Causality and Localization Operators

F. Buscemi, G. Compagno *

INFM, and Dipartimento di Scienze fisiche ed astronomiche dell’Università di Palermo, Via Archirafi 36, 90123 Palermo, Italy

Abstract

The evolution of the expectation values of one and two points scalar field operators and of positive localization operators generated by an instantaneous point source is non local. Non locality is attributed either to zero point vacuum fluctuation, or to non local operations or to the microcausality principle being not satisfied.

Key words: Localization; Causality; Hegerfeldt’s theorem
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1 Introduction

The problem of non locality in quantum mechanics originates from the studies of the propagation effects from time varying sources of electromagnetic (e.m.) field [1,2]. For atomic sources, it has been shown that, some quantities as correlations of excitation between two atoms or of the electromagnetic field are non zero at two spacetime points with spacelike separation [3,4,5]. Instead local quantities, as the excitation of a second atom [6,7], appear to depend causally on the source. Recently causal behavior of local e.m. field operator has been obtained while in the correlation functions is present a non local source independent part, that may be attributed to the zero point field fluctuations [8,9].

Other aspects of non locality are present in the free evolution of an initially localized field configuration [10,11,12]. In this case it appears that, as a consequence of Hegerfeldt’s theorem, the wavefunctions and the average values of some positive local observables differ from zero outside the light cone of the initially localized region [10,11]. Non local terms appear also in the scalar

* Corresponding author

Email address: compagno@fisica.unipa.it (G. Compagno).
product between the state of the evolving system and the eigenstates of local positive operators like the Newton-Wigner’s or Glauber’s [12,13,14]. Here we shall adopt a model previously used by Maiani and Testa to treat the problem of causality in Q.F.T [15]. It consists of a time dependent and localized scalar source linearly interacting with a scalar field. The use of this model avoids any problem relative to the definition of the particle localization due to the traverse character of the e.m field. [16]. Moreover to keep the problem simple and to avoid the question linked to the effective localization of a quantum source [15,17] we shall take the source classical. We take the source localized in an arbitrary small region of space and turned on and then off after an arbitrarily small interval of time. While this model can in principle be treated exactly, for our purposes we shall limit ourselves to second order perturbation theory in the source field coupling constant. We shall then calculate the time evolution of the state of the system and of the expectation values of one, two points and localization operators.

2 The model

In our model a classical scalar source is linearly coupled to a scalar field \( \Phi(x) \). The source is assumed to be localized in an arbitrarily small spacetime region around the spacetime point \( y \equiv (y, y_0) \) so that it is effective for an arbitrarily small time around \( y_0 \). The Hamiltonian that describes our system is then:

\[
H = H_0 + H_{int}
\]

with

\[
H_0 = \frac{1}{2} \int \left( \frac{d^3k}{2\omega} \right) \omega \left( a^\dagger(k)a(k) + a(k)a^\dagger(k) \right)
\]

and

\[
H_{int} = g \int_{-\infty}^{+\infty} d^3x \Phi(x) \delta^4(x - y)
\]

where \( g \) is the field-source coupling constant and \( a(k) \) and \( a^\dagger(k) \) are the usual annihilation and creation operators satisfying the relativistic commutator rules [20]:

\[
[a(k), a^\dagger(k')] = 2\omega \delta^3(k - k').
\]

The field \( \Phi(x) \) may be expanded in terms of the operators \( a(k) \) and \( a^\dagger(k) \) in the standard way. Before the source is turned on, the field is assumed to be in its ground state \( |0\rangle \). In the following we shall use the interaction picture. The state at time \( t \) \( |\Psi\rangle \) will then be given by:

\[
|\Psi\rangle = U(t)|0\rangle
\]

where \( U(t) \) is the interaction picture time evolution operator. \( U(t) \) can be easily obtained by integrating the interaction picture equation of motion arising
from Hamiltonian in eq. (1) and it may be shown to have the form:

\[ U(t) = \exp \left( -i g \Theta(t - y_0) \hat{\Phi}(y) \right) \]  

(6)

with \( \Theta(t) \) is the step function. Under the hypothesis of weak coupling \( (g \ll 1) \), we shall expand the evolution operator given by eq.(6) up to second order in \( g \). Substituting it in eq.(5), the explicit form of the state up to second order then becomes:

\[ |\Psi\rangle = |\Psi(0)\rangle + |\Psi(1)\rangle + |\Psi(2)\rangle \]  

