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ABSTRACT

We revisit the recent limits from LHCb on a Majorana neutrino $N$ in the mass range 250–5000 MeV [1]. These limits are among the best currently available, and they will be improved soon by the addition of data from Run 2 of the LHC. LHCb presented a model-independent constraint on the rate of like-sign leptonic decays, and then derived a constraint on the mixing angle $V_{\mu 4}$ based on a theoretical model for the $B$ decay width to $N$ and the $N$ lifetime. The model used is unfortunately unsound. We revise the conclusions of the paper based on a decay model similar to the one used for the $\tau$ lepton and provide formulae useful for future analyses.

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1 Introduction

One of the important questions in neutrino physics is that of the existence of massive sterile neutrinos. Neutrinos with zero charges under $SU(2) \times U(1)$ could potentially exist at any of a number of different mass scales. For eV-scale masses, massive neutrinos are invoked to explain anomalies in short-baseline neutrino experiments [2–4]. For masses of $10^9$–$10^{12}$ GeV, massive neutrinos are invoked in the seesaw mechanism that leads to small masses for the known neutrinos even in the presence of large Yukawa couplings [5–7]. However, massive sterile neutrinos could potentially exist at any mass scale, provided they are mixed with the known leptons sufficiently weakly.

For a massive neutrino in the GeV mass range, the Belle and LHCb experiments have recently presented new limits based on the possible appearance of such neutrinos in $B$ meson decays. In the processes studied, the massive neutrino $N$ would be a $B$ meson decay product with an observable lifetime that decays to the final state $\pi^\pm \ell^\mp$. Belle has searched for the production of $N$ in $B \to D, D^* + \ell N$, as well as in $B \to X \ell N$ in an analysis valid for $m_N > 2$ GeV, where $X$ includes a series of light-quark mesons [8]. The Belle analysis states a limit on the mixing of the sterile neutrino with the muon neutrino close to $|V_{\mu 4}|^2 < 0.5 \times 10^{-4}$ over the mass range from about 0.7 GeV to 4.5 GeV. We caution that this limit appears to be over-estimated and may be revised upward for $m_N > 3$ GeV as the result of an ongoing re-analysis [9].

The LHCb experiment has published the results of a dedicated search for a massive Majorana neutrino in the lepton-number-violating decay

$$B^- \to \mu^- N , \quad N \to \pi^+ \mu^-$$

and its charge-conjugate process [1]. Searches are performed for both prompt and displaced $N$ decays. The requirement of like-sign leptons and the requirement that the three final charged particle momenta sum to the $B$ mass reduce the background in this hadron collider experiment to about 70 events, scattered in $m_N$ over the window from 0.5 to 5 GeV. The limit obtained on $|V_{\mu 4}|^2$ is weaker than the Belle limit but potentially competitive in the future, especially with the prospect of including events with $e^\pm$ and other production and decay modes, together with a large new data sample from the LHC Run 2.

The analysis in [1] provides model-independent limits on the rate of like-sign leptonic $B$ decays and then interprets these limits in terms of $|V_{\mu 4}|^2$ based on a theoretical model. Unfortunately, the model described in [1] for the rates of the relevant $B^-$ and $N$ decays is unsound, with dependences on $m_N$ that are difficult to defend theoretically. Replacing these expressions with more correct ones significantly changes the quoted limits. In view of the promise of this experiment to probe for
Majorana neutrinos at a very sensitive level with future data expected from the LHC, we felt that it would be useful to present an improved theory of these decays and to compute more correct limits on $|V_{\mu 4}|^2$ based on the results of [1].

2 Decay rates involving $N$

The analysis of [1] proceeds in two stages. First, model-independent limits are placed on the product of branching ratios $\text{BR}(B^- \to \mu^- N) \cdot \text{BR}(N \to \pi^+ \mu^-)$ as a function of the mass $m_N$ and decay rate $\Gamma_N$. These constraints are then reinterpreted as limits on the sterile neutrino mixing angle $|V_{\mu 4}|^2$ using theoretical expressions that relate $\Gamma_N$ and $m_N$. The efficiency for detecting the events as a function of the lifetime of $N$ plays a non-trivial role.

The decay rates of a massive Majorana neutrino into a variety of relevant final states have been computed by Gorbunov and Shaposhnikov [10], and Atre, Han, Pascoli, and Zhang [11]. In the following expressions, we neglect final-state masses for all particles except the $N$, which is sufficient to demonstrate the parametric dependence of the decay rates. The partial widths relevant to this analysis are:

$$\Gamma(B^- \to \mu^- N) = \frac{G_F^2}{8\pi} m_N^2 f_B^2 m_B |V_{\mu 4}|^2 |V_{ub}|^2 (1 - \frac{m_N^2}{m_B^2})^2$$

$$\Gamma(N \to \pi^+ \mu^-) = \frac{G_F^2}{16\pi} m_N^3 f_\pi^2 |V_{\mu 4}|^2 |V_{ud}|^2$$

