Cosmological Backreaction for a Test Field Observer in a Chaotic Inflationary Model

Giovanni Marozzi\textsuperscript{1,2*, 3}†, Gian Paolo Vacca\textsuperscript{3†}, Robert H. Brandenberger\textsuperscript{4‡}

\textsuperscript{1} Collège de France, 11 Place M. Berthelot, 75005 Paris, France.
\textsuperscript{2} Université de Genève, Département de Physique Théorique and CAP, 24 quai Ernest-Ansermet, CH-1211 Genève 4, Switzerland.
\textsuperscript{3} INFN Sezione di Bologna, via Irnerio 46, I-40126 Bologna, Italy.
\textsuperscript{4} Physics Department, McGill University, 3600 University Street, Montreal, QC H3A 2T8, Canada

Abstract

In an inhomogeneous universe, an observer associated with a particular matter field does not necessarily measure the same cosmological evolution as an observer in a homogeneous and isotropic universe. Here we consider, in the context of a chaotic inflationary background model, a class of observers associated with a “clock field” for which we use a light test field. We compute the effective expansion rate and fluid equation of state in a gauge invariant way, taking into account the quantum fluctuations of the long wavelength modes, and working up to second order in perturbation theory and in the slow-roll approximation. We find that the effective expansion rate is smaller than what would be measured in the absence of fluctuations. Within the stochastic approach we study the bounds for which the approximations we make are consistent.

1 Introduction

The setting of this work is inflationary cosmology \cite{1}, specifically a simple large field (or “chaotic”) inflationary model \cite{2} in which the almost constant potential of a slowly rolling scalar field $\Phi$ (the “inflaton”) leads to a phase of almost exponential expansion of space. We know, however, that there must be other matter fields present, in particular those of the Standard Model of particle physics. These fields may play a sub-dominant role in determining the expansion of space during the period of inflation, but they are very relevant to late-time cosmology. In such a framework it is reasonable to consider the possibility of associating to one of these extra fields a class of dynamical observers, a fact which leads to the construction and study of observables which are determined by measurements performed by such a class of observers. In the presence of primordial cosmological fluctuations, which are unavoidably present in the semiclassical approach of the gravity-matter dynamics, such observers will in general not measure exactly the same physics as a canonical observer in a homogeneous universe would. Such an effect is, in general, called the cosmological backreaction.

This is a non-trivial task and we shall consider here a simple toy model setup. Following \cite{3}, we will consider the case where the clock/observer is associated to a light test scalar field $\chi$ (as opposed to the inflaton field), and the inflaton field has the standard quadratic potential of chaotic inflation given by $V(\Phi) = \frac{m^2}{2} \Phi^2$. More specifically, we assume that the light field $\chi$ at the background level does not affect the evolution of the space-time during the inflationary regime but only plays the role of a clock.

\*giovanni.marozzi@unige.ch
\†vacca@bo.infn.it
\‡rhb@physics.mcgill.ca
In general, the issue of backreaction of cosmological perturbations \[4\] can be described as follows. If we start with a background cosmological solution (metric and matter) and add to it small amplitude fluctuations of relative amplitude \(\epsilon\) which satisfy the linear perturbation equations, then due to the non-linearities in the Einstein equations these equations are not satisfied to quadratic order. Second order fluctuations in matter and metric build up. In particular, corrections to the background can be induced. The calculations of \[4\] showed that, to leading order in spatial gradients, super-Hubble linear fluctuation modes induce effects which grow logarithmically in time and act like a negative contribution to the effective cosmological constant. It was then conjectured \[5\] that this effect might lead to a dynamical relaxation mechanism for the cosmological constant, in analogy of the effect which is conjectured to occur in pure quantum gravity due to two loop infrared gravitons \[6\]. In \[4\], the effects were computed as a function of the background time variable \(t\).

However, in \[7\] the challenge was raised as to whether this effect could be measured in terms of a physical clock. Indeed, it was then shown \[8\] that in the case of single field matter, the infrared backreaction is not physically measurable (see \[9\] for the same result obtained within a manifestly gauge invariant, and up to second order in perturbation theory, approach). The Hubble expansion rate measured by the only available physical clock (namely the value of the scalar field) has the same history with and without long wavelength fluctuations (see also \[10\]).

Now, in late time cosmology we measure time in terms of the temperature of the cosmic microwave background (CMB) which at late times is a sub-dominant component of matter. Hence, as said, it is reasonable to ask whether effects of cosmological fluctuations can be measured by a clock which corresponds to a sub-dominant field, a field which in the late time universe would represent the temperature of the CMB. A first step in this direction was made in \[3\] where it was shown that a clock tied to an entropy field will indeed measure a different expansion rate of space in the presence and absence of curvature fluctuations.

On the other hand, in order to give meaningful statements about observables - once the observers have been chosen - these must be constructed in a gauge invariant way, i.e. they should be defined covariantly in a reparameterization invariant way, so that any other spectator (secondary) observer agrees with the definition given, computing it with its preferred reference system. Therefore some care is required since normally one is dealing with observables obtained after averaging procedures. For example, it is crucial to take into account the volume measure. The problem can be naturally analyzed employing the gauge invariant method developed in \[11, 12\] (where the set of effective equations for the averaged geometry given in \[13\] has been generalized in a covariant and gauge invariant form) and recently applied to single field inflationary models \[9, 14\].

Therefore, considering the gauge invariant approach just mentioned, the question we address in this paper is the following: we compare the average expansion rate of a spatial hypersurface characterized by a fixed value of a spectator test scalar field in two space-time manifolds, the first one which has no curvature fluctuations, the second one which contains such perturbations \[2\]. We find that as a consequence of long wavelength fluctuations, the average expansion rate is smaller. This effect is at quadratic order in the amplitude of cosmological fluctuations (at one loop in quantum field theory language) as opposed to the situation in pure gravity where backreaction effects of long wavelength fluctuations occur only at two loop order \[6\]. The backreaction leads to a reduction of the expansion rate. Whether the effect leads to a uniform relaxation of the cosmological constant or simply to a stochastic adjustment is probably difficult to be answered using only our perturbative approach.

The outline of this paper is as follows. In Section 2 we review the covariant and gauge-invariant approach which we shall use, and introduce the “effective Hubble expansion rate” for an observer in an inhomogeneous universe. Section 3 forms the heart of this paper: we introduce our toy model, define the class of observers used (observers associated with a test scalar field \(\chi\)), determine the gauge transformation into a coordinate system in which \(\chi\) is constant on spatial hypersurfaces, and

---

1Note that \(\epsilon\) is not the inflationary slow-roll parameter.

2Let us observe that, in investigations concerning the actual dynamics of the Universe, one may imagine more suitable choices to investigate physical phenomena based on past light-cone observations. In particular, the gauge invariant approach of \[11, 12\] has been generalized to averaging over the past light-cone of a generic observer in \[15\] and this new approach has been applied in \[16\].
evaluate the effective Hubble expansion rate which our observer sees. We also discuss bounds on
the parameters of the model under which our perturbative analysis is valid, estimate the magnitude
of the terms contributing to the backreaction, and determine the equation of state as seen by our
observers. In Section 4 we give a general discussion and provide some numerical results. The final
section contains our conclusions.

