Quasirotational disturbances of linear string baryon configuration

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Abstract

For the linear string baryon model \( q-q-q \) the small disturbances of its rotational motion (quasirotational states) are investigated. The spectrum of these states is obtained in the form of Fourier series and the complex eigenfrequencies are found in this spectrum. So the classic rotational motions of the linear string baryon model are unstable (unlike the similar motions for the string with massive ends). This instability differs from its analog for the three-string baryon model.

Introduction

The string model of the meson is obvious from geometric point of view — this is the relativistic string with massive ends [1]. But for the baryon we have to choose between the following four string models (four types of binding three quarks by relativistic strings) suggested by X. Artru in Ref. [2]: a) the meson-like quark-diquark model \( qqq \); b) the “three-string” model or \( Y \) configuration with three strings from three quarks joined in the fourth massless point [4]; c) the “triangle” model or \( \Delta \)-configuration with pairwise connection of three quarks by three relativistic strings [5]; d) the linear configuration \( q-q-q \) with quarks connected in series [6].

Here the latter model is considered (in comparison with some others). It was not studied quantitatively before Ref. [6] where the initial-boundary value problem for classical motion of this configuration were solved and the stability problem for the rotational motion of this system was investigated. This motion is the uniform rotation of the rectilinear string with the middle quark at rest at a center of rotation [3, 7]). Numerical experiments in Ref. [6] showed that the rotational motions of the system \( q-q-q \) are unstable. Any small asymmetric disturbances grow and result in centrifugal moving away the middle material point (quark) and its complicated motion with quasi-periodical varying of the distance between the nearest two quarks. But the system \( q-q-q \) is not transformed into the quark-diquark \( (q-qq) \) one, as was supposed previously in Ref. [3].

In this paper for the system \( q-q-q \) the result of the numerical experiments in Ref. [6] is proved analytically. For this purpose the spectrum of quasirotational states (small disturbances of the rotational motion) is obtained and compared with the similar spectrum for the string with massive ends.

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After the brief review the classical dynamics for the model $q$-$q$-$q$ in Sect. 1 we consider in Sect. 2 the quasirotational states of the relativistic string with massive ends (the model $q$-$q$ or $q$-$qq$) and then in Sect. 3 the similar states of the linear string baryon model $q$-$q$-$q$.

1. Dynamics and rotational motions of the linear string model $q$-$q$-$q$

Let’s consider an open relativistic string with the tension $\gamma$ carrying three pointlike masses $m_1$, $m_2$, $m_3$ (the masses $m_1$ and $m_3$ are at the ends of the string). The action for this system is

$$S = \int_{\tau_1}^{\tau_2} d\tau \left\{ \int_{\sigma_1(\tau)}^{\sigma_3(\tau)} \left[ (\dot{X}^a X^a - \dot{X}^a X'^a)^{1/2} d\sigma + \sum_{i=1}^{3} m_i \sqrt{\dot{x}_i^2(\tau)} \right] \right\}. $$

Here $X^\mu(\tau,\sigma)$ are coordinates of a point of the string in $D$-dimensional Minkowski space $R^{1, D-1}$ with signature $(+, - , - , \ldots)$, the speed of light $c = 1$, $(\tau, \sigma) \in G = G_1 \cup G_2$ (Fig. 1), $(a, b) = a^\mu b_\mu$ is the (pseudo)scalar product, $\dot{X}^\mu = \partial_\tau X^\mu$, $X'^\mu = \partial_\sigma X^\mu$, $\dot{x}_i^2(\tau) = \frac{d^2}{d\tau^2} X^\mu(\tau, \sigma_i(\tau))$; $\sigma_i(\tau)$ ($i = 1, 2, 3$) are inner coordinates of world lines for quarks shown in Fig. 1. If $m_2 = 0$ the action (1) will describe the relativistic string with massive ends.

Figure 1: Domain of integration in Eq. (1).

The equations of motion of the $q$-$q$-$q$ string and the boundary conditions are derived from the action (1). They take the simplest form if with the help of nondegenerate reparametrization $\tau = \tau(\tilde{\tau}, \tilde{\sigma})$, $\sigma = \sigma(\tilde{\tau}, \tilde{\sigma})$ the induced metric on the world surface of the string is made continuous and conformally-flat, i.e., satisfies the orthonormality conditions

$$\dot{X}^2 + X'^2 = 0, \quad (\dot{X}, X') = 0. $$

Under conditions (2) the equations of motion become linear

$$\ddot{X}^\mu - X''^\mu = 0 $$

and the boundary conditions take the simplest form

$$m_i \frac{d}{d\tau} U_i^\mu(\tau) + \epsilon_i \gamma [X'^\mu + \sigma'_i(\tau) \dot{X}^\mu] \bigg|_{\sigma = \sigma_i(\tau)} = 0, \quad i = 1, 3, $$

$$m_2 \frac{d}{d\tau} U_2^\mu(\tau) - \gamma [X'^\mu + \sigma'_2(\tau) \dot{X}^\mu] \bigg|_{\sigma = \sigma_2(\tau) + 0} + \gamma (X'^\mu + \sigma'_2(\tau) \dot{X}^\mu) \bigg|_{\sigma = \sigma_2(\tau) - 0} = 0. $$

