Highly Dynamic Eddy-Current-Based Sealed Magnetic Bearing Position Measurement With Temperature Drift Correction - “Seeing Through Conductive Walls”

ROSARIO V. GIUFFRIDA (Student Member, IEEE), SPASOJE MIRIĆ (Student Member, IEEE), JOHANN W. KOLAR (Fellow, IEEE), AND DOMINIK BORTIS (Senior Member, IEEE)

Power Electronic Systems Laboratory, ETH Zurich, 8092 Zurich, Switzerland

CORRESPONDING AUTHOR: ROSARIO V. GIUFFRIDA (e-mail: giuffrida@lem.ee.ethz.ch).

This work was supported by the Else und Friedrich Hugel-Fonds for Mechatronik/ETH Foundation for the research on magnetically levitated systems at the Power Electronic Systems Laboratory of ETH Zurich.

ABSTRACT This paper investigates the design of an Eddy Current Sensor (ECS) for position measurement of a moving conductive target located behind a fixed conductive shielding surface. Such a sensor can e.g. be used in completely sealed actuators with magnetically levitated rotor or mover for high purity applications. Starting from the analysis of the sensor’s operating principle, the design of the excitation coil, the achievable sensitivity and bandwidth as well as the temperature stability of the sensor are investigated. Subsequently, a suitable sensor interface, consisting of the driving and signal conditioning electronics, is selected. With this it is possible to distinguish between position and temperature variations, for which the optimal operational frequencies are identified. The results are finally verified with measurements on a hardware sensor prototype, showing that the ECS can achieve a sensitivity of 1 mV/µm, a position resolution of 1 µm, with a measurement bandwidth of 30 kHz and can hence be used to capture the mover’s position in an active magnetic bearing feedback control structure.

INDEX TERMS Eddy Currents, Magnetic Levitation, Position Measurement.

I. INTRODUCTION
In many industrial applications a contactless position sensing of an object is of main importance. In the case of a conductive object, a popular choice are Eddy Current Sensors (ECSs), which due to their non-contact nature offer a clear advantage over e.g. resistive or capacitive sensors [1], [2]. Moreover, ECSs can also operate under harsh conditions, dirty environments or in vacuum, which makes them applicable for Magnetic Bearings (MBs) of magnetically levitated actuator systems [3]–[6], where the ECSs located on the stator are used to capture the position of the levitated mover (cf. Fig. 1(a)). ECSs are also extensively used in Non-Destructive Testing (NDT) for inspection of damages on the surface of conductive materials [7]. There are also applications where the moving conductive target is located behind another conductive material. For example, in entirely sealed actuator systems used for high purity food, medical or chemical applications, the stator and the magnetically levitated mover are fully encapsulated with e.g. a stainless steel housing (cf. Fig. 1(b)). Consequently, the ECS located on the stator has to measure the mover position through the stator housing, which due to its conductivity highly degrades the magnetic coupling between excitation and/or pick-up coil and the mover, and in turn leads to a reduction of the measurement sensitivity, i.e. the mover’s position accuracy. As a further challenge, the sensor is expected to operate under different temperature conditions, as high as 100 ºC due to ohmic losses in the stator. Therefore, it is important to study the sensor’s stability with respect to temperature sensitivity, in order to prevent the magnetic bearings from failing due to thermal drift of the measured mover’s position. In contrast to other magnetic sensors as e.g. Hall effect sensors, which can also be used to estimate
the mover’s position through the stator’s housing exploiting the magnetic field of the mover’s permanent magnets, ECSs typically provide higher resolution, thermal stability and immunity to external disturbing magnetic fields [2], [3]. In the literature [8], such an ECS measuring through a conductive wall is e.g. used for periodical inspections of nuclear power plants by measuring the distance between two conductive tubes. There, a commercial ECS is used, with excitation frequencies up to 16 kHz, which for stationary applications are clearly sufficient. However, for highly dynamic position measurements in entirely sealed MBs, this could result in a too low measurement bandwidth. For instance, in very-high-speed magnetically levitated machines, position control bandwidths up to 1 kHz are needed [9]. Consequently, sensor measurement bandwidths of at least 10 kHz have to be achieved [3], [5], which in turn roughly lead to 10 times higher excitation frequencies of about 100 kHz. Importantly, however, it has also to be considered that above a certain excitation frequency, defined by the material properties and dimensions of the conductive wall, the skin and proximity effects of this intermediate layer start to play an important role, leading in the worst case to a complete loss of any mover position information. Consequently, the ECS’s excitation frequency has to be selected properly, such that on one hand a sufficient position sensor bandwidth is achieved and, on the other hand, the position sensitivity is not compromised too much by the intermediate conductive layer.

This paper extends the previous work of the authors [10], thoroughly presenting and discussing the design of an ECS capable of measuring through conductive walls. In particular, the ECS is used as a position sensor for entirely sealed actuator systems with a magnetically levitated mover, targeting the specifications of Table 1. After shortly summarizing the sensor’s operating principle presented in [10] in Section II, in the first part of this work the modeling and optimal design of the excitation coil, i.e. the sensor head, is analyzed. More specifically, in Section III an equivalent transformer model is introduced, whose circuit elements (impedances) are calculated analytically depending on the coil geometry and material properties. Furthermore, the model is validated by comparison with FEM simulations and measurements on the optimally-designed excitation coil. Afterwards, in Section IV the temperature’s influence on the coil impedance is investigated and quantified, in order to propose a method to measure and/or compensate it.

In the second part of this work, the practical problem of selecting a sensor interface with appropriate signal conditioning, which finally provides a measurable voltage signal, is addressed. This explicitly takes into account the temperature’s influence on the sensor’s output, finally providing a way of measuring both position and temperature as described in detail in Section V. The concept is verified in Section VI with a hardware sensor prototype on which experimental measurements are performed. Finally, Section VII concludes the paper.

II. SUMMARY OF PREVIOUS WORK
In [10], the authors investigate the ECS measuring through a conductive wall starting from the analysis of the conventional ECS and extending it to the new configuration of Fig. 2(a). An ECS is typically realized as a coil carrying a high frequency excitation current, which induces eddy currents in a conductive target. The secondary magnetic field generated by the circulating eddy currents couples with the excitation coil and finally affects its equivalent input impedance $Z_{c,in}$. The variations of $Z_{c,in}(\omega, \delta)$ depend on the chosen excitation frequency $\omega_{exc} = 2\pi f_{exc}$ and, most importantly, on the air gap $\delta$, which is the measurand.
For a conventional ECS (without intermediate shield), a convenient way of describing the variations of $Z_{\text{c.in}}(\omega, \delta) = R_{\text{c.in}}(\omega, \delta) + j\omega L_{\text{c.in}}(\omega, \delta)$ is the transformer model in Fig. 2(b) and results analytically in:

$$R_{\text{c.in}}(\omega, \delta) = R_c + \frac{\omega^2 M_{ct}^2}{R_t^2 + \omega^2 L_t^2} R_t = R_c + R_{\text{c,var}}(\omega, \delta),$$

