Research Article

Optimization of Cable Force Adjustment in Cable-Stayed Bridge considering the Number of Stay Cable Adjustment

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Modern cable-stayed bridges are spatial, multicables systems. The cable force needs to be adjusted during the construction phase and maintenance phase. The existing calculation methods of cable force adjustment mainly considered the rationality of structural force, but only few research studies have been conducted on how to reduce the number of stay cables which need to be adjusted. This study aims to propose an optimization calculation method including the optimization module with the sensitivity analysis and updating design variable module (UDVM), which are used for cable force adjustment in cable-stayed bridges. Based on the finite difference method, the sensitivity analysis is adopted in the optimization module, which can capture the response of structures as design variables vary; the particle swarm optimization method is adopted for structural optimization. The proposed method can dramatically reduce the number of stay cables which need to be adjusted and ensure the main girder stresses remain in a reasonable state during stay cable adjustment progress by UDVM. Moreover, the proposed method can continuously update the objective function, constraint conditions, and design variables. Finally, this proposed optimization calculation method is applied to two different cable-stayed bridges to validate the reliability and feasibility of the method.

1. Introduction

The number of cable-stayed bridges increased rapidly around the world due to their highly evolved construction techniques, superior mechanical performance, and relatively low costs of construction and maintenance. Cable-stayed bridge is a competitive structural form with span ranging from 200 m to 1000 m [1–3]. Notable examples of cable-stayed bridges are the Skarnsund Bridge in Norway with a 530 m main span and the Jingzhou Yangtze River Bridge in China with a 500 m main span [4, 5]. The space dense cable system is a common structure system for modern cable-stayed bridges. However, the illogical adjustment of cable force can easily lead to the overstress in the main beam. And the implementation of conventional calculation method is rarely used in engineering because the number of stay cables which need to be adjusted is large [5].

In recent years, researchers mainly focus on the calculation method of stay cable force adjustment in concrete cable-stayed bridges through the optimization theory [6–14]. Both of the calculation methods proposed by Wang et al. [6] and Liang et al. [7] can determine the cable force adjustment, including the calculation method based on the requirement of vertical displacement or the distribution of bending moment of the main beam. These methods are mainly based on the influence matrix method to calculate the value of cable force adjustment by establishing an unconstrained mathematical optimization model. Martins et al. [8–10] proposed a method for calculating the cable force of concrete cable-stayed bridges as well as designed a discrete direct sensitivity analysis module and a multiobjective optimization calculation program based on such method. By using this method, the initial tension of the stay cables and the optimal cable tension could be obtained as ensuring the forces of structures is reasonable during the construction phase. Based on the minimum potential energy principle, Qin [11, 12] established mechanical equilibrium models for different bridge construction stages. By introducing an unstressed state to the structural components, the relationships between construction phase and the finished phase...
of bridges can be established. The unstressed state control method could rapidly calculate the cable adjustment force by the unstressed cable length. Dan and Yang [13] and Yuan [14] presented a calculating method for the cable adjustment of cable-stayed bridges based on particle swarm optimization calculation method. For specific situation of adjusting cable force, an optimization model for solving the cable force was established, and the particle swarm optimization method with global search capability was used to achieve the optimization calculation.

Nevertheless, these proposed calculation methods regarded all the cables as adjustment variables, and only the force condition of main beam and cables during the cable adjustment phase or finished phase of bridges were analyzed [6–15]. The number of adjustment cables was not taken into account during the cable adjustment process. Sometimes, a large number of stay cables need to be adjusted, which may lead to large construction cumulative errors and complicated operation. Hence, this paper proposes an innovative calculation method to reduce the number of cables while ensuring that the structural stress and cable force of the cable-stayed bridge do not exceed the limits during the cable adjustment process and after construction. This method minimizes the number of stay cables which need to be adjusted on the premise that the order of adjustments is determined.

For cable-stayed bridges, the cable forces need to be adjusted during both the design phase and maintenance phase [4, 5]. The single-time tension of cables in design phrase often leads to overstress in the main beam and the reduction of load capacity. During routine maintenance, the real cable force often deviates from the design value, resulting in unbalanced force in bridge. The calculation of the cable force adjustment in concrete cable-stayed bridges is more difficult than that in steel main girder cable-stayed bridge, since the stress redundancy of the main girder in concrete cable-stayed bridges is less than that in steel main girder cable-stayed bridges. Therefore, this article takes concrete cable-stayed bridge as an example to carry out the analysis of the proposed cable force adjustment calculation method. The proposed method was applied to two concrete cable-stayed bridges (case A and case B), which correspond to the calculation of adjustment cables in design phase (case A) and in maintenance phase (case B), respectively. The structure of this paper is as follows: Section 2 describes the optimization plans; Section 3 analyzes the calculation method to reduce the number of adjustment cables; Section 4 verifies the feasibility and reliability of the proposed method by combining finite element models with two specific engineering examples; and Section 5 summarizes the paper, drawing some conclusions.

### 2. Optimization Plans

The application of the optimization method is necessary for the investigation of the optimal cable adjustment for concrete cable-stayed bridges, including defining the design variables and determining their objective functions and constraint conditions [16].

#### 2.1. Design Variables

This study is conducted on the premise that the bridge dimensions, exterior loads, and the sequence of stay cables adjustment are predetermined. Therefore, the adjustment difference values of each cable that needs to be adjusted are used as design variables, as shown in the following equation:

\[
x = [x_1, x_2, \ldots, x_n]^T,
\]

where \(x_i\) is the adjusted amount of cable force for cable \(i\).

