Analytical model of the inductor system of the device to prevent ice formation on power lines

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Abstract. The article describes the analytical model of the magnetic-pulsed electromechanical energy converter in relation to the device for preventing ice formation on power lines. The inductor system of the device is a flat inductor and a firing pin made of electrically conductive material. The mobility of both the firing pin, and also the inductor is due to the attachment of the device to the pair of power line wires. This feature, in contrast to the fixed position of the inductor traditionally used, required the development of the universal design scheme for the electrodynamic interaction of the inductor and the firing pin. The proposed analytical model takes into account the forces of resistance to the motion of the inductor system caused by the presence of the elastic return mechanisms in the device and attachment to the power line wire, which is a body with a distributed mass. The developed analytical model was tested. The calculation of the device parameters for the transmission line span with the specified mass-dimensional characteristics was performed.

1. Introduction
Magnetic-pulsed electromechanical energy converters (EEC) belong to the class of linear motors and they are increasingly used in solving many technological problems in various branches of technology. They are used in machines that require short-term force action on the material being processed, such as hammers, presses, pumping and compressor units. They are successfully used in the processes where it is necessary to obtain high performance: switching and locking mechanisms, valves. They are used for throwing the solid bodies weighing from several grams [1] to tens of kilograms [2] - [5], they are used in the field of metal processing by pressure [1], in the destruction of arches and cleaning of equipment [6], including cleaning from ice [7], in other fields of science, technology, security and defense [8] - [11]. The use of the device for preventing [12] and removing [13] - [15] ice on power transmission lines as an actuating mechanism is a new field for EEC.

The shock method for preventing ice formation involves the pulse action in the local area of the wire, as a result of which a running wave propagates along the wire in both directions from the place of impact. The amplitude of the wave depends on the time of impact and it may be small relative to the wire diameter. High impact speeds, creating alternating accelerations of large magnitude, that is, the forces
between the wire and the water mass, will cause it to break off, when the force of adhesion of a drop of water to the surface of the wire is exceeded.

The actuators of the impact devices, which were considered by the developers in relation to the prevention of ice formation on power lines [16] - [18], are reduced to one of the systems: mechanical, pneumatic, magnetic or electromechanical. In comparison with others, EECs have a number of advantages. They provide accurate calculated impact parameters in the range of hundreds of microseconds and thousands of newtons. Magnetic pulse EECs can be controlled by the adjustment of the impact force changing the charge voltage. Due to the direct connection with the working tool without intermediate gears, the design of the working machine is simplified, its reliability increases, and its weight and size characteristics are improved. In this case, there is no electrical contact with the secondary element. The disadvantages of magnetic pulse EECs include a small efficiency, the presence of the pulse of electromagnetic interference when the device is triggered, a limited service life of the inductor assembly, as well as the strong dependence of the throwing speed on the electrical conductivity of the material from which the thrown body is made.

The operation of magnetic pulse EECs is based on the occurrence of the electromagnetic force pulse as the result of the interaction of eddy currents induced in the firing pin of the device (the conducting plate) with the magnetic field pulse created by a high-density current in the turns of the inductor coil. It should be noted that, despite the simplicity of the principle, there is a complex relationship between the parameters of electromagnetic and electromechanical transient processes during the discharge of the capacitive energy storage and the parameters of the electromechanical installation: the dimensions of the inductor system (IS), the materials of which it is made, the charge voltage and capacity of the capacitor bank, the resistance force to movement of IS. To calculate the parameters of transient processes, the approaches of the field theory or the theory of electric circuits are used [5], [6], [11], [19]. However, until now, the operation of the IC under the conditions of mobility of the firing pin, but also the inductor has not been considered. Besides, the forces of resistance to the movement of the IC under conditions of attachment to a body with the distributed mass, such as a wire, were not considered. Due to the weight of the wire mass, this force cannot be ignored in relation to the device for preventing ice on power lines.

2. Statement of the problem
We will consider a magnetic pulse EEC, the IC of which is a flat inductor and a firing pin made of an electrically conductive material, shown in figure 1.

