THE OBSERVED \( M-\sigma \) RELATIONS IMPLY THAT SUPER-MASSIVE BLACK HOLES GROW BY COLD CHAOTIC ACCRETION

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ABSTRACT

We argue that current observations of \( M-\sigma \) relations for galaxies can be used to constrain theories of super-massive black holes (SMBHs) feeding. In particular, assuming that SMBH mass is limited only by the feedback on the gas that feeds it, we show that SMBHs fed via a planar galaxy-scale gas flow, such as a disk or a bar, should be much more massive than their counterparts fed by quasi-spherical inflows. This follows from the relative inefficiency of active galactic nucleus feedback on a flattened inflow. We find that even under the most optimistic conditions for SMBH feedback on flattened inflows, the mass at which the SMBH expels the gas disk and terminates its own growth is a factor of several higher than the one established for quasi-spherical inflows. Any beaming of feedback away from the disk and any disk self-shadowing strengthen this result further. Contrary to this theoretical expectation, recent observations have shown that SMBHs in pseudobulge galaxies (which are associated with barred galaxies) are typically under- rather than overmassive when compared with their classical bulge counterparts at a fixed value of \( \sigma \). We conclude from this that SMBHs are not fed by large (100 pc to many kpc) scale gas disks or bars, most likely because such planar flows are turned into stars too efficiently to allow any SMBH growth. Based on this and other related observational evidence, we argue that most SMBHs grow by chaotic accretion of gas clouds with a small and nearly randomly distributed direction of angular momentum.

Key words: galaxies: nuclei – quasars: general

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1. INTRODUCTION

It is now well established that super-massive black holes (SMBHs) reside in the nuclei of many galaxies. The masses of these SMBHs correlate with a range of properties of their host spheroids, including luminosity (e.g., Magorrian et al. 1998) and, consequently, mass (e.g., Marconi & Hunt 2003; Haring & Rix 2004); the concentration of stellar bulge light (e.g., Graham et al. 2001); the deficit of stellar bulge light in ellipticals (e.g., Graham 2004; Ferrarese et al. 2006; Kormendy & Bender 2009); and the gravitational binding energy of the bulge (e.g., Feoli & Mele 2005, 2007; de Francesco et al. 2006; Aller & Richstone 2007).

The correlation between SMBH mass (\( M_{\text{bh}} \)) and the velocity dispersion (\( \sigma \)) of its host spheroid, referred to below as a “\( M_{\text{bh}}-\sigma \) relation,” has been studied by many authors. A power-law fit to the data, \( M_{\text{bh}} \propto \sigma^p \), yielded values of \( p \) in the range of \( p \sim 4-5 \) (see Ferrarese & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002; Gültekin et al. 2009), and is now widely interpreted as the imprint of the self-regulated growth of the SMBH. A more recent extended sample of galaxies shows that the power-law index \( p \) may be as high as \( p \approx 6 \) (Graham et al. 2011).

Furthermore, recent work has highlighted that the \( M_{\text{bh}}-\sigma \) relation one measures will depend on the nature of the galaxy sample. For example, McConnell et al. (2011) present two extremely massive SMBHs, \( M_{\text{bh}} \sim 10^{10} M_\odot \), in the centers of two giant elliptical galaxies. Combined with other previous SMBH mass measurements, these authors obtain \( p \approx 4.5 \) for elliptical and S0 galaxies. A similar but offset to lower SMBH masses result is found for spiral galaxies in the sample. Alternatively, when data for all galaxies are combined in one \( M_{\text{bh}}-\sigma \) relation, \( p \approx 5 \) is found.

Another important conclusion is gradually emerging from the observations; SMBHs in galaxies that show flattened morphology, such as disks, pseudobulges, or bars, appear to be less massive at the same \( \sigma \) than their counterparts in elliptical galaxies. In particular, Hu (2008) showed that pseudobulge galaxies tend to have undermassive SMBHs, and Graham (2008) showed that barred galaxies tend to have SMBHs that are a factor of \( \sim 3-10 \) smaller than elliptical galaxies at the same velocity dispersion. While Kormendy et al. (2011), in variance with other authors, find that SMBHs in pseudobulges and galaxy disks do not even follow an \( M_{\text{bh}}-\sigma \) relation, their results are consistent with that of the other authors in that most of their pseudobulge systems lie below the \( M_{\text{bh}}-\sigma \) relation for ellipticals. Mathur et al. (2011) firmly up these conclusions with a sample of narrow line Seyfert 1 nuclei (all classifiable as pseudobulge systems). A useful compilation of the recent data, summarizing these points, is given in the right panel of Figure 1 in Shankar et al. (2012).

