Feasibility of the experimental study of $D_s^* \to \phi\pi$ decay

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Abstract

The current knowledge on the $D_s^*$ meson are very limited. Besides the dominant electromagnetic decays, the $D_s^*$ weak decays are legal and offer the valuable opportunities to explore the wanted $D_s^*$ meson. In this paper, the $D_s^* \to \phi\pi$ decay was studied with the factorization approach. It is found that the branching ratio $B(D_s^*\to\phi\pi) \sim O(10^{-7})$, which corresponds to several thousands of events at the $e^+e^-$ collider experiments including STCF, SuperKEKB, CEPC and FCC-ee, and several millions of events at the hadron collider experiments, such as LHCb@HL-LHC. It is feasible to experimentally study the $D_s^* \to \phi\pi$ weak decay in the future, even considering the identification efficiency.

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I. INTRODUCTION

The first evidence for the charmed strange mesons $D_s^*$ was observed in the exclusive reaction $e^+e^- \rightarrow F\bar{F}^*$ by the DASP collaboration in the year of 1977 [1], where the symbols of $F$ and $F^*$ were formerly used to denote the $D_s$ and $D_s^*$ particles, respectively. According to the $SU(4)$ quark model assignments, the vector mesons $D_s^*$ are assumed to have the same quark compositions as their twin pseudoscalar partners $D_s$. Both the $D_s^{*+}$ and $D_s^{++}$ mesons are consisting of a quark-antiquark pair $c\bar{s}$, and have the same additive quantum numbers of Charm, Strangeness and Charge, i.e., $C = S = Q = +1$. The different spin configurations of interquark potential make the mass of the ground spin-triplet $1^3S_1$ state for the $D_s^*$ mesons to be above that of the ground spin-singlet $1^1S_0$ state for the $D_s$ mesons [2].

Compared with the pseudoscalar meson $D_s$, the experimental information on the properties of the vector meson $D_s^*$ is still very limited by now [2]. Although there were many measurements of the mass of the $D_s^*$ meson (such as in Refs. [1, 3–9]), only one measurement was solemnly quoted by the Particle Data Group (PDG) until now [2]. The measurement was carried out by the Mark III collaboration in 1987 [6], thirty-five years ago. And the errors of the measurement of mass, $m_{D_s^*} = 2109.3 \pm 2.1 \pm 3.1$ MeV [6], are significantly larger than those of current values of the $D_s$ meson, $m_{D_s} = 1968.35 \pm 0.07$ MeV [2]. For the full width of the $D_s^*$ meson, only the upper limit was given by different experimental groups [2] and the latest and minimal upper limit on the decay width of the $D_s^*$ meson was given by the CLEO Collaboration in 1995 [9], twenty-seven years ago. The natural spin-parity of the $D_s^*$ meson was analyzed to be most likely $J^P = 1^-$ [9], but has not been unambiguously determined experimentally [2].

