Development of new novel constitutive model for deep reservoir sandstone rock for sand production application

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Abstract. Sand production in oil or gas well can cause serious havoc leading to surface and downhole equipment erosion and induced unwanted cost to fix it. Thus, for the completion selection and optimisation, it requires a proper geomechanical analysis. During this process it is very important to use an appropriate failure constitutive model where it should be able to honour the onset failure of the geomaterial and the post failure as well. Currently most of existing failure criteria do not honour full spectrum of stress and strain evolution including post failure (softening). We developed a new elastoplastic constitutive model for weak geomaterial to fill the gap by using single equation to honour both hardening and softening of the geomaterial. Triaxial test has been conducted on rock sample under different confining pressure using a servo controller. The data from lab test has been used to develop the constitutive failure model followed by a validation using the Finite Element Method. Mohr Coulomb failure which is commonly used in the industry has been compared with the new constitutive model (known as ASA model). As the results the newly developed model was able to capture better the full elastoplastic behaviour that includes softening or post failure of the geomaterial mechanical response compared to Mohr Coulomb criterion. This is quite critical for oil and gas applications, especially in the case of sand production. In conclusion, the new constitutive model was able to honour the full spectrum of nonlinear stress strain evolution using single equation and demonstrated superior compared to the commonly used failure criteria in the industry.

1. Introduction

Natural hydrocarbon is still a major energy source globally. This fossil juice normally can be found in different types of rock or geomaterial such as carbonate, sandstone or directly from source rock (organic shale). Approximately 60% of world reserve are coming from sandstone reservoir. Sandstone reservoir is a composite geomaterial where quartz mineral grains are the main component. These quartz grains are bonded or cemented together by cementing material. The hydrocarbon is normally trapped in
between the pores of the geomaterial. The pore pressure in the porous media will reduce during the production, which can lead to failure of the grain to grain bonding due stress and strain increment. This failure may lead to a phenomenon known as sand production. Approximately 50% of the wells in this sandstone reservoir has potential for sand production [1]. As the field/reservoir is maturing, sand production severity and risk factor will increase as well. Thus, good understanding of sandstone mechanical behavior can help the industry to model the reservoir behavior for better field development planning and to justify properly their need for any additional investment requirements for better well completion [2].

There are few failure criteria that have been developed and readily available. Even though most of the developed failure criteria are for civil engineering purposes, petroleum industry adopted the criteria for sub-surface reservoir rocks[3]. Unfortunately, most of the failure criteria are not able to capture the elasto plastic non-linearity of the geomaterial including post failure in single constitutive model. As for petroleum industry applications, rock mechanics plays vital role for wellbore instability, sand production, hydraulic fracturing, compaction, subsidence etc. Unlike civil engineering applications, where zero tolerance for failure is applied in any of the design, petroleum industry has to allow some failure of the rock and manage it during operation. Thus, non-linearity and post failure mechanism are very essential to evaluate the risk and consequences

2. Laboratory experiments

In order to evaluate the rock mechanical behavior link to petroleum applications, the geomaterial has to be tested under different confining pressure to represent the sub-surface downhole condition, also to honors stress changes due to depletion. This test is also known as triaxial compression test. The triaxial test results have been used to formulate the constitutive model. Cylindrical core sample is placed into a jacketed triaxial hook cell apparatus and then loaded until failure and stabilization. We used ASTM Specification D2938 [4] for sample preparation and load rate. The triaxial test was performed in two stages (Figure 1a). During Stage 1, a hydrostatic pressure is applied and during Stage 2, the axial load is applied until the rock fails and strength drops until it reaches the residue strength. A servo controller has been used for this test where the applied axial load is controlled by constant axial strain rate. All test was done in drained condition where the pore pressure is vent to atmospheric pressure. The geomaterial (core) response is monitored and recorded. In general, the deviatoric stress against axial and radial strains plots provides the changes in rock mechanical behavior, where the rock sample undergoes different mechanical regimes changes namely (a) Elastic region: (b) Yield point, (c) Strain hardening (d) Failure and (e) Post Failure or softening. This illustration can be found in Figure 1b which shows a typical response of a triaxial compression test where the rock failing in shear.

