Averaged energy-balance analysis of wavy micro-channels

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Abstract. This study presents numerical simulations of the convective heat transfer on wavy micro-channels to investigate heat transfer enhancement in these systems. The goal is to extend the analysis of our previous work [1, 2], by proposing a methodology based on local and global energy balances in the device instead of the commonly used Nusselt number. The analysis is performed on a single-wave baseline micro-channel model that is exposed to a heat influx. The governing equations for an incompressible laminar flow and conjugate heat transfer are first built, and then solved, for representative models, under several operating conditions, by the finite element technique. From computed velocity, pressure and temperature fields, local and global energy balances based on cross-section-averaged velocities and temperatures enable calculating the heat rate at each section. Results from this study show that this so-called averaged energy-balance methodology enables an accurate assessment of the channel performance.

1. Introduction
Miniaturization and scaling of integrated circuits for high power-density electronic devices has increased the demand of high heat removal capabilities. Although many alternatives have been proposed to solve this problem, an approach based on micro-channels was suggested by Tuckerman and Pease [3] several years ago. Since then, many studies and reviews of micro-channel designs, based on wavy geometries, using single-phase fluids, have been conducted; e.g. [4, 5], with the idea of improving fluid mixing and surface area [6, 7]. Using the Nusselt number as the basis for analysis, recent work by Moon et al. [1] has shown that the addition of harmonic surfaces to the basic wavy topology enhances the heat transfer. On the other hand, by considering a variety of conductive materials for the solid block enclosing the channel, Moon et al. [2] found that the selection of specific material highly impacts the diffusion of heat in the solid, but it is negligible on the Nusselt number for the fluid.

Here we consider a methodology that uses the heat transfer rate instead of the common Nusselt number as the basis for the calculations. To this end, we focus on single-wave micro-channel models and build for them the conjugate heat transfer equations. The model equations are then solved on a representative computational domain using finite elements to obtain velocity-, pressure- and temperature-fields for the entire system. Finally, values of the pressure drop, inlet-to-outlet fluid and the heat rate are used to assess the relative system performance.
2. Problem Description and Mathematical Model
The device is illustrated schematically in Fig. 1. The model consists of a channel of square cross-section that is enclosed by a copper block. Its dimensions are: 500 μm × 500 μm × 20 mm, with two 2 mm inlet- and outlet-straight sections and a 16 mm middle wavy-section. The block is a square duct of thickness 1.5 mm and length 20 mm. A heat flux of 47 W/cm² is uniformly applied at the bottom surface and advected out of the system. The set of micro-channel-block designs, referred to as wavy configuration, has sinusoidal profiles at the top and bottom surfaces, based on the function \( y(x) = A \cdot \sin(2\pi x/\lambda) \), where \( x \) and \( y \) refer to the streamwise and vertical directions, respectively, \( A \) as the wave amplitude, and \( \lambda \) is its wavelength.

![Figure 1. Schematic side view of a wavy micro-channel geometry (dimensions in mm).](image)

The governing equations correspond to a three-dimensional conjugate model for two domains: the channel inside of which water flows \( \Omega_f \), and the solid block \( \Omega_s \) surrounding it. For \( \Omega_f \), we consider the incompressible flow of a Newtonian fluid, with constant properties, in the laminar regime, under steady-state conditions, and without body forces, viscous dissipation and negligible radiative heat transfer, the mathematical model is given by

\[
\begin{align*}
\nabla \cdot \mathbf{u} &= 0 \\
\rho_f (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p + \mu_f \nabla^2 \mathbf{u} \\
\rho_f c_{p,f} (\mathbf{u} \cdot \nabla) T &= k_f \nabla^2 T
\end{align*}
\]

whereas for \( \Omega_s \), the energy equation for a homogeneous solid material is given by

\[
k_s \nabla^2 T_s = 0
\]

where \( \mathbf{u} \), in Eqs. (1)-(3), is the velocity vector with components \( u, v \) and \( w \), in the \( x-, y- \), and \( z \)-direction, respectively; \( p \) is the fluid pressure, \( \rho_f \) is its density, and \( c_{p,f} \) is the specific heat, \( k_f \) the fluid thermal conductivity, \( \mu_f \) is the dynamic viscosity and \( T \) its temperature. For the solid block, \( k_s, \rho_s, c_{p,s} \) and \( T_s \), in Eq. (4), are the thermal conductivity, density and specific heat of the block material, and its corresponding temperature, respectively. The values of properties for both fluid (water) and solid (copper), used here, are: \( k_f = 0.61 \text{ W/m} \cdot \text{K}, c_{p,f} = 4.18 \text{ kJ/kg} \cdot \text{K}, \rho_f = 908.4 \text{ kg/m}^3, \mu = 1.007 \times 10^{-3} \text{ Pa} \cdot \text{s}, k_s = 403.7 \text{ W/m} \cdot \text{K}, c_{p,s} = 0.375 \text{ kJ/kg} \cdot \text{K}, \) and \( \rho_s = 9933 \text{ kg/m}^3 \), all taken at a reference temperature of 300 K.

