A QUANTITATIVE MODEL FOR DRIFTING SUBPULSES IN PSR B0809+74

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ABSTRACT

In this paper, we analyze high time resolution single-pulse data of PSR B0809+74 at 820 MHz. We compare the subpulse phase behavior, undocumented at 820 MHz, with previously published results. The subpulse period changes over time and we measure a subpulse phase jump, when visible, that ranges from 95° to 147°. We find a correlation between the subpulse modulation, subpulse phase, and orthogonal polarization modes. This variety of complicated behavior is not well understood and is not easily explained within the framework of existing models, most of which are founded on the drifting spark model of Ruderman & Sutherland. We quantitatively fit our data with a non-radial oscillation model of Clemens & Rosen and show that the model can accurately reproduce the drifting subpulses, orthogonal polarization modes, subpulse phase jump, and can explain the correlation between all these features.

Key words: asteroseismology – polarization – pulsars: general – pulsars: individual (B0809+74) – stars: neutron – stars: oscillations

1. INTRODUCTION

First discovered by Cole & Pilkington (1968), PSR B0809+74 is a bright, slow pulsar with drifting subpulses that has been continuously studied over the past 40 years. The literature contains a wide range of behavior including changes in subpulse period, subpulse phase behavior, average pulse shape, and orthogonal polarization modes as a function of radio frequency. The average pulse profile and polarization angle histogram for our observations at 820 MHz are shown in Figure 1. Hobbs et al. (2004) have monitored this pulsar for at least six years, measuring a spin period (P1) of 1.292 s and a dispersion measure of 6.116 pc cm−3. A list of basic parameters is given in Table 1.

At low frequencies, from 81.5 to 151 MHz, the measured subpulse period is around 53 ms (Bartel et al. 1981; Davies et al. 1984). The measured subpulse period is the spacing between two adjacent subpulses in the same pulse and is usually referred to as P2. As discussed in Clemens & Rosen (2004), the value of P2 is not an accurate measurement of the underlying fundamental subpulse period, Ptime. At higher frequencies, P2 appears to decrease to 39 ms, 31 ms, and 29 ms at 406 MHz, 1412 MHz, and 1720 MHz, respectively (Davies et al. 1984; Bartel et al. 1981). Our measurements at 820 MHz fall in the middle range of observational frequencies. Bartel et al. (1981) find that while the subpulse period appears to change between 102.5 and 1720 MHz by a factor of 1.8, the time it takes for a subpulse to return to the same longitude, P3, remains constant.

The likely underlying cause for the change in the measured subpulse period is a subpulse phase discontinuity (or jump) that appears at high frequencies but not at low frequencies. The subpulse phase jump is not seen at 328 MHz (Edwards & Stappers 2003; Edwards 2004), 408 MHz (Proszynski & Wolszczan 1986), or at 500 MHz (Wolszczan et al. 1981). At 1380 MHz, Edwards & Stappers (2003) and Edwards (2004) report a phase jump of ∼120° as shown in the middle panel of Figure 2. Furthermore, the subpulse phase jump occurs at approximately 56°5 in pulse longitude (bottom left panel of Figure 2), corresponding to a minimum subpulse amplitude envelope (dark line in the top left panel of Figure 2). As Edwards & Stappers (2003) discuss, in any given pulse with two subpulses, the subpulses generally lie on opposite sides of the subpulse phase jump and appear closer together in pulse phase than they normally would in the absence of the phase jump, resulting in a smaller value of P3.

Most drifting subpulse models are based on the drifting spark model (Ruderman & Sutherland 1975) in which a vacuum gap forms between the stellar surface and co-rotating magnetosphere due to the charge depletion from the emitted particles. To prevent the vacuum gap from growing indefinitely, sparks discharge across the vacuum gap. These sparks are fixed relative to each other and form a carousel that rotates around the magnetic pole at a rate incommensurate with the spin period of the star. The drifting subpulses are the manifestation of these spark discharges. Known as the drifting or (rotating) spark model, this model is the basis for many current models of drifting subpulses (Komesaroff 1970; Backer 1976; Gil & Sendyk 2000).

In Clemens & Rosen (2004, 2008), we proposed a non-radial oscillation model based on asteroseismological techniques (Dziembowski 1977; see also Robinson et al. 1982; Clemens et al. 2000) as an alternative to the drifting spark model. Pulsations in stars are not uncommon: GW Vir, white dwarf stars (DBVs), ZZ Ceti stars, rapidly oscillating AP stars, and delta Scuti stars all show pulsation modes (Kleinman et al. 1998; van Kerkwijk et al. 2000; Winget et al. 1981; Kurtz 1982; Breger 1969). We were not the first proponents of an oscillation model for pulsars; Gold (1968), van Horn (1980), and Strohmayer (1992) all proposed oscillations as an explanation for drifting subpulses. However, these papers did not address the wide range of phenomenology seen in pulsars with drifting subpulses.

