Dynamic analysis on rotor-bearing system with coupling faults of crack and rub-impact

Zhiwei Huang
1
1China Ship Development and Design Center, Wuhan, China
E-mail: hzwhust@aliyun.com

Abstract. Rub-impact and fatigue crack are two important rotor faults. Based on the crack theory, an improved switching crack model is presented. Dynamic characteristics of a rotor-bearing system with imbalance, rub-impact and transverse crack are attempted. Various nonlinear dynamic phenomena are analyzed using numerical method. The results reveal that unstable form of the rotor system with coupling faults is extremely complex as the rotating speed increases and there are some low frequencies with large amplitude. The influence of crack depth and angle on the dynamic behavior of a cracked rotor is important insomuch as the quasi-periodical and chaotic motions in system response will occur along with different parameters. It is indicated that this study can contribute to a further understanding of the nonlinear dynamics of such a rotor system with coupling faults of crack and rub-impact.

1. Introduction
An ever-increasing pursuit of higher power and efficiency has lead to highly stressed condition of rotating machines. Rotors are becoming more flexible and operate under tight clearances and harsh environment. One or more faults are likely to creep in the system under such circumstances. For example, rotor could develop contact with the stator under tighter clearances or fatigue crack under severe thermal and mechanical stresses. Because of rub-impact and fatigue crack’s potential to cause catastrophic failures are a heavy threat to an uninterrupted operation and performance, and they are two important faults in rotating machines [1-2]. Thus, it is imperative to understand the nonlinear behavior of the rotor rub-impact and fatigue crack interaction in the design of the system.

Under fatigue load operating conditions, the stiffness and dynamic behavior of the rotor shaft will change greatly, which may generate huge failure and loss. Recently, Sabnavis [3] has reviewed the literature on cracked shaft detection and diagnosis appeared after 1990, grouping the literature under vibration-based methods, modal testing and non-traditional methods. Digital simulation of a rotating shaft with a transverse crack was investigated by Chan [4]. Zeng [5] and Sekhar [6] calculated stiffness of a simple rotor with a transverse crack and investigated the relationships between the stiffness and some parameters such as the crack location, the crack depth and the slenderness ratio. Not only cracks will cause fundamental frequency vibration, but also probably super-harmonic or sub-harmonic response, and even quasi-periodic or chaotic response [7]. Gómez-Mancilla [8] studied the influence of crack-imbalance orientation and orbital evolution for an extended crack Jeffcott rotor. It

1Zhiwei Huang, China Ship Development and Design Center, Wuhan, People’s Republic of China
results that orbital evolution around 1/2 and 1/3 of the first resonance could be used to detect rotor cracks, even if the crack-imbalance orientation was unknown. Many researchers [9-11] studied imbalance vibration response of the cracked rotors and found that along with increase in 1X frequency component, rotor crack induced periodic steady-state lateral vibrations at 2X and 3X frequency components due to breathing of the crack under gravity loading. On the other hand, investigation of the characteristics of rotor-stator rub-impact was also extensive. Considering the oil film force, Chu [12] has found that a rub-impact rotor system could exhibit very rich forms of motion, e.g. periodic, quasi-periodic and chaotic vibrations. The coupled transient torsional and lateral vibration of imbalanced rotors with rotor-to-stator rubbing has been analyzed by Al-Bedoor [13]. Dai [14] investigated the forced nonlinear vibration of a rotor rubbing with motion-limiting stops and found that the partial rubbing would expand to full rubbing in which the rotor kept contact with the stop with the increase of the amplitudes of the excitation. So the present study is to examine such a situation wherein the effect of three different disturbing functions, namely imbalance, rotor rub-impact and fatigue crack is simulated together. The rotor resulting vibration response due to rotor-stator rub exhibits characteristics typical of any nonlinear system. Presence of crack in rotor system also exhibits nonlinearities. Hence, it is essential to understand the exact nature of nonlinear vibration response of rotor with coupled faults and to understand its unique vibration signature.

