Excited tau lepton contribution to $W \rightarrow \tau \nu_\tau$
decay at the one-loop level

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Abstract

We evaluate the compositeness effects of tau lepton on the vertex
$W \tau \nu_\tau$ in the context of an effective Lagrangian approach. We consider
that only the third family is composed and we get the corrections to the
non universal lepton coupling $g_\tau/g_e$. As the experimental bounds on non
universal lepton couplings in $W$ decays are weak, we find that the excited
particles contributions do not give realistic limits on the excited mass,
since they lead to $\Lambda < m^*$. Owing to the precision reached by experimental results nowadays, we are
able to impose significant constraints to physics beyond the Standard Model
(SM). Compositeness is a very important alternative of new physics which could
solve some problems of SM, among them the family mass hierarchy. Composi-
teness theories lie on the idea that known fermions and perhaps bosons possess
an underlying structure, characterized by the scale $\Lambda$. The fundamental con-
stituents of fermions and bosons could generate the mass spectra if we knew the
confinement mechanism, so the mass hierarchy problem would be solved.

Some attempts to generate a proper confinement mechanism have been done
by Seiberg, Harari, Terazawa[1] and others[2],[3]. However there is not any
satisfactory confinement mechanism able to generate the whole mass spectrum
from preons hitherto, as a consequence we should resort to effective Lagrangian
techniques in order to describe the behavior of excited states[5]. Such states
should become manifest at a certain energy scale $\Lambda$, and the SM is seen as an
effective theory of a more fundamental one.

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Measurements of anomalous magnetic moment of muon and electron have not shown any track of substructure for the first and second lepton families, consequently we will make the assumption that only the third family shows excited states. Therefore, it is considered that the other families are either elementary or exhibit a much higher scale factor i.e. $\Lambda_\tau \ll \Lambda_e, \Lambda_\mu$. From this fact, excited states of the third family have appealed the attention of many collaborations such as L3, DELPHI and OPAL\(^3\) whose analyses are based on an effective $SU(2) \times U(1)$ invariant Lagrangian proposed by Hagiwara et.al.\(^5\). Moreover, theoretical constraints have been derived from the contribution of anomalous magnetic moment of leptons and the $Z$ scale observables at the CERN $e^+e^-$ collider LEP. Constraints in the context of an effective Lagrangian approach have been extracted from leptonic branching ratios as an allowed region on the $(m^*, f/\Lambda)$ — plane, based on different experiments as well as bounds coming from the anomalous weak magnetic moment of the tau lepton and precision measurement on the $Z$ peak\(^6\).

Additionally, another information about excited states could come from the Tevatron experimental limits on lepton universality violation in $W$ decays $(g_\tau/g_e)$ by evaluating the contribution of this new physics to $W\tau\nu_\tau$ vertex. Since electrons could be considered elementary, no corrections from compositeness are made to the coupling $g_e$, then we can evaluate the correction to $\tau$ coupling by the ratio $g_\tau/g_e$ and taking $g_e = g$.

We assume that excited fermions acquire their masses before the $SU(2) \times U(1)$ breaking, so that both left handed and right handed states belong to weak isodoublets (vector-like model). The effective dimension five Lagrangian that describes the coupling of excited leptons with ordinary ones can be written as\(^5\)

$$\mathcal{L}_{\text{eff}} = \frac{g_f}{\Lambda} \bar{L} \sigma^{\mu\nu} \frac{\tau^2}{2} \bar{W}_\mu \gamma_\nu L + \frac{g'f'}{\Lambda} \bar{L} \gamma^\mu Y_B \gamma_\nu W_\mu L + \text{h.c.}$$

(1)

where

$$L = \begin{pmatrix} \nu_e^* \\ \tau^* \end{pmatrix}, \quad l = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

(2)

so, $L$ represents an isodoublet of excited lepton states of the third generation, which is a vector-like multiplet, and $l$, represents a left-handed doublet of ordinary leptons of the third generation. $g$ and $g'$ are the usual $SU(2)$ and $U(1)$ coupling constants respectively, $Y$ is the hypercharge and $\tau$ are the Pauli matrices. There are operators of higher dimensions that can contribute to the excited-ordinary lepton interactions, but they are suppressed by higher powers of the $\Lambda$ scale. On the other hand, since the excited fermions are doublets, they have gauge couplings given by the following renormalizable Lagrangian

$$\mathcal{L}_{\text{eff}} = \bar{L}_\nu \gamma_\mu \left( \partial^\mu - ig_\mu \frac{\tau^2}{2} \bar{W}_\mu - ig'_\mu \frac{1}{2} Y B^\mu \right) L$$

(3)

which is clearly $SU(2) \times U(1)$ gauge invariant.