2 Observables and gauge invariant backreaction

The issue of the backreaction on an observable induced by quantum fluctuations in an inflationary
universe can be cast on solid grounds using a recent covariant and gauge invariant (GI) approach [11,
[12]. One can construct a general non-local observable by taking quantum averages of a scalar field
$S(x)$ over a space-time hypersurface $\Sigma_{A0}$ where another scalar $A(x)$ (assumed to have a time-like
gradient) has constant value $A_0$. Given this second scalar field, we can define a particular (barred)
coordinate system $\bar{x}^\mu = (t, \bar{x})$ in which the scalar $A$ is homogeneous. A physical (and hence GI)
definition of the observable is then given by the following quantity

$$\langle S \rangle_{A0} = \frac{\langle \sqrt{\gamma(t_0, \bar{x})} \bar{S}(t_0, \bar{x}) \rangle}{\langle \sqrt{\gamma(t_0, \bar{x})} \rangle}, \quad (1)$$

where we have denoted by $\sqrt{\gamma(t_0, \bar{x})}$ the determinant of the induced three dimensional metric on
$\Sigma_{A0}$. The natural foliation of spacetime is then defined by the four vector

$$n^\mu = -\frac{\partial^\mu A}{(-\partial^\mu A\partial_\mu A)^{1/2}} \quad (2)$$

which characterizes the class of observers.

We are interested in the dynamics encoded in the effective scale factor $a_{\text{eff}} = \left(\sqrt{\gamma}\right)^{1/3}$, which
satisfies the GI effective cosmological equation [12]

$$\left(\frac{1}{a_{\text{eff}}} \frac{\partial a_{\text{eff}}}{\partial A_0}\right)^2 = \frac{1}{9} \left(\frac{\Theta}{(-\partial^\mu A\partial_\mu A)^{1/2}}\right)^2 A_0, \quad (3)$$

where $\Theta = \nabla_\mu n^\mu$ is the expansion scalar of the timelike congruence $n^\mu$. This effective scale factor
describes the expansion of the space as seen by the class of observers sitting on the hypersurface
$\Sigma_{A0}$. The dynamics can be then extracted solving the Einstein and matter equations of motion in
gauge.

In order to deal with the metric components in any specific frame we employ the standard decom-
position of the metric in terms of scalar, transverse vector ($B_i, \chi_i$) and traceless transverse tensor ($h_{ij}$)
fluctuations up to the second order around a homogeneous Friedmann-Lemaitre-Robertson-Walker
(FLRW) zero order space-time

$$g_{00} = -1 - 2\alpha - 2\alpha^{(2)}, \quad g_{i0} = -\frac{a}{2}(\beta_i + B_i) - \frac{a}{2} \left(\beta_\alpha^{(2)} + B_\alpha^{(2)}\right)$$

$$g_{ij} = a^2 \left[\delta_{ij} \left(1 - 2\psi - 2\psi^{(2)}\right) + D_{ij}(E + E^{(2)}) + \frac{1}{2} \left(\chi_{i,j} + \chi_{j,i} + h_{ij}\right) + \frac{1}{2} \left(\chi_i^{(2)} + \chi_j^{(2)} + h_i^{(2)} + h_j^{(2)}\right)\right] \quad (4)$$

where $D_{ij} = \partial_i \partial_j - \delta_{ij} \nabla^2/3$ and for notational simplicity we have removed a superscript for first order
terms. The Einstein equations connect these fluctuations with those in the matter fields. For example,
an inflaton field can be written to second order as $\Phi(x) = \phi^{(0)}(t) + \phi^{(1)}(x) + \phi^{(2)}(x)$ (where
$x$ is the space-time four vector).

These general perturbed expressions can be gauge fixed. Let us recall some common gauge fixings of
the scalar and vector part: the synchronous gauge (SG) is defined by $g_{00} = -1$ and $g_{i0} = 0$,
the uniform field gauge (UFG) apart from setting $\Phi(x) = \phi^{(0)}(t)$ must be supplemented by other
conditions (one can consider $g_{i0} = 0$), finally the uniform curvature gauge (UCG) is defined by
$g_{ij} = a^2 \left[\delta_{ij} + \frac{1}{2} \left(h_{ij} + h_i^{(2)}\right)\right]$. 


To quadratic order (in the amplitude of the fluctuations), the inhomogeneities effect the effective expansion rate. This is called the “backreaction” effect. Using Eq. (3) we can now define the backreaction on the averaged expansion rate as the one obtained with respect to a class of observers identified by the scalar field $A(t, \vec{x})$. In the long wavelength (LW) limit and neglecting the tensor perturbations, we obtain \[H_{\text{eff}}^2 \equiv \dot{A}^2 \left( \frac{1}{a_{\text{eff}}} \frac{\partial a_{\text{eff}}}{\partial A_0} \right)^2 = H^2 \left[ 1 + \frac{2}{H} \langle \dot{\psi} \dot{\bar{\psi}} \rangle - \frac{2}{H} \langle \dot{\bar{\psi}}^{(2)} \rangle \right], \tag{5}\]

Let us stress here that the choice of a particular class of observers leads to the choice of a particular observable. The observers depend on a preferred reference frame but the associated observable is physical, and hence gauge invariant, in the sense that every other observer associated to a different frame agrees on its value.

In an inflationary background, the long wavelength contribution to the expectation values $\langle \dot{\psi} \dot{\bar{\psi}} \rangle$ and $\langle \dot{\bar{\psi}}^{(2)} \rangle$ will be increasing in time since the phase space of infrared (super-Hubble) modes is growing. The contribution of the ultraviolet (sub-Hubble) modes is constant in time since, given an ultraviolet cutoff which is fixed in physical coordinates (see e.g. \[17\] for a discussion of this point), the phase space of these modes is constant (in the limit of exponential inflation). In the following we will use the stochastic approach to evaluate the above expectation values.

3 Gauge invariant backreaction in a 2-field model

The classical model from which we start is a Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime with an inflaton (scalar) field $\Phi$ and a second light field $\chi$ which we study in the test field approximation (i.e. neglecting its energy density and pressure in the background FLRW equations). We shall admit all possible inhomogeneous scalar quantum fluctuations of the metric and of the scalar matter fields, up to second order in perturbation theory. In particular, the inflaton model we consider here is, for simplicity, given by a free massive scalar field which we shall treat in the slow-roll approximation together with a second free massive (test) field $\chi$, both minimally coupled to gravity. The action is

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} m^2 \Phi^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m^2 \chi^2 \right]. \tag{6}$$

In the rest of this work we shall make an heavy use of the results on the stochastic approach to inflationary dynamics presented in Section 5 of \[18\].

3.1 Perturbative dynamics of the observers

We now choose a special class of observers which will define our observables. Following \[8\], we consider the average expansion rate as computed by the observers "$\chi$", namely by the observers sitting on the spacelike hypersurface defined by $\chi$ equal to a constant. This hypersurface characterizes what we denote the barred gauge. To proceed we need the solutions of the equation of motion in this frame up to second order in perturbation theory (see Eq. \[5\]). These can be obtained with a gauge transformation from the results in the UCG \[18, 19\]. As a consequence we have to find the gauge transformation that goes from the UCG to the barred gauge, where $\chi(x) = \chi(0)(t) + \chi^{(1)}(x) + \chi^{(2)}(x)$ is equal to a constant, i.e. where $\chi^{(1)} = \chi^{(2)} = 0$. To fix the second gauge completely we also impose the condition $\beta = 0$. This gauge, which we denote U$\chi$FG, explicitly depends on the property of the field $\chi$. We stress that this is a new scenario compared to previous analysis in a single field inflationary model \[9, 14\], where the observers were defined in a geometrical way.