(7)

with

\[ |\Psi(0)\rangle = |0\rangle \]

\[ |\Psi(1)\rangle = \int d^3k \alpha(k) a^\dagger(k)|0\rangle \]

\[ |\Psi(2)\rangle = \frac{1}{2} \left[ - \int d^3k \alpha(k) \alpha^*(k)|0\rangle + \int d^3k \int d^3k' \alpha(k) \alpha(k') a^\dagger(k) a^\dagger(k')|0\rangle \right] \]  

(8)

and where

\[ \alpha(k) = -\frac{ig \Theta(t - y_0)}{(2\pi)^{3/2}} \frac{1}{(2\omega)^{1/2}} e^{ik \cdot y}. \]  

(9)

The integrals present in eq.(8) are regularized by the introduction of a cut-off \( \lambda \). This effectively constraints \( k \) to \( |k| \leq \lambda \) and whenever an explicit dependence on \( \lambda \) is present in the matrix elements, we shall eventually consider the limit \( \lambda \rightarrow \infty \). The state, in the second quantized form given in eq.(7), shall be the basis for the calculations of the expectation values we are interested in.

Recently, for the e.m. field generated by an atomic source, it has been examined the possibility of measuring the arrival time of a single particle generated by the source [18]. Therefore, as the next step, we shall extract from the second quantized state \( |\Psi\rangle \) of eq.(7), the first quantization wavefunction \( \Psi(x) \) describing a field quantum. This can be accomplished by projecting the one quantum component of the state \( |\Psi\rangle \), expressed in momentum space, on the one quantum space state \( |x\rangle = a(x)|0\rangle \) [19]. From eqs.(7) and (8) we obtain:

\[ \Psi(x) = \langle 0|a(x)|\Psi\rangle = \langle x| \Psi(1)\rangle = g \Theta(t - y_0) \Delta_+(x - y, t - y_0) \]  

(10)

where \( \Delta_+ \) is the positive frequency propagator \( \Delta \) function, that can be expressed by

\[ [\hat{\Phi}_+(x), \hat{\Phi}_-(y)] = i \Delta_+(x - y) \]  

(11)

with \( \hat{\Phi}_+(\hat{\Phi}_-) \) is the positive (negative) frequency part of the field operator \( \hat{\Phi} \). The explicit form of \( \Delta_+ \) [20] shows that it is not zero for \( x - y \) spacelike. Thus the single particle component of the state generated by the pointlike instantaneous source, develops in a nonlocal way. This result, although surprising,
appears to be in agreement with Hegerfeldt’s theorem and with previous results about the non causal evolution of the first quantization wavefunction [21,22]. We shall comment this result in the following.

3 Expectation values one point field operators

We shall now examine the expectation values of one point operators functions of the scalar field. As first, lets consider directly the expectation value of the scalar field operator \( \hat{\Phi}(x) \) on the state, generated by our instantaneous pointlike source, \(|\Psi\rangle\). Using eq.(8) we get:

\[
\langle \Psi | \hat{\Phi}(x) | \Psi \rangle = g \Delta_{\text{ret}}(x - y)
\]

where \( \Delta_{\text{ret}}(x) \) is the retarded causal propagator function [20]:

\[
\Delta_{\text{ret}}(x) = \Theta(x_0) \Delta(x)
\]

that can be expressed in terms of \( \Delta(x) \), that in terms of commutator of the field is [20]:

\[
[\hat{\Phi}(x), \hat{\Phi}(y)] = i \Delta(x - y)
\]

As because \( \Delta \) is zero for spacelike argument, \( \Delta_{\text{ret}} \) is therefore retarded and zero outside the lightcone centered on the source at spacetime point \( y \). Then the evolution of the expectation value of the field, generated by the instantaneous pointlike source on the vacuum, clearly shows a causal behavior. By using instead the one point field intensity operator \( \hat{\Phi}^2(x) \), we get:

\[
\langle \Psi | \hat{\Phi}^2(x) | \Psi \rangle = \langle 0 | \hat{\Phi}^2(x) | 0 \rangle + g^2 \Delta^2_{\text{ret}}(x - y).
\]