1 We use the term “quark” for brevity, here and below quarks, antiquarks and diquarks are material points on the classic level.
Here $\epsilon_1 = -1$, $\epsilon_3 = 1$ and

$$U_i^\mu(\tau) = \frac{i^\mu(\tau)}{\sqrt{\dot{a}_i^2(\tau)}} = \frac{\dot{X}^\mu + \sigma'_i(\tau) X'^\mu}{\sqrt{X^2 \cdot (1 - \sigma'^2_i)}}|_{\sigma = \sigma_i(\tau)}, \quad i = 1, 2, 3 \quad (6)$$

are the unit $R^{1, D - 1}$-velocity vector of $i$-th quark.

Derivatives of $X^\mu(\tau, \sigma)$ can have discontinuities on the line $\sigma = \sigma_2(\tau)$. However, the function $X^\mu(\tau, \sigma)$ and the tangential derivatives $\frac{d}{d\tau} X^\mu(\tau, \sigma_2(\tau))$ are continuous. In Ref. [6] we showed that without loss of generality one can choose the coordinates $\tau$, $\sigma$ satisfying both the orthonormality conditions (2) and the following restrictions for the endpoints’ inner equations:

$$\sigma_1(\tau) = 0, \quad \sigma_3(\tau) = \pi \implies \sigma \in [0, \pi]. \quad (7)$$

But we can’t fix the remaining function $\sigma = \sigma_2(\tau)$ for the middle quark in general if the conditions (2) and (3) are satisfied.

However, the rotational motion of this configuration (uniform rotation of the rectilinear string segment with the middle quark at the rotational center) is the well known exact solution of equations (2) satisfying all conditions (4) – (7). This solution may be presented in the form

$$X^\mu = X^\mu_{rot}(\tau, \sigma) = \Omega^{-1}[\theta \tau e^\mu_0 + \cos(\theta \sigma + \phi_1) \cdot e^\mu(\tau)], \quad \sigma \in [0, \pi], \quad (8)$$

Here $\Omega$ is the angular velocity, $e^\mu_0$ is the unit time-like velocity vector of c.m. in Minkowski space, $e^\mu(\tau) = e^\mu_1 \cos \theta \tau + e^\mu_2 \sin \theta \tau$ is the unit ($e^2 = -1$) space-like rotating vector directed along the string. The parameters $\theta$ (dimensionless frequency) and $\phi_1$ are connected with the constant speeds $v_i$ of the ends

$$v_1 = \cos \phi_1, \quad v_3 = -\cos(\pi \theta + \phi_1), \quad m_i \Omega / \gamma = v_i^{-1} - v_i, \quad i = 1, 3. \quad (9)$$

The central massive point of the $q-q-q$ system is at rest (in the corresponding frame of reference) at the rotational center. Its inner coordinate is

$$\sigma_2(\tau) = \sigma_2^{rot} = (\pi / 2 - \phi_1) / \theta = \text{const.} \quad (10)$$

In the following two chapters we’ll study small disturbances of the rotational motion (8) (quasirotational states). But before the analysis of the complicated linear string baryon model $q$-$q$-$q$ in Sect. 3 we’ll consider in the following section the more simple system — the relativistic string with massive ends $q$-$q'$.

2. Quasirotational motions of the string with massive ends

The quasirotational states of various string hadron models [7, 8] are interesting due to the following reasons: (a) we are to search the motions describing the hadron states, which are usually interpreted as higher radially excited states and other states in the potential models, in other words, we are to describe the daughter Regge trajectories; (b) the quasirotational states are the basis for quantization of these nonlinear problems in the linear vicinity of the solutions (8), (if they are stable); (c) the quasirotational motions are necessary for solving the important problem of stability of rotational states for all string hadron models.

For the meson string model the quasirotational motions of slightly curved string with massive ends were studied in Refs. [9]. But these authors used very narrow ansatz for searching
these disturbances and the complicated nonlinear form of the string motion equations beyond the conditions (2). Besides they neglected some important dependencies and the boundary conditions (3) so these solutions in Refs. [4] were not correct (details are in Ref. [8]).