The experimental data on the $D_s^*$ mesons are accumulating increasingly. The quantitative study on the $D_s^*$ mesons is coming. Inspired by the potential prospects of high-luminosity-frontier flavor experiments, more and more data of the $D_s^*$ mesons will be available, so more accurate information and more detailed knowledge of the properties of the $D_s^*$ mesons will be accessible. In the $e^+e^-$ colliders, it is promisely expected that there will be a total of about $5 \times 10^{10}$ $c\bar{c}$ pairs at the SuperKEKB [10], about $10^{11}$ $c\bar{c}$ pairs from $10^{12}$ $Z^0$ boson decays at the Circular Electron Positron Collider (CEPC) [11], about $6 \times 10^{11}$ $c\bar{c}$ pairs from $5 \times 10^{12}$ $Z^0$ boson decays at the Future Circular Collider (FCC-ee) [12], where the branching fraction for the $Z^0$ boson decay into the $c\bar{c}$ pair is $\mathcal{B}(Z^0 \rightarrow c\bar{c}) = (12.03 \pm 0.21)\%$ [2]. Considering
the fraction of the charmed quark fragmenting into the $D_s^*$ meson $f(c \rightarrow D_s^*) \simeq 5.5\%$ [13],
these high statistical $c \bar{c}$ pairs correspond to some $6 \times 10^9$, $10^{10}$ and $6 \times 10^{10}$ $D_s^*$ mesons at
the SuperKEKB, CEPC and FCC-ee, respectively. In addition, about $10^{10} D_s^*$ mesons are
expected above the $\psi(4040)$ threshold (see Fig. 6 of Ref. [14]) at both the super $\tau$-charm
factory (STCF) in China [15] and the super charm-tau factory (SCTF) in Novosibirsk, Russia
[16], based on an integrated luminosity of $10 \text{ab}^{-1}$ [15]. In the high-energy hadron colliders,
about $4 \times 10^{13} D_s^*$ mesons [14] are expected to be obtainable with a data sample of target
luminosity $300 \text{fb}^{-1}$ at the LHCb@HL-LHC experiments [17], and more $D_s^*$ mesons will be
accumulated at ALICE and ATLAS [14]. The huge amount of experimental data provide
a tremendous foundation and valuable opportunities for studying and understanding the
properties of $D_s^*$ meson. A brilliant portrait of the characteristics of $D_s^*$ mesons is going to
be unfolded smoothly and completely.

The fit mass of $D_s^*$ meson is $m_{D_s^*} = 2112.2 \pm 0.4$ MeV [2], just below the mass threshold of
the $D S\bar{K}$ pair and above the mass threshold of the $D_s^*\pi$ pair and, i.e., the mass relations $m_{D_{u,d}} + m_K > m_{D_s^*} > m_{D_s} + m_\pi$. Thus the hadronic decays $D_s^* \rightarrow D S\bar{K}$ are strictly forbidden by the
law of conservation of energy. The hadronic decay $D_s^* \rightarrow D_s \pi$ is permissible kinematically,
but violates the the isospin conservation in the strong interactions. The absences of
decay modes induced by the strong interactions make the $D_s^*$ meson to be very narrow.
The natural width of the the $D_s^*$ meson is significantly less than the best experimental
resolution. Here, it should be noted that the $D_{s}^* \rightarrow D_{s}\pi$ decay is suppressed not only by the
phenomenological Okubo-Zweig-Iizuka (OZI) rule [22–24] but also by the extremely limited
phase spaces, due to $m_{D_{s}^*} - m_{D_{s}} - m_{\pi} < 6$ MeV. Thus the electromagnetic decay $D_{s}^* \rightarrow D_{s}\gamma$ is dominant, with the branching ratio $\mathcal{B}(D_{s^*} \rightarrow D_{s}\gamma) = (93.5 \pm 0.7)\%$ exceeding that of hadronic decay $\mathcal{B}(D_{s^*} \rightarrow D_{s}\pi) = (5.8 \pm 0.7)\%$ [2]. In addition, for the $D_{s}^* \rightarrow D_{s}\pi^0$, $D_{s}\gamma$
decays, the final photons are seriously polluted by those from bremsstrahlung radiation,
which will significantly affect the identification efficiency of the accident photon. Besides, the
$D_s^*$ meson can also decay via the weak interactions, although with a very small probability.

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a Within the chiral perturbative theory, it is usually taken for granted that the $D_{s}^* \rightarrow D_{s}\pi$ decay can also
decay through the strong interactions via the $\pi^0$-$\eta$ mixing by assuming a small isoscalar $\eta$ meson component
in the physical $\pi^0$ meson, because the $\eta$ meson can couple to the strange quark in the charmed strange
mesons [18–21].

b The neutral pion decay predominantly through $\pi^0 \rightarrow \gamma \gamma$ with a branching ratio of 98.8\% [2].
The weak decays of the $D_s^*$ meson provide another platform and opportunities to explore and understand the properties of the $D_s^*$ mesons. In this paper, we will evaluate the feasibility of experimentally investigating the $D_s^*$ meson through the weak decay $D_s^* \to \phi\pi$.

