The role of the Hamiltonian in the interpretation of quantum mechanics

Mario Castagnino\textsuperscript{1} and Olimpia Lombardi\textsuperscript{2}

\textsuperscript{1} CONICET-Institutos de Física de Rosario y de Astronomía y Física del Espacio. Casilla de Correos 67, Sucursal 28, 1428, Buenos Aires, Argentina
\textsuperscript{2} CONICET-Universidad de Buenos Aires. Crisologo Larralde 3440, 1430, Buenos Aires, Argentina

E-mail: mariocastagnino@citynet.net.ar, olimpiafilo@arnet.com.ar

Abstract. In this paper we propose a new realist, non-collapse interpretation of quantum mechanics, which moves away from the prevailing trend in the subject by paying special attention to the physical relevance of the interpretation. In particular, our proposal endows the Hamiltonian of the system, systematically ignored in the traditional interpretations, with a central role: it distinguishes between systems and subsystems and is the main ingredient in the selection of the definite-valued observables. We show how this interpretation solves the measurement problem, both in the ideal and in the non-ideal version, and we argue for the physical relevance of the new definite-value assignment.

1. Introduction

In spite of the impressive empirical success of quantum mechanics, its interpretation is still an open problem. During many years since the birth of the theory, instrumentalist interpretations prevailed: the meaning of quantum mechanics was expressed in terms of measurements results. But in the last decades, several attempts to interpret quantum mechanics from a realist viewpoint have been proposed. David Mermin’s “Ithaca interpretation” ([1]) is an example of this new trend. From this realist perspective, Mermin proposes six \textit{desiderata} that any interpretation of quantum mechanics should satisfy:

(i) \textit{The theory should describe an objective reality independent of the observers and their knowledge.}
(ii) \textit{The concept of measurement should play no fundamental role.}
(iii) \textit{The theory should describe individual systems, not just ensembles.}
(iv) \textit{The theory should describe small isolated systems without having to invoke interaction with anything external.}
(v) \textit{Objectively real internal properties of an isolated individual system should not change when something is done in another non-interacting system.}
(vi) \textit{It suffices to base the interpretation of quantum mechanics on the interpretation of objective probabilities.}
As it is well known, the Kochen-Specker theorem (\cite{2}) is an insurmountable barrier to any classical-like interpretation of quantum mechanics, since it shows the contextuality of the theory by proving the impossibility of ascribing precise values to all the observables of a quantum system in a non-contradictory manner. Therefore, any realist, non-collapse interpretation is committed to select a preferred context, which defines the observables that will acquire definite values. This point is so relevant that Bub (\cite{3}) classifies the best known realist non-collapse interpretations in terms of the observable $R$ chosen for defining the preferred context in each one of them. For instance, in Bohr’s complementarity interpretation (\cite{4}), $R$ is defined by the experimental arrangement; in the interpretation of Bohm (\cite{5}), $R$ is simply the position observable; for some modal interpretations (\cite{6}, \cite{7}, \cite{8}), $R$ is given by the biorthonormal decomposition (Schmidt) theorem.

When the many interpretations of quantum mechanics proposed through the years are reviewed from this general viewpoint, it is easy to realize that the Hamiltonian has been systematically ignored in the discussions. In this paper our purpose is to present a new realist interpretation, where the Hamiltonian of the system acquires a central relevance, besides its well known role in representing a constant of motion and in governing the dynamics of the system. As we will see, in this interpretation the Hamiltonian is decisive in the definition of systems and subsystems (Section 2), and in the selection of the definite-valued observables (Section 3). On this basis, the measurement problem receives an adequate answer (Section 4), and several traditional interpretative difficulties can be solved (Section 5). Moreover, the physical relevance of this interpretation can be argued for (Section 6). Finally, we will show that this new interpretation satisfies the six desiderata proposed by Mermin (Conclusions).

2. Systems and subsystems

In order to study the physical world, we have to identify the systems populating that world. We can cut out the physical reality in many different and arbitrary ways, but only when a portion of reality does not interact with others, we have a non-arbitrary, objective criterion to identify that portion as a system. For this reason, we will design our interpretation to account for those pieces of reality non-interacting with other pieces and, so, we will conceive only closed systems as quantum systems. On this basis, and by adopting an algebraic perspective, we define a quantum system $S$ as a pair $(\mathcal{O}, H)$ such that (i) $\mathcal{O}$ is a space of self-adjoint operators on a Hilbert space $\mathcal{H}$, representing the observables of the system, (ii) $H \in \mathcal{O}$ is the time-independent Hamiltonian of the system, and (iii) being $\rho_\mathcal{O} \in \mathcal{O}'$ (where $\mathcal{O}'$ is the dual space of $\mathcal{O}$) the initial state of $S$, it evolves according to the Schrödinger equation in its von Neumann version.

Of course, any quantum system can be decomposed in parts in many ways; however, not any decomposition will lead to parts which are, in turn, quantum systems. This will be the case only when the components’ behaviors are dynamically independent to each other, that is, the time-evolution of each component can be described independently to the remaining ones (see \cite{9}). In other words, a quantum system can be split into subsystems when there is no interaction among the subsystems. On this basis, we will say that a quantum system $S : (\mathcal{O}, H)$ with initial state $\rho_0 \in \mathcal{O}'$ is composite when it can be partitioned into two quantum systems $S^1 : (\mathcal{O}^1, H^1)$ and $S^2 : (\mathcal{O}^2, H^2)$ such that (i) $\mathcal{O} = \mathcal{O}^1 \otimes \mathcal{O}^2$, and (ii) $H = H^1 \otimes I^2 + I^1 \otimes H^1$ (where $I^1$ and $I^2$ are the identity operators in the corresponding tensor product spaces). In this case, the initial states of $S^1$ and $S^2$ are obtained as the partial traces $\rho^1_0 = Tr_2(\rho_0) \in \mathcal{O}^1'$ and $\rho^2_0 = Tr_1(\rho_0) \in \mathcal{O}^2'$, and we will say that $S^1$ with initial state $\rho^1_0$ and $S^2$ with initial state $\rho^2_0$ are subsystems of the composite system $S = S^1 \cup S^2$.

It is worth stressing that this definition of composite system does not imply that the initial state $\rho_0$ of $S$ is the tensor product $\rho^1_0 \otimes \rho^2_0$; this factored or uncorrelated state is a very special kind of state corresponding to independent preparations of the component systems (see...
Therefore, and as a consequence, this means that according to the Schrödinger equation. In fact, since there is no interaction between $S^1$ and $S^2,$

$$[H^1 \otimes I^2, I^2 \otimes H^1] = 0$$

and, as a consequence,

$$\exp \left[ -iHt \right] = \exp \left[ -iH^1 t \right] \exp \left[ -iH^2 t \right]$$

Therefore,

$$\rho^1(t) = T r_{(2)}(t) = T r_{(2)}(e^{-iHt} \rho_0 e^{iHt}) = e^{-iH^1 t} (T r_{(2)}(\rho_0) e^{iH^1 t} = e^{-iH^1 t} \rho_0^1 e^{iH^1 t}$$

$$\rho^2(t) = T r_{(1)}(\rho(t)) = T r_{(1)}(e^{-iHt} \rho_0 e^{iHt}) = e^{-iH^2 t} (T r_{(1)}(\rho_0) e^{iH^2 t} = e^{-iH^2 t} \rho_0^2 e^{iH^2 t}$$

This means that $S^1$ and $S^2$ are dynamically independent: each one of them will evolve under the action of its own Hamiltonian.

It has to be emphasized that, although this definition of composite quantum system is completely general, the decomposition of a quantum system into subsystems is not always possible: it may happen that there is no partition of the whole $S$ such that the total Hamiltonian can be expressed as a sum of component Hamiltonians. In this case, the quantum system is not composite and we will call it elemental. Therefore, this interpretation supplies a precise criterion to distinguish between elemental and composite systems, and such a criterion is based on the system’s Hamiltonian.

