Study of the couplings of QED and QCD from the Adler function

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\( \bar{\alpha}_\mu^{\text{HLO}}(Q^2_{\text{ref}}) \) is dominated by the low \( Q^2 \) region: noisy and long-distance contributions

\[
\bar{\alpha}_\mu^{\text{HLO}}(Q^2_{\text{ref}}) \equiv 4\pi^2 \left( \frac{\alpha}{\pi} \right)^2 \int_{Q^2_{\text{ref}}}^{\infty} dQ^2 \, f \left( Q^2, m_\mu^2 \right) \left[ \Pi(Q^2) - \Pi(Q^2_{\text{ref}}) \right] \xrightarrow{Q^2_{\text{ref}} \to 0} \alpha_\mu^{\text{HLO}}
\]

Integrand is peaked at \( Q^2 \sim m_\mu^2 \)

\( m_\mu^2 \sim 0.01 \text{ GeV}^2 \)

\(~\Rightarrow~\) here we will consider physical quantities sensitive to larger \( Q^2 \) regime
running of QED coupling

$\Delta \alpha_{\text{had}}^{\text{QED}}$
\[ \Delta \alpha_{QED} \]

\[ \alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{QED}(Q^2)} \]

\begin{itemize}
  \item vacuum polarisation: charge screening \(\leadsto\) running of QED coupling
  \item Standard Model (SM) precision tests and sensitivity to new physics requires precise knowledge of \(\Delta\alpha_{QED}(Q^2)\): input parameter of SM
  \item \(\alpha = 1/137.035999074(44) \quad [0.3 \text{ ppb}] \quad \text{[PDG, 2013]}\)
  \item \(\alpha(M_Z^2) = 1/128.952(14) \quad [10^{-4}] \quad \leadsto \quad 10^5 \text{ less accurate} \quad \text{[M. Davier et al., 1010.4180]}\)
  \item uncertainty in \(\alpha(M_Z^2)\) is significantly larger than that of \(M_Z\)
  \item hadronic effects: \(\alpha(Q^2)\) depends strongly on \(Q^2\) at low energies
	hadronic uncertainties propagate...
\end{itemize}
\[ \alpha(Q^2) = \frac{\alpha}{1 - \Delta \alpha_{\text{QED}}(Q^2)} \]

- leading order (LO) contribution

\[ \mu \nu \int d^4x \ e^{iQx} \langle J_{\mu}(x) J_{\nu}(0) \rangle = (Q_{\mu} Q_{\nu} - Q^2 \delta_{\mu\nu}) \Pi(Q^2) \]

\[ J_{\mu}(x) = \sum_{f=1}^{N_f} Q_f \overline{\psi}_f(x) \gamma_{\mu} \psi_f(x) \quad Q_f \in \{-1/3, 2/3\} \]

- \( \Pi(Q^2) \): photon vacuum polarisation function (VPF)
\[ \Delta \alpha_{\text{QED}}(Q^2) = \frac{\alpha}{1 - \Delta \alpha_{\text{QED}}(Q^2)} \]

- **leading order (LO) contribution**

\[ \int d^4x \, e^{iQx} \langle J_{\mu}(x) J_{\nu}(0) \rangle = (Q_{\mu} Q_{\nu} - Q^2 \delta_{\mu\nu}) \Pi(Q^2) \]

\[ J_{\mu}(x) = \sum_{f=1}^{N_f} Q_f \overline{\psi}_f(x) \gamma_{\mu} \psi_f(x) \]

- **\( \Pi(Q^2) \): photon vacuum polarisation function (VPF)**

\[ \Delta \alpha_{\text{QED}}(Q^2) = 4\pi \alpha \left( \Pi(Q^2) - \Pi(0) \right) \]

- **Adler function \( D(Q^2) \):**

\[ \frac{D(Q^2)}{Q^2} = 12\pi^2 \frac{d \Pi(q^2)}{dq^2} = -\frac{3\pi}{\alpha} \frac{d}{dq^2} \Delta \alpha_{\text{had}}(q^2) \quad Q^2 = -q^2 \]
\[ \alpha(Q^2) = \frac{\alpha}{1 - \Delta \alpha_{\text{QED}}(Q^2)} \]

