Scalar Perturbations of Galileon Cosmologies in the Mechanical Approach within the Late Universe

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Abstract—We investigate the Universe at the late stage of its evolution and inside the cell of uniformity, 150–370 Mpc. We consider the Universe to be filled at these scales with dustlike matter, a minimally coupled Galileon field and radiation. We use the mechanical approach. Therefore, the peculiar velocities of the inhomogeneities as well as the fluctuations of other perfect fluids can be considered nonrelativistic. Such fluids are said to be coupled because they are concentrated around the inhomogeneities. We investigate the conditions under which the physical Galileon field, i.e., the field compatible with the results of the latest gravitational wave experiments GW150914, GW151226, GW170104, GW170814, GW170817 and GW170608, can become coupled. We know that at the background level coupled scalar fields behave as two-component perfect fluids: one which mimics a network of frustrated cosmic strings and another one which corresponds to an effective cosmological constant. We found as well a correction for the Galileon field, which behaves like a matter component.

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1. INTRODUCTION

The $\Lambda$CDM model is consistent with observational data, but the energy scale of dark energy (DE) is too low [1, 2]. Therefore, this cosmological constant is not compatible with the cosmological constant originated from the vacuum energy in quantum field theory. Already many models have been proposed to explain the present accelerated expansion of the Universe. If the history of particle physics is any guide, then one can assume that DE is due to a new field. Within this setup, the most popular ones are models based on scalar fields [3].

DE models based on a minimally coupled scalar field are named quintessence [4]. In this case, the equation of state (EoS) parameter $w \in (-1, 0)$. It is as well possible to have phantom scalar fields where $w < -1$. Finally, quintom models are based on scalar fields where there is a crossing of the value $w = -1$. In addition, a dynamical EoS parameter, $w$, can help in solving the coincidence problem.

Another possibility to describe the late-time acceleration of the Universe is to modify the law of gravity from higher-dimensional models that realize the cosmic acceleration through gravitational leakage to extra dimensions. The DGP (Dvali–Gabadadze–Poratti) model belongs to this class.

Mostly inspired by the DGP model, the authors of [8] derived five Lagrangians that lead to field equations invariant under the Galileon symmetry $\partial_\mu \phi \rightarrow \partial_\mu \phi + b_\mu$ (the vectorial parameter $b_\mu$ corresponds to a constant shift) in Minkowski space-time:

\[
L_1 = M^3 \phi, \quad L_2 = (\nabla \phi)^2, \quad L_3 = (\Box \phi)(\nabla \phi)^2 / M^3, \quad L_4 = (\nabla \phi)^2 \left[ 2(\nabla \phi)^2 - 2 \phi_{,\mu \nu} \phi^{\mu \nu} \right. \\
\left. - \frac{1}{2} R(\nabla \phi)^2 \right] / M^6, \quad L_5 = \left( \nabla \phi \right)^2 \left( \left[ (\nabla \phi)^3 - 3 \Box \phi \phi_{,\mu \nu} \phi^{\mu \nu} \right. \\
\left. + 2 \phi_{,\mu} \phi^{\mu \nu} \phi_{,\nu} \phi^{\nu} \right. \\
\left. - 6 \phi_{,\mu} \phi^{\mu \nu} \phi^{,\nu} G_{\nu \rho} \right] / M^9, \quad M^3 = M_p H_0^2.
\]

The scalar field $\phi$ that respects the Galileon symmetry is named the Galileon field [9]. Each of the five terms give origin to a second-order differential equation of motion, which keeps the theory free from unstable spin-2 ghost degrees of freedom. If we carry

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1 Modified gravity models as a mean to explain the late-time acceleration of the universe need to be constructed to recover GR behavior in the regions of high density for the consistency with local gravity experiments. There are two possible ways to do it: the chameleon mechanism and the Vainshtein mechanism [7, 10, 11].

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out the analysis in curved space-time, we need to change the Lagrangians to their covariant form\(^2\).

It is of great importance to suggest a mechanism which can verify the viability of Galileon models. The theory of cosmological perturbations is a powerful tool to investigate cosmological models [13]. In fact, we will study the Universe at late stages of its evolution and deep inside the cell of uniformity. In this setup, there are discrete inhomogeneities in this cell, because galaxies, groups and clusters of galaxies were already formed and can be considered as discrete sources for the gravitational potential. It was shown in previous works that in this case the mechanical approach [14–16] is a powerful tool for studying scalar perturbations. It enables us to get the gravitational potential and to describe the motion of galaxies.

