Light-Ray Evolution Equations and Leading-Twist Parton Helicity-Dependent Nonforward Distributions

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We discuss the calculation of the evolution kernels $\Delta W^\zeta(X,Z)$ for the leading-twist nonforward parton distributions $G^\zeta(X,t)$ sensitive to parton helicities. We present our results for the kernels governing evolution of the relevant light-ray operators and describe a simple method allowing to obtain from them the components of the nonforward kernels $\Delta W^\zeta(X,Z)$.

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I. INTRODUCTION

Applications of perturbative QCD to deeply virtual Compton scattering \cite{1} and hard exclusive electroweak processes \cite{1} require a generalization of usual parton distributions for the case when long-distance information is accumulated in nonforward matrix elements \( \langle p-r|O(0,z)|p \rangle \big|_{z=0} \) of quark and gluon light-cone operators. In refs. \cite{1} it was shown that such matrix elements can be parametrized by two basic types of non-perturbative functions. The double distribution \( F(x,y;t) \) specifies the light-cone “plus” fractions \( xp^+ \) and \( yp^+ \) of the initial hadron momentum \( p \) and the momentum transfer \( r \) carried by the initial parton. Since \( r^+ \) is proportional to \( p^+ \): \( r^+ = \zeta p^+ \), it is possible to introduce the nonforward parton distribution \( \mathcal{F}_\zeta(X:t) \) with \( X = x + y\zeta \) being the total fraction of the initial hadron momentum taken by the initial parton. For processes mentioned above, the parameter \( \zeta = 1 - (p^\prime z)/(pz) \) characterizing the longitudinal momentum asymmetry (“skewness”) of the nonforward matrix element takes the values \( 0 < \zeta < 1 \).

At leading twist, there are two light-ray quark operators \( \bar{\psi}(0)\gamma_\mu E(0,z;A)\psi(z) \) and \( \bar{\psi}(0)\gamma_\mu\gamma_5 E(0,z;A)\psi(z) \), where \( E(0,z;A) \) is the standard path-ordered exponential which makes the operators gauge-invariant. In the forward case, the first operator is related to the spin-averaged distribution functions \( f_a(x) \) while the second one corresponds to the spin-dependent distribution functions \( \Delta f_a(x) \). The nonforward parton distributions related to the \( \bar{\psi}(0)\gamma_\mu E(0,z;A)\psi(z) \) operators were studied in refs. \cite{1,2,3,4,5}. In this paper, we will discuss flavor-singlet parton helicity-dependent nonforward distributions corresponding to quark operators \( \bar{\psi}(0)\gamma_\mu\gamma_5 E(0,z;A)\psi(z) \) and the gluonic operator \( G_{\mu\alpha}(0)E(0,z;A)\bar{G}_{\alpha\nu}(z) \) mixing with each other under evolution.

II. NONFORWARD DISTRIBUTIONS

We define the nonforward quark distributions by writing the relevant matrix element as (cf. \cite{1})

\[
\langle p', s' | \bar{\psi}_a(0)\hat{z}\gamma_5 E(0,z;A)\psi_s(z) | p, s \rangle \big|_{z=0} = \int_0^1 \left( e^{-iX(pz)} G^a_\zeta(X;t) + e^{i(X-\zeta)(pz)} \bar{G}^a_\zeta(X;t) \right) dX \]

where \( t = (p' - p)^2 \), \( a \) denotes the quark flavor (here we consider only the flavor-diagonal distributions), \( M \) is the nucleon mass and \( s, s' \) specify the nucleon polarization. Throughout the paper, we use the “hat” convention \( \hat{z} \equiv z^\mu\gamma_\mu \). In Eq. (1), we explicitly separated quark and antiquark contributions (cf. \cite{1}). This definition corresponds to the parton picture in which the initial quark (or antiquark) takes the momentum \( Xp \) from the hadronic matrix element and “returns” into it the momentum \((X-\zeta)p \). Since the fraction \( X - \zeta \) is positive for \( X > \zeta \) and negative when \( X < \zeta \), the nonforward distributions can be divided into two components. In the region \( X \geq \zeta \), one can treat \( G^a_\zeta(X,t) \) as a generalization of the usual distribution function \( \Delta f_a(x) \). In particular, in the limit \( t \to 0, \zeta \to 0 \), the limiting curves for \( G^a_\zeta(X,t) \) reproduce \( \Delta f_a(X) \):

