Beyond the Zero-Binding Approximation in Quarkonium

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Abstract

The hadronic decays of quarkonium and the B meson inclusive decay into $J/\Psi + X$, if treated in the zero-binding approximation, suggest a value of the strong coupling constant much smaller than the value implied in the standard model by running from measurements at the Z. Thus, assuming the standard model is correct in the low energy region, there must be very substantial relativistic binding corrections to these processes. We discuss the wave function factor appearing in covariant treatments of quarkonium production and decay processes with special attention to the way in which the spin of the quarks carries over into the spin of the bound state. We find that Lorentz covariance requires that the bound state be a superposition of free quark and antiquark spinors notably different from the usual ones. We resolve a superficial apparent paradox suggesting that the relative momentum of the quarks should lie in a plane perpendicular to the spin quantization axis and we calculate the $J/\Psi$ binding corrections to lepton pair decay and to the inclusive B decay.

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1 Introduction

Although perturbative QCD was originally seen as the source of the Zweig rule leading to the narrowness of heavy quarkonium \[1\], measurements of the strong coupling constant at the Z boson would imply, in the non-relativistic or zero-binding approximation (ZBA), hadronic decay rates significantly greater than those observed \[2\]. Thus, if the standard model is correct in the quarkonium region, corrections to these decay rates beyond the ZBA must greatly suppress these rates. A similar problem exists in the treatment of the inclusive B meson decay into \(J/\Psi + X\). After correcting an error in \[3\] noted by \[4\], the color singlet model gives an \(\alpha_s(m_b)\) some 20\% lower than suggested by extrapolating in the standard model from measurements at the \(Z\) boson (i.e., \(0.154 \pm 0.005\) versus \(0.20 \pm 0.01\)). These low values of the strong coupling constant required in the ZBA together with low values \[5\] obtained in other low energy measurements have stimulated interest in the light gluino hypothesis, but direct searches for such light supersymmetric particles have, up to now, all turned out negative. This increases the importance of a thorough investigation of quarkonium wave function effects beyond the ZBA. Quarkonium has a useful analog in positronium \[6\] although the former case is complicated due to the non-Abelian nature of the binding force and due to non-perturbative confinement effects. There is continuing discussion of the much smaller binding effects in positronium \[7\] to which we can compare our results.

For definiteness we restrict our attention to the vector quarkonium states such as \(\Phi, J/\Psi, \Upsilon\), and in particular to the \(J/\Psi\). In a relativistic treatment the production amplitude of such a state with polarization \(\epsilon\) together with other particles from an arbitrary initial state takes the form

\[
\mathcal{A} = \epsilon^\mu M_\mu. \tag{1.1}
\]

In the rest frame of the \(J/\Psi\), the states of spin +1, -1, or zero along the z axis (quantization axis) are described by polarization vectors

\[
\epsilon^\mu(\pm)^R = -\frac{1}{\sqrt{2}} (0, \pm 1, i, 0) \\
\epsilon^\mu(0)^R = (0, 0, 0, 1). \tag{1.2}
\]

(Throughout this article we use the superscript \(R\) to indicate \(J/\Psi\) rest frame values; for phase conventions see \[8\], page 54.)

The production of a \(J/\Psi\) is, of course, equivalent to the production of a quark of momentum \(p\) and spin \(\lambda/2\) together with an antiquark of momentum \(\bar{p}\) and spin \(\bar{\lambda}/2\) in a bound state wave function \(\Psi\). The \(J/\Psi\) four-momentum, \(P\), and the relative momentum, \(K\), are

\[
P = p + \bar{p} \\
K = (p - \bar{p})/2 \tag{1.3}
\]

with

\[
P^2 = M^2 \\
p^2 = \bar{p}^2 = m^2 \tag{1.4}
\]
Thus

\[ K^2 = m^2 - \frac{M^2}{4} \]  

(1.5)