To find the gauge transformation, let us first consider the “infinitesimal” coordinate transformation parametrized by the first-order and second-order generators $\epsilon^{\mu}_{(1)}$ and $\epsilon^{\mu}_{(2)}$, respectively, and given by \[20\]

$$x^{\mu} \rightarrow \tilde{x}^{\mu} = x^{\mu} + \epsilon^{\mu}_{(1)} + \frac{1}{2} \left( \epsilon^{\nu}_{(1)} \partial_\nu \epsilon^{\mu}_{(1)} + \epsilon^{\mu}_{(2)} \right) + \ldots \tag{7}$$
where
\[ \epsilon_{(1)}^\mu = \begin{pmatrix} \epsilon_{(1)}^0, \partial^i \epsilon_{(1)}^i + \epsilon_{(1)}^i \end{pmatrix}, \quad \epsilon_{(2)}^\mu = \begin{pmatrix} \epsilon_{(2)}^0, \partial^i \epsilon_{(2)}^i + \epsilon_{(2)}^i \end{pmatrix} \]
(8)

(we have explicitly separated the scalar part from the pure transverse vector part \( \epsilon_{(1)}^i, \epsilon_{(2)}^i \)). Let us then recall that the associated gauge transformation of a scalar field \( S \) is, to first order,
\[ S^{(1)} \rightarrow \tilde{S}^{(1)} = S^{(1)} - \epsilon_{(1)}^0 \dot{S}^{(0)}, \]
and, to second order,
\[ S^{(2)} \rightarrow \tilde{S}^{(2)} = S^{(2)} - \epsilon_{(1)}^0 \dot{S}^{(1)} - \left( \epsilon_{(1)}^i + \partial^i \epsilon_{(1)}^i \right) \partial_i S^{(1)} + \frac{1}{2} \left[ \epsilon_{(1)}^0 \partial_t \epsilon_{(1)}^0 \dot{S}^{(0)} + \left( \epsilon_{(1)}^i + \partial^i \epsilon_{(1)}^i \right) \partial_i \epsilon_{(1)}^0 \dot{S}^{(0)} - \epsilon_{(2)}^0 \dot{S}^{(0)} \right]. \]
(10)

On neglecting vector perturbations we can fix to zero \( \epsilon_{(1)}^i \) and with \( S \equiv \chi \) the gauge conditions \( \dot{\chi}^{(1)} = 0 \) and \( \dot{\chi}^{(2)} = 0 \) give the following results:
\[ \epsilon_{(1)}^0 = \frac{\chi^{(1)}}{\chi^{(0)}}, \]
(11)
\[ \epsilon_{(2)}^0 = 2 \frac{\chi^{(2)}}{\chi^{(0)}} + \frac{1}{\chi^{(0)}} \left[ -2 \left( \partial^i \epsilon_{(1)}^i \partial^j \chi^{(1)} + \epsilon_{(1)}^0 \chi^{(1)} \right) + \epsilon_{(1)}^i \partial_i \left( \epsilon_{(1)}^0 \dot{\chi}^{(0)} \right) + \left( \partial^i \epsilon_{(1)}^i \partial_i \epsilon_{(1)}^0 \right) \dot{\chi}^{(0)} \right], \]
(12)
and using \( \epsilon_{(1)}^0 \) in \( \epsilon_{(2)}^0 \) one obtains
\[ \epsilon_{(2)}^0 = 2 \frac{\chi^{(2)}}{\chi^{(0)}} - \frac{1}{\chi^{(0)}} \partial^i \epsilon_{(1)}^i \partial_i \chi^{(1)} - \frac{1}{\chi^{(0)} \chi^{(0)}} \chi^{(1)} \chi^{(1)}. \]
(13)

We can now evaluate \( \epsilon_{(1)}^i \) from the second gauge condition of U\( \chi \)FG, namely \( \tilde{\beta} = 0 \). One obtains (from Eq. (3.8) of [21])
\[ \epsilon_{(1)}^i = \int dt \left[ \frac{1}{a^2} \frac{\chi^{(1)}}{\chi^{(0)}} - \frac{\beta}{2a} \right]. \]
(14)

Considering finally the transformation between UCG and U\( \chi \)FG it is straightforward to derive the following gauge transformation for \( \psi \) and \( \bar{\psi}^{(2)} \) (from Eqs. (3.9) and (3.13) of [21]):
\[ \bar{\psi} = H \epsilon_{(1)}^0 + \frac{1}{3} \nabla^2 \epsilon_{(1)}^i \]
(15)
\[ \bar{\psi}^{(2)} = \frac{H}{2} \epsilon_{(2)}^0 + \frac{1}{6} \nabla^2 \epsilon_{(2)}^i - \frac{H}{2} \epsilon_{(1)}^0 \epsilon_{(1)} + \frac{\epsilon_{(2)}^0}{2} \left( \dot{H} + 2H^2 \right) - \frac{H}{2} \partial_i \epsilon_{(1)}^0 \partial^i \epsilon_{(1)} - \frac{1}{6} \Pi_{ij}, \]
(16)
where \( \Pi_{ij} \) is defined in Eq. (3.16) of [21]. Using the above results and working in the long wavelength limit (and neglecting also tensor perturbations) one explicitly obtains
\[ \bar{\psi} = \frac{H}{\chi^{(0)}} \chi^{(1)}, \quad \bar{\psi}^{(2)} = \frac{H}{\chi^{(0)}} \chi^{(2)} - \frac{H}{\chi^{(0)}} \chi^{(1)} \chi^{(1)} - \frac{\chi^{(1)}}{2 \chi^{(0)} \chi^{(0)}} \left( 2H^2 + \dot{H} - H \frac{\dot{\chi}^{(0)}}{\chi^{(0)}} \right). \]
(17)

We have now almost all the ingredients needed to evaluate the different terms involved in the backreaction equation [5]. Let us begin with \( \langle \bar{\psi} \hat{\psi} \rangle \):
\[ \langle \bar{\psi} \hat{\psi} \rangle = \frac{H^2}{\chi^{(0)} \chi^{(0)}} \left[ \frac{\dot{H}}{H} \bar{\chi}^{(0)} + \frac{\dot{\chi}^{(0)}}{\chi^{(0)}} \right] \left( \chi^{(1)} \chi^{(1)} \right). \]
(18)

Following [18] we solve the background equations for our particular model and find the following zero order solution for the test field \( \chi \)
\[ \chi^{(0)}(t) = \chi^{(0)}(t_i) \left( \frac{H(t)}{H(t_i)} \right)^{\alpha}, \]
(19)
where we have defined $\alpha = \frac{m^2}{\omega^2}$.

Let us remark that, for the purpose of this work, based on a perturbative analysis, we need to consider $\chi^{(0)}(t_i) \neq 0$, namely, a dynamical regime such that $\chi$ has non zero vacuum expectation value, a condensate. In such a case we have a scalar with time-like gradient which defines a space-like hypersurface, $\chi(x)$ equal to a constant, up to when the space dependence of the scalar can be seen as a perturbative dependence over a time function $\chi^{(0)}(t)$. Namely, when the stochastic perturbative approach defined in [18] is valid (see, Eqs. (29-31)). Such a configuration is compatible with a dynamical phase of universe expansion such as inflation. Let us further underline as the dynamical approach defined in [18] is valid (see, Eqs. (29-31)). Such a configuration is compatible with a physical clock field which can be used to follow the evolution of space-time.

