Electric Dipole Polarizability of Neutron Rich Nuclei

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Insights into the equation of state of neutron rich matter obtained from the neutron skin thickness of $^{208}$Pb are in sharp conflict with earlier measurements of the electric dipole polarizability. A set of accurately calibrated energy density functionals are used to highlight the tension and to articulate how a highly-intense Gamma Factory may help resolve the present dilemma.

1. Introduction

The quest to determine the equation of state (EOS) of neutron rich matter has been re-energized by recent astronomical discoveries that have opened the brand new era of multi-messenger astronomy. Also playing a critical role in the determination of key parameter of the EOS is laboratory experiments at a variety of terrestrial facilities. Experiments with electroweak probes provide the cleanest connection to the EOS and in particular to the slope of the symmetry energy at nuclear saturation density, a quantity often denoted by $L$. The symmetry energy quantifies the cost in turning symmetric nuclear matter into pure neutron matter. Experiments with electroweak probes constrain the symmetry energy by measuring the neutron skin thickness of $^{208}$Pb ($R_{\text{skin}}^{^{208}\text{Pb}}$) and the electric dipole polarizability of a variety of nuclei. Given the importance of the slope of the symmetry energy in the determination of several neutron-star properties—particularly stellar radii—a concerted community effort has been devoted to a high-precision determination of $L$. Indeed, from a recent compilation of several theoretical predictions and experimental measurements the recommended value of $L$ is $[16]$

$$L = (59.8 \pm 4.1) \text{ MeV} \quad (1)$$

Such relatively small value indicates that the symmetry energy is soft, namely, that the pressure increases slowly with increasing density; see Figure 2 of ref. [16]. However, this finding has been recently brought into question by the PREX collaboration who has reported the following value for the neutron skin thickness of $^{208}$Pb$^{[8]}$

$$R_{\text{skin}}^{^{208}\text{Pb}} = (0.283 \pm 0.071) \text{ fm} \quad (2)$$

Using this value, plus invoking the strong correlation between $R_{\text{skin}}^{^{208}\text{Pb}}$ and the following two key parameters of the symmetry energy, one obtains $[17]$

$$J = (38.1 \pm 4.7) \text{ MeV} \quad (3a)$$

$$L = (106 \pm 37) \text{ MeV} \quad (3b)$$

where $J$ is the symmetry energy at saturation density. The large discrepancy between this result for $L$ and the one quoted in Equation(1) highlights the tension that emerged after the PREX-2 measurement. Given that some of the earlier limits on $L$ were inferred from the analysis of the electric dipole polarizability of $^{208}$Pb, we now proceed to confront those earlier results against the new limits inferred from invoking the newly recommended value for $R_{\text{skin}}^{^{208}\text{Pb}}$. From the experimental side, the commissioning of a Gamma Factory$^{[18,19]}$ provides a unique opportunity to probe with unprecedented precision the electric dipole response of exotic, neutron rich nuclei.

2. Formalism

The theoretical framework implemented in this work has been presented in much greater detail in several references, so we limit ourselves to highlight the most salient points; see for example ref. [20]. The starting point in our calculation of the nuclear response is a covariant energy density functional (EDF) accurately calibrated to properties of finite nuclei and neutron stars. Once the calibration of the functional is completed, one proceeds to compute both ground-state properties as well as the linear response using a self-consistent formalism rooted in the random phase approximation (RPA). In the long wavelength limit, the distribution of isovector dipole strength $R(\omega;E1)$ is directly related to the photoabsorption cross section $\sigma_{\text{abs}}(\omega)$. That is,

$$\sigma_{\text{abs}}(\omega) = \frac{16\pi^3 c^2}{9} \frac{1}{\hbar c} R(\omega;E1) \quad (4)$$