The vibration in crack direction determines the breathing of the crack. In switching crack model, the crack can be considered as either fully open or fully closed when the crack depth is very small. However, partial opening and closing of the crack is present as the crack depth increases [5]. Based on the structure analysis and experimental research, there are three kinds of open-close crack models including the rectangular wave model, the cosine wave model and the synthesized model [15]. The Rectangular wave model reflects the basic features of the crack’s opening and closing wholly, but it does not reflect the transition process from open to close. Although the cosine wave model roughly reflects the process, it is basically applicable in the condition that the crack depth is equal to the radius. Synthesized model can reflect a more detailed process of the varying crack, but it doesn’t take the gravity load into account. Davies [16] applied the method measuring the shaft strain to confirm the response of the crack opening and closing under gravity in the experiment. Correlation between the regularity of crack’s opening and closing and crack depth was found. Therefore, in this investigation, an improved switching crack model that can express the transition is presented.

Most of the operating conditions for the rotor system and its structure are pretty complex. There are many fault factors, some of which will lead to same faults. On the other hand a factor sometimes can lead to multiple failures. The occurrence of a malfunction may cause or aggravate other failures. It can be said that rotor system failures are likely to coexist with a variety of faults, and these faults with each other making the system more complex. Multiple nonlinear factors coupling with others have increasingly become the focus of the fault diagnosis research.

In this paper, an improved switching crack model in a beam is presented, and the nonlinear dynamical behavior of a rotor-bearing system with coupling faults of transverse crack and rub-impact are investigated using a numerical method. Some parametric studies regarding the crack depth, crack angle and clearance are carried out in order to show their influence on the change in dynamic characteristics of the system by Poincaré maps, axis orbits, phase diagrams, time histories and amplitude spectrum and bifurcation diagrams. Various non-linear phenomena compressing periodic, quasi-periodic and chaotic motions are also investigated. The study results could provide important theory references to fault diagnoses, safety operating and vibration control in rotating machinery.

2. Mathematical model
To analyze the influence of the crack and rub-impact, an assumption is proposed that the torsional vibration of rotor is neglected and only the rotor transverse vibration is considered. Therefore, the rotor-bearing system can be modeled as a Jeffcott rotor system, in which the rotor is simplified to one disk with two transverse stiffness and the masses of the shaft equivalent to the disk and two bearings. The model of rotor-bearing system is shown in Fig. 1(a), where \( O_1 \) is the bearing center, \( O_2 \) is the disk...
geometric center, $O_1$ is the disk mass center, $F_s$ and $F_t$ are seal fluid force, $P_s$ and $P_t$ are dimensionless oil force, $m_1$ is the bearing mass, $m_2$ is the disk mass, $c$ is the clearance of bearing, $R$ is the radius of shaft, $a$ is the crack depth in middle of shaft, $k$ is the stiffness of the shaft without crack and $\delta$ is an initial clearance between the rotor and stator.


![Figure 1. A cracked rotor-bearing system](image_url)

**2.1. Governing equations**

In the model of the rotor-bearing system, four degrees of freedom-horizontal and vertical displacements of the rotor are taken into consideration at the disk location $(x_2, y_2)$ and at the journal $(x_1, y_1)$, correspondingly. If $X_1=x/c, Y_1=y/c, X_2=x_2/c$ and $Y_2=y_2/c$, then the dynamic equations of system can be established as follows:

\[
\begin{bmatrix}
\ddot{X}_1 \\
\ddot{Y}_1
\end{bmatrix}
+ \begin{bmatrix}
c_1/c_1m_1 & 0 \\
0 & c_2/c_2m_1
\end{bmatrix}
\begin{bmatrix}
\dot{X}_1 \\
\dot{Y}_1
\end{bmatrix}
+ \begin{bmatrix}
k_{xx} & k_{xy} \\
k_{yx} & k_{yy}
\end{bmatrix}
\begin{bmatrix}
X_1-X_2 \\
Y_1-Y_2
\end{bmatrix}
= \sigma/cm_1
\begin{bmatrix}
P_s(X_1, Y_1) \\
P_t(X_1, Y_1)
\end{bmatrix}
+ \frac{1}{c c_2}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\tag{1}
\]

