Non-linear general instability of ring-stiffened conical shells under external hydrostatic pressure.

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Abstract.

The paper presents the experimental results for 15 ring-stiffened circular steel conical shells, which failed by non-linear general instability. The results of these investigations were compared with various theoretical analyses, including an ANSYS eigen buckling analysis and another ANSYS analysis; which involved a step-by-step method until collapse; where both material & geometrical nonlinearity were considered. The investigation also involved an analysis using BS5500 (PD 5500), together with the method of Ross of the University of Portsmouth. The ANSYS eigen buckling analysis tended to overestimate the predicted buckling pressures; whereas the ANSYS nonlinear results compared favourably with the experimental results. The PD5500 analysis was very time consuming and tended to grossly underestimate the experimental buckling pressures and in some cases, overestimate them. In contrast to PD5500 & ANSYS, the design charts of Ross of the University of Portsmouth were the easiest of all these methods to use and generally only slightly underestimated the experimental collapse pressures. The ANSYS analyses gave some excellent graphical displays.

1. Introduction

It is currently estimated that by the year 2030 the world’s cumulative demand of primary energy will increase by over 40%, based on 2007 usage figures (WEO 2009) [1]. Moreover, despite the promotion of alternative fuels, fossil fuels will continue to account for more than ¾ of fuel consumption growth. At present, the exploration of both land and shallow water is advanced, with oil and natural gas estimated to be reaching peak output. Therefore, the search for additional oil and gas supplies is crucial for ensuring worldwide energy security and economic stability. It is for this reason that the world’s new oil and gas reserves are moving from shallow waters into deeper waters. Deep water exploration has become the key focus of the energy industries strategic outlook, with proven reserves of approximately 10 000 billion tonnes of methane hydrates [2], together with very large reserves of oil. However, oil and gas production in the deep sea face greater challenges due to the increasing depths and higher pressures. This results in inseparable high costs and advanced technical requirements, with the relative cost of drilling land, shallow water and deep water, being approximately 1:10:100. It is therefore imperative that the technological advancements required to meet the challenges associated with such depths are not only practical and cost effective but capable and safe. It is for this reason that the current dated methods of design and testing require
modernisation to aid the development of future technologies without the compromise of accuracy, reliability and safety.
Ultimately it is true to say that most of the resources under the sea remain unknown to mankind. With more humans to having laid a foot on the moon than to have explored the oceans bottoms, there are likely to be limitless opportunities waiting to be found.
Submarine pressure hulls that are designed to withstand uniform external pressure commonly take the shape of cylinders, cones and domes, as shown in Figure 1 [3]. The part of the submarine pressure hull that is of special interest here is the truncated cone.

![Figure 1. The submarine pressure hull.](image)

Under external hydrostatic pressure an unstiffened circular conical shell implodes through shell instability or lobar buckling, at a fraction of the pressure for the same vessel to explode under internal pressure. The shell instability mode of failure is shown in Figure 2.

![Figure 2. Shell instability of thin-walled cones.](image)
One way to improve the structural efficiency of thin-walled cones is to ring-stiffen them in their flanks, as shown in Figure 3.

![Ring-stiffened cones](image_url)

**Figure 3.** Ring-stiffened cones.

If however the ring-stiffeners are not strong enough, the entire ring-shell combinations can buckle bodily in their flanks through general instability, as shown in Figure 4.

![General instability of ring-stiffened cones](image_url)

**Figure 4.** General instability of ring-stiffened cones.

Another mode of failure is called axisymmetric deformation; which is shown in Figure 5. In this mode of failure the vessel implodes inwards, keeping its circular form throughout its collapse.
In the present study, the main collapse mode of interest is plastic general instability.

2. The Methods

The current standard method for designing pressure vessels uses PD5500:2000 (BS5500) [4], Specification for Unfired fusion welded pressure vessels, published by the British Standards Institute. This method is widely regarded as over conservative and as a result can lead to the over engineering of full scale vessels increasing production costs and limiting capability. In addition to this the modified method for ring stiffened conical shell is complex and extremely time consuming. It is based upon early research into general instability by Bryant [5], who produced an approximate formula for calculating the collapse pressure of ring stiffened circular cylinders; with later work by Kendrick [6, 7], producing more accurate predictions.