On the other hand, given any two quantum systems $S^1 : (O^1, H^1)$ and $S^2 : (O^2, H^2)$ with initial states $\rho_0^1 \in O^1$ and $\rho_0^2 \in O^2$ respectively, we can always define a composite system $S : (O, H)$ such that $O = O^1 \otimes O^2,$ $H = H^1 \otimes I^2 + I^1 \otimes H^2 + H_{int},$ where $H_{int}$ is the interaction Hamiltonian, and $\rho_0 = \rho_0^1 \otimes \rho_0^2 \in O'.$ In this case, the initial state $\rho_0$ of $S$ and the initial states $\rho_0^1$ of $S^1$ and $\rho_0^2$ of $S^2$ are still related by a partial trace, since

$$T r_{(2)}(\rho_0) = T r_{(2)}(\rho_0 \otimes \rho_0^2) = \rho_0^1$$

$$T r_{(1)}(\rho_0) = T r_{(1)}(\rho_0 \otimes \rho_0^2) = \rho_0^2$$

However, when the two systems $S^1$ and $S^2$ interact to each other, $H_{int} \neq 0$ and, therefore, $\rho_0^1$ and $\rho_0^2$ do not evolve according to the Schrödinger equation. This means that, strictly speaking, $S^1$ and $S^2$ are not subsystems of $S$ but should be considered as mere “parts” of $S.$ Only in the particular case that $H_{int} = 0,$ $S^1$ and $S^2$ will evolve unitarily as in eqs. (3) and (4), and they will properly be subsystems of $S.$

3. The selection of the preferred context
As we have pointed out in the introduction, the subtler point in any realist interpretation of quantum mechanics is the choice of the preferred context, which will define the definite-valued observables of the system. In this section we will introduce the rule for selecting such a context. However, before explaining the rule, two remarks are in order.

As it is well known, quantum mechanics is a probabilistic theory: by contrast with classical mechanics, it does not ascribe precise values to the observables of the system, but only probabilities to each possible value (see [11]). Moreover, quantum mechanics is intrinsically probabilistic: any attempt to ascribe precise values to all the observables of the system, in such a way that probabilities be interpreted in terms of ignorance about an underlying classical-like state, runs into contradiction, as proved by the Kochen-Specker theorem (see [3], [12]). This means that the rule of definite-value ascription, which selects the preferred context, cannot be
inferred from the formalism, but has to be introduced as an interpretational postulate. Therefore, the adequacy of such a rule has to be assessed in the light of its physical relevance and its ability to solve the interpretation problems of the theory.

The second remark concerns to the interpretation of probability. If, as Mermin proposes, quantum mechanics is a fundamental theory describing individual systems and not ensembles, the Kochen-Specker theorem teaches us that there are not hidden variables explaining an underlying level of reality. As Giere ([13]) points out, only the propensity interpretation of probability takes seriously the “no-hidden variables” results, since it requires that quantum probabilities describe ultimate and irreducible features of physical systems. On this basis, our interpretation conceives probabilities as ontological irreducible propensities, and the time evolution of the quantum state as the time evolution of those propensities. In this interpretational framework, the rule that selects the preferred context is the interpretational postulate that defines, among all the observables of the system, which of them will acquire actual, and not merely possible, values. For this reason, we will call it Actualization Rule.

We have seen that each realist interpretation appeals to a different criterion to select the preferred context: the classicality of the experimental arrangement (Bohr), the assumption of the priority of position (Bohm), the mathematical result of the Schmidt theorem (Dieks-Vermaas). However, the Hamiltonian of the system has never been taken into account. Our Actualization Rule, on the contrary, endows the Hamiltonian, with its particular symmetries, with a central role.

It is widely accepted that the laws of physics are covariant under certain transformations represented by a definite group; in particular, quantum mechanics is covariant under the Galilean transformations, represented by the Galilean group. As it is well known, there are ten symmetry generators $K_i$ of the Galilean group: one time-displacement, three space-displacements, three boost-velocity components and three space-rotations. These generators represent the fundamental dynamical magnitudes of the theory (see [10]): the energy $H$ (time-displacement), the momentum $P$ (space-displacement), the position $Q$ (boost-transformation: boost generator = $mQ$), the total angular momentum $J$ (space-rotation).

Since we have defined a quantum system as a closed system, its energy is constant in time and, then, the Hamiltonian $H$ is time-independent: $H$ is always invariant under time displacement. Nevertheless, in a given quantum system, $H$ may have the remaining symmetries or not. To say that the Hamiltonian is symmetric or invariant under a certain continuous transformation means that

$$e^{iK_\alpha} He^{-iK_\alpha} = H \quad \text{then} \quad [H, K] = 0 \quad (7)$$

where $\alpha$ is the parameter of the transformation and $K$ is the corresponding generator. This means that, when $H$ is invariant under a certain transformation, the generator of that transformation is a constant of motion: each symmetry of $H$ defines a conserved quantity. For instance, invariance of $H$ under space displacement in the direction $\alpha$ implies that the component $P_\alpha$ of the momentum $P$ is a constant of motion; invariance of $H$ under rotation about an axis $\alpha$ implies that the component $J_\alpha$ of the total angular momentum $J$ is a constant of motion.

Moreover, we know that each symmetry of the Hamiltonian leads to an energy degeneracy. In fact, if $H$ is invariant under a continuous transformation with generator $K$, we can write

$$KH |n\rangle = K \omega_n |n\rangle \quad \Rightarrow \quad HK |n\rangle = \omega_n K |n\rangle \quad (8)$$

1 Actually, the generators are proportional to these dynamical magnitudes; for instance, the time-displacement generator is $H/\hbar$. For simplicity, here we are taking $\hbar = 1$.

2 Other kinds of symmetry transformations, like those of the q-integrable systems (see [14], [15]) will be considered elsewhere.
This means that any vector $K |n\rangle$, obtained by applying the operator $K$ on the eigenvector $|n\rangle$, is also an eigenvector of $H$ with the same eigenvalue (see [16]). As a consequence, $H$ can be expressed as

$$H = \sum_n \omega_n P_n$$  \hspace{1cm} (9)

where $P_n$ is the projector operator onto the subspace spanned by the degenerate eigenvectors corresponding to $\omega_n$. Then, we can write explicitly the index $k_n$ corresponding to the degeneracy of the eigenvalue $\omega_n$, in such a way that

$$H |n, k_n\rangle = \omega_n |n, k_n\rangle \Rightarrow H = \sum_{n,k_n} \omega_n |n, k_n\rangle \langle n, k_n| \hspace{1cm} (10)$$

$$K |n, k_n\rangle = \kappa_{k_n} |n, k_n\rangle \Rightarrow K = \sum_{n,k_n} \kappa_{k_n} |n, k_n\rangle \langle n, k_n| \hspace{1cm} (11)$$

The degeneracies with origin in symmetries are called “normal” ([16]) or “systematic” ([17]). On the contrary, degeneracies that have no obvious origin in symmetries are called “accidental” (see [16], [17]). However, deeper study usually shows either that the accidental degeneracy is not exact, or else that a hidden symmetry in the Hamiltonian can be found which explains the degeneracy (see [16]). For this reason it is assumed that, once all the symmetries of the Hamiltonian have been considered, a basis for the Hilbert space $\mathcal{H}$ is obtained and the “good quantum numbers” are well defined.

Now we have all the conceptual elements necessary to present our Actualization Rule. The basic idea is that the Hamiltonian of the system defines actualization; therefore, any observable that does not have the symmetries of the Hamiltonian cannot acquire an actual value, since this actualization would break the symmetry of the system in an arbitrary way. Precisely, given an elemental quantum system $S : (\mathcal{O}, H)$, (i) if the Hamiltonian $H$ is zero, there is no actualization, that is, none observable acquires definite value, and (ii) if the Hamiltonian $H$ is not zero, the definite-valued observables are $H$ and the observables commuting with $H$ and having, at least, the same symmetries as $H$. Let us see how the rule works in different cases:

- The Hamiltonian $H$ does not have symmetries; this means that it is non-degenerate. In this case,

$$H |n\rangle = \omega_n |n\rangle \quad \text{with} \quad \omega_n \neq \omega_n'$$  \hspace{1cm} (12)

where $\{|n\rangle\}$ is a basis of the Hilbert space $\mathcal{H}$. Therefore, $n$ is the only good quantum number: the definite-valued observables of the system are $H$ and all the observables commuting with $H$.

