Split extended supersymmetry

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Abstract

We show how splitting supersymmetry reconciles a class of intersecting brane models with unification. The gauge sector in these models arises in multiplets of extended supersymmetry while matter states are in $N=1$ representations. A deformation of the angles between the branes gives large masses to squarks and sleptons, as well as supersymmetry breaking contributions to other string states. The latter generate at one-loop heavy Dirac masses for Winos and gluinos and can induce a mass term for the Higgsino doublets. We find that this scenario is compatible with gauge coupling unification at high scale for both cases where the gauge sector is $N=2$ and $N=4$ supersymmetric. Moreover a neutralino, combination of neutral Higgsinos and Binos, is a natural candidate for dark matter.

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INTRODUCTION The necessity of a Dark Matter (DM) candidate and the fact that LEP data favor the unification of the three Standard Model (SM) gauge couplings are smoking guns for the presence of new physics at high energies. The latter can take the form of supersymmetry which, if broken at low energies, offers a framework for solving the gauge hierarchy problem. Supersymmetry is also welcome as it naturally arises in string theory, which provides a framework for incorporating the gravitational interaction in our quantum picture of the universe. Recently strong evidence has been accumulated for the presence of a tiny dark energy in the universe. This raises another hierarchy problem which is not solved by any known symmetry. It leads to reconsider our notion of naturalness, possibly also affecting our view of mass hierarchy. It was then proposed to consider that supersymmetry might be broken at high energies without solving the gauge hierarchy problem. More precisely, making squarks and sleptons heavy does not spoil unification and the existence of a DM candidate while at the same time it gets rid of all unwanted features of the supersymmetric SM related to its complicated scalar sector. On the other hand, experimental hints to the existence of supersymmetry persist since there are still gauginos and Higgsinos at the electroweak (EW) scale. This is the so-called split supersymmetry framework \cite{1}. Implementing this idea in string theory is straightforward \cite{2}. However one faces a generic problem: in simple brane constructions the gauge sector comes in multiplets of extended supersymmetry \cite{3,4}. In this work we show that these economical string-inspired brane models allow for unification of gauge couplings at scales safe from proton decay problems. Moreover they provide us with a natural DM candidate.

BRANE MODELS The simplest supersymmetric brane models are obtained as compactification on a six-dimensional torus which is a product of three factorized tori. The states in these models can be assembled into three sets. The first set is made of strings with both ends on the same stack of branes, leading to $N = 4$ vector multiplets. In the following we will also consider a departure from this minimal case, when two chiral adjoint $N = 1$ multiplets are projected out to remain with $N = 2$ vector supermultiplets$^1$. The second set contains strings which stretch between two stacks of branes that intersect only in two out of the three internal tori, giving rise to $N = 2$ hypermultiplets. The last set contains strings which are localized at intersection points of branes in all the three tori and the associated states form $N = 1$ supermultiplets. In this setup of supersymmetric limit, the SM states are identified as follows:

- Gauge bosons emerge as massless modes of open strings with both ends on the same stack of coincident branes. They arise in $N = 2$ or $N = 4$ supermultiplets which are decomposed, for each gauge group factor $G_a$, into one $N = 1$ vector superfield $W_a$ and one or three chiral adjoint superfields $A_a$, respectively.

- Quarks and leptons are identified with massless modes of open strings localized at point-like brane intersections and belong to $N = 1$ chiral multiplets.

- Pairs of Higgs doublets originate from $N = 2$ supersymmetry preserving intersections.

$^1$Note that there are also brane constructions with no extended supersymmetry in the gauge sector based on non-toroidal compactifications.
They are localized in two tori where branes intersect, while they propagate freely in the third torus where the two brane stacks are parallel. Here we assume that all possible additional non-chiral states that may appear in generic string constructions can be made superheavy.

Supersymmetry breaking is then achieved by deforming brane intersections with a small angle \( \Theta \). As a result, a \( D \)-term with \( \langle D \rangle = \Theta M_S^2 \) appears, associated to a corresponding magnetized \( U(1) \) factor with superfield strength \( W \). Here, \( M_S \) is the string scale. Supersymmetry is then broken and soft masses are induced:

- A tree-level mass \( m_0 \propto \sqrt{\Theta} M_S \) for squarks and sleptons localized at the deformed intersections. All other scalars acquire in general high masses of order \( m_0 \) by one loop radiative corrections. Appropriate fine-tuning is needed in the Higgs sector to keep light \( n_H \) doublets.

