Calculation of Shallow Foundation Settlement Taking into Account Lateral Expansion of Soil

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Abstract. Nowadays the prediction of the settlement of shallow foundation is almost carried out by using the result of the oedometer test, in which the lateral expansion of soil mass is excluded. Geotechnical engineering software, such as Plaxis, Geostudio, Flac, Midas-GTS-NX, etc, are capable of solving most geotechnical problems, taking into account the lateral expansion of soil mass. The comparison between the result procured when using the classically analytical method and the result obtained with the help of the geotechnical engineering software show us an enormous divergence. It can be explained by the difference in approaches of solving the question between numerical method and analytical method, in which the effect of lateral expansion of soil mass is neglected. Therefore, the authors suggest a method to improve the prediction of settlement basing on the studies of Russian scientists in the 1960s, taking into account the effect of lateral expansion of soil mass below the shallow foundation. The result of the article can be considered to predict the long-term, short-term settlement as well as the settlement at any moment of soil consolidation with the acceptable difference when comparing with the results given from the nowadays geotechnical engineering software.

1. Introduction

Considering the case when a uniformly distributed load \( p \) of width \( 2a \) is applied on the ground surface, we get the calculation scheme for homogeneous elastic half-space.

![Figure 1. Calculation scheme of stresses in soil mass due to applied load \( p \).](image)
The equations for determination of stresses in the soil mass due to a uniformly distributed load with 2 m width \( p \) at the ground surface are written as the followings:

\[
\sigma_x = \frac{p}{\pi} \left[ \arctg \left( \frac{a-x}{z} \right) + \arctg \left( \frac{a+x}{z} \right) \right] + \frac{2a.p.z.(x^2 - z^2 - a^2)}{\pi \left[ (x^2 + z^2 + a^2)^2 + 4a^2.z^2 \right]} 
\]

(1)

\[
\sigma_z = \frac{p}{\pi} \left[ \arctg \left( \frac{a-x}{z} \right) + \arctg \left( \frac{a+x}{z} \right) \right] - \frac{2a.p.z.(x^2 - z^2 - a^2)}{\pi \left[ (x^2 + z^2 + a^2)^2 + 4a^2.z^2 \right]} 
\]

(2)

\[
\tau_{xc} = \frac{p}{\pi} \frac{4a.x.z^2}{(x-a)^2 + z^2} \left[ (x+a)^2 + z^2 \right] 
\]

(3)

\[
\sigma = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{(1+\nu)(\sigma_x + \sigma_y)}{3} = \frac{2p.(1+\nu)}{3\pi} \left[ \arctg \left( \frac{a-x}{z} \right) + \arctg \left( \frac{a+x}{z} \right) \right] 
\]

(4)

In the consideration of homogeneous isotropic elastic medium, we can use the equations below to determine the deformation of soil mass:

\[
\varepsilon_z = \frac{1}{E} \left[ \sigma_z - \nu(\sigma_x + \sigma_y) \right] 
\]

(5)

\[
\varepsilon_x = \frac{1}{E} \left[ \sigma_x - \nu(\sigma_z + \sigma_y) \right] 
\]

(6)

\[
\varepsilon_y = \frac{1}{E} \left[ \sigma_y - \nu(\sigma_z + \sigma_x) \right] 
\]

(7)

in which \( \varepsilon_z, \varepsilon_x \) and \( \varepsilon_y \) are vertical deformation and lateral deformation of Ox and Oy, respectively; \( \nu \) is Poisson’s ratio; \( \sigma_x, \sigma_y, \sigma_z \) are increasing stresses in soil mass; \( E \) – deformation modulus of soil.

In soil mechanics, the settlement of soil foundation is estimated using results and theory from the oedometer test, in which lateral expansion of soil mass is forsaken, but actually the lateral deformation has an apparent effect on vertical displacement of foundation. Ignoring the lateral deformation when estimating the vertical displacement as well as the stability of soil mass can cause a risk of collapse of soil foundation or wasting resources and money. Therefore, that authors’ suggestion of a method estimating and determining settlement of soil foundation, taking into account the lateral expansion of soil mass, using database of triaxial soil test is vitally necessary.

![Figure 2](image_url)

**Figure 2.** Dependency curve of volume deformation on mean stress (a) and shear deformation on shear stress (b).

Figure 2 presents the increase of vertical load varying from 300 to 500 kPa, thereby, not only the dependency of deformations on the stresses at both loading and unloading states but also shear modulus and bulk modulus can be considered linear. The values of modulus at loading-state are higher than ones at unloading state 5-10 times.
Nikolay Tsytovich proposed the concept of an elastic model for soil foundation applied load at the ground surface, in which modulus deformation \( G_0, K_0 \) are exerted to determine the dependency of deformations on stresses during loading period without separating the deformation into elastic and plastic. This concept nowadays is still used to determine modulus deformation corresponding with linear behavior of soil foundation.

