Modelling the efficiency of infrastructure repair based on the risk process

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Abstract. To improve the efficiency of railway infrastructure repairs, it is proposed to introduce an insurance fund, which performs two functions: it accumulates payments with different frequency and cost for performing different types of repair work; it pays for these works as necessary. To mathematically describe the state of the insurance fund, it is suggested that the risk process of a special kind be used. This approach made it possible to introduce indicators of repairs in the form of resource-cost and financial risks, considering the possibility of non-performance of repairs due to lack of funds. To study these indicators, a modeling program has been created in the MATLAB environment, based on the event-driven approach. Experiments with the modeling program led to the conclusion that, in terms of a minimum of risk values, preference should be given to the option when the shares and frequency of payments should depend on the type of repair work and initial data.

1. Introduction

The purpose of the maintenance and repair system is to manage the technical condition of the equipment within its resource, allowing one to: a) ensure that the equipment is ready for use as intended and operational during operation; b) minimize time, labor and equipment cost for repairs. The importance of introducing mathematical methods in improving the efficiency of repairs of various equipment, as well as in the creation of indicators for evaluating performance management systems, is confirmed by publications [1-3].

Maintenance and repair of railway infrastructure include operation and maintenance of the track, contact network and the infrastructure complex as a whole [3, 4]. Three types of repairs can be identified: 1) current, 2) emergency, 3) major. To make this work more effective, considering the developed system of diagnostics and monitoring [5], they need to be performed according to the actual condition. Repairs by actual condition take into account an important factor of freight transport - functioning under uncertainty and risk [6].

To improve the efficiency of repairs and their evaluation, it is planned to introduce an insurance fund, which performs two functions: 1) accumulation of payments with different frequency and cost; 2) paying for these works, as necessary. At the same time, taking into account the maintenance of the infrastructure according to the actual state in the conditions of uncertainty of freight transport, the time intervals between the repair works (day) and the cost of these works (million rubles) can be considered as random variables with known distribution functions up to the values of their parameters. This approach makes it possible to mathematically describe the state of the insurance fund when...
modeling the effectiveness of infrastructure repair works, using a special type of risk process, which is used in insurance mathematics [7], as well as when assessing the effectiveness of complex equipment repair works [8].

2. Mathematical description of performance indicators based on risk process

Considering that the work investigates three types of repair work, we define the risk process as follows:

\[ R(t) = X_0 + Y_1(t) + Y_2(t) + Y_3(t) - YT(t) - YA(t) - YK(t), \]

(1)

where \( X_0 \) is the initial funds of the insurance fund; \( Y_j(t) \) is the total accumulation of payments by type of work, \( (j=1,2,3) \); \( YT(t) \) is the total cost for current work; \( YA(t) \) is the total cost for emergency situations; \( YK(t) \) is the total cost for major work.

When servicing the railway infrastructure during its operation, the amount of payments for various types of repairs for a year is initially formed (\( X_0 \), million rubles). Further, the annual amount of payments is distributed by type of work:

\[ X^{(1)}_j = c_1 \cdot X, \quad X^{(2)}_j = c_2 \cdot X, \quad X^{(3)}_j = c_3 \cdot X; \quad c_1 + c_2 + c_3 = 1. \]

(2)

Here \( c_1 \) is the coefficient that takes into account a part of the payments for current repair work; \( c_2 \) is the coefficient that takes into account a part of the payments for emergency repair work; \( c_3 \) is the coefficient that takes into account a part of the payments for major repair work; \( X^{(j)}_j \) are annual payments to the insurance fund by type of work, \( (j=1,2,3) \).

The cost of one payment to the insurance fund for the \( j \)-th type repair work is based on (2):

\[ Y_j = h_j \cdot X^{(j)}_j / Tg = c_j \cdot h_j \cdot X / Tg, \quad j = 1,2,3, \]

(3)

where \( Tg \) is the number of days in a year; \( h_j \) is the frequency of payments to the insurance fund (day) for the \( j \)-th type of work.

The total accumulations of payments to the insurance fund for \( j \)-th type repair works both based on (2) and (3) and assumptions of replenishment frequency are equal to:

\[ Y_j(t) = Y_j \cdot N_j(t) = (c_j \cdot h_j \cdot X / Tg) \cdot N_j(t), \quad j = 1,2,3, \]

(4)

where \( N_j(t) \) is the number of payments to the insurance fund over time \( t \) for the \( j \)-th type of work.