The momentum, \( K \), is conjugate to the relative position of the quark and antiquark in the bound state wave function; we understand (1.5), therefore, in the sense of an average over the wave function. The matrix element \( A \) in (1.1) is therefore related to a wave function factor, \( F_w \), which is linear in \( \epsilon^* \) and is a matrix in the Dirac space of the heavy quarks:

\[
A = \int \frac{d^4 k}{(2\pi)^3} \delta \left( \frac{P \cdot K}{M} \right) \tilde{\Psi}(K) (F_w)_{\alpha\beta} \mathcal{O}_{\beta\alpha}.
\]  

(1.6)

Here \( \alpha \) and \( \beta \) are spinor indices and \( F_w \) depends on the quark and antiquark momenta and spins while \( \mathcal{O} \) carries the dependence on other variables in the problem. If the initial and final states contain heavy quarks only in the bound state, then the matrix element takes the form of a Dirac trace over \( F_w \) and other factors from \( \mathcal{O} \). For example, the decay amplitude for \( J/\Psi \) into an electron positron pair is

\[
A = \int \frac{d^4 k}{(2\pi)^3} \delta \left( \frac{P \cdot K}{M} \right) \tilde{\Psi}(K) \left( \text{Tr} F_w \gamma_\mu \right) \pi(p^-) \gamma^\mu v(p^+) \cdot 2m_e \cdot 4\pi \alpha e_q / q^2
\]  

(1.7)

where \( q^2 \) is the virtual photon’s squared four-momentum and

\[
F_w = \gamma_0 F_w^\dagger \gamma_0.
\]  

(1.8)

We use dimensionless spinors normalized to

\[
\pi u = -\pi v = 1.
\]  

(1.9)

The \( J/\Psi \) wave function is strongly affected by non-perturbative confinement effects and is therefore model dependent. In this article we ask what can be said assuming only that the wave function is spherically symmetric (an S wave), that the binding is small compared to the \( J/\Psi \) mass, and that the bound state can be treated as a superposition of on-shell quark and antiquark.

Since the production of a \( J/\Psi \) is the production of a quark-antiquark in the appropriate wave function, we would expect that, if \( \lambda = \lambda \), for suitable quark and antiquark spinors \( \pi \) and \( \pi \),

\[
F_w(\lambda)_{\alpha\beta} = \sqrt{\frac{2}{M}} \sqrt{\frac{2m}{\sqrt{3}}} v(\bar{\lambda})_\alpha \pi(p, \lambda)_\beta
\]  

(1.10)

while in the case of \( \lambda = -\lambda \) we would have the spin projection zero quarkonium state

\[
F_w(0)_{\alpha\beta} = \sqrt{\frac{2}{M}} \sqrt{\frac{2m}{\sqrt{3}}} v(\bar{\pi}, +)_\alpha \pi(p, -)_\beta + v(\bar{\pi}, -)_\alpha \pi(p, +)_\beta.
\]  

(1.11)
It is understood that $F$ contains a unit matrix in the quark color space which, with the $\sqrt{3}$ in eqs. (1.10) and (1.11), puts the quark-antiquark pair into a color singlet state. The traces in eqs. (1.6) and (1.7) include a color space trace. In the ZBA, the normalization factors are derived in [9] and [10]. They carry over without change to the more general case. Here we have used the Fourier transform and normalization identities

$$\Psi(x) = \int \frac{d^3K}{(2\pi)^3} \tilde{\Psi}(K) \exp iK \cdot x$$

$$\int d^3x |\Psi(x)|^2 = 1.$$  \hfill (1.12)

Assuming spherical symmetry and on-shell quarks, in addition to (1.3) we have

$$< K_\mu K_\nu > = \frac{< K^2 >}{3} \left( g_{\mu\nu} - \frac{P_\mu P_\nu}{M^2} \right)$$ \hfill (1.13)

Binding effects in any production or decay process can, therefore, be expressed as a power series in $< K^2 >$.

