Modeling and Simulation of the Effects of Steel Casing Confinements on Formation of Explosively Formed Projectile

Li Ding\textsuperscript{1,2,*}, Jianwei Jiang\textsuperscript{2}, Jianbing Men\textsuperscript{2} and Shuyou Wang\textsuperscript{2}

\textsuperscript{1}School of Mechanical Engineering, Nanjing University of Science \& Technology, Nanjing 210094, Jiangsu, China; \textsuperscript{2}State Key Laboratory of Explosion Science and Technology, Beijing Institute of Technology, Beijing 10086, China

*Corresponding author email: dingli@njust.edu.cn

Abstract. LS-DYNA software is used to model and simulate the effects of thickness and length of steel casing on the forming status of EFP specifically, and then a theoretical model is established to explain the forming mechanism and calculate the velocity of EFP. The following conclusion can be obtained. (1) The thicker steel cased charge can provide more impulse to the liner in EFP's forming process. A formula of $f(M/C)$ is fitted to estimate the steel casing effect on the velocity of EFP; (2) In order to get an EFP with higher solidity it is not necessary for the casing to fully cover the liner and charge, and when $L_c/L_0$ reaches to 35%, the obtained value of $\eta$ can be as much as that when the casing fully cover the liner and charge with acceptable velocity range. And 65\% of the casing mass can be lightened to reduce the weight of EFP charge structure; (3) A theoretical model to calculate the velocity of cased EFP based on the theory of instantaneous detonation is put forward to explain the mechanism of EFP's forming process and calculate the velocity of EFP. The research results are significant in the design of light weight EFP warhead, and the theoretical model is easy and convenient to calculate the velocity of EFP with different steel cased charge structure, which can be used to provide reference for researchers in the design of EFP.

Keywords: EFP; Casing confinements; Numerical simulation; Velocity calculation.

1. Introduction

According to the shaped charge mechanism, Explosively Formed Projectile (EFP) makes use of explosive blasting to mould liner into a preferred penetrator. When detonation products interact with liner, the EFP liner undergoes plastic deformation to flip and form a projectile of high velocity with aerodynamically stable shape, which can be used to defeat armor, masonry and concrete targets effectively from a considerable distance\cite{1}.

The EFP casing provides confinement for the explosive charge. The addition of mass around the explosive and liner increases the duration of the explosive impulse and hence increase the total energy delivered to the liner\cite{2}. Many scholars have carried out research on casing parameter's effect on the forming property of EFP. The parameters include the thickness, material and structure of casing\cite{3-7}. But only few papers talked about the length of casing and analytical model to calculate the impulse of velocity influenced by the casing systematically.

In this paper, LS-DYNA software is used to simulate the forming characteristics of EFP with different casing confinement conditions. A formula about $f(M/C)$ is put forward to evaluate the effect of the casing on EFP’s forming velocity and a theoretical model is derived based on the theory of instantaneous detonation. The theoretical model is easy and convenient to calculate and predict the
velocity of EFP with different steel casings of different M/C ratios, which can be used to provide reference for the researchers in the design of EFP charge structure.

2. Numerical Modeling and Simulation Scheme

As shown in Fig. 1 (a), a typical EFP charge structure is composed of explosive charge, hemispherical liner and casing. The explosive charge is made of JH-2 explosive, with diameter CD is 56mm and length $L_E$ equals to 0.8CD. The material of the liner is OFHC, and the liner is designed with constant wall thickness $h$ which equals to 0.06CD. As the outer curvature and liner curvature are noted as $R_o$ and $R_i$, the wall thickness $h$ equals to the value of $R_o-R_i$. The casing is made of steel, with thickness $\delta$ and length $L_C$ from the bottom of liner. The maximum length of the casing can reach $L_0$, which will cover both the explosive and liner. Center point initiation method is used in the simulation. Fig. 1 (b) illustrates the simulation model of EFP structure.

