Logarithmic correction to the
Cardy-Verlinde formula in Topological
Reissner-Nordström de Sitter Space

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Abstract

In this paper we compute leading order correction due to small statistical fluctuations around equilibrium, to the Cardy-Verlinde entropy formula (which is supposed to be an entropy formula of conformal field theory in any dimension) of a Topological Reissner-Nordstrom black hole in de Sitter space.

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1 Introduction

It is commonly believed that any valid theory of quantum gravity must necessarily incorporate the Bekenstein-Hawking definition of black hole entropy [1, 2] into its conceptual framework. However, the microscopic origin of this entropy remains an enigma for two reasons. First of all although the various counting methods have pointed to the expected semi-classical result, there is still a lack of recognition as to what degrees of freedom are truly being counted. This ambiguity can be attributed to most of these methods being based on dualities with simpler theories, thus obscuring the physical interpretation from the perspective of the black hole in question. Secondly, the vast and varied number of successful counting techniques only serve to cloud up an already fuzzy picture.

de Sitter/Conformal Field Theory correspondence (dS/CFT) may hold the key to its microscopical interpretation. Naively, we would expect dS/CFT correspondence to proceed along the lines of Anti-de Sitter /Conformal Field Theory (AdS/CFT) correspondence because de Sitter spacetime can be obtained from anti-de Sitter spacetime by analytically continuing the cosmological constant to imaginary values. However, local and global properties of dS spacetime lead to unexpected obstructions. Unlike AdS, the boundary of dS spacetime is spacelike and its dual CFT is Euclidean. Moreover, dS spacetime does not admit a global timelike Killing vector. The time dependence of the spacetime metric precludes a consistent definition of energy and the use of Cardy formula to compute dS entropy. Finally dS/CFT duality leads to boundary operators with complex conformal weights, i.e. to a non-unitary CFT. In spite of these difficulties, some progress towards a consistent definition of dS/CFT correspondence has been achieved [3]-[33]

There has been much recent interest in calculating the quantum corrections to $S_{BH}$ (the Bekenestein-Hawking entropy) [34]-[70]. The leading-order correction is proportional to $\ln S_{BH}$. There are, two distinct and separable sources for this logarithmic correction [62, 66] (see also recent paper by Gour and Medved [70]). Firstly, there should be a correction to the number of microstates that is a quantum correction to the microcanonical entropy, secondly, as any black hole will typically exchange heat or matter with its surrounding, there should also be a correction due to thermal fluctuations in the horizon area. In this paper we consider Topological Reissner-Nordstrom black hole in de Sitter space in arbitrary dimension. In section 2 we calculate the corresponding thermodynamical quantities for both cosmological and black hole horizon. In section 3 at first we review the calculation of the logarithmic correction due to the thermal fluctuation to the entropy, then we obtain the corresponding correction to the Cardy-Verlind formula. Last section contain a summary of paper.

2 Thermodynamical quantities of TRNdS black hole

The topological Reissner-Nordström dS black hole solution in $(n + 2)$-dimensions has the following form

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\gamma_{ij}dx^i dx^j,$$

$$f(r) = k - \frac{\omega_n M}{r^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)r^{2n-2}} - \frac{r^2}{l^2},$$

(1)
where
\[ \omega_n = \frac{16\pi G}{n \text{Vol}(\Sigma)}, \quad \phi = -\frac{n}{4(n-1)} \frac{\omega_n Q}{r^{n-1}}, \] (2)

where \( Q \) is the electric/magnetic charge of Maxwell field, \( M \) is assumed to be a positive constant, \( l \) is the curvature radius of de Sitter space, \( \gamma_{ij} dx^i dx^j \) denotes the line element of an \( n \)-dimensional hypersurface \( \Sigma_k \) with the constant curvature \( n(n-1)k \) and its volume \( V(\Sigma_k) \). \( \Sigma_k \) is given by spherical \( (k = 1) \), flat \( (k = 0) \), hyperbolic \( (k = -1) \), \( \phi \) is the electrostatic potential related to the charge \( Q \). When \( k = 1 \), the metric Eq.(1) is just the Reissner-Nordström-de Sitter solution. For general \( M \) and \( Q \), the equation \( f(r) = 0 \) may have four real roots. Three of them are real, the largest one is the cosmological horizon \( r_c \), the smallest is the inner (Cauchy) horizon of black hole, the middle one is the outer horizon \( r_+ \) of the black hole. And the fourth is negative and has no physical meaning. The case \( M = Q = 0 \) reduces to the de Sitter space with a cosmological horizon \( r_c \).

