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Key Points:
- A stage-wise inversion framework is developed by combining the transition probability approach and the deep-learning methods
- The developed framework can identify subsurface sedimentary structures using sparse geological data and flow and transport responses
- The estimated structures can honor the available flow and transport responses and capture the preferential flow pathways

Supporting Information:
Supporting Information may be found in the online version of this article.

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Title:
Stage-Wise Stochastic Deep Learning Inversion Framework for Subsurface Sedimentary Structure Identification

Abstract
The stochastic models and deep-learning models are the two most commonly used methods for subsurface sedimentary structures identification. The results from the stochastic models typically involve uncertainty due to their nature. For the deep-learning models, sufficient structure samples are necessary for training, but they are practically difficult to obtain. This study develops an inversion framework to combine the strength of these two models to overcome the limitations. The stochastic model is first adopted to generate the structure samples required by the deep-learning models by integrating available observations. Then the trained deep-learning model is utilized to reduce the uncertainty of the structures generated by the stochastic models. This integrated framework can successfully estimate the structures using available observations. Importantly, no additional structure training samples are required in the identification process. To summarize, the combination of the stochastic and the deep-learning models shows great advantages in identifying subsurface sedimentary structures.

Plain Language Summary
This study develops an integrated inversion framework that uses sparse data obtained from different geophysical measurements to identify subsurface sedimentary structures. By combining the advantages of physically-based geological models and advanced deep-learning techniques, the developed framework can identify complex subsurface sedimentary structures in a more practical way than is currently possible. The advantage of this method is that the identification process relies only on sparse measurement data and does not need additional structure samples for deep-learning model training. It has significant practical value because most deep-learning-based methods require a large number of training samples, which are practically difficult to obtain. Therefore, the developed framework has great advantages in practical subsurface sedimentary structures identification applications.

1. Introduction
Delineating subsurface sedimentary structure is one of the most significant challenges in earth sciences and energy related applications (Bianchi & Pedretti, 2018; Dogan et al., 2011; Fuku et al., 2019; Lu et al., 2012; Rizzo & de Barros, 2017). The structure estimated by sparse geological data typically involves great uncertainty (J. Chen & Rubin, 2003). This uncertainty may introduce significant biases in numerical simulations of flow and solute transport (Carrera, 1993; Carrera et al., 2005; Dai et al., 2014; Dai et al., 2020; Dentz et al., 2020; Geng et al., 2020; Nowak et al., 2010; Rajaram & Gelhar, 1995; Ritzi & Soltanian, 2015; Soltanian et al., 2019; Vrugt et al., 2005). Therefore, how to effectively use sparse observations, direct or indirect, to obtain a more realistic view of subsurface sedimentary structures in practice is an essential issue (Doherty, 2003; Keating et al., 2010).

Considering the lack of direct facies data (i.e., bodies of sediments and rocks, Soltanian & Ritzi [2014]), inversion or data assimilating methods have been widely adopted to delineate spatial distribution of facies by estimating parameters of the subsurface sedimentary structure model (X. Song et al., 2019; Zhang et al., 2020). The stochastic models (Dai et al., 2019; He et al., 2014; Rajaram, 2016; Soltanian et al., 2020; Tahmasebi, 2017) and the deep-learning-based generative models (Bao et al., 2020; Mosser et al., 2017; Tang et al., 2021) are amongst the most commonly used methods to derive subsurface sedimentary structures. The stochastic models use sparse geological data to estimate the spatial correlation of facies distribution. The correlation is then used to generate a series of possible structures with honoring the conditional data. Considering the stochastic inversion process is often not computationally efficient, Harp et al. (2008) used an analytical solution to estimate the transition probability (TP) (probability of transition from one facies to another) to represent aquifer structures. The solution...
was developed by Dai et al. (2007) and relies on physically-based and observable facies attributes such as volume proportions and mean lengths. The TP model can identify the structure with optimized structural parameters using the indicator cokriging simulation. However, due to the TP based models’ stochastic nature, the uncertainties remain in the generated structures, particularly for three-dimensional (3D) cases (X. Song et al., 2019).

