Estimation of Seismic Moment Tensors Using Variational Inference Machine Learning

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Abstract  We present an approach for rapidly estimating full moment tensors of earthquakes and their parameter uncertainties based on short time windows of recorded seismic waveform data by considering deep learning of Bayesian Neural Networks (BNNs). The individual neural networks are trained on synthetic seismic waveform data and corresponding known earthquake moment-tensor parameters. A monitoring volume has been predefined to form a three-dimensional grid of locations and to train a BNN for each grid point. Variational inference on several of these networks allows us to consider several sources of error and how they affect the estimated full moment-tensor parameters and their uncertainties. In particular, we demonstrate how estimated parameter distributions are affected by uncertainties in the earthquake centroid location in space and time as well as in the assumed Earth structure model. We apply our approach as a proof of concept on seismic waveform recordings of aftershocks of the Ridgecrest 2019 earthquake with moment magnitudes ranging from Mw 2.7 to Mw 5.5. Overall, good agreement has been achieved between inferred parameter ensembles and independently estimated parameters using classical methods. Our developed approach is fast and robust, and therefore, suitable for down-stream analyses that need rapid estimates of the source mechanism for a large number of earthquakes.

Plain Language Summary  Source characteristics of an earthquake can be estimated from seismic waves that have been recorded at seismometers. This process typically involves many assumptions for example: Where was the earthquake located? When did it happen? What are properties of rocks in the underground? These assumptions and a limited number of seismometers inherently affect the outcome of the estimation procedure such that not only a single characteristic earthquake source can explain the observed ground motion at seismometers, but rather a full range of potential earthquake source characteristics. These earthquake source characteristics can be used for example to estimate the amount of expected shaking at the Earth’s surface. Currently, classic methods either do not provide the full range of potential earthquake sources or they are very slow of doing so. Here, we present a machine learning approach that not only rapidly estimates the earthquake source characteristics, but also considers variability in all previously mentioned assumptions. We apply our approach as a proof of concept on seismic recordings of small to medium-sized earthquakes that occurred after the large Mw 7.1 Ridgecrest 2019 earthquake. We compare the estimates of our approach with independent, classically determined earthquake source characteristics and find a very good agreement.

1. Introduction

Robust and fast estimation of the seismic moment tensor (MT), is important for assessing the characteristics of an earthquake. Routine operational monitoring frameworks such as the United States Geological Survey (USGS) and GEOFON provide automatic centroid moment tensor (CMT) point-source solutions within minutes for moderate and large earthquakes (>Mw 4.5), usually in teleseismic distances (Ekström et al., 2012; Hanka & Kind, 1994). However, especially the MTs for smaller regional or local earthquakes are often only analyzed after manual inspection with delay times of up to days. The estimation of the full MT of smaller earthquakes (>Mw 3) can be important for detailed analysis of fore- and aftershock sequences, inference of local stress redistribution and especially, for seismicity monitoring in geotechnical applications (Cesca et al., 2014), where significant non-double-couple (DC) components due to volumetric changes can be expected.
CMTs are usually estimated as solutions to an inverse problem by iterative comparison of synthetic and observed waveform data until a sufficient match is achieved. Forward modeling of synthetics is typically performed by assuming a point source and by considering a range of potential source model parameters and their combinations; whereas the uncertainties of the estimated parameters are quantified by considering data errors and theory errors which are introduced by the measurement and the assumptions in the inverse problem, respectively (Tarantola & Valette, 1982). Uncertainties can be obtained through probabilistic approaches (e.g., Duputel et al., 2012; Kühn et al., 2020; Stähler & Sigloch, 2014, 2016; Vackář et al., 2017; Vasyura-Bathke et al., 2020), but these methods are computationally expensive and the estimation of CMT parameter densities can take tens of minutes to hours. Not only could faster estimates of the earthquake MT increase the catalog density of available MT solutions, but these estimates could also provide currently unavailable uncertainty quantification. These would be important for further down-stream analysis of the earthquake toward tsunamigenic properties (Barrientos, 2018) and the rapid characterization of the tectonic context of an earthquake (Thompson et al., 2020).

