Effects of Mechanical Coupling on Dynamics of Balancing Tasks

Katsutoshi Yoshida · Atsushi Higeta · Shinchi Watanabe

Abstract Coupled human balancing tasks are investigated on the basis of both pseudo-neural controllers characterized by time-delayed feedback with random gain and natural human balancing tasks. It is numerically shown that in comparison to single balancing tasks, balancing tasks coupled by mechanical structures increase the stability of balancing errors in terms of both amplitude and velocity and also improve the tracking ability of the controllers. We then perform an experiment in which the numerical pseudo-neural controllers are replaced with natural human balancing tasks carried out using computer mice. The result shows that the coupling structure generates asymmetric tracking abilities in the subjects whose tracking abilities are nearly symmetric in their single balancing tasks.

Keywords Neural Controller · Mechanical Coupling · Visuomotor Tracking · Stability · Sensitivity

1 Introduction

Competitive and cooperative dynamics can arise when multiple agents (autonomous entities) share common resources and environments. Extensive research has been conducted on such mutual interactions. Research in this area can be broadly classified into two categories: the field of mathematical ecology (Sigmund and Hofbauer, 1998) which finds group behavior models of low degree of freedom, and the field of multi-robot systems (Asama et al., 1996) which develops individual agents generating group behavior. In the ecological approach, individual structures of agents are eliminated in order to develop low degree-of-freedom (DOF) models suitable for nonlinear analysis, while in the robotics approach, it is rather difficult to develop such low DOF models since detailed structures of agents must be designed in order to produce robots. To solve this dilemma, we previously developed the coupled inverted pendula (CIP) model of four DOF (Yoshida and Ohta, 2008), in which nonlinear behavior similar to interspecific competition in an ecosystem (Sigmund and Hofbauer, 1998) can be directly generated by individual mechanical structures of agents. The CIP model proposed by the authors consists of a pair of independently PD-controlled inverted pendula whose tips are connected with a rigid rod.

Our main interest in this work is to replace the PD controllers of the CIP model with human balancing tasks in order to investigate what type of dynamics occurs when neural controllers with random fluctuations, which are typical of human balancing tasks, are coupled with a mechanical structure. From the point of view of statistical mechanics, it has already been reported that random fluctuations arising in single human stick balancing tasks can be accurately modeled as an inverted pendulum with a time-delayed and randomly modulated feedback controller (Cabrera and Milton, 2002). Since such a controller does not simulate the structure of a neural network, it should be referred to as a pseudo-neural control model of human balancing tasks. The most remarkable effect of the pseudo-neural controller is that near stability boundaries, parametric noise can allow the controller to produce corrective movements on time scales shorter than the delay time of the controller under certain suitable conditions, exhibiting the scaling laws typical of self-similarity dynamics of on-off intermittency (Venkataramani et al., 1996). This effect has
also been experimentally confirmed via physical human tasks, such as stick balancing (Cabrera and Milton, 2002, 2004) and visuomotor tracking on a computer screen (Bormann et al, 2004). However, these single balancing models are not directly applicable to considering the coupling effect in balancing tasks. Meanwhile, the mathematical treatment of coupled neural networks has been developed in the field of nonlinear dynamical systems (Campbell et al, 2004, 2006), where the conditions for global stability of trivial solutions and the bifurcation and stability of nontrivial synchronous solutions are derived in a rigorous manner. However, randomly fluctuating coupled structures were not considered in the analysis.

In this paper, we propose a new model consisting of time-delayed and randomly modulated feedback controllers which are coupled with a mechanical structure equivalent to our CIP model. For this purpose, first we derive a linearized reduced–order version of the CIP model, after which we replace the PD controllers with pseudo-neural controllers as developed in the literature (Cabrera and Milton, 2002, 2004; Bormann et al, 2004). Using this model, we demonstrate the effects of coupling manifested as improvement in stability and sensitivity of the corrective motions in the balancing tasks. We also perform an experiment in which we replace the numerical controllers with human visuomotor tracking tasks performed by subjects using computer mice. It is experimentally shown that the coupling structure between two subjects induces asymmetric sensitivity in the corrective motions of the subjects.

