The crystallography of strange quark matter

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Abstract. Cold three-flavor quark matter at large (but not asymptotically large) densities may exist as a crystalline color superconductor. We explore this possibility by calculating the gap parameter $\Delta$ and free energy $\Omega(\Delta)$ for possible crystal structures within a Ginzburg-Landau approximation, evaluating $\Omega(\Delta)$ to order $\Delta^6$. We develop a qualitative understanding of what makes a crystal structure stable, and find two structures with particularly large values of $\Delta$ and the condensation energy, within a factor of two of those for the CFL phase known to characterize QCD at asymptotically large densities. The robustness of these phases results in their being favored over wide ranges of density and though it also implies that the Ginzburg-Landau approximation is not quantitatively reliable, previous work suggests that it can be trusted for qualitative comparisons between crystal structures. We close with a look ahead at the calculations that remain to be done in order to make contact with observed pulsar glitches and neutron star cooling.

We explore properties of quark matter at zero temperature and at densities that may occur within neutron star cores. Absent interactions, each flavor of quark will fill momentum eigenstates up to a Fermi surface. However, this configuration is unstable to the formation of Cooper pairs in the presence of any attractive interaction between quarks. In QCD, the interaction between pairs of quarks that are antisymmetric in color is attractive, and hence the ground state features a diquark condensate that is dominantly antisymmetric in color. This is color superconductivity. We consider condensates antisymmetric in Dirac indices and, consequently, in flavor also, implying that quarks in a pair have different flavors.

At asymptotic densities ($M_s/\mu \to 0$), where the three light quarks can be treated as massless, quark matter exists in the CFL phase \[1\] in which quarks of all three colors and all three flavors form Cooper pairs with zero total momentum, yielding $\langle ds \rangle$, $\langle us \rangle$ and $\langle ud \rangle$ condensates, and in which all fermionic excitations are gapped, with a gap parameter $\Delta_0 \sim 10 - 100$ MeV. However, at densities relevant for neutron star phenomenology, meaning quark chemical potentials at most $\mu \sim 500$ MeV, the strange quark mass $M_s$ cannot be neglected. In neutral unpaired quark matter in weak equilibrium, $M_s$ introduces splitting between the Fermi surfaces for quarks of different flavor which can be taken into account to lowest order in $M_s^2/\mu^2$ by treating the quarks as if they were massless but with chemical potential splittings $\delta\mu_2 \equiv (\mu_u - \mu_s)/2$ and $\delta\mu_3 \equiv (\mu_d - \mu_u)/2$ given by $\delta\mu_2 = \delta\mu_3 = \delta\mu = M_s^2/(8\mu)$. Note that the splitting between unpaired Fermi surfaces increases with decreasing density. In the CFL phase, the Fermi momenta are not given by these optimal values for unpaired quark matter; instead, the system pays a free energy price $\propto \delta\mu^2/\mu^2$ to equalize all Fermi momenta and gains a pairing energy benefit $\propto \Delta_0^2\mu^2$. As a function of decreasing density, there comes a point (at which $\delta\mu \approx \Delta_0/4$ \[2\]) when the system can lower its energy by breaking pairs. Below this density the CFL phase can certainly not be the ground state of matter. Within a spatially homogeneous pairing ansatz, the phase that results when CFL Cooper pairs start to break is the gapless CFL (gCFL) phase of Ref. \[2\]. However, this phase turns out to be unstable to the formation of counter-propagating currents, presumably leading to inhomogeneity, and therefore cannot be the ground state of matter at any density \[3\]. Within the range of densities in which the gCFL phase has lower free energy than the CFL and unpaired phases, the true ground state of dense matter must have lower free energy still. We have recently proposed a candidate phase (actually, two candidate phases) for the ground state of matter over much of this

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density regime and at still lower densities [3]. In these proceedings we describe this proposal. For further
references, in particular to other suggested resolutions of the gCFL instability, see Ref. [4].