Previously identified as a strong isovector indicator$^{[22]}$ the electric dipole polarizability is directly obtained from the inverse-energy-weighted sum of the dipole response:

$$a_D = \frac{\hbar c}{2\pi^2} \int_0^\infty \frac{\sigma_{\text{abs}}(\omega)}{\omega^2} \, d\omega = \frac{8\pi c^2}{9} \int_0^\infty \frac{R(\omega;E1)}{\omega} \, d\omega \quad (5)$$

It is the powerful connection between the photoabsorption cross section, the electric dipole polarizability, and the slope of the symmetry energy that make a highly intense Gamma Factory an ideal facility in the quest to constrain the equation of state.
3. Results

In this section we present results for the electric dipole polarizability of $^{208}$Pb to highlight the disagreement of the PREX-2 informed prediction of $\alpha_2^{208}$ with an earlier measurement at the RCNP facility in Osaka, Japan.[9] A high-precision measurement of the electric dipole response of $^{208}$Pb was obtained from a small-angle $(p, p')$ experiment that fully agreed with earlier photoabsorption measurements.[9] The experimental distribution of strength contains a small non-resonant “quasi-deuteron” (QD) contribution that must be removed before it can be compared against theoretical (RPA) predictions. Once the QD contribution has been removed, one obtained the following estimate for the electric dipole polarizability of $^{208}$Pb:[23]

$$\alpha_2^{208} = (19.6 \pm 0.6) \text{ fm}^3 \quad (6)$$

To assess the impact of PREX-2 on $\alpha_2^{208}$ we display in Figure 1 the inverse-energy-weighted distribution of electric dipole strength as predicted by a set of 13 covariant EDFs used in ref. [17]. The various functionals are consistent with ground state properties of final nuclei, yet are flexible enough in that they span a wide range of values for the neutron skin thickness of $^{208}$Pb, as indicated by the labels in the figure. This flexibility reflects our poor understanding of the density dependence of the symmetry energy. We note that a small artificial width has been included to smear transitions into discrete states. However, transitions into the continuum are treated exactly in our non-spectral Green’s function approach.[20] The distribution of strength displays a significant model dependence, that is better illustrated in the inset of Figure 1. The inset displays the “running (or cumulative) sum” $\alpha_2^0(\omega)$, with the value at the largest excitation energy encoding the prediction for $\alpha_2^{208}$. Note that models with a stiffer symmetry energy, namely those that predict large values for both $R_{\text{skin}}^{208}$ and $L$, generate larger values for $\alpha_2^{208}$.[24] Also included in the inset is the value extracted from the RCNP experiment, which favors smaller values for $\alpha_2^{208}$ and therefore a relatively soft symmetry energy. Unlike the electric dipole polarizability, the classical energy weighted sum rule is largely model independent—with corrections due to charge dependent terms encoded in a parameter commonly referred to as $\kappa$.[25] For the set of covariant EDFs used in the work, we obtain a distribution of $\kappa$-values given by $\kappa = 0.25 \pm 0.02$.

Although the correlation between the electric dipole polarizability and the neutron skin thickness is strong,[22] a far stronger correlation exists between $R_{\text{skin}}^{208}$ and the product of $J$ times the electric dipole polarizability.[26,27] Indeed, insights from the droplet model suggest how the product of $J\alpha_2$ is a strong isovector indicator that could act as a proxy for the slope of the symmetry $L$.[27] Such as strong correlation is indicated in the inset of Figure 2, where the number above the line displays the correlation coefficient and the shaded region indicates the $1\sigma$ error in $R_{\text{skin}}^{208}$.[8] Also shown in Figure 2 are the probability distribution functions for $\alpha_2^{208}$ obtained from both the RCNP experiment and the PREX-2-informed extraction.