\[
G^a_{\zeta=0}(X,t=0) = \Delta f_a(X) ; \quad G^a_{\zeta=0}(X,t=0) = \Delta f_a(X).
\]

On the other hand, in the region \( X < \zeta \), both quarks should be treated as going out of the nucleon matrix element, with momenta \( Xp \) and \((\zeta - X)p \), respectively. Now, one can define \( Y = X/\zeta \) and treat the function \( G^a_\zeta(X) \) as the distribution amplitude \( \Psi^a_\zeta(Y) \). In particular, the \( \bar{G} \)-part in this region can be written as

\[\text{\footnotesize \textsuperscript{\dagger}The off-forward parton distributions introduced by X. Ji \cite{1} (see also \cite{1,2}) and non-diagonal distributions of Collins, Frankfurt and Strikman \cite{1} can be related to nonforward distributions (see \cite{1}) but do not coincide with them.} \]
\[ \zeta \bar{u}(p') \hat{z} u(p) \int_0^1 \left[ e^{-iY(rz)} G_{\zeta}^a(\zeta Y) + e^{-i(1-Y)(rz)} \tilde{G}_{\zeta}^a(\zeta Y) \right] dY = \zeta \bar{u}(p') \hat{z} u(p) \int_0^1 e^{-iY(rz)} \Psi_{\zeta}^a(Y) dY, \]  

(3)

where the distribution amplitude \( \Psi_{\zeta}^a(Y) \) is defined by \( \Psi_{\zeta}^a(Y) = G_{\zeta}^a(Y \zeta) + \tilde{G}_{\zeta}^a(Y \zeta). \) The function \( \Psi_{\zeta}^a(Y) \) gives the probability amplitude that the initial nucleon with momentum \( p \) is composed of the final nucleon with momentum \( p' = p - r \) and a \( \bar{q}q \) pair in which the pair momentum \( r \) is shared in fractions \( Y \) and \( 1 - Y \).

For gluons, the nonforward distribution \( G_{\zeta}^a(X; t) \) is defined through the matrix element

\[ \langle p' \mid z_\mu z_\nu G_{\mu\nu}^a(0) E^{ab}(0, z; A) G_{\nu\sigma}^b(z) \mid p \rangle \bigg|_{z^2 = 0} \]

\[ = \bar{u}(p') \hat{z} \gamma_5 u(p) \int_0^1 \frac{1}{2} \left[ e^{-iX(pz)} - e^{i(X-\zeta)(pz)} \right] G_{\zeta}^a(X; t) dX \]

\[ + \bar{u}(p') \frac{r^2}{M} \gamma_5 u(p) \int_0^1 \frac{1}{2} \left[ e^{-iX(pz)} - e^{i(X-\zeta)(pz)} \right] P_{\zeta}^a(X; t) dX. \]

As usual, \( \tilde{G}_{\alpha\nu} = \frac{1}{4} \epsilon_{\alpha\nu\beta\mu} G_{\beta\mu}^a. \) Since there are no “antigluons”, the exponentials \( e^{-iX(pz)} \) and \( e^{i(X-\zeta)(pz)} \) are accompanied here by the same function \( G_{\zeta}^a(X; t) \). Again, the contribution from the region \( 0 < X < \zeta \) can be written as

\[ i\bar{u}(p') \hat{z} \gamma_5 u(p) (z \cdot r) \int_0^1 e^{-iY(rz)} \Psi_{\zeta}^a(Y; t) dY + \text{"P" term}, \]