(2)

where $f_i$ is the leptonic decay constant for pseudoscalar $i$, $G_F$ is the Fermi constant, and $V_{qq'}$ is the relevant CKM matrix entry. This gives

$$\text{BR}(B^- \to \mu^- N) \cdot \text{BR}(N \to \pi^+ \mu^-) = \tau_B \tau_N \cdot \frac{G_F^4 f_B^2 f_\pi^2 m_B m_N^2}{128\pi^2} |V_{\mu 4}|^4 |V_{ub}|^2 |V_{ud}|^2 (1 - \frac{m_N^2}{m_B^2})^2$$

(3)

where $\tau_B$ and $\tau_N$ are the lifetimes of $B$ and $N$. This result is similar to eq. (1) of [1], which is based on eq. (3.30) of [11]. It differs, however, by a factor $m_N^4/m_B^4$, which gives a substantially smaller signal rate at low values of $m_N$. Of this, one factor of $m_N^2/m_B^2$ comes from the helicity suppression of the first process in (2), while the other factor of $m_N^2/m_B^2$ comes from the reduced phase space for the on-shell $N$ decay. We note that the result in [11] is given only as an estimate for general lepton number violating rare decays.

The $B$ lifetime is known, but the lifetime of $N$ must be computed from theory. The leptonic partial widths are straightforwardly computed. For the hadronic decays, it has been understood for a long time that the total hadronic rate for weak interaction
decays of a lepton is well estimated by the QCD decay to quark-antiquark pairs even for a mass as low as \( m_\tau = 1.78 \) GeV \([12–14]\). This is now the basis of our precision understanding of inclusive \( \tau \) decays \([15]\). The same formalism should apply to compute the lifetime of the \( N \) when the \( N \) has a mass comparable to or higher than \( m_\tau \). Note that, at the same time that we use this QCD estimate to compute the total decay rate of the \( N \), we must still use \( (2) \) to compute the rate for the exclusive decay to \( \pi^+\mu^- \).

For Majorana \( N \), the leptonic charged-current decay rate is

\[
\Gamma(N \to \ell_i^- \ell_j^+ \nu_j) = \frac{G_F^2 m_N^5}{192\pi^3} |V_{ij}|^2 ,
\]

(4)

In this equation, \( i, j = 1, 2, 3 \) run over the three generations, with \( i \neq j \). Also, here and in all cases below, we must add an equal rate for the separate decay to the charge-conjugate channel (here, \( N \to \ell_i^+ \ell_j^- \nu_j \)). The hadronic charged-current decay rate is

\[
\Gamma(N \to \ell_i^- u_j d_j) = 3 \left(1 + \frac{\alpha_s}{\pi} + \cdots\right) \frac{G_F^2 m_N^5}{192\pi^3} |V_{ijd}|^2 .
\]

(5)

Here, \( u_j = (u,c) \), \( d_j = (d,s) \); the third generation is kinematically inaccessible. We can ignore CKM mixing in computing the total rate in the limit of massless 1st and 2nd generation quarks.

The neutral current decay rate to a quark or lepton \( f \) depends on the electric charge \( Q_f \) and the left- and right-handed \( Z \) charges

\[
Q_{ZfL} = \pm \frac{1}{2} - Q_f \sin^2 \theta_w \quad Q_{ZfR} = -Q_f \sin^2 \theta_w ,
\]

(6)

where the + applies to up-type quarks and neutrinos, and the − applies to down-type quarks and charged leptons. Let

\[
S_f = Q_{ZfL}^2 + Q_{ZfR}^2 = \frac{1}{4} - |Q_f| \sin^2 \theta_w + 2Q_f^2 \sin^4 \theta_w .
\]

(7)

Then, for \( f \neq \nu_i \),

\[
\Gamma(N \to \nu_i^- f \bar{f}) = \frac{G_F^2 m_N^5}{192\pi^3} |V_{i4}|^2 S_f ,
\]

(8)

The decay rate for \( N \to \nu_i \nu_j \bar{\nu}_i \) is 2 times the rate for \( N \to \nu_i \nu_j \bar{\nu}_j, i \neq j \). Finally, for the charged lepton decay with \( i = j \),

\[
\Gamma(N \to \nu_i^- \ell_i^+ \ell_j^-) = \frac{G_F^2 m_N^5}{192\pi^3} |V_{i4}|^2 \left(\frac{1}{4} + \sin^2 \theta_w + 2 \sin^4 \theta_w \right) .
\]

(9)

We now evaluate these formulae for the case \(|V_{\mu 4}|^2 \neq 0, |V_{i4}|^2 = 0 \) for \( i \neq \mu \), which is constrained by the LHCb measurement. The charged-current widths to e\( \mu \nu \), e\( e \nu \) and \( \mu \mu \nu \) are, respectively

\[
\Gamma_{e\mu\nu} = \frac{G_F^2 m_N^5}{96\pi^3} |V_{i4}|^2 (1 + 0.13 + 0.59) .
\]

