Bound states induced giant oscillations of the conductance in the quantum Hall regime

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Abstract

We theoretically studied the quasiparticle transport in a 2D electron gas biased in the quantum Hall regime and in the presence of a lateral potential barrier. The lateral junction hosts the specific magnetic field dependent quasiparticle states highly localized in the transverse direction. The quantum tunnelling across the barrier provides a complex bands structure of a one-dimensional energy spectrum of these bound states, \( \epsilon_n(p_x) \), where \( p_x \) is the electron momentum in the longitudinal direction \( y \). Such a spectrum manifests itself by a large number of peaks and drops in the dependence of the magnetic edge states transmission coefficient \( D(E) \) on the electron energy \( E \). E.g. the high value of \( D \) occurs as soon as the electron energy \( E \) reaches gaps in the spectrum. These peaks and drops of \( D(E) \) result in giant oscillations of the transverse conductance \( G_x \) with the magnetic field and/or the transport voltage. Our theoretical analysis, based on the coherent macroscopic quantum superposition of the bound states and the magnetic edge states propagating along the system boundaries, is in a good accord with the experimental observations found in Kang et al (2000 Lett. Nat. 403 59)

Keywords: quantum Hall regime, lateral junction, magnetic edge states, bound states, quantum tunnelling, transmission coefficient

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(Some figures may appear in colour only in the online journal)

1. Introduction

Great interest has been devoted to theoretical and experimental studies of various low-dimensional systems such as tunnel junctions, quantum point contacts (QPC), quantum nanowires, and 2D electron gas based nanostructures, to name a few. These systems show a large variety of fascinating quantum-mechanical effects on the macroscopic scale, e.g. weak localization [2, 3], the quantum Hall effect [4], macroscopic quantum tunnelling [5], the conductance quantization in QPCs [2, 4, 6] etc. Interesting quantum-mechanical phenomena arise in confined quantum Hall systems under dc and ac currents. In particular, nonlinear current voltage characteristics and Hall voltage induced magnetoresistance oscillations occur due to the geometrical resonance in the electron transitions between the Landau levels (the hopping between Landau orbits in the presence of random potential) [7–12] These effects can be observed even in weak magnetic fields.

Another type of magnetic field induced phenomena occur in a 2D electron gas with artificially prepared potential barrier. In such systems the quantum-mechanical dynamics of quasiparticles and the electronic transport become even more complex and intriguing when bound states are present. Indeed, it was shown in [13, 14] that the bound states manifest themselves by narrow drops (the anti-resonance) in the gate voltage dependent conductance of QPCs. It is also well known that the bound states naturally arising on the boundaries of various systems, such as graphene nanoribbons [15], nanowires [16, 17], 2D electron gas under magnetic field, greatly influence the transport properties.

Specific magnetic field dependent bound states can be artificially created if a lateral junction (barrier) is fabricated inside of a 2D electron gas subjected to an externally applied...
magnetic field (see figure 1) [1, 18–21]. These highly localized states are formed due to the quantum-mechanical interference of magnetic edge states occurring on both sides of the barrier. The localization length in the transverse direction \(x\) is of the order of the magnetic length \(\ell_H = \sqrt{\hbar c/(eH)}\) in the regime of a strong externally applied magnetic field, \(H\).

As a 2D electron gas has no boundaries these bound states show delocalized behavior in the longitudinal direction \(y\), and the dynamics of electrons is characterized by conserving the component of the electron momentum \(p_y\). Thus, the lateral barrier forms a 1D channel where the energy spectrum of electrons, \(\varepsilon_{\text{bd}}(p_y)\), contains many bands, and it is shown for two lowest bands in figure 2(A). In the absence of quantum tunnelling across the junction the left–right symmetry of the electron states results in a large number of degenerate states in the electronic spectrum (see figure 2(A), dashed line). The quantum tunnelling provides the lifting of the degeneracy, and the energy spectrum shows a complex structure with bands and gaps (see figure 2(A), solid line). Notice here that in the region of a rather small magnetic field such a spectrum was calculated in [19, 20], and in the region of a strong magnetic field, i.e. in the quantum Hall regime, it was obtained in [1]. Since this spectrum, having an origin in the coherent quantum-interference phenomenon, is extremely sensitive to various interacting effects, the renormalization of the above-mentioned spectrum due to the Coulomb interaction and/or the impurities has been theoretically studied in papers [22–26].

