The Edge of the Galaxy

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ABSTRACT

We use cosmological simulations of isolated Milky Way-mass galaxies, as well as Local Group analogues, to define the “edge” — a caustic manifested in a drop in density or radial velocity — of Galactic-sized haloes, both in dark matter and in stars. In the dark matter, we typically identify two caustics: the outermost caustic located at ~1.4r200m corresponding to the “splashback” radius, and a second caustic located at ~0.6r200m which likely corresponds to the edge of the virialized material which has completed at least two pericentric passages. The splashback radius is ill defined in Local Group type environments where the halos of the two galaxies overlap. However, the second caustic is less affected by the presence of a companion, and seems a more natural definition for the boundary of the Milky Way halo. Curiously, the stellar distribution also has a clearly defined caustic, which, in most cases, coincides with the second caustic of the dark matter. This can be identified in both radial density and radial velocity profiles, and should be measurable in future observational programmes. Finally, we show that the second caustic can also be identified in the phase-space distribution of dwarf galaxies in the Local Group. Using the current dwarf galaxy population, we predict the edge of the Milky Way halo to be 292 ± 61 kpc.

Key words: Galaxy: halo – galaxies: haloes – galaxies: kinematics and dynamics – Local Group – methods: numerical

1 INTRODUCTION

The mass condensations commonly referred to as dark matter haloes in simulations fade gradually into the background matter distribution and have no well-defined edge (e.g. Diemer et al. 2013). Furthermore, haloes are not spherical but have irregular shapes. Nevertheless, definitions of the nominal boundary of a halo such as the “friends-of-friends” radius (Davis et al. 1985), the “virial radius” (e.g. Cole & Lacey 1996) or “r200” abound in the literature. Even the latter is ambiguous, as it is sometimes defined as the radius, r200c, within which the mean density equals 200 times the critical density (e.g Navarro et al. 1996) or as the radius, r200m, within which the mean density equals 200 times the mean cosmic value (e.g. Diemand et al. 2007).

From a practical point of view, the ambiguity regarding the definition of the boundary of a dark matter halo can become troublesome when we want to define the dark matter particles, stars, gas or subhaloes that “belong” to a halo, or when we wish to define the radius at which tracers can escape from a self-bound system (e.g. Leonard & Tremaine 1990; Springel 2005). The physical extent of haloes varies significantly at different mass scales and in different environments (e.g. Navarro et al. 1996, 1997; Bullock et al. 2001; Wechsler et al. 2002) and, when contrast simulations or comparing them to observations, a common definition of halo extent is essential to avoid confusion. In addition, while the backdrop of our current theory of structure formation is cold dark matter, it is just as important to understand how the baryonic components relate to the dark matter, and where observational boundaries lie (e.g. Kravtsov 2013; Shull 2014; Wechsler & Tinker 2018).

Analytical solutions for the collapse of spherical gravitational structures in a cosmological context provide valuable insight into the structure of dark matter haloes. The spherical collapse model, first presented by Gunn & Gott (1972) for an Einstein-de Sitter Universe, describes the evolution of spherical shells of matter around an overdensity (see also Fillmore & Goldreich 1984; Bertschinger 1985). In this model, initially overdense regions gravitationally attract the surrounding matter, causing it to detach from the Hubble flow and collapse, forming larger and larger equilib-

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rium structures. Each successive mass shell collapses onto a deeper potential well and thus has a higher energy and a larger apocentre. Material piles up at these apocentres, giving rise to a singularity or caustic surface. Of particular interest is the outermost caustic, termed the “splashback” radius, which corresponds to the apocentre of material that has most recently completed its first pericentric passage.

The spherical collapse model has served as a motivation for many of the commonly used definitions of halo masses and sizes. Traditionally (see e.g. Binney & Tremaine 2008 Section 9.2.1), an Einstein-de Sitter Universe is assumed, where energy conservation and the virial theorem imply that the “virial” radius (enclosing the mass whose potential energy is twice the negative kinetic energy) occurs at half the turnaround radius. In the Einstein-de Sitter model the overdensity (relative to the critical density) at virialization is \( \Delta_c = \rho_{\text{vir}}/\rho_c = 18\pi^2 = 178 \). This formalism has been generalized for a ΛCDM universe (Lahav et al. 1991; Eke et al. 1998; Bryan & Norman 1998), in which case the overdensity defining the boundary is \( \Delta_c \approx 100 \) at \( z = 0 \), and evolves with redshift.

In the spherical collapse model the virial radius defines the region within which the system is virialized; beyond this radius mass is still collapsing onto the object. N-body simulations suggest that this distinction occurs at \( \Delta_c \approx 200 \) (Cole & Lacey 1996), so a commonly used definition of halo is \( r_{200} \). Another commonly used definition, particularly in studies of the halo occupation distribution of galaxies (e.g. Berlind & Weinberg 2002; Kravtsov et al. 2004), is \( r_{200m} \), which corresponds to \( \Delta_c = 200 \times \Omega_m \approx 60 \) today. For a Milky Way mass halo \(( \sim 1 \times 10^{12} M_\odot) \), these halo boundaries are typically: \( r_{200} \approx 220 \text{kpc} \), \( r_{\text{vir}} \approx 290 \text{kpc} \), and \( r_{200m} \approx 350 \text{kpc} \). Several authors have argued that the splashback radius, predicted by the spherical collapse model, is the most natural definition of the boundary of a halo (e.g. Adhikari et al. 2014; Diemer & Kravtsov 2014; More et al. 2015). For a Milky Way halo the splashback radius is typically \( \approx 500 \text{kpc} \) (assuming the splashback radius lies at \( \sim 1.5 r_{200m} \), see below).

In reality, halo collapse is non-spherical, lumpy and significantly anisotropic. Several works have used N-body simulations to follow this collapse in detail (e.g. Davis et al. 1985; Frenk et al. 1988; Cole & Lacey 1996; Diemand & Kuhlen 2008; Springel et al. 2008) and to compare with the predictions of the spherical collapse model (e.g. Prada et al. 2006; Zavala et al. 2008; Ascasibar et al. 2007; Ludlow et al. 2010). While most studies have concentrated on the inner profiles of dark matter haloes (e.g. Navarro et al. 1996; Moore et al. 1999a; Stadel et al. 2009), more recently, Adhikari et al. (2014), Diemer & Kravtsov (2014) and More et al. (2015) have explored the outer density profiles of dark matter haloes. These studies identify the outer caustic, or splashback radius, as a sharp jump in the density profile. For example, Diemer & Kravtsov (2014) and More et al. (2015) find that the splashback radius falls in the range \( (0.8 \sim 1.0) \tilde{r}_{200} \) for rapidly accreting haloes, and is \( \approx 1.5 \tilde{r}_{200} \) for slowly accreting haloes.