(5)

with the \( Y \leftrightarrow \bar{Y} \) antisymmetric generalized distribution amplitude \( \Psi_{\zeta}^a(Y; t) \) given by

\[ \Psi_{\zeta}^a(Y; t) = \frac{1}{2} \left( G_{\zeta}^a(Y \zeta; t) - \tilde{G}_{\zeta}^a(Y \zeta; t) \right). \]

(6)

In the formal \( t = 0 \) limit, the nonforward distributions \( G_{\zeta}^a(X; t), P_{\zeta}^a(X; t) \) convert into the asymmetric distribution functions \( G_{\zeta}^a(X), P_{\zeta}^a(X) \). Finally, in the \( \zeta = 0 \) limit, \( G_{\zeta}^a(X) \) reduces to the usual polarized gluon density

\[ G_{\zeta=0}^a(X) = X \Delta g(X). \]

(7)

Under pQCD evolution, the gluonic operator

\[ O_g(uz, vz) = z_\mu z_\nu G_{\mu\nu}^a(uz) E^{ab}(uz, vz; A) \tilde{G}_{\nu\sigma}^b(vz) \]

(8)

mixes with the flavor-singlet quark operator

\[ O_Q(uz, vz) = \sum_{a=1}^{N_f} O_{a}^{(+)}(uz, vz) \]

(9)

where

\[ O_{a}^{(+)}(uz, vz) = \frac{1}{2} \left[ \bar{\psi}_a(uz) \hat{z} \gamma_5 E(uz, vz; A) \psi_a(vz) + \bar{\psi}_a(vz) \hat{z} \gamma_5 E(uz, vz; A) \psi_a(uz) \right]. \]

(10)

The nonforward distribution function \( G_{\zeta}^a(X; t) \) for the flavor-singlet quark combination \( \bar{u} \)

\[ \langle p', s' \mid O_Q(uz, vz) \mid p, s \rangle \bigg|_{z^2 = 0} = \bar{u}(p', s') \hat{z} \gamma_5 u(p, s) \int_0^1 \frac{1}{2} \left[ e^{-i\nu X(pz) + i\nu X'(pz)} + e^{i\nu X'(pz) - i\nu X(pz)} \right] G_{\zeta}^a(X; t) dX + \text{"P" term}, \]

(11)

(\( X' = X - \zeta \)) can be expressed as the sum of "a + \bar{a}" distributions:

\[ G_{\zeta}^Q(X; t) = \sum_{a=1}^{N_f} (G_{\zeta}^a(X; t) + \bar{G}_{\zeta}^a(X; t)). \]

(12)
we introduce the flavor-singlet quark distribution amplitude $\Psi_\zeta^{Q}(Y;t)$ which has the symmetry property $\Psi_\zeta^{Q}(Y;t) = \Psi_\zeta^{Q}(\bar{Y};t)$ with respect to the $Y \leftrightarrow \bar{Y}$ transformation.

### III. EVOLUTION EQUATIONS FOR LIGHT-RAY OPERATORS

Near the light cone $z^2 \sim 0$, the bilocal operators $O(uz,vz)$ develop logarithmic singularities $\ln z^2$. Calculationally, these singularities manifest themselves as ultraviolet divergences for operators taken on the light cone. The divergences are removed by a subtraction prescription characterized by some scale $\mu$: $G_\zeta(X;t) \rightarrow G_\zeta(X;t;\mu)$. At one loop, the set of evolution equations for the flavor-singlet light-ray operators has the following form (cf. [9,10]):

$$
\mu \frac{d}{d\mu} O_a(0,z) = \int_0^1 \int_0^1 \sum_b A_{ab}(u,v) O_b(uz,\bar{v}z) \theta(u+v \leq 1) \, du \, dv,
$$

where $a, b = g, Q$ and $\bar{v} \equiv 1-v$, $\bar{u} \equiv 1-u$. For flavor-nonsinglet distributions, there is no mixing, and their evolution is generated by the $QQ$-kernel alone. To calculate the kernels, we incorporated the approach [10] based on the background-field method. Below we present our results in the form similar to that used in refs. [6,4]:

$$
A_{QQ}(u,v) = \frac{\alpha_s}{\pi} C_F \left( 1 + \frac{3}{2} \frac{\delta(u)\delta(v)}{v} + \left\{ \frac{\delta(u)}{v} - \delta(v) \int_0^1 d\tilde{v} \frac{1}{\tilde{v}} \right\} + \{u \leftrightarrow v\} \right),
$$

