Modeling the Transport of Nonthermal Particles in Flares Using Fokker–Planck Kinetic Theory

Joel C. Allred1, Meriem Alaoui2, Adam F. Kowalski3,4,5, and Graham S. Kerr2

1 NASA Goddard Space Flight Center, Solar Physics Laboratory, Code 671, Greenbelt, MD 20771, USA; joel.c.allred@nasa.gov
2 Catholic University of America at NASA Goddard Space Flight Center, Solar Physics Laboratory, Code 671, Greenbelt, MD 20771, USA
3 National Solar Observatory, University of Colorado Boulder, 3665 Discovery Drive, Boulder, CO 80303, USA
4 Department of Astrophysical and Planetary Sciences, University of Colorado, 2000 Colorado Avenue, Boulder, CO 80305, USA
5 Laboratory for Atmospheric and Space Physics, University of Colorado Boulder, 3665 Discovery Drive, Boulder, CO 80303, USA

Received 2020 July 9; revised 2020 August 18; accepted 2020 August 21; published 2020 October 8

Abstract

We describe a new approach for modeling the transport of high-energy particles accelerated during flares from the acceleration region in the solar corona until their eventual thermalization in the flare footpoint. Our technique numerically solves the Fokker–Planck equation and includes forces corresponding to Coulomb collisions in a flux loop with nonuniform ionization, synchrotron emission reaction, magnetic mirroring, and a return current electric field. Our solution to the Fokker–Planck equation includes second-order pitch angle and momentum diffusion. It is applicable to particles of arbitrary mass and charge. By tracking the collisions, we predict the bremsstrahlung produced as these particles interact with the ambient stellar atmosphere. This can be compared directly with observations and used to constrain the accelerated particle energy distribution. We have named our numerical code FP and distributed it for general use. We demonstrate its effectiveness in several test cases.

Unified Astronomy Thesaurus concepts: Solar flares (1496); Solar x-ray flares (1816); Solar flare spectra (1982); Stellar flares (1603); Non-thermal radiation sources (1119); Computational methods (165); Solar magnetic fields (1503)

Supporting material: animation

1. Introduction

During solar and stellar flares, numerous particles are accelerated to high energies. This acceleration likely results from the release of energy during the reconnection of the magnetic field occurring near the tops of magnetic flux loops in the corona. These high-energy particles propagate along magnetic field lines toward the lower atmosphere. We detect their presence through bursts of nonthermal radiation produced by their interactions with the ambient atmosphere during their transport. The bulk of this emission is produced when the particles reach sufficient density, typically in the footpoints, to slow them to thermal speeds. Details of these particles’ energy distributions can be determined by analyzing the spectra of these radiation bursts through either a forward fit of an electron model or inversion of the X-ray spectrum. For example, Holman et al. (2003) used X-rays observed by RHESSI (Lin et al. 2002) of a powerful solar flare to determine the energy spectrum of nonthermal electrons in the flare footpoints. Similar X-ray spectral analyses have been performed in numerous studies (e.g., Kontar et al. 2011; Emslie et al. 2012; Simões & Kontar 2013; Aschwanden et al. 2016; Alaoui et al. 2019, and references therein).

Forces acting during the particles’ transport from loop top to footpoint can significantly alter their energy distributions. If the measured distributions are to be compared to predictions from acceleration models (see, e.g., Aschwanden 2002, and references therein), the effects of these forces must be taken into account. Much progress has been made in constructing models of nonthermal particle transport in the presence of strong forces. A kinetic Fokker–Planck description of the distribution function evolution for particles experiencing Coulomb collisions and other forces has been developed by many authors (Chandrasekhar 1943; Snyder & Scott 1949; Rosenbluth et al. 1957; Spitzer 1962; Trubnikov 1965). In the context of solar flares, Leach & Petrosian (1981) solved a Fokker–Planck equation using a finite difference method for nonthermal electrons including cold-target Coulomb collisions and mirroring due to converging magnetic fields. Here the cold-target assumption means that the speed of the nonthermal electrons greatly exceeds the average speed of thermal particles. McTiernan & Petrosian (1990) improved upon Leach & Petrosian (1981) by accounting for relativistic effects and energy lost due to synchrotron emission. Mauas & Gómez (1997) and Jeffrey et al. (2019) further improved on Leach & Petrosian (1981) by including the momentum diffusion term (D in our notation; discussed in Section 2.2), which is important for modeling the thermalization of lower-energy particles. Battaglia et al. (2012) used a stochastic differential equation approach to model the source sizes of X-rays produced by nonthermal electrons injected at the top of the loop models. Emslie et al. (2018) used a collisional transport model to consider the effects of momentum and pitch-angle diffusion on the energy deposition rate from nonthermal electrons.

In addition to electrons, protons and heavier ions are likely accelerated during flares (e.g., Hurford et al. 2006). Emslie et al. (2012) found the energy in ions to be comparable to that of electrons in several large flares. Tamres et al. (1986) studied energy and momentum deposition in flare loops produced by beams of nonthermal protons. They considered cold-target and warm-target cases. Here warm-target means that the nonthermal protons had velocities between those of the thermal electrons and ions. They found that in the warm-target approximation, protons were able to penetrate to higher column depths than expected in the cold-target approximation. Emslie et al. (1998) considered the hydrodynamic response of flare loops heated by beams of nonthermal protons. Gordovskyy et al. (2005) compared the hydrodynamic response of loops heated by proton beams to those...
heated by electron beams. However, the lack of observational constraints on the accelerated ion distribution has precluded their widespread use in flare modeling, despite their likely presence and importance.

During flares the flux of nonthermal particles injected at loop tops can be very large. For example, Holman et al. (2003) found an injection rate greater than 10$^{16}$ electrons s$^{-1}$ during an X-class flare. This corresponds to a very large electric current that, if unbalanced, would produce a large magnetic field that would choke off the propagation of nonthermal electrons away from the acceleration site. In order to maintain a steady beam of nonthermal particles (flare observations show that particles are accelerated into the lower atmosphere over time scales much longer than the transient electrodynamical and time-of-flight time scales) and charge neutrality, a cospatial counterstreaming return current is required. It is driven by an electric field induced in the plasma by the streaming nonthermal particles (van den Oord 1990). This electric field accelerates ambient plasma electrons but acts to slow the downward-directed nonthermal particles. In fact, this force often dominates over much of the distance traveled by the nonthermal particles. Therefore, it must be included in determining the evolution of the nonthermal particle distribution function.

Considerable progress has been made in developing a theory of return current. Knight & Sturrock (1977) first investigated a nonthermal beam/return current system in one dimension and deduced that the return current can result in additional heating in the upper atmosphere. This was improved by Emslie (1980) to include collisional deceleration. Zharkova & Gordovskyy (2005, 2006) performed numerical simulations of the effects of return current electric force on nonthermal electron beam propagation and calculated the corresponding hard X-ray spectra. These simulations tracked the hydrodynamic evolution of a flux loop to heating from the nonthermal beam, as well as Joule heating produced from the return current. Holman (2012) studied signatures of the return current on the hard X-ray bremsstrahlung emitted during large flares and found that the return current can cause a flattening in X-ray spectra at low energy. Alaoui & Holman (2017) used the return current collisional thick target model (RCCTTM) developed by Holman (2012) and analyzed flattenings in the spectra of several large flares. Under the assumption that these were caused by the return current electric field, they used RCCTTM to fit the observed hard X-ray flux to deduce the injected electron flux density. The RCCTTM separates the effects of return current from Coulomb collisions. The return current is assumed to dominate (and collisional deceleration is neglected) while nonthermal electrons propagate through the relatively low-density corona. Conversely, in the footpoint, collisions are assumed to dominate, and the return current electric force is neglected. This was found to work well for several flares, but in others, it was found that the coronal density is sufficiently high that collisions cannot be neglected in the corona, and the RCCTTM assumptions break down. The authors further concluded that if the injected low-energy cutoff is $\gtrsim 60$ keV (flare- and time-dependent), the return current is stable to current-driven instabilities, but if $E_c \sim \delta k T$ or the area of the loop is smaller than that observed with RHESSI, current-driven instabilities can arise. Battaglia & Benz (2008) found that the difference between the loop-top and footpoint X-ray spectra shows a difference higher than can be explained by the classical cold-target model (Brown 1971) and attributed this difference to return current losses in the corona. They further concluded that the resistivity is enhanced, and current-driven instabilities could be responsible.