The bands and gaps in the energy spectrum of electronic states bounded to the junction, directly manifest themselves in a large amount of interesting effects as the transport in the longitudinal direction is studied. Indeed, the giant oscillations of the longitudinal conductance with the gate voltage, strongly nonlinear current–voltage characteristics and coherent Bloch oscillations under a weak electric fields, have been predicted and theoretically studied [19, 20]. The electric current flowing in the longitudinal direction is directly expressed in terms of this spectrum because the electrons carrying the current are in the proper quantum-mechanical states of the system, and, hence, the conductance along the lateral junction is equal to zero if the Fermi energy is inside one of the energy gaps and it is large if the Fermi energy is inside one of the energy bands.

These qualitative considerations as well as the analytical calculations presented in [20], cannot be used if the current flows perpendicular to the lateral junction because the incoming electrons are not in the above-mentioned proper quantum-mechanical states (the \(p_y\) component of the electronic momentum does not conserve), and, therefore, in order to obtain the transverse conductance one should solve the problem of the electron resonant scattering by the lateral junction.

Qualitatively, the quantum-mechanical dynamics of electrons in a strong magnetic field and in the presence of a lateral junction can be considered as following: the lateral junction serves as a tunable quantum-mechanical scatterer for propagating edge states (see figure 1). E.g. the edge state 1 propagating from the left lead along the lower boundary is scattered by the barrier into the edge state 2 going to the right lead, and into the edge state 3 which reflects from the barrier to the left lead along the upper boundary. The highest probability of such reflection occurs if the propagation of electrons along the junction is allowed. It takes place if the electron energy \(E\) is inside of the energy bands of bound states. In this case the transmission coefficient \(D(E)\) shows minimal values. As the energy of electrons is tuned to gaps in the spectrum of the bound states, the edge states propagation along the junction is forbidden, and a great enhancement of \(D(E)\) is obtained. These oscillations of \(D(E)\) transform in giant oscillations of \(G_x\) under variation of the magnetic field or the dc voltage. Such oscillations of \(G_x\) have been verified experimentally in [1] but the quantitative analysis has not been done. Notice here, that in [27], the electron current flowing perpendicular to

![Figure 1](image1.png)

**Figure 1.** The point contact with a lateral junction in the quantum Hall regime. A scattering process (propagation and reflection) of magnetic edge states on the electron states localized on the lateral junction, is schematically shown. Here, the magnetic edge states 1, 2 and 3 are the incoming, transmitted, and reflected ones, accordingly.

![Figure 2](image2.png)

**Figure 2.** The energy spectrum of electron states bounded to the lateral junction and corresponding semiclassical trajectories. (A) The lowest two bands of the spectrum \(\varepsilon_{\text{bd}}(p_y)\) are shown by dashed (in the absence of tunnelling) and solid (in the presence of tunnelling) lines. (B) The phase trajectories \(P(p_y)\) for different values of the electron energy: (a) \(\varepsilon < \varepsilon_{\text{cr}}(1) = \varepsilon_0 - \Delta\); (b) \(\varepsilon = \varepsilon_{\text{cr}}(1)\); (c) \(\varepsilon = \varepsilon_{\text{cr}}(2) = \varepsilon_0 + \Delta\); (d) \(\varepsilon > \varepsilon_{\text{cr}}(2)\) (here \(2\Delta\) is the energy gap).
the lateral junction in a 2D electron gas under a strong magnetic field was numerically calculated, and similar oscillations of the conductance were obtained.

The Coulomb interaction between electrons considered in [22–26] renormalizes the energy spectrum of electrons bound to the lateral barrier and may close the energy gaps. It enables incoming electrons to skip along the barrier and come back to the left leads. Thus, such an effect can essentially suppress the transverse conductance with respect to the $e^2/h$ value. However, in this paper we show that such a suppression of the transverse conductance observed in the experiment [1] may be explained in terms of the quantum superposition of the bound states and the magnetic edge states propagating along the system boundaries without invoking the Coulomb interaction.

