First and second order phase transitions and magnetic hysteresis for a superconducting plate

G. F. Zharkov

P.N. Lebedev Physical Institute, Russian Academy of Sciences, Moscow, 119991, Russia

The self-consistent solutions of a nonlinear Ginzburg–Landau equations, which describe the behavior of a superconducting plate of thickness $2D$ in a magnetic field $H$ parallel to its surface (provided that there are no vortices inside the plate), are studied. We distinguish two types of superconductors according to the behavior of their magnetization $M(H)$ in an increasing field. The magnetization can vanish either by a first order phase transition (type-I superconductors), or by a second order (type-II). The boundary $S_{\text{I-II}}$, which separates two regions (I and II) on the plane of variables $(D, \kappa)$, is found. The boundary $\zeta(D, \kappa)$ of the region, where the hysteresis in a decreasing field is possible (for superconductors of both type), is also calculated. The metastable $d$-states, which are responsible for the hysteresis in type-II superconductors, are described. The region of parameters $(D, \kappa)$ for type-I superconductors is found, where the supercooled normal metal (before passing to a superconducting Meissner state) goes over into a metastable precursor state ($p$-). In the limit $\kappa \to 1/\sqrt{2}$ and $D \gg \lambda$ (where $\lambda$ is the London penetration depth) the self-consistent $p$-solution coincides with the analytic solution, found from the degenerate Bogomolnyi equations. The critical fields $H_1, H_2, H_p, H_r$ for type-I and type-II superconducting plates are also found.

1. Introduction

The macroscopic Ginzburg–Landau theory of superconductivity (GL) [1] is widely used to describe the superconductors behavior in the external magnetic field. In particular, in [1] the structure of the intermediate region in the vicinity of superconducting ($s$-) and normal ($n$-) semi-infinite metallic phases, brought into contact in magnetic field, was studied. It was found that the free energy $\sigma_{ns}$ of the interface between $n$- and $s$-phases vanishes, when GL-parameter $\kappa$ equals to $\kappa_0 = 1/\sqrt{2}$. On this basis all superconductors are usually divided in two groups [1]: the superconductors with $\sigma_{ns} > 0$ ($\kappa < \kappa_0$) belong to first group, the superconductors with $\sigma_{ns} < 0$ belong to second group. (The value $\sigma_{ns} < 0$ indicates that $s$-phase is unstable in respect to the appearance of vortices in a bulk superconductor [2].)

However, one can use different criterion to divide superconductors in two groups, namely, according to the shape of the magnetization curve, using the formula $\overline{B} = H + 4\pi M$, where $\overline{B}$ is the mean field value in the specimen, $H$ is the external field, $M$ is the magnetization. Such criterion was used in [3] for a cylinder of radius $R$ (assuming no vortices inside the cylinder). On the base of the self-consistent solutions of GL-equations the critical parameter $\kappa_c(R)$, dividing two types of the dependence $M(H)$ in the increasing field, was found. It was shown that for $\kappa < \kappa_c$ (in type-I superconductors) the cylinder magnetization vanishes in a
jump (by a first order phase transition to \(n\)-state). For \(\kappa > \kappa_c\) (in type-II superconductors) the magnetization vanishes gradually, by a second order phase transition. (The difference in critical parameters \(\kappa_0\) and \(\kappa_c(R)\) is due to the different geometries and different criteria for sorting the solutions, assumed in [1,2] and [3].) It was also shown in [3] that the cylinder behavior in a magnetic field is by far nontrivial: there exist several states (the solution branches), the transitions between these states, the hysteresis phenomena, etc. It was noted that the properties of the vortex-free state should be taken into account in interpreting the experiments with sufficiently thin (mesoscopic) samples.

In this paper we consider an infinite plate of thickness \(2D\) in a field \(H\) parallel to its surface, assuming no vortices inside the plate. (Such vortex-free state was previously studied numerically in [4,5], but not in sufficient detail.) We show that even in the simplest case of a vortex-free plate the solutions depend on parameters \((D, \kappa, H)\) in a very complicated way, analogously to the case of a cylinder geometry [3,6,7]. Thus, there exists a critical value \(\kappa_c(D) \neq \kappa_0\), which defines two types of the magnetization behavior, \(M(D, \kappa, H)\). In type-I superconductors (with \(\kappa < \kappa_c\)) the magnetization of the plate vanishes (if the field increases) in a jump (by a first order phase transition into \(n\)-state in some field \(H_1\)). In type-II superconductors (with \(\kappa > \kappa_c\)) the magnetization vanishes gradually (by a second order phase transition into \(n\)-state in a field \(H_2\)). If the thickness is sufficiently small, \(D \sim \lambda\) (\(\lambda\) is the London penetration depth), the superconducting state is destroyed by a second order phase transition at arbitrary \(\kappa\). If \(\kappa = \kappa_c(D)\), the first and second order phase transitions become indistinguishable. We found also, that type-II superconducting plate, which is in the vortex-free Meissner state (with the order parameter \(\psi \sim 1\)), may pass (if the field increases, FI-regime) into a special (also vortex-free) "edge-suppressed" \(e\)-state [5]. The order parameter \(\psi\) in \(e\)-state is strongly suppressed in some layer near the plate surface, so the magnetic field \(B\) in this layer is not screened and practically equals to the external field \(H\). In type-I superconductors such \(e\)-state does not form. (The possibility of transitions into \(e\)-state means that alongside with the usual vortex mechanism [2] there exists the additional "edge" mechanism of the field penetration into a mesoscopic sample.)

The hysteresis \(d\)-states, which appear in type-II superconductors if the field decreases (FD-regime), and the so-called "precursor" states \((p\)-\) in type-I superconductors, which describe the onset of \(s\)-state (in the field \(H_p\)) from the supercooled \(n\)-state, are also studied. We show, that in the limit \(D \gg \lambda\) and \(\kappa \to \kappa_0 = 1/\sqrt{2}\) the hysteresis (metastable) \(p\)-solution coincides with the solution of the degenerate Bogomolnyi equations [8] and can be described analytically [9].