#### 2.2. Objective Function

Objective function defines the optimization direction for the design variables. The first objective function is the total amount of work undertaken in the adjustment of all the cable forces (equation (2)), aiming to minimize the energy consumption [17]. The second objective function is the maximum absolute value (as shown in equation (3)) of the cable adjustment force of each cable, so as to avoid overlarge adjustments for individual cables [13]. By the weighting method, all objective functions are combined into one function to solve the problem, namely, the multiobjective optimization issue is transformed into several single objective optimization issues in sequence (as shown in equation (4)) [18].

\[
\begin{align*}
\min f_1(x) &= \min \left(\sum_{i=1}^n x_i|\Delta l_i| \right) = \min \left(\sum_{i=1}^n \frac{x_i^2 l_i}{2EA_i}\right), \\
\min f_2(x) &= \min \max(|x_1|, |x_2|, \ldots, |x_n|), \\
\min f_3(x) &= \min \left(f_1(x) + \alpha f_2(x)\right),
\end{align*}
\]

where \(\Delta l_i\) is the deformation of cable \(i\) under the action of the cable adjustment force \(x_i\); \(l_i\) and \(A_i\) refer to the unstressed length and section area of cable \(i\); \(E\) is the elastic modulus of the steel wire; and \(\alpha\) is the weighting factor of \(f_2(x)\), varying in different cases.

#### 2.3. Constraint Conditions

Constraint conditions were defined in order to avoid the unreasonable force state of the bridge structure during the cable adjustment process and the finished stage. Concrete main bridge alignment can be adjusted by precamber [19]. Moreover, concrete cable-stayed bridge should regard main girder stress as primary constraint condition because of the relatively low capacity of concrete main girder section with applied stress. When minimizing the number of cable adjustment, the rationality of structure stress is considered to be the main factor. Thus, vertical displacement is not considered as the constraint condition. The design calculation of the pylon is controlled by dynamic conditions because the section stress redundancy under static load is larger than that under dynamic load. As a consequence, the section force of the pylon is not constricted under static load. In addition, the allowable safety factor (2.0) of the stay cable during construction is lower than the allowable safety factor (2.5) during the completion stage [20]. Therefore, the limit value of the stress in the main beam is defined during the cable adjustment.
process as well as the completion state, and the limit value of the cable force is defined for the completion state.

During the cable adjustment process, the main control sections in the main beam are usually fully compressed; therefore, it is sufficient to only consider the restriction of the maximum compressive stress in the main beam section. In the Chinese code “Specifications for Design of Highway Reinforced Concrete and Prestressed Concrete Bridges and Culverts” (JTG3362-2018), the stress value on the section should not exceed the limit value [21]. In order to reduce the number of constraints, some specific conditions of stay cables adjustment can be chosen to constric the stress value of main girder.

\[ 0 < \sigma_{ai}^j \leq 0.5 \cdot f_{ck}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, k, \] (5)

where \( \sigma_{ai}^j \) is the maximum compressive stress in the \( i \)th stress control section of main girder for the \( j \)th condition of cable adjustment; \( f_{ck} \) is the standard value of axial compressive strength of the concrete; \( m \) is the total number of stress control sections; and \( k \) is the total number of cable adjustments.

After all of the cable adjustments have been finished, the stress value of stress control section in the main beam in completion stage should be close to the design value, as shown in the following equation:

\[ |\sigma_{ci} - \sigma_{di}| \leq s, \quad i = 1, 2, \ldots, m, \] (6)

where \( \sigma_{ci} \) is the stress value for the \( i \)th stress control section of the main girder following the completion of the cable adjustment; \( \sigma_{di} \) is the stress design value for the \( i \)th stress control section of the main girder in its bridge finished state; \( S \) is the allowable stress error; and \( m \) is the total number of stress control sections.

The cable force value in the bridge finished phase should be close to the design cable force value, and the error range is generally controlled within 5%, as described in the following equation:

\[ 0.95 \leq \frac{T_i}{T_{di}} \leq 1.05, \quad i = 1, 2, \ldots, n, \] (7)

where \( T_i \) is the cable force of cable \( i \) after the completion of cable adjustment; \( T_{di} \) is the design cable value of cable \( i \) in the completion stage; and \( n \) is the total number of cables.

### 3. Optimization Calculation Method

The optimization calculation method consists of a module for sensitivity analysis and optimization as well as an update module for design variables. The flowchart of the optimization calculation method is illustrated in Figure 1. The value of optimized adjustment cables force can be obtained by the sensitivity analysis and optimization module, which meets the limit value of main girder stress and cable force. This paper innovatively proposes an update module of design variable, which reduces the number of cables requiring adjustment. The meanings of the parameters in the flowchart are described below. The detailed flow of the two modules is described in Sections 3.1 and 3.2.

\[
[x^d_m] = [x^d_{m,1}, x^d_{m,2}, \ldots, x^d_{m,n}]^T,
\]

where \( [x^d_m] \) is the optimal solution (or initial value) of the design variables; \( d \) is the number of optimized calculations in the sensitivity module, which is reset to zero after complete the update module of design variable for one time; and \( m \) is the number of calculation steps. Each time the UDVM is completed, calculation steps increased by one, which means the number of design variables reduced by one; \( i \) is the number of the cables.

\[ x^d_{m,i} = \min(x^d_{m,i}), \] (9)

where \( x^d_{m,i} \) is the minimum value (min(\( x^d_{m,i} \))) of the optimal solution \([x^d_{m,i}]\) obtained by the sensitivity analysis module.