- A Dirac mass \([5]\) is induced through the dimension-five operator

\[
\frac{a}{M_S} \int d^2 \theta W W^a A_a \Rightarrow m_{1/2}^D \sim a \frac{m_0^2}{M_S},
\]

where \( a \) accounts for a possible loop factor. Actually, this operator arises quite generally at one-loop level in intersecting D-brane models with a coupling that depends only on the massless (topological) sector of the theory \([4]\). Note that this mass does not break \( R \)-symmetry and provides an answer to a problem of split supersymmetry related to the mechanism of generating gaugino masses.

**UNIFICATION**

We now study the compatibility of this framework with one-loop unification. In the energy regime between the unification scale \( M_{GUT} \) and the EW scale \( M_W \), the renormalization group equations meet three thresholds. From \( M_{GUT} \) to the common scalar mass \( m_0 \) all charged states contribute. Below \( m_0 \) squarks and sleptons (which do not affect unification), adjoint scalars and \( 2-n_H \) Higgses decouple, while below \( m_{1/2}^D \) the \( N = 2 \) or \( N = 4 \) gluinos and Winos drop out. Finally, at TeV energies Higgsinos (and maybe the Binos) decouple and we are left at low energies with the Standard Model with \( n_H \) Higgs doublets. For the purpose of this computation we use \( M_S \sim M_{GUT} \) and we vary \( a \) between \( a = 1 \) and \( a = 1/100 \). Realistic values for \( M_{GUT} \) and \( m_0 \) are obtained in both \( N = 4 \) and \( N = 2 \) cases. The results are summarized in Table 1.

Notice that we have imposed perfect unification at one-loop. Although this is not necessary from the string theory point of view, it is one of the main motivations of split supersymmetry and can be imposed along the lines of \([2]\). In this case, for \( N = 4 \) there is no solution with \( n_H = 2 \) Higgses at low energies, whereas for \( N = 2 \) we find a solution with either \( n_H = 1 \) or \( n_H = 2 \). In all cases the unification scale is high enough to avoid problems with proton decay. For the two possible cases with one light Higgs (\( N = 2 \) or \( N = 4 \)), \( M_{GUT} \) is very close to the Planck scale so that there should be no need to explain the usual mismatch between these
two scales. Varying the loop factor $a$ from 1 to $1/100$ amounts to an increase by one order of magnitude in the value of $m_0$, but $M_{\text{GUT}}$ and $m_{1/2}^D$ remain stable within $\mathcal{O}(1)$ factors.

The low energy sector of these models contains, besides the SM, just some fermion doublets (Higgsinos) and eventually two singlets (the Binos from the discussion below). It therefore illustrates the fact that only these states are needed for a minimal extension of the SM consistent with unification and DM candidates, and not the full fermion spectrum of split supersymmetry. A similar observation was recently done in the literature [6], without the presence of Dirac gauginos associated to the intermediate scale $m_{1/2}^D$. Here, we do not find these solutions because they do not unify at one-loop.

**DARK MATTER** Another constraint on the models is that they must provide a DM candidate. As usually in supersymmetric theories this should be the lightest neutralino. Pure Higgsinos $(\tilde{H}_1, \tilde{H}_2)^T$ cannot be DM candidates because their mass is of Dirac type: $-\mu \tilde{H}_1 \tilde{H}_2 + h.c.$ Quasi-Dirac Higgsinos would interact inelastically with matter via vector-like couplings as $i(\tilde{H}_- \sigma_\mu \tilde{H}_+ - \tilde{H}_+ \sigma_\mu \tilde{H}_-) \equiv (\tilde{H}_- \sigma_{\mu \nu} \tilde{H}_+) \delta_{\mu \nu}$ where $\tilde{H}_\pm \sim \tilde{H}_1 \pm i \tilde{H}_2$ are the mass eigenstates with mass eigenvalues $\mu \pm \epsilon$ respectively. In Dark Matter direct detection experiments $\tilde{H}_-$ can only scatter inelastically off of a nucleus of mass $m_N$ by transitioning to $\tilde{H}_+$ if $\epsilon < \epsilon_0 \simeq \frac{1}{2} \beta^2 m_N$ [7]. For Ge experiments with $m_N = 73$ GeV and taking a typical escape Dark Matter particle velocity $\beta c \simeq 600$ km/s one obtains $\epsilon_0 \simeq 146$ keV. Since direct detection experiments have ruled out Dirac fermions up to masses of order 50 TeV some mixing coming from the Binos is required in order to break the degeneracies of the two lightest neutralinos and provide a mass difference $\epsilon > \epsilon_0$ preventing inelastic scattering off the nucleus. The mass difference $\epsilon > \epsilon_0$ translates into an upper bound on the Dirac gaugino mass of about $10^5$ GeV, for the required Higgsino mass splitting to be generated through the EW symmetry breaking mixing (of order $m_{1/2}^D/m_{1/2}^D$) described below. This value compared to the values in Table [1] leads to the $N = 2$, $n_H = 1$ case as the only possibility to accommodate it.