The paper is divided into several Sections. In Sec. 2 the basic equations and boundary conditions are written, and necessary notations introduced; in Sec. 3 the critical lines \(S_{I-II}, \zeta\) and \(\pi\), which exist on a plane of parameters \((D, \kappa)\), are calculated and explained; in Sec. 4 the superconductor behavior in the vicinity of the critical lines \(S_{I-II}\) and \(\zeta\) is described; in Sec. 5 the examples of the space profiles of different solutions are given; in Sec. 6 the metastable solutions, which exist in the region of \(p\)-states and on its \(\pi\)-boundary are studied; in Sec. 7 the connection of the self-consistent solutions of GL-equations with the solutions of the degenerate Bogomolnyi equations is discussed; in Sec. 8 the critical fields \(H_1, H_2, H_p\) and \(H_r\), which characterize the behavior of type-I and type-II superconducting plates, are found; in Sec. 9 the results are shortly discussed.
2. Equations

In the case of a plate the system of GL-equations [1] can be written in the following dimensionless form:

\[ \frac{d^2 a}{dx^2} - \psi^2 a = 0, \]  
\[ \frac{d^2 \psi}{dx^2} + \kappa^2 (\psi - \psi^3) - a^2 \psi = 0. \]  

(1)

(2)

Here, instead of the dimensioned potential \( A(x') \), field \( B(x') \), current \( j_s(x') \) and the coordinate \( x' \), the dimensionless quantities \( a(x), b(x) \) and \( j(x) \) are introduced, where

\[ A = \frac{\phi_0}{2\pi \lambda} a, \quad B = \frac{\phi_0}{2\pi \lambda^2} b, \quad b = \frac{da}{dx}, \quad j(x) = j_s/\frac{c\phi_0}{8\pi^2 \lambda^3} = -\psi^2 a \]  

(3)

\( x = x'/\lambda \) is the dimensionless coordinate, \( -D_\lambda \leq x \leq D_\lambda, \) \( D_\lambda = D/\lambda, \lambda = \kappa \xi, \xi \) is the coherence length, \( \kappa \) is the GL-parameter, \( \phi_0 = hc/2e \) is the flux quantum).

We will consider the vortex-free state. In this case the order parameter \( \psi(x) \) is an even real function, and the potential \( a(x) \) (and the current \( j(x) \)) are the odd functions of coordinate: \( a(x) = -a(-x) \), i.e. \( a(0) = 0 \). Consequently, the boundary conditions to Eq. (1) can be written in the form (assuming \( 0 \leq x \leq D_\lambda \)):

\[ a|_{x=0} = 0, \quad \left. \frac{da}{dx} \right|_{x=D_\lambda} = h_\lambda, \]  

(4)

where \( h_\lambda = H/H_\lambda, \) \( H_\lambda = \phi_0/(2\pi \lambda^2) \).

As to the Eq. (2), we will take the usual boundary condition at the external surface [1]: \( \left. d\psi/dx \right|_{x=D_\lambda} = 0 \). The order parameter at the center is maximal, thus, the boundary conditions to Eq.(2) are:

\[ \left. \frac{d\psi}{dx} \right|_{x=0} = 0, \quad \left. \frac{d\psi}{dx} \right|_{x=D_\lambda} = 0. \]  

(5)

The magnetic moment (or magnetization) of the plate, related to the unity volume, is

\[ \frac{M}{V} = \frac{1}{V} \int \frac{B - H}{4\pi} dv = \frac{B_{av} - H}{4\pi}, \quad B_{av} = \frac{1}{V} \int B(r)dv = \frac{1}{S} \Phi(D_\lambda) = \frac{1}{D} A(D), \]  

where \( B_{av} \) is the mean field value inside the superconductor, \( S \) is the plate cross-section in the \((x,y)\)-plane. In normalization (3), denoting \( \bar{b} = B_{av}/H_\lambda, \) \( M_\lambda = M/H_\lambda, \) one finds:

\[ 4\pi M_\lambda = \bar{b} - h_\lambda, \quad \bar{b} = \frac{1}{D_\lambda} a_\lambda, \quad a_\lambda = a(D_\lambda), \quad D_\lambda = \frac{D}{\lambda}. \]  

(6)

The difference of Gibbs free energies of the system in superconducting and normal states, \( \Delta G = G_s - G_n \), can be expressed through its magnetic moment:

\[ \Delta G = F_{s0} - \frac{1}{2} MH, \quad F_{s0} = \frac{H^2}{8\pi} \int \left[ \psi^4 - 2\psi^2 + \xi^2 \left( \frac{d\psi}{dx} \right)^2 \right] dv, \]  

(7)
where $F_{s0}$ corresponds to the superconductor condensation energy, $H_c = \phi_0/(2\pi\sqrt{2}\lambda\xi)$ is thermodynamic critical field. Using (3), one finds from (7) the normalized expression
\[
\Delta g = \Delta G / \left(\frac{H_c^2}{2V}\right) = g_0 - \frac{8\pi M_\lambda h_\lambda}{\varphi^2}, \quad g_0 = \frac{1}{D_\lambda} \int_0^{D_\lambda} dx \left[ \psi^4 - 2\psi^2 + \frac{1}{\varphi^2} \left(\frac{d\psi}{dx}\right)^2 \right]. \tag{8}
\]
The expressions (6)–(8) are used below.

[Note, that the lengths $\xi$ and $\lambda = \varphi \xi$ enter the GL-equations on equal footing, each one may be chosen as the unit of length; for measuring the field the unit may be either $H_\lambda = \phi_0/(2\pi\lambda^2)$, or $H_\xi = \phi_0/(2\pi\xi^2)$ ($H_\xi = \varphi^2 H_\lambda$). In presenting the numeric results various variants of normalization will be used.]

The solutions were found by using the iteration procedure, described in [10]. In the beginning of iterations the trial function was taken as $\psi(x) \approx 1$ in FI-regime, and $\psi(x) \approx 0.01$ in FD-regime. The results do not depend on the choice of the concrete numeric algorithm.

If $\psi \ll 1$ ($B(x) \approx H$), the system of equations (1), (2) reduces to a single $\varphi$-independent linear equation
\[
d^2\psi/dx^2 + (1 - h_\lambda^2 x^2)\psi(x) = 0 \tag{9}
\]
$(-D_\xi \leq x \leq D_\xi, \; D_\xi = D/\xi, \; h_\lambda = H/H_\xi)$, with $\psi(x) = e^{-\xi^2 y} w(y)$, $y = \sqrt{2h_\xi x}$, where the Weber function $w(y)$ satisfies the confluent hypergeometric equation $w'' - y w' - aw(y) = 0$, $a = \frac{1}{2}(1 - 1/h_\xi)$, and can be expressed in a form [11], (2.44):
\[
w(y) = C_1 w_1(y) + C_2 w_2(y), \tag{10}
\]
\[
w_1(y) = 1 + \sum_{\nu=1}^{\infty} \frac{a(a + 2) \cdots (a + 2\nu - 2)}{(2\nu)!} y^{2\nu},
\]
\[
w_2(y) = y \left[ 1 + \sum_{\nu=1}^{\infty} \frac{(a + 1)(a + 3) \cdots (a + 2\nu - 1)}{(2\nu + 1)!} y^{2\nu} \right].
\]
Eq. (9) and the boundary conditions (5) allow to find the critical field of the second order phase transition, when $\psi(x) \to 0$ [12,13]. This critical field does not depend on $\varphi$. However, in the case of first order phase transitions the condition $\psi(x) \ll 1$ is not fulfilled (see below), so in a general case to find the critical fields it is necessary to solve the full system of nonlinear equations (1), (2), (5).