In this work, we present a more comprehensive computational model, FP, that solves the Fokker–Planck equation for the transport of nonthermal particles through magnetic flux loops and including the effects of strong forces on their energy distributions. MacKinnon & Craig (1991) demonstrated that the Fokker–Planck equation is equivalent to a coupled set of stochastic ordinary differential equations. Park & Petrovian (1996) compared the stochastic method with the finite difference method and found that the stochastic method was computationally more expensive. Therefore, in this work, we employ a finite difference method. We have designed this method to be general so that it can model many different types of nonthermal particles (e.g., electrons, protons, and ions), as well as general ambient atmospheric conditions. FP improves upon previous work in several important ways.

1. The injected nonthermal particles can be of arbitrary mass and charge. Our method applies to injected electrons, protons, alphas, and heavier ions.

2. We do not make cold- or warm-target assumptions. Our Coulomb collision operator (Section 2.2) uses general forms for the Rosenbluth potentials (see Equation (5) in Section 2.2) for collisions of nonthermal with thermal particles. These are applicable over large ranges of nonthermal particle energy and ambient plasma temperatures. Note that our own earlier implementation of a Fokker–Planck treatment into a flare numerical code (Allred et al. 2015) also made no target temperature assumption.

3. Our solution to the Fokker–Planck equation includes the return current force (Section 2.4) and self-consistently solves the resulting nonlinear differential equation.

4. Nonthermal particles are injected into loop models produced using the RADYN flare modeling code (Carlsson & Stein 1992, 1995, 1997; Allred et al. 2005, 2015). In detail RADYN models the chromospheric radiative transfer and its coupling to non-LTE hydrogen, calcium, and helium atomic level populations. Our model uses the ionization state number densities from RADYN in calculating the Coulomb collisional operator for collisions with charged (Section 2.2) and neutral (Section 2.3) particles.

5. FP can be used as a plug-in to the X-ray data analysis tool, Object Spectral Executive (OSPEX; https://hesperia.gsfc.nasa.gov/ssw/packages/spex/doc/spex_explanation.htm). OSPEX fits X-ray observations during flares from, for example, RHESSI to a model of X-ray production. Since FP can predict bremsstrahlung resulting from nonthermal particles during their transport, OSPEX+FP can be used to forward fit X-ray observations to obtain constraints on the injected nonthermal electron spectra accounting for the numerous transport effects described in Section 2. In a forthcoming study, we will demonstrate the use of OSPEX+FP to determine injected electron distributions in a large solar flare.

6. FP is designed for general use, and we have released it as an open-source computational tool. This allows FP to be easily incorporated into numerical models of flares driven by nonthermal particles.

---

6 Available for download at https://github.com/solarFP/FP.
As we have discussed previously, one of the main motivations for the development of FP was to more accurately model the transport of nonthermal particles during flares and the resulting impacts on stellar atmospheres. The flare modeling code RADYN previously included a Fokker–Planck solver based on work by McTiernan & Petrosian (1990). This was described in Allred et al. (2015). That version was similar to what we describe, but we have enhanced it to self-consistently handle the force due to return currents and second-order parallel momentum (energy) diffusion. This updated Fokker–Planck treatment is currently being incorporated into RADYN. The predicted heating rates due to nonthermal particles can be very different for models that include return current effects. An example is shown in Section 3.1.

We wish to stress that while FP represents a marked improvement over our prior implementation of the Fokker–Planck treatment in RADYN, that prior effort did include many of the transport effects discussed here. Namely, since Allred et al. (2015), RADYN has made no assumption as to target temperature (there is no cold- or warm-target approximation).

In this paper, we will describe our new Fokker–Planck nonthermal particle transport code, FP. In Section 2 we describe the computational method we employ to solve the Fokker–Planck equation. In Section 3 we demonstrate solutions to several test cases and compare them to results derived analytically. Finally, in Section 4 we present our conclusions and discuss plans for future improvements.

2. Methodology

The semirelativistic Fokker–Planck kinetic equation for Coulomb collisions and external forces can be written using the notation of Trubnikov (1965) as follows,

\[ \frac{\partial f_a}{\partial t} + \nabla_r \cdot (v f_a) + \nabla_p \cdot (F^e f_a) + \sum_b \nabla_p \cdot j^{\mu/b} = 0, \]

(1)

where \( f_a \) is the distribution function for a particle, \( a \), with mass, \( m_a \). That is, \( f_a(r, p, t) d^3r \, d^3p \, dt \) is the number of particles of type \( a \) within the volume \( r + d^3r \) and with momenta between \( p \) and \( p + d^3p \) in the time interval \( t + dt \). Here \( r \) is the phase-space velocity (\( r = p / \gamma m_a \)), \( \gamma \) is the relativistic Lorentz factor, \( F^e \) is the sum of external forces and includes all forces except the Coulomb collision force, and \( j^{\mu/b} / f_a \) is the net force on particles of type \( a \) due to Coulomb collisions with particles of type \( b \). The sum is over all constituent plasma particles. Here \( \nabla_r \) and \( \nabla_p \) are divergence operators in phase space for the geometric and momentum spaces, respectively. In this form, it is evident that Equation (1) is a conservation equation in phase space.

2.1. Loop Geometry

In the low-plasma-beta environment typical of the solar corona, charged particles are constrained to move along the axis of magnetic field lines. Therefore, we follow the example of many previous works (e.g., Fisher et al. 1985; Abbett & Hawley 1999; Allred et al. 2005, 2015; Kašparová et al. 2009; Liu et al. 2009; Petkaki et al. 2012; Kowalski et al. 2015; Rubio da Costa et al. 2016) making the assumption that the motion of beams of nonthermal particles traveling from the acceleration region near the top of magnetic loops toward the footpoints is well characterized by a 1.5D geometry with spatial dimension, \( z \), being the distance along a loop measured from the injection site assumed to be at the loop apex and pitch angle, \( \theta \), being the angle between a particle’s momentum vector and the \( z \)-axis. We assume that the loop has a constant cross-sectional area. In solving Equation (1), we assume no variation in directions perpendicular to the magnetic field axis, which implicitly assumes that we obtain \( f_a \) for positions near the center of a bundle of magnetic loops each receiving equal injection of nonthermal particles. In the momentum space, we employ a standard spherical coordinate system, where \( p \) is the momentum magnitude and \( \theta \) and \( \phi \) are the pitch and azimuthal angles, respectively. For this work, we assume azimuthal symmetry, so \( \partial / \partial \phi = 0 \). We use the momentum space basis vectors \( \hat{p} \), which is a unit vector in the direction of \( p \), and \( \hat{\theta} \), which is perpendicular to \( \hat{p} \) and in the direction of increasing \( \theta \). For convenience, we introduce \( \mu = \cos \theta \).

