Unbound exotic nuclei studied by projectile fragmentation *

Guillaume Blanchon\textsuperscript{a,b}, A. Bonaccorso\textsuperscript{b}, D. M. Brink\textsuperscript{c}, A. García- Camacho\textsuperscript{b} and N. Vinh Mau\textsuperscript{d}.

\textsuperscript{a}Scuola di Dottorato G. Galilei, Pisa.

\textsuperscript{b}INFN, Sez. di Pisa and Dipartimento di Fisica, Università di Pisa, Largo Pontecorvo 3, 56127 Pisa, Italy.

\textsuperscript{c}Department of Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, U. K.

\textsuperscript{d}Institut de Physique Nucléaire, IN2P3-CNRS, F-91406, Orsay Cedex, France.

We call projectile fragmentation of neutron halo nuclei the elastic breakup (diffraction) reaction, when the observable studied is the neutron-core relative energy spectrum. This observable has been measured in relation to the Coulomb breakup on heavy target and recently also on light targets. Such data enlighten the effect of the neutron final state interaction with the core of origin. Projectile fragmentation is studied here by a time dependent model for the excitation of a nucleon from a bound state to a continuum resonant state in a neutron-core complex potential which acts as a final state interaction. The final state is described by an optical model S-matrix so that both resonant and non resonant states of any continuum energy can be studied as well as deeply bound initial states. It turns out that due to the coupling between the initial and final states, the neutron-core free particle phase shifts are modified, in the exit channel, by an additional phase. Some typical numerical calculations for the relevant observables are presented and compared to experimental data. It is suggest that the excitation energy spectra of an unbound nucleus might reflect the structure of the parent nucleus from whose fragmentation they are obtained.

1. Introduction

All theoretical methods used so far to describe breakup rely on a basic approximation to describe the collision with only the three-body variables of nucleon coordinate, projectile coordinate, and target coordinate. Thus the dynamics is controlled by the three potentials describing nucleon-core, nucleon-target, and core-target interactions. In most cases the projectile-target relative motion is treated semiclassically by using a trajectory of the center of the projectile relative to the center of the target $R(t) = b_e + vt\hat{z}$ with constant velocity $v$ in the $z$ direction and impact parameter $b_e$ in the $xy$ plane. This approximation makes our formalism applicable for incident energies above the Coulomb

---

*In memory of Adelchi Fabrocini.
barrier. Along this trajectory the amplitude for a transition from a nucleon state $\psi_i$ bound in the projectile, to a final continuum state $\psi_f$, is given by $[1, 2]$

$$A_{fi} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt \langle \psi_f(r, t)|V(r, R(t))|\psi_i(r, t) \rangle,$$

(1)

where $V$ is the interaction responsible for the transition which will be specified in the following. The probabilities for different processes can be represented in terms of the amplitude as $dP/d\xi = \sum |A_{fi}|^2 \delta(\xi - \xi_f)$ where $\xi$ can be momentum, energy or any other variable for which a differential cross section is measured. Direct one-particle re-arrangement reactions of the peripheral type in presence of strong core-target absorption can be described by an equation like $[2, 3, 4, 5]$

$$\frac{d\sigma_n}{d\xi_f} = C^2S \int dbc \frac{dP_{bup}(bc)}{d\xi_f} P_{ct}(bc),$$

(2)

(see Eq. (2.3) of $[3]$) and $C^2S$ is the spectroscopic factor for the initial single particle state. The core survival probability is defined in terms of a S-matrix function of the core-target distance of closest approach $b_c$. A simple parameterisation is $P_{ct}(bc) = \frac{1}{e^a}$. It takes into account the peripheral nature of the reaction and naturally excludes the possibility of large overlaps between projectile and target. The strong absorption radius $R_s \approx 1.4(A_1^{1/3} + A_1^{1/3})$ fm is defined as the distance of closest approach for a trajectory that is 50% absorbed from the elastic channel and $a=0.6$ fm is a diffuse-ness parameter. The values of $R_s$ thus obtained agree within a few percent with those of the Kox parameterization$[6]$.

2. Projectile Fragmentation

Let us call projectile fragmentation the elastic breakup (diffraction dissociation) reaction, when the observable studied is the neutron-core relative energy spectrum. This kind of observable has been widely measured in relation to the Coulomb breakup on heavy target. Results on light targets have also been presented $[7]$. These data enlighten the effect of the neutron final state interaction with the core of origin, while observables like the core energy or momentum distributions enlighten the effect of the neutron final state interaction with the target.