The purpose of our paper is to show that there is a unified theoretical picture within which this observational trend can be understood, and that the observed \( M_{\text{bh}}-\sigma \) relations place very interesting constraints on the still unknown feeding mode of SMBHs. A well-known difficulty in fuelling active galactic nuclei (AGNs) is that the typical angular momentum of gas in the galactic bulge is very large compared with that of the last stable orbit around a black hole (e.g., Krolik 1999; Combes 2001). One expects that due to circularization shocks, gas will end up in a planar feature, such as a disk or a ring (see simulation S30 shown in Figure 1 and the left panel of Figure 13 in Hobbs et al. 2011). It has been argued that large-scale angular momentum transfer occurs rapidly enough through the action of stellar and gaseous bars (e.g., Shlosman et al. 1990) or via spiral density
waves (Thompson et al. 2005) to overcome this difficulty. These galaxy-wide gas disks (with radial scales of many kpc) could then extend all the way to the inner galaxy and even feed the SMBH. This “planar mode” of feeding is schematically illustrated in the left panel of Figure 1.

However, gas disks tend to become unstable to gravitational fragmentation and the resulting star formation (SF) beyond a self-gravity radius of $\sim 0.1$ pc (e.g., Paczyński 1978; Kolykhalov & Sunyaev 1980; Shlosman & Begelman 1989; Collin & Zahn 1999). In the simplest model, this would result in complete consumption of gas in the disk by SF (Nayakshin et al. 2007) and so there would be no way to fuel the SMBH. However, stellar feedback could heat up the large-scale disk to self-regulate the SF rate in the disk to a level such that a sufficient amount of gas can reach the SMBH, but there is no clear consensus on this point (e.g., Goodman 2003; Thompson et al. 2005).

An alternative solution to the difficulty of fueling AGNs is provided by chaotic feeding (King & Pringle 2007) or ballistic accretion of cold streams when the gas is cold enough to form dense clouds or filaments (King & Pringle 2006; Nayakshin & King 2007; Hobbs et al. 2011). Large-scale cosmological simulations show that cold streams may be important for supplying relatively low angular momentum gas to galaxies (e.g., Kereš et al. 2009; Kimm et al. 2011; Dubois et al. 2011), while higher resolution simulations that resolve the regions nearer to SMBH suggest that similar processes may operate on smaller scales (Levine et al. 2010). In particular, the numerical experiments of Hobbs et al. (2011) show that turbulence (driven by stellar feedback in the bulge) in a quasi-spherical distribution of gas leads to formation of convergent flows that create high-density filaments. The latter can travel almost ballistically through the rest of the bulge, with some filaments arriving in the inner parsecs of the galaxy. Note that most of the gas still ends up in a galaxy-scale disk with a large angular momentum; however, the SMBH receives most of its fuel not from that disk but from the gas filaments arriving at its vicinity directly.

This “chaotic cold stream” feeding mode is schematically shown in the right panel of Figure 1. In this class of model, the orientation of the innermost subparsec scale disk makes a random walk because of the arrival of material with a fluctuating angular momentum. The cancellation of the angular momentum in shocks transfers gas into the SMBH more rapidly than viscous torques can, and so these disks should be more resilient to SF. Furthermore, because the orientation of the inner disk is constantly changing, feedback from the AGN will be quasi-isotropic.

The main point of our paper can be summarized as follows. Large-scale gas disks are better able to withstand AGN feedback than quasi-spherical inflows (Nayakshin & Power 2010). This is so even under the most optimistic assumptions about AGN feedback—that is, feedback acting directly on the disk without being shadowed by the inner disk, and no beaming away from the disk/galaxy plane. We show that, under these most favorable conditions for which AGN feedback is maximal, there is a critical SMBH mass at which its feedback expels the disk that feeds it. We call this a “disk $M_{\text{bh}} – \sigma / \text{relation}” because it arises only for SMBHs accreting from large-scale galaxy disks. This mass is larger than the canonical $M_{\text{bh}} – \sigma$ relation, and any dilution of AGN feedback would increase it even further. Theoretically, we see that if SMBHs were fed by planar accretion, then we could expect SMBHs in pseudobulge systems to lie above the $M_{\text{bh}} – \sigma$ relation for classical bulges. Observationally, there is no strong evidence for this (e.g., Hu 2008; Graham 2008; Kormendy et al. 2011), and so we conclude that most SMBHs do not grow via the planar accretion mode but are much more likely to be fed by quasi-spherical inflows as in the models of King & Pringle (2007) and Hobbs et al. (2011).

To avoid misunderstanding, we note that galaxy-scale gas disks and gas bars are obviously important for galaxy evolution as a whole, e.g., as a birthplace for a significant fraction of all stars, and also by effecting a radial redistribution of gas and stars within the galaxy, but we see no clear observational evidence that these features are able to channel their gas all the way to the SMBHs.