2.1. Flat inductor system
A flat inductor is a spiral inductor filled or pressed with a high-strength insulating material. The inductor has a disk shape and it is wound with a copper bus bar in width $\Delta_s$ and in height $h_s$. The coils of the inductor winding are separated by an insulating layer of width $\Delta_i$.

The outer diameters $D$ and inner diameters $d$ of the inductor and the firing pin are equal. The average diameter of the inductor / firing pin is determined by the formula (1):

$$d_a = \frac{(D + d)}{2} \quad (1)$$

The width of the current band of the inductor is determined by the formula (2):

$$r = \frac{(D - d)}{2} \quad (2)$$

The flat inductor is characterized by the fact that the height of its spiral is less than the width of the current band $h_s < r$, and the values $\zeta$ and $\alpha$, determined by the formulas [6] (3), (4), vary within 0.1...0.5 and 0.2...0.8, respectively. The height of the firing pin is less than the height of the inductor bus.

$$\zeta = \frac{h}{r} \quad (3)$$
\[
\alpha = \frac{r}{d_s}
\]  

(4)

The inductor and the firing pin are characterized by the depth of penetration of a pulsed magnetic field into their thickness, called the skin layer [20]. The skin layer \( \Delta \) depends on the frequency of the supplied current and the physical constants of the material.

In addition to the IC, the magnetic pulse EEC includes a capacitive energy storage device – a pulse capacitor and a switching device.

Figure 1. Basic geometric characteristics of the inductor system.

Figure 2. Double-circuit replacement circuit of the magnetic-pulse EEC.

2.2. Replacement scheme of the magnetic pulse EEC

In accordance with the method of the theory of electric circuits, we present the described magnetic pulse EEC in the form of a two-circuit replacement circuit shown in figure 2.

The parameters of the primary circuit are the inductance \( L_1 \) and active resistance \( R_1 \) of the inductor, the parasitic inductance \( L_0 \) and active resistance \( R_0 \) of the capacitor bank, the switch and the connecting wires. The parameters of the secondary circuit are the inductance \( L_2 \) and active resistance \( R_2 \) of the firing pin connected inductively to the inductor. The electromagnetic force (EMF) acting on the IC is determined by the formula (5):

\[
F = i_1 i_2 \frac{dM}{dz}
\]  

(5)

where \( M \) is the mutual inductance between the inductor and the firing pin, \( i_1, i_2 \) is the current flowing in the primary and secondary circuits, respectively, \( z \) is the coordinate of the movement of the striker. This expression, in contrast to the expression for the single-circuit replacement scheme [6], takes into account the alternating character of the EMF, which is important at high travel speeds.

In general, the mutual inductance of the inductor and the firing pin depends on the number of the coils, the geometric dimensions of the IC, and the distance between the inductor and the firing pin. The limit value of mutual inductance for a flat inductor and the firing pin of the same diameter is calculated by the formula [1], [21] (6):

\[
M_0 = \frac{\mu_i w d_i \Psi(\alpha, \xi)}{4\pi}
\]  

(6)

where \( w = (D - d) / \left[ 2(\Delta_s + 2\Delta_s) \right] \) - the number of turns of the inductor coil, \( \alpha, \xi \) - the values determined by the formulas [21] (4), (7):

\[
\xi = \frac{\xi_0}{d_s}
\]  

(7)
where \( z' = z_0 + 0.5(\Delta_1 + \Delta_2) \) is the equivalent gap between the inductor and the firing pin, \( \Delta_1, \Delta_2 \) is the skin layer of the inductor and the firing pin, respectively, \( \Psi \) is the value determined with the help of the tables or according to the formula given in [21].

The change in mutual inductance depending on the distance between the inductor and the firing pin is approximated [22] by the exponent (8):

\[
M = M_0 e^{-Az/d},
\]

where \( z \) is the distance between the firing pin and the inductor, \( \alpha = 2.3/\alpha^{0.326} \) is a function of the parameter \( \alpha \) in the range 0.2...1.

