A New Minimax Optimization Design of $H_\infty$ LQ Fault-Tolerant Tracking Control for Multi-Phase Batch Processes

JUN MAO$^{1,2}$, HUI YI$^3$, CHAO GAO$^4$, RENYOU ZHANG$^5$, AND FENG JIANG$^6$

1 School of Mathematics and Statistics, Hainan Normal University, Haikou 571158, China
2 Key Laboratory of Computational Science and Application of Hainan Province, Haikou 571158, China
3 College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing 211816, China
4 School of Mathematics and Applied Mathematics, Changchun Normal University, Changchun 130032, China
5 Department of Industrial Engineering, Tsinghua University, Beijing 100084, China
6 School of Physical Education and Humanities, Nanjing Sport Institute, Nanjing 210014, China

Corresponding author: Feng Jiang (2001080091@nsi.edu.cn)

This work was supported in part by the major programs of National Cultivation Project of Nanjing Sport Institute under Grant PY201923, and in part by the National Natural Science Foundation of China under Grant 61773190.

ABSTRACT For multi-phase batch processes with partial failure of the actuator and unknown disturbance, this paper proposes a minimax optimization design method for $H_\infty$ linear quadratic fault-tolerant tracking control based on switching strategy. Firstly, a non-minimum state space model is constructed based on the input-output model given by the batch process, and then a discrete switched model consisting of state errors, tracking errors and new state variables containing tracking errors is constructed. In this model, the anti-disturbances performance index function with terminal constraint is selected, and the controller and the external disturbances are obtained by using the optimization theory. Then, under the acquired control law, the switching signal is designed and the range of uncertainty caused by the fault is given to realize the robustness of the system. The advantage of this design is that the system is tracking fast and the tracking error is small. Finally, by taking the parameters such as speed and pressure in the process of injection molding as examples, and comparing with the traditional control methods, the method presented in this paper is proved to be effective.

INDEX TERMS $H_\infty$ LQ fault-tolerant tracking control, multi-phase batch processes, actuator partial failure, optimization design, switching strategy.

I. INTRODUCTION

Batch production processes have developed particularly rapidly in recent years due to its wide application in industrial process. The related modeling and control technologies have also made great progress [1]–[5]. With the increasingly strict requirements on product quality and operation safety, its control is facing severe challenges [6]. At the same time, for the inevitable uncertainty and disturbance also put forward more stringent requirements for the improved control method [7], [8]. The uncertainty caused by the aging or damage caused by the long-term use of the actuator is common in the batch production process. The actuator action may not be the exact response of the control signal calculated by the controller [9]. In this case, the performance of the control system may deteriorate to an unacceptable degree. In serious cases, not only will affect the production efficiency and product quality, and may even cause major property losses and casualties.

In order to change this situation, the corresponding fault diagnosis technology and fault-tolerant control strategy are put forward for actuator failures. A large number of research results have been available [10]–[14]. There are mainly three kinds of actuator failures: partial failure, complete failure and jammed failure. Among them, the latter two cases will make the process control no longer work, it is not necessary to study the controller design. Therefore, most of the results focus on partial failures, that is, multiplicative failures. Of course, there is also an additive fault in actuator failures, and this part of the research results also exists [14]. In these articles, the faults are considered when designing the controller, and even with the change of fault value, the controller parameters
will change. In batch processes, some researchers have presented their research results. At present, there are two main types of research results. One is to propose different iterative learning fault-tolerant control design methods, namely 2D control theory, according to the repetitive characteristics of batch production. These results are reflected in the single-phase batch process [15]–[18] and the multi-phase process [19], [20]. It can be seen from [18] that the two-dimensional system design combining feedback control and iterative learning control (FLC) has better performance than the one-dimensional design. The design premise of this scheme is to assume that the system information between batches is completely duplicated. But in practice, many batch processes have unrepeatable dynamic characteristics and unknown disturbances. As the action of the physical actuator no longer matches the output of the controller, the iterative learning control is no longer applicable, which produces the new control method. The emergence of time-dependent control strategy design is the one-dimensional theoretical control method. In order to improve the control performance of the system, the minimax optimization model predictive control theory is used. Paper [21] combines the minimax optimization with the maximum-forecast control to solve the strategic and tactical inventory control problems in DCS management. The paper [22] proposes an algorithm to determine the explicit solution of the constrained minimax model predictive control problem. The paper [23] proposes a nonlinear minimax model predictive control based on Volterra model. A new nonlinear control scheme is derived in [24]. In paper [25], a MPC controller based on the minimax optimization is designed to improve the performance of the system. In order to resist the influence of external disturbances and actuator failure, the minimum-maximum optimization theory combined with H\text{\text{\infty}} control method is reproduced.

Throughout the research results under the condition of disturbances mismatch, we find that all the results are based on a certain phase of the design of the corresponding control algorithm. However, the batch production process is a multi-phase process essentially, and the phases affect each other. The poor control of one phase will affect the next phase and even affect the whole process. In this case, it is very necessary to find a good control method. It is a good choice to study the high precision control by using the concept of switched system control. In the case of multi-phase, great achievements have been made in the study of the repetition characteristic of batch processes [28], [29]. In the case of the actuator failure, the design iterative learning fault-tolerant control strategy for the multi-phase batch process has also appeared [30]. Recently, the fault tolerant control of quadratic linear tracking based on the switching strategy has also been studied [31], and extended to random faults of iterative learning control based on switching strategies [32].

Obviously, Most of the existing results assume that the external information matches the system information, but in fact, this is not the case. In the multi-phase batch process, when information does not match, the research results are rare. Based on this, a new design of the minimax H\text{\text{\infty}} tracking fault-tolerant control strategy is proposed by using the extended non-minimum state space model. The reason why H\text{\text{\infty}} tracking fault-tolerant control strategy is proposed is to resist external disturbances. H\text{\text{\infty}} control is an effective control method and some representative results have appeared [33]–[37]. Of course, the method proposed in this paper does not simply extend the single-phase model to multiple phases, such as [38] and [39]. Instead, a new state variable is added to the original model. The advantage of this is that as the number of variables increases and the number of adjustable free variables increases, the system may converge faster and track better. The ultimate goal is to improve the control accuracy of the system. Second, switching from one phase to another of the multi-phase models considered here requires the design of switching conditions. Reasonable switching conditions can avoid system overshoot and other phenomena. After all, excessive overshoot is not allowed in production, which will seriously lead to the scrapping of products. The specific design ideas of the control law are as follows: Firstly, establish the multi-phase extended non-minimum state space model, introduce state errors, output tracking errors and new state variables related to output tracking errors, and make its original system known as an equivalent new model of dimensionality expansion. Then, the quadratic performance index function is constructed, and the function contains external disturbances. To design the optimal control law, to resist the maximum external disturbances and achieve the minimum system performance index, the control law and disturbances are designed by using optimization theory. Then the stability and robustness of the system are analyzed. Finally, by controlling a certain phase and several adjacent phases of batch process as examples, the system simulation analysis from special to general is realized. By comparing with the latest existing control methods, it is concluded that the control method proposed in this paper makes the system output tracking faster, anti-disturbances stronger and closer to the given value. In this paper, the fault under consideration is regarded as a disturbance, which can also include additive faults. In a certain allowable range of faults, the designed control law can realize its control. Although the control law cannot be adjusted with the change of faults, this design is simple and feasible, and the control effect is very good.