3. The state diagram on the plane $(D_\lambda, \varphi)$

The solutions of Eqs. (1)–(5) depend on the co-ordinate $x$ and three parameters $(D_\lambda, \varphi, h_\lambda)$. In Fig. 1 the state diagram on the plane of parameters $(D_\lambda, \varphi)$ is presented. In each point of this plane there exist a set of self-consistent solutions for the order parameter $\psi(x; h_\lambda)$ and the field $b(x; h_\lambda)$. If the representation point $(D_\lambda, \varphi)$ shifts, the corresponding state changes. In particular, the magnitude of the order parameter in the origin of co-ordinates changes, $\psi_0 = \psi(0; h_\lambda)$, and also the mean field value $\bar{b} = h_\lambda + 4\pi M_\lambda$. Such information permits to judge how the solution depends on the external field. It is convenient to imagine a peephole pierced in arbitrary point $(D_\lambda, \varphi)$, what allows to see the dependence of $\psi_0$ (and of the
magnetization, \(-4\pi M_\lambda = h_\lambda - \mathbf{b}\) on the field \(h_\lambda\) in this point. Studying such dependences, one can find on the plane of parameters \((D_\lambda, z)\) three critical lines \((\pi, S_{1-11}\) and \(\zeta)\), which divide this plane into four regions \((I_a, I_b, II, \) and \(II_b)\) with its own characteristic behavior of \(\psi_0\) (and \(-4\pi M_\lambda\) on the field \(h_\lambda\). The meaning of these regions is clarified in Fig. 2.

Fig. 2(a) shows (schematically) the dependence \(\psi_0(h_\lambda)\) in region \(I_a\). Evidently, for small \(h_\lambda\) there exists a stable superconducting state with \(\psi(x) \approx 1 (\psi_0 \approx 1)\), in which a weak external field is almost completely screened and does not penetrate into the bulk of a superconductor (the Meissner state, M-). With the field increasing (FI-regime) the order parameter is gradually suppressed, but when the field \(h_\lambda\) exceeds the critical value \(h_1\) M-solution becomes unstable (small perturbations grow) and passes in a jump \((\delta_1)\) into normal state \((n)\). For \(h_\lambda > h_1\) there is only one stable \(n\)-solution with \(\psi \equiv 0\).

Now, searching for the solution in FD-regime with small values of the trial function \(\psi(x) \ll 1\), one finds that \(n\)-state remains stable at \(h_\lambda < h_1\) down to the restoration field \(h_r\) in Fig. 2(a). In the field interval \(\Delta_n = h_1 - h_r\) two stable states (M- and \(n\)-) exist, but a supercooled \(n\)-state is metastable, because M-state (with \(\psi \approx 1\)) has smaller free energy (due to the negative condensation energy \(g_0\) in \((8)\)). At \(h_\lambda = h_r\) \(n\)-state loses stability (small perturbations grow) with a consequent jump \((\delta_r)\) into M-state. For \(h_\lambda < h_r\) there is only one stable M-solution.

The analogous picture is present in Fig. 2(e), where initial linear grows of the magnetization \((-4\pi M_\lambda)\) corresponds to the Meissner state \((\mathbf{b} \approx 0)\); the transition from M- to \(n\)-state is accompanied by a first order jump \((\delta_1)\); there is a hysteresis loop due to the presence of a supercooled \(n\)-state, and a first order jump \((\delta_r)\) from \(n\)- to M-state.

The characteristic behavior \(\psi_0(h_\lambda)\) in region \(I_b\) (Fig. 1) is depicted schematically in Fig. 2(b). The picture here is analogous to Fig. 2(a): in FI-regime there is a jump \((\delta_1)\) from M- to \(n\)-state; however, a supercooled (metastable) \(n\)-state in the field \(h_p\) passes into a special (also metastable) precursor \((p)\)-state, which looses stability in the field \(h_r\) with a first order jump \((\delta_r)\) to M-state.

The behavior of the magnetization in this region (Fig. 2(f)) is also characterized by a jump \((\delta_1)\); by the presence of a supercooled \(n\)-state in the field interval \(\Delta_n = h_1 - h_p\); by the presence of \(p\)-state (which exists in the field interval \(\Delta_p = h_p - h_r\)), and by a jump \((\delta_r)\) from \(p\)- to M-state in the field \(h_r\). The total width of the hysteresis loop is \(\Delta_{pn} = \Delta_p + \Delta_n = h_1 - h_r\). The width of the interval \(\Delta_p\) diminishes in the vicinity to the critical \(\pi\)-line (Fig. 1), and \(\Delta_p = 0\) on \(\pi\)-line. Thus, \(p\)-states exist only in region \(I_b\).

In region \(II_a\) M-state in FI-regime (see Fig. 2(c)) becomes unstable in the field \(h_1\) and transforms in a jump \((\delta_1)\) into a new stable "edge-suppressed" state \((e)\). In this state the order parameter is strongly suppressed in some layer near the plate boundary, so the magnetic field penetrates this layer practically without screening (see below Fig. 6). If \(h_\lambda\) is further increased, the superconducting \(e\)-state is destroyed gradually \((\psi \rightarrow 0)\), with final transition to \(n\)-state in the field \(h_2\).

The plate magnetization (Fig. 2(g)) is also characterized by the presence of \(e\)-tail and by a second order phase transition to \(n\)-state in the field \(h_2\). If now the field is decreased, \(n\)-state becomes absolutely unstable in the field \(h_2\), where \(e\)-state appears again and for fields \(h_\lambda < h_1\) it passes smoothly into a metastable "depressed" \((d)\)-state (see Fig. 6). Such \(d\)-state (i.e., a supercooled \(e\)-state) is a characteristic feature of region \(II_a\), it exists in the
field interval $\Delta_d = h_1 - h_r$ alongside with M-state and is responsible for a magnetization hysteresis (Fig. 2(g)). In region IIa a supercooled n-state is absolutely unstable, in difference to region Ia (Fig. 2(b,f)) where n-state is metastable in the field interval $\Delta_n = h_1 - h_r$.

If the thickness $D_\lambda$ decreases, remaining in region IIa ($\varkappa = \text{const}$), the jump amplitudes $\delta_r, \delta_l$ (and the interval $\Delta_d = h_1 - h_r$, Fig. 2(c,g)) decrease also, and they vanish ($\delta_r = \delta_l = \Delta_d = 0$) on $\varkappa$-line (Fig. 1). Below $\varkappa$-line (in region IIb) the hysteresis is impossible (Fig. 2(d,h)) and s-state passes into n-state by a second order phase transition.