2 Analytical Model

2.1 Single balancing tasks

Human stick balancing tasks and their scaling properties have been accurately modeled as an inverted pendulum with a time-delayed feedback of random gain in the following form (Cabrera and Milton, 2002):

\[ \ddot{\theta} + \gamma \dot{\theta} - \alpha \sin \theta + \beta R(t) \theta(t - \tau) = 0 \]  

(1)

where \( \tau \) is a time delay representing the latency of neural reflex in human balancing tasks and \( R(t) = 1 + \nu(\xi(t) \) is random feedback gain, \( \xi(t) \) is standard Gaussian white noise, and \( \nu \) represents the strength of the noise. Note that a stick with a length \( l \) and constant linear density is modeled as \( \alpha = 3g/(2l) \), where \( g \) is the gravitational acceleration.

The linearized version of (1):

\[ \dot{\Delta x} + \gamma \Delta \dot{x} - \alpha \Delta x + \beta R(t) \Delta x(t - \tau) = 0 \]  

(2)

can be interpreted as an equation of motion of the relative displacement \( \Delta x := x_T(t) - x_M(t) \), where \( x_T \) is the displacement of the upper end of the stick and \( x_M \) is that of the lower end of the stick in the balancing task. It has been shown in the literature (Bormann et al, 2004) that \( x_T(t) \) and \( x_M(t) \) are governed by the following equations:

\[ \ddot{x}_T + \gamma \dot{x}_T = \alpha \Delta x(t), \]  

(3a)

\[ \ddot{x}_M + \gamma \dot{x}_M = \beta R(t) \Delta x(t - \tau). \]  

(3b)

2.2 Coupled balancing tasks

In this paper, we investigate the type of stability which can arise if two sticks which are balanced independently are linked with a connecting rod at their upper ends. Figure 1 represents a physical example of this situation, where each subject manipulates one of the sticks at the lower end along the mechanical slider with the aim of maintaining the stick in upright position. Let \( q_{i1} \) and \( q_{i2} \) be the horizontal displacement of the upper and lower end of the \( i \)th stick respectively. Then, the presence of the connecting rod can be described by the distance \( l := q_{i1} - q_{i2} \) maintained by the fixed length of the rod. It should be noted that this constant length yields the following equalities:

\[ \dot{q}_{i1} = \dot{q}_{i2} =: \dot{q}_r, \quad \dot{q}_{i1} = \dot{q}_{i2} =: \dot{q}_r. \]  

(4)

For a simpler description of this coupled task, we propose the following model:

\[ 2 \ddot{q}_r + 2 \gamma \dot{q}_r = \alpha \Delta q_1(t) + \alpha \Delta q_2(t), \]  

(5a)

\[ \ddot{q}_{i1} + \gamma \dot{q}_{i1} = u_i(t, \tau). \]  

(5b)

\[ u_i(t, \tau) := \beta R_i(t) \Delta q_i(t - \tau) \quad (i = 1, 2) \]  

(5c)

where \( \Delta q_i := q_T - q_{Mi} \) (\( i = 1, 2 \)) represents the horizontal displacement between the upper and lower ends of the \( i \)th stick respectively, as obtained in independent

![Fig. 1 A physical example of a coupled human balancing task.](image)
balancing, and \( R_i(t) = 1 + \nu \xi_i(t) \) (\( i = 1, 2 \)) are independent random feedback gains, where \( \xi_i \) (\( i = 1, 2 \)) are mutually independent standard Gaussian white noises. The equation (5b) can be reduced to the following relative form:

\[
\ddot{\Delta q}_i + \gamma \dot{\Delta q}_i - \frac{1}{2} \alpha (\Delta q_1 + \Delta q_2) + u_i(t, \tau) = 0 \quad (i = 1, 2).
\]

