Recombination induced thermodynamic Gaussian cosmological baryonic fluctuations.

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ABSTRACT
In some instances, e.g. near phase transitions, thermodynamic fluctuations become macroscopically relevant, and relative amplitudes grow far above the standard $N^{-1/2}$ scale, with $N$ the number of particles. Such large fluctuations are characterised by a scale invariant Gaussian power spectrum. In this letter I show that the abrupt drop in the baryonic sound speed across recombination leads to conditions resulting in such large thermodynamic Gaussian fluctuations in the ionisation fraction of the baryons. Under pressure equilibrium, this will result in a mechanism for generating scale invariant density and temperature fluctuations in the baryonic component, inherent to the thermodynamics of the baryons themselves. Within a ΛCDM framework, this extra random fluctuation source leads to a decoupling of the inflationary relic small wave number spectrum and the amplitude of the Gaussian random fluctuations at frequencies higher than the first acoustic peak, an effect which could explain the mismatch between cosmic microwave background (CMB) inferences and local kinetic determinations of the Hubble constant. Within modified gravity theories in absence of dark matter, the mechanism proposed serves as a source for random Gaussian density fluctuations in the acoustic peak region.

Key words: cosmology:theory — cosmic background radiation.

1 INTRODUCTION
The random Gaussian temperature fluctuations in the cosmic microwave background (CMB) first detected by the COBE satellite in the early 1990s (Smoot et al. 1992), have now been extensively studied and characterised by subsequent space observatories, in particular the WMAP (Bennett et al. 2003) and Plank missions (e.g. Plank Collaboration 2016). It is well established that a random component having a very close to Gaussian power spectrum is present at around the recombination redshift, $z_{rec}$. Assuming an adiabatic equation of state for the baryonic component implies a corresponding density fluctuation spectrum for the baryons. Given the tight coupling between baryons and photons prior to recombination, when the ionisation fraction was close to unity and the effective sound speed was of $(v_s/c)^2 \approx 1/3$, the corresponding pressure ensures the rapid erasing of any baryonic fluctuations at $z > z_{rec}$. Their presence at the last scattering surface is generally understood in the context of a gravitational coupling of the baryonic component having a Gaussian fluctuation spectrum generated during an inflationary phase, and which survives until $z_{rec}$ by virtue of having no coupling to the radiation field beyond gravitational terms.

It is interesting that thermodynamic fluctuations have Gaussian power spectra and a scale invariant character. In some cases, e.g. near phase transitions, anomalous thermodynamic fluctuations grow significantly, to far exceed the usual $N^{-1/2}$ scaling in the relative amplitudes of standard thermodynamic fluctuations, to become macroscopically relevant. By considering the relevant $\Delta \phi$ potential of ionisation fluctuations in the baryonic component near recombination, in this letter I show that a mechanism for generating Gaussian density fluctuations appears associated to the very rapid drop in baryonic sound speed across recombination under pressure equilibrium, inherent to purely baryonic thermodynamical processes.

Previous studies have looked at ionisation fraction fluctuations either in the context of hydrodynamical acoustic instabilities due to coupling of the baryons to the CMB e.g. Shaviv (1998) or Liu et al. (2001) and Singh & Ma (2002) including a three level atom approximation, or ionisation fraction fluctuations resulting from the baryon density fluctuations and the dependence of recombination and ionisation rates on density, Novosyadlyj (2006). In all such studies, the resulting ionisation fluctuations are shown to be negligible.

The appearance of a further fluctuation production mechanism around recombination effectively decouples the baryonic power spectrum in the acoustic peak region from
that observed at shorter wave numbers, thought to bear
the imprint and normalisation of physics at the inflationary
epoch at scales beyond the causal horizon at \( z_c \). This extra
parameter in the modeling of CMB fluctuations might help
to alleviate the present offset of over 3σ between CMB infer-
ences and more local expansion probes in the determination
of the Hubble parameter (e.g. Riess et al. 2016, Casertano et
al. 2017), through allowing for changes in the normalisation
within the acoustic peak region.

