Zweig-rule-satisfying inelastic rescattering in $B$
decays to pseudoscalar mesons

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Abstract

We discuss all contributions from Zweig-rule-satisfying SU(3)-symmetric
inelastic FSI-induced corrections in $B$ decays to $\pi\pi$, $\pi K$, $K\bar{K}$, $\pi\eta(\eta')$, and
$K\eta(\eta')$. It is shown how all of these FSI corrections lead to a simple re-
definition of the amplitudes, permitting the use of a simple diagram-based
description, in which, however, weak phases may enter in a modified way.
The inclusion of FSI corrections admitted by the present data allows an arbi-
trary relative phase between the penguin and tree short-distance amplitudes.
The FSI-induced error of the method, in which the value of the weak phase
$\gamma$ is to be determined by combining future results from $B^+, B^0_d, B^0_s$ decays to
$K\pi$, is estimated to be of the order of $5^\circ$ for $\gamma \approx 50^\circ - 60^\circ$.

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1 Introduction

Most of the analyses of CP-violating effects in $B$ decays deal with quark-diagram short-distance (SD) amplitudes and assume that final state interactions (FSI) are negligible. On the other hand, it has been argued that this neglect is not justified, and that any reliable analysis of $B \rightarrow PP$ decays ($P$ - pseudoscalar mesons) must take rescattering into account $[1,2]$. While only small effects of rescattering through low-lying intermediate states are generally expected, the sequence $B \rightarrow i \rightarrow PP$ with inelastic multiparticle intermediate states $i$ might in principle lead to important corrections $[1]$. The estimates of the size of such inelastic effects are model-dependent, with the contributions from different intermediate states either cancelling in an approximate way, or having random phases $[1]$, or even adding coherently $[3]$.

With our insufficient knowledge of $PP$ interactions at 5.2 $GeV$, there is virtually no hope that the rescattering effects may be reliably calculated. Under the circumstances it seems appropriate to use symmetry-based approaches and to parametrize the rescattering in terms of a few FSI-related parameters. Such an approach has been recently studied in ref. $[4]$, where SU(3) symmetry was used to reduce our ignorance of the inelastic rescattering to a small set of effective SU(3) parameters jointly describing all $i \rightarrow PP$ processes. In the present paper we limit the considerations of ref. $[4]$ to the case when the rescattering amplitudes satisfy Zweig rule.

2 Short-distance amplitudes

Short-distance decay amplitudes lead to $q\bar{q}q\bar{q}$ states which convert to hadron-level two-body states composed of various mesons $M_1$ and $M_2$ (including the pseudoscalar mesons $P_1$ and $P_2$). Transitions to many-body states occur when the $q\bar{q}$ states radiate off further quark-antiquark pairs and gluons leading to the decays of $M_i$. In order to estimate the rescattering through such many-body states, we use unitarity to replace the sum over these states with the contribution from $M_i$ themselves:

$$\sum |M_i \text{ decay products}\rangle \langle M_i \text{ decay products}| = |M_i\rangle \langle M_i|$$

(1)
As in ref.\cite{4}, we restrict our study to the case when FSI (now Zweig-rule-satisfying) are SU(3)-symmetric. Consequently, for the intermediate $M_1M_2$ states it is appropriate to use the basis in which a pair of mesons $M_1M_2$ (in most cases, heavy) forms a state belonging to a definite SU(3) multiplet. In ref.\cite{4} the amplitudes for the short-distance-driven decay of a $B$ meson into such states are expressed in terms of standard SD diagram amplitudes $T, T'$ (tree), $C, C'$ (color suppressed), $P, P'$ (penguin), $E, E'$ (exchange), $A, A'$ (annihilation), $PA, PA'$ (penguin annihilation), $S, S'$ (singlet penguin), $SS, SS'$ (double singlet penguin). As usual, strangeness-conserving $\Delta S = 0$ (strangeness-violating $|\Delta S| = 1$) processes are denoted by unprimed (primed) amplitudes. These amplitudes may be thought to incorporate the electroweak penguin contributions according to $T \rightarrow T + P_{EW}^c$, $P \rightarrow P - P_{EW}^c/3$, $C \rightarrow C + P_{EW}$, $S \rightarrow S - P_{EW}/3\) \cite{5}.

Let us first recapitulate the essential assumption of ref.\cite{4}, which permits parametrization of all FSI effects in terms of a few parameters only. Namely, since at the SD level it is not yet decided whether the particular quark-level state will hadronize as the $PP$ state or one of the heavier $M_1M_2$ states, one expects that quark-level SD amplitudes for $B$ decays into two arbitrary mesons $M_1M_2$ are proportional to the corresponding amplitudes of SD decay into two pseudoscalar mesons $P_1P_2$. The coefficient of proportionality may depend on the type of mesons in the $M_1M_2$ state (i.e. whether $M_1$ ($M_2$) are vector, axial, tensor, etc.), but - by virtue of the SU(3) symmetry - it must be the same for all $M_1, M_2$ within given SU(3) multiplets. Consequently, this coefficient may be absorbed into the FSI amplitude for the process $M_1M_2 \rightarrow P_1P_2$, whose size, due to our ignorance, must be again treated as a free parameter. In other words, $T, C, P, \ldots$-type amplitudes used for SD $B \rightarrow M_1M_2$ decays are normalized to their SD $B \rightarrow P_1P_2$ counterparts. This justifies the use of the same letter $T$ for both $T_{B \rightarrow P_1P_2} \equiv T$ and $T_{B \rightarrow M_1M_2}$, and similarly for other types of diagrams. (A part of the analysis of this paper would go through also if coefficients of proportionality between the $B \rightarrow M_1M_2$ and $B \rightarrow P_1P_2$ amplitudes depended on the type of diagram, i.e. if they were different for tree, penguin, etc. amplitudes. Since this introduces additional parameters, we do not consider this possibility further on.) When the rescattering contributions from all intermediate $M_1M_2$ states are added, they are gathered into a few groups differing in their SU(3) symmetry structure. For each such group, the SU(3) structure is factorized and then
the remaining sum of unknown free parameters is replaced with a single parameter, as discussed in [4].