Projectile fragmentation has also been used to study two neutron halo projectiles $[9]-[24]$. In this case it has been suggested that the reaction might proceed by the simultaneous emission of the two neutrons or by successive emissions $[9]$. The successive emission can be due to a mechanism in which one neutron is stripped by the interaction with the target, as in the one-neutron fragmentation case, while the other is left behind, for example in a resonance state, which then decays. This mechanism has been described by the sudden approximation$[10]$ under the hypothesis that while the first neutron is stripped, the second neutron is emitted at large impact parameters with no final state interaction with the target. The emission can be expected sequential if the two neutrons are not strongly correlated.

If the two neutrons are strongly correlated they will preferentially be emitted simultaneously. If the neutron which is not detected is stripped while the other suffers an
Unbound exotic nuclei studied by projectile fragmentation

Elastic scattering on the target, then in both cases to first order in the interaction the neutron ends-up in a plane wave final state [2]. It can then re-interact with the core which, for example, is going to be $^{10}$Be in the case of the one-neutron halo projectile $^{11}$Be, while it will be $^{12}$Be in the case of the projectile fragmentation of $^{14}$Be, since $^{13}$Be is not bound. Experiments with a $^{14}$B projectile [11] have also been performed, in which the $n$-$^{12}$Be relative energy spectra have been reconstructed by coincidence measurements. In such a nucleus the valence neutron is weakly bound, while the valence proton is strongly bound. Thus the neutron will probably be emitted in the first step and then re-scattered by the core minus one proton nucleus. The projectile-target distances at which this kind of mechanism would be relevant are probably not so large to neglect the effect of the neutron-target interaction.

2.1. Inelastic excitation to the continuum.

To first order the inelastic-like excitations can be described again by the time dependent perturbation amplitude Eq.(1) [1, 2]. In this section also, the potential $V(r, R(t))$, which is the interaction responsible for the neutron transition, moves past on a constant velocity path as described in the previous sections. The radial part $\phi_i(r)$ of the single particle initial state wave function $\psi_i(r, t)$ is calculated in a potential $V_{WS}(r)$ which is fixed in space. The coordinate system and other details of the calculations can be found in Ref.[12]. In the special case of exotic nuclei the traditional approach to inelastic excitations needs to be modified. For example the final state can be eigenstate of a potential $V_1$ modified with respect to $V_{WS}$ because some other particle is emitted during the reaction process as discussed in the introduction. The final state interaction might also have an imaginary part which would take into account the coupling between a continuum state and an excited core. The first order time dependent perturbation amplitude then reads

$$A_{fi} = \frac{1}{i\hbar v} \int_{-\infty}^{\infty} dx dy dz \phi_f^*(x, y, z) \phi_i(x, y, z)e^{iqz} \tilde{V}(x - b_c, y, q),$$

where $\tilde{V}(x - b_c, y, q) = \int_{-\infty}^{\infty} dz V(x - b_c, y, z)e^{iqz}$, and we changed variables and put $z' = z - vt$ or $t = (z - z')/v$, $q = \varepsilon_f - \varepsilon_i/\hbar v$. Here $\varepsilon_f$ is the neutron-core relative energy in the final state.

The role of the target represented by $\tilde{V}$ is just to perturb the initial bound state wave function and to allow the transition to the continuum by transferring some momentum to the neutron. Then it is enough to choose a simplified form of the interaction, such as a delta-function potential $V(r) = v_2 \delta(x)\delta(y)\delta(z)$. The value of the strength $v_2 \equiv [\text{MeV fm}^3]$ used in the calculation is taken equal to the volume integral of the appropriate neutron-target interaction. It is clear that while in the sudden approach the initial and final state overlap is taken in the whole coordinate space, irrespective of the target and of the beam velocity, here the overlap of the initial and final wave functions depends on the core-target impact parameter. The neutron is emitted preferentially on the reaction plane and the $z$-component, being along the relative velocity axis is boosted by a momentum $q$.

Due of the strong core absorption discussed in Sec.1 these calculations are performed using the asymptotic form of the initial and final state wave functions. Introducing the quantization condition [2] the probability spectrum reads

$$\frac{dP_m}{d\varepsilon_f} = \frac{2 v_2^2}{\pi \hbar^2 v^2} C_i^2 m \frac{1}{\hbar k 2l_i + 1} \Sigma_{m_i, m_f} |1 - \tilde{S}_{m_i, m_f}|^2 |I_{m_i, m_f}|^2.$$

(4)
The generalization including spin is given in Appendix B of Ref. [12] and

\[ |I_{m_i,m_f}|^2 = \left| \int_{-\infty}^{\infty} dz e^{i q z} i^{l_i} \gamma h^{(1)}_{l_i} (i \gamma r) Y_{l_i m_i}(\theta,0) k h^{(-)}_{l_f} (kr) Y_{l_f m_f}(\theta,0) \right|^2. \]

The quantity \( \bar{S} = e^{2i(\delta + \nu)} \) is an off-the-energy-shell S-matrix representing the final state interaction of the neutron with the projectile core. It depends on a phase which is the sum of \( \delta \), the free particle n-core phase shift, plus \( \nu \) the phase of the matrix element \( |I| \).

