Short-range force detection using optically-cooled levitated microspheres

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(Dated: August 10, 2010)

We propose an experiment using optically trapped and cooled dielectric microspheres for the detection of short-range forces. The center-of-mass motion of a microsphere trapped in vacuum can experience extremely low dissipation and quality factors of $10^{12}$, leading to yoctonewton force sensitivity. Trapping the sphere in an optical field enables positioning at less than 1 μm from a surface, a regime where exotic new forces may exist. We expect that the proposed system could advance the search for non-Newtonian gravity forces via an enhanced sensitivity of $10^{-9} - 10^{-7}$ over current experiments at the 1 μm length scale. Moreover, our system may be useful for characterizing other short-range physics such as Casimir forces.

PACS numbers: 04.80.Cc,07.10.Pz,42.50.Pq

Since the pioneering work of Ashkin and coworkers in the 1970s [1], optical trapping of dielectric objects has become an extraordinarily rich area of research. Optical tweezers are used extensively in biophysics to study and manipulate the dynamics of single molecules, and in soft condensed-matter physics to study macromolecular interactions [2, 3]. Much recent work has focused on trapping in solution where strong viscous damping dominates particle motion. There has also been interest in extending the regime that Ashkin and coworkers opened, namely, trapping sub-wavelength particles in vacuum where particle motion is strongly decoupled from a room-temperature environment [1, 4].

Recent theoretical studies have suggested that nanoscale dielectric objects trapped in ultrahigh vacuum might be cooled to their ground state of (center-of-mass) motion via radiation pressure forces of an optical cavity [5, 6]. This remarkable result is made possible by isolation from the thermal bath, robust decoupling from internal vibrations, and lack of a clamping mechanism. In fact, a trapped dielectric nanosphere has been predicted to attain an ultrahigh mechanical quality factor $Q$ exceeding $10^{12}$ for the center-of-mass mode, limited by background gas collisions. Such large $Q$ factors enable cavity cooling, for which the lowest possible phonon occupation of the mechanical oscillator is $n_T/Q$, where $n_T$ is the number of room-temperature thermal phonons. Although such $Q$ factors have yet to be observed in experiment, optically levitated microspheres have been trapped in vacuum for lifetimes exceeding 1000 s [11] and electrically levitated microspheres have exhibited pressure-limited damping that is consistent with theoretical predictions down to $\sim 10^{-6}$ Torr [7].

In addition to being beneficial for ground-state cooling and studies of quantum coherence in mesoscopic systems, mechanical oscillators with high quality factors also enable sensitive resonant force detection [8, 9]. Optically levitated microspheres in vacuum have been studied theoretically in the context of reaching and exceeding the standard quantum limit of position measurement [10]. In this paper, we discuss the force sensing capability of a microsphere trapped inside a medium-finesse optical cavity at ultra-high vacuum, and propose an experiment that could extend the search for non-Newtonian gravity-like forces that may occur at micron scale distances. Such forces could be mediated by particles residing in sub-millimeter scale extra spatial dimensions [11] or by moduli in the case of gauge-mediated supersymmetry breaking [12]. The apparatus we propose is also well suited to studying Casimir forces [13], and may be useful for studying radiative heat transfer at the nano-scale [14].

Corrections to Newtonian gravity at short range are generally parameterized according to a Yukawa-type potential

$$V = -\frac{G_N m_1 m_2}{r} \left[1 + \alpha e^{-r/\lambda}\right],$$

where $m_1$ and $m_2$ are two masses interacting at distance $r$, $\alpha$ is the strength of the potential relative to gravity, and $\lambda$ is the range of the interaction. For two objects of mass density $\rho$ and linear dimension $\lambda$ with separation $r = \lambda$, a Yukawa-force scales roughly as $F_Y \sim G_N \rho^2 \alpha \lambda^3$, decreasing rapidly with smaller $\lambda$. For example, taking gold masses, for an interaction potential with $\alpha = 10^5$ and $\lambda = 1 \mu$m, $F_Y \sim 10^{-21}$ N. As we will discuss, the thermal-noise-limited force sensitivity of micron scale, optically levitated silica micro-spheres at 300 K with $Q = 10^{12}$ can be of order $10^{-21}$ N/$\sqrt{\text{Hz}}$, and therefore allows probing deep into unexplored regimes. For instance, current experimental limits at $\lambda = 1 \mu$m have ruled out interactions with $|\alpha|$ exceeding $10^{10}$.