This expectation value is the sum of two terms. The first (where the regularization of the integrals should be be exploited) is independent from the field-source coupling constant. It derives from the zero-point field fluctuations that are always present and in fact is non zero everywhere on the whole space-time. The second term is source dependent and causally retarded. In it there are contributions due all the terms, up to second order, of the state \(|\Psi\rangle\). Thus, in order to examine the causal effects linked to the variations of the source, it is physically obvious from eq.(15) that the vacuum contribution should be subtracted from the total expression in agreement with previous results [6,7,8]. As last we shall consider as one point operator function the field energy density operator:

\[
\mathcal{H}(x) = \frac{1}{2} \left( |\nabla \hat{\Phi}(x)|^2 + \hat{\Phi}(x)^2 + m^2 \hat{\Phi}^2(x) \right).
\]

Proceeding as before, we get for its expectation value on the state \(|\Psi\rangle\) given by eq.(8) and up to the order of \( g^2 \):
\[
\langle \Psi | H(x) | \Psi \rangle = \langle 0 | H(x) | 0 \rangle + \frac{1}{2} g^2 \left( \left( \nabla \Delta_{\text{ret}}(x - y) \right)^2 + \left( \partial_t \Delta_{\text{ret}}(x - y) \right)^2 \right) + m^2 \Delta_{\text{ret}}^2(x - y).
\]
\]

In (17) the energy density expectation value can be again separated in two parts. The first represents the vacuum contribution to energy density. The second, source dependent term, is expressed in terms of powers of \( \Delta_{\text{ret}} \) and of its derivatives. Therefore it is retarded and zero outside the lightcone centered on the source. Thus the field energy density term coming from the source propagates causally.

We may also show that the two point field correlations function does share the same behavior. In fact taking the average of the product \( \hat{\Phi}(x) \hat{\Phi}(x') \) on the state \( |\Psi\rangle \), we obtain:

\[
\langle \Psi | \hat{\Phi}(x) \hat{\Phi}(x') | \Psi \rangle = \langle 0 | \hat{\Phi}(x) \hat{\Phi}(x') | 0 \rangle + g^2 \Delta_{\text{ret}}(x - y) \Delta_{\text{ret}}(x' - y)
\]

with the first term on the right side of (18) representing the field vacuum correlations. It now depends on the separation \( x - x' \) and is moreover not zero at spacelike separation. This is just a property of zero point correlations. For example it is well known that at equal time \( x_0 = x'_0 \) the space dependence of scalar field correlations in vacuum at large distances go as \( 1/r^2 \) [20]. Also the second, source dependent, term is not zero for spacelike intervals \( x - x' \).

However from its structure in eq.(18) one sees that it consists of a product of terms, containing either \( x \) or \( x' \) each causally connected to the source at \( y \). The source dependent correlations in \( x \) and \( x' \) are non zero at spacelike distances, and are expression of the fact that the field at each of these points is correlated, in a causal retarded way, to the source at \( y \).

Non local effects have also been shown to arise during the free evolution of an initially localized field configuration [12,13]. Recently the case has been considered where the state, describing the field generated by a localized source, is subjected to the action of non unitary operator that truncates some of its parts. After proper renormalization of the state, the expectation value of the field intensity operator has been calculated on it showing a non local behavior [13].

Here we shall analogously consider the average value of the one point operator \( \hat{\Phi}^2(x) \) on the state \( |\Upsilon\rangle \) generated from \( |\Psi\rangle \) of eq.(8), by the action of the number operator \( \hat{N} = \int \frac{d^3k}{2\omega} a^\dagger(k) a(k) \). The action of \( \hat{N} \) on \( |\Psi\rangle \) eliminates its zero point part. We obtain, except to a normalization factor:

\[
\langle \Upsilon | \hat{\Phi}^2(x) | \Upsilon \rangle \propto g^2 \Theta^2(t - y_0) |\Delta_{+}(x - y)|^2.
\]