The hydrodynamic approach is a good tool for investigating the growth of structure of the early universe. It works well in the linear approximation. However, it became inapplicable in the strongly non-linear regime. It starts for \( z \) of few dozens. However, on much bigger scales, matter becomes on the average homogeneous and isotropic, with matter in the form of a perfect fluid. It is important to determine theoretically at which scales we should consider the transition from a highly inhomogeneous mechanical distribution to the smooth hydrodynamic one.

The Universe is filled with inhomogeneously distributed discrete structures at the scale of 150–370 MPc. The mechanical approach enables us to obtain the gravitational potential for an arbitrary number of randomly distributed inhomogeneities. We can investigate the relative motion of galaxies and the formation of the Hubble flow with the expression for the gravitational potential. The mechanical approach works well for the ΛCDM model, where the peculiar velocities of the inhomogeneities could be considered to be negligibly small when we compare them with the speed of light. Additionally, we consider scales deep inside the cell of uniformity. Consequently, we can drop the peculiar velocities in the first order of approximation. Such models were also generalized to the case of cosmologies with different perfect fluids, which can play the role of DE and dark matter (DM) [17–21].

The fluctuations of these additional perfect fluids also form their own inhomogeneities. It is supposed in the mechanical approach that the velocities of displacement of such inhomogeneities are of the order of peculiar velocities of the inhomogeneities corresponding to dustlike matter. These types of inhomogeneities are coupled to each other in the sense mentioned in [22]. This means that for the considered models, we investigate the possible existence of such coupled fluids. They can as well play the role of DM, as shown in [21].

In the present paper, we consider a cosmological model with a Galileon field minimally coupled to gravity. The Universe is also filled with dustlike matter and radiation. We study the theory of scalar perturbations [23] for such models and obtain a condition under which the inhomogeneities of the dustlike matter and those of the scalar field can be coupled to each other. We show that this condition imposes rather strong restrictions on the scalar field itself. The coupled scalar field behaves at the background level as a three-component perfect fluid: a cosmological constant, a term which mimics a network of frustrated cosmic strings and a further component which behaves as matter. Though the gravitational wave experiments constrain the Galileon cosmologies, here we provide a further tool based on the mechanical approach to further constrain these theories.

The work is structured as follows: we present the action which we consider in this article, in Section 2; we study the mechanical approach in the next Section 3, and we present the results in the final Section 4.

2. SUMMARY OF THE GALILEON MODELS

Given that nongravitational coupling between the Galileon field and gravity is physically allowed, we will simply consider the following self-interaction term for the Galileon field:

\[
S_I = \alpha \int_M \sqrt{|g|} \Box \phi \partial_\mu \phi \partial^\mu \phi \, d^4x,
\]

where \( \alpha \) is a small parameter, which measures the deviation from the model of a minimally coupled scalar field and has units of volume. We intend to demonstrate the typical behavior of Galileon models using this Lagrangian within the mechanical approach. First of all, we will obtain the energy-momentum tensor for this Lagrangian as

\[
T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_I}{\delta g^{\mu\nu}},
\]

where \( \delta S/\delta g^{\mu\nu} \) is the variational derivative of the action. We then get [24]

\[
T_{\mu\nu} = \alpha 2 \partial_\mu \phi \partial_\nu \phi \Box \phi - 2 \alpha \phi_{,\mu} (\partial_\rho \phi \partial^\rho \phi)_{,\nu} + \alpha g_{\mu\nu} \phi \cdot (\partial_\rho \phi \partial^\rho \phi)^\lambda.
\]

\(^2\)Some of the terms \( L_\alpha \) are already disfavored by the results of the experiment GW170817 [12], but the term which we will consider in our work is fully viable.
We would like to use the following perturbed metric in our computations according to [22]:

\[
d s^2 = a^2 \left[ (1 + 2\Phi) d\eta^2 - (1 - 2\Psi)\gamma_{ij} dx^i dx^j \right].
\]

(2.3)