- The Hamiltonian $H$ has certain symmetries that lead to energy degeneracy. In this case, $H$ can be written as in eq. (9). Therefore, for any index $i_n$ expressing the degeneracy of the eigenvalue $\omega_n$,

$$H |n, i_n\rangle = \omega_n |n, i_n\rangle \quad \Rightarrow \quad H = \sum_{n,i_n} \omega_n |n, i_n\rangle \langle n, i_n| = \sum_{n} \omega_n \sum_{i_n} |n, i_n\rangle \langle n, i_n| = \sum_n \omega_n P_n$$  \hspace{1cm} (13)

Let us consider an observable of the form

$$A = \sum_{n,i_n} a_n |n, i_n\rangle \langle n, i_n| = \sum_{n} a_n \sum_{i_n} |n, i_n\rangle \langle n, i_n| = \sum_n a_n P_n$$  \hspace{1cm} (14)

It is clear that $[H, A] = 0$. Moreover, $A$ has, at least, the same degeneracy of $H$ since the subspace spanned by the degenerate eigenvectors corresponding to $a_n$ is the same as that
spanned by the degenerate eigenvectors corresponding to $\omega_n$. In other words, $A$ has the same symmetries as $H$. Therefore, $H$ and all the observables $A$ commuting with $H$ and having the form of eq. (14) are definite-valued. On the contrary, e.g. observables of the form

$$B = \sum_{n,i} b_{n,i} |n,i\rangle \langle n,i|$$

in spite of commuting with $H$, do not acquire definite values, since the actualization of a particular eigenvalue of $B$ would discriminate among the degenerate eigenvectors corresponding to a single eigenvalue of $H$ and, in this way, would introduce in the system an asymmetry not contained in the Hamiltonian.

As we have said, Bub (\cite{3}) classifies the realist non-collapse interpretations in terms of the observable $R$ by means of which the preferred basis is defined. In particular, each interpretation selects a sublattice $\mathcal{D}(\langle e |, R)$ of the complete lattice of quantum propositions, where $|e\rangle$ is the instantaneous state of the system: in this sublattice, truth values can be assigned and standard Kolmogorov probabilities can be defined. But since, in general, the sublattice depends on the instantaneous state $|e\rangle$, it changes with time. This means that the set of definite-valued observables, that is, the set of properties that acquire actual values, is different at each time as the instantaneous state of the system evolves dynamically. This result not only defies intuitions (a system having, say, position but not momentum at $t$, and momentum but not position at an infinitesimal time later), but also leads to the need of accounting for the dynamics of actual properties (see \cite{18}, \cite{19}). In our interpretation, on the contrary, this step is unnecessary because the dynamics of actual properties is trivial. In fact, since in any case the definite-valued observables commute with the Hamiltonian, they are constants of motion of the system: in spite of the fact that probabilities are continuously evolving, the set of definite-valued observables is time-independent and, thus, completely robust. This supplies a more intuitive picture of the behavior of a quantum system: actualization occurs only once, with the constitution of the system as such, and since then the definite properties are the same at any time, up to the time when the system “disappears” as that particular system by interacting with another system. This picture is consistent with the well known fact that, if the energy of a quantum system is completely definite, time is completely indefinite; therefore, the search of a definite time when actualization occurs in a closed, constant-energy system is senseless.

4. The measurement problem
As we have stressed in the previous section, since the Actualization Rule is an interpretational postulate that cannot be inferred from the formalism, it has to be assessed by its ability to solve the interpretation difficulties of the theory. Among them, the measurement problem is one of the main challenges.

In the standard model (von Neumann), a quantum measurement is conceived as an interaction between a system $S$ and a measuring apparatus $M$. Before the interaction, $M$ is prepared in a ready-to-measure state $|r_0\rangle$, eigenvector of the pointer observable $R$ of $M$, and the state of $S$ is a superposition of the eigenstates $|a_i\rangle$ of an observable $A$ of $S$. The interaction introduces a correlation between the eigenstates $|a_i\rangle$ of $A$ and the eigenstates $|r_i\rangle$ of $R$ (see \cite{20}):

$$|\Psi_0\rangle = \sum_i c_i |a_i\rangle \otimes |r_0\rangle \quad \rightarrow \quad |\Psi\rangle = \sum_i c_i |a_i\rangle \otimes |r_i\rangle$$

The problem consists in explaining why, being the state $|\Psi\rangle$ a superposition of the $|a_i\rangle \otimes |r_i\rangle$, the pointer $R$ acquires a definite value.

In the orthodox collapse interpretation, the pure state $|\Psi\rangle$ is assumed to “collapse” to one of the components of the superposition, say $|a_k\rangle \otimes |r_k\rangle$, with probability $|c_k|^2$ (see \cite{21}):
\begin{equation}
|\Psi\rangle = \sum_i c_i |a_i\rangle \otimes |r_i\rangle \implies |a_k\rangle \otimes |r_k\rangle| \tag{17}
\end{equation}

Then, it is supposed that, after detection, the state of the measuring apparatus is the eigenvector $|r_k\rangle$ of the pointer $R$ and, as a consequence, $R$ acquires the definite value $r_k$, that is, the eigenvalue corresponding to $|r_k\rangle$. In the realist non-collapse interpretations, the problem is to explain the definite reading of the pointer (with its associated probability) without assuming the collapse hypothesis. In our case, the Actualization Rule is what must accomplish this task.

Let us suppose that we want to obtain the coefficients of the state $|\Psi_S\rangle$ of the elemental quantum system $S : (O_S, H_S)$,
\begin{equation}
|\Psi_S\rangle = \sum_i c_i |a_i\rangle \in \mathcal H_S \tag{18}
\end{equation}

where $\{|a_i\rangle\}$ is a basis of $\mathcal H_S$ and $A = \sum_i a_i |a_i\rangle \langle a_i|$. For simplicity, we will assume that the Hamiltonian $H_S$ is non-degenerate: $H_S = \sum_i \omega_{S_i} |\omega_{S_i}\rangle \langle \omega_{S_i}|$. The measuring apparatus is an elemental quantum system $M : (O_M, H_M)$ with an observable $R \in O_M$, which must possess different and macroscopically distinguishable eigenvalues $r_i$ in order to play the role of the pointer: $R = \sum_i r_i |r_i\rangle \langle r_i|$. Since the apparatus is a macroscopic system, its Hamiltonian is the result of the interaction among a huge number of degrees of freedom. It is quite clear that symmetry decreases with complexity: the probability that the Hamiltonian of a macroscopic system have some symmetry is vanishing small. Therefore, it can be assumed, with no loss of generality, that the Hamiltonian $H_M$ of $M$ is non-degenerate: $H_M = \sum_i \omega_{M_i} |\omega_{M_i}\rangle \langle \omega_{M_i}|$. Moreover, by construction of the apparatus, the pointer $R$ commutes with $H_M$ for the eigenvectors $|r_i\rangle$ to be stationary and, therefore, the reading of $R$ to be possible.\textsuperscript{3} $M$ is prepared in a ready-to-measure state $|r_0\rangle$, eigenvector of $R$:
\begin{equation}
|\Psi_M\rangle = |r_0\rangle \in \mathcal H_M \tag{19}
\end{equation}

Then, at time $t_0 = 0$, the Hamiltonian of the composite system $S \cup M$ is $H = H_S \otimes I_M + I_S \otimes H_M$ and its state is
\begin{equation}
|\Psi_0\rangle = |\Psi_S\rangle \otimes |\Psi_M\rangle = \sum_i c_i |a_i\rangle \otimes |r_0\rangle \in \mathcal H_S \otimes \mathcal H_M \tag{20}
\end{equation}

In a second, interaction stage, $S$ and $M$ interact through an interaction Hamiltonian $H_{int}$. The total Hamiltonian of $S \cup M$ is
\begin{equation}
H = H_S \otimes I_M + I_S \otimes H_M + H_{int} \tag{21}
\end{equation}

$H$ introduces a correlation between the $|a_i\rangle$ and the $|r_i\rangle$ in such a way that, when the interaction ends, the state is
\begin{equation}
|\Psi\rangle = \sum_i c_i |a_i\rangle \otimes |r_i\rangle \tag{22}
\end{equation}

In the third stage, the interaction ends and the Hamiltonian is again $H = H_S \otimes I_M + I_S \otimes H_M$. Therefore, in spite of the fact that the initial state of this stage is a correlated, entangled state $|\Psi\rangle$, $S$ and $M$ turn out to be again subsystems of the composite system $S \cup M$ (see Section II); therefore, we can apply the Actualization Rule to each one of them.

