Probing very high energy prompt muon and neutrino fluxes and the cosmic ray knee via underground muons

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Abstract. We calculate event rates and demonstrate the observational feasibility of very high energy muons (1–1000 TeV) in a large-mass underground detector operating as a pair-meter. This energy range corresponds to surface muon energies of $\sim(5–5000$ TeV) and primary cosmic ray energies of $\sim(50$ TeV–$5 \times 10^4$ TeV). Such measurements would significantly assist in an improved understanding of the prompt contribution to $\nu_e$, $\nu_\mu$ and $\mu$ fluxes in present and future ultra high energy neutrino detectors. In addition, they would shed light on the origin of and possible compositional changes at and around the observed ‘knee’ in the cosmic ray spectrum.

Keywords: ultra high energy cosmic rays, cosmic rays, ultra high energy photons and neutrinos

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1. The cosmic ray spectrum and the knee

Cosmic ray studies, with the spectrum extending over ten decades in energy, have proved to be fertile terrain for furthering our knowledge of both astrophysics and particle physics (reviews may be found in [1]–[4]). They have traditionally provided us with clues for the existence of new particles and the physics associated with them, which have later been confirmed by detailed accelerator experiments. In fact, prior to 1950 and the advent of modern accelerator technology, they provided the only means of studying high energy particle production and interactions. Additionally, as a result of our attempts to understand the origin of cosmic rays, they have contributed to our knowledge of acceleration via shocks, and the propagation of charged particles in the galaxy and heliosphere.

The cosmic ray spectrum, characterized by a steeply falling power-law behaviour over its entire range, exhibits two transition regions where the slope changes noticeably:

- A steepening of the spectrum occurs at around $E \approx 5 \times 10^6$ GeV, i.e. the index $\gamma$ describing the power-law behaviour of the differential flux, $dN/dE \sim E^\gamma$, changes from $\gamma \approx -2.7$ to $\gamma \approx -3.1$, leading to the feature called the ‘knee’.
- A flattening of the spectrum occurs at around $E \approx 5 \times 10^9$ GeV, i.e at the ‘ankle’, with the index $\gamma$ changing back to $\approx 2.4$–2.7. Beyond the ankle, in the realm of ultra high energy cosmic rays, data [5]–[7] is sparse and conflicting, but highly intriguing. While we will not address the interesting puzzle in this regime here, a discussion of the various issues may be found in [8,9], and a recent assessment of the shape of the spectrum based on current knowledge can be found in [10].
The physical reason for the existence of the knee is at present an unresolved problem of great significance to understanding the origin of galactic cosmic rays. It is generally believed that the reasons underlying this distinctive shift in the spectrum are astrophysical in nature, as opposed to those stemming from a change in hadronic interactions at these energies which, at present, are not within the reach of existing accelerators. This conclusion is based on the observed correspondence between independent measurements of the muon number spectrum, Cerenkov radiation (CR) and hadronic constituents of air-showers [11]–[15]. While the reasons for the shift in the spectrum are still not understood, these data exhibit an expected co-relation which supports the absence of radically different physics interactions at these energies.

While the case for the existence of new physics being at least partially responsible for a shift in the spectral index is not wholly without motivation\(^2\), we stress that it appears unlikely that this can be empirically corroborated or refuted in the near future in CR measurements. This is because, as we shall discuss below, uncertainties (in the knee region) in the CR composition and prompt muon and neutrino contributions would likely overshadow evidence of such new interactions. In any case, this hypothesis will be thoroughly probed in the near future by the large hadron collider (LHC) at CERN, which will operate at a centre-of-mass energy of \(\sqrt{s} = 14\) TeV.

2. Uncertainties in the muon and neutrino fluxes in the knee region and beyond

As stated earlier, present data [20], when culled together and correlated, appear to favour one or more astrophysical reasons for the existence of the knee. These include it being a rigidity-dependent effect (originally proposed in [21]) related to the (different) maximum acceleration energies for different nuclei either in the cosmic ray source itself or during the propagation process. Data from surface air-showers and optical detectors indicate, without being conclusive, that the average mass of the cosmic ray spectrum nuclei differs before and after the steepening at the knee. In particular, there appears to be some evidence [22,23] that the composition is heavier above the knee region. If this is true, then, as discussed in [24], significant suppression of the very high energy (\(\geq 10^5\) GeV) muon and neutrino fluxes resulting from CR interactions in the atmosphere and in the interstellar medium can occur.

