Dirac Phase interferometer in a plasmonic waveguide

V. Savinov
Optoelectronics Research Centre & Centre for Photonic Metamaterials, University of Southampton SO17 1BJ, UK

N. I. Zheludev
Optoelectronics Research Centre & Centre for Photonic Metamaterials, University of Southampton SO17 1BJ, UK and Centre for Disruptive Photonic Technologies, TPI, Nanyang Technological University, Singapore 637371, Singapore

By viewing plasmon waves in metallic waveguides as propagating electric and magnetic dipoles we show that according to laws of quantum mechanics they will acquire additional phase when propagating through space with static magnetic field. The new effect is physically different from conventional magneto-plasmonic phenomena and is sufficiently strong to observe it under routinely accessible experimental conditions.

The Aharonov-Bohm (AB) effect is both one of the most celebrated and vigorously debated effects in quantum physics [1, 2]. It is a quantum mechanical topological phenomenon in which phase of an electrically charged particle is affected by the presence of magnetic field, even if the particle is confined to a region in which the field is zero. The modern interpretation of this phenomenon is that the dynamics of the electrons are affected by the magnetic vector potential rather than by the magnetic field.

The Aharonov-Bohm effect arises as a result of Dirac’s magnetic phase factor [3–5], the extra phase gained by the electrons propagating in the electromagnetic potential. There are other similar effects that also arise due to Dirac’s phase factor. In particular, in the Aharonov-Casher (AC) effect the phase is gained by the particle with permanent magnetic dipole due to propagation in electric field [6], whilst in the He-McKellar-Wilkens (HMW) effect [7, 8], the phase is gained by the particles with permanent electric dipole that propagate in magnetic field.

All three effects described above have now been experimentally observed. The Aharonov-Bohm effect was demonstrated by Tonomura [9]. In his experiment electrons were made to pass through and around a toroidal solenoid and then their interference pattern was recorded. The Aharonov-Casher effect was tested with neutron [10] and atomic interferometry [11], and was also observed in solid-state systems [12] [13] as well as in Josephson junctions [14] [15]. The He-McKellar-Wilkens effect was only confirmed recently by interfering trains of polarized lithium atoms [16] [17].

The focus of our paper will be the He-McKellar-Wilkens effect illustrated in Fig. 1. Consider a train of traveling particles with permanent electric (p) and magnetic (m) moments. The train is passed through a particle interferometer where it is split in two branches which are routed along different paths, and are then joined back together into a single train. If the interferometer is placed into magnetic field (B), the phase of the particles in the two branches of the train will be shifted by an amount that will depend on the magnetic field strength. This will lead to magnetically tunable particle interference at the output of the interferometer.

In this paper we will show that the He-McKellar-Wilkens effect can be observed with surface plasmon-polaritons propagating in static magnetic field. Surface plasmon-polaritons are coupled oscillations of light and electron plasma in metals. These propagating coupled oscillations can be described as quantum-mechanical particles propagating along the waveguide trajectory. The correspondence between the wave and particle pictures is established by assuming that particles carry electric and
magnetic dipole moments corresponding to the classical polarization ($\mathbf{P}$) and magnetization ($\mathbf{M}$) of the guided wave. We will now show that the phase gain by plasmon-polaritons due to He-McKellar-Wilkens effect will be observable under routinely accessible experimental conditions in a plasmonic waveguide.

The Lagrangian for a neutral particle with permanent electric ($\mathbf{p}$) and magnetic ($\mathbf{m}$) dipole moments moving at velocity $\mathbf{v}$ through electromagnetic field is \[ L = L_{\text{kin}} + L_{\text{self}} + \mathbf{E} \cdot \left( \mathbf{p} + \frac{1}{c^2} \mathbf{v} \times \mathbf{m} \right) + \mathbf{B} \cdot (\mathbf{m} - \mathbf{v} \times \mathbf{p}) \]

Where $\mathbf{E}$ and $\mathbf{B}$ denote the electric and magnetic fields, respectively. Above, $L_{\text{kin}}$ denotes the kinetic part of the Lagrangian and $L_{\text{self}}$ denotes the Lagrangian due to interaction of the dipoles with their fields. These two terms will be of no interest to us in what is to follow, as the effect in question is related to the last two terms in the Lagrangian.