3. THE MECHANICAL APPROACH

Now we can write the energy-momentum tensor for whole action, where we also include a minimally coupled scalar field:

\[
S = \int_M \sqrt{|g|} \left( \frac{1}{2} \partial_{\alpha} \phi \partial^{\alpha} \phi - V(\phi) + \alpha \Box \phi \partial_{\mu} \phi \partial^{\mu} \phi \right) d^3 x.
\]

The characteristic term in Galileon cosmologies is the term \( \Box \phi \), so we need to compute \( \Box \phi \) up to the first order in the perturbative setup. We use the notation \( \phi = \phi_c + \varphi \), where \( \phi_c \) is the background term and \( \varphi \) the perturbed quantity (all derivatives will be with respect to the conformal time):

\[
\Box \phi = \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial \phi}{\partial x} \frac{\partial^2 \varphi}{\partial x^2} + \left[ -\frac{2}{a^2} \frac{\partial^2 \phi}{\partial \varphi^2} + \frac{\varphi''}{a^2} \gamma \right] + \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial x^2}.
\]

(3.1)

where we have defined \( \Delta_{ij} \varphi = \varphi_{ij} \gamma^{ij} \) and \( -\varphi_{ij} \Gamma^k_{ij} \gamma^{ij} = \Box \varphi - \Delta_{ij} \varphi. \) So we can finally compute \( T_{00} \) for \( S_f \):

\[
\tilde{T}^0_0 + \delta T^0_0 = 2\alpha \frac{1}{a^2} a^2 \left( \frac{\varphi''}{a^2} + \frac{\gamma}{a^2} \partial_{\alpha} \phi_c a^2 - \frac{\partial^2 \phi_c}{\partial x^2} \right) - 2\alpha \frac{1}{a^4} \left( \frac{\varphi''}{a^2} - \frac{\partial \phi_c}{\partial x} \frac{\partial^2 \phi_c}{\partial x^2} \right) - \frac{\varphi''}{a^2} - \frac{\gamma}{a^2} \partial_{\alpha} \phi_c a^2 - \frac{\partial^2 \phi_c}{\partial x^2}.
\]

(3.2)

And we must find the variation of the energy momentum tensor:

\[
\delta T^0_0 = 2\alpha \frac{(\phi'_c)^2}{a^2} \left[ -\frac{2}{a^2} \frac{\partial^2 \phi_c}{\partial x^2} + \frac{\varphi''}{a^2} - \frac{\partial \phi_c}{\partial x} \frac{\partial^2 \phi_c}{\partial x^2} + \frac{\gamma}{a^2} \left( -\phi_c' \psi' - 2\Phi \phi'_c - \frac{\varphi''}{a^2} - \frac{\gamma}{a^2} \partial_{\alpha} \phi_c a^2 \right) - \frac{\partial \phi_c'}{\partial x} \frac{\partial^2 \phi_c}{\partial x^2} \right] + 4\alpha \frac{(\phi'_c)^2}{a^2} \frac{\partial \phi_c'}{\partial x} \frac{\partial^2 \phi_c}{\partial x^2}.
\]

(3.3)

\[
\delta T^i_j = \alpha g^i_j \frac{\phi'_c}{a^6} \left[ 4\phi'_c \varphi'' a^2 + 2\varphi'' \phi'_c a^2 - 6\phi_c \varphi' a a' - 8\Phi \phi'_c a^2 \phi'' - 8\Phi \phi'_c a^2 \phi'' - 2\phi'_c a^2 \varphi' \right];
\]
\[ \delta T^0_0 = 2\alpha g^{00}\phi'_c,\phi,\phi_0 \partial_\phi - \alpha g^{00}(\phi_c,\phi,\alpha g^\rho_\phi,\phi_0)_{,i} - \alpha \phi_i g^{00}(\phi_c,\phi,\phi,\alpha g^\rho_\phi)_{,0} \]
\[ = \frac{2}{a^2}\alpha \phi'_c,\varphi, - \frac{\gamma}{a^2}\phi' a' - \frac{\alpha'}{a^3} \left( \frac{\phi'_c}{a^2} - \frac{\gamma}{a^2} \phi' a' - \frac{\alpha'}{a^3} \phi'_c \right) \]
\[ - \alpha g^{00}(\phi_c,\phi,\phi_0 g^\rho_\phi + \phi,\phi_0 g^\rho_\phi) = \frac{2}{a^2}\alpha \phi'_c,\varphi, \left( \frac{\phi'_c}{a^2} - \frac{\gamma}{a^2} \phi' a' - \frac{\alpha'}{a^3} \phi'_c \right) \]
\[ - 2\alpha \left( \frac{\phi'_c}{a^4} \varphi_{,ii} + 2\alpha (\phi'_c)^3 + 2\alpha \varphi_{,i} \right) \left[ - \phi_c,00 + (\phi_c,0)^2 a' a' \right] , \quad (3.3) \]

where \( g^\rho_\phi \) is a derivative of the contravariant metric with respect to conformal time.

