RELIABILITY ASSESSMENT OF AN EXISTING STEEL PLATE GIRDER BRIDGE USING POSTERIOR DISTRIBUTIONS OF MODEL PARAMETERS

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This study shows the applicability of using the posterior distributions of model parameters on the structural reliability analysis of existing bridges. The target here was a steel plate-girder bridge that showed corrosions and deterioration in its girders, bearings, and concrete slab. The mean and variance of resonant frequencies were acquired through multiple measurements of traffic vibration for use as observation data in the Bayesian inference. A nominal finite element (FE) model of the target bridge was then constructed. The prior distributions of model parameters that represented the uncertainties due to the deteriorations were then determined based on the inspection data, and the parameters that had sensitivities to resonant frequencies were extracted by the analysis of variance (ANOVA) to prevent ill-posedness. The posterior distributions of the spring stiffness at the ends of steel girders, which were given to represent performances of the corroded bearings in the FE model and the density of the concrete slab, were estimated by applying surrogate modeling and the Markov Chain Monte Carlo (MCMC). It was then recognized that the uncertainties of those model parameters could be reduced, and those posterior distributions could explain the actual deteriorations. The posterior distributions were used to calculate the structural reliability index $\beta$ of the target bridge. The probability density functions of maximum Mises-stress on the steel girders under the design load were first obtained by the Monte Carlo FE model calculations using the prior and posterior distributions, respectively. In the results, the reliability index against the variability of steel yield stress in the case of posterior distributions became higher than that in the case of the prior. It was then concluded that the procedure presented in this study could provide an understanding of the structural reliability of existing bridges quantitatively.

Key Words: posterior distribution, reliability index, steel plate-girder bridge, corrosion, dynamic measurement data, inspection data

1. INTRODUCTION

Many of the bridges in Japan were constructed in the 1960s and 1970s, thus they are now approaching 50 years of service. Performing proper and efficient maintenance of these bridges has been required, and periodic inspection of all bridges at least once every five years has been started since 2014. Bridge owners sometimes have to decide on countermeasures based on the degree of damage determined in the inspection; however, it is difficult to make decisions on whether the structure should be repaired, reinforced or replaced, or to prioritize cases where there are multiple bridges found to be damaged. Model-based performance evaluation of existing structures is expected to be effective in these decision makings. Here, a numerical model, such as finite element (FE) model, of the target structure is constructed, and the capacity for external load is evaluated based on the structural reliability. For proper performance evaluation of existing bridges, uncertainties in structural properties due to damage and deterioration cannot be ignored, and they must be considered in the stochastic process of the structural reliability analysis.
Some previous studies worked on the structural reliability analysis of bridges considering deterioration. Okasha et al. estimated the amount of thickness reduction in corroded main steel plate-girders using strain measurement data, and the structural reliability index \( \beta \) was obtained using a numerical model where the estimated thickness reduction was represented. Kayser and Nowak predicted aging reduction of structural reliability index \( \beta \) due to corrosion at the end of girder, which typically occurs in steel plate-girder bridges. However, there are few studies that consider the effects of actual damage and deterioration in existing bridges to their current capacity by uncertainty quantification (UQ) using measurement and inspection data. This study approaches this point in the structural performance evaluation of existing bridges.

For accurate model-based performance evaluation, the validity of the numerical model must be ensured considering model parameter uncertainties. It is thus necessary to perform the process of Model V&V (verification & validation). Here, verification is a question of “whether a model is correctly solved.” This includes, for instance, verification of model elements and mesh configuration adopted, and selection of numerical calculation algorithm. Validation is a question of “whether a correct model is solved.” It concerns whether appropriate assumptions are adopted in modeling the target structure and whether validated parameters are assigned. Uncertainties of model parameters are considered in this process; especially, the uncertainty due to deterioration and damage is an issue to consider especially in modeling existing structures. In existing bridges, uncertainties of material properties and boundary conditions increase due to corrosion of steel girders and bearings, crack of concrete slab, and so on. Thus, the author has been working on the uncertainty quantification (UQ) of existing bridges based on Bayesian inference using structural monitoring data. In our previous study, it was shown that the degree of uncertainty reduction in the posterior distribution differed depending on the setting of prior distribution that indicated recognition of the uncertainty of current structural condition of the existing bridge. In addition, another study conducted by Nishio and Hitomi showed that the process of deterioration due to the corrosion of steel bearings can be evaluated by a posterior distributions that can be estimated appropriately by setting prior distributions based on visual evaluation. These results provide the idea for this study aimed at evaluating the structural reliability of existing bridges considering their current structural conditions using the posterior distributions.

This study aims to show the structural reliability evaluation of an existing bridge by deriving the structural reliability index \( \beta \) from the posterior distributions of finite element (FE) model parameters estimated using dynamic monitoring data and visual inspection data. In this paper, the data acquisition of target bridge resonant frequencies is first explained. The data are then used as the observation data for posterior distribution estimation. Next, the construction of an FE model of the target bridge and its verification of mesh convergence are presented. Then, prior distributions, which are the prior uncertainties of model parameters, are set using inspection data. Global sensitivity analysis is conducted to extract the parameters to reduce uncertainties using the measurement data, and the posterior distributions are then estimated by Bayesian inference. This is the Model V&V process, and validities of obtained posterior distributions are evaluated by comparing them with the actual structural condition reported in the visual inspection. The posterior distributions, which are updated uncertainties considering the structural condition, can then be directly used in the Monte Carlo-based structural reliability analysis. Here, reliability index \( \beta \) of the maximum stress of main steel girders under the design traffic load to the yield stress is derived to evaluate the load bearing capacity.

