A tetraquark model for the new $X(1576)$ $K^+K^-$ resonance

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Abstract

We discuss the likely tetraquark interpretation of the $X(1576)$ $K^+K^-$ isovector resonance recently reported by BES with $J^{PC} = 1^{--}$. We point out that if this interpretation as a four-quark state is correct, the $su\bar{s}d$ tetraquark decays might have striking signatures. We also provide predictions for possible analogous tetraquarks involving heavy quarks – $cq\bar{c}q$ and $b\bar{b}q\bar{q}$.

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I. THE NEW ISOVECTOR $K^+K^-$ RESONANCE

We have noticed with interest the new $K^+K^-$ resonance [1] with $J^{PC} = 1^{--}$, and a pole mass at $1576^{+49}_{-55} \text{(stat)}^{+108}_{-91} \text{(syst)} - i \left( 409^{+11}_{-12} \text{(stat)}^{+32}_{-67} \text{(syst)} \right) \text{MeV}$ which seems to be a four-quark state. Since it is produced with a pion from the isoscalar $J/\psi$, which has odd $G$-parity it must be isovector and have even $G$-parity.

Since no nonstrange even-$G$ vector resonances like $\rho^-$ or $\pi^-$ have been reported in this mass range the new $K^+K^-$ resonance seems to contain a strange quark pair. An isovector particle containing a strange quark pair must necessarily contain an additional isovector nonstrange pair to make an isovector. Thus this new state must contain a minimum of two quark-antiquark pairs. It is also worth noting that this state cannot be a $\bar{s}s$ hybrid, since a state of a strange quark pair and gluons is isoscalar and cannot be isovector.

A recent examination of four-quark (tetraquark) states [2] has emphasized the states including heavy quarks near the two-meson threshold may well exist as bound tetraquarks. Applying this approach to the two-kaon system gives two diquark-antidiquark configurations with masses above the two-kaon threshold with ratios of masses to the mass of two kaons of 1.21 and 1.16 respectively. These results are for $S$-wave systems and neglect spin dependence. They are therefore lower than the mass of the new state which must have a $P$-wave to explain the negative parity. They are therefore in a reasonable ball park, since experimentally $m_X/(2m_K) \approx 1.6$.

In contrast with the $1^{--}$ state reported by BES [1], the $S$-wave tetraquark with these flavor quantum numbers probably breaks up so fast that it is too broad to be seen.

At this stage any further calculation of orbital and spin effects will be highly model dependent and unreliable. However, the diquark-antidiquark model [2] makes clear flavor predictions that are easily tested in experiment. We consider here predictions relating the decays of the new $X(1576)$ resonance. Although only $X^0$ has been observed experimentally, existence of $X^+$ and $X^-$ follows from $X$ having $I = 1$. Since decays involving charged pions in the final state are much easier to observe, we focus on these:

We first note that the dominant decay mode of a tetraquark is the “fall-apart mode” in which the two quarks and two antiquarks rearrange into two mesons and separate. No new quark-antiquark pairs are created or destroyed. This immediately leads to a selection rule for a diquark($us$)-antidiquark($\bar{d}\bar{s}$) model for the new $X(1576)$ resonance.

$$BR(X^+ \to \pi^+\pi^0) = 0$$

(1)

This selection rule provides the crucial distinction between tetraquark and $q\bar{q}$ models. A positively charged $q\bar{q}$ must be in an octet of $SU(3)_{flavor}$. The $\pi\pi$ decay amplitude is required by $SU(3)$ symmetry to be proportional to the same reduced matrix element as the $K\bar{K}$ and cannot vanish for a hadron decaying into $K\bar{K}$.

A further prediction for the tetraquark model is obtained by noting that there are two ways in which a $us$ diquark and a $d\bar{s}$ antidiquark can rearrange to make two mesons

$$|us, \bar{d}s\rangle \to |M(u\bar{d}), M(s\bar{s})\rangle; \quad |us, \bar{d}s\rangle \to |M(u\bar{s}), M(s\bar{d})\rangle$$

(2)

These two decay modes must be equal in the flavor-$SU(3)$ symmetry limit since there is no preferred way that a quark in the diquark can choose a particular antiquark in the antidiquark.
$SU(3)_f$ breaking will make the two modes somewhat different, because the color-magnetic spin-dependent interaction depends on the quark masses, but we expect the approximate equality to hold at least as well as most $SU(3)_f$ relations.

The approximate equality of the two modes (2) immediately gives predictions relating the decays of the new $X(1576)$ resonance.

$$
\tilde{\Gamma}(X^+ \rightarrow K^+\bar{K}^*) = \tilde{\Gamma}(X^+ \rightarrow K^{*+}\bar{K}^0) \approx \tilde{\Gamma}(X^+ \rightarrow \pi^+\phi) \approx \tilde{\Gamma}(X^+ \rightarrow \rho^+\eta) + \tilde{\Gamma}(X^+ \rightarrow \rho^+\eta')
$$

(3)

where $\tilde{\Gamma}$ denotes the partial width neglecting phase space corrections, and the conventional nonet description is used for the pseudoscalar mesons with $SU(3)$ flavor symmetry.