Within the context of modified gravity theories not in-
cluding the dark matter hypothesis currently being explored,
e.g. MOND of Milgrom (1983), F(R) modifications such as
Mendoza et al. (2013) or variants reviewed in Capozziello &
de Laurentis (2011), the emergent gravity proposal of Ver-
linde (2016) or the constant bounding curvature criterion of
Hernandez et al. (2017), the requirement for a mechanism
to explain the random component of observed temperature
fluctuations in the CMB becomes critical.

The precise power spectrum before the appearance of
the resonant acoustic peaks will depend on the details of the
gravitational forcing term, either through a standard dark
matter potential, or an enhanced baryonic modified grav-
tational forcing term, either through a standard dark

2 IONISATION FLUCTUATIONS UNDER
PRESSURE EQUILIBRIUM

In general, thermodynamic fluctuations will occur with a
probability, \( \omega \), given by:

\[
\omega = e^{-\Delta \phi/kT},
\]

where \( \Delta \phi \) is the energy associated with the fluctuation be-
ing analysed, \( k \) gives the Boltzmann constant and \( T \) is the
temperature of the system. As it is well known, when ap-
proaching a phase transition, the flatness of the relevant
potential leads the effective \( \Delta \phi \) to tend to zero, and hence
the probability of the fluctuations appearing tends to unity.

This yields the observed anomalous growth of fluctuations
to many orders of magnitude above the usual \( N^{-1/2} \) scale on
approaching critical points, e.g. the Ginsburg-Landau the-
y. Although cosmological recombination is not strictly a
phase transition, the extremely abrupt nature of the pro-
cess, which can be described through a \( x \propto (1 + z)^{12.75} \)
scaling, as first estimated by Jones & Wise (1985), leads to
the expectation of extremely flat potentials associated with
fluctuations in \( x \), the ionisation fraction. Should a \( \Delta \phi = 0 \)
phase appear, we would expect the appearance of large bary-
onic fluctuations near the recombination epoch, intrinsic to
purely baryonic thermodynamics.

If at some point during the recombination epoch a ther-

modynamic fluctuation occurs such that the unperturbed
background yields a small amount of energy to a certain re-
gion resulting in a change of the ionisation fraction within
this region of \( x \rightarrow x + \Delta x \), the ensuing enhanced coupled
to the photon background will result in a slightly enhanced
pressure within this region of \( P \rightarrow P + \Delta P \). Imposing pres-
sure equilibrium implies that the slightly over ionised region
will expand against the background a little, and undergo a
change in its volume of \( V \rightarrow V + \Delta V \). The net energy vari-
ation of the region in question to first order in the fluctuation
will now be:

\[
\Delta \phi = N \Delta x \epsilon_i - P \Delta V.
\]

In the above \( \epsilon_i \) is the 13.6 ev of the ionisation potential of
hydrogen, \( P \) gives the pressure of the background and \( N \)
the number of atoms of hydrogen in the region in question.

We can eliminate \( \Delta V \) in the above equation in favour of \( M, \rho \)
and \( \Delta \rho \), the total mass, density and density fluctuation
of the region in question, through deriving \( \rho = M/V \) at constant
mass:

\[
\Delta V = -\frac{M}{\rho^2} \Delta \rho.
\]

If we now substitute \( M = N m_p \), approximating the total
mass of the perturbed region as the number of protons it
contains, equation (2) reads:

\[
\Delta \phi = N \Delta x \epsilon_i + N m_p \frac{\Delta \rho}{\rho^2}.
\]