Below we present a complete analytical solution of the transverse \textit{ballistic} transport of electrons that allows to clarify both qualitatively and quantitatively experimental features [1] such as the small value of the observed conductance in comparison to the expected Landauer conductance, the oscillations of the conductance as a function of the magnetic field or voltage, the dependence of the conductance on the lateral size of the system, etc. Our analysis based on the quantum-mechanical scattering of propagating magnetic edge states on the bound states is in a good accord with both the qualitative scenario and experimental observations.

The paper is organized as follows: in section 2 we derive the generic equations of the problem and analyze their solutions in the semiclassical approximation. We show that dynamics of electrons in the vicinity of the barrier is determined by a peculiar quantum tunnelling of electrons between semiclassical trajectories. In section 3, we obtain the probability of the quantum tunnelling between the semiclassical trajectories. In section 4 we prove that the transmission probability calculated in section 3 is also the global transparency of the point contact and by making use of the Landauer approach we obtain giant oscillations in the dependence of the transverse conductance on the dc voltage and magnetic field. Section 5 provides conclusions. In the appendices we present the gauge transformation of the electron wave function that makes it possible to derive the Landauer formula for the current.

2. Formulation of the problem

Let us consider a QPC fabricated in a two-dimensional electron gas in the presence of a lateral junction and subject to an externally applied magnetic field. The magnetic field $H$ is perpendicular to the QPC plane. The QPC is characterized by the coordinate-dependent electrostatic potential $V(x, y) = \frac{m\omega_x^2}{2} y^2 + V(x)$, where the last term describes the potential barrier between two parts of the electron gas. The parameter $\omega_x$ characterizes the curvature of the confinement potential of the QPC, $m$ is the electron effective mass.

Quantum dynamics of electrons in the QPC with the lateral junction is described by the wave function $\Psi(x, y)$ satisfying the two-dimensional Schrödinger equation:

\[
-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + \left[ \frac{1}{2m} \left( -i\hbar \frac{\partial}{\partial y} - eH_x/c \right)^2 + \frac{m\omega_y^2}{2} y^2 + V(x) - E \right] \Psi = 0, \tag{1}
\]

where the Landau gauge, i.e. the vector-potential $A = (0, H_x, 0)$, is used. Here, the axis $y$ is parallel to the barrier and the $x$-axis is directed along the QPC (see figure 1). Introducing the dimensionless variables, $\xi = x/\ell_c$ and $\varepsilon = E/(\hbar\omega_c)$ (the $\omega_c$ is the cyclotron frequency), we write the total wave function $\Psi$ as

\[
\Psi(\xi, \varepsilon) = \int_{-\infty}^{\infty} Q(p_z, \xi) \exp\{i\frac{p_z y}{\hbar}\} dp_z,
\]

\[
Q(p_z, \xi) = \sum_{n=0}^{\infty} R(p_z) \varphi_{n, p_z}(\xi). \tag{2}
\]

Here, the partial wave functions $\varphi_{n, p_z}(\xi)$ describe the magnetic field dependent electron states bounded to the lateral junction and satisfy to the equation

\[
\frac{\partial^2 \varphi_{n, p_z}}{\partial \xi^2} - \left[ \frac{p_z \ell_c}{\hbar} - \xi \right]^2 + \nu(\xi) - 2\varepsilon_n(p_z) \varphi_{n, p_z}(\xi) = 0 \tag{3}
\]

with the boundary conditions being $\varphi_{n, p_z}(\xi) \to 0$ at $\xi \to \pm\infty$.

Here, the dimensionless potential of the junction is $\nu(\xi) = 2\omega_b V(\xi, \varepsilon)/\hbar^2$. These bound states are formed from the magnetic edge states propagating along the barrier $\phi_{n, p_z}^{(i)}(\xi)$ (or $\phi_{n, p_z}^{(r)}(\xi)$) on the left (right) parts of the lateral junction (it is shown in figure 1 in red (gray)). In the quantum Hall regime as $E \approx \hbar\omega_c$ these bound states decay in the transverse direction on the distance $\ell_c$. In the absence of quantum tunnelling the electronic spectrum contains a large amount of degenerate states. The quantum-mechanical interference between left and right edge states results in a lifting of this degeneracy, and a peculiar one-dimensional spectrum $\varepsilon_n(p_z)$ with an alternating sequence of narrow energy bands $\sim \sqrt{1 - |\xi|^2} \hbar\omega_c$ and energy gaps, $[1, 19, 20] \Delta_n \sim |\xi| \hbar\omega_c$, where $|\xi|^2$ is the barrier transparency (see figure 2(A)).