However, the accuracy of the theoretical analysis for general instability of these vessels is in question, as vessels commonly buckle plastically at pressures much lower than predictions based on elastic theory. It is therefore usually best to design pressure vessels so that buckling is eliminated and any likely failure would be through axisymmetric yield. This is because the theory is based upon a perfect cylinder, cone or dome and does not take into account slight initial out of circularity of the vessel’s geometry; introduced in the manufacture, together with material non linearity. Previously these effects have been difficult to predict or model, as in practice they grow non-linearly with the increase in uniform external pressure. Eventually this will cause part of the vessel to become plastic thus dramatically reducing the Young’s modulus in this area. Such vessels where the initial imperfections in the geometry cause the experimental buckling pressures to be much lower than that predicted by Kendrick are said to suffer plastic knockdown.

Previous work by Ross et al [8, 9] have produced a design chart incorporating a plastic knock down factor (PKD), that can be applied to the theoretical elastic buckling pressures of Kendrick ($P_{cr}$) Part 1 & 3 [6,7] for perfect vessels, to predict the actual experimental buckling pressure ($P_{exp}$) for general instability of ring stiffened conical shells, where $P_{exp}$ is obtained by dividing the $P_{cr}$ by the PKD as follows:

$$P_{exp} = \frac{P_{cr}}{PKD}$$

To then obtain the design buckling pressure the $P_{exp}$ is then divided by a suitable safety factor (SF);

$$P_{des} = \frac{P_{exp}}{SF}$$
The design charts produced by Ross et al. present fairly linear results and should prove useful to designers. This study plans to use an ANSYS step-by-step non-linear finite element analysis, to validate these design charts and to show the limits of PD5500. This computational analysis allows for both geometrical and material non-linearity of the vessels being tested. It is based on an incremental step-by-step method that applies the total load in relatively small load steps; about 50 steps in these cases. Each time a step is carried out, the resultant stress and deflections are used to calculate the new geometry of the vessel. This new geometry is then used as the initial state of the next load step. The resultant stresses and deflections are also added to the stress and deflection of the previous step, as shown in Figure 6. If the resulting stress in any element exceeds its von Mises yield stress, the Young’s modulus in that element is reduced to \( \frac{1}{50} \) of the elastic Young’s modulus. This is further reduced to \( \frac{1}{100} \) if the yield stress is exceeded by a factor of 1.1. This process is repeated until the vessel fails. Ross [3] gives full details of the non-linear analysis theory and also presents a similar investigation into the axisymmetric failure of ring stiffened cylinders. As the results of Ross’s investigation showed good agreement between the predicted non-linear theoretical buckling pressures and the experimental buckling pressures it may be possible to apply this non-linear method to the design of full scale vessels to predict a more reliable collapse pressure than that of the current BS5500 (PD5500) standard.

![Figure 6](image.png)

**Figure 6.** Incremental step-by-step method.

### 3. Methodology

The methodology contained in this report is split into three main sections:

- Applying ANSYS FEA [10] (Eigenbuckling & the nonlinear method) to the 15 ring stiffened cones.
- Apply Kendrick Parts 1 & 3 formulae [6, 7] to the 15 ring stiffened cones, together with Ross’ Design Charts [3, 8, 9].
- Apply Bryant’s formula together with BS5500 (PD5500) [4] to the 15 ring stiffened cones.
In addition to this the experimental method is also summarised. All methods are detailed below with supporting documentation contained in the annex.

3.1. Details of Ring Stiffened Cones

The theoretical analyses using both BS5500 and the non-linear FEA analysis of ANSYS were based upon cones from previous experimental work by Ross et al [3, 8, 9].

3.2. Experimental Method

The experiments carried out by Ross et al [3, 8, 9] were conducted in a pressurised water tank. The method described used ten circumferential foil strain gauges placed inside the cone at equal intervals along the circumference of the vessel, with an additional gauge positioned near the base of the cone in order to measure the longitudinal strain. The strain indicators were all connected in line with a strain indicator. The vessel was then fixed to the top of the tank as seen in Figure 7, and the tank filled and bled with water. A hydraulic pump was then used to slowly pressurize the water tank in increments of 100psi (6.89bar). At each interval the strain readings were recorded. When the strain values around the circumference became nonlinear the incremental pressure load was decreased to 50psi (3.45bar). This process was continued until each of the cones collapsed through general instability [11, 12].