The proposed model (5) provides a linearized reduced-order counterpart of the CIP model (Yoshida and Ohta, 2008), as schematically shown in Fig. 2. In other words, by assuming \(|\theta_i| \ll 1\), the gravitational restoring force proportional to \( \sin \theta_i \) in the CIP model is approximated as a linear spring force with a negative coefficient \( \alpha < 0 \). This setup is expected to provide the simplest model of coupled human balancing tasks. It is noteworthy that the coupled model (5b) coincides with the single model (2) if \( \Delta q_1 = \Delta q_2 \) and \( \Delta \dot{q}_1 = \Delta \dot{q}_2 \). In what follows, we choose \( \gamma = 50, \alpha = 22, \nu = 0.6 \), and \( \tau = 0.1 \). For convenience, we refer to \( \Delta x \) and \( \Delta q_i \) as balancing errors.

3 Effects of Coupling in the Balancing Model

3.1 On-off intermittency

It is known that the human stick balancing task exhibits on-off intermittency with respect to the balancing error (Cabrera and Milton, 2002; Bormann et al, 2004), as well as that the on-off intermittency occurs when the largest Lyapunov exponent \( \lambda_1 \) is slightly larger than zero (Venkataramani et al, 1996).

Figure 3 shows the largest Lyapunov exponent \( \lambda_1 \) for the single and the coupled balancing task as a function of the feedback gain \( \beta \).

The result for \( \Delta q_2 \) is omitted due to its similarity to that for \( \Delta q_1 \). We choose \( \lambda_1 = 5 \times 10^{-4} \) to be slightly larger than zero in order to produce on-off intermittency of the balancing errors \( \Delta x \) in (2) and \( \Delta q_1 \) in (5), as shown in Fig. 4.

Figure 5(a) and (b) shows double logarithmic plots of the power spectra of the balancing errors \( \Delta x \) and \( \Delta q_1 \) respectively. Both results have two linear slopes representing two power law regimes, which is in good agreement with previous balancing experiments (Cabrera and Milton, 2002, 2004; Bormann et al, 2004), in which the two slopes were interpreted as a sign of on-off intermittency (Venkataramani et al, 1996). From these results, it can be concluded that the coupling term does not change the scaling law typical of the on-off intermittency of balancing tasks.

3.2 Coupling-induced stability

The most remarkable effect of the coupling in (5) is that the balancing error is drastically suppressed by the coupling, as shown in Fig. 4(a) and (b). That is, the error \( \Delta q_1 \) of the coupled balancing is as low as 1% of \( \Delta x \) of that of single balancing in terms of both maximal values and root mean square (RMS) values. Figure 6 shows...
The mean RMS values, represented by the solid lines, are 1.5 × 10⁻³ for the single balancing and 3.0 × 10⁻¹ for the coupled balancing. The result for \( \Delta q_2 \) is omitted due to its similarity to that for \( \Delta q_1 \). It is clearly seen in Fig. 6 that the coupling term reduces the balancing error \( \Delta q_1(t) \) to 0.02% of \( \Delta x \) of the single balancing. This result implies that the mechanical coupling structure can improve the stability of amplitude in human balancing tasks.

In order to examine the velocities of the balancing errors, the ratio of the probability density \( p(\Delta \dot{q}_1) \) divided by \( p(\Delta \dot{x}) \) is shown in Fig. 7, where \( \Delta \dot{q}_1 \) and \( \Delta \dot{x} \) are the velocity errors of the coupled and single balancing respectively. It is seen from Fig. 7 that \( p(\Delta \dot{q}_1) \) is approximately twice as high as \( p(\Delta \dot{x}) \) near the origin. Since in the linear approximation the velocity errors \( \Delta \dot{x} \) and \( \Delta \dot{q}_1 \) are proportional to the respective slant angles of the sticks, the result suggests that the probability of maintaining constant angles is nearly doubled by the mechanical coupling structure.

Therefore, it can be concluded from these results that the mechanical coupling structure possibly reduces the balancing error in terms of both amplitude and velocity under suitable conditions.