\[
\begin{bmatrix}
\ddot{X}_2 \\
\ddot{Y}_2
\end{bmatrix}
+ \begin{bmatrix}
c_2/c_2m_2 & 0 \\
0 & c_1/c_1m_2
\end{bmatrix}
\begin{bmatrix}
\dot{X}_2 \\
\dot{Y}_2
\end{bmatrix}
+ \begin{bmatrix}
k_{xx} & k_{xy} \\
k_{yx} & k_{yy}
\end{bmatrix}
\begin{bmatrix}
X_2-X_1 \\
Y_2-Y_1
\end{bmatrix}
= \frac{e}{c}
\begin{bmatrix}
\cos \omega t \\
\sin \omega t
\end{bmatrix}
+ \frac{1}{c \alpha m_2}
\begin{bmatrix}
F_s(X_2, Y_2) \\
F_t(X_2, Y_2)
\end{bmatrix}
+ \frac{1}{c \alpha f}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\tag{2}
\]

where $c_1$ is damping coefficient of the bearing, $c_2$ is the viscous damping of the rotor disk, $e$ is the eccentricity of the rotor, $\omega$ is the angular speed of the rotor, $\sigma$ is Sommerfeld modifying parameter and $\sigma = \mu \omega R L / c (2R/c)^2$.

**2.2. Rub-impact force**

It is assumed that the heating effects caused by friction are ignored. Compared with one complete rotating period, the time during rub-impact is so short that the contact between the stator and rotor can be regarded as elastic impact. Under these assumptions, the rub-impact model is illustrated in Fig. 1(b). When rubbing happens, the radial impact force $F_n$ and the tangential rub force $F_t$ can thus be expressed [13] as:

\[
\begin{bmatrix}
F_n \\
F_t
\end{bmatrix}
= \begin{cases}
(r-\delta)k_r, & (x \geq \delta) \\
fF_n & 
\end{cases}
\tag{3}
\]

Where $k_r$ is the radial stiffness of the stator, $f$ is the friction coefficient between rotor and stator and $r = \sqrt{x^2 + y^2}/c$ is the radial displacement of the generator rotor. In $xoy$ reference frame, the force $F_s$ and $F_t$ can be written as
\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix}
= -H(r - \delta)(r - \delta)k_r \begin{bmatrix}
1 - f \\
f
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]

(4)

where \( H \), the Heaviside function, can be defined as

\[
H(x) = \begin{cases} 
0 & x < 0 \\
1 & x \geq 0 
\end{cases}
\]

Equation (4) indicates that when \( r < \delta \), there will be no rub-impact interaction and the rub-impact forces are zero. While \( r \geq \delta \), the rub-impact will happen.

2.3. Nonlinear oil-film force

Considering the nonlinear oil-film force model under the assumption of short bearing [12], a dynamic model of the nonlinear oil-film force is established for better accuracy and convergence. The model non-dimensional Reynolds equation is listed as follows:

\[
\frac{\partial}{\partial \tau} \left( \frac{h^3 \partial p}{\partial \theta} \right) + \left( \frac{R}{L} \right)^2 \frac{\partial^2}{\partial z^2} \left( \frac{h^3 \partial p}{\partial z} \right) = \frac{\partial h}{\partial \theta} + 2 \frac{\partial h}{\partial \tau}
\]

(5)

where \( R \) is the radius of bearing, \( L \) is the length of bearing, \( z \) is non-dimensional axial coordinate, \( h \) is non-dimensional oil-film thickness, \( p \) is non-dimensional oil-film pressure, \( \theta \) is the rotor angular, and \( \tau \) is non-dimensional time.

The non-dimensional oil-film pressure equation is given by Eq. (5) and can be written as follows:

\[
p = \frac{1}{2} \left( \frac{L}{D} \right)^2 \frac{(x - 2\dot{y}) \sin \theta - (y + 2\dot{x}) \cos \theta}{(1 - x \cos \theta - y \sin \theta)^2} (4\dot{z}^2 - 1)
\]

(6)

The non-dimension oil-film force is obtained from bearing theory and it can be expressed as [11]

\[
\begin{bmatrix}
P_x \\
P_y
\end{bmatrix}
= \begin{bmatrix}
\frac{(x - 2\dot{y})^2 + (y + 2\dot{x})^2}{1 - x^2 - y^2} [3xV(x, y, \alpha) - \sin \alpha G(x, y, \alpha) - 2 \cos \alpha S(x, y, \alpha)] \\
3yV(x, y, \alpha) + \cos \alpha G(x, y, \alpha) - 2 \sin \alpha S(x, y, \alpha)
\end{bmatrix}
\]

