Cavitation near the oscillating piezoelectric plate in water

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Abstract
It is known that gas bubbles on the surface bounding a fluid flow can change the coefficient of friction and affect the parameters of the boundary layer. In this paper, we propose a method that allows us to create, in the near-wall region, a thin layer of liquid filled with bubbles. It will be shown that if there is an oscillating piezoelectric plate on the surface bounding a liquid, then, under certain conditions, cavitation develops in the boundary layer. The relationship between the parameters of cavitation and the characteristics of the piezoelectric plate oscillations is obtained. Possible applications are discussed.

1. Introduction
It was shown in [1-3] that the coefficient of friction of the surface of a moving object in water is reduced by 20-50% if the contact region is filled with vapor bubbles. Such bubbles are intensely formed in the liquid as a result of cavitation in areas of negative pressure. In this paper, we propose a method for generating cavitation in the near-wall region. We will show that cavitation can be generated in the boundary layer near the wall by performing ultrasonic vibrations. Such oscillations can be excited, for example, by a piezoelectric plate glued to the wall bounding a liquid. Fig. 1 shows a sketch of such a control of drag reduction.

![Fig. 1. Scheme of generation of cavitation by a system of oscillating piezoelectric plates. The vertical arrow shows the amplitude of the oscillations of the piezoelectric plates, the dotted line shows the region of generation of bubbles, it also coincides with the region of damping of the sound wave on the bubbles.](image)
2. Vibrations of the plate and generation of bubbles

Consider generation of sound in water caused by transverse vibrations of an immersed flat plate. In the linear approximation without damping, equations describing the oscillations of a liquid can be written in the form [4]:

\[
\begin{align*}
\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial u'}{\partial x} &= 0, \\
\frac{\partial u'}{\partial t} &= -\frac{1}{\rho_0} \frac{\partial P'}{\partial x}.
\end{align*}
\] (1)

Here \(\rho', P', u'\) are perturbations of density, pressure, and fluid velocity, \(\rho_0 = 10^3\) kg/m\(^3\) is the unperturbed density of water. In the pressure range of -150 to +500 MPa, a linear dependence of the density perturbation on pressure \(\rho' \approx c_s^2 P'\) can be used with an error of 5% or less, where \(c_s \approx 1500\) m/s is the speed of sound in water (for example, see [5,6]). Assuming the plate vibrations to be harmonic \(a(t) = a_0 \sin(\omega t)\), from the no-slip condition (where the normal and longitudinal velocities of the fluid at the boundary equal the oscillatory velocity of the plate and zero, respectively), it follows from (1) that \(\rho', P', u' \propto \cos(\omega t - kx)\), where \(k = \omega / c_s\). The corresponding amplitudes of the normal velocity and pressure of the liquid on the oscillating boundary are \(u_0 = \omega a_0\) and \(P_0 = \rho_0 c_s \omega a_0\), respectively.

The critical negative pressure at which cavitation begins in water is approximately \(P_{cr} \approx -30\) MPa (for example, see the review [7]). The sound waves in the fluid decay with a characteristic attenuation increment \(\gamma \approx \omega^2 \nu / c_s^2\), where \(\nu \approx 10^{-6}\) m\(^2\)/s [8] is the coefficient of kinematic viscosity of water. For ultrasonic frequencies ~1-100 MHz, \(\gamma \approx 10^{-12}\) sec\(^{-1}\) and, consequently, the characteristic spatial scale of the damping is of order \(l \approx c_s / \gamma \approx 10^{-2} - 10^{-3}\) m. The corresponding wavelength of the sound is \(\lambda = 2\pi c_s / \omega \approx 10^{-4} - 10^{-2}\) m, is much less than the characteristic length of the damping. Note that we approximate the oscillating wall as an infinite piston, i.e., we assume that the size of the oscillating plate exceeds essentially the amplitude of its oscillations and the wavelength of its radiated sound. Otherwise, edge effects would become important, the wavefronts could no longer be considered flat and it would be necessary to solve the problem for the near field of the acoustic radiation.