The transverse electronic transport is determined by the partial wave functions $R(p_z)$ which, in turn, are entangled with the bound states.

The generic total quantum-mechanical state is presented as a superposition of basis functions, i.e.

\[
Q_n(p_z, \xi) = \sum_{n=0}^{\infty} \left\{ C_n^{(1)}(\varepsilon) R_n^{(1)}(p_z) \phi_{n, p_z}^{(1)}(\xi) + C_n^{(2)}(\varepsilon) R_n^{(2)}(p_z) \phi_{n, p_z}^{(2)}(\xi) \right\}. \tag{4}
\]

Here, $R_{n, \pm 2} = R_n \pm R_{n+1}$ are normalized to the unit flux density, the energy dependent coefficients $C_n^{(1)}(\varepsilon)$ and $C_n^{(2)}(\varepsilon)$ determine the transmission coefficient $D(E)$ and, therefore, the electronic transport.
The partial wave functions $R_{n}^{(j,l)}(p_y)$ are satisfied to the following set of coupled equations (its derivation is given in the supplementary material (stacks.iop.org/JPhysCM/28/255301/mmedia)):

\[
\begin{align*}
-\alpha^2 \hbar^2 \frac{d^2 \varepsilon_{n}^{(j,l)}(p_y)}{dp_y^2} + \varepsilon_{n}^{(j,l)}(p_y) - \varepsilon \varepsilon_{n}^{(j,l)}(p_y) &= 0; \\
-\alpha^2 \hbar^2 \frac{d^2 \varepsilon_{n}^{(r,l)}(p_y)}{dp_y^2} + \varepsilon_{n}^{(r,l)}(p_y) - \varepsilon \varepsilon_{n}^{(r,l)}(p_y) &= 0.
\end{align*}
\]

Here, $\alpha = \omega_1/\omega_c$, and $\varepsilon_{n}^{(j,l)}(p_y)$ and $\varepsilon_{n}^{(r,l)}(p_y)$ are the energy spectrum of the left (right) edge states in the absence of the tunnelling (dashed line in figure 2(A)), they degenerate, $\varepsilon_{n}^{(j,l)}(p_y) = \varepsilon_{n}^{(r,l)}(p_y)$, at $p_y = p_y^0$; the dimensionless energy gap $\Delta_n = \Delta \hbar/\omega_c \approx -\delta_{n,1}(0)/\sqrt{2}$ is determined by the overlapping of the left and right edge state functions in the middle of the barrier $y = 0$, i.e. by the quantum tunnelling across the barrier. We assume that the dimensionless energy gaps are small. For the sake of simplicity, below we drop the subscript $n$ assuming all the energy gaps to be equal.

As $\alpha \ll 1$ one may solve this set of equations in the semiclassical approximation. Indeed, substituting

\[
R_n^{(j,l)}(p_y) = A_n^{(j,l)}(p_y) \exp\left(\frac{\text{ih} \alpha \tilde{\varepsilon}_n^{(j,l)}(p_y)}{\hbar} \right)
\]

in equation (5) and introducing the classical momenta as $P = dS/dp_y$ one finds that the electron dynamics is determined by the semiclassical phase trajectories $R_n(p_y; \varepsilon)$ as

\[
p_y^2 = \varepsilon - \varepsilon_{n}^{(j,l)}(p_y) + \varepsilon_{n}^{(r,l)}(p_y) \pm \sqrt{\left(\varepsilon_{n}^{(j,l)}(p_y) - \varepsilon_{n}^{(r,l)}(p_y)\right)^2 + \Delta^2}
\]

while the semiclassical parameter is

\[
\kappa = \frac{\omega_1 \ell_y}{\alpha \nu} \approx \left|\varepsilon - \varepsilon_{n}^{(j,l)}(p_y)\right| > 1.
\]