For the random risk process (1), the time instant \( \tau \) (day) is determined when the condition \( R(t) < 0 \) is fulfilled for the first time:

\[ \tau = \min \{ t : R(t) < 0 \}. \]

(5)

A moment of time (5) characterizes the efficiency of repair work in terms of the formation of payments by their types (3), and the probability of a special event is proposed to be considered as a resource-cost risk:

\[ r_\tau = P(\tau < T_\tau), \]

(6)

where \( T_\tau \) is set time (day).

The risk indicator (6) is called resource-cost because it evaluates the “Resource- cost” model for repair work related to the railway infrastructure operation process. In the absence of funds in the insurance fund, there is a risk of non-performance of these works.

In practice, a two-factor model is more widely used, where in addition to the probability of a negative event, the financial consequences of the event are considered. In this regard, the work introduces the concept of financial risk of the form:
\[ R_{\phi} = r_{\tau} \cdot C_{R}, \text{ million rubles}, \]  
\[ \text{where } C_{R} \text{ is the losses from non-performing repairs, million rubles; } r_{\tau} \text{ is the value that characterizes resource-cost risk. Values (6) and (7) are unknown; in simulation they are replaced by point and interval estimates:} \]
\[ \hat{R}_{\tau} = k_{\tau} / n_{0}, \]
where \( k_{\tau} \) is the number of process implementations (1) for which the condition (5) is fulfilled, \( n_{0} \) is the total number of created implementations by simulation method;

\[ \tau_{i} = k_{\tau} / \left[ k_{\tau} + (n_{0} - k_{\tau} + 1) \cdot F_{1}(v_{1}, v_{2}) \right], \]
where \( F_{1}(v_{1}, v_{2}) \) is the critical value for the \( F \)-distribution with \( v_{1} \) and \( v_{2} \) degrees of freedom and confidence probability \( \gamma = 0.95 \); \( v_{1} = 2 \cdot (n_{0} - k_{\tau} + 1), v_{2} = 2 \cdot k_{\tau} \);

\[ \tau_{2} = \frac{(k_{\tau} + 1) \cdot F_{2}(v_{3}, v_{4})}{[n_{0} - k_{\tau} + (k_{\tau} + 1) \cdot F_{2}(v_{3}, v_{4})]}, \]
where \( F_{2}(v_{3}, v_{4}) \) is the critical value for the \( F \)-distribution with \( v_{3} \) and \( v_{4} \) degrees of freedom and confidence probability \( \gamma = 0.95 \); \( v_{3} = 2 \cdot (k_{\tau} + 1), v_{4} = 2 \cdot (n - k_{\tau}) \):

\[ \hat{R}_{\phi} = \hat{R}_{\tau} \cdot C_{R}; \]

\[ R_{i} = \tau_{i} \cdot C_{R}, R_{2} = \tau_{2} \cdot C_{R}. \]  

3. Selecting the original data and the task of a computational experiment

Based on the developed mathematical support described above, a modeling program for the study of rail infrastructure repair work has been created by the simulation method. The software is based on the MATLAB programming language, which has several advantages over other software environments designed to perform scientific and engineering calculations [9].

Table 1 shows the distribution laws and their numerical characteristics for the time intervals between the types of work and the costs of these works used in this study: \( mi, mz \) are the mathematical expectations, \( k_{i} \) is the coefficients of variation. The choice of distribution laws and their numerical characteristics was made with the help of experts and literary sources [10]. The software allows for changing these models.

| Works   | Intervals of time, day | Costs, million rubles |
|---------|------------------------|-----------------------|
| Current | Birnbaum-Saunders       | Lognormal             |
|         | \( mi=15.0 \)           | \( mz=3.5 \)          |
|         | \( k_{i}=0.20 \)        | \( k_{i}=0.20 \)      |
| Emergency | Weibull     | Pareto                |
|         | \( mi=45.0 \)           | \( mz=10.0 \)         |
|         | \( k_{i}=0.30 \)        | \( k_{i}=1.50 \)      |
| Major   | Gamma                  | Normal                |
|         | \( mi=60.0 \)           | \( mz=20.0 \)         |
|         | \( k_{i}=0.25 \)        | \( k_{i}=0.15 \)      |

With the selected mathematical expectations (table 1), the average cost of repairs by type (current, emergency, major) per year is equal to:

\[ P_{j} = T_{g} \cdot mz_{j} / mi_{j}, \ j = 1, 2, 3. \]  

\( P_{j} \text{ is the average cost of repairs by type } j \); \( T_{g} \text{ is the total number of work types } g \).
Considering (13) and the values of Table 1, average expenses are equal to:

\[ P_1 = 84 \text{ million rubles, } P_2 = 80 \text{ million rubles, } P_3 = 120 \text{ million rubles.} \]  

(14)