(7)

Where

\[
V(x_1, y_1, \alpha) = \frac{2 + (y \cos \alpha - x \sin \alpha)G(x_1, y_1, \alpha)}{1 - x^2 - y^2}
\]

\[
S(x_1, y_1, \alpha) = \frac{x \cos \alpha + y \sin \alpha}{1 - (x \cos \alpha + y \sin \alpha)^2}
\]

\[
G(x_1, y, \alpha) = \frac{2}{(1 - x^2 - y^2)^{1/2}} \left[ \frac{\pi}{2} + \arctg \frac{y \cos \alpha - x \sin \alpha}{(1 - x^2 - y^2)^{1/2}} \right]
\]

\[
\alpha = \arctan \frac{y + 2\dot{x}}{x - 2\dot{y}} - \frac{\pi}{2} \text{sgn} \left( \frac{y + 2\dot{x}}{x - 2\dot{y}} \right) - \frac{\pi}{2} \text{sgn}(y + 2\dot{x})
\]

2.4. Crack model

When a cracked rotor rotates slowly under gravity loading, the crack will open and close once per revolution. Therefore the stiffness matrix of the shaft in the disc position is non-linear and the periodic time varies during operation due to the crack effect. The periodic closing and opening of the crack is called “breathing” action. Its breathing motion can be described by the steering function \( f(\psi) \) related to the crack depth [5]. For small cracks when \( A < 0.5 \) (dimensionless \( A = a/R \)), Gash [17] considered the
simple hinge model as a good model. However, as the increase of crack depth, when $A \geq 0.5$, the model can accurately reflect the change of breathing crack by Mayes [16]. Then it is expressed as

$$f(\psi) = \begin{cases} 
1 + 2/3 \cos 3\psi + 2/5 \cos 5\psi & A < 0.5 \\
1/2 (1 + \cos \psi) & A \geq 0.5 
\end{cases}$$

(8)

Where the rotating angle satisfies $\psi = \omega t + \phi + \beta - \phi$, $\beta$ is the phase between the eccentricity and the center of the crack, i.e. crack angle and $\phi$ is expressed as

$$\phi = \begin{cases} 
\arctg(y/x) + 2n\pi & n = 0, \pm 1, \pm 2, \ldots \\
\arctg(y/x) + (2n + 1)\pi & n = 0, \pm 1, \pm 2, \ldots \\
\pi/2 + 2n\pi & n = 0, \pm 1, \pm 2, \ldots 
\end{cases}$$

(9)

From equation (8), it can be concluded that the switching crack model is not continuous for $A = 0.5$. And this model is not related with the depth of crack. So an improved continuous switching crack model taking the depth of crack in account is proposed in this paper, given by

$$f_k(\psi) = (1 - A) f_1(\psi) + Af_2(\psi)$$

(10)

From equation (10), we can find that while $A = 0$, $f_k(\psi) = f_1(\psi)$, and while $A = 1$, $f_k(\psi) = f_2(\psi)$. For a small $A$, the model of $f_1(\psi)$ is similar to $f_1(\psi)$. On the contrary, a large $A$ makes $f_k(\psi)$ is similar to $f_2(\psi)$. The relation of $f_k(\psi)$, $f_1(\psi)$ and $f_2(\psi)$ is shown in Fig. 2. Thus, the proposed model is more suitable to the rule of open and closed crack.

![Figure 2. The relation of $f_k(\psi)$, $f_1(\psi)$ and $f_2(\psi)$](image)

Since the exact model of the “breathing” crack is quite complicated, the variation of stiffness of the crack shaft in the rotating co-ordinate system is often considered in the form

$$\begin{bmatrix}
k_{xx} & k_{xy} \\
k_{yx} & k_{yy}
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\
0 & 1 
\end{bmatrix} - f(\psi) \times \begin{bmatrix}
k_\zeta \cos^2 \omega t + k_\eta \sin^2 \omega t & (k_\zeta - k_\eta) \sin \omega t \cos \omega t \\
(k_\zeta - k_\eta) \sin \omega t \cos \omega t & k_\zeta \sin^2 \omega t + k_\eta \cos^2 \omega t
\end{bmatrix}$$