Deep-learning-based generative models are inspired by the growing popularity of deep learning methods over the last few years, especially the variational autoencoder (Canchumuni et al., 2019; Laloy et al., 2017), generative adversarial network (Chan & Elsheikh, 2019; T.-F Zhang et al., 2019) and the latest—adversarial autoencoder (Mo et al., 2020). These methods can transform low-dimensional latent vectors to high-dimensional subsurface sedimentary structures with geostatistical characters similar to training samples. In contrast to the stochastic models, the deep-learning-based generative models will only generate a structure by feeding a latent vector. By changing the value of each dimension of the vector, different structure details can be modified (Laloy et al., 2018; Nesvold & Mukerji, 2021). Therefore, more accurate subsurface sedimentary structures can be obtained by estimating optimized latent vectors (Zhang et al., 2020). However, the requirement of training samples is the main limitation for such methods, as it is practically difficult to obtain sufficient samples with facies distributions similar to that of the target structure.

In practice, the mentioned limitations of stochastic and deep-learning-based generative models make it difficult to apply these methods to identify subsurface sedimentary structures by using sparse observations. To alleviate these problems, a stage-wise stochastic deep learning inversion framework is developed here. It combines the strengths of the stochastic models facilitating the integration of direct geological observations and the deep-learning-based generative models. The inversion process is split into two stages. The purpose of the first stage is to use the stochastic model to generate a series of possible structures with available observations and use these structures as the training samples of the deep-learning-based models. Then, in the second stage, these possible structures can be further modified by updating the latent vectors of the trained deep-learning-based model. Finally, a more realistic view of subsurface sedimentary structures can be obtained by the developed framework. The identification process is based on sparse available observations without the need for providing additional structure samples.

2. Methods

The developed framework has four modules: (a) the analytical solution of TP model, which uses structural parameters to represent subsurface sedimentary structures, (b) the octave convolution adversarial autoencoder (OCAAE) that transforms latent vectors to corresponding subsurface sedimentary structures, (c) the deep octave convolution residual network (DOCRN) surrogate model for improving the inversion speed, and (d) the iterative local update ensemble smoother (ILUES) for parameter estimation. The workflow of this framework is illustrated in Figure 1a and discussed in Section 2.1. Sections 2.2–2.5 respectively describe the four components in detail.

2.1. Stage-Wise Inversion Framework

We incorporate the TP model, OCAAE generative model, and DOCRN surrogate model into the ILUES to form the framework. The involved steps are decomposed into two major stages:

**Stage-1 Prior** ensembles of structural parameters are generated based on prior information such as geological data. Next, the facies TP model is modified by updating structural parameter ensembles. Using the modified TP model, the subsurface sedimentary structure is estimated by indicator cokriging simulation. Then, the flow and transport simulation is performed using the trained DOCRN surrogate model to obtain the hydraulic head and concentrations at observation points. Based on these estimated responses data, the optimized structural parameters can be obtained by repeatedly update the parameter ensembles with ILUES.

**Stage-2 The** OCAAE is first trained with the subsurface sedimentary structure samples generated by the TP model with the optimized structural parameters. Then, prior ensembles of the latent vector are obtained based on the user-defined distribution (in our implementation, the standard normal distribution is used, more discussion see Text S3 in Supporting Information S1), and the structure is modified by updating latent vector ensembles. The DOCRN is still used here to obtain the estimated observations, and the ensembles are also updated by ILUES.
The steps in stage-1 are similar to the traditional stochastic inversion method (e.g., Dai et al., 2007). The difference is that we incorporate the DOCRN surrogate model to accelerate the inversion process. However, the results of the stochastic inversion method are still very uncertain. As for the steps in stage-2, they are developed based on the popular deep-learning-based identification method recently (e.g., Laloy et al., 2018; Mo et al., 2020), this method can obtain more realistic structures, but enough training subsurface structure samples are required to training the generative model. The developed framework combines the strengths of these two methods and overcomes the limitation of each method. Stage-1 is performed to obtain the optimized structural parameters and provide training samples of the OCAAE. Then, stage-2 can further reduce the uncertainty of subsurface sedimentary structures generated in stage-1. The final product is a more realistic estimation of structure ensembles obtained with the OCAAE using optimized latent vectors.

Figure 1. (a) Flow diagram of stage-wise stochastic inversion process. Illustration of component network structure: (b) octave convolution adversarial autoencoder. (c) Deep octave convolution residual network.

