Investigation of the influence of the electromagnetic interaction between volume bodies on the scattering field characteristics

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Abstract. Using the method of discrete sources, we solve the problem of scattering of an electromagnetic wave by a structure composed of two closely adjacent perfectly conducting triaxial ellipsoids. The solution was realized as a computer code. Using this code, we investigated the influence of the electromagnetic interaction of ellipsoids located at different distances on the scattering field characteristics. Some numerical results are presented.

1. Introduction
It is of considerable interest for researchers to study scattering of electromagnetic waves by a system of perfectly conducting bodies having sizes comparable with the exciting-field wavelength. This interest arises from the need to solve a number of important applied problems, e.g. the electromagnetic compatibility problems, radionavigation, identification of objects, etc. The specificity of electromagnetic processes for structures from several bodies consists of the following. The field scattered by each body induces derived currents on all adjacent bodies, thereby the currents on all bodies of a structure are interconnected. This phenomenon is called “Electromagnetic interaction”. If the bodies of a structure are located at the distances less or compared with the exciting-field wavelength, it is necessary to take into account the electromagnetic interaction, and to investigate a structure as a whole. This fact complicates the solution of considered scattering problem. If the bodies of a structure are located so far one from another that it is possible to neglect of influence of derived currents induced by adjacent bodies, we can represent the field scattered by a structure as a superposition of the fields scattered by single bodies of a structure. In this case for finding of field scattered by a structure it is enough to solve more easier scattering problems for single bodies of a structure. In connection with the reasons mentioned above the investigations aimed at analysis of the influence of the distance between the bodies on their electromagnetic interaction are very interesting.

This paper makes contribution to subject under discussion. In this paper we present some results concerning of the influence of electromagnetic interaction of two ellipsoids located at different distances on their bistatic scattering cross section. The idea of investigations is the following. Firstly, we solve the problem of scattering of an electromagnetic wave by a structure composed of two closely adjacent perfectly conducting ellipsoids with taking into account their electromagnetic interaction. Then for the same exciting field we define the field scattered by the structure as the superposition of two solutions, one of which is the solution of the problem of scattering by the first ellipsoid of the structure in the absence of the second one, while the other is the solution of the problem of scattering by the second ellipsoid in the absence of the first one. The solution received by superposition is compared with one received with taking into account the electromagnetic interaction of ellipsoids.
2. Formulation of the problem and its solution with taking into account the electromagnetic interaction of ellipsoids

The geometry of the problem is shown in Figure 1. We will consider a stationary problem (a time dependence of $\exp(-i\omega t)$ is assumed) of scattering of an electromagnetic field $\{\vec{E}_0, \vec{H}_0\}$ by a structure composed of two disjoint perfectly conducting triaxial ellipsoids $D_1$ and $D_2$ bounded by the Lyapunov surfaces $S_1$ and $S_2$ and located in a homogeneous unbounded medium $D_e$ of dielectric permittivity $\varepsilon_e$ and magnetic permeability $\mu_e$. We wish to find the scattered field $\{\vec{E}_s, \vec{H}_s\}$ in the domain $D_e$.

The mathematical formulation of the problem is as follows:

$$\nabla \times \vec{E}_e = i\omega \mu_e \vec{H}_e, \quad \nabla \times \vec{H}_e = -i\omega \varepsilon_e \vec{E}_e$$

in the domain $D_e$,

$$\vec{n} \times \vec{E}_e = -\vec{n} \times \vec{E}_0$$

on the surfaces $S_1$ and $S_2$, and

$$\{\sqrt{\varepsilon_e} \vec{E}_e; \sqrt{\mu_e} \vec{H}_e\} \times \vec{R}/R + \{\sqrt{\mu_e} \vec{H}_0; -\sqrt{\varepsilon_e} \vec{E}_0\} = O(R^{-1}), R \to \infty,$$

where $\vec{n}$ is a unit vector normal to the surfaces $S_1$ and $S_2$; $R=(x^2+y^2+z^2)^{1/2}$ and $\vec{a} \times \vec{b}$ is a vector product.