3. Applications

3.1. The reaction \( {}^{11}\text{Be} \rightarrow \text{n}+{}^{10}\text{Be} \)

As a test of our model we calculate the relative energy spectrum \( \text{n}+{}^{10}\text{Be} \) obtained by the authors of Ref. [7] in the breakup reaction of \( {}^{11}\text{Be} + {}^{12}\text{C} \rightarrow \text{n}+{}^{10}\text{Be} + \text{X} \) at 69 A.MeV. Only the contributions from an s initial state with spectroscopic factor \( C^2 S = 0.84 \) are calculated. The triangles are the total calculated result after convolution with the experimental resolution function. The dots are the experimental points from [7].

Figure 1. n-\( {}^{10}\text{Be} \) relative energy spectrum, including Coulomb and nuclear breakup for the reaction \( {}^{11}\text{Be} + {}^{12}\text{C} \rightarrow \text{n}+{}^{10}\text{Be} + \text{X} \) at 69 A.MeV. Only the contributions from an s initial state with spectroscopic factor \( C^2 S = 0.84 \) are calculated. The triangles are the total calculated result after convolution with the experimental resolution function. The dots are the experimental points from [7].
neutron-target interaction. We have then folded the calculated spectrum through the experimental resolution function of Fukuda et al. [7], as given in Ref.[8]. The result is shown in Fig.3.1 by the triangles. The full curve is the total spectrum, sum of Coulomb and nuclear breakup. Each individual transition, due to the nuclear interaction only, is also shown. The dots are the experimental points from [7]. The kind of discrepancy between our calculation and the data in the range 1-2 MeV is very similar to that of the calculations in Ref.[8].

3.2. Structure of $^{14}$Be and $^{14}$B

Uncertainties in the interpretation of experimental results as compared to structure calculations were at the origin of our motivations to try to understand whether the neutron-$^{12}$Be relative energy spectra obtained from fragmentation of $^{14}$Be or $^{14}$B would show differences predictable in a theoretical model. If differences will be found in the experimental results with $^{14}$B and $^{14}$Be beams they could be due to an interplay between structure and reaction effects.

The ground state of $^{14}$Be has spin $J^\pi = 0^+$. In a simple model assuming two neutrons added to a $^{12}$Be core in its ground state the wave function is:

$$|^{14}Be > = [b_1(2s_1/2)^2 + b_2(1p_1/2)^2 + b_3(1d_5/2)^2] \otimes |^{12}Be, 0^+ >.$$  (5)

Then the bound neutron can be in a 2s, 1p$_{1/2}$ or 1d$_{5/2}$ state. However, as it has been discussed in the previous section, the situation is much more complicated [18]-[22] and in particular the calculations of Ref. [23] show that there is a large component $(2s_1/2, 1d_5/2) \otimes |^{12}Be, 2^+ >$ with the core in its low energy 2$^+$ state which can modify the neutron distribution.

![Figure 2](image)

Figure 2. Sum of all transitions from the s initial state for the reaction $^{14}$Be+$^{12}$C $\rightarrow n+^{12}$Be+X. Experimental points from [16]. Dashed line is the folding of the calculated spectrum with the experimental resolution curve.

The ground state of $^{14}$B has spin $J^\pi = 2^-$. In a model where it is described as a neutron-proton pair added to a $^{12}$Be core in its $0^+$ ground state with the proton in the 1p$_{3/2}$ shell, its wave function may be written as:

$$|^{14}B > = [a_1(p_{3/2}, 2s_{1/2}) + a_2(p_{3/2}, d_{5/2})] \otimes |^{12}Be, 0^+ >.$$  (6)
Guillaume Blanchon, A. Bonaccorso, D. M. Brink, A. García-Camacho and N. Vinh Mau.