2.3. Electrodynamic interaction of the inductor system

The electrodynamic interaction of the inductor and the firing pin in a magnetic pulse EEC, taking into account the immobility of the inductor, is described by the system of differential equations (9) of the two-circuit replacement scheme, including the equations of the theory of electric circuits and mechanics:

\[
(L_0 + L_1) \frac{di_1}{dt} + (R_0 + R_1)i_1 + M \frac{di_2}{dt} + i_2 \frac{dM}{dt} = U_c
\]

\[
L_z \frac{di_2}{dt} + R_z i_2 + M \frac{di_1}{dt} + i_1 \frac{dM}{dt} = 0
\]

\[
C \frac{dU_c}{dt} = -i_1
\]

\[
\frac{d^2z}{dt^2} = i_1 i_2 \frac{dM}{m} - \frac{f_r(t, z)}{m}
\]

where \( C, U_c \) - the capacitance and voltage of the storage device, \( z \) - coordinate of the movement of the striker, \( m \) - the striker mass, \( f_r(t, z) \) - the resistance force to the movement of the firing pin.

The above system does not take into account the effect of IC heating on the energy conversion process due to the fact that the ice prevention device operates at low ambient temperatures, and the discharge of the capacitor bank is performed no more than once per minute.

Under the given initial conditions (10), the analytical model provides the calculation of the coordinates of the movement of the firing pin.

\[
i_1(0) = i_2(0) = 0; U_c(0) = U_o; z(0) = 0; \frac{dz}{dt}(0) = 0
\]

The electrical parameters of the replacement circuit, the mass-dimensional parameters of the IC and the wire set the boundary conditions. To calculate the electrical parameters \( L_1, R_1, L_2, R_2 \) of the two-circuit replacement circuit, we use the method proposed in [1], which requires a minimum number of empirical formulas and experimental data, but it provides a sufficient degree of accuracy.

To solve the system of differential equations, it must be reduced to the Cauchy normal form and it must be integrated by one of the numerical methods, for example, the fourth-order Runge-Kutta method with automatic selection of the integration step. This method is stable and to get the solution at the next point it is necessary to have the value of the solution at the previous point. Therefore, the integration step can be changed at any stage of the calculation. The advantage of the method is the slow accumulation of the integration error at a relatively high computational speed. The accuracy control and the selection of the integration step are performed by comparing the calculation results at the same point obtained with the steps \( h \) and \( 2h \).

Consider the lateral method of attaching the device to prevent ice formation, shown in figure 3. In this case, the ends of the rods rest against a pair of wires, due to which, when the actuator is triggered, the operation of preventing ice formation is performed immediately on the pair of the wires. The inductor
and the firing pin are mutually repelled by the EMF, however, the springs and the wire resist the movement of the IC.

![Figure 3. Method of attaching the device from the side (top view): 1-inductor; 2-firing pin; 3-rod; 4-power line wire; 5-springs (the shock absorption system); 6-Ampere force; 7-function block.](image)

Thus, it is necessary to create an analytical model that takes into account the mobility of not only the firing pin, but also the inductor, as well as all the forces that resist the movement of the IC.

3. Analytical model
Under the influence of EMF, not only the firing pin moves, but also the inductor, because the inductor in the device for preventing ice formation on the power line wires is not fixed motionless on the ground. In order to take into account the mobility of the IC, we introduce additional variables: \( z_1 \) - the coordinate of the inductor, \( z \) - the coordinate of the firing pin, \( z = z_1 - z_2 \) - the distance between the inductor and the firing pin. The inductor and the firing pin move in opposite directions, their velocities and accelerations being the first and second derivatives of the corresponding time travel coordinates. In this case, the mutual inductance depends on the distance between the inductor and the firing pin.

In the device for preventing ice, the repelled mass is not only the mass of the inductor \( m_1 \) or firing pin \( m_2 \), but also the mass of the rod with the functional block \( m_r \) and the part of the wire to which the rod is attached, that is, it depends on the width of the contact \( 2\delta \) and the linear weight of the wire \( p \).

