Research Article

Influence Analysis of Transmission Lines on a Stable Non-Foster-Loaded Electrically Small Dipole

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Non-Foster-loaded antennas have the advantages of compact size and large bandwidth. Meanwhile, they suffer from two issues: internal instability and simulation inaccuracy resulting from distribution parameters. The most commonly used stability analysis method for microwave circuits, Rollett’s criteria, is not suitable for negative impedance circuits. This paper has explained the reason and proposed an effective method for stability analysis. Transmission lines between lumped components are found to be a main reason of inaccurate simulations, which is analyzed in this paper, and it is concluded that their influence also exists at hundreds of megahertz. In order to solve this problem and improve simulation accuracy, circuit and electromagnetic cosimulation is conducted. Finally, a 320 mm dipole loading with a negative capacitor is fabricated to verify the analysis. Simulated and measured results indicate that the proposed stability analysis is effective and the simulation accuracy is significantly improved. The matched dipole achieves less than −10 dB reflection coefficient from 30 MHz to 580 MHz. Furthermore, a 14 dB gain improvement is obtained in electrically small condition.

1. Introduction

Non-Foster circuits (NFCs) are circuits whose reactance decreases with frequency increases. So the circuits can act as negative inductors and capacitors which are consequently called non-Foster elements [1]. These non-Foster elements have been used in RF and microwave devices to achieve better performance. NFCs can be used in broad impedance matching circuits as they violate the Bode-Fano limitation. NFCs are also employed in electrically small antennas (ESAs) to reduce their Q value and broaden their bandwidth [4–6]. In addition, NFCs are loaded in a waveguide and microstrip for broadband fast-wave propagation and squint-free beam-forming of antenna arrays [7–10].

The first use of NFCs can be traced back to the 1920s when negative impedance converters (NICs) were designed with vacuum tubes to realize non-Foster elements for loss compensation in a telephone cable [2]. As semiconductor technology developed, the performance of transistors got better, and transistor-based NICs were introduced by Linvill in 1953 [3]. After that, different kinds of grounded and floating-point NICs became known. FETs are voltage-controlled current sources which enable transformation of inductance into capacitance. So FET-based non-Foster circuits can transfer inductors into negative capacitors, namely, negative impedance inverters (NIIs). Operational amplifiers can also implement NFCs. This kind of NFCs is simpler than transistor-based NICs, and it is easier to achieve the desired performance, but due to the limited bandwidth gain product of operational amplifiers, their bandwidths are narrower. Of all the NFCs mentioned above, the transistor type is used most commonly.

Even though NFC enables circuits to cancel the reactance, thus broadening the bandwidth, the circuit plays a less than expected role in the previous measurement. Possible reasons are as follows: mismatch and instability. Firstly, mismatch may be due to the performance of NFCs changing with frequency. The negative reactance varies with frequency, and it is not likely to cancel the reactance in a wide frequency band. What is more, the interface transmission lines also have an impact on negative impedance which is often overlooked, resulting in simulation inaccuracy. Instability is one of the most important issues with NFCs because they are potentially unstable two-port networks [11]. Scarification of
Implementing Kirchhoff’s equivalent circuit of the interface transmission line is analyzed and verified by simulation. Finally, an electrically small dipole loaded with a transistor-based NIC for broadband operation was designed and fabricated.

2. Transistor-Based NIC Design

2.1. Circuit Analysis. Transistor-based NICs generally work by maintaining the current of the load and inverting the voltage at the output port. Consider the floating NIC shown in Figure 1. Port 2 is terminated with the device to be matched whose impedance is $Z_a$ while port 1 acts as the output, connecting with a 50-ohm transmission line. The simplified equivalent circuit of the floating NIC is given in Figure 2. Implementing Kirchhoff’s law, assuming a current source $I_i$ flowing in port 1, the following equations can be obtained:

\[
\frac{1}{r_e} V_1 - \frac{1}{r_e} V_4 = -I_i, \\
\left(\frac{1}{r_e} - g_m\right) V_1 + g_m V_2 + \left(\frac{1}{Z_L} - g_m\right) V_3 + \left(g_m - \frac{1}{r_e} - \frac{1}{Z_L}\right) V_4 = 0, \\
g_m V_1 + \left(\frac{1}{r_e} - g_m\right) V_2 + \left(\frac{1}{r_e} - \frac{1}{Z_L}\right) V_3 + \left(\frac{1}{Z_L} - g_m\right) V_4 = 0.
\]