Throughout this paper, the following notations are used: For any matrix $M$, $M > 0$ ($M < 0$) means $M$ is a positive (negative) definite matrix. $M^T$ represents the transpose of matrix $M$. $I$ and $0$ respectively denote the identity matrix and the zero matrix with appropriate dimensions. $\sigma_i^j(\chi)$ is the maximum singular value. $\lambda_i^j_{\text{max}}(\chi)$ and $\lambda_i^j_{\text{min}}(\chi)$ are respectively the minimum eigenvalue and the maximum eigenvalue of $\chi^T$. $\|\cdot\|$ represents the norm of matrix $\cdot$.\)
average model:

\[ A^i(z^{-1})y(z) = B^i(z^{-1})u(z) + B^w_i(z^{-1})w(z) / \Delta \]  

where \( y(z), u(z), \) and \( w(z) \) are the transformation \( z \) of output \( y(k), \) input \( u(k) \) and the disturbance \( w(k) \), respectively; \( \nu(k) = [0, \infty) \rightarrow \tilde{p} = \{1, 2, \ldots p\} \) is a switching signal related to system state and discrete time. It means that each batch is divided into \( p \) phases. Among \( i \in \tilde{p}, \Delta \) is the difference operator, \( A^i(z^{-1}), B^i(z^{-1}), \) and \( B^w_i(z^{-1}) \) are correlation polynomials with appropriate dimensions. They are represented by

\[
A^i(z^{-1}) = 1 + F_1^i z^{-1} + F_2^i z^{-2} + \ldots + F_n^i z^{-n} \\
B^i(z^{-1}) = H_1^i z^{-1} + H_2^i z^{-2} + \ldots + H_m^i z^{-m} \\
B^w_i(z^{-1}) = 1 + L_1^i z^{-1} + L_2^i z^{-2} + \ldots + L_r^i z^{-r}
\] (2)

The \( i^{th} \) phase discrete system can be converted into the following form:

\[ A^i(z^{-1})y^i(z) = B^i(z^{-1})u^i(z) + B^w_i(z^{-1})w^i(z) / \Delta \]  

Select the following state space vector \( x^i(k) \):

\[
x^i(k) = [y^i(k), y^i(k-1), \ldots, y^i(k-n+1), \\
u^i(k-1), u^i(k-2), \ldots, u^i(k-m+1)]^	ext{T}
\] (4)

Then the system state space equation (5), as shown at the bottom of the next page, where \( u^i(k), y^i(k) \) and \( w^i(k) \) is input, output and the disturbance of \( k \) time respectively.

In fact, the established model may be inconsistent with the actual model of the system. In particular, the actual input may be \( u^F(z) \), where

\[ u^F(z) = \alpha u(z) \]  

with \( 0 < \alpha \leq \alpha \leq \bar{\alpha} \) and \( \alpha \leq 1, \bar{\alpha} \geq 1 \).

A representation like (6) is called an actuator failure, \( 0 < \alpha \neq 1 \) is considered partial failure, \( \alpha = 0 \) indicates complete failure of the actuator. The control input in the system is no longer in effect and will not be considered here; \( \alpha = 1 \) represents the system is normal, the actual system is consistent with the modeling system. It is worth noting that here we only consider the multiplicative faults, the additive faults are not considered. In fact, additive faults do not need to be considered. The reason is that multiplicative faults are regarded as interferences in controller design, and then obviously additive faults can be included.

The object is to design the controller based on the established model so that the output of the actual system can track the given output as much as possible, even if the actuators in the system fail.

The batch production process is the essence of a multiple phase. In a batch process system, when the switching condition meets to switch from one phase to another phase, at the instant of the switch, the state between adjacent phase can be expressed as follows:

\[ x^{i+1}(T^i) = \Phi^i x^i(T^i) \]  

\( \Phi^i \) is the state transition matrix. If the states of two adjacent phases have the same dimension, the state transition matrix is equal to the identity matrix.

\( v(k) \) is a state-dependent switch signal related to the state of the system, it can be expressed as follows:

\[ v(k+1) = \begin{cases} v(k) + 1, & M^{(k+1)}(x(k)) < 0 \\ v(k), & \text{other} \end{cases} \] (8)

\( M^{(k+1)}(x(k)) < 0 \) is the switching condition. It is related to the actual process. This chapter is given in the case study. Once this switching condition is satisfied for \( i \) phase, the system switches from \( i \) phase to \( i+1 \) phase. Generally, system switching time is the key factor to determine product quality and product revenue. Based on the known system state, the switching time \( T^i \) can be expressed as follows:

\[ T^i = \min \left\{ k > T^{i-1} | M^i(x(k)) < 0 \right\}, \quad T^0 = 0 \] (9)

The time interval between two adjacent phases shall be satisfied \( T^i - T^{i-1} \geq t^i \), where \( t^i \) is average dwell time.

### III. THE LATEST NON-MINIMUM STATE SPACE LQ CONTROL

At present, according to the current control methods, the following steps can be followed to design the controller.

For a single phase, the state space vector is chosen \( \Delta x^i(k)(i = 1): \)

\[
\Delta x^i(k) = [\Delta y^i(k), \Delta y^i(k-1), \ldots, \Delta y^i(k-n+1), \\
\Delta u^i(k-1), \Delta u^i(k-2), \ldots, \Delta u^i(k-m+1)]^\text{T}
\] (11)

The auto-regressive model based on input-output process data is transformed into a traditional non-minimum state space model (TNMSS):

\[ \Delta x^i(k+1) = A^i \Delta x^i(k) + B^i \Delta u^i(k) + B^w_i \Delta w^i(k) \]  

where \( \Delta u(k), \Delta y(k) \) and \( \Delta w(k) \) are respectively input increment, output increment and disturbance quantity at \( k \) moment, \( A^i, B^i, B^w_i \) is indicated above. \( \Delta w^i(k) = [w^i(k+1) + w^i(k) + \ldots + w^i(k-r+1)]^\text{T} \)