### 3.3 Coupling-induced sensitivity

It has already been reported that near stability boundaries, parametric noise can allow time-delayed feedback controllers to produce corrective movements on a time scale shorter than that of the delayed feedback (Cabrera and Milton, 2002). In order to observe this effect of improvement in sensitivity as a function of time, we consider the short-time cross-correlation coefficient (STCC) in the following manner. Letting \( x(t), y(t) \) be a pair of time series to be compared and \( \Delta t \) be the length of the time interval of short-time averaging, we define the STCC as

\[
R(x, y; \tau) = \frac{C(x - m_x, y - m_y; \tau, t)}{\sigma_x \sigma_y},
\]

where

\[
C(x, y; \tau, t) := \langle x(s) y(s + \tau) \rangle_{[t, t+\Delta t]},
\]

\[
m_x := \langle x(s) \rangle_{[t, t+\Delta t]}, \quad \sigma_x := \left( \langle (x(s) - m_x)^2 \rangle_{[t, t+\Delta t]} \right)^{1/2}.
\]
where $\langle X(s) \rangle_{[a, b]} := (b - a)^{-1} \int_a^b X(s)ds$ is a temporal average of $X(s)$ over the time interval $[a, b]$.

In what follows, we draw a comparison between the velocities of the upper end and the lower end of each stick by using the STCC in (5). More specifically, we focus on $R(\dot{x}_T, \dot{x}_M)$ for the single system (3) and $R(\dot{q}_T, \dot{q}_M)$ ($i = 1, 2$) for the coupled system (5). Moreover, in order to evaluate the sensitivity of the corrective movements of the feedback controllers, we also define the first dominant peak points $\hat{\tau}_x$, $\hat{\tau}_{q_1}$, $\hat{\tau}_{q_2}$ in $R(\dot{x}_T, \dot{x}_M)$, $R(\dot{q}_T, \dot{q}_M)$ ($i = 1, 2$) respectively. These peak points are assumed to evaluate the tracking abilities of the feedback controllers.

Figure 8 shows the realization of the STCCs, $R(\dot{q}_T, \dot{q}_M)$ and $R(\dot{q}_T, \dot{q}_M')$ of the coupled system (5) at $t = 36$ for $\Delta t = 5$. In this plot, there are single peaks at $\hat{\tau}_{q_1} = 0.105$ and $\hat{\tau}_{q_2} = 0.035$. Since the delay time of the controller is $\tau = 0.1$ in our calculations, on average the first controller $u_1(t, \tau)$ corrects the stick movement slower than the delay time, while the second controller $u_2(t, \tau)$ performs the corrections faster than the delay time. Regarding $\hat{\tau}_{q_1}$, $\hat{\tau}_{q_2}$ as the tracking ability of the controller, it can be said that the second controller can track the stick movement $0.105/0.035 = 3$ times faster than the first controller. This indicates that symmetrically placed controllers $u_1$ and $u_2$ having the same specifications can develop asymmetric tracking abilities in short time scales.

The peak points $\hat{\tau}_x$ and $\hat{\tau}_{q_i}$ ($i = 1, 2$) as a function of time $\tau$ are plotted in Fig. 9. The plots are numerically constructed from single realizations of numerical solutions of (3) and (5). It is clear that the peak points randomly fluctuate over time, forming intermittent clusters of points. It is also confirmed that the sensitive region in which the peak point is smaller than the delay time have sufficient length for physical observations. However, the effects of coupling are not clearly observable in Fig. 9, and therefore the differences between the results (a), (b), and (c) in Fig. 9 are hardly observed in these plots.