The proposed experimental setup is shown schematically in Fig. 1. A dielectric microsphere of radius $a = 150$ nm is optically levitated and cooled in an optical cavity of length $L$ by use of two light fields of wavenumbers $k_t = 2\pi/\lambda_{\text{trap}}$ and $k_c = 2\pi/\lambda_{\text{cool}}$, respectively. The silica microsphere has density $\rho = 2300$ kg/m$^3$, dielectric constant $\epsilon = 2$, and is trapped near the position of the closest anti-node of the cavity trapping field to a gold mirror surface. The mirror is a 200 nm thick SiN membrane coated with 200 nm of gold. A source mass of thickness $t = 5 \mu$m
of the trapping light. The trap depth is $U_{\text{ness}}$, we consider a cavity of length $L$. For a sub-wavelength dielectric microsphere of radius $a$ is trapped with light in an optical cavity. The sphere is positioned at an anti-node occurring at distance $z$ from a gold-coated SiN membrane. Light of a second wavelength $\lambda_{\text{cool}} = 2\lambda_{\text{trap}}/3$ is used to simultaneously cool and measure the center of mass motion of the sphere. The sphere displacement $\delta z$ results in a phase shift $\delta \phi$ in the cooling light reflected from the cavity. For the short-range gravity measurement, a source mass is coated with a thin layer of gold to provide an equipotential. (b) Displacement spectral density (blue) due to thermal noise and shot-noise limited displacement sensitivity (flat line, red) for parameters discussed in the text.

FIG. 1: (color online) a) Proposed experimental geometry. A sub-wavelength dielectric microsphere of radius $a$ is trapped with light in an optical cavity. The sphere is positioned at an anti-node occurring at distance $z$ from a gold-coated SiN membrane. Light of a second wavelength $\lambda_{\text{cool}} = 2\lambda_{\text{trap}}/3$ is used to simultaneously cool and measure the center of mass motion of the sphere. The sphere displacement $\delta z$ results in a phase shift $\delta \phi$ in the cooling light reflected from the cavity. For the short-range gravity measurement, a source mass is coated with a thin layer of gold to provide an equipotential. (b) Displacement spectral density (blue) due to thermal noise and shot-noise limited displacement sensitivity (flat line, red) for parameters discussed in the text.

and length 20 $\mu$m with varying density sections of width 2 $\mu$m (e.g., Au and Si) is positioned at edge-to-edge separation $d = 1 \mu$m from the sphere. Below we describe trapping and cooling of the microsphere’s center-of-mass motion, detection of Casimir forces between the microsphere and gold mirror, and the search for gravity-like forces on the microsphere due to the source mass.

Following Ref. [3], the sub-wavelength dielectric particle has a center-of-mass resonance frequency $\omega_0 = \left[ \frac{6k_0^2\hbar}{m}\Re e - \frac{1}{\epsilon + 2} \right]^{1/2}$, where $I_0$ is the intracavity intensity of the trapping light. The trap depth is $U = \frac{3\hbar V}{\pi - 1}$, where $V$ is the volume of the microsphere. For concreteness, we consider a cavity of length $L = 0.15$ m, finesse $F = 200$, driven with a trapping laser of power $P_t = 2$ mW and wavelength $\lambda_{\text{trap}} = 1.5 \mu$m. We choose a cavity mode waist $w = 15 \mu$m. The Gaussian profile of the trapping beam near the mode waist provides transverse confinement, with an oscillation frequency of $\sim 1$ kHz. Tighter transverse confinement could be established by use of a transverse standing wave potential. The cooling light has input power $P_c = 48$ $\mu$W, and an optimized red detuning of $\delta = -0.23\kappa$, where the cavity decay rate is $\kappa = \pi c/L F$. The cooling light causes a slight shift $\delta z$ in the axial equilibrium position of the microsphere, given by $\delta z = \frac{I_c}{2}\frac{L}{k_0^2}\approx 2$ nm, where $I_c$ is the intracavity intensity of the cooling mode. The optomechanical coupling of the cooling mode is $g = \frac{\hbar V}{\pi - 1}\omega_c$, where $V_c = \pi a^2 L/4$ is the cavity mode volume [14], and $\omega_c = k_c c$. The optimum detuning is determined by minimizing the final phonon occupancy $n_f$, which depends on the laser-cooling rate and heating due to photon recoil from light scattered by the sphere. Additional cavity loss due to photon scattering is negligible: $\sim 10^{-3}\kappa$ for our parameters. Values of the trapping and cooling parameters appear in Table I.

A mechanical oscillator with frequency $\sim 37$ kHz and $Q \sim 10^{12}$ will respond to perturbations with a characteristic time scale of $2Q/\omega_0 \sim 10^7$ s. The cooling serves both to damp the $Q$ factor so that perturbations to the system ring down within reasonably short periods of time, and to localize the sphere by reducing the amplitude of the thermal motion. Because of the low cavity finesse, the cooling is not in the resolved sideband regime. Still, for the parameters discussed above the phonon occupation of the microsphere oscillation is reduced by a factor of over $10^5$. This corresponds to operating with an effective $Q_{\text{eff}} \approx 10^5$ and ring down time of $\approx 1$ s. Cooling of the transverse motion is also required, as the rms position spread must be maintained to be less than $\approx 0.1 \mu$m. We imagine this can be done with active feedback to modulate the power of a transverse trapping laser using the signal from a transverse position measurement, for example generated by measuring scattered light incident on a quadrant photodiode. A modest cooling factor of $\approx 1000$ in the transverse directions is sufficient to yield the required localization.

