A simple and universal setup of quasimomocolor gamma ray source

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Abstract

Strict classic 3-D dynamics theory reveals that arbitrarily high center frequency light source can be achieved by flexible application of not-too-strong static electric field and static magnetic field. The magnitudes of the fields are not required to be high.

Even though significant application value has promoted intensive investigation on short-wavelength light source (or X-ray or higher energy photons source) for decades [1-14], there are still considerable difficulty in making a satisfactory achievement on this issue. Because 1 µm (1nm) wavelength radiation means the time cycle being 3.33fs (3.33as), we therefore wish the time cycle of electron motion to be 3.33fs (3.33as). Clearly, the difficulty is just that so rapid electron oscillation, which can be finished hundreds (or more) times per fs, is unavailable. This limits the efficiency of generating X-ray. For example, in the high-order harmonics generation (HHG), if the driving laser is of 1 µm wavelength, the driven atomic dipole moment oscillation is usually of a time cycle ~3.33fs. The HHG is only because the time shape of the driven atomic dipole moment oscillation differs from
\[
\sin \left(\frac{2\pi t}{3.33fs}\right) \text{greatly. This determines that low-order harmonics are dominant.}
\]

According to most HHG experimental results [7-14], low-order harmonics components whose orders are tens can be warranted but components at higher order hundreds are negligible. On the other hand, in free-electron-laser (FEL) [2-5], the efficiency of generating radiations at desired short wavelength is also far from satisfactory because sufficiently rapid electron velocity oscillation is unavailable.

Clearly, if we can directly set up a sufficiently rapid electron velocity oscillation, we will obtain an efficient monochromatic short-wavelength light source at desired short wavelength. If 1nm wavelength is desired, we should set up an electron velocity oscillation whose time cycle is \(\approx 3.33as\). To drive so rapid oscillation usually demands, according to current knowledge, electromagnetic field of very short wavelength, which is unavailable because they are just what we are pursuing. Namely, we are trying to get what we are pursuing by applying what we are pursuing.

We consider a simple configuration containing merely static electric field (along \(x\)-direction) and static magnetic field (along \(z\)-direction). When an electron is input into this configuration, its behavior can be described by dimensionless 3-D relativistic Newton equations (RNEs)

\[
d_s \left[ \Gamma d_s Z \right] = 0,
\]

\[
d_s \left[ \Gamma d_s Y \right] = W_B d_s X
\]

\[
d_s \left[ \Gamma d_s X \right] = -W_B \left[ \eta + d_s Y \right]
\]

\[
\frac{1}{\Gamma} = \sqrt{1 - (d_s X)^2 - (d_s Y)^2 - (d_s Z)^2}.
\]

Moreover, \(E_s\) and \(B_s\) are constant-valued electric field and magnetic one and meet \(E_s = \eta c B_s\), \(\lambda = c/\omega\) and \(\omega\) are reference wavelength and frequency, and \(s = \omega t\), \(Z = \frac{z}{\lambda}\), \(Y = \frac{y}{\lambda}\), \(X = \frac{x}{\lambda}\), \(W_B = \frac{\omega_B}{\omega}\), \(\omega_B = \frac{eB_s}{me}\) is the cyclotron frequency.

Eqs.(1-3) will lead to

\[
d_s Z \equiv 0
\]

\[
\Gamma d_s Y - W_B X = const = C_y;
\]

\[
\Gamma d_s X + W_B \left[ \eta s + Y \right] = const = C_x,
\]
where the values of these constants \( const \) are determined from the initial conditions \((X, Y, Z, d_sX, d_sY, d_sZ)\) \(|s=0=(0, 0, 0, \frac{C_y}{\sqrt{1+C_y^2+C_y^2}}, \frac{C_y}{\sqrt{1+C_y^2+C_y^2}}, 0)\).

Eqs.(5-7) can yield an equation set of \( d_sX \) and \( d_sY \)

\[
(d_sY)^2 = [C_y + W_B X]^2 \star [1 - (d_sX)^2 - (d_sY)^2] \quad (8)
\]

\[
(d_sX)^2 = [C_x - W_B \ast (\eta s + Y)]^2 \star [1 - (d_sX)^2 - (d_sY)^2] \quad (9)
\]

whose solution reads

\[
(d_sX)^2 = \frac{[C_x - W_B \ast (\eta s + Y)]^2}{[1 + [C_y + W_B X]^2 + [C_x - W_B \ast (\eta s + Y)]^2]} \quad (10)
\]

\[
(d_sY)^2 = \frac{[C_y + W_B X]^2}{[1 + [C_y + W_B X]^2 + [C_x - W_B \ast (\eta s + Y)]^2]} \quad (11)
\]

Note that the solutions (10,11) will cause \( \Gamma = \sqrt{1 + [C_y + W_B X]^2 + [C_x - W_B \ast (\eta s + Y)]^2} \) and, with the help of Eqs.(6,7), \( d_s \Gamma = -W_B \eta \ast d_s X \) (i.e. \( m_e c^2 d_s \Gamma = eE d_s X \)).