In this paper, we analyze high quality single-pulse measurements of PSR B0809+74 at 820 MHz. In Section 2, we discuss our observations and conduct a detailed analysis of the data in Section 3. We then explain our model in Section 4 and examine the data in the context of our model in Section 5. Finally, in Section 6, we discuss how our 820 MHz observations compare the observations at other frequencies and explain the single-pulse behavior within the context of a non-radial oscillation model.
2. OBSERVATIONS

The observations of PSR B0809+74 were taken in the spring of 2009 with the 100 m Green Bank Telescope using the new pulsar backend GUPPI in filterbank mode. The dates and lengths of the 10 observations are listed in Table 2. Full-Stokes spectra were acquired in a 200 MHz-wide band centered at 820 MHz radio frequency. The frequency resolution was ∼98 kHz and the spectrum integration time was 160 μs. The filterbank data were then averaged into 1024-bin single-pulse profiles using the ephemeris given in Table 1.

Flux and polarization calibration were performed using the psrchive software package (Hotan et al. 2004). Each of the 10 observations was performed at a different hour angle. The rotation of the source with respect to the telescope was used to solve for the receiver system’s intrinsic polarization cross-coupling matrix, following van Straten (2004). From the calibrated profiles, we determined a rotation measure (RM) of $-12.2 \pm 0.2 \text{ rad m}^{-2}$, consistent with the cataloged value of $-11.7 \pm 1.3 \text{ rad m}^{-2}$ (Manchester 1972). The calibrated single-pulse profiles were then RM-corrected and integrated over the full band for the analysis described in the following sections.

3. DATA ANALYSIS OF PSR B0809+74

3.1. Subpulse Period

The 10 observations we acquired in 2009 April varied in length and morphology, as shown in Table 2. Eight of the ten observations displayed nulling, periods with zero emission. Only two epochs, MJDs 54922 and 54944b, exhibited a single subpulse period in the fast Fourier transform (FFT) of the entire run; the remaining observations had multiple significant periods. The first and last observations (MJDs 54922 and 54961) had the brightest flux. When displaying data and our fits to the data, we use epoch MJD 54922 as it has the second largest flux and is not affected by nulls as are data from MJD 54961.

We find the fundamental subpulse period, $P_{\text{time}}$, to be between 48 and 54 ms, which is consistent with the measured value of $P_2$ by Davies et al. (1984) and Bartel et al. (1981) at low frequencies without the presence of a subpulse phase jump (see Section 5.2). An FFT shows that the subpulse alias peaks at approximately 39 ms. However, the FFT calculates the period of the subpulse based on the separation of the peaks, and the presence of the subpulse phase jump makes the subpulse peaks artificially closer than they would be in the absence of the phase jump (Edwards & Stappers 2003).

The average pulse profile at 820 MHz is consistent with those at 328 MHz and 1380 MHz. As shown in the top panels of Figure 2, the subpulse modulation envelope at 820 MHz (dashed line, top right panel) resembles that at 1380 MHz but not at 328 MHz (top left panel). This difference is the result of the subpulse phase jump (shown in the bottom panels of Figure 2) at 820 MHz and 1380 MHz; we discuss the subpulse phase behavior in Section 3.2.

Figure 3 characterizes the subpulse behavior for each observation. The mean flux in the top panel of Figure 3 is also described in Table 2. The subpulse period can also be characterized by $P_3$, which is the time it takes for a subpulse to return to the same longitude after successive spins of the star. $P_3$ is not sensitive to which alias we chose from the FFT as the subpulse period. To measure $P_3$, we use the longitude resolved fluctuation spectrum (LRFS), a Fourier transform calculated at each spin phase. The value of $P_3$ for each observation is shown in the middle panel in
Figure 2. Top left: the average profile and subpulse longitude envelope for PSR B0809+74 at 328 MHz (dashed lines) and 1380 MHz (solid lines). Top right: the average profile and subpulse longitude envelope at 820 MHz taken on MJD 54922. Middle left: the subpulse phase, plotted multiple times, spaced 360° apart. The white and dark circles are from data collected at 328 MHz and 1380 MHz, respectively. The dotted line shows the phase slope of 25 degrees per degree. Middle right: the subpulse phase for our data at 820 MHz; the dotted line shows the phase slope of 27.8 degrees per degree. Bottom left: the difference between the subpulse phase and that of the phase slope, indicating the magnitude of the phase jump. The phase jump at 1380 MHz (dark circles) is 120° and the phase jump at 328 MHz (white circles) is plotted twice with an offset of 120°. Bottom right: the difference between the subpulse phase and that of the phase slope for our data at 820 MHz, results in a phase jump of 116°. Using an alternative method of fitting two lines on either side of the phase jump (middle, right panel) rather than subtracting the phase slope results in a phase jump of 145° (see Section 3.2). The phase jump at 820 MHz occurs at the same pulse longitude as the phase jump at 328 MHz and 1380 MHz. All the plots on the left panels are reproduced from Edwards & Stappers (2003). Credit: R. T. Edwards and B. W. Stappers, A&A, 410, 961, 2003, reproduced with permission © ESO.