The forces in the Fokker–Planck equation are dependent on the ambient plasma density, temperature, and magnetic flux density, so it is important to have a model for how these quantities vary along the loop. As examples, we use the temperature and density stratification from RADYN loop models that have been evolved into a state of hydrodynamic equilibrium. The magnetic flux density is from a model that assumes 1000 G in the photosphere and exponentially decreases to 75 G at 3 Mm above the photosphere and is constant above that. While it is true that this magnetic flux density model is ad hoc, we note that its most important effect, the magnetic mirroring force, is relatively insensitive to inaccuracies in the overall field strength. Often, the changing magnetic flux density is used as a proxy for estimating changes in the loop cross-sectional area, so that magnetic flux is constant within the loop. Since we have assumed a constant loop cross-sectional area, we are not making this assumption, but we note that it is straightforward to alter our method to do so. When evaluating the divergence of the geometric fluxes in Equation (1) \( (\nabla_r) \), a nonuniform area, \( A(z) \), that scales with magnetic flux density can be used.

Figure 1 shows temperature, density, and magnetic flux density stratification for three loop conditions in hydrodynamic equilibrium obtained from RADYN simulations. Each of these has a half-length, \( L \), of 13 Mm, but their densities and temperatures are quite different. One (HL) has an apex temperature of 34 MK, which is typical of temperatures measured at the peak of large flares (Caspi et al. 2014). The others (CL and CL_CCF) have an apex temperature of 3.4 MK, which is typical of nonflaring active region loops. CL and CL_CCF are identical except for their magnetic flux density stratifications. Here CL has a uniform magnetic flux density throughout the upper coronal portion of the loop, but CL_CCF has a strongly converging coronal field.

For fast particles accelerated during flares, the hydrodynamic transient and time-of-flight time scales are much less than is observable with current instrumentation. Therefore, for this work, we seek a steady-state solution consistent with a flux of nonthermal particles injected at the top of flaring loops.

2.2. Collisions with Charged Particles

The collision term, \( j^{\mu/b} \), can be divided into terms representing dynamic friction \( (F) \) and diffusion \( (D) \) forces. For this work, we consider the collision of nonthermal particles...
with a thermal plasma. Even in superhot flares (∼50 MK), thermal particles are mostly nonrelativistic and well characterized by a Maxwell–Boltzmann distribution with temperature, $T_b$. The collision terms, $F$ and $D$, can be further divided into terms representing collisions with charged ($F_c$ and $D_c$) and neutral ($F_N$ and $D_N$) particles.

Making the assumption that the target particles, $b$, are nonrelativistic, the charged particle collision terms can be written as

$$F_b = -4\pi K^{a/b} \nabla \cdot \nabla \Phi_b,$$
$$D_b = 4\pi K^{a/b} \frac{m_a}{m_b} \nabla \cdot \nabla \Psi_b. \tag{2}$$

Here we make use of the notation that $(A \cdot \nabla_p)$ is the momentum space convective operator for a vector, $A$. The collection of constants, $K^{a/b}$, is defined as

$$K^{a/b} = 4\pi \lambda^{a/b} m_a (e^2 Z_a Z_b)^2,$$  

where $\lambda^{a/b}$ is the Coulomb logarithm; $Z_a$ and $Z_b$ and $m_a$ and $m_b$ are the charge numbers and masses of the particles of type $a$ and $b$, respectively; and $e$ is the electron charge in Gaussian units. The expression for $\lambda^{a/b}$ is from Equation (13) of Allred et al. (2015), which we reproduce here for clarity:

$$\lambda^{a/b} = \ln \left( \frac{M_{ab} v^2}{\hbar} \frac{m_b}{\pi n_b Z_b^2 e^2} \right)^2.$$  

where $M_{ab}$ is the reduced mass of the beam and target particles. Here $\Phi_b$ and $\Psi_b$ are potential functions derived from the distribution function, $f_b$, for particles of type $b$. They are defined as

$$\Psi_b(p) = -\frac{1}{8\pi} \int |p - p'| f_b(p') d^3 p',$$
$$\Phi_b(p) = -\frac{1}{4\pi} \int f_b(p') d^3 p'. \tag{5}$$

For Maxwell–Boltzmann distributions of the target, the potentials have the forms (Equations (17.9) and (17.23) of Trubnikov 1965)

$$\Psi_b = -\frac{n_b}{8\pi \sqrt{m_b}} \sqrt{2m_b kT_b}$$
$$\times \left[ (x_b + 1) \xi'(x_b) + \left( x_b + \frac{1}{2} \right) \xi(x_b) \right], \tag{6}$$

and

$$\Phi_b = -\frac{n_b}{4\pi m_b v} [\xi'(x_b) + \xi(x_b)], \tag{7}$$

where

$$\xi(x_b) = \text{erf}(\sqrt{x_b}) - \xi(x_b),$$
$$\xi'(x_b) = \frac{2}{\pi} e^{-x_b} \sqrt{x_b},$$  

$$x_b = \frac{m_b v^2}{2kT_b} = \frac{m_b p^2}{2m_b kT}.$$  

In deriving Equations (2)–(8), we have not made use of our loop geometry, and they are valid in any 3D geometry. Now imposing azimuthal symmetry and noting that $\Phi_b$ and $\Psi_b$ are functions of only the momentum magnitude greatly simplifies
the expressions for $\mathbf{F}_c$ and $\mathbf{D}_c$, which can be rewritten as

$$
\mathbf{F}_c = -\frac{m_a n_a K^{a/b}}{m_b} \xi_0 \hat{\mathbf{p}},
$$

$$
\mathbf{D}_c = -n_b K^{a/b} 2 \xi_0 \left( \frac{\xi + \xi_0}{2 \xi_0} \right) \frac{\partial f}{\partial \theta} \hat{\mathbf{p}},
$$

where $D_p$ is the parallel momentum diffusion term discussed by Jeffrey et al. (2019), and $D_\theta$ is the pitch-angle diffusion term.

### 2.3. Collisions with Neutral Particles

The stopping force of a neutral gas from collisions with energetic particles is discussed extensively in Evans (1955), and here we make use of those results. The dynamic friction and diffusion terms can be written as (see Equations (2.24) and (2.25), p. 581 of Evans 1955; Snyder & Scott 1949)

$$
\mathbf{F}_N = -\frac{m_a n_a K^{a/N}}{m_e} \xi_0 \hat{\mathbf{p}},
$$

$$
\mathbf{D}_N = -\frac{n_b K^{a/N}}{2 \xi_0} \frac{\partial f}{\partial \theta} \hat{\mathbf{p}},
$$

where $K^{a/N}$ and $K^{a/N}$ are defined as

$$
K^{a/N} = 4 \pi \lambda^{a/N} m_a (e^2 Z_a)^2 Z_N,
$$

$$
K^{a/N} = 4 \pi \lambda^{a/N} m_a (e^2 Z_a)^2 Z_N^2,
$$

where $Z_N$ is the atomic number of the neutral species, and $m_e$ is the electron mass. The Coulomb logarithm, $\lambda^{a/N}$, differs when $a$ represents electrons or ions. They are

$$
\lambda^{e/N} = \ln \left( \frac{m_e c^2 \beta \gamma \sqrt{\gamma - 1}}{I_N} \right),
$$

$$
\lambda^{i/N} = \ln \left( \frac{2 m_e c^2 \beta \gamma^2}{I_N} \right),
$$

for electrons and ions, respectively. Here $\beta$ is the relativistic speed ($\beta = v/c$), and $I_N$ is the ionization energy. The Coulomb logarithm for diffusion from neutral particles is

$$
\lambda^{a/N} = \ln \left( \frac{\gamma \beta}{\sqrt{2} Z_N^2} \right),
$$

where $a$ is the fine structure constant.