A second major factor in determining the enhancement (or lack thereof) of muon and neutrino fluxes above several TeV are the uncertain magnitudes of the prompt fluxes of both (i.e. those resulting from heavy meson decay, notably charm mesons and heavier composites). At low (i.e. \(\sim\)GeV) energies, the cosmic ray induced neutrino and muon fluxes receive their dominant contributions from the decays of \(\pi\) and \(K\) mesons, whose interaction lengths significantly exceed their decay lengths [45]–[48]. (These fluxes are henceforth referred to as the conventional flux in what follows.) This situation changes at \(\sim\)TeV energies, and secondary interactions of these particles become possible, leading to the production of heavy short-lived hadrons. While upper bounds on the fluxes of muons

\(^2\) This region in \(E_p\) (i.e. the energy of the primary particle) corresponds to several TeV in centre-of-mass energies. Thus there are many conjectures for physics beyond the Standard Model which come into play, e.g. SUSY, technicolour, large extra dimensions etc. These could lead (via new particle production and decay) to energy being channelled into muons, neutrinos or other secondary particles in a manner to which present cosmic ray experiments are insensitive, causing the shift in the (unmeasured) energy spectrum [16]–[19].
and neutrinos have been provided by several experiments e.g. LVD [49], AKENO [50] and AMANDA [51], they still allow for a very large possible range of prompt flux magnitudes.

Present phenomenological predictions for the diffuse fluxes of these prompt muons and neutrinos can differ by about two orders of magnitude [48], [52]–[57]. The sources of this large uncertainty lie, to a significant extent, in the choice of charm production models. For instance, differing predictions arise from models based on perturbative QCD (pQCD) with a $K$ factor [48], next-to-leading-order (NLO) pQCD [56,54], quark–gluon string models, and recombination quark–parton models [55] etc. In general, QCD-based models must contend with a large uncertainty associated with the extrapolation of the gluon parton distribution function, $g(x)$, to small fractional momentum, $x < 10^{-5}$. Theoretical models generally assume that

$$xg(x) \sim x^{-\lambda},$$

where $\lambda$ is in the range 0–0.5, and fluxes depend strongly on the chosen value of $\lambda$. We note that, depending on the model, the prompt muon and neutrino fluxes from charm decay exceed the corresponding conventional fluxes (from $\pi$ and $K$ decays) somewhere between (surface) muon energies of a few tens of TeV and a few PeV [56]. Reliable measurements of muon fluxes in this range would thus, at the very least, help in establishing the reliability of a particular class of models.

While we do not give a detailed account of the flux predictions from all the different models, we attempt to give a representative idea in our calculations of the variation possible even within a given charm production model. While the conventional muon flux from $\pi$ and $K$ decays is well understood and fairly firm, the prompt flux predictions are subject to variations resulting from different parton distribution functions and choices of the factorization and renormalization scales. We stress the need for a better empirical determination of the muon (and associated neutrino fluxes) in this region, a topic which is elaborated upon in the next section.

3. The significance of measurements of the muon and neutrino fluxes in the knee region

With very few exceptions, available data on muons above several TeV consists of measurements of the number spectrum as opposed to the energy. This is primarily due to the size and density requirements imposed on detectors by the significant penetration lengths achieved by high energy muons.