(11)

Where $k_\zeta$ is the stiffness in $\zeta$ orientation in rotating co-ordinates, $k_\eta$ is the stiffness in $\eta$ orientation in rotating co-ordinates. The result of calculated $k_\zeta$ and $k_\eta$ by Zeng [5] at different dimensionless crack depth $A$ is shown in Table 1.

| $A$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $k_\zeta$ |  |  |  |  |  |  |  |  |  |  |
| $k_\eta$ |  |  |  |  |  |  |  |  |  |  |
3. Simulation results and analysis

The rotating speed is one of the primary parameters affecting the dynamic characteristics of a rotor system. Dynamic responses of the rotor-bearing system with coupling faults become more complicated when taking rotating speed as a control parameter and the nonlinear oil film force is combined with the nonlinear rub-impact force. Due to the strong non-linearity of the coupled system, numerical analysis is carried out by fourth-order Runge-Kutta method. Note that the time series data of the first 2000 revolutions of the rotor are deliberately excluded from the dynamic behavior investigation to ensure that the data used correspond to the steady state. The main partial parameters of numerical calculation are as follows [18]:

\[ m_1 = 4.0 \text{kg}, \quad m_2 = 32.1 \text{kg}, \quad c_1 = 1050 \text{N} \cdot \text{s} / \text{m}, \quad c_2 = 2100 \text{N} \cdot \text{s} / \text{m}, \quad e = 0.05 \text{mm}, \quad c = 0.11 \text{mm}, \quad R = 25 \text{mm}, \quad L = 12 \text{mm}, \quad \mu = 0.018 \text{Pa} \cdot \text{s}, \quad k = 2.5 \times 10^7 \text{N} / \text{m}, \quad k_r = 4.5 \times 10^7 \text{N} / \text{m}, \quad f = 0.1. \]

If the crack and rotor-stator rub are not taken into account, the natural frequency of the rotor system is obtained as \( \omega_0 = 882.5 \text{rad/s} \).

3.1. Effect of crack depth

The crack depth is one of the important factors affecting nonlinear dynamic characteristics of the coupled system. Fig. 3 presents the bifurcation diagram of the rotor system at different crack depth, in the rotating speed range \( \omega \in [100, 2100] \text{ rad/s} \), where \( \beta = \pi \) and \( \delta = 0.16 \text{mm} \). It exhibits synchronous motion with period-one, double periodic motion, quasi-periodic motion and chaotic motion. Compared with Fig. 3(a) ~ (d), it can be seen that the influence of crack depth surpasses the vibration of the rotor system. The unstable form of the rotor system is Hopf bifurcation when the depth of crack is smaller, for the reason that the nonlinear oil film force plays a dominant role. However, with the increase of the depth of crack, the dynamic responses of the rotor undergo great changes and the unstable form of the coupled system becomes period-doubling bifurcation. Two specific cases are now presented to do some in-depth study investigating of the behavior of the model derived above. One is the region for chaotic motion nearby \( \omega_0 \), the other is the region for quasi-periodic motion about \( 3 \omega_0 / 2 \).

As shown in Fig. 4(a), the Poincaré map and axis orbit indicate that the motion is chaotic on account of the fractal like structure of the attractor. The spectrum for the chaotic motion is not composed of discrete frequencies but displays a continuous, broad band, nature at the lower frequencies. However, with the increase of crack depth, number of the attractors for chaotic motion reduces and the structure of the attractors becomes compact until the motion becomes period-8, and the amplitudes become more and more large at 1/2 frequency component and disappears at some low-frequencies in Fig. 4(b)-(d). The chaotic motions are inhibited about critical speed as the crack depth increasing. At \( \omega = 1340 \text{rad/s} \), it can be observed that motions of the rotor exhibit double-periodic and quasi-periodic alternately due to rotor-stator rubbing. The responses of the rotor system also contain period-2, period-4, period-11 and quasi-periodic motions, as shown in Fig. 5. From which the Poincaré maps, axis orbits, time-histories and amplitude spectrum diagrams on the response of the rotor system for the change of dynamic characteristics at different crack depth are illustrated, respectively.
Figure 3. Bifurcation diagrams of the rotor response with $\omega$ as a control parameter for different crack depth: (a) $A=0.2$, (b) $A=0.4$, (c) $A=0.6$, (d) $A=0.8$. 
Figure 4. Poincaré maps, axis orbits, time-histories and amplitude spectrum diagrams of the rotor response at $\omega=900$rad/s when different crack depth: (a) $A=0.2$; (b) $A=0.4$; (c) $A=0.6$; (d) $A=0.8$.