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The training samples of DOCRN are generated in TOUGHREACT (Xu et al., 2006). It is done by: (a) Generating structural parameter (stage-1) or latent vector (stage-2) ensembles from the prior distribution, (b) Obtaining subsurface sedimentary structure ensembles by TP model (stage-1) or the OCAAE (stage-2), using the corresponding parameter ensembles, and (c) Estimating state variables (e.g., hydraulic head and concentration distribution) by feeding subsurface sedimentary structures to TOUGHREACT.

Evidently, no additional training sample is used in any of the inversion process and the DOCRN training steps. The entire process involving the inversion framework for subsurface sedimentary structure identification just relies on geological data and flow and transport responses.

2.2. Analytical Solution of Transition Probability Model

For ease of inversion, we use the analytical solution of the TP model developed by Dai et al. (2007) to represent the facies spatial correlations:

\[
t_{ij}(h_\phi) = p_i + (\delta_{ij} - p_i) e^{-h_\phi / (L_{i\phi}(1 - p_i))} \quad (\text{for } i, j = 1, N)
\]

where \(t_{ij}(h_\phi)\) is the TP in direction \(\phi\) from facies \(i\) to facies \(j\) with a lag distance of \(h_\phi\), \(p_i\) is the proportion of facies \(i\), \(N\) is the number of facies, \(\delta_{ij}\) is the Kronecker delta, and \(L_{i\phi}\) is the mean length of facies \(i\) in direction \(\phi\). Based on Equation 1, the continuous-lag TP matrix \(T\) in direction \(\phi\) can be obtained as \(T(h_\phi) = [t_{ij}(h_\phi)]_{N \times N}\). Using this matrix, the subsurface sedimentary structure can then be estimated by indicator cokriging simulation (Carle & Fogg, 1997). More detailed information about the simulation process is presented in supporting information.

2.3. Octave Convolution Adversarial Autoencoder

The OCAAE includes three component networks: octave convolution encoder, octave convolution decoder, and discriminator. The octave convolution encoder learns to transform the input subsurface sedimentary structure \(X\) to a low-dimensional latent vector \(Z\). Then octave convolution decoder learns to map the latent vector \(Z\) onto the reconstructed structure \(\hat{X}\) that is close to \(X\). The discriminator is trained to distinguish whether the latent vector arises from encoding distribution \(\psi(Z)\) (the posterior distribution of latent vectors generated from the encoder) or prespecified one \(\phi(Z)\) (the distribution that we want latent vectors to follow). The incorporation of the discriminator makes the decoder generate more realistic subsurface sedimentary structures (see the discussion in Text S4 in Supporting Information S1). After training, the trained octave convolution decoder can generate the structures by feeding latent vectors. More details about input, output features, and parameters of OCAAE are presented in Text S2 in Supporting Information S1.

The OCAAE improves the convolution adversarial autoencoder (Mo et al., 2020) (see Figure 1b) with two major aspects: (a) the traditional convolution layer is replaced by octave convolution in the encoder and decoder. Octave convolution can decompose the input subsurface sedimentary structure into high- and low-frequency information. It can reduce redundant information in training samples and the occupied GPU memory during the training process (Y. Chen et al., 2019), enabling a larger batch-size to improve training speed and accuracy. Additional details on octave convolution can be found in Y. Chen et al. (2019); (b) the residual in residual dense residual block (RRDRB) is used in the encoder and decoder. The RRDRB contains multilevel residual connections, enabling the OCAAE to learn different patterns of subsurface sedimentary structure at multiple levels (Rakotonirina & Rasanoivo, 2020). Thus more realistic structures can be generated. In addition, we extend octave convolution and RRDRB to suit 3D cases and incorporate them to form a novel residual block, the Octave RRDRB (OctRRDRB; additional details are in Supporting Information S1). The OctRRDRB is used as the primary block to build the OCAAE model. The application of OctRRDRB can improve the training accuracy and stability of OCAAE (See the ablation study in Text S5 in Supporting Information S1). The detailed structures of these components are presented in Figures S1–S3 in Supporting Information S1.

2.4. Deep Octave Convolution Residual Network

The DOCRN uses the idea of image transformation to build the surrogate model of transport simulation by establishing the relationship between subsurface structure and the whole state field (image-to-image). Compared to directly mapping the subsurface structure to observations (image-to-point), the DOCRN can make full use of
the convolutional neural network and better consider the influence of the structure's heterogeneity on the state distribution field. In addition, the image-to-image method enables us to use some state-of-the-art super-resolution image generation technologies to establish a more accurate surrogate model. For example, the OctRRDRB can also be applied in the DOCRN to improve the prediction accuracy. As shown in Figure 1c, after training, the subsurface structures are regarded as the image data and input into the DOCRN, then the corresponding state field can be estimated (More information about input and output features of DOCRN see in Text S2 in Supporting Information S1).