The influence of environment, mass accretion rate, and redshift on the splashback radius was investigated by Diemer et al. (2017) and Mansfield et al. (2017) and the splashback radius is now a commonly used, and thoroughly explored halo boundary. Interestingly, there is now considerable evidence that splashback radii have been measured observationally in the outskirts of galaxy clusters (e.g. More et al. 2016; Baxter et al. 2017; Chang et al. 2018; Shin et al. 2019; Contigiani et al. 2019; Zürcher & More 2019; Murata et al. 2020). While the measured splashback radii tend to be smaller than those predicted in ΛCDM simulations, these results are still subject to systematic effects (Busch & White 2017; Xhakaj et al. 2019; Murata et al. 2020).

Often the most relevant, and even the most physical, definition of halo boundary depends on the situation at hand. The term splashback is often used by reference to the population of “backsplash” galaxies, i.e. galaxies that have been inside, but are now outside the virial radius, and may extend well beyond any traditional spherical collapse boundary (e.g. Balogh et al. 2000; Mamon et al. 2004; Gill et al. 2005; Sales et al. 2007; Ludlow et al. 2009; Teyssier et al. 2012; Bahé et al. 2013; Wetzel et al. 2014). The properties of these backsplash galaxies demonstrate that the environmental effects of haloes can extend well beyond the traditional virial radius boundary. However, even if the zone of influence of haloes extends significantly beyond the virial radius, haloes are never isolated systems, and are eventually run into other massive systems. For example, the Milky Way galaxy resides in the Local Group, and is located \( \sim 800 \text{kpc} \) from the roughly equal mass halo of M31. Thus, the splashback radius for a Milky Way mass halo runs into that of M31. In this case, it is perhaps more physical to consider the splashback radius of the entire Local Group, rather than of its individual components. Nonetheless, a physically motivated definition of the extent for the Milky Way is warranted, and will become even more important when the next generation surveys discover many tens of dwarf galaxies in the Local Group.

In this work we explore the boundary of Milky Way mass haloes using high-resolution cosmological simulations. In particular, we use the outer density profiles of the haloes to quantify their extent. We take into account two important characteristics of the Milky Way: (1) its location in the Local Group, and hence its proximity to M31, and (2) the relation between the extent of the stellar distribution and that of the underlying dark matter. This consideration is important for observational probes of the Milky Way halo boundary. In Section 2 we describe the cosmological simulations used in this work. These comprise both collisionless and hydrodynamic simulations, as well as simulations designed to mimic the Local Group. We quantify the “edges” of the dark matter haloes, stellar haloes, and satellite dwarf galaxy populations, and compare these various boundaries in Section 3. Finally, we summarise our main results in Section 4.

## 2 SIMULATIONS

We use a large range of high resolution simulations of Milky Way mass haloes to quantify the edges of Galactic-sized haloes. Below we describe each simulation suite in turn.

### 2.1 ELVIS

The “Exploring the Local Volume in Simulations” (ELVIS) project is a suite of 48 simulations of Galaxy-size haloes (Garrison-Kimmel et al. 2014). These simulations were designed to model the Local Group (LG) environment in a cosmological context. Half of the haloes (24) are in paired configurations similar to the Milky Way and M31. The LG analogues were selected from medium resolution (\( m_p = 9.7 \times 10^7 M_\odot \), force softening 1.4 kpc) cosmological simulations. Twelve halo pairs were selected for resimulation based on phase-space criteria appropriate to the MW/M31 system (e.g. separation, total mass, radial velocity). The resulting zoom simulations are high resolution (\( m_p = 1.9 \times 10^8 M_\odot \), force softening 141 pc) volumes that span 2-5 Mpc in size. The remaining half (24) of the ELVIS suite are isolated, mass-matched.
analogue, which are resimulated at the same resolution as the paired haloes. The resulting sample consists of 48 high-resolution haloes in the mass range \(1 - 3 \times 10^{12} M_\odot\). The ELVIS suite was run with the WMAP-7 cosmology (Larson et al. 2011) with parameters: \(\Omega_m = 0.266, \Omega_\Lambda = 0.734, H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}\).

Subhaloes were identified using the ROCKSTAR halo finder (Behroozi et al. 2013a) and were followed through time with consistent trees (Behroozi et al. 2013b). We define the centre of the host haloes using the position and velocity of the main subhalo calculated in the ROCKSTAR algorithm. Garrison-Kimmel et al. (2014) find that the subhalo sample in ELVIS is complete down to \(M_{\text{sub}} > 2 \times 10^7 M_\odot\) (or \(v_{\text{max}} > 8 \text{ km s}^{-1}\)). The general properties of the ELVIS haloes are described in Garrison-Kimmel et al. (2014) and summarised in their Table 1. This suite has produced a number of results, including predictions for future dwarf galaxy detections (Garrison-Kimmel et al. 2014), the stellar-mass halo relation for LG galaxies (Garrison-Kimmel et al. 2017), the prevalence of dwarf-dwarf mergers and group-infall onto MW mass haloes (Deason et al. 2014b; Wetzel et al. 2015), and insights into the planar alignment of MW satellites (Pawlowski et al. 2017).

### 2.2 APOSTLE

APOSTLE (A Project Of Simulating The Local Environment) is a suite of high resolution, hydrodynamical simulations consisting of 12 halo pairs (Fattahi et al. 2016; Sawala et al. 2016). These pairs were drawn from the medium resolution \((m_p = 8.8 \times 10^6 M_\odot)\) DOVE dark matter-only cosmological simulation described by Jenkins (2013). The candidates were selected to have paired configurations similar to the LG, based on the separation of the pairs, their relative radial and tangential velocities, a Hubble flow constraint, and the combined mass of the pair. The exact selection criteria differ from the ELVIS suite, with the main difference being the total masses of the haloes. The APOSTLE suite has typically lower halo masses, and span the mass range \(0.5 - 2.5 \times 10^{11} M_\odot\). The resolutions span \(2.3 \text{ Mpc}^3\) in size and were run with the same hydrodynamic code as the EAGLE Reference calibration (Schaye et al. 2015; Crain et al. 2015), which includes subgrid prescriptions for star formation, feedback, metal enrichment, cosmic reionization, and AGN. The simulations were performed at three different resolution levels, and we use the “medium” L2 resolution suite which has 10 times better mass resolution than DOVE \((m_p = 6 \times 10^5 M_\odot, \text{ force softening } 307 \text{ pc}), \text{ with a gas particle mass of } 1.2 \times 10^3 M_\odot\). APOSTLE was run with the WMAP-7 cosmology (Komatsu et al. 2011) with parameters: \(\Omega_m = 0.272, \Omega_b = 0.0455, \Omega_\Lambda = 0.728, H_0 = 70.4 \text{ km s}^{-1} \text{ Mpc}^{-1}\).