The amplitudes for $B \to M_1M_2$, calculated in [4], are given here in Tables 4, 2 and 3 (in a normalization adjusted to that used normally for $B \to P_1P_2$). From these Tables the amplitude of, say, a $B^+$ decay into a pair of two mesons $M_1M_2$ in an overall antisymmetric octet state and of total isospin $1/2$ may be read of from the column marked $(8_u, 1/2)$ to be

$$\frac{1}{\sqrt{3}}(T' - C' + 3P' + 3A').$$

3 General FSI amplitudes satisfying Zweig rule

Zweig-rule-satisfying rescattering $M_1M_2 \to P_1P_2$ is described by two types of connected diagrams: the ”uncrossed” diagrams of Fig. 1(u), and the ”crossed” diagrams of Fig. 1(c). By virtue of Bose statistics, the final $P_1P_2$ pair must be in an overall symmetric state.

For the uncrossed $M_1M_2 \to P_1P_2$ diagrams, the requirement of Bose statistics for $P_1P_2$ means that there are two allowed types of SU(3) amplitudes, ie. (using a particle symbol for the corresponding SU(3) matrix):

$$\text{Tr}([M^\dagger_1, M^\dagger_2] \{P_1, P_2\}) \ u_+$$

and

$$\text{Tr}([M^\dagger_1, M^\dagger_2] \{P_1, P_2\}) \ u_-$$

where the requirement in question is reflected through the presence of the anticommutator $\{P_1, P_2\}$ of meson matrices, and $u_\pm$ denote the strength of rescattering amplitudes. Eqs.(2,3) incorporate nonet symmetry for both intermediate and final mesons. Invariance of strong interactions under charge conjugation demands that mesons $M_1$ and $M_2$ belong to multiplets of the same (opposite) C-parities for the first (second) amplitude above. Thus, the subscript of $u_\pm$ may be understood also as the value of the product of the C-parities of mesons $M_1$ and $M_2$.

For the crossed diagrams, the requirement of $P_1 \rightleftharpoons P_2$ symmetry admits only one combination:

$$\text{Tr}(M^\dagger_1 P_1 M^\dagger_2 P_2 + M^\dagger_1 P_2 M^\dagger_2 P_1) \ c$$
Table 1: SD amplitudes into two-meson octet-octet states forming a 27-plet

|       | (27,2)                  | (27,3/2)                | (27,1)                  | (27,1/2)                | (27,0)                  |
|-------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| $B^+$ | $\frac{-1}{\sqrt{2}}(T+C)$ | $\frac{1}{\sqrt{3}}(T'^{'+}C')$ | $\frac{-1}{\sqrt{10}}(T+C)$ | $\frac{2}{\sqrt{15}}(T'+C')$ | $0$                     |
| $B^0_d$| $\frac{-1}{\sqrt{3}}(T+C)$ | $\frac{1}{\sqrt{3}}(T'+C')$ | $0$                     | $\frac{1}{\sqrt{15}}(T'+C')$ | $-\frac{1}{2\sqrt{15}}(T+C)$ |
| $B^0_s$| $0$                     | $-\frac{1}{\sqrt{3}}(T+C)$ | $-\frac{1}{\sqrt{3}}(T'+C')$ | $-\frac{1}{\sqrt{15}}(T+C)$ | $\sqrt{\frac{3}{20}}(T'+C')$ |

Table 2: SD amplitudes into two-meson states in overall octet SU(3) representation for symmetric octet-octet, antisymmetric octet-octet, and (symmetric) octet-singlet combinations

|       | (8s, 1)                  | (8s, 1/2)                | (8s, 0)                  |
|-------|-------------------------|-------------------------|-------------------------|
| $B^+$ | $-\frac{1}{\sqrt{15}}(T+C+5P+5A)$ | $\frac{1}{\sqrt{15}}(T'+C'+5P'+5A')$ | $0$                     |
| $B^0_d$| $\frac{1}{\sqrt{2}}(E-P)$ | $\frac{1}{\sqrt{15}}(3T'-2C'+5P')$ | $-\frac{1}{3\sqrt{15}}(6T'-4C+5P+5E)$ |
| $B^0_s$| $\frac{1}{\sqrt{30}}(3T'^{'}+5E'-2C')$ | $-\frac{1}{\sqrt{15}}(3T'-2C+5P)$ | $\frac{1}{3\sqrt{15}}(3T'-2C'+10P'-5E')$ |