At this point we turn to the evaluation of the contribution of the long wavelength (super-Hubble) modes to the expectation values which appear in Eq. (18). Following [18] we obtain for our particular background the following stochastic solutions up to second order in perturbation theory

$$\langle \chi^{(1)2} \rangle = \frac{3H^2}{8\pi^2m^2(2-\alpha)} (H_i^{4-2\alpha} - H^4 - 2\alpha) - \frac{\alpha^2}{48\pi^2} \frac{\chi^{(0)}(t_i)^2}{M^4_{pl}} \left( \frac{H}{H_i} \right)^{2\alpha} \frac{1}{H^4} (H^2 - H_i^2)^3, \quad (20)$$

$$\langle \chi^{(2)} \rangle = \frac{\alpha}{8\pi^2} \frac{\chi^{(0)}(t_i)}{M^2_{pl}} \left( \frac{H}{H_i} \right)^\alpha \left[ -\frac{H_i^6}{H^2} \frac{1 - \alpha/2}{6} + \frac{H_i^4}{H^2} \frac{4 - \alpha - H H_i}{H^4} + \frac{H_i^2}{H} \frac{1 + \alpha}{4} \right]. \quad (21)$$

Going back to the expression of Eq. (18) we obtain by substitution the following result

$$\langle \dot{\psi} \ddot{\psi} \rangle = \frac{H^2 \dot{H}}{\chi^{(0)2} \dot{H}} \left\{ \frac{3H^{2\alpha}}{8\pi^2m^2(2-\alpha)} \left[ 2 - \frac{H \dot{H}}{H^2} \right] H_i^{4-2\alpha} - \left( 4 - \frac{H \dot{H}}{H^2} \right) H^4 - 2\alpha \right\} - \frac{\alpha^2}{48\pi^2} \frac{\chi^{(0)}(t_i)^2}{M^4_{pl}} \left( \frac{H}{H_i} \right)^{2\alpha} \frac{1}{H^4} \left[ -\frac{H \ddot{H}}{H^2} (H^2 - H_i^2)^3 + \frac{3H^2}{H} (H^2 - H_i^2)^2 \right]. \quad (22)$$

Let us now consider the other term in Eq. (5) which depends on $\langle \dot{\psi}^{(2)} \rangle$. It is easy to show that the leading slow-roll contribution is given by the derivative of the leading term of $\langle \dot{\psi}^{(2)} \rangle$. Indeed, considering the leading part in Eq. (17), then in the slow-roll approximation, and starting from the expression

$$\langle \dot{\psi}^{(2)} \rangle \simeq \frac{\dot{H}}{H} \langle \chi^{(2)} \rangle - \frac{H^2}{\chi^{(0)2}} \langle \chi^{(1)2} \rangle, \quad (23)$$

one obtains that

$$\langle \dot{\psi}^{(2)} \rangle \simeq \frac{\dot{H}}{H} \left( \frac{\dot{H}}{H} - \frac{\chi^{(0)}}{\chi^{(0)}} \right) \langle \chi^{(2)} \rangle + \langle \chi^{(2)} \rangle - 2\langle \dot{\psi} \ddot{\psi} \rangle, \quad (24)$$

from which it follows that the leading value of $\langle \dot{\psi}^{(2)} \rangle$ will be given by

$$\langle \dot{\psi}^{(2)} \rangle \simeq \frac{\dot{H}}{H} \left( -2 \frac{\dot{H}}{H} - \frac{\dot{H}}{H} \right) \langle \chi^{(2)} \rangle - 2\langle \dot{\psi} \ddot{\psi} \rangle. \quad (25)$$

As a consequence, we can rewrite the cosmological backreaction (Eq. (5)) for the expansion rate as seen by an observer comoving with the light test field $\chi$ in the following way

$$H_{eff}^2 \simeq H^2 \left[ 1 + \frac{6}{H} \langle \psi \ddot{\psi} \rangle + \frac{2}{H^2} \left( \frac{2}{H} + \frac{\dot{H}}{H} \right) \langle \chi^{(2)} \rangle \right]. \quad (26)$$

This equation will be the starting point of the analysis which we shall develop in Section 4.
3.2 Bounds

Let us now see which are the constraints that we have to take into consideration to have a consistent description of our problem.

**Test field condition for** \( \chi \). The condition that the gravitational background dynamics is not affected by the field \( \chi \) can be derived from \( \rho_\chi \ll \rho_\Phi \) using the solution in Eq. (19) and, for the whole inflationary period, reads [18]

\[
\chi^{(0)}(t_i)^2 \ll \left[ 1 + \frac{\alpha m^2}{9 H^2} \right]^{-1} \frac{1}{\alpha} \left( \frac{H}{H_i} \right)^{2-2\alpha} \frac{H_i^2}{m^2 M_{pl}^4}.
\]  

(27)

In particular, for the case \( \alpha \ll 1 \), we obtain the following limiting condition at the end of inflation \((H \simeq m)\):

\[
\chi^{(0)}(t_i)^2 \ll \frac{6}{\alpha} M_{pl}^2.
\]  

(28)

**Reliability of the perturbative approach.** We study the quantum fluctuations of the system of fields \( \Phi \) and \( \chi \) in perturbation theory. Such an approach is valid provided that the perturbed quantities are smaller in amplitude than the background values. There are several conditions which need to be satisfied

\[
\left( \langle \phi^{(1)}(2) \rangle \right)^{1/2} \ll 1, \quad \frac{\langle \phi^{(1)} \rangle}{\langle \phi^{(1)}(1) \rangle} \langle \phi^{(1)}(1) \rangle^{1/2} \ll 1, \quad \frac{\langle \phi^{(1)} \chi^{(1)} \rangle}{\langle \phi^{(0)} \chi^{(0)} \rangle} \ll 1, \quad \left( \frac{\langle \chi^{(1)} \rangle}{\langle \chi^{(0)} \rangle} \right)^{1/2} \ll 1, \quad \left( \frac{\langle \chi^{(2)} \rangle}{\langle \chi^{(1)}(2) \rangle} \right)^{1/2} \ll 1.
\]  

(29) - (31)

The first conditions given in (29) are related only to the inflaton field. For the chaotic model under investigation they have been already analyzed in the appendix of [22]. One finds that the perturbative treatment up to second order of the inflaton field is reliable, for all the duration of inflation, only for a value of \( H_i \) less of nearly 130\( m \) (where we consider \( M_{pl} = 10^5 m \)).

It can be shown that the last two conditions (Eq. (31)), which involve only the test field \( \chi \), are the most stringent, and in particular stronger that the one which contains the mixed correlator in Eq. (30).

In Section 4 we shall present a numerical evaluation based only on the constraints in (31), together with the one in Eq. (27). We shall also introduce a bound to control the magnitude of the backreaction (see Eq. (26)). Indeed, to obtain a reliable estimate of the quantum backreaction in a perturbative way, it is natural to study a constraint to limit its value. If the backreaction terms become comparable to the background values, then the perturbative approach will break down.

3.3 Estimates for the backreaction

Before presenting the numerical evaluation of the backreaction effect it is useful to investigate the analytical expressions of the various contributions to the effective Hubble expansion rate in order to extract the leading behaviour in the different phases of the inflationary evolution, in particular during the final stages of inflation. Since the phase space of infrared modes is increasing as we approach the end of inflation, we expect the magnitude of the backreaction terms to increase.

