Abstract

The recent discussions by Kocić and Kogut on the nature of the chiral phase transition are reviewed. The mean-field nature of the transition suggested by these authors is supported in random matrix theory by Verbaarschot and Jackson which reproduces many aspects of QCD lattice simulations. In this paper, we point out physical arguments that favor a mean-field transition, not only for zero density and high temperature, but also for finite density. We show, using the Gross-Neveu model in 3 spatial dimensions in mean-field approximation, how the phase transition is constructed. In order to reproduce the lowering of the $\rho = 0$, $T = 0$ vacuum evaluated in lattice calculations, we introduce nucleons rather than constituent quarks in negative energy states, down to a momentum cut-off of $\Lambda$. We also discuss Brown-Rho scaling of the hadron masses in relation to the QCD phase transition, and how this scaling affects the CERES and HELIOS-3 dilepton experiments.
1 Introduction

Until recently, the finite-temperature chiral restoration transition in QCD has been guided by the concepts of dimensional reduction and universality. Pisarski and Wilczek [1] and Wilczek [2] described the QCD chiral transition with two light quarks near the chiral transition by the (dimensionally reduced) three-dimensional $\sigma$-model. The fluctuations of the order parameter, $\sigma$ and $\pi$, go soft at the transition temperature. Since $\sigma$ and $\pi$ are bosons, they have zero modes $\omega_0 = 0$ in their finite temperature Matsubara decomposition. These zero modes dominate the infrared behavior, and, therefore, the phase transition. The consequence is that the chiral transition of four-dimensional QCD should lie in the same universality class as the three-dimensional $O(4)$ magnet [1, 2].

Kocić and Kogut [3, 4] recently pointed out that the fermionic nature of the quark and antiquark, the constituents of the $\sigma$-meson, can fundamentally affect the nature of the phase transition. This is easily seen by noting that the sum over Matsubara frequencies in the finite-temperature fermion Green’s function brings in the factor

$$F = \left[ \frac{1}{2} - \frac{1}{e^{\omega/T} + 1} \right],$$

which vanishes for $\omega \to 0$. The $\omega = 0$ states of the fermions in the heat bath at finite $T$ are all filled, so it is not possible to introduce an additional $\omega = 0$ fermion into the system. Similar arguments lead Kocić and Kogut to suggest that the QCD chiral restoration transition could be mean-field in nature.

The arguments based on the thermal factor eq. (1.1) would also hold for finite density, except at $T = 0$. Whereas relatively firm arguments about the nature of the phase transition can be made only for zero density and finite $T$, we shall assume that the transition is of mean-field type for finite densities and temperatures. This will be in line with our use of a scalar mean field as order parameter.

Recently, Verbaarschot [5] has shown that the Columbia valence quark mass dependence of the chiral condensate evaluated in QCD lattice simulations with dynamical fermions for a series of couplings close to the critical temperature can be well described by chiral random matrix theory. The randomly drawn matrix elements of the Dirac operator are constants, independent of space or time. The supposition can be made that the chiral restoration transition is governed
by the mean field, also independent of space and time. Jackson and Verbaarschot [6] have formulated a random matrix model which mimics the chiral phase transition with two light flavors. They find mean-field values for the critical exponents $\beta$ and $\delta$. It could well be that movement towards $T_c$ is described well by mean field until $T$ is very close to $T_c$, and then changes character. In the case of BCS superconductivity, the “Ginzburg window” for the phase transition is $t \sim 10^{-14}$ where $t = \frac{T - T_c}{T_c}$; down to this window the transition is well described by mean field. The window for something like the Pisarski-Wilczek behavior to be realized is small, according to our above arguments, so the mean-field assumption will be adequate for us.

As noted by Kocić and Kogut [3, 4] and in earlier references in these works, the Nambu–Jona-Lasinio model has at finite temperature a chiral restoring transition in mean field (with mean-field critical exponents). They point out [4] that one expects meson masses to gradually decrease to zero, and meson radii to increase, with movement in temperature towards symmetry restoration. Furthermore, lattice calculations find that the screening masses of the $\rho$ and $a_1$ mesons go to $2\pi T$ as $T \rightarrow T_c$, whereas they would be

$$m_{scr} = \sqrt{(2\pi T)^2 + m_{dyn}^2}$$  \hspace{1cm} (1.2)

were a dynamical mass $m_{dyn}$ left at $T_c$. It turns out that to the accuracy of the lattice calculations, no $m_{dyn}$ is needed.

2 Walecka Mean Field Theory

Walecka mean-field theory describes well a number of nuclear phenomena. We take this to mean that it works well at nuclear matter density. Several recent developments support this assertion: (1) Gelmini and Ritzi [10] proposed deriving Walecka mean-field theory from chiral mean-field theory, treating both vector and scalar fields as chiral singlets; (2) Brown and Rho [11] showed that Brown-Rho (BR) scaling played a key role in obtaining the Walecka parameters at nuclear matter density $\rho = \rho_o$; (3) it has been shown by Park, Min and Rho [13] that BR scaling can be derived in chiral perturbation theory in medium with the inclusion of multi-Fermi interactions in the chiral Lagrangian and (4) it has been argued by Friman and Rho [14] that the mean-field Lagrangian theory with BR scaling can be mapped to the Landau Fermi
liquid fixed point, with the Landau parameters completely describing low-energy spectroscopic properties of heavy nuclei.

In order to describe density-dependent constituent quark masses in the analysis of dilepton production in the CERN SPS relativistic heavy ion reactions, Li, Ko and Brown \[15\] employed Walecka theory at constituent quark level in the transport equations; i.e., in the relativistic VUU \[16\] equations. A satisfactory treatment, which is thermodynamically consistent (conserving energy, etc.), can be obtained in this way.