The general method of solution of the similar problems was suggested in [1, 2]. We introduce the auxiliary surfaces $S_{e1} = K_{e1} S_1$ and $S_{e2} = K_{e2} S_2$ inside each of ellipsoids such that they are homothetic, with the centers at the points $O_1$ and $O_2$, to the scatterer surfaces $S_1$ and $S_2$. The homothety (similarity) coefficients $K_{e1}$ and $K_{e2}$ characterize the spacing of the auxiliary surfaces and the surfaces of the corresponding ellipsoids, and their values lie in the interval $0 < K_{e1}, K_{e2} < 1$ (if $K_{e1}, K_{e2} = 0$, the auxiliary surface shrinks to a point; if $K_{e1}, K_{e2} = 1$ it coincides with the surface of the corresponding ellipsoids).

We choose a finite set of points $\{M_{n,1}\}_{n=1}^{N_1}$ on auxiliary surface $S_{e1}$ inside ellipsoid $D_1$ and at each point $M_{n,1}$ we locate a pair of independent auxiliary electric dipoles with moments
\[ \vec{p}_{n,1} = p_{n,1} e_{n,1}, \vec{p}_{n,2} = p_{n,2} e_{n,2} \] aligned with the unit vectors \( e_{n,1}, e_{n,2} \), respectively. The dipoles are chosen to lie in a plane tangential to the surface \( S_{e,1} \) at the point \( M_{n,1} \). By analogy, we choose a finite set of points \( \{M_{n,j}\}_{n=1}^{N_1} \) on auxiliary surface \( S_{e,2} \) inside ellipsoid \( D_2 \) and at each point \( M_{n,j} \) we locate a pair of independent auxiliary electric dipoles with moments \( \vec{p}_{n,1} = p_{n,1} e_{n,1}, \vec{p}_{n,2} = p_{n,2} e_{n,2} \) aligned with the unit vectors \( e_{n,1}, e_{n,2} \), respectively. These dipoles are chosen to lie in a plane tangential to the surface \( S_{e,2} \) at the point \( M_{n,2} \). The number of the dipoles on the auxiliary surface \( S_{e,1} \) is \( 2N_1 \); the number of the dipoles on the auxiliary surface \( S_{e,2} \) is \( 2N_2 \). It is supposed that all dipoles radiate in a homogeneous medium with parameters \( \epsilon_e \) and \( \mu_e \).

Now we represent the unknown scattered field \( \{\vec{E}_e, \vec{H}_e\} \) in \( D_e \) as a sum of the fields from the introduced auxiliary dipoles as follows:

\[
\vec{H}_e(M) = \sum_{n=1}^{N_1} \sum_{n=1}^{N_2} \nabla \times (\nabla \times \vec{P}_{n,1}) + \sum_{n=1}^{N_2} \nabla \times (\nabla \times \vec{P}_{n,2}),
\]

\[
\vec{E}_e(M) = \sum_{n=1}^{N_1} \vec{P}_{n,1} + \sum_{n=1}^{N_2} \vec{P}_{n,2}.
\]

(4)

Here \( k_e = \omega \sqrt{\epsilon_e \mu_e} = 2\pi/\lambda \) is the wave number in medium \( D_e \); \( R_{MM_e} \) is the distance from the point \( M_{n,1} \) on \( S_{e,1} \) to the point \( M \) in \( D_e \); \( R_{MM_{e,2}} \) is the distance from the point \( M_{n,2} \) on \( S_{e,2} \) to the same point \( M \) in \( D_e \); \( p_{n,1}^{n,1}(n=1,N_1) \) and \( p_{n,2}^{n,2}(n=1,N_2) \) are known complex constants (dipole moments); \( N_1 \) is the number of points for allocating of dipoles on the auxiliary surface \( S_{e,1} \), and \( N_2 \) is the number of points for allocating of dipoles on the auxiliary surface \( S_{e,2} \).

The field (4) satisfies Eqs. (1) and radiation conditions (3) in the domain \( D_e \). To satisfy boundary conditions (2), we should properly choose the dipole moments \( p_{n,1}^{n,1}, p_{n,2}^{n,1}(n=1,N_1) \) and \( p_{n,1}^{n,2}, p_{n,2}^{n,2}(n=1,N_2) \). To this end, we use the collocation method. Let \( M_{j_1}(j_1=1,2,...,L_1) \) be the collocation points on the surface \( S_{e,1} \), where \( L_1 \) is the number of the collocation points on \( S_{e,1} \), and let \( M_{j_2}(j_2=1,2,...,L_2) \) be the collocation points on the surface \( S_{e,2} \), where \( L_2 \) is the number of collocation points on \( S_{e,2} \). Then, to determine the unknown constants \( p_{n,1}^{n,1}, p_{n,2}^{n,1}(n=1,N_1) \) and \( p_{n,1}^{n,2}, p_{n,2}^{n,2}(n=1,N_2) \), we have the following system of linear algebraic equations:

\[
\vec{n}_h \times \vec{E}_h^{j_1} = -\vec{n}_h \times \vec{E}_0^{j_1}, \quad j_1 = 1, L_1,
\]

\[
\vec{n}_h \times \vec{E}_h^{j_2} = -\vec{n}_h \times \vec{E}_0^{j_2}, \quad j_2 = 1, L_2,
\]

