The concept and the principle of the diagnostic observability of the object in problems of monitoring and non-destructive testing

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Abstract. The design of non-destructive testing systems relates to the solution of the problem of analysis of observability. It is the assessment of the opportunities of the set of measured parameters to diagnose the object, to identify emerging defects and the specific modes of operation. The term ‘observability’ in control theory does not correspond to the tasks of diagnosis and requires the model of the object’s state. The proposed concept of ‘diagnostic observability’ allows to use this term in tasks which aren’t related to the control. It doesn’t require the model of the object’s state, and realizes the estimation on the array of measurements performed for various diagnosed states of the object. The set of diagnostic state variables is determined as coordinates of diagnostic orthogonal space. The proposed principle of diagnostic observability is determined by the method of construction of the space of the diagnostics and by the analysis of the images of states of object. It can become a new basis for building an effective universal adaptive systems of non-destructive testing of complex technical and other objects.

1. Introduction

In control theory [1], the term ‘observability’ is defined for the object, described in the state space:

\[
\begin{align*}
\dot{x} &= f(x,u,t), \quad x \in \mathbb{R}^k, \quad u \in \mathbb{R}^r, \\
y &= F(x,u,t), \quad y \in \mathbb{R}^o,
\end{align*}
\]

where \( x \) – the vector of state variables, \( u \) – the vector of control response, \( y \) – the vector of output measured values. The first equation in (1) – the equation of state of the object, the second – the equation of observation of the object. The system (1) is completely observed [1], if there is \( t_1, \ t < t_1 < \infty \), and measured values \( y(\tau) \) and \( u(\tau) \) on interval \( t \leq \tau \leq t_1 \) can determine state \( x(t) \).

The system (1) for the linear steady-state object is written as follows:

\[
\begin{align*}
\dot{x} &= Ax + Bu, \quad x \in \mathbb{R}^k, \quad u \in \mathbb{R}^r, \\
y &= Cx + Du, \quad y \in \mathbb{R}^o,
\end{align*}
\]

where \( A, B, C, D \) - matrices of appropriate dimensions. The concept of the matrix of observability is
introduced [1]:

\[ H = \begin{bmatrix} C^T & A^T C^T & \ldots & \left(A^T\right)^{k-1} C^T \end{bmatrix}. \]  

(3)

In this case, the object (2) is considered to be completely observable if and only if the rank of the matrix of observability is \( k \): \( \text{rank} \ H = k \).

The equation of state and the equation of observation of the object are necessary for the assessment of the observability in the existing theory. The production of these equations is possible by structural and parametric identification, which in turn is a complex task and depends on the identifiability of the object [1].

The above-stated traditional definition of observability is somewhat different characteristic than it is required in the diagnosis and non-destructive testing. For example, the matrix (3) characterizes the observability of the state variables only in normal operating conditions and in case in (2) it can be assumed that all matrices \( A, B, C, D \) are not changing. In case of non-destructive testing, the observability should be evaluated by other state variables that characterize the appearance or presence of defects, faults in the object. The model (2) is changed because of the defects and faults. In this case, matrices \( A, B, C, D \) of the model (2) can not be considered as constant. The application of the matrix of observability (3) is impossible in this case.

Two principles of probing, passive and active, are realized in the non-destructive testing. The detection of moments, which are the most suitable for diagnostic object by \( y \), occurs in the passive monitoring of parameters of operating complex object; special control actions \( u \) are not formed. With the implementation of the active probing there is a formation of known control action \( u \) and a measurement of the output value \( y \), as the response to this action.

In both cases of the probing, the component of control action \( u \) in the model of a linear steady-state object (2) can be replaced by constants \( \xi_d, \zeta_d \) as follows:

\[
\begin{align*}
\dot{x}_d &= A_d x_d + \xi_d, \quad x_d \in \mathbb{R}^n; \\
y &= C_d x_d + \zeta_d, \quad y \in \mathbb{R}^n,
\end{align*}
\]

(3)

where \( x_d \) - the vector of other diagnostic state variables, \( y \) - the vector of output measured values.