![Figure 1. Scheme and simulation model of EFP structure](image)

LS-DYNA Finite Element Software is chosen to simulate the forming process of EFP. Casing, explosive charge and liner are modeled with Lagrange algorithm in simulation, meshed with axisymmetric solid-area weighted (NO.14) shell elements. The 1/2 model about y axis axisymmetric of 2D numerical simulation is carried out. The mesh size is about 0.5mm per grid. The material models of the EFP charge structure components are listed in Tab.1. The JWL equation of state parameters of JH-2 are listed in Tab.2, and the equation of state and constitutive model parameters are listed in Tab.3 and Tab.4.

| Components | Material | Density $\rho$ (g/cm$^3$) | Equation of State | Constitutive Model | Failure Model |
|------------|----------|---------------------------|-------------------|-------------------|---------------|
| Liner      | OFHC     | 8.96                      | Grüneisen         | Johnson-Cook      | Johnson-Cook  |
| Charge     | JH-2     | 1.71                      | JWL               | High-Explosive-Burn |               |
| Casing     | Steel    | 7.83                      | Grüneisen         | Johnson-Cook      |               |

| Explosive | $D$(m/s) | $A$(GPa) | $B$(GPa) | $R_1$ | $R_2$ | $\omega$ | $E_0$(J/mm$^3$) |
|-----------|----------|----------|----------|-------|-------|----------|-----------------|
| JH-2      | 8315     | 881.45   | 10.459   | 4.80  | 1.0   | 0.32     | 0.087           |

| Material | Gruneisen Coefficient | $C_1$(m/s) | $S_1$ |
|----------|-----------------------|------------|-------|
| OFHC     | 2.02                  | 3490       | 1.489 |

| Material | $\sigma_0$(MPa) | $B$(MPa) | $n$ | $C$ | $m$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ |
|----------|-----------------|----------|-----|-----|-----|-------|-------|-------|-------|-------|
| OFHC     | 90              | 292      | 0.31| 0.025| 1.09| 0.54  | 4.89  | -3.03 | 0.014 | 1.12  |

3. Numerical Results and Analysis

As the inappropriate matching relationship between the liner and shape charge, it is likely to get non-solid EFPs or hollow EFPs in the forming process. To estimate the shapes of EFPs for the
purposes of flight stability and penetration performance, the degree about solidity of an EFP is defined as \( \eta \) which equals to the ratio of solid volume to the volume determined from the maximum outer dimensions. Fig.2(a) shows the cross section sketch of an ideal solid EFP. \( L \) and \( D \) are the length and diameter of the EFP. Fig.2(b) shows the cross section sketch of a generic non-solid EFP. \( L \) and \( D \) are the length and diameter of maximum in outer dimension. \( l \) and \( d \) are the length and diameter of hollow volume. Fig.3 shows the formation of a generic rod shaped EFP. It can be seen from the figure that after 100\( \mu \)s the shape of the EFP remains the same.

![Figure 2. Cross section sketch of generic rod EFPs](image)

![Figure 3. Formation of a generic rod shaped EFP](image)

3.1. Effect of Thickness of Steel Casing on Forming Results of EFP

Formation states of EFPs in steel casing with different relative thickness are listed in Tab.5. The length of casing is set to the maximum \( L_0 \). Typical hemispherical liner with curvature of 1.07CD was chosen in the simulation. The tail of EFP is more likely to fracture in the forming process with bare EFP charge structure. With the increase of casing thickness \( \delta \), the solidity of EFP increases correspondingly. When the thickness of steel casing exceeds a certain value, the EFP is likely to neck and fracture into two parts in axial direction due to the velocity gradient caused by the casing’s excess constraint effect. When the casing is set as fixed boundary, the thickness of the casing is regarded as infinity.