As the running wave propagates through the wire, the rest of the wire is turned into motion. Thus, the mass of the wire, which affects the movement of the firing pin, increases with the speed (11):

\[
\mu = \frac{dm}{dt} \tag{11}
\]

where the attached mass, taking into account the fact that the attachment process occurs on both sides of the device, is defined as (12):

\[
dm = 2pSdx \tag{12}
\]

where \( S \) is the cross-sectional area of the wire, \( dx \) is the increment of the part of the wire included in the movement, per unit of time. Whence the speed of connection of the mass of the wire is (13):

\[
\mu = \frac{2pSdx}{dt} = 2pSb \tag{13}
\]

where \( b \) is the velocity of propagation of the transverse wave in the wire.

Thus, the motion of the inductor and the wire attached to it is considered as the motion of a body with the variable mass, and its mass is determined by the formula (14):

\[
m_i(t) = m_i + m_r + p2\delta + \mu t \tag{14}
\]

The mass of the firing pin at the current moment is determined in a similar way. Then the system of equations (9) is transformed to the form (15):
(15)

\[ \frac{d^2 z_1}{dt^2} = \frac{i_2}{m_1(t)} \frac{dM}{dz} - \frac{f_{z_1}(t, z_1)}{m_1(t)} \]

\[ \frac{d^2 z_2}{dt^2} = \frac{i_2}{m_2(t)} \frac{dM}{dz} - \frac{f_{z_2}(t, z_2)}{m_2(t)} \]

We determine the forces that resist the movement of the inductor or firing pin. The force due to the mass of the wire included in the motion is determined by the formula (16):

\[ \bar{F}_w = \mu \bar{u}_{w(+)} \] (16)

where \( \bar{u}_{w(+)} \) is the speed of the wire relative to the inductor or firing pin. Since the wire, not included in the motion, is stationary relative to the inertial system of reference, its absolute velocity is zero, so the relative velocity in the projection on the axis \( Oz \) is defined as (17):

\[ u_{w(+)} = -v_z \] (17)

In addition, the resistance force to the motion of the inductor or the firing pin includes the force of gravity, the elastic force of the return spring, the friction force of the inductor or the firing pin on the body of the device, and also the power wire resistance caused by the tension of the wire. This force depends on the tension \( N \) and the angles of inclination of the wire \( \gamma_1, \gamma_2 \) at the points of contact with the rod of the device. The equation of motion of the attached body is a matching condition for the wire during the exposure period [20] (18):

\[ m \frac{d^2 z}{dt^2} = N \left( \sin \gamma_1 + \sin \gamma_2 \right) \] (18)

If the angles are equal, the expression (18) is converted to (19):

\[ m \frac{dV}{dt} = 2N \sin \gamma \] (19)

The sine of the angle of inclination of the wire in the places adjacent to the firing pin is defined as (20):

\[ \sin \gamma = \frac{z}{\sqrt{z^2 + (bt)^2}} \] (20)

Thus, for the force of resistance to the motion of the inductor or the firing pin, we make the expression (21):

\[ f_z(t, z) = \mu \bar{u}_{w(+)} + m(t) g + k(z + \Delta z) + \eta m(t) g + 2N \frac{z}{\sqrt{z^2 + (bt)^2}} \] (21)

where \( k \) - the coefficient of elasticity of the return spring, \( \Delta z \) - the pre-tension of the return spring, \( \eta \) - the coefficient of friction of the inductor or the firing pin on the body. Under the given initial conditions
(22), the analytical model (15), (21) provides the calculation of the coordinates of the movement of the IC attached to a pair of wires and moving in the horizontal direction.

\[ i_1(0) = i_2(0) = 0; \ U_1(0) = U_0; \ z_1(0) = z_2(0) = 0; \ \frac{dz_1}{dr}(0) = \frac{dz_2}{dr}(0) = 0 \]  

(22)

4. Experimental and numerical studies

To verify the analytical model, the calculations were performed based on the experimental and numerical data presented in [6]. In the experiment, a stationary inductor made of a copper busbar and an aluminum firing pin are considered. The geometric characteristics of the IC and the electrical parameters of the energy storage device are shown in Table 1.