It appears that to the expectation value of eq.(19) contributes only one term. It depends on the source and it does not appear to be causal, at variance with the source dependent part of the expectation value of the same operator.
given by eq.(15). Although we shall not report here the explicit form, we must however observe the expression of \( \hat{N} \) in terms of the field \( \Phi \) is non local. So, as a matter of the fact, the action of \( \hat{N} \) on the state \( |\Psi\rangle \) is a non local operation that is equivalent to the action of a non local source. This is expected to induce changes over a spacelike region. As a consequence, the expectation values of one point operators on the state \( |\Upsilon\rangle \) may develop non locally in time. This is clearly expressed by the appearance in eq. (19) of the propagator function \( \Delta_+ \) instead of \( \Delta_{\text{ret}} \). It is of interest to observe that in eq. (19) only the first order part of the state \( |\Upsilon\rangle \) contributes. Therefore in order to study possible non local effects it appears to be safer also when one considers the free evolution, to use states generated by the action of unitary evolution operators, representing well localized source, as the one expressed by eq.(6)

4 Expectation values of localization operators

The study of non locality and its connection with causality has often been conducted by analyzing the behavior of local operators. For example some forms of the Hegerfeldt’s theorem refer to operators which are both local and positive [10,11]. The existence of operators satisfying both these requirements has however been questioned [23]. In particular the one point operators \( \hat{\Phi}(x)^2 e^{\mathcal{H}(x)} \), that we have previously used, are indeed also positive. The non local parts, appearing in their expectation values, are source independent and attributable to the vacuum fluctuations. If we want to analyze the local proprieties of the system, we must keep only the physical relevant part and are compelled to subtract the zero point contributions. This is equivalent to take the operators in their normal ordered form. Under this form they do not however satisfy anymore the positivity condition. Their use in the context of Hegerfeldt’s theorem appears so to be unappropriate. In order to define the concepts of localization in quantum mechanics positive operators have previously been used. Among them second quantized form of the Newton-Wigner(NW) operator position \( \hat{\rho}_{\text{NW}}(x) \)\cite{12}, whose first quantized form was initially introduced to define single particle localization \cite{24}, and the Glauber operator \( \hat{\rho}_G(x) \) \cite{25}, initially used in quantum optics in the context of local photon detection. Both operators satisfy the requirement of positivity.

We shall now calculate their expectation values on the state \( |\Psi\rangle \) of eq.(7). The NW operator has the form [12]:

\[
\hat{\rho}_{\text{NW}}(x) = a_{\text{NW}}^\dagger(x) a_{\text{NW}}(x)
\]

(20)

where
Its average value on the state $|\Psi\rangle$, up to second order in the constant coupling $g$, is:

$$
\langle \Psi | \hat{\rho}_{NW}(x) | \Psi \rangle = g^2 \Theta^2 (t - y_0) \left| \frac{1}{(2\pi)^{3/2}} \int \frac{d^3 k}{(2\omega)^{1/2}} e^{i (\mathbf{k} \cdot (\mathbf{x} - \mathbf{y}) - \omega (t - y_0))} \right|^2
$$

$$
= g^2 \Theta^2 (t - y_0) \left| \psi_{\text{NW}, y_0}^\ast (x, t) \psi_{\text{NW}, y_0} (x, t) \right|^2
$$

(22)

where $\psi_{\text{NW}, y_0}^\ast (x, t)$ is the NW first quantized relativistic wavefunction for the positive frequency state localized at $\mathbf{y}$ at time $t = y_0$ [24]. The expectation value of $\hat{\rho}_{NW}(x)$ on the state generated by our pointlike instantaneous source, is therefore proportional to the square modulus of the NW wavefunction. Using the stationary phase method, the asymptotic expression of the NW wavefunction can be shown to be for $|T^2 - r^2| \geq 1$, where $T = (t - y_0)$ and $r = |\mathbf{x} - \mathbf{y}|$:

$$
\psi_{\text{NW}, y_0} (x, t) \propto \begin{cases} 
m \sqrt{T} (T^2 - r^2)^{1/2} e^{-i m \sqrt{T^2 - r^2}}, & T^2 - r^2 > 0 \\
n \sqrt{T} (r^2 - T^2)^{1/2} e^{-i m \sqrt{r^2 - T^2}}, & T^2 - r^2 < 0. 
\end{cases}
$$

(23)

$\psi_{\text{NW}, y_0}$ is non zero for spacelike intervals, thus non local effects show up in the evolution of (22). The Glauber operator for a scalar field is

$$
\hat{\rho}_G (x) = \hat{\Phi}^\ast (x) \hat{\Phi} (x).
$$

(24)