2. RESONANT FREQUENCIES DATA ACQUISITION BY TRAFFIC VIBRATION MEASUREMENT AT TARGET BRIDGE

The target structure in this study was an existing steel plate girder bridge, whose bridge type was the most common one built during the high economic growth period in Japan in the 1960s and 1970s. A visual inspection conducted in November 2014 revealed cracks in the concrete slab, corrosion of the girders, and corrosion of the steel bearings. Resonant characteristics are often used as features to compare properties of structures globally. It was thus expected to show sensitivity to the damages observed at the target bridge. In addition, the resonant characteristics of an actual bridge are relatively easy to acquire just by putting accelerometers on the road. This chapter describes the acquisition of the observation data used for estimating the posterior distributions. The resonant mode characteristics were identified by a modal analysis using acceleration data acquired in the target bridge under normal traffic. The data acquisitions were conducted multiple times, and statistical properties of resonant frequencies were determined for use in the estimation of posterior distributions.
(1) Target bridge and data acquisition

The target bridge in this study is a simply-supported plate-girder bridge located in Yokohama City, which was designed following the Specification for Highway Bridges (published in 1956), and completed in November 1963. It is built over a canal around 200 m distance from the port area. The span length is 11.7 m and the width is 7.2 m, which consists of a single lane with a width of 3.5 m and a sidewalk, as shown in the cross-section view in Fig. 1. The bridge is configured by six I-beam girders and a reinforced concrete (RC) slab. The thicknesses of web, upper flange, and lower flange of the I-beam are 9 mm, 28 mm, and 25 mm, respectively. At both ends of each of the six girders, steel bearings are installed with a steel sole plate measuring 250 mm×500 mm and 25 mm in thickness. The bearings placed on the P1-pier side in Fig. 2 are movable only in the longitudinal direction, and those on the P2-pier side assign the fixed boundary conditions in all directions.

The data of bridge accelerations under normal traffic were acquired for the identification of resonant mode properties of the target bridge. Figure 3 shows the location of six accelerometers in the data acquisition, where each of CH1-CH3 and CH4-CH6 were installed every 1/4 span from the P1 pier side relative to the center side and to the edge side of sidewalk, respectively. The accelerometer used was the piezoelectric sensor (product of PCB 393B04) with the specifications of 3×10^{-5} m/s^2 resolution, frequency range of 0.06 to 450 Hz in ±5%, and acceleration range of ±5,000 Gal. Four data sets and fifteen data sets were acquired on October 13 and October 27, 2015, respectively; therefore, 19 data sets were obtained in total. Each data set was 5 min long with sampling frequency of 2kHz. It can be said that the resonant frequencies identified from each 5min long data are then the averaged dynamic characteristic within the five minutes.

(2) Identification of resonant modes properties

Figure 3 shows the power spectral density (PSD) derived from the acceleration data of CH4 in one of the 19 data sets. It was estimated by the Welch’s method with FFT length of 150,000 (quarter of data length 600,000), 50% overlap averaging, and Hanning windowing. Three peak frequencies were determined by the curve fitting (sixth-order polynomial) within the frequency ranges of ±0.5 Hz around 10, 12, and 18 Hz, respectively. In addition, the mode shapes in the determined three peak frequencies were derived from the phases of cross-power spectrum density between all two channels. The results are shown in Fig. 4. Considering that the sensors of CH1, CH2, and CH3 were placed on the center side in the axial direction of the bridge, the 1st peak frequency is the 1st bending mode because the phases of CH1-CH6 are in-phase. The 2nd peak frequency is the 1st torsion because the mode shape amplitudes in CH4-CH6 are larger than those in CH1-CH3. The 3rd peak frequency is the resonant mode that shows anti-phase between CH1-CH3 and CH4-CH6.

Table 1 shows the resonant frequency mean, standard deviation, and coefficient of variation (COV) identified for the total 19 data sets. The mean was 9.82 Hz for the 1st mode, 11.7 Hz for the 2nd mode, and 18.4 Hz for the 3rd mode. The COV became smaller in the higher order. These means and COVs of resonant frequencies then became the observation data used for the estimations of posterior distributions.

| Mode  | Mean (Hz) | Standard deviation (Hz) | Coefficient of variation (%) |
|-------|-----------|-------------------------|-----------------------------|
| 1st   | 9.82      | 0.148                   | 1.51                        |
| 2nd   | 11.7      | 0.124                   | 1.06                        |
| 3rd   | 18.4      | 0.143                   | 0.781                       |
3. POSTERIOR DISTRIBUTIONS OF FE MODEL PARAMETERS

This section shows the derivation of posterior distributions of the FE model parameters used in the structural reliability analysis. Before estimating the posteriors using the observation data of resonant frequencies acquired in the target bridge, the procedures of FE modeling, prior distributions setting, and sensitivity analysis are required. These processes of Model V&V reduce uncertainties of the existing bridge (uncertainty quantification: UQ).