Although, as explained later, the Walecka mean-field theory cannot be carried blindly all of the way to the chiral restoration transition, we can still gain a valuable insight from it into how chiral restoration is approached. First of all, we remark that the coupling constants of the Walecka linear sigma-omega model, chosen to fit nuclear phenomena, are satisfactory for the constituent quark model. The \( g_{\sigma NN} \approx 10 \) of the linear sigma-omega model \[17\] becomes

\[
g_{\sigma QQ} = \frac{g_{\sigma NN}}{3} \approx \frac{10}{3} \quad (2.1)
\]

for the constituent quark \( Q \). The mean-field Goldberger-Treiman relation is

\[
m_Q = g_{\sigma QQ} f_\pi ,
\]

with \( g_A^Q = 1 \) (implied by the large \( N_c \) argument) which gives

\[
m_Q = 310 \, \text{MeV} \quad (2.3)
\]

using eq. (2.2) and \( f_\pi = 93 \) \, MeV. This seems satisfactory in that \( m_Q \) is about \( 1/3 \) of the nucleon mass. We therefore propose to treat the light hadronic degrees of freedom in terms of light-quark constituent quark degrees of freedom. In doing this confinement is not considered explicitly. When necessary, we will put the confinement effect by hand as will be the case with the calculation of the bag constant.

We now show that the scalar field energy in Walecka theory acts like a bag constant of the hadronic sector as \( T \to T_c \). In other words

\[
\varepsilon_{Field} = \frac{1}{2} m_Q^2 \sigma_W^2 = B_{eff} ,
\]

where \( \sigma_W \) is the Walecka scalar mean field. That \( \varepsilon_{Field} \) behaves like a bag constant can be seen from the expression for the pressure

\[
p = n_B \frac{d\varepsilon}{dn_B} - \varepsilon ,
\]

(2.5)
where $\varepsilon$ is the energy density and $n_B$ the baryon number. Since the field energy is independent of $n_B$, $p = -\varepsilon_{\text{Field}}$, which is just the behavior of the bag constant $B$.

Neglecting the fact that in Walecka theory, the effective mass $m_Q^*$ goes to zero only as the density $\rho \to \infty$ (we shall repair this defect in the next section), let us evaluate eq. (2.4) for $m_Q^* \to 0$:

$$B_{\text{eff}} = \varepsilon_{\text{Field}}(m_Q^* = 0) = \frac{1}{2} m^2_\sigma M_N^2 / g_{\sigma NN}^2 \simeq \frac{1}{2} m^2_\sigma m_Q^2 / g_{\sigma QQ}^2 = \frac{1}{2} m^2_\sigma f_\pi^2 ,$$

(2.6)

where we have used $M_N^* = M_N - g_{\sigma NN} \sigma_W$ from Walecka theory and eqs. (2.2) and (2.4). Now Walecka chooses $m_\sigma = 550$ MeV. In the interpretation of Brown and Rho [8], this $m_\sigma$ should be $m_\sigma^*(\rho_0)$, and the $m_\sigma$ at zero density would be, according to their arguments, $m_\sigma \simeq m_\sigma^*(\rho_0)/0.78 \simeq 705$ MeV. The factor 0.78 is the computed ratio $f_\pi^*(\rho_0)/f_\pi$ and in Brown-Rho scaling [12], the scalar masses scale as $f_\pi$. With this $m_\sigma$

$$B_{\text{eff}} = 280 \text{ MeV/fm}^3 .$$

(2.7)

This is somewhat uncertain in magnitude, because there is no real scalar meson of mass $m_\sigma$; the value 705 MeV (or the in-medium 550 MeV used in nuclear calculations) is an effective value, used in mean-field calculations, to describe an enhancement in the $S = 0$ two-pion-exchange. This may be interpreted as the “dilaton” field that joins the pion field at the phase transition as described in [9]. The value (2.7) may be a bit high, because Walecka mean-field calculations are mostly performed in nuclei at a density $\rho < \rho_0$, so that the extrapolation $\rho = 0$ will not bring in quite as large an $m_\sigma(\rho = 0)$. Corrections for such effects are within our uncertainties.

We suggest that it was no coincidence that Koch and Brown [18] found the change in quark and gluon condensate, as $T$ went upwards through the phase transition at $T_c$

$$\delta B \sim \frac{1}{2} B ,$$

(2.8)

where $B$ is the full gluon condensate [19]

$$B = 471 \text{ MeV/fm}^3 = (254 \text{ MeV})^4 .$$

(2.9)

$\delta B$ is the bag constant which should be used in order to determine the pressure in the quark/gluon phase; i.e., for massless noninteracting quarks and gluons

$$p = \frac{37 \pi^2}{90} T^4 - \delta B .$$

(2.10)
The $\delta B \sim 250$ MeV/fm$^3$ found by Koch and Brown [18] is only $\lesssim$ half of the full gluon condensate [19]. Whereas it is known that only part of the glue melts at the chiral restoration transition (see [20] for a full discussion of this), it has been somewhat of a mystery why just about half of the condensate melts.

We shall show later that, in order to construct a second-order chiral restoring transition, $B_{\text{eff}}$ on the hadron side must equal the $\delta B$ on the quark-gluon side of the phase transition. In other words, in the hadron sector, the vacuum energy must be raised just enough to make the quarks massless, and then the transition can take place. We believe that this determines the necessary fraction of the gluons which have to melt before the phase transition can take place. In fact, the increase in vacuum energy must be given generally as a contribution to $T^{\mu \nu}$ of the stress-energy tensor. *This contribution is expressed below $T_c$ in terms of hadronic degrees of freedom, and above $T_c$ in quark/gluon ones.* It is, thus, not surprising that the bag constants at two sides of the phase transition are equal at $T_c$, because they are the same $T^{\mu \nu}$, expressed in different variables.