The semiclassical phase trajectories in the vicinity of the lowest degenerate energy $\varepsilon_0$ determined by equation (8) at $n = 0$ for different values of the electron energy $\varepsilon$ are shown in figure 2(B). As the well separated phase trajectories approach each other, quantum tunnelling occurs between them. However, in contrast to the standard interband transitions [28], in the case under consideration the tunnelling takes place in the vicinity of Lifshitz’s phase transition [29]. Indeed, as it follows from equation (8) there are two critical energies at which the topology of trajectories changes: at first, in the vicinity of the energy $\varepsilon_{c1}^{(2)} = -\varepsilon_0 + \Delta$ the mutual directions of motion on the two open trajectories vary (see figures 2(Ba) and(Bb)); secondly, a new closed orbit arises at the critical energy $\varepsilon_{c2}^{(2)} = \varepsilon_0 + \Delta$ (see figures 2(Bc) and (Bd)).

As one sees from equation (9) the semiclassical approximation breaks down not only at the turning points but also in the vicinity of the degeneration points $|p_y - p_y^0| \sim \alpha^{2/3}p_y$, where the quantum tunnelling between the trajectories takes place. Therefore, in order to find the transparency $D(\varepsilon)$ of the point contact (which determines its conductance) one should match the above-mentioned semiclassical function solving the set of differential equations equation (5) in the direct vicinity of points $p_y^0$ without use of the semiclassical approximation.

In the next section we solve this problem and find the probability of the quantum tunnelling between the semiclassical trajectories that allows to match the above-mentioned semiclassical functions.

### 3. Quantum tunnelling between semiclassical trajectories in the vicinity of degeneration points

In order to properly elaborate quantum tunnelling between various semiclassical phase trajectories (which takes place near the degeneration points $p_y^0$, see the above section) we solve equation (5) in a region $|p_y - p_y^0| \ll p_y$ that includes the degeneracy point $p_y^0$ and, on the other hand, overlaps with the regions where the semiclassical solutions are valid.

For the sake of simplicity, in this section we consider dynamics of electrons near the lowest degeneration point $p_y^0$. The solution of this problem in the general case of any $p_y^0$ is presented in the supplementary material.

Expanding the bound energy states in equation (5) $\varepsilon_0^{(j,l)}(p_y) \approx \varepsilon_0 \pm \nu p_y$ and introducing the Fourier transformation as

\[
R_0^{(j,l)}(p_y) = \int_{-\infty}^{\infty} g_{1,2}(Y) \exp\left[-\frac{\text{i} \nu Y Y p_y}{\hbar \alpha} \right] dY,
\]

where the Fourier transform $g_{1,2}(Y)$ is presented in the form

\[
g_{1,2}(\zeta) = e^{\pm i \nu (\zeta^2 + 2 \nu (\zeta))},
\]

one gets the following set of equations\(^3\) for the new variable $\zeta = Y [\nu \omega_1/\alpha \nu]^{1/3}$ (here, $\nu = |d\varepsilon_0^{(j,l)}(p_y)/dp_y|$ is the electron velocity):

\[
\begin{align*}
\frac{d\nu_1(\zeta)}{d\zeta} &= -\gamma e^{-\nu_2(\zeta)}, \\
\frac{d\nu_2(\zeta)}{d\zeta} &= +\nu e^{\gamma \nu_2(\zeta)}
\end{align*}
\]

where the parameters

\[
\gamma = \Delta \left[\frac{\nu_1 \omega_1}{\alpha \nu}\right]^{2/3} \gamma, \quad \nu = \nu_0 - \varepsilon \left[\frac{\nu_1 \omega_1}{\alpha \nu}\right]^{2/3}.
\]

Here, the parameter $\eta$ controls the topology of the phase trajectories (see figure 2(B)) and the parameter $\gamma$ determines the probability of quantum tunnelling between trajectories.