The desirability of improved and statistically significant muon energy measurements in the region of a few TeV to a few hundred TeVs stems from (at least) three reasons:

- As mentioned above, the physical origin and composition of cosmic rays in this energy range is currently obscured by a paucity of data on very high energy (VHE) muon and neutrino fluxes. Observations would thus certainly illuminate the current debate on the reason for the occurrence of and compositional changes at the knee.
- Also, as discussed above, QCD-related theoretical uncertainties dominate the predictions of the prompt contribution to muon and neutrino fluxes. As emphasized in [56,58], the measurement of down-going muon fluxes would provide a valuable handle in their reduction.
Both the conventional and prompt muon fluxes at these energies are closely related to the associated neutrino fluxes. For prompt contributions, this is because the kinematics of charmed particle decay and the corresponding semi-leptonic branching ratios ensure that the $\nu_e$ and $\nu_\mu$ fluxes are identical, up to a few per cent, to the muon fluxes in this energy range, regardless of the choice of the charm production model or of $\lambda$. The conventional neutrino flux, on the other hand, is about 10% of the conventional muon flux. At the energies of interest here, neutrinos resulting from cosmic ray interactions in the atmosphere and in the inter-stellar medium constitute the most important background to searches for diffuse fluxes of ultra high energy (UHE) neutrinos \cite{32}–\cite{38} from cosmological sources (e.g. active galactic nuclei, gamma-ray bursts etc) in neutrino telescopes like AMANDA \cite{39}, ICECUBE \cite{40} and NEMO \cite{41}. Thus, they are an important obstacle to the much-anticipated detection of such energetic point sources in these detectors. Empirical data on down-going muons from cosmic rays would prove invaluable in understanding this background to the UHE neutrino signal.

In the context of the points above, it is relevant to stress the importance of being able to disentangle the prompt (due to the decay of produced heavy mesons) and conventional (from $\pi$ and $K$ decays) and diffuse UHE (from extra-galactic sources) contributions to the neutrino fluxes. Methods to enable this have been studied, and are based on the differences in zenith angle, depth and spectral dependence between these fluxes \cite{42,43} and on the isolating capability of showering $\nu_e$ charged current events via a break in the spectrum \cite{44}.

Having emphasized the importance of muon energy measurements in the range of several TeV to several hundred TeV, we proceed in the next section to study the potential of the pair meter method \cite{25}–\cite{27} as applied to such measurements made in a large iron calorimeter (50 kT)\textsuperscript{3}. Since individual muon energies will become measurable using this technique, it will be possible to augment the sparse existing data on cosmic ray muons in the important range where they have surface energies of $\approx$5–5000 TeV. Furthermore, these observations can be combined with balloon-based experiments (e.g. TRACER \cite{29}) and upcoming hybrid air-shower experiments (e.g. KASCADE-Grande \cite{30} and LOPES \cite{31}) to enhance our understanding of the issues discussed above. We mention here that this range in muon surface energy roughly corresponds to a range of 50 TeV–$5 \times 10^4$ TeV in primary cosmic ray energy, which is crucial to an enhanced understanding of the origin of the knee.

In the remainder of the paper, we first provide a discussion of the pair-meter technique and the pair production cross section which results in the observed cascades. This is followed by a brief description of a typical large-mass iron calorimeter. Subsequent to this, we summarize the interactions and losses of muons in matter en route to an underground detector, and their incorporation into our calculation. We then calculate anticipated event rates for a 50 kT detector and demonstrate that even after accounting for energy losses in the surrounding rock, event rates can be appreciably large for the 1–1000 TeV range, corresponding to surface muon energies in the range of several TeV to several PeV.

\textsuperscript{3} Such a detector is currently being planned for location in India \cite{28}. 

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pair production
bremsstrahlung
photonuclear

Figure 1. Differential cross section $v \frac{d\sigma}{dv}$ versus $v^{-1}$ (inverse of the relative energy transfer) for pair production (solid) [59], bremsstrahlung (dotted) [60] and photonuclear (dashed) [61] processes.

4. The pair-meter method and the associated pair production cross section

Due to the penetrating power of muons, their energy measurements require techniques which differ from those employed for photons, hadrons and electrons. Furthermore, muon energy measurement methods which work well in the GeV range (magnetic spectrometry or measuring Cerenkov radiation) are rendered impractical in the TeV range, primarily due to the requirements of size imposed by the combination of high energies and a steeply falling spectrum.