Let us begin with the first term of the cosmological quantum backreaction terms in Eq. (26), the one proportional to \( \langle \bar{\psi} \psi \rangle \). At the end of inflation, the leading value of the second term in Eq. (22) is negligible with respect to the leading value of the first one, for \( \alpha < 2 \) which is the region of interest for our model since we want to describe light fields, provided that

\[
\chi^{(0)}(t_i)^2 \ll \frac{81}{\alpha^2} \frac{2}{2-\alpha} \frac{M_{pl}^2 m^2}{H_i^2}.
\]  

(32)
Note that this condition is different from the condition \((28)\). If we consider the particular case \(\alpha \ll 1\) and require that \((28)\) implies \((32)\), we obtain the following condition on \(\alpha\):

\[
\alpha < \frac{27 \, m^2}{2 \, H_i^2}.
\]  

(33)

In such a case the leading result is given by

\[
\frac{6}{H} \langle \bar{\psi} \dot{\bar{\psi}} \rangle = \frac{\dot{H}}{\chi^{(0)} H} \frac{9H^{2\alpha}}{2\pi^2 m^2 (2 - \alpha)} H_i^{1 - 2\alpha}.
\]  

(34)

Note that the sign of this contribution is negative. Hence, this backreaction term will contribute to a decrease in the measured local expansion rate, an effect conjectured in \([4, 5]\). The contribution is increasing as inflation proceeds, an effect which is due to the increasing phase space of infrared modes, before being attenuated at the end of inflation by an \(H^2\) factor (coming from the \(1/\chi^{(0)}\)).

Returning to the full expression of Eq. \((22)\), let us note that at the beginning of inflation the second term in the square parentheses of the first line dominates over the first, and that therefore the sign is opposite, and thus the contribution to the effective Hubble expansion rate starts out positive.

We are left with the task of analyzing the magnitude of the second term in Eq. \((26)\), proportional to \(\langle \chi^{(2)} \rangle\). Starting from Eq. \((21)\), one can see that this term reaches its maximum value at the end of inflation or more generally for \(H \ll H_i\). Working in this limit, assuming \(\alpha \ll 2\), considering the long wavelength limit and calculating in leading slow-roll approximation, one finds

\[
\frac{2}{\chi^{(0)}} \left( 2 \frac{\dot{H}}{H} + \frac{\ddot{H}}{H} \right) \langle \chi^{(2)} \rangle \simeq -\frac{1}{12\pi^2} \frac{H_i^6}{M_{pl}^2 H^4} \left( 1 - \frac{\alpha}{2} \right).
\]  

(35)

A first interesting observation is that this contribution \((35)\) is always negative (as well as the full expression of \(\langle \chi^{(2)} \rangle\) in Eq. \((21)\)) and once again goes in the direction of decreasing the effective expansion rate of the Universe given in Eq. \((26)\), namely the expansion rate measured by the observer "sitting" on \(\chi\). Furthermore, this contribution is of the same order of magnitude of the one found in \([14]\) for the case of a single field model. In fact, in \([14]\) we found, at leading order in the slow-roll approximation and in the long wavelength limit, a backreaction effect on the expansion rate of the Universe as seen by an isotropic observer, of the following magnitude:

\[
\frac{\dot{H}}{H^2} \frac{\langle \phi^{(1/2)} \rangle}{M_{pl}^2} \simeq -\frac{1}{24\pi^2} \frac{H_i^6}{M_{pl}^2 H^4}.
\]  

(36)

which is indeed of the same order as \((35)\).

We have now seen that both backreaction terms in Eq. \((26)\) reduce the effective expansion rate of the Universe as seen by an observer located on constant \(\chi\) surfaces. We now turn to an estimate of the magnitude of the backreaction effect in the region of parameter space in which the bounds presented in the previous subsection apply. We consider only the final stage of the inflationary era \((H \ll H_i)\) and, for simplicity, maintain the condition of Eq. \((32)\).

Using Eq. \((19)\) and the slow-roll approximation \(\dot{H} \simeq -m^2/3\), the expression in Eq. \((34)\) becomes

\[
\frac{6}{H} \langle \bar{\psi} \dot{\bar{\psi}} \rangle \simeq -\frac{1}{\chi^{(0)}(t_i)^2} \frac{27}{2\pi^2} \frac{1}{\alpha^2(2 - \alpha)} \frac{H^2 H_i^4}{m^4}.
\]  

(37)

If we now consider, in order to be consistent, the test field condition \((28)\) (considering the limit \((33)\) in the case when this implies the condition \((32)\)) we find a lower bound to the above contribution

\[
\left| \frac{6}{H} \langle \bar{\psi} \dot{\bar{\psi}} \rangle \right| \gg \frac{9}{4\pi^2} \frac{1}{\alpha(2 - \alpha)} \frac{H_i^2 H_i^4}{M_{pl}^2 m^3}.
\]  

(38)

This is to be compared with the other contribution to the backreaction explicitly given in Eq. \((35)\) for the case \(H \ll H_i\).
Concerning their order of magnitude we note that, under the condition considered here, while the contribution \( \langle \psi \dot{\psi} \rangle \) depends, for the given model, on the zeroth order value \( \chi_i(0) \) and on the mass \( m_\chi \) of the test field, the contribution \( \langle \psi \dot{\psi} \rangle \) is almost independent of the features of the test field, provided that it is light \((\alpha \ll 1)\). Furthermore, for typical values of our parameters we will have

\[
\left| \frac{6}{H} \langle \psi \dot{\psi} \rangle \right| \gg \left| \frac{2}{\chi(0)(t_i)} \left( \frac{2\ddot{H}}{H} + \frac{\dddot{H}}{H} \right) \langle \chi^{(2)} \rangle \right|
\]

and the first contribution will be the leading one. For example, if we consider a light test field at the end of inflation, and the following typical value for a chaotic model of inflation \( H_i = \mathcal{O}(10)m \) with \( m = 10^{-5}M_{pl} \), we obtain (at the end of inflation when \( H \sim m \)) the following order of magnitude (from Eqs. (38) and (35))

\[
\left| \frac{6}{H} \langle \psi \dot{\psi} \rangle \right| \gg \frac{1}{\alpha(2-\alpha)} \mathcal{O}(10^{-6})
\]

and

\[
\left| \frac{2}{\chi(0)(t_i)} \left( \frac{2\ddot{H}}{H} + \frac{\dddot{H}}{H} \right) \langle \chi^{(2)} \rangle \right| \simeq \mathcal{O}(10^{-6}).
\]

During the early stages of inflation the behaviour of \( \frac{6}{\tilde{H}} \langle \psi \dot{\psi} \rangle \) is more complicated, but in general it will be larger in amplitude than \( \frac{\alpha}{\chi(0)} \left( \frac{2\ddot{H}}{H} + \frac{\dddot{H}}{H} \right) \langle \chi^{(2)} \rangle \). In particular, as said, looking at the dominant contribution in the first line of (22), one can see that it is positive when the second term in the square bracket overcomes the first one. This happens approximately up to the point where \( H \simeq 2^{1/(4-2\alpha)}H_i \). The ratio between the maximum positive backreaction contribution to \( H^2_{eff} \) and the absolute value of the maximum negative contributions is monotonically decreasing from approximately 3.7 \((\alpha \ll 1)\) to 3.2 \((\alpha = 1)\). In Fig. 1 we plot these two different contributions, to the effective Hubble expansion rate, without making any approximation, and for typical values of our parameter for which the backreaction is still perturbative and our results consistent.

