Comparison of Frames: Jordan vs Einstein Frame for a Non-minimal Dark Energy Model

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Abstract

We construct a dark energy model where a scalar field non-minimally coupled to gravity plays the role of the dark component. We compare cosmological consequences of this non-minimal coupling of the scalar field and gravity in the spirit of the dark energy paradigm in Jordan and Einstein frames. Some important issues such as phantom divide line crossing, existence of the bouncing solutions and the stability of the solutions are compared in these two frames. We show that while a non-minimally coupled scalar field in the Jordan frame is a suitable dark energy component with capability to realize phantom divide line crossing, its conformal transformation in the Einstein frame has not this capability. The conformal transformation from Jordan frame to Einstein frame transforms the equation of state parameter of the dark energy component to its minimal form with a redefined scalar field and in this case it is impossible to realize a phantom phase with possible crossing of the phantom divide line.

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1 Introduction

Recently, evidences from supernova searches data [1,2], cosmic microwave background (CMB) results [3-5] and also Wilkinson Microwave Anisotropy Probe (WMAP) data [6,7], indicate an positively accelerating phase of the cosmological expansion today and this feature shows that the simple picture of universe consisting of a pressureless fluid is not enough. In this regard, the universe may contain some sort of additional negative-pressure dark energy. Analysis of the five year WMAP data [8-11] shows that there is no indication for any significant deviations from Gaussianity and adiabaticity of the CMB power spectrum and therefore suggests that the universe is spatially flat to within the limits of the observational accuracy. Further, the combined analysis of the three-year WMAP data with the supernova Legacy survey (SNLS) [8], constrains the equation of state $w_{de}$, corresponding to almost 74% contribution of dark energy in the currently accelerating universe, to be very close to that of the cosmological constant value. Moreover, observations appear to favor a dark energy equation of state, $w_{de} < -1$ [12,13]. Therefore a viable cosmological model should admit a dynamical equation of state that might have crossed the value $w_{de} = -1$, in the recent epoch of cosmological evolution. In fact, to explain positively accelerated expansion of the universe, there are two alternative approaches: incorporating an additional cosmological component to the matter part of the Einstein equations or modifying gravity (the geometric part of the Einstein field equations) at cosmological scale. Multi-component dark energy with at least one non-canonical phantom field is a possible candidate of the first alternative. This viewpoint has been studied extensively in literature (see [14] and references therein).

On the other hand, despite of several successes, the standard model of cosmology suffers from a series of problems. The most serious of these problems is the problem of initial singularity because the laws of physics break down at the singularity point. In order to avoid this lawlessness, there is a huge interest in the solutions that do not display divergencies. These solutions could be obtained at a classical level or by quantum modifications. Most of the efforts in quantum gravity is devoted to reveal the nature of the initial singularity and to understand the origin of matter, non-gravitational fields, and the very nature of the spacetime. In recent analysis done within the loop quantum cosmology, the Big Bang singularity is replaced by a quantum Big Bounce with finite energy density of matter. This scenario has strong quantum effects at the Planck scale too. Another motivation to remove the initial singularity is the initial value problem. A sound gravitational theory needs to have a well-posed Cauchy problem. Due to the fact that the gravitational field diverges at the singularity, we could not have a well-formulated Cauchy problem as we cannot set the initial values at that point.

With these preliminaries, the purpose of the present paper is to study some currently important cosmological issues such as phantom divide line crossing, avoiding singularities by realization of the bouncing solutions and the stability of these solutions in a non-minimally coupled scalar field model of universe in Jordan and Einstein frames and in the spirit of dark energy model. We compare cosmological consequences of the non-minimal coupling between scalar field and gravity in the spirit of the dark energy scenario in these two frames. Some important issues such as phantom divide line crossing, existence of the bouncing solutions
and the stability of the solutions are compared in these two frames. Especially, we analyze the parameter space of the model numerically to show that which frame allows for stability of the solutions in the separate regions of the $\omega - \omega'$ phase-plane. We show that with only one scalar field non-minimally coupled to gravity, crossing of the phantom divide line can be realized just in the Jordan frame. By transforming to Einstein’s frame, we show that this model cannot account for crossing of the phantom divide line. We show that while a non-minimally coupled scalar field in the Jordan frame is a suitable dark energy component with capability to realize phantom divide line crossing, its conformal transformation in the Einstein frame has not this capability. In fact, conformal transformation from Jordan frame to Einstein frame transforms the equation of state parameter of dark energy component to its minimal form with a redefined scalar field and in this case it is impossible to realize a phantom phase with possible crossing of the phantom divide line.