Haloes are identified using a friends-of-friends (FOF) algorithm (Davis et al. 1985), and subhaloes belonging to each FOF halo were identified using the SUBFIND algorithm (Springel et al. 2001). We use the position and velocity of the main (sub)halo calculated in SUBFIND to define the centre of the host halo. Note that this definition of halo centre is different to the one used in ELVIS, which is based on ROCKSTAR. For a comparison of the SUBFIND and ROCKSTAR subhalo finding algorithms, see e.g. Knebe et al. (2011). Sawala et al. (2016) showed that the satellite luminosity function of APOSTLE L2 is complete down to \(M_{\text{star}} \sim 10^5 M_\odot\), and they used the APOSTLE suite to address apparent small-scale problems in the ΛCDM cosmology. In particular, they showed that the simulations match the abundance of observed dwarf satellites in the Milky Way and M31, thus solving the apparent “missing satellites” (Moore et al. 1999b) and “too-big-to-fail” (Boylan-Kolchin et al. 2011) problems. Several other works have used the APOSTLE suite to investigate a wide range of topics. These include, probing the nature and properties of dark matter (Lovell et al. 2017; Sawala et al. 2017), the tidal stripping of dwarf galaxies and formation of the stellar halo (Starkenburg et al. 2017; Fattahi et al. 2018), and tests of observational mass estimates of dwarf galaxies (Campbell et al. 2017; Genina et al. 2018, 2019).

### 2.3 Auriga

The Auriga suite consists of cosmological hydrodynamical zoom-in simulations of isolated Milky Way-mass haloes (Grand et al. 2017). Candidates for resimulation were selected from the 100 cMpc dark matter only cube of the EAGLE simulation (Schaye et al. 2015). The sample of Auriga haloes was chosen to be relatively isolated at \(z = 0\), with no objects with masses greater than half of the parent halo closer than 1.37 Mpc. The initial sample of 30 haloes was selected in the mass range \(1 - 2 \times 10^{12} M_\odot\), and a further 10 lower mass (\(0.5 - 1 \times 10^{12} M_\odot\)) haloes were more recently added to the suite (Grand et al. 2019b). The zoom resimulations were performed with the AREPO code, which follows magnetohydrodynamic and collisionless components in a cosmological context. At the resolution used in this work (L4) the gravitational softening is 370 pc and the typical particle/cell masses are \(3 \times 10^5 M_\odot\) and \(5 \times 10^5 M_\odot\) for the dark matter and gas, respectively. The Auriga galaxy formation model includes subgrid prescriptions for several important physical processes, such as star formation, supernova feedback, gas cooling, metal enrichment and magnetic fields (see Grand et al. 2017 for more details). The Auriga suite was run with the Planck cosmology Planck Collaboration et al. (2014) with parameters: \(\Omega_M = 0.307, \Omega_b = 0.048, \Omega_\Lambda = 0.693, H_0 = 67.77 \text{ km s}^{-1} \text{ Mpc}^{-1}\).

Subhaloes in the Auriga haloes are identified using the SUBFIND algorithm, and we use the position and velocity of the main subhalo calculated by SUBFIND to define the centre of the host. The Auriga galaxies match well a number of observed properties of disc galaxies, such as their sizes, rotation curves, stellar masses, chemistry and star formation rates (Grand et al. 2016, 2017; Marinacci et al. 2017; Grand et al. 2018). In addition, the suite has been used to study the stellar haloes of disc galaxies (Monachesi et al. 2016, 2019), interpret the assembly history of the Milky Way halo (Deason et al. 2017; Fattahi et al. 2019b; Belokurov et al. 2019), study the quenching of satellite galaxies (Simpson et al. 2018), and measure the total mass of the Galaxy (Deason et al. 2019; Grand et al. 2019a; Callingham et al. 2019).

### 3 THE EDGE OF MILKY MASS HALOES

We identify the “edges” of Milky Way-mass haloes in the ELVIS, APOSTLE and Auriga simulations using both the density and the radial velocity profile as a function of radius. The former is motivated by the work by Diemer & Kravtsov (2014), who used the slope of the logarithmic density profile to identify the outer edges of dark matter haloes. Here, we apply a similar formalism, but also apply this to the stars and subhaloes. We use the radial velocity profiles in a similar manner.

Throughout this work we give radii in units of \(r_{200\text{m}}\), defined as the radius at which the density of a halo falls to 200 times the universal matter density at \(z = 0\) \((\rho_m = \Omega_m \rho_c)\). We also give radial velocities in units of \(v_{200\text{m}}\), where \(v_{200\text{m}} = \sqrt{G M_{200\text{m}}/r_{200\text{m}}}\).
Diemer & Kravtsov (2014) show that \( r_{200m} \), rather than the commonly used \( r_{200c} \), is a more natural choice to scale haloes at large radii. However, as we will show, \( r_{200m} \) (or even \( r_{200c} \), Eke et al. 1998; Bryan & Norman 1998) may be a more natural choice to define the edges of Milky Way mass haloes. Note, for a typical NFW profile with concentration, \( c = 10 \), \( r_{200m} \approx 1.6 r_{200c} \).

### 3.1 Dark Matter

We first focus on the dark matter profiles of the haloes. For the radial density profiles we use 40 evenly spaced bins in \( \log(r/r_{200m}) \) between \(-1.0 \) and 0.6. The logarithmic slope profile, \( d \log(\rho)/d \log(r) \), is computed using the fourth-order Savitzky-Golay smoothing algorithm over the 15 nearest bins (Savitzky & Golay 1964). This choice of smoothing length allows us to identify the strongest features in the profile, and removes most of the noise (cf. Diemer & Kravtsov 2014).