Furthermore, the earthquake source mechanism can be used to determine stress (Michael, 1987) and to help to infer possible aftershock distribution patterns (Kilb et al., 2000; King et al., 1994). Machine learning algorithms have been shown to be helpful and fast for seismic signal detection and localization (Kriegrowski et al., 2019; Smith et al., 2021), phase picking (Ross et al., 2018; Mousavi et al., 2020) as well as initial characterization of the seismic source (e.g., Käufl et al., 2014; van den Ende & Ampuero, 2020). P wave first-motion polarity can be used to determine the MT of earthquakes assuming a DC source, which has been shown to be fast and reliably to enhance MT catalogs using deep learning (Hara et al., 2019; Ross et al., 2018; Uchide, 2020). Recently, deep learning has been used to train the so-called Focal Mechanism Network (FMNet) to determine pure DC MTs based on full waveform synthetics Kuang et al. (2021). The FMNet has 16 trainable layers and was applied to four 2019 Ridgecrest earthquakes with a magnitude larger than Mw 5.4. The network was trained on subjectively predefined Gaussian distributions as labels, describing the assumed distribution of the DC parameters strike, dip, and rake.

Here, we present a flexible machine learning framework using several Bayesian Neural Networks (BNNs) and variational inference. Comprehensible consideration of errors are especially important for estimates obtained from unsupervised machine learning algorithms, as these are often treated and used as black boxes. Our BNNs are trained on synthetic waveforms with the aim to estimate rapidly MT parameters considering errors in measurement and theory. We validate our approach on a subset of earthquakes from the aftershocks of the Californian Ridgecrest 2019–2020 sequence (Ross et al., 2019), as the Ridgecrest area is exceptionally well monitored with a dense station distribution, both in azimuth and distance (Figure 1a). The main shock of the 2019 Ridgecrest sequence was the Mw 7.1 July 6, 2019 03:19:52 earthquake, preceded by several foreshocks of which the largest was the Mw 6.4 July 4, 2019 17:33:49 earthquake. The following months several hundred aftershocks >Mw 3 were recorded (Ross et al., 2019). For the subset of earthquakes from the 2019 Ridgecrest sequence, we investigate earthquakes with moment magnitudes $M_W$ between 2.7 and 5.5. We choose that particular magnitude range in the frame of our proof of concept, because independently determined cataloged MT solutions of that magnitude range are available for this area provided by the Southern California Earthquake Data Center (SCEDC). Thus, we can compare our estimations with those independent MT solutions.

2. Variational Inference Neural Network Estimation of MTs

Our main goal is to infer the radiation pattern and the orientation of the earthquake source. We train location-specific neural networks for each point of a predefined grid of potential hypo-centers based on full sets of synthetic waveforms with associated source model parameters to be learned. We use a set of 41 broadband stations within a range up to 150 km around the center of our grid (Figure 1a). The grid (Figure 1b) extends horizontally 10.5 by 10.5 km, with a step size of 1.5 km. The vertical extent ranges from 2 to 10 km depth, in 2 km steps.

As prior information, our proposed framework needs a detection of an earthquake and the associated approximate centroid time. Furthermore, an approximate earthquake location can be considered. Nevertheless, it has already been demonstrated that detection and location of earthquakes are timely deliverable by
other established machine learning-based algorithms (Kriegerowski et al., 2019; Mousavi et al., 2020). Our approach does not estimate earthquake moment magnitudes and, because of our choices of settings it is indirectly limited to a range of magnitudes (e.g., between Mw 3 and 5) as the network training depends on signal processing parameters. The magnitude of earthquakes can be readily estimated in real-time by other approaches (van den Ende &amp; Ampuero, 2020).

2.1. Input

As input, we use synthetic displacement waveform data calculated for a specific earthquake source and for all considered stations in E, N, and Z components. Training on synthetic data has several advantages compared to training on recorded data sets. The procedure is applicable to regions with low seismicity, and furthermore, the use of synthetic waveforms allows exploring the full range of possible CMTs. Consequently, the training is not restricted by a biased set of catalog mechanisms from available observations, but it can be assured that the complete parameter space has been explored. For fast simulation of synthetic waveforms we use pre-calculated Green’s functions (GF) stores from the Pyrocko software framework (Heimann et al., 2017, 2019). These GF stores are based on 1-D layered Earth structure models computed by using the reflectivity-type wavenumber integration method implemented in QSEIS (Wang, 1999). We calculate three different GF stores based on 1-D velocity profiles (Figure S1 in Supporting Information S1) (a) for the entire Mojave Region used by the USGS and the SCEDC, (b) for the Coso Geothermal area (Wu & Lees, 1999), and (c) for a regional shallow velocity profile based on Crust2.0 (Bassin et al., 2000).