### 3 Review of the Chiral Restoring Transition in Walecka Theory

The chiral restoring transition has been constructed in Walecka theory [21]. Even though the Walecka theory is not manifestly chiral (although it can be made so at mean-field level by change of variables [10, 11, 13]), it is useful to review this work, because it bears on the nature of the transition, in that the $B_{\text{eff}}$ from Walecka theory does give approximately the increase in the hadronic vacuum energy necessary to make a second-order phase transition.

As constructed by Theis et al. [21], the transition at baryon chemical potential equal to zero is one from massive nucleons and antinucleons making a transition to massless ones. These authors find a transition with zero latent heat for the minimum coupling

\[(g_{\sigma NN})_{\text{min}} \approx 10.8 \tag{3.1}\]

necessary to effect a phase transition. (We consider this essentially the same as the $g_{\sigma NN} \approx 10$ of the linear sigma-omega model; the authors of [21] changed somewhat the parameters of the linear sigma-omega model.) Because of the zero latent heat, Theis et al. call the transition with $(g_{\sigma NN})_{\text{min}}$ a second-order one. Since $M_N^*$ changes from a small, but finite value to zero going
through the phase transition, and $M_N^*$ (or $m_Q^*$) is our order parameter, this transition would be (perhaps weakly) first-order in our classification.

The nucleon effective mass in Walecka theory does not go to zero at finite temperature. This is because Walecka theory includes only positive energy nucleon (quark) states, and the nature of the phase transition of the theory will be changed once negative energy states are taken duly into account.

4 The Gross-Neveu model

Kocić and Kogut [3, 4] have discussed and made lattice calculations with the higher dimensional Gross-Neveu model. On the basis of these, they suggested mean-field critical exponents, not only for this model but as a possibility for QCD. The works of Verbaarschot [5] and Jackson and Verbaarschot [6] support these points.

The Lagrangian of the Gross-Neveu model is

$$L = \bar{\psi} \left( i \partial \! / \! \! \! \! \! \! \! \! / + g\sigma \right) \psi - \frac{1}{2} m_\sigma^2 \sigma^2$$ \hspace{1cm} (4.1)

which we shall consider for three spatial dimensions. Here $\sigma$ is an auxiliary (scalar) field. Its vacuum expectation value can be obtained by by setting the variation of the expectation value of $L$ to zero:

$$\frac{\delta \langle L \rangle}{\delta \sigma} = 0$$ \hspace{1cm} (4.2)

from which we obtain

$$\sigma = \frac{g}{m_\sigma^2} \langle \bar{\psi} \psi \rangle$$ \hspace{1cm} (4.3)

ignoring quantum fluctuations in the $\psi$ field. Eq. (4.1) looks very much like Walecka theory at mean-field level, except that negative-energy (quark) states are included in the $\langle \bar{\psi} \psi \rangle$ of eq. (4.3) and one must take their kinetic energy into account. For our purposes, the Gross-Neveu model is essentially the Nambu–Jona-Lasinio theory without pions. Therefore, it has only $Z_2$ symmetry. However, the dynamical mass generation is basically the same as in NJL.

The quark mass is

$$m_Q = -g\sigma,$$ \hspace{1cm} (4.4)
at \( \rho = 0, T = 0 \). From eq. (2.2) we see that \( g \) can be identified with \(-g_{\sigma QQ}\), and \( \sigma = f_\pi \) at \( \rho = 0, T = 0 \). The field \( \sigma \) is the order parameter, and at finite \( T \) and/or \( \rho \), \( \sigma \to \sigma^* \), so that \( m_Q \to m_Q^* \).

The chiral restoration transition occurs when \( \sigma^* \to 0 \), or equivalently \( f_{\pi}^* = \sigma^* \to 0 \). This will be the same point at which \( m_Q^* \to 0 \). (In mean field, we do not have variation in \( g_{\sigma QQ} \).)

We can easily write down the gap equation for finite density, from eqs. (4.3) and (4.4):

\[
m_Q^* = 12 \frac{g_{\sigma QQ}^2}{m_\sigma^2} \left\{ \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \frac{m_Q^*}{\sqrt{k^2 + m_Q^*}} - \int_{k_F}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{m_Q^*}{\sqrt{k^2 + m_Q^*}} \right\}
= 12 \frac{g_{\sigma QQ}^2}{m_\sigma^2} \int_{k_F}^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{m_Q^*}{\sqrt{k^2 + m_Q^*}}.
\tag{4.5}
\]

The first term in the curly bracket comes from quarks in negative energy states, the second term from those in positive energy states.

If we assume a second-order finite density phase transition we can find the transition point by dividing eq. (4.5) by \( m_Q^* \) and solving the resulting equation for \( m_Q^* = 0 \). This leads to a critical Fermi momentum of

\[
k_F^c = \sqrt{\Lambda^2 - \frac{\pi^2 m_Q^2}{3g_{\sigma QQ}^2}} = \Lambda \sqrt{1 - \sqrt{1 + \frac{m_Q^2}{A^2} \ln \left( \frac{\Lambda}{m_Q} \right) + \frac{\Lambda^2}{m_Q^2}}},
\tag{4.6}
\]

where the second equality follows from the vacuum gap equation.

Our construction here gives the simplest description of the \( T = 0 \) phase transition and it is not clear that this somewhat schematic description holds in detail. It is well known that the finite density transition depends on details of the interparticle interaction. Depending on the parameters we may have to add a vector interaction term to the Lagrangian eq. (4.1) in order to stabilize the non-trivial solutions of eq. (4.5). Thus the type of the phase transition (first order or second order) may depend on the vector coupling. A vector term in the Lagrangian, however, does not alter eq. (4.5).

