A Dual-Function Radar Communication System Using Index Modulation

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Abstract—Dual-function radar communication (DFRC) systems implement both sensing and communication using the same hardware. Such schemes are typically more efficient in terms of size, power, and cost, over using distinct radar and communication systems. Since these functionalities share resources such as spectrum, power, and antennas, DFRC methods typically entail some degradation in both radar and communication performance. In this work we propose a multi-carrier agile phased array radar (MAPAR) DFRC, which extends the previously proposed frequency agile radar and multi-carrier agile radar to a phased array antenna operating with constant modulus waveforms. The inherent spatial and spectral randomness of the proposed MAPAR is utilized to convey digital messages in the form of index modulation. The resulting communication scheme naturally coexists with the radar functionality, and thus does not come at the cost of reduced radar performance. We carry out a theoretical analysis of the radar and communication performances, quantifying the radar beam pattern and the achievable bit rate. Our numerical results demonstrate that the proposed MAPAR yields an effective range-spatial beam pattern, similar to that achievable with traditional phased array radar, while its communications bit rate is comparable to utilizing an independent communication module.

I. INTRODUCTION

Recent years have witnessed a growing interest in dual-function radar communication (DFRC) systems. Natural applications that implement both sensing and communications include autonomous vehicles, commercial flight control, and military radar systems [1]. Jointly implementing radar and communication contributes to reducing the number of antennas, system size, weight, and power consumption [2], as well as alleviating concerns for electromagnetic compatibility (EMC) and spectrum congestion issues [3]. In the context of autonomous vehicles, DFRC systems can improve connectivity by providing an additional communications medium integrated in the radar functionality. In such joint radar and communications models, the DFRC system acts as the radar transmitter, receiver, and communications transmitter simultaneously. In such scenarios, radar is regarded as the primary function and communications as the secondary one, sharing the power and bandwidth of the radar system [4].

Since DFRC systems implement both radar and communications using a single hardware device, these functionalities inherently share some of the system resources, such as spectrum, antennas, and power. To facilitate their coexistence, many different DFRC approaches have been proposed in the literature. In a single antenna radar or traditional phased array radar that transmits a single waveform, a common scheme is to utilize the communication signal as the radar probing waveform [5]. Such dual-function waveforms include phase modulation, as well as orthogonal frequency division multiplexing (OFDM) signaling [5], [6]. The design of such waveforms to fit a given beam pattern was studied in [7]. However, this approach may reduce radar performance [4], [8]. Furthermore, transmitting non-constant modulus communication waveforms may result in low power efficiency when using non-linear amplifiers.

Another common DFRC approach is to utilize different signals for radar and communications, designing the functionalities to coexist by mitigating their cross interference. Multiple-input multiple-output (MIMO) radar systems in which a subset of the antenna array is allocated to radar and the rest to communications were studied in [8], along with the setup in which both functionalities utilize all the antennas. Methods for treating the effect of spectrally interfering separate radar and communications systems were studied in [9], [10]. Coexistence in MIMO DFRC systems can be realized using beamforming, namely, by generating multiple beams with different waveforms towards radar targets and communication users at diverse directions [11]. The drawback of this approach, particularly when radar is the primary functionality, is that communications interferes with radar, either via spectral interference, power sharing, or by reducing the number of available antennas, resulting in an inherent tradeoff between radar and communication performance [12].

Frequency agile radar (FAR) is a radar scheme which randomly varies the carrier frequencies from pulse to pulse [13]. Frequency agility results in excellent EMC performance [13], spectral efficiency [14], and is able to synthesize an effectively large bandwidth while using a small instantaneous bandwidth [15]. Furthermore, the inherent randomness of FAR can be utilized to convey information using index modulation, which is the focus of current work.

In this work we consider a phased array antenna, extending the standard FAR architecture to a multi-carrier agile phased array radar (MAPAR). We then show how the MAPAR approach is capable of conveying information to a remote receiver using index modulation. Index modulation methods use the indices of the building blocks (e.g., frequencies and/or antennas) to convey additional information bits [16]. Integrating index modulation into a radar system leads to high spectral and energy efficient DFRC system, without degrading radar performance. Unlike previous DFRC systems [5]–[9], which use dedicated independent waveforms and/or antennas for communication, in our proposed MAPAR the ability to convey information is an inherent byproduct of the radar scheme. Consequently, communication is naturally obtained from the radar design, eliminating cross interference.