The present experimental information [25] on $^{14}$B is that the neutron is in a state combination of s and d-components with weights 66% and 30% respectively, while shell model calculations show a similar mixture and no component with an excited state of the core. There are two possibilities for the reaction mechanism. One is that a proton is knocked out in the reaction with the target. The remaining $^{13}$Be would be left in an unbound s-state with probability $|a_1|^2$, in a d$_{5/2}$-state with probability $|a_2|^2$. These unbound states would decay showing the s-wave threshold and d-wave resonance effects. As mentioned in the introduction, the second possibility is that the neutron is knocked out first due to its small separation energy and that the proton is stripped from the remaining $^{13}$B.

To give another example of a possible comparison with available data, we show in Fig. 3.2 the experimental points from H. Simon et al. [16] for the reaction $^{14}$Be+$^{12}$C → n+$^{12}$Be+X at 250 A.MeV. The normalization factor of the data to mb/MeV is 0.843. The solid line gives the sum of all transitions from the s initial state with $\varepsilon_f=-1.85$ MeV (solid line), renormalized with a factor 2.4. The dashed line is the folding of the calculated spectrum with the experimental resolution curve. Therefore the calculation underestimate the absolute experimental cross section by a factor of 2. In view of the incertitude in the strength of our n-target $\delta$-potential and on the initial state spectroscopic factor which has been taken as unit, we can consider our absolute cross sections quite reasonable. A more detailed account of these calculations is given in [12].

4. Conclusions and Outlook

The field of Rare Isotopes Studies is very active, growing steadily and rapidly. Some recent achievements in the reaction theory for elastic breakup have been presented. From the structure point of view, in the search for the dripline position, a very important role is played by the study of nuclei unstable by neutron emission. This is one of the most important subjects which need to be adressed and further developed in the near future and for which some suggestions have been presented.

REFERENCES

1. K. Alder and A. Winther, Electromagnetic Excitation, North-Holland, 1975.
2. A. Bonaccorso and D.M. Brink, Phys. Rev. C 38 (1988) 1776; Phys. Rev. C 43 (1991) 299; Phys. Rev. C 44 (1991) 1559.
3. A. Bonaccorso, Phys. Rev. C 60 (1999) 054604.
4. J. Margueron, A. Bonaccorso and D.M. Brink, Nucl. Phys. A 703 (2002) 105; Nucl. Phys. A 720 (2003) 337.
5. R. A. Broglia and A. Winther, Heavy Ion Reactions, Benjamin, Reading, Mass, 1981.
6. S. Kox et al., Phys. Rev. C 35 (1987) 1678.
7. N. Fukuda et al., Phys. Rev. C 70, (2004) 054606.
8. P. Capel, D. Baye, Phys. Rev. C70 (2004) 064605.
9. F.M. Marqués et al., Phys. Rev. C 64 (2001) 061301(R). N.Orr, Prog. Theor. Phys. Suppl. 146 (2003) 201.
10. G.F. Bertsch, K. Hencken and H. Esbensen, Phys. Rev. C 57 (1998) 1366.
11. J.L. Lecouey, Few Body Syst. 34 (2004) 21-26.
12. G. Blanchon, A. Bonaccorso, D.M. Brink, A. García-Camacho and N. Vinh Mau, Nucl. Phys. A. (2006), in press.
13. G. Blanchon, A. Bonaccorso and N. Vinh Mau, Nucl. Phys. A 739 (2004) 259.
14. M. Thoennessen et al., Phys. Rev. C 59 (1999) 111; Phys. Rev. C 60 (1999) 027303; Proceedings the Erice Int. School of Heavy-Ion Physics, 4th Course. Eds. R.A. Broglia and P.G. Hansen. World Scientific, Singapore 1998, pag.269.
15. M. Labiche et al. Phys. Rev. Lett. 86 (2001) 600.
16. H. Simon et al., Nucl. Phys. A734 (2004) 323 and private communication.
17. A.A. Korsheninnikov et al., Phys. Lett. B 343 (1995) 53.
18. G.F. Bertsch and H. Esbensen, Ann. Phys. (N.Y.) 209 (1991) 327.
19. I.J. Thompson and M.V. Zhukov, Phys. Rev. C 53 (1996) 708.
20. N. Vinh Mau and J.C. Pacheco, Nucl. Phys. A 607 (1996) 163.
21. J.C. Pacheco and N. Vinh Mau, Phys. Rev. C 65 (2002) 044004.
22. P. Descouvemont, Phys. Lett. B 331 (1994) 271; Phys. Rev. C 52 (1995) 704.
23. T. Tarutina, I.J. Thompson, J.A. Tostevin, Nucl. Phys. A 733 (2004) 53.
24. M. Labiche, F.M. Marques, O. Sorlin and N. Vinh Mau, Phys. Rev. C 60 (1999) 027303.
25. V. Guimarães et al., Phys. Rev. C61 (2000) 064609.