Notice that a positive \( \Delta x \) fluctuation will result in a positive
\( \Delta \rho \) fluctuation, expansion and hence a dilution, a negative
\( \Delta \rho \) fluctuation hence allowing for a \( \Delta \phi = 0 \) point in the
above relation. This is in fact the trend found in Vennum-
hav & Hirata (2015), where positive \( x \) fluctuations corre-
spond to rarefactions in the density field. Such a \( \Delta \phi = 0 \)
condition will be met provided:

\[
\frac{\epsilon_i}{m_p} = -\frac{P}{\rho^2} \frac{\partial \rho}{\partial x},
\]

where I have taken the limit in the change in density of the
fluctuation resulting from changes in the ionisation fraction
only. Since \( N \) has cancelled out, fluctuations of all scales will
appear if the above equation is satisfied. A symmetric situa-
tion appears for negative fluctuations, which will appear as
both enhancements and drops with respect to the mean ion-
isation level. Indeed, it is a generic feature of thermodynamic
fluctuations near critical points that a scale invariant Gauss-
ian power spectrum of fluctuations results, e.g. Landau &
Lifshitz (1980).

We can now divide both sides of the previous equation
by the square of the speed of light and approximate the
adiabatic sound speed of the baryons as \( v_s^2 = P/\rho \) such that
the \( \Delta \phi = 0 \) condition yields:

\[
\frac{\epsilon_i}{m_p c^2} = -\left( \frac{v_s}{c} \right)^2 \frac{1}{\rho} \frac{\partial \rho}{\partial x}.
\]

The left hand side of the above clearly dimensionless rela-
tion has a constant value given by dividing the 13.6 ev of
the ionisation potential of hydrogen by the 938 MeV of the
proton mass, which gives \( 1.45 \times 10^{-8} \). Whilst the left hand
side of equation (6) is clearly constant, the right hand side
changes dramatically across the recombination region, where
the \( (v_s/c)^2 \) term changes from close to 1/3 at the beginning

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of recombination to about $10^{-10}$ towards the end where $v_s$ settles to close to 3 km/s e.g. Longair (2008).

We can now estimate $\partial \rho / \partial x$ as usual from the background properties, by taking the total derivative of $\rho$ as the sum of its partial derivatives, $d\rho = (\partial \rho / \partial x) dx + (\partial \rho / \partial z) dz$ and dividing by $dx$ to obtain:

$$\frac{\partial \rho}{\partial x} = \frac{d\rho}{dx} \frac{\partial z}{\partial x}$$

(7)

Given the small total change in density and the large total change in $x$ over recombination, we can neglect the first term on the right hand side of the above relation to get the $\Delta \phi = 0$ condition as:

$$1.45 \times 10^{-8} = \left( \frac{v_s}{c} \right)^2 \frac{3 \, dz}{z \, dx}$$

(8)

where in the above, since all this is occurring at redshift values of $z \approx 1000$, we have taken the baryon density as $\rho = \rho_0 (1 + z)^3 \approx \rho_0 z^3$.

Although during the rapidly evolving phase of the recombination process $x$ scales with $(1 + z)^{12.73}$ (Jones & Wyse 1985), we can consider a redshift, $z_i$, at the start of the process where the ionisation fraction begins to drop and a behaviour close to $x = z_i^{n_f}$ will appear, with $z_i = z/1000$ and $0 \leq n_i \leq 1$, whilst towards the end of recombination, where the ionisation fraction begins to settle to the residual value of about $10^{-3}$ a behaviour close to $x = 10^{-3} + z_f^{n_f}$ with $n_f \geq 1$ will appear. A schematic representation of the above is shown in figure (1).