(1)\hspace{1cm} L_{\text{c.in}}(\omega, \delta) = L_c - \frac{\omega^2 M_{ct}^2}{R_t^2 + \omega^2 L_t^2} L_t = L_c + L_{\text{c,var}}(\omega, \delta),$$

(2) where $M_{ct} = k_{ct}\sqrt{L_c L_t}$ is the mutual inductance between coil (c) and target (t), with the coupling factor $k_{ct}(\omega, \delta)$. In order to obtain significant variations in $R_{\text{c.in}}$ and $L_{\text{c.in}}$ with the air gap $\delta$, the excitation frequency has to be selected above a coupling-independent angular cutoff frequency

$$\omega_{\text{co}} = 2\pi f_{\text{co}} = R_t/L_t,$$

(3) which can be found as the frequency for which $R_{\text{c.in}}$ and $L_{\text{c.in}}$ experience half of their total variations

$$R_{\text{c,var}}(\omega \to \infty, \delta) = \left( \frac{M_{ct}}{L_t} \right)^2 R_t = k_{ct}(\omega, \delta)^2 L_c \omega_{\text{co}},$$

(4)\hspace{1cm} L_{\text{c,var}}(\omega \to \infty, \delta) = -\left( \frac{M_{ct}}{L_t} \right)^2 L_t = -k_{ct}(\omega, \delta)^2 L_c.$$

(5) When a second intermediate conductive shield is added, eddy currents are also induced in it, thus affecting the variations of $Z_{\text{c.in}}(\omega, \delta)$ i.e. the position sensitivity. A general understanding of the resulting effect on $Z_{\text{c.in}}(\omega, \delta)$ can be gained as follows. Considering the case where $\delta \to \infty$, i.e. when the target is far away. This corresponds to the conventional ECS with the shield acting like a target, and hence the same considerations as before apply, but now $\omega_{\text{co},s} = 2\pi f_{\text{co},s} = R_s/L_s$ is defined by the shield. When the target approaches the shield, it offers a parallel circulation path for the eddy currents, thus reducing the original $R_s$ of the shield alone. The secondary-side resistance can be substituted by an equivalent resistance $R_{\text{eq}}$, which can be seen as some special target at some equivalent position having mixed material properties of shield and target. The resulting $R_{\text{eq}}$ is smaller than $R_s$ and continuously reduces with the target approaching the shield. Consequently, also the cutoff frequency $\omega_{\text{co},s}$ shifts to lower frequencies. For the extreme case with $\delta = 0 \text{ mm}$, assuming that shield and target have the same coupling to the coil, the minimum cutoff frequency is roughly given by the parallel connection of $R_s$ and $R_t$ as

$$\omega_{\text{co},\text{min}} = 2\pi f_{\text{co},\text{min}} = R_s R_t/(R_s + R_t)/L_s.$$  

(6) This air gap-dependent shift of the cutoff frequency affects the variations of resistance $R_{\text{c.var}}$ and inductance $L_{\text{c.var}}$ as shown in the exemplary curves of Fig. 3, obtained with FEM simulations in [10] for different shield/target material combinations.

In particular, $L_{\text{c,var}}$ is mostly sensitive to variations of the air gap $\delta$ in the frequency range between $f_{\text{co,min}}$ and $f_{\text{co},s}$ and, as derived in [10], the optimal excitation frequency $f_{\text{opt,L}}$ that maximizes the position sensitivity of $L_{\text{c,var}}$ lies approximately in the center of this interval, i.e. the geometric mean of the two boundary frequencies

$$f_{\text{opt,L}} \approx \sqrt{f_{\text{co},\text{min}} \cdot f_{\text{co},s}}.$$  

(7) The cutoff frequency shift also reduces the total resistance variation, as it appears from (4). Therefore, the frequency range for which $R_{\text{c,var}}$ is mostly sensitive to variations of $\delta$ lies above $f_{\text{co},s}$. Nevertheless, in order to ensure that the excitation field penetrates the shield and eddy currents are induced in the target, the excitation frequency must be kept below a certain skin-depth-related frequency

$$f_{\text{sk},s} = \frac{1}{\pi \mu_0 \mu_{\text{rt},s} \sigma_s d_s^2},$$

(8) where $\mu_0$ is the magnetic permeability of vacuum and $\mu_{\text{rt},s}$, $\sigma_s$ and $d_s$ are the relative permeability, conductivity and thickness of the shield, respectively. In this work, only non-magnetic materials ($\mu_{\text{rt},s} = 1$) are considered. As it appears from (8), ferromagnetic materials (such as ferritic stainless steel) would reduce the penetration depth of the excitation field, resulting in lower sensitivity. Beyond $f_{\text{sk},s}$, no sensitivity to $\delta$ can be achieved. Thus, the optimal excitation frequency $f_{\text{opt,R}}$ lies approx. in the center between $f_{\text{co},s}$ and $f_{\text{sk},s}$, i.e. again the geometric mean

$$f_{\text{opt,R}} \approx \sqrt{f_{\text{sk},s} \cdot f_{\text{co},s}}.$$  

(9) With these results, the authors provide some general design guidelines for the ECS measuring through conductive walls and identify the optimal excitation frequencies:

- In order to achieve a high measurement sensitivity, for both resistance and inductance variations, the shield resistance $R_s$ should be much larger than the target resistance $R_t$, which is either obtained by selecting different materials or by using different thicknesses.
of the lumped shield resistance and inductance $R_s$ and $L_s$ and a tertiary side with the lumped target parameters. Consequently, also the mutual inductances $M_{cs} = k_{cs} \sqrt{L_s L_t}$ and $M_{st} = k_{st} \sqrt{L_t L_s}$ are introduced, where $k_{cs} = k_{cs}(\omega, \delta)$ and $k_{st} = k_{st}(\omega, \delta)$ are the coil-shield and shield-target coupling factors. It can be observed that $k_{cs}$ is always larger than $k_{ct}$. By circuitual analysis, an expression for the input impedance of the excitation coil $Z_{in}(\omega, \delta)$ can be provided. This is slightly more complicated with respect to the conventional ECS case, cf. (1) and (2), and is hence reported in the Appendix. The description of $Z_{in}$ provided by the transformer model is completed with the analytical expressions for all its parameters as derived in the following:

1) EXCITATION COIL (PRIMARY-SIDE) PARAMETERS

Expressions for the coil resistance $R_c$ and inductance $L_c$ can usually be found in the literature for different coil geometries [11], [12]. A common choice for ECSs is a PCB-integrated planar spiral coil, as it allows to obtain a relatively large inductance already with a single-layer coil. This allows to minimize the inter-layer parasitic capacitance of the coil, thus allowing to use higher excitation frequencies. Minimizing the parasitic capacitance $C_{par}$ is needed to guarantee that the coil’s Self-Resonant Frequency (SRF) is well above the chosen excitation frequency. This is a stringent requirement for the correct operation of the ECS and the validity of the introduced transformer model. For the scope of this analysis it is sufficient to consider at least the following relations linking the coil geometry to its electrical properties. For the coil’s DC resistance

\[ R_c \propto \frac{d_{c, avg} N}{\sigma_{cu}}, \]  

(10)

whereas for the inductance, in general

\[ L_c \propto \mu_0 d_{c, avg} N^2 \]  

(11)

where $d_{c, avg}$ is the average diameter of the excitation coil.