Theoretically, the charm-flavor-changing decay $D_s^{*-} \to \phi\pi^+$ is actually induced by the quark transition $c \to s + W^{*+}$ at the tree level in the standard model (SM) of elementary particles. Here, it is assumed that the vector $\phi$ meson consists of the pure $s\bar{s}$ quark pair with neither possible $u\bar{u}$ nor $d\bar{d}$ components, i.e., that the mixing between the $\phi$-$\omega$ system is ideal. Clearly, this decay mode is the Cabibbo-favored one and its amplitudes are proportional to the Cabibbo-Kobayashi-Maskawa (CKM) matrix [25, 26] element $|V_{cs}| \sim \mathcal{O}(1)$. This decay would have a relatively large branching ratio among the $D_s^*$ meson weak decays, and hence should have a high priority to be studied. In addition, the charm quark is somewhat massive and can be regarded as one bridge between the perturbative and nonperturbative regimes. The charm quark decays offer a laboratory to test various phenomenological models and study the behaviors of the strong interactions near the scale of $\mathcal{O}(m_c)$.

Experimentally, the curved tracks of the charged pion and kaon plunged into magnetic field will be unambiguously detectable by the highly sensitive detectors. So, the final states are easily identified for the $D_s^* \to \phi\pi$ decays, where $\phi$ and $\pi$ mesons with a definite momentum are back-to-back in the center-of-mass frame of the $D_s^*$ meson, and the $\phi$ meson can be well reconstructed from the kaon pairs. It is expected to have a higher signal-to-background ratio and a better identification efficiency, and have a big competitive advantage over both the pure leptonic decays $D_s^* \to \ell\bar{\nu}$ and semileptonic decays $D_s^* \to \phi\ell\bar{\nu}$ which suffer from the additional complications caused by the final neutrinos.

In this paper, we will study the $D_s^* \to \phi\pi$ decay within SM by using the phenomenological factorization approach [27], and estimate the branching ratio in order to provide a ready reference for future experimental analysis. This paper is organized as follows. The amplitudes for the $D_s^*$ decay in question using the factorization approximation is given in Sec. II. Branching ratio and event numbers of the $D_s^* \to \phi\pi$ decay are listed in Sec. III. Section IV devotes to a summary.
II. THE THEORETICAL FRAMEWORK

At the quark level, the effective Hamiltonian responsible for the nonleptonic decay $D^*_s \to \phi \pi$ can be written as [28],

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left\{ C_1 O_1 + C_2 O_1 \right\} + \text{h.c.},$$  \hspace{1cm} (1)

where the Fermi constant $G_F$ is the weak interaction coupling coefficient, $G_F \approx 1.166 \times 10^{-5}$ GeV$^{-2}$ [2]. $V_{cs}^* V_{ud}$ is the product of CKM matrix elements, which has been determined precisely by experiments, $|V_{ud}| = 0.97370(14)$ and $|V_{cs}| = 0.987(11)$ [2]. The Wilson coefficients $\vec{C} = \{C_1, C_2\}$ can be obtained with the renormalization group equation,

$$\vec{C}(\mu_c) = U_{4f}(\mu_f, \mu_i) M(m_b) U_{5f}(m_b, m_W) \vec{C}(m_W),$$  \hspace{1cm} (2)

where $\mu_c \sim O(m_c)$ is the scale for the charm quark decays. $m_b$ and $m_W$ are the mass of the bottom quark and the charged $W$ gauge boson, respectively. $U_{4f}(\mu_f, \mu_i)$ and $M(m_b)$ are the evolution matrix and threshold matching matrix, respectively. The expressions of $\vec{C}(m_W)$, $U_{4f}(\mu_f, \mu_i)$ and $M(m_b)$ can be found in Ref. [28]. The effective operators are defined as follows.

