A topology and sizing optimization method for frame structures combined with an orthogonal-maximin Latin hypercube design (LHD) method

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Abstract. This work presents a new strategy for improving the global searching capability of the two-level approximation method. By using an orthogonal-maximin latin hypercube design (LHD) method, five initial designs reflecting more information of the feasible domain can be generated. Due to their good representation for the feasible domain of the five latin hypercube designs, it is more likely to obtain a global optimum or a near global optimum which is better than before. Numerical examples of frame structures are established to illustrate the effectiveness of this method. From the results, it can be seen that a series of better optimal solutions can be obtained by using the strategy of orthogonal-maximin LHD method and the number of structure analysis required in the optimization process is dozens of times or fewer, even comparable to size-variable optimization.

1. Introduction
Frame structures have been widely applied in the engineering field, e.g., aerospace and mechanical fields. Since the design and requirement of these structures are increasingly complex and various, frame topology optimization has been attracted much attention over the last decades[1][2]. Two main fields are researched about these optimization problems. One is the continuous topology optimization, which is usually applied for entity beam models and optimized by using material density as a variable. Starting from an initial continuous solid structure, the element density will be optimized based on the two-phase field method which determines the existence of the corresponding element[3]. However, there is a problem that due to presence or absence of the element, the boundary of the structure will not be continuous, which must be smoothed in the following process. A level-set method is presented with three field method to solve this problem[4], yet a large number of iterations is required in the optimization process. Another main field is discrete-component topology optimization, which is the primal problem for topology optimization. Starting from a ground structure, the optimal solution will be obtained with various discrete optimization methods. These methods can be involved into two types. i.e. meta heuristics and global optimization methods. As for meta heuristics, genetic algorithms (GAs) which imitate the evolutionary process of living organisms have been attracted more attention and widely used in the past thirty years[5][6], yet a considerable deal of computation cost is required when using these methods. Global optimization methods are established based on a branch of mathematical programming, and among these methods, branch-and-bound algorithms are the most frequently used method. By formulating the original problem as a mixed 0-1-linear/conic program, the optimal result
can be obtained with branch-and-bound algorithms\(^7\). However, the computational time of these methods is still large. In recent studies, Li\(^8\) presented a two-level approximate method combined with an improved GA for solving truss topology optimization. Starting from an initial ground structure, the topology configuration is determined with GA and its related size variables are optimized by tackling a series of approximate problems. Because of utilization of the approximate concept, the number of structural analyses is quite small. Subsequently, An and Huang\(^9\) developed this method to solve frame topology optimization problems, and specific cross sectional dimensions were directly treated as variables instead of areas. However, since the optimal result is likely to be relevant to the initial design, a satisfactory solution may not be obtained without a number of initial designs are attempted. In this work, in order to enhance the global searching capability of the above approximate method, an orthogonal-maximin latin hypercube design (LHD) method is used\(^10\). Before the approximate optimization process starts, five initial designs are generated by using orthogonal-maximin LHD method. Then the approximate method is executed for five times separately with different initial designs. Compared with standard LHD methods, the orthogonal-maximin LHD method adds the procedures of optimizing the pairwise correlations or the inter-site distances, which is conducive to finding good LHDs. Meanwhile, since the number of structural analyses is small for the original approximate method, the total computational cost in this work is still not very large. Numerical examples are established to illustrate feasibility of this method as well as high efficiency, and the optimization result show that a satisfactory lightweight structure can be obtained with five LHDs.