- In the apparatus $M$, the initial state of the third stage, $\rho_M = Tr_S |\Psi\rangle \langle \Psi|$, evolves unitarily under the action of $H_M$. Nevertheless, the preferred context is time-invariant: since $H_M$ is the non-degenerate Hamiltonian of a macroscopic system and $[H_M, R] = 0$, both $H_M$ and $R$ are definite-valued.

\textsuperscript{3} In the highly improbable case that $H_M$ were degenerate, $H_M = \sum_i \omega_{M_i} |\omega_{M_i}, \mu\rangle \langle \omega_{M_i}, \mu|$, it would be sufficient that the pointer have the same symmetries (the same degeneracy) as $H$: $R = \sum_i r_i |r_i, \mu\rangle \langle r_i, \mu|$. 

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• The system $S$ also evolves unitarily under the action of its Hamiltonian $H_S$, and the preferred context is defined by $H_S$. However, two cases have to be distinguished. If $[H_S, A] = 0$, then both $H_S$ and $A$ may have definite values. But if $[H_S, A] \neq 0$, the observable $A$ is not definite-valued.

The fact that in certain situations the observable $A$ of the system $S$ may have no definite value turns out to be non-problematic when we recall that the goal of a quantum system is not to "discover" the value of the observable $A$, but to know the coefficients $|c_i|^2$ of the system's state $|\Psi_S\rangle$. In fact, the $|c_i|^2$ can be obtained by registering the frequencies of detection of each eigenvalue of $R$, since

$$\rho_M = Tr(S(|\Psi\rangle\langle\Psi|) = \sum_{ij} c_i c_j^* |r_i\rangle\langle r_i|$$

and

$$Pr(r_i) = \langle r_i | \rho_M | r_i \rangle = |c_i|^2$$

In summary, according to our interpretation, no matter whether the system’s observable acquires a definite value or not, the apparatus’ pointer is always definite-valued, and the frequencies of those definite values provide us the correct coefficients of the system’s state.4

5. Solving old problems

Having rejected the collapse hypothesis to solve the measurement problem, many non-collapse interpretations were specifically designed to supply an alternative answer to the problem. Usually, those attempts work with no difficulties in the case of ideal measurements, but run into troubles when faced to non-ideal measurements. This fact cannot be ignored, since ideal measurement is a situation that can never be achieved in practice: the interaction between the measured system and the measuring apparatus never introduces an absolutely perfect correlation. In spite of this, successful measurements are commonly performed in real experiments. So, we are committed to show that our interpretation is able to account for quantum measurements even in non-ideal situations.

Let us consider the general case where the correlation introduced by $H_{int}$ is not perfect; then, the initial state of $M$ at the third stage of the measurement process is not given by eq. (22), but by

$$|\Psi\rangle = \sum_{ij} d_{ij} |a_i\rangle \otimes |r_j\rangle$$

In the interpretations where the selection of the preferred basis depends on the instantaneous state of the system, the imperfect correlation may represent a very serious challenge. For instance, the rule of property ascription based on the Schmidt theorem ([8], [18]), when applied to non-ideal measurements, leads to results that disagree with those obtained in the orthodox collapse interpretation (see [22], [23]). This fact has been considered as a “silver bullet” for killing the interpretations based on the Schmidt decomposition (see [24]).

In the case of our interpretation, on the contrary, the imperfect correlation is not an obstacle for the definite value of the pointer $R$. In fact, the initial state of $M$ at the third stage of the process reads

$$\rho_M = Tr(S)|\Psi\rangle\langle\Psi| = \sum_{ij} \rho_{Mij} |r_i\rangle\langle r_j|$$

4 When the initial state of $S$ is not pure but mixed, $\rho_S = \sum_{ij} \rho_{ij} |a_i\rangle\langle a_j|$, this procedure will supply only the diagonal coefficients $|\rho_{ii}|^2$. If we want to know the remaining coefficients $\rho_{ij}$ with $i \neq j$, we have to perform further measurements with different experimental arrangements (see [10]).
where
\[ \rho_{Mij} = \sum_n d_{ni} d_{nj}^* \]  

(27)

Nevertheless, since \( H_M \) and \( R \) are non-degenerate and \( [H_M, R] = 0 \), both \( H_M \) and \( R \) are definite-valued. In this case, the probability of each eigenvalue \( r_i \) of \( \rho \) is computed as
\[ \Pr(r_i) = \langle r_i | \rho_M | r_i \rangle = \rho_{Mii} = \sum_n |d_{ni}|^2 = |d_{ii}|^2 + \sum_{n \neq i} |d_{ni}|^2 \]  

(28)

which reduces to the ideal measurement case when \( d_{ni} = 0 \), for \( n \neq i \). In the non-ideal case, two situations can be distinguished. When the \( d_{ni} \), with \( n \neq i \), are small (\( \sum_{n \neq i} |d_{ni}|^2 \ll |d_{ii}|^2 \)), then \( \rho_{Mii} \simeq |d_{ii}|^2 \) and the diagonal coefficients of the system’s state can be approximately obtained. But when the \( d_{ni} \), with \( n \neq i \), are not small, the frequencies of the pointer’s values do not supply the desired coefficients and the measurement is non-reliable.

Summing up, our interpretation can account for the fact that perfect correlation is not a necessary condition for “good” measurements: if the reliability condition of small cross-terms is satisfied, the coefficients of the system’s state at the beginning of the process can be approximately computed even when the correlation is not perfect. Nevertheless, both in the reliable and in the non-reliable case, in each measurement a definite reading of the pointer is obtained.

Another argument that stresses the difficulties introduced by non-ideal measurements is that posed by Elby (25) in the context of the Stern-Gerlach experiment. This argument points to the fact that the wavefunctions in \( z \) variable typically have infinite “tails” that introduce non-zero cross-terms; therefore, the “tail” of the wavefunction of the “down” beam may produce detection in the upper detector, prepared to detect \( p_+ \), and vice versa.

Let us consider this new argument in detail by supposing that the imperfection is due to a non-perfect collimation of the incoming beam. In this case, with the magnetic field still turned off, we would obtain a diffuse spot instead of a definite point on the screen. Therefore, the perfect ready-to-measure state \(|r_0\rangle\) has to be replaced with a narrow gaussian \(|\varphi_0(z)\rangle\). As a consequence, the measurement process turns out to be expressed as
\[ |\Psi_0\rangle = (c_1 | \uparrow \rangle + c_2 | \downarrow \rangle) \otimes |\varphi_0(z)\rangle \longrightarrow |\Psi\rangle = c_1 | \uparrow \rangle \otimes |\varphi_+ (z)\rangle + c_2 | \downarrow \rangle \otimes |\varphi_-(z)\rangle \]  

(29)

where now \(|\varphi_+(z)\rangle\) and \(|\varphi_-(z)\rangle\) are gaussians that do not need to be as narrow as the initial one. Let us call the widths of the upper and the lower detectors \( \Delta z_+ \) and \( \Delta z_- \) respectively. Thus, the long tail of the gaussian \(|\varphi_+(z)\rangle\) arrives to \( \Delta z_- \) and the long tail of the gaussian \(|\varphi_-(z)\rangle\) arrives to \( \Delta z_+ \). We can compute the probabilities corresponding to the four possible cases:

\[ \text{Pr}(\uparrow, +) = \langle (|\uparrow \rangle \otimes \langle \varphi_+(z)|)|\Psi\rangle^2 = |c_1|^2 \int_{\Delta z_+} ||\varphi_+(z)||^2 \, dz = |c_{11}|^2 \]  

