Kinematic Model for Shear Assessment of RC Short Columns Subjected to Frost Damage

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Abstract
Few reports have described practice-based models assessing ultimate strengths of existing reinforced concrete (RC) members subjected to frost damage. This paper presents a kinematic model for shear assessment of damaged RC short columns based on the upper bound theorem. Without regressive functions, the developed model predicts the shear strength contribution of damaged concrete when the displacement field is divided into undamaged and damaged zones based on damage depths obtained from core sampling. The model accuracy is verified by comparison of its predictions with those of earlier test results of 14 RC columns presenting shear failure after freeze–thaw exposure. The analytical predictions show good agreement with experimentally obtained results within error of 20%. Shear strength predictions for different damage depths are presented for an existing RC bridge pier with severe frost damage. Rational shear assessment was achieved because the kinematic analysis directly correlates the damage depth with shear strength reduction.

1. Introduction
Frost damage develops non-uniformly on reinforced concrete (RC) structures, strongly influencing their structural performance (Matthews and Bigaj-van Vliet 2013). Most earlier studies examining frost damage have specifically assessed plain concrete or mortar (Hasan et al. 2004; Li et al. 2017): RC members with frost damage have received little attention (Duan et al. 2017; Petersen et al. 2007; Xu et al. 2016). Rational structural assessment is quite difficult to conduct in mechanical terms. Strength assessment is often replaced by durability assessment based on the relative dynamic modulus of elasticity (RDME) (Duan et al. 2014; Qin et al. 2016), which corresponds well with the mechanical properties of plain concrete. Results obtained using various assessment models do not correlate with information obtained from on-site inspections. For example, if a field engineer were to sample concrete cores from an existing structure, then the internal distribution of frost damage would not be predictable in a three-dimensional finite element model (Berto et al. 2015; Gong and Maekawa 2018; Hanjari et al. 2013). A practice-based model for strength assessment is crucially important to produce a mechanically supported decision about a future course of action.

Frost damage, depending on its depth, alters RC member failure modes from flexure to shear-domination (Hayashida et al. 2014; Qin et al. 2017). For shear assessment of such existing RC structural members, design equations are not necessarily appropriate (fib 2010; Su et al. 2019) because of their inherent conservative character, which stems from the lower bound analysis. In fact, several well-established theories for shear (Mihaylov et al. 2013; Muttoni and Fernández Ruiz 2008; Yang et al. 2016b) are based not only on the force equilibrium, but on the failure kinematics. The former and latter respectively correlate to lower and upper bound analyses. This study therefore presents a kinematic model combined with damage depth of freeze–thaw action to ascertain the actual shear capacity and to avoid extra conservativeness.

The present analysis enables direct correlation between the shear strengths of RC short columns and the damage depth obtained from on-site inspections, under the physically consistent framework of the upper bound theorem (Nielsen and Braestrup 1978) (Fig. 1). A typical displacement field for shear failure is divisible into damaged and undamaged zones. That division enables the shear component derivation of the damaged concrete based on its internal work. Subsequently, the developed analysis is validated by comparison of its predictions with experimental data of 14 RC columns currently available in the literature (Rong et al. 2020; Yang et al. 2016a; Zhang et al. 2019). Finally, the validated model predicts the shear strength of an existing RC bridge pier with severe frost damage. Information obtained from a concrete core sampled from the pier is used for shear assessment. Results indicate a quantitative relation between the damage depth and shear strength. They underscore the possibility of rational shear assessment according to the damage depth.

2. Shear strength analysis combined with frost damage depth

2.1 Analytical framework
To calculate shear strength based on the upper bound...
theorem, we introduce the following assumptions (Nielsen and Braestrup 1978):
(a) Concrete and steel reinforcements are in a plane stress state.
(b) Dowel action of the longitudinal reinforcements is neglected.
(c) Perfect bonding exists between steel reinforcement and concrete.
(d) Concrete does not carry tensile stress.