The pair-meter technique [25]–[27] skirts some of the disadvantages of traditional muon detectors by relying on a somewhat indirect method, i.e. the measurements of the energy and frequency of electron–positron pair cascades produced by the passage of a high energy muon in dense matter. A reliable reconstruction of the muon energy in this method is based on the following:

- The cross section for $e^+e^-$ pair production by a muon with energy $E_\mu$ with energy transfer above a threshold $E_0$ grows as $\ln^2(2m_eE_\mu/m_\mu E_0)$, where $m_\mu$ and $m_e$ are the muon and electron masses, respectively.
- Defining $v = E_0/E_\mu$, above $v^{-1} = 10$ this cross section dominates those for other muon energy loss processes which generate observable cascades in its passage through dense matter, e.g. $\mu-N$ inelastic scattering and bremsstrahlung emission. This is demonstrated in figure 1, where we compare the differential cross sections for these various interactions as a function of $v^{-1}$.
- The energy lost to each cascade resulting from $e^+e^-$ pair production is a very small fraction (about $10^{-2}$) of the muon energy for the range of $v^{-1}$ which we focus on here.
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- The dependence of the pair production cross section on $E/\mu /E_0$ then allows one to infer the muon energy by counting the number of interaction cascades $N$ in the detector with energies above a threshold $E_0$.

We now make the above statements more precise. In the approximations

$$v = \frac{E_0}{E_\mu} \ll \frac{2m_e}{m_\mu}$$

and

$$E_0 \gg 2m_e 189 \sqrt{eZ}^{-1/3} \simeq 0.3 Z^{-1/3} \text{GeV},$$

(where $Z =$ atomic number $= 26$, for iron) both of which are valid for the choice of $E_0$ and $E_\mu$, for which we present results below, the expression for the differential pair production cross section is given by

$$\frac{d\sigma}{dv} \simeq \frac{14 \alpha}{9 \pi t_0} \ln \left( \frac{\kappa m_e E_\mu}{\epsilon m_\mu} \right),$$

(1)

where $\alpha = 1/137$ and $\kappa \simeq 1.8$. $t_0$ is the radiation length (rl) which is given by

$$t_0 = \left( \frac{4Z(Z+1)}{A_W} \alpha r_0^2 N_A \ln(189 Z^{-1/3}) \right)^{-1}.$$

(2)

Here, $A_W$ is the atomic weight, $r_0$ is the classical electron radius, and $N_A$ is the Avogadro number. For iron, this gives $t_0 = 13.75 \text{ g cm}^{-2}$.

The average number of interaction cascades $M$ above a threshold $E_0$ for $v \leq 10^{-3}$ is given by

$$M(E_0, E_\mu) = T t_0 \sigma(E_0, E_\mu),$$

(3)

where $T$ is the thickness of the target in units of $t_0$ and $\sigma(E_0, E_\mu)$ is the integrated cross section (in units of cm$^2$ g$^{-1}$):

$$\sigma(E_0, E_\mu) \simeq \frac{7 \alpha}{9 \pi t_0} \left( \ln^2 \left( \frac{\kappa m_e E_\mu}{E_0 \epsilon m_\mu} \right) + C \right),$$

(4)

where $C \simeq 1.4$.

The calculations which follow are performed for a 50 kT iron calorimeter. Our prototype is based on the suggested design for INO; see [28] for details. The dimensions of a 50 kT detector of this type would correspond to (approximately) 15 m $\times$ 15 m $\times$ 45 m. A muon traversing a 20 m path in this detector corresponds to a path-length of $\sim 1145$ rl. In what follows, we assume a (conservative) ‘average’ path-length of 1000 rl for the typical muon and calculate the number of observable cascades produced by it, for different cascade thresholds and muon energies. Figure 2 shows the average number of cascades above a threshold energy $E_0$ produced by a muon entering the detector with energy $E_\mu$ and $T = 1000$ rl, for three different choices of $E_0$, i.e., 1, 10 and 100 GeV. Quantitatively, we note that this leads to a $E_\mu = 100 \text{ TeV}$ muon that generates approximately 40 cascades, each of energy greater than $E_0 = 10 \text{ GeV}$, and 10 cascades with energy in excess of 100 GeV. By counting the cascades for several choices of thresholds for a traversing muon, one obtains a reliable estimate of its energy.
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![Figure 2. Average number of cascades above a threshold $E_0$ versus muon energy for $E_0 = 1$ GeV (solid line), 10 GeV (dotted) and 100 GeV (dashed), with $T$ fixed to 1000 rl.](image)