In order to evaluate the effects of coupling on the tracking ability, the probability densities of the peak points $\hat{\tau}_x$ and $\hat{\tau}_{q_i}$ are shown in Fig. 10 as averaged over 100 realizations over the time interval $[0, 1200]$ of the numerical solutions of (3) and (5). The result for $\hat{\tau}(\dot{q}_2)$ is omitted because it is quite similar to that for $\hat{\tau}(\dot{q}_1)$. It is clearly observed in Fig. 11 that the density of coupled balancing, represented as small rectangles, produces a simple peak which is $38\%$ higher than that of the single balancing, where the peak is placed at a time scale of $\hat{\tau}$ which is $20\%$ shorter than that of the single balancing. From these results, it can be concluded that the mechanical coupling structure increases the probability of occurrence of faster corrective movements and improves the tracking ability of the controllers.

4 Experiment of Coupled Visuomotor Tracking on a Computer Screen

4.1 Experimental setup

We perform an experiment in which the numerical pseudo-neural controllers in (5) are replaced with natural human balancing tasks as shown in Fig. 11 (a). In practice, the variable $x_M$ in the single model (5a) is replaced with the movement of the mouse manipulated by a subject, and $q_{M1}$, $q_{M2}$ in (5a) are replaced with those of two subjects. Each subject is presented with combinations of a thick and a thin line on a computer screen, as shown in Fig. 11 (b). The thick lines represent the upper ends $x_T$, $q_T$, as calculated from the numerical models (5a) and (5a), while the thin lines represent the lower ends...
**Fig. 11** Experimental setup of the coupled human balancing tasks.

$x_M$, $q_{Mi}$ manipulated by the subjects. The screen resolution is 1200 × 600 (pixels), where the range of the displacement $[-3, 3]$ in the numerical model maps to the horizontal range of pixels $[1, 1200]$ on the screen. The movements performed by the subjects $x_M$, $q_{Mi}$ are recorded and substituted into the numerical models (3a) and (5a) with a sampling rate of 50Hz, and the set of lines on the screen is animated with the same rate.

The experimenter issues the following instructions to the subjects:

- Each subject should collaborate with their partner to maintain the assigned pendulum, provided as an overhead view on the computer screen, in the upright position by manipulating the lower end of the pendulum, which is represented as a thin line.
- The upper ends, represented as thick lines, are assumed to be connected by a rigid rod in such a way that a constant distance is maintained between the thick lines.

According to these instructions, after the countdown, the numerical simulation is started and the experiment begins. The experiment ends after ten minutes or when any of the lines leaves the visible range.

The experiment begins after a countdown performed by the experimenter, and ends after ten minutes or when any of the lines leaves the visible range.

The subject can abort the experiment at anytime.
Fig. 13 Experimental velocities of the target $\dot{q}_T$ (thick line) and of the subject’s movement $\dot{q}_M$ (thin line), and power spectra of the balancing error $\Delta q_i$ in the coupled balancing by the pair of subjects A ($i=1$) and B ($i=2$).

Fig. 14 Experimental short-time cross-correlation coefficients $R$ in single balancing independently performed by subjects A and B.

Fig. 15 Experimental short-time cross-correlation coefficients $R$ in coupled balancing performed by the pair (A,B).

4.2 Experimental results

First, we focus on subjects A and B and the pair (A,B). Figure 12 shows the experimental results for the single balancing independently performed by subjects A and B. It is clearly seen that the thick line, which corresponds to the velocity $\dot{x}_T$ of the target (the upper end of the stick), slightly precedes the thin line, which corresponds to the subject’s velocity $\dot{x}_M$, due to a combination of time delays from neural reflex and computer processing. It is also clear in the case of both subjects that the power spectrum $S(\omega)$ of the balancing error $\Delta x$ exhibits power laws typical of the neural controllers, which have two different power laws with an exponent $\approx -1/2$ in the lower frequency range (Cabrera and Milton, 2002).

On the other hand, Fig. 13 shows experimental results for the case where the same subjects are coupled by a rigid rod of length $l$ in the numerical model. It should be noted that the velocities of both targets (thick lines) coincide as a result of the equalities (4). It is again clear that the thick line, which corresponds to the target velocity $\dot{q}_T$, slightly precedes the thin line, which corresponds to the subject’s velocities $\dot{q}_M1$ and $\dot{q}_M2$, as well as that the balancing errors $\Delta q_1$, $\Delta q_2$ are governed by a power law with an exponent of $\approx -1/2$ in the lower frequency range. From these results, it can be concluded that the mechanical coupling structure between the two balancing tasks maintains the properties of time delays and scaling laws typical of the independently performed single balancing tasks.