An output tracking error is introduced as follows:

\[ e^i(k) = y^i(k) - y^i_f(k) \] (13)

where \( y^i(k) \) is the output at \( k \) moment of \( i \) phase. \( y^i_f(k) \) is the set value at \( k \) moment of \( i \) phase.
As can be seen from (12) and (13):
\[
e^{i}(k+1) = e^{i}(k) + C^{i}A^{i} \Delta x^{i}(k) + C^{i}B^{i}u^{i}(k) + C^{i}B^{w}_{i}\Delta w^{i}(k)
\]
(14)
Then the augmented model can be deduced from (12) and (14):
\[
\begin{align*}
\dot{x}_{i}^{j}(k+1) &= A_{1}^{i}x_{i}^{j}(k) + B_{1}^{i}u^{i}(k) + B_{w1}^{i}w^{i}(k) \\
\dot{y}_{i}^{j}(k+1) &= C_{1}^{i}x_{i}^{j}(k+1)
\end{align*}
\]
(15)
where
\[
x_{i}^{j}(k) = \begin{bmatrix} \Delta x_{i}^{j}(k) \\ e^{j}(k) \end{bmatrix}, \quad A_{1}^{i} = \begin{bmatrix} A^{i} & 0 \\ C^{i}A^{i} & I^{i} \end{bmatrix}, \quad B_{1}^{i} = \begin{bmatrix} B^{i} \\ C^{i}B^{i} \end{bmatrix}, \quad B_{w1}^{i} = \begin{bmatrix} B_{w}^{i} \\ C^{i}B_{w}^{i} \end{bmatrix}, \quad C_{1}^{i} = \begin{bmatrix} 0 & I^{i} \end{bmatrix}.
\]
(16)
\[
\begin{align*}
J^{i}(\Delta u, \Delta w) &= \sum_{k=k_{0}}^{k_{1}-1} \left[ (x_{i}^{j}(k))^{T} \hat{Q}_{i}^{j}x_{i}^{j}(k) + (\Delta u^{j})^{T} \hat{R}_{i}^{j}\Delta u^{j} \\
&\quad - \gamma^{j}_{i}(\Delta w^{i}(k))^{T} \Delta w^{i}(k) \\
&\quad + (x_{i}^{j}(k_{f}))^{T} \hat{Q}_{i}^{j}x_{i}^{j}(k_{f}) \right]
\end{align*}
(17)
where \( \hat{Q}_{i}^{j}, \hat{R}_{i}^{j} \) and \( \hat{Q}_{i}^{j} \) are given positive definite symmetric matrices.

Consider the following optimization problem:
\[
\min_{\Delta u(k)} \max_{\Delta w(k)} J^{i}(k)
\]
(18)
\[
[k_{0}, k_{f}] \text{ is the optimization time range, } \hat{Q}_{i}^{j} > 0, \hat{Q}_{i}^{j} > 0, \hat{R}_{i}^{j} > 0 \text{ represent the weighted matrix, } \gamma^{j}_{i} > 0 \text{ is a constant value associated with } \Delta w^{i}(k).
\]

**IV. EXTENDED NEW NON-MINIMUM STATE SPACE LQ CONTROL**

**A. ESTABLISHMENT OF THE EQUIVALENT MODEL**

In order to stabilize the tracking error along the time direction as soon as possible, so that the system can reach the set value more quickly, a new extended state variable is introduced under the control method proposed above. The model is described as follows:
\[
\ddot{x}(k+1) = \dot{x}(k) + e^{j}(k)
\]
(19)
Thus, a new extended non-minimum state space model with uncertainty is obtained:
\[
\ddot{z}(k+1) = A_{m}^{i} \dot{z}(k) + B_{m}^{i} \Delta u(k) + B_{w}^{i} \Delta w(k)
\]
(20)
\[
\dot{z}(k) = \begin{bmatrix} \Delta x(k) \\ \dot{x}(k) \end{bmatrix}, \quad A_{m}^{i} = \begin{bmatrix} A^{i} & 0 \\ C^{i}A^{i} & I^{i} \end{bmatrix}, \quad B_{m}^{i} = \begin{bmatrix} B^{i} \\ C^{i}B^{i} \end{bmatrix}, \quad B_{w}^{i} = \begin{bmatrix} B_{w}^{i} \\ C^{i}B_{w}^{i} \end{bmatrix}.
\]
For the above model, the switched system is represented as
\[ z(k + 1) = A_m^{v(k)}z(k) + B_m^{v(k)}u(k) + B_{wm}(k) w(k) \]  
(20)
where \( v(k) \in \{ 1, 2, \ldots, p \} \). For the system in phase \( i \) mentioned above, the corresponding optimized performance indicators are selected as follows:
\[
\bar{J}(\Delta u, \Delta w^i) = \sum_{k=0}^{k_f-1} [(A_m^i(z(k)) + (\Delta u^i)^T R^i \Delta u^i \\
- \gamma_i^2(\Delta w^i(k))^T \Delta w^i(k)] + (z_i^T(k+1) - \gamma_i^2 k_f (z_i^T(k))] 
\]  
(21)
where \( Q^i, R^i \) and \( Q_f \) are given positive definite symmetric matrices. Obviously, the dimension of positive definite matrix here increases with the introduction of new state variables, so the adjustable variables also increase. By adjusting more state variables, we still expect the following optimization problem to be realized under its constraints. And the performance index is less than (16).
\[
\min_{\Delta u(k)} \max_{\Delta w(k)} \bar{J}(\Delta u, \Delta w^i) 
\]  
(22)
Next, in order to achieve the above goals, the following ideas can be followed: (1) Design the controller; (2) Analyze the asymptotic stability of the system; (3) When a switching performance index is less than (16).

Thus the necessary conditions of the optimal control law can be obtained as follows:
\[
p_k^i = \frac{\partial H_k^i}{\partial z_i^T(k)} = 2Q_i^T(k) + (A_m^i)^T p_{k+1}^i \]  
(26)
\[
p_{k_f}^i = 2Q_f^T(k) \]  
(27)
\[
0 = \frac{\partial H_k^i}{\partial \Delta u^i(k)} = 2R_i^T \Delta u^i(k) + (B_m^i)^T p_{k+1}^i \]  
(28)
\[
0 = \frac{\partial H_k^i}{\partial \Delta w^i(k)} = -2\gamma_i^2 \Delta w^i(k) + (B_{wm}^i)^T p_{k+1}^i \]  
(29)

Design
\[
p_k^i = 2M_{k,k_f}^i z_i^T(k) \]  
(30)
If equation (30) is substituted into equation (28), the following can be obtained:
\[
\frac{\partial H_k^i}{\partial \Delta u^i(k)} = 2R_i^T \Delta u^i(k) + (B_m^i)^T p_{k+1}^i 
= 2R_i^T \Delta u^i(k) + 2(B_m^i)^T M_{k+1,k_f}^i z_i^T(k+1) \]  
(31)
Then it can be deduced that:
\[
\frac{\partial^2 H_k^i}{\partial \Delta u^i(k)} = 2R_i^2 + (B_m^i)^T M_{k+1,k_f}^i B_m^i \]  
(32)
It is obviously that \( \frac{\partial^2 H_k^i}{\partial \Delta u^i(k)} > 0 \), for \( k = k_0, \ldots, k_f - 1 \).