![Figure 7](image_url)

**Figure 7.** Ring stiffened cone attached to the inside of the pressure tank

3.3. ANSYS - Finite Element Analysis

For this investigation it was necessary to carry out two types of finite element analysis using ANSYS. The first was an Eigen buckling analysis that gives a theoretical predicted buckling pressure ($P_{cr}$) based upon the elastic theory of a perfect conical shell. The second was a non linear analysis that uses plastic theory, accounting for geometrical and material non-linearity of the vessel. For both these methods it was first necessary to create a computer model of the experimental cones in ANSYS as shown in Figure 8. This was done using the design dimensions of the models [3,8, 9], however, this was slightly modified to incorporate a lid on the cone. The thickness of the lid was taken as 10 times
the shell thickness. A detailed step by step process for building the model and running both the Eigen and Non-linear analysis was carried out.

For the shell of the cone, the element ‘Shell93’ was used as it is particularly well suited to model curved shells. The element has six degrees of freedom at each node, translations in the nodal x, y & z directions and rotations about the nodal x, y & z axes. The element has plasticity, stress stiffening, large deflections and large strain capabilities. The element is defined by eight nodes, for thickness and the orthotropic material properties. The element may vary in thickness, however, if the thickness remains constant throughout the element, then only the thickness at only one node needs to be input.

For the ring stiffeners of the cone, element ‘Beam44’ was used. It is a uniaxial element with tension, compression, torsion and bending capabilities. It also has six degrees of freedom, translations in the nodal x, y & z directions and rotations about the nodal x, y & z axes. An ANSYS eigen buckling analysis for one of the cones is shown in Figure 8.

![Figure 8](image1.png)

**Figure 8.** ANSYS model showing the elements for a ring stiffened cone.

![Figure 9](image2.png)

**Figure 9.** ANSYS results showing deformation of a buckled ring stiffened cone.
4. Results

The ANSYS nonlinear results for this paper are presented in a tabulated format to allow easy comparison of each of the individual cones and the various methods of analysis, as shown in Table 1 together with the other results.

The computation was carried out in the following order: (1) ANSYS eigen buckling, (2) ANSYS nonlinear, (3) PD 5500, (4) Ross’ design method. The difficulty of doing the analyses is shown in the following order, with the most difficult being shown first: (1) PD 5500, (2) ANSYS nonlinear, (3) ANSYS eigen buckling, (4) Ross; design methods.

Table 1 presents the theoretical predicted collapse pressures ($P_{cr}$) for the Eigen and Non linear analysis compared against those determined from PD5500 (which is based upon Bryant’s Formula), Bryant’s (from BRYANT.EXE) and also the experimentally determined pressures ($P_{exp}$). The
Theoretical collapse pressure \( (P_{cr}) \) determined from the computer programs, KENDRIC1.EXE & KENDPT3, were then plotted on the design charts produced by Ross [3] using the thinness ratios \((\lambda')\) of each of the cones. A comparison of results is presented in Table 1 below.

**Table 1.** Theoretical collapse pressures \((P_{cr})\) compared with experimental values \((P_{exp})\)

| Cone | Eigen (MPa) | Non Linear (MPa) | PD5500 (MPa) | Ross1 (MPa) | Ross3 (MPa) | \(P_{exp} [1]\) (MPa) | \(1/\lambda'\) |
|------|-------------|-----------------|--------------|-------------|-------------|---------------------|-----------|
| 1    | 3.00        | 3.92            | 0.76         | 1.73        | 1.10        | 2.98                | 0.582     |
| 2    | 4.15        | 3.92            | 2.27         | 1.53        | 1.33        | 3.93                | 0.706     |
| 3    | 6.48        | 4.13            | 2.25         | 1.50        | 1.00        | 4.10                | 0.787     |
| 4    | 7.72        | 7.7             | 9.52         | 3.41        | 3.52        | 5.18                | 0.746     |
| 5    | 8.50        | 8.25            | 8.08         | 3.59        | 3.50        | 5.18                | 0.822     |
| 6    | 10.01       | 8.0             | 9.76         | 5.06        | 5.33        | 5.47                | 1.107     |
| 7    | 24.79       | 11.6            | 9.426        | 7.31        | 6.93        | 10.35               | 1.164     |
| 8    | 27.27       | 12.0            | 11.066       | 7.92        | 6.38        | 11.10               | 1.234     |
| 9    | 30.35       | 12.2            | 14.096       | 9.04        | 7.01        | 11.72               | 1.346     |
| 10   | 60.19       | 7.5             | 14.86        | 13.30       | 12.83       | 14.3                | 1.614     |
| 11   | 66.90       | 9.0             | 18.58        | 14.69       | 14.07       | 15.8                | 1.729     |
| 12   | 74.49       | 12.0            | 23.33        | 16.66       | 15.03       | 18.8                | 1.838     |
| 13   | 28.27       | 13.2            | 15.32        | 9.61        | 9.54        | 14.07               | 1.325     |
| 14   | 31.04       | 13.0            | 19.30        | 9.00        | 8.73        | 14.55               | 1.434     |
| 15   | 34.06       | 13.2            | 25.74        | 10.44       | 9.87        | 14.48               | 1.504     |