So, the perturbed part of the Einstein equations reads [33]:

\[ \Delta \Phi - 3H(\Phi' + H\Phi) + 3K\Phi = \frac{\kappa}{2}a^2(\delta \epsilon_{\text{dust}} + \delta \epsilon_{\text{rad}}) + \frac{\kappa}{2} \left[ - (\phi'_c)^2 \Phi + \phi'_c \Phi' + a^2 \frac{dV}{d\phi}(\phi_c) \Phi \right. \]
\[ \left. + 2\alpha \gamma \frac{\phi'_c}{a^2}(\phi'_c,\Phi' - 3\phi' a') - 2\alpha \frac{\phi'_c}{a^2} \varphi \right] , \quad (3.4) \]

\[ \partial_t \Phi' + H\partial_i \Phi = \frac{\kappa}{2} \left[ \phi' \partial_i \varphi + 2\alpha \phi'_c \partial_i \partial' \phi \left( \frac{\phi'_c}{a^2} - \frac{\gamma}{a^2} \phi' a' - \frac{\alpha'}{a^3} \phi'_c \right) - 2\alpha \frac{\phi'_c}{a^2} \varphi,0i \right] \]
\[ + \frac{2\alpha}{a^2} \left( \phi'_c \right)^3 + 2\alpha \frac{\varphi_{,i} \Phi}{a^2} \left[ \phi_{c,00} (\phi_c,0) + (\phi_c,0)^2 a' a' \right] , \quad (3.5) \]

\[ \frac{2}{a^2} \left\{ \Phi'' + 3H \Phi' + \Phi(2\frac{a''}{a} - H^2 - K) \right\} = \kappa \left[ \delta \rho_{\text{rad}} - \frac{(\phi'_c)^2 \Phi}{a^2} + \phi'_c \Phi' + a^2 \frac{dV}{d\phi}(\phi_c) \Phi \right. \]
\[ \left. - \frac{\alpha \phi'_c}{a^6} \left[ 4\phi'_c a^2 + 4\phi'' a^2 - 6\phi'' a' a' - 8\phi'_c a^2 \phi'_c + 8\Phi(\phi'_c)^2 a' a' - 2(\phi'_c)^2 a^2 \Phi' \right] \right\} . \quad (3.6) \]

We obtain the field equation for \( \phi \) by variation of the action, and the result reads

\[ -2\alpha(\partial_\phi)^2 + 2\alpha \nabla^\mu \nabla_\mu \nabla_\nu \nabla_\nu \phi + 2\alpha \nabla^\mu \phi \nabla_\nu \phi R_{\mu \nu} - \partial_\phi - \frac{dV}{d\phi} = 0. \quad (3.7) \]

After plugging in the definition of the energy–momentum tensor, we have

\[ \bar{\epsilon}_\varphi = -8\alpha \left( \frac{\phi'_c}{a^2} \right)^3 + 2\alpha \frac{\phi''}{a^4} + \frac{1}{2a^2} \left( \phi'_c \right)^2 + V(\phi_c) , \quad (3.8) \]

\[ \bar{p}_\varphi = -6\alpha \frac{\phi'_c}{a^4} + 60 \frac{a'}{a^5} \left( \phi'_c \right)^3 + \frac{1}{2a^2} \left( \phi'_c \right)^2 - V(\phi_c) , \quad (3.9) \]

where \( \bar{\epsilon}_\varphi \) and \( \bar{p}_\varphi \) are the energy density of the Galileon field at the background level.