### 2.4. External Forces

The external forces considered in this work are the reaction force from synchrotron radiation, $\mathbf{F}_S$: reflecting due to magnetic mirroring, $\mathbf{F}_M$; and, importantly, the return current force, $\mathbf{F}_{RC}$. The first two of these are discussed in McTiernan & Petrosian (1990), and we simply quote their expressions here:

$$
\mathbf{F}_S = -S \beta (\gamma^2 - 1 - \mu^2) \hat{\mathbf{p}} + \mu \sqrt{1 - \mu^2} \mathbf{\hat{\theta}},
$$

where $S$ is defined by

$$
S = \frac{2}{3} \left( \frac{Z_a e}{m_e c^2} \right)^3 B^2,
$$

and $B$ is the magnetic flux density in gauss. The magnetic mirroring force is given by

$$
\mathbf{F}_M = \frac{1}{2} m_a \nu^2 \sqrt{1 - \mu^2} \frac{d \ln B}{dz} \hat{\mathbf{\theta}}.
$$

Including the return current force is the main task in this work. We assume that the return current is carried by drifting thermal electrons moving to neutralize the beam current. We neglect instabilities (Papadopoulos 1977; Benz 2002; Karlický 2009) and the presence of runaway electrons (Holman 1985; Rowland & Vlahos 1985); therefore, we implicitly assume that the electric field is much less than the Dreicer field (Dreicer 1960). We make the assumption that, in the steady state, the return current density, $J_{RC}$, cancels the beam current density, $J_{beam}$, at each point in space. The beam current density is obtained from the distribution function, $f_a$,

$$
J_{RC} = -J_{beam} = -Z_a e \mathcal{F},
$$

where $\mathcal{F}$ is the number flux of beam particles. Because of azimuthal symmetry, only the field-aligned component has a net contribution to $\mathcal{F}$, which is given by

$$
\mathcal{F} = \int f_a d^4 p = \int \mu \partial f_a d^3 p \, \hat{\mathbf{\theta}}.
$$

For time scales much longer than the collisional timescale, van den Oord (1990) found Ohm’s law to be valid. Since we are considering the steady state, we assume the plasma obeys Ohm’s law. That is, $E_{RC} = \eta \mathbf{F}_{RC}$, where $\eta$ is the plasma resistivity. The force on beam particles from $E_{RC}$ is given by

$$
F_{RC} = -(Z_a e)^2 \eta \mathcal{F} (\mu \hat{\mathbf{\theta}} - \sqrt{1 - \mu^2} \mathbf{\hat{\theta}}),
$$

where we have made use of $\hat{\mathbf{\theta}} = \mu \hat{\mathbf{\theta}} - \sqrt{1 - \mu^2} \mathbf{\hat{\theta}}$. We use an expression for $\eta$ that includes the collisions of drifting ambient electrons with ionized hydrogen (Spitzer 1962) and helium (Hirshman 1977) and neutral hydrogen. The drifting electron/neutral hydrogen collision frequency is from Martínez-Sykora et al. (2012).

### 2.5. Steady State

We rewrite Equation (1) in a steady-state form by setting $\partial f_a / \partial t = 0$, evaluating the divergences, and collecting terms with similar derivatives of $f_a$. After considerable algebraic manipulation, we obtain

$$
A^{(p)} \frac{\partial f_a}{\partial \mathbf{p}} + A^{(p)} \frac{\partial f_a}{\partial \mu} + A^{(p)} \frac{\partial f_a}{\partial \mathbf{\hat{\theta}}},
$$

where

$$
A^{(p)} = -\sum_b n_b K^{a/b} \frac{\xi}{2 \xi_0}.
$$
\[
A^{(p)} = \sum_b \left( \frac{n_b K^{a/b}}{p^3} \left( \frac{\xi}{2x_b} - \xi' - \frac{m_a}{m_b} \xi \right) \right) - \sum_n \left( \frac{m_a n_n K^{a/N}}{m^2} \right) - S(\gamma_k^2 - \mu^2) - (Z_a e)^2 \eta \mu \mathcal{F},
\]
(22b)
\[
A^{(\mu)} = C^{(p)} (1 - \mu^2),
\]
(22c)
\[
A^{(c)} = \mu \beta \xi,
\]
(22d)
\[
A^{(f)} = - \sum_b \left( \frac{n_b K^{a/b} (\xi + \xi' - \frac{\xi}{2x_b})}{p^3} \right) - \sum_n \left( \frac{n_k K^{a/N}}{2p^3} \right).
\]
(22g)

In deriving these expressions, we have neglected terms proportional to \(\partial \lambda / \partial \mu\) because the Coulomb logarithms vary slowly with respect to \(\mu\).

2.6. Numerical Method

We solve the time-independent Fokker–Planck equation (Equation (21)) by discretizing the phase space, approximating derivatives as finite differences, and iterating the solution until the residuals are below a threshold value. In our field-aligned loop approximation, there are three phase-space dimensions: the \(z\)-axis ranging from \(z = 0, L\), the \(\theta\)-axis ranging from \(\theta = 0, \pi\), and the \(p\)-axis ranging from \(p = [p_{\min}, p_{\max}]\). Here \(p_{\min}\) and \(p_{\max}\) are configurable. It is possible to choose \(p_{\min} = 0\), but that results in slow convergence to the steady state. A better choice is \(p_{\min} = \sqrt{2m_e k T_e}\), where \(T_e\) is the ambient electron temperature. This allows the solution to model the thermalization of beam particles while still maintaining good convergence. Here \(p_{\max}\) should be chosen to be sufficiently large that the number of particles with momenta above it is negligible. A reasonable choice for flare conditions is \(p_{\max} = 100 m_e c\). Within these ranges, the phase space is divided into grid cells. The \(z\)-axis grid is provided by the loop model and must have sufficient resolution to resolve temperature and density gradients. For loop models provided by RADYN, there are typically 191 grid cells along the \(z\)-axis. RADYN uses an adaptive grid, allowing it to resolve steep gradients in the transition region. The location of \(z\)-axis grid cells for the loop models HL and CL are shown using the “+” symbol in Figure 1. The \(\theta\) and \(p\)-axes are divided into evenly and logarithmically spaced grid cells, respectively. The number of cells in each of these dimensions is configurable, but 60 cells in the \(\theta\)-axis and 100 cells in the \(p\)-axis are typical numbers based on our numerical experiments. On these grids we construct three-point (second-order accurate) first and second derivative stencils. In solving Equation (21), we employ an upwind scheme, so first derivative stencils are one-sided in the upwind direction, except in computing the residual, when we use a centered stencil.

Equation (21) can be represented as a nonlinear matrix equation,
\[
A_{\mu} = b,
\]
(23)
where \(A\) is a function of \(f_a\) through the factor, \(\mathcal{F}\), in the return current force. The right-hand side, \(b\), is nonzero only at the boundaries, which are discussed in Section 2.7.