Crystalline color superconductivity [5, 6] naturally permits pairing between quarks living at split Fermi
surfaces by allowing Cooper pairs with nonzero net momentum. In three-flavor quark matter, this allows
pairing to occur even with the Fermi surfaces split in the free-energetically optimal way as in the absence
of pairing [3, 4, 3]. This is the origin of the advantage that crystalline color superconducting phases have
over the CFL and gCFL phases at large values of the splitting $\delta \mu$. For example, by allowing $u$ quarks with
momentum $p + q_3$ to pair with $d$ quarks with momentum $-p + q_3$, for any $p$, we can pair $u$ and $d$ quarks
along rings on their respective Fermi surfaces [5, 6]. In coordinate space, this corresponds to a condensate
of the form $|\psi_{\alpha} C \gamma^5 \psi_{\beta}\rangle \propto \sum_{I} \epsilon_{I \alpha \beta} \epsilon_{I \gamma j} \Delta_I \sum_{q_3} \exp(2i q_3^i \cdot r)$ with the $q_3^i$ chosen from
some specified set $\{q_3\}$. This is a condensate modulated in position space in some crystalline pattern, with
the crystal structure defined by $\{q_3\}$. In this two-flavor context, a Ginzburg-Landau analysis reveals that
the best $\{q_3\}$ contains eight vectors pointing at the corners of a cube, say in the $(\pm 1, \pm 1, \pm 1)$ directions
in momentum space, yielding a face-centered cubic structure in position space [4].

We use the following ansatz for the three-flavor crystalline color superconducting condensate [3]:

$$\langle \psi_{\alpha} C \gamma^5 \psi_{\beta}\rangle \propto \sum_{I} \epsilon_{I \alpha \beta} \epsilon_{I \gamma j} \Delta_I \sum_{q_3} \exp(2i q_3^i \cdot r).$$

This is antisymmetric in color $(\alpha, \beta)$, spin, and flavor $(i, j)$ indices and is thus a generalization of the CFL
condensate to crystalline color superconductivity. We set $\Delta_1 = 0$, neglecting $(ds)$ pairing because the $d$
and $s$ Fermi surfaces are twice as far apart from each other as each is from the intervening $u$ Fermi surface.
Hence, $I$ can be taken to run over 2 and 3 only. $\{q_2\}$ and $\{q_3\}$ define the crystal structures of the $\langle us\rangle$
and $\langle ud\rangle$ condensates respectively. We only consider crystal structures in which all the vectors in $\{q_2\}$ are
equivalent to each other, and same for $\{q_3\}$, as this justifies our simplifying assumption that the $\langle us\rangle$
and $\langle ud\rangle$ condensates are each specified by a single gap parameter ($\Delta_2$ and $\Delta_3$ respectively), avoidin having to
introduce one gap parameter per $q$. We furthermore only consider crystal structures which are exchange
symmetric, meaning that $\{q_2\}$ and $\{q_3\}$ can be exchanged by some combination of rigid rotations and
reflections applied simultaneously to all the vectors in both sets. This simplification, together with $\delta \mu_2 = \delta \mu_3$
an approximation corrected only at order $M_\Lambda^4/\mu^4$, guarantees that we find solutions with $\Delta_2 = \Delta_3$.

We analyze and compare candidate crystal structures by evaluating the free energy $\Omega(\Delta_2, \Delta_3)$ for each
crystal structure in a Ginzburg-Landau expansion in powers of the $\Delta$'s. This approximation is controlled if
$\Delta_2, \Delta_3 \ll \Delta_0, \delta \mu$, with $\Delta_0$ the gap parameter in the CFL phase at $M_\Lambda^2/\mu = 0$. The terms in the Ginzburg-
Landau expansion must respect the global $U(1)$ symmetry for each flavor, meaning that each $\Delta_j$ can only appear in the combination $|\Delta_j|^2$. (The $U(1)$ symmetries are spontaneously broken by the condensate, but not explicitly broken.) Therefore, $\Omega(\Delta_2, \Delta_3)$ is given to sextic order by

$$\Omega(\Delta_2, \Delta_3) = \frac{2\mu^2}{\pi^2} \left[ P_2 \alpha_2 |\Delta_2|^2 + P_3 \alpha_3 |\Delta_3|^2 + \frac{1}{2} \left( \beta_2 |\Delta_2|^4 + \beta_3 |\Delta_3|^4 + \beta_{23} |\Delta_2||\Delta_3|^2 \right) \right. + \left. \frac{1}{3} \left( \gamma_2 |\Delta_2|^6 + \gamma_3 |\Delta_3|^6 + \gamma_{23} |\Delta_2|^2 |\Delta_3|^4 + \gamma_{233} |\Delta_2|^4 |\Delta_3|^2 \right) \right],$$

where we have chosen notation consistent with that used in the two flavor study of Ref. [4], which arises as a special case of $\{q_1\}$ if we take $\Delta_2$ or $\Delta_3$ to be zero. $P_j$ is the number of vectors in the set $\{q_j\}$. The form of the Ginzburg-Landau expansion $\{q_1\}$ is model-independent, whereas the expressions for the coefficients $\alpha_1$, $\beta_1$, $\beta_{1J}$, $\gamma_1$, and $\gamma_{1JJ}$ for a specific crystal structure are model-dependent. For exchange symmetric crystal structures, $\alpha_2 = \alpha_3 \equiv \alpha$, $\beta_2 = \beta_3 \equiv \beta$, $\gamma_2 = \gamma_3 \equiv \gamma$ and $\gamma_{233} = \gamma_{322}$. 