By substituting equation (30) into equation (29), we can get:
\[
\frac{\partial H_k^i}{\partial \Delta w^i(k)} = -2\gamma_i^2 \Delta w^i(k) + 2(B_{wm}^i)^T M_{k+1,k_f}^i z_i^T(k+1) \]  
(33)
Then it can be derived:
\[
\frac{\partial^2 H_k^i}{\partial \Delta w^i(k)} = -2\gamma_i^2 + 2(B_{wm}^i)^T M_{k+1,k_f}^i B_{wm}^i \]  
(34)

On the basis of equations (28) and (29), the state variables at \( k + 1 \) moment can be obtained:
\[
z_i^T(k+1) = A_m^i z_i^T(k) + \frac{1}{2} (B_m^i (R_i^2 - 1)(B_m^i)^T \\
+ (y_i^T)^2 B_{wm}^i (B_{wm}^i)^T) p_{k+1}^i \]  
(35)
By substituting equation (35) into equation (30) we can get:
\[
p_k^{i+1} = 2M_{k+1,k_f}^i z_i^T(k+1) 
= 2M_{k+1,k_f}^i A_m^i z_i^T(k) + M_{k+1,k_f}^i (-B_m^i (R_i^2 - 1)(B_m^i)^T \\
+ (y_i^T)^2 B_{wm}^i (B_{wm}^i)^T) p_{k+1}^i \]  
(36)
Therefore
\[
p_k^i = 2 \left[ I^T + M_{k+1,k_f}^i (B_m^i (R_i^2 - 1)(B_m^i)^T \\
- (y_i^T)^2 B_{wm}^i (B_{wm}^i)^T) \right]^{-1} \times M_{k+1,k_f}^i A_m^i z_i^T(k) \]  
(37)
According to equations (26) and (39), the following forms
\[ \begin{align*}
\Delta u^i(k) &= -(R^i)^{-1}(B^i_m)^T(P^i_{k+1,k})^{-1}A^i_mz^i(k) \\
\Delta w^i(k) &= (y^i)^{-2}(B^i_{wm})^T(P^i_{k+1,k})^{-1}A^i_mz^i(k)
\end{align*} \] (47)

Under boundary conditions: \( P^i_{k,k} = (Q^i_{k})^{-1} + \Pi^i \)

Equations (23) and (24) are redefined as:
\[ \begin{align*}
\Delta u^i(k) &= -(R^i)^{-1}(B^i_m)^T(P^i_{k+1,k})^{-1}A^i_mz^i(k) \\
\Delta w^i(k) &= (y^i)^{-2}(B^i_{wm})^T(P^i_{k+1,k})^{-1}A^i_mz^i(k)
\end{align*} \] (48)

The stability of the closed-loop system is as follows:

\[ H^i = -(R^i)^{-1}(B^i_m)^T \left[ I^i + Q^i \Pi^i \right]^{-1}Q^i A^i_m, \]

If \( Q^i \geq (A^i_m)^T Q^i \left[ I^i + \Pi^i Q^i \right]^{-1} A^i_m + Q^i \) (49)

So \( J^i(z^i(k), 0, k_f) \) satisfies the following monotonicity:
\[ J^i(z^i(k), 0, k_f + 1) \leq J^i(z^i(k), 0, k_f) \] (50)

\[ M^i_{k,k_f} \leq M^i_{k,k_f + 1} \] (51)

(2) Condition (49) can be rewritten as:
\[ P^i_{k,k_f} \leq (A^i_m)^{-1} P^i_{k,k_f} \left[ I^i + (A^i_m)^{-T}Q^i(A^i_m)^{-1}P^i_{k,k_f} \right]^{-1} \times (A^i_m)^{-T} + \Pi^i \] (52)

where \( P^i_{k,k_f} \geq P^i_{k+1,k_f} \)

(3) Assuming \( (A^i_m, B^i_m) \) controllable, \( A^i_m \) and \( B^i_m \) are non-singular matrices, if the control law in (47) meets equation (52), then the closed-loop system is asymptotically stable under the condition of \( k_f \geq 1 \).

Proof: 
\[ J^i \left( z^i(k), 0, k_f + 1 \right) \] (53)

\[ J^i \left( z^i(k), 0, k_f \right) \] (54)

\[ \Delta u^i_1(k) \Delta w^i_1(k) \text{ and } \Delta u^i_2(k) \Delta w^i_2(k) \text{ are the corresponding saddle points. } \Delta u^i(k) \text{ substitutes } \Delta u^i_1(k) \text{ in performance index. } J^i \left( z^i(k), 0, k_f + 1 \right) \text{ is the optimal performance, the following can be obtained:} \]

\[ \sum_{k=k_0}^{k_f} \left[ z^i(k)^T Q^i z^i(k) + \Delta u^i_1(k)^T R^i \Delta u^i_1(k) \right] \] (55)
where \( J^*(z(k), 0, k_f) \) is also optimal performance, similarly, we can know: 
\[
\sum_{k=0}^{k_f-1} \left[ z^T(k) Q^i z^i(k) + \Delta u^2(k) R^i \right]^{T} \Delta u^2(k) \\
- \gamma^2 \Delta w^2(k) \\
+ \gamma^2 (k_f)^T Q^a z^a(k_f) \\
\geq \sum_{k=0}^{k_f-1} \left[ z^T(k) Q^a z^a(k) + \Delta u^2(k) R^a \right]^{T} \Delta u^2(k) \\
- \gamma^2 \Delta w^2(k) \\
+ \gamma^2 (k_f)^T Q^a z^a(k_f) \\
(56)
\]

From (55) minus (56), we can get (50), if the following formula is true:
\[
Q^i_z \geq Q^z + (H^T R H)^{T} \Pi^z + (A^i_z)^T Q^a A^i_z \\
\text{where}
A^i_z = A^i_m + B^i_m H^i + B^i_{wm} \Gamma^i \\
\Gamma^i = (\gamma^2)^2 (B^i_{wm})^2 \left( (\gamma^2)^2 A^i_m \\
\Lambda^i = I^i + Q^a \Pi^z \\
H^i = -(R^i)^{-1} (B^i_m)^T \left( I^i + Q^a \Pi^z \right)^{-1} \left( Q^a A^i_m \\
\right)
(57)
\]

Through matrix inversion, it can be known that:
\[
\left[ \Pi^z Q^a \right]^T = \Pi^z - \Pi^z \left( Q^a \Pi^z + I^i \right)^{-1} Q^a \\
(59)
\]

The optimal solutions of (53) and (54) are respectively
\[
z^i(k)^T M^i_{k+1, k} z^i(k) \\
z^z(k)^T
\]

Therefore, we can be obtained equation (51).