2. Problem Description

The optimization model is established based on the ground structural method. As the dimension and existence of each bar member should be determined simultaneously, two types of variables are introduced in this work. Continuous size variables are used to optimize specific properties of the beam, while discrete 0/1 variables are utilized to represent its presence (1) and absence (0). In order not to remove the absent components in the FEM model, the dimensions of unnecessary components are set as small values which are marginal for the structure consisting of no absent members. The topology optimization problem is formulated as follows.

\[
\begin{align*}
\text{Find} & \quad X = \{x_1, x_2, \ldots, x_m\}^T \\
& \quad \alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}^T \\
\text{Min} & \quad f(X, \alpha) \\
\text{s.t.} & \quad g_j(X, \alpha) \leq 0 \quad j = 1, \ldots, J_0 \\
& \quad \alpha_i^k = 0 \text{ or } \alpha_i^k = 1 \quad i = 1, \ldots, n, \quad k = 1, \ldots, m \\
& \quad \alpha^k_i x_i^k + (1-\alpha^k_i) x_i^U \leq x_i \\
& \quad x_i \leq \alpha^k_i x_i^U + (1-\alpha^k_i) x_i^L 
\end{align*}
\]

Here, \(X\) is the discrete size variable vector consisting of \(m\) variables, and \(\alpha\) represents the discrete 0/1 variable vector with \(n\) variables. \(x_i^L\) and \(x_i^U\) are the lower and upper bounds of the \(k-th\) variable \(x_i\) if the related structural component is retained, respectively, while \(x_i^e\) is a small value to represent the size variable of unnecessary component. \(\alpha^k_i\) is the \(i-th\) variable in \(\alpha\) which represents the existence of \(x_i\). \(f(X, \alpha)\) and \(g_j(X, \alpha)\) denote the objective and normalized structural constraint functions, respectively, and \(J_0\) is the number of total constraints.

3. Optimization Method

3.1. The first-level approximate problem
The constraints in Eq.(1) are implicit and nonlinear in general, in order to formulate the constraints explicitly, a sequence of first-level approximate problems are established. In the $p-th$ stage, it can be constructed as follows.

$$
\begin{align*}
\text{Min} & \quad f^{(p)}(X,\alpha) \\
\text{s.t.} & \quad g_j^{(p)}(X,\alpha) \leq 0 \quad j = 1,\ldots,J_i \\
& \quad \alpha_k^i = 0 \text{ or } \alpha_k^i = 1 \quad i = 1,\ldots,n, \quad k = 1,\ldots,m \\
& \quad \alpha_k^i x_{kip}^{jU} + (1-\alpha_k^i) x_{kip}^{jL} \leq x_k \\
& \quad x_k \leq \alpha_k^i x_{kip}^{jU} + (1-\alpha_k^i) x_{kip}^{jL} \\
& \quad x_{kip}^{jU} = \min\{x_k^U, \bar{x}_{kip}^{jU}\} \\
& \quad x_{kip}^{jL} = \max\{x_k^L, \bar{x}_{kip}^{jL}\}
\end{align*}
$$

(2)

Here, $f^{(p)}(X,\alpha)$ and $g_j^{(p)}(X,\alpha)$ represent the approximate objective and constraint functions which can be formulated with a branched multipoint approximation (BMA) function, respectively. $J_i$ is the number of active constraints in the current topology configuration. $\bar{x}_{kip}^{jU}$ and $\bar{x}_{kip}^{jL}$ denote the move limits of $x_k$ during the $p-th$ stage, respectively.

The effectiveness of the BMA function has been proved in many literatures$^{[11]}$, in this work, the function is established as follows.

$$
\bar{w}_i^{(p)}(X,\alpha) = \sum_{i=1}^{n} \bar{w}_i(X,\alpha)h_t(X,\alpha) 
$$

(3)

where

$$
\bar{w}_i(X,\alpha) = w(X,\alpha) + \sum_{i=1}^{n} \Delta w_{i,i}^{(p)}(X,\alpha)
$$

(4)

$$
\Delta w_{i,i}^{(p)}(X,\alpha) = \left\{
\begin{array}{ll}
\frac{1}{r_{i,i}} \frac{\partial w(X,\alpha)}{\partial x_i} (x_i^{jU} - x_i^{jL}) & \text{if } \alpha = 1 \\
\frac{1}{r_{i,i}} \frac{\partial w(X,\alpha)}{\partial x_i} (1 - e^{-(x_i^{jU} - x_i^{jL})}) & \text{if } \alpha = 0
\end{array}
\right.
$$