The boundary conditions are: \( u = u_{in} \) at the channel inlet; zero-pressure and zero-viscous stresses at the outlet; no-slip and no-penetration conditions at the channel walls. In addition, \( T_{in} = 300 \text{ K} \) at the inlet, \( T = T_s \) and \( k_f \partial T/\partial n = k_s \partial T_s/\partial n \) at the solid-fluid interface-walls; and \( \partial T/\partial n = 0 \) at the outlet. The heat-generating chip is modeled as uniform heat influx of \( q'' = 47 \text{ W/cm}^2 \), at the bottom surface (on a 10 mm × 1.5 mm area); all other surfaces of the solid are considered adiabatic. System symmetry is prescribed at the mid-plane \( z = 0 \).
3. Solution Method and Grid Independence
To solve Eqs. (1)-(4), these are first discretized on the channel $\Omega_f$, and solid-block $\Omega_s$, domains using finite elements, and then solved with COMSOL Multiphysics (http://www.comsol.com). We use unstructured meshes with four-node tetrahedral elements for the fluid temperature, velocity and pressure, and the temperatures within the solid. Hexahedral elements are used close to the walls for sharp representation of the fluid-solid interface. The resulting system is solved iteratively, with a relative tolerance of $10^{-6}$. After testing various grid sizes in the (2.9-4.2) million-element range, mesh independence is ensured with a grid with 3.6 million elements.

4. Hydrodynamics and Heat Transfer
Numerical solutions are provided in Figs. 2 and 3 for $A = \{0, 50, 100, 150, 200\} \mu m$, and $Re = D_h u_{in} \rho_f / \mu_f = \{50, 100, 150\}$, with $\lambda = 2 \text{mm}$ and $q'' = 47 \text{ W/cm}^2$. Figure 2 shows a comparison of the results for the pressure drop $\Delta p$, as function of $A$ and $Re$, to those of Gong et al. [6]. From the figure it can be observed that our solutions not only qualitatively follow the same trend as those of [6], but quantitatively they are also very close; the maximum absolute value of the percentage difference is found to be less than 5%. On the other hand, Fig. 3, illustrates the results as streamlines, temperature contours and isotherms, all computed at the mid-plane $z = 0$, for the entire device, for $A = 150 \mu m$ and $Re = 100$ (results for other values of the parameters show similar behavior). From Fig. 3(a), it is seen that the uniform flow at the inlet develops into periodic patterns, and then into a new uniform flow near its outlet. Figures 3(b) and 3(c) show that the temperature field transitions from a uniform profile and develops – but not in a periodic fashion – as the fluid travels along the channel. Also, the temperature contours in the copper block are spanwise homogeneous due to its high thermal conductivity, but they increase along the $x$-direction, due to the advective process from the cooling water.

5. Averaged Energy Balance Equations
To develop the methodology, we follow the investigations by refs. [8]–[10], but here we consider that the thermal device deals with a constant heat influx $Q_{in} = q'' A_{base}$. Thus, under this consideration, the results for the cross-sectional-averaged pressures $\overline{p}$, streamwise flow velocities $\overline{u}$, and the fluid temperatures $\overline{T}$, derived from the local values of the $p$-, $u$-, and $T$-fields, are

$$\overline{p} = \frac{1}{A} \int_A p \, dA; \quad \overline{u} = \frac{1}{A} \int_A (u \cdot n) \, dA; \quad \overline{T} = \frac{\int_A (uT \cdot n) \, dA}{\int_A (u \cdot n) \, dA},$$

(5)
where \( A \) is the cross-sectional area normal to the unit vector \( \mathbf{n} \), associated with the surface of interest \( dA = dy
dz \), at any point in the streamwise direction \( x \).

