External Fields and Color Confinement

P. Cea\textsuperscript{a}, L. Cosmai\textsuperscript{b}

\textsuperscript{a}Dept. of Physics and INFN - Bari - Italy
\textsuperscript{b}INFN - Bari - Italy

U(1), SU(2), and SU(3) lattice gauge theories in presence of external fields are investigated both in (3+1) and (2+1) dimensions. The free energy of gauge systems has been measured. While the phase transition in compact U(1) is not influenced by the strength of an external constant magnetic field, the deconfinement temperature for SU(2) and SU(3) gauge systems in a constant abelian chromomagnetic field decreases when the strength of the applied field increases. The dependence of the deconfinement temperature on the strength of an external constant chromomagnetic field seems to be a peculiar feature of non abelian gauge theories.

1. Introduction

Color confinement is still a puzzling problem notwithstanding the large mess of numerical investigations aimed to understand the nature of the QCD vacuum. Therefore we feel that it is a good task to explore new paths that possibly may suggest new ideas towards understanding confinement.

In a seminal paper \[1\] R. P. Feynman argued that in non abelian gauge theories long range correlation between gluonic degrees of freedom can destroy confinement. This suggested us that a uniform constant background field (that in this case should restore long range correlations between gluonic degrees of freedom) could influence the deconfinement temperature for non abelian gauge theories.

In order to investigate vacuum structure of lattice gauge theories at zero temperature a lattice gauge invariant effective action $\Gamma[\vec{A}]$ for an external background field $\vec{A}$ was introduced in Refs. \[2,3\]. For a gauge theory at finite temperature $T = 1 / (aL_t)$ in presence of an external background field, the relevant quantity is the free energy functional defined as (for more details and discussions, see Refs. \[4,5\])

\[
\mathcal{F}[\vec{A}] = -\frac{1}{L_t} \ln \left\{ \frac{Z_T[\vec{A}]}{Z_T[0]} \right\}.
\]

$Z_T[\vec{A}]$ is the thermal partition functional in presence of the background field $\vec{A}$, and is defined as

\[
Z_T[\vec{A}] = \int \mathcal{D}U e^{-S_W}.
\]

In Eq. (2) the spatial links belonging to the time slice $x_t = 0$ are constrained to the value of the external background field, the temporal links are not constrained.

2. Deconfinement Temperature vs. $gH$

We study vacuum dynamics for U(1), SU(2), and SU(3) lattice gauge theories under the influence of an Abelian chromomagnetic background field. In our previous studies we found that SU(3) deconfinement temperature depends on the strength of an applied external constant Abelian chromomagnetic field \[4\]. We would like to corroborate our findings with further investigations, in particular we would like to ascertain if the dependence of the deconfinement temperature on the strength of an applied external constant Abelian chromomagnetic field is a peculiar feature of non abelian gauge theories.

In the continuum a static constant Abelian chromomagnetic field is given by:

\[
\vec{A}_a^\text{ext}(\vec{x}) = \vec{A}^\text{ext}(\vec{z}) \delta_{a,3}, \quad A_k^\text{ext}(\vec{x}) = \delta_{k,2} x_1 H.
\]
Figure 1. SU(3) in (3+1) dimensions. $T_c/\sqrt{\sigma}$ vs. the applied field strength in units of string tension.

On a lattice with hypertoroidal geometry the magnetic field turns out to be quantized (i.e.: $a^2 gH/2 = 2\pi/L_x, n_{\text{ext}}$ integer). The free energy $F[A^{\text{ext}}]$ is proportional to spatial volume $V = L_s^3$ and the relevant quantity is the density $f[A^{\text{ext}}]$ of free energy. We evaluate the $\beta$-derivative of $f[A^{\text{ext}}]$ at fixed external field strength $gH (\Omega = L_s^3 \times L_t)$:

$$f'[A^{\text{ext}}] = \left\langle \frac{1}{\Omega} \sum_{x, \mu<\nu} \frac{1}{3} \text{Re} \text{Tr} U_{\mu\nu}(x) \right\rangle_0 - \left\langle \frac{1}{\Omega} \sum_{x, \mu<\nu} \frac{1}{3} \text{Re} \text{Tr} U_{\mu\nu}(x) \right\rangle_{A^{\text{ext}}},$$

where the subscripts on the averages indicate the value of the external field.

As is well known pure SU(3) gauge system undergoes a deconfinement phase transition by increasing temperature. We can evaluate the critical temperature $T_c$ by locating the peak of $f'[A^{\text{ext}}]$ as a function of $\beta$ for different lattice temporal sizes $L_t$. We vary the strength of the applied external Abelian chromomagnetic background field to study quantitatively the dependence of $T_c$ on $gH$. To extract the critical field strength in physical units we compute $T_c/\sqrt{\sigma}$ vs. $\sqrt{gH}/\sqrt{\sigma}$, where the string tension at $\beta^*(L_t = 8, n_{\text{ext}})$ is obtained from Eq. (4.4) in Ref. [7]. We obtain that the critical temperature decreases by increasing the external Abelian chromomagnetic field. If the magnetic length $a_H \sim 1/\sqrt{gH}$ is the only relevant scale of the problem for dimensional reasons one expect that $T_c/\sqrt{\sigma} \sim \sqrt{gH}/\sqrt{\sigma}$. Indeed we get a good linear fit to our data with the critical field $\sqrt{gH}/\sqrt{\sigma} = 2.63 \pm 0.15$ (see Fig. 1).