When \( k = 0 \) or \( k = -1 \), there is only one positive real root of \( f(r) \), and this locates the position of cosmological horizon \( r_c \).

In the case of \( k = 0 \), \( \gamma_{ij} dx^i dx^j \) is an \( n \)-dimensional Ricci flat hypersurface, when \( M = Q = 0 \) the solution Eq.(1) goes to pure de Sitter space
\[ ds^2 = \frac{r^2}{l^2} dt^2 - \frac{l^2}{r^2} dr^2 + r^2 dx_n^2, \] (3)
in which \( r \) becomes a timelike coordinate.

When \( Q = 0 \), and \( M \rightarrow -M \) the metric Eq.(1) is the TdS (Topological de Sitter) solution [30], which have a cosmological horizon and a naked singularity.

In the BBM prescription [22], the gravitational mass, subtracted the anomalous Casimir energy, of the TRNdS solution is
\[ E_c = -M = -\frac{r_c^{n-1}}{\omega_n} \left( k - \frac{r_c^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_c^{n-2}} \right). \] (4)

The Hawking temperature \( T_{cTRNdS} \) and entropy \( S_{cTRNdS} \) associated with the cosmological horizon are
\[ T_{cTRNdS} = \frac{-f'(r_c)}{4\pi} = \frac{1}{4\pi r_c} \left( -(n-1)k + (n+1) \frac{r_c^2}{l^2} + \frac{n\omega_n^2 Q^2}{8r_c^{2n-2}} \right), \]
\[ S_{cTRNdS} = \frac{r_c^n \text{Vol}(\Sigma)}{4G}, \] (5)
where \( V_c = r_c^n \text{Vol}(\Sigma) \) is area of the cosmological horizon. The AD mass of TRNdS solution can be expressed in terms of black hole horizon radius \( r_+ \) and charge \( Q \),
\[ E_b = M = \frac{r_+^{n-1}}{\omega_n} \left( 1 - \frac{r_+^2}{l^2} + \frac{n\omega_n^2 Q^2}{8r_+^{2n-2}} \right). \] (6)

The black hole horizon Hawking temperature \( T_{bTRNdS} \) and entropy \( S_{bTRNdS} \) are given by
\[ T_{bTRNdS} = \frac{-f'(r_+)}{4\pi} = \frac{1}{4\pi r_+} \left( (n-1) - (n+1) \frac{r_+^2}{l^2} - \frac{n\omega_n^2 Q^2}{8r_+^{2n-2}} \right), \]
\[ S_{bTRNdS} = \frac{r_+^n \text{Vol}(\Sigma)}{4G}, \] (7)
where \( r = r_+ \) is black hole horizon and \( V_+ = r_+^n \text{Vol}(\Sigma) \) is area of it in \((n+2)\)–dimensional asymptotically dS space. Therefore, there are two kinds of temperatures corresponding to two horizon, the system is not thermodynamically stable. However, the system should be adiabatic since one can define the temperature in the vicinity of each horizon.

Making use of the fact that the metric for the boundary CFT can be determined only up to a conformal factor, we rescale the boundary metric for the CFT to be of the following form

\[
d s_{\text{CFT}}^2 = \lim_{r \to \infty} \left[ \frac{l^2}{r^2} d s_{n+2}^2 \right] = -dt^2 + l^2 \gamma_{ij} dx^i dx^j. \tag{8}
\]

Then the thermodynamic relations between the boundary CFT and the bulk TRNdS are given by

\[
E_{\text{CFT}} = M \frac{l}{r}, \quad \Phi_{\text{CFT}} = \Phi \frac{l}{r},
\]

\[
T_{\text{CFT}} = T_{\text{TRNdS}} \frac{l}{r}, \quad S_{\text{CFT}} = S_{\text{TRNdS}}, \tag{9}
\]

The specific heat of the black hole is given by

\[
C_{c,b} = \frac{d E_{c,b}}{dT} = \frac{4 \pi r_+^2 (8k(1-n)l^2 r_{c,+}^{n-2} + 8(n+1) r_{c,+}^n + n \omega_n^2 l^2 r_{c,+}^{-n} Q^2)}{\omega_n (8l^2 (n-1) k + 88 r_{c,+}^2 (n+1) + (1-2n)l^2 \omega_n^2 r_{c,+}^{2n} Q^2)}. \tag{10}
\]