|       | (8a, 1)                  | (8a, 1/2)                | (8a, 0)                  |
|-------|-------------------------|-------------------------|-------------------------|
| $B^+$ | $-\frac{1}{\sqrt{3}}(T-C+3P+3A)$ | $\frac{1}{\sqrt{3}}(T'-C'+3P'+3A')$ | $0$                     |
| $B^0_d$| $-\frac{1}{\sqrt{2}}(2T+3P-3E)$ | $\frac{1}{\sqrt{3}}(T'+3P')$ | $-\frac{1}{\sqrt{3}}(E+P)$ |
| $B^0_s$| $-\frac{1}{\sqrt{6}}(T'-3E')$ | $-\frac{1}{\sqrt{3}}(T+3P)$ | $\frac{1}{\sqrt{2}}(T'+2P'-E')$ |

|       | (8_{s1}, 1)             | (8_{s1}, 1/2)            | (8_{s1}, 0)             |
|-------|-------------------------|-------------------------|-------------------------|
| $B^+$ | $-\frac{1}{\sqrt{3}}(T+C+2P+2A+3S)$ | $\frac{1}{\sqrt{3}}(T'+C'+2P'+2A'+3S')$ | $0$                     |
| $B^0_d$| $\frac{1}{\sqrt{6}}(-2P+2E-3S)$ | $\frac{1}{\sqrt{3}}(C'+2P'+3S')$ | $-\frac{1}{3\sqrt{2}}(2C+2P+2E+3S)$ |
| $B^0_s$| $\frac{1}{\sqrt{6}}(C'+2E')$ | $-\frac{1}{\sqrt{3}}(C+2P+3S)$ | $-\frac{1}{3\sqrt{2}}(-C'-4P'+2E'-6S')$ |

Table 3: SD amplitudes into two-meson octet-octet and singlet-singlet states forming an overall singlet

|       | (1_{ss}, 0)             | (1_{11}, 0)              |
|-------|-------------------------|-------------------------|
| $B^+$ | $0$                     | $0$                     |
| $B^0_d$| $\frac{1}{6}(3T-C+8P+8E+12PA)$ | $\frac{1}{3\sqrt{2}}(2C+2P+2E+3PA+6S+SS)$ |
| $B^0_s$| $\frac{1}{6}(3T'-C'+8P'+8E'+12PA')$ | $\frac{1}{3\sqrt{2}}(2C'+2P'+2E'+3PA'+6S'+SS')$ |
where \( c \) denotes the strength of the amplitude. This combination, symmetric under \( M_1 \leftrightarrow M_2 \), is charge-conjugation invariant if \( M_1 \) and \( M_2 \) have C-parities of the same sign. (If \( M_1 \) and \( M_2 \) have opposite C-parities, charge-conjugation-invariance requires that the ”+” sign in Eq.(4) be changed into ”−”. This leads to an expression antisymmetric under \( P_1 \leftrightarrow P_2 \), in violation of the requirement of Bose statistics.)

4 Rescattering contributions to B decays

From Eqs.(2,3,4) one can evaluate the contribution of \( u \)-type FSI diagrams with intermediate two-meson states in each of the different SU(3) representations: 27, \( 8_s \), \( 8_a \), \( 8\{81\} \), \( 1\{88\} \), and \( 1\{11\} \). The set consisting of \( 8_s \), \( 8\{81\} \), \( 1\{88\} \), and \( 1\{11\} \) originates from the intermediate states in which mesons \( M_1 \) and \( M_2 \) have the same C-parity, while the other set (ie. \( 8_a \) - in which they have opposite C-parities. Their respective sizes are measured by \( u_+ \) and \( u_- \). In the following we will use

\[
\begin{align*}
    u & \equiv \frac{(u_+ + u_-)}{2} \\
    d & \equiv u_+ - u_-
\end{align*}
\]

It was argued that it is the sum over many intermediate states that might lead to significant FSI effects. This is represented by \( u \). The terms proportional to \( d \) represent the difference of contributions from the \( C_1 C_2 = +1 \) and \( C_1 C_2 = -1 \) states. While such difference for the lowest-lying \( P_1 P_2 \) (\( C_1 C_2 = +1 \)) and \( P_1 V_2 \) (\( C_1 C_2 = -1 \), \( V \)-vector meson) states may be important in itself, these are just two of many possible intermediate states. Thus, hopefully, the contribution of the lowest states to the difference in question is not large. For heavier \( M_1 M_2 \) states, one may expect that the difference between the contributions from many neighbouring and overlapping \( C_1 C_2 = +1 \) and \( C_1 C_2 = -1 \) intermediate states is small. Thus, unless the few lowest-lying intermediate states strongly violate the expected approximate equality of the \( C_1 C_2 = +1 \) and \( C_1 C_2 = -1 \) contributions, \( |d| \) should be much smaller than \( |u| \).

Summation over all SU(3) representations in the s-channel yields expressions for the combined contributions induced by all uncrossed and all crossed FSI diagrams. For the decays \( B \rightarrow \pi\pi, \pi K, K\bar{K} \), these contributions are given in Tables 4 and 5.