The cooling light is also used to detect the position of the sphere. The phase of the cooling light reflected from the cavity is modulated by microsphere motion through the optomechanical coupling $\partial \omega_c/\partial z = 2k_c g$. Photon shot-noise limits the minimum detectable phase shift to $\delta \phi \approx 1/(2\sqrt{T})$ where $T = I_c/(\omega_0)$ [15]. The corresponding photon shot-noise limited displacement sensitivity is $\sqrt{S_z(\omega)} = \frac{k_g}{\sqrt{1 + \frac{k_t^2}{\kappa}}}$ for an impedance matched cavity. This displacement sensitivity is generally well below the thermal noise limited sensitivity, as shown in Fig. 1(b). We assume that substrate vibrational noise, electronics noise and laser noise can be controlled at a level comparable to the photon shot noise. The minimum detectable force due to thermal noise at temperature $T_{\text{eff}}$ is $F_{\text{min}} = \sqrt{\frac{4k_t k_g T_{\text{eff}} b}{\omega_0 Q_{\text{eff}}}}$, where $k$ is the center-of-mass mode spring constant, and $b$ is the bandwidth of the measurement. We assume an initial center-of-mass temperature $T_{\text{CM}} = 300$ K, and that $Q \approx \omega_0/\gamma_g$ is limited by background gas collisions, with loss rate $\gamma_g = 16P_{\text{gas}}/(\pi a^2)$ [16], for a background air pressure of $P_{\text{gas}} = 10^{-10}$ Torr and rms gas velocity $\bar{v}$. Cooling the center-of-mass mode to $T_{\text{eff}} = 0.9$ mK results in $F_{\text{min}} \approx 10^{-21}$ N/$\sqrt{\text{Hz}}$ as shown in Table I. In this regime $F_{\text{min}}$ scales linearly with the sphere radius $a$. 
The microsphere absorbs optical power from both the trapping and cooling light in the cavity, which results in an increased internal temperature $T_{\text{int}}$. Assuming negligible cooling due to gas collisions, the absorbed power is re-radiated as blackbody radiation. $T_{\text{int}}$ is listed in Table I for fused silica with dielectric response $\epsilon = \epsilon_1 + i\epsilon_2$, with $\epsilon_1 = 2$ and $\epsilon_2 = 1.0 \times 10^{-5}$ as in Ref. [17], and $\epsilon_{\text{th}} = 0.1$ as in Ref. [15], for an environmental temperature $T_{\text{ext}} = 300$ K. We assume $T_{\text{int}}$ and $T_{\text{CM}}$ are not significantly coupled over the time scale of the experimental measurements at $P_{\text{gas}} \sim 10^{-10}$ Torr.

**Casimir Force.** The Casimir force between a dielectric sphere and metal plane can be written using the proximity force approximation (PFA) as $F_c = -\eta \pi^2 a b c/\epsilon_0 \lambda_{\text{trap}}$ in the limit that $(z-a) \ll a$. The prefactor $\eta$ characterizes the reduction in the force compared with that between two perfect conductors [18]. For $z \gg a$, the force takes the Casimir-Polder [19] form $F_{\text{cp}} = -\eta \pi^2 a b c/\epsilon_0 \lambda_{\text{trap}}$, where $\alpha_V = 3\epsilon_0 V z/\epsilon_2$ is the electric polarizability. Our setup is capable of probing the unexplored transition between these two regimes, and of testing the PFA, which is expected to be valid for $z-a \lesssim a$ [20]. To estimate $\eta$, we adopt a similar approach to that taken in Ref. [18] to determine the force between a metal and dielectric plate. We assume dielectric spheres separated from a metal mirror will have a similar pre-factor. Taking an infinite plate with $\epsilon = 2$ and thickness $2a$, and another with gold of thickness 200 nm, we find $\eta \approx 0.13$ at $(z-a) = 225$ nm.