The conventional limit analysis assumes rigid, perfectly plastic behavior for concrete and steel reinforcements (Nielsen and Braestrup 1978). Although the effectiveness factor \( v_c \) is determined empirically in such analyses, \( v_c \) for frost-damaged RC elements is not studied fully because of mechanical anisotropy, which is dependent on the reinforcement ratio (Kanazawa and Sato 2018). We use the effective stress–strain relation proposed by Vecchio and Collins (1986) (Fig. 2 and Eqs. 1a and 1b) for the present study because \( v_c \) is related to the principal tensile strain \( \varepsilon_1 \), as determined by the strain compatibility under the framework of the upper bound theorem.

\[
\sigma_2 = v_c f_c (\varepsilon_2 / \varepsilon_c) - (\varepsilon_2 / \varepsilon_c)^2 \tag{1a}
\]

\[
v_c = \frac{1}{(0.8 - 0.34 \varepsilon_1 / \varepsilon_c)} \tag{1b}
\]

In those equations, \( \sigma_2 \) and \( \varepsilon_2 \) respectively represent principal compressive stress and strain, \( \varepsilon_c \) is defined in Fig. 2. Stresses and strains in compression are negative; those denoted with a prime have positive compression.

Elastic–perfectly plastic behavior is assumed for longitudinal and web reinforcements, as presented in Fig. 3. We introduce the Mohr–Coulomb model proposed by Pujol et al. (2016) as the failure criterion. This model incorporates consideration of \( \sigma_t \) and \( \sigma_a \) in Eq. (3), respectively representing axial stress of longitudinal reinforcement and confinement stress of web reinforcements. Eq. (3) is the solution for which Mohr’s circle reaches Coulomb’s criterion of Eq. (4), as portrayed in Fig. 4.

\[
\sigma_s = \frac{N + T}{A_t} \tag{2a}
\]

\[
T = \frac{1}{4} A_t \sigma_{s0} \left( 1 - \frac{N}{0.33 A_t} \right) \tag{2b}
\]

\[
\sigma_t = \rho_{st} \sigma_{sat} \tag{2c}
\]

Therein, \( N \) and \( T \) respectively denote the external axial force and the resultant force in longitudinal reinforcements. In addition, \( A_t \) represents the cross-sectional area except for concrete cover, \( A_y \) represents the area of longitudinal reinforcements, and \( \sigma_{s0} \) and \( \sigma_{sat} \) respec-
respectively stand for the yield strength of longitudinal and web reinforcements. In addition, $\rho_{sw}$ is the web reinforcement ratio.

$$\tau = \frac{1}{5} \left( \frac{2}{3} f'_e + 4\sigma_s - \sigma_t \right) \left( \frac{2}{3} f_e' - \sigma_s + 4\sigma_t \right)$$

(3)

$$\tau_{\cos} = \frac{1}{6} f_e' + \frac{3}{4} \sigma'$$

(4)

In this equation, 1/6 and 3/4 respectively correspond to $k_1$ and $k_2$ in Fig. 4.

Figure 5 depicts displacement fields combined with damage depths. We assumed the following:

(e) The displacement field is divisible into undamaged and damaged zones, as defined by damage depths $D_1$ and $D_3$.

(f) Inclination angles of the respective zones vary as $\alpha_1 \leq \alpha_2 \leq \alpha_3$ and $\beta_1 \leq \beta_2 \leq \beta_3$.

(g) Longitudinal strains of web reinforcements develop along the $y$-axis in Fig. 5.

Assumption (f) is derived from the geometrical constraints. The possible ranges of the angles of inclination, $\alpha_i$ and $\beta_i$, are presented in Table 1. It is noteworthy that each zone is denoted by subscript $i$ hereinafter. $i = 1, 3$ and $i = 2$ respectively denote damaged and undamaged zones. In addition, $L$ and $l$ in Fig. 5 respectively represent the clear span and the net one, which the latter contributes to internal work. The mechanical boundary condition of antisymmetric moment is assumed for Fig. 5. The cantilever specimens with shear span of $a$ are therefore analyzed as beams having a clear span of $L = 2a$, as presented in Fig. 5.