(30)

\[ \text{Pr}(\uparrow, -) = \langle (|\uparrow \rangle \otimes \langle \varphi_-(z)|)|\Psi\rangle^2 = |c_1|^2 \int_{\Delta z_-} ||\varphi_-(z)||^2 \, dz = |c_{12}|^2 \]  

(31)

\[ \text{Pr}(\downarrow, +) = \langle (|\downarrow \rangle \otimes \langle \varphi_+(z)|)|\Psi\rangle^2 = |c_2|^2 \int_{\Delta z_+} ||\varphi_+(z)||^2 \, dz = |c_{21}|^2 \]  

(32)

\[ \text{Pr}(\downarrow, -) = \langle (|\downarrow \rangle \otimes \langle \varphi_-(z)|)|\Psi\rangle^2 = |c_2|^2 \int_{\Delta z_-} ||\varphi_-(z)||^2 \, dz = |c_{22}|^2 \]  

(33)

These cases can be read as follows:

- \(|c_{11}|^2\) is the probability that \(|\uparrow\rangle\) be detected by \(\Delta z_+\)

\[ |c_{11}|^2 \text{ is the probability that } |\uparrow\rangle \text{ be detected by } \Delta z_+ \]
If the reliability condition $|c_{21}|^2 << |c_{11}|^2$ and $|c_{12}|^2 << |c_{22}|^2$ holds, then the collimation, even if not perfect, is good enough for the measurement since $|c_{11}|^2 \simeq |c_{12}|^2$ and $|c_{22}|^2 \simeq |c_{2}|^2$. If the original gaussian is not very narrow or the screen is placed too far from the magnet, the measurement will be non-reliable since the $c_{ij}$, with $i \neq j$, are not small enough. Nevertheless, in our interpretation we always obtain a definite reading of the pointer (a definite detection in $\Delta z_+$ or $\Delta z_-$) because the preferred basis is always the eigenbasis of $H_M$, and it commutes with the pointer.

6. Physical relevance

During the last decades, the research on the formal properties of the mathematical structure of quantum mechanics has shown a great advance: many results, unknown by the founding fathers of the theory, have been obtained, and this work has greatly improved the understanding of the deep obstacles that any interpretation must face. However, this interest in the features of the formalism has led to forget the physical content of quantum mechanics. In fact, in the last times, the realist interpretations usually rely on mathematical results and focus their attention mainly on the formal model of the measurement problem. But quantum mechanics is a physical theory that has been applied to many well known systems and by means of which an impressive amount of experimental evidence has been accounted for. Therefore, a “good” interpretation of quantum mechanics should not only face the traditional interpretational challenges of the theory, but also show its agreement with the orthodox practice of physics.

In this sense, our position moves away from the present trend in the research on the interpretation of quantum mechanics, since it places an element with a clear physical meaning, the Hamiltonian of the system, at the heart of the interpretation. This is what will allow us to argue for the physical relevance of our approach, by showing that it is consistent with many experimental results accounted for by the theory. In this section we will show how our Actualization Rule agrees with the best known results obtained in quantum experiments.

6.1. Free pointlike particle

The Hamiltonian $H$ of the free particle reads

$$H = \frac{P^2}{2m} = \frac{P_x^2 + P_y^2 + P_z^2}{2m}$$

(34)

where $P$ is the momentum operator and $m$ is the mass of the particle. This system is invariant under the full Galilean group and, in particular, under space-displacement: the three components $P_x$, $P_y$ and $P_z$ of the momentum are the generators of the symmetry and, as a consequence, constants of motion of the system. Therefore, the Hamiltonian is degenerate.

According to our Actualization Rule, $H$ acquires a definite value, and also $P^2$ since it is proportional to $H$ and, then, has the same space-displacement symmetry. Nevertheless, $P_x$, $P_y$ and $P_z$ are not definite-valued because, being the generators of the symmetry, the actualization of their values would break the symmetry of the free particle.

Of course, the three components of $P$ can be used for the theoretical description of the free particle; in fact, usually any two of them are added to $H$ to constitute a CSCO=$\{H, P_x, P_y\}$, $\{H, P_x, P_z\}$ or $\{H, P_y, P_z\}$ that defines a basis of the Hilbert space (given the functional dependence among the four magnitudes, the CSCO=$\{P_x, P_y, P_z\}$ can be equivalently used). But if we want to know the value of those observables, we have to perform a measurement on the particle. As we have seen, measurement always involves an interaction with the measured
object, which breaks the symmetry of the original system by modifying its Hamiltonian. This means that, under measurement, the particle is not free anymore, and the symmetry breaking introduced by the interaction with the measuring apparatus is what allows us to have empirical access to an observable that was a symmetry generator of the original free system.

6.2. Free particle with spin
Spin is an internal contribution to the total angular momentum and, therefore, adds further degrees of freedom to the particle (see [10], [17]): the Hilbert space is now $\mathcal{H} = \mathcal{H}_f \otimes \mathcal{H}_s$, where $\mathcal{H}_f$ is the Hilbert space of the free particle and $\mathcal{H}_s$ is the Hilbert space of the spin. In this case, the Hamiltonian is

$$H = \frac{p^2}{2m} + E_0$$

(35)

where $E_0$ can only be a multiple of $S^2$ and, then, may be conceived as an internal contribution to the energy (see [10]). But in this case the system is not elemental, because it can be decomposed into two subsystems which, by definition of subsystem (see Section 2), do not interact to each other: a free particle with Hamiltonian $H_f = \frac{p^2}{2m}$, and a spin system with Hamiltonian $H_s = kS^2$, with $k = const$. Then, the Actualization Rule has to be applied independently to each elemental subsystem.

The rule applies to the free particle subsystem, with its symmetries, as explained above. In the spin subsystem, on the contrary, $H_s$ is invariant under space-rotation: the generators of this symmetry are the three components $J_x$, $J_y$ and $J_z$ of the total angular momentum $J$. But since in this case the orbital angular momentum $L$ is zero, the total angular momentum $J = L + S$ turns out to be simply $J = S$, and the three components $S_x$, $S_y$ and $S_z$ of the spin $S$ are the generators of the space-rotation symmetry. Analogously to the case of the free particle, according to our Actualization Rule in this case $H_s$ acquires a definite value, and also $S^2$ since it is proportional to $H_s$; nevertheless, $S_x$, $S_y$ and $S_z$ are not definite-valued since they are the generators of the symmetry of the Hamiltonian.

Again, we have no direct access to the spin of a free particle. If we want to know the value of the spin, we have to introduce a magnetic field $\mathbf{B}$ of modulus $B$ in some direction, say $z$, that breaks the isotropy of space and, then, the space-rotation symmetry of the system: the Hamiltonian $H_s$ is not invariant under space-rotation anymore because now it includes the interaction $-\gamma BS_z$ that privileges a particular direction of space. In other words, we only have experimental access to the spin component $S_z$ by means of a measurement that breaks the space-rotation symmetry of the original free system: this is the usual way in which a spin component is measured in a Stern-Gerlach experiment.