2. THE GAS WEIGHT ARGUMENT FOR A SPHERICAL AGN FEEDBACK

We briefly review existing ideas of AGN feedback acting on quasi-spherical distributions of gas before moving on to the problem of AGN feedback acting on a gas disk. Silk & Rees
(1998) assumed feedback in the form of an energy-conserving outflow and obtained a scaling \( M_{\text{bh}} \propto \sigma^2 \). Subsequently, Fabian (1999), King (2003, 2005), and Murray et al. (2005) solved the momentum equations for momentum-conserving AGN feedback acting on a spherical shell of gas in an isothermal bulge potential (see below) and showed that it leads to an \( M_{\text{bh}}-\sigma \) relation:

\[
M_{\text{bh}} = \frac{f_c k}{\pi G^2} \sigma^4, \tag{1}
\]

where we follow King’s analysis; here, \( k \) is the opacity, which is assumed to be dominated by the electron scattering, and \( f_c \) is the baryon fraction, which is assumed to be equal to the initial cosmological value of \( f_c \approx 0.16 \) (see Spergel et al. 2007). We note that all of the aforementioned theoretical approaches used a singular isothermal sphere potential (e.g., Section 4.3.3b in Binney & Tremaine 2008) for simplicity. For such a potential, the one-dimensional (1D) velocity dispersion is a constant independent of radius, \( \sigma = (GM_{\text{total}}/2R)^{1/2} \), where \( M_{\text{total}}(R) \) is the total enclosed mass including dark matter inside the radius \( R \) (the distance from the center of the galaxy). The enclosed gas mass, \( M(R) = f_g M_{\text{total}}(R) = 2f_g \sigma^2 R/G \), is proportional to \( R \). The gas density at radius \( R \) for such a potential is \( \rho_g(R) = f_g \sigma^2/(2\pi G R^2) \) for reference.

King (2003, 2005) assumed that the SMBH luminosity is limited by the Eddington value \( L_{\text{Edd}} \), and the momentum outflow rate produced by radiation pressure (King & Pounds 2003) is of the order of

\[
\dot{\Pi}_{\text{SMBH}} \approx \frac{L_{\text{Edd}}}{c} = \frac{4\pi GM_{\text{bh}}}{\kappa}. \tag{2}
\]

We can recover the result of King (2003, 2005) using a simpler “weight argument,” which requires that the momentum output produced by the SMBH (\( \dot{\Pi}_{\text{SMBH}} \)) should just balance the weight of the gas in the bulge, \( W(R) = GM(R)/M_{\text{total}}(R)/R^2 \), which turns out to be independent of radius:

\[
W = \frac{GM(R)M_{\text{total}}(R)}{R^2} = \frac{4f_g \sigma^4}{G}. \tag{3}
\]

To order of magnitude, Equation (3) holds for any potential if estimated at a radius which encloses most of the gas potentially available for SMBH fueling, e.g., such as the virial radius for the Navarro et al. (1997) potential. Balancing the outward force of the outflow (Equation (2)) with the weight of the gas in the bulge (Equation (3)) then leads naturally to Equation (1).

The momentum feedback model is attractive in its physical simplicity. The model is underpinned by observations of fast \( v \sim 0.1-0.3c \) outflows from bright AGNs and quasars (e.g., Pounds et al. 2003a, 2003b). Studying the blueshifted ionized absorption features in the 7–10 keV energy range in a large sample of AGNs with XMM-Newton, Tombesi et al. (2010) concluded that such fast AGN outflows appear in \( \sim 40\% \) of the sample. As these authors emphasize, their uniform sample overcomes “publication bias” and shows that fast AGN outflows are widespread. It also requires the outflows to be wide-angled (rather than jet-like), which is beneficial in spreading the influence of AGN feedback as broadly over the galaxy bulge as possible.

Equation (1) contains no free parameters, but nevertheless it is very close to the observed \( M_{\text{bh}}-\sigma \) relation. Similar reasoning helps to explain both the observed \( M_{\text{bh}}-\sigma \) relation for Nuclear star Clusters (NCs; see McLaughlin et al. 2006) and the observation that NCs are preferentially found in low-mass (low \( \sigma \)) galaxies (Nayakshin et al. 2009). The model has been used recently by Zubovas et al. (2011) to explain the two \( \sim 10 \) kpc scale symmetric lobes apparently filled with cosmic rays detected by Fermi Large Area Telescope in the Milky Way (Su et al. 2010). To explain the particular geometry of the bubbles, the only adjustment to the basic King (2005) model required by Zubovas et al. (2011) has been an addition of a dense disk of molecular gas, known as the Central Molecular Zone, found in the central \( \sim 200 \) pc of our Galaxy (Morris & Serabyn 1996).