Table 1. Parameters of the IC and the energy storage device.

| Parameter | The value | Parameter | The value | Parameter | The value |
|-----------|-----------|-----------|-----------|-----------|-----------|
| \( D \)   | 78 mm     | \( \Delta_c \) | 0.9 mm    | \( R_0 \) | 0.031 ohms |
| \( d \)   | 30 mm     | \( w \) | 21 pieces | \( l_1 \) | 26.2 mcg   |
| \( h_s \) | 2.8 mm    | \( L_0 \) | 0.66 mcg  | \( C \) | 200 icF    |

For calculations under the condition of immobility of the inductor, its mass in the analytical model is given much more than the mass of the firing pin. Figure 4 shows graphs of the change in time of the current in the inductor and the movement of the firing pin.

In addition, the model was verified in comparison with its own experiment [23]. In this case, the inductor is also stationary, the firing pin is connected to the wire through the rod. The device has a side mount to the wire. The geometric characteristics of the IC and the electrical parameters of the energy storage device are shown in Table 2. The results of the calculations obtained using the developed analytical model are compared with the experimental data and the results of the calculations performed using the developed numerical model [24]. The graphs of the movement of the wire at the attachment point of the firing pin rod are shown in figure 5.

Table 2. Parameters of the IC and the energy storage device.

| Parameter | The value | Parameter | The value | Parameter | The value |
|-----------|-----------|-----------|-----------|-----------|-----------|
| \( D \)   | 55 mm     | \( l_1 \) | 7.86 mcg  | \( N \)   | N1=23 N, N2=49 N |
| \( d \)   | 20 mm     | \( C \) | 900 icF   | \( b \)   | V1=63 m / s, |
\[ k = 3.6 \text{ mm} \quad U_0 = 136 \text{ V} \pm 5 \% \quad V_2 = 43 \text{ m/s} \]
\[ \Delta_s = 1.2 \text{ mm} \quad U_2 = 172 \text{ V} \pm 5 \% \quad 2\Delta = 6 \text{ mm} \]
\[ w = 14 \text{ pieces} \quad U_3 = 230 \text{ V} \pm 5 \% \quad P = 0.012 \text{ kg/m} \]
\[ L_0 = 3.5 \text{ mcg} \quad m = 0.079 \text{ kg} \quad \rho = 7881.7823 \text{ kg/m}^3 \]
\[ R_0 = 0.0215 \text{ ohms} \quad k = 0.5 \quad S = 1.5248 \text{ mm}^2 \]

**Figure 5.** The graphs of the movement of the wire at the attachment point of the rod: a, b, c—gravity of 49 N, voltage-U1, U2, U3, respectively; d, e—gravity of 23 N, voltage-U1, U2, respectively.

The numerical experiments were carried out when the device was attached to the side between the wires of the AC120/19 power line with a span length of 200 m and tension of 3.93 kN without the device. The masses on the side of the firing pin and inductor are 4 kg, the coefficient of elasticity of the return spring is 47.099 N/mm, the pre-tension of the return spring is 0.69 mm, the coefficient of friction is 0.5. In accordance with the method [25], the optimal impact parameters that can cause the removal of water droplets along the entire length of the span without damaging the wire are calculated: the impact force is 20 kN, the impact time is 250 microseconds.

**Figure 6.** The graphs of wire movement in the place of rod attachment, span of 200 m.
Then, in accordance with the methodology, using the developed analytical model, the parameters of the magnetic pulse EEC are calculated, which can provide the corresponding impact parameters. With an external and internal IC diameter of 0.2 and 0.1 m, respectively, and inductor inductance of 6.83 mcg, the energy storage capacity should be 1500 icF, and the charge voltage should be 800 V. In this case, the parasitic resistance should not exceed 0.009 ohms, and the parasitic inductance - 2.5 mcg. The results of the calculations on the deviation of the wire at the attachment point of the device rod using the analytical and numerical models are shown in figure 6.

5. Conclusion
The developed analytical model is a universal design scheme for the electrodynamic interaction of an inductor and the firing pin in a magnetic pulse EEC. The possibility of taking into account the mobility of the elements of the inductor system and the various forces that affect its dynamics makes it possible to use an analytical model to calculate the dynamics of the device for preventing ice formation on power lines. It should be noted that the accuracy of the results significantly depends on the method of calculating the electrical parameters of the inductor system. The analytical model can be used as part of the methodology for determining the parameters of the device for preventing the formation of ice on power lines.

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