Phase velocity to distance conversion in machine location of array recorded teleseismic events

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ABSTRACT. In on-line deployment of small computer systems with limited machine capability, the object of locating seismic sources at least in the teleseismic distance range essentially requires, besides azimuth, accurate deduction of epicentral distance from apparent phase velocity in minimum operational steps. It has been shown that a seventh order polynomial in positive powers of apparent $P$-velocity yields reasonably precise distance estimates. The merits of this optimum relation have also been discussed in the light of its application to the Gauribidanur Array real-time processing system.

1. Introduction

The phase velocity due to a plane wave traversing a group of detectors placed in one horizontal plane, viz., an array of seismometers, varies non-linearly with epicentral distance. The velocity tends to increase, generally, with increasing distance owing to decreasing angle of emergence at the surface. For example, the phase velocity of $P$ wave due to a nearly surface-focus source increases from about 10 km/sec at 20° to about 24.5 km/sec at 100° (Herrin 1968).

In off-line analysis, the epicentral distance corresponding to a known value of phase velocity is obtained from the digitized distance-velocity curve with reasonable precision. However, in on-line applications using systems with limited machine capability, e.g., the TDC 12 real-time processing system at Gauribidanur Array (Ram Datt and Dilip Kumar 1973), it is desirable to express the distance-velocity function as a polynomial which can be easily incorporated in the form of a library routine to reproduce the distances as best as possible.

Further, to economise on machine time, an optimum polynomial is sought for where-in the low order of the function is compromised with a satisfactory limit of conversion accuracy throughout the range of distance. In the present case, better than 1/2° accuracy is considered quite satisfactory for our application for distances ranging between 20° and 100°.

An attempt has been made to quantitatively assess the implications of fitting polynomials both in positive and negative powers of phase velocity beginning with a least-squares line up to 12th order expressions. On the basis of merits of the fit, it has been found that the 7th order polynomial in positive powers of velocity represents optimum conditions beyond which order no significant reduction in error (departure from true values) in the deduced distance is achieved. Under this conversion scheme, the coefficient system converges fast so that the source location procedure becomes relatively more reliable and economical in terms of computer time.

2. Phase velocity across the array

The basic parameter in the source location is the pair of time lags along the lines of equi-spaced (2.5 km) seismometers of the L-shaped medium-aperture (about 25 km × 25 km) Gauribidanur Array (GBA). Obviously, the closer to the true values the computed lag pair is, the higher is the accuracy of epicentral location for teleseismic events. A lag is governed by the component, along the arm, of the velocity of propagation in the plane of the array as well as on the interelement spacing $D$. The horizontal velocity, different from the actual velocity with which the wave travels in the earth’s interior, is the apparent phase velocity $V_a$ we are referring to.

An expression for $V_a$ in terms of the two lags $T_R$ and $T_B$ is given by (Arora 1967, 1971):

$$V_a = \frac{V_R}{V_R + V_B} \left(\frac{V_R^2 + V_B^2}{2}\right)^{-1/2}$$

where $V_R$ and $V_B$ are the component velocities along the $R$- and the $B$-arm respectively. From the point of view of velocity and azimuthal discrimination of signals (Birtill and Whiteway 1965) the reliability factor tends to diminish as the incident wave approaches to coincide with any one of the arms, a zero lag being associated with infinite apparent speed.