![Figure 1](image-url)

**Figure 1:** We show \( \frac{6}{\tilde{H}} \langle \psi \dot{\psi} \rangle \) (left plot) and \( \frac{2}{\chi(0)} \left( \frac{2\ddot{H}}{H} + \frac{\dddot{H}}{H} \right) \langle \chi^{(2)} \rangle \) (right plot) as functions of \( H/m \) with \( H_i = 10m \), \( \alpha = 0.2 \) and \( \chi_0 = 0.7M_{pl} \).

Furthermore, for a very light test field, the contribution of the first term \( \sim \frac{6}{\tilde{H}} \langle \psi \dot{\psi} \rangle \) may result (since it contains a \( 1/\chi^{(0)} \) enhancement factor (see Eq. (22))) in a value which is even greater in magnitude than 1. This may happen even if the perturbative expansion used to study the dynamics is still valid. But the perturbative approach to study backreaction will have broken down. We shall make more comment on this fact in Section 4.

### 3.4 Gauge invariant backreaction on the effective equation of state

A valuable piece of information, which can be obtained without much effort, is given by the effective equation of state with respect to observers which measure the effective Hubble expansion rate discussed here. For this purpose, we take advantage of the fact that one of the quantum contributions (the second one) to the effective expansion rate in Eq. (26) can be written as \( c \left( \frac{H}{H^2} \right) \langle \varphi^2 \rangle / M_{pl}^2 \) with \( c = 2 - \alpha \) (see Eqs. (35, 36)). Let us start from the general result obtained in [14], valid for any class
of observers and slow-roll inflationary models, where we allow for a further second order contribution $H^2B$ in the effective expansion rate, beyond the typical terms $\sim \langle \varphi^2 \rangle$,

$$
\dot{\chi}^{(0)} = \frac{1}{a_{eff}} \left( \frac{\partial a_{eff}}{\partial \chi_0} \right) = \frac{8\pi G}{3} \rho_{eff} = H^2 \left[ 1 + \left( c \frac{\dot{H}}{H^2} + d \frac{\dot{H}^2}{H^4} + \mathcal{O} \left( \frac{\dot{H}^3}{H^6} \right) \right) \frac{\langle \varphi^2 \rangle}{M_{pl}^2} + B(t) \right].
$$

(42)

In the single field model [14], the parameters $c$ and $d$ encode the typical non zero backreaction effects at first and second order in the slow-roll approximation and $B(t)$ encodes other possible effects (in our case $c = 2 - \alpha$ while $d$ is not fixed). Since this latter quantity is of second order in perturbation theory we have to consider only linear terms in any perturbative expression. Then, from the consistency between the effective equations for the averaged geometry (see [19] [14]), one obtains

$$
- \frac{1}{a_{eff}} \dot{\chi}^{(0)} \frac{\partial}{\partial \chi_0} \left( \chi^{(0)} \frac{\partial a_{eff}}{\partial \chi_0} \right) = \frac{4\pi G}{3} \left( \rho_{eff} + 3p_{eff} \right) = -\dot{H} - H^2 - H^2 \left\{ \left[ c \frac{\dot{H}}{H^2} + \left( d - \frac{3}{2} c \right) \frac{\dot{H}^2}{H^4} \right. \right.

\left. \left. + c \frac{\dot{H}}{H^3} + \mathcal{O} \left( \frac{\dot{H}^3}{H^6} \right) \right] \frac{\langle \varphi^2 \rangle}{M_{pl}^2} + B(t) + \frac{\dot{H}}{2H^2} \left[ B(t) + \dot{B}(t) \right] \right\}.
$$

(43)

From these relations it is easy to see that the effective equation of state $w_{eff} = p_{eff}/\rho_{eff}$, to first non trivial order, is given by

$$
w_{eff} = \frac{p_{eff}}{\rho_{eff}} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2} + \left\{ \left[ 5 \frac{\dot{H}^2}{H^4} - 2 \frac{\dot{H}}{3} \frac{\dot{H}}{H^3} + \mathcal{O} \left( \frac{\dot{H}^3}{H^6} \right) \right] \frac{\langle \varphi^2 \rangle}{M_{pl}^2} + \frac{\dot{H}}{3H^2} \left[ B(t) - \frac{1}{3H} \dot{B}(t) \right] \right\},
$$

(44)

where the $d$ dependence disappears in the leading order correction. In the case under investigation

$$
B = \frac{6}{H} \langle \tilde{\psi} \tilde{\psi} \rangle.
$$

(45)

4 Discussions and numerical results

In the previous section we have defined our model, and studied the main observables in a GI way, giving their analytical expressions for the quantum corrections. Having also provided the general criteria for the reliability of our model within a perturbative approach, we want now to analyze numerically the magnitude of the overall backreaction effects experienced by our observers due to quantum fluctuations.

Let us first begin with some general considerations. As seen, in the evaluation of the backreaction from Eq. (5), the quantities $\tilde{\psi}$ and $\tilde{\psi}^{(2)}$ are connected to $\chi^{(1)}$ and $\chi^{(2)}$ by the gauge transformation given in Eqs. (11) [13] [14]. Therefore, the backreaction terms will be given by combinations of test field perturbations multiplied by factors like $1/\chi^{(0)}$ and $(1/\chi^{(0)})^2$. These factors replace the similar factors $1/\phi$ and $(1/\phi)^2$ which are present in the backreaction terms obtained in [14] for the single field model. On the other hand, even in comparison with the slow rolling of the background inflaton field, the background test field $\chi^{(0)}$ changes relatively slowly for $\alpha \ll 1$ (see Eq. (19)) and hence the above factors can give a strong enhancement of the backreaction compared to the effect of the factors $(1/\phi)^2 \sim 1/M_{pl}^2$ in the case of single field inflation. So, contrary to [18], where it was shown that for a chaotic model the mean square gauge-invariant inflaton fluctuations grow faster than those of any test field with nonnegative mass, the backreaction on the effective expansion rate, measured with respect to the observer comoving with the light test field in a two field model, is at least of the same order, and in general bigger, than the typical backreaction that we can obtain in a single field case.

The relative magnitude of the backreaction terms in the expression for the effective Hubble expansion rate has already been given in Fig. 1. We now want to numerically study the parameter space where our analysis is under perturbative control.

Concerning the validity of the expressions for the quantum corrections (backreaction) found in the previous section, we have a safe perturbative expansion for the observables only for $\frac{\hat{H}}{H} \langle \tilde{\psi} \tilde{\psi} \rangle \ll 1$, since this appears in the leading terms both in Eq. (26) and in Eq. (44).
In the following we give the limit of validity of our results imposing the condition that the backreaction contributes a fixed maximal amount to the effective Hubble expansion rate, i.e. $|H^2_{\text{eff}} - H^2| / H^2 \ll 1$. In the numerical analysis we consider the condition $|H^2_{\text{eff}} - H^2| / H^2 < 10^{-2}$. To do this we join the test field condition (27) (with the $\ll$ sign replaced by $< 10^{-2}$) and conditions on the validity of the perturbative expansion of the fields. The latter can be obtained by requiring the condition given in Eq. (31) (with the $\ll$ sign replaced by $< 10^{-1}$), which is the most stringent one among (29)-(31).

In Fig. 2 we consider the cases of $H_i = 10m$ and $H_i = 20m$, and show the region in the two-dimensional parameter space labelled by $\alpha$ and by the initial value of the $\chi$ field for which all of the conditions are satisfied. The shaded region is the allowed one. For very small initial values of the $\chi$ field the backreaction becomes too large in amplitude, for very large values the test field condition is satisfied only for $\alpha \ll 1$ (see also Fig 3). This leaves us with the indicated region. For higher values of $H_i$ the region narrows more and more until it disappears around a value of $H_i \simeq 50m$.