The solutions behavior in the vicinity of critical lines $\pi, S_{\text{I-II}}$ and $\zeta$ will be studied in more details below, here we note the following. As is clear from Figs. 2(e-h), a superconductor in the increasing field passes to n-state either by a first order jump, or there is a tail of e-states on the magnetization curve and n-state appears by a second order phase transition. Accordingly, we will distinguish type-I and type-II superconductors. The boundary between two types of superconductors is represented by the curve $S_{\text{I-II}}$ (Fig.1). This boundary (or, equivalently, the critical value $\varkappa_c(D_\lambda)$) depends on the plate thickness, what does not coincide with the simple criterion $\varkappa_0 = 1/\sqrt{2}$ [1], used usually for dividing superconductors into two groups. This disagreement (as in the case of a cylinder [3]) is caused by several reasons.

First, in [1] the case is considered of an infinite superconductor, while we consider a superconducting plate of finite thickness. Second, our superconductor boarders a vacuum, while in [1] a case is considered of two contacting semi-infinite $s$- and $n$-metals. Third, we divide two types of superconductors according to the shape of their magnetization curves, while in [1] the division is made using different criterion: according to the sign of the surface free-energy $\sigma(\pi)$ of the interface (with $\sigma = 0$ at $\pi = 1/\sqrt{2}$ [1]). Thus, the mentioned disagreement is due to different settings of the problem.

4. $S_{\text{I-II}}$- and $\zeta$-boundaries; G-point

In this section the results of a self-consistent calculations for magnetization $-4\pi M_\lambda(h_\lambda)$ and $\psi_0(h_\lambda)$ are given, in a case of a superconducting plate with the parameters $D_\lambda, \varkappa$ laying near $S_{\text{I-II}}$-boundary in Fig. 1.

Fig. 3 represents the case $D_\lambda = 7$ for three values of $\varkappa$. In Figs. 3(a,b) ($\varkappa = 0.9$, a peephole is in region I$_b$) there are: M-state, a jump $\delta_l$ from M- to n-state (in the field $h_1$, FI-regime); a supercooled n-state ($\Delta_n$); a metastable p-state; a jump $\delta_r$ from p- to M-state (in the field $h_r$, FD-regime).

In Figs. 3(c,d) ($\varkappa = 0.9193$, a peephole is on $S_{\text{I-II}}$-boundary) a supercooled n-state has vanished already, but the tail of e-states did not yet appear; this special p-state (or marginal $\mu$-state) attains maximum amplitude in FD-regime at the restoration field $h_r$ (with a jump to M-state).

In Figs. 3(e,f) ($\varkappa = 0.95$, a peephole is in region IIa) there is a jump ($\delta_l$) from M- to e-state, a supercooled n-state is absent, however, a metastable $d$-state appears which ends in a jump ($\delta_r$) from $d$- to M-state in the field $h_r$.

There is a hysteresis loop on all the curves in Fig. 3; solid lines correspond to FI-regime, dotted lines correspond to FD-regime.

Fig. 4 illustrates what happens for smaller plate thickness ($D_\lambda = 2$). The value $\varkappa = 0.93$ belongs to region I$_b$ in Fig. 1. The value $\varkappa = 0.953$ corresponds to the boundary $S_{\text{I-II}}$. The
value \( \varkappa = 0.98 \) corresponds to region \( \Pi_a \) (the field interval \( \Delta_d \), where a hysteresis \( d \)-state exists, diminishes in moving closer to \( \zeta \)-boundary). The value \( \varkappa = 1.03 \) lies on \( \zeta \)-boundary, here the jumps vanish \( (\delta_1 = \delta_r = \Delta_d = 0) \) and the curves become hysteresis-less, having a vertical tangent in point \( i_0 \). For \( \varkappa = 1.05 \) (region \( \Pi_b \)) there is no hysteresis, but the inflexion point \( i \) with finite derivative remains on the curves. If \( \Delta_\lambda \) diminishes further, the inflexion point lowers and the curves become monotonous, without inflexion.

It is interesting also to trace what happens in the region of \( G \)-point in Fig. 1. The critical lines \( S_{I-II}, \zeta \) and \( \pi \) merge in this point \( (\varkappa_G \approx 0.915, \Delta_G \approx 1.51) \) and for \( \Delta_\lambda < \Delta_G \) there is a single critical curve. Above it (in region \( \Pi_a \)) the destruction (and nucleation) of superconductivity is accompanied by jumps \( \delta_1 \) (and \( \delta_r \)). Below it (in region \( \Pi_b \)) there is a smooth second order phase transition. Thus, for sufficiently small thicknesses all type-I superconductors (with \( \varkappa < \varkappa_G \)) become, in fact, type-II superconductors.

To the same conclusion (basing on different considerations) arrived Ginzburg [14], who noticed that type-I superconductors (with \( \varkappa \ll 1 \)) behave in magnetic field as type-II superconductors. Therefore, \( G \)-point may be referred to as the Ginzburg point. [Apart \( G \)-point there exist the so called tricritical Landau points (\( L- \)) [15], where the distinction vanishes between the critical fields, which correspond to the supercooled, equilibrium and superheated states of a superconductor [16] (i.e., where the hysteresis vanishes). The hysteresys-less critical line \( \zeta \) in Fig. 1 consists, in fact, of \( L \)-points.]

Fig. 5(a) shows the dependence of \( \psi_0 \) and \(-4\pi M_\lambda \) on the field \( h_\lambda \) in \( G \)-point. These curves end by a second order phase transition to \( n \)-state, having vertical tangent at the transition point \( h_2 \). Fig. 5b illustrates the dependence \( \psi_0(h_\lambda) \) in the vicinity of \( G \)-point \( (D_G \approx 1.51) \): curve 1 corresponds to \( \varkappa = 0.8 \) (region \( \Pi_a \) in Fig. 1); curve 2 corresponds to \( \varkappa_G = 0.915 \) \( (G \)-point); curve 3 corresponds to \( \varkappa = 1.0 \) (region \( \Pi_b \)). In region \( \Pi_a \) (curve 1) there exists a supercooled \( n \)-state \( (\Delta_n) \), there are first order jumps \( (\delta_1 \) and \( \delta_r) \) and the hysteresis loop. On curve 2 the derivative \( d\psi_0/dh_\lambda = \infty \) at the second order phase transition field \( h_2 \). The hysteresis-less curve 3 with finite derivative at the transition point \( h_2 \) corresponds to region \( \Pi_b \).

