Multi-dimensional object estimation by its deviation from goal

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Abstract. In the paper, an approach for estimation of objects relative to specified target that was proposed by Wierzbicki A. P. is analyzed. It is asserted that this approach does not contradict with using utility (value) functions and weight coefficients at generalizing function for solving the same task. This assertion is justified by a mutual inverse relationship between functions estimating the goal achievement and deviation from a goal. It is shown that the deviation functions additionally allow solving such problems as ordering objects by penalties and bonuses, as well as by deviation from the norm. Using a simple medical example, the results of ranking patients by deviation from the norm, obtained by the criterion of minimax optimization and the additive generalizing function, are compared with equal and different values of the weighting coefficients of the indicators.

1. Introduction

In the introduction to paper [1], the following was stated: “Any point in the objective space – no matter whether it is attainable or not, ideal or not – can be used instead of weighting coefficients to derive scalarizing functions which have minima at Pareto points only”. And then “entire basic theory of multiobjective optimization – necessary and sufficient conditions of optimality and existence of Pareto-optimal solutions, etc. – can be developed with the help of reference objectives instead of weighting coefficients or utility (value) functions”. The author bases his denial of utility functions on the statement that each individual consumer does not maximize an aggregated utility function; instead of that the consumer follows some “reference objective which is usually determined partly subconsciously, not fully rationally”.

The last statement cannot be considered as valid. Of course, any individual does not maximize an aggregated utility function numerically. But he/she subconsciously maximizes the aggregated utility targeting at the wanted values of partial indicators (note that these values can be non-compatible with each other). The ‘utility’ concept is inherent as for averaged [1] and for each individual consumer. At that the consumer defines the priority of each indicator individually, and maximal utility is realized indirectly via specified target value of the indicator (also known as ‘reference objective’). The value \( c_j = \arg \max u(y_j) \) is the target value of the \( j \)-th indicator.

An individual defines the reference objective as a vector \( c = (c_1, \ldots, c_j, \ldots, c_n) \) consisting of wanted values of indicators that specify attributes of an object to be chosen. This vector \( c \) defines the desired
point at \( n \)-dimensional space of indicators. This point corresponds to the radius vector, whose angles of inclination to the coordinate axes reflect the weight coefficients of the generalizing function [2].

In this way we do not see contradictions between model of multi-dimension estimation of objects in utility theory and the model proposed in [1], even if we take into consideration that in this model the utility of indicator values is not specified explicitly. The fundamental difference between the models lies in the method of solving the problem. Author of the paper [1] contraposes maximization in utility theory and the model proposed in [1], even if we take into consideration that in this model the points at coordinate axes reflect the weight coefficients of the generalizing function [2].

This implies an obvious relationship between the estimation functions used in the utility (value) theory and the function of deviation from the goal, proposed in [1]. In this regard, purpose of this paper is to find a connection between these functions and define the place of this approach in the theory of multidimensional optimization. To solve this problem, it is necessary to answer the following questions:

1. How does the indication of a certain point in the \( n \)-dimensional space of indicators agree with the definition of an objective reference by an individual consumer?
2. How is the function estimating the deviation from a goal related to value and utility functions?
3. Is there any connection between any specified point in \( n \)-dimensional space and weight coefficients in generalizing function (GF)?
4. And, finally, what is the usage scope for functions of deviation from the goal?

2. Concept of the goal

As the key word of the paper is “goal”, we should clarify the concept in order to further analyze utility functions and functions of deviation from the goal. According to [3], the preference relation \( R_{pr,j} \) specified for \( j \)-th indicator is transformed to criterion by specification of a target value \( c_j \) as the comparison base for any value \( y_j = f_j(x) \) at this indicator’s scale defined as range \( Y_j = [y_{j,\min}, y_{j,\max}] \). The target value \( c_j \) is specified either at a bound of the scale (\( c_j = y_{j,\max} \) or \( c_j = y_{j,\min} \)) or at any internal point of the scale (\( y_{j,\min} < c_j < y_{j,\max} \)). The targets at the boundaries of the scale correspond to the criteria \( y_j \to \max \) and \( y_j \to \min \). Note that we use symbol \( y \to \text{extr} \) to designate the requirement to search for an extremal (largest or smallest) value of \( y \) on some set. In paper [3] such criteria have been called “goal criteria” and targets related to them have been called “ideal goals”.