(1) Initial FE model construction with verification

The FE model of the target bridge was constructed using the general purpose FEA software Abaqus (ver. 6.14-1). Here, the steel girders and the RC slab were modeled using the shell element. The nominal material properties of steel and reinforced concrete were assigned as the initial values of the model parameters. The simply-supported boundary condition was applied to the center point of the sole plate in each end of the six main girders. The boundary conditions at the movable bearings on the P1-pier side were set free only in the longitudinal direction, and those on the other side were fixed in all directions. Although the RC slab and the girder were not jointed in the design, the bridge did not behave as completely unjointed during the normal service; therefore, those two structural members were jointed in the FE model.

The selection of element type and mesh size should be verified in the viewpoint of Model V&V. Here, the 1st-order 4-node quadrilateral shell element with reduced integration was adopted considering the computational cost. The mesh configuration was determined by investigating the convergence of the purpose outputs; i.e., resonant frequencies. The convergence of resonant frequency only in a I-girder was firstly investigated, and the number of element 7,988 was determined. Next, an FE model of the entire bridge with the determined number of elements at six I-girders was constructed, and the convergence of resonance frequency was investigated while increasing the number of elements of the RC slab. From the results shown in Fig. 5, the FE model with 124,108 elements in total was determined as the initial one because all of the 1st to 3rd mode frequencies converged. Figures 6 (a), (b), and (c) show the mode shapes of the 1st to 3rd resonant modes calculated in the initial FE model. It could be confirmed that the mode shapes of the initial FE model showed agreement with those identified from measurement data shown in Fig. 4.

(2) Prior distributions of FE model parameters

a) Model parameters uncertainties in existing structures

One of the uncertainties in the structural calculation model parameters is due to randomness in material properties and dimensions. In most cases, those uncertainties are evaluated by statistical data. For instance, a previous study, which verified the applicability of load and resistance factors determined based on the structural reliability to the design of steel bridges, considered the uncertainties due to randomness of structural properties using past statistical data. In existing bridges, however, it is considered that the uncertainties due to deterioration and damages are more significant on the output response. The prior distributions in this study were set to the FE model parameters that related to the uncertainties due to the corrosions at the steel girders and bearings and the cracks in the RC slab. Here, the periodic inspection data provided from the bridge owner were used to extract the parameters with particularly high uncertainties.

Figure 7 shows pictures of the girders and slab recorded in the periodic inspection of the target bridge. Figures 7 (a) and (b) show the corrosion spread at the bottom surfaces of the lower flanges and the lower side of the web of the steel girders. The cracks locally distributed in the RC deck can be seen in Fig. 7 (d). It was also reported that corrosion was observed in the steel bearings, as shown in Fig. 7 (f).
Corrosion was also observed in the steel piers in the target bridge; however, the parameters of bridge piers were not considered in this study because the data represented the dynamic response under traffic vibration. From these points recognized in the visual inspection, the corrosion of the main girders and the crack in the RC slab were considered by the thickness reduction of shell elements for steel plate girders and the Young’s modulus reduction of shell elements of the slab part, respectively. Also, the movability of corroded steel bearing was considered by the increase in linear spring stiffness, which was assigned in the lower surface of the sole plate elements at the ends of the main girders. The uncertainties of those FE model parameters were then verified.

b) Priors of FE model parameters

The prior distribution adopted to the selected FE model parameters was the uniform distribution with upper and lower ranges. First, in the steel main girders, the corrosion and paint cracking can be recognized on the bottom of the lower flange, as shown in Figs. 7 (a) and (b). Corrosion is also seen on the lower side of the web in Fig. 7 (b); however, it is not as bad as that of the bottom surface of the lower flange. Among the six main girders, there is hardly any corrosion on both the inside and outside of the web at the outside main girders G1, as shown in Fig. 7 (c). From these observations, the upper and lower ranges of the uniform distribution in terms of the thickness reduction at the main girders due to corruptions were determined based on previous studies.7, 8 That investigated the thickness reductions of steel members in actual corroded bridges. These studies showed that even in the highly corroded areas of the lower flange of the I-girder, the thickness reduction was no more than 2 mm. Therefore, the upper and lower ranges for the plate thickness of the corroded parts in the lower flange of main girders G2-G6 were determined to be 25mm and 23mm, respectively. That is, the maximum thickness reduction at the corroded parts was set to 2mm. In the target bridge, the corrosion at the web plate was not heavy as that in the lower flange. This observation was also recognized in previous studies7, 8; there, it was presented that the thickness reduction at the lower flange was from 1.1 to 3.2 times greater than the reduction at the web. In this study, therefore, the thickness reduction of the lower side of the web was set to 50% of that of the bottom surface of the flange. Only in girder G1, where any corrosion was not observed in the web, the effect of corrosion was considered in the flange, and the thickness of web was fixed to the initial value. Figure 8 (b) shows the location of areas with thickness reduction within the I-girder cross-section in the FE model. The cracks in the RC deck cause the bending stiffness reduction of the slab; therefore, this effect was considered by the Young’s modulus reduction in the FE model. However, there was no previous study or data that quantitatively showed the degree of stiffness reduction in the RC slab with cracks. This can be regarded as uncertainty due to lack of knowledge. The uniform distribution with the range of ±10% against the initial Young’s modulus of RC slab was thus adopted for considering relatively high uncertainty.

