Influence of non-linearity of medium on the laser induced filamentation instability in magnetized plasma

Sepideh Dashtestani, Akbar Parvazian

Department of Physics, Isfahan University of Technology, Isfahan 84156-83111, Iran

Hamidreza Mohammadi

Department of Physics, University of Isfahan, Isfahan, Iran and
Quantum Optics Group, University of Isfahan, Isfahan, Iran

(Dated: August 11, 2018)

Abstract

The effects of the non-linearity of the medium on the growth rate of filamentation instability in a magnetized plasma interacting with an intense laser pulse, is investigated. The non-linearity of the medium, modeled by Kerr non-linearity, is an important factor, which determines the rate of instability growth. Sensitivity of the rate of filamentation growth, to the Kerr non-linear coefficient could be adjusted by the external magnetic field and laser intensity.
I. INTRODUCTION

Laser-induced instabilities in plasma is a vast studied subject in the field of laser-plasma interaction. Among these instabilities, filamentation, a large-scaled phenomenon propagation along the direction of the laser light, is one of the most important. First time the filamentation and spectral (red) broadening phenomena were reported by Shimizu et. al\textsuperscript{[1]}. Authors of Ref. \textsuperscript{[2]} have been showed that a powerful femtosecond laser pulse could uncover a sub-field of applied physics, namely, filamentation non-linear optics. Controlling this phenomenon (filamentation) can be used to improve the performance of the laser lightning protection devices \textsuperscript{[3]}, self channeling and selfguiding laser system \textsuperscript{[4, 5]} and etc. The filamentation in various media such as air, water and gases have been heavily studied in recent years \textsuperscript{[6, 7]}. The filamentation phenomenon originates from competition between self-focusing and plasma defocusing of an intense laser propagating in Kerr medium \textsuperscript{[8]}. In this paper we investigate formation of the filamentation induced by laser in magnetized plasma, by relating the instability growth rate of the filamentation to the controllable parameters of the medium, the laser and also the external magnetic field. In a laser plasma interaction the presence of the magnetic field was seen to impress the filamentation very remarkably \textsuperscript{[9, 10]}. The magnetic field can influence the transverse size and growth rate of filaments. The output results of 30 years investigation in this field reveal that the laser induced multiple filamentation initialization stems to random noise existing in the input beam profile \textsuperscript{[11]}. Despite this explanation, some works states that multiple filamentation was created due to vectorial effects \textsuperscript{[12-14]}. Numerical simulations show that when the input beam is sufficiently powerful, vectorial effects lead to multiple filamentation. According to the results of this works, non-linear response of the medium to the laser pulse, appears to be a strong determinant of multiple filamentation. The effects of the non-linearity of the magnetized plasma on the formation and the growth of filamentation are not considered yet. In this paper we study the growth rate of the laser induced filamentation in a magnetized plasma and bold the effects of non-linearity of the medium on the filamentation formation. In this way we compare the growth rate of the filamentation in linear and non-linear mediums. The system under consideration is a plasma which is deriving by an external magnetic field, directed in $+z$-direction and interacted by an intense pulsed laser light propagates in the same direction. The laser field assumed to has right-circular polarization (RCP).
The results show that beside the noise effects the non-linear response of the medium has a significance effects on the construction and growth rate of the filamentation. We assume that plasma medium is influenced by a magnetic field $\vec{B} = (0, 0, B)$ and parametrized by: $\omega_p = (\frac{4\pi n_0 e^2}{m})^{\frac{1}{2}}$ and $\omega_c = \frac{eB}{mc}$ as plasma frequency and cyclotron frequency of plasma with electron density $n_0$, respectively. Here, -e, m are the charge and mass of the electron, respectively. We consider a circularly polarized laser beam propagated in the direction of external magnetic field: $\vec{E}_0 = (\hat{x} + i\hat{y})A_{10}e^{-i(kz-\omega t)}$ where, $k = \frac{\omega}{c}\left[1 - \frac{\omega_p^2}{\omega^2(1 - \omega_c^2)}\right]^{\frac{1}{2}}$. Where $A_{01}$ is a real constant proportional to square root of laser intensity. We amuse that the laser intensity has an instability modeled by considering a circularly polarized ripple in the electric field amplitude, $\vec{E} = \vec{E}_0 + \vec{E}_1$. Where, $\vec{E}_1 = (\hat{x} + i\hat{y})A_1(x, z)e^{-i(\omega t - kz)}$ and $A_{10}$ is a complex number. The laser pulse imparts the electron to oscillate with velocity:

$$\vec{v} = \frac{e\vec{E}}{mi(\omega - \omega_c)}. \quad (1)$$

The pondermotive force is [15]:

$$\vec{f}_p = -e\frac{\vec{v} \times \vec{B}}{c} - m(\vec{v} \cdot \vec{\nabla})\vec{v}. \quad (2)$$

The time independent part of $\vec{f}_p$ can be written as

$$\vec{f}_p = \frac{-e^2}{4m(\omega - \omega_c)^2} \vec{\nabla}(\vec{E}\vec{E}^*)^*, \quad (3)$$

where $^*$ represents complex conjugate. Using Eq. (1) for velocity $\vec{v}$ and $\vec{B} = \vec{\nabla} \times \vec{E}$, the pondermotive force is given by

$$\vec{f}_p = \frac{-e^2}{4m(\omega - \omega_c)^2} \vec{\nabla}A_{10}(A_1 + A_1^*)^*, \quad (4)$$

and the corresponding pondermotive potential is

$$\phi_p = \frac{-e}{4m(\omega - \omega_c)^2} A_{10}(A_1 + A_1^*). \quad (5)$$

The electrons was expelled away from the regions of higher electric field by this pondermotive potential while ions remain stationary due to their large inertia [15]. An electrostatic field $\vec{E} = -\vec{\nabla}\phi$, which is induced by space charge, pulls back the electrons causes plasma oscillation. Under the quasisteady state condition and cold plasma approximation we can following [15] and write $\phi = -\phi_p$. The modified electron density could be calculated from the Poisson’s equation, $\nabla^2 \phi = 4\pi e(n_e - n_0)$, is:

$$n_e = n_0 - \frac{1}{4\pi e} \nabla^2 \phi_p. \quad (6)$$
In the magnetized plasma the effective dielectric constant for the right circularly polarized extraordinary mode by use the Lorentz-Durde model can be written as [16]:

$$\varepsilon_+ = 1 - \frac{n_e \omega_p^2}{n_0 \omega^2 (1 - \frac{\omega_p}{\omega})}. \quad (7)$$

Substituting for $n_e$ from Eq. (5) in Eq. (6), now we have

$$\varepsilon_+ = \varepsilon_{0+} + \psi_+(A_{10}(A_1 + A_1^*)) \quad (8)$$

Now we can consider linear and non-linear part of the dielectric constant as follows [15]:

$$\varepsilon_{0+} = 1 - \frac{\omega_p^2}{\omega^2 (1 - \frac{\omega_p}{\omega})},$$

$$\psi_+(A_{10}(A_1 + A_1^*)) = -\frac{\epsilon^2 \vec{\nabla}^2 A_{10}(A_1 + A_1^*)}{4m^2\omega^5(1 - \frac{\omega_p}{\omega})^3}.$$

Where $\psi_+(A_{10}(A_1 + A_1^*))$ is the non-linearity induced by electric field ripple of the dielectric constant and $\varepsilon_{0+}$ is the linear response of the medium. The response of the medium to the laser light is encapsulated in dielectric constant $\varepsilon$ or electric susceptibility $\chi$. The electric constant can be calculated by the concept of pondermotive force and Maxwell equations. propagation of an electromagnetic wave in a dielectric medium is governed by the vector wave equation extracted from Maxwell equations. The vector wave equation for $\vec{E}$ in a homogeneous medium can be expressed as [17, 18]:

$$\nabla^2 \vec{E} - \vec{\nabla} (\vec{\nabla} \cdot \vec{E}) = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{4\pi}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2} + \frac{4\pi}{c^2} \frac{\partial \vec{J}}{\partial t}, \quad (9)$$

where $\vec{P} = \vec{P}^{(1)} + \vec{P}_{NL}$ and $\vec{J} = -n_e e \vec{v}$.