5. Results & Analysis

- **ANSYS Eigen Analysis**

The ANSYS Eigen results obtained in this study were key to the continuity of the nonlinear investigation with previous work by Ross et al [3, 8, 9]. The small percentage differences in the predicted collapse pressures in addition to the matching number of predicted lobes \((n)\) indicated that the cones modelled in this report were accurate models to that of previous work by Ross et al[3, 8, 9], and therefore validated their use for further investigations and comparisons.

- **ANSYS Non Linear Analysis**

The ANSYS non linear analysis results in this paper showed good agreement between all the cones, with the collapse pressures increasing as expected with each increase in the stiffener geometry. However, the nonlinear analysis tended to overestimate the collapse pressures for Cones 4, 5 & 6 and to underestimate the collapse pressures for Cones 10, 11 & 12. PD 5500 tended to be unreliable, while Ross’ design method appeared to be the most reliable. It was also evident from the graphical results that each of the cones collapsed plastically through general instability with matching values of the circumferential collapse lobes, \(n\), similar to that of the Eigen analysis. However on closer inspection of the results there was also evidence of axisymmetric deformation in the first bay, between the bottom bulkhead and the first stiffener. Plotting the results for the previous sub step confirmed this theory as can be seen in Figure 12. Having then identified that the onset of this mode of failure occurred earlier than that of general instability, it was determined that the point at which plastic deformation began on the graph of ‘Load factor/Displacement’ was the pressure at which this axisymmetric deformation began. The pressure at this point being the equivalent to that of the
pressure calculated with PD5500 for which stress in between stiffeners reach’s yield point pressure (P_{yc}). This theory was further supported due to the fact that this pressure remained constant between all the cones, whilst the overall collapse pressure was varied. From the ANSYS nonlinear graphs this was calculated as 11MPa in comparison to P_{ys} = 10.5MPa calculated with PD5500.

6. Conclusions.

This paper has successfully investigated various methods of predicting the theoretical collapse pressures for 15 ring stiffened conical vessels. The ANSYS nonlinear analysis and Ross’ design method were good. This was unlike PD 5500; which was unreliable. It was believed that PD 5500 was unreliable because the same curve is used for un-stiffened and ring stiffened circular cylinders and also cones. The results of these methods were then compared to design charts produced by Ross [3,8,9] to determine the accuracy and reliability of the charts and even evaluate their application to full scale submarines.

![Figure 12. Axisymmetrical deformation of ring stiffened cone 9.](image)

The results presented in the study have highlighted the erratic and unreliable results of the current standard practice of PD5500. The author found that of the 3 main methodologies tested, ANSYS non linear, Ross/Kendrick and PD5500, the last was not only complex and time consuming, but it lacked any consistency in its predicted collapse pressures. The wide range of results produced meant that an appropriately large safety factor would have to be applied to the desired design stress of a vessel when applying this method. This could essentially result in the over engineering and reduction of capability of a vessel, both of which would have an associated cost factor. The ANSYS non linear buckling analysis carried out, produced the most accurate results than ANSYS eigen. The authors also found this method was most time consuming; with over 150 h of training and familiarisation with the software required before being able to successfully carry out the analysis for this investigation.

The main finding of this report showed that the results of Ross/Kendrick produced were not only simple and quick to determine, but they were also consistent and gave safe predictions for all 15 cones. This demonstrated the successful application of the Ross [3, 8, 9] design charts to the theoretical collapse pressure of Kendrick Parts 1 & 3 (the results of Kendrick 1 & 3 alone being over optimistic). The authors did also find the Ross/Kendrick results for all the cones tested, were slightly conservative, so that we were in a win-win situation.
It should be pointed out that for vessels with initial geometrical imperfections, such as in cases of initial out-of-circularity, caused by manufacture, there can be a serious drop in the experimentally obtained buckling pressures; even though these initial geometrical imperfections may be small.

7. References

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