(10)
The charged-current and neutral-current quark contributions, treating $u, d, s, c$ as massless, give

$$\Gamma_{qq} = \frac{G_F^2 m_N^5}{96\pi^3} |V_{\mu 4}|^2 \cdot (8.24). \tag{11}$$

The neutral-current widths to 3 neutrinos gives

$$\Gamma_N \approx \frac{G_F^2 m_N^5}{96\pi^3} |V_{i 4}|^2 \cdot (1.00). \tag{12}$$

The sum of these gives the total width,

$$\Gamma_N \approx \frac{G_F^2 m_N^5}{96\pi^3} |V_{i 4}|^2 \cdot (10.95). \tag{13}$$

The last numerical factor becomes (12.08) when we include decays to $\tau$ while ignoring the $\tau$ mass. The dependence on the mass of the $N$ is always $m_N^5$. This contrasts with eq. (2) of [1], which has the same structure for the leptonic decay width but behaves as $m_N^8$ for the hadronic part of the decay width. Our formula (13) is considerably smaller at large values of $m_N$ than the width employed in the LHCb interpretation.

For small values of the mass of the $N$, the QCD estimate for the total decay rate will break down, and the decay rate must be computed as the sum of exclusive decays to charged and neutral hadrons. We have computed the total decay rate for $N$ in this way, summing over charged and neutral current hadronic decays to $\nu_i\pi$ and $\nu_i\rho$. This approach gives values within 15% of the QCD formula in the region $1.0 \text{ GeV} < m_N < 1.5 \text{ GeV}$. As we discuss below, the transition makes little difference to the final result since, in this mass range, the limit on $|V_{\mu 4}|^2$ is insensitive to the $N$ lifetime.

3 Results

Using the above results, we convert the model-independent LHCb limits on the rate of like-sign lepton decays of $B^-$ into limits on $|V_{\mu 4}|^2$ as a function of $m_N$, assuming $|V_{i 4}|^2 = 0$ for $i = e, \tau$. In our analysis, we compute the $N$ production rate using (3). To compute the total width $\tau_N^{-1}$ of the $N$, we model the hadronic decays using the quark model for $m_N \geq 1.5 \text{ GeV}$ as in (13), and we sum over exclusive hadronic decay modes for $m_N < 1.5 \text{ GeV}$. In this computation, we restore all final-state masses in the expressions for the $B^-$ and $N$ decay rates [16], which were neglected above to simply the analytic expressions.

Ref. [1] provides limits on the product of branching ratios $\text{BR}(B^- \rightarrow \mu^- N) \cdot \text{BR}(N \rightarrow \pi^+ \mu^-)$ for certain values of the lifetime $\tau_N$; we interpolate between the given values, and for very large $\tau_N$, where the mean decay length is far outside the
detector, we use a decay acceptance inversely proportional to the lifetime. For each value of $m_N$, we iteratively scan through values of $|V_{\mu 4}|^2$, determining the mixing angle for which the computed $N$ production rate is equal to the LHC constraint on $\text{BR}(B^- \to \mu^- N) \cdot \text{BR}(N \to \pi^+ \mu^-)$ for the lifetime corresponding to $|V_{\mu 4}|^2$. To evaluate (3), we use the same values as LHCb to facilitate comparison: $f_B = 0.19$ GeV, $f_\pi = 0.131$ GeV, $|V_{ub}| = 0.004$, $|V_{ud}| = 0.9738$, $M_B = 5.279$ GeV, $\tau_B = 1.671$ ps. The uncertainties in these quantities have only a small effect on the quoted limits.

The effect of the updated analysis, shown in Fig. 1, is substantial. To understand this, first note that $\text{BR}(N \to \pi^+ \mu^-)$ includes the factor $\Gamma_N^{-1}$ and so is linearly proportional to $\tau_N$. With this in mind, the differences between our result and that of [1] come from two effects: At low values of $m_N$ (below 2 GeV), the change in eq. (3) leads to a substantially smaller event rate at low values of $m_N$ (below 2 GeV). In this region, the limit on the mixing angle is largely insensitive to the lifetime $\tau_N$. The reason for this is that the decay length is sufficiently long that the decay acceptance is inversely proportional to $\tau_N$, cancelling the factor of $\tau_N$ from $\text{BR}(N \to \mu^+ \pi^-)$. At high values of $m_N$ (above 3 GeV), the updated $\tau_N$ is significantly larger than before, leading to a larger $\text{BR}(N \to \mu^+ \pi^-)$. In this region, most $N$ decays occur inside the detector and so this change is mainly reflected in a larger signal rate predicted by theory and, consequently, a stronger limit.
We look forward to a substantial improvement in the limits on $|V_{\mu d}|^2$ from LHCb using the large data sets that will be available from the LHC Run 2 and beyond.

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