The goal criterion is characterized by uniformity and monotony of preference at whole scale of \( j \)-th indicator. The preference is performed by specification of the target value \( c_j \) from any of bounds of \( j \)-th indicator’s scale as comparison base in the predicate \( Pr(y_j, c_j) \). The comparison base \( c_j = y_{j,\min} \) corresponds to minimization, i. e. \( y_j \to \max \Leftrightarrow Pr_{\geq}(y_j, y_{j,\min}) \), and the comparison base \( c_j = y_{j,\max} \) corresponds to maximization, i. e. \( y_j \to \min \Leftrightarrow Pr_{\leq}(y_j, y_{j,\max}) \), because estimation of fulfillment of a requirement, for example, maximization, is realized by comparing the \( y_j \) value with the smallest possible value \( y_{j,\min} \).

The target specified between \( j \)-th indicator’s scale bounds, is called ‘real’ and related criterion is called ‘constraint criterion’ in the analogy for optimization model of mathematical programming. When the preference relation \( Pr(y_j, c_j) \) contains the equality relation \( y_j = c_j \) is realized, the constraint criteria “great or equal” (“lower restriction”) \( Pr_{\geq}(y_j, c_j) \) and “less or equal” (“upper restriction”) \( Pr_{\leq}(y_j, c_j) \) are correspondingly formed. In infix form they are written as \( y_j \geq c_j \) and \( y_j \leq c_j \) respectively. When the compliance relation \( Pr_{\geq}(y_j, c_j) \) with comparison base \( y_{j,\min} < c_j < y_{j,\max} \) is realized, the point constraint criterion \( Pr_{\geq}(y_j, c_j) \) is formed, and with comparison base \( [c_{j,l}, c_{j,u}] \subseteq Y_j \) the interval constraint criterion \( Pr_{\geq}(y_j, [c_{j,l}, c_{j,u}]) \) is formed. Here \( c_{j,l} \) is the lower \( (y_{j,\min} < c_{j,l}) \) and \( c_{j,u} \) is the upper \( (c_{j,u} < y_{j,\max}) \) bound of a range of required values of \( j \)-th indicator.
3. Estimation functions

In paper [4], an approach based on replacement of utility functions with targets is proposed. But actually specifying target values does not contradict to specifying the utility value at indicator’s scale as it will be demonstrated below.

According to [3], we suggest the following functions as estimation functions $f_c(y_j)$ having the scale of $j$-th indicator $Y_j = [y_{j,min}, y_{j,max}]$, $j = 1, n$ as domain:

1. value function $v: Y_j \rightarrow [0, 1],$
2. utility function $u: Y_j \rightarrow [-1, 1],$
3. plan function $s: Y_j \rightarrow [0, 200\%],$
4. membership function (of object to $l$-th class by $j$-th indicator) $\mu_l: Y_j \rightarrow [0, 1], l = 1, k$.

Here and below $y_j = f(x)$ is a value of $j$-th indicator for object $x$. The codomains of estimation functions (EF) are specified at the right. The value function (VF) and utility function (UF) are normed according to $j$-th indicator’s scale. Codomain of the plan function (PF) is upper limited by double implementation (fulfillment) of the plan. Regarding non-fulfillment and over-fulfillment of the plan, range of PF values can be presented by absolute bipolar scale $[-1, 1]$. Failure to fulfill the plan $0\% < s(y_j) < 100\%$ corresponds to the interval $[-1, 0)$, and the over-fulfillment of the plan $100\% < s(y_j) \leq 200\%$ corresponds to the interval $(0, 1]$. Membership function’s (MF) codomain is defined by fuzzy logic axioms. Each the EF in its own way reflects preference relation $R_{pr,j}$ specified at $Y_j$ to the codomain.