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Table 4: Contributions to $\Delta S = 0$ decays $B \to \pi\pi, \pi K, K\bar{K}$

| decay            | SD                  | u-type FSI diagrams | c-type FSI diagrams |
|------------------|---------------------|---------------------|---------------------|
| $B^+ \to \pi^+\pi^0$ | $-\frac{1}{\sqrt{2}}(T + C)$ | 0                   | $-\frac{1}{\sqrt{2}}(T + C) \cdot 2c$ |
| $B^+ \to K^+\bar{K}^0$ | $-P$                | $-(C \cdot 2u + (T + 3P)d)$ | 0                   |
| $B_d^0 \to \pi^+\pi^-$ | $-(T + P)$          | $-((T + 2P) \cdot 2u + (T + 3P)d)$ | $-C \cdot 2c$       |
| $B_d^0 \to \pi^0\pi^0$ | $-\frac{1}{\sqrt{2}}(C - P)$ | $\frac{1}{\sqrt{2}}((T + 2P) \cdot 2u + (T + 3P)d)$ | $-\frac{1}{\sqrt{2}} T \cdot 2c$ |
| $B_d^0 \to K^+K^-$ | 0                   | $(T + 2P) \cdot 2u$            | 0                   |
| $B_d^0 \to K^0\bar{K}^0$ | $-P$                | $-(2P \cdot 2u + (T + 3P)d)$ | 0                   |
| $B_s^0 \to \pi^+K^-$ | $-(T + P)$          | $-(T + 3P)d$            | $-C \cdot 2c$       |
| $B_s^0 \to \pi^0\bar{K}^0$ | $-\frac{1}{\sqrt{2}}(C - P)$ | $\frac{1}{\sqrt{2}}(T + 3P)d$ | $-\frac{1}{\sqrt{2}} T \cdot 2c$ |

Relationship between $u_+, u_-, c$, and the parameters defined in ref.[4] is as follows:

\[
\begin{align*}
    f_{27} &= 2c \\
    f_s &= \frac{5}{3} u_+ - \frac{4}{3} c \\
    f_a &= -5u_- \\
    f_{88} &= \frac{16}{3} u_+ - \frac{2}{3} c \\
    f_{18} &= \frac{10}{3} u_+ + \frac{10}{3} c \\
    f_{11} &= \frac{4}{3} u_+ + \frac{4}{3} c
\end{align*}
\]

or, equivalently,

\[
\begin{align*}
    \Delta_1 &= 5d \\
    \Delta_2 &= 15d - 10c \\
    \Delta_3 &= 10u \\
    \Delta_4 &= 20u + 10d - 10c \\
    \Delta_5 &= 60u + 30d - 20c
\end{align*}
\]

In the $\Delta S = 0$ decays, we keep only the terms proportional to $T$, $C$, and $P$, since these are expected to be the leading ones. In the $|\Delta S| = 1$ decays, the contributions from the singlet penguin $S'$ are shown as well.

For completeness, it is appropriate to discuss the decays into $\pi\eta, \pi\eta', K\eta$, and $K\eta'$ as well. FSI-induced contributions to these decays are gathered in Tables 6 and 7.
Table 5: Contributions to $|\Delta S = 1|$ decays $B \to \pi\pi, \pi K, K\bar{K}$

| decay          | SD     | u-type FSI diagrams                                         | c-type FSI diagrams  |
|----------------|--------|-------------------------------------------------------------|----------------------|
| $B^+ \to \pi^+ K^0$ | $-P'$  | $-(T' + 3P' + S')d + (C' + S') \cdot 2u$                   | $-S' \cdot 2c$       |
| $B^+ \to \pi^0 K^+$ | $\frac{1}{\sqrt{2}}(T' + C' + P')$ | $\frac{1}{\sqrt{2}}((T' + 3P' + S')d + (C' + S') \cdot 2u)$ | $\frac{1}{\sqrt{2}}(T' + C' + S') \cdot 2c$ |
| $B^0_d \to \pi^- K^+$ | $T' + P'$ | $(T' + 3P' + S')d + S' \cdot 2u$ | $(C' + S') \cdot 2c$ |
| $B^0_d \to \pi^0 K^0$ | $\frac{1}{\sqrt{2}}(C' - P')$ | $-\frac{1}{\sqrt{2}}((T' + 3P' + S')d + S' \cdot 2u)$ | $\frac{1}{\sqrt{2}}(T' - S') \cdot 2c$ |
| $B^0_s \to \pi^+\pi^-$ | 0      | $-(T' + 2P') \cdot 2u$                                     | 0                    |
| $B^0_s \to \pi^0\pi^0$ | 0      | $\frac{1}{\sqrt{2}}(T' + 2P') \cdot 2u$                  | 0                    |
| $B^0_s \to K^+ K^-$ | $T' + P'$ | $(T' + 2P' + S') \cdot 2u + (T' + 3P' + S')d$ | $(C' + S') \cdot 2c$ |
| $B^0_s \to K^0 K^0$ | $-P'$  | $-(2P' + S') \cdot 2u + (T' + 3P' + S')d$                  | $-S' \cdot 2c$       |

For $\eta$ and $\eta'$ we used

$$\eta = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} - s\bar{s})$$  \hspace{1cm} (18)

$$\eta' = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} + 2s\bar{s})$$  \hspace{1cm} (19)

corresponding to an octet-singlet mixing angle of $\theta = -19.5^\circ$ (as generally assumed, see eg. \[3, 4, 5, 6\]). When calculating expressions in Table 3, as in Table 4 we have assumed that the SD singlet penguin amplitude is negligible. The large branching ratios of $B \to K\eta'$ decays seem to require significant $S'$. Thus, we have kept $S'$ in Table 7 not only in the SD contribution but also, as in Table 4, in the FSI part.