$$\nabla^2 \vec{E} - \vec{\nabla} (\vec{\nabla} \cdot \vec{E}) = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{4\pi}{c^2} \frac{\partial^2 \vec{P}^{(1)}}{\partial t^2} + \frac{4\pi}{c^2} \frac{\partial \vec{J}}{\partial t} + \frac{4\pi}{c^2} \frac{\partial^2 \vec{P}_{NL}}{\partial t^2}. \quad (10)$$

In the following, in section 2, the filamentation formation rate in a linear medium ($\vec{P}_{NL} = 0$) in the presence of laser noise is calculated. Then, the effects of non-linearity of the medium are illustrated in section 3. Ultimately, the relation between the growth rate of the filamentation and the system and environment parameters is presented in section 4. Finally, in section 5, a discussion concludes the paper.
II. LINEAR RESPONSE OF THE MEDIUM

In the low-intensity regime the response of the medium (magnetized plasma) is linear, i.e. the refractive index is intensity-independent. Thus for linear medium we have $P_{NL} = 0$ and we can write $[19]$: \[ p^{(1)} = \frac{1}{4\pi} \chi^{(1)} E, \] where the constant of proportionality $\chi^{(1)}$, is known as the linear susceptibility, $\varepsilon_0$ is the permittivity of Vacuum and $n_0$ is the linear refractive index of medium. By considering $1 + \chi^{(1)} = \varepsilon$ from Eq. $[9]$ we have:

\[ \nabla^2 \vec{E} - \vec{\nabla} (\vec{\nabla} \cdot \vec{E}) + \frac{\omega^2}{c^2} \varepsilon \vec{E} - \frac{4\pi}{c^2} \frac{\partial J}{\partial t} = 0. \] (12)

By using $\frac{\partial E_x}{\partial z} = -(\frac{1}{\varepsilon_{zz}})[(\frac{\partial}{\partial x})(\varepsilon_{xx} E_x + \varepsilon_{xy} E_y) + (\frac{\partial}{\partial y}) \times (-\varepsilon_{xy} E_x + \varepsilon_{xx} E_y)]$ in $[15]$, we obtain the following differential equation for the field amplitude, $A_+ = (A_{10} + A_1) e^{i(\omega t - qz)}$:

\[ \frac{\partial^2 A_+}{\partial z^2} + \frac{1}{2} (1 + \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) \nabla^2 A_+ + \frac{\omega^2}{c^2} (\psi_+ A_+) + \frac{\omega^2 \omega_p^2}{c^2 \omega^2 (1 - \frac{\omega_p}{\omega})} A_+ = 0. \] (13)

Here, subscript $\perp$ stands for traverse to the $z$-direction. Finally, the wave equation for perturbed field given by:

\[ 2ik \frac{\partial A_1}{\partial z} + \frac{1}{2} (1 + \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) \nabla^2 A_1 + \frac{\omega^2}{c^2} (\psi_+ A_{10}) + \frac{\omega^2 \omega_p^2}{c^2 \omega^2 (1 - \frac{\omega_p}{\omega})} A_1 = 0. \] (14)

This is the famous Non-Linear Schrödinger Equation (NLSE). For the purpose of calculating the growth rate of the filament, we separate the real and imaginary parts of Eq. (11). By defining $A_1 = A_{1r} + A_{1i}$, we have:

\[-2k \frac{\partial A_{1i}}{\partial z} + \frac{1}{2} (1 + \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) \nabla^2 A_{1r} - \frac{4m^2 \omega^2}{c^2 (1 - \frac{\omega_p}{\omega})^3} + \frac{\omega^2 \omega_p^2}{c^2 \omega^2 (1 - \frac{\omega_p}{\omega})} A_{1r} = 0, \]

\[ 2k \frac{\partial A_{1r}}{\partial z} + \frac{1}{2} (1 + \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) \nabla^2 A_{1i} + \frac{\omega^2 \omega_p^2}{c^2 \omega^2 (1 - \frac{\omega_p}{\omega})} A_{1i} = 0, \]

where $A_{1i}, A_{1r} \approx exp[i(q||z + q_\perp x)]$. By replacing $\nabla^2 \rightarrow -q^2_\perp$ and $\frac{\partial}{\partial z} \rightarrow iq||$ in the above equations the following coupled equations for $A_{1r}$, and $A_{1i}$ has been achieved:

\[-2ikq|| A_{1i} - \frac{1}{2} (1 + \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) q^2_\perp A_{1r} + \frac{\alpha^2 A_{1r}(q^2_\perp + q^2_\parallel)}{2(1 - \omega_c/\omega)} + \frac{\omega^2 \omega_p^2}{c^2 \omega^2 (1 - \frac{\omega_p}{\omega})} A_{1r} = 0, \]
2ikq_{\parallel}A_{1r} - \frac{1}{2}(1 + \varepsilon_{0+}/\varepsilon_{zz})q_{\perp}^2 A_{1i} + \frac{\omega^2 \omega_p^2}{c^2 \omega^2 (1 - \omega_c/\omega)} d A_{1i} = 0.

Where \( a_0 = \frac{\varepsilon A_{10}}{m c^2} \) is the normalized laser field amplitude, \( a^2 = \frac{a_0^2}{(1 - \omega_c/\omega)^2} \) and \( d = \frac{n_e}{n_0} \).

\[
\left[ \frac{4k^2}{1/2(1 + \varepsilon_{0+}/\varepsilon_{zz})q_{\perp}^2 - \frac{\omega^2 \omega_p^2}{c^2 \omega^2 (1 - \omega_c/\omega)} } + \frac{a^2}{2(1 - \omega_c/\omega)} \right] q_{\parallel}^2 \\
+ \left[ \frac{a^2}{2(1 - \omega_c/\omega)} - 1/2(1 + \varepsilon_{0+}/\varepsilon_{zz}) \right] q_{\perp}^2 + \frac{\omega^2 \omega_p^2}{c^2 \omega^2 (1 - \omega_c/\omega)} d = 0.
\]

These equations gives the spatial growth rate \( \Gamma \) as,

\[
\Gamma = i q_{\parallel} = \left[ \left( \frac{a^2}{2(1 - \omega_c/\omega)} - 1/2(1 + \varepsilon_{0+}/\varepsilon_{zz}) \right) q_{\perp}^2 + \frac{\omega^2 \omega_p^2}{c^2 \omega^2 (1 - \omega_c/\omega)} \right]^{1/2} \\
\times \left[ \frac{(1 - \omega_c/\omega)(1 + \varepsilon_{0+}/\varepsilon_{zz}) q_{\perp}^2 - \frac{2\omega^2}{c^2 \omega_p^2} \omega^2 d}{8k^2(1 - \omega_c/\omega) + 1/2a^2(1 + \varepsilon_{0+}/\varepsilon_{zz}) q_{\perp}^2 - \frac{\omega^2}{c^2 (\omega_p^2/\omega^2)} da^2/(1 - \omega_c/\omega)} \right]^{1/2}.
\]

Solid lines in Figure 1 illustrate the normalized growth rate \( \Gamma_{nor}(\Gamma c/\omega) \) versus normalized transverse wave number \( Q_L(q_{\perp} c/\omega) \) for some special fixed parameters.

### III. NON-LINEAR RESPONSE OF THE MEDIUM

Where the laser power exceeds the critical power \( P_c = \frac{3.77 \lambda^2}{8\pi n_0 n_2} \) [20] the non-linear effects appears due to self-focusing phenomenon. In this regime the refractive index of the medium depends on the laser intensity and kerr phenomenon occurs. This critical laser power can be easily achieved by the ultra short femtosecond lasers. The response of our system, magnetic plasma, to such intense laser is governed by Eq. (10) with \( \vec{P}_{NL} \neq 0 \).

\[
\nabla^2 \vec{E} - \nabla(\nabla \cdot \vec{E}) + \frac{\omega^2}{c^2} \varepsilon \vec{E} + \frac{4\pi}{c^2} \omega^2 \vec{P}_{NL} - \frac{4\pi}{c^2} \frac{\partial \vec{J}}{\partial t} = 0, \quad (16)
\]