2) SHIELD/TARGET (SECONDARY-SIDE) PARAMETERS

Providing expressions for the lumped shield or target resistance $R_{s,t}$ and inductance $L_{s,t}$ requires some preliminary consideration on the geometry of the eddy current circulation paths. Following [13], for a circular excitation coil the volume of the target carrying significant eddy currents (the effective volume) can be segmented into a certain number $K$ of concentric rings. Each of them can be considered insulated with respect to the other, as it can be shown that the current density vector field $\mathbf{J}$ inside the target, expressed in polar coordinates, has no radial component. The simplest approximation, corresponding to $K = 1$, is to describe the effective volume with a circular single-turn image coil of rectangular cross section, with inner and outer radii $r_{in}$ and $r_{out}$, as indicated in purple in Fig. 4(b). The thickness of such image coil can be assumed to be equal to the shield or target thickness $d_{s,t}$, as long as the eddy currents penetrate the material completely. However, for higher frequencies, the penetration depth reduces due to the well-known skin effect, according to the expression for the

VOLUME 3, 2022

255
skin depth

$$\delta_{k,s,t}(\omega) = \frac{2}{\mu_0 \mu_r \sigma_{s,t}(\omega) \omega}$$  \hfill (12)

When eventually $\delta_{k,s,t}(\omega)$ becomes smaller than $d_{s,t}$, the eddy currents are distributed only in a portion of the total shield or target thickness. Consequently, the effective height of the image coil is

$$h_{s,t}(\omega) = \begin{cases} d_{s,t}, & \text{for } \delta_{k,s,t}(\omega) \geq d_{s,t} \\ \delta_{k,s,t}(\omega), & \text{otherwise} \end{cases}$$  \hfill (13)

which decreases for higher frequencies. However, a reduction of the penetration depth is only allowed for the target. The excitation frequency $f_{\text{exc}}$ should always be lower than the skin-depth frequency of the shield. Consequently, thickness and the selected material of the shield immediately limit the excitation frequency range and the analysis thereof.

This way, $R_{s,t}$ and $L_{s,t}$ can be found as the resistance and the self-inductance of the respective equivalent image coil. For the resistance $R_{s,t}$, it is sufficient to integrate the resistance of an infinitely thin loop in the volume of the image coil, obtaining

$$R_{s,t}(\omega) = \frac{2\pi}{\sigma_{s,t} h_{s,t}(\omega) \ln \left( \frac{r_{\text{out}}}{r_{\text{in}}} \right)}.$$  \hfill (14)

For the inductance $L_{s,t}$, instead, the empirical formula from [14] for a circular coil of rectangular cross section can be used

$$L_{s,t}[\mu\text{H}] \approx \frac{40 r_{\text{avg}}^2}{8 r_{\text{avg}} + 11 (r_{\text{out}} - r_{\text{in}})},$$  \hfill (15)

where $r_{\text{avg}} = (r_{\text{out}} + r_{\text{in}})/2$ is the average radius of the image coil. In a first approximation, $r_{\text{in}}$ and $r_{\text{out}}$ can coincide with the inner and outer radii of the excitation coil, and hence $r_{\text{avg}} = d_{c,\text{avg}}/2$. In this case, FEM simulations reveal that 80% of the eddy currents are induced in this volume for both shield and target. The volume enclosed by an image coil with the same average radius $r_{\text{avg}}$ but a 50% wider cross section (i.e. 1.5 · $(r_{\text{out}} - r_{\text{in}})$, as the ones represented in Fig. 4(b)) would instead consider up to 95% of the total eddy currents.

3) COUPLING FACTORS

Finally, the coupling factors can be obtained from the mutual inductions $M_{0,s}$, $M_{0,s}$ and $M_{0,t}$ according to the formula $k_{12} = M_{12}/\sqrt{M_{11} M_{22}}$. With the introduced image coils, it is possible to use formulas for the mutual inductance of coaxial disk coils that can be found in the literature [15], [16]. As the model is highly sensitive to these three parameters, it is recommended to obtain or verify them with a FEM simulation.

B. OPTIMAL PCB-EMBEDDED COIL DESIGN CONSIDERATIONS

Once the transformer model and all the expressions for its parameters are introduced, they can be used to optimally design the excitation coil. As mentioned, this is typically realized as a PCB- or flex-PCB-embedded spiral coil, hence there are only a few main parameters to be determined, i.e. the coil diameter $d_c$ and the number of turns $N$, which can be distributed in $m$ layers. The optimal design procedure is summarized with the flow diagram of Fig. 5. The specified shield material with its conductivity $\sigma_s$ and thickness $d_s$ defines the skin-depth related frequency $f_{k,s}$ (cf. (8)). From the required controller bandwidth, the needed excitation frequency $f_{\text{exc}}$ is defined, which coincides with $f_{\text{opt},R}$. Based on (9), it is hence possible to calculate the shield crossover frequency $f_{\text{co},s}$, which gives the ratio between $R_s$ and $L_s$ (cf. (3)). These two electrical parameters allow to select the dimensions of the image coil (average, inner and outer radii $r_{\text{avg, in}}$ and $r_{\text{out}}$, cf. (14) and (15)) and, consequently, of the excitation coil. With the maximum diameter $d_c$, the maximum number of turns $N$ of the excitation coil can be calculated, which gives a high excitation coil’s self-inductance $L_e$ and, in turn, a high position sensitivity (cf. (4) and (5)). The number of turns $N$ can be further increased by using multiple PCB layers, but special care must be taken, as this particularly increases the parasitic capacitance $C_{c,\text{par}}$, thus lowering the coil’s SRF. Another way of further increasing $L_c$, compatibly with the available space in the stator, is to place a layer of ferromagnetic material on top of the excitation coil. As an additional benefit, this provides shielding against external disturbing magnetic fields.

Therefore, for the given case, the excitation coil’s diameter is maximized to the available $d_c = 11.5$ mm, with which the minimum track width and distance of 150 $\mu$m yields a maximum number of turns per layer of $N = 13$. A picture of the realized PCB-embedded excitation coil is shown in Fig. 6(a). In order to ensure that the coil’s SRF is above 10 MHz, (i.e. well above the range of the considered excitation frequencies), the number of PCB layers is limited to $m = 3$. In addition, a high-frequency ferrite core is added on top of the coil (cf. Fig. 6(b)) to further increase $L_c$, resulting in a total $L_c = 13$ $\mu$H.

C. EXPERIMENTAL IMPEDANCE MEASUREMENTS AND MODEL VERIFICATION

The impedance variations of the ECS measuring through conductive walls are verified with the realized PCB-embedded excitation coil in the studied planar geometry of Fig. 2(a). This is realized with a collection of square samples of aluminum.