In the case of digitized seismic data, the expression for $V_a$ reduces to (Ram Datt et al. 1969):
| $\Delta$ | $V_a$   | $\Delta$ | $V_a$   | $\Delta$ | $V_a$   | $\Delta$ | $V_a$   |
|-------|--------|---------|--------|---------|--------|---------|--------|
| 20.0  | 9.8494 | 20.5    | 10.0625| 21.0    | 10.2849| 21.5    | 10.5113|
| 22.0  | 10.7312| 22.5    | 10.9431| 23.0    | 11.1520| 23.5    | 11.3565|
| 24.0  | 11.5518| 24.5    | 11.7533| 25.0    | 11.8953| 25.5    | 12.0347|
| 26.0  | 12.1538| 26.5    | 12.2532| 27.0    | 12.3339| 27.5    | 12.5958|
| 28.0  | 12.4307| 28.5    | 12.4602| 29.0    | 12.4843| 29.5    | 12.4579|
| 30.0  | 12.5087| 30.5    | 12.5402| 31.0    | 12.5752| 31.5    | 12.6113|
| 32.0  | 12.6409| 32.5    | 12.6792| 33.0    | 12.7148| 33.5    | 12.7535|
| 34.0  | 12.7938| 34.5    | 12.8358| 35.0    | 12.8787| 35.5    | 12.9227|
| 36.0  | 12.9690| 36.5    | 13.0170| 37.0    | 13.0693| 37.5    | 13.1230|
| 38.0  | 13.1776| 38.5    | 13.2390| 39.0    | 13.2861| 39.5    | 13.3396|
| 40.0  | 13.3931| 40.5    | 13.4472| 41.0    | 13.5021| 41.5    | 13.5382|
| 42.0  | 13.6148| 42.5    | 13.6722| 43.0    | 13.7310| 43.5    | 13.7908|
| 44.0  | 13.8519| 44.5    | 13.9140| 45.0    | 13.9768| 45.5    | 14.0405|
| 46.0  | 14.1060| 46.5    | 14.1736| 47.0    | 14.2434| 47.5    | 14.3149|
| 48.0  | 14.3887| 48.5    | 14.4584| 49.0    | 14.5294| 49.5    | 14.6096|
| 50.0  | 14.6695| 50.5    | 14.7395| 51.0    | 14.8094| 51.5    | 14.8794|
| 52.0  | 14.9408| 52.5    | 15.0705| 53.0    | 15.0926| 53.5    | 15.1650|
| 54.0  | 15.2410| 54.5    | 15.3167| 55.0    | 15.3929| 55.5    | 15.4697|
| 56.0  | 15.5481| 56.5    | 15.6335| 57.0    | 15.7106| 57.5    | 15.7941|
| 58.0  | 15.8790| 58.5    | 15.9612| 59.0    | 16.0438| 59.5    | 16.1257|
| 60.0  | 16.2061| 60.5    | 16.2849| 61.0    | 16.3626| 61.5    | 16.4395|
| 62.0  | 16.5159| 62.5    | 16.5535| 63.0    | 16.6373| 63.5    | 16.7265|
| 64.0  | 16.8416| 64.5    | 16.9285| 65.0    | 17.0166| 65.5    | 17.1056|
| 66.0  | 17.1950| 66.5    | 17.2832| 67.0    | 17.3729| 67.5    | 17.4629|
| 68.0  | 17.5694| 68.5    | 17.6542| 69.0    | 17.7551| 69.5    | 17.8618|
| 70.0  | 17.9779| 70.5    | 18.1029| 71.0    | 18.2314| 71.5    | 18.3611|
| 72.0  | 18.4912| 72.5    | 18.6185| 73.0    | 18.7412| 73.5    | 18.8894|
| 74.0  | 18.9756| 74.5    | 19.0012| 75.0    | 19.0963| 75.5    | 19.3231|
| 76.0  | 19.4434| 76.5    | 19.5603| 77.0    | 19.7030| 77.5    | 19.8456|
| 78.0  | 19.9890| 78.5    | 20.1331| 79.0    | 20.2803| 79.5    | 20.4297|
| 80.0  | 20.5783| 80.5    | 20.7204| 81.0    | 20.8766| 81.5    | 21.0298|
| 82.0  | 21.1833| 82.5    | 21.3500| 83.0    | 21.5457| 83.5    | 21.7462|
| 84.0  | 21.9506| 84.5    | 22.1513| 85.0    | 22.3445| 85.5    | 22.5323|
| 86.0  | 22.6854| 86.5    | 22.8366| 87.0    | 22.9746| 87.5    | 23.1077|
| 88.0  | 23.2997| 88.5    | 23.3613| 89.0    | 23.4747| 89.5    | 23.5772|
| 90.0  | 23.6076| 90.5    | 23.7474| 91.0    | 23.8292| 91.5    | 23.8888|
| 92.0  | 23.9351| 92.5    | 24.0198| 93.0    | 24.0892| 93.5    | 24.1345|
| 94.0  | 24.1823| 94.5    | 24.2234| 95.0    | 24.2577| 95.5    | 24.2869|
| 96.0  | 24.3108| 96.5    | 24.3394| 97.0    | 24.4369| 97.5    | 24.5666|
| 98.0  | 24.3008| 98.5    | 24.3014| 99.0    | 24.3619| 99.5    | 24.3619|

$\Delta$ = Epicentral distance (degrees)

\[
V_a = \frac{D}{\delta t} (N_R^2 + N_B^2)^{-1/2}
\]

(2)

where $\delta t$ is the sampling interval, $N_R$ and $N_B$ are integer numbers proportional to the corresponding lags and satisfy the relations:

\[
T_B = N_B \delta t \quad T_B = N_B \delta t
\]

(3)

It follows therefore that in the digital approach while the resolution in $V_a$ depends upon the sampling rate, the precision in $V_a$ estimate depends upon the accuracy with which $N_R$ and $N_B$ are determined and hence upon the signal correlation across the array. Improvements in the lag estimates can be achieved by using an interpolation method so as to obtain an epicentre from the continuous distribution of points on the globe. In the GBA on-line system, once the arrival of a seismic signal is detected, control is transferred to a background program which estimates the lag pair and
computes the epicycral distance together with the probable error in the computed distance.