In fact one could have an order of magnitude suppression of the induced Dirac mass for Binos

| $N$ | $n_H$ | $a$ | $M_{\text{GUT}}$ | $m_0$ | $m_{1/2}^D$ |
|-----|------|-----|----------------|-------|------------|
| 2   | 1    | 1   | $2.8 \times 10^{18}$ | $4.5 \times 10^{12}$ | $7.2 \times 10^6$ |
|     | 1    | 1/100 | $3.8 \times 10^{18}$ | $3.2 \times 10^{13}$ | $2.7 \times 10^6$ |
|     | 2    | 1    | $4.5 \times 10^{16}$ | $1.1 \times 10^{13}$ | $2.7 \times 10^9$ |
|     | 2    | 1/100 | $4.5 \times 10^{16}$ | $8.6 \times 10^{13}$ | $1.6 \times 10^9$ |
| 4   | 1    | 1    | $9.7 \times 10^{18}$ | $8.5 \times 10^{16}$ | $7.4 \times 10^{12}$ |
|     | 1    | 1/100 | $10^{19}$ | $6.8 \times 10^{16}$ | $3.4 \times 10^{12}$ |

Table 1: Values for the unification scale $M_{\text{GUT}}$, scalar masses $m_0$ and Dirac gaugino masses $m_{1/2}^D$ in GeV for $N = 2, 4$ supersymmetric gauge sector, $n_H = 1, 2$ light Higgses, and varying the loop factor $a$. 

\[2\text{In this case and to destroy the Dirac nature of mass eigenstates the Higgs sector should be arranged (as quarks and leptons do) in $N = 1$ multiplets.}\]
relative to the other gauginos, which is not unreasonable to assume in brane constructions.

In the other two models, the required suppression factor is much higher and the above mechanism would be very unnatural. However, since Binos play no rôle for unification as they carry no SM charge, we could imagine a scenario where \( m_{1/2}^D \) vanishes identically for Binos, but not for the other gauginos. For instance consider the case where Dirac masses from the operator \( \frac{b^2}{M_3^2} \), generated by loop diagrams involving \( N = 2 \) hypermultiplets with supersymmetric masses of order \( M_{GUT} \) and a supersymmetry breaking splitting of order \( \Theta \). It is then possible to choose these massive states such that they carry no hypercharge. In that case Binos can only have Majorana masses. Such masses are in general induced by a dimension-seven effective operator, generated at two-loop level [8]:

\[
\frac{b^2}{M_3^2} \int d^2 \theta W^2 \text{Tr} W^2 \Rightarrow m_{1/2}^M \sim \frac{b^2 m_0^4}{M_3^2},
\]

where \( b \) is another loop factor. Putting numbers in the above formula, we get for the \( N = 4 \ n_H = 1 \) model \( m_{1/2}^M \sim 5 \times 10^6 \) GeV, which is the upper bound for DM with Majorana Bino mass \(^3\). On the other hand, for the \( N = 2 \ n_H = 2 \) model, we get \( m_{1/2}^M \sim 100 \) GeV which is obviously acceptable. Finally, for the \( N = 2 \ n_H = 1 \) case, \( m_{1/2}^M \sim 10 \) keV which does not play any rôle if there is also a Dirac mass, as we assume and discussed above. Thus, the constraint of a viable DM candidate leaves us with two possibilities: (a) \( N = 2 \) with \( n_H = 1 \) and Dirac masses for all gauginos and (b) \( N = 4 \) with \( n_H = 1 \), or \( N = 2 \) with \( n_H = 2 \) and Majorana mass Binos.

**Higgsino Masses** The Higgsinos themselves must acquire a mass of order the EW scale. This is induced by the following dimension-seven operator, generated at one-loop level [8, 4]:

\[
\frac{c}{M_3^2} \int d^2 \theta W^2 \overline{D}^2 \bar{H}_1 \bar{H}_2 \Rightarrow \mu \sim \frac{c m_0^4}{M_3^2},
\]

where \( c \) is again a loop factor. The resulting numerical value is of the same order as \( m_{1/2}^M \) of Eq. (2). Thus, such an operator can only give a sensible value of \( \mu \) for the \( N = 2 \ n_H = 2 \) model. In the other two cases, \( N = 4 \) or \( N = 2 \) with \( n_H = 1 \), \( \mu \) remains an independent parameter.