2 A Ricci-Coupled Scalar Field Model in the Jordan Frame

For a model universe with a non-minimally coupled scalar field as matter content of the universe, the action in the absence of other matter sources in the Jordan frame can be written as follows

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{k_4^2} \alpha(\phi) R[g] - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right],$$

(1)

where we have included an explicit and general non-minimal coupling of the scalar field and gravity as $\alpha(\phi)$. For simplicity, from now on we set $k_4^2 \equiv 8\pi G_N = 1$. Variation of the action with respect to the metric gives the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \alpha^{-1} T_{\mu\nu}.$$  \hspace{1cm} (2)

$T_{\mu\nu}$, the energy-momentum tensor of the scalar field non-minimally coupled to gravity, is given by

$$T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla \phi)^2 - g_{\mu\nu} V(\phi) + g_{\mu\nu} \Box \alpha(\phi) - \nabla_\mu \nabla_\nu \alpha(\phi),$$

(3)

where $\Box$ shows 4-dimensional d’Alembertian. For FRW universe with line element defined as

$$ds^2 = -dt^2 + a^2(t) d\Sigma_k^2,$$

(4)

where $d\Sigma_k^2$ is the line element for a manifold of constant curvature $k = +1, 0, -1$, the equation of motion for scalar field $\phi$ is

$$\nabla^\mu \nabla_\mu \phi = V' - \alpha' R[g],$$

(5)
where a prime denotes derivative of any quantity with respect to $\phi$. This equation can be rewritten as

$$\ddot{\phi} + \frac{3}{a} \frac{\dot{a}}{a} \phi + \frac{dV}{d\phi} = \alpha' R[g].$$

(6)

where a dot denotes the derivative with respect to cosmic time $t$ and Ricci scalar is given by

$$R = 6 \left( \dot{H} + 2H^2 + \frac{k}{a^2} \right).$$

(7)

With this non-minimally coupled scalar field as matter content of the universe, cosmological dynamics are described by

$$\frac{\ddot{a}}{a^2} = -\frac{k}{a^2} + \frac{\rho}{3},$$

(8)

and

$$\frac{\dot{a}}{a} = -\frac{1}{6}(\rho + 3p).$$

(9)

In these equations, the effect of the non-minimal coupling of the scalar field and gravity is hidden in the definition of $\rho$ and $p$. We assume that scalar field, $\phi$, has only time dependence and using (3), we find

$$\rho = \alpha^{-1} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) - 6\alpha' H \dot{\phi} \right),$$

(10)

$$p = \alpha^{-1} \left( \frac{1}{2} \dot{\phi}^2 - V(\phi) + 2 \left( \alpha' \dot{\phi} + 2H \alpha' \dot{\phi} + \alpha'' \dot{\phi}^2 \right) \right),$$

(11)

where $H = \frac{\dot{a}}{a}$ is Hubble parameter. Now, equation (9) takes the following form

$$\frac{\ddot{a}}{a} = -\frac{1}{6} \alpha^{-1} \left( 2\dot{\phi}^2 - 2V(\phi) + 6 \left( \alpha' \dot{\phi} + H \alpha' \dot{\phi} + \alpha'' \dot{\phi}^2 \right) \right),$$

(12)

and dynamics of the equation of state parameter is given by

$$w \equiv \frac{p}{\rho} = \frac{\dot{\phi}^2 - 2V(\phi) + 4 \left( \alpha' \ddot{\phi} + 2H \alpha' \dot{\phi} + \alpha'' \dot{\phi}^2 \right) \right)}{\dot{\phi}^2 + 2V(\phi) - 12\alpha' H \dot{\phi}}.$$

(13)