3. Generation of cavitation bubbles by the sound waves

It is known that cavitation can occur in the rarefaction regions of a sound wave. An expression for the nucleation rate in the negative pressure region is as follows [9]:

\[
\frac{dn}{dt} = \Gamma = \frac{3}{V_b 4\pi R_b^3} \frac{k_B T}{2\pi \hbar} \exp\left(-\frac{W_b}{k_B T}\right).
\] (2)

Here \(V_b = 4\pi R_b^3 / 3\), \(W_b\), \(n_b\), are the volume of a spherical bubble of radius \(R_b\), the energy necessary for bubble formation, and the number of such bubbles per unit volume; \(T\) is the temperature in liquid; \(k_B, \hbar\) are the Boltzmann and reduced Planck constants. Note that in [10,11], other expressions for the pre-exponential coefficient in (2) are proposed, but change the
result insignificantly. Assuming the "Lorentzian" dependence of the surface tension coefficient \( \sigma \) on the bubble radius \( R_b \) [9]

\[
\sigma = \frac{\sigma_0 R_b^2}{(R_b^2 + \delta_b^2)},
\]

(3)

where \( \sigma_0 = 0.072 \text{N/m} \) is the surface tension coefficient of water [12], \( \delta_b \) is a parameter similar to the Tolman parameter [13], which takes into account the smallness of the bubble size, when the scale of the transition layer at the inner boundary of the bubble is comparable with the radius of the bubble.

The corresponding energy of bubble formation [9]

\[
W_b = -\frac{4\pi}{3} P |R_b^3 + 4\pi R_b^2 \sigma_0 \left( 1 - \frac{\delta_b^2}{R_b^2} \ln \left( 1 + \frac{R_b^2}{\delta_b^2} \right) \right)|,
\]

(4)

where \( P \) is the negative pressure in the liquid. From (4) we find the value of the bubble radius \( R_{cr} \) at which \( W_b \) reaches a maximum at fixed value of \( |P| \):

\[
R_{cr} = \frac{\sigma_0}{|P|} + \sqrt{\left( \frac{\sigma_0}{|P|} \right)^2 - \delta_b^2}.
\]

(5)

It follows from (5) that appeared cavitation bubble grows indefinitely at \( R_b > R_{cr} \), and it collapses when \( R_b < R_{cr} \).

From (2) we can estimate the negative pressure \( P_{cr} \) at which \( W_b \sim k_B T \), and the probability of the appearance of a cavitation nanopore (cavitation microbubble) is large enough. Note that, as shown in [9], an anomalously large critical pressure \( P_{cr} \approx -1000 \text{ MPa} \) is obtained at \( \delta_b = 0 \), whereas the characteristic experimental value of \( P_{cr} = -30 \text{MPa} \) is obtained at \( \delta_b \approx 2.26 \text{ nm} \). Due to the exponential dependence of \( \Gamma \) on \( W_b \), it follows that when the negative pressure in the liquid slightly exceeds the threshold \( W_{cr} = W_b(P_{cr}) \), the number of bubbles of critical size in a unit volume arising in a few nanoseconds can exceed the number of water molecules present. A limiting mechanism of the cavitation generation was considered in [9,14]. Let us assume that the volume in which a supercritical negative pressure occurs is of the order of \( l_\perp^3 \). The number of nanopores that appear in the negative pressure region \( l_\perp^3 \) over a characteristic time of pressure equilibration \( \tau_\perp = l_\perp / c_s \) is equal to \( N \approx l_\perp^3 \int_0^{\tau_\perp} \Gamma dt \). Accordingly, the relative change in volume in the region where nanopores are emerging is:

\[
\frac{\Delta V}{V} \approx \frac{V_{cr} N}{l_\perp^3} = V_{cr} n_b, \quad V_{cr} = \frac{4}{3} \pi R_{cr}^3.
\]

(6)

The corresponding change in pressure in the liquid is:
\[ \delta P = \frac{c_s^2}{\delta} \rho = V_{cr} n_b c_s^2 \rho_0 . \]  

(7)

From (7) follows the estimate for the number of the cavitation pores that appeared in the unit of volume. Since the total pressure in the bubble generation region should not exceed the threshold value,

\[ n_{b,\text{sat}} \approx \frac{|P_{cr}|}{V_{cr} c_s^2 \rho_0} , \]  