For isotropic and homogeneous Kerr medium we have [21, 22]:

\[
\vec{P}_{NL} = \frac{4n_0 n_2}{4\pi(1 + \gamma)} [ (\vec{E} \cdot \vec{E}^*) \vec{E} + \gamma(\vec{E} \cdot \vec{E}) \vec{E}^* ], \quad (17)
\]

where \( n_2 \) is the Kerr coefficient, \( \vec{E}^* \) is the complex conjugate of \( \vec{E} \) and \( \gamma \) is a positive constant whose value depends on the physical origin of the Kerr effect [12].

\[
\nabla \cdot \vec{E} = -\frac{4\pi}{n_0^2} \nabla \cdot \vec{P}_{NL} - 4\pi \nabla . \vec{F}^{(1)}.
\]

(18)
The wave equation for perturbed field is given by

\[ \nabla^2 \tilde{E} + \nabla (\nabla \cdot \tilde{E}) + \frac{\omega^2}{c^2} \varepsilon \tilde{E} \]

\[ + \frac{\omega^2}{c^2} \frac{4n_0 n_2}{(1 + \gamma)} [(\tilde{E} \cdot \tilde{E}^*) \tilde{E} + \gamma (\tilde{E} \cdot \tilde{E}^*) \tilde{E}^*] + i \omega c^2 J = 0, \]  \hspace{1cm} (19)

\[ \frac{\partial^2 A_+}{\partial z^2} + \frac{1}{2} (1 + 2 \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) \nabla^2 \nabla A_+ + \frac{\omega^2}{c^2} \varepsilon_0 A_+ + \frac{\omega^2 \frac{4n_0 n_2}{c^2}}{1 + \gamma} [(A_+ \cdot A_+) A_+ + \gamma (A_+ \cdot A_+) A_+] \]

\[ + \frac{\omega^2}{c^2} \frac{\omega_p^2}{\omega^2} \frac{d}{(1 - \omega_c/\omega)} A_+ = \frac{-4n_2}{n_0 (1 + \gamma)} [\nabla (\nabla \cdot (A_+ \cdot A_+) A_+ + \gamma (A_+ \cdot A_+) A_+)]. \]  \hspace{1cm} (20)

The wave equation for perturbed field is given by

\[ 2i k \frac{\partial A_1}{\partial z} + \frac{1}{2} (1 + 2 \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) \nabla^2 A_1 + \frac{\omega^2}{c^2} (\psi_+ A_{10}) + \frac{\omega^2 \frac{4n_0 n_2}{c^2}}{1 + \gamma} [(A_{10})^2 A_1 + \gamma (A_{10})^2 A_1^*] \]

\[ + \frac{\omega^2 \omega_p^2}{c^2} \frac{d}{\omega^2 (1 - \omega_c/\omega)} A_1 = \frac{-4n_2}{n_0 (1 + \gamma)} [\nabla (\nabla \cdot (A_{10})^2 A_1 + \gamma (A_{10})^2 A_1^*)]. \]  \hspace{1cm} (21)

Separating the real and imaginary part we have:

\[ -2k \frac{\partial A_{1r}}{\partial z} - \frac{1}{2} (1 + 2 \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) q_{1r} A_{1r} + \frac{a^2}{2(1 - \omega_c/\omega)} q_{1r}^2 A_{1r} + \frac{a^2}{2(1 - \omega_c/\omega)} q_{1r}^2 A_{1r} \]

\[ + \frac{\omega^2}{c^2} \frac{4n_0 n_2}{(1 + \gamma)} [(A_{10})^2 + \gamma (A_{10})^2] A_{1r} + \frac{\omega^2 \omega_p^2}{c^2} \frac{d}{\omega^2 (1 - \omega_c/\omega)} A_{1r} \]

\[ + \frac{4n_2}{n_0 (1 + \gamma)} [(A_{10})^2 \nabla^2 + \gamma (A_{10})^2 \nabla^2] A_{1r} = 0, \]  \hspace{1cm} (22)

\[ 2k \frac{\partial A_{1i}}{\partial z} - \frac{1}{2} (1 + 2 \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) q_{1i}^2 A_{1i} + \frac{\omega^2}{c^2} \frac{4n_0 n_2}{1 + \gamma} [(A_{10})^2 - \gamma (A_{10})^2] A_{1i} \]