## 2 The spin paradox

We are free to choose any quantization axis along which to measure the spin of the $J/\Psi$. If we make the conventional choice of the $z$ axis as quantization axis, one might attempt to use the standard Bjorken-Drell [11] spinors

$$u(p, \lambda) = \frac{1}{\sqrt{2m(p^0 + m)}} \begin{pmatrix} m + p^0 + m \end{pmatrix} \begin{pmatrix} \chi_\lambda \\ 0 \end{pmatrix}$$

$$v(p, \lambda) = \frac{1}{\sqrt{2m(p^0 + m)}} \begin{pmatrix} m - p^0 + m \end{pmatrix} \begin{pmatrix} 0 \\ i\sigma_2 \lambda \chi \bar{\lambda} \end{pmatrix}$$ \hfill (2.1)

with

$$\chi_\lambda = \frac{(1 + \lambda \sigma_3)}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$ \hfill (2.2)

In the case $\bar{\lambda} = \lambda$ we have

$$\begin{pmatrix} 0 \\ i\sigma_2 \chi_\lambda \end{pmatrix} (\chi_\lambda \\ 0) = i\rho_2 (1 + \rho_3) (\lambda \sigma_1 - i \sigma_2) / 4$$

$$= 2 \gamma^R (\lambda) \frac{M + P^R}{2\sqrt{2}M}$$ \hfill (2.3)

where the $R$ refers to the rest frame values of (1.2) so that $P^R = M \gamma_0$. In (2.3) we have used the common decomposition of the four-by-four Dirac matrices into two two-by-two spaces

$$\gamma_0 = \rho_3,$$ \hfill (2.4)

$$\gamma^i = i\rho_2 \sigma^i.$$ \hfill (2.5)
Inserting (2.7) into (1.10,1.11) one would obtain

\[
(F_w)_{\alpha\beta} = \frac{1}{2M \sqrt{3M(p_0 + m)(\overline{p}_0 + m)}} \left( (m - \overline{p}) \varphi^R(\lambda)(M + P^R)(m + \varphi)I_0 \right)_{\alpha\beta}
\]

(2.6)

where \(I_0\) is the unit matrix in the three-by-three quark color space. In general this naive use of (2.7) leads to a non-covariant \(F_w\), as can be seen by noting the explicit appearance of the \(J/\Psi\) rest frame values of \(\epsilon\) and \(P\). Problems with Lorentz covariance have been discussed by [7] and attributed to non-perturbative effects. The non-covariance of (2.6) is connected with a non-zero relative momentum \(K\). The rest frame values of \(\varphi\) are related to the values in a general frame where the \(J/\Psi\) has four-momentum \(P^\mu = (P^0, \vec{P})\) by

\[
\varphi^R = \varphi - \epsilon_0 \frac{P + P^R}{P_0 + M}.
\]

(2.7)

Only if \(K = 0\) does (2.6) take the covariant form

\[
F_w = \frac{1}{2\sqrt{3M}} \varphi^R (P + M).
\]

(2.8)

Although binding effects in QCD may be inherently non-perturbative, we see this problem not as one to be resolved by assuming non-perturbative effects, but as requiring consideration of what spinors can be covariant in a bound state problem. Going beyond the ZBA requires a more careful choice of the Dirac spinors \(u\) and \(v\). Another symptom of the problem is associated with the way the spin of the quarks is carried over to the spin of the bound state, as we discuss shortly.

The spin operator along a quantization axis \(\vec{n}\) appropriate to act on a Dirac spinor of momentum \(p\) is

\[
S_{\vec{n}} = \frac{\varphi}{2m} \gamma_5 \vec{n}
\]

(2.9)

provided \(p \cdot \vec{n} = 0\) and \(\vec{n}^2 = -1\). Conventionally, one takes

\[
\vec{n}^\mu = (0, 0, 0, 1),
\]

(2.10)

i.e., pure spacelike in the z direction. However, this spin operator acting on the spinors of (2.7) gives the appropriate \(\lambda/2\) or \(\bar{\lambda}/2\) only if

\[
p_3 = \overline{p}_3 = 0.
\]

(2.11)