We train our neural networks on pure synthetic waveforms without adding noise, because the characteristics of the noise would be learned as well by the networks. We filter the waveforms with a causal butterworth band-pass filter of fourth order between 0.8 and 2.4 Hz to avoid poor long-period response and weak
Table 1
Lane Parameter Definitions and Chosen Discretization for Constructing the Training Data Set

| Parameter | Interpretation          | Min. value | Max. value | Step size |
|-----------|-------------------------|------------|------------|-----------|
| \( \kappa \) | Strike angle            | 0          | 2\( \pi \) | 0.1\( \pi \) |
| \( \sigma \) | Rake angle              | -\( \pi \) | \( \pi \) | 0.2       |
| \( h \)   | Dip angle               | 0          | 1          | 0.2       |
| \( w \)   | Lune latitude           | -\( \frac{3}{8} \pi \) | \( \frac{3}{8} \pi \) | 0.2       |
| \( v \)   | Lune co-longitude       | -\( \frac{1}{3} \) | \( \frac{1}{3} \) | 0.02      |

long-period signals below the corner frequency of Mw 3.5 earthquakes (Aki & Richards, 2002). We assume a triangular source time function of fixed duration of 0.5 s, representative of earthquakes in the magnitude range 3–3.5 (Aki & Richards, 2002). Therefore, our trained networks are restricted to specific frequencies. This implies that our trained networks are only valid for a predefined magnitude range and that for studying earthquakes of different magnitudes, additional specific networks would need to be trained.

This choice is taken to match the available SCEDC catalog but can be changed by users as a setting. For each source grid point location and the given 1-D Earth structure model we use the expected theoretical travel times to extract a snippet of waveform data 1 s before and 4 s after the theoretical first phase arrival at all stations. This also means that our extracted waveform snippets are relative in time and that they can be used for all possible centroid times in the training phase. To cut out real data, however, this means that the centroid time needs to be known. The length of the time window is chosen for the local station setting of this proof of concept, and it could be configured for time windows of different duration; e.g., long duration for stations in teleseismic distances. Waveforms need to have arrived at all stations for our approach to work. As some stations are located in up to 150 km distance for this Ridgecrest 2019 aftershock sequence, our approach is unsuitable for real-time warning applications, in its current implementation. We only consider body-waves and disregard surface waves, for which at local and regional distances the impact of miss-modeling due to incomplete Earth structure models can be significant.

We convert the extracted waveform snippets around the P wave onset to form a 2D input image such that the rows represent the waveforms that are grouped first by channels (E, N, and Z) and second by stations; the columns represent the samples over time. Finally, we normalize and re-scale the image by the absolute maximum amplitude of the full image such that all values fall between the closed interval of 0 and 1, where 0.5 indicates zero in the original waveform amplitudes as well as missing data. Due to this normalization, all synthetics can be calculated for one single (but arbitrary) magnitude. The order of stations needs to be consistent for each image and must not change. Here, we chose an alphabetical order according to the station codes as arranging by azimuth or distance would be different for each considered source location and would cause artificial patterns which in turn would make efficient training of the networks difficult.

### 2.2. Labels
For each set of synthetic waveforms forming an input image, we know the parameters of the underlying source. These are the output labels that our networks predict. The common MT parameterization with six independent components (Aki & Richards, 2002; Madariaga, 2007) seems a natural choice for describing the seismic source. However, a uniform sampling in this parameter space does not yield a uniform unique distribution of samples in moment-tensor space (Tape & Tape, 2015). Such a nonuniform and nonunique mapping would lead to bias in learned patterns for our networks. This problem can be solved by using spherical coordinates on the unit sphere of the fundamental lune description of the MT (Tape & Tape, 2012b).

Moreover, this parameterization allows for a uniform sampling of moment-tensors, with the advantage of only five independent parameters to describe the full spectrum of MTs. These five independent parameters (Table 1) are: \( \kappa \) as the strike-angle equivalent, \( \sigma \) as the rake-angle equivalent of the MT slip angle, \( h \) as the dip-angle equivalent and the nonisotropic components \( v \) and \( w \) as the lune latitude and co-longitude, respectively (Tape & Tape, 2015). This parameterization of the MT clearly separates the radiation pattern from the source orientation. We choose a discretization of 0.1\( \pi \) for \( \kappa \), 0.2 for \( \sigma, h, w \) and 0.02 for \( v \). This results to 171,600 synthetic waveform datasets that we use for training for each single location grid point.