Experimental confirmation of the selection rule (1) and the predictions (3) would provide unambiguous support for the tetraquark description, since other models relate the observed $K\bar{K}$ decay very differently to these other two-pseudoscalar decays.

The first equality in eq. (3) is model independent and follows from $G$-parity. The remaining equalities have symmetry breaking corrections which can be averaged out in some approximation by rewriting this relation as

$$
\tilde{\Gamma}(X^+ \rightarrow K^+\bar{K}^*) + \tilde{\Gamma}(X^+ \rightarrow K^{*+}\bar{K}^0) \approx \tilde{\Gamma}(X^+ \rightarrow \pi^+\phi) + \tilde{\Gamma}(X^+ \rightarrow \rho^+\eta) + \tilde{\Gamma}(X^+ \rightarrow \rho^+\eta')
$$

(4)

The prediction (3) relates OZI-allowed decays to OZI-forbidden decays. Although the equality may not be exact, because of flavor symmetry breaking, there is clearly no OZI suppression predicted.

Further support for a tetraquark description of the $X(1576)$ would be a strong $\phi\pi$ branching ratio, since the $\phi\pi$ decay is OZI-forbidden for a normal quark-antiquark resonance.

Recent data on the decay $J/\psi \rightarrow \phi\pi\pi$ show no $\phi\pi$ resonance near the new $X(1576)$ resonance [3]. If this leads to a strong disagreement with the prediction (4), it will pose a serious difficulty for most models. A model containing an isovector $q\bar{q}$ pair and no additional quarks must be in a flavor $SU(3)$ octet which requires comparable $K^+K^-$ and $\pi^+\pi^-$ decays. A model which contains both an isovector pair and a strange quark pair can decay by quark rearrangement into $\phi\pi$. There is no known OZI rule for tetraquarks to suppress this decay.

II. OZI VIOLATION AS THE KEY TO MULTIQUARK STATES

Violation of the OZI rule provides a useful signature to distinguish between decays of normal mesons and tetraquarks. In the two ways (2) in which a $us$ diquark and a $\bar{d}\bar{s}$ antidiquark can rearrange to make two mesons both ways have the same topology and the two are equivalent. When these final states are produced by the decay of a normal $q\bar{q}$ isovector meson, the transition to the state $|M(u\bar{d}), M(s\bar{s})\rangle$ is forbidden by the OZI rule while the transition to the state $|M(u\bar{s}), M(s\bar{d})\rangle$ is allowed.

This flavor-exchange symmetry can thus be used as a test of tetraquark models for other anomalous quarkonium states including heavy quarks. All isovector states that decay into heavy quark pairs are generally recognizable as multiquark states, since such a decay of a
nonstrange isovector state by the creation of a heavy $\bar{q}q$ is expected to be suppressed. The analogues of eq. (3) for charm and bottom are:

$$\tilde{\Gamma}(X^+_c \rightarrow D^+ D^{*0}) = \tilde{\Gamma}(X^+_c \rightarrow D^{*+} D^0) \approx \tilde{\Gamma}(X^+_c \rightarrow \pi^+ J/\psi) \approx \tilde{\Gamma}(X^+_c \rightarrow \rho^+ \eta_c) \quad (5)$$

$$\tilde{\Gamma}(X^+_b \rightarrow B^+ B^{*0}) = \tilde{\Gamma}(X^+_b \rightarrow B^{*+} B^0) \approx \tilde{\Gamma}(X^+_b \rightarrow \pi^+ \Upsilon) \approx \tilde{\Gamma}(X^+_b \rightarrow \rho^+ \eta_b) \quad (6)$$

In analogy with eq. (3) the first equalities in eqs. (5) and (6) are model independent and follow from $G$-parity. The remaining equalities have symmetry breaking corrections which can be averaged out in some approximation by rewriting these relations as

$$\tilde{\Gamma}(X^+_c \rightarrow D^+ D^{*0}) + \tilde{\Gamma}(X^+_c \rightarrow D^{*+} D^0) = \tilde{\Gamma}(X^+_c \rightarrow \pi^+ J/\psi) + \tilde{\Gamma}(X^+_c \rightarrow \rho^+ \eta_c) \quad (7)$$

$$\tilde{\Gamma}(X^+_b \rightarrow B^+ B^{*0}) + \tilde{\Gamma}(X^+_b \rightarrow B^{*+} B^0) = \tilde{\Gamma}(X^+_b \rightarrow \pi^+ \Upsilon) + \tilde{\Gamma}(X^+_b \rightarrow \rho^+ \eta_b) \quad (8)$$

One should keep in mind, however, that even with this averaging relations (7) and (8) receive significant corrections due to $m_b, m_c \gg m_u, m_d$.

