The Impact of Rotation on Cluster Dynamics

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Abstract. The evolution of rotating, isolated clusters of stars up to core-collapse is investigated with n-body numerical codes. The simulations start off from axisymmetric generalisations of King profiles, with added global angular momentum. In this contribution we report on results obtained for two sets of single-mass cluster simulations. These confirm the more rapid evolution of even mildly-rotating clusters. A model is presented with rotational energy comparable to ωCentauri’s; it reaches core-collapse in less than half the time required for non-rotating clusters.

1. Background

Star clusters are self-gravitating Newtonian systems of choice where to brew complex gravitational dynamics (Meylan & Heggie 1997 for a review). Observations of old, globular, stellar clusters have led to the formulation of spherically symmetric dynamical models of equilibria. The most successful and universally studied one-integral spherical models are the King (1966) profiles. However, surveys of up to 100 Milky Way clusters have found small but significant departures from spherical symmetry (White & Shawl 1987): fits to their projected isophotes yield ellipticities \( <\epsilon> \equiv 1 - <a/b> \approx 0.07 \pm 0.01 \). A study of 173 clusters in M31 found \( <\epsilon> = 0.09 \pm 0.04 \) (Staneva, Spassova & Golev 1996). Observations of young clusters in the Large Magellanic Cloud revealed isophotal contours with ellipticities as large as \( \epsilon = 0.3 \) (Elson, Fall & Freeman 1987; Kontizas et al. 1990). This raises the possibility that clusters are formed as strongly flattened structures which then evolve towards rounder configurations (cf. Frenk & Fall 1982; Boily, Clarke & Murray 1999; Theis & Spurzem 1999), and brings up important theoretical issues concerning processes which may drive this evolution.

Rotation stretches any stellar association along a preferred axis: observations of the clusters ωCentauri and M13 have shown that they are flattened by rotation (Meylan & Mayor 1986; Merritt, Meylan & Mayor 1997; Lupton, Gunn & Griffin 1987). Thus angular momentum, measured or possibly lost during evolution, offers a way to account for the morphology of clusters. Yet to date there are few evolutionary models of clusters with initial angular momentum. We have started on a project to develop three-dimensional dynamical models of rotating star clusters. In this articles results for two n-body models of isolated clusters are presented. Previous modelling of rotating clusters is reviewed first.
2. Gas and Fokker-Planck Models of rotating clusters

Agekian (1958) considered the effects of angular momentum diffusion on the equilibria of rotating fluid masses of uniform density. In his analysis, concentric spheroids rotating about their minor axis become rounder in time when the spheroids have initially an ellipticity \( \epsilon = 1 - a/b \leq 0.735 \), where \( a \) and \( b \) are! the minor and major axes. Shapiro & Marchant (1976) integrated the equations of motion for this fluid in the limit of adiabatic (slow) diffusion of momentum. Angular momentum losses are driven by mass elements moving in the direction of the stream leaving the system at a rate higher than those moving in the opposite direction. The energy required for escape comes from ‘heat’, attributed to two-body encounters. Thus, angular momentum losses are accrued over a local two-body relaxation timescale, \( t_{\text{col}} \), which is inversely proportional to the mass density (\( \propto 1/\rho_* \)). In practice this hinders applications of the results to actual clusters, which show centrally peaked density profiles (Meylan & Heggie 1997). Nevertheless, the framework set by Agekian provides a start in linking rotating bodies and observed (non-rotating) globular clusters.

With zero rotation, the central region of a cluster evolves towards a cusp in density during what is known as the gravothermal catastrophe. Does rotation stop the formation of a cusp? Hachisu (1979, 1982) discussed the time-evolution of self-gravitating cylindrical distributions of gas with angular momentum. He predicted a runaway collapse of the central region whenever angular momentum is expelled faster than a critical rate (see also Lagoute & Longaretti 1996). Hachisu dubbed this the ‘gravo-gyro catastrophe’, by analogy with the non-rotating case. These were until recently the only evolutionary models of rotating clusters. Two-dimensional orbit-averaged Fokker-Planck methods have now also been developed to address this issue. Following Goodman’s (1983) approach, Einsel & Spurzem (1999) integrated the Fokker-Planck equation in energy-momentum space \([E, J_z]\). Their initial configurations are truncated King models with added bulk motion. This velocity field takes the form of a Maxwellian distribution such that the mean velocity scales in proportion to radius away from the centre, then drops off at large radii. Their adopted axisymmetric distribution function (cf. Lupton, Gunn & Griffin 1987)

\[
f(E, J_z) \propto \exp\left(-\beta \Omega_o J_z\right) \cdot \left[\exp\left(-\beta E\right) - 1\right]
\]

where \( \beta \) is the inverse square central velocity dispersion and \( \Omega_o \) an angular velocity. The initial conditions are fixed by specifying the dimensionless parameters

\[
\omega_o = \Omega_o/\sqrt{9G\rho_c/(4\pi)} \quad \text{and} \quad W_o,
\]

ie, the scales of angular momentum and gravitational potential, respectively. The latter is the King parameter.