As one can see the above specific heat is positive in the case \( k = -1, k = 0 \), for \( k = 1 \), \( C_{c,b} \) is positive only with following condition

\[
8(n+1)r_{c,+}^n + n \omega_n^2 l^2 r_{c,+}^{-n} Q^2 > 8k(1-n)l^2 r_{c,+}^{n-2} \tag{11}
\]

3 Logarithmic correction to the entropy and Cardy-Verlinde formula

At first we review the calculation of the logarithmic correction to the entropy (see the paper by Das et al. [37]), the partition function in the canonical ensemble is given by

\[
Z(\beta) = \int_0^\infty \rho(E) e^{-\beta E} dE, \tag{12}
\]

where \( T = 1/\beta \) is the temperature in units of the Boltzmann constant \( k_B \). The density of states can be obtained from (12) by doing an inverse Laplace transform (keeping \( E \) fixed)

\[
\rho(E) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} Z(\beta) e^{\beta E} d\beta = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{S(\beta)} d\beta, \tag{13}
\]

where

\[
S(\beta) = \ln Z(\beta) + \beta E \tag{14}
\]

is the exact entropy as a function of temperature, not just its value at equilibrium. The complex integral can be performed by the method of steepest descent around the saddle point \( \beta_0(= 1/T_0) \), such that \( S_0' := (\partial S(\beta)/\partial \beta)_{\beta = \beta_0} = 0 \). \( T_0 \) is the equilibrium temperature,
such that the usual equilibrium relation $E = -(\partial \ln Z(\beta)/\partial \beta)_{\beta=\beta_0}$ is obeyed. Expanding $S(\beta)$ around $\beta = \beta_0$, we get

$$S = S_0 + \frac{1}{2}(\beta - \beta_0)^2 S''_0 + \cdots ,$$

(15)

Substituting (15) in (13):

$$\rho(E) = \frac{e^{S_0}}{\sqrt{2\pi S''_0}} .$$

(16)

Note that the density of states $\rho(E)$ and $S''_0$ have dimensions of inverse energy and energy squared respectively. Henceforth, we set the Boltzmann constant $k_B$ to unity. The logarithm of the density of states $\rho(E)$ is then the microcanonical entropy

$$S = \ln \rho(E) = S_0 - \frac{1}{2} \ln S''_0 + \text{(higher order terms)} .$$

(17)

Now, we will estimate $S''_0$, without assuming any specific form of $S(\beta)$. From Eq.(14), it follows that

$$S''(\beta) = \frac{1}{Z} \left( \frac{\partial^2 Z(\beta)}{\partial \beta^2} - \frac{1}{Z^2} \left( \frac{\partial Z}{\partial \beta} \right)^2 \right) .$$

(18)

This means that $S''_0$ is nothing but the fluctuation squared of energy from the equilibrium, i.e.,

$$S''_0 = \langle E^2 \rangle - \langle E \rangle^2 ,$$

(19)

where, by the definition of $\beta_0$, $E = \langle E \rangle = -(\partial \ln Z/\partial \beta)_{\beta=\beta_0}$. It immediately follows that

$$S''_0 = T^2 C$$

(20)

where $C \equiv (\partial E/\partial T)_{T_0}$ is the dimensionless specific heat. Substituting for $S''_0$ from (20) in (17), we get:

$$S = \ln \rho = S_0 - \frac{1}{2} \ln \left( C T^2 \right) + \cdots$$

(21)

This equation can only be directly applied if the specific heat is non negative. Therefore the entropy always has the logarithmic correction due to thermal fluctuation. To get entropy as a logarithm of dimensionless quantity, we can multiply the density of state with $E$ which has the dimension of energy [59], then the correction to the entropy becomes

$$S = S_0 + \ln \frac{E}{\sqrt{CT^2}} + \cdots$$

(22)

we can set the scale $E$ to be the temperature $T$ of the system, this is because we have temperature as the only available scale in canonical ensemble. Therefore we have

$$S = S_0 - \frac{1}{2} \ln C + \cdots$$

(23)

When $r_{c,+}^2 \gg l^2, C \simeq nS_0$, in this case we have

$$S = S_0 - \frac{1}{2} \ln S_0 + \cdots$$

(24)
It is now possible to drive the corresponding correction to Cardy-Verlinde formula. The Casimir energy $E_c$, defined as

$$E_{c,b} = (n+1)E^{c,b} - nT^{c,b}S^{c,b} - n\phi^{c,b}Q,$$  

(25)

in this case, is found to be

$$E_{c,b} = \frac{-2nkT^{c,b}Vol(\Sigma)}{16\pi G},$$  

(26)

which is valid for both cosmological and black hole horizon. One can see that the entropy Eqs. (5,7) of the cosmological and black hole horizon can be written as

$$S_{c,b} = \frac{2\pi l}{n} \sqrt{\frac{E_{c,b}}{k}}((2E_{c,b} - E_{q,b}) - E_{c,b}),$$  