4.3 Effects of coupling on stability and sensitivity

As already discussed in Section 3.3, the correlation time (the first dominant peak in STCC) allows us to evaluate the sensitivity of the corrective movements of the subjects, in other words, the tracking ability of the subjects. Experimental correlation times in single balancing independently performed by the two subjects (denoted as A and B again) are shown in Fig. 14 as peak points $\hat{\tau}_{xy}$ of 0.14, 0.12 of STCC. The differences between the correlation times become slightly larger when the mechanical coupling structure is placed between the two balancing tasks, as shown in Fig. 15 i.e., $\hat{\tau}_{xy} = 0.12$, 0.18. The correlation times obtained from different trials of the experiment and their averages are listed in Table 1. It is clearly seen from the averages in Table 1 that the coupling structure increases the individual differences between the subjects. That is, their single balancing tasks yield similar tracking abilities ($\langle \hat{\tau} \rangle = 0.132$, 0.136), while they become asymmetric ($\langle \hat{\tau} \rangle = 0.128$, 0.168) when they are coupled by the mechanical structure. In the same manner, Table 2 lists the RMS of the balancing errors and their averages to estimate the stabilities of the balancing. It is clearly shown from the averages in Table 2 that the coupling has the opposite effect on RMS as compared with the STCC, that is, the coupling structure decreases the individual differences.
in the RMS of the balancing errors from \( \langle \text{RMS} \rangle = 3.62 \), 5.34 for the single task to \( \langle \text{RMS} \rangle = 3.02 \), 2.50 for the coupled task.

The results in Table 1 and 2 imply that the coupling structure increases the individual differences between the tracking abilities (STCC), while it decreases those of the stabilities of balancing (RMS). This effect on the six subjects can be seen in Fig. 16 which shows experimental plots of the correlation time \( \tau \) versus the RMS of the balancing errors over all trials of the single balancing tasks performed by subjects A, B, C, D, E, and F and the coupled balancing tasks performed by the pairs (A,B), (C,D), and (F,D). The green plots indicate the results of the single tasks, and the orange plots indicate those of the coupled tasks. The oval is centered at the average point of the corresponding plots and the radius in the horizontal and the vertical direction represents the standard deviation of the correlation time and of the RMS of balancing errors respectively. By comparing the single tasks (green oval) with the coupled tasks (orange oval), it can be seen that the coupling structure decreases the individual differences in correlation time, while it increases those of the RMS. This result statistically reconfirms the coupling effect arising in the two subjects as already shown in Table 1 and 2.

One possible explanation of these effects of coupling on the individual differences might be the separation of roles assumed by the subjects in the case that requires cooperation. However, similar asymmetrical properties can arise in the numerical controller \( u_i(t, \tau) \) in (6c), which has no intelligence to cooperate. Moreover, the numerical controllers provide feedback of the combination of their own past state and the shared state, i.e.,

\[
\Delta q_i = q_T - q_{Mi},
\]

in the following form:

\[
u_i = u_i(\Delta q_i) = \beta R_i(t) \Delta q_i(t - \tau) \quad \text{in (6c).} \tag{7}
\]

On the contrary, the human subjects can also receive visual feedback of their partner’s state via the shared computer screen in Fig. 11 and as a result their controller model may appear to have the following structure:

\[
u_i = u_i(\Delta q_1, \Delta q_2) \quad (i = 1, 2).
\]

Developing precise descriptions of such a human controller model will provide the first step toward exploring the coupling effects, such as the changes in the individual differences. However, the detailed explanation of these effects is beyond the scope of this paper and will be discussed elsewhere.