5. Examples of the co-ordinate dependences

Figs. 1–5 illustrate the solutions behavior on parameters \( (\varkappa, D_\lambda, h_\lambda) \). In Fig. 6 the self-consistent solutions \( \psi(x) \) and \( b(x) \) are depicted as functions of the reduced co-ordinate \( x/D_\lambda \), when the representation point crosses the plane of parameters \( (\varkappa, D_\lambda) \) in Fig. 1 along the lines \( D_\lambda = 7 \) and \( D_\lambda = 2 \).

Fig. 6a shows the space profile of the order parameter \( \psi(x) \) for \( D_\lambda = 7 \) and several values of \( \varkappa \). Curve \( M_\mu \) \( (\varkappa = 1.1, \text{region } \Pi_a \text{ in Fig. 1}) \) is the Meissner state in the field \( h_l = 0.9742 \), which precedes the jump to \( \epsilon \)-state \( (h_\lambda = 0.9743) \). The order parameter of \( \epsilon \)-curve is suppressed near the plate boundary. When the field \( h_\lambda \) is further increased, the amplitude of \( \epsilon \)-state tends to zero and vanishes finally at \( h_2 = 1.2114 \). Curve \( p \) \( (\varkappa = 0.8, \text{region } \Pi_l) \) is a precursor state in the field \( h_r = 0.5676 \), which precedes the jump to the Meissner state \( M_p \) \( (h_\lambda = 0.5675) \). Curve \( \mu \) \( (\varkappa = 0.9193) \), the peephole is on \( S_{I-II} \)-boundary) corresponds to the marginal \( p \)-state, which attains the maximal amplitude \( (\psi_r = 0.988) \) before transforming into \( M_\mu \)-state \( (\psi_0 = 0.9999) \) at \( h_r = 0.6520 \). The depressed \( d \)-state forms in FD-regime from
e-state (which exists at \( h_{\lambda} = 0.9743, \kappa = 1.1 \)) by gradual transformation of e-solution profile and ends (at \( h_{r} = 0.7797 \)) in a jump to the Meissner \( M_d \)-state (\( h_{\lambda} = 0.7796 \)).

The corresponding profiles of \( b(x) \) are depicted in Fig. 6(b).

The solutions \( \psi(x) \) and \( b(x) \) for a plate of smaller thickness (\( D_{\lambda} = 2 \)) are depicted in Figs. 6(c,d). Shown are: e-solution at the point of transition from \( M_e \)- to e-state (\( \kappa = 0.97 \), region \( \Pi_e \)); d-solution at the point of transition from \( d^- \) to \( M_{d^-} \) (\( \kappa = 0.97 \), region \( \Pi_d^- \)); \( \mu \)-solution in the field of transition from \( \mu^- \) to \( M_{\mu^-} \) (\( \kappa = 0.953 \), the boundary \( S_{I-II} \)); \( p \)-solution in the field of transition from \( p^- \) to \( M_{p^-} \) (\( \kappa = 0.93 \), region \( \Pi_0 \)). The solution \( i_0 \) is also shown, which lies on \( \zeta \)-boundary (\( \kappa = 1.030, h_{\lambda} = 1.076 \)) with \( M' = \infty \) at the inflexion point, and the solution \( \pi \) (\( \kappa = 1.05 \), region \( \Pi_b, h_{\lambda} = 1.1 \)) with finite value \( M' \) at the inflexion point.

### 6. Region of \( p \)-states, \( \pi \)-boundary

In this Section the precursor solutions \( \psi(x) \) and \( b(x) \) are shown at the points (\( \kappa, D_{\lambda} \)) belonging to region \( \Pi_b \) of Fig. 1.

The space profiles \( \psi(x) \) and \( b(x) \) for a plate with \( D_{\lambda} = 10 \) and various \( \kappa \) are depicted in Figs. 7(a,b). The value \( \kappa = 0.922 \) corresponds to \( S_{I-II} \) boundary (at \( D_{\lambda} = 10 \)). At this point the jump from \( M \) - to \( n \)-state happens in the field \( h_{1} = 0.8511 \) (FI-regime, see Fig. 3(c)). In FD-regime a supercooled \( n \)-state does not exist (it is absolutely unstable), but a superconducting \( p \)-solution appears, having the amplitude \( \psi_0 \) which grows with the field diminishing and reaches the maximum value in the field \( h_{r} = 0.6529 \). Such hysteresis \( p \)-state, which belongs to \( S_{I-II} \) boundary, will be named as the marginal \( \mu \)-state. It exists in the field interval \( \Delta_{p} = h_{1} - h_{r} = 0.1982 \) simultaneously with \( M \)-state and is depicted in Fig. 7(a) by curve \( \mu \) at the field of a jump (\( h_{r} \)) to the Meissner state \( M_{\mu} \).

If one moves from \( S_{I-II} \) boundary into region \( \Pi_b \), a superconducting \( p \)-state begins nucleating from a supercooled \( n \)-state at the field \( h_{p} \). The amplitude of \( p \)-state reaches the maximum at the field \( h_{p} \), after that the jump to \( M \)-state occurs. Such hysteresis \( p \)-state (for \( \kappa = 0.8 \), region \( \Pi_b \)) is represented by curve \( p_{r} \) at the point of a jump (\( h_{r} = 0.5664 \)) to \( M_{p^-} \)-state (not shown); \( p \)-state with \( \kappa = 0.8 \) exists in the field interval \( \Delta_{p} = h_{p} - h_{r} = 0.0744 \).

The field interval \( \Delta_{p} \), where \( p \)-state exists, diminishes rapidly with \( \kappa \) diminishing. (At \( \kappa = 0.708 \), region \( \Pi_b \), we have \( h_{p} = 0.5018, h_{r} = 0.5013, \Delta_{p} = 5 \cdot 10^{-4} \); at \( \kappa = 0.707 \), region \( \Pi_a \), we have \( \Delta_{p} = 0 \), with \( M \)-state restoring from a supercooled \( n \)-state in a first order jump without forming \( p \)-state preliminary.) The last of \( p \)-states existing in region \( \Pi_b \) corresponds to \( \pi \)-boundary of Fig. 1. Such \( \pi \)-state is represented by curve \( \pi \) in Fig. 7(a) (at the field of a jump into corresponding \( M_{\pi} \)-state, \( h_{r} = 0.5013 \)).

The profiles of \( b(x) \) for the same states (\( D_{\lambda} = 10 \)) are shown in Fig. 7(b).

If \( D_{\lambda} \) diminishes, the interval of \( \kappa \) in region \( \Pi_b \) (where the hysteresis \( p \)-states exist) diminishes also (Fig. 1) and \( \pi \)-boundary curve merges with the curves \( S_{I-II} \) and \( \zeta \) at the same point \( G \). There are no \( p \)-states in region \( \Pi_a \).