Another constraint on these models may come from the life-time of the extra states. We first consider the case of \( N = 2 \). Scalars can decay into gauginos, Dirac gluinos decay through squark loops sufficiently fast and Dirac Winos and Binos decay into Higgses and Higgsinos. On the other hand, for \( n_H = 2 \), there are two Majorana Binos at low energies. However, as mentioned above, in the models we consider the two Higgs doublets form an \( N = 2 \) hypermultiplet. It follows that there is an \( N = 2 \) coupling between both Binos with Higgses and Higgsinos, implying that the only stable particle is the usual lightest sparticle (LSP). Finally, in the \( N = 4 \)

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\(^3\)This higher value for the upper bound on Majorana (\( \sim 5 \times 10^6 \) GeV) versus Dirac Bino mass (\( \sim 10^5 \) GeV) we used before is due to the fact that the induced Higgsino mass splitting through EW symmetry breaking mixing is further suppressed by the weak angle in the Dirac case.
model, scalars still decay into gauginos, but we have now two Dirac gluinos, Winos and Binos; half of them decay as before, either through scalar loops or into Higgs-Higgsinos, while the other half can only decay through string massive states. Their lifetime is then estimated by $\tau \sim (M_S/10^{13} \text{GeV})^4 (10^2 \text{GeV} / m_{\tilde{g}})^3 \tau_U$, where $m_{\tilde{g}}$ is the gaugino mass and $\tau_U$ is the lifetime of the universe. For gluinos and Winos there is no problem, but Binos are very long lived although still safe, with a life-time of order $\tau_U/10$.

To summarize, at low energies we end up with two distinct scenarios after all massive particles are decoupled: i) $n_H = 1$ with light Higgsinos (models with $N = 2$ and $N = 4$ gauge sector and $n_H = 1$), and; ii) $n_H = 2$ with light Higgsinos and Binos (model with $N = 2$ gauge sector and $n_H = 2$). In the $n_H = 1$ scenario the DM candidate is mainly Higgsino, although the much heavier Bino is light enough to forbid any vector couplings. The relic density can be estimated (using for example the DarkSUSY program [9]) and reproduces the actual WMAP results for $\mu \sim 1.1$ TeV.

The $n_H = 2$ scenario is more interesting since there are more particles at low energies. The $N = 2$ coupling between Binos-Higgs-Higgsinos leads, after EW symmetry breaking to the neutralino mass matrix,

$$
\begin{pmatrix}
M & 0 & m_z s_w c_\beta & m_z s_w s_\beta \\
0 & M & -m_z s_w s_\beta & m_z s_w c_\beta \\
m_z s_w c_\beta & -m_z s_w s_\beta & 0 & -\mu \\
m_z s_w s_\beta & m_z s_w c_\beta & -\mu & 0
\end{pmatrix}
$$

in the basis $(\tilde{B}_1, \tilde{B}_2, \tilde{H}_1, \tilde{H}_2)$ and where $M = m_{1/2}^M$ stands for the Bino Majorana masses. The mass matrix can be diagonalized to obtain:

$$m_\chi = 1/2 \left[ (M + \epsilon_1 \mu) - \epsilon_2 \sqrt{(M - \epsilon_1 \mu)^2 + 4m_z^2 s^2_w} \right]$$

where the four different mass eigenvalues are labeled by $\epsilon_{1,2} = \pm 1$.

The values of $\mu$ and $M$ can then be chosen in such a way as to reproduce the WMAP observed DM density, as previously studied [10]. There are then three cases:

- $M \ll \mu$: the Bino does not interact strongly enough to annihilate and will in general overclose the universe.
- $M \gg \mu$: this model converges to the $n_H = 1$ scenario and WMAP results require $\mu \sim 1.1$ TeV.
- $M \sim \mu$: the lightest neutralino ($\chi$) is in general a mixture of Higgsinos and Binos and is a natural candidate for DM. Low values of $\mu$ are now possible.