### 2.3. Network Design
Instead of using deterministic network layers where scalar weights and biases are learned, we use their probabilistic expression with distributions of weights and biases. Each distribution is assumed to be
Gaussian with mean $\mu$ and a standard deviation $\hat{\sigma}$ (e.g., Blundell et al., 2015; Graves, 2011; Wen et al., 2018). A neural network designed with such probabilistic layers (i.e., flipout layers) forms a Bayesian Neural Network (BNN) and can be considered as representing an ensemble of deterministic neural networks trained several times on the same input data. These BNNs allow representing epistemic uncertainty in their inherent predictions due to limited training data and they yield a likelihood value to each drawn sample. Consequently, rather than predicting the same set of output labels given the same input data, repeated forward pass yields a distribution of output labels, i.e., uncertainties in lune parameters. This can vary for each BNN learned for the grid points, as the significance of specific seismic stations toward the source will vary.

Each single training iteration of a BNN consists of a forward pass and a back-propagation pass (Wen et al., 2018). In the forward pass, a single sample is drawn from the output labels. During a backwards pass the gradients of the layer weights and bias distributions (i.e., means $\mu$ and standard-deviations $\hat{\sigma}$) are calculated with automatic differentiation and $\mu$ and $\hat{\sigma}$ are then updated to maximize an objective function depending on the input and output labels (Wen et al., 2018).

Our goal is to use a simple neuronal network architecture to avoid over-fitting and to allow for straightforward interpretation of the individual training steps. The network design (Figure 2a) is similar in rationale to Kriegerowski et al. (2019). We use three 2-D convolutional flipout hidden layers. The first two hidden layers are sensitive to the information over time only (Figure 2a). The first hidden layer has 8 filters and a 1 by 2 kernel and the second layer has 10 filters and a 1 by 30 kernel. The last 2-D hidden layer collects information over the station components with 12 filters and a 3 by 1 kernel. We use a dropout of 0.2 between convolutional flipout layers to robustly handle data errors and missing waveform data.

We downsample the output data of the convolutional flipout layers with a 2-D max pooling layer with a 3 by 4 kernel (example activations see Figure 2c) followed by flattening the data into a vector and feeding them into a fully connected dense flipout layer. The relatively simple network design allows for visual inspection of the activations in each layer (Figure S2 in Supporting Information S1). All convolutional flipout layers are activated using a Rectified Linear Unit function (RELU; Glorot et al., 2011). Finally, a nonactivated lambda distribution layer is used to hold the resulting distributions of predicted source parameters. As objective function (loss function), we use the negative log-likelihood and as optimizer the Adam algorithm (Kingma & Ba, 2014).

2.4. Variational Inference From Multiple BNN

The probabilistic output of the BNNs allows combining inferences at several likely locations and centroid times of the earthquake’s source. Each evaluation of a network with inputs yields a single prediction of the 5 MT parameters and the associated negative log-likelihood. The inferences from all these individual evaluations of networks can be combined and the source’s errors in both centroid location and time can be propagated to uncertainties in MT parameter marginals through variational inference, yielding an ensemble of possible source mechanisms.

We consider an error in location within an ellipse around the assumed centroid location and evaluate the respective BNNs with the given input (Figure 3). Note that waveform snippets are extracted differently from the waveform input according to theoretical arrival times at each receiver location (Section 2.1).
Errors in centroid time result in shifts of the predicted theoretical arrival times and the extracted waveform snippets. We assume uniform distributed errors in timing and draw random samples within the timing errors and therefore, all BNNs at the considered grid points are evaluated several times. For the test data set of the Ridgecrest 2019 area, we consider the centroid time error as provided by the SCEDC. However, our framework also allows handling time errors at individual stations, which would be a typical feature of phase arrival picks. Consequently, we get different likelihoods to the differently extracted waveform snippets. In classical approaches in seismology this corresponds roughly to shifting the waveforms to find the maximum correlation (e.g., Kühn et al., 2020).

Finally, errors in the Earth structure model can be taken into account for robust inference on the estimated source mechanism (Vasyura-Bathke et al., 2021). The theory error from the choice of the 1-D Earth structure model can be included in our framework by training and evaluating BNNs on each grid point for each Earth structure. This requires calculation of the full set of synthetic waveforms for different Earth structures; in our case, three structures (Figure S1 in Supporting Information S1). This results in total to over 100 Million waveform datasets on which the 588 BNNs (196 grid points times three Earth structures) are trained. The calculation of synthetic waveforms and the network training was done in parallel on several machines with a total of 128 CPUs over a period of 3 months. By using GPUs this time could be drastically reduced to a few days.