It is interesting to watch how the profiles \( \psi(x) \) and \( b(x) \) of \( \pi \)-states change, while moving along \( \pi \)-boundary in Fig. 1. This is shown in Fig. 8(a,b) where \( \pi \)-solutions for \( D_{\lambda} = 10 \) (\( \kappa_{\pi} = 0.708, h_{r} = 0.5013 \)), \( D_{\lambda} = 7 \) (\( \kappa_{\pi} = 0.708, h_{r} = 0.5014 \)), \( D_{\lambda} = 5 \) (\( \kappa_{\pi} = 0.708, h_{r} = 0.5015 \)) and \( D_{\lambda} = 3 \) (\( \kappa_{\pi} = 0.775, h_{r} = 0.6152 \)) are presented. [In distinction to Fig. 7(b) where \( b = B/H_{\lambda} \), the fields \( b_{\xi} = B/H_{\xi} \) in Fig. 8(b) are normalized to \( H_{\xi} = \kappa^2 H_{\lambda} \).]
The dotted curve in Fig. 8(a) is the solution \((W)\) of the linear equation \((9)\), normalized to the maximum value of curve 3. It is evident, that the self-consistent solutions \(\psi(x)\) are described at \(\psi_0 \ll 1\) by the Weber functions \((10)\). Simultaneously, \(b_\xi \approx 1\) and \(H \approx H_\xi\), so the linear equation \((9)\) can be used for finding the minimal supercooling field \(H_r(D) \rightarrow H_\xi\) at \(D \gg 1\).

One can see from Fig. 8(a), that when \(x \rightarrow 1/\sqrt{2}\) and \(D_\lambda \gg 1\) the \(\pi\)-states profiles take a characteristic shape of the interface between \(s\)- and \(n\)-half-spaces \([1]\). We show below that in this special case the metastable (hysteresis) \(\pi\)-state coincides with the degenerate Bogomolnyi state \([8]\) and can be described analytically \([9]\).

### 7. Connection with the Bogomolnyi equations

As was shown by Bogomolnyi \([8]\), at \(x = 1/\sqrt{2}\) the GL-equations for the infinite superconductor degenerate and can be reduced to a system of two nonlinear first order differential equations, which have the analytic solutions \([8,9]\). If \(\psi\) is a real function only of one Cartesian co-ordinate \(x\), the solution is given by the implicit formula \([9]\):

\[
\int_{\psi}^{\psi_0} \frac{dy}{y \sqrt{y^2 - (1 + \ln y^2)}} = \pm x, \quad b_\xi^2(x) = 1 - \psi^2(x), \quad b_\xi(x) = \frac{B(x)}{H_\xi},
\]

with \(\psi \rightarrow 1\) if \(x \rightarrow -\infty\), and \(\psi \rightarrow 0\) if \(x \rightarrow +\infty\). Point \(x = 0\) is defined by the condition \(d^2\psi/dx^2 = 0\) (the inflexion point of \(\psi(x)\)), i.e., by the equation \(\psi^2 - 1 - \ln \psi = 0\) with the root \(\psi_i = 0.451\).

The solid line in Fig. 9 is the solution \(\psi(x)\) found from the full system of GL-equations (\(\pi\)-solution in Fig. 8(a) at \(D_\lambda = 10, \, x = 0.708\)). The dotted line is the analytic solution \((11)\) \((-\infty < x < +\infty, \, x = 1/\sqrt{2}\), the inflexion points of both solutions are superimposed). Evidently, the self-consistent \(\pi\)-solution in the limit \(D_\lambda \gg 1\) coincides with the Bogomolnyi solution. The self-consistent field \(b_\xi(x)\) (see the solution with \(D_\lambda = 10\) in Fig. 8(b)) also matches the formula \((11)\) \((b_\xi^2 = 1 - \psi^2)\). [In this connection see \([17]\), where the solutions of the Bogomolnyi equations for a single vortex in the infinite superconductor are discussed.]

In Fig. 7 the profiles of \(p\)-solutions \(\psi(x)\) and \(b(x)\) were shown at the point of a jump \(h_r\) from \(p\)- to \(M\)-state. Fig. 10 illustrates the behavior of \(p\)-solutions as function of \(h_\xi\) for \(D_\lambda = 10\) and various \(x\). Figs. 10(a,b) demonstrate: (a) – the mean value \(\overline{\psi}\) and (b) – the free energy \(\Delta g\) \((b)\) in \(\mu\)-state \((x_\mu = 0.922\), the peephole is on the boundary \(S_{1-11}\). The field interval \(\Delta_p = h_p - h_r\) where \(\mu\)-state exists is \(\Delta_p = 0.2331\) (normalized by \(H_\xi\)). In Figs. 10(c,d) \((x_\rho = 0.8\), the peephole is inside region \(I_b\)) this interval is \(\Delta_p = 0.1163\). In Figs. 10(e,f) \((x_\pi = 0.708\), the peephole is almost on \(\pi\)-boundary) \(\Delta_p = 0.001\).

From Fig. 10 it follows: 1) the degenerate Bogomolnyi solution \((B\)-state, which exists at \(x = 1/\sqrt{2}, \, D_\lambda \rightarrow \infty\) is a special case of \(p\)-states nucleating in the hysteresis FD-regime from a supercooled \(n\)-state; 2) \(B\)-state exists only in the field \(h_1 = h_p = h_r = 1\) (i.e., \(H = H_\xi\)) when the superconductivity simultaneously originates \((\overline{\psi}_p \approx 0)\) and reaches the maximal amplitude \((\psi(0) = 1, \, \overline{\psi} = 0.5\), see Fig. 9); 3) \(B\)-state is metastable because \(M\)-state \((\overline{\psi} = 1)\) of smaller energy exists as well.

Notice, that Fig. 10 indicates also to the nonanalyticity of GL-solutions at the degeneration point \((x_\sigma = 1/\sqrt{2}, \, D = \infty, \, h_\xi = 1)\). Indeed, at \(x > x_\sigma\) \(p\)-solutions exist, having a form
very similar to the degenerate Bogomolnyi solution (with $\bar{\psi} \approx 0.5$, see Fig. 7). However, at $\kappa < \kappa_0$ only the absolutely stable M-state remains with $\bar{\psi} \approx 1$, and absolutely unstable $n$-state with $\bar{\psi} \equiv 0$. In another words, there is a termination point of $p$-solutions on $\pi$-boundary, i.e., nonanalyticity at the point $\kappa = \kappa_0$.

8. Critical fields (phase diagrams)

As was mentioned, in an arbitrary point of the state diagram (Fig. 1) the critical fields exist ($h_1$ and $h_2$ in FI-regime, or $h_p$ and $h_r$ in FD-regime) which are represented schematically in Fig. 2. The dependence of the critical fields on the plate thickness may be seen, if one makes a mental cut of the plane of states in Fig. 1 along a line $\kappa = \text{const}$. The picture seen is presented in Fig. 11 by a number of phase diagrams (in the co-ordinates $h_\xi = H/H_\xi$, $D_\xi = D/\xi$) for plates with different $\kappa$.