![Figure 1. The Triaxial Test loading process (a) and typical stress strain response from the test (b).](image-url)
2.1. Triaxial test data analysis

In order to eliminate human error in picking the key mechanical properties from the stress strain curve to develop our constitutive modeling, a Mathematica code was developed. This code enabled us to consistency analyze the data and avoid any error in picking the desired point from the stress-strain curves. Elastic properties of the geomaterial should be drawn taken for elastic region. Therefore, we need first to estimate the yielding point of the geomaterial during loading. The point of initial yielding of the geomaterial can be detected by plotting the deviation value $S^2$. The deviation can be computed based on the intermediate points from the elastic straight line which is drawn between the origin and each last point. Figure 2 shows the $S^2$ and the yield point pick by the code.

$$S^2 = \frac{\sum_{i=1}^{N} [y_i - (a + bx_i)]^2}{N}$$  \hspace{1cm} (1)

where the constant a and b are defined by the equation of straight line between the origin (0) and each last point.

![Graph 1](Image)

(a) Yield point pick for S2 computation  
(b) Yield point in stress strain curve

**Figure 2.** (a) Analysis using Equation (1) for selecting proper yield point and (b) the yield point on the stress strain curves.

To get accurately the range of Young’s Modulus (E) and Poisson’s Ratio (PR), we estimated the instantaneous Young’s Modulus for each stress strain points as shown in Figure 3a. Taking this range as guideline, from the stress strain curve, the average Young’s Modulus and Poisson’s Ratio can be estimated as shown in Figure 3b.

![Graph 2](Image)

(a) Instantaneous Young’s Modulus, E plots to assist finding best region for elastic properties, (b) Young Modulus, E and Poisson’s Ratio estimation from stress-strain curves
As for the peak strength value, it was computed from the maximum axial stress the sample has seen before the failure occurs. Table 1 summarizes the lab test results for a set of rock that has been tested.

**Table 1.** Summary of Set-1 Sandstone mechanical properties from the lab test results

| Sample Name | Confining Pressure | Young’s Modulus | Poisson's Ratio | Yield Strength | Peak Strength |
|-------------|--------------------|-----------------|----------------|----------------|--------------|
|             | MPa                | Gpa             | unitless       | MPa            | MPa          |
| CG-5        | 3.45               | 6.08            | 0.36           | 40.36          | 49.29        |
| CG-10       | 10.00              | 7.04            | 0.33           | 53.01          | 64.78        |
| CG-20       | 20.00              | 9.83            | 0.33           | 70.05          | 96.12        |
| CG-30       | 30.00              | 9.98            | 0.30           | 83.13          | 110.94       |
| CG-40       | 40.00              | 10.38           | 0.28           | 81.18          | 120.93       |

### 3. Development of the constitutive model

First, in order to develop the non-linear constitutive model of the geomaterial (sandstone), we need to differentiate the various types of strain during loading. Figure 4 illustrates the stress and strain curve for one of the tested samples. The strains have been divided between elastic ($\varepsilon^e_{1or3}$) and plastic strain ($\varepsilon^p_{1or3}$). The plastic stain is further divided, during hardening ($\varepsilon^{hard}_{1or3}$) and post failure softening ($\varepsilon^{soft}_{1or3}$).

![Figure 4. Stress and Strain Curves and types of strains](image)

The newly developed constitutive model has been named as ASA Model (Assef-Surej-Ariffin) throughout this paper. We adopted the irreversible thermodynamic framework processes for our effort. The literature is rich in relation to this concept. Many researchers have adopted similar approach following the thermodynamic framework to model plasticity for different geomaterials [5-11]. We engaged the small strain concept here the incremental total strain tensor $d\varepsilon$ is decomposed into two components namely reversible elastic component $d\varepsilon^e$ and irreversible nonlinear plastic component $d\varepsilon^p$:

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p$$

(2)

The existence of thermodynamic potential of plastic materials has been hypothesized and the potential thermodynamic potential can be written as:

$$\Psi = \frac{1}{2}(\varepsilon - \varepsilon^p):C(\varepsilon - \varepsilon^p) + \Psi_p(\gamma_p)$$

(3)

where $C$ is the fourth order elastic tensor and it is function of the geomaterial shear and bulk modulus.
\( \Psi_p(\gamma_p) \) is known as the plastic energy in the plastic hardening and softening. The final equation of state can be derived from the standard derivation of this thermodynamic potential with respect to elastic strain tensor and can be written as:

\[
\bar{\sigma} = \frac{\partial \Psi}{\partial \bar{\varepsilon}} = C : (\bar{\varepsilon} - \bar{\varepsilon}^p) \quad \cdots \cdots \cdots \cdots \quad (4)
\]

As we are dealing with sandstone reservoir rock, we may assume that the material is isotropic in nature. Adopting Hill’s notation [12], the elastic stiffness tensor can be written as:

\[
C = 2\mu K + 3\mu \] \quad \cdots \cdots \cdots \cdots \quad (5)
\]

where \( \mu \) and \( K \) are the material bulk and shear moduli, respectively. \( K \) and \( J \) are fourth order deviatoric and spherical tensors.