By partitioning the wavy-section of the channel into \( N \) subsections (here \( N = 8 \)) of length \( \Delta L \), and using the averaged values \( \overline{u} \) and \( \overline{T} \), an energy balance provides the net heat rate \( Q \) advected by the fluid in the specific subsection as

\[
Q_i = \rho_f \overline{u} A_{p,f} \left[ \overline{T}(x_i) - \overline{T}(x_{i-1}) \right],
\]

where \( x_{i-1} = x([i - 1] \Delta L) \) and \( x_i = x(j \Delta L) \), for \( i = 1, 2, \ldots, N \), represent the location of the cross-section plane of the \( i \)-th subsection and \( Q_i \) is the corresponding heat rate. The total heat transferred by the device, \( Q_T \), is given by

\[
Q_T = \left( \sum_{i=1}^{N=8} Q_i \right) + Q_{L_{x-in}} + Q_{L_{x-out}} = \rho_f \overline{u} A_{p,f} \left[ \overline{T}_{out} - \overline{T}_{in} \right],
\]

where \( T_{in} \) and \( T_{out} \), are the terminal temperatures of the entire device, and \( Q_{L_{x-in}} \) and \( Q_{L_{x-out}} \) are the heat transfer contributions at the entrance and exit straight-sections.

![Figure 4](image.jpg)  
Figure 4. Fraction of inlet heat rate transferred at each section, for \( A = 150 \mu m \).

![Figure 5](image.jpg)  
Figure 5. Spanwise-averaged fluid temperature distribution, \( \overline{T}(x) \).

Results from the methodology are shown in Fig. 4 as a fraction of the inlet heat rate \( Q/Q_{in} \) (with \( Q_{in} = q'' A_{base} = 7.05 \text{ J} \)), advected by the fluid at each section of the micro-channel – and its cumulative values – for a sample amplitude of \( A = 150 \mu m \) and \( Re = \{50, 100, 150\} \), and in Fig. 5 in terms of the spanwise-averaged fluid temperature distribution \( \overline{T}(x) \), for \( A = \{0, 50, 100, 150, 200\} \mu m \), and the same set of \( Re \) numbers. From Fig. 4, it can be observed that, as expected, the fraction \( Q/Q_{in} \) at each section varies both along the streamwise direction, and with \( Re \) number. Notably, the largest values occur at the inlet section \( (Q/Q_{in} \approx 20\%) \) and those in the middle sections with \( Q/Q_{in} \approx 10 - 13\% \), the smallest values appearing in sections downstream in the channel \( (Q/Q_{in} < 8.5\%) \), although \( Q_{in} \), is applied in the region of sections 2-7. Moreover, the \( Q/Q_{in} \)-distribution changes with \( Re \) number; e.g., a smooth decline in \( Q/Q_{in} \) appears for \( Re = 50 \), and a decrease-increase-decrease transition arises for \( Re = 100 \) and, more pronounced, for \( Re = 150 \). This behavior is also reflected in the distribution of the cumulative values of \( Q/Q_{in} \) (see Fig. 4), as they approach the value of unity at the outlet section of the channel, due to the larger fluid capability to advect energy as \( Re \) increases.
The aforementioned behavior of $Q/Q_{in}$ in the device is confirmed in Fig. 5, as two important features for $T$ can be identified: (1) for a fixed $Re$, and all amplitudes considered, the temperature difference between inlet and outlet at each section, $\Delta T$, have larger values in sections closer to the inlet and smaller ones closer to the channel outlet; and (2) although the trends are similar; for a fixed $x$-location, and a fixed amplitude $A$, smaller values of $\bar{T}$ take place for larger values of $Re$. This shows that the fluid leaving at a lower temperature has the potential to advect more energy, thus positively affecting the block temperature. It is to note that similar results occur for other values of $A$. However, surprisingly, the values of $\bar{T}$ do not change significantly with wave amplitude, maybe due to the averaging process of the local fluid-temperature field.

6. Conclusions
In this study, we have considered the hydrodynamic and heat transfer characteristics of single-wave wavy micro-channels by introducing a methodology of analysis based on local and global energy balances, from three-dimensional velocity and temperature fields, to design more efficient devices. The approach is useful in providing accurate and clear information, at the expense of some generality, to better understand the conjugate heat transfer processes in these devices. Using the procedure, along with the concept of fraction of the total heat rate input onto the specific device, the numerical data have shown that, it is possible to obtain a distribution of both the average fluid temperature along the device, and the corresponding fraction of the inlet heat transfer rate at specific sections to assess its relative performance.

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