\[ + \frac{\omega^2 \omega_p^2}{c^2} \frac{d}{\omega^2 (1 - \omega_c/\omega)} A_{1i} + \frac{4n_2}{n_0 (1 + \gamma)} [(A_{10})^2 \nabla^2 - \gamma (A_{10})^2 \nabla^2] A_{1i} = 0, \]  \hspace{1cm} (23)

\[ -2i k q_{\parallel} A_{1i} - \frac{1}{2} (1 + 2 \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) q_{\perp}^2 A_{1i} + \frac{a^2}{2(1 - \omega_c/\omega)} q_{\perp}^2 A_{1i} + \frac{a^2}{2(1 - \omega_c/\omega)} q_{\parallel}^2 A_{1r} \]

\[ + \frac{\omega^2 \omega_p^2}{c^2} \frac{d}{\omega^2 (1 - \omega_c/\omega)} A_{1r} + \frac{\omega^2}{c^2} \frac{4n_0 n_2 (A_{10})^2 A_{1r} - 4n_2}{3n_0} (A_{10})^2 q_{\perp}^2 A_{1i} = 0, \]  \hspace{1cm} (24)

\[ 2i k q_{\parallel} A_{1r} = \frac{1}{2} (1 + 2 \frac{\varepsilon_{0+}}{\varepsilon_{zz}}) q_{\parallel}^2 A_{1i} + \frac{\omega^2}{c^2} \frac{4n_0 n_2}{3} (A_{10})^2 A_{1r} - \frac{4n_2}{3n_0} (A_{10})^2 q_{\perp}^2 A_{1i} + \frac{\omega^2 \omega_p^2}{c^2} \frac{d}{\omega^2 (1 - \omega_c/\omega)} A_{1i} = 0. \]  \hspace{1cm} (25)
In the following, we define $b^2 = 4n_2(A_{10})^2$ for the purpose of simplification.

$$
\frac{4k^2}{3n_0} q_\perp^2 + \frac{1}{2} \left( 1 + 2 \xi_{zz}^0 \right) q_\perp^2 - \frac{\omega^2 k^2 n_0}{3c^2} - \frac{\omega^2}{c^4} \left( \frac{d}{1 - \omega_c/\omega} \right) q_\perp^2 + \frac{a^2}{2(1 - \omega_c/\omega)} q_\perp^2 + \frac{1}{2} \left( 1 + 2 \xi_{zz}^0 \right) q_\perp^2 - \frac{b^2}{n_0} q_\perp^2 = 0. \tag{26}
$$

This equation gives the spatial growth rate $\Gamma$ as:

$$
\Gamma = iq\parallel q\parallel = \frac{\omega^2}{c^2} \left( \frac{d}{1 - \omega_c/\omega} + n_0 b^2 \right) + \left[ -\frac{1}{2} \left( 1 + 2 \xi_{zz}^0 \right) q_\perp^2 + \frac{a^2}{2(1 - \omega_c/\omega)} - \frac{b^2}{n_0} q_\perp^2 \right]^{\frac{1}{2}}.
\tag{27}
$$

FIG. 1. (Color online) normalized growth rate $\Gamma_{nor}(c/\omega)$ versus normalized transverse wave number $Q_{\perp} (q_\perp c/\omega)$ for $\omega_p^2/\omega^2 = 0.1$, $\omega_c/\omega = 0.2$, $a_0 = 1$, $d = \frac{1}{2}$ and $\gamma = \frac{1}{2}$.