#### 3.1. Sensitivity Analysis and Optimization Module

It is difficult to formulate the accurate structural response expressions for design variables in complicated structures such as cable-stayed bridges [22]. In equations (5)–(7), the response of the stress and the cable force corresponding to the varying design variables are obtained by the finite difference method. If the design variables vary slightly approaching to the initial value \( (\Delta x = [\Delta x_1, \Delta x_2, \ldots, \Delta x_n]^T) \), then the structural response should be expanded by using the first-order Taylor series near the initial value, as shown in equations (10)–(12).

\[ 0 < \sigma_{ai}^{j(0)} + \sum_{l=1}^{n} \frac{\partial \sigma_{ai}^{j}}{\partial x_l} \Delta x_l \leq 0.5 f_{ck}, \] (10)

\[ i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, k, \]

where \( \sigma_{ai}^{j(0)} \) is the stress value for the \( i \)th stress control section in the main girder for the \( j \)th cable adjustment condition when the design variable is the initial value and \( (\partial \sigma_{ai}^{j}/\partial x_l) \) is the value of sensitivity about \( \sigma_{ai}^{j} \) to \( x_l \), which is estimated by the finite difference method.

\[ \sigma_{ai(0)} + \sum_{l=1}^{n} \frac{\partial \sigma_{ai}}{\partial x_l} \Delta x_l - \sigma_{ai} \leq s, \quad i = 1, 2, \ldots, m, \] (11)

where \( \sigma_{ai(0)} \) is the value of stress for the \( i \)th stress control section of the main girder in the completion stage while the design variable is the initial value and \( (\partial \sigma_{ai}/\partial x_l) \) is the value of sensitivity about \( \sigma_{ai} \) to \( x_l \).

\[ 0.95 T_{ai(0)} \leq T_{ai(0)} + \sum_{l=1}^{n} \frac{\partial T_{ai}}{\partial x_l} \Delta x_l \leq 1.05 T_{ai(0)}, \quad i = 1, 2, \ldots, n, \] (12)

where \( T_{ai(0)} \) is the cable force for the \( i \)th cable in bridge finished phase when the design variable is the initial value or the iterative value and \( (\partial T_{ai}/\partial x_l) \) is the value of sensitivity about \( T_{ai} \) to \( x_l \).

Corresponding objective function could be rewritten as follows:
It can be seen from equation (14) that the objective function is an implicit expression of the design variables \([x]\). Currently, large quantities of methods can be adopted for optimization, such as particle swarm optimization (PSO), Structural analysis
Finite element formal analysis
Sensitivity analysis
Composition optimization functions (including equations (8)–(10), and (13))

Figure 1: The flowchart of optimization.

$$\text{min } f_1(x) = \sum_{i=1}^{n} \frac{(x_{0i} + \Delta x_i)^2}{2EA_i}, \quad (13)$$

$$\text{min } f_2(x) = \max(|x_1 + \Delta x_1|, |x_2 + \Delta x_2|, \ldots, |x_n + \Delta x_n|), \quad (14)$$

$$\text{min } f_3(x) = \min[f_1(x) + 0.1f_2(x)]. \quad (15)$$
genetic algorithm (GA), bidirectional search, and Java algorithm. Previous research studies regarding cable force optimization revealed that the PSO featured high accuracy and low convergence error. According to references [13, 23, 24], the maximum convergence error is approaching 5% for optimization despite processing PSO 100 times. Thus, PSO is adopted in this paper and the convergence error is set as 5%.

Previous research studies demonstrated that the number of iterative steps should not exceed 100 when structure size and the values of cable forces are used as design variables with sensitivity analysis processing structural optimization [10, 22, 25]. And the requirement of iterative steps needs to be fewer for structural optimization, when only the value of cable force is used as design variable with sensitivity analysis [26]. The maximum number of iterative steps is set as 100, which means the number of iterative steps could satisfy the optimization calculation of cable-stayed bridges. If the number of iterative steps exceeds 100 without optimal solution, the result of previous step should be adopted directly.

It means within the allowed number of iterations (100 times), when the error of the value of objective function between two adjacent iteration points is less than 5% (equation (15)), the calculation will stop.

As shown in the flowchart of the sensitivity analysis and optimization module in Figure 1, for one given initial value \([x_0^d] = [x_{1,1}^d, x_{1,2}^d, \ldots, x_{1,n}^d]^T\), the finite element analysis results and sensitivity analysis results could be obtained. The optimization function is established by combining equations (10)–(12) and (15), and the optimal solution \(x_{opt}^d = [x_{m,1}^d, x_{m,2}^d, \ldots, x_{m,n}^d]^T\) is obtained by the optimization calculation method. Take this optimal solution as the initial value of the design variable and repeat the above steps; then, the updated optimal solution \(x_{opt}^{d+1} = [x_{m,1}^{d+1}, x_{m,2}^{d+1}, \ldots, x_{m,n}^{d+1}]^T\) can be obtained subsequently. The process of iteration would be stopped when the termination criterion is met, and then the next module starts and the optimal solution is recorded.