### 5 Conclusion

Coupled human balancing tasks are investigated on the basis of pseudo-neural controllers modeled by using timedelayed feedback with random gain. It is numerically shown that when compared with the case of single balancing tasks, the coupling structure increases the stability of balancing errors in terms of both amplitude and velocity and also improves the tracking ability of the controllers. We then perform an experiment in which the pseudo-neural controller in the numerical model is replaced with natural human balancing tasks carried out using computer mice. The result shows that the mechanical coupling structure increases the individual differences in tracking abilities between the subjects, while it decreases that of the stabilities of balancing errors.

---

**Table 1** Correlation times at different trials for the pair (A,B).

| Subject | Index of trial | Average (\(\tau\)) |
|---------|----------------|-------------------|
| Single: | 1  | 2  | 3  | 4  | 5  | \(\langle \tau \rangle\) |
| A       | 0.12 | 0.14 | 0.14 | 0.12 | 0.14 | 0.132 |
| B       | 0.14 | 0.12 | 0.14 | 0.14 | 0.14 | 0.136 |
| Coupled: | 1  | 2  | 3  | 4  | 5  | \(\langle \tau \rangle\) |
| A       | 0.12 | 0.12 | 0.12 | 0.12 | 0.16 | 0.128 |
| B       | 0.18 | 0.16 | 0.16 | 0.18 | 0.16 | 0.168 |

**Table 2** RMS of the balancing errors at different trials for the pair (A,B).

| Subject | Index of trial | Average (RMS) |
|---------|----------------|---------------|
| Single: | 1  | 2  | 3  | 4  | 5  | \(\langle \text{RMS} \rangle\) |
| A       | 2.4 | 2.9 | 3.8 | 5.5 | 3.5 | 3.62 |
| B       | 4.9 | 4.9 | 5.8 | 5.0 | 6.1 | 5.34 |
| Coupled: | 1  | 2  | 3  | 4  | 5  | \(\langle \text{RMS} \rangle\) |
| A       | 2.9 | 3.5 | 2.4 | 3.9 | 2.4 | 3.02 |
| B       | 2.4 | 2.8 | 2.6 | 2.7 | 2.0 | 2.50 |

**Fig. 16** Experimental plots of the correlation time \( \tau \) versus the RMS of the balancing errors over all trials of the single balancing tasks performed by subjects A, B, C, D, E, and F and the coupled balancing tasks performed by the pairs (A,B), (C,D), and (F,D).
Our model and experimental method is expected to provide the simplest way to understanding the cooperative behavior between humans sharing mechanical contacts as well as new insights into welfare engineering and related fields. In future work, we plan to perform model identification of human visuomotor tracking tasks in order to characterize the trade-off mechanism between tracking abilities and stabilities produced by coupled human subjects.

References

Asama H, Fukuda T, Arai T, Endo I (eds) (1996) Distributed Autonomous Robotic Systems 2. Springer-Verlag
Bormann R, Cabrera JL, Milton JG, Eurich CW (2004) Visuomotor tracking on a computer screen—an experimental paradigm to study the dynamics of motor control. Neurocomputing 58(60):517–523
Cabrera JL, Milton JG (2002) On-off intermittency in a human balancing task. PRL 89(15):158,702
Cabrera JL, Milton JG (2004) Human stick balancing: Tuning lévy flights to improve balance control. Chaos 14(3):691–698
Campbell SA, Edwards R, van den Driessche P (2004) Delayed coupling between two neural network loops. SIAM Journal on Applied Mathematics 65(1):316–335
Campbell SA, Ncube I, Wu J (2006) Multistability and stable asynchronous periodic oscillations in a multiple-delayed neural system. Physica D 214:101–119
Sigmund K, Hofbauer J (1998) Evolutionary Games and Population Dynamics. Cambridge University Press
Venkataramani SC, Jr TMA, Ott E, Sommerer JC (1996) On-off intermittency: Power spectrum and fractal properties of time series. Physica D pp 66–99
Yoshida K, Ohta H (2008) Coupled inverted pendula model of competition and cooperation. Journal of System Design and Dynamics 2(3):727–737