Next, to represent the movability reduction of the steel bearing due to corrosion, a linear spring was applied to the P1-pier side of each girder, as shown in Fig. 8 (a). Here, notice that $K_1$-$K_6$ are the spring stiffness in girders G1-G6, respectively. Each linear spring was placed at the center of the bottom surface of the sole plate element in the FE model. This linear
spring can reproduce the movability reduction by increasing the spring stiffness. However, as this is a virtual element in the FE model, the spring stiffness cannot be determined based on any physical property related to the movability of corroded bearings. Therefore, the range of prior uniform distribution for the spring stiffness was determined by a pre-calculation using an FE model of the I-beam main girder. Here, the lower and upper limits of spring stiffness, which showed the same resonant characteristics as those in the simply-supported and fixed-end boundary conditions, respectively, were determined by plotting resonant frequencies of the I-beam by increasing the spring stiffness. Figure 9 shows the plot of the 1st mode resonant frequency by changing spring stiffness. Figure 9

Table 2 Prior distributions of FE model parameters.

| Member | Model parameters (Unit) | Prior distribution |
|--------|-------------------------|--------------------|
| Girder G1 | Thickness of lower flange (mm) | Tb1 23 25 |
|         | Bearing spring stiffness (N/m) | K1 10^4 10^13 |
| Girder G2 | Thickness of lower web (mm) | Tb2 8 9 |
|         | Thickness of lower flange (mm) | K2 10^4 10^13 |
| Girder G3 | Thickness of lower web (mm) | Tb3 8 9 |
|         | Thickness of lower flange (mm) | K3 10^4 10^13 |
| Girder G4 | Thickness of lower web (mm) | Tb4 8 9 |
|         | Thickness of lower flange (mm) | K4 10^4 10^13 |
| Girder G5 | Thickness of lower web (mm) | Tb5 8 9 |
|         | Thickness of lower flange (mm) | K5 10^4 10^13 |
| Girder G6 | Thickness of lower web (mm) | Tb6 8 9 |
|         | Thickness of lower flange (mm) | K6 10^4 10^13 |
| RC slab | Young’s modulus (GPa) | Ec 22.5 25.0 |
|         | Density (kg/m^3) | Dc 2160 2640 |

RC slab dead load was caused by the random variation RC deck thickness, which was around 6% COV, from a previous study that mentioned the statistics of the steel plate-girder bridge properties. In addition, considering the variabilities of weights of the pavement and the road accessories, such as curbs and hand railings, shown in the same literature, the prior distribution of the density of slab element was determined in the uniform distribution with upper and lower limits of ±10% against the initial value. The determined prior distributions of all target FE model parameters are summarized in Table 2.

c) Global Sensitivity Analysis

In the estimation of posterior distributions, when there are FE model parameters whose uncertainties are not dependent on the observation data of resonant frequencies, the estimation problem becomes an ill-posed problem. In other words, the FE model param-
eter with low sensitivity to the resonant frequencies cannot be included in the estimation parameters. To prevent becoming such an ill-posed estimation problem, the global sensitivity analysis (GSA) was conducted to remove the parameters that had relatively low contribution to the resonant frequencies. The simplest GSA is the Analysis of Variance (ANOVA) based on the design of experiment method. The ANOVA determines the contribution degree of variation level $j$ of each influential factor (i.e., FE model parameter) to the total variation of variable $y$ (i.e., resonant frequencies). The measure of contribution degree is the coefficient of determination $R^2$:

$$R^2 = 1 - \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij} - \bar{y})^2}{\sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij} - \bar{y})^2},$$

(1)

where $y_{ij}$ is $i$-th data ($i=1$-$m$) in level $j$ ($j=1$-$n$), and $\bar{y}$ is the total mean of all data. Response data $y_{ij}$ were the 1st to 3rd modes resonant frequencies of the bridge FE model assigned $i$-th sample of model parameters created by the design-of-experiment (DoE) sampling. Here, the two-level full-factorial design was adopted for the DoE sampling method. The range of upper and lower limits of the prior distribution of FE model parameter in Table 2 becomes the variation level $j$.

In this study, considering the calculation cost, the 14 FE model parameters in Table 2 were divided into two groups. Here, Group #1 was composed of the thickness of lower flange in the six girders, the Young’s modulus of slab, and the density of slab, while Group #2 was composed of the spring stiffnesses in the girders that have the large mode amplitudes show high contributions to the resonant frequencies. In the 3rd mode, $R^2$ values of the six stiffnesses $K_1$ to $K_6$ are relatively high compared to the slab density $D_c$. In particular, for the 1st mode resonance frequency, the $R^2$ value of $D_c$ contributes only slightly less than 20% of the total variance, and the total $R^2$ values of the six stiffnesses $K_1$ to $K_6$ contribute more than 80%. To the higher mode frequencies, however, the variation in slab density $D_c$ shows higher contributions, i.e., near 40% in the 2nd mode and near 60% in the 3rd mode. In seeing the relationships within $R^2$ values of the six stiffnesses, the values gradually become higher in the outer girders, $K_1$ and $K_6$, both in the 1st and 2nd modes. In the 3rd mode, the $R^2$ values in the outer ($K_1$ and $K_3$) and inner girders ($K_3$ and $K_5$) are relatively high, and there are few sensitivities in $K_2$ and $K_5$. These $R^2$ value distributions within $K_1$ to $K_6$ are considered to be related to the mode shape in each mode, which are shown in Fig.6. The variations in spring stiffnesses in the girders that have the large mode amplitudes show high contributions to the resonant frequency of the corresponding mode.