While it is clear that the exact \( M_{\text{bh}}-\sigma \) relation derived analytically is affected by the choice of the potential, we note that the main conclusion of our paper—the fact that it is harder to expel a disk feeding the SMBH rather than a quasi-spherical shell—remains unchanged, as it is based on a simple geometrical argument (see below). Furthermore, gravitational potentials of elliptical and lenticular galaxies appear to be relatively well approximated by a constant \( \sigma \) value inside their effective radii. Cappellari et al. (2006) find that the 1D velocity dispersion of stars within the galaxies in their sample depends on radius \( R \) (within the galaxy) as a very weak power law: \( \sigma(R) \propto R^{-0.06} \). This implies that \( \sigma \) drops on average by a factor of \( \sim 1.16 \) when \( R \) changes by an order of magnitude, and \( \sigma^4 \) varies by a factor of \( \sim 1.8 \).

Indeed, McQuillin & McLaughlin (2012) consider SMBH momentum feedback for potentials other than isothermal, e.g., the Navarro–Frenk–White (NFW) potential (Navarro et al. 1997), Hernquist (1990), and Dehnen & McLaughlin (2005) potentials. In all cases, momentum-driven outflow produces relations of the form \( M_{\text{bh}} \propto V_m^4 \), where \( V_m \) is the maximum circular velocity in the potential. The normalization of the relation is slightly smaller than for the simpler isothermal potential we use here, which implies that escape from the isothermal potential is slightly more demanding than for all the others. To quote a numerical factor here, McQuillin & McLaughlin (2012) find a change in the critical SMBH mass (only) by a factor of order unity only for the NFW potential, as an example, if one defines \( \sigma \) at the peak of the rotation curve of the potential as \( \sigma = V_m/\sqrt{2} \).

No analytical treatment for the complex problem at hand can be expected to be accurate within better than a factor of \( \sim 2 \), and therefore we feel that the singular isothermal potential is an appropriate approximation to use here.

3. THEORETICAL DISK \( M_{\text{bh}}-\sigma \) RELATION

As shown by Nayakshin & Power (2010), the above argument does not quite work for a non-spherical geometry. To see this, consider the simplest example—an axisymmetric rotating galaxy. The gas settles in the plane of symmetry of the galaxy and forms a rotationally supported disk. Let \( H \) be the local (i.e., at a given \( R \)) vertical disk scale height. Let us consider a disk at radius \( R \). The maximum momentum flux from the SMBH outflow striking the disk is

\[
\dot{\Pi}_{\text{disk}} = \frac{H}{R} L_{\text{Edd}}/c. \tag{4}
\]

This is an absolute maximum given by the total momentum outflow rate (Equation (2)) times the fraction of the solid angle the disk subtends as seen from the SMBH location. In reality, some of the momentum outflow from the AGN could be intercepted (shadowed) by the disk at smaller radii (see the left panel of Figure 1), and also the outflow may be beamed along
the direction perpendicular to the disk. These individual effects actually strengthen the conclusion we draw because they act to lessen the effect of feedback on the disk and therefore should increase the mass of the SMBH.

We first assume that the disk mass is dominated by gas, and we correct for the presence of stars within the disk later on. The effective weight of the gas disk is given by

\[ W_{\text{eff}} \sim GM_{\text{total}}(R)M_{\text{disk}}/R^2, \]

where \( M_{\text{disk}}(R) \equiv \pi R^2 \Sigma(R) \) is the disk mass and \( \Sigma(R) = 2H(R)\rho_c(R) \) is the gas disk surface density, with \( \rho_c(R) \) being the mid-plane disk density. We note here that a disk in circular rotation actually has a zero weight because gravity is exactly balanced by the centrifugal force. However, within a factor of a few, the effective weight defined above still applies\(^3\) if we wish to expel the disk to infinity (where the centrifugal force becomes negligible if the angular momentum of the material is conserved). Therefore, by requiring that momentum flux balances weight (\( P_{\text{disk}} \lesssim W_{\text{eff}} \)), we limit the SMBH mass to

\[
M_{\text{bh}} = \frac{\kappa}{4\pi} \frac{M_{\text{total}}(R)M_{\text{disk}}}{RH} = \frac{\kappa \sigma^2}{2\pi G} \frac{M_{\text{disk}}}{H}. \tag{5}
\]

A casual look at this equation would seem to suggest that, because \( H \) and \( M_{\text{disk}} \) may vary with radius \( R \) and from system to system, one should not expect any robust correlation between \( M_{\text{bh}} \) and \( \sigma \)\(^4\) to arise.

