Phase-Aligned Space-Time Coding for a Single Stream MIMO system

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Abstract—We present a phase-aligned space-time coding scheme that expands the original Alamouti codeword to three or four transmit antennas \(N_t = 3\) or \(4\) with phase alignment. With 1 2 bits feedback for the phase information, the fundamental performance penalty of \(10\log_{10}(N_t)\) dB of orthogonal space-time coding compared to the optimum beamforming is reduced by 1 dB (for \(N_t = 3\)) or 2 dB (for \(N_t = 4\)) on average. With the proposed scheme, the full diversity order of \(N_t\) is achievable, whereas the receiver architecture remains the same as the legacy Alamouti decoding with codeword size of two, since the spatial expansion is transparent to the receiver. Our results show the proposed scheme outperforms open-loop space-time coding for three or four transmit antennas by more than 3 dB.

Index Terms—MIMO, STBC, Beamforming, Feedback

I. INTRODUCTION

Space-Time Block Coding (STBC) is a keyword to obtain the diversity gain by using the multi-input multi-output (MIMO) system in digital communication. Since Alamouti codeword was proposed \[1\] for two transmit antenna transmission, many transceiver techniques have been developed to realize the full diversity gain for more than two transmit antennas. However, the open-loop orthogonal transceiver design with full-rate and full diversity gain for arbitrary complex channels is proven to be limited to the case of two transmit antennas \[2\]. For more than two antennas, many space-time coding schemes have been introduced to achieve the full diversity gain, however, with less than full-rate \[3\].

There was another attempt to achieve full-rate but with partial diversity by adopting quasi-orthogonal STBC (QO-STBC) structure \[4\]. While QO-STBC can extend Alamouti pairs to four transmit antennas easily, it often requires an extensive receiver design, such as Maximum-Likelihood (ML) technique \[5\], to suppress the self-interference caused by non-orthogonal structure. However, with an aid of partial channel information feedback, the full diversity gain is shown to be achievable for four transmit antenna system with rate one \[6\]. It also shows the feedback of only a few number of bits can achieve nearly full orthogonal STBC diversity gain, thereby cheaper minimum-mean-square-error (MMSE) receiver can match the performance of ML receiver.

However, there is still fundamental performance penalty, \(10\log_{10}(N_t)\) dB, even for orthogonal STBC \[6\], \[7\], compared to closed-loop beamforming, where \(N_t\) is the number of transmit antennas. In order to reduce the gap, it is recommended to obtain an aid of partial channel information combined with STBC with minimal feedback overhead, e.g., 1 2 bits, otherwise beamforming is still an attractive solution when larger feedback overhead is allowed \[8\]. Transmit antenna selection can reach the same goal of full transmit diversity gain \[9\], \[10\], but it suffers from non-linearity distortion at high power transmission, which is a typical scenario to extend the transmission range with STBC. Achieving full diversity gain by introducing new constellation for symbols \[11\] requires a special constellation optimized for re-designed STBC, which is not applicable for legacy systems with Gray code labeling \[12\].

In this paper, our goal is to improve an STBC scheme which can be detected by a legacy Alamouti decoder with codeword size of two, by introducing only a few number of bits for the feedback. We achieve the goal with full transmit antenna diversity by expanding STBC codewords over three or four transmit antennas. The spatial expansion matrix is found to align the phase of Alamouti pairs, whose information is obtained by channel measurement and feedback performed by the receiver. We present a closed form solution for the phase alignment and show the proposed scheme with 1 2 bits feedback performs to within only fractional dB of performance of the same system with full knowledge of the channel. Note \[6\] eliminates the off-diagonal interference terms in the effective channel, however the proposed scheme in this paper maximizes the diagonal terms in the effective channel since it retains orthogonality with the spatial expansion matrix. The proposed scheme is transparent to the receiver, so a legacy Alamouti decoder can still be applied without knowledge on whether this new scheme is applied at the transmitter.

II. PHASE-ALIGNED STBC

Alamouti codeword is a space-time coding scheme that can achieve full diversity gain when the number of transmit antennas is two. When the number of transmit antennas is more than two, we can spread the output of STBC encoder over the \(N_t\) space by adopting a spatial mapping matrix \(Q\) with dimension of \(N_t \times 2\) as shown in Figure \[1\]. An Alamouti codeword for a single stream is

\[
S = \begin{bmatrix}
x_k & x_{k+1} \\
x_{k+1} & -x_k
\end{bmatrix},
\] (1)
where row represents an antenna index and column represents a time instance for STBC. Superscript * denotes complex conjugate. \( x_k \) is the original input constellation symbol at \( k^{th} \) original sequence.

