Effect of strong correlation on the study of renormalization group flow diagram for Kondo effect in a interacting quantum wire.

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We present the study of Kondo effect in an interacting quantum wire. We mainly emphasize the effect of strong electronic correlations in the study of renormalization group flow diagram and the stability analysis of fixed points for both magnetic and nonmagnetic impurities. We observe that the behavior of the system is either in the single channel or in the two channel Kondo effect depending on the initial values of coupling constants and strong correlations.

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I. INTRODUCTION

One dimensional interacting electron systems are interesting in their own right. In one dimension, interaction between the electrons has large impact on the low-energy properties of the electron liquids. The basic physics of electron liquid is changing from the Fermi liquid (FL) to the Luttinger liquid (LL) physics. Such an one dimensional LL can be realized in a very narrow quantum wire. In this work, we are interested in the study of Kondo effect in a quantum wire, where both magnetic and nonmagnetic impurities are present. Kondo effect is one of the interesting phenomena of correlated electronic systems since its discovery [1, 2, 3]. This effect arises from the exchange interactions between an impurity spin and an electron gas in three dimensions. In this case interaction changes from weak to strong coupling as one decrease the temperature or energy scale. At T=0 the impurity spin is completely screened by the conduction electrons and the basic physics can be understood from the local FL [1, 2, 3]. It was also observed that when the channel (N) exceeds twice of the impurity spin the system is governed by the physics of non-Fermi liquid theory [4, 5].

In the conventional theory of Kondo effect, people have worked on an independent electron picture because in three dimensions the interacting electron system can be described by the noninteracting quasiparticle (FL). However this scenario is drastically changing in one dimension when the basic physics is govern by the LL. The Kondo effect in Luttinger liquid was first discussed by Lee and Toner [6]. Furusaki and Nagaosa [7] have derived the scaling equations for the Kondo couplings in the weak coupling regime by using the poor-man’s scaling method, which preserves the SU(2) symmetry. They predicted that at low temperature the system is governed by the strong coupling fixed point, where the impurity spin is screened completely. They observed an important difference with conventional Kondo physics is that Kondo coupling flows to the strong coupling regime for both antiferromagnetic and ferromagnetic coupling because the local backward scattering potential is relevant perturbation in the LL. But we shall see in our study that strong correlation effect has not taken into account properly, i.e., the weak coupling RG flow diagram is the same for noninteracting, repulsive and attractive LL. The problem of a spin-1/2 magnetic impurity in a LL has also been largely studied in the literature [8, 9, 10, 11, 12, 13, 14]. It is well known that the repulsive interaction due to nonmagnetic impurity breaks the wire at the impurity site and treats the residual tunneling through the barrier as a perturbation. Here we study the two (dual) models that describe the impurity. The plan of the work is the following: In section II, we present the weak coupling renormalization group flow diagram with necessary physical analysis for a one dimensional interacting electron system with only magnetic impurity. In sec. III, we present weak-coupling RG flow diagram of an one dimensional system with both magnetic and nonmagnetic impurities are present. The section IV is devoted for conclusions.

II. WEAK COUPLING RENORMALIZATION GROUP STUDY OF A QUANTUM WIRE WITH ONLY MAGNETIC IMPURITY

Here we consider a magnetic impurity of spin-1/2 at the origin of one dimensional interacting systems. It is well known that away from half-filling the basic physics of this type of system is governed by the spin sector of the Hamiltonian. We only consider the exchange coupling between the impurity spin and conduction electrons as follows

$$H_1 = \frac{J_1}{2} \sum \left[ \psi_{Ra}^\dagger \sigma_{\alpha\beta} \psi_{Ra} + \psi_{La}^\dagger \sigma_{\alpha\beta} \psi_{La} \right]$$

$$+ \frac{J_2}{2} \sum \left[ \psi_{Ra}^\dagger \sigma_{\alpha\beta} \psi_{La} + \psi_{La}^\dagger \sigma_{\alpha\beta} \psi_{Ra} \right],$$

where $J_1$ and $J_2$ are respectively the forward and backward Kondo scattering coupling, $\sigma$ is the pauli matrix. $\psi_{Ra}$ and $\psi_{La}$ are respectively the fermionic right and left mover field operators with spin $\alpha$. The field operators of right and left going electrons with spin $\sigma$
are, \( \psi_{R\sigma}(x) = \frac{\eta}{\sqrt{2\pi a}} e^{i\phi_R(x)}, \psi_{L\sigma}(x) = \frac{\eta}{\sqrt{2\pi a}} e^{-i\phi_L(x)}, \)
where \( \phi \) is the bosonic field. \( \eta \) is the Klein factor that preserve the anticommutativity of fermionic field.