6.3. Harmonic oscillator
By definition, an harmonic oscillator is a system with a quadratic potential energy, which produces a restoring force against any displacement from equilibrium that is proportional to the displacement (see [10], [17]). In one dimension, the Hamiltonian of this system is

$$H = \frac{p^2}{2m} + \frac{m\Omega^2Q^2}{2}$$

(36)

where $Q$ is the position observable and $\Omega$ is the frequency of oscillation. If the dimensionless position and momentum operators $q = (\frac{m\Omega}{\hbar})^{1/2}Q$ and $p = (\frac{1}{m\Omega})^{1/2}P$ are introduced in eq. (36), the Hamiltonian reads

$$H = \frac{1}{2}\hbar\Omega (q^2 + p^2)$$

(37)
In turn, if the observable number of modes $N = a^\dagger a = \left(\frac{q-ip}{\sqrt{2}}\right)\left(\frac{q+ip}{\sqrt{2}}\right)$ is used,

$$H = \hbar \Omega \left(N + \frac{1}{2}\right)$$  \hspace{1cm} (38)

As it is well known, in this case the energy spectrum of the system and the spectrum of the observable $N$ can be obtained algebraically:

$$N \ |n\rangle = n \ |n\rangle \quad \text{with} \quad n = 0, 1, 2, ... \hspace{1cm} (39)$$

$$H \ |n\rangle = \omega_n \ |n\rangle \quad \text{with} \quad \omega_n = \hbar \Omega \left(n + \frac{1}{2}\right) \hspace{1cm} (40)$$

In fact, since $H$ has no symmetries, it is non-degenerate: the CSCO=$\{H\}$ defines a basis of the Hilbert space of the system. According to our Actualization Rule, $H$ acquires a definite value and, due to its non-degeneracy, the preferred context that it defines is the preferred basis $\{|n\rangle\}$: any observable commuting with $H$ (that is, whose eigenvectors are vectors of the preferred basis) are also definite-valued. In particular, the number of particles $N$ acquires a definite value since $[N, H] = 0$.

The harmonic oscillator has a central relevance in quantum mechanics because it provides a model for many kinds of vibrating systems. In particular, the electromagnetic field can be decomposed in terms of linearly independent modes, each one of which behaves as a harmonic oscillator usually associated to a particle; in this case, $N$ is conceived as the observable number of particles. But the point to stress here is that, in all of those vibrating phenomena, the energy of the system is the relevant physical magnitude, whose values are experimentally accessible, and our Actualization Rule accounts for this fact.

6.4. Free Hydrogen atom

The Hydrogen atom is a two-particles system consisting of an electron and a proton interacting to each other by means of a Coulombian interaction. In this case, the Hamiltonian reads

$$H = \frac{P_e^2}{2m_e} + \frac{P_p^2}{2m_e} - \frac{e^2}{|Q_e - Q_p|}$$  \hspace{1cm} (41)

where the subindexes $e$ and $p$ refer to the electron and the proton respectively. The usual strategy for solving the energy eigenvalue equation in coordinate representation is to refer the Hamiltonian to the center of mass of the system by means of a canonical transformation, and to write the resulting equation in spherical coordinates $(r, \theta, \phi)$. As it is well known, with this strategy the solution of the equation can be expressed as the product of two functions, one only dependent on $r$ and the other only dependent on the angular coordinates (see [26]):

$$\Psi(r, \theta, \phi) = R(r) \ Y(\theta, \phi)$$  \hspace{1cm} (42)

By solving the radial and the angular equations, three “good” quantum numbers are obtained: the principal quantum number $n$, the orbital angular momentum quantum number $l$ and the magnetic quantum number $m_l$. These quantum numbers correspond to the eigenvalues of the observables $H$, $L^2$ and $L_z$ respectively, where $L$ is the orbital angular momentum, $L_x$, $L_y$, $L_z$ are its components, and $L^2$ is the Casimir operator of the Lie group generated by these components:

$$H \ |n, l, m_l\rangle = \omega_n \ |n, l, m_l\rangle$$  \hspace{1cm} (43)

$$L^2 \ |n, l, m_l\rangle = l(l+1) \hbar^2 \ |n, l, m_l\rangle$$  \hspace{1cm} (44)

$$L_z \ |n, l, m_l\rangle = m_l \hbar \ |n, l, m_l\rangle$$  \hspace{1cm} (45)
with \( n = 0, 1, 2, \ldots, l < n \) and \(-l \leq m_l \leq l\). In particular, the energy eigenvalues are computed as
\[
\omega_n = -\frac{\mu e^4}{2\hbar^2 n^2}
\]
where \( \mu = (1/m_e^2 + 1/m_p^2)^{-1} \) is the reduced mass of the atom. Therefore, the Hydrogen atom is described in terms of the basis \( \{|n, l, m_l\}\) defined by the CSCO=\( \{H, L^2, L_z\} \), and the quantum numbers \( n, l \) and \( m_l \) label the solutions \( \Psi_{nlm} \) of the energy eigenvalue equation.

It is clear that \( L^2 \) and \( L_z \) are constants of motion since they commute with \( H \). However, the Hamiltonian is degenerate due to its space-rotation invariance: since in this case \( J = L \), then \( L_x, L_y \) and \( L_z \) are the generators of the symmetry, and \( L^2 \) is the Casimir operator of this group. In other words, although \( l \) and \( m_l \) are good quantum numbers, the eigenvalues \( \omega_n \) of the Hamiltonian do not depend on them due to the symmetry of \( H \): the values of \( L^2 \) and \( L_z \) have no manifestations in the energy spectrum. According to our Actualization Rule, this means that \( L^2 \) and \( L_z \) do not acquire definite values: the only definite-valued observables of the system are \( H \) and the observables with the same space-rotation symmetry (at least, the same degeneracy) as \( H \).

In quantum chemistry, the states \( \Psi_{nlm} \) of the atom (orbitals) are labeled as \( X_\alpha \), where \( X \) is the principal quantum number \( n \), and \( \alpha \) is replaced with \( s, p, d, f, \) etc., that is, with letters corresponding to the value of the angular momentum quantum number \( l \): \( 1s, 2s, 2p, 3s, 3p, 3d, \) etc. The number \( m_l \) is not included in those labels because there is no experimental evidence for it in the free Hydrogen atom. In fact, although \( \Psi_{nlm} \) depends on the three quantum numbers, the space-rotation symmetry of the system makes the selection of \( L_z \) for completing the basis a completely arbitrary decision: we can choose \( L_x \) or \( L_y \) for obtaining an equally legitimate description of the atom. This fact justifies the prescription, derived from our Actualization Rule, of not selecting \( L_z \) as a definite-valued observable.

Nevertheless, the value of the orbital angular momentum quantum number \( l \) can be inferred from the observed energy spectrum of the Hydrogen atom, and this might be interpreted as a symptom of the definite-valuedness of \( L^2 \). However, the manifestation of the value of \( l \) requires the interaction between the atom and an electromagnetic field. The usual explanation runs as follows. Since energy transitions involve the absorption or emission of a photon (spin 1), conservation of the angular momentum requires that the atom experiences a change of 1 in its orbital angular momentum \( L \). For this reason, when a photon is absorbed by an atom in an \( s \) orbital, the atom acquires orbital momentum and makes a transition to a \( p \) orbital; when absorbed by an atom in a \( p \) orbital, the orbital momentum increases (\( p \to d \) transition) or decreases (\( p \to s \) transition), depending on the relative orientations of the photon and the atom angular momenta. But transitions \( s \to d \) or \( p \to f \) are forbidden. It is clear, then, that the value of \( l \) is obtained as a consequence of an interaction; then, the system is not the free Hydrogen atom anymore. The new system has a Hamiltonian of the form
\[
H = H_{at} + H_{em} + H_{int}
\]
where \( H_{at} \) is the Hamiltonian of the free Hydrogen atom, \( H_{em} \) is the Hamiltonian of the electromagnetic field (which can be computed as the infinite sum of the Hamiltonians of the independent harmonic oscillators corresponding to the infinite modes of the field), and \( H_{int} \) is the interaction Hamiltonian (see [10], p. 548)
\[
H_{int} \simeq H_D = -DE
\]
where \( D \) is the dipole moment observable of the atom, and \( E \) is the electric field observable. The interaction removes the energy degeneracy in the quantum number \( l \) of the free atom: this fact is what leads to the manifestation of the value of \( l \) in the energy spectrum.
This does not mean that the eigenvalues of the observable $L^2$ never become actual in a free atom. The particular features of the Hydrogen atom strongly depend on the Coulombian potential generated by its one-proton nucleus. In more complex atoms, the potential is not perfectly Coulombian, and this fact removes the degeneracy of the Hamiltonian in $l$: the energy eigenvalues $\omega_{nl}$ turn out to be functions of both the quantum numbers $n$ and $l$. This means that $L^2$ does not discriminate among the degenerate eigenvalues corresponding to a single energy eigenvalue anymore, but rather it removes the degeneracy of the Coulombian case. According to our Actualization Rule, this means that in this case $L^2$ is a definite-valued observable of the system.

