Effect of magnetic field on intersubband polaritons in a quantum well: Strong to weak coupling conversion

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We investigate theoretically the effect of a magnetic field on intersubband polaritons in an asymmetric quantum well placed inside an optical resonator. It is demonstrated that the field-induced diamagnetic shift of electron subbands in the well increases the broadening of optical lines corresponding to intersubband electron transitions. As a consequence, the magnetic field can switch the polariton system from the regime of strong light-matter coupling to the regime of weak one. This effect paves a way to the effective control of polaritonic devices with a magnetic field.

In recent decades, the significant technological progress in fabrication of optical resonators (microcavities) with high quality factor was achieved. This led to both experimental and theoretical development of the concept of strong light-matter coupling, which covers various electron-photon processes accompanying the oscillatory energy exchange between an electromagnetic field confined in a microcavity and electrons in condensed-matter structures placed inside the microcavity\textsuperscript{13–15}. Currently, the regime of strong light-matter coupling has been realized experimentally in a variety of condensed-matter structures, including quantum wells (QWs)\textsuperscript{16–18}, quantum dots\textsuperscript{19–22}, quantum wires\textsuperscript{23–25}, and others. Particularly, the energy exchange between electrons in QW and a confined electromagnetic field in a microcavity leads to the oscillating transitions between different electron subbands in QW. These periodical optical transitions of electrons can be described formally as a composite electron-photon quasi-particle called “intersubband polariton”\textsuperscript{24–31}. Immediately after experimental discovery of the intersubband polaritons, they attracted great attention of research community. This was caused by their unique physical properties which meet the needs of various modern optoelectronic devices operating in a wide frequency range from infrared to terahertz domains, including quantum cascade lasers\textsuperscript{32–33}, light emitting devices\textsuperscript{34–35} and photodetectors\textsuperscript{36–37}. In order to control the devices, the frequency tuning of intersubband polariton should be elaborated. Currently, the tuning is realized with the control of light-matter coupling constant by the electrical gating applied to QW\textsuperscript{38–39}. In the present study, we propose the alternative method of the frequency tuning by an external magnetic field. In what follows, we will show theoretically that the magnetic field can effectively control the intersubband polaritons by switching the electron-photon system in QW from the regime of strong light-matter coupling to the weak one.

Generally, the regime of strong light-matter coupling is realized when the coherent light-matter interaction overcomes the characteristic damping. In this regime, the energy is periodically exchanged between the “light part” and “matter part” of the light-matter system with the Rabi frequency, $\Omega_R$, which is given by the expression\textsuperscript{1}:

$$\hbar \Omega_R = \sqrt{g_R^2 - \Gamma^2}, \quad (1)$$

where $g_R$ is the characteristic light-matter coupling constant, and $\Gamma$ is the characteristic linewidth depending on the damping in the system. It follows from Eq. (1) that the condition of strong light-matter coupling can be written in the most general form as $g_R > \Gamma$. The opposite case, $g_R \leq \Gamma$, corresponds to the regime of weak light-matter coupling, when there is no oscillating processes in the light-matter system.

Let us apply the aforesaid to analyze electron-photon processes in a QW placed inside a planar microcavity (see Fig. 1a). We will assume the confining potential of QW, $U(z)$, to be asymmetric (see Fig. 1b) and restrict the consideration by two first electron subbands in QW (see Fig. 2). In the considered system, the regime of strong light-matter coupling corresponds to the intersubband Rabi oscillations of electrons with the frequency $\Omega_L$, where $\Gamma$ is the characteristic linewidth of the intersubband transitions induced by cavity photons. Since the characteristic value of the photon momentum is very small, these optical transitions can be pictured approximately as vertical (see red arrows in Fig. 2). As to the electron-photon coupling constant, it is

$$g_R = g(q_0) \sqrt{S n_e}, \quad (2)$$

where

$$g(q) = \sqrt{\frac{|d_{i2}|^2 \Delta^2}{\hbar \epsilon_0 c L \omega_0(q)} \frac{q^2}{(\pi/L)^2 + q^2}} \quad (3)$$