2.5. Moment Tensor Ensemble Similarity

To assess the similarity between the predicted ensemble of MTs and a reference solution, e.g. from a catalog, we use the omega angle measure (Tape & Tape, 2012a). The omega angle has the advantage that focal mechanisms with opposite polarities are considered most dissimilar in contrast to other measures, e.g., the Kagan angle (Cesca et al., 2014; Tape & Tape, 2012a). The normalized omega angle distance \( d_\omega \) (Cesca et al., 2014; Tape & Tape, 2012a) between two MTs \( U_1 \) and \( U_2 \) with components I and J is calculated by:

\[
d_\omega = \frac{1}{2} \left[ 1 - \frac{U_1 \cdot U_2}{\|U_1\| \|U_2\|} \right] = \frac{1}{2} \left[ 1 - \frac{\sum_{i,j=1}^J U_{1i} \cdot U_{2j}}{\left(\sum_{i,j=1}^J U_{1i}^2\right)^{\frac{1}{2}} \left(\sum_{i,j=1}^J U_{2j}^2\right)^{\frac{1}{2}}} \right]
\]

\( d_\omega \) is defined between 0 and 1, for identical and opposite seismic radiation patterns between the two compared MTs, respectively. Note, that in order to calculate \( d_\omega \), we need to convert our predicted MT ensemble from the Lune parameterization to the North-East-Down coordinate system (Aki & Richards, 2002).

3. Application to the Ridgecrest 2019 Earthquake Aftershock Sequence

We train our networks for an area South of the Coso geothermal field (Figure 1), which is known to host both induced and tectonic earthquakes (Monastero et al., 2005; Schoenball et al., 2015). Significant non-DC components can be expected for earthquakes in this region (Ichinose et al., 2003), potentially also for tectonic earthquakes, due to the influence of the geothermal reservoir. To test the performance of our framework we use recorded waveform data of the aftershocks that occurred between July 2019 and December 2020 to the Mw 7.1 Ridgecrest earthquake.

For these aftershocks, eight full MT solutions and 198 pure DC MT solutions (Figure 1b) are calculated (Hauksson & Unruh, 2007; Jordan & Maechling, 2003) and made publicly available by the SCEDC.
We compare the MT estimates of our approach to the MTs as determined independently by USGS and SCEDC (Hutton et al., 2010).

We download the waveform data for all events and for the 41 stations from the Southern California Seismic Network (California Institute of Technology and United States Geological Survey Pasadena, 1926). Missing waveform data for any station and time period are replaced by zero values in the waveform data, which are then mapped to 0.5 values in the normalized input images. This, as well as the dropout in the neural network architecture, allows for robust handling of station outages or discontinuation of station installations, which may typically occur in observational practice in a seismic network. Measured waveform data are treated in the same way as our synthetic waveforms (Section 2.1), i.e., data is restituted to ground displacement and down-sampled to match the GF sampling rate of 14 Hz.

For each aftershock, we evaluate the BNNs for a total of 6,000 samples. However, the number of activated BNNs depends on the uncertainties in centroid location and time as provided by the SCEDC catalog. The location uncertainty in horizontal and vertical position as given by the SCEDC is increased 10 times, as reported uncertainties are in the order of few hundreds of meters. The total ensemble of samples is then obtained by evaluating the activated BNNs equally.

### 3.1. Inferences for Full Moment Tensors

We focus primarily on eight aftershocks for which a full MT solution is available in the SCEDC catalog. We refer to these solutions as “reference” in the following. We first evaluate only the waveform input with the BNN's trained using synthetics based on the Mojave Earth structure, which is the same as used by the SCEDC to determine their focal mechanisms (Figure S1 in Supporting Information S1). Consequently, the reference and predicted MTs should be consistent in their epistemic uncertainty as the same Earth structure model and (mostly the same) data set is used. We use the SCEDC catalog values for source position and centroid time. For the comparison, we only consider uncertainty in centroid location. We find very good agreement of our predicted ensembles to most of the eight reference MTs, with most of the omega angle distances $\omega$ being below 0.1 (Figures 4a–4h). Histograms of $\omega$ show their maximum mostly within the first few bins. Only two ensembles of predicted MTs show small systematic errors (Figures 4f and 4g). For those also the histograms of $\omega$ show their maxima at distances above zero.