**Table 5.** Formation state of EFPs in steel casing with different relative thickness \( \delta/CD \)

| \( \delta/CD \) | Forming Result | \( \delta/CD \) | Forming Result | \( \delta/CD \) | Forming Result |
|----------------|----------------|----------------|----------------|----------------|----------------|
| 0              | 0.018          | 0.018          | 0.027          | 0.027          | 0.027          |
| 0.045          | 0.063          | 0.063          | 0.089          | 0.089          | 0.089          |
| 0.089          | 0.107          | 0.107          | 0.134          | 0.134          | 0.134          |
| 0.161          | 0.179          | 0.179          | \( \infty^* \) | \( \infty^* \) | \( \infty^* \) |

*Infinite thickness is modeled by adding the fixed boundary to outer edge of the thick casing.
Fig. 4. Velocity and impulse gains of EFP at different relative thicknesses of steel casing.

With the increase of steel casing thickness $\delta$, the velocity $v$ and impulse $I$ increase respectively. Compared with the EFP bare charge structure, the maximum gain of velocity and impulse of the cased EFP can reach over 45%. It can be concluded that because of the existence of steel casing, more impulse is delivered to the EFP liner. The thicker steel cased charge can provide more impulse to the liner in EFP’s forming process.

Similar to the Gurney equation, assuming the impulse and the velocity of EFP are related to the ratio of $M/C$, where $M$ is the mass of casing covering the cambered surface of cylinder charge and $C$ is the mass of charge, $I_c$ is the impulse delivered to the liner with casing and $I_b$ is the impulse delivered to the liner with bare charge structure. Function form of $f(M/C)$ represents the casing effect, and $f(M/C) \geq 1$, then,

$$I_c = I_b \cdot f(M/C)$$  \hspace{1cm} (1)

As the mass of liner stays constant in the forming process, the velocity $v$ of EFP can be derived, where $v_c$ is the velocity of EFP with cased charge and $v_b$ is the velocity with bare charge structure.

$$v_c = v_b \cdot f(M/C)$$  \hspace{1cm} (2)

Assuming $f(M/C)$ has the form below,

$$f(M/C) = \left(\frac{a + bM}{C} / a\right)^n$$  \hspace{1cm} (3)

Then, $v_c$ has the form below,
\[ v_C = v_B \left( \left( a + b \frac{M}{C} / a \right) / a \right)^n \] (4)

In the formula, \( a \), \( b \) and \( n \) are constants depending on the properties of casing material, which can be obtained using user defined formula curve fitting in the Origin software. In this paper, \( a = 0.19117 \), \( b = 1.56484 \), \( n = 0.10615 \). Fig.5 illustrates the velocity data in simulation and the fitted curve of \( v(M/C) \), it can be seen that the fitted curve agrees well with the simulation data.

### 3.2. Effect of Steel Casing Length on Forming Results of EFP

Formation shapes of EFPs with different relative casing lengths \( L_C/L_0 \) are listed in Tab.6. In the simulation models, the steel casing thickness \( \delta \) is 0.045CD and the wall thickness of liner \( h \) is 0.006CD. The outer curvatures of the hemisphere liners are 0.893CD, 0.982CD and 1.071CD. The EFP structures with different liners could all form EFPs, though non-solid EFPs or fractures of EFPs exist.

**Table 6.** Formation shapes of EFPs with different relative casing lengths \( L_C/L_0 \)

| \( L_C/L_0 \) | Forming Results |
|--------------|----------------|
|              | \( R_o = 0.893\)CD | \( R_o = 0.982\)CD | \( R_o = 1.071\)CD |
| 0            |                |                |                |
| 25           |                |                |                |
| 50           |                |                |                |
| 75           |                |                |                |
| 100          |                |                |                |

The velocity gains of EFP at different \( M/C \) ratios are shown in Fig.6. As the increase of \( M/C \) ratio which indicates the increase of relative casing length \( L_C/L_0 \), the velocity gain increases gradually and reaches the maximum value when the relative length \( L_C/L_0 \) equals to one. And the gains of fitted curve \( f(M/C) \) are in good agreement with the velocity gain data.