3.2 Updating Design Variable Module (UDVM). The optimal cable force values are obtained by the sensitivity analysis and optimization module. However, in order to reduce number of cables needed to be adjusted, an innovative updating design variable module (UDVM) is proposed. For the optimal solution \(x_{opt}^d = [x_{m,1}^d, x_{m,2}^d, \ldots, x_{m,n}^d]^T\) of the \(m\)th calculation step, the minimum value \(x_{m,t}^d = \min(x_{m,t}^d)\) is sought and set as \(x_{m,t}^d = 0\). Because \(x_{m,t}^d\) is the minimum value of \(x_{m,t}^d\), it should have little influence on the value of objective equation (4). For constraint conditions (for example, constraint expressions equations (5)–(7)), Taylor expansion is derived for the optimal solution \(x_{opt}^d\), and then the value of the neighborhood \([x_{m,t}^d] = [x_{m,1}^d, \ldots, x_{m,t-1}^d, 0, x_{m,t+1}^d, x_{m,n}^d]^T\) was substituted, as shown in the following equations:

\[
T_i(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) = T_i(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) - \frac{\partial T_i(x_{m,1}, x_{m,2}, \ldots, x_{m,n})}{\partial x_{m,t}^d} x_{m,t}^d + \frac{\partial^2 T_i(x_{m,1}, x_{m,2}, \ldots, x_{m,n})}{2 \partial x_{m,t}^d} x_{m,t}^d^2, \quad (16)
\]

\[
\sigma_{ii}(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) = \sigma_{ii}(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) - \frac{\partial \sigma_{ii}(x_{m,1}, x_{m,2}, \ldots, x_{m,n})}{\partial x_{m,t}^d} x_{m,t}^d + \frac{\partial^2 \sigma_{ii}(x_{m,1}, x_{m,2}, \ldots, x_{m,n})}{2 \partial x_{m,t}^d} x_{m,t}^d^2, \quad (17)
\]

\[
\sigma_{ij}(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) = \sigma_{ij}(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) - \frac{\partial \sigma_{ij}(x_{m,1}, x_{m,2}, \ldots, x_{m,n})}{\partial x_{m,t}^d} x_{m,t}^d + \frac{\partial^2 \sigma_{ij}(x_{m,1}, x_{m,2}, \ldots, x_{m,n})}{2 \partial x_{m,t}^d} x_{m,t}^d^2, \quad (18)
\]

In equations (16)–(18), the values of second-order partial derivative and higher order partial derivative are relatively small, which have small influence on subsequent calculation when nonlinearity of structure system is not high. Equations (19)–(21) are easier to establish when \(x_{m,t}^d\) is relatively small. That is, it is easier to satisfy constraint condition by eliminating the design variable corresponding to the minimum \(x_{m,t}^d\) value than eliminating other design variables. And it is easier to satisfy constraint condition (optimal solution) by optimizing the remaining design variables.

When \(x_{m,t}^d\) is relatively small:

\[
T_i(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) \approx T_i(x_{m,1}, x_{m,2}, \ldots, x_{m,n}), \quad (19)
\]

\[
\sigma_{ii}(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) \approx \sigma_{ii}(x_{m,1}, x_{m,2}, \ldots, x_{m,n}), \quad (20)
\]

\[
\sigma_{ij}(x_{m,1}, x_{m,2}, \ldots, x_{m,n}) \approx \sigma_{ij}(x_{m,1}, x_{m,2}, \ldots, x_{m,n}). \quad (21)
\]
In other words, it is easy to satisfy or approximate constraint conditions compared with eliminating other designed variables \( x^d_m = [x^d_1, x^d_2, \ldots, x^d_n]^T \).

Based on the above analysis, the entire analysis sequence of the optimization calculation method is described in the flowchart of Figure 1:

Step 1: determine the initial values \( [x^0_1] = [x^0_1, x^0_2, \ldots, x^0_n]^T \). Then, the initial values are substituted into the sensitivity analysis and the optimization module for iterative calculations. The iterative calculations would be stopped if the termination criterion is met, and then the next module starts and the optimal solution is recorded (detailed steps have been described in Section 3.1).

Step 2: proceed to the next calculation step (the next module) and set \( m = m + 1 \).

Step 3: search for \( x^d_m = \min(x^d_m) \) in \( [x^d_m] \) and set \( x^d_{m,t} = 0 \).

Step 4: update the design variables as \( x = [x_1, \ldots, x_{m-1, j}, x_{m, j}, \ldots, x_{n, t}]^T \). Then, the objective function (equation (4)) and constraint conditions (equations (5)–(7)) are changed corresponding to the updated design variables.

Step 5: update the initial values as \( [x^0_{m+1}] = [x^0_{m,1}, \ldots, x^0_{m,t}, x^0_{m+1,0}, x^0_{m+1,t}]^T \). Then, repeat step 2.

Step 6: repeat steps 2 to 5 if the optimal solution exists; output the result of the final calculation \( x^d_m \) if the iterations exceed the maximum iteration set in the sensitivity analysis module. The result \( x^d_m \) can be verified by the finite element model.

4. Case Studies

Two actual engineering cases, case A and case B, are selected for the calculation of the cable force adjustment in the design phase and the completion phase, respectively.

4.1. Case A

4.1.1. Overview. Case A is a two-pylon, three-span, prestressed reinforced concrete cable-stayed bridge. During the bridge’s preliminary design stage, the cable force of the bridge is determined according to the minimum sum of the bending energy of the main pylon and the main beam under the dead load equilibrium state, and the area of the stay cable is determined based on the cable force. In order to optimize the structural stress state, the cross section size of the main girder is designed to be comparatively small. And the initial cable tension is low (the vertical component of the initial tensile force is close to the deadweight of the corresponding section, ensuring that the cross section of the main beam is under pressure during the construction stage). Therefore, after the closure of the main girder, it is necessary to increase the cable adjustment condition and adjust the cable force to the design value. In this section, combined with an example analysis, the proposed calculation method is used to calculate the cable force.