We solve Equation (23) using an alternating direction implicit iterative scheme. We split the operator, \(A\), into components along each axis, that is, \(A = (A_d + A_p + A_{\mu} + A_z)\), where \(A_d\) contains the diagonal term of \(A\) and \(A_p\), \(A_{\mu}\), and \(A_z\) contain the off-diagonal terms in the \(p\), \(\mu\), and \(z\)-directions, respectively. Therefore, we can construct implicit matrix equations in each direction separately. For example, in the \(p\)-direction, Equation (23) is rewritten as \((A_d + A_p) f_a^{n+1} + b = A_{\mu} + A_z f_a^n\), where \(f_a^n\) and \(f_a^{n+1}\) are the current and next iterative solutions. In each grid cell, the direction of the flux is evaluated using \(f_a^n\), and the correct upwind stencil is chosen. Thus, Equation (23) is approximated as a series of pentadiagonal matrix equations. The pentadiagonal systems are easily solved in sweeps along the \(p\), \(\mu\), and \(z\)-directions. After a complete iteration, \(\mathcal{F}\) is reevaluated, and the return current components of \(A\) are updated. Sweeps are performed until the residual and relative changes between \(f_a^n\) and \(f_a^{n+1}\) are below threshold values. The residual of an equation in the form of Equation (23) is typically defined by \(||A f_a - b||/||b||\), where \(||\cdot||\) is the \(L^2\)-norm. But since \(b\) is zero in the interior of our computational domain, we cannot use that. Instead, we define a normalized residual, \(r\), that measures how closely the components of \(A f_a\) sum to zero. We use
\[
r = \frac{||A f_a||}{||(|A_d f_a| + |A_{\mu} f_a| + |A_p f_a| + |A_z f_a|)||}.
\]
(24)

2.7. Boundary Conditions

Since Equation (23) is homogeneous, the boundary conditions determine the particular solution to which our algorithm converges. Because of azimuthal symmetry, \(\partial f_a / \partial \mu = 0\) at the \(\mu = \pm 1\) boundaries. At \(p = p_{\min}\), we assume \(\partial f_a / \partial p = 0\). This is applicable, since in the low-energy limit, \(f_a\) approaches a thermal distribution. Since \(p_{\max}\) is chosen to be sufficiently high that the number of particles above it is negligible, we assume \(f_a(p_{\max}) = 0\). At the \(z = L\) boundary, we have implemented a nonreflecting boundary condition so that particles reaching that depth simply pass through. However, we note that for loop models extending into the subphotosphere, the densities are high, and very few particles reach the \(z = L\) boundary.

The injected particle flux distribution, \(\mathcal{F}_0(E, \theta)\), is specified at the \(z = 0\) boundary. Numerous observations have shown that \(\mathcal{F}_0\) is well represented by a power law in energy, \(E\). So \(\mathcal{F}_0\) is
written as
\[ f_0(E, \theta) = \begin{cases} \frac{\delta(\delta - 1)}{E_c} M_0(\theta) \left( \frac{E}{E_c} \right)^{-\delta} & \text{for } E \geq E_c, \\ 0 & \text{for } E < E_c \end{cases} \] (25)
where \( N_0 \) is the total number flux density, \( E_c \) is a cutoff energy below which no particles are injected, and \( \delta \) is the power-law index. The pitch-angle distribution of the injected particles, \( M_0(\theta) \), remains largely unconstrained by current observations.

To account for this, FP allows users to select from three general forms for \( M_0(\theta) \). Those are fully beamed (\( \theta = 0 \) for all injected particles), isotropic in the forward hemisphere, and a Gaussian centered at \( \theta = 0 \) and with a standard deviation of \( \theta_0 \).

2.8. Initial Estimate

Since Equation (23) is solved iteratively, we must provide an initial estimate of the solution, \( f_0^0 \). We note that often in flare loops, collisional deceleration dominates other forces. So we make an initial estimate using the analytic solution to a much simpler form of Equation (21) in which \( A^{(3)} = A^{(2)} = A^{(0)} = 0 \) and forces in \( A^{(p)} \) other than the collision force are neglected. This simple first-order differential equation has a solution, \( f_0^0(z, \mu, p) = f_0^0(z = 0, \mu, p(z)) \), where \( p(z) = p - \int_0^z A^{(p)}/A^{(c)} \, dz \). In other words, this solution is just the \( z = 0 \) boundary condition but shifted by the cumulative effect of collisional decelerations.

2.9. Heating Rate on the Ambient Plasma

Much of the energy lost by the nonthermal particles during their transport is gained as heat by the ambient plasma. FP models this heating and momentum deposition. This is useful for dynamic flare simulations, such as those produced using RADYN. We assume that the energy and momentum lost due to Coulomb collisions and the return current are gained by the local ambient plasma. Since synchrotron emission is usually optically thin, we assume that it has no local heating effect. Using the solution to the distribution function, the heating rate, \( Q \), and the momentum deposition rate, \( G \), are obtained from
\[ Q = -\int (F_t \cdot \nu)f_0^0 \, d^3p, \] (26)
\[ G = -\int (F_t \cdot \hat{z})f_0^0 \, d^3p \hat{z}, \] (27)
where \( F_t \) is the total of the Coulomb collision and return current forces.

3. Test Cases

In this section, we demonstrate the performance of FP and compare its predictions to previous work using several test cases.

3.1. Collisional Cold Thick Target

The collisional cold thick target model (CCTTM; Brown 1971) has been of fundamental importance to the study of flares. Briefly, the model assumes that fast electrons accelerated in the corona travel toward the chromosphere. The electrons are slowed by Coulomb collisions, but because they are fast compared to the ambient thermal speed, the cold-target approximation is valid. Return currents and other transport forces are ignored by the CCTTM. Using the CCTTM, Emslie (1978, hereafter E78) derived an analytic model for how nonthermal electrons (or protons) deposit energy into the ambient atmosphere assuming an arbitrary but uniform ionization fraction. Hawley & Fisher (1994, hereafter HF94) extended that analytic expression to include a nonuniform ionization fraction over the course of the electrons’ transport. We have performed a test to compare how FP performs compared to the E78+HF94 heating rate. For the first step, we configured FP to mimic the assumptions of the CCTTM (labeled FP_CC_only). That is, all forces are switched off except Coulomb collisions of nonthermal electrons with ambient electrons and neutral hydrogen. We modeled the transport of electrons injected into CL (Figure 1) with a power-law flux distribution (Equation (25)) characterized by \( E_c = 20 \text{ keV} \), \( \delta = 4 \), and an injected energy flux of \( 1 \times 10^{11} \text{ erg cm}^{-2} \text{s}^{-1} \). These are typical values measured during M- and X-class solar flares (e.g., Holman et al. 2003; Milligan et al. 2014; Warmuth & Mann 2016). The results are shown in Figure 2. The top left panel shows the temperature and density stratification in our loop model. The bottom left panel shows the volumetric heating computed by FP_CC_only (black line) compared to the E78+HF94 heating rate (gray dashed line). FP_CC_only reproduces the E78+HF94 analytic model well.

For the second step of this test case, we have switched on the return current, magnetic mirroring, and synchrotron reaction forces (labeled FP). We have injected electrons into the CL loop model plotted in Figure 1, which includes hydrogen and helium ionization fractions. The same power-law distribution was injected. The blue line in the bottom left panel of Figure 2 shows the resulting volumetric heating rate predicted by FP. More than an order of magnitude more heat is deposited in the coronal portions of the loop than predicted in the E78+HF94 model; since the density is low, this can dramatically raise the coronal temperature. The bulk of the coronal portion is due to Joule heating caused by the return current. The peak of the energy deposition is 0.14 Mm higher and primarily in the transition region and top of the chromosphere rather than the deep chromosphere, as predicted by the E78+HF94 model. The small bump near 10 Mm is due to the magnetic mirror causing a spreading in pitch angle resulting in a slight increase in column depth and thus more heating. Simulations of flare loops and the resulting emission (e.g., Allred et al. 2005) will behave very differently in response to heating computed using the E78+HF94 model compared to FP.