Dashed line $\kappa = 0.3$ in Fig. 11(a) corresponds to the critical field $h_1$ (FI-regime); below this line lies the region of superconducting M-phase ($\bar{\psi} \sim 1$), above this line lies the region of $n$-phase ($\bar{\psi} \equiv 0$). Solid line $W$ corresponds to the critical field $h_r$ (FD-regime); above this line lies the region of metastable $n$-phase, below this line lies the region of M-states. In region I$_a$ (Fig. 1) a supercooled $n$-state becomes absolutely unstable in the field $h_r$ where the jump to M-state occurs. A supercooled $n$-state exists in the field interval $\Delta_n = h_1 - h_r$ where the hysteresis is possible. The interval $\Delta_n$ diminishes with the plate thickness and at some $D_\xi$ the lines $h_1$ and $h_r$ merge ($\Delta_n = 0$). This point in Fig. 1 (at $\kappa = 0.3$) is represented by the value $D_\lambda = 1.14$ ($D_\xi = \kappa D_\lambda = 0.342$), which lies on $\zeta$-boundary of the hysteresis region. For smaller thicknesses (region II$_r$ in Fig. 1) there exists unique critical field $W$, in which the superconducting state is destroyed (FI) or originates (FD) without hysteresis by a second order phase transition. (Notice, that for small $\kappa$ the hysteresis interval $\Delta_n$ seems to increase, because the field scale diminishes, $H_\xi = \kappa^2 H_\lambda$.)

In Fig. 11(a) the critical fields $h_1$ and $W$ are also drawn for the values $\kappa = 0.5$ and $\kappa = 0.7071$ (which lie to the left of the line $\kappa_0 = 1/\sqrt{2}$ in Fig. 1). Evidently, the critical field $h_1$ (dashed line) diminishes with $\kappa$ increasing. The fields $h_r$ (normalized to $H_\xi$) are represented for all $\kappa$ [12] by a single curve $W$ (which corresponds to the stability boundary of a supercooled $n$-phase).

The picture changes in passing to $\kappa > \kappa_0$. Because in region I$_b$ (Fig. 1) there are metastable $p$-states, we have three critical fields here ($h_1, h_p, h_r$ in Figs. 2(b,f)). These fields are represented in Fig. 11(a) (for $\kappa = 0.8$) by three curves $h_1, W, h_r$. The field $h_1$ (dashed line) corresponds again to the maximum field, at which the jump from M- to $n$-state occurs. At the field $h_p$ a supercooled $n$-state becomes absolutely unstable and a superconducting (metastable) $p$-state of small amplitude originates (this field is presented by the same curve $W$ as for $\kappa < \kappa_0$). At the field $h_r$ (dotted line) the metastable $p$-state becomes absolutely unstable and a first order transition from $p$- to M-state occurs. In the field interval $\Delta_p = h_p - h_r$ the metastable $p$-states exist and the hysteresis is possible.

The field interval $\Delta_p$ diminishes with the plate thickness, so crossing $\pi$-boundary (Fig. 1) we get from region I$_b$ into region I$_a$ where $p$-states are absent but the supercooled $n$-state exists. (For $\kappa = 0.8$ $\pi$-boundary is crossed at $D_\lambda = 2.7$ or $D_\xi = 2.16$. The dotted line $h_r$ in Fig. 11(a) merges with the curve $W$ at point $\alpha$.) In region I$_a$ there are only two critical
fields \((h_1 \text{ and } W)\). Further decreasing \(D\) we get from region I\(_a\) into region I\(_b\) crossing the hysteresis \(\zeta\)-boundary (for \(\kappa = 0.8\) this happens at \(D_\xi = 1.2\), point \(\beta\) in Fig. 11(a), when the curves \(h_1\) and \(W\) merge). In region I\(_b\) \((D_\xi < 1.2)\) no hysteresis is possible and in Fig. 11(a) remains only one critical curve \(W\) which describes the states with \(\psi \to 0\).

In Fig. 11(b) the critical fields are depicted for \(\kappa = 1; 1.2; 2\). If \(\kappa = 1\) (the mental cut in the plane of Fig. 1 lies inside region I\(_a\)) there are three critical fields: \(h_r, h_1, h_2\) (see Fig. 2(c,g)). The tail of \(e\)-states with \(\overline{\psi} \ll 1\) either vanish (FI) or appears (FD) in the field \(h_2\). This field is marked in Fig. 11(b) by a letter \(W\) (solid curve). In the field \(h_1\) (dashed line) the jump from \(M\)- to \(e\)-state occurs (in FI-regime) and \(d\)-state appears (in FD-regime). The metastable \(d\)-states (and the corresponding hysteresis) exist down to the field \(h_r\) where the jump from \(d\)- to \(M\)-state occurs. The width of the hysteresis region \(\Delta_d = h_1 - h_r\) depends on the plate thickness. The points \(\Delta_d = 0\) in Fig. 11(b) (or the Landau points, \(L\)) correspond to the intersection of the line \(\kappa = \text{const}\) with \(\zeta\)-boundary in Fig. 1. (For \(\kappa = 1\) \(\Delta_d = 0\) at \(D_\xi = 2.0\).) In region I\(_b\) the hysteresis is absent, so here exists only one critical field \(W\) which describes the reversible second order phase transition.

Phase diagrams at \(\kappa = 1.2\) and \(\kappa = 2\) are analogous to the case \(\kappa = 1\). The second order phase transition curve \(W\) (as well as the curve \(W\) in Fig. 11(a)) is the same for all \(\kappa\), it can be found from the linearized equation (9) and expressed through the Weber functions \(w\) (10). However, to find the first order phase transition fields \((h_1\) and \(h_r\)) it is necessary to solve full system of GL-equations.

Note, that the interval between \(L\)-point (where \(\Delta_d = 0\)) and \(W\)-curve diminishes with \(\kappa\), so the curves of Fig. 11(b) transform continuously into the curves of Fig. 11(a).

9. Conclusion

Note in conclusion, that the vortex-free states, studied above, may be realized in mesoscopic samples with characteristic dimension \(D\) of several \(\lambda\). With \(D\) increasing the uniform (one-dimensional) edge-suppressed \(e\)-state (as well as \(d\)- and \(p\)-states) may become unstable relative breaking the boundary region into separate vortices, with forming subsequently the regular vortex lattice [2]. However, the detailed study of such inhomogeneous states demands the solution of partial differential equations what is outside the scope of the present investigation. (In this connection see, for instance, Refs. [18,19] where some of such problems are considered to explain the experiments [20–24] with thin superconducting discs of various form in a perpendicular magnetic field.) Besides, many of the theoretical results happen to depend rather weakly on the sample geometry, so the predictions obtained for the plate (or the cylinder [3]) on the base of one-dimensional equations, have, probably, more general value and may be used in discussing the details of the concrete experiments.