### FIGURE 5.
Flow diagram illustrating the optimal PCB-embedded coil design. From the specified shield material and control bandwidth, it is possible to define the characteristic frequencies, which are linked to the coil’s geometry through the secondary-side parameters $R_s$ and $L_s$. Once the coil’s dimensions are obtained, its self-inductance $L_c$ is maximized.
FIGURE 6. (a) Realized PCB-embedded excitation coil. (b) On top of the excitation coil, a high-frequency ferrite disk is added to increase the self-inductance $L_c$. (c) Realized exemplary shield/target and spacer samples, made of stainless steel or aluminum and PTFE, respectively. (d) PCB coil buildup implementing the studied planar geometry, held together by a PTFE fixture.

and stainless steel (304, IE 1.4301, austenitic) with thicknesses 0.5 mm and 0.2 mm, to be used as shield or target (cf. Fig. 6(c)). The air gap between the two samples is fixed with spacers, realized with a non-conductive, temperature-resistant material (PTFE) and different thicknesses. This way, it is possible to fix the air gap in steps of 0.25 mm, which is the thickness of the thinnest spacer. The selected shield sample, spacer(s) and target sample are stacked up, with the excitation coil PCB on top, as shown in Fig. 6(d). All layers are also pressed together, thus reducing the bending of the metallic samples or the spacers. This measure is important to ensure correct results, especially considering the high sensitivity and the targeted resolution in the $\mu$m range.

The measurements are performed with the Omicron Lab Bode100 impedance analyzer for the 0.2 mm thick stainless steel shield and 0.5 mm thick stainless steel target configuration and for the 0.2 mm thick stainless steel shield and 0.5 mm thick aluminum target configuration. The results are compared to the corresponding FEM simulations in Fig. 7, showing good agreement between the two. In particular, the optimal frequencies reported in Table 2 match closely. Also the values calculated with the analytical transformer model are reported and matching. The model allows to obtain the complete impedance curves, which however were found to be in agreement only until around $f_{sk,s}$. For the sake of this analysis and the optimal coil design, calculating the characteristic frequencies is sufficient, and therefore the impedance curves are not reported.

IV. TEMPERATURE’S INFLUENCE ON THE SENSOR’S OUTPUT

An important aspect to investigate for many sensors and particularly for the studied ECS is its behavior under different operating temperatures. Ideally, for a constant position, the sensor output has to stay constant throughout the entire range of operating temperatures. In the considered case, this problem is of particular interest. In fact, the temperature inside the sealed stator (where the ECS is located) varies in the range of 25°C to 100°C, since the actuator’s winding, located inside the stator, heats up during operation. Consequently, a thermal drift of the sensor’s output in active magnetic bearings would translate in a certain offset in the controlled position of the levitated mover with respect to the geometric center of the machine, which can lead in the worst case to failure of the levitation control. For these reasons, it is important to quantify the effect of temperature on the sensor’s output and, possibly, take specific actions to limit or compensate it.

As a starting point for this investigation, it should be considered that the conductivity $\sigma$ exhibits a temperature dependency. Another effect is thermal expansion, which however is not expected to be prominent in this case for thin shields or targets and is therefore neglected. As a consequence of the dependency on the conductivity $\sigma$, it has to be expected that the electrical properties of all the elements in the studied ECS configuration, i.e. coil, shield and target, are affected by temperature. The temperature dependency is usually given for the resistivity $\rho = \sigma^{-1}$ with the approximated linear relationship

$$\rho(T) = \rho_0 [1 + C_T (T - T_0)],$$

where $\rho_0$, $C_T$, and $T_0$ are the resistivity at a reference temperature, the temperature coefficient, and the reference temperature, respectively.
TABLE 3  Conductivity and Temperature Coefficient of the Considered Materials

| Material      | Conductivity at 25 °C σ₀ | Temperature Coeff. (Cₜ) |
|---------------|---------------------------|-------------------------|
| Copper        | 59.6 MS                   | 3900 ppm/°C             |
| Aluminum      | 37.7 MS                   | 3800 ppm/°C             |
| Stainless Steel| 1.37 MS                   | 850 ppm/°C              |

where T is the temperature, ρ₀ is the resistivity of the material at the reference temperature T₀ (e.g. 25°C) and Cₜ is the temperature coefficient of the material, expressed in °C⁻¹.

The values of ρ₀ and Cₜ for copper, aluminum and stainless steel are reported in Table 3. For the considered operating temperature range of 75°C, the resistivity of copper increases by about 30%.

Based on the analytical model previously introduced in Section III, it is already possible to predict qualitatively how the temperature influences Zₗ in. The most prominent effect, given the high Cₜ of copper, is the increase of Rₗ (cf. (10)), which offsets the total Rₗ var with respect to the nominal DC resistance at 25°C Rₗ DC(25°C). Secondly, also Rₗ and Rₛ in increase with temperature (cf. (14)), according to their materials’ properties. This reflects into an increase of the cutoff frequencies fₗopt,s (cf. (3)) and fₗopt,min (cf. (6)), which can affect the final sensitivity (cf. (4)) and alters the optimal excitation frequencies. Some influence has also to be expected for higher frequencies, as the skin depth δₗ,s depends on σ₀ as well. On the contrary, no relevant effect has to be expected on Lₗ (cf. (11)) nor on Lₗ var (cf. (5)). This is the main reason why conventional ECSs rely on Lₗ var to measure δ and disregard Rₗ var [17].

These considerations are verified by measuring the impedance of the realized PCB-integrated excitation coil at 25°C and 100°C. The measurements are conducted for the shield/target combination with 0.2 mm thick stainless steel shield and 0.5 mm thick stainless steel target, as well as 0.2 mm thick stainless steel shield and 0.5 mm thick aluminum target. The latter are shown in Fig. 8. With these results, it is possible to observe and quantify the influence of temperature on the equivalent input impedance of the excitation coil. As expected, this is much more prominent on Rₗ in, with variations of about 30% of the value of Rₗ DC(25°C) at the optimal excitation frequencies, compared to Lₗ in, which differs by less than 6% of the value of Lₗ DC(25°C) and is hence negligible. In particular, the increased DC resistance of the sensing coil Rₛ represents the main difference component of Rₗ var, which is about 30% larger for all frequencies (cf. Fig. 8(b)). Additionally, the crossover frequencies fₗopt,min and fₗopt,s are shifted slightly towards higher frequencies as a result of the increased Rₛ and Rₗ, causing the largest differences in Lₗ var in Fig. 8(b). The prominent effect of temperature on Rₗ var leads to an error in the measured position, which can be visualized in Fig. 8(c) for different values of the air gap δ when exciting the coil at e.g. fₗopt,R. If the sensor is calibrated for T = 25°C, in the extreme case of T = 100°C the error δₗerr on the measured position can be as large as 1.1 mm. This is quite concerning for the stability of the magnetic bearings and needs to be explicitly corrected. On the other hand, if the same is checked for Lₗ var at the optimal excitation frequency fₗopt,L, δₗerr results smaller than 2 µm, i.e. temperature does not have a significant influence on the measured position. However, it has to be considered that fₗopt,L is smaller than fₗopt,R, even by an order of magnitude in the case of Fig. 8(a), and it might not be sufficient for highly dynamic applications as e.g. very-high-speed magnetically levitated machines. In such a case, it is therefore necessary to be able to distinguish between impedance variations caused by position or temperature variations. This is only possible if full knowledge about the impedance Zₗ in is gained, which means that not only Rₗ in or Lₗ in but both values have to be determined. In order to achieve this, an appropriate sensor interface is needed, as discussed in the next section. There it is shown that in this way, the ECS measuring through conductive walls allows to measure accurately the air gap δ, since variations in temperature T are compensated.