For the final step of this test case, we use FP to compute the nonthermal bremsstrahlung emitted by the transporting electrons and compare it to the CCTTM prediction calculated from the FP_CC_only case. The bremsstrahlung cross section is from Haug (1997), as implemented in the SSW IDL routine BREMCROSS. The result is shown in the right panel of Figure 2. As a check to demonstrate that FP_CC_only behaves similar to the CCTTM, we also plot the predicted spectrum computed using the SSW IDL routine BREM2_NUI, which implements a nonuniform ionization extension to the CCTTM (Su et al. 2011). The CCTTM case predicts nearly an order of magnitude more flux than the FP model. This is because the return current force slows the nonthermal electrons, so that much of their energy has been lost by the time they reach the thick target. Clearly, inverting observed X-ray spectra with models that do not account for the return current force will infer...
very different injected electron distributions compared with models that include it. To remedy this, we have incorporated FP into the X-ray inversion model, OSPEX.

### 3.2. Return Current Model

For this test case, we demonstrate the performance of FP in conditions where the return current force dominates and compare it to the analytic return current model, RCCTTM, developed by Holman (2012). The plasma resistivity, $\eta$, increases with decreasing temperature, and Coulomb collisions decrease with decreasing density, so the return current force dominates in low-temperature, low-density environments.

For comparison with RCCTTM, we put FP into a mode in which it makes similar assumptions, and then, by relaxing those assumptions, we demonstrate their range of applicability. The RCCTTM is a 1D model, assuming that all nonthermal electrons have zero pitch angle. The electrons are decelerated by the return current electric field, and other forces are neglected by RCCTTM. Once the nonthermal electrons have been reduced in energy to a thermalization energy, $E_{th}$, they are lost from the electron beam. Here $E_{th}$ is a free parameter of the RCCTTM and often taken to be $(\delta + 1)kT$ (see Equation (28)).

We label FP configured to run in a mode that mimics RCCTTM as FP_RC_only_1D. We relax the 1D assumption by allowing the electron pitch angle to change in response to the return current force but still with all other forces switched off. In this mode, FP is labeled FP_RC_only. Finally, FP including all forces described in Section 2 keeps the label FP. An advantage of FP is that, since it includes the dynamic friction and energy diffusion components of the Coulomb collision force, it is able to directly model thermalization. Therefore, the parameter $E_{th}$ is not needed, and FP naturally produces a Maxwell–Boltzmann distribution as the nonthermal particles undergo many collisions.

As in previous test cases, we model the injection of nonthermal electrons with $E_{x} = 20$ keV, $\delta = 4$, and an energy flux of $1 \times 10^{15}$ erg cm$^{-2}$ s$^{-1}$ into the CL loop model. The results of the comparison are shown in Figure 3. In the top panel, we plot the heating rates obtained using the RCCTTM model (red dashed line) compared to FP_RC_only_1D (gray line), FP_RC_only (black line), and FP (blue line). As expected, the heating rates predicted by FP_RC_only_1D and RCCTTM are essentially identical. Here FP_RC_only predicts a smaller heating rate throughout the coronal portion of the loop. Interestingly, FP also predicts a lower heating rate, even though it includes deceleration from Coulomb collisions in addition to return current. A reason for this is that in this low-density loop, there is relatively little energy loss from the Coulomb collisions, but the collisions do cause some pitch-angle scattering. This decreases the beam collimation, resulting in a smaller current density and return current force.

This effect can be seen in the bottom panel of Figure 3, which compares the electron flux distributions obtained from FP and RCCTTM. In FP_RC_only, and even more markedly in FP, the flux at lower energies is directed upward (i.e., in the negative $\hat{z}$ direction), resulting in a relatively smaller $\mathcal{F}$. For particles moving upward, the return current produces an acceleration. This important effect is not captured in 1D models, thus demonstrating the need for FP’s pitch-angle treatment. Some numerical diffusion across the sharp low-energy cutoff is evident in the FP_RC_only_1D case. Increasing the energy resolution can reduce that effect. We have compared the predicted heating rates and bremsstrahlung spectra from FP_RC_only_1D and RCCTTM and found them to be nearly identical, so we conclude that the current resolution is sufficient.
3.3. Magnetic Mirroring

During flares, nonthermal hard X-rays are occasionally seen near loop tops in addition to the footpoints (e.g., Dennis et al. 2018). A likely explanation is that a strongly converging coronal magnetic field forms a magnetic mirror that traps a fraction of the injected particles and produces a coronal X-ray source (Kong et al. 2019). In this test case, we use FP to demonstrate this trapping effect. We use a flux of nonthermal electrons similar to the previous test cases, that is, a power law with $E = E_0 \cdot 20 \text{ keV}$, $\delta = 4$, and an energy flux of $1 \times 10^{11} \text{ erg cm}^{-2} \text{s}^{-1}$ were injected into the CL loop model. (Bottom panel) Electron flux distributions at 7 Mm for the same models in the top panel. The dotted black line is the injected flux distribution.

3.4. Energy Diffusion

For this test case, we consider the importance of parallel momentum (i.e., energy) diffusion. This is the $\hat{p}$ component of $D_c$ (Equation (9)) and gives rise to the second derivative, $\partial^2 f_e / \partial p^2$, in Equation (21). To understand when energy diffusion is important relative to dynamic friction, we take the ratio

$$\frac{D_c}{F_c} = \frac{\partial^2 f_e}{2\epsilon_e f_n} = -\frac{(\delta + 1)kT}{E},$$

where we have assumed that the flux is a power law with respect to energy (i.e., has a form like Equation (25)), so $f_e$ is a power law with index $-2(\delta + 1)$ with respect to momentum. Here we consider the collision of nonthermal electrons on thermal electron targets, so that the mass ratio appearing in $F_c$ cancels. In these circumstances, $D_c$ becomes dominant for energies less than $(\delta + 1)kT$. This is closely related to the warm-target approximation discussed in the proton beam test case (Section 3.5). Flares often heat a large emission measure of plasma to greater than 30 MK. Here $(\delta + 1)kT = 13 \text{ keV}$ for $\delta = 4$ and $T = 30 \text{ MK}$, which is comparable to cutoff energies deduced from the transition energy between the transitions.
Figure 4. (Top panel) Density of nonthermal electrons with energies between 35 and 50 keV as a function of distance from the loop top injected into the CL (black line) and CL_CCF (red line) loop models. In each case, a power-law spectrum characterized by $E_c = 20$ keV, $\delta = 4$, and an energy flux of $1 \times 10^{11}$ erg cm$^{-2}$ s$^{-1}$ is injected. (Bottom panel) X-ray bremsstrahlung flux between 35 and 50 keV integrated over 2 Mm distance intervals resulting from the nonthermal electron densities shown in the top panel. The X-ray flux has been normalized to the value in the footpoint interval (11–13 Mm).

dimensional components of X-ray spectra in many flares. Here $D_e$ is in the positive $\hat{p}$ direction, meaning that these low-energy electrons experience a net acceleration, distorting $f_\alpha$ away from a power law. The steady state is achieved when $D_e$ balances $F_\alpha$, in which case $f_\alpha$ is a Maxwell–Boltzmann distribution and the electrons are thermalized.