- Combine experimental data and perturbation theory (PT)

\[ \Delta \alpha_{\text{had}}^{(5)}(M_Z^2) = \Delta \alpha_{\text{had}}^{(5)}(-M_0^2)^{\text{exp}} \]
\[ + \left[ \Delta \alpha_{\text{had}}^{(5)}(-M_Z^2) - \Delta \alpha_{\text{had}}^{(5)}(-M_0^2) \right]^{\text{pQCD}} \]
\[ + \left[ \Delta \alpha_{\text{had}}^{(5)}(M_Z^2) - \Delta \alpha_{\text{had}}^{(5)}(-M_Z^2) \right]^{\text{pQCD}} \]

\[ M_0^2 = (2.5 \text{ GeV})^2 \approx 6 \text{ GeV}^2 \]

\[ \geq [1\%] \]

- Can lattice QCD reach similar precision for \( \Delta \alpha_{\text{QED}}^{\text{had}} \)?
lattice setup

$\mathbf{a^2 \ [fm^2]}$

$\mathbf{\frac{1}{L} \ [fm^{-1}]}$

$\mathbf{N_f = 2 \ \mathcal{O}(a) \ improved \ Wilson \ fermions \ [CLS]}$

strange and charm are quenched: $s_\Phi, c_\Phi$

only quark-connected contributions

increased statistics
Adler function: lattice spacing dependence

\[ D(Q^2) = \frac{3\pi}{\alpha} \left( \frac{d}{d\log(q^2)} \Delta\alpha_{\text{QED}}(q^2) \right) \Delta \alpha_{\text{had}}(q^2) \]

\[ a \approx 0.08 \text{ fm} \]
\[ a \approx 0.06 \text{ fm} \]
\[ a \approx 0.05 \text{ fm} \]

\[ M_{\text{PS}} \approx 330 \text{ MeV} \]

combined fits [talk by Hanno Horch]

lattice artefacts: dominant systematic effect for \( Q^2 \gtrsim 1 \text{ GeV}^2 \)
Adler function: lattice spacing dependence

\[ D(\hat{Q}^2) = \frac{3\pi}{\alpha} \frac{d}{d \log(q^2)} \Delta\alpha_{\text{QED}}(q^2) \]

\[
\hat{Q}^2 = 2.0 \text{ GeV}^2; \quad M_{\text{PS}} \approx 330 \text{ MeV}
\]

combined fits [talk by Hanno Horch]

lattice artefacts: dominant systematic effect for \( Q^2 \gtrsim 1 \text{ GeV}^2 \)
Adler function: light-quark mass dependence

\[ D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta_{\text{QED}}(q^2) \]

\( u, d \)

\[ \hat{Q}^2 = 1.3 \text{ GeV}^2 \quad \text{C.L.} \]

\[ a \approx 0.05 \text{ fm} \]

\[ a \approx 0.06 \text{ fm} \]

\[ a \approx 0.08 \text{ fm} \]

\[ \hat{Q}^2 = 2.4 \text{ GeV}^2 \quad \text{C.L.} \]

\[ a \approx 0.05 \text{ fm} \]

\[ a \approx 0.06 \text{ fm} \]

\[ a \approx 0.08 \text{ fm} \]
Adler function: strange quark

\[ D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta_{\text{QED}}(q^2) \]

\[ \hat{Q}^2 = 2.4 \text{ GeV}^2 \]

\[ a \approx 0.05 \text{ fm} \]
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\[ M_{PS} \approx 330 \text{ MeV} \text{ C.L.} \]

lattice artefacts: dominant systematic effect for \( Q^2 \gtrsim 1 \text{ GeV}^2 \)
Adler function: strange quark

\[ D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\text{QED}}^\text{had}(Q^2) \]

\[ M_{\text{PS}} \approx 330 \text{ MeV} \quad \text{C.L.} \]

\[ \hat{Q}^2 = 2.4 \text{ GeV}^2 \quad \text{C.L.} \]

lattice artefacts: dominant systematic effect for \( Q^2 \gtrsim 1 \text{ GeV}^2 \)
Adler function: charm quark

\[ D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\text{QED}}(q^2) \]

\[ \hat{Q}^2 = 2.6 \text{ GeV}^2 \quad a \approx 0.05 \text{ fm} \quad \rightarrow \quad \hat{Q}^2 \approx 0.05 \text{ fm} \]

\[ \hat{Q}^2 \approx 0.06 \text{ fm} \]

\[ M_{\text{PS}} \approx 330 \text{ MeV} \quad \text{C.L.} \]

lattice artefacts: dominant systematic effect for \( Q^2 \gtrsim 1 \text{ GeV}^2 \)
Adler function: charm quark

\[ D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\text{QED}}(Q^2) \]

\[ \hat{Q}^2 = 2.6 \text{ GeV}^2 \quad \text{C.L.} \]