---

\(^3\) An analogous equation defines the magnetic breakdown probability near the touching points of classical orbits of an electron under a strong magnetic field [30].
that conserves the projection of the electron at $2\hbar$, is the Airy function \[31\] (see supplementary material for details). The origin of these oscillations is the quantization of equation (2). In the opposite regime at $E \gg \hbar \omega_c$, the fast oscillations are partially washed out at finite temperature (see appendix A). However, the obtained wave functions are the solution of Schrödinger’s equation (1) written in the Landau gauge $\mathbf{A} = (0, Hx, 0)$ that does not allow its direct using for description of the transport properties of the point contact along the $x$-axis. In appendix B we show how this wave function may be presented in the form convenient for the transport description:

$$x \rightarrow -\infty; \Psi_E^{(1)}(x, y) = e^{-i\hbar E/\hbar x}$$

$$\times \sum |n\rangle \left[ C_{in}^{(1,n)} + i C_{in}^{(2,n)} \right] \chi_n^{(+)}(y)$$

$$+ C_{out}^{(1,n)} \exp \left[ -i\frac{p^{(n)\perp}_x}{\hbar} \right] \chi_n^{(-)}(y) \right)$$

$$x \rightarrow +\infty; \Psi_E^{(2)}(x, y) = e^{-i\hbar E/\hbar x}$$

$$\times \sum |n\rangle \left[ C_{out}^{(1,n)} + i C_{out}^{(2,n)} \right] \chi_n^{(+)}(y)$$

$$+ C_{in}^{(1,n)} \exp \left[ -i\frac{p^{(n)\perp}_x}{\hbar} \right] \chi_n^{(-)}(y) \right).$$

The wave functions inside the square brackets are solutions of Schrödinger’s equation (1) at $|x| \gg R$, in which the gauge of the vector potential is changed to $\mathbf{A} = (-Hy, 0, 0)$ that conserves the projection of the electron momentum $p_x$ parallel to the longitudinal direction of the junction (see, e.g. [32]):

$$\chi_n^{(+)}(y) = \exp \left[ -\frac{y^2}{2\ell_c} \right] H_n \left( \frac{y_n^{\perp}}{\ell_c} \right)$$

where $p^{(n)\perp}_x = \sqrt{2m^*\left[E - \hbar^2 \omega_c (n + 1/2)\right]}$, $m^* = m/\ell_c^2$ is the re-normalized mass, and $y_n^{\perp} = y \pm p^{(n)\perp}_x/\hbar H$.

The coefficients of the incoming, $C_{in}^{(r,l,n)}$, and outgoing wave functions, $C_{out}^{(r,l,n)}$, (which are normalized to the unity flux) are determined by the relations

$$|C_{out}^{(r,l,n)}|^2 = D_n |C_{in}^{(r,l,n)}|^2,$$
The typical dependence of $G(x)$ is determined by equations (20) and (21). This spectrum contains an alternating factor $\gamma_{\omega}/2\pi$. The two reasons determine the energy of degeneration of the left and right edge states and (21) in which $\omega_{n}$ are the values of the magnetic field used in panels (a) and (b), respectively. The chosen ratio of the magnetic fields corresponds to the experimental data presented in [1].

while
\[ D_{0}(E) = \gamma_{\omega}^{2/3}2\pi^{2/3}A_{\text{d}}(2^{2/3}/\omega_{n})^{2/3} \]  
and
\[ \gamma_{\omega} \approx \Delta \left( \frac{\ell_{c}^{2/3}}{\alpha_{\nu n}} \right); \quad \eta_{n} \approx (\varepsilon_{n} - \varepsilon)\left( \frac{\ell_{c}^{2/3}}{\alpha_{\nu n}} \right)^{2/3}, \]  
where $\nu_{n} = d\varepsilon_{n}/dp_{x}$ taken at $p_{y} = p_{y}^{(n)}$ (see supplementary materials, section III, for details).

Using equations (16) and (17) together with equation (19) one finds the current flowing along the junction as follows:

\[ I(V) = \frac{2e}{h} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \int_{-\infty}^{\infty} dE_{n}(E - E_{k}^{n}-iV) \times [f(E + eV/2) - f(E - eV/2)], \]  
where $D_{0}(E - E_{k}^{n}(V))$ is determined by equations (20) and (21) in which $\varepsilon_{n}$ is changed to $E_{k}^{n}(V)$, the latter being the energy of degeneration of the left and right edge states under applied voltage drop $V$, that is $E_{k}^{n}(V) \equiv E_{k}^{n}(p_{x}) + eV/2 = E_{k}^{n}(p_{x}) - eV/2$.

The differential conductance $G(x)$ is determined as $G_{x} = dI/dV$. The typical dependence of $G_{x}(V)$ determined by equations (15) and (22) is shown in figure 4.