$$O_1 = \left[ \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\alpha \right] \left[ \bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\beta \right],$$  \hspace{1cm} (3)

$$O_2 = \left[ \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\beta \right] \left[ \bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\alpha \right],$$  \hspace{1cm} (4)

where $\alpha$ and $\beta$ are the color indices. Because the $D^*_s \to \phi \pi$ decay is an external $W$ emission process, there are only two current-current operator $O_{1,2}$ and without the penguin operators, and the contributions from new physics beyond SM to this decay are negligible.

The initial and final states are hadrons, while the operators are the specific combinations of four quarks. The influence of the long-distance strong interactions on the transitions between quarks and hadrons makes the predictions of nonleptonic decays notoriously difficult. To obtain the decay amplitudes for the $D^*_s \to \phi \pi$ decay, the remaining work is to evaluate the hadronic matrix elements (HMEs) $\langle \phi \pi | O_i | D^*_s \rangle$.

Phenomenologically, one of the most frequently used methods to deal with HME is the naive factorization (NF) approach [27]. The NF approach is based on the color transparency hypothesis [29] that a nearly collinear and relativistic light quark-antiquark pair originating from the heavy quark decays might be approximated as a color singlet before its
hadronization and complete separation from the interaction points. According to the color transparency hypothesis, it is possible to replace the product of the quark currents in the effective Hamiltonian of Eq.(1) by product of the corresponding hadron currents, and express the color singlet quark currents in terms of the participating hadron fields [30]. The outgoing light hadrons of two-body decays are back-to-back and energetic in the heavy quark limit, and fly away far from each other before the interference with the soft gluons. It may be a good approximation to neglect the final state interactions for the moment. In addition, the asymptotic freedom property of the strong interactions implies that the creation of quark pairs of high energy from the vacuum by hard virtual gluon is highly suppressed [31], i.e., it is believed that the \( W \)-annihilation amplitudes for the nonleptonic heavy-flavored hadron decays might be much smaller than the \( W \)-emission amplitudes.

Under the assumption of factorization, the decay amplitudes are written as,

\[
\mathcal{A}(D_s^* \to \phi \pi) = \langle \phi \pi | \mathcal{H}_{\text{eff}} | D_s^* \rangle
\]

\[
= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} a_1 \langle \phi \pi | (\bar{s} c)_H (\bar{u} d)_H | D_s^* \rangle
\]

\[
= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} a_1 \langle \pi | (\bar{u} d)_H | 0 \rangle \langle \phi | (\bar{s} c)_H | D_s^* \rangle,
\]

where \( (\bar{s} c)_H \) and \( (\bar{u} d)_H \) are the color singlet \( V-A \) hadron currents, and the subscript \( H \) is introduced to indicate the change to hadron currents and distinguish with quark currents of Eq.(3) and Eq.(4). The effects from the color exchanges are embodied into the coefficient \( a_1 = C_1 + \xi C_2 \). It is expected \( \xi = 1/N_c = 1/3 \) from color matching. \( \xi \) or \( a_1 \) sometimes is regarded as a parameter for different factorization approaches, because of the uncertain contributions of color octet current product and nonfactorizable contributions. The approximation of \( a_1 \approx 1.1 \) is frequently used in many phenomenological studies of nonleptonic decays for charmed hadron mesons, such as Refs. [30–41].