First, we consider stacked density profiles of each simulation suite (ELVIS, APOSTLE, and Auriga) at various position angles. We split each halo into wedges with a narrow range in position angle (0.2 width in \( \cos(\theta) \)) and compute the radial density profile in each wedge. We then calculate the median stacked density profile in each wedge for the entire halo sample. For isolated haloes, this position angle is arbitrary and we define \( \cos(\theta) = x/r \), where \( x \) is a randomly chosen axis in the simulation box. For the paired haloes, the position angle is defined relative to the vector joining the two haloes, so \( \cos(\theta) = 1 \) is directly towards the neighbouring halo. In Fig. 1 we show the slopes of the median stacked profiles. The different coloured lines show ten equally spaced wedges in \( \cos(\theta) \), and the thick orange line shows the logarithmic slope profile of the median density profile over all position angles. For the logarithmic slope profile of median density profile (thick orange line) we take the median density in each radial bin (over all haloes and position angles) and then compute the logarithmic slope profile. This is not the same procedure as taking the median of logarithmic slope profiles for each position angle (shown with the coloured lines), so the median profile does not always lie in the middle of these lines. The same procedure is used in subsequent plots when we show the slope profile of the median density. The dotted vertical lines indicate the most prominent minima of \( \log(\rho)/d \log(r) \) for each position angle. Note these minima are chosen to have \( \log(\rho)/d \log(r) < -2.5 \) to minimize the effect of noise. The location of these minima, \( R_{\text{caustic}}/r_{200m} \), which we use to define the caustics, are shown as a function of position angle in the adjacent panels. Note that although we show stacked profiles over several haloes, the profiles in each wedge are subject to the effects of substructure. When averaging over all position angles, we can account for this (see below). However, here we explicitly check that removing substructures from the analysis does not significantly affect the results.

Previous work (e.g. Adhikari et al. 2014; Diemer & Kravtsov 2014; More et al. 2015; Diemer et al. 2017) has used the location of these minima, \( R_{\text{caustic}} \), in dark matter haloes to define the so-called “splashback” radius, which is predicted in spherical models of secondary collapse (e.g. Fillmore & Goldreich 1984; Bertschinger 1985). For isolated haloes (ELVIS-Iso, Auriga) the location of this...
We have\(^{(2014,\text{Mansfield et al.}})\) fitted the logarithmic slope profile, \(d\log(\rho)/d\log(r)\), of the dark matter density profiles for the isolated ELVIS and Auriga haloes. Here, we show three bins of recent mass accretion rate, \(\Gamma\), increasing from left to right. The black lines show individual halo profiles, and the thick orange line indicates the logarithmic slope of the median density profile for each mass accretion rate bin. The feature we have termed the second caustic, which is a less pronounced than the splashback radius and is located at smaller radii, becomes more evident for low mass accretion rates.

The location of the minimum in the paired haloes is less clear than in the isolated haloes. Here there is more variation in \(R_{\text{Caustic}}\) and the overall median stacked profile (solid orange line) appears to have two minima (see below). The variation in the location of \(R_{\text{Caustic}}\) is not random. For position angles directly towards and away from the neighbouring halo \(R_{\text{Caustic}}\) is significantly smaller \((R_{\text{Caustic}}/r_{200m} \sim 0.6)\) than in other directions. It is unsurprising that the caustic towards the neighbour is affected: here, the typical splashback radius \((\sim 1.4r_{200m})\) runs into the neighbouring halo. However, it is less obvious why the directly opposite direction should be affected. For paired haloes the dynamics of the particles are governed by the effective potential of the two massive haloes, and there is a “saddle point” in the potential at \(\cos(\theta) = 1\). Our interpretation is that along this direction particles can only accrete from a limited distance due to the presence of the neighbour. This material will then have less time to accelerate before it reaches apocentre due to its smaller starting distance, and thus will reach a smaller apocentre on the opposite side (i.e. at \(\cos(\theta) = -1\)). Another possibility is that distribution of mass in the \(\cos(\theta) = -1\) direction is shaped by the presence of the Lagrange points of the effective potential that are expected in that direction.

The location of a second caustic at smaller radii has been seen in previous work (see e.g. figs 10, 13, 14 in Diemer & Kravtsov\(^{(2014)}\) and has been demonstrated explicitly in (Adhikari et al.\(^{(2014,\text{Mansfield et al.}})\), see their fig. 9). Adhikari et al.\(^{(2014)}\) show that for slowly accreting haloes, the stream of splashback material is separated from the rest of the virialized matter in the halo, and the location of the second caustic becomes more pronounced. The majority of Milky Way-mass haloes are slowly accreting (especially relative to cluster-sized haloes), so it is particularly intriguing that we detect this feature here. Curiously, the typical location of this second caustic corresponds to \(r_{200c}\), rather than \(r_{200m}\) (as \(r_{200m} \sim 1.6 \times r_{200c}\)). We first noted this secondary feature in the paired haloes, however, this feature is also apparent in the individual profiles of the isolated haloes (see below). This feature can be difficult to see in the stacked profiles in Fig. 1 as there is considerable halo-to-halo scatter, and the signal is relatively weak (especially relative to the splashback radius for isolated haloes). In Fig. 2 we show the logarithmic slope profiles for individual haloes in the isolated ELVIS and Auriga runs. Here, we separate the haloes into three bins with increasing (recent) mass accretion rate from left to right. The thick orange lines show the logarithmic slope profile of the median density profiles in each bin (where the differential profile is computed after finding the median density in each radial bin, as described above). We use the definition given by Diemer & Kravtsov\(^{(2014)}\) to define mass accretion rate:

\[
\Gamma = \frac{\log M_{\text{dm}}(z_1) - \log M_{\text{dm}}(z_2)}{\log(a_1) - \log(a_2)}
\]

where \(z_1 = 0\) and \(z_2 = 0.5\). Note when computing the individual halo profiles we compute the median value over 10 equally spaced wedges in position angle (i.e. 0.2 width in \(\cos(\theta)\)) for each radial bin. This procedure has the advantage of minimizing the effect of substructure in the profile (Mansfield et al.\(^{(2017)}\)). We have checked that explicitly removing (bound) substructures produces very similar results, however we do caution that there are other in-homogeneities present in the density that could effect the results, but we expect that our procedure will account for the most prominent irregularities. Fig. 2 illustrates two important points. First, as mentioned above, there is wide range in halo-to-halo scatter, particularly, for any second caustic features. Second, the second caustic becomes more prominent at lower mass accretion rates, as predicted by Adhikari et al.\(^{(2014)}\). We now explore this feature further by analysing individual haloes in more detail.