IV. RESULTS

Since $\Gamma$ depends on the parameters involved, it cannot be determined without knowing the values of the parameters. The effects of each parameter can be seen by fixing the other parameters. The results are depicted in figures 135. Fig. 1 illustrates the normalized growth rate $\Gamma$, $\Gamma_{\omega}^\xi$, versus normalized transverse wave number $Q_{\perp}$, $Q_{\perp} (q_\perp c/\omega)$, for different values of Kerr parameters, $b^2$ and fixed other parameters. The results reveal that for a given $b^2$, there is an optimum $Q_{\perp}$, $Q_{\perp}^{\text{opt}}$, which $\Gamma$ is maximum, $\Gamma_{\text{max}}$, at that point. This figure shows $\Gamma_{\text{max}}$ increases as $b^2$ increases. Furthermore, the maximum is occurred in higher $Q_{\perp}^{\text{opt}}$.
for higher values of $b^2$. The parameter $b^2$ is determined by non-linearity of the medium and the intensity of laser pulse. For example: for a typical peta-wat femtosecond laser pulse with $\lambda = 800\text{nm}$ and $|A_{10}|^2 \sim 10^{15}$, the value of the parameter $n_2$ is obtained as order as $10^{-19}\text{cm}^2\text{W}^{-1}$ for gaseous media [23]. This leads to the values 1-5 for the $b^2$ parameter. Another important parameter is $\omega_c$ which can be easily adjusted by the external magnetic field, $B_0$. Figs. 2 and 3 depict the mutual influence of the parameters $\omega_c$ and $b^2$, for two values of $Q_\perp$ ($Q_\perp=0.5$ and $Q_\perp=0.75$). The results show that: for a specific values of $Q_\perp$, there exist

![Fig. 2](image1)

FIG. 2.  (Color online) normalized growth rate $\Gamma_{nor}(\Gamma_c/\omega)$ versus $b^2$ for $\omega_p^2/\omega^2 = 0.1$, $a_0 = 1$, $d = \frac{1}{2}$, $Q_\perp = (2a : 0.5, 2b : 0.75)$ and $\gamma = \frac{1}{2}$.

![Fig. 3](image2)

FIG. 3.  (Color online) normalized growth rate $\Gamma_{nor}(\Gamma_c/\omega)$ versus $\frac{\omega}{\omega}$ for $\omega_p^2/\omega^2 = 0.1$, $a_0 = 1$, $d = \frac{1}{2}$, $Q_\perp = (3a : 0.5, 3b : 0.75)$ and $\gamma = \frac{1}{2}$.

an optimum Kerr parameters, $b^2_{opt}$. The loci of $b^2_{opt}$ can be adjusted by magnetic field via
\( \omega_c \). \( b_{opt}^2 \) is smaller for larger values of \( \omega_c \). The behavior of growth rate with respect to \( \omega_c \) and \( b^2 \), is heavily depends on the value of \( Q_\perp \). The results may reveal that the magnetic effects could be competes by the non-linear effects. Drawing normalized growth rate \( \Gamma^c_\omega \) versus \( \omega_c \) for different values of \( b^2 \), show that normalized growth rate \( \Gamma^c_\omega \) is not a smooth function of \( \omega_c \), i.e. it is not a differentiable and single value function for all values of \( \omega_c \) e.g. for an interval \( \omega_c \in [\omega_c1, \omega_c2] \). The value of \( \omega_{c1} \) and \( \omega_{c2} \) is depends on other parameters, particularly \( b^2 \) and \( Q_\perp \). The figures resemblance the phase transition phenomenon. The behavior of normalized growth rate \( \Gamma c/\omega \) before and after critical region is very different. This strange behavior is also for variation of plasma density which indicated by \( \omega^2 \). This fact is depicted in Figs. 4a-4c. The result shows that the critical region and the behavior of normalized growth rate \( \Gamma^c_\omega \) before and after of this region, is heavily depends on the parameters involved. The figure 5 illustrate the normalized growth rate versus normalized laser field amplitude (\( \sqrt{\text{intensity}} \)) for some media. To formalize the effect of medium Kerr

![Normalized growth rate vs. \( \omega_c \) and \( b^2 \)](image-url)
non-linearity i.e. \( n_2 \) we write \( b^2 = \alpha a_0^2 \). The numerical value of \( \alpha \) is considered as \( \alpha = 0 \) (for linear medium), \( \alpha = 1 \) (for medium with weak non-linearity) and \( \alpha = 10 \) (for medium with high non-linearity). The result show that the filamentation is not occurs in low intensity of laser for linear mediums. Because each medium shows non-linearity when the power of the driven laser exceeds a critical value, the filamentation is observed only for \( a_0 > a_{0c} \).