For the following system:
\[
z^i(k+1) = (A^i_m)^{-T} z^z(k) + (A^i_m)^{-T} (Q^a)^{1/2} u^i(k) \\
(60)
\]

And the LQ performance index function is:
\[
\tilde{J}^i(z(k), 0, k_f) = \sum_{k=k_0}^{k_f-1} \left[ z^T(k) \Pi^z z^z(k) + u^i(k)^T u^i(k) \right]^{T} \\
+ \gamma^2 (k_f)^T P^i_{k, k_f} z^i(k_f) \\
(61)
\]

The optimal performance index function is as follows:
\[
\tilde{J}^i_w(z^i(k), 0, k_f) = z^i(k)^T P^i_{k, k_f} z^i(k_f) \\
(62)
\]

Compared with standard LQ, \((A^i_m)^{-T} \) and \((A^i_m)^{-T} (Q^a)^{1/2} \) are the system matrices, \( \Pi^i \), \( I^i \) and \( P^i_{k, k_f} \) are the parameters in the performance indicator function. Considering that the adjoint system of the original system (19) with the control law (47) and (48), it has as follows:
\[
z^i(i+1) = \left[ A^i_m - \Pi^i (P^i_{k+1, k_f})^{-1} A^i_m \right]^{T} z^i(i) \\
(63)
\]

Define a scalar function:
\[
V^i \left( \tilde{z}^z(i) \right) = z^i(i)^T (A^i_m)^{-1} P^i_{k+1, k_f} (A^i_m)^{-1} T z^i(i) \\
(64)
\]

Formula (46) can be changed to:
\[
P^i_{k, k_f} = (A^i_m)^{-T} P^i_{k+1, k_f} \left[ I^i + (A^i_m)^{-T} Q^a (A^i_m)^{-1} P^i_{k+1, k_f} \right]^{-1} \\
\geq (A^i_m)^{-T} \left[ P^i_{k+1, k_f} - P^i_{k+1, k_f} (A^i_m)^{-T} Q^a (A^i_m)^{-1} P^i_{k+1, k_f} \right] I^i \\
\geq \left( (Q^a)^{1/2} (A^i_m)^{-1} P^i_{k+1, k_f} (A^i_m)^{-T} Q^a (A^i_m)^{-1} P^i_{k+1, k_f} \right)^{1/2} + I^i \\
= (A^i_m)^{-T} P^i_{k+1, k_f} (A^i_m)^{-T} + \Pi^z + \Xi^i \\
(65)
\]

From (63) and (65):
\[
V^i \left( \tilde{z}^z(i) \right) - V^i \left( \tilde{z}^z(i+1) \right) = z^i(i+1)^T \left[ P^i_{k+1, k_f} - 2 \Pi^i + \Pi^i (P^i_{k+1, k_f})^{-1} \Pi^z \right] z^z(i) \\
- \tilde{z}^z(i)^T [P^i_{k, k_f}] - \Pi^z + \Xi^i \tilde{z}^z(i+1) \\
= \tilde{z}^i(i+1)^T \left[ P^i_{k+1, k_f} - P^i_{k, k_f} - \Pi^i + \Pi^i (P^i_{k+1, k_f})^{-1} \Pi^z \\
\Xi^i \tilde{z}^z(i+1) \right) \right) \\
(66)
\]

Because of \( P^i_{k, k_f} > P^i_{k+1, k_f} \), and \( \Xi^i \) is semi-positive definite, it can be known that:
\[
V^i(\tilde{z}^z(i)) - V^i(\tilde{z}^z(i+1)) \leq \tilde{z}^i(i+1)^T \left[ \Pi^z - \Pi^i (P^i_{k+1, k_f})^{-1} \Pi^z \right] \tilde{z}^z(i+1) \\
\]

where \( \Pi^i - \Pi^i (P^i_{k+1, k_f})^{-1} \Pi^z \) is positive definite.
\[
\Pi^i - \Pi^i (P^i_{k+1, k_f})^{-1} \Pi^z > 0 \\
\Leftrightarrow \pi^i \left[ I^i - (\Pi^z)^{1/2} (P^i_{k+1, k_f})^{-1} (\Pi^i)^{1/2} \right] (\Pi^z)^{1/2} > 0 \\
\geq \left[ I^i - (\Pi^z)^{1/2} (P^i_{k+1, k_f})^{-1} (\Pi^i)^{1/2} \right] > 0 \\
\geq \left[ I^i - (P^i_{k+1, k_f})^{-1} (\Pi^z)^{1/2} (\Pi^i)^{1/2} \right] > 0 \\
\geq P^i_{k+1, k_f} - \Pi^i > 0 \Leftrightarrow (M^i_{k+1, k_f})^{-1} > 0 \right) \\
(68)
\]

This indicates that the closed-loop system (63) grows exponentially, and therefore the closed-loop system (20) controlled by (47) and (48) drops exponentially.
C. ROBUSTNESS ANALYSIS

In this case, the controller designed under normal conditions must have a certain robustness to resist the influence brought by the failure, and the tracking error should converge or approach zero under the proposed optimization algorithm. The robust stability criterion is given below.