In case A, the main span of the bridge is 250 m, the two side spans are 119 m, and the height of the main pylon is 98.1 m. To accommodate uneven geological conditions, the pile foundation’s depth is 37.8 m at the north side and 78.7 m at the south side. The bridge contains 160 stay cables, which consist of parallel steel wires with a standard tensile strength of 1770 MPa. Among the cables, the cable types of M1–M3 (S1–S3), M4–M8 (S4–S8), M8–M12 (S8–S12), M13–M16 (S13–S16), and M17–M20 (S17–S20) are PES7-163, PES7-199, PES7-223, PES7-283, and PES7-313, respectively. The main girders are made of C55 concrete with \( f_{ck} = 35.5 \) MPa (\( f_{ck} \) means the standard value of axial compressive strength). Figure 2 illustrates the layout of this bridge and the number of the cables. The two shortest cable and the two longest cables of each pylon in the north side span are denoted by NS1 and NS20, respectively. For the 20 stay cables in the north side span, they are notated as NS1 to NS20 according to the length of cables. The two shortest cables and the two longest cables of each pylon in the north midspan are denoted by NM1 and NM20, respectively. For the 20 stay cables in the north midspan, they are notated as NSM1 to NSM20 according to the length of cables. The method of notation of stay cables of the south side span and midspan is the same as that on the north side. The cross sections of the main structural components are shown in Figure 3. The characteristics and the limit stress value of the cross sections are listed in Table 1 (the \( x \)-axis is the longitudinal direction of the bridge, the \( y \)-axis is the transverse direction of the bridge, and the direction of gravity corresponds with the \( z \)-axis).

The cable force is adjusted from the short cable to the long cable by referring to the previous study and several cases regarding sequence of stay cable adjustment [9]. In order to meet the construction conditions and guarantee the load balance of the single-sided main pylon during the cable adjustment process, the cable adjustment scheme is designed as follows: firstly, the forces of the two stay cables on the north side span (NS1) are adjusted, and then the forces of the two stay cables on the north midspan (NM1) are adjusted. The sequence of the stay cables adjustment of the north side is NS1-NM1-NS2-NM2-· · ·-NS19-NM19-NS20-NM20; secondly, the sequence of stay cable adjustment of south side is the same as that of north side, namely, SS1-SM1-SS2-SM2-· · ·-SS19-SM19-SS20-SM20.

4.1.2. Finite Element Model. The three-dimensional finite element model of this bridge was established in ANSYS. Although the superstructure is symmetrical, the whole structure of the bridge was modeled due to the large discrepancy between the pile foundation depths on two sides. The Beam 4 element (Beam 4 is three-dimensional elastic element) is adopted to simulate the main girder and pylon, and the Link 10 element (Link 10 is three-dimensional elastic bar element only considering tension and compression) is used for the stay cables [18, 27]. The materials’ constitution is set as linear elastic, and the influence of geometric non-linearity and pile-soil interaction are taken into account. The elastic modulus of the stay cable varies with the change of the
Figure 2: General layout of the bridge (m). Note: NS means north side span, NM means north midspan, SS means south side span, and SM means south midspan.

Figure 3: Cross sections of main stress components (cm). (a) Girder. (b) Pile foundation. (c) Stay cable of parallel wires. (d) Tower (top-bottom).
section stress, and thus it is corrected according to the Ernst
formula [4],
\[
E_{eq} = \frac{E}{1 + \left( \frac{\gamma \cdot L}{\alpha E} \right)^2 \cos(12\alpha^2)},
\]
(22)
where \(E_{eq}\) is the equivalent cable modulus of elasticity; \(E\) is
the effective cable material modulus of elasticity; \(\gamma\) is the
specific weight of the cable material; \(L\) is the length of the
chord; \(\alpha\) is the angle between the cable chord and the
horizontal direction; and \(\sigma\) is the tension stress in the cable.

The finite element model simulates the construction
process by using the element birth and death method [28].

The prestressed tendons of the main girder are simulated by
truss elements, and prestress is applied by means of tem-
terature loads. Cable force is also applied by means of
temperature loads [27, 29]. The shrinkage and creep effects
of concrete should not be considered due to the short du-
ration of the adjustment process. Node coupling is used as
connection of pylon to stay cables, pylon to stay cables, and
pylon to main girder in longitudinal direction, but no coupling
is used as connection of pylon to main girder in transverse
direction. The diagram of the three-dimensional finite element
model is shown in Figure 4, and the boundary
conditions of the model are listed in Table 2. Only constant
load is considered in this model without live load (wind,
earthquake, and truck loads): self-weight (26 kN/m³), sec-
ondary load (110 kN/m), and prestressed load.

4.1.3. Optimization. Similar to Section 2, the specific pa-
rameters in this case study are selected as follows.

The design variables are the adjusted value of each cable
force \(x_1, x_2, \ldots, x_{80}\); \(x_1 \sim x_{20}\) represents cable 20 to cable 1
on the northern side span; \(x_{21} \sim x_{40}\) represents cable 1 to
cable 20 on the northern middle span; \(x_{41} \sim x_{60}\) represents
cable 20 to cable 1 on the southern middle span; \(x_{61} \sim x_{80}\)
represents cable 1 to cable 20 on the southern side span. In
order to verify the reliability of the calculation method, the
initial value \(x_i\) is randomly selected within a certain range
because the optimal solution may be influenced by the initial
value. In this case, \(x_i \in [-1000, 1000], i = 1, 2, \ldots, 80\).