In Fig. 3 we show two example haloes. The left panels show the dark matter distribution of Auriga-1 (an isolated halo), and the right panels show APOSTLE V10 (a paired halo: in Fig. 3 the coordinate system is centred at \((x, y, z) = (61.948, 24.230, 48.305)\) Mpc in the V10 system, see Table A1 in Fatnati et al.\(^{(2016)}\)). The top panels show a 2D projection of the dark matter distribution, the middle panels the density profile and logarithmic slope profile, and the bottom panels the radial velocity profile and corresponding log-
Our Milky Way is located in the Local Group and neighbours a massive halo, so the definition of \( r_{200m} \) is. The feature looks similar to the second caustic features shown in Adhikari et al. (2014), and we suggest that this feature relates to the edge of the material in the halo at the position where particles have completed at least two passages through pericentre. The splashback radius is related to the material infalling onto the haloes for the first time, and the second caustic relates to the edge of the virialized material, which has undergone at least two orbital passages through pericentre. The caustics defined in density or velocity space are closely related, albeit with some scatter (see Fig. 4).

Figure 3. Two example haloes from Auriga (left panels) and APOSTLE (right panels). Here we show the density of dark matter in \((x, y)\) projection (top panels), the radial density profiles (middle panels), and the radial velocity profiles (bottom panels). The shading in the top and bottom (left) panels shows 200 × 200 pixels saturated at the 95th percentile of the 2D histogram. In addition to the density and radial velocity profiles, we also show the logarithmic slope profiles of these quantities: \( \frac{d \log \rho}{d \log r} \) and \( \frac{d \log v_r}{d \log r} \). These logarithmic slope profiles are used to identify caustics in the dark matter. The radial velocity profiles (the solid pink lines show the median profile, and the dotted pink line indicates the zero level for reference) suggest that the splashback radius is related to the material infalling onto the haloes for the first time, and the second caustic relates to the edge of the virialized material, which has undergone at least two orbital passages through pericentre. The caustics defined in density or velocity space are closely related, albeit with some scatter (see Fig. 4).

The second caustic can also be seen in the radial velocity profile. Here, we use the local minimum of \( d(v_r)/d \log r \) to identify the caustics. The velocity and density caustics typically align on average, but there is some scatter (see Fig. 4). The radial velocity profile allows us to see more clearly what the second caustic is. The feature looks similar to the second caustic features shown in Adhikari et al. (2014), and we suggest that this feature relates to the edge of the material in the halo at the position where particles have completed at least two passages through pericentre. The splashback radius is located where material is outgoing for the first time, and particles have only completed one pericentric passage. The existence of two caustics, each defining different regions of the halo, begs the question: which should we use to define the edge of the halo? This question is particularly relevant for low mass accreting haloes, where the splashback and second caustic are well separated (Adhikari et al. 2014). Our Milky Way is located in the Local Group and neighbours a massive halo, so the definition of splashback radius is less clear (and indeed overlaps with the halo of M31). For this reason, we suggest that the most meaningful radius for the Milky Way is the second caustic. We will show in Sec-
function 3.2 that this definition is also applicable to the stellar material. Note, however, that although we have defined this interesting feature as the “second caustic”, this does not necessarily correspond to the classical definition of second caustic from spherical (or ellipsoidal) collapse models (as seen in Adhikari et al. 2014). In particular, the wide halo-to-halo scatter, and the apparent correlation with the stellar distribution (see following section), could point to a merger origin, i.e. from the apocentre of the last major merger. The actual origin of this feature will require further investigation, ideally with particle evolution tracking.

In Fig. 4 we show the positions of the dark matter caustics for individual haloes in isolated (top panels) and paired (bottom panels) environments. The caustics are identified as minima in the $d\log(\rho)/d\log(r)$ and $d(\Gamma)/d\log(r)$ profiles. We consider the two most prominent (outer) caustics, and only consider features with $d\log(\rho)/d\log(r) \sim -2.5$ and $d(\Gamma)/d\log(r) \sim -0.25$, respectively. In addition, for every individual halo we visually inspect the profiles to ensure we are not confusing noise with a real caustic. The left panels show the position of the velocity caustics against the density caustics. The filled circles show the splashback radii and the open squares the second caustics. Note that for isolated haloes the splashback radius can be identified in almost all of the haloes; however, even with a restriction on position angle, this can be harder to detect in the paired haloes. Over all paired haloes in ELVIS and APOSTLE 21 percent have no detectable splashback radius in density or velocity. Moreover, the density and velocity caustics are not as closely aligned in the paired environments. On the other hand, the detection efficiency of the second caustic is very similar between isolated and paired haloes of similar mass (e.g. by com-paring ELVIS Isolated and Paired haloes). There is no discernible second caustic in 16 percent of the haloes (over all haloes in ELVIS, APOSTLE and Auriga), and the non detections are typically more massive haloes with higher recent accretion rates (see below and Fig. 2).

The symbols in Fig. 4 are coloured according to the recent mass accretion rate (see Eqn. 1). The majority of haloes have quite low recent mass accretion rates ($\Gamma < 1$), as expected for Milky Way mass haloes. The middle panels of Fig. 4 show how the positions of the caustics relate to $\Gamma$. The caustics in the isolated haloes are typically at smaller radii for haloes with higher recent mass accretion rates (as shown in Diemer et al. 2017 over a wider mass range). However, this trend is less obvious in the paired environments, particularly for $\Gamma > 1.5$. This is likely because the splashback radius and the second caustic run into each other at higher...
mass accretion rates, and are harder to distinguish. Furthermore, $\Gamma$ is poorly defined in paired environments where the outer profiles of the neighbouring haloes overlap. Finally, we show the location of the caustics as a function of halo mass in the right-hand panels. We see very little dependence between $R_{\text{Caustic}}/r_{200m}$ and halo mass. Indeed, analytical models predict that mass accretion rate, rather than halo mass, is the more important physical quantity the determines the splashback radius (e.g. Adhikari et al. 2014).

### 3.2 Stars

We now turn our attention to the stellar material in Milky Way-sized haloes. We analyse the APOSTLE and Auriga simulations which include baryonic material. In Fig. 5 we show the logarithmic slope of the stellar density profiles of the Auriga (left) and APOSTLE (right) haloes. We use the same bin sizes and smoothing technique as for the dark matter. As in Fig. 1, the median stacked profiles are shown, and the different colours show ten different wedges in position angle. The solid orange line shows the logarithmic slope of the median density profile for all haloes over all position angles. The thick orange line indicates the logarithmic slope profile of the median density profile for all haloes over all position angle. The solid orange line shows the logarithmic slope profile of the median density profile for all haloes over all position angles. The thick orange line indicates the logarithmic slope profile of the median density profile for all haloes over all position angles. The thick orange line indicates the logarithmic slope profile of the median density profile for all haloes over all position angles.