6.5. Zeeman effect
When an external magnetic field is applied to the atom, the spectral lines split into multiple closely spaced lines. First observed by P. Zeeman in 1896, this phenomenon is known as Zeeman effect.

As we have seen, in a free atom the energy eigenvalues are not functions of the magnetic quantum number $m_l$, either in the Coulombian or in the non-Coulombian potential case: in both situations, $m_l$ corresponds to a degeneracy of the Hamiltonian resulting from its rotation-symmetry. It is precisely this symmetry what makes the selection of $L_z$ for completing the basis of the Hilbert space, instead of $L_x$ or $L_y$, an arbitrary decision. The arbitrariness in the selection of the $z$-direction agrees with the fact that there is no experimental evidence of $m_l$ in the energy spectrum.

A magnetic field $B$ along the $z$-axis breaks the isotropy of space and, as a consequence, the complete space-rotation symmetry of the Hamiltonian. In turn, the breaking of the symmetry removes the degeneracy of $H$ in $m_l$: now $L_z$ is not arbitrarily chosen but it is selected by the direction of the magnetic field. But now the atom is not free anymore; the Hamiltonian of the new system is approximately (see [10], [27])

$$H = H_{at} + \frac{e}{2m_e c}BL \tag{49}$$

The original degeneracy of the $(2l+1)$-fold multiplet of fixed $n$ and $l$ is now removed: the energy levels turn out to be displaced by an amount

$$\Delta \omega_{nlm_l} = \frac{\hbar B}{2m_e c}m_l \tag{50}$$

This means that the Hamiltonian, with eigenvalues $\omega_{nlm_l}$, is non-degenerate and, therefore, it constitutes by itself the CSCO=$\{H\}$ that defines the preferred basis $\{|n,l,m_l\}\}$. According to our Actualization Rule, in this case $H$ and all the observables commuting with $H$ are definite-valued: since this is the case for $L^2$ and $L_z$, both observables will acquire definite values, in agreement with the experimental evidence of the Zeeman effect.

6.6. Fine structure
When the spectral lines of the Hydrogen atom corresponding to $n > 1$ are examined at a very high resolution, they are found to be closely spaced doublets. This splitting was one of the first experimental evidences of electron spin. In fact, the energy levels of the atom are affected by the interaction between the electron spin $S$ and the orbital angular momentum $L$. Now the Hamiltonian of the system reads

$$H = H_{at} + H_S + H_{s-o} \tag{51}$$
where $H_{el}$ is again the Hamiltonian of the free Hydrogen atom, $H_s$ is the Hamiltonian of the spin, and $H_{s-o}$ is the Hamiltonian representing the spin-orbit interaction, function of the product $L.S$. When the spin-orbit interaction is taken into account, the observables $L_z$ and $S_z$ no longer commute with $H$ and, therefore, they are not constants of motion: it is usually said that $m_l$ and $m_s$ are not good quantum numbers anymore (see [27, 28]). Nevertheless, the system is still invariant under spatial rotation: the components $J_x$, $J_y$ and $J_z$ of the total angular momentum $J$ are the generators of this symmetry, and $J^2$ is the Casimir operator of this group. In turn, $J$ is the vectorial sum of the orbital angular momentum $L$ and the spin angular momentum $S$:

$$J = L + S \quad m_j = m_l + m_s$$  \hspace{1cm} (52)

So, now $m_j$ is a good quantum number. But we also know that

$$J^2 = (L + S)^2 \quad \Rightarrow \quad L.S = \frac{J^2 - L^2 - S^2}{2}$$  \hspace{1cm} (53)

Therefore, $H_{s-o}$ is a function of $J^2$, $L^2$ and $S^2$, and the corresponding quantum numbers $j$, $l$ and $s$ are also good quantum numbers. As a consequence, the eigenvalues of the total Hamiltonian have the general form (see [16], pp. 181-183)

$$\omega_{nljs} = \omega_{nl} + \xi(nl) [j(j + 1) - l(l + 1) - s(s + 1)]$$  \hspace{1cm} (54)

where the $\omega_{nl}$ represent the energy eigenvalues with no spin-orbit coupling, and $\xi$ is a function of $nl$. Then, the basis of the Hilbert space of the system is defined by the CSCO=$\{H, L^2, J^2, S^2, J_z\}$, where

$$H |n, l, j, s, m_j\rangle = \omega_{nljs} |n, l, j, s, m_j\rangle$$  \hspace{1cm} (55)

$$L^2 |n, l, j, s, m_j\rangle = l(l + 1) \hbar^2 |n, l, j, s, m_j\rangle$$  \hspace{1cm} (56)

$$J^2 |n, l, j, s, m_j\rangle = j(j + 1) \hbar^2 |n, l, j, s, m_j\rangle$$  \hspace{1cm} (57)

$$S^2 |n, l, j, s, m_j\rangle = s(s + 1) \hbar^2 |n, l, j, s, m_j\rangle$$  \hspace{1cm} (58)

$$J_z |n, l, j, s, m_j\rangle = m_j \hbar |n, l, j, s, m_j\rangle$$  \hspace{1cm} (59)

It is quite clear that the spin-orbit coupling removes the original degeneracy of the eigenvalues $\omega_{nl}$ of the atom with no coupling. Therefore, in this case our Actualization Rule selects $L^2$, $J^2$ and $S^2$ as definite-valued observables, because all of them commute with $H$ and have the same symmetry in $m_j$ as $H$. But the rotation symmetry, still present in the system, leads to a degeneracy manifested by the fact that the energy eigenvalues $\omega_{nljs}$ do not depend on $m_j$. Then, according to our Actualization Rule, although in this case $m_j$ is a good quantum number, $J_z$ does not acquire a definite value, and this result agrees with the arbitrariness in the selection of the $z$-direction for $J_z$.

As we have seen, when a magnetic field is applied to the atom, the spectral lines split in different ways. The “normal” Zeeman effect explained above is observed in spin 0 states where, obviously, the spin-orbit coupling has no effect. In the states where the spin-orbit coupling is effective, the action of the magnetic field produces a further splitting of the energy levels known as “anomalous” Zeeman effect. Nevertheless, the explanation of the “anomalous” effect is the same as that of the “normal” effect: the action of the magnetic field breaks the full space-rotation symmetry of the system by selecting a $z$-direction, and this leads to the removal of the original degeneracy of the Hamiltonian in the quantum number $m_j$ (instead of the quantum number $m_l$ as in the “normal” effect). In this case, our Actualization Rule prescribe that $J_z$ will be also definite-valued.
6.7. Definite position

Our Actualization Rule endows the Hamiltonian of the system with the role of selecting the preferred context and, therefore, the energy of the system always acquires an actual definite value. But this does not mean that the momentum becomes actual in any case, since it does not always commute with the Hamiltonian. In fact, when a system is affected by a scalar field, its Hamiltonian has the general form

\[ H = \frac{p^2}{2m} + V(Q) \] (60)

When the mass of the system is small, the kinetic term prevails over the potential term, and the Hamiltonian approximately commutes with the momentum operator. In turn, for very large masses, the kinetic term can be neglected and \( H \) approximately commutes with the position operator. So, according to our rule, "small" systems actualize in momentum and "large" systems actualize in position. In this sense, our interpretation supports the usual physical assumption that electrons have definite momentum but not position, and nucleus have definite position but not momentum. In general, our interpretational postulate explains the fact that macroscopic systems always posses a definite value of position.

This point has a particular relevance in molecular chemistry, where the description of molecules is based on the adiabatic separation of electron and nuclear motions. As it is well known, the Born-Oppenheimer approximation conceives the nuclei as classical-like particles, that is, as precisely localized objects. This approximation strategy of holding the nucleus at rest in a definite position can be thought off as formally arising from making the masses of the nuclei infinite. However, from a strictly quantum mechanics viewpoint, without a rule for selecting the definite-valued observables of the system, the assumption of infinite nuclear masses does not explain yet why the nucleus can be treated as having a definite value of position. As Primas says ([29], p. 13), “we hardly understand why the Born-Oppenheimer picture is compatible with the concepts of quantum mechanics” (see also [30], [31]).