Figure 2: We plot the region where all three conditions (validity of the perturbation theory, test field approximation, both defined in the text, and requiring the backreaction to be at most 1%) are satisfied, for the values $H_i = 10m$ (left plot) and $H_i = 20m$ (right plot). The horizontal axis denotes $\chi(0)(t_i)/M_{\text{pl}}$ and the vertical axis $\alpha$. We chose the value of $m$ given by $M_{\text{pl}} = 10^5m$ to be consistent with the observed value of cosmological perturbations.

We have still not yet discussed the change in magnitude of the backreaction as our parameters vary. In the region where the backreaction is small, then for the whole duration of inflation we can study numerically how it depends on the free parameters of the model (initial condition $\chi^{(0)}(t_i)$ and the mass $m_{\chi}$). We find it convenient to show in Fig. 3 how the constrained region in parameter space, obtained requiring a certain amount of maximum backreaction, changes as the limit on the amplitude of backreaction is changed. In each of the plots, the right dashed line separates the region where the test field approximation is valid (left of the line) from the region where it is not (right of the line). The left dashed line separates the region where the backreaction effect on the observed Hubble parameter is smaller than the limit indicated (right of the line) from that where it is not (left of the line). One can see that the relative amplitude of backreaction on the local Hubble expansion rate increases for smaller values of the mass $m_{\chi}$ and for smaller $\chi(0)(t_i)$.

In both the two plots of Fig. 3 we give also further informations related to the different bounds. Each bound is associated to a region which has a colour: yellow (perturbation theory valid), blue (test field approximation) and light red (backreaction with an upper bound). Where two regions overlap the colours change as follow: blue+yellow is gray, yellow + light red is orange. Where all the bounds are satisfied, all regions overlap, the colour is darker red.

It may be interesting to follow the results obtained in our perturbative analysis up to the point when the backreaction effects are of the same order (positive or negative) of the background, while maintaining the other bounds (test field approximation and validity of the cosmological perturbation theory for metric and matter fluctuations). We want to stress that this is of course not formally a safe choice since the backreaction on the observables might have important perturbative contributions at
higher order. However, it may be useful to gain some qualitative indication of the effects.

We have therefore studied, for several values of the parameters $\alpha$ and $\chi^{(0)}(t_i)$, $H_{\text{eff}}$ and $w_{\text{eff}}$ defined by the observer comoving with the light test field. It is important to stress that even assuming that such observations might be of a physical interest during inflation, they are not connected in a simple way to late time observations, unlike the direct relation which exists in ordinary cosmological perturbation theory for the spectrum of the cosmological fluctuations.

We present in the following just one case, giving the results in Fig. 4, for $H_i = 10m$ and the parameter values $\alpha = 0.0085$ and $\chi_0 = 0.5M_{\text{pl}}$. In this specific case one can see, in the left plot of Fig. 4, that the quantum backreaction on the effective expansion scalar is positive at the beginning of the inflationary period and diminishes monotonically with the effective number of e-foldings $N_{\text{eff}} = \int dt H_{\text{eff}}$ up to the point where it has become negative enough to stop the observed accelerated expansion earlier than what would occur in the absence of backreaction. For the parameters chosen in this simulation this happens at $N_{\text{eff}} \approx 89$ compared to the case without backreaction when $N = 148.5$. Here, the end point of inflation is taken to correspond to $H(t_f) = m$. Therefore, the observer comoving with the moduli field is experiencing an apparently shorter inflationary period.

The effective equation of state is affected in the following way: $w_{\text{eff}}$ receives a positive backreaction at the beginning which leads it farther away from a de Sitter like equation of state, but during the evolution the difference decreases. As shown in the figure, the background value is always more de Sitter like. The observer experiences a kind of “apparent inverted chaotic inflation (AICI)”, in the sense that the evolution of the effective equation of state is somehow reversed.

5 Conclusions

In this work we focused on the backreaction of long wavelength quantum fluctuations produced during inflation. We have adopted a computational scheme, recently developed [11, 12], which is covariant and has guided us to define gauge invariant observables in the sense that any observer, in whatever reference system, agrees on them.

Since in late time cosmology time is measured in terms of the temperature of the CMB, which is subdominant at later times compared to other matter, one might try to measure, even during an inflationary regime, an effective expansion rate in terms of a clock associated to a test field which is not directly affecting the background geometry. This measure nevertheless may be affected by strong quantum fluctuations of the gravity-matter system which must be taken into account in a gauge invariant way.

Taking advantage of the results obtained in [18], for a chaotic inflationary model with the presence
of a test field, and using a stochastic approach in the slow-roll regime, we have thus studied a particular class of observables which are related to measurements performed by an observer comoving with a test field. In particular, we have analyzed the average effective expansion rate and the effective equation of state as seen by such class of observers. We find that - except for a short period after the onset of inflation - the long wavelength fluctuations lead to a reduction of the expansion rate of space compared to what an observer in an unperturbed space-time would see. This leads to a reduction in the measured length of the inflationary period. Throughout our work we have taken into account the constraints on the parameter space of the model in which the dynamics can be studied at second order in perturbation theory in a reliable way.

As we have shown explicitly, then during the final stages of the inflationary phase, and in the case of a sufficiently light test field, the quantum corrections of the average expansion rate, as measured by our comoving field, are dominated by the term proportional to the inverse of the test field mass to the fourth power times the square of the homogeneous initial value of the test field. As a consequence, for an extremely light test field, the backreaction may even overcome the value given by the classical dynamics, even if cosmological perturbation theory for the field fluctuations and the test field approximation are both valid. In such a case one should employ a non perturbative approach to study the observables. One may argue that in such a regime these observables are very peculiar. In a sense they feel the quantum effects in an amplified way.

We have investigated the region of parameter space where the quantum backreaction effects are of the same order of magnitude as the background quantities and we have shown that these effects can stop the “observed” inflation much earlier. We have also noted that the measured effective equation of state has an inverted time evolution starting farther from but getting closer to a de Sitter like phase with the evolution. We have called this kind of observed behavior “apparent inverted chaotic inflation (AICI)”. Let us point out that this “apparent” dynamics cannot be mimicked by another chaotic inflationary model (even with an higher mass) since the effective equation of state has a completely different “inverted” dynamics.

A way to give a physical explanation for such possibly non-perturbative effects of backreaction, as seen by observers comoving with the test field $\chi$, comes from looking at the relative gauge transformation that connects a general gauge to the gauge $U\chi FG$. Starting from Eqs. (11,13) it is easy to see as such a gauge transformation corresponds to a big time shift (from Eq.(7)) when $\chi^{(0)} \ll 1$, and a non-perturbative effect of the backreaction is possible. Namely, we find that non-perturbative backreaction corresponds to a gauge trasformation which is no longer infinitesimal.

Note also that we have made a test field approximation for the scalar $\chi$ associated to the observer measuring $H_{\text{eff}}$. This means that the dynamics (solution of the semiclassical equations of motion) of the inflaton and of the metric is not affected by the presence of $\chi$. This fact helps to understand that the results found for the observables considered here are therefore non related to an absolute concept of quantum backreaction on the dynamics of the expansion of the Universe, which is probably a very
difficult concept to define. It is related instead to observations which the specific class of dynamical observers can make.