For more than two transmit antennas, we simply introduce a spatial mapping \( Q \) after STBC encoder. In order to preserve the orthogonality for the matched filter, i.e., \( Q^H Q = I \), where \( I \) is an identity matrix, it is easy to show that \( (q)_{lm} = 0 \) when \( i + m \) is an odd integer without loss of generality, where \( (q)_{lm} \) is the \( (i, m)^{th} \) component of \( Q \). With per-antenna power constraint to avoid non-linearity distortion at high transmit power, we can choose each element of \( Q \) matrix with unity magnitude.

We consider four transmit antennas as a simple example. With a \( 4 \times 2 \) spatial mapping matrix \( Q \), we can distribute the Alamouti codeword over 4 transmit antennas. The spatial mapping matrix \( Q \) we choose is

\[
Q = \begin{bmatrix}
1 & 0 & e^{-j\theta} & 0 \\
0 & 1 & 0 & e^{-j\theta}
\end{bmatrix}^T,
\]

where \( \theta \) is determined by the channel information and superscript \( ^T \) denotes transpose. Herein, the scale factor is moved to the channel in (5). Equivalently, the new STBC codeword with \( Q \) matrix to be transmitted over four transmit antennas is

\[
S' = \begin{bmatrix}
x_k \\
x_{k+1}
\end{bmatrix} \times
\begin{bmatrix}
x_k \\
x_{k+1}
\end{bmatrix}^T + \begin{bmatrix}
N_1 \\
N_2
\end{bmatrix}.
\]

Assuming the channel is not varying over the size of the codeword, i.e., \( h_{im}(k) = h_{im}(k+1) \) where \( k \) is a time index, the received signal for the STBC codeword in (4) with \( N_r \) receive antennas is given by

\[
H = \begin{bmatrix}
H_1^T \\
H_2^T \\
\vdots \\
H_{N_r}^T
\end{bmatrix} = \begin{bmatrix}
H_1 \\
H_2 \\
\vdots \\
H_{N_r}
\end{bmatrix}^T
\]

\[
H_m = \begin{bmatrix}
1 & \frac{1}{\sqrt{N_t}} \\
-\frac{1}{\sqrt{N_t}}
\end{bmatrix} \begin{bmatrix}
h_{1m} + e^{-j\theta}h_{3m} & h_{2m} + e^{-j\theta}h_{4m} \\
h_{2m} - e^{-j\theta}h_{4m} & h_{1m} + e^{j\theta}h_{3m}
\end{bmatrix}
\]

\[
R_m = \begin{bmatrix}
r_{m,k} \\
r_{m,k+1}
\end{bmatrix}^T
\]

\[
N_m = \begin{bmatrix}
n_{m,k} \\
n_{m,k+1}
\end{bmatrix}^T.
\]

For an example of \( N_t = 3 \), (2) becomes

\[
Q = \begin{bmatrix}
1 & 0 & e^{-j\theta} \\
0 & 1 & 0
\end{bmatrix}^T,
\]

thereby the last row of \( S' \) in (3) does not exist. Also, (5) and (2) can be rewritten as

\[
H_m = \begin{bmatrix}
h_{1m} + e^{-j\theta}h_{3m} & h_{2m} \\
-h_{2m} & h_{1m} + e^{j\theta}h_{3m}
\end{bmatrix}
\]

\[
\Sigma = \sum_{m=1}^{N_r} \left| h_{1m} + e^{-j\theta}h_{3m} \right|^2 + \left| h_{2m} \right|^2
\]

\[
\alpha = \sum_{m=1}^{N_r} \left( h_{1m}h_{3m} + h_{2m}h_{4m} \right).
\]

Note that the effective channel has a nice orthogonal structure; there is no need to operate additional interference cancellation process. Finally, the closed form solution for \( \theta \) to maximize the magnitude of \( \Sigma \) by matching the phase of \( \alpha \) is

\[
\theta = \angle \alpha,
\]

where \( \angle \{ \} \) denotes the phase of the argument.