In Ref. [8] another approach for obtaining the quasirotational solutions was suggested. It includes the orthonormality conditions (2) and, hence, the linear equations of motion (3) with their general solution

\[ X^\mu(\tau, \sigma) = \frac{1}{2}[\Psi^\mu_+(\tau + \sigma) + \Psi^\mu_-(\tau - \sigma)]. \]  

(11)

So the problem is reduced to the system of ordinary differential equations resulting from the boundary conditions (2). The unknown function may be \( \Psi^\mu_+(\tau), \Psi^\mu_-(\tau) \), or unit velocity vectors of the endpoints \( U^\mu_1(\tau) \) or \( U^\mu_2(\tau) \) of the string with massive ends — this is equivalent due to the relations (11)

\[ \Psi^\mu_+(\tau \pm \sigma_i) = m_i \gamma^{-1}[\sqrt{-U^\mu_2(\tau)U^\mu_1(\tau)} \mp (-1)^i U^\mu_i(\tau)]. \]  

(12)

The expression (8) describes the rotational motion not only for the baryon \( q\bar{q}q \) model but also for the relativistic string with massive ends (where \( \sigma_2 = \pi \) and \( v_3 \) in Eq. (9) should be substituted by \( v_2 \)). In Ref. [8] the boundary conditions (3) for this model with one infinitely heavy (fixed) end with \( m_2 \to \infty \) on the basis of relations (11), (12) were reduced to the ordinary differential equation with respect to the vector function \( U^\mu_1(\tau) \).

For the case with two non-zero finite masses \( 0 < m_i < \infty \) the generalization of this equation resulting from Eqs. (11), (12) takes the form

\[
\begin{align*}
U^\mu_1(\tau) &= m_2 m_1^{-1}[\delta^\mu_\nu - U^\mu(\tau) U_{1\nu}(\tau)] \sqrt{-U^\mu_2(\tau - \pi) U^\mu_1(\tau - \pi) - U^\mu_2(\tau - \pi)}, \\
U^\mu_2(\tau) &= m_1 m_2^{-1}[\delta^\mu_\nu - U^\mu(\tau) U_{2\nu}(\tau)] \sqrt{-U^\mu_1(\tau - \pi) U^\mu_1(\tau - \pi) - U^\mu_2(\tau - \pi)},
\end{align*}
\]  

(13)

where \( \delta^\mu_\nu = \begin{cases} 1, & \mu = \nu, \\
0, & \mu \neq \nu. \end{cases} \) This system of ordinary differential equations with shifted arguments exhaustively describes the classic dynamics of the string with massive ends.

If the vector-function \( U^\mu_1(\tau) \) (or \( U^\mu_2(\tau) \)) is given in the segment \( I = [\tau_0, \tau_0 + 2\pi] \) (the values \( \gamma/m_i, U^\mu_1(\tau_0 + \pi) \) are also given) one can determine the functions \( U^\mu_1(\tau) \) for \( \tau > \tau_0 \) from the system (13). Then we may obtain the world surface \( X^\mu(\tau, \sigma) \) with the help of the relations (14) and (11). So we may conclude that the function \( U^\mu_1(\tau) \) or \( U^\mu_2(\tau) \) given in the segment \( I \) contains all information about this motion of the system [8].

For the rotational motion (8) the velocities \( U^\mu_1 \) of the moving quark satisfying Eqs. (13) may be written in the form

\[
U^\mu_1 = U^\mu_{1(\mathrm{rot})}(\tau) = \Gamma_1[e^\mu_0 + v_1 \hat{e}^\mu(\tau)], \quad U^\mu_2 = U^\mu_{2(\mathrm{rot})} = \Gamma_2[e^\mu_0 - v_2 \hat{e}^\mu(\tau)], \quad \Gamma_i = (1 - v_i^2)^{-1/2}. \]  

(14)

Here the unit space-like rotating vectors \( \hat{e}^\mu \) and \( e^\mu \)

\[
e^\mu(\tau) = e^\mu_0 \cos \theta \tau + e^\mu_1 \sin \theta \tau, \quad \hat{e}^\mu = \theta^{-1} \frac{d}{d\tau} e^\mu(\tau) = -e^\mu_1 \sin \theta \tau + e^\mu_2 \cos \theta \tau \]  

(15)

consist the moving basis in the rotational plane. The four vectors \( e^\mu_0, e^\mu(\tau), \hat{e}^\mu(\tau), e^\mu_3 \) will be used below as the orthonormal tetrad in the Minkowski space \( R^{1,3} \).