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Because setting one of the \( \Delta_I \) to zero reduces the problem to one with two-flavor pairing only, we can obtain \( \alpha, \beta \) and \( \gamma \) via applying the two-flavor analysis described in Ref. [6] to either \( \{q_2\} \) or \( \{q_3\} \) separately. Using \( \alpha \) as an example, we learn that

\[
\alpha_I = \alpha(q_I, \delta \mu_I) = -1 + \frac{\delta \mu_I}{2q_I} \log \left( \frac{q_I + \delta \mu_I}{q_I - \delta \mu_I} \right) - \frac{1}{2} \log \left( \frac{\Delta_{2SC}^2}{4(q_I^2 - \delta \mu_I^2)} \right),
\]

Here, \( q_I = \lvert q_I \rvert \) and \( \Delta_{2SC} \) is the gap parameter for the 2SC (2-flavor, 2-color) BCS pairing obtained with \( \delta \mu_I = 0 \) and \( \Delta_I \) nonzero with the other two gap parameters set to zero. Assuming that \( \Delta_0 \ll \mu \), it is given by \( \Delta_{2SC} = 2^{\frac{2}{3}} \Delta_0 [10] \). In the Ginzburg-Landau approximation, in which the \( \Delta_I \) are assumed small, we must first minimize the quadratic contribution to the free energy, and only then investigate the quartic and sextic contributions. Minimizing \( \alpha_I \) fixes the length of all the vectors in the set \( \{q_I\} \), yielding \( q_I = \eta \delta \mu_I \) with \( \eta = 1.1997 \) the solution to \( \frac{1}{2} \log \left( (\eta + 1)/(\eta - 1) \right) = 1 [5] \). Upon setting \( q_I = \eta \delta \mu_I \), [4] becomes

\[
\alpha_I(\delta \mu_I) = -\frac{1}{2} \log \left( \frac{\Delta_{2SC}^2}{4 \delta \mu_I^2 (\eta^2 - 1)} \right).
\]

Furthermore, the only dimensionful quantities on which the quartic and sextic coefficients can depend are then the \( \delta \mu_I [6, 4] \), meaning that for exchange symmetric crystal structures and with \( \delta \mu_2 = \delta \mu_3 = \delta \mu \) we have \( \beta = \beta/\delta \mu^2 \), \( \beta_{32} = \beta_{32}/\delta \mu^2 \), \( \gamma = \gamma/\delta \mu^4 \) and \( \gamma_{322} = \gamma_{322}/\delta \mu^4 \) where the barred quantities are dimensionless numbers which depend only on \( \{q_2\} \) and \( \{q_3\} \) that must be evaluated for each crystal structure. Doing so requires evaluating one loop Feynman diagrams with 4 or 6 insertions of \( \Delta_I \)'s. Each insertion of \( \Delta_I \) \( (\Delta_I^*) \) adds (subtracts) momentum \( 2q_I^* \) for some \( a \), meaning that the calculation consists of a bookkeeping task (determining which combinations of 4 or 6 \( q_I^* \)'s are allowed) that grows rapidly in complexity with the complexity of the crystal structure and a loop integration that is nontrivial because the momentum in the propagator changes after each insertion. See Ref. [4], where this calculation is carried out explicitly for 11 crystal structures in a mean-field NJL model upon making the weak coupling \( (\Delta_0 \ll \delta \mu) \) approximation. Note that in this approximation neither the NJL cutoff nor the NJL coupling constant appear in any quartic or higher Ginzburg-Landau coefficient, and they appear in \( \delta \mu \) only within \( \Delta_0 \). Hence, the details of the model do not matter as long as one thinks of \( \Delta_0 \) as a parameter, kept \( \ll \mu \).