In addition to uncertainty in centroid location we consider in the following uncertainties in the centroid time, which are also provided by the SCEDC catalog for each event. These uncertainties differ from earthquake to earthquake but they do not exceed 0.4 s for the considered aftershocks. When uncertainties in centroid times are considered the widths of some of the $d_\omega$ histograms increase for some ensembles of MT predictions (Figures 4b and 4f) confirming the quality of the absolute centroid times of these aftershocks determined by SCEDC. However, it is worth mentioning that the widths of some of the $d_\omega$ histograms also decrease for some ensembles of MT predictions (Figures 4c, 4g, and 4h) suggesting biased absolute centroid times for those aftershocks in the catalog. Finally, in addition to uncertainties in centroid location and time we consider uncertainties in Earth structure. We evaluate the BNNs that have been trained on the synthetics from three different Earth structures (Figure S1 in Supporting Information S1). The expected arrival times and thus extracted waveform snippets will be systematically different for each Earth structure. For some of the inferred MT ensembles the spread in $d_\omega$ histograms increase and some show values of up to 0.5 (Figures 4f and 4g). For those events, the choice of the Earth structure model has a strong impact on the spread of predicted mechanisms. For other MT ensembles (Figures 4a, 4d, 4e, and 4h), the spread in $d_\omega$ histograms decreases or stays similar, meaning that the uncertainties in Earth structure are less crucial for those events. Nevertheless, the maximum a posterior (MAP) solution still shows good agreement between extracted waveform data snippets and synthetic waveforms calculated from the predicted source parameters (Figures 5, 6, and Figures S8–S13 in Supporting Information S1) and the resulting ensembles of predicted MTs also comprise the solutions of considering only location uncertainty (Figure 7 and Figures S5–S7 in Supporting Information S1). The range of uncertainty of predicted waveforms can be quite different for the same station for two different earthquakes. While the range of uncertainty is low for example at station CI.TIN for one event (July 6, 2019 Figure 5) it is high especially around the onset for another event (July 26, 2019 Figure 6). Also, the fit of horizontal and vertical waveform components often differs qualitatively.
Figure 4.
3.2. Inferences for Double-Couple Moment-Tensors

The SCEDC catalog also contains 198 pure DC focal mechanisms for events that occurred in the area of interest, which we refer to as reference in the following. Without visual inspection, we let for the waveform data of each of those events our BNNs infer ensembles of 6,000 MT solutions considering centroid location and time uncertainty. We compare the 198 reference focal mechanisms with our ensembles of MT parameter predictions from our framework by setting the predicted \( v \) and \( w \) values to zero, representing a pure DC source (Figure 8a). We also show \( d_{\omega} \) between the reference mechanism and the predicted full seismic MT ensembles (Figure 8b). The additional degree of freedom of full MT solutions versus DC constrained solutions results in broadening and a slight shift of the histogram toward higher \( d_{\omega} \) (Figure 8b).

With decreasing earthquake magnitude the spread of \( d_{\omega} \) of the trained networks is increasing comparing the predicted ensembles of MT for the 198 earthquakes (Figure 8c). This spread is expected as the signal-to-noise ratio decreases with lower magnitude and larger \( d_{\omega} \) values are expressions of an increase in uncertainty of the MT ensembles. However, the bulk part of \( d_{\omega} \) shows distances below 0.1 and the predicted ensembles are in good agreement with the reference solutions across different magnitudes 2.7–4.5 (Figures 8d–8g). We also notice a slight increase in the omega angle distances between reference and predicted source mechanisms for the largest of the 198 earthquakes. This might indicate a need for incorporating non-DC components in the source mechanism; whereas these components are missing in the catalog descriptions.