In the 2D Fokker-Planck models core-collapse proceeds on a much shorter timescale than in the non-rotating case, confirming Hachisu’s early intuition. However the central angular velocity does not increase at the high rates expected during the on-set of a gravo-gyro catastrophe; however near the end of core-collapse the central velocity dispersion bears the same relation to the central density as in the non-rotating self-similar collapse. This leaves open the question
of what controls the final phase of evolution in these systems, i.e., whether or not rotation truly survives up to core-collapse. We chose to approach this problem using three-dimensional numerical integration; the setup is summarised below, followed by results and a discussion.

3. Basic properties

Self-consistent n-body realisations of the distribution function were obtained from the equilibrium Fokker-Planck code FOPAX developed by Christian Einsel. The models are fully specified once values are assigned to \((W_0, \omega_o)\). Figure 1 illustrates the properties of a set of models of 10,000 particles with \(W_0 = 6.0\) and four values of \(\omega_o\). The model clusters rotate about the z-axis and the equator lies in the x-y plane of a Cartesian coordinate system. Rotation causes the cluster in equilibrium to flatten down the z-axis and this is shown from computing the components of the inertia tensor \(I_{ij}\) for a series of twenty concentric spherical shells of equal mass \(dM\). We define

\[
\eta[r_k] = 1 - \frac{2 I_{zz}}{I_{xx} + I_{yy}}, \quad \forall \text{ particles in } r_k - dr < r < r_k + dr.
\]  

(2)

The parameter \(\eta = 0\) when the mass within a shell is distributed isotropically; \(\eta < 0\) (or, \(> 0\)) when the distribution is anisotropic oblate (or, prolate). For the spherical model \(\omega_o = 0\) we found indeed near-zero values of \(\eta\) at all radii. Models with rotation have \(\omega_o \neq 0\) and a range of values for \(\eta\) increasing with it. Note that all models, save one with \(\omega_o = 0.8\), have values of \(\eta\) compatible with sphericity at the centre. At larger radii, the models are all distinguished from one another.

Fast-rotating models need be more compact in order to sustain the accrued centrifugal force,

\[
\text{centrifugal force} = \frac{v_0^2}{r} = r \Omega^2,
\]

which must always be smaller than the gravitational force, giving the condition

\[
\Omega^2 \leq \frac{GM}{r_s^3}.
\]  

(3)

Thus at constant mass \(M\) the system radius \(r_s\) must be smaller to allow for larger angular speed \(\Omega\). This is illustrated on figure 1, which displays \(\eta\) and \(\Omega\) computed from the same set of particles. All models show \(\Omega\) decreasing with radius. Note that the curves are consistent with solid-body rotation in the core-region. Further out \(\Omega\) declines to near-zero, in a trend opposite that of \(\eta\). The core remains roundish despite the large angular speed because the gravity is relatively stronger there than near the edge, and so random motion of the particles dominate over streaming motion. Overall the fraction of kinetic energy invested in streaming motion ranges from 0% to 4%, 14% and 26% in increasing order of \(\omega_o\). For comparisons, the cluster \(\omega\text{Centauri}\) invests perhaps as much as 22% of its kinetic energy in rotation (Merritt, Meylan & Mayor 1997).
Figure 1. Initial profile of models with \( W_o = 6.0 \) and four different values of \( \omega_o \). The parameter \( \eta \) defined in (2) is a measure of anisotropy, while \( \Omega \) is the angular speed at radius \( r \) (in model units). Both quantities are averaged over spherical shells.

4. N-body simulations

The code NBODY6++ is an Aarseth-type integration code based on a Hermite expansion of the variables in time (Aarseth 1999). It has been ported to parallel architecture (Spurzem 2000); the calculations were performed on CRAY computers linked up with MPI library. The code treats particles as point-masses and stellar evolution options were switched off. The chain-regularisation algorithm for hierarchical stellar encounters as well as the standard ‘KS’ regularisation (Mikkola & Aarseth 1998; Aarseth 1999) ensures high-precision integration during close interactions. Only simulations with \( N = 5,000 \) equal-mass particles will be discussed. There are no external tides.