Figure 5. The logarithmic slope profile, $\frac{d\log(\rho)}{d\log(r)}$, of the stellar density profiles of the Auriga (left) and APOSTLE (right) haloes. Here, 40 evenly spaced bins in $\log(r/\text{r}_{200m})$ have been used in the range $\log(r/\text{r}_{200m}) = [-1.0, 0.6]$. The logarithmic profile is computed using the fourth-order Savitzky-Golay smoothing algorithm over the 15 nearest bins (Savitzky & Golay 1964). The thick orange line indicates the logarithmic slope profile of the median density profile, and the coloured lines show the slope profiles along wedges in position angle. Ten wedges are equally spaced in $\cos(\theta)$ (we take $\cos(\theta) = x/r$). For the isolated haloes, the position angle is arbitrary. For pairs of haloes, the position angle is defined relative to the vector joining the two haloes (i.e. $\cos(\theta) = 1$ is directly towards the neighbouring halo). The dotted vertical lines show the minimum, defined as $R_{\text{Caustic}}$, of the logarithmic slope profile in each wedge. The adjacent panels show $R_{\text{Caustic}}$ as a function of position angle. The colours of the filled squares correspond to the coloured lines. The caustics for paired and isolated haloes are similar, and are typically located at $0.6r_{200m}$.

Figure 6. The logarithmic slope profile, $\frac{d\log(\Sigma)}{d\log(R)}$, of the stellar surface density profiles of the Auriga (left) and APOSTLE (right) haloes. Here, 40 evenly spaced bins in $\log(R/r_{200m})$ have been used in the range $\log(R/r_{200m}) = [-1.0, 0.6]$. The logarithmic profile is computed using the fourth-order Savitzky-Golay smoothing algorithm over the 15 nearest bins (Savitzky & Golay 1964). The three line styles show the stacked profiles for three (random) projections. For comparison, the logarithmic slope profile of the 3D stellar density is shown with the dotted red line (see Fig. 5). A well-defined edge is also seen in the (stacked) projected stellar density profiles, although this is a weaker feature than in the 3D case.
we examine the stellar caustics of individual haloes.

Figure 7. Two example haloes from Auriga (left panels) and APOSTLE (right panels). These are the same haloes shown in Fig. 3. Here, we show the density of stars in the (x, y) projection (top panels), the radial density profiles (middle panels), and the radial velocity profiles (bottom panels). The shading in the top and bottom (left panels) shows 200 × 200 pixels saturated at the 90th percentile of the 2D histogram. In addition to the density and radial velocity profiles, we also show the logarithmic slope profiles of these quantities: \( \frac{d \log \rho}{d \log r} \) and \( \frac{d (v_\text{stellar})}{d \log r} \). The stellar caustics are identified as minima in the logarithmic slope profiles, and are indicated with the vertical solid lines.

prominent caustics at smaller radii, which are associated with apocentres of past accretion events (these can be seen in both dark matter and stars, see e.g. figs 3 and 7). Such a feature has already been seen in the Milky Way halo at \( r \sim 20 \) kpc, and is likely related to the apocentre of the Gaia-Sausage/Enceladus event (Deason et al. 2013, 2018). In this work we are interested in the caustic that defines the edge of the stellar material, and is hence associated with the last (significant) accretion event. The radial velocity profiles suggest that the location of this stellar caustic coincides with the edge of the material that has completed at least two pericentric passages, similarly to the second caustic in the dark matter (see below). We find no obvious difference between the isolated and paired haloes, which is unsurprising as the location of the stellar caustic (0.6/200m − 200c/200m ∼ 220 kpc) does not generally overlap with the neighbouring halo.

In Fig. 8 we examine the stellar caustics of individual haloes in more detail. We are able to identify a stellar caustic in over 90 percent of the haloes. Those cases where we cannot clearly identify a feature (in either density or velocity) are typically cases where there is very recent accretion and the outer density profiles are messy. Note we typically only consider stellar caustics with \( \frac{d \log (\rho)}{d \log (r)} < 5 \) or \( \frac{d (v_\text{stellar})}{d \log (r)} < -1.0 \), which we choose to be distinct from the noise level. The left panel of Fig. 8 relates the positions of the velocity and density caustics of the stars. These caustics generally coincide but there is significant scatter. The points are colour coded according to the recent (total) mass accretion rate, \( \Gamma(z = 0.5) \) (see Eqn. 1). In the middle (density) and right-hand (velocity) panels we relate the stellar caustics to the dark matter caustics. Solid filled symbols are used for the splashback radii of the dark matter and open squares for the second caustic of the dark matter. As mentioned earlier, the stellar caustics are strongly related to the second caustic in the dark matter. Note the dashed line indicates the one-to-one relation; this is not a fit! This relation holds for \( \sim (0.3 - 0.8)/200m \), but seems to break down at larger radii. This discrepancy at large radii is likely for two reasons. Firstly, when \( r_\text{Caustic} \) is large the stellar caustic can be closer to the splashback radius, or even somewhere between the second caustic and the splashback radius. Secondly, the stellar caustic is harder
Deason et al. discussed how the second caustic of the dark matter coincides with the second caustic in the dark matter. This may be especially true in relatively major mergers, where almost all of the dark matter has already been peeled away. However, with the varying smooth to lumpy mass accretion rate, the Mauna Kea (2011; Moster et al. 2010; Genel et al. 2010; Genel et al. 2010), will likely lead to different relations at higher and lower masses.

We speculate that to lose stars to tidal forces subhaloes must typically pass through at least two pericentres, and thus the “edge” of the stellar material coincides with the second caustic in the dark matter. This may be especially true in relatively major mergers, which typically dominate the mass budget of the accreted stellar halo (see e.g. Purcell et al. 2007; Cooper et al. 2010; Deason et al. 2016; D’Souza & Bell 2018), when such passages lead to a loss of angular momentum and shrinking of the pericentre. Finally, we remark that the relation between the “edges” of stars and dark matter may vary at different mass scales. Here, we have focused on Milky Way-mass halos, but the non-linear stellar mass to dark matter mass relation (Behroozi et al. 2010; Moster et al. 2010; Read et al. 2017), and the varying smooth to lumpy mass accretion rate (Genel et al. 2010), will likely lead to different relations at higher and lower masses.