What is missing in the above analysis is the possibility of SF in the galaxy disk (this was also not included in the numerical experiments of Nayakshin & Power 2010). Massive cool gas disks become gravitationally unstable when the Toomre parameter approaches unity from above (Toomre 1964):

\[
Q = \frac{\kappa \Omega c_s}{\pi G \Sigma} \lesssim 1, \tag{6}
\]

where \( \kappa \Omega \) is the epicyclic frequency. Noting that \( \kappa \Omega = \sqrt{2}\Omega \) for the singular isothermal sphere potential, \( \Omega^2 = GM_{\text{total}}/R^3 \), and using the hydrostatic balance condition \( H = c_s \Omega^{-1} \), we find that

\[
Q = \frac{\sqrt{2}H M_{\text{total}}(R)}{R M_{\text{disk}}}. \tag{7}
\]

Note that \( M_{\text{disk}} \), defined just below Equation (4), is a function of \( R \) in the above equation.

It is widely believed that self-gravitating gas disks self-regulate their SF rates to maintain marginally stable states. As a disk becomes self-gravitating, stars form on a local dynamical timescale (i.e., extremely rapidly) and the energy and momentum released by the stars heat the disk up; this increases \( Q \) and the SF rate is reduced to an equilibrium rate that maintains quasi-equilibrium. Generally, the disk first becomes unstable when \( Q \approx 1.5 \), and therefore we arrive at the constraint

\[
\frac{M_{\text{disk}}}{H} \approx \frac{M_{\text{total}}(R)}{R}. \tag{8}
\]

Using this now in Equation (5), we arrive at the “disk \( M_{\text{bh}}-\sigma \) relation,”

\[
M_{\text{bh}} = \frac{\kappa f_d}{\pi G^2} \sigma^4, \tag{9}
\]

where we introduce \( f_d < 1 \) as the gas mass fraction in the disk to account for the fact that a realistic galaxy disk contains gas and stars as well. According to this definition, the fraction \( 1 - f_d \) of the disk mass is in the recently formed stars or very dense massive molecular clumps on the way to collapsing and forming stars.

We see that the disk \( M_{\text{bh}}-\sigma \) relation is of exactly the same form as the “spherical” \( M_{\text{bh}}-\sigma \) relation (Equation (1)) derived by King (2003, 2005). However, we note that the coefficient in front of the latter is smaller than that in the former. The difference is difficult to quantify exactly, but would seem to be at least a factor of a few to 10. Physically, it is due to (1) the factor \( f_d = 0.16 \) in front of the spherical relation and \( f_d \) for the disk relation. It is unlikely that \( f_d \) is significantly smaller than unity during the early gas-rich epoch of galaxy formation. Furthermore, in principle a similar in meaning coefficient—taking into account that part of the spherical shell is already converted into stars—may be introduced in the spherical \( M_{\text{bh}}-\sigma \) relation, in which case the difference between the two relations would be exactly equal to \( f_d \). (2) When deriving the disk \( M_{\text{bh}}-\sigma \) relation, we took the most optimistic assumptions about the AGN feedback on the disk. As discussed below Equation (4), realistically, a part of the feedback can be shadowed by the inner disk (the innermost part of which is very hard to affect by the feedback, see below). Beaming away from the disk plane may decrease the momentum feedback on the disk further yet.

Another important point is that although our arguments were presented for a disk geometry, they are also applicable to a bar-mediated gas inflows, such as those discussed by Shlosman et al. (1990). These authors argued that “bars within bars” may effectively channel the gas into the central regions of galaxies to fuel AGNs. However, bars present even less of an effective solid angle than disks for AGN feedback to work on: barred gas inflows are squashed geometrically, not only to the galaxy mid-plane, as disk inflows, but also in the azimuthal direction within the galaxy plane. Therefore, the momentum striking a bar would be even less than that for the disk (Equation (4)), and thus we would expect even more massive SMBHs if they were limited by their feedback on the gas bars only.

The derivation in this section is applicable to regions well beyond the black hole’s influence radius, \( R_h = GM_{\text{bh}}/2c_s^2 \), which is typically of the order of a few to tens of pc. In the Appendix, we show that a gas disk at \( R \ll R_h \) is actually able to withstand the momentum feedback of the AGN. This is why the sketch in Figure 1 shows small-scale disks unaffected by the feedback.

4. WHY DO OBSERVED SMBHs NOT FOLLOW THE DISK \( M_{\text{bh}}-\sigma \) RELATION?

Kormendy & Gebhardt (2001) found that SMBHs do not correlate with circular velocities of galaxy disks. Hu (2008) showed that SMBHs in pseudobulge galaxies are underweight by a factor of about 10 with respect to their cousins in classical bulge systems. Graham (2008) showed that barred galaxies tend to have SMBHs that are a factor of ~3–10 smaller than elliptical galaxies at the same velocity dispersion. Kormendy et al. (2011) found that observed SMBHs do not correlate well with velocity dispersions of either galaxy disks or pseudobulges.