From this equation, when $\dot{\phi} = 0$, we obtain $p = -\rho$. In this case $\rho$ is independent of $a$ and $V(\phi)$ plays the role of a cosmological constant. In the minimal case when $\dot{\phi}^2 < V(\phi)$, using (9) we obtain $p < -\frac{\rho}{3}$ which shows an accelerated expansion which is driven by cosmological constant. However, cosmological constant is not a good candidate for dark energy since it suffers from several conceptual problems such as its unknown origin and also need to huge amount of fine-tuning. In non-minimal case the cosmological dynamics depends on the value of the non-minimal coupling. In order to solve the Friedmann equation (8), we choose the following scalar field potential

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

(14)
With this potential, a possible solution of our basic equations, (6), (8) and (10) is as follows (see [15] for a similar argument)

\[ \phi = \sqrt{C_0 \cos(mt)} \quad (15) \]

where \( C_0 \) is a parameter with the dimension of mass squared describing the oscillating amplitude of the fields. Here we assume \( \alpha(\phi) = \frac{1}{2}(1 - \xi \phi^2) \). We note that there are two critical values of \( \phi \) given by \( \phi_c = \pm \frac{1}{\sqrt{\xi}} \) that should be avoided to have well-defined field equations. Using (14) and (15) in (10), we find

\[ \rho = \frac{m^2 C_0 - 12 \xi H C_0 \sin(mt) \cos(mt) \left( 1 - \xi C_0 \cos^2(mt) \right)}{1 - \xi C_0 \cos^2(mt)}, \quad (16) \]

where those values of the cosmic time coordinates that lead to singular energy density are excluded from our considerations. Therefore, for a flat spatial geometry, Friedmann equation (8) can be rewritten as follows

\[ H = \frac{\left( 6 \cos(mt) \sin(mt) C_0 \xi \pm \sqrt{36 \cos(mt)^2 \sin(mt)^2 C_0^2 \xi^2 + C_0 - \xi C_0^2 \cos^2(mt)^2} \right) m}{-1 + \xi C_0 \cos^2(mt)}, \quad (17) \]

We avoid imaginary values of the Hubble parameter in which follows.

### 2.1 Crossing of the phantom Hubble parameter

Non-minimal coupling of the scalar field and gravity in the Jordan frame provides a suitable framework for explanation of the late-time accelerated expansion [16]. On the other hand, after substituting corresponding relations for \( \phi, H \) and \( V \) in equation (13), dynamics of the equation of state parameter for a non-minimally coupled scalar field is given by

\[ \omega(t) = \left\{ -8 \xi \epsilon \sin \left( \frac{mt}{2} \right) \cos \left( \frac{mt}{2} \right) \sqrt{-36 \left( \frac{1}{36} + C_0 \xi^2 \cos^4(mt) - \xi C_0 \left( \xi - \frac{1}{36} \right) \cos^2(mt) \right) C_0} ight. \\
+48 \xi C_0 \left( \frac{1}{24} + \xi \right) \cos^4(mt) + \left( 2 + 52 C_0 \xi^2 \right) \cos^2(mt) - 1 + 4 \xi \right\} \times \\
\left[ -12 \xi \epsilon \sin(mt) \cos(mt) \sqrt{-36 \left( \frac{1}{36} + C_0 \xi^2 \cos^4(mt) - \xi C_0 \left( \xi - \frac{1}{36} \right) \cos^2(mt) \right) C_0 - 1} \\
+72 C_0 \xi^2 \cos^4(mt) - 72 \xi \left( \frac{1}{72} + \xi \right) C_0 \cos^2(mt) \right]^{-1} \quad (18) \]

where \( \epsilon = \pm 1 \). Figure 1 shows the crossing of the phantom horizon with equation of state parameter of this non-minimally coupled scalar field. Nevertheless, as figure 2 shows, in the case of \( \xi = 0 \), that is a single and minimally coupled scalar field, there is no crossing of the phantom horizon, as has been emphasized by other literature such as [14]. We note that our analysis shows that for negative values of \( \xi \) (corresponding to anti-gravitation) and any values of \( \epsilon \) (\( \epsilon = -1 \) or \( \epsilon = +1 \)), it is impossible to realize the phantom horizon crossing in this setup.
for $x_i > 0$ and $\epsilon > 0$

Figure 1: A non-minimally coupled scalar field in Jordan frame has the capability to realize crossing of the phantom divide line by its equation of state parameter in a suitable subspace of the model parameter space.