The extension of eq. (4.5) to finite temperature is easily made by integrating over all positive and negative energy quark states with inclusion of fermion temperature weighting factors. For this, we follow the work of Bernard et al. \[23\].
The bag constant can be written \[20\] as

\[
B = 12 \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_Q^2} - \frac{3\Lambda^4}{2\pi^2} - 6 \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \frac{m_Q^2}{\sqrt{k^2 + m_Q^2}},
\]

(4.7)

where the first term on the right-hand side is the negative of the energy of constituent quarks in negative energy states, the second term subtracts off the massless quark energies they would have at chiral restoration, and the final term is easily shown to be equal to \(-\frac{1}{2}m_\sigma^2\sigma^2\). The result of eq. (4.7) strongly depends on the constituent quark mass \(m_Q\). In order to reproduce the value given by eq. (2.8) we need a rather large mass of \(m_Q \simeq 500\) MeV. Then for a cut-off \(\Lambda = 560\) MeV, which roughly leads to the correct values for \(f_\pi\) and the quark condensate (we find \(f_\pi = 91\) MeV and \(\langle \bar{u}u \rangle = -(239\) MeV\(^3\)), the bag constant is

\[
B = 219\text{ MeV/fm}^3,
\]

(4.8)

in good agreement with eq. (2.8). Then from eq. (4.6) we find that the phase transition takes place at about 2.5 times nuclear matter density. This is much too low a density to be trusted.

Although quark masses of \(\sim 500\) MeV can be found in the literature this value seems to be somewhat high. On the other hand we would expect such large masses from the gap equation

\[
m_Q = \frac{-g_{\sigma QQ}^2}{m_\sigma^2} \langle \bar{u}u + \bar{d}d \rangle,
\]

(4.9)

if we use \(g_{\sigma QQ} = 10/3\) and \(m_\sigma = 705\) MeV, as argued in section 2. Then for \(\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -(239\) MeV\(^3\)) we would find \(m_Q = 610\) MeV, even larger than the 500 MeV we needed to get the result eq. (4.8).

However, if we insist on smaller quark masses, there is no chance to reproduce the bag constant given by eq. (2.8). If we choose, for instance, \(m_Q = m_N/3 = 313\) MeV we find \(B = 74\) MeV/fm\(^3\) for \(\Lambda = 650\) MeV which is needed to reproduce the correct value of \(f_\pi\). For \(m_Q = 400\) MeV, as suggested by a constituent quark picture of \(\Delta, \rho\) and \(\omega\), we get \(B = 135\) MeV/fm\(^3\) for \(\Lambda = 600\) MeV.

At this point we should note that at \(\rho = 0, T = 0\) the proper variables are nucleons, as in the original Nambu–Jona-Lasinio paper, and not quarks. The mass of the baryon which moves is \(m_N\), not \(m_Q\). Thus \[24\]

\[
B = -E_{vac} = 4 \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_N^2} - \frac{\Lambda^4}{2\pi^2} - 2 \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \frac{m_N^2}{\sqrt{k^2 + m_N^2}},
\]

(4.10)
where, again, the last term is $-\frac{1}{2}m^2\sigma^2$. Basically the procedure in [24] was an extension of the Walecka theory to include the negative energy condensate. Since each nucleon contains 3 quarks, we can make the connection between the nucleon and quark scalar densities as

$$\langle \bar{u}u \rangle + \langle \bar{d}d \rangle = 3\langle \bar{N}N \rangle = -12 \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \frac{m_N}{\sqrt{k^2 + m_N^2}}.$$

Equation (4.11)

Taking $\Lambda = 550$ MeV, we find

$$\langle \bar{u}u \rangle = (-249 \text{ MeV})^3.$$

Equation (4.12)

In this case the phase transition takes place at about 4.5 times nuclear matter density. Calculation of the bag constant gives

$$B = 278 \text{ MeV/fm}^3.$$

Equation (4.13)

This is rather close to $\delta B$.

As a reconciliation of this $B$, the Walecka $B$ and the $\delta B$ found by Koch and Brown [18] we will adopt

$$B_{eff} \approx 250 \text{ MeV/fm}^3.$$

Equation (4.14)

By extending Walecka theory to include negative energy nucleon states down to a cut-off $\Lambda$ we have been able to understand the value of $\delta B$ found in lattice calculations [18]. It has been pointed out by Koch et al. [25] a long time ago that Walecka theory including negative energy nucleon states does not saturate at any finite densities. The authors repaired this defect of the model by introducing another term to the Lagrangian which is to mimic higher-order processes. In the mean-field approximation, this corresponds to a density dependent scalar coupling constant and the effect of the extra term vanishes for $\rho = 0$. Therefore it does not change eq. (4.7) for the bag constant.

5 The Chiral Restoration Transition

In this section we construct the $\rho = 0$, finite-temperature transition. Our basic premise is that the transition will occur as soon as the pressure $p$ can be brought to zero (from below). This is based on the observation that within their accuracy, lattice gauge simulations [26] find the pressure to be zero. Taking eq. (2.10), and setting $p = 0$, we find

$$\frac{37}{90} \pi^2 T_c^4 = B_{eff},$$

Equation (5.1)
where we have used the pressure from noninteracting quarks and gluons. We discuss corrections of order $\alpha_S$ below. Using eq. (4.14), we have

$$T_c = 147 \text{ MeV}$$

(5.2)

in good agreement with lattice results [27, 26].

Now the pressure must be the same (zero) on the hadronic side of the phase transition. Brown, Jackson, Bethe and Pizzochero [28] have shown how this can be accomplished. Although hadrons, other than the Goldstone bosons, go massless at $T_c$, the effective number that can enter into the pressure equation must be the same as the number of degrees of freedom in the quarks and gluons. In [28] this is brought about by including excluded volume effects in the hadron gas. The philosophy here is analogous to that invoked for Debye theory for phonons.