2. Prediction of settlement of soil foundation taking into account lateral expansion of soil mass

2.1. Determination of long-term settlement of soil foundation

In the case, when the lateral expansion of soil mass is considered \( (\varepsilon_x = \varepsilon_y \neq 0) \), performing some transformations of expressions (5), (6) and (7) we get:

\[
\varepsilon_x = \frac{1}{2G}(\sigma_x - \sigma) + \frac{3}{K}\sigma
\]

\[
\varepsilon_y = \frac{1}{2G}(\sigma_y - \sigma) + \frac{3}{K}\sigma
\]

\[
\varepsilon_z = \frac{1}{2G}(\sigma_z - \sigma) + \frac{3}{K}\sigma
\]

in which \( G, K \) are shear modulus and bulk modulus of soil, respectively:

\[
G = \frac{E}{2(1 + \nu)}; \quad K = \frac{E}{3(1 + \nu)}
\]

\( \sigma \) is determined by using expression (4).

Vertical displacement of homogeneous isotropic soil foundation with \( h \) (m) thickness is defined from expression (10)

\[
\Delta s = h\left[\frac{\sigma_z - \sigma}{2G} + \frac{3\sigma}{K}\right]
\]

Integrating expression (13) of \( z \) from 0 to \( h \), we get the total vertical displacement of soil foundation.

\[
S = \int \left(\frac{\sigma_z - \sigma}{2G} + \frac{3\sigma}{K}\right) dz = \int \frac{\sigma_z - \sigma}{2G} dz + \int \frac{3\sigma}{K} dz
\]

and alternatively, we have \( S = S_v + S_γ \)

in which \( S_v \) and \( S_γ \) are volume and shear displacement of soil, respectively.

\[
S_v = \int_0^h \frac{3\sigma}{K} dz; \quad S_γ = \int_0^h \frac{\sigma_z - \sigma}{2G} dz
\]

where \( h_v \) and \( h_γ \) are the values of thickness of compressed and shear soil layer.

Volume and shear displacement of soil is obtained after integrating expressions (15) of \( z \) from 0 to \( h_v \) and \( h_γ \).

\[
S_v = \frac{4p(1+\nu)}{\pi K} \int_0^{h_v} \text{arcctg} \frac{h_v}{a} + a \ln \frac{a^2 + h_v^2}{a^2}; \quad S_γ = \frac{p}{3\sigma G} \left(1 - 2\nu\right)h_γ \text{arcctg} \frac{h_γ}{a} + (2 - \nu)a \ln \frac{a^2 + h_γ^2}{a^2}
\]

Corresponding to decreasing \( \sigma \) and \( \sigma_z \) of \( z \) the values of \( h_v \) and \( h_γ \) is defined as shown at figures 3 and 4.
2.2. Determination of short-term settlement of soil foundation

In the case of fully saturated soil, when applying a uniformly distributed load at the ground surface, the calculation increasing stresses on soil foundation requires the Terzaghi Theory of effective stress written as following.

\[ \sigma = \sigma' + u_w \]  \hspace{1cm} (18)

where \( \sigma \) is total mean stress; \( \sigma' \) is effective stress and \( u_w \) is pore water pressure.

Proposing a concept of deformation for three-phase soil, in which we accept an equilibrium:

\[ \varepsilon = \varepsilon_s + n \varepsilon_w \]  \hspace{1cm} (19)

where \( \varepsilon = \frac{\sigma}{K_u} \); \( \varepsilon_s = \frac{\sigma'}{K_s} \); \( \varepsilon_w = \frac{u_w}{K_w} \) \hspace{1cm} (20)

in which \( n \) is the soil porosity, \( \varepsilon_s \) is the volume deformation of soil skeleton; \( \varepsilon_w \) is the volume deformation of pore water; \( K_s \) is the bulk modulus of soil skeleton; \( K_w \) is the bulk modulus of pore water; \( K_u \) is the total bulk modulus of soil in the condition of the undrained state.

Putting expression (20) into (18) we get:

\[ K_u = K_s + \frac{K_w}{n} \]  \hspace{1cm} (21)

\[ u_w = \sigma \beta_0 \]  \hspace{1cm} (22)

where \( \beta_0 \) is the coefficient of pore water pressure and \( \beta_0 = \frac{K_w}{K_w + K_s n} \) \hspace{1cm} (23)

Coefficient of pore water pressure \( \beta_0 \) is required to separate the total stress \( \sigma \) into effective stress \( \sigma' \) and pore water pressure \( u_w \). The moisture level of soil \( S_r \) is included in the expression determining \( \beta_0 \), thereby, that soil is considered as a three-phase system consisting of soil solids, voids containing pore water and empty voids filled with air can describe effectively distribution of effective stress and pore water pressure in soil foundation. Quoting reference [15,19] we get:
\[ K_w = \frac{K_{wg}K_g}{K_{wg}(1-S_r) + K_gS_r} \] (24)