As for constraint equation (5), quarter point on the two
side spans and one eighth point on the middle span (total 13
sections) are selected as stress-controlled sections. The stress
constraints of the section need to be defined when the stay
cable forces at these 13 control sections are adjusted. The
stress constraints of the section do not need to be defined
when the cable force is not carried to the 13 stress-controlled
main beam sections.

| Control section   | A (m²) | \(I_x\) (m⁴) | \(I_y\) (m⁴) | \(E\) (MPa) | \(L\) (MPa) |
|-------------------|-------|-------------|-------------|------------|-----------|
| Main girder       | 24    | 21.7        | 3571.4      | 35500      | 17.8      |
| Main pylon (top)  | 15.4  | 59.7        | 27.3        | 33500      | 16.2      |
| Main pylon (bottom)| 38.1 | 254.9       | 165.9       | 33500      | 16.2      |
| Pile foundation   | 3.8   | 1.15        | 1.15        | 30000      | 10.05     |

Note: \(A\) = cross-sectional area; \(I_x\) = moment of inertia for the x-axis; \(I_y\) = moment of inertia for the y-axis; \(E\) = elastic modulus; \(L\) = limit value of the stress.

Two optimization object functions are proposed as
\(f_1(x)\) and \(f_2(x)\). \(f_3(x)\) is proposed subsequently as a new
optimization object function by the linear weighting
method. Generally, the weighted factors need to be defined
for each object function according to both magnitude and
importance degree. The importance degrees for both two
optimization object functions as \(f_1(x)\) and \(f_2(x)\) are
considered as the same, and thus the weighted factors are
determined according to the magnitudes of the two functions.

The weighted factors are obtained by the following
equation:
\[
w_k = \frac{1}{f_k} (X^*), \quad k = 1, 2,
\]
(26)
where \(f_k(X^*)\) is the optimal solution of single objective
optimization problem composed by the first and the second
subobjective functions.

In the first optimization calculation including overall
design variables: only \(f_1(x)\) or \(f_2(x)\) is chosen as the object
function to obtain the first optimal solution, and the other
one is selected as the object function to obtain the other
optimal solution. The optimal solutions of \(f_1(x)\) and \(f_2(x)\)
are 53 kJ and 635 kN, respectively. And \(w_1 = 0.018, w_2 ≈ 0.0016, \) and \(w_2/w_1 ≈ 0.1\) (dimensionless method).
Then, the weight \(\alpha\) is set as 0 and 1.0, and the optimal
solutions are substituted into \(f_3(x)\), which is 115.1 and
116.5, respectively. Hence, the weighted factors have little

Table 1: Characteristics of cross sections.
influence on the object function. The weight $\alpha$ is set as 0.1 for the second object function.

### 4.1.4. Optimization Results

The distribution of cable forces prior to adjustment is presented in Figure 5. Some cable forces deviate significantly from their design values; therefore, the additional adjustment is necessary.

In total, 20 optimization calculations are conducted. For each optimization process, the optimal result is obtained after approximately 40 to 50 calculation steps. After completing one calculation step, the number of design variables is reduced by one. The change of objective function value (equation (4)) during the first 20 optimization calculation steps is shown in Figure 6. Figure 6 illustrates that fewer number of stay cables need to be adjusted with calculation steps increasing (the value of the objective function increases gradually with the number of calculation steps, indicating that as optimization progressed, the number of cables that need to be adjusted reduced). After the 20th calculation step, due to differences in the design variables of each optimization process, the gap between the various objective functions has gradually increased. The total work is less and the values of stay cable forces which need to be adjusted are indeed relatively low when taking the whole stay cables as design variables. The optimal solution can be also conducted when taking part of the stay cables as design variables. At this time, although the work required to adjust the cable force increases and the cable force to be adjusted increases, the number of cables to be adjusted decreases dramatically, which is very important in practical application.

The results of the 20 optimization calculations are different, and the cables that need to be adjusted in each result are also different. Table 3 shows the number of occurrences for each stay cable in the results of 20 optimization calculations (for example, “ZBS1:20” indicates that cable ZBS1 appeared in the result of all 20 optimization calculations). Figure 7 presents the values of six heavily adjusted cables from the results of 20 optimization calculations. The blue lines represent the mean value of the results of 20 calculations and the red lines are the 68.3% confidence interval (mean ± standard deviation). From Figures 5 and 7, it is clear that the results of 20 optimization calculations remain similar and have a low level of discreteness, suggesting that the proposed calculation method is reliable and stable, and the calculation results afterwards are credible [13].

It can be seen that the number of stay cables which need to be adjusted for the 14th optimization result is the least (only 27 pairs of cable need to be adjusted). On the premise of satisfying the constraint conditions, the adjustment amount required for the 14th optimization result is the least,

#### Table 2: Boundary constraint conditions of finite element model.

| Position                                      | $D_x$ | $D_y$ | $D_z$ | $R_x$ | $R_y$ | $R_z$ |
|-----------------------------------------------|-------|-------|-------|-------|-------|-------|
| Intersection node between girder and tower (boundary 1) | Free  | Coupled | Coupled | Coupled | Free | Free |
| Bottom of the tower (boundary 2)             | Fixed | Fixed | Fixed | Fixed | Fixed | Fixed |
| Bottom of the tower (boundary 3)             | Fixed | Fixed | Fixed | Fixed | Fixed | Fixed |
| Side span (boundary 4)                       | Free  | Fixed | Fixed | Fixed | Free  | Fixed |
| Intersection node between cable and girder   | Coupled | Coupled | Coupled | Coupled | Coupled | Coupled |
| Intersection node between cable and tower    | Coupled | Coupled | Coupled | Coupled | Coupled | Coupled |

Note: the unit of $D_x$, $D_y$, and $D_z$ is N/m; the unit of $R_x$, $R_y$, and $R_z$ is kN⋅m/rad.
Figure 5: Continued.
suggesting that 54/160 cables require adjustment. The values of stay cable adjustment are shown in Table 4, which were obtained based on a predetermined sequence of cable adjustments.