If noting \( \Gamma \) can be formally expressed as \( \Gamma = \sqrt{1 + C_y^2 + C_x^2 - W_B \eta \ast X} \), which agrees with Takeuchi’s theory [15], we can find that the electronic trajectory can be expressed as

\[
\left[\sqrt{1 + C_y^2 + C_x^2 - W_B \eta \ast X}\right]^2 = 1 + [C_y + W_B X]^2 + [C_x - W_B \ast (\eta s + Y)]^2 , \quad (12)
\]

or

\[
(1 - \eta^2) \left[ X + \frac{(\eta + \nu_y) \Gamma_0}{1 - \eta^2 \frac{W_B}{W_B}} \right]^2 + \left[ (Y + \eta s) - \nu_{x0} \frac{\Gamma_0}{W_B} \right]^2 = \frac{[(\eta + \nu_y)^2 + (1 - \eta^2) \nu_{x0}^2]}{1 - \eta^2} \left( \frac{\Gamma_0}{W_B} \right)^2 , \quad (13)
\]

where \( \Gamma_0 = \sqrt{1 + C_y^2 + C_x^2}, \nu_{x0} = \frac{C_x}{\Gamma_0} \) and \( \nu_{y0} = \frac{C_y}{\Gamma_0} \).

There will be an elliptical trajectory for \( \eta < 1 \) and a hyperbolic one for \( \eta > 1 \) [15,16]. The time for an electron travelling through an elliptical trajectory can be exactly calculated by re-writing Eq.(10) as [15]

\[
\pm ds = \frac{1}{W_B} \Gamma_0 - \eta \ast X \sqrt{aX^2 + bX + c} dX = \frac{\eta}{\sqrt{-a}} \frac{X_N - X}{\sqrt{b^2 - 4ac^2}} (X + \frac{b}{2a})^2 dX , \quad (14)
\]
where $a = (\eta^2 - 1)$, $b = -2[\eta \Gamma_0 - C_y]\frac{1}{W_B}$, $c = C_x^2 \left( \frac{1}{W_B} \right)^2$ and $X_N = \frac{1 - \frac{1}{\eta} \Gamma_0}{\eta}$. The equation can be written as a more general form

$$\pm \, ds = \frac{M - u}{\sqrt{1 - u^2}} \, du$$

(15)

where $u = \frac{X + \frac{b}{2a}}{\sqrt{\frac{b^2 - 4ac}{4a^2}}}$, $X_L = \min(\frac{-b - \sqrt{b^2 - 4ac}}{2a}, \frac{b + \sqrt{b^2 - 4ac}}{2a})$ and $X_R = \max(\frac{-b + \sqrt{b^2 - 4ac}}{2a}, \frac{b + \sqrt{b^2 - 4ac}}{2a})$. In addition, $\sigma = \sqrt{\frac{a}{\eta}} \sqrt{\frac{b^2 - 4ac}{4a^2}}$ and $M = \frac{X_N + \frac{b}{2a}}{\sqrt{\frac{b^2 - 4ac}{4a^2}}}$.

It is easy to verify that for $\eta^2 - 1 < 0$, there is $M = \frac{1 + \eta v_{y0}}{\eta \sqrt{((1 + \eta^2) + (1 - \eta^2)v_{x0}^2)}} > 1$. Initially, $(X, Y)|_{s=0} = (0, 0)$ and hence

$$u_{st} = u|_{s=0} = \frac{0 + \frac{b}{2a}}{\sqrt{\frac{b^2 - 4ac}{4a^2}}} = \frac{-\eta v_{x0}}{\sqrt{(1 + \eta^2)v_{x0}^2}}. \quad \text{From strict solution}$$

$$\pm \, s(u) = \sigma \left\{ M * \arcsin(u) + \sqrt{1 - u^2} \right\} + \text{const},$$

(16)

we can find the time for an electron travelling through an elliptical trajectory to meet $s_{cycle} = \omega T_c = 2 \star [\sigma M \pi]$ and hence a time cycle $T_c = \frac{2\pi}{\omega B \sqrt{1 - \eta^2}}$. Namely, the oscillation along the elliptical trajectory will have a circular frequency $\leq \omega_B$. Moreover, it is interest to note that $(v_{x0}, v_{y0}) = (0, -\eta)$ will lead to $\frac{[(1 + \eta^2)v_{x0}]^2 + (1 - \eta^2)v_{y0}^2}{1 - \eta^2} = 0$ and hence a straight-line trajectory $(X(s), Y(s)) = (0, -\eta s)$.