As shown here, the GSA based on ANOVA indicates the contribution of each parameter variations to the output response. From the results of Fig.10, it can be said that the parameter variation in slab density $D_c$ cannot be ignored within the variations in the FE model parameters under the prior distributions shown in Table 2. Then, the parameters with little sensitivity compared to $D_c$ are removed from the list of applicable parameters for the uncertainty quantification. In conclusion, the parameters of $K_1$-$K_6$, which considered the movability reduction of the corroded steel bearings, and the slab density $D_c$ were decided to be

![Fig.10 Global sensitivities of uncertain FE model parameters.](image-url)
applied in the estimation of posterior distributions.

(3) Posterior of FE model parameters

a) posteriors estimation by MCMC

The posterior distribution in the Bayesian inference is obtained by multiplying the prior distribution by the likelihood function of observation data. The posterior distributions can be derived when the parameters are few in number, and some probability models, such as the normal distribution, are applicable to the prior distribution and the likelihood function. In other cases, a Markov Chain Monte Carlo (MCMC) or other sampling-based methods must be applied to obtain the posterior distribution. Furthermore, it should be noticed that the parameters estimating the posterior distributions were not directly measured; i.e., the observation data were not the FE model parameters but the resonant frequencies. In the case where the M-H algorithm was directly applied, the FE model calculation had to be performed at every sampling step, which could reach more than 10,000 times, and thus significantly raise the computational cost.

In this study, the surrogate modeling was adopted to address this issue of computational cost. The Bayesian inference based on the Gaussian process (GP) modeling, which was presented in a previous study, was used, which had also been applied in authors’ previous related studies. Here, measurement data \( y \) is expressed as in the following equation:

\[
y = \eta(\theta) + \delta + \varepsilon,
\]

where \( \eta(\theta) \) and \( \delta \) are the simulation output and the modeling error in model parameter \( \theta \) (multivariate vector \( \theta \)), respectively, and \( \varepsilon \) is the white noise. The GP model is then applied to \( \eta \) and \( \delta \) and the covariance matrix is configured using correlation coefficient \( \rho_\eta, \rho_\delta \), and mean \( \mu_\eta, \mu_\delta \) (=0). The posterior distribution is then represented as:

\[
\pi(\theta, \mu, \lambda_\eta, \rho_\eta, \lambda_\delta, \rho_\delta | \mathcal{D}) \propto L(\mathcal{D} | \theta, \mu, \lambda_\eta, \rho_\eta, \lambda_\delta, \rho_\delta, \Sigma, \eta) \\
\times \pi(\theta) \times \pi(\mu) \times \pi(\lambda_\eta) \times \pi(\rho_\eta) \times \pi(\lambda_\delta) \times \pi(\rho_\delta)
\]

The left side of the equation is the posterior distribution, and the right side is a formula, in which prior distributions \( \pi(.) \) of the target model parameters \( \theta \) and the surrogate model hyperparameters \( \mu, \lambda_\eta, \rho_\eta, \lambda_\delta, \rho_\delta \) are multiplied by likelihood function \( L(.) \).

The likelihood function \( L(.) \) is a conditional probability distribution of data \( \mathcal{D} \) under \( \theta \) and \( \mu, \lambda_\eta, \rho_\eta, \lambda_\delta, \rho_\delta \). When the GP is assigned to \( \eta \) and \( \delta \), the covariance matrix can be configured by the hyperparameters, and combined with the covariance matrix of measurement data. In that, the data \( \mathcal{D} \) is then configured as in Eq. (4).

\[
\mathcal{D} = (y^T, \eta^T)^T
\]

This is a vector or matrix composed of measurement data \( y \) (number of measurements \( n \)) and numerical data \( \eta \), composed of response outputs calculated under the model parameter \( \theta \) sampled from the prior distribution by the design of experiment (DoE) with number of samples \( m \). Therefore, the observation data \( \mathcal{D} \) becomes a vector or matrix of \( n+m \) rows. Since the relationship of the model parameters \( \theta \) to the numerical output \( y \) is considered in the covariance matrix in the likelihood function \( L(\mathcal{D}) \) and the hyperparameters are included in the estimation parameters, it is possible to avoid conducting numerical analysis in each step of MCMC. However, the configuration of data \( \mathcal{D} \) significantly affects the posterior distributions.

In this study, 500 samples were created by Latin hypercube sampling (LHS) from the FE model parameter prior distributions. The output response of the 1st to 3rd mode resonant frequencies was calculated in each sample and configured as \( \eta \). The observation data \( \mathcal{D} \) were then prepared by combining \( \eta \) with measurement data \( y \), which was configured by the 1st to 3rd mode resonant frequencies acquired in the measurement with number of data \( n = 19 \). The posterior distribution was then derived by the M-H algorithm of MCMC with 5000 steps burn-in and five steps thinning, and 50,000 steps were adopted as the posterior distributions. The autocorrelation function was used to confirm whether the adopted posteriors became the stationary process or not.

b) Results of posterior distributions

Figure 11 shows the observation data \( \mathcal{D} \) configured by 19 measurement data \( y \) and 500 numerical data \( \eta \). Since the likelihood function \( L(\mathcal{D}) \) is determined in the data of 519 rows, it should be confirmed that the distribution of the measurement data and numerical data overlaps in order to perform the appropriate posterior distribution estimation.

In Fig. 11, the average of measurement data is indicated in a solid line to compare two variabilities of measurement and numerical data. It was confirmed that the distributions of measurement and numerical data overlapped in each resonant mode.