For $N$ lattice points in a crystal there are $3N$ degrees of freedom. The sum over phonons is truncated at $3N$ phonons, because the number of effective degrees of freedom cannot exceed the number of fundamental ones. Thus, there are 37 effective hadron degrees of freedom at $T_c$. Since, as outlined earlier, the $B_{eff}$ in the hadron sector is equal to the $\delta B$, which follows from decondensation of quarks and gluons, the pressure is continuous (and zero) across the phase transition.

Koch et al. [39] fit the Bethe-Salpeter wave function of the $\pi$- and $\rho$-mesons involving quarks and antiquarks propagated in the spatial direction in hot QCD for $T \gtrsim T_c$. The quarks had dynamically generated mass zero. The time direction in Euclidean space is periodic in a box of width $T^{-1}$, so it is compressed with increasing $T$. Already at $T = T_c$, dimensional reduction seems to work. A “funny space” was introduced by interchanging $z$ and $t$; i.e., $z' = t$ and $t' = z$, where the primes refer to the funny space. Thus the funny (imaginary) time is zero and $z'$ is periodic with period $T^{-1}$. In the funny space, the wave functions are two-dimensional in $x$ and $y$, constant in the (compressed) $z'$; i.e., they are two-dimensional “slab” wave functions.

Given the two-dimensionality, only the components of the spin perpendicular to the slab in the $z'$-direction survive integration over the $\pi$- and $\rho$- wave functions. Thus, the hyperfine splitting goes as $\sigma_1 \sigma_2$, and the helicity-zero $\rho$ meson and pion wave functions become degenerate, both being lowered in energy from the helicity-one $\rho$-meson wave functions by the hyperfine interaction. The effective dimensional reduction thus leads to the longitudinal component of the
ρ-meson being “spit-out” as the ρ goes massless, and forming a doublet with the pion. This is reminiscent of the Georgi vector limit \[34\] in which chiral symmetry is realized simultaneously in the Wigner-Weyl and Nambu-Goldstone mode. A microscopic model of fully polarized instantons \[35\] reproduces features consistent with the Georgi vector limit discussed here.

Brown and Rho \[33\] describe lattice gauge calculations \[36\] that show that the hadronic vector coupling $g_V$ goes to zero as $T \to T_c$, giving way to the colored vector gluon exchange of QCD. Together with $m_V^* \to 0$, where $m_V^*$ is the in-medium vector meson mass, this provides yet another requirement for the Georgi vector limit.

Lattice calculations \[37\] show from the scalar susceptibility that the scalar $\sigma$-meson mass goes to zero as $T \to T_c$. From these calculations and an (unpublished) extension, the $T = 1$ scalar $\delta$-meson mass goes to zero a few MeV above $T_c$. The work of Verbaarschot \[3\] shows that the $\delta$-meson mass actually goes to zero at $T_c$, once the bare quark mass is taken to zero.

Since the $T = 1$ scalar and $T = 0$ scalar become degenerate at $T = T_c$, it is reasonable to assume that $\pi$ and $\eta$ also become degenerate. This may mean the restoration of $U_A(1)$-invariance. Indeed, the work of Koch et al. \[39\] can be interpreted as the Bethe-Salpeter wave functions being states of free quark and antiquark correlated by the Ampère’s law magnetic interaction. Here $\alpha_S = 0.20$ was used. Thus, to order zero in $\alpha_S$, the dynamics is trivial, i.e., that of free quarks, correlated in order $\alpha_S$. Since a quark can move about from being correlated with another, the correlations being formed at order $\alpha_S$, effectively deconfinement – in the sense outlined – accompanies chiral restoration (even though $\sim 50\%$ of the gluon condensate remains at $T \sim T_c$).

In summary, the dynamics above $T_c$ are those of free quarks, correlated in order $\alpha_S$, and, therefore, are rather trivial. The Georgi vector limit is that of nearly free quarks with, however, nontrivial dynamics. The nontrivial aspect is that the hadronic vector coupling $g_V$ goes to zero *Although we favor the Georgi vector symmetry near $T_c$ – and our discussion throughout this paper are made with that structure in mind, we must admit that there is nothing in lattice data or in real experiments so far performed that rules out that the phase slightly above $T_c$ is in the standard Wigner phase in which $SU(n_f) \times SU(n_f)$ is restored. What would differentiate the two is the “decay constant” $f_\pi$ (and $f_\delta$) which would be non-zero in the Georgi vector symmetry phase and zero in the standard Wigner phase. If pseudoscalar ($\pi$) and scalar ($\delta$) mesons do persist above the critical temperature, then the Georgi vector symmetry will be favored over the Wigner symmetry.*
as $T \to T_c$, whereas the scalar $\sigma$ and pseudoscalar pion couplings continue smoothly through $T_c$. The continuity of the scalar and pseudoscalar degrees of freedom was easy to understand in the Pisarski-Wilczek picture where the $\sigma$ and $\pi$ are order parameters of the second-order phase transition. In our interpretation of the transition as mean field, the order parameter is the chiral partner of the scalar meson, so it is not surprising that these degrees of freedom carry on through the transition.

6 Density and Temperature Dependent Masses

The medium dependence of vector meson masses is of great interest these days because of the dilepton experiments at SPS at CERN. Calculations with medium-dependent masses have been carried out treating the vector mesons as composed of constituent quarks and antiquarks, and then implementing the density dependence of the constituent quark/antiquark by Walecka theory. In this way, Brown/Rho scaling is realized as

$$\frac{m_N^*}{m_N} \approx \frac{m_\rho^*}{m_\rho} \approx \frac{m_\omega^*}{m_\omega} .$$

(6.1)

The $a_1$ meson is included by using $m_{a_1}^* = \sqrt{2}m_\rho^*$. In fact, with nonperturbative corrections, Brown-Rho scaling should be

$$\frac{m_N^*}{m_N} = \frac{\sqrt{g_A^* m_\rho^*}}{\sqrt{g_A m_\rho}} .$$

(6.2)

but this has not yet been included in the calculations of heavy-ion processes.