To study the small disturbances of the rotational motion (8) we consider arbitrary small disturbances of this motion or of the vector (14) in the form

\[
U^\mu_1(\tau) = U^\mu_{1(\mathrm{rot})}(\tau) + u^\mu_1(\tau), \quad |u^\mu_1| \ll 1. \]  

(16)
For the exhaustive description of this quasirotational state the disturbance \( u_\mu^\nu(\tau) \) may be given in the initial segment \( I = [\tau_0, \tau_0 + 2\pi] \). It is small so we neglect in the linear approximation the second order terms. The equality \( U_i^\mu(\tau) = 1 \) for both vectors \( U_i^\mu \) and \( U_{i(\text{rot})}^\mu \) leads in the linear approximation to the condition

\[
U_{i(\text{rot})}^\mu(\tau) u_{i\mu}(\tau) = 0. \tag{17}
\]

When we substitute the expressions (14) into the system (13) and omit the second order terms we obtain the linearized system of equations describing the evolution of small arbitrary disturbances \( u_{i\mu}^\nu \). Considering projections of these two vector equations onto the basic vectors \( e_0, e, \bar{e}, e_3 \), we obtain the following system of equations with respect to projections of \( u_{i\mu}^\nu \):

\[
\begin{align*}
\tau_{10} u_{10}(\tau) &+ Q_1 u_{1e}(\tau) - \bar{e} Q_1 u_{1e}(\tau) = M_0[u'_{20} - Q_2 u_{20} + \Gamma Q_2 u_{2e}], \\
u_{1e}(\tau) + Q_1 u_{1e}(\tau) + \theta \nu_{1e}(\tau) = M_1^{-1}[- u'_{2e} - Q_1 u_{2e} + N_1^* u'_{20} + N_2 u_{20}], \\
u_{20} + Q_2 u_{20} + \Gamma Q_2 u_{2e} = M_0^{-1}u_{10}(\tau) - Q_1 u_{10}(\tau) - \Gamma Q_1 u_{1e}(\tau) - Q_2 u_{2e}, \\
u_{2e} + Q_2 u_{2e} - \theta \nu_{2e} - u_{10}(\tau) = -M_1[- u'_{1e}(\tau) - Q_2 u_{1e}(\tau) + N_1^* u'_{10}(\tau) - N_1 u_{10}(\tau)], \\
u_{1e}(\tau) + Q_1 u_{10}(\tau) + \bar{e} Q_1 u_{10}(\tau) = (m_1/m_2)[u'_{2e} + Q_2 u_{2e}], \\
u_{2e} + Q_2 u_{2e} = (m_1/m_2)[u'_{1e}(\tau) + Q_1 u_{1e}(\tau)].
\end{align*} \tag{18}
\]

Here \( Q_i = \Gamma_i \theta v_i = \text{const}, \ (\tau - \pi) \equiv (\tau - 2\pi) \), the functions

\[
u_{i0}(\tau) = (e_0, u_i), \quad \nu_{i\mu}(\tau) = (e, u_i), \quad \nu_{i0}(\tau) = (e, u_i) \tag{19}
\]

are the projections of the vectors \( u_{i\mu}^\nu(\tau) \) onto the mentioned basis. The projections of \( u_{i\mu}^\nu \) onto \( \bar{e} \mu \) may be expressed through \( u_{i0} \): \( (\bar{e}, u_i) = (1)^i u_{i0}^\nu \) due to the equality (17). Arguments \( (\tau - \pi) \) of the functions \( u_{20}, u_{2e}, u_{2e} \) are omitted. The constants in Eqs. (18) are

\[
M_0 = m_2 Q_1/(m_1 Q_2), \quad M_1 = m_1 \Gamma_1/(m_2 \Gamma_2), \quad N_i^* = (-1)^i(1 + Q_3 - i Q_4)/\Gamma_i, \quad N_i = (-1)^i(Q_3 - i Q_4)/\Gamma_i, \quad \kappa_i = 1 + v_i^{-2}.
\]

We shall search solutions of the linearized system (18) in the form

\[
u_{i\mu}^\nu(\tau) = A_{i\mu}^\nu e^{-i\omega \tau}. \tag{20}
\]