**Theorem 3:** For the batch process (3) described by the actuator (6), if the LQ controller is based on the nominal system can be written

\[
\dot{z}(k)^T \left[ (A_m^i - K_j^i)^T P^i (A_m^i - K_j^i) - \dot{z}(k)^T P^i \right] \dot{z}(k) \\
\leq -\lambda_{\min}^i (W^i) \left\| \dot{z}(k) \right\|^2
\]

(78)

For

\[
z^i(k)^T (A_m^i - K_j^i)^T P^i (A_m^i - K_j^i) z(k) \\
+ z(k)^T (A_m^i - B_m^i K_j^i)^T P^i (A_m^i - K_j^i) z(k) < 2\sigma_{\max}^i (A_m^i - K_j^i)^T \left\| \Delta A_m^i - \Delta B_m^i K_j^i \right\| \left\| z(k) \right\|^2
\]

(79)

and

\[
z^i(k)^T (A_m^i - B_m^i K_j^i)^T P^i (A_m^i - B_m^i K_j^i) z(k) \\
+ z^i(k)^T (A_m^i - B_m^i K_j^i)^T P^i (A_m^i - B_m^i K_j^i) z(k) < 2\sigma_{\max}^i (A_m^i - K_j^i)^T \left\| \Delta A_m^i - \Delta B_m^i K_j^i \right\| \left\| z(k) \right\|^2
\]

(80)

From (78) - (80):

\[
\Delta V^i(z(k)) \\
\leq \left\| \dot{z}(k) \right\|^2 \left( -\lambda_{\min}^i (W^i) + 2\sigma_{\max}^i (A_m^i - K_j^i)^T \lambda_{\max}^i (P^i) \right) \\
\times \left\| \Delta A_m^i - \Delta B_m^i K_j^i \right\|^2 + \lambda_{\max}^i (P^i) \left\| \Delta A_m^i - \Delta B_m^i K_j^i \right\|^2
\]

(81)

Obviously, if the following conditions are met:

\[
-\sigma_{\max}^i (A_m^i - B_m^i K_j^i) - \sqrt{(\sigma^i)_{\max}^i (A_m^i - K_j^i) + \lambda_{\min}^i (W^i) / \lambda_{\max}^i (P^i)} < \left\| \Delta A_m^i - \Delta B_m^i K_j^i \right\|
\]

(82)

Then the following formula is true:

\[
-\lambda_{\min}^i (W^i) + 2\sigma_{\max}^i (A_m^i - K_j^i)^T \lambda_{\max}^i (P^i) \left\| \Delta A_m^i - \Delta B_m^i K_j^i \right\|^2 + \lambda_{\max}^i (P^i) \left\| \Delta A_m^i - \Delta B_m^i K_j^i \right\|^2 < 0
\]

(83)

Then \( \Delta V^i(z(k)) < 0 \) set up. For \( \forall O \in [T^i, T^{i+1}] \) from (76):

\[
V^i(z(T^i)) < (z^i)^{T - T^i} V^i(z(T^i))
\]

(84)

If the switching condition is satisfied, the following can be obtained according to condition (71a):

\[
V^i(z(T^i)) \leq \dot{\vartheta}_i V^{i-1}(z(T^i))
\]

(85)
From (84) and (85):

\[
V^{\nu(T)}(z(k)) < (\varsigma^i)^{O-T} V^{\nu(T)}(z(T^i)) \\
\leq \theta^i (\varsigma^i)^{O-T} V^{\nu(T)}(z(T^i)) \\
\vdots \\
\leq \prod_{i=1}^{p} ((\theta^i)^{1/T^i}(\varsigma^i))^{T^i(f,O)} V^{\nu(T)}(z(T^i)) \\
= \prod_{i=1}^{p} ((\theta^i)^{1/T^i}(\varsigma^i))^{T^i(f,O)} V^{\nu(T)}(z(T^1))
\]

(86)

For \(0 < \varsigma^i < 1, \ln \varsigma^i < 0\), According to the conditions (71b):

\[
(\theta^i)^{1/T^i}(\varsigma^i) = \exp \left( \ln \left( \exp \left( \frac{1}{T^i} \ln \theta^i + \ln \varsigma^i \right) \right) \right) \\
= \exp \left( \frac{1}{T^i} \ln \theta^i + \ln \varsigma^i \right)
\]

(88)

Remark 1: Two kinds of disturbance, internal disturbance and external one are considered. The external disturbance in this paper has been taken into account when designing the performance index function, as shown in Formula (21), while the internal disturbance, that is, the uncertainty caused by the fault has been given in Theorem 3, as shown in Formula (69). It can be seen from Equation (69) that the uncertainty caused by fault is \(\Delta A^i_m, \Delta B^i_m\), and the norm of their sum satisfies a certain bound. The fault \(\alpha^i\) here may be a variable fault.

\[\Delta A^i_m = \begin{bmatrix}
0 & 0 & \cdots & 0 & H^i_2 \alpha^i - H^i_2 & \cdots & H^i_{m-1} \alpha^i - H^i_{m-1} & H^i_m \alpha^i - H^i_m & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[\Delta B^i_m = \begin{bmatrix}
H^i_1 \alpha^i - H^i_1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & H^i_1 \alpha^i - H^i_1 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
\end{bmatrix}
\]

V. CASE STUDIES

Injection molding has been more and more widely used in the production of plastic products due to its advantages of fast production speed, high efficiency, accurate product size and easy replacement.

The injection molding process is a typical multi-phase batch process. Figure 1 shows the three important phases of injection molding, namely injection, pressure holding, and cooling. In the injection phase, as shown in figure 1(a), the plastic cylinder is uniformly plasticized, and the melt material is driven by the screw rod to be injected into the mold cavity until the cavity is completely filled. After the completion of filling, the system enters into the pressure-holding state, and its purpose is to cool and solidify the material, as shown in figure 1(b). The injection speed and nozzle pressure will change during the filling and maintaining filling phases. At this phase, the melting material will be transported forward through the rotating screw. As the melting material at the screw head increases, the pressure in the cavity will increase accordingly. At this moment, the screw stops rotating and the completion of plasticization is shown in figure 1(c). The materials in the cavity are cooled to complete solidification.
output fluctuation is small, the tracking is fast, and even zero tracking error is realized. Figure 2 (b) as input curve, it can see from the picture using the new type of control method, the input is smooth. Figure 2(c) is the tracking error curve. In addition to the step point, the curve has a mutation. At other moments, the error curve obviously changes little or even approaches zero. Figure 2(d) represents the disturbance signal curve. Because the parameters of Q and R in the performance index function can be adjusted, the process state and tracking error can be adjusted, which results in more smooth control input into the system. The proposed method can also suppress the disturbances so that it becomes smooth through the system process and has strong anti-disturbance capability. Figure 3 shows various curves of the system under time-varying faults. Obviously, under the influence of time-varying faults, all kinds of curves fluctuate to some extent, which indicates that under time-varying faults, the control effect of the system becomes worse, but it is still within the allowable range and better than existing control methods obviously.