Figures 2 & 3 illustrate the time-evolution of the models. The central density, total angular momentum and mean and core radii are plotted as function of time in units of the two-body relaxation time \( t_{col} \) (see Meylan & Heggie 1997; Casertano & Hut 1985). (Note: the lengths were normalised to their initial values.) The top panels show evolution for the case of \( \omega_o = 0.5 \), the bottom set for \( \omega_o = 0.8 \). Looking at these diagrams we find an evolution of the central density similar to the standard case with no rotation: the contraction of the central region leads to more close encounters and ejection, hence further contraction ensues, etc, until \( t \simeq 5 \) \( t_{col} \) when the density peaks sharply, indicating core-collapse: at the end of the simulations \( r_c \approx 0.03 \) and 0.015, respectively, for \( \omega_o = 0.5 \) and 0.8. At constant energy, core-contraction drives the expansion of the outer envelope and hence the mean radius expands rapidly at core-collapse. Note a subtle but noticeable difference between the two simulations, namely that the cluster with initially more rotation evolves faster; this is particularly visible in a comparison of radii at fixed time. A more convincing demonstration...
See GIF figures attached

Figure 2. Time-evolution of the central density (left-most panels), total z-angular momentum (middle) and the mean- and core-radii (right-hand panels) for runs with $W_0 = 6.0$.

of the fast evolution of such clusters follows if we recall that in this unit of time, clusters without rotation reach core-collapse in around $12 \ t_{\text{col}}$, which is more than twice as long.

To appreciate how many stars might be lost to galactic tides, were a tidal field present, we imagine the cluster orbiting the galaxy on a circular orbit. A tidal radius may then be defined from the initial configuration, by computing the radius $r_t$ at which the mean density at time $t$ equates the initial mean density:

$$r_t(t) = \langle r \rangle_0 \times (M[t]/M[0])^{1/3}.$$ 

If we label as escapers all stars found outside $2 \ r_t(t)$, we obtain an estimate of the number of stars likely to leave the cluster on a timescale short compared with $t_{\text{col}}$: the angular momentum they carry with them is deduced from summing up all the momenta of the stars left behind, and comparing with the initial value. Implementing this algorithm, we found the run with $\omega = 0.5$ (top panels) would have lost 3.7% of the initial angular momentum over the time of evolution, but only 1.0% (51:5000) of its mass. The second model, with more rotation, would have lost 0.96% (48:5000) of its mass, but only 1.4% of its total momentum to such escapers. This shows how the cluster redistributes angular momentum efficiently within itself, such that the core evolves towards core-collapse despite added rotational support. Evolution in the core is faster, the faster the core rotates initially, since the cluster is more compact (cf. Eq. 3 and Fig. 1) which speeds up two-body effects.

Figure 3 graphs the specific angular momentum of the $\omega_0 = 0.8$ cluster as a function of radius $r$ for three different times. For comparison, two components are given: the z-axis component about which the cluster rotates; and the x-
See GIF figures attached

Figure 3. Distribution of specific angular momentum components for three time-slots. See text for explanation of the black squares.

axis component. Dividing the cluster in ten concentric shells, we computed $L = r \times v$ and summed up the momenta in each shell: the result is the series of black squares shown on the figure. Initially the (net) $z$-angular momentum increases steadily from the centre, outwards; the symmetry of the figure would make the sum over $L_z$'s cancel out, and the black squares have been left out for this quantity. Evolution is monotonic, with the net $z$-momenta inside $r = 2$pc decreasing, from which we deduce that an increasing fraction of the momentum is transferred to the volume $> 2$pc. Notice on figure [3] that the stars form a core around $r = 0.5$pc in the final stage of the simulation (right-most panels). It is not clear whether this is the result of an $m = 1$ (lopsided) instability, attributable to the dynamics of the system (from a d.f. point of view), or a case of core-wandering, likely due to the small number of particles inside $r = 0.5$pc (Sweatman 1993).

5. Conclusion

The faster evolution of clusters with rotation has been illustrated with two sample runs. The time to core-collapse we found from three-dimensional n-body simulations are in agreement with two-dimensional Fokker-Planck calculations (Einsel & Spurzem 1999): the collapse time of $5.4 \ t_{col}$ obtained for the $\omega_o = 0.8$ agrees with the Fokker-Planck solution of $5.6 \ t_{col}$ for these parameters. This increases confidence in the results up to core collapse, obtained with two different algorithms.
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Discussion

A. Eckart: What happens to the rotation of the cluster after core-collapse?
C. M. Boily: I stopped my simulations precisely at core-collapse. What you measure in the envelope depends a lot on e.g. the galactic tide, which is absent in these simulations. In the core region I expect evolution will proceed much as in the standard case without rotation, since the cusp is isotropic and carries little net momentum. More detailed modelling is needed here.
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