During our continuum field theoretical calculations, we consider the following relations of the bosonic fields as
\[ \phi_c(x) = \frac{1}{2} [\phi_{L1} + \phi_{L1} - \phi_{R1} - \phi_{R1}], \]
\[ \theta_c(x) = \frac{1}{2} [\phi_{L1} + \phi_{L1} - \phi_{R1} - \phi_{R1}], \]
\[ \phi_s(x) = \frac{1}{2} [\phi_{L1} - \phi_{L1} + \phi_{R1} - \phi_{R1}]. \]
\[ \theta_s(x) = \frac{1}{2} [\phi_{L1} - \phi_{L1} - \phi_{R1} + \phi_{R1}]. \]

Finally we get the bosonized Hamiltonian
\[ H = \frac{J_1 S^{-}}{4\pi a} [e^{i(\theta_c(0) - \phi_s(0))} + e^{-i(\theta_c(0) + \phi_s(0))}] + H.C \]
\[ - \frac{J_1 S^{-}}{\pi} \partial_x \phi(x) + \frac{J_2 S^{-}}{2\pi a} \cos(\phi_c(0) - \theta_s(0)) \]
\[ + \frac{J_2 S^{+}}{2\pi a} \cos(\phi_c(0) + \theta_s(0)) \]
\[ - \frac{J_2 S}{\pi a} \sin(\phi_c(0)) \sin(\phi_s(0)). \]  \( (2) \)

The scaling dimension for the forward scattering terms and backward scattering terms are respectively 1 and \( \frac{1}{2}(1 + K_c) \). So the backward scattering term is relevant for \( K_c < 1 \), where \( K_c \) is the LL parameter of the charge sector. One can derive the renormalization group equation for one loop order by using poor-man’s scaling method.

The RG equations are the following \( (3, 12) \):

\[ \frac{dJ_1}{dl} = \frac{1}{2\pi a} (J_1^2 + J_2^2), \]
\[ \frac{dJ_2}{dl} = \frac{1}{2}(1 - K_c) J_2 + \frac{2J_1 J_2}{2\pi a} \]

These RG equations have only trivial fixed point, \( (J_1^*, J_2^*) = (0, 0) \). We do the linear stability analysis to check the stability of these fixed points (FP). After the linear stability analysis RG equations reduce to

\[ \frac{d}{dl} A_1 = \frac{1}{2\pi a} B_1 A_1, \]  \( (4) \)

where

\[ A_1 = \begin{pmatrix} J_1 & J_2 \\ J_2 & J_2 \end{pmatrix} \]

and

\[ B_1 = \begin{pmatrix} J_1^* & J_2^* \\ J_2^* & (1 - K_c) \end{pmatrix} \]

At the trivial fixed point, \( \frac{dJ_1}{dl} = 0 \) \( J_1 \) and \( \frac{dJ_2}{dl} = (1 - K_c) J_2 \). The equations for \( J_1 \) and \( J_2 \) will be stable and unstable when the right hand side coefficient of \( J_1 \) and \( J_2 \) are negative and positive respectively. The couplings are marginal when the coefficients are zero. If we look at the next order term for the marginal case, i.e., \( \frac{dJ_1}{dl} = a J_1^2 \) \( (a > 0) \), we say that FP at \( J_1 = 0 \) is stable on the \( x < 0 \) side and unstable on the \( x > 0 \) side. We get from the linear stability analysis of \( J_2 \) that the fixed point is stable when \( K_c > 1 \), is marginal for \( K_c = 1 \) and is unstable for \( K_c < 1 \).

In Fig. 1, we present the RG flow diagram for the repulsive quantum wire, i.e, \( K_c = 1/4 \). The Kondo couplings \( J_1 \) and \( J_2 \) are renormalized towards strong coupling irrespective of the couplings are antiferromagnetic or ferromagnetic Kondo couplings. This observation is in contrast with the three dimensional Kondo effect. We observe from our study that the RG flow takes a large range of initial conditions to the FP at \( (0, 0) \). For all other initial conditions, we see that there are two directions along which the Kondo couplings flow to strong coupling \( \frac{dJ_1}{dl} = 1 \) and \( \frac{dJ_2}{dl} = -1/2 \). This explicit study of the phase diagram was absent in the previous studies of the Kondo effect in an one dimensional interacting quantum wire \( (2, 12) \). The one-loop RG equations suggest that apart from the trivial FP \( (0, 0) \), there are other three FPs, \( (J_1^*, J_2^*) = (+ \infty, +\infty), (+ \infty, 0), (+ \infty, -\infty) \). Here we want to discuss the FP \( (+ \infty, 0) \) . It corresponds to the two channel Kondo problem. When \( J_2 = 0 \), the spin and the charge sectors are decoupled in the bosonized Hamiltonian. The impurity spin is interacting with \( \phi_s \) and \( \theta_s \) only and the Hamiltonian of the spin sector is equivalent to the two-channel Kondo problem with the right and left going electrons correspond to two channel.