(1)

The solution to the equations are

\[
\begin{align*}
V_1 &= (2g_mZ_Lr_e - Z_a - Z_L - 2r_e)I_i, \\
V_2 &= -Z_aI_i, \\
V_3 &= -(Z_a + r_e)I_i, \\
V_4 &= (2g_mZ_Lr_e - Z_L - Z_a - r_e)I_i.
\end{align*}
\]

(2)

Thus, the input impedance $Z_m$ seen from port 1 is

\[
Z_m = \frac{V_1}{I_i} = 2g_mZ_Lr_e - Z_a - Z_L - 2r_e,
\]

(3)

where $r_e$ is the emitter resistance of the transistor itself and $g_m$ is the transconductance, affected by the operating point of the transistor. Therefore, by selecting a suitable transistor and employing it at an appropriate operating point, the impedance $Z_a$ can be used to cancel the impedance $Z_a$ or the unwanted part of $Z_a$. It needs to be pointed out that the DC bias network of the transistor is not shown in Figures 1 and 2. Moreover, the distribution parameters of the circuit are not considered in the previous analysis. So when (3) is used to design NICs, more work is needed for better performance which will be introduced later in the following sections.

A negative capacitor is designed and loaded on a dipole to verify the effect of the NIC, as shown in Figure 3. The dipole operates at 450 MHz with a 40 MHz bandwidth. It is well known that the dipole is capacitive in an electrically small case, so the load of NIC should be a capacitor. In addition, the used NIC is an unbalanced structure while the dipole is balanced. So a wideband balun is placed between the dipole and circuit. NPN epitaxial silicon transistor NE68139e is chosen for the circuit as it has advantages such as low noise and up to 2 GHz cut-off frequency. Simulation result shows that the NIC loading dipole has $-10$ dB reflection coefficient from 24 MHz to 828 MHz. The imaginary part of the matched dipole is between $-10$ and 17 ohms, indicating that the reactance of the dipole is mostly cancelled out. This results indicate that NICs have the ability to match impedance in broadband.

2.2. Stability Analysis. The most commonly used stability determination method for microwave circuits is Rollett’s criteria. Rollett’s conditions for 2-port unconditional stability are [12]

\[
K = 1 - \frac{|S_{11}^2 - |S_{22}^2 + |\Delta|^2|}{2|S_{12}S_{21}|} > 1, \quad |\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1.
\]

(4)

An additional condition must also be satisfied wherein the ports must be either open-circuit stable (OCS) or short-circuit stable (SCS). However, this criterion may give an incorrect result when applied to negative impedance circuits [13]. The K factor only analyzes the S-parameters of the circuit, but does not consider the unstable hidden mode.
inside the circuit. The circuit with two non-Foster elements and two positive resistors shown in Figure 4 is a simple example to demonstrate our opinion. In this circuit, if \( R_1, R_2, L, \) and \( C \) satisfy the following conditions:

\[
R_1 = R_2 = R, \\
L = R^2 C < 0, \\
\]

then

\[
Z_{11} = Z_{22} = \frac{(R_1 + Ls)(R_2 + 1/sC)}{R_1 + Ls + R_2 + 1/sC} = R. \\
\]

Using Rollett’s criteria, the circuit is unconditionally stable as its input and output impedance are constant \( R \) in the whole frequency band. But the series RL and RC will oscillate, so the circuit is practically unstable. Actually, the \( K \) factor only considers the input and output port stability of the circuit, and it is enough for most active microwave circuits like RF amplifiers. For the non-Foster case, the circuits are internal unstable, so other ways for stability analysis should be employed.