From the point of view of achieving the specified target, all EF are subject to maximization. They differ from each other, first of all, by a method to specify the target $c_j$ at $j$-th indicator’s scale using the criterion formulated as some two-place preference predicate $P_{r\Theta}(y_j, c_j)$. The target value $c_j$ is used here as comparison base for value $y_j$. The value $c_j$ is specified either at bounds of the scale ($c_j = y_{j,max}$ or $c_j = y_{j,min}$) or internal point of the scale ($y_{j,min} < c_j < y_{j,max}$). Striving to achieve the ideal goal is expressed by goal criteria $y_j \rightarrow$ max and $y_j \rightarrow$ min.

Relative to the real target the following preference types $\Theta$ are used: ‘$\geq$’ (not less), ‘$\leq$’ (not more), ‘$=$’ (equal), ‘[ ]’ (in the range). When the indicator value comes near the target, the EF’s value increases, and in the point $c_j$ the EF must have the acceptable value $f_c(c_j) = u^f_j$. For an ideal goal $u^f_j = 1$ and for a real target $0 < u^f_j \leq 1$.

When indicator scales are not equal and preferences at all values of the scale are homogeneous, degree of achievement of a goal by object $x$ is estimated by linear norming function. For the preference $y_j \geq c_j$ $f''_{e,max}(y_j)$ in range $[y_{j,min}, c_j]$ is defined as following:

$$f''_{e,max}(y_j) = \frac{y_j - y_{j,min}}{c_j - y_{j,min}} + b,$$

Exceeding the target $c_j$ in the range $[c_j, y_{j,max}]$ is described by the function $f''_{e,max}(y_j)$:

$$f''_{e,max}(y_j) = \frac{y_j - c_j}{y_{j,max} - c_j} + b.$$  

In equation (2), the lower bound $b$ of function $f_{e,max}$ is $b = -1$, and in equation (3) $b = 0$. In Figure 1 piecewise linear function $f_{e,max}(y_j)$ is presented by lines 1’ and 1”. In [3] the norming function of a criterion is understood as linear form of an utility function $u(y_j)$. In Figure 1 $u(c_j) = 0$, that, according to [5] corresponds to the decision maker’s (DM) refusal to participate in the lottery. A variant of EF $f_{e,min}(y_j)$ that norms the indicator in correspondence with criterion $y_j \leq c_j$ can be created in the same way.

Taking into account the propensity of the DM to take risks, EF $f_{e,max}(y_j)$ is transformed into UF $u^f(y_j)$ in the range $[y_{j,min}, c_j]$ and taking into account the propensity of the DM to refuse risks, into UF $u^r(y_j)$ in the range $(c_j, y_{j,max})$. The non-linearity is formed by raising the function to the appropriate power $n \neq 1$. In Figure 1, the curve 2’ demonstrates the propensity of the DM to take risks at scope of
goal achievement and the curve 2" demonstrates the propensity of the DM to refuse risks at scope of goal exceeding.

For the range compliance, the condition of target compliance \( f_e(y_j) = 1 \) is \( y_j \in [c_{j,l}, c_{j,u}] \). The decrease in the degree of target compliance as the value of \( y_j \) moves away from the boundaries of the interval (to the left of \( c_{j,l} \) and to the right of \( c_{j,u} \)) is described by equations (2) and (3) with \( b = 0 \) and substitution of these values instead of \( c_j \). In Figure 1 they are presented by left (line 1') and right (line 1") fronts of the trapezium. In case \( c_j = c_{j,l} = c_{j,u} \) the interval compliance is transformed to point compliance \( y_j = c_j \) and the trapezium is transformed to a triangle. These functions correspond to trapezium and triangle membership functions in the fuzzy logic.