Table 2
Summary of Our Observations at 820 MHz

| Date    | Epoch | Length of Observations | Length of Nulls | Multiple Periods | Mean Flux (mJy) |
|---------|-------|------------------------|----------------|------------------|-----------------|
| 2009 Apr 1 | 54922 | 232                    | 2, 2           | no               | 310.3           |
| 2009 Apr 22 | 54943 | 466                    | 0              | yes              | 91.9            |
| 2009 Apr 23 | 54944a | 397                    | 3              | yes              | 191.9           |
| 2009 Apr 23 | 54944b | 476                    | 2              | no               | 189.9           |
| 2009 Apr 27 | 54948a | 475                    | 3, 2, 2        | yes              | 126.5           |
| 2009 Apr 27 | 54948b | 475                    | 3              | yes              | 171.7           |
| 2009 Apr 27 | 54948c | 476                    | 2, 2, 2, 2     | ??               | 114.3           |
| 2009 Apr 27 | 54948d | 476                    | 0              | yes              | 60.2            |
| 2009 Apr 27 | 54948e | 477                    | 3              | yes              | 40.3            |
| 2009 May 10 | 54961 | 475                    | 8, 4           | yes              | 374.6           |

Notes. The first and last observations (MJDs 54922 and 54961) have the brightest flux. When displaying data and our fits to the data, we use epoch MJD 54922 as it has the second largest flux and does not display nulls.
Figure 3. Top panel: the mean flux as a function of epoch. Middle panel: the period $P_3$ at each epoch. Bottom panel: the subpulse phase jump; see Section 3.2 for our methodology on calculating the phase jump.

Figure 4. Top: the LRFS from MJDs 54922 (left) and 54961 (right), where $P_3 \sim 11.31 P_1$. The left panels show the integrated pulse profile; the maximum on the plot is 1100 mJy. The bottom panels show the power in mJy$^2$ Hz$^{-1}$. Bottom: the same data set folded at $P_3$.

Figure 3. The bottom panel displays the subpulse phase jump; the method for calculating the subpulse phase jump is discussed in Section 3.2.

The top plots in Figure 4 show the LRFS of two different data sets taken at different epochs; the bottom panels of the top plots show a peak at $P_3 = 14.61$ s, or $\sim 11.31 P_1$. The bottom plots of Figure 4 show the corresponding drift-band plot, a contour plot of the data folded at $P_3$. Figure 3 shows $P_3$, calculated from the LRFS, for all the different epochs.
3.2. Subpulse Phase and Orthogonal Polarization Modes

Each of the 10 data sets shows a subpulse phase jump at a pulse longitude of 56°:5 (offset by 3:5 from the maximum in the pulse profile), as shown in the right panels of Figure 2. We calculate the subpulse phase using the amplitude and phase of the peak of \( P_3 \) in the LRFS. Figure 2 shows the subpulse modulation envelope, the subpulse phase, and the subpulse phase with the nominal phase slope of 28.7 degrees per degree removed (bottom right panel). Instead of fitting a line to the subpulse phase jump like Edwards & Stappers (2003), we fit two lines: one on each side of the phase jump. The solid lines in the bottom right panel of Figure 2 illustrates this fitting. The difference between the two parallel solid lines at pulse longitude 56°:5 is the subpulse phase jump; in Figure 2, the jump is 145°:1. The middle panel of Figure 3 shows the subpulse phase jump for each epoch and the bottom panel shows the average slope (taken from the two lines fit on either side of the jump). The magnitude of the jump, present in seven of the 10 data sets, ranges from 95° to 147°.

The orthogonal polarization modes have distinctly different behavior on the leading and trailing edge of the pulse profile; the behavior changes at 56°:5 as well, coincident with the subpulse phase jump. As shown in Figure 1, the left side of the pulse profile appears to be dominated by a single polarization mode; the right side of the pulse profile shows a combination of two orthogonal polarization modes. This is consistent with the polarization behavior described by Edwards (2004) at 1380 MHz.