### Table 1 Possible ranges of inclination angles.

| Angles | Upper limit | Lower limit |
|--------|-------------|-------------|
| $\alpha_1$ | $\pi/2$ | $\alpha_2$ |
| $\alpha_2$ | $\alpha_1$ | $\alpha_3$ |
| $\alpha_3$ | $\alpha_2$ | 0 |
| $\beta_1$ | $\beta_2$ | $\tan^{-1} \left( \frac{D_1}{L - D_1 \cot \beta_1 - D_2 \cot \beta_2} \right)$ |
| $\beta_2$ | $\beta_3$ | $\tan^{-1} \left( \frac{D_2}{L - D_2 \cot \beta_3} \right)$ |
| $\beta_3$ | $\pi/2$ | $\tan^{-1} \left( \frac{b}{L} \right)$ |

In the developed displacement field (Fig. 5), the displacements of $u_\sin$ and $u_\cos$ are respectively imposed on the longitudinal and transverse directions of the column. It follows that the equilibrium between external and internal works is expressed as Eq. (5). One obtains shear strength $V$ by arranging Eq. (5) with $Vu$ and by then differentiating it with respect to $u$.

$$Vu \cos \alpha + Nu \sin \alpha = \sum_{i=1}^{N} (W_{ci} + W_{di} + W_{wi})$$

(5)

In this equation, $W_{ci}$, $W_{di}$, and $W_{wi}$ respectively represent the internal work of concrete, longitudinal, and web reinforcements.

![Fig. 4 Mohr–Coulomb failure criterion.](image1)

![Fig. 5 Developed displacement field with damage depth.](image2)
(1) Concrete internal work

**Figure 5** presents the displacements, strains, and the field width as presented below.

\[ u_{ii} = u \cos \alpha_i \sin \beta_i, \quad u_{iy} = u \cos \alpha_i \cos \beta_i, \]
\[ e_{ii} = 0 \]  \hspace{1cm} (6)

\[ e_{iy} = \frac{u}{l} \cos \alpha_i \cot \beta_i \]  \hspace{1cm} (7a)

\[ \gamma_i = \frac{u}{l} \cos \alpha_i \]  \hspace{1cm} (7c)

\[ \delta_i = l \sin \beta_i \]  \hspace{1cm} (8)

In those equations, \( \gamma_i \) denotes the engineering shear strain. Eqs. (7a)–(7c) represent the principal strains as presented below.

\[ \cos \cot \nu_i i u \]
\[ \sin \cot \nu_i i u \]
\[ \gamma_i = \frac{u}{l} \cos \alpha_i \]  \hspace{1cm} (7a)

\[ \cos \nu_i i u \]
\[ \sin \nu_i i u \]
\[ \delta_i = l \sin \beta_i \]  \hspace{1cm} (8)

One obtains the concrete internal work per unit volume \( w_{ci} \) through integration of Eq. (1).

\[ w_{ci} = \int \sigma_{ii} d e_{ii} = \frac{\nu_i f_{d,l} e_{ii}^2}{e_{ii}} \left( 1 - \frac{\nu_i}{3 e_{ii}} \right) \]  \hspace{1cm} (10)

The volumetric work is given by multiplying \( b_{ii,j} l \) by Eq. (10) as shown below.

\[ w_{ci} = b_{ii,j} l \nu_i f_{d,l} e_{ii}^2 \left( 1 - \frac{\nu_i}{3 e_{ii}} \right) \]  \hspace{1cm} (11)

In this equation, \( b_{ii} \) and \( j_i \) respectively stand for the net width without cover depths and the height of each zone, which equals \( D_2 \) (Fig. 5) in the undamaged zone \( (i = 2) \). Subtracting cover depths from \( D_1 \) gives \( j_{1,3} \) in the damaged zones.

(2) Internal work of longitudinal reinforcements

The strain of longitudinal reinforcements is determined by its elongation of \( u_{ii} \).

\[ e_{ii} = \frac{u \sin \alpha_i}{\delta_i / \sin \beta_i} = \frac{u \sin \alpha_i}{l} \]  \hspace{1cm} (12)

Eq. (13) gives the internal work per unit volume as

\[ w_{ci} = p_{ii,j} \int \sigma_{ii} d e_{ii} \]  \hspace{1cm} (13)

where \( p_{ii,j} \) is the longitudinal reinforcement ratio, \( \sigma_{ii} \) and \( e_{ii} \) respectively denote stress and strain in the respective zones.