In this example, the injection phase is defined as the first phase and the pressure holding phase is defined as the second phase in the process. In the injection phase, the injection velocity (IV) model corresponding to the valve opening (VO) can be described as:

\[
\begin{align*}
(1 - 0.9291z^{-1} - 0.03191z^{-2}) IV \\
= (8.687z^{-1} - 5.617z^{-2}) VO + z^{-1}w(z)
\end{align*}
\] (93)

And the nozzle pressure (NP) model corresponding to the injection speed is as follows:

\[
(1 - z^{-1}) NP = 0.1054 IV
\] (94)

Similarly, in the pressure holding section, the nozzle pressure model corresponding to the valve opening is:

\[
(1 - 1.317z^{-1} + 0.3259z^{-2}) NP \\
= (171.8z^{-1} - 156.8z^{-2}) VO + z^{-1}w(z)
\] (95)

(93) and (94) constitute the state space model according to the previous form, and the state is regarded as \(x^1(k)\), and the switching condition between phases is \(M^1_s(k) = 350 - [0 \ 0 \ 1]x^1(k) < 0\). When the nozzle pressure is greater than 350pa, the switch occurs. In case 2, the control effect is analyzed in three categories. (1) Perfect matching (no faults); (2) Constant fault is assumed to occur when \(k = 150\); (3) Time-varying fault \(\alpha = 0.6 + 0.01\sin(k)\), and the occurrence time is the initial time.

In the case of (1), the control effect is shown in figure 4. As can be seen from figure 4(a), under the control algorithm proposed in this paper, the system output can quickly track the given output with little fluctuation, as can be seen from figure 4(c). Figure 4(b) and 4(d) represent the input curve and disturbances curve, which are relatively smooth due to the introduction of new control methods. Figure 5 shows various
the time-varying fault has a great influence on the control performance of the system, but it is still within the allowable range and better than the existing control methods.

VI. CONCLUSION

For the multi-phase batch processes with uncertainties, this paper puts forward an improved minimax linear quadratic tracking fault-tolerant control. Through comparing with extension information system, the new control method using the extended dynamic model can make the process more stable and efficient operation. The output value track to the set value faster, system response speed is faster and more stable when switching. The error is also smaller, which means higher control precision.

REFERENCES

[1] B. Corbett, B. Macdonald, and P. Mhaskar, “Model predictive quality control of polymethyl methacrylate,” IEEE Trans. Control Syst. Technol., vol. 23, no. 2, pp. 687–692, Mar. 2015.

[2] L. Wang, B. Li, J. Yu, R. Zhang, and F. Gao, “Design of fuzzy iterative learning fault-tolerant control for batch processes with time-varying delays,” Optim. Control Appl. Meth., vol. 39, no. 10, pp. 1–17, Jul. 2018.

[3] R. Zhang and Q. Jin, “Design and implementation of hybrid modeling and PFC for oxygen content regulation in a coke furnace,” IEEE Trans. Ind. Informat., vol. 14, no. 6, pp. 2335–2342, Jun. 2018.

[4] S. Y. Mo, “From one-time dimensional control to two-time dimensional hybrid control in batch processes,” Ph.D. dissertation, Dept. Chem. Biol., Hong Kong Univ. Sci. Technol., Hong Kong, 2013.

[5] L. Wang, J. Yu, R. Zhang, P. Li, and F. Gao, “Iterative learning control for multiphase batch processes with asynchronous switching,” IEEE Trans. Syst., Man, Cybern. Syst., early access, May 24, 2019, doi: 10.1109/TSMC.2019.2916006.

[6] L. Wang, Y. Shen, B. Li, J. Yu, R. Zhang, and F. Gao, “Hybrid iterative learning fault-tolerant guaranteed cost control design for multi-phase batch processes,” Can. J. Chem. Eng., vol. 96, no. 2, pp. 521–530, Feb. 2018.

[7] C.-J. Seo and B. K. Kim, “Robust and reliable H∞ control for linear systems with parameter uncertainty and actuator failure,” Automatica, vol. 32, no. 3, pp. 465–467, Mar. 1996.

[8] S. J. Qin and T. A. Badgwell, “A survey of industrial model predictive control technology,” Control Eng. Pract., vol. 11, no. 7, pp. 733–754, Jul. 2003.

[9] M. Villani, M. Tursini, G. Fabri, and L. Castellini, “High reliability permanent magnet brushless motor drive for aircraft application,” IEEE Trans. Ind. Electron., vol. 59, no. 5, pp. 2073–2081, May 2012.

[10] Y. Jiang, S. Yin, and O. Kaynak, “Data-driven monitoring and safety control of industrial cyber-physical systems: Basics and beyond,” IEEE Access, vol. 6, pp. 47374–47384, 2018.

[11] S. Yin, J. J. Rodriguez-Andina, and Y. Jiang, “Real-time monitoring and control of industrial cyberphysical systems: With integrated plant-wide monitoring and control framework.” IEEE Ind. Electron. Mag., vol. 13, no. 4, pp. 38–47, Dec. 2019, doi: 10.1109/MIE.2019.2938925.

[12] H. Y. Yang and S. Yin, “Descriptor observers design for Markov jump systems with simultaneous sensor and actuator faults,” IEEE Trans. Autom. Control, vol. 61, no. 10, pp. 3045–3051, Oct. 2016.

[13] S. Yin, H. Yang, H. Gao, J. Qiu, and O. Kaynak, “An adaptive NN-based approach for fault-tolerant control of nonlinear time-varying delay systems with unmodeled dynamics,” IEEE Trans. Neural Netw. Learn. Syst., vol. 28, no. 8, pp. 1902–1913, Aug. 2017.

[14] H. Yang, Y. Jiang, and S. Yin, “Adaptive fuzzy fault tolerant control for Markov jump systems with additive and multiplicative actuator faults,” IEEE Trans. Fuzzy Syst., early access, Jan. 13, 2020, doi: 10.1109/TFUZZ.2020.2965884.

[15] Y. Q. Wang, J. Shi, D. H. Zhou, and F. R. Gao, “Iterative learning fault-tolerant control for batch processes,” Ind. Eng. Chem. Res., vol. 45, no. 26, pp. 9050–9560, Nov. 2006.

[16] L. Wang, S. Mo, D. Zhou, F. Gao, and X. Chen, “Robust delay dependent iterative learning fault-tolerant control for batch processes with state delay and actuator failures,” J. Process Control, vol. 22, no. 7, pp. 1273–1286, Aug. 2012.
[17] M. Gao, L. Sheng, D. Zhou, and F. Gao, “Iterative learning fault-tolerant control for networked batch processes with multirate sampling and quantization effects,” Ind. Eng. Chem. Res., vol. 56, no. 9, pp. 2515–2525, Feb. 2017.

[18] L. Wang, L. Sun, and W. Luo, “Robust constrained iterative learning predictive fault-tolerant control of uncertain batch processes,” Sci. China Inf. Sci., vol. 62, no. 11, Apr. 2019. Art. no. 219201.

[19] L. M. Wang, L. M. Sun, J. X. Yu, R. D. Zhang, and F. R. Gao, “Robust iterative learning fault-tolerant control for multiphase batch processes with uncertainties,” Ind. Eng. Chem. Res., vol. 56, no. 36, pp. 10090–10109, Jul. 2017.