A quantum mechanical particle described by the Lagrangian from Eq. (1) will gain phase as a result of propagation. The path-specific expression for the phase gain ($\Delta \phi$) due to transition from point $\mathbf{r}_a$ at time $t_a$ to point $\mathbf{r}_b$ at time $t_b$ can be written in terms of action ($S$) divided by reduced Planck constant ($\hbar$):

\[ \Delta \phi = S[\mathbf{r}] / \hbar = \frac{1}{\hbar} \int_{t_a}^{t_b} dt \, L = \Delta \phi_{\text{kin}} + \Delta \phi_{\text{self}} + \frac{1}{\hbar} \int_{t_a}^{t_b} dt \, (\mathbf{p} \cdot \mathbf{E} + \mathbf{m} \cdot \mathbf{B}) + \frac{1}{\hbar} \int_{\mathbf{r}_a}^{\mathbf{r}_b} d\mathbf{r} \cdot \left( \frac{1}{c^2} \mathbf{m} \times \mathbf{E} - \mathbf{p} \times \mathbf{B} \right) \]

In Eq. (2) the fourth term includes phase gained due to increased or reduced energy of the electric and magnetic dipoles in the electromagnetic field. The last term of Eq. (2) differs significantly from the preceding terms in being expressed through an integral along a path rather than a temporal integral. It is relevant to both the He-McKellar-Wilkens and the Aharonov-Casher phenomena. Indeed, due to lack of elementary particles with electric dipole, the recent observations of this effect have been obtained by inducing a polarization in otherwise non-polarized charge-current distribution of the lithium atoms [16–19]. We argue that it should be possible to observe the He-McKellar-Wilkens effect in the traveling waves of induced polarization in metals, i.e. surface plasmon-polaritons. For simplicity we shall consider plasmon waves in a V-groove plasmonic waveguide, as is shown in Fig. 2. Since in a linear waveguide the induced polarization scales with amplitude of the guided mode, one should expect the He-McKellar-Wilkens phase shift in the waveguide to be a function of the power of the guided mode.

For the following analysis we will adopt a specific waveguide geometry with opening angle of 25° and depth of 3 µm (see Fig. 3). We will also assume that the waveguide is made of gold (dielectric constant taken from Ref. [21]) and that the ambient environment is vacuum (or air). Furthermore we shall assume that the free-space wavelength of the light propagating along the waveguide is $\lambda_0 = 1.31$ µm and that the power carried by the waveguide is $P = 1$ µW. These assumptions are in no way mandatory for observation of the effect, instead they represent a set of conditions that can be easily accessed in the experiment.

We proceed by breaking up the continuous traveling charge ($\rho$) and current ($\mathbf{J}$) density waves into individual half-periods, and finding the electric ($\mathbf{p}$) and magnetic
The waveguide

Asymmetry of Vacuum Gold

$\lambda$ is the dielectric constant of gold at

write the full distribution of the electric field as

Consequently, the expressions for the single-half-period

in the xy-plane, the plane perpendicular to the direc-

E

well as the distribution of the electric field (refractive index of the mode $\tilde{n}_{\text{eff}}$) allows to determine the effective

guide, shown in Fig. 3, allows to determine the effective

electric field as

Numerical simulation of the guided mode in the ex-

emplary waveguide. The main plot shows the log-scale colourmap of the intensity of guided radiation in the wave-

guide. The waveguide is made of gold and is operating in free-

space environment. The free-space wavelength of the guided

mode is $\lambda_0 = 1.31 \mu m$. The inset on the bottom left-hand side

shows the details of the waveguide geometry.

(m) dipole moments of each half-period using [22]:

where the integral is taken over the volume occupied

by a single half-period.

Numerical simulation of the mode guided by the wave-

guide, shown in Fig. 3 allows to determine the effective

refractive index of the mode $\tilde{n}_{\text{eff}} = 1.027 - i0.00195$ as well as the distribution of the electric field ($E_{\text{mode}}(x,y)$) in the xy-plane, the plane perpendicular to the direc-

tion of mode propagation (along the z-axis). Ignoring

the losses in the waveguide ($n_{\text{eff}} = \Re(\tilde{n}_{\text{eff}})$) we can write the full distribution of the electric field as $E(r) = E_{\text{mode}}(x,y) \times \exp(-i \frac{2\pi}{\lambda_0} n_{\text{eff}} z)$ and the full current

density distribution as $J = i \omega \epsilon_0 \left( \varepsilon_r^{(Au)} - 1 \right) E$, where $\epsilon_0$ is

the free-space permittivity and $\varepsilon_r^{(Au)} = -79.1 - i7.02$ is the dielectric constant of gold at $\lambda_0 = 1.31 \mu m$ [21]. Consequently, the expressions for the single-half-period

dipoles become:

Numerically evaluating the integrals in Eq. (4,5) we

find, up to a complex phase, $p = (9.8 \times 10^{-27} \text{C.m}) \hat{x}$ and $m = (4.1 \times 10^{-17} \text{J/T}) \hat{y}$. One should note that by

the nature of the guided mode in the plasmonic wave-

guide the two dipoles are perpendicular to each other

($p \perp m$), and to the direction of propagation, at all

times ($p, m \perp v$, where $v$ is the velocity of the guided

mode).