Our interpretation provides a simple answer to this conceptual problem. For large masses, the Hamiltonian is approximately invariant under boost transformation and, therefore, it approximately commutes with position. As a consequence, according to the Actualization Rule the position observable acquires a definite value: this provides a conceptual justification to the Born-Oppenheimer assumption. Of course, masses are never infinite: this is what makes the Born-Oppenheimer strategy an approximation and not a precise method. But also in this sense our interpretation agrees with the usual assumptions: since the Hamiltonian perfectly commutes with position only in the infinite mass limit, only in this limit we can say with absolute precision that position actualizes; in real situations, the observable that acquires definite-value will be an observable very similar to position, but which becomes indistinguishable from position for increasing masses.

6.8. Decoherence

As it is widely accepted, quantum correlations are the main obstacle to classicality. Therefore, if the classical world has to be explained in terms of quantum mechanics, the first step is to find a process leading to the stable diagonalization of the operator describing the system: this process is known as decoherence. The subtler point of the theory of decoherence is to find a precise definition of the so-called “pointer basis”, that is, the basis of diagonalization, since the observables defined by such a basis are the candidates for classical observables.

In previous papers ([32], [33], [34], [35], [36], [37]) we have developed the self-induced approach to decoherence (SID), according to which a system with continuous or quasi-continuous energy spectrum decoheres by destructive interference. Here we will not develop the complete formalism.
of our approach: we only want to stress the compatibility between SID and the proposed interpretation of quantum mechanics. In fact, SID is a process that leads the expectation value of any physically relevant observable \( O \) of the system to a value that can be computed as if the system were in a state represented by the diagonal functional \( \rho_* \):

\[
\lim_{t \to -\infty} \langle O \rangle_{\rho(t)} = \langle O \rangle_{\rho_*}
\] (61)

Decoherence is the first step towards the classical limit when it is applied to macroscopic systems where, as we have argued above (see Section 4), we have good reasons to assume the non-degeneracy of the Hamiltonian. Under this assumption, \( \rho_* \) reads in the discrete case

\[
H |n\rangle = \omega_n |n\rangle \rightarrow \rho_* = \sum_n \rho_n |n\rangle \langle n|
\] (62)

and in the continuous case

\[
H |\omega\rangle = \omega |\omega\rangle \rightarrow \rho_* = \int \rho(\omega) |\omega\rangle \langle \omega| d\omega
\] (63)

where \( \{|\omega|\} \) is the cobasis of \( \{|\omega\rangle\langle \omega|\} \). The pointer basis is, then, the preferred basis \( \{|\omega|\} \), and it is always the eigenbasis of the Hamiltonian. This result agrees with the prescription of our Actualization Rule which, in the non-degenerate case, selects \( H \) and all the observables commuting with \( H \) as the definite-valued observables of the system.

### 6.9. Classical limit

If quantum mechanics is a fundamental theory, it should account for the behavior of any kind of systems, not only microscopic systems but also macroscopic bodies governed by classical mechanics. The problem of explaining the classical description of macroscopic systems in terms of quantum mechanics is known as “the problem of the classical limit”, and it has been one of the challenges for the interpretation of quantum mechanics since its birth.

Let us begin with considering the classical description of a system in phase space. If the system is integrable, its trajectory is defined; but when the system is non-integrable, the only observables we can guarantee to be definite-valued at any time are the global constants of motion. As we have shown in previous works ([38], [39]), the quantum correlates of the global constants of motion of classical mechanics are the observables of a CSCO containing the Hamiltonian and, therefore, they are also quantum constants of motion. In fact, given a quantum system endowed with a CSCO \( \{O_0, ..., O_N\} \), with \( O_0 = H \), by definition \( [O_i, O_j] = 0 \), for \( i, j = 0 \) to \( N \). But we know that the Wigner transformation \( symb \) turns the quantum commutator into the Moyal bracket \( \{\}_{mb} \), where (see [40])

\[
\{A(\phi), B(\phi)\}_{mb} = \{A(\phi), B(\phi)\}_{pb} + 0(\hbar^2)
\] (64)

If we call \( O_i(\phi) = symb O_i \), with \( \phi \in \mathbb{R}^{2(N+1)} \), the Moyal brackets corresponding to \( [O_i, O_j] = 0 \) are

\[
symb [O_i, O_j] = i\hbar \{O_i(\phi), O_j(\phi)\}_{mb} = i\hbar \left[ \{O_i(\phi), O_j(\phi)\}_{pb} + 0(\hbar^2) \right] = 0
\] (65)

Therefore, in the limit \( \hbar \to 0 \), \( \{O_i(\phi)\} \) is a complete set of constants of motion in involution, globally defined all over the phase space \( \mathbb{R}^{2(N+1)} \).

Nevertheless, as we have argued above, in general a macroscopic system has no symmetries and the only constant of motion is the non-degenerate Hamiltonian. In previous papers ([41], [42], [43]) we have proved that, in the non-degenerate case, when the diagonal \( \rho_* \) resulting from
decoherence (see eqs. (62) and (63)) is Wigner transformed and the classical limit $\hbar \to 0$ is applied, we obtain a classical distribution $\rho_c(\phi)$

$$\rho_c(\phi) = \int_\omega \rho(\omega) \delta(H(\phi) - \omega) d\omega$$  \hspace{1cm} (66)

where $H(\phi) = \text{symb}H$ is the Wigner transformation of the Hamiltonian. The distribution $\rho_c(\phi)$ can be interpreted as an infinite sum of classical densities infinitely peaked in the classical trajectories defined by the values $\omega$ of the classical constant of motion $H(\phi)$, and weighted by their corresponding probabilities $\rho(\omega)$.

Of course, from the classical viewpoint we can expect that the classical Hamiltonian $H(\phi)$ involved in the definition of $\rho_c(\phi)$ be definite-valued. But from the quantum viewpoint, without a rule of definite-value assignment, there are no grounds for supposing that the quantum observable $H$ acquires a definite value. Our Actualization Rule provides the desired grounds by positing that the quantum Hamiltonian is always a definite-valued observable. Therefore, our interpretation may explain the fact that the quantum Hamiltonian, after decoherence and in the macroscopic limit, manifests itself as a classical constant of motion in the phase space representation of the system.

7. Conclusions
In this paper we have proposed a new realist, non-collapse interpretation of quantum mechanics, which moves away from the prevailing trend in the subject by paying special attention to the physical relevance of the interpretation. In particular, our proposal endows the Hamiltonian of the system, systematically ignored in the traditional interpretations, with a central role: it distinguishes between systems and subsystems, and is the main ingredient in the selection of the definite-valued observables. We have shown how this interpretation, through its Actualization Rule, solves the measurement problem, both in its ideal and in its non-ideal version. Moreover, we have argued for the physical relevance of the new interpretational rule.

Finally, we want to stress the realist nature of our interpretation by showing that it satisfies the six desiderata posed by Mermin:

(i) The theory should describe an objective reality independent of the observers and their knowledge. In fact, our interpretation is based on the definition of quantum system, with no reference to observers.

(ii) The concept of measurement should play no fundamental role. In our interpretation, measurement is not a departing point, but it is explained in terms of an interpretational postulate.

(iii) The theory should describe individual systems, not just ensembles. In our interpretation, states and probabilities apply to individual systems.

(iv) The theory should describe small isolated systems without having to invoke interaction with anything external. In our interpretation, the basic entity is the quantum system, closed with respect to any interaction with other systems.

(v) Objectively real internal properties of an isolated individual system should not change when something is done in another non-interacting system. According to our interpretation, this is the case even in subsystems with entangled states.

(vi) It suffices to base the interpretation of quantum mechanics on the interpretation of objective probabilities. In fact, in our interpretation probabilities measure ontological propensities, non-reducible in terms of ignorance.

Of course, this presentation does not exhaust all the questions involved in the interpretation of quantum mechanics. Nevertheless, we consider that the advantages shown by this perspective open up a new field of research that deserves to be studied.
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