### 2.5. Iterative Local Update Ensemble Smoother

The inversion process is accomplished by the ILUES method developed by Zhang et al. (2018) As opposed to the ensemble Kalman filter (Kollat et al., 2008; Xu et al., 2021), the ILUES can assimilate all observation data simultaneously to update parameter ensembles. The update scheme is expressed by Equation 2:

\[
p_{k+1} = p_k + C_{PM} C_{MM}^{-1} N_{iter} C_D -1 \left[ d + \sqrt{N_{iter} e_v} - f(G(p_k)) \right]
\]

where \(p_k\) is the \(k\)th updated parameter sample after the \(n\)th iteration, \(C_{PM}\) is the cross-covariance between the surrogate model estimations and the updated parameters (structural parameters (stage-1) or latent vectors (stage-2)) of the \(n\)th iteration, and \(C_{MM}\) denotes the auto-covariance of the estimated parameters. \(N_{iter}\) is the number of iterations, \(C_D\) represents the covariance of observation error, \(e_v\) denotes the \(k\)th observation error, \(f\) represents the surrogate model, and \(G\) refers to the TP model (in stage-1) and octave convolution decoder (in stage-1), respectively.

### 3. Application Examples

The performance of the inversion framework was illustrated by two synthetic examples, including a two-dimensional (2D) and a three-dimensional (3D) case. The synthetic structures were obtained by indicator cokriging simulation, using quantifiable physical attributes of facies (see Table 1: true parameters) and conditional data. The size of the 2D and 3D subsurface sedimentary structures are 80 m × 40 m and 80 m × 60 m × 20 m, respectively. The distribution of facies, locations of conditional data, observation data, and source zone for solute transport are presented in Figures 2a and 2b.

The aquifer contains two facies, namely high- and low-permeability (k) facies. The high-k facies is two orders of magnitude more conductive than the low-k facies. In the 2D case, the total simulation time is a year. The right and left boundaries are as constant hydraulic head boundaries with the hydraulic gradient of \(5 \times 10^{-3}\). Other boundaries are no-flux. A continuous release of tracer at the rate of 0.04 kg/s was from the full thickness near the left boundary for the first two months. The concentrations at six discrete points over the released period and the hydraulic head at the end of simulation time were monitored. As for the 3D case, the tracer test was simulated for five days. The set of boundaries and the hydraulic gradient are the same as those in the 2D case. Tracer was continuously released from a 6 m × 6 m × 2 m source area with the rate of 0.5 kg/s on the first day. Observation data used in the 3D case include seven discrete time points concentrations and the final hydraulic head over the released period. The total number of observations for the 2D and 3D cases are 336 and 800, respectively, and 5% noise is imposed on the observations.

| Table 1 |
| The Structural Parameters, Corresponding Prior Distributions of Low-k Facies, and Connectivity Analysis Results of High-k Facies for the True and Estimated Structures |
|-------|-------|-------|-------|-------|-------|-------|
|      | \(P\) | \(L_x\) (m) | \(L_y\) (m) | \(L_z\) (m) | \(N_{CC}\) | \(N_{LC}\) | \(R_{LC}\) |
| 2D estimated | 0.4079 | 19.84 | – | 4.56 | 5.115 | 1815 | 0.5671 |
| 2D true | 0.4 | 20 | – | 4.5 | 8 | 1733 | 0.5415 |
| 2D prior distribution | [0.35, 0.45] | [10, 80] | – | [4, 5] | – | – | – |
| 3D estimated | 0.4576 | 39.27 | 30.85 | 2.30 | 7.527 | 13,580 | 0.5658 |
| 3D true | 0.45 | 40 | 30 | 2.3 | 15 | 13,100 | 0.5458 |
| 3D prior distribution | [0.4, 0.5] | [10, 80] | [10,60] | [2, 2.5] | – | – | – |
4. Results and Discussion

Based on the conditional and observation data, the inversion process is implemented according to the steps described in Section 2.1. The detailed information of the inversion process is described in Supporting Information S1 (ILUES: Text S2 in Supporting Information S1, OCAAE, and DOCRN: Text S3 in Supporting Information S1). The performance evaluation of OCAAE and DOCRN is also conducted and presented in Texts S4 and S5 in Supporting Information S1.