The posterior distributions of FE model parameters are shown in Fig. 12. These are the histograms of adopted 50,000 samples of joint probability distributions of seven parameters selected in the GSA. The range of each histogram indicates the upper and lower limits of prior distribution. One of the aut-cor-
relation functions for spring stiffness $K_1$ is also shown in Fig. 13. It can be seen that the value almost converges to exceed 30 lags. The same convergence performances were observed in other parameters; therefore, it can be said that the obtained posterior distributions can be used for the considerations.

Since each of the posterior distributions in Fig. 12 shows the profile with biases or peak, it can be thus said that the uncertainty in each FE model parameter was somehow reduced. In the posterior distributions of bearing spring stiffnesses $K_1$-$K_6$, high probabilities are distributed in the range of high values. On the other hand, in the slab density $D_c$, the distribution profile is biased in the range of lower values. The profiles of the posterior distributions of bearing spring stiffnesses were considered by referring to the relationships between the resonant frequencies and the value of spring stiffness shown in Fig. 9. In spring stiffnesses $K_1$ and $K_6$, which indicate the movability performances of steel bearings in the outer girders G1 and G6, the probabilities in the range over the value of $1 \times 10^9$ N/m, where the bearing movability is not functioning, are relatively low. In both posterior distributions of $K_1$ and $K_6$, although the peak is observed within the range from $1 \times 10^6$ to $1 \times 10^7$ N/m, where the movability is functioning, high probabilities are distributed in the range from $1 \times 10^6$ to $1 \times 10^{10}$ N/m, where there is a possibility of fixation tendency. On the other hand, for the spring stiffness of the inner girders $K_2$-$K_5$, the high probability is biased in the range of high values exceeding $1 \times 10^{13}$ N/m, where the girder boundary conditions are fixed. From these points, it can be said that there is a possibility of slight fixation tendency in the outer girders G1 and G6, and the movabilities of bearings at the inner girders G2 to G5 may not function due to the corrosions.

Seeing the conditions of steel bearings in the movable side recorded in the inspection data, which are shown in Figs. 7 (e) and (f), the corrosions are not found in the outer girders as shown in Fig. 7 (e).
is due to the rain washing effect of attached airborne sea salt on the girder. On the other hand, the heavy corrosion was recognized in the bearing of inner girder G2, as shown in Fig.7 (f). It was also confirmed that the corrosion were pointed out in the inspection data of movable bearings at all inner girders G2-G5.

From these considerations, it can be concluded that obtained posterior distributions of the FE model parameters showed consistencies with the observations acquired in the inspection. The posterior distributions, which were the uncertainties quantified by the measurement data, updated the prior uncertainties, which were represented by the uniform distribution in that they provided knowledge about the current structural condition stochastically. The following structural reliability analysis will show how uncertainties of structural conditions influence the reliability index by comparing cases that use prior and posterior distributions.

4. STRUCTURAL RELIABILITY ANALYSIS USING POSTERIOR DISTRIBUTIONS

The structural reliability analysis using the posterior distributions, where the uncertainty is reduced by measurement data, is expected to realize the performance evaluation considering current structural conditions in the existing structures. In this chapter, the probability distribution of maximum stress in the steel main girder under design load is compared between cases derived from prior distribution and posterior distribution. The structural performance evaluation of reducing uncertainties of structural conditions is then verified by deriving the reliability index \( \beta \) of the steel yield stress.

(1) Procedure of structural reliability analysis

The structures lose their function when the response to the combination of loads exceeds the defined limit state. The structural reliability is the stochastic measure of structural performance considering both the uncertainty in response to the load and that of the limit state. Here, the probability of failure is examined by a performance function that represents the stochastic distance between these uncertainties. The variables in the performance function should be described as random variables for the strict calculation; however, it is often difficult in practice. There are actually some approximate solutions for the structural reliability analysis. In this study, one of those methods, the first-order second-moment (FOSM) method was adopted to derive the structural reliability index \( \beta \).

In the FOSM method, when proof stress \( R \) and load action \( S \) are independent of each other, and both follow normal distributions, the performance function \( Z=R−S \) also follows the normal distribution. The mean \( \mu_Z \) and standard deviation \( \sigma_Z \) of \( Z \) can then be obtained by the formulas below using mean \( \mu_R, \mu_S \) and standard deviation \( \sigma_R, \sigma_S \) of \( R \) and \( S \).

\[
\mu_Z = \mu_R - \mu_S \\
\sigma_Z^2 = \sigma_R^2 + \sigma_S^2
\]

(5)

Also, if proof stress \( R \) and load action \( S \) are independent of each other, and both follow the log-normal distribution, the performance function \( Z = \ln(R/S) \) will also follow normal distribution. At this time, the means of \( \ln R \) and \( \ln S \), \( \mu_{LR} \) and \( \mu_{LS} \), and the standard deviations, \( \sigma_{LR} \) and \( \sigma_{LS} \), can be described as Eqs. (7) and (8).

\[
\mu_{LR} = \ln \left( \frac{\mu_R^2}{\sqrt{\sigma_R^2 + \mu_R^2}} \right), \quad \mu_{LS} = \ln \left( \frac{\mu_S^2}{\sqrt{\sigma_S^2 + \mu_S^2}} \right)
\]

(7)

\[
\sigma_{LR} = \sqrt{\ln \left( \frac{\sigma_R}{\mu_R} \right)^2 + 1}, \quad \sigma_{LS} = \sqrt{\ln \left( \frac{\sigma_S}{\mu_S} \right)^2 + 1}
\]

(8)