It is easy to show that for exchange symmetric crystal structures any extrema of \( \Omega(\Delta_2, \Delta_3) \) in \( (\Delta_2, \Delta_3) \)-space must either have \( \Delta_2 = \Delta_3 = \Delta \), or have one of \( \Delta_2 \) and \( \Delta_3 \) vanishing [4]. It is also possible to show that the three-flavor crystalline phases with \( \Delta_2 = \Delta_3 = \Delta \) are electrically neutral whereas two-flavor solutions in which only one of the \( \Delta \)'s is nonzero are not [4]. We therefore analyze only solutions with \( \Delta_2 = \Delta_3 = \Delta \). In marked contrast to the two-flavor results of Ref. [6] which (ignoring the requirement of neutrality) show that many two-flavor crystal structures have negative \( \gamma \) and hence sextic order free energies that are unbounded from below, we find that \( \Omega(\Delta, \Delta) \) is positive for large \( \Delta \) for all the crystal structures that we investigate [4]. This allows us to minimize \( \Omega(\Delta, \Delta) \) with respect to \( \Delta \), thus evaluating \( \Delta \) and \( \Omega \).

We begin with the simplest three-flavor “crystal” structure in which \( \{q_2\} \) and \( \{q_3\} \) each contain only a single vector, making the \( \langle u \rangle \) and \( \langle d \rangle \) condensates each a single plane wave. This simple condensate yields a qualitative lesson which proves helpful in winnowing the space of multiple plane wave crystal structures [4]. For this simple “crystal” structure, all the coefficients in the Ginzburg-Landau free energy can be evaluated analytically [4]. The terms that occur in the three-flavor case but not in the two-flavor case, namely \( \beta_{32} \) and \( \gamma_{322} \), describe the interaction between the two condensates, and depend on the angle \( \phi \) between \( q_2 \) and \( q_3 \). For any angle \( \phi \), both \( \beta_{32} \) and \( \gamma_{322} \) are positive. And, both increase monotonically with \( \phi \) and diverge as \( \phi \to \pi [4] \). This divergence tells us that choosing \( q_2 \) and \( q_3 \) precisely antiparallel exactlys an infinite free energy price in the combined Ginzburg-Landau and weak-coupling limit in which \( \Delta \ll \delta \mu, \Delta_0 \ll \mu \), meaning that in this limit if we chose \( \phi = \pi \) we find \( \Delta = 0 \). Away from the Ginzburg-Landau limit, when the pairing rings on the Fermi surfaces widen into bands, choosing \( \phi = \pi \) exacts a finite price meaning that \( \Delta \) is nonzero but smaller than that for any other choice of \( \phi \). The high cost of choosing \( q_2 \) and \( q_3 \) precisely antiparallel can be understood qualitatively as arising from the fact that in this case the ring of states on the \( u \)-quark Fermi surface that “want to” pair with \( d \)-quarks coincides precisely with the ring.
that “wants to” pair with $s$-quarks \cite{9}. This simple two plane wave ansatz has been analyzed upon making the weak-coupling approximation but without making the Ginzburg-Landau approximation \cite{9}. All the qualitative lessons learned from the Ginzburg-Landau approximation remain valid and we learn further that the Ginzburg-Landau approximation always underestimates $\Delta$ \cite{9}.

The analysis of the simple two plane wave “crystal” structure, together with the observation that in more complicated crystal structures with more than one vector in \{${\bf q}_2$\} and \{${\bf q}_3$\} the Ginzburg-Landau coefficient $\beta_{32}$ ($\gamma_{322}$) is given in whole (in part) by a sum of many two plane wave contributions, yields one of two rules for constructing favorable crystal structures for three-flavor crystalline color superconductivity \cite{11}: \{${\bf q}_2$\} and \{${\bf q}_3$\} should be rotated with respect to each other in a way that best keeps vectors in one set away from the antipodes of vectors in the other set. The second rule is that the sets \{${\bf q}_2$\} and \{${\bf q}_3$\} should each be chosen to yield crystal structures which, seen as separate two-flavor crystalline phases, are as favorable as possible. The 11 crystal structures analyzed in Ref. \cite{4} allow one to make several pairwise comparisons that test these two rules. There are instances of two structures which differ only in the relative orientation of \{${\bf q}_2$\} and \{${\bf q}_3$\} and in these cases the structure in which vectors in \{${\bf q}_2$\} get closer to the antipodes of vectors in \{${\bf q}_3$\} are disfavored. And, there are instances where the smallest angle between a vector in \{${\bf q}_2$\} and the antipodes of a vector in \{${\bf q}_3$\} are the same for two different crystal structures, and in these cases the one with the more favorable two-flavor structure is more favorable. These considerations, together with explicit calculations, indicate that two structures, which we denote “$2\text{Cube}45z$” and “$\text{Cube}X$”, are particularly favorable.