Again its expectation value on the state $|\Psi\rangle$ is:

$$
\langle \Psi | \hat{\Phi} (x) \hat{\Phi} (x) | \Psi \rangle = g^2 \Theta^2 (t - y_0) \Delta_+ (x - y) \Delta_+ (x - y)
$$

(25)

where $\Delta_+ (x) = \Delta_+^\ast (x)$. Again from the proprieties of $\Delta_- (\Delta_+)$ we see that the expectation value (25) is also non zero outside the lightcone centered at $y$. It is clear that the appearance of non local effects in $\langle \hat{\rho}_{NW}(x) \rangle$ and $\langle \hat{\rho}_G (x) \rangle$ on the state $|\Psi\rangle$, cannot be attributed to the presence of vacuum fluctuations. In fact the expectation values of these observables on the vacuum state $|0\rangle$ is zero. The results of eqs. (22) and (25) thus may seem to show evidence of non local effects generated by the source, while our previous results of eqs.(12), (15) e (17), obtained using field operators, suggest the contrary. However in studying non locality, one should use operators that do not introduce by their same definition non local effects. In particular it is a standard requirement
in relativistic quantum field theory [26] that any local operators $\hat{O}(x)$ should satisfy the principle of microcausality. That is $\hat{O}(x)$ must satisfy:

$$[\hat{O}(x), \hat{O}(y)] = 0$$

(26)

for spacelike $(x - y)$ intervals. Now using $\hat{\rho}_{NW}(x)$ and $\hat{\rho}_{G}(x)$ in place of $\hat{O}(x)$ in eq.(26), we obtain:

$$\left[\hat{\rho}_{NW}(x), \hat{\rho}_{NW}(y)\right] = -2\left\{ \left( \partial_1 \Delta_+(x-y) \right) a_{NW}(x) a_{NW}^\dagger(y)
+ \left( \partial_1 \Delta_-(x-y) \right) a_{NW}(y) a_{NW}^\dagger(x) \right\}$$

(27)

and

$$\left[\hat{\rho}_{G}(x), \hat{\rho}_{G}(y)\right] = \Phi_-(x) \Phi_+(y) \Delta_+(x-y) + \Phi_-(y) \Phi_+(x) \Delta_-(x-y).$$

(28)

The appearance of the function $\Delta_+$ and $\Delta_-$ and of their derivatives makes immediately clear that $\hat{\rho}_{NW}(x)$ and $\hat{\rho}_{G}(x)$ do not satisfy the microcausality principle. This may induce non local effects at spacelike distances. A manifestation of this is given by the fact that two NW localized states at two spacetime points $x$ and $y$, that are eigenstates of the NW operator $\hat{\rho}_{NW}(x)$, are in general not orthogonal [27].

5 Conclusions

To investigate the non local effects that appear in the propagation of quantum field from time varying sources, we have used a model consisting of a quantum scalar field linearly interacting with a classical instantaneous pointlike source. In our model there are not present spurious effects due to the difficulty to localize a quantum mechanical source [15,17] or to define single particle localized states for traverse fields [16]. Our results of the expectation values of one and two point operators are obtained using second order perturbation theory in the field-source coupling constant. Because of the simplicity of the model all the expectation values can be expressed in terms of the propagator functions $\Delta_s$ whose lightcone properties are well known.

We have found, in agreement with previous results [6,7,8], that non locality appears both in the expectation value of some single point operators functions of the field and in the two point correlation function. However the non local terms are source independent and due to the effect of the field zero point fluctuations.

Non local behavior appears also in the one particle part of the state that evolves under the action of the source. We have shown that the appearance
of this non local behavior can be traced to the fact that the one particle wavefunction corresponds to take a part of the complete state obtained by unitary evolution under the action of the source. This is equivalent to the action of an extended non local source.

At the end we have studied the expectation values evolution of the second quantized Newton-Wigner and Glauber operators. Here are present, source dependent, non local effects. However we have suggested that in this case the appearance of non locality can be traced to the fact that these operators do not satisfy the microcausality principle.

In conclusion for our system and within our approximations, as long as we make local measurement of operators satisfying the microcausality principle, do not appear non local effects, except for the ones due to vacuum fluctuations.

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