In order to create simple in calculations, \( K_g \) is assumed as constant, in which \( K_g = p_a = 101.3 \) kPa, where \( p_a \) is the atmospheric pressure. Increasing pore water pressure in the soil mass due to uniformly distributed load at ground surface is determined by dint of expression (4), (18) and (23)

\[ u_w(x,z,0) = \beta_0 \cdot \frac{2p(1+\nu)}{3\pi} \left( \arctg \frac{a-x}{z} + \arctg \frac{a+x}{z} \right) \] (25)

In case of undrained state of soil, short-term settlement, which also is immediate settlement of soil foundation can be calculated as follows:

\[ S(0) = S_v(0) + S_f(0) = \int_0^h \frac{3\sigma'_f}{K_s} dz + \int_0^h \frac{\sigma_z - \sigma}{2G} dz \] (26)

where \( K_s \) is the bulk modulus, in case of undrained state of soil, obtained from triaxial soil test.

Solving equation (26) we can determine settlement at the moment application of load \( p \) in case of undrained state of soil. In this article, the authors solve equation (26) by means of using computer software Mathcad.

2.3. Determination of consolidation settlement

In order to estimate the stress-strain state of soil foundation, taking into account its saturation, the use of consolidation equation is considered:

\[ \frac{\partial u_w}{\partial t} = c_v \left( \frac{\partial^2 u_w}{\partial x^2} + \frac{\partial^2 u_w}{\partial z^2} \right) + \beta_0 \frac{\partial \sigma}{\partial t} \] (27)

in which \( c_v \) is the coefficient of consolidation.

Taking into account expression (25), the value of pore water pressure on axis Oz of strip foundation \( u_w(0,t) = 0 \) and constant of uniformly distributed load, the solution of equation (27) can be written as follows:

\[ u_w(t) = \frac{1}{4\pi c_v t} \int_{-\infty}^{\infty} \int_{0}^{h} u_w(\xi,\eta,0) \left[ e^{-\frac{a-x}{4c_v t}} - e^{-\frac{a-x}{4c_v t}} \right] d\eta d\xi \] (28)

in which

\[ u_w(\xi,\eta,0) = \beta_0 \cdot \frac{2p(1+\nu)}{3\pi} \left( \arctg \frac{a-\xi}{\eta} + \arctg \frac{a+\xi}{\eta} \right) \] (29)

The equation for consolidation settlement of soil foundation

\[ S_v(t) = \int_0^h \left[ \frac{3(\sigma - u_w(t))}{K_s} \right] dz \] (30)

\[ S(t) = S_v(t) + S_f(0) \] (31)

Considering an example:

A considered 4m-width strip foundation with a uniformly distributed load \( p=300 \) kPa is applied on the ground surface and the soil foundation below has the following parameters, such as \( S_r=0.98, c_v=0.04 \) m²/day, \( E=3000 \) kPa, \( \beta_0=0.9 \) and \( \nu=0.3 \). Putting these parameters into expression (28) by means of Mathcad we get the results shown in the following figure in cases of different consolidation times. The obtained isobars of pore water pressure corresponding with cases of consolidation times a) \( t= 0 \) days, b) \( t = 30 \) days, c) \( t = 100 \) days, and finally d) \( t = 1000 \) days are presented as follows:
Figure 5. Isobars of pore water pressure in soil mass.

The values of consolidation settlement of the soil below the strip foundation, on which the effect of creep in soil mass is neglected are described in figure 6. The results are obtained by using PLAXIS 2D and expression (31).

Figure 6. Dependency curve of vertical displacement at the on axis Oz of strip foundation on time.
3. Conclusion
The article shows the studied results of the method estimating settlement of soil below the shallow foundation, taking into account its lateral expansion corresponding with elastic model for soil. Besides, long-term, short-term settlement and consolidation settlement are also performed so that all states of soil foundation before being applied load can be predicted and controlled. Furthermore, this study can be exerted in order to solve similar problems for foundation resting on heterogeneous soils, rock and coral.

Analyzing figure 5, in case of fully saturated soil, we can draw that a point where pore water pressure achieves the maximum value exists in soil mass, and over time this point gradually moves down with decreasing amplitude as a result of vertical expulsion pore water.

The value of immediate settlement procured by using expression (31) is quite large due to consideration of lateral expansion of soil mass, in some cases it can account for 60-70% of the permanent settlement, as a result, the time for achievement of permanent settlement is reduced. Moreover, basing on the values of hv and hγ achieved from figure 4 and pore water pressure in figure 6, engineers can estimate the depth of soil foundation required for reinforcement. The question of determination of reinforcement depth show a vital significance, in a case when the width of strip foundation and the depth of weak soil are small.

Data presented in figure 7 indicates that the calculation results gained by using expression (31) are quite close to the results achieved from PLAXIS. The article requires more studies in theoretical basis and field test in order to improve on method of estimating settlement in cases of non-linear behaviour of soil, heterogeneous soils and 3D model.

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