Influence of a time-dependent Chern-Simons term on the London penetration depth of the Type-I superconductor

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The effects of a time-dependent Chern-Simons (CS) term have been widely studied in the theoretical physics, particularly in the particle physics. The aim of this paper is to estimate a London penetration depth of the Type-I superconductor using Maxwell’s equations including the time-dependent CS term. The CS term act on the London penetration depth. The effect of the CS term to be insignificant. However, the effect in the case of a special material that has very small \( n_s \) may be detected.

INTRODUCTION

Maxwell-Chern-Simons theory is the extension of Maxwell’s electromagnetic theory that include the Chern-Simons term. The CS term expresses the topologically massive photon. There are the varieties of the massive photon theory, and the many authors study it. In this paper, the topologically massive model due to the CS term is deal. Chiral Magnetic Effect (CME) [1–3] is a well known topologically induced electromagnetic effect in the presence of the time-dependent CS term. The literatures [4, 5] reported the CS terms effect. In this connection, it is probable that the CS term has effects on the properties of the matter [6]. A number of more detailed studies [7–10] has addressed the role of the CS term in a superconductor. However, there have been few reports about the time dependence of the topological phase.

In variety possibilities, the London penetration depth is focused. The superconductor has perfect diamagnetism that is called the Meissner effect. Since this effect, the magnetic field does not penetrate toward deep inside the superconductor. The London moment explains the Meissner effect. The London moment acquired by a rotating superconductor, that was predicted by Becker, Heller and Sauter [11] in 1933, just before the Meissner effect was discovered. The paper [12] gives more details on these things. The London moment is explained by the London moment. The London moment is the magnetic field which is proportional to the angular verosity of the rotation \( \omega \)

\[
\text{B} = \frac{2m}{e\omega} \mu, \quad \text{(6)}
\]

exists in the rotating superconductor. From this equation, we get

\[
\text{B} = -\frac{m}{e^2 n_s} (\nabla \times \mathbf{j}), \quad \text{(7)}
\]
and (7) represents the perfect diamagnetism, where $\nabla \times (\omega \times r) = 2\omega$, $v = \omega \times r$, and the current density $j = (-en_s)^{-1}v$ are adopted.

It is important to note that the magnetic field becomes zero at deep inside the superconductor, to estimate the London penetration depth.

$$\lim_{x \to \infty} B = 0. \quad (8)$$

From (4), in MCS electrodynamics, the current generate the magnetic field not only at the right direction but also at the horizontal direction. Namely, it has to be made sure that the calculation under the conditions that little different from the typical. This paper shows a simple method that derives the modified London penetration depth.

Let us now take a rotation to the both sides of (5). The left hands side is

$$\nabla \times (\nabla \times B) = -\nabla^2 B + \nabla (\nabla \cdot B) = -\nabla^2 B. \quad (9)$$

On the right hands side, we write it as

$$\nabla \times \left( j + \frac{\partial e^2}{4\pi^2} B \right) = \nabla \times j + \alpha \nabla \times B \quad (10)$$

where we define $\alpha = (e^2 \theta)/(4\pi^2)$, and $\partial E = 0$, thus we find

$$-\nabla^2 B = \nabla \times j + \alpha \nabla \times B. \quad (11)$$

Note that the second term depends on the topological phase $\theta$.

Next, The following conditions are considered. First, the superconductor is placed in the region $x > 0$. Second, the magnetic field, and the current only depends on $x$ direction. Then, the magnetic field $B$, and the current $j$ is expressed as $B = B_y(x)e_y + B_z(x)e_z$, and $j = j_y(x)e_y + j_z(x)e_z$, respectively in Cartesian coordinates. From the equations (7), the components of $e_y$, and $e_z$ of the equation (11) are rewritten as

$$-\frac{\partial^2 B_y(x)}{\partial x^2} = -\beta B_y(x) - \alpha \frac{\partial}{\partial x} B_z(x) \quad (12)$$

$$-\frac{\partial^2 B_z(x)}{\partial x^2} = -\beta B_z(x) + \alpha \frac{\partial}{\partial x} B_y(x) \quad (13)$$

where $\beta = (n_s e^2)/(m)$ is defined. Here, we define that the ratio of $B_y$, $B_z$ is

$$\frac{B_z(x)}{B_y(x)} = \tan \eta. \quad (14)$$

Note that the equation (12), and (13) do not loss of generality. Substituting $B_z(x) = B_y(x) \tan \eta$ to (12), and (13), we get the equation:

$$\beta (1 + \tan \eta) B_y(x) = (1 + \tan \eta) \frac{\partial^2}{\partial x^2} B_y(x)$$

$$+ \alpha (1 - \tan \eta) \frac{\partial}{\partial x} B_y(x) \quad (15)$$

by the sum of both sides. Dividing both sides of (15) by $(1 + \tan \eta)$, we get