In the rest frame of the bound state this implies that

\[
K_3 = 0
\]

(2.12)

but, if the \(J/\Psi\) has unit spin along the z axis and there is no orbital angular momentum, then the constituent quark and antiquark must each have a unique spin \(1/2\) along this axis. This leads to the apparent paradox that the relative momentum, \(K\), in the \(J/\Psi\) rest frame must lie in the plane perpendicular to the quantization axis, contrary to the requirement that an s-wave state must have a spherically symmetric distribution in the relative momentum.
3 Recovering covariance; resolving the spin paradox

A free fermion spinor can only depend on the particle momentum and the spin axis as in (2.1). However, in order to recover covariance in the bound state and resolve the spin paradox one needs to employ not the usual Bjorken-Drell spinors (2.1) but instead

\[
u(p, \lambda) = \frac{m + p}{\sqrt{2m(M/2 + m)}} \frac{M + P}{\sqrt{2M(P_0 + M)}} \left( \begin{array}{c} \chi_\lambda \\ 0 \end{array} \right),
\]

\[
u(p, \lambda) = \frac{m - p}{\sqrt{2m(M/2 + m)}} \frac{M - P}{\sqrt{2M(P_0 + M)}} \left( \begin{array}{c} 0 \\ i\sigma_2 \chi_\lambda \end{array} \right).
\]

With these spinors in place of (2.1) the wave function factor becomes

\[F_w(\lambda + \lambda_0 / 2) = m \sqrt{\frac{8}{3M}} v(p) \bar{u}(p, \lambda) \bar{u}(p, \lambda) = \frac{(m - p) \chi^*(M + P)(m + p)I_0}{\sqrt{3MM(M + 2m)}}.
\]

For the spin projection zero state, the central expression is the symmetrized combination as in (1.11).

Our result agrees with that found in a recent work [12] that has appeared since we began this research. While confirming their result, the current article emphasizes the covariance requirement of bound state fermion spinors depending not only on the fermion momentum but also on the bound state momentum. In addition we consider in detail how the spin structure of the constituents carries over to the spin of the bound state. A comparison with other earlier work is given below.

In calculating a quarkonium production or decay process including first order corrections beyond the ZBA, the usual prescription is to expand the matrix element in the relative momentum and use the expectation values (1.5,1.13) as we will do in the next section for the quarkonium lepton pair decay rate and the \(B\) meson to \(J/\Psi + X\) rate. The remainder of this section is devoted to justifying the above choice of spinors in the quarkonium bound state.

One should note first that the quark and antiquark spin projections along any axis only need to sum to the quarkonium spin along that axis after performing the integration over relative momentum, \(K\). For any fixed value of \(K\) there can be an orbital angular momentum that comes into the equation. Further, the usual spin states are defined by reference to axes in the particle rest frame, and axis directions are not Lorentz invariant. Bjorken and Drell [11] specified their spinors for nonzero fermion three-momentum in terms of the Lorentz transformation taking \((m, 0, 0, 0)\) to \((E, \vec{p})\), but they did not point out that such a transformation is not unique: any rotation in the rest frame can be applied first. Such a rotation is the difference between the spinors we propose in (3.1) and those of (2.1); such a rotation also is the difference between a single Lorentz boost and the same boost realized via two noncollinear boosts. To construct the quark and antiquark spinors with the proper spin structure we first boost each from their differing rest frames into the rest frame of the \(J/\Psi\), since the conventional use of eqs.(1.2) implies the choice of the \(z\) axis in the rest frame of the quarkonium for the defining quantization axis.
Any spinor in the rest frame of a particle of mass $m$ is boosted into the frame where this particle has four-momentum $p^\mu$ by the Lorentz transformation matrix

$$T_p = \frac{m + p\gamma_0}{\sqrt{2m(m + p_0)}}. \quad (3.3)$$

Any covariant matrix, $A$, in Dirac space is boosted between these two frames by sandwiching the matrix in the particle rest frame between $T_p$ on the left and