(5)

$$
\bar{h}_i(X,\alpha) = \frac{\bar{h}_i(X,\alpha)}{\sum_{i=1}^{n}} \quad t = 1,\ldots,H
$$

(6)

$$
\bar{h}_{\sum_{i=1}^{n}}(X,\alpha) = \prod_{i=1}^{n} (X - X_i)^T (X - X_i)
$$

(7)

Here, $w^{(p)}(X,\alpha)$ is the BMA function which consists of two parts -- approximate value function $\bar{w}_i(X,\alpha)$ and weighted function $\bar{h}_i(X,\alpha)$. In $\bar{w}_i(X,\alpha)$, the real constraint $w(X,\alpha)$ are utilized to preserve the accurate constraint value at the point $(X,\alpha)$, and $\Delta w_{i,i}^{(p)}(X,\alpha)$ is a branched analogical partial derivative function which is used to enhance approximate accuracy in the region surrounding $(X,\alpha)$. $\bar{h}_i(X,\alpha)$ satisfies all conditions for the weighted function in interpolation functions.

Meanwhile, $r_{i,i}$ and $r_{i,i}$ are the adaptive parameters which are employed to control the nonlinearity of the BMA function and determined with the least square method, shown as follows.

$$
\begin{align*}
\min & \quad \sqrt{\sum_{h=1}^{H}[w(X_h,\alpha_h) - \bar{w}_i(X_h,\alpha_h)]^2} \\
\text{s.t.} & \quad r_{i,i}^L \leq r_{i,i} \leq r_{i,i}^U, \quad k = 1,\ldots,H
\end{align*}
$$

(8)
where $l^r_w$, $u^r_w$, and $r^l_w$ are the lower and upper bounds of $r_w$ and $r_{sw}$, respectively. As the approximate function of the new design point is controlled with the adaptive parameters, the nonlinearity of the approximation can be dominated and its accuracy will be improved. Since the first-level approximate problem involves both discrete and continuous variables, it is quite difficult to be tackled by using general mathematical programming methods. To solve this problem, a layered optimization strategy is constructed\([9]\). Discrete 0/1 variables are determined with genetic algorithms (GAs) in the external layer, and continuous variables are optimized in the internal layer by establishing the second-level approximation problems. The second-level problems can be tackled with a dual method.

3.2. GA for discrete variables
A standard GA is used here to optimize discrete variables. Starting from an initial population randomly generated with 0 or 1, it is more likely to be preserved for the ones owing larger fitness functions by using the simulated roulette wheel selection method. Then, with the crossover and mutation operators, a new population will be reproduced and the preceding operation will be executed repeatedly until the convergence conditions are satisfied. The fitness function in this work is formulated as follows.

$$F = \begin{cases} 
 f^{(p)}(X_\alpha, a_\epsilon) - \{f^{(p)}(X, a)\} & \text{if } (X_\alpha, a_\epsilon) \text{ is feasible}, \\
 \frac{1}{n} \sum_{j=1}^{n} \frac{g_j^{(p)}(X_\alpha, a_\epsilon)}{1 + \sum_{j=1}^{n} \bar{g}_j^{(p)}(X, a_\epsilon)} - 1 & \text{otherwise}. 
\end{cases} \quad (9)$$

where

$$\bar{g}_j^{(p)}(X, a_\epsilon) = \begin{cases} 
 f^{(p)}(X_\alpha, a_\epsilon) & \text{if } f^{(p)}(X_\alpha, a_\epsilon) > \{f^{(p)}(X, a)\}, \\
 \{f^{(p)}(X, a)\} & \text{otherwise}. 
\end{cases} \quad (10)$$

Here, $\{f^{(p)}(X, a)\}$ denotes the average of the approximate objective functions in the current population, while $\bar{g}_j^{(p)}$ represents the average violation of the $j$-th constraint. $f^{(p)}(X_\alpha, a_\epsilon)$ and $g_j^{(p)}(X_\alpha, a_\epsilon)$ are the approximate objective and constraint functions with respect to the point $(X_\alpha, a_\epsilon)$, respectively, where $X_\alpha$ represents the vector of optimal size variables obtained from the second-level approximation problem and $a_\epsilon$ is its given layout. $\epsilon$ is a small value used for rationalizing the proportion of the maximum constraint in the fitness function when the design point $(X_\alpha, a_\epsilon)$ satisfies all the constraint requirements.