4. Discussion

In general, we find a good agreement between the ensemble of predicted MTs and the independently determined and unseen MT solutions from the SCEDC. Only a few predicted MT ensembles show systematic differences (Figures 4f and 4g), which could be due to several reasons, e.g., differences in the station configurations. Another reason could be unmodeled source complexity that can be significant for moderate-sized earthquakes (e.g., Boatwright, 2007; Pacor et al., 2016). In this case, our assumed point source MT with a source time function of 0.5 s duration might not fully explain the source mechanism and may produce a biased source mechanism prediction (Steinberg et al., 2020). Other systematic differences between MT predictions and reference partly vanished by including also uncertainty in centroid time into the variational inference scheme (Figure 4g). As we estimate the full seismic MT the distribution and density of the non-DC components from the predicted ensemble can be inferred (Figures S3 and S4 in Supporting Information S1). The main regions of high probability of solutions are consistent considering different sources of theory error. However, larger uncertainties for both the CLVD as well as the isotropic components, i.e. the lune \( v \) and \( w \) parameters, can be observed when additionally considering errors in Earth structure models (Figure S3 in Supporting Information S1). It has been shown that an error in Earth's structure is often compensated by increased CLVD and isotropic MT components (Hejrani & Tkalčić, 2020; Valentine & Woodhouse, 2010; Vasyura-Bathke et al., 2021). This is apparently also the case here, comparing lune diagrams of the MT ensembles without errors in Earth structure (Figure S4 in Supporting Information S1) to those ensembles comprising errors in Earth structure (Figure S3 in Supporting Information S1). Another source of theory error is source-receiver-specific propagation path effects caused by lateral small-scale velocity heterogeneity distorting the radiation pattern of \( P \) and \( S \) waves (Kobayashi et al., 2015; Takemura et al., 2009). Those scattering effects can have significant impact on the determined MT (Burgos et al., 2016). However, in our simulated waveforms, the scattering effects are not explicitly modeled as we only assume a layered 1-D
Figure 5. Exemplary waveform fits between observed waveforms (black) and synthetic waveforms based on the ensemble of estimated MT parameters (brown) with the MAP in red, for the Mw 3.8 earthquake on July 6, 2019 12:00:05. Note that the waveforms are in displacement and waveform windows have a duration of 5 s.
Figure 6. Same as in Figure 5, but for the Mw 4.74 earthquake on July 26, 2019 00:42:48.
Earth structure. Nevertheless, they are accounted for to some extent by our approach of varying the Earth structure model.

We note that we evaluate the prediction accuracy of our framework by comparison with SCEDC cataloged MTs. These solutions, however, could potentially also be biased, deviating from the unknown “true” earthquake source. Variance reduction could be used to estimate the precision with respect to the real waveform data. We observe larger $d_{\omega}$ angle distances between the predicted MT ensemble and reference MTs when considering the inferences from several Earth structure models (Figures 4a–4h). This is not unexpected, because the reference solutions are estimated with only one of the Earth structure models. However, it is also possible that the “true” unknown solution is better represented by our ensemble of predictions considering other Earth structure models. In regions with well-known structure this approach likely overestimates the parameter uncertainties, but in regions with poorly known structure it might provide a more realistic representation of parameter uncertainties (Vasyura-Bathke et al., 2021).

The observation of a relation between spread of inferred parameter uncertainties with magnitude is a result of parameter selections before training, such as filter and time window length, as well as decreasing signal-to-noise ratios for lower magnitudes. Our considered filter frequencies are optimal for earthquakes with magnitudes Mw 3 to 4, of which hundreds occurred during the 2019 Ridgecrest sequence (Ross et al., 2019). The station distribution around the Ridgecrest area and the good quality of the waveform data due to mostly remote station locations are exceptional and together with the statistically significant number of earthquakes this study area is bench-marking showcase to demonstrate the robustness and performance of our approach. It remains to be evaluated whether our approach performs equally well in areas with a sparse station network under worse noise conditions.

The novelty of our proposed framework lies in the estimation of ensembles of the full seismic MTs yielding uncertainties in parameter estimates based on seismic waveforms. A shortcoming in our approach is the currently limited transferability of the trained BNNs to other study areas, unlike P wave first motion polarity-based approaches (e.g., Ross et al., 2018). We point out that the proposed framework does have a delay time in the order of tens of seconds between earthquake and prediction output. Main factors that influence this response time are: (a) Our algorithm considers a waveform window of 5 s. (b) The safe restitution of the waveform data into displacement to avoid filter effects requires that at least several seconds of data are available (around 2 s for the chosen frequencies in the case study). (c) In its current form, our approach requires the detection and location of an earthquake, which can be used to infer a centroid time and optionally, its uncertainty as prior knowledge. However, these can be delivered fast by other deep learning methods (Kriegerowski et al., 2019). (d) Finally, the evaluation of the waveform data by a single trained BNN takes a few hundred milliseconds and can be done in parallel for several BNNs at the same time. Hence, approaches based on P wave first motion polarity only (Hara et al., 2019; Ross et al., 2018; Uchide, 2020) will likely outperform our proposed framework in terms of response time. Thus, the approach in its current implementation is not suitable for real-time applications. Nevertheless, these time factors are not of importance for already cataloged data in a database, which can be searched fast by keeping the recorded waveform data in memory.