It is also relevant to remark here that the relative energy measurement error, $\delta E_\mu / E_\mu$, in the pair meter is given by

$$\delta E_\mu / E_\mu = \sqrt{\frac{9\pi}{28\alpha T}} \simeq \sqrt{\frac{137}{T}}.$$  \hspace{1cm} (5)

For $v = (10^{-3}–10^{-2})$, which is the range on which we focus here, this allows a liberal tolerance for errors in the measurements of individual cascade energies. We note also that the errors do not worsen with increasing muon energy, which is an important advantage of the pair-meter technique.

5. Surface muon energy determination for underground events

It is important to correlate the measured muon energies in an underground detector to their surface energies, which we take to be those that would be observed were our detector placed on the surface of the earth. This requires a calculation of the energy loss as the muon traverses the rock between the earth’s surface and the detector.

These losses originate from ionization, bremsstrahlung, pair production and photonuclear interactions. They can be effectively parametrized \[59\]–\[61\] for $E_\mu \geq 1$ TeV, since the average loss increases predominantly linearly with energy,

$$\left\langle \frac{dE}{dX} \right\rangle = -\alpha - \beta E,$$  \hspace{1cm} (6)

where $\alpha$ parametrizes the contribution from the ionization of muons and $\beta$ encapsulates the contribution from bremsstrahlung, pair production and photonuclear processes. Note
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\[ \langle E_\mu(X) \rangle = \left( E_\mu^s + \frac{\alpha}{\beta} \right) e^{-\beta X} - \frac{\alpha}{\beta}, \tag{7} \]

where \( E_\mu^s \) is the initial surface muon energy. One may use this to write down the minimum surface energy required for a muon to reach a depth \( X \) as,

\[ E_{\min}^s = \frac{\alpha}{\beta} \left( e^{\beta X} - 1 \right). \tag{8} \]

From equation (7), we get the relation between the initial energy, \( E_\mu^s \), and the degraded energy, \( E_\mu \), of the muon after travelling a distance \( X \) as,

\[ E_\mu^s = \left( E_\mu + \frac{\alpha}{\beta} \right) e^{\beta X} - \frac{\alpha}{\beta}. \tag{9} \]

The differential muon flux at a depth \( X \) is given by,

\[ \frac{dN}{dE_\mu} = \frac{dN}{dE_\mu^s} e^{\beta X}, \tag{10} \]

where \( (dN/dE_\mu^s) \) is the initial muon flux with surface muon energy \( E_\mu^s \).

The \( \alpha \) and \( \beta \) in the analytical expression can be obtained for standard rock. The depth relevant to the INO detector’s proposed location is \( 3.5 \times 10^5 \text{ g cm}^{-2} \). The values are,

\[ \beta = 4 \times 10^{-6} \text{ cm}^2 \text{ g}^{-1}, \quad \frac{\alpha}{\beta} = 675 \text{ GeV}. \tag{11} \]

Figure 3 shows the degraded muon energies (i.e. those measured for muons entering the detector after traversing the rock) versus their corresponding surface energies after losses are accounted for in the manner described above. We note that, typically, \( E_\mu^s \simeq (2–5) \times E_\mu \).
Table 1. PRS parameters for the prompt muon and anti-muon fluxes. $m_c$ is the mass of the charm quark.

| Flux model | PDF  | Scales | $a$  | $b$  | $c$  | $d$  |
|------------|------|--------|------|------|------|------|
| PRS1       | CTEQ3| $\tilde{M} = \tilde{\mu} = m_c$ | 5.37 | 0.0191 | 0.156 | 0.0153 |
| PRS2       | CTEQ3| $\tilde{M} = 2\tilde{\mu} = 2m_c$ | 5.79 | 0.345  | 0.105 | 0.0127 |
| PRS3       | D    | $\tilde{M} = 2\tilde{\mu} = 2m_c$ | 5.91 | 0.290  | 0.143 | 0.0147 |

6. Muon fluxes

Extensive predictions and studies [45]–[48], [52]–[57] for prompt cosmic ray muon fluxes at very high energies exist in the literature, as mentioned earlier. For our representative calculations of muon event rates, we have used the relatively conservative predictions for charm induced fluxes given in [48, 54]. The large variation in muon rates possible due to flux uncertainties, even when these fluxes are used, is amply reflected in our results, most noticeably in table 2. One would expect much larger variations if the full range of prompt flux models available is used to calculate event rates.