We analyze the individual performance of both radar and
communications using MAPAR. We first characterize the beam pattern of the radar waveform, which uses constant modulus signals. We show that the beam pattern exhibits only a slight increase in the sidelobe and grating lobe levels compared to that achievable using a full-band waveform, and demonstrate that the mean beam pattern of the proposed waveform is identical to that of the latter waveform. For the communications functionality, we show that the maximal number of bits which can be conveyed in each pulse grows linearly with the number of transmit antennas and logarithmically with the number of available carrier frequencies. We numerically evaluate the proposed DFRC system, demonstrating its ability to achieve wideband mean beam pattern. We also numerically demonstrate that the proposed MAPAR achieves comparable communication rates to using dedicated communication antennas, without affecting the radar performance and resources.

The rest of the paper is organized as follows. Section II presents the proposed MAPAR. Section III studies the radar performance, while Section IV is devoted to communication analysis. Numerical results are provided in Section V.

Throughout the paper we use $\mathbb{C}$, $\mathbb{R}$ to denote the sets of complex and real numbers, respectively. We use $| \cdot |$ for the magnitude or cardinality of a scalar value or a set, respectively. Uppercase and lowercase boldface letters denote matrices and vectors, respectively. The stochastic expectation, complex conjugate, transpose, and Hermitian transpose are denoted by $\mathbb{E}[\cdot]$, $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$, respectively.

II. Multi-Carrier Agile Phased Array Radar

In this work, we propose a MAPAR system, which is capable of conveying information to a remote receiver. The concept of multi-carrier agile radar [17] extends FAR [13] by increasing the number of frequencies utilized simultaneously, thus improving the range-Doppler recovery performance. In particular, our proposed MAPAR extends the scheme of [17] to use different sub-arrays for selected frequencies, while avoiding the envelop fluctuation problem of multiband signals while allowing to embed information bits into the radar waveform. Thus, we first review FAR in Subsection II-A, and present the radar and communication schemes in Subsections II-B- II-C.

A. Preliminaries of FAR

FAR [13] is a technique for enhancing the EMC performance of radar systems. The original formulation of FAR changes the frequencies from pulse to pulse [13]. To formulate FAR, define the set of available carrier frequencies via

$$\mathcal{F} := \{ f_c + m\Delta f | m = 0, 1, \ldots, M-1 \}, \quad (1)$$

where $f_c$ is the initial carrier frequency, $\Delta f$ is the frequency step, and $M$ is the number of available frequencies. Let $f_n \in \mathcal{F}$ denote the carrier frequency of the $n$th pulse. Radar pulses are repeatedly transmitted, starting from time instance $nT_r$ to $(n+1)T_r$, $n = 0, 1, \ldots, N-1$, where $T_r$ and $T_p$ represent the pulse repetition interval and pulse duration, respectively. $T_r > T_p$. The transmitted pulse at time index $t$ can be written as $a_n(t) = \frac{t-nT_r}{T_p} e^{j2\pi f_c(t-nT_r)}$, where $a_n(\cdot)$ is the baseband signal for the $n$-th pulse. To obtain high power efficiency, constant modulus waveforms are preferred as radar base-band signals. For simplicity, in the sequel we use the rectangular envelope pulse signal, $a_n(t) = \text{rect}(t)$, where $\text{rect}(t) = 1$ for $t \in [0, 1)$ and zero otherwise. In FAR, $f_n$ is randomized in each pulse. Typically, $\{f_n\}$ is a set of i.i.d. random variables.

In comparison with a wideband radar that transmits and receives all bands in $\mathcal{F}$, the agility in carrier frequencies simplifies the hardware system by replacing the instantaneous wideband transceiver with a narrowband one, and enhances the survivability of FAR in complex electromagnetic environments at the cost of reducing the number of radar measurements in the frequency domain. To increase the spectral resolution, FAR was recently extended in [17] to utilize multiple carriers simultaneously instead of a monotone. This extension requires large instantaneous bandwidth and leads to high envelope fluctuation and low amplifier efficiency. To overcome these issues, we next introduce MAPAR.