In the $2\text{Cube}45z$ crystal, \{${\bf q}_2$\} and \{${\bf q}_3$\} each contain eight vectors pointing at the corners of a cube. If we orient \{${\bf q}_2$\} so that its vectors point in the ($\pm1, \pm1, \pm1$) directions in momentum space, then \{${\bf q}_3$\} is rotated relative to \{${\bf q}_2$\} by $45^\circ$ about the $z$-axis. In this crystal structure, the $\langle ud \rangle$ and $\langle us \rangle$ condensates are each given by the most favored two-flavor crystal structure \cite{11}. The relative rotation maximizes the separation between any vector in \{${\bf q}_2$\} and the nearest antipodes of a vector in \{${\bf q}_3$\}.

We arrive at the $\text{Cube}X$ structure by reducing the number of vectors in \{${\bf q}_2$\} and \{${\bf q}_3$\}. This worsens the two-flavor free energy of each condensate separately, but allows vectors in \{${\bf q}_2$\} to be kept farther away from the antipodes of vectors in \{${\bf q}_3$\}. We have not analyzed all structures obtainable in this way, but we have found one and only one which has a condensation energy comparable to that of the $2\text{Cube}45z$ structure. In the $\text{Cube}X$ structure, \{${\bf q}_2$\} and \{${\bf q}_3$\} each contain four vectors forming a rectangle. The eight vectors together point toward the corners of a cube. The 2 rectangles intersect to look like an “X” if viewed end-on. The color, flavor and position space dependence of the $\text{Cube}X$ condensate is given by

\begin{equation}
\epsilon_{2\alpha\beta}\epsilon_{2ij} \frac{2\Delta}{a} \left[ \cos \frac{2\pi}{a} (x + y + z) + \cos \frac{2\pi}{a} (-x - y + z) \right] \\
+ \epsilon_{3\alpha\beta}\epsilon_{3ij} \frac{2\Delta}{a} \left[ \cos \frac{2\pi}{a} (-x + y + z) + \cos \frac{2\pi}{a} (x - y + z) \right],
\end{equation}

where $a = \sqrt{3}\pi/q = 4.536/\delta\mu = \mu/(1.764M^2)$ is the lattice spacing. For example, with $M^2/\mu = 100, 150, 200$ MeV the lattice spacing is $a = 72, 48, 36$ fm. We depict this condensate in Fig. \[5\].

In Figs. 2 and 3, we plot $\Delta$ and $\Omega$ versus $M^2/\mu$ for the most favorable crystal structures that we have found, namely the $\text{Cube}X$ and $2\text{Cube}45z$ structures described above. We have taken the CFL gap parameter $\Delta_0 = 25$ MeV in these figures, but they can easily be rescaled to any value of $\Delta_0 \ll \mu$ \cite{11}. Fig. 2 shows that the gap parameters are large enough that the Ginzburg-Landau approximation is at the edge of its domain of reliability. However, results obtained for simpler crystal structures suggest that the Ginzburg-Landau calculation underestimates $\Delta$ and the condensation energy and that, even when it breaks down, it is a good qualitative guide to the favorable structure \cite{9}. We therefore trust the result, evident in Fig. 3, that these crystalline phases are both impressively robust, with one or other of them favored over a wide swath of $M^2/\mu$ and hence density. We do not trust the Ginzburg-Landau calculation to discriminate between these two structures, particularly given that although we have a qualitative understanding of why these two are favorable we have no qualitative argument for why one should be favored over the other.
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Figure 1. The CubeX crystal structure of Eq. 6. The figure extends from 0 to \(a/2\) in the \(x\), \(y\) and \(z\) directions. Both \(\Delta_2(r)\) and \(\Delta_3(r)\) vanish at the horizontal plane. \(\Delta_2(r)\) vanishes on the darker vertical planes, and \(\Delta_3(r)\) vanishes on the lighter vertical planes. On the upper (lower) dark cylinders and the lower (upper) two small corners of dark cylinders, \(\Delta_2(r) = +3\Delta\) (\(\Delta_2(r) = -3\Delta\)). On the upper (lower) lighter cylinders and the lower (upper) two small corners of lighter cylinders, \(\Delta_3(r) = -3\Delta\) (\(\Delta_3(r) = +3\Delta\)). The largest value of \(|\Delta_I(r)|\) is \(4\Delta\), occurring along lines at the centers of the cylinders. The lattice spacing is \(a\) when one takes into account the signs of the condensates; if one looks only at \(|\Delta_I(r)|\), the lattice spacing is \(a/2\).