Using the parameterization for amplitude in Eq.(5), the decay widths can be given in terms of measurable physical HMEs. The HMEs of hadron currents in Eq.(5) are related to the decay constants and hadron transition form factors. The one-body HMEs are relevant to decay constants of hadrons,

\[
\langle 0 | \bar{d} \gamma_\mu u | \pi^+(p) \rangle = 0,
\]

\[
\langle 0 | \bar{d} \gamma_\mu \gamma_5 u | \pi^+(p) \rangle = i f_\pi p_\mu.
\]

The charged pion decay constant has been well determined from numerical lattice QCD.
simulations, $f_\pi = 130.2 \pm 1.2$ MeV (See Ref. [2] for a summary review). With the conventions of Refs. [42], the form factors are defined as,

$$
\langle \phi(\epsilon_2, p_2) | \bar{s} \gamma_\mu c | D_s^*(\epsilon_1, p_1) \rangle \\
= - (\epsilon_1 \cdot \epsilon_2^*) \left\{ P_\mu V_1(q^2) - q_\mu V_2(q^2) \right\} - (\epsilon_1 \cdot q) \epsilon_2^* \mu V_5(q^2) + (\epsilon_2 \cdot q) \epsilon_1 \cdot V_6(q^2) \\
+ \frac{(\epsilon_1 \cdot q) (\epsilon_2 \cdot q)}{m_{D_s^*}^2 - m_\phi^2} \left\{ [P_\mu - \frac{m_{D_s^*}^2 - m_\phi^2}{q^2} q_\mu] V_3(q^2) + \frac{m_{D_s^*}^2 - m_\phi^2}{q^2} q_\mu V_4(q^2) \right\},
$$

(8)

$$
\langle \phi(\epsilon_2, p_2) | \bar{s} \gamma_5 \gamma_\mu c | D_s^*(\epsilon_1, p_1) \rangle \\
= -i \varepsilon_{\mu \nu \alpha \beta} \epsilon_1^\alpha \epsilon_2^\beta \left\{ [P_\nu - \frac{m_{D_s^*}^2 - m_\phi^2}{q^2} q_\nu] A_1(q^2) + \frac{m_{D_s^*}^2 - m_\phi^2}{q^2} q_\nu A_2(q^2) \right\} \\
- i \frac{\varepsilon_{\mu \nu \alpha \beta} P_\mu q_\nu}{m_{D_s^*}^2 - m_\phi^2} \left\{ (\epsilon_2 \cdot q) \epsilon_1^\alpha A_3(q^2) - (\epsilon_1 \cdot q) \epsilon_2^\alpha A_4(q^2) \right\},
$$

(9)

where $\epsilon_i$ denotes the polarization vector of the vector mesons. The momentum $P = p_1 + p_2$ and $q = p_1 - p_2$. At the pole $q^2 = 0$, there is,

$$
V_3(0) = V_4(0),
$$

(10)

$$
A_1(0) = A_2(0).
$$

(11)

The values of formfactors for the $D_s^* \to \phi$ transition have been obtained with the light front approach [42], for example, $A_1(0) = 0.65$, $V_1(0) = 0.71$, $V_4(0) = 0.28$, $V_5(0) = 1.54$, and $V_6(0) = 0.86$.

Finally, the decay amplitude can be expressed by three invariant amplitudes. They are defined by the decomposition,

$$
\mathcal{A}(D_s^* \to \phi \pi) \\
= a \left( \epsilon_{D_s^*}^\alpha \epsilon_\phi^\beta \right) + \frac{b}{m_{D_s^*} m_\phi} (\epsilon_{D_s^*}^\alpha p_\pi) (\epsilon_\phi^\beta p_\pi) + \frac{c}{m_{D_s^*} m_\phi} \varepsilon_{\mu \nu \alpha \beta} \epsilon_\phi^\alpha \epsilon_{D_s^*}^\beta p_\pi^\mu (p_{D_s^*} + p_\phi)^\nu \\
= \epsilon_{D_s^*}^\alpha \epsilon_\phi^\beta \left\{ a g_{\alpha \beta} + \frac{b}{m_{D_s^*} m_\phi} p_{\pi, \alpha} p_{\pi, \beta} + \frac{c}{m_{D_s^*} m_\phi} \varepsilon_{\mu \nu \alpha \beta} p_\pi^\mu (p_{D_s^*} + p_\phi)^\nu \right\},
$$