For a sphere located at the position of the first anti-node of the trapping field, the Casimir force displaces the equilibrium position by approximately $-3$ nm. The gradient of the Casimir force near the static mirror surface produces a fractional shift in the resonance frequency of the sphere given by $|\delta \omega_0/\omega_0| = |\partial F_c/\partial z|/(2k)$. A similar frequency shift has been measured for an atomic sample [21]. The shift is shown in Fig. 2 as a function of mirror separation $(z-a)$ for $\eta = 0.13$. The minimum detectable frequency shift due to thermal noise is given by $|\delta \omega_0/\omega_0|_{\min} = \sqrt{k_b T_{\text{trap}} b/k_{\text{B}} \omega_0 Q_{\text{int}} z_{\text{rms}}^2}$. For $z_{\text{rms}} = 5$ nm, $|\delta \omega_0/\omega_0| \approx 10^{-7}$ can be detected in $\sim 1$ s. Other sources of systematic frequency shifts near the surface, for example from variation of the cavity finesse with bead position or from diffuse scattered light on the gold surface, would need to be experimentally characterized. Also, surface roughness of the microsphere can modify the Casimir force [22]. Rotation of the microsphere may lead to an effective averaging over surface inhomogeneities.

*Search for non-Newtonian gravity.* To generate a modulation of any Yukawa-type force at the resonance frequency of the center-of-mass mode along $z$, the source mass is mounted on a cantilever beam that undergoes a lateral tip displacement of $2.6\ \mu$m at a frequency of $\omega_0/3$. The mechanical motion occurs at a sub-harmonic of the microsphere resonance to avoid direct vibrational coupling. To estimate the force on the sphere a numerical integration over the geometry of the masses is performed. For $b = 10^{-5}$ Hz, the estimated search reach is shown in Fig. 3 (a). Several orders of magnitude of improvement are possible between 0.1 nm and a few microns, due to the proximity of the masses and high force sensitivity.

The source mass surface is coated with 200 nm of gold in order to screen the differential Casimir force, depending on which material is directly beneath the microsphere. Following the method of Ref. [18], the differential Casimir force is $9 \times 10^{-24}$ N, which is comparable to the sensitivity of the experiment at $10^{-5}$ Hz bandwidth. The gold coating on the cavity mirror membrane attenuates this Casimir interaction even further. Patch potentials on the mirror surface and any electric charge on the sphere can produce spurious forces on the sphere.
for the system we consider would yield a pressure-limited
extrapolating the results of Ref. [7] at 10
and similar techniques may work for the setup proposed
successful at optically trapping 1
individual microspheres and precisely control their posi-
tation near a surface. Previous experimental work has been
been successful at optically trapping 1.5 \mu m radius spheres [4],
and similar techniques may work for the setup proposed here. Extrapolating the results of Ref. [7] at 10^{-6} Torr
for the system we consider would yield a pressure-limited
Q \sim 10^9. In the absence of additional damping mecha-
nisms, we expect that Q \approx 10^{12} could be achieved at
lower pressure. Further improvements in force sensitiv-
ity may be possible in a cryogenic environment.

We thank John Bollinger and Jeff Sherman for a care-
ful reading of this manuscript. AG and SP acknowledge
support from the NRC.

\begin{figure}[h]
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\includegraphics[width=\textwidth]{fig3.png}
\caption{(color online) Experimental constraints [28]
and theoretical predictions [30] for short-range forces
due to an interaction potential of Yukawa form \( V = \frac{Q \alpha e^{-r/\lambda}}{1 + \alpha e^{-r/\lambda}} \). Lines (a) and (b) denote the pro-
jected improved search reach for microspheres of radius \( a = 150 \) nm and \( a = 1500 \) nm, respectively.
}
\end{figure}

translating the position of the optical trap along the surface,
these and other backgrounds, e.g. vibration, can be
distinguished from a Yukawa-type signal, as any Yukawa-
type signal should exhibit a spatial periodicity associated
with the alternating density pattern, similar to the sys-
tem discussed in Ref. [28].

Increasing the radius of the sphere can significantly
enhance the search for non-Newtonian effects at longer
range. Curve (b) in Fig. 3 shows the estimated search reach
that would be obtained by scaling the sphere size by a factor of 10 and positioning it at edge-to-edge separa-
tion of 3.8 \mu m from a source mass with thickness \( t = 10 \) \mu m consisting of sections of width 10 \mu m driven at an am-
plitude of 13 \mu m. Such a larger sphere could be trapped
in an optical lattice potential with the incident beams at
a shallow angle, instead of in an optical cavity, to enable
sub-wavelength confinement. In this case cooling could
be performed by use of active feedback. Alternatively it
may be possible to trap the larger 1.5 \mu m sphere in a cav-
ity by use of longer wavelength light (e.g., \( \lambda_{\text{trap}} = 10.6 \) \mu m)
by choosing a sphere material such as ZnSe with
lower optical loss at this wavelength.

The experiment we have proposed may allow improve-
ment by several orders of magnitude in the search for
non-Newtonian gravity below the 10 \mu m length scale.
An experimental challenge will be to capture and cool
dividual microspheres and precisely control their posi-
tion near a surface. Previous experimental work has been
been successful at optically trapping 1.5 \mu m radius spheres [4],
and similar techniques may work for the setup proposed here. Extrapolating the results of Ref. [7] at 10^{-6} Torr
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