We are confident that 2Cube45z is the most favorable structure obtained by rotating one cube relative to another. We are not as confident that CubeX is the best possible structure with fewer than 8+8 vectors. Regardless, the 2Cube45z and CubeX crystalline phases together make the case that three-flavor crystalline color superconducting phases are the ground state of cold quark matter over a wide range of densities. If even better crystal structures can be found, this will only further strengthen this case.

Fig. 3 shows that over most of the range of \(M_s^2/\mu\) where it was once considered a possibility, the gCFL phase can be replaced by a much more favorable three-flavor crystalline color superconducting phase. It also shows that it is hard to find a crystalline phase with lower free energy than the gCFL phase at the lowest values of \(M_s^2/\mu\) (highest densities) in the “gCFL window”. This narrow window where the gCFL curve remains the lowest in Fig. 3 is thus the most likely place in the QCD phase diagram to find the gCFL phase augmented by current-carrying meson condensates described in Refs. [11]. Except within this window, crystalline color superconducting phases with the CubeX or the 2Cube45z crystal structures provide an attractive resolution to the instability of the gCFL phase.

The three-flavor crystalline color superconducting phases with CubeX and 2Cube45z crystal structures are the lowest free energy phases that we know of, and hence candidates for the ground state of QCD, over a wide range of densities. One or other is favored over the CFL, gCFL and unpaired phases for \(2.9\Delta_0 < M_s^2/\mu < 10.4\Delta_0\), as shown in Fig. 3. For \(\Delta_0 = 25\) MeV and \(M_s = 250\) MeV, this translates to \(240\) MeV \(< \mu < 847\) MeV. With these choices of parameters, then, the lower part of this range of \(\mu\) (higher part of the range of \(M_s^2/\mu\)) is certainly superseded by nuclear matter. And, the high end of this range extends far beyond the \(\mu \sim 500\) MeV characteristic of the quark matter at the densities expected at the center of compact stars. If compact stars have quark matter cores, then, it is reasonable to include the possibility that the entire quark matter core could be in a crystalline color superconducting phase on the menu of options that must ultimately be winnowed by confrontation with astrophysical observations. If \(\Delta_0\) is larger, say \(\sim 100\) MeV, the entire quark matter core could be in the CFL phase.

Now that we have two candidates for the crystal structure of the three-flavor crystalline color superconducting phase of cold quark matter, favorable over a very wide range of intermediate densities, and a qualitative guide to the scale of \(\Delta\) and \(\Omega(\Delta)\), we can look ahead toward the calculation of astrophysically relevant observables. The heat capacity of the CubeX and 2Cube45z phases should be only quantitatively suppressed relative to that of unpaired quark matter, but their neutrino emissivity may be more significantly suppressed [4]. Both can now be calculated, yielding an estimate of the effects of a crystalline quark matter core on the rate at which a neutron star cools by neutrino emission. A crystalline color superconducting core, being both crystalline and superfluid, could be a region within which rotational vortices are pinned and hence (some) pulsar glitches originate [4]. Or, the presence of crystalline quark matter within neutron stars...
could be ruled out if it predicts glitch phenomenology in qualitative disagreement with that observed. The two microphysical properties of crystalline quark matter that must be estimated before glitch phenomenology can be addressed are the vortex pinning force and the shear modulus. Glitches occur if rotational vortices are pinned and immobile while the spinning pulsar’s angular velocity slows over years, with the glitch triggered by the catastrophic unpinning of long-immobile vortices. Immobilizing vortices requires sufficient pinning force and shear modulus. Estimating the pinning force will require analyzing how the CubeX and 2Cube45z phases respond when rotated. The shear modulus is related to the coefficients in the effective theory that describes the phonon modes of the crystal \[12\]. Analyzing the phonons in the three-flavor crystalline color superconducting phases with the 2Cube45z and CubeX crystal structures is thus a priority. These are the prerequisites to determining whether observations of pulsar glitches can be used to rule out (or in) the presence of quark matter in a crystalline color superconducting phase within neutron stars.

We thank the LBNL Nuclear Theory Group for hospitality. Research supported in part by the U.S. DOE under contract #DE-AC02-05CH11231 and cooperative research agreement #DF-FC02-94ER40818.

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