(5)

where \( \vec{n}_h \) and \( \vec{n}_h \) are the unit normal vectors at points \( M_{j_1} \) and \( M_{j_2} \), respectively; \( \vec{E}_h^{j_1}, \vec{E}_h^{j_2} \) and \( \vec{E}_0^{j_1}, \vec{E}_0^{j_2} \) are the electric components of the scattered field (4) and the exciting field at the same points. Solution of the system (5) is found by minimizing the functional.
The electromagnetic interaction of the ellipsoids on the bistatic scattering cross section is composed of two triaxial ellipsoids with centers on the intersection plane (xz plane) consisting of two cross-sectional halves: \( \theta = 0^\circ \) and \( \theta = 180^\circ \). Curves 1 in Figures 3–5 show the results of the solution of the problem of scattering by two ellipsoids within the framework of a full-wave electrodynamic formulation. Curves 2 in Figures 3–5 show the results of the superposition of two solutions, one of which is the solution of the problem of scattering by the first ellipsoid of the structure in the absence of the second one, while the other is the solution of the problem of scattering by the second ellipsoid in the absence of the first one. Clearly, such an approach does not allow for the electromagnetic interaction of the structure scatterers. Comparison of curves 1 and 2 allows us to estimate the influence of the electromagnetic interaction of the ellipsoids on the bistatic scattering cross sections. The distance between the ellipsoids was varied by moving the second ellipsoid along the x axis. The parameter presented in the Figure 2 is the least distance, expressed in terms of the exiting-field wavelength \( \lambda \), between the points of surfaces of the ellipsoids.

\[
\Phi = \sum_{j=1}^{N_1} \bar{n} \times (\vec{E}_e + \vec{E}_o) \right|_{\theta=0}^2 + \sum_{j=1}^{N_2} \bar{n} \times (\vec{E}_e + \vec{E}_o) \right|_{\theta=180}^2. 
\] (6)

After solving the minimization problem (finding the unknown dipole moments \( p_{n1}, p_{n2} (n = 1, N_1) \) and \( p_{n1}, p_{n2} (n = 1, N_2) \), the required scattered-field characteristics are determined from Eqs. (4).

The scattering problem for each single ellipsoid, supposing that other ellipsoid is absent, we solve by using that the same method.

3. Numerical results

Based on the above-stated method, we developed a computer code for calculating the scattered field. The initial data of the code are the coordinates of the ellipsoid’s centers, the orientation and length of their semiaxes (measured in wavelength), the exiting field \( \{\vec{E}_e, \vec{H}_o\} \), the similarity parameters \( K_{e1} \) and \( K_{e2} \), the numbers \( N_1 \) and \( N_2 \) of points in which dipoles are located, and the numbers \( L_1 \) and \( L_2 \) of the collocation points for each of the ellipsoids. Outgoing data of the code are components \( E_{e,\theta} = (\vec{E}_e, 0) \) and \( E_{e,\varphi} = (\vec{E}_e, \varphi) \) of scattered field, and bistatic cross-section

\[
\sigma(\theta, \varphi) = \lim_{R \to \infty} 4\pi R^2 \left| \left| \vec{E}_{e,\theta} \right| \right|^2 + \left| \left| \vec{E}_{e,\varphi} \right| \right|^2. 
\] (7)

Minimization of functional (6) is performed by the conjugate gradient method.

The code allows to find the field components scattered by each of ellipsoids, supposing that other ellipsoid is absent, too.