For the last two equations (18) (they form the closed subsystem) solutions in the form (20) exist only if the dimensionless frequency \( \omega \) satisfies the transcendental equation

\[
\frac{\omega^2 - Q_1 Q_2}{Q_1 + Q_2} \omega = \cot \pi \omega, \tag{21}
\]

but for the subsystem of the first 4 equations (18) the corresponding frequencies \( \omega = \tilde{\omega} \) are roots of another equation

\[
\frac{\tilde{\omega}^2 - Q_1^2 \kappa_1(\tilde{\omega}^2 - Q_2^2 \kappa_2) - 4 Q_1 Q_2 \tilde{\omega}^2}{2\tilde{\omega}[Q_1(\tilde{\omega}^2 - Q_2^2 \kappa_2) + Q_2(\tilde{\omega}^2 - Q_1^2 \kappa_1)]} = \cot \pi \tilde{\omega}, \tag{22}
\]

One can numerate the roots \( \omega = \omega_n \) of Eq. (21) and \( \tilde{\omega} = \tilde{\omega}_n \) for Eq. (22) in order of increasing so that \( \omega_0 = \tilde{\omega}_0 = 0, \ n - 1 < \omega_n, \tilde{\omega}_n < n \) for \( n \geq 1 \). The system of functions \( \exp(-i\omega_n \tau) \) or \( \exp(-i\tilde{\omega}_n \tau), \ n = 0, \pm 1, \pm 2, \ldots \) is the full system in the class \( C(I) \) [3, 10] so an arbitrary
Using this expansion for the disturbance (16) \( u^\mu_i \) of the velocity vectors (14) we obtain with the help of Eqs. (11), (12) the following expression for an arbitrary quasirotational motion of the string with massive ends (11):

\[
X^\mu(\tau, \sigma) = X^\mu_{rot}(\tau, \sigma) + \sum_{n=-\infty}^{\infty} \left\{ e_n^\mu \cos(\omega_n \sigma + \phi_n) \exp(-i\omega_n \tau) + \beta_n [e_n^\mu f_0(\sigma) + e_n^\mu f_\perp(\sigma) + i\epsilon^\mu(\tau) f_r(\sigma)] \exp(-i\tilde{\omega}_n \tau) \right\}. \tag{24}
\]

Each term in Eq. (24) describes the string oscillation that looks like the stationary wave with \( n \) nodes. There are two types of these stationary waves: (a) orthogonal oscillations along \( z \) or \( e_3 \)-axis at the frequencies proportional to the roots \( \omega_n \) of equation (21), and (b) planar oscillations (in the rotational plane \( e_1, e_2 \)) at the dimensionless frequencies \( \tilde{\omega}_n \) satisfying the equation (22) with the following expressions for \( f_0, f_\perp, f_r \):

\[
\begin{align*}
   f_0(\sigma) &= \frac{1}{2}(Q_1 \kappa_1 \tilde{\omega}_n - Q_1^{-1} \tilde{\omega}_n) \cos \tilde{\omega}_n \sigma - \sin \tilde{\omega}_n \sigma, \\
   f_\perp(\sigma) &= \Gamma_1(\Theta_n \tilde{\omega}_n - h_n v_1) C_\theta C_\omega - v_1^{-1} C_\theta S_\omega - \Gamma_1 \theta \Theta_n S_\theta S_\omega + h_n S_\theta C_\omega, \\
   f_r(\sigma) &= \Gamma_1(\Theta_n \tilde{\omega}_n - h_n v_1) S_\theta S_\omega - v_1^{-1} S_\theta C_\omega + \Gamma_1 \theta \Theta_n C_\theta C_\omega - h_n C_\theta S_\omega.
\end{align*}
\]

Here \( \Theta_n = \frac{2\theta}{\omega^2_n - \theta^2}, \quad h_n = \frac{1}{2} \left[ \frac{\theta}{\omega_n} \left( \frac{1}{v_1} + v_1 \right) + \frac{\tilde{\omega}_n}{\theta} \left( \frac{1}{v_1} - v_1 \right) \right], \quad C_\theta(\sigma) = \cos \theta \sigma, \quad S_\theta(\sigma) = \sin \theta \sigma, \quad C_\omega(\sigma) = \cos \tilde{\omega}_n \sigma, \quad S_\omega(\sigma) = \sin \tilde{\omega}_n \sigma.

The frequencies \( \omega_n \) and \( \tilde{\omega}_n \) from Eqs. (21) and (22) are real numbers so the rotations (8) of the string with massive ends are stable in the linear approximation. One may consider the expansion (24) for an arbitrary quasirotational motion as the basis for further quantization of this system in the linear vicinity of the solution (8).