The cable force and the stress on the main girder in the completion stage, as shown in Figures 5 and 8, are calculated using the adjustment cable forces listed in Table 4, indicating that after adjusting the stay cable forces, the cable forces are relatively close to their design values, with only a 5% difference in magnitude. The stresses of main girder after cables adjustment are within the range of error, with a maximum stress of 14 MPa. The maximum compressive stress produced by live load of this bridge is approximately 3 MPa during the maintenance phase according to previous design experience [4, 5]. Therefore, the maximum compressive stress the bridge is approximately 17 MPa during the maintenance phase, which satisfies the design requirement (i.e., for a bridge in operation, the maximum compressive stress is 0.5 times the standard value of the axial compressive strength of concrete).

In addition, attempts are made to eliminate several design variables corresponding to the least optimal solution simultaneously in each optimization circulation progress. It is revealed from the final results that 32, 34, and 37 design variables are obtained when eliminating 3, 5, and 8 design variables corresponding to the least optimal solutions simultaneously in each optimization circulation progress, respectively. Thus, it can be concluded that if only the design variable corresponding to the least 1 optimal solution in each optimization circulation progress is eliminated, the number of design variables will be the minimum in the final result.

4.2. Case B. Case B is a single-pylon, three-span, prestressed reinforced concrete cable-stayed bridge located in an urban transportation hub, which has been in service for 20 years. During the service stage, the cable force values have deviated
Figure 6: Objective function varying with calculation step.

Table 3: Number of occurrences for each cable.

| Cable | Occurrence |
|-------|------------|
| NS1   | 20         |
| NM1   | 20         |
| NM2   | 20         |
| NM3   | 20         |
| NM7   | 8          |
| NS8   | 19         |
| NS9   | 20         |
| NM9   | 19         |
| NS10  | 18         |
| NM10  | 12         |
| NM12  | 4          |
| NS13  | 18         |
| NM13  | 9          |
| NM14  | 5          |
| NM15  | 5          |
| NM16  | 7          |
| NM18  | 19         |
| NS19  | 20         |
| NM19  | 20         |
| NS20  | 20         |
| NM20  | 20         |
| SS1   | 20         |
| SM1   | 20         |
| SM2   | 20         |
| SM3   | 19         |
| SM4   | 3          |
| SS5   | 4          |
| SM5   | 2          |
| SS6   | 8          |
Table 3: Continued.

| Cable  | Occurrence |
|--------|------------|
| SM6    | 6          |
| SS7    | 16         |
| SM7    | 12         |
| SS8    | 17         |
| SS9    | 20         |
| SM9    | 19         |
| SM10   | 8          |
| SS12   | 20         |
| SS13   | 20         |
| SM13   | 15         |
| SM17   | 3          |
| SM18   | 18         |
| SM19   | 20         |
| SS20   | 20         |
| SM20   | 20         |

Note: cables which do not require force adjustment (zero occurrence) are not listed in the table.

Figure 7: Continued.
from the original design values, and the deviations of several cable forces are severe. Case B aims to verify applicability of the proposed method during the maintenance phase.

The main span of the bridge in case B is 180 m, the side span is 150 m, the height of the main pylon is 111 m, and the depth of the pile foundation is 42 m. Each side span (S1–S26) and each main span (P1–P26) contain 26 pairs of cables (104 cables in total). The notation of the pairs of stay cables in side span and main span are S1 to S26 and P1 to P26, respectively, with length of a pair of stay cable increasing. For instance, the shortest pair of stay cables in side span is denoted as S1, the longest pair of stay cables is denoted as S26, the shortest pair of stay cables is denoted as P1, and the longest pair of stay cables is denoted as P26. Detailed information is demonstrated in Figure 9. The cables are made of parallel steel strands with a standard

Table 4: The adjusted values of cable forces (14th calculation) (kN).

| The sequence of stay cable adjustment | Cable | The adjustment value of cable force | The force of stay cable after adjustment |
|--------------------------------------|-------|------------------------------------|----------------------------------------|
| 1                                    | NS1   | 316                                | 2645                                   |
| 2                                    | NM1   | 376                                | 2658                                   |
| 3                                    | NM2   | 174                                | 2632                                   |
| 4                                    | NM3   | 95                                 | 2835                                   |
| 5                                    | NS8   | −78                                | 4161                                   |
| 6                                    | NS9   | −174                               | 4251                                   |
| 7                                    | NM14  | 176                                | 5308                                   |
| 8                                    | NM17  | 354                                | 5600                                   |
| 9                                    | NM18  | 684                                | 5715                                   |
| 10                                   | NS19  | 681                                | 5694                                   |
| 11                                   | NM19  | 751                                | 5726                                   |
| 12                                   | NS20  | 948                                | 5854                                   |
| 13                                   | NM20  | 1134                               | 5711                                   |
| 14                                   | SS1   | 428                                | 2762                                   |
| 15                                   | SM1   | 343                                | 2628                                   |
| 16                                   | SM2   | 173                                | 2633                                   |
| 17                                   | SM3   | 48                                 | 2788                                   |
| 18                                   | SS4   | 276                                | 3484                                   |
| 19                                   | SS6   | 159                                | 3766                                   |
| 20                                   | SM6   | 251                                | 3845                                   |
| 21                                   | SS9   | −171                               | 4220                                   |
| 22                                   | SS12  | −84                                | 4790                                   |
| 23                                   | SS13  | −263                               | 4597                                   |
| 24                                   | SM18  | 591                                | 5682                                   |
| 25                                   | SM19  | 779                                | 5650                                   |
| 26                                   | SS20  | 286                                | 5233                                   |
| 27                                   | SM20  | 1031                               | 5602                                   |

Note: the sequence of stay cables is predetermined.
tensile strength of 1670 MPa. The main girders are made of C50 concrete with $f_{ck} = 32.5$ MPa ($f_{ck}$ is the standard value of axial compressive strength). This bridge has been in service for 20 years and has experienced the shrinkage and creep of the concrete, the retraction of the anchor head, and the relaxation of the steel wire. The actual forces of the stay cables have deviated from the design values. And the value of stay cable forces obtained from the field test is shown in Figure 10.