$$T_p^\dagger T_p = \frac{m + \gamma_0 p}{\sqrt{2m(m + p_0)}}. \quad (3.4)$$
on the right. The spinors (3.1) may be written by transforming the quark and antiquark spinors from their rest frame, where they have spin projections $\lambda/2$ and $\bar{\lambda}/2$ respectively, first to the quarkonium rest frame where they have momenta $p^R$ and $\bar{p}^R$, and then transforming to the general frame where the quarkonium has momentum $P^\mu$. Thus

$$u(p, \lambda) = T_p T_p^{r\kappa} \frac{1 + \lambda \gamma_5 \bar{\not{p}}}{2} u_0$$

$$v(p, \bar{\lambda}) = T_p T_p^{r\kappa} \frac{1 + \bar{\lambda} \gamma_5 \not{p}}{2} v_0. \quad (3.5)$$

Here,

$$u_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (3.6)$$

$$v_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}. \quad (3.7)$$

Note that the spinors of (2.1) are

$$\begin{pmatrix} \chi_\lambda \\ 0 \end{pmatrix} = \frac{1 + \lambda \gamma_5 \bar{\not{p}}}{2} u_0$$

$$\begin{pmatrix} 0 \\ i\sigma_2 \chi_{\bar{\lambda}} \end{pmatrix} = \frac{1 + \bar{\lambda} \gamma_5 \not{p}}{2} v_0. \quad (3.8)$$

The quark and antiquark momenta, $p^R$ and $\bar{p}^R$, in the quarkonium rest frame are related to their values $p$ and $\bar{p}$ in the general frame where the quarkonium has four-momentum $P^\mu = (P^0, \vec{P})$ by

$$p^R_0 = \frac{P \cdot p}{M} = \frac{M}{2}$$

$$\bar{p}^R = \bar{p} - \frac{\vec{P}(p_0 + M/2)}{P_0 + M} \quad (3.10)$$
and equivalent equations for $\overline{\psi}^R$.

The $\gamma_0$’s in the $T_p$ and $\overline{T}_p$ of eqs. (3.3) commute with $\gamma_5 \overline{\psi}$ and give +1 and −1 respectively acting on $u_0$ and $v_0$. We may then use

\begin{equation}
T_p (m + \not{p}^R) \overline{T}_p = (m + \not{p})
T_p (m - \not{p}^R) \overline{T}_p = (m - \not{p})
\end{equation}

or

\begin{equation}
T_p (m + \not{p}^R) = (m + \not{p}) T_p
T_p (m - \not{p}^R) = (m - \not{p}) T_p.
\end{equation}

Again, the $\gamma_0$ in $T_p$ acting on $u_0$ or $v_0$ gives ±1 respectively so the spinors of (3.5) become identical to those of (3.1). Combining $v$ and $\overline{\sigma}$ and using (2.3) we have

\begin{equation}
F_w = m \sqrt{\frac{8}{3M}} v \overline{\sigma} = \frac{(m - \not{p})}{(M/2 + m)} T_p \psi^R (M + \not{P}^R) \overline{T}_p (m + \not{p}) I_0.
\end{equation}

Then, using

\begin{equation}
T_p \psi^R (M + \not{P}^R) \overline{T}_p = \psi^R (M + \not{P}),
\end{equation}

we have (3.2). This result agrees with that of [12]. One might ask, however, why the conventional spinors (2.1) lead to a non-covariant result while the spinors of (3.1) or (3.5) lead to a satisfactory covariant result. The answer lies in the following observation. A quark spinor is given by a covariant factor times a rest frame spinor. The rest frame spinors combine to a covariant form in the meson rest frame (2.3). From there one would have to boost to the general frame via $T_p$. But such a spinor would not satisfy the on-shell condition. The combination (2.3) sandwiched with $T_{pR}$ and $\overline{T}_{pR}$ also provides a covariant form in the meson rest frame. This boosted by the $T_p$ gives a covariant form in the general frame and also satisfies the Dirac on-shell conditions due to (3.12).