In Section 3.1 we discussed how the second caustic of the dark matter, which we now see coincides with the stellar caustic, may be the most relevant definition of the edge of the Milky Way. This means that the edge of our own Galaxy is, potentially, observable in the stellar distribution. Currently, the density profile of the stellar halo has only been mapped out to ~50 – 100 kpc (e.g. Deason et al. 2011; Sesar et al. 2011; Deason et al. 2014a; Xue et al. 2015; Slater et al. 2016; Hernitschek et al. 2018). Moreover, radial velocities of stars are only available, in any significant numbers, out to similar distances (e.g. Mauron et al. 2004; Deason et al. 2012; Bochanski et al. 2014; Cohen et al. 2017). However, with upcoming wide-field photometric and spectroscopic facilities like the Rubin Observatory Legacy Survey of Space and Time (LSST, Ivezić et al. 2019), WFIRST (Spengler et al. 2015), the Mauna Kea Spectroscopic Explorer (MSE, Bauman et al. 2016) and the Subaru Prime Focus Spectrograph (FPS, Takada et al. 2014) on the horizon, exploring these extreme distances will be feasible in the near future.

Figure 8. The positions of the stellar caustics in Auriga and APOSTLE haloes. The left-hand panel shows the radii of the stellar density caustics against the radius of the stellar radial velocity caustics. The middle and right-hand panels show the radii of the stellar density (middle) and velocity (right) caustics against those of the dark matter caustics. The filled circles and open squares indicate the dark matter splashback and second caustic radii. The symbols are coloured according to the mass accretion rate, $\Gamma (z = 0.5)$. The dashed lines show the one-to-one relation. Note that the DM caustics at large radii appear discretized owing to the logarithmic binning. Over a wide range in radii (out to ~0.8$\,r_{200m}$) the stellar caustics correspond to the second caustic in the dark matter. In a few cases where $R_{\text{Caustic}}^{\text{STAR}}$ is large, the stellar caustic can lie in between the dark matter caustics, and can even be closer to the splashback radius.

Finally, it is worth discussing how the concept of galaxy edge is relevant to studies that require a definition of where the halo ends. For example, when using the escape velocity of local halo stars to estimate the total mass of the Galaxy, the definition of the radius of “escape” is an important element of the analysis. Indeed, Deason et al. (2019) used a radius of $2r_{200c}$ (~1.25$r_{200m}$), which is at the extreme end for the Auriga haloes. However, while this approach is conservative in that it does not allow for radii where stars can potentially escape, our results suggest that a smaller radius is likely more applicable. For example, the median stellar caustic radius of the Auriga simulations is 0.7$r_{200m}$, which is approximately 1.2$r_{200c}$. If this distance is used in the Deason et al. (2019) analysis to define the radius beyond which stars have escaped, then the total mass of the Milky Way is revised upwards by 20 percent. Interestingly, this is approximately the change that Grand et al. (2019a) found was required to correct the mass estimates when the procedure is applied to the Auriga haloes. In particular, Grand et al. (2019a) suggest that the mass estimates are underestimated because the local stars do not reach out to $2r_{200c}$. Here, we show that this is indeed the case. However, as a cautionary note, we should use the observed $R_{\text{Caustic}}^{\text{STAR}}$ rather than the median value of the Auriga haloes, which does not necessarily coincide with the true Milky Way value (see end of Section 3.3).
In the previous subsections, we have focused on the distribution of dark matter and stars. Now we apply a similar analysis to the subhalo population. In this case, the number of discrete tracers is much lower than for the dark matter or star particles. For this reason, we concentrate only on the caustics defined in velocity space, where it is easier to identify features associated with caustics when there are low numbers of tracers. It is worth noting that there is no division into position angle sectors here (cf. the dark matter and stars), which makes the subhalo-based profiles sensitive to substructure. Thus, although this analysis is a valuable first step, we plan to apply more sophisticated techniques tailored towards highly discretely sampled distributions in future work.

We use the (dark matter only) ELVIS suite to study the general subhalo population, and APOSTLE and Auriga to analyse the “dwarf” population. Here we do not distinguish between isolated and paired environments and, in the paired cases, only consider subhaloes with \( \cos(\theta) < 0.6 \) to identify caustics. We define subhaloes as all bound substructures with \( M_{\text{DM}}^{\text{Sub}} > 10^{7.5} M_\odot \). This is the convergence limit for subhaloes found by Garrison-Kimmel et al. (2014). In APOSTLE and Auriga, subhaloes with at least one star particle are identified as luminous dwarfs. This approximately corresponds to subhaloes with \( M_{\text{Star}}^{\text{Sub}} > 10^5 M_\odot \).

For each individual halo we use the logarithmic slope of the radial velocity profile to define the caustics in the subhalo population. Note that for the dwarf galaxies, where the numbers of objects are typically low (O(100) per halo), we change the binning in logarithmic radius to have 25 equally spaced bins in the range \( \log(r/\text{200m}) \in [-1.0, 0.5] \) and use the same smoothing kernel as in the previous subsections. Due to the small numbers we only identify the most prominent caustic and do not attempt to find two distinct caustics.

### 3.3 Subhaloes and Dwarf Galaxies

The resulting caustics are shown in Fig. 9 as a function of the (two) dark matter caustics (computed in Section 3.1). Caustics can be identified for the majority of subhalo populations, but in several cases (30 percent) a caustic could not be identified in the dwarf population, mainly as a result of small numbers. The filled gray circlesynthetic in Fig. 9 indicate the splashback radius in the dark matter and the filled red squares the second caustic in the dark matter. Interestingly, we find that the caustic in the subhalo population corresponds to the splashback radius (left panel), while the caustic in the luminous dwarfs’ population coincides with the second caustic in the dark matter (right panel). This is perhaps unsurprising as the subhalo population traces the dark matter, while the luminous dwarfs are more closely related to the accretion of the more massive subhaloes, and hence the stellar halo.

We show two examples for the dwarf galaxy population in Auriga-16 (top panel) and APOSTLE-V5 (bottom panel) in Fig. 10. The left panels show the radial velocities of the dwarfs as a function of radius. For the paired halo dwarfs with \( |\cos(\theta)| > 0.6 \) (i.e. close in angle to the line joining the two haloes) are indicated in red. The right-hand panels show the logarithmic slope profiles of the radial velocities. The vertical dashed line indicates the caustic.