4. OUR NON-RADIAL OSCILLATION MODEL

In previous papers, we developed an oblique pulsator model (Kurtz 1982) for pulsars in which drifting subpulses are produced by non-radial oscillations whose periods are incommensurate with the spin period of the pulsar (Clemens & Rosen 2004, 2008). The non-radial modes of our model are aligned to the magnetic axis, so in addition to the drifting time-like pulses, our model produces longitude stationary variations caused by nodal lines rotating past our line of sight (Clemens & Rosen 2004). Nodal lines are places of unmodulated emission and are described by the zeros in the spherical harmonic in Equation 1. The emission on either side of a nodal line is out of phase and the subpulse period is defined as the rate at which a region between two nodal lines emits radio emission.

The pulsations cause displacements of stellar material which modulate the linearly polarized emission, as might be produced by curvature radiation. These displacements have a transverse component which points toward the magnetic pole and follows the rotating vector model of Radhakrishnan & Cooke (1969, see Figure 5). This mode of polarization, the displacement polarization mode, can be described as (Clemens & Rosen 2008):

\[
A_{\text{VPM}}(t) = a_{\text{VPM}} + a_{1\text{VPM}} \Psi_{l,m=0}(\theta_{\text{mag}}) \cos(\omega t - \psi_0 - \psi_{\text{delay}}),
\]

where \( \Psi_{l,m=0} \) is a spherical harmonic of high \( l \) and \( m = 0 \). The variable \( \theta_{\text{mag}} \) refers to the magnetic co-latitude, because the pulsations in our model are aligned to the magnetic pole. The subpulse period, \( \omega_s \), is related to the fundamental subpulse period, \( P_{\text{time}} = 2\pi/\omega_s \). The phase term \( \psi_0 \) allows for the arbitrary phase of the drifting subpulses. The \( \psi_{\text{delay}} \) term allows for a time lag between the maximum amplitude of the pulsations and emission maximum; for non-adiabatic oscillations, the thermal maximum can lag in phase and \( \psi_{\text{delay}} \) allows for this effect (Clemens & Rosen 2008). The amplitudes \( a_{\text{VPM}} \) and \( a_{1\text{VPM}} \) are to be fitted to the data, as well as to \( P_{\text{time}}, \psi_0 \), and \( \psi_{\text{delay}} \).

The pulsational displacements and their associated velocities move in the plane of the magnetic field. Thus, the induced electric field as a result of the velocities \( \vec{E} = \vec{v} \times \vec{B} \) is naturally orthogonal to the Radhakrishnan and Cooke vector (see Figure 5). The polarization mode that results from the induced electric field due to the velocities is the velocity polarization mode and is expressed as

\[
A_{\text{VPM}}(t) = a_{\text{VPM}} \frac{\partial \Psi_{l,m=0}}{\partial \theta_{\text{mag}}} \sin(\omega t - \psi_0),
\]

which incorporates the time derivative and the \( \theta_{\text{mag}} \) derivative of Equation (1), as appropriate for horizontal pulsation velocities. This equation is analogous to the \( V_{\theta} \) in Equation (3) of Dziembowski (1977).

Thermal and field emission from the neutron star surface can accelerate electrons along open field lines with the formation of a vacuum gap (Jessner et al. 2001). Strohmayer (1992) proposed that neutron star oscillations could modulate the radio intensity if greater quantities of plasma are injected into the magnetosphere during pulsation maxima, when local heating of the stellar surface is greatest. As discussed in Clemens & Rosen (2008), we assume that the amplitude of the displacement polarization mode follows surface thermal variations caused by non-radial oscillations. This means that for non-adiabatic oscillations, the thermal maximum can lag the displacements in phase, parameterized by \( \psi_{\text{delay}} \) in Equation (1).

The parameters in our model can be divided into two groups: geometrical parameters and pulsational parameters (Rosen & Clemens 2008). The geometrical parameters, discussed in Section 5.1, are largely independent; only \( \beta \), the angle between the magnetic pole and our line of sight, and \( \ell \), the degree of the spherical harmonic, are related. There are seven pulsational parameters—amplitudes, periods, and phases and all of these are fit within our model.
5. QUANTITATIVE FITTING OF PSR B0809+74

The fitting process occurs in two steps: we fit the polarization angle swing to determine the pulsar geometry and then we fit the data to our non-radial oscillation model to determine the pulsational parameters. Since we do not expect the geometry of the star to change over time, we added the polarization angle from each of the data sets together and determined the geometry from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit. Because our model does not incorporate circular polarization, we removed the Stokes parameter $V$ from a single fit.

5.1. Pulsar Geometry

We fit the polarization angle swing to determine the pulsar geometry, namely $\alpha$ (the offset in rotation and magnetic axes), $\beta$ (the angle between the magnetic pole and our line of sight), $\phi_o$ (the rotational longitude of the magnetic pole), and $\chi_o$ (the position angle of the linear polarization at $\phi_o$). The only pulsational parameter that interacts with these parameters is the spherical harmonic degree $\ell$ because the positions of the nodal lines that encircle the magnetic pole are related to $\ell$, and the path our sightline crosses through these nodal lines is related to $\beta$.