B. The Proposed Radar Scheme

Here, we introduce MAPAR, which is a DFRC scheme building upon the concepts of FAR and multi-carrier agile radar. We consider a radar system equipped with $L_A$ antenna elements, uniformly located on the antenna array with distance $d$ between two adjacent elements. In the $n$-th pulse of the MAPAR, multiple carrier frequencies, denoted by the set $\Omega_n$, are randomly selected from $\mathcal{F}$, $\Omega_n \subset \mathcal{F}$. We assume that the cardinality $|\Omega_n|$ is constant, $|\Omega_n| = K$, and write the elements of this set as $\Omega_n = \{ \Omega_{n,0}, \ldots, \Omega_{n,K-1} \}$. A sub-array is allocated for each frequency, and all antenna array elements are utilized for transmission. Denote by $f_{n,l} \in \Omega_n$ the frequency used by the $l$-th antenna array element, whose waveform when transmitting the $n$-th pulse is expressed as $\phi(f_{n,l}, t-nT_r)$, where $\phi(f, t) := \text{rect}(t/T_p) e^{j2\pi f t}$. In order to direct the antenna beam towards a desired angle $\theta$, the signal transmitted is weighted by the function $w(t, f_{n,l}) \in \mathbb{C}$, which is set to [18] $w(t, f_{n,l}) = e^{j2\pi f_{n,l} t \sin \theta / c}$, where $c$ is the speed of light. The transmission of the $l$th element is thus

$$[\mathbf{x}(n,t)]_l = w_l(\theta, f_{n,l}) \phi(f_{n,l}, t-nT_r). \quad (2)$$

The vector $\mathbf{x}(n,t) \in \mathbb{C}^{L_A}$ in (2) denotes the transmission of the full array. Conventional FAR with a phased array antenna can be considered as a special case with $K = 1$ and $f_{n,l}$ independent of $l$. The transmitted signal (2) can also be expressed by grouping the array elements which use the same frequency $\Omega_{n,k}$, $k = 0, \ldots, K-1$. Let $\mathbf{x}_k(n,t) \in \mathbb{C}^{L_A}$ represent the portion of $\mathbf{x}(n,t)$, which utilizes $\Omega_{n,k}$, i.e.,

$$\mathbf{x}(n,t) = \sum_{k=0}^{K-1} \mathbf{x}_k(n,t). \quad (3)$$

where $\mathbf{P}(n,k) = \{0, 1 \}^{L_A \times L_R}$ is a diagonal selection matrix with diagonal $\mathbf{p}(n,k) \in \{0, 1 \}^{L_A}$, whose entries equal one for the $l$-th array element using $\Omega_{n,k}$ and zero otherwise, i.e., $[\mathbf{P}(n,k)]_{l,l} = [\mathbf{p}(n,k)]_{l,l} = 1$ when $[\mathbf{x}_k(n,t)]_{l} \neq 0$.

When receiving the $n$th returned pulse, each antenna element only receives signals at its corresponding frequency $f_{n,l}$, operating as a narrowband radar receiver. In Section III we analyze the performance of this radar scheme, showing that,
while it requires a simple narrowband hardware, it achieves the same mean beam pattern as costly wideband radar.

C. MAPAR as a DFRC System

The inherent randomness in the selection of carrier frequencies and their allocation among the transmit antennas can be exploited to convey information in the form of index modulation. Index modulation refers to the embedding of information bits through indexes of parameters involved in the transmission [16], such as OFDM subcarrier indexes [19] or MIMO antenna selection [16]. Our proposed MAPAR implements a DFRC system by embedding bits in the index corresponding to a set of carrier frequencies, and in the permutation of the selected frequencies among the antennas.

The proposed information embedding method is applied identically on each pulse. To formulate the embedding method, we only consider a single pulse. Accordingly, we simplify our notations as follows: \( \Omega := \Omega_n \), \( P_k := P(n, k) \), \( p_k := p(n, k) \), \( x(t) := x(n, t) \), \( x_k(t) := x_k(n, t) \), and \( w_k := w(\theta, \Omega_n, k) \).

Before transmitting the dual function waveform, MAPAR first selects frequencies and then allocates array elements to each frequency. In particular, \( K \) out of \( M \) frequencies in \( \mathcal{F} \) are used for each pulse, and each selection represents a different codeword. The set of possible frequency selections is

\[
\mathcal{U} := \left\{ \Omega^{(i)} \mid \Omega^{(i)} \subset \mathcal{F}, i = 0, 1, 2, \ldots \right\},
\]

where the superscript (i) stands for the i-th codeword in the set \( \mathcal{U} \). The number of codewords is \( |\mathcal{U}| = \binom{M}{K} = \frac{M!}{K!(M-K)!} \).

Once the carrier frequencies are selected, each antenna element uses a single frequency to transmit its monotone beam pattern introduced by a wideband phased array radar, since the inherent randomness in the selection of carrier frequencies among the antennas.

Let \( I \) denote the diagonal selection matrices \( L = P_k \), indicating the theoretical benefits of using MAPAR with large-scale antenna arrays.