V. SENSOR INTERFACE AND SIGNAL CONDITIONING

Once the sensor concept is analyzed, it is possible to understand its operating principle and which quantities contain information on the measurand. The next problem to address is how to extract such information and convert it into a usable signal. For this purpose, a measurement circuit (or sensor
A. OVERVIEW OF THE PROPOSED MEASUREMENT CIRCUIT

An overview of the proposed measurement circuit is given in Fig. 9. It consists of three main parts, which implement the three aforementioned functions.

The excitation frequency \( \omega_{\text{exc}} = 2\pi f_{\text{exc}} \) directly determines the sensor’s position (and temperature) sensitivity, as it affects the penetration depth of the primary magnetic field in the shield-target combination. Therefore, it is reasonable to select a measurement circuit which operates with a fixed excitation frequency. Consequently, measurement circuits that perform an impedance-to-frequency conversion, commonly used for conventional ECS [18], have to be excluded. With a fixed excitation frequency, \( \Delta Z_{c,\text{in}}(\delta, T, \omega) \), in order to be able to distinguish and measure both variations of the air gap \( \delta \) and of the temperature \( T \).

The excitation voltage \( U_{\text{exc}} \) is then a sinusoidal voltage signal. The most suitable candidate circuit performing such conversion is the AC Wheatstone bridge, often found in the literature as an ECS interface [19]. The bridge is excited by a sinusoidal voltage signal \( U_{\text{exc}} \) with frequency \( f_{\text{exc}} \) and its output is the voltage \( U_{\text{br.out}} \). This circuit is particularly well suited to measure unbalances between the impedances of its two legs. Therefore, in many cases, the excitation coil of the ECS is included in the bridge together with a replica, purposely manufactured and used as a reference coil, kept in nominal conditions (e.g. at the nominal distance from the target). This solution is used for high precision sensors, capable e.g. to measure in the subnanometer range [20]. An additional advantage, in fact, is that any changes which are common to both coils (like e.g. the offset in \( R \) due to temperature found in Fig. 8), are canceled. In the considered application, due to the axial symmetry of the machine, it is advantageous to implement the ECS as a differential sensor, which is obtained by placing two identical copies of the excitation coil \( ECS_+ \) and \( ECS_- \) at the two opposite sides of the levitated rotor. By including both sensing coils in the AC Wheatstone bridge, it can be shown that the output voltage \( U_{\text{br.out}} \) depends on the differential impedance \( Z_{\text{diff}} = Z_{c,\text{in}+} - Z_{c,\text{in}-} \), which finally gives a measurement of the differential position \( \delta_{\text{diff}} = \delta_+ - \delta_- \). It can be proved that differential sensing improves the sensitivity and the linearity of the sensor’s output, even though the original variations of the measurand are not linear [21].

As a result of the variations of both the air gap \( \delta \) and temperature \( T \), the output voltage \( U_{\text{br.out}} \) is finally an attenuated and phase-shifted version of the excitation voltage \( U_{\text{exc}} \). More specifically, the variable air gap \( \delta_{\text{diff}}(t) \) modulates in amplitude and phase \( U_{\text{br.out}} \). Therefore, in order to recover \( \delta_{\text{diff}}(t) \) and simultaneously gain full information (real and imaginary parts) on \( U_{\text{br.out}} \) (and hence on the impedance \( Z_{\text{diff}} \)) quadrature demodulation is used. In case only \( \delta_{\text{diff}}(t) \) is of interest, with e.g. minimal influence of \( T \), it is sufficient to use only one demodulation channel.

B. ANALYSIS AND SELECTION OF THE AC WHEATSTONE BRIDGE CONFIGURATION

The AC Wheatstone bridge can be configured in many different ways: for example, the two excitation coils can be placed either in the same bridge leg or in two different legs. Additionally, it is possible to add a capacitor \( C_{\text{res}} \) in series to the excitation coil to compensate its inductive reactance at a specific resonant frequency. In this case, it would also be possible to excite the bridge with a square wave voltage, which is much simpler to realize in practice.

Three configurations of most interest are analyzed. In configuration B1 Fig. 10(a.i), the two excitation coils make part of two different bridge legs. This is the most basic configuration, and the value of the bridge resistors \( R_{\text{br}} \) is optimized to yield the largest magnitude of \( U_{\text{br.out}} \). In configuration B2 Fig. 10(b.i), a capacitor \( C_{\text{res}} \) is added in series to each coil, and chosen to resonate with the value of \( L_{c,\text{in}} \) for \( \delta = 1 \text{ mm} \), which is the nominal air gap. The resonant frequency is also optimized, together with the value of \( R_{\text{br}} \), to yield the largest magnitude of \( U_{\text{br.out}} \). Finally, configuration B3 (Fig. 10(c.ii)) is a variant of B2 realized with only one capacitor in series with both bridge legs. Bridge configurations where both the excitation coils are in the same leg were studied as well, but provide in general slightly worse sensitivity to \( \delta_{\text{diff}} \), hence they are not reported. In order to select the most appropriate bridge circuit, together with the corresponding optimal excitation frequency \( f_{\text{opt}} \), each configuration is investigated analytically. By simple AC circuital analysis, the transfer function \( G(s) \) from the input voltage \( U_{\text{exc}} \) to the output voltage \( U_{\text{br.out}} \) is derived. With this, it is possible to obtain \( U_{\text{br.out}} = G(s) U_{\text{exc}} \) as a
complex-valued phasor for a given array of frequencies and differential positions, starting from the measured impedance of the realized PCB-embedded coil. The influence of temperature discussed in Section IV can also be easily included in this analysis. The values of $R_{c,in}$ and $L_{c,in}$ used are the ones measured for the most sensitive shield/target combination, i.e. 0.2 mm thick stainless steel shield and 0.5 mm thick aluminum target. In Fig. 10, for each bridge configuration, $U_{br,out}$ is visualized on the complex plane for different frequencies, values of the differential position $\delta_{diff}$ and temperatures $T = 25^\circ C$ and $100^\circ C$.

