Are four dimensions enough; a note on ambient cosmology.

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Abstract

The group of homothetic symmetries in the conformal infinity (the 4-dimensional “ambient boundary”) of a 5-dimensional spacetime restricts the choice of topology to a topology under which the group of homeomorphisms of a spacetime manifold is the group of homothetic transformations. Since there are such spacetime topologies in the class of Zeeman-Göbel, under which the formation of basic contradiction present in proofs of singularity theorems is impossible, an important question is raised: why should one construct a 5-dimensional metric, in order to return back such a topology to its 4-dimensional conformal boundary, while such topologies, like those ones in the Zeeman-Göbel class, are already considered as more “natural” topologies for a spacetime, rather than the artificial (according to Zeeman) manifold topology?

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1 On Ambient Cosmology and Spacetime Singularities.

In [1] the authors create a model of ambient cosmology, the “Ambient Space - Ambient Boundary” pair, inspired by previous approaches on braneworlds and holographic ideas, with the motivation to describe the spacetime singularities that are predicted in the theory of general relativity, showing that under this model the singularities disappear and the proposition of cosmic censorship becomes valid. The authors start from a fixed metric in the boundary and then they consider the conformal structure of this boundary for constructing a 5-dimensional metric that will return a suitable 4-dimensional metric to the conformal boundary. In this way, 4-dimensional relativistic manifold is examined as an asymptotic and holographic limit of a 5-dimensional structure, the “ambient space” \((M \times \mathbb{R}, g_+)\) which satisfies the 5-dimensional Einstein equations with fluid sources and is defined, in a local manner, in an open neighbourhood of the “ambient boundary” (the 4-dimensional spacetime \((M, g)\) that we live in).

In [2] and [3] the authors show that, by its construction, the ambient metric \(g_+\) defines a homothetic symmetry on the ambient boundary \(M\) and, since in [9] it has been proved that the Fine topology (as well as other topologies of the class Zeeman-Göbel) admits the property that the group of homeomorphisms of a spacetime (under this topology) is the group of all homothetic transformations of the spacetime (in other words, a homeomorphism is an isometry)), it should be that the unique topology on the ambient boundary, with some physical meaning, should be the Fine topology. In [5] it is mentioned that there is an erratum in article [3], where there is a claim that the Fine topology does not admit “Euclidean-open balls with their Euclidean metric” (see page 5 of [3]) - here we should mention that open balls should be defined by an appropriate Riemann metric, to be more correct, since we refer to spacetime manifolds--; the Fine topology, as a finer topology than the manifold topology (by its construction, see [8] and [9]) contains all the open sets of the manifold topology. So, the assertion that “all sequences will be Zeno sequences” is not valid and, hence, the assertion that the Limit Curve Theorem does not hold, under the Fine topology, is not valid, as well.

In the next section, we show that there are actually topologies in the class of Zeeman-
Goebel, where the Limit Curve Theorem fails to hold, but for a different reason.

2 The Path Topology and the Convergence of Causal Curves.

In [6], the author shows that in the Path topology (see [7]) the Limit Curve Theorem fails to hold. Since the Path topology is finer than the manifold topology, every manifold-open set is also open in the Path topology; hence the argument in [2] that the convergence of causal curves depends on the existence of Euclidean- (manifold-, more correctly) open balls is not valid. The Limit Curve Theorem (under the manifold topology) states that if $\gamma_n$ is a sequence of causal curves, $x_n$ is a point on $\gamma_n$ for each $n$, and if $x$ is a limit point of $\{x_n\}$, then there is an endless causal curve $\gamma$, passing through $x$, which is a limit curve of the sequence $\gamma_n$. The failure of this theorem, in this sense under the Path topology, is very important, because it avoids basic contradiction arguments that are present in the proofs of all singularity theorems. In [3], the authors highlight that such contradictions appear when one assumes the existence of a causal curve whose length is greater than some maximum that starts from a spacelike Cauchy surface with negative curvature, downwards to the past. Thus, if a limit curve $\gamma$ cannot be extracted as an appropriate limit of convergence of causal curves, one cannot speak of geodesic incompleteness.

Since the Path topology has been shown to be the general relativistic analogue of a topology that has been suggested by Zeeman in [8] (see [11], [10] as well as [12]) and since according to Goebel in topologies of this class the group of homeomorphisms of a spacetime manifold is the group homothetic symmetries, then a topology which could justify the construction of the Ambient Boundary - Ambient Space pair, in [1] could certainly be the Path topology.

We have one more objection here, though, that we will express in the next section.
3 Discussion: Are Spacetime Singularities a Topological Effect?

In the previous two sections we first mentioned that the existence or not of manifold-open sets is not linked to the validity of the Limit Curve Theorem and also that the group of homothetic symmetries of the Ambient Boundary does not restrict our choice of an “appropriately natural” topology to the Fine topology; a candidate topology, where one cannot talk about convergence of causal curves, could be the Path topology. Since the Path topology, though, is a challenging alternative of the manifold topology, as it has been strongly suggested at least by Zeeman, Göbel, Hawking, King and McCarthy in the above mentioned papers, as it embodies the causal, differential and conformal structures of spacetime, the objection on why one should need a model of Ambient Cosmology, adding extra dimensions, at least from a topological perspective becomes more and more stronger. It is evident that, since the convergence of causal curves depends on the choice of a topology for a spacetime, the singularity problem as a whole can be placed within a topological frame exclusively: spacetime singularities seem to be a topological effect. It is the topology of the spacetime which will determine the validity of singularity theorems or cosmic censorship and not the examination of a 4-dimensional spacetime from a perspective of extra dimensions.

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