3. Quasirotational motions of the linear string model \( q-q-q \)

The stability problem for the rotational motion (8) of the linear system \( q-q-q \) was studied numerically in Ref. [3]. Now we present the analytical investigation of this problem based upon the method developed in the previous section for the string with massive ends. In particular, we may express the world surface of the linear \( q-q-q \) configuration through the unit velocity vectors (4) \( U^\mu_1(\tau) \) and \( U^\mu_3(\tau) \) of the massive end using the generalized formula (12):

\[
\Psi^\mu_{1\pm}(\tau \pm \sigma_2) = m_1 \gamma^{-1} \left[ \sqrt{-U^2_1(\tau \pm \sigma_2)} U^\mu_1(\tau \pm \sigma_2) \pm U^\mu_1(\tau \pm \sigma_2) \right], \tag{26}
\]

\[
\Psi^\mu_{2\pm}(\tau \pm \sigma_2) = m_3 \gamma^{-1} \left[ \sqrt{-U^2_3(\tau \pm \sigma_2 \mp \pi)} U^\mu_3(\tau \pm \sigma_2 \mp \pi) \mp U^\mu_3(\tau \pm \sigma_2 \mp \pi) \right].
\]

Here

\[
X^\mu(\tau, \sigma) = \frac{1}{2} [\Psi^\mu_{\mp}(\tau + \sigma) + \Psi^\mu_{\pm}(\tau - \sigma)], \quad (\tau, \sigma) \in G_i, \tag{27}
\]

\footnote{It is interesting that the same equation (21) describes the spectrum of states for the relativistic string with massive ends with linearizable boundary conditions [10].}
is the general solution of the string equation (8) (generalization of Eq. (11)) for this system. It is described by the different functions $\Psi_{1+}^\mu$ and $\Psi_{2+}^\mu$ in the domains $G_1$ and $G_2$ in Fig. 1, because $X^\mu(\tau, \sigma)$ is not continuous on the line $\sigma = \sigma_2(\tau)$, dividing these domains. However, as was mentioned above the function $X^\mu(\tau, \sigma)$ and the tangential derivatives $\frac{d}{d\tau}X^\mu(\tau, \sigma_2(\tau))$ are continuous. The latter fact results in the equality

$$(1 + \sigma_2')(\Psi_{1+}^\mu(+)) + (1 - \sigma_2')\Psi_{1-}^\mu(-) = (1 + \sigma_2')\Psi_{2+}^\mu(+)) + (1 - \sigma_2')\Psi_{2-}^\mu(-).$$  

(28)

Here $+ \equiv (\tau + \sigma_2(\tau))$, $- \equiv (\tau - \sigma_2(\tau))$, the conditions (7) are assumed.

If we substitute the general solution (27) into the boundary condition of the middle quark (6) it will take the form

$$m_2 \frac{d}{d\tau}U_2^\mu(\tau) = \gamma[\delta^\mu_\nu - U_2^\mu(\tau)U_{2
u}(\tau)][(1 + \sigma_2')\Psi_{2+}^\nu(\tau + \sigma_2) + (1 - \sigma_2')\Psi_{1-}^\nu(\tau - \sigma_2)].$$  

(29)

The analog of the system (13) for the model $q-q-q$ may be obtained if we substitute the expressions (20) into the boundary conditions (28) and (29). This system of equations (it is too cumbersome so is isn’t written here explicitly) (29), (28), (20) connects the functions $U_i^\mu(\tau)$, $i = 1, 2, 3$ and $\sigma_2(\tau)$. For analysis of the quasirotational states and the stability problem for the motion (8) of the $q-q$ model we substitute into the mentioned system of equations the small disturbances of the velocity vectors $U_i^\mu$ in the same form (16), $i = 1, 2, 3$ omitting the 2-nd order terms of $u_i^\mu(\tau)$. Besides (this is specific feature of the model $q-q-q$) one should consider the small disturbance of the function $\sigma_2(\tau)$ in this system

$$\sigma_2(\tau) = \sigma_2^{\text{rot}} + \delta\sigma_2(\tau).$$

Here for the rotational motion (8) $\sigma_2^{\text{rot}}$ is the value (10) and the velocities $U_i^{\mu(\text{rot})}$ are described by the slightly modified formula (14)

$$U_i^{\mu(\text{rot})}(\tau) = \Gamma_i[e^0_\mu - \epsilon_i\nu_i^\mu(\tau)]; \quad \epsilon_1 = -1, \quad \epsilon_3 = 1,$$

where $\nu_2 = 0$, because the middle quark with the mass $m_2$ is at rest at the rotational center.