The distances and radial velocities are converted to Galactocentric coordinates, assuming a circular velocity of \( v_c(t_0) = 235 \text{ km s}^{-1} \) at the position of the Sun \( (t_0 = 8.1 \text{ kpc}) \), and a peculiar solar motion of \( (U_\odot, V_\odot, W_\odot) = (11.1, 12.24, 7.25) \text{ km s}^{-1} \) ( Schönrich et al. 2010). We use 22 radial bins equally spaced in \( \log(r) \) between 1.0 and 3.3. As we did previously, the logarithmic slope profile is computed using the fourth-order Savitzky-Golay smoothing algorithm over the 15 nearest bins (Savitzky & Golay 1964). We indicate in the
Figure 11. Left panel: the radial velocities (in Galactocentric coordinates, $V_{\text{GSR}}$) of observed Local Group dwarf galaxies. Dwarfs with $|\cos(\theta)| > 0.6$ (i.e. close in angle to the line joining the Milky Way and M31) are indicated in red. The solid black line indicates the median radial velocity profile, and the shaded region indicates the dispersion (defined as $1.4826\times$ the median absolute deviation) calculating using a bootstrap method. Right panel: the logarithmic slope profile of the median radial velocity. The vertical dashed line indicates the caustic that defines the edge of the Galaxy. This lies at 290 kpc and approximately corresponds to $0.8r_{200m}$ (or $-1.0r_{\text{vir}}$, $\sim 1.3r_{200c}$), assuming the Milky Way mass estimated by Callingham et al. (2019).

We identify a minimum in the observed population of dwarfs at $\sim 290$ kpc. Using a bootstrap method to estimate the uncertainty, we find $R_{\text{edge}} = 292 \pm 61$ kpc. If we assume the Milky Way halo mass recently measured by Callingham et al. (2019) and a typical halo concentration ($\sim 10$ for Milky Way-mass haloes; Neto et al. e.g. 2007; Ludlow et al. e.g. 2014; Klypin et al. e.g. 2016), this radius corresponds to $0.8r_{200m}$ (or $1.3r_{200c}$). Interestingly, this radius (292 kpc) lies at exactly the “virial radius” defined by the fitting formulae in Bryan & Norman (1998). Moreover, this also coincides with the radius where the HI content of Local Group satellites sharply drops (around 270 kpc, Greveich & Putman 2009). Given the rather large uncertainty in the measurement, these could simply be coincidences, however, it is worth noting that we are probing an interesting radial regime of the Galactic halo.

We can also use this measured radius to independently estimate the mass of the Milky Way using the escape velocity analysis described by Deason et al. (2019). As mentioned in Section 3.2, this technique depends on the definition of the “outer boundary” of the halo stars. If we use a boundary of 290 kpc, rather than a fixed fraction of $r_{200c}$, like Deason et al. (2019), we find a mass of $M_{200c} \sim 1.1 \times 10^{12} M_\odot$. Although there is considerable uncertainty in this definition of halo edge, it is reassuring that this mass is in excellent agreement with the recent measurements by Callingham et al. (2019) and Cautun et al. (2019).

While we suggest that the edge of the Milky Way halo lies at 290 kpc, this remains a tentative result for two important reasons. Firstly, the value is strongly dependent on Leo I (located at 250 kpc): there is a significant gap between the most distant satellite of the Milky Way and the nearest dwarfs in the Local Group. Second, and perhaps most importantly, our census of local dwarfs is far from complete and we have made no attempt to correct for selection effects or observational biases. Indeed, as recently predicted by Fattahi et al. (2019a), there are troves of local group dwarfs waiting to be discovered by future wide-field imaging surveys.

4 CONCLUSIONS

In this work we have analysed three different suites of simulated Milky Way-mass haloes (ELVIS, APOSTLE and Auriga) to explore the “edge” of Galactic-sized haloes. We use the logarithmic slope profiles of the density and radial velocity distributions to identify the location of caustics in the halo. These features, which correspond to the build up of particles at apocentre, are used to define the edges of the dark matter, stars, and subhalo population. Our main conclusions are summarised as follows:

- We typically identify two distinct caustics in the outer dark matter profiles. The outermost caustic, called the “splashback” radius, is the boundary at which accreted dark matter reaches its first orbital apocentre after turnaround. This lies at approximately $\sim 1.4r_{200m}$ for Milky Way-mass haloes. We suggest that the second caustic, which is located at a smaller radius ($\sim 0.6r_{200m} \approx r_{200c}$) and is typically less prominent than the caustic at the splashback radius, corresponds to the edge of the material which has passed through at least two pericentric passages.

- In Local Group-like environments, the splashback radius of one of the haloes is poorly defined, as it often overlaps with the other halo. However, the second caustic in the dark matter is less affected by the companion and appears to be a more natural choice for the definition of the halo boundary of the Milky Way.

- We identify a prominent caustic in the stellar distribution in both the radial density and velocity profiles. This typically lies at $0.6r_{200m}$ and, in the majority of cases, coincides with the second caustic of the dark matter. This feature can potentially be identified in the Milky Way using future observational facilities, such as LSST and MSE. Moreover, there is scope to measure this edge in external galaxies, either by stacking profiles, or by obtaining deeper and wider images with forthcoming facilities such as WFIRST.

- The outer caustic, corresponding to the splashback radius, can be identified in the phase-space distribution of the subhalo population. If we consider only luminous dwarfs (with $M_{\text{star}} > 10^4 M_\odot$) the best defined caustic coincides with the second caustic in the dark matter (and hence with the stellar caustic).

- We applied our analysis to the currently known population of dwarf galaxies in the Local Group. We predict that the edge of the Milky Way (defined as the second caustic in the dark matter) lies at $\sim 290$ kpc. For the total Milky Way mass measurement by Callingham et al. (2019), this radius coincides approximately with the value of $r_{\text{vir}}$ obtained from the fitting formula of Bryan & Norman (1998), albeit with significant uncertainty. This is a tentative measurement of the Galactic edge, but will greatly improve with future discoveries of more Local Group dwarfs.

In many analyses of the Milky Way halo its outer boundary is a fundamental constraint. Often the choice is subjective, but as we have argued, it is preferable to define a physically and/or observationally motivated outer edge. Here we have linked the boundary of the underlying dark matter distribution to the observable stellar halo and the dwarf galaxy population. There is great hope that future data will provide a more robust and accurate measurement of the edge of the Milky Way and nearby Milky Way-mass galaxies than the one we have presented here. In this work we have focused on Milky Way mass haloes in a $\Lambda$CDM cosmology, but a similar analysis can be extended to wider mass scales and applied to different cosmologies or dark matter models.