**COLLIDER PHENOMENOLOGY** In the $n_H = 1$ scenario with $\mu \sim 1.1$ TeV, the heavy spectrum will be hardly observable at LHC while a Linear Collider with center of mass energy of around 2.5 TeV will be needed to detect a possible signature.
In the $n_H = 2$ scenario, the main collider signature is through the production of charginos. Their mass is given by $m_{\chi^\pm} = \mu + \delta \mu$, where $\delta \mu$ is due to electromagnetic contributions and is of order 300 to 400 MeV. The produced charginos will decay into the neutralino, mainly through emission of a virtual $W^\pm$ which gives rise to lepton pairs or pions depending on its energy. This decay is governed by the mass difference $\Delta m_{\chi} = m_{\chi^\pm} - m_{\chi^0}$, which is a function of the two parameters $M$ and $\mu$. Because charginos are produced through EW processes, LHC will mainly be able to explore the case of very light charginos, which exist only in the limited area of the parameter space with $M \sim \mu$. Unlike in low energy supersymmetry, the absence of cascade decays in this case will make it difficult to separate the signal from similar events produced by Standard Model $W^\pm$ production processes.

The search at future $e^+e^-$ colliders is more promising, and can be discussed either as a function of the model parameters $(M, \mu)$, or as a function of the low energy observables $(m_{\chi^\pm}, \Delta m_{\chi})$. For most of the $(M, \mu)$ parameter range, $\Delta m_{\chi}$ is small, at most of order a few GeV. Because the value of $\delta \mu$ is not small enough to make the chargino long-lived as to produce visible tracks in the vertex detectors, we have to rely on its decay products. This degeneracy implies that the produced leptons or pions are very soft and it would typically be difficult to disentangle them from the background due to emission of photons from the beam. The strategy is then to look for $e^+e^- \to \gamma + E_T$. A proper cut on the transverse momentum of the photon allows to eliminate the background of missing energy due to emission of $e^+e^-$ pairs along the beam, as the conservation of transverse momentum implies now a simultaneous detection of electrons or positrons $^{[11]}$. The best possible scenario is when $M$ and $\mu$ are of the same order since, as soon as $M$ starts to be greater than $\mu$, the Binos quickly decouple and this model converges to the $n_H = 1$ scenario with $\mu \sim 1.1$ TeV. The Higgs sector of these models will also give a signal at LHC. The case with just one Higgs at low energies predicts a mass $\lesssim 160$ GeV$^{[10]}$ depending on the exact value of $m_0$. In the case of two Higgses, one of them will have this bound whereas the mass of the other doublet is controlled by $m_A$ as in the MSSM. The result is that one Higgs will always be discovered at LHC whereas the others will depend on the value of $m_A$, which is a free parameter. In any case it will be impossible to distinguish these models from another type of split supersymmetry or from a general two Higgs model just at LHC. A careful measurement of Higgs coupling will be needed to disentangle between the different models that could be achieved at the ILC.

**CONCLUSION** Before closing, we would like to make a few comments concerning the number of parameters and fine-tuning issues. Dirac gaugino masses, and consequently scalar masses related through Eq. (1), are fixed by the one-loop unification condition according to Table I. In the two $n_H = 1$ models (with $N = 4$ or $N = 2$ gauge sector) the Higgsino mass $\mu$ is fixed by the DM constraint to $\sim 1.1$ TeV. Since the supersymmetry breaking scale $m_0$ is high, a fine-tuning is needed like in split supersymmetry to keep one Higgs scalar light. On the other hand, in the $n_H = 2$ model with $N = 2$, $\mu$ is determined to the right scale by the effective operator (3) and the required Majorana component of Bino mass by (2). Obviously in this case one needs more fine-tuning conditions in order to keep both eigenvalues of the Higgs mass-
squared matrix light. One may wonder that for a two-by-two symmetric matrix, this implies three conditions on all its elements. However, it is easy to see that only two conditions are really needed. The reason is that the off-diagonal element is protected by two global low energy symmetries, namely Peccei-Quinn and $R$-invariance. As a result, if its tree-level value vanishes, quantum contributions will be proportional to $\mu M_{1/2}^M \sim \mathcal{O}(\text{TeV})^2$.

In summary, we presented three viable models of high scale supersymmetry that naturally emerge in simple string constructions with intersecting branes and are compatible with gauge coupling unification and the existence of a dark matter candidate. Gauginos come in multiplets of extended supersymmetry and get high scale Dirac masses by dimension-five effective operators without breaking $R$-symmetry, consistently with gauge coupling unification. The low energy sector contains besides the Standard Model particle content a Higgsino pair providing, in general through mixing with the Bino, a natural Dark Matter candidate.

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