Total cross section of neutron-proton scattering at low energies in quark-gluon model

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Abstract

We show that analysis of nonrelativistic neutron-proton scattering in a framework of relativistic QCD based quark model can give important information about QCD vacuum structure. In this model we describe total cross section of neutron-proton scattering at kinetic energies of projectile neutron from 1 eV up to 1 MeV.

1. Numerous experimental data are collected in low energy nuclear physics, substantial part of which demands rethinking in a framework of modern understanding of strong interaction physics. In particular, in wide range of nonrelativistic kinetic energies of projectile neutron the total cross section of neutron-proton scattering is constant and is approximately equal to 20 barn [1], see also [http://www.nndc.bnl.gov/sigma/].

The theoretical explanation of the total cross section constancy [2] is based on $S$-wave scattering where total cross section is given by the following formula

$$\sigma_{tot}^{(t)} = \frac{4\pi \hbar^2}{M} \frac{1}{E + E_d},$$

(1)

where $M$ – nucleon mass, the deuteron binding energy $E_d \approx 2.23$ MeV. The upper index in (1) means that the scattering takes place in triplet state. However, if one substitutes all numerical values into (1) (which has no free parameters), one obtains $\sigma_{tot}^{(t)} \simeq 2$ barn at energies $E \ll E_d$, this is one order

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of magnitude smaller than the experimental value. So in order to describe the experimental data one has to introduce contribution from neutron-proton system singlet state with an unclear phenomenological parameter $E_s$ – energy of this system virtual state.

2. Quantum chromodynamics is a microtheory of strong interactions. QCD self-consistently describes processes in the region of “hard” physics, where running coupling constant $\alpha_s$ is small and perturbation theory can be applied. In the region of “soft” physics, where distances are large and transferred momenta are small, there is no justification for QCD diagrams applicability. But, using our previous experience of strong interactions analysis in colorless particles language, where the Feynman diagrams allowed to explain inelastic processes structure at high energies, we will consider QCD diagrams in this region.

3. The example of QCD Feynman diagrams successful use in “soft” physics is the description of hadron-hadron interactions cross sections in wide range of ultrarelativistic energies [3]. In this work we have successfully described total cross sections of $pp$, $\bar{p}p$, $np$, $\bar{n}p$, $\pi^\pm p$, $K^\pm p$ scatterings by the same parameters in the framework of one approach. The main idea of this approach lies in fact that constant part of raising cross sections is described by one gluon exchange between components with only valence quarks (Fig. 1). The convolution of this diagram with its adjoint diagram (Fig. 2a) gives contribution to the total cross section of nucleon-nucleon scattering. Since color objects do not fly out, the $F_n$ amplitudes correspond to processes which take place with probability equals to 1. Therefore the diagram from Fig. 2a is equivalent [3] to diagram of the Low-Nussinov two-gluon pomeron [4]. Namely such diagram gives constant contribution to total cross section.

![Figure 1: One-gluon exchange (wavy line) between hadrons components containing only valence quarks (thin solid lines). Thick solid lines correspond to hadrons both in initial and final states.](image)

4. The diagram of two-gluon pomeron for meson-meson scattering was calculated in [5] with $\pi$-meson model form factor as in [6]. Constant contri-
Figure 2: a) convolution of one-gluon exchange diagram with its adjoint diagram; b) two-gluon exchange diagram – Low-Nussinov two-gluon pomeron.

The constant contribution to $\pi\pi$ scattering is equal to

$$\sigma_{\text{tot}}^{\pi\pi(\text{const})} = \frac{128}{9} \pi \frac{\alpha_s^2}{m_\rho^2}, \quad (2)$$

where $m_\rho$ – $\rho$-meson mass. Using Levin-Frankfurt relations of total cross sections ratios for hadron-hadron scattering [7]

$$\sigma_{\text{tot}}^{\pi\pi} : \sigma_{\text{tot}}^{\pi p} : \sigma_{\text{tot}}^{p p} = 4 : 6 : 9, \quad (3)$$

the last two are in good agreement with experiment, we obtain the expression for constant contribution to neutron-proton scattering at high energies

$$\sigma_{\text{tot}}^{np(\text{const})} = 32 \pi \frac{\alpha_s^2}{m_\rho^2}. \quad (4)$$

Given the value of $pp$, $\bar{p}p$, $np$, $\bar{p}n$ total cross sections constant part $\sigma_0 = 21.63$ [3] we can substitute it in (4) and thus we obtain

$$\alpha_s = 0.576, \quad \alpha_s^2 = 0.332. \quad (5)$$

5. The two-gluon exchange amplitude (Fig. 2b), which is defined in ultrarelativistic region, must also exist for nonrelativistic energies.

First, since non particle degrees of freedom (e.g. instantons) exist in QCD, particle degrees of freedom – quarks and gluons also must exist. Therefore two-gluon diagrams, generally speaking, must give contribution also in non-relativistic region.

Second, in ultrarelativistic case of “soft” scattering, where the most part of hadron-hadron total cross section is gathered, no noticeable contributions to interaction through non particle degrees of freedom are seen experimentally. Also, non particle degrees of freedom effects are not taken into account.
in various fits of total cross sections. (We think that non particle degrees of freedom affect QCD vacuum.)

Third, the constant contribution to total cross sections is provided only by vector exchange, i.e. only two-gluon diagram can give constant value of neutron-proton scattering.

Therefore we accept that two-gluon diagram gives contribution in non-relativistic region.