In [48] (henceforth referred to as the TIG flux), the conventional and prompt fluxes have been parametrized as

$$\frac{dN}{dE} = \frac{N_0 E^{-\gamma-1}}{1 + AE}$$

for $E < E_a$, and as

$$\frac{dN}{dE} = \frac{N'_0 E^{-\gamma'-1}}{1 + AE}$$

for $E > E_a$. For the conventional muon flux, $N_0 = 0.2$, $N'_0 = 0.2$, $\gamma = 1.74$, $\gamma' = 2.1$, $E_a = 5.3 \times 10^5$, and $A = 0.007$.

For the prompt muon flux, $N_0 = 1.4 \times 10^{-5}$, $N'_0 = 4.3 \times 10^{-4}$, $\gamma = 1.77$, $\gamma' = 2.01$, $E_a = 9.2 \times 10^5$, and $A = 2.8 \times 10^{-8}$.

The second set of representative prompt muon fluxes that we use are calculated in [54] (henceforth referred to as the PRS1, PRS2 and PRS3 fluxes). The differences in the three fluxes originate in different choices of parton distribution functions (PDF) and factorization ($\tilde{M}$) and renormalization scales ($\tilde{\mu}$) of the theory. These fluxes can be conveniently parametrized [54] as follows:

$$\frac{dN}{dE} = 10^{-a+bx+cx^2-dx^3},$$

where $x = \log_{10}(E/\text{GeV})$, with $a, b, c$ and $d$ as in table 1.

In figure 4, we show the conventional (TIG) and prompt (TIG and PRS) surface muon fluxes. Uncertainties in the conventional flux, unlike the prompt case, are not major, hence we have shown only the TIG parametrization. We note that, depending on the flux model, the prompt fluxes rise above the conventional flux for (surface) muon energies between 200 and 1000 TeV. In terms of (degraded) muons entering the detector, we see from figures 3 and 5 that this corresponds to measured muon energies of several tens of TeV and several hundreds of TeV. Thus, we note that underground muon measurements in this range will
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Figure 4. \( (E_{\mu}^{s} \times \text{flux}) \) versus surface muon energy \( E_{\mu}^{s} \) for total TIG flux (solid), conventional (dotted), TIG prompt (short dashed), and prompt fluxes corresponding to PRS1 (short spaced dots), PRS2 (large dashed) and PRS3 (large spaced dots).

Table 2. Number of muons per solid angle entering the detector over five years for various energies of the entering muon, \( E_{\mu} \).

| \( E_{\mu} \) (TeV) | Conv. + TIG | Conv. | TIG | PRS1 | PRS2 | PRS3 |
|-------------------|--------------|--------|-----|------|------|------|
| 1                 | \( 1.035 \times 10^7 \) | \( 1.03 \times 10^7 \) | 37461 | 55482 | 95489 | 136871 |
| 10                | 52486        | 51282  | 1204 | 2952  | 5341  | 10443 |
| 50                | 770          | 696    | 74   | 236   | 431   | 1104  |
| 100               | 127          | 106    | 21   | 73    | 134   | 387   |
| 200               | 22           | 16     | 6    | 22    | 40    | 129   |
| 300               | 8            | 5      | 3    | 11    | 19    | 66    |
| 400               | 4            | 2      | 2    | 6     | 11    | 41    |
| 500               | 2            | 1      | 1    | 4     | 7     | 28    |
| 600               | 1.5          | 1      | 1    | 3     | 5     | 20    |
| 700               | 1            | 0.5    | 0.5  | 4     | 7.5   | 31    |
| 800               | 0.8          | 0.35   | 0.5  | 1.5   | 3     | 12    |
| 900               | 0.65         | 0.25   | 0.37 | 1.25  | 2.5   | 10    |
| 1000              | 0.5          | 0.2    | 0.3  | 1     | 2     | 4     |
| 10000             | 0.0025       | 0.0003 | 0.0022 | 0.007 | 0.013 | 0.08 |