3.3. Second-level approximate problem for continuous variables
In Section 3.2, the topology layout for each individual in GA is determined, recorded as $a_\epsilon$. For the components which are removed from the ground structure, their size variables are fixed with small values. Therefore, only such size variables corresponding to present components need to be optimized in the first-level approximate problem. For simplification, a second-level approximate problem is established to approach the primal problem in Section 3.1. In the second-level problem, the original approximate objective and constraint functions are reconstructed with respect to continuous variables and their reciprocal variables, respectively. For each individual with a fixed layout in GA, this approximate problem is formulated as follows.
where $\mathbf{X}$ represents the vector of remained continuous size variables containing $I$ members. As some structural components are removed, a portion of constraints are not considered and will be deleted by using temporal deletion techniques (For more details, please see the previous work [9] and [12]). $J_2$ represents the number of retained constraints, while $\mathbf{g}^{(q)}(\mathbf{X}, \mathbf{a}_s)$ and $\mathbf{f}^{(q)}(\mathbf{X}, \mathbf{a}_s)$ are the constraint and objective functions of the point $(\mathbf{X}, \mathbf{a}_s)$, respectively. $x_{i(k)}^L$ and $x_{i(k)}^U$ are the upper and lower bound of $x_i$, and $\bar{x}_{i(k)}^L$ as well as $\bar{x}_{i(k)}^U$ denote the move limit of $x_i$.

The dual problem of the above approximate problem can be stated as

$$
\begin{align*}
\max & \quad I(\lambda) = \min_{\mathbf{x}, \lambda \in \mathbf{X}} \left[ \mathbf{f}^{(q)}(\mathbf{X}, \mathbf{a}_s) + \sum_{j=1}^{J_2} \lambda_j \mathbf{g}^{(q)}(\mathbf{X}, \mathbf{a}_s) \right] \\
\text{s.t.} & \quad \lambda_j \geq 0, \quad j = 1, \ldots, J_2
\end{align*}
$$

where $R(\mathbf{X}) = \{ \mathbf{x} \mid x_{i(k)}^L \leq x_i \leq x_{i(k)}^U, \quad k = 1, \ldots, I \}$. By solving the above problem, continuous size variables can be obtained.

3.4. Orthogonal-maximin LHD method

In order to find more better solutions based on the above topology optimization method, a new method which can generate space-filling samples is employed, recorded as orthogonal-maximin LHD method. This method is developed from traditional LHD methods. LHD methods have been widely used in computer experiments. With several LHD samples, the property of the whole feasible domain can be represented in general. However, a randomly generated LHD may be highly correlated or may not have good space-filling properties. For this situation, an orthogonal-maximin LHD method was presented. Starting from a random LHD $X$. A perturbation $X_{\text{py}}$ of a design $X$ is generated by interchanging two elements within a chosen column in $X$. The perturbation $X_{\text{py}}$ replaces $X$ if it leads to an improvement. Otherwise, it will replace $X$ with probability $\pi = \exp[-(\phi_p(X_{\text{py}}) - \phi_p(X))/t]$, where $t$ is a preset parameter known as “temperature”. The two elements chosen in $X$ are determined with the multi-objective criterion, which consists of both $\rho^2$ and $\phi_p$, written as $w_1\rho^2 + w_2\phi_p$. Here, $\rho^2$ is the parameter of the combination for all linear correlations between two columns, and $\phi_p$ is a function involving all inter-site distances of the above corresponding columns. For more details, please see the previous work in [10].