(12)

and the invariant amplitudes $a$, $b$, and $c$ describe the $s$-, $d$-, and $p$-wave contributions.

$$
a = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi (m_{D_s^*}^2 - m_\phi^2) a_1 V_1(0),
$$

(13)

$$
b = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi m_{D_s^*} m_\phi a_1 \left\{ V_5(0) - V_6(0) - V_4(0) \right\},
$$

(14)

$$
c = - \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi m_{D_s^*} m_\phi a_1 A_1(0).
$$

(15)
In the rest frame of the $D_s^*$ meson, branching ratio is defined as,

$$
B(D_s^* \rightarrow \phi \pi) = \frac{1}{24 \pi} \frac{p_{\text{c.m.}}}{m_{D_s^*}^2 \Gamma_{D_s^*}} |\mathcal{A}(D_s^* \rightarrow \phi \pi)|^2
$$

$$
= \frac{1}{24 \pi} \frac{p_{\text{c.m.}}}{m_{D_s^*}^2 \Gamma_{D_s^*}} \left\{ |a|^2 (2 + x^2) + |b|^2 (x^2 - 1)^2 + |2 c|^2 2 (x^2 - 1) - 2 \text{Re}(a b^*) x (x^2 - 1) \right\},
$$

(16)

where the center-of-mass momentum of final states is of magnitude,

$$
p_{\text{c.m.}} = \frac{\lambda(x,y,z)}{2 m_{D_s^*}},
$$

(17)

the parameter $x$ is defined as,

$$
x = \frac{p_{D_s^*} \cdot p_{\phi}}{m_{D_s^*} m_{\phi}} = \frac{E_{\phi}}{m_{\phi}} = \frac{m_{D_s^*}^2 + m_{\phi}^2 - m_{\pi}^2}{2 m_{D_s^*} m_{\phi}},
$$

(18)

$$
\lambda(x,y,z) = x^2 + y^2 + z^2 - 2 x y - 2 y z - 2 z x,
$$

(19)

$$
p_{\text{c.m.}}^2 = m_{\phi}^2 (x^2 - 1).
$$

(20)

### III. NUMERICAL RESULTS AND DISCUSSION

The total decay width $\Gamma_{D_s^*} < 1.9$ MeV was set at the 90% confidence level by the CLEO collaboration in 1995 [9]. A quantitative and concrete result currently comes from theoretical estimations. Because of the lion’s share $B(D_s^* \rightarrow \gamma D_s) = (93.5 \pm 0.7)\%$ [2], an approximation $\Gamma_{D_s} \approx \Gamma(D_s^* \rightarrow \gamma D_s)$ is often used in theoretical calculation. The decay width for the magnetic dipole transition is [14],

$$
\Gamma(D_s^* \rightarrow D_s \gamma) = \frac{4}{3} \alpha_{\text{em}} k_{\gamma}^3 \mu_{D_s}^2 \approx 0.36 \text{ keV},
$$

(21)

where $\mu_{D_s \gamma}$ is the magnetic dipole moment and $k_{\gamma}$ is the momentum of photon.

Using Eq.(16), we can obtain branching ratio,

$$
B(D_s^* \rightarrow \phi \pi) \approx 2.4 \times \frac{0.36 \text{ keV}}{\Gamma_{D_s^*}} \times 10^{-7},
$$