As noted earlier, the Walecka theory lets $m_Q^* \to 0$ only as $\rho \to \infty$. This is a consequence of the restriction to positive energy states. The Walecka theory has the advantage that its parameters at $\rho \sim \rho_o$ can be obtained from nuclear phenomena. In the case of the Gross-Neveu model, which more properly describes the phase transition, the order parameter $\sigma^*$ moves smoothly to zero. The Gross-Neveu model has not, however, yet been employed in the relativistic transport calculations. In order to use it, one would have to transport negative energy quarks, describing their collisions and mean fields in detail assuming equilibrium during the fireball expansion. The accuracy of equilibration was evaluated by Cassing et al. for dilepton production using bare meson masses. Comparison with results by Li et al. who assumed equilibration, gave little difference in the results. In the case of medium-dependent masses there are many more degrees
of freedom, and equilibration should be even better. Using the Gross-Neveu model is unlikely to make a large difference, however, because dileptons from $\rho$-meson effective masses $m_\rho^* < 2m_\pi$ will chiefly come out in the low-energy dilepton peak dominated by the background of Dalitz pairs, and it will not be easy to distinguish the low-energy dileptons from the latter.

Koch and Brown [18] found that the increase in entropy in the region of $T = T_c$ in lattice calculations could be well described by the Nambu–Jona-Lasinio dependence

$$ \frac{m^*}{m} = \frac{\langle \bar{\psi}\psi \rangle^*}{\langle \bar{\psi}\psi \rangle}, $$

(6.3)

rather than $m^*/m$ scaling as $(\langle \bar{\psi}\psi \rangle^*/\langle \bar{\psi}\psi \rangle)^{1/3}$ as naively suggested by Brown and Rho [12]. Now following Kocić and Kogut [4], we propose to make the following argument. Define two relevant critical exponents $\beta$ and $\nu$ by

$$ \langle \bar{\psi}\psi \rangle^* \propto t^\beta, \quad M^* \propto t^\nu $$

(6.4)

for small $t = |T - T_c|/T_c$, where $M^*$ is some physical scale like a dynamically generated quark or hadron mass. Near $T_c$ one can do dimensional reduction with $M^*$ as the only scale. Then the following relation between the order parameter and the scale holds:

$$ \langle \bar{\psi}\psi \rangle^* \propto M^{*\beta/\nu}. $$

(6.5)

The mean-field critical exponents are

$$ \beta = \frac{1}{2}, \quad \nu = \frac{1}{2} $$

(6.6)

so that $\beta/\nu = 1$. Of course, what we call Nambu–Jona-Lasinio dependence comes from a mean-field calculation, so the result is readily understandable.

The three-dimensional $O(4)$ magnet suggested in [8] and [2] has $\beta/\nu = 3/2$ so that $M^* \propto ((\langle \bar{\psi}\psi \rangle^*)^{2/3}$. This dependence could not have been ruled out by Koch and Brown [18], but $\beta/\nu = 1$ was definitely favored. Thus, although the relation (6.1) of Brown/Rho scaling appears to hold at mean-field level, as already suggested by Brown and Rho [4], the proportionality $m^*/m = ((\langle \bar{\psi}\psi \rangle^*/\langle \bar{\psi}\psi \rangle)^{1/3}$ is probably not right.

7 The Softest Point

In our description of the chiral transition occurring when the hadron masses become zero, and our limitation of the number of hadrons involved to the number of underlying degrees of freedom,
it became clear that the phase transition would take place at the lowest temperature where the pressure could be brought non-negative; i.e., at zero pressure. Lattice gauge simulations show this quite accurately to be true [26]. The energy density just above the phase transition is, then,

\[ \varepsilon = 37 \frac{\pi^2}{30} T^4 + B_{\text{eff}} = 4B_{\text{eff}} \]  

(7.1)
in the approximation of free quarks and gluons. For \( B_{\text{eff}} \sim 250 \text{ MeV/fm}^3 \), this gives \( \varepsilon \sim 1 \text{ GeV/fm}^3 \) in agreement with what was found by the MILK collaboration [26].

In relativistic heavy ion collisions, a finite baryon number chemical potential must be introduced. The addition to the pressure is

\[ \delta p = T_c^2 \mu_B^2 + \frac{\mu_B^4}{2\pi^2}. \]  

(7.2)
The \( \mu_B \) would be the baryon chemical potential for the (massless) nucleons in the beam. Again, the phase transition will take place once the temperature is reached such that the pressure is non-negative; i.e., at

\[ \frac{37\pi^2}{90} T_c^4 + \delta p(T_c) - B = 0 \]  

(7.3)
where \( \delta p \) is the additional pressure (7.2) from the finite baryon number. In calculations to date, substantial additional pressure is produced by the repulsion from vector meson exchange between the baryons. As noted earlier, Brown and Rho [33] have discussed lattice calculations of the quark number susceptibility which show that hadronic vector exchange cedes to perturbative colored gluon exchange at \( T = T_c \). We have interpreted this to be in accord with chiral symmetry being realized in the Georgi vector limit [34].