4. Numerical examples

4.1. Example 1: Five-beam frame structure
The ground structure of five-beam structure is shown in Figure 1, which is a typical example in [11]. The two lower fulcrums are fixed, while the lumped mass shown as solid black balls is $454\text{kg}$. All beams are composed of solid round cross sections. The material density is $2770\text{kg/m}^3$, and the Young’s modulus is $6.89\times10^9\text{Pa}$ with a Possion’s ratio of 0.33. This example is expected to find the lightest structure while its fundamental frequency should be more than $15.76\text{Hz}$. Each beam treats its cross-sectional radius as an independent size variable whose initial value is generated by using orthogonal-maximin LHD method and given in Table 1. The upper and lower bounds of each size variable are set as 0.2m and 2.5m, respectively. For GA parameters, the population size and the maximum generation number are both 50.

With the proposed method, five optimization results are shown in Table 2 and the final topology configuration in Figure 2, while their objective iteration history are shown in Figure 3. It is noteworthy that because of symmetry, the topology configuration of all the results is same. From Table 2, it can be seen that with five LHD points, a satisfactory structure which is even lighter than the one obtained from the original approximate method[11] can be derived, in this work it is the result in Case 2. Meanwhile, by using approximate concepts, the total number of structural is still not very large. An advantage of the proposed method is that the initial design is not necessary to be considered. With five LHD points, a satisfactory result can be obtained without a considerable deal of computational cost. According to the above analysis, it can be seen that the proposed method can approach reasonable results with high efficiency.

![Figure 1. Ground structure of five-beam](image)

| Case   | Case 2 | Case 3 | Case 4 | Case 5 |
|--------|--------|--------|--------|--------|
| $R_1$,m| 0.4347 | 0.2089 | 0.3739 | 0.4656 | 0.423 |
| $R_2$,m| 0.3115 | 0.335  | 0.4179 | 0.3121 | 0.3632 |
| $R_3$,m| 0.4834 | 0.2408 | 0.3127 | 0.2139 | 0.4227 |
| $R_4$,m| 0.3439 | 0.3082 | 0.3747 | 0.3873 | 0.3172 |
| $R_5$,m| 0.2439 | 0.2596 | 0.2724 | 0.3658 | 0.4087 |

| Huang [11] | Case 1 | Case 2* | Case 3 | Case 4 | Case 5 |
|------------|--------|---------|--------|--------|--------|
| $R_1$,m    | 0.2744 | 0.21471 | 0.26672 | 0.27441 | 0.25602 | 0.22962 |
| $R_2$,m    | 0.2394 | 0.30997 | 0.4108  | 0.39793 | 0.28726 | 0.2663 |
| $R_3$,m    | 0.2171 | 0.38356 | 0.2     | 0.2     | 0.2     | 0.31151 |
| $R_4$,m    | 0.4626 | 0.40542 | 0.35751 | 0.36718 | 0.45464 | 0.45569 |
| $R_5$,m    | 0     | 0       | 0      | 0      | 0      | 0      |
mass, kg
38384
41517
36655
36946
38225
40928

frequency, Hz
15.75
15.63
15.69
15.73
15.66
15.85

No. of S.A.
14
15
30
31
22
16

*: Final result for this work

4.2. Example 2: Twelve-beam frame structure
A twelve-beam frame structure is considered in this example, and its ground structure and geometric dimensions are shown in Figure 4. The lumped mass shown as a black ball is 454 kg, and the cross section of each beam is solid round. The material density is $2770 \text{ kg/m}^3$, with an elastic modulus of 68.9 GPa and a Possion’s ratio of 0.33. Under the constraint that the natural frequency should be more than 14.13 Hz (the value is 14 Hz in [13]), the optimal result is to be found by minimizing the structural mass. Each radius is treated as an independent variable, while its initial values obtained with the LHD are shown in Table 3. The upper and lower bounds are 2.5 m and 0.2 m, respectively. Both the population size and the maximum generation number in GA are 60.