An analogous result has been found for SU(2) l.g.t., where the critical coupling $\beta^*(L_t, n_{\text{ext}})$ has been evaluated on a $64^3 \times 8$ lattice, and the string tension is obtained according to Ref. [8]. In this case the critical field is $\sqrt{gH}/\sqrt{\sigma} = 5.33 \pm 0.33$ (see Fig. 2). It is worthwhile to note that, using an effective approach within dual superconductor picture, it was suggested [9] that $gH_c/\sigma = 1$ for SU(2) and $gH_c/\sigma = 3/4$ for SU(3).

To ascertain if the effect we have found is peculiar of non Abelian gauge theories we have simulated U(1) lattice gauge theory at zero temperature in order to test a possible dependence of the confinement-Coulomb phase transition on the strength of an applied constant magnetic field. Indeed the critical coupling does not depend on the applied magnetic field strength, in agreement with Ref. [10] (see Fig. 3).

Finally, to see if this color Meissner effect is a generic feature of non Abelian gauge theories, we
also consider gauge systems in (2+1) dimensions. Gauge theories in (2+1) dimensions possess a dimensionful coupling, namely \( g^2 \) has dimension of mass and so provides a physical scale.

In (2+1) dimensions the chromomagnetic field \( H^a \) is a (pseudo)scalar
\[
H^a = \frac{1}{2} \epsilon_{ij} F^a_{ij} = F^a_{12}.
\]

For SU(3) gauge theory a constant abelian chromomagnetic field \( H^3 \) can be obtained with
\[
U_1^{\text{ext}}(\vec{x}) = 1,
U_2^{\text{ext}}(\vec{x}) = \begin{bmatrix}
\exp(i \frac{g H^3_{12}}{2}) & 0 & 0 \\
0 & \exp(-i \frac{g H^3_{12}}{2}) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Again we find that \( \frac{T_c}{\sqrt{\sigma}} \) depends linearly on the applied field strength. A preliminary estimate, with \( L_t = 4 \), gives \( \frac{T_c}{\sqrt{\sigma}} = 1.073(87) \), \( g H_{c3} \sqrt{\sigma} = 5.5 \pm 3.7 \), where the string tension has been taken from Ref. [11]. The critical temperature value from Ref. [12] is \( \frac{T_c}{\sqrt{\sigma}} = 0.972(10) \).

3. Conclusions

We have investigated U(1) and SU(2) in (3+1) dimensions, and SU(3) both in (3+1) and (2+1) dimensions. Our numerical simulations were performed on APE machines in Bari. For non abelian gauge theories we found that there is a critical field \( g H_c \) such that for \( g H > g H_c \) the gauge systems are in the deconfined phase. Such an effect is generic for non Abelian gauge theories and it is not shared by U(1) gauge theory.

As argued in Ref. [1] gauge invariance of the confining vacuum disorders the system in such a way that there are not long range color correlations. On the other hand strong enough chromomagnetic fields do introduce long range color correlations such that the system gets deconfined. In conclusion it is worthwhile to observe that in general the existence of a critical chromomagnetic field could be explained if the confining vacuum behaves as a condensate of color charged fields whose mass is proportional to the inverse of the magnetic length.

REFERENCES
1. R.P. Feynman, Nucl. Phys. B188 (1981) 479.
2. P. Cea, L. Cosmai and A.D. Polosa, Phys. Lett. B392 (1997) 177, [hep-lat/9601010]
3. P. Cea and L. Cosmai, Phys. Rev. D60 (1999) 094506, [hep-lat/9903005]
4. P. Cea and L. Cosmai, JHEP 02 (2003) 031, [hep-lat/0204023]
5. P. Cea, L. Cosmai and M. D’Elia, JHEP 02 (2004) 018, [hep-lat/0401020]
6. D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. 53 (1981) 43.
7. R.G. Edwards, U.M. Heller and T.R. Klassen, Nucl. Phys. B517 (1998) 377, [hep-lat/9711003]
8. M.J. Teper, (1998), [hep-th/9812187]
9. M.N. Chernodub, Phys. Lett. B549 (2002) 146, [hep-ph/0208105]
10. M.N. Chernodub, E.M. Ilgenfritz and A. Schiller, Phys. Rev. Lett. 88 (2002) 231601, [hep-lat/0112048]
11. M.J. Teper, Phys. Rev. D59 (1999) 014512, [hep-lat/9804008]
12. J. Engels et al., Nucl. Phys. Proc. Suppl. 53 (1997) 420, [hep-lat/9608099]