A. Received Radar Signal Model

In order to formulate the received radar signal, the location of the target from which the waveform is reflected must be specified. We consider a single ideal scattering point at direction \( \theta_s \), range \( r_s \) with unit scattering coefficient. Under the far-field assumption, the radar signal that reaches the scattering point is a summation of transmissions from all the antenna array elements. By defining \( \tilde{v}_P := w(\theta, \Omega_n, k) \in \mathbb{C}^{L_n} \), i.e., \( \tilde{v}_P = e^{j2\pi\Omega_n,k \cdot \ldots} \), letting \( t_p = r_s/c \) denote the propagation delay, and recalling that \( P_k \) represents the sub-array used for acquiring the k-th the frequency band, the reflected signal acquired by the radar receiver can be expressed as

\[
y_S(t) = \sum_{k=0}^{K-1} P_k \tilde{v}_P^* \tilde{v}_P^H x_k(t - 2t_p).
\]

The received signal is sampled at time instances \( t = T_p, 2T_p, \ldots, [T_r/T_p]T_p \), which correspond to different range cells, respectively. The discrete received vector is given by

\[
y_S := \sum_{k=0}^{K-1} P_k \tilde{v}_P^* \tilde{v}_P^H P_k \tilde{v}_P e^{-j4\pi\Omega_n,k(r/c-t_p)} \in \mathbb{C}^{L_n},
\]

where \( t_p := \lfloor t_p/T_p \rfloor T_p \). The vector (7) is digitally processed for target detection, and is used to analyze radar performance.

B. Beam Pattern

As shown in (6), the received radar echoes depend on both range and direction. The beam pattern corresponding to \( (\theta_s, r_s) \) is defined as the normalized inner product between the received signal \( y_S \) and the reference \( y_{ref} \).

\[
y_{ref}^H y_S / ||y_{ref}||_2 / ||y_S||_2, \text{ where the reference } y_{ref} \text{ is the radar return from a hypothetical scattering point at direction } \theta \text{ and range } r.
\]

Substituting \( \Theta = \theta \) and \( \Theta = r \) into (7) yields

\[
y_{ref} = \sum_{k=0}^{K-1} P_k \tilde{v}_P^* \tilde{v}_P^H P_k \tilde{v}_P e^{-j4\pi\Omega_n,k(r/c-t_p)}.
\]

Then, the beam pattern becomes

\[
\frac{y_{ref}^H y_S}{||y_{ref}||_2} \approx \frac{1}{L_R} \sum_{k=0}^{K-1} \sum_{l=0}^{M-1} \tilde{v}_P^T P_k \tilde{v}_P^H P_k \tilde{v}_P e^{-j4\pi\Omega_n,k(c-r_s)}.
\]

When the bandwidth is fairly small compared to the carrier frequency, i.e., \( M\Delta f \ll f_c \), the steering vector is approximately frequency invariant, thus \( e^{j4\pi\Omega_n,k \cdot \ldots} \approx e^{j4\pi f_c \cdot \ldots} \) [18]. In such cases, it can be shown that (9) depends only on the difference of the direction sines and ranges, i.e., \( \sin(\theta - \sin(\theta_s)) \) and \( r - r_s \), instead of the actual values. By defining \( a := 2f_c (\sin(\theta - \sin(\theta_s)) / c \) and \( b := 2\Delta f (r - r_s) / c \), the beam pattern can be written as a function of \( a, b \): \n
\[
\chi(a, b) := \frac{1}{L_R} \sum_{k=0}^{K-1} \sum_{l=0}^{M-1} \tilde{v}_P^T e^{j2\pi\Omega_n,k \cdot \ldots} e^{j2\pi f_c \cdot \ldots}.
\]

By repeating the above steps, one can obtain the ideal beam pattern introduced by a wideband phased array radar, in which each antenna element transmits all the M sub-bands simultaneously. The resulting absolute value of the optimal beam pattern can be shown to be given by

\[
|\chi_{opt}(a, b)| = \frac{1}{M} \left| \frac{\sin(\pi a L_R)}{\sin(\pi a)} \frac{\sin(\pi b M)}{\sin(\pi b)} \right|.
\]
The difficulty in comparing the magnitude of \((10)\) to the optimal value in \((11)\) stems from the fact that, due to the randomness of MAPAR, the beam pattern \((10)\) is a random quantity. In Section V we numerically show that the instantaneous beam pattern \((10)\) exhibits increased sidelobe and grating lobe levels in both range and angle domains. However, the mean beam pattern coincides with \((11)\), as stated in the proposition 1, given without proof due to space limitations:

**Proposition 1.** If the codewords are uniformly distributed over \(\mathcal{D} \times \mathcal{P}\), then the mean beam pattern of MAPAR is given by

\[
E[\chi(a, b)] = |\chi_{\text{opt}}(a, b)| e^{j\pi(a(L_R - 1) + b(M - 1))}.
\]

The envelope of \((12)\) is identical to \((11)\), indicating that when the number of pulses is sufficiently large such that the average beam pattern approaches its ergodic value, the performance of MAPAR approaches that of a wideband phased array radar, while the system complexity is significantly reduced.

IV. COMMUNICATION ANALYSIS

We next analyze the communication functionality of the DFRC system. We first model the received signal in Subsection IV-A, and characterize the achievable rate in Subsection IV-B.

A. Received Communication Signal Model

Let \(L_C\) be the number of antennas of the receiver. We consider a memoryless additive white Gaussian noise channel. Here, the channel output observed by the receiver is given by

\[
y_C(t) = \sum_{k=0}^{K-1}HX_k(t) + n_C(t),
\]

where \(n_C(t) \in \mathbb{C}^{L_C}\) is additive noise and \(H \in \mathbb{C}^{L_C \times L_R}\) is the channel matrix between DFRC system and the receiver.

After down-conversion with \(e^{-j2\pi f_T t}\), each receiver samples the signal at time instances \(iT_T\), where \(T_T\) is the sampling interval, and \(i = 0, 1, \ldots, \lfloor T_p/T_T \rfloor\), resulting in \(L_T := \lfloor T_p/T_T \rfloor + 1\) outputs per pulse. By letting \(Y_C, N_C \in \mathbb{C}^{L_C \times L_T}\) denote the sampled channel output and noise, respectively, the received signal corresponding to a single pulse is given by

\[
Y_C = \sum_{k=0}^{K-1}HP_kw_k\psi_{c_k}^T + N_C.
\]

In \((14)\), we define \(c_k := (\Omega_{n,k} - f_c)/\Delta f \in \{0, \ldots, M - 1\}\) as the frequency code corresponding to \(\Omega_{n,k}\), and \(\psi_{c_k} := [1, e^{j2\pi c_k\Delta f T_T}, \ldots, e^{j2\pi (M-1)c_k\Delta f T_T}]^T \in \mathbb{C}^{L_T}\) as the baseband signal corresponding to the frequency code \(c_k\). We assume that the receiver knows the channel \(H\), the number of frequencies \(K\), and the steering vectors \(\{w_k\}\). Under the above signal model, we study the achievable rate and low-complexity designs in the sequel.

B. Achievable Rate Analysis

In order to analyze the proposed communication scheme, we first characterize its achievable rate, namely, the maximal number of bits which can be reliably conveyed to the receiver at a given noise level in each pulse. To facilitate the analysis, we assume that each discrete-time channel output represents a single pulse, i.e., \(L_T = 1\). It is emphasized that the following analysis can also be extended any positive integer value of \(L_T\). Under this model, for each pulse, \((14)\) is given by

\[
y_C = Hx + n_C,
\]

where \(x = \sum_{k=0}^{K-1}P_kw_k\), and \(n_C\) is additive white Gaussian noise with covariance \(\sigma^2I_{L_C}\), independent of \(x\).

Based on the transmission scheme detailed in Subsection II-C, we define a set \(X \subset \mathbb{C}^{L_C}\) that contains all the possible transmitted signal vectors \(x\), whose carnality is \(|X| = \pi(M-1)\Delta f T_T/2\) and assume that \(x\) is uniformly distributed over \(X\). The channel output \(y_C\) thus obeys a Gaussian mixture (GM) distribution with equal priors. In particular, by letting \(f_{G_{L_C}}(u; m, C)\) denote the probability density function (PDF) of an \(L_C \times 1\) proper-complex Gaussian vector with mean \(m\) and covariance matrix \(C\), evaluated at \(u\), the PDF of \(y_C\) is

\[
\begin{align*}
&f_{y_C}(u) = \frac{1}{|X|} \sum_{x \in X} f_{G_{L_C}}(u; Hx(i), \sigma^2I_{L_C}).
\end{align*}
\]

Using the input-output relationship of the channel, we can characterize the achievable rate. Let \(h(\cdot)\) denote the differential entropy. Since the channel \((15)\) is memoryless and the noise is proper complex Gaussian, its achievable rate is given by

\[
R_C = h(y_C) = L_C \cdot \log_2 (\pi \cdot \sigma^2).
\]