(15)

$$
A_{gQ}(u,v) = \frac{\alpha_s}{\pi} C_F \left( \delta(u)\delta(v) - 2 \right),
$$

(16)

$$
A_{Qg}(u,v) = \frac{\alpha_s}{\pi} N_f (1-u-v),
$$

(17)

$$
A_{gg}(u,v) = \frac{\alpha_s}{\pi} N_c \left( 4(1-u-v) + \frac{\beta_0}{2N_c} \delta(u)\delta(v) + \left\{ \delta(u) \left( v^2 \frac{1}{v} - \delta(v) \int_0^1 d\tilde{v} \frac{1}{\tilde{v}} \right) + \{u \leftrightarrow v\} \right\} \right).
$$

(18)

Independently, these kernels were calculated by Blumlein, Geyer and Robaschik [11,12]. Their results agree with ours.

### IV. EVOLUTION EQUATIONS FOR NONFORWARD DISTRIBUTIONS

Inserting the light-ray evolution equations (14) between chosen hadronic states and parametrizing matrix elements by appropriate distributions, one can get the “old” DGLAP [13,14] and BL-type [16,17] evolution kernels as well as calculate the new kernels $\Delta W_\zeta^{ab}(X,Z)$ governing the evolution of nonforward parton distributions:

$$
\mu \frac{d}{d\mu} G_\zeta^{ab}(X;t;\mu) = \int_0^1 \sum_b \Delta W_\zeta^{ab}(X,Z) G_\zeta^{b}(Z;t;\mu) \, dZ.
$$

(19)

Extracting $\Delta W_\zeta^{ab}(X,Z)$ from the light-ray kernels $A_{ab}(u,v)$, one should take into account the extra $(pz)$ factor in the rhs of the gluon distribution definition, which under the Fourier transformation with respect to $(pz)$ results in the differentiation $\partial/\partial X$. Thus, it is convenient to introduce first the auxiliary kernels $\Delta M_\zeta^{ab}(X,Z)$ directly related to the light-ray kernels $A(u,v)$ by

$$
\Delta M_\zeta^{ab}(X,Z) = \int_0^1 \int_0^1 A_{ab}(u,v) \delta(X - \bar{u}Z + v(Z - \zeta)) \theta(u+v \leq 1) \, du \, dv.
$$

(20)
The $\Delta W$-kernels are obtained from the $\Delta M$-kernels using

$$
\Delta W^{gq}_\zeta(X, Z) = \Delta M^{gq}_\zeta(X, Z), \quad \Delta W^{QQ}_\zeta(X, Z) = \Delta M^{QQ}_\zeta(X, Z),
$$  

(21)

$$
\frac{\partial}{\partial X} \Delta W^{gq}_\zeta(X, Z) = -\Delta M^{gq}_\zeta(\bar{X}, Z) d\bar{X}, \quad \Delta W^{Qq}_\zeta(X, Z) = -\frac{\partial}{\partial X} \Delta M^{Qq}_\zeta(X, Z).
$$  

(22)

Hence, to get $\Delta W^{gq}_\zeta(X, Z)$ we should integrate $\Delta M^{gq}_\zeta(X, Z)$ with respect to $X$. We fix the integration constant by the requirement that $\Delta W^{gq}_\zeta(X, Z)$ vanishes for $X > 1$. Then

$$
\Delta W^{gq}_\zeta(X, Z) = \int^1_X \Delta M^{gq}_\zeta(\bar{X}, Z) d\bar{X}.
$$  

(23)

Integrating the delta-function in eq. (21) produces four different types of the $\theta$-functions, each of which corresponds to a specific component of the kernel governing the evolution of the nonforward distributions.