Eq. (13) becomes Eqs. (14a) and (14b), respectively, before and after yielding of longitudinal reinforcements.

Eq. (14b) shows that the gray zone of **Fig. 3** is subtracted from the total work after yielding.

\[ w_{ci} = \frac{p_{ii,j} E_{ii} e_{ii}^2}{2} \left( \cdot \sigma_{ii} = E_{ii} e_{ii} \right) \]  \hspace{1cm} (14a)

\[ w_{ci} = p_{ii,j} \sigma_{ii} \left( \cdot \sigma_{ii} = \frac{\sigma_{ii}}{2 E_{ii}} \right) \left( \cdot \sigma_{ii} = \sigma_{ii} \right) \]  \hspace{1cm} (14b)

In those equations, \( E_{ii} \) represents the modulus of elasticity of longitudinal reinforcements.

Internal work before and after yielding (Eqs. (15a) and (15b)) is given by multiplying \( b_{ii,j} l, \) respectively, to Eqs. (14a) and (14b).

\[ w_{ci} = \frac{p_{ii,j} E_{ii} j_{ii} l}{2} \left( \sin \alpha_i \right)^2 \]  \hspace{1cm} (15a)

\[ w_{ci} = p_{ii,j} E_{ii} \sigma_{ii} \left( \sin \alpha_i, \frac{1}{2 E_{ii}} \right) \]  \hspace{1cm} (15b)

(3) Internal work of web reinforcements

To help elucidate the deformational behavior, one can explain the strains of web reinforcements \( e_{sw} \) develop along the \( \gamma_i \) axis in each zone, as depicted in **Fig. 6**, in which the displacement field is not divided. One obtains Eq. (16) by dividing the elongation of \( u_{ii} = u \cos \alpha_i \cos \beta_i \) by the field width of \( \delta_i \).

\[ e_{sw} = \frac{u \sin \alpha_i \cos \beta_i}{l} \]  \hspace{1cm} (16)

The internal work is calculable using the same expansion as that used for longitudinal reinforcements.

Replacing the subscript \( ii \) by \( sw, \) one obtains Eqs. (17a) and (17b), respectively denoting the volumetric internal work before and after yielding.

\[ W_{sw,i} = \frac{b_{sw,j} E_{sw} j_{sw} l}{2} \left( \cos \alpha_i \cos \beta_i \right)^2 \]  \hspace{1cm} (17a)

\[ W_{sw,i} = b_{sw,j} E_{sw} \sigma_{sw} \left( \cos \alpha_i \cos \beta_i \right) \]  \hspace{1cm} (17b)

2.3 Derivation of shear strength

Arranging Eq. (5) with \( V_u \) gives Eq. (18). Differentiating

![Fig. 6 Schematic drawing of the deformational behavior of web reinforcements.](image-url)
with respect to \( u \) yields the shear strength \( V \).

\[
V_u = \frac{1}{3} \left( \frac{W_{c_1} + W_{c_2} + W_{c_3}}{\cos \alpha_i} \right) - Nu \tan \alpha
\]  

(18)

The shear component of concrete \( V_{c_1} \) is found by differentiating the concrete internal work of Eq. (11):

\[
V_{c_1} = b_i j \left( \frac{1}{\varepsilon_{s_1}^2} \right) \left[ \left( \frac{d\varepsilon_{s_1}}{du} \right) + \frac{2v_1}{u} \left( \frac{1 - \varepsilon_{s_1}}{3\varepsilon_{s_1}} \right) - \frac{v_1\varepsilon_{s_1}}{3u\varepsilon_{s_1}} \right] 
\]  

(19)

In this equation, \( \frac{d\varepsilon_{s_1}}{du} \) equals \( \frac{d\varepsilon_{c_1}}{du} = 0.34\varepsilon_{c_1}^2 / \varepsilon_{c_1} \) by Eq. (1); it becomes Eqs. (20a) and (20b).

