On the Coulomb Interaction in a Nucleus for Odd J States

Larry Zamick
Department of Physics and Astronomy,
Rutgers University, Piscataway, New Jersey 08854

May 17, 2018

1 Abstract
With a certain approximation for the Coulomb matrix elements in a single j shell of protons and neutrons it is found that wave functions of states of odd angular momentum J in an even-even nucleus are not strongly affected by their presence.

2 Introduction
We make a simple estimate of the matrix elements of the Coulomb interaction and then calculate the isospin mixing of the lowest J = 2^+ T = 0 and lowest J = 2^+ T = 1 states in ^{44}\text{Ti}.

3 The Calculation.
If one examines the Coulomb 2 particle matrix elements in a single j shell one notices that the main effect of this presence is to add a repulsive term to the J = 0 matrix element, i.e. like anti-pairing. What are the consequences of assuming this simple model?

Let us start with 2 protons and 2 neutrons in a single j shell. The wave functions can be written as follows:
\[ \psi_{\alpha J} = \sum D^{\alpha J}(J_p J_n) [(jj)^{J_p} (jj)^{J_n}]^J \]
where \[ \sum D^{\alpha J}(J_p J_n) \] is the probability amplitude that the 2 protons couple to \[ J_p \] and the 2 neutrons to \[ J_n \].

To satisfy the Pauli principle \[ J_p \] must be even and likewise \[ J_n \]. Odd J \((T=0)\) states can only exist in the proton-neutron channel.

Note that for odd J it is easy to see that \[ J_p \] cannot be equal to zero. If it were zero then J would equal \[ J_n \]. But \[ J_n \] is even. Hence the odd J wave
function is insensitive to the 2 particle J=0 Coulomb matrix element, which in our approximation is the only non-vanishing Coulomb matrix element.

Let us contrast the above with what happens if we add a constant to all J=0 matrix elements - pp, nn and np. In that case there would be contributions from the pp channel, the nn channel and the np channel. For the pp and nn channels the expression is easy. Concerning the latter the expression for a matrix element os a 2p-2n system is

\[ < (J'_{p'} J'_{n'}) J_{p} J_{n} | V | (J_{p} J_{n}) J_{p} J_{n} > = \delta_{J'_{p'} J_{p}} \delta_{J'_{n'} J_{n}} ((E(J_{p}) + E(J_{n})) + 4 \Sigma((jj)^{J_{p}} | (jj)^{J_{A}} (jj)^{J_{B}}) \Sigma((jj)^{J_{n}} | (jj)^{J_{A}} (jj)^{J_{B}}) < (jj)^{J_{B}} | V_{np} | (jj)^{J_{p}} > \]

Although J_B has to be even J_A can be even or odd. Hence we can have a contribution where

J_B is equal to zero and J_A is equal to J. However in the Coulomb case we do not have this neutron-proton contribution.

With this approximation of the Coulomb matrix elements there is also no mixing for states of even J with high spin. If the highest J for 2 identical nucleons is J_p then states of 2 protons and 2 neutrons with J greater than J_p will not have any components with J_p equal to zero. For example, in the f_{7/2} shell the maximum J for 2 protons is 6 and so states with J= 8, 10 and 12 will not have components with J_p equal to zero.

For an even J less than J_m the coupling matrix element between a T=0 and T=1 state in this approximation is

\[ C \frac{D^{T=0} (0, J) D^{T=1} (0, J)}{E(T=0) - E(T=1)} \]

As an illustration we show the lowest J=2 T=0 and J=2 T=1 states as calculated with the MBZE wave functions [1]. Note that the MBZE 2 body matrix elements are different from those in earlier works [2,3]. This is especially true for the T=0 matrix 2 body elements.

Table I Wave functions of J=2^+ states in 44Ti with the MBZE interaction - f_{7/2} shell. E(T=0) 1.1631 MeV, E(T=1)=5.2366 MeV.

| J_p J_n | D^{T=0} (J_p J_n) | D^{T=1} (J_p J_n) |
|---------|------------------|------------------|
| 0,2     | 0.6099           | -0.6370          |
| 2,0     | 0.6099           | 0.6370           |
| 2,2     | -0.3538          | 0                |
| 2,4     | 0.2416           | -0.2962          |
| 4,2     | 0.2416           | 0.2932           |
| 4,4     | -0.0591          | 0                |
| 4,6     | 0.0613           | -0.0909          |
| 6,4     | 0.0613           | 0.0909           |
| 6,6     | 0.0563           | 0                |

Using the approximate formula the mixing amplitude between these 2 J=2^+ states is C 0.6099* -0.6370/(1.1631-5.2366) = 0.09537 C. Taking a resonable estimate
C=1.0 MeV we find the probability of a T=1 admixture in a basically J=2+
T=0 state is 0.0091.

References

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