V. BL-TYPE EVOLUTION KERNELS

When $\zeta = 1$, $G_\zeta(X)$ reduces to a distribution amplitude whose evolution is governed by the BL-type kernels:

$$
\Delta W^{ab}_{\zeta=1}(X, Z) = V^{ab}(X, Z).
$$  

(24)

Taking $\zeta = 1$ in Eq. (20) we obtain

$$
\Delta M^{ab}_{\zeta=1}(X, Z) \equiv U^{ab}(X, Z) = \int^1_0 \int^1_0 A_{ab}(u, v) \delta(X - \bar{u}Z - v(1 - Z)) \theta(u + v \leq 1) du \, dv.
$$  

(25)

In fact, the BL-type kernels appear as a part of the nonforward kernel $W^{ab}_\zeta(X, Z)$ even in the general $\zeta \neq 1, 0$ case. As explained earlier, if $X$ is in the region $X \leq \zeta$, then the function $G_\zeta(X)$ can be treated as a distribution amplitude $\Psi_\zeta(Y)$ with $Y = X/\zeta$. For this reason, when both $X$ and $Z$ are smaller than $\zeta$, the kernels $W^{ab}_\zeta(X, Z)$ simply reduce to the BL-type evolution kernels $V^{ab}(X/\zeta, Z/\zeta)$. Indeed, the relation (20) can be written as

$$
\Delta M^{ab}_\zeta(X, Z) = \frac{1}{\zeta} \int^1_0 \int^1_0 A_{ab}(u, v) \delta(\frac{X}{\zeta} - \bar{u}Z - v(1 - Z/\zeta)) \theta(u + v \leq 1) du \, dv.
$$  

(26)

Comparing this expression with the representation for the $U^{ab}(X, Z)$ kernels, we conclude that in the region where $X/\zeta \leq 1$ and $Z/\zeta \leq 1$, the kernels $\Delta M^{ab}_\zeta(X, Z)$ are given by

$$
\Delta M^{ab}_\zeta(X, Z)|_{0 \leq \{X, Z\} \leq \zeta} = \frac{1}{\zeta} U^{ab}(X/\zeta, Z/\zeta).
$$  

(27)

Now, using the expressions connecting the $\Delta W$- and $\Delta M$-kernels, we obtain the following relations between the nonforward evolution kernels $\Delta W^{ab}_\zeta(X, Z)$ in the region $0 \leq \{X, Z\} \leq \zeta$ and the BL-type kernels $V^{ab}(X, Z)$:

$$
\Delta W^{QQ}_\zeta(X, Z) = \frac{1}{\zeta} V^{QQ}(X/\zeta, Z/\zeta); \quad \Delta W^{gq}_\zeta(X, Z) = V^{gq}(X/\zeta, Z/\zeta);
$$

$$
\Delta W^{Qq}_\zeta(X, Z) = \frac{1}{\zeta} V^{Qq}(X/\zeta, Z/\zeta); \quad \Delta W^{gg}_\zeta(X, Z) = \frac{1}{\zeta} V^{gg}(X/\zeta, Z/\zeta).
$$  

(28)

The kernels $V^{ab}(X, Z)$, in their turn, are derived from the auxiliary kernels $U^{ab}(X, Z)$. Due to the symmetry property $A_{ab}(u, v) = A_{ab}(v, u)$ the kernels $U^{ab}(X, Z)$ satisfy $U^{ab}(X, \bar{Z}) = U^{ab}(X, Z)$. Hence, it is sufficient to know the $U$-kernels in the $X \leq Z$ region only:

$$
U^{ab}(X, Z) = \theta(X \leq Z) U^{ab}_{0}(X, Z) + \theta(Z \leq X) U^{ab}_{0}(\bar{X}, \bar{Z}),
$$