In total, these expenses are equal to the annual amount of payments \( X \). Then the share of payments for the types of repairs (2) can be determined as:

\[ c_j = \frac{P_j}{X}, \quad j = 1, 2, 3; \quad X = \sum_{j=1}^{3} P_j. \]  

(15)

If we put in the formula (15) value (14), we will get the following shares of payments:

\[ c_1 = 0.296; \quad c_2 = 0.282; \quad c_3 = 0.422. \]  

(16)

Values (3) with a view (13) equal:

\[ Y_j = h_j \cdot P_j / T g, \quad j = 1, 2, 3. \]  

(17)

Values (4) per year are equal to:

\[ Y_j(T g) = Y_j \cdot T g / h_j, \quad j = 1, 2, 3. \]  

(18)

The amount of payments for the year (18) and the amount of expenses for the year (13) for each type of repair should be, on average, equal to each other:

\[ Y_j(T g) = P_j, \quad j = 1, 2, 3. \]

In risk theory, it has been proven that it is necessary to have an excess of income over expenses [6]. In our case, the condition must be met:

\[ X_0 + \sum_{j=1}^{3} Y_j(T g) > \sum_{j=1}^{3} P_j. \]  

(19)

In the work [8], it is shown that the excess of the revenue side over the expenditure side is carried out at the expense of the initial annual funds of the insurance fund \( X_0 \).

In this paper, it is proposed to link the frequency of payments (\( h_j \)) with the mathematical expectations of the time intervals between types of work (\( m_{ij} \), Table 1):

\[ q_j = h_j / m_{ij}, \quad j = 1, 2, 3. \]  

(20)

To obtain practical results, using the created modeling program, three options were investigated. For each option, three cases are considered, when \( X_0 \) is equal to 5, 10 and 20 percent of annual payments of \( X = 284.0 \) million rubles: a) \( X_0 = 14.2 \) million rubles; b) \( X_0 = 28.4 \) million rubles; c) \( X_0 = 56.8 \) million rubles. This research includes three options:

- option A, when the share of payments is equal (16), the frequency of payments based on (20) is equal: \( h_1 = 10 \) days (\( q_1 = 0.67 \)), \( h_2 = 30 \) days (\( q_2 = 0.67 \)), \( h_3 = 90 \) days (\( q_3 = 1,50 \));

- option B, when the payment shares are the same: \( c_1 = 0.333; \quad c_2 = 0.333; \quad c_3 = 0.334; \) the frequency of payments based on (20) is equal: \( h_1 = 10 \) days (\( q_1 = 0.67 \)), \( h_2 = 60 \) days (\( q_2 = 0.67 \)), \( h_3 = 90 \) days (\( q_3 = 1,50 \));

- option C, when the share of payments is equal (16), the frequency of payments based on (20) is equal: \( h_1 = 20 \) days (\( q_1 = 1.33 \)), \( h_2 = 60 \) days (\( q_2 = 1.33 \)), \( h_3 = 90 \) days (\( q_3 = 1,50 \)).

The considered options are modeled for four values of the quantity \( T g \): 30 day, 90 day, 180 day, 360 day.
4. The results of a computational experiment
Table 2 shows the results of a computational experiment.
Analysis of the values in Table 2 allows us to conclude that, in terms of reducing resource-cost and financial risks, the share of payments should depend on the type repair work and determined by the formula (15) (for the selected initial data, they are equal (16)). The frequency of payments should depend on the mathematical expectations of the time intervals between the types of work, the coefficients of proportionality (20) should be close to each other. For the selected initial data, option C has the best frequency of payments. This option, when the shares of payments are equal (16), has the lowest risks in all cases.

The confidence of this conclusion is also justified by the fact that differences in point estimates are confirmed not by the intersection of confidence intervals of risks. This fact allows us to choose a statistical hypothesis that the risks are less for option C than for option B and option A.