The vanishing of the hadronic vector interaction is necessary in order to understand [40] the “cool” kaons observed in preliminary data of the E814 and E877 experiments [42]. Confirmation of these experiments by further data would give support to our scenario here. We plan to carry out the fireball evolution quantitatively for the AGS energies. Note that for a nearly stationary fireball to be formed, the vector mean field must be zero as in the Georgi vector limit. If it were not so, there would not be enough energy in the AGS experiments for the colliding nuclei to get on top of each other, since the effective bag constant energy \( 4B_{\text{eff}} \) uses up, all by itself, the total AGS energy. (See eq.(7.1).)

The addition of \( \delta p(T_c) \) in eq. (7.3) will lower the temperature \( T_c \) of the phase transition as one can see in a model with dilated quarks in heat bath [13]. Our previous argument still
goes through that the energy density will be $4\delta B$. We have, however, restricted our theoretical considerations here to $SU(2) \times SU(2)$ and experiments show that a substantial amount of strangeness is produced. Correcting for this will raise the energy density $\varepsilon$, somewhat above 1 GeV/fm$^3$.

For the central Si + A collisions at Brookhaven AGS, including the E814 and E877 experiments, a rather complete set of hadron yields has been analyzed. The authors find a high degree of equilibration in the products with a freezeout temperature of

$$T_{fo} \sim 120 - 140 \text{ MeV}. \quad (7.4)$$

Not only do these small systems seem to equilibrate before freezeout, which has surprised many workers in the field, but there is no indication that strangeness is not in chemical equilibrium. It seems therefore highly plausible that strange particles are nearly equilibrated.

In order to reach a high degree of equilibration, our scenario of dropping masses is definitely helpful although perhaps not indispensable. When the negative energy condensate is broken up in a relativistic heavy-ion collision, it no longer acts as a coherent source of scalar meson field to give the hadron masses. The mass of the scalar degrees of freedom is $m_{\sigma} \sim 500 \text{ MeV}$, so the time of collision with exchange of these degrees of freedom is

$$\tau \sim \hbar/m_{\sigma}c \sim 0.4 \text{ fm}/c. \quad (7.5)$$

In a time somewhat longer than this, the hadrons should go (nearly) massless independently of equilibration. In fact, in our scenario, the hadrons should go massless more quickly than they equilibrate. Furthermore going massless in turn will speed up equilibration because there is a large increase in the number of hadrons, as we have discussed in connection with the phase transition.

In the case of strange particles, Brown and Rho showed that the scalar mean fields act on the nonstrange quarks and antiquarks in them very much as in nonstrange hadrons. Additionally, considerable $\langle \bar{s}s \rangle$ is produced in the heavy-ion collision, and this brings down the mass of the strange quark. Thus, although not massless, the strange particles have much reduced masses in the fireball formed in the relativistic heavy-ion collisions, so we can understand how they more or less equilibrate.
The analysis of Braun-Munzinger et al. \cite{44} uses a bag constant of $B \approx 260 \text{ MeV/fm}^3$, essentially the value we arrived at. In our theory, this bag constant is just the scalar field energy which raises the vacuum energy smoothly as the hadrons become massless as $T \to T_c$. Even though the growth in field energy is smooth, most of it takes place at $T$ very near $T_c$ as indicated in lattice calculations.

On the hadron side of the chiral transition, Braun-Munzinger et al. do not bring the nucleons massless – in fact, they are still essentially nonrelativistic at the phase transition, going over to massless particles above $T_c$. Thus, a full analysis carried out in our scenario, where the hadrons go massless at $T_c$, would give somewhat different values for the various parameters. In particular, baryon number densities in the hadron gas will be closer to those in the quark-gluon plasma if nucleons in both phases go massless at $T_c$.

8 Discussions

The Walecka theory is not manifestly chirally invariant, so near the chiral phase transition, it may require non-trivial modification to be reliable. Nonetheless, it is effectively chirally invariant at long-wavelength regime in the sense that the $\omega$ and $\sigma$ degrees of freedom entering in Walecka theory are present in chiral Lagrangians in the form of multi-Fermi interactions. In any event, it reproduces well a large amount of nuclear phenomenology. Thus, the parameters of the Walecka theory, when interpreted correctly, could be close to the chiral ones even when it comes to giving a description of the chiral phase transition.

We find that the Gross-Neveu model, essentially Nambu–Jona-Lasinio without the pions, in mean-field approximation, is useful in making the description of the chiral transition.

Amusingly, the bag constant we need for the chiral transition is neither the MIT bag constant nor that from the trace anomaly, but has a value $\sim 1/2$ of the latter. We find that it is just the field energy in Walecka theory would result if $m_N^*$ were brought to zero by the scalar field. We give a chiral description of it in the Gross-Neveu model, where the value turns out to be about the same. In all cases, our treatment is based on the lattice gauge calculations which show that about half of the condensate is melted as the temperature $T$ goes upward through $T_c$. This $\sim 50$ \% is not only a property of the calculation with dynamical quarks, as analyzed by Koch and
Brown [18], but the same $\sim 50\%$ was found for the quenched case by Adami et al. [46]. Thus, the behavior found for the condensate is presumably a property of the vacuum.

As we discussed earlier, the field energy in the Walecka linear sigma-omega model nicely mocks up the $\sim 50\%$ of the condensate. We were able to parameterize the Gross-Neveu model which contains the dynamical symmetry breaking and mass generation so as to contain the same $\sim 50\%$. It is clear from this that we need two scales for the glue: “hard” glue and “soft” glue. The hard glue stays rigid, like epoxy, through the phase transition, whereas the soft glue is melted. The division into these two components must be $\sim 50/50$. We shall return to a more formal discussion of this below.