(8)

a more accurate estimate of the total number of bubbles has the form [9]:

\[ n_{b,\text{sat}} = \frac{\ln \left( 1 + \frac{c_s^2 \rho_0 V_{cr}^2}{k_B T} \Gamma \left| \frac{P_{cr}}{k_B T} \delta_\tau \right| \right)}{c_s^2 \rho_0 V_{cr}^2 / k_B T} , \]  

(9)

where \( \delta_\tau \) is the characteristic time of generation of bubbles. Thus, at, \( \delta_b \approx 2.26 \text{ nm} \) and \( P_{cr} = -30 \text{ MPa} \), the critical radius is \( R_{cr} = 3.2 \text{ nm} \). Fig. 2 shows the dependence of the nanopore number density on time without (8) and with (9) the saturation effect, for the case of \( \delta_b = 2.26 \text{ nm} \) and constant pressure \( P = P_{cr} \).

![Graph showing nanopore number density on time](image)

**Fig. 2.** The dependence of emerging nanopore number density on time. Line 1 – without considering the effect of saturation, 2 – with saturation (8). \( \delta_b = 2.26 \), \( P_{cr} = -30 \text{ MPa} \).

For example, at a plate oscillation frequency, \( F = 5 \text{ MHz} \), the period and length of sound waves are \( \tau_s = 2 \times 10^{-7} \text{ s} \) and \( \lambda_s = 3 \times 10^{-4} \text{ m} \), respectively. Since the pressure at a given point in the fluid
changes as \( P = P_0 \cos(\omega t) \), the characteristic time of pressure equilibration (the characteristic time of the bubble generation), shown in Fig. 2, is

\[
\delta t_c = 2 \arccos\left(\frac{P_{cr}}{P_0}\right) / \omega = 6 \cdot 10^{-8} \text{ s}.
\] (10)

Additionally, when the amplitude of the plate oscillations correspond to a pressure amplitude of \( P_0 = 50 \text{ MPa} \), \( \delta t_c \approx 60 \text{ ns} \). In accordance with Fig. 1, the number density of bubbles generated during the time \( \delta t_c = 60 \text{ ns} \) in the negative pressure region is \( n_{p,satur} \approx 10^3 \mu\text{m}^{-3} \). Without saturation, the bubble density would exceed the number of water molecules in the negative pressure region. Outside the bubble generation region (see Fig. 3), bubbles collapse, leading to heating of the liquid. Note that heating in the cavitation region due to bubble collapse also contributes to thinning of the boundary layer since viscosity (both dynamic and kinematic) decreases with increasing of water temperature (for example, see [16]).

![Fig. 3. Distribution of pressure in a sound wave in time. Curve 1 corresponds to the pressure amplitude \( p_0 < |p_{cr}| = 30 \text{ MPa} \) curve 2: \( p_0 = 50 \text{ MPa} \). \( \delta t_c, \delta t_d \) are the time interval of bubble generation (10) and dissipation.](image)

4. Expansion of bubbles

In the negative pressure region, the generated cavitation bubbles increase in size. We will show that when cavitation generation conditions are met, cavitation bubbles can grow to submicron sizes. The average distance between cavitation bubbles is of the order of \( l_b \sim n_b^{-1/3} \). For the above example, \( l_b \sim 0.1\mu\text{m} \), about 30 times larger than the critical radius of the cavitation nanopores. Therefore, the pores can be assumed to be noninteracting among themselves and therefore considered as being in a self-consistent field of pressure. Since \( R_b \ll \lambda_s \), to describe the pore dynamics, we can use the Rayleigh equation [15,9] for a spherically symmetric pore in a constant negative pressure field.
\[
\frac{d}{dt}\left( R_b^2 \left( \frac{dR_b}{dt}\right)^2 \right) = \frac{2}{\rho_0} \left( -P - \frac{2\sigma_0}{R_b(1+\delta^2/R_b^2)} \right) R_b^2 \frac{dR_b}{dt} .
\]  \hspace{1cm} (11)