Summarizing our findings, from our analysis we therefore deduce that a gedanken experiment with observers of the kind discussed here may be useful to extract "amplified" quantum corrections in the associated observables. Insisting on considering such observables as physically relevant, one faces in general a computability problem. If the backreaction on the GI observable associated to the effective expansion rate is small then one can proceed with our perturbative approach which stops at second order and obtain a reasonable estimate. We have found this is possible only in a bounded parameter region. When outside of this region, the perturbative analysis for the observer level could fail. We lack other tools to perform a different investigation. Therefore in such a case we are not sure that the results found can be extrapolated, at least qualitatively, to a non-perturbative region.

Acknowledgements
We wish to thank Gabriele Veneziano for stimulating discussions. GPV thanks the Cosmology Group of the University of Geneva for its kind hospitality. GM is supported by the Marie Curie IEF, Project NeBRiC - “Non-linear effects and backreaction in classical and quantum cosmology”. The research of RB is supported by an NSERC Discovery Grant and by funds from the Canada Research Chair program.

References

[1] A. H. Guth, “The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems,” Phys. Rev. D 23, 347 (1981).

[2] A. D. Linde, “Chaotic Inflation,” Phys. Lett. B 129, 177 (1983).

[3] G. Geshnizjani and R. Brandenberger, “Back reaction of perturbations in two scalar field inflationary models,” JCAP 0504, 006 (2005) [hep-th/0310265].

[4] L. R. W. Abramo, R. H. Brandenberger and V. F. Mukhanov, “The energy-momentum tensor for cosmological perturbations,” Phys. Rev. D 56, 3248 (1997) [arXiv:gr-qc/9704037];
V. F. Mukhanov, L. R. W. Abramo and R. H. Brandenberger, “On the back reaction problem for gravitational perturbations,” Phys. Rev. Lett. 78, 1624 (1997) [arXiv:gr-qc/9609020];
L. R. W. Abramo and R. P. Woodard, “One loop back reaction on chaotic inflation,” Phys. Rev. D 60, 044010 (1999) [arXiv:astro-ph/9811430].

[5] R. H. Brandenberger, “Back reaction of cosmological perturbations and the cosmological constant problem,” [hep-th/0210165]

[6] N. C. Tsamis and R. P. Woodard, “Relaxing The Cosmological Constant,” Phys. Lett. B 301, 351 (1993);
N. C. Tsamis and R. P. Woodard, “The Physical basis for infrared divergences in inflationary quantum gravity,” Class. Quant. Grav. 11, 2969 (1994);
N. C. Tsamis and R. P. Woodard, “Strong infrared effects in quantum gravity,” Annals Phys. 238, 1 (1995);
N. C. Tsamis and R. P. Woodard, “The quantum gravitational back-reaction on inflation,” Annals Phys. 253, 1 (1997) [arXiv:hep-ph/9602316];
N. C. Tsamis and R. P. Woodard, “Quantum Gravity Slows Inflation,” Nucl. Phys. B 474, 235 (1996) [arXiv:hep-ph/9602315];
N. C. Tsamis and R. P. Woodard, “One Loop Graviton Self-Energy In A Locally De Sitter Background,” Phys. Rev. D 54, 2621 (1996) [arXiv:hep-ph/9602317];
N. C. Tsamis and R. P. Woodard, “A Phenomenological Model for the Early Universe,” Phys. Rev. D 80, 083512 (2009) [arXiv:0904.2368 [gr-qc]].

[7] W. Unruh, “Cosmological long wavelength perturbations,” astro-ph/9802323.
[8] G. Geshnizjani and R. Brandenberger, “Back reaction and local cosmological expansion rate,” Phys. Rev. D 66, 123507 (2002) [arXiv:gr-qc/0204074].

[9] F. Finelli, G. Marozzi, G. P. Vacca and G. Venturi, “Backreaction during inflation: A Physical gauge invariant formulation,” Phys. Rev. Lett. 106, 121304 (2011) [arXiv:1101.1051 [gr-qc]].

[10] L. R. Abramo and R. P. Woodard, “No one loop back-reaction in chaotic inflation,” Phys. Rev. D 65, 063515 (2002) [arXiv:astro-ph/0109272].

[11] M. Gasperini, G. Marozzi and G. Veneziano, “Gauge invariant averages for the cosmological backreaction,” JCAP 0903, 011 (2009) [arXiv:0901.1303 [gr-qc]].

[12] M. Gasperini, G. Marozzi and G. Veneziano, “A Covariant and gauge invariant formulation of the cosmological ‘backreaction’,” JCAP 1002, 009 (2010) [arXiv:0912.3244 [gr-qc]].

[13] T. Buchert, “On average properties of inhomogeneous fluids in general relativity. 1. Dust cosmologies,” Gen. Rel. Grav. 32, 105 (2000) [gr-qc/9906015].

[14] G. Marozzi and G. P. Vacca, “Isotropic Observers and the Inflationary Backreaction Problem,” Class. Quant. Grav. 29 (2012) 115007 [arXiv:1108.1363 [gr-qc]].

[15] M. Gasperini, G. Marozzi, F. Nugier and G. Veneziano, “Light-cone averaging in cosmology: Formalism and applications,” JCAP 1107, 008 (2011) [arXiv:1104.1167 [astro-ph.CO]].

[16] I. Ben-Dayan, M. Gasperini, G. Marozzi, F. Nugier and G. Veneziano, “Backreaction on the luminosity-redshift relation from gauge invariant light-cone averaging,” JCAP 1204, 036 (2012) [arXiv:1202.1247 [astro-ph.CO]]; I. Ben-Dayan, M. Gasperini, G. Marozzi, F. Nugier and G. Veneziano, “Do stochastic inhomogeneities affect dark-energy precision measurements?,” Phys. Rev. Lett. 110, 021301 (2013) [arXiv:1207.1286 [astro-ph.CO]]; I. Ben-Dayan, G. Marozzi, F. Nugier and G. Veneziano, “The second-order luminosity-redshift relation in a generic inhomogeneous cosmology,” JCAP 1211, 045 (2012) [arXiv:1209.4326 [astro-ph.CO]].

[17] W. Xue, K. Dasgupta and R. Brandenberger, “Cosmological UV/IR Divergences and de-Sitter Spacetime,” Phys. Rev. D 83, 083520 (2011) [arXiv:1103.0285 [hep-th]].

[18] F. Finelli, G. Marozzi, A. A. Starobinsky, G. P. Vacca and G. Venturi, “Stochastic growth of quantum fluctuations during slow-roll inflation,” Phys. Rev. D 82, 064020 (2010) [arXiv:1003.1327 [hep-th]].

[19] F. Finelli, G. Marozzi, G. P. Vacca and G. Venturi, “Energy momentum tensor of cosmological fluctuations during inflation,” Phys. Rev. D 69, 123508 (2004) [arXiv:gr-qc/0310086].

[20] M. Bruni, S. Matarrese, S. Mollerach and S. Sonego, “Perturbations of space-time: Gauge transformations and gauge invariance at second order and beyond,” Class. Quant. Grav. 14, 2585 (1997) [arXiv:gr-qc/9609040].

[21] G. Marozzi, “The cosmological backreaction: gauge (in)dependence, observers and scalars,” JCAP 1101, 012 (2011) [arXiv:1011.4921 [gr-qc]].

[22] F. Finelli, G. Marozzi, A. A. Starobinsky, G. P. Vacca and G. Venturi, “Generation of fluctuations during inflation: Comparison of stochastic and field-theoretic approaches,” Phys. Rev. D 79, 044007 (2009) [arXiv:0808.1786 [hep-th]].