As well as monotonic function of the goal achievement (see plot 1 in Figure 1, section \( a \)), the non-monotonic function of the target compliance (see plot 1 in Figure 1, section \( b \)) can be transformed to nonlinear function basing on combination of propensity of the DM to take and to refuse risks (see curves 2' and 2" in Figure 1, section \( b \)). All the described functions are implemented at program system of choice and ranking SVIR [6].

**Fig. 1.** EFs for preferences ‘\( \geq \)' (\( a \)), ‘\( [\]’ (\( b \)).

4. **Goal deviation functions**

In [1] the goal deviation function value was defined as a measure of non-achievement of the goal. In the present paper the concept “deviation from the goal” is extended to goal exceeding and applied to UFs and VFs. According to the principle “by contradiction”, the value of the deviation from the goal function (DF) complements the value of the goal achievement function or the goal fit function to 1. Graphically this principle is illustrated via symmetric of graphics of the corresponding EFs. In Figure 1, DF graphics are presented by dashed lines. The noted complement principle allows using DFs in order to solve problems of ranking or classification of objects together with functions of achievement and compliance of the target.

The DF value \( d(y_j) \) is the measure of non-compliance to the target (norm). The further in any direction from the target value \( c_j \) or from the boundaries of the norm \([c_{j,l}, c_{j,u}]\) is the value \( y_j \) of the j-th indicator, the greater is the value of the function \( d(y_j) \). The DF \( d(y_j) \) is equal to 0 either if the \( y_j \) fully complies to the target \( y_j = c_j \), or belongs to the range of the norm \([c_{j,l}, c_{j,u}]\). Thus the objects estimated by degree of not-achievement of a goal or non-compliance to a norm are ranked using the criterion \( d(y_j) \rightarrow \min \).

Especially interesting is the case when non-achievement and exceeding of a real target are taken into account separately. In paper [7], it was proposed to estimate degree of non-achievement of a target by positive numerical *penalties* and exceeding of the target, by negative numerical *bonuses*. In the Figure 2, at section \( a \), the piecewise linear DF from target value \( c_j \) for restriction “\( \geq \)” is shown, and at section \( b \), for restriction “\( \leq \)”.
Fig. 2. Deviation functions: a – for restriction ‘$\geq$’ (not less), b – for restriction ‘$\leq$’ (not more).

At section $a$ in Figure 2, for value $y_j < c_j$ the penalty is defined as $d_{p,j}$ and at section $b$ the bonus is defined as $-d_{p,j}$. In the same way, for value $y_j > c_j$ at section $b$ the bonus is defined as $-d_{p,j}$, and the penalty is defined as $d_{p,j}$.

The relative deviations from the target for the constraint “not less” (“lower”) are calculated by the following rule:

$$d^l(y_j) = \begin{cases} \frac{c_j - y_j}{y_j,\text{max} - c_j}, & \text{if} \quad y_j > c_j, \\ \frac{c_j - y_j}{c_j - y_j,\text{min}}, & \text{if} \quad y_j < c_j. \end{cases}$$ (4)

The upper line of equation (4) reflecting the exceeding of the target value, is the bonus function $d_b$ having values at negative part of codomain of $d^l(y_j)$, and lower line is the penalty function $d_p$ having values at positive part of codomain.

Bonus and penalty functions of DF for the constraint “not more” (“upper”) are the following:

$$d^u(y_j) = \begin{cases} \frac{y_j - c_j}{c_j - y_j,\text{min}}, & \text{if} \quad y_j < c_j, \\ \frac{y_j - c_j}{y_j,\text{max} - c_j}, & \text{if} \quad y_j > c_j. \end{cases}$$ (5)

As result, the DF $d(y_j)$ consists of two mutual functions: penalty function $d_p(y_j)$ and bonus function $d_b(y_j)$ that are defined so that $d(y_j) = d_p(y_j) + d_b(y_j)$ and if $d_p(y_j) \neq 0$, then $d_b(y_j) = 0$ and vice versa. This feature creates the additional abilities for objects estimation.