\[ a \approx 0.05 \text{ fm} \]

\[ a \approx 0.06 \text{ fm} \]

\[ M_{\text{PS}} \approx 330 \text{ MeV C.L.} \]

lattice artefacts: dominant systematic effect for \( Q^2 \gtrsim 1 \text{ GeV}^2 \)
Adler function: flavour contributions

\[ D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d \log(q^2)} \Delta_{\text{QED}}^{\text{had}}(q^2) \]

\[ a \to 0 \quad M_\pi^{\text{phys}} \]

\[ u, d, s_Q, c_Q \]

\[ u, d, s, \ldots \]

stat. error only
Adler function: flavour contributions

\[ D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d \log(q^2)} \Delta \alpha_{\text{QED}}(q^2) \]

\[ a \rightarrow 0 \quad M_{\pi}^{\text{phys}} \]

\[ u, d, s_Q, c_Q \]

\[ u, d, s, \ldots \]

\[ \text{stat. error only} \]

Pheno. model \( u, d \) [D. Bernecker & H. Meyer, 1107.4388]
running QED coupling: $\Delta \alpha_{\text{QED}}^\text{had}(Q^2)$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta \alpha_{\text{QED}}(Q^2)}$$

$$\Delta \alpha_{\text{QED}}^\text{had}(Q^2) = 4\pi \alpha \left( \Pi(Q^2) - \Pi(0) \right)$$
running QED coupling: $\Delta \alpha_{\text{QED}}^{\text{had}}(Q^2)$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta \alpha_{\text{QED}}(Q^2)}$$

$$\Delta \alpha_{\text{QED}}^{\text{had}}(Q^2) = 4\pi \alpha \left( \Pi(Q^2) - \Pi(0) \right)$$

[PRELIMINARY]

- $\Delta \alpha_{\text{QED}}^{\text{had}}(1 \text{GeV}^2)$
  - $u, d$: $2.95(04)(05)$ [2%]
  - $u, d, s_Q$: $3.36(04)(06)$ [2%]
  - $u, d, s_Q, c_Q$: $3.50(05)(09)$ [3%]
  - $u, d, s, c, b$: $3.64(04)$ [1%]

Pheno. [alphaQED package, F. Jegerlehner]

- difference: $\Delta \alpha_{\text{QED}}^{\text{had}}(4 \text{GeV}^2) - \Delta \alpha_{\text{QED}}^{\text{had}}(1 \text{GeV})$
  - $u, d$: $1.97(02)(06)$ [3%]
  - $u, d, s_Q$: $2.33(02)(08)$ [3%]
  - $u, d, s_Q, c_Q$: $2.65(02)(15)$ [6%]
comparison to perturbative QCD

$\alpha_s$
Operator Product Expansion (OPE)

Matching of lattice determinations of the VPF and Adler function to perturbation theory

Non-singlet and singlet contributions

\[ D^{(N_f)}(\hat{Q}^2, \alpha_s) = \sum_f Q_f^2 \, D_{\text{con}}(\alpha_s, \hat{Q}^2, m_f) + \sum_{f,f'} Q_f Q_{f'} \, D_{\text{disc}}(\alpha_s, \hat{Q}^2, m_f, m_{f'}) \]

\[ D_{\text{OPE}}^{\text{con}}(\hat{Q}^2, \alpha_s, m_f) = D_0(\alpha_s, \hat{Q}^2, \mu^2) \]

\[ + \, D_2^m(\alpha_s, \hat{Q}^2, \mu^2) \, \left( \frac{m_f[\hat{Q}^2]}{\hat{Q}^2} \right)^2 \]

\[ + \, D_4^\overline{}(\alpha_s, \hat{Q}^2, \mu^2) \, \frac{m_f \langle \overline{\psi}_f \psi_f \rangle}{\hat{Q}^4} \]

\[ + \, D_4^G(\alpha_s, \hat{Q}^2, \mu^2) \, \frac{\langle O_{\text{OPE}}^{(4)} \rangle}{\hat{Q}^4} \]

\[ + \, \mathcal{O}\left( \frac{1}{\hat{Q}^6} \right) \]
Operator Product Expansion (OPE)

- only quark-connected contributions both in PT and lattice

- Wilson coefficients $D_0$, $D_2^m$, $D_4^F$, $D_4^G$ are computed in PT

  
  $D_0: \mathcal{O}(\alpha_s^4)$, \quad $D_2^m: \mathcal{O}(\alpha_s^2)$, \quad $D_4^F: \mathcal{O}(\alpha_s^2)$, \quad $D_4^G: \mathcal{O}(\alpha_s)$