The dependence of $G_{x}(V)$ displays giant oscillations with the period $\delta E_{2} \approx \hbar \omega_{n}$. The two reasons determine the appearance of peaks in the $G_{x}(V)$ dependence: at first, the applied voltage distorts the spectrum of bound states to $e_{f}(p_{y}, eV)$, and secondly, the voltage tunable energy of electrons $E = \mu + eV/2$ traces the energy gaps $\Delta_{n}$ in the spectrum of bound states. However, notice here that the transverse conductance is much smaller than the conductance quantum $2e^{2}/\hbar$ at all values of $\mu + eV/2$.

The consistent experimental study of the conductance of a 2D electron gas biased in the quantum Hall regime and in the presence of a lateral junction has been carried out in [1]. Our quantitative analysis presented in figures 3 and 4 shows all important features observed in [1], namely a great enhancement and fast oscillations of the linear conductance $G_{x}(0)$ as the magnetic field was decreased (a decrease of the magnetic field results in an effective increase of the chemical potential $\mu$), and unique giant oscillations of $G_{x}(V)$.

5. Conclusion

In conclusion, we have shown that in the quantum Hall regime a lateral junction formed in the QPC serves as a unique quantum-mechanical scatterer for propagating magnetic edge states. Such a lateral junction hosts the electrons bound states having a peculiar magnetic field dependent 1D spectrum $\varepsilon_{f}(p_{x})$. This spectrum contains an alternating sequence of narrow energy bands and gaps. These band and gaps manifests themselves by giant oscillations of the transmission coefficient $D(E)$ on the electron energy $E$ and the voltage dependent conductance $G_{x}(V)$. Our theoretical analysis based on the coherent quantum-mechanical superposition of localized and delocalized magnetic edge states is in a good accord with the experimental observations [1]. In particular, the zero bias peaks in the conductance $G_{x}(0)$ are essentially suppressed in respect to the $e^{2}/\hbar$ value that resolves this puzzling observation without taking into account the electron–electron interaction. Such a generic approach (see the equations (5) and (15)) can be applied to the variety of solid state systems where the physical properties are determined by the coherent quantum dynamics of interacting quantum-mechanical objects, e.g. the magnetic edge states propagation in graphene based nanostructures [21] or the superconducting quantum metamaterials [33].

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Appendix A. Asymptotic of the electron wave functions at $x \gg R_{1}$: the particular gauge $A = (0, H_{x}, 0)$

Equation (19) allows to connect the asymptotic of the electron wave functions both in the momentum and coordinate spaces for any number of incoming modes $n$.
Taking the semiclassical solutions of equation (5) in the region $|p_r| \gg (eH/c)R_b = p_c = \sqrt{2mE}$, i.e. far from the barrier, one finds the Fourier factor $Q$ (see equation (4)) as follows:

$$Q^{(l,r)}(x,p_r) = \frac{1}{\sqrt{p_{l}^{(n)}}} \times \left[ C^{(l,r)}_{in} \exp \left( i \frac{E_l^{(n)}}{c} p_{l}^{(n)} x \right) \right]$$

$$\times \exp \left[ \frac{(x + c\ell_p/\ell c)^2}{2c^2} \right] \frac{H_n \left( \frac{x + c\ell_p/\ell c}{\ell c} \right)}{H_n (\ell c)} \right),$$

(A.1)

where the sum is over all magnetic edge state modes $n$ taken at a given electron energy $\varepsilon_n$; $H_n(x)$ is the Hermite polynomial; the superscripts $(l)$ and $(r)$ denote the regions $p_x < 0$ and $p_x > 0$, respectively.