The studied configurations are compared according to a few selection criteria. The most important requirement is high sensitivity to the differential position $\delta_{diff}$, which translates in the largest magnitude of the phasor $U_{br,out}$. The second aspect to take into consideration is the sensitivity to the temperature $T$. As mentioned, the designer can choose whether measuring $T$ is of interest, or whether it is more important to minimize its influence on the $\delta_{diff}$ measurement. The normalized sensitivity curves in Fig. 11 are extracted from the data in Fig. 10 for the exemplary configuration B1. The $\delta_{diff}$-sensitivity curve is given by the magnitude of $U_{br,out}$ for the largest $\delta_{diff} = 1$ mm versus frequency and normalized to its maximum. For the $T$-sensitivity curve, instead, the magnitude of the difference vector $U_{br,out,100^\circ C} - U_{br,out,25^\circ C}$ versus frequency is considered, again for $\delta_{diff} = 1$ mm and normalized to its maximum. With the sensitivity curves two optimal frequencies can be identified, for two scenarios:

1) in case combined sensitivity to $\delta_{diff}$ and $T$ is desired, the optimal frequency $f_{opt,comb}$ can be found, which corresponds to the peak of the product sensitivity curve shown in Fig. 11, which in case of configuration B1 leads to an optimal frequency $f_{opt,comb} = 299$ kHz. The corresponding $U_{br,out}$ obtained at this frequency on the complex plane is highlighted in blue in Fig. 10(a.ii).

2) in case only $\delta_{diff}$ is of interest, it is possible to select the frequency $f_{opt,\delta}$, which offers the optimal compromise between sensitivity to $\delta_{diff}$ and to $T$. This corresponds to the condition for which the sensitivity to $\delta_{diff}$ dominates the most with respect to the sensitivity to $T$, i.e. to the peak of the ratio sensitivity curve as shown in Fig. 11. In case of configuration B1 this leads to an optimal excitation frequency $f_{opt,\delta} = 563$ kHz. The corresponding $U_{br,out}$ obtained at this frequency on the complex plane is highlighted in orange in Fig. 10(a.ii).

Besides sensitivity, another important requirement is the measurement bandwidth of the sensor, which has to be sufficiently high. This is directly related (approx. one decade before) to the selected $f_{exc}$. Finally, linearity has the least priority, as the non-linear sensor readout can eventually be corrected (e.g. in firmware).
With the results of Fig. 10, the analyzed bridge configurations can be compared and some trade-offs outlined. The sensitivity to $\delta_{\text{diff}}$ is higher for the alternatives with a resonant capacitor $C_{\text{res}}$. For $B_2$ (max 0.26 $\text{V mm}^{-1}$) it is up to 40% larger than $B_1$. The enhanced sensitivity comes at the cost of a visibly higher non-linearity. For $B_3$, instead, it is only 20% larger than $B_1$. As mentioned, the value of $C_{\text{res}}$ is optimized together with the resonant frequency to yield the largest possible $U_{\text{br, out}}$. Interestingly, during the analysis it was found that this design yields a sensitivity above 0.2 $\text{V mm}^{-1}$ for a large range of frequencies, from 100 kHz to 1.4 MHz for $B_2$ and from 150 kHz to 600 kHz for $B_3$. Therefore, these configurations give the freedom of adjusting $f_{\text{exc}}$ according to the designer’s preferences, also allowing to obtain larger bandwidths. Nevertheless, in $B_2$ the two resonant capacitors have to be precisely matched for good results, which can be impaired by the components’ tolerances and that would require cumbersome capacitive trimming. In this sense, the advantage of $B_1$ is that no additional component is required. Furthermore, this simple configuration still offers a sensitivity comparable to e.g. $B_3$ and it allows to use higher excitation frequencies.

C. AMPLIFICATION AND DEMODULATION STAGE

Before processing $U_{\text{br, out}}$ further, a difference amplifier is employed at the output of the bridge. Typically an instrumentation amplifier is used, due to its high input impedance and common mode rejection ratio. The amplified signal is then demodulated with coherent demodulation, in order to preserve information on its sign. Coherent demodulation is realized by multiplying $U_{\text{br, out}}$ with the excitation signal $U_{\text{exc}}$ (the carrier). Finally, the resulting $2 \cdot f_{\text{exc}}$ components in the multiplied signal are removed by a low-pass filter $H(s)$, with a cutoff frequency chosen e.g. a decade before $f_{\text{exc}}$. This way, only the low frequency information about $\delta_{\text{diff}}(t)$ is retained. Importantly, provided that all the previous stages are designed to achieve a sufficiently high bandwidth (e.g. at least one decade larger than $f_{\text{exc}}$), the final measurement bandwidth of the ECS is only defined by the cutoff frequency of the low-pass filter. Quadrature demodulation is realized with an additional channel, which employs a 90° phase-shifted version of the carrier. With the combined information from the in-phase ($I$-) and quadrature ($Q$-) channels, the modulating signal can be fully recovered. In fact, it can be easily verified that the $I$- and $Q$- voltages $U_I$ and $U_Q$ correspond to the real and imaginary components of the complex phasor $U_{\text{br, out}}$ scaled by a factor 0.5 and hence

$$
\delta_{\text{diff}}(t) \propto |U_{\text{br, out}}|(t) = 2\sqrt{U_I^2(t) + U_Q^2(t)}. \quad (17)
$$

Finally, the demodulated $U_I$ and $U_Q$ are the signals that have to be measured or sampled by an ADC. If the gain of the instrumentation amplifier is adjusted to fully utilize the input range of the ADC, the final resolution of the ECS is defined by the number of bits of the ADC. Clearly, bridge configurations that offer a large sensitivity will result in a better signal-to-noise ratio.

D. REALIZED MEASUREMENT/EVALUATION BOARD

The proposed measurement circuit is implemented with a hardware evaluation board prototype, shown in Fig. 12. The excitation stage consists of a stimulus generator followed by a driving stage, which can supply the bridge with sufficient current. In order to allow for the maximum flexibility during commissioning of the ECS, the stimulus generator is the Direct Digital Synthesizer AD9833 by Analog Devices. This IC is very simple to use and it can be programmed via SPI to generate sine, square or triangular voltage waveforms with frequencies up to 12.5 MHz and a resolution of 0.1 Hz. The generated signal is pre-processed with a RC high-pass filter (15 Hz cutoff) for DC removal, and then pre-amplified to an amplitude of 1 V, which matches the input specifications of the analog multiplier. In fact, this is the signal used later as demodulation carrier. The 90° phase-shifted carrier for the $Q$-channel is also generated exactly in the same way, with a second AD9833. The driving amplifier is configured with a gain of 2.5, thus outputting the excitation voltage $U_{\text{exc}}$ with an amplitude of 2.5 V. The AC differential Wheatstone bridge follows, implemented in such a way that all the possible bridge configurations (cf. Fig. 10) can be realized. Although monolithic solutions exist, the instrumentation amplifier is realized with three op-amps in order to guarantee high bandwidth (above 10 MHz). The gain of the instrumentation amplifier is adjustable with a 2 kΩ precision trimmer resistor, in order to match the input voltage levels of the analog multiplier ($\pm 1$ V) for the largest $\delta_{\text{diff}} = \pm 1$ mm. For each of the two demodulation channels, the analog multiplier takes the amplified version of $U_{\text{br, out}}$ and the pre-amplified $U_{\text{exc}}$ as inputs. Each product signal is then filtered with an active 4th order low-pass filter in a Multi-Feedback configuration [22]. The chosen cutoff frequency is 30 kHz. Additionally, this stage features a gain of 2 to compensate the 0.5 factor introduced by demodulation and a 2.5 V level shift to obtain a positive output signal. This way, the two demodulated $I$- and $Q$- voltages can be sampled with the 12-bit ADC LTC2313, which has an input...
Experimental verification of the evaluation board

2.5 V = δ mm and = Q 20 = f (a) 750 kHz is about 33 %

VI. MEASUREMENTS AND RESULTS

The impedance variations were verified with the realized PCB-embedded excitation coil in Section III for different values of the air gap δ. In this section, the proposed measurement circuit, implemented with the realized evaluation board, is verified instead.