To investigate how the quark and antiquark spins add to the quarkonium spin we consider the momentum dependent spin operators for quark and antiquark

\begin{equation}
S_i(p) = T_p T_{pR} \frac{\gamma_0 \gamma_5 \gamma_i}{2} \overline{T}_{pR} \overline{T}_p
\end{equation}

The $S_i(p)$ satisfy the $SU(2)$ algebra for any $p$. In the quark rest frame we have

\begin{equation}
\frac{\gamma_0 \gamma_5 \gamma_i}{2} = \frac{\not{p}}{2m} \gamma_5 \not{e}_i = \frac{\sigma_i}{2}
\end{equation}

where $e_i$ is a pure space-like unit vector along the $i$‘th axis. Without forcing the relative momentum to lie in the $x, y$ plane, $S_3(p)$ and $S_3(\overline{p})$ have the proper eigenvalues $\lambda/2$ and $\overline{\lambda}/2$ acting on the spinors of (3.1). In addition we note that, in the quarkonium rest frame where $T_P = 1$,

\begin{equation}
S_3(p) = S_3(p^R) = \frac{\not{p}^R}{2m} \gamma_5 \not{p}^R
\end{equation}
where
\[ \hat{p}^R = T_{p^R} \bar{p} \mathcal{T} p^R = \vec{n} + \frac{\vec{n} \cdot \vec{p}^R}{m} p^R + m \gamma_0 \frac{p^R}{p^0_R + m}. \] (3.17)

\( p^R_0 \), of course, is merely \( M/2 \). After averaging over \( K \) directions \( p^R \) is in the \( \vec{n} \) direction. For on-shell quarks \( p^R \) is, after averaging, precisely \( \vec{n} \). Thus this construction insures that in the quarkonium rest frame the quark and antiquark spins are taken with respect to the same \( \vec{n} \) direction after averaging over relative momenta. It is this property that ensures covariance of the \( \bar{v}u \) bound state and ensures that the spins of the quark and antiquark carry over properly to the spin of the quarkonium.

In the general frame, where the quarkonium has four-momentum \((P^0, \vec{P})\),
\[ S_3(p) = \frac{\not{p}}{2m} \gamma_5 \not{p} \] (3.18)

where
\[ n^\mu = n^\mu_P - \frac{p \cdot n_P}{m} Q^\mu \] (3.19)

and \( Q^\mu \) is the dimensionless four-vector
\[ Q^\mu = \frac{m P^\mu + M p^\mu}{M(M/2 + m)} \] (3.20)

\( n_P \) is the four-vector that is \( \vec{n} \) in the quarkonium rest frame.
\[ n_P = \Lambda(P) \vec{n} = \left( \frac{\vec{n} \cdot \vec{P}}{M}, \vec{n} + \vec{P} \frac{\vec{P} \cdot \vec{n}}{M (P^0 + M)} \right) \] (3.21)

where \( \Lambda(P) \) is the Lorentz transformation matrix that relates the quarkonium rest frame to the general frame. The four-dimensional spin axis \( \vec{n} \) for the antiquark is given by eqs. 3.19 and 3.20 replacing \( p \) by \( \bar{p} \). Only in the \( K = 0 \) limit or after averaging over \( K \) are these axes equal. Note that \( p \cdot n = \bar{p} \cdot \bar{n} = P \cdot n_P = 0 \) as required by Lorentz invariance. Using these orthogonalities and the fact that
\[ (m + \not{p}) \gamma_5 Q(M + P) = 0 \] (3.22)

it is easy to show that
\[ S_3(p) u(p, \lambda) = \frac{\lambda}{2} u(p, \lambda) \]
\[ S_3(\bar{p}) v(\bar{p}, \bar{\lambda}) = \frac{\bar{\lambda}}{2} v(\bar{p}, \bar{\lambda}) \] (3.23)

where the \( u \) and \( v \) spinors here are those of (3.1). Therefore,
\[ S_3(\bar{p}) F_w + F_w S_3(p) = \frac{\lambda + \bar{\lambda}}{2} F_w. \] (3.24)
This makes manifest the addition of the quark and antiquark spin projections to compose that of the meson for arbitrary values of $K$.