To determine the geometry, we use the polarization angle histogram rather than the individual polarization angle measurements (Everett & Weisberg 2001). To do this, we compute a single histogram of the polarization angle using all 10 data sets. We then discard all polarization angles less than 90°, so that we fit a single polarization mode. Since the polarization angle is a histogram, each bin has a distinct number of counts. Using the counts in each bin (not counting the discarded polarization angles less than 90°), we calculated the standard deviation for the counts per bin. We fit only the polarization angle bins in the histogram that have counts greater than $2\sigma$. The top panel of Figure 6 shows the average number of counts per bin for each polarization angle. The dotted line represents our $2\sigma$ cut off; we only fit values above this line. This process prevents our fits from being dominated by the noise.

To fit our data, we convert each count in each bin in the polarization angle histogram above the $2\sigma$ threshold to $x$–$y$ values. If a bin at a given polarization angle has 20 counts, we create 20 $x$–$y$ pairs at that value. The more counts per bin, the more $x$–$y$ pairs are created at the polarization angle. We do this for all the bins above the threshold and then fit all the $x$–$y$ points.

In Rosen & Clemens (2008), we found that the four geometrical parameters ($\alpha$, $\beta$, $\chi_o$, and $\phi_o$) cannot be fit independently and it is the ratio of $\alpha$ to $\beta$ that is significant; for any value of $\alpha$ and $\chi_o$, corresponding values of $\beta$ and $\phi_o$ could be found with approximately the same goodness of fit. Therefore, to determine the geometry, we fit $\alpha$ and $\chi_o$ for various values of $\beta$ and $\phi_o$. Figure 7 shows a map of the standard error for all trial values of $\beta$ and $\phi_o$. We determined the best geometry from the combination of these two parameters that has the smallest standard error, denoted by the circle in Figure 7.

We find that a $\beta$ of $-3:1$ and a $\phi_o$ of $76:75$ had the smallest standard error. For these values of $\beta$ and $\phi_o$, the best-fit results in an $\alpha$ of $16:81$ and a $\chi_o$ of $-199:29$. Interestingly, the fit places the closest crossing of the magnetic pole, $\phi_o$, on the trailing edge of the pulse profile, offset from the maximum intensity. This is similar to B0656+14: the average polarization angle shows a shallow slope slightly more curved at the tail end of the pulse profile (Everett & Weisberg 2001). Everett & Weisberg (2001) fit the polarization angle and find $\phi_o$ to be offset from the maximum in the total intensity by 14°:9 in pulse longitude; in our case, $\phi_o$ is offset from the maximum in the total intensity by 19°:17.

These values differ from that of Rankin (1993b), who found $\alpha$ and $\beta$ to be 9° and 4°:5, respectively. Rankin (1993a) determines the geometry based on the half-power width of the pulse profiles and the beam radius at 1 GHz. The difference between the values

![Figure 6](#)
of Rankin (1993b) and our fits is most likely due to the different approach in determining the geometry and to the change in geometry with respect to the observing frequency (Smits et al. 2006). We find that the value of $\phi_o$, while at the trailing edge of the pulse profile, is consistent with the standard polar cap size. Using similar analysis to that of Weltevrede & Wright (2009), we find that the emission height is about 0.08$R_{LC}$, where $R_{LC}$ is the light cylinder radius, and that the size of the magnetic cap is between the magnetic axis and the last open field line. Even if we assume that the steepest polarization angle swing coincides with the center of the pulse profile and half of the emission region is missing, the polar cap size does not extend past the last open field line.

At the bottom panel of Figure 6, we show the portion of the histogram data that we use to fit the polarization angle as well as our fit. We also show the polarization angle using the values in Rankin (1993b) and assuming $\phi_o$ is at the maximum of the pulse profile ($\sim 18^\circ$ in Figure 7 or 56.5 in pulse longitude) and $\chi_o = -134^\circ$. However, for the fit using the values in Rankin (1993b), we flipped the sign on $\beta$ for internal consistency (Everett & Weisberg 2001).

5.2. Pulsational Parameters

To verify that a longer subpulse period, consistent with the literature at low frequencies, matches the data better than a period of 39 ms, we fit our model to the data that we discuss in detail in Section 5.2. For each of the 10 data sets, we fit the full range of observed subpulse aliases from 31.5 to 51.5 ms. For nine out of 10 scans, a smaller root mean square residual is obtained for a subpulse period in the range from 48 to 54 ms than the 39 ms where the power in the FFT peaks. We fix the subpulse period in our fits rather than letting it be a free parameter because several data sets show that the peak due to the subpulse period has a secondary peak of lesser amplitude at a spacing several time larger than the resolution in the FFT. We speculate that the presence of nulls in some of the data sets causes the subpulse period to wander throughout the observations (see Table 2). van Leeuwen et al. (2003) show similar behavior where the subpulse drift rate in PSR B0809+74 changes after nulling.