The Lagrangian in eq. (1) could be rewritten in the following way

$$\mathcal{L}_{\text{eff}} = \sum_{V = \gamma, Z, W} T_{VL} \bar{L} \sigma^{\mu\nu} P_L \partial_\mu V_\nu + i \sum_{V = \gamma, Z} Q_{VL} \bar{L} \sigma^{\mu\nu} P_L W_\mu V_\nu + \text{h.c.}$$

(4)
where the coupling constants $T_{VLL}$ and the quartic interaction couplings $Q_{VLL}$ are given by

\[
\begin{align*}
T_{\gamma\tau^*\tau} &= \frac{e}{\Lambda} (f + f') , \\
T_{\gamma\nu^*\nu} &= \frac{e}{\Lambda} (f' - f) , \\
T_{Z\tau^*\tau} &= \frac{e}{\Lambda} (f \cot \theta_W - f' \tan \theta_W) , \\
T_{Z\nu^*\nu} &= -\frac{e}{\Lambda} (f \cot \theta_W + f' \tan \theta_W) , \\
T_{W\tau^*\nu} &= -T_{W\nu^*\tau} = -\frac{\sqrt{2}e}{\Lambda \sin \theta_W} f , \\
Q_{\gamma\tau^*\nu} &= -Q_{\gamma\nu^*\tau} = -\frac{\sqrt{2}e^2}{\Lambda \sin \theta_W} f , \\
Q_{Z\tau^*\nu} &= -Q_{Z\nu^*\tau} = -\frac{\sqrt{2}e^2 \cos \theta_W}{\Lambda \sin^2 \theta_W} f , \\
Q_{W\nu^*\nu} &= -Q_{W\tau^*\tau} = -\frac{e^2}{\Lambda \sin^2 \theta_W} f .
\end{align*}
\]

(5)

Now, the Lagrangian in eq. (3) could be rewritten in the following way

\[
\begin{align*}
\mathcal{L}_{\text{ren}} &= \sum_{V=\gamma,Z,W} A_{VLL} F_{\gamma}^\mu V_{\mu} F \\
\end{align*}
\]

(6)

where $F = \nu^*_\tau, \tau^*$ and the coupling constants are given by

\[
\begin{align*}
A_{\gamma\tau^*\tau} &= -e , \quad A_{\gamma\nu^*\nu} = 0 , \quad A_{Z\tau^*\tau} = \frac{(2 \sin^2 \theta_W - 1) e}{2 \sin \theta_W \cos \theta_W} , \\
A_{Z\nu^*\nu} &= \frac{e}{2 \sin \theta_W \cos \theta_W} , \quad A_{W\tau^*\nu} = \frac{\sqrt{2}e}{\sqrt{2} \sin \theta_W} .
\end{align*}
\]

(7)

In this calculation we have used the on-shell renormalization scheme and the dimensional regularization which is a gauge-invariant method. Working in $D = 4 - 2\epsilon$ dimensions, we identify the poles at $D = 2$ with quadratic divergencies and the poles at $D = 4$ with logarithmic divergencies. These divergencies are related to the cutoff scale $\Lambda$ by the following relations

\[
\begin{align*}
4\pi\mu^2 \left( \frac{1}{\epsilon - 1} + 1 \right) &= \frac{\Lambda^2}{\mu^2} , \\
\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 1 &= \ln \frac{\Lambda^2}{\mu^2} .
\end{align*}
\]

(8)

The final result for the one-loop calculation can be written as

\[
\delta = \delta_{\text{finite}} + \delta_1 \ln \frac{\Lambda^2}{\mu^2} + \delta_2 \frac{\Lambda^2}{4\pi\mu^2}
\]

(9)
where
\[
\delta_{\text{finite}} = \lim_{\epsilon \to 0} \left[ \delta(\epsilon) - \delta_1(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 1) - \delta_2(\frac{1}{\epsilon - 1} + 1) \right]
\]  
(10)

and \(\delta_1(2)\) are the residues of the poles at \(\epsilon = 0(1)\).

We write the \(W_{\tau\nu}\) vertex as
\[
\frac{ig}{\sqrt{2}} \rho_{W_{\tau\nu}, \bar{u}(\tau)} \gamma_{\mu} P_L v(\nu) \epsilon^\mu_W
\]  
(11)

where \(\rho_{W_{\tau\nu}}\) has the universal contributions which are independent on final fermion states as well as the non-universal ones, which are dependent on the fermion wave function and vertex contribution.

Figure 1: Feynman diagrams relevant in the calculation of \(W_{\tau\nu}\) vertex.