It is important when the output measured value is scalar response function \( y(t) \) measured in interval \([t, t + \Delta t]\) in the context of diagnosis and non-destructive testing. In this case, the equation of observation (3) can be written as follows:

\[ y = C_d x_d + \zeta_d, \quad y \in \mathbb{R} \].

(4)

The production of the equation of state (3) is not required in the diagnosis. The evaluation of the value of vector \( x_d \) of diagnostic of state variables is only performed using equation (4). The space of value \( x_d \) will be called diagnostic states space \( D = \{x_d\} \).

2. The methods for evaluating the diagnostic states space

The construction of diagnostic space \( D \) is based on the mathematical techniques of reduction of measured signals \( y \) in (4), in particular, signals of a response to a probing action. Under the term ‘reduction’ of measurement \( y \) we understand the decrease of the dimension of space of measurement \( V = \{y\} \) with the identification of generalized parameters – diagnostic state variables \( x_d \). Variables \( x_d \) must retain a significant variability of measurements \( y \) during the transition from one diagnosed state to another. It is important to have a stable unique dependence between the change of the diagnostic state variables \( x_d \) and the diagnosed change in object's state. The presence of significant unique dependence corresponds to the diagnostic observability of object.
Mathematical reduction of dimension of space \( V = \{ y \} \) can be carried out by using different approaches: the traditional spectral analysis [2-4]; the correlation analysis of signals [4-7]; the artificial neural networks [6,8]; the statistical approaches and models [8,9]; the wavelet transform signals [3-5,10]; the decomposition modeling method [7] and others.

Analysis showed that the use of the decomposition modeling method is superior to others. The optimal orthogonal decomposition of signals \( y \) on the adaptively adjusted basis based on training sample \( \{ y \} \) is realized for the construction of diagnostic states space \( D \). Thus, obtained space \( D \) allows one to analyze signals \( y \) for signs of defects more effectively. This analysis can be used as a criterion of the quality of measurements \( y \). It requires replacement of the measured signal and measurement principles in case of low-quality detection of defects in the needed group.

This approach has been tested and has shown the ability of effective using in different diagnostic systems: the non-destructive testing of mechanical structures and mechanisms with wave low-frequency probing effects [11]; the determining of corrosive destruction [5]; the electrochemical systems of diagnosis and control of metal alloys [7]; the on-line diagnostics of the autonomous power sources [7]; the diagnostic of modes of operation of large power systems [7]; the analysis of the mechanical loading of items bearing structures [11]. Its more expanded use is possible for detection of damaged blades of wind turbines [8]; for non-destructive testing of materials in engineering [6]; for detection of foreign bodies and its classification with ultrasound using [10]; for fault location with using of environment vibration [2] and others.

3. The principle of reduction in the construction of a diagnostic state space on the basis of the decomposition method of modeling

For simplicity, we consider the one-dimensional case. The output measured value is scalar response function \( y(t) = f(t), \ t \in [0, T_0] \) to probing effect \( F \). Time interval \( T_0 \) must be sufficient for stable identification of emerging diagnosed conditions. The shock action generating surface waves acts as the probing one of low-frequency non-invasive diagnosis of metal structures.

The learning sample is formed from separate functions \( f_i(t) \) obtained for different types of defects in the construction, but in case of one type of non-destructive probing effects \( F \). Each function \( f_i(t) \) can be considered as the vector of real values

\[
{f_i} = [f_{i1}, f_{i2}, f_{i3}, \ldots, f_{iN}]^T \in \mathbb{R}^N
\]

of time changing of signal response. Sampling increment \( \Delta t \) or the number of counts \( N \) per observation interval is selected according to the sampling theorem [1].

