Massive scalar field quasinormal modes of a Schwarzschild black hole surrounded by quintessence

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Abstract. We present the quasinormal frequencies of the massive scalar field in the background of a Schwarzschild black hole surrounded by quintessence with the third-order WKB method. The mass of the scalar field $u$ plays an important role in studying the quasinormal frequencies, the real part of the frequencies increases linearly as mass $u$ increases, while the imaginary part in absolute value decreases linearly which leads to damping more slowly and the frequencies having a limited value. Moreover, owing to the presence of the quintessence, the massive scalar field damps more slowly.
1. Introduction

The quasinormal modes of black holes have drawn much attention in recent years. Vishveshwara first put forward the concept of quasinormal modes in calculations of the scattering of gravitational radiation by a Schwarzschild black-hole[1] and Press proposed the term quasinormal frequencies[2]. The quasinormal frequencies are an important characteristic of a black hole, because the frequencies only depend on the black hole parameters rather than the initial perturbation. In addition, the properties of quasinormal modes have been explored in the Ads/CFT correspondence[3] and loop quantum gravity[4].

On the other hand, a large number of astronomical observations, such as type Ia supernovae [5], CMB[6] and large scale structure[7], indicate that the expansion of the universe is speeding up rather than slowing down. To explain this accelerated expansion, the universe is regarded as being dominated by an exotic component with large negative pressure called ”dark energy” which constitutes about 70% of the energy density of the universe. There are several candidates for dark energy: the cosmological constant[8] and dynamic candidates(for example phantom[9], quintessence[10], k-essence[11] and quintom[12]). The difference of these candidates for dark energy is the size of the parameter $\omega_q$, namely, the ratio of the pressure and energy density of the dark energy and for quintessence $-1 \leq \omega_q \leq -\frac{1}{3}$.

The quasinormal modes of different fields perturbation around different black holes have been widely investigated[13]-[32], especially the massless scalar field[33]. Although the massive field was studied in different black holes[34]-[37], the massive field quasinormal modes still have gaps. Kiselev[38] recently considered Einstein’s field equations for a black hole surrounded by quintessence and obtained a new solution related to state parameter $\omega_q$ of the quintessence. In this paper, we use the third-order WKB method to explore the quasinormal modes of massive scalar field perturbation around a Schwarzschild black hole surrounded by quintessence.

The outline of this paper is as follows: in section 2, we evaluate the quasinormal frequencies of the low overtones quasinormal modes. The discussion and summary are presented in section 3.

2. Massive scalar field quasinormal modes of a Schwarzschild black hole surrounded by Quintessence

Kiselev[38] stated a new static spherically-symmetric exact solution of Einstein equations describing a black hole charged or not and surrounded by the quintessence under the condition of additivity and linearity in the energy-momentum tensor. For the Schwarzschild black hole, the metric is given by[39]:

$$ds^2 = \left(1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}}\right)dt^2 - \left(1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q+1}}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$ (1)

where $M$ is the mass of the black hole, $\omega_q$ is the quintessential state parameter, $c$ is the normalization factor dependent on $\rho_q = -\frac{\epsilon}{2^{3\omega_q+1}p_{m}^2}$, and $\rho_q$ is the density of quintessence.
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The massive scalar field in a curved background is governed by the Klein-Gordon equation:

\[ \Box \Phi - u^2 \Phi = \frac{1}{\sqrt{-g}} \left( g^{\mu \nu} \sqrt{-g} \Phi_{,\nu} \right)_{,\mu} - u^2 \Phi = 0, \tag{2} \]

where \( \Phi \) is the scalar field.

After substituting equation (1) into equation (2) and separating angular and time variables, we obtain the radial equation:

\[ \left( \frac{d^2}{dr_*^2} + \omega^2 - V(r) \right) \Phi(r) = 0, \tag{3} \]

where:

\[ V(r) = (1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q + 1}})(l(l + 1) + \frac{2M}{r^2} + \frac{c(3\omega_q + 1)}{r^{3\omega_q + 3}} + u^2), \tag{4} \]

\[ dr_* = \frac{dr}{1 - \frac{2M}{r} - \frac{c}{r^{3\omega_q + 1}}}, \tag{5} \]

and \( l = 0, 1, 2, 3... \) parameterizes the field angular harmonic index. The effective potential \( V(r) \) approaches to a constant both at the event horizon and at spatial infinity. It is clear that the effective potential relates to the value of \( r \), angular harmonic index \( l \), the state parameter \( \omega_q \), the scalar field mass \( u \), the normalization factor \( c \) and the mass of the black hole \( M \). However, in this paper, we only want to investigate the relationship between the state parameter \( \omega_q \) or the scalar field mass \( u \) and the quasinormal modes. Therefore, taking \( M = 1 \) and \( c = 0.001 \), we compute the quasinormal frequencies stipulated by above potential using the third-order WKB method developed by Schutz, Will and Iyer[40]-[42]:

\[ \omega^2 = [V_0 + (-2V_0')^{1/2}] \Lambda - i(n + \frac{1}{2})(-2V_0')^{1/2}(1 + \Omega), \tag{6} \]

where

\[ \Lambda = \frac{1}{(-2V_0')^{1/2}} \left\{ \frac{1}{8} \left( \frac{V_0^{(4)}}{V_0''} \right) (\frac{1}{4} + \alpha^2) - \frac{1}{288} \left( \frac{V_0'''}{V_0''} \right)^2 (7 + 60\alpha^2) \right\}, \tag{7} \]

\[ \Omega = \frac{1}{(-2V_0')^{1/2}} \left\{ \frac{5}{6912} \left( \frac{V_0'''}{V_0''} \right)^4 (77 + 188\alpha^2) \right. \\
- \frac{1}{384} \left( \frac{V_0'^4}{V_0''} \right) (51 + 100\alpha^2) + \frac{1}{2304} \left( \frac{V_0'^4}{V_0''} \right)^2 (67 + 68\alpha^2) \\
+ \frac{1}{288} \left( \frac{V_0''^6}{V_0''^2} \right) (19 + 28\alpha^2) - \frac{1}{288} \left( \frac{V_0''^6}{V_0''^2} \right) (5 + 4\alpha^2) \right\}, \tag{8} \]

and

\[ \alpha = n + \frac{1}{2}, \tag{9} \]

\[ V_0^{(n)} = \left. \frac{d^n V}{dr_*^n} \right|_{r_* = r_p}, \tag{10} \]

where \( n \) is the overtone number.
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Table 1. Values of the quasinormal frequencies for low overtones in the Schwarzschild black hole without quintessence ($c = 0$) for fixed $l = 5$, $u = 0.2$

| $\omega(n = 0)$ | $\omega(n = 1)$ | $\omega(n = 2)$ | $\omega(n = 3)$ | $\omega(n = 4)$ |
|------------------|------------------|------------------|------------------|------------------|
| 1.065754-0.095396i | 1.055414-0.287524i | 1.036163-0.483127i | 1.010110-0.683090i | 0.9790981-0.887095i |

Table 2. Values of the quasinormal frequencies for low overtones in the Schwarzschild black hole surrounded by quintessence ($c = 0.001$) for fixed $l = 5$, $u = 0.2$

| $3\omega_q + 1$ | $\omega(n = 0)$ | $\omega(n = 1)$ | $\omega(n = 2)$ | $\omega(n = 3)$ | $\omega(n = 4)$ |
|------------------|------------------|------------------|------------------|------------------|------------------|
| 0.0              | 1.064163-0.095203i | 1.053847-0.286942i | 1.034639-0.482147i | 1.008645-0.681703i | 0.977700-0.885292i |
| -0.2             | 1.063768-0.095151i | 1.053458-0.286786i | 1.034262-0.481884i | 1.008282-0.681329i | 0.977355-0.884806i |
| -0.4             | 1.062374-0.095089i | 1.052972-0.286597i | 1.033790-0.481566i | 1.007828-0.680878i | 0.976921-0.884220i |
| -0.6             | 1.062656-0.095104i | 1.052364-0.286373i | 1.033200-0.481186i | 1.007261-0.680339i | 0.976380-0.883517i |
| -0.8             | 1.061885-0.094943i | 1.051606-0.286107i | 1.032465-0.480738i | 1.006556-0.679700i | 0.975710-0.882638i |
| -1.0             | 1.060920-0.094825i | 1.050659-0.285798i | 1.031552-0.480214i | 1.005685-0.678952i | 0.974885-0.881701i |
| -1.2             | 1.059714-0.094710i | 1.049480-0.285446i | 1.030420-0.479613i | 1.004615-0.678084i | 0.973881-0.880551i |
| -1.4             | 1.058205-0.094581i | 1.048013-0.285053i | 1.029026-0.478931i | 1.003310-0.677091i | 0.972671-0.879211i |
| -1.6             | 1.056320-0.094443i | 1.046191-0.284623i | 1.027316-0.478172i | 1.001373-0.675948i | 0.971230-0.877652i |
| -1.8             | 1.053963-0.094300i | 1.043933-0.284170i | 1.025229-0.477343i | 0.999850-0.674662i | 0.969539-0.875834i |
| -2.0             | 1.051018-0.094162i | 1.041143-0.283715i | 1.022701-0.476462i | 0.997622-0.673216i | 0.967584-0.873701i |

Substituting the effective potential (4) into the formula above, we can get the quasinormal frequencies for the massive scalar field in the Schwarzschild black hole surrounded by quintessence background and the quasinormal frequencies are shown in table 1, table 2, table 3 and figures 1-2.