The DFs corresponding to compliance criteria are symmetrical relative to target (norm), so any deviations from the norm are estimated only by penalty function $d_p(y_j)$. If these deviations are linear the DF is triangular or trapezium, as it is presented in Figure 3. Note that these DFs are complementary for corresponding EFs shown in Figure 1, section $b$.

The scopes of the scale where indicator values do not agree with the corresponding restriction are shown by dash lines on the abscissa axis. For all such values the penalty proportional to the deviation value is calculated. For these constraints, the relative deviation of value $y_j$ from target value $c_j$ or norm range $[c_{j,l}, c_{j,u}]$ is always positive (see the Figure 3).

For the constraint “equal”, the relative deviation from the target is calculated using equation (6), which consists of two penalty functions (lower $d_{p,l}$ and upper $d_{p,u}$):
The deviation functions are defined as follows:

\[ d_{eq}(y_j) = \begin{cases} 
\frac{y_j - c_j}{c_j - y_j,\min} & \text{if } y_j < c_j, \\
\frac{y_j - c_j}{y_j,\max - c_j} & \text{if } y_j > c_j.
\end{cases} \]

(6)

For the range constraint \( y_j \in [c_{j,l}, c_{j,u}] \) the relative deviation into the range is 0 (i.e. \( d_{eq}(y_j) = 0 \)), if all points into the range are equivalent. When the value \( y_j \) exceeds any bound of \([c_{j,l}, c_{j,u}]\), the relative deviation is calculated using the equation (6), where the lower bound \( c_{j,l} \) is substituted instead of \( c_j \) to the upper line and the upper bound \( c_{j,u} \) of the range \([c_{j,l}, c_{j,u}]\) is substituted to the lower line.

For the range constraint \( y_j \in [c_{j,l}, c_{j,u}] \) the relative deviation into the range is 0 (i.e. \( d_{eq}(y_j) = 0 \)), if all points into the range are equivalent. When the value \( y_j \) exceeds any bound of \([c_{j,l}, c_{j,u}]\), the relative deviation is calculated using the equation (6), where the lower bound \( c_{j,l} \) is substituted instead of \( c_j \) to the upper line and the upper bound \( c_{j,u} \) of the range \([c_{j,l}, c_{j,u}]\) is substituted to the lower line.

Note that DFs also can be transformed to non-linear form as well as EFs from the Figure 1. To do that it is necessary to complement the values of corresponding non-linear EF to 1.

In the paper [1], the requirement of maximization of indicator’s utility value was opposed to minimization of distance to the specified point in a multidimensional space, and the additive generalizing function (AGF) was opposed to minimax criterion used for choosing an object with lower deviation from maximal deviations from the specified target. This opposition has allowed making a conclusion that weight coefficients at AGF are not needed. But the AGF can be also used for finding a generalized deviation from the vector of targets in cases where it is necessary to take into account the importance of indicators. The result deviation \( d^*(x) \) of object \( x \in X \) from the target using all the types of restriction can be calculated by equation (7):

\[ d^*(x) = \sum_{j=1}^{l} w_j \cdot d_{p,j}^*(y_j) + \sum_{j=1}^{u} w_j \cdot d_{u,j}^*(y_j) + \sum_{j=1}^{s} w_j \cdot d_{s,j}^*(y_j) + \sum_{j=1}^{e} w_j \cdot d_{e,j}^*(y_j), \]

(7)

Here \( d_{\Theta}^*(y_j) \) is the DF specified for \( j \)-th indicator according to criterion formulated for this indicator, \( \Theta \in \{l, u, eq, rg\} \), and \( y_j = f_j(x) \) is the value of the object \( x \in X \) for \( j \)-th indicator. Upper indexes of the sums correspond to numbers of indicators with the corresponding type of restriction, at that \( l + u + s + e = n \), and \( n \) is the total number of indicators. Indicator weights here agree with the following norming condition: \( \sum_{j=1}^{n} w_j = 1 \).