After establishing the pulsar geometry, we fit the pulsational parameters, namely, the spherical degree, $\ell$, the amplitudes of the displacement, and velocity polarization modes ($a_{1DPM}$, $a_{0DPM}$, and $a_{0VPM}$), the arbitrary phase of the drifting subpulses ($\psi_o$), and the phase that allows for a time lag between the maximum amplitude of the pulsations and emission maximum ($\psi_{delay}$). Because the subpulse period has a small secondary peak in some of the data sets (see Section 3.1), we fix the subpulse period based on the subpulse alias in the FFT (see below). The subpulse period is similar but unique for each data set.

As with our fits to the polarization angle, we fit each data set to our model for various values of $\ell$. For each data set, the value of $\ell$ that returns the smallest $\chi^2$ is the best fit to the data. We treat each data set separately in determining the goodness of fit because of the variation of stochastic pulse amplitudes. The data sets that have small variations in the pulse amplitude will have a smaller standard errors compared to data sets that have large variations. We estimate the noise $\sigma_i$ in our data using the radiometer equation:

$$\sigma_i = \frac{T_{sys}}{\sqrt{B\tau}},$$

where $T_{sys}$ is the system equivalent flux density in mJy ($\sim 14$–18 Jy depending on the epoch), $B$ is the bandwidth (200 MHz), and $\tau$ is the integration time (160 $\mu$s).

The top panel of Figure 8 shows total $\chi^2$ for all the data; the bottom panel shows the individual $\Delta\chi^2$ for each epoch as a function of spherical degree, $\ell$. Each dot represents one trial, integer value of $\ell$. For all data sets, $\ell$ is either 18 or 19; the difference is within the error of our fit. The best value of $\ell$ based on the total $\chi^2$ is 19.

We use a method similar to that for finding the best value of $\ell$ to find the best subpulse period. Since our model fits the
data in real space ($I$, $Q$, and $U$) rather than Fourier space, and a subpulse phase jump is a natural part of the model due to the presence of a nodal line, fitting the data to our model is more accurate method for determining the subpulse period than using the FFT. For each data set, we choose a subpulse alias from the FFT and fit our model to the data, calculating $\chi^2$. We repeat this for every subpulse alias from 31.5 to 58.5 ms for each data set, using the value of the subpulse period with the smallest $\chi^2$ as the best fit. The top panel of Figure 9 shows the total $\chi^2$ for values of the subpulse period using different subpulse aliases in the FFT from 31.5 to 51.5 ms; each dot is one alias from the FFT. The bottom panel shows the individual $\Delta \chi^2$ for each epoch. The subpulse period for all data sets except for one is between 48 and 54 ms; the best subpulse period based on the total $\chi^2$ is 51.5 ms.

Once we determine the best value of $\ell$ and the subpulse period for each data set, we use Equations (1) and (2) to fit our single-pulse data to the model. From Equations (1) and (2), we can create the primed Stokes parameters, the Stokes parameters in the non-rotating frame of the star (Rosen & Clemens 2008). However, instead of defining an emission window as we did in Rosen & Clemens (2008), we multiply the Stokes parameter $I$ by the average pulse shape. We then used Equations (7)–(9) from Rosen & Clemens (2008) to transform the primed Stokes parameters into the observer’s frame of reference (unprimed space), and similarly multiplied $L$, which is invariant under this transformation, by the average linear polarization. For each epoch, we use the best value of $\ell$ and the subpulse period, as determined in the process outlined above, and fit the remaining parameters ($a_{1\text{DPM}}$, $a_{0\text{DPM}}$, $a_{0\text{VPM}}$, $\psi_o$, and $\psi_{\text{delay}}$). Figure 10 shows the reduced $\chi^2 (\chi^2_{\nu})$ for each epoch. The value of $\chi^2_{\nu}$ strongly correlates with the mean flux (see the top panel of Figure 3). Table 3 lists the best fit for the geometrical and pulsational parameters for MJD 54922.
Figure 10. Data fitted to our model, where the amplitudes and phases are the free parameters. We set the values of $\ell$ and the subpulse period based on the methodology described in the text. The plot shows reduced $\chi^2 (\nu)$ resulting from our fit for each epoch.

