Developing learning trajectory for teaching statistics at junior high school using RME approach

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Abstract. We developed a learning trajectory (LT) for teaching statistics at Year 8 junior high school using Realistic Mathematics Education (RME) approach. The research aimed at investigating the validity and practicality of the LT as well as its influence on students’ statistical reasoning abilities. This was a design research that consisted of a cyclic process of preparing and conducting the experiment, and retrospective analysis. The subjects of the research were 30 Year 8 students in a junior high school in Indonesia. To collect the data, we used observations, interviews, checklist, videotaping, and analyzing the students’ works. In preparing for the experiment, it was designed the LT for teaching statistics that met the criteria of validity (relevance and consistency). During one-to-one, small group, and field test evaluations we discovered that the LT could work as intended (met the criteria of practicality). We also found that the LT for teaching statistic using RME approach could help the students to reinvent the concepts about central tendency by themselves. Finally, we found the growths in the students’ statistical reasoning abilities.

1. Introduction
Almost every aspect of our life is related to numbers and data. When doing activities at home, reading a newspaper, watching television, or buying something in a supermarket, we deal with numbers and data. Bailey [1] mentioned that we are drowning in data, while Steen [2] argued that the world of the twenty-first century is a world awash in numbers. To be able to deal with the data appropriately, people need to be statistically literate and have good statistical reasoning. Watson [3] said that the ability to provide reasonable evidence based arguments and to evaluate data based claims critically are important skills that all citizens should have.

As the need to improve statistical literacy and statistical reasoning grow, they become part of the mainstream school curriculum in many countries [4]. We can even find the topics of statistics in the primary school curriculum in many countries, including Indonesia. According to Garfield and Ben-Zvi [5], this condition is not surprising because statistics instruction at all educational levels is gaining more students and drawing more attention.

Although the topics of statistics have been introduced since early grades, many studies showed that the students still find difficulties when they learn similar concepts in the higher school levels [6-9]. There were some causes of this condition. Firstly, the focus of teaching statistics was more on applying formulas and computations [10,11]. Secondly, statistical concepts presented in mathematics textbooks were not sequenced in a meaningful way [12,13]. These causes were crucial because it also influenced the way the teachers teach in the classrooms that tend to be mechanistic [14-16]. To
overcome these problems, we developed a learning trajectory (LT) for teaching Statistics in junior school using realistic mathematics education (RME) approach.

A learning trajectory is the sequences of activities and tasks that designed by mathematics teachers or researchers to guide students to achieve a specific instructional goal [17]. Gravemeijer [18] argued that neither teachers nor researchers could rely on fixed teaching sequences, because they continuously have to adapt to the actual thinking and learning of her students. Therefore, in the beginning, an LT is designed in the form of a hypothetical learning trajectory (HLT). An HLT consists of the learning goal, the learning activities, and the hypothetical learning processes; a prediction and anticipation of how the students’ thinking and understanding will evolve in the context of the learning activities [17,19]. After the HLT is tested through a cyclic process in the classroom, it becomes an LT; a theory that can be used to teach the topic by other teachers. The theory is called a local instructional theory (LIT) [20,21] or a domain specific theory [22] that explain about both the process of learning a particular topic and the means to support that learning [18]. In this research, we developed a local instructional theory to teach statistics for the students at the Year 8 junior high school.

Many researchers and math educators have developed learning trajectories [23-27] that covered various topics such as statistics, social arithmetic, optimization, and logarithm. These studies showed that the LT’s could work not only for teaching mathematics in the primary schools but also in the colleges. The results of the research mostly indicated that the LT’s could help the students to learn mathematical concepts in many meaningful ways [24,28].

The idea of learning mathematics in an LT is in line with the idea of horizontal and vertical mathematization in RME approach, as can be seen in Figure 1. Therefore, the LT that we designed in this research was based RME approach.

![Figure 1. Processes of horizontal and vertical mathematization [19]](image-url)

As we can see in Figure 1, learning mathematics using RME approach is started by giving contextual problems that can stimulate students to use their informal knowledge and their strategies to solve the problems. Gravemeijer [19] called this process as horizontal mathematization. Step by step, and after experiencing similar processes, students will use more formal knowledge, symbols, or strategies for solving the problems. It means they start experiencing vertical mathematization, until they use formal mathematical language or algorithm to solve contextual problems.

In designing the LT for teaching statistics, we also referred to the key principles of RME, namely guided reinvention through progressive mathematization, didactical phenomenology, and emerging models [29,30]. In implementing the LT in the classrooms, we adopted the characteristics of RME for teaching and learning [31-34]. Although the RME approach has been established since the 1970’s, especially in the Netherlands, it still powerful to solve current problems in mathematics education. Many researchers from different countries [35-38] recently used the approach to stimulate the students’ thinking and their mathematical abilities. In this research, we investigate the effect of the RME approach on the students’ statistical reasoning ability. To focus the research, we formulated a
research question as follows: what are the characteristics of the learning trajectory for teaching Statistics using RME approach which are valid, practical, and effective to stimulate students’ statistical reasoning ability?

2. Method
The LT for teaching statistics was developed by using design research approach mentioned by Gravemeijer and Cobb [19] because it gave more insight about the interrelatedness between teaching and learning to improve teaching [18]. The design research approach was implemented in a cyclic process of preparing for the experiment, conducting the experiment, and retrospective analysis, as illustrated in Figure 2.