In the sample of Kormendy et al. (2011), only two SMBHs in pseudobulge systems appear to lie above the classical \( M_{\text{bh}}-\sigma \) relation for bulges, with the rest being somewhat below the relation, in a qualitative agreement with Hu (2008).

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\(^3\) This can be seen by observing that escape velocity from distance \( R \) to infinity for a circularly rotating disk is smaller by a factor of \( \sqrt{2} \) than that from a static point within the same potential.
If these observations are confirmed in the future, then this is clearly very strongly at odds with the theory developed above: because disks are difficult to affect by AGN feedback simply due to geometrical arguments (see also simulations by Nayakshin & Power 2010), an SMBH fed by a disk should be able to grow more massive by at least a factor of a few (compare Equation (9) to Equation (1)) than an SMBH embedded in a bulge. In fact, theoretically, one may expect disk SMBHs to be even more massive: as noted below Equation (4), this is the absolute minimum to which the SMBH can grow if fed through the disk because, in general, AGN feedback impacting a given disk radius can be diluted at smaller radii or be beamed away from the disk (neither of these effects is included in Equation (9)).

The simplest interpretation of this result is to accept that SF in large-scale disks is too efficient a process, and so significant gas flow into the subparsec vicinity of SMBH is suppressed, leading to a quenching of SMBH fueling (see the arguments by Goodman 2003; Sirko & Goodman 2003; Nayakshin et al. 2007). Therefore, we conclude that these disks do not extend to the SMBH sphere of influence (between 1 and 10 pc, depending on SMBH mass), as suggested in the right panel of Figure 1.

Thompson et al. (2005) proposed a starburst disk model which appears to overcome the difficulties noted by Goodman (2003). Specifically, they argue that the SF rate in the disk is limited by the action of energy and momentum feedback from massive stars and supernovae, allowing some fuel to trickle down all the way to the nucleus. However, one problem with the Thompson et al. (2005) argument applied to the inner galaxy is a mismatch of timescales. Gravitational collapse of the disk is expected to take place on the dynamical timescale (Gammie 2001),

$$t_{\text{dyn}} = \frac{R^{3/2}}{G^{1/2} M_{\text{bh}}^{1/2}} \sim 10^3 \frac{R_{\text{pc}}^{3/2} M_8^{-1/2}}{\text{years}},$$

where $M_8$ is the SMBH mass in units of $10^8 M_\odot$ and $R_{\text{pc}}$ is the radius in units of parsec. Thompson et al. (2005) show that the disk cooling time is much shorter than the dynamical time in their model. On the other hand, massive stars dominating the feedback release energy on a timescale of a few million years. Therefore, for radii where $t_{\text{dyn}}$ is much shorter than the lifetime of the massive stars, the feedback might be released too slowly, i.e., only when the disk would already have collapsed gravitationally.

In conclusion, we believe that the observations of Hu (2008), Graham (2008), and Kormendy et al. (2011) imply that SMBHs do not grow by large-scale (e.g., tens of pc to $\gtrsim$ kpc) disks.

5. CHAOTIC/BALLISTIC FEEDING OF SMBHs

Surveys show little evidence for correlation between the orientation of jets from AGNs and the large-scale structure of their host galaxies (Schmitt et al. 2002; Verdoes Kleijn & de Zeeuw 2005, for example). This shows that the inner $\lesssim$ pc scale gas disks feeding AGNs are also uncorrelated with the large-scale disks of host galaxies. In our own Galaxy, the observed young stellar disks in the Galactic center (Paumard et al. 2006) are inclined at very large angles to the Galactic plane. This indicates that the disk stars originated from a deposition of one or two gas clouds with angular momentum directions very different from that of the Galactic disk (Bonnell & Rice 2008; Hobbs & Nayakshin 2009).

An attractive physical picture for AGN feeding that explains these observations and the results of our paper is that SMBHs in general are fueled by stochastic deposition of gas clouds with randomly oriented angular momentum, as sketched in the right panel of Figure 1.

Numerical simulations by Hobbs et al. (2011) demonstrated that this situation may be realized in an initially coherently rotating gas sphere if strong turbulence (driven by supernova explosions due to SF in the bulge, for example) is present. Convergent turbulent gas flows lead to the formation of high-density regions traveling through the rest of the gas nearly ballistically. A small fraction of such regions will be on nearly radial orbits that impact the innermost parsecs of the SMBH directly without going through a large-scale disk. Nayakshin & King (2007) argued that such flows create small (pc-scale) warped disks with a fluctuating orientation. These disks are able to feed the SMBH at much higher rates than the flat disks (see Goodman 2003) without becoming self-gravitating, effectively due to a much faster transfer of gas through the inner parsecs.