To demonstrate this effect, we consider the injection of a power-law distribution of nonthermal electrons into our hot loop model, HL, which has an apex temperature of 34 MK. We choose the relatively low value of $E_c = 15$ keV to highlight the effects of energy diffusion. As in previous cases, we choose $\delta = 4$ and an energy flux of $1 \times 10^{11}$ erg cm$^{-2}$ s$^{-1}$. At 34 MK, $(\delta + 1)kT = 15$ keV. The results are shown in Figure 5. In this loop, the coronal density is high ($>10^{11}$ cm$^{-3}$), and the column depth (gray dashed line in the top panel) quickly becomes large. Above 1.25 Mm, $f_\alpha$ (black solid line in the bottom panel) begins to resemble a Maxwell–Boltzmann distribution (red line in the bottom panel)—albeit much hotter than the ambient temperature—for energies less than 5 keV. The column depth continues to increase as the electrons move down the loop, and by 9 Mm, electrons with $E < 10$ keV are essentially fully thermalized. The lower-energy side of $f_\alpha$ is well fit by a Maxwell–Boltzmann distribution with temperature comparable to the ambient electron temperature. In the transition region, $f_\alpha$ is best fit at low energy by two Maxwell–Boltzmann distributions, as shown in Figure 6. The cooler distribution (dashed blue line in the bottom panel) has a temperature comparable to that of the local ambient plasma, and the hotter distribution (dashed green line in the bottom panel) has $T \sim 10$ MK and represents the heat flux carried by the injected electrons moving through the transition region. The vertical black dashed line in the bottom panel indicates the energy, $E = (\delta + 1)kT$. This is a good indicator of the energy above which $f_\alpha$ deviates from a thermal distribution.

3.5 Proton Beams

In this test case, we demonstrate how FP models the transport of nonthermal protons injected into the CL loop model (Figure 1). We consider the injection of protons with a power-law energy distribution for two different cutoff energies, 100 keV and 1 MeV. These values were chosen to demonstrate the warm- and cold-target regimes. The results are shown in the top and bottom panels of Figure 7, respectively. In the CL loop model, the corona is at a temperature of 3.4 MK, so the speed of 100 keV protons is slower than the thermal electron speed but faster than the thermal proton speed. That is, $x_e = 0.19$ and $x_p = 344$ (Equation (8)). This is the “warm-target” regime as defined by Tamres et al. (1986). In that work, they obtained an analytic expression (their Equation (38)) for the heating rate produced by the injection of proton beams on a warm target. We have plotted their analytic expression (T86; red dashed line) in Figure 7 and compared it to the result from E78 for cold-target collisions (gray dashed line) and the heating rates predicted by FP with only the Coulomb collision force on (FP_CC_only; black line) and FP with all forces on (blue line). We emphasize that both FP and FP_CC_only do not make the
Figure 5. (Top panel) Temperature (solid black line) and column depth (dashed gray line) in the HL loop model. The vertical dashed line indicates the \( z \) position of the distribution function plotted in the lower panel. (Bottom panel) Distribution function, \( f_{E_z}(E, \mu = 0, z) \) (solid black line), for nonthermal electrons injected into the HL loop model at the position indicated in the top panel. For comparison, the injected distribution function, \( f_{E_z}(E, \mu = 0, z = 0) \), is also plotted (dashed–dotted black line). The lower-energy portion of \( f_{e} \) was fitted to a Maxwell–Boltzmann function (red line and shaded area). The temperature of the fit and ambient electron temperature are stated in the panel. The energy, \( (b + 1)kT \), is indicated by the dashed vertical black line. This figure is available as an animation. The animation follows the sequence from an ambient temperature of 34.14 to 0.10 MK. The real-time duration of the video is 22 s. (An animation of this figure is available.)

Figure 6. Same as Figure 5 but for a different position along the loop. In the transition region, the low-energy portion of \( f_{e} \) is best fit by two Maxwell–Boltzmann functions. Their sum, the hotter component, and the cooler component are indicated by the solid red, dashed green, and dashed blue lines, respectively.
warm- or cold-target assumptions but rather use a full form of the Coulomb collision operator and are applicable in both extremes and between them. This is important for modeling the transport of particles as they move from one regime into the other. For example, in the top panel of Figure 7, the 100 keV protons are in the warm-target regime as they transport through the corona, but as they move into the transition region, the cold-target regime becomes more applicable. FP smoothly captures the collisions in both regions. Interestingly, since the heating rates predicted by FP and T86 are much less than that of E78 in the corona, it is evident that collisions in the warm-target regime are much less effective at slowing particles than predicted by cold-target theory. This is important in predicting to what column depth particles will penetrate.

For the case of 1 MeV protons, $x_e = 1.9$ in the corona, and the cold-target theory is more applicable throughout the protons’ transport. This is shown in the bottom panel of Figure 7. The cold-target heating rate (gray dashed line) closely matches the FP_CC_only rate throughout the entire loop. Interestingly, since the heating rates in the FP_CC_only and FP cases are similar, it is evident that the return current has only a small effect on these nonthermal protons. The return current causes a potential drop of a few keV, but that is relatively small compared to their injected energies. The increased heating rate between 10 and 12 Mm in the FP compared to FP_CC_only cases is due to the magnetic mirror causing an increase in pitch angle and thus an increase in column depth.

4. Conclusions

We have developed a method, called FP, to model the transport of nonthermal particles injected at the top of magnetic loops in stellar atmospheres. The nonthermal particles are widely believed to be a dominant source of energy transport in flares, so accurately modeling their transport is critical for understanding flare energetics. Their interactions with the ambient atmosphere are responsible for producing the ubiquitous hard X-ray bremsstrahlung and radio synchrotron (in the case of nonthermal electrons) and $\gamma$-ray line emission (in the case of nonthermal protons). By inverting hard X-ray, radio, and $\gamma$-ray observations during flares, details of the injected particles can be inferred. By accounting for numerous transport effects, FP can be a powerful tool for performing these inversions. We have incorporated FP into the X-ray data analysis tool, OSPEX. In a forthcoming study, we will describe using FP combined with OSPEX to determine injected electron distributions during solar flares.

Our method solves the steady-state Fokker–Planck equation accounting for the effects of Coulomb collisions including dynamic friction and energy diffusion, return currents, magnetic mirroring, and synchrotron emission. These are the dominant forces acting on the particles during their transport. We have demonstrated the performance of FP using several test cases. We showed that in cool loops, the return current can be a dominant force on nonthermal electrons. The return current force significantly alters the location of heating compared to models that neglect the return current. We have used FP to
show an example of a strongly converging magnetic field in the corona trapping energetic electrons and producing a loop-top nonthermal X-ray source. We have demonstrated when energy diffusion can be significant in comparison to the dynamic friction and showed how including both allows our model to capture the thermalization process. We have demonstrated that for $\sim$100 keV protons, warm-target collisions are much less effective at slowing the protons than predicted from cold-target theory. This allows them to penetrate deeper. For protons, we have shown that return current is not a significant effect, since the return current causes a potential drop that is a very small fraction of their kinetic energy.

We have taken care to make FP as accurate as possible. However, there are a few limitations to our method, listed below.

1. Our method neglects “self-collisions.” That is, we neglect the effect of nonthermal particles colliding with other nonthermal particles in the same distribution, $f_n$. We expect this effect to be small because the number density of nonthermal particles is typically much less than that of the ambient plasma. But in future work, we plan to remedy this limitation. The method is straightforward. When iteratively solving Equation (23), the current solution, $\mathbf{J}_n$, is used to evaluate the Rosenbluth potentials (Equation (5)), and the corresponding forces are calculated using Equation (2) and added to the net collision force.

2. Our model of the return current does not include a component of ambient electrons accelerated by the return current electric field into the runaway regime (Holman 1985). We are currently developing a model to include runaways, and in future work, we will incorporate that into FP.