5
with the basic function $U_{0}^{ab}(X, Z) \equiv \theta(X \leq Z) U_{0}^{ab}(X, Z)$ given by

$$
U_{0}^{ab}(X, Z) = \frac{1}{Z} \int_{0}^{X} A_{ab}(\bar{v} - (X-v)/Z, v) dv .
$$  (29)

Using Eqs.(13)-(18), the $A \to U_{0}$ conversion formulas

$$
\delta(u) \delta(v) \to \delta(Z - X) , \quad 1 \to \frac{X}{Z} , \quad \delta(u) \frac{\bar{v}}{v} \to 0 , \quad \delta(u) \left(\frac{\bar{v}}{v}\right)^{2} \to 0 ,
$$
$$
\delta(v) \frac{\bar{u}}{u} \to \frac{X}{Z - X} , \quad \delta(v) \frac{\bar{u}^{2}}{u} \to \left(\frac{X}{Z}\right)^{2} \frac{1}{Z - X} , \quad u + v \to \frac{X}{Z} \left(1 - \frac{X}{2Z}\right)
$$
and Eqs.(20)-(24), (28) we get the BL-type kernels

$$
V_{Q\bar{Q}}(X, Z) = \frac{\alpha_{s}}{\pi} C_F \left\{ \left[ \frac{X}{Z} \left(1 + \frac{1}{Z - X}\right) \theta(X < Z) \right]_{+} + \{X \to \bar{X}, Z \to \bar{Z}\} \right\}
$$
$$
V_{Qg}(X, Z) = \frac{\alpha_{s}}{\pi} N_f \left\{ -\frac{X}{Z^{2}} \theta(X < Z) + \frac{X}{Z^{2}} \theta(X > Z) \right\} , \tag{32}
$$
$$
V_{g\bar{Q}}(X, Z) = \frac{\alpha_{s}}{\pi} C_F \left\{ \frac{X^{2}}{Z} \theta(X < Z) - \frac{X^{2}}{Z} \theta(X > Z) \right\} , \tag{33}
$$
$$
V_{gg}(X, Z) = \frac{\alpha_{s}}{\pi} N_c \left\{ \frac{2X^{2} - X - Z}{Z^{2}} \theta(X < Z) + \left[ \frac{\theta(X < Z)}{Z - X} \right]_{+} + \{X \to \bar{X}, Z \to \bar{Z}\} \right\}
$$
$$
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + \frac{\beta_{0}}{2N_c} \delta(X - Z) \right\} , \tag{34}
$$
calculated originally in [14] for flavor-singlet pseudoscalar meson distribution amplitudes. With respect to integration over $0 \leq X \leq 1$, the “plus”-prescription for a function $V(X, Z)$ is defined by

$$
[V(X, Z)]_{+} = V(X, Z) - \delta(X - Z) \int_{0}^{1} V(Y, Z) dY . \tag{35}
$$

The BL-type kernels also govern the evolution in the region corresponding to transitions from a fraction $Z$ which is larger than $\zeta$ to a fraction $X$ which is smaller than $\zeta$. Indeed, using the $\delta$-function to calculate the integral over $u$, we get

$$
\Delta M_{\zeta}^{ab}(X, Z)|_{X \leq \zeta \leq Z} = \frac{1}{Z} \int_{0}^{X/\zeta} A_{ab} \left[ |1 - X/Z - v(1 - \zeta/Z)|, v \right] dv , \tag{36}
$$
which has the same analytic form (23) as the expression for $M_{\zeta}^{ab}(X, Z)$ in the region $X \leq Z \leq \zeta$. For $Q\bar{Q}, gg$ and $Qg$ kernels, this automatically means that $\Delta W_{\zeta}^{ab}(X, Z)$ is given by the same analytic expression as $\Delta W_{\zeta}^{ab}(X, Z)$ for $X < Z < \zeta$. Because of integration in Eq.(23), to get $\Delta W_{\zeta}^{g\bar{Q}}(X, Z)$ one should also know $\Delta M_{\zeta}^{g\bar{Q}}(X, Z)$ in the region $\zeta \leq X \leq Z$. However, our explicit calculation confirms that $\Delta W_{\zeta}^{g\bar{Q}}(X, Z)$ in the transition region $X \leq \zeta \leq Z$ is given by the same expression as $\Delta W_{\zeta}^{g\bar{Q}}(X, Z)$ for $X < Z \leq \zeta$.