3. Distance deduction : Polynomial fitting

We seek to obtain an nth order polynomial of the form:

\[ \Delta = A_0 + A_1 V_a + \ldots + A_n V_a^n \]  

(4)

where \( \Delta \) is the epicycral distance, and \( A_0, A_1, \ldots, A_n \) are \( n+1 \) real coefficients. The set of a total of \( N, N=161 \), data points \([\Delta, (V_a)]\) in the range \( 20^\circ < \Delta < 100^\circ \) are derived (Arora and Krishnan 1970) from the Herrin's surface-focus \( P \) travel-times at \( \frac{1}{2}^\circ \) interval and are shown in Table 1. The sum of error square function \( S \) of reproduction of \( \Delta \) is given by:

\[ S = \sum_{i=1}^{N} [A_0 + A_1 (V_a) + \ldots + A_n (V_a^n)]^2 - \Delta_i^2 \]  

(5)

For minimising \( S \), we take partial d.c.'s w.r.t. each coefficient and equate each of them to zero. Thus

\[ A_0 \sum_i (V_a) + A_1 \sum_i (V_a^2) \ldots + A_n \sum_i (V_a^n) = \sum_i \Delta_i \]  

(6)

The set of Eqns. (6) can be solved by inverting the non-singular square matrix of order \( n+1 \)

\[
\begin{bmatrix}
N \\
\sum_i (V_a) \\
\sum_i (V_a^2) \\
\sum_i (V_a^n)
\end{bmatrix}
\]

and pre-multiplying it with the column vector (8). The solution matrix is the vector of \( n+1 \) elements (9):

\[
\begin{bmatrix}
\sum_i \Delta_i \\
\sum_i (V_a) \Delta_i \\
\ldots \\
\sum_i (V_a^n) \Delta_i
\end{bmatrix}
\]

(8)

\[
\begin{bmatrix}
A_0 \\
A_1 \\
\ldots \\
A_n
\end{bmatrix}
\]

(9)

A computer routine SOLVER, which incorporates matrix inversion and matrix multiplication, has been written for the BBSM-6 computer to perform the above job. Polynomials up to 12th order have been fitted through the entire range \( 20^\circ < \Delta < 100^\circ \) in powers of \( V_a \) as well as \( 1/V_a \) and the results showing the errors in computed \( \Delta \) together with the standard deviation in this error are presented in Table 2.

4. Discussion

As shown in Table 2, we have calculated departures from the true values in the reproduced values of \( \Delta \) over three distinct ranges, viz., \( 20^\circ < \Delta < 28^\circ \), \( 28^\circ < \Delta < 88^\circ \) and \( 88^\circ < \Delta < 100^\circ \), of the Herrin's velocity function using \( V_a \) as well as \( 1/V_a \) data. From these computations it appears that a seventh order polynomial (Fig. 1) in \( V_a \) is reasonably satisfactory for our purpose. In comparison, the optimum polynomial in \( 1/V_a \) goes to eighth order. Besides, the chain of coefficients in the \( V_a \) case is found to be highly convergent while that in the \( 1/V_a \) case is highly divergent (Table 3). Under the circumstances the seventh order polynomial in \( V_a \) becomes more tempting to use, particularly when a single polynomial is needed for use in small on-line computer systems to yield best possible results throughout the range.

Although the polynomial fitting ideally serves to give \( \Delta \) to within \( \frac{1}{2}^\circ \) accuracy, the errors in basic parameter \( V_a \) generally creeps in due to following main reasons which may affect the source location.

(i) Surface-focus assumption, (ii) Limited directional response of the medium-aperture I-pattern of the array, (iii) Lateral inhomogeneities in the receiver crust and the possible effects, though small, of a layered crust on the arrival angles and hence on the apparent velocity (Nuttli 1964, Hasegawa 1971, Brown and Enayatollah 1973).