We therefore suggest that the absence of an observed $M_{\text{bh}}-\sigma$ relation for pure disk galaxies or pseudobulges provides observational support for the chaotic/ballistic mode of SMBH feeding. In our interpretation of the data, the lack of classical bulges in these galaxies shuts down the most efficient channel by which SMBH grows—-the cold chaotic accretion—and therefore led to their SMBHs growing mainly by planar (disk or bar) accretion, which we argue is inefficient. Having said this, we note that it remains to be seen if $M_{\text{bh}}-\sigma$ relation for chaotic streams conforms to the relations obtained by King (2003).

6. DEFICIENCIES OF OUR WORK

We shall now remark on limitations of our work. First of all, we only considered the momentum feedback here, whereas more complete treatments show that the SMBH outflow switches from momentum driving to energy driving at larger radii (King 2005; Ciotti et al. 2010). However, this is not likely to affect our conclusions significantly because it is the momentum-driven part of the AGN feedback process that constitutes the bottle neck for removing the gas to infinity (King et al. 2011). Furthermore, numerical simulations of Zubovas & Nayakshin (2012) that include both the momentum and energy feedback forms confirm the points made here on the difficulty of expelling a massive dense gas disk for the particular case of the Milky Way.

In common with other analytical papers on the subject (e.g., Silk & Rees 1998; Fabian 1999; King 2003, 2005; McLaughlin et al. 2006), we have also used an isothermal potential. However, the discussion in the end of Section 2 shows that this has little effect on momentum-driven outflows studied here. Zubovas & King (2012) have extended this result to large-scale energy-driven outflows.

A more serious worry is that our theoretical model predicts $M_{\text{bh}} \propto \sigma^4$, whereas different authors now find that $\sigma$ ranges from somewhat below four to as high as six (see references in the Introduction). One possible explanation for this within our momentum-driven feedback model is that the steeper slope for samples that combine different types of galaxies actually results from a superposition of several $M_{\text{bh}} \propto \sigma^4$ relations for different galaxies vertically offset in mass (K. Zubovas & A. R. King 2012, in preparation). Future theoretical and observational work is needed to resolve this issue.

Finally, given our SMBH feeding focus, we do not attempt to relate the gas disks we consider here to exponential stellar disks observed in real galaxies. This would require an additional
multi-parameter model for transformation of gas into stars and is beyond the scope of our paper.

7. DISCUSSION

Previous analytical treatments of the \( M_{\text{bh}}-\sigma \) relation have modeled the impact of momentum-driven AGN feedback on a spherical gas distribution (e.g., King 2003, 2005). We have shown analytically that if SMBH is fed by a large-scale gas disk, then there exists a “disk \( M_{\text{bh}}-\sigma \) relation.” The latter has the same shape as the “spherical” \( M_{\text{bh}}-\sigma \) relation (King 2003, 2005) but is offset to higher masses by a factor of a few to 10, realistically.

We emphasize that the precise value of the offset is model dependent, but the general trend is undeniable even for AGN feedback models not considered here, such as energy-driven models (e.g., Silk & Rees 1998). This follows simply from the geometrical argument first emphasized by Nayakshin & Power (2010): a dense flat disk, and even more so a bar, presents a much smaller target for AGN feedback to act upon than a quasi-spherical gas distribution, and therefore much more massive SMBHs could be built if their masses were limited by a feedback argument. We may further allude here to the analogy from the field of massive SF, where radiation pressure was once thought to limit stellar growth; instead, recent simulations by Krumholz et al. (2009) show that the accretion through a disk (which we note is stable to self-gravitational instabilities on the scales of the simulations) is very efficient as radiation is channeled away from the disk. If a similar situation is applied in galaxy formation, then SMBHs would grow as long as there is fuel in the massive galaxy-wide disks or bars. One could then expect SMBH becoming comparable in mass to galaxy gas disks themselves, which is clearly observationally not the case.

Reconciliation of this theoretical result with observations leads to interesting and significant conclusions about the dominant mode by which SMBHs grow in galaxies. Hu (2008) showed that pseudobulge galaxies tend to have undermassive SMBHs, and Graham (2008) showed that barred galaxies tend to have SMBHs that are a factor of \( \sim 3-10 \) smaller than elliptical galaxies at the same velocity dispersion. While Kormendy et al. (2011) argue, in variance with the above authors, that SMBHs do not even correlate with velocity dispersions of galaxy disks and pseudobulges, we note that their results also show that SMBHs in pseudobulge systems are typically undermassive with respect to their classical bulge cousins (e.g., see Figure 2 in Kormendy et al. 2011).

We suggest that these observations hint that SMBHs in most galaxies (1) do not grow by accretion of gas through \( \sim \)kpc gas disks, most likely because gas is turned into stars too efficiently, and (2) grow via a mode directly linked to existence of a classical quasi-spherical bulge. In this case, it is natural that the SMBHs in galaxies lacking the classical bulge (pseudobulge galaxies) are undermassive, and it is also natural that SMBHs do not correlate with galaxy disks. For the SMBH accretion modes physically linked to classical bulges, we suggest the stochastic (King & Pringle 2007) or ballistic (Hobbs et al. 2011) accretion modes in which the bulge is the source of low and fluctuating-in-direction angular momentum material.