A. MEASUREMENT CIRCUIT VERIFICATION

The functionality of the measurement circuit is verified with both static and dynamic measurements.

1) STATIC MEASUREMENTS FOR DIFFERENT TEMPERATURES

As the name suggests, the static measurements are realized for fixed values of the differential position δdiff. By sweeping the excitation frequency and plotting on the complex plane the measured values of \( U_i \) and \( U_Q \), it is possible to obtain the experimental version of the bridge output plots of Fig. 10, scaled by the constant gain of the instrumentation amplifier. Also the temperature \( T \) is varied from 25°C to 100°C. With the following set of measurements, the functionality of the measurement circuit (excitation, differential bridge configuration, demodulation) is verified completely.

First of all, multiple sample buildups are prepared as described in Section III for the differential positions \( \delta_{diff} = \{0.5, 0.75, 1\} \) mm and placed in an oven with controllable temperature. They are connected through a hole in the wall of the oven to the evaluation board, which is placed outside of the oven as close as possible. For each measurement, all the sample buildups are first heated up to the desired temperature. Then, a pair of sample buildups is connected to the evaluation board and a frequency sweep from 10 kHz to 10 MHz and 40 points per decade is automatically performed. This is done by commanding the desired frequency to the signal generators and then acquiring 100 samples with both I- and Q- channels ADCs, which are then averaged. In the process, some intermediate signals, like e.g. \( U_{exc} \) and the output of the instrumentation amplifier are monitored with an oscilloscope. When the measurement is completed, the next pair of sample buildups is connected, a certain time is waited in order for their temperature to settle again to the desired value and finally the routine is restarted. The results are visualized in Fig. 13 for the prototypical case with 0.2 mm thick stainless steel shield and 0.5 mm thick aluminum target and the bridge configuration B1. The 2.5 V level shift introduced by the low-pass stage is removed by measuring the ADC readings for no excitation and removing such offset from the measured data. Additionally, in order to recover the original \( U_{br, out} \) and allow a more direct comparison with Fig. 10(a,ii), the measurements are scaled down by the gain of instrumentation amplifier, which is measured experimentally for a test excitation and is equal to \( G_{inA} = 1.5 \text{ V/V} \) in this case, and by a factor 2.5 because of the ±2.5 V \( U_{exc} \). Fig. 13(a) shows the sensitivity curves introduced in Fig. 11, which are extracted from the measured data plotted on the complex plane in Fig. 13(b). The optimal excitation frequency \( f_{opt,comb} = 355 \text{ kHz} \) is close to the expected value of 299 kHz. However, \( f_{opt, \delta} = 750 \text{ kHz} \) is about 33 % off. This can be explained by the fact that the ratio-sensitivity curve in Fig. 13(a) is relatively flat, and hence its maximum can deviate strongly from the predicted one in Fig. 11. Also the temperature’s influence is visible and measurable if two demodulation channels are used. With the additional points for 50°C and 75°C, it can also be verified that the voltage varies smoothly within the two boundary curves.

2) DYNAMIC MEASUREMENTS AND BANDWIDTH VERIFICATION

As a final step, the measurement bandwidth of the realized ECS is verified with an experimental Bode plot. As mentioned, the low-pass filter of the demodulation stage is the one defining the final ECS bandwidth. For this reason, this verification can be conducted electrically with a signal generator. In particular, the \( U_{br, out} \) resulting from a target moving sinusoidally can be emulated by a sinusoidal carrier at the frequency \( f_{exc} \) with double-sideband amplitude modulation (DSB-AM). The carrier’s frequency is fixed to \( f_{exc} = 300 \text{ kHz} \) and its amplitude is 0.4V, chosen together with the gain of the instrumentation amplifier \( G_{inA} = 2.5 \text{ V/V} \) to get a ±1V output. The frequency of the modulating sine wave is swept from...
Experimental Bode plot verifying the transfer function $H(s)$ of the low-pass stage used after the analog multiplier for amplitude demodulation. The blue line is obtained from the analytic transfer function. The measurements confirm the expected DC gain, cutoff frequency and a fourth-order low-pass characteristic.

![Experimental Bode plot](image)

100 Hz to 100 kHz with 11 points per decade. The ±1V carrier used for demodulation by the multiplier is provided by the same external generator, in order to guarantee synchronization between the signals. The output of the low-pass filter is a sine wave with the same frequency as the modulating signal, as expected. For each frequency, its amplitude is measured (which, compared to the unitary voltage input directly gives the gain of $H(s)$), together with the phase-shift with respect to the DSB-AM modulating signal. Fig. 14 shows the obtained experimental Bode plot. The analytic transfer function for the Multi-Feedback active filter configuration is given in [22]. Two of such second-order filter stages are cascaded, with the values of the passive components selected to yield a DC gain $H_{DC} = 6$ dB, a cutoff frequency of $f_{LPF} = 30$ kHz and an overall fourth-order low-pass characteristic with −80 dB slope beyond $f_{LPF}$. All these characteristics are verified by the measurements. Consequently, the final ECS measurement bandwidth lies approx. one decade before $f_{LPF}$, i.e. $f_{ECS} \approx 3$ kHz.

VII. CONCLUSION
This paper discusses the design of an eddy current sensor (ECS) measuring the position of a moving conductive target located behind another fixed conductive shield. The analysis previously started with the aid of FEM simulations in [10] is now extended and completed with a full analytical transformer model, which describes the impedance variations of the sensor’s excitation coil for varying frequency and air gap and allows to calculate the optimal excitation frequencies, thus allowing to design optimally the excitation coil to maximize e.g. position sensitivity. Then an investigation on the temperature’s influence on the sensor’s output is conducted, which is a relevant problem given the underlying application. Temperature particularly affects the resistive part of the impedance variations, so it has to be taken into account also while selecting a sensor interface. The most suitable measurement circuit is a differential AC Wheatstone bridge, which is analyzed thoroughly to find the optimal excitation frequencies which result in the maximum output voltage. Finally, a measurement setup consisting of a hardware prototype of the evaluation board, the excitation coil and various thin metallic samples is realized. The results verify the functionality of the measurement circuit, of the sensor concept itself and its achievable bandwidth. With a 30 kHz measurement bandwidth, a sensitivity of 1 mV/µm⁻¹ and 1 µm resolution, the studied ECS is applicable as a highly-dynamic position measurement system for entirely sealed MBs. In future work, stable operation of AMBs using the proposed ECS can be demonstrated experimentally with a hardware prototype of a sealed actuator. In this context, the exact number of sensors required and their placement will be analyzed. Such experimental setup would also allow to investigate the impact of disturbing magnetic fields coming from the stator’s winding on the proposed ECS.