In order to meet the construction conditions on site and ensure the load balance of the main pylon during the cable adjustment progress, the scheme for the cable adjustment is an alternating predetermined adjustment between the side and the middle span just as case A. Firstly, adjust the force of the shortest pair of stay cables in the side span (S1) and then adjust the force of the shortest pair of stay cables in the main span (P1). Secondly, adjust the second shortest pair of stay cables in the side span (S2) and then adjust the second shortest pair of stay cables in the main span (P2). Subsequently, repeat the adjustment with the above method. The stay cable adjustment sequence of side span and main span is presented as S1-P1-S2-P2-$\cdots$-S25-P25-S26-P26.

**Figure 8:** Stress state of the main girder at the completion of bridge. (a) Upper edge stress of the main girder. (b) Bottom edge stress of the main girder.
Figure 9: General layout of the bridge (m).

Figure 10: Continued.
The finite element model was established for analysis by ANSYS program, and the measured cable force is used as the initial cable force before the bridge cable is adjusted. The optimal design and calculation method of the bridge are the same as those in the previous section (case A). Only dead loads are considered in this model without live loads (wind, earthquake, and truck loads): self-weight (26 kN/m²), secondary load (110 kN/m), and prestressed load. Then, the best set of optimal cable from the 20 calculation results (6th calculation) is directly given, as shown in Table 5. This set of results indicates that only 44 (44/104) stay cables need to be adjusted. The cable force value after adjustment is demonstrated in Figure 10. The maximum error value of cable force after adjustment is about 5%. The stress value of the main beam after the adjustment is shown in Figure 11.

Two engineering cases of concrete cable-stayed bridges (case A and case B) revealed that the proposed method for calculating the cable adjustment force regarding the cable-stayed bridges satisfies cable-stayed bridges in the design phase (case A) and in the maintenance phase (case B).
Figure 11: Stress state of the main girder at the completion of bridge. (a) Upper edge stress of the main girder. (b) Bottom edge stress of the main girder.

Table 6: Summary of cable adjustment in engineering case.

| Case | Bridge state | Bridge type            | Structure type                  | Total stay cables | Number of adjustment cables | Proportion of adjustment cables (%) |
|------|--------------|------------------------|---------------------------------|------------------|-----------------------------|-----------------------------------|
| A    | Design stage | Cable-stayed bridge    | Double pylons with three spans  | 160              | 54                          | 34                                |
| B    | Operation stage | Cable-stayed bridge | Single pylon with three spans  | 104              | 44                          | 42                                |
Besides, the proposed method significantly reduces the number of stay cables which need to be adjusted. The number of stay cables which need to be adjusted is 34% of the total stay cables in case A and 42% of the total stay cables in case B. The detailed data are shown in Table 6.

5. Conclusions

In this study, an innovative optimization method of stay cable adjustment is proposed for cable-stayed bridges considering the minimum number of adjustment cables. The following conclusions can be drawn:

(1) This paper proposed an original calculation method for reducing the numbers of cables while ensuring that the structural stress of the cable-stayed bridge does not exceed the limit value during the cable adjustment progress and the completion stage. The proposed optimization calculation method includes a sensitivity analysis and optimization module and an updating design variable module (UDVM). The sensitivity analysis and optimization module is used to optimize the mechanical performance of the cable-stayed bridge structure, and the updating design variable module (UDVM) can really optimize the number of stay cable adjustment.

(2) It can be seen from case A and case B that the proposed method not only satisfies the structural load requirements in the cable-stayed bridge design phase (case A) and maintenance phase (case B) but also greatly reduces the number of stay cables which need to be adjusted by 66% and 58%, respectively.

(3) The calculation results of the cable adjustment force of case A and case B are obtained based on a pre-determined sequence. Calculation results may be different for different cable adjustment sequences. In the application of engineering, the adjustment sequence can be defined for further optimization according to the actual situation.

(4) The proposed calculation method of cable adjustment force can be used for cable-stayed bridges of any material type: not only for concrete cable-stayed bridges, but also for steel main beam cable-stayed bridges and composite beam cable-stayed bridges. Different types of cable-stayed bridges are reflected in diverse constraints during optimization. For example, the concrete cable-stayed bridge needs to restrain the stress value of the main beam during cable adjustment. However, the steel main girder cable-stayed bridge can be unconstrained because of the higher stress redundancy in the main girder.

The main point of the follow-up research is to optimize the calculation method of updating design variable module and determine the optimal sequence of adjustment cables simultaneously. By optimizing the calculation method of updating design variable module, the calculation efficiency could be improved. Different cable adjustment sequences may lead to different number of stay cables which need to be adjusted. The follow-up research aims to find an optimal sequence of cable adjustment corresponding to the minimum number of stay cables which need to be adjusted automatically.

Data Availability

The data supporting the conclusions in the manuscript come from numerical analysis.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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