(22)

and the corresponding partial decay width, $\Gamma(D_s^* \rightarrow \phi \pi) \approx 0.86 \times 10^{-13}$ GeV, is more than twice as large as the recent estimate using the QCD light cone sum rules in Ref. [43] where a relatively smaller coefficient $a_1 \approx 1.0$ is used.
We will make two comments on branching ratio. (1) There are many factors which influence the numerical results, such as the final state interactions. It is foreseeable that there will very large theoretical uncertainties. For example, using a much smaller decay width $\Gamma_{D_s^*} \approx 0.07$ keV from the lattice QCD simulations \[44\], branching ratio will be increased five times. Our focus is whether there is feasible to explore the $D_s^*$ meson via the $\phi\pi$ final states at the future experiments. A rough estimate rather than precise calculation on branching ratio is enough. (2) For the tree-dominated and color-favored nonleptonic heavy flavored meson decays arising from the external $W$ emission weak interaction, there is a consensus that NF approximation does hold and can give a reasonable and correct magnitude order estimation on branching ratio. In this sense, $B(D_s^*\to\phi\pi) \sim O(10^{-7})$ seems credible.

Based on the above analysis, it can be conclude that the $D_s^* \to \phi\pi$ decay should be measurable in the future experiments, such as STCF, SuperKEKB, CEPC, FCC-ee and LHCb. The potential event numbers of the $D_s^*$ mesons and the $D_s^* \to \phi\pi$ decays are listed in Table I. It is clearly seen from Table I that the natural properties of the $D_s^*$ meson can be investigated via the $D_s^* \to \phi\pi$ weak decays, particularly in the future FCC-ee and LHCb experiments.

### TABLE I: The potential event numbers of the $D_s^*$ meson available and the $D_s^* \to \phi\pi$ decays in the future experiments, with the branching ratio $B(Z^0 \to c\bar{c}) \approx 12\%$ [2] and $B(D_s^*\to\phi\pi) \approx 3 \times 10^{-7}$, the fragmentation fraction $f(c\to D_s^*) \approx 5.5\%$ [13] and the identification efficiency $\epsilon \sim 20\%$.

| experiment       | $N_{D_s^*}$ | $N_{D_s^*\to\phi\pi}$ | $\epsilon \times N_{D_s^*\to\phi\pi}$ | remarks                           |
|------------------|-------------|------------------------|--------------------------------------|----------------------------------|
| STCF [15, 16]    | $10^{10}$ [14] | 3000                   | 600                                  | with 10 $ab^{-1}$ data           |
| SuperKEKB [10]   | $5.5 \times 10^9$ | 1600                   | 300                                  | with $5 \times 10^{10}$ charm quark pairs |
| CEPC [11]        | $1.3 \times 10^{10}$ | 4000                   | 800                                  | from $10^{12}$ $Z^0$ boson decays |
| FCC-ee [12]      | $6.6 \times 10^{10}$ | $2 \times 10^4$       | 4000                                 | from $5 \times 10^{12}$ $Z^0$ boson decays |
| LHCb@HL-LHC [17] | $4 \times 10^{13}$ | $10^7$                 | $2 \times 10^6$                      | with 300 $fb^{-1}$ data          |

### IV. SUMMARY

Inspired by the inadequate understanding of the properties of $D_s^*$ meson, and the promisingly experimental prospects of investigating the $D_s^*$ meson in the future high-luminosity...
experiments, the $D_s^* \rightarrow \phi \pi$ decay was studied by using the NF approach within SM. The nonleptonic $D_s^* \rightarrow \phi \pi$ weak decay offers a fresh arena and a tempting opportunity to explore the wanted $D_s^*$ meson, although with a very tiny occurrence probability of $\sim \mathcal{O}(10^{-7})$. The final states of the $D_s^* \rightarrow \phi \pi$ decay have the relatively larger momenta than those of the predominant electromagnetic decays $D_s^* \rightarrow D_s^* \gamma$ and $\rightarrow D_s \pi$, and can be more easily identified by the sensitive detectors. It is found that several thousands of events for the $D_s^* \rightarrow \phi \pi$ decay are expected to be accessible at the STCF, SuperKEKB, CEPC and FCC-ee experiments, several millions of events at LHCb@HL-LHC experiments. It is practicable to experimentally study the $D_s^* \rightarrow \phi \pi$ weak decay in the future.

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