The He-McKellar-Wilkens phase shift of the guided

plasmonic mode can be detected in a plasmonic inter-

ferometer as is shown in Fig. 3. Using the fact that the

magnetic dipole, electric dipole and the velocity are al-

ways mutually perpendicular ($p \perp m \perp v$) the expres-
The phase difference $\Delta \phi_{\text{HMW}}$ scales linearly with magnetic field ($B$), with length difference of the interferometer arms ($\Delta D$), and with magnitudes of the two dipole moments ($m$ and $p$). The dipole magnitudes, in turn, scale as the square-root of the power ($P$) guided by the plasmonic interferometer. One can therefore quantify the He-McKellar-Wilkens effect in a plasmonic interferometer in terms of a dimensional constant:

$$\Lambda_{\text{HMW}} = \frac{\Delta \phi_{\text{HMW}}}{\Delta D \cdot B \cdot \sqrt{P}} \approx 1200 \text{ rad/(nm.T.$\sqrt{W}$)}$$

It is important to note that in plasmonic He-McKellar-Wilkens effect the transmission of the interferometer depends both on the applied magnetic field and on the power of the guided mode, consequently this effect cannot be ascribed to Faraday effect or to any other linear, i.e. power-independent, magneto-optical phenomena. It should also be noted that, in contrast to conventional magneto-optical effects, in the plasmonic He-McKellar-Wilkens effect the mechanism of modulation of interferometer transmission is not linked to material used to implement the waveguide, instead the modulation arises as an intrinsic property of the charge carriers that give rise to the plasmon waves. Finally we address the scaling of the observable effect with optical power. To remain within the quasi-particle model, we shall assume that the asymmetry of the interferometer is small ($\cos(\Delta \phi_{\text{HMW}}) \approx 1 - \frac{1}{2} \Delta \phi_{\text{HMW}}^2$), and thus the observable interferometer disbalance due to He-McKellar-Wilkens effect will scale linearly with guided power.

In conclusion, we have proposed and analyzed a plasmonic version of the He-McKellar-Wilkens effect. We have shown that this effect can be observed under routinely accessible strengths of magnetic field and power of electromagnetic radiation, using a plasmonic waveguide interferometer. The proposed effect will allow the development of a new generation of compact magneto-optical modulators and may be used for tuning active plasmonic devices such as spaser.

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ring interferometers,” *Phys. Rev. Lett.*, vol. 107, p. 016802, 2011.

[14] W. J. Elion, J. J. Wachtets, L. L. Sohn, and J. E. Mooij, “Observation of the Aharonov-Casher effect for vortices in Josephson-Junction arrays,” *Phys. Rev. Lett.*, vol. 71, no. 14, pp. 2311–2314, 1993.

[15] I. M. Pop et al., “Experimental demonstration of Aharonov-Casher interference in a Josephson junction circuit,” *Phys Rev. B*, vol. 85, p. 094503, 2012.

[16] S. Lepouvre, A. Gauguet, G. Trénc, M. Bückner, and J. Vigué, “He-McKellar-Wilkens topological phase in atom interferometry,” *Phys. Rev. Lett.*, vol. 109, p. 120404, 2012.

[17] J. Gillot, S. Lepouvre, A. Gauguet, M. Buchner, and J. Vigué, “Measurement of the He-McKellar-Wilkens topological phase by atom interferometry and test of its independence with atom velocity,” *Phys. Rev. Lett.*, vol. 111, p. 030401, 2013.

[18] S. Lepouvre, A. Gauguet, M. Bückner, and J. Vigué, “Test of the He-McKellar-Wilkens topological phase by atom interferometry. I. Theoretical discussion,” *Phys. Rev. A*, vol. 88, p. 043627, 2013.

[19] ———, “Test of the He-McKellar-Wilkens topological phase by atom interferometry. II. The experiment and its results,” *Phys. Rev. A*, vol. 88, p. 043628, 2013.

[20] J. Anandan, “Classical and quantum Interaction of the dipole,” *Phys. Rev. Lett.*, vol. 85, no. 7, pp. 1354–1357, 2000.

[21] P. B. Johnson and R. W. Christy, “Optical constants of the noble metals,” *Phys. Rev. B*, vol. 6, p. 4370, 1972.

[22] J. D. Jackson, *Classical Electrodynamics*. Wiley, New York ed. 3, 1999.

[23] D. J. Bergman and M. I. Stockman, “Surface plasmon amplification by stimulated emission of radiation: quantum generation of coherent surface plasmons in nanosystems,” *Phys. Rev. Lett.*, vol. 90, p. 027402, 2003.

[24] M. I. Stockman, “Spasers explained,” *Nature Photon.*, vol. 2, pp. 327–329, 2008.

[25] N. I. Zheludev, S. L. Prosvirnin, N. Papasimakis and V. A. Fedotov, “Lasing spaser,” *Nature Photon.*, vol. 2, pp. 351–354, 2008.