We turn now to a comparison of our result with that of other authors. We convert all results to those appropriate to production of a quarkonium state as opposed to a decay process and we ignore normalization factors.

In one of the seminal early papers on quarkonium processes [9], there appears the expression

$$F_w \sim (m - \not{p}) \not{\epsilon}^*(\gamma^0 + 1)(m + \not{p}).$$  (3.25)

These authors did not attempt to treat the system beyond the ZBA and noted that the above result held in the bound state rest frame, but did not deal with the apparent non-covariance of the result.

In [13], the wave function factor was given as

$$F_w \sim (m - \not{p}) \not{\epsilon}^*(m + \not{p}).$$  (3.26)

This paper omitted the factor, $(M + \not{P})$, which is important in going beyond the ZBA. The same form appears in [14].

In [8], the form

$$F_w \sim \not{\epsilon}^*(m + \not{P})$$  (3.27)

is given without attempting an analysis beyond the ZBA.

In [7], in addition to the form (3.26), there appears the non-covariant form

$$F_w \sim \not{\epsilon}^*(\gamma^0 + 1).$$  (3.28)

Again, these authors did not deal with the apparent non-covariance, preferring to restrict their considerations to the meson rest frame.

Repeating our analysis above, it is straightforward to write the wave function factor for a bound state of different quark species. For a quark of mass $m$ and antiquark of mass $\overline{m}$, the appropriate generalization is

$$F_w(\lambda) = \frac{(m - \overline{p}) \not{\epsilon}^*(M + P)(m + \not{p}) I_0}{M \sqrt{12M(p_0^R + m)(\overline{p}_0^R + \overline{m})}}.$$  (3.29)

Here the quark and antiquark energies in the rest frame of the bound state are

$$p_0^R = \frac{M}{2} + \frac{m^2 - \overline{m}^2}{2M},$$

$$\overline{p}_0^R = \frac{M}{2} - \frac{m^2 - \overline{m}^2}{2M}.$$  (3.30)

In the case of non-equal masses, the $\delta$ function in (1.6) is modified to

$$\delta\left(\frac{P \cdot K}{M} - \frac{m^2 - \overline{m}^2}{M}\right).$$  (3.31)
4 Decay rates beyond the zero-binding approximation

We would now like to use the bound state formalism to calculate the decay rates \( J/\Psi \to l^+l^- \) and \( B \to J/\Psi + X \) including the first corrections beyond the ZBA. In the decay of a \( J/\Psi \) as opposed to the production one uses the conjugate wave function factor (1.8). Using (1.10) in the trace of (1.7) and averaging over the relative momenta using (1.5,1.13) we have

\[
\text{Tr}(F_w\gamma_\mu) = \sqrt{3}(2m + M) \left( 1 - \frac{4K^2}{3(2m + M)^2} \right) \sim \sqrt{12}\epsilon_\mu \left( 1 + \frac{2K^2}{3M^2} \right). \quad (4.1)
\]

Thus, to lowest order in \( K^2 \),

\[
\Gamma(J/\Psi \to l^+l^-) = 16\pi\alpha^2\epsilon_q^2 \frac{|\Psi(0)|^2}{M^2} \left( 1 + \frac{4K^2}{3M^2} \right). \quad (4.2)
\]

Using the Particle Data Group \([15]\) charm quark mass \( m = (1.25 \pm 0.1) \text{ GeV} \) and \( J/\Psi \) mass \( M = 3.097 \text{ GeV} \) we see that the first order correction beyond the ZBA decreases the leptonic pair decay rate of the \( J/\Psi \) by a factor of \( 0.88 \pm 0.04 \). This correction has been also found in \([7]\). The effect goes in the wrong direction to explain the suppression of the hadronic \( J/\Psi \) decays relative to the leptonic pair decay. However, important contributions come from the binding corrections to the three gluon decay. Although the first order correction does reduce the three gluon decay rate, the authors of \([12]\) note an apparent slow convergence of the \( K^2 \) expansion indicating that further study is needed.