In application to parton distributions related to nonforward matrix elements, X. Ji was the first [11] who calculated analogous kernels $P'(x, \xi)$ which govern the evolution of his off-forward parton distributions $H(x, t; \mu)$ in the $-\xi/2 < x < \xi/2$ region (in our variables this region corresponds to $0 < X < \zeta$). He used a direct momentum-representation approach in the light-cone gauge. After proper redefinitions (discussed in [1]), we reproduced his expressions for the first three kernels. For the gluon-gluon kernel, our result formally differs from that obtained by X. Ji [3]. However, due to the symmetry properties of the gluon distribution in the X. Ji approach, the relevant integral vanishes and the difference does not contribute to the evolution. Blumlein et al. [13] derive the “extended”
VI. GENERALIZED DGLAP KERNELS

When \( X > \zeta \), we can treat the asymmetric distribution function \( G_{\zeta}^a(X) \) as a generalization of the usual distribution function \( \Delta f_\mu(X) \) for a skewed kinematics. Hence, evolution in the region \( \zeta < X \leq 1, \zeta < Z \leq 1 \) is close to that generated by the DGLAP equation. In particular, it has the basic property that the evolved fraction \( X \) cannot be larger than the original fraction \( Z \). The relevant kernels are given by

\[
\Delta M^{ab}_{\zeta}(X, Z)|_{\zeta \leq X \leq Z} \equiv \frac{Z - X}{Z Z'} \int_0^1 \alpha_a \left( \bar{w} \left( 1 - X/Z \right), \bar{w} \left( 1 - X'/Z' \right) \right) dw, \tag{37}
\]

where \( X' \equiv X - \zeta \) and \( Z' \equiv Z - \zeta \) are the “returning” partners of the original fractions \( X, Z \). Note, that since \( Z - X = Z' - X' \), the kernels \( \Delta M^{ab}_{\zeta}(X, Z) \) are given by functions symmetric with respect to the interchange of \( X, Z \) with \( X', Z' \). Using the table for transition from the \( \alpha_a \)-kernels to the \( \Delta M^{ab} \)-kernels in the region \( \zeta \leq X \leq Z \leq 1 \)

\[
\delta(u) \delta(v) \to \delta(Z - X) \quad ; \quad (u + v) \to \frac{Z - X}{Z Z'} \left[ \frac{X}{Z} + \frac{X'}{Z'} \right] ;
\]

\[
\left( \frac{\delta(u)}{v} + \frac{\delta(v)}{u} \right) \to \frac{Z - X}{Z Z'} \left[ \frac{X}{Z} + \frac{X'}{Z'} \right] ;
\]

and Eqs.\((21), (22)\), we obtain the kernels \( \Delta P^{ab}_{\zeta}(X, Z) \equiv \Delta W^{ab}_{\zeta}(X, Z)|_{\zeta \leq X \leq Z} \leq 1 \):

\[
\Delta P^{QQ}_{\zeta}(X, Z) = \frac{\alpha_s}{\pi} C_F \left\{ \frac{1}{Z - X} \left[ 1 + \frac{X X'}{Z Z'} \right] \theta(X < Z) + \delta(X - Z) \left[ \frac{3}{2} - \int_0^1 \frac{du}{u} - \int_0^1 \frac{dv}{v} \right] \right\} \to \frac{1}{Z} \Delta P^{QQ}(X/Z), \tag{39}
\]