The basic Fortran compiler of the TDC-12 with 12-bit word length permits usage of number
### Table 2

Comparative estimate of departure from true values in computed $\Delta$ using a single polynomial fit with (i) positive powers of $V_{a}$ and (ii) negative powers of $V_{a}$

- $n =$ Order of polynomial; $\sigma(a)_{a,b} =$ True minus computed $\Delta$ in the range $a$ to $b$ of $\Delta$

| $n$ | $\sigma(n)_{a,28}$ | $\sigma(n)_{b,28}$ | $\sigma(n)_{a,100}$ | With powers of $V_{a}$ | With powers of $1/V_{a}$ |
|-----|--------------------|--------------------|----------------------|-----------------------|------------------------|
| 1   | $-0.51 \pm 2.19$  | $1.37 \pm 3.59$   | $-3.37 \pm 2.00$    | $3.17 \pm 8.01$    | $-1.08 \pm 0.88$     |
| 2   | $0.41 \pm 4.62$   | $-0.14 \pm 1.68$  | $0.66 \pm 2.72$    | $-0.71 \pm 4.36$  | $0.03 \pm 1.74$     |
| 3   | $-0.28 \pm 4.76$  | $0.14 \pm 1.64$   | $16.58 \pm 2.67$   | $-0.98 \pm 2.24$  | $0.17 \pm 1.39$     |
| 4   | $0.91 \pm 1.89$   | $0.15 \pm 0.96$   | $-0.23 \pm 1.33$   | $-0.67 \pm 2.60$  | $-0.11 \pm 0.84$    |
| 5   | $-0.80 \pm 1.65$  | $0.13 \pm 0.90$   | $-0.22 \pm 1.57$   | $-0.57 \pm 1.23$  | $0.06 \pm 0.85$     |
| 6   | $-0.29 \pm 2.01$  | $0.05 \pm 0.33$   | $-0.11 \pm 0.12$   | $-0.59 \pm 1.29$  | $0.06 \pm 0.85$     |
| 7   | $-0.47 \pm 1.55$  | $0.05 \pm 0.62$   | $-0.01 \pm 0.87$   | $-0.19 \pm 1.77$  | $0.04 \pm 0.52$     |
| 8   | $-0.48 \pm 1.32$  | $0.07 \pm 0.62$   | $-0.07 \pm 0.96$   | $-0.28 \pm 0.85$  | $0.06 \pm 0.48$     |
| 9   | $-0.29 \pm 1.10$  | $0.02 \pm 0.42$   | $0.05 \pm 0.73$    | $-0.28 \pm 0.85$  | $0.06 \pm 0.48$     |
| 10  | $-0.25 \pm 1.16$  | $0.03 \pm 0.39$   | $-0.14 \pm 0.75$   | $-0.19 \pm 1.12$  | $0.05 \pm 0.41$     |
| 11  | $-0.04 \pm 1.06$  | $2.17 \pm 2.42$   | $12.90 \pm 1.27$   | $1.28 \pm 1.14$   | $0.27 \pm 0.35$     |
| 12  | $0.90 \pm 0.96$   | $14.33 \pm 17.53$ | $96.40 \pm 9.29$   | $23.95 \pm 14.32$ | $3.74 \pm 3.32$     |

*Most optimum order of polynomial in powers of $V_{a}$

† Most optimum order of polynomial in powers of $1/V_{a}$

### Table 3

Values of coefficients pertaining to the optimum polynomial (see Table 2), in powers of $V_{a}$ and $1/V_{a}$, beginning from 0th order term

| Coefficient | Value Base 10 exp. | Function of $V_{a}$ | Value Base 10 exp. | Function of $1/V_{a}$ |
|-------------|--------------------|----------------------|--------------------|-----------------------|
| $A_{0}$     | $-4.65892$ 3       | $1.046101$ 5        | $A_{0}$            | $-4.65892$ 3       |
| $A_{1}$     | $2.58129$ 3       | $-1.21803626$ 7     | $A_{1}$            | $2.58129$ 3       |
| $A_{2}$     | $-6.738$ 2        | $0.5818902880$ 8    | $A_{2}$            | $-6.738$ 2        |
| $A_{3}$     | $0.738$ 1         | $-8.853200073666$ 10| $A_{3}$            | $0.738$ 1         |
| $A_{4}$     | $-4.55$ 0         | $-3.4339045615737$ 11| $A_{4}$            | $-4.55$ 0         |
| $A_{5}$     | $1.8$ 1           | $-8.4967139703572$ 12| $A_{5}$            | $1.8$ 1           |
| $A_{6}$     | $-3.8$ 0          | $-8.902280118289496$ 13| $A_{6}$            | $-3.8$ 0          |
| $A_{7}$     | $3.4$ 0           | $-1.121374611833774$ 14| $A_{7}$            | $3.4$ 0           |

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