[20] L. Wang, B. Liu, J. Yu, P. Li, R. Zhang, and F. Gao, “Delay-range-dependent hybrid iterative learning fault-tolerant guaranteed cost control for multiphase batch processes,” Ind. Eng. Chem. Res., vol. 57, no. 8, pp. 2932–2944, Feb. 2018.

[21] A. Alessandri, M. Gaggero, and F. Tonelli, “Min-max and predictive control for the management of distribution in supply chains,” IEEE Trans. Control Syst. Technol., vol. 19, no. 5, pp. 1075–1089, Sep. 2011.

[22] Y. Gao and K. T. Chong, “The explicit constrained min-max model predictive control of a discrete-time linear system with uncertain disturbances,” IEEE Trans. Autom. Control, vol. 57, no. 9, pp. 2373–2378, Sep. 2012.

[23] K. J. Gruber, D. R. Ramirez, D. Limon, and T. Alamo, “Computationally efficient nonlinear min–max model predictive control based on volterra series models—Application to a pilot plant,” J. Process Control, vol. 23, no. 4, pp. 543–560, Apr. 2013.

[24] L. Wang, C. Zhu, J. Yu, L. Ping, R. Zhang, and F. Gao, “Fuzzy iterative learning control for batch processes with interval time-varying delays,” Ind. Eng. Chem. Res., vol. 56, no. 14, pp. 3993–4001, Apr. 2017.

[25] R. Zhang, S. Wu, Z. Cao, J. Lu, and F. Gao, “A systematic min–max optimization design of constrained model predictive tracking control for industrial processes against uncertainty,” IEEE Trans. Control Syst. Technol., vol. 26, no. 6, pp. 2157–2164, Nov. 2018.

[26] L. Wang, F. Liu, J. Yu, P. Li, R. Zhang, and F. Gao, “Iterative learning fault-tolerant control for injection molding processes against actuator faults,” J. Process Control, vol. 59, pp. 59–72, Nov. 2017.

[27] L. Wang, Y. Shen, J. Yu, P. Li, R. Zhang, and F. Gao, “Robust iterative learning control for multi-phase batch processes: An average dwell-time method with 2D convergence indexes,” Int. J. Syst. Sci., vol. 49, no. 2, pp. 324–343, Jan. 2018.

[28] Y. Wang, D. Zhou, and F. Gao, “Iterative learning model predictive control for multi-phase batch processes,” J. Process Control, vol. 18, no. 6, pp. 543–557, Jul. 2008.

[29] L. Wang, X. He, and D. Zhou, “Average dwell time-based optimal iterative learning control for multi-phase batch processes,” J. Process Control, vol. 40, pp. 1–12, Apr. 2016.

[30] L. M. Wang, R. D. Zhang, and F. R. Gao, Iterative Learning Stabilization and Fault-Tolerant Control for Batch Processes. Singapore: Springer, 2020.

[31] W. Luo, L. Wang, R. Zhang, and F. Gao, “2D switched model-based infinite horizon LQ fault-tolerant tracking control for batch process,” Ind. Eng. Chem. Res., vol. 58, no. 22, pp. 9540–9551, May 2019.

[32] L. Wang, B. Li, R. Zhang, and F. Gao, “Design of a switching control strategy for time-varying delay batch processes using fault probability-based average dwell time method,” Ind. Eng. Chem. Res., vol. 59, no. 11, pp. 5087–5102, Feb. 2020.

[33] Y. C. Chang and B. S. Chen, “A nonlinear adaptive H∞ tracking control design in robotic systems via neural networks,” IEEE Trans. Control Syst. Technol., vol. 5, no. 1, pp. 13–29, Jan. 1997.

[34] K. Back Kim, J.-W. Lee, and W. Hyun Kwon, “Intervalwise receding horizon H$_{\infty}$ tracking control for discrete linear periodic systems,” IEEE Trans. Autom. Control, vol. 45, no. 4, pp. 747–752, Apr. 2000.

[35] D. Zhai, X. Liu, C. Xi, and H. Wang, “Adaptive reliable H$_{\infty}$ control for a class of T-S fuzzy systems with stochastic actuator failures,” IEEE Access, vol. 5, pp. 22750–22759, 2017.

[36] D. Ye, L. Su, J.-L. Wang, and Y.-N. Pan, “Adaptive reliable H$_{\infty}$ optimization control for linear systems with time-varying actuator fault and delays,” IEEE Trans. Syst., Man, Cybern., Syst., vol. 47, no. 7, pp. 1635–1643, Jul. 2017.

[37] Y. Wei, J. Qiu, H.-K. Lam, and L. Wu, “Approaches to T-S fuzzy-affine model-based reliable output feedback control for nonlinear i/o stochastic systems,” IEEE Trans. Fuzzy Syst., vol. 25, no. 3, pp. 569–583, Jun. 2017.

[38] R. Zhang, R. Lu, A. Xue, and F. Gao, “New minmax linear quadratic fault-tolerant tracking control for batch processes,” IEEE Trans. Autom. Control, vol. 61, no. 10, pp. 3045–3051, Oct. 2016.

[39] R. Zhang and F. Gao, “A new synthetic minmax optimization design of H∞ LQ tracking control for industrial processes under partial actuator failure,” IEEE Trans. Rel., vol. 69, no. 1, pp. 322–333, Mar. 2020.

JUN MAO received the Ph.D. degree from Hunan University. He is currently a Lecturer of statistics with Hainan Normal University. His research interests include the warning control model and data analysis.

HUI YI received the B.E. and Ph.D. degrees from the College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, in 2005 and 2012, respectively. In 2012, he joined the College of Electrical Engineering and Control Science, Nanjing Tech University, as a Lecturer, where he is currently an Associate Professor. His research interests include batch process control and fault diagnosis.

CHAO GAO received the bachelor’s degree from the College of Mathematics and Applied Mathematics, Changchun Normal University. She is currently working as a Math Teacher with Hangzhou Yulan High School. Her current research interests include batch process control and fault-tolerant control.

RENYOU ZHANG received the B.Sc. degree in safety engineering from the Beijing University of Chemical Technology, China, in 2012, the M.Sc. degree in process safety and loss prevention from The University of Sheffield, U.K., in 2014, and the Ph.D. degree in safety and reliability engineering from the University of Aberdeen, U.K., in 2018. He is currently a Postdoctoral Research Associate with the Department of Industrial Engineering, Tsinghua University, Beijing, China. His major research interests include industrial safety and reliability modeling and analysis.

FENG JIANG received the master’s degree in sports psychology from the Nanjing Sport Institute. He is currently pursuing the Ph.D. degree in logic with Nanjing University. His research interests include cognitive science, decision analysis, and artificial intelligence.