for $x_i=0$ and $\epsilon < 0$ or $\epsilon > 0$

Figure 2: Equation of state parameter of a single, minimally coupled scalar field (with $\xi = 0$), cannot explain crossing of the phantom divide line.
2.2 Bouncing behavior of the model

A bouncing universe with an initial contraction to a non-vanishing minimal radius and a subsequent expanding phase provides a possible solution to the singularity problem of the standard Big Bang cosmology. For a successful bounce, it can be shown that within the framework of the standard 4-dimensional Friedmann-Robertson-Walker (FRW) cosmology with Einstein gravity, the null energy condition (NEC) is violated for a period of time around the bouncing point. Moreover, for the universe entering into the hot Big Bang era after the bouncing, the EoS of the matter content \( w \) in the universe must transit from \( w < -1 \) to \( w > -1 \) [17].

The model proposed to understand the behavior of dark energy with an EoS of \( w > -1 \) in the past and \( w < -1 \) at present, has been supported by the observational data [18]. This is a dynamical model of dark energy and it differs from the cosmological constant, Quintessence, Phantom, K-essence and so on in the determination of the cosmological evolution. Here we study the possibility of realization of the bouncing solution in a model universe dominated by the nonminimally coupled scaler field as matter content. We start with a detailed examination of the necessary conditions required for a successful bounce. During the contracting phase, the scale factor \( a(t) \) is decreasing, i.e., \( \dot{a}(t) < 0 \), and in the expanding phase we have \( \dot{a}(t) > 0 \). At the bouncing point, \( \dot{a}(t) = 0 \), and around this point \( \ddot{a}(t) > 0 \) for a period of time. Equivalently, in the bouncing cosmology the Hubble parameter \( H \) runs across zero from \( H < 0 \) to \( H > 0 \) and \( H = 0 \) at the bouncing point. In our model this behavior is shown in figure 3. A successful bounce requires that around this point

\[
\dot{H} = -4\pi G \rho (1 + w) > 0.
\] (19)

From equation (18) one can see that \( w < -1 \) in a neighborhood of the bouncing point. After the bounce the universe needs to enter into the hot Big Bang era, otherwise the universe filled with the matter with an EoS \( w < -1 \) will reach the big rip singularity as what happens to the Phantom dark energy which violates the null energy condition [19]. This requires the EoS of the matter to transit from \( w < -1 \) to \( w > -1 \).

By solving the Friedmann equation (17), we can study the scale factor versus the cosmic time, \( t \). If we integrate equation (17), we find

\[
a(t) = a_0 \exp \left[ \int \left( \frac{6 \cos (mt) \sin (mt) C_0 \xi \pm \sqrt{36 (\cos (mt))^2 (\sin (mt))^2 C_0^2 \xi^2 + C_0 - \xi C_0^2 (\cos (mt))^2}}{1 + \xi C_0 (\cos (mt))^2} \right) \frac{m}{dt} \right].
\] (20)

Using equation (20), we plot the behavior of the scale factor versus the cosmic time. One can see from Figures 3 (left) and 4 that a nonsingular bounce happens when the Hubble parameter \( H \) runs across zero with a minimal and non-vanishing scale factor \( a \).

2.3 Stability of the model

Now we study the stability of our model. In order to study the classical stability of our model, we analyze the behavior of the model in the \( \omega - \omega' \) plane where \( \omega' \) is the derivative
Figure 3: Variation of the Hubble parameter $H$ relative to cosmic time $t$. The left hand side figure shows that the universe can switch alternatively between expanding and contracting phases.

Figure 4: Variation of the scale factor $a$ relative to cosmic time $t$. Note that the initial conditions have been set so that integrand of equation (20) has non-vanishing derivative at the minimum point.
of $\omega$ with respect to the logarithm of the scale factor (see [20-23] for a similar analysis for other interesting cases)

$$\omega' \equiv \frac{d\omega}{d\ln a} = \frac{d\omega}{dt} \frac{dt}{d\ln a} = \frac{\dot{\omega}}{H}.$$  \hspace{1cm} (21)

The sound speed expresses the phase velocity of the inhomogeneous perturbations of the quintessence field. We define the function $c_a$ as

$$c_a^2 \equiv \frac{\dot{p}}{\dot{\rho}}.$$  \hspace{1cm} (22)