is the matrix element of electron-photon coupling, $S$ is the area of cavity, $n_e$ is the density of electrons in QW,

$$d_{ij} = \langle \psi_i(z) | z | \psi_j(z) \rangle$$

is the matrix element of dipole moment, $\Delta$ is the intersubband gap, $\epsilon_0$ is the vacuum permittivity, $\epsilon$ is the
dielectric permittivity, \( L \) is the length of the cavity,

\[
\omega_0(q) = \frac{c}{n} \sqrt{q^2 + \left( \frac{\pi}{L} \right)^2}
\]  

(4)
is the frequency of cavity photon with the wave vector \( q = (q_x, q_y) \), \( n \) is the refractive index of the cavity, and \( q_0 \) is the resonance wave vector of cavity photons, which is defined by the equality \( h\omega_0(q_0) = \Delta \) and marked at the insert in Fig. 1.

In the absence of a magnetic field, the energy spectrum of electron subbands in QW consists of a set of equidistant parabolas, \( \varepsilon_n(k) = \varepsilon_n + \hbar^2 k^2 / 2m^* \), where \( m^* \) is the effective mass of electron and \( \mathbf{k} = (k_x, k_y) \) is the electron wave vector. \( \varepsilon_n \) is the frequency of cavity photon with the wave vector \( q = (q_x, q_y) \), \( n \) is the refractive index of the cavity, and \( q_0 \) is the resonance wave vector of cavity photons, which is defined by the equality \( h\omega_0(q_0) = \Delta \) and marked at the insert in Fig. 1.

In the presence of an in-plane magnetic field, \( \mathbf{B} = (0, B) \), the energy spectrum of electron subbands in the QW reads

\[
\varepsilon_n(k) = \varepsilon_n + \frac{\hbar^2 k^2}{2m^*} + \frac{\hbar k_x d_{nn} |B|}{m^*},
\]  

(5)

where the last term describes the diamagnetic shift of electron energy. Due to this term, the subbands in

\[ \varepsilon_{\perp}(k_x) \]

are shifted relative to each other by the wave vector

\[
\Delta k = \frac{|d_{11} - d_{22}| B}{\hbar},
\]  

(6)

It should be stressed that this shift arises from the inequality \( d_{11} \neq d_{22} \) and, therefore, takes place only in QWs with an asymmetric confining potential, \( U(z) \). It is seen in Fig. 2 that the shift results in different energies of intersubband optical transitions for different wave vectors \( k_x \). As a consequence, the linewidth of the intersubband transitions achieves the additional magnetic-induced broadening, \( \Gamma_B \), which can be approximately written as

\[
\Gamma_B \approx \frac{2\hbar k_F B |d_{11} - d_{22}|}{m^*},
\]  

(7)

where \( k_F = \sqrt{2\pi n_e} \) stands for the Fermi wave vector of electrons. Therefore, the conversion of the regime of strong light-matter coupling into the regime of weak light-matter coupling is defined by the condition \( g_R = \Gamma_0 + \Gamma_B \). Since the broadening grows with increasing magnetic field, this condition is satisfied at the critical magnetic field

\[
B_0 \approx \frac{m^* (g_R - \Gamma_0)}{2\hbar k_F |d_{11} - d_{22}|}.
\]  

(8)

As a result, the range of weak magnetic fields, \( B < B_0 \), corresponds to the regime of strong light-matter coupling, whereas the range of strong magnetic fields, \( B \geq B_0 \), corresponds to the regime of weak light-matter coupling. One can expect that the field-induced switching between these two different regimes will be crucial for physical properties of intersubband polaritons. To concretize this, let us discuss the effect of magnetic field on polaritonic dispersion.
To calculate the dispersion of intersubband polaritons, we will approximate the confining potential of QW by the triangular well,

$$ U(z) = \begin{cases} \infty, & z \leq 0 \\ eEz, & z > 0 \end{cases}, \quad (9) $$

