Homological mirror symmetry of Fermat polynomials

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Abstract

We discuss homological mirror symmetry of Fermat polynomials in terms of derived Morita equivalence between derived categories of coherent sheaves and Fukaya-Seidel categories (a.k.a. perfect derived categories \cite{Kon09} of directed Fukaya categories \cite{Sei08, Sei03, AurKatOrl08}), and some related aspects such as stability conditions, (kinds of) modular forms, and Hochschild homologies.

1 Introduction

Homological mirror symmetry was introduced by Kontsevich \cite{Kon95} as the categorical equivalence of so-called A- and B- models of topological field theories. For a pair of manifolds $X$ and $Y$ possibly with group actions, homological mirror symmetry is an equivalence of triangulated categories of coherent sheaves of $X$ and Lagrangians of $Y$ with natural enhancements to differential graded (dg) categories (see \cite{Kel06}).

We have seen such equivalences for elliptic curves \cite{PolZas}, the quartic surface case \cite{Sei03}, degenerating families of Calabi-Yau varieties and abelian varieties \cite{KonSoi01}. The framework has been extended to Fano varieties and singularities in \cite{AurKatOrl08, AurKatOrl06, Sei01}. See \cite{HKKPTVVZ} for a comprehensive source of references.

Let $X_n$ be the function $F_n : \mathbb{C}^n \to \mathbb{C}$ for the Fermat polynomial $F_n := x_1^n + \cdots + x_n^n$, and $Y_n$ be $F_n : \mathbb{C}^n/G_n \to \mathbb{C}$ with the group $G_n := \{ (\xi_k)_{k=1\ldots n} \in \mathbb{C}^n | \xi_k^n = 1 \}$ acting on $\mathbb{C}^n$ as $(x_k)_{k=1\ldots n} \in \mathbb{C}^n \mapsto (\xi_k x_k)_{k=1\ldots n} \in \mathbb{C}^n$. For Morsifications of our functions, we define Fukaya-Seidel categories as perfect derived categories of $A_\infty$ categories (see \cite{Kel01}) of ordered vanishing cycles (Lagrangian spheres) \cite{Sei08, Sei03, AurKatOrl08}.

For $X_n$, we define $D^b(\text{Coh} X_n)$ as the perfect derived category of coherent sheaves on its fiber over zero in the projective space of $\mathbb{C}^n$ (so, it gives the

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bounded derived category of the coherent sheaves of the Fermat hypersurface in \( \mathbb{P}^{n-1} \). By the same logic, we define \( D^b(\text{Coh } Y_n) \), which gives the bounded derived category of coherent sheaves on the Fermat hypersurface in \( \mathbb{P}^{n-1} \) as the stack with respect to the action of \( H_n \coloneqq G_n/\text{diagonals} \).

Between derived categories such as \( D^b(\text{Coh } Y_n) \) and \( \text{FS}(X_n) \), we prove derived Morita equivalences in the sense that we take compact generating objects on both sides and dg categories (derived endomorphisms) of the objects in some dg enhancements, and find them isomorphic.

Homological mirror symmetry comes with the canonical mirror equivalence. We put \( \text{FS}(Y_n) \), as a quotient of \( \text{FS}(X_n) \) by the dual group \( \hat{H}_n \) of \( H_n \). In \( \text{FS}(X_n) \), the \( A_\infty \) category of ordered vanishing cycles for a Morsification of \( X_n \) is \( A_\infty \) isomorphic to a dg category. We take the dg category of a compact generating object consisting of these vanishing cycles such that \( \hat{H}_n \) acts as an automorphism group on the category of the dg category. We put Fukaya-Seidel category \( \text{FS}(Y_n) \) as the perfect derived category of the \( \hat{H}_n \)-orbit category of the dg category.

We take the dg category of a \( H_n \)-invariant compact generating object of \( D^b(\text{Coh } X_n) \) in some dg enhancement and find the derived Morita equivalence between \( D^b(\text{Coh } X_n) \) and \( \text{FS}(Y_n) \), comparing the dg \( \hat{H}_n \)-orbit category and the dg category. The following is a summary:

\[
\begin{align*}
D^b(\text{Coh } X_n) & \overset{\cong}{\rightarrow} \text{FS}(X_n) \\
\uparrow \text{equivariance by } H_n & \downarrow \text{quotient by } \hat{H}_n \\
D^b(\text{Coh } X_n) & \cong \text{FS}(Y_n).
\end{align*}
\]

For recent discussion on some aspects of homological mirror symmetry, see [KapKreSch, Kon09] for references.