Searching oscillatory solutions of this linearized system (the analog of Eqs. (13)), we substitute into it the small disturbances $\delta\sigma_2(\tau)$ and $u_i^\mu(\tau)$ in the form (20)

$$\delta\sigma_2(\tau) = \delta_0e^{-i\omega\tau}, \quad u_2^\mu(\tau) = [A_2^0e^\mu(\tau) + A_2^1\epsilon^\mu(\tau) + A_2^2\epsilon^\mu_3]e^{-i\omega\tau},$$

$$u_i^\mu(\tau) = [A_i^0e_\nu^\mu + A_i^\epsilon\nu_i^\mu(\tau) - \epsilon_i\nu^{-1}A_i^0\epsilon^\mu(\tau) + A_i^3\epsilon^\mu_3]e^{-i\omega\tau}, \quad i = 1, 3.$$  

(30)

Here the conditions (17) are taken into account. This results in the following system of linear equations with respect to the complex amplitudes $A_i^\nu$, $\delta_0$:

$$K_1(Q_1S_1 + \omega C_1)A_1^\mu = K_3(Q_3S_3 + \omega C_3)A_3^\mu + \omega\mu_2A_2^\mu = 0, \quad K_j(Q_jC_j - \omega S_j)A_j^\mu = A_j^\mu;$$

$$K_1(Q_1\kappa_1C_1 - \omega S_1)A_1^0 + i\omega Q_1^{-1}C_1A_1 = K_3(Q_3\kappa_3C_3 - \omega S_3)A_3^0 - i\omega Q_3^{-1}C_3A_3, \quad i\epsilon_jK_j(Q_j\kappa_jS_j + \omega C_j)A_j^\mu - \omega Q_j^{-1}C_jA_j = A_j - i\omega\delta_0, \quad j = 1, 3,$$

$$i\omega A_2 + \theta A_2^\pm = 0, \quad \epsilon_jK_j^\ast(\omega S_j - Q_jC_j)A_j^0 - i(v_j\Gamma_j)^{-1}S_jA_j = A_j^\pm + \theta\delta_0,$$

$$\mu_2(\theta^2 - \omega^2)A_2^\pm + K_j^\ast[(Q_j^2\kappa_j - \omega^2)C_j - 2\omega\theta S_j]A_j^0 = K_3^\ast[(Q_3^2\kappa_3 - \omega^2)C_3 - 2\omega\theta S_3]A_3^0.$$  

Here $C_j = \cos\omega(\sigma_j - \sigma_2^{\text{rot}})$, $S_j = \epsilon_j\sin\omega(\sigma_j - \sigma_2^{\text{rot}})$, $Q_j = \Gamma_j\theta v_j$, $K_j = (1 - v_j^2)/(\theta v_j)$, $K_j^\ast = K_j/(v_j\Gamma_j)$, $\Gamma_j = (1 - v_j^2)^{-1/2}$, $\kappa_j = 1 + v_j^{-2}$, $\mu_2 = m_2K_1/m_1 = m_2K_3/m_3$.  

7
Non-trivial solutions of this system exist only if the value $\omega$ is a root of the equations

$$
\mu_2 \omega + \frac{(Q_1 Q_3 - \omega^2) \sin \pi \omega + (Q_1 + Q_3) \cos \pi \omega}{(Q_1 C_1 - \omega S_1)(Q_3 C_3 - \omega S_3)} = 0; \quad (31)
$$

$$
\mu_2 \tilde{\omega}^2 - \frac{\theta^2}{\tilde{\omega}^2 + \theta^2} = \frac{(Q_1^2 \kappa_1 - \tilde{\omega}^2) C_1 - 2\tilde{\omega} Q_1 S_1}{(Q_1^2 \kappa_1 - \tilde{\omega}^2) S_1 + 2\tilde{\omega} Q_1 C_1} + \frac{(Q_3^2 \kappa_3 - \tilde{\omega}^2) C_3 - 2\tilde{\omega} Q_3 S_3}{(Q_3^2 \kappa_3 - \tilde{\omega}^2) S_3 + 2\tilde{\omega} Q_3 C_3}. \quad (32)
$$

They generalize correspondingly Eqs. (21) and (22) — the last equations are limits of Eqs. (31), (32) if $\mu_2 = m_2 = 0$. The roots $\omega = \omega_n$ of Eq. (31) correspond to oscillations of the rotating system $q$-$q$-$q$ in $z$- or $e_3$-direction, and the roots $\tilde{\omega}_n$ of Eq. (32) describe oscillations in the rotational plane.