help to reduce the present uncertainties in deducing the charm contributions to muon and neutrino fluxes. Our calculations provide a quantitative estimate of the potential of these measurements to accomplish this.
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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{(\(E_\mu^3 \times \text{Flux}\)) versus energy of muon entering the underground detector \(E_\mu\) (for the flux models listed in the previous figure) after passing through a rock distance of \(3.5 \times 10^5\) g cm\(^{-2}\). The labelling of plots is identical to figure 4.}
\end{figure}

7. Results and discussion

We are now in a position to calculate the expected cascade events for a 50 kT detector in the energy range of interest discussed above. While an entering muon in this energy range will produce observable cascades, the number entering the detector over a given period is limited by the sharply falling fluxes at these energies. It is thus pertinent to obtain a quantitative measure of this by estimating \(n_\mu\), the number of muons above a given threshold entering the detector per steradian for an exposure of \(t\) years:

\[ n_\mu = \int_{E_{\text{th}}}^{\infty} dE_\mu \frac{dN}{dE_\mu} A \times t, \]

where \(A\) is the exposed area of a 50 kT iron detector. This is shown in figure 6 and table 2. We note that, while the number of entering muons for the lowest energy in table 2, i.e. 1 TeV, is very large, one also obtains an observable number, i.e. one to three events after integrating over a solid angle (considering that there is no ‘background’ as such for such events) over the five year period, even for \(E_\mu = 1000\) TeV for the most conservative flux choice (TIG). These energies delineate the muon energy range that is accessible. The number of entering muons for all choices of PRS fluxes will be substantially higher, as shown. Even for the most conservative (TIG) flux choice, one expects good observational capability up to several hundred TeV.\(^4\)

The number of cascades per muon above \(E_0 = 5, 10, 20, 50, 100, 300, 500, 1000, \) and 5000 GeV respectively, using equations (3) and (4), are tabulated in table 3. Table 3 also

\(^4\) We have given the per-steradian rates here. In order to predict the rate integrated over angle, predictions must take into account the depth dependence of the fluxes for a particular detector and its surrounding topography.
lists the surface muon energy $E_{\mu}^s$ corresponding to the underground muon energies $E_{\mu}$. While these are sample choices, it is clear that they can be further optimized, based on the muon energy that one wants to observe to good statistical accuracy.

The number of expected cascade events per steradian $N_c(E_0)$ is given by

$$N_c(E_0) = M(E_{\mu}, E_0)n_{\mu},$$

where $M(E_{\mu}, E_0)$ is the cascade number calculated above in section 2.1. From tables 2 and 3, we observe that there is a considerable number of cascade events per solid angle produced in a 50 kT INO detector over five years of running time for different thresholds ranging from 5 to 5000 GeV. For example, if we consider a conventional + TIG flux model, then at muon energies of 1000 TeV one can produce 51 events per solid angle for a threshold of 5 GeV to one event per solid angle for a threshold of 5000 GeV. Similarly, PRS models will show more events compared to the above estimates.

We have used the TIG flux as a benchmark to establish observability, since it leads to the most conservative event rate predictions. All other flux parametrizations lead to higher predictions. We note that, even though TIG and PRS are not vastly different from each other in a qualitative sense since both are based on perturbative QCD inputs, their event rates in a large-mass pair meter differ significantly. Indeed, the variations between fluxes in the same family (PRS1, PRS2, PRS3) are also large. Thus, the muon event rate can act as a soft discriminator (i.e. not definitive, given the large uncertainties in the QCD predictions) between various prompt flux models and provide pointers to the physics input that should guide their development. Similarly, this rate provides a tool to better understand the present spectral uncertainties in the cosmic ray knee origin.
Table 3. Number of cascades above thresholds $E_0 = 5, 10, 20, 50, 100, 300, 500, 1000,$ and $5000$ GeV per muon. Here, $E_\mu$ is the energy of the muon in TeV entering the detector, and $E_s^\mu$ is its corresponding energy in TeV at the surface of the earth, assuming it traversed a depth of rock corresponding to $3.5 \times 10^5$ g cm$^{-2}$.