Many researchers implement time-domain approach for NFC stability analysis, applying a Gaussian pulse excitation and observing the time-domain response of the signal. The circuit is stable if the energy of the signal decays as time goes by. The problem is that transients can be very long in NIC circuits. Additionally, the width of the Gaussian pulse should be designed for operating frequency based on the Fourier transform which is complicated.

Another stability test that has been widely recognized for NICs is the normalized determinant function (NDF) test. In this way, all the possible feedback effects, including those resulting from parasitic elements inside the device models, are taken into account [14]. NDF calculates the open-loop impedance and closed-loop impedance of the circuit. The circuit is stable only if there are no zeros of NDF in the right half of the complex frequency s-plane (RHP). To simplify the calculation, a Nyquist plot is usually used to check the number of RHP zeros which can avoid solving the complex formula. The number of clockwise encirclements of the origin indicates the number of RHP zeros that were produced by the circuit. However, the NDF of the NFCs may be inaccurate because the parasitic parameters and transmission lines have an impact on NFCs which cannot be accurately evaluated. The NDF will also become complicated after considering these factors.

In this paper, we test the loop gain of the circuit for stability analysis. This method determines whether the transmission function will be infinite in some frequencies, which is essentially consistent with the NDF method, but it is not as complicated as the NDF method. For a NFC system,
assuming that the open-loop gain of the system is \( A(s) \) and the function of the feedback network is \( \beta(s) \), then the transfer function of the system is

\[
F(s) = \frac{A(s)}{1 - \beta(s)A(s)}. \tag{7}
\]

The product \( \beta(s)A(s) \) is the loop gain of the system. Note that the denominator of (7) is not a sum, because the NFCs are positive-feedback networks. The circuit is unstable if there are points where the magnitude of the loop gain is greater than 1, the phase is 0, and the phase is decreasing with increasing frequency. Because the transfer function of the system tends to be infinite, the zero input may result in an infinite output in such a case [15]. A commercial simulator, ADS is used to calculate the loop gain and analyze circuit stability. A device in ADS called “OscTest” is inserted in the circuit. This device can evaluate the closed loop, small signal gain of the circuit, which is exactly needed for our stability analysis.

Stability analysis of the NIC shown in Figure 3 is conducted, and it is found that the circuit is unstable below 100 MHz. The magnitude of the loop gain is close to one and the phase is 0 at 80 MHz. To stabilize the NIC, a parallel 15-ohm resistor and 20 pf capacitor network was added in the emitter of the transistor connected to the output port, wherein the resistor improves the stability of the circuit, and the capacitor shorts the resistor at high frequencies, reducing signal loss. Simulated loop gain of the circuit is shown in Figure 5. It can be observed that the magnitude of loop gain after stabilizing becomes further away from 1 and the phase of which is less than 0 from 15 MHz to 1 GHz, indicating that the circuit is stable in this frequency range.

2.3. Transmission Line Analysis. Though the VHF band wavelength is much longer than the distance of the device in NFCs, simulations and experiments indicate that the influence of the transmission line (TL) between devices cannot be ignored. Specifically, the interface TL of the antenna and NFC has great impact on the output performance. Figure 6 compares the reflection coefficient of the circuit in Figure 3 at different interface TL lengths. The result shows that even in tens of megahertz, TL will have an impact on the overall reflection coefficient, not to mention hundreds of megahertz or gigahertz. Generally, the interface TL of the antenna and NFC is two SMA connectors, which will introduce a transition from coaxial to microstrip. The impedance of the antenna with interface TL is different from the original one, resulting in simulation inaccuracy and mismatch. A good
way to minimize the influence of the interface TL is integrating the antenna and NFC on a printed circuit board to eliminate the use of SMA connectors and connecting the dipole directly to the balun without TLs.