In order to evaluate \((17)\), one has to compute the differential entropy of \(y_C\). While there is no closed-form analytic expression for the differential entropy of GM random vectors [22], the following lower bound on \((17)\) can be obtained:

**Proposition 2.** The achievable rate of the proposed communication scheme is lower bounded by

\[
R_C \geq -\frac{1}{|X|} \sum_{x(i) \in X} \log_2 f_{y_C}(Hx(i)) - L_C \cdot \log_2 (\pi \cdot \sigma^2).
\]

**Proof:** The proposition follows from [22, Thm. 2].

A trivial upper bound on \(R_C\) is obtained by noting that \(x\) is uniformly distributed over the discrete set \(X\), thus, \(R_C \leq h(x) = \log_2 |X|\). This upper bound implies that the number of bits conveyed on each pulse cannot be larger than the number of bits needed for representing the different codewords.

V. SIMULATIONS

In this section we numerically evaluate the performance of the radar and communication functions. We begin with the fundamental limits of the proposed system, compared to using different waveforms for communication and radar.

We consider a \(4 \times 4\) channel, i.e., \(L_R = L_C = 4\), where the channel matrix \(H\) models spatial exponential decay, i.e., \([H]_{ij} = e^{-|i-j|}h\). The parameters of the proposed system are set to \(f_c = 1.9\ GHz\), \(\Delta f = 10\ MHz\), \(\theta = \pi/4\), \(d = 10\frac{\lambda}{2}\), the number of available frequencies is \(M = 10\), and the number of frequencies utilized at each pulse is \(K = 2\). The overall number of codewords here is \(|X| = 270\), i.e., the maximal number of bits per pulse is \(\log_2 |X| \approx 8.1\). In Fig. 1 we depict the lower bound on the achievable rate of Proposition 2 versus SNR \(\hat{\Delta} = 1/\sigma^2\), compared to the rate achievable when one of the antennas is dedicated only for communications, neglecting the cross interference between radar and communications. Since all channels here observe the same SNR, the selection of the antenna does not affect the rate.
Fig. 1. Achievable rate comparison.

Observing Fig. 1, we note that in relatively low SNR, our proposed scheme is capable of achieving higher rates compared to using a dedicated communications antenna without impairing the radar performance. As the SNR increases, using a dedicated waveform outperforms our proposed system as more bits can be reliably conveyed in a single channel symbol. However, it is emphasized that by allocating an antenna element for communications, the radar performance, which is considered as the primary user in our case, is degraded. Furthermore, in order to avoid coexistence issues, which we did not consider here, the communications and radar signals should be orthogonal, e.g., use distinct bands, thus reducing the radar bandwidth. Finally, the computation of the achievable rate with a dedicated antenna does not account for the need to utilize constant modulus waveforms, and is in fact achievable using Gaussian signaling. The fact that, in addition to the practical benefits of our proposed scheme, it is also capable of achieving rates comparable to using dedicated communication waveforms, illustrates the gains of our proposed DFRC system.

We next numerically evaluate the beam pattern of MAPAR. In particular, we consider a system with $M = 7$ sub-bands and $L_R = 8$ transmit antennas. In each pulse, $K = 2$ sub-bands are transmitted. The beam pattern is calculated according to (10), and the expectation is obtained using (12). An example for a realization of the beam pattern with a randomly selected $p_k$ is shown in Fig. 2a, while its expectation $|E[\chi(a,b)]|$ is depicted in Fig. 2b. The beam pattern in Fig. 2a is range angular dependent, and has a noise-like sidelobe and grating lobe. Next, we observe the expected beam pattern depicted in Fig. 2b, which governs the radar performance when a large number of independent pulses are transmitted. It is noted that its resolutions in terms of normalized range and angular are close to $\frac{1}{R_R}$ and $\frac{1}{\gamma}$, respectively, which are the same with the wideband phased array radar that simultaneously transmits all sub-bands using each single antenna element.

VI. CONCLUSIONS

In this paper, we proposed MAPAR - a DFRC system which combines frequency and spatial agility. MAPAR exploits its inherent randomness to convey information to a remote receiver using index modulation without introducing coexistence issues. The radar scheme was shown to approach the beam pattern of wideband phased array radar in the mean sense while operating with monotone waveforms. The achievable rate was shown to be comparable to that of dedicated communication waveforms without interfering with radar.

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