| $E_\mu$ (GeV) | $E_s^\mu$ (GeV) | 5   | 10  | 20  | 50  | 100 | 300 | 500 | 1000 | 5000 |
|--------------|----------------|-----|-----|-----|-----|-----|-----|-----|------|------|
| 1            | 6.1            | 3.08| 2.56| 3.78|     |     |     |     |      |      |
| 10           | 40.26          | 17.28| 10.99| 6.43| 3.08| 2.56|     |     |      |      |
| 20           | 83.16          | 25.3 | 17.28| 10.99| 5.34| 3.08|     |     |      |      |
| 50           | 205            | 38.58| 28.26| 19.67| 10.99| 6.43| 2.78| 2.56|      |      |
| 100          | 407.58         | 50.63| 38.58| 28.26| 17.28| 10.99| 4.58| 3.08| 2.56|      |
| 200          | 813            | 64.43| 50.63| 38.58| 25.30| 17.28| 8.11| 5.34| 3.08|      |
| 300          | 1218           | 73.3 | 58.49| 45.42| 30.8  | 21.76| 10.99| 7.46| 4.19|      |
| 400          | 1624           | 79.96| 64.43| 50.63| 35.06| 25.3  | 13.39| 9.33| 5.34|      |
| 500          | 2029           | 85.33| 69.24| 54.89| 38.58| 28.26| 15.45| 10.99| 6.43| 2.56|      |
| 600          | 2435           | 89.85| 73.3 | 58.49| 41.58| 30.8  | 17.28| 12.47| 7.46| 2.58|      |
| 700          | 2841           | 93.76| 76.83| 61.64| 44.21| 33.05| 18.91| 13.82| 8.43| 2.6 |      |
| 800          | 3246           | 97.23| 79.96| 64.43| 46.56| 35.06| 20.4 | 15.06| 9.33| 2.72|      |
| 900          | 3652           | 100.33| 82.77| 66.95| 48.69| 36.9  | 21.76| 16.21| 10.18| 2.89|      |
| 1000         | 4057           | 103.16| 85.33| 69.24| 50.63| 38.58| 23.02| 17.28| 10.99| 3.08|      |
| 10000        | 40554          | 174.84| 151.24| 129.38| 103.16| 85.33| 60.63| 50.63| 38.58| 17.28|      |

8. Conclusions

Our main results are presented in figures 6 and 2 and tables 2 and 3. From these, we (conservatively) conclude that underground muon energy measurements for an energy range of $E_\mu$ of 1–1000 TeV are possible with a 50 kT iron detector running for five years. This will give a better handle on the very high energy muon fluxes between several TeV to about 5 PeV, and consequently illuminate our estimates of the background muon and neutrino fluxes for ultra high energy neutrino detectors and lessen the present uncertainties in charm production models. As emphasized earlier, the prompt muon flux is a measure of the prompt $\nu_e$ and $\nu_\mu$ flux, hence its importance to ultra high energy neutrino astronomy cannot be underestimated.

The observable muon energy range discussed in our results also corresponds to a range of 50 TeV–50 PeV in primary cosmic ray energies. This range is crucial to an understanding of the origin of the knee and our calculations demonstrate the feasibility and potential that results from muon measurements for a better understanding of the origin of the knee.

A detailed and comprehensive set of predictions for a given large-mass detector necessarily requires a much more elaborate calculation of the muon losses than what is presented here, since local topography plays an important role in determining the surface muon energy corresponding to a measured muon energy. Our aim in this paper has been more to demonstrate the observational feasibility rather than to make precise predictions.

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5 As stated earlier, we have used a depth corresponding to the proposed INO site for specificity, however the results can be easily generalized to other depths, as should be obvious.
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Thus the calculations here show that very high energy muon measurements are possible in a large iron calorimeter and can aid in illuminating three important outstanding questions that address partially overlapping issues, one in cosmic ray physics, the second in theoretical QCD, and the third in ultra high energy neutrino astronomy.

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