Thus, it is reasonable to expect that around both $z_i$ and $z_f$, $(dz/dx)$ will be of order unity. With this last condition we can estimate the value of the right hand side of equation (8) both around $z_i$ and $z_f$. Taking $v_s(z_i) \approx 3^{1/2} c$, $v_s(z_f) \approx 3 \, km/s$ and $z_i \approx z_f \approx 1000$ give for the right hand side of equation (8) at $z = z_i$ a value of $3 \times 10^{-3}$, five orders of magnitude greater than the left hand side of that relation. Similarly for $z = z_f$ we get a value of $3 \times 10^{-13}$ for the right hand side of equation (8), this time five orders of magnitude below the constant value of the left hand side of this equation. Thus, although the $\Delta \phi = 0$ condition is not met, by many orders of magnitude, neither at the start nor at the end of the recombination period, the clear change from the right hand side of equation (8) dominating near $z_i$ to it being the left hand side of that equation which dominates towards $z_f$, implies by the intermediate value theorem, that there will necessarily be at least one point where the condition $\Delta \phi = 0$ is satisfied for a critical $z_c$ such that $z_c < z_i < z_f$.

Notice also that although a number of approximations have been introduced (beyond the ones already explicitly mentioned, including the rich atomic physics phenomenology beyond ground-level recombination, e.g. the three-level approximation of hydrogen and helium atoms -Matsuda et al. 1971, Krolik 1990 or Hummer & Storey 1998- or the multilevel structure considered by e.g. Seager et al. 2000), the overwhelming difference between the left and right sides of equation (8), of five orders of magnitude, and in opposite directions, near the start and end of recombination makes it inevitable that the condition $\Delta \phi = 0$ will be met at some point during the recombination process. At that point, anomalous fluctuations in the ionisation fraction will appear, accompanied by corresponding density fluctuations in the baryonic density field of:

$$\frac{\Delta \rho}{\rho} = \frac{\epsilon_i \rho}{m_p P} \Delta x,$$

as implied by equation (5), or within the $v_s^2 = P/\rho$ approximation,

$$\frac{\Delta \rho}{\rho} = \frac{\epsilon_i}{m_p c^2} \left( \frac{c}{v_s} \right)^2 x \left( \frac{\Delta x}{x} \right)^2,$$

(10)

Towards $z_f$ where $x \approx 10^{-3}$ and $v_s \approx 3 \, km/s$, obtaining $\Delta \rho / \rho \approx 10^{-5}$ requires only $\Delta x / x \approx 10^{-4}$, i.e. $x = 10^{-3} \pm 10^{-7}$, extremely small ionisation fluctuations. Under the usual adiabatic assumption, these density fluctuations will give rise to the observed Gaussian component in the temperature fluctuations in the CMB, of the same order.

It is clear that in the development presented the energy fluctuation giving rise to $x \rightarrow x + \Delta x$ and the resulting volume increase of $V \rightarrow V + \Delta V$ do not occur sequentially, but adiabatically and simultaneously. The efficiency of the mechanism proposed will hence sensitively depend on the rate at which the $\Delta \phi = 0$ condition is crossed, being this efficiency maximal for a very tangential and gradual crossing which allows time for the fluctuations described to develop at various scales. If, on the other hand, the $\Delta \phi = 0$ condition is only very briefly met, then the fluctuations described will only begin to develop at the smallest scales.

3 CONCLUSIONS

I have shown that in a process akin to the growth of fluctuations in the vicinity of phase transitions, near the extremely fast phase of cosmological recombination, the relevant potential for ionisation fraction fluctuations under pressure equilibrium will have a zero point and hence lead to significant thermodynamic fluctuations in the baryonic density field. As a generic feature of thermodynamic fluctuations, these will be characterised by a scale invariant Gaussian power spectrum, as it is inferred from satellite observations of the CMB. Baryonic thermodynamical processes during

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recombination lead to naturally arising random fluctuations of the type observed in the CMB.

Within the context of a GR ΛCDM cosmology, this constitutes a further mechanism for producing baryonic fluctuations close to the surface of last scattering. Within the context of modified gravity theories modelling the universe without the dark matter hypothesis, the mechanism presented here allows for an understanding of the small temperature fluctuations detected in the CMB, in the absence of a hypothetical dominant dark matter component carrying the imprint of a remote inflationary phase in the form of density fluctuations shielded from photon damping by the assumed lack of interaction between dark matter and radiation.

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