To estimate the total penalty \( d^p(x) \) for object \( x \in X \), corresponding penalty functions from (5), (6) are used as multipliers at first two sums in (7) the, and to estimate total bonus \( d^b(x) \), the corresponding bonus functions are used. At that only two first sums of (7) are used, because only the restrictions “lower” and “upper” can have the non-zero values of bonus function \( d_b(y_j) \):

\[ d^b(x) = \sum_{j=1}^{l} w_j \cdot d_{b,j}(x_j) + \sum_{j=1}^{u} w_j \cdot d_{b,j}(x_j), \]

(8)
The objective function corresponds to the requirement of minimizing deviation from the target: 
\[ d^*(x) \rightarrow \min. \]

Consider the typical problems of ordering objects by deviation functions.

1. **Compliance with the norm.** The object \( x \) agrees with all restrictions, if \( d^*_p(x) = 0 \), this corresponds to the solution of the choice problem or a positive control result.

2. **Deviation from the target.**
   a. **Weighted average deviation.** It is calculated by equation (7) with multipliers at first sums calculated by equations (5) and (6) for penalties and bonuses.
   b. **The best of the worst.** The deviation is calculated by equation (1).

3. **Ranking.**
   a. **By penalties.** Total penalty is calculated by equation (7) using only penalties from (5), (6) at first two sums, \( d^*_p(x) \rightarrow \min. \)
   b. **By bonuses.** Total bonus is calculated by equation (7) using only bonuses (4), (5) at first two sums, \( d^*_b(x) \rightarrow \min. \)

The ordering by penalties and bonuses can be used for the estimation of ideal goal achievement, if the target value \( c_j \) is assigned with scale bounds: \( c_j = y_{j,\text{min}} \), if \( y_j \geq c_j \), and \( c_j = y_{j,\text{max}} \), if \( y_j \leq c_j \) for penalties and at opposite bounds for bonuses.

4. **Example**
Consider the following example of using the DFs for parametric control problems solved in medicine. One of the tasks of this type is the task of checking the patient’s health based on the results of various laboratory tests, for example, clinical or biochemical blood tests. The compliance of the patient’s condition with the clinical norm is set by a set of reference values (a medical term used in conducting and evaluating laboratory tests, which is determined on the basis of the average value of a certain laboratory indicator obtained as a result of mass examinations of healthy population) of measured parameters. A clinical blood test allows assessing the hemoglobin content in the red blood system, the number of erythrocytes, the color index, the number of leukocytes and platelets, considering the leukogram and measuring the erythrocyte sedimentation rate (ESR). With this analysis, you can identify anemia, inflammation, etc.

Using a simplified example, let us estimate the deviation of the health state from the clinical norm for five conditional patients according to such basic indicators as the quantitative content of blood cells in the venous blood, as well as the content of hemoglobin and ESR. The initial data are given in Table 1.

| Patients | Leukocytes | Erythrocytes | Hemoglobin | Hematocrit | Platelets | Lymphocytes | ESR |
|----------|------------|--------------|------------|------------|-----------|-------------|-----|
| Patient #1 | 3.9        | 4.4          | 100        | 40         | 160       | 35          | 9.0 |
| Patient #2 | 4.2        | 5.0          | 36         | 42         | 152       | 26          | 7.0 |
| Patient #3 | 7.7        | 2.7          | 18         | 50         | 390       | 41          | 6.5 |
| Patient #4 | 9.4        | 5.3          | 120        | 27         | 100       | 30          | 8.0 |
| Patient #5 | 6.6        | 7.2          | 143        | 41         | 400       | 25          | 8.3 |

| Reference Values | [4, 9] | [4.3, 5.5] | [20, 140] | [39, 49] | [150, 400] | [25, 40] | [0, 8] |
|------------------|--------|-----------|----------|--------|-----------|--------|-------|
| Scales of Indicators | [0, 10] | [0, 10] | [0, 200] | [0, 100] | [0, 500] | [0, 100] | [0, 50] |