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Figures captions

Fig. 1. The state diagram on the plane \((D_\lambda, \kappa)\). Curve \(S_{1-II}\) is the boundary between first and second order phase transitions from \(s\) - to \(n\)-state in the increasing field (FI-regime); curve \(\pi\) is the boundary of metastable \(p\)-states in the decreasing field (FD-regime); curve \(\zeta\) is the hysteresis boundary, below \(\zeta\)-line the hysteresis is absent. The asymptotics of \(\zeta\)-boundary are: \(D_\lambda = 1.13\) for \(\kappa \to 0\); \(D_\lambda = 2.43\) for \(\kappa > 6\).

Fig. 2. The order parameter \(\psi_0\) and magnetization \((-4\pi M_\lambda)\) versus field \(h_\lambda\) in different regions of Fig. 1 (schematically, see the text).

Fig. 3. The dependences of \(\psi_0\) and \(-4\pi M_\lambda\) on \(h_\lambda\) in the vicinity of \(S_{1-II}\)-boundary in Fig. 1 \((D_\lambda = 7\), the values of \(\kappa\) are given in the figure.) \(M\) is the Meissner state; \(p\) is the precursor state \((\text{region } I_b)\); \(\mu\) is the marginal \(p\)-state \((\text{laying on the critical } S_{1-II}\text{-boundary})\); \(d\) is the metastable depressed \(d\)-state \((\text{region } \Pi_a)\). Dotted lines correspond to FD-regime.

. 4. The dependences of \(\psi_0\) and \(-4\pi M_\lambda\) on \(h_\lambda\) at \(D_\lambda = 2\) and various \(\kappa\): \(\kappa = 0.93\) \((\text{region } I_b \text{ in Fig. 1})\), \(\kappa = 0.953\) \((\mu\)-state on \(S_{1-II}\)-boundary), \(\kappa = 0.98\) \((\text{region } \Pi_a)\), \(\kappa = 1.03\) \((\zeta\)-boundary), \(\kappa = 1.05\) \((\text{region } \Pi_b)\); \(p\), \(\mu\), \(d\) are the hysteresis \((\text{metastable})\) states. On the curves \(\kappa = 1.05\) the points of inflexion \(i\) are marked.

. 5. (a) – The dependences \(\psi_0(h_\lambda)\) and \(-4\pi M_\lambda(h_\lambda)\) in \(G\)-point in Fig. 1. (b) – The dependences \(\psi_0(h_\lambda)\) in the vicinity of \(G\)-point \((D_G = 1.51)\): \(1 - \kappa = 0.8\) \((\text{region } I_a)\); \(2 - \kappa = 0.915\) \((G\)-point\); \(3 - \kappa = 1.0\) \((\text{region } \Pi_b)\).

. 6. (a) – The order parameter \(\psi(x)\) and \((b) – \text{the field } b_\lambda(x) (D_\lambda = 7)\) in different states: \(e - \kappa = 1.1\), \(h_1 = 0.9743\); \(p - \kappa = 0.8\), \(h_r = 0.5676\); \(\mu - \kappa = 0.9193\), \(h_r = 0.6520\); \(d - \kappa = 1.1\), \(h_r = 0.7796\). The corresponding M-states \((\text{see the text})\) are also shown.

(c) and (d) – The analogous curves for \(D_\lambda = 2\): \(e - \kappa = 0.97\), \(h_1 = 1.0172\); \(p - \kappa = 0.93\), \(h_r = 0.9612\); \(\mu - \kappa = 0.953\), \(h_r = 0.9903\); \(d - \kappa = 0.97\), \(h_r = 1.072\); \(i_0 - \kappa = 1.030\), \(h_\lambda = 1.076\); \(i - \kappa = 1.05\), \(h_\lambda = 1.1\) \((\text{see the text})\).

Fig. 7. The precursor states: \((a) – \psi(x)\) and \((b) – b_\lambda(x)\) for \(D_\lambda = 10\) and different \(\kappa\): \(\mu - (S_{1-II}\text{-boundary})\) \(\kappa = 0.922\), \(h_r = 0.6529\); \(p_r - \kappa = 0.8\), \(h_r = 0.5664\); \(\pi - \kappa = 0.708\), \(h_r = 0.5013\) \((\text{see the text})\).

Fig. 8. The space profiles of \(p\)-states, laying on \(\pi\)-boundary at the field of a jump \((h_r)\) from \(p\)- to M-state \((\text{for } D_\lambda = 5; 7; 10)\). The values of \(\kappa\) and \(h_r\) \((\text{normalized to } H_\xi)\) are given in the text. The dotted line in \(a)\) is the normalized solution \(W\) of the linear equation \((9)\).

Fig. 9. Solid lines are the self-consistent solutions for \(\psi(x)\) and \(b(x)\) \((\text{normalized to } H_\xi)\) for \(D_\lambda = 10\), \(\kappa = 0.708\). Dashed line is the degenerate Bogomolnyi solution \((11)\). The solutions are superimposed at the inflexion points \(\psi_i = 0.451\).

Fig. 10. The mean value \(\bar{\psi}\) \((a)\) and the free energy \(\Delta g\) \((b)\) as functions of \(h_\xi\) for the states existing in the plate of thickness \(D_\lambda = 10\) and various \(\kappa\) \((\text{shown in figure})\). Solid lines are \(p\) - and \(n\)-states, dashed lines are the precursor states \((\mu, p, \pi)\). Points \(B\) on \(\pi\)-curves correspond to the degenerate Bogomolnyi solution with \(\bar{\psi} \approx 0.5\).

Fig. 11. The critical fields for different \(\kappa\) \((\text{shown in figure})\): (a) – type-I superconductors, \(\kappa < \kappa_c\); (b) – type-II superconductors, \(\kappa > \kappa_c\). Dashed lines are the fields \(h_1\) in which M-state becomes absolutely unstable \((\text{FI})\); solid line \(W\) is the field in which \(n\)-state becomes absolutely unstable \((\text{FD})\); dotted lines are the fields \(h_r\) in which M-state restores \((\text{FD})\).
Fig. 2
Fig. 4
Fig. 5

\[ D_0 = 1.51 \]
\[ \kappa_0 = 0.915 \]
Fig. 6
Fig. 7
Fig. 8
$D_{\lambda} = 10$
$\kappa = 0.708$

Fig. 9
Fig. 10
Fig. 11