In fig. 1 we show the Feynman diagrams relevant in this calculation at the one loop-level. The contributions of all diagrams lead to a shift in the \(W_{\tau\nu}\)
coupling which can be written as

\[-\frac{ig}{\sqrt{2}}(1 + \delta_{NP})\bar{u}(\tau)\gamma_\mu P_L v(\nu_\tau)\epsilon_W^\mu\]  \hspace{1cm} (12)

where \(\delta_{NP}\) contains the radiative corrections of compositeness contributions. Then, the effective coupling for \(W\tau\nu_\tau\) is given by:

\[g_\tau = g(1 + \delta_{NP}).\]  \hspace{1cm} (13)

We do not consider oblique contributions (universal) because they are cancelled out in the \(g_\tau/g_e\) quotient. The most important contributions come from the diagrams containing two excited leptons into the loop and correspond to the \((c)\) diagrams. The \((g)\) and \((h)\) diagrams are the self-energy contributions and provide the wave-function renormalization. After summing all Feynman diagrams the quadratic divergencies cancel out and only logarithmic ones appear in the final result.

![Figure 2: Contourplots of constant \(\delta_{NP}\) in the \(m^* - \Lambda\) plane. The curve for \(\delta_{NP} = 0.036\) correspond to the intersection of \(\delta_{NP}\) with the upper experimental limit.](image)

In the following analysis we take \(m_{\tau}^* = m_{\nu_\tau}^* = m^*\) since the doublet of excited particles in eq. \(\mathbf{2}\) is vector-like. Further, we assume for the sake of simplicity that \(f = f' = 1\). In this case we do not write down the radiative corrections as Veltman-Passarino functions because there are amplitudes with Feynman diagrams that involve four momenta in the numerator, see figs. \(\mathbf{1}(c), (d)\). The expression for \(\delta_{NP}\) is very long and does not contain any illuminating
information. Instead of that, we display in fig. 2 the contour plots of constant \( \delta_{NP} \) in the \( m^* - \Lambda \) plane. The curve for \( \delta_{NP} = 0.036 \) correspond to intersection of \( \delta_{NP} \) with the upper experimental limit for \( g_\tau/g_e \): 

\[
g_\tau/g_e = 1.004 \pm 0.019 \pm 0.026. \tag{14}\n\]

This sets correlate bounds for the excited lepton mass and \( \Lambda \). For completeness, we also include the asymptotic limit for \( \delta_{NP} \) when \( m^* \) goes to infinity

\[
\delta_{NP} = \left[ \frac{g^2}{16\pi^2} \frac{f^2 m^*}{\Lambda^4} \right] \left[ -0.16 + 0.54 \log \left( \frac{m^*}{\Lambda^2} \right) \right]. \tag{15}\n\]

Since in our calculations quadratic divergencies cancel out, we find that the new physics decouples in the limit \( \Lambda \to \infty \), because \( \delta_{NP} \approx (\ln \Lambda^2)/\Lambda^2 \) vanishes for large \( \Lambda \).

In fig. 3, we plot the quotient \( g_\tau/g_e - 1 \) as a function of \( m^* \) for different values of the scale \( \Lambda \). The horizontal lines correspond to the experimental allowed range for this quotient.

Figure 3: Plots for the quotient \( g_\tau/g_e - 1 \) as a function of \( m^* \) for different values of the scale \( \Lambda \). The horizontal lines correspond to the experimental allowed range for this quotient.

In fig. 3 we plot the quotient \( g_\tau/g_e - 1 \) as a function of \( m^* \) for different values of the scale \( \Lambda \). Moreover, the two solid horizontal lines, represent the experimental allowed range for the quotient \( g_\tau/g_e \).

We can see that compositeness radiative corrections do not give severe limits on \( m^* \) as a consequence of the weak experimental bounds on lepton universality violation in \( W \) decays. The experimental uncertainty have not imposed significant constraints on \( m^*/\Lambda \) yet. For example, for \( f = 1 \), we obtain from eq. 15.
the $1\sigma$ bound of $m^*/\Lambda \approx 3.4$, getting that $m^*$ is larger than $\Lambda$. It is natural to await that new physics decouples from standard physics when the scale $\Lambda$ goes to infinity. As $\delta_{NP}$ is basically dominated for the ratio $m^*/\Lambda$ then we have to impose the condition that $\Lambda > m^*$ to get a realistic bound. This is not satisfied by our results.

In conclusion, we evaluated the contributions to $W\tau\nu_\tau$ coming from compositeness effects at the one loop level using an effective dimension five Lagrangian in the decoupling scenario, where this effective Lagrangian is valid for energies less than $\Lambda$. As a consequence, the heavy degrees of freedom $\nu^*$, $\tau^*$ should have a $m^*$ lower than $\Lambda$. However, we find that $m^* > \Lambda$ from the current precision measurements of non universal lepton couplings of $W$ decays, so it is impossible to obtain a realistic bound on the excited states. The intersection of $\delta_{NP}$ with the upper experimental bound is positive (0.036), then it is clear from eq. (15) that we require $m^* > \Lambda$ in order to get a positive value for $\delta_{NP}$.

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