Light emission by free electrons in photonic time-crystals

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Abstract: We study emission of electrons moving in photonic time-crystals, and find exponentially enhanced Cherenkov emission in the momentum gap drawing energy from the modulation, and suppressed emission in the opposite direction due to avoided crossing. © 2021 The Author(s).

A breakthrough in our understanding of the free-electron interactions with dielectric media was the 1934 discovery of the Vavilov-Cherenkov radiation[1]. Three years later, Frank & Tamm[2] explained the radiation as the outcome of electrons moving faster than the speed of light in the material, creating an electromagnetic (EM) shockwave. This breakthrough led to the 1958 Nobel Prize in Physics and initiated a field of electron-light interactions in (or near) materials. Since its discovery, a vast amount of theory and experiments had been developed, including quantum analysis and inverse Cherenkov effects[3], with applications for X-ray sources, particle accelerators, and detectors[4].

A significant development in the research of electron-light interactions is the discovery of the Smith-Purcell effect[5], where a medium with a spatially periodic permittivity exchanges momentum with the moving electrons to produce radiation without having to surpass the Cherenkov velocity threshold. The fundamental importance of this idea was the usage of the spatial periodicity of the material for generation of light from electrons. Our work proposes to instead use the temporal periodicity of materials for a novel mechanism of light emission.

Here, we present a quantum electrodynamic (QED) study of the radiation emission by free electrons moving in a Photonic Time-Crystal (PTC): a medium in which the refractive index is strongly modulated periodically in time, thus inducing a gap in the momentum[6]. We find that the electrons emit exponentially amplified radiation for wavevectors k residing in the momentum bandgap of the PTC. Moreover, we find that the radiation is emitted below and above the Cherenkov threshold, and displays a phase transition when crossing the Cherenkov threshold. The QED analysis of the system with strong electron-photon coupling reveals a new effect where the emission to the momentum-gap of the PTC and the PTC bands experience avoided-crossing, prohibiting spontaneous emission at the crossing point. The process of free-electron emission in PTCs, and especially the exponential amplification driven by the modulated refractive index, offers a plethora of new phenomena and suggest novel applications such as X-ray lasers created by periodically-driven materials and novel tunable particle detectors.

Consider a free charged particle, an electron in this case, moving with velocity v = cβz in a PTC. A PTC is a dielectric crystal whose permittivity, here it is ε(t) = εr + εs cos(Ωt), varies significantly on time scales comparable to the cycle of the EM field interacting with it (few-fs in the optical range), thereby inducing a gap in the momentum. A schematic diagram of this is shown in Fig. 1(a), where the electron moves in a PTC and radiates. To describe the evolution of the system, we find the Hamiltonian:

\[ H_{\text{en}} = H_{\text{EM}}(t) + H_r + H_f = \sum_{\sigma} \frac{\hbar}{2m} \left( \frac{1 + \epsilon(t)}{\epsilon(t)} \right) a_{\sigma}^\dagger a_{\sigma} + \frac{1}{4} \left( 1 - \frac{\epsilon(t)}{\epsilon(t)} \right) \left( a_{\sigma}^\dagger a_{\sigma}^\dagger a_{\sigma} a_{\sigma} + a_{\sigma} a_{\sigma}^\dagger a_{\sigma}^\dagger a_{\sigma} \right) + \frac{\mathbf{P}^2}{2m} - \frac{e}{m} \mathbf{P} \cdot \hat{A} \]  

Fig. 1. Free-electron radiation in a photonic time-crystal (PTC). (a) Scheme of an electron with velocity \( \beta \mathbf{v} \) in a PTC with \( \epsilon(t) = \epsilon_r + \epsilon_s \cos(\Omega t) \). The electron emits photons related to energy exchange with the PTC (pink), in addition to regular Cherenkov radiation (purple), and the spontaneous momentum-gap radiation (orange). (b) k-space diagram of the three emission processes: Cherenkov radiation (purple), momentum-gap radiation (orange), and electron-PTC radiation (pink). (c) The average number of photons emitted by the electron for the wavevector k marked with a blue circle (point A) in (b). We compare the emission of the electron with a PTC (blue) and without it (red), also plotting their difference (yellow). The addition to Cherenkov emission in the PTC is exponentially growing (fit of 2.27e-4RMSE, dashed black) when compared to the case with a blue circle (point A) in (b). We compare the emission of the electron with a PTC (blue) and without it (red), also plotting their difference (orange), and electron-PTC radiation (pink). (c) The average number of photons emitted by the electron for the wavevector k marked with a blue circle (point A) in (b). We compare the emission of the electron with a PTC (blue) and without it (red), also plotting their difference (yellow). The addition to Cherenkov emission in the PTC is exponentially growing (fit of 2.27e-4RMSE, dashed black) when compared to the case with a blue circle (point A) in (b). We compare the emission of the electron with a PTC (blue) and without it (red), also plotting their difference (orange), and electron-PTC radiation (pink).
This expression consists of the Hamiltonians for the EM field $H_{EM}$, the electron energy $H_a$, and the interaction term $H_i$, where $n_\epsilon$ is the ambient refractive index, $\mathbf{k}$ and $\sigma$ are the wavenumber and polarization, $\epsilon(\mathbf{r})$ is the modulated permittivity, $a_{\mathbf{k}\sigma}^{\dagger}, a_{\mathbf{k}\sigma}$ are the creation and annihilation operators of a photon with $\mathbf{k}$ and $\sigma$, $\mathbf{P}$ is the momentum operator and $\mathbf{A}$ is the vector potential operator. In a time-modulated medium without an electron (only $H_{EM}$ is non-zero), we can arrange the states in independent ladders of pairs of photons with $\mathbf{k}$ and $-\mathbf{k}$ wavevector (e.g. $|0_{\mathbf{k}}, 0_{-\mathbf{k}}\rangle$, $|1_{\mathbf{k}}, 1_{-\mathbf{k}}\rangle$, $|2_{\mathbf{k}}, 2_{-\mathbf{k}}\rangle$, etc.). The right term of $H_{EM}$ couples these states by adding (or subtracting) exactly one pair of $\mathbf{k}$ and $-\mathbf{k}$ photons, thus not changing the overall momentum. Eventually, the number of photons with $\mathbf{k}$ values in the PTC k-gap increases (Fig. 1(c)-orange), even if initially there were no photons of the same kind in the medium, i.e., these photons can appear spontaneously from vacuum fluctuations [7] – a pure QED effect.