\[
\Delta P^{Qg}_{\zeta}(X, Z) = \frac{\alpha_s}{\pi} N_f \frac{1}{Z Z'} \left\{ \frac{X}{Z} + \frac{X'}{Z'} - 1 \right\} \to \frac{1}{Z^2} \Delta P^{Qg}(X/Z), \tag{40}
\]

\[
\Delta P^{gQ}_{\zeta}(X, Z) = \frac{\alpha_s}{\pi} C_F \left( \frac{X}{Z} + \frac{X'}{Z'} - \frac{X X'}{Z Z'} \right) \to \frac{1}{Z} \Delta P^{gQ}(X/Z), \tag{41}
\]

\[
\Delta P^{gg}_{\zeta}(X, Z) = \frac{\alpha_s}{\pi} N_c \left\{ \left( \frac{2}{Z} + \frac{X'}{Z'} \right) \frac{Z - X}{Z Z'} + \frac{1}{Z - X} \left[ \frac{X}{Z} \right]^2 + \frac{X'}{Z} \right\} \theta(X < Z) + \delta(X - Z) \left( \frac{\beta_0}{2N_c} - \int_0^1 \frac{du}{u} - \int_0^1 \frac{dv}{v} \right) \right\} \to \frac{1}{Z^2} \Delta P^{gg}(X/Z). \tag{42}
\]

The formally divergent integrals over \( u \) and \( v \) provide here the usual “plus”-type regularization of the \( 1/(Z - X) \) singularities. The prescription following from Eqs.\((37), (38)\) is that combining the \( 1/(Z - X) \) and \( \delta(Z - X) \) terms into \([G_{\zeta}(Z) - G_{\zeta}(X)]/(Z - X)\) in the convolution of \( \Delta P_{\zeta}(X, Z) \) with \( G_{\zeta}(Z) \) one should change \( u \to 1 - X/Z \) and \( v \to 1 - X'/Z' \).

As expected, the \( \Delta P^{ab}_{\zeta}(X, Z) \) kernels have a symmetric form. The arrows indicate how the nonforward kernels \( \Delta P^{ab}_{\zeta}(X, Z) \) are related to the DGLAP kernels in the \( \zeta = 0 \) limit when \( Z = Z' \) and \( X = X' \). Deriving these relations, one should take into account that the gluonic asymmetric distribution function \( G_{\zeta}(X) \) reduces in the \( \zeta \to 0 \) limit to \( X \Delta g(X) \) rather than to \( \Delta g(X) \).

\[\text{We are grateful to J. Blumlein who informed us that the authors of ref. (11) agree with our results.}\]
After the appropriate redefinitions, we managed to reproduce from our results all four kernels $\Delta P_{ab}(x, \xi)$ (relevant to the $x > \xi/2$ region) calculated by X. Ji [3].

Note, that in the region $Z > \zeta$ the evolved fraction $X$ is always smaller than $Z$. Furthermore, if $Z \leq \zeta$ then also $X \leq \zeta$, i.e., distributions in the $X > \zeta$ regions are not affected by the distributions in the $X < \zeta$ regions. Hence, information about the initial distribution in the $Z > \zeta$ region is sufficient for calculating its evolution in this region. This situation may be contrasted with the evolution of distributions in the $Z < \zeta$ regions: in that case one should know the nonforward parton distributions in the whole domain $0 < Z < 1$.

VII. CONCLUSIONS

In this letter, we discussed the calculation of the evolution kernels $\Delta W_{\zeta}(X, Z)$ for nonforward parton distributions $G_{\zeta}(X, t)$ sensitive to parton helicities. We presented the evolution kernels for the relevant light-ray operators and demonstrated how one can obtain from them the components of the nonforward kernels $\Delta W_{\zeta}(X, Z)$. Our results have a transparent relation with DGLAP and BL-type kernels and a compact form convenient for further practical applications such as numerical studies of the evolution of nonforward distributions.

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