The evolution of the plastic strain tensor \( \bar{\varepsilon}^p \) is accumulated from the point of yielding to post failure until the rock reaches the residual strength. Our laboratory experiments illustrate clearly that this evolution is dependent on material strength of sandstone and is influenced by confining pressure. Therefore, the classical constitutive models with linear failure surface become contemptible to pronounce the mechanical behavior of materials of such type. It is thus crucial to use a non-linear failure surface. Encouraged by this, we developed a relation of the failure surface which can be written as below:

\[
F = q' - g(\theta)UCS \left[ A \left( C + \frac{p'}{UCS} \right) \right]^n = 0 \quad \cdots \cdots \cdots \cdots \quad (6)
\]

where: \( p' \) is the effective mean stress; \( q' \) is the deviatoric stress defined by \( q' = \sqrt{3} \frac{1}{2} \bar{S}^t : \bar{S}^t \); \( \bar{S}^t \) is the deviatoric stress tensor expressed by \( \bar{S}^t = \bar{\sigma}^t - \frac{1}{3} tr(\bar{\sigma}^t) \delta \); \( \theta \) is Lode’s angle; \( UCS \) is the uniaxial compressive strength; \( A \) represents the coefficient of internal friction; \( C \) describes the material cohesion; \( n \) describes the dependency of material failure with effective mean stress and \( \delta \) is Kronecker tensor. Lode’s angle \( \theta \) is defined within the interval \((-\pi/6, \pi/6)\). To consider the influence of the third invariant of stress on the shape of the surface, the function \( g(\theta) \) was introduced into the failure surface. Inspired by the previous work of William et al. [13] and applied by Mohamad-Hussein et al. [10], the following function was adopted:

\[
g(\theta) = \frac{2(1 - K^2) \cos\left(\frac{\pi}{6} + \theta\right) + (2K - 1) \sqrt{4(1 - K^2) \cos\left(\frac{\pi}{6} + \theta\right) + 5K^2 - 4K}}{4(1 - K^2) \cos\left(\frac{\pi}{6} + \theta\right) + (2K - 1)^2} \quad \cdots \cdots \cdots \cdots \quad (7)
\]

where, \( K \) designates the ratio between the tensile and compressive meridian.

3.1. Plastic deformation

Plastic yield surface, plastic potential, hardening law, softening law and a failure surface will control the plastic deformation of sandstone. The failure surface is supposed to represent the position of plastic loading surface at peak strength of the material. Therefore, a hardening/softening law \( a(\gamma_p) \) has been introduced in our constitutive model to obtain the plastic yield surface. The obtained equation of the plastic yield surface can be written as:

\[
f = q' - a(\gamma_p)g(\theta)UCS \left[ A \left( C + \frac{p'}{UCS} \right) \right]^n = 0 \quad \cdots \cdots \cdots \cdots \quad (8)
\]

Both hardening and softening behaviour of sandstone were governed by plasticity. For sake of
simplicity, a unified hardening and softening law \( \alpha(\gamma_p) \) has been induced in the plastic distortion \( \gamma_p \) function. This allows us to capture the plastic hardening and softening which is computed from the standard derivation of the locked plastic energy with respect to the internal variable of plastic hardening and softening \( \gamma_p \):

\[
\alpha(\gamma_p) = (\alpha_0 - \alpha_r) \left[ 1 - H \left( \frac{\gamma_p - \frac{1}{B_1}}{} \right) \right] + \alpha_r
\]

\[
+ (1 - \alpha_0) \left[ 1 - H \left( \frac{\gamma_p - \frac{1}{B_1}}{} \right) \right] \sqrt{B_1 \gamma_p (2 - B_1 \gamma_p)}
\]

\[
+ (1 - \alpha_r) H \left( \gamma_p \right) \exp \left[ -B_2 \left( \frac{\gamma_p - \frac{1}{B_1}}{} \right)^2 \right]
\]

Figure 5 shows the variation of the parameter \( \alpha \) with generalized plastic shear strain \( \gamma_p \). The geomaterials exhibit a changeover from plastic compressibility to dilatancy. We have seen such transition behavior evidence in laboratory tests with the increase of deviatoric stress. This motivates for the new development on the constitutive model and the necessity of using a non-associated plastic flow rule is vital for petroleum industry. Finally, plastic potential was adopted can be written as:

\[
Q = q' + g(\theta) (\alpha - \beta) UCS \left[ \left( C + \frac{P'}{UCS} \right)^\mu \right] \]

where \( \beta \) is a model parameter the outlines the transition from contractancy to dilatancy. It splits the stress space into two regions, respectively corresponding to plastic compressibility \( (\alpha < \beta) \) and dilatancy \( (\alpha > \beta) \).