In principle, if FSI are important, the decay $B \to K\eta'$ may be described also when the short-distance singlet penguin $S'$ is small. Indeed, neglecting all terms in the FSI contributions but those proportional to $P'$, and putting $d = 0$ for simplicity, we observe that an effective singlet penguin amplitude of size $S'_{eff} \approx P' \cdot 2c$ is generated in the formulas for $B^+, B^0_d \to K\eta, K\eta'$ amplitudes (with the $P'$ amplitude defined from $B^+ \to \pi^+ K^0$ and $B^0_d \to \pi^- K^+$ decays, see Table 3). Thus, a part of the singlet penguin amplitude, phenomenologically required in $B^+, B^0_d \to K\eta'$ decays, may have its origin in final state interactions. A positive value of $c$ around +0.24 brings about a constructive interference between $P'$ and $S'_{eff}$, and permits a fit to the data \[3, 10\]. In fact, if a large part of the effective singlet penguin amplitude originates from FSI, one expects its phase to be real with respect to that
of the regular penguin: the \( qq\bar{q}q \) structure of \( s \)-channel states in the crossed diagrams should entail real \( c \).

Let us now analyse the formulas given in Tables 6 and 7 to see what general FSI pattern is generated, and whether something can be said not only about \( c \) but also about the values of \( u \), and \( d \), and the extraction of the SD amplitudes from the data. To this end, in Tables 8 and 9 we rewrite the content of Tables 6 and 7 in terms of redefined amplitudes

\[
\tilde{T}' = T' + C' \cdot 2c \tag{20}
\]

\[
\tilde{C}' = C' + T' \cdot 2c \tag{21}
\]

\[
\tilde{P}' = P' + S' \cdot (2c + 2u) + (T' + 3P' + S')d \tag{22}
\]
\[ \tilde{S}' = S' + P' \cdot 2c \]  

(23)

and (for \( S = 0 \))

\[ \tilde{T} = T + C \cdot 2c \]  

(24)

\[ \tilde{C} = C + T \cdot 2c \]  

(25)

\[ \tilde{P} = P + (T + 3P)d \]  

(26)

\[ \tilde{S} = P \cdot 2c. \]  

(27)

Note that for \( d \neq 0 \) the redefined amplitude \( \tilde{P} \) depends on two weak phases: \( \beta \) and \( \gamma \). This may affect the methods of \( \gamma \) determination (see below). The analysis of Tables 8 and 9, and of the general short-distance expressions for decay amplitudes (eg. ref. [11, 4]) shows that all inelastic FSI effects marked in the Tables as ”observable FSI modifications” have the pattern of effective annihilation \( \tilde{A} \), exchange \( \tilde{E} \), and penguin annihilation \( \tilde{P}A \) amplitudes

\[ \tilde{A} = C \cdot 2u \]  

(28)

\[ \tilde{E} = T \cdot 2u \]  

(29)

\[ \tilde{P}A = 2P \cdot 2u \]  

(30)

and

\[ \tilde{A}' = C' \cdot 2u \]  

(31)

\[ \tilde{E}' = T' \cdot 2u \]  

(32)

\[ \tilde{P}A' = 2P' \cdot 2u \]  

(33)

Thus, for SU(3)-symmetric FSI it is impossible to distinguish between pure SD and FSI-corrected amplitudes on the basis of experimental data alone. Only if the short-distance \( A, E, ... \) amplitudes are known to be negligible, may one attempt to deduce the size of FSI effects. In that case, \( u \) might be estimated as follows. From \( B^+ \rightarrow \pi^+ K^0 \), neglecting the \( C' \cdot 2u \) term, one extracts \( \tilde{P}' \approx 4.15 \) (with the amplitude squares giving \( B \) decay branching ratios in units of \( 10^{-6} \)). If terms proportional to \( d \) and \( S' \) in Eqs(22,26) may be neglected, one finds \( |\tilde{P}| \approx |V_{td}/V_{ts}||\tilde{P}'| \approx 0.179|\tilde{P}'| = 0.74 \) with an error of ±0.06. Neglecting \( \tilde{C} \) with respect to \( \tilde{T} \) from the \( B^+ \rightarrow \pi^+ \pi^0 \) decays one obtains that \( |\tilde{T}| \) is 3.38. Various analyses tend to give a slightly smaller
central value: $|\tilde{T}| = 2.7 \pm 0.6$ 
then, from

$\Gamma(B^{0}_s \rightarrow \pi^{+}\pi^{-}) \approx \Gamma(B^{+} \rightarrow \pi^{+}K^{0}) |_{max}$

one might deduce the phase $\delta_{u}$ of $u$. Finally, the size of $\Gamma(B^{0}_d \rightarrow K^{+}K^{-})$, with $|u|$ known and $|\tilde{T}|$ being comparable to $2|\tilde{P}|$, puts a constraint on the relative phase of $\tilde{T}$ and $\tilde{P}$.