From the point of view of the stability problem the most important fact is the presence (if $m_2 \neq 0$) of the imaginary root in Eq. (32). It may be easily found after the substitution $\omega = i\xi$, the corresponding value $\xi = \xi^* \in (0, \pi)$. The spectrum of an arbitrary quasirotational motion of the $q$-$q$-$q$ configuration has the form similar to Eq. (24) and contains all oscillatory modes with frequencies $\omega_n$ and $\tilde{\omega}_n$, which are roots of Eqs. (31), (32). If this motion has no certain symmetry, the exponentially growing mode with the factor $\exp(-i\omega \tau) = \exp(\xi^* \tau)$ is in this spectrum. This proves the conclusion made in Ref. [3] about instability of the rotation (8) of the system $q$-$q$-$q$ in Lyapunov’s sense — any small asymmetric perturbation is growing.

**Conclusion**

In the present work the we proved that the classical rotational motions (8) of the linear string baryon model $q$-$q$-$q$ are unstable. This analysis doesn’t allow to describe the future evolution of this instability when the amplitudes of growing disturbances are not small. But this process was studied previously in Ref. [8] with using the suggested numerical methods based upon determination of an arbitrary classical motion of the $q$-$q$-$q$ system if its initial position in Minkowski space $R_1$ and initial velocities of string points are given. These numerical experiments showed the picture of the instability (any arbitrarily small asymmetric disturbances are growing) and demonstrated that the result of its evolution is the complicated motion with quasi-periodical varying of the distance between the nearest two quarks. However the minimal value of the mentioned distance $\Delta R$ does not equal zero, in other words, the system $q$-$q$-$q$ is not transformed in quark-diquark ($q$-$q$) one, as was supposed in Ref. [3].

This picture radically differs from that for the relativistic string with massive ends. For the latter model both the numerical experiments in Ref. [8] and the analysis in Sect. 2 of this paper demonstrate the stability of the rotational motion (8) (in the linear approximation).

For the rotational motions of the “three-string” model or Y configuration we also see instability both in the numerical calculations Ref. [8] and in the analytical investigations Ref. [11] of spectra for quasirotational states. This spectrum contains the branch of oscillatory states, whose dimensionless frequencies $\tilde{\omega}_n$ are roots of the equation

$$
2Q_1 \tilde{\omega}(\theta^2 - \tilde{\omega}^2) - i(\tilde{\omega}^2 - Q_1^2 \kappa_1)(\tilde{\omega}^2 + \theta^2) = \cot \pi \tilde{\omega}.
$$

These roots are obligatory complex numbers (except for $\tilde{\omega} = \pm \theta$). Imaginary parts of them are always positive so the disturbances of this class (branch) are exponentially growing in accordance with the factor $\exp(-i\omega_n \tau) = \exp(-i\Re \omega_n \tau) \exp(3\omega_n \tau)$. Arbitrary quasirotational states, whose dimensionless frequencies $\tilde{\omega}_n$ are always positive so the disturbances of this class (branch) are exponentially growing in accordance with the factor $\exp(-i\omega_n \tau) = \exp(-i\Re \omega_n \tau) \exp(3\omega_n \tau)$. Arbitrary quasirotational states, whose dimensionless frequencies $\tilde{\omega}_n$ are always positive so the disturbances of this class (branch) are exponentially growing in accordance with the factor $\exp(-i\omega_n \tau) = \exp(-i\Re \omega_n \tau) \exp(3\omega_n \tau).$

\[^3\]It is the model of the meson $q$-$\bar{q}$ or the baryon in the form $q$-$qq$. 
motion of the system $Y$ may also be expanded in the Fourier series of the type (8) with harmonics of all classes described above. So only for the disturbances with the special symmetry (when all amplitudes of the modes (33) equal zero) these disturbances do not grow exponentially, in other words — the rotational motion for the three-string configuration is unstable even in the linear approximation.

The evolution of this instability was calculated numerically in Ref. 8 where we showed that the picture of motion is qualitatively identical for any small asymmetric disturbance. Starting from some point in time the junction begins to move. The distance between the junction and the rotational center increases and the lengths of the string segments vary quasiperiodically unless one of the material points inevitably merges with the junction.

This picture differs from that for the linear string baryon model $q-q-q$. It is, in particular, connected with different properties of the complex roots of Eq. (32) and Eq. (33).

Instability of the rotations for the string baryon models $q-q-q$ and $Y$ is, of course, their drawback. But it is not “fatal” drawback, the classic instability is only one of the features for choosing the most adequate string baryon model among the existing four ones. The most important consequence of the rotational instability is in impossibility to quantize the quasirotational states. This procedure can be developed for the states (24) (the Fourier series) for the string with massive ends in the stable case. But for the unstable models $q-q-q$ and $Y$ this procedure is not permitted.

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