1. Introduction

A direct generalization of the Standard-Model Extension\(^1\) (SME) follows by taking into account nonrenormalizable operators, that is, operators with mass dimension higher than four. Such program has been successfully implemented in the photon sector,\(^2\) fermion sector\(^3\) and more recently in the linearized sector of gravity.\(^4\) An earlier work of Myers and Pospelov focuses on dimension-five operators with approximately cubic dispersion relations.\(^5\) Alternatively, Lorentz-invariance violation with higher-order operators may be realized with higher-order coupling terms.\(^6\)

Quantum field theory with higher-order operators may lead to an indefinite metric in the Hilbert space. The extended inner product introduced by the indefinite metric $\eta$ allows for negative norm states or ghosts and produces a pseudo-unitary condition for the $S$ matrix, i.e., $S\eta S = \eta$. As shown by Lee and Wick, by imposing the boundary condition in which only positive norm states appear in the asymptotic Hilbert space, it is possible to preserve unitarity order by order in perturbation theory.\(^7\) In this work, in the light of the Lee-Wick studies, we show how unitarity can be conserved in a nonminimal Lorentz-invariance violating QED model.
2. Lee-Wick theory

In 1969 Lee and Wick\(^7\) proposed a modified QED with the advantage of being finite but leading to an indefinite metric in Hilbert space. The origin of the negative metric is an extra field introduced by hand, which may be seen to arise from a higher-order operator as well.\(^8\) Several issues regarding stability and unitarity were solved using what is now called the Lee-Wick prescription.

The point of departure from usual quantum theory is the definition of the inner product. In an indefinite metric theory the inner product of two states \(|\phi\rangle\) and \(|\psi\rangle\) is defined by \(\langle \phi | \psi \rangle = \phi^*_i \eta_{ij} \psi^*_j\) where the metric \(\eta_{ij}\) can take negative values. In this way one has negative norm states in the theory. In particular, the eigenstates of the self-adjoint Hamiltonian operator can be states with positive norm and real eigenvalues or states with zero norm and complex eigenvalues.\(^9\) In this way the Hilbert space contains states with positive norm which oscillate in time and zero norm states which grow or decay. The Lee and Wick prescription consists of excising the growing or decaying modes from the asymptotic Hilbert space and to modify the Feynman diagrams diagram by diagram to allow for stability and unitarity of the \(S\) matrix.

3. The QED model

Our starting point is the Myer and Pospelov lagrangian\(^5\)

\[
\mathcal{L} = \bar{\psi} (i\slashed{D} - m) \psi + g \bar{\psi} \slashed{n} (n \cdot \partial)^2 \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},
\]

with \(n\) a privileged four-vector and \(g\) a small parameter. We choose \(n = (1, 0, 0, 0)\) which yields the dispersion relation \((p_0 - gp_0^2 - E^2) = 0\) with \(E = \sqrt{\vec{p}^2 + m^2}\). The Hamiltonian is

\[
H = \sum_s \int \frac{d^4p}{(2\pi)^3} \left( \omega_1 a_p^s a_p^s + \omega_2 b_p^s b_p^{s\dagger} - W_1 c_p^s c_p^s - W_2 d_p^s d_p^{s\dagger} \right),
\]

where the four solutions are given by

\[
\omega_1 = \frac{1 - \sqrt{1 - 4gE}}{2g}, \quad \omega_2 = \frac{1 - \sqrt{1 + 4gE}}{2g}, \quad W_1 = \frac{1 + \sqrt{1 - 4gE}}{2g}, \quad W_2 = \frac{1 + \sqrt{1 + 4gE}}{2g}.
\]

The first two solutions \(\omega_1, \omega_2\) correspond to modifications to the usual solutions \(E, -E\), respectively, and the next two \(W_1, W_2\) correspond to Lee-Wick modes associated to a negative metric, as seen from the Hamiltonian (2).
4. Stability and unitarity

Our goal is to verify the optical theorem which is basically the equality of the sum over final states of the amplitude with the imaginary part of the loop diagram. We follow the Lee-Wick prescription in order to prove the equality of the matrix elements in the scattering process \( e^+ (k_1) + e^- (k_2) \to e^+ (k) + e^- (k') \). Some central points to satisfy the perturbative constraint are: (i) the sum over physical states in the amplitude diagram must be carried only over positive metric states, (ii) in the loop diagram a suitable prescription for the path \( C \) in needed to avoid the poles and to compute the residues, (iii) the previous prescription has to reproduce well the usual case in a limit and have the exact discontinuities in the physical sheet in order to produce the correct imaginary part. Finally, by comparing both sides one is able to prove the unitarity constraint for the considered one-loop scattering process.

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