Using this code, we carried out a series of computational tests aimed at analysis of the influence of electromagnetic interaction of two ellipsoids located at different distances on their bistatic scattering cross section. Some numerical results for the structure shown in Figure 2 are given in Figures 3–5. The structure is composed of two triaxial ellipsoids with centers on the x axis. The center of the first ellipsoid coincides with the origin of the Cartesian coordinate system. The ellipsoid semiaxes \( k_e a, k_e b \) and \( k_e c \) are aligned with the x, y and z axes, respectively, and are equal to \( k_e a_1 = 1, k_e b_1 = 1.5 \) and \( k_e c_1 = 2 \) for the first ellipsoid, and to \( k_e a_2 = 2, k_e b_2 = 1.5 \) and \( k_e c_2 = 1 \) for the second ellipsoid. The structure are exited by a linearly polarized plane wave propagating along the z axis and having the vector \( \vec{E}_0 \) aligned with the x axis. The results are shown in the E-plane (xz plane) consisting of two cross-sectional halves: \( \varphi = 0^\circ \) and \( \varphi = 180^\circ \). Curves 1 in Figures 3–5 show the results of the solution of the problem of scattering by two ellipsoids within the framework of a full-wave electrodynamic formulation. Curves 2 in Figures 3–5 show the results of the superposition of two solutions, one of which is the solution of the problem of scattering by the first ellipsoid of the structure in the absence of the second one, while the other is the solution of the problem of scattering by the second ellipsoid in the absence of the first one. Clearly, such an approach does not allow for the electromagnetic interaction of the structure scatterers. Comparison of curves 1 and 2 allows us to estimate the influence of the electromagnetic interaction of the ellipsoids on the bistatic scattering cross sections. The distance between the ellipsoids was varied by moving the second ellipsoid along the x axis. The parameter \( \lambda \) presented in the Figure 2 is the least distance, expressed in terms of the exiting-field wavelength \( \lambda \), between the points of surfaces of the ellipsoids.
Figure 2. The structure under consideration

Figure 3. Bistatic scattering cross-sections in the $E$-plane of the structure shown in Figure 2 for the case where the distance between the ellipsoids $\Delta l = 0.01\lambda$. Curve 1 shows the results obtained with taking into account electromagnetic interaction of ellipsoids; curve 2 shows the results obtained by superposition of two solutions.
When obtaining the results discussed, we chose the following parameters of the method. The similarity coefficients $K_{r,1}$, $K_{r,2}$ were equal to 0.6. For both ellipsoids the numbers $N_1$, $N_2$ of points in which dipoles are located were chosen identical and equal to 168, and the numbers of the collocation points $L_1$, $L_2$ were equal to 336.

Figures 3–5 show that if the distance between scatterers is small (\(\Delta l \leq 0.1\lambda\)), the interaction of the scatterers strongly affects the bistatic scattering cross sections, resulting in a qualitative change of the scattering patterns. For these distances it is necessary to take into account the electromagnetic interaction between the scatterers. As the distance between scatterers increases their interaction decreases gradually. At \(\Delta l \approx 0.5\lambda\), the differences of the bistatic scattering cross-sections calculated with and without allowance for the interaction between scatterers do not exceed 2 dB. These differences are due to both the influence of the interaction between scatterers and the computer errors. Hence the differences of the bistatic scattering cross sections, which are due to the influence of the interaction, are actually smaller than 2 dB. Thus, if the distance between the bodies in the structure

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**Figure 4.** The same as in Figure 3, but for the case where the distance between the ellipsoids $\Delta l = 0.1\lambda$.

**Figure 5.** The same as in Figure 3, but for the case where the distance between the ellipsoids $\Delta l = 0.5\lambda$. 
\( \Delta l > 0.5 \lambda \), one may neglect the interaction of scatterers when calculating the power scattering characteristics.

4. Summary

In this paper we present some results concerning of the influence of electromagnetic interaction of two ellipsoids located at different distances on their bistatic scattering cross section. The idea of investigations is the following. Firstly, we solve the problem of scattering of an electromagnetic wave by a structure composed of two closely adjacent perfectly conducting ellipsoids with taking into account their electromagnetic interaction. Then for the same exciting field we define the field scattered by the structure as the superposition of two solutions, one of which is the solution of the problem of scattering by the first ellipsoid of the structure in the absence of the second one, while the other is the solution of the problem of scattering by the second ellipsoid in the absence of the first one. The solution received by superposition is compared with one received with taking into account the electromagnetic interaction of the ellipsoids. The comparison of two solutions shows that if the distance between the scatterers less 0.5\( \lambda \), it is necessary to take into account the electromagnetic interaction of the scatterers.

References

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