In Fig. 2, we present the RG flow diagram for \( K_c = 3/2 \), i.e, the system of attractive LL. We observe that the flow diagram and the two channel Kondo regime is the same as repulsive LL \( (K_c = 1/4) \). It also reveals from our study that noninteracting RG
flow diagrams show the same behavior as the attractive and repulsive LL. So we conclude that the behavior of the RG flow diagram is the characteristic of the one dimensional system, interactions (different values of LL parameter) have no effect. This explicit study of strong electronic correlation on the study of Renormalization Group flow diagram was absent in the previous study [7,12]. The RG equations of a quantum wire with a magnetic impurity are not sufficient to show up the strong correlation effects because there is no breaking of quantum wire. Therefore one has to consider the presence of strong nonmagnetic scatterer, which we will discuss in the next section.

**RENORMALIZATION GROUP STUDY OF A QUANTUM WIRE WITH MAGNETIC IMPURITY AND NONMAGNETIC LOCAL POTENTIAL**

Here we present the RG study of an interacting wire with magnetic ($J_1$) and nonmagnetic ($J_2$) local potential. We study this part following Ref. 15. We consider the situation where the strength of local impurity potential is larger than the antiferromagnetic exchange $J_1$. For this case one can first diagonalize the Hamiltonian with only local potential. It is well known that for repulsive interaction due to nonmagnetic impurity cutting the wire at the impurity site. Here we consider the tunneling $t$ as a perturbation. The Hamiltonian of the system is the following,

$$H_2 = \frac{J_1}{2} S. \sum_{i=R,L} \sum_{\alpha, \beta} \psi_{i\alpha}^{\dagger} \sigma_{\alpha\beta} \psi_{i\alpha} + \frac{J_2}{2} S.[\psi_{R\alpha}^{\dagger} \sigma_{\alpha\beta} \psi_{L\beta} + H.C] + t \sum_{\alpha} (\psi_{R\alpha}^{\dagger} \psi_{L\alpha} + H.C),$$

(5)

Where R and L correspond to the right and left side of the impurity, $S$ is the impurity spin-1/2 operators and $\sigma$ is the spin operator of the conduction electrons. The first term represents the exchange interaction of the leads with the impurity spin. The second term represents the tunneling process with spin-flip. The third term presents the tunneling of electrons without spin flip. This model Hamiltonian represents the two channel Kondo model when $J_2$ and $t$ are absent. The presence of $J_2$ and $t$ will introduce anisotropy between two channels. In Ref. 15, the RG equation for this problem has derived. We think that this derivation is not the complete one, i.e., there is one more RG equation for the tunneling without spin flip. Now we present the derivation of that RG equation very briefly. One can write the boundary field operator $[15]$ as

$$\psi_{i1/1} = \frac{\eta_{i1/1}}{\sqrt{2\pi a}} e^{\sqrt{\frac{\pi a}{2}} \cos \frac{\phi_{is}}{\sqrt{2}}},$$

(6)

where $\phi_{is} = \frac{1}{\sqrt{2}} (\phi_{i1} - \phi_{i1})$ and $\phi_{ic} = \frac{1}{\sqrt{2}} (\phi_{i1} + \phi_{i1})$. $K_s$ is the LL parameter of the spin sector. Using these expressions for fermionic and bosonic fields, one can write the bosonized version of the tunneling without spin flip component of the Hamiltonian as

$$\sum \frac{2}{\pi a} \cos \frac{\phi_{Rs} - \phi_{Rs}}{\sqrt{2 K_s}} \cos \frac{\phi_{Rs} - \phi_{Rs}}{\sqrt{2 K_s}}.$$  

The scaling dimension of this term is $\frac{1}{2} \left( \frac{1}{\sqrt{K_c}} + \frac{1}{\sqrt{K_s}} \right)$ and there is no term alike $t$ from the operator product expansion of the first two terms of the Hamiltonian. So the one-loop RG equation is

$$\frac{dt}{dl} = (1 - \frac{1}{2} \left( \frac{1}{K_c} + \frac{1}{K_s} \right)) t$$

(7)

Therefor the total RG equations are

$$\frac{dJ_1}{dl} = \frac{1}{2 \pi v_F} (J_1^2 + J_2^2),$$

$$\frac{dJ_2}{dl} = (\frac{1}{2} (1 - \frac{1}{K_c}) J_2 + \frac{2 J_1 J_2}{2 \pi v_F}),$$

$$\frac{dt}{dl} = (1 - \frac{1}{2} \left( \frac{1}{K_c} + \frac{1}{K_s} \right)) t$$