The optimization results are shown in Table 4 and Figure 5, while their objective iteration curve is shown in Figure 6. From Table 4, it can be seen that by introducing the orthogonal-maximin LHD method into the TMA-GA method, a satisfactory result can be obtained and is even better than the one derived with TMA-GA. It also can be seen that the total number of structural analyses for the proposed method is still not very large, about 170 times.
Table 3. Initial values for size variables

| LHD  | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 |
|------|--------|--------|--------|--------|--------|
| R_1,m | 0.2598 | 0.3305 | 0.4188 | 0.302 | 0.3218 |
| R_2,m | 0.2154 | 0.33 | 0.3102 | 0.3792 | 0.4528 |
| R_3,m | 0.5018 | 0.2767 | 0.2886 | 0.3194 | 0.3429 |
| R_4,m | 0.398 | 0.3202 | 0.447 | 0.3086 | 0.3918 |
| R_5,m | 0.2335 | 0.2708 | 0.3156 | 0.4493 | 0.3867 |
| R_6,m | 0.2875 | 0.3904 | 0.2816 | 0.5053 | 0.2486 |
| R_7,m | 0.3683 | 0.4189 | 0.3383 | 0.2256 | 0.3414 |
| R_8,m | 0.2042 | 0.5063 | 0.3134 | 0.3393 | 0.2498 |
| R_9,m | 0.2334 | 0.2604 | 0.4315 | 0.274 | 0.4188 |
| R_10,m | 0.5123 | 0.3761 | 0.3383 | 0.43812 | 0.5124 |
| R_11,m | 0.402 | 0.376 | 0.3425 | 0.43812 | 0.4068 |
| R_12,m | 0.3032 | 0.4168 | 0.3937 | 0.3502 | 0.4471 |

Table 4. Optimization result for Example 2

| TMA-GA [9] | This work |
|------------|-----------|
|            | Case 1 | Case 2* | Case 3 | Case 4 | Case 5 |
| R_1,m      | 0.5391 | 0.42407 | 0.5416 | 0.50798 | 0.41056 | 0.46093 |
| R_2,m      | 0.2 | 0.3117 | 0.2 | 0.2 | 0.43221 | 0.47067 |
| R_3,m      | 0.3337 | 0.55 | 0.3555 | 0.32258 | 0.43812 | 0.46818 |
| R_4,m      | 0.4262 | 0.54244 | 0.44842 | 0.46187 | 0.2102 | 0.23256 |
| R_5,m      | 0.2188 | 0.28605 | 0.25361 | 0.31169 | 0.42973 | 0 |
| R_6,m      | 0.2 | 0.37076 | 0.20476 | 0.2183 | 0.35943 | 0.2544 |
| R_7,m      | 0.5105 | 0.49073 | 0.47015 | 0.4764 | 0.28754 | 0.31931 |
| R_8,m      | 0 | 0.46588 | 0 | 0 | 0.40085 | 0.39495 |
| R_9,m      | 0 | 0 | 0 | 0 | 0.3843 | 0.38485 |
| R_10,m     | 0.4133 | 0.46998 | 0.37857 | 0.38434 | 0 | 0 |
| R_11,m     | 0 | 0 | 0 | 0 | 0 | 0 |
| R_12,m     | 0 | 0 | 0 | 0 | 0 | 0 |
| mass, kg   | 105160 | 163354 | 1042034 | 102654 | 116044 | 107124 |
| nat.freq, Hz | 14.08 | 15.22 | 14.01 | 14.04 | 14.12 | 14.08 |
| No. of S.A. | 52 | 47 | 20 | 49 | 35 | 24 |

*: Final result for this work
5. Conclusion
This work introduces an orthogonal-maximin LHD strategy to the original two-level approximate method. Five initial designs are generated with the orthogonal-maximin LHD method, which can obtain an even better result compared with the preceding approximate method. Two numerical
examples are established to illustrate the effectiveness as well as efficiency of the proposed method for frame topology optimization, and the optimization result show that a more satisfactory lightweight structure can be obtained.

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