![Figure 5. Variation of \( \alpha \) in function of generalized plastic shear strain \( \gamma_p \) illustrating hardening and softening phases](image)

4. Constitutive model validation

The laboratory test results from Table 1 and Error! Reference source not found. were used to regulate the best parameters to fit all the test point using the ASA constitutive model. The unconfined compressive strength (UCS) was determined from the peak strength of samples where no confining pressure was applied. Figure 6 shows the yield and failure stress points for Set 1 and Set 2 of sampled sandstone. The Failure and Yield surfaces (continuous lines) are also plotted on the same figure which was model using the new ASA constitutive model i.e. equations (6) and (8). As the results the ASA model seems able to capture the non-linearity of the material as confining pressure increases.
4.1. Validation using Finite Element Model

The mechanical behavior of the rock has been further validated using a FEM code. We developed a FEM code and incorporated the newly proposed elastoplastic constitutive model (ASA) for this validation process [14]. The results from FEM then compared with actual lab test triaxial test for each sample.

Figure 8 shows the comparisons between the numerical simulation and the triaxial test experimental data for various confining pressures. As can been seen, the proposed constitutive model able to predict the main feature of sandstone material such as elasticity, yielding, strain hardening, failure and softening behaviour. In addition to this, the ASA model also able to capture important features i.e. predicting the brittle behaviour at lower confining pressures and ductility at higher confining pressures. Thus, overall it gives confidence that the new ASA constitutive model able to simulate the mechanical behaviour of sandstone geomaterial. Furthermore, such constitutive model is very useful in petroleum geomechanics applications not limited to sand production but also for wellbore stability analysis and reservoir geomechanics where it is capable to simulate as boundary value problems where the non-linear response.

Attempt also made to compare the ASA model against the commonly used Mohr Coulomb model with modified hardening parameter to honour plasticity[15, 16]. Figure 7 shows the comparison between numerical response using Mohr Coulomb model and ASA model against the actual laboratory test data, for the sandstone. Classical Mohr Coulomb criteria was unable to reproduce the softening behaviour of the material but ASA model able to capture such mechanical behaviour.
Figure 8. Lab and FEM ASA constitutive model stress strain curves comparison for Set-1 sandstone for different confining pressures (a) 5MPa (b) 10 MPa, (c) 20 MPa, (d) 30 MPa

5. Conclusions

A constitutive model that can predict the overall mechanical behaviour of the experimental data can be used for any engineering applications not limited to sand production and wellbore stability analysis. In the context of petroleum sector where the post failure behaviour is extremely critical to evaluate the severity and stabilisation. Experimental test has been conducted to formulate the new constitutive model. The proposed new ASA constitutive model has been implemented in a Finite Element Method (FEM) code. The consistency of the mathematical formulations of the model and the procedure for determining the model parameters have been validated by comparison between the numerical response of the model and the triaxial test data of sandstone samples.

Data analytic process has been implemented to extract rock mechanical properties using a Mathematica code in order to reduce any induced human error. The laboratory test on the sandstone rock shows that the rock can be either brittle or ductile dependent on the confining pressure applied to the rock. This is very important for petroleum industry applications as the geomaterial is expose at very high confining state. Thus, a comprehensive rock mechanical behaviour model is key to predict the life cycle of the rock stress and strain i.e. elasto plastic and post failure. The ASA model now can be used to predict sand production initiation and stabilization eventuality for amount of sand production (volume) and its severity.

Normally sand production is not a continuous process during oil/gas production, where the sand production stabilized at one stage as the stress stabilization (post failure behaviour and stress relaxation). Thus, the newly developed constitutive model (ASA) will be able to capture full spectrum of the sandstone mechanical behaviour. Therefore, knowing when, where and how much sand will be produced can help for optimum completion design. Operators can make decision on their investment based on the severity due to geomechanical aspect.
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