3. The FP does not include beam- and current-driven instabilities and so is not applicable in regimes where those instabilities become dominant.

4. The FP models “half-loops” from loop top to footpoint with the assumption that a full loop is symmetric about the loop top. Thus, FP is not able to model full loops with asymmetric legs.

We have demonstrated that FP predicts significantly different heating rates than the analytical approach (E78 and E78+HF94) and the Coulomb collision–only (CC_only) approaches that are commonly used in solar/stellar flare modeling. It is currently being merged with the flare radiation hydrodynamic model RADYN. Using this, we will investigate, in a future work, the resulting impact of this more physically accurate heating rate on flare dynamics.

J.C.A. acknowledges funding from NASA’s Heliophysics Innovation Fund and NASA’s Heliophysics Supporting Research program. M.A. was supported by NASA’s Heliophysics Supporting Research fund. A.F.K. acknowledges funding from NSF AGS Solar-Terrestrial Award No. 1916511, and he thanks Prof. Lyndsay Fletcher for helpful discussions about velocity diffusion in solar flares. G.S.K. was funded by an appointment to the NASA Postdoctoral Program at Goddard Space Flight Center, administered by USRA through a contract with NASA, and by the Heliophysics Innovation Fund. This research benefited from discussions held at a meeting of Graham Kerr and Vanessa Polito’s International Space Science Institute team: “Interrogating Field-Aligned Solar Flare Models: Comparing, Contrasting and Improving.”

ORCID iDs
Joel C. Allred https://orcid.org/0000-0003-4227-6809
Meriem Alaoui https://orcid.org/0000-0003-2932-3623
Adam F. Kowalski https://orcid.org/0000-0001-7458-1176
Graham S. Kerr https://orcid.org/0000-0001-5316-914X

References
Abbott, W. P., & Hawley, S. L. 1999, ApJ, 521, 906
Alaoui, M., & Holman, G. D. 2017, ApJ, 851, 78
Alaoui, M., Krucker, S., & Saint-Hilaire, P. 2019, SoPh, 294, 105
Allred, J. C., Hawley, S. L., Abbott, W. P., & Carlsson, M. 2005, ApJ, 630, 573
Allred, J. C., Kowalski, A. F., & Carlsson, M. 2015, ApJ, 809, 104
Aschwanden, M. J. 2002, SSRv, 101, 1
Aschwanden, M. J., Holman, G., O’Flannagain, A., et al. 2016, ApJ, 832, 27
Battaglia, M., & Benz, A. O. 2008, A&A, 487, 337
Battaglia, M., Kontar, E. P., Fletcher, L., & MacKinnon, A. L. 2012, ApJ, 752, 4
Benz, A. 2002, Plasma Astrophysics (2nd ed.; Dordrecht: Kluwer)
Brown, J. C. 1971, SoPh, 18, 489
Carlsson, M., & Stein, R. F. 1992, ApJL, 397, L59
Carlsson, M., & Stein, R. F. 1995, ApJL, 440, L29
Carlsson, M., & Stein, R. F. 1997, ApJ, 481, 500
Caspi, A., Krucker, S., & Lin, R. P. 2014, ApJL, 781, 43
Chandrasekhar, S. 1943, RvMP, 15, 1
Dennis, B. R., Duval-Poo, M. A., Piana, M., et al. 2018, ApJ, 867, 82
Dreicer, H. 1960, PhRv, 117, 329
Emslie, A. G. 1978, ApJ, 224, 241
Emslie, A. G. 1980, ApJ, 235, 1055
Emslie, A. G., Bian, N. H., & Kontar, E. P. 2018, ApJ, 862, 158
Emslie, A. G., Dennis, B. R., Shih, A. Y., et al. 2012, ApJ, 759, 71
Emslie, A. G., Mariska, J. T., Montgomery, M. M., & Newton, E. K. 1998, ApJ, 498, 441
Evans, R. D. 1955, The Atomic Nucleus (New York: McGraw-Hill)
Fisher, G. H., Canfield, R. C., & McClymont, A. N. 1985, ApJ, 289, 414
Gordovskyy, M., Zharkova, V. V., Voitenko, Y. M., & Goossens, M. 2005, AdSpR, 35, 1743
Haug, E. 1997, A&A, 326, 417
Hawley, S. L., & Fisher, O. H. 1994, ApJ, 426, 387
Hirshman, S. P. 1977, PhFl, 20, 589
Holman, G. D. 1985, ApJ, 293, 584
Holman, G. D. 2012, ApJ, 745, 52
Holman, G. D., Sui, L., Schwartz, R. A., & Emslie, A. G. 2003, ApJL, 595, L97
Hurford, G. J., Krucker, S., Lin, R. P., et al. 2006, ApJL, 644, L93
Jeffrey, N. L. S., Kontar, E. P., & Fletcher, L. 2019, ApJ, 880, 136
Karlický, M. 2009, ApJ, 690, 189
Kasparová, J., Varady, M., Heinzl, P., Karlický, M., & Moravec, Z. 2009, A&A, 499, 923
Knight, J. W., & Sturrock, P. A. 1977, ApJ, 218, 306
Kong, X., Guo, F., Shen, C., et al. 2019, ApJL, 887, L37
Kontar, E. P., Brown, J. C., Emslie, A. G., et al. 2011, SSRv, 159, 301
Kowalski, A. F., Hawley, S. L., Carlsson, M., et al. 2015, SoPh, 290, 3487
Leach, J., & Petrovian, V. 1981, ApJL, 251, 781
Lin, R. P., Dennis, B. R., Hurford, G. J., et al. 2002, SoPh, 210, 3
Liu, W., Petrovian, V., & Mariska, T. J. 2009, ApJ, 702, 1553
MacKinnon, A. L., & Craig, I. J. D. 1991, A&A, 251, 693
Martínez-Sykora, J., & Rostoker, G. 2012, ApJ, 753, 161
Mauas, P. J. D., & Gómez, D. O. 1997, ApJ, 483, 496
McTiernan, J. M., & Petrovian, V. 1990, ApJL, 359, 524
Milligan, R. O., Kerr, G. S., Dennis, B. R., et al. 2014, ApJL, 793, 70
Papadopoulos, K. 1977, RvGSP, 15, 113
Park, B. T., & Petrovian, V. 1996, ApJL, 463, 255
Petak, P., Del Zanna, G., Mason, H. E., & Bradshaw, S. J. 2012, A&A, 547, A25
Rosenbluth, M. N., MacDonald, W. M., & Judd, D. L. 1957, PhRv, 107, 1
Rowland, H. L., & Vlahos, L. 1985, A&A, 142, 219
Rubio da Costa, F., Klein, L., Petrovian, V., Liu, W., & Allred, J. C. 2016, ApJ, 827, 38
Simões, P. J. A., & Kontar, E. P. 2013, A&A, 551, A135
Snyder, H. S., & Scott, W. T. 1949, PhRv, 76, 220
Spitzer, L. 1962, Physics of Fully Ionized Gases (Dordrecht: Kluwer)
Su, Y., Holman, G. D., & Dennis, B. R. 2011, ApJ, 731, 106
Tamres, D. H., Canfield, R. C., & McClymont, A. N. 1986, ApJ, 309, 409
Trubnikov, B. A. 1965, RvPP, 1, 105

van den Oord, G. H. J. 1990, A&A, 234, 496
Warmuth, A., & Mann, G. 2016, A&A, 588, A115
Zharkova, V. V., & Gordovskyy, M. 2005, A&A, 432, 1033
Zharkova, V. V., & Gordovskyy, M. 2006, ApJ, 651, 553