Consequently, the Friedmann and Raychaudhuri equations read

\[ H^2 = \frac{\kappa a^2}{3} \left[ \epsilon_{\text{dust}} + \epsilon_{\text{rad}} + \frac{1}{2a^2} \left( \phi'_c \right)^2 + V(\phi_c) - \frac{8a' \left( \phi'_c \right)^3}{a^5} + 2\alpha \left( \phi'_c \right)^2 a^4 \right] - K ; \quad (3.10) \]

\[ H' = -\frac{\kappa a^2}{6} \left[ \epsilon_{\text{dust}} + 16 \frac{\phi'_c}{a^4} + 10a' \frac{a'}{a^5} \left( \phi'_c \right)^3 + 2 \frac{\left( \phi'_c \right)^2}{a^2} - 2V(\phi_c) \right] . \quad (3.11) \]

We continue now in two steps: we compute the derivative of \( \Phi' + H\Phi \), and we plug the second Einstein equation (3.5) to the third Einstein equation (3.6):

\[ \Phi' + H\Phi = \frac{\kappa}{2} \left\{ \phi'_c \varphi + 2\alpha \phi'_c \varphi' \left( - \frac{\gamma}{a^2} \phi' a' - \frac{\alpha'}{a^3} \phi'_c \right) - \frac{2}{a^2} \alpha \left( \phi'_c \right)^2 a' \varphi' \right. \]
\[ \left. + 2\alpha \frac{\phi}{a^2} \frac{\phi'_c}{a^4} \left( \phi'_c \right)^3 \right\} , \quad (3.12) \]

\[ \Phi'' + H'\Phi + H\Phi' = \frac{\kappa}{2a^4} \left\{ \phi''_c a^4 \varphi + \phi'_c a^4 \varphi' - 12\alpha \phi''_c \phi'_c a a' \varphi \right. \]
So, after plugging to the third Einstein equation (3.6), we have

\[
\delta \varepsilon_{\text{rad}} = \frac{-1}{3a^6} \left[ -36a^6 \phi'' \phi' + 18aH' \phi' \phi'' + 18aH \phi' \phi'' + 12a^6 \phi'' \phi' \phi' + 18 \phi'' a'' \phi' + 18a^6 \phi'' \phi' a - 18 \phi'' a'' \phi' - 30a^6 \phi' \phi'' a' a + 36a^6 \phi' \phi'' a' a \phi' + 3a^6 \delta \varepsilon_{\text{dust}} + 4a^6 \delta \varepsilon_{\text{rad}} \right].
\]

We can plug this expression into the first Einstein equation (3.4):

\[
\frac{2}{\kappa} \Delta \Phi + \frac{6 \kappa}{\kappa} \phi'' \phi - a \left[ \frac{\partial \phi}{\partial a} + 3 \frac{\partial \Phi}{\partial a} \right] - 36aH \phi' \phi'' \phi' a^2 + 6a^2 \phi'' \phi' a + 6 \kappa^2 \phi' \phi'' \phi + (\phi' \phi'' \phi a^2)
\]

\[
+ 3a^2 \delta \varepsilon_{\text{dust}} + 4a^2 \delta \varepsilon_{\text{rad}} - \left[ 3H \phi' + \frac{2 a^2}{a^4} \phi' \phi'' + 36 \phi'' a'' \phi' + 18a^6 \phi'' \phi' a - 18 \phi'' a'' \phi' - 30a^6 \phi' \phi'' a' a + 36a^6 \phi' \phi'' a' a \phi' + 3a^6 \delta \varepsilon_{\text{dust}} + 4a^6 \delta \varepsilon_{\text{rad}} \right].
\]

4. RESULTS

Similarly to [22], we suppose that \( \Omega = \Omega(r) \) (where \( \Phi = \Omega/r \), and \( \Omega \) is a function of \( a \) and spatial coordinates), which means that \( \phi' = \text{const} \):

\[
\phi_c(\eta) = \beta \eta + \omega,
\]

where \( \omega \) and \( \beta \) are constants. Then we get from the EoM (3.7) that

\[
6a \beta^2 a'' + 2a' \beta + \frac{dV}{d\phi} a^5 = 0.
\]