The mean \( \mu_Z \) and standard deviation \( \sigma_Z \) of the performance function \( Z \) are determined as follows:

\[
\mu_Z = \mu_{LR} - \mu_{LS} \\
\sigma_Z^2 = \sigma_{LR}^2 + \sigma_{LS}^2
\]

(9)

(10)

Once \( \mu_Z \) and \( \sigma_Z \) of the performance function \( Z \) are determined, the reliability index \( \beta \) can be obtained as follows:

\[
\beta = \frac{\mu_Z}{\sigma_Z}
\]

(11)

In this study, the probability distribution of load action \( S \), which was the distribution of maximum stress of the steel girder under design live load, was derived by the prior or posterior distributions. Here, the maximum stress under the design load was identified at the area of girder ends, and its probability distribution was obtained by Monte Carlo calculation for the bridge FE model assigning the model parameters sampled from the prior or posterior distributions. The structural reliability index \( \beta \) was then derived by the proof stress \( R \) of the probability distribution of steel yield stress.

(2) Evaluated performance

The input load in the FE analysis was the design live load defined in the Japanese road bridge specification 12, as shown in Table 3 and Fig.14. The output response was the distribution of the von Mises stress.

\[
\begin{align*}
\mu_Z &= \mu_{LR} - \mu_{LS} \\
\sigma_Z^2 &= \sigma_{LR}^2 + \sigma_{LS}^2 \\
\beta &= \frac{\mu_Z}{\sigma_Z}
\end{align*}
\]

Table 3

Fig.14
Table 3: Applied design live load in the Japanese standard.

| Length D (m) | Vehicle load \( p_1 \) (kN/m²) | Pedestrian load \( p_2 \) (kN/m²) | IM \( i \) |
|--------------|---------------------------------|-------------------------------|------|
| 10           | 10                              | 3.5                           | 3.5  |
|              | \( L \leq 80 \)                 | \( L \leq 80 \)               | \( i = \frac{20}{50 + L} \) |

Table 4

| Unit: MPa |
|-----------|
| 90.0      |
| 87.5      |
| 85.0      |
| 82.5      |
| 80.0      |
| 77.5      |
| 75.0      |
| 72.5      |
| 70.0      |
| 67.5      |
| 65.0      |
| 62.5      |
| 60.0      |
| 57.5      |
| 55.0      |
| 52.5      |
| 50.0      |
| 47.5      |
| 45.0      |
| 42.5      |
| 40.0      |
| 37.5      |
| 35.0      |
| 32.5      |
| 30.0      |
| 27.5      |
| 25.0      |
| 22.5      |
| 20.0      |
| 17.5      |
| 15.0      |
| 12.5      |
| 10.0      |
| 7.5       |
| 5.0       |
| 2.5       |
| 1.0       |
| 0.5       |

The output of maximum Mises stress was verified, and the mesh configuration was determined as shown in Fig.15 (b). Here, since the degree of corrosion was different in each girder, the maximum stress in the twelve ends of all six girders, was used for the reliability analysis of the target bridge.

(3) Probability density function of load effect \( S \)

a) Sampling of FE model parameters from the prior and posterior distributions

The FE model parameters to be considered were the bearing spring stiffnesses \( K_1-K_6 \) and the slab density \( D_c \). Two cases were compared: one is the case that used the prior distributions (Case 1), which were the initial uncertainty, and the other is the case that used the posterior distributions (Case 2), which were the uncertainties updated by the measurement data. Five hundred samplings were generated in each case. In Case 1, the Latin hypercube sampling was applied to generate the samples from the uniform distributions, and in Case 2, the 500 steps were just extracted from the stationary process of posterior distributions in the MCMC.

b) PDF of maximum stress at the ends of girders

The distribution of maximum Mises stress in the area of the girder end under the design live load was obtained by the FE model calculation by assigning each of the 500 samples. Figures 16 (a) and (b) are the histograms of 500 maximum Mises stress values in Case 1 and Case 2, respectively. The mean, standard deviation, coefficient of variation (COV), logarithmic mean and logarithmic standard deviation of each histogram are shown in Table 4. The table shows no significant difference in the mean between the two cases; however, the standard deviation in Case 2 is reduced compared to that in Case 1.

In order to apply the FOSM method to derive the reliability index \( \beta \), it is required to assume the normal or the lognormal distribution against the obtained distributions of maximum stress. For this purpose, the Kolmogorov-Smirnov test 13), which was one of non-parametric tests, was performed. The null hypothesis here was that the cumulative distribution function (CDF) of the distribution of maximum Mises stress was equal to the normal or lognormal distributions. Thus, in the assumption of the lognormal distribution in Case 1, the null hypothesis was not rejected because the p-value was 0.746 and the significance level was larger than 1% (0.01). In Case 2 also, the null hypothesis was not rejected in the same assumption because the p-value was 0.0257, which was larger than the significance level 1% (0.01). It was concluded that the lognormal distribution could be assumed both in Cases 1 and 2, and the logarithm means and logarithm standard deviations shown in Table 4 could be used in the FOSM method.
The lognormal distribution could be assigned to the yield stress of steel used in the target bridge (SS400 in the JIS standard). Here, the proof stress $R$, which was the yield stress of steel used in the design of new structures, was calculated from the expected value and yield stress of steel, and its mean and standard deviation was 0.0812. The logarithmic mean here was 19.5 and the logarithmic standard deviation was 0.0812, in the calculation of Eqs. (9) and (10). The value $\beta$ was then derived by Eq. (11). As a result, the reliability index $\beta$ became 3.44 in Case 1 and 3.94 in Case 2. In both cases, the $\beta$ value exceeded 2.5, which meant that the probability of failure was less than 1%; therefore, it can be said that structural safety of the target bridge is high enough.