The number of vectors \( f_i \) in learning sample \( \{f_1, f_2, f_3, \ldots, f_n\} \in \mathbb{R}^{N \times m} \) is determined by the number of defects \( l \) recognizable in construction \( n = l \cdot \beta \), where \( \beta \) - number of vectors \( f_i \) corresponding to one type of the defect. Initially, a set of vectors \( \{f_1, f_2, f_3, \ldots, f_n\} \) is linearly dependent. It is explained by excess dimension \( N \) of vectors \( f_i \), proximity of shapes of graphs \( f_i \), and it is confirmed by the value of rank of matrix

\[
{f} = [f_1, f_2, f_3, \ldots, f_n]^T \in \mathbb{R}^{N \times N}
\]

for which it is true that \( \text{rank}(f) < N \).

Construction of diagnostic state space \( D = \{x_d\} \) is finding orthogonal transformation \( \Xi \in \mathbb{R}^{N \times m} \) of matrix \( f \in \mathbb{R}^{N \times N} \) into matrix \( A \in \mathbb{R}^{m \times m} \) of form \( A = \Xi f \), excluding excess dimension \( f \) associated with uninformative, often random variations in graphs \( f_i \). This indirectly follows from equation (4). Thus, matrix

\[
\Xi = [\xi_1, \xi_2, \xi_3, \ldots, \xi_m]
\]

defines this linear subspace in \( \mathbb{R}^N \) for which it is true that a set of possible linear combinations of its vectors is also a linear space or a linear span:

\[
\text{span} \{\xi_1, \xi_2, \xi_3, \ldots, \xi_m\} = \left\{ \sum_{j=1}^{m} \beta_j \xi_j : \beta_j \in \mathbb{R} \right\}
\]
In this case, vectors \( \xi_1, \xi_2, \xi_3, \ldots, \xi_m \) form an orthonormal basis in \( \mathbb{R}^N \), and formula \( \Xi^T \Xi = I_m \) is true for matrix \( \Xi \). Conventionally, matrix \( \Xi \) can be considered as a matrix of orthogonal compression of linear space \( \mathbb{R}^N \) into space \( \mathbb{R}^m \). Vectors of responses \( f_i \in \mathbb{R}^N \) are converted into images \( A_i \in \mathbb{R}^m \), \( m << N \) with the help of \( \Xi \), and herewith matrix response \( \mathbf{f} \) is converted into the matrix of images \( \mathbf{A} = [A_1, A_2, A_3, \ldots, A_n]^T \in \mathbb{R}^{m \times n} \). In general, the orthogonal decomposition of original vectors \( f_i \in \mathbb{R}^N \) on the basis of \( \Xi \) can be expressed in the form of \( f_i = \Xi A_i + A_0 \), where \( A_0 \) - the constant component of the transformation. At the same time, it is true that \( x_{di} = A_i \) and \( \mathbf{D} = \mathbf{A} \).

At the same time, orthonormal basis \( \xi_1, \xi_2, \xi_3, \ldots, \xi_m \) is adaptively configurable, trained and dependent on recognizable sample \( \{f_1, f_2, f_3, \ldots, f_n\} \in \mathbb{R}^{m \times N} \). The complex of optimization problems is solved in determining of the basis, in particular:

1) The best reproducibility is \( \| \mathbf{f} - \mathbf{A} \Xi^T \mathbf{A}_0 \|_2 \to \min \), where \( \mathbf{A}_0 \in \mathbb{R}^{m \times N} \) - the matrix of constant components of conversion consisting of vectors \( A_0 \).

2) Property of basis orthonormality: \( \| \Xi^T \Xi - I_m \|_2 \to \min \).

3) The property is the best discernibility: \( d^2 (\mathbf{A}) = \frac{1}{m^2 - m} \sum_{i,j=1}^{n} \| A_i - A_j \|_2 \to \max \).

The implementation of all these tasks forming the space of diagnostic state variables \( \mathbf{D} \) is observed when the decomposition method of modeling [7] is used for this purpose.