To study the emission of an electron in such a medium, we add the two terms, $H_0$ and $H_1$. We find that a novel type of radiation emerges, which does not resemble Cherenkov radiation. This radiation is caused by the exchange of energy quanta $\hbar\omega$ between the electron and the PTC (Fig. 1(a)-pink). We confirm our prediction by classical EM FDTD simulations [8]. In addition to this new type of radiation, we find that when the speed of the electron crosses the speed of light in the medium, a phase transition in the radiation occurs: the electron-PTC radiation becomes superluminal, and the electron starts emitting the ordinary Cherenkov (shockwave) radiation at angle $\theta_{cs} = \cos^{-1}(1/\beta n_\epsilon(k))$ (Fig.1(b)-purple).

We calculate the Cherenkov emission rate in the PTC momentum-gap (Fig. 1(c)) and find that it increases exponentially (yellow) compared to the emission of photons without the PTC (red). The exponential growth is shown by fitting the difference in emission to an exponential function (dotted black). This finding stands in sharp contrast to suppressed (prohibited) emission of photons in the bandgap of spatial photonic crystals [9]. We believe this is a general feature of emission into a time-varying medium: the emission rate is always higher in the momentum gap created by the time modulation (which gives rise to the PTC).

Furthermore, we find that in the same superluminal regime, at the angle opposite to the "conventional" Cherenkov angle, the electron states emitting a photon through the electron-PTC interaction become degenerate with pairs of photons that are spontaneously created by vacuum fluctuations. This momentum and energy degeneracy is lifted by the interaction term and causes avoided crossing between the two [Fig 2(a),(b)]. At the crossing point that occurs at the k-gap, the electron suppresses the spontaneous k-gap emission (Fig. 2(c)), but triggers increased emission at two different wavevectors, $k_{\text{gap}} \pm \Delta k$, where $\Delta k$ depends on the electron-photon interaction (Fig. 2(d)).

Altogether, free-electron emission in a PTC can occur in experimentally-accessible systems and can be measured, for example, in electron microscopes using electron energy loss spectroscopy [10]. Remarkably, epsilon-near-zero materials with strong nonlinear effects were recently developed [11] and shown to possess very fast (fs) and strong (order of ~1) modulation of the permittivity in response to an ultrafast pulse. A train of such pulses will result in time-periodic permittivity – creating a PTC. This scheme can pave the way to new physics, such as radiation by relativistic dipoles that was shown to create new effects such as the superlight inverse Doppler effect. Looking forward, these ideas could lead to X-ray lasers created from periodically-driven materials and novel tunable particle detectors.

![Fig. 2. Avoided crossing of electron-PTC radiation and momentum-gap (k-gap) spontaneous emission.](image)

(a) Calculated emission rates (via the QED formalism) for a region (marked with B) in k-space presented in Fig. 1(b). The radiation for k-values inside the gap is marked with orange and the electron-PTC radiation is marked with pink. We notice an avoided crossing of electron-PTC emission and k-gap emission. (b) Emission probability vs. time for the cross-section marked in (a). We notice suppression of radiation exactly at the k-vectors of the gap and increased probability for a different wavevector, $k_{\text{gap}} + \Delta k$. (c) The number of photons for wavevectors $k$, $N_{k,\mathbf{w}}$ (blue), and $-k$, $N_{-k,\mathbf{w}}$ (red), vs. time at the k-gap of the PTC. The Cherenkov emission in the PTC (yellow) corresponds to the difference $N_{k,\text{w}} - N_{-k,\text{w}}$ and to the number of photons with wavevector $k$ (or $-k$) in a PTC without an electron (dashed black). The spontaneous k-gap emission (dashed black) is suppressed by the presence of the moving electron (yellow). (d) The same as (c) but for the wavevector $k_{\text{gap}} + \Delta k$. The presence of the electron increases the photon emission for $k_{\text{gap}} + \Delta k$.

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