6. Unlike the ultrarelativistic case, where all valence quarks in the upper block of Fig. 1 move in one direction and in the lower block in another (we consider scattering in center-of-mass frame), in the nonrelativistic case the situation is different. We deal with almost rest neutron and proton. We also accept that valence quarks in neutron and proton have current masses of a few MeV. If one supposes that in a rest hadron quarks momenta are of the same magnitude as transverse momenta of quarks in the ultrarelativistic case, since transverse part of momentum does not change under Lorentz transformation, then the value of rest hadron quark momentum is about 0.5 – 0.6 GeV/c. In this case we have to consider relativistic model of quarks interaction even in rest hadron.

7. It is easy to show that additional contribution exists in almost rest neutron and proton, which is equal to contribution from the two-gluon diagram in the ultrarelativistic region. It is connected to movement of quarks in the upper block of diagram in Fig. 1 in negative direction while quarks in the lower block move in positive direction. This contribution decreases like $4M^2/s$ as the energy increases. Then the two-gluon exchange contribution to the amplitude of elastic scattering in nonrelativistic region has the following form

$$\sigma_{np}^{\text{tot}} = 32\frac{\alpha_S^2}{m_p^2} \left( 1 + \frac{4M^2}{s} \right).$$

(6)

8. We accept the hypothesis of “freezing” of running constant $\alpha_S$ and assume that for nonrelativistic neutron-proton scattering $\alpha_S$ is equal to the value, obtained from total cross sections fittings in the ultrarelativistic region, $\alpha_S = 0.576$. Substituting this value in (6) we obtain in the nonrelativistic case $\sigma_{tot}^{np} \simeq 43$ mb. This is 50 times lower than the experimental value of $\sigma_{tot}^{np}$. It is possible to assume that the value of $\alpha_S$ is large in the region of nonrelativistic scattering, we will consider this question in future works. But here we suppose that the running constant $\alpha_S$ is freezed and thus we find a physical mechanism that increases total cross section $\sigma_{tot}^{np}$. 


9. In vacuum quark fluctuations light quarks with sufficiently small momenta $k < \Lambda$ move on distances larger than the confinement radius. In this case color electric field string appears between them. If quarks did not carry spin, tension of string would rotate quarks in direction to each other not allowing them to move further than the confinement radius. But quarks are fermions. Therefore, in order to change the momentum direction, either chirality must be changed or it is needed to rotate quark spin. So Casher [11] has proposed that quark fluctuations in vacuum are not isolated. Thus it is possible to replace quarks from correlated fluctuations instead of rotating spin. In hadrons $q \bar{q}, qqq$ all the time valence quarks come closer to hadron boundary they are replaced by quarks from quark condensate. There is no such effect in the ultrarelativistic case since contribution from vacuum fluctuations can be neglected there.

10. At low kinetic energies of colliding neutron and proton there are no secondary particles coming from color strings decay. Therefore one-gluon exchange amplitude is unlike the diagram in Fig. 1 and is shown in Fig. 3a. Its modulus squared directly gives two-gluon diagram from Fig. 2b. We take into account the effect of valence quark replacement with a quark from quark condensate by considering diagrams in Fig. 3b, 3c. Presumably, interference between quark on mass shell in hadron and virtual quark from quark-gluon condensate is not possible. Therefore we take moduli squared of diagrams in Fig. 3b, 3c. Diagram in Fig. 3d, describing interaction of quarks from condensate, must be neglected.

The interaction radius of quarks in nucleon with quarks from vacuum condensate is defined by the correlation length, i.e. the distance, where
quarks from vacuum fluctuations can replace quarks in nucleons. We define this distance as \( R_{cor} = 1/m_G \). Therefore, by analogy with the ultrarelativistic case we write down the phenomenological formula

\[
\sigma_{np}^{tot} = 32\pi \alpha_s^2(m^2_\rho) \left[ \frac{1}{m^2_\rho} \left( 1 + \frac{2M}{E_{kin}^n M} \right) + \frac{2}{m_G^2} \frac{\sqrt{2} m_\Lambda}{\sqrt{E_{kin}^n M + 2m^2_\Lambda}} \right]. \tag{7}
\]

We explain formula (7). The first term in square bracket is evident, it follows from (6). Factor \( 2/m_G^2 \) in the second term gives contribution from interaction of valence quark from one of the nucleons with quark from vacuum fluctuation, replacing valence quark. Evidently, vacuum fluctuations contribution has to decrease as energy of projectile neutron increases (i.e. square of total energy in center-of-mass system increases). It is taken into account by factor \( \frac{\sqrt{2} m_\Lambda}{\sqrt{E_{kin}^n M + 2m^2_\Lambda}} \) in the second term.

11. We fitted the experimental data on neutron-proton total cross section [1] in kinetic energy range from 1 eV to 1 MeV using formula (7) and obtained values of \( m_G = 35 \text{ MeV} \) and \( m_\Lambda = 4.7 \text{ MeV} \), the result is shown in Fig. 4.

![Figure 4: Fitting of \( \sigma_{np}^{tot} \) by formula (7).](image-url)
12. In conclusion we repeat the logical scheme of our approach.

i. We suppose that quarks are relativistic in rest hadrons.

ii. We accept that the QCD Feynman diagrams can be used for phenomenological description at low nonrelativistic energies of colliding nucleons.

iii. The constant contribution to total cross section of neutron-proton scattering comes from the two-gluon exchange in imaginary part of elastic scattering amplitude.

iv. There is effect of infrared “freezing” of the running coupling constant $\alpha_s$ and its value can be prolonged in region of low nonrelativistic energies.

v. At low energies there is replacement of valence quarks in nucleons by quarks from vacuum fluctuations – Casher mechanism.

With these assumptions it is possible to describe the value of neutron-proton scattering total cross section in neutron energy range from 1 eV to 1 MeV.

In our opinion, the proposed phenomenological model has right to exist and it is not worse than description of cross section $\sigma_{np}$ which is given in nuclear theory. In future works we will give more detailed justification of our assumptions, based on experimental data.

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