### 2 Derived Morita equivalence

Let us say equivalence instead of derived Morita equivalence. Let \( A_{n-1} \) be the Dynkin quiver of \( A \) type with \( n-1 \) vertices and one-way arrows. The perfect derived category \( D^b(\text{mod } A_{n-1}) \) is equivalent to the category of graded B-branes \( D\text{GrB}(x^n) \) (see [Ori, KaiSaiTak]), which has the natural dg enhancement in terms of graded matrix factorizations.

Hochschild cohomology of the path algebra of \( A_{n-1} \) is trivial [Hap], so is that of their tensor products. For Auslander-Reiten transformation \( \tau \) of \( \text{DGrB}(x^n) \) and the projective-simple module \( E \) of \( \text{mod } A_{n-1} \), \( \text{DGrB}(x^n) \) is equivalent to the perfect derived category of the dg category \( A_{n-1} := \text{End}^* (\bigoplus_{\mu=0}^{n-1} \tau^{-\mu}(E)) \), which is generated by degree-zero idempotents and, if any, degree-one nilpotents.

For the function \( x^n : \mathbb{C} \rightarrow \mathbb{C} \), the perfect derived category of \( A_{n-1} \) is equivalent to \( \text{FS}(x^n) := \text{FS}(x^n : \mathbb{C} \rightarrow \mathbb{C}) \) [Sei01, 2B]. Ordered objects \( E, \ldots, \tau^{\frac{n}{2}}(E) \) represent ordered vanishing cycles for a Morsification of the function. Let us mention that they may represent ones for the Morsification of \( x^n + y^2 : \mathbb{C}^2 \rightarrow \mathbb{C} \) in [ACa] (see Sei01 Section 3). The object \( \tau^{1-n}(E) \) represents a connected sum of vanishing cycles.
Up to $A_{\infty}$ isomorphisms, the tensor product of the graded commutative algebra $A_{n-1}^{\otimes n}$ admits no non-trivial $A_{\infty}$ structures. Künneth formula for vanishing cycles of Fukaya-Seidel categories on morphisms and compositions [AurKatOrl08 Proposition 6.3, Lemma 6.4] implies that the perfect derived category of $A_{n-1}^{\otimes n}$ is equivalent to a full subcategory of $FS(X_n)$. By the classical theory of singularity, vanishing cycles represented as objects of $A_{n-1}^{\otimes n}$ compose a generating object of $FS(X_n)$ (see [AurKatOrl08 Conjecture 1.3] and [Oka, SebTho] for related studies).

Let us explain an equivalence between $D^b(Coh Y_n)$ and $D^b(mod A_{n-1})^{\otimes n}$. By [Orl], $D^b(Coh X_n)$ is equivalent to the category of graded B-branes $DGrB(F_n)$, which has the natural dg enhancement in terms of graded matrix factorizations and the shift functor $\tau$ such that $\tau^n \cong [2]$. Let $\Omega_{n-1} := \oplus_{\mu=0}^{n-1} \tau^{-\mu}(O_{X_n})$.

With explicit descriptions given by graded matrix factorizations for each summand of $\Omega_{n-1}$ (see [Asp07 Section IV]), we have the following. The object $\Omega_{n-1}$ is isomorphic to the compact generating object $\oplus_{\mu=0}^{n-1} \Omega_{p-1}^{\mu}(n-\mu)(-\mu)|X_n$ of $D^b(Coh X_n)$. The object $\Omega_{n-1}$ is invariant under $H_n$ actions up to canonical isomorphisms $H_\mu$, and so $H_n$ equivariant objects of $\Omega_{n-1}$ compose a generating object of $D^b(Coh Y_n)$. For the dg category of $\Omega_{n-1}$ in the natural dg enhancement of $DGrB(X_n)$, the $H_n$-equivariant dg category of the dg category of $\Omega_{n-1}$ coincides with $A_{n-1}^{\otimes n}$.