The first term on the right-hand side of (11) corresponds to the time dependence of the pressure of the acoustic wave, the second to the surface tension forces with the assumed "Lorentzian" dependence of the surface tension coefficient on the bubble radius (3). In the region of bubble generation, \( P \approx P_{cr} < 0 \), and the right-hand side is positive. Since the expansion of the bubble causes surface tension forces to drop, as an approximation, we neglect the second term on the right-hand side of (11). In this case, the radius of the cavitation bubble increases linearly with time:

\[
R_b \sim v_b t = \left( \frac{2|P_{cr}|}{3\rho_0} \right)^{1/2} t .
\]  \hspace{1cm} (12)

When \( P_{cr} = -30 \text{ MPa} \), the pore expansion rate is \( v_b \sim 150 \text{ m/s} \). Obviously, bubbles can grow until their volume will not become comparable with the volume of the displaced water

\[
\xi = \frac{4}{3} \pi R_b^3 n_{b,satur} = \frac{4}{3} \pi R_b^3 n_{b,satur} < 1 .
\]  \hspace{1cm} (13)

Multiplying the volume of the cavitation region by \(|P_{cr}|\), we obtain an estimate of the work done by the negative pressure creating bubbles in a unit volume of liquid:

\[
W_b \approx \xi |P_{cr}| .
\]  \hspace{1cm} (14)

The estimate (14) is valid, while \( W_b \) is smaller than the energy density of the sound wave:

\[
W_s = \frac{\rho_0 \omega^2 a_0^2}{2} = \frac{P_0^2}{2 \rho_0 c_s^2} ,
\]  \hspace{1cm} (15)

that is

\[
\eta = \frac{W_b}{W_s} = \frac{2 \rho_0 c_s^2}{P_0^2} \xi |P_{cr}| < 1 .
\]  \hspace{1cm} (16)

From (16) follows:

\[
\xi = \frac{P_0^2}{2 \rho_0 c_s^2 |P_{cr}| \eta} = 1.5 \cdot 10^{-5} P_0^2 \eta ,
\]  \hspace{1cm} (17)

where \( P_0 \) in MPa.

It follows from (17) that if the amplitude of the pressure in the radiated sound wave is \( P_0 < 367 \text{ MPa} \) (\( |P_{cr}| = 30 \text{ MPa} \)) then the work of negative pressure on the creation of bubbles becomes comparable with the energy of the sound wave (\( \eta = 1 \)) before the total volume of bubbles becomes comparable with the volume of water (\( \xi = 1 \)). Figures 4a-4c show the time dependences \( \xi \) and \( \eta \) for different values of \( P_0 \). At \( P_0 = 50 \text{ MPa} \) (Fig. 4a), \( \delta t_\perp = 59 \text{ ns} \), \( \delta t_d = 141 \text{ ns} \), then the work done by negative pressure creating bubbles becomes comparable with the energy
of the sound wave at $t = t_{\text{s,d}} \approx 0.1$ ns; (4b) - $P_0 = 200$ MPa, $t = t_{b,\text{grow}} \approx 0.55$ ns , $\delta t_c = 90$ ns, $\delta t_d = 110$ ns, $\delta t_{s,d} \approx 0.36$ ns, at $t_{b,\text{grow}} \approx 0.53$ ns. In both cases (a) and (b), the volume occupied by the bubbles is insignificant compared to the volume occupied by water. However, at $P_0 = 370$ MPa (Fig. 4c) $\delta t_c = 95$ ns, $\delta t_d = 105$ ns, $t_{s,d} \approx t_{b,\text{grow}} \approx 0.5$ ns, the volume occupied by the bubbles in the cavitation region becomes equal to the volume of the displaced water.

![Graphs](image)

**Fig. 4.** The time dependences $\xi$ and $\eta_c$ at $P_{cr} = -30$ MPa, $f = \omega / 2\pi = 5$ MHz. (a) - $P_0 = 50$ MPa , b) - $P_0 = 200$ MPa , c) - $P_0 = 370$ MPa .