$$\beta B_y(x) = \frac{\partial^2}{\partial x^2} B_y(x) + \alpha \frac{1 - \tan \eta}{1 + \tan \eta} \frac{\partial}{\partial x} B_y(x)$$

$$= \frac{\partial^2}{\partial x^2} B_y(x) + 2\alpha \Gamma(\eta) \frac{\partial}{\partial x} B_y(x) \quad (16)$$

where the constant $2\Gamma(\eta) = (1 - \tan \eta)/(1 + \tan \eta)$ that composed by the ratio $\eta$ is defined. Assuming as $\lim_{x \to 0} B_y(x) = B_{y0}$, The simple magnetic solution of (16)

$$\frac{1}{\lambda_{CS}} = \sqrt{\beta + \alpha^2 \Gamma(\eta)^2} + \alpha \Gamma(\eta) \quad (17)$$

is derived, where the condition (5) is adopted.

In (17), grant that $\theta = 0$, and $\alpha = 0$. Then the modified London penetration depth becomes $\lambda_{CS} = \beta^{-1/2} = \sqrt{\mu_0/m}/(n_s e^2) = \lambda_L$. It means that be in agreement with usual London penetration depth $\lambda_L$. Therefore, this solution is regarded as the London penetration depth of the Type-I superconductor at the presence of the CS term.

**DISCUSSIONS**

From the results, the effect of the CS term is estimated. Take SI units, the London penetration depth and the magnetic field strength become

$$B(x) = B_{y0} e^{-\frac{x}{\lambda_{CS}}} \quad (18)$$

$$\frac{1}{\lambda_{CS}} = \sqrt{\mu_0 m \frac{n_s^2 e^2}{\hbar^2} + \Gamma(\eta)^2 \frac{\mu_0^2 e^4 \hbar^2}{(2\pi)^4} + \frac{\mu_0^2 \hbar^2}{(2\pi)^2}} \quad (19)$$

Here, assuming that the magnetic field ratio $\eta \to 0$, namely $2\Gamma \to 1$ that implies the external magnetic filed is only y component, that derives

$$\frac{1}{\lambda_{CS}} = \sqrt{\mu_0 m \frac{n_s^2 e^2}{\hbar^2} + \frac{\mu_0^2 e^4 \hbar^2}{4(2\pi)^4} + \frac{\mu_0 e^2 \hbar^2}{(2\pi)^2}} \quad (20)$$

Presence of the CS term would make a difference the London penetration depth. Granted that $\hat{\theta}$ is very small value, so expanding $\lambda_{CS}$ with $\hat{\theta}$, the form of the easy to understand correction:

$$\lambda_{CS} = \sqrt{\frac{m}{e^2 \mu_0 n_s}} + \frac{\hat{\theta} \mu_0}{4\pi^2 \hbar n_s} + \sqrt{\frac{\mu_0}{32\pi^3 \hbar^2}} \sqrt{\frac{1}{n_s^3}} \hat{\theta}^2 + O(\hat{\theta}^3) \quad (21)$$

is deviated. Assuming as $\hat{\theta}^2 < 10^{-2}$ and the superconducting electron density of Niobium $n_s(Nb) \sim 2.1 \times 10^{33} \text{ m}^{-3}$ (This value is obtained from the normal London penetration depth $\lambda_L = \sqrt{m}/(e^2 \mu_0 n_s)$, where the value of $\lambda_L$ of the literature [14] is used.), the first term value is $10^{-8} \text{ m}$, the second term value is $10^{-34} \text{ m}$, and
the third term value is $10^{-61}$ m. There is a difference of $10^{-27}$ m in the values of first two terms. That is to say, it seems that the effect of the CS term found to be too insignificant to measure.

As other possibilities, it is found that the second term in \[ (21) \] only depends on density of superconducting electrons $n_s$ except $\dot{\theta}$ as physical variable. If $n_s$ has a very small value, $\lambda_{CS}$ comes to be short. The multiplier of $n_s$ in the second term is larger than it in the first term. Therefore, there is some possibility of detecting the CS term effect on superconductor that has small $n_s$ (see FIG.\[1\]). In addition, a direct detection of the $\dot{\theta}$ may be able to be carried out using this effect. However, this condition is not realistic, so we would like to emphasize the difficulty lies in detecting this effect.

![FIG. 1. A plot presenting the modified London penetration depth $\lambda_{CS}$, and the relationship between $n_s$ and $\lambda_{CS}$. The thick line express the case of $\dot{\theta} = 0$, and the dashed line express the case of $\dot{\theta} \neq 0$.](image)

CONCLUSION

The London penetration depth about the Type-I superconductor was calculated from the MCS equation in the presence of the time-dependent CS term. From the result, the effect of the CS term found to be too insignificant to measure. However, the effect in the case of a special material that has a minuscule $n_s$ may be detected. The derivation method would be also effective in the other massive photon models.

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