Let us explain an equivalence between $D^b(Coh X_n)$ and $FS(Y_n)$. Morphism spaces of the dg category of $\Omega_{n-1}$ are representations of $H_n$ and compositions of morphisms are multiplications of matrices. In the sense of [CibMar], by the action of $H_n$, morphism spaces of the dg category of $\Omega_{n-1}$ are $H_n$-graded. For a finite group $G$ of dg functors of a dg category $A$ inducing an equivalence on $H^0(A)$, we have the dg orbit category [Kel05] $B = A/G$ such that dg categories $A$ and $B$ have the same objects and for objects $X$ and $Y$ in $B$, $B(X,Y) = \text{colim}_{g \in G} \oplus_{f \in G} A(f(X), g \circ f(Y))$. By applying [CibMar Proposition 3.2], the $H_n$-orbit category of the category of $A_{n-1}^{\otimes n}$ is equivalent to the category of the dg category of $\Omega_{n-1}$. Differentials have not been altered by taking dg equivariance and orbits. The dg category of $\Omega_{n-1}$ in $DGrB(X_n)$ is equivalent to the dg $H_n$-orbit category of $A_{n-1}^{\otimes n}$. Explicitly, we define $FS(Y_n)$ as the perfect derived category of the dg $H_n$-orbit category. The perfect derived category of the dg category of $\Omega_{n-1}$ in $DGrB(X_n)$ is equivalent to $D^b(Coh X_n)$, and by the construction, $FS(Y_n)$ is equivalent to $D^b(Coh X_n)$. So, we have the following.

**Theorem 2.1.** For $n > 0$, $D^b(Coh Y_n)$ is derived Morita equivalent to $FS(X_n)$ and $D^b(Coh X_n)$ is derived Morita equivalent to $FS(Y_n)$.

### 3 Discussion

Let us start off with some aspects of the dg category of $\Omega_{n-1}$, which is generated by degree-zero idempotents representing summands and, if any, degree-one closed differential forms. The dg category of $\Omega_{n-1}$ together with its deformation theory would be related to the notion of Gepner model in physics (see...
In \cite[Section 5.1]{HorWal}, explicit forms of deformations of summands of the dg category of $\Omega_4$ have been computed along the deformation flow which is compatible with the actions of $(\tilde{\xi}_1, \ldots, \tilde{\xi}_5)$ in $H_5$ subject to $\tilde{\xi}_1 \cdots \tilde{\xi}_5 = 1$.

Each summand of $\Omega_{n-1}$ by the definition represents a Lagrangian among $\hat{H}_n$ isomorphic ones. In $FS(x^n)$, in the notation of the previous section, we observe that we count the intersection of the connected sum $\tau^{1-n}(E)$ with vanishing cycles $E$ and $\tau^{2-n}(E)$. We have non-trivial homomorphisms from $\tau^{1-n}(E)[1]$ to $E$ and from $\tau^{2-n}(E)$ to $\tau^{1-n}(E)[1]$ and trivial ones with $\tau^{-\mu}(E)$ for $0 < \mu < n-2$.

For $0 \leq \mu < \mu' \leq n-1$ and Euler paring $\chi(\tilde{\tau}^{-\mu}(O_{X_5})[\mu], \tilde{\tau}^{-\mu'}(O_{X_5})[\mu'])$ counts intersections for each Lagrangian represented by $\tilde{\tau}^{-\mu}(O_{X_5})$ with Lagrangians represented by $\tilde{\tau}^{-\mu'}(O_{X_5})$.

### 3.1 Two types of stability conditions

Let us discuss the notion of stability conditions on triangulated categories \cite{Bri, GorKulRud, KonSoi08}. From a viewpoint, a stability condition on a triangulated category is a collection of objects, which are called stable and has a partial order with indices called slopes, such that for consecutive stable objects $E \geq F$ with a non-trivial homomorphism from $E$ to $F[1]$ we have consecutive stable objects $F[1] \geq E$. For reasonable such collections, we have taken extension-closed full subcategories consisting of some consecutive stable objects to study wall-crossings, moduli problems, Donaldson-Thomas type invariants, and so on.

For the quintic, a few types of stability conditions have been studied in mathematics and physics. First type of stability conditions consists of variants of Gieseker-Maruyama stabilities such as \cite{Bay, Ina, Tod}. Slopes of stable objects are given by appropriate approximations of Hilbert polynomials. These stability conditions are compatible with the tensoring by the ample line bundle, which is called the monodromy around the large radius limit (see \cite[Section F]{Asp07}).

Second type of stability conditions has been studied in terms of Douglas’ II-stabilities for Gepner point \cite{AspDou, Dou02, Dou01}. With our viewpoint, the simplest stability condition compatible with $\tilde{\tau}$, which is called the monodromy around Gepner point (see \cite[Section F]{Asp07}), and with complexified numerical classes of vanishing cycles, is as follows. For $\mu \in \mathbb{Z}$, stable objects consist of objects $\tilde{\tau}^{-\mu}(O_{X_5})$ and the order of them is given by the clockwise order of $\exp(-\mu \frac{2\pi i}{5})$.