At the bottom of the Table 1 are the reference values corresponding to the normal (healthy) state of the patient, and the scales of indicators indicating the theoretically possible spread of the values of each of the indicators. Taking into account different scales and units of measurement of blood parameters, we calculate the relative deviations from the norm. Taking into account that all...
requirements are specified only by interval norms, to calculate the deviations we will use the function shown in Figure 6, section b, and equation (6) for the lower and upper bounds of the norm. There is no deviation from the norm, i.e., \( d(y_j) = 0 \), if the norm \([c_{j,l}, c_{j,u}]\) specified by the corresponding reference values is met.

**Experiment 1.** Let us evaluate the state of health of patients by all indicators, applying the generalizing function to the partial deviations from the norm (1). The maximum deviation from the norm for each patient is marked in bold in Table 2. They are duplicated in the \( d^*_{mm}(x) \) column.

| Patient | \( d(y_1) \) | \( d(y_2) \) | \( d(y_3) \) | \( d(y_4) \) | \( d(y_5) \) | \( d(y_6) \) | \( d(y_7) \) | \( d^*_{mm}(x) \) | Rank |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|----------------|------|
| Patient #1 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.03 | 2 |
| Patient #2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1 |
| Patient #3 | 0.00 | 0.37 | 0.10 | 0.02 | 0.00 | 0.02 | 0.00 | 0.37 | 3 |
| Patient #4 | 0.40 | 0.00 | 0.00 | 0.31 | 0.33 | 0.00 | 0.00 | 0.40 | 5 |
| Patient #5 | 0.00 | 0.38 | 0.05 | 0.00 | 0.00 | 0.00 | 0.01 | 0.38 | 4 |

In the last column of the Table 2 patients are ordered in the direction of increasing maximum deviations from the norm. Compliance with the norm for all indicators was found only for patient #2. The advantage of using the generalizing function (1) is the emphasis of the DM on the worst indicator for each object.

**Experiment 2.** Let us estimate the state of health of patients by all indicators, applying the generalizing function to the partial deviations from the norm (7). Taking into account that all requirements are specified only by interval norms, this equation uses only the last sum of partial deviations from the norm. In the absence of expert information on the importance of indicators for estimating the state of health of patients, the weight coefficients of all indicators in equation (7) are taken equal: \( w_j = 1/7, \ j = 1, \ldots, 7 \). The results of the estimation are given in Table 3. The results of ordering patients by minimax and averaging generalizing functions differ in the places of patient #3 and patient #5.

| Patient | \( d(y_1) \) | \( d(y_2) \) | \( d(y_3) \) | \( d(y_4) \) | \( d(y_5) \) | \( d(y_6) \) | \( d(y_7) \) | \( d^*_{av}(x) \) | Rank |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|----------------|------|
| Patient #1 | 0.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.007 | 2 |
| Patient #2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.000 | 1 |
| Patient #3 | 0.00 | 0.37 | 0.10 | 0.02 | 0.00 | 0.02 | 0.00 | 0.073 | 4 |
| Patient #4 | 0.40 | 0.00 | 0.00 | 0.31 | 0.33 | 0.00 | 0.00 | 0.149 | 5 |
| Patient #5 | 0.00 | 0.38 | 0.05 | 0.00 | 0.00 | 0.00 | 0.01 | 0.062 | 3 |

The slight difference in the ratings of patients obtained allows us to principally agree with the assertion of the author of paper [1] about the acceptability of replacing generalizing functions with weighting coefficients by a minimax generalizing function. However, the exclusion of indicators for which deviations from the norm are not maximum from consideration affects the generalized estimation of their health. This should be taken into account even more when diagnosing a disease that depends on the different importance of indicators.