Figure 5. Poincaré maps, axis orbits, time-histories and amplitude spectrum diagrams of the rotor response at $\omega=1340$rad/s for different crack depth: (a) $A=0.2$; (b) $A=0.4$; (c) $A=0.6$; (d) $A=0.8$.

3.2. Effect of crack angle
Crack angle is also important for the vibration of the cracked rotor system. During the operating process, a practical crack angle will be changed unavoidably due to the effects. Therefore, it is valuable to investigate various dynamic characteristics of the rotor-bearing system for different crack angle. Fig. 6 shows the bifurcation diagrams of the rotor response with $\omega$ as the control parameter, where $A=0.7$ and $\delta=0.16$mm. Fig. 7 illustrates the Poincaré maps, phase diagrams, time-histories and amplitude spectrum diagrams of the response of the rotor system for different crack angle and rotating speed. At $\beta=0$ and $\beta=\pi/4$, the bifurcation diagrams (Fig. 6(a)-(b)) of the rotor system are very similar to that of Fig. 3(a). It can be observed that for the rotor a sequence of Hopf bifurcations lead the system into chaotic regime, and then it experiences a sequence of reverse Hopf bifurcations. Fig. 6(c) shows the bifurcation diagram of $\omega$ on the response of the rotor system at $\beta=3\pi/4$. The bifurcation diagram of the rotor system around critical speed is divided to two islands caused by the crack. And motion of the rotor becomes period-13 from period-5 when leaving period-2, which is shown in Fig.
7(a). While the rotating speed is increased, the dynamic response of the rotor system becomes extremely complicated. Fig. 7(b) shows the dynamic responses of the rotor at \( \omega = 1900 \text{rad/s} \). All of these prove that the motion is quasi-periodic. At \( \beta = \pi \), as shown in Fig. 6(d), the bifurcation diagram of the rotor system in critical speed range is divided to four islands and there is a long region for period-3 in supercritical speed range due to the nonlinear oil film force and crack. Fig. 7(c) and (d) illustrate the Poincaré maps, phase diagrams, time-histories and amplitude spectrum diagrams on the response of the rotor system at \( \omega = 900 \text{rad/s} \) and \( \omega = 1850 \text{rad/s} \), respectively. From amplitude spectrum diagrams of the rotor in Fig. 7, it can be observed that the spectrums for the chaotic and quasi-periodic motions have some discrete nature at the lower frequencies and amplitudes are larger than that of 1X frequency component at some conditions.

![Figure 6. Bifurcation diagrams of the rotor response with \( \omega \) as a control parameter for different crack angle: (a) \( \beta = 0 \), (b) \( \beta = \pi/4 \), (c) \( \beta = 3\pi/4 \), (d) \( \beta = \pi \).]
Figure 7. Poincaré maps, phase diagrams, time-histories and amplitude spectrum diagrams of the rotor response for different crack angle and rotating speed: (a) $\beta=3\pi/4$ and $\omega=1300\text{rad/s}$; (b) $\beta=3\pi/4$ and $\omega=1900\text{rad/s}$; (c) $\beta=\pi$ and $\omega=900\text{rad/s}$; (d) $\beta=\pi$ and $\omega=1850\text{rad/s}$

