Supporting Information:
The Statistics of Bulk Segregant Analysis
Using Next Generation Sequencing

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Derivation of Log-Normal Distribution from $E[G']$ and $\text{Var}[G']$

If $G'$ is log-normally distributed, then:

$$E[G'] = e^{\mu + \frac{1}{2} \sigma^2}$$
$$\text{Var}[G'] = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$$

Given $E[G']$ and $\text{Var}[G']$ (see text), we can estimate the parameters of the log-normal distribution as:

$$\mu = \ln(E[G']) - \frac{1}{2} \left( 1 + \frac{\text{Var}[G']}{E[G']^2} \right)$$

$$\sigma^2 = \ln \left( 1 + \frac{\text{Var}[G']}{E[G']^2} \right)$$

Expected shape of $G$ around a causal SNP

Assume the parental genotypes at two loci are $AB$ and $ab$, and that there is a single causal allele, $A$, at the $A$ locus. If $p_A$ is the frequency of allele $A$ in a bulk of $F_2$ segregants, the expected allele frequency of $B$ is $p_B = p_A(1 - 2r_{AB}) + r_{AB}$, where $r_{AB}$ is the recombination rate between $A$ and $B$. The frequency of allele $b$ is $p_b = 1 - p_B$. Ignoring sampling variation, and assuming the expected frequency of each allele is 0.5, $G$ is proportional to:

$$p_{B,l} \ln p_{B,l} + p_{b,l} \ln p_{b,l} + p_{B,h} \ln p_{B,h} + p_{b,h} \ln p_{b,h} + 2 \ln 2$$

If $A$ is fixed in a bulk, then $p_B = 1 - r_{AB}$ and $p_b = r_{AB}$. If we assume that the allele $A$ is fixed in one bulk, and at the expected frequency in the other bulk than $G$ should be proportional to:

$$r_{AB} \ln r_{AB} + (1 - r_{AB}) \ln(1 - r_{AB}) + \ln 2$$
A scaled and normalized representation of allelic bias ($p_B - 0.5$) and the corresponding normalized $G$ around a causal SNP are illustrated in Figure 1, with respect to map distance (after converting map distance to recombination rate using Haldane’s formula).

![Graph showing Allelic Bias and G statistic](image)

Figure 1: The allelic bias (dashed line) and $G$ statistic around a causal SNP, not accounting for sampling variation.

One interesting feature to note is that the full-width at half maximum (FWHM) for $G$ is significantly smaller than that of allelic bias, indicating that the signal for $G$ falls of more rapidly around the causal SNP than the comparable signal for allelic bias. Assuming sufficient SNP density, this is a useful aspect of $G$ because it implies relatively smaller support intervals.
Supplementary Tables

Table 1: A summary of repeated simulations of the null hypothesis with 100,000 replicates per parameter set. For all simulations, the window size was 20 cM with 10 SNPs per cM. The mean coverage per bulk, C, was varied from half $n_s$ to five times $n_s$ for each value of $n_s$. 

| nS | C  | Mean G' Observed | Predicted | Variance of G' Observed | Predicted |
|----|----|------------------|-----------|-------------------------|-----------|
| 20 | 10 | 1.38             | 1.25      | 0.14                    | 0.13      |
| 20 | 20 | 1.56             | 1.50      | 0.42                    | 0.45      |
| 20 | 40 | 2.05             | 2.00      | 1.66                    | 1.71      |
| 20 | 100| 3.57             | 3.50      | 10.12                   | 10.53     |
| 50 | 25 | 1.29             | 1.25      | 0.11                    | 0.13      |
| 50 | 50 | 1.52             | 1.50      | 0.41                    | 0.45      |
| 50 | 100| 2.02             | 2.00      | 1.60                    | 1.73      |
| 50 | 250| 3.54             | 3.50      | 9.88                    | 10.61     |
| 100| 50 | 1.27             | 1.25      | 0.11                    | 0.13      |
| 100| 100| 1.51             | 1.50      | 0.41                    | 0.45      |
| 100| 200| 2.02             | 2.00      | 1.57                    | 1.73      |
| 100| 500| 3.50             | 3.50      | 9.53                    | 10.64     |
| 200| 100| 1.27             | 1.25      | 0.11                    | 0.13      |
| 200| 200| 1.51             | 1.50      | 0.41                    | 0.45      |
| 200| 400| 2.02             | 2.00      | 1.57                    | 1.73      |
| 200| 1000| 3.51          | 3.50     | 9.73                    | 10.65     |