For the unavoidable TLs between lumped components (TLBLCs) in NFC, their impact on the circuit must be analyzed and minimized. In order to reduce the transmission mismatch, the TLs of the signal path in the circuit should be a 50-ohm microstrip line. The characteristic impedance $Z_0$ of the microstrip line is determined by the equivalent permittivity $\varepsilon_r$, thickness of the printed circuit board $h$, and line width $w$:

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\varepsilon_r}} \ln \left( \frac{8h + w}{w + 4h} \right), & \frac{w}{h} \leq 1, \\ \frac{120\pi}{\sqrt{\varepsilon_r}} \left[ \frac{w}{h} + 1.393 + 0.667 \ln \left( \frac{w}{h} + 1.444 \right) \right], & \frac{w}{h} \geq 1. \end{cases}$$

If the line width is wider, the difficulty of layout design increases, and the size of the printed circuit board also becomes larger. After comprehensive consideration, a 0.5 mm thick FR4 board with a relative permittivity of 4.4 is used. The 0.95 mm wide 50-ohm microstrip line is convenient for layout design. With the layout of the NFC attained, the influence of the TLBLCs on NFC can be analyzed. In this paper, only the TLBLCs of the signal path is discussed, which is also the part that has the greatest impact on the circuit. The NIC with its signal path layout is shown in Figure 7. A sinusoidal signal source was added at port 1 to see the influence of the TLBLCs on NIC. The voltages at nodes 1, 2, and 3 in the circuit were tested, which would be the same without TLBLCs. The result shown in Figure 8 reveals that the magnitudes of the voltages at these three nodes are almost the same, but the voltage at node 1 produces a significant phase shift with 500 MHz input signal. It indicates that the TLBLCs of the signal path with the characteristic impedance of 50 ohms will not cause circuit mismatch but will obviously affect the phase of the signal. Comparing the phase of reflection coefficient of the NFC with and without TLBLCs, the maximum phase deviation exceeds 30 degrees in the range of 50–500 MHz. It indicates that the effect of the TLBLCs on the phase of the NFC is not only determined by its length. For example, the phase of the NFC with TLBLCs at 100 MHz is 34 degrees higher than the one without, and the phase difference is equivalent to a line length of 283.3 mm, which is much more...
than the length of the signal path. Furthermore, it can be found that the influence of the phase is less when the length of each branch is equal, because phase shift can be compensated to some extent in cross-coupling. So, when designing NFCs, not only should the distance of the components be as small as possible to make the length of the entire signal path short enough, but the length of the TLBLC of each branch should also be kept as uniform as possible.

TLs also influence the stability of the NFCs as they may shift the poles of NDF to RHP, resulting in instability [16]. From the previous analysis, TLs have a great influence on the phase of the NFCs, so when implementing loop gain stability analysis, the TLs should not be ignored too. The actual circuit layout needs to be included in the simulation to obtain a reliable result. In this paper, we use the EM model of TLBLCs and the schematic model of the other lumped components for cosimulation. Simulation result indicates that the circuit with the dipole terminated is stable in the frequencies from 30 MHz to 1000 MHz.

3. Experimental Verification

A prototype of the non-Foster circuit terminated with a dipole is fabricated as shown in Figure 9. The circuit part is $35 \times 35 \times 0.5 \text{mm}^3$ in size, integrated with a 320 mm dipole. No oscillations are observed in the spectrum analyzer when the DC bias is turned on. The NIC-loaded dipole is stable as predicted by previous simulation. It is worth mentioning that the method of circuit stability analysis we use is for a known terminal load, so the terminal load should be cleared before the analysis is conducted.