An additional indirect evidence supporting our conclusions comes from recent semianalytical models of galaxy formation by Shankar et al. (2012). These authors show that semianalytical models which turn off SMBH feeding during pseudobulge growth episodes driven by secular instabilities (bars) provide a much better fit to the data than models that allow a concurrent growth of SMBH and pseudobulges (see Figure 1 in Shankar et al. 2012). We here thus provide a physical interpretation of this empirical result: the “fragmentation catastrophe” of the outer cold and massive regions of AGN disks makes them too susceptible to SF, so that not enough gas trickles down to the SMBH (Goodman 2003; Nayakshin et al. 2007).

To avoid confusion, we emphasize that we do not argue that small (e.g., subparsec) scale accretion disks are unimportant for SMBH growth. In the chaotic accretion picture, there will always be a non-vanishing amount of angular momentum associated with the accretion flow and this results in the formation of “small”, by galactic standards, disks \( R < 0.01-0.1 \) pc, depending on the SMBH mass and the accretion rate, see Goodman (2003) that are resilient to AGN feedback (as we show in the Appendix). Rather, our key point is that the origin of material that feeds these small disks is distinct from the larger-scale (with respect to the SMBH’s sphere of influence) disk or a galactic bar, and the orientation of these smaller-scale AGN disks is likely to be chaotic because of the manner in which the quasi-spherical inflows that feed these disks fluctuate in time.

We also note that large-scale galaxy-wide gas disks and gas bars are obviously important for galaxy formation as a whole, likely accounting for a significant fraction of all stars formed and also for a radial redistribution of gas and stars within the galaxy, but we see no clear evidence that these features are able to channel their gas all the way to the SMBHs.

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APPENDIX

FEEDBACK RADIUS

We have argued above that AGN feedback can affect a large-scale gas disk, expelling the gas outward just as it can for a spherical distribution of gas. We now show that there exists a critical radius within which this argument no longer holds: the disk there is resilient to AGN feedback.

Inside the SMBH’s sphere of influence, marked by the radius \( R_b = G M_{\text{bh}}/\sigma^2 \), the gravitational potential is dominated by the SMBH and so we can replace \( M(R) \) with \( M_{\text{bh}} \). For an SMBH on the \( M_{\text{bh}}-\sigma \) relation, the SMBH’s radius of influence is given by

\[ R_b \approx 10 \text{ pc} \ M_{\text{bh}}^{\sigma^2} 200. \]  

As the result, the critical momentum flux striking the edge of the disk must be

\[ \frac{H}{R_c} \frac{L_{\text{edd}}}{c} \geq \frac{G M_{\text{bh}}}{R^2} M_{\text{disk}}. \]  

Using \( M_{\text{disk}} \approx M_{\text{bh}}(H/R) \) for a marginally self-gravitating disk again, we arrive at the requirement that

\[ \kappa \Sigma_{\text{BH}} = \kappa \frac{M_{\text{bh}}}{\pi R^2} \lesssim 4. \]  

This can be satisfied at radii greater than

\[ R \approx \left( \kappa M_{\text{bh}}/4\pi \right)^{1/2} \approx 25 \text{ pc} \ M_{\text{bh}}^{1/2}. \]  

where \( M_{\text{bh}} \) is the SMBH mass in units of \( 10^8 M_{\odot} \). Recalling that we assumed that \( R \leq R_b \), we conclude that inside the “feedback
radius$^*$ $R_{fb}$ given by

$$R_{fb} = \min \left[ R_h, \left( \kappa M_{bh}/4\pi \right)^{1/2} \right], \quad (A5)$$

momentum feedback by the SMBH is unable to affect a self-gravitating disk.

Physically, the significance of this feedback radius can be appreciated by considering a major gas mass deposition event—say, the infall of a very massive giant molecular cloud or a merger with a gas-rich galaxy that dumps a significant amount of gas into the inner galaxy. This gas will settle into a disk because it is very likely to have a non-zero net specific angular momentum. At $R \ll R_{fb}$, the disk cannot be expelled by the AGN, whereas at $R \ll R_{fb}$ it may as shown in the main body of the paper.

Somewhat counterintuitively, gas closest to the SMBH is actually hardest to expel to infinity (although this becomes clear if one considers that SMBH’s gravity is enormous in the inner ∼pc). Deposition of a small cloud at $R \ll R_{fb}$ may be much more promising for AGN feeding than having a vast amount of gas at $R \gg R_{fb}$, as the latter is more easily affected by AGN feedback and SF.

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