If the matter is considered as a perfect fluid, this function would be the adiabatic sound speed of this fluid. We note that with scalar fields that do not obey perfect fluid form necessarily, this quantity is not actually a sound speed. In which follows, we demand that $c_a^2 > 0$ in order to avoid the big rip singularity at the end of the universe evolution. The conservation of the quintessence field effective energy density can be stated as

$$\frac{d\rho_{\text{quintessence}}}{dt} + 3H(\rho_{\text{quintessence}} + p_{\text{quintessence}}) = 0$$  \hspace{1cm} (23)

Since the dust matter obeys the continuity equation and the Bianchi identity keeps valid, total energy density satisfies the continuity equation. From above equation, we have

$$\dot{\rho}_{de} = -3H\rho_{de}(1 + \omega_{de})$$  \hspace{1cm} (24)

Using equation of state $p_{de} = \omega_{de}\rho_{de}$, we obtain

$$\dot{p}_{de} = \dot{\omega}\rho_{de} + \omega_{de}\dot{\rho}_{de}$$  \hspace{1cm} (25)

Therefore, the function $c_a^2$ could be rewritten as

$$c_a^2 = \frac{\dot{\omega}_{de}}{-3H(1 + \omega_{de})} + \omega_{de}$$  \hspace{1cm} (26)

In this situation, the $\omega - \omega'$ plane is divided into four regions defined as follows

\begin{align*}
I : \quad & \omega_{de} > -1, \quad \omega' > 3\omega(1 + \omega) \quad \Rightarrow \quad c_a^2 > 0 \\
II : \quad & \omega_{de} > -1, \quad \omega' < 3\omega(1 + \omega) \quad \Rightarrow \quad c_a^2 < 0 \\
III : \quad & \omega_{de} < -1, \quad \omega' > 3\omega(1 + \omega) \quad \Rightarrow \quad c_a^2 < 0 \\
IV : \quad & \omega_{de} < -1, \quad \omega' < 3\omega(1 + \omega) \quad \Rightarrow \quad c_a^2 > 0
\end{align*}  \hspace{1cm} (27)

As one can see from these relations, the regions I and IV have the classical stability in our model. We plot the behavior of the model in the $\omega - \omega'$ phase plane and identify the regions mentioned above in figure 5.
Figure 5: Bounds on $\omega'$ as a function of $\omega$ in $\omega - \omega'$ phase plane. The stable regions are I and IV.

3 A Ricci-Coupled Scalar Field in the Einstein Frame

Now we extend our study of the mentioned cosmological issues to the Einstein frame by adopting a conformal transformation. The action (1) in the Jordan frame can be rewritten as follows

$$S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \xi R \phi^2 - V(\phi) \right]$$

(28)

where we assumed $k_4^2 = 1$ and $\alpha(\phi) = \frac{1}{2}(1 - \xi \phi^2)$ and $\xi$ is a non-minimal coupling. The metric signature convention is chosen to be $(+ - - -)$ with spatially flat Robertson-Walker metric as follows

$$ds^2 = dt^2 - a(t)^2 \delta_{ij} dx^i dx^j.$$  

(29)

To obtain the fundamental background equations in Einstein frame, we perform the following conformal transformation

$$\hat{g}_{\mu \nu} = \Omega g_{\mu \nu}, \quad \Omega = 1 - \xi \phi^2.$$  

(30)

Here we use a hat on a variable defined in the Einstein frame. The conformal transformation gives (see for instance [16] and references therein)

$$S = \int d^4 \hat{x} \sqrt{-\hat{g}} \left[ \frac{1}{2} \hat{R} - \frac{1}{2} \hat{F}^2(\phi) \hat{g}^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \hat{V}(\phi) \right].$$  

(31)
where by definition

\[ F^2(\phi) \equiv \frac{1 - \xi \phi^2(1 - 6\xi)}{(1 - \xi \phi^2)^2} \]  

(32)

and

\[ \dot{V}(\phi) \equiv \frac{V(\phi)}{(1 - \xi \phi^2)^2}. \]

(33)