![Emerging Local Instructional Theory](image)

**Figure 2.** The cyclic process of design research [19].

In preparing for the experiment, we determined the endpoints of the instructions in which the students will reinvent the concepts about central tendencies by themselves. Therefore, we analyzed the essential concepts about statistics in the curriculum, the literature about teaching statistics, and the characteristics of the students such as their hobbies, their preferences in learning, and their preferences activities. Based on the analysis, we designed the three components of HLT [17,19] for teaching statistics using the RME approach. Then, three experts in mathematics education and one expert in instructional design validated the HLT to judge its validity (relevance and consistency) [39].

After the revision, the HLT was tried out in three cycles, namely one-to-one, small group, and field test. The first two cycles respectively involved three and six Year 8 students in a junior high school in Indonesia. To analyze the results of the tried out, we conducted the retrospective analysis and re-designed processes that involved the research team, a teacher, and an observer. The revised version of the HLT was implemented during the field test to 30 Year 8 students in the same school. To collect the data we used observations, interviews, checklist, videotaping, and analyzing the students' works. Collected data were analyzed using a descriptive technique.

3. Result and discussion
Based on the primary analysis during the preparing for experiment phase, it was designed the HLT for teaching statistics using RME approach. The learning activities in the HLT can be described as follows. The first meeting was about exploring the concept of mode in which the students collected the data about students’ preference of candy flavors. Then, they were asked to present the data in various representations. By analyzing the data representations, the students can reinvent the concept of mode as the most favorite candy flavor. Besides, the students will also observe the concept through different representations. In the second meeting, the students reinvent the concept of median by exploring contextual problems about a company that wants to establish its headquarters at one of the branch
offices on a highway. To reinvent the concept of mean, the students will get involved in the activities of dividing marbles, the yield of maize, and estimating the number of large quantities. We chose the activity about fair share to reinvent the concept of mean due to the opinion of Franklin and Kader [40]. They stated the mean as fair share and balance, therefore to find the mean concept informally, students can divide fairly. In these activities, the students will get an opportunity to use their informal knowledge and strategies to explore the concepts.

The relevancy of the HLT to the state of the art knowledge and the consistency of each component were validated by three experts in mathematics education and one expert in instructional design. After the revision processes, the HLT reached the criteria of validity [39] with the characteristics: 1) the HLT reflected the key principles and characteristics of RME, 2) the activities provided in the HLT could give a meaningful learning experience for the students to reinvent the concepts of central tendency, and 3) the components in the HLT consistently supported each other. Moreover, the results of one-to-one and small group evaluation revealed that, in general, the HLT could work as intended in the classrooms. Based on the results of the evaluations, the HLT was revised until it fulfilled the criteria of practicality [39].

In teaching experiment during the field test, it was observed that the HLT could help the students to reinvent the concept of mode by exploring the contextual problems. In this case, they discover by themselves the concept of mode as the most favorite candy flavor by listing the result of data collection in a table. They also could recognize the concept of mode when the data represented in various diagrams by referring to the frequency of the highest bar (in bar diagram), the biggest part (in pie diagram), or the highest point (in a line diagram). This finding indicates one of RME’s principles namely guided reinvention [20] in which the contextual problems in the HLT could guide the students to reinvent the concept of mode. An example of a student's answer can be seen in Figure 3.

![Figure 3. Student’s answer when exploring the concept of mode using diagrams](image)

To reinvent the concept of median, the students were given the next contextual problem. A company has five branch offices that located along the main street between Padang and Painan. The first branch office is located in Padang, while the other offices are located 20 km, 28 km, 58 km, 89 km respectively from the first office. The director of the company wants to establish its headquarters at one of the branch offices so that it has the shortest distance to other branch offices. Which branch office the director must choose? Explain your answer.

In solving this problem, most students used two different strategies. In the first strategy (see Figure 4), the students tried to put each branch office in logical order, and then calculate the distance among the branch offices. By using this strategy, the students concluded that the director must choose the branch office located in the middle as its headquarters. This experience helped the students to understand the concept of median as the middle of a sorted list of data.
In the other strategy (see Figure 5), the students calculated the difference of the distance between one branch office to another. They decided to choose branch office C as the headquarters because total deviation (in distance) to other branch offices was the smallest. This strategy leads the students to discover a characteristic of median as the smallest total deviation [41].

Based on the results of the experiments and retrospective analysis phases we can argue that the activities designed in the LT could facilitate the students to reinvent the concepts in statistics. The activities were well sequenced so that the students could learn the concepts in a meaningful way. The predictions about the students’ thinking (written in the LT) mostly accurate so that the teachers could use the right probing questions (that already prepared) to guide the students to achieve the learning goals. These findings strengthen the results from previous studies that the LT was very helpful in stimulating the students to reinvent mathematical concepts [23-29]. RME approach also played an important role in building the students' understanding in this research. The contextual problems designed in the LT gave the opportunities to the students to use their informal knowledge and strategies to solve the problems. These conditions related to RME characteristics namely students’ contribution and students’ free production [20,29,32]. By experiencing these characteristics, the students built their confidence in learning mathematics because they can give a contribution to the learning process. Finally, during the implementation of the LT, we gradually observed the growths in students’ statistical reasoning ability.
4. Conclusion
In this research, we designed a learning trajectory for teaching statistics using RME approach. The LT reached the criteria of validity, as it reflected the key principles and characteristics of RME. The activities provided in the HLT could give a meaningful learning experience for the students to reinvent the concepts of central tendency, while the components in the HLT consistently supported each other. The LT also fulfill the criteria of practicality, as it could work as intended in the classrooms. The students had more confidence to use their own strategies in solving contextual problems. The most important thing, we discovered the growth in the students’ statistical reasoning.

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