The solidity \( \eta \) of EFPs with different liner curvatures at different relative casing length \( L_C/L_0 \) are shown in Fig.7. Even though the curvatures of the liners are different in the three cures, the patterns stay the same. As the increase of \( L_C/L_0 \) below 30%, the solidity \( \eta \) of EFP increases gradually; while when \( L_C/L_0 \) arrives at nearly 35%, the value of \( \eta \) almost reaches to the maximum, with \( M/C \) ratio exceeding 0.65 and \( v_C/v_B \) reaching the maximum in this condition; and with further enhancement of \( L_C/L_0 \), the value of \( \eta \) stays stable around the maximum.

So it can be concluded that, in order to get an EFP with higher solidity it is not necessary for the casing to fully cover the liner and charge, and when \( L_C/L_0 \) reaches to 35%, the obtained value of \( \eta \) can be as much as that when the casing fully cover the liner and charge with acceptable velocity range. When comes to the requirement of constrained space or light weapon system, it is significant and important to reduce the casing length to fulfill the purpose of light weight design. When the length casing reduced to 35%, 65% of the casing mass can be lighted or be replaced by other light material, thus the weight of EFP charge structure is significantly reduced.
Figure 6. Velocity gains of EFP at different M/C ratios

Figure 7. Solidity of EFPs with different liner curvatures at different relative casing lengths LC/L0

4. Theoretical Analysis

Li Bihong et al.[10] came up with an engineering model to calculate the velocity of EFP with bare charge based on the theory of instantaneous detonation. As shown in Fig.8, R is the radius of explosive charge, A is an arbitral infinitesimal liner element, with the coordinate of (x, y). The bottom point of the liner in axial direction is set as coordinate origin O, x0 is the maximum coordinate value of the liner in x axis. ρ0 is the density of explosive charge, ρ is the density of liner, y=f(x) is the liner curve formula, δ=δ(x) is the wall thickness of the liner, α is the angle between the pressing velocity and the horizontal line. The theoretical procedures are described as follows.

Figure 8. Theoretical model of bare EFP charge structure

4.1. Impulse Delivered to Infinitesimal Liner Element

For an arbitral infinitesimal liner element A, assuming that detonation pressure interacts with liner at the time 0 and vanishes at τ, then the effective duration of instantaneous detonation pressure is τ. \( P_H \) is the instantaneous detonation pressure, ds is the area of element A. Then the impulse \( I(x) \) delivered to the element A can be obtained,

\[
I(x) = \bar{P}_H \tau ds
\]  

(5)

\( \bar{P}_H \) and \( \tau \) can be calculated as below,

\[
\bar{P}_H = \frac{1}{2} P_H = \frac{\rho_0 D^2}{2(K+1)}
\]  

(6)

\[
\tau = (R-y)/W
\]  

(7)

W is the wave front velocity of detonation products, \( \mu \) is the velocity of detonation products. \( Q_V, D \) and \( K \) are the explosion heat, detonation velocity and isentropic exponent of explosive charge, respectively.

\[
W = \bar{P}_H / (\rho_0 \mu)
\]  

(8)
When formula (8) and (9) are substituted, the formula (5) can be transformed below,

$$I(x) = \int \tau ds = \rho_0 \sqrt{Q_v} (R-y) ds$$

(10)

As the element is in the shape of symmetrical cirque,

$$ds = 2\pi y dx$$

(11)

Then,

$$I(x) = 2\pi \rho_0 \sqrt{Q_v} (R-y) y ds$$

(12)

**4.2. Resultant Velocity of Liner Element**

For element A, the initial impulse is zero, and the final impulse is $m(x) \cdot v(x)$. According to the law of momentum, it can be obtained that,

$$\int I(x) = 2\pi \rho_0 \sqrt{Q_v} (R-y) y ds = m(x) v(x)$$

(13)

And,

$$m(x) = \rho ds \delta(x) = 2\pi y \rho \delta(r) dx$$

(14)