In practice with a digital communication system, the phase information \( \theta \) to be used back needs to be quantized. The feedback information can be obtained by quantizing the angle within \([0, 2\pi]\) range, and the most simplest form with 1-bit or 2-bits feedback can be found as \( e^{-j\theta} = 1 \) or \(-1\), or \( e^{-j\theta} = 1, -1, j \) or \(-j\), respectively, which are found by quantizing the angle in (13).
Beamforming. For a sum of signal power for all transmit antennas is unity for each transmit antenna has unity power for STBC while the dB gain over an orthogonal STBC scheme. Since symbols at full knowledge of the channel. Since beamforming typically requires more than 18 bits feedback to nearly achieve the performance with infinite bits feedback [3], [4], the phase-aligned STBC with 2-bits feedback has only 4 dB penalty on average for the output SNR with 90% percentile saving on the feedback overhead, compared to beamforming.

Most of research on the topic of STBC assumed the channel does not change within the STBC codeword. While it is a reasonable assumption for slow fading channels, it becomes a more strong assumption when the size of the STBC codewords is larger. Note QO-STBC [4] and orthogonal STBC [6] assume \( h_{im}(k+l) = h_{im}(k) \) for \( l = 1, 2 \) and 3, but the proposed scheme assumes it only for \( l = 0 \) and 1, because it maintains the size of codewords to two. In reality, however, the channel response can vary within a codeword, which results in self-interference in the matched filter operation. Figure 3 demonstrates how much sensitive the MMSE performance of the proposed scheme is, compared with other STBC schemes, when the channel is varying within the STBC codeword. In this plot, the correlation \( \rho \) represents the difference of the channel at each time instance; \( h_{im}(k+1) = \rho h_{im}(k) + \sqrt{1 - \rho^2} g_{im}(k) \) where both of \( h_{im}(k) \) and \( g_{im}(k) \) are i.i.d. Gaussian random variables. Then, the MMSE output SNR is calculated and the gap to the output SNR of QO-STBC when channels vary within the STBC codewords is plotted. When \( \rho \) decreases from 1, all of STBC schemes suffer from self-interference. Especially, the impact on orthogonal STBC is larger due to additional loss on the stale feedback information. However, the proposed scheme maintains the loss less than 1.5 dB when \( \rho = 0.8 \), which is smaller than QO-STBC and orthogonal STBC with codeword size of four.

Figure 4 shows the average bit-error-rate (BER) performance comparison when the information bits are not coded. For this simulation, 16 quadrature amplitude modulation (QAM) is used with i.i.d. complex Gaussian channels. It demonstrates beamforming, the proposed scheme and orthogonal STBC achieves full diversity order of four transmit antennas. The proposed scheme reduces the gap to beamforming

When \( N_t > 4 \), as long as \( N_t \) is an even number, we can easily expand this scheme with another angles, \( \theta_n \) with \( n = 2, 3, \cdots, N_t/2 - 1 \), by adding multiple pairs of Alamouti codeword over the transmit antennas. When \( N_t \) is an odd number, the same expansion of even number of transmit antennas with \( N_t + 1 \) is applied, but the last channel response, \( h_{N_t+1,m} \) is replaced with zero. To generalize this scheme for arbitrary configuration is out of scope for this paper.

III. SIMULATION AND COMPARISON

We consider a 4-transmit, 1-receive antenna (4 \( \times \) 1) system with independent and identically distributed (i.i.d.) complex Gaussian channels to demonstrate the benefits of the proposed scheme. It is well known that beamforming has 10 log\(_{10}(N_t)\) dB gain over an orthogonal STBC scheme, since symbols at each transmit antenna has unity power for STBC while the sum of signal power for all transmit antennas is unity for beamforming. For a 4\( \times \)1 system, singular value decomposition (SVD) is used for beamforming and [6] is used for orthogonal STBC scheme as the optimum solution, assuming full channel information is available at the transmitter.

Figure 2 shows the output signal-to-noise ratio (SNR) gain of an MMSE receiver over the orthogonal STBC scheme [6] when the noise power is 10 dB less than the received signal power. It demonstrates exactly 6 dB difference between the SVD beamforming and the orthogonal STBC scheme. With QO-STBC [4], it shows performance penalty compared to orthogonal STBC due to the non-orthogonal structure with interference. On the other hand, the proposed phase-aligned STBC shows the output SNR gain from 0 dB to 3 dB over orthogonal STBC; the maximum gain is obtained when two channel products are naturally co-phased to begin with, or no gain when even and odd channel pairs are out of phase (\( \alpha = 0 \)). Interestingly, with 2-bits feedback on quantized value of \( \theta \), the output SNR curve is very tight to the curve with full knowledge of the channel. Since beamforming typically
by 2 dB, compared with orthogonal STBC. 1 ∼ 2 bits quantization performs to within less than 1 dB of the optimum performance with full resolution feedback.