- connection of $\alpha_s$ to $\Lambda_{\overline{MS}}^{(N_f=2)}$ via the 4-loop $\beta$-function

- fit of lattice data to PT: range of validity of PT vs. discretisation effects

- fit parameters: $\alpha_s(\mu = 2 \text{ GeV})$, $\langle \mathcal{O}_{\text{OPE}}^{(4)} \rangle$

  and 2 parameters for lattice artefacts

  chiral condensate $\langle \overline{\psi} \psi \rangle_{\overline{MS}; \mu = 2 \text{ GeV}}$ from [FLAG. 1310.8555]

earlier lattice studies [JLQCD, 0807.0556, 1002.0371]
fit to PT: \( a = \{0.05, 0.06, 0.08\} \text{ fm} \)

ongoing studies of systematic effects: lattice artefacts, \( Q^2 \) interval,

order in OPE and perturbative expansions, \ldots
fit to PT: comparison

$a \to 0$

$M_{\pi}^{phys}$

$D(\hat{Q}^2)$

$\Lambda_{MS}^{(N_f=2)}: [\sim 20\%]$ stat.

$\Lambda_{MS}^{(N_f=2)}: [\sim 20\%]$ stat.
conclusions

- Adler function $\sim a_{\mu}^{\text{had}}, \Delta\alpha_{\text{QED}}(Q^2), \alpha_s$

- good prospects for accurate determination of $\Delta\alpha_{\text{QED}}(Q^2)$ on the lattice

- matching to perturbation theory: suffers both from statistical and systematic uncertainties

in view of future experimental results for $a_{\mu}$ . . . or to address e.g. the $e^+e^- - \tau$ difference

→ improve precision and accuracy

- combination of standard and “mixed representation” methods $\sim a_{\mu}^{\text{had}}$

  [talk by Anthony Francis]

- quark-disconnected diagrams

  [talk by Vera Gülpers]

- variance reduction techniques

  [poster by Eigo Shintani]

- . . .
Lattice VPF

Local current

\[ J^{(l, f)}_{\mu}(x) = Z_V \bar{\psi}_f(x) \gamma_\mu \psi_f(x) \]

Conserved-local correlator

\[
\alpha^6 \left\langle \sum_{f=1}^{N_f} \left( Q_f J^{(ps, f)}_{\mu}(x) \right) \sum_{f'=1}^{N_f} \left( Q_{f'} J^{(l, f')}_{\nu}(0) \right) \right\rangle
\]

\[
\Pi_{\mu\nu}(\hat{Q}) = \alpha^4 \sum_x e^{i\hat{Q}(x+a\hat{\mu}/2)} \left\langle J^{(ps)}_{\mu}(x) J^{(l)}_{\nu}(0) \right\rangle \sim \Pi(\hat{Q}^2)
\]

\[ \hat{Q}_{\mu} = \frac{2}{a} \sin \left( \frac{aQ_\mu}{2} \right) \]
Adler function: combined fit

Adler function:

\[ D(Q^2) = -12 \pi^2 Q^2 \frac{d\Pi(Q^2)}{dQ^2} \]

▶ fit form:

\[ D(Q^2) = \text{Padé}(Q^2) \left[ 1 + \text{discr.} + \text{mass} \right], \]

\[ D(Q^2) = Q^2 \left( p_0 + \frac{p_1}{(p_2 + Q^2)^2} + \frac{p_3}{(p_4 + Q^2)^2} \right) \times \]

\[ \left[ 1 + \left( d_1 d^n + d_2 (aQ)^n \right) + \left( \frac{c_1}{c_2 + Q^2} \right) (M_{PS}^2 - M_{\pi}^2) \right]. \]

\[ n = \{1, 2\} \]

▶ consider 11 ensembles with different \( a, M_{PS} \)

▶ \( u, d, s, q \) and \( c_q \)
Adler function: light-quark mass dependence

\[ D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\text{QED}}(q^2) \]

\[ u, d \approx 0.06 \text{ fm} \]

\[ M_{\text{phys}} \approx 190 \text{ MeV} \]

\[ M_{\text{PS}} \approx 280 \text{ MeV} \]

\[ M_{\text{PS}} \approx 330 \text{ MeV} \]

\[ M_{\text{PS}} \approx 460 \text{ MeV} \]
Adler function: light-quark mass dependence

\[ D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\text{QED}}(q^2) \]

\[ u, d \approx 0.06 \text{ fm} \]
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