In order to verify the performance of the negative capacitance generated by the NIC, S-parameters are measured using a vector network analyzer. Figure 10 compares the reflection coefficient of measured, simulated NIC-loaded dipole and the unmatched one. It can be observed that the NIC greatly broadens the bandwidth of the antenna, especially at low frequencies. The simulation results especially the cosimulation considering the influence of TLs are consistent with the measured result. At the frequencies above 400 MHz, the advantages of cosimulation are pronounced. Although the trend of reflection coefficient changes with frequency is similar, the simulated one without TLs is 5 dB lower than the measured one while the cosimulated one with TLs is almost the same as the measured one. This means that NICs may be applied to a higher frequency using cosimulation. The measured result shows that the operating frequency of the matched dipole ranges from 30 MHz to 580 MHz, 14 times

![Figure 8: Voltages of nodes 1, 2, and 3 with 500 MHz sinusoidal signal source added in port 1.](image)

![Figure 9: Fabricated NIC-loaded dipole.](image)

![Figure 10: Reflection coefficient of measured, simulated NIC-loaded dipole and unmatched dipole.](image)
the bandwidth of the one without matching. The imaginary part of the input impedance is shown in Figure 11(b), which indicates that the capacitive part of the electrically small dipole has been canceled well. The mismatch problem caused by the capacitive impedance of the electrically small antenna has been greatly improved.

Note that the loss of the NFC may be greater than its improvement of the matching. It can be seen in Figure 11(a) that the resistance of the NIC matched dipole is around 50 ohms while the unmatched one is almost 0 in the electrically small case. The NFC cannot enhance the radiation impedance of the antenna, so the increased resistance should be the loss resistance. We found that the NIC in this paper itself brings approximately 10 dB extra loss, which weakens the bandwidth benefit of the NIC. The measured gain of the NIC-loaded dipole at 450 MHz is −10 dBi while the unloaded dipole is 1.78 dBi. However, in the case of 100 MHz where the wavelength is 0.11λ (λ is the operating wavelength of the dipole), the gain of the matched one becomes −18 dBi and that of the unmatched one is −32 dBi. A 14 dB gain improvement is obtained in the electrically small case even though the NIC introduces 10 dB loss. The total efficiency (including reflection efficiency and radiation efficiency) of the NIC matched and unmatched dipole is shown in Figure 12. It indicates that the efficiency of the matched dipole increases by 10 percent compared to that of the unmatched one below 300 MHz. The benefit of match improvement is better than the loss of the NIC. However, the efficiency of the NIC matched dipole in 450 MHz is 20 percent lower than that of the unmatched one, indicating that the loss of the NIC is better than the match improvement. Thus, the NIC is more suitable for a serious mismatch case like electrically small antennas, rather than broadening the original antenna band because of its loss. A reasonable approach to tackle the issue of loss could be to add a low-noise amplifier after the NIC for compensation.

4. Conclusion

This paper analyzes the impedance matching capability, stability, and TL influence of the NFCs. Circuit characteristics of Linvill’s negative impedance converter is given by means of Kirchhoff’s law. Difference stability analysis methods including Rollett’s criteria, NDF, and time-domain analysis are compared and inapplicability of some of the methods for NFCs is demonstrated. On the other hand, the loop gain method used in this paper is proved to be effective in NFC stability analysis. TLs have an impact on the phase of signal and stability of NFCs. In order to reduce uncertainty of interface TLs, the NIC and antenna are integrated together to avoid the transition from coaxial to microstrip. A significant method to improve simulation accuracy is to add the layout...
TLBLCs into the schematic and cosimulate the whole circuit. The more consistent the added TLBLCs is with the actual layout, the higher the simulation accuracy is, especially in high frequency. The NIC is applied to match a 320 mm electrically small dipole. Measured and simulated results show good agreement with each other. The measurement result shows that the matched dipole achieves less than $-10\,\text{dB}$ reflection coefficient from 30 MHz to 580 MHz. A 14 dB gain improvement is obtained in the electrically small case, verifying that the NIC is effective.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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