In principle, the presented method can be made independent of the particular station configuration at the expense of computational cost. This could be accomplished by calculating the synthetic waveforms for a distance-depth grid of locations and shifting the source and receiver relatively or by assuming a location
The actual station locations can then be mapped to such an abstract receiver grid by interpolation or nearest neighbor. However, we do not expect that the framework could be made transferable to other regions, because of the characteristics of the assumed Earth structure models that are learned by the BNNs.

The choice of training a BNN for each considered grid point instead of training a single large neural network with waveforms from all possible locations, such as in Kuang et al. (2021), is a key point in our approach which allows us for estimating MT parameter uncertainties considering uncertainty in centroid time and location as well as uncertainty in Earth structure. Training a single large neural network with waveforms from all potential source locations would require to estimate additionally three location parameters (latitude, longitude, and depth) as labels. This significantly increases the nonlinearity of the problem and consequently increases the required complexity of the neural network architecture, i.e., the number of trained filter weights and biases. In our view, a simple network architecture with few trainable parameters is favorable (Mignan & Broccardo, 2019) and, therefore, we chose to train multiple, but individually rather simple networks.

As a by-product of our approach it turns out that our BNNs also learned to be sensitive to the centroid location. Assuming that an earthquake occurred somewhere in the grid of BNNs, each BNN can be queried to return the log-likelihoods for the input data. The highest log-likelihoods should stem from BNNs learned for grid locations close to the true centroid location. We test this assumption for a Mw 3.9 earthquake included in the SCEDC catalog and indeed find a correlation of the log-likelihood values with distance to the centroid location (Figure 9). As prior information only the centroid time and optionally its uncertainty is needed.
5. Conclusions

We demonstrated that variational inference based on deep learning of BNNs shows the capability to not only reproduce optimum parameter estimates of classical full MT inversion, but it also yields uncertainties of the inferred MT parameters rapidly. Our presented approach is flexible enough to optionally account for various cases of theory error that are well known to affect MT parameter estimates, i.e., errors in centroid location and time as well as errors in the assumed Earth structure.

The presented method has been successfully applied on local scale using field data of a subset of the Ridgecrest 2019 aftershock sequence, comprising 206 earthquakes with magnitudes Mw 2.7–5.5. The developed framework is adaptable to other magnitude ranges. The inferred ensembles of MT parameters have been compared to independently determined source mechanisms by the SCEDC.

One limitation of the presented approach is the nontransferable nature of the trained networks as they are trained for user-defined specific Earth structure models, station setups, frequency filters, and phase arrival time windows. However, these parameters could be configured for other setups e.g., regional or teleseismic distances that require longer waveform windows.

Our approach demonstrates the capabilities and the potential of machine learning for rapid earthquake source mechanism estimation with associated uncertainties. These are important information for hazard assessment and for deriving other products that are based on earthquake source analysis, e.g., shake map scenarios (Dahm et al., 2018), which in turn are of relevance for decision makers and the public. The presented framework has the potential to be expanded upon and to be used in standardized automatic operational procedures.

Figure 9. Earthquake centroid location inference. The grid points are colored by the negative log-likelihood values as inferred from evaluation of the BNNs for the real waveform data of the Mw 3.9 at July 6, 2019 17:59:15. The map view shows grid points at 4 km depth, whereas side views left and bottom show the grid points at depth versus latitude and longitude along the profiles outlined with gray rectangles in the map view, respectively. The black star marks the centroid location as given by SCEDC for this earthquake.
Data Availability Statement

Data from regional seismometers are available via FDSN services from GEOFON and IRIS. The Caltech/USGS Southern California Seismic Network (SCSN) earthquake catalog, along with metadata and other ancillary data, such as MTs and focal mechanisms has been used and are available at https://scedc.caltech.edu/index.html. The GF stores used here are uploaded on Zenodo under https://doi.org/10.5281/zenodo.4643478. The authors make the code available and only use open-access waveform data for testing. The authors make the software available as Jupyter notebook in the supplement and with pre-calculated example data as well on Zenodo under https://doi.org/10.5281/zenodo.4646666.

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