Figure 2. Distribution of facies indicating clay (red), sand (blue), locations of conditional facies data (light green), observation data (dark green), and contaminated area (yellow): (a) 2D; (b) 3D. The mean absolute error between observations and estimations versus the number of iterations for the 2D (c) and the 3D (d) cases. True subsurface sedimentary structure: (e) 2D; (h) 3D. low-k facies probability maps after the first inversion stage: (f) 2D; (i) 3D. Second inversion result: (g) 2D; (j) 3D.
Four different criteria is used here to evaluate the quality of inversion results:

(1) The discrepancy between the observations and estimations based on modified structures [see Figure (c) and (d)]. The discrepancy was measured using mean absolute error (MAE):

\[
MAE = \frac{1}{N_{obs}N_e} \sum_{i=1}^{N_{obs}} \sum_{j=1}^{N_e} |f_i(X_j) - d_i|
\]  

(3)

where \(N_{obs}\) and \(N_e\) denote the number of observation points and parameter samples, respectively. \(X_j\) is \(j\)th modified structures. \(d_i\) represents the observations at the \(i\)th observation point, \(f_i\) is the DOCRN estimations at the \(i\)th observation point with \(j\)th parameter sample.

(2) The facies probability map, which is generated from the sample mean of the indicator variables of low-k facies [see Figure (f), (g), (i), and (j)]. The marginal probability of low-k facies at one location is expressed as follows

\[
p_{\text{low-k}}(x) = \Pr \{I_{\text{low-k}}(x) = 1\} = \frac{1}{M} \sum_{i=1}^{M} I_{\text{low-k},i}(x)
\]

(4)

where \(I_{\text{low-k}}(x) = 1\) if low-k facies occurs at location \(x\), and \(I_{\text{low-k}}(x) = 0\) otherwise. The \(M\) is the number of samples.

(3) The uncertainty analysis of trace breakthrough curves (BTC) in three selected wells [see Figure 2a and 2b: well 1–3]. The comparison of BTC can reflect the discrepancy in solute transport responses between the estimated structure and the real one. The smaller the discrepancy, the more accurate solute transport prediction based on the estimated structure. The uncertainty analysis was conducted through TOUGHREACT by feeding the subsurface structure ensembles. The results are shown in Figure 3 for the 2D case and Figure S4 in Supporting Information S1 for the 3D case.

(4) The connectivity of high-k facies. It is essential for solute transport simulation in subsurface sedimentary structures (Bianchi & Pedretti, 2017; Fiori & Jankovic, 2012; Henri et al., 2015; Poeter & Townsend, 1994; Renard & Allard, 2013; Rizzo & de Barros, 2017; Tyukhova & Willmann, 2016; Zinn & Harvey, 2003), as solute can transfer in the highly permeable channels at velocities much larger than the mean. To measure the connectivity of high-k facies, the number of independent high-k facies zone \(N_{CC}\), the size (grids number) of the largest high-k facies zone \(N_{LC}\), and the rate of the largest high-k facies zone’ size to the that of the whole structure \(R_{LC}\) are analyzed by using a Fortran code: CONNEC3D (Pardo-Igúzquiza & Dowd, 2003), and are presented in Table 1.

In both 2D and 3D cases, the MAE decreases as the iteration proceeds, and the converged values of stage-1 are much smaller than those of stage-1, indicating that more realistic subsurface sedimentary structures were obtained through stage-2 inversion. In stage-1, as the aquifer with binary facies considered here has the low-k facies embedded within high-k facies, we are only required to estimate the structural parameters of low-k facies, including facies proportion (\(p\): the rate of grid number of low-k facies to total grid number) and mean length in \(x, y, z\) directions (\(L_x, L_y, L_z\)). More information about the prior distribution of the structural parameters is presented in Text S2 in Supporting Information S1. The inversion results are shown in Table 1. The mismatch between the true and estimated parameters (the mean based on all posterior structures) is relatively small, confirming that accurate structural parameters can be obtained in stage-1. However, as observed from the probability map of low-k facies in Figures 2f and 2i, some uncertainties remain in the subsurface sedimentary structures generated with the optimized transition probability model, especially for the 3D case. Although the uncertainty of the BTC based on the posterior structures in stage-1 (Figure 3b) is smaller than that of prior structures (Figure 3a). It still varies widely due to the stochastic nature of the TP-based realizations. Therefore, it is necessary to carry out the stage-2 inversion developed in this study.