The extension-closed full subcategory of the consecutive stable objects $O_{X_5} > \tilde{\tau}^{-1}(O_{X_5})$ of the stability condition of the second type gives the graded generalized Kronecker quiver with degree-one arrows from $O_{X_5}$ to the other, together with the same number of degree-two arrows in the opposite way and a degree-three loop on each vertex. For some vanishing cycle represented by $O_{X_5}$, the number of degree-one arrows represents the number of intersections of the vanishing cycle with vanishing cycles represented by the other object and coincides with the Donaldson-Thomas invariant of the moduli space of the numerical class which is the sum of the ones of the two stable objects. The one of the three
consecutive stable objects $O_{X_5} > \tilde{\tau}^{-1}(O_{X_5}) > \tilde{\tau}^{-2}(O_{X_5})$ gives two copies of the quiver glued, together with degree-two arrows from $O_{X_5}$ to $\tilde{\tau}^{-2}(O_{X_5})$ and the same number of degree-one arrows in the opposite way and a degree-three loop on each vertex. The objects $O_{X_5}$, $\tilde{\tau}^{-1}(O_{X_5})$, and $\tilde{\tau}^{-2}(O_{X_5})$ make cluster collections [KonSoi09, KonSoi08].

The stability condition of the second type is equipped with the consecutive stable objects $\tilde{\tau}^{-1}(O_{X_5})[1] > O_{X_5}$. The extension-closed full subcategory of the pair gives the graded generalized Kronecker quiver with degree-zero arrows in one way, together with the same number of degree-three arrows in the opposite way and a degree-three loop on each vertex, which coincides with that of the consecutive stable objects $O_{X_5}(1) > O_{X_5}$ of the simplest stability condition of the first type on the sequence $O_{X_5}(\mu)$ for $\mu \in \mathbb{Z}$.

3.2 (kinds of) Modular forms

For a stability condition of either type, objects in the extension-closed full subcategory consisting of stable objects of a slope are said to be semistable. Let us count semistable objects in the extension-closed full subcategory consisting of a stable object for the stability condition of the second type. The same counting logic applies when we have a single stable spherical object of some slope.

As in [MelOka], we are interested in (kinds of) modular forms and Donaldson-Thomas type invariants. For the motive of the affine line $L$, we represent the formal symbol $L^1$ by $-q^{1/2}$, which conjecturally corresponds to so-called graviphoton background in the supergravity theory [DimGuk, DimGukSoi]. We obtain $J_m(q) := \frac{1}{m^2} \frac{(-q)^{m-2}}{(1-q^m)}$ for $m > 0$ as the coefficient of $x^m$ in the formal logarithm of the quantum dilogarithm $\sum_{m \geq 0} \frac{(-q)^{m}}{(1-q^m)} x^m$ [KonSoi09, KonSoi08].

The generating function $\sum_{n>0} m^{1-k} J_m(q) = \sum_{m,r>0} m^{-k} q^{mr} - m^{-k} q^{mr}$ consists of Eisenstein series for negative odd $k$, or Eichler integrals [LawZag] of Eisenstein series for positive odd $k$. In particular, for $k = -1$, $q = \exp(2\pi i z)$, and Eisenstein series $E_2(z)$, we obtain the quasimodular form $E_2(z) - E_2(2z)$ [KanZag].

3.3 Hochschild homologies

In $\mathbb{P}^{n-1}$, $Y_n$ is the same as $\mathbb{P}^{n-2}$ given by $x_1 + \cdots + x_n = 0$ with the orbifold structures of degree $n$ along coordinate hypersurfaces. By computing Poincaré polynomial of $Y_n$, which can be partitioned according to degrees of orbifold structures, we obtain $P(Y_n, q) = \sum_{2 \leq j \leq n, 2 \leq k \leq j} n^{n-j} \binom{n}{j} (-1)^{j-k} P(\mathbb{P}^{k-2}, q)$.

For example, we have $1$, $q^7 + 7q^2 + 13q + 67$, and $q^3 + 21q^2 + 181q + 821$. In particular, by computing Euler characteristics in terms of Hochschild homologies (see [Kel98, HocKosRos]), we obtain $P(Y_n, 1) = (n-1)^n$. 

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