In the absence of $B^{0}_s$ decay data, the best that can be done is to place an upper limit on the size of $|u|$, eg. by measuring the branching ratios for the $B^{0}_d \rightarrow K^{+}K^{-}(K^{0}K^{0})$ decays 

$\Gamma(B^{0}_d \rightarrow K^{0}K^{0}) \approx \Gamma(B^{+} \rightarrow \pi^{+}K^{0}) |_{max}$

The estimate of $|u|$ from the correction term $(T + 2P) \cdot 2u$ in $B^{0}_d \rightarrow K^{+}K^{-}$ can be hampered if $T$ and $P$ interfere destructively, as might be the case 

In Fig. 2 we show the present bounds on the size of $|u|$ and the SD tree-penguin relative phase $\phi_{T} - \phi_{P}$, obtained from the $B^{0}_d \rightarrow \pi^{+}\pi^{-}$ entry in Table 8 when the central value of 4.4 is assumed for the corresponding branching ratio.

The dependence of rescattering parameter $|u|$ on the tree-penguin relative phase $\phi_{T} - \phi_{P}$ is given there for several cases:

a) for central values $|\tilde{P}| = 0.74$ and $|\tilde{T}| = 2.7$ - thick solid line for cos $\delta_{u} = -1$, thick dashed line for cos $\delta_{u} = -0.85$,

b) for $|\tilde{P}|_{max} = 0.80$, $|\tilde{T}|_{min} = 2.10$, and cos $\delta_{u} = -1$ - short dashed line (approximate lower range),

c) for $|\tilde{P}|_{min} = 0.68$, $|\tilde{T}|_{max} = 3.30$, and cos $\delta_{u} = -0.85$ - long dashed line (approximate upper range),

d) the upper bound on $|u|$ from $B \rightarrow K^{+}K^{-}$ branching ratio (using central values for $|\tilde{T}|$ and $|\tilde{P}|$) - thin line: $B.R.(B \rightarrow K^{+}K^{-}) = 1.9 \cdot 10^{-6}$ (present experimental value); dotted line: $B.R.(B \rightarrow K^{+}K^{-}) = 0.1 \cdot 10^{-6}$.

Fig. 2 shows that $|u|$ of the order of 0.15 is consistent with the present data for any value of $|\phi_{T} - \phi_{P}|$. If $|u|$ is close to 0, the data seem to indicate a cancellation between the tree and penguin diagrams. (Although Fig. 2 shows that for $|\tilde{T}| = 2.7,$
|\bar{P}| = 0.74 and the $B_d^0 \rightarrow \pi^+\pi^-$ branching ratio of 4.4 there is no solution for $|\phi_T - \phi_P|$ above 150°; such a solution does exist if errors on $\bar{T}$, etc. are admitted.)

5 FSI-induced errors in the extraction of $\gamma$

Let us now comment on the relationship between the FSI effects in $B^+ \rightarrow K^+K^0$, $B^+ \rightarrow \pi^+K^0$, $B_d^0 \rightarrow \pi^-K^+$, and $B_s^0 \rightarrow \pi^+K^-$ discussed in ref.[4]. It was argued there that the FSI effects may affect the determination of the CP-violating angle $\gamma$ from the latter three decays. These conclusions followed from the neglect of FSI-induced terms originating from SD-driven diagrams other than $P$, $P'$, $T$, and $T'$, and from the subsequent discussion of the four relevant amplitudes in the case when FSI corrections proportional to $T$ are kept, while those proportional to $T'$ are neglected.

When Zweig rule is maintained in FSI, the relevant formulas may be read from Tables [4, 5] to be

\[
W(B^+ \rightarrow K^+\bar{K}^0) = -\bar{P} - Td \tag{36}
\]

\[
W(B_s^0 \rightarrow \pi^+K^-) = -\bar{P} - \bar{T} + 2Td \tag{37}
\]

\[
W(B^+ \rightarrow \pi^+K^0) = -\bar{P}' \tag{38}
\]

\[
W(B_d^0 \rightarrow \pi^-K^+) = \bar{P}' + \bar{T}' \tag{39}
\]

where $T^{(\cdot)} = T^{(\cdot)}(1 + 3d)$, $\bar{P}^{(\cdot)} = P^{(\cdot)}(1 + 3d)$. We observe that the FSI effects discussed in [4] are proportional to $d$ representing the difference between the overall contributions from the $C_1C_2 = +1$ and $C_1C_2 = -1$ states. Since, as discussed earlier, $d$ is probably much smaller than $u$, the FSI effects referred to in ref.[4] should not affect the determination of $\gamma$ too much.

In order to get a theoretical feeling for what might be the absolute size of $d$, let us consider the following. As discussed earlier, a nonzero value of $d$ arises most probably from the few lowest-lying intermediate states. Consequently, $d$ should be (very roughly) of the order of a contribution from a single intermediate channel, the best representative being the $PP$ state composed of two pseudoscalar mesons. A rough estimate of this contribution was made in ref.[3] in the framework of a Regge exchange model. From Eqs.(10) and (16) of ref.[3] one may deduce that the
contribution to the octet Argand amplitude arising from the "uncrossed" Regge exchange ($d$ measures deviation from the $C_1C_2 = +1 \iff C_1C_2 = -1$ symmetry for uncrossed diagrams) is

$$a_8(\text{uncrossed}) = -0.030 + 0.033i$$  \hspace{1cm} (40)$$

Since for small rescattering corrections the FSI-induced modification of the decay amplitude is proportional to $S^{1/2} - 1 \approx ia$ we expect that $|d|$ should be of the order of

$$|d| \approx |a_8| = 0.045$$  \hspace{1cm} (41)$$

While the Regge approach may not be the most reliable one, the above estimate can certainly provide an educated guess as to the order of magnitude of $d$. Note that in this case $|d| \ll 2|c|, 2|u|$ if the values of $|c| \approx 0.24$ and some average $|u|$ around 0.1 or 0.15 (cf. Fig. 2) are accepted.