The motion on an elliptical trajectory is very inhomogeneous. The time for finishing the $\eta X > 0$ half might be very short while that for the $\eta X < 0$ half might be very long. We term two halves as fast-half and slow-half respectively. If $\eta$ is fixed over whole space, a fast-half is always linked with a slow-half and hence makes the time cycle for finishing the whole trajectory being at considerable level.

For convenience, our discussion is based on the parameterized ellipse. For the case $(v_{x0} = 0, v_{y0} = -\eta - \Delta)$, (where $\Delta$ is small-valued positive), the starting position $X = 0$ is the left extreme of the ellipse and hence corresponds to $u = -1$. The time required for an acute-angled rotation from $u = -1$ to $u = -1 + \xi$, (where $\xi$ is small-valued positive), will be $\sigma M \left\{ \arcsin(-1 + \xi) - \frac{\pi}{2} \right\} + \sigma \sqrt{2\xi - \xi^2}$, which is $= 0$ if $\xi = 0$. 

4
It is interesting to note that if there is \( B = 0 \) at the region \( u > -1 + \xi \), the electron will enter from \( (E, B = \frac{E}{\eta c}) \)-region into \( (E, B = 0) \)-region with an initial velocity whose \( x \)-component is \( v_x = \frac{1}{\sigma} \sqrt{\frac{1}{M} - \left(\frac{-1 + \xi}{\sqrt{1 - (E_c B)^2}}\right)^2} > 0 \) and \( y \)-component \( v_y \) is \( \neq 0 \). Then, the electron will enter into the \( (E, B = 0) \)-region a distance because of \( v_x > 0 \). After a time \( T_{tr} = \frac{2\sqrt{1 - v_x^2 / v_y}}{E \sqrt{1 - (E_c B)^2}} \), the electron will return into the \( (E, B = \frac{E}{\eta c}) \)-region and the returning velocity will have a \( x \)-component \( -v_x \). During this stage, the electron will move \( v_y T_{tr} \) along the \( y \)-direction. Then, the motion in the \( (E, B = \frac{E}{\eta c}) \)-region can be described by an acute-angled rotation along the ellipse \( u = -1 + \xi \rightarrow u = -1 \). Thus, a complete closed cycle along the \( x \)-direction is finished even though the motion along the \( y \)-direction is not closed. Repeating this closed cycle will lead to an oscillation along the \( x \)-direction.

Clearly, the time cycle of such an oscillation is \( T_x = T_{tr} + 2\sigma M \left[ \arcsin \left( -1 + \xi \right) - \frac{\pi}{2} \right] + 2\sigma \sqrt{2\xi - \xi^2} \). Under fixed values of \( \Delta, E \) and \( B \), the smaller \( \xi \) is, the smaller \( T_x \) is. There will be \( T_x = 0 \) at \( \xi = 0 \). In principle, arbitrary value of \( T_x < T_c \) can be achieved by choosing suitable value of \( \xi \). Namely, arbitrary high center frequency (> \( \omega_B \)) oscillation can be achieved by choosing suitable value of \( \xi \). Although the time history of \( x(t) \) might cause its Fourier spectrum being of some spread, the center frequency will be \( \frac{1}{T_x} \).

This result implies a simple and universal method of setting up quasi-monocolor light source at any desirable center wavelength. That is, applying vertically static electric field \( E = E_x \) and static magnetic field \( B = B_z \) and on purpose letting a \( B = 0 \) region existing and the ratio \( \frac{E}{cB} < 1 \), then injecting electron along the \( y \)-axis with a velocity slightly above \( \frac{|E|}{cB} \) and close to the boundary line between the \( B = 0 \) region and the \( B \neq 0 \) region. As shown in Fig.1, adjusting the distance \( D = \xi \sqrt{\frac{\nu^2 - 4\alpha c^2}{4\alpha^2}} \) can lead to a quasi-monocolor oscillation source with any desired center frequency up to \( \gamma \)-ray level.

In conclusion, we have described a simple and universal method of achieving quasi-monocolor light source at any desirable center wavelength. The kernel of this method is to utilize the motion of fast electron near the surface of magnetic field. Here, fast electron means its velocity being \( \frac{E}{cB} \). Actually, it is to adopt the fast-half of an ellipse and replace the time-consuming slow-half with a faster orbit governed by \( E \) only.
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Figure Caption

Fig.1. The sketch of experimental setup.
$E$ is rightward, $B$ is inward to paper, the dashed line is the boundary of $B=0$ region. The unperturbed electron trajectory is arranged to be of a distance $D$ from the boundary line.