(27)

where

$$E_{q,b} = \frac{1}{2}\phi^{c,b}Q = -\frac{n}{8(n-1)}\frac{\omega_nQ^2}{r^{n-1}}.$$  

(28)

For the present discussion, the total entropy is assumed to be of the form Eq.(24), where the uncorrected entropy, $S_0$ correspond to that associated in Eqs. (5,7). It then follows by employing Eqs.(4,5,6,7) that the Casimir energy Eq.(25) can be expressed in term of the uncorrected entropy. (Following expressions are valid for both cosmological and black hole horizon, then for simplicity we omit the subscript $c$ and $b$)

$$E_C = \frac{-2nr^{n-1}Vol(\Sigma)}{16\pi G} + \frac{nT}{2}LnS_0,$$  

(29)

Then we obtain

$$\frac{2\pi l}{n} \sqrt{\frac{E_C}{k}}((2E - E_q) - E_C) \simeq S_0 - \frac{r^{2}_{c,+}}{2r^{2}_{c,+} + 1}\frac{\pi l^{2}T}{LnS_0}$$  

(30)

In the limit where the correction is small, the coefficient of the logarithmic term on the right-hand side of Eq.(30) can be expressed in terms of the energy and Casimir energies

$$\frac{E - E_q - E_c/2}{E_c/2} = -\frac{r^{2}_{c,+}}{kl^2}.$$  

(31)

Using the following equation

$$\frac{(r^{2}_{c,+} + 1)}{2r^{2}_{c,+}}\frac{\pi l^{2}}{4\pi r^{n}_{c,+}}\frac{(n+1)E - E_c - 2nE_q)\omega_n}{4\pi r^{n}_{c,+}} = (1 - \frac{E_c/2}{E - E_q - E_c/2})\frac{2nE_q + E_c - (n+1)E}{4E_c},$$  

(32)

we may conclude, therefore that in the limit where the logarithmic corrections are sub-dominant, Eq.(30) can be rewritten to express the entropy in terms of the energy and Casimir energy.

$$S_0 = \frac{2\pi l}{n} \sqrt{\frac{E_C}{k}}((2E - E_q) - E_c)$$  

(33)

\[+ \frac{2nE_q + E_c - (n+1)E}{4E_c} \left( \frac{E - E_q - E_c}{E - E_q - E_c/2} \right) \ln \left( \frac{2\pi l}{n} \sqrt{\frac{E_C}{k}}((2E - E_q) - E_c) \right) \]
and consequently, the total entropy Eq.(24) to first order in the logarithmic term, is given by

\[
S \simeq \frac{2\pi l}{n} \sqrt{\frac{E_c}{k}} |(2(E - E_q) - E_c) + \frac{E_q[(3n + 1)E - 2nE_q + (1 - 2n)E_c] + E[nE_c - (n + 1)E_c]}{4E_c(E - E_q - E_c/2)}
\]

\[
Ln \left( \frac{2\pi l}{n} \sqrt{\frac{E_c}{k}} |(2(E - E_q) - E_c) \right)
\]  

(34)

Therefore taking into account thermal fluctuations defines the logarithmic corrections to both cosmological and black hole entropies. As a result the Cardy-Verlinde formula receive logarithmic corrections in our interest TRNdS black hole background in any dimension, in the way similar to the Cardy-Verlinde formula for the SAdS and SdS black holes in 5-dimension [69, 71]

4 Conclusion

For a large class of black hole, the Bekenstein-Hawking entropy formula receives additive logarithmic corrections due to thermal fluctuations. On the basis of general thermodynamic arguments, Das et al [37] deduced that the black hole entropy can be expressed as the Eq.(21). In this paper we have analyzed this correction of the entropy of TRNdS black hole in any dimension in the light of dS/CFT. We have obtain the logarithmic correction to both cosmological and black hole entropies. Then using the form of the logarithmic correction Eq.(24) we have derived the corresponding correction to the Cardy-Verlinde formula which relates the entropy of a certain CFT to its total energy and Casimir energy in arbitrary dimension.

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