Turning to the decay \( B \to J/\Psi + X \), the Dirac structure of the invariant amplitude is

\[
\mathcal{A} \sim \pi(p_b)\gamma_\mu P_L F_w\gamma^\nu P_L u(p_\Psi) \left( -g_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2} \right) \frac{1}{M_W^2 - k^2} \quad (4.3)
\]

where \( P_L \) is the left handed chiral projection and

\[
k_\mu = p_\Psi \mu - p_\mu = p_{sb} + \mathbf{p}_\mu. \quad (4.4)
\]

It is easy to see that, to lowest order in \( \frac{1}{M_W^2} \) and to first order in \( \frac{K^2}{M^2} \), the decay rate is modified by the identical factor \( 1 + \frac{4K^2}{3M^2} \) that occurs in lepton pair decay. Thus, measured relative to lepton pair decay or, equivalently, using the leptonic rate as a measure of \( |\Psi(0)|^2 \), the relativistic corrections do not change the inclusive \( J/\Psi \) decay rate of the \( B \) meson at zeroth order in the strong coupling constant.

In summary, we have shown how the recovery of Lorentz covariance requires the use of quark spinors in the bound state that depend not only on the quark momentum but also on the bound state momentum. We have also noted how the relevant spin operators for quark and antiquark refer to momentum dependent axes that are only equal in an average sense. We have confirmed earlier recent results for the wave function factor and lepton-pair decay mode of quarkonium, and have extended the beyond-ZBA analyses to the inclusive \( J/\Psi \) decay of the \( B \) meson.

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References

[1] T. Appelquist and H. Politzer, Phys. Rev. Lett. 34, 43 (1973)
   A. DeRujula and S. L. Glashow, Phys. Rev. Lett. 34, 46 (1973)

[2] L. Clavelli and P. W. Coulter, Phys. Rev. D51, 1117 (1995)

[3] P. H. Cox, S. Hovater, S. Jones, and L. Clavelli, Phys. Rev. D32, 1157 (1985)
   S. T. Jones and P. H. Cox, Phys. Rev. D35, 1064 (1987)

[4] G. Bodwin, E. Braaten, T. C. Yuan, and G. P. Lepage, Phys. Rev. D46, 3703 (1992)

[5] M. Shifman, Int. J. of Mod. Phys. A11, 3195 (1996)

[6] E. Salpeter, Phys. Rev. 87 328 (1952)
   R. Barbieri, R. Gatto, and E. Remiddi, Phys. Lett. 61B, 465 (1976)

[7] G. Lopez Castro, J. Pestieau, and C. Smith, hep-ph/0004209
   G. Lopez Castro, J. Pestieau, C. Smith, and S. Trine, hep-ph/0006016, hep-ph/0006018
   J. Pestieau, C. Smith, and S. Trine, Int. J. Mod. Phys. A17, 1355 (2002) (hep-ph/0105034)

[8] Franz Gross, Relativistic Quantum Mechanics and Field Theory, (Wiley, New York, 1993)

[9] J. H. Kühn, J. Kaplan, and E. G. Safiani, Nucl. Phys. B157, 125 (1979)

[10] V. Barger and R. Phillips, Collider Physics, (Perseus, Cambridge, 1991)

[11] J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics, (McGraw-Hill, New York, 1964)

[12] G. Bodwin and A. Petrelli, hep-ph/0205210

[13] B. Guberina, J. H. Kühn, R. D. Peccei, and R. Rückl, Nucl. Phys. B174, 317 (1980)

[14] A. Petrelli, M. Cacciari, M. Greco, F. Maltoni, and M. L. Mangano, Nucl. Phys. B514, 245 (1998) (hep-ph/9707223)

[15] Particle Data Group, D. E. Groom et al., European Phys. J. C15, 1 (2000)