(8)

Here the FPs are the trivial ones, i.e., $(J_1^*, J_2^*, t^*) = (0,0,0)$. We do linear stability analysis of the FPs to
study the nature of the FPs. After the linear stability analysis the RG equations are reduced to

$$\frac{d}{dl} A_2 = \frac{1}{2\pi v_F} B_2 A_2,$$

(9)

where

$$A_2 = \begin{pmatrix} J_1 \\ J_2 \\ t \end{pmatrix}$$

and

$$B_2 = \begin{pmatrix} J_1^* & \frac{1}{2} (1-K_c) J_2^* & 0 \\ J_2^* & J_1^* + \frac{1}{2} (1-K_c) t & 0 \\ 0 & 0 & \frac{1}{2} (1-K_c) \end{pmatrix}$$

At the trivial fixed point, $\frac{dJ}{dl} = 0 \times J_1$, $\frac{dJ}{dl} = \frac{1}{2} (1-K_c) \times J_2$, and $\frac{dt}{dl} = \frac{1}{2} (1-K_c) t$. We understand from the analysis of these RG equations that for study of the $J_1$ vs. $J_2$ flow diagram, the first two RG equations are sufficient to draw the phase diagram. At the large length scale, the FP is approached as $J_1 \sim \frac{1}{\ln(T_K/T)}$ and $J_2 \sim \left( \frac{T}{T_K} \right)^{1/(1-K_c)}$. Here $T_K$ is the Kondo temperature. The behavior of $J_1$ and $J_2$ at large length scale in contrast to the behavior of FL. In this case the Eq. 8, can be solved in terms of the linear combinations of, $J_1$ and $J_2$ such that, $J_1 - J_2$ and $J_1 + J_2$. Here the fixed point is also at $(J_1^*, J_2^*) = (0, 0)$ with $J_1 \sim \frac{1}{\ln(T_K/T)}$ and $J_2 \sim \left( \frac{T}{T_K} \right)^{2}$. We see that $J_2$ approaches zero faster than $J_1$ for both FL and LL but for the LL, it goes to zero much faster power of $T$. The above analysis is valid when neither $J_1 - J_2$ nor $J_1 + J_2$ is exactly equal to zero. If one of them is exactly equal to zero and the other is not, then both the couplings go as $\frac{1}{\ln(T_K/T)}$.

This explicit study was absent in Ref. [15].

Before we proceed further with the analysis of the RG flow diagrams, we first want to study the bosonized version of the Hamiltonian to get a better understanding of our results. The bosonized version of this model Hamiltonian has discussed in Ref. 15. We have only borrowed the final results. $J_{1\perp}$ (XY component of $J_1$) is always relevant and $J_{2\perp}$ is relevant only if $1/2 < K_c < 1$. When both $J_{1\perp}$ and $J_{2\perp}$ are relevant the low temperature FP behavior is the same as the single channel Kondo effect. Two channel Kondo behavior is stable for $K < 1/2$. We are now presenting the RG flow diagram for different values of LL parameters. We see from our RG study that for $1/2 < K_c < 1$, a certain region of the parameter space system shows the two channel and single channel Kondo behavior as we see in Fig. 2 and Fig. 1. When $K_c > 1$, i.e., the system is in the attractive regime, the RG flow diagram is the same as Fig. 1 and Fig. 2. Therefore we conclude that in this regime the spin-flip tunneling process is healing the quantum wire. When $K_c < 1/2$, $J_{2\perp}$ term is irrelevant and the system is in the two-channel Kondo behavior. It reveals from our study that for a certain region of the initial values of the coupling constants the system shows the single channel Kondo behavior. As we understand from Fig. 3 and Fig. 4 that the region of two channel Kondo behavior increases as one reduce the value of $K_c$, i.e.,
for the higher values of repulsive local potential. In Fig.
3 and Fig. 4, the region A shows the single channel
Kondo behavior and the rest of the region of the flow
diagram shows the two channel Kondo behavior. In the
two channel Kondo effect there is no healing of quantum
wire. This explicit study of the effect of strong electronic
correlations was absent in the previous study [15].

CONCLUSIONS

We have revisited the problem of magnetic and non-
magnetic impurity in a quantum wire. We have empha-
sized mainly the RG study of this problem. The RG
flow diagram of the previous studies is the schematic one
[7,12,15]. The Abelian bosonization study only reveals
that whether a coupling term is relevant or not. The
RG flow diagram shows us explicitly for which initial val-
ues of coupling constants the systems flows to the single
channel or two-channel Kondo problem.

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