\[
\frac{d\varepsilon_{s_1}}{du} = 0.34\varepsilon_{c_1}^2 \quad \text{(when } v_1 > 1) 
\]

(20a)

\[
\frac{d\varepsilon_{s_1}}{du} = 0 \quad \text{(when } v_1 < 1) 
\]

(20b)

Shear components of longitudinal and web reinforcements are shown, respectively, in Eqs. (21a)–(21b) and Eqs. (22a)–(22b). The latter (Eqs. (21b) and (22b)) are those after yielding of reinforcements.

\[
V_{s_1} = b_s \rho_{s_1} l, E_{s_1} \left( \frac{u \sin^2 \alpha_i}{l} \right) 
\]

(21a)

\[
V_{s_2} = b_s \rho_{s_2} l, E_{s_2} \sin \alpha_i 
\]

(21b)

\[
V_{w_1} = b_w \rho_{w_1} E_{w_1} \left( \frac{u \cos^2 \alpha_i \cot^2 \beta_i}{l} \right) 
\]

(22a)

\[
V_{w_2} = b_w \rho_{w_2} E_{w_2} \left( \frac{u \cos \alpha_i \cot \beta_i}{l} \right) 
\]

(22b)

### 3. Model validation

#### 3.1 Analysis details

(1) **List of specimens**

Table 2 consists of the 14 currently available tests of RC short columns presenting shear failure. The results include 11 specimens subjected to frost damage and three undamaged ones (SW-1, SW-9, and ZD-0). We assume temporarily that \( \varepsilon_{c_1} \) equals -0.003 and -0.002, respectively, for damaged and undamaged zones because the original data were not reported. We also assume that \( \varepsilon_{c_2} \) becomes larger because of frost damage (Hasan et al. 2004).

It is noteworthy that the reinforcement ratios of \( \rho_{s_1} \) and \( \rho_{w_2} \) presented in Table 2 were ascertained based on the cross-sectional area without cover depth. They differed from the original data. All the longitudinal reinforcements are included in \( \rho_{s_1} \). For this reason, it differs from the conventional tensile reinforcement ratio flexural analysis.

(2) **Application of failure criteria**

**Figures 7 and 8** respectively display details and the failure pattern of the representative specimen (SW-15). Experiment results showed that shear strength was sustained after spalling of the concrete cover (damaged zones). The present analysis terminates when the undamaged zone satisfies Eq. (23).

\[
\frac{\tau - \tau_{con}}{\tau_{con}} \leq 0.01
\]

(23)

In that expression, \( \tau \) and \( \tau_{con} \) are defined respectively by Eqs. (3) and (4). To calculate \( \tau_{con} \), the normal stress on the failure surface (\( \sigma \)) was found to be the following (Fig. 9):

![Image](image-url)
The developed analysis does not incorporate consideration of shear reversal effects, but load–displacement responses under repeated loading coincide approximately with that under monotonic loading to a degree after yielding of longitudinal reinforcements (Ozcebe 1989). The peak strengths after yielding taken from the cyclic response envelope are compared with analytical results.

(3) Damage depth estimation

There is little information about damage depth developed inside RC members (Petersen et al. 2004; Qin et al. 2017). The relations presented in Fig. 10 were obtained from three depths of a square column with cross-sectional area of 200 mm × 200 mm (Qin et al. 2017). Results show that RDME values at depths of 0.065 h and 0.185 h decreased more than 60% (threshold value for frost-damaged concrete according to ASTM C666/C666M-03 (2008)) during 100 freeze–thaw cycles.

Table 3 presents the accuracy of model prediction ($V_{ana}/V_{exp}$) and the coefficient of variation (C.O.V.) obtained from four levels of damage depth: $D_{1,3}/h = 0.150$, 0.175, 0.200, and 0.225. It is noteworthy that the results presented in Table 3 are the average of 11 damaged specimens in Table 2. The present analysis yields accurate predictions of the experimentally obtained shear strengths within error of 6% on average. Results also demonstrate that the damage depth of $D_{1,3}/h = 0.200$ shows the best agreement. That of $D_{1,3}/h = 0.225$ presents the smallest coefficient of variation. We assume that all

$$\sigma' = \frac{D_1}{h} N \sin \beta_1 - V_{c2} \cos \beta_2$$

(24)

$V_{ana}$ and $V_{exp}$ respectively represent the shear strength obtained from experiments and analysis.