**Experiment 3.** Let us rank the patients according to the degree of resistance of the organism to infections. Of the indicators adopted for experiments to determine the body’s immunity to infections, the doctor pays attention, first of all, to the compliance with the norms of leukocytes and lymphocytes. We assign the weight coefficients for them, respectively, \( w_1 = 0.5 \) and \( w_6 = 0.4 \), leaving the weight coefficients of other factors equal, \( w_j = 0.02 \). The results of ordering patients according to the state of
their immunity using the weighted average estimates calculated by the equation (7) are given in Table 4. Under these conditions, the patient #5 moved from the third to the second place, and patient #1 from the second place to the third, due to the fact that patient #1 has a greater deviation from the norm for the most important factor “leukocyte level”, and patient #5, despite the presence of deviations from the norm for less important factors, has zero deviation from the norm for both of the most important factors.

| Patient | $d(y_1)$ | $d(y_2)$ | $d(y_3)$ | $d(y_4)$ | $d(y_5)$ | $d(y_6)$ | $d(y_7)$ | $d^*(x)$ | Rank |
|---------|----------|----------|----------|----------|----------|----------|----------|----------|------|
| Patient #1 | 0.03     | 0.00     | 0.00     | 0.00     | 0.00     | 0.00     | 0.02     | 0.013    | 3    |
| Patient #2 | 0.00     | 0.00     | 0.00     | 0.00     | 0.00     | 0.00     | 0.00     | 0.000    | 1    |
| Patient #3 | 0.00     | 0.37     | 0.10     | 0.02     | 0.00     | 0.02     | 0.00     | 0.017    | 4    |
| Patient #4 | 0.40     | 0.00     | 0.00     | 0.31     | 0.33     | 0.00     | 0.00     | 0.213    | 5    |
| Patient #5 | 0.00     | 0.38     | 0.05     | 0.00     | 0.00     | 0.00     | 0.01     | 0.009    | 2    |

Consideration of this example shows the broad possibilities of practical application of methods of deviation from the goal, which contradicts the statements of the author of paper [1]. First, deviation methods are not limited to the use of superiority. Secondly, the minimax generalized estimation does not exclude, but complements the averaging and weighted average generalizing functions.

Thus, the application of the methods of deviation from the goal expands the range of problems that can be solved using the methods of multidimensional estimation of objects of any nature and purpose. When interpreting objects by time intervals using deviation methods, the problem of arrhythmia identification can be solved, in particular.

6. Conclusion

The deviation method proposed by A.P. Wierzbicki does not require finding non-dominated objects to determine the best object among them. Its finding is replaced with a multidimensional estimation of the deviation of objects from the target specified in the $n$-dimensional space of indicators. The fundamental difference with the methods of the theories of utility and value lies in the non-use of utility functions (value) for evaluating objects. But the multi-objective optimization method does not require their application either. Nevertheless, utility (value) functions contain more information about the preferences of the DM.

The paper shows that in the $n$-dimensional space of utility (value) functions, the estimation of the achievement of a given target is inverse to the estimation of the deviation from the target. In this sense, the application of the deviation functions gives the same results as the application of the goal achievement functions. The peculiarity of the functions of deviation from the real target is manifested in the possibility of separate ordering of objects in relation to non-achieving the specified target and exceeding it. For this, it is proposed to introduce scales of fines and incentives.

Proposed by A.P. Wierzbicki, the minimax criterion for multidimensional evaluation of objects, due to its simplicity, does not take into account non-extreme indicator values and their importance. They are taken into account using the weighted average generalizing function. The similarity and difference between the results of using the minimax and weighted average generalizing functions is shown using the example of deviations from the norm in patients’ blood parameters. It is shown that varying the weight coefficients of the weighted average generalizing function expands the range of problems that can be solved using the deviation functions. This example shows the complementarity of both approaches in multidimensional object estimation.

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