Table 3
Geometrical and Pulsational Parameters Resulting from Our Fit of the Data from 54922 to the Model

| Parameter | Value |
|-----------|-------|
| $\alpha$  | 16.61 |
| $\beta$   | −3.1  |
| $\phi_0$  | 76.75 |
| $\chi_0$  | −199.29 |
| $\omega_{\text{DPM}}$ | 1.201 |
| $a_{1\text{DPM}}$ | 0.733 |
| $a_{0\text{VPM}}$ | 0.055 |
| $\psi_0$  | −85.66 |
| $\psi_{\text{delay}}$ | 113.857 |
| $P_2$ | 51.503 ms |
| $I$ | 19 |
| $\chi^2_{\nu}$ | 7.24 |

Using the fitted parameters, we create synthetic light curves and compare them to the data. Figure 11 shows data from MJD 54922 and a simulation using our fitted parameters. The data show a phase jump of 145.1, while the data produced by our model have a phase jump of 187.7. The fact that our model does not reproduce the phase jump exactly is not surprising; the phase jump is fit indirectly in our model. Our model fits $I$, $Q$, and $U$ and the phase is computed secondarily. The phase jump occurs at 56.5, coincident with the nodal line, as a result of our best fit of $\ell$. Figure 12 shows the LRFS and drift-bands of the data from MJD 54922 and simulation. The harmonic in the simulation of 0.14 Hz is more pronounced than that of the data. This is mainly due to the fact that the data neither contain pure sinusoids nor a perfect window function, while our simulation does. The two-phase terms in our fitting, $\psi_o$ and $\psi_{\text{delay}}$, are most prominent in the drift-band plot. In the data, the maximum intensity occurs between a pulse phase of 60° and 65°; altering $\psi_o$ changes the location of the maximum intensity in both pulse phase and $P_3$. In the simulation, the drift-band has a feature at a pulse phase of 65° and period of 14 s. This small feature, less obvious in the data, is due to the velocity polarization mode. The value of $\psi_{\text{delay}}$ dictates the spacing of between the two polarization modes. Figure 13 shows single pulses of the same data and simulations as in Figure 11. In the simulations, the subpulses on the leading edge of the profile are due to the velocity polarization mode and are slightly weaker than those due to the displacement polarization mode.

6. DISCUSSION

6.1. Observations in the Context of a Non-radial Oscillation Model

A non-radial oscillation model explains the wide range of behaviors seen in slow pulsars and is substantially simpler and more cohesive model than those based on drifting sparks. We expect that the subpulse period should not change with radio frequency. We also expect that a nodal line must correspond with a region of zero modulated emission and a subpulse phase jump.

The radio frequency dependence of $P_2$ is a matter of debate. Davies et al. (1984) found that for PSR B0809+74, the subpulse spacing ($P_2$) appears to change with frequency, but $P_3$ does not. Izvekova et al. (1993) found similar results for four additional pulsars. However, the average pulse shape also changes with frequency (Izvekova et al. 1993), resulting from the divergence of the magnetic field and the change in the radial distance from the magnetic pole. High frequencies originate close to the stellar surface, so the average pulse profile is narrow, but emission at these frequencies originates further from the magnetic pole radially than low frequencies (Komesaroff 1970; Cordes 1978; Smits et al. 2006). This scaling law, called radius-to-frequency mapping, can be mistakenly applied to $P_2$ in two ways. As the average pulse profile broadens at lower frequencies, the spacing between the subpulse increases as well (Ruderman & Sutherland 1975; Izvekova et al. 1993; Wolszczan et al. 1981; Bartel et al. 1981; Gil &
Figure 11. Top panels: the average pulse profile (solid line), the average linear polarization (dashed-dotted line), and the subpulse modulation envelope (dotted line) of the data from MJD 54922 (left) and model (right). Middle panels: the polarization angle histogram for both the data and model. Bottom panel: the subpulse phase, calculated from the LRFS (see Section 3.2). The solid line is a linear fit to both components of the subpulse phase.

Figure 12. Left column: the LRFS (top) and drift-band of data from MJD 54922. The left panel shows the integrated pulse profile; the maximum on the plot is 1100 mJy. The bottom panel shows the power in mJy$^2$ Hz$^{-1}$. Right column: the LRFS and drift-band of simulated data using the best parameters as determined by our fit of the data to our model. The panels in the LRFS have the same scale and units as the data on the top left plot.
Krawczyk 1996). However, as Edwards & Stappers (2002) describe, the change in $P_2$ with frequency is a result of the change in the window function: as the average pulse profile broadens (or narrows) the window function that modulates the subpulse period changes as well, affecting the apparent spacing between subpulses. Secondly, as illustrated in PSR B0809+74, the average pulse profile and subpulse phase behavior can change quite noticeably between different frequency. The appearance of a subpulse phase jump will change the measured value of $P_2$ (see Section 3.1). Both the subpulse period and phase envelope should be independent of frequency (Edwards & Stappers 2002; Clemens & Rosen 2004).