4. Conclusions
Rotor rub-impact and fatigue crack are two important rotor faults. They have detrimental effects on health and reliability of the rotating machinery. This paper presents the case that multiple faults are considered together in a rotor-bearing system including imbalance, fatigue crack and rub-impact. According to opening and closing of the crack related to crack angle and depth, an improved switching crack model in a beam is presented. Especially the influence of the crack depth and crack angle, on the response of the coupled system is investigated. Numerical simulations are carried out and vibration characteristics of the rotor with different parameters are analyzed by Poincaré maps, axis orbits, phase diagrams, time histories, frequency spectra and bifurcation diagrams. The rotor response exhibits rich forms of periodic, double-periodic, quasi-periodic and chaotic motions. The research results show that the unstable form of the rotor system is Hopf bifurcation when the depth of crack is smaller. The influence on the response of the system increasing along with the depth of crack and the unstable form is period-doubling bifurcation. The critical rotation speed of the system decreases so that the rotor system is unstability. The effect of cracking angle on the dynamic behavior of a cracked rotor is important insomuch as the periodic and quasi-periodic motions in system response will occur along with different cracking angles in the vicinity of the supercritical speed. Additionally, the clearance has also an important impact on the stability of cracked rotor system.

References
1. Wan FY, Xu QY, Li ST. Vibration analysis of cracked rotor sliding bearing system with rotor-stator rubbing by harmonic wavelet transform. Journal of Sound and Vibration 2004; 271:507-518.
2. Patel TH, Darpe AK. Vibration response of a cracked rotor in presence of rotor-stator rub. Journal of Sound and Vibration 2008; 317:841-865.
3. Sabnavis G., Kirk RG, Kasarda M. Quinn D. Cracked shaft detection and diagnostics: a literature review. The Shock and Vibration Digest 2004; 36:287-296.
4. Chan RKC, Lai TC. Digital simulation of a rotating shaft with a transverse crack. Applied Mathematical Modeling 1995; 19:411-420.
5. Zeng F, Wu ZT. Calculating stiffness of a simple rotor with a transverse. Mechanical Science and Technology 1999; 18(5):745-747 (in Chinese).
6. Sekhar AS, Dey JK. Effects of cracks on rotor system instability. Mechanism and Machine Theory 2000; 35:1657-1674.
7. Fu YM, Zheng YF, Zhu SJ. Analysis of the chaotic motion for a rotor system with a transverse crack. Acta Mechanica Solid a Sinica 2003; 16(1):74-80(in Chinese).
8. Gómez-Mancilla J, Sinou JJ, Nosov VR, Thouverez F, Zambrano A. The influence of crack-
imbalance orientation and orbital evolution for an extended crack Jeffcott rotor. Comptes Rendus Mecanique 2004; 332(12):955-962.

9. Yang JD, Xu PM, Wen BC. Dynamic behavior of a flexible rotor with transverse crack. Journal of Vibration Engineering 2002; 15(1):93-97(in Chinese).

10. Zhou T, Sun ZC, Xu JX, Han WH. Experimental analysis of cracked rotor. Journal of Dynamical System, Measurements, and Control 2005; 127:313-320.

11. Sinou JJ, Lees AW. The influence of cracks in rotating shafts. Journal of Sound and Vibration 2005; 285:1015-1037.

12. Chu F, Zhang Z. Periodic, quasi-periodic and chaotic vibrations of a rub-impact rotor system supported on oil film bearings. International Journal of Engineering Science 1997; 35:963-973.

13. Al-Bedoor BO. Transient torsional and lateral vibrations of imbalanced rotors with rotor-to-stator rubbing. Journal of Sound and Vibration 2000; 229(3):627-645.

14. Farahmandian M, Sadeghi M H. Theoretical and Experimental Study on Vibration of Cracked Shafts Using Order Analysis. International Journal of Advanced Design and Manufacturing Technology, 2014; 7(2): 77.

15. Lin YL, Chu FL. Stiffness models for the cracked shaft of the rotor system. Chinese Journal of Mechanical Engineering 2008; 44(1):114-120(in Chinese).

16. Mayes IW, Davies WGR. Analysis of the response of a multi-rotor-bearing system containing a transverse crack in a rotor. Journal of Vibration Stress and Reliability in Design 1984; 106:183-192.

17. Dimarogonas AD, Paipetis SA, Chondros TG. Dynamics of cracked shafts. Springer Netherlands, 2013; 145-161.

18. Luo YG, Zhang SH, Wen BC. Study on nonlinear characteristics of rotor-bearing system with coupling faults of crack and rub-impact. Journal of Vibration and Shock 2005; 24(3):43-46(in Chinese)