It can be seen from (16) that for a sound wave pressure amplitude below several hundred megapascals, it is impossible during a single oscillation period to create a region with a significant volume of bubbles and thereby reduce the friction coefficient of the liquid on the plate surface. However, we will show that a region filled with bubbles can be created even at comparatively low amplitudes of sound waves pressure. At harmonic oscillations of the piezo plate, the phase of negative pressure is interchanged by a positive pressure phase (Fig. 3), in which the bubbles are compressed at approximately the same rate (12) with which they expand at negative pressure.

Fig. 5a shows the elementary segment of the piezoelectric plate system shown in Fig. 1. To avoid forced compression of the bubbles, the plate must quickly shift down and then slowly return to its initial position, so that the pressure profile has the form shown in Fig. 5b. If, with such a pressure profile, the bubble size $R_b$ grows large enough that the saturated vapor pressure $P_n$ can compensate the surface tension, $R_b > 2\sigma_0 / P_n$, then the bubbles will not collapse during the period of oscillation of the plate and their volume will increase proportionally to the number of plate oscillations $N_{oscil}$.

From (17), we define the volume occupied by bubbles formed during one cycle of the negative pressure phase caused by the displacement of the plate. Assuming all the excess potential energy of the stretched liquid is expended on the formation of cavitation bubbles ($\eta = 1$), we find the relative volume $\Omega$ occupied by the bubbles formed during $N_{oscil}$ oscillations.
\[
\Omega = \varepsilon N_{\text{oscil}}^2 = \frac{P_0^2}{2 \rho_0 c_s^2 |P_{cr}|} N_{\text{oscil}}.
\]  
(18)

Assuming \( \Omega \approx 1 \), we get

\[
N_{\text{oscil}} = \frac{2 \rho_0 c_s^2 |P_{cr}|}{P_0^2} \frac{|P_{cr}|}{W_s}.
\]  
(19)

The power emitted per unit surface of the piezoelectric plate to maintain bubble saturation density in the near-wall region \( \Theta \) \((W/m^2)\) can be estimated by knowing the energy density in a sound wave \( W_s \) in the vicinity of the oscillating wall, the time of energy transfer from sound vibrations to bubble creation \( t_{s,d} \), the number of oscillations in the pulse train \( N_{\text{oscil}} \), and the characteristic time for bubbles to convect away from the generation region \( t_0 = l/u \), where \( l \) is the longitudinal dimension of the plate, \( u \) is the flow velocity.

\[
\Theta \sim W_s c_s t_{s,d} N_{\text{oscil}} \frac{u}{l} \sim 4.5 \cdot 10^9 t_{s,d} \frac{u}{l}.
\]  
(20)

In the numerical coefficient in (20), we substituted the expression for \( N_{\text{oscil}} \) from (19), and the value of the speed of sound in water. Taking, for example, \( u \sim 10 \text{ m/s} \), \( l = 0.01 \text{ m} \), \( t_{s,d} \approx 0.5 \text{ ns} \), we get, \( \Theta \sim 22 \text{ kW/m}^2 \).

**Fig. 5** (a) – oscillations of the boundary surface of the piezoelectric plate. Positions 1 and 2 correspond to the initial position and the maximum displacement of the plate, respectively. When the plate moves downward, a tensile stress appears in the liquid. While moving in the opposite direction, the tensile stress decreases. Since the plate moves slowly in the opposite (upward) direction, a positive pressure does not appear. (b) – the time dependence of the pressure in the liquid near the plate’s surface due to its oscillation.

The estimate (19) is an upper limit, since the flow velocity in the boundary layer near the plate is much lower than in the main flow. Needless to say, the results presented here are estimates, since
we neglect the change in the mechanical properties of the liquid due to a large number of bubbles rising. The present work is of a qualitative, evaluative nature and does not claim to describe accurately the processes of cavitation bubble expansion, fusion and motion induced in a liquid by the oscillations of the plate with the pressure amplitude $P_0 > |P_{cr}|$.

We have shown that ultrasound generated by oscillating piezoelectric plates placed on a surface bounded by a liquid flow can be a source of cavitation in the boundary layer. In turn, following the works [1-3], it can be expected that the emerging cavitation vapor bubbles will lead to a significant decrease in the coefficient of friction. Future experimental studies and further development of the theory will make possible to apply this approach practical applications.

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