Table 4 Statistics of maximum Mises stress distributions.

|        | Mean (MPa) | STD (MPa) | COV (%) | Log-mean | Log-STD |
|--------|------------|-----------|---------|----------|---------|
| Case1  | 147        | 29.2      | 19.9    | 18.8     | 0.197   |
| Case2  | 148        | 24.3      | 16.4    | 18.8     | 0.163   |

**Fig. 16** Probability distribution of maximum Mises stress.

**Fig. 17** Comparison to the steel yield stress distribution.

(4) Structural reliability index $\beta$

The reliability index $\beta$ was derived by the FOSM method using the distributions of load action $S$ and proof stress $R$, which was the yield stress of steel used in the target bridge (SS400 in the JIS standard). Here, the lognormal distribution could be assigned to the yield stress of steel, and its mean and standard deviation were calculated from the expected value and COV indicated in an existing literature. The logarithmic mean here was 19.5 and the logarithmic standard deviation was 0.0812. Figure 17 shows the relationship between the distribution of steel yield stress $R$ and that of maximum Mises stress $S$, in each of Case 1 (prior) and Case 2 (posterior). Furthermore, the maximum stress in the initial FE model, 85.9 MPa, is also indicated by a chain line. Both Case 1 and Case 2 considered the effects of observed damages in the target bridge even in the different description of uncertainties. Therefore, the maximum stress in both cases are distributed in the higher range than the values in the initial FE model. This indicates that the performance of existing structures must be evaluated considering the effects of local damage using the detailed FE models. This is different from the structural performance evaluation in the design of new structures.

In Case 1 for example, logarithmic mean $\mu_{LS}=18.8$ and logarithmic standard deviation $\sigma_{LS}=0.197$ were used with those of the steel yield stress, $\mu_{R}=19.5$ and $\sigma_{R}=0.0812$, in the calculation of Eqs. (9) and (10). The value $\beta$ was then derived by Eq. (11). As a result, the reliability index $\beta$ became 3.44 in Case 1 and 3.94 in Case 2. In both cases, the $\beta$ value exceeded 2.5, which meant that the probability of failure was less than 1%; therefore, it can be said that structural safety of the target bridge is high enough.

**5. CONCLUSION**

In this study, the posterior distributions for the FE model parameters of an existing bridge were estimated by the Bayesian inference using vibration measurement data. This procedure reduces uncertainties regarding current structural conditions. The structural reliability index was then shown in one case using prior distributions, with uncertainties before updating by measurement data, and another case using posterior distributions. The conclusions are summarized as follows:

1. The posterior distributions of the FE model parameters that satisfied the stationarity of MCMC could be estimated using the data of resonant frequencies. The posterior distributions of the spring stiffnesses, which were applied to represent the movabilities of corroded steel bearings, showed consistency with actual corroded conditions recorded in the visual inspection.

2. The maximum stress under the design live load near the bearing at the end of the main girder was influenced by the variation in the bearing spring stiffness. It was recognized that, in the performance evaluation of the existing structures, the response to be evaluated might be different from that...
which was evaluated in designing new structures.

3. The distributions of maximum Mises stress that occurred at the end of the steel girder were compared by the hypothesis test between the distributions derived using the prior distributions and the posterior distributions. The lognormal distribution was accepted with the appropriate significance level. The logarithmic standard deviation in the posterior distribution became lower than that in the prior distribution. This indicates that the uncertainties of structural properties related to the corroded conditions were reduced by the measurement data in the posterior distribution.

4. The reliability index $\beta$ in using the posterior distribution of FE model parameters became higher than that which used the prior distribution. This indicates that it is possible to prevent evaluating the structural performance from being too low by using the posterior distribution, with the updated uncertainties using the measurement data.

From the above-mentioned points, it was concluded that the performance of the target bridge was evaluated considering the effects of local corrosions at the end of the girder by applying the measurement data for the estimation of posterior distributions. The process shown in this study is applicable to the performance evaluation of existing bridges, where any decision to repair or reinforcement is required because the damages are found during the visual inspection.

There are several future works to be considered. The first point is that the numerical modeling strategies in order to represent and evaluate the effects of damages in the existing structure should be verified. In the existing structure, most damages, the effect of which on the structural performance should be evaluated, are local and diverse; therefore, the detail modeling and its V&V process will become important. At the same time, the data acquisition strategy to evaluate the effects of damages on the structural performance should be verified. In this study, the resonant frequencies of global modes were used to estimate the posterior distributions. However, those resonant frequencies were not influenced by the local thickness reduction of steel members due to corrosion, but by the spring stiffness, which represented the movability of bearings as the boundary conditions. The data acquisition strategy should be considered more so based on the global sensitivity that indicates the contribution of each parameter uncertainty to the purpose output response. This will become helpful to more effective data acquisition in the structural health monitoring (SHM) in civil structures. This study showed the effectiveness of structural reliability analysis using the posterior distributions of model parameter for the performance evaluation of existing structure considering the structural condition. The assimilation of structural monitoring data and inspection data to the structural reliability calculation may become an important contribution to the maintenance and operation of existing bridges.

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