4. Cluster and functional interpretation of diagnostic observability

The diagnostic observability can be interpreted with the using of images \( A_i = (a_{i1}, a_{i2}, \ldots, a_{im}) \in \mathbb{R}^m \) of response function \( f_i(t) \) from the learning sample of measurements \( \mathbf{f} \), which includes all states of the object that are necessary to diagnose. The images are located in diagnostic state space \( \mathbf{D} \). Typically, this space is low-dimensional, \( m << N \). In the simplest case, this space is two-dimensional \( A = (a_1, a_2) \in \mathbb{R}^2 \), or space, which can be analyzed as a set of two-dimensional spaces - the projections. In space \( \mathbf{D} \), the boundaries of clusters \( C_j, j = 1, \ldots, 8 \) of images corresponding to each of the individual diagnosed object’s state are defined. Figure 1 shows four clusters for the seven possible diagnosed object’s states. In this two-dimensional projection on the plane of the first two diagnostic state variables \( a_{i1}, a_{i2} \) only four cluster object’s states \( C_1, C_2, C_3, C_4, C_5, C_6, C_7 \) are clearly distinguished. Five states \( (C_1, C_2) \) and \( (C_4, C_5, C_6) \) form only two discernible clusters, so state variables \( a_{i1}, a_{i2} \) are not enough for separate diagnosis of all the seven existing conditions. In this case, diagnostic observability is incomplete - four from seven. It is necessary to use a greater number of state variables \( a_{i1}, a_{i2}, \ldots, a_{im} \) and to consider other two-dimensional projection of images on orthogonal planes for discernibility of all seven states.

Perhaps other projections allow one to exclude indistinguishability of these states. A decomposition modeling method arranges state variables \( a_{i1}, a_{i2}, \ldots, a_{im} \) in the order of their diagnostic informativeness, which is very convenient in the analysis of the diagnostic observability. At the same time, diagnostic informativeness is understood as further variation \( S_N^2(a_j) \) of variable \( a_j \) in diagnostic state space \( \mathbf{D} \):

\[
S_N^2(a_j) = \frac{1}{n} \sum_{i=1}^{n} (a_{ij} - \frac{1}{n} \sum_{i=1}^{n} a_{ij})^2.
\]
If the increase of the dimension of the diagnostic space by the number of state variables does not lead to a complete diagnostic observability, therefore, it is necessary to change the set of measured signals . The selected set does not meet the principle of diagnostic observability. All of the above-mentioned determines the diagnostic of observability in the cluster interpretation.

Besides the visual presentation of diagnostic observability in a cluster interpretation, quantitative estimation is also possible. The well-known clustering quality criteria [12,13] can be used as a quantitative assessment. Specifically, for each state (cluster) ratio of scatter images in the cluster to minimum distance from the cluster to the nearest one in the diagnostic space may be estimated:

\[ \eta_j = \frac{D_{C_j}}{\min_{y \in \mathcal{Y}} R_{jy}}. \]

Functional interpretation of diagnostic observability allows us to evaluate how the changing of the value of one or more diagnosed parameters (such as size, location, shape of the defect or otherwise) can be described as functional dependence from the changing of the values of state variables – coordinate of images \( A_i \in \mathbb{R}^n \) of the learning sample in diagnostic state space \( \mathcal{D} \):

\[ P_{ci} = F(a_{i1}, a_{i2}, \ldots, a_{im}). \]

If the construction of this regressive functional dependence is possible, then an assessment of its adequacy can be regarded as quantitative characteristic of diagnostic observability in the functional interpretation.

Finally, the term diagnostic observability is an assessment of the possibility of an available set of measured parameters by their number and the type which detects defects adequately and accurately, diagnoses and identifies the special emergency mode operation of the object.

5. Conclusion
The customizable orthonormal diagnostic state space for diagnostic systems built with the use of the decomposition simulation method allows us to evaluate the observability of a set of measured data in terms of completeness of diagnostics of the object.

The concept and the principle of the diagnostic observability of the object by the set of measures allowing one to formalize the evaluation of the completeness of the diagnosis of the object in the cluster and functional interpretations have been suggested.
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