In Fig. 3 we show errors induced by admitting nonzero $|d|$ in the method of the determination of $\gamma$ considered in refs. [14]. Solid lines represent the effect discussed by Chiang and Wolfenstein [15], ie. the influence of a nonzero value of CP-violating angle $\beta$ upon the extracted value of $\gamma$. When the calculations of ref. [15] are extended to include the effect of the term proportional to $Td$ in Eq. (37), one obtains the following counterparts of Eqs (7) and (8) from [15]:

$$K = 1 + 2\lambda^2 \frac{\sin \beta \cos \gamma}{\sin(\beta + \gamma)} + \lambda^4 \left( \frac{\sin \beta}{\sin(\beta + \gamma)} \right)^2$$  \hspace{1cm} (42)$$

$$KR_d = 1 + r^2 + 2r \cos \delta \cos \gamma$$  \hspace{1cm} (43)$$

$$KR_s = \lambda^2 + \left( \frac{r}{\lambda} \right)^2 - 2r \cos \delta \cos \gamma + 2|d| \cos \delta_d \left[ \left( \frac{r}{\lambda} \right)^2 + 3\lambda^2 - 4r \cos \gamma \right]$$  \hspace{1cm} (44)$$

where $\delta = \phi_T - \phi_P$ is the relative phase between the tree and penguin amplitudes, $r$ is their ratio, and $\lambda = 0.22$. As in [15] phase $\delta$ is set to zero. The only difference, when compared to ref. [15], is the presence of the rightmost term in Eq. (44). This term leads to additional errors on $\gamma$. In Fig. 3 we show the relevant error bands for $|d| = 0.01$ and $|d| = 0.05$ and $\cos \delta_d = \pm 1$. Thus, for $\gamma$ around 50$^\circ$ to 60$^\circ$ the expected FSI-induced errors are of the order of $\pm 5^\circ$. 


6 Conclusions

This paper shows explicitly how and under what assumptions all inelastic SU(3)-symmetric rescattering effects reduce to the redefinition of initial SD amplitudes, thus permitting the use of a simple diagram-based description, albeit with certain modifications. If FSI are important, the phenomenologically extracted diagram amplitudes do not have to be equal to those of the SD approaches. If only $T$, $P$, $C$ ($P'$, $T'$, $C'$, $S'$) SD amplitudes are nonnegligible, SU(3) symmetric rescattering can 1) generate effective annihilation, exchange and penguin annihilation amplitudes, 2) violate the expected relation $\tilde{P} = V_{td}/V_{ts}\tilde{P}'$, and 3) complicate the way in which weak phases are attributed to penguin-like amplitudes. It is estimated that the FSI-induced error made when extracting weak phase $\gamma$ from the $B \to K\pi$ amplitudes may be of the order of $\pm 5^\circ$ for $\gamma$ around $50^\circ - 60^\circ$.

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Table 8: Effective amplitudes for $\Delta S = 0$ decays. Contributions of standard form (including some FSI effects) and possibly observable FSI-induced modifications are shown separately.

| decay               | standard form | observable FSI modifications |
|---------------------|---------------|-----------------------------|
| $B^+ \rightarrow \pi^+ \pi^0$ | $-\frac{1}{\sqrt{2}}(\tilde{T} + \tilde{C})$ | 0 |
| $B^+ \rightarrow K^+ \bar{K}^0$ | $-\tilde{P}$ | $-C \cdot 2u$ |
| $B_d^0 \rightarrow \pi^+ \pi^-$ | $-(\tilde{T} + \tilde{P})$ | $-(T + 2P) \cdot 2u$ |
| $B_d^0 \rightarrow \pi^0 \pi^0$ | $-\frac{1}{\sqrt{2}}(\tilde{C} - \tilde{P})$ | $\frac{1}{\sqrt{2}}(T + 2P) \cdot 2u$ |
| $B_d^0 \rightarrow K^+ K^-$ | 0 | $(T + 2P) \cdot 2u$ |
| $B_d^0 \rightarrow K^0 \bar{K}^0$ | $-\tilde{P}$ | $-2P \cdot 2u$ |
| $B_s^0 \rightarrow \pi^+ K^-$ | $-(\tilde{T} + \tilde{P})$ | 0 |
| $B_s^0 \rightarrow \pi^0 K^0$ | $-\frac{1}{\sqrt{2}}(\tilde{C} - \tilde{P})$ | 0 |
| $B^+ \rightarrow \pi^+ \eta$ | $-\frac{1}{\sqrt{3}}(\tilde{T} + \tilde{C} + 2\tilde{P} + \tilde{S})$ | $-\frac{2}{\sqrt{3}}C \cdot 2u$ |
| $B^+ \rightarrow \pi^+ \eta'$ | $-\frac{1}{\sqrt{6}}(\tilde{T} + \tilde{C} + 2\tilde{P} + 4\tilde{S})$ | $-\frac{2}{\sqrt{6}}C \cdot 2u$ |
| $B_d^0 \rightarrow \pi^0 \eta$ | $-\frac{1}{\sqrt{6}}(2\tilde{P} + \tilde{S})$ | $\frac{2}{\sqrt{6}}T \cdot 2u$ |
| $B_d^0 \rightarrow \pi^0 \eta'$ | $-\frac{1}{\sqrt{3}}(\tilde{P} + 2\tilde{S})$ | $\frac{1}{\sqrt{3}}T \cdot 2u$ |
| $B_s^0 \rightarrow K^0 \eta$ | $-\frac{1}{\sqrt{3}}(\tilde{C} + \tilde{S})$ | 0 |
| $B_s^0 \rightarrow K^0 \eta'$ | $-\frac{1}{\sqrt{6}}(\tilde{C} + 3\tilde{P} + 4\tilde{S})$ | 0 |
Table 9: Effective amplitudes for $|\Delta S = 1|$ decays. Contributions of standard form (including some FSI effects) and possibly observable FSI-induced modifications are shown separately.