This reasoning can be extended to identify isoscalar heavy-quark tetraquark states. We note that the flavor-exchange symmetry relates the two ways in which a $\bar{d}Q$ antidiquark can rearrange to make two mesons, even though one is OZI allowed and one is OZI forbidden for the decay of a light quark meson.

$$\langle dQ, \bar{d}Q \rangle \rightarrow \langle M(d\bar{d}), M(Q\bar{Q}) \rangle; \quad \langle dQ, \bar{d}Q \rangle \rightarrow \langle M(d\bar{Q}), M(Q\bar{d}) \rangle \quad (9)$$

This immediately gives predictions identifying possible anomalous isoscalar strangonium, charmonium and bottomonium states with even parity and even charge conjugation, denoted as $X^{(I=0)}_s$, $X^{(I=0)}_c$ and $X^{(I=0)}_b$, as cryptoexotic multiquark states by the absence of OZI suppression:

$$2 \tilde{\Gamma}(X^{(I=0)}_c \rightarrow D^+ D^-) \approx \tilde{\Gamma}(X^{(I=0)}_c \rightarrow \eta \eta_c) + \tilde{\Gamma}(X^{(I=0)}_c \rightarrow \eta' \eta_c) \quad (10)$$

$$2 \tilde{\Gamma}(X^{(I=0)}_b \rightarrow B^+ B^-) \approx \tilde{\Gamma}(X^{(I=0)}_b \rightarrow \eta \eta_b) + \tilde{\Gamma}(X^{(I=0)}_b \rightarrow \eta' \eta_b) \quad (11)$$

$$2 \tilde{\Gamma}(X^{(I=0)}_c \rightarrow K^{*+} \bar{K}^{*-}) \approx \tilde{\Gamma}(X^{(I=0)}_c \rightarrow \phi \omega) \quad (12)$$

$$2 \tilde{\Gamma}(X^{(I=0)}_c \rightarrow D^{*+} \bar{D}^{*-}) \approx \tilde{\Gamma}(X^{(I=0)}_c \rightarrow J/\psi \omega) \quad (13)$$

$$2 \tilde{\Gamma}(X^{(I=0)}_b \rightarrow B^{*+} \bar{B}^{*-}) \approx \tilde{\Gamma}(X^{(I=0)}_b \rightarrow \Upsilon \omega) \quad (14)$$

The additional factor of 2 on the l.h.s. of eqs. (10)–(14) results from the that $X^{(I=0)}_Q$ has both a $dQ\bar{d}Q$ and a $uQ\bar{u}Q$ component, so that eq. (9) should have a companion equation with $d$ replaced by $u$. Both of these amplitudes contribute to the right hand side of eqs. (10)–(14). Only one contributes to the left hand side. The other one contributes to the final state with isospin partners of the $q\bar{Q}$ mesons, such as $D^0\bar{D}^{*0}$, etc.

The direct analog of eqs. (10) and (11) for strangeonium decays to the $\eta$ and $\eta'$ are more complicated because of the $\eta - \eta'$ mixing which does not exist for $\eta_c$ and $\eta_b$. For the transitions to the strange and nonstrange pseudoscalar mesons denoted by $\eta_s$ and $\eta_n$ without flavor mixing, we have

$$\tilde{\Gamma}(X^{(I=0)}_s \rightarrow K^+ \bar{K}^-) = \tilde{\Gamma}(X^{(I=0)}_s \rightarrow \eta_s \eta_s) \quad (15)$$
III. ALTERNATIVE TETRAQUARK MODEL - A $K\bar{K}$ MOLECULE

If there is a strong disagreement with the prediction (4), it will pose a serious difficulty for most models.

One model that might explain the absence of a $\phi\pi$ decay mode or a violation of the prediction (4) is a $K^+K^-$ molecule where the two kaons are sufficiently far apart so that they cannot exchange quarks. However, if the $K^+$ and $K^-$ are that far apart they are very likely also too far apart to annihilate a $u\bar{u}$ pair and create a $d\bar{d}$ pair to make a $K^0\bar{K}^0$ molecule. In this case the $X$ is not an isospin eigenstate but is a pure $K^+K^-$ molecule. This is an isospin mixture with equal isovector and isoscalar components. It should be produced in $J/\psi$ decays not only with the isovector pion but also via its isoscalar component with the isoscalar $\eta$ and $\eta'$ or the isoscalar $\omega$. These final states should also be seen if the $X$ is indeed an isovector but has an isoscalar partner, analogous to the $\rho$ and $\omega$ doublet.

IV. CONCLUSIONS

We present experimental predictions for decays of tetraquarks that can distinguish between tetraquark and other models for hadronic states. These predictions provide a serious test for the new $X(1576)$ resonance. An isovector $K^+K^-$ resonance which does not decay into $\pi^+\pi^-$ cannot be a flavor $SU(3)$ octet and must contain a strange quark pair in addition to a nonstrange isovector pair. If its decays also violate the tetraquark predictions, it must have a new multiquark structure like a molecule.

ACKNOWLEDGEMENTS

We thank Bingsong Zou for correspondence on the BES data and for pointing out an error in one of the channels considered in the previous version of this paper. The research of M.K. was supported in part by a grant from the Israel Science Foundation administered by the Israel Academy of Sciences and Humanities. The research of H.J.L. was supported in part by the U.S. Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38.

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