Fig. 7 Dimensions and reinforcement details (SW-15, unit: mm).

Fig. 8 Typical failure pattern of analyzed specimens (SW-15).

Fig. 9 Normal stress on failure surface (undamaged zone).

Fig. 10 Typical failure pattern of analyzed specimens (SW-15).
specimens in Table 2 have damage depth of 0.200h because $D_{i,j}/h = 0.225$ might overestimate the actual damage depths. Although comparison with a broader range of test results is necessary to verify this assumption, the sufficient accuracy of strength prediction (Table 3) is achieved by such simplification of the damage depth. The results suggest that the present model would be easily incorporated into maintenance practice even if the actual damage depth could not be ascertained exactly during on-site inspections. Actually, an existing RC bridge with severe frost damage has damage depth of approximately 0.200h, as presented in the following section.

3.2 Discussion of model validity
As a depiction of the analytical process, Fig. 11 presents the total internal work dependence on inclination angle $\beta_1$ at damaged zone ($j = 1$) of SW-13. This response was obtained at $u = 36.5$ mm, which gives the upper bound solution. Results demonstrate that the total internal work is minimized with respect to inclination angle $\beta_1$. The same tendency was observed when the other angles of inclination varied within their possible ranges (Table 1). The present analysis calculated the internal work for all possible angles at a given displacement $u$, determining the shear strength by the minimum work. Displacement $u$ is increased until upper bound solutions are derived by satisfying Eq. (23).

Figure 12 presents the shear strength–displacement response of specimen SW-13. Although the conventional upper bound analysis yields ultimate strength only (Nielsen and Braestrup 1978), the present model provides load–displacement curves because of the incremental analysis for $u$. The upper bound solution is presented as the white circle ($V_{ana}$), which satisfies both strain compatibility and the failure criterion. Figure 13 portrays a comparison between analytical and experimental shear strengths ($V_{exp}$). The black plots present the undamaged specimens (SW-1, SW-9, and ZD-0). These results were obtained under the displacement field with no damaged zone of Fig. 5. Analytical solutions for SW-9 and ZD-0 agree very closely with test results. Results also demonstrate that $V_{exp}$ of SW-1 was not well predicted. Yang et al. (2016) reported that the flexural deformation of SW-1 became twice as great as the shear deformation. This experimental evidence might lead to a discrepancy because the displacement field of Fig. 5 is applicable to shear-dominant behavior.

In Fig. 13, close agreement for damaged specimens is readily apparent within error of 20%. The model accuracy is estimated by assuming that a minimum upper bound solution ($V_{ana}$) coincides with an exact solution ($V_{exp}$) (Ashour 2000). Results also indicate that $V_{exp}$ of SW-15 was not well predicted by the analysis. $V_{ana}$ of SW-15 was calculated when the minimum value of Eq. (23) was 0.038 because Eq. (23) did not become smaller than 0.01. The largest axial force imposed on SW-15 might elicit this discrepancy. The axial force ratio defined by Eq. (25) was equal to 0.18 for SW-15. Generally, this value is smaller for an RC bridge pier because of its
large cross-sectional area. In fact, the existing bridge pier studied in the following section has \( n = 0.01 \). Therefore, we assume that this failure criterion is applicable to existing RC bridge piers.

\[
N = \frac{b}{bd'f'c^2}
\]

(25)

Therein, \( b \) represents the total width including concrete cover.

4. Shear assessment of an existing RC bridge pier with severe frost damage

4.1 Description of the Current Condition

Figure 14 portrays schematic drawings of the analyzed RC bridge pier (Pier A). Pier A was constructed in the 1950s. The frost damage accelerated scaling so severely that hoop reinforcements were exposed and corroded. To obtain shear strength information based on the upper bound theorem, the original cross-section was conceived as a rectangle having equivalent cross-sectional area to that presented in Fig. 14. We are not concerned here with the size effect on shear strength (Bazant and Kim 1984) because emphasis is put on the incorporation of damage depths. However, the present model can be improved, for example, by considering the “size effect factor” (Bentz et al. 2006).