The skew-normal probability distribution informed by the PREX-2 result is obtained by relying on the nearly perfect correlation between $J\alpha_2^{208}$ and $R_{\text{skin}}^{208}$ displayed in the inset—and also between $J$ and $R_{\text{skin}}^{208}$.[17] In this manner one is able to generate two normal distributions, one for $J$ and another one for $J\alpha_2^{208}$, from which the skew-normal distribution may be readily deduced. Following the standard practice of quoting a 68% ($1\sigma$) estimate,
we obtain the following value for the PREX-2-informed electric dipole polarizability:

\[ \alpha_{D}^{208} = (21.8^{+1.1}_{-1.4}) \text{ fm}^3 \]  

where \( \alpha_{D}^{208} = 21.8 \text{ fm}^3 \) is the median of the distribution. The figure clearly illustrates the incompatibility of the two results—at least at the 1\( \sigma \) level. Indeed, the overlap region in Figure 2—estimated as the area shared by the two probability distributions—amounts to less than 25%.

We conclude the section by providing a prediction for the electric dipole polarizability of the unstable, doubly-magic nucleus \(^{132}\text{Sn}\). While a measurement of the electric dipole response of \(^{132}\text{Sn}\)—both in the low- (“Pygmy”) and high-energy (“Giant”) regions—has been reported in ref. [28], a precise value of the electric dipole polarizability is not yet available. Often portrayed as an oscillation of the excess neutrons against the isospin symmetric core, the emergence of a low-energy dipole mode has been found to correlate strongly with the development of a neutron rich skin.\(^{[29]}\) Indeed, the enhancement of the soft dipole mode is clearly discernible in Figure 3. This finding provides a compelling connection between the electric dipole polarizability and the neutron skin thickness—two critical observables used in the determination of the slope of the symmetry energy. A precise measurement of the electric dipole polarizability in \(^{132}\text{Sn}\) is well motivated. It was observed in ref. [30] that the neutron skin thickness and the electric dipole polarizability of \(^{132}\text{Sn}\) are both strongly correlated to the respective quantities in \(^{208}\text{Pb}\). This suggests that a measurement of \( R_{\text{Skin}}^{132}\) (if feasible) should reflect the PREX-2 result and yield a correspondingly large neutron skin, suggesting that the symmetry energy is stiff. On the other hand, if one follows the \( \alpha_{D} \) correlation, then the relatively low value of \( \alpha_{D}^{208} \) reported by the RCNP experiment suggests a correspondingly low value for \(^{132}\text{Sn}\), and hence a soft symmetry energy. It should also be mentioned that in a recent publication it was discovered that theoretical models that reproduce the electric dipole polarizability of \(^{208}\text{Pb}\) fail to do so along the isotopic chain in tin.\(^{[14]}\) In this context, the Gamma Factory could prove invaluable in solving these dilemmas.

4. Conclusions

We have highlighted a tension between two different experimental techniques that are used to extract a fundamental parameter of the equation of state: the slope of the symmetry energy \( L \). Given that \( L \) correlates strongly to a host of neutron-star observables, the resolution of the tension is of utmost importance. Indeed, laboratory experiments constitute the first rung in a “density ladder” that aims to determine the equation of state of neutron star matter. We note that whereas laboratory experiments are of little value in constraining the maximum neutron star mass because of the huge extrapolations, the situation is different for the case of stellar radii that are sensitive to the EOS at “only” two-to-three time nuclear saturation density. Naturally, one would like to see a significant reduction in the experimental uncertainty. However, the prospect of a more precise electroweak determination of \( R_{\text{Skin}}^{208}\) in the near future is slim, given that the PREX campaign is now over. The prospects for improving the precision in the determination of the electric dipole polarizability are significantly better. The challenges in this arena are associated with the production of unstable nuclei with large neutron skins together with the determination of the electric dipole response over a wide range of energies. With photon fluxes that are more than a million times more intense than the HI\( \gamma \)S facility at the campus of Duke University, the Gamma Factory could play a vital role in the determination of the photoabsorption cross section of exotic nuclei with unprecedented precision. In doing so, the Gamma Factory could provide important constraints on the equation of state of neutron rich matter as we enter in earnest the era of multi-messenger astronomy.

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Conflict of Interest

The author declares no conflict of interest.

Data Availability Statement

Figures are included in the article and the associated data can be shared upon request.

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