where $E \approx e_n e \varepsilon_0$ is the effective electric field in QW. Applying the approach based on Green’s functions technique\textsuperscript{27,29} to the system in question, the formation of an intersubband polariton in QW can be described by the infinite sum of the diagrams corresponding to the absorptions and emissions of the cavity photons, which is pictured schematically in the first line of Fig. 3. This series can be reduced to the Dyson equation for the renormalized Green’s function of cavity photon, $G(\omega, \mathbf{q})$, which corresponds to the second line of Fig. 3. The solution of this equation reads

$$ G(\omega, \mathbf{q}) = \frac{G_0(\omega, \mathbf{q})}{1 - g^2(q)G_0(\omega, \mathbf{q})\Pi(\omega, \mathbf{q})}, \quad (10) $$

where

$$ G_0(\omega, q) = \frac{2\hbar \omega_0(q)}{\hbar^2 \omega^2 - \hbar^2 \omega_0^2(q) + 2i\hbar \Gamma_0 \omega_0(q)} \quad (11) $$

is the Green’s function of bare photon,

$$ \Pi(\omega, q) = -2i \sum_k \int \frac{d\nu}{2\pi} G(\nu + \omega, \mathbf{k} + \mathbf{q})G(\nu, \mathbf{k}) = 2 \sum_k \frac{1}{\hbar \omega + \varepsilon_1(\mathbf{k}) - \varepsilon_2(\mathbf{k} + \mathbf{q}) + i\hbar \Gamma_0} \quad (12) $$

stands for the polarization operator of intersubband transition, where the summation is over filled electron states $k$ under the Fermi energy, $\varepsilon_\nu$. The sought polaritonic dispersion, $\omega(\mathbf{q})$, corresponds to poles of the Green’s function\textsuperscript{10} and can be easily found numerically from the equation

$$ 1 - g^2(q)G_0(\omega, \mathbf{q})\Pi(\omega, \mathbf{q}) = 0. \quad (13) $$

As expected, Eqs. (10)–(13) at $B = 0$ turn into the known expressions describing intersubband polaritons in the absence of magnetic field\textsuperscript{27}.

![Fig. 3: Dyson equation for intersubband polaritons in QW.](image)

Solving the transcendental equation (13), we arrive at the polariton dispersion which is plotted in Fig. 4. In the absence of magnetic field, the regime of strong light-matter coupling is realized. In this case, the characteristic feature of the dispersion is the energy gap between the two polariton branches. This gap, $\hbar \Omega_R$, takes place at the resonant photon wave vector, $q_0$, which corresponds to the intersection of the intersubband resonant energy, $\Delta$, and the cavity photon dispersion, $\hbar \omega_0(q)$ (see the insert in Fig. 4b)). Since the gap arises from the intersubband Rabi oscillations of electrons, it describes the Rabi splitting of the polariton branches and depends on the Rabi frequency, $\Omega_R$. If a magnetic field is strong enough ($B \geq B_0$), the regime of light-matter coupling is switched to weak one. In this case, the intersubband Rabi oscilla-
FIG. 5: Dependence of the critical magnetic field, $B_0$, on the nature broadening of cavity photons, $\Gamma_0$, and the cavity length, $L$, for GaAs-based QW with the electron density $n_e = 10^{11}$ cm$^{-2}$.

tions of electrons are broken and the Rabi splitting vanishes. The critical magnetic field, $B_0$, which corresponds to the transition between these two regimes, depends on parameters of the cavity and QW (see Fig. 5).

Summarizing the aforesaid, we can conclude that a magnetic field can switch a polariton system in a quantum well between the regimes of strong light-matter coupling and weak light-matter coupling. As a consequence, the field crucially changes the polariton dispersion. Namely, in the regime of strong coupling the polaritonic branches are split because of intersubband Rabi oscillations, whereas in the regime of weak coupling this Rabi splitting vanishes. Since all physical properties of polaritons depend on the dispersion, magnetic field can serve as an effective tool to control polaritonic devices.

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