APPENDIX
The expression of the excitation coil’s input impedance $Z_{c, in}(\omega, \delta)$ obtained from the equivalent transformer model is reported:

\[
Z_{c, in}(\omega, \delta) = \frac{Z_{NUM}(\omega, \delta)}{Z_{DEN}(\omega, \delta)}
\]  

(18)

\[
Z_{NUM}(\omega, \delta) = R_1 R_2 R_3 + j (L_1 R_2 R_3 + L_2 R_1 R_3 + L_3 R_1 R_2) \omega + \ldots
\]

\[
- (L_1 L_2 R_3 + L_1 L_3 R_2 + L_2 L_3 R_1 - L_1 L_2 R_3 k_{12}^2)
\]

\[
- (L_1 L_2 R_3 k_{13}^2 - L_2 L_3 R_1 k_{23}^2) \omega^2 + \ldots
\]

\[
- (L_1 L_2 L_3 - j L_1 L_2 k_{12}^2 - j L_1 L_2 L_3 k_{23}^2)
\]

\[
- j L_1 L_2 L_3 k_{13}^2 + 2 j L_1 L_2 L_3 k_{12} k_{23} k_{32} \omega^3
\]

\[
Z_{DEN}(\omega, \delta) = R_2 R_3 + j (L_2 R_3 + L_2 R_2) \omega
\]

\[
+ L_2 L_3 (k_{23}^2 - 1) \omega^2
\]

(19)

References
[1] A. J. Fleming, “A review of nanometer resolution position sensors: Operation and performance,” Sensors Actuators A: Phys., vol. 190, pp. 106–126, 2013.
[2] B. George, Z. Tan, and S. Nihitianov, “Advances in capacitive, eddy current, and magnetic displacement sensors and corresponding interfaces,” IEEE Trans. Ind. Electron., vol. 64, no. 12, pp. 9595–9607, Dec. 2017.
[3] J. Boehm, R. Gerber, and N. R. C. Kiley, “Sensors for magnetic bearings,” IEEE Trans. Magn., vol. 29, no. 6, pp. 2962–2964, Nov. 1993.
[4] L. Xi’nan, W. Fengxiang, and W. Baoguo, “Application of eddy-current sensor for air gap detection in magnetic suspension motors,” in Proc. 5th Int. Conf. Elect. Mach. Syst., 2001, pp. 326–329.
[5] S. Miric, R. Giuffrida, D. Bortis, and J. Kolar, “Dynamic electromechanical model and position controller design of a new high-precision self-bearing linear actuator,” IEEE Trans. Ind. Electron., vol. 68, no. 1, pp. 744–755, Jan. 2021.
[6] T. Wellledieck, “The bearingless pump for high fluid temperatures,” (in German), Ph.D. dissertation, Dept. Inf. Technol. Electr. Eng., ETH Zurich, Zurich, Switzerland, 2017.
[7] A. Sophian and M. Fan, “Pulsed eddy current non-destructive testing and evaluation: A review,” Chin. J. Mech. Eng., vol. 30, pp. 1474–1474, Nov. 2017.
[8] S. Shokralla, S. Sullivan, J. Morelli, and T. W. Krause, “Modelling and validation of eddy current response to changes in factors affecting pressure tube to calandria tube gap measurement,” NDT&E Int., vol. 73, pp. 15–21, 2015.

[9] T. Baumgartner, R. M. Burkart, and J. W. Kolar, “Analysis and design of a 300-W 500000-r/min slotless self-bearing permanent-magnet motor,” IEEE Trans. Ind. Electron., vol. 61, no. 8, pp. 4326–4336, Aug. 2014.

[10] R. V. Giuffrida, S. Mišić, D. Bortis, and J. W. Kolar, “Looking through walls—actuator position measurement through a conductive wall,” in Proc. 23rd Int. Conf. Elect. Mach. Syst., 2020, pp. 1649–1654.

[11] S. S. Mohan, M. del Mar Hershenson, S. P. Boyd, and T. H. Lee, “Simple accurate expressions for planar spiral inductances,” IEEE J. Solid-State Circuits, vol. 34, no. 10, pp. 1419–1424, Oct. 1999.

[12] J. Zhao, “A new calculation for designing multilayer planar inductors,” Elect. Des. News, vol. 55, no. 14, pp. 37–40, 2010.

[13] D. Vydrobal, “Impedance of the eddy-current displacement probe: The transformer model,” IEEE Trans. Instrum. Meas., vol. 53, no. 2, pp. 384–391, Apr. 2004.

[14] H. A. Wheeler, “Simple inductance formulas for radio coils,” Proc. Inst. Radio Engineers, vol. 16, no. 10, pp. 1398–1400, 1928.

[15] C. Akyl, S. Babic, and S. Kincic, “New and fast procedures for calculating the mutual inductance of coaxial circular coils (circular coil-disk coil),” IEEE Trans. Magn., vol. 38, no. 5, pp. 2367–2369, Sep. 2002.

[16] W. G. Hurley, M. C. Duffy, T. Zhang, L. Lope, B. Kunz, and W. H. Wölflle, “A unified approach to the calculation of self- and mutual-inductance for coaxial coils in air,” IEEE Trans. Power Electron., vol. 30, no. 11, pp. 6155–6162, Nov. 2015.

[17] M. R. Nabavi and S. N. Nihtianov, “Design strategies for eddy-current displacement sensor systems: Review and recommendations,” IEEE Sensors J., vol. 12, no. 12, pp. 3346–3355, Dec. 2012.

[18] P. Kejšk, C. Klusier, R. Bischofberger, and R. S. Popovic, “A low-cost inductive proximity sensor for industrial applications,” Sensors Actuators, A: Phys., vol. 110, no. 1–3, pp. 93–97, Feb. 2004.

[19] G. Zhao, J. Yin, L. Wu, and Z. Feng, “Ultradense and low-noise self-compensation method for circuit thermal drift of eddy current sensors based on analog multiplier,” IEEE Trans. Ind. Electron., vol. 67, no. 10, pp. 8851–8859, Oct. 2020.

[20] V. Chaturvedi, J. G. Vogel, K. A. Makinwa, and S. Nihtianov, “A 19.8-nW Eddy-current displacement sensor interface with sub-nanometer resolution,” IEEE J. Solid-State Circuits, vol. 53, no. 8, pp. 2273–2283, Aug. 2018.

[21] R. Pallas-Areny and J. Webster, Sensors and Signal Conditioning, 2nd ed. Hoboken, NJ, USA: Wiley, 2001.

[22] J. Karki, “Active low-pass filter design,” Texas Instruments, Dallas, TX, USA, Tech. Rep. SLOA049B, Sep. 2002.