**Table 2. Results of point and interval risk estimates**

| V₁ | V₂ | Tₛ | kₛ | $\bar{R}_τ$ | $τ_1$ | $τ_2$ | $\bar{R}_φ$ | $R_1$ | $R_2$ |
|----|----|----|----|-----------|------|------|-----------|------|------|
| a  |    | 30 | 45 | 0.002     | 0.002| 0.003| 0.563     | 0.432| 0.721|
|    | 90 | 1303 | 0.065 | 0.062 | 0.068 | 16.288 | 15.575 | 17.023|
|    | 180| 2973 | 0.149 | 0.145 | 0.153 | 37.163 | 36.131 | 38.213|
|    | 360| 5442 | 0.272 | 0.267 | 0.277 | 68.025 | 66.731 | 69.331|
| A  | b  | 30 | 33 | 0.002     | 0.001| 0.002| 0.412     | 0.302| 0.551|
|    | 90 | 653 | 0.033 | 0.031 | 0.035 | 8.162  | 7.652  | 8.698 |
|    | 180| 1668 | 0.083 | 0.080 | 0.087 | 20.850 | 20.051 | 21.671|
|    | 360| 3583 | 0.179 | 0.175 | 0.184 | 44.788 | 43.675 | 45.917|
|    | c  | 30 | 12 | 0.001     | 0.001| 0.001| 0.150     | 0.087| 0.243|
|    | 90 | 223 | 0.011 | 0.010 | 0.012 | 2.788  | 2.489  | 3.113 |
|    | 180| 637 | 0.032 | 0.030 | 0.034 | 7.963  | 7.458  | 8.492 |
|    | 360| 1582 | 0.079 | 0.076 | 0.082 | 19.775 | 18.995 | 20.577|
| B  | b  | 30 | 88 | 0.004     | 0.004| 0.005| 1.100     | 0.915| 1.313|
|    | 90 | 2538 | 0.127 | 0.123 | 0.131 | 31.725 | 30.761 | 31.709|
|    | 180| 4514 | 0.226 | 0.221 | 0.231 | 56.425 | 55.211 | 57.654|
|    | 360| 6637 | 0.332 | 0.326 | 0.337 | 82.962 | 81.592 | 84.342|
|    | c  | 30 | 85 | 0.004     | 0.004| 0.005| 1.063     | 0.881| 1.272|
|    | 90 | 2083 | 0.104 | 0.101 | 0.108 | 26.038 | 25.154 | 26.943|
|    | 180| 3638 | 0.182 | 0.177 | 0.186 | 45.475 | 44.356 | 46.611|
|    | 360| 5212 | 0.261 | 0.255 | 0.266 | 65.150 | 63.874 | 66.439|
| C  | b  | 30 | 64 | 0.003     | 0.003| 0.004| 0.800     | 0.643| 0.985|
|    | 90 | 1408 | 0.070 | 0.067 | 0.073 | 17.600 | 16.861 | 18.362|
|    | 180| 2297 | 0.115 | 0.111 | 0.119 | 28.712 | 27.790 | 29.656|
|    | 360| 3083 | 0.154 | 0.150 | 0.158 | 38.538 | 37.491 | 39.603|
|    | a  | 30 | 63 | 0.003     | 0.003| 0.004| 0.787     | 0.632| 0.971|
|    | 90 | 1413 | 0.071 | 0.068 | 0.074 | 17.663 | 16.923 | 18.425|
|    | 180| 3114 | 0.156 | 0.151 | 0.160 | 38.925 | 37.874 | 39.995|
|    | 360| 5632 | 0.282 | 0.276 | 0.287 | 70.400 | 69.092 | 71.720|
|    | c  | 30 | 38 | 0.002     | 0.001| 0.002| 0.475     | 0.356| 0.622|
|    | 90 | 684  | 0.034 | 0.032 | 0.036 | 8.550  | 8.028  | 9.097 |
|    | 180| 1692 | 0.085 | 0.081 | 0.088 | 21.150 | 20.346 | 21.977|
|    | 360| 3552 | 0.176 | 0.172 | 0.181 | 44.038 | 42.932 | 45.160|
|    | b  | 30 | 16 | 0.001     | 0.001| 0.001| 0.200     | 0.125| 0.304|
|    | 90 | 236  | 0.012 | 0.011 | 0.013 | 2.950  | 2.643  | 3.284 |
|    | 180| 671  | 0.034 | 0.031 | 0.036 | 8.388  | 7.870  | 8.930 |
|    | 360| 1633 | 0.082 | 0.078 | 0.085 | 20.413 | 19.621 | 21.226|
Additionally, it can be argued that with an increase in the initial value, the resource-cost risk decreases, and with an increase in time $T_\tau$, it increases. The latter is logically obvious, but the simulation analysis makes it possible to quantify these changes.

5. Conclusion
A study of the efficiency of railway infrastructure repairs has been carried out in the conditions of the insurance fund. The state of this fund is described by the risk process. A program based on an event approach has been created to model this process. The modeling program creates a sample of the moments of time when there are no funds for equipment repairs. Sampling values are processed according to the proposed algorithms for assessing resource-cost and financial risks. These indicators allow us to assess the efficiency of railway infrastructure repairs and to obtain practically important results. For example, computational experiments show that the criteria for minimum risk assessments should be preferred to the option when the share of payments should depend on the type of repair work, and the frequency of payments should depend on the mathematical expectations of the time intervals between the types of work.

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