| decay                  | standard form                  | observable FSI modifications |
|------------------------|--------------------------------|-----------------------------|
| $B^+ \rightarrow \pi^+ K^0$ | $-\tilde{P}'$                  | $-C' \cdot 2u$             |
| $B^+ \rightarrow \pi^0 K^+$  | $\frac{1}{\sqrt{2}}(\tilde{T}' + \tilde{C}' + \tilde{P}')$ | $\frac{1}{\sqrt{2}}C' \cdot 2u$                   |
| $B^0_d \rightarrow \pi^- K^+$ | $\tilde{T}' + \tilde{P}'$       | 0                           |
| $B^0_d \rightarrow \pi^0 K^0$ | $\frac{1}{\sqrt{2}}(\tilde{C}' - \tilde{P}')$ | 0                           |
| $B^0_s \rightarrow \pi^+ \pi^-$ | 0                             | $-(T' + 2P') \cdot 2u$    |
| $B^0_s \rightarrow \pi^0 \pi^0$ | 0                             | $\frac{1}{\sqrt{2}}(T' + 2P') \cdot 2u$          |
| $B^0_s \rightarrow K^+ K^-$  | $\tilde{T}' + \tilde{P}'$       | $(T' + 2P') \cdot 2u$  |
| $B^0_s \rightarrow K^0 \bar{K}^0$ | $-\tilde{P}'$                  | $-2P' \cdot 2u$             |
| $B^+ \rightarrow K^+ \eta$  | $\frac{1}{\sqrt{3}}(\tilde{T}' + \tilde{C}' + \tilde{S}')$ | 0                           |
| $B^+ \rightarrow K^+ \eta'$ | $\frac{1}{\sqrt{6}}(\tilde{T}' + \tilde{C}' + 3\tilde{P}' + 4\tilde{S}')$ | $\frac{3}{\sqrt{6}}C' \cdot 2u$                   |
| $B^0_d \rightarrow K^0 \eta$  | $\frac{1}{\sqrt{3}}(\tilde{C}' + \tilde{S}')$ | 0                           |
| $B^0_d \rightarrow K^0 \eta'$ | $\frac{1}{\sqrt{6}}(\tilde{C}' + 3\tilde{P}' + 4\tilde{S}')$ | 0                           |
| $B^0_s \rightarrow \pi^0 \eta$ | $-\frac{1}{\sqrt{6}}\tilde{C}'$ | $\frac{2}{\sqrt{6}}T' \cdot 2u$              |
| $B^0_s \rightarrow \pi^0 \eta'$ | $\frac{1}{\sqrt{3}}\tilde{C}'$ | $\frac{1}{\sqrt{3}}T' \cdot 2u$               |
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FIGURE CAPTIONS

Fig. 1 Types of rescattering diagrams: (u) uncrossed, (c) crossed

Fig. 2 Dependence of rescattering parameter $|u|$ on tree-penguin relative phase $|\phi_t - \phi_p|$ (a) for central values $|\tilde{P}| = 0.74$ and $|\tilde{T}| = 2.7$ (thick solid line for $\cos \delta_u = -1$, thick dashed line for $\cos \delta_u = -0.85$). (b) approximate lower range - short-dashed line (for $|\tilde{P}|_{max} = 0.80$, $|\tilde{T}|_{min} = 2.10$, and $\cos \delta_u = -1$) (c) approximate upper range - long-dashed line (for $|\tilde{P}|_{min} = 0.68$, $|\tilde{T}|_{max} = 3.30$, and $\cos \delta_u = -0.85$) (d) upper bound on $|u|$ from $B \to K^+K^-$ branching ratio - thin line: $B.R.(B \to K^+K^-) = 1.9 \cdot 10^{-6}$ (present experimental value); dotted line: $B.R.(B \to K^+K^-) = 0.1 \cdot 10^{-6}$.

Fig. 3 Dependence of $\gamma$ on $\beta$. Estimates of FSI-induced errors for the method of extracting $\gamma$ from $B^+, B^0_d, B^0_s \to K\pi$ decays: (a) $R_d = 0.8, R_s = 0.78$; (b) $R_d = 0.85, R_s = 0.73$; (c) $R_d = 0.9, R_s = 0.68$; (d) $R_d = 1.15, R_s = 0.43$
Fig. 1. Types of rescattering diagrams: (u) uncrossed, (c) crossed.

Fig. 2. Dependence of $|u|$ on $|\varphi_t - \varphi_p|$. 
Fig. 3 (a), (b), (c), (d). Dependence of $\gamma$ on $\beta$. 