Inserting equation (A.1) in equation (2) of the main text one finds the asymptotic of the wave function $\Psi(x,y)^{(l,r)}$ at $|x| \gg R_b$ in the left part $(l)$, $x < 0$, and the right part $(r)$, $x > 0$, of the system as follows:

$$\Psi^{(l)}(x,y) = \exp \left( -\frac{eHl}{c} xy \right) \sum_{n} \left[ C^{(l,n)}_{in} \exp \left( -\frac{eHl}{c} p_{l}^{(n)} x \right) \right]$$

\[
\times \int_{-\infty}^{+\infty} \exp \left( -q^2/2\sigma \right) \exp \left( \left( \frac{p_{l}^{(n)}}{p_{l}^{(n)}} + \frac{y}{h} \right) q \right) H_n \left( \frac{q}{\sqrt{\sigma}} \right) dq 
\]

\[
+ C^{(l,n)}_{out} \exp \left( -\frac{eHl}{c} p_{l}^{(n)} x \right) \int_{-\infty}^{+\infty} \exp \left( -q^2/2\sigma \right) \exp \left( i\frac{q}{\sqrt{\sigma}} \right) dq 
\]

(A.2)

and

$$\Psi^{(r)}(x,y) = \exp \left( -\frac{eHl}{c} xy \right) \sum_{n} \left[ C^{(r,n)}_{in} \exp \left( -\frac{eHl}{c} p_{l}^{(n)} x \right) \right]$$

\[
\times \int_{-\infty}^{+\infty} \exp \left( -q^2/2\sigma \right) \exp \left( \left( \frac{p_{l}^{(n)}}{p_{l}^{(n)}} + \frac{y}{h} \right) q \right) H_n \left( \frac{q}{\sqrt{\sigma}} \right) dq 
\]

\[
+ C^{(r,n)}_{out} \exp \left( -\frac{eHl}{c} p_{l}^{(n)} x \right) \int_{-\infty}^{+\infty} \exp \left( -q^2/2\sigma \right) \exp \left( i\frac{q}{\sqrt{\sigma}} \right) dq 
\]

(A.3)

where $p_{l}^{(n)} = \sqrt{2m(E - h\omega_0\sigma(n + 1/2))}$, $p_{c} = m\hbar\omega_0$, $\sigma = ehH/c$.

In the next section we show that these wave functions may be presented in the form convenient for calculations of the current flowing in the transverse direction through the point contact.

### Appendix B. Gauge transformation of the wave functions

The wave functions determined by equations (A.2) and (A.3), are the solutions of Schrödinger’s equation (1) of the main text in which the gauge $\mathbf{A} = (0, Hx, 0)$ is used. In this gauge the $p_c$-projection of the momentum of the incoming and outgoing magnetic edge states does not conserve and hence one can not directly use these functions to find the current flowing along the point contact. In this subsection we recast the wave functions equations (A.2) and (A.3) into the form that corresponds to the gauge $\mathbf{A} = (-Hy, 0, 0)$ in which $p_y$ conserves.

We use the generating function of the Hermite polynomials (see, e.g. [34])

$$\exp \left( -s^2 + 2sq \right) = \sum_{k=0}^{\infty} \frac{s^k}{k!} H_k(q).$$

(B.1)

Multiplying the both sides of it by $\exp \left( -q^2/2 + i\xi q \right)$ and integrating with respect to $q$ one gets

$$\int_{-\infty}^{\infty} \exp \left( -q^2/2 + i\xi q \right) \exp \left( -s^2 + 2sq \right) dq = \sum_{k=0}^{\infty} \frac{s^k}{k!} \int_{-\infty}^{\infty} \exp \left( -q^2/2 + i\xi q \right) H_k(q) dq;$$

(B.2)

Integrating the left-hand side one easily finds

$$\sum_{k=0}^{\infty} \frac{s^k}{k!} \int_{-\infty}^{\infty} \exp \left( -q^2/2 + i\xi q \right) H_k(q) dq = \sqrt{2\pi} e^{-\xi^2/2} \exp \left( -(\xi)^2 + 2(\xi)\bar{y} \right).$$

(B.3)

Applying equation (B.1) to the right-hand side of this equation one gets the following:

$$\sum_{k=0}^{\infty} \frac{s^k}{k!} \left\{ \int_{-\infty}^{\infty} \exp \left( -q^2/2 + i\xi q \right) H_k(q) dq \right\} = 0.$$

(B.4)

As $s$ is arbitrary one gets the following equation:

$$\int_{-\infty}^{\infty} \exp \left( -q^2/2 + i\xi q \right) H_k(q) dq = i^k \sqrt{2\pi} H_k(\bar{y}).$$

(B.5)

Now applying the equality equation (B.5) to equations (A.2) and (A.3) one gets the wave functions equations (16) and (17) of the main text.

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