$ increasing, $N^*$, expected system length($L(N^*)$) are decreasing and minimum expected cost($T(N^*)$) is increasing.
- As $C_m$, $C_s$, $C_t$ increasing, $N^*$, expected system length($L(N^*)$) and minimum expected cost($T(N^*)$) are increasing.
- As $C_0$, $C_i$ are increasing, no effect on $N^*$, expected system length($L(N^*)$) and increasing minimum expected cost($T(N^*)$).
- As $C_r$ increasing, $N^*$, expected system length($L(N^*)$) are increasing and minimum expected cost($T(N^*)$) is decreasing.
8. Conclusion
In this model the explicit expressions for the system length of a queuing system whenever the server being in
vacation, startup, timeout, in batch service, in individual service and breakdown. Sensitivity analysis made for
and numerical values are presented for the different values of monetary and non-monetary parameters to
illustrate the validity of the proposed model.

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