It is interesting to compare our scenario with that of the instanton model of Shuryak and collaborators [47] which also predicts an $\sim 50\%$ disappearance of the condensate at the chiral restoration transition. In this model of the vacuum which for $T \ll T_c$ chiefly involves the “random instanton liquid,” for $T \sim T_c$, about half of the condensate is in the random instanton liquid (soft glue) and about half in “instanton-anti-instanton molecules” (hard glue): as $T$ approaches $T_c$, the liquid changes from random to include an $\sim 50\%$ component organized into molecules. Now the latter hard component which leaves chiral symmetry unaffected, remains as $T \to T_c$. In fact the molecules are highly polarized [18]. The fact that an instanton and anti-instanton pair can be laid around a torus in the time direction at $T \sim T_c$, so that they just touch, makes the polarized molecule a particularly favorable configuration, and helps to explain why $\sim 50\%$ of the system is in molecules for this temperature.

Whereas the random instanton liquid disappears at $T = T_c$, the molecules will be slowly squeezed out with increasing temperature as $1/T$ becomes smaller. This squeezing out should follow the scenario of Gross, Pisarski and Yaffe [49] and the scale at which they disappear should be $\sim 3T_c$. Thus from the point of view of what happens at $T_c$, the glue in instanton molecules is hard (“epoxy”). Above $T_c$ the quarks in the instanton molecules have the thermal energies given by the Matsubara frequencies, $\omega_n = (2n + 1)\pi T$.

From the standpoint of physics at temperature $T \lesssim T_c$ (we do not go far below $T_c$ because nothing happens there, at least for $\rho = 0$), the glue represented by the trace anomaly

$$\theta_\mu^\nu = \frac{\beta(g)}{2g} \text{Tr} G_\mu^\nu G^{\mu\nu}$$  \hspace{1cm} (8.1)
may, in analogy to the instanton picture with “random instanton liquid” and “instanton molecules” components, be split into a soft part and a hard part

\[ \theta_\mu = (\theta_\mu)_{\text{soft}} + (\theta_\mu)_{\text{hard}}. \]  

(8.2)

In order to make a connection with Brown-Rho scaling which was formulated in terms of an effective chiral Lagrangian implemented with trace anomaly, we should identify

\[ (\theta_\mu)_{\text{soft}} \approx (\chi^*)^4 \]  

(8.3)

where \( \chi^* \) is the soft part of the effective scalar field, to be identified with the dilaton \( \sigma \) at the point of Weinberg’s mended symmetry. Its vacuum expectation value, denoted simply as \( \chi^* \), was used by Brown and Rho for scaling; thus

\[ f_\pi^* = \left( \frac{\chi^*}{\chi_0} \right) f_\pi \]  

(8.4)

and masses were scaled similarly. It was assumed that a vacuum potential \( V(\chi^*/\chi_0) \) ensured that \( \chi^* \) had the correct expectation value. We do not know how to compute the vacuum potential from the first principles but we conjecture that the role of the potential is played by the bag constant in the Walecka or Gross-Neveu model; i.e., these models involve effective masses \( m^* \) etc., so through (8.4) they are functions of \( (\chi^*/\chi_0) \).

The hard glue – “epoxy” – in the instanton molecules does not scale appreciably in the region of \( T \sim T_c \). On the other hand, it does not involve quark zero modes, so it does not participate in the chiral symmetry breaking or restoration.

As we argued, the Walecka field energy or equivalently effective bag constant (4.14) (or the Gross-Neveu \( B_{\text{eff}} \), constructed to roughly equal (4.14)) is just the increase in vacuum energy needed to bring the hadrons massless. This is equivalent to the condensation energy of the random instanton liquid. Because of the reorganization of some of the random liquid into molecules, this is less than if the entire system were random liquid. In both cases, the chiral restoration transition involves the hadrons going massless.

We do not have such a detailed picture for density effects. However, from the chiral Lagrangian and chiral identities, we can calculate variations of relevant parameters with density

\[ ^1 \text{A similar splitting was studied by other authors in a different, though related, context.} \]
\[ f^*_\pi(\rho_0) = \frac{f^*_\pi(\rho_0)}{f_\pi} = 0.78. \]  

(8.5)

We expect that the same argument about effective potential \( V(\chi*/\chi_0) \) holds here.

Recently the work by Verbaarschot and Jackson and Verbaarschot has been substantially extended [51][52]. The success of the random drawing of chiral matrix elements to describe the QCD chiral restoration transition implies that the situation is sufficiently complicated that spatial variations average out. Since the drawn quark matrix elements are constants, independent of space and time, the description arrived at by random drawing is clearly a mean field theory. Indeed, the random matrix model can be shown to correspond to Nambu–Jona-Lasinio theory in mean field approximation with the kinetic energy terms and Matsubara modes other than the lowest one being neglected [53].

As practised, the random drawing includes many more matrix elements than the zero modes, which enter into the random instanton liquid. The latter, in themselves, would give an NJL mean field description by the methods of ref. [53]. Basically, only the symmetries and the randomness are needed. The hard component of the glue, described by instanton molecules, cannot be obtained by random drawing, but must be put in by hand [51].

We argue for nearly trivial dynamics of the quarks and gluons correlated in order \( \alpha_S \sim 0.2 \), once \( T \) exceeds \( T_c \). Our arguments are, however, based on the mean-field description of the phase transition and a somewhat rough interpretation of not-too-accurate lattice data, so we may well have missed more subtle phenomena. We argue that effectively deconfinement takes place at the chiral transition, in that quarks can move relatively freely, although there is a premium for them to be correlated in color singlets [39].

We find in relativistic heavy ion collisions that the “softest point,” that of essentially zero pressure, should correspond to an energy density \( \varepsilon \gtrsim 1 \text{ GeV/fm}^3 \), just about what is reached in AGS collisions at 11 – 15 GeV/N. Analysis of the CERES and HELIOS-3 dilepton experiments [15] shows in some detail how Walecka theory applied with the parameters used here can reproduce the results.
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