Therefore, one may redefine the scalar field as follows

\[ \frac{d\hat{\phi}}{d\phi} = F(\phi) = \frac{\sqrt{1 - \xi \phi^2(1 - 6\xi)}}{1 - \xi \phi^2}. \]

(34)

When we investigate the dynamics of universe in the Einstein frame, we should transform our coordinates system to make the metric in the Robertson-Walker form

\[ \dot{a} = \sqrt{\Omega}a, \quad dt = \sqrt{\Omega}dt, \]

(35)

and we obtain

\[ d\hat{s}^2 = d\hat{t}^2 - \hat{a}^2(\hat{t})\delta_{ij}dx^i dx^j. \]

(36)

Note that the physical quantities in Einstein frame should be defined in this coordinate system. Now the field equations can be written as follows

\[ \dot{H}^2 = \frac{1}{3} \left[ \frac{1}{2} \left( \frac{d\hat{\phi}}{dt} \right)^2 + \dot{V}(\hat{\phi}) \right] = \frac{\dot{\rho}}{3}, \]

(37)

\[ \frac{d^2\hat{\phi}}{dt^2} + 3H \frac{d\hat{\phi}}{dt} + \frac{d\dot{V}}{d\phi} = 0 \]

(38)

where \( \dot{H} = \frac{\dot{a}}{a} \).

We assume that scalar field \( \hat{\phi} \) has only time dependence and we find dynamics of equation of state as follows

\[ \dot{\omega}_\phi = \frac{\hat{\rho}}{\rho} = \frac{\frac{1}{2} \left( \frac{d\hat{\phi}}{dt} \right)^2 - \dot{V}(\hat{\phi})}{\frac{1}{2} \left( \frac{d\hat{\phi}}{dt} \right)^2 + \dot{V}(\hat{\phi})}. \]

(39)

This is an interesting result: while a non-minimally coupled scalar field in the Jordan frame is a suitable dark energy component with capability to realize phantom divide line crossing, its conformal transformation in the Einstein frame has not this capability. In fact, conformal transformation from Jordan frame to Einstein frame transforms the equation of state parameter of dark energy component to its minimal form with a redefined scalar field and in this
case it is impossible to realize a phantom phase with possible crossing of the phantom divide line. In a minimal coupling case with \( \alpha = 0 \), one can obtain equation of state parameter \( \omega = 1 - 2\cos^2(mt) \). As figure 2 shows, this minimally coupled scalar field cannot explain the crossing of the phantom divide line. However, we note that multi component minimally coupled scalar fields can realize this crossing [14].

To study bouncing behavior of the solutions in the Einstein frame with above mentioned redefined scalar field, we see that Hubble parameter and scale factor have no possibility to run across zero. This behavior has been shown in figure 6 using ansatz presented in the previous section. Therefore, in the Einstein frame we have not any possibility to realize bouncing solutions with this redefined scalar field.

### 4 Summary

According to existing literature on scalar field dark energy models, a minimally coupled scalar field is not a good candidate for dark energy since its equation of state parameter has not the capability to realize crossing of the phantom divide line. On the other hand, a scalar field non-minimally coupled to gravity in the Jordan frame has the capability to be a suitable candidate for dark energy which provides crossing of the phantom divide line and other required facilities. Here we have shown that while a non-minimally coupled scalar field in the Jordan frame is a suitable dark energy component with capability to realize phantom divide line crossing, its conformal transformation in the Einstein frame has not this capability. In fact, conformal transformation from Jordan frame to the Einstein frame transforms the equation of state parameter of dark energy component to its minimal form with a redefined scalar field and in this case it is impossible to realize a phantom phase with possible crossing of the phantom divide line. On the other hand, one of the most serious shortcomings of the standard cosmology is the problem of initial (and possibly final) singularity. In recent
analysis done within the loop quantum cosmology, the Big Bang singularity is replaced by a quintessence Big Bounce with finite energy density of the scalar field. As we have shown here, in the Jordan frame with a non-minimally coupled quintessence field, one achieve a phenomenologically viable framework to overcome initial singularity with possible realization of the bouncing solutions. We have studied the stability of this bouncing solutions too. As a result, there are appropriate regions of the $\omega - \omega'$ phase plane that solutions are stable in the Jordan frame. However, the redefined scalar field in the Einstein frame cannot realize these bouncing solutions.

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