Because the oscillation mode is a fundamental property of the star, we expect the subpulse period to be independent of observing frequency. However, the subpulse period can be distorted by both the presence of a nodal line and the width of the average profile, and therefore the apparent spacing of $P_2$ is not an accurate measurement of a stable, underlying clock. $P_{\text{true}}$ (Clemens & Rosen 2004). In the case of PSR B0809+74, since the change in frequency effectively changes our sightline traverse, $\beta$ (Smits et al. 2006), the subpulse phase jump that appears at higher frequencies can be attributed to a nodal line moving into our line of sight due to the change in effective geometry. The subpulse period would remain the same, but the apparent spacing changes. We see this in the period of PSR B0809+74: the FFT incorrectly determines subpulse spacing to be around 39 ms, but fitting the single pulses more accurately measures the subpulse period to be 48–54 ms. While our measurements of the subpulse period appear to be consistent with previous published results (Bartel et al. 1981; Davies et al. 1984; Edwards & Stappers 2003), the true test to determine if the subpulse period is independent of frequency is to conduct simultaneous, multifrequency observations.

The subpulse phase jump appears to change with observing frequency. Edwards (2004) and Edwards & Stappers (2003) report a phase jump of approximately 45° and 120° in the total intensity at 328 MHz and 1380 MHz. Using the methodology of Edwards & Stappers (2003), we find the subpulse phase jump to be 116° at 820 MHz; using a slightly different technique described in Section 3.2, we find the phase jump to be 145°. Regardless, the appearance of the phase jump at 820 MHz and above is consistent with a nodal line moving into our line of sight. This hypothesis is borne out in the subpulse modulation envelope shown in the left upper panel of Figure 2. The dotted lines in the top left panel show the average profile and subpulse modulation envelope at 328 MHz; the solid lines are the same for 1380 MHz. The subpulse modulation envelope at 328 MHz is a single peak, while the modulation envelope at 1380 MHz shows a minimum which would be consistent with the presence of a nodal line.

In our model, the choice of $\ell = 19$ places a nodal line at 56.5° and produces the phase jump at the same location. The combination of a nodal line and $\phi_o$ offset from the center of the pulse profile is responsible for the presence of two orthogonal polarization modes on the trailing edge of the pulse profile. While the amplitude of the displacement and velocity polarization modes are constant throughout the simulation, the asymmetry due to $\ell$ and $\phi_o$ causes the velocity polarization mode to be suppressed on the leading edge.

This value of $\ell$ is significantly smaller than the value we calculated for PSR B0943+10 ($\ell = 75$) (Rosen & Clemens 2008). In Clemens & Rosen (2004), we estimated the magnification

![Figure 13](image-url)
factor to map the $\ell$ we observe to the surface of the star. We found that at 1 GHz, the magnification factor was $\sim 7$, and thus in our qualitative model an $\ell$ of 85 translated to an $\ell$ of 600 on the stellar surface. In Rosen & Clemens (2008), we found $\ell$ to be 75 using 430 MHz archival data (Deshpande & Rankin 2001). Assuming that observations at lower frequencies are at higher altitudes from the surface of the star, and thus the magnetic field is more divergent, the magnification factor is then a lower limit and the $\ell$ of 75 translates to a lower limit of $\ell = 525$. In these data, with an $\ell$ of 18 or 19, the lower limit becomes an $\ell$ of 130 on the stellar surface. We note here that the value of $\ell = 75$ for PSR B0943+10 was not rigorously determined using the methodology outlined in this paper, and smaller values for $\ell$ for that star are possible.

7. CONCLUSIONS

This paper shows the second quantitative fit of our non-radial oscillation model to single-pulse data of a pulsar (Rosen & Clemens 2008). In this paper, we

1. show the subpulse period and subpulse phase jump, previously unpublished at 820 MHz.
2. show the subpulse phase jump and subpulse period changes with epoch.
3. are able to quantitatively determine the best value of $\ell$ and the subpulse period. Our method for determining the subpulse period is more accurate than using the FFT as it determines the subpulse period in real space rather than frequency space, which can be affected by the subpulse phase jump. Our fits using $I$, $Q$, and $U$ account for the subpulse phase jump since it is a natural part of the model.
4. are able to quantitatively fit single-pulse data to our model and determine a goodness of fit using $\chi^2$ and $\chi^2$, statistics.
5. can create simulations based on our fitted parameters which accurately reproduce the subpulse period, subpulse phase jump, and orthogonal polarization modes.

The morphology of PSR B0809+74 is explained easily and naturally within a pulsational model. Our non-radial oscillation model is based on established asteroseismological principles that have explained white dwarf variations for the past 40 years (Dziembowski 1977). This model is a viable alternative to the drifting spark model and can provide physical insight to the emission mechanism and physical structure of neutron stars.

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