We also simulate the average BER performance with a coding for a 4 × 1 system based on the orthogonal frequency-division-multiplexing (OFDM) link model for 802.11n system [14]. The MMSE receiver is employed with an assumption of perfect channel estimation and convolutional codes. Figure 5 shows the BER results as a function of the average received SNR for 802.11n channel B which has 25 nsec rms delay spread with 0.5 antenna correlation [15]. For reference, the BER performance of single-input single output (SISO), 2 × 1 Alamouti STBC, 4 × 1 with cyclic-shift delay (CSD) are also plotted, where all transmit symbols with CSD and STBC schemes are scaled properly with 1/√ν, for fair comparison. In the 802.11n system [14], CSD values are fixed to [0, −50, −100, −150]nsec for four transmit antennas. In addition, Frequency Switched Transmit Diversity (FSTD) [16] is simulated. STBC-FSTD basically chooses two transmit antennas out of available transmit antennas for transmission with an alternating fixed fashion over tones; the first and the second transmit antennas for tone 1, the first and the third transmit antennas for tone 2, and so on. This scheme typically looks for frequency selective effects to obtain more coding gain and saves transmit power by half by nulling the other antennas. STBC with spatial mapping is the proposed scheme without angle information feedback; θ in (2) is fixed to zero. Finally, the proposed phase-aligned STBC is simulated with 1-bit, 2-bits and infinite bits (full resolution) angle feedback to take into account the quantization effect, while both beamforming based on SVD and orthogonal-STBC are assumed to have full channel knowledge at the transmitter for the best performance.

The simulation results show that the proposed scheme with 2-bits feedback nearly achieves the BER performance with infinite bits angle feedback, which has 2 dB gain over orthogonal-STBC and 4 dB loss to beamforming on average. Compared to existing open-loop solutions, e.g., CSD and QO-STBC schemes, it has more than 4 dB gain.

Figure 6 shows BER performance results for more frequency selective channels, 802.11n channel D, which has 50 nsec rms delay spread with 0.3 antenna correlation [15]. It also demonstrates the proposed scheme has 4 dB loss to beamforming and 2 to 3 dB gain over orthogonal STBC and QO-STBC. Note the performance of CSD scheme is worse in this plot, compared to results with results in Figure 5, since the CSD scheme loses its gain with frequency selective channels.

In order to demonstrate the performance of the proposed scheme for other antenna configuration, performance simulation for a 3 × 1 system is also performed. Figure 7 shows the BER performance of an uncoded system with 16 QAM and i.i.d. complex Gaussian channels. We observe significant performance improvement by employing the proposed phase-aligned STBC scheme. For a realistic system, Figure 8 shows...
the BER performance of 4 QAM and convolutional coding rate of 3/4 with 802.11n channel D. With three transmit antennas, CSD values are fixed to $[0, -100, -200]$, but orthogonal STBC and QO-STBC are not available. The results show the fundamental gap of $10\log(3)$ dB to the ideal beamforming is reduced by 1 dB, and the proposed scheme has the performance gain of 3 dB over open-loop schemes.

IV. CONCLUSIONS

We presented phase-aligned space-time coding scheme that spreads Alamouti codewords over the space with phase rotation. The rotation factor is found to compensate the phase of sum of channel products to maximize the diversity gain. The proposed phase-aligned scheme outperforms orthogonal STBC [6], QO-STBC [4] and STBC-FSTD [16] by 2 to 4 dB for a 4 x 1 antenna configuration. For a 3 x 1 system, it outperforms STBC-FSTD by 3 dB. The proposed scheme with 3 or 4 transmit antennas also maintains the performance loss of 4 dB to beamforming with 90% feedback overhead saving. Simulation for an i.i.d. complex Gaussian channel (flat fading) and 802.11 channel B and D (frequency selective fading) shows that the 2-bits feedback scheme performs to within a few fractional dB of the performance with infinite bits feedback. Note the proposed scheme is applied only to the transmitting station, but does not require any new detection algorithm on the legacy Alamouti decoder with codeword size of two, except partial channel feedback capability.

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