After stage-2, by comparing the probability maps of low-k facies (Figures 2g and 2j) and the uncertainty analysis of BTC (Figure 3c), the uncertainty of the subsurface sedimentary structure was significantly reduced through the stage-2 inversion. And the BTC of the estimated structures is very close to that of the true structure. It indicates that our inversion framework can obtain reasonable structures which honor the available flow and transport
responses data. As shown in Table 1, the average $N_{LC}$ and $R_{LC}$ of the estimated structures are also close to those of the true structure for both 2D and 3D cases, only the $N_{CC}$ is underestimated, but the size of the largest high-k facies zone accounts for more than 92% of the total size of high-k facies, just some small zones were not identified. Therefore, preferential flow pathways could be captured by our framework. Furthermore, more than 90% of the estimated facies grid is correct according to the true structure in the 3D case and 95% in the 2D case. The possible reasons for some differences in local characteristics are as follows: (a) high observation noise; (b) slight difference between predicted values of the surrogate model and that of the transport model; (c) discrepancy between optimized structural parameters and predefined ones; and (d) most of the observation data point are outside the solute plume, especially for the 3D case, not providing effective concentration data. Thus, it can be concluded that the developed framework can generate relatively accurate subsurface sedimentary structures using sparse conditional facies and observation data.

Though this study uses examples of binary facies for verification, this stage-wise inversion framework can be easily extended to the inversion in cases involving multiple facies and at different scales (e.g., multiscale facies in Soltanian et al., 2015a, 2015b), only need to estimate more structural parameters at stage-1. In addition to subsurface sedimentary structure identification, the developed framework also has the potential to be used

Figure 3. Ensembles of tracer breakthrough curves in the 2D case: (a) prior ensemble before inversion, posterior ensemble after inversion: (b) stage-1, (c) stage-2.
for other purposes. For example, the inversion framework can be used to optimize the locations of monitoring wells to improve the accuracy of subsurface sedimentary structure identification or parameter estimation (Mo et al., 2019; X. Song et al., 2019). The steps of stage-2 are also suitable for the inversion of continuous parameter fields by replacing the training samples of the OCAAE with corresponding training images of continuous parameter distribution (Mo et al., 2020; Zhang et al., 2020). Furthermore, the OCAAE and DOCRN can be widely used in other geophysical domains, which require repeated utilization of the geological and transport models (Kang et al., 2021; Tang et al., 2021). The developed framework can also be tested for other subsurface problems (e.g., multiphase and reactive transport) where subsurface heterogeneity exerts a strong control on solute transport processes (e.g., Lu et al., 2012; Soltanin et al., 2016, 2019). In the developed work, we use the max number of iterations (Niter) as the stopping criteria, and a large Niter, according to the experience of previous experiments, is chosen to ensure the convergence. Indeed, it may be more reasonable to use the stopping criteria based on the error function (e.g., the MAE expressed by Equation 4), and this criterion is also worthy of further study.

5. Conclusions

A stage-wise stochastic deep learning inversion framework is developed for subsurface sedimentary structure identification. With this framework, preferential flow pathways can be captured relying on available observations. The breakthrough curves derived from the estimated structure are close to that from the true one, indicating the two structures are similar. Compared to the existing deep-learning-based methods, our developed framework does not require any additional subsurface sedimentary structure training samples. While compared to stochastic inversion methods, the uncertainty of the subsurface sedimentary structures is significantly reduced. As long as some geological data are available, this framework can provide probabilistic estimations of the facies distribution based on flow and transport responses of the structure. Therefore, the developed framework has great advantages in practical subsurface sedimentary structure identification. In addition, some advanced deep-learning technologies, such as octave convolution and RRDRB, have been used in the developed framework to increase the efficiency and accuracy of the inversion process. With the development of related technologies, the more powerful stochastic models or deep-learning-based models can readily be incorporated into this framework to improve its performance in the future.

Data Availability Statement

The virtual borehole data of synthetic subsurface sedimentary structures are extracted from an alluvial aquifer in the Maules Creek Valley, Australia. The corresponding ASCII file can be found at https://doi.org/10.5281/zenodo.5641299.

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