Now,

\[
V(\eta) = \frac{\beta^2}{a^2} + V_{\infty} + \alpha f(a),
\]
and the goal is to obtain the dependence \( f(\eta) \) in this previous relation, when we know the dependence \( V(a) = \frac{a^2}{\sigma^2} + V_{\infty} \) for the pure scalar field [22]. So we take the equation
\[
6\alpha^3 \beta^3 \frac{a''}{a} + 2a' \beta^2 + a^2 V' = 0
\]
calculate the derivate of the previous equation (4.1) with respect to \( \eta \) and obtain
\[
V' a^2 = -2\beta^2 \frac{a'}{a} + \alpha \frac{df}{da} a^2. \tag{4.2}
\]
Now we combine these two equations:
\[
6\alpha^3 \beta^3 \frac{a''}{a} + \alpha \frac{df}{da} a^4 = 0. \tag{4.3}
\]
We use that \( a''/a = H' + H^2 \) and substitute \( H \) and \( H' \) from (3.10) and (3.11). We finally obtain
\[
2\alpha^3 \beta \left[ \frac{\dot{\rho}_c}{2a^3} + \frac{3\beta^2}{2a^2} + 2V_{\infty} + 2\alpha f(a) - 13\alpha \frac{\alpha' \beta^3}{a^5} \right] + \alpha \frac{df}{da} a^2 = 0. \tag{4.4}
\]
We should solve this equation for \( f(a) \), and we use the Taylor expansion. We must find the solution of the equation (3.10) for \( H \):
\[
0 = H^2 + \frac{8\kappa \alpha^3}{3a^2} - \frac{\alpha^2}{3} \left[ \epsilon_{\text{dust}} + \epsilon_{\text{rad}} + \frac{3\beta^2}{a^2} + \alpha f(a) + V_{\infty} \right]. \tag{4.5}
\]
It is a quadratic equation. We ignore the negative solution as a nonphysical one, and we obtain for the positive one the following expression:
\[
H(\alpha) = -\frac{4\alpha^3 \beta \kappa}{3a^2} + \left[ \frac{16}{9} \left( \frac{\alpha^2 \beta^3}{a^2} \right)^2 + \frac{\alpha^2}{3} \left( \epsilon_{\text{dust}} + \epsilon_{\text{rad}} + \frac{3\beta^2}{a^2} + \alpha f(a) + V_{\infty} \right) \right]^{1/2}. \tag{4.6}
\]
Expanding this expression in \( \alpha \), we get:
\[
H(\alpha) \bigg|_{\alpha=0} = \sqrt{\frac{\kappa a^2}{3}} \left( \epsilon_{\text{dust}} + \epsilon_{\text{rad}} + \frac{3\beta^2}{a^2} + V_{\infty} \right) + O(\alpha^2). \tag{4.7}
\]
We obtain for Eq. (4.4) in the first-order approximation
\[
0 = 2\alpha^3 \beta \left[ \frac{\dot{\rho}_c}{2a^3} + \frac{3\beta^2}{2a^2} + 2V_{\infty} \right] + \alpha \frac{df(a)}{da}
\]
and see after some simplifications that
\[
\frac{df(a)}{da} = \frac{-2\beta^3 \kappa}{a^5} \left[ \frac{\dot{\rho}_c}{2a^3} + \frac{3\beta^2}{2a^2} + 2V_{\infty}a^3 \right] \left[ H_0^2 \Omega_r + \frac{\kappa \rho_c}{3} - \frac{\alpha f(a)}{2} \right]^{-1/2}. \tag{4.8}
\]
So \( f(a) \) behaves like matter:
\[
f(a) \sim 1/a^3. \tag{4.9}
\]

5. CONCLUSION

We have been studying Galileon cosmologies and used the mechanical approach because we worked deeply inside the cell of uniformity, 150–370 Mpc, and in the late universe. We filled the Universe with a minimally coupled Galileon field and also dustlike matter in the form of discrete distributed galaxies and groups of galaxies. We further included radiation.

All types of inhomogeneities have nonrelativistic velocities. Different types of inhomogeneities do not run away massively during the universe evolution in this case. Fluctuations of the energy density of such perfect fluids are usually concentrated around the inhomogeneities of dustlike matter. Therefore we call those perfect fluids coupled [21]. They can even screen the gravitational potential of the galaxies [17] or they can play a role in dark matter flattening the rotation curves of dwarf galaxies [18]. In the present work, we have investigated the possibility for a Galileon scalar field to be coupled with the galaxies in the late universe. For such Galileon scalar fields to exist, we have shown that they have to meet certain conditions. At the background level, such a scalar field behaves as a three-component perfect fluid: a network, which mimics the behavior of frustrated cosmic strings with the EoS parameter \( w = -1/3 \), a cosmological constant and some matter component, (4.10). This is the main result of our work.

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