Combined with formula (13) and (14), it can be obtained that,

$$v(x) = \frac{\rho_0 \sqrt{Q_v} (R-y)}{\rho \delta(x)}$$

(15)

$v(x)$ is the pressing velocity of the liner element, And,

$$Q_v = D^2 \big/ \left(2(k^2 - 1)\right)$$

(16)

Formula (15) can be expressed as,

$$v(x) = \frac{\rho_0 (R-y)}{\sqrt{k^2 - 1} \cdot \rho \delta(x)} D$$

(17)

$\alpha$ satisfies the equation below,

$$\cot \alpha = f'(x)$$

(18)

The express velocity $v$ can be divided in $x$ and $y$ axis, then velocity components can be obtained,

$$v_x(x) = \frac{\rho_0 (R-y)}{\sqrt{k^2 - 1} \cdot \rho \delta(x)} D \cos \alpha$$

(19)

$$v_y(x) = \frac{\rho_0 (R-y)}{\sqrt{k^2 - 1} \cdot \rho \delta(x)} D \sin \alpha$$

(20)

**4.3. Forming Velocity of EFP**

As the symmetrical model of EFP, the resultant velocity in $y$ axis equals to 0, and only the velocity component in $x$ axis matters. Thus the velocity of EFP in $x$ axis can be integrated below,

$$\int_0^x m(x) v(x) = \int_0^x m(x)$$

(21)
\[
v = \left( \int_{x_0}^{x_1} m(x)v(x) \right) / \left( \int_{x_0}^{x_1} m(x) \right)
\]

(22)

\[
\int_{x_0}^{x_1} m(x) \text{ is the mass of liner, then combined with formula (14) and (19)},
\]

\[
v = \frac{\rho D}{\rho \sqrt{k^2 - 1}} \int_{x_0}^{x_1} y(R - y) \cos \alpha dx
\]

(23)

As \(y = f(x)\), \(\alpha = \arccot f'(x)\), then

\[
v = \frac{\rho D}{\rho \sqrt{k^2 - 1}} \int_{x_0}^{x_1} f(x)(R - f(x)) \cos(\arctan f(x)) dx
\]

(24)

For the hemispherical liner with constant wall thickness, \(y = f(x) = \sqrt{R_0^2 - (R_0 - x)^2}\), \(\tan \alpha = \frac{y}{R_0 - x} = \frac{\sqrt{R_0^2 - (R_0 - x)^2}}{R_0 - x}\), \(\delta = \delta(x) = \text{constant}\), \(x_0 = R_0 - \sqrt{R_0^2 - R^2}\), \(k = 3\), \(L_m\) is the length of generatrix,

\[
x_0 = R_0(1 - \cos \frac{L_m}{R_0}). \text{ Defining } \xi = \frac{R_0}{CD} = \frac{R_0}{2R}, \text{ then formula (24) can be simplified.}
\]

\[
v = \frac{\rho D R}{\rho \delta \sqrt{k^2 - 1} 3\pi - (3/\xi)(1 - (1/(2\xi))^3 - 6\arcsin(1-(1/(2\xi))^2))}
\]

(25)

The velocity of EFP in formula (25) depends only on the radius \(R\) of the charge, wall thickness \(\delta\) and density \(\rho\) of the liner and the geometric ratio of \(\xi\). The length to diameter ratio of charge should be considered, then a correction factor \(k\) and a function of \(L_e/CD\) are introduced to revise the velocity of bare EFP charge structures with different length to diameter charge ratios.

\[
v_p = k \cdot g(L_e/CD) \cdot v
\]

(26)

\[
g(L_e/CD) = (L_e/(2R))^{36}
\]

(27)

According to semi-empirical formula in[11, 12] that, the length to diameter ratio of charge’s effect can be expressed as formula (27), where \(k = 1.1\).