The date of table 1 is the quasinormal frequencies of a Schwarzschild black hole without quintessence and under the quintessence is given in table 2. Figure 1 shows that the real part and the imaginary part of the quasinormal frequencies change as the quintessential state parameter $\omega_q$ changes for fixed mass $u$. Comparing the table 1 with the table 2, we find the real part and the magnitude of imaginary part in the Schwarzschild space-time without quintessence are larger. It means that due to the presence of quintessence, the oscillations damp more slowly. Furthermore, the imaginary part in absolute value and the real part decrease as the value of $\omega_q$ decreases, as shown in figure 1 and table 1.

We present the quasinormal frequencies for different values of the mass of scalar field $u$ in figs 2 and table 3. Notice that, The real part of the quasinormal frequencies grows with increasing of the mass field $u$, while the imaginary part of quasinormal frequencies in absolute value falls down. Moreover, the frequencies change linearly as $u$ changes as shown in figure 2.
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Figure 1. Quasinormal frequencies of the black hole surrounded by quintessence for $c = 0.001, n = 0, u = 0, 0.1, 0.2, 0.3, 3\omega_q + 1$ runs the values from 0 to -2.0 at intervals of -0.2. (a): $l = 2$, (b): $l = 3$, (c): $l = 4$, (d): $l = 5$.

3. Discussion and summary

We have thoroughly investigated the quasinormal modes for massive scalar field perturbation in a Schwarzschild black hole surrounded by quintessence background. The paper proposes the quasinormal modes are greatly influenced by the quintessence and the mass of scalar field, because the introduction of the quintessence and the mass $u$ leads to less damping of the quasinormal modes. Actually $c$ may be too smaller than 0.001 to neglect the influence of the quintessence. However, if the density of quintessence surrounding the black hole is high enough to influence distinctly the quasinormal modes, we can study the character of quintessence by the experimental date.

Another new phenomena found here is for given $l$, $n$, and $\omega_q$, the real part of the frequencies linearly increase, while the magnitude of imaginary part linearly decrease as the mass of the scalar field $u$ increases. As we know, the mass of the scalar field $u$ has a maximum
Table 3. Values of the quasinormal frequencies for fixed $l = 3, n = 0$, in the Schwarzschild black hole surrounded by quintessence ($c = 0.001$) for different values of mass $u$.

| $3\omega_q + 1$ | $u = 0$          | $u = 0.1$        | $u = 0.2$        | $u = 0.3$        |
|-----------------|------------------|------------------|------------------|------------------|
| 0.0             | 0.674185-0.096319i | 0.676547-0.095748i | 0.683650-0.094028i | 0.695548-0.091132i |
| -0.2            | 0.673930-0.096267i | 0.676292-0.095696i | 0.683395-0.093977i | 0.695291-0.091082i |
| -0.4            | 0.673611-0.096204i | 0.675973-0.095634i | 0.683074-0.093915i | 0.694968-0.091022i |
| -0.6            | 0.673212-0.096129i | 0.675572-0.095559i | 0.682671-0.093842i | 0.694561-0.090952i |
| -0.8            | 0.672711-0.096041i | 0.675071-0.095472i | 0.682167-0.093757i | 0.694051-0.090871i |
| -1.0            | 0.672084-0.095940i | 0.674442-0.095372i | 0.681534-0.093660i | 0.693411-0.090780i |
| -1.2            | 0.671299-0.095825i | 0.673655-0.095258i | 0.680740-0.093552i | 0.692607-0.090680i |
| -1.4            | 0.670317-0.095698i | 0.672670-0.095134i | 0.679746-0.093434i | 0.691597-0.090575i |
| -1.6            | 0.669089-0.095562i | 0.671438-0.095001i | 0.678501-0.093312i | 0.690331-0.090471i |
| -1.8            | 0.667553-0.095425i | 0.669897-0.094868i | 0.676944-0.093193i | 0.688745-0.090377i |
| -2.0            | 0.665636-0.095295i | 0.667972-0.094745i | 0.674997-0.093089i | 0.686761-0.090308i |

Figure 2. Quasinormal frequencies of the black hole surrounded by quintessence for $c = 0.001, n = 0, u = 0, 0.1, 0.2, 0.3, 3\omega_q + 1 = -1.0$ (a): $l = 2$, (b): $l = 3$.

value dependent on the mode under consideration[43]. That is mean, the quasinormal frequencies have a limited value. Thereby, the introduction of the the quintessence and the mass $u$ has enriched the quasinormal frequencies of the Schwarzschild black hole.
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