This figure yields \( a_0 \sim 1 \) leads to the value \( P_c \sim 10^{15} \frac{W}{cm^2} \), which is in agreement with the result of other works. Also, this figure reveres that the growth rate of instability increases as \( a_0 \) increase and there exist a saturation effect. In fact, when filamentation is constructed, increasing the laser intensity, increases the rate of filamentation growth. But after an specific value of \( a_0 \) (intensity), the filamentation growth rate reaches a constant value i.e. it does not increase with the laser intensity. The physical region of this phenomenon is may be related to the fact that after the above mentioned laser intensity the filamentation converts the absorbed energy of the laser to increase its temperature. This fact ceases the growth of the instability. Also this figure show that the intensity which the filamentation stops its growth is lower for higher value of \( \alpha \) i.e. higher values of Kerr coefficient.
V. CONCLUSION

The present study investigated the growth rate of filamentation instability under the effect of non-linear polarization in magnetized plasma. The results show that the non-linearity of medium has a significant influence on the formation rate of the filamentation. The growth rate of filamentation in non-linear medium can be controlled by external adjustable parameters such that magnetic field, laser frequency and laser intensity and also by varying internal parameters of the medium such that electron density profile and Kerr coefficient. The results reveal that there is an optimum range for the parameters involved. Also, for some specific and critical range of parameters the growth rate of filamentation change in undefined manner, reminds the phase transition phenomenon. Size of the critical region depends on the parameter involved specially, $\omega_c$ and $\omega_p$. Also, plotting the growth rate of the filamentation versus normalized laser field amplitude, $a_0$ show that the growth rate of instability could be controlled by $a_0$ within an specific range. For $a_0$ below this range there is $a_0$ filamentation and above this region the filamentation growth rate becomes intensity-independent. The Kerr-coefficient of the medium determines the start and final point of this region. The results could be employed for the formation of long-distance plasma channels and there for is applicable in the fields of interaction of the intense laser pulses with rare gases.

[1] J. Reintjes, R. L. Garman and F. Shimizu, Phys.Rev.A. 8, 1486 (1973).
[2] S. L. Chin, F. Théberge, W. Liu, Filamentation non-linear optics. Appl.Phys.B. 86, 477 (2007).
[3] J. Kasparian et. al., Optics Express 16, 5757 (2008); E. Schubert et. al., Optic Express 23, 28640 (2015).
[4] J. Kasparian, In progress in ultrafast intense laser. pp 301-318. science , Springer 2007.
[5] A. Braun et. al., Optics Letters 20, 37 (1995).
[6] W. Liu, S. L. Chin, O. Kosareva, I. S. Golubtsov and V. P. Kandidov, Opt. Commun. 225, 193-209 (2003).
[7] P. Béjot et. al., Opt. Commun. 380, 245 (2016).
[8] M. Hashemzadéh, Physics of plasma. 25, 012304 (2018).
[9] R. L. Stenzel, Phys. Fluids 19, 865 (1976).
[10] R. L. Stenzel, Phys. Fluids 19, 857 (1976).
[11] V. I. Bespalov, V. I. Talanov, JETP Lett. 3, 307 (1966).
[12] G. Fibich and B. Ilan, Opt. Lett. 26, 840-842 (2001).
[13] G. Fibich and B. Ilan, Physica. D. 157, 112-146 (2001).
[14] G. Fibich and B. Ilan, Phys. Rev. lett. 89, 013901-013904 (2002).
[15] M. S. Sodha, A. K. Ghatak and V. K. Tripathi, in Progress in Optics, 13th ed., edited by E. Wolf (North Holland, Amsterdam, 1976), P. 169.
[16] J. R. Reitz, F. J. Milford, R. W. Christy, Foundations of electromagnetic Theory 3rd ed. Addison-Wesley, Reading, MA, (1979).
[17] J. C. Maxwell, Philos Mag. 21, 161 (1861).
[18] L. Berge, S. Skupin, R. Nuter, J. Kas Parian and J. P. Wolf, Rep. Prog. Phys. 70(10), 1633 (2007).
[19] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, Pergamon Press 1960.
[20] S. L. Chin, T. J. Wang, C. Marceau et al., Laser Phys. 22, 1 (2012).
[21] P. D. Maker and R. W. Terhune, Phys. Rev. 148, 990 (1966).
[22] P. D. Maker and R. W. Terhune and C. M. Savage, Phys. Rev. Lett. 16, 832 (1964).
[23] D. Wang, Y. Leng, Optics Communications. 285, 5462 (2012).