Detailed inspections were conducted twice for pier A. Concrete cores were drilled through the pier (Fig. 14). Their compressive mechanical responses and RDME values were obtained along its depth. Figure 15 presents the RDME distribution along the depth from the pier’s surface obtained from the latest inspection. In addition, \( D_1 \) and \( D_3 \) were set respectively to 320 mm and 280 mm according to the definition of the damage depth (RDME \( \leq 60\% \) according to ASTM C666/ C666M-03 (2008)). Results also demonstrate that the inner part of the core has small RDME. This result can be attributed to poor casting because the use of superplasticizer was not generalized in the 1950s. Figure 16 presents stress–strain curves of drilled cores obtained from different depths. The elastic modulus decreases considerably as the sampled depth becomes shallow. This marked reduction is a general tendency of concrete that is subjected to freeze–thaw cycles (Hasan 2004). We inferred the inputs to the present analysis as presented in Table 4 based on these data. The elastic moduli and yield strengths of the steel reinforcements were obtained from tensile tests conducted on the actual samples.

![Fig. 14 Schematic drawing of the existing RC bridge pier (unit: mm).](image)

![Fig. 15 RDME distribution of the concrete core taken from Pier A.](image)

![Fig. 16 Stress–strain curves of the concrete core taken from Pier A.](image)
4.2 Results and discussion

Figure 17 depicts the shear strength – displacement response of pier A. The current shear strength exhibits 12% reduction relative to the intact strength ($V_{\text{int}} = 4229$ kN) calculated from the displacement field without the damaged zones. Results also demonstrate that the damaged concrete has a slight contribution ($V_{c1} + V_{c3}$). Actually, that of hoop reinforcement ($V_{sw}$) is the greatest. The contributions of the respective shear components are tabulated in Table 5. The undamaged concrete component ($V_{c2}$) is not significant because the effectiveness factor ($v_{c2}$) decreases considerably, as portrayed in Fig. 18.

The present analysis enables shear strength prediction for probable cases of damage depths. Figure 19 presents correlation between the shear strength reduction and the damage depth ($D_{1,3}/h$). The vertical axis of Fig. 19 is normalized with respect to the shear strengths under an intact condition ($V_{\text{int}}$). The reason for the marked reduction in the shear strength in the range of $0.25h – 0.30h$ requires further investigation. The developed analysis directly correlates the shear strength with probable damage depths. Therefore, rational assessment of mechanical performance can ensure that shear strength is maintained so as not to decrease the flexural strength.

5. Conclusions

A kinematic model combined with frost-damage depth was presented for use in shear assessment. The developed model was validated by comparison of three ex-
Experimental campaigns including 14 RC short columns. Thereafter, based on data obtained from core sampling, the validated model was used to predict the current shear strength of an existing RC bridge column with severe frost damage.

For ease of use in engineering practice, the developed model adopted the simple assumptions that the damage depths were 0.20h for all specimens, and that uniform deterioration was introduced to the damaged zones. Although the former assumption gave the best agreement with 11 damaged specimens (Table 3), it should be compared with many experimentally obtained results. Despite the simplifications, model validation revealed that the experimental shear strengths were predicted within error of 20%. Moreover, the model enabled derivation of shear components based on the internal work of

\[ \text{von Mises stress} \]

within error of 20%. Moreover, the model enabled derivation of shear components based on the internal work of each. Further study is necessary for the failure criterion of 0.18 ≤ n (axial load index) because a large axial load might disturb the convergence of Eq. (23). The range of n for bridge piers is, however, smaller than that level. Additionally, model simplification should be undertaken to assess existing RC members. The present analysis is not applicable if the dowel action of longitudinal reinforcements (Zarate Garnica 2018) and the bond deterioration attributable to frost damage (Kanazawa et al. 2017) play a non-negligible role in shear behavior. Within the range of applicability, the developed model enables direct computation of shear strength based on the damage depth. A rational shear assessment can be achieved in a mechanical sense using core sampling during field inspections.

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