As the existence of casing, the duration of detonation pressure increases, then the impulse delivered to the liner increases remarkably, so it is reasonable to add the \(f(M/C)\) effect to fully depict the velocity of casing EFP charge structure.

\[
v_p = k \cdot g(L_e/(2R))^{36} \cdot f((a + b(M/C))/a) \cdot v
\]

(28)

In this paper, with the \(L/D = 0.8\) of charge structure and the steel casing structure, \(k = 1.1\), and the fitted value of \(a\) and \(b\) are still suitable in the theoretical model.

5. Conclusions

(1) With the existence of steel casing, more impulse is delivered to the EFP liner. The thicker steel cased charge can provide more impulse to the liner in EFP’s forming process. A formula about \(f(M/C)\) is put forward to estimate the steel casing’s effect on the velocity of EFP, in which the fitted curve agrees well with the simulation data both in the thickness and length effects of the steel casing.

(2) In order to get an EFP with higher solidity it is not necessary for the casing to fully cover the liner and charge, and when \(L_c/L_0\) reaches to 35%, the obtained value of \(\eta\) can be as much as that when the casing fully cover the liner and charge with acceptable velocity range. When the length casing reduced to 35%, 65% of the casing mass can be lightened, thus the weight of EFP charge structure is significantly reduced. When comes to the requirement of constrained space or constrained space or
light weapon system, it is significant and important to reduce the casing length to fulfill the purpose of light weight design.

(3) A theoretical model to calculate the velocity of cased EFP based on the theory of instantaneous detonation is put forward to explain the mechanism of EFP’s forming process and calculate the velocity of EFP.

Acknowledgements
This work is supported by the National Natural Science Foundation of China (Grant No. 11802142) and the opening project of State Key Laboratory of Explosion Science and Technology (Beijing Institute of Technology, the opening project number is KFJJ20-08M).

Reference
[1] Y. Shao-qin, Smart Munition Engineering[M]. Beijing: National Defense Industry Press, 2010.
[2] S. Shu-yuan, W. Shu-shan, Terminal Effects[M]. Beijing: National Defense Industry Press, 2000.
[3] K. Weimann. Research and development in the area of explosively formed projectiles charge technology[J]. Propellants Explosives Pyrotechnics, 1993, 18 (5):294-298.
[4] S. Pappu, L.E. Murr. Hydrocode and microstructural analysis of explosively formed penetrators[J]. Journal of materials science, 2002, 37 (2):233-248.
[5] T. Gang, H. Zhu, L. Shi. THE INFLUENCE OF THE CASING ON EFP[J]. Journal of Ballistics, 1994.
[6] J. Jianwei, Y. Jun, M. Jianbing, et al. Numerical Simulation for the Parameter Study of Explosively Formed Projectile with Aluminum Case[J]. Transactions of Beijing Institute of Technology, 2004, (11):939-941+965.
[7] T.L. Teng, Y.A. Chu, F.A. Chang, et al. Design and implementation of a high-velocity projectile generator[J]. Combustion Explosion & Shock Waves, 43 (2):233-240.
[8] J.G. R, C.W. H, A constitutive model and data for metals subjected to large strains, high strain-rates and high temperatures, in: Proceedings of the Seventh International Symposium on Ballistics, Hague, Netherlands, 1983, pp. 541.
[9] J.G. R, C.W. H. Fracture characteristics of three metals subjected to various strains, strain rates, temperatures, and pressres[J]. Engineering Fracture Mechanics, 1985, 21 (1):31-48.
[10] W. Arnold, E. Rottenkolber, Penetrator/Shaped Charge System Part II: Influence of Design Parameters, in: 23rd International Symposium on Ballistics, Tarragona, Spain, 2007, pp. 271-278.
[11] X. Zhou, L. Yuan, Y. Zhu. Influence of Case Qualities on EFP Performance[J]. Journal of Projectiles Rockets Missiles & Guidance, 2008.
[12] L. Bihong. Application of EFP technology in crumbling concrete obstacles[D]. Central South University, 2004.